Outage Performance of A Unified Non-Orthogonal Multiple Access Framework

Xinwei Yue, Zhijin Qin, Yuanwei Liu, Xiaoming Dai, and Yue Chen

*Beihang University, Beijing, China
†Lancaster University, Lancaster, UK
‡Queen Mary University of London, London, UK
§University of Science and Technology Beijing, Beijing, China

Abstract—In this paper, a unified framework of non-orthogonal multiple access (NOMA) networks is proposed, which can be applied to code-domain NOMA (CD-NOMA) and power-domain NOMA (PD-NOMA). Since the detection of NOMA users mainly depend on efficient successive interference cancellation (SIC) schemes, both imperfect SIC (ipSIC) and perfect SIC (pSIC) are taken into consideration. To characterize the performance of this unified framework, the exact and asymptotic expressions of outage probabilities as well as delay-limited throughput for CD/PD-NOMA with ipSIC/pSIC are derived. Based on the asymptotic analysis, the diversity orders of CD/PD-NOMA are provided. It is confirmed that due to the impact of residual interference (RI), the outage probability of the n-th user with ipSIC for CD/PD-NOMA converges to an error floor in the high signal-to-noise ratio (SNR) region. Numerical simulations demonstrate that the outage behavior of CD-NOMA is superior to that of PD-NOMA.

I. INTRODUCTION

To enhance spectrum efficiency and massive connectivity, non-orthogonal multiple access (NOMA) [1, 2] has been identified as one of the key technologies for the fifth generation (5G) networks. The pivotal feature of NOMA is its capability of sharing the same physical resource element (RE), where multiple users’ signals are linearly superposed over different power levels by using the superposition coding scheme. To get the desired signal, multi-user detection algorithm [3], i.e., successive interference cancellation (SIC) or message passing algorithm (MPA) is carried out at the receiver.

More particularly, based on spreading signature of multiple access (MA), NOMA schemes can be divided into two categories: power-domain NOMA (PD-NOMA) and code-power NOMA (CD-NOMA). In [4], two evaluation metrics of PD-NOMA networks including outage probability and ergodic rate have been proposed, where the outage behaviors of users and ergodic rate have been discussed by applying stochastic geometry. From a practical perspective, the authors in [5] studied the performance of PD-NOMA for the two-user case with imperfect channel state information (CSI), where the closed-form expressions of outage probability were derived. When NOMA users have similar channel conditions, the authors of [6] proposed a PD-NOMA based multicast-unicast scheme and verified that the spectral efficiency of PD-NOMA based multicast-unicast scheme is higher than that of orthogonal multiple access (OMA) based one.

As a further advance, CD-NOMA is viewed as a special extension of PD-NOMA, in which the data streams of multiple users are directly mapped into multiple REs (or subcarriers K) through the sparse matrix/codebook or low density spread sequence. Actually, CD-NOMA mainly include sparse code multiple access (SCMA), pattern division multiple access (PDMA), multi-user sharing access (MUSA), etc. In [7], the authors proposed a sub-optimal design approach to design the sparse codebook of SCMA. On the condition of the fixed sparse pattern matrix, the authors of [8] evaluated the link level performance of PDMA and confirmed that PDMA can achieve the higher spectrum efficiency than OMA. In [9], MUSA is capable of adopting a grant-free scheme to support Internet of Things (IoT) scenario. However, up to now, there is no work investigating the performance of the unified NOMA framework.

Driven by this, we investigate the outage performance of the unified NOMA framework by invoking stochastic geometry. Since the detection of NOMA users mainly depend on efficient SIC schemes, both imperfect SIC (ipSIC) and perfect SIC (pSIC) are taken into consideration. We derive the exact expressions of outage probability for a pair of NOMA users (i.e., the n-th user and m-th user) in the unified framework. To obtain deep insights, we further derive the asymptotic outage probability of two users and attain the corresponding diversity orders. Due to the impact of residual interference (RI), the outage behavior of the n-th user with ipSIC for CD/PD-NOMA (CD-NOMA and PD-NOMA) converges to an error floor. Furthermore, we confirm that the outage behavior of CD-NOMA is superior to that of PD-NOMA. Additionally, we analyze system throughput for CD/PD-NOMA in the delay-limited transmission.

II. NETWORK MODEL

A. Network Descriptions

Consider a unified NOMA downlink transmission scenario, where a base station (BS) transmits the information to M randomly users. The BS directly maps the data streams of multiple users into K subcarriers or REs by utilizing one sparse spreading matrix $G_{K \times M}$, in which there are a number of non-zero entries within it and satisfies the relationship $M > K$. For simplicity, we assume that the BS and NOMA users are equipped with a single antenna,
respectively. Assuming that the BS is located at the center of circular cluster denoted as \( D \), with radius \( R_D \) and the \( M \) NOMA users are uniformly distributed within circular cluster \([10]\). To facilitate analysis, we assume that \( M \) users are divided into \( M/2 \) orthogonal pairs, in which the distant user and the nearby user can be distinguished based on their disparate channel conditions. Furthermore, each pair of users is randomly selected to carry out the NOMA protocol \([4, 11]\). A bound pass model is employed to model the channel coefficients in networks from the BS to users. Meanwhile, these wireless links are disturbed by additive white Gaussian noise (AWGN) with mean power \( N_0 \). Without loss of generality, the effective channel gains between the BS and users are sorted as \( \| h_M \|_2^2 > \cdots > \| h_n \|_2^2 > \cdots > \| h_1 \|_2^2 \) \([12, 13]\) with the assistance of order statistics. In this paper, we focus on the \( m \)-th user paired with the \( n \)-th user for NOMA transmission.

### B. Signal Model

Regarding the unified NOMA downlink transmission scenario, the BS transmits the superposed signals to multiple users, where the data stream of each user spreads over one column of sparse matrix. Hence the observation at the \( \varphi \)-th user over \( K \) subcarriers \( y_\varphi = [y_{\varphi,1} y_{\varphi,2} \cdots y_{\varphi,K}]^T \) is given by

\[
y_\varphi = \text{diag}(h_{\varphi})(g_n \sqrt{P_s a_m x_n} + g_m \sqrt{P_s a_m x_m}) + n_\varphi,
\]

where \( \varphi \in \{n, m\} \), \( x_n \) and \( x_m \) are the normalized unique power signals for the \( n \)-th and \( m \)-th users, respectively, i.e., \( \mathbb{E}\{x_n^2\} = \mathbb{E}\{x_m^2\} = 1 \). We assume the fixed power allocation coefficients satisfy the condition that \( a_m > a_n \) with \( a_m + a_n = 1 \), which is for fairness considerations. \( P_s \) denotes the normalized transmission power at BS. The sparse indicator vector of the \( \varphi \)-th user is denoted by \( g_\varphi = [g_{\varphi,1} g_{\varphi,2} \cdots g_{\varphi,K}]^T \), which is one column of \( G_{K \times M} \). More specifically, \( g_{\varphi,k} \) is the subcarrier index, where \( g_{\varphi,k} = 1 \) indicates the signals are mapped into the corresponding RE, otherwise, \( g_{\varphi,k} = 0 \). Let \( h_{\varphi} = [h_{\varphi,1} h_{\varphi,2} \cdots h_{\varphi,K}]^T \) denotes the channel vector between the BS and \( \varphi \)-th user occupying \( K \) subcarriers with \( h_{\varphi,k} = \frac{\mathcal{C}_n h_{\varphi,k}}{\sqrt{\mathbb{E}\{h_{\varphi,k}^2\}}} \), where \( h_{\varphi,k} \sim \mathcal{CN}(0, 1) \) is the Rayleigh fading channel gain between the BS and \( \varphi \)-th user occupying the \( k \)-th subcarrier, \( \eta \) is a frequency dependent factor, \( \alpha \) is the path loss exponent and \( d \) is the distance from BS to \( \varphi \)-th user. \( n_\varphi \sim \mathcal{CN}(0, N_0 I_K) \) denotes the AWGN.

To maximize signal-to-noise ratios (SNRs) and diversity orders, we employ maximal ratio combiner (MRC) at the \( \varphi \)-th user over \( K \) subcarriers. Let \( u_\varphi = \frac{\text{diag}(h_{\varphi})}{\|\text{diag}(h_{\varphi})\|_2} \|_{K \times 1} \), and then the received signal at the \( \varphi \)-th user can be written as

\[
y_\varphi = u_\varphi \text{diag}(h_{\varphi})(g_n \sqrt{P_s a_m x_n} + g_m \sqrt{P_s a_m x_m}) + u_\varphi n_\varphi.
\]

Based on aforementioned assumptions, the signal-plus-interference-to-noise ratio (SINR) at the \( n \)-th user to detect the \( m \)-th user’s signal \( x_m \) is given by

\[
\gamma_{n \rightarrow m} = \frac{\rho \|\text{diag}(h_n)\|_2^2 a_m}{\rho \|\text{diag}(h_n)\|_2^2 a_n + 1},
\]

where \( \rho = \frac{P_s}{N_0} \) denotes the transmit SNR. For the sake of simplicity, assuming that \( g_m \) and \( g_n \) have the same column weights for \( G_{K \times M} \).

By applying SIC \([14]\), the SINR of the \( m \)-th user, who needs to decode the information of itself is given by

\[
\gamma_m = \frac{\rho \|\text{diag}(h_m)\|_2^2 a_m}{\rho \|\text{diag}(h_m)\|_2^2 a_m + 1},
\]

where \( \varpi = 0 \) and \( \varpi = 1 \) denote the pSIC and ipSIC operations, respectively. Note that \( h_{1K} = [h_{11} h_{12} \cdots h_{1K}]^T \) denotes the RI channel vector at \( K \) subcarriers with \( h_{1k} \sim \mathcal{CN}(0, \Omega_k) \).

The SINR of the \( m \)-th NOMA user to decode the information of itself can be expressed as

\[
\gamma_m = \frac{\rho \|\text{diag}(h_m)\|_2^2 a_m}{\rho \|\text{diag}(h_m)\|_2^2 a_m + 1}.
\]

### III. PERFORMANCE EVALUATION

In this section, the outage probability for a pair of NOMA users is selected as a metric to evaluate the performance of the unified downlink NOMA networks.

#### A. The outage probability of the \( m \)-th user

The outage event of the \( m \)-th user is that the \( m \)-th user cannot detect its own information. Hence the outage probability of the \( m \)-th user for CD-NOMA can be expressed as

\[
P_{m,CD} = \Pr(\gamma_m < \varepsilon_m),
\]

where \( \varepsilon_m = 2^{R_m} - 1 \) and \( R_m \) is the target rate of the \( m \)-th user. The following theorem provides the outage probability of the \( m \)-th user.

**Theorem 1.** The outage probability of the \( m \)-th user for CD-NOMA is given by

\[
P_{m,CD} \approx \phi_m \sum_{p=0}^{M-m} \binom{M-m}{p} \frac{(-1)^p}{p+m+1} \left[ \sum_{u=1}^{U} b_u \left( 1 - e^{-\frac{\tau \xi u}{\eta}} \right) \right]^{m+p},
\]

where \( \tau = \frac{\rho \varepsilon_m}{\rho a_m a_n \|\text{diag}(h_n)\|_2^2} \) with \( a_m > \varepsilon_m a_n \), \( \phi_m = \frac{\binom{M-m}{m} \binom{m-p}{m-p}}{\binom{M-m}{m} \binom{m-p}{m-p}} \), \( b_u = \frac{\pi}{2U} \left( 1 - \frac{U}{\theta_u} \right) \), \( \theta_u = \cos\left( \frac{2u-1}{2U} \pi \right) \) and \( U \) is a parameter to ensure a complexity-accuracy tradeoff.

Proof: See Appendix A.

**Corollary 1.** For the special case with \( K = 1 \), the outage probability of the \( m \)-th user for PD-NOMA is given by

\[
P_{m,PD} \approx \phi_m \sum_{p=0}^{M-m} \binom{M-m}{p} \frac{(-1)^p}{p+m+1} \left[ \sum_{u=1}^{U} b_u \left( 1 - e^{-\frac{\tau \xi u}{\eta}} \right) \right]^{m+p}.
\]
B. The outage probability of the n-th user

As stated in [4, 15], the outage for the n-th user can happen in the following two cases: 1) The n-th user cannot decode the message of the m-th user; and 2) The n-th user can decode the message of the m-th user, and then carries out SIC operations, but cannot decode the information of itself. Hence the outage probability of the n-th user can be expressed as

\[ P_{n,CD} = \Pr \{ \gamma_n < \rho_m \} + \Pr \{ \gamma_n > \rho_m, \gamma_n \leq \rho_n \}, \]  

where \( \rho_n = 2R_n - 1 \) with \( R_n \) being the target rate at the n-th user to detect the m-th user. The following theorem provides the outage probability of the n-th user with ipSIC for CD-NOMA.

**Theorem 2.** The outage probability of the n-th user with ipSIC for CD-NOMA is given by (10), where \( \beta = \frac{\rho_n}{\rho_m} \) and \( \varpi = 1 \).

Proof: See Appendix B.

Substituting \( \varpi = 0 \) into (10), the outage probability of the n-th user with pSIC for CD-NOMA is given by

\[ P_{\nu,CD} = \phi_n \sum_{p=0}^{M-n} \binom{M-n}{p} \frac{(-1)^p}{n+p} \left[ \sum_{u=1}^{U} b_u \left( 1 - \frac{\varpi u}{\eta} \right)^{n+p} \right]. \]  

**Corollary 2.** For the special case with \( K = 1 \), the outage probability of the n-th user with pSIC for PD-NOMA is given by

\[ P_{\nu,PD} = \frac{\phi_n}{\Omega} \sum_{p=0}^{M-n} \binom{M-n}{p} \frac{(-1)^p}{n+p} \left[ \sum_{u=1}^{U} b_u \left( 1 - \frac{\varpi u}{\eta} \right)^{n+p} \right]. \]  

Substituting \( \varpi = 0 \) into (12), the outage probability of the n-th user with pSIC for PD-NOMA is given by

\[ P_{\nu,PD} = \phi_n \sum_{p=0}^{M-n} \binom{M-n}{p} \frac{(-1)^p}{n+p} \left[ \sum_{u=1}^{U} b_u \left( 1 - \frac{\varpi u}{\eta} \right)^{n+p} \right]. \]  

C. Diversity Order Analysis

To obtain deep insights, diversity order analysis is present, which highlights the slope of the curves for outage probabilities varying with the SNRs. The definition of diversity order is given by

\[ d = - \lim_{\rho \to \infty} \frac{\log \left( P^\infty (\rho) \right)}{\log \rho}, \]

where \( P^\infty (\rho) \) denotes the asymptotic outage probability at high SNR region.

**Corollary 3.** The asymptotic outage probability of the m-th user for CD-NOMA is given by

\[ P_{m,CD}^\infty = \frac{M!}{(M-m)!m!} \left[ \sum_{u=1}^{U} b_u \left( \frac{\tau c_u}{\eta} \right)^K \right]^m. \]  

**Proof:** By definition, \( \Theta_1 = 1 - e^{-\frac{\tau c_u}{\eta}} \sum_{i=0}^{K-1} \frac{1}{i!} \left( \frac{\tau c_u}{\eta} \right)^i \).

Applying power series expansion, the summation term \( \Theta_2 \) can be rewritten as \( \Theta_2 = e^{-\frac{\tau c_u}{\eta}} - \sum_{i=K}^{\infty} \frac{1}{i!} \left( \frac{\tau c_u}{\eta} \right)^i \). Substituting \( \Theta_2 \) into \( \Theta_1 \), when \( x \to 0 \), \( \Theta_1 \) with the approximation of \( e^{-x} \approx 1 - x \) is formulated as \( \Theta_1 \approx \frac{1}{K!} \left( \frac{\tau c_u}{\eta} \right)^K \). Furthermore, substituting \( \Theta_1 \) into (7) and taking the approximation term (\( p = 0 \)), we obtain (15). The proof is completed.

For the special case with \( K = 1 \), the asymptotic outage probability of the m-th user for PD-NOMA is given by

\[ P_{m,PD}^\infty = \frac{M!}{(M-m)!m!} \left[ \sum_{u=1}^{U} b_u \left( \frac{\tau c_u}{\eta} \right)^K \right]^m. \]

**Remark 1.** Upon substituting (15) and (16) into (14), the diversity orders of the m-th user for CD-NOMA and PD-NOMA are \( nk \) and \( m \), respectively.

**Corollary 4.** The asymptotic outage probability of the n-th user with pSIC for CD-NOMA is given by

\[ P_{n,CD}^\infty = \phi_n \left( \frac{\tau c_u}{\eta} \right)^K \sum_{p=0}^{M-n} \binom{M-n}{p} \frac{(-1)^p}{n+p} \int_0^\infty e^{-\frac{\tau c_u}{\eta} y^{K-1}} dy. \]  

Substituting \( \varpi = 0 \) into (17), the asymptotic outage probability of the n-th user with pSIC for PD-NOMA is given by

\[ P_{n,PD}^\infty = \frac{M!}{(M-n)!m!} \left[ \sum_{u=1}^{U} b_u \left( \frac{\tau c_u}{\eta} \right)^K \right]^n. \]

**Remark 2.** Upon substituting (17) and (18) into (14), the diversity orders of the n-th user with ipSIC/pSIC for CD-NOMA are \( 0 \) and \( nk \), respectively.

**Corollary 5.** For the special case with \( K = 1 \), the asymptotic outage probability of the n-th user with ipSIC for PD-NOMA is given by

\[ P_{n,PD}^\infty = \phi_n \left( \frac{\tau c_u}{\eta} \right)^K \sum_{p=0}^{M-n} \binom{M-n}{p} \frac{(-1)^p}{n+p} \int_0^\infty e^{-\frac{\tau c_u}{\eta} y^{K-1}} dy. \]  

(19)
Substituting $\varpi = 0$ into (19), the asymptotic outage probability of the $n$-th user with pSIC for PD-NOMA is given by

$$
P_{\text{PD}, n} \approx \frac{M-n}{\Omega_f^K} \sum_{p=0}^{M-n} \binom{M-n}{p} \left( \frac{1}{n+p} \right)^{\frac{M-n}{n+p}} \int_0^\gamma y^{K-1} e^{-\frac{y}{\eta}} \left[ \sum_{u=1}^U b_u \left( 1 - e^{-\frac{c_u (\varphi + \beta)}{\eta}} \right) \right]^{n+p} dy.
$$

(10)

**Remark 3.** Upon substituting (19) and (20) into (14), the diversity orders of the $n$-th user with ipSIC/pSIC for PD-NOMA are zero and $n$, respectively.

**D. Throughput Analysis**

In this subsection, the system throughput of the unified NOMA framework is characterized in delay-limited transmission mode. In this mode, the BS transmits information at a constant rate $R$, which is subject to the effect of outage probability. Hence the system throughput of CD/PD-NOMA with ipSIC/pSIC is given by

$$
R_\psi = (1 - P_{m,\phi}) R_n + \left( 1 - P_{n,\phi}^\psi \right) R_m,
$$

(21)

where $\psi \in \{\text{ipSIC}, \text{pSIC}\}$, $\phi \in \{CD, PD\}$, $P_{m,\phi}$ and $P_{n,\phi}$ are given by (7) and (8), respectively. $P_{\text{ipSIC}, n, CD}$, $P_{\text{ipSIC}, n, CD}$, $P_{\text{n,PD}}$, and $P_{\text{n,PD}}$ are given by (10) and (11), (12) and (13), respectively.

**IV. NUMERICAL RESULTS**

In this section, simulation results are presented to verify the analytical results derived in the above sections. In this unified framework considered, we assume the power allocation coefficients of a pair of users are $a_m = 0.8$ and $a_n = 0.2$, respectively. The target rates are set to be $R_n = R_m = 0.01$ BPCU, where BPCU is short for bit per channel use. Setting the pathloss exponent to $\alpha = 2$ and the system carrier frequency is equal to 1 GHz. The complexity-vs-accuracy tradeoff parameter is set to $N = 15$. Without loss of generality, the OMA is selected to be a benchmark for comparison purposes. Note that NOMA users with low target data rate can be applied to the IoT scenarios, which require low energy consumption, small packet size and so on.

Fig. 1 plots the outage probability of a pair of NOMA users (the $m$-th and $n$-th user) versus the transmit SNR with ipSIC/pSIC, where $K = 2$. The exact analytical curves for the outage probability of the $m$-th user is plotted according to (7). Furthermore, the exact analytical curves for the outage probability of the $n$-th user with pSIC/pSIC are plotted based on (10) and (11), respectively. Obviously, the exact outage probability curves match perfectly with the Monte Carlo simulations results. We observed that the outage behavior of conventional OMA is inferior to the $n$-th user with pSIC and superior to the $m$-th user. This is due to the fact that NOMA is capable of providing better fairness since multiple users are served simultaneously, which is the same as the conclusions in [4, 16].

Additionally, as can be seen from Fig. 1, the dashed curves represent the asymptotic COP of the $m$-th user and $n$-th user with pSIC, which can be obtained by numerically evaluating (15) and (18). One can observe that the asymptotic outage probabilities are approximated to the analytical results in the high SNR region. The dotted curves represent the asymptotic outage probabilities of the $n$-th user with ipSIC, which are calculated from (17), respectively. It is shown that the outage performance of the $n$-th user converges to an error floor and obtain zero diversity order. Due to the influence of RI, the outage behavior of the $n$-th user with ipSIC is inferior to OMA. This is because that the RI signal from imperfect cancellation operation is the dominant impact factor. With the value of RI increasing from $-30$ dB to $-20$ dB, the outage behavior of the $n$-th user is becoming more worse and deteriorating. Hence the design of effective multiuser receiver algorithm is significant to improve the performance of NOMA networks in practical scenarios.

Fig. 2 plots the outage probability versus SNR with the different number of subcarriers (i.e., $K = 1$ and $K = 3$). For the special case with $K = 1$, the unified framework of NOMA becomes PD-NOMA. The exact outage probability curve of the $n$-th user for PD-NOMA is plotted according to (8). The exact outage probability curves of the $n$-th user with pSIC/pSIC are given by Monte Carlo simulations and precisely match with the analytical expressions, which have been derived in (12) and (13), respectively. The asymptotic outage probabilities of this pair of users for PD-NOMA are also approximated with the analytical results in the high SNR region. We observe that CD-NOMA has a more steep slope and can provide better outage performance than PD-NOMA. This is due to the fact that CD-NOMA is capable of achieving the higher diversity orders.

Fig. 3 plots the outage probabilities versus SNR for different user target rates. We observe that with increasing target rates, the lower outage probabilities are achieved. This is due to the fact that the achievable rates are directly related to the target SNRs. It is beneficial to detect the superposed signals for the selected user pairing with smaller target SNRs. It is worth pointing out that the impact of practical scenario parameter frequency dependent factor $\eta$ has been taken into account in the unified NOMA framework. Furthermore, the incorrect choice of $R_n$ and $R_m$ will lead to the improper outage behavior for the unified framework.

Fig. 4 plots the system throughput versus SNR in the delay-
limited transmission mode. The solid black curves represent throughput of CD/PD-NOMA with ipSIC/pSIC, which can be obtained from (21). The dash-dotted blue curves represent throughput of CD-NOMA and PD-NOMA with ipSIC for the different values of RI. We observe that CD-NOMA attains a higher throughput compared to PD-NOMA, since CD-NOMA has the smaller outage probabilities. This is due to that CD-NOMA is capable of attaining the larger diversity order than that of PD-NOMA. Another observation is that increasing the values of RI from $-30$ dB to $-20$ dB will reduce the system throughput in high SNR region. This is because that CD/PD-NOMA converge to the error floors in the high SNR region.

V. CONCLUSIONS

This paper has investigated the outage performance of a unified NOMA framework insightfully by invoking stochastic geometry. The exact expressions for outage probability of a pair of users with ipSIC/pSIC have been derived. It has been observed that the diversity orders of the $m$-th user for CD/PD-NOMA are $mK$ and $m$, respectively. However, due to the influence of RI, the diversity orders achieved by the $n$-th user with ipSIC are zeros for CD/PD-NOMA. On the basis of analytical results, we observed that the outage behaviors of CD-NOMA is superior to that of PD-NOMA. Additionally, the system throughput of CD/PD-NOMA with ipSIC/pSIC has been discussed in the delay-limited transmission mode.

APPENDIX A: PROOF OF THEOREM 1

The proof starts by assuming $g_m$ and $g_n$ have the same column weights for $G_{K \times M}$. That is to say that $\|diag(b_m)g_m\|^2_2$ and $\|diag(b_n)g_n\|^2_2$ follow the same distribution. Hence based on (5), the expression for outage probability of the $m$-th user is rewritten as 

$$
P_{m,CD} = \Pr \left( \frac{\varepsilon_m}{\rho(a_m - \varepsilon_ma_n)} < \frac{\Delta}{\tau} \right),$$

(A.1)
where $Z_m = \|\text{diag}(h_m)g_m\|^2 = \frac{\eta}{\pi} \sum_{k=1}^{K} |g_{mk}h_{mk}|^2$. It is observed that $Y = \frac{\sum_{k=1}^{K} |g_{mk}h_{mk}|^2}{\sum_{k=1}^{K} \frac{1}{\pi}}$ is subject to a Gamma distribution with the parameters of $(K, 1)$. The corresponding CDF of $Y$ is given by $F_Y(y) = 1 - e^{-\frac{y}{\pi}} \sum_{i=0}^{K-1} \frac{y^i}{\pi^i}$.

In addition, on the basis of order statistics [12], the CDF of the sorted channel gains between the BS and users over $K$ subcarriers has a specific relationship with the unordered channels, which can be expressed as follows:

$$F_{Z_m}(z) = \phi_m \sum_{p=0}^{M-m} \left( \frac{M-m}{p} \right) \left( \frac{-1}{p} \right) m+p \left( F_{Z}(z) \right)^{m+p},$$  \quad \text{(A.2)}$$

where $F_{Z}(z)$ denotes the CDF of unordered channels for the $m$-th user. Due to the assumption of homogeneous PPPs [10] for randomly users and applying polar coordinate conversion, the CDF $F_{Z}(z)$ is given by

$$F_{Z}(z) = \frac{2}{R_{D}} \int_{0}^{R_{D}} \left[ 1 - e^{-z \left(1+\alpha^2\right)} \right] \sum_{i=0}^{K-1} \frac{1}{i!} \left( \frac{z}{\eta} \right)^i \cdot \mathrm{d}r.$$

\text{(A.3)}$$

Obviously, it is difficult to obtain effective insights from the above integral. We employ the Gaussian-Chebyshev quadrature to provide an approximation of (A.3) and rewrite it as follows:

$$F_{Z}(z) \approx \sum_{u=1}^{U} b_u \left( 1 - e^{-\frac{z u}{\eta}} \sum_{i=0}^{K-1} \frac{1}{i!} \left( \frac{z u}{\eta} \right)^i \right).$$ \quad \text{(A.4)}$$

Substituting (A.4) and (A.2) into (A.1), we can obtain (7) and complete the proof.

**APPENDIX B: PROOF OF THEOREM 2**

Denote $Z_{n} = \|\text{diag}(h_n)g_n\|^2 = \|\text{diag}(h_n)g_m\|^2$ and $Y_{1} = \|h_{1}\|^2$, respectively. Substituting (3) and (4) into (9), the COP of $Z_{n,CD}$ can be expressed as

$$\begin{align*}
\rho_{n,CD}^{pSIC} &= \text{Pr} \left( \frac{\rho Z_{n,m}}{\rho Z_{n,m} + 1} < \varepsilon \right) \\
+ &\text{Pr} \left( \frac{\rho Z_{n,m}}{\rho Z_{n,m} + 1} > \varepsilon, \frac{\rho a_m Z_{n}}{\rho P Y_{1} + 1} < \varepsilon \right),
\end{align*}$$ \quad \text{(B.1)}$$

where $J_1 = F_{Z_{n}}(\tau)$, $\tau = \varepsilon a_m / \rho Z_{n,m} + 1$ with $a_m > \varepsilon a_m$ and $\varepsilon = 1$. Noting that $Y_{1}$ is also subject to a Gamma distribution with the parameters of $(K, 1)$ and the corresponding PDF $f_{Y_{1}}$ is given by

$$f_{Y_{1}}(y) = \frac{y^{K-1} e^{-\frac{y}{\eta}}}{(K-1)! \Omega_{K}^{2}}.$$ \quad \text{(B.2)}$$

After some mathematical manipulations, $J_2$ is calculated as

$$J_2 = \text{Pr} \left( \tau < Z_n < \rho Y_{1} + \beta \right).$$