Viscous self interacting dark matter and cosmic acceleration

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Abstract. Self interacting dark matter (SIDM) provides us with a consistent solution to certain astrophysical observations in conflict with collision-less cold DM paradigm. In this work we estimate the shear viscosity ($\eta$) and bulk viscosity ($\zeta$) of SIDM, within kinetic theory formalism, for galactic and cluster size SIDM halos. To that extent we make use of the recent constraints on SIDM cross-section for the dwarf galaxies, LSB galaxies and clusters. We also estimate the change in solution of Einstein’s equation due to these viscous effects and find that $\sigma/m$ constraints on SIDM from astrophysical data provide us with sufficient viscosity to account for the observed cosmic acceleration at present epoch, without the need of any additional dark energy component. Using the estimates of dark matter density for galactic and cluster size halo we find that the mean free path of dark matter $\sim$ few Mpc. Thus the smallest scale at which the viscous effect start playing the role is cluster scale. Astrophysical data for dwarf, LSB galaxies and clusters also seems to suggest the same. The entire analysis is independent of any specific particle physics motivated model for SIDM.

Keywords: dark energy theory, dark matter theory

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1 Introduction

The self interacting dark matter (SIDM) is a lucrative alternative to the standard collision-less cold dark matter (CDM) paradigm. The motivation for SIDM comes from the small scale observations of the Universe which are inconsistent with the CDM predictions. There are four major problems viz, core-cusp problem, diversity problem, missing satellites problem and too-big-to-fail problem. SIDM paradigm has the potential to provide a unifying solution to these problems. We refer to [1] for a detailed review on SIDM and its possible resolution to the above mention problems.

An important implication of self interactions in the dark matter (DM) sector would be a non-zero equation of state (EoS). It has been argued that one should attribute thermodynamic properties like internal energy with DM owing to its self-interactions [2]. In addition, DM can also be provided with a polytropic EoS [3]. Both the above attributes are capable of explaining the Type Ia supernovae (SNe-Ia) data without any additional dark energy component in cosmic fluid. The condition of negative pressure is naturally satisfied in these models. There are indications already that dark sector may have a negative equation of state (EoS) at cluster scale [4] and at cosmological scale [5].

Another standard approach to obtain a dark energy like feature is to assign a non-zero bulk viscosity to the dark sector [6–8]. For an homogeneous, isotropic fluid with bulk viscosity $\zeta$, the pressure is $p_v = p + \Pi_b$, where $p_v$ is the pressure of viscous fluid and $\Pi_b = -3\zeta H$. For sufficiently large $\zeta$, the pressure would be negative thus mimicking dark energy. The source of bulk viscosity has been attributed to neutrinos [9], exotic scalar fields [10] or to the decay of cold dark matter into relativistic particles [11]. However these models are severely constrained by observations of the large scale structure formation [12–14]. The reason being that a large bulk viscosity leads to decay of the gravitational potential during structure formation [12].

A SIDM particle has the potential to avoid these constraints on the viscosity as it behaves like a standard CDM candidate at large scales. One can understand this by looking at the scattering rate of SIDM,

$$R_{\text{scat}} = \frac{\langle \sigma v \rangle \rho_{\text{SIDM}}}{m},$$

where $m$ is the mass of SIDM particle, $\langle \sigma v \rangle$ is the velocity weighted cross-section of SIDM particle, $\rho_{\text{SIDM}}$ is the density of the dark matter. As the density falls, the scattering rate goes to zero and dark matter approaches the standard collision-less CDM behaviour, however at small scales i.e. near the central region of dark matter halo, the density is large and
consequently the scattering rate is higher. It is thus expected that these scattering processes would lead to non-zero viscosity within the dark matter halo. It has recently been argued that the viscosity of cosmic fluid ameliorates the tension between the LSS and plank data [15]. For recent discussions on the role of cosmic viscosity on early (Inflationary epoch) and the late time of the Universe see review [16].

In this work we estimate the shear and bulk viscosity of dark matter within the kinetic theory framework. Assuming the velocity distribution of SIDM to be Maxwellian we obtain the expressions for the shear and the bulk viscosity of SIDM in terms of its velocity weighted cross-section to mass ratio, $\langle \sigma v \rangle / m$, and the average velocity, $\langle v \rangle$, of the dark matter halo. The estimates of $\langle \sigma v \rangle / m$ were obtained in [17] by utilizing the astrophysical data from dwarf galaxies, Low Surface Brightness (LSB) galaxies and clusters. Assuming local thermalization in the dark matter halo, we use these estimates to infer the shear and bulk viscosity in the galaxy as well as in the cluster size dark matter halo. We find that the viscous coefficients increase by two orders of magnitude from galactic to cluster scales.

As an application of these results we explore the scenario discussed in ref. [18]. There it was argued that the local velocity perturbations in cosmological fluid depend on the shear and the bulk viscosity of the dark sector. If the back-reaction of these velocity perturbations is large enough then they contribute substantially to the energy dissipation. This modifies the solution to Einstein’s equations and one could then explain the accelerated cosmic expansion without any dark energy component. We test this assertion using our results for the shear and bulk viscosity of the SIDM. We estimate the contribution of the SIDM viscosity to the energy dissipation and find that the dissipative effects are sufficient to account for the observed cosmic acceleration.

The organization of the paper is as follows. In section 2 we will discuss the calculation of bulk and shear viscosity of the dark matter using kinetic theory and present our estimates of the shear and bulk viscosity at the galactic and cluster scales. In section 3 we estimate the energy dissipation in viscous hydrodynamics and use Einstein equations to argue that the dissipative effects due to viscosity can lead to cosmic acceleration. We present our results on scattering cross-section of SIDM and cosmic acceleration in section 4 and conclude in section 5.

2 Shear and bulk viscosity from kinetic theory

In this section we first outline the formalism of extracting the shear and bulk viscosity within the kinetic theory formalism and then use it to estimate the same for SIDM. We work in natural units. Within the hydrodynamics, the stress energy tensor can, in general, be decomposed in two parts: $T_{\text{ideal}}^{\mu\nu}$ and $T_{\text{diss}}^{\mu\nu}$, where $T_{\text{diss}}^{\mu\nu}$ is the dissipative part of energy momentum tensor. In a local Lorentz frame it can be written as

$$T_{\text{diss}}^{ij} = -\eta \left( \frac{\partial u^i}{\partial x^j} + \frac{\partial u^j}{\partial x^i} - \frac{2}{3} \frac{\partial u^k}{\partial x^k} \delta^{ij} \right) - \zeta \frac{\partial u^k}{\partial x^k} \delta^{ij},$$  (2.1)

where $u$ is the fluid velocity, $\eta$ and $\zeta$ are the shear viscous and bulk viscous coefficients respectively. To estimate viscous coefficients we use the kinetic theory formalism and obtain the expression for the shear and bulk viscosity of SIDM in terms of its distribution function, and velocity weighted cross-section [19, 20].

The starting point of kinetic theory is the Boltzmann equation,

$$\frac{\partial f_p}{\partial t} + v_p \frac{\partial f_p}{\partial x^i} = I\{f_p\},$$  (2.2)
where \( v_p \) is single particle velocity, \( f_p \) is the distribution function and \( I\{f_p\} \) is collisional integral describing the rate of change of \( f_p \) due to collisions. One can evaluate this integral under certain approximations and we use the “relaxation time approximation”. In this approximation it is assumed that the variation of distribution function is slow in space and time or equivalently the system is close to thermal equilibrium. Thus one can locally assign thermodynamic quantities like temperature and energy to the system. In addition it is also assumed collisions are effective in bringing the system to thermal equilibrium within a characteristic time, termed as collision time (\( \tau \)) of the system.

Under these assumptions the collision term can be approximated by the linearized form

\[
I\{f_p\} \simeq -\frac{\delta f_p}{\tau},
\]

(2.3)

where \( \delta f_p \equiv (f_p - f^0_p) \) is the deviation of distribution function from the equilibrium distribution function \( f^0_p \). In a general setup \( \tau \) can be a function of energy. Eq. (2.3) in conjunction with eq. (2.2) gives

\[
\delta f_p = -\tau \left( \frac{\partial f^0_p}{\partial t} + v_p \frac{\partial f^0_p}{\partial x^i} \right).
\]

(2.4)

Since we assume local equilibrium, we can define the average energy density (\( T^{00}_0 \)) and momentum density (\( T^{0i}_0 \)) of the system using the distribution function. Extending the definition to the \( ij \) component of \( T_{\mu\nu} \), we can define \( T_{ij} \) as

\[
T_{ij} = \int \frac{d^3p}{(2\pi)^3} v^i p^j f_p.
\]

(2.5)

Using \( f_p = f^0_p + \delta f_p \), with \( \delta f_p \) given by eq. (2.4), we get the \( T_{ij}^{\text{diss}} \) as

\[
T_{ij}^{\text{diss}} = -\int \frac{d^3p}{(2\pi)^3} \tau v^i p^j \left( \frac{\partial f^0_p}{\partial t} + v_p \frac{\partial f^0_p}{\partial x^i} \right).
\]

(2.6)

Let us now consider the fluid motion along, say, \( x \) axis with fluid velocity \( u_x(y) \), i.e. \( u = (u_x(y), 0, 0) \). In this case eq. (2.1) reduces to \( T^{xy} = -\eta \partial u_x / \partial y \). Using the equilibrium distribution function to be of the form \( f_p = \exp(-p^\mu u_\mu / T) \) in eq. (2.6) and comparing with \( T^{xy} \) written above we get

\[
\eta = \frac{1}{15T} \int \frac{d^3p}{(2\pi)^3} \tau p^i \frac{\partial f^0_p}{\partial E_p}.
\]

(2.7)

For bulk viscosity we have to take the trace of eq. (2.1) and compare with trace of eq. (2.6). Using \( T^{\mu\nu}_\text{diss} = 0 \) and some manipulation one can obtain the expression for the bulk viscosity to be

\[
\zeta = \frac{1}{T} \int \frac{d^3p}{(2\pi)^3} \tau \left[ E_p C_n^2 - \frac{p^2}{3E_p} \right]^2 f^0_p,
\]

(2.8)

where \( C_n = \frac{\partial P}{\partial \epsilon} \big|_n \) is the speed of sound at constant number density. Eq. (2.7) and (2.8) are relativistic expression for the shear bulk viscosity from kinetic theory. To estimate these quantities for dark matter we need to estimate the relaxation time \( \tau \). We may approximate relaxation time with average relaxation time \( \tilde{\tau} \) (this average is over energy distribution), given by

\[
\tilde{\tau}^{-1} = n_s v,
\]

(2.9)
where \( n, \langle \sigma v \rangle \) represent average number density and velocity weighted cross-section average. Since we are working in the cold SIDM paradigm we have to estimate \( \eta, \zeta \) in the non-relativistic limit, but before this we need to check whether the relaxation time approximation, which is used to derive eq. (2.7) and (2.8), holds for the SIDM halos. Thus we should compare the relaxation time (\( \tau \)) with the age of the DM halo, \( t_{\text{age}} \). If \( t_{\text{age}}/\tau \approx 1 \), then it would mean that within a halo the SIDM particles must have interacted at least once. Using eq. (1.1) and (2.9), we note that \( t_{\text{age}}/\tau \approx 1 \) translates to \( R_{\text{scat}} \times t_{\text{age}} \approx 1 \). We take the age of DM halo to be 5 and 10 Gyr for the cluster and galactic size halo respectively [17]. The expression for \( R_{\text{scat}} \), given by eq. (1.1), can be written as

\[
R_{\text{scat}} = 0.1 \text{Gyr}^{-1} \times \left( \frac{\rho_{\text{DM}}}{0.1 M_{\odot} \text{pc}^{-3}} \right) \left( \frac{v}{50 \text{km/s}} \right) \left( \frac{\sigma/m}{1 \text{cm}^2/\text{g}} \right) \tag{2.10}
\]

For dwarf galaxies \( \rho \sim 0.1 M_{\odot} \text{pc}^{-3}, v \sim 50 \text{ km/s} \)[21] and \( \sigma/m \sim 2 \text{ cm}^2/\text{g} \)[17], thus \( R_{\text{scat}} \times t_{\text{age}} \approx 2 \). For cluster size SIDM halo, core density \( \rho \sim 2 \times 10^{-2} - 5 \times 10^{-1} M_{\odot} \text{pc}^{-3} \)[22, 23], \( v \sim 1000 \text{ km/s} \) and \( \sigma/m \sim 0.1 \text{ cm}^2/\text{g} \)[17], thus \( R_{\text{scat}} \times t_{\text{age}} \approx 0.2 - 5 \). This clearly indicates that the relaxation time approximation is valid at the galactic and cluster size DM halo and hence we may assume that the DM halo are in local thermal equilibrium.

We thus use the non-relativistic Maxwell-Boltzmann distribution in fluid rest frame, with eq. (2.9), in the non-relativistic limit of eq. (2.7). This gives us

\[
\eta = \left( \frac{m}{\langle \sigma v \rangle} \right) \left( \frac{T}{m} \right). \tag{2.11}
\]

Since we are able to assign local thermal equilibrium in dark matter halos, we can use equipartition of energy to relate root mean square velocity with the temperature \( T \), i.e. \( \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} T \). Also for Maxwellian velocity distribution \( \sqrt{\langle v^2 \rangle} = 1.08 \langle v \rangle \). Eq. (2.11) thus takes the form

\[
\eta = \frac{1.18 m \langle v \rangle^2}{3 \langle \sigma v \rangle}. \tag{2.12}
\]

A similar exercise for eq. (2.8) gives the bulk viscosity as

\[
\zeta = \frac{5.9 m \langle v \rangle^2}{9 \langle \sigma v \rangle}. \tag{2.13}
\]

We can now estimate the values of shear and bulk viscosity for galactic and cluster size dark matter halos. In figure 1 we plot \( \eta \) and \( \zeta \) as functions of \( \langle v \rangle \). In these plots we have converted the results to SI units while the eq. (2.12) and (2.13) are derived using natural units. It is clear from eq. (2.12) and (2.13) that the viscous coefficient \( \eta \) and \( \zeta \) depend on the mass \( m \) of DM candidate, its velocity weighted cross-section average \( \langle \sigma v \rangle \), and on \( \langle v \rangle \). Also we note that the bulk viscosity contribution is slightly larger than the shear contribution at each scale. If we take \( \langle \sigma v \rangle / (\langle v \rangle m) \) as an estimation of \( \langle \sigma \rangle / m \) at a given scale, then \( \eta \) and \( \zeta \) are proportional to \( \langle v \rangle \) in a dark matter halo. From galactic to cluster scale \( \langle \sigma \rangle / m \) changes from \( \sim 1 \text{ cm}^2\text{g}^{-1} \) to \( \sim 0.1 \text{ cm}^2\text{g}^{-1} \)[17] and \( \langle v \rangle \) goes from \( 10^2 \) to \( 10^3 \) km/s. This would mean that viscosity (\( \eta, \zeta \) both) increase by two orders of magnitude as we go from galactic to cluster scale, see figure 1.
3 Effect of viscosity on cosmic expansion

In this section we explore the consequence of dark viscosity on the evolution of the universe. It was argued in [18] that sufficient viscous contribution from the dark sector can modify the solution of Einstein’s equation and may explain the observed accelerated expansion. We use the estimates of $\eta$ and $\zeta$, obtained above, to study the extent of this effect.

To solve for Einstein’s equations we need the energy momentum tensor for a viscous fluid, which, in Landau frame ($u^\mu T_{\mu\nu} = -u^\nu \epsilon$), is given by the expression

$$T_{\mu\nu} = \epsilon u^\mu u^\nu + (P + \pi_b) \Delta_{\mu\nu} + \pi_{\mu\nu}. \quad (3.1)$$

Here $\Delta_{\mu\nu} = u_\mu u_\nu + g_{\mu\nu}$ is the projection operator orthogonal to the fluid velocity, $\pi_b$ is the bulk stress and shear stress, $\pi_{\mu\nu}$, satisfies the conditions $u_\mu \pi_{\mu\nu} = 0$, $\pi_{\mu\mu} = 0$. In the first order hydrodynamics the bulk and shear stresses are given by the expressions

$$\pi_b = -\zeta \nabla_\mu u^\mu, \quad (3.2a)$$

$$\text{and} \quad \pi_{\mu\nu} = -\eta \left[ \Delta^\alpha\beta \Delta_{\mu\alpha} + \Delta^\mu\beta \Delta_{\nu\alpha} - \frac{2}{3} \Delta_{\mu\nu} \Delta^{\alpha\beta} \right] \nabla_\alpha u_\beta. \quad (3.2b)$$

As seen from figure 1, viscous coefficients increase by two order of magnitude while going from galactic to cluster scale. It is thus important to check whether first order hydrodynamics is sufficient or we need to consider second or higher order contributions to the viscous tensor. We argue that contribution of higher order terms will be small. In non-relativistic hydrodynamics, shear viscosity contribution is given by $\eta \nabla^2 v$ term in the Navier Stokes equation. While going from a galactic to a cluster scale the viscosity increases by 2 orders of magnitude whereas the velocity changes by 1 order ($100 \text{km/s}^{-1} - 1000 \text{km/s}^{-1}$). However, the length scales change by 2 orders of magnitude ($O(10 \text{kpc}) - O(\text{Mpc})$). Thus $\nabla^2 \sim 1/L^2$ decreases by 4 orders of magnitude. This leads to a net suppression of order 1 in $\eta \nabla^2 v$ term, while going from galactic to cluster scale. Thus we see that the effect of gradients, and hence the viscous contributions, mellow down as we go to larger scales. It is then safe to truncate the hydrodynamics at the first order in gradient expansion.

In addition to $G_{\mu\nu} = -8\pi G T_{\mu\nu}$, we have the covariant energy momentum conservation $\nabla_\mu T^{\mu\nu} = 0$. For the metric we consider the conformal FRW metric with scalar perturbations. In the limit of small metric perturbations and small velocities ($v^2 \ll 1$), we get the energy
conservation equation as \[18\]
\[
\frac{1}{a} \dot{\epsilon} + 3H(\langle \epsilon \rangle + \langle P \rangle - 3\zeta H) = D,
\]
(3.3a)
where \[ D = \frac{1}{a^2} \left\langle \eta \left[ \partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3} \partial_i v_i \partial_j v_j \right] \right\rangle + \frac{1}{a^2} \left\langle \zeta [\vec{\nabla} \cdot \vec{v}]^2 \right\rangle + \frac{1}{a} \left\langle \vec{v} \cdot \vec{\nabla} (P - 6\zeta H) \right\rangle
\]
(3.3b)
is the dissipation term. In the above equations \(A\) is the spatial average of the quantity \(A\). As is evident from equation (3.3a), that the evolution of average energy density depends crucially on the dissipative term \(D\). Only at late times the velocity gradients acquire non zero values hence it is at late times that the dissipative term becomes important and contribute significantly to the average energy evolution equation. This in-turn affects the evolution of Universe through Einstein’s equations.

To estimate the effect of viscous term on the evolution of the Universe, eq. (3.3a) needs to be solved in conjunction with the suitably averaged Einstein’s equation. The simplest choice is the traced equation \(\langle \nabla \mu \rangle = -8\pi G \langle T^\mu_\mu \rangle\), which gives
\[
\frac{\dot{a}}{a^3} = \frac{H}{a} + 2H^2 = \frac{4\pi G(\langle \epsilon \rangle)}{3} (1 - 3\dot{\omega}_{\text{eff}})
\]
(3.4)
where, \(\dot{\omega}_{\text{eff}} = \frac{\langle P \rangle}{\langle \epsilon \rangle} + \frac{\langle \pi_b \rangle}{\langle \epsilon \rangle}\) is the effective equation of state. We note here that our definition of \(\dot{\omega}_{\text{eff}}\) is the same as the one used in ref. \[18\].

Using eq. (3.3a) with eq. (3.4) one can find equation for deceleration parameter (defined as \(q = -1 - \frac{\dot{H}}{aH^2}\)) to be \[18\]
\[
- \frac{dq}{d\ln a} + 2(q - 1) \left( q - \frac{1 + 3\dot{\omega}_{\text{eff}}}{2} \right) = \frac{4\pi GD(1 - 3\dot{\omega}_{\text{eff}})}{3H^4}
\]
(3.5)
The condition for accelerating universe \((q < 0)\) is satisfied if
\[
\frac{4\pi GD}{3H^4} > \frac{1 + 3\dot{\omega}_{\text{eff}}}{1 - 3\dot{\omega}_{\text{eff}}}
\]
(3.6)
Thus a sufficiently large and positive \(D\) can lead to the accelerated expansion, if \(\dot{\omega}_{\text{eff}} \neq 1/3\). In eq. (3.3a) and (3.4) the averaging is done on a length scale where viscous effects, and hence self-interactions, are important. Next, we find out the appropriate EoS for the SIDM. We have already defined the “effective equation of state”, \(\dot{\omega}_{\text{eff}} = \frac{\langle P \rangle}{\langle \epsilon \rangle} + \frac{\langle \pi_b \rangle}{\langle \epsilon \rangle}\). For a non-relativistic system having a Maxwell-Boltmann distribution one can write the relationship between the kinetic pressure and the number density \(P = nT\) which can be written using equipartition arguments \(P = \epsilon \langle v \rangle^2 / 3\). Considering \(\langle v \rangle \sim 10^{-3}\), we find that value of \(\langle P \rangle / \langle \epsilon \rangle\) is \(\sim 10^{-6}\) for the cluster DM halo. For the second term in \(\dot{\omega}_{\text{eff}}, \langle \pi_b \rangle = -\zeta (\langle \nabla \cdot \vec{v} \rangle + 3\zeta H_0)\). To estimate gradients we approximate \(\partial_i \sim 1 / L\) where \(L\) is the size of the DM halo. For core density, \(\epsilon \sim 5 \times 10^{-1} \text{M}_\odot \text{pc}^{-3}\) \[22, 23\] and \(L \sim 1\) Mpc on the cluster DM halo, we find \(\langle \pi_b \rangle / \langle \epsilon \rangle \sim 10^{-7}\). Hence we find \(\dot{\omega}_{\text{eff}} \ll 1\) on the cluster DM halo and one can therefore write the condition for accelerating Universe using eq. (3.6), to be \(\frac{4\pi GD}{3H^4} > 1\). From observations, \(q \approx -0.6\) at present epoch \[24\]. Thus the dissipation term (assuming \(\left| \frac{dq}{d\ln a} \right| \ll 1\) and \(\dot{\omega}_{\text{eff}} = 0\) is estimated to be \(\frac{4\pi GD}{3H^4} \approx 3.5\) \[18\]. Using the results obtained in the previous sections we
would like to come up with an estimate of $D$ and check the extent of this effect. We use the following simplifying assumptions: (i) We assume that $\eta, \zeta$ do not vary over the spatial region which is averaged. (ii) Gradients and curls of velocity fields are prominent at a scale $L$, thus we approximate $\partial_i \sim 1/L \equiv 1/(R_H \alpha)$, where $R_H = H^{-1}$ is the Hubble size and $\alpha = L/R_H$ is the fraction of Hubble size where the derivatives are prominent. (iii) We confine ourselves to the present epoch, thus we set $H = H_0$ and scale factor $a \equiv a_0 = 1$. (iv) For $\dot{\psi}_\text{eff} = 0$, $\langle P \rangle = -\langle \pi_b \rangle = \zeta((\nabla \cdot \vec{v}) + 3H_0) = \zeta \left( \left( \frac{H_0 \langle \dot{v} \rangle}{\alpha} \right) + 3H_0 \right)$. Thus last term in eq. (3.3b) becomes

$$\langle \vec{v} \cdot \nabla (P - 6\zeta H) \rangle = \zeta \left( \frac{H_0 \langle \dot{v} \rangle}{\alpha} \right)^2 \left[ 1 - 3 \frac{\alpha}{\langle \dot{v} \rangle} \right] \sim \zeta \left( \frac{H_0 \langle \dot{v} \rangle}{\alpha} \right)^2$$

since $\alpha/\langle \dot{v} \rangle \ll 1$. With these assumptions and with the use of eq. (2.12) and (2.13) in the eq. (3.3b), we get the value of dissipative term at present epoch as

$$D = \frac{16.32 \langle \dot{v} \rangle^4}{9} \left( \frac{m}{\sigma \dot{v}} \right) \left( \frac{H_0 \langle \dot{v} \rangle}{\alpha} \right)^2.$$  

(3.7)

Hence we need to calculate dissipative term $D$, which depends on $\langle \sigma \dot{v} \rangle/m$, $\langle \dot{v} \rangle$ and the size $(H_0/\alpha)$ of SIDM halo. An important consideration here is the scale of averaging (galactic or cluster). We know, from observations, that the accelerated cosmic expansion manifests itself at super-cluster or larger scales, thus intuitively the averaging doesn’t seem appropriate for a SIDM halo of galactic size. To settle the issue of averaging scales we estimate the mean free path of the SIDM. The idea behind this exercise is that for the hydrodynamic description to be valid (which allows us to write eq. (3.1) in the first place) the mean free path should be smaller than, or at least be the order of, the averaging scale.

As the densities for galaxy and clusters are traditionally quoted in the units of $M_\odot/kpc^{-3}$, we leave natural units for estimations of mean free path. From eq. (2.12) we have $D \sim \eta \left( \frac{\nabla \cdot \vec{v}}{\dot{v}} \right)^2$. For $\eta$ we use the textbook expression for viscosity of a dilute gas viz. $\eta = \rho v \lambda/3$. With approximations $\partial_i \sim 1/L$ and $v \sim \langle \dot{v} \rangle$ we get $D \sim \rho \lambda \langle \dot{v} \rangle^3/3L^2$. Equating with eq. (3.7) we get $\lambda \sim 5 \left( \frac{m}{\sigma} \right)^{1/3} \rho$. Since $\sigma/m$ is expressed in units of $cm^2/g$, we convert it to $kpc^2/M_\odot$. In these units the expression for mean free path is

$$\lambda \sim 5 \times 10^{10} \left( \frac{m}{\sigma} \right)^{1/3} \rho \text{ (in kpc)},$$

(3.8)

where $\sigma/m$ corresponds to the numerical value in the units of $cm^2/g$ and $\rho$ is numerical value in units of $M_\odot/kpc^{-3}$. For galactic scale $\sigma/m \sim 2 \text{ $cm^2/g$ [17]}$. So for the SIDM mean free path of $\sim 10$ kpc would require that the density at the galactic scales be $2.5 \times 10^7 M_\odot/kpc^{-3}$. At galactic scale the densities are much lower e.g for Milky way the estimates for DM density (with NFW profile) are $\sim 10^6 - 10^7 M_\odot/kpc^{-3}$ [25] which are much lower. For LSB galaxies the peak density for isothermal case are $\sim 10^7 M_\odot/kpc^{-3}$ [26]. For dwarf galaxies the dark matter concentration can be a bit larger ($\sim 10^8 M_\odot/kpc^{-3}$) [21], but the typical sizes are smaller than 10 kpc.

For cluster scale halos, $\sigma/m \sim 0.1 \text{ $cm^2/g$ [17]}$, thus for $\lambda \sim 1$ Mpc we need the halo density to be $\sim 5 \times 10^8 M_\odot/kpc^{-3}$ [from eq. (3.8)]. Such high densities have been estimated in cluster size dark matter halos using isothermal profile for dark matter [22, 23]. It thus seems appropriate to argue that the hydro description and consequently the averaging in dissipation term is not appropriate for galactic scale SIDM halos but for cluster scale at the least.
20 kpc
30 kpc
50 kpc
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2
1
2
5
10
20
q
s
m
H
cm
2
gL
1 Mpc
3 Mpc
5 Mpc
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2
0.1
0.2
0.5
1.0
2.0
5.0
10.0
q
s
m
H
cm
2
gL

Figure 2. The plot between $\sigma/m$ vs $q$ for different $\langle v \rangle$ and $\alpha$ at galactic (left) and cluster (right) scale. The velocities are taken to be 100 kms$^{-1}$ and 1000 kms$^{-1}$ for galactic and cluster scales. Larger negative value of $q$ is supported by smaller values of $\sigma/m$.

Figure 3. Plot of $\langle \sigma v \rangle/m$ vs $\langle v \rangle$ for galactic (left) and cluster (right) size dark halo for $q = -0.6$. The data points are from [17].

4 Results and discussions

The evolution of the deceleration parameter $q$ is governed by eq. (3.5), with $D$ given by eq. (3.7). Assuming $\left| \frac{dq}{dt} \right| \ll 1$, we get $\langle \sigma v \rangle/m$ as

$$\frac{\langle \sigma v \rangle}{m} = \frac{65.28 \pi \langle v \rangle^4}{27(q-1)(2q-1)H_0 m_{\text{pl}}^2 \alpha^2}, \quad \text{where } m_{\text{pl}}^2 = \frac{1}{G}. \quad (4.1)$$

Dividing both sides by $\langle v \rangle$ provides us with an estimate of $\sigma/m$ for different halo sizes. The result is presented in figure 2. For plotting we choose $\langle v \rangle$ to be 100 kms$^{-1}$ for galactic scale while for a cluster size halo we take $\langle v \rangle \sim 10^3$ kms$^{-1}$ as a representative number. It is evident that present observed deceleration parameter ($q \approx -0.6$) can be obtained for a dark matter halo of roughly 50 – 30 kpc size with $\sigma/m \sim 1 - 2$ cm$^2$/g. Similarly at cluster scale the dark matter halo of 3 – 5 Mpc size can provide us with correct $q$ values with $\sigma/m \sim 0.1$ cm$^2$/g. The $\sigma/m$ values of $1 - 2$ cm$^2$/g at galactic scale and $0.1$ cm$^2$/g at cluster scale were deduced from astrophysical data in [17]. To check it further, we plot $\langle \sigma v \rangle/m$ vs $\langle v \rangle$ for $q = -0.6$, at galactic and cluster scales, and compare with the $\langle \sigma v \rangle/m$ values inferred from the astrophysical data [17] in figure 3.
The large dispersion in $\langle \sigma v \rangle / m$ at the galactic scale, as compared to the cluster scale, is an important feature of figure 3. The size of dwarf galaxies is almost 5 kpc and the LSBs are also of size smaller than 10 kpc. Eq. (4.1) thus provides a poor fit to the data as can be seen in figure 3. However for cluster scale halos the scatter is comparatively less and the fit is better than the galactic scale. This can be thought of as the better applicability of the averaging process at the cluster scale as compared to the galactic scale as discussed at the end of previous section.

5 Conclusions

In this work we have estimated the shear and bulk viscosity due to dark matter self interactions within kinetic theory formalism. Assuming local thermalization in dark matter halos, we determined the relation between viscous coefficients ($\eta$, $\zeta$), velocity weighted cross-section to mass ratio ($\langle \sigma v \rangle / m$) of dark matter and $\langle v \rangle$ of dark matter halos. The estimates suggest that $\eta$ and $\zeta$ change by roughly two orders of magnitude from the galactic to cluster scale. We also looked at the effect of dissipation due to these viscous effects, on cosmic acceleration. We find that $\eta$ and $\zeta$ estimated from astrophysical data for $\sigma / m$ of SIDM might account for the observed cosmic acceleration. To get a better understanding of the scales where these effects might be important, we also estimated the mean free path of SIDM. We find that the mean free path $\sim$ few Mpc for a cluster size dark matter halo is supported by the astrophysical data. We thus conclude that smallest scale for viscous effects to play a role in dynamics of the Universe should be cluster scale. We set our theoretical understanding against the astrophysical data and find that data also points in the same direction.

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