A Review of $h_c(1P_1)$, $\eta_c(1S)$ and $\eta_c(2S)$

Jianming Bian

School of Physics and Astronomy
University of Minnesota, Minneapolis, MN 55455

Presented at the 5th International Workshop on Charm Physics
Honolulu, Hawai‘i, May 14–17, 2012

Abstract

Recent experimental results on charmonium $h_c(1P_1)$, $\eta_c(1S)$ and $\eta_c(2S)$ from Belle, BaBar, CLEO and BESIII are reviewed. $h_c$ production and properties, the $\eta_c(1S)$ lineshape and the observation of $\eta_c(2S)$ in $\psi'$ decays are discussed.

PACS numbers: 14.40.Gx, 13.25.Gv, 13.20.Gd, 12.38.Qk
I. INTRODUCTION

The charmonium family is a great laboratory for precision tests of the quark model, because of their relative immunity from complications like relativistic effects and the large value of the strong coupling constant $\alpha_s$. In this talk, a brief review of recent experimental results on charmonium $h_c(1^1P_1)$, $\eta_c(1S)$ and $\eta_c(2S)$ is presented. Although these states were predicted just after the discovery of $J/\psi$, their properties were not very clear for a long period. $h_c$ is the mostly recently discovered charmonium states; recent studies uncovered its production and properties. $\eta_c(1S)$ is the lowest-lying $S$-wave spin-singlet charmonium state and has been observed through various processes. Its “inconsistent” lineshapes in different production modes inspired a couple of precise measurements in the last few years. The $\eta_c(2S)$ is the first radial excitation of the $\eta_c$ charmonium ground state. After 30 years of searching, it was recently observed in charmonium transitions, having been observed in $B$ decays and $\gamma\gamma$ fusion previously. Results in this talk come from BESIII, CLEO, BaBar and Belle. BESIII and CLEO study charmonium from $\psi'$ decays and $e^+e^-$ annihilation near $D\bar{D}$ threshold. They provide very clean and simple environments. BaBar and Belle are $B-$factories, which produce charmonium via $\gamma\gamma$ fusion and $B$ decays. The advantage of studying charmonium in a $B-$factory is the relatively large statistics and reconstruction efficiency.

II. $h_c$

Of the charmonium states below $D\bar{D}$ threshold, the $h_c(1^1P_1)$ is experimentally the least accessible. That is because it cannot be produced directly in $e^+e^-$ annihilation, or appear in the electric dipole transition process of a $J^{PC} = 1^{--}$ charmonium state. Statistics and photon detection also made it very challenging for early experiments to observe $h_c$ in charmonium transitions.

Information about the spin-dependent interaction of heavy quarks can be obtained from precise measurement of the $1P$ hyperfine mass splitting $\Delta M_{hf} \equiv \langle M(1^3P) \rangle - M(1^1P_1)$, where $\langle M(1^3P_J) \rangle = (M(\chi_{c0}) + 3M(\chi_{c1}) + 5M(\chi_{c2})) / 9 = 3525.30 \pm 0.04$ MeV/c$^2$ [1] is the spin-weighted centroid of the $3P_J$ mass and $M(1^1P_1)$ is the mass of the singlet state $h_c$. A non-zero hyperfine splitting may give indication of non-vanishing spin-spin interactions in charmonium potential models [2].

The first evidence of the $h_c$ state was reported by the Fermilab E760 experiment [3] and was based on the process $p\bar{p} \rightarrow \pi^0 J/\psi$. This result was subsequently excluded by the successor experiment E835 [4], which investigated the same reaction with a larger data sample. E835 also studied the process $pp \rightarrow h_c \rightarrow \gamma\eta_c(1S)$, in this case finding an $h_c$ signal. Soon after this the CLEO collaboration observed the $h_c$ and measured its mass [5, 6] by studying the decay chain $\psi(3686) \rightarrow \pi^0 h_c, h_c \rightarrow \gamma\eta_c(1S)$ in $e^+e^-$ collisions. CLEO subsequently presented evidence for $h_c$ decays to multi-pion final states [7]. Since these data were collected in 2009, BESIII has put lots of effort into measuring the properties of $h_c$.

To study the decay $\psi' \rightarrow \pi^0 h_c, h_c \rightarrow \gamma\eta_c(1S)$, three methods have been used:

- Inclusive: In the inclusive mode, only the $\pi^0$ is detected and the $h_c$ are recognized as a peak in the $\pi^0$ recoil mass spectrum. The $\pi^0$ momentum in $\psi' \rightarrow \pi^0 h_c$ is about 85
MeV. The inclusive yield is used to extract the absolute branching ratio of $\psi' \rightarrow \pi^0 h_c$. This mode has the largest background.

- E1-tagged: Detecting the $\pi^0$ and the E1 transition $\gamma$ from the $h_c \rightarrow \gamma \eta_c(1S)$ (500 MeV). The E1-tagged signal yield is proportional to the product branching ratio $B(\psi' \rightarrow \pi^0 h_c) \times B(h_c \rightarrow \gamma \eta_c)$. Combining with the inclusive measurement, E1-tagging also provides the absolute branching ratio of $h_c \rightarrow \gamma \eta_c(1S)$. The background for this mode is smaller than that for the inclusive mode.

- Exclusive: Reconstructing $\pi^0$, E1$\gamma$ and all of the decay products of the $\eta_c(1S)$ in $\psi' \rightarrow \pi^0 h_c, h_c \rightarrow \gamma \eta_c(1S)$. Here all final-state particles are detected and energy-momentum conserving kinematic fits can be used to improve the resolution. This method has small background and provides the best $h_c$ mass and width measurement. The yield is proportional to $B(\psi' \rightarrow \pi^0 h_c) \times B(h_c \rightarrow \gamma \eta_c) \times B(\eta_c \rightarrow X_i)$, where the $X_i$ refer to specific final states in the $\eta_c(1S)$ decay.

The observation of $h_c$ from CLEO used the E1-tagged and exclusive modes. Using inclusive and E1-tagged modes, BESIII first measured the absolute branching ratios $B(\psi(3686) \rightarrow \pi^0 h_c) = (8.4 \pm 1.3 \text{(stat.)} \pm 1.0 \text{(syst.)}) \times 10^{-4}$ and $B(h_c \rightarrow \gamma \eta_c) = (54.3 \pm 6.7 \pm 5.2)\%$ [8]. These results are consistent with theoretical expectations and make it possible to extract absolute $h_c$ cross sections/branching ratios [10–13]. In the same paper, BESIII also determined the mass and width of $h_c$ to be $M(h_c) = 3525.40 \pm 0.13 \pm 0.18$ MeV and $\Gamma(h_c) = 0.73 \pm 0.40 \pm 0.50$ MeV (90% confidence level upper limit is 1.44 MeV), which agree with CLEO’s observation. With this mass value, the $P$-wave hyperfine splitting is $-0.10 \pm 0.22$ MeV, consistent with zero. Fig. 1 shows the $h_c$ signals and fits in the inclusive and E1-tagged $\pi^0$ recoil mass spectrum in $\psi'$ decays [8].

A detailed study of 16 exclusive channels is in progress at BESIII [9]. The aim is to obtain the most precise $h_c$ resonance parameters and study $\eta_c(1S)$ line-shape parameters in the E1 transition $h_c \rightarrow \gamma \eta_c(1S)$. Preliminary results of this study are $M(h_c) = 3525.31 \pm 0.11 \pm 0.15$ and $\Gamma(h_c) = 0.70 \pm 0.28 \pm 0.25$ MeV. The summed fit results are shown in Fig. 2.

CLEO has confirmed BESIII’s inclusive measurement result [14]. By rejecting very asymmetric $\pi^0 \rightarrow \gamma \gamma$ decays, CLEO also observed $h_c$ signal in the inclusive $\pi^0$ spectrum in $\psi'$ decays (Figure 3). $B(\psi' \rightarrow \pi^0 h_c)$ is determined to be $(9.0 \pm 1.5 \pm 1.3) \times 10^{-4}$, which is consistent with the BESIII measurement.

Beyond $\psi' \rightarrow \pi^0 h_c$, CLEO has made an important discovery of $h_c$ production in $e^+e^- \rightarrow \pi^+\pi^- h_c$ at $\sqrt{s} = 4170$ MeV using 586 pb$^{-1}$ of $e^+e^-$ annihilation data. 10σ signal for $h_c$ was found in the decay $e^+e^- (4170) \rightarrow \pi^+\pi^- h_c, h_c \rightarrow \gamma \eta_c, \eta_c \rightarrow 12$ decay modes. This result demonstrates a new prolific source of $h_c$ and has inspired the Belle collaboration to search for $h_b(1P, 2P)$ in $e^+e^-$ annihilations at $= 10.685$ GeV using the same technique. CLEO also finds evidence for $e^+e^- \rightarrow \eta h_c(1P)$ at 4170 MeV at the 3σ level, and sees hints of a rise in the $e^+e^- \rightarrow \pi^+\pi^- h_c(1P)$ cross section at 4260 MeV [15]. The $\pi^+\pi^- h_c$ cross sections measured by CLEO at different center-of-mass energies are summarized in Fig. 4.

III. $\eta_c(1S)$

$\eta_c(1S)$ is the lowest-lying $S$-wave spin-singlet charmonium state. Although it has been known for about thirty years [16], its resonance parameters are still interesting.
For a long period, the measurements of the $\eta_c(1S)$ width from $B-$factories and from charmonium transitions were inconsistent. In PDG10 [17], confidence level of the global fit is only 0.0018 for the $\eta_c(1S)$ mass and only 0.0001 for the $\eta_c(1S)$ width. These discrepancies can be attributed to poor statistics and inadequate consideration of interference between $\eta_c(1S)$ decays and non-resonant backgrounds. The experimental confusion introduced difficulties in the determination of the charmonium 1S mass hyperfine $M(J/\psi) - M(\eta_c)$. With old $\eta_c(1S)$ parameters in PDG10, 1S hyperfine mass splitting is 116.6 ± 1.2 MeV, away from theoretical predictions [18].

Recent studies by Belle, BaBar, CLEO, and BESIII [19–22], with large data samples and careful consideration of interference, obtained similar $\eta_c(1S)$ width and mass results in two-photon-fusion production and $\psi'$ decays. In 2009, CLEO observed a distortion in the $\eta_c(1S)$ line shape in $\psi' \rightarrow \gamma \eta_c(1S)$. CLEO concluded that the distortion is caused by photon-energy dependence of the magnetic dipole transition rate (hindered-M1 transition) [21]. This observation inspired BESIII’s $\eta_c(1S)$ line shape study via $\psi' \rightarrow \gamma \eta_c(1S)$ with a 106M-event $\psi'$ sample [22]. In BESIII’s $\eta_c(1S)$ analysis, $\eta_c(1S)$ is reconstructed with six decay modes: $K_S K^+ \pi^-$, $K^+ K^- \pi^0$, $\pi^+ \pi^- \eta$, $K_S K^+ \pi^- \pi^+ \pi^-$, $K^+ K^- \pi^+ \pi^- \pi^0$, and $3(\pi^+ \pi^-)$. A simultaneous fit to these channels is performed. The $\eta_c(1S)$ Breit-Wigner is weighted by an $E_\gamma^2$ factor to account for the energy dependence of the hindered-M1 transition. Interference with background from non-resonant $\psi'$ decays is also considered. The new BESIII mass and width values are $M(\eta_c) = 2984.3 \pm 0.6 \pm 0.6$ MeV/c$^2$ and $\Gamma(\eta_c) = 32.0 \pm 1.2 \pm 1.0$ MeV. They agree well with results from $B-$factories. Using only the new BESIII $\eta_c(1S)$ mass value, the $J/\psi - \eta_c(1S)$ hyperfine mass splitting is 112.6 ± 0.8 MeV, which agrees better with theory calculations. Figure 5 shows the data and fit for each channel in this analysis. The result from BESIII provides strong evidence that
FIG. 2: The $\pi^0$ recoil mass spectrum in $\psi(3686) \rightarrow \pi^0 h_c, h_c \rightarrow \gamma \eta_c(1S), \eta_c \rightarrow X_i$ summed over the 16 final states $X_i$ in BESIII’s $h_c$ exclusive study. The dots with error bars represent the $\pi^0$ recoil mass spectrum in data, The solid line shows the total fit function and the dashed line is the background component of the fit.

FIG. 3: Inclusive/E1-tagged $h_c$ measurement from CLEO: (a) Fit to the inclusive $\pi^0$ recoil mass spectrum of $\psi'$. (b) As in (a) but with the background fit from (a) subtracted.
previous inconsistent $\eta_c(1S)$ parameters in radiative charmonium decays and two-photon collisions $B-$meson decays are caused by the hindered-$M1$ transition and non-resonant interference.

There are also new measurements of $\eta_c(1S)$ from $B-$factories. With a data sample of 535 million $B\bar{B}$-meson pairs, Belle measured the $\eta_c(1S)$ lineshape via $B^+ \to \eta_c, \eta_c \to K_SK^+\pi^-$. Compared to the two-photon process, the advantages of studying $\eta_c(1S)$ in $B$ decays are the relatively large reconstruction efficiency, small background, and the fixed quantum numbers of the initial state. In Belle’s analysis, a 2D-fit to the $M(K_SK\pi)$ and $\cos\theta$ distributions is performed to obtain the interference contributions, where $\cos\theta$ is defined in Fig. 6. To reduce the uncertainty from the interference, P- and D-waves are separated from the S-wave in the non-resonant background. Fig. 7 shows projections of the fit. $\eta_c(1S)$ parameters measured in this analysis are $M(\eta_c) = (2985.4 \pm 1.5^{+0.5}_{-2.0})$ MeV/$c^2$ and $\Gamma(\eta_c) = (35.1 \pm 3.1^{+1.0}_{-1.8})$ MeV/$c^2$, which are consistent with recent results from BESIII [22] and BaBar [20].

The $h_c \to \gamma\eta_c(1S)$ transition can provide a new laboratory to study $\eta_c(1S)$ properties. The $\eta_c(1S)$ line shape in the $E1$ transition $h_c \to \gamma\eta_c(1S)$ should not be as distorted as in two-photon production at $B-$factories and in other charmonium decays, because non-resonant interfering backgrounds to the dominant transition are small. BESIII is trying to use this method to extract $\eta_c(1S)$ parameters [3].

IV. $\eta_c(2S)$

The $\eta_c(2S)$ is the first radial excitation of the $\eta_c$ charmonium ground state. It was first observed by the Belle in $B$ decays [23]. Since then, it has been confirmed and studied in $B-$factories via two-photon fusion, double-charmonium production and $B$ decays [23]-[27]. In early days the only known decay mode of $\eta_c(2S)$ was $K_SK^+\pi^-$ [17].
FIG. 5: $\eta_c(1S)$ lineshape measurement in $\psi \to \gamma \eta_c(1S)$ from BESIII: The $M(X_i)$ invariant mass distributions for the decays $K_SK^+\pi^-$, $K^+K^-\pi^0$, $\pi^+\pi^-\eta$, $K_SK^+\pi^+\pi^-$, $K^+K^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^+\pi^-$, respectively, with the fit results (for the constructive solution) superimposed. Points are data and the various curves are the total fit results. Signals are shown as short-dashed lines, the non-resonant components as long-dashed lines, and the interference between them as dotted lines. Shaded histograms are in red/yellow/green for continuum/$\pi^0X_i$/other $\psi'\to\eta'X_i$ decays backgrounds. The continuum backgrounds for $K_SK^+\pi^-$ and $\pi^+\pi^-\eta$ decays are negligible.

![Diagram](https://example.com/diagram.png)

FIG. 6: The decay $B^\pm \to K^\pm \eta_c \to K^\pm (K_SK\pi)^0$.

At the present time, Belle’s and BaBar’s efforts on $\eta_c(2S)$ have been moved to measuring its mass and width and looking for new decays other than $K_SK\pi$. Using the same technique in the study of $\eta_c(1S)$, Belle tried to extract $\eta_c(2S)$ resonance parameters. They found that the interference of signal and non-resonant background was very important in the $\eta_c(2S)$ case: with interference, $M(\eta_c(2S)) = 3636.1^{+16.6}_{-14.9}(stat.+model)_{+1.3}^{+0.8} MeV/c^2$ and $\Gamma(\eta_c(2S)) = 6.6^{+4.4}_{-1.1}(stat.+model)_{+2.6}^{+0.9} MeV/c^2$; without interference, $M(\eta_c(2S)) = 3646.5^{+3.7}_{-2.5} MeV/c^2$ and $\Gamma(\eta_c(2S)) = 41.1 \pm 12.0_{-10.9}^{+6.4} MeV/c^2$. Figure 8 shows projections of the fit in Belle’s analysis with the consideration of interference.

BaBar studied $\eta_c(1S)$ and $\eta_c(2S)$ in the two-photon processes $\gamma\gamma \to K_SK^+\pi^-$ and $\gamma\gamma \to
FIG. 7: $\eta_c(1S)$ lineshape measurement via $\eta_c \rightarrow K_SK_\pi$ in $B^\pm \rightarrow K^\pm(K_SK_\pi)^0$ from Belle: Projections of the fit in $K_SK_\pi$ invariant mass in the $\eta_c(1S)$ mass region (left) and $\cos \theta$ in the $\eta_c(1S)$ invariant mass signal (center) and sideband (right) regions. The combinatorial background is subtracted. The gap near 3.1 GeV/c$^2$ is due to the $J/\psi$ veto. The bin size along the $\cos \theta$ axis is 0.2. Along the $M(K_SK_\pi)$ axis the bin size is 10 MeV/c$^2$ in the signal region and 150/130 MeV/c$^2$ in the left/right sideband region.

FIG. 8: $\eta_c(2S)$ lineshape measurement via $\eta_c \rightarrow K_SK_\pi$ in $B^\pm \rightarrow K^\pm(K_SK_\pi)^0$ from Belle: Projections of the fit in $K_SK_\pi$ invariant mass in the $\eta_c(2S)$ mass region (left) and $\cos \theta$ in the $\eta_c(2S)$ invariant mass signal (center) and sideband (right) regions. The combinatorial background is subtracted. The gap near 3.5 GeV/c$^2$ is due to the $\chi_{c1}$ veto. The bin size along the $\cos \theta$ axis is 0.2. Along the $M(K_SK_\pi)$ axis the bin size is 16 MeV/c$^2$ in the signal region and 130 MeV/c$^2$ in the sideband region.

$K^+K^-\pi^+\pi^-\pi^0$ using a data sample of 519.2 fb$^{-1}$ near the $\Upsilon(nS)$ ($n = 2, 3, 4$) resonances. $\eta_c(2S) \rightarrow K^+K^-\pi^+\pi^-\pi^0$ was found with a significance of $5.3\sigma$ [20]. BaBar also obtained the $\eta_c(2S)$ mass and width by performing to a fit to the $M(K_SK_\pi^\pm\pi^-)$ spectrum (Fig. 9), and obtained the values $M(\eta_c(2S)) = 3638.5 \pm 1.5 \pm 0.8$ MeV/c$^2$ and $\Gamma(\eta_c(2S)) = 13.4 \pm 4.6 \pm 3.2$ MeV/c$^2$.

On the other hand, the search for $\eta_c(2S)$ through a radiative transition from the $\psi'$ is very hard and has been stuck for many years. The difficulty comes from the detection of the low-energy radiative photon in the $\psi' \rightarrow \gamma\eta_c(2S)$. The branching ratio and mechanism of this process has been predicted by many papers, but the absence of experimental result had made it impossible to discriminate among them [28]–[30]. For a long period, this transition has been looked for by Crystal Ball [31], BES [32], CLEO [33] and BESIII [34], but none of them could provide a convincing observation. Most recently, BESIII made the
FIG. 9: $\eta_c(1S)$ and $\eta_c(2S)$ decays to the $K_SK^+\pi^-$ and the $K^+K^-\pi^+\pi^-\pi^0$ in two-photon interactions from Babar: Fit to (a) the $K_SK^+\pi^-$ and (c) the $K^+K^-\pi^+\pi^-\pi^0$ mass spectrum. The solid curves represent the total fit functions and the dashed curves show the combinatorial background contributions. The background-subtracted distributions are shown in (b) and (d), where the solid curves indicate the signal components.

first observation of this process using the 106M $\psi'$ sample. Analyses of $\psi' \to \gamma \eta_c(2S)$ with $\eta_c(2S) \to K_SK^+\pi^-$ and $K^+K^-\pi^0$ gave a significance greater than 10$\sigma$. In addition to the excellent low energy photon detection, smart use of the kinematic fitting plays a key role in this observation. Fig. 10 shows data and fits. Numerical results are $M(\eta_c) = 3637.6 \pm 2.9 \pm 1.6$ MeV/$c^2$, $\Gamma(\eta_c(2S)) = 16.9 \pm 6.4 \pm 4.8$ MeV and $B(\psi' \to \gamma \eta_c(2S)) \times B(\eta_c(2S) \to K\bar{K}\pi) = (1.30 \pm 0.20 \pm 0.30) \times 10^{-5}$. The branching ratio of $\psi' \to \gamma \eta_c(2S)$ is determined to be $B(\psi' \to \gamma \eta_c(2S)) = (6.8 \pm 1.1 \pm 4.5) \times 10^{-4}$. These results are consistent with results from $B$–factories.
V. SUMMARY

In summary, experimental studies of $h_c$, $\eta_c(1S)$ and $\eta_c(2S)$ have made major progress and still face nontrivial challenges:

- $h_c$
  The key branching ratios $\psi' \to \pi^0 h_c$ and $h_c \to \gamma \eta_c$ have been nailed down, so the absolute $h_c$ cross sections/branching ratios are available. A new prolific production mode of $h_c$ has been found: $e^+ e^- \to \pi^+ \pi^- h_c$. Because the $B(h_c \to \gamma \eta_c)$ is about 50%, the remaining decays of $h_c$ should be large enough to be observed. Further measurement of these unclear decays will be helpful to understand the property of $h_c$ and the transition mechanism between $h_c$ and other charmonium.

- $\eta_c(1S)$
  The mass and width are more consistent in $\psi'$ decays, $B$ decays and $\gamma \gamma$ production than previously, and the charmonium $1S$ mass hyperfine splitting from experiments agrees better with theory. The $\eta_c$ lineshape in $h_c$ is not as distorted as in charmonium/$B$ decays and $\gamma \gamma$ fusion, because of the small non-resonant interfering background. Ultimately, with a large $\psi'$ sample, this channel will be best suited to determine $\eta_c$ resonance parameters.

- $\eta_c(2S)$
  Finally, $\eta_c(2S)$ was observed in charmonium transitions after thirty years of searching. Finding $\eta_c(2S)$ has proved hard enough, but understanding its decay properties and measuring its lineshape are proving more difficult. Due to the lower production rate, $\eta_c(2S)$ is hard to reconstruct and is affected by interference more than $\eta_c(1S)$. Further measurement of $\eta_c(2S)$ will require large statistics, a convincing theoretical model and a sophisticated fitting method.

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