Impact of nuclear deformation on collective flow observables in relativistic U+U collisions

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A Multi-Phase Transport (AMPT) model is used to investigate the efficacy of several flow observables to constrain the initial-state deformation of the Uranium nuclei in U+U collisions at nucleon-nucleon center-of-mass energy $\sqrt{s_{NN}} = 193$ GeV. The multiparticle azimuthal cumulant method is used to investigate the sensitivity of (I) a set of quantities that are sensitive to both initial- and final-state effects as well as (II) a set of dimensionless quantities that are more sensitive to initial-state effects to the Uranium nuclei quadrupole shape deformation. We find that the combined use of the flow harmonics, flow fluctuations and correlations, linear and non-linear flow correlations to the quadrangular flow harmonic, and the correlations between elliptic flow and the mean-transverse momentum could serve to constrain the nuclear deformation of the Uranium nuclei. Therefore, a comprehensive set of measurements of such observables can provide a quantifying tool for the quadrupole shape deformation via data-model comparisons.

Keywords: Collectivity, correlation, shear viscosity, transverse momentum correlations

I. INTRODUCTION

The quark-gluon plasma (QGP) – a new state of matter, is produced in ultra-relativistic heavy-ion collisions [1,2]. Understanding the QGP’s specific shear viscosity $\eta/s (T, \mu_B)$ dependence on temperature ($T$) and baryon chemical potential ($\mu_B$) is being investigated at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC). This transport property describes the QGP’s power to transfer initial-state anisotropies due to collision geometry into final-state momentum anisotropies. Anisotropic flow ($v_n$) measurements have played a major role in elucidating this final-state effect since they originate from the viscous hydrodynamic response to the eccentricity ($\varepsilon_n$) of the energy-density distribution produced in the initial stages of the collision [3,4].

A possible deformation of the colliding nuclei can also influence $\varepsilon_n$ and consequently, $v_n$ [4,6]: the initial-state profile for each of the colliding nuclei can be characterized with the woods-Saxon distribution for the nuclear density as:

$$\rho(r) = \rho_0 \frac{1}{1 + \exp \left( -|r-R|/a \right)},$$

$$R'(\theta, \phi) = R[1 + \beta_2 Y_2^0(\theta, \phi) + \beta_3 Y_3^0(\theta, \phi) + \beta_4 Y_4^0(\theta, \phi) + ...],$$

where $\rho_0$ is the nucleon density in the center of the nucleus, $a$ is the skin depth (i.e., the surface thickness), $R$ is the nuclear radius and is taken to be $1.2 A^{1/3}$ [44], and $R'(\theta, \phi)$ is the nuclear surface which contains the relevant axial symmetric quadruple ($\beta_2$) octupole ($\beta_3$) and hexadecapole ($\beta_4$) deformations [42,43,45,51]. Within the quantum many-body system, the deformation is a fundamental property of the atomic nucleus that provides the correlated nature of the dynamics of nucleons. Many atomic nuclei exhibit a quadrupole or octupole deformation, which could influence the flow coefficients’ magnitude, fluctuations, and correlations. Indeed, recent measurements employing detailed comparisons between Au+Au and U+U collisions [52] as well as Pb+Pb and Xe+Xe collisions [53,54] have indicated signatures compatible with nuclear deformation. However, the degree to which flow measurements can provide constraints useful for detailed characterization of the deformation of colliding nuclei is still unclear.

Prior works have shown that the elliptic and triangular flow coefficients $v_2$ and $v_3$, are linearly related to the initial-state eccentricities, $\varepsilon_2$ and $\varepsilon_3$ [15,55–61]. The higher-order flow harmonics $v_{n>3}$ not only arise from a linear response to the same-order initial-state anisotropies, but also from a non-linear response to the lower-order eccentricities $\varepsilon_2$ and $\varepsilon_3$ [15,55–61]. The non-linear contributions encode the correlations between different symmetry planes $\Psi_n$ [56] which provides a constraint for the initial-stage dynamics [54,57,58]. The correlation between the flow harmonics $v_n$ and the event mean transverse momentum $⟨p_T⟩$:

$$\rho(v_n^2, ⟨p_T⟩) = \frac{\text{cov}(v_n^2, ⟨p_T⟩)}{\sqrt{\text{Var}(v_n^2)} \sqrt{\text{Var}(⟨p_T⟩)}},$$

which indicates different sensitivities to the initial- and final-states of the collisions [64,65] has also been shown to be sensitive to nuclear deformation, albeit with added sensitivity to the $p_T$ selection, event shape selection, $\eta$ selection, and the centrality definition [55,58]. Here, we use detailed simulations with the AMPT model to investigate supplemental measures that are sensitive to initial-state deformation which could be used to constrain nuclear deformation.

In the current work, we investigate the influence of the nuclear quadrupole deformation ($\beta_2 > 0.0$) in U+U collisions at nucleon-nucleon center-of-mass energy...
$\sqrt{s_{NN}} = 193$ GeV on the $v_n\langle k \rangle_{22}$, $v_2\langle 2 \rangle/v_2\langle 4 \rangle$, the normalized symmetric cumulants (NSC(2,3)), the linear and non-linear contributions to the $v_4$, the coupling constant ($\chi_{4,22}$), and the correlations between different order flow symmetry planes ($\rho_{4,22}$), and the correlation $\rho(v_2^2, \langle p_T \rangle)$ [40, 51, 87, 88]. Here, an important objective is to develop a more stringent constraint for initial-state deformation by simultaneously leveraging the response of several correlators to nuclear quadrupole deformation of the Uranium nuclei.

The paper is organized as follows. Section II summarizes the theoretical model used to investigate the $\beta_2$ dependence on the flow quantities and the details of the analysis method employed. The results from the model studies are presented in Sec. III followed by a summary in Sec. IV

II. METHODOLOGY

A. The AMPT model

The current study is performed with simulated events for U+U collisions at $\sqrt{s_{NN}} = 193$ GeV, generated using the AMPT model [89]. Analysis was performed for charged hadrons in the transverse momentum range of $0.2 < p_T < 2.0$ GeV/c and the pseudorapidity acceptance $|\eta| < 1.0$. In addition, the simulated events were partitioned into several collision centrality classes established on the collision’s impact parameter.

The AMPT model [89] is widely used to study the physics of the relativistic heavy-ion collisions at LHC and RHIC [89, 90]. In this work, events were generated with the AMPT model with the string melting option. In such a scenario initial conditions are given using the Glauber model, hadrons are created using the HIJING model and converted to their valence quarks and anti-quarks, and their space-time evolution is evaluated via the ZPC Parton cascade model [91]. The fundamental elements of the AMPT model are (i) the HIJING model [92, 93] initial Parton-production stage, (ii) a Parton-scattering stage, (iii) hadronization via coalescence then (iv) a hadronic interaction stage [100]. In the Parton-scattering stage the utilized Parton-scattering cross-sections are evaluated using:

$$\sigma_{pp} = \frac{9\pi\alpha_s^2}{2\mu^2}, \quad (3)$$

where $\alpha_s$ is the QCD coupling constant and $\mu$ is the screening mass in the partonic matter. They typically give the expansion dynamics of the A-A collision systems [94]. In this work, U+U collisions at $\sqrt{s_{NN}} = 193$ GeV, were simulated for a fixed value of $\alpha_s = 0.47$, and $\mu = 3.41$ fm$^{-1}$ [101, 102].

The U+U collisions are implemented in the AMPT model by parametrizing the nucleon density distribution as a deformed Woods-Saxon profile $\beta_3$ that is given in Eq. 2. In U+U collisions the projectile and target nuclei are rotated randomly event-by-event along the polar and azimuthal directions. The nucleon density distribution Eq. 1 as well as the initial state eccentricities, can be varied by adjusting the values of: (i) The parameter $a$ that is generally used to distinguish the nucleon sampling from Woods-Saxon distribution between protons and neutrons, (ii) The $\beta_2$ which describes the overall quadrupole deformation, (iii) The $\beta_3$ that controls the overall octupole deformatiions, and (iv) The $\beta_4$ that describes the hexadecapole deformation. Prior investigations inspire the model parameters in this work [44, 104]. Note that the $\beta$ parameters influence the magnitude of the eccentricity (and its fluctuations) in central collisions.

In the current work $a$ is fixed to $a = 0.44$, and the deformation parameters $\beta_2$, $\beta_3$, and (iii) the Subevent cumulants methods [109]. The AMPT-set $\beta_2$, $\beta_4$ for Uranium are given in Tab. 1.

| AMPT-set | $\beta_2$ | $\beta_4$ |
|----------|----------|----------|
| Set-1    | 0.00     | 0.00     |
| Set-2    | 0.00     | 0.20     |
| Set-3    | 0.28     | 0.093    |
| Set-4    | 0.40     | 0.093    |

TABLE I. The summary of the AMPT sets used in this work.

B. Analysis Method

The two- and multi-particle cumulants methods are used in this work. The cumulants method was initially introduced in Refs 106, 107 and was extended using (i) the Q-cumulants [108], (ii) the Generic framework [67], and (iii) the Subevent cumulants methods [109]. The two- and multi-particle cumulants can be constructed in terms of $n^{th}$ flow vectors ($Q_n$) magnitude. The $Q_n$ are given as:

$$Q_{n,k} = \sum_{i=1}^{M} \omega_{i}^{k} e^{i\nu_{i}}, \quad (4)$$

where $M$ is the total number of particles in an event and $\omega_{i}$ is the $i^{th}$ particle weight, note that for uniform acceptance $\omega_{i} = 1$. Also the sum over the particles weight is introduced as:

$$S_{p,k} = \left[ \sum_{i=1}^{M} \omega_{i}^{k} \right]^{p}. \quad (5)$$

Using Eqs. 4, 5 the two-, three-, and four-particle correlations were constructed using the two-subevents cumulant methods [109], with $|\Delta\eta| = |\eta_a - \eta_b| > 0.7$ ($\eta_a > 0.35$ and $\eta_b < -0.35$).
The two-particle correlations:

\[ v_{2}^{2}(2) = \langle (2) \rangle_{n}, \]
\[ = \langle \cos(n\phi_{1} - n\phi_{2}) \rangle, \]
\[ = \sum_{i=1}^{N_{ev}} (M_{2})_{i} \langle 2 \rangle_{n,i} / \sum_{i=1}^{N_{ev}} (M_{2})_{i}, \]
\[ \langle (2) \rangle_{n} = \frac{Q_{2}^{0}}{M_{2}} \left( \frac{Q_{1}^{0}}{M_{1}} \right)^{*}, \]
\[ M_{2} = S_{1,1}^{0} S_{1,1}^{0}, \]

where \( \langle (\rangle \rangle \) is the average over particles, then a multiplicity weighted average over events.

The three-particle correlations:

\[ \langle (3) \rangle_{kmn} = \langle (cos(k\phi_{1} - n\phi_{2} - m\phi_{3}) \rangle, \]
\[ = \sum_{i=1}^{N_{ev}} (M_{3})_{i} \langle (3) \rangle_{kmn,i} / \sum_{i=1}^{N_{ev}} (M_{3})_{i}, \]

where \( k = n + m, \)

\[ \langle (3) \rangle_{kmn} = \frac{\left( Q_{n,1}^{0} Q_{m,1}^{0} Q_{m,2}^{0} - Q_{m,2}^{0} \right)}{M_{3}} \left( \frac{Q_{1}^{0}}{M_{1}} \right)^{*}, \]
\[ M_{3} = \left( S_{2,1}^{0} - S_{1,2}^{0} \right) S_{1,1}^{0}. \]

The four-particle correlations:

\[ \langle (4) \rangle_{nm} = \langle (cos(n\phi_{1} + m\phi_{2} - n\phi_{3} - m\phi_{4}) \rangle, \]
\[ = \sum_{i=1}^{N_{ev}} (M_{4})_{i} \langle (4) \rangle_{nm,i} / \sum_{i=1}^{N_{ev}} (M_{4})_{i}, \]
\[ \langle (4) \rangle_{nm} = \frac{\left( Q_{n,1}^{0} Q_{m,1}^{0} Q_{m,2}^{0} - Q_{m,2}^{0} \right)}{M_{4}} \left( \frac{Q_{1}^{0}}{M_{1}} \right)^{*}, \]
\[ M_{4} = \left( S_{2,1}^{0} | S_{1,2}^{0} \right) \left( S_{2,1}^{0} | S_{1,2}^{0} \right). \]

Using the two- and four-particle correlations we can write the four-particle \( n^{th} \) flow harmonics, and the normalized symmetric cumulants as:

\[ v_{n}^{4}(4) = 2 \langle (2) \rangle_{n} - \langle (4) \rangle_{nm}, \]
\[ NSC(n, m) = \frac{\langle (4) \rangle_{nm} - \langle (2) \rangle_{n} \langle (2) \rangle_{m}}{\langle (2) \rangle_{n} \langle (2) \rangle_{m}}. \]

The benefit of using the two-subevents technique is that it assists the reduction of the near-side non-flow correlations resulting from resonance decays, Bose-Einstein correlations, and the fragments of individual jets [95].

a. Linear and non-linear contributions:

The inclusive \( v_{4} \) which contains the linear and non-linear contributions is given by the two-particle correlations Eq. (9):

\[ v_{4}^{\text{Inclusive}} = v_{2}^{2}(2). \]

The non-linear contribution to \( v_{4} \) can be given as [65, 110]:

\[ v_{4}^{\text{NonLinear}} = \frac{\langle (3) \rangle_{22}}{\sqrt{\langle (4) \rangle_{22}}}, \]
\[ \sim \langle v_{4} \cos(4\Psi_{4} - 2\Psi_{2} - 2\Psi_{2}) \rangle, \]
and the linear contribution to \( v_{4} \) [64, 93] can be expressed as:

\[ v_{4}^{\text{Linear}} = \sqrt{(v_{4}^{\text{Inclusive}})^{2} - (v_{4}^{\text{NonLinear}})^{2}}, \]

The non-linear response coefficient \( \chi_{4,22} \), which quantify the mode-coupling contributions to the \( v_{4} \), is defined as:

\[ \chi_{4,22} = \frac{v_{4}^{\text{NonLinear}}}{\sqrt{\langle (4) \rangle_{22}}}. \]

The correlations between different order flow symmetry planes \( \rho_{4,22} \) can be given as:

\[ \rho_{4,22} = \frac{\langle (3) \rangle_{22}}{\langle (4) \rangle_{22}^{\langle (2) \rangle_{4}} \sim \langle v_{4} \cos(4\Psi_{4} - 4\Psi_{2}) \rangle. \]

b. The flow transverse-momentum correlations:

The \( \rho(v_{2}^{2}, \langle p_{T} \rangle) \) correlation coefficient (Eq. 2) contains the \( v_{n} \) and \( \langle p_{T} \rangle \) variances and covariances that utilize the two- and multi-particle correlations.

The \( v_{2}^{2} \) variance can be given as:

\[ \text{Var}(v_{2}^{2}) \sim v_{2}^{2}(2)^{4} - v_{2}^{2}(4)^{4}, \]

where \( v_{2}^{2}(2) \) and \( v_{2}^{2}(4) \) are the two- and four-particle elliptic flow using the subevent method [109] (see Eqs. 6 and 11).

The variance of the \( \langle p_{T} \rangle \) is evaluated in the range \( |y| < 0.35 \) using the two particle correlation methods [112], given as:

\[ c_{k} = \left\langle 1 - \sum_{B', B \neq B} \frac{1}{N_{pair}} (\langle p_{T,B} \rangle - \langle p_{T} \rangle) (\langle p_{T,B'} \rangle - \langle p_{T} \rangle) \right\rangle, \]

where \( \langle \rangle \) is an average over all events. The condition \( B' \neq B \) is used to remove self-correlations. The event mean \( p_{T} \), is given as,

\[ \langle p_{T} \rangle = \sum_{i=1}^{M_{B}} p_{T,i} / M_{B}, \]

where \( M_{B} \) is the number of tracks in subevent \( B \).

The correlation of \( v_{2}^{2} \) and the \( \langle p_{T} \rangle \) (\( \text{cov}(v_{2}^{2}, \langle p_{T} \rangle) \)) are calculated through the three-subevents methods [114, 115] as,

\[ \text{cov}(v_{2}^{2}, \langle p_{T} \rangle) = \text{Re} \left( \sum_{A,C} e^{i2(\phi_{A} - \phi_{C})} \langle p_{T} \rangle - \langle \text{Re} \rangle_{B} \right). \]

The \( \rho(v_{2}^{2}, \langle p_{T} \rangle) \) coefficient [72, 75, 80, 82] can be given using Eqs. 13 and 21:

\[ \rho(v_{2}^{2}, \langle p_{T} \rangle) = \frac{\text{cov}(v_{2}^{2}, \langle p_{T} \rangle)}{\sqrt{\text{Var}(v_{2}^{2})} / c_{k}}. \]
III. RESULTS AND DISCUSSION

Figure 1 shows a comparison of the centrality dependence of $v_2(2)$ (a), $v_2(4)$ (b), $v_2(6)$ (c), and $v_2(4)/v_2(2)$ (d) computed with the AMPT model sets Tab. I for U+U collisions at $\sqrt{s_{NN}} = 193$ GeV. The bands represent the experimental data reported in Refs. [117, 118].

![Figure 1](image1)

FIG. 1. Centrality dependence of $v_2(2)$ (a), $v_2(4)$ (b), $v_2(6)$ (c), and $v_2(4)/v_2(2)$ computed with the AMPT model sets Tab. I for U+U collisions at $\sqrt{s_{NN}} = 193$ GeV. The bands represent the experimental data reported in Refs. [117, 118].

Figure 2 shows a comparison of the centrality dependence of $\text{Var}(v_2)$, $\sqrt{C_4}/\langle p_T \rangle$, $\text{cov}(v_2, \langle p_T \rangle)$, and $\rho(v_2, \langle p_T \rangle)$ obtained with the AMPT model sets shown in Table I. Panels (a)-(c) show that $v_2(2)$ is much more sensitive to deformation than $v_2(4)$ and $v_2(6)$ and shows a sizable increase with the quadrupole deformation given by $\beta_2$. Such observations reflect the $v_2(2)$ and $v_2(4)$ dependence on $\beta_2^2$ and $\beta_2^4$ respectively as suggested by Refs. [42, 43, 77, 119]. In contrast, $v_2(6)$ shows no sensitivity to the hexadecapole deformation given by $\beta_4$, confirming that $v_2(6)$ depends only on $\beta_2$. The bands in panels (a), and (b) show the experimental measurements constructed from Refs. [117, 118]. The AMPT calculations show poor quantitative agreement with the experimental measurements.

![Figure 2](image2)

FIG. 2. Comparison of the centrality dependence of $\text{Var}(v_2)$ (a), $\sqrt{C_4}/\langle p_T \rangle$ (b), $\text{cov}(v_2, \langle p_T \rangle)$ (c) and $\rho(v_2, \langle p_T \rangle)$ (d) computed from the AMPT model sets given in Tab. I for U+U collisions at $\sqrt{s_{NN}} = 193$ GeV. The bands represent the experimental data reported in Refs. [117, 118].

The ratio $v_2(4)/v_2(2)$ gives the strength of the elliptic flow fluctuations [21, 120, 121]. Note that $v_2(4)/v_2(2) \approx 1.0$ suggests small, if any, fluctuations, whereas $v_2(4)/v_2(2) < 1.0$ implies more fluctuations as this ratio decreases. In addition, the elliptic flow fluctuations are sensitive to fluctuations [21, 120, 121]. Note that $v_2(4)/v_2(2) \approx 1.0$ suggests small, if any, fluctuations, whereas $v_2(4)/v_2(2) < 1.0$ implies more fluctuations as this ratio decreases. In addition, nuclear deformation increases the initial-state fluctuations. The results show that the elliptic flow fluctuations increase with the initial-state fluctuations caused by the increase in the $\beta_2$ value. The results for $v_2(k)$ are sensitive to initial-state effects (i.e., initial-state eccentricity and initial-state eccentricity fluctuations), and final-state effects (i.e., viscous attenuation). Therefore, they are more suitable for constraining the interplay between final- and initial-state effects. By contrast, the elliptic flow fluctuations are sensitive to the initial-state eccentricity and its fluctuations. Therefore, combining the results of the $v_2(k)$ and the ratio $v_2(4)/v_2(2)$ can be used to add a simultaneous constraint on the initial and final-state effects.

In prior work, the correlation between $v_2$ and the event mean $p_T$ ($\rho(v_2, \langle p_T \rangle)$) has been shown to be sensitive to the
nuclear deformation [31, 76, 78, 81, 122, 123]. Fig. 2 compares the β dependence of \( \text{Var}(v^2) \) (a), \( \sqrt{\epsilon_k/(p_T)} \) (b), \( \text{cov}(v^2, \langle p_T \rangle) \) (c) and \( \rho(v^2, \langle p_T \rangle) \) (d) respectively for U+U collisions at \( \sqrt{s_{NN}} = 193 \) GeV from the AMPT model. The results indicate that in central collisions, \( \text{Var}(v^2) \) (panel (a)) increases with \( \beta_2 \) while \( \text{cov}(v^2, \langle p_T \rangle) \) (panel (c)) and \( \rho(v^2, \langle p_T \rangle) \) (panel (d)) decrease with \( \beta_2 \) and even become negative in more central collisions. Such observations can be explained by considering the core of the \( \text{cov}(v^2, \langle p_T \rangle) \) dependence on \( \beta_2 \) [16, 124] (i.e., \( \text{cov}(v^2, \langle p_T \rangle) \sim a_0 - a_1 \beta_2^2 \)). These results agree qualitatively with the recent STAR experiment preliminary measurements [123]. The values for \( \sqrt{\epsilon_k/(p_T)} \) (panel (b)) are relatively insensitive to \( \beta_2 \), which does not agree with the preliminary measurements of the STAR experiment [122]. In addition, these results show no sensitivity to the \( \beta_4 \) variations given by AMPT Set-2. The \( \text{Var}(v^2) \) and \( \text{cov}(v^2, \langle p_T \rangle) \) results in Fig. 2 are sensitive to the interplay between final- and initial-state effects. However, the \( \rho(v^2, \langle p_T \rangle) \) is suggested to leverage the correlation between the eccentricity-driven \( v^2 \) and the transverse size of the overlap region given by the \( \langle p_T \rangle \) [124]. Therefore, the combined measurements of \( v_2(2), v_2(4)/v_2(2) \) and \( \rho(v^2, \langle p_T \rangle) \) would be expected to provide even more stringent constraints for \( \beta_2 \) and the interplay between final- and initial-state effects.

Supplemental constraints for the initial-state deformation can be obtained in tandem via the SC(2, 3) and the NSC(2, 3) correlators. Figure 3 shows a comparison of the centrality dependence of \( v_4 \) (panel (a)) and NSC(2, 3) (panel (b)). The \( v_2 \) values from the AMPT simulations show a clear sensitivity to \( \beta_2 \) as in Fig. 1. By contrast, \( v_2 \) shows the expected insensitivity to \( \beta_2 \). Both quantities show no sensitivity to the \( \beta_4 \) variations given by the AMPT Set-2. The bands in panel (a) represent the experimental data [114, 118] that shows similar trend to the AMPT calculations presented. The SC(2, 3) (panel (c)) which contains the \( \epsilon_2 \) and the \( \epsilon_3 \) variances and correlations indicate a negative value that increase with \( \beta_2 \). Similarly, the dimensionless flow harmonic correlations, NSC(2, 3) (panel (c)) indicate an anti-correlation between \( v_2 \) and \( v_3 \) which grows with \( \beta_2 \), suggesting that its sensitivity to the initial-state deformation can be employed as a supplemental constraint. These NSC(2, 3) trends, in tandem with the results shown in Figs. 1 and 2, could provide more stringent constraints for the influence of nuclear deformation on the initial-state correlations and fluctuations as well as its interplay with final-state effects.

The centrality dependence of the inclusive, linear and non-linear \( v_4 \) (panels (a)–(c)) as well as the non-linear response coefficients, \( \chi_{4,22} \) (panel (d)), and the correlations of the event plane angles, \( \rho_{4,22} \) are shown in Fig. 4. The results indicate that, the inclusive \( v_4 \) depends on \( \beta_2 \) and \( \beta_4 \) (\((v_4^{\text{inclusive}})^2 \sim a_3 \beta_2^2 + a_4 \beta_4^2\)) which in line with the results presented in Ref. [12]. In addition, the AMPT calculations of the linear \( v_4 \) shows a sensitivity to the \( \beta_4 \) variation in central collisions. On other hand, the non-linear contribution of \( v_4 \), which has the weakest contribution to the inclusive \( v_4 \) in central collisions, has a sizable dependence on \( \beta_2 \). Therefore, using the linear \( v_4 \) can add a constraints on the \( \beta_4 \) values and the non-linear \( v_4 \) can add a constraints on the \( \beta_2 \) values.

Figure 4(c) shows that the \( \chi_{4,22} \) indicate a weak centrality dependence for the non-deformed U+U collisions (\( \beta_2 = 0.0 \)) and a modest centrality dependence, in central collisions, for deformed U+U collisions (\( \beta_2 > 0.0 \)). Such an observation suggests that \( \chi_{4,22} \) depends on initial-state effects, which disagrees with Ref. [63]. In addition, the \( \rho_{4,22} \), shows stronger event plane correlations in peripheral collisions, Fig. 4(d) indicate sizable dependence on \( \beta_2 \) in central collisions. The \( \rho_{4,22} \) results suggest that it depends on \( \beta_2 \) similarly to the non-linear \( v_4 \). Also, the results of \( \chi_{4,22} \) and \( \rho_{4,22} \) show no sensitivity to the \( \beta_4 \) variations given by AMPT Set-2. The dimensionless coefficients \( \chi_{4,22}(\rho_{4,22}) \) that show

FIG. 4. Comparison of the centrality-dependent inclusive, linear and non-linear \( v_4 \) panels (a)–(c), \( \chi_{4,22} \) panel (d) and \( \rho_{4,22} \) panel (e) obtained from the AMPT model sets given in Tab. I for U+U collisions at \( \sqrt{s_{NN}} = 193 \) GeV.
a sizable dependence on $\beta_2$ in central collisions (note the indicated increase/decrease of $p_{4,22}(\chi_{4,22})$ with $\beta_2$) suggesting their value as supplemental constraints to the nuclear deformation effects in U+U collisions.

The results presented in Figs. 1–4 as well as in prior studies \cite{22, 21, 20} indicate that the correlators studied fall into two broad categories: (I) correlators that are sensitive to initial and final-state effects ($V_{\alpha}(\{k\})$, linear/non-linear $v_4$, $\text{Var}(v_2^\text{dyn})$, $\sqrt{\chi}$, and $\text{cov}(v_2^\text{dyn}, \langle p_T \rangle)$). These correlators are sensitive to both initial- and final-state effects which makes them less constraining for pinning down the initial-state deformation. (II) Dimensionless correlators from category (II) to constrain the effects of the interplay between initial- and final-state effects ($\chi_{4,22}$, $\rho_{4,22}$, and $v_2$) and the initial-state angular correlations ($\rho_{4,22}$). Therefore, a possible route to constraining the nuclear deformation in U+U collisions would be to: (a) use the correlators from category (I) to constrain the final-state effects and its interplay with the initial-state effects, (b) use the correlators from the first category to constrain the initial-state effects and its interplay with the initial-state effects.

IV. CONCLUSION

In summary, we have made systematic investigations of the effects of nuclear deformation on quantities that are affected by the interplay between the initial- and final-state effects as well as quantities that are more sensitive to the initial-state effects. In the framework of the AMPT model we presented the $\beta_2$ and $\beta_4$ dependence of the $v_2\{2\}$, $\rho(v_2^2, \langle p_T \rangle)$, flow fluctuations and correlations, linear and non-linear contributions to the $V_2$, $\chi_{4,22}$ and $\rho_{4,22}$ in U+U collisions at $\sqrt{s_{NN}} = 193$ GeV. The model predicts characteristic patterns (mostly in central collisions) for the different presented coefficients consistent with the nuclear deformation effects given by the $\beta_2$ and $\beta_4$ values. These predictions suggest that a precise set of measurements for $v_2\{2\}$, $\rho(v_2^2, \langle p_T \rangle)$, $\chi_{4,22}$, $\rho_{4,22}$ together can be used to constrain the nuclear deformation effects in U+U collisions via data-model comparisons.

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