Positive Periodic solution of a discrete Lotka-volterra commensal symbiosis model with Michaelis-Menten type harvesting

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Abstract: - A non-autonomous discrete Lotka-volterra commensal symbiosis model with Michaelis-Menten type harvesting is proposed and studied in this paper. Under some very simple and easily verified condition, we show that the system admits at least one positive periodic solution.

Key-Words: - Commensal symbiosis model, Positive periodic solution, Michaelis-Menten type harvesting.

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1 Introduction
The aim of this paper is to investigate the positive periodic solution of the following discrete commensal symbiosis model with Hassell-Varley type functional response

\[ N_1(k+1) = N_1(k) \exp \left\{ a_1(k) - b_1(k)N_1(k) + c_1(k)N_2(k) \right\}, \]
\[ N_2(k+1) = N_2(k) \exp \left\{ a_2(k) - b_2(k)N_1(k) - \frac{q(k)E(k)}{m_1(k)E(k) + m_2(k)N_2(k)} \right\}, \]

where \( N_1(k) \) and \( N_2(k) \) represent the densities of the first and second species of \( k \)-generation, respectively. In view of seasonal factors, e.g., mating habits, availability of food, weather conditions, harvesting, and hunting, etc., we assume that the coefficients of the system (1) are all periodic sequences with a common integer period. More precisely, we assume that the coefficients of the system (1) satisfies (H1) \{b_1(k)\}, \{b_2(k)\}, \{m_1(k)\}, \{m_2(k)\}, \{c_1(k)\}, \{q(k)\}, \{E(k)\} are all positive \( \omega \)-periodic sequences, \( \omega \) is a fixed positive integer, \{a_i(k)\} are \( \omega \)-periodic sequences, which satisfies

\[ \bar{a}_i = \frac{1}{\omega} \sum_{k=0}^{\omega-1} a_i(k) > 0, i = 1, 2. \]

In the past several years, many scholars paid their attention to study the dynamic behaviors of the commensal symbiosis model, see [1]-[30] and the references cited therein. However, only recently did scholars ([24]-[30]) began to study the influence of harvesting to commensalism model. It is well known that Michaelis-Menten type harvesting ([24]-[26], [29]-[30], [33]-[37]) is more appropriate than the linear harvesting and constant harvesting, and recently, several scholars ( [24]-[26], [29]-[31]) began to study the influence of Michaelis-Menten type harvesting to commensalism model, however, most of them were studied the autonomous ones, and only Liu et al [31] and Xue et al [50] began to investigate the nonautonomous case.

In [31], Liu et al proposed the following nonautonomous Lotka-Volterra commensalism model with Michaelis-Menten type harvesting

\[ \frac{dN_1(t)}{dt} = N_1(t) \left( a(t) - b(t)N_1(t) + c(t)N_2(t) \right), \]
\[ \frac{dN_2(t)}{dt} = N_2(t) \left( d(t) - e(t)N_2(t) \right) - \frac{q(t)E(t)N_2(t)}{m_1(t)E(t) + m_2(t)N_2(t)}. \]

Under the assumption that all the coefficients are continuous positive periodic functions with a common period, the authors obtained a set of sufficient conditions which ensure the existence of at least one positive periodic solution of the system.

It is well known that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have nonoverlapping generations. Hence, corresponding to system (2), we propose the discrete type of Lotka-Volterra commensalism model with
Michaelis-Menten type harvesting, i.e., system (1).
To the best of our knowledge, this is the first time that
the model is proposed. We will focus our attention to
the existence of positive periodic solution of system
(1).

2 Main Result
In the proof of our existence theorem below, we will use
the continuation theorem of Gaines and
Mawhin([32]).

Lemma 2.1 (Continuation Theorem) Let \( L \) be a
Fredholm mapping of index zero and let \( N \) be \( L \)
compact on \( \Omega \). Suppose
(a) For each \( \lambda \in (0, 1) \), every solution \( x \) of \( Lx = \lambda Nx \) is such that \( x \notin \partial \Omega \);
(b) \( QN \neq 0 \) for each \( x \in \partial \Omega \cap Ker L \) and
\[ \deg \{ QN, \Omega \cap Ker L, 0 \} \neq 0. \]
Then the equation \( Lx = Nx \) has at least one solution
lying in \( DomL \cap \Omega \).

Let \( Z, Z^+, R \) and \( R^+ \) denote the sets of all integers,
nonnegative integers, real unumbers, and non-
negative real numbers, respectively. For convenience,
in the following discussion, we will use the notation
below throughout this paper:
\[ I = \{ 0, 1, \ldots, \omega - 1 \}, \quad g = \frac{1}{\omega} \sum_{k=0}^{\omega-1} g(k), \]
\[ g^ua = \max_{k \in I_u} g(k), \quad g^lb = \min_{k \in I_u} g(k), \]
where \( \{ g(k) \} \) is a \( \omega \)-periodic sequence of real numbers
defined for \( k \in Z \).

Lemma 2.2\([40]\) Let \( g : Z \rightarrow R \) be \( \omega \)-periodic, i.e.,
\( g(k + \omega) = g(k) \). Then for any fixed \( k_1, k_2 \in I_u \), and
any \( k \in Z \), one has
\[ g(k) \leq g(k_1) + \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|, \]
\[ g(k) \geq g(k_2) - \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|. \]

Lemma 2.3 Assume that \( \bar{\sigma}_2 > \left( \frac{q}{m_1} \right) \) hold, any solution \((u_1^*, u_2^*)\) of the system of algebraic equations
\[ \bar{a}_1 - \bar{b}_1 \exp\{u_1\} + \bar{c}_1 \exp\{u_2\} = 0, \]
\[ \bar{a}_2 - \bar{b}_2 \exp\{u_2\} \]
\[ - \frac{1}{\omega} \sum_{k=0}^{\omega-1} \frac{q(k)E(k)}{m_1(k)E(k) + m_2(k)\exp\{u_2\}} = 0, \]
satisfies
\[ \ln \frac{\bar{a}_1}{b_1} \leq u_1^* \leq \frac{\bar{a}_1 + \bar{c}_1}{b_2}, \]
\[ \frac{\bar{a}_2}{b_2} - \frac{\bar{q}}{m_1} \leq u_2^* \leq \ln \frac{\bar{a}_2}{b_2}. \]

Proof. From the second equation of (3), it immediately follows that
\[ \bar{a}_2 - \bar{b}_2 \exp\{u_2\} \geq 0. \]
Thus,
\[ u_2 \leq \ln \frac{\bar{a}_2}{b_2}. \]
From the second equation of (3) we also have
\[ \bar{a}_2 - \bar{b}_2 \exp\{u_2\} - \frac{1}{\omega} \sum_{k=0}^{\omega-1} \frac{q(k)E(k)}{m_1(k)E(k)} \leq 0, \]
So,
\[ u_2 \geq \ln \frac{\bar{a}_2}{b_2}. \]
From the first equation of system (3) we have
\[ \bar{a}_1 - \bar{b}_1 \exp\{u_1\} \leq 0, \]
thus
\[ u_1 \geq \frac{\bar{a}_1}{b_1}. \]
From the first equation of system (3) and (6), we also have
\[ 0 = \bar{a}_1 - \bar{b}_1 \exp\{u_1\} + \bar{c}_1 \exp\{u_2\} \]
\[ \leq \bar{a}_1 - \bar{b}_1 \exp\{u_1\} + \bar{c}_1 \exp\left\{ \ln \frac{\bar{a}_2}{b_2} \right\} \]
\[ = \bar{a}_1 + \bar{c}_1 \frac{\bar{a}_2}{b_2} - \bar{b}_1 \exp\{u_1\}. \]
Thus
\[ u_1 \leq \ln \frac{\bar{a}_1 + \bar{c}_1 \bar{a}_2}{b_1 b_2}. \]

This ends the proof of Lemma 2.3.

We now reach the position to establish our main result.

Theorem 2.1 Assume that
\[ \bar{\sigma}_2 > \left( \frac{q}{m_1} \right) \]
hold, system (1) admits at least one positive \( \omega \)-periodic solution.
Proof. Let
\[ N_i(k) = \exp\{u_i(k)\}, \quad i = 1, 2, \]
so that system (1) becomes
\[ \begin{align*}
& u_1(k + 1) - u_1(k) \\
& = a_1(k) - b_1(k) \exp\{u_1(k)\} + c_1(k) \exp\{u_2(k)\}, \\
& u_2(k + 1) - u_2(k) \\
& = a_2(k) - b_2(k) \exp\{u_2(k)\} \\
& - \frac{q(k)E(k)}{m_1(k)E(k) + m_2(k)\exp\{u_2(k)\}}.
\end{align*} \]

Define
\[ l_2 = \left\{ y = (y(k)), y(k) = (y_1(k), y_2(k))^T \in R^2 \right\}. \]

For \( a = (a_1, a_2)^T \in R^2 \), define \( |a| = \max\{|a_1|, |a_2|\} \). Let \( l^a \subset l_2 \) denote the subspace of all \( \omega \) sequences equipped with the usual normal form \( \|u\| = \max_{k \in \omega} |u(k)| \). It is not difficult to show that \( l^a \) is a finite-dimensional Banach space. Let
\[ l_0^a = \left\{ u = (u(k)) \in l^a : \sum_{k=0}^{\omega-1} u(k) = 0 \right\}, \]
\[ l_2^a = \left\{ u = (u(k)) \in l^a : u(k) = h \in R^2, k \in Z \right\}, \]
then \( l_0^a \) and \( l_2^a \) are both closed linear subspace of \( l^a \), and
\[ l^a = l_0^a \oplus l_2^a, \quad \dim l_2^a = 2. \]

Now let us define \( X = Y = l^a, (Lu)(k) = u(k + 1) - u(k) \). It is trivial to see that \( L \) is a bounded linear operator and
\[ \begin{align*}
& Ker L = l_2^a, \quad Im L = l_0^a, \\
& \dim Ker L = 2 = \text{Codim Im L}.
\end{align*} \]

Then it follows that \( L \) is a Fredholm mapping of index zero. Let
\[ N(u_1, u_2)^T = (N_1, N_2)^T := N(u, k), \]
where
\[ \begin{align*}
N_1 &= a_1(k) - b_1(k) \exp\{u_1(k)\} + c_1(k) \exp\{u_2(k)\}, \\
N_2 &= a_2(k) - b_2(k) \exp\{u_2(k)\} \quad - \frac{q(k)E(k)}{m_1(k)E(k) + m_2(k)\exp\{u_2(k)\}}.
\end{align*} \]

Suppose that \( u = (u_1(k), u_2(k))^T \in X \) is an arbitrary solution of system (14) for a certain \( \lambda \in (0, 1) \). Summing on both sides of (14) from 0 to \( \omega - 1 \) with
respect to $k$, we reach
\[ \sum_{k=0}^{\omega-1} \left[ a(k) - b(k) \exp \{ u_1(k) \} \right] \]
\[ + c_1(k) \exp \{ u_2(k) \} \] = 0,
\[ \sum_{k=0}^{\omega-1} \left[ a_2(k) - b_2(k) \exp \{ u_2(k) \} \right] \]
\[ - \frac{\lambda}{m_1(k) E(k) + m_2(k) \exp \{ u_2(k) \} } \] = 0.

That is,
\[ \sum_{k=0}^{\omega-1} b_1(k) \exp \{ u_1(k) \} \]
\[ = \bar{a}_1 \omega + \sum_{k=0}^{\omega-1} c_1(k) \exp \{ u_2(k) \}, \] (15)
\[ \sum_{k=0}^{\omega-1} \left( b_2(k) \exp \{ u_2(k) \} \right) \]
\[ + \frac{\lambda}{m_1(k) E(k) + m_2(k) \exp \{ u_2(k) \} } \] = \bar{a}_2 \omega. (16)

From (14) and (16), we have
\[ \sum_{k=0}^{\omega-1} |u_1(k+1) - u_1(k)| \]
\[ = \lambda \sum_{k=0}^{\omega-1} |a_1(k) - b_1(k) \exp \{ u_1(k) \} \]
\[ + c_1(k) \exp \{ u_2(k) \} | \]
\[ \leq \sum_{k=0}^{\omega-1} |a_1(k)| \]
\[ + \sum_{k=0}^{\omega-1} \left( b_1(k) \exp \{ u_1(k) \} + c_1(k) \exp \{ u_2(k) \} \right) \]
\[ = \sum_{k=0}^{\omega-1} |a_1(k)| + \bar{a}_1 \omega \]
\[ + 2 \sum_{k=0}^{\omega-1} c_1(k) \exp \{ u_2(k) \} \]
\[ = (\bar{A}_1 + \bar{a}_1) \omega + 2 \sum_{k=0}^{\omega-1} c_1(k) \exp \{ u_2(k) \}, \] (17)
\[ \sum_{k=0}^{\omega-1} |u_2(k+1) - u_2(k)| \]
\[ = \lambda \sum_{k=0}^{\omega-1} |a_2(k) - b_2(k) \exp \{ u_2(k) \} \]
\[ - \frac{\lambda}{m_1(k) E(k) + m_2(k) \exp \{ u_2(k) \} } \] \leq \sum_{k=0}^{\omega-1} |a_2(k)| + \sum_{k=0}^{\omega-1} b_2(k) \exp \{ u_2(k) \}
\[ + \frac{\lambda}{m_1(k) E(k) + m_2(k) \exp \{ u_2(k) \} } \]
\[ \leq \sum_{k=0}^{\omega-1} |a_2(k)| + \bar{a}_2 \omega \]
\[ \leq (\bar{A}_2 + \bar{a}_2) \omega. \] (18)

where \( \bar{A}_1 = \frac{1}{\omega} \sum_{k=0}^{\omega-1} |a_1(k)|, \bar{A}_2 = \frac{1}{\omega} \sum_{k=0}^{\omega-1} |a_2(k)|. \)

Since \( \{u(k)\} = \{(u_1(k), u_2(k))^T\} \in X \), there exist \( \eta_i, \delta_i, i = 1,2 \) such that
\[ u_1(\eta_i) = \min_{k \in I_\omega} u_1(k), u_i(\delta_i) = \max_{k \in I_\omega} u_i(k). \] (19)

By (16), we have
\[ \exp \{ u_2(\eta_2) \} \sum_{k=0}^{\omega-1} b_2(k) \leq \bar{a}_2 \omega. \]

So
\[ u_2(\eta_2) \leq \ln \frac{\bar{a}_2}{b_2}. \] (20)

It follows from Lemma 2.2, (18) and (20) that
\[ u_2(k) \leq u_2(\eta_2) + \sum_{k=0}^{\omega-1} |u_2(k+1) - u_2(k)| \]
\[ \leq \ln \frac{\bar{a}_2}{b_2} + (\bar{A}_2 + \bar{a}_2) \omega, \] (21)

From (16) we also have
\[ \exp \{ u_2(\delta_2) \} \sum_{k=0}^{\omega-1} b_2(k) \geq \bar{a}_2 \omega - \sum_{k=0}^{\omega-1} \left( \frac{\lambda}{m_1(k) E(k) } \right), \]
and so
\[ u_2(\delta_2) \geq \ln \frac{\bar{a}_2 - \left( \frac{\lambda}{m_1} \right)}{b_2}. \] (22)
It follows from Lemma 2.2, (18) and (22) that
\[
u_2(k) \geq u_2(\delta_2) - \sum_{k=0}^{\omega-1} |u_2(k+1) - u_2(k)| \geq \ln \left( \frac{q}{b_2} \right) - (\bar{A}_2 + \bar{a}_2)\omega, \tag{23}
\]
which together with (21) leads to
\[
|u_2(k)| \leq \max \left\{ \left| \ln \frac{\bar{a}_2}{b_2} + (\bar{A}_2 + \bar{a}_2)\omega \right|, \right. \\
\left. \left| \ln \left( \frac{q}{b_2} \right) - (\bar{A}_2 + \bar{a}_2)\omega \right| \right\} \overset{\text{def}}{=} H_2. \tag{24}
\]
It follows from (17) and (21) that
\[
\sum_{k=0}^{\omega-1} |u_1(k+1) - u_1(k)| \leq (\bar{A}_1 + \bar{a}_1)\omega + 2 \sum_{k=0}^{\omega-1} c_1(k) \exp \{ u_2(k) \}
\]
\[
\leq (\bar{A}_1 + \bar{a}_1)\omega + 2 \sum_{k=0}^{\omega-1} c_1(k) \exp \{ \ln \frac{\bar{a}_2}{b_2} + (\bar{A}_2 + \bar{a}_2)\omega \}
\]
\[
\leq (\bar{A}_1 + \bar{a}_1)\omega + 2c_1 \frac{\bar{a}_2}{b_2}\omega \exp \{(\bar{A}_2 + \bar{a}_2)\omega\} \overset{\text{def}}{=} \Gamma_1, \tag{25}
\]
and so,
\[
u_1(\eta_1) \leq \ln \frac{\Delta_1}{b_1}, \tag{26}
\]
where
\[
\Delta_1 = \bar{a}_1 + c_1 \frac{\bar{a}_2}{b_2} \exp \{(\bar{A}_2 + \bar{a}_2)\omega\}. \]

It follows from Lemma 2.2, (25) and (26) that
\[
u_1(k) \leq u_1(\eta_1) + \sum_{k=0}^{\omega-1} |u_1(k+1) - u_1(k)| \leq \ln \frac{\Delta_1}{b_1} + \Gamma_1 \overset{\text{def}}{=} M_1. \tag{27}
\]
It follows from (15) that
\[
u_1(k) \geq \sum_{k=0}^{\omega-1} b_1(k) \exp \{ u_1(\delta_1) \}
\]
\[
\geq \bar{a}_1\omega + \sum_{k=0}^{\omega-1} c_1(k) \exp \{ u_2(k) \}
\]
\[
\geq \bar{a}_1\omega, \tag{28}
\]
and so,
\[
u_1(\delta_1) \geq \ln \frac{\bar{a}_1}{b_1}, \tag{29}
\]
It follows from Lemma 2.2, (25) and (28) that
\[
u_1(k) \geq u_1(\delta_1) - \sum_{k=0}^{\omega-1} |u_1(k+1) - u_1(k)| \leq \ln \frac{\bar{a}_1}{b_1} - \Gamma_1 \overset{\text{def}}{=} M_2. \tag{30}
\]
It follows from (27) and (29) that
\[
u_1(k) \leq \max \left\{ \left| M_1 \right|, \left| M_2 \right| \right\} \overset{\text{def}}{=} H_1. \tag{31}
\]
Clearly, \( H_1 \) and \( H_2 \) are independent on the choice of \( \lambda \). Already, in Lemma 2.3, we had showed that under the assumption (12) hold, any solution \( (u_1^*, u_2^*) \) of the system of algebraic equations
\[
\bar{a}_1 - \bar{b}_1 \exp \{ u_1 \} + c_1 \exp \{ u_2 \} = 0,
\]
\[
\bar{a}_2 - \bar{b}_2 \exp \{ u_2 \}
\]
\[
- \frac{1}{\omega} \sum_{k=0}^{\omega-1} \frac{q(k)E(k)}{m_1(k)E(k) + m_2(k)} \exp \{ u_2 \} = 0. \tag{32}
\]
satisfies
\[
\ln \frac{\bar{a}_1}{b_1} \leq u_1^* \leq \ln \frac{\bar{a}_1 + c_1 \bar{a}_2}{b_1},
\]
\[
\ln \frac{\bar{a}_2}{b_2} \leq u_2^* \leq \ln \frac{\bar{a}_2}{b_2}, \tag{33}
\]
Let \( H = H_1 + H_2 + H_3 \), where \( H_3 > 0 \) is taken
sufficiently enough large such that
\[
H_3 > \left| \ln \frac{a_1}{b_1} + \ln \frac{a_2 + \epsilon_1}{b_2} \right| + \left| \ln \frac{\bar{a}_2}{b_2} \right|,
\]
and define
\[
\Omega = \left\{ u(t) = (u_1(k), u_2(k))^T \in X : ||u|| < H \right\}.
\]

It is clear that \( \Omega \) verifies requirement (a) in Lemma 2.1. When \( u \in \partial \Omega \cap \text{Ker L} = \partial \Omega \cap R^2 \), \( u \) is constant vector in \( R^2 \) with \( ||u|| = B \). Then
\[
QN_u = \begin{pmatrix}
\bar{a}_1 - \bar{b}_1 \exp\{u_1\} + \bar{c}_1 \exp\{u_2\} \\
\Delta
\end{pmatrix} \neq 0.
\]
where
\[
\Delta = \bar{a}_2 - \bar{b}_2 \exp\{u_2\}
\]
\[
- \frac{1}{\Omega} \sum_{k=0}^{\Omega-1} \frac{q(k)E(k)}{m_1(k)E(k) + m_2(k)\exp\{u_2\}}.
\]

In order to compute the Brouwer degree, let us consider the homotopy
\[
H_{\mu}u = \mu QN_u + (1 - \mu)Gu, \quad (2.31)
\]
where
\[
Gu = \begin{pmatrix}
\bar{a}_1 - \bar{b}_1 \exp\{u_1\} + \bar{c}_1 \exp\{u_2\} \\
\bar{a}_2 - \bar{b}_2 \exp\{u_2\}
\end{pmatrix}.
\]

From the definition of \( H \), it follows that \( 0 \notin H_{\mu}(\partial \Omega \cap \text{Ker L}) \) for \( \mu \in [0, 1] \). In addition, one can easily show that the algebraic equation \( Gu = 0 \) has a unique solution in \( R^2 \). Note that \( J = 1 \) since \( \text{Im}Q = \text{Ker L} \), by the invariance property of homotopy, direct calculation produces
\[
\text{deg}(JQN, \Omega \cap \text{Ker L}, 0)
\]
\[
= \text{deg}(QN, \Omega \cap \text{Ker L}, 0)
\]
\[
= \text{deg}(G, \Omega \cap \text{Ker L}, 0)
\]
\[
= \text{sgn}(\Gamma) = 1 \neq 0,
\]
where
\[
\Gamma = \bar{b}_1 \bar{b}_2 \exp\{u_1^*\} \exp\{u_2^*\}
\]
and \( \text{deg}(\cdot, \cdot, \cdot) \) is the Brouwer degree. By now we have proved that \( \Omega \) verifies all requirements in Lemma 2.1. Hence (13) has at least one solution \( (u_1^*(k), u_2^*(k))^T \) in \( \text{Dom}L \cap \Omega \). And so, system (1) admits a positive periodic solution \( (N_1^*(k), N_2^*(k))^T \), where \( N_i^*(k) = \exp\{u_i^*(k)\}, i = 1, 2 \). This completes the proof of Theorem 2.1.

3. Example

Now let us consider the following example.

**Example 3.1.**

\[
N_1(k + 1) \\
= N_1(k) \exp \left\{ 0.5 - 0.25 \cos(\pi k) \right\} - (1 + 0.5 \sin(n + \frac{\pi}{4}))N_1(k)
\]
\[
+ (0.5 + 0.3 \sin(\pi k + \frac{\pi}{4}))N_2(k);
\]
\[
N_2(k + 1) \\
= N_2(k) \exp \left\{ 1.5 + 0.5 \sin(\pi k + \frac{\pi}{4}) \right\} - (1 + 0.3 \cos(\pi k + \frac{\pi}{4}))N_2(k)
\]
\[
0.5 + 0.2 \sin(\pi k + \frac{\pi}{4}) \right\}.
\]

Here, corresponding to system (1), we take
\[
a_1(k) = 0.5 - 0.25 \cos(\pi k),
\]
\[
b_1(k) = 1 + 0.5 \sin(\pi n + \frac{\pi}{4}),
\]
\[
c_1(k) = 0.5 + 0.3 \sin(\pi k + \frac{\pi}{4}),
\]
\[
a_2(k) = 1.5 + 0.5 \sin(\pi k + \frac{\pi}{4}),
\]
\[
b_2(k) = 1 + 0.3 \cos(\pi k + \frac{\pi}{4}),
\]
\[
q(k) = 0.5 + 0.2 \sin(\pi k + \frac{\pi}{4}),
\]
\[
E(k) = 1, m_1(k) = 2, m_2(k) = 1.
\]

Obviously, in system (33)
\[
q = 1.5 > 0.25 = \left( \frac{q}{m_1} \right).
\]
It follows from Theorem 2.1 that system (33) admits at least one positive 2-period solution.
3 Conclusion

In this paper, we propose a discrete Lotka-volterra commensalsymbiosismodel with Michaelis-Menten type harvesting, it seems that this is the first time such kind of modelling was proposed. We show that under some suitable condition, the system could admits at least one positive periodic solution, which means that two species could coexist in a fluctuation state.

We will investigate the persistent property and stability property of the system in the future.

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**Contribution of individual authors to the creation of a scientific article (ghostwriting policy)**

All authors reviewed the literature, formulated the problem, provided independent analysis, and jointly wrote and revised the manuscript.

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