Quark orbital angular momentum: can we learn about it from GPDs and TMDs?

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It is known how to access information on quark orbital angular momentum from generalized parton distribution functions, in a certain specified framework. It is intuitively expected, that such information can be accessed also through transverse momentum dependent distribution functions, but not known how. Now quark models provide promising hints. Recent results are reviewed.

Keywords: nucleon spin structure, quark models, orbital angular momentum

1. Introduction

Transverse parton momentum dependent distribution functions (TMDs) and generalized parton distribution functions (GPDs) describe complementary aspects of the transverse nucleon structure. TMDs\textsuperscript{1–3} describe the momentum distribution of partons in the transverse plane. GPDs\textsuperscript{4–7} describe their spatial distribution in the transverse plane\textsuperscript{8} (and much more\textsuperscript{9}).

It is known how to learn from GPDs about orbital angular momentum of partons in the nucleon, namely\textsuperscript{10} (using impact parameter presentation\textsuperscript{8})

\begin{equation}
L^q = \int dx \int d^2b \left( xH^q(x, b) + xE^q(x, b) - \tilde{H}^q(x, b) \right)
\end{equation}

where $\int d^2b \tilde{H}^q(x, b)$ is the helicity distribution and $b$ the impact parameter. This decomposition has the advantage that all spin contributions are measurable quantities. Other decomposition schemes exist\textsuperscript{11} and give, in gauge theories, in general different results.\textsuperscript{12}

In this way, one obtains from the spatial distribution of partons in the transverse plane information about orbital angular momentum. TMDs con-
tains information on the parton momenta $p_T$ in the transverse plane. This is in some sense complimentary to GPDs, see Fig. 1. Intuitively, one would therefore expect TMDs to contain also information about orbital angular momentum. However, so far no rigorous connection of orbital angular momentum and TMDs could be established.

Recent results from quark models could indicate a possible connection, and the key to that is “pretzelosity.” We review the recent developments.

2. The key TMD: pretzelosity

The light-front correlator (with a process-dependent gauge-link $W$)

$$
\phi(x, \vec{p}_T)_{ij} = \int \frac{dz d^2 \vec{z}_T}{(2\pi)^3} e^{ipz} \langle N(P, S) | \bar{\psi}_j(0) W \psi_i(z) | N(P, S) \rangle |_{z^+ = 0, p^+ = xP} + (\vec{p}_T, \vec{p}_T' - \frac{1}{2} \vec{p}_T^2 \delta_{jk}) S_{kT} \frac{h_{LT}}{M_N^2}.
$$

All TMDs in (3) depend on $x$ and $p_T = |\vec{p}_T|$, and can be accessed in semi-inclusive deeply inelastic scattering (SIDIS) in combination with the Collins fragmentation function by measuring various azimuthal (sin-
ngle spin) asymmetries. In particular

\[ A_{UT}^{\sin(3\phi - \phi_5)} \propto \sum_q e_q^2 \frac{h_{1T}^{\perp q} \otimes H_1^{\perp q}}{\sum_q e_q^2 f_q^{1T} \otimes D_1^{1T}}. \]  

(4)

Positivity bounds\(^{14}\) constrain \( |h_{1T}^{\perp q}(x, p_T)| \leq \frac{1}{2} (f_q^1(x, p_T) - g_q^1(x, p_T)) \) with the (1)-moment defined as \( h_{1T}^{\perp q}(x, p_T) \equiv p_T^2/(2M^2) h_{1T}^{\perp q}(x, p_T). \)

In the limit of a large number \( N_c \) of colors in QCD\(^{13}\) pretzelosity behaves as \( (h_{1T}^{\perp u} + h_{1T}^{\perp d})/(h_{1T}^{\perp u} - h_{1T}^{\perp d}) \sim 1/N_c. \) Interesting aspects are\(^{16,17}\) that it describes the “non-sphericity” of the spin distribution of quarks in a transversely polarized nucleon, and requires\(^{16,17}\) the presence of nucleon wave-function components differing by two units of orbital angular momentum. At large \( x \) it is predicted\(^{17,18}\) to behave as \( h_{1T}^{\perp q} \sim (1 - x)^3. \) That is all that is known about this function model-independently.

What raised much interest about this TMD are results from quark models. The following relation was found in the bag model in Ref. [19]

\[ h_{1T}^{\perp q}(x, p_T) = g_q^1(x, p_T) - h_{1}^{\perp q}(x, p_T). \]  

(5)

This relation is supported also in other\(^{20-27}\) though not all\(^{27}\) quark models, and is broken when gauge field degrees of freedom are present,\(^{21}\) see [28] for a review. Notice that \( h_{1T}^{\perp q}(x, p_T) \equiv p_T^2/(2M^2) h_{1T}^{\perp q}(x, p_T) \), and if we recall that the difference of \( q_1 \) and \( h_1 \) vanishes in the non-relativistic limit,\(^ {29}\) we see that this (1)-moment of pretzelosity is a ‘measure of relativistic effects’ in the nucleon. This is not surprising because in this limit\(^ {23}\)

\[ \lim_{\text{non-rel}} h_{1T}^{\perp q}(x, p_T) = -\frac{N_c^2}{2} P_u \delta \left( x - \frac{1}{N_c} \right) \delta^{(2)}(p_T) \]  

(6)

where \( P_u = \frac{4}{3} \), \( P_d = -\frac{1}{3} \), and similarly for other TMDs, i.e. in the non-relativistic limit the (1)-moments of all TMDs vanish. Another interesting model relation is the remarkable \textit{non-linear relation} first observed in the covariant parton model\(^{23}\) connecting all T-even, chiral-odd, twist-2 TMDs:

\[ \frac{1}{2} \left[ h_{1T}^{\perp q}(x, p_T) \right]^2 \overset{\text{model}}{=} -h_{1}^{q}(x, p_T) h_{1T}^{\perp q}(x, p_T). \]  

(7)

This relation implies an attractive prediction. The transversity distribution gives rise to the single spin asymmetry \( A_{UT}^{\sin(\phi + \phi_5)} \propto \sum_q e_q^2 h_{1T}^{\perp q} \otimes H_1^{\perp q} \) whose sign is known experimentally.\(^ {30}\) From (7) it immediately follows that the pretzelosity asymmetry in (4) must have opposite sign,\(^ {28}\) i.e.

\[ \text{sign} \left[ A_{UT}^{\sin(3\phi - \phi_5)} \right] = (-1) \cdot \text{sign} \left[ A_{UT}^{\sin(\phi + \phi_5)} \right], \]  

(8)

which is expected\(^ {28}\) to hold in the valence-\( x \) region, where this prediction is confirmed by COMPASS for negative hadrons from a deuteron target.\(^ {31}\)
3. Pretzelosity and Orbital Angular Momentum

Many more quark model relations among TMDs were found, see [25] for the derivation of a complete set of relations in the bag model, and it is understood why they are widely supported in a large class of quark models.\textsuperscript{32}

The pretzelosity-relation (5) plays a particularly important role in what follows for the following reason. Namely, it was shown in the light-cone SU(6) quark-diquark model\textsuperscript{33} that the contribution to the nucleon spin from the orbital angular momentum of quarks is related to the difference of transversity and helicity distributions, i.e. to the right-hand-side of the pretzelosity-relation (5) found in.\textsuperscript{19} It was subsequently shown that the pretzelosity-relation (5) is valid also in the light-cone SU(6) quark-diquark model.\textsuperscript{24} In other words, the (1)-moment of pretzelosity is, in this quark model, a measure for the contribution of quark orbital angular momentum to the nucleon spin. This exciting finding was subsequently confirmed in the bag model\textsuperscript{25} and the covariant parton model.\textsuperscript{26}

More precisely, three different quark models\textsuperscript{24–26} support the relation

\[ L^q_z = -\int dx h_{1\perp}^{(1)q}(x) \]  

(9)

(in principle, there is a fourth model where it holds: in the non-relativistic limit one has the consistent result\textsuperscript{23} \( L^q_z = -\int dx h_{1\perp}^{(1)q}(x) = 0 \)).

An interesting question in this context: how can chiral-odd (pretzelosity) and chiral-even (orbital angular momentum) quantities be related?

The answer in the bag model is as follows. Here the quark wave-function has an upper-(s-wave-)component and a lower-(p-wave-)component. The expectation value of the orbital angular momentum operator in the s-wave is zero, i.e. only the p-wave contributes. Next, we know\textsuperscript{16,17} pretzelosity requires \( \Delta L = 2 \) which in the bag model is possible only through interference of the \( L_z = \pm 1 \) components of the p-wave, i.e. again only the p-wave contributes. Finally, knowing that only the p-wave (i.e. the lower component of the Dirac-spinor) matters, we can “replace” in the operator of pretzelosity \( \gamma^0 = \text{diag}(1, -1) \) (in Bjorken-Drell notation) by \((-1)\times(\text{unit matrix})\). This changes the number of gamma-matrices by one unit, and “transforms” a chiral-odd operator into a chiral-even one.\textsuperscript{25}

From this exercise we learn: the relation between pretzelosity and orbital angular momentum is at best at the level of matrix elements. In other words, there is no operator-identity between these quantities — not even in quark models. It is interesting to stress that in quark models the result for \( L^q_z \) does not depend on which orbital angular momentum definition\textsuperscript{10,11} is used.\textsuperscript{12}
4. How does pretzelosity look like, and how to access it?

Having discussed that in quark models pretzelosity is related to orbital angular momentum, it is interesting to ask what quark models actually predict. Fig. 2a shows the bag model predictions for $h_{1T}^{|1}\perp q(x)$ in comparison to $h_1^{|1}q(x)$ and $g_1^{|1}q(x)$. The negative sign and the large magnitude of $h_{1T}^{|1}\perp q(x)$ can be understood from the non-relativistic limit, where pretzelosity is enhanced by the factor $\frac{1}{2}N_c^2$ compared to $h_1^{|1}q(x)$ or $g_1^{|1}q(x)$, see Sec. 2. Relativistic models (like the bag model) preserve this enhancement. Concerning the $p_T$-dependence of pretzelosity (and other TMDs), we remark that in the bag and covariant parton model it is approximately Gaussian which is supported by phenomenology.

Note that $h_{1T}^{|1}\perp q(x)$ is not constrained by positivity, but $h_{1T}^{|1}\perp q(x)$ is, see Sec. 2. Moreover, $h_{1T}^{|1}\perp q$ enters cross sections with a prefactor of $O(p_T^2/M^2)$ from the correlator (2). So in observables effectively the (1)-moment of pretzelosity is relevant, and the latter is not large in the bag or light-cone constituent model. In fact, the latter predicts a small asymmetry (4).

The covariant parton model predicts large $h_{1T}^{|1}\perp q(x)$, see Fig. 2b, and hence a sizable pretzelosity asymmetry (4) shown in Fig. 2c for the kinematics of the CLAS experiment with 12 GeV upgrade. The predictions are consistent with preliminary SIDIS data showing a zero within error bars effect. Recent COMPASS data show a small but non-vanishing effect and confirm the signs predicted for $h_{1T}^{|1}\perp q$, as was discussed in Sec. 2.
One has to bear in mind that so far the exciting relation (9) between TMDs and orbital angular momentum is established only in quark models. An important question is: to what extent can we trust such models? One way to address this question consists in reproducing (SI)DIS spin observables within a given quark model. On the basis of the comparison of model results and data it was found\textsuperscript{34} that (light-cone constituent) quark models work in the valence $x$-region $0.2 \lesssim x \lesssim 0.6$ with an accuracy of $(10–30\%)$.

In this context we recall that the absence of gauge degrees of freedom implies in quark models certain relations among TMDs (called “LIRs”), which hold approximately in QCD upon the neglect of quark-gluon-quark correlators and current quark mass terms.\textsuperscript{38} For the collinear twist-3 parton distribution function $g^q_T(x)$ such an approximation works reasonably well\textsuperscript{39} but it needs to be tested for other TMDs.\textsuperscript{40} If LIRs were confirmed to be reasonably good approximations, this would be a necessary (not sufficient) condition for quark model predictions of the type (5, 9) to work similarly.

5. Conclusions

The concept of quark orbital angular momentum is difficult to address rigorously in gauge field theories. GPDs and TMDs, which describe complementary aspects of the nucleon structure, give rise to a dual (in quark models equivalent) picture of quark orbital angular momentum as follows

$$L^q_{QCD+models} = \int dx \int d^2 b \ GPDs(x, b)$$

and

$$L^q_z \text{ quark models} = \int dx \int d^2 p_T \ TMDs(x, p_T)$$

where $GPDs = xH^q + xE^q - \bar{H}^q$ and $TMDs = -h_T^{\perp(1)}$. The first relation (10) holds exactly in QCD and in consistent models.\textsuperscript{10} The second relation (11) holds in a large class of relativistic quark models.\textsuperscript{24–26} Quark models catch important features of QCD, and could provide useful insights also in the context of GPDs, TMDs and orbital angular momentum, provided one uses them responsibly within their range of applicability. Recent advances\textsuperscript{41} allow us to test these relations in lattice QCD, where due the presently often practioned omission of disconnected diagrams valence quarks are probed.

Note added

In the discussion following the talk it was pointed out that in [42] the predictions from quark models were shown to be compatible with phenomenology and lattice data, resolving the “spin crisis” from quark model point of view.
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