Position weighted backpressure intersection control for connected urban networks

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Abstract

This paper proposes a position weighted backpressure (PWBP) control policy for connected network traffic. This is a decentralized policy that considers the spatial distribution of vehicles along the length of network roads to account for spill-back dynamics. The approach can be implemented in real time, without a priori knowledge of network traffic demands, and can scale to large urban networks. The capacity region of the network is formally defined in the context of urban network traffic and it is proved, when exogenous arrival rates are within the capacity region, that PWBP control is network stabilizing. We perform comparisons against fixed signal timing, standard backpressure, and the capacity-aware backpressure control policies using a calibrated microscopic simulation model of Abu Dhabi Island in the United Arab Emirates. We demonstrate the superiority of PWBP over the three other policies in terms of capacity region, network-wide delay, congestion propagation speed, recoverability from heavy congestion (outside of the capacity region), and response to incidents.

Keywords: Decentralized control, backpressure, connected vehicles, traffic network, intersection control

1. Introduction

Various approaches have been proposed to optimize signal timing for isolated intersections, including mixed-integer linear models, rolling horizon approaches, and store-and-forward models based on model predictive control; see (Yann et al., 2011; Gartner, 1983; Tettamanti and Varga, 2010;...
Mirchandani and Head, 2001; You et al., 2013; Ma et al., 2013) for examples. On the one hand, isolated intersection approaches fail to account for spillback from adjacent road segments, which can eventually lead to gridlock throughout a road network (Cervero, 1986). On the other hand, centralized techniques that include coordination between intersection controllers (Heung et al., 2005; Gettman et al., 2007) are not scalable and difficult to implement in real-world/real-time settings (Papageorgiou et al., 2003). For example, ACS-Lite (Gettman et al., 2007) can handle no more than 12 intersections in real-time.

Decentralized control techniques have been proposed to overcome the scalability issues associated with network control optimization. These techniques expect intersection controllers to be able to measure/estimate local traffic information in real-time. This information includes expected traffic demand at the intersection in the next cycle for heuristic approaches, e.g., (Smith, 1980; Lämmer and Helbing, 2008, 2010; Smith, 2011) or the queue sizes along the intersection arcs in max pressure based approaches (Wongpiromsarn et al., 2012; Varaiya, 2013; Xiao et al., 2014; Le et al., 2015). According to (De Gier et al., 2011), control strategies that use traffic conditions along both upstream and downstream arcs are more efficient and reliable than those that utilize upstream traffic conditions only. Recently, backpressure (BP) control techniques have been adopted to signal control; they were first independently proposed in (Wongpiromsarn et al., 2012) and (Varaiya, 2013) based on seminal work in communications networks (Tassiulas and Ephremides, 1992) (see (Neely et al., 2005; Georgiadis et al., 2006; Neely, 2010) for more details).

In general, BP based approaches are scalable and come with theoretical guarantees of network stability. However, as it was originally developed for packet queueing in communications networks, the assumptions are not tailored to traffic problems and in some cases the assumptions are not suitable for traffic networks. Specifically, these models assume point queues and (more critically) infinite queue size capacity. As a result, the models do not account for the spatial distribution of the queues and (more importantly) cannot account for spillback. Another drawback is loss of work conservation, in which no flow is allowed (all red) despite the availability of capacity in the outbound arcs. For example, Fig. 1 shows three cases in which BP control favors the eastbound approach ($Q_a$ to $Q_b$), despite that no vehicles could actually pass the intersection from this approach. Recognizing this problem, (Gregoire et al., 2015) proposed an improvement, referred to as capacity aware back pressure (CABP) control. Their approach can avoid the case illustrated in Fig. 1a, but not the two depicted in Fig. 1b and Fig. 1c.
(in the former, the queue is concentrated at the ingress of the arc). This is due to the fact that their approach models traffic as a point queue.

Another feature of the dynamics of communications networks that does not apply to (road) traffic networks is separate queues for different commodities (corresponding to vehicles with different turning desires in traffic) and no interference between commodities. Shared lanes, which are very common, are one example where this assumption is violated in traffic networks. Even when there are no shared lanes, road widening near
the egresses intersection inbound arcs, also very common geometrical features in urban networks, can create bottlenecks at the lane-branching point. Different turning movements (commodities) interact at the bottleneck, and one queue may block another if it gets too long as illustrated in Fig. 2. In

![Fig. 2: Bottleneck at the lane-branching point.](image)

(road) traffic networks, one has to consider start-up and clearance lost times (i.e. the green lost time) and avoid frequent phase switching. These features do not exist in communications networks and existing BP control approach applied to traffic flow has not considered such green lost time.

This paper proposes decentralized intersection control techniques that build upon macroscopic traffic theory and overcome the issues described above. We refer to this approach as position-weighted backpressure (PWBP). PWBP considers the spatial distribution of vehicles along the road, applying higher weights to queues that extend to the ingress of the road, thereby accounting for the possibility of spillback. Flow rates entering the intersection depend on both the control and the traffic density, thereby capturing diminished flows at phase startups (startup lost times). We perform comparisons in a network setting against fixed intersection control, standard BP, and CABP and demonstrate superiority of PWBP in terms of capacity region, delay, congestion propagation speed, recoverability from heavy congestion and response to an incident.

The type of control proposed can be applied to intersection signal control, i.e., today’s traffic lights. But it can also be thought of as a prioritization
scheme for connected vehicles at network intersections that can guarantee network stability. In both cases, when accurate measurement of the distribution of vehicles along the roads is not possible, one may employ a light-weight traffic state estimation technique. We refer to (Seo et al., 2017; Jabari and Liu, 2013; Zheng et al., 2018) for recent examples.

The remainder of this paper is organized as follows: Sec. 2 describes the traffic dynamics model, macroscopic intersection control, and the proposed PWBP control policy. Sec. 3 rigorously demonstrates the network-wide stability properties of the PWBP approach using Lyapunov drift techniques. Simulation experiments and results are provided in Sec. 4 and Sec. 5 concludes the paper.

2. Problem formulation

2.1. Notation

Consider an urban traffic network represented by the directed graph $G = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is a set of network nodes, representing intersections and $\mathcal{A} \subset \mathcal{N} \times \mathcal{N}$ is a set of network arcs, representing road segments. Each element of $\mathcal{A}$ is in one-to-one correspondence with an ordered pair of elements in $\mathcal{N}$. For each node, $n \in \mathcal{N}$, $\Pi_n$ and $\Sigma_n$ denote, respectively, the set of (predecessor) arcs terminating in $n$ and the set of (successor) arcs emanating from $n$. We also use $\Pi(a) \subseteq \mathcal{A}$ to denote the set of predecessor arcs to arc $a \in \mathcal{A}$. That is, if $n$ is the ingress node of arc $a$, then $\Pi(a) = \Pi_n$. Similarly, $\Sigma(a)$ is the set of successor arc to arc $a$.

Fictitious source arcs are appended to the network to represent exogenous network arrivals. A new junction with indegree zero and outdegree one is created for each exogenous inflow and the new source arc connects this new node to the network boundary node; see Fig. 3. When exogenous inflows occur at the interior of the network (i.e., at a junction with non-zero
in-degree) representing, for example, a parking ramp/lot, the associated arc can be broken into two arcs with a new node placed at the position of the merge; see Fig. 4. Source arcs will be assumed to have infinite jam densities (i.e., they serve as fictitious reservoirs), but the flow rates in and out of these arcs are assumed to be finite (i.e., finite capacities). They shall also be assumed to have zero physical length. Therefore, the traffic states associated with fictitious source arcs are point queues concentrated at the source node. We shall denote the set of (fictitious) source arcs by $A^{src} \subset A$.

### 2.2. Dynamics

We denote the length of each arc $a \in A$ by $l_a$. With slight notation abuse, the upstream-most position (the entrance position) for each arc $a$ in the network is $x = 0$, while the downstream-most position (the arc exit position) is $x = l_a$ (that these coordinates pertain to arc $a$ only should be understood implicitly). We consider a multi-commodity framework, where $\rho^b_a(x, t)$ denotes the traffic density at position $x$ along arc $a$ that is destined to outbound arc $b \in \Sigma(a)$ at time instant $t$. Similarly, $q^b_a(x, t)$ denotes the flow rate at $x$ along $a$ that is destined to $b$ at time $t$. We have the following conservation equation: for each $a \in A$ and $b \in \Sigma(a)$

\[
\frac{\partial \rho^b_a(x, t)}{\partial t} = -\frac{\partial q^b_a(x, t)}{\partial x} \quad x \in [0, l_a], t \geq 0.
\]

Let $p_{a,in}(t)$ and $p_{a, out}(t)$ denote the upstream and downstream control state at boundaries of arc $a$. The boundary flows are written as $q^b_a(0-, t) = q^b_{a,in}(p_{a,in}(t))$ and $q^b_a(l_a+, t) = q^b_{a, out}(p_{a, out}(t))$, where $q^b_{a,in}$ and $q^b_{a, out}$ are boundary flux functions, which depend on the (boundary) control variables and, implicitly, on the node dynamics (for instance, $q^b_{a,in}$ and $q^b_{a, out}$...
cannot exceed local supplies and demands at the arc boundaries). We omit dependence on system state to minimize clutter in our notation.

For (fictitious) source arcs, we assume random arrivals; for commodity \( a \in \mathcal{A}^{\text{src}} \) and \( b \in \Sigma(a) \), let \( A^b_a(t) \) be a random (cumulative) arrival process with (instantaneous) rate \( \lambda^b_a(t) = \mathbb{E} \frac{dA^b_a(t)}{dt} \). We, thus have that

\[
q^b_a(0^-, t) = \frac{dA^b_a(t)}{dt} \quad \text{for} \quad a \in \mathcal{A}^{\text{src}}, b \in \Sigma(a). \tag{2}
\]

At the arc boundaries the conservation law (1) is given, for \( a \in \mathcal{A}/\mathcal{A}^{\text{src}} \), by

\[
\frac{\partial \rho^b_a(x, t)}{\partial t} = - \frac{\partial q^b_a(x, t)}{\partial x} = \begin{cases}
q^b_a(p_{a, \text{in}}(t)) - q^b_a(0, t) & x = 0 \\
q^b_a(l_a, t) - q^b_a(p_{a, \text{out}}(t)) & x = l_a
\end{cases} \quad \tag{3}
\]

and for \( a \in \mathcal{A}^{\text{src}} \) by

\[
\frac{d\rho^b_a(t)}{dt} = - \frac{d q^b_a(t)}{dx} = \frac{dA^b_a(t)}{dt} - q^b_a(p_{a, \text{out}}(t)). \tag{4}
\]

**Remark 1.** We make no assumptions about the relationship between flux and density. The proposed approach is equally valid in first and second order contexts. The only assumptions we make are (i) flow conservation, (ii) probabilistic upper bounds on flux and density, and (iii) that arc parameters do not change along the length of the arc. The last assumption is easy to honor in a general network by splitting arcs with varying parameters into more than one arc.

### 2.3. Junction control

For each node \( n \in \mathcal{N} \), let \( \mathcal{M}_n \) denote the set of allowed movements between inbound and outbound road segments. The set \( \mathcal{M}_n \) consists of ordered pairs \((a, b)\) such that \( a \in \Pi_n \) and \( b \in \Sigma_n \), i.e., \( \mathcal{M}_n \subseteq \Pi_n \times \Sigma_n \). The set of all network movements is denoted by \( \mathcal{M} \equiv \mathcal{M}_1 \sqcup \cdots \sqcup \mathcal{M}_{|\mathcal{N}|} \). A signal phase consists of junction movements that do not conflict with one another. We denote by \( \mathcal{P}_n \subseteq 2^{\mathcal{M}_n} \) the set of allowable phases and by \( \mathcal{P} \subseteq \bigotimes_{n \in \mathcal{N}} \mathcal{P}_n \) the set of allowable network phasing schemes. Essentially, an allowable phase is one that does not allow crossing conflicts and only allows merging conflicts between a protected movement and a permitted movement. Example allowable phases are depicted in Fig. 5. We denote by \( q_{a,b}(p_{b,\text{in}}(t)) \) or

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1The processes \( A^b_a \) may have jumps. To be more accurate in such situations, one defines \( \Lambda^b_a(t_1, t_2) = \mathbb{E} \int_{t_1}^{t_2} A^b_a(t) \, dt = \int_{t_1}^{t_2} \lambda^b_a(t) \, dt \). The boundary flux is then given by \( q^b_a(0^-, t-) = \lim_{\Delta t \downarrow 0} \int_{t-\Delta t}^{t-} A^b_a(t') \, dt' \) and \( q^b_a(0^+, t+) = \lim_{\Delta t \downarrow 0} \int_{t}^{t+\Delta t} A^b_a(t') \, dt' \).
equivalently \( q_{a,b}(p_{a,\text{out}}(t)) \) the rate of flow that departs arc \( a \in \Pi(b) \) into arc \( b \) at time \( t \). These are related to the commodity flows at the arc boundaries as follows:

\[
q_{a,b}^b(p_{a,\text{in}}(t)) = \pi_{a,b}(t) \sum_{c \in \Pi(a):(c,a) \in M} q_{c,a}(p_{a,\text{in}}(t))
\]

and

\[
q_{a,b}^b(p_{a,\text{out}}(t)) = q_{a,b}(p_{a,\text{out}}(t)),
\]

where \( \pi_{a,b}(t) \) is the percentage of flow into \( a \) at time \( t \) that is destined to adjacent arc \( b \in \Sigma(a) \).

Under any network-wide phasing scheme, \( p \in P \), the traffic network can “support” arrival processes with certain rates. Beyond these arrival rates, queues will grow indefinitely. For each \( p \in P \), we say that the network can support an arrival rate vector \( \lambda(p) = [\lambda_a(p) \cdots \lambda_{|A|}(p)]^\top \) if

\[
\lim_{T \to \infty} \frac{1}{T} \int_0^T \left( \lambda_a(p) + q_{a,\text{in}}(p) - q_{a,\text{out}}(p) \right) dt \leq 0,
\]

where with slight abuse of notation, \( q_{a,\text{in}}(p) \) and \( q_{a,\text{out}}(p) \) are the inflow and outflow rates obtained when the network phasing scheme \( p \) is active. This is interpreted as follows: the phasing scheme \( p \) is such that the total arc outflow exceeds the total arc inflow in the long run.

In accord with (7), each \( p \in P \) defines a set of admissible arrival rates; denote these (convex) polytopes by \( \Omega(p) \). Taking the union of these sets, \( \bigcup_{p \in P} \Omega(p) \), we get the vectors of all possible arrival rates that the network can support under all \( p \in P \). This is formally defined next.
Definition 1 (Maximal throughput region). The maximal throughput region (a.k.a., capacity region) of the network, denoted by $\Lambda$, is the convex hull of all sets of admissible flows. That is,

$$\Lambda \equiv \text{Conv}\left( \bigcup_{p \in P} \Omega(p) \right).$$

Arrival rates that lie in $\Lambda$ but not in $\bigcup_{p \in P} \Omega(p)$ are interpreted as arrival rates that can be supported by switching between phasing schemes that lie in the latter (i.e., time-sharing). A control policy that can support all possible arrival rates in $\Lambda$ is referred to as a throughput-maximal control policy. We denote a control policy by a vector of network control states: at time $t$ the network control state is denoted by $p(t) \equiv [\cdots p_{a,\text{in}}(t) \; p_{a,\text{out}}(t) \; \cdots]^T$, a policy is an entire curve $p(\cdot)$.

2.4. Position-weighted back-pressure (PWBP)

For any intersection $n \in \mathcal{N}$, we assume that controllers are capable of assessing the (average) movement fluxes associated with any possible phase $p \in \mathcal{P}_n$. That is, for any $(a, b) \in \mathcal{M}_n$, $E(q_{a,b}(p)|\rho(t)) \equiv E^{p(t)}q_{a,b}(p)$ is known or can be estimated by the controller (locally). Drops in flux as a result of start-up lost times are captured by utilizing demand functions with diminishing flux at higher densities (see (Jabari, 2016) and references therein). The stochasticity in the flows captures supply uncertainty. It can be modeled using parametric uncertainty as presented in (Jabari et al., 2014; Zheng et al., 2018).

For each $n \in \mathcal{N}$ and each $(a, b) \in \mathcal{M}_n$, we define the weight variable

$$w_{a,b}(t) = c_{a,b} \int_0^{l_a} \left( \frac{x}{l_a} \right) \rho^b_a(x, t) \, dx - \int_0^{l_b} \left( \frac{l_b - x}{l_b} \right) \sum_{c \in \Sigma(b); \atop \{b,c\} \in \mathcal{M}} c_{b,c} \pi_{b,c}(t) \rho^b_c(x, t) \, dx,$$

which depends on the (commodity) density curves along arcs $a$ and $b$. To interpret this, first note that

$$\int_0^{l_a} \rho^b_a(x, t) \, dx$$

is just the total traffic volume (queue size) along arc $a$ that is destined to arc $b$. Then the first integral inside the square brackets in (9) can be interpreted as a weighted queue size, where traffic densities at the downstream end of arc $a$ (at $x = l_a$) have the (maximal) weight of one, while traffic densities at the upstream end of $a$ (at $x = 0$) have a weight of zero. In between,
the weights increase linearly with \( x \). Similarly, the second integral inside the square brackets in (9) can also be interpreted as a weighted queue size, but with the weights decreasing linearly with \( x \). Hence, the weight associated with movement \((a, b)\) decreases as the traffic densities at upstream end (ingress) of arc \( b \) increase and vice versa, it increases when the traffic densities are concentrated at the downstream end of arc \( a \) and vice versa. The movement constants \( c_{a,b} \) in (9) allow for assigning higher weights to certain movements. For example, in our experiments, we use this to assign higher weights to left-turn movements: if \((a, b)\) is a left-turning movement, \( c_{a,b} \equiv \left( \frac{\#\text{thru}}{\#\text{left}} \right)^2 \), where \#thru is the number of through lanes and \#left is the number of left lanes. The phase that is active at node \( n \) at time \( t \) under PWBP control, denoted \( p_{\text{PWBP}}(t) \), is given by

\[
p_{\text{PWBP}}(t) \equiv \arg \max_{p \in P_n} \sum_{(a,b) \in M_n} w_{a,b}(t) \mathbb{E} q_{a,b}(t).
\] (11)

Since the number of possible phases at any intersection tend to be small (typically four-eight), (11) can be easily solved by direct enumeration. This allows for real-time distributed implementation of the proposed approach. It can also be demonstrated that PWBP control is network stabilizing. This is proved formally in Sec. 3.

**Remark 2.** In the proposed set-up, the density curves \( \rho^b_a(\cdot, t) \) and the splits \( \pi_{a,b}(t) \) are random quantities that are to be estimated or measured. In a fully automated system, these quantities may degenerate, that is, it is easy to imagine that they can be measured with high accuracy and become deterministic quantities. In present day settings these quantities need to be estimated. The setting envisaged in this paper is one with mixed automated/connected and traditional vehicles. Connected vehicles announce their turning desires upon entering arc \( a \) and may serve as probes to allow the controller to estimate traffic conditions along the arc and the split variables. In such settings, the proposed control techniques need to be coupled with traffic state estimation tools; we refer to (Zheng et al., 2018) and references therein for examples of traffic state estimation in urban traffic settings.

3. Network stability

3.1. Lyapunov functional and stability

The traffic network is said to be strongly stable if (Neely, 2010, Definition 2.7):

\[
\limsup_{T \to \infty} \frac{1}{T} \int_0^T \sum_{(a,b) \in M} \mathbb{E} \left( \int_0^t |\rho^b_a(x, t)| dx \right) dt < \infty.
\] (12)
Since the network traffic densities depend (implicitly) on the control decisions at the network nodes, strong stability implies that the control in place ensure that the network densities do not grow without bound in the long run. This section demonstrates that as long as such a control policy exists\(^2\), the PWBP algorithm ensures strong stability.

Consider the network-wide energy functional \( V : D \to \mathbb{R} \) with domain \( D \) being an appropriately defined \(|\mathcal{A}|\)-dimensional set of curves. \( V \) is defined as

\[
V(\rho(t)) = \frac{1}{2} \sum_{(a,b) \in \mathcal{M}} c_{a,b} \int_0^{l_a} \int_0^{l_b} \left| \frac{l_a - x - x'}{l_a} \right| \rho_a^b(x',t) \rho_a^b(x,t) dx' dx ,
\]

where \( \{c_{a,b}\}_{(a,b) \in \mathcal{M}} \) are non-negative finite constants and \( \rho(t) \) is a vector of network traffic density curves\(^3\). It can be easily shown that \( V \) is a Lyapunov functional: (i) \( V(\rho(t)) \geq 0 \) almost surely since traffic densities are non-negative (with probability 1) and (ii) \( V(\rho) = 0 \) if and only if \( \rho_a^b(x,t) = 0 \) almost surely for all \((a,b) \in \mathcal{M} \) and all \( x \in [0,l_a] \).\(^4\) Lemma 1 below provides a sufficient condition for strong stability using the definition of Lyapunov functionals. A notational convention used below is \( \dot{V} \equiv \frac{dV}{dt} \).

For source nodes, \( l_a = 0 \) (no physical length); consequently, source arc commodities (i.e., when \( a \in \mathcal{A}_{\text{src}} \)) contribute \( \rho_a^b(t) \) only to the Lyapunov function, where dependence on position is dropped as an argument since these are point queues.

**Lemma 1.** For the Lyapunov functional (13), suppose \( \mathbb{E}V(\rho(0)) < \infty \). If there exist constants \( 0 < K < \infty \) and \( 0 < \epsilon < \infty \) such that

\[
\mathbb{E}^\rho(t) \dot{V}(\rho(t)) \leq K - \epsilon \sum_{(a,b) \in \mathcal{M}} \mathbb{E} \int_0^{l_a} |\rho_a^b(x,t)| dx
\]

holds for all \( t \geq 0 \) and all possible \( \rho(t) \), then the traffic network is strongly stable.

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\(^2\)Otherwise, there does not exist a control policy capable of stabilizing the network. Hence, this is a feasibility assumption.

\(^3\)We use the ‘·’ notation as a function argument to represent the entire curve in the dimension in which it is used. In other words, \( \rho_a^b(\cdot,t) \) denotes the traffic density curve along arc \( a \) destined to adjacent arc \( b \) at time instant \( t \).

\(^4\)One can construct pathological density curves with non-zero density spikes, where \( V = 0 \). However, such densities occur with probability zero. Technically, these are overcome by using equivalence classes of density curves, but we shall avoid this level of technicality to promote readability.
PROOF. We first integrate both sides of (14) over the interval \([0, T]\) and take expectation of both sides of the inequality to obtain
\[
\mathbb{E} \int_0^T \mathbb{E} \rho(t) \dot{V}(\rho(t)) \, dt \leq KT - \varepsilon \int_0^T \sum_{(a,b) \in \mathcal{M}} \mathbb{E} \int_0^{l_a} |\rho_a(x, t)| \, dx \, dt.
\] (15)

Reversing the order of the (outer) expectation and the integral, the left-hand side becomes
\[
\int_0^T \mathbb{E} \rho(t) \dot{V}(\rho(t)) \, dt = \mathbb{E} \int_0^T \dot{V}(\rho(t)) \, dt = \mathbb{E} V(\rho(T)) - \mathbb{E} V(\rho(0)).
\] (16)

Combining this with (15), we get the inequality
\[
\mathbb{E} V(\rho(T)) - \mathbb{E} V(\rho(0)) \leq KT - \varepsilon \int_0^T \sum_{(a,b) \in \mathcal{M}} \mathbb{E} \left( \int_0^{l_a} |\rho_a^b(x, t)| \, dx \right) \, dt.
\] (17)

Dividing both sides by \(Te\) and noting that \(\mathbb{E} V(\rho(T)) \geq 0\) (by definition of \(V\)), we get the inequality
\[
\frac{1}{T} \int_0^T \sum_{(a,b) \in \mathcal{M}} \mathbb{E} \left( \int_0^{l_a} |\rho_a^b(x, t)| \, dx \right) \, dt \leq \frac{K}{\varepsilon} + \frac{1}{\varepsilon T} \mathbb{E} V(\rho(0)).
\] (18)

Noting that \(\mathbb{E} V(\rho(0)) < \infty\), the result follows immediately upon taking the limit on both sides as \(T \to \infty\). \(\square\)

3.2. Stability of PWBP

According to Lemma 1, finding (finite) constants \(K\) and \(\varepsilon\) that satisfy the condition (14) will ensure strong stability of the dynamics at the network level. The constant \(K\) is established using the boundedness of the fluxes \(q_a(\cdot, \cdot)\), which is a property of traffic flow (i.e., a physical property that must be ensured by any model). On the other hand, \(\varepsilon\) depends on the intersection control polices. Lemma 2 provides a necessary ingredient that will be used later to establish the constant \(K\).

Lemma 2. Let \(a \in \mathcal{A}/\mathcal{A}^{\text{src}}\) and suppose there exist constants \(0 \leq \underline{q}_a < \infty\) and \(0 \leq \overline{q}_a < \infty\) such that \(\mathbb{P}(q_a^b(x, t) \leq \underline{q}_a) = 1\) and \(\mathbb{P}(q_a^b(x, t) \leq \overline{q}_a) = 1\) for any \((a, b) \in \mathcal{M}\), any \(x \in [0, l_a]\), and any \(t \geq 0\). Then, there exist constants \(0 \leq K_a^{(1)} < \infty\) and \(0 \leq K_a^{(2)} < \infty\) such that, with probability 1,
(i) for any \((x_1, x_2) \subseteq [0, l_a]\)

\[
\mathbb{E}^{\rho(t)} ( q_a^b(l_a, t) \int_{x_1}^{x_2} \rho_a^b(x, t) \, dx ) \leq K_a^{(1)}
\]

and

(ii) for any \((x_1, x_2) \subseteq (0, l_a)\) and any \((x_3, x_4) \subseteq [0, l_a]\

\[
- \mathbb{E}^{\rho(t)} \int_{x_3}^{x_4} \int_{x_1}^{x_2} \rho_a(x, t) \frac{\partial q_a(x', t)}{\partial x} \, dx' \, dx \leq K_a^{(2)}.
\]

**Proof.** First, note that

\[
- \mathbb{E}^{\rho(t)} \int_{x_3}^{x_4} \int_{x_1}^{x_2} \rho_a(x, t) \frac{\partial q_a(x', t)}{\partial x} \, dx' \, dx = \mathbb{E}^{\rho(t)} q_a^b(x_1, t) - \mathbb{E}^{\rho(t)} q_a^b(x_2, t) \leq \overline{q}_a
\]

with probability 1. By the boundedness properties of \(\rho_a^b(\cdot, \cdot)\) for \(a \in \mathcal{A}/\mathcal{A}^\text{src}\), it holds that

\[
\mathbb{P} \left( \int_{x_3}^{x_4} \rho_a^b(x, t) \, dx \leq l_a \overline{q}_a \right) = 1.
\]

Hence, with probability 1, we have that

\[
- \mathbb{E}^{\rho(t)} \int_{x_3}^{x_4} \int_{x_1}^{x_2} \rho_a(x, t) \frac{\partial q_a(x', t)}{\partial x} \, dx' \, dx = - \left( \int_{x_3}^{x_4} \rho_a(x, t) \, dx \right) \mathbb{E}^{\rho(t)} \int_{x_1}^{x_2} \frac{\partial q_a^b(x, t)}{\partial x} \, dx \leq l_a \overline{q}_a \overline{q}_a.
\]

The bound in (20) follows immediately and (19) follows from (22) along with the boundedness of \(q_a^b(x, t)\) and then applying the Cauchy-Schwartz inequality.

**Corollary 1.** Let \(a \in \mathcal{A}/\mathcal{A}^\text{src}\) and assume the probabilistic bounds of Lemma 2.

Then, there exists a constant \(0 \leq K < \infty\) such that

\[
\sum_{(a,b) \in \mathcal{M}: a \neq \mathcal{A}^\text{src}} c_{a,b} \left[ \mathbb{E}^{\rho(t)} ( q_a^b(l_a, t) \int_{0}^{l_a} \left| \frac{x}{l_a} \right| \rho_a^b(x, t) \, dx ) - \mathbb{E}^{\rho(t)} \left( \int_{0}^{l_a} \int_{0}^{l_a} \left| \frac{l_a - x - x'}{l_a} \right| \rho_a^b(x, t) \frac{\partial q_a^b(x', t)}{\partial x} \, dx' \, dx \right) \right] \leq K.
\]
ensures strong stability of the traffic network.

Theorem 3.1 (Stability of PWBP). Assume that the boundedness conditions of Lemma 2 hold for all \( a \in \mathcal{A}/\mathcal{A}^{arc} \), assume that arrival rates lie in \( \Lambda \), that is, there exists a control policy \( p^* (\cdot) \) that can stabilize the network in the sense defined in Lemma 1, and let \( w_{a,b}(\cdot) \) for each movement \( (a,b) \in M \) be as defined in (9). Then the policy

\[
P_{\text{PWBP}}(t) \equiv \arg \max_{p \in \mathcal{P}_a} \sum_{(a,b) \in \mathcal{M}_a} w_{a,b}(t) \mathbb{E}^{p(t)} q_{a,b}(p) \quad \forall n \in \mathcal{N},
\]

ensures strong stability of the traffic network.

Proof.

\[
\mathbb{E}^{p(t)} V(\rho(t)) = \frac{1}{2} \mathbb{E}^{p(t)} \sum_{(a,b) \in \mathcal{M}} c_{a,b} \frac{d}{dt} \int_0^{l_a} \int_0^{l_a} \left| \frac{l_a - x - x'}{l_a} \right| \rho^b_a(x',t) \rho^b_a(x,t) dx' dx
\]

\[
= \frac{1}{2} \sum_{(a,b) \in \mathcal{M}} c_{a,b} \mathbb{E}^{p(t)} \int_0^{l_a} \int_0^{l_a} \left| \frac{l_a - x - x'}{l_a} \right| \frac{\partial}{\partial t} \left( \rho^b_a(x',t) \rho^b_a(x,t) \right) dx' dx
\]

\[
= - \sum_{(a,b) \in \mathcal{M}} c_{a,b} \mathbb{E}^{p(t)} \int_0^{l_a} \int_0^{l_a} \left| \frac{l_a - x - x'}{l_a} \right| \rho^b_a(x,t) \frac{\partial q^b_a(x',t)}{\partial x} dx' dx,
\]

where the last equality follows from (1). For each \( a \), the integrals inside the expectation can be decomposed as

\[
\frac{\partial q^b_a(0,t)}{\partial x} \int_0^{l_a} \left| \frac{l_a - x}{l_a} \right| \rho^b_a(x,t) dx + \frac{\partial q^b_a(l_a,t)}{\partial x} \int_0^{l_a} \left| \frac{x}{l_a} \right| \rho^b_a(x,t) dx
\]

\[
+ \int_0^{l_a} \int_0^{l_a} \left| \frac{l_a - x - x'}{l_a} \right| \rho^b_a(x,t) \frac{\partial q^b_a(x',t)}{\partial x} dx' dx.
\]
By assumption, we have that there exist constants $0 < K^* < \infty$ such that

$$\mathbb{E}^{\rho(t)} \mathcal{V}(\rho(t)) = \sum_{(a,b) \in \mathcal{M} : a \in \mathcal{A}^{\text{src}}} c_{a,b} \mathbb{E}^{\rho(t)} \left( \frac{dA_a^b(t)}{dt} \rho_a^b(t) - q_{a,\text{out}}^b(p_{a,\text{out}}(t)) \rho_a^b(t) \right)$$

$$+ \sum_{(a,b) \in \mathcal{M} : a \in \mathcal{A}^{\text{src}}} c_{a,b} \mathbb{E}^{\rho(t)} \left( q_{a,\text{in}}^b(p_{a,\text{in}}(t)) \int_0^{l_a} \left| \frac{l_a - x}{l_a} \right| \rho_a^b(x,t) dx \right)$$

$$- \sum_{(a,b) \in \mathcal{M} : a \in \mathcal{A}^{\text{src}}} c_{a,b} \mathbb{E}^{\rho(t)} \left( q_{a,\text{out}}^b(p_{a,\text{out}}(t)) \int_0^{l_a} \left| \frac{x}{l_a} \right| \rho_a^b(x,t) dx \right)$$

$$+ \sum_{(a,b) \in \mathcal{M} : a \in \mathcal{A}^{\text{src}}} c_{a,b} \left[ \mathbb{E}^{\rho(t)} \left( \int_0^{l_a} \left| \frac{x}{l_a} \right| \rho_a^b(x,t) dx \right) \right. \left. - \mathbb{E}^{\rho(t)} \left( \int_0^{l_a} \frac{l_a - x - x'}{l_a} \rho_a^b(x,t) \frac{\partial q_{a,\text{out}}^b(p_{a,\text{out}}(t))}{\partial x} dx' dx \right) \right]. \tag{29}$$

Appeal to Corollary 1 and noting that the third sum on the right-hand side is non-negative (and does not involve control variables), we have that there exists a constant $0 < \bar{K} < \infty$ such that

$$\mathbb{E}^{\rho(t)} \mathcal{V}(\rho(t)) \leq \bar{K} - \sum_{(a,b) \in \mathcal{M}} c_{a,b} \mathbb{E}^{\rho(t)} \left[ \int_0^{l_a} \left| \frac{x}{l_a} \right| \rho_a^b(x,t) q_{a,\text{out}}^b(p_{a,\text{out}}(t)) dx \right.$$

$$\left. - \int_0^{l_a} \left| \frac{l_a - x}{l_a} \right| \rho_a^b(x,t) \left( q_{a,\text{in}}^b(p_{a,\text{in}}(t)) + \frac{dA_a^b(t)}{dt} \right) dx \right], \tag{30}$$

where $q_{a,\text{in}}^b(p_{a,\text{in}}(t)) \equiv 0$ for $a \in \mathcal{A}^{\text{src}}$ and $dA_a^b(t)/dt \equiv 0$ for $a \in \mathcal{A}/\mathcal{A}^{\text{src}}$.

Also, for $a \in \mathcal{A}^{\text{src}}$ the traffic density is concentrated at $x = 0$, i.e., $\int_0^{l_a} \left| (l_a - x)/l_a \right| \rho_a^b(x,t) dx = \rho_a^b(t)$. Upon re-arranging terms on the right-hand side of (30) and utilizing the properties of conditional expectation, we have that

$$\mathbb{E}^{\rho(t)} \mathcal{V}(\rho(t)) \leq \bar{K} - \sum_{(a,b) \in \mathcal{M}} \int_0^{l_a} \rho_a(x,t) \mathbb{E}^{\rho(t)} \left[ c_{a,b} \left| \frac{x}{l_a} \right| q_{a,\text{out}}^b(p_{a,\text{out}}(t)) \right.$$

$$\left. - c_{a,b} \left| \frac{l_a - x}{l_a} \right| q_{a,\text{in}}^b(p_{a,\text{in}}(t)) - c_{a,b} \left| \frac{l_a - x}{l_a} \right| \frac{dA_a^b(t)}{dt} \right] dx. \tag{31}$$

By assumption, we have that there exist constants $0 < K^* < \infty$ and $\epsilon^* > 0$. 

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associated with the policy \( p^*(\cdot) \) such that
\[
\mathbb{E}^p(t) \mathcal{V}(\rho(t)) \leq K^* - \epsilon^* \sum_{(a,b) \in \mathcal{M}} \mathbb{E} \int_0^{l_a} |\rho^b_a(x, t)| \, dx.
\] (32)

By definition, we have for each \( t \geq 0 \) that
\[
\epsilon^* \sum_{(a,b) \in \mathcal{M}} \int_0^{l_a} \rho^b_a(x, t) \, dx \leq \max_{p \in \mathcal{P}} \sum_{(a,b) \in \mathcal{M}} \int_0^{l_a} \rho^b_a(x, t) \, dx \left[ c_{a,b} \frac{l_a - x}{l_a} q^b_{a,\text{out}}(p) - c_{a,b} \frac{l_a - x}{l_a} q^b_{a,\text{in}}(p) \right] \, dx.
\] (33)

Hence, setting \( K \equiv \max(K^*, \tilde{K}) \), we have by appeal to Lemma 1 that the control policy, \( p^*(\cdot) \in \mathcal{P} \), which maximizes the right-hand side of (33) for each \( t \geq 0 \) is also network stabilizing.

It remains to show that is equivalent to (26)-(9). We have from (33) that
\[
\arg \max_{p \in \mathcal{P}} \sum_{(a,b) \in \mathcal{M}} c_{a,b} \int_0^{l_a} \rho^b_a(x, t) \, dx \left[ \frac{l_a - x}{l_a} q^b_{a,\text{out}}(p) - \frac{l_a - x}{l_a} q^b_{a,\text{in}}(p) \right] \, dx
\] (34)
for each \( t \geq 0 \) is network stabilizing. (The last term on the right-hand side of (33) is dropped from the optimization problem since it constitutes an additive constant to the problem.) From (5) and (6), the objective function can be written as
\[
\sum_{(a,b) \in \mathcal{M}} c_{a,b} \int_0^{l_a} \rho^b_a(x, t) \, dx \left[ \frac{l_a - x}{l_a} q^b_{a,\text{out}}(p) - \frac{l_a - x}{l_a} q^b_{a,\text{in}}(p) \right] \, dx
\] (35)
which upon re-arranging terms and the orders of summation and integration becomes
\[
\sum_{(a,b) \in \mathcal{M}} \left( c_{a,b} \int_0^{l_a} \frac{x}{l_a} \rho^b_a(x, t) \, dx \right.
\]
\[
- \int_0^{l_b} \left( \sum_{c \in \Pi(a) \cap \Sigma(b) : (b,c) \in \mathcal{M}} c_{b,c} \pi_{b,c}(t) \rho^b_b(x, t) \, dx \right) \mathbb{E}^p(t) q^b_{a,\text{in}}(p).
\] (36)
Since intersection movements do not interact across nodes instantaneously, the optimization problem naturally decomposes by intersection. That is, maximizing (36) is equivalent to solving the $|\mathcal{N}|$ problems
\[
\arg \max_{p \in \mathcal{P}_n} \sum_{(a,b) \in \mathcal{M}_n} w_{a,b}(t) \mathbb{E}^{p(t)} q_{a,b}(p) \quad \forall n \in \mathcal{N},
\]
(37)
where
\[
w_{a,b}(t) \equiv c_{a,b} \int_0^{l_a} \left| \frac{x}{l_a} \right| \rho_{a}^{b}(x,t)dx - \int_0^{l_b} \left| \frac{l_b - x}{l_b} \right| \sum_{c \in \Sigma(b): (b,c) \in \mathcal{M}_c} c_{b,c} \pi_{b,c}(t) \rho_{b}^{c}(x,t)dx.
\]
(38)

This completes the proof. \qed

4. Simulation experiments

**Network description.** We utilize a microscopic traffic simulation network of a part of the city of Abu Dhabi in the United Arab Emirates (UAE) consisting of eleven signalized intersections but also containing unsignalized intermediate junctions. The network layout is shown in Fig. 6. We
compare PWBP control with three other control policies: fixed time, standard BP control, and CABP control. The fixed timing plans are optimized and include optimal offsets (i.e., signal coordination). BP, CABP, and PWBP are all implemented using a software interface. To simplify the experiments, we utilize a uniform demand at the boundaries, which we vary to gauge the capacity region of the network. Using a uniform (average) demand level allows us to use a single number (namely the demand) as a way to gauge the capacity region.

**Average network delay and network capacity region.** Fig. 7 shows the total network delay under different demand scenarios (ranging from 500 to 1800 veh/h on average) for BP, CABP, and PWBP using two types of phasing schemes: one with four phases (“4-phase” scheme) and a scheme with eight phases (“8-phase scheme”). The 4-phase scheme includes phases 1-4 in Fig. 5, while the 8-phase scheme is all eight phases in Fig. 5. We observe that 40 s/veh is a threshold delay, beyond which the delay increases dramatically. We can hence treat 40 s/veh as indicative of reaching the boundary of the capacity region. From Fig. 7, with the 8-phase scheme, we see that delays begin to increase rapidly at a higher average demand levels for the PWBP: 1620 veh/h for the 8-phase scheme vs. 1580 veh/h for the 4-phase scheme. However, this is not the case for BP and CABP control, since
they do not distinguish left-turning and through queues, which results in blocking at the points where roads widen (left-turn lane addition). This indicates that BP and CABP have a wider capacity region using a 4-phase scheme compared to the 8-phase scheme. All subsequent experiments use an 8-phase scheme with PWBP and 4-phase schemes with BP and CABP. The demands at which delays begin to increase quickly for fixed signal timing, BP, CABP, and PWBP are 1225, 1555, 1570, and 1620 veh/h, respectively. Fig. 8 shows a comparison of network delays for the four control policies under varying demands.

![Network Delays](image)

Fig. 8: Network delays associated with different control policies.

**Congestion propagation.** In the following experiments, we set the demand levels to the deterioration bounds of the control policies and compare how congestion levels propagate over time. Since the deterioration bounds for BP and CABP are close, we just use CABP’s bound (1570 veh/h); we, hence, compare three demand scenarios. Fig. 9 – 10 show how the speeds of all vehicles within the network are distributed under demand levels 1225, 1570 and 1620 veh/h. The horizontal axes in these figures are time and the vertical axes are percentage of vehicles traveling at or below the color-coded speeds. Under the different demand levels, the network eventually becomes grid-locked (at different levels for the different control policies). Specifically, it takes about four hours until total network gridlock under a fixing timing plan when the demand reaches 1225 veh/h, under BP and CABP it takes approximately six hours (at 1570 veh/h) until gridlock, and for PWBP, it takes approximately seven hours. This indicates that PWBP is
more resilient than the other policies. **Fig. 11** shows how the total number
Fig. 10: Network speed evolution, (a) CABP under a demand level of 1570 veh/h, (b) PWBP under a demand level of 1570 veh/h, (c) fixed timing under a demand level of 1620 veh/h, and (d) BP under a demand level of 1620 veh/h, (e) CABP under a demand level of 1620 veh/h, (f) PWBP under a demand level of 1620 veh/h.
of vehicles (stuck) in the network evolves with time.

![Graph showing the evolution of total numbers of vehicles in the network under different control policies and demand levels.](image)

**Fig. 11:** Evolution of total numbers of vehicles in the network under different control policies and demand levels of (a) 1225 veh/h, (b) 1570 veh/h, and (c) 1620 veh/h.
Recoverability from congestion. Fig. 12 shows how different control policies recover from congestion. The total simulation time is eight hours,
the time interval from $t = 120$ min to $t = 240$ min is set as a congested period, during which demand levels are set to the deterioration bounds. We set a demand of 1000 veh/h for the remainder of the eight-hour simulation time. Fig. 12a, b and c only differ in the demand levels during the congested period. The congested period demand levels are 1225, 1570 and 1620 veh/h in Fig. 12a, b and c, respectively. According to Fig. 12, for all tested scenarios, PWBP outperforms the other three control policies in terms of both delay and recovery time. Even when the peak demand reaches 1620 veh/h, PWBP only needs 30 min to recover from the congestion, while fixed timing needs about 90 min to recover with a peak demand of 1225 veh/h. Note that when the peak demand reaches 1570 and 1620 veh/h, the delay levels under fixed timing becomes too high and hence cannot be shown in Fig. 12b and c. We also see that using fixed timing, the network does not eventually recover from congestion.

Response to an incident. We investigate the performance of PWBP in the presence of an incident located at the yellow spot in Fig. 6. The incident is located half-way between intersections A and B, along a 3-lanes arc. We test scenarios where one lane and two lanes are blocked for a duration of one and two hours, and under different demand levels. Fig. 13 shows the results for one-lane blocked cases when demand is 1500 veh/h. Fixed timing is not included here since 1500 veh/h is beyond its capacity region.
and the delays will only increase without bound. Dotted lines represent the non-incident cases, while dashed and solid lines represent the incident cases with one and two hour durations, respectively. The incident starts at the 60th min in both cases. When the incident duration is one hour, we see that the network recovers within 30 minutes after the incident is cleared under BP, CABP and PWBP. However, when the incident duration is two hours, PWBP only needs one hour to completely recover, while congestion in the network persists for significantly longer under BP and CABP: the effects of the incident are still felt in the network three hours after the incident is cleared (compared to the no-incident scenarios).

Fig. 14 shows the two-lanes-blocked cases when demand is 1200 veh/h. The network fails to recover under fixed timing, BP and CABP control when the incident blocks two of the three lanes. The delays increase sharply and the whole network becomes gridlocked. In contrast, using PWBP control the incident hardly has any impact at all on network delay.

The reason of the performance difference between BP, CABP and PWBP originates from how the model deals with scenarios in Fig. 1b and Fig. 1c. With an incident located half-way between intersections A to B, the incident results in congested conditions (queueing) between the incident location and intersection A and low volume traffic between incident location and intersection B. When the queue spills back to intersection A (similar to Fig. 1b), PWBP will forbid the movements from A to B, while BP and

Fig. 14: Delays associated with different policies with two lanes blocked by the incident under a demand level of 1200 veh/h.
CABP fail to capture the spillback dynamics. In addition, PWBP does not allocate green time at intersection B to the movement from A when there are actually no vehicle near the stop line (similar to Fig. 1c), while BP and CABP may still allocate green time to this movement.

5. Conclusion and outlook

Backpressure (BP) based intersection control is a control policy that was originally developed for communications networks. Many of the assumptions made in the original theory were adopted in the BP applications to traffic networks despite them not being applicable to vehicular traffic. Specifically, infinite arc capacities, point queues, independence of commodities (turning movements), and there being no analogue for start-up lost times in communications networks. These are critical features in intersection control. To accommodate these features, we develop a backpressure control technique that is based on macroscopic traffic flow, which we refer to as position-weighted backpressure (PWBP). PWBP considers the spatial distribution of vehicles when calculating the backpressure weights.

The proposed PWBP control policy is tested using a microscopic traffic simulation model of an eleven-intersection network in Abu Dhabi. Comparisons against coordinated and optimized fixed signal timing, standard BP, and a capacity-aware variant of BP (CABP) were carried out. The results indicate that PWBP can accommodate higher demand levels than the other three control policies and outperforms them in terms of total network delay, congestion propagation speed, recoverability from heavy congestion, and response to an incident.

This paper has focused on prioritization of movements at network intersections. As a possible future research direction, this can be extended to include real-time route guidance. Another possible avenue for future research is a combined perimeter/interior control policy. Perimeter control (Yang et al., 2017; Chiabaut et al., 2018; Ortigosa et al., 2014; Ambühl et al., 2018; H. et al., 2018; Yang et al., 2018) is emerging as a useful tool for network control at a macroscopic level. A study of the trade-offs between the capacity region of an intersection control policy and perimeter control could serve as a powerful network-wide control tool.

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