Numerical and Experimental Research on Nonlinear Vibration of High Line Speed Friction Pairs of Vehicle Clutch

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Abstract. By analyzing friction pairs surface structural characteristics and using theory of contact mechanics, contact mathematic model of microscopic units of thermal distorsion friction pairs is built. Using mathematical statistics and normalization method, microscopic model can be transferred to macroscopic mathematic model, and then the elastic contact characteristics of friction pairs can be calculated. When Kelvin-Voigt model was inputted into elastic contact characteristics, and viscoelastic contact differential operator was led in equation, viscoelastic contact mathematic model that includes the relation of stress to strain is built. So colligating above all the simulation results, axial vibration mathematic model is built, and the nonlinear vibration characteristics and curves of high line speed friction pairs are obtained by simulation. For verifying simulation results, nonlinear vibration experiment of friction pairs of clutch is tested on conditions of various rotational speed. Result shows that the accuracy of the model is 87%, which is meaningful on both theory and engineering design. It promotes study on vibration characteristics to improve friction pairs serviceability and life.

1. Introduction
In the case of high line speed, the friction pairs of shift clutch of vehicle transmission system will produce thermal deformation in the process of binding separation, which will cause vibration and seriously affect the performance of friction element and others related components [1-3]. The thermal deformation of the friction pairs and its vibration are essential to the design of the clutch of vehicle power-shift steering transmission device [4]. Therefore, it is of important theoretical and engineering significance to study deeply on the vibration characteristics of the friction pairs at high line speed, and to put forward an effective vibration control method to improve the service life of the friction pairs.

By applying the theory and technology of surface structure characteristics and contact mechanics of friction pairs, from the viewpoint of micro mechanics, a microscopic normal element contact model of thermal deformation is established. Then making full use of mathematical statistics and normalization methods, the transformation relationship between micro and macro is established. Thus, a macro contact mathematical model between two pairs of frictional pairs is obtained. At the same time, with introducing Kelvin-Voigt model and increasing the viscoelastic contact differential operator into mathematical model, the viscoelastic contact property including stress and strain is constructed, and a viscoelastic contact mathematic model of the friction pairs can be obtained. At last, the mathematical model of nonlinear vibration is obtained by mechanical analysis, and its vibration characteristics are simulated. The nonlinear vibration characteristics are tested under different rotational speed and lubricating oil, the accuracy of the simulation model is verified.
2. Mathematical model of nonlinear vibration in sliding process

2.1. Dynamic model

A wet clutch friction pair of vehicle transmission system is composed of alternating contact the friction disk and the toroidal dual steel plate. The vibration model of friction pairs of clutch may be divided into several sub vibration models [5,6], which means, the model is made up of several sub vibration models in series in Figure 1, and each sub vibration model is related to each other by the moment M and the axial force Fn, and each sub model contains only one pair of friction pairs. Based on the thesis of dynamics and contact mechanics, sub dynamic model can be shown in Figure 2. There, function Fnc between two contact faces is normal contact force which is made up of scalar sum of elastic contact force and viscoelastic contact force, and function Fkn is structural stiffness elastic force, and Fc is damping force.

According to the actual working conditions of experiment, the initial boundary condition of model is set to include an annular dual steel plate with initial static state and a friction disk with a fixed angular velocity. For making the results to be comparable between simulation and experiment, that is to ensure the consistency of boundary conditions of simulation and test, we suppose that a dual steel plate that would be pressed by the axial force in the friction sliding process, would not be able to rotate around the central line, only to be vibration in the normal direction of the micro displacement[7]. Whereas, in process of sliding, the friction disk not only can rotate, but also the micro displacement vibration of rotating disk can be carried out along the axis direction in the compression process. On condition of axial pressure, when friction pairs are completely combined, the friction disk of model will be fixed in the normal position and cannot rotate around the central line.

\[
\begin{align*}
F_{kn} &= F_p - K_n x - C_n \dot{x} - F_{nc} \\
F_{nc} &= F_e
\end{align*}
\]

(1)

There, \(F_p\) is the preload normal force, \(F_{kn}=F_p K_{es}\), \(F_{nc}=C_n \dot{x}\), \(x\) is axial direction. From formula (1), it can be seen that the contact force vector \(F_{nc}\) is the key to obtain the exact solution of the equation.

2.2. Normal contact model of micro bulge

At high line speed, the surfaces of friction pairs which has contacted each other will produce local spots at high temperature and high pressure [8,9], and its structure shows the shape of micro bulge as a result of thermoelastic deformation [10]. Therefore, contact model is shown as the form of thermal deformation micro bulge contact. From the contact region center to the two surfaces direction along the contact normal, not only two surfaces of contact micro bulges are interactive, but also the vector direction of Hertz contact force is non-orthogonal with the direction of surface, and its direction is \(\alpha\) angle to normal. According to the Hertz contact force formula, the contact force of micro bulge is

\[
f = \frac{4}{3} E\beta(r)^{3/2} w^{3/2}
\]

(2)
There, $E$ is the mixed elastic modulus, $\beta(r)$ is Mixed curvature radius, $w$ is the interference in the contact region along the contact normal from the center to the surface, $r$ is the offset distance of the two contact micro bulges in the tangent direction. Thus, when $r$ is 0, two micro bulges are in interference with each other along the normal line from the center to the plane; when $r$ is not 0, two micro bulges are interference along a diagonal line. When two micro bulges curve are in tangent, $r$ is the maximum.

Similar to the equivalent curvature in [11], the interference occurs at the center of the contact position, approximately on the middle of the cross of the undeformed micro bulge. By calculating and deriving, the curvature radius and interference at any point of the micro bulge curve may be expressed as

$$
\beta(r) = \beta\left(1 + \frac{r^2}{4\beta^2}\right)^{\frac{1}{2}}
$$

$$
w = \left(z_1 + z_2 - h_0 - \frac{r^2}{4\beta}\right) \left(1 + \frac{r^2}{4\beta^2}\right)^{-\frac{1}{2}}
$$

In equation (2), the micro bulge contact force is defined as extending along the normal direction to the contact region, and mainly contains normal and tangent elements. Let $z = z_1 + z_2$, which is defined as the sum of the height of two micro bulges. When equation (3) and (4) are substituted into equation (2), the contact force $f_n$ of normal element of micro bulge can be expressed as

$$
f_n = \frac{4}{3}E\beta^{\frac{1}{2}} \left(z - h_0 - \frac{r^2}{4\beta}\right) \left(1 + \frac{r^2}{4\beta^2}\right)^{-\frac{1}{2}}
$$

2.3. Normal elastic contact mathematical model

In the process of frictional pair sliding vibration, all normal units in variable contact force $dF_{ns}$ can be considered as parallel. Therefore, the normal resultant force of a friction surface to another friction surface can be obtained by geometric addition of the mathematical statistics mean value method. It is assumed that the sum of the heights of two surface micro bulges obeys the Gauss distribution [12], that is, $z \sim N(h, \sigma)$. Taking the standard deviation as the normalized parameter and normalizing all the parameters, the normal surface elastic contact force distribution function $F_{ne}$ of friction pair is

$$
F_{ne}(h, \beta) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{z^2}{2\sigma^2}} drdz
$$

In here, $\Sigma$ represents the whole surface contact region of friction pair, and axial range is $(h-\infty)$, and circumferential range is $(0-\sqrt{4\beta}(s-h))$, and $\lambda$ represents the number of micro bulge per unit nominal area on the contact surface, and the contact nominal area $An$ is the geometric function $\pi(R_0^2 - R_i^2)$, $R_i$ and $R_0$ represent the inside radius and outside radius of the friction disc. Considering all above, the expression of normal elastic contact force of friction pairs can be expressed as

$$
F_{ne}(h, \beta) = \frac{8\pi E}{3\sqrt{2\pi}} \beta^{\frac{1}{2}} \lambda^2 A_n \sigma^4 \int_{-\infty}^{\infty} \int_{0}^{\sqrt{4\beta}(s-h)} e^{-\frac{r^2}{2\sigma^2}} r^2 drdz
$$

2.4. Normal viscoelastic contact mathematical model

In the relative sliding process of friction pairs, there are two main factors that affect the normal viscoelastic contact characteristics [13,14]. One is the changing rate $\dot{h}$ of clearance, the other is the relative sliding velocity $\dot{r}$. For the thermal deformable micro bulge, if two shoulders of the vibration model are in contact with each other, the normal viscoelastic contact force $F_{nv}$ of model can be expressed as

$$
F_{nv} = F_{nvh} + F_{nvr}
$$

There, $F_{nvh}$ and $F_{nvr}$ represent the normal viscoelastic contact force related to dimensionless parameters $h$ and $\dot{r}$. In our study of contact friction model, on the basis of elastic modulus $E$, a loss
factor $E_v$ is added to model, and a viscoelastic contact property with relation of stress and strain is constructed. Then the elastic modulus can be changed to

$$E' = E + E \frac{\partial}{\partial t} = E \left(1 + \varepsilon_v \frac{\partial}{\partial t}\right)$$

(9)

Here, $\varepsilon_v$ is the coefficient of viscoelastic contact differential operator. By substituting the upper formula into the equation (2), the Hertz contact force, which consists of elastic force and viscoelastic force, can be obtained. For axial vibration, the differential operator part in the contact force equation should be the normal viscoelastic contact force.

### 3. Simulation on vibration characteristic

In the simulation process of vibration characteristics of friction pairs, the normalization method is still used to solve the vibration equation. When the normal elastic contact properties, viscoelastic contact characteristics (including the clearance rate and friction speed characteristics), the preload characteristics are substituted into kinetic equation, vibration equation of the friction slipping can be converted to

$$\ddot{x} + 2\zeta_0 \omega_0 \dot{x} + \omega_0^2 x = f_p - f_{ne} - f_{nvh} - f_{nvr}$$

(10)

There, $\omega_0$ is circular frequency, $\zeta$ is damping ratio, $f_i$ (i=p, ne, nvh, nvr) is normalized contact force or load. For equation (12), in once process of bonding friction pair, the numerical simulation of vibration characteristics of friction disc is carried out by the Runge-Kutta method. When the dimensionless parameter $h=1$ and $\beta=100$, the results of simulation are shown in Figure 3.

![Time Simulation Curve](image1)

![Amplitude Frequency Curve](image2)

Figure 3. Simulation Characteristics

From simulation results of Figure 3(a), we can know that, under the action of force, the friction pairs shock reciprocally, and under the action of damping, the vibration system tends to be stable. At this time, the system converges to the initial origin and the vibration amplitudes is between -1mm and 1mm. And from Figure 3(b), in the process of sliding of the friction pairs, the vibration energy is mainly concentrated in the first two orders of resonance, that is less than 100 Hz.

### 4. Experiment test of vibration

The object of this work is to test the vibration of a 405 wet shifting clutch of heavy load vehicle during the process of bonding/separating under different working conditions. The power end and loading end of testbench are motor control units, and its maximum power is 315 KW. The left of test table is axially loaded with force, and keeps synchronous motion with outer hub of clutch (its inner teeth mesh with outer teeth of dual steel plate). The right of test table is rotating power end with maximum speed of 4000 r/min, and keeps synchronous rotation rate with inner hub of clutch. Because the evaluation index of experiment is that the line speed must be more than 70 m/s, only when the rotation rate is greater than 3000 r/min, the experiment will begin to collect data. The experimental results are shown in Figure 4.
Figure 4. Test of Two Bonding / Separating Process (3000 r/min)

The amplitude frequency characteristics from 3000-3400r/min is shown in figure 5.

![Amplitude Frequency Characteristics](image_url)

(a) 3000 r/min  
(b) 3100 r/min  
(c) 3300 r/min  
(d) 3400 r/min

Figure 5. Amplitude Frequency Characteristics

According to the amplitude frequency characteristics of above curves, the amplitudes are between -1.5 and 1.5mm, and the vibration energy is basically concentrated in the first two order resonance frequencies. By PSD calculation, the first two order resonance energy accounts from 86 to 88 percent of the total energy, and the frequency occurring maximum amplitude is from 38.9 to 47.71 Hz, and the frequency response is from 32 to 41dB.
The amplitude frequency characteristics at different speeds are represented by curves at the resonance points, as shown in Figure 6 and 7.

As can be seen from Figure 6, the resonant frequencies of each order at different speeds are basically in the range of respective resonance band. However, with the increase of order, the deviation increases gradually. In the first order, maximum deviation is 8.79 Hz, and maximum deviation of the second order is 17.59 Hz, and 22.6 Hz in the third order, and 35.2 Hz in the fourth order. In Figure 7, the response amplitude is gradually decreased with the increase of the order, and the maximum deviation occurs in the first order. And the deviation is 9.63 dB, and 0.297 dB in the second order, 0.599 dB in the third order, 0.647 dB in the fourth order.

5. Model accuracy verification
The simulated resonant frequency and amplitude frequency response are compared with the experimental data, and the results are shown in Figure 8 and 9.

As can be seen from Figure 8 and 9, the simulation results are all within the range of experimental test, and it further proves that the simulation model is reliable. The deviation rate of each order of experiment and simulation is calculated respectively, and the maximum deviation rate is obtained. The result shows that the accuracy of the simulation model is 87%.

6. Conclusions
Based on the theoretical and experimental analysis of the vibration characteristics of high line speed frictional pairs, the following conclusions can be obtained:
(1) In the process of bonding/separating, the vibration energy is mainly concentrated in the first two orders resonance, that is, resonant frequency is less than 100Hz, and the first two orders resonance energy accounts for about 87% of the total energy;

(2) The frequency of the maximum amplitude is between 38.9 and 47.71 Hz. The frequency response is from 32 to 41dB. With the increase of frequency, the amplitude decreases at the rate of about 42 dB per decade;

(3) The frequency doubling of each order is 1.89, 3.11 and 5.1, respectively, and the maximum error rate is no more than 7.3%;

(4) Comparing the simulation with experimental results, the accuracy of simulation model is 87%, which shows that the simulation model is reliable.

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