Higgs boson bounds in non-minimal
supersymmetric standard models. ∗

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Abstract

In the minimal supersymmetric standard model (MSSM),
when radiative corrections are included, the mass of
the $CP = +1$ lightest Higgs boson is bounded by
$\sim 110 \text{ GeV}$ for $m_t < 150 \text{ GeV}$ and a scale of supersymmetry breaking $\sim 1 \text{ TeV}$. In non-minimal supersymmetric standard models (NMSSM) upper bounds
on the mass of the corresponding scalar Higgs boson
arise if the theory is required to remain perturbative up
to scales $\gg G_F^{-1/2}$. We have computed those bounds
for two illustrative NMSSM: i) A model with an arbitrary number of gauge singlets; ii) A model with three $SU(2)_L$ triplets with $Y = 0, \pm 1$. We have integrated numerically the corresponding renormalization group
equations (RGE), including the top and bottom quark
Yukawa couplings, and added one-loop radiative corrections. For $m_t > 91 \text{ GeV}$ the absolute bounds are
$\sim 140 \text{ GeV}$ for both models.

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1 Introduction

One of the most constraining features of the MSSM is the existence of an absolute upper bound on the tree-level mass of the lightest scalar ($CP = +1$) Higgs boson

$$m_h \leq m_Z | \cos 2\beta |,$$

where $\tan \beta \equiv v_2/v_1$, $v_i \equiv \langle H_i^o \rangle$. Therefore a negative result in the Higgs search at LEP-200 would seem to exclude the MSSM making its search at NLC/LHC/SSC unnecessary. However, before ruling out phenomenological supersymmetry we should consider all possible effects spoiling the relation (1): radiative corrections and the introduction of models with an enlarged Higgs sector. As we will see both effects allow to overcome the bound (1) but nevertheless there are still very constraining bounds in general models.

2 Radiative Corrections

They have been computed by different groups using different methods: standard diagrammatic techniques\(^2\), the one-loop effective potential\(^3\) and the RGE approach\(^4,5\). The latter approach is reliable provided that $\Lambda_S^2/m_W^2 \gg 1$, where $\Lambda_S$ is the
scale of supersymmetry breaking, since it amounts to a resum-
mation of all leading logarithms in the effective potential.

Radiative corrections depend on tan $\beta$, $m_t$ and $\Lambda_S$. In the
MSSM the absolute upper bound on $m_h$, for $|\cos 2\beta| = 1$,
is modified by radiative corrections to a $\Lambda_S$ and $m_t$-dependent
bound. For $\Lambda_S = 1$ TeV we have parametrized it as $^5$:

$$m_h(GeV) = 111 + 80x + 34x^2,$$

(2)

where $x = m_t/150$ GeV $- 1$.

Radiative corrections can be sizeable, and even larger than
the tree-level mass, thus putting doubts on the validity of the
perturbative expansion. The RGE approach has been used to
study the stability of one-loop versus two-loop corrections $^5$. It
has been estimated analytically, and checked numerically, that

$$\frac{\Delta^{2-loop}m_h^2}{\Delta^{1-loop}m_h^2} \sim -0.1 \frac{\alpha}{\pi \sin^2 \theta_W} \left( \frac{m_t}{m_W} \right)^2,$$

(3)

which guaranties the validity of perturbative expansion. From
eq (2) we see that for $m_t < 150$ GeV, $m_h < 111$ GeV which is
the range allowed by radiative corrections in the MSSM.
3 Non-minimal Supersymmetric Standard Models

The tree-level bound (1) does no longer hold in supersymmetric models with extra Higgs fields, i.e. in NMSSM. We will study in this section Higgs bounds in a general class of NMSSM.

The first (obvious) enlargement of the Higgs sector consists in adding pairs of Higgs doublets $H^{(j)}_1, H^{(j)}_2, j = 1, \ldots, N$. These models have been analyzed and their lightest scalar Higgs boson shown to have the tree-level bound (1).

Consider now NMSSM with Higgs doublets $H_1, H_2$ and neutral scalar fields $N^{(i)}_{12}, N^{(i)}_{11}, N^{(j)}_{22}$ (either $SU(2)_L \times U(1)_Y$ singlets or making part of higher dimensional representations) with a cubic superpotential $f = g + f_{YUK}$

$$g = \bar{\lambda} \cdot \vec{N}_{12} H_1^0 H_2^0 + \sum_{i=1}^2 \vec{\chi}_i \cdot \vec{N}_{ii}(H_i^0)^2, \quad (4)$$

where $\bar{\lambda} \cdot \vec{N} \equiv \sum_j \lambda_j N^{(j)}$ and $f_{YUK}$ contains all Yukawa couplings giving mass to fermions. Then, the lightest scalar Higgs boson mass has an upper bound given by

$$\frac{m_h^2}{v^2} \leq \frac{1}{2} (g^2 + g'^2) \cos^2 2\beta + \bar{\lambda}^2 \sin^2 2\beta + \bar{\chi}_1^2 \cos^4 \beta + \bar{\chi}_2^2 \sin^4 \beta, \quad (5)$$

where $g, g'$ are the $SU(2)_L \times U(1)_Y$ couplings and $v^2 \equiv v_1^2 + v_2^2$. 

3
The bound for the MSSM is recovered from (5) when $\vec{\lambda} = \vec{\chi}_1 = \vec{\chi}_2 = 0$. However in NMSSM some of the Yukawa couplings in (4) can be non-zero. In that case the upper bound on the lightest scalar Higgs boson mass comes from the requirement that the supersymmetric theory remains perturbative up to some scale $\Lambda$, in the energy range where the theory holds.

We will keep in $f_{YUK}$ the top and bottom quark Yukawa couplings, \textit{i.e.}

$$f_{YUK} = h_t Q \cdot H_2 U^c + h_b Q \cdot H_1 D^c,$$

with boundary conditions

$$h_t = \frac{g}{\sqrt{2}} \frac{m_t}{m_W} (1 + \cot^2 \beta)^{1/2}, \quad h_b = \frac{g}{\sqrt{2}} \frac{m_b}{m_W} (1 + \tan^2 \beta)^{1/2}.$$  

$m_t$ in (7) will be considered as a variable while $h_b$ is fixed by $m_b$, which is taken to be $m_b(2m_b) = 5 \text{ GeV}$. For $\tan \beta \gg 1$, $h_b$ can become important. In particular it is comparable to $h_t$ for $\tan \beta \sim m_t/m_b$. $h_\tau$ will be neglected since it is given by $h_b(m_\tau/m_b)$ for all values of $\tan \beta$. The cubic $g$-superpotential in (4) and so the tree-level mass in (5) are model dependent. The latter depends on the couplings $\vec{\lambda}, \vec{\chi}_i$ allowed by the perturbative requirement.

Radiative corrections will be included using the RGE approach \textsuperscript{4}. This procedure is universal in the sense of assuming
that the standard model holds below $\Lambda_S$ and the supersymmetric theory beyond $\Lambda_S$. We have taken here $\Lambda_S = 1 \, TeV$. The radiative mass $\Delta m_r^2(\beta)$ is $\beta$-dependent and has to be added to (5). In the following we will consider two generic NMSSM.

3.1 NMSSM with an arbitrary number of singlets

These models are defined by a Higgs sector containing $H_1$, $H_2$ and $n$ singlets $S_i$ ($i = 1, \ldots, n$) with a cubic superpotential

$$g = \bar{\lambda} \cdot \bar{S}H_1 \cdot H_2 + \frac{1}{6} \sum_{i,j,k} \chi_{ijk} S_i S_j S_k.$$  \hspace{1cm} (8)

The model with $n = 1$ has been studied in great detail in the literature 6–9. The tree-level upper bound on the mass of the lightest scalar Higgs boson for the case of arbitrary $n$ can be written as7,10:

$$m^2_h \leq (\cos^2 2\beta + \frac{2\bar{\lambda}^2 \cos^2 \theta_W}{g^2} \sin^2 2\beta) m_Z^2.$$  \hspace{1cm} (9)

The relevant one-loop RGE are

$$4\pi^2 \ddot{\lambda}^2 = \{-\frac{3}{2}g^2 - \frac{1}{2}g'^2 + 2\lambda^2 + \frac{3}{2}(h_t^2 + h_b^2)\} \dot{\lambda}^2 + \frac{1}{4} \lambda_i \lambda_j tr(M_i M_j),$$

$$8\pi^2 \ddot{M}_k = 3\lambda_k \bar{M} \cdot \dot{\lambda} + \frac{3}{4} tr(\bar{M} \cdot M_k) \cdot \bar{M},$$

$$8\pi^2 \ddot{h}_t = \{-\frac{3}{2}g^2 - \frac{13}{18}g'^2 - \frac{8}{3}g_s^2 + \frac{1}{2} \bar{\lambda}^2 + 3h_t^2 + \frac{1}{2} h_b^2\} h_t,$$

$$8\pi^2 \ddot{h}_b = \{-\frac{3}{2}g^2 - \frac{7}{18}g'^2 - \frac{8}{3}g_s^2 + \frac{1}{2} \bar{\lambda}^2 + \frac{1}{2} h_t^2 + 3h_b^2\} h_b.$$
\[ 16\pi^2 \dot{g} = g^3, \]
\[ 16\pi^2 \dot{g'} = 11g'^3, \]
\[ 16\pi^2 \dot{g}_s = -3g_s^3, \]

where \( g_s \) is the \( SU(3) \) gauge coupling and \( (M_k)_{ij} \equiv \chi_{ijk} \). The key observation to maximize \( \tilde{\lambda}^2 \), and so the bound (9), is the property \( \lambda_i \lambda_j tr(M_i M_j) \geq 0 \) which follows trivially from the definition of \( M_j \).

Assuming that the theory remains perturbative up to the scale \( \Lambda = 10^{17} \) GeV, integrating numerically the RGE and including radiative corrections for \( \Lambda_S = 1 \) TeV we find the upper bound shown in Fig. 1 in the \( (m_h, m_t) \)-plane.

We see from Fig. 1 that the detailed functional dependence of \( m_h \) on \( m_t \) is parametrized by the value of \( \tan \beta \). The dashed curve where the solid lines stop correspond to values of \( m_t \) such that the Yukawa coupling \( h_t \) becomes non-perturbative. (For \( \tan \beta > 30 \) the corresponding lines would follow very close to the \( \tan \beta = 20 \) curve in Fig. 1, but stopping at lower values of \( m_t \) because of the large values of \( h_b \).) The dotted curve on the top of the figure is the enveloping for all values of \( \tan \beta \) and can therefore be considered as the absolute upper bound. Of course once the top quark will be discovered, and its mass known, the bound on \( m_h \), and its \( \tan \beta \)-dependence, will become
more dramatic. For instance, for \( \tan \beta \gg 1 \) the bound becomes undistinguishable with that in the MSSM. Using the constraints \( \tan \beta \geq 1 \) and \( m_t \geq 91 \text{ GeV} \) we obtain from Fig. 1, \( m_h \leq 140 \text{ GeV} \).

### 3.2 NMSSM with \( Y = 0, \pm 1 \) \( SU(2)_L \) triplets

This model can be considered as an example where \( \vec{\chi}_i \neq 0 \) (\( i = 1, 2 \)) in (4). It is the supersymmetric extension of a non-supersymmetric standard model with Higgs triplets\(^{11}\) which do not break the custodial symmetry at the tree-level provided there is a particular relation between the vacuum expectation values of their neutral components.

The Higgs content is \( H_1, H_2 \) and \( \Sigma, \Psi_1, \Psi_2 \), which are \( SU(2)_L \) triplets with hypercharges 0, \( \pm 1 \), respectively, field content

\[
\Sigma = \begin{pmatrix}
\frac{\xi^o}{\sqrt{2}} & -\xi_2^+ \\
\xi^- & -\frac{\xi^o}{\sqrt{2}}
\end{pmatrix},
\]

\[
\Psi_1 = \begin{pmatrix}
\psi_1^+ / \sqrt{2} & -\psi_1^{1+} \\
\psi_1^o & -\psi_1^{+} / \sqrt{2}
\end{pmatrix},
\Psi_2 = \begin{pmatrix}
\psi_2^- & -\psi_2^o \\
\psi_2^- & -\psi_2^- / \sqrt{2}
\end{pmatrix},
\]

and a cubic superpotential

\[
g = \lambda_1 H_1 \cdot \Sigma H_2 + \lambda_2 tr \Sigma \Psi_1 \Psi_2 + \chi_1 H_1 \cdot \Psi_1 H_1 + \chi_2 H_2 \cdot \Psi_2 H_2. \tag{12}
\]
The tree-level bound on the mass of the lightest scalar Higgs boson can be written as:

\[ m_h^2 \leq \left\{ \cos^2 2\beta + \left[ \lambda_1^2 \sin^2 2\beta + 2(\chi_1^2 \cos^4 \beta + \chi_2^2 \sin^4 \beta) \right] \frac{\cos^2 \theta_W}{g^2} \right\} m_Z^2. \]

(13)

The one-loop RGE are:

\[ 8\pi^2 \lambda_1 = \{-7/2 g^2 - 1/2 g'^2 + 2\lambda_1^2 + 1/2 \lambda_2^2 + 3\chi_1^2 + 3\chi_2^2 + 3(h_t^2 + h_b^2)\} \lambda_1, \]

\[ 8\pi^2 \lambda_2 = \{-6g^2 - 2g'^2 + 1/2 \lambda_1^2 + 3/2 \lambda_2^2 + \chi_1^2 + \chi_2^2\} \lambda_2, \]

\[ 8\pi^2 \chi_1 = \{-7/2 g^2 - 3/2 g'^2 + 3/2 \lambda_1^2 + 1/2 \lambda_2^2 + 7\chi_1^2 + 3h_b^2\} \chi_1, \]

\[ 8\pi^2 \chi_2 = \{-7/2 g^2 - 3/2 g'^2 + 3/2 \lambda_1^2 + 1/2 \lambda_2^2 + 7\chi_2^2 + 3h_t^2\} \chi_2, \]

\[ 8\pi^2 h_t = \{-3/2 g^2 - 1/18 g'^2 - 8/3 g_s^2 + 3/4 \lambda_1^2 + 3\chi_2^2 + 3h_t^2 + 1/2 h_b^2\} h_t, \]

\[ 8\pi^2 h_b = \{-3/2 g^2 - 7/18 g'^2 - 8/3 g_s^2 + 3/4 \lambda_1^2 + 3\chi_1^2 + 3h_b^2 + 1/2 h_t^2\} h_b, \]

\[ 16\pi^2 g = 7g^3, \]

\[ 16\pi^2 g' = 17g'^3, \]

\[ 16\pi^2 g_s = -3g_s^3. \]

(14)

We have integrated numerically the RGE assuming that the theory remains perturbative up to the scale \( \Lambda = 10^{14} \text{GeV} \) (the scale where the gauge coupling constants become non-perturbative) and included the radiative corrections for \( \Lambda_S = 1 \text{ TeV} \). We find
the upper bound on $m_h$ as a function of $m_t$ for $\tan\beta \leq 20$ in Fig. 2, and for $\tan\beta > 20$ in Fig. 3. The dashed curves correspond again to the region where $h_t$ becomes non-perturbative. We see that the maximum upper bound, $m_h \sim 140$ GeV, corresponds to values of $\tan\beta$ much larger than one. For $\tan\beta > 50$ $h_b$ becomes non-perturbative, and the corresponding curves would fall, very rapidly with increasing $m_t$. 
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Figure Captions

**Fig. 1** Upper bounds on the lightest scalar Higgs boson in NMSSM with singlets.

**Fig. 2** Upper bounds on the lightest scalar Higgs boson in NMSSM with $Y = 0, \pm 1$ triplets for $\tan \beta \leq 20$.

**Fig. 3** Upper bounds on the lightest scalar Higgs boson in NMSSM with $Y = 0, \pm 1$ triplets for $\tan \beta \geq 30$. 