Analytic Impulsive Time-Domain UTD Coefficient for Pyramid-Vertex Diffraction

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Abstract—We introduce our solution for the diffraction of a pulsed ray field, with spherical wavefront, by the vertex (tip) of a pyramid. Within the Uniform Geometrical Theory of Diffraction (UTD) we improve the time domain (TD) solutions available in the literature by introducing the field diffracted by a perfectly conducting faceted structure made by interconnected flat plates, for source and observation points at finite distance from the tip. The proposed closed form expression for an exciting impulsive source has been calculated by employing the one-sided inverse Fourier transform of the frequency domain solution. The solution obtained is able to compensate for the discontinuities of the field predicted by standard TD-UTD, i.e., time domain geometrical optics (TD-GO) combined with the TD-UTD wedge singly diffracted rays.

Index Terms—Asymptotic diffraction theory, electromagnetic diffraction, electromagnetic transient scattering, geometrical theory of diffraction, time domain (TD) uniform theory of diffraction, transient propagation, transient scattering, vertex diffraction.

I. INTRODUCTION

Transient wave phenomena have been extensively studied in recent years primarily due to the increasing interest in ultra-wide band (UWB) technologies.

In the framework of the UTD [1], a first time domain (TD) solution for a straight perfectly conducting (PEC) wedge was obtained in [2], via the application of inverse Laplace transform theory to the corresponding frequency domain UTD wedge diffraction coefficient. This result was extended in [3] to a curved edge formed by the truncation of curved surfaces, including the case of astigmatic time impulsive wavefront. In the last decade new TD-UTD solutions were developed to describe single wedge slope diffraction [4], double diffraction by a pair of coplanar skew edges [5], the scattering from perfectly conducting smooth convex surfaces [6], the radiation of a pulsed antenna placed on PEC smooth convex surfaces [7]. A preliminary expression for TD-UTD vertex diffraction was introduced in [8] for real time, whereas the formulation with complex time has been outlined in [9].

In this paper we introduce the new contribution to the TD-UTD solutions present in the literature, by introducing the TD-UTD solution for the EM scattering by the tip of a pyramid, formed by PEC interconnected flat plates and illuminated by a source with spherical wavefront. Such result is obtained by calculating the analytic time transform of the frequency domain (FD) UTD solution recently developed in [10]. As a consequence, the transient fields propagate along the ray paths of the UTD. In particular, we derived the TD-UTD vertex diffraction coefficient for a tip illuminated by an impulsive field. The diffraction coefficient presented here guarantees the compensation of the discontinuities at GO and UTD wedge diffracted field [3] planar and conical shadow boundaries (SBs), that are typical in vertex diffraction phenomena [10], and, thus, to uniformly describe the total field. Since the UTD is a high-frequency asymptotic theory, its results in the frequency domain remain accurate for moderate to high frequencies; the corresponding TD ray solution, therefore, is valid only for “early to intermediate times”. As a consequence, each TD-UTD ray contribution will be the most accurate in the neighborhood of the ray arrival time, i.e., the time required to traverse its geometric ray path length from the source to the observer. However, in the analysis of high frequency high pass communication systems, when the exciting signal is dominated by high frequencies and has negligible low frequency components, the range of validity of the resulting impulse response is extended to later observation times behind the wavefront.

The impulse response dyadic TD-UTD vertex diffraction coefficient is described in Section II.

II. FORMULATION

Let us consider an infinite PEC pyramid, as depicted in Fig. 1, having $M$ edges and $M$ faces. Edges are counted counterclockwise observing the pyramid from the tip $V$; the face $S_m$ tagged by $m$ is delimited by the edges tagged by $m$ and $m+1$, with $m = 1, ..., M$, and $M + 1$ intended “modulo $M$” (i.e., $M + 1 ≡ 1$). The pyramid results from the superposition of $M$ wedges (the $m$th edge belonging to the $m$th wedge) sharing a common face, all intersecting at the vertex $V$. The pyramid is illuminated by an impulsive source, located at $P'$, which radiates a field with a spherical wavefront.

The analytic time transform (ATT) [11]–[13] of the incident GO ray electric field $\hat{E}_r(t)$ generated at $P'$ and evaluated at
the tip $V$ has the following expression

$$\mathbf{e}_v^+(t) = \mathbf{E}_{10} A_i^+ \left(t - \tau_v\right)$$  \hspace{1cm} (1)$$

where $\mathbf{E}_{10}$ is a constant vector; $A_i = 1/r'$, with $r' = |V - P|$, is the spherical wavefront spreading factor; $\tau_i = r'/c$ is the incident GO ray arrival time at $V$, where $c$ is the speed of light in the medium surrounding the pyramid; and $\delta(t)$ is the analytic delta function defined as in [3].

The TD vertex diffracted field $\mathbf{e}_v^+(t)$ introduced in this paper is in the framework of a general UTD description, hence, the total TD field at a generic point $P$ is represented as

$$\mathbf{e}_{tot}^+(t) = \mathbf{e}_i^+(t) + \mathbf{e}_r^+(t) + \mathbf{e}_d^+(t) + \mathbf{e}_v^+(t).$$ \hspace{1cm} (2)$$

Here $\mathbf{e}_i^+(t)$ is the incident analytic TD field evaluated at $P$; $\mathbf{e}_r^+(t)$ is the analytic TD reflected field by the pyramid faces according to GO; $\mathbf{e}_d^+(t)$ is the analytic TD-UTD field accounting of edge diffraction at the edges of the pyramid. In (2) $\mathbf{e}_d^+(t)$ is the analytic TD field contribution arising from diffraction at the tip $V$ and evaluated at $P$.

The TD-UTD analytic impulse response for the vertex diffracted field at the observation point $P$ is given by

$$\mathbf{d}_v^+(t) = \mathbf{d}_v^+ (\tau_v) \cdot \mathbf{E}_{10} A_i A_v,$$ \hspace{1cm} (3)$$

where $\tau_v$ is a propagation delay, $A_v = 1/r$, with $r = |P - V|$, is the vertex diffraction spreading factor. The term $\mathbf{d}_v^+$ is the dyadic TD-UTD analytic impulse response vertex diffraction coefficient. It can be conveniently calculated by using the ray-fixed reference system defined in [10], and it is expressed as

$$\mathbf{d}_v^+(t) = \sum_{m=1}^{M} \mathbf{d}_{v,m}^+ (t).$$ \hspace{1cm} (4)$$

Notably, each term in (4) contains the analytical TD-UTD canonical transition function for vertex diffraction, which has the form

$$\mathbf{T}_{GFI}^+(\hat{b}, \hat{a}, t) = c \sqrt{\frac{a}{\alpha}} \left[ \frac{\left(\frac{a}{\alpha} + b\right)}{\pi \left(\frac{a}{\alpha} + b + t\right)} \right]$$

$$\log \left[ \frac{\sqrt{j}b + \sqrt{j} (b + t)}{\sqrt{j}b - j (b + t)} \right] + 2j \arctan \left( \frac{\sqrt{a}}{b} \right)$$ \hspace{1cm} (5)$$

and guarantees the uniform description of the total field at the planar and conical shadow boundaries of GO and UTD wedge singly diffracted rays. This analytic TD transition functions is the one-side inverse Fourier transform of the frequency domain vertex transition function in [10]. During the conference, a detailed analysis of the dyadic diffraction coefficient will be provided along with various representative numerical results, which show the validity and the effectiveness of the proposed solution.

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