Anyonic molecules in atomic fractional quantum Hall liquids:

a quantitative probe of fractional charge and anyonic statistics

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Anyons: quasiparticles which are neither bosons nor fermions can exist in 2D.
**Fractional statistics**

**Anyons:** quasiparticles which are neither bosons nor fermions can exist in 2D

\[ \psi(r_2, r_1) = e^{i\pi \alpha} \psi(r_1, r_2) \]

\[ |\psi(r_1, r_2)|^2 = |\psi(r_2, r_1)|^2 \]
Fractional statistics

Anyons: quasiparticles which are neither bosons nor fermions can exist in 2D

$\psi(r_1, r_2) = e^{2\pi i \alpha} \psi(r_1, r_2)$

$e^{2\pi i \alpha} = 1$

$|\psi(r_1, r_2)|^2 = |\psi(r_2, r_1)|^2$

$\alpha$: statistical parameter

\(\alpha = 0\) (bosons)
\(\alpha = 1\) (fermions)
**Fractional statistics**

**Anyons**: quasiparticles which are neither bosons nor fermions can exist in 2D

\[ \psi(r_1, r_2) = e^{2\pi i \alpha} \psi(r_1, r_2) \]

\[ e^{2\pi i \alpha} = 1 \]

\[ |\psi(r_1, r_2)|^2 = |\psi(r_2, r_1)|^2 \]

\[ \alpha = 0 \]

(bosons)

\[ \alpha = 1 \]

(fermions)

\[ \alpha: \text{statistical parameter} \]

This is topologically equivalent to not moving the particles at all.

\[ \psi(r_1, r_2) = e^{2\pi i \alpha} \psi(r_1, r_2) \]

**Initial**  **Final**
Anyons: quasiparticles which are neither bosons nor fermions can exist in 2D

\[ \psi(r_1, r_2) = e^{2i\pi \alpha} \psi(r_1, r_2) \]

\[ e^{2\pi i \alpha} = 1 \]

\[ |\psi(r_1, r_2)|^2 = |\psi(r_2, r_1)|^2 \]

\[ \psi(r_1, r_2) = e^{2i\pi \alpha} \psi(r_1, r_2) \]

Initial \hspace{2cm} Final

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\[ \alpha \]: statistical parameter
Fractional statistics

**Anyons**: quasiparticles which are neither bosons nor fermions can exist in 2D

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{2\pi i \alpha} \psi(\mathbf{r}_2, \mathbf{r}_1)$$

$$|\psi(\mathbf{r}_1, \mathbf{r}_2)|^2 = |\psi(\mathbf{r}_2, \mathbf{r}_1)|^2$$

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$$e^{2\pi i\alpha} = 1$$

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\[ e^{2\pi i \alpha} = 1 \]

\[ |\psi(r_1, r_2)|^2 = |\psi(r_2, r_1)|^2 \]

\[ \alpha = 0 \quad \text{(bosons)} \]

\[ \alpha = 1 \quad \text{(fermions)} \]

\[ \alpha: \text{statistical parameter} \]

This is topologically equivalent to not moving the particles at all
**Fractional statistics**

**Anyons:** quasiparticles which are neither bosons nor fermions can exist in 2D

\[
\psi(r_1, r_2) = e^{2\pi i \alpha} \psi(r_1, r_2) \quad \Rightarrow \quad e^{2\pi i \alpha} = 1
\]

\[
|\psi(r_1, r_2)|^2 = |\psi(r_2, r_1)|^2
\]

\[
\psi(r_1, r_2) = e^{2\pi i \alpha} \psi(r_1, r_2)
\]

**3D**

- \( \alpha = 0 \) (bosons)
- \( \alpha = 1 \) (fermions)

**2D**

- Now, the process is **NOT** topologically equivalent to not moving the particles at all

\[
This is topologically equivalent to not moving the particles at all
\]

\[
\alpha: \text{statistical parameter}
\]

\[
\text{Initial} \quad \text{Final}
\]
Fractional statistics

**Anyons**: quasiparticles which are neither bosons nor fermions can exist in 2D

\[ \psi(r_1, r_2) = e^{2i\pi \alpha} \psi(r_1, r_2) \rightarrow e^{2\pi i \alpha} = 1 \]

\[ |\psi(r_1, r_2)|^2 = |\psi(r_2, r_1)|^2 \]

\[ \psi(r_1, r_2) = e^{2i\pi \alpha} \psi(r_1, r_2) \]

\( \alpha \): statistical parameter

- \( \alpha = 0 \) (bosons)
- \( \alpha = 1 \) (fermions)

This is topologically equivalent to not moving the particles at all

\[ \alpha \in [0, 2) \quad e^{i\pi \alpha} \in U(1) \]

**Braiding** 2 particles produces a unitary transformation on the system

- But it’s **trivial**: global phase for wavefunction

**ABELIAN ANYONS**

[Leinaas and Myrheim, Il Nuovo Cimento B 1977]
[Wilczek, PRL 1982]

Now, the process is **NOT** topologically equivalent to not moving the particles at all.
Anyons in real life

2D electron gas

- Ansatz for ground state
- Excitations: quasi-holes (QHs) and quasi-particles (QPs) with
  - Fractional charge
  - Fractional statistics

Willett et al., PRL 1987
Laughlin, PRL 1983
De Picciotto et al., Nature 1997
Anyons in real life

2D electron gas
+ Strong transverse magnetic field
Anyons in real life

- 2D electron gas
- Strong transverse magnetic field
- Low disorder

Fractional quantum Hall liquid [Willett et al., PRL 1987]

Filling fraction $\nu$
Anyons in real life

2D electron gas
+ Strong transverse magnetic field
+ Low disorder

**Fractional quantum Hall liquid**  
[Willett et al., PRL 1987]

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  ▪ Fractional charge [De Picciotto et al., Nature 1997]
  ▪ **Fractional statistics**

Fractional quantum Hall liquid [Willett et al., PRL 1987]

Filling fraction $\nu$

Shot-noise / interferometric experiments in this setup are difficult to modellize and perform
Anyons in real life

**2D electron gas**
- Strong transverse magnetic field
- Low disorder

**2D synthetic material**

**Fractional quantum Hall liquid** [Willett et al., PRL 1987]

- Ansatz for ground state [Laughlin, PRL 1983]
- **Excitations**: quasi-holes (QHs) and quasi-particles (QPs) with
  - Fractional charge [De Picciotto et al., Nature 1997]
  - **Fractional statistics**

Shot-noise / interferometric experiments in this setup are difficult to modellize and perform
Anyons in real life

2D electron gas + Strong transverse magnetic field + Low disorder

Fractional quantum Hall liquid

2D synthetic material + Strong interactions

Ultracold atoms
▪ Tunable interactions (Feshbach resonance)
▪ Rotation: Coriolis force = Lorentz force
▪ Synthetic charge
▪ Synthetic magnetic field
▪ Photons in twisted cavity
▪ Control laser
▪ Excitation to Rydberg state
▪ Cavity Rydberg polaritons: strong interactions
▪ Probe twisted laser
▪ Artificial gauge field

[Clark et al., arXiv 2019]
[Dalibard, RMP 2011]

• Ansatz for ground state [Laughlin, PRL 1983]

• **Excitations**: quasi-holes (QHs) and quasi-particles (QPs) with
  ▪ Fractional charge [De Picciotto et al., Nature 1997]
  ▪ **Fractional statistics**

Shot-noise / interferometric experiments in this setup are difficult to modellize and perform
Anyons in real life

2D electron gas
+ Strong transverse magnetic field
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Fractional quantum Hall liquid  \[ \text{[Willett et al., PRL 1987]} \]

2D synthetic material
+ Strong interactions
+ Synthetic magnetic field

Fractional quantum Hall liquid

- Ansatz for ground state  \[ \text{[Laughlin, PRL 1983]} \]
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  - **Fractional statistics**?

Shot-noise / interferometric experiments in this setup are difficult to modelise and perform
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Fractional quantum Hall liquid

• Ultracold atoms
  ▪ Tunable interactions (Feshbach resonance)
  ▪ Rotation: Coriolis force = Lorentz force
    • Synthetic charge
    • Synthetic magnetic field [Dalibard, RMP 2011]

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  ▪ Fractional charge [De Picciotto et al., Nature 1997]
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Shot-noise / interferometric experiments in this setup are difficult to modellize and perform
Anyons in real life

| 2D electron gas | 2D synthetic material |
|-----------------|-----------------------|
| + Strong transverse magnetic field | + Strong interactions |
| + Low disorder | + Synthetic magnetic field |

**Fractional quantum Hall liquid**  
[Willett *et al.*, PRL 1987]

- Ansatz for ground state  
  [Laughlin, PRL 1983]
- **Excitations**: quasi-holes (QHs) and quasi-particles (QPs) with
  - Fractional charge  
  [De Picciotto *et al.*, Nature 1997]
  - **Fractional statistics**

![Diagram of Filling fraction $\nu$](image)

**Ultracold atoms**
- Tunable interactions (Feshbach resonance)
- Rotation: Coriolis force = Lorentz force
  - Synthetic charge
  - **Synthetic magnetic field**
  [Dalibard, RMP 2011]

**Photons in twisted cavity**  
[Clark *et al.*, arXiv 2019]

- Control laser
  - Excitation to Rydberg state
  - Cavity Rydberg polaritons: **strong interactions**
- Probe twisted laser
  - **Artificial gauge field**

Shot-noise / interferometric experiments in this setup are difficult to modellize and perform
Impurities inside FQH atomic liquid

\[ B = B u_z \]

| Impurities | Atoms |
|------------|-------|
| Number     | \( N \) | \( n \gg N \) |
| Charge     | \( Q \)  | \( q \)    |
| Mass       | \( M \)  | \( m \)    |
System and model

\[ B = B u_z \]

Hamiltonian:

\[ T_a(\{r_j\}) = \sum_{j=1}^{n} \frac{1}{2m} \left[ -i \nabla_{r_j} - gA(r_j) \right]^2, \]

\[ T_i(\{R_j\}) = \sum_{j=1}^{N} \frac{1}{2M} \left[ -i \nabla_{R_j} - QA(R_j) \right]^2, \]

\[ V_{aa}(\{r_j\}) = g_{aa} \sum_{i<j}^{n} \delta(\mathbf{r}_i - \mathbf{r}_j), \]

\[ V_{ia}(\{r_j\}, \{R_j\}) = \sum_{i=1}^{n} \sum_{j=1}^{N} v_{ia}(\mathbf{r}_i - \mathbf{R}_j), \]

\[ V_{ii}(\{R_j\}) = \sum_{i<j}^{N} v_{ii}(\mathbf{R}_i - \mathbf{R}_j). \]
System and model

Hamiltonian = \[ T_a(\{r_j\}) = \sum_{j=1}^{n} \frac{1}{2m} \left[ -i \nabla_{r_j} - q A(\mathbf{r}_j) \right]^2, \]
\[ + T_i(\{\mathbf{R}_j\}) = \sum_{j=1}^{N} \frac{1}{2M} \left[ -i \nabla_{\mathbf{R}_j} - Q A(\mathbf{R}_j) \right]^2, \]
\[ + V_{aa}(\{r_j\}) = g_{aa} \sum_{i<j}^{n} \delta(\mathbf{r}_i - \mathbf{r}_j), \]
\[ + V_{ia}(\{r_j\}, \{\mathbf{R}_j\}) = \sum_{i=1}^{n} \sum_{j=1}^{N} v_{ia}(\mathbf{r}_i - \mathbf{R}_j), \]
\[ + V_{ii}(\{\mathbf{R}_j\}) = \sum_{i<j}^{N} v_{ii}(\mathbf{R}_i - \mathbf{R}_j). \]
**System and model**

Hamiltonian =

\[
T_a(\{r_j\}) = \sum_{j=1}^{n} \frac{1}{2m} \left[ \mathbf{-i} \nabla_{r_j} - g A(r_j) \right]^2 ,
\]
\[
+ T_i(\{R_j\}) = \sum_{j=1}^{N} \frac{1}{2M} \left[ \mathbf{-i} \nabla_{R_j} - Q A(R_j) \right]^2 ,
\]
\[
+ V_{aa}(\{r_j\}) = g_{aa} \sum_{i<j}^{n} \delta(\mathbf{r}_i - \mathbf{r}_j) ,
\]
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+ V_{ii}(\{R_j\}) = \sum_{i<j}^{N} v_{ii}(\mathbf{R}_i - \mathbf{R}_j) .
\]

**Impurities inside FQH atomic liquid**

| Impurities inside FQH atomic liquid |
|-------------------------------------|
| \( B = B u_z \)                   |
| Hamiltonian                         |

**Number**

| Number | \( N \) |
|--------|---------|

**Charge**

| Charge | \( Q \) |
|--------|---------|

**Mass**

| Mass   | \( M \) |
|--------|---------|

**Repulsive atom-atom interaction**

**Repulsive impurity-atom interaction**
System and model

\[
\text{Impurities inside FQH atomic liquid}
\]

\[
B = B u_z
\]

Hamiltonian

\[
= T_a(\{r_j\}) + T_i(\{R_j\}) + V_{aa}(\{r_j\}) + V_{ia}(\{r_j\}, \{R_j\}) + V_{ii}(\{R_j\})
\]

\[
= \sum_{j=1}^{n} \frac{1}{2m} \left( [\mathbf{i} \nabla_{r_j} - q \mathbf{A}(r_j)]^2 \right)
\]

\[
+ \sum_{j=1}^{N} \frac{1}{2M} \left( [\mathbf{i} \nabla_{R_j} - Q \mathbf{A}(R_j)]^2 \right)
\]

\[
+ g_{aa} \sum_{i<j}^{n} \left( \delta(\mathbf{r}_i - \mathbf{r}_j) \right)
\]

\[
+ \sum_{i=1}^{n} \sum_{j=1}^{N} v_{ia}(\mathbf{r}_i - \mathbf{R}_j)
\]

\[
+ \sum_{i<j}^{N} v_{ii}(\mathbf{R}_i - \mathbf{R}_j)
\]

Impurities

- Number: \(N\)
- Charge: \(Q\)
- Mass: \(M\)

Atoms

- Number: \(n \gg N\)
- Charge: \(q\)
- Mass: \(m\)
System and model

Hamiltonian = \[ T_a(\{r_j\}) = \sum_{j=1}^{n} \frac{1}{2m} \left[ -i \nabla_{r_j} - q A(\mathbf{r}) \right]^2, \]
\[ + T_i(\{R_j\}) = \sum_{j=1}^{N} \frac{1}{2M} \left[ -i \nabla_{R_j} - Q A(\mathbf{R}) \right]^2, \]
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\[ + V_{ii}(\{R_j\}) = \sum_{i<j}^{N} v_{ii}(\mathbf{R}_i - \mathbf{R}_j). \]
• **Total wavefunction:**  (under BO approx.)

\[
\psi(\{r_i\}, \{R_i\}, t) = \varphi_{\{R_i\}}^{(0)}(\{r_i\}, t) \chi(\{R_i\}, t)
\]
• **Total wavefunction:**  (under BO approx.)

\[
\psi(\{r_i\}; \{R_i\}; t) = \varphi^{(0)}(\{r_i\}; t) \chi(\{R_i\}; t)
\]

Ground state of  \( H_{BO} = T_a + V_{aa} + V_{ia} \)

• Laughlin’s Ansatz for FQH liquid
• QHs at impurities positions
• **Total wavefunction:** (under BO approx.)

\[
\psi(\{r_i\}, \{R_i\}, t) = \varphi_{\{r_i\}}^{(0)}(\{r_i\}, t) \chi(\{R_i\}, t)
\]

Ground state of \( H_{BO} = T_a + V_{aa} + V_{ia} \)
- Laughlin’s Ansatz for FQH liquid
- QHs at impurities positions

• **Effective Hamiltonian** acting on \( \chi(R) \)

\[
H_{\text{eff}} = \langle \varphi_R^{(0)} | H | \varphi_R^{(0)} \rangle
\]
• **Total wavefunction:** (under BO approx.)
\[ \psi(\{r_i\}, \{R_i\}, t) = \varphi^{(0)}_{\{R_i\}}(\{r_i\}, t) \chi(\{R_i\}, t) \]

Ground state of \( H_{BO} = T_a + V_{aa} + V_{ia} \)
- Laughlin’s Ansatz for FQH liquid
- QHS at impurities positions

• **Effective Hamiltonian** acting on \( \chi(R) \)
\[ H_{\text{eff}} = \langle \varphi^{(0)}_R | H | \varphi^{(0)}_R \rangle \]

• **Mass renormalization for 1 impurity + 1 QH**
\[ M = M + \Delta M \]
\[ \varphi_R(r, t) \simeq \varphi^{(0)}_R(r) + \varphi^{(1)}_R(r, t) \]

1st correction to BO approx.

[Scherrer et al., PRX 2017]
**Born-Oppenheimer approximation**

- **Total wavefunction**: (under BO approx.)
  \[ \psi(\{r_i\}, \{R_i\}, t) = \varphi_{\{R_i\}}^{(0)}(\{r_i\}, t) \chi(\{R_i\}, t) \]

- **Ground state of** \( H_{BO} = T_a + V_{aa} + V_{ia} \)
  - Laughlin’s Ansatz for FQH liquid
  - QHs at impurities positions

- **Effective Hamiltonian** acting on \( \chi(\mathbf{R}) \)
  \[ H_{\text{eff}} = \langle \varphi^{(0)}_{\mathbf{R}} | H | \varphi^{(0)}_{\mathbf{R}} \rangle \]

- **Mass renormalization for 1 impurity + 1 QH**
  \[ \mathcal{M} = M + \Delta M \]
  \[ \varphi_{\mathbf{R}}(r, t) \simeq \varphi^{(0)}_{\mathbf{R}}(r) + \varphi^{(1)}_{\mathbf{R}}(r, t) \]

  **1st correction to BO approx.**

  [Scherrer et al., PRX 2017]

\[ \Delta M \frac{M}{M} \simeq m \frac{\omega_{\text{cycl}}}{M \Delta \omega_{-1}} \]

\[ \omega_{\text{cycl}} = qB/m \]
• 1 impurity bound to 1 quasihole
Single impurity

1 impurity **bound to 1 quasihole**

Dynamics of impurity governed by effective Hamiltonian acting on $\chi(R)$

$$H_{\text{eff}} = \langle \varphi_R^{(0)}(r) | H | \varphi_R^{(0)}(r) \rangle = \frac{[-i \nabla_R - (Q - \nu q) A(R)]^2}{2M}$$

$$A(R) = \frac{-1}{2eB} u_z \times R \\
B = B u_z$$
Single impurity

- 1 impurity **bound to 1 quasihole**

- Dynamics of impurity governed by effective Hamiltonian acting on $\chi(R)$

$$H_{\text{eff}} = \langle \varphi_R^{(0)}(r) | H | \varphi_R^{(0)}(r) \rangle = \frac{[-i\nabla_R - (Q - \nu q) A(R)]^2}{2M}$$

$$A(R) = -\frac{1}{2eB} \mathbf{u}_z \times R \quad B = B \mathbf{u}_z$$

- **Effective charge** of anyonic molecule:

$$Q = Q - \nu q$$

i.e. impurity charge + QH charge
Single impurity

- 1 impurity **bound to 1 quasi-hole**
- Dynamics of impurity governed by effective Hamiltonian acting on \( \chi(R) \)
  \[
  H_{\text{eff}} = \left< \varphi_R^{(0)}(r) \right| H \left| \varphi_R^{(0)}(r) \right> = \frac{[-i \nabla_R - (Q - \nu q) A(R)]^2}{2M}
  \]
  \[
  A(R) = -\frac{1}{2l_B^2} u_z \times R \quad B = B u_z
  \]
- **Effective charge** of anyonic molecule:
  \[
  Q = Q - \nu q
  \]
  i.e. impurity charge + QH charge
- **Proposed experiment**
  - Give molecule momentum kick \( p = Mv \)
  - Cyclotron orbit with \( r_{\text{cycl}} = \frac{Mv}{QB} \)
Single impurity

1 impurity **bound to 1 quasihole**

Dynamics of impurity governed by effective Hamiltonian acting on \( \chi(R) \)

\[
H_{\text{eff}} = \langle \varphi_{R}^{(0)}(r) | H | \varphi_{R}^{(0)}(r) \rangle = \frac{[ -i \nabla_{R} - (Q - \nu q) A(R)]^{2}}{2M}
\]

\[
A(R) = \frac{-1}{2eB} u_{z} \times R \quad B = B u_{z}
\]

**Effective charge** of anyonic molecule:

\[
Q = Q - \nu q \quad \text{i.e. impurity charge + QH charge}
\]

**Proposed experiment**

- Give molecule momentum kick \( p = \mathcal{M} v \)
- Cyclotron orbit with \( r_{\text{cycl}} = \frac{\mathcal{M} v}{Q B} \)
- Image impurity’s position at different times after deterministic preparation
- Reconstruct trajectory
- Measure \( \mathcal{M}, Q \)
Two impurities

• 1 QH bound to each impurity

\[ \alpha = \nu \] (bosons), \quad \alpha = 1 + \nu \] (fermions)
Two impurities

- 1 QH bound to each impurity
  \[ \alpha = \nu \] (bosons), \[ \alpha = 1 + \nu \] (fermions)

- Impurities in bulk of FQH droplet
- Large inter-impurity distance
  Use \( M \) from the single-impurity calculation
Two impurities

- 1 QH bound to each impurity
  \[ \alpha = \nu \text{ (bosons), } \alpha = 1 + \nu \text{ (fermions)} \]

- Impurities in bulk of FQH droplet
- Large inter-impurity distance
  - Use \( \mathcal{M} \) from the single-impurity calculation

**Effective Hamiltonian:**

\[
H_{\text{eff}} = \sum_{j=1}^{2} \left\{ -i \nabla_{R_j} - QA(R_j) + \frac{A_{\text{stat},j}(\{R_k\})}{2\mathcal{M}} \right\}^2 + V_{\text{int}}(R_1, R_2)
\]

- Long-range **Aharonov-Bohm interaction**

\[
A_{\text{stat},j}(\{R_k\}) = (-1)^j \nu \frac{u_z \times R_{\text{rel}}}{R_{\text{rel}}^2}
\]
Two impurities

- 1 QH bound to each impurity
  \[ \alpha = \nu \quad \text{(bosons)}, \quad \alpha = 1 + \nu \quad \text{(fermions)} \]

- Impurities in bulk of FQH droplet
- Large inter-impurity distance
  \[ \text{Use } \mathcal{M} \text{ from the single-impurity calculation} \]

- Effective Hamiltonian:
  \[ H_{\text{eff}} = \sum_{j=1}^{2} \left[ -i \nabla_{R_j} - QA(R_j) + A_{\text{stat},j}(\{R_k\}) \right]^2 \frac{2M}{2M} + V_{ii}(R_1, R_2) \]

- Long-range Aharonov-Bohm interaction
  \[ A_{\text{stat},i}(\{R_k\}) = (-1)^j \frac{\nu}{R_{\text{rel}}} u_z \times R_{\text{rel}} \]

- 2D scattering between 2 anyonic molecules

- Solve Schrödinger equation
- Differential scattering cross section
  \[ \frac{d\sigma}{d\phi} \]
2D scattering: results

Indistinguishable bosonic impurities

Dipolar

\[ k\alpha = 0.1 \]

\[ k\alpha = 5 \]

Hard-disk

\[ k\alpha = 0.1 \]

\[ k\alpha = 5 \]
2D scattering: results

Indistinguishable bosonic impurities

Dipolar $ka = 0.1$

Hard-disk $ka = 0.1$

Dipolar $ka = 5$

Hard-disk $ka = 5$

Proposed experiment for bosonic impurities
Indistinguishable bosonic impurities

2D scattering: results

Indistinguishable bosonic impurities

Proposed experiment for bosonic impurities

Without FQH droplet

Dipolar

Hard-disk

Proposed experiment for bosonic impurities

Without FQH droplet

Dipolar

Hard-disk

Proposed experiment for bosonic impurities

Without FQH droplet

Dipolar

Hard-disk

Proposed experiment for bosonic impurities

Without FQH droplet
2D scattering: results

Indistinguishable bosonic impurities

Proposed experiment for bosonic impurities
2D scattering: results

Indistinguishable bosonic impurities

Proposed experiment for bosonic impurities

Without FQH droplet  
With FQH droplet  
Always here

Dipolar  
\( ka = 0.1 \), \( ka = 5 \)

Hard-disk  
\( ka = 0.1 \), \( ka = 5 \)
2D scattering: results

Indistinguishable bosonic impurities

Proposed experiment for bosonic impurities

Without FQH droplet

With FQH droplet

→ interpolate

Always here
Conclusions

• Fractional charge and statistics from quantum dynamics of impurities in FQH droplet
Conclusions

• Fractional charge and statistics from quantum dynamics of impurities in FQH droplet

• Single molecule
  ▪ Renormalization of mass and charge
  ▪ Cyclotron orbit

• Future perspectives
  ▪ Molecules with different anyonic statistics
  ▪ FQH fluids with non-Abelian excitations
Conclusions

• Fractional charge and statistics from quantum dynamics of impurities in FQH droplet

• Single molecule
  ▪ Renormalization of mass and charge
  ▪ Cyclotron orbit

• 2 molecules
  ▪ Fractional statistics gives rise to long-range Aharonov-Bohm interaction
  ▪ Differential scattering cross section displays oscillatory pattern
  ▪ Fractional statistics is observable from rigid shift in its angular dependence
Conclusions

• Fractional charge and statistics from quantum dynamics of impurities in FQH droplet

• Single molecule
  ▪ Renormalization of mass and charge
  ▪ Cyclotron orbit

• 2 molecules
  ▪ Fractional statistics gives rise to long-range Aharonov-Bohm interaction
  ▪ Differential scattering cross section displays oscillatory pattern
  ▪ Fractional statistics is observable from rigid shift in its angular dependence

• Future perspectives
  • FQH fluids with non-Abelian excitations

[Nayak, RMP 2008]