Computer simulation of processes of changing the reliability characteristics of systems with recoverable and non-recoverable elements based on the Simulink Dynamic Simulation Package

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Abstract. The paper presents a methodology and software package of mathematical and computer simulation of changes in the reliability characteristics and integrated indicators of system reliability during operation using Simulink. The software package is an application (Toolbox) to the standard Simulink package. The theoretical basis of the proposed algorithms is the method of analyzing structural diagrams of system reliability. The author proposes an original diagram of building a mathematical model of reliability with both non-recoverable and recoverable elements. The diagram involves a broad interpretation of the concepts of a “recoverable element” and a “recoverable element failure”. This approach found reflection in the wide range of proposed simulation tools. The advantage of the proposed products is the unification and relative ease of building models of systems of a sufficiently high level of complexity, and a wide range of intermediate simulation results available for analysis. The methodology and software package can be used during the system design stage to predict their reliability performance. The paper presents some simulation results.

1. Introduction
A method of determining the reliability of systems using reliability structural diagrams has become widespread. The main advantages of the method are its relative simplicity, high clarity and information value in terms of analyzing the reliability of the system and the various factors influencing it. By virtue of these advantages, methods based on the analysis of the structural reliability diagram are not only widely used in practice, but are actually accepted as basic in training courses on system reliability theory [1-3].

A whole range of specialized software is proposed to assess the reliability of systems (information systems for determining and analyzing system reliability based on statistics are not considered in this case). It is worthy of note that most specialized software packages use different methods of reliability assessment and are usually focused on a certain range of tasks or systems, have a closed algorithm and closed source code. These factors make it impossible to upgrade them at the algorithm level and make it difficult to apply “unique” problems to the solution without the developer's participation. The use of specialized software packages in the teaching and learning process on reliability theory is also not justified enough and is not fully consistent with the objectives of educational process (vocational education). To handle reliability sub-tasks, perform specific calculations and solve problems within the educational process, computer programs created by means of different programming systems and
languages are commonly used. This paper proposes a common approach to reliability problems based on the analysis of reliability structural diagrams using Simulink (a component of the Matlab software package).

### 2. Methodology basics and general approaches

The general principles and basics of the proposed methodology are presented in [4].

Calculating the reliability of a technical system using a reliability structural diagram is based on simulation processes of changing the probabilities of the no-failure operation $P$ or the failure $Q$ of the system, depending on time $t$. The initial data are the parameters that determine the nature of the change in the probabilities of the no-failure operation $P$ or the failure $Q$ of the individual elements of the system. All other characteristics of the system's reliability, such as the mean time between failures, the number of operative systems and subsystems at any given point in time, and others, are derived from the values of $P$ or $Q$.

Simulink's visual simulation software environment is a powerful and flexible dynamic simulation tool. This paper proposes a method of building an extension of Simulink (Toolbox) specialized for solving reliability problems. The extension is a set of specialized Simulink visual blocks (subprograms) grouped by purpose, which allow one to combine a mathematical model and a program describing the functioning of any reliability structural diagram.

The reliability structural diagram and parametric or statistical (non-parametric) characteristics of reliability of all system elements are considered as initial data. As a rule, parametric reliability characteristics are understood as the law of probability density distribution of no-failure operation of $i$-element of the system (or its failure) and parameters or any other relations that allow determining the probability of no-failure operation $P_i(t)$ at any specific time $t$. By non-parametric characteristics we mean particularly $P_i(t)$ in some form. If statistical (a priori) data act as the reliability characteristics of the $i$-element, then it is advisable to represent them as, for example, an approximation algorithm $P_i(t)$, $i=1,...,N$.

Here $i$ is a number of the considered system element; $N$ is the total number of the system’s elements.

Typical operations of analysis of reliability structural diagrams can be divided into groups:

1. Operations for calculating the probabilities of no-failure operation of the elements of the reliability structural diagram based on the specified initial data and mathematical models of these particular elements.

2. Operations for calculating the probabilities of no-failure operation of groups of elements that form various diagrammatic connections. The use of operations of this group in accordance with the algorithm developed on the basis of the reliability structural diagram allows one to reduce the structural diagram of any system of $N$ elements to a system of one element, which is, in fact, a mathematical model of the reliability of the system as a whole.

3. Operations for calculating the reliability characteristics specified for the determination for the system as a whole, its subsystems or individual elements. For example, determining the mean time between failures of a system or subsystems.

Each elementary operation is represented as a standard Simulink block with the necessary initial input data and the result of the mathematical implementation of the established operation as the block output.

### 3. Calculation of the probabilities of no-failure operation of the elements of the reliability structural diagram

In the case of a parametric assessment of the reliability of a system element, formulas composed on the basis of the employed distribution law are used to calculate the probability of no-failure operation of the element $P_i(t)$. To assess reliability, the following distributions are most often used: exponential,
normal, uniform, logarithmically normal, binomial, Weibull’s and Poisson’s distribution, gamma
distribution, and others [3, 5]. Their compositions, as well as multidimensional distributions, can be
used. All these distributions are used in the proposed software program in the form of separate
functional blocks. As an example, let us present a methodology for forming a mathematical model of
a block with a normal law of distribution of the system element (block)’s no-failure operation
probability. In the case of the normal law of distribution, the element’s no-failure operation probability
is determined by the well-known expression:

\[ P(t) = 1 - \Phi \left( \frac{t - \bar{T}}{\sigma_t} \right), \]  

(1)

where \( t \) is the current time value at a scale of the parameter being specified (for example, time
to failure); \( \bar{T} \) is the mathematical expectation of time to failure (a parameter of a place of normal
distribution); \( \sigma_t \) is the mean square deviation of time to failure (a parameter of a form of normal
distribution); \( \Phi \left( \frac{t - \bar{T}}{\sigma_t} \right) \) is Laplace’s function defined by the expression

\[ \Phi \left( \frac{t - \bar{T}}{\sigma_t} \right) = \frac{1}{\sqrt{\sigma_t \sqrt{2\pi}}} \exp \left( -\frac{(t - \bar{T})^2}{2\sigma_t^2} \right) dt. \]

The Simulink algorithm and a block design option are shown in figures 1 and 2. The Laplace's
function is determined directly by numerical integration in the program.

Programs are developed and blocks are designed in a similar way for other options of parametric
and nonparametric methods for setting the reliability characteristics of a system element.

4. Calculation of the probability of no-failure operation of complex elements in conditions of
insufficient a priori information

Simulating the functioning of technical systems and technology processes, in the context of
determining the system’s reliability characteristics, has a number of peculiar features.

First, as a rule, each element of the system is a complex (in terms of system analysis) technical
system or technology subprocess. Failures of the very different nature and manifestation forms may
occur in these elements. The complete distribution law of the probability of no-failure operation of
such elements is certainly not simple and can be described by the “classical” laws of the distribution of
the probability density of no-failure operation or failure with unsatisfactory certainty. Attempts to find
an adequate distribution law lead to the creation of increasingly complex model laws which, however,
do not solve the problem in general.

![Simulink algorithm for calculating the element’s no-failure operation probability in the case of a normal distribution of the probability density of failure.](image)

Figure 1. A Simulink algorithm for calculating the element’s no-failure operation probability in the case of a normal distribution of the probability density of failure.
Figure 2. General view of the software block for calculating the element’s no-failure operation probability in case of a normal distribution of the probability density of failure.

Secondly, complete information about the element’s reliability characteristics is usually not available or is estimative (for instance, determined during tests under conditions that do not completely coincide with real-life ones).

In this case an approach is proposed on the basis of the following hypothesis.

Let us assume that all the failures characteristic of any element of the system can be conditionally divided into groups by the physical reasons for these failures and, accordingly, by approximations to some laws of the probability density distribution of no-failure operation or failure. As a rule, it is customary to describe quite definite physical reasons for failures by quite definite distribution laws. For example, failures due to wear of system elements with a high degree of confidence can be described by the normal distribution law. Based on this approach, the concepts of the assigned life and the service life of the product are introduced. Another group is sudden failures occurring from a wide range of reasons that are difficult to identify or non-identifiable. In this case, the system’s no-failure operation probability is usually approximated with the help of the exponential law of the probability density distribution of no-failure operation or failure. Reliability characteristics are determined experimentally while in operation or based on analogy (operating experience of a prototype or analogue) using the concepts of failure occurrence rate and mean time between failures. Naturally, one can also distinguish other groups of failures of an element of the reliability structural diagram with known or specified characteristics.

Let us make the assumption that there is no interrelationship between the facts of the occurrence of failures of different groups. In accordance with this approach, we assume that the probability of occurrence of failures of one group does not depend on the occurrence or characteristics of the laws of distribution of failures of other groups. In this case, on the reliability structural diagram the considered element of the system can be represented as a series connection of conventional elements, each of which determines the probability of failure $P_{G_1}(t)$ and $P_{G_2}(t)$ of the first and second groups, respectively. Thus, it is postulated that an $i$-element of the system (for example, element 1) is considered failed if any group for this element fails:

$$P_i = P_{G_1}(t) \cdot P_{G_2}(t).$$  \hspace{1cm} (2)

5. Calculation of the probability of no-failure operation of elements forming typical diagrammatic connections

The basis for the analysis of the system reliability structural diagram is the grouping of elements according to typical connections and the calculation of the probability of no-failure operation of elements in groups. The main typical connections of elements are: serial connection of elements; parallel connection of two or more elements; bridge circuit of connecting elements; switch circuits; connection of $m$ from $n$ elements, etc. [1, 3, 5]. The software product implements blocks for all typical connections of elements.

An example of the analysis and construction of an algorithm for a typical connection of five elements by a bridge circuit (figures 3 and 4) in a fairly complete manner demonstrates the principles and capabilities of the proposed software package (Simulink extensions).
The creation of a mathematical model of the complex reliability structural diagram is a sequential grouping of elements with the application of typical connections to the groups of blocks (in case of a bridge connection, only blocks of serial and parallel connections were used).

6. The reliability model of the recoverable element

From the viewpoint of the synthesis of a mathematical model of the system reliability, taking into account the possibility of recovering some elements during its functioning, a broader interpretation of the concept of “failure” is proposed. An element is considered failed if it stopped working for any reason. The following options are designated and implemented in the program:

1. The element literally failed, i.e. an event occurred that consists in the malfunction of the element.
2. The element is made inoperative (turned off) for any reason.

The main reasons implemented in the program can be:

1. The signs of a possible failure of the element occurred. In this case, the element can be turned off for the purpose of “recording” the moment of transition to an inoperative state and control over the operation of the object.
2. The element of the no-failure operation probability system \( P_i(t) \) achieved the minimum acceptable value. In this case, the element is turned off for the period of work performed to recover the probability of no-failure operation \( P_i(t) \) to a permissible value. This approach is implemented in the case of using an operation strategy according to an acceptable reliability level.
3. The element reached the operating time or service life established by a resource of any type. The element turns off for the period of work performed to recover the resource. This approach is implemented in the case of using a schedule preventive operation strategy.

The element is considered failed for the entire period of non-operability. It should be noted that when the probability of no-failure operation of \( P_i \) is recovered due to the accumulation of unidentified defects and damages, a complete recovery of the probability to the value of \( P_i = 1 \) is practically impossible.

![Diagram](image)

**Figure 3.** General view of the reliability structural diagram for a bridge circuit of connecting elements in the main (a) and estimated (b) versions of the image.

If the element is operated according to the permissible level of reliability, then the diagram of changing its probability of no-failure operation can be represented by a graphical analogue (figure 5). Here \( t_{\text{start}} \) is the moment of the start of the element’s “operation”; \( P_i \) is the minimum (limit) value of the probability of the element’s no-failure operation; \( \Delta P \) is the value of the probability of no-failure operation that is not returned when the element is recovered (for example, by the repair method); \( \Delta t_{\text{rec}} \) is the object recovery time (the time it was in an inoperative state); \( t_{\text{over}} \) is the time of overhaul maintenance or replacement of the element with a new one (time of full recovery of the probability of the element’s no-failure operation).

All these parameters, along with the reliability characteristics during the operative state of the element, are considered as initial data for the corresponding program block, which describes the element under consideration. The considered algorithms are implemented using the Simulink StateFlow application.
Figure 4. The bridge-connection Simulink program for five elements.

Figure 5. The diagram of changing the probability of no-failure operation of the recoverable element.
7. Assessment of the technical reliability of a typical area of independent power supply

The study object is the reliability characteristics of a typical area of independent power supply (Figure 6). The main characteristic of reliability is understood as the probability of providing consumers with electricity of a specified available (maximum possible) capacity.

The area is characterized by a variety of utilized sources of electricity, both in type and capacity. The reliability characteristics of each element of the system were determined according to the methodology presented in this work. Firstly, according to the documentation based on the established overhaul and assigned resources and service life, with the assumption of the normal distribution law of the probability of failure, taking into account the fact that the probability of failure until the end of the overhaul and (or) assigned resource isn't included into the “three sigma” zone with respect to the mathematical expectation of the moment of failure. Secondly, on the basis of statistical (and constantly updated during operation) data on failures and time between failures of each integral unit, taking into account the configuration by type, operating time, maintenance frequency.

The diagram of the autonomous power supply area utilizes PAES-2500 – the mobile automated gas-turbine electrical power plant with a rated power of 2.5 MW; URAL-6000 – the gas-turbine block-modular electrical power plant (GTEPP) with a rated power of 6 W; SATURN (GTPU-6RM) – the gas-turbine integral unit with a rated power of 6 MW; GTE-6 / 6,3 – the gas-turbine electrical power plant (GTEPP), with a rated power of 6 MW; EG-12 (GTEPP-12P) – the gas-turbine block-modular electrical power plant, with a rated power of 12 MW; CSG-6kV – the closed switchgear, with a rated input voltage of 6 kV; ISB – incoming switchboards; MSG – the main switchgear; switch; TP-6/35 – the mobile transformer substation; CSG-35kV – the closed switchgear, with a rated voltage of 35 kV.

The first 5 objects were accepted to be recoverable. Each such object is represented by blocks of normal and exponential distributions connected in series. The parameters of the distribution laws and recovery processes are determined by the known data on scheduled replacement and maintenance (Table 1) and statistical data on the reliability of equipment (Table 2). Figure 7 shows a typical Simulink algorithm for a recoverable element. The simulation results for the cases of timely recovery of objects and without recovery are shown in figures 8 and 9.

| Table 1. Terms of scheduled replacement and maintenance of electrical power plants. |
|----------------|----------------|----------------|
| Designation of object | Scheduled maintenance time (h) | Duration of scheduled maintenance (h) | Scheduled replacement time (h) |
| PAES | 500 | 22 | 5000 |
| URAL | 1500 | 120 | 8000 |
| SATURN | 1500 | 120 | 8000 |
| GTE | 2500 | 120 | 8000 |
| EG-12 | 2500 | 192 | 8000 |

| Designation of object | Exponential distribution parameter | Normal distribution parameter | Mathematical expectation of normal distribution | The percentage of degradation of the object during recovery | Specified probability | Time of recovery | Time of overhaul maintenance | Initial operating time |
|----------------|------------------------------|-------------------------------|---------------------------------|-----------------|----------------|----------------|----------------|-------------------|
| PAES | 0.0008664 | 100 | 0.005 | 0.68 | 22 | 5000 | 100 |
| URAL | 0.0002888 | 300 | 0.005 | 0.7 | 120 | 8000 | 50 |
| SATURN | 0.0002888 | 300 | 0.005 | 0.8 | 120 | 8000 | 200 |
| GTE | 0.0001733 | 500 | 0.005 | 0.6 | 120 | 8000 | 20 |
| EG-12 | 0.0001733 | 500 | 0.005 | 0.5 | 192 | 8000 | 0 |
Figure 6. Structural diagram of the typical area of the autonomous electricity supply.
Figure 7. A Simulink algorithm of a recoverable object.

Figure 8. The probability of the system's no-failure operation with the recovery of elements.

Figure 9. The probability of the system's no-failure operation without the recovery of elements.
8. Conclusions
The proposed software makes it possible to develop mathematical models of reliability structural diagrams of any complexity, to perform a computational experiment in order to analyze their characteristics and evaluate the effectiveness of measures to improve the reliability of systems. The software allows one to modify mathematical models in a fairly simple way, organize the software interface, and process simulation results.

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