A note on the ambipolar diffusion in superfluid neutron stars

E. M. Kantor *, M. E. Gusakov
Ioffe Physical-Technical Institute of the Russian Academy of Sciences, Polytekhnicheskaya 26, 194021 St.-Petersburg, Russia

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ABSTRACT
We address the problem of magnetic field dissipation in the neutron star cores, focusing on the role of neutron superfluidity. Contrary to the results in the literature, we show that in the finite-temperature superfluid matter composed of neutrons, protons, and electrons, magnetic field dissipates exclusively due to Ohmic losses and non-equilibrium beta-processes, and only an admixture of muons restores (to some extent) the role of particle relative motion for the field dissipation. The reason for this discrepancy is discussed.

Key words: stars: neutron – stars: interiors – stars: magnetic field

1 INTRODUCTION
Since the pioneering works on the magnetic field dissipation in the neutron star (NS) cores by Baym et al. (1969); Haensel et al. (1990); Goldreich & Reisenegger (1992) a significant progress has been made (e.g., Shalybkov & Urpin 1995; Urpin & Shalybkov 1995; Thompson & Duncan 1996; Urpin & Shalybkov 1999; Hoyos et al. 2008, 2010; Glampedakis et al. 2011; Beloborodov & Li 2016; Passamonti et al. 2017; Castillo et al. 2017), but many questions have still remained unanswered. One of these questions regards the effects of nucleon superfluidity and superconductivity. How do they affect the magnetic field evolution? In this short note we partly address this question by analyzing the effects of neutron superfluidity. As we argue, this analysis is more accurate (and simple), than the previous treatments (Glampedakis et al. 2011; Passamonti et al. 2017), which are only valid at stellar temperatures $T$ much smaller than the neutron critical temperature $T_{cn}$, and it leads us to interesting conclusions. In essence, we show that the neutron superfluidity drastically modifies the fluid dynamics, imposing an additional (in comparison to the nonsuperfluid matter) constraint on the velocities of different particle species. As a result, ambipolar diffusion becomes completely irrelevant for the magnetic field evolution in the superfluid neutron-proton-electron ($npe$) matter, once the stellar temperature $T$ falls (even slightly) below $T_{cn}$. Then only Ohmic decay and dissipation due to non-equilibrium beta-processes remain active. An admixture of muons ($\mu$) introduces additional degree of freedom so that dissipation due to particle relative motion can again play a role.

2 MAGNETIC FIELD ENERGY DISSIPATION
2.1 Our assumptions and superfluid equation
We follow the general strategy outlined in Goldreich & Reisenegger (1992). For simplicity, we consider a Newtonian non-rotating NS with the superfluid core composed of relativistic finite-temperature $npe$ (or $npe\mu$) matter, where neutrons are superfluid, and protons are normal (nonsuperconducting). In the absence of magnetic field $B$ the star is in full thermodynamic and hydrostatic equilibrium, the velocities of all particle species vanish. We assume that the magnetic field is the only mechanism that drives the star out of diffusive and beta-equilibrium. Since the evolution occurs on a very long timescale (Goldreich & Reisenegger 1992), it proceeds through a set of quasistationary states, which means that the time derivatives in the “Euler equations” (Eqs. 3, 4–8 and 22–24, see below), as well as in the continuity equations, can be neglected. Next, the magnetic field is considered as a small perturbation, hence the induced particle velocities are also small and the terms

* kantor@mail.ioffe.ru

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depending on square of these velocities in the dynamic equations can be omitted. For simplicity, we also ignore all surface integrals which could appear in the formulas; they can be easily written out if necessary.

The magnetic energy dissipates when the system evolves towards equilibrium. The rate of the magnetic energy change,

$$\dot{E}_B = \int_V \frac{B}{4\pi} \cdot \dot{B} \, dV,$$

(1)

can be presented as (e.g., Goldreich & Reisenegger 1992)

$$\dot{E}_B = -\int_V E \cdot j \, dV,$$

(2)

where $E$ is the electric field; $j$ is the charge current density and $V$ is the system volume.

Because neutrons are assumed to be in the superfluid state, there is an additional degree of freedom in the system – the neutron superfluid velocity $V_{sn}$, which is related to the wave function phase $\Phi_n$ of the neutron Cooper-pair condensate by the condition $V_{sn} = \nabla \Phi_n/(2m_n)$. Generally, this velocity differs from the velocity of normal neutron component, $u_n$ (i.e., the velocity of neutron thermal Bogoliubov excitations). At $T < T_{cn}$ both normal and superfluid components contribute to the neutron current density, $j_n = n_sn V_{sn} + (n_n - n_sn)u_n$, where $n_n$ is the neutron number density and $n_sn$ is the number density corresponding to superfluid neutron component. In the absence of rotation (no Feynman-Onsager vortices) $V_{sn}$ obeys the standard “superfluid” equation, which is valid at arbitrary $T < T_{cn}$ (e.g., Putterman 1974; Khalatnikov 1989; Gusakov & Andersson 2006; note that the quadratically small velocity-dependent terms in this equation, as well as the bulk viscosity terms, are already neglected),

$$m_n \frac{\partial V_{sn}}{\partial t} + \nabla \mu_n^\infty = 0,$$

(3)

where $m_n$ is the neutron bare mass and $\mu_n^\infty$ is the redshifted relativistic neutron chemical potential. The latter is given by $\mu_n^\infty = \mu_n e^{\phi/c^2}$, where $\mu_n$ is the neutron chemical potential, $c$ is the speed of light, and $\phi$ is the gravitational potential. In a Newtonian star ($\phi \ll c^2$) $\nabla \mu_n^\infty$ can be represented as $\nabla \mu_n + \mu_n \nabla \phi/c^2$.

Equation (3) can be further simplified by neglecting inertia term which, as we have already discussed above, is small for a quasistationary evolving NS. Then it reduces to

$$\nabla \mu_n^\infty = 0 \quad \Leftrightarrow \quad \nabla \mu_n + \mu_n \nabla \phi/c^2 = 0 \quad \text{(for a Newtonian star)}.$$

(4)

Equation (4), which is valid at arbitrary $T < T_{cn}$, deserves a comment. In the NS literature it is customary to use a different form of this equation (see, e.g., equation 3 in Glampedakis et al. 2011 and equation 1 in Passamonti et al. 2017) with a friction force density $F_{fr}$ in its right-hand side,

$$\nabla \mu_n^\infty = \frac{F_{fr}}{n_n}.$$

(5)

The force density $F_{fr}$ describes friction of neutrons with electrons and protons, and is usually chosen to be equal to (see, e.g., equations 3, 18, and 40 in Glampedakis et al. 2011 and equations 1, 20, and 21 in Passamonti et al. 2017),

$$F_{fr} = J_n (u_n - u_{e}) + J_{np} (u_p - u_{e}),$$

(6)

where $u_n$ and $u_p$ are the electron and proton velocities, respectively; $u_{e} \equiv j_n/n_n$ is the velocity of neutron liquid as a whole (note that, generally, $V_{sn} \neq V_{en} \neq u_n$); and the ‘friction’ coefficients $J_n$ and $J_{np}$ are defined in Sec. 2.2.

Generally, the ‘neutron’ equation in the form (5) contradicts Eq. (4). Both equations coincide only in the limit $T \ll T_{cn}$, when $J_n$ and $J_{np}$ are suppressed by the neutron superfluidity (e.g., Glampedakis et al. 2011), so that $F_{fr}$ in Eq. (5) can be neglected. So, which equation is correct?

On the one hand, Eq. (4) is obtained from the neutron superfluid equation (3), which has a standard form (e.g., Putterman 1974). 2 Note that, this equation remains unchanged even for mixtures of nonrotating superfluid and normal liquids, as is clearly demonstrated, e.g., in the monograph by Khalatnikov (1989), who analyzed dissipative hydrodynamic equations for solutions of superfluid helium-II and normal 3 He taking into account the diffusion effects (see Chapter 24 and, in particular, equations 24.37 and 24.30 in that reference). On the other hand, Eq. (5) is (as far as we are aware) presented without detailed derivation and, as we believe, is the result of unjustified application of zero-temperature superfluid hydrodynamics to the case of finite temperatures. Thus, we conclude, that equation (5) is inaccurate at $T \lesssim T_{cn}$ and should be disregarded.

1 The expression (6) is written for our simplified problem, i.e., assuming that protons and electrons are normal (nonsuperconducting).

2 Note that any substantial modification of this equation is forbidden by the basic principles of the theory of superfluidity. For example, introduction of a friction force in its right-hand side,

$$m_n \frac{\partial V_{sn}}{\partial t} + \nabla \mu_n^\infty = \frac{F_{fr}}{n_n},$$

violates the potentiality condition for superfluid velocity, $\nabla \times V_{sn} = 0$, which must be satisfied in a nonrotating star (e.g., Khalatnikov 1989).
The equations of motion for electrons and nonsuperconducting protons take the form (Iakovlev & Shalybkov 1991)

\( -e (E + \frac{1}{c} u_e \times B) - \nabla \mu_e - \frac{\mu_e}{c^2} \nabla \phi - \frac{J_{ep}}{n_e} (u_e - u_p) - \frac{J_{en}}{n_e} (u_e - u_n) = 0, \) \hspace{1cm} (7)

\( e (E + \frac{1}{c} u_p \times B) - \nabla \mu_p - \frac{\mu_p}{c^2} \nabla \phi - \frac{J_{ep}}{n_p} (u_p - u_e) - \frac{J_{np}}{n_p} (u_p - u_n) = 0, \) \hspace{1cm} (8)

where \( e \) is the proton electric charge; \( \mu_e \) and \( \mu_p \) are the electron and proton relativistic chemical potentials, respectively. Further, \( n_i, \) is the number density of particle species \( i = p, e \) and \( J_{ik} = J_{ki} \) is the symmetric coefficient related to the effective relaxation time \( \tau_{ik} \) for scattering of particles \( i \) on particles \( k \) by the formula: \( \tau_{ik} = n_i \mu_i / (c^2 J_{ik}) \) (see Iakovlev & Shalybkov 1991).

In Eqs. (7) and (8) thermo-diffusion terms are neglected.

Equations (4)–(8) should be supplemented by the total force balance equation, which, for the problem in question, takes the standard form (the same for superfluid and nonsuperfluid liquids),

\[ \frac{j \times B}{c} = \nabla P + \left( \frac{P + \epsilon}{c^2} \right) \nabla \phi = \sum_{i = n, p, e} \left( n_i \nabla \mu_i + n_i \frac{\mu_i}{c^2} \nabla \phi \right), \]

where \( P \) and \( \epsilon \) are the pressure and energy density, and \( j = e n_p u_p - e n_e u_e. \) Let us now compose the following combination, \( n_e \times (7) + n_p \times (8) - n_n \times (4) - (9). \) Taking into account the quasineutrality condition, \( n_e = n_p, \) we get

\[ J_{en} (u_e - u_n) + J_{np} (u_p - u_n) = 0. \] (10)

This equation imposes an additional (in comparison to the non-superfluid matter) constraint on the velocities \( u_i. \) For example, if we neglect collisions between neutrons and electrons \( (J_{en} = 0), \) we obtain \( u_p = u_n. \)

Now, summing up \( n_e \times (7) + n_p \times (8) + n_n \times (4), \) we arrive at

\[ \frac{j \times B}{c} = -n_p \nabla \Delta \mu_e - n_p \frac{\Delta \mu_e}{c^2} \nabla \phi = -e^{-\phi/c^2} n_p \nabla \left( \frac{\Delta \mu_e}{c^2} \right) \]

\[ e^{\phi/c^2}, \] \hspace{1cm} (11)

where \( \Delta \mu_e = \mu_e - \mu_p - \mu_e. \) In the Newtonian limit \( n_p (\Delta \mu_e / c^2) \nabla \phi \ll n_p \nabla \Delta \mu_e, \) and we have

\[ \frac{j \times B}{c} = -n_p \nabla \Delta \mu_e. \] (12)

This is a Grad-Shafranov type equation for the magnetic field, as in the case of magnetic equilibria in barotropic fluids. Note that the Lorentz force density \( j \times B/c \) depends on the gradient of only one scalar function (in contrast to the non-superfluid matter, where it depends on the gradients of two scalar functions, see Glampedakis & Lasky 2016). This means that only very specific magnetic field configurations can restore hydrostatic equilibrium when neutrons are superfluid (Glampedakis & Lasky 2016). Thus, once NS temperature drops below the neutron critical temperature \( T_{cn} \) at a given point, the magnetic field has to rearrange itself to meet the new hydrostatic equilibrium condition (12). This rearrangement may result in magnetar activity and effective dissipation of the magnetic field energy on a typical timescale of NS cooling.3

To find the dissipation rate \( \dot{E}_B \) (2), we express \( \dot{B} \) from Eq. (8) for protons,

\[ E = -\frac{u_p \times B}{c} + \frac{\nabla \mu_p}{c^2} - \frac{J_{np} (u_p - u_n) + J_{en} (u_p - u_e)}{en_p}. \] (13)

The second and third terms in Eq. (13) are potential 4 and thus do not contribute to the magnetic field dissipation (see this to see this, integrate Eq. 2 by parts and use the continuity equation, \( \nabla \cdot j = 0 \). Thus,

\[ \dot{E}_B = -\int_V \left[ -\frac{u_p \times B}{c} + \frac{J_{np} (u_p - u_n) + J_{en} (u_p - u_e)}{en_p} \right] \cdot j \, dV. \] (14)

The first term here can be rewritten as

\[ \int_V \left( \frac{u_p \times B}{c} \right) \cdot j \, dV = -\int_V \left( \frac{j \times B}{c} \right) \cdot u_p \, dV. \] (15)

Substituting now (12) into (15)

\[ -\int_V \left( \frac{j \times B}{c} \right) \cdot u_p \, dV = \int_V \left( n_p \nabla \Delta \mu_e \right) \cdot u_p \, dV, \] (16)

and integrating by parts, one finds (the integral over the distant surface is omitted)

\[ \int_V \left( \frac{u_p \times B}{c} \right) \cdot j \, dV = \int_V -\nabla \left( n_p u_p \Delta \mu_e \right) \, dV. \] (17)

3 As results of Glampedakis & Lasky (2016) indicate, the same conclusion also applies if protons are superconducting.
4 In the approximation of a Newtonian star \( (\phi/c^2 \ll 1), \) employed in this paper, these terms can be presented as \( \nabla \mu_p / c + \mu_p \nabla \phi / (c^2 e) \approx \nabla (\mu_p e^\phi / c^2) / e \) and hence are indeed potential. It is interesting to note that fully relativistic calculation would not change this result.

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The divergence term here can be expressed with the help of the continuity equation for protons, \( \nabla \cdot (n_p \mathbf{u}_p) = \Delta \Gamma \), where the source \( \Delta \Gamma \) accounts for the non-equilibrium beta-processes. When \( \Delta \mu_p \ll k_B T \), \( \Delta \Gamma \) can be approximated as \( \Delta \Gamma \approx \lambda_c \Delta \mu_p \) (\( \lambda_c \) is the density and temperature-dependent coefficient and \( k_B \) is the Boltzmann constant), so that (17) reduces to

\[
\int_V \left( \mathbf{u}_n \times \frac{\mathbf{B}}{c} \right) \cdot \mathbf{j} \, dV = \int_V -\lambda_c \Delta \mu_n^2 \, dV. \tag{18}
\]

Returning now to Eq. (14), it can be represented as

\[
\dot{E}_B = -\int_V \mathbf{E} \cdot \mathbf{j} \, dV = \int_V \left[ -\lambda_c \Delta \mu_n^2 - J_{en}(\mathbf{u}_e - \mathbf{u}_n)^2 - J_{ep}(\mathbf{u}_e - \mathbf{u}_p)^2 - J_{np}(\mathbf{u}_n - \mathbf{u}_p)^2 \right] \, dV
+ \int_V (\mathbf{u}_e - \mathbf{u}_n) \left[ J_{en}(\mathbf{u}_e - \mathbf{u}_n) + J_{np}(\mathbf{u}_n - \mathbf{u}_p) \right] \, dV, \tag{19}
\]

or, in view of Eq. (10),

\[
\dot{E}_B = -\int_V \mathbf{E} \cdot \mathbf{j} \, dV = \int_V \left[ -\lambda_c \Delta \mu_n^2 - J_{en}(\mathbf{u}_e - \mathbf{u}_n)^2 - J_{ep}(\mathbf{u}_e - \mathbf{u}_p)^2 - J_{np}(\mathbf{u}_n - \mathbf{u}_p)^2 \right] \, dV. \tag{20}
\]

Clearly, magnetic field dissipates due to particle mutual transformations and relative motion (diffusion). In principle, the same expression can also be derived for non-superfluid \( npe \) matter (Gusakov et al., in preparation), but now the velocities are related by the constraint (10). Using this constraint, Eq. (20) can be rewritten as

\[
\dot{E}_B = \int_V \left[ -\lambda_c \Delta \mu_n^2 - \frac{\mathbf{J}_n^2}{\sigma_0} \right] \, dV, \tag{21}
\]

where \( \sigma_0 = e^2 n_e^2 / (J_{ep} + J_{en} / \Delta \mu_n) \) is the electrical conductivity in the absence of a magnetic field. We come to an important conclusion that in superfluid \( npe \) matter there is no magnetic field dissipation due to ambipolar diffusion: Magnetic field dissipates exclusively due to Ohmic decay and non-equilibrium particle transformations. The former is extremely inefficient in neutron stars\(^5\), while the latter strongly depends on the rate of non-equilibrium beta-processes in NS matter. A typical dissipation timescale associated with these processes can be estimated as \( t_{\text{reactions}} \sim B^2/\left(4\pi \lambda_c \Delta \mu_n^2\right) \sim 4\pi n_e^2 / (\lambda_c B^2) \). In the case of modified Urca (mUrca) processes this estimate gives\(^6\) \( t_{\text{reactions}} \gtrsim 3 \times 10^{14} T_k^{-2} B_{14}^{-2} \text{ y} \), too much to affect the magnetic field evolution. In turn, for the direct Urca (dUrca) process we get \( t_{\text{reactions}} \gtrsim 2 \times 10^9 T_k^{-1} B_{14}^{-2} \text{ y} \), i.e., dUrca can be effective dissipation agent for sufficiently strong magnetic fields.

It should be emphasized that the fact that magnetic field does not dissipate through the ambipolar diffusion in the NS region where neutrons are superfluid, was clearly realised long ago by Urpin & Shalybkov (1995, 1999). However, these authors only considered the case of vanishing temperature (\( T = 0 \)), when all neutrons condense in Cooper pairs and simply cannot scatter off the protons. In contrast, here we argue that ambipolar diffusion is not important for any temperature, even slightly smaller than the critical temperature \( T_{cn} \) (when almost all neutrons are unpaired). Note that, this conclusion is in contrast to the generally held view (e.g., Glampedakis et al. 2011; Passamonti et al. 2017) about the possible important role of ambipolar diffusion at temperatures \( T \) comparable to \( T_{cn} \), which is based on the analysis of equations, strictly valid only at \( T \ll T_{cn} \) (see Sec. 2.1 for details).

### 2.3 \( npe\mu \)-matter

An admixture of muons introduces an additional degree of freedom into the system. The dynamic equations for superfluid \( npe\mu \) mixture consist of superfluid (neutron) Eq. (4) together with the three equations on the charged components,

\[
-\mathbf{e} \left( \mathbf{E} + \frac{1}{c} \mathbf{u}_e \times \mathbf{B} \right) - \nabla \mu_e - \mu_e \frac{c^2}{\mathbf{u}_e} \nabla \phi - \frac{J_{en}}{n_e} (\mathbf{u}_e - \mathbf{u}_n) - \frac{J_{ep}}{n_p} (\mathbf{u}_e - \mathbf{u}_p) - \frac{J_{np}}{n_p} (\mathbf{u}_n - \mathbf{u}_p) = 0, \tag{22}
\]

\[
-\mathbf{e} \left( \mathbf{E} + \frac{1}{c} \mathbf{u}_p \times \mathbf{B} \right) - \nabla \mu_p - \mu_p \frac{c^2}{\mathbf{u}_p} \nabla \phi - \frac{J_{en}}{n_e} (\mathbf{u}_e - \mathbf{u}_n) - \frac{J_{np}}{n_p} (\mathbf{u}_n - \mathbf{u}_p) = 0, \tag{23}
\]

\[
\mathbf{e} \left( \mathbf{E} + \frac{1}{c} \mathbf{u}_p \times \mathbf{B} \right) - \nabla \mu_p - \mu_p \frac{c^2}{\mathbf{u}_p} \nabla \phi - \frac{J_{en}}{n_e} (\mathbf{u}_e - \mathbf{u}_n) - \frac{J_{np}}{n_p} (\mathbf{u}_n - \mathbf{u}_p) = 0. \tag{24}
\]

In Eqs. (22)–(24) \( \mathbf{u}_e \) and \( \mathbf{u}_p \) are the muon velocity and relativistic chemical potential, respectively. In analogy with Eq. (12) one can derive the force balance equation for superfluid \( npe\mu \) matter,

\[
\mathbf{j} \times \frac{\mathbf{B}}{c} = -n_p \nabla \Delta \mu_e - n_p \nabla \Delta \mu_p, \tag{25}
\]

and find that the Lorentz force density is determined by the gradients of two scalars, \( \nabla \Delta \mu_e \) and \( \nabla \Delta \mu_p \), where \( \Delta \mu_e \equiv \mu_e - \mu_p - \mu_\mu \). Therefore, comparing to superfluid \( npe \)-matter, there is more freedom to choose possible magnetic field configurations.

\(^5\) A typical timescale is \( t_{\text{Ohmic}} = 4\pi L^2 \sigma_0 / c^2 \sim 10^{14} T_k^{-5/3} \text{ yrs} \), where \( L \) is the lengthscale of the magnetic field variation (we take \( L \sim 10^6 \text{ cm} \)) and \( T_k \) is the temperature of the NS core, normalized to \( 10^8 \text{ K} \) (Shpertin 2008).

\(^6\) The coefficient \( \lambda_c \) is estimated using the formulas given in the review by Yakovlev et al. (2001).
Proceeding in a similar way as in the case of npe matter, one can show that the magnetic field dissipation rate is given by

\[ \dot{E}_B = \int_V \left[ -\lambda_e \Delta \mu_e^2 - \lambda_\mu \Delta \mu_\mu^2 - \frac{1}{2} \sum_{i,k \neq k} J_{ik}(\mathbf{u}_i - \mathbf{u}_k)^2 \right] dV, \tag{26} \]

while the velocities \( \mathbf{u}_i \) are related by (compare this result with the constraint 10)

\[ J_{en}(\mathbf{u}_e - \mathbf{u}_n) + J_{n\mu}(\mathbf{u}_n - \mathbf{u}_\mu) + J_{n\mu}(\mathbf{u}_n - \mathbf{u}_\mu) = 0. \tag{27} \]

In Eq. (26) the indices \( i, k \) run over \( n, p, e, \mu; \lambda_e \) has the same meaning as \( \lambda_e \), but is defined for Urca-reactions involving muons. Generally, as follows from Eqs. (26) and (27), the magnetic field dissipation in \( npe\mu \) matter is {
anyone associated exclusively with the Ohmic decay and non-equilibrium beta-processes. Even if we neglect the lepton interactions with neutrons (\( J_{en} = J_{n\mu} = 0 \)), and find that \( \mathbf{u}_n = \mathbf{u}_\mu \) from Eq. (27) (i.e., normal neutrons move with protons), the relative motion between electrons and muons will still enter the dissipation rate. The effect of such motion on the magnetic field dissipation can be relatively small, especially at large densities (when muons more and more resemble electrons). But this should be checked by direct calculation which is beyond the scope of the present short note. Concluding, the presence of muons (or other, more exotic, particle species) complicates the things and may, in principle, affect the magnetic field evolution in superfluid NSs.

3 CONCLUSIONS

In this note we considered a simplified illustrative problem of the magnetic field evolution in a NS, whose core is composed of superfluid neutrons, nonsuperconducting protons and electrons with possible admixture of muons. A star is assumed to be non-rotating, i.e., it does not have neutron vortices in its interiors. Perturbation of the system by the magnetic field is assumed to be small and the NS evolution is supposed to be quasistationary – standard assumptions (e.g., Goldreich & Reisenegger 1992), that allow us to neglect time derivatives and velocity-dependent nonlinear terms in the Euler-type and particle continuity equations. Bearing in mind simplifications described above, we arrived at the following conclusions:

1. Ambipolar diffusion is irrelevant for the magnetic field dissipation in superfluid npe matter at temperatures \( T \) even slightly smaller than \( T_{cn} \). Magnetic field in this case dissipates only because of Ohmic losses (inefficient mechanism in the neutron star cores) and non-equilibrium Urca processes (can be efficient if the direct Urca process is open). This result is in contrast to the results of Glampedakis et al. (2011) and Passamonti et al. (2017) who, as we argue in Sec. 2.1, used a superfluid dynamic equation for neutrons, which is correct only in the limit \( T \ll T_{cn} \).

2. Since only very specific magnetic field configurations can support hydrostatic equilibrium in superfluid npe matter (see section 2 and the work by Glampedakis & Lasky 2016), magnetic field should “feel” expansion of the superfluid region upon NS cooling and reorganize itself accordingly on a cooling timescale. This may result in an increased magnetic activity, e.g., in magnetars.

3. An admixture of muons will restore, to some extent, the role of ambipolar diffusion for the magnetic field evolution, although the magnetic energy dissipation rate (26) will differ from that in non-superfluid matter because of (i) suppression of \( np, ne, \) and \( n\mu \) collisions by neutron superfluidity (e.g., Yakovlev & Shalybkov 1991) and (ii) an additional constraint (27) relating normal particle velocities \( \mathbf{u}_i \) (\( i = n, p, e, \mu \)). In particular, if we neglect the lepton-neutron collisions, there will be no relative motion between normal neutrons and protons, \( \mathbf{u}_n = \mathbf{u}_p \).

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7 The same expression is also valid for non-superfluid \( npe\mu \) matter, but then it is not constrained by Eq. (27).

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