Modeling and Design of a Transmission Coil and Four Cascaded Receiving Coils Wireless Charging Structure With Lateral Misalignments

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ABSTRACT Magnetic resonance wireless power transfer (WPT) is an emerging technology that may create new applications for wireless power charging. However, the mutual inductance fluctuations resulting from lateral misalignments are the major factors hindering the promotion of this technology. In this paper, a structure of a transmission coil and four cascaded receiving coils for wireless charging systems is proposed to suppress fluctuations in mutual inductance with lateral misalignments. The mathematical model of the proposed structure with lateral misalignments is built based on the equivalent circuit method. The expressions of the output voltage and the efficiency are then derived by solving the system equivalent equations. In addition, a method of compensation capacitance is proposed to suppress the effect of the cross-coupling mutual inductance on the voltage and efficiency. The primary advantage of the proposed structure is that the sum of the mutual inductances between the receiving coils and the transmission coil is nearly constant with lateral misalignments. Therefore, the output voltage and efficiency can also be kept nearly constant by using the proposed structure with lateral misalignments. Simulation and experimental results validating the proposed method are shown.

INDEX TERMS Wireless power transfer (WPT), constant mutual inductance, novel coil structure, magnetic resonance coupling.

I. INTRODUCTION

Magnetic resonance wireless power transfer (WPT) systems are widely used in many applications [1]–[3], such as consumer electronics, medical devices, automatic guided vehicles, and battery charging of electric vehicles due to the advantages of high convenience, safety, and reliability [4], [5]. However, in practical application, inevitable lateral misalignments between the transmission coil and the receiving coil lead to variations in mutual inductance between the transmission coil and the receiving coil [6]–[8]. These variations in mutual inductance result in decreased efficiency and fluctuations in output voltage [9].

To improve the efficiency with lateral misalignments, a method that adjusts an equivalent load resistor [10] has been proposed. In addition, the constant output voltage is obtained by adjusting the equivalent load resistor. However, the equivalent load resistor corresponding to the maximum efficiency is not the same as that corresponding to the constant output voltage for a given mutual inductance [11], [12]. To simultaneously achieve high efficiency and constant output voltage for wireless charging systems, a maximum efficiency point tracking control scheme has been proposed [13]. When the mutual inductance between the transmission coil and the receiving coil is changed, the minimum DC input current can be tracked by real-time changes to the duty cycle of the DC-DC converter (the minimum input current is an equivalent condition of the maximum efficiency when the input voltage and the output voltage are kept constant). The output voltage is kept constant by changing the duty cycle of another DC-DC converter. However, since two DC-DC converters are added to the wireless charging systems, the system efficiency is inevitably decreased because these DC-DC converters suffer losses.
The adverse effect of misalignments on the efficiency and output voltage can be perfectly solved if the mutual inductance remains constant as the lateral misalignment occurs. There are methods using the structure of magnetic coupler coils to solve this problem. There are two main kinds of magnetic coupler coil structures. The first kind is a long-track coupler coil (the transmission coil of the long track coupler coil can be as simple as just two wires), including I-type [14] and S-type [15] couplers. These structures provide a good misalignment tolerance. However, the problem with long-track coupler coils is that the receiving coil covers only a small portion of the transmission coil, which makes the coupling coefficient very small. This poor coupling coefficient leads to low efficiency [3].

The other kind of structure is a segment coupler coil that has been proposed [3] to improve efficiency. The segment coupler coil structure includes a transmission coil and a receiving coil. The transmission coil is nearly the same size as the receiving coil, thus promoting high efficiency. However, the effect of lateral misalignment on the mutual inductance between the transmission coil and the receiving coil is considerable compared with that of the traditional two-coil structure. To address the problem, an extremely asymmetric coil structure with a large transmission coil and a small receiving coil has been proposed [16]. The misalignment tolerance is improved by using this extremely asymmetric coil structure. However, the rate of change in output voltage reaches 29%.

In conclusion, the fluctuations of the mutual inductance and output voltage need to be further reduced. In this paper, a structure consisting of a transmission coil and four receiving coils in a cascaded formation is proposed. The sum of mutual inductances between the receiving coil and the transmission coil is nearly constant with lateral misalignments. Therefore, the output voltage can also be kept nearly constant, and high efficiency can be obtained by using the proposed structure.

The remainder of the paper is organized as follows. Section II describes the two-coil WPT system. Section III describes the proposed structure and the model of the proposed system. Section IV proposes a method of compensation capacitance. Section V presents simulation results, the experimental setup, and the measurement results with lateral misalignments between the transmission coil and the receiving coils. Section VI offers a conclusion.

II. TWO-COIL SYSTEM MODEL

The two-coil WPT system with lateral misalignments is composed of two resonance coils, namely, a transmission coil and a receiving resonance coil, labeled as Tx and Rx, as shown in Fig. 1. D is the distance between Tx and Rx, C1 and C2 are the external compensation capacitances.

The two-coil WPT system can be represented in terms of lumped circuit elements (L, C, and R), as shown in Fig. 2. \( V_s \) is the voltage of the power source, \( R_1 \) is the parasitic resistor of Tx, \( R_2 \) is the parasitic resistor of Rx, \( R_s \) is the internal resistor of the power source, \( R_L \) is the load resistor, \( L_1 \) is the inductance of Tx, \( L_2 \) is the inductance of Rx, and \( M \) is the mutual inductance between Tx and Rx.

By applying Kirchhoff’s voltage law (KVL), the two-coil WPT system is presented as follows:

\[
\begin{align*}
(R_s + Z_1)I_{Tx} + j\omega MI_{Rx} &= V_s \quad (1) \\
& \quad \text{and} \quad (2)
\end{align*}
\]

where \( I_{Tx} \) is the current of Tx, \( I_{Rx} \) is the current of Rx, and \( \omega \) is the operating angular frequency [17], [18].

The currents of Tx and Rx can be obtained by solving (1) and (2).

\[
\begin{align*}
I_{Tx} &= \frac{V_s(R_L + Z_2)}{(R_s + Z_1)(R_L + Z_2) + (\omega M)^2} \\
I_{Rx} &= -\frac{j\omega MV_s}{(R_s + Z_1)(R_L + Z_2) + (\omega M)^2}
\end{align*}
\]

The efficiency of the two-coil WPT system is as follows:

\[
\eta = \frac{I_{Rx}^2 R_L V_s}{V_s I_{Tx}} = \frac{(\omega M)^2 R_L}{(R_s + Z_1)(R_L + Z_2)^2 + (\omega M)^2(R_L + Z_2)}
\]

The output voltage of the two-coil WPT system is as follows:

\[
V_L = \frac{-j\omega MV_s R_L}{(R_s + Z_1)(R_L + Z_2) + (\omega M)^2}
\]

When the two-coil WPT system operates at a resonant state, we have \( Z_1 = R_1 \) and \( Z_2 = R_2 \). The output voltage of the two-coil WPT system can be simplified as follows:

\[
V_L = \frac{-j\omega MV_s R_L}{(R_s + R_1)(R_L + R_2) + (\omega M)^2}
\]

As shown in (6), the voltage of the power source \( V_s \), the internal resistor of the power source \( R_s \), and the load
resistor $R_L$ are nearly constant, and the parasitic resistors of the coil $R_1$ and $R_2$ are also nearly constant for a given coil. The output voltage is changed as the mutual inductance is changed (lateral misalignments between Tx and Rx result in variations in mutual inductance).

III. PROPOSED STRUCTURE AND MODEL

To keep the mutual inductance constant with lateral misalignments, a structure of a transmission coil and four cascaded receiving coils for wireless charging systems is proposed. This structure is composed of a transmission resonance coil and four receiving resonance coils, labeled as Tx, Rx_1, Rx_2, Rx_3, and Rx_4, as shown in Fig. 3.

The transmission coil is larger than the receiving coils in size. In addition, the dimension of each receiving coil is the same. Rx_1 and Rx_3 are symmetric with respect to the Y-axis, Rx_2 and Rx_4 are symmetric with respect to the Y-axis, Rx_1 and Rx_2 are symmetric with respect to the Z-axis, and Rx_3 and Rx_4 are also symmetric with respect to the Z-axis. The four receiving resonance coils are placed on the same plane. The transmitter and receiver are placed in parallel. $O_0$ is the center point of the transmitter, $O_0'$ is the center point of the receiver, and $O_i$ (i=1, 2, 3, 4) is the center point of each receiving coil, respectively. $\Delta_1$ is the lateral misalignment distance between $O_1$ (or $O_3$) and the Z-axis, and $\Delta_2$ is the lateral misalignment distance between $O_2$ (or $O_4$) and the Z-axis. $a$ is the radius of Tx, and $b$ is the radius of the receiving coil. $D$ is the transmission distance between the transmitter and the receiver. Owing to the symmetry of the receiver, the lateral misalignment is only analyzed to the right (Y-axis).

Fig. 4 shows that the receiver is moved in the Y-axis direction. $\Delta$ is the lateral misalignment distance between $O_0$ and the Z-axis. When $\Delta < b$, the larger the value of $\Delta$ is, the smaller the value of $\Delta_1$ becomes, the mutual inductance between Tx and Rx_1 (or Rx_3) increases due to the decrease in $\Delta_1$; the larger the value of $\Delta$ is, the larger the value of $\Delta_2$ becomes, and the mutual inductance between Tx and Rx_2 (or Rx_4) decreases because of the increase in $\Delta_2$.

Therefore, the sum of mutual inductances between Tx and the receiving coils may change smoothly.

A. STRUCTURE OF TRANSMISSION COIL AND FOUR CASCADED RECEIVING COILS

In this section, a structure of a transmission coil and four cascaded receiving coils for wireless charging systems is proposed to address variations in mutual inductance and output voltage with lateral misalignments.

The proposed structure is composed of a transmission resonance coil, four receiving resonance coils, a power source, AC-DC rectifiers (four full-bridge rectifiers), and a load resistor, as shown in Fig. 5. The power source, via the compensation capacitance $C_1$, is connected in series to the transmission coil Tx. The power is transferred from the transmitter to the receiver using magnetic fields. The voltage of the receiving coil...
coil is converted to DC by the full-bridge rectifier. Then, the output voltage of each full-bridge rectifier is connected in series. The voltage across load is equal to the sum of the voltages of each full-bridge rectifier.

B. MATHEMATICAL MODEL OF THE PROPOSED STRUCTURE

The proposed WPT system can be represented in terms of lumped circuit elements \((L, C,\) and \(R)\), as shown in Fig. 6.

\(R_1\) is the parasitic resistor of \(Tx\), \(R_2\) is the parasitic resistor of \(Rx_1\), \(R_3\) is the parasitic resistor of \(Rx_2\), \(R_4\) is the parasitic resistor of \(Rx_3\), \(R_5\) is the parasitic resistor of \(Rx_4\), \(R_s\) is the internal resistor of the power source, and \(R_L\) is the load resistor. \(L_1\) is the inductance of \(Tx\), \(L_2\) is the inductance of \(Rx_1\), \(L_3\) is the inductance of \(Rx_2\), \(L_4\) is the inductance of \(Rx_3\), \(L_5\) is the inductance of \(Rx_4\). \(C_1\) is the external compensation capacitance of \(Tx\), \(C_2\) is the external compensation capacitance of \(Rx_1\), \(C_3\) is the external compensation capacitance of \(Rx_2\), \(C_4\) is the external compensation capacitance of \(Rx_3\), and \(C_5\) is the external compensation capacitance of \(Rx_4\). \(M_{12}\) is the mutual inductance between \(Tx\) and \(Rx_1\), \(M_{13}\) is the mutual inductance between \(Tx\) and \(Rx_2\), \(M_{14}\) is the mutual inductance between \(Tx\) and \(Rx_3\), \(M_{15}\) is the mutual inductance between \(Rx_1\) and \(Rx_2\), \(M_{23}\) is the mutual inductance between \(Rx_1\) and \(Rx_2\), \(M_{24}\) is the mutual inductance between \(Rx_1\) and \(Rx_3\), \(M_{25}\) is the mutual inductance between \(Rx_1\) and \(Rx_4\), \(M_{34}\) is the mutual inductance between \(Rx_2\) and \(Rx_3\), \(M_{35}\) is the mutual inductance between \(Rx_2\) and \(Rx_4\), and \(M_{45}\) is the mutual inductance between \(Rx_3\) and \(Rx_4\).

The equivalent circuit model for the proposed structure consists of an AC-AC module and AC-DC module, as shown in Fig. 6. The red dotted line rectangle is the AC-AC module, and the blue dotted line rectangle is the AC-DC module.

By applying Kirchhoff’s voltage law (KVL), the AC-AC module of the proposed WPT system is presented as follows:

\[
\begin{align*}
(R_5 + Z_1)I_1 + j\omega M_{12}I_2 + j\omega M_{13}I_3 + j\omega M_{14}I_4 + j\omega M_{15}I_5 & = V_s \\
\langle j\omega M_{12}I_1 + Z_2I_2 + j\omega M_{23}I_3 + j\omega M_{24}I_4 + j\omega M_{25}I_5 \rangle & = V_2 \\
\langle j\omega M_{13}I_1 + j\omega M_{23}I_2 + Z_3I_3 + j\omega M_{34}I_4 + j\omega M_{35}I_5 \rangle & = V_3 \\
\langle j\omega M_{14}I_1 + j\omega M_{24}I_2 + j\omega M_{34}I_3 + Z_4I_4 + j\omega M_{45}I_5 \rangle & = V_4 \\
\langle j\omega M_{15}I_1 + j\omega M_{25}I_2 + j\omega M_{35}I_3 + j\omega M_{45}I_4 + Z_5I_5 \rangle & = V_5
\end{align*}
\]  

(7)

where \(I_1\) is the current of \(Tx\), \(I_2\) is the current of \(Rx_1\), \(I_3\) is the current of \(Rx_2\), \(I_4\) is the current of \(Rx_3\), and \(I_5\) is the current of \(Rx_4\). \(V_s\) is the voltage of \(Rx_1\), \(V_2\) is the voltage of \(Rx_2\), \(V_3\) is the voltage of \(Rx_3\), and \(V_4\) is the voltage of \(Rx_4\). We disregard the loss of the full-bridge rectifier, according to the law of conservation of energy; the AC-DC module of the proposed WPT system is presented as follows:

\[
\begin{align*}
|V_{2I_2} \cos \alpha_2| &= |V_{o2}I_o| \\
|V_{3I_3} \cos \alpha_3| &= |V_{o3}I_o| \\
|V_{4I_4} \cos \alpha_4| &= |V_{o4}I_o| \\
|V_{5I_5} \cos \alpha_5| &= |V_{o5}I_o| \\
|V_{o2}I_o + V_{o3}I_o + V_{o4}I_o + V_{o5}I_o| &= |V_oI_o|
\end{align*}
\]  

(9)

where \(V_{oi}\) (\(i = 2, 3, 4, 5\)) is the output voltage of each full-bridge rectifier. \(V_i\) (\(i = 2, 3, 4, 5\)) is the input voltage of each full-bridge rectifier. \(V_{oi}\) is the output voltage of the load resistor. \(I_o\) is the current of the load resistor. \(\alpha_2\) is the phase angle between \(V_2\) and \(I_2\), \(\alpha_3\) is the phase angle between \(V_3\) and \(I_3\), \(\alpha_4\) is the phase angle between \(V_4\) and \(I_4\), and \(\alpha_5\) is the phase angle between \(V_5\) and \(I_5\).

When the WPT system operates at a resonant state, we have

\[
\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0
\]  

(10)

Because the output terminal of each full-bridge rectifier is connected in series, the relationship among the currents of the receiving coils can be obtained as follows:

\[
I_2 = I_3 = I_4 = I_5
\]  

(11)

The input voltage of each full-bridge rectifier \((V_2, V_3, V_4, V_5)\) is a square wave. Fig. 7 shows a square wave. \(U_{2_m}\) is the amplitude of the square wave. \(T\) is the period of the square wave. According to the Fourier series, the square wave can be
expressed as follows:

\[
U_2(t) = \frac{4U_{2,m}}{\pi} \left[ \sin \omega t + \frac{1}{3} \sin 3\omega t + \ldots \right] + \frac{1}{n} \sin n\omega t \quad (n = 2k + 1, k = 1, 2, 3, \ldots) \quad (12)
\]

where \( \omega \) is the fundamental angular frequency.

According to (9)-(12), the relationship between the RMS of the input voltage of each full-bridge rectifier \( V_i \) (\( i = 2, 3, 4, 5 \)) and the maximum of the input voltage of each full-bridge rectifier \( V_{i,m} \) can be obtained as follows:

\[
\begin{align*}
V_2 &= 0.9V_{2,m} \\
V_3 &= 0.9V_{3,m} \\
V_4 &= 0.9V_{4,m} \\
V_5 &= 0.9V_{5,m}
\end{align*}
\]

(13)

Because the input voltage of each full-bridge rectifier \( V_2, V_3, V_4, V_5 \) is a square wave, the relationship between the input voltage of each full-bridge rectifier \( V_2, V_3, V_4, V_5 \) and the output voltage of each full-bridge rectifier \( V_{o2}, V_{o3}, V_{o4}, V_{o5} \) can be obtained as follows:

\[
\begin{align*}
|V_{o2}| &= |V_{2,m}| \\
|V_{o3}| &= |V_{3,m}| \\
|V_{o4}| &= |V_{4,m}| \\
|V_{o5}| &= |V_{5,m}|
\end{align*}
\]

(14)

According to (9)-(14), the relationship between \( I_o \) and \( I_2 \) is as follows:

\[
|I_o| = 0.9|I_2| \quad (15)
\]

According to (7)-(15), the system model of the proposed structure can be expressed as follows:

\[
\begin{align*}
(R_S + Z_L)I_1 + j\omega M_{12}I_2 + j\omega M_{13}I_2 + j\omega M_{14}I_2 \\
\quad + j\omega M_{15}I_2 &= V_s \\
(j\omega M_{12} + j\omega M_{13} + j\omega M_{14} + j\omega M_{15})I_1 \\
\quad + (Z_2 + j\omega M_{23} + j\omega M_{24} + j\omega M_{25})I_2 \\
\quad + (Z_3 + j\omega M_{23} + j\omega M_{24})I_2 \\
\quad + (Z_4 + j\omega M_{24} + j\omega M_{24})I_2 + (Z_5 \\
\quad + j\omega M_{25} + j\omega M_{35} + j\omega M_{45})I_2 + (8\pi^2)I_2R_L &= 0
\end{align*}
\]

(16)

The current expressions can be obtained by solving (16).

\[
\begin{align*}
I_1 &= \frac{V_oB}{Z_1B - A^2} \\
I_2 &= -\frac{V_oA}{Z_1B - A^2}
\end{align*}
\]

(17)

where \( A = j\omega M_{12} + j\omega M_{13} + j\omega M_{14} + j\omega M_{15} \)

\[
B = Z_2 + j\omega M_{23} + j\omega M_{24} + j\omega M_{25} + Z_3 + j\omega M_{23} + j\omega M_{24} + j\omega M_{24} + j\omega M_{24} + j\omega M_{25} + j\omega M_{25} + j\omega M_{35} + j\omega M_{45} + \frac{8}{\pi^2}R_L
\]

The output voltage of the load resistor can be obtained as follows:

\[
V_o = |I_oR_L| = \left| \frac{0.9R_LV_o}{Z_1B - A^2} \right| \quad (18)
\]

According to (17) and (18), the expressions of output power and the efficiency can be obtained as follows:

\[
P_{out} = V_o^2/R_L = \left| \frac{0.9V_o}{Z_1B - A^2} \right|^2R_L \quad (19)
\]

\[
\eta = \frac{P_{out}}{P_{in}} = \left| \frac{0.81R_LA^2}{(Z_1B - A^2)^2} \right| \quad (20)
\]

According to (19) and (20), the output power and the efficiency can be calculated.

**IV. METHOD OF COMPENSATION CAPACITANCE**

In practical applications, the cross-coupling mutual inductance of each receiving coil cannot be neglected. The resonance frequency may be changed, the system efficiency may be decreased, and the output voltage may be changed when the cross-coupling mutual inductance of each receiving coil is considered.

In this section, a method of compensation capacitance of each receiving coil is used to suppress the effect of the cross-coupling mutual inductance on the output voltage and efficiency. The cross-coupling mutual inductance and self-inductance can be compensated by using compensation capacitance. According to (7) and (8), (21) can be obtained as follows:

\[
\begin{align*}
\frac{1}{j\omega L_2} + 1/j(\omega C_2) + j\omega M_{23} + j\omega M_{24} + j\omega M_{25} &= 0 \\
\frac{1}{j\omega L_3} + 1/j(\omega C_3) + j\omega M_{34} + j\omega M_{35} &= 0 \\
\frac{1}{j\omega L_4} + 1/j(\omega C_4) + j\omega M_{45} &= 0 \\
\frac{1}{j\omega L_5} + 1/j(\omega C_5) + j\omega M_{25} + j\omega M_{35} + j\omega M_{45} &= 0
\end{align*}
\]

(21)

According to (21), the compensation capacitance of each receiving coil is as follows:

\[
\begin{align*}
C_2 &= \frac{1}{\omega^2(L_2 + M_{23} + M_{24} + M_{25})} \\
C_3 &= \frac{1}{\omega^2(L_3 + M_{34} + M_{35})} \\
C_4 &= \frac{1}{\omega^2(L_4 + M_{24} + M_{34} + M_{45})} \\
C_5 &= \frac{1}{\omega^2(L_5 + M_{25} + M_{35} + M_{45})}
\end{align*}
\]

(22)
Substituting (22) into (7), the current expression (17) can be simplified as

\[
\begin{align*}
I_1 &= \frac{V_s(R_2 + R_3 + R_4 + R_5 + 0.81R_L)}{(R_1 + R_3 + R_4 + R_5 + 0.81R_L) - \omega M_{12} + j\omega M_{13} + j\omega M_{14} + j\omega M_{15}} \\
I_2 &= -\frac{V_s(j\omega M_{12} + j\omega M_{13} + j\omega M_{14} + j\omega M_{15})^2}{(R_1 + R_3 + R_4 + R_5 + 0.81R_L) - \omega M_{12} + j\omega M_{13} + j\omega M_{14} + j\omega M_{15}}
\end{align*}
\] (23)

The output voltage of the load resistor can be simplified as

\[
V_o = \frac{0.9R_LV_s}{R_1(R_2 + R_3 + R_4 + R_5 + 0.81R_L) - A^2} \quad (24)
\]

When the parameters of each coil are given, \(V_s, R_1, R_2, R_3, R_4, R_5, \) and \(R_L\) are nearly constant, and the output voltage of the load resistor \(V_o\) is only dependent on \(A\). When the receiving coils are moved in the \(Y\)-axis direction and the lateral misalignment \(\Delta < b\) (\(b\) is the radius of each receiving coil), \(A\) may be kept nearly constant; therefore, the output voltage of the load resistor is nearly constant with the lateral misalignment \(\Delta < b\).

The expression of output power (19) can be simplified as

\[
P_{out} = \frac{0.9V_s(j\omega M_{12} + j\omega M_{13} + j\omega M_{14} + j\omega M_{15})^2 R_L}{(R_1 + R_3 + R_4 + R_5 + 0.81R_L) - \omega M_{12} + j\omega M_{13} + j\omega M_{14} + j\omega M_{15}} \quad (25)
\]

The expression of efficiency (20) can be simplified as

\[
\eta = \frac{0.81R_L(j\omega M_{12} + j\omega M_{13} + j\omega M_{14} + j\omega M_{15})^2}{[R_1(R_2 + R_3 + R_4 + R_5 + 0.81R_L) - \omega M_{12} + j\omega M_{13} + j\omega M_{14} + j\omega M_{15}^2]} \times \frac{R_2 + R_3 + R_4 + R_5 + 0.81R_L}{R_1(R_2 + R_3 + R_4 + R_5 + 0.81R_L)} \quad (26)
\]

According to (24)-(26), the efficiency, output voltage, and output power are independent of the cross-coupling mutual inductance of each receiving coil when using the proposed compensation capacitance method.

V. SIMULATION & EXPERIMENTAL VERIFICATION
A. EXPERIMENTAL SETUP
To validate the proposed structure, a prototype model of the system has been built, as shown in Fig. 8. This model is composed of the DC voltage source, a transmission resonance coil, four receiving resonance coils, an H-bridge inverter, four full-bridge rectifiers, and the load. The value of the DC voltage source is 48 V. The overall setup is shown in Fig. 8(a). The transmission resonance coil is shown in Fig. 8(b). The receiving resonance coils are shown in Fig. 8(c). The diameter of the transmission resonance coil is 38 cm with a pitch of 0 cm for approximately 8 turns. Tx is made from 500-strand AWG 38 Litz-wire. The radius of each receiving resonance coil is 15 cm with a pitch of 0 cm for approximately 15 turns. All receiving coils are made from 300-strand AWG 38 Litz-wire. An H-bridge inverter is used at the transmitter side to provide AC excitation, as shown in Fig. 8(d). The H-bridge inverter contains four SiC MOSFETs (C2M0080120D). Four full-bridge rectifiers are used at the receiver side to convert AC to DC. In future research, the power level of the prototype will be increased, and these full-bridge rectifiers will continue to be used in the high-power system. The full-bridge rectifier contains four diodes (D83-06) with a voltage rating of 60 V. The parameters of each coil can be calculated [19]–[21]. Then, an impedance analyzer is used to extract the parameters.
in (7) and (8). The original resonance frequency is set to be 85.0 kHz. The load resistor is set to be 15.0 Ω. The transmission distance is 15 cm. The parameters of the resonance coils are listed in Table 1.

| Symbol | Quantity | Value   |
|--------|----------|---------|
| L₁     | the inductance of Tx | 51.8 µH |
| L₂     | the inductance of Rx₁ | 104.6 µH |
| L₃     | the inductance of Rx₂ | 104.9 µH |
| L₄     | the inductance of Rx₃ | 104.1 µH |
| L₅     | the inductance of Rx₄ | 105.0 µH |
| C₁     | compensation capacitance of Tx | 67.68 nF |
| C₂     | compensation capacitance of Rx₁ | 30.34 nF |
| C₃     | compensation capacitance of Rx₂ | 30.22 nF |
| C₄     | compensation capacitance of Rx₃ | 30.40 nF |
| C₅     | compensation capacitance of Rx₄ | 30.21 nF |
| R₁     | the parasitic resistor of Tx | 0.08 Ω |
| R₂     | the parasitic resistor of Rx₁ | 0.14 Ω |
| R₃     | the parasitic resistor of Rx₂ | 0.14 Ω |
| R₄     | the parasitic resistor of Rx₃ | 0.14 Ω |
| R₅     | the parasitic resistor of Rx₄ | 0.14 Ω |
| f₀     | the original resonance frequency | 85.0 kHz |

**B. MUTUAL INDUCTANCE RESULTS**

The moving direction of the receiver is denoted on the Y-axis, and the coordinates are marked in Fig. 4 (owing to the symmetry of the receiver, the lateral misalignment is only analyzed to the right (Y-axis)). The parameters of each receiving coil are nearly the same.

Fig. 9(a) shows the simulated and measured mutual inductances between the transmission coil and each receiving coil with different lateral misalignments. It can be seen that the mutual inductance $M_{13}$ is changed from 2.52 µH (coupling coefficient equals to 0.0342) to 0.43 µH (coupling coefficient equals to 0.0058) as the misalignment is varied from 0 mm to 240 mm. $M_{12}$ is changed from 2.52 µH (coupling coefficient equals to 0.0343) to 5.27 µH (coupling coefficient equals to 0.0718) as the misalignment is varied from 0 mm to 160 mm and from 5.27 µH (coupling coefficient equals to 0.0718) to 4.11 µH (coupling coefficient equals to 0.0560) as the misalignment is varied from 160 mm to 240 mm.

Fig. 9(b) shows the sum of the simulated and measured mutual inductances $M_{12}$, $M_{13}$, $M_{14}$, and $M_{15}$. The mutual inductance is changed from 17.62 µH to 11.37 µH when the misalignment is varied from 0 mm to 200 mm. The rate of change in mutual inductance is 35.2%. The rate of change in mutual inductance with the traditional structure is larger than that with the proposed structure. Table 2 shows the simulated and measured mutual inductances among the receiving coils. The measured results agree with the simulated results.

**FIGURE 9.** Measured and simulated mutual inductances.


### TABLE 2. Mutual inductances among receiving coils.

| Symbol | Measured mutual inductances (µH) | Simulation mutual inductances (µH) |
|--------|----------------------------------|-----------------------------------|
| $M_{23}$ | 4.97                             | 5.06                              |
| $M_{24}$ | 4.92                             | 5.07                              |
| $M_{34}$ | 1.08                             | 1.16                              |
| $M_{35}$ | 1.23                             | 1.16                              |
| $M_{45}$ | 4.93                             | 5.06                              |
| $M_{45}$ | 5.04                             | 5.06                              |

### FIGURE 10. Input voltage and input current of Tx.

(a) Input voltage and input current of Tx with the traditional compensation capacitance method  
(b) Input voltage and input current of Tx with the proposed method

### C. VERIFICATION OF THE PROPOSED METHOD

We test the input voltage and input current of Tx with the traditional compensation capacitance method and the proposed method, respectively. When the traditional compensation capacitance method is used, Fig. 10(a) shows the waveform of the input voltage and the input current of the coil Tx. It can be clearly seen that the input current is not in phase with the input voltage because the resonance frequency is changed by the cross-coupling mutual inductance of each receiving coil. Whereas the proposed method is used, Fig. 10(b) shows the waveform of the input voltage and the input current of Tx. It is clearly seen that input current is in phase with input voltage because the effect of the cross-coupling mutual inductance on the resonance frequency is compensated by the proposed method. The validity of the proposed method is thus verified.

The misalignment $\Delta$ is set to 0 mm, 40 mm, 80 mm, 120 mm, 160 mm, 200 mm and 240 mm, respectively. We test the input voltage, input current, and the output voltage, respectively. Fig. 11 shows the output voltage versus misalignments. As shown in Fig. 11, the output voltage is equal to 94.2 V with $\Delta = 0$ mm, the maximum output voltage equals to 100.7 V with $\Delta = 240$ mm; whereas the minimum output voltage is equal to 90.0 V with $\Delta = 160$ mm. These results are consistent with the calculation results of (18). The variation in output voltage is only 6.90% when the misalignment is varied from 0 mm to 240 mm, whereas the output voltage variation is only 4.48% when the misalignment is varied from 0 mm to 200 mm.

Based on the values of the input voltage, input current, and output voltage with different misalignments, the efficiency can be calculated. Fig. 12 shows the efficiency versus misalignments. It is interesting to note that the efficiency is approximately 77.2% when the misalignment is varied from 0 mm to 240 mm.

A performance comparison between other references and our work is shown in Table 3. Compared with lateral misalignments in references [14]–[16], the Y-direction misalignment is the largest in our manuscript. And the X-direction misalignment is the same as that of the Y-direction. The rates of change in mutual inductance, efficiency, and output power...
are the smallest. In addition, the sizes of Tx and Rx are the smallest. And the output voltage is kept nearly constant without the voltage regulator with misalignments. According to the SAE J2954 standard, the X-direction misalignment is 100 mm and the Y-direction misalignment is 75 mm. Therefore, the X-direction and Y-direction misalignments meet the SAE J2954 standard in the proposed structure.

VI. CONCLUSION

In this paper, a structure of a transmission coil and four cascaded receiving coils for wireless charging systems is proposed. A mathematical model of the proposed structure with lateral misalignments is built based on the equivalent circuit method. Then, the expressions of the output voltage and the efficiency are derived by solving the system equivalent equations. Moreover, the effect of the cross-coupling mutual inductance on the output voltage and efficiency is suppressed via the proposed compensation capacitance method.

The advantage of the proposed structure is that the sum of the mutual inductances between the receiving coils and the transmission coil is nearly constant with different lateral misalignments. Therefore, the output voltage and efficiency can also be kept nearly constant without any auxiliary equipment under different lateral misalignments. It is interesting to note that the maximum misalignment can be achieved at 240 mm, which is larger than the radius of Tx. Therefore, the proposed structure can be applied not only to static wireless charging systems but also to dynamic wireless charging systems for electric vehicles or automated guided vehicles. Future studies should optimize the parameters of the proposed structure to obtain larger and more constant mutual inductance with different lateral misalignments.

REFERENCES

[1] F. Musavi and W. Eberle, “Overview of wireless power transfer technologies for electric vehicle battery charging,” IET Power Electron., vol. 7, no. 1, pp. 60–66, Jan. 2014.
[2] C. C. Mi, G. Buja, S. Y. Choi, and C. T. Rim, “Modern advances in wireless power transfer systems for roadway powered electric vehicles,” IEEE Trans. Ind. Electron., vol. 63, no. 10, pp. 6533–6545, Oct. 2016.
[3] S. Li and C. Chris Mi, “Wireless power transfer for electric vehicle applications,” IEEE J. Emerg. Sel. Topics Power Electron., vol. 3, no. 1, pp. 4–17, Mar. 2015.
[4] S. Y. Choi, B. W. Gu, S. Y. Jeong, and C. T. Rim, “Advances in wireless power transfer systems for roadway-powered electric vehicles,” IEEE J. Emerg. Sel. Topics Power Electron., vol. 3, no. 1, pp. 18–36, Mar. 2015.
[5] Z. Li, C. Zhu, J. Jiang, K. Song, and G. Wei, “A 3-kW wireless power transfer system for sightseeing car supercapacitor charge,” IEEE Trans. Power Electron., vol. 32, no. 5, pp. 3301–3316, May 2017.
[6] Y. Zhang, K. Chen, F. He, Z. Zhao, T. Lu, and L. Yuan, “Closed-form oriented modeling and analysis of wireless power transfer system with constant-voltage source and load,” IEEE Trans. Power Electron., vol. 31, no. 5, pp. 3472–3481, May 2016.
[7] S. Huang, Z. Li, Y. Li, X. Yuan, and S. Cheng, “A comparative study between novel and conventional four-resonator coil structures in wireless power transfer,” IEEE Trans. Magn., vol. 50, no. 11, pp. 1–4, Nov. 2014.
[8] K. Fotopoulou and B. W. Flynn, “Wireless power transfer in loosely coupled links: Coil misalignment model,” IEEE Trans. Magn., vol. 47, no. 2, pp. 416–430, Feb. 2011.
[9] J. Wang, J. Li, S. L. Ho, W. N. Fu, Y. Li, H. Yu, and M. Sun, “Lateral and angular misalignments analysis of a new PCB circular spiral resonant wireless charger,” IEEE Trans. Magn., vol. 48, no. 11, pp. 4522–4525, Nov. 2012.
[10] S. D. Huang, Z. Q. Li, and Y. Li, “Transfer efficiency analysis of magnetic resonance wireless power transfer with intermediate resonant coil,” J. Appl. Phys., vol. 115, no. 17, May 2014, Art. no. 17A336.
[11] Y. Zhang, T. Lu, Z. Zhao, F. He, K. Chen, and L. Yuan, “Employing load coils for multiple loads of resonant wireless power transfer,” IEEE Trans. Power Electron., vol. 30, no. 11, pp. 6174–6181, Nov. 2015.
[12] Y. Zhang, T. Lu, Z. Zhao, K. Chen, F. He, and L. Yuan, “Wireless power transfer to multiple loads over various distances using relay resonators,” IEEE Microw. Wireless Compon. Lett., vol. 25, no. 5, pp. 337–339, May 2015.
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