Formation of bremsstrahlung in an absorptive QED/QCD medium

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The radiative energy loss of a relativistic charge in a dense, absorptive medium can be affected significantly by damping phenomena. The effect is more pronounced for large energies of the charge and/or large damping of the radiation. This can be understood in terms of a competition between the formation time of bremsstrahlung and a damping time scale. We discuss this competition in detail for the absorptive QED and QCD medium, focusing on the case in which the mass of the charge is large compared to the in-medium mass of the radiation quanta. We identify the regions in energy and parameter space, in which either coherence or damping effects are of major importance for the radiative energy loss spectrum. We show that damping phenomena can lead to a stronger suppression of the spectrum than coherence effects.

PACS numbers: 12.38.Mh, 25.75.-q, 52.20.Hv
Keywords: radiative energy loss, radiation formation time, damping of radiation, density effect, LPM effect, Ter-Mikaelian effect

I. INTRODUCTION

The quenching of jets associated with the strong suppression of the yields of high transverse momentum hadrons in comparison with proton-proton collisions has been observed experimentally in relativistic heavy-ion collisions at the Relativistic Heavy Ion Collider at BNL [1, 2] and at the Large Hadron Collider at CERN, cf. [3] for a recent review. Originally, such a signal was predicted by Bjorken [4] as an evidence for the formation of a deconfined plasma state of QCD matter, in which propagating partons suffer from an enhanced in-medium energy loss.

Considered as dominant contribution, the radiative energy loss of relativistic partons due to medium-induced gluon radiation was, for instance, studied in [5–13]. Important for the discussion of bremsstrahlung in a dense medium are possible coherence effects. These lead to a suppression of the radiation spectrum compared to the spectrum due to incoherent scatterings. This was first realized by Landau, Pomeranchuk [14] and Migdal [15] who studied in QED the possibility of a destructive interference of radiation amplitudes, which stem from multiple scatterings of an electric charge traversing dense matter, within the formation time of radiation (LPM effect). This effect was later generalized to QCD in [16, 17].

Likewise important are modifications of the radiation spectrum due to the dielectric polarization of the medium, which is known as the Ter-Mikaelian (TM) effect [18]. The medium polarization results in a change of the dispersion relation of the radiated quanta. In this way, the spectrum becomes regulated in the soft and collinear regions. In QCD, the TM effect on the radiation spectrum was investigated by considering bremsstrahlung gluons with a finite in-medium mass [19].

The concept of a formation time (or length) of radiation turned out to be extremely fruitful for the discussion of radiative energy losses [7]. It allows for a semi-quantitative understanding of the pattern of the radiation spectra. The spectrum reduction due to the LPM effect, for example, was qualitatively analyzed in this way for QED in [19, 20] and for QCD in the review [21]: If the radiation formation length, which depends on the energy ω of the emitted quantum and on the properties of the dense medium (e.g. the transport coefficient ̂q), is large compared to the mean free path in the medium, multiple scatterings will contribute coherently to the emission of a single radiation quantum. This represents the LPM regime of coherent radiation. In the opposite case, radiation quanta will be formed independently at each individual scattering. This represents the Bethe-Heitler (BH) regime of incoherent radiation.

In this work, we extend this analysis in order to include damping phenomena within an absorptive medium into the discussion. The latter are responsible for a reduction of the radiative energy loss of the charge and the associated radiative energy loss spectrum as was recently studied in [22]. We stress that this behaviour is not a simple consequence of the rather trivial reduction of any radiation spectrum in the far distance due to the absorption of already formed radiation but a result of the effect which damping has on the creation of the radiation itself. Just as the TM effect alters the radiation probability in the soft part of the spectrum [23], damping mechanisms lead also to a modification of the probability for emitting bremsstrahlung. This can be understood by viewing the formation of bremsstrahlung to be hampered by damping effects, in particular, when formation times become large.

The article is organized as follows: In Sec. II, our results [22] for the radiative energy loss spectrum per unit length of a relativistic charge in an absorptive, electromagnetic plasma are reviewed. We highlight that for a
semi-quantitative of this spectrum both the radiation formation time, as discussed in Sec. IIIA, and the time scale associated with damping effects are important. Their competition is analyzed in detail in Sec. IIIB.

This allows for an identification of the regions in energy and parameter space, in which damping phenomena are of importance. A physical discussion in terms of the spectra is given in Sec. IIC.

In Sec. IV we apply the methods of Sec. III in order to study phenomenologically the influence of damping effects on the gluon bremsstrahlung spectrum in the hot QCD plasma and advocate some possible physical consequences in the conclusions in Sec. V, where our results are summarized. Throughout this work natural units are used, i.e. $\hbar = c = 1$ with $\hbar c \simeq 0.197$ GeV·fm.

II. RADIATIVE ENERGY LOSS SPECTRUM IN AN ABSORPTIVE PLASMA

The radiative energy loss spectrum per unit length of an asymptotic, relativistic charge $q$ with energy $E$ and mass $M$ (Lorentz-factor $\gamma = E/M$) traversing a polarizable and absorptive plasma was found in [22] to be substantially reduced by both medium polarization and damping effects. This study was performed for an electromagnetic plasma, where small fractional photon energies $\omega/E$ were considered. In [22], the dense medium was modelled by a complex squared index of refraction,

$$n^2(\omega) = 1 - \frac{m^2}{\omega^2} + \frac{2\Gamma}{\omega},$$

with $m$ and $\Gamma$ accounting for the in-medium mass and damping rate of the radiation quanta, respectively.

Considering an infinite medium with permeability $\mu(\omega) = 1$, one finds for the radiative energy loss spectrum for positive $\omega$

$$-\frac{d^2W}{dzd\omega} \simeq \frac{\alpha \hat{q} \omega}{6\pi E^2} \int_0^{\infty} d\tilde{t} \mathcal{F}(\tilde{t}) \times \sin \left[ \omega\tilde{t} \left( 1 - |n_\gamma|\beta \right) + \frac{\omega |n_\gamma|\hat{q}}{12E^2} \tilde{t}^2 \right]$$

(2)

in linear response theory [22]. Here, $\alpha = q^2/(4\pi)$ is the coupling, $\beta = \sqrt{1 - 1/\gamma^2} \simeq 1 - 1/(2\gamma^2)$, $n_\gamma$ is the real part of $n(\omega) = n_e(\omega) + i n_\gamma(\omega)$ and $\hat{q}$ denotes the mean accumulated transverse momentum squared of the deflected charge per unit time. We note that in [22] $\hat{q}$ denoted only one-half of the mean accumulated transverse momentum squared of the deflected charge per unit time, see also [23].

The factor $\mathcal{F}(\tilde{t})$ in Eq. (2) enters the spectrum for every charge trajectory defining $\Delta\tilde{r} \equiv \tilde{r}(t) - \tilde{r}(t')$ with $t = t - t'$ and reads $\mathcal{F} \equiv \exp[-\omega|n_\gamma|\Delta\tilde{r}]$, cf. [22, 24]. Since $\Delta\tilde{r} \geq 0$, it introduces a genuine damping factor in the spectrum Eq. (2) for an absorptive medium which is related to the absolute value of the imaginary part of $n(\omega)$. A specific form for $\mathcal{F}(\tilde{t}) \equiv \exp[-\omega|n_\gamma|\beta \tilde{t} (1 - \hat{q}\tilde{t}/(12E^2))]$ was used in [22]. This form is a consequence of the particular trajectory considered in [14] yielding $\langle \Delta\tilde{r} \rangle \simeq \beta \tilde{t} [1 - \hat{q}\tilde{t}/(12E^2)]$, which takes explicitly into account the effect of multiple scatterings in the approximation of small deflection angles accumulated within the time-duration $\tilde{t}$. The made approximation is, however, strictly valid only for $\tilde{t} \ll t_{\text{diff}} = 6E^2/\hat{q}$ [23]. For larger times a different dependence than the one used in [22] is to be expected since, physically, $\mathcal{F}(\tilde{t})$ should decrease monotonically. Since for $\tilde{t} < t_{\text{diff}}$ the term quadratic in $\tilde{t}$, being proportional to $\tilde{t}/t_{\text{diff}}$, represents only a minor correction to $\omega|n_\gamma|\beta \tilde{t}$ in $\mathcal{F}(\tilde{t})$ and since $t_{\text{diff}}$ is indeed a very large time scale for relativistic charges, in the following it suffices to consider only the linear time-dependence in the exponential damping factor, i.e.

$$\mathcal{F}(\tilde{t}) \simeq \exp[-\tilde{t}/t_d],$$

(3)

where we define

$$t_d \simeq (\omega|n_\gamma|\beta)^{-1}$$

(4)
as damping time scale.

In the limit $n_\gamma = 1$ and $n_\gamma = 0$, Eq. (2) becomes

$$-\frac{d^2W}{dzd\omega} \simeq \frac{\alpha \hat{q} \omega}{6\pi M^2} \int_0^{\infty} d\tilde{t} \sin \left[ \omega\tilde{t} (1 - \beta) + \omega\tilde{t}/12E^2 \tilde{t}^2 \right],$$

(5)

which agrees with the result for the radiative spectrum reported in [14] if $\hat{q}$ is properly identified with the parameters used therein. Substituting in Eq. (5) $\tilde{t}$ by $u/\omega(1 - \beta)$, one obtains

$$-\frac{d^2W}{dzd\omega} \simeq \frac{\alpha \hat{q}}{3\pi M^2} \int_0^\infty du \sin \left[ u + \mathcal{A}u^2 \right],$$

(6)

where $\mathcal{A} = \hat{q}E^2/(3\omega M^4)$. The latter integral may be evaluated analytically in terms of the known Fresnel-integrals $\mathcal{S}(y)$ and $\mathcal{C}(y)$ resulting in

$$-\frac{d^2W}{dzd\omega} \simeq \frac{\alpha \hat{q}}{3\pi M^2} \sqrt{\frac{\pi}{2\mathcal{A}}} \left\{ \cos \left( \frac{1}{4\mathcal{A}} \right) [1 - 2\mathcal{S}(y)] + \sin \left( \frac{1}{4\mathcal{A}} \right) [2\mathcal{C}(y) - 1] \right\}$$

(7)

with $y^{-1} = \sqrt{2\mathcal{A}}$. For small $\mathcal{A}$, one finds formally from Eq. (7) that $d^2W \simeq d^2W_{\text{BH}}$ with

$$-\frac{d^2W_{\text{BH}}}{dzd\omega} = \frac{\alpha \hat{q}}{3\pi M^2},$$

(8)

cf. [25], which is equivalent to the BH-result for the bremsstrahlung spectrum from incoherent scatterings, cf. e.g. [19, 21, 22]. More precisely, the relative deviation of Eq. (8) from Eq. (10) is less than 10% for $\mathcal{A} < 0.026$. This case is achieved for large $\omega$, whenever $E \ll 3M^4/\hat{q}$. The $\omega$-independent result in Eq. (8) provides a particularly suitable reference point for an analysis of in-medium effects on the radiative energy loss spectrum.
In [22], it has been shown that for non-zero \( n \) the differential spectrum in Eq. (2) is reduced with increasing \( E \) (for fixed \( \Gamma \)) or with increasing \( \Gamma \) (for fixed \( E \)). This is a consequence of the increasing influence of \( \mathcal{F}(\hat{t}) \) in Eq. (2) with increasing \( \Gamma \), but also with decreasing \( \omega / E \), as was discussed in [25]. The behaviour of the spectrum can be understood by analyzing the competition of two different time scales: The formation time of radiation for negligible damping \( t_f \), and the damping time, \( t_d \), at which the radiation amplitude in Eq. (2) is essentially suppressed due to \( \mathcal{F}(\hat{t}) \).

The formation time of radiation in the limit \( \Gamma \ll m \ll \omega \), i.e. in the case of negligible damping effects, can be found, following [14], from a condition for the phase of the oscillating function in Eq. (2) reading

\[
 t_f [\omega - k(\omega)\beta] + t_f^2 \frac{qk(\omega)\beta}{12E^2} \simeq 1
\]

with \( k(\omega) = \sqrt{\omega^2 - m^2} \). The first term, which is linear in \( t_f \), is specific for a single scattering process, while the second term, which is quadratic in \( t_f \), is genuine for multiple scatterings [14]. Considering Eq. (9) either the linear or the quadratic term in \( t_f \) as the dominant contribution, \( t_f \) may be estimated by \( t_f \simeq \min\{t_f^{(s)}, t_f^{(m)}\} \), i.e. the minimum of two time scales \( t_f^{(s)} \) (the photon formation time in incoherent (single) scatterings) and \( t_f^{(m)} \) (the photon formation time in coherent (multiple) scatterings). These time scales are defined as

\[
 t_f^{(s)} \approx \frac{1}{\omega - k(\omega)\beta}, \quad t_f^{(m)} \approx \sqrt{\frac{12E^2}{qk(\omega)\beta}}.
\]

With these time scales, it is possible to rewrite Eq. (2) normalized with respect to the BH-result in Eq. (8) as

\[
 -\frac{d^2W}{dzd\omega} \left/ \left( -\frac{d^2W_{BH}}{dzd\omega} \right) \right. \simeq \frac{1}{t_{BH}} \text{Im} \int_0^\infty d\tilde{t} \exp \left[ -\frac{\bar{t}}{t_d} + i \left( \frac{\bar{t}}{t_f^{(s)}} + \frac{\bar{t}^2}{t_f^{(m)} t_f^{(m)}} \right) \right]
\]

with \( t_{BH} \simeq 2\gamma^2 / \omega \). The integral in Eq. (12) can be evaluated exactly [27], leading to

\[
 -\frac{d^2W}{dzd\omega} \left/ \left( -\frac{d^2W_{BH}}{dzd\omega} \right) \right. \simeq \frac{t_f^{(m)}}{2t_{BH}} \text{Im} \left[ \sqrt{\pi} e^{i\zeta^2} \left( 1 - \text{erf}(\sqrt{\zeta}) \right) \right],
\]

with \( \zeta = (1/t_d - i/t_f^{(s)}) t_f^{(m)} / 2 \) and \( \text{erf}(\xi) = \sqrt{4/\pi} \int_0^\xi du e^{-u^2} \). This result is valid for either \( 1/t_d \neq 0 \) or \( 1/t_f^{(m)} \neq 0 \), while for \( 1/t_d = 1/t_f^{(m)} = 0 \) the integral in Eq. (12) needs additional regulation.

The behaviour of the full spectrum relative to the BH-result can be reproduced semi-quantitatively by examining the limiting expressions of Eq. (13) obtained if one of the time scales \( t_d, \) \( t_f^{(s)} \) or \( t_f^{(m)} \) is much smaller than the other two. In the case \( t_d \gg t_f^{(s)} \) and \( t_d \gg t_f^{(m)} \), which comprises the special case \( t_d \to \infty \), one finds from Eq. (13)

\[
 -\frac{d^2W}{dzd\omega} \left/ \left( -\frac{d^2W_{BH}}{dzd\omega} \right) \right. \simeq \kappa \frac{t_f}{t_{BH}}
\]

with \( \kappa = 1 \) if \( t_f^{(s)} \ll t_f^{(m)} \) and \( \kappa = \sqrt{\pi / 8} \) if \( t_f^{(m)} \ll t_f^{(s)} \) instead. This particular limit comprises the LPM effect as discussed in [12]. Accordingly, we define

\[
 -\frac{d^2W_{LPM}}{dzd\omega} \simeq -\frac{d^2W_{BH}}{8 t_{BH}} \frac{\pi t_f^{(m)}}{t_f^{(s)}}.
\]

Moreover, it turns out that Eq. (14) is also valid if \( t_d \) is large compared to either \( t_f^{(s)} \) or \( t_f^{(m)} \) only.

In case \( t_d \) is small with respect to \( t_f^{(s)} \) and \( t_f^{(m)} \), one may expand the integrand in Eq. (12) formally as

\[
 \text{Im} \left\{ \exp \left[ -\frac{\bar{t}}{t_d} + i \left( \frac{\bar{t}}{t_f^{(s)}} + \frac{\bar{t}^2}{t_f^{(m)} t_f^{(m)}} \right) \right] \right\} =
 e^{-i/t_d} \left\{ \frac{\bar{t}}{t_f^{(s)}} + \frac{\bar{t}^2}{t_f^{(m)} t_f^{(m)}} + O(\bar{t}^3) \right\}
\]

and evaluate the simple integrals realizing that higher-order terms in \( \bar{t} \) give rise to subdominant contributions. In the regime, in which \( t_d \ll t_f^{(s)} \ll t_f^{(m)} \), one finds

\[
 -\frac{d^2W}{dzd\omega} \left/ \left( -\frac{d^2W_{BH}}{dzd\omega} \right) \right. \simeq \frac{t_d^3}{t_f^{(s)} t_f^{(m)}}.
\]

In case \( t_d \ll t_f^{(m)} \ll t_f^{(s)} \), one has to distinguish between \( t_d \ll t_f^{(m)} t_f^{(s)} \), in which case Eq. (17) is also found, and \( t_d \gg t_f^{(m)} t_f^{(s)} \), which leads to

\[
 -\frac{d^2W}{dzd\omega} \left/ \left( -\frac{d^2W_{BH}}{dzd\omega} \right) \right. \simeq \frac{2 t_d^3}{t_f^{(m)} t_f^{(m)} t_f^{(s)}}.
\]

In order to highlight the influence of an absorptive medium on coherence effects in that medium, one may normalize the full spectrum in Eq. (2) rather with respect to the spectrum expression \( -d^2W_{LPM} \) relevant in the regime, in which coherence effects dominate \( t_f^{(m)} \ll t_f^{(s)} \). The quantity \( -d^2W / (-d^2W_{LPM}) \) is shown in Fig. 1 as a function of \( t_d/t_f^{(m)} \) for the case \( t_f^{(m)} = t_f^{(s)}/20 \). As evident from Fig. 1 the full spectrum is significantly suppressed compared to \( -d^2W_{LPM} \) for small \( t_d \), i.e. when damping effects are large. Moreover, Fig. 1 demonstrates that the limiting expressions in Eqs. (13), (17)
FIG. 1: (Color online) The ratio \(-d^2W/(-d^2W_{LPM})\) – that is Eq. (2) divided by Eq. (15) – as a function of \(t_d/t_f^{(m)}\) (solid curve) illustrating the impact of damping effects on coherence effects in an absorptive medium. Here, the special case of \(t_f^{(m)}\) small compared to \(t_f^{(s)}\) is considered, taking \(t_f^{(s)} \equiv t_{\text{BH}} = 20 t_f^{(m)}\), see text for details. For large \(t_d/t_f^{(m)}\), damping mechanisms do not significantly affect the spectrum. The vertical lines separate regions characterized by different hierarchies in time scales, where the acronyms SD, MD and MU are explained below in Fig. 3 and Sec. III C.

and (18) account fairly well for the behaviour of the radiative energy loss spectrum in an absorptive medium in the different physical regimes.

The above scaling laws show impressively the importance of the parameter-sensitive time scales \(t_f^{(s)}\), \(t_f^{(m)}\) and \(t_d\): From their interplay, the structure of the radiative energy loss spectrum in comparison with the BH-result as reference spectrum can be obtained semi-quantitatively. In the next section, we will study these time scales and their competition in detail and discuss the meaning of our findings for the spectra.

III. ANALYSIS OF THE REGIMES IN THE QED-CASE

A. Time scales in the absence of damping

In the following analysis, we adopt a different notation and reformulate the expressions for the different time scales in terms of the fractional bremsstrahlung quantum energy \(x = \omega/E\). Then, Eqs. (10) and (11) become

\[
\begin{align*}
t_f^{(s)} &\approx \frac{1}{x E - k(x)\beta}, \\
t_f^{(m)} &\approx \sqrt{\frac{12 E^2}{q k(x)\beta}}
\end{align*}
\]

with \(k(x) = \sqrt{x^2 E^2 - m^2}\). The method of approximating \(t_f\) by the minimum of \(t_f^{(s)}\) and \(t_f^{(m)}\) is sketched in Fig. 2 using selected values for the entering quantities. The corresponding exact numerical solution of Eq. (9) in terms of \(x\) agrees with this estimate for small \(x\) and for \(x\) close to 1, while it is typically somewhat smaller in the intermediate-\(x\) region.

Estimating the formation time by \(\min\{t_f^{(s)}; t_f^{(m)}\}\) allows for a suitable analysis of the functional behaviour of \(t_f\) with \(x\) in dependence of \(\gamma\) and of the parameter values for \(M\) and \(\tilde{q}\). Moreover, in this way the interplay of \(t_f\) with the damping time \(t_d\) can be discussed easily. The results of the following analysis are summarized graphically in Figs. 3 and 4 and the appearing variables and their definitions are collected in Tab. I.

According to Eq. (19) at leading order in \(1/\gamma\) and for \(m \ll xE\), one finds from Eq. (19)

\[
t_f^{(s)} \approx \frac{2 \gamma M}{x^2 M^2 + m^2}
\]

while from Eq. (20)

\[
t_f^{(m)} \approx \sqrt{\frac{12 \gamma M}{q x}}
\]
follows. The function in Eq. (21) exhibits a maximum at $x^* = m/M$ with $t_f^{(s)}(x^*) \simeq \gamma/m$ (cf. Fig. 2). Moreover, it approaches $2/m$ for $\gamma \gg 1$ when $x$ approaches the minimal allowed value $x^*/\gamma$ which arises from the absence of radiation in the plasma for $xE < m$, cf. [22]. We note that the exact expression for $t_f^{(s)}$ in Eq. (19) approaches $1/m$ instead.

In the following, we restrict the discussion to the case $m \ll M$ such that $x^* \ll 1$. Then, one finds that $t_f^{(s)}(x) < t_f^{(m)}(x)$ over the whole $x$-range if $\gamma < \gamma_c^{(1)} \sim m M^2/\bar{q}$. In this case, the formation time will be given by $t_f^{(s)}$ only.

For $\gamma > \gamma_c^{(1)}$, instead, one finds two values of $x$ as shown in Fig. 2 for which $t_f^{(s)}(x) = t_f^{(m)}(x)$. These intersection points are given by $x_1 \sim x^*(\gamma_c^{(1)}/\gamma)^{1/3}$ for small $x$ and by $x_2 \sim x^*/\gamma/\gamma_c^{(1)}$ for larger $x$. We note that for $\gamma \rightarrow \gamma_c^{(1)}$ one finds $x_{1,2} \rightarrow x^*$. The formation times at the intersection points read $t_f(x_1) \sim (\gamma/m)(\gamma_c^{(1)}/\gamma)^{1/3}$ and $t_f(x_2) = t_f^{\text{max}}$, where

$$
t_f^{\text{max}} \sim \gamma_c^{(2)}/M \tag{23}
$$

with $\gamma_c^{(2)} \sim M^3/\bar{q}$. The scale $t_f^{\text{max}}$ is independent of $\gamma$ and represents the minimal amount of time necessary to radiate a photon through a multiple scattering process.

To obtain the intersection points and the corresponding formation times, we have approximated $t_f^{(s)}$ in Eq. (21) either by $t_f^{(s)} \simeq 2x\gamma M/m^2$ for small $x \ll x^*$, cf. also [17], or by $t_f^{(s)} \simeq 2\gamma/(xM) \simeq t_{BH}$ for larger $x \gg x^*$.

Between $x_1$ and $x_2$, $t_f^{(s)}(x) > t_f^{(m)}(x)$ and therefore $t_f^{(m)}$ determines the formation time of radiation, whereas outside of this region radiation is still dominated by single scatterings, in particular for $x < x_1$, cf. also [15]. The behaviour observed for $m \neq 0$ is in striking contrast to the $m = 0$ case. In this case, the expressions for the formation times become $t_f^{(s)} \rightarrow t_{BH} \simeq 2\gamma^2/(xE)$ and $t_f^{(m)} \rightarrow \sqrt{12E/(\bar{q}x)}$, which are equivalent to the known results [14] for bremsstrahlung photons from incoherent (single) and coherent (multiple) scatterings, respectively.

In the region $0 \leq x \lesssim x_2$ one finds $t_f^{(s)}(x) > t_f^{(m)}(x)$ such that $t_f$ is also for small $x$ determined by $t_f^{(m)}$, cf. [14]. The difference stems from the evident reduction of $t_f^{(s)}$ compared to $t_{BH}$ in the small-$x$ region due to polarization effects when $m \neq 0$.

The lower bound $x_1$ decreases with $\gamma^{-1/3}$, while the upper bound $x_2$ increases linearly with $\gamma$. Consequently, the region in which multiple scattering processes become effective increases with increasing $\gamma$ (or increasing energy $E$ for fixed $M$) as evident from Fig. 3. For $\gamma \rightarrow \gamma_c^{(2)}$, $x_2$ tends to 1. Thus, for $\gamma > \gamma_c^{(2)}$ one would expect to find only one intersection point, for which $t_f^{(s)}(x) = t_f^{(m)}(x)$, so that for $x \sim 1$ the BH-spectrum in Eq. (5) could not be recovered, cf. [21]. However, with increasing $x$ towards 1 corrections, which are not present in Eq. (1), have to be taken into account. These are discussed later in Sec. IV A in the case of QCD. Generically, such corrections lead to a reduction of both $t_f^{(s)}$ and $t_f^{(m)}$ for $x$ close to 1, where $t_f^{(s)}$ becomes stronger reduced. This implies that the region in which multiple scattering processes dominate effectively shrinks and the Bethe-Heitler bremsstrahlung spectrum is necessarily recovered for $x \sim 1$. As long as $m \ll M$ and $\gamma < \gamma_c^{(2)}$, one may refrain from including corrections for $x$ close to 1. Otherwise, these corrections would become mandatory, which is, however, beyond the scope of this section.

**B. Competition between photon formation and damping**

In an absorptive medium, damping of radiation effects can modify the aforementioned picture significantly. Considering $m \ll xE$ and $\Gamma \ll xE$, the exponential damping factor in Eq. (4) becomes $\mathcal{F}(t) \simeq e^{-t\Gamma}$. This gives rise to a damping time, $t_d \simeq 1/\Gamma$ (cf. Eq. (11)), as highlighted in Fig. 2 by long-dashed horizontal lines, at which, compared to the undamped case, the amplitude in Eq. (2) is reduced by a factor $1/e$.

In case $t_d$ is large compared to the formation time of radiation, damping mechanisms will not significantly affect the spectrum, as evident from Fig. 1. This situation is also illustrated by the upper long-dashed horizontal line in Fig. 2. If, in contrast, $t_d \lesssim t_f$ in a specific re-
For such $x$, damping will be of significance in that region. In particular, it implies that any coherent interference of radiation amplitudes happens only during $t_{d}$ rather than $t_{f}$, because the formation process of the radiation quantum will be hampered by damping mechanisms. Strictly speaking, any finite value of $\Gamma$ enters $\gamma \sim 3/\gamma - \Gamma$ and thus modifies Eq. (3). The above parametric discussion of $t_{f}$, however, is still valid qualitatively as long as we demand $\Gamma < m$.

The relevant scales for discussing the influence of damping mechanisms are $t_{d}$ and $t_{f}^{\text{onset}} \sim M^{2}/\hat{q}$ or, equivalently, $\Gamma$ and $\hat{q}/M^{2}$. In case $t_{d} > t_{f}^{\text{onset}}$ or $\Gamma > \hat{q}/M^{2}$, one expects to find a region in $\gamma$-$x$-space, in which coherence effects are of importance. In the opposite case, however, i.e. if $t_{d} < t_{f}^{\text{onset}}$ or $\Gamma > \hat{q}/M^{2}$, damping effects will be significant and coherence effects will play no role. This stems from the definition of $t_{f}^{\text{onset}}$, see the comment below Eq. (3). Again, we do not include corrections necessary for $x$ close to 1 in this discussion.

Considering first damping rates $\Gamma < \hat{q}/M^{2}$ (case sketched in panel (a) of Fig. 4), one finds that damping plays a negligible role for $\gamma < \gamma_{c}^{(1)}(\sim m/\Gamma)\sqrt{q}/(\Gamma M^{2})$. For such $\gamma$, $t_{d} > t_{f}(x_{1})$ and, thus, $t_{f}$ is larger than $t_{d}$ for any $x$. It implies, in particular, that for $\gamma < \gamma_{c}^{(1)}$ undamped single scatterings dominate the formation of radiation as before because $\gamma_{c}^{(1)} < \gamma_{c}^{(1)}$ by definition. For $\gamma > \gamma_{c}^{(1)}$, instead, damping mechanisms become relevant in a region between $x_{3}$ and $x_{4}$ (region I) in panel (a) of Fig. 4. These boundary points are determined from the conditions $t_{d} = t_{f}^{(s)}(x_{3})$ for $x_{3} \ll x^*$ and $t_{d} = t_{f}^{(m)}(x_{4})$ (leading to $x_{4} > x_{1}$), respectively. They read $x_{3} \sim x^{*}m/\Gamma_{1} \sim x^{*}\gamma_{d}^{(1)}(\sim \Gamma M^{2}/\hat{q})$ and $x_{4} \sim \Gamma^{2}\gamma M/\hat{q}$, cf. also [19]. This situation is also illustrated by the middle long-dashed horizontal line in Fig. 4.

The lower bound $x_{5} \sim \gamma^{-1}$ decreases faster than $x_{1}$ with increasing $\gamma$ but tends to $x_{1}$ as $\gamma \sim 3/\gamma - \Gamma$, while the upper bound $x_{4} \sim \gamma$ increases slower than $x_{2}$ with increasing $\gamma$ and tends to 1 as $\gamma \sim 3/\gamma - \Gamma$. This leads to the situation exhibited in Fig. 4 (panel (a)). For $\gamma > \gamma_{c}^{(2)}$, damping mechanisms are significant for the whole $x$-range above $x_{3}$, while for $\gamma_{c}^{(2)} > \gamma > \gamma_{c}^{(1)}$, a region for $x > x_{4}$ emerges, in which undamped multiple scattering processes become relevant. In case $\gamma_{c}^{(2)} > \gamma > \gamma_{c}^{(1)}$, this picture will be modified to the extent that, in addition, a region for $x > x_{5}(> x_{4})$ exists, in which undamped single scatterings dominate the formation of radiation again.

In the special case $\Gamma \sim \hat{q}/M^{2}$, one finds that $\gamma_{d}^{(1)} \sim \gamma_{c}^{(1)}$ and $\gamma_{d}^{(2)} = \gamma_{c}^{(2)}$, where $x_{4} \rightarrow x_{2}$. This implies, in particular, that for any $\gamma > \gamma_{c}^{(1)}$ coherence effects are superseded by damping mechanisms and, thus, play no significant role for the formation of radiation anymore.

For even larger damping rates $\Gamma > \hat{q}/M^{2}$ (case sketched in panel (b) of Fig. 4), damping effects will be influential already for $\gamma < \gamma_{c}^{(1)}$: As $t_{f}$ is given by $t_{f}^{(s)}$ for all $x$ in this case, another scale $\gamma_{d}^{(3)} \sim m/\Gamma_{1} \sim \gamma_{c}^{(1)}$ arises from assuming that $t_{d} = t_{f}^{(s)}(x_{5})$. Accordingly, damping mechanisms become important for $\gamma > \gamma_{c}^{(3)}$ in a region between $x_{3}$ and $x_{5} \sim \Gamma_{1}/\gamma_{M}$ (region II) in panel (b) of Fig. 4, while outside of this region radiation is still formed by undamped single scatterings. The boundary point $x_{5}$ is determined from the condition $t_{d} = t_{f}^{(s)}(x_{5})$ for $x_{5} \gg x^*$. This picture does not change for $\gamma > \gamma_{c}^{(1)}$. Because already $t_{d} < t_{f}^{(s)} = t_{f}(x_{2})$, damping effects are also in this case important in the region between $x_{3}$ and

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**FIG. 4:** (Color online) (a): Sketch of the region (shaded region (I)) in $\gamma$-$x$-space, which is influenced by damping effects for fixed $\Gamma < \hat{q}/M^{2}$. As evident, for a fixed $\Gamma$ damping plays an increasing role with increasing $\gamma$. (b): As in panel (a) but for fixed $\Gamma > \hat{q}/M^{2}$ (shaded region (II)). The acronyms SU, MU, MD and SD stand for single undamped, multiple undamped, multiple damped and single damped, respectively, as explained in detail in Sec. III C and $x_{cr} \sim \sqrt{m^{2}/M^{2} \Gamma - \hat{q}}$. The other depicted variables are summarized in Tab. [III] cf. also text for details.
TABLE I: Summary of the definitions of the variables appearing in the text and depicted in Figs. 2, 3 and 4 as determined from these definitions.

| \( \gamma \)-scales | noticeable points | conditions |
|---------------------|------------------|------------|
| \( \gamma_c^{(1)} \sim mM^2/\hat{q} \) | \( x^* = m/M \) | \( t_f^{(s)}(x_1) = t_f^{(m)}(x_1), \ x \ll x^* \) |
| \( \gamma_c^{(2)} \sim M^3/\hat{q} \) | \( x_1 \sim x^*(\gamma_c^{(1)}/\gamma)^{1/3} \sim (m^4/(\gamma M\hat{q}))^{1/3} \) | \( t_f^{(s)}(x_2) = t_f^{(m)}(x_2), \ x \gg x^* \) |
| \( \gamma_d^{(1)} \sim \sqrt{qM^2/(\Gamma M^2)} \) | \( x_2 \sim x^*\gamma_c^{(1)} \sim \gamma_\gamma^{(2)} \sim \gamma\hat{q}/M^3 \) | \( t_4 = t_f^{(s)}(x_3), \ x \ll x^* \) |
| \( \gamma_d^{(2)} \sim \hat{q}/(\Gamma M) \) | \( x_3 \sim x^*\gamma_\gamma^{(3)} \gamma \sim M^2/(\gamma\Gamma) \) | \( t_4 = t_f^{(m)}(x_4) \) |
| \( \gamma_d^{(3)} \sim \Gamma \) | \( x_4 \sim \gamma_\gamma^{(2)} \sim \gamma\Gamma T^2/\hat{q} \) | \( t_4 = t_f^{(s)}(x_5), \ x \gg x^* \) |
| \( \gamma_d^{(4)} \sim M/\Gamma \) | \( x_5 \sim \gamma_\gamma^{(4)} \sim \gamma\Gamma/M \) | |

\( x_5 > x_2 \) (as also illustrated by the lower long-dashed horizontal line in Fig. 4). Therefore, damping mechanisms hamper the formation of radiation in a large region of \( \gamma\gamma\)-space for \( \Gamma > \hat{q}/M^2 \). We note that \( x_5 \) tends to 1 as \( \gamma \to \gamma_d^{(4)} \sim M/\Gamma \).

C. Physical implications

The above analysis discussed the competition of the time scales \( t_f \), \( t_f^{(m)} \) and \( t_d \). In this section, we want to interpret our findings in the context of the spectrum. In case of negligible damping, i.e. \( t_d \) large compared to the other time scales, the spectrum behaves in line with Eq. (14). In case of a non-negligible damping rate, however, we identified three different regimes, in which, depending on \( x \) and \( \gamma \), damping effects can be important: (a) when \( t_d \ll t_f^{(s)} \ll t_f^{(m)} \), (b) when \( t_d \ll t_f^{(m)}/t_f^{(s)} \ll t_f^{(m)} \ll t_f^{(s)} \), and (c) when \( t_f^{(m)}/t_f^{(s)} \ll t_d \ll t_f^{(m)} \ll t_f^{(s)} \).

Among these regimes, regime (b) is particularly interesting. From our discussion of the competition of the different time scales in Secs. III B and III C one could understand the physical process taking place as the hampering of the formation of radiation in a multiple scattering process, because without damping in the medium radiation would be formed coherently (\( t_f^{(m)} \ll t_f^{(s)} \)). In view of the spectra, however, the regimes (a), in which \( t_f^{(s)} \ll t_f^{(m)} \), and (b) yield equivalent results, cf. the discussion around Eq. (17). This observation can be understood by interpreting that in the case \( t_d \ll t_f^{(m)2}/t_f^{(s)} \), the influence of damping effects is so strong that effectively the radiation formation is hampered already after a single scattering of the charge in the absorptive medium even if \( t_f^{(m)} \ll t_f^{(s)} \) in the absence of damping.

This particular physical situation is realized in parts of the regions (SD) highlighted in Fig. 4, in which single damped scatterings dominate the physics. More precisely, the conditions for regime (b) are met in the shaded region between \( x_1 \) and \( x_{cr} \sim \sqrt{m^2\Gamma/(M^2 - \hat{q})} \) in panel (a) of Fig. 4 and between \( x_1 \) and \( x_2 \) in panel (b) of Fig. 4. Here, \( x_{cr} \) represents a \( \gamma \)-independent critical \( x \)-value determined from assuming \( t_d = t_f^{(m)2}/t_f^{(s)} \) which, in case of \( \Gamma < \hat{q}/M^2 \), allows for a discrimination of the nature of the physical process taking place in the \( \gamma\gamma \)-space regions, in which damping plays a role (i.e. in region (I) of Fig. 4 panel (a)). With increasing \( \Gamma \to \hat{q}/M^2 \), one finds \( x_{cr} \to \infty \).

Corresponding to this interpretation, the physical picture simplifies tremendously: For \( \Gamma < \hat{q}/M^2 \) (see panel (a) of Fig. 4) and any fixed \( x < x_{cr} \), damping effects are strong enough to influence the radiation formation significantly already after a single scattering of the charge (shaded region (SD) between \( x_3 \) and \( x_{cr} \)), while for fixed \( x \geq x_{cr} \) damping effects influence the formation of radiation in multiple scattering processes only (shaded region (MD) between \( x_{cr} \) and \( x_4 \)), provided \( \gamma \) is large enough. For \( x < x_{cr} \), one passes therefore with increasing \( \gamma \) from single undamped (region (SU) below the \( x_3 \)-curve) to single damped (shaded region (SD)) scattering processes, which determine the spectrum. For \( x > x_{cr} \), instead, one passes from single undamped (region (SU) below the \( x_1 \)-\( x_2 \)-curve) to multiple undamped (region (MU) in between the \( x_4 \)-curve and the \( x_1 \)-\( x_2 \)-curve) to multiple damped (shaded region (MD)) scatterings with increasing \( \gamma \).

For \( \Gamma \geq \hat{q}/M^2 \) (see panel (b) of Fig. 4), damping effects are so strong that effectively the radiation formation is hampered for any \( x \) after a single scattering of the charge (shaded region (SD) between \( x_3 \) and \( x_5 \)) as long as \( \gamma \) is large enough. Otherwise, single undamped scatterings determine the spectrum (region (SU) below the \( x_3 \)-\( x_5 \)-curve), and there is no room left for coherence effects.

IV. APPLICATION TO QCD PHENOMENOLOGY

We now extend our study to the consideration of damping phenomena in a strongly interacting medium. To our best knowledge, the influence of the damping of radiation has not yet been investigated in the context of radiative energy loss in QCD matter. Given the universality of the possible physical processes, it is reasonable to assume that dissipative effects lead equally to a damping of the radiation spectrum in QCD matter, which may formally be written down similar to Eq. (12) up to a different normalization factor and essential differences in the entering time scales due to the non-Abelian
character of QCD. In Sec. [IV A] we start our discussion, as in Sec. [III A] by analyzing the functional behaviour and the parametric dependence of the formation time \( t_f \) of gluon bremsstrahlung by ignoring possible damping mechanisms. In this case, estimates for \( t_f \) exist, cf. e.g. [28]. Then, in Sec. [IV B] we include damping phenomena into the discussion in the same way as done in Sec. [III B] by introducing a competing time scale \( t_d \). The phenomenological consequences for the gluon bremsstrahlung spectrum are analyzed in Sec. [IV C] in analogy to Sec. [III C] and a discussion of possible physical mechanisms leading to the damping of the gluon radiation is found in Sec. [IV D]

A. Analysis of the gluon bremsstrahlung formation time in the absence of damping

The formation time of gluon bremsstrahlung in the QCD plasma can be estimated, following [28], from the inverse expectation value of the energy imbalance between the final state (a relativistic on-mass-shell parton with momentum \( \vec{p} \) plus an on-mass-shell bremsstrahlung gluon with momentum \( \vec{k} \)) and a relativistic on-mass-shell single parton state carrying the same total momentum \( \vec{P} = \vec{p} + \vec{k} \). This leads to the condition [28]

\[
t_f \left[ \frac{\langle p_B^2 \rangle + x^2 m_s^2 + (1-x)m_g^2}{2x(1-x)E} \right] \approx 1,
\]

where \( m_s \) and \( m_g \) denote the masses of the emitting color charge and of the emitted bremsstrahlung gluon, respectively, while \( \langle p_B^2 \rangle \) is defined as \( \langle p_B^2 \rangle = (p_{k_{\perp}} - k_{p_{\perp}})/P \) and \( E \approx P \).

In order to make a connection with Sec. [IIIA] we consider in Eq. (24) first the special case in which \( k_{\perp} \) vanishes, while the emitting charge experiences small deflections through soft, elastic scatterings in the medium. Accordingly, \( \langle p_B^2 \rangle \approx x^2 t_f \hat{q}_g \), where \( \hat{q}_g \) is the mean squared transverse momentum per unit time picked up by the charge, and Eq. (24) renders into

\[
t_f \frac{x}{2E} \left[ \frac{m_s^2}{(1-x)} + \frac{m_g^2}{x^2} \right] + t_f^2 \frac{x \hat{q}_g}{2E(1-x)} \approx 1.
\]

It is interesting to note that in the small \( x \ll 1 \) limit this condition equation will be equivalent to the condition Eq. (9) discussed in detail above, apart from a factor \( \hat{q} = 6 \hat{q}_g \), if in Eq. (9), now written in terms of \( x \), the function \( k(x) \) is expanded for \( m \ll xE \), only terms of \( \mathcal{O}(1/\gamma) \) are kept, and \( M \) and \( m \) are identified with \( m_s \) and \( m_g \), respectively.

The dominant contribution to the radiative energy loss of a relativistic parton in the hot QCD plasma stems, however, from the rescatterings of the radiated gluon in the medium [7]. Thus, we consider in Eq. (24) rather the case in which the bremsstrahlung gluon undergoes soft, elastic scatterings during its formation, while any deflection of the emitting color charge is assumed to be of minor importance. This leads to \( \langle p_B^2 \rangle \approx (1-x) x^2 m_g^2 \approx (1-x)^2 t_f \hat{q}_g \), where \( \hat{q}_g \) is the mean squared transverse momentum per unit time picked up by the radiated gluon.

Then, for the condition for the gluon formation time Eq. (24) becomes

\[
t_f \left[ \frac{x^2 m_g^2 + m_s^2 (1-x)}{2x(1-x)E} \right] + t_f^2 \frac{(1-x) \hat{q}_g}{2x E} \approx 1.
\]

We note that the above equation is strictly applicable only in the limit of highly energetic gluons, i.e. when \( k_{||} \approx x E \). For gluon energies of the order of the gluon mass, in contrast, it has to be generalized. This can be achieved conveniently by considering rather the imbalance of parallel momenta in the above mentioned scattering process, while the energy is conserved. The resulting condition equation for the gluon formation length (the formation length translates easily into a formation time) looks similar to Eq. (20), where one has to replace only \( m_g^2 / (2x E) \) by \( m_g^2 / (\omega + k_{||}) \) and \( (1-x) \hat{q}_g \) by \( 2 \hat{q} / (\omega - k_{||}) - \hat{q}_g \). As a self-consistency condition, the gluon energy must satisfy

\[
\omega \geq \sqrt{m_g^2 + \langle k_{||}^2 \rangle}.
\]

These modifications introduce, however, merely numerical changes of order \( \mathcal{O}(1) \) in the prefactors so that Eq. (20) is completely sufficient for the parametric discussion we aim at in the following.

As in Sec. [IIIA] the functional behaviour of the formation time for gluon bremsstrahlung \( t_f \) with \( x \) depending on \( \gamma = E/m_s \) and on the parameter values for \( m_s \), \( m_g \) and \( \hat{q}_g \) can be analyzed conveniently by estimating \( t_f \) by the minimum of \( t_f^{(s)} \) and \( t_f^{(m)} \) defined as

\[
t_f^{(s)} \simeq \frac{2x(1-x)\gamma m_s}{x^2 m_g^2 + m_g^2 (1-x)},
\]

\[
t_f^{(m)} \simeq \frac{2x \gamma m_s}{\sqrt{(1-x) \hat{q}_g}},
\]

which both increase with \( \gamma \), \( t_f^{(s)} \) showing the stronger dependence. Similar to the previous sections, \( t_f^{(s)} \) denotes the gluon formation time in incoherent (single) scatterings, while \( t_f^{(m)} \) represents the formation time of bremsstrahlung gluons in a coherent (multiple) scattering process. This procedure for determining \( t_f \) is shown in Fig. 5. We stress here that the above expression for \( t_f^{(s)} \) is only applicable for finite \( m_g \), while in the limit of a vanishing in-medium gluon mass it has to be modified as is discussed below. The results of the following analysis are summarized graphically in Figs. 6 and 7, while the definitions of the appearing variables are listed in Tab. 1.

Compared to Eqs. (22) and (21), corrections for \( x \) close to 1 are now taken into account in both expressions for
$t_f^{(s)}$ and $t_f^{(m)}$. These modify the behaviour of the time scales for $x \sim 1$. Apart from these modifications, $t_f^{(s)}$ is rather similar to the expression in Sec. IIIA, while the functional form of $t_f^{(m)}$ is significantly different. The function $t_f^{(s)}$ in Eq. (28) exhibits a maximum at $m_g/(m_s + m_g)$. In the following, we focus our discussion on the case $m_g < m_s$, such that the position of this maximum becomes $x^* = m_g/m_s$ with $t_f^{(s)}(x^*) \approx \gamma/m_g$ (cf. Fig. 5), similar to Sec. IIIA. Moreover, $t_f^{(s)}$ from Eq. (28) may be approximated by $t_f^{(s)} \approx 2x \gamma m_s/m_g^2$ for small $x \ll x^*$ and by $t_f^{(s)} \approx 2 \gamma/(1 - x)/(x m_s)$ for $x \gg x^*$.

In the parametric analysis, one has to distinguish between two distinct cases, namely $m_3^3 > \tilde{q}_g$ and $m_3^3 < \tilde{q}_g$. In perturbative QCD (pQCD), $m_3^3 \gg \tilde{q}_g$ because the in-medium gluon mass $m_g \sim gT$ \cite{17, 18, 29}, while $\tilde{q}_g \sim g^4 T^3$ \cite{31} for small QCD running coupling $g \ll 1$ at large temperatures $T$. We start our discussion with this case. Nonetheless, we do not want to restrict the analysis of the parameter space to the case $m_3^3 > \tilde{q}_g$ only, because the opposite case could be satisfied in the non-perturbative (strong coupling) regime.

For $m_3^3 > \tilde{q}_g$, the self-consistency condition Eq. (27) implies that the minimal allowed $x$-value is given by $x^*/\gamma$, at which $t_f^{(s)} < t_f^{(m)}$. In case $\gamma < \gamma_c^{(1)} \sim m_g^3/\tilde{q}_g$, $t_f^{(s)}(x) < t_f^{(m)}(x)$ for all $x$. Then, the formation time will be determined by $t_f^{(s)}$ over the whole $x$-range. In contrast, for $\gamma > \gamma_c^{(1)}$ a region exists, in which $t_f^{(s)}(x) > t_f^{(m)}(x)$. This region is bound by $x_1 \sim x^* m_g^3/(\gamma \tilde{q}_g)$ and $x_2 \sim C/(1 + C)$ with $C = (\gamma \tilde{q}_g/m_g)^{1/3}$, at which $t_f^{(s)} = t_f^{(m)}$. This situation is illustrated in Fig. 6. The formation times at the intersection points read

$$t_f(x_1) = t_f^{\text{inset}} \approx m_g^2/\tilde{q}_g,$$

which is independent of $\gamma$ and represents the minimal amount of time necessary for gluon formation in a multiple scattering process, and $t_f(x_2) \approx (\gamma^2/\tilde{q}_g)^{1/3}$, which increases with $\gamma^{2/3}$.

In the case $m_3^3 < \tilde{q}_g$, in contrast, the picture changes. Here, the self-consistency condition Eq. (27), supplemented by $\langle k_\perp^2 \rangle = \tilde{q}_g t_f^{(m)}$, implies that the minimal allowed $x$-value is rather given by $x_{1,S} \sim \tilde{q}_g^{1/3}/(\gamma m_s)$.

This implies that gluons cannot be formed at rest with intermediate mass $m_g$ due to their reinteractions within the medium. At $x_{1,S}$ one finds $t_f^{(s)} > t_f^{(m)}$, with

$$t_f^{(m)}(x_{1,S}) = t_f^{\text{onset}} \approx \tilde{q}_g^{-1/3}.$$  

For any possible $\gamma$, $t_f^{(m)}$ is smaller than $t_f^{(s)}$ in the entire $x$-region between $x_{1,S}$ and $x_2$, so that the formation time is determined by $t_f^{(m)}$ in this region. In contrast, for $x > x_2$ one finds $t_f^{(s)}(x) < t_f^{(m)}(x)$.

In the BDMPS-approach \cite{8}, incoherent scatterings determine the gluon radiation spectrum in hot QCD matter for $x < x_{\text{LPM}}$, where $x_{\text{LPM}} \sim \lambda_\mu^2/(\gamma m_s)$. Here, $\lambda_\mu \sim (g^2 T)^{-1}$ is the gluon mean free path between successive elastic scatterings in the medium, while $\mu \sim gT$ is the typical momentum transfer to the gluon in a single scattering. For $x > x_{\text{LPM}}$, instead, the spectrum is suppressed $\propto x^{-1/2}$ due to the LPM effect analogon. In our approach, we find in the case $m_3^3 > \tilde{q}_g$ that $x_1 \sim x_{\text{LPM}}$, when we insert for the quantities entering $x_i$ their parametric dependencies as known from pQCD. Moreover, we find parametrically that $t_f(x_1) \approx m_3^3/\tilde{q}_g \sim \lambda_\mu$. This analogy between BDMPS and the case $m_3^3 > \tilde{q}_g$ is, however, only possible because $m_g$ is proportional to $\mu$ in pQCD. Naively, one would rather expect that the above discussed case $m_3^3 < \tilde{q}_g$ resembles the BDMPS-results, which are derived for $m_g = 0$. Nonetheless, one cannot associate this case with BDMPS, because in our approach at vanishing $m_g$ coherence effects dominate for small $x$-values, even though the corresponding formation length is smaller than $\lambda_\mu$. This unphysical situation can be cured by adding the scale $\mu^3$ - understood as a minimal value for $\langle p_\perp^2 \rangle$ in Eq. (24) - in the denominator of $t_f^{(s)}$ in Eq. (28). Then, one finds $t_f^{(s)} < t_f^{(m)}$ for $x < \mu^3/(\tilde{q}_g^2 \gamma m_s)$, where $\mu t_f^{(s)}(\tilde{q}_g \gamma m_s) \sim x_{\text{LPM}}$ in pQCD. This analysis shows that the case $m_3^3 < \tilde{q}_g$ does not have to be understood as $m_3^3$ alone being small compared to $\tilde{q}_g$ but rather as the case of a $\tilde{q}_g$ that is large compared to both $m_3^3$ and $\mu^3$. 

![Diagram](https://via.placeholder.com/150)  

FIG. 5: (Color online) Visualization of the formation time $t_f$ for gluon bremsstrahlung as a function of $x$ for selected parameter values and fixed $\gamma$. Short-dashed and dotted curves exhibit $t_f^{(s)}$ and $t_f^{(m)}$, respectively, as defined in Eqs. (28) and (29), while the solid curve depicts our estimate for $t_f$ given by the minimum of $t_f^{(s)}$ and $t_f^{(m)}$. Moreover, long-dashed horizontal lines indicate a fixed damping time $t_d$, where damping increases from top to bottom, cf. text for details.
We now proceed by including damping effects into our considerations. We recall that our main symbols are defined in Tab. II. We assume that the damping mechanisms impose a competing time scale $t_d \simeq 1/\Gamma$ for the formation of gluon bremsstrahlung similar to Sec. III B. Then, they become influential if $1/\Gamma \lesssim t_f$. In Fig. 6 we illustrate where in $\gamma$-x-space damping effects are important in case $m_g^3 > \hat{q}_g$. In Fig. 6 a larger value of $\hat{q}$ was chosen, such that $m_g^3 < \hat{q}_g$. For negligible $\Gamma$, coherence effects dominate the radiation formation in the region between $x_1$ and $x_2$ in case $m_g^3 > \hat{q}_g$ (cf. Fig. 6), and between $x_1,S$ and $x_2$ in case $m_g^3 < \hat{q}_g$ (cf. Fig. 6). These regions increase with increasing $\gamma$ (or increasing energy $E$ for fixed $m_s$). For a non-negligible $\Gamma$, however, the situation changes.

Considering first the case $m_g^3 > \hat{q}_g$, one finds the following picture: For fixed damping rates $\Gamma < \hat{q}_g/m_g^2$ (case sketched in panel (a) of Fig. 6), which implies $t_d > t_f(x_1) = t_f^{(\text{onset})}$, damping mechanisms become only important for $\gamma > \gamma_d^{(1)} \sim \sqrt{\hat{q}_g/\Gamma^3}$, i.e., when $t_f^{(\text{onset})} < t_d < t_f(x_2)$. The scale $\gamma_d^{(1)}$ is by definition larger than $\gamma_c^{(1)}$ but decreases with increasing $\Gamma$. For $\gamma > \gamma_d^{(1)}$, damping effects are of significance in a region between $x_3$ and $x_4$ (region (I), see also in Fig. 6 the long-dashed horizontal line in the middle), where $x_3 \sim \hat{q}_g/(\Gamma^2 \gamma m_s)$.

| $\gamma$-scales | noticeable points | conditions |
|------------------|------------------|------------|
| $\gamma_c^{(1)} \sim m_g^3/\hat{q}_g$ | $x^* = m_g/m_s$ | $t_f^{(\text{onset})}(x_1) = t_f(x_1)$, $x \ll x^*$ |
| $x_S^{(1)} \sim \sqrt{\hat{q}_g/\Gamma^2}$ | $x_1 \sim x^*/(\gamma m_s \hat{q}_g)$ | $t_f^{(\text{onset})}(x_2) = t_f(x_2)$, $x \gg x^*$ |
| $\gamma_d^{(2)} \sim m_g/\Gamma$ | $x_1, S \sim x_S^{(1)}/\gamma$ | $t_d = t_f^{(\text{onset})}(x_3)$ |
| | $x_2 \sim C/(1 + C) \sim (\gamma \hat{q}_g)^{1/3}/(m_s + (\gamma \hat{q}_g)^{1/3})$ | $t_d = t_f^{(\text{onset})}(x_4)$, $x \gg x^*$ |
| | $x_3 \sim \hat{q}_g/(\Gamma^2 \gamma m_s)$ | $t_d = t_f^{(\text{onset})}(x_5)$, $x \ll x^*$ |
| | $x_4 \sim \gamma \Gamma/(m_s + \Gamma \Gamma)$ | |
| | $x_S^{(2)} \sim m_g^3/(\Gamma \gamma m_s)$ | |

**TABLE II:** Summary of the definitions of the variables appearing in the text and depicted in Figs. 6, 7, 8 and 9 as determined from these definitions, where $C = x^*/(\gamma/\gamma_c^{(1)})^{1/3}$.

**B. Competition between gluon bremsstrahlung formation and damping**

We now proceed by including damping effects into our considerations. We recall that our main symbols are defined in Tab. II. We assume that the damping mechanisms impose a competing time scale $t_d \simeq 1/\Gamma$ for the formation of gluon bremsstrahlung similar to Sec. III B. Then, they become influential if $1/\Gamma \lesssim t_f$. In Fig. 6 we illustrate where in $\gamma$-x-space damping effects are important in case $m_g^3 > \hat{q}_g$. In Fig. 6 a larger value of $\hat{q}$ was chosen, such that $m_g^3 < \hat{q}_g$. For negligible $\Gamma$, coherence effects dominate the radiation formation in the region between $x_1$ and $x_2$ in case $m_g^3 > \hat{q}_g$ (cf. Fig. 6), and between $x_1,S$ and $x_2$ in case $m_g^3 < \hat{q}_g$ (cf. Fig. 6). These regions increase with increasing $\gamma$ (or increasing energy $E$ for fixed $m_s$). For a non-negligible $\Gamma$, however, the situation changes.

Considering first the case $m_g^3 > \hat{q}_g$, one finds the following picture: For fixed damping rates $\Gamma < \hat{q}_g/m_g^2$ (case sketched in panel (a) of Fig. 6), which implies $t_d > t_f(x_1) = t_f^{(\text{onset})}$, damping mechanisms become only important for $\gamma > \gamma_d^{(1)} \sim \sqrt{\hat{q}_g/\Gamma^3}$, i.e., when $t_f^{(\text{onset})} < t_d < t_f(x_2)$. The scale $\gamma_d^{(1)}$ is by definition larger than $\gamma_c^{(1)}$ but decreases with increasing $\Gamma$. For $\gamma > \gamma_d^{(1)}$, damping effects are of significance in a region between $x_3$ and $x_4$ (region (I), see also in Fig. 6 the long-dashed horizontal line in the middle), where $x_3 \sim \hat{q}_g/(\Gamma^2 \gamma m_s)$.
and $x_4 \sim \gamma \Gamma / (m_g + \gamma \Gamma)$. These boundary points are determined from the conditions $t_d = t_f^{(m)}(x_3)$, and from $t_d = t_f^{(s)}(x_4)$ for $x_4 \gg x^*$, respectively.

The lower bound $x_3$ decreases with $\gamma^{-1}$ (but slower than $x_1$) and will approach $x_1$ only if $\Gamma \to \hat{q}_g / m_g^2$, for which also $\gamma_d^{(1)} \to \gamma_c^{(1)}$. The upper bound $x_4$, instead, increases with $\gamma$ (but faster than $x_2$). Thus, the $x$-region in which damping mechanisms are of importance increases with increasing $\gamma$ and/or increasing $\Gamma$.

In contrast, for larger $\Gamma > \hat{q}_g / m_g^2$ (case sketched in panel (b) of Fig. 6) one finds $t_d < t_f^{\text{onset}}$ (see also in Fig. 5) the lower long-dashed horizontal line). Then, damping mechanisms become important already for $\gamma > \gamma_d^{(2)} \sim m_g / \Gamma$, where $\gamma_d^{(2)} < \gamma_c^{(1)}$, in a region between $x_5 \sim m_g^2 / (\Gamma m_g x_4)$ and $x_4$ (region (II)). The lower bound $x_5$ is determined from the condition $t_d = t_f^{(s)}(x_5)$ for $x_5 \ll x^*$. We note that $x_5$ decreases faster than $x_3$ with increasing $\gamma$. Thus, we find for the case $\Gamma > \hat{q}_g / m_g^2$ that damping mechanisms influence the formation of gluon bremsstrahlung in a large region of $\gamma$-$x$-space, implying a negligible role of coherence effects on the radiative energy loss spectrum.

Considering now the case $m_g^3 < \hat{q}_g$, i.e. when $t_f^{(m)} < t_f^{(s)}$ at $x_1$, we find the situation depicted in Fig. 7. For fixed damping rates $\Gamma < \hat{q}_g / m_g^3$, damping effects are of importance for $\gamma > \gamma_d^{(3)}$ in a region between $x_3$ and $x_4$ (region (I)) as in the above case $m_g^3 > \hat{q}_g$ (cf. panel (a) of Fig. 6). We conclude that, also in the $m_g^3 < \hat{q}_g$ case, damping effects become more important for the formation of gluon bremsstrahlung with increasing $\gamma$ and/or $\Gamma$. As $\Gamma \to \hat{q}_g / m_g^3$, one would find that $x_3 < x_1$ for any $\gamma$ and, thus, damping effects would be of significance in almost the entire $\gamma$-$x$-space. However, here we refrain from considering damping rates $\Gamma$ as large as $\Gamma > \hat{q}_g / m_g^3$ or even larger. This is because for $\Gamma \to \hat{q}_g / m_g^3$ one finds $\Gamma > m_g$ such that the underlying assumption, that the inclusion of damping phenomena does not qualitatively influence the above $\Gamma$-independent discussion of $t_f$, would become invalid. In any case, we expect the effect of damping to be more pronounced with increasing $\Gamma$ in this regime, too.

C. Impact on the spectra

The above formation and damping time analysis allows us to discuss qualitatively the influence of different in-medium effects such as damping on the radiation spectrum in QCD. Given the generic structure of the spectrum discussed in Sec. II we make use of the scaling laws determined there. However, here we want to analyze the behaviour of the radiative power spectrum $dI/dx$ relative to the soft, i.e. $x \ll x^*$, Gunion-Bertsch (GB) power spectrum limit of gluon radiation off massive partons $dI_{\text{GB}}/dx$ from incoherent scatterings. The latter was determined within scalar QCD in \[31\] as an extension of the result by Gunion and Bertsch \[32\] for massless partons. With this normalization, the analogon of the scaling law in Eq. (14) reads as

$$\frac{dI}{dI_{\text{GB}}} \simeq \kappa t_f / t_{GB}$$

with $t_f \simeq \min\{t_f^{(s)}, t_f^{(m)}\}$. The scale $t_{GB} \simeq 2 \pi m_g^2 / m_g^2$ is the formation time $t_f^{(s)}$ of soft gluon radiation in a single scattering process.

Considering first the case $m_g^3 > \hat{q}_g$, the spectra ratio is $dI/dI_{\text{GB}} = 1$ for small $x$ as seen in the small-$x$ regions (SU) in Fig. 8 because $t_f$ is given by $t_f^{(s)}$ for such $x$ (cf. also Fig. 5). With increasing $x$, the ratio $dI/dI_{\text{GB}}$ is reduced, where the reduction is proportional to a specific power of the gluon fractional energy in line with the appropriate scaling law.

For negligible damping rates $\Gamma$, as illustrated by solid curves in Fig. 8 panels (a) and (b), $dI/dI_{\text{GB}} \propto (x(1-x))^{-1/2}$ if $t_f^{(m)} < t_f^{(s)}$, which holds for $x_1 < x < x_2$. This includes, in particular, the known BDMPZ-Z suppression of the power spectrum \[4, 9\] $dI/dI_{\text{GB}} \propto x^{-1/2}$ for $x < 1$. If, instead $x > x_2$, i.e. $t_f^{(s)} < t_f^{(m)}$, then $dI/dI_{\text{GB}} \propto (1-x)/x^2$ is found. This behaviour is a consequence of the finite parton mass $m_g$ in Eq. (25) and, thus, a feature specific to heavy quarks, cf. \[20\].

For non-negligible damping rates $\Gamma$, one finds at most four different physical regions and sizeable parts of the spectrum can become additionally reduced. For $\Gamma <
The spectrum suppression is dominated by multiple undamped scatterings, then by multiple undamped (MD) processes, where single undamped scatterings are either (SU) in the absence of damping effects. For $x_{cr} < x < x_2$, damping effects are so strong that effectively the formation of radiation is hampered already after a single scattering process, even though $t_f^{(m)} < t_f^{(s)}$ in this region. Here, $x_{cr}$ is determined from $t_d = t_f^{(m)} / t_f^{(s)}$, and follows, assuming $x_{cr} > x^*$, as $x_{cr} \sim 1/[1 + m_s \sqrt{\Gamma / \gamma_g}]$. Passing from small-$x$-values towards $x = 1$, the spectrum is dominated first by single undamped scatterings, then by multiple undamped scattering processes with the known BDMPS-Z suppression, followed by multiple damped (MD) processes, where the spectrum suppression is $\propto (1 - x) / x^2$ in line with

$$\frac{dI}{d\Omega} \approx \frac{t_f^3}{t_f^{(m)} t_f^{(s)}}. \quad (33)$$

For $x_{cr} < x < x_2$, single damped (SD) scatterings dominate the spectrum with a modification factor $\propto 1/(1 - x) + m_g^2/(x^2 m_g^2)$ in line with

$$\frac{dI}{d\Omega} \approx \frac{t_f^2}{t_f^{(s)} t_f^{(GB)}}. \quad (34)$$

and for $x > x_2$ a region (SU) follows with $dI/d\Omega \propto (1 - x)/x^2$. The spectrum suppression in the damped regions (MD) and (SD) is evidently stronger than the suppression $\propto x^{-1/2}$ in the BDMPS-Z regime.

With increasing $\Gamma$, the regions dominated by damping mechanisms increase. For $\Gamma > \hat{\gamma}_g/m_g^2$, the physical situation changes significantly (cf. panel (b) in Fig. 8). In a large $x$-region between $x_5$ and $x_4$ (cf. also Fig. 6 panel (b)) single damped scatterings determine the power spectrum according to Eq. (34), and there is no room left for coherence effects. It is noteworthy, however, that for large $x$ the term $\propto 1/(1 - x)$ dominates in Eq. (34) such that the effect of a power spectrum suppression due to damping becomes reduced.

The opposite case $m_g^2 < \hat{\gamma}_g$ is shown in Fig. 8. For small $x$, multiple undamped scattering processes dominate the power spectrum and $dI/d\Omega < 1$ already in this region. For negligible $\Gamma$, the physical situation changes only for $x > x_2$, where single undamped scatterings determine the spectrum. For non-negligible $\Gamma < \hat{\gamma}_g^{1/3}$, damping effects become influential at intermediate $x$, where for $x < x_{cr}$ a region of multiple damped scatterings and for $x > x_{cr}$ a region of single damped scatterings emerges. Again, the spectrum suppression in the regions (MD) and (SD) is stronger than in the regime (SU), in which coherence effects dominate the radiation spectrum.

D. Discussion of possible damping mechanisms

Damping phenomena specific to hot QCD matter become important for the radiative energy loss of an energetic parton, when they start to influence the formation process of the bremsstrahlung gluons. The gluon damping rate is related to the imaginary part of the poles in
the corresponding in-medium propagator $\hat{g}$. In perturbative QCD approaches, it has been calculated both for collective gluon modes in the plasma as well as for hard gluons. For collective gluons with momenta and energies of $O(gT)$ both elastic and inelastic processes contribute equally to a damping rate $\Gamma \sim g^2T$. In case of a hard gluon with either momentum or energy of $O(T)$ or larger only elastic processes have been considered in the evaluation of the gluon damping rate in [30], yielding $\Gamma \sim g^2T \ln(1/g)$ for $g \ll 1$. However, elastic rescatterings of a bremsstrahlung gluon during its formation are already taken into account by $\hat{q}_g$ in our considerations. The nature of the damping mechanisms we have in mind here is different: As possible inelastic processes leading to a damping of gluon radiation, one might consider either quark–anti-quark pair production or secondary bremsstrahlung creation from a gluon during its formation process.

For estimating the parametric dependence of the latter process, one might view the emission of the secondary bremsstrahlung gluon in the dense medium as a BDMPS-mechanism. In doing so, a preformed gluon with fractional energy $x > x_{LPM}$, which is, however, small compared to 1, emits a secondary bremsstrahlung gluon of fractional energy $x'$ in an inelastic scattering process. In order to be of influence on the formation of the preformed gluon, the formation time of the secondary gluon must be small compared to the formation time of its emitter, which can be realized if $x' \ll x$, cf. Fig. 5. The production rate for the secondary gluon as a measure for the damping rate of the preformed emitter gluon is obtained by integrating the weighted BDMPS-radiation spectrum. Neglecting the part in the integral, which is suppressed with $x^{-3/2}$, one finds a gluon damping rate $\Gamma \sim g^4T$ up to corrections of order $\ln(1/g)$. We note that a similar parametric dependence was found in [37] for the gluon production rate, which is of interest for the chemical equilibration of the hot QCD plasma. For such a damping rate, the case $\Gamma < \hat{q}_g/m_g^2$ would be realized in pQCD, leading to the situation depicted in panel (a) of Fig. 6.

V. CONCLUSION

Damping phenomena in a dense, absorptive plasma can influence substantially the radiative energy loss of an energetic charge traversing this medium. They manifest themselves in a non-trivial reduction of the associated radiation spectrum off that charge as compared to the spectrum from incoherent scatterings in non-absorptive matter. The effect is more pronounced for large energies $E$ (or equivalently large Lorentz-factors $\gamma$) of the charge and/or large damping rates $\Gamma$ of the radiated quanta in the medium. This behaviour can be understood semi-quantitatively by making use of the concept of a formation time for radiation: Damping mechanisms reduce the radiative energy loss spectrum if the typical time scale for the damping of radiation quanta with fractional energy $x = \omega/E$ is small compared to their formation time such that these effects influence already the creation of the radiation.

In this article, we analyzed systematically the interplay between these competing time scales for an absorptive QED and QCD medium. We started with the case of a polarizable and absorptive, infinite electro-magnetic plasma, for which analytical results for the radiative energy loss spectrum per unit length have been derived in [22, 24]. Then, we extended the phenomenological discussion to QCD matter. This is the first time that the consequences of damping are studied in QCD as in perturbative QCD approaches damping phenomena were so far considered as negligible, higher-order effects.

In both cases, i.e. QED and QCD, we identified parametrically the regions in $\gamma$-x-space, in which either coherence or damping effects significantly influence the associated radiation spectrum, cf. Figs. 4 6 and 7. We showed that, generically, damping effects become important in an intermediate-$x$ regime, which grows with increasing $\gamma$ and/or $\Gamma$. Any suppression of the spectrum in this regime has to be attributed to damping phenomena rather than coherence effects. We showed that the radiation spectrum is stronger suppressed through damping effects than it is through coherence effects. Restricting our analysis for QCD matter to the case of heavy quarks, we visualized this feature in Figs. 8 and 9 by using typical values for the entering parameters. The utilized concept should, however, be generalizable to the study of light partons as well.

As damping effects become pronounced for large $\gamma$, the study of hadronic correlations at high transverse momenta $p_T$ might open the avenue for experimentally measuring the absorptive properties of hot QCD mat-
ter. Moreover, the effect could be a key ingredient for the understanding of the observed heavy meson spectra: If \( f_{qT} \) becomes the dominant scale, the radiation spectra turn out to be mass-independent, which would constitute a step towards solving the non-photonic single-electron puzzle [38]. Corresponding phenomenological investigations are presented in [39], where the quark-mass independence of the heavy-flavor meson quenching at large \( p_T \) is quantified and a comparison with the available experimental data from the ALICE collaboration on the \( D \) meson quenching suggests a rather strong damping rate \( \Gamma/T \approx 0.75 \). Also, we expect our results to be of relevance in heavy-quark tagged jet physics: On the one hand, either the finite in-medium gluon mass \( m_g \) or the parameter \( q^{1/3}_0 \) represent a natural lower energy cutoff for bremsstrahlung gluons in the soft region of the spectrum. Damping effects, on the other hand, hamper the formation of hard or intermediate hard gluons, providing effectively an upper limit for the radiative energy loss of an energetic projectile, similar to coherence effects but with a stronger impact on the power spectra.

In this work, we neglected the consideration of any dependence on a finite path length \( L \). Nevertheless, whether the energetic charge is created in the remote past or originating from the interior bulk of a thick medium, only quanta with a formation length smaller than the distance \( L \) travelled by the charge can contribute to the medium-induced radiative energy loss, cf. [26]. This implies that in the regions of \( \gamma-x \)-space, in which damping effects are relevant, finite path length effects play no additional role for the radiation spectrum off these charges. Following the heuristic arguments presented in [8], we expect that the averaged radiative energy loss of such partons in the hot QCD plasma is proportional to \( L/\Gamma \) in case \( 1/\Gamma < L \). Only for partons stemming from the outer crust of the medium of thickness \( 1/\Gamma \) finite path length effects are important and the averaged radiative energy loss is proportional to \( L^2 \). We considered, moreover, \( x \)-independent damping rates in our analysis. In general, however, \( \Gamma \) should depend on the energy of the radiation quanta. We leave such studies for future investigations.

Acknowledgements

We acknowledge valuable discussions with Yu. L. Dokshitzer, E. Iancu, B. Kämpfer, S. Peigné and M. H. Thoma. We also thank Yu. A. Markov for drawing our attention to Ref. 19. The work is supported by the European Network I3-HP2 Toric, the ANR research program “Hadrons@LHC” (grant ANR-08-BLAN-0093-02) and the “Pays de la Loire” research project TOGETHER.
125008.

[31] P. B. Gossiaux, J. Aichelin, T. Gousset, and V. Guiho, J. Phys. G 37 (2010) 094019; J. Aichelin, P. B. Gossiaux, and T. Gousset, Phys. Rev. D 89 (2014) 074018.

[32] J. F. Gunion and G. Bertsch, Phys. Rev. D 25 (1982) 746.

[33] H. A. Weldon, Phys. Rev. D 28 (1983) 2007.

[34] E. Braaten and R. D. Pisarski, Phys. Rev. D 42 (1990) 2156.

[35] M. H. Thoma in Quark-Gluon plasma 2 / editor, R. C. Hwa (World Scientific, Singapore, 1995).

[36] R. D. Pisarski, Phys. Rev. D 47 (1993) 5589.

[37] T. S. Biro, E. van Doorn, B. Müller, M. H. Thoma, and X.-N. Wang, Phys. Rev. C 48 (1993) 1275.

[38] M. Djordjevic, J. Phys. G 32 (2006) 5333.

[39] P. B. Gossiaux, M. Nahrgang, M. Bluhm, T. Gousset, and J. Aichelin, Nucl. Phys. A 904-905 (2013) 992c; M. Nahrgang, M. Bluhm, P. B. Gossiaux, and J. Aichelin, J. Phys. Conf. Ser. 422 (2013) 012016.