The brane as a Higgs domain wall: ideas and issues

Raymond R. Volkas

School of Physics, Research Centre for High Energy Physics, The University of Melbourne, Victoria 3010 Australia

Abstract. The most obvious field-theoretic model for a brane is a scalar field domain wall or kink. I discuss how this idea can be connected with spontaneous internal symmetry breaking via a mechanism called the “clash of symmetries”. Compatibility with Randall and Sundrum’s warped metric alternative to compactification is then demonstrated. I end with brief remarks about open questions.

Keywords: brane, extra dimension, kink, domain wall, scalar field, symmetry

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BACKGROUND

In field theory, a brane can be inserted into a higher-dimensional action as a fundamental object, or it can emerge dynamically as a scalar-field soliton [1]. In this talk, I shall discuss the latter hypothesis, and also specialise to one extra dimension with the soliton being a domain wall or kink. The brane is thus smooth and of finite thickness.

In the standard model, scalar fields are used to spontaneously break the electroweak gauge symmetry. We are thus led to explore the possibility that the scalar field composing the brane also plays a role in some internal symmetry breaking (not necessarily electroweak). This can be achieved in a toy-model sense through domain wall configurations displaying a feature dubbed the “clash of symmetries” [2], a generalization of the simple $Z_2$ kink.

We first briefly review the $Z_2$ kink. Take a scalar field in a spacetime of any dimensionality $d$ and give it the potential energy $V = \lambda (\phi^2 - u^2)^2$, invariant under the discrete $Z_2$ symmetry $\phi \rightarrow -\phi$, and with $\lambda > 0$. The global minima are $\phi = +u$ and $\phi = -u$. They are degenerate and separated by a potential barrier; the vacuum “manifold” is simply two disconnected points, reflecting the spontaneous breaking of the $Z_2$ symmetry. A kink configuration is a static, topologically-stable solution of the Euler-Lagrange equations that depends on one of the spatial dimensions only, and has the two global minima as asymptotic boundary conditions. For the quartic potential case quoted, the solutions are

$$\phi(w) = \pm u \tanh(\sqrt{2\lambda}uw)$$

where $w$ is the extra-dimension coordinate and $\pm$ refers to kink and antikink, respectively. The kink has width $1/\sqrt{2\lambda}u$, and becomes a step function as $\lambda \rightarrow \infty$. It is a fuzzy $d-1$-dimensional brane-like entity.
The clash of symmetries [2] generalises the above by using a vacuum manifold that consists not just of disconnected points, but rather a set of disconnected copies of a non-trivial manifold. Take a Higgs model with a continuous symmetry $G$ that spontaneously breaks to $H$ at the global minima of the potential, and that also features a spontaneously broken discrete symmetry lying outside of $G$. The vacuum manifold is a set of disconnected copies of the coset space $G/H$, with the number of copies given by the discrete symmetry breaking pattern.

Each point within a $G/H$ corresponds to a differently embedded $H$ subgroup. But each such point is now a potential boundary condition for a generalized kink configuration. We define a kink in this context as a static 1-dimensional solution to the scalar-field Euler-Lagrange equations that interpolates between a given point in one $G/H$ and a certain point in a $G/H$ disconnected from the first. The straightforward analogue of the $Z_2$ kink requires these two points to correspond to identically embedded subgroups $H$. In this case, the “instantaneous” unbroken subgroup as a function of $w$ is always the same subgroup $H$. A clash of symmetries kink has the two boundary condition points as corresponding to differently embedded (but isomorphic) $H$ subgroups. At finite $w$, the configuration displays an unbroken symmetry $H_{\text{finite}}$ that is smaller than $H$, typically just the intersection $H(w = -\infty) \cap H(w = +\infty)$. At precisely $w = 0$, the odd-function components of the Higgs configuration vanish, leading to an instantaneously enhanced symmetry we shall call $H_0$ (not isomorphic to $H$ in general). We have thus achieved a spatially-dependent symmetry breaking pattern.

It has been speculated that a hierarchical symmetry breaking cascade $G \to H_0 \to H_{\text{finite}}$ may be felt by degrees of freedom that are almost $\delta$-function confined near $w = 0$ [3]. If the confinement was precisely of $\delta$-function form, then the brane localized effective theory would display $G \to H_0$, assuming local interactions between the brane and bulk fields. But given the finite thickness of the brane, one would rather expect localization of finite width, in which case the further breaking $H_0 \to H_{\text{finite}}$ would feature in the effective theory at a lower energy scale driven by the localization width. No explicit realization of this idea has yet been found. The well-known Yukawa-style localization of 4-d fermion zero modes, at least in its simplest form, appears to not be what is needed to realize this idea [4].

**$O(10)$ KINKS: BREAKING TO $SU(3) \otimes SU(2) \otimes U(1)^2$**

I now display some explicit $O(10)$ adjoint-Higgs kinks displaying the clash of symmetries [3]. Writing the adjoint Higgs $\Phi$ as a $10 \times 10$ antisymmetric matrix, the most general quartic potential has the terms $Tr(\Phi^2)$, $(Tr(\Phi^2))^2$ and $Tr(\Phi^4)$. The invariant cubic term $Tr(\Phi^3)$ is identically zero, so there is an accidental $\Phi \to -\Phi$ discrete symmetry also, and it lies outside of $O(10)$. The minimisation of this Higgs potential was studied in Ref.[5]. The first step is to use an $O(10)$ transformation to bring the VEV into the standard form $\Phi = \text{diag}(a_1 \varepsilon, a_2 \varepsilon, a_3 \varepsilon, a_4 \varepsilon, a_5 \varepsilon)$, where $\varepsilon = i\sigma_2$ is the $2 \times 2$ antisymmetric matrix. For a certain range of parameters, the global minima are

$$a_i^2 = \text{const.} \equiv a_{\text{min}}^2 \forall i,$$  (2)
where the constant is a certain combination of Higgs potential parameters. The unbroken subgroup is $U(5)$.

We now seek kink solutions. Let us choose $\Phi(-\infty) = -a_{\text{min}} \operatorname{diag}(\varepsilon, \varepsilon, \varepsilon, \varepsilon, \varepsilon)$ as the boundary condition at $w = -\infty$, where the overall minus sign is related to the $\Phi \rightarrow -\Phi$ breaking. At $w = +\infty$ there are three sensible choices:

$$
\Phi(+\infty) = \begin{cases} 
  a_{\text{min}} \operatorname{diag}(\varepsilon, \varepsilon, \varepsilon, \varepsilon) \\
  a_{\text{min}} \operatorname{diag}(\varepsilon, \varepsilon, -\varepsilon, -\varepsilon) \\
  a_{\text{min}} \operatorname{diag}(\varepsilon, -\varepsilon, -\varepsilon, -\varepsilon)
\end{cases},
$$

(3)

giving three kinds of kinks: symmetric, asymmetric and super-asymmetric, respectively. (There must be an odd number of relative minus signs between the $-\infty$ and $+\infty$ boundary conditions to ensure that they cannot be transformed into each other under $O(10)$.) The kink configurations are of the form $\Phi_k(w) = \alpha(w)\Phi(-\infty) + \beta(w)\Phi(+\infty)$ leading to $H_{\text{finite}} = U(5)$, $U(3) \otimes U(2)$, $U(4) \otimes U(1)$ respectively. The asymmetric and super-asymmetric kinks display the clash of symmetries phenomenon. Since $U(3) \otimes U(2) \cong G_{\text{SM}} \otimes U(1)'$, asymmetric $O(10)$ kinks show some model-building promise. For the parameter slice where the coefficient of the $(Tr(\Phi^2))^2$ term vanishes, there is an analytic solution:

$$
a_1(w) = a_2(w) = a_3(w) = a_{\text{min}} \tanh(\mu w), \quad a_4(w) = a_5(w) = -a_{\text{min}},
$$

(4)

where $1/\mu$ is the width of the domain wall. For this parameter slice, the super-asymmetric kink has the lowest energy density. The symmetric and asymmetric kinks would therefore be expected to be unstable to evolution to the lowest energy configuration. It is unknown whether or not the asymmetric configuration has the lowest energy density in other regions of parameter space, where the functional forms of $a_{1,2,3}(w)$ and $a_{4,5}(w)$ would be different from the simple analytic solution quoted above. There is also no a priori need to stick with a quartic potential.

Notice that at $w = 0$, the odd-functions $a_{1,2,3}$ vanish, leading to $H_0 = O(6) \otimes U(2) \cong SU(4) \otimes SU(2) \otimes U(1)$. It would be nice to find a way to realise the hierarchical breaking cascade

$$
O(10) \rightarrow SU(4) \otimes SU(2) \otimes U(1) \rightarrow G_{\text{SM}} \otimes U(1)'
$$

(5)

as per the speculative idea discussed previously.

**RANDALL-SUNDRUM GRAVITY**

Consider a $U(1) \otimes U(1)$ model with two complex Higgs fields $\Phi_{1,2}$ minimally coupled to 5-d Einstein gravity. Adopting the Randall-Sundrum style warped metric ansatz $d\tilde{s}_5^2 = dw^2 + e^{2f(w)} d\tilde{s}_4^2$ with a Minkowski brane [6, 7], the coupled Einstein-Higgs equations yield the solution

$$
\Phi_{1,2}(w) = \frac{u}{\sqrt{2}} \sqrt{1 \pm \tanh \beta w}, \quad f(w) = -\frac{u^2}{12\kappa} \ln(\cosh \beta w)
$$

(6)
for a certain sextic Higgs potential \cite{8}. The parameter $u$ is a VEV, $\beta$ is the inverse domain wall width, while $\kappa$ is related to the 5-d Planck mass. The warp factor exponent is a smooth analogue of $-k|w|$ familiar from the RS2 $\delta$-function brane case. An RS2-like \cite{7} limit is reached when $\beta \to \infty$ and $u \to 0$ such that $u^2 \beta$ is kept finite. Notice that this is a strange limit in this context, because the kink amplitude $u$ shrinks to zero.

The $\Phi_{1,2}$ kinks display a primordial clash of symmetries feature, because asymptotically a different $U(1)$ group is exact on opposite sides of the wall, while both $U(1)$’s are broken at all finite $w$. This construction shows that the clash of symmetries is compatible with a smoothed out version of RS2 gravity.

**PROSPECTS, ISSUES, QUESTIONS**

Ultimately, one would want to construct a Higgs kink brane-world model with a phenomenologically successful 4-d effective standard model or extension thereof on the domain wall. Hopefully the clash of symmetries or a related idea could be used to connect brane formation with at least some of the required internal symmetry breaking, for example $SO(10)$ GUT breaking. The resulting model should also have a successful 4-d effective cosmology (see \cite{10} and references therein for an introduction to kink-brane cosmology).

To achieve this end, we need to simultaneously localize fermions, gravitons \cite{7}, gauge bosons \cite{9} and possibly also some Higgs bosons to the domain wall. Mechanisms exist for all these disparate fields, but they need to be non-trivially combined so as to yield successful particle and cosmological 4-d phenomenology. This is an interesting prospect and challenge.

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