A Deeply Virtual Compton Scattering Amplitude

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Abstract

A factorized Regge-pole model for deeply virtual Compton scattering is suggested. The use of an effective logarithmic Regge-Pomeron trajectory provides for the description of both “soft” (small $|t|$) and “hard” (large $|t|$) dynamics. The model contains explicitly the photoproduction and the DIS limits and fits the existing HERA data on deeply virtual Compton scattering.

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1 Introduction

Interest in deeply virtual Compton scattering (DVCS) $ep \to e\gamma p$ is related to the prospects to use it as a tool in studies of Generalized Parton Distributions (GPD) [1, 2].

At HERA the DVCS cross-section has been measured [3–4], in diffractive $ep$ interactions, as a function of $Q^2$, $W$ and $t$ that are respectively the photon virtuality, the invariant mass of the $\gamma^* p$ system and the squared 4-momentum transferred at the proton vertex; the diagram in Fig. 1a shows the production of a real photon at HERA.

The $Q^2$ evolution of the DVCS amplitude has been studied in several papers, mainly in the context of perturbative quantum chromodynamics (QCD) [5–6] and recently in [7]. The $t$ dependence in many papers was introduced by a simple factorized exponential in $t$, which however differs from the Regge pole theory. Since the electron-proton scattering at HERA is dominated by a single photon exchange, the calculation of the DVCS scattering amplitude reduces to that of the $\gamma^* p \to \gamma p$ amplitude, which at high energies, in the Regge pole approach, is dominated by the exchange of positive-signature Reggeons, associated with the Pomeron and the $f$-trajectories [8]. This DVCS amplitude is shown in Fig. 1b.
in a Regge-factorized form. In the figure $q_{1,2}$ are the four-momenta of the incoming and outgoing photons, $p_{1,2}$ are the four-momenta of the incoming and outgoing protons, $r$ is the four-momentum of the Reggeon exchanged in the $t$ channel, $r^2 = t = (q_1 - q_2)^2$ and $s = W^2 = (q_1 + p_1)^2$ is the squared centre-of-mass energy of the incoming system.

Figure 1: a) Diagram of a DVCS event at HERA; b) DVCS amplitude in a Regge-factorized form.

Unless specified (as in the deep inelastic scattering (DIS) limit, discussed in Sec. 3), $q_2^2 = 0$, and hence, for brevity, $q_1^2 = -Q^2$. In the upper vertex $V_1$, Fig. 1b, a virtual photon with 4-momentum $q_1$, and a Reggeon (e.g. Pomeron) with 4-momentum $r$, enter and a real photon, with 4-momentum $q_2 = q_1 + r$ appears in the final state as an outgoing particle. The vertex $V_1$ depends on all the possible invariants constructed with the above 4-momenta, $V_1[q_1^2, r^2, q_1 \cdot r]$, where $r^2 = t \leq 0, q_1^2 = -Q^2 \leq 0$. The three invariants are not independent since the mass-shell condition for the outgoing photon, $q_2^2 = (q_1 + r)^2 = 0$, provides the relation

$$q_1 \cdot r = \frac{-q_1^2 - r^2}{2} = \frac{Q^2 - t}{2}. \quad (1)$$

Hence, the vertex can be considered as a function of the invariants $[Q^2, q_1 \cdot r]$ or $[t, q_1 \cdot r]$. This does not mean that the variables cannot appear separately but it could also happen that $q_1 \cdot r$ become a scaling variable, and consequently the vertex will finally depend on $q_1 \cdot r$ only. It depends on the dynamics of the process and, for the moment, we prefer to keep $t$, apart from $Q^2$, as the second independent variable.

Electroproduction of a vector meson gives another example since in this case $(q_1 + r)^2 = M_V^2$, and the variable $q_1 \cdot r$ becomes

$$q_1 \cdot r = \frac{M_V^2 - q_1^2 - r^2}{2} = \frac{M_V^2 + Q^2 - t}{2}. \quad (2)$$

The interplay of the $Q^2$- and $t$-dependence in the DVCS amplitude was recently discussed in Ref. [9], where the existence of a new, universal variable $z$ was suggested. The basic idea is that $Q^2$ and $t$, both having the meaning of a squared mass of a virtual particle (photon
or Reggeon), should be treated on the same footing, by means a new variable, defined as
\[ z = q_1^2 + t = -Q^2 + t, \] (3)
in the same way as the vector meson mass squared is added to the squared photon virtuality, giving \( \tilde{Q}^2 = Q^2 + M_V^2 \) in the case of vector meson electroproduction [10] [11].

In this paper we examine an explicit model for DVCS with \( Q^2 \)- and \( t \)-dependences determined by the \( \gamma^* p\gamma \) vertex. We suggest the use of the new variable defined in Eq. 3 with its possible generalization to vector meson electroproduction,
\[ z = t - (Q^2 + M_V^2) = t - \tilde{Q}^2 \] (4)
or virtual photon (lepton pair) electroproduction,
\[ z = t - (Q_1^2 + Q_2^2), \] (5)
where \( Q_2^2 = -q_2^2 \). However, differently from Ref. [9], here we introduce the new variable only in the upper, \( \gamma^* p\gamma \) vertex, to which the photons couple.

In the next Section we introduce the model. Its viability is supported by the correct photoproduction- (\( Q^2 = 0 \)) and DIS- (\( Q^2 > 0 \) and \( t \to 0 \)) limits, demonstrated in Sec. 3. Fits to the data are presented in Sec. 4, while discussions and conclusions are in Sec. 5.

2 The model

According to Fig. 1b, the DVCS amplitude can be written as
\[ A(s, t, Q^2)_{\gamma^* p \to \gamma p} = -A_0 V_1(t, Q^2)V_2(t)(-is/s_0)^{\alpha(t)}, \] (6)
where \( A_0 \) is a normalization factor, \( V_1(t, Q^2) \) is the \( \gamma^* p\gamma \) vertex, \( V_2(t) \) is the \( pIp p \) vertex and \( \alpha(t) \) is the exchanged Pomeron trajectory, which we assume in a logarithmic form:
\[ \alpha(t) = \alpha(0) - \alpha_1 \ln(1 - \alpha_2 t). \] (7)

Such a trajectory is nearly linear for small \( |t| \), thus reproducing the forward cone of the differential cross section, while its logarithmic asymptotics provides for the large-angle scaling behavior [12] [13], typical of hard collisions at small distances, with power-law fall-off in \( |t| \), obeying quark counting rules [12] [14] [15]. Here we are referring to the dominant Pomeron contribution plus a secondary trajectory, e.g. the \( f \)-Reggeon. Although we are aware of the importance of this subleading contribution at HERA energies, nevertheless we cannot afford the duplication of the number of free parameters, therefore we include it effectively by rescaling the parameters. Ultimately, the Pomeron and the \( f \)-Reggeon have the same functional form, differing only by the values of their parameters. Furthermore, the Pomeron itself is unlikely to be a single term, so instead of including several Regge terms with many free parameters, it may be reasonable to comprise them in a single term, called ”effective Reggeon” or ”effective Pomeron”, depending on the kinematical region of interest.
Although the parameters of this effective Reggeon (Pomeron) (e.g. its intercept and slope) can be close to the "true" one (whose form is at best a convention), for the above reason they never should be taken as granted.

For convenience, and following the arguments based on duality (see Ref. [17] and references therein), the t dependence of the pIPp vertex is introduced via the α(t) trajectory: $V_2(t) = e^{b\alpha(t)}$ where b is a parameter. A generalization of this concept will be applied also to the upper, $\gamma^* p\gamma$ vertex by introducing the “trajectory”

$$\beta(z) = \alpha(0) - \alpha_1 \ln(1 - \alpha_2 z),$$

where the value of the parameter $\alpha_2$ may be different in $\alpha(t)$ and $\beta(z)$ (a relevant check will be possible when more data will be available). Hence the scattering amplitude (6), with the correct signature, becomes

$$A(s, t, Q^2)_{\gamma^* p \rightarrow \gamma p} = -A_0 e^{b\alpha(t)} e^{b\beta(z)} (-is/s_0)^{\alpha(t)} = -A_0 e^{(b+L)\alpha(t)+b\beta(z)},$$

where $L \equiv \ln(-is/s_0)$.

The model contains a limited number of free parameters. Moreover, most of them can be estimated a priori. The product $\alpha_1 \alpha_2$ is just the forward slope $\alpha'$ of the Reggeon ($\approx 0.2$ GeV$^{-2}$ for the Pomeron, but much higher for f and/or for an effective Reggeon) $^1$. The value of $\alpha_1$ can be estimated from the large-angle quark counting rules $^{[12][14][15]}$. For large $t$ ($|t| \gg 1$ GeV$^2$) the amplitude goes roughly (a detailed treatment of this point can be found in Refs. $^{[12][13]}$) like $\sim e^{-\alpha_1 \ln(-t)} = (-t)^{\alpha_1}$ where the power $\alpha_1$ is related to the number of quarks in a collision $^{[12][14][15]}$, e.g. their number minus one. Various versions of the counting rules suggest different combinatorics giving slightly different values for this power. We set $\alpha_1 = 1$, and hence $\alpha_2 = \alpha'$. For the intercept of our effective Reggeon, dominated by the Pomeron, we set $\alpha(0) = 1.25$ as an “average” over the “soft+hard” Pomerons $^2$. The above values of the parameters should not be taken as granted, they should be considered as starting values in the fitting procedure presented in Sec. 4.

From Eq. (10) the slope of the forward cone is

$$B(s, Q^2, t) = \frac{d}{dt} \ln |A|^2 = 2 \left[b + \ln \left(\frac{s}{s_0}\right)\right] \frac{\alpha'}{1 - \alpha_2 t} + 2b \frac{\alpha'}{1 - \alpha_2 z},$$

which, in the forward limit, $t = 0$ reduces to

$$B(s, Q^2) = 2 \left[b + \ln \left(\frac{s}{s_0}\right)\right] \alpha' + 2b \frac{\alpha'}{1 + \alpha_2 Q^2}.$$

Thus, the slope shows shrinkage in s and antishrinkage in $Q^2$.

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$^1$As emphasized in a number of papers, e.g. in Ref. [18], the wide-spread prejudice of the "flatness" of the Pomeron in electroproduction is wrong for at least two reasons: one is that it was deduced by fitting data to a particular "effective Reggeon" (see the relevant discussion above) and the second is that the Pomeron is universal, and its nonzero slope is well known from hadronic reactions.

$^2$This is an obvious simplification and we are fully aware of the variety of alternatives for the energy dependences, e.g. that of a dipole Pomeron, as in Ref. [17], a "soft" plus a "hard" one, as e.g. in Ref. [9]. Ultimately, from QCD’s BFKL equation $^{[16]}$ an infinite number of Pomeron singularities follows unless simplifications are used. For the present study in term of the new, z, variable the simplest “supercritical” Pomeron $^5$ with an effective intercept is suitable.
3 Photoproduction- and DIS limits

In the $Q^2 \rightarrow 0$ limit the Eq. (9) becomes

$$A(s, t) = -A_0 e^{2\alpha(t)}(-is/s_0)^{\alpha(t)}$$

where we recognize a typical Regge-behaved photoproduction (or, for $Q^2 \rightarrow m_H^2$, on-shell hadronic ($H$)) amplitude. The related deep inelastic scattering structure function is recovered by setting $Q^2_2 = Q^2_1 = Q^2$ and $t = 0$, to get a typical elastic virtual forward Compton scattering amplitude:

$$A(s, Q^2) = -A_0 e^{b(\alpha(0)-\alpha_1 \ln(1+\alpha_2 Q^2))} e^{(b+\ln(-is/s_0))\alpha(0)} \propto -(1 + \alpha_2 Q^2)^{-\alpha_1}(-is/s_0)^{\alpha(0)}.$$  (13)

In the Bjorken limit, when both $s$ and $Q^2$ are large and $t = 0$ (with $x \approx Q^2/s$ valid for large $s$), the structure function is given by:

$$F_2(s, Q^2) \approx \frac{(1-x)Q^2}{\pi\alpha_e} \Im A(s, Q^2)/s,$$  (14)

where $\alpha_e$ is the electromagnetic coupling constant and the normalization is $\sigma_t(s) = \frac{4\pi}{s} \Im A(s, Q^2)$. The resulting structure function has the correct (required by gauge invariance) $Q^2 \rightarrow 0$ limit and approximate scaling (in $x$) behavior for large enough $s$ and $Q^2$.

It should be noted, however, that the Regge behavior has a limited range of validity in $Q^2$. The smooth transition to DGLAP evolution was studied in Ref. [19], while a relevant explicit model was developed in Ref. [20].

4 Fits to the $ep \rightarrow e\gamma p$ data

A standard procedure for the fit to the HERA data on DVCS [3, 4] based on Eq. (9) has been adopted. A detailed analysis of the data would require a sum of a Pomeron plus an $f$-Reggeon contribution:

$$A = A^P + A^f.$$  (15)

To avoid the introduction of too many parameters, given the limited number of experimental data points, we use a single Reggeon term, as already discussed in Sec. 2, which can be treated as an effective Reggeon. The parameters $\alpha(0)$, $\alpha_1$ and $\alpha'$ have been fixed to 1.25, 1.0 and 0.38 GeV$^{-2}$ respectively and the values of the fitted parameters $A_0$ and $b$, described in Eq. (9) are listed in Table I. The value of $\alpha'$ has been determined in an exploratory fit with this parameter left free to vary between 0.2 and 0.4 GeV$^{-2}$.

The ZEUS measurements have been rescaled to the $W$ and $Q^2$ values of the H1 measurements. The mean value of $|t|$ has been fixed to 0.17 GeV$^2$ according with the H1 measurements of the differential cross-section in the range (0.1-0.8)GeV$^2$ for H1 [3] taking into account the value 6.02GeV$^{-2}$ for the slope $B$ as determined by the experiment.
Table 1: The values of the fitted parameters quoted in Eq. (9).

| parameter | $\sigma_{DVCS}$ vs $Q^2$ | $\sigma_{DVCS}$ vs $t$ | $\sigma_{DVCS}$ vs $W$ |
|-----------|--------------------------|------------------------|------------------------|
| $|A_0|^2$  | 0.08 $\pm$ 0.01          | 0.11 $\pm$ 0.24        | 0.06 $\pm$ 0.01        |
| $b$       | 0.93 $\pm$ 0.05          | 1.04 $\pm$ 0.91        | 1.08 $\pm$ 0.10        |
| $\chi^2/n dof$ | 0.57                  | 0.15                   | 1.15                   |

The results of the fits to the HERA data on DVCS are shown in Fig. 2. The cross-section $\sigma(\gamma^*p \to \gamma p)$ as a function of $Q^2$ and $W = \sqrt{s}$ are presented respectively in Fig. 2a and Fig. 2b. The differential cross-section $d\sigma(\gamma^*p \to \gamma p)/dt$, given by

$$
\frac{d\sigma}{dt}(s,t,Q^2) = \frac{\pi}{s^2}|A(s,t,Q^2)|^2,
$$

is presented in Fig. 2c.

The quality of the fits is satisfactory; in particular our model fits rather well the cross-sections as a function of $Q^2$ and the cross-section differential in $t$. Although the present HERA data on DVCS are well within the “soft” region, the model potentially is applicable for much higher values of $|t|$, dominate by hard scattering.

Figure 2: The $\gamma^* p \to \gamma p$ cross section as a function of $Q^2$ (a), of $W$ (b) and the cross-section differential in $t$ (c) measured by H1 and ZEUS experiments [3, 4]. The ZEUS measurements have been rescaled to the $W$ and $Q^2$ H1 values. The lines show the results of the fits obtained from Eq. (16) to the data.
Figure 3: The $Q^2$- and $s$ dependence of the local slope described in Eq. 10 (dotted and dashed line) and Eq. 11 (solid line). The triangles show the experimental measurements of $H1$.

Finally, Fig. 3 shows antishrinkage in $Q^2$ and shrinkage in $W$ of the forward cone, according to Eqs. 10 and 11. The curves are compared with the H1 experimental results.
5 Conclusions and discussion

The model presented in this paper may have two-fold applications. On one hand, it can be used by experimentalists as a guide. The fits to the data could be improved, when more data are available, by accounting for the Pomeron(s) and $f$-Reggeon contributions separately as well as by using expressions for Regge trajectories which take exactly into account analyticity and unitarity. On the other hand, the model can be used to study various extreme regimes of the scattering amplitude in all the three variables it depends on. For that purpose, however, the transition from Regge behavior to QCD evolution at large $Q^2$ should be accounted for. A formula interpolating between the two regimes (Regge pole and asymptotic QCD evolution) was proposed in Ref. [20] for $t = 0$ only. Its generalization to non zero $t$ value is possible by applying the ideas and the model presented in this paper. The applicability of the model in both soft and hard domains can be used to learn about the transition between perturbative (QCD) and non-perturbative (Regge poles) dynamics.

Independently of the pragmatic use of this model as a instrument to guide experimentalists, given its explicit form, it can be regarded also as an explicit realization of the corresponding principle [21] of exclusive-inclusive connection in various kinematical limits.

Last but not least, the simple and feasible model of DVCS presented in this paper can be used to study general parton distributions (GPD). As emphasized in Ref. [22], in the first approximation, the imaginary part of the DVCS amplitude is equal to a GPD. The presence of the Regge phase in our model can be used for restoring the correct phase of the amplitude, for which the interference experiments (with Bethe-Heitler radiation) are designed.

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