An investigation of the influence of the reduced elasticity modulus on strength of pultruded FRP members in bending

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Abstract. The polymer composite known as Fibre Reinforced Polymer (FRP) has several advantages over conventional materials. It has been slowly incorporated into civil infrastructures, because the general regulations have not yet been officially introduced. Pultruded FRP manufactures have internal standards of organizations and Guidelines. Therefore, the design identifies issues that require clarification. This study presents influence of the reduced elasticity modulus on strength of pultruded FRP members in bending with a symmetrical cross-section. The position of the neutral line and calculation of strength and deflections with and without reduced modulus of elasticity are determined in the study. Comparison of the results of calculations showed that when taking into account the reduced modulus of elasticity, the deformability is reduced by up to 15 %. The significance of the results obtained for the construction industry consists in the possibility of a refined calculation of the flexural pultruded FRP members taking into account the reduced modulus of elasticity in order to effectively use the strength reserves and reducing material consumption.

Keywords: construction, reduced elasticity modulus, strength, pultruded FRP beam, mechanical characteristic, deflection.

1 Introduction
Currently, FRP is used for temporary buildings and structures. They are widely used in the chemical, oil and mining industries, as well as in aviation, shipbuilding and mechanical engineering [1-2], their use in construction is very relevant [3-5]. Particular interesting ones among them are pultruded FRP which have strength characteristics close to steel and wood [6]. FRP has a number of advantages over other materials: low specific gravity, high strength, corrosion resistance, fast installation and low maintenance cost [7-9]. For the calculation and design of structures pultruded FRP LLC “Tatneft-Presscomposite” developed guidelines [10], which describes the method and algorithm for calculating compressed, stretched and bent elements, shear, torsion and bolted connections.

One of the main members in building structures is beams which in most cases work on a bend. FRP has different elastic modules under tension and compression. Bank [11] has performed extensive research in the field of material characterization of pultruded FRP beams and has suggested a method to determine Young’s and shear moduli of pultruded beams by the consideration of Timoshenko beam theory in conjunction with the three-point bending test of beams. Based on this method, other experimental programs were conducted by Netoand Rovere[12], Brooks and Turvey [13], Robert and Ubaidi [14] for measurement of Young’s and shear moduli. They have compared the results of stiffness measured from this bending test with stiffness from tensile testing of coupons as well as analytical equations. Robert and Ubaidi [14] have performed the uniform and nonuniform torsion test for measurement of shear modulus and rigidity. In another study, Howard [15] has performed the three-point bending and torsional tests for measurement of beam stiffness. Results are close to those obtained from approximate classical lamination beam theory (CLBT). Minghini et al. [16] have proposed the different position of loading in four-point bending test of beams, to measure flexural and shear rigidity. Correia [17] and Correia et al. [18] have determined Young’s modulus of beams by tensile and flexural testing of coupons.
Few researches have determined elastic properties of pultruded beams by micromechanics approach. They have measured the stiffness of all plies of flanges and web; then, combined the stiffness to obtain the full section properties. Nagaraj and GangaRao [19] have presented an approximate classical lamination theory. They have predicted the anisotropic modulus in the structural coordinate system that takes into account the lamina properties in the transverse directions. Feraboli and Kedward [20] have performed the four-point bending test of laminates to examine the inter-laminar shear strength of cross-ply and quasi-isotropic laminates.

The main objective of this paper is to present the calculation pultruded FRP members in bending in two ways according guidelines. In the first case the minimum modulus of elasticity is used, i.e. the modulus of elasticity in compression. In this case the inaccuracy of the calculation presupposes an additional margin of strength. In the second case it is necessary to calculate the reduced modulus of elasticity. The guidelines does not give specific recommendations, but there is a separate condition, which indicates that “If there is a specialized software tool that allows the work of a multi modular anisotropic material, the calculation should be performed taking into account the reduced modulus of elasticity”. Therefore, the study performed the calculation of the neutral line position, the calculation of pultruded FRP members work on bending, computer modeling of bending members with the setting of the modulus of elasticity.

2Methods
2.1Calculation of the neutral line position of the pultruded FRP members in bending
For calculation of the reduced modulus of elasticity it is necessary to determine the position of the neutral line. Pultruded FRP differently works on tension and compression. In this case we can use the bilinear deflection model review pultruded FRP I-beams (see Figure 1).

Since for pultruded FRP the elastic modulus of tension and compression are different, the neutral line in the section will not pass through the center of gravity. Let the dimensions \( h_1 \), \( h_2 \) determine the dimensions of the compressed and stretched cross section zones, respectively, then we will express one through the other:

\[
h_1 = h - h_2
\]

![Figure 1. Pultruded FRP I-beam.](image)

According to the Bernoulli hypothesis displacements are defined by the relation:

\[
\varepsilon = k \cdot y
\]

The stresses are determined by Hooke’s law:

\[
\sigma = E \cdot \varepsilon
\]

Then to find the position of the neutral line we write the equation of equilibrium of all forces:

\[
\int \sigma dA = \int E \varepsilon dA = \int E k y dA = 0
\]

We write the equation of the balance of forces of the compressed and stretched zone separately:

\[
\int E^- k y dA^- + \int E^+ k y dA^+ = 0
\]
where \( A^- \), \( A^+ \) – areas of the compressed and stretched area, \( E^- \), \( E^+ \) – modulus of elasticity in compression and tension, respectively. Excluding \( k \) from the equation (5) we get:

\[
\int E^- ydA^- + \int E^+ ydA^+ = 0
\]  

(6)

Based on:

\[
\int ydA = S
\]  

(7)

We get:

\[
E^- S^- + E^+ S^+ = 0
\]  

(8)

For the ratio of the elastic modulus of the compressed zone to the elastic modulus of the stretched zone we take \( \omega \) and get:

\[
m = \frac{E^-}{E^+}
\]  

(9)

Solving the system of equations (1) and (10) we get the value of \( h_1 \) and \( h_2 \) distance to the neutral line in the I-section beam. To determine the neutral line in other types of sections we solve the system of equations (1) and (8).

According to the guidelines reduced modulus of elasticity is based on the equation:

\[
E_{pr} = E_{pr}^L + E_{pr}^H
\]  

(11)

The resulting reduced modulus of elasticity is used in further calculation of pultruded FRP beams for bending.

2.2 Numerical investigation

According to the guidelines for calculating bending elements design value of the element’s strength is taken as the smallest, obtained from calculations strength in transverse bending \( M_Q \) (bending from transverse forces), in longitudinal bending \( M_N \) (bending from longitudinal forces) and local stability in longitudinal bending \( M_{st}^{loc} \):

\[
M_{st} = \min \{ M_Q; M_N; M_{st}^{loc} \}
\]  

(12)

When calculating building structures made of pultruded FRP, the displacement condition must also be met:

\[
f \leq f_u
\]  

(13)

where \( f \) – deflection (bend) of the structural element; \( f_u \) – limit deflection (bend), set by SP 20.13330.2016.

The calculation is performed for a beam with a cross section of 200x200x15 mm with the concentrated load in the middle of span (see Figure 2). Mechanical characteristics of pultruded FRP are accepted according to the guidelines.

![Figure 2. Design scheme for numerical studies.](image)

Equating the calculated bending moment and the moment of strength, the calculation of the maximum concentrated loads under the conditions of strength and deformability is made. The spans of beams of 3,4,5, and 6 meters are considered, while the load \( P_1 \) corresponds to the maximum strength in
transverse bending, P2 to the strength in longitudinal bending, P3 to the local stability in longitudinal bending.

3 Results and discussion
The calculation results are shown in table 1. The main result of the calculations is the reveal that the determining condition in all cases is displacements.

| Length L, m | 3  | 4  | 5  | 6  |
|-------------|----|----|----|----|
| **Ultimate concentrated load when E=E<sub>c</sub> for three conditions** |
| P1, kN      | 84.19 | 63.14 | 50.52 | 42.1 |
| P2, kN      | 47.15 | 22.4  | 12.9  | 8.4  |
| P3, kN      | 50.77 | 38.07 | 30.46 | 25.39 |
| **Ultimate concentrated load when E=E<sub>red</sub> for three conditions** |
| P1, kN      | 85.85 | 64.39 | 51.51 | 42.9 |
| P2, kN      | 47.15 | 22.4  | 12.9  | 8.4  |
| P3, kN      | 44.93 | 33.7  | 26.96 | 22.47 |
| **Ultimate concentrated load when calculating displacements** |
| E<sub>c</sub>, kN    | 17.148 | 9.1  | 5.683 | 3.616 |
| E<sub>red</sub>, kN | 19.328 | 10.348 | 6.491 | 4.142 |

Comparison of calculation results using the minimum module of elasticity and reduced modulus of elasticity is given in table 2.

| Length L, m | 3  | 4  | 5  | 6  |
|-------------|----|----|----|----|
| **Strength** |
| P1, %       | 1.97 | 1.97 | 1.96 | 1.9 |
| P2, %       | 0   | 0   | 0   | 0   |
| P3, %       | -11.503 | -11.47 | -11.49 | -11.5 |
| Deflections, % | 12.712 | 13.71 | 14.21 | 14.54 |

Thus, the reduced modulus of elasticity in the calculation of the deformability is reduced to 14.5 %, the minimum modulus of elasticity in the calculations is made in the margin of strength. However, due to the displacement of the neutral axis relative to the center of gravity the distance from the neutral axis to the extreme cross-section fibers increases, which leads to a decrease in the bearing capacity for local stability by 11.5 % (the third strength condition).

This factor plays a significant role in the case of the calculation of the minimum of three moments obtained from strength calculations in transverse bending M<sub>st</sub><sup>Q</sup> (bending from transverse forces), in longitudinal bending M<sub>st</sub><sup>N</sup> (bending from longitudinal forces) and local stability in longitudinal bending M<sub>st</sub><sup>loc</sup>, the latter is the determining factor.

3.1 Computer modeling of bending elements with setting modulus of elasticity
To confirm the obtained results of numerical investigation, a computer model was created in the “ANSYS” software package in three variants: with a given modulus of elasticity under compression, reduced modulus of elasticity, and taking into account the different performance of the material under tension and compression.
Variant 1. The material is specified with a compression modulus of elasticity. The load is set as 9.1 kN (see Figure 3). The results of numerical investigation and computer modeling are the same, while the deflection value in the simulation is 23.76 mm the difference is less than 2%.

![Figure 3. Isofield of displacements in the calculation with the modulus of elasticity in compression.](image)

Variant 2. The material is specified with a reduced modulus of elasticity obtained by numerical investigation. The load is set as 9.1 kN (see Figure 4). The displacement is 20.6 mm less than when setting the elastic modulus in compression by 14%, which coincides with the numerical calculation.

![Figure 4. Isofield of displacements in the calculation with the reduced modulus of elasticity.](image)

Variant 3. “ANSYS” has catalogs with the specified characteristics of materials including fiberglass. However, the calculations in the study are based on mechanical characteristics of the pultruded FRP of OOO “Tatneft-Presscomposite”. None of the methods used in the program to set the material could meet the requirements. As the result, we need to set 6 elastic modulus, two in each direction. This can only be done as an anisotropic material, but there is not enough raw data for this. The value of displacements was greater by 17% than when setting the minimum modulus of elasticity (see Figure 5).
Figure 5. Isofield of displacements in the calculation with the modulus of elasticity in compression and tension specified by the command

4 Conclusions
1. An algorithm for calculating the position of the neutral axis in beams with a symmetrical cross-section relative to the horizontal axis of pultruded FRP in flat bending has been developed.
2. Numerical investigations and computer simulations were performed on the “ANSYS” software package in three variants: with a given modulus of elasticity under compression, reduced modulus of elasticity, and taking into account the different performance of the material under tension and compression.
3. When comparing the results obtained, the refined calculation taking into account the reduced modulus of elasticity allows reducing the deformability up to 15%.
4. When calculating the pultruded FRP with the displacement of the neutral line the strength for local stability is 11.5 % less then when calculating using the minimum module of elasticity.

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