Higgs boson mass in supersymmetry to three loops

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Within the minimal supersymmetric extension of the Standard Model, the mass of the light CP-even Higgs boson is computed to three-loop accuracy, taking into account the next-to-next-to-leading order effects from supersymmetric Quantum Chromodynamics. We consider two different scenarios for the mass hierarchies of the supersymmetric spectrum. Our numerical results amount to corrections of about 500 MeV which is of the same order as the experimental accuracy expected at the CERN Large Hadron Collider (LHC).

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I. INTRODUCTION

Supersymmetry is currently the most-studied extension of the Standard Model (see, e.g., Ref. \[1\]). It provides solutions to some profound theoretical problems of the Standard Model: the fine tuning of the Higgs mass, the (non-)unification of gauge couplings, a mechanism for spontaneous symmetry breaking, and a Cold Dark Matter candidate.

The minimal supersymmetric extension of the Standard Model (MSSM) is based on a two-Higgs-doublet model (2HDM) with five physical Higgs bosons: two CP-even $h/H$, one CP-odd $A$ (also named the “pseudo-scalar” Higgs), and two charged scalars $H^{\pm}$. Each particle of this 2HDM receives a SUSY partner of opposite spin-statistics, where left- and right-handed components of a Standard Model Dirac fermion are attributed with separate scalars $f_{L/R}$ which mix to the physical mass eigenstates $f_{1/2}$.

Compared to the Standard Model, the MSSM Higgs sector is described by two additional parameters, usually chosen to be the pseudo-scalar mass $M_A$ and the ratio of the vacuum expectation values of the two Higgs doublets, $\tan \beta = v_2/v_1$. The masses of the other Higgs bosons are then fixed by SUSY constraints. In particular, the mass of the light CP-even Higgs boson, $M_h$, is bounded from above. At tree-level, it is $M_h < M_Z$. Radiative corrections to the Higgs pole masses raise this bound substantially to values that were inaccessible by LEP \[2, 3, 4\]. The large numerical impact is due to a contribution $\sim \alpha t M_t^2 \sim M_t^4$ coming from top- and stop quark loops ($M_t$ is the top quark mass and $\sqrt{\alpha_t}$ is proportional to the top Yukawa coupling).

The one-loop corrections to the Higgs pole masses are known without any approximations \[5, 6, 7, 8\]. They show that the bulk of the numerical effects can be obtained in the so-called effective-potential approach in the limit of vanishing external momentum. Motivated by this observation, all presumably relevant two-loop terms have since been evaluated in this approach (for reviews, see e.g. Refs. \[9, 10\]). More recently there has been quite some activity in the context of the MSSM with complex parameters which can lead to sizeable effects (see, e.g., Ref. \[11\]). The two-loop results are implemented in the numerical programs FeynHiggs \[12\] and CPsuperH \[13, 14\] using on-shell particle masses, and in SoftSusy \[15\], SPheno \[16\], and Suspect \[17\] using DR parameters, that is, dimensional reduction with minimal subtraction. The influence of terms that go beyond the approximation of vanishing external momentum has been investigated in Ref. \[18\].

Based mostly on the renormalization scale and scheme dependence, the theoretical uncertainty on the prediction of the light Higgs boson mass $M_h$ has been estimated to 3-5 GeV \[10, 19\]. This is to be compared with the expected experimental uncertainty of a Higgs mass measurement at the LHC of the order of 100-200 MeV \[20\]. At an International Linear Collider, this goes even down to roughly 50 MeV \[21\]. These numbers clearly show the need for three-loop corrections to the SUSY Higgs bosons masses in order to fully exploit the physics potential of these colliders.

In fact, quite recently the leading and next-to-leading logarithmic terms in $\ln(M_{\text{SUSY}}/M_t)$ at three-loop level have been obtained, where $M_{\text{SUSY}}$ is the typical scale of SUSY particle masses \[22\]. In this letter, we want to present the first genuine three-loop calculation of the lightest Higgs boson mass, focusing on a few simplifying limiting cases for the sake of brevity. In particular, we consider effects of order $\alpha t \alpha_t^2$, keep only the leading terms $\sim M_t^4$, and neglect all mixing effects in the stop sector. More general results and their detailed phenomenological impacts shall be deferred to a later publication.

II. THE HIGGS BOSON MASS IN THE MSSM

At tree-level, the mass matrix of the neutral, CP-even Higgs bosons $h, H$ has the following form:

\[ M_{H,\text{tree}}^2 = \frac{\sin 2\beta}{2} \times \begin{pmatrix} M_2^2 \cot \beta + M_A^2 \tan \beta & -M_Z^2 - M_A^2 \\ -M_Z^2 - M_A^2 & M_2^2 \tan \beta + M_A^2 \cot \beta \end{pmatrix} \tag{1} \]
The diagonalization of $\mathcal{M}_{H,\text{tree}}^2$ gives the tree-level result for $M_1$ and $M_H$, and leads to the well-known bound $M_1 < M_Z$ which is approached in the limit $\tan \beta \to \infty$.

Quantum corrections to the Higgs boson masses are incorporated by evaluating the poles of the Higgs boson propagator at higher orders. As mentioned in the Introduction, the numerically dominant contributions can be obtained in the approximation of zero external momentum (see, e.g., Refs. [22]) which we will adopt in the following. Furthermore, we will only consider corrections of order $\alpha_0^2 \alpha_t^2$. Apart from the quark, squark, and gluino masses, there is another parameter with mass dimension, the trilinear coupling of the soft SUSY breaking terms, $A_t$. Before renormalization, we express it through the stop masses $M_{\tilde{t}_1}$, $M_{\tilde{t}_2}$, the stop mixing angle $\theta_t$, and the bilinear Higgs parameter $\mu_{\text{SUSY}}$ as follows:

$$2M_1 A_t = (M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2) \sin 2\theta_t + 2M_t \mu_{\text{SUSY}} \cot \beta.$$  \hspace{1cm} (2)

The mass matrix $\mathcal{M}_H^2$ is obtained from the quadratic terms in the Higgs boson potential constructed from the fields $\phi_1$ and $\phi_2$. They are related to the physical Higgs mass eigenstates via a mixing angle $\alpha$. Since $\phi_1$ does not couple directly to top quarks, it is convenient to perform the calculations of the Feynman diagrams in the $(\phi_1, \phi_2)$ basis. Including higher order corrections, one obtains the Higgs boson mass matrix

$$\mathcal{M}_H^2 = \mathcal{M}_{H,\text{tree}}^2 - \left( \frac{\Sigma_{\phi_1 \phi_1}}{\Sigma_{\phi_1 \phi_2}} \right),$$  \hspace{1cm} \hspace{1cm} (3)

which again gives the physical Higgs boson masses upon diagonalization. The renormalized quantities $\Sigma_{\phi_1}$, $\Sigma_{\phi_2}$ and $\Sigma_{\phi_1 \phi_2}$ are obtained from the self energies of the fields $\phi_1$, $\phi_2$, $A$, evaluated at zero external momentum, as well as from tadpole contributions of $\phi_1$ and $\phi_2$ (see, e.g., Ref. [9]). Let us remark that if one sets $M_{\tilde{t}_1} = M_{\tilde{t}_2}$ and $A_t = 0$, and evaluates only the leading contribution of $\Sigma_{\phi_2}$, then only $\Sigma_{\phi_2} \neq 0$ and the matrix $\mathcal{M}_H^2 - \mathcal{M}_{H,\text{tree}}^2$ is diagonal. On the other hand, if we allow for non-zero $A_t$, also $\Sigma_{\phi_1}$ and $\Sigma_{\phi_1 \phi_2}$ contribute in general.

The calculation of $\Sigma_{\phi_2}$ is organized as follows: All Feynman diagrams are generated with QCRAF [24]. In order to properly take into account the Majorana character of the gluino, the output is subsequently manipulated by a PERL script which applies the rules given in Ref. [25]. The various diagram topologies are identified and transformed to FORM [27] with the help of q2e and exp [27, 28]. The program exp is also used in order to apply the asymptotic expansion (see, e.g., Ref. [24]) in the various mass hierarchies. The actual evaluation of the integrals is performed with the package MATAD [30], resulting in an expansion in $d-4$ for each diagram, where $d$ is the space-time dimension. The total number of three-loop diagrams amounts to about 16,000.

At three-loop level we need to renormalize the top quark mass, the top squark mass, and the stop mixing angle at the two-loop order. In addition, the one-loop counterterm of the gluino mass is needed for the renormalization of the two-loop expression. We implement Dimensional Reduction (DRED) with the help of the so-called $\epsilon$-scalars which appear for the first time at two loops. The renormalization of the $\epsilon$-scalar mass is performed in the on-shell scheme, requiring that the renormalized mass is equal to zero. In the literature this is referred to as DIR scheme.

The one-loop on-shell counterterms are well-known (see, e.g., Refs. [8, 31, 32, 33]). As far as the two-loop counterterms for the squarks and quarks are concerned, one can find the results in Refs. [31, 32]. However, it is rather tedious to extract the results for the mass hierarchies we are interested in. Thus, we re-computed the corresponding corrections.

To our knowledge, the two-loop counterterm for the stop mixing angle is not yet available in the literature. It turns out that in our approximation, where $M_{\tilde{t}_1} = M_{\tilde{t}_2}$ and $A_t = 0$, only the one-loop counterterm of the mixing angle enters the three-loop result.

As a cross check for our calculation, we recalculated the exact two-loop result (in the limit of vanishing external momentum) and find perfect agreement with the literature [24, 30].

Furthermore, the expansion of the exact expressions confirms the limiting cases discussed below. Both the two- and three-loop calculations are performed for a general QCD gauge parameter $\xi_8$. The independence of the final results on $\xi_8$ serves as another welcome check on the correctness of our result.

We use anti-commuting $\gamma_5$ which is allowed for fermion traces which involve an even number of $\gamma_5$ matrices. It turns out that all traces involving an odd number of $\gamma_5$ vanish because they contain less than four gamma matrices.

In the following we discuss three different cases for the mass hierarchy. In all cases we set the light quark masses to zero.

(i) Supersymmetric limit, i.e., $M_t = M_{\tilde{t}}$ and the gluino and other squarks are massless: $M_{\tilde{g}} = M_{\tilde{g}} = 0$. The quantum corrections to the Higgs boson mass vanish in this case, as required by supersymmetry. Still, the individual diagrams are different from zero and thus the calculation imposes a strong check on our setup.

(ii) Massless gluino, $M_{\tilde{g}} = 0$. Expanding in the limit $M_t \ll M_{\tilde{t}} = M_{\tilde{g}} \equiv M_{\text{SUSY}}$, we obtain for the leading
term of this expansion
\begin{align}
\hat{\Sigma}_{\phi_2} &= \frac{3G_F M_t^4}{\sqrt{2\pi^2} \sin^2 \beta} \left\{ L_{tS} + \frac{\alpha_s}{\pi} \left[ -1 - 4L_{tS} + 2L_{tS}^2 \right] \right.
onumber \\
&\quad + \left( \frac{\alpha_s}{\pi} \right)^2 \left\{ - \frac{593}{27} - \frac{3}{4} L_{\mu \tau} + \frac{23}{81} \pi^2 + \frac{401}{18} \zeta_3 \right\} L_{tS} \nonumber \\
&\quad \left. + \left( \frac{47}{4} - 3L_{\mu \tau} + \frac{4}{9} \pi^2 - \frac{4}{9} \pi^2 \ln 2 \right) L_{tS} \right\} \left[ L_{tS} + \frac{5}{2} L_{tS}^2 \right] \bigg] \bigg\}, \tag{4} 
\end{align}

with $L_{\mu \tau} = \ln(\mu^2/M_{\phi_2}^2)$ and $L_{tS} = \ln(M_t^2/M_{\phi_2}^2)$. (iii) Common SUSY mass. In this scenario we assume $\alpha_s(\mu_t) = 0.0926$ for a common SUSY mass $M_{\text{SUSY}} = M_{\tilde{q}}$. It is interesting to mention that large cancellations occur among the cubic, quadratic, linear and non-logarithmic term of $\hat{\Sigma}_{\phi_2}$ at three-loop order. E.g., for our default input values of $\mu_t = 1 \text{ TeV}$ and $\tan \beta = 30$, the size of the overall corrections by about 30% with respect to the one-loop result.

III. NUMERICAL RESULTS

In the remainder of this letter, we discuss the numerical effect of our result, restricting ourselves to $A_t = 0$. We adopt the on-shell scheme for the quark, squark and gluino masses.

We choose $\mu = M_t$ as the default value for the renormalization scale. First we compute $\alpha_s(M_t)$, defined in the DR scheme and the full SUSY theory, from the SM input value $\alpha_s(M_Z) = 0.1189$ [27] which is given within five-flavour QCD. We follow the procedure outlined in Ref. [38] which includes three-loop running and two-loop matching effects. As a result we obtain, e.g., $\alpha_s(M_t) = 0.0926$ for a common SUSY mass $M_{\text{SUSY}} = 1 \text{ TeV}$. The SM input parameters are given as $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$, $M_Z = 91.1876 \text{ GeV}$ [39], $M_t = 170.9 \text{ GeV}$ [40]. For the heavy squark mass ($\tilde{q} \neq t$) we use $M_{\tilde{q}} = 2 \text{ TeV}$.

In order to evaluate the tree-level approximation of the Higgs boson mass we also need the parameters $M_A$ and $\tan \beta$. If not stated otherwise we adopt the values $M_A = 1 \text{ TeV}$ and $\tan \beta = 40$. Since these parameters do not enter the corrections considered in this paper, they only have minor influence on the plots presented in the following.

In Figs. [1] and [2] we discuss the difference between the Higgs boson mass evaluated with $i$-loop approximation and the tree-level result,

$$\Delta M_h^{(i)} = M_h^{(i-\text{loop})} - M_h^{\text{tree}}. \tag{6}$$

Fig. [1] shows $\Delta M_h^{(i)}$ for $i = 1$ (dotted), $i = 2$ (dashed) and $i = 3$ (solid line) as a function of $M_{\text{SUSY}}$ in the range between 200 GeV and 2 TeV. As is well known, the one-loop corrections are large, increasing $M_h$ by up to 46 GeV. The two-loop effects are negative, reducing the size of the overall corrections by about 30% with respect to the one-loop result.
The three-loop terms are much smaller and clearly stabilize the perturbative behaviour. At \( \mu = M_t \), for example, they lead to a further reduction of \( \Delta M_h^{(3)} \) by about 400 MeV for \( M_{SUSY} = 300 \) GeV and an enhancement of about 500 MeV for \( M_{SUSY} = 2 \) TeV. Note that the numerical impact is larger than the precision on the lightest Higgs boson mass as expected at the LHC.

In order to estimate the size of the higher order corrections, we consider the dependence of the result on the choice of the renormalization scale. In Fig. 2 we plot \( \Delta M_h^{(1)} \) as a function of \( \mu \) which is varied from 50 GeV to 500 GeV. The two-loop results show a variation of more than 1 GeV over this range. The error band derived in this way nicely covers the three-loop result, which itself varies by less than 135 MeV. For other values of \( M_{SUSY} \) the variation can reach up to 100 MeV. The three-loop curve in Fig. 2 shows a shallow minimum close to \( \mu = M_t \) which in turn is close to the intersection point of the two- and three-loop result. This justifies the choice \( \mu = M_t \) as default value.

IV. CONCLUSIONS

To summarize, in this letter the three-loop corrections to the lightest Higgs boson mass have been computed in three different limits of the SUSY parameter space. For the phenomenologically interesting case where the gluino and top squarks have about the same mass and the remaining squarks are heavier, we observe effects of approximately 500 MeV. The dependence of the three-loop result on the renormalization scale indicates that the residual theoretical uncertainty matches the expected accuracy for a Higgs mass measurement at LHC and possibly even at a future linear collider.

It remains to say that the calculational setup which was used to obtain the results of this paper is not restricted to the specific MSSM parameter points considered here. A more comprehensive study is in preparation and will be presented elsewhere.

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