Transfer Learning Across Patient Variations with Hidden Parameter Markov Decision Processes

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Abstract

Due to physiological variation, patients diagnosed with the same condition may exhibit divergent, but related, responses to the same treatments. Hidden Parameter Markov Decision Processes (HiP-MDPs) tackle this transfer-learning problem by embedding these tasks into a low-dimensional space. However, the original formulation of HiP-MDP had a critical flaw: the embedding uncertainty was modeled independently of the agent’s state uncertainty, requiring an unnatural training procedure in which all tasks visited every part of the state space—possible for robots that can be moved to a particular location, impossible for human patients. We update the HiP-MDP framework and extend it to more robustly develop personalized medicine strategies for HIV treatment.

1 Introduction

Due to physiological variation, patients diagnosed with the same condition may exhibit divergent, but related, responses to the same treatments. To develop optimal treatment or control policies for a patient, it is undesirable and ineffectual to start afresh each time a new individual is cared for. However, each patient may still require a tailored treatment plan as “one-size-fits-all” treatments can introduce more risk in aggressive diagnoses. Ideally, an agent tasked with developing an optimal health management policy would be able to leverage the similarities across separate, but related, instances while also customizing treatment for the individual. This paradigm of learning introduces a compelling regime for transfer learning.

The Hidden Parameter Markov Decision Process (HiP-MDP) [9] formalizes the transfer learning task in the following way: first it assumes that any task instance can be fully parameterized by a bounded number of latent parameters \( w \). That is, we posit that the dynamics dictating a patient’s physiological response can be expressed as \( T(s'|s, a, w_b) \) for patient \( b \). Second, we assume that the system dynamics will not change during a task and an agent would be capable of determining when a change occurs (e.g. a new patient). Doshi-Velez and Konidaris [9] show that the HiP-MDP can identify the dynamics of a new task instance and flexibly adapt to the variations present therein. However, the original HiP-MDP formulation had a critical flaw: the embedding uncertainty of the latent parameter space was modeled independently from the agent’s state uncertainty. This assumption required the agent to have the ability to visit every part of the state space before identifying the variations present in the dynamics of the current instance. While this may be feasible in robotic systems, it is not generally available to domains in healthcare.

We present an alternative HiP-MDP formulation that alleviates this issue via a Gaussian Process latent variable model (GPLVM). This approach creates a unified Gaussian Process (GP) model for both inferring the transition dynamics within a task instance but also in the transfer between task instances [5]. Steps are taken to avoid negative transfer by selecting the most representative examples of the prior instances with regards to the latent parameter setting. This change in the model allows for better uncertainty quantification and thus more robust and direct transfer. We ground our approach...
with recent advances in the use of GPs to approximate dynamical systems and in transfer learning as well as discuss relevant Reinforcement Learning (RL) applications to healthcare (Sec. 2). In Sec. 3 we formalize the adjustments to the HiP-MDP framework and in Sec. 5 we present the performance of the adjusted HiP-MDP on developing personalized treatment strategies within HIV simulators.

2 Related Work

The use of RL (and machine learning, in general) in the development of optimal control policies and decision making strategies in healthcare [22] is gaining significant momentum as methodologies have begun to adequately account for uncertainty and variations in the problem space. There have been notable efforts made in the administration of anesthesia [18], in personalizing cancer [24] and HIV therapies [11] and in understanding the causality of macro events in diabetes management [16]. Marivat et al. [15] formalized a routine to accommodate multiple sources of uncertainty in batch RL methods to better evaluate the effectiveness of treatments across a subpopulations of patients. We similarly attempt to address and identify the variations across subpopulations in the development treatment policies. We instead, attempt to account for these variations while developing effective treatment policies in an approximate online fashion.

GPs have increasingly been used to facilitate methods of RL [19, 20]. Recent advances in modeling dynamical systems with GPs have led to more efficient and robust formulations [7, 8], most particularly in the approximation and simulation of dynamical systems. The HiP-MDP approximates the underlying dynamical system of the task through the training of a Gaussian Process dynamical model [6, 27] where only a small portion of the true system dynamics may be observed as is common in partially observable Markov Decision Processes (POMDP) [12]. In order to facilitate the transfer between task instances we embed a latent, low-dimensional parametrization of the system dynamics with the states. By virtue of the GP [13, 25], this latent embedding allows the HiP-MDP to infer across similar task instances and provide a better prediction of the currently observed system.

The use of GPs to facilitate the transfer of previously learned information to new instances of the same or a similar task has a rich history [2, 12, 19]. More recently, there have been advances in organizing how the GP is used to transfer, being constrained to only select previous task instances where positive transfer occurs [5, 14]. This adaptive approach to transfer learning helps to avoid previous instances that would otherwise negatively affect effective learning in the current instance. By selecting the most relevant instances of a current task for transfer, learning in the current instance becomes more efficient.

3 Model

The HiP-MDP is described by a tuple: \(\{S, A, \Theta, T, R, \gamma, P_0\}\), where \(S\) and \(A\) are the sets of states \(s\) and actions \(a\) (eg. patient health state and prescribed treatment, respectively), and \(R(s, a)\) is the reward function mapping the utility of taking action \(a\) from state \(s\). The transition dynamics \(T(s' | s, a, \theta_b)\) for each task instance \(b\) depends on the value of the hidden parameters \(\theta_b \in \Theta\) (eg. patient physiology). Where the set of all possible parameters \(\theta_b\) is denoted by \(\Theta\) and where \(P_0\) is the prior over these parameters. Finally, \(\gamma \in (0, 1]\) is the factor by which \(R\) is discounted to express how influential immediate rewards are when learning a control policy. Thereby, the HiP-MDP describes a class of tasks; where particular instances of that class are obtained by independently sampling a parameter vector \(\theta_b \in \Theta\) at the initiation of a new task instance \(b\). We assume that \(\theta_b\) is invariant over the duration of the instance, signaling distinct learning frontiers between instances when a newly drawn \(\theta_b\) accompanies observed additions to \(S\) and \(A\).

The HiP-MDP presented in [9] provided a transition model of the form:

\[
(s_d' - s_d) \sim \sum_k z_{kad} w_{kb} f_{kad}(s) + \epsilon
\]

\[
\epsilon \sim \mathcal{N}(0, \sigma_{mod}^2)
\]

which sought to learn weights \(w_{kb}\) based on the \(k^{th}\) latent factor corresponding to task instance \(b\), filter parameters \(z_{kad} \in \{0, 1\}\) denoting whether the \(k^{th}\) latent parameter is relevant in predicting dimension \(d\) when taking action \(a\) as well as task specific basis functions \(f_{kad}\) drawn from a GP. While this formulation is expressive, it presents a problematic flaw when trained. Due to the independence
of the weights \( w_{kb} \) from the basis functions \( f_{kad} \), training the HiP-MDP requires canvassing the state space \( S \) in order to infer the filter parameters \( z_{kad} \) and learn the instance specific weights \( w_{kb} \) for each latent parameter.

We bypass this flaw by applying a GPLVM [13] to jointly represent the dynamics and the latent weights \( w_b \) corresponding to a specific task instance \( b \). This leads to providing as input to the GP, with hyperparameters \( \psi \), the augmented state \( \tilde{s} =: [s^T, a, w_b]^T \). The approximated transition model then takes the form of:

\[
\begin{align*}
    s'_d &\sim f_d(\tilde{s}) + \epsilon \\
    f_d &\sim GP(\psi) \\
    w_b &\sim \mathcal{N}(\mu_b, \Sigma_b) \\
    \epsilon &\sim \mathcal{N}(0, \sigma_{bd})
\end{align*}
\]

This approach enables the HiP-MDP to flexibly infer the dynamics of a new instance by virtue of the statistical similarities found in the learned covariance function between observed states of the new instance and those from prior instances. Another feature of formulating the HiP-MDP after this fashion is that we are able to leverage the marginal log likelihood of the GP to optimize the weight distribution and thereby quantify the uncertainty [3, 4] of the latent embedding of \( w_b \) for \( \theta_b \). These two features of reformulating the HiP-MDP as a GPLVM allows for more robust and efficient transfer.

**Demonstration** We demonstrate a toy example (see Fig. 1) of a domain where an agent is able to learn separate policies according to a hidden latent parameter. Instances inhabiting a “blue” latent parametrization can only pass through to the goal region over the blue boundary while those with a “red” parametrization can only cross the red boundary. After a few training instances, the HiP-MDP is able to separate the two latent classes and develops individualized policies for each. Due to the flexibility enabled by embedding the latent parametrization into the system’s state, the GPLVM identifies which class the current instance belongs to within the first couple of training episodes. In total, this example took approximately 30 minutes to develop optimal policies for 20 task instances. We place an unclassified survey point in the top left quadrant to gather information about the policy uncertainty given the two latent classes.

![Figure 1: Toy Problem](image)

**Figure 1:** Toy Problem: (a) Schematic outlining the domain, (b) learned policy for “red" parametrization, (c) learned policy for “blue" parametrization, (d) uncertainty measure for input point according to separate latent classes.

4 **Inference**

**Parameter Learning and Updates** We deploy the HiP-MDP when the agent is provided a large amount of batch observational data from several task instances (e.g. patients) and tasked with quickly performing well on new instances. With this observational data the GP transition functions \( f_d \) are learned and the individual weighting distributions for \( w_b \) are optimized. However, the training of the \( f_d \) requires computing inverses of matrices of size \( N = \sum_b n_b \) where \( n_b \) is the number of data points collected from instance \( b \). To streamline the approximation of \( T \) we choose a set of support points...
Control Policy  A control policy is learned for each task instance \( b \) following the procedure outlined in [7] where a set of tuples \((s, a, s', r)\) are observed and the policy is periodically updated (as is the latent embedding \( w_b \)) in an online fashion, leveraging the approximate dynamics of \( T \) via the \( f^*_d \) to create a synthetic batch of data from the current instance. This generated batch of data from \( b \) is then used to improve the current policy via the Double Deep Q Network variant of fitted-Q using prioritized experience replay [17, 21, 26]. Multiple episodes are run from each instance \( b \) to optimize the policy for completing the task under the hidden parameter setting \( \theta_b \). After doing so, the hyperparameters of the GP defining the \( f_d \) are updated before learning for another randomly manifest task instance.

5 Experimental Results

Baselines  We benchmark the HiP-MDP framework in the HIV domain by observing how an agent would perform without transferring information from prior patients to aid in the efficient development of the treatment policy for a current patient. We do this by representing two ends of the precision medicine spectrum; a "one-size-fits-all" approach that learns a single treatment policy for all patients by using all previous patient data together and a "personally tailored" treatment plan where a single patient’s data is all that is used to train the policy. We represent these baselines in environments where a model is present (with the simulators) or absent (utilizing the GP approximation).

HIV Treatment  Ernst, et.al. [11] leverage the mathematical representation of how a patient responds to HIV treatments [1] in developing an RL approach to find effective treatment policies using methods introduced in [10]. The learned treatment policies cycle on and off two different types of anti-retroviral medication in a sequence that maximizes long-term health. By perturbing the underlying system parameters one can simulate varied patient physiologies. We leverage these variations via the GPLVM augmentation to the HiP-MDP to efficiently learn treatment policies that match the naive "personally tailored" baseline but with reliance on much less data. The HiP-MDP also outperforms the "one-size-fits-all" baseline, as expected. (see Fig. 2 for representative results). We see that the GPLVM driven HiP-MDP is capable of immediately taking advantage of the prior information from previously learned data even in the face of unique physiological characteristics. The robust and efficient manner in which the HiP-MDP achieves such results in the HIV domain is promising and, in turn, motivates further inquiry into a more generalized learning agent for the development of other individualized medical treatment plans.

Figure 2: Representative results of applying the GPLVM aided HiP-MDP model to the HIV treatment simulator as provided from [11]. The HiP-MDP learned treatment policy (blue) matches or improves on the naive baseline policy development strategies.
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