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Joachim Cohen, W. Clarke Smith, Michel H. Devoret, and Mazyar Mirrahimi

1QUANTIC project-team, INRIA Paris, France
2Department of Applied Physics, Yale University, New Haven, CT 06520, USA
3Yale Quantum Institute, Yale University, New Haven, CT 06520, USA

A central requirement for any quantum error correction scheme is the ability to perform quantum non-demolition measurements of an error syndrome, corresponding to a special symmetry property of the encoding scheme. It is in particular important that such a measurement does not introduce extra error mechanisms, not included in the error model of the correction scheme. In this letter, we ensure such a robustness by designing an interaction with a measurement device that preserves the degeneracy of the measured observable. More precisely, we propose a scheme to perform continuous and quantum non-demolition measurement of photon-number parity in a microwave cavity. This corresponds to the error syndrome in a class of error correcting codes called the cat-codes, which have recently proven to be efficient and versatile for quantum information processing. In our design, we exploit the strongly nonlinear Hamiltonian of a high-impedance Josephson circuit, coupling a high-Q cavity storage cavity mode to a low-Q readout one. By driving the readout resonator at its resonance, the phase of the reflected/transmitted signal carries directly exploitable information on parity-type observables for encoded cat-qubits of the high-Q mode.

By encoding a qubit in a superposition of coherent states of a harmonic oscillator, one benefits from the redundancy provided by the infinite dimensional Hilbert space of the system to realize a quantum error correction (QEC) protocol. In a set of theoretical and experimental results, various aspects of encoding [1, 2], manipulation [3–6], error syndrome measurement [7] and full quantum error correction [8, 9] with these states have been explored. Most spectacularly, a recent experiment [9] demonstrated an enhancement of the error-corrected cat-code’s lifetime with respect to all system components. The performance of the error correction is however limited by uncorrected error channels such as deterministic relaxation of the coherent states amplitude, dephasing induced by cavity’s inherited anharmonicity, and most significantly the propagating errors from the ancillary transmon [10] used for error syndrome measurements.

In an effort towards a fault-tolerant and scalable architecture for quantum information processing, we recently proposed a framework based on non-linear drives and dissipations to dynamically protect a degenerate manifold spanned by two or four coherent states against some of these error channels [3]. Indeed, by engineering a non-linear coupling to a driven bath where the exchange of photons occurs mainly in pairs (or quadruples) of photons, one can stabilize a manifold spanned by two (resp. four) coherent states $M_{2,\alpha} = \text{span}\{|\pm \alpha\rangle\}$ (resp. $M_{4,\alpha} = \text{span}\{|\pm \alpha\rangle\}$). This stabilization suppresses, exponentially in $|\alpha|^2$, the phase-flip errors of a logical qubit given by $|0\rangle_L = |C^{(0\text{mod}4)}_\alpha\rangle$, $|1\rangle_L = |C^{(2\text{mod}4)}_\alpha\rangle$ (resp. $|0\rangle_L = |C^{(3\text{mod}4)}_\alpha\rangle$, $|1\rangle_L = |C^{(1\text{mod}4)}_\alpha\rangle$) where, $|C^{(\pm)}_\alpha\rangle = \mathcal{N}_\pm(|\alpha\rangle \pm |\alpha\rangle)$, $|C^{(0\text{mod}4)}_\alpha\rangle = N_0(|C^{(1\text{mod}4)}_\alpha\rangle + |C^{(3\text{mod}4)}_\alpha\rangle)$, $|C^{(1\text{mod}4)}_\alpha\rangle = N_1(|C^{(2\text{mod}4)}_\alpha\rangle - |C^{(0\text{mod}4)}_\alpha\rangle)$, $|C^{(2\text{mod}4)}_\alpha\rangle = N_2(|C^{(3\text{mod}4)}_\alpha\rangle - i|C^{(1\text{mod}4)}_\alpha\rangle)$, $|C^{(3\text{mod}4)}_\alpha\rangle = N_3(|C^{(0\text{mod}4)}_\alpha\rangle + i|C^{(2\text{mod}4)}_\alpha\rangle)$, and $N_{\pm,0,1,2,3}$ are normalization constants near $1/\sqrt{2}$. One therefore deals with logical qubits that are only susceptible to bit-flip errors. These errors can next be suppressed to first order by photon-number parity measurements as in [9]. Also, one can achieve higher-order correction through a register of such logical qubits and performing joint parity measurements between adjacent ones.

While initial experiments with two-photon driven dissipation [11] illustrate the viability of such a framework, many theoretical and experimental improvements are required in order to achieve a fully fault-tolerant architecture. One very important improvement concerns the quantum non-demolition (QND) measurement protocols for parity-type observables. Such single-mode or two-mode photon number parity measurements have been performed using an ancillary transmon and a Ramsey interferometry type scheme [6, 7, 12]. These measurements however are not fault-tolerant and represent the main limitation in QEC [9]. In this letter, we propose a new framework to perform QND measurement of various parity-type observables which could be integrated in a fault-tolerant architecture.

The current measurement schemes [6, 7] are based on a dispersive coupling of the cavity mode to a transmon through a Hamiltonian of the form $-\hbar \chi |e\rangle \langle e| a^\dagger a$. The parity measurement is performed by initializing the

\* Electronic address: joachim.cohen@inria.fr
\$ Electronic address: clarke.smith@yale.edu
\$ Electronic address: michel.devoret@yale.edu
\$ Electronic address: mazyar.mirrahimi@inria.fr
transmon in the superposition \((|g⟩ + |e⟩)/√2\) and waiting for a time \(\pi/\chi\). The \(|e⟩\) state of transmon acquires a \(\pi\) phase only for odd cavity Fock states. A measurement of the transmon, distinguishing between \((|g⟩ + |e⟩)/√2\) and \((|g⟩ − |e⟩)/√2\) indicates the photon number parity. Nevertheless, a \(T_1\) error of the transmon during the evolution propagates to the cavity mode inducing photon dephasing. Indeed, such a measurement protocol is not fault-tolerant as the eigenstates of the measured observable (here parity cat states) get entangled to the ancillary system during the measurement protocol, making them vulnerable to the ancilla’s errors (here \(T_1\) errors): a cat state \(|C_T⟩\) evolves to \((|C_T⟩ ⊗ |g⟩ + |C_T⟩ ⊗ |e⟩)/√2\). A fault-tolerant parity measurement could be for instance achieved through an effective Hamiltonian of the form \(h\chi|e⟩⟨e|\cos(\pi\varphi_a|a⟩)\). A cat state \(|C_T⟩\) would then evolve to \((|C_T⟩ ⊗ (|g⟩ + e^{±i\chi t}|e⟩))/√2\), without entangling to the transmon.

While the engineering of a highly degenerate Hamiltonian of the form \(h\chi\cos(\pi\varphi_a|a⟩)\) seems to be a complicated task, that we show in presence of two-photon or four-photon driven dissipation, it could be effectively achieved with the help of quantum Zeno dynamics \([13]\). By confining the dynamics to the manifold \(\mathcal{M}_{2, a}\), a physical Hamiltonian \(H\) acts as a projected one \(\hat{H}|\mathcal{M}_{2, a}⟩ = \Pi|\mathcal{M}_{2, a}⟩ H|\mathcal{M}_{2, a}⟩\), where \(\Pi\) represents the projector on \(\mathcal{M}_{2, a}\). To achieve an effective parity Hamiltonian, one requires a physical Hamiltonian \(H\), such that its projection on \(\mathcal{M}_{2, a}\) is equivalent to the projection of \(h\chi\cos(\pi\varphi_a|a⟩)\) on the same manifold. This means \(\hat{H}|\mathcal{M}_{2, a}⟩ = h\chi\Pi|\mathcal{M}_{2, a}⟩ \cos(\pi\varphi_a|a⟩)\Pi|\mathcal{M}_{2, a}⟩ = h\chi\sigma_z^a\), where \(\sigma_z^a\) is the Pauli operator along the z-axis of the logical qubit defined by \(|C^a⟩\) and well-approximated by \(|\alpha⟩⟨-\alpha| + |\alpha⟩⟨\alpha|\). Therefore, \(H\) should couple the two coherent states \(|±\alpha⟩\). In the context of quantum superconducting circuits, such a Hamiltonian can be achieved by strongly coupling a high impedance cavity mode to a Josephson junction \([14–16]\). Indeed, considering a cavity mode with frequency \(\omega_a\) coupled capacitively to a Josephson junction, and assuming that other modes (including the junction mode) are never excited, the effective Hamiltonian in the interaction picture will be of the form

\[
\hat{H}_{\text{int}}(t) = -\frac{E_J}{2} (\mathbf{D}|β(t)⟩ + \mathbf{D}^†|β(t)⟩), \quad β(t) = i\varphi_a e^{i\omega_a t}.
\]

Here \(E_J\) is the effective Josephson energy and \(\varphi_a = \sqrt{Z_a/2R_Q}\), where \(Z_a\) is the impedance of the cavity mode seen by the junction and \(R_Q = (2e)^2/h\) is the superconducting resistance quantum. Moreover, \(\mathbf{D}|β(t)⟩\) is the displacement operator defined by \(\mathbf{D}|β(t)⟩ = e^{β(t)|a⟩ - β^†(t)|a⟩}\). For \(\varphi_a ≈ 2|α|\), this Hamiltonian couples the two coherent states \(|±\alpha⟩\). While a practical realization of such a high impedance cavity mode is discussed later, we provide here a precise analysis of the effective Hamiltonian.

In the limit \(\hbar\omega_a \gg E_J\), we can apply a rotating wave approximation (RWA) to \(\hat{H}_{\text{int}}(t)\) \([17–19]\), leading to

\[
\hat{H}^{\text{RWA}} = -E_J e^{-\frac{2\varphi_a^2}{\Omega_a}} \sum_n L_n(\varphi_a^2)\langle n|n\rangle, \quad (1)
\]

where \(L_n(\cdot)\) is the Laguerre polynomial of order \(n\). In the presence of two-photon loss, and under the condition \(\varphi_a ≈ 2|α|\), the effective Hamiltonian takes the form of the parity Hamiltonian, i.e

\[
\hat{H}_{\mathcal{M}_{2, a}}^{\text{RWA}} = -\frac{\hbar\Omega_a}{2} \sigma_z^a + \mathcal{O}(E_J e^{-\frac{2\varphi_a^2}{\Omega_a}}), \quad (2)
\]

where \(\sigma_z^a = |C^+⟩⟨C^+_a| - |C^-⟩⟨C^-_a|\), and \(\Omega_a\) is a function of \(E_J\), \(\varphi_a\) and \(α\). It is well approximated by \(\Omega_a = E_J e^{-\frac{1}{2}(\varphi_a^2)/\pi|\alpha|^2}\) \([20]\). This result is illustrated in Fig. 1(a) and Fig. 1(b).
From the construction of a single-mode parity Hamiltonian, acting as a $\sigma^Z$ Pauli operator in the logical basis, stems an immediate route to build a joint-parity Hamiltonian of two cavity modes $a$ and $b$ both subject to two-photon dissipation. Considering two cavity modes $a$ and $b$ coupled to a Josephson junction, the interaction Hamiltonian reads $H_{int}(t) = -E_J \cos[\varphi_a (ae^{-i\omega_a t} + c.c.) + \varphi_b (be^{-i\omega_b t} + c.c.)]$. The mode frequencies $\omega_a$ and $\omega_b$ are off-resonant so that we can apply the RWA (one needs to also avoid high-order resonances)

$$H^{RWA} = -E_J e^{-\frac{\varphi_a^2}{2} + \varphi_b^2} \sum_{n_a, n_b} L_{na}(\varphi_a^2)L_{nb}(\varphi_b^2)|n_a, n_b\rangle\langle n_a, n_b|.$$

If both $a$ and $b$ are high-impedance modes and are subject to two-photon loss, one can choose $|\alpha| \approx \varphi_a/2$ and $|\beta| \approx \varphi_b/2$, such that the confined Hamiltonian takes the form $H^{RWA}_{M_{a,\alpha}} = -\hbar \Omega_{a,b} \sigma^a_{L} \otimes \sigma^b_{L}$, where $\Omega_{a,b} = \hbar \Omega_a/2E_J$, $\sigma^b_{L} = |C^+_{\alpha(\beta)}\rangle \langle C^+_{\alpha(\beta)}| - |C^-_{\alpha(\beta)}\rangle \langle C^-_{\alpha(\beta)}|$, and $\xi = (\sqrt{2} - 1)^2 \approx 0.17 [20]$. The undesired term that scales as $e^{-|\alpha|^2}$ lifts the degeneracy within the parity subspaces. This non-degeneracy is however suppressed exponentially with cavity size $|\alpha|^2$, while the effective Hamiltonian strength decreases only linearly in $|\alpha|^{-1}$ (Fig. 1(d)). Therefore for large enough $\alpha$’s we still achieve an effective parity Hamiltonian. The perfect degeneracy, for cat states of smaller amplitude, can also be achieved by introducing more junctions providing more degrees of freedom [20].

Following the same idea as in the usual dispersive measurements of superconducting qubits [22], one can perform a continuous QND measurement of the above observables, $\sigma^Z$ and $\sigma^x \otimes \sigma^x$ for the two-photon dissipation scheme, and $\pi_{4ph}$ for the four-photon dissipation. This can be done by coupling an extra off-resonant readout mode to the same junction. This mode is then driven at its resonance (Fig. 2) and the measurement outcome is imprinted on the phase of the reflected signal. More precisely, by coupling a driven readout mode $c$ to the junction, and in the case of $\varphi_c \sqrt{\pi c} \ll 1$ (here $n_c$ denotes the average number of readout photons and this requirement is equivalent to assuming $n_c \ll n_{crit}$, the critical number for dispersive approximation [23]), we achieve the effective Hamiltonians:

$$H^{disp}_{M_{a,\alpha}} \approx -\frac{\hbar \Omega_a}{2} \sigma^a_{Z} + \frac{\hbar \chi_a}{2} \sigma^a_{Z} c^\dag c + H_{\text{drive}}(t),$$

$$H^{disp}_{M_{a,b}} \approx -\frac{\hbar \Omega_{a,b}}{2} \sigma^a_{Z} \sigma^b_{Z} + \frac{\hbar \chi_{a,b}}{2} \sigma^a_{Z} \sigma^b_{Z} c^\dag c + H_{\text{drive}}(t),$$

Here $H_{\text{drive}}(t) = \hbar (\epsilon_c(t) c^\dag c + \epsilon_c^*(t) c)$, $\tilde{\Omega}_a = e^{-\varphi_a^2/2} \Omega_a$, $\tilde{\Omega}_{a,b} = e^{-\varphi_{a,b}^2/2} \Omega_{a,b}$, $\chi_a = \Omega_{a,c} \chi_{a,c}$, $\chi_{a,b} = \Omega_{a,b} \chi_{a,b}$. The first terms in the above Hamiltonians simply induce deterministic rotations in the associated parity subspaces, whereas the second terms correspond to frequency pulls on mode $c$ that depend on the values of associated observables. By driving the mode $c$ at resonance, the measurement outcome is imprinted on the phase of the pointer coherent state. Taking $\kappa_c$ to be the dissipation rate of $c$ induced by its coupling to a transmission line, the measurement rate is optimal when $\kappa_c = \chi_a$ ($\chi_{a,b}$ for joint-parity measurement) [24]. This optimal rate is given by (see Fig. 3)

$$\Gamma_{m}^{a} = \bar{n}_c \chi_a = \bar{n}_c \kappa_c^2 e^{-\frac{\varphi_a^2}{2} + \varphi_b^2} E_J e^{-\frac{1}{2}(\varphi_a - 2|\alpha|)^2}{\hbar}\sqrt{|\alpha|^2 |\varphi_a|^2}.$$  

(4)

$$\Gamma_{m}^{a,b} = \bar{n}_c \chi_{a,b} = \bar{n}_c \kappa_c^2 e^{-\frac{\varphi_a^2}{2} + \varphi_b^2} E_J e^{-\frac{1}{2}(\varphi_a - 2|\alpha|)^2}{\hbar}2\pi\sqrt{|\beta|^2 |\varphi_a|^2}.$$  

Practical realization of a high impedance cavity mode, satisfying $\varphi_a \approx 2|\alpha|$, poses a notable challenge. To
Another limitation concerns the Zeno approximation. The projected Hamiltonian $H_{\text{RWA}}^{\text{M}_2}$ corresponds to a first order Zeno dynamics approximation in $\epsilon_{\text{Zeno}} = E_J/\hbar\kappa_{2ph}$ [29, 30]. The second order correction in $\epsilon_{\text{Zeno}}$ induces a dephasing in the basis $\{|C_{n_{\sigma}}^{\pm}\rangle\}$ occurring at a rate $\Gamma_Z = r(\alpha, \phi_a)\epsilon_{\text{Zeno}}^{2}\kappa_{2ph}$, where the numerical factor $r(\alpha, \phi_a)$ can be derived from [30]. This could be seen as an inefficiency in the measurement, where a constant part (independent of the number of readout photons) of the measurement signal is lost through the two-photon decay channel. Here, we analyze numerically this second order effect by simulating the master equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_{\text{RWA}}, \rho] + \kappa_{2ph}D[a^2 - \alpha^2](\rho)$$

where the single mode Hamiltonian, $H_{\text{RWA}}$, is given by (1) and $\rho(0) = \frac{1}{2}(|C_{0}^{+}\rangle + |C_{0}^{-}\rangle)(\langle C_{0}^{+}| + \langle C_{0}^{-}|$). Taking $\alpha = 2$, and varying $\phi_a$ and the Zeno parameter $\epsilon_{\text{Zeno}}$, we look at the decay of purity with time. This corresponds to a dephasing due to higher order Zeno dynamics well-approximated by $\Gamma_Z$. We illustrate in Fig. 3 (dashed lines, right axis) the value of $\eta_{\text{ph}} = \Gamma_{m}^{1ph}/(\Gamma_{m}^{1ph} + \Gamma_Z)$ corresponding to the measurement efficiency with number of readout photons fixed to $n_c = 1$. As $\Gamma_Z$ does not depend on the number of readout photons while $\Gamma_m$ increases linearly in $n_c$, this efficiency improves for higher number of readout photons. We observe that for a given Zeno parameter $\epsilon_{\text{Zeno}}$, the point $\phi_a \approx 2\alpha$ corresponding to the optimal measurement rate $\Gamma_m$, is also a local optimum of the efficiency. We do not account for the readout mode $\epsilon_c$ in these simulations, as it does not contribute to higher-order Zeno approximations and therefore to $\Gamma_Z$.

This measurement inefficiency is the only detrimental effect of such higher-order dynamics. As the Hamiltonian $H_{\text{RWA}}$ is diagonal in the Fock states basis, it does not change the parity and therefore do not lead to any bit-flip type error of the logical qubit. The coupling, through the novel Josephson circuit, between the storage cavity to modes that would be anomalously excited, will generate another source of inefficiency. Note that such couplings, in presence of nonlinear dissipation, have no effect more serious than simply unread parity measurements, and do not lead to uncorrectable errors.

We can perform a similar analysis for the two-mode joint-parity measurement protocol. While higher order Zeno dynamics cannot lead to any change of photon number parities (Hamiltonian is diagonal in the Fock states basis), in principle, it can lead to a dephasing for each logical qubit. However, to a very good approximation (exponentially precise in $|\alpha|^2$), such a dephasing occurs in a correlated manner, giving rise to a dissipation channel of the form $\sigma_{yz}^c \otimes \sigma_{yz}^b$ [20]. Therefore such higher order effects do not induce any decoherence within a given joint-parity subspace. We thus deal with a QND measurement (with non-unit efficiency) of joint parity.

We have shown how to achieve continuous QND measurement of three parity-type observables for harmonic oscillators. We focus on the case of multi-photon driven
dissipative systems previously introduced for universal quantum computation with cat-qubits [3]. The three observables consist of \( \sigma_z^a = |C_\alpha^+\rangle \langle C_\alpha^-| - |C_\alpha^-\rangle \langle C_\alpha^+| \) for a single-mode under two-photon process, joint-parity \( \sigma_z^a \otimes \sigma_z^b \) for two modes under two-photon process, and \( \pi_{\text{4ph}} = |C_\alpha^{(0\text{mod}4)}\rangle \langle C_\alpha^{(0\text{mod}4)}| + |C_\alpha^{(2\text{mod}4)}\rangle \langle C_\alpha^{(2\text{mod}4)}| - |C_\alpha^{(1\text{mod}4)}\rangle \langle C_\alpha^{(1\text{mod}4)}| - |C_\alpha^{(3\text{mod}4)}\rangle \langle C_\alpha^{(3\text{mod}4)}| \) under four-photon process. We also propose a possible implementation of these measurements through the high-impedance coupling of the cavity mode(s) to a Josephson junction. The fault-tolerant aspect of this measurement protocol plays a crucial role in the lifetime improvement through a QEC protocol. Indeed, using experimental parameters that are within the reach of current experiments, we predict that this fault-tolerance leads to a lifetime improvement of at least 2 orders of magnitude higher than protocols based on non-fault-tolerant parity measurement [20].

This scheme could also be adapted to non-dissipative cases such as [31]. In presence of strong Kerr type non-linearities, the Hamiltonian perturbation due to high-impedance coupling to a Josephson circuit results in the creation of a parity Hamiltonian. More precisely, considering a cavity subjected to strong self-Kerr effect and a two-photon drive, the Hamiltonian, in the interaction picture, is given by \( H_0 = -\hbar K(a^2 - \bar{\varepsilon}_p^2)(a^2 - \bar{\varepsilon}_p^2) \), with \( K \) the self-Kerr coefficient, and \( \bar{\varepsilon}_p \) the two-photon drive strength [31]. The 2D-manifold \( M_{2,\alpha} = \text{span}(|C_\alpha^\alpha\rangle) \) is a doubly degenerate eigenspace of \( H_0 \), separated from the other eigenspaces by an energy gap of order \( 4\hbar|\bar{\varepsilon}_p| \).

Considering the first order effect of a perturbative Hamiltonian \( H_1 = H_{\text{RWA}} \) (see expression (1)) with \( |H_{\text{RWA}}| \ll 4\hbar|\bar{\varepsilon}_p| \), we lift the degeneracy of \( M_{2,\alpha} \), leading to two non-degenerate eigenstates approximately given by \( |C_\alpha^\alpha\rangle \) and \( |C_\alpha^-\rangle \). This implies that we have achieved an effective \( \sigma_z \) Hamiltonian on the logical basis of cat states.

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