From vertices to vortices in magnetic nanoislands

Cite as: Phys. Fluids 35, 017101 (2023); https://doi.org/10.1063/5.0131158
Submitted: 18 October 2022 • Accepted: 05 December 2022 • Accepted Manuscript Online: 08 December 2022 • Published Online: 03 January 2023

Michael Saccone, Jack C. Gartside, Kilian D. Stenning, et al.

COLLECTIONS

This paper was selected as Featured

ARTICLES YOU MAY BE INTERESTED IN

Hybrid method for wall local refinement in lattice Boltzmann method simulation
Physics of Fluids 35, 017103 (2023); https://doi.org/10.1063/5.0130467

Some exact solutions that describe an ideal fluid flow with a free boundary
Physics of Fluids 35, 017104 (2023); https://doi.org/10.1063/5.0132100

A perspective on physical reservoir computing with nanomagnetic devices
Applied Physics Letters 122, 040501 (2023); https://doi.org/10.1063/5.0119040
From vertices to vortices in magnetic nanoislands

Cite as: Phys. Fluids 35, 017101 (2023); doi:10.1063/5.0131158
Submitted: 18 October 2022 · Accepted: 5 December 2022 · Published Online: 3 January 2023

Michael Saccone,1,2 Jack C. Gartside,3 Kilian D. Stenning,3 Will R. Branford,3,4 and Francesco Caravelli1,4

AFFILIATIONS
1Theoretical Division (T4), Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
2Center for Nonlinear Studies (T-Division), Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
3Blackett Laboratory, Imperial College London, London SW7 2AZ, United Kingdom
4London Centre for Nanotechnology, University College London, London WC1H 0AH, United Kingdom

Author to whom correspondence should be addressed: caravelli@lanl.gov

ABSTRACT
Recent studies in magnetic nanoarrays show that a variety of complex magnetic states and textures emerge as a function of a single magnetic nanoisland’s aspect ratio. We propose a model that, in addition to fitting experiments, predicts magnetic states with continuous symmetry at particular aspect ratios and reveals a duality between vortex and vertex states. Our model opens new means of engineering novel types of artificial spin systems, and their application to complex magnetic textures in devices and computing.

Published under an exclusive license by AIP Publishing. https://doi.org/10.1063/5.0131158

I. INTRODUCTION
Remarkable advances in nanofabrication have led to the possibility of engineering designer artificial magnetic systems. In particular, Artificial Spin Ices (ASI) are arrays of magnetically frustrated nanoislands with low-energy state degeneracy, which emulate ice models.1,2 Currently, the literature on ASI is expanding from the study of novel phases of matter in many-body physics,3 to diverse applications, including reconfigurable magnonics,4 neuromorphic,5 and reservoir computing.6 The magnetic islands (MIs) of the original Artificial Square Ice1 had an aspect ratio typically above four, meaning they were four times longer than wide, causing the island to magnetize in a collinear state along the longitudinal direction (a “macrospin” state). Islands with lower aspect ratios can also support the emergence of equilibrium spin textures that point approximately azimuthally around the center of an MI, vortex domains, or a combination of two vortices, a double-vortex domain, both with zero net magnetization.7–14 It is also worth stressing that our model fits only the emergence of these states as a function of aspect ratios, but that other geometrical aspects could affect the populations, for instance thickness, which is constant in our experiments. Nonetheless, understanding the emergence of vortex domains in single islands typically requires the use of slow and costly micromagnetic simulations,1 which numerically integrate the Landau–Lifschitz–Gilbert equation—precluding study of large many-island arrays. The emergence of vortices occurs when the island aspect ratio is around two to three. This has been recently used experimentally to amplify the number of configurations that arrays of nanoislands can have, in particular, granting an advantage for reservoir computing via enhanced microstate space and hence computational richness. Increasingly numerous island states introduce an enriched microstate space and emergent metamaterial memory dynamics.15,16 Given this growing interest, we developed a simple yet feature-rich model to explore the emergence of vortices in magnetic nanoislands. Such model will guide the ASI field in developing islands and their interactions that encourage the formation of complex and dynamic magnetic textures. It is important to stress that our model is computationally efficient, enabling study of large many-body arrays without the need for hundreds to thousands of variables per spin required by the equivalent micromagnetic simulations.

Artificial spin ices can be used to engineer phases of matter. Since at large aspect ratios, the interaction between fully magnetized (macrospin) nanoislands can be approximated using the dipolar interaction model: collinear islands behave ferromagnetically, while orthogonal ones antiferromagnetically. Since each island in an ASI has two possible magnetic states, one can map a single vertex in a spin ice as an Ising model, with \( s_{\mu}, s_{\nu}, s_{\delta} \) and \( s \) the Ising states of the up, right, down, and left island, respectively, with \( s = \pm 1 \). If we call \( J_{\perp} \) the coupling for perpendicular spins, and \( J_{\parallel} \) for collinear ones, the Hamiltonian of a coordination-four spin ice vertex can be written in the form

\[
H_{\text{vert}} = \frac{1}{2} (J_{\parallel} (s_{\mu} - s_{\nu}) (s_{\mu} - s_{\delta}) - J_{\perp} (s_{\mu} s_{\nu} + s_{\mu} s_{\delta})).
\]  

These interaction patterns can be repeated in any ASI, which is a decimation of a square ice. However, as this model fails to
II. MODEL

We employ a generalized bipartite mean field model\(^{17}\) to multiple domains. We partition our system in magnetic domains composed of large numbers of Ising spins. For a magnetic system such a spin ice, one can either choose the magnetic domains to be those of a fully magnetized island, e.g., a macrospin, or those internal to an island. We assume that our system is partitioned into \(K\) subsystems composed of \(N\) spins, based on the symmetry of the problem. The first approximation is that each subsystem is composed of perfectly aligned Ising spins, an approximation which is justified for island’s aspect ratios above one. The spins in domain \(A\) interact with the spins in the domain \(B\), with a certain exchange interaction \(J_{AB}\), and among themselves with a certain effective interaction \(J_{AA}\). The graph that describes the interaction between the sets is described by an adjacency matrix \(A_{AB}\), \(A = 1, \ldots, K\). Each subset of spins is of size \(N_1, \ldots, N_K\). After a brief calculation, we can see that, in the spirit of the bipartite case, the Hamiltonian for a larger set of mean field spins is given by

\[
H = \sum_{i=1}^{K} \sum_{j=1}^{K} J_{AB} A_{i} A_{j} \left( \sum_{s \in i} \sqrt{N_s} \right) \left( \sum_{s \in j} \sqrt{N_s} \right) + \sum_{i=1}^{K} \sum_{j=1}^{K} h_i \left( \sum_{s \in i} \sqrt{N_s} \right),
\]

where \(A\) has only zeros and ones, while \(B\) has only zeros and ones, while \(J_{AB}\) are coupling constants. As in the case of the bipartite mean field model, we introduce for every set of spins a mean field parameter \(m_A = \frac{1}{N} \sum_{s \in A} s\), and a Lagrange multiplier \(\lambda_A\) to enforce the constraints. Rescaling all the couplings by \(J_{AB} \rightarrow J_{AB} / \sqrt{N_i N_f}\), the partition function reads (derivation in the supplementary material)

\[
Z \propto \int \mathcal{D} \mathbf{m} e^{-\beta H_{\text{eff}}},
\]

where

\[
H_{\text{eff}} = -\frac{J_A}{\beta} \sum_{\alpha} \lambda_{\alpha} m_{\alpha} - \sum_{A} \frac{J_{AA}}{2} m_{A}^2
- \sum_{A} \frac{J_{AB} A_{AB}}{\xi} m_{A} m_{B} + \sum_{A} \log 2 \cosh(\lambda_{A}),
\]

where \(\xi\) is a phenomenological parameter, \(m_{A}\) is the mean field, and \(\lambda_{A}\) a conjugate field. As we can see, the exponent is proportional to \(N\). We then obtained, via Steepest Descent in the thermodynamic limit, the emerging mean field equations, given by

\[
m_{A} = \tanh \left( \sum_{\beta} \beta Q_{AB} m_{B} + \beta h_{A} \right),
\]

in terms of effective couplings \(Q_{AB} = J_{AB} A_{AB}\).

It is interesting at this point to note that the fixed points of these equations are, for sufficiently low temperature, the minima of the Hamiltonian

\[
\mathcal{H} = \frac{1}{2} \mathbf{m}^{T} \mathbf{Q} \mathbf{m},
\]

but whether the minima may be reached or not depends on the type of dynamics. As we show in the supplementary material, this model is able to encode the ground states of various spin ice models. One important comment is that if we study spin ice vertices, the effective matrix \(\mathbf{Q}\) has the same structure as the Hamiltonian of a vertex. The key difference is that while the vector \(\mathbf{s}\) is composed of discrete states, the vector \(\mathbf{m}\) is composed of continuous magnetic states \(-1 \leq m_{i} \leq 1\). As such, the minimum of the Hamiltonian of Eq. (4) can be studied using its eigenvectors and eigenvalues. When the \(m_i\) represent subdomains, the key advantage over micromagnetic simulations is that stable states become a tractable eigenvector problem.

III. MAGNETIC ISLAND AND EIGENVALUES OF 4 \(\times\) 4 STRUCTURED MATRICES

Let us now restrict our discussion to the behavior of a single MI, which is the building block of all artificial spin ices. We consider a partitioning of the island as in Fig. 1(a). We assume that the island can be divided in four subdomains \(m_1, \ldots, m_4\) assumed to be oriented to the right when positive. By symmetry considerations, we can then introduce three couplings \(J_f\) between the domain 1 and 4 and 3 and 2 across the diagonal, \(J_f\) for the domains 1 and 2 and 3 and 4, and then \(J_f\) for 1 and 3, and 2 and 4. Since our effective spins are continuous, one can also introduce a diagonal component \(q\). While this interaction is important for the dynamics, since by symmetry considerations \(q\) is identical for all the spins, it is not relevant in our considerations below.

The interaction matrix is then given by

\[
\mathbf{Q} = \begin{pmatrix}
0 & J_f & J_f & J_f \\
J_f & J_d & J_d & J_f \\
J_f & J_d & J_d & J_f \\
J_f & J_f & J_f & 0
\end{pmatrix}.
\]

What is remarkable is that for this matrix, the eigenvectors do not depend on the particular values of \(J_d, J_f, J_f\), although clearly the eigenvalues do. The eigenvalues \(\eta_1\) and eigenvectors of this matrix are \(\eta_1 = J_d - J_f - J_f, \eta_2 = -J_d + J_f - J_f, \eta_3 = -J_d - J_f + J_f, \eta_4 = J_d + J_f,\) with corresponding eigenvectors, \(\vec{\eta}_1 = (1, -1, 1, -1), \vec{\eta}_2 = (-1, -1, 1, 1), \vec{\eta}_3 = (-1, 1, -1, 1), \vec{\eta}_4 = (1, 1, 1, 1).\) Clearly, also the states with opposite signs are valid eigenvectors.

As it turns out, if we look at the graphics of Fig. 1(a), these correspond, respectively, to (1) an antiferromagnetic order state, which we interpret as a two-vortex state; (2) a domain-wall non-magnetized state; (3) a one-vortex state; and (4) a fully magnetized state. To show that this nomenclature is justified, we initialized micromagnetic simulations (OOMMF package [https://math.nist.gov/oommf]) of islands in each of these four-domain states and allow them to relax to their stable configurations, shown in Fig. 2. We find the relaxed four-domain states correspond accurately to the various macrospin, vortex, and double-vortex textures states, matching experimental observations. We have used random initial magnetization conditions for an oval shape matrix of aspect ratio 2 with the material parameters of Permalloy.

Our simulations were performed on a grid of 100 \(\times\) 50 points, with 2 nm\(^2\) cell size, using the parameters for Permalloy. As we can see in Fig. 2, these initial states relax into the fully magnetized, one-vortex, and two-vortex states, respectively (the red lines), showing that our interpretation of the paper is correct from a micromagnetic perspective.

If one considers this a squashed square ice vertex, there is a duality between “vertex type” states and the single island eigenvectors. In fact type I vertices, correspond to the two-vortex state. Type II vertices may be constructed from the one-vortex and fully magnetized states. Type IV vertices, highly energetic magnetic charge states, are non-magnetized state equivalent of the domain wall type. The remaining
type III vertices are superpositions of these eigenvectors, fittingly as they exist as excitations from the spin ice ground state. This simple model incorporates immediately, only via symmetry considerations, the experimentally observed magnetic states of a single MI and those relevant to square ice studies. Interestingly, only four degrees of freedom analytically capture what previously required micromagnetics, despite only crudely approximating the shape of vortices. Finer structures would emerge if more degrees of freedom were allowed through the introduction of a subdivided grid, but these alone present a compelling correspondence to nature. Whether the underlying magnetic interactions are due to exchange, dipolar interactions, or a combination of the two, is irrelevant in this model insofar as the appropriate behaviors are captured. Despite the fact that the entire field of ASI requires this feature, it is often given for granted.

Focusing on a single island, what is then important is which eigenvalue is the minimum one as a function of the parameters $J_f, J_a, J_d$. Since we know that one of the states has to be fully magnetized, we can safely set $J_f < 0$ and study the minimum eigenvalue
problem as a function of $J_a$ and $J_d$ only. The result is shown in the diagram of Fig. 1(b). In the top right region $J_a > 0, J_d > 0$, we have as the ground state the one-vortex state. In the top left region $J_a < J_d$, we have the two vortex states. The bottom left of the diagram is given by fully magnetized states, while the bottom right by a non-magnetized state, which is however rarely observed. The interface lines between two regions $i, j$ imply that we have eigenvalue degeneracy in the ground state. Thus, on those particular lines $J_a = \pm J_d$, or $J_d = \pm J_f$, we have that the magnetic ground state can be written in a continuous superposition of the form $\hat{v}_i(\theta) = \cos \theta \hat{v}_i + \sin \theta \hat{v}_j$, $i \neq j$ [U(1) symmetry]. At the interface between three regions $i, j, k$, the degeneracy becomes triple, and the corresponding eigenvector at that particular point can be written in terms of Euler angles as $\hat{v}_{ijk}(\theta, \phi) = \cos \theta \cos \phi \hat{v}_i + \sin \theta \cos \phi \hat{v}_j + \sin \phi \hat{v}_k$ [O(3) symmetry].

IV. EXPERIMENTS

Let us now use the model above to interpret the experimental data on islands with various aspect ratio. Permalloy nanomagnets of thickness 25 nm were fabricated via electron beam lithography with 1 µm inter-island spacing such that dipolar interaction between neighboring islands is negligible and islands behave as isolated. 1350 nanoislands were fabricated with lengths of 460–715 nm, widths of 225–275 nm, and aspect ratios of 0.5–3.0. The islands were effectively annealed via an AC demagnetization protocol, initially saturating islands at 65 mT (well above their coercive fields) and then oscillating down to 0 field in 0.5 mT steps. We then counted the different occurrence of magnetic texture states (macrospin, vortex, and double-vortex). A subset of the islands imaged via magnetic force microscopy (MFM) are shown in Fig. 3(a). As we can see, as a function of the aspect ratio we only observe states given by a macrospin and single vortex. Thus, in the phase diagram of Fig. 1(b), we must be in a region around $J_a \approx -J_f$ and $J_d \approx J_f$. Because the nanomagnets are at room temperature (the energy barrier between states is much higher than the thermal energy at 300 K), the system can be trapped in a metastable state. To reproduce these results, we introduce a form of dynamics for the Hamiltonian of Eq. (4). Note that for $r = 1$ the majority of the population is in the one-vortex state, and for $r = 2$ we are predominantly in the macrospin state. At intermediate aspect ratios, we observe an increasing population of two-vortex states.

As a result, we hypothetize that parametrically, we are moving along a curve $J_a = -E_d/J_f (r + r^2)$, $J_d \propto E_d/J_f^2$. If $E_d = J_f$, then at $J_a = J_f$, we are at the triple point, which allows the coexistence of all states. Such function is shown in Fig. 1(b), which is the yellow dashed curve. A sample of the experimental results is shown in Fig. 3(a), and the vertex populations for different aspect ratios are shown in Fig. 3(b), averaged over 1350 experiments, with light lines. The theoretically obtained populations after a Metropolis annealing are shown in the same figure (averaged over 100 samples per point), and are given by hard dashed lines. We see a good match between the experimental and the theoretical results, with fitting parameters $E_d = 0.6$, $E_a = 0.6$, $r = 1.2$, all of order one.

V. CONCLUSIONS

We provided a simple model of the emergent states of magnetic nanoislands, as a function of its aspect ratio, showing that these are the eigenvectors of a $4 \times 4$ matrix representing the couplings. This seemingly arbitrary coarse graining of a nanoisland not only explained previously seen vortices but located a triple-point region in parameter space where macrospin, vortex, and double-vortex magnetic textures are tristable in zero-field and hosted reconfigrurably in a specific nanostructure. Our experimental results then support this, highlighting the efficacy of our model in engineering designer system energetics and microstate landscapes. It is worth mentioning that single-vortex and double-vortex, but our experiments support the population switching predicted by our model.

Simultaneously, this justifies the lack of domain wall states and opens the possibility of observing degenerate continuous ground states, which is a prediction of this paper. By simple symmetry
considerations, we attained a mapping between ASI vertex states and MIs states that generated new insight into why magnetic domains form. As shown in the supplementary material (see App. F, Sec. V), our model also reproduces the low energy regime of known spin ices. While micromagnetic simulations are sufficiently fast for understanding properties of 1–10 nanomagnets before an experiment, this mean-field model excels at handling 100–10,000 nanomagnets and grasping their non-Ising behaviors. Emergent states grant higher memory capacity, analogies to more complex models in statistical physics,19 and the potential to embed a wider class of computational problems within patterned nanomagnet arrays. Our model also enables dynamical analysis to explain observed emergent phenomena such as avalanches20 and coupling to magnon modes.21

SUPPLEMENTARY MATERIAL
See the supplementary material for more details on the experimental results, derivations of the mean field theory, and further numerical studies corroborating the validity of the multi-vertex states and of the low energy states obtained via the mean field theory.

ACKNOWLEDGMENTS

The work of F.C. and M.S. was carried out under the NNSA of the U.S., DoE at LANL, Contract No. DE-AC52-06NA25396 (LDRD Grant No. PRD20190195). W.R.B. and J.C.G. were supported by the Leverhulme Trust (No. RPG-2017-257) and the Imperial College London President’s Excellence Fund for Frontier Research. J.C.G. was supported by the Royal Academy of Engineering under the Research Fellowship programme.

AUTHOR DECLARATIONS

Conflict of Interest
The authors have no conflicts to disclose.

Author Contributions

Michael Saccone: Investigation (equal); Methodology (equal); Writing – review & editing (equal). Jack Carter-Gartside: Data curation (equal); Investigation (equal); Writing – review & editing (equal). Kilian Stenning: Data curation (equal); Will R Branford: Supervision (equal); Writing – review & editing (equal). Francesco Caravelli: Conceptualization (equal); Funding acquisition (equal); Investigation (equal); Software (equal); Supervision (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES
1. R. Wang, C. Nisoli, R. Freitas, J. Li, W. McConville, B. Cooley, M. Lund, N. Samarth, C. Leighton, V. Crespi et al., “Artificial ‘spin ice’ in a geometrically frustrated lattice of nanoscale ferromagnetic islands,” Nature 439, 303 (2006).

S. H. Skjervø, C. H. Marrows, R. L. Stamps, and L. J. Heyderman, “Advances in artificial spin ice,” Nat. Rev. Phys. 2, 13 (2019).

A. Farhan, M. Saccone, C. F. Petersen, S. Dhuey, R. V. Chopdekar, Y.-L. Huang, N. Kent, Z. Chen, M. J. Alava, T. Lipperi et al., “Emergent magnetic monopole dynamics in macroscopically degenerate artificial spin ice,” Sci. Adv. 5, eaav6380 (2019).

S. Gliga, E. Iacocca, and O. G. Heinonen, “Dynamics of reconfigurable artificial spin ice: Toward magnonic functional materials,” APL Mater. 8, 040911 (2020).

E. Iacocca, S. Gliga, R. L. Stamps, and O. Heinonen, “Reconfigurable wave band structure of an artificial square ice,” Phys. Rev. B 93, 134420 (2016).

M. Saccone, F. Caravelli, K. Holthuis, S. Parchenko, Y. A. Birkholzer, S. Dhuey, A. Klebert, S. Van Dijken, C. Nisoli, and A. Farhan, “Direct observation of a dynamical glass transition in a nanomagnetic artificial Hopfield network,” Nat. Phys. 18, 517 (2022).

T. C. Gartside, K. D. Stenning, A. Vanstone, H. H. Holder, D. M. Arroo, T. Dion, F. Caravelli, H. Kurebayashi, and W. R. Branford, “Reconfigurable training and reservoir computing in an artificial spin-ice vortex via spin-wave fingerprinting,” Nat. Nanotechnol. 17, 460 (2022).

K. D. Stenning, J. C. Gartside, L. Mannesci, C. T. S. Cheung, T. Chen, A. Vanstone, J. Love, H. H. Holder, F. Caravelli, K. Everschor-Sitte et al., “Adaptive programmable networks for in materia neuromorphic computing,” arXiv:2211.06373 (2022).

S. Jain, A. Adeleye, and N. Singh, “Spin re-orientation in magnetostatically coupled NiFe20ellipsoidal nanomagnets,” Nanotechnology 21, 285702 (2010).

H. Zhang, Y. Liu, M. Yan, and R. Hertel, “Azimuthal spin wave modes excited in an elliptical nanomagnet with vortex pair states,” IEEE Trans. Magn. 46, 1675 (2010).

G. Leaf, H. Kaper, M. Yan, V. Novosad, P. Vavassori, R. Camley, and M. Grimsditch, “Dynamic origin of stripe domains,” Phys. Rev. Lett. 96, 017201 (2006).

F. Montoncello, L. Giovannini, F. Nizzoli, P. Vavassori, M. Grimsditch, T. Ono, G. Gubbiotti, S. Tacchi, and G. Carlotti, “Soft spin waves and magnetization reversal in elliptical Permalloy nanodots: Experiments and dynamical matrix results,” Phys. Rev. B 76, 024426 (2007).

P. Vavassori, N. Zaluzec, V. Meltshkio, V. Novosad, B. Ilie, and M. Grimsditch, “Magnetization reversal via single and double vortex states in sub-micron Permalloy ellipses,” Phys. Rev. B 69, 214404 (2004).

A. Imre, G. Csaba, L. Ji, A. Orlov, G. Bernstein, and W. Porod, “Majority logic gate for magnetic quantum-dot cellular automata,” Science 311, 205 (2006).

N. C. Keim, J. D. Paulsen, Z. Zeravcic, S. Sastry, and R. S. Nagel, “Memory formation in matter,” Rev. Mod. Phys. 91, 035002 (2019).

C. Merrigan, D. Shohat, C. Sirote, Y. Lahini, C. Nisoli, and Y. Shokef, “Majority logic gate for magnetic quantum-dot cellular automata,” Nat. Nanotechnol. 2, 460 (2007).

E. Iacocca, S. Gliga, R. L. Stamps, and O. Heinonen, “Reconfigurable wave band structure of an artificial square ice,” Phys. Rev. B 93, 024426 (2007).

A. Imre, G. Csaba, L. Ji, A. Orlov, G. Bernstein, and W. Porod, “Majority logic gate for magnetic quantum-dot cellular automata,” Science 311, 205 (2006).

N. C. Keim, J. D. Paulsen, Z. Zeravcic, S. Sastry, and R. S. Nagel, “Memory formation in matter,” Rev. Mod. Phys. 91, 035002 (2019).

C. Merrigan, D. Shohat, C. Sirote, Y. Lahini, C. Nisoli, and Y. Shokef, “Emergent disorder and mechanical memory in periodic metamaterials,” arXiv:2204.04000 (2022).

I. Gallo and P. Contacci, “Bipartite mean field spin systems. Existence and solution,” Math. Phys. Electron. J. 14, 1–22 (2008).

A. Ehrmann and T. Blachowicz, “Vortex and double-vortex nucleation during magnetization reversal in Fe nanodots of different dimensions,” J. Magn. Magn. Mater. 475, 727 (2019).

P. Schiffer and C. Nisoli, “Artificial spin ice: Paths forward,” Appl. Phys. Lett. 118, 110501 (2020).

G.-W. Chern, C. Reichhardt, and C. Reichhardt, “Avalanches and disorder-induced criticality in artificial spin ices,” New J. Phys. 16, 063051 (2014).

S. Urazhdin, V. E. Demidov, H. Ulrichs, T. Kendziorczyk, T. Kuhn, J. Leuthold, G. Wilde, and S. O. Demokritov, “Nonmagnetic devices based on the spin-transfer torque,” Nat. Nanotechnol. 9, 509 (2014).

S. Lendinez and M. Jungfleisch, “Magnetization dynamics in artificial spin ice,” J. Phys.: Condens. Matter 32, 013001 (2020).