Galaxy rotation curves in de Sitter space

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The observed positive cosmological constant $\Lambda$ and Hubble constant $H_0$ introduce a background of de Sitter gravitons of mass $m_0 = \sqrt{\Lambda} c/\hbar$ at a non-relativistic temperature $k_B T_{dS} = a_H \hbar c/\pi e^c$, where $a_H = H_0 c$, where $h$ denotes the Planck constant and $c$ the velocity of light. In this cosmology, gravitational interactions are parameterized by the inverse temperature $\beta = T_{dS}/T$ of the vacuum. The high and low $\beta$ limits produce an acceleration $a = \sqrt{\Lambda}/a_H$, $a_0 = 2a_H/(1 + \beta_{dS}) \approx 1.37 \times 10^{-8}$ cm s$^{-2}$, observed in observed galaxy rotation curves and, respectively, Newton’s law. Gravitation may be anomalously weak in a transition about $2.17 \, M_\odot^{1/2}$ kpc around a galaxy of $M = M_\odot 10^{11} M_\odot$. 

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The observed positive cosmological constant $\Lambda$ \cite{1,2} has broad implications for cosmology and gravitation. The graviton acquires a finite mass $m_0 = \sqrt{\Lambda} c/\hbar$, e.g., by coupling to the Ricci tensor in the nonlinear wave equations for the Riemann-Cartan connections \cite{4}. The total energy $h\omega$, where $h$ denotes Planck’s constant, in terms of momentum $h k$ of a graviton with wave number $k$ hereby satisfies the dispersion relation

$$\omega = c\sqrt{k^2 + \Lambda}, \quad (1)$$

where $c$ denotes the velocity of light. This relation persists also in theories of massive gravitation, that seek to preserve gauge invariance by including Stueckelberg fields \cite{5,6} and references therein. Gravitons thus attain a rest mass energy $\epsilon_0 = h c/\sqrt{\Lambda} a$. In the presence of a deceleration parameter $q_0 = -q_0 H^2 = H^2 + H$, the generalized Higuchi constraint $m^2 \geq 2(H^2 + H)$ \cite{6,7} reduces to $\Omega_\Lambda \geq -\frac{1}{2} q_0$. Based on supernovae and baryon-acoustic oscillations in the CMB, $q_0 \approx -0.53$ \cite{14}, while the ESSENCE supernova survey alone suggests $q_0 = -0.788$ \cite{15}. The observed three-flatness of the Universe \cite{2} further implies $q_0 = \Omega_M/2 - \Omega_\Lambda \approx -0.5$, which gives some credence to the former. Hence, $q_0 > -1$ appears secure.

The Planck observation of the vacuum-energy density parameter $\Omega_\Lambda \approx 2/3$ in a three-flat universe to within 1 $\sigma$ uncertainty \cite{3}. Consequently, $H_0 \approx \sqrt{\Lambda}/2 c$ and the vacuum attains a finite de Sitter temperature $k_B T_{dS} = H_0 h / 2 \pi$ \cite{10,11} of

$$k_B T_{dS} \approx \frac{\sqrt{\Lambda} c}{2\pi \sqrt{2}}, \quad (2)$$

where $k_B$ denotes the Boltzmann constant. Consequently, the gravitons are warm and assume a non-relativistic temperature. With $k_B T_0 = m_0 c^2$, we have $\beta_{dS} = T_0 / T_{dS}$ satisfying

$$\beta_{dS} = 2\pi \sqrt{2} \quad (3)$$

To an inertial observer, $T_{dS}$ appears in the form of isotropic radiation coming from all directions. An accelerating observer experiences additional radiation at an Unruh temperature $k_B T_U = \hbar c/(2 \pi c)$ of with momenta along the direction of acceleration by equivalence in Rindler and Schwarzschild space times \cite{12,13}. The momenta of Unruh radiation, $p_U = k_B T_U / c$ and the isotropic momenta $p_{dS} = k_B T_{dS} / c$ of de Sitter background radiation are hereby uncorrelated. The average net momentum in magnitude hereby satisfies $p = \sqrt{p_U^2 + p_{dS}^2}$, giving rise to an apparent net temperature \cite{16,18}

$$\tilde{T} = \sqrt{1 + T^2_{dS}}, \quad (4)$$

where the hat refers to normalization with respect to $T_{dS}$. The associated radiation energy $e = k_B T$ hereby satisfies the same “smile” as (1) with $k_B T_{dS}$ setting a minimum temperature \cite{19}. In holography \cite{20,21}, the phase space of space-time is parametrized by information on a boundary specified by a two-dimensional screen, attributed to internal degrees at a finite temperature. In the approximation of a vanishing background temperature, the screen temperature satisfies

$$T = \left( \frac{\partial S}{\partial E} \right)^{-1}, \quad (5)$$

where $E$ denotes the enclosed total energy and $S$ is the entropy of the screen defined by its phase space. Neglecting $T_{dS}$, it recovers $T_U$ above with Newton’s law of gravitation as an entropic force \cite{24}. The latter can be obtained from Gibbs’ principle applied to deformations of light cones and the apparent event horizons of black holes alike, when endowed with the Bekenstein-Hawking entropy \cite{24}. Here, we extend the application of Gibbs’ principle to holography in a de Sitter background. In the imaging of a particle, let

$$E = m_0 c^2 + e \quad (6)$$

denote the enclosed mass-energy in terms of an associated rest mass energy $m_0 c^2$ and internal energy $e$ attributed to
In planetary motion, the latter would be absorbed in a corresponding radius is about a shift in acceleration on the order of Milgrom’s law \[22, 23\],

\[ a/a \]

\[ = \frac{\partial S}{\partial e} \]

\[ (\partial S/\partial e)^{-1}, \]

where \([T] = T - T_{\text{ds}}\).

We now consider holographic imaging by excitation of on-shell gravitational modes satisfying \(1\) at the temperature \(4\). Consequently, \(e = mc^2/(1 + \beta)\) and \(m_0c^2 = \beta mc^2/(1 + \beta)\), where \(\beta = T_0/T\) generalizes \(3\). For a spherical screen of finite radius \(r\) with \(N = 4\pi r^2/l_p^2\) Planck sized surface elements, \(7\) implies

\[ e = \frac{1}{2} N [k_B T]. \]

In the limit of arbitrarily large \(r\), \([k_B T]\) approaches zero and \(\beta\) increases to its maximum \(\beta_{\text{ds}}\) in \(3\).

The holographic formulation \(4\) and \(8\) implicitly defines \(T_U\) as a function of \(N\). Here, \(N\) is readily expressed in terms of \(T_N = a_d H/(2\pi c)\), parametrizing the Newtonian acceleration \(a_N = Gm/r^2\) by an enclosed baryonic mass \(m\). Explicitly, we have

\[ T_U = \frac{1}{2}T + \frac{1}{2}\sqrt{T^2 + 4\beta_{\text{ds}}}, \]

\[ \tilde{T} = \tilde{T}_N + 1 - \beta_{\text{ds}}. \]

Fig. 1 shows the observed acceleration \(a = 2\pi k_B T_U c/\hbar\) as a function of \(a_N\) and current data on galaxy rotation curves, where the abscissa is the normalized acceleration \(a/a_H\), \(a_H = H_0 c\).

Fig. 1 shows a high \(\beta\) limit at low-acceleration, known as Milgrom’s law \(22, 23\),

\[ a_+ = \sqrt{a_N a_0} \quad (a_N << a_H), \]

where

\[ a_0 = a_H \frac{2}{1 + \beta_{\text{ds}}} \approx 1.37 \times 10^{-8} \text{ cm s}^{-2}. \]

The low \(\beta\) limit for which \(e \approx (1/2)Nk_B T\) gives

\[ a_- = a_N - 2\pi \sqrt{2} a_H + O(a_H/a). \]

Here, the leading order term is the Newtonian gravitational attraction of \(24\) and the second order term is a shift in acceleration on the order of \(-6 \times 10^{-7} \text{ cm s}^{-2}\).

In planetary motion, the latter would be absorbed in a shift outwards in the orbit, and hence essentially unobservable.

Our model \(9\) shows a minimum \(a/a_N \approx 0.32\) at \(a_N/a_H \approx 4.6\), when \(T_U\) is similar to \(T_{\text{ds}}\). The corresponding radius is about

\[ r_0 \approx \frac{1}{2} \sqrt{R_g R_H} = 2.17 M_{11}^{1/2} \text{ kpc} \]

for a galaxy of mass \(M = M_{11} 10^{11} M_\odot\), where \(a_N \approx 3 \times 10^{-7} \text{ cm s}^{-2}\). Milgrom and Newton’s law hold asymptotically in, respectively, \(r >> r_0\) and \(r << r_0\).

Based on solely on the observed dark energy and Hubble constant, Fig. 1 shows that holography in a de Sitter background of warm gravitations produces the observed rotation curves to within about 10% at high \(\beta\), while essentially preserving Newton’s law \(12\) at low \(\beta\).

The observational agreement of our model with the data is remarkable in the absence of any fine-tuning. It gives novel justification for the otherwise mysterious nature of holographic imaging \(20, 21\) and the holographic origin of phase space \(20\). The present formalism applies to low redshifts \(z\) only. At high high \(z\), \(a_N >> a_H\) by scaling of co-moving distances, leaving \(\beta << 1\) everywhere with no implications for early cosmology.

The high and low beta limits are separated by a transition region about a characteristic radius \(13\). It extends over about one order of magnitude in radius where gravity is anomalously weak, as shown in the dip of the model curve in Fig. 1. It appears that this transition region is relatively sparsely sampled by current data. Conceivably, it can be probed further by enlarging the sample of galaxies, to ascertain if gravity drops below \(a_N\).

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