Calculation of Dip Features in the SIS Conductance of D-Wave Superconductors
Using Eliashberg Formalism

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Recent SIS tunneling conductance measurements on a range of Bi2212 crystals with varying oxygen doping have revealed new information on the behavior of a dip feature in the tunneling conductance. Calculations presented here show that the observed variation in position of the dip can be generated in an Eliashberg formalism for a d-wave superconducting state based on a single peak spectral weight function.

I. INTRODUCTION

Recent measurements [1] of the SIS tunneling conductance of a series of Bi-2212 superconducting crystals, with oxygen doping ranging from overdoped to underdoped, have revealed a variation with doping in the position of a dip relative to the main tunneling conductance peak. The separation between the main conductance peak and the dip, expressed in meV [1], agrees closely with neutron scattering measurements of the energy of a magnetic resonance mode in these superconductors.

Theoretical work on the origin of dip features in tunneling and photoemission in the cuprates has been in existence for several years [2]. An example of more recent calculations is that of [3] which, using a strong coupling method, explored the role of spin fluctuations in producing a d-wave symmetry superconducting state. In that work, which shows a dip in the calculated density of states, the experimentally measured resonance mode was associated with a peak in the calculated spin fluctuation spectrum near the Q vector (π , π). The recent experimental observations, that the energy separation between the main SIS conductance peak and the dip minimum agrees closely with the measured mode energy in Bi-2212 [1], present a good test of strong coupling scenarios based on a pairing mechanism arising from spin fluctuations.

As a reminder of what to look for in the density of states, consider a strong coupling Eliashberg calculation for an s-wave symmetry superconductor with a spectral weight, $\alpha^2 F(E)$, consisting of a single peak at an energy $E_{\text{mode}} = 1.0$ as depicted in figure (1). This will lead to a dip feature in the resulting superconducting density of states. The dip has an onset at an energy $E = \Delta + E_{\text{mode}}$, with the position of the dip minimum occurring at a higher energy [4] as illustrated in figure (2). For a coupling strength $\lambda = 2 \int \alpha^2 F(E)/E$ of the order of 2.0, for example, the position of the dip minimum is located noticeably further out from the onset threshold of the dip in the density of states. Consequently, for the s-wave strong coupling case, the energy separation between the exact position of the dip minimum and the main density of states peak will not be an accurate measure of the energy of the original peak in the underlying $\alpha^2 F(E)$. This is the case whether the density of states is studied, or the corresponding SIS conductance curves which result from a convolution of two such s-wave density of states.

The aim of calculations presented here, which are based on conventional Eliashberg strong coupling theory, is to study variations in the position of dip features in the density of states resulting from a model spectral weight with single peak at $E = E_{\text{mode}}$ for a d-wave symmetry superconductor. The use of a spectral weight with a single peak is motivated by the recent neutron scattering measurements of the magnetic resonance mode and speculation that it could be at the origin of d-wave superconductivity in the cuprates. The results are compared with recent SIS tunneling measurements [1].

II. METHOD

The spectral weight function used in this work is given by

$$\alpha^2 F(\omega) = [c_S + c_D \cos(2(\phi - \phi'))] f(\omega)$$

where $f(\omega)$ is a single peaked Lorentzian, a typical example of which is depicted in Figure 1 where $E = \hbar \omega$.

$\phi$ represents the angular position on a two dimensional Fermi surface. This type of interaction which combines an s-wave coupling ($c_S$) and a d-wave coupling ($c_D$) has been studied in the context of mixed s and d-wave superconducting...
states [3,4]. In [3], the d-wave phase was preferred for sufficiently small values of $c_S/c_D$. In the present study, this ratio is varied from 0 up to 0.4.

Assuming a strong coupling renormalization $Z(\omega)$ that is independent of angle $\phi$ and a d-wave superconducting gap function that can be written as $\Delta(\omega) \cos(2\phi)$, the conventional Eliashberg strong coupling equations [1] can be written in the form

$$\omega Z(\omega) = \omega - \int_0^\infty d\omega' \int_0^{2\pi} d\phi' \frac{c_S \omega' K_{+-}(\omega',\omega)}{2\pi \sqrt{\omega'^2 - \Delta^2(\omega') \cos^2(2\phi')}}$$

(2)

and

$$\Delta(\omega) = \frac{1}{Z(\omega)} \int_0^\infty d\omega' \int_0^{2\pi} d\phi' c_D \Delta(\omega') \cos^2(2\phi') K_{+-}(\omega',\omega)$$

(3)

where

$$K_{+-}(\omega',\omega) = \int_0^\infty d\omega'' \alpha^2 F(\omega'') \left( \frac{1}{\omega + \omega' + \omega'' + i\delta} - \frac{1}{\omega - \omega' - \omega'' + i\delta} \right)$$

(4)

The density of states is given by

$$N(\omega) = \frac{1}{2\pi} \int_0^{2\pi} d\phi \text{Real} \frac{\omega}{\sqrt{\omega^2 - \Delta^2(\omega) \cos^2(2\phi)}}$$

(5)

and the SIS tunneling conductance curves are calculated from $dI/dV$ where

$$I = \int_{-\infty}^\infty d\omega \ N(\omega) \ N(\omega + eV) \ [f(\omega) - f(\omega + eV)]$$

(6)

III. DISCUSSION

Results for d-wave SIS conductance ($dI/dV$) curves are presented in figures 3 and 4. Tables I and II provide a list of the values of the gap parameter, $\Delta$, the dip position $E_{\text{dip}}$ obtained from the SIS conductance curves and the corresponding parameter values for $c_S$, $c_D$ and $E_{\text{mode}}$. For the results shown in figure 3, the position of the peak in the spectral weight function $f(E)$ of equation (1) is kept fixed at $E_{\text{mode}} = 1.0$, the coupling constant $c_S$ is increased from 0 to 0.4 and $c_D$ is kept fixed at 1.0. For figure 4, the position of the peak in the spectral weight function $f(E)$ is decreased from $E_{\text{mode}} = 0.7$ to 0.4, the coupling constant $c_S$ is increased from 0.25 to 0.4 and $c_D$ is kept fixed at 1.0.

The results shown in figures 3 and 4 show variations in the calculated position of the dip in the calculated SIS conductance curves as the ratio $c_S/c_D$ is changed. In calculating these curves, the experimental measurements [1], which give values for the ratio $E_{\text{mode}}/\Delta$ ranging from 0.5 (underdoped) up to 2.0 (overdoped), provide a useful guide for the overall magnitude of $\alpha^2 F(\omega)$ in equations (2) and (3). For comparison, for an s-wave superconducting state to yield values of $E_{\text{mode}}/\Delta$ in the same range as the experiments on the Bi-2212 crystals, a strong coupling calculation based on a spectral weight function of the type shown in figure 1 would require a $\lambda = 2 \int \alpha^2 F(E)/E$ of the order of 2.0. This would result in the type of density of states $N(E)$ curve shown in figure 2.

In the $dI/dV$ curves shown in figures 3 and 4, the energy separation between the main conductance peak at $2\Delta$ and the dip at $E_{\text{dip}}$, which is listed in column 3 of Tables I and II, is approximately equal to the energy of the mode in the spectral weight function $f(E)$. This approximate equality begins to break down somewhat as $c_S$ gets larger in both sets of results.

As the energy of the peak in $f(E)$, $E_{\text{mode}}$, is decreased and $c_S$ is increased in the calculations leading to figure 4, the superconducting gap decreases from 0.42 to slightly less than 0.19 and the dip feature moves away from the main tunneling conductance peak while weakening in strength. The energy separation between the conductance peak at $2\Delta$ and the dip at $E_{\text{dip}}$ decreases and is close to the mode energy in all four curves of figure 4, with the largest deviation occurring for curve D. Furthermore, the ratio of the mode energy to the superconducting gap energy ($E_{\text{mode}}/\Delta$ ) ranges from 1.67 up to 2.1. The variation for $E_{\text{mode}}/\Delta$ seen in the curves of figure 4 and the approximate equality between the mode energy and the conductance peak-dip energy separation are consistent with the recent tunneling data on the Bi-2212 crystals [4].

A broadening of the main SIS conductance peak is evident in some of the curves of Figures 3 and 4. The peak is at its sharpest when $c_S$ is largest (curve E of figure 3 and curve D of figure 4). The decrease in the overall magnitude
of the imaginary part of the gap, $\text{Im}\Delta(\omega)$, which is shown in figure 5, for curves A and D of figure 4, is seen to be at the origin of the sharpening of the main $dI/dV$ peaks in figure 4. Similar behavior is present in the results leading to figure 3. A sharpening of the main SIS tunneling conductance peak with decreasing superconducting gap is also observed in the results of [1].

Finally, the effect of choosing a more sharply peaked $f(E)$ in the spectral weight function of equation (1) is depicted in Figure 6 where the superconducting density of states is labelled as Curve A and the corresponding SIS conductance as Curve B. The $f(E)$ function for these curves is depicted in Figure 7.

IV. CONCLUSION

Results are presented for the SIS conductance for a d-wave symmetry superconducting state using a conventional Eliashberg strong coupling formalism incorporating the model spectral weight of equation (1). The aim is to study the position of a dip feature which is a signature of the single peak in the spectral weight.

It is possible, using the model spectral weight of equation (1), to arrange for the energy difference between the positions of main SIS conductance peak and the dip minimum to be comparable to the energy of the spectral weight peak, which is denoted by $E_{\text{mode}}$ in this work. Recent SIS tunneling measurements [1] have noted this correspondence. The results presented here are a consequence of variations in the ratio of the coupling constants, $c_S/c_D$, of the model spectral weight function of equation (1) and are used to model the influence of oxygen doping variations on the electronic properties of the Bi2212 crystals in the experiment [1]. To more fully exploit the recent SIS experimental observations [1], more sophisticated strong coupling calculations incorporating the tight band structure appropriate to the cuprates are required in which the effect of variations in the chemical potential $\mu$ on the properties of the spin fluctuation spectral weight and the resulting d-wave superconductivity can be studied. Of particular interest would be the variation in position of the dip feature in the density of states and the relationship of this position to the energy of the $(\pi, \pi)$ peak in the spin fluctuation susceptibility.

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### TABLE I. Figure 3

| Curve | $2\Delta$ | $E_{Dip}-2\Delta$ | $E_{mode}$ | $C_S$ | $C_D$ | $E_{mode}/\Delta$ |
|-------|------------|-------------------|------------|-------|-------|-------------------|
| A     | 0.96       | 0.8               | 1.0        | 0.0   | 1.0   | 2.08              |
| B     | 0.96       | 0.88              | 1.0        | 0.1   | 1.0   | 2.08              |
| C     | 1.08       | 0.92              | 1.0        | 0.2   | 1.0   | 1.85              |
| D     | 0.72       | 1.12              | 1.0        | 0.3   | 1.0   | 2.78              |
| E     | 0.56       | 1.2               | 1.0        | 0.4   | 1.0   | 3.57              |

### TABLE II. Figure 4

| Curve | $2\Delta$ | $E_{Dip}-2\Delta$ | $E_{mode}$ | $C_S$ | $C_D$ | $E_{mode}/\Delta$ |
|-------|------------|-------------------|------------|-------|-------|-------------------|
| A     | 0.84       | 0.72              | 0.7        | 0.25  | 1.0   | 1.67              |
| B     | 0.78       | 0.67              | 0.6        | 0.3   | 1.0   | 1.54              |
| C     | 0.55       | 0.61              | 0.5        | 0.35  | 1.0   | 1.81              |
| D     | 0.39       | 0.55              | 0.4        | 0.4   | 1.0   | 2.1               |
FIG. 1. Typical $f(E)$ function used in the $\alpha^2 F(E)$ of equation (1). The position of the peak in $f(E)$ is denoted by $E_{\text{mode}}$ in the text.

FIG. 2. Superconducting density of states $N(E)$ for an s-wave superconductor with a single peak $\alpha^2 F(E)$ at $E_{\text{mode}} = 1.0$ in the strong coupling limit. Point A denotes the onset of the dip and point B denotes the dip minimum.
FIG. 3. SIS conductance curves for the d-wave superconducting state. The position of the spectral weight peak is held fixed in these curves. See Table 1.
FIG. 4. SIS conductance curves for the d-wave superconducting state. The position of the spectral weight peak is varied in these curves. See Table 2.
FIG. 5. The imaginary part of the superconducting gap for curves A and D of figure 4
FIG. 6. The quasiparticle density of states (Curve A) and the corresponding SIS conductance (Curve B) for a d-wave symmetry state with a narrow spectral weight peak function $f(E)$ depicted in Figure 7.

FIG. 7. The $f(E)$ function used in the $\alpha^2 F(E)$ of equation (1) to generate the results shown in Figure 6.