Temperature Dependence of the Order Parameter of the Polar Phase of Liquid $^3$He in a Nematic Aerogel

I. A. Fomin*,**

Kapitza Institute for Physical Problems, Russian Academy of Sciences, ul. Kosygina 2, Moscow, 119334 Russia

*e-mail: fomin@kapitza.ras.ru
**e-mail: igor_fomin@list.ru

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Abstract—It is proved that in the polar phase of superfluid $^3$He stabilized by a nematic aerogel the temperature dependence of the gap in the spectrum of Fermi excitations must be the same as that in the bulk polar phase without foreign impurities when the liquid quasiparticles are specularly reflected from the aerogel strands. The analogy with the Anderson theorem for conventional superconductors is discussed.

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1. INTRODUCTION

The experimental realization of the polar phase of superfluid $^3$He [1] and the results of its further study [2] became an interesting and important event in the physics of quantum liquids. All superfluid $^3$He phases result from the formation of Cooper pairs in the states with an orbital angular momentum $l=1$ and spin $s=1$. The spin structure of the order parameter of the polar phase is the same as that for the A-phase. The specificity of the new phase manifests itself in the orbital part—the state in which all pairs have an orbital angular momentum projection $l_z=0$ onto the preferential direction corresponds to the polar phase. At temperatures below the Cooper pairing temperature $T_c$ the order parameter of the polar phase is one of the extrema of the free energy of bulk superfluid $^3$He [3], but this extremum is energetically less favorable than the extrema corresponding to the order parameters of the A- and B-phases, which also contain other orbital angular momentum projections.

In bulk $^3$He the polar phase is unstable. Dmitriev and his coworkers [1] were able to suppress the Cooper pairing for “superfluous” angular momentum projections and to stabilize the polar phase in liquid $^3$He filling the space between the strands of a highly porous nematic nafen aerogel. Nafen is a rigid structure [4] that consists of nearly parallel Al$_2$O$_3$ strands. The average diameter of the strands is 8–9 nm, while the average distance between them in experiments [1] for different samples varied from 18 to 64 nm. In these experiments the nafen strands were coated with a $^4$He film with a thickness of 2.5–3 atomic layers. Such a film prevents the formation of a layer of solid paramagnetic $^3$He on the strands. The efficiency of the polar phase stabilization by nafen was confirmed by the experiments carried out in other laboratories [5, 6].

In the earlier experiments of Dmitriev’s group [7], where another nematic aerogel (Obninsk) was used instead of nafen, the polar phase was not observed despite the fact that the anisotropy of the mean free path for this aerogel exceeded the anisotropy sufficient for the polar phase stabilization according to theoretical estimates [8] by tens of times.

Further experiments with nafen [2] also showed that the pattern of scattering of Fermi quasiparticles by the nafen strands plays a significant role. In these experiments the pattern of scattering could be changed by varying the thickness of the $^4$He film coating the strands. The polar phase was observed only for sufficiently thick coatings, where the reflection of quasiparticles from the strands was nearly specular, at least at low pressures [9].

At present, there is no universal theory that would satisfactorily describe the influence of various highly porous aerogels on the properties of the superfluid $^3$He phases. A generalization of the theory of superconducting alloys [10, 11] to the case of $p$-wave pairing provides good qualitative agreement with the experiment. This approach, in the spirit of the mean-field theory, is sometimes called a homogeneous scattering model [12]. This theory disregards the fluctuations in the arrangement of impurities. The aerogels that are used in experiments with liquid $^3$He, including nafen, do not satisfy the condition of applicability of the mean-field theory. Less universal models that take into account the specificity of concrete aerogels were proposed for a better description of the results of real experiments [12].
A distinctive property of nafen is its strong anisotropy. This property is taken into account in the recently proposed idealized model of nafen [13]. In this model the nafen strands are assumed to be straight lines parallel to one another. In a plane perpendicular to the strands they are arranged randomly and the Fermi quasiparticles are specularly reflected from the strands. The latter assumption implies that when the excitations are scattered by the strands, the longitudinal (with respect to the strands) components of their momenta are conserved.

It was shown previously [13] that for this model of nafen the transition of liquid $^3$He filling the space between the strands from the normal phase to the polar one containing only one orbital angular momentum projection $l_z = 0$ occurs at a higher temperature than does the transition to other superfluid phases containing also $l_z = \pm 1$, with the temperature of the transition to the polar phase coinciding with the temperature of the transition of bulk $^3$He to the superfluid state. The latter assertion is analogous to one of the assertions for the thermodynamic properties of superconductors do not change when nonmagnetic impurities are injected into them. In particular, nonmagnetic impurities do not change the temperature dependence of the superconducting gap. An analogous assertion for the thermodynamic properties of the polar phase was not proved in [13], although it was used in the experimental identification of the polar phase [1] and in the experimental proof of the existence of a line of zeros in its gap [6].

The goal of this paper is to formally prove the validity of the assertion that nafen has no effect on the temperature dependence of the gap in the spectrum of polar phase excitations for the idealized model of nafen.

2. PROOF

The reasoning used in [13] is suitable only for calculating the transition temperature. The complete system of equations of the Abrikosov–Gorkov theory of superconducting alloys [10, 11] formulated for triplet $p$-wave pairing, as was done by Larkin [15], should be used to find the temperature dependence of the gap. In this case, the order parameter, i.e., the anomalous average $F_{\alpha\beta}(\mathbf{k})$, is a symmetric $2 \times 2$ spin matrix dependent on direction $\mathbf{k}$ in the space of wave vectors. It can be written as a combination of Pauli matrices $\sigma^\mu_{\alpha\beta}$:

$$ F_{\alpha\beta}(\mathbf{k}) \sim d_\mu(\mathbf{k}) \sigma^\mu_{\alpha\beta} \sigma^\nu_{\gamma\delta}. $$

The index $\mu$ runs over three values: $x, y, \text{ and } z$. The polar phase belongs to the family of phases for which the vector $d_\mu$ is real and its direction does not depend on $k$. The orientation of $d_\mu$ affects the gap only to the extent of a very weak dipole–dipole interaction and this effect may be neglected. It is convenient to assume that the vector $d_\mu$ is directed along the $y$ axis. Then, $F_{\alpha\beta}(\mathbf{k}) \sim \delta_{\alpha\beta}$ and the basic equations of the theory can be written as scalar equations for the normal, $G(\mathbf{k})$, and anomalous, $F^\dagger(\mathbf{k})$, Green functions:

$$ i\omega_n - \xi - \overline{G}_\alpha(\mathbf{k})G(\mathbf{k}, \omega_n) + |\Delta(\mathbf{k}) + \overline{F}_\alpha(\mathbf{k})|F^\dagger(\mathbf{k}, \omega_n) = 1, $$

$$ i\omega_n + \xi + \overline{G}_\alpha(\mathbf{k})F^\dagger(\mathbf{k}, \omega_n) + |\Delta^*(\mathbf{k}) + \overline{F}_\alpha(\mathbf{k})|G(\mathbf{k}, \omega_n) = 0. $$

The written equations contain the self-energy parts $\overline{G}_\alpha(\mathbf{k})$, $\overline{F}_\alpha(\mathbf{k})$, and $\overline{F}^\dagger(\mathbf{k})$ whose explicit form depends on the quasiparticle–impurity interaction potential, $U(\mathbf{r})$. In the idealized model of nafen this potential is assumed to be independent of the coordinate $z$ along the direction of the strands:

$$ U(\mathbf{r}) = \sum_\alpha u(\rho - \rho_\alpha), $$

where $\rho(\mathbf{x}, \mathbf{y})$ is a two-dimensional vector. The index $\alpha$ numbers the strands. The equations include the Fourier transform of the potential:

$$ U(\mathbf{k}) = 2\pi \delta(k_\perp)u(\mathbf{k})\sum_\alpha \exp(-i\kappa \rho_\alpha), $$

where $\kappa = (k_x, k_y)$ is a two-dimensional wave vector and $u(\mathbf{k}) = \int u(\rho)\exp(i\kappa \rho)d^2\rho$. For this model

$$ \overline{G}_\alpha(\mathbf{k}) = n_2 \int |u(\mathbf{k} - \mathbf{k}_\alpha)|^2 G(\mathbf{k}_\alpha, k_z) \frac{d^2 k_{\perp}}{(2\pi)^2}, $$

$$ \overline{F}_\alpha^\dagger(\mathbf{k}) = n_2 \int |u(\mathbf{k} - \mathbf{k}_\alpha)|^2 F^\dagger(\mathbf{k}_\alpha, k_z) \frac{d^2 k_{\perp}}{(2\pi)^2}. $$

Here, $n_2$ is the two-dimensional density of the strands.

The system of equations (1)–(5) differs from that considered in [15] by the form of the impurity potential (3). The presence of $\delta(k_\perp)$ in this potential implies that the longitudinal component $k_z$ of the wave vector is conserved when the quasiparticles are scattered by the strands. As a consequence of this conservation, Eqs. (1)–(5) have a solution $\Delta(\mathbf{k}) = \Delta(T)\mathbf{k}_\perp$ that contains no other components of $\mathbf{k}$. This solution was previously shown to correspond to the superfluid phase with the highest temperature of the transition to the normal phase [13]. To find the explicit form of this
solution, we should split $G_\omega(\tilde{k})$ into even and odd (in $\omega$) parts:

$$g_\omega(\omega, \tilde{k}) = \frac{1}{2}[G_\omega(\tilde{k}) + G_\omega(-\tilde{k})],$$

$$g_\omega(\omega, \tilde{k}) = \frac{1}{2}[G_\omega(\tilde{k}) - G_\omega(-\tilde{k})],$$

and then introduce new variables:

$$i\tilde{\omega}_n = i\omega_{n}(\omega, \tilde{k}), \quad \tilde{\xi} = \xi + \tilde{\omega}_e(\omega, \tilde{k}),$$

$$\tilde{\Delta}^\dagger = \Delta^* + F^\dagger(\tilde{k}).$$

In these variables Eqs. (1) and (2) take a simple form:

$$(i\tilde{\omega}_n - \tilde{\xi})G(\tilde{k}, \omega_n) + \Delta^\dagger F^\dagger(\tilde{k}, \omega_n) = 1,$$  \hspace{1cm} (6)

$$\Delta^\dagger G(\tilde{k}, \omega_n) + (i\tilde{\omega}_n + \tilde{\xi})F^\dagger(\tilde{k}, \omega_n) = 0.$$  \hspace{1cm} (7)

They are easily solved for $G(\tilde{k}, \omega_n)$ and $F(\tilde{k}, \omega_n)$:

$$G(\tilde{k}, \omega_n) = -\frac{i\tilde{\omega}_n + \tilde{\xi}}{(\omega_n)^2 + \tilde{\xi}^2 + |\Delta|^2},$$

$$F^\dagger(\tilde{k}, \omega_n) = -\frac{\Delta^\dagger}{(\omega_n)^2 + \tilde{\xi}^2 + |\Delta|^2}. \hspace{1cm} (8)$$

Substituting these expressions into Eqs. (4) and (5) gives equations for $G$, $F^\dagger$, and their combinations. Thus,

$$g_\omega(\omega, \tilde{k}) = -n_2\int u(\kappa - \kappa_1)^2 \frac{\tilde{\xi}}{(\omega_n)^2 + \tilde{\xi}^2 + |\Delta|^2} \frac{d^2\kappa_1}{(2\pi)^2}. \hspace{1cm} (9)$$



The integral on the right-hand side formally diverges at large $\tilde{\xi}$, i.e., the value of the integral is determined by the contributions of states far from the Fermi level. As in the case of $s$-wave pairing [17], the addition $g_\omega(\omega, \tilde{k})$ to the variable $\tilde{\xi}$ may be included in the renormalization of the chemical potential and $\tilde{\xi}$ may be deemed a new integration variable reckoned from the renormalized Fermi energy.

The gap and its temperature dependence are found from the equation

$$\Delta^\dagger(\tilde{k}) = -T \sum_n \sum_{\tilde{k}'} V(\tilde{k}, \tilde{k}') F^\dagger(\tilde{k}', \omega_n). \hspace{1cm} (10)$$

The interaction responsible for the pairing in Eq. (10) is usually written as

$$V(\tilde{k}, \tilde{k}') = 3g(\tilde{k} \cdot \tilde{k}'). \hspace{1cm} (11)$$

Equation (10) is a closed equation for the gap if the function $F(\tilde{k}, \omega_n)$ is expressed via the original variables $\omega_n$ and $\Delta^\dagger(\tilde{k})$. For the model of nafen being considered here the following relations can hold:

$$i\tilde{\omega}_n = i\omega_{n}\eta(\kappa, \omega), \quad \tilde{\Delta}^\dagger = \Delta^\dagger\eta(\kappa, \omega)$$

with the same function $\eta(\kappa, \omega)$. The form of the function is found by substituting these relations into the definitions of $i\omega_n$ or $\tilde{\Delta}^\dagger$:

$$\eta(\kappa, \omega) = 1 + \frac{m^*n_2}{4\pi} \int |u(\kappa - \kappa_1)|^2 d\varphi, \hspace{1cm} (12)$$

where $m^*$ is the effective excitatory mass. The integration here is over the angle $\varphi$ between $\kappa_1$ and $\kappa$.

Substituting $F(\tilde{k}, \omega_n)$ into Eq. (10) gives

$$\Delta^\dagger(\tilde{k}) = -T \sum_n \sum_{\tilde{k}'} 3g(\tilde{k} \cdot \tilde{k}') \frac{\Delta^\dagger(\tilde{k}')\eta}{\omega_n^2 + |\Delta(\tilde{k}')|^2 + \tilde{\xi}^2}. \hspace{1cm} (13)$$

The summation over $\tilde{k}'$ is replaced in the conventional way by integration.

The solution $\Delta(\tilde{k}) = \Delta(T)\tilde{k}_c$ should be substituted into Eq. (13). After the change of the integration variable $\tilde{\xi} = \omega_1$, the function $\eta$ is eliminated from the integrand in the equation for $\Delta(T)$. The change has an effect only on the upper limit of the integral over $d\tilde{\xi}$. This change leads to corrections $1/\kappa_r$, which fall outside the accuracy range of the written equations. When integrating over $d\varphi$, the terms of the scalar product $(\tilde{k} \cdot \tilde{k}')$ that contain the transverse components $\tilde{k}_c$ and $\tilde{k}_c'$ make no zero contribution, while the remaining terms are multiplied by $2\pi$. As a result, we obtain an equation that does not contain the cross section for the scattering of quasiparticles by the nafen strands:

$$1 = \frac{3g^*k_r}{\pi^2} \int_0^1 x^2 dx \times \sum_n \int_0^{T/T} \frac{d\nu}{(2n+1)^2 + \nu^2 + (\Delta(T)x)^2}, \hspace{1cm} (14)$$

where $\Delta = \Delta(T)/\pi T$ and $\nu = \eta\tilde{\xi}/\pi T$. Equation (14) coincides with the equation defining the temperature dependence of the gap in the impurity-free polar phase of superfluid $^3$He. The upper limit of integration $u_{max}$ in this equation can be expressed via the observed quantities $T_c$ and $\Delta_0 = \Delta(T = 0)$. The standard methods of analyzing Eq. (14) in the limits $T \to T_c$ and $T \to 0$ are described in the literature [16].

As $T \to T_c$, the fraction in the integrand should be expanded in powers of $(\Delta(T)x)^2$. Following the reasoning in the monograph [16], we find the asymptotic expansion

$$\ln \frac{T}{T_c} = -\frac{37}{58} \zeta(3) \frac{(\Delta/\pi T)^2}{\Delta} + \frac{393}{7128} \zeta(5) \frac{(\Delta/\pi T)^4}{\Delta} - ..., \hspace{1cm} (15)$$

where $\zeta(z)$ is the Riemann $\zeta$ function. Equation (15) differs from the corresponding equation for the $s$-wave case by the additional coefficients $3/5$ and $3/7$ in front of $(\Delta/\pi T)^2$ and $(\Delta/\pi T)^4$, respectively. This difference
results from the angular dependence of the gap. The temperature \( T_c \) here coincides with the temperature of the transition from the normal phase to the superfluid one for bulk \(^3\)He.

In the limit \( T \to 0 \) it is more convenient first to perform the summation over \( n \) in Eq. (12) and to use the following equation as the initial one:

\[
1 = \frac{3g_{m^*}k_F}{2\pi^2} \int_0^1 x^2 \, dx \\
\times \int_0^{\infty} \frac{dv}{\sqrt{v^2 + (\Delta x)^2}} \tanh \frac{\sqrt{v^2 + (\Delta x)^2}}{2},
\]

(16)

where \( x = k_x \). The asymptotic solution of this equation for \( T \to 0 \) is discussed in detail in the Supplementary Material of [6] and there is no need to reproduce this reasoning here.

Thus, the idealized nematic aerogel does not affect not only the temperature of the transition from the normal phase to the polar one, but also the temperature dependence of the superconducting gap and other thermodynamic properties of the polar phase, much as nonmagnetic impurities do not affect the thermodynamic properties of conventional superconductors.

3. DISCUSSION

The possibility of applying the assertion proved above for the interpretation of experiments is limited by the accuracy with which the idealized model describes the real nafen. The assumption about the specular pattern of reflection of quasiparticles from the nafen strands built into the model can raise questions. This assumption is important, because the longitudinal components of the quasiparticle momenta are conserved in the case of specular reflection, with this conservation law holding for all realizations of the system.

The question about the compatibility of a particular form of the order parameter with specified Fermi-liquid perturbations was discussed in the literature in connection with the theory of so-called multi-orbital superconductors [17–19]. Superfluid \(^3\)He falls into this category, because the Cooper pairs here have the same orbital angular momentum \( l_z = 1 \), but they can be a combination of states with different angular momentum projections onto the preferential direction, \( l_z = 0 \) and \( l_z = \pm 1 \). In bulk \(^3\)He the transition temperature \( T_c \) is the same for all three projections, but the impurity potential (4) reduces the system’s symmetry and can lead to transition splitting.

The first step toward allowance for the influence of a perturbation on the normal phase is to determine the correct functions of the zeroth approximation. When the quasiparticles are specularly reflected in the basis \( l_z = 0 \) and \( l_z = \pm 1 \), the perturbation matrix (3) is block-diagonal. In terms of [17] this means that the polar phase is the result of intra-band Cooper pairing and that the potential (3) for it is not a pair-breaking one. When a diffuse scattering component is added, inter-band Cooper pairing becomes possible. The coupling between the states with \( l_z = 0 \) and \( l_z = \pm 1 \) arising in this case can change the form and symmetry of the order parameter by adding the transverse components to it. Generally speaking, this should lower the superfluid transition temperature.

In this paper we have dealt with only the potential scattering of quasiparticles by the nafen strands. However, as the thickness of the \(^4\)He film coating the nafen strands in the experiments [2] decreases, the magnetic scattering of quasiparticles is inevitably switched on due to the emergence of a direct contact between the bulk liquid \(^3\)He and the layer of a paramagnetic solid phase forming on the strands. The possible role of this Cooper pair destruction mechanism in the polar phase was discussed in several papers [2, 13, 20]. The question about the magnetic scattering is also interesting as applied to other \(^3\)He phases and other aerogels [21]. At present, it is unclear, however, how to separate experimentally the influence of magnetic scattering on the phase diagram of superfluid \(^3\)He from the influence of a change in the degree of specularity of the potential scattering of excitations by the strands.

This paper was written for the special issue of JETP devoted to the centenary of academician A.S. Borovik-Romanov. In the 1980s I happened to work in close contact with the group of experimenters from the Kapitza Institute for Physical Problems led by A.S. Borovik-Romanov. In addition to Borovik-Romanov, this group included Yu.M. Bunkov, V.V. Dmitriev, and Yu.M. Mukharskii. We worked with great enthusiasm and the almost daily discussions of current results, in which A.S. Borovik-Romanov invariably took part, are still one of my most vivid memories.

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TEMPERATURE DEPENDENCE OF THE ORDER PARAMETER

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