Numerical investigation of entropy generation in microchannels heat sink with different shapes

A A Alfaryjat1, D Stanciu1, A Dobrovicescu1, V Badescu1 and M Aldhaidhawi1
1Faculty of Mechanical Engineering and Mechatronics, University Politehnica of Bucharest, Bucharest, Romania
E-mail: altayyeb81@yahoo.com

Abstract. Entropy generation of 3D cross sections circular, square, and hexagon shapes microchannel heat sinks (MCHS) were numerically performed. The governing equations (continuity, momentum and energy) along with the boundary conditions and the study state conjugate heat transfer problem were solved using the finite volume method (FVM). The Reynolds number in the range of 100 to 1600 and heat flux of 125, 150, 175 and 200 kW/m² were covered in this study. The overall entropy generation rate and entropy generation number are obtained by integrating the volumetric rate components over the entire heat sink. The results indicated that entropy generation decreases with increases of the Reynolds number. Decreasing the heat flux led to decreasing entropy generation. The square microchannel heat sink has the lowest entropy generation and entropy generation number.

1. Introduction
Cooling been used for maintaining the performance of an extensive variety of products such as car engines, powered electronics and high-powered lasers. In recent years, electronic devices became more advanced and smaller, thus in highly technical devices cooling present one of the top technical to decrease heat loads. For example, electronics manufacturing has provided computers with smaller sizes, faster speeds and expanded features, this leading to rising heat loads, heat fluxes, and localized hot spots at the chip and package levels. The perfect way to improve heat transfer in thermal systems is to increase the surface area of the heat transfer of cooling devices for ejecting heat to the coolant by adding fins or by using the system’s size. The first to place microchannel cooling technology in 1981 were Tuckerman and Pease [1]. They managed to circulate water in microchannels that were fabricated in silicon chips. The heat flux flexibly reached 790W/cm² without a penalty on phase change in a pressure to drop of 1.94 bars.

Increasing entropy generation or reducing thermal efficiency always affects thermal devices by irreversible losses. Thus, in the design and engineering of heat removal devices and in energy optimization problems, it is essential to find the entropy generation or exergy destruction due to thermal and friction entropy as a faction of the variable design selected for the optimization.

Chen [2] investigated the entropy generation of the flow in a microchannel for different boundary conditions. It was found that the convection effect on entropy and fluid temperature is negligible when the peclet number < 0.1. Based on the direction of the mass and heat flow, the entropy generation rate can be positive or negative for the subsystems.

Abbassi [3] investigated entropy generation in a uniformly heated rectangular microchannel. The thermal properties of the flow are incompressible, fully developed and laminar. It was noticed that the
best thermal performance was achieved when the Nusselt number is high and entropy generation is low. The thermal conductivity ratio of the fluid increased the thermal entropy generation, but it has no influence on frictional entropy generation.

The viscous effect of the entropy generation in a single phase circular fully developed forced convection microchannel has been investigated by Hung [4], [5]. He discovered that when increasing the Brinkman number, a high amount of heat is transferred from the channel wall to the fluid, and dimensionless entropy generation decreases.

Khan et al [6] numerically reported the overall performance of entropy generation minimization in a rectangular microchannel heat sink. The assumption’s properties were slip flow, laminar flow, and incompressible fluid, and fully developed. The results show that decreasing the Knudsen number and increasing the volume flow rate led to decreasing the pressure drop and thermal resistance in the microchannel. The total entropy generation rate increased when heat transfer increased and fluid friction decreased. Also, it was found that a large number of the microchannels and low thermal conductivity of the heat sink will show good performance in terms of the entropy generation rate.

The entropy generation of the open-end and closed-end of the electro-osmotic microchannel was numerically simulated by Zhuo and Liu [7]. It was discovered that in the centre of the microchannel, the maximum volumetric entropy generation existed and near the wall, the volumetric entropy generation appeared. The heat entropy generation conduction of electro-osmotic flow showed the major percent in the total entropy generation, when the fluid temperature raised due to Joule heating is smaller than the temperature difference between top wall and inlet.

Ibanez and Cuevas [8] studied numerically the rising in the dissipative processes in electromagnetic microchannel which is appeared in the MHD (magnetohydrodynamic) micropump. It was found that the global entropy generation can be effected under different physical properties which are related to microchannel walls, fluid properties, and geometry particular.

Slip gaseous flow in silicon trapezoidal microchannels has been investigated by Kuddus [9]. The temperature and velocity of the continuum approach at the walls is practical for expanding the mathematical model of the trapezoidal microchannel. It was discovered that when the Brinkman number increases, average entropy generation increases. Moreover, the irreversibility is high near the adiabatic wall corners and low at the middle. Viscous heating, wall heat transfer, friction from the fluid and entropy generation can all influenced the irreversibilities.

Guo et al [10], [11] numerically studied the curved square microchannel thermodynamic performance which was affected by viscous dissipation and temperature dependent viscosity. The hydraulic diameters range from 1 µ to 1 mm and the flow is laminar with a Re of less than 1500. The results appeared that, when the fluid heated, the entropy increased with an increase in the Reynolds number and frictional and heat transfer entropy generation decreased for temperature dependent viscosity. Also, at low specific heat and high dynamic viscosity, the Brinkman number of the fluid became large.

Yazdi et al [12] studied numerically the exergy losses in the magneto-hydrodynamics (MHD) flow at a prescribed surface temperature (PST) in moving non-slip surface embedded parallel microchannels. The local entropy generation along the surface width was also investigated. It was found that decreasing the heat transfer rate and wall shear stress was caused by decreases in the heat transfer and friction of the irreversibility. They also reported the slip moving surface embedded parallel microchannels [13]. They have shown that the decrease in entropy generation is caused by increasing the width of the microchannels and enhancing the slip effects.

Matin and Khan [14] investigated numerically the mass and heat transfer of the entropy generation in slit microchannels. They reported that the minimum local entropy generation is located in the middle of the channels, and it increases at the walls.

The entropy generation of a microchannel with fan-shaped reentrant cavities and trapezoidal ribs, fan-shaped reentrant cavities and triangular ribs, and fan-shaped reentrant cavities and rectangular ribs has been numerically investigated by Zhai et al [15]. The results showed that the microchannel with trapezoidal ribs was the best performing configuration when Re was less than 300, while the circular
A rib microchannel shows better performance when the Re is larger than 300, depending on the range of the Reynolds number.

From the literature review, it is obvious that entropy generation in MCHS was studied generally but there is no extensive study to examine the effect of the channel shape (square, hexagon and circular) on entropy generation in MCHS. The point of this study is to examine numerically the effect of entropy generation and entropy generation number in various microchannels. The Reynolds number for all channel shapes varies between 100 – 1600.

2. Problem definition and mathematical formulation

MCHS is applied for heat removal from integrated circuits and electronic chips surfaces. To simulate the 3D thermal and flow fields, a commercial code Fluent was employed. The flow field was considered as laminar, steady, single Phase, and incompressible under low pressure conditions. Figure 1 depicts typical MCHS cross sections (circular, square and hexagon). By circulating coolant through channel, the heat is leached from the solid walls. The dominant mode of transport is by conduction through solids and interfacial convection. The width of the heat sink is \( W_{hs} \), the height is \( H_{hs} \) and the length is \( L_{hs} \). Over a number of microchannels situated along the Z direction, the coolant passes and takes the heat from the top of the heat sink. For the symmetry advantage, a control volume (channel 14) is selected to study the entropy generation model. The dimensions of the microchannel heat sink and the three different channels of each cross-section shape are given in Table 1. For all different cross sections, the hydraulic diameter is 278 µm. The top surface of all control volumes is heated with a constant heat flux of \( (125, 150, 175 \text{ and } 200) \text{ kW/m}^2 \). The control volume bottom is thermally insulated and the side walls are symmetrical. The inlet temperature of the water is 290K. The Reynolds number for all channel cross sections is varies between 100 – 1600.

![Figure 1](image_url)

**Figure 1.** (a) Schematic diagram of the computational domain, (b) sections of the hexagon, circular, and square cross-section MCHS.

The non-dimensional governing equations of continuity, momentum and energy are [16]:

The **Continuity equation** is:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0. \tag{1}
\]
The X-Momentum equation is:

\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} = -\frac{dp}{dX} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right). \] (2)

The Y-Momentum equation is:

\[ U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} = -\frac{dp}{dY} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} \right). \] (3)

The Z-Momentum equation is:

\[ U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} = -\frac{dp}{dZ} + \frac{1}{Re} \left( \frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right). \] (4)

The energy equation is:

\[ U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} + W \frac{\partial \theta}{\partial Z} = -\frac{1}{Re.Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} \right). \] (5)

The dimensionless parameters are:

\[ X = \frac{x}{D_h}, Y = \frac{y}{D_h}, Z = \frac{z}{D_h}, U = \frac{u}{u_m}, V = \frac{v}{u_m}, W = \frac{w}{u_m}. \]

where \( U, V, \) and \( W \): Dimensionless velocity in \( x, y, z \) coordinates.

In the convection process, the overall rate of volumetric entropy generation equation is linked as.

\[ \dot{S}_{gen} = \iiint_{\Omega} \dot{S}_{gen}^m d\Omega. \] (6)

where \( \Omega \) represent the computing volume, obviously it consists of fluid and solid parts. For the solid parts only the thermal term of entropy generation equation is computed [17]:

\[ \dot{S}_{gen}^m = \dot{S}_{gen,th} + \dot{S}_{gen,f}. \] (7)

\[ \dot{S}_{gen,th} = \frac{k}{T^2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right]. \] (8)

\[ \dot{S}_{gen,f} = \frac{\mu}{T} \phi. \] (9)

where \( \dot{S}_{gen}^m \): total Generation entropy (W/K), \( \dot{S}_{gen,th}^m \): Heat transfer entropy generation (W/K), \( \dot{S}_{gen,f}^m \): Frictional entropy generation (W/K), and \( \phi \) is the frictional dissipation function and its computed as follow [17]:

\[ \phi = \frac{1}{2} \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 \] (10)

The dimensionless entropy generation numbers are defined as follows:
\[ N_S = \frac{\dot{Q}}{Q^* / T_{in}} \]  

where \( \dot{Q} \): Heat transfer rate and \( T_{in} \): Inlet temperature.

The solid and water physical properties used in the computation are

\[ \rho = 998.2 \text{ kg/m}^3, \quad C_p = 4182 \text{ J/(kg.K)}, \quad \mu = 0.001003 \text{ kg/(m.s)}, \quad k = 0.6 \text{ W/(m.K)}, \quad \text{and} \quad k_s = 202.4 \text{ W/(m.K)}. \]

where: \( \rho \), Density, \( C_p \), Specific heat, \( \mu \), Viscosity, \( k \), Thermal conductivity, and \( k_s \), Solid thermal conductivity.

**Table 1:** Geometry data of the microchannel heat sink.

|                      | Circular – Hexagon - Square |
|----------------------|-----------------------------|
| Heat sink length, \( L_{hs} \) (\( \mu \)m) | 10,000                      |
| Heat sink width, \( W_{hs} \) (\( \mu \)m) | 22,000                      |
| Heat sink height, \( H_{hs} \) (\( \mu \)m) | 1500                        |
| Hydraulic diameter, \( D_h \) (\( \mu \)m) | 278                         |
| Microchannel width, \( W_{ch} \) (\( \mu \)m) | 788                         |

3. **Numerical procedures**

The calculations pertaining to the numerical simulations were carried out by solving the governing conservation equations (Equations (1) – (5)) using the finite volume method (FVM) together with the corresponding boundary conditions [18]. The equations used for both the solid and fluid phase were solved simultaneously as a single domain conjugate problem. The MCHS’ flow field was solved using the SIMPLEC algorithm [19]. The geometry of the CFD region was determined and the mesh was generated using the GAMBIT program. By applying the CFD techniques, geometrical parameters’ effects on the MCHS were investigated, water being used as the base fluid. For the convective terms, the second-order upwind differencing scheme was considered. Finding the velocity components required solving the momentum equation. The continuity equation was used to update the pressure. The continuity equation does not have any pressure, but it can be easily transformed into a pressure correction equation. The convergence criterion required that the relative maximum mass residual dependent on the inlet mass be smaller than \( 1 \times 10^{-12} \) and the velocity components did not change from iteration to iteration.

4. **Results**

4.1. **Entropy generation**

The entropy generation for square, hexagon and circular cross sections MCHS of different heat fluxes and various Reynolds numbers is illustrated in figure 2 a, b, c. For these three shapes, the total entropy generation decreases when the Re increases. It exhibits a stable behaviour between Reynolds number 900 to 1200, then a slight apparent increase in the entropy generation. Clearly, when the heat fluxes increase, entropy generation increases. The heat flux at 125 kW/m² shows the lowest entropy generation.

Figure 3 shows the total entropy generation rate for circular, hexagon and square cross section shapes at a heat flux \( q \) of 200kW/m² and a Reynolds number range of 100 to 1500. The total entropy generation is function of the volume flow rate. Moreover, square MCHS appear to have the lowest entropy generation rate among all cross section microchannel shapes.
Figure 2. Entropy generation versus Re, (a) square cross section MCHS, (b) circular cross section MCHS, (c) hexagon cross section MCHS.

Figure 3. Entropy generation for different cross section shapes.
4.2. Entropy Generation Number

The total entropy generation number for square, hexagon and circular cross section MCHS at different heat fluxes (125, 150, 175, 200 kW/m²) and various Reynolds numbers are given in figure 4 a, b, c. While the heat flux increases, the entropy generation number increases. The heat flux 125 kW/m² exhibits the minimum value. As inlet temperatures of working fluids they remain the same, so the entropy generation number can be understood as the entropy generation rate per unit heat transfer rate.

![Figure 4: Entropy generation number versus Re, (a) square cross section MCHS, (b) circular cross section MCHS, (c) hexagon cross section MCHS.](image)

The variations of the total entropy generation number with the Reynolds number at \( q_w = 200 \) kW/m² for different shapes (square, hexagon and circular) presented in figure 5 which illustrates that total entropy generation number monotonously decreases with an increase of the Reynolds number. Square MCHS shows the lowest value of the entropy generation number.
4. Conclusion
Numerical simulation of entropy generation in MCHS using water as a base fluid was carried out in this paper. Three different single shapes (square, hexagonal and circular) were examined. Based on the above simulated results, the following conclusions can be drawn: total entropy generation and the entropy generation number decrease when the Reynolds number increases; increasing the heat flux caused an increase in the total entropy generation and entropy generation number; the square MCHS has the lowest value of the total entropy generation and entropy generation numbers, lower than the hexagonal and circular MCHS.

5. References
[1] Tuckerman D B and Pease R F 1981 High performance heat sinking for VLSI IEEE Electron devices Lett EDL 2 pp 126-129
[2] Chen k 2005 Second-law analysis and optimization of microchannel flows subjected to different thermal boundary conditions International Journal Energy Res 29 pp 249–263
[3] Abbassi H 2007 Entropy generation analysis in a uniformly heated microchannel heat sink Energy 32 pp 1932–1947
[4] Hung Y M 2008 Viscous dissipation effect on entropy generation for non-Newtonian fluids in Microchannels International Communications in Heat and Mass Transfer 35 pp 1125–1129
[5] Hung Y M 2009 A comparative study of viscous dissipation effect on entropy generation in single phase liquid flow in microchannels International Journal of Thermal Sciences 48 pp 1026–1035
[6] Khan W A, Culham J R, Member IEEE and Yovanovich M M 2009 Optimization of Microchannel Heat Sinks Using Entropy Generation Minimization Method IEEE Transactions on components and packaging technologies 32 pp 2
[7] Zhao L and Liu H L 2010 Entropy generation analysis of electro-osmotic flow in open-end and closed-end micro-channels International Journal of Thermal Sciences 49 pp 418–427
[8] Ibáñez G and Cuevas S 2010 Entropy generation minimization of a MHD (magnetohydrodynamic) flow in a microchannel Energy 35 pp 4149-4155
[9] Kuddus L 2011 First and second law analysis of fully developed gaseous slip flow in trapezoidal silicon microchannels considering viscous dissipation effect International Journal of Heat and Mass Transfer 54 pp 52–64
[10] Guo J, Xu M, Cai J and Huai X 2011 Viscous dissipation effect on entropy generation in curved square microchannels Energy 36 pp 5416-5423
[11] Guo J, Xu M, Cai J and Huai X 2012 The effect of temperature-dependent viscosity on entropy generation in curved square microchannel, Chemical Engineering and Processing 52 pp 85–91
[12] Yazdi M H, Abdullah S, Hashim I and Sopian K 2012 Second Law Analysis of MHD Flow over Open Parallel Microchannels Embedded in a Micropatterned Surface Advances in Fluid Mechanics And Heat & Mass Transfer
[13] Yazdi M H, Abdullah S, Hashim I and Sopian K 2013 Reducing Entropy Generation in MHD Fluid Flow over Open Parallel Microchannels Embedded in a Micropatterned Permeable Surface Entropy 15 pp 4822-4843
[14] Matin M H and Khan W A 2013 Entropy generation analysis of heat and mass transfer in mixed electrokinetically and pressure driven flow through a slit microchannel Energy 56 pp 207-217
[15] Zhai Y L, Xia G D, Liu X F and Li Y F 2014 Heat transfer in the microchannels with fan – shaped reentrantcavities and different ribs based on field synergy principle and entropy generation analysis International Journal of Heat and Mass Transfer 68 pp 224–233
[16] Alfaryjat A A, Mohammed H A, Adam N M, Ariffin M K A and Najafabadi M I 2014 Influence of geometrical parameters of hexagonal, circular, and rhombus microchannel heat sinks on thethermohydraulic characteristics International Communications in Heat and Mass Transfer 52 pp 121–131
[17] Li J and Kleinstreuer C 2010 Entropy Generation Analysis for Nanofluid Flow in Microchannels Journal of Heat Transfer ASME 132 pp 122401-1
[18] Patankar S V 1980 Numerical Heat Transfer and Fluid Flow Hemisphere (New York)
[19] Anderson J D Computational Fluid Dynamic: The Basics with Applications (McGraw-Hill, New York).