Effects of Scalar Leptoquarks in $b \rightarrow s$ Transitions

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Abstract. We investigate the effect of scalar leptoquarks on the recent anomalies observed in rare semileptonic $B$ meson decays involving the quark level transition $b \rightarrow s$. The leptoquark parameter space is constrained by using the measured branching ratio of $B_s \rightarrow \mu^+\mu^-$ process. We estimate the branching ratios of $B \rightarrow K^{(*)}\mu^+\mu^-(\nu\bar{\nu})$ processes, using the constrained leptoquark couplings. We also compute forward-backward asymmetry, polarization fractions of $K^*$ and $P_\mu$ observable in the $B \rightarrow K^*\mu^+\mu^-$ process. The $R_K$ anomaly in the $B \rightarrow K^{(*)}\mu^+\mu^-$ process is also studied. Furthermore, we predict the branching ratios of the lepton flavour violating decays, such as $B_s \rightarrow l_i^+l_j^-$, $B \rightarrow K^{(*)}l_i^+l_j^-$ and $B \rightarrow \phi l_i^+l_j^-$, which are found to be within the experimental reach of LHCb and the upcoming Belle II experiments.

1. Introduction
The study of rare semileptonic decays of $B$ mesons involving the flavour-changing neutral current (FCNC) transition $b \rightarrow s$, plays an important role to critically test the standard model (SM) and to look for the possible existence of new physics (NP). Such rare processes are highly suppressed in the SM as they occur at one-loop level (penguin and box diagrams). Recently LHCb has reported a $3\sigma$ discrepancy in $B \rightarrow K^\ast \mu^+\mu^-$ angular observables, mainly in the decay rate $[1]$ and $P_\mu^*$ $[2]$. Furthermore, the lepton non-universality parameter in the $B \rightarrow K^{(*)}\ell^+\ell^-$ decay is found to be $R_K^{\text{HCB}} = \frac{\text{Br}(B^+ \rightarrow K^+\mu^+\mu^-)/\text{Br}(B^{(*)} \rightarrow K^{(*)}e^+e^-)}{R_K^{\text{SM}}} = 0.745^{+0.09}_{-0.074} \pm 0.036$ $[3]$ in the low $q^2$ bin ($1 \leq q^2 \leq 6$) GeV$^2$, which has a $2.6\sigma$ deviation from the SM prediction $R_K^{\text{SM}} = 1.0003 \pm 0.0001$.

In this article, we intend to study the effect of scalar leptoquark (LQ) on the branching ratios as well as other asymmetry parameters of the $B \rightarrow K^{(*)}\mu^+\mu^-(\nu\bar{\nu})$ processes. We also investigate the lepton flavour violating (LFV) decays, such as $B_s \rightarrow l_i^+l_j^-$, $B_{(s)} \rightarrow K^{(*)}(\phi)l_i^+l_j^-$ mediated via the scalar LQ. Leptoquarks are color triplet bosonic particles which can couple to a quark and a lepton simultaneously and can occur in various extensions of the SM, e.g., grand unified theory, Pati Salam model, composite scenarios, etc. We consider the simple renormalizable scalar LQ model, for which the bounds from proton decays may not be relevant, and LQ may give signatures in other low-energy processes. In this work, we would like to see whether the scalar LQ model can accommodate some of the recent anomalies observed at LHCb.

The outline of the paper is follows. In section 2 we discuss the new physics contributions to the SM values due to the exchange of scalar LQ and the constraint on the leptoquark couplings from $B_s \rightarrow \mu^+\mu^-$ process. The branching ratios and other recent anomalies in the $B \rightarrow K^{(*)}\mu^+\mu^-$ process are computed in section 3. In section 4 we estimate the branching ratio of the $B \rightarrow K^{(*)}\nu\bar{\nu}$ process. The rare LFV decays are studied in section 5 and section 6 contains the summary and conclusion.
2. New physics contributions due to the leptoquark exchange

In the SM, the effective Hamiltonian for processes involving $b \rightarrow s l^+ l^-$ quark level transition is given by [4]

$$H_{\text{eff}} = -\frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^\ast \sum_{i=1}^6 \left( C_i(\mu) \mathcal{O}_i + C_7 \frac{e}{16 \pi^2} \left( \bar{s} \sigma_{\mu\nu} (m_s P_L + m_b P_R) b \right) F^{\mu\nu} \right. $$

$$\left. + C_9^{\alpha\gamma} \frac{\alpha}{4\pi} (\bar{s} \gamma^\mu P_L b) \gamma_\mu l + C_{10} \frac{\alpha}{4\pi} (\bar{s} \gamma^\mu P_L b) \gamma_5 l \right), $$

(1)

where $G_F$ is the Fermi constant, $V_{qq'}$ are the CKM matrix elements, $\alpha$ denotes the fine structure constant, $P_{L,R}$ are the projection operators and $C_i$'s are the Wilson coefficients.

The SM effective Hamiltonian [1] can be modified in the scalar leptoquark model due to the exchange of LQ. Here we will consider two minimal renormalizable scalar leptoquark multiplets $X = (3,2,7/6)$ and $(3,2,1/6)$, which are invariant under the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ and do not allow proton decay. The interaction Lagrangian of $X = (3,2,7/6)$ leptoquark with the SM fermion bilinear is given by [4]

$$\mathcal{L} = -\lambda_{ij} \bar{q}_{iR} (V_\alpha e^j_L - Y_{i\alpha} \nu^j_L) - \lambda_{\alpha} \bar{e}^j_R \left( V^j_L u^j_{\alpha L} + Y^j_{\alpha} d^j_{\alpha L} \right) + h.c.,$$

(2)

which after performing the Fierz transformation and then comparing with the SM effective Hamiltonian [1] gives additional Wilson coefficients to the $b \rightarrow s l^+ l^-$ processes as

$$C_9^{NP} = C_{10}^{NP} = -\frac{\pi}{2\sqrt{2} G_F \alpha} \lambda_{ij}^{32} \lambda_{ij}^{22} \frac{\lambda_{ij}^{32} \lambda_{ij}^{22}}{M_\chi^2} .$$

(3)

Similarly the $X = (3,2,1/6)$ LQ also provide new primed Wilson coefficients $C_{9,10}^{NP}$ corresponding to the semileptonic electroweak penguin operators $\mathcal{O}_{9,10}^{NP}$. These Wilson coefficients will give additional contributions to the leptonic decay rate $B_s \rightarrow \mu \mu$. Now comparing the SM predicted branching ratio of $B_s \rightarrow \mu^+ \mu^-$ process from [5], with the corresponding experimental result [6], we obtain the constraint on the combination of leptoquark couplings. The detailed formalism of the constraints on leptoquark coupling can be found in [4], therefore, here we will simply quote the results, as

$$0 \leq \frac{\lambda_{ij}^{32} \lambda_{ij}^{22}}{M_\chi^2} \leq 5 \times 10^{-9} \text{ GeV}^{-2} \quad \text{for} \quad \pi/2 \leq \phi^{NP} \leq 3\pi/2 ,$$

(4)

where $M_\chi$ is the mass of scalar leptoquark. Similarly for $B_s \rightarrow e^+ e^-$ process, the bound on the product of leptoquark coupling is found as $|\lambda_{31} \lambda_{21}^*|/M_\chi^2 < 2.54 \times 10^{-5}$.

3. $B \rightarrow K^{(*)} \mu^+ \mu^-$ process

In this section, we study the anomalies in the $B \rightarrow K^{(*)} \mu^+ \mu^-$ process in the scalar leptoquark model. The differential decay distribution with respect to the lepton-pair invariant mass ($q^2$) after integration over all three angles ($\theta_K$, $\theta_l$ and $\phi$) is given by [7] [8]

$$\frac{d\Gamma}{dq^2} = \frac{3}{4} \left( J_1 - J_2 \frac{2}{3} \right),$$

(5)

where the coefficients $J_{1,2}$ are functions of the dilepton invariant mass.
The various combinations of $J_i$, for $i = 1, ..., 9$ coefficients will give additional interesting observables to look for new physics, such as forward-backward asymmetry, isospin asymmetry, polarization fraction of $K^*$ and $P'_{4,5,6}$ observables. The forward backward asymmetry ($A_{FB}$), longitudinal polarization fraction of $K^*$ ($F_L$) and the form-factor-independent (FFI) observable ($P'_5$) are defined as [7] 

$$A_{FB}(q^2) = -\frac{3}{8} \frac{J_6}{dJ/dq^2}, \quad F_L(q^2) = \frac{3J^c_6 - J^c_2}{4dJ/dq^2}, \quad P'_5(q^2) = \frac{J_5}{2\sqrt{-J^c_2J^c_6}}. \quad (6)$$

In Fig. 1, we show the $q^2$ variation of branching ratio (top left panel), $A_{FB}$ (top right panel) and $P'_5$ observable (bottom panel) in the $(3, 2, 7/6)$ LQ model. The integrated values of branching ratio of $B \rightarrow K^{*}\mu^+\mu^-$ process in both SM and LQ model are found to be [7] 

$$\text{Br}(B \rightarrow K^{*}\mu^+\mu^-)|_{\text{SM}} = (7.74 \pm 0.46) \times 10^{-7}, \quad \text{Br}(B \rightarrow K^{*}\mu^+\mu^-)|_{\text{LQ}} = (6.88 - 8.73) \times 10^{-7}.$$ 

and the predicted value of $A_{FB}$, $F_L$ and $P'_5$ in the SM and LQ model are [7] 

$$\langle A_{FB}\rangle_{\text{SM}} = -(0.09 \pm 0.005), \quad \langle F_L\rangle_{\text{SM}} = 0.71 \pm 0.043, \quad \langle P'_5\rangle_{\text{SM}} = -0.204 \pm 0.012,$n \n$$
$$\langle A_{FB}\rangle_{\text{LQ}} = -0.11 \rightarrow 0.004, \quad \langle F_L\rangle_{\text{LQ}} = 0.7 \rightarrow 0.8, \quad \langle P'_5\rangle_{\text{LQ}} = -0.42 \rightarrow 0.13. \quad (8)$$

Another interesting observable is the lepton non-universality parameter ($R_K$) in the $B \rightarrow K l^+l^-$ process, which is the ratio of branching fractions of $B \rightarrow Kl^+l^-$ decays into dimuons over dielectrons. In these processes, the intermediate region is dominated by the charmonium resonance background induced by the decays $B \rightarrow K(c\bar{c}) \rightarrow Kl^+l^-$, where $c\bar{c} = J/\psi, \psi'$. Therefore, we calculate the value of $R_K$ in both low and high $q^2$ regions except the intermediate region around $q^2 \sim m^2_{J/\psi}$ and $m^2_{\psi'}$. In Fig. 2, we show the variation of lepton non-universality parameter with respect to low $q^2$ (left panel) and high $q^2$ (right panel) in leptoquark model. The predicted values of $R_K$ in $q^2 \in [1, 6]$/GeV$^2$ bin is $(0.62 - 0.96)$ [4] and in high $q^2$ bin ($q^2 \geq 14.18$) GeV$^2$ is $(0.75 \rightarrow 1.0)$ [10].

### 4. $B \rightarrow K^{(*)}\nu\bar{\nu}$ process

The $B \rightarrow K^{(*)}\nu\bar{\nu}$ process is mediated by $b \rightarrow s\nu\bar{\nu}$ transition and the effective Hamiltonian in the SM is given by [10] [11] 

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V^*_{ts} (C'_{L} \mathcal{O}_L^* + C'_{R} \mathcal{O}_R^*) + h.c.. \quad (9)$$

In the SM, the $C'_{R}$ Wilson coefficient is zero and can only be generated by the new physics. The new contribution to the SM effective Hamiltonian [9] due to the exchange of $(3, 2, 1/6)$ scalar leptoquark is given by [10] 

$$\mathcal{H}_{LQ} = \frac{\lambda^{32}_{\mu} \lambda^{22*}_{\nu}}{M^2_{\nu}} (\bar{s}\gamma^\mu P_R b)(\bar{\nu}\gamma^\mu (1 - \gamma_5)\nu), \quad (10)$$

which contributes $C'_{R}$ Wilson coefficient as 

$$C'_{R}|_{LQ} = -\frac{\pi}{2 \sqrt{2} G_F a V_{tb} V^*_{ts}} \frac{\lambda^{22}_{d} \lambda^{32*}_{d}}{M^2_{\nu}}. \quad (11)$$

The decay distribution of $B \rightarrow K\nu\bar{\nu}$ process with the di-neutrino invariant mass is given by 

$$\frac{d\Gamma}{ds_B} = \frac{G_F^2 a^2}{256\pi^5} |V_{ts} V_{tb}|^2 m_B^5 \lambda^{3/2}(s_B, \tilde{m}_K^2, 1)|f_K(s_B)|^2 |C'_{L} + C'_{R}|^2, \quad (12)$$
The $q^2$ variation of the branching ratio (top left panel), forward-backward asymmetry (top right panel) and $P_{K}^{LQ}$ observable (bottom panel) of the $B \rightarrow K^* \mu^+ \mu^-$ process in the scalar leptoquark model [7]. The gray bands represent the theoretical uncertainties arising from the input parameters in the SM.

$P_{K}^{LQ} (q^2)$

Figure 2. The variation of lepton non-universality parameter ($R_K$) with respect to low $q^2$ (left panel) and high $q^2$ (right panel) in the scalar leptoquark model [4].

where $\tilde{m}_i = m_i/m_B$ and $s_B = s/m_B^2$. The differential decay rate for $B \rightarrow K^* \nu \bar{\nu}$ process is

$$\frac{d^2\Gamma}{ds_B d\cos \theta} = \frac{9}{4} m_B^2 (|A_{\perp}|^2 + |A_{\parallel}|^2) \sin^2 \theta + \frac{9}{2} m_B^2 |A_0|^2 \cos^2 \theta,$$

where the transversity amplitudes $A_{\perp, \parallel, 0}$ are given in [10][11].

Now using the constrained LQ coupling from Eqn. (4), the predicted branching ratios of $B \rightarrow K^{(*)} \nu \bar{\nu}$ processes (in units of $10^{-6}$) both in the SM and LQ model respectively are [10]

$$Br(B_d \rightarrow K \nu \bar{\nu})_{SM} = (4.9 \pm 0.29), \quad Br(B_d \rightarrow K \nu \bar{\nu})_{LQ} = (3.6 - 5.2),$$

(14)

$$Br(B_d \rightarrow K^* \nu \bar{\nu})_{SM} = (9.54 \pm 0.57), \quad Br(B_d \rightarrow K^* \nu \bar{\nu})_{LQ} = (7.0 - 10.13),$$

(15)
and the variation of the branching ratios of $B \to K\nu\bar{\nu}$ (left panel) and $B \to K^*\nu\bar{\nu}$ (right panel) processes with $s_B$ in the leptoquark model is shown in Fig. 3.

**Figure 3.** The variation of the branching ratios of $B \to K\nu\bar{\nu}$ (left panel) and $B \to K^*\nu\bar{\nu}$ (right panel) processes with respect to $s_B$ in the scalar leptoquark model [10]. The gray bands represent the theoretical uncertainties coming from the input parameters in the SM.

5. Lepton flavour violating decays

In this section, we will discuss the lepton flavour violating $B$ meson decays mediated through the exchange of scalar leptoquarks. The LFV processes are extremely rare in the SM as they either proceed through box diagrams or are two-loop suppressed with tiny neutrino mass in one of the loop. However, in the LQ model they can occur at tree level. The effective Hamiltonian for $b \to s l^+l^-$ LFV decays in the LQ model is given by [12]

$$
H_{LQ} = \left[ G_{LQ} (\bar{q} \gamma^\mu P_L b) (\bar{l}_i \gamma_5 (1 + \gamma_5) l_j) + H_{LQ} (\bar{q} \gamma^\mu P_L b) (\bar{l}_j \gamma_5 (1 + \gamma_5) l_i) \right],
$$

where the constant coefficients $G_{LQ}$ and $H_{LQ}$ are

$$
G_{LQ} = \frac{\lambda^3_i \lambda^{kj*}}{8M_Y^2}, \quad H_{LQ} = \frac{\lambda^i_k \lambda^{3j*}}{8M_Y^2}.
$$

Now, in order to compute the required leptoquark coupling, we used the coupling given in Eqn. (4) as basis value and assumed that the leptoquark couplings between different generation of quark and lepton follow the simple scaling law, i.e. $\lambda^{ij} = (m_i/m_j)^{1/4} \lambda^{ii}$ with $j>i$. Using this ansatz, we compute the branching ratios of LFV decays, such as $B_s \to l_i^+l_j^-$, $B^+ \to K^{(*)+}l_i^+l_j^-$ and $B_s \to \phi l_i^+l_j^-$, and the predicted values are listed in Table-I, which are consistent with present experimental upper limits [6]. We show the plots for the branching ratios of $B^+ \to K^{(*)+}\mu^+\mu^-$ (left panel), $B^+ \to K^{(*)+}\tau^+\tau^-$ (middle panel) and $B^+ \to K^{(*)+}\mu^+\nu\bar{\nu}$ (right panel) decays in Fig. 4.

6. Conclusion

In this paper we have studied the recent anomalies in the rare semileptonic $B$ meson decays in scalar leptoquark model. We constrained the new LQ parameter space using the recent measurements on $B_s \to \mu^+\mu^-$ process. Using such constrained LQ couplings, we computed the branching ratios of the $B \to K^{(*)+}\mu^+\nu\bar{\nu}$ processes. We also estimated the forward-backward asymmetry, polarization fractions of $K^*$ and the FFI observable $P_5'$. We found that the observed $R_K$ anomaly can be explained in the LQ model. We then predicted the branching ratios of the
Figure 4. The variation of the branching ratios of $B^+ \rightarrow K^+ \mu^+ e^-$ (left panel), $B^+ \rightarrow K^+ \tau^+ e^-$ (middle panel), and $B^+ \rightarrow K^+ \tau^+ \mu^-$ (right panel) processes with respect to $q^2$ in the scalar leptoquark model [12].

Table 1. The predicted branching ratios for $B_s \rightarrow l_i^+ l_j^-$, $B_{(s)}^+ \rightarrow K^{(*)+}(\phi) l_i^+ l_j^-$ lepton flavour violating decays, where $l = e, \mu, \tau$ in the leptoquark model [12].

| Decay process | Predicted Br | Expt. limit [6] |
|---------------|--------------|-----------------|
| $B_s \rightarrow \mu^+ e^-$ | $< 1.5 \times 10^{-11}$ | $< 1.1 \times 10^{-8}$ |
| $B_s \rightarrow \mu^+ \tau^+$ | $< 1.2 \times 10^{-8}$ | ... |
| $B_s \rightarrow e^+ \tau^+$ | $< 8.5 \times 10^{-10}$ | ... |
| $B^+ \rightarrow K^+ \mu^+ e^-$ | $< 1.36 \times 10^{-9}$ | $< 1.3 \times 10^{-7}$ |
| $B^+ \rightarrow K^+ \tau^+ \mu^-$ | $< 8.8 \times 10^{-9}$ | $< 2.8 \times 10^{-5}$ |
| $B^+ \rightarrow K^+ \tau^+ e^-$ | $< 1.12 \times 10^{-9}$ | $< 1.5 \times 10^{-5}$ |
| $B^+ \rightarrow K^{*+} \mu^+ e^-$ | $< 1.4 \times 10^{-9}$ | $< 9.9 \times 10^{-7}$ |
| $B^+ \rightarrow K^{*+} \tau^+ \mu^-$ | $< 1.56 \times 10^{-9}$ | ... |
| $B^+ \rightarrow K^{*+} \tau^+ e^-$ | $< 2 \times 10^{-9}$ | ... |
| $B_s \rightarrow \phi\mu^+ e^-$ | $< 8.2 \times 10^{-10}$ | ... |
| $B_s \rightarrow \phi\tau^+ \mu^-$ | $< 1.1 \times 10^{-8}$ | ... |
| $B_s \rightarrow \phi\tau^+ e^-$ | $< 1.42 \times 10^{-9}$ | ... |

LFV decays such as $B_s \rightarrow l_i^+ l_j^-$, $B_{(s)}^+ \rightarrow K^{(*)+}(\phi) l_i^+ l_j^-$, which are found to be well below the present experimental limits.

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