Comparison of different methods for optical gain spectroscopy

Dominic J. Kunzmann*, Matthias Wachs, Lukas Uhlig, and Ulrich T. Schwarz ©

Chemnitz University of Technology, Experimental Sensor Science, 09126 Chemnitz, Germany
E-mail: dominic.kunzmann@physik.tu-chemnitz.de

Received December 22, 2018; revised February 1, 2019; accepted February 8, 2019; published online April 26, 2019

The internal losses of green and blue laser diodes are challenging to determine because of the narrow longitudinal mode spacing. Furthermore, the internal losses of state-of-the-art blue and green laser diodes are in the range of only a few inverse centimeter. Therefore the dynamical range given by the maxima and minima of the longitudinal mode spectrum is very large, even for moderate optical gain. Under these conditions, the usually employed, so-called Hakki–Paoli method to determine the optical gain becomes inaccurate. Now, we compare this with two other methods, the Cassidy method and an evaluation based on a Fourier transformation for a green laser diode. An error estimation as well as a correction of the systematic error caused by the spectral resolution of the setup were established. The overall highest gain was measured with the Cassidy method in the range of the lasing wavelength, as this method is least affected by the spectral resolution. In comparison of all methods, the highest gain for the wavelengths above the lasing wavelength is observed for one variation of the Fourier method, because background noise has the least influence on this method. For wavelengths below lasing wavelength we see similar optical gain for all methods.

© 2019 The Japan Society of Applied Physics

1. Introduction

Laser diodes made from GaN are important for many applications, for example for laser projectors, augmented and virtual reality and automotive lighting.1,2) For the optimization and simulation of these laser diodes it is necessary to characterize the optical gain spectra as function of the driving current.1,3–11) To get information about the optical gain, the common method is to measure the modulation depth of the longitudinal mode spectrum of the laser diode, as shown in Fig. 1. The formation of the longitudinal mode spectrum can be described as interference in a Fabry–Pérot etalon.12) From the finesse of the peaks, it is possible to calculate the gain with the method of Refs. 3, 4.

The optical gain is an important parameter for the quality of a laser diode, because it is possible to determine the internal losses, differential gain, and gain dispersion.6,7) For state-of-the-art blue and green laser diodes, there occur some challenges. The mode spacing is only about 0.06–0.08 nm, so a spectrometer with extremely high resolution is required to measure modulation depth of the peaks.11) Additionally, the laser diodes have a broad spectrum below threshold, which lead to an other challenge, the intensities for the minima and the regions outside the peak wavelength are comparably small and therefore the signal to noise ratio becomes as well.13)

For known reflectivities and resonator length, the modal gain \( g_l \) can be calculated from the maxima and minima of the measured longitudinal mode spectrum.

\[
g_l = \frac{1}{L} \ln \left( \frac{R_1 + 1}{R_1 - 1} \right) + \frac{1}{2L} \ln (R_1R_2) \tag{1}
\]

with

\[
P_i = \frac{I_{\text{max},i}}{I_{\text{min},i}} \sqrt{\frac{\lambda_{\text{max},i}}{\lambda_{\text{min},i}}} \tag{2}
\]

and \( L \) is the resonator length, \( R_1 \) and \( R_2 \) are the mirror reflectivities and \( I_{\text{max}} \) and \( I_{\text{min}} \) is the intensity in the maxima and minima of the longitudinal mode spectrum, respectively.

The determination of the minima is a significant challenge and strongly dependent on the noise level. The wavelength resolution influences the spectrum in a way that the maxima are decreased while the peaks are widened. To mitigate the influence of the resolution, Cassidy suggests to take into account integrated intensity of a peak instead of the maxima to minima ratio.14) This method is intended to be less dependent from the resolution of the spectrometer, as the integrated intensity does weakly depend on the resolution

\[
P_i = \frac{\int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} I(\lambda) d\lambda}{I_{\text{min}}(\lambda) \cdot 2x} \tag{3}
\]

In this equation the numerator stands for the area below the peak and the denominator for the area between the minimum and the x-axis, respectively.

A third way to determine the optical gain is to use a Fourier transformation of parts of the spectrum. This Fourier spectrum consists of a series of equidistant peaks. For high gain, the longitudinal gain spectrum is similar to a series of delta peaks, and the amplitude of the peaks in Fourier space are slowly decreasing with increasing order. For low gain the

---

Content from this work may be used under the terms of the Creative Commons Attribution 4.0 license. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

SCCC05-1 © 2019 The Japan Society of Applied Physics
measure the spectrum at the output slit with a photodiode. The layout of the monochromator is drawn in Fig. 3. We use a SPEX 1404 double spectrometer of 0.85 m focal length and a very high spectral resolution. The last lens (L2) focuses the beam on the entry slit of the monochromator (Mono) and photodiode (PD); the beam is guided to the monochromator. There it is analyzed and measured using a photodiode.

2.1. Setup

Our setup for the measurement consists of the laser diode, lenses to collimate and focus the beam, a polarization filter, a chopper wheel and a pinhole. The setup is shown in Fig. 2. The last lens (L2) focuses the beam on the entry slit of the monochromator, which is a SPEX 1404 double spectrometer. The foldable mirrors vary the used entrance and output. The concave mirrors guide the beam to the gratings, which diffract the light and the central slits suppress stray light.

2.2. Measurements

The measurements were done with a generic commercial green laser diode with a resonator length of 600 μm and the reflectivity of the mirror is given with 0.85. The threshold current of this diode is in the range of 39 mA and the slope efficiency is about 0.45 W A⁻¹.

First, we measure the dark spectrum which depends on the sensitivity of the lock-in amplifier as described above. It is important to subtract the dark spectrum from the measured spectra to correct the light of the environment and the dark current of the photodiode without light. In the final step, we measure the longitudinal mode spectra below threshold.

Small wavelength sections of the longitudinal mode spectrum shown in Fig. 1 for wavelengths below, at, and above peak gain are presented in Fig. 4. The methods of Hakki–Paoli and Cassidy are illustrated at the lasing wavelength range. For an accurate measurement it is necessary that light from only one transverse mode of the laser cavity is collected. Any emission into other modes, out of plane, or of wrong polarization will contribute to background, reduce the observed modulation of the spectrum and thus decrease the measured optical gain. Therefore, a linear polarization filter is inserted to suppress TM modes. Furthermore, a 200 μm pinhole is inserted as spatial filter to select the light from the waveguide and suppress the influence of spontaneous emission into substrate. L1 and L3 form a telescope with a 1.25 × magnification. The pinhole therefore selects a area which is small compared to the size of the substrate, and most of the light scattered from rough sidewalls and bond interface towards the spectrometer can thus be suppressed. The double spectrometer has an additional slits between the two stages to suppress scattered light intensity and therefore improve the dynamic range.

2.3. Validation

A comparison with the measurement done with a low temperature laser diode is shown in Fig. 5. For an accurate measurement it is necessary that light from only one transverse mode of the laser cavity is collected. Any emission into other modes, out of plane, or of wrong polarization will contribute to background, reduce the observed modulation of the spectrum and thus decrease the measured optical gain. Therefore, a linear polarization filter is inserted to suppress TM modes. Furthermore, a 200 μm pinhole is inserted as spatial filter to select the light from the waveguide and suppress the influence of spontaneous emission into substrate. L1 and L3 form a telescope with a 1.25 × magnification. The pinhole therefore selects a area which is small compared to the size of the substrate, and most of the light scattered from rough sidewalls and bond interface towards the spectrometer can thus be suppressed. The double spectrometer has an additional slits between the two stages to suppress scattered light intensity and therefore improve the dynamic range.

References

15, 16 showed that the slope of the decrease of the peak amplitudes deviates from this expected exponential behavior. Therefore, the zeroth Fourier coefficients on logarithmic scale is proportional to the modal gain. However, due to the measurement errors, the amplitude of the peaks deviates from this expected behavior. The other peaks depend on the shape of the longitudinal mode spectrum and derive the optical gain have been compared to the modal gain. However, due to the measurement errors, the amplitude of the peaks deviates from this expected exponential behavior. Therefore, the zeroth Fourier coefficients which are being used to calculate the gain by Fourier method. In particular, the zeroth Fourier peak is influenced by a constant background and by noise. The other peaks depend on the shape of the longitudinal mode peaks, and are therefore influenced by the resolution of the spectrometer (Fig. 5).

Different methods to evaluate the longitudinal mode spectrum and derive the optical gain have been compared for laser diodes in the telecommunication wavelength range. Here, we compare these methods for a laser diode in the green spectral range.
of the spectrometer: the peaks of higher order decreased in Fourier faster than linear. Above $\lambda_{\text{cusp}}$ the low signal-to-background ratio rises the zeroth order peak in Fourier space and lower dependence on the resolution. Because of these reasons, the slope of the linear fit to the peaks in Fourier space depend on the number and order of the peaks taken into account. We compared three versions of the Fourier method. In the first case, only the zeroth and first peak in Fourier space were taken into account. This minimizes the impact of the spectral resolution, however, it is most sensitive to background noise contributing to the zeroth peak. The second method takes the first and second peak into account. The error due to spectral resolution is still small, and the effect of background noise is mostly eliminated. For the third method, the Fourier peaks 1–4 were taken into account in the linear fit.

3. Error discussion

3.1. Error due to finite resolution

The resolution of the spectrometer can be determined, when we measure a single longitudinal mode of the laser diode spectrum above threshold. As the spectral width of the mode is small compared to the spectrometer resolution, the shape of the measured peak reflects the spectral resolution. We fit this spectral response function by a Gaussian function, where the full width half maximum (FWHM) can be calculated from the standard deviation $\sigma$ with $\text{FWHM} = \sigma \cdot \sqrt{2 \cdot \ln 2}$. For a slit width of 30 $\mu$m, we obtain a spectral resolution of about 9 pm.

To estimate the error caused by the finite resolution of the spectrometer for the different methods, we simulate longitudinal mode spectra by convolution of an Airy function for given $\text{finesse}$ with a Gaussian of given FWHM. Then the
error is determined by calculating the optical gain from these simulated curves for all three methods. In Fig. 7 the calculated optical gain is plotted as function of the finesse. For all methods the derived gain is lower than the ideal gain which was used as input for the simulated longitudinal mode spectra. For better visibility, this systematic error of the determined optical gain is plotted in Fig. 8, again as function of the ideal finesse. As expected, Cassidy is less sensitive to the resolution than Hakki–Paoli and Fourier.14) For Hakki–Paoli and Cassidy the influence of the resolution differs for different fineses, while for Fourier the influence is constant for all fineses, but depends on the number of Fourier orders taken into account to calculate the optical gain.

The resolution of the spectrometer decreases the calculated gain for all methods. With the results shown in Fig. 6 we can correct this systematic error, which is caused by the resolution. The inaccuracy of the fit with the Gaussian function results in a secondary error. This is less than the systematic error and can decrease or increase the gain. It is therefore added to the standard deviation of the measured gain.

### 3.2 Standard deviations for dark spectra, minima and maxima

Another factor for the calculation of the gain is the accuracy of the used measuring devices. Besides resolution, the detector noise is an important source for measurement errors. This intensity noise depends on the settings of the sensitivity of the lock-in amplifier. This noise leads to an error for the mean of the dark spectra, for the determined maxima, minima and also for the integral used in the Cassidy method. Consequently we have three contributions to statistical error, where the variations affect the calculated gain for both Hakki–Paoli and Cassidy and one parameter (the dark spectrum) for Fourier including the zeroth peak. We calculate the standard deviation for the measured dark spectra and weighted it for the different parameters of influence. $\sigma_{\text{weighted}} = \frac{\sigma}{\sqrt{N}}$, where $N$ is the number of points, for which we need the standard deviation. To keep the error of the mean of the dark spectra small, we take the mean for 2000 points. In deriving the error bars for the different methods, we take into account that the background noise level is added to only one point for the maximum in Hakki–Paoli method, but for all points contributing to the integral in the Cassidy method. Thus we multiply this error with the number of points between two neighboring peaks. The error of the minima is determined in a different way, as we fit points of the minima applying a parabolic fit, as shown in Fig. 4. The standard deviation of this fit is also divided by the square root of the number of points fitted.

We did some longer dark measurements over several hours with up to 40 000 points to prove that we observe a statistic error with Gaussian normal distribution (see Fig. 9). We divided these spectra in pieces of equal length and compared the resolved distributions for each part with a Gaussian distribution and see a very good agreement. The differences between the mean values, standard deviations and weighted deviations for the parts are relatively small. We also measure the dark spectra for the same settings before and after the measurement and do not observe significant differences. If we compare dark measurements from different times of the working process for the same settings of the sensitivity, we also get similar results.

### 3.3 Error calculation

We made an error calculation for all three methods with errors as determined in Sect. 3 and thus derived error bars for the gain.

We can divide the gain spectrum in three parts, one part with the wavelengths below the lasing wavelength $\lambda_{\text{lase}}$, one part in the range of the lasing wavelength, and the last part for wavelengths above the lasing wavelength (Fig. 10). We see some differences for the used methods at the different parts.
For short wavelengths the influence of the resolution of the spectrometer increases for increasing wavelength towards $\lambda_{\text{lase}}$. This leads to lower gain for the Fourier methods and the highest gain for Cassidy. At the lasing wavelength Hakki–Paoli is about 0.3 cm$^{-1}$ less than Cassidy and Fourier with the zeroth and first peak is about 1.0 cm$^{-1}$ less and Fourier first to fourth peak is 1.2 cm$^{-1}$ less than Cassidy. We suppose that Cassidy reflects the gain best, because of the smallest influence of the resolution (Fig. 8). At a certain wavelength above $\lambda_{\text{lase}}$ there is a crossover of the gain spectra as calculated by the different methods. Above this crossover wavelength the two Fourier methods without the zeroth peak produce a higher gain than the other three methods. Above this wavelength the differences increase until the lasing wavelength. The Fourier methods without the zeroth peak result in the lowest gain.

In the range of the lasing wavelength the correction of the resolution error leads to a convergence of all Fourier methods and Hakki–Paoli to Cassidy method without reaching the level of Cassidy. Therefore, we assume that the Cassidy method is the best method for this part of the spectrum. The Fourier methods without the zeroth peak result in the highest gain for wavelengths above the crossover point. Hakki–Paoli, Cassidy and Fourier with the zeroth peak decrease much more due to the increasing noise and background level. In a Fourier transformed spectrum only the zeroth peak is influenced by the noise and background, thus the methods without this peak are not decreased.

We conclude that Fourier with the zeroth peak results in the smoothest curve below lasing wavelength. At lasing wavelength Cassidy obtains the best result because of the lowest influence of the resolution of the spectrometer and for wavelengths above the lasing wavelength Fourier method without respect to the zeroth peak evaluate the best gain curve because this is not influenced by the noise and background.

4. Conclusion

For wavelengths below the lasing wavelength all methods result in similar gains. For increasing wavelength the differences increase until the lasing wavelength. The Fourier methods without the zeroth peak result in the lowest gain.

In the range of the lasing wavelength the correction of the resolution error leads to a convergence of all Fourier methods and Hakki–Paoli to Cassidy method without reaching the level of Cassidy. Therefore, we assume that the Cassidy method is the best method for this part of the spectrum. The Fourier methods without the zeroth peak result in the highest gain for wavelengths above the crossover point. Hakki–Paoli, Cassidy and Fourier with the zeroth peak decrease much more due to the increasing noise and background level. In a Fourier transformed spectrum only the zeroth peak is influenced by the noise and background, thus the methods without this peak are not decreased.

We conclude that Fourier with the zeroth peak results in the smoothest curve below lasing wavelength. At lasing wavelength Cassidy obtains the best result because of the lowest influence of the resolution of the spectrometer and for wavelengths above the lasing wavelength Fourier method without respect to the zeroth peak evaluate the best gain curve because this is not influenced by the noise and background.

ORCID iDs

Ulrich T. Schwarz © https://orcid.org/0000-0002-1889-2188

1) T. Meyer, H. Braun, U. T. Schwarz, S. Tautz, M. Schillgalies, S. Lutgen, and U. Strauss, Opt. Express 16, 6833 (2008).
2) K. Kojima, U. T. Schwarz, M. Funato, Y. Kawakami, S. Nagahama, and T. Mukai, Opt. Express 15, 7730 (2007).
3) B. W. Hakki and T. L. Paoli, J. Appl. Phys. 44, 4113 (1973).
4) B. W. Hakki and T. L. Paoli, J. Appl. Phys. 46, 1299 (1975).
5) S. Nakamura, M. Senoh, S. Nagahama, N. Iwasa, T. Yamada, T. Matsushita, Y. Sagimoto, and H. Kiyoku, Appl. Phys. Lett. 69, 1568 (1996).
6) T. Lermer et al., Appl. Phys. Lett. 98, 021115 (2011).
7) Y. Barbarin, E. A. I. M. Bente, G. Servantion, L. Mussard, Y. S. Oei, R. Nötzel, and M. K. Smit, Appl. Opt. 45, 9007 (2006).
8) Y. S. Kim, A. Kaneta, M. Funato, Y. Kawakami, T. Kyono, M. Ueno, and T. Nakamura, Appl. Phys. Express 4, 052103 (2011).
9) C.-S. Chang, S. L. Chuang, J. T. Mich, W.-C. W. Fang, Y. K. Chen, and T. Tanbun-Ek, IEEE J. Sel. Top. Quantum Electron. 1, 1100 (1995).
10) T. Deguchi, T. Azuhata, T. Sota, S. Chichibu, M. Arita, H. Nakanishi, and S. Nakamura, Semicond. Sci. Technol. 13, 97 (1998).
11) B. Witzigmann et al., Appl. Phys. Lett. 88, 021104 (2006).
12) W. G. Scheibenzuber, U. T. Schwarz, T. Lermer, S. Lutgen, and U. Strauss, Appl. Phys. Lett. 97, 021102 (2010).
13) U. T. Schwarz, E. Sturm, W. Wegscheider, V. Kümmler, A. Lell, and V. Härl, Appl. Phys. Lett. 83, 4095 (2003).
14) D. T. Cassidy, J. Appl. Phys. 56, 3096 (1984).
15) D. Hofstetter and J. Faist, IEEE Photonics Technol. Lett. 11, 1372 (1999).
16) D. Hofstetter and R. L. Thornton, IEEE J. Quantum Electron. 34, 1914 (1998).
17) H. Wang and D. T. Cassidy, IEEE J. Quantum Electron. 41, 532 (2005).
18) M. Zhang, D. Liu, and T. Makino, IEEE Photonics Technol. Lett. 25, 1122 (2013).