Physics of Higgs Boson Family

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Abstract

In the Standard Model, there is the single Higgs field, $\phi$, which gives rise to constituent quark and lepton masses. The Yukawa coupling is a highly complex set of $3 \times 3$ matrices, resulting in many textures of quark and lepton masses.

In this talk, I present a model which transfers the complexity of the Yukawa coupling matrices to a family of Higgs fields, so that the Yukawa coupling itself becomes a simple interaction.

In the context of this Enriched Standard Model, we introduce a new $r$-symmetry in the extended $SU(2)_L \times U(1)_Y \times U(1)_R$ model and show how the 125 GeV and 750 GeV resonances may be identified with $H$ and $H'$, the key members of the Higgs family, with $H$ being in every way identified with the SM Higgs. There are interesting consequences of their $2\gamma$ decay widths.

Keywords: Enriched Standard Model, $r$-symmetry, Yukawa coupling, family of Higgs fields

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1 Introduction

There has been great excitement over the Christmas holidays barely two months ago when CERN announced an intriguing hint of a new resonance at 750 GeV. In my talk today, I would like to reflect on the implications that such a new resonance at 750 GeV would have for the proposal of the Enriched Standard Model (ESM).\[1\]

In the Standard Model, the quarks acquire mass through the vacuum expectation value of a single $SU(2)$ Higgs doublet, $\phi^\alpha$. The complexity of the texture of quark mass matrices\[4\] is attributed to the Yukawa coupling matrix\[\]

\[
L_Y = -Y_u^{ij} \bar{u}_R^i q_L^j \epsilon_{\alpha\beta} \phi^\beta + h.c. \\
- Y_d^{ij} \bar{d}_R^i q_L^j \phi^\alpha + h.c. 
\]

where we have introduced for later convenience the convention

\[
\phi^\alpha \equiv (\phi^\alpha)^\dagger
\]  

Much of the Standard Model phenomenology is dedicated to the determination of the magnitudes and phases of the $CKM$ matrix that can be derived from the $u, d$ Yukawa coupling matrices. In particular, there are many unexplained hierarchies in the quark masses, and the Wolfenstein hierarchies in the associated $CKM$ matrices.

In what I call the ESM, we transfer the complexity of the Yukawa coupling to the $SU(3)$ family of Higgs fields, so that the Yukawa interaction is now

\[
L_Y = \begin{align*}
- h_u & \bar{u}_R^i q_L^j \epsilon_{\alpha\beta} \phi^\beta + h.c. \\
- h_d & \bar{d}_R^i q_L^j \tilde{\phi}_{ij\alpha} + h.c.
\end{align*}
\]

Note that in so doing we have also added the distinction between the Higgs fields, $\tilde{\phi}_{ij\alpha}$, coupled to the down-quark family versus the original $\phi^\alpha_{ij}$ associated with the up-quark family. From a certain point of view, it looks

§ Here $i, j$ range over the values 1, 2, 3. We have also suppressed, throughout this paper, the $SU(3)_c$ indices on the quarks. The Higgs fields are as usual taken to be color neutral.
counter-intuitive to double down on the number of Higgs fields when pheno-
momenologically there was such a great difficulty in even finding one Higgs.
The pay-off comes when you realize there is a greater symmetry that results
from so doing.

2 r-Symmetry

To make the model more attractive, we make the simple requirement

\[ h_u = h_d \]  

(4)

so that

\[ \mathcal{L}_Y = -h_q \left( \overline{u}^i_R q^\beta_i L \phi^\alpha_{ij} \epsilon_{\alpha\beta} + \overline{d}^i_R q^\alpha_i \hat{\phi}^\alpha_{ij} \right) + h.c. \]  

(5)

This requirement may look odd, as everyone knows that the down quark in
each family is much lighter than the up quark. But, for this Enriched Stan-
dard Model, with the two families of Higgs fields, \( \phi^\alpha \), and \( \hat{\phi}_{\alpha} \), the difference
in the physical up and down quark masses may be attributed to the difference
in vacuum expectation values of the corresponding Higgs fields.

To implement this requirement we extend the enriched Standard Model to
the gauge group \( SU(2)_L \times U(1)_Y \times U(1)_R \) and impose a new \( r \)-symmetry on
the full Lagrangian.

|          | \( Y_R \) | \( Y' \) | \( (I_3)_L \) | \( Y_R \) | \( Y' \) | \( (I_3)_L \) |
|----------|-----------|-----------|-------------|-----------|-----------|-------------|
| \( \phi^{ij}_+ \) | +1/2      | +1/2      |             |           |           | +1/2        |
| \( \phi^{ij}_- \) | +1/2      | -1/2      |             |           |           | -1/2        |
| \( \hat{\phi}^{+ij} \) | +1/2      | +1/2      |             |           |           | +1/2        |
| \( \hat{\phi}^{\alpha ij} \) | +1/2      | -1/2      |             |           |           | +1/2        |
The covariant derivatives for the quark & Higgs fields now read as

\[ D_\mu q^i_L^\alpha = \partial_\mu q^i_L^\alpha + i \frac{g}{2} \left( \vec{\tau} \cdot \vec{W} \right)_\beta^{i\beta} q^i_L^\alpha + i \frac{g'}{6} B'_\mu q^i_L^\alpha \]

\[ D_\mu q^a_R^i = \partial_\mu q^a_R^i + i g_R \left( \tau_3 B_R^\mu \right)_b^a q^b_R^i + i \frac{g'}{6} B'_\mu q^a_R^i \]

\[ D_\mu \phi^\alpha_{ij} = \partial_\mu \phi^\alpha_{ij} + i g \left( \vec{\tau} \cdot \vec{W} \right)_\beta^{\alpha\beta} \phi^\beta_{ij} + i \frac{g_R}{2} B_R^\mu \phi^\alpha_{ij} \]

\[ D_\mu \hat{\phi}^\alpha_{ij} = \partial_\mu \hat{\phi}^\alpha_{ij} - i g \left( \vec{\tau} \cdot \vec{W} \right)_\beta^{\alpha\beta} \hat{\phi}^\beta_{ij} - i \frac{g_R}{2} B_R^\mu \hat{\phi}^\alpha_{ij} \]

The full Lagrangian is invariant under the \( r \)-symmetry

\[ \left\{ \begin{array}{l} u_{Ri} \rightarrow d_{Ri} \\
\phi^\alpha_{ij} \rightarrow -\epsilon^{\alpha\beta} \hat{\phi}^\beta_{ij} \\
d_{Ri} \rightarrow -u_{Ri} \end{array} \right\} \]

and for the gauge fields

\[ \left\{ \begin{array}{l} B_R^\mu \rightarrow -B_R^\mu \\
\hat{W}_\mu \rightarrow +\hat{W}_\mu \\
B'_\mu \rightarrow +B'_\mu \end{array} \right\} \]

This extension of the Standard Model parallels the \( SU(2)_L \times SU(2)_R \times U(1)_Y \)

\[ \text{of Mohapatra & Senjanovic}^2 \]. Instead of the full set of \( SU(2)_R \) gauge bosons, however, we have only the neutral \( B_R \) gauge boson. To have the correct neutrino phenomenology, we introduce a set of very heavy Higgs fields, \( \Delta^{R}_{ab} \).

### 3 Include Leptons

To include leptons, we introduce the compact notation for the Higgs fields

\[ \phi^\beta_{a,ij} = \begin{pmatrix} \hat{\phi}^{(o)}_{ij} & \phi^{(z)}_{ij} \\ \phi^{-}_{ij} & \phi^{(o)}_{ij} \end{pmatrix} \]

\[ \phi^\alpha_{ij} \]

Here \( a, b = 1, 2 \) refer to the flavor of the right-handed quark families \( q_R^a \), while \( \alpha = 1, 2 \) refer as usual to the flavor of the left-handed \( q_L^i \) quark families, and \( i, j = 1, 2, 3 \) refer to the families.
using the convention that

\[
\phi^i_{\alpha j} \equiv \left(\phi^\alpha_{ij}\right)^\dagger \\
\hat{\phi}^\alpha_{ij} \equiv \left(\hat{\phi}_\alpha^i j\right)^\dagger
\]  

(13)

Here in addition to the Dirac mass terms for the leptons, we introduce the Majorana mass terms for the leptons. The complete fermion Yukawa Lagrangian now reads

\[
\mathcal{L}_Y = -h_q q^i R a \bar{q}^j L \epsilon_{\alpha \beta} \phi^\beta_{bij} \epsilon^{ab} + h.c. \\
- \bar{h}_l \ell^i R a \bar{\ell}^j L \epsilon_{\alpha \beta} \phi^\beta_{bij} \epsilon^{ab} + h.c. \\
+ \frac{1}{2} \ell^a R i \ell^b R j \Delta_{ab}^{R,ij} + h.c.
\]  

(14)

Under r-symmetry, the fermion fields transform as

\[
\begin{align*}
q^a R_i &\to (i\sigma_2)^a b q^b R_i \\
\bar{q}^j L &\to \bar{q}^j L \\
\ell^a R_i &\to (i\sigma_2)^a b \ell^b R_i \\
\bar{\ell}^j L &\to \bar{\ell}^j L \end{align*}
\]  

(15)

while the Higgs fields transform as

\[
\begin{align*}
\phi^\beta_{bij} &\to (-i\sigma_2)^c b \phi^\beta_{cij} \\
\Delta_{ab}^{R,ij} &\to (i\sigma_2)^c d \Delta_{cd}^{R,ij} (i\sigma_2)^d b
\end{align*}
\]  

(16)

or, specifically for the new \( \Delta_{ab}^{R,ij} \) fields under r-symmetry

\[
\begin{align*}
\Delta_{ab}^{R,ij} &\leftrightarrow \Delta_{22}^{R,ij} \\
\Delta_{12}^{R,ij} &\leftrightarrow -\Delta_{21}^{R,ij}
\end{align*}
\]  

(17)

The covariant derivatives of the Higgs fields now read in totality

\[
\begin{align*}
D_{\mu} \phi^\beta_{ij a} &= \partial_{\mu} \phi^\beta_{ij a} + ig R R a \left(\bar{\tau} \cdot \tilde{W}_{\mu}\right) \phi^\alpha_{ij a} - ig R R a \left(\sigma_3\right)_{a'} B_{R\mu} \phi^\alpha_{ij a'} \\
D_{\mu} \Delta_{Rab}^{ij} &= \partial_{\mu} \Delta_{Rab}^{ij} - ig R R a \left(\sigma_3\right)_{a'} B_{R\mu} \Delta_{Rab'}^{ij} \\
&\quad - ig R R a \left(\sigma_3\right) \bar{B}_{R\mu} \Delta_{Rab'}^{ij} + ig R R a \Delta_{Rab}^{ij}
\end{align*}
\]  

(18)
Following Mohapatra & Senjanovic[2], we implement the symmetry breaking pattern such that the vacuum expectation values of the different Higgs fields

\[ < \Delta^{ij}_{Rab} > = \begin{pmatrix} v^{ij}_R & 0 \\ 0 & 0 \end{pmatrix}_{ab} \]  

\[ < \phi^{\beta}_{bij} > = \begin{pmatrix} \hat{v}^{ij}_b & 0 \\ 0 & v^{ij}_b \end{pmatrix}_{\beta} \]  

possess the hierarchy

\[ v_R \gg v \gg \hat{v} \]  

where we have used the simplified notation

\[ \begin{align*}
    v^2 &= v^{ij} v^{ij} \\
    \hat{v}^2 &= \hat{v}^{ij} \hat{v}^{ij} \\
    v^2_R &= v^{ij}_R v^{ij}_R
  \end{align*} \]  

4 Gauge Bosons Acquire Mass

From the covariant derivatives in eq.(18), we arrive at the physical gauge bosons \( (v^2_t \equiv v^2 + \hat{v}^2) \)

\[ Z_{\mu} = \cos \theta_W W^3_{\mu} - \sin \theta_W \frac{(g' B_{R\mu} + g_R B_{\mu})}{\sqrt{g_R^2 + g'^2}} + O\left(\frac{v^2_t}{v^2_R}\right) \]  

\[ A_{\mu} = \sin \theta_W W^3_{\mu} + \cos \theta_W \frac{(g' B_{R\mu} + g_R B_{\mu})}{\sqrt{g_R^2 + g'^2}} + O\left(\frac{v^2_t}{v^2_R}\right) \]  

\[ Z_{R\mu} = \frac{g_R B_{R\mu} - g' B_{\mu}}{\sqrt{g_R^2 + g'^2}} + O\left(\frac{v^2_t}{v^2_R}\right) \]  

with the masses given by

\[ m^2_W = \frac{1}{2} g^2 v^2_t \]  

\[ m^2_Z = \frac{m^2_W}{\cos^2 \theta_W} \]  

\[ = \frac{v^2_t}{2} (g^2 + g'^2) \]  

\[ m^2_{Z_R} = 2 (v^2_R) (g_R^2 + g'^2) + O(v^2) \]
where

\[ g'_s = \frac{g_R g'_s}{\sqrt{g'^2_R + g'^2_s}} \]  \hspace{1cm} (28)

or

\[ \frac{1}{g'^2_s} = \frac{1}{g'^2_R} + \frac{1}{g'^2} \]  \hspace{1cm} (29)

From eq. (26) we see that the \( g'_s \) is actually the \( U(1)_Y \) coupling of the Standard Model, and eq. (29) gives its relationship with the couplings of the extended \( SU(2)_L \times U(1)_Y \times U(1)_R \) model.

These relationships are a manifestation of the decoupling theorem of Georgi-Weinberg\[3\]. In eq. (23) we see how in the limit of \( \nu_t/\nu_R \to 0 \),

\[ \frac{g'B_R + g'Y}{\sqrt{g'^2_R + g'^2}} \to B_{Y\mu} \]  \hspace{1cm} (30)

where \( B_{Y\mu} \) is the \( U(1)_Y \) gauge field of the usual \( SU(2)_L \times U(1)_Y \) group.

5 A Simple Higgs Potential

As noted already by Mohapatra and Senjanovic\[2\], neutrino phenomenology requires that the Higgs fields \( \Delta^{ij}_{Rab} \) be associated with a mass scale that is much higher than the Higgs \( \phi^\beta_{a_{ij}} \). For our purposes, the Georgi-Weinberg decoupling theorem\[3\] enables us to focus on the low energy phenomenology associated with the \( \phi^\beta_{a_{ij}} \).

Rather than work with a most general for the Higgs potential, We turn to a particularly simple form for the Higgs potential.

We consider the potential

\[ V = V_\phi + V_\Delta \]  \hspace{1cm} (31)

where \( V_\phi \) involves the lighter \( \phi^\alpha_{ij} \) and \( \bar{\phi}^{\alpha \eta} \) fields, while \( V_\Delta \) involves the heavy \( \Delta^{ij}_{Rab} \) fields. For the general non-degenerate case, we introduce the three
coupling constants, $\lambda_1, \lambda_2, \lambda_3$ in the maximally symmetric potential

$$V_\phi = + \frac{\lambda_1}{2} \left( \epsilon_{\alpha\beta} \phi^\alpha_{a,ij} \phi^\beta_{b,kl} \right) \left( \epsilon^{\alpha'\beta'} \phi^{a',ij}_{a',i'} \phi^{b',kl}_{b',k'} \right)$$

$$+ \frac{\lambda_2}{2} \left( \phi^\alpha_{a,ij} \phi^{b',kl}_{b',k'} \phi_\alpha^{a',ij}_{a',i'} - \frac{\lambda_3}{4} \left( \phi^\alpha_{a,ij} \phi^{a,ij}_\alpha \right)^2 \right)$$

The symmetry is broken through the vacuum expectation values of the $\phi^\alpha_{a,ij}$ fields, as given in eq.(22) above.

The Higgs potential $V_\phi$ around the new vacuum takes the form

$$V_\phi = + \frac{\lambda_1}{2} \left( \epsilon_{\alpha\beta} \left[ \phi^\alpha_{a,ij} \phi^{\beta}_{b,kl} - \nu_{a,ij} \nu_{b,kl} \right] \right) \times \left( \epsilon_{\alpha'\beta'} \left[ \phi^{a',ij}_{a',i'} \phi^{b',kl}_{b',k'} - \nu^{a',ij}_{a',i'} \nu^{b',kl}_{b',k'} \right] \right)$$

$$+ \frac{\lambda_2}{2} \left( \left[ \phi^\alpha_{a,ij} \phi^{b',kl}_{b',k'} - \nu^\alpha_{a,ij} \nu^{b',kl}_{b',k'} \right] \right) \times \left[ \left[ \phi^{\beta}_{b,kl} \phi^{a,ij}_{a,ij} - \nu^{\beta}_{b,kl} \nu^{a,ij}_{a,ij} \right] \right)$$

$$- \frac{\lambda_3}{4} \left( \phi^\alpha_{a,ij} \phi^{a,ij}_\alpha - \nu^2 \right)^2$$

Likewise, the Higgs potential involving the heavy fields takes the form

$$V_\Delta = + \lambda_4 \left( \Delta^{ij}_{R_{ab}} \Delta^{R_{ab}}_{ijkl} - \nu^2_R \right)^2$$

$$+ \lambda_5 \left( \Delta^{ij}_{R_{ab}} \Delta^{R_{ab}}_{R_{cd}} \epsilon^{ac} \epsilon^{bd} \right) \left( \Delta^{R_{ab}}_{ijkl} \Delta^{R_{ab}}_{ijkl} \epsilon^{a'b'c'd'} \epsilon^{a'c'e'b'd'} \right)$$

$$+ \lambda_6 \nu^2_R \left[ \Delta^{ij}_{R_{ab}} \left( \sigma^a_1 \Delta^{R_{ab}}_{ijkl} (\sigma^b_1)_{ij} \right)$$

$$- \Delta^{ij}_{R_{ab}} \left( i\sigma^a_2 \Delta^{R_{ab}}_{ijkl} (i\sigma^b_2)_{ij} \right)$$

By construction, this Higgs potential is stable about the broken vacuum with $\nu_{ij}$ and $\nu_{kl}$.

6 Quark Mass Diagonal (QMD) Basis

To work out the mass spectrum for the Higgs potential of eq.(33), we need to go to the basis where the physical quark fields have diagonal mass matrices,
where

\[ \bar{u}_R^i = \bar{u}_R^j (V_{uR}^*)^i_j \]
\[ u_L^j = (V_{uL})^j_i u_L^i \]
\[ \bar{d}_R^i = \bar{d}_R^j (V_{dR}^*)^i_j \]
\[ d_L^j = (V_{dL})^j_i d_L^i \]

For simplicity, we make the assumption

\[ V_{uL} = V_{uR} \]  \hspace{1cm} (35)
\[ V_{dL} = V_{dR} \]  \hspace{1cm} (36)

so that the CKM matrix takes the form

\[ V_i^j = \left( (V_u)^\dagger V_d \right)^i_j \]  \hspace{1cm} (37)

In this QMD basis, the Higgs fields take the form

\[ \phi_{ij}^{(o)} = \left( \tilde{V}_u \right)^{i'}_i \Phi_{ij'}^{(o)} (V_d^\dagger)^{j'}_j \]  \hspace{1cm} (38)
\[ \phi_{ij}^{(+)} = \left( \tilde{V}_u \right)^{i'}_i \Phi_{ij'}^{(+)} (V_d^\dagger)^{j'}_j \]  \hspace{1cm} (39)
\[ \hat{\phi}_{ij}^{(o)} = \left( V_{d}^* \right)^i_{i'} \hat{\Phi}_{ij'}^{(o)} (V_d)^{j'}_j \]  \hspace{1cm} (40)
\[ \hat{\phi}_{ij}^{(+)} = \left( V_{d}^* \right)^i_{i'} \hat{\Phi}_{ij'}^{(+)} (V_d)^{j'}_j \]  \hspace{1cm} (41)

and the \( \Phi_{ij}, \hat{\Phi}_{ij} \) fields have vacuum expectation values \( w_{ij}, \hat{w}_{ij} \) that are diagonal, with

\[ < \Phi_{ij}^{(o)} > = w_{ij} = \begin{pmatrix} v_1 & 0 & 0 \\ 0 & v_2 & 0 \\ 0 & 0 & v_3 \end{pmatrix} \]  \hspace{1cm} (42)

and

\[ < \hat{\Phi}_{ij}^{(o)} > = \hat{w}_{ij} = \begin{pmatrix} \hat{v}_1 & 0 & 0 \\ 0 & \hat{v}_2 & 0 \\ 0 & 0 & \hat{v}_3 \end{pmatrix} \]  \hspace{1cm} (43)
The Higgs potential in eq. (33) gives rise to a rich mass spectrum. It involves the full complexity of the texture of the vacuum expectation values, and poses a daunting task for the timid explorer. In my earlier works, I had explored the spectrum in the leading hierarchy by simply setting \( \nu_{33} = \nu, \hat{\nu}_{33} = \hat{\nu} \), and letting all the other vacuum expectation values be zero.

In this talk, I will set forth a more complete analysis of the mass spectrum for the general mass hierarchy.

### 7 Neutral Higgs Boson Spectrum

For the neutral Higgs sector, we express the resulting spectrum in terms of the hermitian fields, \( h_{ij} \) and \( z_{ij} \), with

\[
\Phi^0_{ij} = \frac{h_{ij} - iz_{ij}}{\sqrt{2}} \\
\hat{\Phi}^0_{ij} = \frac{\hat{h}_{ij} - i\hat{z}_{ij}}{\sqrt{2}}
\]

(44)

The flavor-diagonal family of neutral scalar Higgs bosons may be expressed in terms of the orthonormal basis \( h \) eigenstates

\[
\begin{align*}
h_a &= (v_3 h_{33} + \frac{v_2}{v_\perp} (v_2 h_{22} + v_1 h_{11})) / v \\
h_b &= (-v_\perp h_{33} + \frac{v_2}{v_\perp} (v_2 h_{22} + v_1 h_{11})) / v \\
h_c &= (-v_1 h_{22} + v_2 h_{11}) / v_\perp \\
\tilde{h}_a &= (\hat{\nu}_3 \tilde{h}_{33} + (\hat{\nu}_2 \tilde{h}_{22} + \hat{\nu}_1 \tilde{h}_{11})) / \hat{\nu} \\
\tilde{h}_b &= (-\hat{\nu}_\perp \tilde{h}_{33} + \frac{\hat{\nu}_3}{\hat{\nu}_\perp} (\hat{\nu}_2 \tilde{h}_{22} + \hat{\nu}_1 \tilde{h}_{11})) / \hat{\nu} \\
\tilde{h}_c &= (-\hat{\nu}_1 \tilde{h}_{22} + \hat{\nu}_2 \tilde{h}_{11}) / \hat{\nu}_\perp
\end{align*}
\]

(45)

where

\[
\begin{align*}
v &= \sqrt{v_3^2 + v_2^2 + v_1^2} \\
v_\perp &= \sqrt{v_2^2 + v_1^2} \\
\hat{\nu} &= \sqrt{\hat{\nu}_3^2 + \hat{\nu}_2^2 + \hat{\nu}_1^2} \\
\hat{\nu}_\perp &= \sqrt{\hat{\nu}_2^2 + \hat{\nu}_1^2}
\end{align*}
\]

(46)
Likewise, the flavor-diagonal pseudoscalar Higgs bosons may be expressed in terms of the orthonormal basis $z$ eigenstates

$$
\begin{align*}
z_a &= \left( \nu_3 z_{33} + \frac{(\nu_2 z_{22} + \nu_1 z_{11})}{\nu} \right) / \nu \\
z_b &= \left( -\nu_3 z_{33} + \frac{\nu_3}{\nu} (\nu_2 z_{22} + \nu_1 z_{11}) \right) / \nu \\
z_c &= \left( -\nu_1 z_{11} + \nu_2 z_{11} \right) / \nu \\
\tilde{z}_a &= \left( \hat{\nu}_3 \tilde{z}_{33} + \frac{(\hat{\nu}_2 \tilde{z}_{22} + \hat{\nu}_1 \tilde{z}_{11})}{\hat{\nu}} \right) / \hat{\nu} \\
\tilde{z}_b &= \left( -\hat{\nu}_3 \tilde{z}_{33} + \frac{\hat{\nu}_3}{\hat{\nu}} (\hat{\nu}_2 \tilde{z}_{22} + \hat{\nu}_1 \tilde{z}_{11}) \right) / \hat{\nu} \\
\tilde{z}_c &= \left( -\hat{\nu}_1 \tilde{z}_{11} + \hat{\nu}_2 \tilde{z}_{11} \right) / \hat{\nu} \\
\end{align*}
$$

(47)

In terms of these basis states, the spectrum of neutral Higgs mass eigenstates is given by

| Neutral Higgs boson | ( Mass )$^2$ |
|--------------------|-----------|
| $H = (v h_a + \hat{\nu} \hat{h}_a) / v_t$ | $m_1^2$ |
| $H' = (-\hat{\nu} h_a + v \hat{h}_a) / v_t$ | $m_2^2$ |
| $G^0 = (v z_a + \hat{\nu} \hat{z}_a) / v_t$ | $m_Z^2$ |
| $A = (-\hat{\nu} z_a + v \hat{z}_a) / v_t$ | $\lambda_1 \nu_t^2$ |

Table I

$h_b, h_c, h_{ij}$ for $i \neq j$

$z_b, z_c, z_{ij}$ for $i \neq j$

$\tilde{h}_b, \tilde{h}_c, \tilde{h}_{ij}$ for $i \neq j$

$\tilde{z}_b, \tilde{z}_c, \tilde{z}_{ij}$ for $i \neq j$

where, neglecting terms of order $\tilde{\nu}^4 / \nu^2$,

$$
\begin{align*}
m_1^2 &= (2\lambda_2 - \lambda_3) \nu^2 + \left[ \lambda_1 + \frac{(\lambda_1 - \lambda_3)^2}{2\lambda_2 - \lambda_1 - \lambda_3} \right] \tilde{\nu}^2 \\
m_2^2 &= \lambda_1 \nu^2 + \left[ 2\lambda_2 - \lambda_3 - \frac{(\lambda_1 - \lambda_3)^2}{2\lambda_2 - \lambda_1 - \lambda_3} \right] \tilde{\nu}^2
\end{align*}
$$

(48) (49)

Here I have adopted the notation as employed by the $2HDM$ and $MSSM$ literature. The masses for the Goldstone bosons are given in the 't Hooft gauge, ($\xi = 1$).
and we have, finally, the notation

\[ \nu_t \equiv \sqrt{v^2 + \nu_t^2} \quad (50) \]

Note that the vacuum expectation values are

\[ \langle h_{33} \rangle = \sqrt{2} \nu_3 \]
\[ \langle \hat{h}_{33} \rangle = \sqrt{2} \nu_3 \]

so that

\[ m_t = h_q \nu_3 \]
\[ m_b = h_q \nu_3 \]

8 Charged Higgs Boson Spectrum

\[
\Phi_{\ell}^{'+} = \left( \nu_3 \Phi_{33}^{'+} + \left( \nu_2 \Phi_{22}^{'+} + \nu_1 \Phi_{11}^{'+} \right) \right) / \nu
\]
\[
\Phi_{b}^{'+} = \left( -\nu_{\perp} \Phi_{33}^{'+} + \frac{\nu_3}{\nu_{\perp}} \left( \nu_2 \Phi_{22}^{'+} + \nu_1 \Phi_{11}^{'+} \right) \right) / \nu
\]
\[
\Phi_{c}^{'+} = \left( -\nu_{\perp} \Phi_{22}^{'+} + \nu_2 \Phi_{11}^{'+} \right) / \nu_{\perp}
\]

(51)

with the notation

\[ \Phi_{ij}^{'+} = \Phi_{i\ell}^{+} (V^{\dagger})_{\ell j} \]

(52)

\( (V) \) being the CKM matrix defined in eq.(37).

\[
\tilde{\Phi}_{a}^{'+} = \left( \tilde{\nu}_3 \tilde{\Phi}_{33}^{'+} + \left( \tilde{\nu}_2 \tilde{\Phi}_{22}^{'+} + \tilde{\nu}_1 \tilde{\Phi}_{11}^{'+} \right) \right) / \tilde{\nu}
\]
\[
\tilde{\Phi}_{b}^{'+} = \left( -\tilde{\nu}_{\perp} \tilde{\Phi}_{33}^{'+} + \frac{\tilde{\nu}_3}{\tilde{\nu}_{\perp}} \left( \tilde{\nu}_2 \tilde{\Phi}_{22}^{'+} + \tilde{\nu}_1 \tilde{\Phi}_{11}^{'+} \right) \right) / \tilde{\nu}
\]
\[
\tilde{\Phi}_{c}^{'+} = \left( -\tilde{\nu}_{\perp} \tilde{\Phi}_{22}^{'+} + \tilde{\nu}_2 \tilde{\Phi}_{11}^{'+} \right) / \tilde{\nu}_{\perp}
\]

(53)

\[ \tilde{\Phi}_{ij}^{'+} \equiv \tilde{\Phi}_{i\ell}^{+} (V^{\dagger})_{\ell j} \]

(54)

In terms of these basis states, we can now give the spectrum of charged Higgs mass eigenstates in the 't Hooft gauge (\( \xi = 1 \))
Table II

| Charged Higgs boson | ( Mass )$^2$ |
|---------------------|------------|
| $G^+ = \left( v \Phi_a^+ + \bar{v} \hat{\Phi}_a^+ \right) / v_t$ | $m_W^2$ |
| $H'^+ = \left(-\bar{v} \Phi_a^+ + v \hat{\Phi}_a^+ \right) / v_t$ | $\lambda_2 v_t^2$ |
| $\Phi_b^+, \Phi_c^+, \Phi^+_{ij}$ for $i \neq j$ | $\lambda_1 v^2 + \lambda_2 \bar{v}^2$ |
| $\hat{\Phi}_b^+, \hat{\Phi}_c^+, \hat{\Phi}^+_{ij}$ for $i \neq j$ | $\lambda_1 \bar{v}^2 + \lambda_2 v^2$ |

9 Higgs couplings to fermions

Before considering the phenomenological implications of this Enriched Standard Model, we list here the couplings of the neutral Higgs mesons to the top and bottom fermions.

\[
\mathcal{L}_Y = -\frac{h_q v_3}{\sqrt{2}} \bar{t} t \left( v H - \bar{v} H' \right) + i \frac{h_q v_3}{\sqrt{2}} \bar{t} t \gamma_5 \left( v G^o - \bar{v} A \right) + i \frac{h_q v_3}{\sqrt{2}} \bar{b} b \left( \bar{v} H + v H' \right) + i \frac{h_q v_3}{\sqrt{2}} \bar{b} b \gamma_5 \left( \bar{v} G^o + v A \right) \]

(55)

where $G^o$ is the Goldstone bosons in the 't Hooft gauge with mass $M_Z$. Note that here $H$ and $H'$ are scalar fields, while $A$ is a pseudoscalar field.

10 Phenomenological Implications

There has been a lot of excitement in the world of particle physics ever since the announcement of the discovery of a 125 GeV Higgs-like boson on July 4, 2012, followed by the hint in late 2015 of a new resonance at 750 GeV. When I first proposed the ESM model in the context of the discovery of the 125 GeV Higgs, for simplicity of parameters, I restricted myself to the fully degenerate case of $\lambda_1 = \lambda_2 = \lambda_3$. Now that there is the exciting possibility of a new 750 GeV Higgs, I present here a more general case of $\lambda_1 = \lambda_2$. In a forthcoming paper, I will discuss the most general case, with all three different couplings, so that the Higgs family masses will have more fine structure.

**For brevity, we have omitted here the Yukawa couplings involving the $u, d, c, s$ quarks. Because of the hierarchy $v_3 >> v_2 >> v_1, \ldots$ they have negligible effect on the leading order phenomenology.
11 **H field couplings same as those in SM**

Among the enriched family of neutral Higgs bosons, only the $H$ field develops a vacuum expectation value $\sqrt{2} v_t$. Its trilinear and quartic couplings to the electroweak gauge fields and its Yukawa coupling to the matter fields are the same as in the Standard Model. It can thus be identified with the SM Higgs field.

12 **Only H is in $W, Z$ fusion & associated production**

While the $VVH$ coupling is the usual SM coupling, eq.9 leads to the complete trilinear decoupling of all the rest of the Higgs family. By this I mean that $VVh_{ij},VV\hat{h}_{ij},VVz_{ij},VV\hat{z}_{ij}$, couplings for $(ij) \neq (33)$ are absent. In the vector boson fusion & associated production processes, therefore only $H$ is produced.

The absence of $VVH'$ coupling does not, however, imply a complete decoupling of $H'$ from the gauge fields. For they are coupled to the gauge bosons through the quartic coupling with the same coupling strengths as noted below

$$\mathcal{L}_{VVHH} = \mathcal{L}_{VVH'H'} = \mathcal{L}_{VVH'H'}$$

and likewise for the quartic coupling of the rest of the family of Higgs fields.

13 **Normal vs. Inverted Scenario**

Having determined the properties of $H$ versus $H'$, we now come to the interesting question of which of the two resonances is $H$, and which is $H'$. For this discussion, we restrict ourselves to the case $\lambda_1 = \lambda_2$.

13.1 **Normal Scenario**

If we identify the neutral $H$ state with the observed 125 GeV, and the $H'$ state with the 750 GeV, then the mass eigenvalues imply the values of the coupling constants
\[ \begin{align*}
    m_H &= 125 \text{ GeV} \quad \lambda_1, \lambda_2 = 18.59 \\
    m_{H'} &= 750 \text{ GeV} \quad \lambda_3 = 36.66
\end{align*} \] (56)

With this scenario, all the rest of the family of neutral as well as charged Higgs are crowded around 750 GeV.

### 13.2 Inverted Scenario

It is interesting to note the inverted scenario, where \( H \) is the heavier resonance, while \( H' \) is the lighter one. In this case, we have

\[ \begin{align*}
    m_H &= 750 \text{ GeV} \quad \lambda_1, \lambda_2 = 0.516 \\
    m_{H'} &= 125 \text{ GeV} \quad \lambda_3 = -17.56
\end{align*} \] (57)

With this scenario, all the rest of the family of neutral as well as charged Higgs are crowded around 125 GeV.

In a forthcoming paper, we shall consider the case where \( \lambda_1 \neq \lambda_2 \), for which the rest of the family of Higgs will, in the normal scenario, be spread out above 750 GeV, while in the inverted scenario, the rest of the family is now spread below 125 GeV.

### 14 \( H, H', A \to \gamma\gamma \) decay rates

Preliminary data on the new 750 GeV resonance indicates a width for its \( \gamma\gamma \) decay that is much wider than the \( \gamma\gamma \) decay width for the 125 GeV. With the coupling constants as determined in eq.(56, 57), it is possible to make a leading order calculation of the predicted widths. For this, we need to derive the trilinear Higgs self-coupling from \( V_\phi \) in eq.(33)

\[ \mathcal{L}_{HHH} = \frac{-2\lambda_1 - \lambda_3}{2} \sqrt{2} v t H \left( G^+ G_- + H'^+ H'^' \right) \]
\[ -\frac{2\lambda_1 - \lambda_3}{2} \sqrt{2} v t H \sum_{(ij) \neq (33)} \left( H'^{ij} H'^{ij} + \hat{H}'^{ij} \hat{H}'^{ij} \right) \] (58)

Based on eq.(55, 58), we can proceed to make a leading order estimate of the decay rates in both the normal and inverted scenarios. Using eq.(A4) of ref.[6], the decay widths are
Table III

| Scenario                              | $\Gamma(H \rightarrow \gamma\gamma)$ | $\Gamma(H' \rightarrow \gamma\gamma)$ | $\Gamma(A \rightarrow \gamma\gamma)$ |
|---------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| $m_H = 125 \text{ GeV}, m_{H'} = 750 \text{ GeV}$ | 10 $\text{ keV}$                      | 0.7 $\text{ keV}$                      | 0.5 $\text{ keV}$                      |
| $m_H = 750 \text{ GeV}, m_{H'} = 125 \text{ GeV}$ | 45.6 $\text{ MeV}$                    | 1.4 $\text{ keV}$                      | 1.3 $\text{ keV}$                      |

What is interesting about the Inverted Scenario is the large width for $H \rightarrow \gamma\gamma$ compared with the $2\gamma$ decay width of the smaller mass $H'$ Higgs.

15 Conclusion

There has been a lot of excitement in the world of particle physics ever since the announcement of the discovery of Higgs boson on July 4, 2012, and, over the Christmas break of 2015, the hint of a new resonance at 750 GeV. Now that we contemplate an Enriched Standard Model, a natural question that arises would be which of the two is the $H$ and $H'$ of our family of neutral Higgs. The answer becomes clear when we take into account the production and decay channels involved.

For at high energies, the leading production processes predominantly involve top quark loops. Therefore, the production of the family Higgs, $h_{ij}$, $z_{ij}$, $\hat{h}_{ij}$, $\hat{z}_{ij}$, $\Phi'^+_{ij}$, $\Phi'^{+}\hat{u}_{ij}$, with $(ij) \neq (33)$, are suppressed.

Among the family of Higgs, $H$, plays a special role. It behaves like the single Higgs field of the Standard Model, with the same trilinear coupling to the gauge bosons as the standard Higgs. It is produced via associative production with $W, Z$ or Vector Boson fusion. Its production cross-section is identical to that of the Standard Model.

In contrast, the orthogonal Higgs bosons, $H'$ and $A$, do not have trilinear couplings to the gauge bosons. They are produced through coupling to the top and bottom quarks, see eq. (55). While we have not yet pursued a full-blown program of calculating all the widths, the partial result involving the $2\gamma$ decay encourages us to look for dramatic difference in the widths.
There is much work that remains to be done to explore the consequences of this proposal. I welcome your comments and suggestions.

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