Four- and Five-Body Scattering Calculations of Exotic Hadron Systems

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We study the five-quark system \( uudd\bar{s} \) in the standard non-relativistic quark model by solving the scattering problem. Using the Gaussian Expansion Method (GEM), we perform the almost precise multi-quark calculations by treating a very large five-body modelspace including the NK scattering channel explicitly. Although a lot of pseudostates (discretized continuum states) with \( J^\pi = \frac{1}{2}^\pm \) and \( J^\pi = \frac{3}{2}^\pm \) are obtained within the bound-state approximation, all the states in \( 1.4 - 1.85 \) GeV in mass around \( \Theta^+(1540) \) melt into non-resonant continuum states through the coupling with the NK scattering state in the realistic case, i.e., there is no five-quark resonance below \( 1.85 \) GeV. Instead, we predict a five-quark resonance state of \( J^\pi = \frac{1}{2}^- \) with the mass of about \( 1.9 \) GeV and the width of \( \Gamma \approx 2.68 \) MeV. Similar calculation is done for the four-quark system \( c\bar{c}q\bar{q} (q = u, d) \) in connection with \( X^0(3872) \).

§1. Introduction: importance of the wave function of multi-quarks

From the viewpoint of Elementary Particle Physics, the lattice QCD calculation is one of the most standard approaches to investigate hadrons including multi-quarks. However, it is rather difficult to extract the “wave function” of hadrons in the lattice QCD calculation because it is based on the path-integral, where all contributions of possible states are summed up and only vacuum expectation values can be obtained.

Needless to say, the wave function is one of the most important quantities in quantum physics, and, of course, also in Quark-Hadron Physics. In particular for multi-quark systems, the analysis with the quark wave function is necessary to clarify whether the multi-quark hadron is an exotic resonance state or a two-hadron scattering state. Thus, to extract the state information (the wave function) of hadrons, we need a reliable calculational method for multi-quark systems instead of lattice QCD. One of the attractive methods is the constituent quark-model calculation, since it seems workable up to several hundred MeV excitation, and its precise calculation for multi-quark systems can be done with the Gaussian Expansion Method (GEM).1,2

In Nuclear Physics, a precise calculational method for bound and scattering states of various few-body systems using GEM1 has been developed by two of the present authors (E. H. and M. K.) and their collaborators, and has successfully been applied to light nuclei, light hypernuclei, exotic atoms/molecules and so on.1

Using GEM within the framework of a constituent quark model, Hiyama et al.2 investigated for the first time scattering and resonance states of the five-quark system \( uudd\bar{s} \) under the explicit NK scattering boundary condition. It was made clear that there appears no \( \frac{1}{2}^\pm \) resonance state around the reported energy of \( \Theta^+(1540) \).3–5

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In this paper, we perform the similar precise calculation of the five-quark system \( uudd\bar{s} \) with \( J^\pi = \frac{1}{2}^\pm \) and \( J^\pi = \frac{3}{2}^\pm \) using the linear confinement potential, which is more appropriate as indicated by lattice QCD calculations\(^{6,7}\). Furthermore, we apply the same method to the four-quark system \( c\bar{c}q\bar{q} \) in connection with \( X^0(3872) \)\(^8\) and discuss about possibility of any bound/resonance state near the \( D^0\bar{D}^{*0} \) threshold.

\section{Five-body quark-model calculation for \( \Theta^+(1540) \)}

We study the five-quark system \( uudd\bar{s} \) by solving the five-body Schrödinger equation \((H - E)\Psi_{J^\pi M} = 0\) including the NK scattering channel explicitly, as was done in Ref. 2). We here take a standard non-relativistic quark-model Hamiltonian with the linear-type confining potential \( V_{\text{conf}} \) and the color-magnetic potential \( V_{\text{CM}} \):

\[
V_{\text{conf}}(r_{ij}) = -\frac{\lambda^a_i}{2} \frac{\lambda^a_j}{2} \left( \frac{3}{4} \sigma r_{ij} + v_0 \right), \quad V_{\text{CM}}(r_{ij}) = -\frac{\lambda^a_i}{2} \frac{\lambda^a_j}{2} \xi_\alpha \frac{\sigma_i \cdot \sigma_j}{m_i m_j} e^{-r^2_{ij}/\beta^2}. \tag{2.1}
\]

Using the parameters listed in Table I, we can well reproduce masses of ordinary baryons and mesons as shown in Tables II and III, and well reproduce or predict hadron properties, e.g., \( \mu_p \simeq 2.75 \text{ nm} \) (exp. 2.78 nm), \( \mu_n \simeq -1.80 \text{ nm} \) (exp. -1.91 nm), \( \mu_\Lambda \simeq -0.60 \text{ nm} \) (exp. -0.61 nm), \( \mu_{\Sigma^0} \simeq 0.81 \text{ nm} \), \( \mu_{\Omega} \simeq -1.84 \text{ nm} \) (exp. -2.02 nm) for the magnetic moment. Note here that the form of the linear-type confining potential \( V_{\text{conf}} \) is appropriate as an approximation of the \( Y \)-type linear three-quark potential indicated by lattice QCD\(^{6,7}\) although the adopted value of the string tension \( \sigma \simeq 0.56 \text{ GeV/fm} \) is smaller than the standard value \( \sigma \simeq 0.89 \text{ GeV/fm} \).

Now, we perform an almost precise quark-model calculation with GEM for multi-quark systems and clarify whether each obtained state is a resonance or a continuum scattering state through the phase-shift analysis in the model calculation. For the

\begin{table}[h]
\centering
\begin{tabular}{lllllll}
\hline
parameter & \( m_u \) & \( m_d \) & \( m_s \) & string tension \( \sigma \) & \( \beta \) & \( v_0 \) & \( \xi_\alpha \) \\
\hline
adopted value & 330 MeV & 500 MeV & 0.56 GeV/fm & 0.5 fm & -572 MeV & 240 MeV & \( m_u^2 \) \\
\hline
\end{tabular}
\caption{Parameters of the present constituent quark model with a linear-type confining potential.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{llllllll}
\hline
baryon & \( \Lambda \) & \( \Sigma \) & \( \Xi \) & \( \Omega \) & \( \Lambda^* \) & \( \Sigma^* \) & \( \Xi^* \) & \( \Omega^* \) \\
\hline
\text{calculated mass [MeV]} & 939 & 1235 & 1064.8 & 1129 & 1324 & 1539.5 & 1462 \\
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\hline
\end{tabular}
\caption{Calculated masses of typical baryons obtained with the present constituent quark model.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{llllllll}
\hline
meson & \( \omega \) & \( \rho \) & \( K^* \) & \( K \) & \( \pi \) \\
\hline
\text{calculated mass [MeV]} & 759 & 759.3 & 864.4 & 457.9 & 152 \\
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\end{tabular}
\caption{Calculated masses of typical mesons obtained with the present constituent quark model.}
\end{table}
complete set of five-body eigenstates, \( \{ \Phi_\nu \} \). The amplitude is expanded using a nearly complete set in the finite interaction region. The second term describes five-body degrees of freedom in the interaction-region amplitude which vanishes asymptotically. The amplitude is expanded using a nearly complete set in the finite interaction region.

\[
\Psi_{J M}(E) = \Psi_{J M}^{(NK)}(E) + \sum_{\nu=1}^{\nu_{\text{max}}} b_{J}^{(\nu)}(E) \Phi_{J M}^{(\nu)}(E). \tag{2.2}
\]

The first term is the NK scattering component expressed by

\[
\Psi_{J M}^{(NK)}(E) = A_{1234} \{ [\phi^{(N)}_{1/2}(123) \phi^{(K)}_{0}(45)] \chi L(R_1) \} . \tag{2.3}
\]

The second term describes five-body degrees of freedom in the interaction-region amplitude which vanishes asymptotically. The amplitude is expanded using a nearly complete set of five-body eigenstates, \( \{ \Phi_{J M}^{(\nu)}(E) \} \), constructed by diagonalizing the total Hamiltonian as \( \langle \Phi_{J M}^{(\nu)}(E) | H | \Phi_{J M}^{(\nu')} (E) \rangle = E_\nu \delta_{\nu \nu'} \). Here, each of \( \Phi_{J M}^{(\nu)}(E) \) is described as a superposition of \( L^2 \)-type five-body Gaussian basis functions written in all the Jacobi coordinates \( c = 1 - 5 \) as shown in Fig. 1. By employing 15,000 five-body basis functions, i.e., \( \nu_{\text{max}} = 15,000 \), the eigenfunction set \( \{ \Phi_{J M}^{(\nu)}(E) \} \) forms a nearly complete set in the finite interaction region.

The eigenfunctions \( \Phi_{J M}^{(\nu)}(E) \) stand for discretized continuum states of the five-body system, and are called “pseudostates” in the scattering theory. It is known that most of the pseudostates do not actually represent resonance states but melt into non-resonant continuum states when the scattering boundary condition is imposed to the total wave function.

We actually perform the precise quark-model calculation including all the pseudostate terms in (2.2), and calculate the phase shift \( \delta \) in the NK elastic scattering \( N + K \rightarrow N + K \) for \( J^\pi = \frac{1}{2}^\pm, \frac{3}{2}^\pm \) channels, respectively. For \( J^\pi = \frac{1}{2}^\pm \), all the pseudostates located at \( 0 - 450 \) MeV above the NK threshold melt into non-resonant continuum states when the NK scattering boundary condition is imposed, which means the strong coupling between those pseudostates and the NK scattering state. We show in Fig. 2 the calculated phase shift \( \delta \) and find no resonance for \( 0 - 450 \) MeV above the NK threshold, i.e., \( 1.4 - 1.85 \) GeV in mass region around \( \Theta^+(1540) \). We find a sharp resonance for \( J^\pi = \frac{1}{2}^- \) and a broad one for \( J^\pi = \frac{1}{2}^+ \) around 1.9 GeV in Fig. 2, although their energies are too high to be identified with the \( \Theta^+(1540) \). For the \( J^\pi = \frac{3}{2}^\pm \) states, we find no resonance up to 500 MeV above the NK threshold.

Note that these results on the absence of low-lying pentaquark resonances with \( J^\pi = \frac{1}{2}^\pm, \frac{3}{2}^\pm \) are consistent with the lattice QCD results in Refs. 9) and 10). As
Fig. 2. The calculated phase shift $\delta$ for the five-quark system with $J^\pi = \frac{1}{2}^-$ (left) and $J^\pi = \frac{3}{2}^+$ (right) in the NK scattering (N+K $\rightarrow$ N+K) as the function of the energy measured from the NK threshold $E_{\text{th}} = m_N + m_K$. No resonance is seen around the energy of $\Theta^+(1540)$. Instead, there is a sharp resonance state with $J^\pi = \frac{1}{2}^-$ and the mass of about 1.9 GeV.

for the presence of $\frac{1}{2}^-$ penta-quark resonance around 1.9 GeV, Ref. 11) shows a consistent lattice QCD result indicating a $\frac{1}{2}^-$ penta-quark resonance around 1.8 GeV.

To conclude, there is no five-quark resonance with $J^\pi = \frac{1}{2}^\pm, \frac{3}{2}^\pm$ below 1.85 GeV. Instead, the quark-model calculation predict a five-quark resonance state of $J^\pi = \frac{1}{2}^-$ with the mass of about 1.9 GeV and the width of $\Gamma \approx 2.68$ MeV.

§3. Four-body quark-model calculation for $X_0^0(3872)$

With GEM, we analyze four-quark systems $c\bar{c}q\bar{q}$ for $I = 0, 1$ in a quark model to investigate $X_0^0(3872),^8$ which may have the tetraquark structure as $c\bar{c}u\bar{u}.^12$ Note here that, if $X_0^0(3872)$ is an $I = 1$ state, it is manifestly exotic and there should exist its isospin partners $X^\pm (c\bar{c}ud\bar{d}$ and $c\bar{c}d\bar{u})$ around 3.87 GeV. For the calculation of four-quark charmed systems, we adopt the quark-quark interaction of Ref. 13, which effectively includes the exchange effect of Nambu-Goldstone bosons. With $m_c \approx 1752$ MeV, $m_{u,d} \approx 313$ MeV and $m_s \approx 555$ MeV, the quark model leads light hadron masses as $m_\rho \approx 772.8$ MeV, $m_\omega \approx 696.3$ MeV and $m_\pi \approx 148.7$ MeV, and the calculated (experimental) masses of charmed mesons are $m_D \approx 1897.6(1867)$ MeV, $m_{D^*} \approx 2017.1(2008)$ MeV, $m_{J/\psi} \approx 3096.5(3097)$ MeV and $m_{\eta_c} \approx 2989.1(2980)$ MeV.

For the four-quark calculation, we employ all the 18 sets of the Jacobi coordinates of the four-body system.1) We show in Fig. 4 the three important sets describing the $c\bar{u}$ and $\bar{c}u$ correlations ($c = 1$), the $c\bar{c}$ and $u\bar{u}$ ones ($c = 2$) and the $cu$ and $\bar{c}\bar{u}$ ones ($c = 3$). The four-body Gaussian basis functions are prepared in the Jacobi sets $c = 1 - 18$ and the four-body pseudostates $\Phi^{(\nu)}_{J^\pi M}(E_\nu)$ are obtained by diagonalizing the four-quark Hamiltonian using nearly 13,000 Gaussian basis functions. For the phase shift calculation, we consider the scattering channels $D^0 + \bar{D}^{*0}$ and $J/\psi + \omega$ for $I = 0, J^\pi = 1^+$, and $D^0 + \bar{D}^{*0}$ and $J/\psi + \rho$ for $I = 1, J^\pi = 1^+$.

For the $I = 0$ states, we find a very sharp resonance, dominantly having the $D^0\bar{D}^{*0}$ component, slightly below the $D^0\bar{D}^{*0}$ threshold. In this calculation, however,
the $J/\psi \omega$ threshold energy is much lower than the experimental value by 86 MeV. We find that if the quark-quark interaction is artificially adjusted so as to reproduce the experimental value of the $J/\psi \omega$ threshold, neither bound nor resonance state appears. In fact, the result seems rather sensitive to the quark-quark interaction, and we need better interaction to obtain definite conclusions for $I = 0$ states.

For the $I = 1$ states, we obtain several pseudostates near the $D^0\bar{D}^{*0}$ threshold region, but all of them disappear when the scattering boundary condition is switched on. The resultant scattering phase shift by the full coupled-channel calculation is given in Fig. 3, which shows no resonance behavior. We thus find no $c\bar{c}q\bar{q}$-type tetraquark resonance with $I = 1$ in mass region of 3.87−4.0 GeV.

In this way, the almost precise quark-model calculation using the Gaussian Expansion Method (GEM) is a powerful tool to clarify the state properties of multi-quark systems, which is a theoretical search for exotic hadron resonances.

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