High speed quantum gates with cavity quantum electrodynamics

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(Dated: April 16, 2009)

Cavity quantum electrodynamic schemes for quantum gates are amongst the earliest quantum computing proposals. Despite continued progress and the recent demonstration of photon blockade, there are still issues with optimal coupling and gate operation involving high-quality cavities. Here we show that dynamic cavity control allows for scalable cavity-QED based quantum gates using the full cavity bandwidth. This technique allows an order of magnitude increase in operating speed, and two orders reduction in cavity Q, over passive systems. Our method exploits Stark shift based Q switching, and is ideally suited to solid-state integrated optical approaches to quantum computing.

PACS numbers: 03.67.Lx, 42.60.Da, 42.60.Gd, 32.80.Qk

I. INTRODUCTION

Quantum information technology enables new forms of communication and computation that are more efficient than existing classical approaches. Applications as diverse as secure communication [1], simulating quantum systems [2], and factoring [3] have already been identified, and experimental techniques essential to realize these and other applications are advancing steadily. Recent, notable results include those based on optical [4] and trapped-ion systems [5], and architectures have been proposed that incorporate quantum error correction and address various issues associated with scalability [6].

Amongst the variety of physical systems being explored, cavity quantum electrodynamics (QED) systems have shown great promise. A high quality (Q-factor), small volume (V) optical cavity represents an almost ideal environment for achieving coherent atomic manipulation at the single-quantum level with minimal dissipation. Such high-Q/V cavities allow for deterministic atom-photon coupling in a near or far-off resonant configuration that elicit linear or Kerr-like nonlinear atomic response. These processes are central to some of the pioneering proposals – single photon sources [2], quantum bus protocols [3], quantum computation and communication schemes [9] – and demonstrations of quadrature-phase gate [10], quantum memory [11] and photon blockade [12]. Cavity-assisted photonic networks for preparing 2D and 3D cluster states have also been discussed [13].

Despite these promising applications, the fundamental time-bandwidth relation of passive high-Q cavities limits operational speeds and ultimately practicality. Worse still, oscillations in the photon intensity due to poorly matched pulses are serious source of error. Here we show that the use of appropriate dynamic control breaks these limitations and reduces required cavity Q by two orders of magnitude compared with passive schemes, and at the same time realizes high-fidelity faster quantum gate.

The dynamic control approach used here modifies the cavity-waveguide coupling, from high Q during confinement to low Q during in/out-coupling, with a tuning time much shorter than the photon lifetime of the cavity [14]. This approach can also be exploited for active pulse shap-
ing. Although $Q$ switching is a standard practice for classical lasers, there are few schemes [15, 16] that work at the quantum level and are compatible with solid-state cavities such as photonic-band-gap (PBG) structures.

Within this context, we report the first theoretical study of a controlled phase (CZ) gate between photonic (control) and matter-based (target) qubits using active $Q$ switching. The CZ gate is a two-qubit entangling gate which induces a phase flip if the control and the target are in a specified state, i.e. for two arbitrary qubits, it performs the transformation $|11⟩ → −|11⟩$. Specifically, the state of an atom inside a cavity is controlled by the photonic occupation of the cavity via usual Jaynes-Cummings interaction in the strong coupling and dispersive regime. Using a two-cavity setup, we show that quantum interference allows high reflectivity and optimal photonic confinement of the interaction cavity. Through an adiabatic process controllable by the switch, ultra low and relatively fast transfer of excitations between the interaction cavity and waveguide is possible. To generate a scalable architecture for quantum computing, this form of CZ gate is integral to coupling between matter-based qubits where stationary-to-flying qubit interconversion is used. Our scheme is a powerful enabler in preparing cluster states through operator measurement across multiple photonic qubits [13] and for integrated optics at the quantum level. This scheme is discussed generically, but is pertinent for strong coupling systems, e.g. semiconductor quantum dots in photonic crystal cavities [17] and circuit-QED [18]. PBG structures in diamond are also developed to exploit the vast resource of optically active color centers for such applications [19].

II. IMPLEMENTING A CONTROLLED PHASE GATE

The proposed CZ gate is shown schematically in Fig. II(a). To realize the $Q$ switch, a two-level atom $q$ is coupled to the cavity mode $q$, detuned by $Δ_q$, with single photon Rabi frequency $Ω_q$. In the storage cavity, the target qubit (atom-$s$) is a three level system with ground $|g⟩_s$, metastable $|f⟩_s$ and excited $|e⟩_s$ states. $|g⟩_s ↔ |f⟩_s$ is the qubit transition that can be controlled via resonant RF coupling. To achieve a linear gate, we operate in the dispersive regime where the $|g⟩_s ↔ |e⟩_s$ transition couples to the cavity mode $s$ with Rabi frequency $Ω_s ≪ Δ_s$. This interaction is preferred over more complicated resonant schemes which suffer from increased sensitivity to timing noise. To illustrate CZ operation, we consider, without loss of generality, the starting states $|±⟩ ≡ (|f⟩_s ± |g⟩_s)/\sqrt{2}$, and the gate is described by the unitary operator $U ≈ \exp[-i(Ω_q t/Δ_s)|1⟩_s (1) ∗ |g⟩_{ss}(s)]$ where $|1⟩_s$ denotes a photon in mode $s$. After time $T_r ≈ πΔ_s/Ω_s^2$, a phase flip is induced on $|g⟩_s$ so that $|±⟩_s ↔ |∓⟩_s$.

Before continuing with the construction of the gate, we note that without switching, i.e. only the coupled atom-cavity system on the left of Fig. II(a), we need a cavity of low bandwidth and strong atom-cavity coupling to effect a phase-flip via scattering. To illustrate our points, we consider an implementation in the optical regime with nitrogen-vacancy (NV) diamond defect center in a PBG cavity fabricated in diamond. The center is placed at the maximum of the cavity mode with wavelength near the zero-phonon line resonance of the center $λ = 638$ nm and frequency $2.95$ PHz. This corresponds to the transition from the excited spin triplet state $|3⟩$ to the $m = 0$ sublevel of the triplet ground state $|3⟩$. Assuming that the cavity is sub-micrometer in dimension with $V ≃ λ^3$, then $Ω_s ≈ 10$ GHZ [13]. In this regime, the phonon sidebands will be suppressed [20], justifying the three-state approximation of the center where $|e⟩_α ≡ |3⟩, |g⟩_α ≡ |3⟩, 0⟩$ and $|f⟩_s ≡ |3⟩, ±1⟩$ for $α = s, q$. A $2.88$ GHZ RF field allows a complete control over its ground state transitions [21]. To effect a gate under these conditions implies a $0.1$ μs pulse and a technically-challenging static $Q$ of $10^8$ for a gate error rate of $10^{-3}$. When $Q = 10^6$ is used, the photon only succeeds in inducing $0.14π$-phase shift. In contrast, active switching can achieve a gate fidelity of $0.993$ with nanosecond photon pulse, nanosecond gate time and more modest $Q ≃ O(10^5 - 10^6)$. This fidelity is sufficient to implement the topological error correction scheme described in Ref. [27] using the cluster state preparation network introduced in Ref. [13]. Cavities in this range have been demonstrated in silicon [22]. Recently, cavity modes in diamond photonic crystal cavities near $638$ nm with $Q = 585$ has also been demonstrated [23] and there exist suitable diamond cavity designs [24] to achieve the required $Q/V$. To implement schemes which demand higher fidelity gates a higher $Q/V$ cavity will be required.

The coupled-cavity system, in the dipole and rotating-wave approximations, is governed by the Hamiltonian,

$$\mathcal{H} = Δ_s|e⟩_s⟨e| + Δ_q - iΔq|a⟩_q⟨a| + \left( Δ_q + Δ_s - iΔq^2 \right)|s⟩_s⟨s|,$$

$$× |e⟩_q⟨e| + \int_{-∞}^{∞} ω b^\dagger(ω)b(ω)dω + Ω_sσ^+_s a_s + κa^+_qa_q + \Omega_qσ^+_qa_q + \int_{-∞}^{∞} \sqrt{γ/2π} b^\dagger(ω)a_qdω + h.c.,$$

where $σ^+_s ≡ |e⟩_s⟨a|, a_α (a^+_α)$ is the annihilation (creation) operator of the cavity, $b(ω)$ is its counterpart for a photon of frequency $ω$ in the waveguide, and satisfy the Heisenberg equation of motion [26],

$$\dot{b}(ω) = -iωb(ω) + \sqrt{γ/2π} a_q,.$$

where $γ ≡ ω_c/(2Q)$ is the cavity decay rate, where $ω_c$ is the cavity resonant frequency. The resonant frequency of the cavities differs by $Δ_q$. In the tight-binding regime, the cavities are coupled with photon-hopping rate $κ$.

To treat decoherence, we introduce $γ_τ$, the spontaneous emission rate of atom $q$ and $γ_q$, the transverse cavity de-
cay rate. Decoherence of the atomic qubit becomes negli-
gible in the dispersive regime as the emission rate from
|e⟩_s scales with (Ω_q/Δ_q)^2. It is implicit in our treat-
ment that transverse decay rate of the left cavity is weak
compared to the required confinement time T_c.

In the one-quantum manifold, from ψ = −iHψ, it is
straightforward to obtain a set of differential equa-
tions for the probability amplitudes (C^e_q, D^e_q) for
states, namely ⟨ξ, 1⟩_s|g⟩_q|vac⟩ (C^e_q), |e⟩_s|g⟩_q|vac⟩
(D^e_q), ⟨ξ, 0⟩_s|g⟩_q|vac⟩ (C^e_q), ⟨ξ, 0⟩_s|e⟩_q|vac⟩ (D^e_q), and
⟨ξ, 0⟩_s|g⟩_q|Φ⟩ (C^out_q), where ξ identifies the dressed basis
of atom-s and the third ket indicates the photonic state
in the waveguide. In turn, the amplitude of the output
pulse f^ξ_{out} is related to the input (f^ξ_{in}) by the standard
input-output relation f^ξ_{out} = √N C^e−ξ − f^ξ_{in}[26].

These equations can be solved for the system dynamics
numerically. To optimally couple the photon to the cavity
and perform a gate requires three steps. These are:

Step A – Loading. Near-resonance coupling of atom q
with mode q is best expressed in the well-known dressed
basis |±⟩_q [26]. Since the eigenenergies vary with atomic
frequency, the system can be made transmissive (at reso-
nance) to an incoming pulse on the waveguide. Sweeping
the switch through this resonance by tuning Δ_p permits
mode-matched transfer of a left-travelling photon into
mode s. This occurs when we evolve the joint storage-
switch system along a particular energy eigenstate that
has a standard form for a dressed A-atom system [27],

Φ = 1/ N 1/ (κΩ_q|1⟩_s + Ω_q|1⟩_q| + |E(E − δ_q) − κ^2|e⟩_q) (3)

normalized with N, where E is its eigenenergy. In partic-
ular, |Φ⟩ ≈ |1⟩_s when the storage is weakly coupled to the
switch for some Δ_q = Δ_q^n and |Φ⟩ ≈ |1⟩_q when mode s
is resonant with |±⟩_q for Δ_q = Δ_q^n ≡ (−δ_q^2 + Ω_q^2)/δ_q.
Thus as we spectrally tune atom q, we effect coherent
transfer from the waveguide to mode s via the switch.
The evolution requires adiabaticity parameterized by,

A = |⟨Φ'|H|Φ⟩|/|⟨Φ'|H|Φ⟩ − ⟨Φ|H|Φ⟩|^2 ≪ 1. (4)

where |Φ⟩ is the eigenstate closest to |Φ⟩ in energy. The
position of storage-switch resonance is set up with
detuning δ_q. The use of large |δ_q| lengthens the switching
time T_sw as the matrix element for photon hopping
|±⟩|a⟩_s weakens, whereas small |δ_q| implies that atom q
must be tuned over an increasing range as off-
and on-resonance points become farther apart [Fig. 1(c)].

Step B – Gate. Once the photon is inside the cavity, we
deouple mode s from the waveguide for a duration of T_g
to enact |+⟩_s − |−⟩_s. High gate fidelity F ≡ |C^out_q|^2 is
achievable, as we show, when T_sw ≪ T_g. However, one
can improve the gate time and fidelity with an additional
control to perform an adiabatic modulation of dispersive
interaction, where the rate of phase shift is controlled
via tuning of atom-s. The idea is to use a suitable Δ_g to
induce rapid phase flip over (shorter) T_g, and a large Δ_s
during switching improves the fidelity by avoiding over-
rotation. We now further develop the underlying phys-
es for light confinement and identify the ideal value for Δ_q^n.

The coupled-cavity solution of our setup is a spatial
analogue of electromagnetically-induced transparency in
a A-system [28]. At exact two-photon resonance between
mode s and atom q for Δ_q = Δ_q^n ≡ −δ_q, |Φ⟩ is maxi-
mal decoupled from |1⟩_q, and a favoured population in
|1⟩_s when Ω_q ≫ κ. In this configuration, the storage-
waveguide coupling is minimized, offering both optimal
photon confinement and reflectivity to any incoming
light [Fig. 1(c)]. Although some finite overlap with |e⟩_q
leads to leakage, this probability scales with (κ/Ω_q)^2.

Step C – Unloading. The photon is removed from the
cavity using a time-reversed Stark tuning. Being the
time-reversal of Step A, this final step restores the ini-
tial photon and leaves the atom in its desired state.

III. RESULTS AND DISCUSSION

With above formalism, we simulate the operation of the
CZ gate with realistic parameters in Fig. 2. An input
single-photon pulse is switched into the storage using
a linear shift Δ_q over a time scale of 10^−1. In this case
(A ≈ 10^−3), the success probability for adiabatic transfer
P_out = 1 − O(10^−4). While the optimal input amplitude
has been constructed as the complex conjugate time-
reversal of a switched photon, it is a Gaussian pulse eas-
ily prepared with a conventional coherent source. Such

FIG. 2: (color online) (a) Simulation of the CZ gate. The
matter qubit (atom-s) is initialized in +⟩. Step A: A single
photon of shape |f_{in}^ξ|^2 (red dotted) is switched into the storage
cavity adiabatically (|C^out_q|^2, red dashed). Step B: Disper-
sive interaction results a phase shift from +⟩ to −⟩; (black dashed). Step C: The photon is returned to the waveguide.
The gate fidelity F ≡ |C^out_q|^2 = 0.993 (black solid). |f_{out}^ξ|^2 (black dotted) is the pulse envelope of the output photon.
(b) Corresponding atomic detuning Δ_p(t) in unit of κ, that varies between Δ_q^n where the switch has minimal transmit-
tivity and Δ_q^n where |1⟩_s is resonant with |−⟩_q. Parameters are κ = γ, Ω_q = 5κ, Ω_q = 20κ, δ_q = 10κ, Δ_q = 10^3κ, and γ_q, γ_e = 0.
pulses are standard for cavity-QED gate systems, being ideal for fiber transmission and Hong-Ou-Mandel interferometry \[20\]. When the waveguide is decoupled, the leakage error is \(10^{-5}\) for the choice \(\Omega_\text{q} = 20\kappa\). The likelihood of single-photon absorption by atom-s must be \(\eta \ll 1\). Here since \(\Delta_s = \Omega_\text{q}/\sqrt{\gamma}\) for \(\eta = O(10^{-5})\), it is an unlikely source of error. Immediately after the photon is removed, the qubit is phase-flipped with \(F = 0.993\). Most importantly, we have demonstrated that the storage time is much longer than both the photon lifetime of the stand-alone device and the time-bandwidth of the travelling photon.

Proceeding with further simulations, we first ignore the decoherence and study the effect of the cavity decay rate \(\gamma\) on success probability \(P_{\text{out}}\) for light transfer from mode \(s\) to waveguide. Fig. 3(a) shows that this probability peaks at \(\gamma/\kappa = O(1)\) for a fixed switching time. This is contrary to the expectation that a large \(\gamma/\kappa \gg 1\) should suggest the light field would be dumped immediately from mode \(q\) after the adiabatic transfer.

Thus far, leakage before switching is an inherent error due to state mismatch between \(\Phi\) and \(|1\rangle_s\). In Fig. 3(b), we show \(F\) versus \(\Omega_\text{q}/\kappa\) where the error rates due to imperfect adiabatic transfer and absorptions are explicitly excluded. It proves that the overlap \(q\langle e|\Phi\rangle\) leads to this error, and thus can be suppressed by increasing \(\Omega_\text{q}\) with higher \(Q/V\) cavities. For instance, \(F \sim 0.99999\) is feasible when \(\Omega_\text{q}/\kappa = 100, \Delta_s = 10^4\kappa\) and decoherence is suppressed accordingly. At last, we take decoherence at the switch into consideration. In Fig. 3(c), we expect that the fidelity is fundamentally limited by premature leakage, and degrades as the decoherence rate \(\sim 10^{-2}\kappa\).

When realized with NV centers \((\gamma_e = 10\ \text{MHz})\) in PBG cavities of \(Q = 10^6\), this gate operates with \(\sim 20\ \text{ns}\) pulse and a gate time of 200 ns for \(F \approx 0.989\). However, an extra control \(\Delta_s(t)\) realizes \(F = 0.996\) and a gate time \(\sim 2T_{\text{sw}} = 40\ \text{ns}\). Strong tuning of isolated centers via an external control field has been demonstrated \[21\]. The tuning range from the early work of Redman et al. of order 1 THz \[31\] is more than enough for the proposed scheme. In the microwave regime, when superconducting qubits \((\gamma_e = 1\ \text{MHz}, \omega_c \sim 1\ \text{GHz})\) are coupled to stripline cavities with rate 0.1 GHz \[18\], \(Q \sim 10^2\) suffices for a similar fidelity with 2 \(\mu\text{s}\) pulse and \(\sim 4\ \mu\text{s}\) gate time.

IV. CONCLUSION

Advances in fabrication have led to the development of ultra-small, low loss, solid-state cavities, with obvious potential for quantum information applications. However passive devices suffer from the unavoidable time-bandwidth relation that severely limits their performance. Active \(Q\) switching breaks this nexus and allows the full bandwidth of the cavities be used. We have shown that dynamic Stark-shifting with coupled cavities permits high speed single-photon \(Q\) switching, realizing an order of magnitude faster two-qubit (CZ) gate with less stringent \(Q\) requirement. This is a significant step in improving the prospects for solid-state cavity-QED based quantum logic, and motivates the further experimental effort in coupled-cavity QED.

ACKNOWLEDGMENTS

We thank A. M. Stephens, Z. W. E. Evans, C. D. Hill and S. J. Devitt for valuable discussions. WJM and KN acknowledge the support of QAP, MEXT, NICT and HP. CHS, ADG and LCLH acknowledge the support of Quantum Communications Victoria, funded by the Victorian Science, Technology and Innovation (STI) initiative, the Australian Research Council (ARC), and the International Science Linkages program. ADG and LCLH acknowledge the ARC for financial support (Projects No. DP0880466 and No. DP0770715, respectively).

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