Multisources Risk Management in a Supply Chain under Option Contracts

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1. Introduction

Supply chain risk management has been a burgeoning area of research in the last two decades [1–3]. Uncertainty in supply chain can occur in various forms, but the most prominent ones are demand and supply uncertainties [4–6]. Particularly, uncertainties in product demand, component yield, and spot price are key to many industrial settings and they are usually explicitly incorporated [7]. For managers, the major challenge is what mechanisms can be adopted to hedge the risks and minimize losses or maximize profits. A spectrum of flexibility contracts have been demonstrated as incentive mechanisms for coordinating the agents’ decisions and improving the profit of all supply chain parties [8, 9]. One of the most popular contracts is the option contract which is derived from the real option in financial field [7, 10, 11].

Luo and Chen [12] investigated the value of option contracts in a supplier-manufacturer system. The manufacturer orders key components from the supplier who is subject to random yield and processes/assembles the components to meet a deterministic market demand. Apart from the contract market, they assumed that there is a spot market in which both the manufacturer and the supplier can buy (replenish) or sell the components at a random spot price. They derived the optimal ordering policy of the manufacturer and the optimal production policy of the supplier and compared these policies under the contexts of with and without option contracts. They pointed out that option contracts can coordinate the decisions of the manufacturer and the supplier, and the optimal system performance can be achieved under coordinated contracts. However, they limited the problem setting with deterministic demand. Intuitively, there comes a question whether the conclusion still holds if the demand is random? Taking this question, we in this paper revisit similar problem as Luo and Chen [12] and consider that the manufacturer faces a stochastic market for the final products. The main purpose of this paper is to examine the value of option contracts in hedging the yield, demand, and spot price risks in the supply chain. Our major contributions are as follows.

(1) We expand the models developed by Luo and Chen [12] to the case of stochastic demand. The optimal
ordering and production policies are obtained in closed-form expression and the impact of options on the decisions and performances of the supply chain under different contract parameters, the variability of yield, demand, and spot price is analysed.

(2) We demonstrate that, under random yield, demand, and spot price, both the manufacturer and the supplier can benefit from introducing option contracts, but the supply chain cannot be coordinated by a single option contract. It can be concluded that option contracts can coordinate the channel in the study of Luo and Chen [12], which however does not hold in our setting.

(3) To coordinate such supply chain, we propose a protocol to combine with the option contracts. Further, the explicit condition for coordination under the proposed contracts is identified. Our paper casts light on designing coordination mechanism to improve the profit of all parties in the context of random yield, demand, and spot price.

2. Literature Review

In literature, there is a rich body of research investigating supply chain management with option contracts. For example, Xu [13] examined the flexibility of options contracts in a multiperiod procurement policy under capacity constraints. Assuming that a buyer has limited capital and can obtain credit support from a financial institution, Feng et al. [14] studied the impact of option contracts on the buyer’s optimal ordering strategy. Considering customer returns, Wang et al. [15] examined the effect of option contracts on a firm’s pricing and order decisions. These papers focus on the flexibility of options contracts from the buyer-perspective. With option contracts and external financing, Hu et al. [16] investigated the portfolio procurement policies under the case that the budget is constrained. Considering the customer returns, Wang et al. [17] studied the impact of bidirectional option contracts and refund prices on order decisions. As to the buyer-seller perspective, Cachon and Lariviere [18] explored the role of option contracts in the coordination of a supplier-manufacturer system where the demand forecast is allowed to be shared between the two firms. Barnes-Schuster et al. [19] built demand-based option game models to study the ordering policy of manufacturers, the production policy of suppliers, and the strategy for supply chain coordination. Wang and Liu [20] studied risk sharing and channel coordination in a retailer-led system. In a manufacturer-retailer system, Zhao et al. [21] used cooperative game to investigate the coordination problem with option contracts. In an attempt to reduce the demand risk and improve the quality of customer service, Chen and Shen [22] pioneered the research framework of integrating option game and service level constraint. The ordering, production, and coordination polices of the supply chain with service level constraint are addressed. Chen et al. [23] considered coordinating a supply chain with a risk-neutral supplier and a risk-averse retailer with options. Nosoohi and Nookabadi [24] studied how the manufacturer uses option contracts to coordinate its customer-oriented production system. Under price-dependent demand, Hu et al. [25] studied the supply chain coordination with joint pricing. These studies assessed the effectiveness of option contracts from the buyer-seller perspective but only considered the context of random demands.

Taking spot market into account, under random demand and spot price, Wu and Kleindorfer [26] developed a framework to analyse the equilibrium structure of the buyer’s optimal portfolios procurement and the sellers’ pricing for reservation and execution. Martinez-de-Albéniz and Simchi-Levi [27] characterized a manufacturer’s multiperiod procurement strategy. They revealed that this policy has a simple structure. Fu et al. [10] studied the procurement problem of a buyer who orders from several suppliers through wholesale contracts or/and option contracts under the context that the spot price and demand are independent or correlated. They also studied how different demands affect the optimal strategies. Lee et al. [28] used dynamic programming approach to build an options and spot portfolio procurement model for the buyer with capacity constraints and firm order cost constraints. Fu et al. [5] investigated the optimal inventory replenishment policy and the dynamic pricing policy of a firm in a periodic-review system with price-dependent demands. Boyabatlı [29] investigated the procurement management of a primary input which generates two outputs in a fixed proportion. By comparative statics, they find that lower demand variability, higher input spot price variability, and higher demand correlation may benefit the processor. From supply chain perspective, VafaArani et al. [30] pointed out that a retailer-manufacturer system can be coordinated by a European call option with a revenue-sharing mechanism. With option contract and spot market, Wan and Chen [31] investigated the multiperiod replenishment problem. It is worth noting that all the researches mentioned above are under the assumptions of deterministic yield.

With random yield and demand, Xu [32] studied the manufacturer’s procurement policy as well as the supplier’s production policy under the assumption that the manufacturer can place option orders and place instant orders for the components from the unreliable supplier. In a one manufacturer and one retailer system, under the assumption that the excess demand is partially backordered, Hu et al. [33] further investigated the procurement policy as well as the production policy under option contracts. Kaki et al. [34] developed a scenario-based framework to study optimal procurement strategy of a manufacturer who orders goods from an unreliable supplier with wholesale price contracts and an additional reliable supplier with options contracts. Cai et al. [35] showed that a vendor-managed inventory (VMI) supply chain can be coordinated by an option contract with subsidy contract. However, they only consider the contract market but exclude the spot market. The most related study to this paper was made by Luo and Chen [12], but they considered that the market demand is deterministic.

In all, the previous studies on the supply chains management with option contracts consider only one or two risks posed by the random yield, random demand, or random spot price but none of them take the three uncertainties into account at a time. In this paper, we will fill this gap by integrating random yield and demand with random spot
Table 1: Summary of related and latest literature.

| Author                                      | Risks | Decision-making perspective |
|---------------------------------------------|-------|-----------------------------|
| Feng et al., 2014; Wang et al., 2017; Hu et al., 2018; Wang et al., 2019 | ✓✓   | ✓                           |
| Zhao et al., 2010; Chen and Shen, 2012; Chen et al., 2014; Nosoohi and Nookabadi, 2014; Hu et al., 2018 | ✓      | ✓ ✓ ✓                      |
| Lee et al., 2013; Fu et al., 2012; Boyabatlı, 2015; Vafa Arani et al., 2016; Wan et al., 2018 | ✓ ✓ ✓ | ✓ ✓ ✓                      |
| Xu, 2010; Hu et al., 2014                   | ✓✓   | ✓                           |
| Kaki et al., 2017                          | ✓✓   | ✓                           |
| Cai et al., 2017                           | ✓✓   | ✓                           |
| Luo and Chen, 2017                         | ✓✓   | ✓                           |
| Current study                              | ✓✓   | ✓                           |

price into a supply chain model under option contracts. We focus on the optimal ordering and production decisions from buyer-seller perspective as well as the supply chain coordination. Therefore, our study casts light on supply chain management with revenue sharing contracts in the context of stochastic demand and random yield. To better elaborate the position of our research, we refer to Table 1 to summarize the latest related literature.

The next section presents model formulation and assumptions. Sections 4 and 5 develop models for the manufacturer’s problem and the supplier’s problem under option contracts, respectively. Section 6 investigates the impacts of option contracts and uncertainties on the decisions and performances of the supply chain. Section 7 addresses the supply chain coordination. Section 8 summarizes our findings and suggests future research directions.

3. Model Formulation and Assumptions

Consider one component-supplier and one end-product manufacturer. The supplier suffers from random yield risk in the production process. The manufacturer may obtain components from the supplier through firm orders or option orders and then assembles the components into the end-product to meet the random market demand. Apart from the contract market, we assume that there is a spot market in which both the manufacturer and the supplier can buy (replenish) or sell the components. Specifically, the manufacturer can purchase the components from the spot market to cover its residual demand and sell its surplus components from over-firm-order on it as well. For the supplier, it can not only sell the remaining components on the spot market but also buy the shortage from it to fulfill the contracts when the realized yield is low. The major notations are presented in Table 2.

The supply chain scenario in this paper is depicted in Figure 1. The timing of the events consists of two stages. At the first stage, the manufacturer places a firm order and option order when the demand and spot price are not known. Upon knowing the manufacturer’s order, the supplier decides how many units are to be put into production. Then, production takes place and the output is revealed. At the second stage, the demand and the spot price are observed; the supplier delivers components to the manufacturer according to the contracts and purchases the shortage or deals with the remainder part on the spot market if necessary. Then the manufacturer replenishes its unmet demand or sells the residual components on the spot market.

Moreover, some assumptions are made to develop the proposed model as follows.

(i) The capacity of the spot market is not limited; the two firms can always buy the shortage on it, but owing to the “imperfect access risk” [26] when they need to deal with the surplus components, they may not be able to find an appropriate customer on the spot market. Wu and Kleindorfer [26] used $\alpha$ ($0 \leq \alpha < 1$) to capture the risk. $\alpha$ can be taken as the percentage of the surplus part that can be sold on the day. In this paper, we will follow the suit.

(ii) The manufacturer first exercises its options and then buys the components on the spot market. The actual exercising price for each option is $\min(e, p_s)$; that is, when $p_s \geq e$, the options are exercised at the price $e$; when $p_s < e$, the options are exercised at the spot price $p_s$.

(iii) To ensure that the manufacturer can earn profit and place firm as well as option orders, we assume that $r > \frac{\bar{p}_s}{p_s} > o + e > w > e$; to ensure that the supplier can earn profit and to eliminate the case that the supplier
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Table 2: Notation.

| Symbol | Description |
|--------|-------------|
| $T$    | Supplier’s random yield rate, which is characterized by $\Phi(t)$ (CDF) and $\varphi(t)$ (PDF). $T \in [a, b]$ ($0 \leq a < b \leq 1$), $E(T) = \mu$. |
| $D$    | End-product’s market demand, which is a random variable with $F(x)$ (CDF) and $f(x)$ (PDF). $D \in (0, \infty)$, $E(x) = \delta$. |
| $w$    | Wholesale price ($) |
| $\sigma$ | Option price ($) |
| $e$    | Exercise price ($) |
| $Q$    | Supplier’s production quantity |
| $q^0$  | Manufacturer’s firm order quantity |
| $q^1$  | Manufacturer’s option order quantity |
| $P_s$  | Spot market price ($), which is a random variable with $V(p_s)$ (PDF) and $V(p_s)$ (CDF). $P_s \in [A, B]$, $E(p_s) = \bar{p}_s$. |
| $c$    | Production cost for the component ($) |
| $r$    | Retail price for the end-product ($) |

![Figure 1: Supply chain scenario.](image)

(iv) Produces unlimited, we assume that $o + e > w > c/\mu > \alpha \bar{p}_s$.

4. Manufacturer’s Problem

With option contracts, the manufacturer’s problem is to determine the firm order $q^0$ and option order $q^1$ to maximize its expected profit $\Pi^m(q^0, q^1)$. The expected profit of the manufacturer is

$$\Pi^m(q^0, q^1) = rED - wq^0 - oq^1 - E \min(e, p_s) \min[(D - q^0)^+, q^1]$$

$$- p_s E(D - q^0 - q^1)^+ + \alpha p_s E(q^0 - D)^+. \tag{1}$$

The first term of (1) is the expected revenue from selling final goods. The next four terms are the costs incurred by firm orders, purchase and exercise of options, and the spot purchase for the components, respectively. The last term is the expected revenue from dealing with the residual components on the spot market.

For convenience, let $\nabla(e) = \int_e^b(p_s)dp_s$, $p_{se} = \int_e^b p_s v(p_s) dp_s$, and $\bar{p}_{se} = \int_e^b p_s v(p_s) dp_s$. Then, we can rewrite the above equation as follows:

$$\Pi^m(q^0, q^1) = (r - \bar{p}_s)\delta + (\bar{p}_s - w)q^0$$

$$- (e \nabla(e) + p_{se} - \alpha \bar{p}_s) \int_0^{q^0} F(x) \, dx + (\bar{p}_{se} - e \nabla(e) - o)q^1$$

$$- (\bar{p}_{se} - e \nabla(e)) \int_0^{q^0+q^1} F(x) \, dx. \tag{2}$$

Hence, the optimization problem of the manufacturer can be characterized as the following proposition.

Proposition 1. $q^{0*} = F^{-1}((e \nabla(e) + p_{se} - w + o)/(e \nabla(e) + p_{se} - \alpha \bar{p}_s))$ and $q^{1*} = F^{-1}((\bar{p}_{se} - e \nabla(e) - o)/(\bar{p}_{se} - e \nabla(e))) - F^{-1}((e \nabla(e) + p_{se} - w + o)/(e \nabla(e) + p_{se} - \alpha \bar{p}_s))$.

Proof. Equation (2) shows that

$$\frac{\partial \Pi^m(q^0, q^1)}{\partial q^0} = (\bar{p}_s - w)$$

$$- (e \nabla(e) + p_{se} - \alpha \bar{p}_s) F(q^0 + q^1)$$

$$- (e \nabla(e) + p_{se} - \alpha \bar{p}_s) F(q^0),$$

$$\frac{\partial \Pi^m(q^0, q^1)}{\partial q^1} = (\bar{p}_{se} - e \nabla(e) - o)$$

$$- (\bar{p}_{se} - e \nabla(e)) F(q^0 + q^1).$$
Proof of Corollary 2. Let \( q^* \) be the manufacturer's overall optimal order quantity; that is, \( q^* = q^{0*} + q^{1*} \). Then, we have Corollary 2.

**Corollary 2.** \( q^{0*} \) is decreasing in \( w \) and increasing in \((o, e)\); \( q^{1*} \) is decreasing in \((o, e)\) but is independent of \( w \).

**Proof of Corollary 2.** From Proposition 1, taking the first derivatives w.r.t \( o, e, \) and \( w \), respectively, we obtain

\[
\frac{dq^{1*}}{do} = -\frac{1}{f(q^{0*} + q^{1*}) (\overline{p}_e - eV(e))} \left( eV(e) + p_e - \alpha \overline{p}_s \right) < 0,
\]

and

\[
\frac{dq^{1*}}{dw} = \frac{1}{(eV(e) + p_e - \alpha \overline{p}_s)} > 0.
\]

So \( q^{1*} \) is decreasing in \( o \) and \( e \) and increasing in \( w \). From \( q^* = F^{-1}((\overline{p}_e - eV(e) - o)/(\overline{p}_e - eV(e))) \), we have \( dq^*/do = -(1/\overline{p}_e - eV(e))) \cdot (1/f[F^{-1}(B)]) < 0 \), \( dq^*/dw = -(o(V(e) + p_e) - eV(e))/(\overline{p}_e - eV(e))^2 \cdot (1/f[F^{-1}(B)]) < 0 \). So \( q^* \) is decreasing in \((o, e)\) but has nothing to do with \( w \).

This corollary suggests that if the wholesale price is high, the manufacturer will set a low firm order and a high option order, but the overall order quantity is fixed; if the option price or the exercise price are high, the manufacturer will place a high firm order, a low option order, and a low total order. Clearly, a large option order will provide more flexibility to respond to the uncertainty in demand and spot market.

**5. Supplier's Problem**

In this section, we focus on the supplier's production decision. Having received the manufacturer's orders \((q^{0*}, q^{1*})\), the supplier's concern is to decide its production quantity \( Q \) to maximize its expected profit \( \Pi^*(Q) \). Then

\[
\Pi^*(Q) = wq^{0*} + q^{1*} + E \min(e, p_s) \left( (D - q^{0*}), q^{1*} \right)
\]

\[
+ \alpha E \left( TQ - q^{0*} - \min \left( (D - q^{0*}), q^{1*} \right) \right)
\]

\[
- cQ - E \left( q^{0*} + \min \left( (D - q^{0*}), q^{1*} \right) - TQ \right)
\]

The first four terms of (6) are the expected revenue. The last two terms are the costs.

Further, (6) can be rewritten as follows:

\[
\Pi^*(Q) = (1 - \alpha) \overline{p}_e \int_0^{q^{0*} + q^{1*}} F(x) dx + \left( TQ - q^{0*} - q^{1*} \right) \varphi(t) dt - (1 - \alpha)
\]

\[
\cdot \overline{p}_s \int_{q^{0*} + q^{1*}}^{q^{0*} + q^{1*}} F(x) dx
\]
Therefore, we have the following proposition.

**Proposition 3.** $Q^*$ is given by

$$\int_{q_0}^{q^*} F(x) dx + (w - \alpha \bar{P}_s) q_0^* + o \sqrt{v}(e)
+ \bar{P}_{se} + o - \alpha \bar{P}_s) q_1^* - (c - \alpha \bar{P}_s, \mu) Q.$$  \hspace{1cm} (7)

Proof. Equation (7) shows that

$$\frac{d\Pi'(Q)}{dQ} = (1 - \alpha) \bar{P}_s \int_0^{q_0/Q} F(tQ) \theta(t) dt + (1
- \alpha) \bar{P}_s \int_0^{q_0/Q} F(tQ) \theta(t) dt - (c - \alpha \bar{P}_s, \mu),$$

$$\frac{d^2\Pi'(Q)}{dQ^2} = (1 - \alpha) \bar{P}_s \int_0^{q_0/Q} f(tQ) t^2 \theta(t) dt$$

$$+ (1 - \alpha) \bar{P}_s \int_0^{q_0/Q} f(tQ) t^2 \theta(t) dt
- (1 - \alpha) \bar{P}_s \left[ \frac{(q_0 + q_1)^2}{Q^3} \phi \left( \frac{q_0 + q_1}{Q} \right) F(q_0 + q_1) \right] + \frac{q_0^2}{Q^3} \phi \left( \frac{q_0}{Q} \right) F(q_0) < 0.$$  \hspace{1cm} (8)

It follows from formulas above that $\Pi'(Q)$ is strictly concave in $Q$, and a unique components production quantity $Q^*$ which maximizes $\Pi'(Q)$ exists. By the first-order optimality condition, we can get Proposition 3.

This proposition indicates that the supplier's optimal production quantity $Q^*$ exists, and it is determined by the manufacturer's optimal order quantity $(q_0^*, q_1^*)$, the distributions of yield rate $\theta(t)$ and market demand $F(x)$, the production cost $c$, the mean of spot price $\bar{P}_s$ and the risk factor $\alpha$. In addition, from Proposition 3, it can also be found that $dQ^*/d\bar{P}_s > 0$, $dQ^*/da > 0$, and $dQ^*/dc < 0$, which means that the supplier's optimal production is increasing in the mean spot price $\bar{P}_s$ and the risk factor $\alpha$, while it is decreasing in the production cost $c$. That is, the higher the mean spot price and the value of risk factor or the lower the production cost, the more the production level the supplier will set.

6. **Effect of Options**

Propositions 1 and 3 suggest that under the circumstance of random yield, demand, and spot price there is a unique equilibrium between the manufacturer's order quantity and the supplier's production quantity in the context of with option contracts. In this section, based on the research findings, we resort to numerical experiments to investigate the effect of options on manufacturer's optimal ordering policies and the supplier's optimal production policies as well as all parties' expected profits by comparing the supply chain with option contracts and that without.

Let $q^1$ be 0 in (1) and (6); it can get that the optimal order quantity (denoted by $q_0^*$) and the optimal production quantity (denoted by $Q_0^*$) in the context of without option order are determined by $q_0^* = F^{-1}((\tilde{P}_s - w)/(1 - \alpha)\bar{P}_s)$ and $\int_0^{q_0^*} \theta(t) dt = (c - \alpha \bar{P}_s, \mu)/(1 - \alpha)\bar{P}_s$, respectively. In our experimental setting, the combination of yield rate, demand, and spot price distributions are assumed as [Uniform, Normal, and Uniform]. Throughout this numerical study, we consider a base example with $T \sim U(0.2, 1), D \sim N(100, 30)$, and $P_s \sim U(4, 9), c = 2.58, w = 68, \alpha = 0.68, e = 5.78$, $r = 108$, and $\alpha = 0.3$.

First, we compare the optimal decisions and expected profits under different option prices and exercise prices. For the case of without option contracts, we can get the optimal order quantity $q_0^* = 63$ and the production quantity $Q_0^*$, respectively; the expected profits of the supplier and the manufacturer are $\pi'(Q_0^*) = 71.5$ and $\pi''(q_0^*) = 388.5$, respectively.

As shown in Table 3, the firm order quantity of the manufacturer decreases as the option price decreases and the exercise price increases, while the optimal option order, total order, and production quantity and the expected profits of all parties increase. The reason why the manufacturer's profit increases lies in the fact that the lower option price entices the manufacturer to place relatively more options and less firm order, which makes it more flexible to the uncertain market demand. For the supplier, the increase of the total order quantity leads to the growth of production, which dampens out the double marginalization effect and improves the supply chain's overall performance, posing a positive effect on the supplier's profit.

Observing the Tables 1 to 4, we summarize some of the key properties in the following two remarks.

**Remark 4.** Under random yield, demand, and spot price, with option contracts, the optimal firm order quantity of the manufacturer is lower than that of without, but the overall order quantity and the optimal production quantity of the supplier are higher than that of without.

Remark 4 shows that the option contracts can enable the manufacturer to reduce the firm order while increasing its overall order, thereby promoting the production level of the supplier. This occurs because if the manufacturer can purchase options, it will lower the firm order to mitigate overage risk while exercising the options to mitigate underage risk. Thanks to the flexibility of option contracts, the manufacturer is willing to order more (overall quantity), while the supplier will set higher production quantity to mitigate the high spot price risk and ensure sufficient supply.
Table 3: Optimal decisions and expected profits under different option costs.

| (o, e)   | q^o* | q^i* | q^* | Q^* | π^m* | π^s* |
|----------|------|------|-----|-----|------|------|
| (0.9, 5.4) | 46   | 37   | 83  | 115 | 389.1| 79.3 |
| (0.8, 5.5) | 41   | 46   | 87  | 117 | 389.9| 81.1 |
| (0.7, 5.6) | 34   | 57   | 91  | 119 | 391.0| 85.2 |
| (0.6, 5.7) | 19   | 77   | 96  | 122 | 392.2| 88.4 |

Table 4: Comparison of optimal decisions under the variability of yields, demands, and spot prices.

|         | With option | Without option |
|---------|-------------|----------------|
|         | q^o* | q^i* | q^* | Q^* | q^o* | Q^* |
| T~ U    |      |      |     |     |      |     |
| (0.35, 0.85) | 19   | 77   | 96  | 137 | 63   | 98  |
| (0.3, 0.9) | 19   | 77   | 96  | 132 | 63   | 94  |
| (0.25, 0.95) | 19   | 77   | 96  | 127 | 63   | 91  |
| (0.2, 1.0) | 19   | 77   | 96  | 122 | 63   | 88  |
| D~ N    |      |      |     |     |      |     |
| (100,10) | 73   | 26   | 99  | 134 | 87   | 122 |
| (100,20) | 45   | 52   | 97  | 128 | 75   | 105 |
| (100,30) | 19   | 77   | 96  | 122 | 63   | 88  |
| (100,40) | 6    | 89   | 95  | 118 | 52   | 73  |
| P_s ~ U |      |      |     |     |      |     |
| (4.6,8.4) | 47   | 43   | 90  | 117 | 63   | 88  |
| (4.4,8.6) | 43   | 49   | 92  | 119 | 63   | 88  |
| (4.2,8.8) | 35   | 59   | 94  | 120 | 63   | 88  |
| (4.0,9.0) | 19   | 77   | 96  | 122 | 63   | 88  |

Remark 5. Under random yield, demand, and spot price, the optimal expected profit of the manufacturer and the supplier and option contracts are larger than without.

This remark implies that with option contracts the manufacturer and the supplier can effectively manage their multisource risks (random demand and spot price for the manufacturer, random yield, demand, and spot price for the supplier) and earn more expected profits than without. That is, both firms can benefit from option contracts in the case of random yield, demand, and spot price.

Next, we compare the optimal decisions and expected profits under variability of yields, demands, and spot prices. We fix the mean demand δ = 100; let the standard deviation be 10, 20, 30, and 40, respectively; fix the mean yield rate μ = 0.5; let the standard deviation be the values of 0.25, 0.3, 0.35, and 0.4; fix the mean spot price P_s = 6.5$; let the standard deviation be 1.9, 2.1, 2.3, and 2.5. The results are summarized in Table 4 and Figures 2–4. Note that in these Figures the solid line and the dotted line stand for the cases of with and without options, respectively.

Table 4 and Figure 2 show that the manufacturer’s optimal order and expected profit will not change with the fluctuation of the variance of yield rate, whereas both the optimal production and the expected profit of the supplier decrease as the yield variability increases under both cases (with or without options). Although the supplier’s expected profit with option contracts is also larger than that without, the increment decreases as the yield variability increases. The reason lies in the fact that under forced contracts compliance the firm order and the option order can be fully satisfied whatever the supplier’s actual output is, so the manufacturer’s decision and profit are not affected by the yield uncertainty. For the supplier, high yield variability means high underproduction or overproduction risks, so it will reduce the production level and turn to the spot market to fulfill the contract. In a word, the yield randomness has no influence on the manufacturer under forced contract compliance, but it is bad for the supplier.

From Table 4 and Figure 3, it can be seen that as the demand variability increases the manufacturer’s optimal firm and total order quantity as well as the supplier’s optimal production quantity decrease under the two cases, while the optimal option order increases. It is consistent with the intuition that the higher the demand variability, the lower the firm order and the higher the option order the manufacture will place. However, since components can be obtained from the spot market, the increment in option order will be less than the cut in firm order as the demand variability increases, which eventually leads to the decrease in the total order and the production quantity of the supplier. Moreover, we find that the two firms’ expected profits with options contracts are also larger than that of without, and the profit increment increases as the demand variability increases. From the discussion above it can be concluded that option contracts are more valuable when the demands are more volatile.

Table 4 and Figure 4 show that without option contracts the decision and the expected profit of both firms will not change with the fluctuation of the variance of spot price; in contrast, when option contracts are adopted, as the yield
Figure 2: Comparison of expected profits under yield variability.

Figure 3: Comparison of expected profits under demand variability.

Figure 4: Comparison of expected profits under spot price variability.
variability increases, the option order, the total order, and the production increase while the firm order decreases. It is consistent with the intuition that high variation of the spot price will urge the manufacturer to purchase more contracts while reducing spot market purchase. Moreover, from Figure 4, we find that as the spot price variability increases the expected profit of the manufacturer increases and the expected profit of the supplier first decreases and then increases at a slow pace. Importantly, the expected profit of the manufacturer with option contracts is no longer larger than that without when the spot price variability is small. It reveals that the option contracts are only valuable for the manufacturer when the spot price variability is high.

In a word, we can draw the following conclusion from Figures 2–4.

Remark 6. The profit increment derived from option contracts is greater for the manufacturer and the supplier when the demand and spot price are more volatile but is greater when the yield is less volatile for the supplier.

This remark depicts the effects of yield, demand, and spot price uncertainty on the value of the option contracts for the manufacturer and the supplier. It suggests that, for the manufacturer, the more the demand volatility or the spot price volatility, the greater the value of the option contracts. For the supplier, the more the yield volatility, the lower the value of the option contracts, the more the demand volatility or the spot price volatility, and the greater the value of the option contracts.

Remarks 5 and 6 show that by adopting option contracts the manufacturer and the supplier can effectively deal with their multisource risks. More importantly, the bigger the demand risk or the spot price risk, the greater the value of option contracts for the manufacturer and the supplier.

7. Supply Chain Coordination

Here comes an interesting question, whether the double marginalisation effect can be completely eliminated, that is, whether the supply chain can be coordinated with options under our setting. First, denoting products of the centralized system as $Q^c$, then the expected profit of the centralized system can be written as

$$\Pi^c(Q^c) = rED + \alpha \overline{P}_s E(TQ^c - D)^+ - cQ$$

(9)

The following proposition characterizes the optimal components production problem of the centralized system.

Proposition 7. $\Pi^c(Q^c)$ is concave in $Q^c$, and $Q^c$ satisfies

$$\int_0^1 F(tQ^c) t \phi(t) dt = \frac{-\overline{P}_s \mu - c}{1 - \alpha} \overline{P}_s.$$  

(10)

Defining the total expected profit of the decentralized system under option contracts as $\Pi(q_{0s}^o, q_{1s}^s, Q^c)$, then

$$\Pi(q_{0s}^o, q_{1s}^s, Q^c) = \Pi^m(q_{0s}^o, q_{1s}^s) + \Pi^c(Q^c)$$

$$= rED + \alpha \overline{P}_s E\left\{TQ^c - q_{0s}^o - \min\left[(D - q_{0s}^o)^+, q_{1s}^s\right]\right\}^+$$

$$+ \alpha \overline{P}_s (q_{0s}^o - D)^+$$

$$- \overline{P}_s E\left\{q_{0s}^o + \min\left[(D - q_{0s}^o)^+, q_{1s}^s\right] - TQ^c\right\}^+$$

$$- \overline{P}_s E(D - q_{0s}^o - q_{1s}^s)^+ - cQ^c.$$  

(11)

Similar to the approach of Luo and Chen [6], we assume that the supplier chooses $Q = Q^c$ to produce. The results of comparing the maximum expected profits of the decentralized with the centralized show that $\Pi(q_{0s}^o, q_{1s}^s, Q^c) < \Pi^c(Q^c)$ when $TQ^c < x < q_{0s}^o, x < TQ^c < q_{0s}^o, tQ^c > x > q_{0s}^o + q_{1s}^s$, and $x > TQ^c > q_{0s}^o + q_{1s}^s$ (the proof is in Appendix). It reveals that under the option contracts the decentralized supply chain cannot be coordinated. Hence, we have the following proposition.

Proposition 8. With random yield, demand, and spot price, the traditional option contracts cannot achieve the supply chain coordination any more.

The revenue loss of the decentralized supply chain, which is caused by the different transaction risks between buying and selling the components on the spot market, explains Proposition 8. To avoid the problem, we propose a protocol to be added to the options contracts: (1) the manufacturer first replenishes the unmet demands from the supplier and then on the spot market when both the supplier’s actual output and the realized demand are more than the manufacturer’s total order quantity (sum of the two-type order quantity); (2) the quantity the supplier should deliver to the manufacturer is the maximum of the actual output and the realized demand when both the supplier’s actual output and the realized demand are less than the manufacturer’s firm order quantity.

With the proposed protocol, use $q_{1s}^o$ and $q_{1s}^s$ to denote option order quantity and use $Q_s$ to denote production quantity; we can express the manufacturer’s and the supplier’s expected profit as follows:

$$\Pi^m(q_{1s}^o, q_{1s}^s, Q_s) = rED - \omega \max\{\min(q_{0s}^o, D),$$

$$\min(q_{0s}^o, TQ_s)\} - q_{1s}^s - (eU(e) + \bar{p}_{ce}) \cdot E$$

$$\cdot \min\left[(D - q_{1s}^o)^+, q_{1s}^s\right] - \overline{P}_s E(D - q_{1s}^o - q_{1s}^s)^+$$

$$+ \alpha \overline{P}_s \min\left[(q_{1s}^o - D)^+, (TQ_s - D)^+\right].$$

(12)
Proposition 9. $Q_1^*$ is given by

$$q_{1s}^* + \left( e V (e) + \bar{p}_s \right) \int_0^{\hat{q}_1} f (x) dx + \left( w - \bar{p}_s \right) q_1^* + (1 - \alpha)$$

$$\frac{\partial \Pi^*(Q_1)}{\partial Q_1} = \left( e V (e) + \bar{p}_s \right) \int_0^{\hat{q}_1} f (x) dx + \left( w - \bar{p}_s \right) q_1^* + (1 - \alpha)$$

$$\int_0^{\hat{q}_1} \left( (x - \hat{q}_1) f (x) dx \right) \phi (t) dt + \left( e V (e) \right)$$

$$\frac{\partial \Pi^*(Q_1)}{\partial Q_1} = \left( e V (e) + \bar{p}_s \right) \int_0^{\hat{q}_1} f (x) dx + \left( w - \bar{p}_s \right) q_1^* + (1 - \alpha)$$

$$\int_0^{\hat{q}_1} \left( (x - \hat{q}_1) f (x) dx \right) \phi (t) dt + \left( e V (e) \right)$$

It shows that

$$d \Pi^*(Q_1)$$

$$= (w - \bar{p}_s) \int_0^{q_1^*} F (t Q_1) t \varphi (t) dt$$

$$- (1 - \alpha) \bar{p}_s \int_0^{\hat{q}_1} F (t Q_1) t \varphi (t) dt - (c - \bar{p}_s \mu)$$

and

$$\frac{d^2 \Pi^*(Q_1)}{d (Q_1)^2}$$

$$= (w - \bar{p}_s) \int_0^{q_1^*} f (t Q_1) t^2 \varphi (t) dt$$

$$- (1 - \alpha) \bar{p}_s \int_0^{\hat{q}_1} f (t Q_1) t^2 \varphi (t) dt$$

$$- (w - \bar{p}_s) \left( \frac{q_0^*}{Q_1} \right)^2 F \left( \frac{q_0^*}{Q_1} \right) \phi \left( \frac{q_0^*}{Q_1} \right)$$

Because $\frac{\partial \Pi^*(Q_1)}{\partial Q_1} > w > \frac{\partial \Pi^*(Q_1)}{\partial Q_1}$,

$$\frac{d^2 \Pi^*(Q_1)}{d (Q_1)^2} < \left( w - \bar{p}_s \right) \int_0^{\hat{q}_1} f (t Q_1) t^2 \varphi (t) dt$$

$$- (1 - \alpha) \bar{p}_s \int_0^{\hat{q}_1} f (t Q_1) t^2 \varphi (t) dt$$

$$- (w - \bar{p}_s) \left( \frac{q_0^*}{Q_1} \right)^2 F \left( \frac{q_0^*}{Q_1} \right) \phi \left( \frac{q_0^*}{Q_1} \right)$$

$$< 0.$$
Proof. To achieve system coordination, it must be ensured that $Q_1^* = Q^*$. From (14), we have

$$
(1 - \alpha)\bar{p}_s \int_0^1 F (tQ^*) t\varphi(t) \, dt 
- (w - \alpha \bar{p}_s) \int_0^{\bar{q}_1^*/Q^*} F (tQ^*) t\varphi(t) \, dt 
= \bar{p}_s \mu - c
$$

Since $\int_0^1 F(t(tQ^*) \varphi(t) \, dt = (\bar{p}_s \mu - c) / (1 - \alpha) \bar{p}_s$, the above equation still holds when $w = \alpha \bar{p}_s$. Further, from (12), we can get

$$
(eV(e) + p_{se} - \alpha \bar{p}_s) F(q_1^{0*}) - (w - \alpha \bar{p}_s) 
\cdot \int_0^{q_1^*/Q^*} \left[ F(q_1^{0*}) - F(tQ^*) t \frac{dQ^*}{dq_1^{0*}} \right]_{q_1^*=q_1^{0*}} 
\cdot \varphi(t) \, dt 
= eV(e) + p_{se} + o - w;
$$

$$
q_1^{1*} = F\left(\frac{p_{se} - eV(e) - o}{p_{se} - eV(e)}\right) - q_1^{0*}.
$$

If $w = \alpha \bar{p}_s$, then $q_1^{1*} = 0$, which implies that $eV(e) + p_{se} + o = w$. \qed

Proposition 10 reveals that by varying the contract parameters $w$, $e$, and $o$ according to (18) the coordination of the supply chain and the allocation profit between the two firms will be achieved in many different ways. Compared with the noncoordinating scenario, there are sufficient coordination contracts to achieve a win-win situation for the manufacturer and the supplier.

8. Conclusions and Future Research

This paper considers a system with one component supplier who suffers from random yield and one end product manufacturer facing stochastic market demands. The manufacturer can obtain goods from the supplier via firm order contracts and option contracts. Apart from the contract market, there is a spot market where both firms can buy and sell the components. The main purpose of the paper is to explore the flexibility of the option contracts when yield, demand, and spot price are all random in a supply chain. Our research shows that under the circumstance of random yield, demand, and spot price, a unique equilibrium exists between the manufacturer's order quantity and the supplier's production quantity in the decentralized system with option contracts. It suggests that the option contracts can facilitate the order of the manufacturer and the production level of the supplier, meanwhile bringing the increase in profit of both firms. Our findings also show that the value of option contracts for the supplier and the manufacturer is related to the volatility of the random variables (yield, demand, and spot price). It also shows that the channel coordination cannot be achieved by the single option contract. However, when combined with a protocol, it is able to coordinate the supply chain.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Additional Points

Study Limitations. Our model also has some limitations. First, this paper is motivated by the phenomenon of random yield in semiconductor industry; we adopt multiplicative yield model (also called stochastically proportional yield model) which is frequently used in the literatures (see Yano and Lee, 1995; Grosfeld-Nir and Gerchak, 2004) to depict the yield risk. However, the additive yield model may be more suitable for the agriculture-related industries, that is, to use $Q + T$ ($T \in [a, b]$; $-Q \leq a \leq 0, b \geq 0$) as the actual output for an input $Q$ of the supplier (see Keren, 2009). Therefore, to make this study more applicable in real life, the additive yield models should be addressed in direct extension. Second, to simplify the models and provide some interesting insights, we assume that the spot price is independent from yields and demands, but it should be known that the yield or/and demand will often impact the spot price. So, considering the spot price to be correlated with the yield or/and demand is another key extension. Another interesting extension includes considering the supply chain members' risk attitudes (e.g., loss aversion), multiple suppliers, or/and multiple manufacturers.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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