A solution of the spacetime singularity problem in relativistic cosmology by using an additional variable

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Abstract

The spacetime singularity in relativistic cosmology is cancelled by using an additional variable. That is, the singularity-free models for an expanding universe are obtained from general relativity.
1 Introduction

Many physicists believe that the Friedmann models [1-3] describe the expanding phenomena of the space of the universe, and they accept the models as a standard cosmological model. However, the models have many problems. The scale factor \( R \) in the models has no exact meaning and is an unobservable quantity, that is, \( R \) is not an astrophysical quantity but only a mathematical one. Thus the models do not agree with Hubble’s observations [4, 5] completely. The Hubble’s law \((V = H_0 D, \text{Fig. 1})\) states that the distance between two neighbouring galaxies increases with time. If the universe is homogeneous and isotropic, the increasing distance means that the radius or the diameter of the universe is increasing. Therefore, the scale of the radius of the universe depends on the distance between two neighbouring galaxies. However, the Friedmann models do not represent this fact.

The energy tensors \((T_{ik})\) in the Friedmann models only have a contribution from the mass density of the galaxies. Therefore, the models should only describe the expansion of the present universe. However, the models are also used to expansion of the early universe.

An expansion age of the universe estimated by the Friedmann model does not agree with the experimental data [6], that is, the age of the Friedmann universe is smaller than that of the components (stars or galaxies) of the real universe. Recent measurements using the Hubble Space Telescope [7] and the Canada-France-Hawaii Telescope [8] have given the Hubble constant \( H_0 = 80 \text{ Kms}^{-1}\text{Mpc}^{-1} \) and \( 87 \text{ Kms}^{-1}\text{Mpc}^{-1} \), respectively. According to the Friedmann model these values give an expansion age of \( 8 \times 10^9\text{yrs} \) and \( 7 \times 10^9\text{yrs} \), respectively. Theses ages
lie below the age of globular clusters \((10 - 18 \times 10^9 \text{yrs})\) obtained from stellar evolution theory [9].
The most important problem in the Friedmann models is the spacetime singularity. First, it is very difficult to imagine a universe that has a zero radius at the time zero. That is, with a gravitation theory we cannot explain the creation of the space of the universe, since the theory describes the interaction of matter. We must define the concept of the universe. The universe is a very large group of \(10^{11}\) galaxies in an infinite space, and the galaxies in the components of the universe, i.e. local group, cluster and supercluster, are connected through gravitation. The space of the universe is just a space filled by these galaxies. Therefore, the space of the universe is only a subspace of the infinite space and cannot have been created by the big bang or a physical process. However, the spacetime singularity confuses this concept of the universe. Thus a few astronomers [10,11] proposed the steady state theory for an expanding universe.
Second, general relativity cannot be applied to the early universe in particle form, since the particle is the source of neither the weak nor the strong field. However, the spacetime singularity compelled us to apply the theory to the early universe. Third, according to the big bang model the temperature of the universe was nearly infinite at the big bang epoch. However, the universe should have an upper limit in temperature. Fourth, the singularity gave rise to the horizon and the flatness problems in the initial phase of the universe. The inflationary universe [12,13,14] is an alternative model for a solution of these problems. However, the model cannot fundamentally solve the singularity problem.
If a model for an expanding universe agrees completely with the
Hubble’s law, the model should have no singularity, since the Hubble’s law has no singularity and is valid only for the expanding phenomena of the present universe. If the singularity is due to general relativity [15], the theory is not a complete theory, since a gravitation theory cannot describe the creation and the annihilation of the space of the universe. However, the singularity is due not to the incompleteness of general relativity but to a careless investigation of the spacetime geometry. We can avoid the singularity if we study the geometry carefully.

2 New geometrical investigation of the present universe

The main purpose of relativistic cosmology is to describe how $10^{11}$ galaxies move relative to the centre of the universe. We assume that the galaxies are isotropically and homogeneously distributed in the space of the universe [16], and we take the Weyl postulate [17] for the simplicity of calculations. Under these assumptions, the line element in four-dimensional spacetime coordinates $x^0, x^1, x^2$ and $x^3$ has the following form

$$ds^2 = (dx^0)^2 - d\sigma^2,$$

(1)

where $d\sigma^2 = \sum g_{ik} dx^i dx^k$ (i,k=1,2,3) and is the metric on one of the spherical hypersurfaces orthogonal to the world line of galaxies.

We must express the metric $d\sigma^2$ in spherical coordinates. To do this, we consider a hypersphere of the radius R and embed it in the Cartesian coordinates. In terms of the Cartesian coordinates $x_1, x_2, x_3$ and $x_4$, the equation of the surface of the
four-dimensional hypersphere with constant positive curvature is given by

\[ x_1^2 + x_2^2 + x_3^2 + x_4^2 = R^2, \]  

(2)

where the radius \( R \) of the hypersphere is a constant in the Cartesian coordinates and corresponds to the radius of universe in the spacetime coordinates. If we choose the following coordinates

\[
\begin{align*}
x_1 &= R \sinh \alpha \cos \theta, \\
x_2 &= R \sinh \alpha \sin \theta \cos \phi, \\
x_3 &= R \sinh \alpha \sin \theta \sin \phi, \\
x_4 &= R \cosh \alpha,
\end{align*}
\]

with \( z = \sinh \alpha \) we get the metric

\[ d\sigma^2 = R^2 \left[ \frac{dz^2}{1 - z^2} + z^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]. \]  

(3)

d\( \sigma^2 \) is the metric on the surface of the hypersphere of the radius \( R \). In order to obtain a time-dependent metric \( d\sigma^2 \) in the Cartesian coordinates, we must change the radius \( R \). However, we cannot directly insert \( R(t) \) instead of \( R \) in eq. (3), since \( R \) is a constant in the Cartesian coordinates. We must distinguish a hypersphere in the Cartesian coordinates from one in the spacetime coordinate. The radius \( R \) should be a constant in the Cartesian coordinates. Therefore, we must choose a hypersphere with the radius \( \neq R \) and find it in the real universe represented by the spacetime coordinates. That is, we must find a time-dependent variable that is proportional to the radius of the universe. According to the Hubble’s law, the variable is the distance \( r \) between two neighbouring galaxies. It can be geometrically realized as follows.

Let us consider a cross section that contains the centre of the hypersphere in the Cartesian coordinates. The cross section is a circle of the radius \( R \) which corresponds to the cross section that contains the centre of the universe in the spacetime coordinates. We must consider the distribution of galaxies in the
cross section. Since we assumed an isotropic and homogeneous
distribution of the galaxies, the relation between R and r can be
approximated in the cross section as follows (see Fig. 2)

\[ R = Ar + B, \]  

(4)

where A is the number of galaxies on an axis and B is the product
of A and D (diameter of galaxy). We must note that R is not a
constant in the spacetime coordinates.

We return to the Cartesian coordinates. Let us consider a cross
section of the radius r in the spacetime coordinates as in Figure
2. The cross section of the radius r corresponds to that of the
hypersphere of the radius r in the Cartesian coordinates. From
now on we consider a hypersphere of the radius r instead of one
of the radius R. We embed the hypersphere of the radius r in
the Cartesian coordinates. Inserting r(t) instead of R in eq. (3),
we obtain

\[ d\sigma^2 = r^2(t)\left[ \frac{dz^2}{1 - z^2} + z^2(d\theta^2 + \sin^2\theta d\phi^2) \right]. \]  

(5)

d\sigma^2 is the time-dependent metric on the surface of the hyper-
sphere of the radius r. r depends only on t and corresponds
to the distance between two neighbouring galaxies in the space-
time coordinates. Since there is no relation between r and R in
the Cartesian coordinates, R is a constant. With the curvature
parameter k we obtain a time-dependent line element

\[ ds^2 = c^2dt^2 - r^2(t)\left[ \frac{dz^2}{1 - kz^2} + z^2(d\theta^2 + \sin^2\theta d\phi^2) \right]. \]  

(6)

With the line element we can solve the field equations of Hilbert

\[ R_{ik} - \frac{1}{2}g_{ik}R = -kT_{ik}. \]  

(7)
Setting the spacetime coordinates \((x^0, x^1, x^2, x^3)\) and the Cartesian coordinates \((ct, z, \theta, \phi)\) as follows

\[
x^0 = ct, \quad x^1 = z, \quad x^2 = \theta, \quad x^3 = \phi,
\]

we get the nontrivial equations from the equations with \(\dot{r} = dr/dt\)

\[
\frac{2\ddot{r}}{r} + \frac{\dot{r}^2 + kc^2}{r^2} = \frac{8\pi\gamma T^k}{c^2} \quad (k = 1, 2, 3),
\]

\[
\frac{\dot{r}^2 + kc^2}{r^2} = \frac{8\pi\gamma T^0}{3c^2}.
\]

The energy tensors of matter have the following forms

\[
T^k_k = -p, \quad T^0_0 = \epsilon.
\]

In the case of the system of galaxy behaving like dust the energy tensors (11) have specific forms

\[
p = 0, \quad \epsilon = \rho c^2,
\]

where \(\rho\) only has the contribution from the mass density of the galaxies which is given by

\[
\rho = \rho_0 r^3_0 / r^3.
\]

\(\rho_0\) and \(r_0\) are present values of \(\rho\) and \(r\). With the energy tensors eq. (9) and eq. (10) become

\[
\frac{2\ddot{r}}{r} + \frac{\dot{r}^2 + kc^2}{r^2} = 0,
\]

\[
\frac{\dot{r}^2 + kc^2}{r^2} = \frac{8\pi\gamma \rho_0 r^3}{3r^3}.
\]

In the case of positive curvature \((k=+1)\) eq. (15) can be rewritten

\[
\dot{r}^2 = c^2 \left( \frac{\beta}{r} - 1 \right),
\]
with $\beta$ given by

$$\beta = \frac{2q_0 c}{(2q_0 - 1)^{3/2} H_0}. \quad (17)$$

$H_0$ is the Hubble constant and $q_0$ is the deceleration parameter.

Eq. (16) has the solution

$$r = \beta \sin^2 \frac{\theta}{2}. \quad (18)$$

This is the distance between two neighbouring galaxies. The distance increases with time. This is the reason why the universe expands. The radius $R$ of the universe is obtained by inserting the solution in eq. (4)

$$R = A\beta \sin^2 \left(\frac{\theta}{2}\right) + B. \quad (19)$$

The radius oscillates between minimum value $B$ and maximum $A\beta + B$, that is, the galaxies go first away from the centre of the universe and then move for the centre (Fig.3a).

In the cases of $k=0$ and $k=-1$ eq. (15) can be solved. In these cases the radius of the universe is illustrated in Fig.3b and Fig.3c. The universe expands forever. The spacetime singularity exists no more in the radius of the universe.

3 Discussion

We solved the spacetime singularity problem in relativistic cosmology by using an additional variable $r$. The singularity is due not to the incompleteness of general theory but to a careless investigation of the spacetime geometry. The relativistic cosmology becomes complete with the variable $r$. 
Although the spacetime singularity was removed in the radius of the universe, the singularity still exists in the distance between two galaxies. The singularity means that the distance was zero after formation of the galaxies and that they collide with each other again. However, in the real universe the galaxies should have a minimum distance after their formation, that is, the universe is expanding from the minimum. Therefore, the singularity should be also removed. However, it is impossible since general relativity is a relativistic theory of gravitation in four dimensions.

The spacetime geometry does not explain why the distance between two galaxies increases or decreases, since general relativity is a purely mathematical theory of gravitation. We need a non-relativistic and dynamical theory of gravitation for a more exact description of the expanding and contracting phenomena of our universe [19]. The physics of the early universe should be reconstructed in a reasonable and scientifical manner.

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Figure captions
Fig.1 Hubble’s law. (a) The recession velocity $V$ of a galaxy is proportional to its distance $D$ from the Milky Way $\oplus$. (b) Distribution of galaxies at a later time; the distance between two neighbouring galaxies increased.
Fig.2 Distribution of galaxies on an axis of the cross section that contains the centre of the universe in the spacetime coordinates.
Fig.3a Radius $R$ of the universe as a function of cosmic time $t$ for $k=1$.
Fig.3b Radius $R$ of the universe as a function of cosmic time $t$ for $k=0$.
Fig.3c Radius $R$ of the universe as a function of cosmic time $t$ for $k=-1$. 
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