Symmetry Energy, Temperature, Density and Isoscaling Parameter as a Function of Excitation energy in A ~ 100 mass region

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The symmetry energy, temperature, density and isoscaling parameter, in $^{58}\text{Ni} + ^{58}\text{Ni}$, $^{58}\text{Fe} + ^{58}\text{Ni}$ and $^{58}\text{Fe} + ^{58}\text{Fe}$ reactions at beam energies of 30, 40 and 47 MeV/nucleon, are studied as a function of excitation energy of the multifragmenting source. It is shown that the decrease in the isoscaling parameter is related to the near flattening of the temperature in the caloric curve, and the decrease in the density and the symmetry energy with increasing excitation energy. The decrease in the symmetry energy is mainly a consequence of decreasing density with increasing excitation rather than the increasing temperature. The symmetry energy as a function of density obtained from the correlation is in close agreement with the form, $E_{\text{sym}}(\rho) = 31.6 (\rho/\rho_0)^{0.69}$.

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Due to its vast implications ranging from how nucleons clusterize into nuclei at low densities to the structure and stability of neutron stars at high density, the interest in understanding the behavior of nuclear matter at temperatures, densities and isospin (neutron-to-proton asymmetry) away from those of normal nuclear matter ($T \approx 0$ MeV; $\rho_0 \approx 0.16$ fm$^{-3}$; $N \approx Z$) has gained tremendous importance [1].

Experimentally, the multifragmentation reaction [2, 3, 4, 5, 6], where a highly excited nucleus expands to a low density region and disassembles into many light and heavy fragments, provides the best possible means of studying nuclear matter at non-normal densities, temperatures and isospin. It has been shown from various experiments [7] that the temperature as a function of excitation energy (caloric curve) in multifragmentation reactions shows a near flattening, or a plateau-like region, at higher excitation energies. There are also indications that the density of the system decreases with increasing excitation energy [8, 9]. Recently, it has been shown that the isoscaling parameter, obtained from the fragment yield distribution, shows a decrease with increasing beam energy [10, 11]. A decrease in the symmetry energy has also been experimentally observed [11, 12].

In this paper, we seek to understand the correlation between the temperature, density and symmetry energy of a multifragmenting system as it evolves with excitation energy. Such a correlation is important for constructing the nuclear matter equation of state and studying the density dependence of the symmetry energy; a key unknown in the equation of state of asymmetric nuclear matter.

We make use of the fragment yield distributions measured [11, 13, 14] in $^{58}\text{Ni}$, $^{58}\text{Fe} + ^{58}\text{Ni}$, $^{58}\text{Fe}$ reactions at 30, 40 and 47 MeV/nucleon to determine the isoscaling parameter $\alpha$, as a function of the excitation energy of the fragmenting source. The parameter $\alpha$’s were obtained by taking the ratio of the isotopic yields for two different pairs of reactions, $^{58}\text{Fe} + ^{58}\text{Ni}$ and $^{58}\text{Ni} + ^{58}\text{Ni}$, and $^{58}\text{Fe} + ^{58}\text{Fe}$ and $^{58}\text{Ni} + ^{58}\text{Ni}$ as described in Ref. [11, 13]. The excitation energy of the source for each beam energy was determined by simulating the initial stage of the collision dynamics using the Boltzmann-Nordheim-Vlasov (BNV) model calculation [15]. The results were obtained at a time around 40 - 50 fm/c after the projectile had fused with the target nuclei and the quadrupole moment of the nucleon coordinates (used for identification of the deformation of the system) approached zero. These excitation energies were compared with those obtained from the systematic calorimetric measurements (see Ref. [1]) for systems with mass ($A \sim 100$), and similar to those studied in the present work, and are in good agreement. Fig. 1 shows the experimental isoscaling parameter $\alpha$, as a function of the excitation energy obtained in the present study for both the pairs of reactions. A systematic decrease in the absolute values of the isoscaling parameter with increasing excitation energy is observed for both pairs. The $\alpha$ parameters for the $^{58}\text{Fe} + ^{58}\text{Fe}$ and $^{58}\text{Ni} + ^{58}\text{Ni}$ are about twice as large compared to those for the $^{58}\text{Fe} + ^{58}\text{Ni}$ and $^{58}\text{Ni} + ^{58}\text{Ni}$ pair of reactions. It is interesting to note that the difference in $Z/A$, i.e. $\Delta(Z/A)^2$, of the composite systems in the $^{58}\text{Fe} + ^{58}\text{Fe}$ and $^{58}\text{Ni} + ^{58}\text{Ni}$ pair is also twice as large compared to the $^{58}\text{Fe} + ^{58}\text{Ni}$ and $^{58}\text{Ni} + ^{58}\text{Ni}$ pair.

The experimental isoscaling parameter was compared to the predictions of the Statistical Multifragmentation Model (SMM) [4, 16] calculations to study the dependence on the excitation energy and the isospin content. The initial parameters such as, the mass, charge and excitation energy of the fragmenting source for the calculation, was obtained from the BNV calculations as discussed above. To account for the possible uncertainties in the source parameters due to the loss of nucleons during pre-equilibrium emission, the calculations were also performed for smaller source sizes. The break-up density in the calculation was taken to be multiplicity-dependent and was varied from approximately 1/2 to 1/3 the saturation density. This was achieved by varying the free volume with the excitation energy as shown in Ref. [4]. The form of the dependence was adopted from the work of Bondorf et al., [5, 17] (and shown by the solid curve in Fig. 4). It is known that the multiplicity-dependent

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FIG. 1: Experimental isoscaling parameter $\alpha$, as a function of excitation energy for the Fe + Fe and Ni + Ni pair of reaction (inverted triangles), and Fe + Ni and Ni + Ni pair of reactions (solid circles) for the 30, 40 and 47 MeV/nucleon. The solid and the dotted curves are the statistical multifragmentation model calculations as discussed in the text.

Fig. 1 shows the comparison between the SMM calculated and the measured $\alpha$ for both pairs of systems. The dotted curves correspond to the calculation for the primary fragments and the solid curves to the secondary fragments. The width in the curve is the measure of the uncertainty in the inputs to the SMM calculation. It is observed that, within the given uncertainties, the decrease in the $\alpha$ values with increasing excitation energy and decreasing isospin difference $\Delta(Z/A)^2$, of the systems is well reproduced by the SMM calculation. One also notes that the effect of sequential decay effect on the isoscaling parameter is small as has been observed in several other studies using statistical models [18, 21].

We show in Fig. 2, the temperature as a function of excitation energy (caloric curve) obtained from the above SMM calculation that uses the excitation energy dependence of the break-up density to explain the observed isoscaling parameters. These are shown by the solid and inverted triangle symbols. Also shown in the figure are the experimentally measured caloric curve data compiled by Natowitz et al. [7] from various measurements for this mass range. The data from these measurements are shown collectively by solid star symbols and no distinction is made among them. The Fermi-gas model predictions with inverse level density parameter $K_o = 10$ (solid and dashed lines), is also shown. It is evident from the figure that the temperatures obtained from the SMM calculations are in good agreement with the overall trend of the caloric curve. Somewhat lower value for the temperature is observed when the break-up density of the system is kept constant at 1/3 the normal nuclear density. By allowing the break-up density to evolve with the excitation energy, a near plateau that agrees with the experimentally measured caloric curves is obtained. This assures that the input parameters used in the SMM calculation for comparing with the data are reasonable.

The symmetry energies obtained from the statistical model comparison of the experimental isoscaling parameter $\alpha$, are as shown in Fig. 3. A steady decrease in the

break-up density, which corresponds to a fixed interfragment spacing and constant pressure at break-up, leads to a pronounced plateau in the caloric curve [18, 21]. A constant break-up density would lead to a steeper temperature versus excitation energy dependence. We will return to this point later in this paper.

The symmetry energy in the calculation was varied until a reasonable agreement between the calculated and the measured $\alpha$ was obtained. It has been shown [6, 18, 19], that the symmetry energy in the statistical model calculations is related to the isoscaling parameter through the relation,

$$\alpha_{prim} = \frac{4G_{sym}}{T} \left[ (Z/A)^2 - (Z/A)^2_2 \right]$$

(1)

where $\alpha_{prim}$ is the isoscaling parameter for the hot primary fragments, i.e., before they sequentially decay into cold secondary fragments. $Z_1, A_1$ and $Z_2, A_2$ are the charge and the mass numbers of the fragmenting systems. $T$ is the temperature of the systems and $C_{sym}$ is the symmetry energy. In the above equation, the entropic contribution to the symmetry free energy is assumed to be small (the contribution becomes important at densities below 0.008 $fm^{-3}$ [20]), the symmetry energy can therefore be reliably substituted for the free energy.

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symmetry energy with increasing excitation energy is observed for both pairs of systems. Such a decrease has also been observed in several other studies \[11, 12, 22, 23\]. We have also estimated the effect of the symmetry energy evolving during the sequential de-excitation of the primary fragments \[11, 24\]. These are reflected in the large error bars shown in Fig. 3.

The phase diagram of the multifragmenting system is two dimensional and hence the excitation energy dependence of the temperature (the caloric curve) must take into account the density dependence too. Often this dependence is neglected while studying the caloric curve. In the following, we attempt to extract the density of the fragmenting system as a function of excitation energy using the relation

\[ T = \sqrt{\frac{K(\rho) E^*}{K_o(\rho/\rho_o)^{2/3} E^*}} \tag{2} \]

In the above expression, the momentum and the frequency dependent factors in the effective mass ratio are taken to be one as expected at high excitation energies and low densities studied in this work \[21, 27, 28\].

The resulting densities for the two pairs of systems are shown in Fig. 4 by the solid circles and inverted triangles. For comparison, we also show the break-up densities obtained from the analysis of the apparent level density parameters required to fit the measured caloric curve by Natowitz \[8\] and those obtained by Viola \[9\] from the Coulomb barrier systematics that are required to fit the measured intermediate mass fragment kinetic energy spectra. One observes that the present results obtained by requiring to fit the measured isoscaling parameters and the caloric curve are in good agreement with those obtained by Natowitz \[8\] et al. The figure also shows the fixed freeze-out density of 1/3 (dashed line) and 1/6 (dotted line) the saturation density assumed in various statistical model comparisons.

It is evident from figures 1, 2, 3 and 4 that the decrease in the experimental isoscaling parameter \(\alpha\), symmetry energy, break-up density, and the flattening of the temperature with increasing excitation energy are all correlated. One can thus conclude that the expansion of the system during the multifragmentation process leads to a decrease in the isoscaling parameter, decrease in the symmetry energy and density, and the flattening of the temperature with excitation energy. Table I shows the correlated quantities obtained using Eqs. 1 and 2 for both pairs of systems.

From the above correlation between the symmetry energy as a function of excitation energy, and the density as a function of excitation energy, we obtain the symmetry energy as a function of density. This is shown in Fig. 5. for both pairs of systems. The temperature in the present work remains nearly constant for the range of excitation energies studied. The observed decrease in the symmetry energy with increasing excitation energy is therefore a consequence of decreasing density. This is also supported
TABLE I: Primary isoscaling parameter, temperature, density and symmetry energy obtained for Fe + Fe and Ni + Ni pair (top row), and Fe + Ni and Ni + Ni pair (bottom row) of systems at various excitation energies using Eqs. 1 and 2.

| $E^*$ (MeV/nucleon) | $\alpha_{prim}$ | Temp. (MeV) | $\rho/\rho_o$ | Symmetry Energy (MeV) |
|---------------------|-----------------|-------------|---------------|-----------------------|
| 5.0                 | 0.44            | 5.8         | 0.56          | 20.0                  |
| 7.0                 | 0.35            | 6.4         | 0.45          | 17.5                  |
| 9.5                 | 0.30            | 7.0         | 0.38          | 16.5                  |
| 5.0                 | 0.22            | 5.7         | 0.52          | 20.0                  |
| 7.0                 | 0.17            | 6.4         | 0.45          | 17.0                  |
| 9.5                 | 0.15            | 6.8         | 0.34          | 16.0                  |

FIG. 5: Symmetry energy as a function of density for the Fe + Fe and Ni + Ni pair of reaction (inverted triangles), and Fe + Ni and Ni + Ni pair of reactions (solid circles) for the 30, 40 and 47 MeV/nucleon. The solid square corresponds to those from Ref. [30]. The solid curve is the dependence obtained in Ref. [32, 33].

by microscopic calculations which shows an extremely slow evolution of the symmetry energy with temperature [1, 29]. The evolution is practically negligible for the temperature range studied in this work. Also shown in Fig. 5 is the symmetry energy value of Khoa et al. [30], (solid square) obtained by fitting the experimental differential cross-section data in a charge exchange reaction using the isospin dependent optical potential. The solid curve corresponds to the dependence, $E_{sym}(\rho) = 31.6 (\rho/\rho_o)^{0.69}$, obtained by comparing the present data with the Antisymmetrized Molecular Dynamic (AMD) calculation [31], in previous work [17, 32, 33]. The similarities in the density dependence of the symmetry energy obtained from the present statistical model approach and the AMD model approach is intriguing. It should be noted that the symmetry energy shown by the solid curve in the figure relates to the volume part of the symmetry energy as in infinite nuclear matter, whereas, the symmetry energy obtained from the present study (solid circles and inverted triangles) relates to the fragment that are finite and has surface contribution. The similarity between the two can be understood in terms of the weakening of the surface symmetry free energy when the fragments are being formed. During the density fluctuation in uniform low density matter, the fragments are not completely isolated and continue to interact with each other, resulting in a decrease in the surface contribution as predicted by Ono et al. [34]. The present observation therefore lends credence to the fact that it is possible to directly obtain the properties of infinite nuclear matter from the fragments produced in the multifragmentation process [34]. More studies are required to illustrate this point further.

In summary, we have studied the isoscaling parameter, symmetry energy, temperature and density as a function of excitation energy in reactions populating $A \sim 100$ mass region. It is observed that the decrease in the experimental isoscaling parameter $\alpha$, symmetry energy, breakup density, and the flattening of the temperature with increasing excitation energy are related to each other. The observed decrease in the symmetry energy with increasing excitation energy appears to be mainly a consequence of decreasing density. The symmetry energy as a function of density obtained from the present study is consistent with those obtained from the dynamical AMD calculation, indicating that the surface contribution to the symmetry energy could be small.

I. ACKNOWLEDGMENT

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[1] *Isospin Physics in Heavy Ion Collisions at Intermediate Energies*, edited by B.-A. Li and W. Schroder (Nova Science, New York, 2001).

[2] D.H.E. Gross, Rep. Prog. Phys. 53, 1122 (1990).

[3] W.A. Friedman, Phys. Rev. C 42, 667 (1990).

[4] J.P. Bondorf, A.S. Botvina, A.S. Iljinov, I.N. Mishustin and K. Sneppen, Phys. Rep. 257, 133 (1995).

[5] J.P. Bondorf, R. Donangelo, I.N. Mishustin and H. Schultz, Nucl. Phys. A 444, 460 (1985).

[6] A.S. Botvina, O.V. Lozhkin, and W. Trautmann, Phys. Rev. C 65, 044610 (2002).

[7] J.B. Natowitz, R. Wada, K. Hagel, T. Keutgen, M. Murray, A. Makeev, L. Qin, P. Smith, and C. Hamilton, Phys. Rev. C 65, 034618 (2002); and references therein.

[8] J.B. Natowitz, K. Hagel, Y. Ma, M. Murray, L. Qin, S. Shlomo, R. Wada, and J. Wang, Phys. Rev. C 66, 031601 (2002).

[9] V.E. Viola, K. Kwiatkowski, J.B. Natowitz, and S.J. Yennello, Phys. Rev. Lett. 93, 132701 (2004).

[10] D.V. Shetty et al., Phys. Rev. C 70, 011601 (2004).

[11] J. Iglio, D.V. Shetty, S.J. Yennello, G.A. Souliotis, M. Jandel, A. Keksis, S. Soisson, B. Stein and S. Wenzschel, Phys. Rev. C (submitted) (2006).

[12] D.V. Shetty, A.S. Botvina, S.J. Yennello, A. Keksis, E. Martin and G.A. Souliotis, Phys. Rev. C 71, 024602 (2005).

[13] D.V. Shetty, S.J. Yennello, E. Martin, A. Keksis, and G.A. Souliotis, Phys. Rev. C 68, 021602 (2003).

[14] E. Ramakrishnan et al., Phys. Rev. C 57, 1803 (1998).

[15] V. Baran, M. Colonna, M. Di Toro, V. Greco, M. Zielinska-Pfabe and H.H. Wolter, Nucl. Phys. A 703, 603 (2002).

[16] A.S. Botvina and I.N. Mishustin, Phys. Rev. C 63, 061601 (2001).

[17] J.P. Bondorf, A.S. Botvina and I.N. Mishustin, Phys. Rev. C 58, 27 (1998).

[18] M.B. Tsang *et al.*, Phys. Rev. C 64, 054615 (2001).

[19] M.B. Tsang *et al.*, Phys. Rev. Lett. 86, 5023 (2001).

[20] C.J. Horowitz and A. Schwenk, nucl-th/0507033 (2005).

[21] W.P. Tan *et al.*, Phys. Rev. C 64, 061901 (2001).

[22] A. LeFevre *et al.*, Phys. Rev. Lett. 94, 162701 (2005).

[23] D. Henzlova, A.S. Botvina, K.H. Schmidt, V. Henzi, P. Napolitani, M.V. Ricciardi, nucl-ex/0507003 (2005).

[24] N. Buyukcizmeci, R. Ogul, and A.S. Botvina, Eur. Phys. J. 25, 57 (2005).

[25] L.G. Sobotka, R.J. Charity, J. Toke and W.U. Schroder, Phys. Rev. Lett. 93, 132702 (2004).

[26] R. Hasse and P. Schuck, Phys. Lett. B 179, 313 (1986).

[27] S. Shlomo and J.B. Natowitz, Phys. Lett. B 252, 187 (1990).

[28] S. Shlomo and J.B. Natowitz, Phys. Rev. C 44, 2878 (1991).

[29] B.A. Li and L.W. Chen, nucl-th/0605002 (2006).

[30] D.T. Khoa and H.S. Than, Phys. Rev. C 71, 044601 (2005).

[31] A. Ono, P. Danielewicz, W.A. Friedman, W.G. Lynch and M.B. Tsang, Phys. Rev. C 68, 051601 (2003).

[32] D.V. Shetty et al., nucl-ex/0505011 (2005).

[33] D.V. Shetty et al., nucl-ex/0603016 (2006).

[34] A. Ono, P. Danielewicz, W.A. Friedman, W.G. Lynch and M.B. Tsang, Phys. Rev. C 70, 041604 (2004).