\(N = 2\) supersymmetric Yang-Mills theory and the superparticle: twistor transform and \(\kappa\)--symmetry

D.V. Uvarov*

Kharkov Institute of Physics and Technology
61108 Kharkov, Ukraine

Abstract

Lagrangian and Hamiltonian dynamics of de Azcarraga-Lukierski \(N = 2\) massive superparticle is considered in the framework of twistor-like Lorentz-harmonic approach. The emphasis is on the study of the interaction with external Abelian gauge superfield. The requirement of preservation of all gauge symmetries of the free model including \(\kappa\)--symmetry yields correct expressions for the superfield strength constraints and determines the form of nonminimal interaction. We also show that for de Azcarraga-Lukierski \(N = 2\) massive superparticle the pullback of field strength 2-superform to the superworld line is not integrable in contrast to the massless superparticle.

1 Introduction

Supersymmetric Yang-Mills theories find the wide range of applications from a search for possible extensions of the Standard Model to string theory. The peculiar feature of superfield formulations of supersymmetric Yang-Mills theories (as well as supergravity theories) is the necessity of the constraints imposition on superfield strengths (supertorsion 2-form) to eliminate numerous auxiliary fields and achieve agreement with the component formulations [1], [2]. The justification of the choice of correct constraints is a subtle matter. It turns out that the investigation of the interaction of supersymmetric particle models [3], [4] can yield proper superfield constraints for super Yang-Mills theories [5], [6], [7], [8], [9] because the minimal interaction is introduced as the pullback of the superpotential 1-form onto the particle’s world line.

*E-mail address: uvarov@kipt.kharkov.ua, d_uvarov@hotmail.com
Another interesting feature of massless superparticles discovered by Witten \[10\], \[7\] is that they provide the necessary framework for the twistor transform of the supersymmetric Yang-Mills theories. This transform is based on the integrability of the superfield strength 2-form pullbacked on the light-like superparticle’s trajectory in superspace. For comparison the twistor transform of selfdual Yang-Mills theory involves 2-complex dimensional null planes \[11\]. So, one needs to have a proper description of supersymmetric light-like lines that possess one bosonic and \(n\) fermionic dimensions and are the trajectories of massless superparticles in the target superspace. The Grassmann dimensionality of supersymmetric light-like lines equals to half of that for the target superspace and is related to the partial breaking of global supersymmetry and local \(\kappa\)–symmetry.

It is explained by the invariance of the massless Brink-Schwarz superparticle \[3\] with an arbitrary number \(N\) of supersymmetries, as well as, the massive de Azcarraga-Lukierski superparticle \[4\] with extended \(N > 1\) supersymmetry under the local \(\kappa\)–symmetry transformations \[4\], \[12\] which allow to gauge away half of the Grassmann coordinates \(\theta\). In the original formulation this symmetry is infinitely reducible and the way to remedy this drawback is to introduce auxiliary Lorentz-harmonic variables \[13\], \[14\], \[15\], \[16\], \[17\], \[18\], \[19\], \[20\] that generalize those harmonic variables advanced in \[21\] to describe theories with extended supersymmetry in superspace. Lorentz harmonic approach is in fact the component version of the more general superembedding approach (for review see e.g. \[22\]) that treats branes as supersurfaces embedded into a target superspace and traces back to the Lund-Regge-Omnes geometric approach to string theory \[23\], \[24\].

Here we develop the Lorentz-harmonic formulation for massive \(N = 2\) de Azcarraga-Lukierski superparticle\(^1\) in the super Yang-Mills background. This allows to realize \(\kappa\)–symmetry in the irreducible form, where its world-line nature is more transparent. The condition for the model to preserve the \(\kappa\)–symmetry after the transition to the super Yang-Mills background requires introduction of nonminimal interaction terms to yield the desired constraints on the superfield strengths. This nonminimal interaction amounts to taking into account the superparticle’s anomalous magnetic moment with the value equal to \(\mu = \frac{e}{2m}\) which is fixed by the \(\kappa\)–symmetry invariance. In \[26\] these results were obtained in the framework of the Hamiltonian approach.

We start from the Lagrangian \[26\] and analyse the Noether identities applying the above-mentioned Lorentz harmonic technique. The correspondence between the triviality of superfield

\(^1\)Superparticles based on de Azcarraga-Lukierski model \[4\] but with the harmonics \[21\] related to the automorphisms group of extended supersymmetry rather than the Lorentz group were considered in \[25\].
strength on the superworld line and the \( \kappa \)--symmetry preservation in the presence of interaction is also examined.

## 2 Lagrangian formulation

de Azcarraga-Lukierski massive superparticle in \( D = 4 \) \( N = 2 \) target superspace is described by the action

\[
S = -m \int \sqrt{-\omega^m_\tau \omega_{\tau \mu}} + im \int \left( \theta^a_\alpha \dot{\theta}^I_\alpha - \bar{\theta}^{\dot{I}} \dot{\bar{\theta}}^{\alpha \dot{I}} \right)^2
\]

where \( \omega^m_\tau = \dot{x}^m + i \theta^a_\alpha \sigma^m_{a\beta} \dot{\theta}^\beta_1 - i \bar{\theta}^{\dot{a}}_1 \sigma^m_{a\beta} \dot{\theta}^{\beta}_{\dot{a}} \), \( \dot{\theta}^I_\alpha, \dot{\bar{\theta}}^{\dot{I}} \) are the pullbacks onto the world line, parametrized by \( \tau \), of the \( D = 4 \) \( N = 2 \) supersymmetric Cartan forms. The automorphisms group of \( N = 2 \) superalgebra is chosen to be \( SU(2) \) and \( \theta^I_\alpha, \theta^{\dot{I}}_\alpha \) belong to its fundamental representation. The second term in (1) is the 1d Wess-Zumino term ensuring the local \( \kappa \)--symmetry invariance of the action. In the first order formalism action (1) is presented as

\[
S = \int d\tau \left[ p_m \omega^m_\tau - \frac{1}{2} \kappa (p^2 + m^2) \right] + im \int d\tau \left( \theta^a_\alpha \dot{\theta}^I_\alpha - \bar{\theta}^{\dot{I}} \dot{\bar{\theta}}^{\alpha \dot{I}} \right), \tag{2}
\]

where \( p_m \) is particle’s momentum and \( \kappa (\tau) \) the Lagrange multiplier imposing the mass-shell constraint \( p^2 + m^2 = 0 \).

In order to solve it in the manifestly covariant way and gain irreducible description of the symmetries of superparticle action we introduce appropriate Lorentz-harmonic variables \( V = (v^{(\mu)}_\alpha, v^{(\dot{\mu})}_\dot{\alpha}) \in SL(2, C) \) defined by the harmonicity conditions

\[
\Xi = \frac{1}{2} \theta^a_\alpha v^{(\mu)}_\alpha - 1 = 0, \quad \bar{\Xi} = \frac{1}{2} \bar{\theta}^{\dot{a}}_{\dot{\alpha}} v^{(\dot{\mu})}_{\dot{\alpha}} - 1 = 0 \tag{3}
\]

which results in \( detv^{(\mu)}_\alpha = detv^{(\dot{\mu})}_{\dot{\alpha}} = 1 \). The inverse harmonics can be expressed as \( V^{-1} = (\epsilon^{\alpha \beta} \epsilon_{(\mu)(\nu)} v^{(\nu)}_\beta, \epsilon^{\dot{\alpha} \dot{\beta}} \epsilon_{(\dot{\mu})(\dot{\nu})} v^{(\dot{\nu})}_{\dot{\beta}}) \in SL(2, C) \), where \( \epsilon^{\alpha \beta}, \epsilon^{\dot{\alpha} \dot{\beta}} \) and \( \epsilon_{(\mu)(\nu)}, \epsilon_{(\dot{\mu})(\dot{\nu})} \) are \( SL(2, C) \) invariant antisymmetric unit tensors. Vector Lorentz harmonics are defined as bilinear combinations of the spinor Lorentz harmonics \( u^{(n)}_m = \frac{1}{2} u^{(\mu)}_\alpha \sigma_{(\mu)(\dot{\nu})} v^{(\nu)}_\beta \sigma^{(n)}_{(\dot{\mu})(\dot{\nu})} = \frac{1}{2} u^{(\mu)}_\alpha \sigma_{m\beta} v^{(\nu)}_\beta \sigma^{(n)}_{(\dot{\mu})} \) and are orthonormal \( u^{(n)}_m u^{(n)}_k = \delta^{(n)}(k) \) as the consequence of the harmonicity conditions (3).

In the presence of massive particle \( SL(2, C)_R \) group acting on indices in brackets of harmonics is isomorphic to \( SO(3) \simeq SU(2) \) so the spinor harmonics can be presented as \( v^{(\mu)}_\alpha = v^{i}_\alpha, v^{(\dot{\mu})}_{\dot{\alpha}} = v^{\dot{i}}_{\dot{\alpha}}, v^{(\nu)}_\beta = v^{\nu}_\beta, v^{(\dot{\nu})}_{\dot{\beta}} = v^{\dot{\nu}}_{\dot{\beta}} \), where index \( i \) corresponds to the fundamental representation

\[
^2\text{Note that the Wess-Zumino term can also be taken of the form } m \int (\theta^a_\alpha \dot{\theta}^I_\alpha + \bar{\theta}^{\dot{I}} \dot{\bar{\theta}}^{\alpha \dot{I}}). \text{ We use another expression since the factor } i \text{ that supersymmetric Cartan forms contain then drops out from the equations of motion for } \theta^a_\alpha, \theta^{\dot{I}} \alpha \dot{I}.\]

\[
\]
of $SU(2)$. They parametrize the coset space $SL(2, C)/SO(3) \simeq SL(2, C)/SU(2)$. Accordingly vector harmonics can be divided into tangential
\begin{equation}
\begin{aligned}
u_m^{(i)} &= -\frac{1}{2}v_\alpha^j\sigma_{\beta}^{\alpha}v_{\beta}^{j} = -\frac{1}{2}v_\alpha^j\sigma_{\beta}^{\alpha}v_{\beta}^{j}, \\
u_m^{(0)} &= \frac{1}{2}v_\alpha^j\sigma_{\beta}^{\alpha}v_{\beta}^{j} = -\frac{1}{2}v_\alpha^j\sigma_{\beta}^{\alpha}v_{\beta}^{j}, \\
u_m^{(0)} \cdot u^{(0)} &= -1
\end{aligned}
\end{equation}

and orthogonal
\begin{equation}
\begin{aligned}
u_m^{(i)} &= -\frac{1}{2}v_\alpha^j\sigma_{\beta}^{\alpha}v_{\beta}^{j} = -\frac{1}{2}v_\alpha^j\sigma_{\beta}^{\alpha}v_{\beta}^{j}, \\
u_m^{(i)} &= \frac{1}{2}v_\alpha^j\sigma_{\beta}^{\alpha}v_{\beta}^{j} = -\frac{1}{2}v_\alpha^j\sigma_{\beta}^{\alpha}v_{\beta}^{j}, \\
u_m^{(i)} \cdot u^{(j)} &= \delta^{(i)(j)}
\end{aligned}
\end{equation}
to the particle’s trajectory sets. In (4) and (5) for brevity summation over Minkowski vector indices was denoted by $\cdot$ and $\tau$-matrices were introduced: $\tau^{(i)}_{j} = \sigma_{\alpha\beta}/\delta^{(i)(j)}$. Indices of the $3-$representation of $SU(2)$ are denoted by small Latin letters in brackets to distinguish from the fundamental representation indices.

The above analysis suggests that the mass-shell constraint $p^2 + m^2 = 0$ can be solved by putting $p_m = mu_m^{(0)}$. This relation is the key to the construction of harmonic formulation for the massive superparticle. The action (2) then reads
\begin{equation}
S = m \int \nu_m^{(0)} \omega^{m}_{\tau} + i m \int (\theta_\alpha^{(i)} \dot{\theta}^{(i)}_{\alpha} - \theta_{\alpha I} \dot{\theta}^{(i)}_{\alpha I})
\end{equation}

Analogous Lorentz-harmonic formulation for $D0-$brane in $D = 10$ $N = 2A$ superspace was considered in [27], using the generalized action approach [28].

The general variation of the action (6) is
\begin{equation}
\delta S = -m \int \nu_m^{(0)} \omega^{m}(\delta) + m \int \delta u_m^{(0)} \omega^{m}_{\tau} + 2 i m \int (v_\alpha^j \dot{\theta}^{(i)}_{\alpha I} + v_\beta^{j} \dot{\theta}^{(i)}_{\beta I}) \delta \theta^{+j}_{I}
\end{equation}

where $\delta \theta^{+j}_{I} = v_\alpha^{ij} \delta \theta_{\alpha I} + v_\beta^{ij} \delta \theta_{\beta I}$, $(\delta \theta^{+j}_{I})^* = -\delta \theta^{+j}_{I}$, and $\omega^{m}(\delta) = \delta x^m + i \theta_\alpha^{(i)} \sigma_{\alpha\beta}^{(i)} \theta^{\beta I} - i \delta \theta_\alpha^{(i)} \sigma_{\alpha\beta}^{(i)} \theta^{\beta I}$. Variation for harmonics requires care since they satisfy orthonormality conditions (3). The most general variation that does not violate them is given by
\begin{equation}
\delta u_m^{(n)} = \Omega^{(n)}_{(k)}(\delta) u_m^{(k)}: \quad \delta u_m^{(0)} = \Omega^{(0)(i)}(\delta) u_m^{(i)}, \quad \delta u_m^{(i)} = \Omega^{(0)(i)}(\delta) u_m^{(0)} + \Omega^{(i)(j)}(\delta) u_m^{(j)}.
\end{equation}

For detailed discussion see [18]–[20]. Expression (7) does not enter the counterpart of $\delta \theta^{+j}_{I}$:
\begin{equation}
\delta \theta^{-j}_{I} = v_\alpha^{ij} \delta \theta_{\alpha I} - v_\beta^{ij} \delta \theta_{\beta I}, \quad (\delta \theta^{-j}_{I})^* = \delta \theta^{-j}_{I}
\end{equation}
so that $\frac{\delta S}{\delta \theta^{+j}_{I}} = 0$. This indicates the invariance of the action (6) under $\kappa-$symmetry transformations in their irreducible version
\begin{equation}
\omega^{m}(\delta) = 0, \quad \delta u_m^{(0)} = 0, \quad \delta \theta^{+j}_{I} = 0 \Rightarrow \delta \theta_\alpha^{(i)} = -\kappa_\alpha^{(i)} v^{(i)}_\alpha, \quad \delta \theta_\alpha^{(i)} = \kappa_\alpha^{(i)} v^{(i)}_\alpha
\end{equation}
with local parameters $(\kappa_\alpha^{(i)})^* = \kappa_\alpha^{(i)}$. Other Noether identities $u_m^{(0)} \frac{\delta S}{\delta \omega_m} = 0, \frac{\delta S}{\delta \Omega^{(0)(j)}} \equiv 0$ correspond to reparametrization invariance and local $SU(2)-$symmetry respectively.
To introduce minimal interaction with external Abelian \( N = 2 \) superpotential \( A_{\alpha}(x, \theta, \bar{\theta}) \) in \(^2\) one has to shift the coefficients at the pullbacks of Cartan forms by the corresponding components of superpotential

\[
p_m \to p_m + ieA_m(x, \theta, \bar{\theta}), \\
m\theta^\alpha_i \to m\theta^\alpha_i - eA^\alpha_i(x, \theta, \bar{\theta}), \\
m\theta_{\dot{\alpha}I} \to m\theta_{\dot{\alpha}I} + eA_{\dot{\alpha}I}(x, \theta, \bar{\theta}).
\]

This yields the following action

\[
S = \int d\tau \left[ p_m \omega^m_{\tau} - \frac{1}{2} v(p^2 + m^2) \right] + im \int d\tau (\theta^\alpha_i \dot{\theta}^I_{\alpha} - \theta_{\dot{\alpha}I} \dot{\theta}^{\dot{\alpha}I}) \\
+ ie \int (\omega^m_{\tau} A_m(x, \theta, \bar{\theta}) + \dot{\theta}^\alpha_i A^I_{\alpha}(x, \theta, \bar{\theta}) + \dot{\theta}^{\dot{\alpha}I} A_{\dot{\alpha}I}(x, \theta, \bar{\theta})).
\]

Note that in \(^1\) we shifted Grassmann coordinates \((\theta^\alpha_i, \theta_{\dot{\alpha}I})\) rather than conjugate momenta \((\pi^\alpha_I, \pi^{\dot{\alpha}I})\) that can be expressed \(^3\) in terms of the coordinates \((\theta^\alpha_i, \theta_{\dot{\alpha}I})\) and \(p_m\) and hence are not independent phase-space variables. However, as was argued in \(^2\) such action is not \(\kappa\)--invariant.

The way to restore \(\kappa\)--symmetry is to introduce nonminimal interaction by rescaling the mass entering the mass-shell constraint

\[
m \to mF,
\]

where \(F(x, \theta, \bar{\theta})\) is a gauge-invariant function of the superfield strengths \(^2\). The action \(^1\) then acquires the form

\[
S = \int d\tau \left[ p_m \omega^m_{\tau} - \frac{1}{2} v(p^2 + (mF)^2) \right] + im \int d\tau (\theta^\alpha_i \dot{\theta}^I_{\alpha} - \theta_{\dot{\alpha}I} \dot{\theta}^{\dot{\alpha}I}) \\
+ ie \int (\omega^m_{\tau} A_m(x, \theta, \bar{\theta}) + \dot{\theta}^\alpha_i A^I_{\alpha}(x, \theta, \bar{\theta}) + \dot{\theta}^{\dot{\alpha}I} A_{\dot{\alpha}I}(x, \theta, \bar{\theta})).
\]

The solution to the generalized mass-shell constraint \(p^2 + (mF)^2 = 0\) is obtained in the same way as for the free model \(p_m = mFu^m_0\) and can be viewed as the rescaling of the vector harmonic \(u^m_0\). Accordingly taking into account the relation \(^4\) between vector and spinor harmonics it is natural to assume that \(F = k\bar{k}\), where the complex conjugate factors \(k(x, \theta, \bar{\theta})\) and \(\bar{k}(x, \theta, \bar{\theta})\) tending to unity when \(e \to 0\) rescale spinor harmonics \(v^\alpha_i\) and \(v_{\dot{\alpha}I}\).

So, the starting point of our analysis is the following action

\[
S = m \int F u^m_0 \omega^m_{\tau} + im \int (\theta^\alpha_i \dot{\theta}^I_{\alpha} - \theta_{\dot{\alpha}I} \dot{\theta}^{\dot{\alpha}I}) + ie \int (\omega^m_{\tau} A_m + \dot{\theta}^\alpha_i A^I_{\alpha} + \dot{\theta}^{\dot{\alpha}I} A_{\dot{\alpha}I}).
\]

The variation of the action \(^1\) is

\[
\delta S = - \int \left[ m \frac{d}{d\tau} (F u^m_0) \omega^m_{\tau} (\delta) - 2i Fu^m_0 \delta \theta^\alpha_i \sigma^m_{\alpha\beta} \dot{\theta}^{\beta I} + 2i Fu^m_0 \delta \theta^{\dot{\alpha}I} \sigma^m_{\dot{\alpha}\dot{\beta}} \dot{\theta}^{\dot{\beta}I} \right] \\
+ m \int \omega^m_{\tau} \delta (F u^m_0) - 2im \int (\dot{\theta}^\alpha_i \delta \theta^\alpha_i - \dot{\theta}_{\dot{\alpha}I} \delta \theta^{\dot{\alpha}I}) + ie \int E^A E^B (\delta) F_{BA},
\]
where \( E^A = (\omega^m, \dot{\theta}^\alpha, \dot{\vartheta}^\alpha) \), \( E^B(\delta) = (\omega^n(\delta), \delta \dot{\theta}^\beta, \delta \dot{\vartheta}^\beta) \) and \( F = dA = \frac{1}{2} E^A E^B F_{BA} \) is the field strength 2-superform. Introducing harmonic variables through the completeness relations \( v^\alpha_i v^\beta_j = \delta^\alpha_\beta, v^\alpha_i v^\alpha_i = \delta^\alpha_\beta \) we express \([15]\) via \( \delta \dot{\theta}^{i-1}_i = k v^\alpha_i \delta \dot{\theta}^\alpha_i + \bar{k} v^\alpha_i \delta \dot{\theta}^\beta_i \) and \( \delta \dot{\theta}^{i+}_i = k v^\alpha_i \delta \dot{\theta}^\alpha_i - \bar{k} v^\alpha_i \delta \dot{\theta}^\beta_i \). Thus, \([13]\) acquires the form

\[
\delta S = \int \left[ -m F u^{(0)}_m + i e \omega^m_\tau (F_{mn} + \frac{im}{e} (u^{(0)}_m \partial_n F - u^{(0)}_n \partial_m F)) \right. \\
+ i e \dot{\theta}^\alpha_i (F_{m\alpha} + \frac{im}{e} u^{(0)}_m D^\alpha F) + i e \dot{\vartheta}^\alpha_i (F_{m\dot{\alpha}} + \frac{im}{e} u^{(0)}_m D^\alpha F) \left. \right] \omega^m(\delta) \\
- \frac{ie}{2} \int \left[ \omega^m_\tau (k^{-1} v^\alpha_\bar{i} (F_{\bar{i}\bar{\alpha}} + \frac{im}{e} u^{(0)}_m D^\alpha F) - \bar{k}^{-1} v^\beta_i (F_{\bar{i}\bar{\beta}} + \frac{im}{e} u^{(0)}_m D^\alpha F)) \right. \\
+ \dot{\theta}^\alpha_i (k^{-1} v^\beta_i (F^{\bar{i}\alpha} - \bar{v}^\alpha_i (F^{\bar{i}\beta} - \bar{k}^{-1} F)\varepsilon_{\alpha\beta} \varepsilon^{ff})) \right. \\
- \frac{ie}{2} \int \left[ \omega^m_\tau (k^{-1} v^\alpha_i (F_{\bar{i}\bar{\alpha}} + \frac{im}{e} u^{(0)}_m D^\alpha F) + \bar{k}^{-1} v^\beta_i (F_{\bar{i}\bar{\beta}} + \frac{im}{e} u^{(0)}_m D^\alpha F)) \right. \\
+ \dot{\vartheta}^\beta_i (k^{-1} v^\alpha_i (F^{\bar{i}\alpha} + \frac{im}{e} u^{(0)}_m D^\alpha F) + \bar{k}^{-1} v^\beta_i (F^{\bar{i}\beta} - \bar{k}^{-1} F)\varepsilon_{\alpha\beta} \varepsilon^{ff})) \right) \delta \theta^{i+}_i \\
- \frac{ie}{2} \int \left[ \omega^m_\tau (k^{-1} v^\alpha_i (F_{\bar{i}\bar{\alpha}} + \frac{im}{e} u^{(0)}_m D^\alpha F) + \bar{k}^{-1} v^\beta_i (F_{\bar{i}\bar{\beta}} + \frac{im}{e} u^{(0)}_m D^\alpha F)) \right. \\
+ \dot{\theta}^\alpha_i (k^{-1} v^\beta_i (F^{\bar{i}\alpha} + \frac{im}{e} u^{(0)}_m D^\alpha F) + \bar{k}^{-1} v^\beta_i (F^{\bar{i}\beta} - \bar{k}^{-1} F)\varepsilon_{\alpha\beta} \varepsilon^{ff})) \right) \delta \theta^{i-}_i \\
+ m \int F_{\omega \tau} \cdot u^{(0)}_i \Omega^{(0)(i)}(\delta).
\]

From the variation \([16]\) we derive the equations of motion

\[
\frac{\delta S}{\delta \Omega^{(0)(i)}} = \omega_\tau \cdot u^{(i)} = 0,
\]

\[
\frac{\delta S}{\delta \omega_m} = -m F u^{(0)}_m + i e \omega^m_\tau (F_{mn} + \frac{im}{e} (u^{(0)}_m \partial_n F - u^{(0)}_n \partial_m F)) \\
+ i e \dot{\theta}^\alpha_i (F_{m\alpha} + \frac{im}{e} u^{(0)}_m D^\alpha F) + i e \dot{\vartheta}^\alpha_i (F_{m\dot{\alpha}} + \frac{im}{e} u^{(0)}_m D^\alpha F) = 0,
\]

\[
\frac{2}{ie} \frac{\delta S}{\delta \theta^{i+}_i} = \omega^m_\tau (k^{-1} v^\alpha_i (F^{\bar{i}\alpha} + \frac{im}{e} u^{(0)}_m D^\alpha F) - \bar{k}^{-1} v^\beta_i (F^{\bar{i}\beta} + \frac{im}{e} u^{(0)}_m D^\alpha F)) \\
+ \dot{\theta}^\alpha_i (k^{-1} v^\beta_i (F^{\bar{i}\alpha} + \frac{im}{e} u^{(0)}_m D^\alpha F) + \bar{k}^{-1} v^\beta_i (F^{\bar{i}\beta} - \bar{k}^{-1} F)\varepsilon_{\alpha\beta} \varepsilon^{ff})) = 0,
\]

\[
\frac{2}{ie} \frac{\delta S}{\delta \theta^{i+}_i} = \omega^m_\tau (k^{-1} v^\alpha_i (F^{\bar{i}\alpha} + \frac{im}{e} u^{(0)}_m D^\alpha F) + \bar{k}^{-1} v^\beta_i (F^{\bar{i}\beta} + \frac{im}{e} u^{(0)}_m D^\alpha F)) \\
+ \dot{\vartheta}^\beta_i (k^{-1} v^\alpha_i (F^{\bar{i}\alpha} + \frac{im}{e} u^{(0)}_m D^\alpha F) - \bar{k}^{-1} v^\beta_i (F^{\bar{i}\beta} - \bar{k}^{-1} F)\varepsilon_{\alpha\beta} \varepsilon^{ff})) = 0.
\]

The Noether identities for the reparametrization symmetry and local \( SU(2) \)–invariance have the form

\[
u^{(0)}_m \frac{\delta S}{\delta \omega_m} + i e (u^{(n)(i)} F_{nm} u^{m(0)} + \frac{im}{e} u^{m(i)} \partial_m F) \frac{\delta S}{\delta \Omega^{(0)(i)}} + \\
\frac{1}{\omega_\tau \cdot u^{(i)}} [(k \theta^\alpha_i v^\alpha_i + \bar{k} \theta^\alpha_i v^\alpha_i) \frac{\delta S}{\delta \theta^{i+}_i} + (k \vartheta^\beta_i v^\beta_i - \bar{k} \vartheta^\beta_i v^\beta_i) \frac{\delta S}{\delta \theta^{i-}_i}] = 0,
\]
\[
\frac{\delta S}{\delta \Omega^{(i)(j)}} \equiv 0,
\]
(22)

and hold for any values of the background superfield strength and functions \(k\) and \(\bar{k}\). When the interaction is switched off \(\frac{\delta S}{\delta \theta} \equiv 0\) that reflects the presence of \(\kappa\)–symmetry of the action (3).

Thus, for the action (14) to be invariant under \(\kappa\)–symmetry transformations in the presence of external background we require (20) to be satisfied identically for any values of \(x^m, \theta^I, \theta_{\alpha I}, \phi^I, \psi_{\dot{\alpha}i}\) when the other equations of motion are taken into account, i.e. we consider equations (20) as restrictions for the background superfield strengths. In this way we obtain from the last two lines of (20) that contain the lowest dimension \([L]^{-1}\) superstrength components

\[
F^{I \dot{J}}_{\alpha \dot{\beta}} = \frac{2m}{e} (1 - k^2) \varepsilon_{\alpha \beta} \varepsilon^{I \dot{J}} = 0, \quad F^{\dot{I} \dot{J}}_{\dot{\alpha} \dot{\beta}} = \frac{2m}{e} (1 - \bar{k}^2) \varepsilon_{\dot{\alpha} \dot{\beta}} \varepsilon^{I \dot{J}} = 0, \quad F^{I \dot{J}}_{\alpha \dot{\beta}} = 0.
\]
(23)

To analyse the content of the constraints (23) we decompose spinor-spinor components of superfield strengths on the \(SU(2)\)–irreducible parts

\[
F^{I \dot{J}}_{\alpha \dot{\beta}} = -\varepsilon_{\alpha \beta} \varepsilon^{I \dot{J}} \bar{W} + \tau(I) \bar{W} F^{I}_{\alpha \dot{\beta}}, \quad F^{\dot{I} \dot{J}}_{\dot{\alpha} \dot{\beta}} = -\varepsilon_{\dot{\alpha} \dot{\beta}} \varepsilon^{I \dot{J}} W + \tau(I) \dot{W} F^{(I)}_{\dot{\alpha} \dot{\beta}}.
\]
(24)

Substituting these expansions back into (23) yields

\[
1 + \frac{eW}{2m} = \bar{k}^2, \quad 1 + \frac{e\bar{W}}{2m} = k^2, \quad F^{(I)}_{\alpha \dot{\beta}} = 0, \quad F^{(I)}_{\dot{\alpha} \dot{\beta}} = 0
\]
(25)

so that

\[
k = \sqrt{1 + \frac{eW}{2m}}, \quad \bar{k} = \sqrt{1 + \frac{e\bar{W}}{2m}}, \quad F = \sqrt{\left(1 + \frac{eW}{2m}\right) \left(1 + \frac{e\bar{W}}{2m}\right)}.
\]
(26)

Solution for the Bianchi identities for superfield strengths satisfying the constraints \(F^{(I)}_{\alpha \dot{\beta}} = 0, F^{(I)}_{\dot{\alpha} \dot{\beta}} = 0\) and \(F^{I \dot{J}}_{\alpha \dot{\beta}, J} = 0\) yields [29] that the superfields \(W\) and \(\bar{W}\) are chiral \(D_{\dot{\alpha} I} W = D_{\alpha I} \bar{W} = 0\) and spinor-vector superfield strength components are of the form

\[
F^I_{\alpha \beta} = -\frac{i}{4} \sigma_{\alpha \beta} \partial^I W, \quad F^I_{\dot{\alpha} \dot{\beta}} = -\frac{i}{4} \sigma_{\dot{\alpha} \dot{\beta}} D^I \bar{W}.
\]
(27)

Thus, the first line of (20) turns to 0 identically. Note that for the minimal interaction \(F = 1\) there remains additional algebraic constraint \(\sqrt{\left(1 + \frac{eW}{2m}\right) \left(1 + \frac{e\bar{W}}{2m}\right)} = 1\) that eliminates physical degrees of freedom of the superfields \(W\) and \(\bar{W}\).

3 Hamiltonian formulation

The Noether identities of the Lagrangian description are in one-to-one correspondence with the first-class constraint in the Hamiltonian picture. To discover expressions for these constraints
we perform the Hamiltonian analysis of the model. To this end we introduce canonical momenta conjugate to coordinates \( Q = (x^m, \theta^I_\alpha, \dot{\theta}^I_\alpha, v^i_\alpha, \dot{v}^i_\alpha) \)

\[ P = \frac{\partial L}{\partial \dot{Q}} \]  
(28)

and find the set of the primary constraints

\[ \Phi_m = \mathcal{P}_m - mF_u^{(0)} \approx 0, \]  
(29)

\[ V^I_\alpha = \pi^I_\alpha + ip_{\alpha\dot{\alpha}}\dot{\theta}^I_\dot{\alpha} + im\theta^I_\alpha - ieA^I_\alpha \approx 0, \quad V_{\dot{\alpha}I} = \pi_{\dot{\alpha}I} + i\dot{\theta}^I_\alpha p_{\alpha\dot{\alpha}} - im\theta_{\dot{\alpha}I} - ieA_{\dot{\alpha}I} \approx 0, \]  
(30)

\[ P^i_\alpha \approx 0, \quad P_{\dot{\alpha}i} \approx 0, \]  
(31)

where generalized momentum is defined as \( \mathcal{P}_m = p_m - ieA_m \). In harmonic sector there are two extra constraints. To exclude them from the list of constraints one can utilize covariant momenta in harmonic sector

\[ \Pi^{(k)(l)} = v^{(\mu)}_\alpha \sigma^{(k)(l)}_{(\mu)} P^{(\nu)}_\alpha - P^{(\mu)}_{(\nu)} \tilde{\sigma}^{(k)(l)(\nu)}_{(\mu)} v^{(\mu)}_\alpha \approx 0. \]  
(32)

These \( 6=\dim SO(1, 3) \) constraints commute with harmonicity conditions and form the \( SO(1, 3) \) algebra on the Poisson brackets. Covariant momenta are decomposed on \( SU(2) \)–covariant parts as follows

\[ \Pi^{(0)(i)} = -\frac{1}{2} v^{(i)}_\alpha \tau^{(i)} j P^a_j - \frac{1}{2} P^{\dot{\alpha}i \tau^{(i)} j} v_{\dot{a}j} \approx 0, \]  
(33)

\[ \Pi^{(i)(j)} = -v^{(i)}_\alpha \tau^{(i)(j)} j P^a_j + P^{\dot{a}i \tau^{(i)(j)} j} v_{\dot{a}j} \approx 0. \]  
(34)

As we shall see below constraints belong to the second class, whereas the constraints to the first class and generate on the Poisson brackets \( SU(2) \)–symmetry transformations.

To evaluate Poisson brackets of the primary constraints we adopt the following definitions

\[ \{p_m, x^n\} = -i\delta^a_m, \quad \{\pi^I_\alpha, \theta^j_\beta\} = -i\delta^\beta_j \delta^I_\alpha, \quad \{\pi_{\dot{a}I}, \dot{\theta}^j_\beta\} = -i\delta^\beta_j \delta^I_\alpha, \]  
(35)

\[ \{P^i_\alpha, v^j_\beta\} = -i\delta^\beta_j \delta^i_\alpha, \quad \{P_{\dot{a}i}, \dot{v}^j_\beta\} = \delta^\beta_j \delta^i_\alpha \]  
(36)

and find

\[ \{\Phi_m, \Phi_n\} = -ef_{mn} - im(v^{(0)}_m \partial_n F - v^{(0)}_n \partial_m F), \]  
(37)

\[ \{\Phi_m, V^I_\alpha\} = -ef_{m\alpha} - imv^{(0)}_m D^I_\alpha F, \quad \{\Phi_m, V_{\dot{a}I}\} = -ef_{m\dot{a}} - imv^{(0)}_m D_{\dot{a}I} F, \]  
(38)

\[ \{V^I_\alpha, V^J_{\beta}\} = 2m\varepsilon_{\alpha\beta}^{IJ} \left( 1 + \frac{eW}{2m} \right), \quad \{V^I_\alpha, \dot{V}^J_{\beta}\} = 2m\varepsilon_{\dot{a}\dot{a}}^{IJ} \left( 1 + \frac{eW}{2m} \right), \]  
(39)

\[ \{V^I_\alpha, V_{\dot{a}J}\} = 2\delta^J_\alpha P_{\dot{a}J}. \]  
(40)
In harmonic sector we have
\[ \{ \Pi^{(i)(j)} , \Pi^{(l)(j')} \} = i ( \delta^{(j)(j')} \Pi^{(i)(j)} - \delta^{(i)(j')} \Pi^{(j)(j)} ) , \]  
\( \{ \Pi^{(0)(i)}, \Pi^{(j)(j')} \} = i \delta^{(0)(j')} \Pi^{(0)(j)} - i \delta^{(i)(j')} \Pi^{(0)(j)} , \quad \{ \Pi^{(0)(i)}, \Pi^{(0)(j)} \} = i \Pi^{(i)(j)} , \]  
\[ \{ \Pi^{(0)(i)}, \Phi_m \} = i m F u^{(i)}_m , \quad \{ \Pi^{(i)(j)}, \Phi_m \} = 0 , \]  
\[ \{ \Pi^{(0)(i)}, V'_{\alpha} \} = 0 , \quad \{ \Pi^{(i)(j)}, V'_{\alpha} \} = 0 . \]  
Canonical Hamiltonian is defined by the expression
\[ H_0 = \dot{x}^m p_m + \dot{\theta}^i \pi^I_{\alpha} + \dot{\theta}^\lambda \pi_{\alpha I} + \Omega^{(0)(i)} \Pi^{(0)(i)} + \Omega^{(i)(j)} \Pi^{(i)(j)} - L , \]  
where \( \Omega^{(0)(i)} \) and \( \Omega^{(i)(j)} \) are the pullbacks onto the world line of Cartan forms conjugate to covariant momenta. Following the Dirac method we add to \( H_0 \) a linear combination of the primary constraints with arbitrary Lagrange multipliers to obtain the total Hamiltonian
\[ H = \lambda^I_{\alpha I} V'_{\alpha} + \lambda^{\alpha I} V'_{\alpha I} + a^m \Phi_m + \eta^{(i)(j)} \Pi^{(i)(j)} \approx 0 . \]  

The next step is the exploration of the conservation conditions for the set of the primary constraints \[ 29, 30, 33, 34 \] \( \dot{f} = i \{ f , H \} \approx 0 . \) As the outcome the total Hamiltonian is presented as a linear combination of the first-class constraints with arbitrary Lagrange multipliers
\[ H = \lambda_{i I} V'^{i I} + a T + \eta^{(i)(j)} \Pi^{(i)(j)} \approx 0 , \]  
where
\[ V'^{i I} = k^{-1} v^{i I} V'_{\alpha} + \bar{k}^{-1} v^{\alpha I} V'_{\bar{\alpha}} + \frac{1}{F} \Pi^{(0)(i)} \tau^{(i)} \frac{1}{4} (v^{\alpha j} D^I_{\alpha} \bar{k} - v^{\alpha j} D^I_{\bar{\alpha}} k) \approx 0 \]  
is the \( \kappa \)–symmetry generator,
\[ T = u^{n(0)} \Phi_m + \frac{1}{m F} (i e u^{m(i)} F_{mn} u^{n(0)} - m u^{n(i)} \partial_m F) \Pi^{(0)(i)} + \frac{i}{4} (v^{\alpha i} D^I_{\alpha} k - v^{\alpha i} D^I_{\bar{\alpha}} k) (k^{-1} v^I_{\beta I} \bar{k} - \bar{k}^{-1} v^I_{\beta I} \bar{k} + \frac{1}{F} \Pi^{(0)(i)} \tau^{(i)} j (v^I_{\beta I} D^I_{\beta I} k - v^I_{\beta I} D^I_{\beta I} k)) \approx 0 \]  
generates reparametrizations and \( \Pi^{(i)(j)} \) local SU(2)–transformations.

The set of the second-class constraints can be chosen as
\[ D^{i I} = k^{-1} v^{i I} V'_{\alpha} - \bar{k}^{-1} v^{\alpha I} V'_{\bar{\alpha}} \approx 0 , \quad \Phi^{(i)} = u^{(i)} \cdot \mathcal{P} \approx 0 , \quad \Pi^{(0)(i)} \approx 0 . \]  

Local symmetries of the action \[ 14 \] are generated on the Poisson brackets by the first-class constraints. \( \kappa \)–Symmetry transformations have the form
\[ \delta \theta^I_{\alpha} = k^{-1} \kappa^I v_{\alpha} , \quad \delta \theta^I_{\bar{\alpha}} = \bar{k}^{-1} \kappa^I v^I_{\bar{\alpha}} , \quad \omega^m (\delta) = 0 , \]  
\[ \delta u^{(i)}_m = F^{-1} k^I u^{(i)}_m \tau^{(i)} j (v^I_{\beta I} D^I_{\beta I} k - v^I_{\beta I} D^I_{\beta I} k) \]
with four anticommuting local parameters $\kappa^i_I(\tau)$, $\kappa^i_I)^* = \kappa^I_i$. For reparametrization symmetry we find the following transformation laws

$$
\delta \theta^I_a = \frac{i e a}{16mFk} h^I_i \nu^{\alpha i}, \quad \delta \theta_{\dot{\alpha}I} = -\frac{i e a}{16mFk} h^I_{\dot{\alpha}i} \nu_{\dot{\alpha}i}, \quad \omega^m(\delta) = -a u^m(0),
$$

$$
\delta u^m_{(0)} = \frac{a}{mFk} u^m_{(i)} (m u^{n(i)} \partial_n F - i e u^{n(i)} F_{np} \nu^{p(0)}) + \frac{ie a}{16} h^I_i \tau^{(i)}_I (v^\beta_{\dot{\beta}1k} + v^\dot{\beta}_{\beta1k})),
$$

where $h^I_i = \bar{W} = kv^{\alpha i} D^I_{\alpha} k$, with the local parameter $a(\tau)$. Finally the $SU(2)-$symmetry with parameters $b^{(i)(j)}(\tau)$ affects only harmonics

$$
\delta v^{i}_{\alpha} = v^j_{\alpha} b^{(i)(j)} \tau^{(i)(j)}_j, \quad \delta v^i_{\dot{\alpha}} = v^j_{\dot{\alpha}} b^{(i)(j)} \tau^{(i)(j)}_j \Rightarrow \delta u^m_{(0)} = 0.
$$

4 Integrability of the superfield strength on the particle’s superworld line and $\kappa-$symmetry

Let us now consider the property of integrability of superfield strength 2–form on the particle’s superworld line. As is known the triviality of $N = 1 D = 3, 4, 6, 10$ super Yang-Mills field strength pullbacked to the supersymmetric world line of massless superparticle, is in one-to-one correspondence with the superfield constraints $F_{\alpha \beta} = 0$ \[10\], \[7\]. This phenomenon is intimately related to $\kappa-$symmetry invariance of the massless superparticle that is also preserved in the background of $N = 1$ super Yang-Mills theory and constitutes the basis for the twistor transform.

In the case of the massive particle its superworld line is parametrized by the single bosonic coordinate $\tau$ and four fermionic coordinates $\eta^I_i (\eta_i^{I*} = \eta_i^I)$. As a result of the geometric properties of the (1, 4)–supermanifold induced by the superworld line embedding \[22\], the einbein one-superform $e^\dot{\alpha} = d\zeta m e^{\dot{\alpha}}(\zeta) = (e^\tau, e^I_i)$ can be chosen to be flat and the superconnection one-form $\hat{\Omega}^i_j$ may be identified with the correspondent form from the harmonic sector. The analysis of the embedding coditions

$$
\omega \cdot u^{(0)} = -e^\tau, \quad \omega \cdot u^{(i)} = 0, \quad \frac{1}{2}(v^{\alpha i} d\theta_{\alpha I} - v^{\dot{\alpha}i} d\theta_{\dot{\alpha}I}) = e^I_i
$$

results in the general decompositions for the target superspace one-forms

$$
\omega_m = e^\tau u^{(0)}_m, \quad d\theta^I_a = v^i_{\alpha} e^I_i, \quad d\theta_{\dot{\alpha}I} = v_{\dot{\alpha}i} e^I_i.
$$

\[3\] For the treatment of supergravity constraints as integrability conditions see \[30\].
Then one observes that the field strength 2–superform pullbacked to the massive particle’s superworld line is nonzero

\[ F|_{M(1|4)} = \frac{i}{4} e^I e^J (v^\alpha_i D^I_{\alpha} W + v^\alpha_i D^I_{\alpha} \bar{W}) + \frac{1}{2} e^I e^J (W + \bar{W}) \neq 0, \]

(56)

where we used the superfield constraints (23), (25), (27). On the contrary it can be verified that the field strength 2–superform pullbacked to the superworld line of \( N = 2 \) massless superparticle is trivial on the mass shell. However, the action of the massless superparticle coupled to \( N = 2 \) Abelian superpotential is not \( \kappa \)–invariant. Indeed, the rank of the Grassmann constraints’ matrix \( G_0 \) for free \( N = 2 \) massless superparticle

\[ V^I_{\alpha} = \pi^I_{\alpha} + ip_{\alpha\dot{\alpha}} \theta^{\dot{\alpha}I} \approx 0, \quad V_{\dot{\alpha}I} = \pi_{\dot{\alpha}I} + i \theta_{\dot{\alpha}I} p_{\alpha\dot{\alpha}} \approx 0 \]

(57)

\[ G_0 = \delta^I_0 \begin{pmatrix} 0_2 & \rho_{\alpha\dot{\alpha}} \\ \rho^{\dot{\alpha}\beta} & 0_2 \end{pmatrix} \]

(58)

is halved because of the mass shell constraint \( p^2 \approx 0 \). Whereas in the \( N = 2 \) super Yang-Mills background this is not the case due to the nontrivial contribution from \( F^{IJ}_{\alpha\dot{\alpha}} = -\varepsilon_{\alpha\beta} \varepsilon^{IJ} \bar{W} \) and

\[ F^{IJ}_{\dot{\alpha}\dot{\beta}} = -\varepsilon_{\dot{\alpha}\dot{\beta}} \varepsilon^{IJ} W \]

\[ G_{int} = \delta^I_0 \begin{pmatrix} \delta^\beta_\alpha eW & \mathcal{P}_{\alpha\dot{\alpha}} \\ \mathcal{P}^{\dot{\alpha}\beta} & -\delta^\beta_\alpha eW \end{pmatrix}, \]

(59)

where \( \mathcal{P}^2 \approx 0 \).

5 Conclusion

We have considered de Azcarraga-Lukierski massive \( N = 2 \) superparticle in the twistor-like Lorentz harmonic formulation nonminimally coupled to external Abelian Maxwell supermultiplet both in Lagrangian and Hamiltonian approaches. In the Lagrangian approach local symmetries of the superparticle are encoded in the Noether identities. The introduction of harmonic variables allows to realize these symmetries (in particular, \( \kappa \)–symmetry) in the irreducible form. We established that in the presence of the interaction the requirement of preservation of all the symmetries (Noether identities) of the free superparticle identifies the proper constraints on the superfield strengths that single out \( N = 2 \) extended Maxwell supermultiplet through the solution of the Bianchi identities. It also fixes the form of the gauge-invariant nonminimal interaction.
In the Hamiltonian approach we deduced irreducible first-class constraints that generate on the Poisson brackets $\kappa-$symmetry, world-line reparametrizations and local $SU(2)-$transformations.

In the last section we addressed the issue of an interplay between the triviality of the Abelian field strength 2-superform, when restricted to the particle’s superworld line, and $\kappa-$symmetry. In case of $N = 1 \; D = 3, 4, 6, 10$ massless particle the condition of triviality of the superfield strength on the superworld line, that can be viewed as the supersymmetric generalization of ordinary light-like line, is equivalent to conventional constraints on superfield strengths. It is also well known that the same constraints can be obtained when considering the massless superparticle in the background of external superpotential and requiring the action to be $\kappa-$invariant. However, when dealing with $N = 2$ super Yang-Mills theory the situation changes. Though the superfield strength remains trivial on the superworld line of the $N = 2$ massless superparticle the action is no more $\kappa-$invariant in the background of $N = 2$ superpotential. The possibility to preserve $\kappa-$symmetry exists for massive superparticle and is based on the introduction of gauge invariant nonminimal interaction. As was shown in [26] it actually corresponds to the interaction caused by superparticle’s anomalous magnetic moment of the magnitude $\mu = \frac{e}{2m}$. Therefore, in the case of $N = 2$ supersymmetry the $\kappa-$symmetry requirement is not compatible with the triviality condition for the Abelian superfield strength 2-form independently whether the superparticle is massless or massive.

It is also interesting to compare the dilaton dependence of the action [13, 14] with that for the $D0-$brane in $D = 10 \; N = 2A$ supergravity background [31]. There the role of superpotential $A_A(x, \theta, \bar{\theta})$ is played by the RR 1-superform minimally coupled to $D0-$brane. It has nonzero limit when the interaction is switched off producing 1d Wess-Zumino term. Its superfield strength components depend on dilaton superfield $\phi(x, \theta)$ and its derivatives to compensate the $\kappa-$variation of the kinetic Dirac-Born-Infeld term containing the exponent of the dilaton superfield as the scaling in analogy with the function $F$ in our case. $F$ is the function of gauge-invariant superfields $W = \frac{1}{4} F^{\alpha I}_{\alpha I}$ and $\bar{W} = -\frac{1}{4} F^{\alpha I}_{I\alpha}$ which leading component is the complex dilaton-axion scalar-field $z(x) = \phi(x) + i a(x)$ of the $D = 4 \; N = 2$ Maxwell supermultiplet. The fact that $D = 10 \; N = 2A$ supergravity theory is the dimensional reduction of $D = 11$ supergravity suggests that the nonminimal action [13, 14] (or its generalization to include supergravity background) can also be obtained by dimensional reduction of $D = 5 \; N = 1$ massless superparticle model minimally coupled to a Maxwell superfield or $D = 5$
Another line for further investigation is to extend the above considered analysis of $N = 2$ super Yang-Mills theory to the most interesting case of $N = 4$ super Yang-Mills theory due to the AdS/CFT-correspondence conjecture \cite{33}. In this case it should be noted, however, that the superfield constraints like (23), (25), (27) put the theory on the mass shell.

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