Next-to-Minimal SOFTSUSY

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Abstract

We describe an extension to the SOFTSUSY program that provides for the calculation of the sparticle spectrum in the Next-to-Minimal Supersymmetric Standard Model (NMSSM), where a chiral superfield that is a singlet of the Standard Model gauge group is added to the Minimal Supersymmetric Standard Model (MSSM) fields. Often, a $\mathbb{Z}_3$ symmetry is imposed upon the model. SOFTSUSY can calculate the spectrum in this case as well as the case where general $\mathbb{Z}_3$ violating (denoted as $\mathbb{Z}_3^\text{v}$) terms are added to the soft supersymmetry breaking terms and the superpotential. The user provides a theoretical boundary condition for the couplings and mass terms of the singlet. Radiative electroweak symmetry breaking data along with electroweak and CKM matrix data are used as weak-scale boundary conditions. The renormalisation group equations are solved numerically between the weak scale and a high energy scale using a nested iterative algorithm. This paper serves as a manual to the NMSSM mode of the program, detailing the approximations and conventions used.

Keywords: sparticle, NMSSM, Higgs

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1. Program Summary

Program title: SOFTSUSY
Program obtainable from: http://softsusy.hepforge.org/
Distribution format: tar.gz
Programming language: C++, fortran
Computer: Personal computer.
Operating system: Tested on Linux 3.x
Word size: 64 bits.
External routines: None.
Typical running time: A few tenths of a second per parameter point.
Nature of problem: Calculating supersymmetric particle spectrum and mixing parameters in the next-to-minimal supersymmetric standard model. The solution to the renormalisation group equations must be consistent with boundary conditions on supersymmetry breaking parameters, as well as on the weak-scale boundary condition on gauge couplings, Yukawa couplings and the Higgs potential parameters.

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Solution method: Nested iterative algorithm and numerical minimisation of the Higgs potential.

Restrictions: SOFTSUSY will provide a solution only in the perturbative regime and it assumes that all couplings of the model are real (i.e. CP-conserving). If the parameter point under investigation is non-physical for some reason (for example because the electroweak potential does not have an acceptable minimum), SOFTSUSY returns an error message.

CPC Classification: 11.1 and 11.6.

Does the new version supersede the previous version?: Yes.

Reasons for the new version: Major extension to include the next-to-minimal supersymmetric standard model.

Summary of revisions: Added additional supersymmetric and supersymmetry breaking parameters associated with the additional gauge singlet. Electroweak symmetry breaking conditions are significantly changed in the next-to-minimal mode, and some sparticle mixing changes. An interface to NMSSMTools has also been included. Some of the object structure has also changed, and the command line interface has been made more user friendly.
2. Introduction

While TeV-scale supersymmetric particles have not yet been found at the LHC \([1,2]\), searches for them continue along with continuing strong theoretical interest in supersymmetric (SUSY) models. This is a testament to the theoretical successes of weak-scale supersymmetry: chiefly the resolution of the technical hierarchy problem, improvement of the apparent unification of Standard Model (SM) gauge couplings and the provision of a potential dark matter candidate. In order to pursue SUSY phenomenology, a long calculational chain is required \([3]\). Typically, this chain begins with the calculation of the supersymmetric spectrum, including the couplings of the various sparticles and Higgs bosons. Currently, in the Minimal Supersymmetric Standard Model (MSSM), there are several spectrum generators: ISASUSY \([4]\), SOFTSUSY \([5]\), SPheno \([6]\), SUSEFLAV \([7]\) and SUSPECT \([8]\). Information from these spectrum generators is then passed to other programs (for example those that calculate decays, that simulate collider events, or that calculate the thermal relic density of dark matter) via data in the SUSY Les Houches Accord format \([9]\).

Recently a boson was discovered in the CMS and ATLAS experiments at over the 5–σ level \([10,11]\) with properties consistent with the SM Higgs boson. Using 4.8 fb\(^{-1}\) of 7 TeV data and 20.7 fb\(^{-1}\) of 8 TeV data, ATLAS measures the mass to be \(m_h = 125.5 \pm 0.2^{+0.5}_{-0.6}\) GeV by combining the \(H \to \gamma\gamma\) and \(H \to ZZ\) decay channels \([12]\). In CMS, these channels give the combined constraint \(m_h = 125.3 \pm 0.4 \pm 0.5\) GeV in 5.1 fb\(^{-1}\) of 7 TeV data and 5.3 fb\(^{-1}\) of 8 TeV data. In the MSSM, one can often obtain a CP even Higgs that couples in a similar way to the Standard Model Higgs boson. At tree-level, its mass is bounded by \(m_{\tilde{g}} < M_{\tilde{g}}\), at odds with the LHC experiments’ mass measurements. However, the radiative corrections to the CP even Higgs mass can be sizeable, particularly those from stops. The corrections are larger if the stops are heavy, and if they are heavily mixed. Indeed, the MSSM has enough flexibility \([13]\) such that the experimental values of \(m_{\tilde{g}}\) are achievable with TeV-scale stops and large mixing. On the other hand, these relatively heavy stops reintroduce the little hierarchy problem, requiring cancellation (at the level of one in several tens) between apparently unrelated parameters in the MSSM Higgs potential. Thus, we have the well known correlation \([14]\) between a higher Higgs mass \(m_{\tilde{g}} > 106\) GeV and a higher level of apparently unnatural cancellation. In several well-studied simple models of supersymmetry breaking mediation, the problem is much exacerbated \([15]\).

In order to reduce the unnatural cancellations implied by the Higgs mass measurement \([16,17,18,19,20]\), one can augment the MSSM by a gauge singlet chiral superfield \([21,22,23]\). This model is referred to as the Next-to-Minimal Supersymmetric Standard Model (NMSSM) \([24]\). We shall distinguish between a version where an extra symmetry is assumed (often a \(Z_3\) symmetry) and a version where it is not \([25,26,27]\). In the MSSM, (based on a two Higgs doublet version of the SM with softly broken \(N = 1\) global supersymmetry) the tree-level bound upon the Higgs mass comes from the fact that the quartic Higgs couplings are related to the electroweak gauge couplings by supersymmetry. The Higgs potential is modified by the addition of a gauge singlet, and the resulting lightest CP even Higgs boson can receive additional positive corrections to its mass at tree-level. In addition, the neutral Higgs potential (now a function of three fields rather than two in the MSSM) is heavily modified, with associated potential reductions in the unnatural cancellations. Along with other factors this had lead to considerable interest in the NMSSM in the recent literature and benchmarks points with a 125 GeV Higgs have already been proposed \([28]\). It is therefore essential for the research community to have access to a variety of reliable computational tools to calculate the relevant NMSSM observables.

As mentioned above in the MSSM case, the initial step in a calculational chain is typically spectrum and couplings calculation. Currently, there is one out-of-the-box package NMSSM \([29]\) which calculates the spectrum of the NMSSM, matching weak-scale data with theoretical boundary conditions on supersymmetry breaking and Higgs potential parameters. However, one can also marry SARAH \([30,31,32,33]\) with SPheno \([6]\) in order to be able to calculate the spectrum after setting up the model. The NMSSM was included in an extended version of the SUSY Les Houches Accord \([34]\) so that this information may be passed to programs performing other calculations. For instance, NMHDECAY \([35]\) is capable of calculating the NMSSM Higgs decays, and NMSSDECAY \([37,38]\) calculates sparticle decays. PYTHIA \([39]\) is then capable of simulating particle collisions in the NMSSM and, in addition, micrOMEGAs \([40]\) can calculate the thermal dark matter relic density.

Having several public spectrum generators for the MSSM has proved fruitful for the community. As well as comparisons and bug-finding, the various generators have different levels of approximations and are able to calculate

\[\text{In some cases, lower bounds of 1 TeV or more have been placed upon gluinos and squarks by LHC experiments.}\]

\[\text{This has also been done in some non-NMSSM contexts — for a recent example see \([15]\).}\]

\[\text{3}\]
in different generalisations of the MSSM. For example, some are easier (or harder) to use for certain assumptions about supersymmetry breaking mediation. The advantages of having several supported, publicly available spectrum generators naturally also extends to the case of the increasingly popular NMSSM. The extension of SOFTSUSY to include the NMSSM will hopefully aid the accuracy and feasibility of a variety of NMSSM studies.

In the present paper, we focus on the recent components that have been added to SOFTSUSY in order to include the effects of the gauge singlet superfield. Up-to-date versions of this manual (along with other SOFTSUSY manuals) will be released along with the code in the doc/c subdirectory. The other manuals in this subdirectory detail the standard $R$–parity conserving MSSM [5], the $R$–parity violating MSSM [41] and the loop-level neutrino mass computation in the $R$–parity violating MSSM [42]. The remainder of the paper proceeds as follows: in section 3 we introduce the NMSSM supersymmetric parameters and the soft supersymmetry breaking parameters using our conventions. In section 4 we describe the algorithm employed to calculate the spectrum of masses and couplings of NMSSM particles, detailing our level of approximation for various parts of the calculation. More technical information is relegated to the appendices. In section Appendix A we explain how to run the program. The class structure, along with the names used in the program code are shown explicitly in section Appendix C. Finally, in section Appendix D we reproduce the renormalisation group equations of the NMSSM to two-loops including the full 3 by 3 flavour structure.

3. NMSSM Parameters

In this section, we introduce the NMSSM parameters in the SOFTSUSY conventions. The translations to the variable names used in the program code are shown explicitly in section Appendix C.

3.1. Supersymmetric parameters

The chiral superfield particle content of the NMSSM has the following $SU(3)_c\times SU(2)_L\times U(1)_Y$ quantum numbers

$$
\begin{align*}
L & : (1, 2, -\frac{1}{2}), & \bar{E} & : (1, 1, 1), & Q & : (3, 2, \frac{\bar{\epsilon}}{6}), & \bar{U} & : (\bar{3}, 1, -\frac{\bar{\epsilon}}{2}), \\
\bar{D} & : (\bar{3}, 1, \frac{\bar{\epsilon}}{2}), & H_1 & : (1, 2, -\frac{1}{2}), & H_2 & : (1, 2, \frac{1}{2}), & S & : (1, 1, 0).
\end{align*}
$$

$S$ is the gauge singlet chiral superfield that is particular to the NMSSM. $L$, $Q$, $H_1$, and $H_2$ are the left-handed doublet lepton and quark superfields and the two Higgs doublets. $\bar{E}$, $\bar{U}$, and $\bar{D}$ are the lepton, up-type quark and down-type quark right-handed superfield singlets, respectively. Note that the lepton doublet superfields $L^\mu$ and the Higgs doublet superfield $H_1$ coupling to the down-type quarks have the same SM gauge quantum numbers. We denote an $SU(3)$ colour index of the fundamental representation by $\{x, y, z\} \in \{1, 2, 3\}$. The $SU(2)_L$ fundamental representation indices are denoted by $\{a, b, c\} \in \{1, 2\}$ and the generation indices by $\{i, j, k\} \in \{1, 2, 3\}$. $\epsilon_{xyz} = \epsilon^{abc}$ and $\epsilon_{ab} = \epsilon^{ab}$ are totally antisymmetric tensors, with $\epsilon_{123} = 1$ and $\epsilon_{12} = 1$, respectively. Currently, only real couplings in the superpotential and Lagrangian are included.

The full renormalisable, $R$–parity conserving superpotential is given by

$$
W_{Z_3} = \epsilon_{ab} \left( (Y_E)_i L^i H_1^a \bar{E}_j + (Y_D)_i Q^i H_1^a H_1^b \bar{D}_{jx} \right) + (Y_U)_i Q^i \bar{H}_1^a H_2^b \bar{U}_{jx} + (A_S + \mu)(H_2^a H_1^a) + \xi_F S + \frac{\mu'}{2} S^2 + \frac{\kappa}{3} S^3
\tag{2}
$$

where $(Y_U,D,E)_{ij}$ and $\lambda, \kappa$ are dimensionless Yukawa couplings, $\mu$ and $\mu'$ are supersymmetric mass terms, and $\xi_F$ encodes the effects of the supersymmetric tadpole term. We use the subscript $Z_3$ to reflect the fact that this superpotential contains terms which violate the $Z_3$ symmetry that is commonly imposed on the NMSSM. Imposing the $Z_3$ symmetry restricts the superpotential to

$$
W_{Z_3} = \epsilon_{ab} \left( (Y_E)_i L^i H_1^a \bar{E}_j + (Y_D)_i Q^i H_1^a \bar{D}_{jx} \right) + (Y_U)_i Q^i \bar{H}_1^a H_2^b \bar{U}_{jx} + \lambda S (H_2^a H_1^a) + \frac{\kappa}{3} S^3
\tag{4}
$$

The $Z_3$-NMSSM superpotential Eq. (5) contains no explicit mass parameter, thereby allowing a solution to the $\mu$-problem when the singlet field acquires a Vacuum Expectation Value (VEV) and generates an effective $\mu$ term of the
3.3. Higgs potential and electroweak symmetry breaking

At tree-level, the Higgs potential is given by

\[ V_{\text{Higgs}} = V_{\mu}^H + V_{D}^H + V_{\text{soft}}^H = V_{\mu,0}^{\mu,0} + V_{HN}^{HN} + V_{\text{soft}}^{HN}, \]

where all \( \mathcal{X}_3 \) terms are included in

\[ V_{\mathcal{X}_3} = \xi S + \frac{m_3^2}{2} S^2 + \epsilon_{dG} m_3^2 H_3^2 H_1^2 + \frac{\kappa A_s}{3} S^3 + V_{\mathcal{X}_3}, \]

(6)

Expressions for the trilinear scalar interaction potential \( V_3 \) and scalar bilinear SUSY breaking potential \( V_2 \) of the MSSM are given in Sect. 2.2 of the SOFTSUSY manual [5] for the \( R \)-parity conserving MSSM. The notation \( V_{\mu,0}^{\mu,0} \) indicates that the \( \mathcal{X}_3 \) soft bilinear mass \( m_3^2 \) present in \( V_2 \) is set to zero to avoid double counting with the third term in Eq. (7).

3.3. Higgs potential and electroweak symmetry breaking

At tree-level, the Higgs potential is given by

\[ V_{\text{Higgs}} = V_{\mu}^H + V_{D}^H + V_{\text{soft}}^H = V_{\mu,0}^{\mu,0} + V_{HN}^{HN} + V_{\text{soft}}^{HN}, \]

(8)

where

\[ V_{\mu,0}^{\mu,0} = m_3^2 |S|^2 + (\frac{\kappa A_s}{3} S^3 + \frac{m_3^2}{2} S^2 + \xi S + \epsilon_{dG} m_3^2 H_3^2 H_1^2 + \frac{\kappa A_s}{3} S^3 + \xi S + \text{h.c.}), \]

(9)

The three neutral Higgs fields then pick up VEVs

\[ \langle H_1^0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_2^0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle S \rangle = \frac{1}{\sqrt{2}} s, \]

(12)

which are related to the soft masses via the minimization conditions

\[ m_{H_1}^2 = -M_Z^2 \cos(2\beta) - \frac{\lambda^2}{2} v_1^2 + (m_3^2)_{\text{eff}} \tan\beta - \mu_{\text{eff}}^2, \]

(13)

\[ m_{H_2}^2 = -M_Z^2 \cos(2\beta) - \frac{\lambda^2}{2} v_1^2 + (m_3^2)_{\text{eff}} \tan\beta - \mu_{\text{eff}}^2, \]

(14)

\[ m_S^2 = -\kappa^2 s^2 - \frac{\lambda^2}{2} v_1^2 + \kappa A_s v_1 v_2 + A_A \frac{v_2 v_1}{\sqrt{2} s} - \kappa A_s m_s - m_3^2 - \mu_s^2 + 2k \xi_F - 3k s \mu' , \]

(15)

where \( M_Z^2 = \frac{1}{2} g^2 (v_1^2 + v_2^2) \) and \( \beta = (g_2^2 + g'^2)^{1/2} \) for gauge couplings \( g_2, g' \) and \( \frac{\sqrt{3} S}{\xi_F} \) of \( U(2)_L \) and (unnormalised) \( U(1) \) interactions respectively. We have \( \tan\beta = v_2/v_1 \) and for simplicity we have introduced

\[ (m_3^2)_{\text{eff}} \equiv \frac{\lambda s}{\sqrt{2}} B_{\text{eff}} + \bar{m}_3^2, \]

(16)

and

\[ \mu_{\text{eff}} \equiv \mu + \frac{\lambda s}{\sqrt{2}} B_{\text{eff}} + \bar{m}_3^2, \quad B_{\text{eff}} \equiv \kappa s, \quad \bar{m}_3^2 \equiv m_3^2 + A \left( \frac{\mu_s}{\sqrt{2}} + \xi_F \right). \]

(17)
### 3.4. Tree-level masses

The chargino and sfermion masses are obtained by substituting $\mu \to \mu_{\text{eff}}$ into the MSSM expressions. The neutralino mass matrix is contained in the Lagrangian term $\frac{1}{2} \bar{\phi}^0 M_{\phi} \phi^0 + \text{h.c.}$, where $\bar{\phi}^0 = (-ib, -i\omega^3, \tilde{h}_1, \tilde{h}_2, s)^T$ and

$$M_{\phi} = \begin{pmatrix}
  M_1 & 0 & -M_{Z\bar{c}bsW} & M_{Z\bar{c}\mu cW} & 0 \\
  0 & M_2 & M_{Z\bar{c}\mu cW} & -M_{Z\bar{\mu} cW} & 0 \\
  -M_{Zc\bar{b}sW} & M_{Zc\mu cW} & 0 & -\mu & -\lambda V_2 \\
  M_{Zc\bar{b}sW} & -M_{Zc\mu cW} & -\mu & 0 & -\lambda V_1 \\
  0 & 0 & 0 & 0 & 2\mu + \mu' 
\end{pmatrix}. \quad (18)$$

We use $s$ and $c$ for sine and cosine, so that $s_{\beta} \equiv \sin \beta$, $c_{\beta} \equiv \cos \beta$ and $s_{\omega}(c_{\omega})$ is the sine (cosine) of the weak mixing angle. The 5 by 5 neutralino mixing matrix is an orthogonal matrix $O$ with real entries, such that $O^T M_{\phi} O$ is diagonal. The neutralinos $\chi_i^0$ are defined such that their absolute masses increase with increasing $i$. Note that some of their mass values can be negative.

The CP-even gauge eigenstates $(H^0)^T = (H_1^0, H_2^0, S)$ are rotated into mass eigenstates $(h^0)^T = (h_1, h_2, h_3)$ by a mixing matrix $R$,

$$h^0 = RH^0. \quad (19)$$

The mass matrix $M_{h^0}$ is obtained by expanding $H_{1,2}$ and $S$ about their VEVs $(12)$ and identifying terms $-(H^0)^T M_{h^0} H^0$ in the Lagrangian. Typically, the resulting matrix elements $(M_{h^0})_{ij}$ are simplified by using the tree-level electroweak symmetry breaking (EWSB) conditions $(13,15)$ in order to eliminate the soft terms $m_{H_1}, m_{H_2}, m_{\tilde{\chi}}$. This is equivalent to defining

$$(M_{h^0})_{ij} \equiv \frac{\partial^2 V}{\partial v_i \partial v_j} = \delta_{ij} \frac{\partial V}{\partial v_i} \quad \text{with} \quad v_3 \equiv s, \quad (20)$$

and under this prescription we find

\[
\begin{align*}
(M_{h^0})_{11} &= M_{Zc_{\beta}}^2 + \left(\frac{\lambda s}{\sqrt{2}} B_{\text{eff}} + \bar{m}_{\tilde{\chi}}^2\right) \tan \beta, \\
(M_{h^0})_{12} &= (4\lambda^2 - \frac{\lambda s^2}{\sqrt{2}}) \frac{v_2 v_1}{4} - \frac{\lambda s}{\sqrt{2}} B_{\text{eff}} - \bar{m}_{\tilde{\chi}}^2, \\
(M_{h^0})_{13} &= \lambda \left[2\mu_{\text{eff}} \frac{v_1}{\sqrt{2}} - (B_{\text{eff}} + \kappa s + \mu') \frac{v_2}{\sqrt{2}}\right], \\
(M_{h^0})_{22} &= M_{Z\bar{c}\mu cW}^2 + \left(\frac{\lambda s}{\sqrt{2}} B_{\text{eff}} + \bar{m}_{\tilde{\chi}}^2\right) \frac{\tan \beta}{\sqrt{2}}, \\
(M_{h^0})_{23} &= \lambda \left[2\mu_{\text{eff}} \frac{v_2}{\sqrt{2}} - (B_{\text{eff}} + \kappa s + \mu') \frac{v_1}{\sqrt{2}}\right], \\
(M_{h^0})_{33} &= \lambda \left[\frac{\lambda s}{\sqrt{2}} (A_4 + \mu') \frac{v_3 v_1}{s} + \frac{\kappa s}{\sqrt{2}} (A_4 + 4\kappa s \frac{\sqrt{2}}{2} + 3\mu') - \sqrt{2}(\xi_S + \xi_F \mu') / s\right].
\end{align*}
\]

The three imaginary components of the neutral Higgs fields $(H^0)^T = (H_1^0, H_2^0, S')$ mix to give the two physical CP odd bosons $A_{1,2}$ and the Goldstone boson $G^0$. A mixing matrix $P$ relates the two bases

$$a = PH^0, \quad (27)$$

where $a^T = (G_0, A_1, A_2)$. Here, $P$ matches the conventions of $(43)$, while deleting the first row from $P$ produces the 2 by 3 mixing matrix for the physical CP-odd Higgs bosons in SLHA2 conventions $(35)$. Following EHT $(22)$, the
entries of the 3 by 3 mass matrix $M^2_F$ in the $H^i$ basis read

$$
(M^2_F)_{11} = \left(\frac{\lambda s}{\sqrt{2}} B_{\text{eff}} + \bar{m}^2 \right) \tan \beta,
$$

$$
(M^2_F)_{12} = \left(\frac{\lambda s}{\sqrt{2}} B_{\text{eff}} + \bar{m}^2 \right),
$$

$$
(M^2_F)_{13} = \lambda v_a (A_4 - 2 k s - \mu'),
$$

$$
(M^2_F)_{22} = \left(\frac{\lambda s}{\sqrt{2}} B_{\text{eff}} + \bar{m}^2 \right) / \tan \beta,
$$

$$
(M^2_F)_{23} = \lambda v_a (A_4 - 2 k s - \mu'),
$$

$$
(M^2_F)_{33} = \lambda (B_{\text{eff}} + 3 k s + \mu') \frac{v_d v_d}{s} - 3 k A_s s - 2 m_Z^2 - k \mu' s - \xi_F \left(4 k + h' \right) - \frac{\xi_s}{s}.
$$

where tree-level EWSB has been imposed.

Note that — as in the MSSM — the mixing of the Goldstone boson $G^0$ depends only on $\tan \beta$. As shown in EHT, this can be seen by first performing a rotation by $\beta$, which converts $M^2_F$ to be block diagonal. The resulting 2 by 2 submatrix may then be diagonalised. Therefore the CP-odd mixing can be stored as a single mixing angle.

Finally, the charged Higgs fields in the mass basis contain one massless charged Goldstone boson $G^\pm$ and a charged Higgs, $H^\pm$ with mass

$$
m_{H^\pm}^2 = \left(\frac{\lambda s}{\sqrt{2}} B_{\text{eff}} + \bar{m}^2 \right) \left(\tan \beta + \cot \beta \right) + M_W^2 - \frac{\lambda^2 v^2}{2}.
$$

4. Calculation Algorithm

We now describe the algorithm used to perform the calculation. The full iterative algorithm to determine the mass spectrum is shown schematically in Fig. 4. Here we will provide a detailed description of this procedure and specify all contributions that are included in the calculation.

As in MSSM SOFTSUSY, the SM fermion and gauge boson masses, and the couplings $\alpha(M_Z)$, $G^\mu$, and $\alpha_s(M_t)$ act as low energy constraints. Below $M_Z$, the evolution of these input parameters proceeds in the manner described in Sect. 3.1 of the MSSM SOFTSUSY manual. Similarly, the initial guess for the SUSY preserving $\overline{DR}$ parameters at $m_t$ follows the procedure outlined in Sect. 3.2 of [5], with the additional NMSSM parameters $\{\lambda, \kappa, s, \xi_F, \mu'\}$ either initially set to their (user specified) input values, or to zero in the case when $\kappa$ and $s$ are treated as outputs from the EWSB conditions (section 4.2).

4.1. Running of NMSSM couplings

Following the initial guess at $m_t$, the two-loop $\beta$ functions of the $\mathbb{F}_5$-NMSSM are used to evolve the SUSY preserving parameters to a user specified scale $M_X$. If gauge unification has been specified as a boundary condition, $M_X$ is revised to leading-log order to provide a more accurate value upon the next iteration:

$$
M_X^{\text{new}} = M_X \exp \left( \frac{g_2(M_X) - g_1(M_X)}{g_3'(M_X) - g_2'(M_X)} \right),
$$

where primes denote derivatives calculated to two-loop order.

In all stages of the calculation, the evolution of the NMSSM parameters is governed by three family, two-loop renormalization group equations (RGEs), whose form [4, 13] for a general, $N = 1$ semi-simple SUSY gauge theory is known. From these general results, it is possible to derive the explicit expressions of the RGEs in a chosen model (e.g. the work of Martin and Vaughn [4, 13] provides a complete list of the RGEs for the MSSM).

3SOFTSUSY does this internally by storing $\theta_{\phi \gamma}$ in the $\text{aPhysical1}$ object (see Eq. [Appendix C.4]). Note that the SLHA output gives the 3 by 2 mixing matrix and thus matches SLHA2 conventions.
1. SUSY radiative corrections to $g_i(M_Z)$.

2. SUSY radiative corrections to $h_{i,b,t}(M_Z)$.

3. Run to $M_{SUSY}$.

4. EWSB with iterative solution for $\mu_{eff}$, outputs $\{s, \kappa, m_S\}$ in $\mathbb{Z}_3$-NMSSM and $\{\mu, m_3^2, \xi_S\}$ in $\mathbb{Z}_3$-NMSSM.

5. Run to $M_X$. Apply soft breaking and NMSSM SUSY boundary conditions.

6. Run to $M_Z$.

7. Calculate Higgs and sparticle pole masses at $M_{SUSY}$. Run to $M_Z$.

Figure 1: Iterative algorithm used to calculate the NMSSM spectrum. The initial step is the uppermost one. $M_{SUSY}$ is the scale at which the EWSB conditions are imposed, as discussed in the text. $M_X$ is the scale at which the high energy SUSY breaking boundary conditions are imposed. Although Higgs and sparticle masses are calculated at $M_{SUSY}$, the empirical values of electroweak boson and quark/lepton masses are imposed at $M_Z$. It is the SOFTSUSY convention to evolve $\overline{\text{DR}}$ couplings to $M_Z$ as the final step, although in the SLHA2 output [35], various couplings at $M_{SUSY}$ are output.
In the case of the NMSSM considered here, it is a simple task to generalize the MSSM expressions \[44\] to include contributions due to superpotential parameters such as \( \lambda \) and their soft SUSY-breaking counterparts \( a_i \). (Naturally, the RGEs for such parameters must be derived separately.) The two-loop RGEs for the \( \tilde{T}_3 \)-NMSSM are presented in the review by EHT \[22\], with the third family approximation

\[
Y_U \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}, \quad Y_B \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad Y_E \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_e \end{pmatrix},
\]

imposed to simplify the resulting expressions. However, in SOFTSUSY the whole calculation is performed with quark flavor-mixing between all three families, so it is necessary to derive the additional NMSSM contributions from the general RGEs \[44, 45\]. The resulting expressions are collected in section Appendix D and in each case we have found agreement with the results of EHT \[22\] once the third family approximation Eq. (36) is enforced. Note that in the SOFTSUSY conventions, all \( \beta \) functions are real. We also incorporate the two-loop running for \( \tan \beta \) and the Higgs VEVs \( v_{1,2} \) and \( s \). Here, we make use of the results obtained by Sperling et al. \[46, 47\], where the pure NMSSM contributions are reproduced in section Appendix D. The program can be made to run faster by switching off the two-loop renormalization of the scalar masses and tri-linear scalar couplings. Once the user-supplied boundary conditions are applied at \( M_X \), the whole ensemble of NMSSM soft breaking and SUSY preserving couplings are evolved to \( M_Z \). The inclusion of radiative corrections to the gauge and Yukawa couplings (steps 1 and 2 in Fig. 1), and NMSSM renormalization (step 3) is analogous to MSSM SOFTSUSY — for details we refer the reader to sections 3.3 and 3.4 of the SOFTSUSY manual \[5\].

4.2. Low energy boundary conditions and electroweak symmetry breaking

The electroweak symmetry breaking (EWSB) conditions \[13, 15\] allow one to constrain three model parameters of the theory. With the central value for the Z pole mass \( M_Z \) taken as input, we rewrite Eqs. (13) and (14) in terms of \( \mu_{\text{eff}} \) and \( (m_{3\text{eff}})^2 \), as in the MSSM. By including tadpole corrections \( t_i \) and the transverse self energy \( \Pi_{ZZ}^T \) of the Z boson, we find

\[
\mu_{\text{eff}}^2(M_{\text{SUSY}}) = \frac{m_{3\text{eff}}^2(M_{\text{SUSY}}) - m_{\text{eff}_3}(M_{\text{SUSY}}) \tan^2 \beta(M_{\text{SUSY}})}{\tan^2 \beta(M_{\text{SUSY}}) - 1} - \frac{1}{2} M_Z^2 \Pi_{ZZ}^T(M_{\text{SUSY}}),
\]

(37)

\[
(m_{3\text{eff}}^2(M_{\text{SUSY}}) = \frac{\sin 2\beta(M_{\text{SUSY}})}{2} \left( m_{H_u}^2(M_{\text{SUSY}}) + m_{H_d}^2(M_{\text{SUSY}}) + 2 m_{\text{eff}_3}^2(M_{\text{SUSY}}) \right) + \frac{M_Z^2}{g^2} (M_{\text{SUSY}}) \right),
\]

(38)

where \( m_{H_u}^2 = m_{H_u}^2 - t_i/v_i \), \( M_Z^2(M_{\text{SUSY}}) = M_Z^2 + \Re \Pi_{ZZ}^T(M_{\text{SUSY}}) \) is the \( \overline{\text{DR}} \) running (mass)\(^2 \) of the Z boson. Through Eqs. (37) and (38) we can fix \( \mu_{\text{eff}} \) and \( (m_{3\text{eff}}^2) \) in a similar manner to the MSSM. Note however, that in this case these are effective parameters constructed from several model parameters, so we must select which of the latter are fixed. In the \( \tilde{T}_3 \)-NMSSM, we fix \( s \) via Eq. (37) and \( \kappa \) via Eq. (38) and use the third EWSB condition \[15\] to fix \( m_3^2 \). In the \( \tilde{T}_3 \)-NMSSM, we have more freedom and can choose to fix \( \mu \) and \( m_3^2 \) — as in the MSSM — and use the third EWSB condition to fix \( \xi_5 \). Alternatively, the EWSB conditions \[13, 15\] can be used to fix the soft Higgs masses \( m_{H_1}^2 \), \( m_{H_2}^2 \) and \( m_{H_3}^2 \); see Appendix A.

The full one-loop tadpole corrections from \[43\] are implemented, along with NMSSM two-loop \( O(\alpha_3, \alpha_i) \) and \( O(\alpha_3^2, \alpha_3) \) contributions \[43\] to the tadpoles. The two loop corrections from the MSSM are used for \( O(\alpha_3^2) \), \( O(\alpha_3) \), \( O(\alpha_2) \), \( O(\alpha_3 \alpha_2) \), \( O(\alpha_3^2 \alpha_2) \), and \( O(\alpha_3^2 \alpha_1) \), though it should be noted that these are not complete in the NMSSM. In both one-loop and two-loop cases, the tadpole corrections themselves depend on the output from the EWSB conditions, therefore an iteration is employed to find a self consistent solution. After the EWSB iteration converges, the whole set of NMSSM parameters are run to \( m_Z \). As detailed in Section 3.3 of \[5\], the gauge couplings \( g_1, g_2 \) and \( g_3 \) (where \( g_1 \) is the GUT normalised gauge coupling of \( U(1)_Y \) and third family \( \overline{\text{DR}} \) Yukawa couplings, \( y_e, y_b \) and \( y_t \) are fixed, including the precision corrections at \( m_Z \). Note however, that the expressions for the one-loop self energies of the gauge bosons and fermions are modified to match those given in \[43\] for the NMSSM.

4 We thank Pietro Slavich for kindly supplying us with the FORTRAN files.
SOFTSUSY calculates corrections to $\sin \theta_W$ following the procedure outlined in \[48\]. We use the same procedure in the NMSSM, with expressions for the MSSM self energies \[48\] generalised to include NMSSM contributions \[43\]. In the Higgs sector, we only consider contributions from the lightest NMSSM Higgs, since contributions from heavy Higgs states are negligible \[48\]. This is achieved by taking the Higgs state whose mass and coupling produces the contributions listed in \[48\] once the MSSM limit is taken. Note that this ensures a simple MSSM limit for threshold corrections.

In the $\mathbb{Z}_3$-NMSSM, the parameters $\kappa$, $s$, $\xi_F$ and $\mu'$ are reset to their input values at $M_Z$. The parameters are then evolved back to $M_{\text{SUSY}}$ where $M_Z^2$ and $\tan \beta$ are predicted as part of a consistency check. If the user has specified that any of the parameters $\lambda$, $\kappa$, $s$, $\xi_F$ and $\mu'$ are to be input at the SUSY scale rather than the default option of inputting them at the GUT scale, then they are set here.

In general, the scalar Higgs potential (in both the $\mathbb{Z}_3$- and $\mathbb{Z}_2$-NMSSM) can possess several local minima \[22\], so we include a test at $M_{\text{SUSY}}$ to determine whether the chosen parameter space point corresponds to a global minimum (as done in the NMSPEC \[25\] CHECKMIN routine). The test works by comparing the value of the physical potential at the VEVs $v_q,v_u,s$ against scenarios where two or more VEVs are zero. We include one-loop radiative corrections to the effective potential from third generation quarks and squarks; corrections from other sfermions are negligible due to their small Yukawa couplings. The parameters are then evolved back up to $M_Z$ and the procedure is repeated until convergence is achieved, as shown in Fig. 1 (If the iteration does not converge to the desired accuracy, SOFTSUSY outputs a $\theta_0$ convergence warning message — see also Appendix C in \[5\].)

4.3. NMSSM spectrum

After the iteration has converged we calculate the pole masses. The Higgs pole masses are calculated using one-loop self energies from Degrassi and Slavich \[43\], with additional $\mathbb{Z}_3$ contributions to the triple Higgs couplings included (see Appendix A of EHT \[22\]). Two-loop corrections \[43\] of $O(\alpha s\alpha t)$ and $O(\alpha s\alpha s)$ are incorporated via the $\text{FORTRAN}$ files provided by Pietro Slavich. Contributions of order $O(\alpha^2)$, $O(\alpha s\alpha t)$, $O(\alpha s^2)$ and $O(\alpha s^2)$ are included from the MSSM $\text{FORTRAN}$ files (also supplied by Pietro Slavich), but we note that these expressions receive additional NMSSM contributions which are currently unavailable. Consequently, our calculation is not corrected to this order, but rather to $O(\alpha s\alpha t)$ and $O(\alpha s\alpha s)$. Nevertheless, the higher order MSSM contributions provide (a) a good approximation in the vicinity of the MSSM limit, and (b) easier comparisons against MSSM results.

The sfermions, neutralinos and charginos also receive new NMSSM corrections to their self energies. To the best of our knowledge, the required expressions are presented only in \[49\]. However, we found a number of typographical errors in the published results \[49\], whose origin was due to the need to manually condense the auto-generated $\text{LaTeX}$ output from \textsc{SARAH} \[30\] \[31\] \[32\] \[33\]. In particular, the self energy expressions generated by \textsc{SARAH} do not contain these errors. Therefore, we used a combination of results listed in \[49\], auto-generated $\text{LaTeX}$ output from \textsc{SARAH} for the self energies, plus individual checks of our own. Finally, all one-loop self energies, tadpole corrections, and two-loop RGEs were unit tested against code pieces auto-generated from \textsc{FlexibleSUSY} \[50\], an in development \textsc{Mathematica} package for generating C++ code which makes use of the aforementioned \textsc{SARAH} package.

Acknowledgments

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We thank Pietro Slavich for supplying us with the NMSSM $\text{FORTRAN}$ files with two-loop $O(\alpha s\alpha t)$ and $O(\alpha s\alpha s)$ contributions to the Higgs masses and also a $\text{FORTRAN}$ file with one-loop self energies (which we used as a cross check), as well as his helpful explanations on how to use them. We thank Florian Staub for responding quickly to our questions regarding \[49\], and on questions and bug reports when comparing against \textsc{SARAH} and \textsc{FlexibleSUSY}. We also thank Ben Farmer for providing useful feedback after using a pre-release version of the code.

PA and AV thank Dominik Stöckinger for many helpful comments and discussions regarding the precision corrections included here, and also thank both him and Jae-hyeon Park for listening to a number of discussions about this

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5 See \[Appendix A\] for details on how to do this.
6 F. Staub, private communication.
Appendix A. Running SOFTSUSY

SOFTSUSY produces an executable called softpoint.x. For the calculation of the spectrum of single points in parameter space, we recommend the SUSY Les Houches Accord 2 (SLHA2) \[35\] input/output option. The user must provide a file (e.g. the example file included in the SOFTSUSY distribution \texttt{rpvHouchesInput}), that specifies the model dependent input parameters. The program may then be run with

\[
./softpoint.x \text{ leshouches < mssmHouchesInput}
\]

NMSSM-SOFTSUSY accepts input files compliant with the SLHA2 format given in Ref. \[35\] and supports the setting of all SLHA2 input blocks associated with non-complex couplings. The set of input parameters which also exist in the MSSM are entered as described in \[9\], just as for the MSSM version of SOFTSUSY, while the new NMSSM parameters are all given in the \texttt{EXTPAR} block as outlined in \[35\]. For example, in the $Z_3$-NMSSM one can set:

Block EXTPAR # $Z_3$ violating NMSSM
\[
\begin{array}{ll}
\# & 23 \quad 100 \quad \# \text{mu} \\
\# & 24 \quad 1000 \quad \# m_3 / (\cos(\beta) \times \sin(\beta)) \\
& 61 \quad 0.1 \quad \# \lambda(MX) \\
& 62 \quad 0.1 \quad \# \kappa(MX) \\
& 63 \quad 1000 \quad \# A(\lambda)(MX) \\
& 64 \quad 1000 \quad \# A(\kappa)(MX) \\
& 65 \quad 500 \quad \# (\lambda(MX) * s)(MX) \\
& 66 \quad 100 \quad \# \xi_F(MX) \\
\# & 67 \quad 1000 \quad \# \xi_S(MX) \\
& 68 \quad 1000 \quad \# \mu'(MX) \\
& 69 \quad 1000 \quad \# m'_S^2(MX) \\
& 70 \quad 1000 \quad \# m_S^2(MX)
\end{array}
\]

The parameters
\[
\{\mu, m_3^2 / (\cos\beta \times \sin\beta), \xi_S\} : \quad Z_3\text{-NMSSM} \tag{A.1}
\]

must not be set here, because they are output by the EWSB conditions, see section \[4,5\]. In the $Z_3$-NMSSM one must set all $Z_3$ violating parameters ($\mu, m_3^2, \xi_F, \xi_S, \mu', m'_S^2$) to zero or comment them out. The parameters
\[
\{\kappa, s, m_S^2\} : \quad Z_3\text{-NMSSM} \tag{A.2}
\]

are then output from the EWSB conditions, as in section \[4,2\] and they should therefore not be set either. One is then left with the following three free parameters:

Block EXTPAR # $Z_3$ symmetric NMSSM
\[
\begin{array}{ll}
61 \quad 0.1 \quad \# \lambda(MX) \\
63 \quad 1000 \quad \# A(\lambda)(MX) \\
64 \quad 1000 \quad \# A(\kappa)(MX)
\end{array}
\]

By default all parameters are input at the scale $M_X$, defined either by (a) entry 0 in block EXTPAR or (b) as the gauge coupling unification scale where $g_1 = g_2$ (determined iteratively) when entry 0 in block EXTPAR is not set.

Should the user desire to input the parameters $\lambda, \kappa, \lambda s/ \sqrt{2}, \xi_F$ and $\mu'$ at $M_{SUSY}$, a corresponding $-1$ entry in the block EXTPAR has to be given:

\footnote{Unset parameters are assumed to be zero.}
Note that $M_{\text{SUSY}}$ is the scale where EWSB conditions are fixed, which is (by default) defined as $M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}}$ and is re-calculated at each step in the iteration. This matches SLHA conventions but in SOFTSUSY there is also a special option to vary this scale by setting entry 4 of block SOFTSUSY to give the value of QEWSB, which alters $M_{\text{SUSY}}$ as described in section A.1 of the MSSM manual [5].

Instead of choosing the default EWSB output parameters (A.1-A.2) it is also possible to output the soft scalar Higgs masses. The EWSB conditions in Eqs. (13-15) will then determine $m_{H_1}^2$, $m_{H_2}^2$ and $m_S^2$ when entry 18 in block SOFTSUSY is set to 1:

Block SOFTSUSY

18 1 # use soft Higgs masses as EWSB output

In this case, the default EWSB output parameters (A.1-A.2) must be given in the SLHA file or they will be set to zero by default. Since all $\lambda_3$ parameters are inputs in this case, setting any of them to a non-zero value implies that the point considered belongs to the $\lambda_3$-NMSSM. In this particular case, entries 23, 24 and 67 in block EXTPAR may now be set for a $\lambda_3$-NMSSM point since they are no longer EWSB outputs.

For the SLHA2 input option, the output will also be given in SLHA2 format. Such output can be used for input into other programs which subscribe to the accord, such as PYTHIA [39] (for simulating sparticle production and decays at colliders), for example. For further details on the format of the input and output files, see Refs. [35] and [9].

An alternative input option for SOFTSUSY is to input the parameters via the command-line interface. As of SOFTSUSY 3.4, the command line interface of softpoint.x has changed, see softpoint.x --help. For the NMSSM, the syntax is

```
./softpoint.x nmssm <susy-breaking-model> [NMSSM flags] [NMSSM parameters] [general options]
```

where sugra is the only currently available susy-breaking model. The general options are listed in Ref. [5] and the NMSSM flags and parameter options are listed in Table A.1.

Appendix B. Calculating decays with NMSSMTools

SOFTSUSY has a compatibility mode which interfaces with NMSSMTools to calculate sparticle decays in the NMSSM. To enable it, the user has to first install NMSSMTools and then run the setup_nmssmtools.sh script

```
$ cd /path/to/NMSSMTools/
$ wget \protect\vrule widthOpt\protect\href{http://www.th.u-psud.fr/NMHDECAY/NMSSMTools_4.1.2.tgz}{NMSSMTools_4.1.2.tgz}\}
$ tar xf NMSSMTools_4.1.2.tgz
$ cd /path/to/softsusy/
$ ./setup_nmssmtools.sh \
   --nmssmtools-dir=/path/to/NMSSMTools/NMSSMTools_4.1.2 \ 
   --compile
```

The setup_nmssmtools.sh script copies nmProcessSpec.f and Makefile.nmssmtools from the SOFTSUSY directory to the main/ directory within the NMSSMTools folder. If the --compile flag is provided, NMSSMTools is recompiled. Afterwards the user can generate a NMSSM spectrum with SOFTSUSY and use NMSSMTools to calculate the decays. The softsusy_nmssmtools.x script combines these two steps:

8though in the SLHA papers this scale is named $M_{EWSB}$. 

12
$ ./softsusy_nmssmtools.x leshouches < slhaInput > slhaOutput

Here slhaInput is an SLHA input file with the SOFTSUSY block entry 15 set to 1. Additional NMSSMTools specific flags can also be used with entries 16 and 17, which are past to NMSSMTools as MODSEL blocks 9 and 10 respectively, following the NMSSMTools convention.

Block SOFTSUSY

| Block | Entry | Value | Description |
|-------|-------|-------|-------------|
| 15    | 1     |       | # NMSSMTools compatible output (default: 0) |
| 16    | 4     |       | # Select Micromegas option for NMSSMTools |
|       |       | (default: 0) | 0=no, 1=relic density only |
|       |       | 2=direct detection + relic density, |
|       |       | 3=indirect detection + relic density |
|       |       | 4=all |
| 17    | 1     |       | # 1: sparticle decays via NMSDECAY (default: 0) |

After softsusy_nmssmtools.x is called, the following three output files can be found in the NMSSMTools directory NMSSMTools_4.1.2/main/. The file nmProcessSpec-decay contains the sparticle decays in form of SLHA DECAY blocks, nmProcessSpec-omega will contain the output from micrOMEGAS if entry 16 is selected to be non-zero and nmProcessSpec-spectr contains the spectrum calculated by NMSSMTools.

Appendix C. Class Structure

We now go on to sketch the NMSSM class hierarchy. Only methods and data which are deemed of possible importance for prospective users are mentioned here, but there are many others within the program itself.

Appendix C.1. General structure

To implement the NMSSM (and other non-minimal supersymmetric models), the SOFTSUSY class hierarchy was generalized with the following requirements in mind:
• The class of supersymmetric parameters (gauge couplings, superpotential parameters and VEVs), whose beta functions are independent of soft-breaking parameters, should be at the top of the class hierarchy. This makes them usable independently of the soft-breaking parameters, for example during the initial guess.

• One should be able to reuse as much MSSM code as possible, for example by inheriting from existing MSSM classes.

The above requirements were implemented by the following changes:

1. The class of the soft breaking MSSM parameters and their beta functions was converted into the class template `SoftPars<Susy>`. The template parameter represents the class of supersymmetric parameters, from which `SoftPars<Susy>` inherits. The class which contains all MSSM parameters and beta functions, `SoftParsMssm`, was made a typedef for `SoftPars<MssmSusy>`, where `MssmSusy` is the class that contains the supersymmetric MSSM parameters and beta functions.

   ```cpp
template <class Susy>
   class SoftPars : public Susy {
   // implementation of soft breaking MSSM parameters
   // and their beta functions
   
   
   
   
   
   
   typedef SoftPars<MssmSusy> SoftParsMssm;
   ```

   This approach makes it possible to have a class of soft breaking MSSM parameters but with a different set of supersymmetric parameters. This mechanism is used in the NMSSM, see Section [Appendix C.3](#).

2. The class which organises the MSSM mass spectrum calculation was converted into the class template `Softsusy<SoftPars>`. The template parameter represents the class of all model parameters and beta functions, from which `Softsusy<SoftPars>` inherits. `MssmSoftsusy` was made a typedef for `Softsusy<SoftParsMssm>`.

   ```cpp
template <class SoftPars>
   class Softsusy : public SoftPars {
   // organisation of MSSM mass spectrum calculation
   // using model parameters in SoftPars
   ```
This approach makes it possible to have a MSSM spectrum calculation class but with an arbitrary set of model parameters. This mechanism is used in the NMSSM, see \[\text{Appendix C.4}\].

**Appendix C.2. NmssmSusy class**

The class of supersymmetric NMSSM parameters and beta functions, \texttt{NmssmSusy}, inherits from \texttt{MssmSusy} to reuse the MSSM parameters and beta functions, see Figure \[\text{C.2}\]. It adds data members and access methods for the new supersymmetric NMSSM parameters, which can be found in Table \[\text{C.2}\].

**Appendix C.3. SoftParsNmssm class**

To implement the class of soft-breaking NMSSM parameters, \texttt{SoftParsNmssm}, the \texttt{SoftPars<Susy>} template is instantiated using \texttt{NmssmSusy} as template parameter. Thereby one obtains the class of MSSM soft-breaking beta functions, using supersymmetric NMSSM parameters. \texttt{SoftParsNmssm} then inherits from \texttt{SoftPars<NmssmSusy>} to add extra NMSSM contributions to the soft-breaking beta functions:

\begin{verbatim}
class NmssmSusy : public MssmSusy {
    // implement supersymmetric NMSSM parameter beta functions
    // by reusing MSSM ones
};

class SoftParsNmssm : public SoftPars<NmssmSusy> {
    // implement soft-breaking NMSSM parameter beta functions
    // by reusing MSSM ones
};
\end{verbatim}

Furthermore, \texttt{SoftParsNmssm} adds new soft-breaking NMSSM data members and access methods, which are listed in Table \[\text{C.3}\].

**Appendix C.4. NmssmSoftsusy class**

To create the NMSSM spectrum calculation class, \texttt{NmssmSoftsusy}, the \texttt{Softsusy<SoftPars>} template class is instantiated using \texttt{SoftParsNmssm} as template parameter. Thereby one obtains an NMSSM spectrum calculator, which uses NMSSM parameters and beta functions. \texttt{NmssmSoftsusy} then inherits from \texttt{Softsusy<SoftParsNmssm>} and overwrites MSSM functions to account for the extra NMSSM particles:

\begin{verbatim}
class NmssmSoftsusy : public Softsusy<SoftParsNmssm> {
    // organise NMSSM spectrum calculation reusing MSSM functions
};
\end{verbatim}
data variable methods
---
double alambda, akappa | trilinear soft parameters | displayTrialambda
A_1 | a_1/\lambda | displaySoftAlambda
A_k | a_k/\kappa | displaySoftAkappa
double mSpsq | bilinear soft parameters | displayMspSquared
double mSsq | soft scalar mass parameters | displayMsSquared
double xiS | linear soft parameters | displayXiS

| data variable | description |
|---------------|-------------|
| DoubleVector mh0, mA0 | vectors of neutral Higgs masses \( m_{h_0}^{\pm}, m_{A_0}^{\pm} \) (MSSM: \( n = 2, m = 1 \), NMSSM: \( n = 3, m = 2 \)) |
| double mHpm | charged Higgs mass \( m_H \) |
| DoubleVector msnu | vector of \( m_{\tilde{\nu}_i} \) masses (MSSM: \( n = 4 \), NMSSM: \( n = 5 \)) |
| double mGluino | gluino mass \( m_{\tilde{g}} \) |
| DoubleMatrix mixNeut | orthogonal neutralino mixing matrix \( O \) (MSSM: 4 by 4, NMSSM: 5 by 5) |
| double thetaL, thetaR | \( \theta_{L,R} \) chargino mixing angles |
| double thetaI, thetab | \( \theta_{I,b} \) sparticle mixing angles |
| double thetaI0 | \( \theta_{I0} \) particle mixing angle |
| double thetaH | CP-even Higgs mixing angle \( \alpha \) in the MSSM |
| double thetaA0 | CP-odd Higgs mixing angle \( \theta_{A0} \) in the NMSSM |
| DoubleMatrix mixh0 | orthogonal CP-even Higgs mixing matrix \( R \) in the NMSSM |
| DoubleMatrix mu, md, me | (2 by 3) matrices of up squark, down squark and charged slepton masses |

Table C.3: SoftParsNmssm class data and accessor methods

To implement the NMSSM pole masses and mixing matrices, the aPhysical structure had to be generalized, as in Table C.4.

Appendix D. Renormalization Group Equations for the NMSSM

In this section, we present the components of the one- and two-loop renormalization group equations (RGEs) which belong exclusively to the NMSSM. Our expressions have been derived in the \( \overline{\text{DR}} \) scheme from existing results \cite{44, 45} for general SUSY gauge theories. The complete RGEs are then obtained by combining the expressions below with those for the MSSM \cite{44}.

Appendix D.1. Yukawa Couplings

For \( t = \ln Q \), the trilinear superpotential parameter \( Y^{ijk} \) evolves according to the general expression \cite{44}:

\[
\frac{d}{dt} Y^{ijk} = Y^{i\beta p} T_p^{\gamma} + Y^{k\gamma p} T_p^{\beta} + Y^{\alpha \beta p} T_p^{\gamma},
\]  
(D.1)
where
\[ \Gamma_i^j = \frac{1}{16\pi^2} \gamma_i^{(1)j} + \frac{1}{(6\pi^2)^2} \gamma_i^{(2)j}, \] (D.2)
and \( \gamma_i^{(1,2)j} \) are the one- and two-loop anomalous dimensions respectively. Note that the 3 × 3 Yukawa matrices \( Y_{U,D,E} \) are obtained by identifying indices in Eq. (D.1) with the relevant chiral superfields in the superpotential.

At two-loop order, all the gauge-Yukawa contributions from \( d \) arising in the NMSSM are simply given by
\[ y^{(2)j}_{i} \left|_{\xi} = \lambda^2 (Y_E Y_i) \right|, \] (D.4)
where the one- and two-loop expressions for the singlet anomalous dimension are given by
\[ y^{(1)H_1}_{i} \left|_{\xi} = \lambda^2 \right. \] (D.3)
and \( y^{(2)H_1}_{i} \left|_{\xi} = \lambda^2 \right. \) (D.10).

At one-loop order, the only addition to the MSSM expressions [44] for the \( Y_{U,D,E} \) RGEs is the inclusion of \( \lambda^2 \) terms which originate from the Higgs anomalous dimensions
\[ \gamma_{H_1}^{(1)} \left|_{\lambda} = \lambda^2 \right. \] (D.3)
and \( \gamma_{H_2}^{(1)} \left|_{\lambda} = \lambda^2 \right. \) (D.3).

In a similar manner, the RGEs for \( \lambda \) and \( \kappa \) are obtained from Eq. (D.11), with
\[ \frac{d}{dt} \lambda = \lambda (\gamma_{H_1}^{H_1} + \gamma_{H_2}^{H_2} + \gamma_{S}^{S}), \] (D.11)
and
\[ \frac{d}{dt} \kappa = 3 \kappa \gamma_{S}^{S}, \] (D.12)
where the one- and two-loop expressions for the singlet anomalous dimension are given by
\[ \gamma_{S}^{(1)} = 2 \lambda^2 + 2 \kappa^2, \] (D.13)
\[ \gamma_{S}^{(2)} = -4 \lambda^4 - 8 \kappa^2 \lambda^2 - 8 \kappa^4 - 6 \lambda^2 \text{Tr}(Y_U Y_U^i) - 6 \lambda^2 \text{Tr}(Y_Y Y_Y^i) - 2 \lambda^2 \text{Tr}(Y_E Y_E^i) + \frac{g_a^2}{2} \lambda^2 + 6 g_a^2 \lambda^2. \] (D.14)

Appendix D.2. Gauge Couplings

In the NMSSM, the one-loop RGEs for the gauge couplings \( g_a \) are identical to those for the MSSM. At two-loop order however, the \( \lambda \) coupling appears through the term
\[ \frac{d}{dt} \lambda \right|_{\xi} = -\frac{g_a^3}{(16\pi^2)^2} Y_{ijk} Y^{ijk} C_{a}(k) / d(G_a), \] (D.15)
where \( d(G_a) \) is the dimension of the adjoint representation of gauge group \( G_a \). The result is
\[ \frac{d}{dt} g_a \right|_{\xi} = -\frac{g_a^3}{(16\pi^2)^2} \Lambda_a^{(2)}(2), \] (D.16)
where we have taken into account the additional factor of 2 which arises from tracing over \( SU(2) \) group indices in Eq. (D.15).

---

9For example, for \( k = H_2 \) we have \( Y_{ij}^{H_2} \equiv (Y_U)^i \).
Appendix D.3. Gaugino Mass Parameters

As for the gauge couplings above, we need only consider the addition of the $\lambda^2$ terms arising from

$$\frac{d}{dt} M_a = \frac{2g_a^2}{(16\pi^2)^2} \left( T^{ijk}_A - M_a Y^{ijk}_A C_a(k) \right),$$

(D.17)

where $T^{ijk}_A$ is a trilinear soft SUSY-breaking parameter. By evaluating the summations in Eq. (D.17), we find

$$\frac{d}{dt} M_a \bigg|_a = \frac{2g_a^2}{(16\pi^2)^2} (\lambda a) \Lambda_a^{(2)} M_a,$$

(D.18)

with $\Lambda_a^{(2)}$ as given in (D.16).

Appendix D.4. $\mu$ Parameters

The general expression [44, 45] for the SUSY-conserving bilinear terms is given by

$$\frac{d}{dt} \mu = \mu \Gamma_\mu + \mu \Gamma_p,$$

(D.19)

from which we obtain

$$\frac{d}{dt} \mu = \mu (H_i + \bar{H}_i),$$

$$\frac{d}{dt} \mu' = 2 \mu \Gamma_\lambda^\mu$$

(D.20)

Appendix D.5. Trilinear Couplings

If we denote $T^{ijk}_A$ as a soft SUSY-breaking trilinear, then the evolution at two-loop is given by

$$\frac{d}{dt} T^{ijk}_A = \frac{1}{16\pi^2} [\beta^{(1)}_{T}^{ij}]^{jk} + \frac{1}{16\pi^2} [\beta^{(2)}_{T}^{ij}]^{jk},$$

(D.21)

where the explicit expressions for the $\beta$ functions can be found in [44]. For $T = U, D, E$, the $\lambda$ contribution to the one-loop $\beta$ function arises from the following factor

$$[\beta^{(1)}_{T}]^{ij} \equiv \frac{1}{2} (T_A)^{ij} Y_{H_{mn}} Y_{\text{mhlv}} + (Y_{\lambda})^{ij} Y_{H_{mn}} T^{mn}_{A} T^{\text{mhlv}},$$

(D.22)

where there is no summation over $\alpha = 1, 2$, with the index determined by the choice of $T$ (e.g. if $T = U$ then $\alpha = 2$). Expanding the indices leads to

$$[\beta^{(1)}_{T}]^{ij} \bigg|_{\lambda} = (T_A)^{ij} \lambda^2 + (Y_{\lambda})^{ij} 2 \lambda a_i.$$

(D.23)

The two-loop expressions involve a large number of summations so to minimize the proliferation of generation indices we choose to express our results in terms of $3 \times 3$ matrices:

$$\beta^{(2)}_{U_A}|_{\lambda} = -\lambda^2 U_A [3 \lambda^2 + 2 \alpha^2 + 3 \text{Tr}(Y_D Y_U^i) + \text{Tr}(Y_E Y_U^i)] - \lambda^2 [5 Y_U Y_{\mu} U_A + 4 U_A Y_{\mu} Y_{U} + Y_D Y_{\mu} U_A + 2 D_A Y_{\mu} Y_{\mu}],$$

$$-2 \lambda a_i Y_{U} [3 \lambda^2 + 2 \alpha^2 + 3 \text{Tr}(Y_D Y_U^i) + \text{Tr}(Y_E Y_U^i)] - 2 \lambda^2 Y_{U} [3 \lambda a_i + 2 \alpha a_i + 3 \text{Tr}(D_A Y_{\mu}^i) + \text{Tr}(E_A Y_{\mu}^i)],$$

$$-2 \lambda a_i [3 Y_U Y_{\mu} Y_{U} + Y_D Y_{\mu} Y_{U}],$$

(D.24)

$$\beta^{(2)}_{D_A}|_{\lambda} = -\lambda^2 D_A [3 \lambda^2 + 2 \alpha^2 + 3 \text{Tr}(Y_E Y_U^i)] - \lambda^2 [5 Y_U Y_{\mu} D_A + 4 D_A Y_{\mu} Y_{D} + 2 U_A Y_{\mu} Y_{D} + Y_U Y_{\mu} D_A],$$

$$-2 \lambda a_i Y_{D} [3 \lambda^2 + 2 \alpha^2 + 3 \text{Tr}(Y_E Y_U^i)] - 2 \lambda^2 Y_{D} [3 \lambda a_i + 2 \alpha a_i + 3 \text{Tr}(U_A Y_{\mu}^i)] - 2 \lambda a_i [3 Y_U Y_{\mu} Y_{D} + Y_U Y_{\mu} Y_{D}],$$

(D.25)

$$\beta^{(2)}_{E_A}|_{\lambda} = -\lambda^2 E_A [3 \lambda^2 + 2 \alpha^2 + 3 \text{Tr}(Y_E Y_U^i)] - \lambda^2 [5 Y_U Y_{\mu} E_A + 4 E_A Y_{\mu} Y_{E} - 2 \lambda a_i Y_{E} [3 \lambda^2 + 2 \alpha^2 + 3 \text{Tr}(Y_D Y_U^i)] - 2 \lambda^2 Y_{E} [3 \lambda a_i + 2 \alpha a_i + 3 \text{Tr}(U_A Y_{\mu}^i)] - 6 \lambda a_i Y_{E} Y_{\mu} Y_{E}.$$

(D.26)
For $a_i$, the one-loop $\beta$ function reads in full
\[
\beta_{a_i}^{(1)} = \frac{1}{2} a_i (Y_{H_{1,mn}} Y_{H_{1,mn}}^{\text{ann}} + Y_{H_{2,mn}} Y_{H_{2,mn}}^{\text{ann}} + Y_{S_{mn}} Y_{S_{mn}}^{\text{ann}}) + \lambda (Y_{H_{1,mn}} T_{A}^{\text{ann}} H_{1} + Y_{H_{2,mn}} T_{A}^{\text{ann}} H_{2} + Y_{S_{mn}} T_{A}^{\text{ann}} S) \\
- 4 \sum_{\alpha = 1, 2, 3} \left( a_\alpha - 2 M_\alpha \right) g_\alpha^2 C_\alpha (H),
\]
(D.27)

from which the various sums immediately yield
\[
\beta_{a_i}^{(1)} = a_i \left[ 3 \text{Tr}(Y_U Y_U^\dagger) + 3 \text{Tr}(Y_D Y_D^\dagger) + \text{Tr}(Y_U Y_U^\dagger) + 12 \lambda^2 + 2 \kappa^2 - \frac{6}{5} g_1^2 - 3 g_2^2 \right]
+ \lambda \left[ 6 \text{Tr}(U_A Y_U^\dagger) + 6 \text{Tr}(D_A Y_D^\dagger) + 2 \text{Tr}(E_A Y_E^\dagger) + 4 a_\kappa + \frac{3}{5} g_1^2 M_1 + 6 g_2^2 M_2 \right].
\]
(D.28)

The two-loop expression is given by
\[
\beta_{a_i}^{(2)} = -50 \lambda^4 a_1 - 36 \lambda^2 \lambda^2 \text{Tr}(U_A Y_U^\dagger Y_U Y_U^\dagger) - 36 \lambda^2 \text{Tr}(D_A Y_D^\dagger Y_D^\dagger) - 12 \lambda^2 \text{Tr}(E_A Y_E^\dagger Y_E^\dagger) - 9 a_1 \left[ \text{Tr}(Y_U Y_U^\dagger) + \text{Tr}(U_A Y_U^\dagger \lambda \text{Tr}(Y_U Y_U^\dagger) + \text{Tr}(D_A Y_D^\dagger Y_D^\dagger) \right]
\]
- 18 \lambda^2 \left[ (a_1 \lambda) \lambda \text{Tr}(Y_U Y_U^\dagger) + \text{Tr}(U_A Y_U^\dagger \lambda \text{Tr}(Y_U Y_U^\dagger) \right]
- 6 \lambda^2 \lambda \left[ (a_1 \lambda) \lambda \text{Tr}(Y_U Y_U^\dagger) + \text{Tr}(Y_U Y_U^\dagger) \right]
- 24 \lambda^2 \lambda \left[ (a_1 \lambda) \lambda \text{Tr}(Y_U Y_U^\dagger) - 12 \lambda \left[ \text{Tr}(U_A Y_U^\dagger Y_U Y_U^\dagger) + \text{Tr}(D_A Y_D^\dagger Y_D^\dagger) \right]
\]
- 3 \lambda^2 a_1 \left[ 3 \text{Tr}(Y_U Y_U^\dagger) + 3 \text{Tr}(Y_D Y_D^\dagger) + 3 \text{Tr}(Y_U Y_U^\dagger) + \frac{1}{4} g_1^2 \lambda \left[ (a_1 \lambda) - \lambda M_1 \right]
+ \frac{5}{5} g_1^2 a_1 \left[ 2 \text{Tr}(Y_U Y_U^\dagger) - \text{Tr}(D_A Y_D^\dagger Y_D^\dagger) + 12 g_1^2 \lambda \left[ \frac{1}{4} a_1 - \lambda M_2 \right]
\]
+ 32 g_1^2 a_1 \left[ (a_1 \lambda) + 2 \text{Tr}(Y_U Y_U^\dagger) + 3 \text{Tr}(D_A Y_D^\dagger Y_D^\dagger) + 3 \text{Tr}(Y_U Y_U^\dagger) + \frac{1}{4} g_1^2 \lambda \left[ 15 (a_1 \lambda) - 60 M_2 \right] + \frac{5}{5} g_1^2 \lambda \left[ (a_1 \lambda) - 2 (M_1 + M_2) \right]\right.
\]
(D.29)

For $a_\kappa$, the one-loop calculation is similar to that of $a_i$, with the result
\[
\beta_{a_\kappa}^{(1)} = 18 a_\kappa \lambda^2 + 12 a_\kappa \lambda \left[ \lambda M_1 + 6 a_\kappa \right].
\]
(D.30)

At two-loop we have
\[
\beta_{a_\kappa}^{(2)} = -120 \lambda^4 a_\kappa - 12 \lambda^2 a_\kappa - 48 \lambda^4 \lambda \left[ (a_1 \lambda) \lambda \text{Tr}(Y_U Y_U^\dagger) - 24 \lambda^2 \lambda \text{Tr}(Y_U Y_U^\dagger) \right]
- 36 \lambda^2 \lambda \left[ \text{Tr}(U_A Y_U^\dagger Y_U Y_U^\dagger) + \text{Tr}(Y_D Y_D^\dagger) \right]
- 12 \lambda^2 \lambda \left[ \text{Tr}(Y_U Y_U^\dagger) - 3 \text{Tr}(Y_U Y_U^\dagger) + \frac{1}{4} g_1^2 \lambda a_1 \left[ (a_1 \lambda) - \lambda M_1 \right]
+ 36 g_1^2 \lambda \left[ (a_1 \lambda) - \frac{1}{4} a_1 \lambda \right].
\]
(D.31)

Appendix D.6. Higgs Masses

To determine the $\lambda$ and $\kappa$ contributions to the Higgs masses, it is useful to define the following quantities
\[
M_1 = m_{H_1}^2 + m_{H_2}^2 + m_3^2 + a_3^2 / \lambda^2,
\]
\[
M_2 = 3 m_3^2 + a_3^2 / \kappa^2,
\]
\[
M_3 = \text{Tr}(m_3^2 Y_U Y_U^\dagger) + \text{Tr}(m_3^2 Y_D Y_D^\dagger) + \text{Tr}(m_3^2 Y_U Y_U^\dagger) + \text{Tr}(U_A Y_U^\dagger)
\]
\[
M_4 = \text{Tr}(m_3^2 Y_U Y_U^\dagger) + \text{Tr}(m_3^2 Y_D Y_D^\dagger) + \text{Tr}(m_3^2 Y_U Y_U^\dagger) + \text{Tr}(D_A Y_D^\dagger)
\]
\[
M_5 = \text{Tr}(m_3^2 Y_E Y_E^\dagger) + \text{Tr}(m_3^2 Y_D Y_D^\dagger) + \text{Tr}(m_3^2 Y_E Y_E^\dagger) + \text{Tr}(E_A Y_E^\dagger)
\]
(D.32)

Both the up- and down-type Higgs masses $m_{H_1}$ and $m_{H_2}$ receive the same $\lambda$ contribution at one-loop order,
\[
\beta_{m_{H_1,2}}^{(1)} = 2 \lambda^2 M_1^2, \quad \alpha = 1, 2.
\]
(D.33)
The two-loop expressions for $m_{H_t}^2$ are

$$\beta_{m_{H_t}^2}^{(2)} = -12t^4[M_\lambda^2 + (a_/\lambda)^2] - 6t^2[M_\lambda^2 + M_\lambda^2\text{Tr}(Y_d Y_d^\dagger) + 2(a_/\lambda)\text{Tr}(D_A Y_d^\dagger)]$$

$$- 2t^4[M_\kappa^2 + M_\kappa^2\text{Tr}(Y_e Y_e^\dagger) + (2a_/\lambda)\text{Tr}(E_A Y_e^\dagger)] - 4t^2\kappa^2[M_\lambda^2 + M_\kappa^2 + 2(a_/\lambda)(a_/\kappa)]$$

$$+ \frac{6}{\lambda}g_1^2\lambda^2(m_{H_t}^2 - m_{H_t}^2). \quad (D.34)$$

with a similar result for $m_{H_u}^2$,

$$\beta_{m_{H_u}^2}^{(2)} = -12t^4[M_\lambda^2 + (a_/\lambda)^2] - 6t^2[M_\lambda^2 + M_\lambda^2\text{Tr}(Y_u Y_u^\dagger) + 2(a_/\lambda)\text{Tr}(U_A Y_u^\dagger)]$$

$$- 4t^2\kappa^2[M_\lambda^2 + M_\kappa^2 + 2(a_/\lambda)(a_/\kappa)] - \frac{6}{\lambda}g_1^2\lambda^2(m_{H_u}^2 - m_{H_u}^2). \quad (D.35)$$

For the singlet mass $m_S$, the one-loop result is

$$\beta_{m_S}^{(1)} = Y_{S pq} Y^{pq S} m_S^2 + 2Y_{S pq} Y^{pq S} (m_l^2) + h_{S pq} h^{pq}, \quad (D.36)$$

where

$$Y_{S pq} Y^{pq S} (m_l^2) = 2t^2(m_{H_u}^2 + m_{H_t}^2) + 4\kappa^2 m_S^2,$$

$$h_{S pq} h^{pq} = 4a_1^2 + 4a_2^2,$$

and thus Eq. $(D.36)$ becomes

$$\beta_{m_S}^{(1)} = 4t^2 M_\lambda^2 + 4\kappa^2 M_\kappa^2. \quad (D.38)$$

At two-loop we get

$$\beta_{m_S}^{(2)} = -16t^4[M_\lambda^2 + (a_/\lambda)^2] - 32t^2[M_\lambda^2 + (a_/\lambda)^2] - 12t^2[M_\lambda^2\text{Tr}(Y_u Y_u^\dagger) + M_\lambda^2 + 2(a_/\lambda)\text{Tr}(U_A Y_u^\dagger)]$$

$$- 12t^2[M_\lambda^2\text{Tr}(Y_e Y_e^\dagger) + M_\alpha^2 + (2a_/\lambda)\text{Tr}(D_A Y_e^\dagger)] - 4t^2[M_\lambda^2\text{Tr}(Y_d Y_d^\dagger) + M_\lambda^2 + 2(a_/\lambda)(a_/\kappa)]$$

$$+ 12g_1^2\lambda^2[M_\lambda^2 - 2M_1[(a_/\lambda) - M_1)] \quad (D.39)$$

Appendix D.7. Squark and Slepion Masses

The squark and slepton masses only receive contributions from $\lambda, \kappa$ at two-loop order. The results are listed below, where $1$ is a $3 \times 3$ unit matrix.

$$\beta_{m_{\tilde{u}_R}^2}^{(2)} = -\lambda^2[2Y_d u_R^2 Y_u + m_{\tilde{u}_R}^2 Y_u + 2m_{H_t} Y_u Y_u + 2U_A U_A^\dagger + 2M_2 Y_u Y_u + 2a_/\lambda(Y_u U_A^\dagger + U_A Y_u)]$$

$$- \lambda^2[2Y_d u_R^2 Y_d + m_{\tilde{u}_R}^2 Y_d + 2m_{H_t} Y_d Y_d + 2D_A D_A^\dagger + 2M_2 Y_d Y_d + 2a_/\lambda(Y_d D_A^\dagger + D_A Y_d)]$$

$$+ \frac{6}{\lambda}g_1^2\lambda^2(m_{\tilde{u}_R}^2 - m_{\tilde{u}_R}^2)1, \quad (D.40)$$

$$\beta_{m_{\tilde{d}_R}^2}^{(2)} = -2t^2[2Y_d m_{\tilde{d}_R}^2 Y_u + m_{\tilde{d}_R}^2 Y_u + 2m_{H_t} Y_u Y_u + 2U_A U_A^\dagger + 2M_2 Y_u Y_u + 2a_/\lambda(Y_u U_A^\dagger + U_A Y_u)]$$

$$- \lambda^2[2Y_d m_{\tilde{d}_R}^2 Y_d + m_{\tilde{d}_R}^2 Y_d + 2m_{H_t} Y_d Y_d + 2D_A D_A^\dagger + 2M_2 Y_d Y_d + 2a_/\lambda(Y_d D_A^\dagger + D_A Y_d)]$$

$$+ \frac{6}{\lambda}g_1^2\lambda^2(m_{\tilde{d}_R}^2 - m_{\tilde{d}_R}^2)1, \quad (D.41)$$

$$\beta_{m_{\tilde{e}_L}^2}^{(2)} = -2t^2[2Y_d m_{\tilde{e}_L}^2 Y_u + m_{\tilde{e}_L}^2 Y_u + 2m_{H_t} Y_u Y_u + 2U_A U_A^\dagger + 2M_2 Y_u Y_u + 2a_/\lambda(Y_u U_A^\dagger + U_A Y_u)]$$

$$- \lambda^2[2Y_d m_{\tilde{e}_L}^2 Y_d + m_{\tilde{e}_L}^2 Y_d + 2m_{H_t} Y_d Y_d + 2D_A D_A^\dagger + 2M_2 Y_d Y_d + 2a_/\lambda(Y_d D_A^\dagger + D_A Y_d)]$$

$$+ \frac{6}{\lambda}g_1^2\lambda^2(m_{\tilde{e}_L}^2 - m_{\tilde{e}_L}^2)1, \quad (D.42)$$

20
\[
\beta_{m_{\text{IR}}}^{(2)} = -\lambda^2 \{2Y_Y Y_Y + m_Y^2 Y_Y + Y_Y Y_Y + 2m_{H_u}^2 Y_Y + 2E_A E_A + 2m_{H_u}^2 Y_Y + 2(\alpha_Y/Y_YY_Y + E_A Y_Y)\}
\]
\[
- \frac{\lambda^2}{2} g^2 \lambda^2 (m_{H_u}^2 - m_{H_u}^2) 1,
\]
(D.43)
\[
\beta_{m_{\text{IR}}}^{(2)} = -2\lambda^2 \{2Y_Y Y_Y + m_Y^2 Y_Y + Y_Y Y_Y + 2m_{H_u}^2 Y_Y + 2E_A E_A + 2m_{H_u}^2 Y_Y + 2(\alpha_Y/Y_YY_Y + E_A Y_Y)\}
\]
\[
+ \frac{\lambda^2}{2} g^2 \lambda^2 (m_{H_u}^2 - m_{H_u}^2) 1. 
\]
(D.44)

Appendix D.8. Tadpole Terms

The general RGE for a SUSY-conserving term reads
\[
\frac{d}{dt} L = L^\Gamma_p,
\]
and thus for \( i = S \) one has
\[
\frac{d}{dt} S = \xi_p L^\Gamma_S.
\]
For the soft SUSY-breaking terms \( \xi_S \), we use the general RGE from [45] because Martin and Vaughn [44] do not include the tadpole as part of \( L_{\text{soft}} \). The relevant RGE reads
\[
\frac{d}{dt} \xi_S = \frac{1}{16\pi^2} \beta_1 \xi_S + \frac{1}{(16\pi^2)^2} \beta_{(2)} \xi_S,
\]
where the one-loop \( \beta \) function is given by
\[
\beta_{\xi_S}^{(1)} = 2(\lambda^2 + \kappa^2) \xi_S + 4(\lambda a_\lambda + \kappa a_\kappa) \xi_F + 2\mu(2\lambda m_{H_u}^2 + \kappa m_{H_u}^2)
\]
\[
+ 4(\lambda a_\lambda m_{H_u}^2 + \kappa m_{H_u}^2) y + 4a_\lambda m_{H_u}^2 + 2a_\kappa m_{H_u}^2.
\]
(D.47)

At two-loop we obtain
\[
\beta_{\xi_S}^{(2)} = -4\lambda^4 [\xi_S + 4(\alpha_\lambda/\lambda) \xi_F] - 8\kappa^4 [\xi_S + 4(\alpha_m/\kappa) \xi_F] - 6\lambda^2 [\xi_S \text{Tr}(Y_Y Y_Y) + 2[(\alpha_m/\lambda) \text{Tr}(Y_Y Y_Y) + \text{Tr}(U_A Y_Y)] \xi_F
\]
\[
- 6\lambda^2 [(x_\lambda/\lambda) \text{Tr}(Y_Y Y_Y) + 2[(\alpha_m/\lambda) \text{Tr}(Y_Y Y_Y) + \text{Tr}(U_A Y_Y)] \xi_F
\]
\[
- 2\lambda^2 [(x_\lambda/\lambda) \text{Tr}(Y_Y Y_Y) + 2[(\alpha_m/\lambda) \text{Tr}(Y_Y Y_Y) + \text{Tr}(E_A Y_Y)] \xi_F
\]
\[
- 8\lambda^2 [x_\lambda x_\lambda + 2(\alpha_m/\lambda) + (\alpha_m/\kappa) \xi_F]
\]
\[
- 12\lambda [m_{\xi_S}^2 [(\alpha_m/\lambda) + \mu] \text{Tr}(Y_Y Y_Y) + m_Y^2 \text{Tr}(U_A Y_Y) + \mu [M_Y^2 + [(\alpha_m/\lambda) + \mu] \text{Tr}(U_A Y_Y) + m_{H_u}^2 + m_{H_u}^2] \text{Tr}(Y_Y Y_Y)]
\]
\[
- 12\lambda [m_{\xi_S}^2 [(\alpha_m/\lambda) + \mu] \text{Tr}(Y_Y Y_Y) + m_Y^2 \text{Tr}(U_A Y_Y) + \mu [M_Y^2 + [(\alpha_m/\lambda) + \mu] \text{Tr}(U_A Y_Y) + m_{H_u}^2 + m_{H_u}^2] \text{Tr}(Y_Y Y_Y)]
\]
\[
- 4\lambda [m_{\xi_S}^2 [(\alpha_m/\lambda) + \mu] \text{Tr}(Y_Y Y_Y) + m_Y^2 \text{Tr}(U_A Y_Y) + \mu [M_Y^2 + [(\alpha_m/\lambda) + \mu] \text{Tr}(U_A Y_Y) + m_{H_u}^2 + m_{H_u}^2] \text{Tr}(Y_Y Y_Y)]
\]
\[
- 8\lambda^2 [m_{\xi_S}^2 [(\alpha_m/\lambda) + \mu] \text{Tr}(Y_Y Y_Y) + m_Y^2 \text{Tr}(U_A Y_Y) + \mu [M_Y^2 + [(\alpha_m/\lambda) + \mu] \text{Tr}(U_A Y_Y) + m_{H_u}^2 + m_{H_u}^2] \text{Tr}(Y_Y Y_Y)]
\]
\[
- 8\lambda^2 [m_{\xi_S}^2 [(\alpha_m/\lambda) + \mu] \text{Tr}(Y_Y Y_Y) + m_Y^2 \text{Tr}(U_A Y_Y) + \mu [M_Y^2 + [(\alpha_m/\lambda) + \mu] \text{Tr}(U_A Y_Y) + m_{H_u}^2 + m_{H_u}^2] \text{Tr}(Y_Y Y_Y)]
\]
\[
- 8\kappa^2 [m_{\xi_S}^2 [(\alpha_m/\lambda) + \mu] \text{Tr}(Y_Y Y_Y) + m_Y^2 \text{Tr}(U_A Y_Y) + \mu [M_Y^2 + [(\alpha_m/\lambda) + \mu] \text{Tr}(U_A Y_Y) + m_{H_u}^2 + m_{H_u}^2] \text{Tr}(Y_Y Y_Y)]
\]
\[
- 8\lambda^2 [m_{\xi_S}^2 [(\alpha_m/\lambda) + \mu] \text{Tr}(Y_Y Y_Y) + m_Y^2 \text{Tr}(U_A Y_Y) + \mu [M_Y^2 + [(\alpha_m/\lambda) + \mu] \text{Tr}(U_A Y_Y) + m_{H_u}^2 + m_{H_u}^2] \text{Tr}(Y_Y Y_Y)]
\]
\[
- 8\kappa^2 [m_{\xi_S}^2 [(\alpha_m/\lambda) + \mu] \text{Tr}(Y_Y Y_Y) + m_Y^2 \text{Tr}(U_A Y_Y) + \mu [M_Y^2 + [(\alpha_m/\lambda) + \mu] \text{Tr}(U_A Y_Y) + m_{H_u}^2 + m_{H_u}^2] \text{Tr}(Y_Y Y_Y)]
\]
\[
+ 2\lambda^2 [3m_{\xi_S}^2 [(\alpha_m/\lambda) + \mu] \text{Tr}(Y_Y Y_Y) + m_Y^2 \text{Tr}(U_A Y_Y) + \mu [M_Y^2 + [(\alpha_m/\lambda) + \mu] \text{Tr}(U_A Y_Y) + m_{H_u}^2 + m_{H_u}^2] \text{Tr}(Y_Y Y_Y)]
\]
\[
+ 3\lambda^2 [3m_{\xi_S}^2 [(\alpha_m/\lambda) + \mu] \text{Tr}(Y_Y Y_Y) + m_Y^2 \text{Tr}(U_A Y_Y) + \mu [M_Y^2 + [(\alpha_m/\lambda) + \mu] \text{Tr}(U_A Y_Y) + m_{H_u}^2 + m_{H_u}^2] \text{Tr}(Y_Y Y_Y)].
\]
(D.49)
Appendix D.9. Additional Parameters

Here we list the $\lambda$ and $\kappa$ contributions to the RGEs for the scalar masses $m_j^2 \equiv B\mu$ and $m_3^2 \equiv B'\mu'$, and the evolution of the Higgs VEVs $v_{1,2}$. For the former, we get at one-loop

$$\beta^{(1)}_{m_j^2} = 3\lambda \beta^{(1)} + 2\beta \lambda m_j^2. \quad (D.50)$$

At two-loop we have

$$\beta^{(2)}_{m_j^2} = \beta^{(1)}_{m_j^2}, \quad \beta^{(1)}_{m_j^2} = 2\lambda(3\lambda m_j^2 + 2\mu_1) + 2\lambda \kappa m_j^2. \quad (D.51)$$

For $m_3^2$, the one-loop $\beta$ function reads

$$\beta^{(1)}_{m_3^2} = 4\lambda(\lambda m_3^2 + 2\mu' a_3) + 8\kappa(\kappa m_3^2 + \mu' a_3) + 8\lambda \kappa m_3^2. \quad (D.52)$$

At two-loop we have

$$\beta^{(2)}_{m_3^2} = -8\lambda^4(\lambda m_3^2 + 2\mu' a_3) + 16\lambda^2(\lambda m_3^2 + 2\mu' a_3) - 16\lambda^4 m_3^2 + (3\lambda^2 + 2\lambda \kappa) m_3^2. \quad (D.53)$$

At one-loop, the up- and down-type Higgs VEVs $v_{u,d}$ receive additional contributions solely from $\lambda$ \cite{46},

$$\beta^{(1)}_{v_{\alpha}} = -v_{\alpha} \lambda^2, \quad \alpha = 1, 2, \quad (D.54)$$

while the $\beta$ function for the singlet VEV $s$ is given by

$$\beta^{(1)}_s = -\lambda (\lambda s^2 + \kappa^2). \quad (D.55)$$

At two-loop, the $\beta$ functions are given by \cite{46,47}

$$\beta^{(2)}_{v_{\alpha}} = v_{\alpha} \left( \gamma^{(2)H_{\alpha}}_{H} - \left( \frac{3}{8} s^2 + \frac{3}{8} g_2^2 \right) \left[ 3 \text{Tr}(Y_{d} Y_{d}^\dagger) + 3 \text{Tr}(Y_{u} Y_{u}^\dagger) + \lambda^2 \right] + \frac{9}{8} s^2 \right), \quad (D.56)$$

$$\beta^{(2)}_s = v_{\alpha} \left( \gamma^{(2)S}_{H} - \left( \frac{3}{8} s^2 + \frac{3}{8} g_2^2 \right) \left[ 3 \text{Tr}(Y_{d} Y_{d}^\dagger) + \lambda^2 \right] + \frac{9}{8} s^2 \right), \quad (D.57)$$

$$\beta^{(2)}_{\tan \beta} = \gamma^{(2)H_{\tan \beta}}_{H} - \left( \frac{3}{8} s^2 + \frac{3}{8} g_2^2 \right) \left[ 3 \text{Tr}(Y_{d} Y_{d}^\dagger) + \lambda^2 \right] + \frac{9}{8} s^2 \right), \quad (D.58)$$

The one-loop $\beta$ function for $\tan \beta$ is the same in the NMSSM as the MSSM. At two-loop, one has

$$\beta^{(2)}_{\tan \beta} = \tan \beta \left( \gamma^{(2)H_{\tan \beta}}_{H} - \left( \frac{3}{8} s^2 + \frac{3}{8} g_2^2 \right) \beta^{(1)}_{\tan \beta} \right). \quad (D.59)$$
