A superstructure over the Farhi - Susskind Technicolor model

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Abstract

We suggest the model with the gauge group ... $\otimes SU(6) \otimes SU(5) \otimes SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)$. This group is the infinite continuation of the gauge group $SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)$ of Farhi - Susskind model. The constructed model contains fermions from the fundamental representations of any $SU(N)$ subgroups of the gauge group. In the construction of the model we use essentially the requirement that it possesses an additional discrete symmetry $\mathcal{Z}$ that is the continuation of the $\mathbb{Z}_6$ symmetry of the Standard Model. It has been found that there exists such a choice of the hypercharges of the fermions that the chiral anomaly is absent while the symmetry $\mathcal{Z}$ is preserved.

1 Introduction

Recently we have shown [1] that the $\mathbb{Z}_6$ symmetry of the Standard Model [2, 3, 4, 5, 6] (SM) can be continued to the Technicolor models (TC). It was shown that among various models only a few ones possess the new discrete symmetry $\mathcal{Z}$. In particular, for the Farhi - Susskind model with Technicolor group $SU(N_{TC})$ there are two possibilities: $N_{TC} = 2$ and $N_{TC} = 4$. It is worth mentioning that the $SU(2)$ Farhi - Susskind model [7] suffers from the vacuum alignment problems. That’s why we do not consider it as realistic and the only possibility remains that is the $SU(4)$ Farhi - Susskind model. This model (together with the Standard Model) has the gauge group $SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)$. The hypercharge assignment for the technifermions is fixed by the additional discrete symmetry up to an integer number.
The structure of the gauge group $SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)$ prompts its possible continuation as the infinite sequence of $SU(N)$ subgroups:

$$G = \ldots \otimes SU(6) \otimes SU(5) \otimes SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)/\mathcal{Z},$$

(1)

where $\mathcal{Z}$ is the discrete group to be specified below.

The question arises: is it possible to continue the $Z_6$ symmetry of the Standard Model to this sequence. In the present paper we construct such a continuation. We arrange the fermions of the model in the fundamental representations of $SU(N)$ subgroups of (1). In general case the model with the gauge group (1) suffers from chiral anomalies of different types. It is not obvious a priori that there exists the hypercharge assignment of the fermions such that the chiral anomalies are absent while the additional discrete symmetry is preserved. Below we show that it is possible to satisfy both requirements simultaneously.

The Technicolor interaction alone serves only as a source of Electroweak gauge symmetry breaking. Usually in order to make Standard Model fermions massive extra gauge interaction is added, which is called Extended Technicolor (ETC) [8, 9, 10, 11, 12, 13, 14, 15]. In this gauge theory the Standard Model fermions and technifermions enter the same representation of the Extended Technicolor group. Standard Model fermions become massive because they may be transformed into technifermions with ejecting of the new massive gauge bosons. The ETC models suffer from extremely large flavor - changing amplitudes and unphysically large contributions to the Electroweak polarization operators [8, 9, 10]. The possible way to overcome these problems is related to the behavior of chiral gauge theories at large number of fermions or for the higher order representations [16, 17].

In the present paper we do not concretize the mechanism of fermion mass generation. It may be either of the ETC type or some unknown mechanism related to a higher energy scale. Whatever mechanism is it defines (together with the chiral symmetry breaking) the correspondence between the left handed and the right - handed fermions that is identified with parity conjugation of spinors. Our model as well as the SM and the Farhi - Susskind model contains left handed doublets and right handed singlets. That’s why the mentioned correspondence connects pairs of right - handed singlets with the components of certain left - handed doublets. In particular, the component of the left - handed doublet that corresponds to the right - handed electron is called left - handed electron. Let us denote the left - handed doublet of the 1-st generation SM leptons by $\Theta$. The given correspondence can
be written as: $\Omega^i_1 \Theta^i = \nu_L; \Omega^i_2 \Theta^i = e_L$, where we introduce the auxiliary field $\Omega \in SU(2)$. The same field $\Omega$ applied to the other left-handed doublets gives the parity conjugated partners to the remaining right-handed singlets. The physical quantities do not depend on $\Omega$ and it can always be chosen equal to unity. This choice corresponds to the unitary gauge. The mass term generated via the ETC (or other) interactions gives the transition amplitudes between the right-handed singlets and their parity partners. At the same time the dynamical fermion terms contain mixing. The requirement that $\Omega$ is the same (up to the gauge transformation) for all left-handed doublets is necessary for the correct realization of the Electroweak symmetry breakdown. It is worth mentioning that in the $SU(4)$ Farhi-Susskind model with ETC interaction considered as a perturbation the vacuum alignment works properly and leads to the correct Electroweak symmetry breaking [19]. We also notice that the field $\Omega$ is not dynamical, i.e. there is no integration over $\Omega$ in the functional integral that defines correlation functions. Fixing of $\Omega$ means that one of the equivalent vacua is chosen during the spontaneous breakdown of Electroweak symmetry.

Extra $SU(N)$ ($N > 4$) gauge interactions present in (1) may be observed in principle at the energies above the Technicolor scale. We briefly concern their properties at the end of the paper. Throughout the paper we call $SU(N)$ subgroups for $N > 4$ the Hypercolor groups. We also feel it appropriate to refer to the sequence (1) as to the Hypercolor tower.

2 The model

In our approach the theory contains $U(1)$ gauge group and the groups $SU(N)$ with any $N$. So, the gauge group of the theory is (1). Next, we suppose, that in the theory any fermions are present that belong to the fundamental representations of the $SU(N)$ subgroups of $G$. So, the possible fermions are right-handed $\Psi^{\alpha i_{kN} \ldots i_{k3}i_{k2}}_{A,Y}$ and left-handed $\Theta^{i_{kN} \ldots i_{k3}i_{k2}}_{\beta A,Y}$, where $\alpha$ and $\beta$ are spinor indices, $A$ enumerates generations while index $i_k$ belongs to the subgroup $SU(k)$. Here $Y$ is the $U(1)$ charge of the given fermion. In particular, the fermions $\Psi_{A,Y}$ are present that have no indices and the only subgroup that acts on $\Psi_{A,Y}$ is $U(1)$. Moreover, we suppose that the fermions are present such that $G$ does not act on them at all. We denote them $\Psi_{A,0}$. All fermions in the theory are two-component spinors. We also suppose from the very beginning that the $SU(2)$ group acts on the left-handed
spinors only. The action of parity conjugation on them will be considered later. For the simplicity we omit below both spinor and generation indices. So, our fermions are

\[
\begin{align*}
U(1) : & \quad \Psi_0, \Psi_{Y'_1}, \Psi_{Y'_2}, \ldots; \\
U(1), SU(2) : & \quad \Theta^{i_2}_{Y_2}, \Theta^{i_2}_{Y'_2}, \ldots; \\
U(1), SU(3) : & \quad \Psi^{i_3}_{Y_3}, \Psi^{i_3}_{Y'_3}, \ldots; \\
U(1), SU(2), SU(3) : & \quad \Theta^{i_3 i_2}_{Y_3 Y_2}, \Theta^{i_3 i_2}_{Y'_3 Y'_2}, \ldots; \\
U(1), SU(4) : & \quad \Psi^{i_4}_{Y_4}, \Psi^{i_4}_{Y'_4}, \ldots; \\
U(1), SU(2), SU(4) : & \quad \Theta^{i_4 i_2}_{Y_4 Y_2}, \Theta^{i_4 i_2}_{Y'_4 Y'_2}, \ldots; \\
U(1), SU(3), SU(4) : & \quad \Psi^{i_4 i_3}_{Y_4 Y_3}, \Psi^{i_4 i_3}_{Y'_4 Y'_3}, \ldots; \\
U(1), SU(2), SU(3), SU(4) : & \quad \Theta^{i_4 i_3 i_2}_{Y_4 Y_3 Y_2}, \Theta^{i_4 i_3 i_2}_{Y'_4 Y'_3 Y'_2}, \ldots;
\end{align*}
\]

Here in each row we list the subgroups of \(G\) that act on the fermions listed in the row. In each row the allowed values of \(U(1)\) charge are denoted by \(Y, Y', \ldots\).

Let us consider the first row. Here in order to reproduce the Standard Model we restrict ourselves by the values of \(Y\) equal to 0 and \(-2\). Next, the second row must contain the only element with \(Y = -1\). The third row contains two elements with \(Y = \frac{4}{3}\) and \(Y = -\frac{2}{3}\). In the forth row we have the only element with \(Y = \frac{1}{3}\). This row completes the Standard Model and we enter the rows related to its ultraviolet completion.

Now let us consider the second four rows in (2). We suggest them in the form that represents \(SU(4)\) Farhi - Susskind model of Technicolor [7]. In [1] we have derived the hypercharge assignment for the technifermions such that the chiral anomaly is absent while the additional discrete symmetry is preserved. As a result the hypercharge assignment is the following. In the 5-th row there are two elements with \(Y_4 = \frac{1}{2} - 6K + 1\) and \(Y'_4 = \frac{1}{2} - 6K - 1\) (were \(K\) is an arbitrary integer number). In the 6-th row we have the only element with \(Y_{42} = \frac{1}{2} - 6K\), where \(K\) is the same as in the previous row. In the 7-th row there are two elements with \(Y_{43} = -\frac{1}{3} - \frac{6K}{3} + 1\) and \(Y'_{43} = -\frac{1}{3} - \frac{6K}{3} - 1\). The 8-th row contains the only element with \(Y_{432} = -\frac{1}{3} - \frac{6K}{3}\). Again, in these two rows \(K\) is the same as before.
Let us specify how parity conjugation $\mathcal{P}$ acts on the fermions. If only two fermions $\chi^\alpha$ and $\eta_\alpha$ are present, then $\mathcal{P}\chi^\alpha(t, \bar{r}) = i\eta_\alpha(t, -\bar{r})$; $\mathcal{P}\eta_\alpha(t, \bar{r}) = i\chi^\alpha(t, -\bar{r})$. In our case we require that for any configuration of $SU(N)$ ($N > 2$) indices there exist two right-handed spinors and one $SU(2)$ doublet. The parity conjugation connects each of the right handed spinors with a component of the $SU(2)$ doublet. Thus

$$\begin{align*}
\mathcal{P}\Psi_0(t, \bar{r}) &= i\Omega^1_{i_3}(t, -\bar{r})\Theta^{i_2}_{1}(t, -\bar{r}); \mathcal{P}\Psi_{-2} = i\Omega^2_{i_2}\Theta^{i_3}_{1}; \\
\mathcal{P}\Psi^\frac{1}{3} &= i\Omega^1_{i_2}\Theta^{i_1}_{3}; \mathcal{P}\Psi^\frac{1}{3} = i\Omega^2_{i_2}\Theta^{i_1}_{1}; \\
\mathcal{P}\Psi^{i_1} &= i\Omega^1_{i_2}\Theta^{i_1}_{1}; \mathcal{P}\Psi^{i_1} = i\Omega^2_{i_2}\Theta^{i_1}_{1}; \\
\mathcal{P}\Psi^{i_1i_3} &= i\Omega^1_{i_2}\Theta^{i_1i_3}_{1}; \mathcal{P}\Psi^{i_1i_3} = i\Omega^2_{i_2}\Theta^{i_1i_3}_{1}; \\
\mathcal{P}\Psi_{Y_4} &= \mathcal{P}\Psi_{Y_4} = \mathcal{P}\Psi_{Y_4} = \mathcal{P}\Psi_{Y_4}.
\end{align*}$$

(3)

Here $\Omega$ is the auxiliary $SU(2)$ field mentioned in the Introduction. The physical sense of the field $\Omega$ is that it peeks up the parity partner for each right-handed spinor.

The correspondence between our notations and the conventional ones is the following (we list here the case $K = 0$ for the first generation only):

$$\begin{align*}
\Psi_0 &= \nu_R; \Psi_{-2} = e_R; \mathcal{P}\Psi_0(t, \bar{r}) = i\nu_L(t, -\bar{r}); \mathcal{P}\Psi_{-2} = ie_L; \\
\Psi^\frac{1}{3} &= u_R; \Psi^\frac{1}{3} = d_R; \mathcal{P}\Psi^{\frac{1}{3}} = iu_L; \mathcal{P}\Psi^{\frac{1}{3}} = id_L; \\
\Psi^{i_1} &= N_R; \Psi^{i_1} = E_R; \mathcal{P}\Psi^{i_1} = iN_L; \mathcal{P}\Psi^{i_1} = iE_L; \\
\Psi^{i_1i_3} &= U_R; \Psi^{i_1i_3} = D_R; \mathcal{P}\Psi^{i_1i_3} = iU_L; \mathcal{P}\Psi^{i_1i_3} = iD_L.
\end{align*}$$

(4)

It is worth mentioning that the fermions of the first generation listed here do not diagonalize the mass matrix. Instead the certain linear combinations of the listed fermions diagonalize the mass matrix thus giving rise to mixing angles and flavor changing amplitudes.

Before dealing with the next rows let us remind what we call the additional $Z_6$ symmetry in the Standard Model and how can it be continued to the Hypercolor interactions.

3 \space \mathcal{Z} \space symmetry

Within the Standard Model for any path $\mathcal{C}$, we may calculate the elementary parallel transporters $\Gamma = P \exp(i \int_\mathcal{C} C^\mu dx^\mu), U = P \exp(i \int_\mathcal{C} A^\mu dx^\mu), e^{i\theta} =$
exp\left( i \int_C B^\mu dx^\mu \right), where C, A, and B are correspondingly SU(3), SU(2) and U(1) gauge fields of the Standard Model. The parallel transporter correspondent to each fermion of the Standard Model is the product of the elementary ones listed above. Therefore, the elementary parallel transporters are encountered in the theory only in the following combinations: \( e^{-2i\theta}; U e^{-i\theta}; \Gamma U e^{i\theta}; \Gamma e^{\frac{2i}{3}\theta}; \Gamma e^{\frac{4i}{3}\theta}. \)

It can be easily seen [5] that all the listed combinations are invariant under the following \( Z_6 \) transformations: \( U \rightarrow U e^{i\pi N}, \theta \rightarrow \theta + \pi N, \Gamma \rightarrow \Gamma e^{(2\pi i/3)N}, \) where \( N \) is an arbitrary integer number. This symmetry allows to define the Standard Model with the gauge group \( SU(3) \times SU(2) \times U(1)/Z_6 \) instead of the usual \( SU(3) \times SU(2) \times U(1). \)

In [1] we have suggested the way to continue this symmetry to the Technicolor extension of the Standard Model. Now we generalize the construction of [1] and suggest the following discrete symmetry:

\[
\begin{align*}
U & \rightarrow U e^{i\pi N}, \\
\theta & \rightarrow \theta + \pi N, \\
\Gamma & \rightarrow \Gamma e^{(2\pi i/3)N}, \\
\Pi_4 & \rightarrow \Pi_4 e^{(2\pi i/4)N}, \\
\Pi_5 & \rightarrow \Pi_5 e^{(2\pi i/5)N}, \\
\Pi_6 & \rightarrow \Pi_6 e^{(2\pi i/6)N}, \\
\ldots
\end{align*}
\]

Here \( \Pi_K \) is the \( SU(K) \) parallel transporter. We construct our model in such a way that the parallel transporters correspondent to the new fermions of the theory are invariant under (5). The resulting symmetry is denoted by \( \mathcal{Z} \) and enters expression (1).

Let us also point out how our model can be embedded, in principle, into a ETC model. Let \( U(N_{ETC}), N_{ETC} \rightarrow \infty \) be the Unified gauge group. The breakdown pattern is \( U(N_{ETC}) \rightarrow \ldots \otimes SU(5) \otimes SU(4) \otimes SU(3) \times SU(2) \times U(1)/\mathcal{Z}. \) We may suppose, for example, that at low energies the \( U(N_{ETC}) \) parallel transporter has the form:
The form of this parallel transporter demonstrates naturally that the symmetry (5) is indeed preserved. The fermions of each generation \( \Psi_{i1} \ldots i_N \) carry indices \( i_k \) of the fundamental representation of \( U(N_{ETC}) \) and the indices \( j_k \) of the conjugate representation. They may be identified with the Standard Model fermions and Farhi - Susskind fermions as follows (we consider here the first generation only):

\[
\begin{align*}
\Psi^1 &= e_R; \quad \Psi^1_1 = \nu_R; \quad \Psi^{i2} = \left( \frac{\nu_L}{e_L} \right); \\
\Psi^{i3} &= d_{i3,R}; \quad \Psi^{i3}_1 = u_{i3,R}; \quad \Psi^{i2i3} = \left( \frac{u^i_L}{d^i_L} \right); \\
\Psi^{i4} &= E_{i4,R}; \quad \Psi^{i4}_1 = N_{i4,R}; \quad \Psi^{i2i4} = \left( \frac{N^i_L}{E^i_L} \right); \\
\Psi^{i3i4} &= D_{i3i4,R}; \quad \Psi^{i3i4}_1 = U_{i3i4,R}; \quad \Psi^{i2i3i4} = \left( \frac{U^{i3i4}_L}{D^{i3i4}_L} \right)
\end{align*}
\]

\( (i_2 = 1, 3; \quad i_3 = 4, 5, 6; \quad i_4 = 7, 8, 9, 10) \); (7)

The other fermions of our model can be arranged in the representations of \( U(N_{ETC}) \) in a similar way. From (7) it follows that all Standard model fermions can be transformed into the technifermions with ejecting of the \( U(N_{ETC}) \) gauge bosons. This is necessary for them to acquire masses. Of course, the given ETC scheme does not describe the appearance of the realistic masses for the known particles. However, it gives an example of how, in principle, the model given in the present paper may be incorporated with the Extended Technicolor. We omit here the details of the ETC symmetry breakdown. We do not describe how does this breakdown occur and what
mechanism washes out the unnecessary fermions. We also do not consider the anomalies in the given ETC model. We consider all these issues to be out of the scope of the present paper.

4 $SU(N)$ groups with $N > 4$

The next step of our investigation is the analysis of the sequence (2) in the form (4). Let us notice that the second two rows are actually the copy of the first two rows supplemented by an additional $SU(3)$ index. Next, the second four rows are again the copy of the first four rows supplemented by an additional $SU(4)$ index. Let us suppose that this process is repeated infinitely. Then the sequence of fermions has the form:

\[
\begin{align*}
U(1), SU(5) : & \quad \Psi^{i_5}_{Y_5}, \Psi^{i_5}_{Y'_5}, \\
U(1), SU(2), SU(5) : & \quad \Theta^{i_2}_{Y_2}, \\
U(1), SU(3), SU(5) : & \quad \Psi^{i_3}_{Y_3}, \Psi^{i_3}_{Y'_3}, \\
U(1), SU(2), SU(3), SU(5) : & \quad \Theta^{i_2}_{Y_2}, \\
U(1), SU(4), SU(5) : & \quad \Psi^{i_4}_{Y_4}, \Psi^{i_4}_{Y'_4}, \\
U(1), SU(2), SU(4), SU(5) : & \quad \Theta^{i_2}_{Y_2}, \\
U(1), SU(3), SU(4), SU(5) : & \quad \Psi^{i_3}_{Y_3}, \Psi^{i_3}_{Y'_3}, \\
U(1), SU(2), SU(3), SU(4), SU(5) : & \quad \Theta^{i_2}_{Y_2}, \\
& \quad \vdots \\
U(1), \ldots, SU(K) : & \quad \Psi^{i_K}_{Y_K}, \Psi^{i_K}_{Y'_K}, \\
U(1), SU(2), \ldots, SU(K) : & \quad \Theta^{i_2}_{Y_2}, \\
& \quad \vdots 
\end{align*}
\]

Below we derive the hypercharge assignment for all fermions of our model. We require that the chiral anomaly is absent and the additional $Z$ symmetry is preserved. Actually, the fact that there exists such a solution is nontrivial. A priori it is not clear that it is possible to satisfy both requirements simultaneously.

The anomaly cancellation is always necessary for the model to be well defined. At the same time the requirement that the $Z$ symmetry is preserved
must be considered as additional. Of course, at the present moment we do not have any reason to impose this symmetry but the intuition. So, our reason to consider the extension of the \( Z_6 \) symmetry of the Standard Model is the supposition that it does not appear accidentally. That’s why we suppose that it is to be the manifestation of a more general symmetry. Our choice of \( Z \) here is only one of the possible ways to generalize the \( Z_6 \). (We hope, however, that this is one of the most natural ways.) Below it will be shown that \( Z \) symmetry gives an important limitation on the choice of fermion hypercharges and almost fix them (up to the set of integers). Besides, it was shown in \([1]\) that the additional discrete symmetry has an important consequence in the monopole pattern of the Unified model.

Now we require that the chiral anomaly is absent while the gauge group is \([1]\), where \( Z \) is defined by \([5]\). Below we prove that the necessary hypercharge assignment is

\[
Y_2 = -1
\]

\[
Y_{ij...i_{M-1}i_M} = -1 + 2(1 - \frac{1}{i_M}) + 2 \sum_{k=1}^{M-1} [\theta(i_k - i_{k+1} - 1) - \frac{1}{i_k}] + 2N_{i_1i_2i_3...i_{M-1}i_M}^2
\]

\[
Y_{ij...l} = Y_{ij...l} + 1; \quad Y'_{ij...l} = Y_{ij...l} - 1
\]

where \( \theta(x) = 1 \) for \( x > 0 \); \( \theta(x) = 0 \) for \( x \leq 0 \). In the second row \( M \geq 1 \). For any \( K \) integer numbers \( N_{i_1i_2i_3...i_{M-1}i_M} \) entering \([9]\) must satisfy the equation

\[
\sum_{K>i>j>...>l>2} ij...l N_{Kij...l} = 0
\]

Here the sum is over any (unordered) sets of different integer numbers \( i, j, ..., l \) such that \( 2 < i, j, ..., l < K \).

The proof is as follows. First of all, if \([5]\) is the symmetry of the theory then the recursion relations take place:

\[
Y_{Kij...l} = Y_{ij...l} - \frac{2}{K} + 2 M_{Kij...l}; \quad Y'_{Kij...l} = Y_{Kij...l} + 1; \quad Y'_{Kij...l} = Y_{Kij...l} - 1
\]

where \( M_{Kij...l} \) is an integer number.

Let us require that for any \( K \)

\[
\sum_{K>i>j>...>l>2} ij...l Y_{Kij...l} = 0
\]
This means that the chiral anomaly is absent even if the sequence (11) is ended at the $SU(K)$ factor with any value of $K$.

Namely, there may appear the new anomalies of the following types [20]:

1) $SU(N) - SU(N) - SU(N)$, $N > 2$
2) $SU(N) - SU(N) - U(1)$, $N > 2$
3) $SU(2) - SU(2) - U(1)$
4) $U(1) - U(1) - U(1)$ (13)

The anomaly of the first type vanishes because the number of left-handed fermions is equal to the number of the right-handed ones while both types of fermions belong to the fundamental representation of $SU(N)$. The anomalies of the second type vanish because $Y_{ij...l} = Y_{ij...l} + 1$; $Y'_{ij...l} = Y_{ij...l} - 1$.

The anomalies of the third and the fourth types vanish if the sum of the hypercharge over left-handed doublets is zero. This leads to (12).

Below we prove that for any $K$ integer numbers $M_{Kij...l}$ can be chosen in such a way, that (12) is satisfied. Let $\sum_{K' > i > j > ... > l > 2} ij...l Y_{K'ij...l} = 0$ for $K' < K$ (this was demonstrated already for $K' = 4$). Then

$$\sum_{K > i > j > ... > l > 2} ij...l Y_{Kij...l} = -2\frac{K!}{3!K} + 2 \sum_{K > i > j > ... > l > 2} ij...l M_{Kij...l}$$

(14)

Here we used the identity $\sum_{K > i > j > ... > l > 2} ij...l = \frac{K!}{3!}$.

Let us introduce the following notations:

$$M_{Kij...l} = M'_{Kij...l} + 1, \text{ for } K - 1 > i > j > ... > l > 2;$$
$$M_{Kij...l} = M'_{Kij...l}, \text{ for } K - 1 = i > j > ... > l > 2$$

(15)

Then

$$-\frac{K!}{3!K} + \sum_{K > i > j > ... > l > 2} ij...l M_{Kij...l} = \sum_{K > i > j > ... > l > 2} ij...l M'_{Kij...l}$$

(16)

The relations that define the fermion hypercharges can be rewritten in the following way:

$$Y_{Kij...l} = Y_{Kij...l} + 1; Y'_{Kij...l} = Y_{Kij...l} - 1; Y_{Kij...l} = Y_{ij...l} - \frac{2}{K} + 2M'_{Kij...l}$$

(for $K - 1 > i > j > ... > l > 2$, or $K = 3$);

$$Y_{Kij...l} = Y_{ij...l} - \frac{2}{K} + 2M'_{Kij...l}$$

(17)
Here integer numbers $M_{Kij...l2}$ are chosen in such a way that $\sum_{K>i>j>...>l>2} M_{Kij...l2} = 0$.

Finally we come to the solution of (12) in the form (9). In particular, the choice $N_{i1i2i3...i_{M-1}i_{M2}} = 0$ corresponds to $Y_{i1i2i3...i_{M-1}i_{M2}} = -1 + 2(1 - 1_{iM}) + 2 \sum_{k=1}^{M-1} [\theta(i_k - i_{k+1} - 1) - \frac{1}{i_k}]$. Thus the additional symmetry (5) fixes the hypercharge assignment up to the choice of integer numbers $N_{i1i2i3...i_{M-1}i_{M2}}$ such that (10) is satisfied.

It is worth mentioning that if the $Z$ symmetry is not imposed, then the hypercharge assignment is defined by the anomaly cancellation only. In this case the hypercharge assignment is given by (9), where $N_{i1i2i3...i_{M-1}i_{M2}}$ are not necessarily integer.

## 5 Discussion

The dynamics of Technicolor is related in a usual way to the number of fermions $N_f$. Namely, the beta-function in one loop approximation has the form: $\beta_{SU(4)}(\alpha) = -\frac{11K-2N_f}{6\pi} \alpha^2$ where $\alpha = \frac{g^2_{SU(4)}}{4\pi}$. If $N_f < \frac{11}{2}K$, the one loop calculation indicates asymptotic freedom. The two-loop calculations [21, 22] indicate that the chiral symmetry breaking occurs at $N_f < N_c \sim K^{100K^2-66}_{25K^2-15} \sim 4K$. This is required for the appearance of gauge boson masses.

In our model we have three generations of Farhi-Susskind technifermions. Therefore, their number is $24 > 4N_{TC} = 16$. However, it is important that only such technifermions are relevant, the masses of which are of the order of $\Lambda_{TC}$ and smaller ($\Lambda_{TC}$ is the $SU(4)$ analogue of $\Lambda_{QCD}$). Therefore, we suppose that the masses of the third generation technifermions and, probably, the masses of some of the second generation technifermions are essentially larger, than the Technicolor scale. We also assume that the masses of the fermions that carry the indices of higher Hypercolor groups are essentially larger than the Technicolor scale. So, they do not affect the Technicolor dynamics. Thus the $SU(4)$ interactions lead to the chiral symmetry breaking and provide $W$ and $Z$ bosons with their masses.

If the number of fermions approach $N_c \sim 4N_{TC}$, then the behavior of the model becomes close to conformal. In this case the effective charge becomes walking instead of running [23]. So, in our case (two generations of fermions for $N_{TC} = 4$) the behavior of the technicolor may be close to conformal.

As for the higher Hypercolor groups, already for $SU(5)$ interactions the
number of the first generation hyperfermions (fermions carrying \(SU(5)\) index) is \(2(1+3+4+12) = 40 > \frac{55}{2} = 27.5\). We suppose their masses are close to each other. That’s why the Hypercolor forces at \(K > 4\) are not asymptotic free, and do not confine. As a result the Landau pole is present in their effective charges. This means that our model does not have a rigorous continuum limit, and should be considered as a finite cutoff model. At the energies of the order of this cutoff the new theory should appear that incorporates the Hypercolor tower as an effective low energy theory. In principle, this scale may be extremely large, even of the order of Plank mass depending on the value of \(g_{SU(5)}^2\) at low energies. Very roughly this scale (as given by the \(SU(5)\) effective charge) can be estimated as \(\Lambda_h = e^{\frac{6\pi}{g_{SU(5)}(1\text{ Tev})}}\) Tev. Say, if three generations are involved, and \(\alpha_{SU(5)}(1\text{ Tev}) = \frac{1}{300}\), then the Landau pole occurs in the \(SU(5)\) gauge coupling at \(\Lambda_h \sim 10^{13}\text{ Tev} \sim 10^{-3}M_{\text{Plank}}\).

At the energies much less than \(\Lambda_h\) the \(SU(5)\) interactions can be taken into account perturbatively just like in QED. However, the description of possible effects due to Hypercolor \(SU(5)\) (and due to the other Hypercolor interactions \(SU(N)\) for \(N > 4\)) is out of the scope of the present paper.

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