Excited bottom and bottom-strange mesons in the quark model

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In order to understand the possible $qar{q}$ quark-model assignments of the $B_1(5840)$ and $B_1(5960)$ recently reported by the LHCb Collaboration, we evaluate mass spectra, strong decays, and radiative decays of bottom and bottom-strange mesons in a nonrelativistic quark model. Comparing these predictions with the relevant experimental results, we suggest that the $B_1(5840)$ and $B_1(5960)$ can be identified as $B(2^1 S_0)$ and $B(1^3 D_3)$, respectively, and the $B(5970)$ reported by the CDF Collaboration can be interpreted as $B(2^3 S_1)$ or $B(1^5 D_3)$. Further precise measurements of the widths, spin and decay modes of the $B(5970)$ are needed to distinguish these two assignments. These predictions of bottom and bottom-strange mesons can provide useful information to further experimental investigations.

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I. INTRODUCTION

Heavy-light mesons composed of one heavy quark and one light quark act as the hydrogen atoms of hadron physics and are the ideal laboratory for the understanding of strong interactions in the non-perturbative regime[1–3]. In the past several years, significant progress has been achieved in studying the charmed and charmed-strange states experimentally[4–8]. It is widely accepted that the ground charmed and charmed-strange mesons such as $D(1S)$, $D'(1P)$, $D_s(1S)$, and $D_s(1P)$ have been established[4], and some candidates for higher radial and orbital excitations have also been reported, which have stimulated many theoretical investigations on these excitations[9–28].

Recently, the LHCb Collaboration studied $B^+\pi^−$ and $B^0\pi^-$ invariant mass distributions by analysing $pp$ collision data at centre-of-mass energies of 7 and 8 TeV[29]. Precise masses and widths of the $B_1(5721)$ and $B'_1(5747)$ are measured, and two excited bottom mesons $B_1(5840)^{0,+}$ and $B_1(5960)^{0,+}$ are observed, whose masses and widths are also studied with various quantum number hypotheses. The measured masses and widths of neutral $B_1(5840)$ and $B'_1(5960)$ under different spin-parity hypotheses are listed in Table I. In 2013, the CDF Collaboration studied the $B^0\pi^+$ and $B^+\pi^-$ invariant mass distributions using the data from $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV[30]. A new resonance $B(5970)$ is found both in the $B^0\pi^+$ and $B^+\pi^-$ mass distributions, whose mass and width of the neutral state are $5978\pm5\pm12$ MeV and $70^{+30}_{-20}\pm30$ MeV, respectively. Since the $B(5970)$ can decay into $B\pi$ final state, it should be a natural spin-parity state.

Unlike the prosperity of charm sector, experimental information on excited bottom and bottom-strange mesons is scarce. Therefore, the above excited $B$ mesons reported by the LHCb and CDF Collaborations provide a good platform to study the low-lying excited bottom and bottom-strange mesons. Some theoretical predictions on masses and widths of bottom and bottom-strange mesons have been performed in different approaches such as constituent quark model[31–35], chiral quark model[11], $3P_0$ model[36–38], heavy meson effective theory[39, 40] and other approaches[41, 42]. These theoretical predictions are not completely consistent with each other. In order to under the natures of the $B_1(5840)$, $B'_1(5960)$, and $B(5970)$, further test calculations against the experimental measurements are required.

The main purpose of this work is to discuss the possible quark-model assignments of the $B_1(5840)$, $B'_1(5960)$, and $B(5970)$. We shall calculate the masses of excited bottom and bottom-strange mesons in a nonrelativistic quark model and the corresponding strong decay behaviors in the $3P_0$ model. The relevant radiative transitions are also evaluated.

This work is organized as follows. In Sec. II, we calculate the bottom and bottom-strange meson masses in a nonrelativistic quark model. In Sec. III, we evaluate the two-body OZI allowed strong decays of the bottom and bottom-strange mesons in the $3P_0$ model with the realistic wave functions from the quark model employed in Sec. II. In Sec. IV, we give the $E1$ and $M1$ radiative decays of the bottom and bottom-strange mesons. A summary is given in the last section.

II. MASSES

To obtain the bottom and bottom-strange meson spectroscopy, we calculate their masses in a nonrelativistic quark model proposed by Lakhina and Swanson, which can describe the heavy-light meson and heavy quarkonium masses with reasonable accuracy[43]. We have employed this model to evaluate the open-charm mesons masses in Ref.[18]. In this model, the total Hamiltonian can be written as

$$H = H_0 + H_{sd} + C_{q\bar{q}}$$

(1)

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TABLE I: The neutral charge resonances observed by the LHCb Collaboration with different spin-parity hypotheses\,[29]. The N and UN stand for the natural spin-parity $[P = (−1)^{J+1}]$ and unnatural spin parity $[P = (−1)^{J+1}]$, respectively.

| Hypothesis  | $M_{B_J(5840)^0}$ (GeV) | $Γ_{B_J(5840)^0}$ (MeV) | $M_{B_J(5960)^0}$ (GeV) | $Γ_{B_J(5960)^0}$ (MeV) |
|-------------|--------------------------|--------------------------|--------------------------|--------------------------|
| I           | 5862.9 ± 5.0 ± 6.7 ± 0.2 MeV | 127.4 ± 16.7 ± 34.2 MeV | 5969.2 ± 2.9 ± 5.1 ± 0.2 MeV | 82.3 ± 7.7 ± 9.4 MeV |
| II          | 5889.7 ± 22.0 ± 6.7 ± 0.2 MeV | 107.0 ± 19.6 ± 34.2 MeV | 6015.9 ± 3.7 ± 5.1 ± 0.2 MeV | 81.6 ± 9.9 ± 9.4 MeV |
| III         | 5907.8 ± 4.7 ± 6.7 ± 0.2 ± 0.4 MeV | 119.4 ± 17.2 ± 34.2 MeV | 5993.6 ± 6.4 ± 5.1 ± 0.2 MeV | 55.9 ± 6.6 ± 9.4 MeV |

where $H_0$ is the zeroth-order Hamiltonian, $H_{sd}$ is the spin-dependent Hamiltonian, and $C_{qq}$ is a constant. The $H_0$ is

$$H_0 = \frac{p^2}{2m} - \frac{4\alpha_s}{3} \frac{e^{2\alpha_s(1-s^2)}}{\sqrt{\pi m q m_q}} S_q \cdot S_{\bar{q}},$$

where $p$ is the center-of-mass momentum, $r$ is the $q\bar{q}$ separation, $M_r = 2m_qm_{\bar{q}}/(m_q + m_{\bar{q}})$; $m_q$ and $m_{\bar{q}}$ are the masses of quark $q$ and antiquark $\bar{q}$, respectively; $S_q$ and $S_{\bar{q}}$ are the spins of the quark $q$ and antiquark $\bar{q}$, respectively. The spin-dependent part $H_{sd}$ can be expressed as

$$H_{sd} = \left( \frac{S_q}{2m_q^2} + \frac{S_{\bar{q}}}{2m_{\bar{q}}^2} \right) \cdot \frac{L}{2} \left( \frac{1}{r} \frac{dV_2}{dr} + \frac{2}{r} \frac{dV_1}{dr} \right) + \left( \frac{S_q}{m_q m_{\bar{q}}} + \frac{S_{\bar{q}}}{m_{\bar{q}} m_q} \right) \cdot \frac{S}{r} \cdot \frac{V_4}{3m_q m_{\bar{q}}} + \frac{\left( \frac{S_q}{m_q^2} - \frac{S_{\bar{q}}}{m_{\bar{q}}^2} \right) + \frac{S}{m_q m_{\bar{q}}} \cdot L V_4, (3)}{m_{\bar{q}}^2} \right) \cdot \frac{S}{r} \cdot \frac{V_4}{3m_q m_{\bar{q}}}$$

where $L$ is the relative orbital angular momentum of the $q\bar{q}$ system, and

$$V_2 = -\frac{4\alpha_s}{3} \frac{e^{2\alpha_s(1-s^2)}}{r} \left( \frac{1}{r} \frac{dV_2}{dr} + \frac{2}{r} \frac{dV_1}{dr} \right),$$

$$V_3 = -\frac{4\alpha_s}{3} \frac{e^{2\alpha_s(1-s^2)}}{r^3} \left( \frac{1}{r} \frac{dV_2}{dr} + \frac{2}{r} \frac{dV_1}{dr} \right),$$

$$V_4 = \frac{1}{\pi} \frac{\ln \left( \frac{m_q}{m_{\bar{q}}} \right)}{r^3}.$$

Here $\gamma_E = 0.5772$ and the scale $\mu$ is set to 1.1 GeV.

The parameters used in this work are $\alpha_s = 0.5$, $b = 0.14$ GeV$^2$, $\sigma = 1.17$ GeV, $C_{dB} = 0.003$ GeV, $C_{dh} = 0.051$ GeV. The constituent quark masses are taken to be $m_u = m_d = 0.45$ GeV, $m_s = 0.55$ GeV, and $m_b = 4.5$ GeV.

The spin-orbit term included in the $H_{sd}$ can be decomposed into symmetric part $H_{sym}$ and antisymmetric part $H_{anti}$. These two parts can be written as

$$H_{sym} = \frac{S \cdot L}{2} \left( \frac{1}{2m_q^2} + \frac{1}{2m_{\bar{q}}^2} \right) \left( \frac{1}{r} \frac{dV_2}{dr} + \frac{2}{r} \frac{dV_1}{dr} \right) + \frac{2}{m_q m_{\bar{q}}} \left( \frac{1}{m_q^2} - \frac{1}{m_{\bar{q}}^2} \right) V_4,$$

$$H_{anti} = \frac{S \cdot L}{2} \left( \frac{1}{2m_q^2} + \frac{1}{2m_{\bar{q}}^2} \right) \left( \frac{1}{r} \frac{dV_2}{dr} + \frac{2}{r} \frac{dV_1}{dr} \right) + \left( \frac{1}{m_q^2} - \frac{1}{m_{\bar{q}}^2} + \frac{2}{m_q m_{\bar{q}}} \right) V_4.$$
the $H_{sd}$ as the perturbative term. The obtained bottom and bottom-strange meson masses are shown in Table II and III. The predictions of some other quark models[32–35] are also listed.

### TABLE II: The B meson masses in MeV from different quark models.

The mixing angles of $B_{l} - B_{l}'$ obtained in this work are $\theta_{lP} = -34.6^\circ$, $\theta_{2P} = -36.1^\circ$, $\theta_{1D} = -39.6^\circ$, $\theta_{2D} = -39.7^\circ$, $\theta_{1F} = -41.0^\circ$. A dash denotes that the corresponding mass was not calculated in the corresponding reference.

| State   | This work ZVR[32] DE[33] EFG[34] LNR[35] |
|---------|------------------------------------------|
| $B(1^{1}S_{0})$ | 5280 5280 5279 5280 5277 |
| $B(1^{3}S_{1})$ | 5329 5330 5324 5326 5325 |
| $B(2^{1}S_{0})$ | 5910 5830 5886 5890 5822 |
| $B(2^{3}S_{1})$ | 5939 5870 5920 5906 5848 |
| $B(3^{1}S_{1})$ | 6369 6210 6320 6379 6117 |
| $B(3^{3}S_{1})$ | 6391 6240 6347 6387 6136 |
| $B(1^{1}P_{0})$ | 5683 5650 5706 5749 5678 |
| $B(1^{1}P_{1})$ | 5729 5690 5700 5723 5686 |
| $B(1^{3}P_{0})$ | 5768 5710 5714 5741 5704 |
| $B(2^{1}P_{1})$ | 6145 6060 6163 6221 6010 |
| $B(2^{3}P_{2})$ | 6185 6100 6175 6209 6022 |
| $B(2^{3}P_{0})$ | 6241 6100 6194 6281 6028 |
| $B(3^{1}P_{1})$ | 6253 6120 6188 6260 6040 |
| $B(1^{3}D_{1})$ | 6095 5970 6025 6119 6005 |
| $B(2^{1}D_{2})$ | 6004 5960 5985 6103 5920 |
| $B(2^{3}D_{2})$ | 6113 5988 6038 6121 5955 |
| $B(3^{1}D_{3})$ | 6014 5970 5993 6091 5871 |
| $B(2^{3}D_{1})$ | 6497 − − 6534 6248 |
| $B(2^{3}D_{2})$ | 6435 6310 − 6528 6179 |
| $B(3^{3}D_{2})$ | 6513 6320 − 6554 6207 |
| $B(1^{3}F_{3})$ | 6444 6320 − 6542 6140 |
| $B(1^{1}F_{2})$ | 6383 6190 6264 6412 − |
| $B(1^{3}F_{1})$ | 6236 6180 6220 6391 − |
| $B(2^{3}F_{3})$ | 6393 6280 6271 6420 − |
| $B(1^{3}F_{4})$ | 6243 6180 6226 6380 − |

### TABLE III: The $B_s$ meson masses in MeV from different quark models.

| State   | This work ZVR[32] DE[33] EFG[34] LNR[35] |
|---------|------------------------------------------|
| $B_s(1^{1}S_{0})$ | 5362 5370 5373 5372 5366 |
| $B_s(1^{3}S_{1})$ | 5413 5430 5421 5414 5417 |
| $B_s(2^{1}S_{0})$ | 5977 5930 5985 5976 5939 |
| $B_s(2^{3}S_{1})$ | 6003 5970 6019 5992 5966 |
| $B_s(3^{1}S_{1})$ | 6415 6310 6421 6467 6254 |
| $B_s(3^{3}S_{1})$ | 6435 6340 6449 6475 6274 |
| $B_s(1^{1}P_{0})$ | 5756 5750 5804 5833 5781 |
| $B_s(1^{1}P_{1})$ | 5801 5790 5805 5831 5795 |
| $B_s(1^{3}P_{0})$ | 5836 5800 5842 5865 5805 |
| $B_s(1^{3}P_{1})$ | 5851 5820 5820 5842 5815 |
| $B_s(2^{3}P_{0})$ | 6203 6170 6264 6318 6143 |
| $B_s(2^{3}P_{1})$ | 6241 6200 6278 6321 6153 |
| $B_s(2^{3}P_{2})$ | 6297 6210 6296 6345 6160 |
| $B_s(2^{3}P_{2})$ | 6309 6220 6292 6359 6170 |
| $B_s(3^{1}D_{1})$ | 6142 6070 6127 6209 6094 |
| $B_s(3^{3}D_{1})$ | 6087 6070 6095 6189 6043 |
| $B_s(3^{3}D_{2})$ | 6159 6080 6140 6218 6067 |
| $B_s(3^{3}D_{3})$ | 6096 6080 6103 6191 6016 |
| $B_s(2^{3}D_{1})$ | 6527 − − 6629 6362 |
| $B_s(2^{3}D_{2})$ | 6492 6410 − 6625 6320 |
| $B_s(2^{3}D_{2})$ | 6542 6420 − 6651 6339 |
| $B_s(2^{3}D_{3})$ | 6500 6420 − 6637 6298 |
| $B_s(1^{3}F_{2})$ | 6412 6300 6369 6501 − |
| $B_s(3^{3}F_{2})$ | 6313 6280 6332 6468 − |
| $B_s(3^{3}F_{1})$ | 6422 6310 6376 6515 − |
| $B_s(1^{3}F_{4})$ | 6319 6290 6337 6475 − |

This equation gives an upper limit of the relativistic kinetic energy term. The equality in Eq. (10) holds if the extremum is taken in $M$. Based on this effective mass expansion of the relativistic kinetic energy term, Jacek and Durand explain the success of Martin’s nonrelativistic descriptions for the spectra of the relativistic light-light and heavy-light mesons[47]. In fact, when an extremum is taken already in the spectrum, the resulting procedure is just a variational method, which establishes a connection between the nonrelativistic potential model and the relativistic potential model. This method, based on the original work[48] and also known as the auxiliary or einbein field method, has proven to be rather accurate in various calculations for the relativistic systems[49] and has been applied to the light-light and heavy-light mesons, glueballs, and hybrids[49–55]. This suggests that one can describe the relativistic heavy-light mesons with formally nonrelativistic formulæ.

When we discuss the possible assignments of the observed bottom states based on the mass information, we use the mass ranges from different quark models including the nonrelativistic and relativistic models rather than
only from the nonrelativistic model (1).

The predicted mass ranges from different quark models and the observed bottom states are shown in Fig. 1. It is shown that the ground states $B$ and $B^*$ can be well described. For the $P$ wave bottom mesons, $B_1(5721)$ and $B_2^*(5747)$ lie within the $1P$ mass range. Hence, the $B_1(5721)$ can be regarded as $B_1(1P)$ or $B_2(1P)$, and $B_2^*(5747)$ is identified as $B(1^3P_2)$. The observed $B_J(5840)$, $B_J(5960)$, and $B(5970)$ lie close to the mass ranges of the $B(2^1S_0)$, $B(2^3S_1)$, $B_2(1D)$, and $B(1^3D_3)$ states. Considering the spin-parity and masses, we tentatively identify $B(5970)$ as the $B(2^3S_1)$ or $B(1^3D_3)$ state. For the $B_J(5840)$ and $B_J(5960)$, all spin-parity hypotheses should be considered. The bottom-strange mesons $B_s$ and $B_s^*$ are well established. $B_{s1}(5830)$ and $B_{s2}^*(5840)$ can be clarified into $P$ wave bottom-strange mesons. The assignments for these observed bottom and bottom-strange states are listed in Table IV. Below, we shall focus on these possible assignments. Since the mass information alone is insufficient to identify these states, hence the strong decay behaviors also need to be investigated in the $^3P_0$ model.

III. STRONG DECAYS

A. $^3P_0$ model

In this work, we adopt the $^3P_0$ model to evaluate the Okubo-Zweig-Iizuka-allowed two-body strong decays of the bottom and bottom-strange mesons. The $^3P_0$ model, also called as quark pair creation model, has been wildly applied to study hadron strong decays with considerable success[56–59]. In this model, the meson decay occurs through a quark-antiquark pair with the vacuum quantum number[60]. Here we give a brief review of the $^3P_0$ model. The transition operator $T$ of the decay $A \rightarrow BC$
in the $^3P_0$ model can be written as\cite{61}

$$T = -3\gamma \sum_m (1m1-m|00) \int d^3p_3 d^3p_4 \delta^3(p_3+p_4) \chi_{34}^{LM}(p_3) \chi_{34}^{LM}(p_4),$$

where $\gamma$ is a dimensionless $q_3\bar{q}_4$ pair-production strength, and $p_3$ and $p_4$ are the momenta of the created quark $q_3$ and antiquark $\bar{q}_4$, respectively. $\chi_{34}^{LM}$ are the flavor, color, and spin wave functions of the $q_3\bar{q}_4$, respectively. The solid harmonic polynomial $\chi_{34}^{LM}(p) \equiv |p|^1 Y_{LM}^m(\theta_p, \phi_p)$ reflects the momentum-space distribution of the $q_3\bar{q}_4$.

The partial wave amplitude $\mathcal{M}^{LS}_{J}(P)$ can be expressed as

$$\mathcal{M}^{LS}_{J}(P) = \sum_{M_1J_1M_2J_2M_3J_3} \langle LM_{L},SM_{S}|J_{A}M_{J_{A}}\rangle \
\times \langle J_{B}M_{J_{B}}J_{C}M_{J_{C}}|SM_{S}\rangle \
\times \int d\Omega \chi_{LM}^{M_1J_1J_2J_3}(P),$$

where $\mathcal{M}^{M_{J_{A}}M_{J_{B}}M_{J_{C}}}(P)$ is the helicity amplitude and defined as

$$\langle BC|T|A \rangle = \delta^3(P_{A}-P_{B}-P_{C}) \mathcal{M}^{M_{J_{A}}M_{J_{B}}M_{J_{C}}}(P)$$

The $|A\rangle$, $|B\rangle$, and $|C\rangle$ denote the mock meson states and the mock meson $|A\rangle$ is defined by\cite{62}

$$|A\rangle = n_{A}^{2S_{A}+1} \sum_{M_{L_{A}},M_{S_{A}}} \langle LM_{L_{A}},SM_{S_{A}}|J_{A}M_{J_{A}}\rangle \
\times \int d^3p_{A} \chi_{n_{A}L_{A}}^{M_{L_{A}}M_{S_{A}}}(p_{A}) \chi_{34}^{LM}(p_{A}) \omega_{A}^{12} \
\times \langle q_{1}(m_{1}+m_{2})P_{A}+p_{A}\rangle \bar{q}_{2}(m_{1}+m_{2})P_{A}-p_{A}\rangle,$$

where $m_1$ and $m_2$ ($p_1$ and $p_2$) are the masses (momenta) of the quark $q_1$ and the antiquark $\bar{q}_2$, respectively; $P_{A} = p_{1} + p_{2}$, $P_{A} = m_{1}P_{1}+m_{2}P_{2}$; $\chi_{34}^{LM}(p_{A})$ and $\omega_{A}^{12}$ are the spin, flavor, color, and space wave functions of the meson $A$ composed of $q_1\bar{q}_2$ with total energy $E_{A}$, respectively.

Because of different choices of the pair-production vertex, phase space conventions, employed meson wave functions, various $^3P_0$ models exist in literatures. In this work, we restrict to the simplest vertex as introduced originally by\cite{60}, which assumes a spatially constant pair-production strength $\gamma$, adopt the relativistic phase space as Ref.\cite{61}, and the realistic meson wave functions from the quark model\cite{1}. With the relativistic phase space, the decay width $\Gamma(A \to BC)$ can be expressed in terms of the partial wave amplitude Eq. (12)

$$\Gamma(A \to BC) = \frac{\pi P}{4M_{A}} \sum_{LS} |\mathcal{M}^{LS}(P)|^{2},$$

where $P = |P| = \sqrt{M_{A}^{2}-(M_{B}+M_{C})^{2}}/2M_{A}$, and $M_{A}$, $M_{B}$, and $M_{C}$ are the masses of the mesons $A$, $B$, and $C$, respectively.

We take the light nonstrange quark pair creation strength $\gamma = 7.6$ by fitting to the total width of the $B^+_2(5747)$ as the $B(1^3P_0)$ state. The $\gamma$ and strange quark pair creation strength $\gamma_{s\bar{s}}$ can be related by $\gamma_{s\bar{s}} = \gamma_{u\bar{u}}\gamma_{d\bar{d}}$\cite{33}, where the constituent quark masses $m_{u}$ and $m_{d}$ are the same as used in the mass estimators in the quark model\cite{1}. Our value of $\gamma$ is higher than that used by other groups such as\cite{22,59} by a factor of $\sqrt{96\pi}$ due to different field conventions. The mixing angles $\theta_{nL}$ are taken from Table II and III.

### Table IV: Possible assignments for the observed bottom and bottom-strange states based on masses and spin-parity.

| State      | Possible assignments                        |
|------------|--------------------------------------------|
| $B^+_2(5721)$ | $B^+_1(1P), B'_1(1P)$                     |
| $B^+_2(5747)$ | $B^+_1(3P_2)$                             |
| $B_{1}(5970)$ | $B^+_1(3S_1), B^+_1(3D_3)$                |
| $B^+_1(5840)$ | $B^+_1(2S_0)$                             |
| $B^+_1(5960)$ | $B^+_1(2D_2)$                             |
| $B_{1}(5840)$ | $B^+_1(2S_0)$                             |
| $B_{1}(5960)$ | $B^+_1(2D_2)$                             |
| $B_{1}(5840)$ | $B^+_1(2S_0)$                             |
| $B_{1}(5960)$ | $B^+_1(2D_2)$                             |
| $B_{1}(5840)$ | $B^+_1(2S_0)$                             |
| $B_{1}(5960)$ | $B^+_1(2D_2)$                             |

### Table V: Decay widths of the $B(1^3P_0)$ in MeV.

| $B^+_1\pi$ | 153.80 |
| $B^+_1\eta$ | 76.62 |
| Total | 230.43 |

For the $B(1^3P_0)$ state, the predicted mass is above the $B\pi$ threshold. The decay widths of the $B(1^3P_0)$ are shown in Table V. No experimental data of the $B(1^3P_0)$ exist, but some theoretical estimations also give a broad width\cite{9,36}, which is consistent with our result.

In Table VI, we give the decay widths of the $B_{2}^+(5747)$. The $\gamma$-independent ratio is predicted to be

$$\frac{\Gamma(B_{2}^+(5747)^{0} \to B^+ \pi^{-})}{\Gamma(B_{2}^+(5747)^{0} \to B^+ \pi^{-})} = 0.95,$$

which is consistent with experimental data of $0.10 \pm 0.02$\cite{64} and $0.71 \pm 0.14 \pm 0.30$\cite{29}.
The decay widths of the $B_1(5721)$ as the $B_1(1P)$ and $B_1'(1P)$ are listed in Table VII. With the $B_1(1P)$ assignment to $B_1(5721)$, the total decay width is expected to be about 200 MeV, much larger than the experiment, hence this assignment can be totally excluded. With the $B_1'(1P)$ assignment, the total width of the $B_1(5721)$ is 40.63 MeV, consistent with 30.1 $\pm$ 1.5 $\pm$ 3.5 given by the LHCb Collaboration[29].

The dependence of the $B_1(5721)$ total width on the mixing angle $\theta_{1P}$ is depicted in Fig 2. The predicted mixing angle from the quark model (1) is $\theta_{1P} = -34.6^\circ$. With this angle, the $B_1(1P)$ decay width is much broader than that of the $B_1'(1P)$, which is consistent with other theoretical predictions[9, 36]. In the heavy quark effective theory, the $P$ wave heavy-light mesons can be divided into the $(0^+, 1^+)_j = \frac{1}{2}$ and $(1^+, 2^+)_j = \frac{3}{2}$ doublets, where $j$ is the total angular momentum of the light quark. In the heavy quark limit, the $B_1(5721)$ corresponds to the $1^+$ bottom meson belonging to the $(1^+, 2^+)_j = \frac{3}{2}$ doublet.

C. $B_J(5840)$ and $B_J(5960)$

For the $B_J(5840)$ and $B_J(5960)$, three spin-parity hypothesis exist, which classifies these states into different possible assignments. In the following, we will consider these assignments one by one.

In Table VIII, we list the decay widths of the $B_J(5840)$ and $B_J(5960)$ under the hypothesis I. The predicted total width of the $B_J(5840)$ as the $B(2^1S_0)$ is 106.13 MeV, consistent with the experimental data of 107.0 $\pm$ 19.7 $\pm$ 34.2 MeV. However, the predicted total width of the $B_J(5960)$ as the $B_2(1D)$ state is much larger than the experimental data of 82.3 $\pm$ 7.7 $\pm$ 9.4 MeV.

The predicted decay widths of the $B_J(5840)$ and $B_J(5960)$ under the hypothesis II are presented in Table IX. The predicted total width of the $B_J(5840)$ as the $B(2^1S_0)$ state is 106.13 MeV, consistent with the experimental data of 107.0 $\pm$ 19.7 $\pm$ 34.2 MeV. However, the predicted total width of the $B_J(5960)$ as the $B_2(1D)$ state is much larger than the experimental data of 81.6 $\pm$ 9.9 $\pm$ 9.4 MeV.

The decay widths of the $B_J(5840)$ and $B_J(5960)$ under the hypothesis III are shown in Table X. The predicted width of the $B_J(5840)$ as the $B(2^1S_0)$ is about 134 MeV, consistent with the measured result of 119.4 $\pm$ 17.2 $\pm$ 34.2 MeV. If the $B_J(5960)$ is the $B(2^3S_1)$, the $B_J(5960)$ total width is expected to be 131.97 MeV, far away from the measured width of 55.9 $\pm$ 6.6 $\pm$ 9.4 MeV. If the $B_J(5960)$ is the $B(1^3D_3)$, the $B_J(5960)$ total width is expected
TABLE IX: Decay widths of the $B_J(5840)$ and $B_J(5960)$ under the hypothesis II in MeV. A dash indicates that a decay mode is forbidden.

| $B_J(5840)$ | $B_J(5960)$ |
|----------------|----------------|
| $B(2^3S_1)$ | $B(2^1D_3)$ |
| $B^+\pi^-$ | 20.43 | – |
| $B^0\pi^0$ | 10.26 | – |
| $B^0\pi^-$ | 46.68 | 59.21 |
| $B^-\pi^-$ | 23.39 | 29.70 |
| $B(1^3P_0)^+\pi^-$ | 0.002 | 84.63 |
| $B(1^3P_0)^0\pi^0$ | 2.9 $\times$ 10$^{-4}$ | 42.42 |
| $B_2^+ (5747)^+\pi^-$ | – | 0.91 |
| $B_2^0 (5747)^0\pi^0$ | – | 0.006 |
| $B_3^+ (1P)^+\pi^-$ | 0.21 | 0.05 |
| $B_3^0 (1P)^0\pi^0$ | 0.10 | 0.02 |
| $B_4^+ (1P)^+\pi^-$ | 0.05 | 0.54 |
| $B_4^0 (1P)^0\pi^0$ | 0.02 | 0.26 |
| $B_5^+$ | 2.61 | – |
| $B_5^-$ | 0.98 | 16.79 |
| $B_6$ | 1.40 | – |
| $B_7$ | 24.14 | – |
| Total width | 106.13 | 257.79 |
| Experiment | 107.0 $\pm$ 19.6 $\pm$ 34.2 | 81.6 $\pm$ 9.9 $\pm$ 9.4 |

TABLE X: Decay widths of the $B_J(5840)$ and $B_J(5960)$ under the hypothesis III in MeV. A dash indicates that a decay mode is forbidden.

| $B_J(5840)$ | $B_J(5960)$ |
|----------------|----------------|
| $B(2^3S_1)$ | $B(2^1D_3)$ |
| $B^+\pi^-$ | – | 14.39 | 15.08 |
| $B^0\pi^0$ | – | 7.25 | 7.48 |
| $B^+\pi^-$ | 86.24 | 38.20 | 16.17 |
| $B^0\pi^0$ | 43.21 | 19.20 | 8.03 |
| $B^+ (1^3P_0)^+\pi^-$ | 4.1 $\times$ 10$^{-4}$ | – | – |
| $B^0 (1^3P_0)^0\pi^0$ | 4.0 $\times$ 10$^{-5}$ | – | – |
| $B_2^+ (5747)^+\pi^-$ | 0.05 | 1.62 | 0.35 |
| $B_2^0 (5747)^0\pi^0$ | 0.02 | 0.76 | 0.16 |
| $B_3^+ (1P)^+\pi^-$ | – | 0.37 | 0.15 |
| $B_3^0 (1P)^0\pi^0$ | – | 0.18 | 0.07 |
| $B_4^+ (1P)^+\pi^-$ | – | 2.41 | 0.04 |
| $B_4^0 (1P)^0\pi^0$ | – | 1.14 | 0.02 |
| $B_5^+$ | – | 6.74 | 0.44 |
| $B_5^-$ | 4.57 | 12.06 | 0.23 |
| $B_6$ | – | 11.81 | 0.26 |
| $B_7$ | 15.83 | 0.08 |
| Total width | 134.09 | 131.97 | 48.55 |
| Experiment | 119.4 $\pm$ 17.2 $\pm$ 34.2 | 55.9 $\pm$ 6.6 $\pm$ 9.4 |

to be 48.55 MeV, in good agreement with the data of 55.9 $\pm$ 6.6 $\pm$ 9.4 MeV.

To sum up, with the hypothesis III, the total widths of the $B_J(5840)$ and $B_J(5960)$ can be reproduced simultaneously. The strong decay behaviors combined with masses indicate that the $B_J(5840)$ and $B_J(5960)$ can be identified as the $B(2^1S_0)$ and $B(1^3D_3)$, respectively. The assignment of the $B_J(5840)$ as the $B(2^1S_0)$ state, is also suggested by the LHCB Collaboration [29]. The main decay modes of the $B(2^1S_0)$ are expected to $B^+\pi$ and $B^+\eta$. The main decay modes of the $B(1^3D_3)$ are expected to be $B\pi$ and $B^+\pi$.

D. $B(5970)$

The decay widths of the $B(5970)$ as the $B(2^1S_1)$ and $B(1^3D_3)$ are listed in Table XI. Since the $B(5970)$ mass is close to the $B_J(5960)$ mass, the results for the $B(2^1S_1)$ and $B(1^3D_3)$ are similar with those in Table X. However, because of the large uncertainty of the $B(5970)$ total width, both the $B(2^1S_1)$ and $B(1^3D_3)$ assignments are favored by the experimental data [30]. Ref. [36] interprets the $B(5870)$ as the $B(2^1S_0)$ state while Ref. [9] assigns the $B(5970)$ as the $B(1^3D_3)$ state [9]. The main decay modes of the $B(2^1S_1)$ are expected to be $B\pi$, $B^+\pi$, $B\eta$, $B^+\eta$, $B_s K$, and $B^* K$, while the $B(1^3D_3)$ is expected to mainly decay to $B\pi$, $B^+\pi$. Further precise measurements of the width, spin and decay modes are needed to distinguish these two assignments.

E. $B(1^3D_3)$, $B_2(1D)$ and $B_2^*(1D)$

Given the bottom masses and spin-parity, no experimental candidates exist for the $B(1^3D_1)$, $B_2(1D)$ and $B_2^*(1D)$ states. Our predicted masses for these three states are 6095 MeV, 6004 MeV, and 6113 MeV, respectively. With these masses as inputs, their total decay widths are listed in Table XII.

It is shown that all these three states have large total widths more than 200 MeV. The decay modes of these states are different, mainly due to the quantum number conservation and the threshold.
In heavy quark limit, the mixing angle is \( \theta_{1D} = -50.8^\circ \)\[65\]. Our predicted \( \theta_{1D} = -39.6^\circ \) is close to \(-50.8^\circ \). With this mixing angle, the total decay width of \( B_2(1D) \) is broader than \( B_2'(1D) \), which indicate that \( B_2(1D) \) and \( B_2'(1D) \) corresponds to the \( \{1^-, 2^-\}_j = \frac{3}{2} \) and \( \{2^-, 3^-\}_j = \frac{7}{2} \) doublets, respectively.

### F. \( B_s(1P) \) states

For the \( B_s(1^3P_0) \) state, the predicted mass is below the \( BK \) threshold, which is consistent with some other studies\[32, 66\]. Hence, there is no OZI-allowed strong decay pattern and the dominant decay mode may be \( B_s \pi \). This situation is analogous to the charmed-strange partner \( D_{s0}^* \) (2317), whose decay width is mainly due to OZI-violated \( D_s \pi \) channel. Based on higher mass of \( B_s(1^3P_0) \) state, some theoretical calculations give a broad decay width\[9, 36\]. Further experimental search for the \( B_s(1^3P_0) \) state will distinguish these two predictions.

The decay widths of the \( B_s^*(5840) \) are listed in Table XIII. The predicted total decay width is 1.99 MeV, in good agreement with LHCb experimental data of 1.56 ± 0.13 ± 0.47 MeV[4, 67] and the CDF result of 1.4 ± 0.4 ± 0.2 MeV[30]. The predicted ratio

\[
\frac{\Gamma(B_s^{*+}(5840) \rightarrow B^{*}K^-)}{\Gamma(B_s^{*0}(5840) \rightarrow B^{*}K^-)} = 0.086
\]

is independent with the \( \gamma \) and in agreement with the LHCb experimental data of 0.093 ± 0.013 ± 0.012[4, 67].

In analogous to the \( B_1(5721) \), the \( B_{s1}(5830) \) can be \( B_{s1}(1P) \) or \( B_{s1}'(1P) \). With the predicted mixing angle \( \theta_{1PS} = -34.9^\circ \), the total widths of \( B_{s1}(1P) \) and \( B_{s1}'(1P) \) are expected to be 162.76 MeV and 21.35 MeV, respectively, both much larger than the CDF data of \( 0.5 \pm 0.3\) MeV\[30\]. The dependence of the total widths of \( B_{s1}(1P) \) and \( B_{s1}'(1P) \) versus the mixing angle are shown in Fig 3. It can be seen that when the mixing angle varies in the range of \( (-59.2 \sim -50.4)^\circ \), the total width of the \( B_{s1}(1P) \) is consistent with the observed width. In the heavy quark effective theory, the ideal value of the mixing angle is \( \theta_{1P} = -54.7^\circ \), lying in the range of \( (-59.2 \sim -50.4)^\circ \). The extremely narrow total width of the \( B_{s1}(5830) \) suggests that it can be identified as the \( B_{s1}'(1P) \) state belonging to the \( \{1^+, 2^+\}_j = \frac{5}{2} \) doublet.

### G. \( B_s(2S) \)

Our predicted masses of the \( B_s(2^1S_0) \) and \( B_s(2^3S_1) \) are 5977 MeV and 6003 MeV, respectively. The decay widths of the \( B_s(2^1S_0) \) and \( B_s(2^1S_1) \) are listed in Table XIV. It is shown that the \( B_s(2^1S_0) \) state mainly decays into \( B^*K \), and the main decay modes of the \( B_s(2^3S_1) \) state are \( BK \) and \( B^*K \).

### Table XII: Decay widths of the \( B(1^3D_1), B_2(1D) \) and \( B_2'(1D) \) states in MeV. A dash indicates that a decay mode is forbidden.

| State          | \( B(1^3D_1) \) | \( B_2(1D) \) | \( B_2'(1D) \) |
|----------------|-----------------|----------------|----------------|
| \( B^+\pi^- \) | 26.44           | -              | -              |
| \( B^0\pi^0 \) | 13.31           | -              | -              |
| \( B^+\pi^- \) | 15.45           | 60.57          | 61.08          |
| \( B^0\pi^0 \) | 7.76            | 30.38          | 30.44          |
| \( B(1^3P_0)^+\pi^- \) | 4.52 | 81.79          | 3.51          |
| \( B(1^3P_0)^0\pi^0 \) | 2.21 | 49.93          | 1.76          |
| \( B_2'(5747)^+\pi^- \) | - | 0.01          | 0.20          |
| \( B_2'(5747)^0\pi^0 \) | - | 0.005         | 0.10          |
| \( B_1(1P)^+\pi^- \) | 10.30 | 0.03          | 0.18          |
| \( B_1(1P)^0\pi^0 \) | 5.20            | 0.02          | 0.09          |
| \( B_2(1P)^+\pi^- \) | 74.77           | 0.42          | 5.57          |
| \( B_2(1P)^0\pi^0 \) | 37.71           | 0.20          | 2.74          |
| \( B_\eta \) | 14.18           | -             | -             |
| \( B_\eta' \) | 7.34            | 15.58         | 4.64          |
| \( B_\eta_s \) | 32.67           | -             | -             |
| \( B_\eta_s' \) | 15.06           | 20.75         | 5.04          |
| \( B_\omega \) | 10.64           | -             | 48.37         |
| \( B_\omega' \) | 5.27            | -             | 24.04         |
| \( B_\omega'' \) | -               | -             | 4.02          |
| \( B_{s0}(1P)^+\pi^- \) | - | 2.01          | -             |
| \( B_{s0}(1P)^0\pi^0 \) | 4.20 | 21.08         | -             |
| \( B_{s0}(1P)\pi^0 \) | - | -             | 0.58          |
| \( B_{s0}(1P)^+\pi^- \) | 0.52 | -             | -             |
| \( B_{s0}'(1P)^+\pi^- \) | 0.24 | -             | -             |
| \( B_{s0}'(1P)^0\pi^0 \) | 0.05 | -             | 0.01          |
| \( B_{s0}'(1P)\pi^0 \) | - | 0.02          | -             |
| Total width   | 287.85          | 250.69         | 215.47         |

### Table XIII: Decay widths of the \( B_{s2}^*(5840) \) in MeV.

| Decay Mode   | Width (MeV) |
|--------------|-------------|
| \( B^+K^- \) | 1.09        |
| \( B^0K^0 \) | 0.86        |
| \( B^{*}\bar{K}^- \) | 0.09 |
| \( B^{*0}\bar{K}^0 \) | 0.05 |
| Total width  | 1.99        |

### Table XIV: Decay widths of the \( B_s(2^1S_0) \) and \( B_s(2^3S_1) \) in MeV.

| Decay Mode   | Width (MeV) |
|--------------|-------------|
| \( B_s(2^1S_0) \) | \( B_s(2^3S_1) \) |
| \( B^+K^- \) | - 35.67     |
| \( B^{*0}K^0 \) | - 35.67     |
| \( B^{*}\bar{K}^- \) | 106.62 70.71 |
| \( B^{*0}\bar{K}^0 \) | 105.36 70.29 |
| \( B_s\eta \) | - 5.32      |
| \( B_s^{*}\eta \) | 1.51 4.24   |
| Total width  | 213.38 221.90 |
heavy-light mesons. We evaluate the $E1$ and $M1$ radiative partial widths between the $v = n^{2S+1}L_J$ and $v' = n^{2S'+1}L'_J$, states using\cite{22, 68, 69}

\[ \Gamma_{E1}(v \rightarrow v' + \gamma) = \frac{4\alpha e_Q^2}{3} C_{fi} \delta_{SS'} |<v'|\gamma|v>|^2 \frac{E_\gamma E_f}{M_i} \]

\[ \Gamma_{M1}(v \rightarrow v' + \gamma) = \frac{\alpha e_Q^2}{3} 2J' + 1 \frac{E_\gamma E_f}{M_i} \delta_{LL'} \delta_{SS'} |<v'|j_0(\frac{E_\gamma r}{2})|v>|^2 \frac{E_\gamma E_f}{M_i} \]

where $e_Q = \frac{m_q Q_Q + m_b Q_b}{m_q + m_b}$, $e'_Q = \frac{m_q Q'_Q + m_b Q'_b}{m_q + m_b}$, $Q_i$, and $Q_q$ stand for the charges of the quark $b$ and $q$ in units of $|e|$, respectively. $\alpha = 1/137$ is the fine-structure constant, $E_\gamma$ is the photon energy, $E_f$ is the energy of final heavy-light meson, $M_i$ is the mass of initial state, and the angular matrix element $C_{fi}$ can be expressed as

\[ C_{fi} = \text{Max}(L, L')(2J' + 1) \left\{ \begin{array}{c} L' \ J' \ S \\ J \ L \ 1 \end{array} \right\}^2. \]

The wavefunctions obtained from the quark model (1) are used to calculate the $E1$ and $M1$ radiative partial widths. To determine the photon and final state energies, the masses of these initial and final states should be involved. For the well established $B$, $B^*$, $B_1$, and $B_2^*$, the masses are taken from PDG\cite{4}. For the $B'(1P)$, $B'(1P')$, $B'(2P)$, $B'(2P')$, $B'^*(1P)$, $B'^*(1P')$, $B'^*(2P)$, $B'^*(2P')$, their masses are taken to be the $B_1(5721)$, $B_2^*(5747)$, $B_1'(5840)$, $B_{11}'(5960)$, $B_{31}'(5830)$, and $B_{21}'(5840)$ masses, respectively. For other states, their masses are taken from the predictions of the quark model(1). The $E1$ and $M1$ transitions widths of the neutral charge open-bottom states together with the photon energies are listed in Tables XVI, XVII, XVIII, XIX.

From Tables XVI, it can be seen that the $B(1^3P_1)\gamma$, $B_1(1P)\gamma$, and $B_2'(1P')\gamma$ channels are essential to discriminate the $B(2^3S_1)$ and $B(1^3D_3)$ interpretations for the $B(5970)$, since these these decay mode are forbidden for the $B(1^3D_3)$ state while allowable for the $B(1^3D_3)$ state. The $B(1^3P_1)\gamma$, $B_1(1P)\gamma$ final state for these two assignments has sizable decay widths, and can be observed experimentally.
TABLE XVII: E1 transitions widths of the neutral charge bottom mesons, $E_\gamma$ in MeV and $\Gamma$ in keV.

| Multiplets  | Initial meson | Final meson | $E_\gamma$ | $\Gamma$ |
|-------------|---------------|-------------|------------|----------|
| $B(2S) \rightarrow B(1P)$ | $B(2^3S_1)$ | $B(1^3P_0)$ | 250 21.4 |  |
| $B(2S)$ | $B(2^3S_1)$ | $B(1^3P_0)$ | 196 51.6 |  |
| $B(2^3S_1)$ | $B_1(1P)$ | 206 11.67 |  |  |
| $B(2^3S_1)$ | $B'_1(1P)$ | 210 25.9 |  |  |
| $B(2^3S_1)$ | $B_1(1P)$ | 176 49.6 |  |  |
| $B(2^3S_5)$ | $B'_1(1P)$ | 180 25.2 |  |  |
| $B(1P) \rightarrow B(1S)$ | $B(1^3P_0)$ | $B(1^3P_0)$ | 347 116.9 |  |
| $B(1^3P_0)$ | $B(1^3P_0)$ | 400 177.7 |  |  |
| $B_1(1P)$ | $B(1^3P_0)$ | 390 53.1 |  |  |
| $B'_1(1P)$ | $B(1^3S_1)$ | 386 108.5 |  |  |
| $B_1(1P)$ | $B(1^3S_1)$ | 432 130.2 |  |  |
| $B'_1(1P)$ | $B(1^3S_1)$ | 428 60.4 |  |  |

TABLE XVI: E1 transitions widths of neutral charge bottom-strange mesons, $E_\gamma$ in MeV and $\Gamma$ in keV.

| Multiplets  | Initial meson | Final meson | $E_\gamma$ | $\Gamma$ |
|-------------|---------------|-------------|------------|----------|
| $B(1D) \rightarrow B(1P)$ | $B(1^3D_1)$ | $B(1^3P_2)$ | 248 127.0 |  |
| $B(1^3D_1)$ | $B(1^3P_2)$ | 345 9.3 |  |  |
| $B(1^3D_1)$ | $B(1^3P_2)$ | 398 283.5 |  |  |
| $B(1^3D_1)$ | $B_1(1P)$ | 355 49.0 |  |  |
| $B(1^3D_1)$ | $B'_1(1P)$ | 359 106.2 |  |  |
| $B_2(1D)$ | $B(1^3P_2)$ | 258 14.5 |  |  |
| $B_2(1D)$ | $B_1(1P)$ | 269 143.1 |  |  |
| $B_2'(1D)$ | $B'_1(1P)$ | 273 0.1 |  |  |
| $B'_2(1D)$ | $B(1^3P_2)$ | 362 57.2 |  |  |
| $B'_2(1D)$ | $B_1(1P)$ | 372 8.6 |  |  |
| $B'_2'(1D)$ | $B'_1(1P)$ | 376 356.3 |  |  |

TABLE XVIII: M1 transitions widths of the neutral charge bottom mesons, $E_\gamma$ in MeV and $\Gamma$ in keV.

| Initial Multiplet | Initial meson | Final meson | $E_\gamma$ | $\Gamma$ |
|-------------------|---------------|-------------|------------|----------|
| $B_1(1S)$ | $B(1^3S_1)$ | $B(1^3S_0)$ | 45 0.1 |  |
| $B_2(1S)$ | $B(1^3S_1)$ | $B(1^3S_0)$ | 623 8.0 |  |
| $B(1^3S_1)$ | $B(1^3S_0)$ | 31 0.05 |  |  |
| $B(1^3S_1)$ | $B(1^3S_0)$ | 554 0.9 |  |  |
| $B(1P)$ | $B_1(1P)$ | $B(1^3P_0)$ | 46 0.03 |  |
| $B'_1(1P)$ | $B(1^3P_0)$ | 42 0.01 |  |  |
| $B(1^3P_0)$ | $B_1(1P)$ | 11 1.4x10$^{-3}$ |  |  |
| $B(1^3P_0)$ | $B_1(1P)$ | 15 1.7x10$^{-3}$ |  |  |

TABLE XIX: M1 transitions widths of the neutral charge bottom-strange mesons, $E_\gamma$ in MeV and $\Gamma$ in keV.

| Initial Multiplet | Initial meson | Final meson | $E_\gamma$ | $\Gamma$ |
|-------------------|---------------|-------------|------------|----------|
| $B_{1s}(1S)$ | $B(1^3D_1)$ | $B(1^3S_0)$ | 50.1 |  |
| $B_2(1S)$ | $B(1^3S_1)$ | $B(1^3S_0)$ | 603 4.0 |  |
| $B(1^3S_1)$ | $B(1^3S_0)$ | 25.9 0.02 |  |  |
| $B(1^3S_1)$ | $B(1^3S_0)$ | 553 0.1 |  |  |
| $B_1(1P)$ | $B(1^3P_0)$ | 45 0.02 |  |  |
| $B'_1(1P)$ | $B(1^3P_0)$ | 72 0.05 |  |  |
| $B(1^3P_0)$ | $B_1(1P)$ | 39 0.04 |  |  |
| $B(1^3P_0)$ | $B'_1(1P)$ | 11 5.2x10$^{-4}$ |  |  |
| $B_1(1D)$ | $B(1^3D_1)$ | $B(1^3D_2)$ | 55 0.1 |  |
| $B_2'(1D)$ | $B(1^3D_1)$ | 17 1.3x10$^{-3}$ |  |  |
| $B_2'(1D)$ | $B(1^3D_2)$ | 63 0.2 |  |  |

V. SUMMARY

In this paper, we calculate the bottom and bottom-strange meson spectroscopy in a nonrelativistic quark model proposed by Lakhina and Swanson. Our predictions, combined with the results from some other quark models, give the mass ranges of bottom and bottom-strange mesons. With these predictions, we give the possible quark-model assignments for these bottom mesons observed by LHCb and CDF Collaborations. Furthermore, the strong and radiative decay behaviors of these bottom and bottom-strange mesons are investigated with the realistic meson wave functions from our employed nonrelativistic quark model. The $B(5721)$ and $B'_1(5747)$ can be classified into the $B_1(1P)$ and $B(1^3P_2)$, respectively. The $B_1(5830)$ and $B'_1(5840)$ can be identified as the $B'_1(1P)$ and $B(1^3P_2)$ states, respectively. The $B_1(5840)$ and $B_2(5960)$ can be explained as the $B(2^1S_0)$ and $B(1^3D_3)$, respectively. The $B(5970)$ can be interpreted as the $B(2^3S_1)$ or $B(1^3D_3)$. The properties of other states are also predicted, which will be helpful to search for these states experimentally.
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