Supplementary Information for:
'Spontaneous Skyrmion Ground States in Magnetic Metals'

U.K. Rößler and A.N. Bogdanov

IFW Dresden, P.O. Box 270116, D-01171 Dresden, Germany

C. Pfleiderer

Physik Department E21, Technische Universität München, James-Franck-Strasse, D-85748 Garching, Germany

(Dated: April 26, 2018)

Abstract

Supplementary information for our manuscript, entitled 'Spontaneous Skyrmion Ground States of Magnetic Metals', cond-mat/0603103 is presented. The physical nature of the gradient terms of our generalized micromagnetic model for ferromagnets with softened longitudinal fluctuations is explained. The relationship of our micromagnetic model with the spin fluctuation theory of itinerant-electron magnets is discussed. Experimental estimates of the parameter $\eta$, which accounts for an effective reduced longitudinal stiffness, are presented for real materials from published polarized neutron scattering experiments on EuS, Ni and MnSi. The available experimental data clearly show that $\eta$ is significantly reduced for the latter two systems. It is suggested that particle-hole excitations are at the root of this longitudinal softness in itinerant-electron ferromagnets. The current status of the experimental evidence supporting spontaneous, amorphous skyrmion textures in MnSi and other materials is reviewed. Finally, we also address the general potential of skyrmion textures in chiral magnets for other fields of physics.
I. OUTLINE

An important ingredient for the prediction of spontaneous skyrmion ground states in magnetic metals with chiral interactions in our manuscript [cond-mat/0603103] is the assumption that the longitudinal magnetic stiffness is particularly soft. In section II we consider the theoretical underpinning of this assumption. The physical nature of the gradient terms of our generalized micromagnetic model for ferromagnets with softened longitudinal fluctuations is explained. As part of the discussion we also address the relationship of our generalized micromagnetic model and the spin-fluctuation theory of weak itinerant-electron magnets. In section III we review the experimental evidence for $\eta < 1$ in real materials. This is followed by a discussion in section IV of available evidence that supports a spontaneous skyrmion ground state in the itinerant-electron magnet MnSi. The supplement concludes with a discussion of the broader implications of spontaneous skyrmion ground states in magnetic metals in section V.

II. THEORETICAL UNDERPINNING OF THE LONGITUDINAL STIFFNESS

In the following we address from a theoretical point of view the possibility of reductions of the longitudinal stiffness in magnetic materials and their importance. For this purpose we consider the ratio $\eta$ of the longitudinal to the transverse magnetic stiffness, where $\eta$ is identical to that in Eqn. (1) in the main text. In our discussion we focus on the ordered state of isotropic ferromagnets. However, the arguments can be generalised in principle to all magnetic materials. Before entering a more technical level we present an intuitive explanation.

It is instructive to consider $\eta$ at first for isotropic local moment ferromagnets (LMFM). The conventional description of LFM leads to a continuum theory for the magnetization distribution with $\eta \equiv 1$, which is derived as the continuum limit of a Heisenberg model (for an elementary derivation see, e.g., [2]). As shown by Patashinskii and Pokrovskii [3], magnon-magnon scattering in the ordered state forces the longitudinal stiffness to vanish for vanishing wavevector $q$. This effect originates alone in the broken continuous symmetry of the isotropic Heisenberg magnet, where $\eta = 1$ for $q \to 0$ (see also, Zwerger [4]). On more general grounds we note that the conservation of total spin, based on the SU(2) symmetry
of the spin system, requires that a reduction of magnetisation amplitude has to occur as a higher order process that involves at least two transverse processes. The finite lifetime of transverse excitations in turn results in a reduced longitudinal stiffness. If there are microscopic processes limiting the lifetime of transverse excitations in addition to magnon-magnon scattering, the longitudinal stiffness should be reduced further. Examples for such processes include: (i) interactions of magnons with either intraband or interband particle-hole excitations in itinerant magnets, (ii) interactions of magnons with defects, and (iii) the interaction of magnons with dipolar components of the magnetisation.

The stiffness of a magnetic state is a consequence of the correlations introduced by these various microscopic processes. In an ordered or field-polarized state, the correlation lengths transverse and longitudinal to the local polarization direction, $\xi_\perp$ and $\xi_\parallel$, respectively, are generally different. These correlation lengths are a direct measure of the stiffness. Because the decay of longitudinal excitations requires higher-order transverse processes, we expect that $\xi_\parallel$ is always shorter than $\xi_\perp$. Thus, the longitudinal stiffness is expected to be lower than the transverse stiffness, i.e., in real materials we expect $\eta < 1$. This leads us to conclude that the longitudinal stiffness in real materials is always reduced below pure magnon-magnon scattering ($\eta = 1$), whence $\eta < 1$. This conclusion is strongly supported by the experimental data reviewed in section III which gives in the local moment ferromagnet EuS ($\eta_{\text{EuS}} = 0.925$), and the itinerant-electron ferromagnets Ni ($\eta_{\text{Ni}} = 0.65$) and MnSi ($\eta_{\text{MnSi}} = 0.4$).

We now enter the more technical part of the discussion. It is helpful to clarify at first the terminology we use. When we refer to ‘micromagnetic models’ we refer to a language that is in the tradition of the continuum theory of magnetism founded mainly by Landau’s school in the 1940s. Micromagnetic models are used in the field of technical magnetism and magnetic domains for length scales down to the nanometer scale. Traditionally micromagnetic models consider only changes of the magnetization direction, while the amplitude is fixed by the saturation magnetization. Our generalised micromagnetic model for the magnetic stiffness is hence outside the tradition of micromagnetism. The field-theory underlying conventional micromagnetic models is the non-linear $\sigma$ model (NL$\sigma$M). In contrast, the description developed for spin fluctuations in itinerant-electron magnets is referred to as spin fluctuation theory. Spin fluctuation theory is based on Fermi liquid theory and the spontaneous spin polarisation of the metallic state. Traditionally, it considers materials where the magnetization is free to change direction and amplitude. Both, micromagnetic models and spin
fluctuation theory can be represented in terms of a Ginzburg-Landau functional of a 3-vector model for the magnetization distribution.

The task of this discussion is to bring out how to modify micromagnetic models for a description of itinerant-electron magnets. The discussion this way also aims to clarify the nature of the gradient term in $\eta$. In the following we first consider the relationship of the conventional action of spin fluctuation theory with the NL$\sigma$M as the low temperature field-theoretical approximation that corresponds to micromagnetic models. This allows to illustrate how to change micromagnetic models to become appropriate for itinerant-electron magnets. We then provide arguments that mechanisms beyond magnon-magnon processes, which additionally reduce the longitudinal stiffness, indeed lead to a significantly reduced ratio $\eta < 1$ in real materials.

We begin with an outline how to derive a corresponding micromagnetic model from spin-fluctuation theories for weak itinerant-electron ferromagnets. The appropriate Ginzburg-Landau functional in spin fluctuation theory is developed in a classical vector $m(x)$ with three-components [5], where the action (or reduced Hamiltonian) [7] is given by

$$S(m) = \int d^d x \left[ A \left( \sum_\alpha \partial_\alpha m \right)^2 + r |m|^2 + b |m|^4 \ldots \right].$$

Eqn. (1) reduces to the NL$\sigma$M when the amplitude is kept constant, $m(x) \equiv m_0 = \text{constant}$, i.e., the fluctuations of the modulus $m$ are neglected. It follows

$$S_0(n) = \int d^d x \left[ A m_0^2 \left( \sum_\alpha \partial_\alpha n \right)^2 \right],$$

where constant terms have been dropped. This is the low-temperature field-theoretical approximation used in micromagnetism. To go beyond this approximation, we have to
include fluctuations in $m(x)$. Considering that the longitudinal and transverse stiffness may differ in the ordered state, we use two different parameters for the two gradient terms in Eqn. (2). This is achieved by introducing the parameter $\eta$.

$$S_1(m, n) = \int d^d x \left[ A m^2(x) (\sum_{\alpha} \partial_\alpha n)^2 + A \eta \sum_{\alpha} (\partial_\alpha m(x))^2 + r m(x)^2 + b m(x)^4 \ldots \right]. \quad (4)$$

As shown in the following, $\eta$ measures the ratio of longitudinal to transverse stiffness. To bring out the nature of the gradient terms in the context of our micromagnetic Ansatz, Eqn. (4) can be rewritten as a functional of the magnetization vector $m$ and amplitude $m$

$$S_1(m, m) = \int d^d x \left[ A (\sum_{\alpha} \partial_\alpha m)^2 - (1 - \eta) A \sum_{\alpha} (\partial_\alpha m(x))^2 + r m(x)^2 + b m^4 \ldots \right]. \quad (5)$$

For $\eta < 1$, the form of the action $S_1(m, m)$ implies that fluctuations of the amplitude lead to energy gains as compared to the case $\eta \equiv 1$. In other words, the action $S_1(m, n)$ (Eqn 4) describes softened longitudinal stiffness for $\eta < 1$.

The fixed modulus version of the action given in Eqn. (3), which may be derived by approaching the low-temperature limit by taking $r \to -\infty$, $b \to \infty$, and $r/b = \text{const}$, has been analysed earlier in the context of critical phenomena and phase transitions [8, 9]. In the present context, we are interested in the gradient energy within the magnetically ordered state at an intermediate length, which is set by the competition between microscopic direct exchange and the chiral Dzyaloshinskii-Moriya exchange. It has been shown for the long-distance behaviour that only the usual gradient energy describing the rotation of a fixed-length magnetisation is relevant as used in micromagnetic theory in Eqn. (3) [3, 8, 9, 10]. More specifically, in the renormalization group analysis one finds that the gradient term in $(1 - \eta)$ in Eqn. (5) is irrelevant for the critical properties of ferromagnets [10], i.e., in the limit of diverging correlation length at $T_C$. The analysis moreover stresses that only the transverse fluctuations are relevant to drive the phase-transition from the ordered state into the isotropic paramagnetic state. At the same time it is found that the irrelevant term with factor $(1 - \eta)$ has a different scaling behaviour as compared to the transverse stiffness.

These renormalization group results [8, 9] assert that the generalised gradient energy in our model does not change the expected universal behaviour of an isotropic ferromagnet with collinear ordering. This is the formal proof that we may freely use a generalized version of the gradient energy with $\eta < 1$. As the two gradient terms have differing scaling behaviour, a coarse-grained effective action after a suitable renormalization procedure will
always lead to gradient terms with $\eta \neq 1$ for the ratio of effective longitudinal and transverse stiffness. Therefore, this ratio will also depend on the scale considered under coarse-graining. When taken together, these theoretical results justify the assumption that we can represent the two different longitudinal and transverse correlations lengths in the ordered state by the generalization of the gradient energy, which is embodied in the two parameters $A$ and $\eta$. For arbitrary values of $\eta > 0$ the gross qualitative properties of the paramagnetic to ferromagnetic transition and the ordered state are unchanged. However, the term arising for $\eta < 1$ in the phenomenological theory given by Eqn. (4) is the most important and simplest term to account for effects due to two different lengths for the stiffness of the ordered state. This term changes only non-universal properties. Essentially, it leads only to a shifted ordering temperature. Therefore, the effects due to a softened longitudinal stiffness for $\eta < 1$ are difficult to trace in the properties of a conventional collinear ferromagnet. However, the generalization for $\eta \leq 1$ becomes important at intermediate lengths in the ordered state when competing magnetic couplings are present. In the case of chiral magnets considered in Ref. [1], a twisting length arises due to the competition between Dzyaloshinskii-Moriya couplings and direct exchange. This gives an additional length scale. The chiral couplings have a tendency to prompt the formation of inhomogeneous multiply twisted states. This is associated with the shift of the ordering temperature, according to the temperature scale $T_D$ as defined in Eqn. (3) of Ref. [1].

For the resulting multiply twisted non-collinear magnetization distribution, the difference of longitudinal and transverse stiffness becomes crucial. The formation of inhomogeneous non-collinear magnetization structures is facilitated by local shifts of the transition temperature that are enabled through a softened longitudinal stiffness. This is the reason why the generalized gradient term of our model with $\eta < 1$ has to be taken into account. We note that the term we consider is the leading-order correction needed to generalize the gradient term in a continuum model for the magnetization. Thus the conclusions of our study are intuitive and of the greatest generality possible.

While our work was going on, it has been suggested that higher order gradient terms may also stabilise multiply twisted structures [11, 12]. However, the possible existence of these terms is specific to certain materials and difficult to justify microscopically, so that the conclusions are not physically transparent and lack general relevance.

Having justified the gradient term in $\eta$ purely from a technical point of view, we now
consider how this term evolves under coarse graining in a general context. This serves to certify that the mechanisms listed above, which in principle may reduce the longitudinal stiffness, indeed will generate an effective model with $\eta < 1$. Following convention, a generalised Ginzburg-Landau theory may be written in terms of transverse and longitudinal fluctuating fields, $\phi_\perp(x)$ and $m(x)$, respectively. The longitudinal component or modulus may be further decomposed as $m(x) = M + m'(x)$ with a constant expectation value of the magnetization $M$ and fluctuations of amplitude $m'(x)$. This leads to the action

$$
S_e(m', \phi_\perp) = \int d^4x \left[ A \left( \sum_\alpha \partial_\alpha \phi_\perp \right)^2 + A \eta \sum_\alpha \left( \partial_\alpha m'(x) \right)^2 
+ r M^2 + b M^4 \ldots 
+ r_\parallel m'(x)^2 + r_\perp |\phi_\perp(x)|^2 
+ w_1 m'(x) |\phi_\perp(x)|^2 + w_2 m'(x)^3 + + b (m'^2(x) + |\phi_\perp(x)|^2)^2 \ldots \right] (6)
$$

with constants $r_\parallel = r + 32 b M^2$, $r_\perp = r + 8 b M^2$ $w_1 = w_2 = 16 b M^2$. We note that the action given in Eqn. (6) is formally the analogue to that considered for itinerant-electron magnets at the border of ferromagnetic order. The limit of a self-consistent coupling of finite $q$ modes to the uniform magnetisation ($q = 0$) of this model has been studied extensively by Moriya, Lonzarich and others. An important assumption of their analysis is that the response of the system is that of a Fermi liquid in the random phase approximation (RPA).

To establish the relationship of our micromagnetic model, given by Eqn. (1) in the manuscript with Eqn. (6) it is in principle necessary to carry out a renormalisation of the action $S_e(m', \phi_\perp)$ up to the length scales addressed in the micromagnetic model. Unfortunately, a rigorous renormalization analysis below the ordering temperature for isotropic $n$-vector models has not been achieved due to great technical difficulties in treating mode-mode coupling correctly. These difficulties may be traced to the criticality of the transverse field components along the whole coexistence line below $T_C$ and the associated system of Goldstone-modes. The apparent non-analytic form of the gradient energy for $\eta < 1$, as seen in Eqn. (4), is likewise expected because of the divergencies related to transverse fluctuations in the ordered state.

While it appears a major challenge to perform a rigorous renormalisation of isotropic $n$-vector models in the ordered state that is well beyond the scope of the work reported here, it is nevertheless possible to track the evolution of $\eta$ under renormalisation in a Gedanken calculation. We begin by noting that in a perturbation expansion beyond one-loop-order,
the renormalization of the fields $\phi_\perp$ and $m'$ scales as powers of the correlation lengths $\xi_\perp^{-d_\phi}$ and $\xi_\parallel^{-d_\phi}$, respectively, where the exponent $d_\phi$ is also known as anomalous dimension of the fields [18]. This implies that renormalizations would, in principle, break the isotropy of the model [13]. To guarantee the isotropy of the model under renormalisation, the longitudinal and transverse field components should instead be rescaled consistently by the same factor $\xi_\parallel^{d_\phi}$. Thus the action has to be re-paramaterised under renormalisation, which is achieved by forcing the term in $\eta$ to receive a renormalization by an additional factor $(\xi_\parallel/\xi_\perp)^{d_\phi}$. We note, that along or near the co-existence line, the anomalous dimension $d_\phi$ is to be considered as an effective exponent of a cross-over region [15]. Therefore, it will depend on distance to the critical points at temperatures $T = T_C$ and at $T = 0$. Because the longitudinal correlation length on general grounds is shorter than the transverse correlation length, as discussed above, $\eta$ must remain less than one also after integrating out degrees of freedom for the construction of effective coarse-grained theories.

A rigorous proof along the lines of our argument is difficult, because the transverse correlation length diverges in zero magnetic field, $\xi_\perp \to \infty$, due to the system of transverse Goldstone modes. To avoid the problems stemming from the divergence of the correlation lengths, we may assume that the divergencies at long lengths are cut-off by inclusion of an arbitrary small field in longitudinal direction. This field serves also to define the longitudinal and transverse directions. Then, the scaling properties under coarse-graining can be connected with spin-fluctuation theories of an isotropic itinerant electron ferromagnet in the presence of finite polarization, which generally also yields different perpendicular and longitudinal correlation lengths as discussed in Ref. [19, 20]. The form of the spin-fluctuation theory reported in Ref. [19, 20] takes mode-mode coupling into account, and the approach assumes that the squared modulus of spin-fluctuations is held constant. Thus, it can be viewed as an approximation for an itinerant-electron ferromagnet with maximal longitudinal rigidness, similar to the case of a Heisenberg ferromagnet with local moments (constant modulus). From Refs. [19, 20] we have that $(\xi_\parallel/\xi_\perp) < 1$ everywhere below the ordering temperature at finite polarization, and $(\xi_\parallel/\xi_\perp) = 1$, asymptotically at $T_C$. The relation $(\xi_\parallel/\xi_\perp) < 1$ given from the spin-fluctuation theory of an itinerant-electron ferromagnet in the ordered state is the expected result from our general consideration on transverse and longitudinal correlation lengths. Consequently, following our scaling argument, $\eta < 1$ must hold below the transition temperature as a leading order (irrelevant) correction in a coarse-
grained theory for the itinerant-electron ferromagnets. We note, that this result does not depend on the detailed electronic structure, say, whether a single-band or multiband system is considered.

In summary we conclude that the assumption \( \eta < 1 \) should hold quite generally in effective micromagnetic theories, for the ordered magnetic state whenever amplitude fluctuations of the magnetisation are important. For fixed moment magnets that are well described on a microscopic scale by a Heisenberg model, the effect may be rather weak as it involves higher-order perturbation contributions. If the underlying microscopic processes in a material allow for longitudinal fluctuations on short lengths scales, an effective term \( \eta < 1 \) is always expected.

III. THE LONGITUDINAL STIFFNESS IN REAL MATERIALS

In the following we review the transverse and longitudinal magnetic stiffness observed experimentally by polarised neutron scattering at the border of magnetic order in real materials. In the context of our micromagnetic model we are interested, if the longitudinal stiffness is reduced on intermediate lengthscales. For this purpose we consider the ratio \( \eta \) of the longitudinal to the transverse magnetic stiffness, where we refer to section II for a more detailed theoretical underpinning of the analysis. As materials we consider the cubic systems EuS, Nickel and MnSi, which exemplify local-moment ferromagnetism, itinerant-electron ferromagnetism and weak itinerant-electron ferromagnetism, respectively. In MnSi we the presence of weak chiral interactions do not alter the outcome of these estimates.

In the three compounds considered here the wavevector dependent susceptibility follows the Ornstein-Zernicke form \( \chi^{-1}_i(q) \approx c_i(\kappa_i^2 + q^2) + \ldots \) over major portions of the Brillouin zone, where the index \( i \) denotes transverse \((i = \perp)\) and longitudinal components \((i = ||)\), respectively. This form is consistent with the generalised spin fluctuation model given in Eqn. (6). The parameter \( \kappa_i = 1/\xi_i \) is the inverse correlation length and the parameter \( c_i \) the stiffness. The stiffness ratio \( \delta = c_{||}/c_\perp \) permits a comparison of different materials without need for complicated normalisation procedures of the data. For the cubic systems considered here the total susceptibility near \( T_C \) is expected to be consistent with a 3-component vector model \[22\], where \( \chi_{tot} = 2\chi_\perp + \chi_{||} \) accounts for 2 transverse and 1 longitudinal mode. For a total susceptibility of the Ornstein-Zernicke form it follows that \( \chi_{tot}^{-1} = c_{tot}(\kappa_{tot}^2 + q^2) \), where
transverse and longitudinal contributions are given by $\chi^{-1}_\perp = c_\perp(\kappa^2_\perp + q^2) = (3/2)\chi_{\text{tot}}$ and $\chi^{-1}_\parallel = c_\parallel(\kappa^2_\parallel + q^2) = 3\chi_{\text{tot}}$, respectively. Thus, when the longitudinal and transverse stiffness are the same it follows $\delta = 2$. The parameter $\eta$ introduced above is given by $\eta = \delta/2$, i.e., $\eta$ is the stiffness ratio per mode.

By definition $\chi(q)$ is equivalent to the frequency integrated scattering intensity. To obtain the stiffness ratio of EuS we have taken the slopes of the inverse integrated scattering intensities, and thus $\chi^{-1}_i(q)$, shown in Fig. 6 in Ref. [23]. Here we obtain $\delta_{\text{EuS}} = c_\parallel/c_\perp \approx 1.85$. This corresponds to $\eta_{\text{EuS}} \approx 0.925$. Taking likewise the ratio of the slopes of the integrated scattering intensity for the itinerant-electron ferromagnet Nickel shown in Fig. 10 in Ref. [26] we obtain $\delta_{\text{Ni}} \approx 1.3$ or $\eta_{\text{Ni}} \approx 0.65$ Finally, for MnSi the available spin-polarised neutron data yield $\delta_{\text{MnSi}} \approx 0.8$ [24, 25], i.e., $\eta_{\text{MnSi}} \approx 0.4$. Hence, in the range where data are available, the longitudinal stiffness in Ni and MnSi is reduced by 35% and 60%, respectively. Because the $q$-range considered in Ni and MnSi is in a regime where neutron scattering shows an abundance of spin-flip particle-hole excitations, we conclude that particle-hole excitations are dominant in causing the reduction of the longitudinal stiffness of Ni and MnSi.

The conjecture that particle-hole excitations are dominant in reducing $\eta$ is supported further by the characteristic frequency dependence, which reflects on the mechanisms that determine the transverse and longitudinal stiffness. For local moment systems such as EuS, it is given by the Résibois-Piettes function. In contrast for itinerant-electron magnets, an additional softening has been noticed (see, e.g., Fig. 13 in Ref. [26] for Ni). A simple measure of the frequency dependence and thus $\eta$ may be the spin fluctuation temperature, $T_{sf}$, defined from the width of the spin fluctuation relaxation frequency spectrum. The ratio of Curie temperature to spin fluctuation temperature, $\tau = T_C/T_{sf}$, may be used as an estimate for the magnitude of $\eta$, notably, the smaller the ratio $\tau$ the smaller $\eta$. This suggestion is consistent with the experimentally observed reduced longitudinal stiffness in Ni and MnSi, discussed above.

When taken together, the experimental evidence establishes beyond doubt that many materials exist, which may be described by the regime $\eta < 1$ of our model. Moreover, the experimental data identifies MnSi, which supports chiral interactions, as a prime candidate for a spontaneous skyrmion ground state, as discussed in section IV. Interestingly the experimental data even shows $\eta < 1$ in the local moment magnet EuS. This is not a suprise, since a multitude of mechanism exists that reduce the longitudinal stiffness. As a consequence,
spontaneous skyrmion phases may exist in chiral local moment magnets, albeit they may be much less stable.

IV. POSSIBLE EVIDENCE OF SKYRMION GROUND STATES IN MNSI

As the main result of our study we predict the spontaneous formation of a skyrmion ground state in a temperature interval prior to helical order in all materials with soft magnetisation amplitude and weak chiral interactions. Bulk compounds which develop long-wavelength helical order of the kind considered here are, for instance, MnSi\textsuperscript{27}, FeGe\textsuperscript{28} and (Fe,Co)Si\textsuperscript{29}, where the modulation wavelengths are $\lambda_{\text{MnSi}} \approx 180\text{Å}$, $\lambda_{\text{FeGe}} \approx 700\text{Å}$ and $\lambda_{(\text{Fe,Co})\text{Si}} \approx 300\text{Å}$, respectively. In these materials the helical modulation is driven by Dzyaloshinsky-Moriya chiral interactions that originate in the lack of inversion symmetry in their cubic B20 crystal structure. Being transition metal compounds the magnetic order is due to itinerant electrons, although this has only been shown experimentally in MnSi\textsuperscript{30}. As discussed above polarised neutron scattering data exist for MnSi\textsuperscript{24} that establish a reduced longitudinal stiffness ($\eta_{\text{MnSi}} = 0.4$). Thus at least MnSi meets all the requirements for the formation of the predicted spontaneous skyrmion ground states.

The helical order in MnSi, FeGe and (Fe,Co)Si may be seen most easily in small-angle neutron scattering (SANS) experiments, where sharp magnetic satellites were observed. In a small temperature interval above the onset of helical order, SANS data for MnSi and FeGe show neutron scattering intensity located on the surface of a sphere in reciprocal space as an additional feature\textsuperscript{28, 31}. The radius of this sphere corresponds to the same modulation length as the helical order. The sphere of scattering intensity has therefore traditionally been interpreted as fluctuating helical order. However, in these studies it was not considered that the scattering intensity may also be related to a frozen-in form of order. In this respect it is interesting to note, that the scattering intensity on the surface of the sphere is perfectly consistent with an amorphous array of randomly oriented cylindrical skyrmion tubes.

To better identify of the ring of scattering intensity we present a rough estimate of the temperature interval $\Delta T = T_D - T_C$, for which the skyrmion ground state is expected in MnSi. The lattice constant of MnSi is $a_{\text{MnSi}} = 4.55\text{Å}$. There are four formula units per unit cell, so that the number-density is $N/V = n = 4.246 \cdot 10^{26}\text{m}^{-3}$. The effective moment
determined from the Curie-Weiss temperature dependence of the susceptibility is $p_{\text{eff}} = 2.2 \mu_B$ and the susceptibility $\chi = C_{\text{CW}}/(T - T_C)$ where $C_{\text{CW}} = n\mu_0\mu_B^3(p_{\text{eff}}^2)/(3k_B) = 0.534$ K. The upper limit of the skyrmion transition temperature is $T_h = T_C + (1/2)(D/(2A))(D/a)$ as given in Eq.(2) in Ref. [1]. These parameters have to be taken from experimental data, notably Ref. [33]. The parameter $a$ is given by the initial susceptibility and may be estimated from the Curie-Weiss susceptibility: $a = 1/(2C_{\text{CW}}) = 0.936$ K$^{-1}$. The modulus of the helical propagation vector is $q_0 = (D/(2A)) = 0.039$ Å$^{-1}$. In the following we use temperature as unit of energy-density. The exchange stiffness $A$ may then be estimated from $A \approx (a_{\text{MnSi}}^2)T_C = 50 \cdot 10^{-3}$ eV Å$^2 = 587$ K Å$^2$. The strength of the DM-coupling $D$ may be estimated from $q_0$ as $D = 1.9 \cdot 10^{-3}$ eV Å= 22 K Å. These parameters finally result in $T_h = T_C + (1/2)q_0(D/a) = T_C + q_0DC_{\text{CW}} = (29.5 + 0.46)$ K. The ordering temperature of the skyrmion phase (according to Eq.(3) in Ref. [1]) is obtained by doubling the difference between $T_C$ and $T_h$ so that $\Delta T = T_D - T_C = 0.9$K. This value of $\Delta T$ gives an order-of-magnitude estimate for the temperature interval of the stable skyrmion phase in MnSi as shown in the phase diagram of Fig. 2 in our manuscript [1].

Detailed studies of the specific heat (Fig. 6 in Ref. [34, 35]) and thermal expansion [36] of the paramagnetic to helical transition in MnSi show the presence of a spike at $T_h$ and a broad shoulder in a temperature interval of order $\sim$1 K above this transition. Detailed SANS studies of the ring of intensity as function of temperature suggest that the ring extends over the same temperature interval where the shoulder in the specific heat is seen [37]. Interestingly the temperature interval corresponds fairly well with that predicted for the skyrmion phase. In combination, the broad shoulder in the specific heat clearly signals the presence of quasi-static order already above $T_h$ that may be either related to randomly oriented helical domains or an amorphous texture of skyrmions. An important additional aspect that has been emphasized in [34] is, that the shape of the specific heat anomaly in very high purity samples is quite different, yet, still supports the possible existence of a precursor phase. However the shoulder is observed for samples with largely varying residual resistivity ratio [35]. It is possible that the dependence on extreme sample purity signals that the stabilisation of the quasistatic order sensitively depends on residual defects in the material. The spontaneous formation of skyrmion ground states allows to explain many of the mysterious properties of MnSi at high pressure and low temperatures. It was recently
reported by one of us (CP and collaborators), that the metallic state of MnSi changes as function of pressure from a quadratic temperature dependence of a weakly spin-polarized Fermi liquid to a $T^{3/2}$ resistivity that is stable over a remarkably large pressure range $38, 39, 40, 41$. Three aspects appear to be inconsistent with the present day understanding of metallic magnetism: First, as pointed out in Ref. $38$ a $T^{3/2}$ resistivity is normally observed in spin glasses and amorphous ferromagnets, where it is explained by a diffusive motion of the charge carriers $42$. However, for the high-purity MnSi single-crystals investigated the behaviour of a moderately enhanced Fermi liquid is expected by all accounts. It was therefore emphasised that the diffusive charge carrier motion would have to be intrinsic. Second, the $T^{3/2}$ resistivity is extremely stable as function of pressure, where it has been seen up to at least $45\,\text{kbar} \approx 3p_c$. This suggests the formation of an extended new phase in contrast to a cross-over phenomenon normally observed at quantum phase transitions. Third, the $T^{3/2}$ resistivity is extremely stable as function of magnetic field, where experiment shows that it collapses abruptly only above a certain critical field related to a so-called itinerant-electron metamagnetic transition (MMT) $43$. At the highest pressures the $T^{3/2}$ resistivity extends at least up to $1\,\text{T}$ (cf Fig. 3 in Ref. $40$) which substantially exceeds the field needed to collapse helical order $B_c \approx 0.6T$. We note that $B_c$ studied by various magnetic probes is insensitive to pressure.

As a possible resolution to these contradictions we suggest that the Fermi liquid to non-Fermi liquid transition in MnSi may signal a transition between helical order and an amorphous skyrmion ground state. A spontaneous amorphous skyrmion ground state provides a simple intrinsic mechanism that explains why the transport properties of an amorphous ferromagnet are seen even in a high purity material in the absence of disorder or frustration. The leading order effect of pressure in this scenario would be to generate a slight mode softening that reduces the stability of helical order, but not to collapse the magnetic moment on local scales completely as normally assumed in quantum phase transitions. Because the skyrmions form as a genuine, stable ground state, it would finally not be difficult to explain, a priori, the extent of the $T^{3/2}$ resistivity behaviour in the temperature-pressure phase-diagram. Also, it has been shown that the core of cylindrical skyrmion tubes is extremely stable against external fields $44$. Thus, if the $T^{3/2}$ resistivity is due to an amorphous skyrmion texture it is expected that the behaviour survives to much higher fields. The signature of such an amorphous skyrmion ground state in neutron scattering would be intensity
on the surface of a small sphere in reciprocal space, similar to that seen at ambient pressure above $T_h$.

Recent neutron scattering studies of the magnetism as function of pressure indeed show that large ordered moments survive deep into the phase where the $T^{3/2}$ resistivity is observed \[45\]. The ordered moments are organised such that they lead to scattering intensity on the surface of a sphere in reciprocal space. There is, however, a very important difference with the intensity on a sphere seen at ambient pressure near $T_h$. The ambient pressure intensity is isotropic, while broad maxima are observed for $\langle 110 \rangle$ at high pressure. The $\langle 110 \rangle$ direction is unusual for this cubic magnet. This is so, because (i) the leading magnetocrystalline anisotropy favours easy magnetic axes only in $\langle 111 \rangle$ or $\langle 100 \rangle$ directions. And, (ii), in itinerant d-electron magnets as MnSi with very weak anisotropy, it is very unlikely that higher-order anisotropy contributions do play any role. In fact, the easy axis of the helical order in MnSi is $\langle 111 \rangle$ and in FeGe $\langle 100 \rangle$. Thus, if there was randomly oriented helical order at high pressure in MnSi, it would be very difficult to explain the broad maxima along $\langle 110 \rangle$. In contrast, for a condensate of skyrmion tubes the odd maxima for $\langle 110 \rangle$ are easily explained, if we suppose that the easy magnetic axis of MnSi, $\langle 111 \rangle$, does not change as function of pressure, while the magnetism changes from a helical modulation along $\langle 111 \rangle$ to an amorphous texture with cylindrical skyrmion tubes having their axes preferentially along $\langle 111 \rangle$. For cylindrical skyrmion tubes that are stratified along $\langle 111 \rangle$ neutron scattering intensity in reciprocal space is expected on great circles perpendicular to $\langle 111 \rangle$ directions. As all $\langle 111 \rangle$ directions produce intensities along such great circles, these will intersect and produce maxima of intensity in the $\langle 110 \rangle$ directions. The available experimental data are consistent with this possibility.

However, in contrast to the remarkable extent of the $T^{3/2}$ resistivity under pressure and magnetic field, the sphere of scattering intensity under pressure is seen only in a fairly small portion of the phase diagram. A possible explanation discussed in Ref. \[45\] is that the textures are dynamically destabilized and become 'invisible' in the neutron scattering experiment. Such a slowly meandering form of magnetic order could persist on a time scale that it still much slower than the time scale relevant to the electrical resistivity. A liquid instead of a frozen skyrmion phase in this pressure range provides a natural explanation for the difference in the transport measurements and the visible static magnetic order. Such a liquid skyrmion phase would not be unusual and may be analogous to liquid vortex phases in superconductors \[46\]. It will, in any case, require a host of novel experimental
techniques to settle this issue unambiguously. For instance, real space imaging of helical order in thin (Fe,Co)Si layers was recently reported using Lorentz force microscopy (LFM) [47]. However, samples that can be studied with LFM have to be atomically thin so that uncontrolled mechanical strains and their interplay with the magnetism represents a major concern. As regards LFM in MnSi we note, moreover, that even state-of-the-art LFM does not offer sufficient resolution yet to image the magnetic structure with sizes of the twisting length 18 nm. We believe that, as an important alternative to real-space imaging, the most promising method in the immediate future will be full three-dimensional polarisation analysis for neutron scattering. This is presently under development under the synonym mu-PAD.

V. ON THE BROADER IMPLICATIONS OF SPONTANEOUS SKYRMION GROUND STATES DUE TO CHIRAL INTERACTIONS

Skyrmion states in field theories have attracted great interest because they represent stable, particle-like objects with a nontrivial topology that originates in fundamental symmetries [48, 49]. For instance, Skyrme’s field theory for elementary nuclear particles has been applied to condensates of a few skyrmions [50] and extended lattice structures [51] as models of ordinary nuclei and compact stars, respectively. Our prediction of spontaneous skyrmion ground states in chiral magnets is connected to the fundamental questions about the relation between continuum theories, the countable nature of particles and the possibility to relate both aspects by topology in extended matter systems. In this more general context the role of chirality and chiral interactions, which are possible in condensed matter systems [52], has not yet been appreciated. Chiral interactions in condensed matter systems offer an exceptionally rich setting, because they exist in many different contexts, e.g., (i) spin-orbit interactions in non-centrosymmetric magnetic materials, also referred to as Dzyaloshinsky-Moriya (DM) interactions [53, 54], (ii) in non-centrosymmetric ferroelectrics [55, 56], (iii) for certain structural phase transitions [57, 58], (iv) in chiral liquid crystals [59], or (v) in the form of Chern-Simons terms in gauge field theories [60, 61].

The example of an extended magnetic skyrmion texture shown in Figure 4 of Ref. [1] demonstrates the condensation of particle- or string-like localized objects. There are, however, further intriguing possibilities beyond these extended ground-states. Magnetic skyrmions may also arise as non-linear localized excitations both in homogeneously ordered
systems and in the paramagnetic state. This possibility has not yet been considered in theories on the metallic state of systems with broken inversion symmetry. In particular, the description of skyrmion excitations on a paramagnetic background would be very similar to that of particles generated in a vacuum state. While being large and localised, they are subject to an intrinsic length scale related to the twisting length of the system. This length-scale implies non-dispersive behaviour unlike propagating waves. Moreover, skyrmions in condensed matter systems may allow to study processes like particle generation and annihilation, scattering between particles, or particle confinement.

Fundamental processes of this kind are usually the subject of field-theories in particle physics only. Images of such processes for elementary particles can therefore be inferred at best only indirectly. In chiral magnetic condensed matter, these fundamental processes take place in a continuum distribution of magnetization that may be visualized in a solid state laboratory by modern magnetic real- and reciprocal space imaging methods. In fact, because the spontaneous skyrmion ground states predicted here represent large (several hundreds Å), but localized non-linear spin-fluctuations, they would provide the first cases of skyrmions in a condensed matter system, where detailed imaging of such processes may be possible. With this perspective in mind, it is gratifying that novel chiral crystals have recently been found, where superconductivity coexists with magnetic order, as in the novel heavy-fermion superconductors CePt$_3$Si [62] and UIr [63]. In these materials different types of long-range order coexist, so that it may be possible to study interactions between skyrmionic excitations in the associated order-parameter fields which in turn have different symmetries. In fact, chiral condensed matter systems could eventually be used to mimick a universe with different types of particles and their interactions.

In spite of their low symmetry, metallic magnets with broken inversion symmetry are rather common. In particular, Dzyaloshinsky-Moriya interactions are induced by surfaces in all magnetic nanostructures [64, 65]. Thus magnetic nanostructures can endow science with an arena, where creation, interactions, and condensation of particle-like states in a continuous field can be engineered for experiment.
References

[1] U. K. Rößler, A. N. Bogdanov, C. Pfleiderer, Spontaneous Skyrmion Ground States in Magnetic Metals. [http://xxx.lanl.gov/abs/cond-mat/0603103](http://xxx.lanl.gov/abs/cond-mat/0603103) (2006).

[2] A. I. Akhiezer, V. G. Bar’yakhtar, S. V. Peletminskii, Spin Waves (North Holland Publ., Amsterdam 1968).

[3] A. Z. Patashinskii, V. L. Pokrovskii, Longitudinal susceptibility and correlations in degenerate systems. *Sov. Phys. JETP* **37**, 733-736 (1974).

[4] W. Zwerger, Anomalous Fluctuations in Phases with a Broken Continuous Symmetry. *Phys. Rev. Lett.* **92**, 027203/1-4 (2004).

[5] G. G. Lonzarich, L. Taillefer, Effect of spin fluctuations on the magnetic equation of state of ferromagnetic or nearly ferromagnetic metals. *J. Phys. C: Solid State Phys.* **18**, 4339-4371 (1985).

[6] T. Moriya, *Spin Fluctuations in Itinerant-Electron Magnetism* (Springer Series in Solid State Sciences vol. 56, Springer-Verlag 1985).

[7] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Clarendon Press, Oxford 1997, 3rd edition) p.668f.

[8] E. Brézin, J. Zinn-Justin, J. C. Le Guillou, Anomalous dimensions of composite operators near two dimensions for ferromagnets with O(n) symmetry. *Phys. Rev. B* **14**, 4976-4977 (1976).

[9] R. A. Pelcovits, D. R. Nelson, Bicritical points in $2 + \epsilon$ dimensions. *Phys. Lett. A* **57**, 23-25 (1976).

[10] In Refs. [8] the irrelevance of “dimension-2 operators” is remarked. This means that there can be two parameters in the gradient energy (of dimension-2) of a generalized NL$\sigma$M [9]. These are the terms in Eqn.(2) of Ref. [9] with parameters $f \propto A$ and $g_1$, which plays the role of $1 - \eta$. By recursion relations, it is shown that for the ferromagnetic ordering, the terms related to $g_1$ are irrelevant (see Eqs.(4) and (5) in [9]).

[11] S. Tewari, D. Belitz, T.R. Kirkpatrick, Blue Quantum Fog: Chiral Condensation in Quantum Helimagnets. *Phys. Rev. Lett.* **96**, 047207/1-4 (2006).

[12] B. Binz, A. Vishwanath, V. Aji, Theory of the helical spin crystal: a candidate for the partially
ordered state of MnSi. [http://xxx.lanl.gov/abs/cond-mat/0602529](http://xxx.lanl.gov/abs/cond-mat/0602529) (2006).

[13] D.R. Nelson, Coexistence-curve singularities in isotropic ferromagnets. *Phys. Rev. B* **13**, 2222 (1976).

[14] E. Frey, F. Schwabl, Critical dynamics of magnets, *Adv. Phys.* **43**, 577 (1994).

[15] D. O’Connor, C.R. Stephens, Renormalization group theory of crossovers. *Phys.Rep.* **363**, 425 (2002).

[16] I.D. Lawrie, Goldstone mode singularities in specific heats and non-ordering susceptibilities of isotropic systems. *J. Phys. A: Math. Gen.* **18**, 1141 (1985).

[17] I.D. Lawrie, Goldstone modes and coexistence in isotropic N-vector models. *J. Phys. A: Math. Gen.* **14**, 2489 (1981).

[18] See, e.g., M. LeBellac, *Quantum and Statistical Field Theory* (Clarendon Press, Oxford 1991) chap. 3.5.3, p.102-108.

[19] Y. Takahashi, Quantum spin fluctuation theory of the magnetic equation of state of weak itinerant-electron ferromagnets. *J. Phys.: Condens. Matter* **13**, 6323 (2001).

[20] Y. Takahashi, H. Nakano, Temperature and magnetic field dependence of the heat capacity in itinerant electron ferromagnets. *J. Phys.: Condens. Matter* **16**, 4505 (2004).

[21] See the expressions for inverse correlation lengths, $y$, $y_z$, for $T < T_C$ in [20], p. 4512 and appendix A.

[22] J. Als-Nielsen, O.-W. Dietrich, O.-W., L. Passel, Neutron scattering from the Heisenberg ferromagnets EuO and EuS. II. Static critical properties. *Phys. Rev. B*, **14**, 4908 (1976); note in particular p. 4915.

[23] P. Böni, B. Roessli, D. Görlitz, and J. Kötzler, Damping of spin waves and singularity of the longitudinal modes in the dipolar critical regime of the Heisenberg ferromagnet EuS. *Phys. Rev. B*, **65**, 144434/1-9 (2002).

[24] F. Semadeni, P. Böni, Y. Endoh, B. Roessli, G. Shirane, Direct observation of spin-flip excitations in MnSi. *Physica B* **267-268**, 248-251 (1999).

[25] P. Böni, C. Pfleiderer, Excitations of magnetic metals, unpublished (2006).

[26] P. Böni, J. L. Martinez, J. M. Tranquada, Longitudinal spin fluctuations in Ni. *Phys. Rev. B* **43**, 575-584 (1991).

[27] B. Lebech, in *Recent Advances in Magnetism of Transition Metal Compounds* (Eds. A. Kotani, N. Suzuki) 167-178 (World Scientific, Singapore, 1993).
[28] B. Lebech, J. Bernhard, T. Freltoft, Magnetic structures of cubic FeGe studied by small-angle neutron scattering. *J. Phys.: Condens. Matter* **1**, 6105 (1989).

[29] J. Beille, J. Voiron, M. Roth, Long period helimagnetism in the cubic B20 Fe$_x$Co$_{1-x}$Si and Co$_x$Mn$_{1-x}$Si alloys. *Solid State Commun.* **47**, 399 (1983).

[30] L. Taillefer, G. G. Lonzarich, P. Strange, The band magnetism of MnSi. *J. Magnet. Magnet. Mater.* **54-57**, 957 (1986).

[31] B. Lebech, Magnetic Ordering in Nearly Ferromagnetic Antiferromagnetic Helices, in *Recent Advances in Magnetism of Transition Metal Compounds* (Eds. A. Kotani, N. Suzuki) 167-178 (World Scientific, Singapore, 1993).

[32] H. Yasuoka, V. Jaccarino, R. C. Sherwood, J. H. Wernick, NMR and susceptibility studies of MnSi above $T_C$. *J. Phys. Soc. Japan* **44**, 842 (1978).

[33] S. V. Grigoriev, S. V. Maleyev, A. I. Okorokov, Y. O. Chetverikov, R. Georgii, P. Böni, D. Lamago, H. Eckerlebe, K. Pranzas, Critical fluctuations in MnSi near $T_C$: A polarized neutron scattering study. *Phys. Rev. B* **72**, 134420 (2005).

[34] C. Pfleiderer, Experimental studies of weakly magnetic transition metal compounds. *J. Magn. Magn. Mater.* **226-230**, 23 (2001).

[35] C. Pfleiderer, unpublished (1997).

[36] M. Matsanuga, Y. Ishikawa, T. Nakajima, Magneto-volume effect in the weak itinerant ferromagnet MnSi. *J. Phys. Soc. Jpn.* **51**, 1153 (1982).

[37] D. Lamago, PhD thesis, Technische Universität München (2006).

[38] C. Pfleiderer, S. R. Julian, G. G. Lonzarich, Non-Fermi-liquid nature of the normal state of itinerant-electron ferromagnets. *Nature* **414**, 427 (2001).

[39] C. Pfleiderer, Non-Fermi liquid puzzle of MnSi at high pressure. *Physica B* **328**, 100 (2003).

[40] N. Doiron-Leyraud, et al., Fermi-liquid breakdown in the paramagnetic phase of a pure metal. *Nature* **425**, 595-599 (2003).

[41] P. Pedrazzini, et al., Probing the extended non-Fermi liquid regimes of MnSi and Fe. cond-mat/0509772.

[42] N. Rivier, A. Adkins: Resistivity of spin glasses. *J. Phys. F* **5**, 1745 (1975).

[43] C. Thessieu, C. Pfleiderer, A. N. Stepanov, J. Flouquet, Field dependence of the magnetic quantum phase transition in MnSi. *J. Phys.: Condensed Matter* **9**, 6677 (1997).

[44] A. Bogdanov and A. Hubert, Thermodynamically stable magnetic vortex states in magnetic...
crystals. J. Magn. Magn. Mater. 138, 255-269 (1994).

[45] C. Pfleiderer, D. Reznik, L. Pintschovius, H. v. Löhneysen, M. Garst, A. Rosch, Partial order in the non-Fermi-liquid phase of MnSi. Nature 427, 227-231 (2004).

[46] G. Blatter, M. V. Feigel’man, V. B. Geshkenbein, A.I. Larkin, V.M. Vinokur, Vortices in high-temperature superconductors. Rev. Mod. Phys. 66, 1125-1388 (1994).

[47] M. Uchida, Y. Onose, Y. Matsui, Y. Tokura, Real-Space Observation of Helical Spin Order. Science 311, 359-361 (2006).

[48] See, e.g., V.G. Makhankov, Y.R. Rybakov, V.I. Sanyuk, The Skyrme Model (Springer Series in Nuclear and Particle Physics. Springer-Verlag, Berlin 1993).

[49] E.Witten, Global aspects of current algebra. Nuclear Physics B, 223, 422-432 (1983).

[50] E. Braaten, L. Carson, Deuteron as a Soliton in the Skyrme Model. Phys. Rev. Lett. 56, 1897-1900 (1986).

[51] M. Kutschera, C.J. Pethick, D.G. Ravenhall, Dense Matter in the Chiral Soliton Model. Phys. Rev. Lett. 53, 1041-1044 (1984).

[52] A. Bogdanov, New localized solutions of the nonlinear field-equations. JETP Lett. 62, 247-251 (1995).

[53] I. E. Dzyaloshinskii, Theory of helicoidal structures in antiferromagnets. Sov. Phys. JETP 19, 960-971 (1964).

[54] Yu. A. Izyumov, Modulated, or long-periodic, magnetic structures of crystals. Soviet Physics — Uspekhi 27, 845-867 (1984) [Usp. Fiz. Nauk. 144, 439 (1984)].

[55] I. Sosnowska, T. Peterlin-Neumaier, E. Steichele, Spiral magnetic-ordering in bismuth ferrite. J. Phys. C 15, 4835-4846 (1982).

[56] P. S. Halasyamani, K. R. Poeppelmeier, Noncentrosymmetric oxides. Chemistry of Materials 10, 2753-2769 (1998).

[57] E.M. Lifshitz, Zh. Eksp. Teor. Fiz. 11 255 (1941).

[58] J.C. Toledano, P. Toledano, The Landau Theory of Phase Transitions (World Scientific, Singapore, 1987).

[59] D. C. Wright, N. D. Mermin, Crystalline liquids — the blue phases. Rev. Mod. Phys. 61, 385-432 (1989).

[60] G. Murthy, R. Shankar, Hamiltonian theories of the fractional quantum Hall effect. Rev. Mod. Phys. 75, 1101-1158 (2003).
[61] W. Chen, M. Li, Driving Operators Relevant: A Feature of the Chern-Simons Interaction. *Phys. Rev. Lett.* **70**, 884-887 (1993).

[62] E. Bauer, G. Hilscher, H. Michor, C. Paul, E. W. Scheidt, A. Gribanov, Y. Seropegin, H. Noel, M. Sigrist, P. Rogl, Heavy fermion superconductivity and magnetic order in noncentrosymmetric CePt3Si. *Phys. Rev. Lett.* **92**, 027003 (2004).

[63] T. Akazawa, H. Hidaka, T. Fujiwara, T. C. Kobayashi, E. Yamamoto, Y. Haga, R. Setti, and Y. Onuki, Pressure-induced superconductivity in ferromagnetic UIr without inversion symmetry. *J. Phys.: Condens. Matter* **16**, L29-L32 (2004).

[64] A. Fert, Metallic Multilayers. in: A. Chamberod, J. Hillairat (Eds.), Materials Science Forum **59-60**, 439 (1990).

[65] A. N. Bogdanov and U. K. Rößler, Chiral symmetry breaking in magnetic thin films and multilayers. *Phys. Rev. Lett.* **87**, 037203 (2001).