Analysis of mathematics textbook use: An argument for combining horizontal, vertical, and contextual analyses

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Abstract. This paper contributes to a larger endeavour of understanding how mathematics textbooks are used in classrooms and what shapes the specific forms of their use. Three types of mathematics textbook analysis have been previously identified, namely, horizontal, vertical, and contextual analysis. Based on literature review, we discuss how these three types of analysis can be usefully combined, and explain why contextual analysis, where data on textbook use is collected in classrooms, provides the most revealing insights. We argue that because of its relevance for textbook design as well as for teacher professional development, researchers should consider including the contextual analysis when analysing textbooks.

1. Introduction

Textbooks are the main curriculum resource in many countries and an important element in education [1-3]. They connect knowledge domains to lesson content [2] and are often used to determine the content of classroom teaching [3,4]. Because textbooks provide lesson contents for students and guidance for teachers in classroom teaching, textbooks influence students’ learning opportunities [1,3]. Textbooks are also often used as an instrument to translate curriculum policies into teaching and learning activities [3]. Consequently, conceptualizing relationships between textbooks and curriculum policy [5] and understanding how this impacts the enactment of the policies in specific classrooms are both important. This may involve understanding how curriculum content is presented in textbooks, how textbooks guide teachers in conducting classroom teaching, and how they might thus shape classroom learning. For these types of research questions, in addition to examining textbooks, the usage of textbooks in classroom teaching also need to be analysed.

Mathematics textbooks analyses present in the literature can be classified into three types, namely, horizontal, vertical, and contextual analyses [6]. Horizontal analysis focuses on the attributes present in a mathematics textbook. Vertical analysis focuses on how mathematics textbooks or textbooks series develop certain mathematical concepts across longer periods of time. Contextual analysis focuses on how mathematics textbooks are used in mathematics classroom activities. This paper aims to elaborate why and how the contextual analysis adds to the horizontal and vertical analyses of textbooks when the aim of research is to understand and/or evaluate the role of textbooks in
communicating specific policy ideas to teachers and making these ideas a reality in mathematics classrooms.

2. Methods
This is a conceptual paper based on a review of literature on textbook analysis. This paper is based on classification of mathematics textbook analyses developed by Charalambous and colleagues [6] into three types, namely, horizontal, vertical, and contextual analyses. Literature review was then conducted to explore how different types of textbook analyses contribute to understanding of teachers’ work with textbooks [7].

3. Purposes for and types of textbook analyses
What researchers explore in different textbook analyses, as well as the specific criteria they examine, depend on the purposes of the research study. We first provide examples of the research uses of the horizontal and vertical analyses, and then elaborate how additional type of analysis is needed to extend our understanding beyond the textbook in its own right to how the textbook accomplishes different roles for which it was designed.

The horizontal and the vertical analyses can be used to examine the objectives of textbook authors or publishers and to identify different textbook features by which authors aimed to construct student learning opportunities. For example, Charalambous and colleagues [6] conducted the horizontal and the vertical analyses with these aims in three stages. Firstly, these authors reviewed literature related to student errors and learning difficulties. Based on the initial review, the authors examined existing frameworks for analysing how textbooks may contribute to addressing the identified errors and difficulties. Secondly, the authors established several criteria to analyse textbook features for the horizontal analysis and several criteria to analyse the evolution of the mathematical content in the textbooks for the vertical analysis. Finally, the authors grouped the criteria into several categories. The criteria for the horizontal analysis were grouped in two categories, namely, ‘background information’ and ‘overall structure of the textbook’, while the criteria for the vertical analysis were grouped into three categories, namely ‘communicated to students’, ‘required of students’, and ‘connections’. Given that learning mathematics might comprise understanding the content of a topic and advancing certain skills and attitudes, in this research, the authors divided the categories for the vertical analysis into sub-categories. The ‘communicated to students’ category were divided into three sub-categories, namely, ‘mathematical content’, ‘mathematical practices’, and ‘attitudes toward the subject’. The ‘required of students’ category were divided into two sub-categories, namely, ‘potential cognitive demands’ and ‘types of responses’. The ‘connections’ category were divided into three sub-categories namely, ‘connections within and between strands’, ‘connections between classroom instruction and textbooks’, and ‘connections to situations outside of school’.

The vertical analysis examines the focus of analysis across different chapters or books in which the authors explore the progression of the themes found in the horizontal analysis. For example, In the horizontal analysis of ‘the potential cognitive demands of procedures’, the researchers explore such demands as presented in individual tasks and summary in each chapter of a book. In the vertical analysis of ‘the potential cognitive demands of procedures’, the researchers examine such demands across different chapters or books.

To compare, Wijaya and colleagues [8] conducted an analysis to learn about opportunity-to-learn context-based tasks in mathematics textbooks. Following Charalambous and colleagues, these authors applied the horizontal analysis to analyse textbook features and the vertical analysis to analyse mathematical content. They used the horizontal analysis to examine physical characteristics and instructional components of the textbooks, and the vertical analysis to examine the evolution of the task characteristics of the textbooks. The horizontal analysis explored the physical characteristics consisting of page size, number of pages, and page surface area, and the instructional components of the textbooks consisting of number of sections, number of tasks in worked example sections, and number of tasks in task sections. The vertical analysis then explored the task characteristics of the
textbooks consisting of type of context, purpose of context in a contextualised task, type of information, and type of cognitive demand.

Another study by Hadar and colleagues [9] in Israel explored the extent to which examined textbooks reflected the expectations by curriculum policy makers. They found that textbooks were not in line with what was expected by curriculum. This study had two stages. First, the authors identified the types and levels of understanding performances required by each task in the four textbooks by applying qualitative coding and using an Understanding Performance (UP) framework in mathematics for characterizing cognitive demands of a task. Second, they computed the codes that expressed the understanding level and complexity of each task. As the implication for practice, teachers, school leaders, and decision makers could use the UP framework as a tool to examine opportunities to learn in cognitive domain in mathematics textbooks when they chose a mathematics textbook among many choices. This research study found that the opportunities to learn in cognitive domain, which was expected by Israel national mathematics curriculum varied in the four mathematics textbooks analysed. The authors concluded that the mathematics textbooks did not fully support national mathematics curriculum applied in this country.

These examples from literature illustrate how the horizontal and vertical analyses can be useful to pursue research questions regarding the elements of textbooks that could support teachers in the implementation of certain teaching approaches in their classrooms, and the extent to which the textbooks are in line with the curriculum policy. These types of textbook analyses are conducted only through examining textbooks and cannot address questions regarding the implementation of curriculum policies via use of textbooks in classroom teaching.

3.1. A case for adding contextual analysis

The contextual analysis is used in research studies that aim to explore textbook use in teaching activities [6], including those that investigate issues related to the implementation of curriculum in classrooms. Remillard [10] identified four conceptualisations of teachers’ textbook use (i.e. curriculum resource use) present in the literature, namely curriculum use as following or subverting texts, curriculum use as drawing on texts, curriculum use as the interpretation of texts, and curriculum use as participation with texts. In research studies that conceptualise curriculum use as following or subverting texts, textbooks are viewed as a representation of enacted curriculum, the teachers as enactors of curriculum, teacher-curriculum relationship as teachers’ fidelity towards curriculum, and the focus of research studies is on textbooks as the agent of curriculum influencing teachers’ textbook use [10]. The studies in this category often examine the compatibilities between the content of textbooks and the content of classroom teaching without considering teacher characteristics that also influence teachers’ textbook use [11]. Furthermore, teachers’ fidelity towards textbook, as viewed in teacher-curriculum relationship in this conceptualization, can only be achieved in ideal conditions that occur rarely (if ever) [11].

In the research studies that conceptualise curriculum use as drawing on texts, textbooks are viewed as one of resources that teachers use in shaping enacted curriculum, the teachers as active designers of enacted curriculum, and as the agent in the teacher-curriculum relationship, and the focus of the studies is on how teachers’ agency in relation to their curriculum influences their textbook use [10]. It is notable that studies that fit within this conceptualization (e.g., Sosniak and Stodolsky [12]) did not consider that textbook features also influenced teachers’ textbook use and underplayed the possibility that teachers might not follow textbooks in enacting curriculum.

In the research studies that conceptualise curriculum use as the interpretation of texts, textbooks are viewed as a representation of tasks and concepts, the teachers as interpreters of written curriculum based on their beliefs and experiences, teacher-curriculum relationship as teachers’ infidelity towards curriculum, and the focus is on teachers’ interpretations of textbooks and how these interpretations result in particular implementations in classrooms [10]. This conceptualization does not analyse deeply teacher-curriculum relationship if the research studies applying this conceptualization do not investigate how teachers enact their interpretations in classrooms (see e.g., Collopy [13]).
Finally, in research studies that conceptualise curriculum use as participation with texts, textbooks are viewed as artefacts or tools, the teachers as collaborators with textbooks to design enacted curriculum, teacher-curriculum relationship as participatory relationship influenced by both teacher and curriculum, and the focus is on this participatory relationship [10]. In this conceptualization, teachers and curriculum have interrelationship, in which both teachers and curriculum are viewed as influencing one another, and the ways how teachers enact curriculum in classrooms are the result of their interpretation of textbooks see e.g. Remillard [14].

The four outlined conceptualisations of teachers’ textbook use illustrate various views that were in past adopted by researchers in examining the usage of textbooks. Importantly, the textbook-related research questions raised in these studies could not be investigated by the horizontal and the vertical analysis. How teachers plan classroom activities and what is the role of textbook in their planning as well as how teachers teach in their classrooms and what is the role of textbook in their teaching can be investigated by using the contextual analysis [6]. Given the existing analyses and critiques of conceptualisations of teacher-textbook relationships, conceptualising such relationship as participation with texts [10] appears important when aim is to understand the contribution of textbook to the endeavour of implementation of curriculum policies in classrooms. When data collection is guided by this conceptualisation, the researchers are led to examine teachers’ activities in using textbooks (e.g., the selection of the content of classroom teaching, planning for, and teaching in the classrooms) as well as consider both teacher and textbook characteristics that shape teachers’ textbook use.

4. Conclusion
The contextual analysis is an analytical tool that contributes to development of more complete picture of how teachers’ use specific mathematics textbooks in their classrooms, and what shapes their textbook use. Researchers should conduct the contextual analysis in addition to the horizontal and vertical analyses when aiming to understand textbooks as a means by which curriculum policies shape students’ classroom learning.

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