Developing a Local Instruction Theory for Learning the Concept of Solving Quadratic Equation Using Babylonian Approach

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Abstract. The purpose of this research is to develop a set of learning instructions and investigate how the Old Babylonian Geometric Method: Naïve Geometry can support students' understanding about the concept of solving quadratic equations. This study aims to investigate on how students relate the Babylonian Geometric approach with the solving of the quadratic equation especially on how student bring their geometric solution into algebraic form. This research was conducted at a state junior high school in Indonesia. Design research was chosen as the method to reach the purpose. The instructional activities designed to achieve the learning objectives in this research consists of several activities, namely manipulating geometric form to solve the problem, using the Babylonian geometric method to solve the problem, linking geometric problems to algebra, and finding common formulas to solve quadratic equations. Through these activities, students acquire the idea of solving quadratic equations using algebraic completing perfect square method by manipulating and reshaping a rectangle into a square. However, our important finding is that only students who have high mathematics ability who reach the learning objective until the last stage of activity, which is to reinvent the common algebraic formula in solving the quadratic equation.

1. Introduction
In this research, we developed a local instruction theory for learning the concept of solving quadratic equation using a historical phenomenology. This was inspired by Radford's study namely Second Degree Equations in the Classroom: A Babylonian Approach. The Old Babylonian geometric approach, preserved in a tablet known as BM 13901 Problem I, used to build the concept of quadratic equation through a progressive instruction which starts with the use of manipulative and evolves through an investigative problem-solving process that combines the numerical and geometrical experiences [1]. But, we try to modify the approach that has used before. Here we also use geometric approach preserved in BM 13901 Problem II to build the learning instruction for learning the concept of the quadratic equation. Furthermore, we present some reasons that led us to conduct this study.

Some researchers have conducted studies on learning quadratic equation and noted that it is an important and interesting research object [1-4]. Some of those studies indicate that there are many students make errors in term of solving the quadratic equation. The errors, for instances, around inability to uncover the relationship between variables and roots, determining the appropriate quadratic equation for a given set of roots and vice versa [5]. In addition, O’Connor and Norton [6] found and categorized learners’ errors into four patterns: lack of fractional reasoning, lack of conceptual understandings related
to quadratic form and algebraic process, algebraic misconception, and inefficient procedural techniques of finding solutions. Zakaria & Maat [4] argue that such kinds of errors and difficulties may be caused by lack of emphasis by teachers on understanding the language of mathematics and the skills needed by the students. These show the importance of improving the quality of learning quadratic equation.

Effort on supporting students’ understanding on solving quadratic equations have been conducted through a variety of approaches, such as by employing technology-assisted learning using spreadsheet [7] and applying constructivist-oriented learning with inquiry approach [8]. Another approach which may be used is employing historical perspectives of quadratic equation. This is due to the reasons that historical development of a science is very important not only from a cultural and humanistic perspective but also because it facilitates understanding the concept of a topic [9]. Hence integration of aspects of history in learning will be very helpful, either for students or teachers. Thus, problems from history mathematics are arguably potential as sources of context to make a meaningful mathematics learning.

In general, researchers suggest to integrate mathematics learning with the history of mathematics [1,10]. Radford states that the historical construction of mathematical concepts can supply us with a better understanding of the ways in which our students construct their knowledge of mathematics [11]. Moreover, Grugnetti explains that by knowing the history of how a concept was invented or developed, will support and improve the skills about that concept [12]. In particular, based on the historical perspective, the concept of solving quadratic equations was built by the geometric foundation, such as by employing the so-called ‘naïve geometry’[13]. Therefore, using naïve geometry to develop the learning instruction for learning the concept of the quadratic equation is regarded helpful for students in tackling the quadratic equation problems.

2. Theoretical Background

2.1. Naïve Geometry

The Old Babylonian geometric method is a geometric method that can be used to solving a quadratic equation. This method was identified by J. Hǿyrup and he called it Naïve Geometry [14]. In order to show the method, let we discuss one of the Babylonian problems, problem II of a tablet preserved at the British museum and known as BM 13901. The statement of the problem, that is to find the length of the square’s side, is the following:

My confrontation inside of the surface I have torn out: 14˚30’

Meanwhile, the statement of solution appears in the tablet is the following:

My confrontation inside of the surface I have torn out: 14˚30’. 1 the wasitum; You pose. The moiety (half) of 1 you break, 30’ and 30’ you make span; 15’ to 14˚30’ you append: 14˚30’15’ makes 29˚30’ equilateral. 30’ Which you have made span to 29˚30’ you append; 30 the confrontation. (Hǿyrup, 1990b)

note: 14˚30’ = 870, 30’ = \frac{1}{2}, 15’ = \frac{3}{4}, 14˚30’15’ = 870\frac{3}{4}.

The statement of the problem is, "My confrontation inside of the surface I have torn out: 14˚30’. 1 the wasitum? Hǿyrup [10] explained that the "Confrontation" is a side of the square and the "surface" is a square. The side is not a simple side, but as the side along with us as a side provided with a canonical projection that forms, along with the side (rectangle form) [1]. The “wasitum” is means something going out, including something projecting from a building [14]. The problem simply is to find the value of the square’s side if it is known that the area of the square minus its side is 870. To get a better understanding of the statement in the tablet, Hǿyrup [14] interpret the geometric shapes as shown below.
Figure 1. Geometric Interpretation of BM 13901 Problem II

Figure 1 shows that the problems and resolution steps written by the Babylonian mathematicians look simpler if interpreted into the algebraic symbol. The basic idea of the method used by Babylonian mathematicians on the problem found in BM 13901 No.2, i.e. reshaping and completing rectangle into a perfect square. Geometric interpretation also makes the problem is easier to understand. The scribes did not know any algebraic symbol, but based on the geometric and algebraic interpretations presented in figure 1, it can be concluded that they knew the formula of quadratic equations and how to find solutions.

2.2. Local Instruction theory
Gravemaijer [11] states that Local Instruction Theory (LIT) is a description of the conjecture of learning activity on a particular topic. Gravermeijer & Cobb [16] stated that LIT consists of conjecture a learning process also switch alleged guesswork about the possibility of facilities that can support the learning process. Gravemeijer [15] explains that LIT is a theory that offers a plan which can be used as guidance by teachers to choose learning activities and compose HLT.

3. Method
The core of this type of research is formed by classroom teaching experiments that centre on the development of instructional sequences and the local instructional theories that underpin them [15]. There are three main phases undertaken in this research, preliminary design, teaching experiment, and retrospective analysis [15]. In the first phase, we formulated a Hypothetical Learning Trajectory (HLT) for learning the concept of quadratic equation that is made up of learning goals for students, planned instructional activities and a conjectured learning process in which one anticipates how students’ thinking and understanding could evolve when the instructional activities are used in the classroom [17].

3.1. Participants
On teaching experiment, the participants were 32 students consisting of 15 male students and 17 female students. Meanwhile, on the pilot experiment involving 5 students from the same school.

3.2. Data collection and analysis
Data were collected through video documentation, student work photos, field notes, and interviews. Then, we conducted triangulation to test the HLT that had been developed. HLT were tested through two phases, pilot experiment, and teaching experiment by analysing the students’ work and interviewing.

4. Result
In preliminary design, we developed four instruction activities to be implemented in the classroom or we call it HLT. The following is a brief explanation of our HLT. In the first meeting, learning started by introducing the context and steps must be followed so that students can solve the problem using
geometric method (ignoring algebraic procedure). The second problem, students have to solve the second problem (determining the length and width of a rectangle) using the same method they used in the first meeting. However, the step needed is the opposite way. In first meeting (first activity), to solve the problem they must reshape square into a rectangle but in second meeting (second activity) is the opposite. Then, in third and fourth meeting the problem used is similar (determining the side of a square), they also need to reshaping rectangle into a square but the difference is that concrete number numbers are given neither for the area nor base of the rectangle. In the last activity, we focus on how student brings the geometric procedure to the algebraic form and understand the concept of solving quadratic equation through Naïve Geometry.

**The teaching sequences**

Here, we try to show the brief explanation and analysis of teaching sequences developed and students work (in teaching experiment).

1) **Part 1. Manipulating geometric form to solve the problem**

In part 1, students are presented with the following problem:

*The area of a rectangular garden is 32 ha. Around the garden, there are 120 Palm Trees which the distance of each tree is 20 meters. If the owner of the garden wants to put up an iron fence on the front side of the garden, help the owner to determine the length of the fence has to be purchased!*

![Figure 2. Problem 1](image)

Students are asked to solve the problem using any method. Most all groups use the trial and error method to solve the problem. By using a cutting paper as props, the teacher shows them the technique of Babylonian approach or known as Naïve Geometry.

Then, the following is the second problem:

*What should the length and width of a rectangle if the area is 84 units and the circumference is 40 units?*

By solving the problem, students understand that by performing geometric manipulation of naive geometry methods, the length of the rectangle is obtained by adding the square side formed at the beginning with the cut sides of the square (disposed to match the area of the desired rectangle). Because the square’s side formed at the beginning is 10 units and the square side that removed is 4 units, the rectangle dimensions are 10 + 4 = 14 (length) and 10 - 4 = 6 (width) (see figure 3).

![Figure 3. Students’ Geometric Manipulation](image)

Here is the last problem of part 1:

On the last problem, to help students achieve a better understanding of the deeper level of naive geometry, we give a problem that leads students confronted with a conflict if the area to be disposed of is not square (if expressed by an integer). In this case, some groups can solve it well while others failed.
2) **Part 2. Using the Babylonia geometric method to solve the problem**

In part 2, the students are presented with a problem that needs different use of naive geometry method. The problem is the following:

*Determine the dimensions of a rectangle if known that the area is 117 unit and the difference between its length and width is 4 units?*

In this case, the geometric manipulation performed by the students to solve the problem is in accordance with the conjecture on the HLT. The steps they use are almost the same as Geometric Interpretation of BM 13901 Problem II (see figure 1). But, they met some problems when we asked to work on a description of the steps to follow in order to solve this type of problem.

3) **Linking geometric problems to algebra, and finding common formulas to solve quadratic equations**

In part 3, the first problem must be solved by the students is to determine the side of the square if some of the area, which is rectangular in shape (width is 2 unit, length is the same with the square side), is removed so that the area becomes 24 unit area. The idea of giving this problem is to build student understanding that the geometry problems they face are a quadratic equation problem (that is to solve \( x^2 - 2x = 24 \)). By the naive geometry manipulation method, the geometric form becomes a simple algebraic equation, \((x - 1)^2 = 25\), that can be solved easier without involving complex algebraic manipulation.

The second problem given in this part is the same type as the previous. The difference is we did not provide a concrete number. In this part, we gave the following problem to the students.

*The length of a square is ‘x’ units. If a part of its regions, that is rectangular shape with the width b unit and the same length as the square side, is removed, its area becomes c units. Determine a) The algebraic form of the problem. b) A formula or steps (in the form of algebraic symbols) to determine the length of the sides of the square! (Use naive geometry method).*

This section guides students to link the geometry problems they solved with algebra where students focus on how to interpret geometric manipulation into algebraic symbols. The following is the result of student work in determining the side of a square (value of x) if the shaded area is c (see figure 4).

![Figure 4. Problem in fourth activity and the student’s work](image)

5. **Discussion and Conclusion**

Based on the implementation of the HLT in teaching experiment, the use of Naïve Geometry method can be an alternative method for learning the concept of the quadratic equation because the conjecture in HLT that has been made of is quite in accordance with the result of teaching experiment. In common, the learning instruction we developed based on Old Babylonian Geometric Method: Naïve Geometry can support students’ understanding of the concept of solving quadratic equations. Based on that learning conducted in the second cycle of teaching experiment, we also conclude that through these activity students can realize the idea of Naïve Geometry can be used to solve quadratic equations and finding a general form to solve quadratic equations. Through the geometric idea (reshaping into a square), also supports students to perform symbol operations are meaningful because they are familiar with the context involved. However, from the implementation of learning activities, we found that only students with high mathematics ability who reach the learning objective until the last stage of activity, which is to reinvent the common algebraic formula in solving the quadratic equation.

In common, through activities that have been designed in HLT, students learned to understand algebraic problems and how to solve them through geometric approaches that they are familiar (contextual). Thus the students get a chance to learn how to find the solution of quadratic equations
meaningfully. Implementation of HLT this time can be concluded in line with Freudenthal statement [18] that learning will happen when it is meaningful for students.

In general, we recommend teachers to integrate the history of mathematics in learning activities, which is in line with other researchers’ recommendations [5,19]. On the other hand, we recommend that similar research will be conducted for other learning topics. Hopefully, it can help and enrich teachers’ references in using the history of mathematics in learning.

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