Semi-aligned two Higgs doublet model

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Abstract

In the left-right symmetric model based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry, there appear heavy neutral scalar particles mediating quark flavor changing neutral currents (FCNCs) at tree level. We consider a situation where such FCNCs give the only sign of the left-right model while $W_R$ gauge boson is decoupled, and name it “semi-aligned two Higgs doublet model” because the model resembles a two Higgs doublet model with mildly-aligned Yukawa couplings to quarks. We predict a correlation among processes induced by quark FCNCs in the model, and argue that future precise calculation of meson-antimeson mixings and CP violation therein may hint at the semi-aligned two Higgs doublet model and the left-right model behind it.

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I. INTRODUCTION

The left-right symmetric model based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry and the left-right parity (symmetry under the Lorentzian parity transformation accompanied by the exchange of $SU(2)_L$ and $SU(2)_R$) is a well-motivated extension of the Standard Model (SM). In the model, the chiral nature of the SM is beautifully attributed to spontaneous breaking of $SU(2)_R \times U(1)_{B-L}$ gauge symmetry. More importantly, the model offers a new look into the strong CP problem; since the left-right parity demands the Yukawa couplings to be Hermitian and forbids gluon $\theta$ term at tree level, if one could find a symmetry-based reason that the two vacuum expectation values (VEVs) of the $SU(2)_L \times SU(2)_R$ bi-fundamental scalar are both made real, the strong CP problem would be solved. Put another way, the original strong CP problem, which is about a miraculous cancellation between the $\theta$ term in QCD and the quark mass phases in the theory of electroweak symmetry breaking, is simplified to the issue of why the scalar potential does not break CP spontaneously. Thus, the strong CP problem becomes more tractable, although the left-right model by itself does not solve it.

The first experimental hint of the left-right model would probably come in the form of quark flavor changing neutral currents (FCNCs) mediated by heavy neutral scalar particles at tree level, because the $SU(2)_L \times SU(2)_R$ bi-fundamental scalar necessarily has two unaligned Yukawa couplings both contributing to up and down-type quark masses to accommodate the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and the resultant quark FCNCs and CP violation are efficiently searched for through meson-antimeson mixings. On the other hand, recent studies have centered on the possibility of a direct measurement of $W_R$ gauge boson at the LHC, by elaborating a scalar potential where $SU(2)_R \times U(1)_{B-L}$ breaking VEV $v_R$ is below $\sim$5 TeV (hence $W_R$ mass being several TeV), while the heavy scalar particles have masses above $\sim$20 TeV to evade the bounds on FCNCs. Such a scalar potential contains $O(1)$ quartic couplings that quickly become non-perturbative along renormalization group (RG) evolutions, and even if this is circumvented, there is no theoretical reason that favors having specially heavy scalar particles. Conversely, if $W_R$ gauge boson has a mass similar to or larger than the heavy scalar particles (e.g. $(W_R \text{ mass}) \gtrsim (\text{heavy scalar mass}) = 100 \text{ TeV}$, so that the model evades the constraints from meson-antimeson mixings), then $W_R$ gauge boson leaves no direct or indirect experimental...
signature and it is the heavy scalar particles that allow us to probe the left-right model through FCNCs they mediate.\footnote{For a collider study of the heavy scalar particles in the left-right model, see Ref. \cite{8}.}

In this paper, therefore, we pose the following question: If quark FCNCs mediated by the heavy neutral scalars give the only sign of the left-right model, can we test the model? To answer this, we extract the bi-fundamental scalar part of the model, assuming that $W_R$ gauge boson is decoupled from phenomenology, and systematically study its indirect signatures and their correlation. We coin the term “semi-aligned two Higgs doublet model (semi-aligned 2HDM)” to describe the bi-fundamental scalar part, in light of the fact that the bi-fundamental scalar contains two $SU(2)_L$ doublet scalars and their quark Yukawa couplings are mildly aligned reflecting the smallness of CKM mixing angles.

Our analysis starts by realizing that the flavor and CP-violating couplings of the heavy scalar particles are uniquely determined as follows: Since the search for neutron electric dipole moment (EDM) has put a severe bound on the strong CP phase, we may concentrate on the limit with a vanishing spontaneous CP phase for the bi-fundamental scalar VEVs $\Phi$.\footnote{In this paper, we focus on phenomenological consequences of a vanishing spontaneous CP phase and do not discuss its theoretical origin. For attempts to derive the vanishing spontaneous CP phase in the framework of the left-right model, see Ref. \cite{10}.} In this limit, the quark mass matrices are Hermitian and the mixing matrix for right-handed quarks is identical with the CKM matrix, which allows one to express the couplings of the heavy scalar particles to quarks in terms of the SM quark masses and CKM matrix, without free parameters. The amplitudes for various FCNC processes are then calculated as functions of just one free parameter, that is, the nearly degenerate mass of the heavy scalar particles, and we thus predict a correlation among various FCNC processes. We will show that indirect CP violation in kaon system, $\Re \epsilon$, the $B_d^0$ mass splitting, $\Delta M_{B_d}$, and the $B_s^0$ mass splitting, $\Delta M_{B_s}$, are the most sensitive probes for the semi-aligned 2HDM, and if uncertainties in their calculation are reduced and the prediction including the contributions of heavy scalar particles converges to the experimental values for some unique value of the heavy scalar mass, it is evidence for the semi-aligned 2HDM and the left-right model behind it.

A novelty of our study compared to previous works $\cite{2, 3}$ is that we concentrate on the limit with decoupled $W_R$ gauge boson, which enables us to compute all amplitudes with only one free parameter and investigate their correlation. Also, we pay attention to the
fact that new physics contributions can distort the determination of the CKM matrix. To avoid this, when we derive the current bound on the semi-aligned 2HDM, we refit the CKM matrix, attempting to fit the experimental data with SM+new physics contributions and using a tension in the fitting to constrain the model. When we make a prediction for FCNC processes, we assume that the CKM matrix is determined beforehand in a way unaffected by new physics contributions.

This paper is organized as follows: In Section II, we describe the semi-aligned 2HDM induced from the left-right model, and calculate the mass and couplings of the heavy scalar particles. In Section III, we review the procedure for computing amplitudes for $\Delta F = 2$ processes. In Section IV, we derive the current bound on the heavy neutral scalar mass. Section V presents our main results, which are $\text{Re } \epsilon$, $\Delta M_{B_d}$ and $\Delta M_{B_s}$ expressed in terms of one parameter. Section VI summarizes the paper.

II. MODEL

We start from the left-right symmetric model based on $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry and the left-right parity, whose full expression is in Appendix A. The VEV of the $SU(2)_R$ triplet scalar, $v_R$, breaks $SU(2)_R \times U(1)_{B-L}$ into hypercharge $U(1)_Y$ and also breaks the left-right parity. In this paper, we focus on a limit where $v_R$ is much larger than the electroweak scale, and at the same time, the quartic couplings between two $SU(2)_R$ triplets and two bi-fundamentals are much smaller than 1, namely,

$$v_R \gg v \simeq 246 \text{ GeV}, \quad |\alpha_1|, |\alpha_{2R}|, |\alpha_{2I}|, |\alpha_3| \ll 1,$$

(1)

where $\alpha_1$, $\alpha_{2R}$, $\alpha_{2I}$, $\alpha_3$ are defined in the Lagrangian of Appendix A. In the limit with Eq. (1), the low-energy theory at scales below $v_R$ (we call it the semi-aligned 2HDM) contains the SM fermions+three right-handed neutrinos+the bi-fundamental scalar and possesses SM $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry. However, the Yukawa couplings and self-couplings of the bi-fundamental scalar respect global $SU(2)_R$ symmetry and the left-right parity, which are remnants of the left-right model, and hence have highly constrained structures. In Table[I] we summarize the fields in the semi-aligned 2HDM and their charges in $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group and global $SU(2)_R$ group. Here, the
TABLE I. Field content and charge assignments. \( i = 1, 2, 3 \) is the flavor index.

| Field | Lorentz | \( SO(1, 3) \) | \( SU(3)_C \) | \( SU(2)_L \) | global \( SU(2)_R \) | \( U(1)_Y \) |
|-------|---------|----------------|----------------|----------------|-----------------|----------------|
| \( q^i_L \) | \((2, 1)\) | 3 | 2 | 1 | 1/6 |
| \( q^i_R = \begin{pmatrix} u^i_R \\ d^i_R \end{pmatrix} \) | \((1, 2)\) | 3 | 1 | 2 | \begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix} |
| \( \ell^i_L \) | \((2, 1)\) | 1 | 2 | 1 | -1/2 |
| \( \ell^i_R = \begin{pmatrix} \nu^i_R \\ \ell^i_R \end{pmatrix} \) | \((1, 2)\) | 1 | 1 | 2 | \begin{pmatrix} 0 \\ -1 \end{pmatrix} |
| \( \Phi = (i\sigma_2 H^*_u, i\sigma_2 H^*_d) \) | 1 | 1 | 2 | 2 | \((-1/2, 1/2)\) |

SM right-handed fermions form doublets of global \( SU(2)_R \) symmetry, \( q_R \) and \( \ell_R \). The bi-fundamental scalar \( \Phi \) is expressed as a \( 2 \times 2 \) matrix transforming under a \( SU(2)_L \times SU(2)_R \) gauge transformation as

\[
\Phi \rightarrow e^{i\tau_a \theta^a_L} \Phi e^{-i\tau_a \theta^a_R}, \quad \theta^a_L, \theta^a_R : \text{gauge parameters}, \quad \tau^a \equiv \sigma^a/2, \tag{2}
\]

which is then decomposed into two \( SU(2)_L \) doublet scalars with hypercharge \( Y = \pm 1/2 \), \( H_u \) and \( H_d \), as \( \Phi = (i\sigma_2 H^*_u, i\sigma_2 H^*_d) \). The Lagrangian of the semi-aligned 2HDM is given by

\[
-\mathcal{L} = (Y_q)_{ij} \bar{q}^i_L \Phi q^j_R + (\bar{Y}_q)_{ij} \bar{q}^i_L \Phi^\dagger q^j_R + (Y_\ell)_{ij} \bar{\ell}^i_L \Phi \ell^j_R + (\bar{Y}_\ell)_{ij} \bar{\ell}^i_L \Phi^\dagger \ell^j_R + \text{H.c.} \tag{3}
\]

\[
+ \frac{v_R}{\sqrt{2}} (Y_M)_{ij} N^i_{R}^{T} N^j_{R} + \text{H.c.} \tag{4}
\]

\[
+ m_1^2 \text{tr} [\Phi^\dagger \Phi] + m_2^2 R \text{tr} [\Phi^\dagger \Phi + \Phi \Phi^\dagger] + m_2^2 i \text{tr} [\Phi^\dagger \Phi - \Phi \Phi^\dagger] + m_3^2 \text{tr} \left[ \Phi^\dagger \Phi \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right] + \lambda_1 \text{tr} [\Phi^\dagger \Phi]^2 + \lambda_2 \left( \text{tr} [\Phi^\dagger \Phi]^2 + \text{tr} [\Phi \Phi^\dagger]^2 \right) + \lambda_3 \text{tr} [\Phi^\dagger \Phi] \text{tr} [\Phi \Phi^\dagger] + \lambda_4 \text{tr} [\Phi^\dagger \Phi] \text{tr} [\Phi^\dagger \Phi^\dagger], \tag{5}
\]

with \( \bar{\Phi} \equiv i\sigma_2 \Phi^\dagger i\sigma_2 \),

where \( m_1^2, m_2^2, m_2^2 R, m_3^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) are all real, and the Yukawa coupling matrices are Hermitian, \( Y^+ = Y_\ell, \bar{Y}^+ = \bar{Y}_\ell \). Notice that \( m_2^2 R \) softly breaks the left-right parity, and \( m_3^2 \) softly breaks \( SU(2)_R \) symmetry. Both result from the spontaneous left-right symmetry breaking and are proportional to \( v_R^2 \).
Φ develops a VEV to break $SU(2)_L \times U(1)_Y$ symmetry. Through a $SU(2)_L$ plus $\sigma_3$ part of $SU(2)_R$ symmetry transformation, the VEV is made into the following form, with one VEV having a CP phase $\alpha$:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \sin \beta & 0 \\ 0 & -e^{i\alpha} v \cos \beta \end{pmatrix}, \quad v \simeq 246 \text{ GeV}, \quad \sin \beta > 0, \quad \cos \beta > 0. \quad (6)$$

The mass matrices for up-type quarks, $M_u$, down-type quarks, $M_d$, charged leptons, $M_e$, are given by

$$M_u = \frac{v}{\sqrt{2}} (Y_q \sin \beta + \tilde{Y}_q \cos \beta e^{-i\alpha}), \quad (7)$$
$$M_d = -\frac{v}{\sqrt{2}} (Y_q \cos \beta e^{i\alpha} + \tilde{Y}_q \sin \beta), \quad (8)$$
$$M_e = -\frac{v}{\sqrt{2}} (Y_\ell \cos \beta e^{i\alpha} + \tilde{Y}_\ell \sin \beta). \quad (9)$$

The spontaneous CP phase $\alpha$ has already been severely constrained by the search for neutron EDM. Since $W_R$ is decoupled and the coupling of scalar particles to up and down quarks is Yukawa-suppressed, perturbative corrections to neutron EDM are negligible, and the experimental bound is directly translated into a bound on $\arg \det(M_u M_d)$ and hence on $\alpha$. In the limit of neglecting the quark flavor mixing (but no assumptions are made on $\beta$ or $\alpha$), we obtain, from Eqs. (7, 8), the following formula:

$$\arg \det(M_u M_d) \simeq \theta_{ud} + \theta_{cs} + \theta_{tb},$$
$$\sin \theta_{ud} = -\frac{m_u^2 - m_d^2}{2m_u m_d} \tan(2\beta) \sin \alpha, \quad (u,d) \rightarrow (c,s), \quad (u,d) \rightarrow (t,b). \quad (10)$$

The current experimental bound [11] roughly gives

$$10^{-10} > |\bar{\theta}| = |\arg \det(M_u M_d)| \simeq |\theta_{ud} + \theta_{cs} + \theta_{tb}|, \quad (11)$$

where it should be reminded that the QCD $\theta$ term is prohibited at tree level by the left-right parity. $\alpha$ is thus constrained to be much below 1, and based on this fact, we fix $\alpha = 0$ in the rest of the paper.

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3 This symmetry transformation generates a phase for $v_R$, but this can be negated by a $U(1)_{B-L}$ symmetry transformation.

4 Deriving $\alpha = 0$ theoretically is equivalent to solving the strong CP problem, which we do not attempt in this paper.
The physical scalar particles after the electroweak symmetry breaking are a charged scalar, $H^\pm$, a CP-odd scalar, $A$, a lighter CP-even scalar which we identify with the SM Higgs particle, $h$, and a heavier CP-even scalar, $H$. The $H^\pm$ and $A$ masses read

$$m_{H^\pm}^2 = \frac{m_3^2}{\cos 2\beta}, \quad m_A^2 = \frac{m_3^2}{\cos 2\beta} - (4\lambda_2 - 2\lambda_3)v^2,$$

The $H$ and $h$ masses expanded to the order of $O(v^2/|m_3^2|)$ are found to be

$$m_H^2 \simeq \frac{m_3^2}{\cos 2\beta} + (4\lambda_2 + 2\lambda_3)\cos 2\beta v^2, \quad m_h^2 \simeq 2\lambda_1 v^2 + (4\lambda_2 + 2\lambda_3)\sin^2 2\beta v^2 + 4\lambda_4 \sin 2\beta v^2.$$

The bi-fundamental scalar $\Phi = (i\sigma_2 H_u^*, i\sigma_2 H_d^*)$ can be decomposed into the physical scalar particles $H^\pm, A, H, h$ and Nambu-Goldstone bosons, $G^\pm, G^0$, in the following way:

$$H_u = \begin{pmatrix}
-\sin \beta & G^+ + \cos \beta H^+ \\
\frac{1}{\sqrt{2}} (\sin \beta v + \cos \gamma h + \sin \gamma H - i \sin \beta G^0 + i \cos \beta A) 
\end{pmatrix},$$

$$H_d = \begin{pmatrix}
-\sin \beta & G^- + \sin \beta H^- \\
\frac{1}{\sqrt{2}} (\cos \beta v - \sin \gamma h + \cos \gamma H + i \cos \beta G^0 + i \sin \beta A) 
\end{pmatrix}, \quad (16)$$

where $\gamma$ is the mixing angle of the CP-even scalars satisfying

$$\tan 2\gamma = \frac{m_3^2 - 2v^2(\lambda_1 + 2\lambda_2 + \lambda_3)\cos 2\beta - 4v^2\lambda_4 \cot 2\beta}{m_3^2 - 2v^2(\lambda_1 - 2\lambda_2 - \lambda_3)\cos 2\beta} \tan 2\beta, \quad 0 > \gamma > -\pi/2. \quad (17)$$

Since the $A$ and $H$ masses are experimentally constrained to be above $\sim 10$ TeV, we work in the decoupling limit with $v^2/|m_3^2| \to 0$ to realize $m_H^2, m_A^2 \gg m_h^2$, in which case the masses of $H^\pm, A, H$ and the CP-even scalar mixing angle $\gamma$ satisfy

$$m_{H^\pm}^2 = m_A^2 = m_H^2, \quad (18)$$

$$\gamma = \beta - \frac{\pi}{2}. \quad (19)$$

---

Since $\alpha = 0$, CP is not broken spontaneously and the CP-even and odd scalar particles can be defined.
Equation (76) induces Yukawa couplings for quarks and $H^\pm$, $H$ and $A$, given by

$$-\mathcal{L}_{\text{yukawa}} = (Y_q \sin \beta - \bar{Y}_q \cos \beta)_{ij} u^i_L d^j_R H^+ + (\bar{Y}_q \cos \beta + Y_q \sin \beta)_{ij} \bar{d}^i_L u^j_R H^- + \text{H.c.}$$

$$+ \frac{1}{\sqrt{2}} (Y_q \cos \beta + \bar{Y}_q \sin \beta)_{ij} u^i_L d^j_R A + \frac{i}{\sqrt{2}} (Y_q \sin \beta - \bar{Y}_q \cos \beta)_{ij} \bar{d}^i_L u^j_R A$$

$$+ \frac{1}{\sqrt{2}} (Y_q \sin \gamma + \bar{Y}_q \cos \gamma)_{ij} \bar{u}^i u^j H + \frac{1}{\sqrt{2}} (Y_q \cos \gamma - \bar{Y}_q \sin \gamma)_{ij} \bar{d}^i \gamma_5 d^j H \quad (20)$$

$$\simeq -\sqrt{2} \frac{(M_u + M_d \sin 2\beta)_{ij}}{v \cos 2\beta} \bar{u}^i u^j H^+ + \sqrt{2} \frac{(M_d + M_u \sin 2\beta)_{ij}}{v \cos 2\beta} \bar{d}^i u^j H^- + \text{H.c.}$$

$$+ \frac{(M_d + M_u \sin 2\beta)_{ij}}{v \cos 2\beta} \bar{u}^i u^j H + \frac{i}{v \cos 2\beta} (M_d + M_u \sin 2\beta)_{ij} \bar{u}^i \gamma_5 u^j A$$

$$+ \frac{(M_u + M_d \sin 2\beta)_{ij}}{v \cos 2\beta} \bar{d}^i d^j H - \frac{i}{v \cos 2\beta} (M_u + M_d \sin 2\beta)_{ij} \bar{d}^i \gamma_5 d^j A, \quad (21)$$

where Eq. (19) has been used. We diagonalize the quark mass matrices by the rotation,

$$u_{L,R} \rightarrow V_u u_{L,R}, \quad d_{L,R} \rightarrow V_d d_{L,R},$$

(22)

to obtain

$$V_u^\dagger M_u V_u = m_u^D, \quad V_d^\dagger M_d V_d = m_d^D,$$

(23)

$$m_u^D = \text{diag}(s_u m_u, s_c m_c, s_t m_t), \quad m_d^D = \text{diag}(s_d m_d, s_s m_s, s_b m_b),$$

(24)

where $m_u^D$ and $m_d^D$ are diagonalized mass matrices for up and down-type quarks, respectively, and $s_f = \pm 1(f = u, c, t, d, s, b)$, which reflects sign uncertainty of the mass eigenvalues. Note that the left and right-handed quarks are rotated with the same unitary matrix, since $\alpha = 0$ and the mass matrices are Hermitian. Accordingly, the Yukawa couplings become

$$-\mathcal{L}_{\text{yukawa}} = -\sqrt{2} \frac{(m_u^D V + V^\dagger m_u^D \sin 2\beta)_{ij}}{v \cos 2\beta} \bar{u}^i L d^j R H^+ + \sqrt{2} \frac{(m_d^D V^\dagger + V m_u^D \sin 2\beta)_{ij}}{v \cos 2\beta} \bar{d}^i L u^j R H^- + \text{H.c.}$$

$$+ \frac{(V m_u^D V^\dagger + m_u^D \sin 2\beta)_{ij}}{v \cos 2\beta} \bar{u}^i u^j H + i \frac{(V m_d^D V^\dagger + m_d^D \sin 2\beta)_{ij}}{v \cos 2\beta} \bar{u}^i \gamma_5 u^j A$$

$$+ \frac{(V^\dagger m_u^D V + m_u^D \sin 2\beta)_{ij}}{v \cos 2\beta} \bar{d}^i d^j H - i \frac{(V^\dagger m_d^D V + m_d^D \sin 2\beta)_{ij}}{v \cos 2\beta} \bar{d}^i \gamma_5 d^j A, \quad (25)$$

where we have defined the CKM matrix, $V$, as

$$V = V_u^\dagger V_d.$$

(26)

We find that the following part in Eq. (25) induces FCNCs at tree level by the exchange of heavy neutral scalars $H, A$, with the strength controlled by the CKM matrix multiplied by
quark masses:

\[-L_{\text{yukawa}} \supset \frac{(V m_d^D V^\dagger)_{ij}}{v \cos 2\beta} (\bar{u}^i u^j H + i \bar{u}^i \gamma_5 u^j A) + \frac{(V^\dagger m_u^D V)_{ij}}{v \cos 2\beta} (\bar{d}^i d^j H - i \bar{d}^i \gamma_5 d^j A).\]  (27)

Flavor violation is suppressed by off-diagonal components of the CKM matrix, and possibly by light quark masses, which is a characteristic property of the model.

Since the Yukawa couplings always appear in combination with the factor $1/\cos 2\beta$, hereafter we redefine the heavy scalar masses as

new $m^2_{H^\pm} \equiv m^2_{H^\pm} \cos^2 2\beta$,  
new $m^2_{H} \equiv m^2_{H} \cos^2 2\beta$,  
new $m^2_{A} \equiv m^2_{A} \cos^2 2\beta$.  

(28)

As a reference, we present the absolute values of the flavor-violating part of the Yukawa couplings for $s_f = +1 (f = u, c, t, d, s, b)$:

For up $-$ type:

\[|V m_d^D V^\dagger|_v = \begin{pmatrix}
0.000037 & 0.000080 & 0.000059 \\
\ast & 0.00038 & 0.00068 \\
\ast & \ast & 0.017
\end{pmatrix},\]  

(29)

For down $-$ type:

\[|V^\dagger m_u^D V|_v = \begin{pmatrix}
0.00032 & 0.0013 & 0.0060 \\
\ast & 0.0058 & 0.027 \\
\ast & \ast & 0.70
\end{pmatrix}.\]  

(30)

The off-diagonal components in the above matrices indicate the strength of FCNCs mediated by the neutral scalars. Here, the CKM matrix components are obtained from the Wolfenstein parameters reported by the CKMfitter, which read

\[\lambda = 0.22548, \quad A = 0.810, \quad \bar{\rho} = 0.145, \quad \bar{\eta} = 0.343.\]  

(31)

We comment in passing that the Yukawa couplings for leptons and $H^\pm$, $H$ and $A$ are obtained by a simple replacement: $d \rightarrow e$, $u \rightarrow \nu$, $M_d \rightarrow M_e$, $M_u \rightarrow M_D$ in Eq. (21), with $M_D$ being the neutrino Dirac mass involved in the seesaw mechanism. Due to our ignorance of the seesaw scale, we cannot predict the strength of the flavor-violating couplings for charged leptons.

\[6\] If we require $\tan \beta \sim m_t/m_b$ so that the top and bottom quark mass ratio is derived without fine-tuning, we have $\cos^2 2\beta \approx 1$ and this redefinition becomes trivial.
III. $\Delta F = 2$ AMPLITUDES

We give formulae for $\Delta F = 2$ amplitudes to analyze flavor observables in the quark sector. In particular, mass differences for $K, B_d^0$ and $B_s^0$, and a CP violating observable in kaon system are given. The effective Hamiltonian contributing to $\Delta S = 2$ processes is given as follows,

$$H_{\Delta S=2}^{\text{FCNH}} = \frac{G_F}{\sqrt{2}} (C_S O_S + C_P O_P) + \text{H.c.},$$

\[C_S = -\frac{1}{m_H^2} \left[ \sum_{k} \lambda_k^d (m_D u)^k \right]^2, \quad C_P = \frac{1}{m_A^2} \left[ \sum_{k} \lambda_k^d (m_D u)^k \right]^2, \quad \lambda_{ij}^k = V_{ki}^* V_{kj},\]  

(32)

Proper replacement of the indices in Eqs. (32-34) enables us to write the effective Hamiltonian in $\Delta B = 2$ processes. The Hamiltonian in Eq. (32) represents the contribution of a tree-level diagram in Figure III.1 arising from the exchange of a heavy neutral scalar particle $H, A$, which we denote “flavor changing neutral Higgs (FCNH)”. When the heavy scalar particles are degenerate, one can obtain a further simplified Hamiltonian. Including the SM contribution, we can write,

$$H_{\Delta S=2} = H_{\Delta S=2}^{\text{SM}} + H_{\Delta S=2}^{\text{FCNH}},$$

(35)

$$H_{\Delta S=2}^{\text{SM}} = C_{1}^{\text{VLL}} Q_{1}^{\text{VLL}} + \text{H.c.},$$

(36)

$$H_{\Delta S=2}^{\text{FCNH}} = \sum_{i=1}^{2} C_{i}^{\text{LR}} Q_{i}^{\text{LR}} + \text{H.c.},$$

(37)

where we follow the notation of Ref. [13]. In Eq. (37), we do not include operators, $Q_{1}^{\text{SLL}}, Q_{2}^{\text{SLL}}$ and other chirality-flipped ones given in Ref. [13], since they do not arise from the...
FCNH diagram. Furthermore, $Q_{1}^{SLL}$ and $Q_{2}^{SLL}$ are decoupled from the mixing with the other operators so that we omit these contributions. The Wilson coefficients and the operators in Eqs. (36, 37) are given as,

$$
C_{VLL}^{1} = \frac{G_{F}^{2}M_{W}^{2}}{(4\pi)^{2}}4\tilde{S}, \quad C_{1}^{LR}(\mu_{H}) = 0, \quad C_{2}^{LR}(\mu_{H}) = -\frac{2\sqrt{2}G_{F}}{m_{H}^{2}}\left[\sum_{k}\lambda_{k}^{sd}(m_{u}^{D}k)\right]^{2},
$$

$$
Q_{1}^{VLL} = \bar{s}^{\alpha}\gamma_{\mu}P_{L}d^{\alpha}s^{\beta}\gamma_{\mu}P_{L}d^{\beta}, \quad Q_{2}^{LR} = \bar{s}^{\alpha}\gamma_{\mu}P_{L}d^{\alpha}\tilde{s}^{\beta}\gamma_{\mu}P_{R}d^{\beta}, \quad Q_{2}^{LR} = \bar{s}^{\alpha}P_{L}d^{\alpha}\tilde{s}^{\beta}P_{R}d^{\beta},
$$

where $P_{R(L)} = (1 \pm \gamma_{5})/2$ denotes chirality projection operators while $\alpha$ and $\beta$ represent color indices. In Eq. (38), $C_{VLL}^{1}$ stands for the contribution within the SM, and the Inami-Lim function [14] is given as,

$$
\tilde{S} = \eta_{1}(\lambda_{c}^{sd})^{2}S(x_{c}) + \eta_{2}(\lambda_{t}^{sd})^{2}S(x_{t}) + 2\eta_{3}\lambda_{c}^{sd}\lambda_{t}^{sd}S(x_{c}, x_{t}), \quad x_{i} = \frac{m_{i}^{2}}{M_{W}^{2}} (i = c, t),
$$

$$
S(x_{i}, x_{j}) = x_{i}x_{j}\left[\frac{1}{x_{i} - x_{j}}\left(\frac{1}{4} - \frac{3}{2x_{i}} - \frac{1}{4(x_{i} - 1)^{2}}\right)\ln x_{i}
- \left(\frac{1}{4} - \frac{3}{2x_{j}} - \frac{1}{4(x_{j} - 1)^{2}}\right)\ln x_{j}
- \frac{3}{4(x_{i} - 1)(x_{j} - 1)}\right],
$$

$$
S(x_{i}) = \frac{4x_{i} - 11x_{i}^{2} + x_{i}^{3}}{4(x_{i} - 1)^{2}} + \frac{3}{2}\left(\frac{x_{i}}{x_{i} - 1}\right)^{3}\ln x_{i},
$$

where NLO QCD correction factors within the SM, $(\eta_{1}, \eta_{2}, \eta_{3})$, have been calculated in Ref. [13]. To be precise, one should multiply Eq. (40) by an overall factor which accounts renormalization scale of lattice QCD calculation.

As for the $\Delta B = 2$ processes, we only take account of the contribution of internal top quarks, and the corresponding NLO QCD correction is obtained through the method in Ref. [13]. The formulae for an anomalous dimension matrix including two-loop contribution are given in Ref. [16]. As remarked in the literature [13], this renormalization group effect drastically enhances $C_{2}^{LR}$ while it does not significantly change $C_{1}^{LR}$. In our analysis, new world averages of the QCD scale obtained by PDG [17] are used.
The matrix elements of the $\Delta F = 2$ transition are parametrized as,

$$
\langle \bar{K}^0 | Q^{\text{VLL}}_1(\mu) | K^0 \rangle = \frac{1}{3} M_K f_K^2 B_1^{\text{VLL}}(\mu), \quad (43)
$$

$$
\langle \bar{K}^0 | Q^{\text{LR}}_1(\mu) | K^0 \rangle = -\frac{1}{6} M_K f_K^2 B_1^{\text{LR}}(\mu) \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2, \quad (44)
$$

$$
\langle \bar{K}^0 | Q^{\text{LR}}_2(\mu) | K^0 \rangle = \frac{1}{4} M_K f_K^2 B_2^{\text{LR}}(\mu) \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2, \quad (45)
$$

$$
\langle B^0_q | Q^{\text{VLL}}_1(\mu) | B^0_q \rangle = \frac{1}{3} M_{B_q} f_{B_q}^2 B_1^{\text{VLL}}(\mu), \quad (46)
$$

$$
\langle B^0_q | Q^{\text{LR}}_1(\mu) | B^0_q \rangle = -\frac{1}{6} M_{B_q} f_{B_q}^2 B_1^{\text{LR}}(\mu) \left( \frac{M_{B_q}}{m_b(\mu) + m_q(\mu)} \right)^2, \quad (47)
$$

$$
\langle B^0_q | Q^{\text{LR}}_2(\mu) | B^0_q \rangle = \frac{1}{4} M_{B_q} f_{B_q}^2 B_2^{\text{LR}}(\mu) \left( \frac{M_{B_q}}{m_b(\mu) + m_q(\mu)} \right)^2. \quad (48)
$$

where $q = d, s$. In this normalization, kaon decay constant is given as $f_K = 156.1$ MeV.

The matrix elements in Eqs. (43)-(48) are written in terms of bag parameters, which represent the deviation from vacuum saturation approximation. For these parameter, we use the data which are calculated by the ETM collaboration [18, 19]. Their results are obtained in $\overline{\text{MS}}$ scheme, and extracted from the result in the supersymmetric basis. The correspondence between bag parameters in the operator basis in Eq. (39) and ones in the supersymmetric basis is given in Ref. [13].

The mass differences of neutral meson system are obtained as follows,

$$
\Delta M_K = 2 \text{Re} M_{12}, \quad \Delta M_{B_q} = 2 |M_{12}^q|, \quad (49)
$$

$$
M_{12} = M_{12}^{\text{SM}} + M_{12}^{\text{FCNH}}, \quad M_{12}^q = M_{12}^{\text{SM}} + M_{12}^{\text{FCNH}}, \quad (50)
$$

where $M_{12}^{(q)}$ is divided into the SM part and the new physics part,

$$
M_{12}^{\text{SM}} = \langle \bar{K}^0 | \mathcal{H}_{\Delta S=2}^{\text{SM}} | K^0 \rangle^*, \quad M_{12}^{\text{FCNH}} = \langle \bar{K}^0 | \mathcal{H}_{\Delta S=2}^{\text{FCNH}} | K^0 \rangle^*, \quad (51)
$$

$$
M_{12}^{q\text{SM}} = \langle B^0_q | \mathcal{H}_{\Delta B=2}^{\text{SM}} | B^0_q \rangle^*, \quad M_{12}^{q\text{FCNH}} = \langle B^0_q | \mathcal{H}_{\Delta B=2}^{\text{FCNH}} | B^0_q \rangle^*. \quad (52)
$$

Moreover, indirect CP violation in kaon system is characterized by

$$
\epsilon = \frac{e^{\frac{i\pi}{4}}}{\sqrt{2}} \text{Im} M_{12}. \quad (53)
$$

**IV. CURRENT BOUND ON THE MODEL**

In this section, we obtain the bound on mass of heavy Higgs through flavor observables. First, $\sin 2\beta_{\text{eff}}$, which represents CP violation of interference in $B_d^0 - \bar{B}_d^0$ mixing and $B_d^0 \to
$J/\psi K_S$, is analyzed. As discussed later, the bounds on Higgs mass are determined by p-values of CKM fitting.

In Table II, input data which appear in the numerical analysis are summarized.

**TABLE II.** Input data used in the analysis. The third column corresponds to references on which the data are based. For $K_L - K_S$ mass difference, the fitting result w/ CPT assumption is given. Re $\epsilon$ is extracted from asymmetry of semi-leptonic decay rates given by PDG.

|     | Value                  | Reference |
|-----|------------------------|-----------|
| $\eta_1$ | 1.32$^{+0.21}_{-0.23}$ | [13]      |
| $\eta_2$ | 0.57$^{+0.00}_{-0.01}$ | [13]      |
| $\eta_3$ | 0.47$^{+0.03}_{-0.04}$ | [13]      |
| $\Lambda^{(6)}_{\overline{MS}}$ | $(87 \pm 7)$ MeV | [17]      |
| $\Lambda^{(5)}_{\overline{MS}}$ | $(210 \pm 15)$ MeV | [17]      |
| $\Lambda^{(4)}_{\overline{MS}}$ | $(291 \pm 19)$ MeV | [17]      |
| Re $\epsilon$ | $(1.66 \pm 0.03) \times 10^{-3}$ | [17]      |
| $\Delta M_K$ | $3.484 \pm 0.006 \ [10^{-12} \text{ MeV}]$ | [17]      |
| $\Delta M_{B_d}$ | $0.5064 \pm 0.0019 \ [\text{ps}^{-1}]$ | [20]      |
| $\Delta M_{B_s}$ | $17.757 \pm 0.0021 \ [\text{ps}^{-1}]$ | [20]      |
| sin $2\beta_{\text{eff}}$ | $0.691 \pm 0.017$ | [20]      |
| $B_1^{\text{VLL}}(3 \text{ GeV})$ | $0.506 \pm 0.017$ | [18]      |
| $B_1^{\text{LR}}(3 \text{ GeV})$ | $0.49 \pm 0.04$ | [18]      |
| $B_2^{\text{LR}}(3 \text{ GeV})$ | $0.78 \pm 0.05$ | [18]      |
| $f_{B_d}\sqrt{B_1^{\text{VLL}}(m_b)}$ | $174 \pm 8 \text{ MeV}$ | [19]      |
| $f_{B_d}\sqrt{B_1^{\text{LR}}(m_b)}$ | $229 \pm 14 \text{ MeV}$ | [19]      |
| $f_{B_d}\sqrt{B_2^{\text{LR}}(m_b)}$ | $185 \pm 9 \text{ MeV}$ | [19]      |
| $f_{B_s}\sqrt{B_1^{\text{VLL}}(m_b)}$ | $211 \pm 8 \text{ MeV}$ | [19]      |
| $f_{B_s}\sqrt{B_1^{\text{LR}}(m_b)}$ | $285 \pm 14 \text{ MeV}$ | [19]      |
| $f_{B_s}\sqrt{B_2^{\text{LR}}(m_b)}$ | $220 \pm 9 \text{ MeV}$ | [19]      |
A. sin 2βeff measured in $B_d^0 \to J/\psi K_S$ decay

Throughout this paper, we assume that direct CP violation in $B_d^0 \to J/\psi K_S$ decay is negligible. Within this approximation, the time-dependent CP asymmetry in $B_d^0 \to J/\psi K_S$ decay is given as follows,

$$A_{CP}(t) \simeq S_{J/\psi K_S} \sin(\Delta M_{B_d} t), \quad (54)$$

$$S_{J/\psi K_S} \simeq \text{Im} \left( \frac{q A_{J/\psi K_S}}{p A_{J/\psi K_S}} \right), \quad (55)$$

where $q/p$ is a mixing parameter [21] in $B^0_d$ system while $A_{J/\psi K_S}(\bar{A}_{J/\psi K_S})$ represents decay amplitude of $B_d^0(\bar{B_d}^0) \to J/\psi K_S$. In the semi-aligned 2HDM, the decay amplitude in Eq. (55) does not deviate from the SM prediction, since a diagram of the charged scalar exchange gives rise to minor modification due to smallness of Yukawa couplings. We do not take account of such negligible contribution for the decay amplitude. Furthermore, the correction coming from penguin pollution in the SM is also small [22] because of the suppression for the CKM and the loop factor, and hence we ignore this effect. Meanwhile, the mixing parameter for $B_d^0$ system, $q/p$, and one for kaon system are modified due to the diagrams for FCNH exchange. Hence, the parameter in Eq. (55) is given as [23, 24],

$$\sin 2\beta_{\text{eff}} = S_{J/\psi K_S} = \sin \left[ 2\beta + \text{arg} \left( 1 + \frac{M_{d_{\text{FCNH}}}^{12}}{M_{d_{\text{SM}}}^{12}} \right) - \text{arg} \left( 1 + \frac{M_{d_{\text{FCNH}}}^{12}}{M_{d_{\text{SM}}}^{12}} \right) \right], \quad (56)$$

$$\beta = \text{arg} \left( -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right), \quad (57)$$

where the definitions of $M_{12}^{(d)SM}$ and $M_{12}^{(d)FCNC}$ are given in Eqs. (51, 52). Thus, the experimental observable deviates from $\sin 2\beta$ due to modification of the mixing parameters.

In Figure IV.1 in order to illustrate the FCNH mass dependence of $\sin 2\beta_{\text{eff}}$, we plot,

$$\text{arg} \left( 1 + \frac{M_{d_{\text{FCNH}}}^{12}}{M_{d_{\text{SM}}}^{12}} \right), \quad (58)$$

where for the CKM matrix components, we used the values in Eq. (31). The numerical behavior of the argument in Eq. (58) can be understood in the following way: If one neglects masses of up and charm quarks, a phase for $M_{d_{\text{FCNH}}}^{12}$ is determined as $-\left( V^t m_u D V \right)_{bd} \propto -(V^t_{tb} V_{td})^2$. This factor is the same as the box diagram in the SM up to its sign. Therefore,
FIG. IV.1. Argument of \( 1 + M_{12}^{\text{FCNH}}/M_{12}^{\text{SM}} \).

FIG. IV.2. Model prediction for \( \sin 2\beta_{\text{eff}} \) with (a) \((s_u, s_c, s_t) = (+, +, +)\) and (b) \((s_u, s_c, s_t) = (+, -, +)\).

\( M_{12}^{\text{FCNH}}/M_{12}^{\text{SM}} \) is a negative real number, approximately. Owing to this fact, we find

\[
\begin{align*}
\arg(1 + M_{12}^{\text{FCNH}}/M_{12}^{\text{SM}}) &= 0, \quad (-1 < M_{12}^{\text{FCNH}}/M_{12}^{\text{SM}} < 0) \\
\arg(1 + M_{12}^{\text{FCNH}}/M_{12}^{\text{SM}}) &= \pm \pi, \quad (M_{12}^{\text{FCNH}}/M_{12}^{\text{SM}} < -1)
\end{align*}
\]  

(59)

where for \( M_{12}^{\text{FCNH}}/M_{12}^{\text{SM}} < -1 \), sign of the argument depends on sign of tiny imaginary part in the argument. The relations in Eq. (59) indicate that the argument in Eq. (58) vanishes if \( m_H \) is sufficiently large. From Figure IV.1, we find that the argument in Eq. (59) does not affect \( \sin 2\beta_{\text{eff}} \) for \( m_H \geq 12 \text{ TeV} \). Note that in fact, charm quark mass slightly contributes to the above quantity so that the exact behavior in Figure IV.1 is not identical to Eq. (59).

We should also note that quark mass signs in Eq. (24) give rise to numerical difference in the argument in Eq. (58). However, choice of \( s_u = \pm 1 \) yields a minor difference since
up quark mass is negligible. Thus, the two representative cases, \((s_u, s_c, s_t) = (+, +, +)\) and \((s_u, s_c, s_t) = (+, -, +)\) are sufficient since the relative sign for \(s_c\) and \(s_t\) almost determines the coupling for \(\Delta F = 2\) processes in Eq. (38).

In Figure IV.2 we show the prediction for \(\sin 2\beta_{\text{eff}}\) with \((s_u, s_c, s_t) = (+, +, +)\) and \((s_u, s_c, s_t) = (+, -, +)\). In the plot, one can find that \(\sin 2\beta_{\text{eff}}\) shows non-trivial dependence on Higgs mass since both \(K\) and \(B^0_d\) systems affect the observable. For \(m_H \gg 50\) TeV, the neutral Higgs decouples from \(\sin 2\beta_{\text{eff}}\) so that it becomes identical to the SM prediction asymptotically.

**B. Bound on \(m_H\) from CKM fitting**

Now that we have obtained the formulae for flavor-violating observables, we derive bounds on FCNH mass.

It should be noted that in the presence of FCNH, the values of CKM matrix components are altered from those found in the literature, since the FCNH exchange process modifies the theoretical formulæ for \(\Delta M_{B_d}, \Delta M_{B_s}, \Re \epsilon\) and \(\sin 2\beta_{\text{eff}}\). In order to derive the bounds, we utilize p-values in the CKM fitting for flavor-violating observables. The analysis is performed in the following way: We carry out \(\chi^2\) fittings with fixed \(m_H\). For each value of \(m_H\), p-values are given to specify a disfavored range of Higgs mass. Given the stringent experimental constraint on \(\Re \epsilon\), the FCNH must be sufficiently heavy. Thus, \(m_H \geq 20\) TeV is considered in the fittings.

For completeness, we show the Wolfenstein parametrization of the CKM matrix [25],

\[
V = \begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix},
\]  

(60)

where terms of \(O(\lambda^4)\) are ignored. In Eq. (60), \((\rho, \eta)\) are redefined in terms of phase convention independent parameters, \((\bar{\rho}, \bar{\eta})\) [26],

\[
\rho + i\eta = \frac{(\bar{\rho} + i\bar{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1 - \lambda^2[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]}}.
\]  

(61)
In addition, angles which constitute the unitarity triangle are defined as,

\[ \alpha = \arg \left( -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right) , \tag{62} \]

\[ \gamma = \arg \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right) , \tag{63} \]

where the other angle, \( \beta \), is given in Eq. (57). We note that \((\alpha, \beta, \gamma)\) can be written in terms of the Wolfenstein parameters, \((\lambda, A, \bar{\rho}, \bar{\eta})\).

In carrying out the analysis, the following facts are considered:

- Measurements of \(|V_{ud}|, |V_{us}|, |V_{ub}|, |V_{cd}|, |V_{cs}| \) and \(|V_{cb}|\) are unchanged in the presence of FCNH. This is because these absolute values are determined through semi-leptonic decays, while the model predicts minor correction for semi-leptonic processes.

- Angle \( \alpha \), which is measured in time-dependent processes for \( B \to \pi \pi, B \to \pi \rho \) and \( B \to \rho \rho \) decays, is altered by FCNH, since these observables are sensitive to \( B_d^0 - \bar{B}_d^0 \) mixing. However, as stated in the previous subsection, the FCNH contribution to \( B_d^0 - \bar{B}_d^0 \) mixing does not affect \( q/p \) for \( m_H \geq 12 \) TeV so that we decouple this effect.

- Measurement of angle \( \gamma \) is carried out in \( B^\pm \to DK^\pm \) and \( B^0 \to D^{(*)\pm} \pi^\mp \) decays. These tree-level processes are not significantly modified by the FCNH exchange.

In the fitting, we include uncertainties of the Wolfenstein parameters \(^{12}\), the bag parameters \(^{18, 19}\) and NLO QCD correction factors for the \( \Delta S = 2 \) process denoted by \( \eta_1, \eta_2 \) and \( \eta_3 \) \(^{15}\). As for the \( \Delta B = 2 \) processes, uncertainty in short-distance QCD correction factors is not considered because they are more precisely determined than those for the \( \Delta S = 2 \) process. We use the central value of this QCD correction calculated through the method of Ref. \(^{13}\).

In the CKM fitting, a statistic,

\[ (\lambda^2)_{\text{fixed } m_H} = \sum_{i,j} \left( \frac{|V_{ij}|_{\text{Th}} - |V_{ij}|_{\text{Exp}}}{\sigma_{|V_{ij}|}_{\text{Exp}}} \right)^2 + \sum_{k=\alpha, \gamma, \sin 2\beta_{\text{eff}}} \frac{(k_{\text{Th}} - k_{\text{Exp}})^2}{(\sigma_k)^2_{\text{Exp}}} \]

\[ \qquad + \sum_{l=d,s} \frac{(\Delta M_{Bl,\text{Th}} - \Delta M_{Bl,\text{Exp}})^2}{(\sigma_{\Delta M_{Bl}})^2_{\text{Th}} + (\sigma_{\Delta M_{Bl}})^2_{\text{Exp}}} + \frac{(\text{Re} \epsilon_{\text{Th}} - \text{Re} \epsilon_{\text{Exp}})^2}{(\sigma_{\text{Re} \epsilon})^2_{\text{Th}} + (\sigma_{\text{Re} \epsilon})^2_{\text{Exp}}} , \tag{64} \]

is minimized with \((i, j) = (u, d), (u, s), (c, d), (c, s), (c, b), (u, b)\). Experimental data on the
absolute values of the CKM matrix elements and $\alpha, \gamma$ are provided by PDG as,

$$\begin{align*}
|V_{ud}|_{\text{Exp}} &= 0.97417 \pm 0.00021, \\
|V_{us}|_{\text{Exp}} &= 0.2248 \pm 0.0006, \\
|V_{cd}|_{\text{Exp}} &= 0.220 \pm 0.005, \\
|V_{cs}|_{\text{Exp}} &= 0.995 \pm 0.016, \\
|V_{cb}|_{\text{Exp}} &= (40.5 \pm 1.5) \times 10^{-3}, \\
|V_{ub}|_{\text{Exp}} &= (4.09 \pm 0.39) \times 10^{-3}, \\
\alpha_{\text{Exp}} &= (87.6 \pm 3.4)^\circ, \\
\gamma_{\text{Exp}} &= (73.2 \pm 6.7)^\circ,
\end{align*}$$

where we have taken averages of errors for $\alpha_{\text{Exp}}$ and $\gamma_{\text{Exp}}$, which are originally given as asymmetric forms. The data of $(\sin 2\beta_{\text{eff}})_{\text{Exp}}, (\text{Re } \epsilon)_{\text{Exp}}, (\Delta M_{B_d})_{\text{Exp}}, (\Delta M_{B_s})_{\text{Exp}}$ are extracted from Table II. In the r.h.s of Eq. (64), note that Higgs mass is fixed and the Wolfenstein parameters are adjustable to minimize $(\chi^2)_{\text{fixed } m_H}$. Furthermore, in Eq. (64), the theoretical errors for the bag parameters in $\Delta M_{B_d}, \Delta M_{B_s}$ are added to experimental ones in quadrature while for $\text{Re } \epsilon$, the errors of perturbative QCD factors ($\eta_1, \eta_2, \eta_3$) are also accounted. Note that our fitting analysis is different from the one performed by the CKMfitter group [12], which is based on $R$fit [27], another frequentist approach to include theoretical uncertainty.

In Figure V.1, p-values obtained for $20 \text{ TeV} \leq m_H \leq 350 \text{ TeV}$ are presented. One observes a difference between $s_u s_c = +1$ and $s_u s_c = -1$. From Figure V.1, we derive lower bounds on the FCNH mass in which p-values are disfavored by $3\sigma$ and $5\sigma$. For the two cases of sign choice, the bounds are

$$\begin{align*}
\text{For } (s_u, s_c, s_t) &= (+, +, +), \quad m_H > 84 \text{ TeV (}3\sigma\), \quad m_H > 64 \text{ TeV (}5\sigma\), \quad (65) \\
\text{For } (s_u, s_c, s_t) &= (+, -, +), \quad m_H > 75 \text{ TeV (}3\sigma\), \quad m_H > 56 \text{ TeV (}5\sigma\). \quad (66)
\end{align*}$$

V. PREDICTION FOR $\text{Re } \epsilon, \Delta M_{B_d}$ AND $\Delta M_{B_s}$

In this section, we present the prediction for observables to illustrate the pattern of deviation in the model. For this purpose, the Wolfenstein parameters are estimated from
observables which are not affected by FCNH. Hence, we consider the following statistic,

$$
\chi^2 = \sum_{i,j} \frac{(|V_{ij}|_{\text{Th}} - |V_{ij}|_{\text{Exp}})^2}{(\sigma_{|V_{ij}|})^2_{\text{Exp}}} + \frac{(\gamma_{\text{Th}} - \gamma_{\text{Exp}})^2}{(\sigma_{\gamma})^2_{\text{Exp}}}.
$$

Through the minimization of Eq. (67), one can extract ($\lambda, A, \bar{\rho}, \bar{\eta}$).

The Belle II experiment announces that the expected integrated luminosity is 50 ab$^{-1}$ in five years of running. Motivated by this, we consider Case I and Case II described below.

- Case I:

  The errors of $|V_{ub}|$ and $\gamma$ are reduced by 1/3 and 1/7, respectively, without changing their central values. The errors and central values of the other quantities, including $|V_{cb}|$, remain the same. Under this circumstance, the parameters are estimated as

  $$
  \lambda = 0.22547 \pm 0.00050, \quad A = 0.797 \pm 0.030, \\
  \bar{\rho} = 0.1262 \pm 0.093, \quad \bar{\eta} = 0.418 \pm 0.021,
  $$

  (68)
where a correlation matrix for \((\lambda, A, \bar{\rho}, \bar{\eta})\) is
\[
\begin{pmatrix}
1 & -0.12 & -0.032 & -0.048 \\
* & 1 & -0.50 & -0.74 \\
* & * & 1 & 0.59 \\
* & * & * & 1
\end{pmatrix},
\] (69)

On the basis of the Wolfenstein parameters in Eq. (70), the prediction for \((\text{Re} \epsilon, \Delta M_{B_d}, \Delta M_{B_s})\) is given in Figure V.1. We also present the result for the case in which errors of bag parameters and decay constants are three times as small as the ones calculated by the ETM collaboration.

- Case II:
  Case II is an ideal situation in which the errors of \(|V_{cb}|, |V_{ub}|\) and \(\gamma\) are reduced by 1/7 (the other quantities remain the same). In this case, the Wolfenstein parameters and their correlation matrix are,

\[
\lambda = 0.22547 \pm 0.00050, \quad A = 0.7967 \pm 0.0055,
\]
\[
\bar{\rho} = 0.1262 \pm 0.071, \quad \bar{\eta} = 0.4180 \pm 0.0065,
\] (70)
\[
\begin{pmatrix}
1 & -0.65 & -0.042 & -0.15 \\
* & 1 & -0.044 & -0.16 \\
* & * & 1 & -0.063 \\
* & * & * & 1
\end{pmatrix}.
\] (71)

For Case II, the prediction for \((\text{Re} \epsilon, \Delta M_{B_d}, \Delta M_{B_s})\) is presented in Figure V.2.

With future precision of input data and reduction of theoretical uncertainty as in Case II, \(\text{Re} \epsilon, \Delta M_{B_d}\) and \(\Delta M_{B_s}\) computed in the SM may deviate from the experimental values. Under this circumstance, it can be the case that theoretical calculations including the FCNH contributions are consistent with the experimental values. For example, suppose that the input parameters for \(\text{Re} \epsilon\) are precisely determined without changing their central values. A region of interest is then \(m_H \sim 100\) TeV and \(s_c s_t = -1\), because the central value of \(\text{Re} \epsilon\) is consistent with the experimental value in Figure V.1. If the model prediction for \(\Delta M_{B_d}\) and \(\Delta M_{B_s}\) should converge to the current central values, then, for \(m_H = 100\) TeV
and \( s_c s_t = -1 \), the prediction and the experimental value of \( \Delta M_{B_d} \) would be separated by more than \( 2\sigma \) whereas the prediction for \( \Delta M_{B_s} \) would be consistent with the experiment. However, it is possible that theoretical calculations of \( \Delta M_{B_d} \) and \( \Delta M_{B_s} \) converge to different values, independently of each other and of \( \text{Re} \, \epsilon \). If the theoretical inputs for \( B_d^0 - \bar{B}_d^0 \) and \( B_s^0 - \bar{B}_s^0 \) are determined as

\[
f_{B_d} \sqrt{B_1^{dVLL}(m_b)} = 158.5 \, \text{MeV}, \quad f_{B_s} \sqrt{B_1^{sVLL}(m_b)} = 211.9 \, \text{MeV},
\]

\( \Delta M_{B_d} \) and \( \Delta M_{B_s} \) will be in agreement with the experiment. In this case, the deviation between the measurements and calculations of \( \text{Re} \, \epsilon, \Delta M_{B_d} \) and \( \Delta M_{B_s} \) hints at the semi-aligned 2HDM.
FIG. V.1. Model predictions for $\text{Re} \, \epsilon$ and $\Delta M_{B_d}$, $\Delta M_{B_s}$ in Case I. In these plots, the Wolfenstein parameters used are estimated through observables which are not affected by FCNH. Red (orange) bands represent the model predictions in $1\sigma$ ($2\sigma$) CL. Yellow bands stand for the model predictions in which the errors of decay constant or bag parameters are three times as small as the one given by the ETM collaboration. For comparison, the experimental data of PDG and HFLAV are shown.
FIG. V.2. Same figure as Fig. V.1 in Case II.
In the following, we comment on other flavor-violating observables. Since a charged scalar exchange alters $b \to cl^- \nu$ decay rate at tree level, this model might be able to address the anomaly in $R_{D^{(*)}}$. However, this is not the case because the absolute value of the $bcH^-$ coupling is small up to $\sqrt{2}|V_{cb}m_b|/v \sim O(10^{-3})$, which does not exceed the SM $bcW^-$ coupling, $|V_{cb}| \sim O(10^{-2})$.

We examine the correction to $b \to ss\bar{d}$ decay. In the SM, this proceeds via the box diagram and is highly suppressed by the Glashow-Iliopoulos-Maiani mechanism as $\text{Br}^{\text{SM}}[b \to ss\bar{d}] = O(10^{-12})$ [28, 29]. For the corresponding exclusive mode, experimental searches have been performed [30] and recently, the LHCb collaboration has reported $\text{Br}[B^- \to K^-K^-\pi^+] < 1.1 \times 10^{-8}$ (90% CL) [31]. In Ref. [32], the FCNH contribution to the decay width is calculated in the Type-III 2HDM and is found to be

$$\Gamma_{\text{FCNH}}[b \to ss\bar{d}] = \frac{m_b^5}{3072(2\pi)^3} \sum_{i,j}^{u,c,t} \left| \lambda^{bd}_{i} \lambda^{sd}_{j} \left( \frac{m_D^{(D)}}{v^2} \right) \right|^2 \left[ 11 \left( \frac{1}{m_H^4} + \frac{1}{m_A^4} \right) + \frac{2}{m_H^2 m_A^2} \right].$$

Using the above formula, we verify that the correction to $b \to ss\bar{d}$ decay in the semi-aligned 2HDM is given by $\text{Br}^{\text{FCNH}}[b \to ss\bar{d}] = O(10^{-14} - 10^{-15})$ for $m_H = 10$ TeV, which is much smaller than the SM prediction.

VI. SUMMARY

We have focused on a situation where the left-right symmetric model with the left-right parity is probed only through quark flavor changing neutral currents mediated by the heavy neutral scalar particles (flavor changing neutral Higgses (FCNHs)) arising from the $SU(2)_L \times SU(2)_R$ bi-fundamental scalar. We have extracted the bi-fundamental scalar part of the left-right model, named it “semi-aligned two Higgs doublet model”, and investigated its phenomenology by focusing on various $\Delta F = 2$ processes.

First, we have derived the current lower bound on the mass of FCNH, by calculating $\Delta M_{B_d}$, $\Delta M_{B_s}$, $\text{Re} \varphi$ and $\sin 2\beta_{\text{eff}}$ with the inclusion of the FCNH exchange contribution, and fitting them and other observables with the CKM matrix. The tension in the fitting, represented by p-values, has given the bounds on the FCNH mass, which read: $m_H > 75 - 84$ TeV for $3\sigma$ and $m_H > 56 - 64$ TeV for $5\sigma$.

Secondly, we have made a prediction for $(\text{Re} \varphi, \Delta M_{B_d}, \Delta M_{B_s})$ in terms of only one free parameter, i.e. the FCNH mass, under the assumption that uncertainties in the measurement
of $|V_{ub}|$ and $\gamma$ are reduced from the current estimates, allowing us to determine the CKM matrix without being affected by the FCNH exchange. We have revealed that if the precision of $|V_{ub}|$ and $\gamma$ is improved and the SM prediction disagrees with the experimental data in such a way that the inclusion of FCNH contributions (for some unique value of the FCNH mass) is mandatory to fit the data, it hints at the semi-aligned two Higgs doublet model and the left-right model.

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APPENDIX: LEFT-RIGHT SYMMETRIC MODEL WITH LEFT-RIGHT PARITY

We present the left-right symmetric model [1] with the left-right parity. The gauge symmetry is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, and the full field content is shown in Table III. The model is invariant under the left-right parity transformation, in which $SU(2)_L$ and $SU(2)_R$ gauge groups are interchanged and the fields transform as

$$
\Phi \leftrightarrow \Phi^\dagger, \quad \Delta_L \leftrightarrow \Delta_R, \quad q_L \leftrightarrow q_R, \quad \ell_L \leftrightarrow \ell_R.
$$

(74)

We write the bi-fundamental scalar $\Phi$ and triplet scalars $\Delta_L, \Delta_R$ as $2 \times 2$ matrices that transform under a $SU(2)_L \times SU(2)_R$ gauge transformation as

$$
\Phi \rightarrow e^{i\tau^a \theta^a_L} \Phi e^{-i\tau^a \theta^a_R}, \quad \Delta_L \rightarrow e^{i\tau^a \theta^a_L} \Delta_L e^{-i\tau^a \theta^a_L}, \quad \Delta_R \rightarrow e^{i\tau^a \theta^a_R} \Delta_R e^{-i\tau^a \theta^a_R},
$$

(75)

$\theta^a_L, \theta^a_R$: gauge parameters, \hspace{1cm} $\tau^a \equiv \sigma^a/2$.

The Yukawa couplings and the scalar potential of the left-right symmetric model with
TABLE III. Full field content of the left-right symmetric model. $i = 1, 2, 3$ is the flavor index.

| Field | Lorentz $SO(1,3)$ | $SU(3)_{C}$ | $SU(2)_{L} \times SU(2)_{R}$ | $U(1)_{B-L}$ |
|-------|-------------------|-------------|-----------------------------|--------------|
| $q^i_L$ | $(2, 1)$ | $3$ | $(2, 1)$ | $1/3$ |
| $q^i_R$ | $(1, 2)$ | $3$ | $(1, 2)$ | $1/3$ |
| $\ell^i_L$ | $(2, 1)$ | $1$ | $(2, 1)$ | $-1$ |
| $\ell^i_R$ | $(1, 2)$ | $1$ | $(1, 2)$ | $-1$ |
| $\Phi$ | $1$ | $1$ | $(2, 2)$ | $0$ |
| $\Delta_L$ | $1$ | $1$ | $(3, 1)$ | $2$ |
| $\Delta_R$ | $1$ | $1$ | $(1, 3)$ | $2$ |

the left-right parity are found to be

$$- \mathcal{L} = (Y_q)_{ij} \bar{q}^i_L \Phi q^j_R + (\bar{Y}_q)_{ij} \bar{q}^i_L \bar{\Phi} q^j_R + (Y_\ell)_{ij} \bar{\ell}^i_L \Phi \ell^j_R + (\bar{Y}_\ell)_{ij} \bar{\ell}^i_L \bar{\Phi} \ell^j_R + \text{H.c.}$$

$$+ (Y_M)_{ij} \left( \ell^i_L \bar{\epsilon} \Delta_L \ell^j_R + \ell^i_R \bar{\epsilon} \Delta_R \ell^j_R \right) + \text{H.c.}$$

$$+ \mu_1^2 \text{tr} \left[ \Phi^\dagger \Phi \right] + \mu_2^2 \text{tr} \left[ \Phi^\dagger \bar{\Phi} + \bar{\Phi} \Phi \dagger \right] + \mu_3^2 \text{tr} \left[ \Delta^\dagger_L \Delta_L + \Delta^\dagger_R \Delta_R \right]$$

$$+ \lambda_1 \text{tr} \left[ \Phi^\dagger \Phi \right]^2 + \lambda_2 \left( \text{tr} \left[ \Phi^\dagger \bar{\Phi} \right]^2 + \text{tr} \left[ \bar{\Phi} \Phi \dagger \right]^2 \right) + \lambda_3 \text{tr} \left[ \Phi^\dagger \bar{\Phi} \right] \text{tr} \left[ \bar{\Phi} \Phi \dagger \right]$$

$$+ \lambda_4 \text{tr} \left[ \Phi^\dagger \Phi \right] \text{tr} \left[ \Phi^\dagger \Phi + \Phi \Phi \dagger \right]$$

$$+ \rho_1 \left( \text{tr} \left[ \Delta^\dagger_L \Delta_L \right]^2 + \text{tr} \left[ \Delta^\dagger_R \Delta_R \right]^2 \right) + \rho_2 \left( \text{tr} \left[ \Delta_L \Delta_L \right] \text{tr} \left[ \Delta^\dagger_L \Delta^\dagger_L \right] + \text{tr} \left[ \Delta_R \Delta_R \right] \text{tr} \left[ \Delta^\dagger_R \Delta^\dagger_R \right] \right)$$

$$+ \rho_3 \text{tr} \left[ \Delta^\dagger_L \Delta_L \right] \text{tr} \left[ \Delta^\dagger_R \Delta_R \right] + \rho_4 \left( \text{tr} \left[ \Delta_L \Delta_L \right] \text{tr} \left[ \Delta^\dagger_R \Delta^\dagger_R \right] + \text{tr} \left[ \Delta_R \Delta_R \right] \text{tr} \left[ \Delta^\dagger_L \Delta^\dagger_L \right] \right)$$

$$+ \alpha_1 \text{tr} \left[ \Phi^\dagger \Phi \right] \text{tr} \left[ \Delta^\dagger_L \Delta_L + \Delta^\dagger_R \Delta_R \right]$$

$$+ \alpha_{2R} \left( \text{tr} \left[ \Phi^\dagger \Phi \right] \text{tr} \left[ \Delta^\dagger_L \Delta_L \right] + \text{tr} \left[ \Phi \Phi^\dagger \right] \text{tr} \left[ \Delta^\dagger_R \Delta_R \right] + \text{tr} \left[ \Phi^\dagger \Phi \right] \text{tr} \left[ \Delta^\dagger_R \Delta_R \right] + \text{tr} \left[ \Phi^\dagger \Phi \right] \text{tr} \left[ \Delta^\dagger_L \Delta_L \right] \right)$$

$$+ i \alpha_{2L} \left( \text{tr} \left[ \Phi^\dagger \Phi \right] \text{tr} \left[ \Delta^\dagger_L \Delta_L \right] + \text{tr} \left[ \Phi \Phi^\dagger \right] \text{tr} \left[ \Delta^\dagger_R \Delta_R \right] - \text{tr} \left[ \Phi^\dagger \Phi \right] \text{tr} \left[ \Delta^\dagger_L \Delta_L \right] - \text{tr} \left[ \Phi^\dagger \Phi \right] \text{tr} \left[ \Delta^\dagger_R \Delta_R \right] \right)$$

$$+ \alpha_3 \left[ \Phi \Phi^\dagger \Delta^\dagger_L \Delta_L + \Phi^\dagger \Phi \Delta^\dagger_R \Delta_R \right]$$

$$+ \beta_1 \left[ \Phi \Delta_R \Phi^\dagger \Delta^\dagger_L + \Phi^\dagger \Delta_L \Phi \Delta^\dagger_R \right] + \beta_2 \left[ \Phi \Delta_R \Phi^\dagger \Delta^\dagger_L + \Phi^\dagger \Delta_L \Phi \Delta^\dagger_R \right] + \beta_3 \left[ \Phi \Delta_R \Phi^\dagger \Delta^\dagger_L + \Phi^\dagger \Delta_L \Phi \Delta^\dagger_R \right]$$

(78)

with $\bar{\Phi} \equiv i \sigma_2 \Phi^* i \sigma_2$.

where $Y_q, \bar{Y}_q, Y_\ell, \bar{Y}_\ell$ are Hermitian matrices, and $Y_M$ is a complex-valued symmetric matrix.
The mass terms $\mu_1^2, \mu_2^2, \mu_3^2$ and the coupling constants $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \rho_1, \rho_2, \rho_3, \rho_4, \alpha_1, \alpha_2, \tilde{\alpha}_2, \alpha_3, \beta_1, \beta_2, \beta_3$ are all real.

Through a $SU(2)_R \times U(1)_{B-L}$ symmetry transformation, one can set the VEV of $\Delta_R$ in the following form:

$$\langle \Delta_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \quad v_R > 0.$$  \hspace{1cm} (79)

Through a subsequent $SU(2)_L$ and $\sigma_3$ part of $SU(2)_R$ symmetry transformation, one can set

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & -e^{i\alpha}v_2 \end{pmatrix}, \quad v_1 > 0, \quad v_2 > 0,$$  \hspace{1cm} (80)

which gives rise to a phase for the $\Delta_R$ VEV, but this can be negated by a $U(1)_{B-L}$ symmetry transformation.

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