Δ resonances and charged ρ mesons in neutron stars

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Abstract. We study the equation of state of cold and dense baryon matter within the relativistic mean-field framework with hadron masses and coupling constants dependent on the mean scalar field. We include Δ(1232) isobars into previously developed models with hyperons and consider the possibility of charged ρ-meson condensation. The Δ-isobars, being included with realistic values of the attractive in-medium potential, do not lead to a strong decrease of the maximum predicted neutron star (NS) mass, and our models thus resolve the “Δ-resonance puzzle”. The charged ρ−-meson condensation leads to a substantial maximum NS mass decrease. However, the observational constraint on the minimal value of the maximum NS mass remains fulfilled in all our models under consideration. The decrease of NS mass can be lowered, if one assumes a limited decrease of the ρ-meson effective mass at high densities.

1. Introduction

The description of NS properties requires the knowledge of the equation of state (EoS) of cold hadronic matter, which can be conveniently constructed within the relativistic mean-field (RMF) framework. A realistic phenomenological EoS has to fulfill a large set of experimental constraints [1]. The most challenging problem is to pass the constraint on the pressure in the isospin-symmetrical matter (ISM) (so-called “flow constraint” [2]), which favors a rather soft EoS, and the constraint that the maximum NS mass should be above the measured mass $M = 2.01 ± 0.04 M_\odot$ [3] ($M_\odot$ is the solar mass) of the pulsar PSR J0348+0432, that favors a stiff EoS in beta equilibrium matter (BEM). In standard RMF models hyperons and Δ-isobars may appear in NS cores, which results in a decrease of the maximum NS mass below the observed limit, cf. [4]. As was shown in [5], the softening of an EoS of NS matter can also be the result of a ρ−-meson condensation. In [6, 7] we proposed a RMF model with scaled hadron masses and coupling constants (labeled there as MKVOR), which allows to solve the hyperon puzzle and fulfills the maximum mass constraint, the flow constraint and various other constraints, but the Δs and meson condensates were not included. In [8] we included Δ into consideration. Here we show the results of the MKVOR-based model with hyperons together with the inclusion of Δs and the condensation of ρ− mesons.

2. The RMF model with scaled hadron masses and couplings

The framework we use [9] is a generalization of the non-linear Walecka model, in which the effective coupling constants $g_{mb}^* = g_{mb}(\sigma)$ and hadron masses $m_i^* = m_i(\sigma)$ depend on the scalar field $\sigma$ through the scaling functions $\chi_{mb}(\sigma)$ and $\Phi_i(\sigma)$. Here $m = \{\sigma, \omega, \rho, \phi\}$ denotes
mesons, \( b = (N, H, \Delta) \) lists baryon species, where \( N \) denotes nucleons \( \{ p, n \} \), \( H \) stands for hyperons \( \{ \Lambda, \Sigma, \Xi \} \) and \( \Delta \) denotes \( \Delta \)-isobars, and index \( i \) runs through all hadrons \( (m, b) \). The expression for the energy density is obtained in the standard mean-field approximation, and disregarding the possibility of a charged \( \rho \)-meson condensation it renders as:

\[
E[\{n_b\}, \{m_i\}, f] = \sum_b E_{\text{kin}}(p_{F,b}, m_b \Phi_b(f), s_b) + \sum_{i=e,\mu} E_{\text{kin}}(p_{F,i}, m_i, s_i) + \frac{m_N^4 f^2}{2 C_\sigma^2} \eta_\sigma(f) + \frac{1}{2 m_N^2} \left[ \frac{C_\omega^2 n_V^2}{\eta_\omega(f)} + \frac{C_\rho^2 n_I^2}{\eta_\rho(f)} + \frac{C_\sigma^2 n_S^2}{\eta_\sigma(f)} \right],
\]

\[
E_{\text{kin}}(p_{F}, m, s) = (2s + 1) \int_0^{p_{F}} \frac{p^2 dp}{2 \pi^2} \sqrt{p^2 + m^2},
\]

where \( f = g_N \chi_N \sigma(m, \sigma) / m_N \) is the dimensionless scalar field variable, and we define the coupling constant ratios \( x_{mb} = g_{mb} / g_{mN} \). The isospin projection of baryon \( b \) is \( t_{3b} \), and \( p_{F,j} = (6 \pi^2 n_j / (2 s_j + 1))^{1/3} \) is the Fermi momentum of a fermion \( j \), where \( s_j \) is the fermion spin and \( n_j \) denotes the density of a species \( j, j = (b, l), l \) labels leptons. If the finite size effects are negligible (infinite matter), the baryon coupling constants, masses and scaling functions enter the energy density only in combinations \( C_M = g_{3BN} n_N / m_M, M = \sigma, \rho, \omega, C_\phi = g_{\omega N} m_N / m_\phi \).

Here the scaling function \( \eta_\sigma(f) \) is expressed via the self-interaction potential \( U(\sigma) \) entering the Lagrangian of the model. Explicit expressions for the scaling functions \( \eta_m(f) \) and values of the parameters are given in [6, 7, 8]. The version of the MKVOR model suitable for the inclusion \( \Delta_s \) was labeled in [8] as MKVOR*. We use \( \chi_{MB} = \chi_{MN}, \chi_{\phi H} = 1 \).

The vector-meson coupling ratios with baryons \( x_{\omega B} \) and \( x_{\rho B} \) are chosen following the quark SU(6) symmetry. The baryon coupling ratios with the scalar field \( x_{\sigma B} \) are deduced from the potentials \( U_B(n_0) = C_\omega^2 m_N^{-2} x_{\omega B} n_0 \eta_\omega(f(n_0)) - x_{\sigma B} (m_N + m_{\Sigma} n_0) \) in ISM at the saturation baryon density \( n = n_0 \). There are large uncertainties in fixing of the \( \Delta \) potential \( U_\Delta(n_0) \equiv U_\Delta \) in the literature. We use \( U_\Delta = -50 \text{MeV} \), which results from calculations [10]. The details on parameters of all included baryon species can be found in [8].

### 3. Inclusion of \( \rho^- \) condensation

The \( \rho^- \) contribution to the Lagrangian is [5, 9]

\[
\mathcal{L}_\rho = -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2} m_\rho^2 \Phi_\rho^2 \tilde{\rho}^\mu \tilde{\rho}_\mu - \sum_b g_{\rho b} \chi_{\rho b} \tilde{\rho}_{j_{1,b}}, \quad \tilde{\rho}_j = \tilde{\rho}_j = \tilde{\rho}_j, \quad \tilde{F}_{\mu\nu} = \tilde{\rho}_\mu \tilde{\rho}_\nu - \tilde{\rho}_\mu \tilde{\rho}_\nu + g_\rho^\kappa \chi_\rho(f)[\tilde{\rho}_\mu \times \tilde{\rho}_\nu] + \mu_{\chi\rho} \delta_{\nu 0} [\tilde{\rho}_3 \times \tilde{\rho}_\mu] - \mu_{\chi\rho} \delta_{\mu 0} [\tilde{\rho}_3 \times \tilde{\rho}_\nu],
\]

where \( \tilde{\rho}_3^{a} = \delta^{a3} \) is the unit vector in the isospin space, \( \mu_{\chi\rho} \) is the chemical potential for charged mesons, and \( g_\rho^\kappa \) and \( \chi_\rho(f) \) are the non-Abelian coupling constant and its scaling function, respectively. From the hidden local symmetry arguments it follows that \( g_\rho^\kappa = g_\rho m_N \), which we adopt here, and for simplicity we consider \( \chi_{\rho}(f) = 1 \). The new ansatz for the \( \rho \) meson field includes non-zero \( \rho_0^{(3)} \) and \( \rho_3^{(i)} = \left( \rho_1^{(1)} + \rho_2^{(2)} \right) / \sqrt{2} \), \( i = 1, 2, 3 \) components. It can be shown that the condition \( \rho_1^{(+)} \rho_2^{(-)} - \rho_2^{(+) \rho_1^{(-)}} = 0 \) is required for minimization of the energy. This implies that the ratio \( \rho_1^{(+)} / \rho_1^{(-)} \) is constant and independent of the spatial index \( i \). Thus we assume \( \rho_1^{(-)} = a_i \rho_c \) and \( \rho_1^{(+)} = a_i \rho_c^\dagger \), where \( a_i = \{ a_i \} \) is the spatial unit vector, and
\( \rho_c \) is a complex amplitude of the charged \( \rho \) meson field. With this ansatz the equations for minimization of the thermodynamic potential for the \( \rho_0^{(3)} \) and \( \rho_c \) fields have two solutions. One of them is the traditional one with only \( \rho_0^{(3)} \) being non-zero. The second solution is

\[
\rho_0^{(3)} = \frac{\mu_{\rho_{0}^{(3)}} - m_{\rho} \Phi_{\rho}}{g_{\rho} n_{\rho}^{0}}, \quad |\rho_c|^2 = \frac{(-n_{I} - n_{p})(-n_{I} - n_{p})}{2 m_{\rho} n_{\rho}^{0} x_{\rho}}, \quad \text{where } n_{\rho} = a (m_{\rho} \Phi_{\rho} - \mu_{\text{ch}, \rho}), \quad a = \frac{m_{\rho}^{2} n_{\rho}^{0} x_{\rho}^{1/2}}{\epsilon_{\rho}^{0} x_{\rho}^{0}}.
\]

The \( \rho \) charged density is \( n_{\text{ch}, \rho} = -2m_{\rho} \Phi_{\rho}(f)/|\rho_c|^2 < 0 \). The contribution of the charged \( \rho^- \) meson condensate to the energy density is then given by

\[
\Delta E_{\text{ch}, \rho}(n_{b}; f) = -\frac{C_{\rho}^{2}}{2 m_{\rho}^{2} n_{\rho}^{0}}(n_{I} + n_{\rho})^{2} \theta(-n_{I} - n_{\rho}) - \mu_{\text{ch}, \rho} n_{\text{ch}, \rho},
\]

where \( \theta(-n_{I} - n_{\rho}) = 1 \) for \( n_{I} + n_{\rho} < 0 \) and zero otherwise. The charge neutrality condition is \( \sum_b Q_{b} n_{b} = n_{e} - n_{\mu} + n_{\text{ch}, \rho} = 0 \) and the relations between chemical potentials in BEM are \( \mu_{e} = \mu_{\mu} = \mu_{\text{ch}, \rho}; \mu_{b} = \mu_{n} - Q_{b} \mu_{l} \). All equations are solved self-consistently with the equation of motion for the scalar field \( \partial E/\partial f = 0 \). The pressure is given by \( P = \sum_{j=b,l}(\mu_{j} n_{j} - E) \).

4. Numerical results

In Fig. 1 we show the results of the inclusion of \( \Delta s \) into our model (MKVOR) with hyperons. In the BEM the appearance of \( \Delta^- \) becomes favorable at \( n \approx 2.5 n_{0}, 1.73 n_{0} \) for \( U_{\Delta} = -50 \text{MeV}, -100 \text{MeV}, \) correspondingly. We find that despite the considerable amount of \( \Delta s \) in NS matter at large densities, the reduction of the NS mass does not exceed 0.2 \( M_{\odot} \), and the decrease of the maximum NS mass is smaller than 0.05 \( M_{\odot} \). The mass-radius relation is also affected moderately. The radius of a 1.5 \( M_{\odot} \) star decreases by 0.5 km if \( \Delta s \) are included.

We study the \( \rho^- \) condensation for various \( \rho^- \) meson effective mass scaling functions \( \Phi_{\rho}(f) \), shown in the left panel of Fig. 2. They are labeled by the minimum values \( \Phi_{\rho, \text{min}} \). Case \( \Phi_{\rho, \text{min}} = 0 \) corresponds to universal scaling, \( \Phi_{m}(f) = \Phi_{N}(f) = 1 - f \). For this case the equilibrium concentrations are shown in the middle panel of Fig. 2. We use \( U_{\Delta}(n_{0}) = -50 \text{MeV} \). There exists a region with multiple solutions for equilibrium concentrations at a given density, so the \( \rho^- \) condensation in our model appears by a 1st order phase transition (PT). Such an early and strong PT results in a substantial decrease of the NS mass down to 2.03 \( M_{\odot} \) for \( \Phi_{\rho, \text{min}} = 0 \), but the maximum NS mass constraint is still satisfied within the experimental error-bars.

If one assumes that \( \Phi_{\rho}(f) \) has the same decrease rate as \( \Phi_{N} \) at low scalar field values (corresponding to low densities), but saturates then at some value \( \Phi_{\rho, \text{min}} \) (see curves \( \Phi_{\rho, \text{min}} = 0.3, 0.5, 0.7 \)), the effect of the \( \rho^- \) condensation is diminished. The critical density for the \( \rho^- \)
condensation increases, and the density jump in the Maxwell construction decreases. This leads to an increase of the NS maximum mass up to 2.06, 2.16 and 2.21 $M_\odot$ (Fig. 2, right panel) for $\Phi_{\rho,\text{min}} = 0.3, 0.5, 0.7$, respectively. For $\Phi_{\rho,\text{min}} = 0.7$ there is already no $\rho^-$ condensate in NSs.

5. Conclusion
We studied the possibility of appearance of $\Delta$ isobars and $\rho^-$-meson condensate within a realistic RMF model with scaled hadron masses and coupling constants. We have shown that the $\Delta$s are energetically favorable to appear in NS matter, but do not lead to a substantial decrease of the maximum predicted NS mass, if one uses a realistic value for the $\Delta$ potential at saturation in ISM. Thus our model resolves the “$\Delta$-resonance puzzle”. The $\rho^-$ condensation in our model appears by the I$^\text{st}$ order PT and leads to a substantial decrease of the maximum NS mass, but it remains above the observational limit. The decrease can be lowered, if one assumes a lower rate of decrease of the $\rho$-meson effective mass than that of the nucleon at large densities.

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References
[1] Klähn T et al. 2006 Phys. Rev. C 74 035802.
[2] Danielewicz P, Lacey R and Lynch W G 2002 Science 298 1592.
[3] Antoniadis J et al. 2013 Science 340 6131.
[4] Drago A et al. 2014 Phys. Rev. C 90 065809.
[5] Voskresensky D N 1997 Phys. Lett. B 392 262.
[6] Maslov K A, Kolomeitsev E E and Voskresensky D N 2015 Phys. Lett. B 748 369.
[7] Maslov K A, Kolomeitsev E E and Voskresensky D N 2016 Nucl. Phys. A 950 64.
[8] Kolomeitsev E E, Maslov K A and Voskresensky D N 2017 Nucl. Phys. A 961 106.
[9] Kolomeitsev E E and Voskresensky D N 2005 Nucl. Phys. A 759 373.
[10] Riek F, Lutz M F M and Korpa C L 2009 Phys. Rev. C 80 024902.