Simulation of the action of a three-dimensional nonlinear spiral phase plate in the near diffraction zone

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Abstract. The report investigates the action of a nonlinear spiral phase plate in the near diffraction zone, taking into account the three-dimensional structure of the optical element. Simulation of the diffraction of a Gaussian beam with linear polarization is performed on the basis of the finite difference in the time domain method. Numerical comparison of the action of linear and nonlinear phase plates showed a difference in their action at distancing from the optical element.

1. Introduction

A spiral phase plate [1] is an optical element with a complex transmission function \( \exp(im\varphi) \), where \( \varphi \) is the polar angle, and \( m \) is the topological charge of the formed vortex beam. In 1992, it was shown that optical vortex beams with a spiral wave front have an orbital angular momentum (OAM) \([2]\), and the fabricated spiral phase plate (SPP) was used as a “phase rotor filter” for the optical implementation of the radial Hilbert transformation \([3]\). Since then, SPPs have become the most common and popular device of forming optical vortex beams \([4]\).

There are two main options for implementing traditional SPP: 1) a "refractive" plate with a monotonous increase of phase from 0 to \( 2\pi m \) \([5]\), 2) a "diffractive" or sector plate divided into \( m \) sectors with a phase change from 0 to \( 2\pi \) in each of the sectors \([6]\). The first variant is called "refractive", since the height of the optical element increases with the growth of the topological charge \( m \). For example, in order to form a vortex beam with \( m = 50 \) and a wavelength of 532 nm, it is required to fabricate SPP from fused silica with a height difference of 58 \( \mu \)m. This complicates the manufacturing process of such an element \([7-10]\), however, the same element can be used for radiation with different wavelengths in order to vary the topological charge of the beam \([8]\), including with fractional values \([11]\), as well as pulsed radiation with a wide spectral range \([12-14]\).

The use of "diffractive" sector SPP does not require the formation of a high relief, but their production requires a greater transverse resolution \([15, 16]\). It is known that the beams formed by diffractive SPP strongly transform when the wavelength of the illuminating radiation changes, "falling apart" into \( m \) beams with a single topological charge \([17, 18]\). An unconventional SPP with a phase distribution with a nonlinear dependence, described as \( \exp(i m \varphi^2) \), was presented \([19]\). The vortex beams formed in this case have a spiral shape with an intensity and phase gradient. Subsequently, the propagation of such vortices in free space and during their focusing was studied \([20]\). Recently, the properties of
optical vortices formed by generalized SPPs have been considered [21]. The phase function of such optical elements is described by an arbitrary monotonic function. The unique structure of such vortex beams causes the energy flow directed in spirals, which can be used in the field of laser surface structuring [22-27]. In this case, it is important to know the distribution of the field that is formed in the near diffraction zone. The report investigates the effect of a "refractive" spiral phase plate in the near diffraction zone, taking into account the three-dimensional structure of the optical element. Simulation of the diffraction of a Gaussian beam with linear polarization is performed on the basis of the finite difference in the time domain method.

2. Results and discussion
The finite difference in the time domain (FDTD) method was used to model the action of an optical element taking into account the three-dimensional structure. As an optical element, both a linear and a nonlinear phase plate are considered. It is known that the height of the optical element must be consistent with the wavelength \( \lambda \) of the incident beam. Relations for calculating the height of the element \( h \) has the following form:

\[
h(x, y) = \psi(x, y) / k(n-1), \text{ where } \psi(x, y) \text{ – optical element phase, } k = 2\pi/\lambda \text{ – wave number, } n \text{ – refractive index of manufactured element material.}
\]

Consider a nonlinear SPP of the following type: 

\[
\tau(x, y) = \exp[i\psi(x, y)], \quad \psi(x, y) = \alpha(\tan^{-1}(y/x))^q,
\]

where \( \alpha \) is a real number, and \( q \) is a positive integer. For \( q=1 \) and integer \( \alpha \), \( \tau(x, y) \) corresponds to the classical linear SPP.

We calculate the action of a three-dimensional optical element (Table 1) with a phase defined by the function \( \psi(x, y) \).

**Table 1.** Modeling of SPP with parameters: \( \alpha=1, n=1.5, h=3 \, \mu\text{m}, R=5 \, \mu\text{m}, q=1 \) (a) \( q=2 \) (b) \( q=3 \) (c)

| Profile | q=1 | q=2 | q=3 |
|---------|-----|-----|-----|
| 3D      | ![Profile 3D q=1](image) | ![Profile 3D q=2](image) | ![Profile 3D q=3](image) |
| XY      | ![Profile XY q=1](image) | ![Profile XY q=2](image) | ![Profile XY q=3](image) |
| XZ      | ![Profile XZ q=1](image) | ![Profile XZ q=2](image) | ![Profile XZ q=3](image) |
| YZ      | ![Profile YZ q=1](image) | ![Profile YZ q=2](image) | ![Profile YZ q=3](image) |

To simulate the action of a spiral phase plate, it is necessary to direct a light beam at the optical element and calculate the intensity distribution formed in a certain area.

Let's set the input field as a Gaussian beam, which can be described by the formula: 

\[
G_\sigma(x, y) = \exp\left[-\frac{(x^2 + y^2)}{\sigma^2}\right], \text{ where } \sigma \text{ – the radius of the beam waist (Table 1).} \]
Table 2. Light Gauss beam. Parameters of beam: $\lambda = 1.5 \, \mu m$, $(x,y) \in [-5, 5] \, \mu m$.

| Transverse section XY | 3 | 4 | 5 | 6 | 7 |
|-----------------------|---|---|---|---|---|

Let us simulate the propagation of a Gaussian beam through an SPP in free space and fix the distribution in some $z$ plane. For clarity of the SPP action, we show the phase in the $z$ plane for different values of $\sigma$ and $q$ (Table 3).

Table 3. Modeling of SPP action. Parameters of Gaussian beam: $\lambda = 1.5 \, \mu m$. Parameters of SPP: $\alpha = 1$, $n = 1.5$, $h = 3 \, \mu m$, $R = 5 \, \mu m$. Parameters of environment: $(x,y) \in [-5, 5] \, \mu m$, $z \in [0,20] \, \mu m$, $n_c = 1$

| $q$ | $\sigma$ | 1 | 2 | 3 |
|-----|----------|---|---|---|
| 3   | 1        |   |   |   |
|      | 2        |   |   |   |
|      | 3        |   |   |   |

| 5   | 1        |   |   |   |
|      | 2        |   |   |   |
|      | 3        |   |   |   |

| 7   | 1        |   |   |   |
|      | 2        |   |   |   |
|      | 3        |   |   |   |

From table 3 we can see that the most obvious result was obtained for $\sigma = 3 \, \mu m$. The phase distribution calculated in this numerical experiment corresponds to the vortex structure of the beam and is consistent with the theoretical basis. We show in detail the longitudinal and transverse intensity distribution during the propagation of the beam in free space (Table 4).

Table 4. Modeling of SPP action. Parameters of Gaussian beam: $\lambda = 1.5 \, \mu m$, $\sigma = 3 \, \mu m$. Parameters of SPP: $\alpha = 1$, $n = 1.5$, $h = 3 \, \mu m$, $R = 5 \, \mu m$. Parameters of environment: $(x,y) \in [-5, 5] \, \mu m$, $z \in [0,20] \, \mu m$, $n_c = 1$

| $q$ | $z$ | Longitudinal section YZ | Transverse section XY | Longitudinal section YZ | Transverse section XY | Longitudinal section YZ | Transverse section XY |
|-----|-----|-------------------------|-----------------------|-------------------------|-----------------------|-------------------------|-----------------------|
|     | 3   |                         |                       |                         |                       |                         |                       |
|     | 10  |                         |                       |                         |                       |                         |                       |
|     | 20  |                         |                       |                         |                       |                         |                       |
It can be seen that at a very small distance, the intensity distribution is similar to a ring with a gap. When moving away from the element, a qualitative difference appears: with a linear plate, the cross-section shape approaches the ring, and with a non-linear one, it remains an arc. A study was also conducted of the SPP action for a linearly polarized laser beam (Table 5).

**Table 5.** Modeling of SPP action. Parameters of polarized Gaussian beam: $\lambda=1.5 \, \mu m$, $\sigma=3 \, \mu m$. Parameters of SPP: $\alpha=1$, $n=1.5$, $h=3 \, \mu m$, $R=5 \, \mu m$. Parameters of environment: $(x,y) \in [-5, 5] \, \mu m$, $z \in [0,20] \, \mu m$, $n=1$

| Type of polarization | X–polarization | Y–polarization |
|---------------------|----------------|----------------|
| Longitudinal section YZ | ![Image](image1.png) | ![Image](image2.png) |
| Transverse section XY ($z=15 \mu m$) | ![Image](image3.png) | ![Image](image4.png) |
| Transverse section XY ($z=10 \mu m$) | ![Image](image5.png) | ![Image](image6.png) |

Table 5 presents the numerical simulation results of the SPP action for X and Y linearly polarized light. It can be seen from the pictures that the SPP under illumination by a linearly polarized beam makes it possible to obtain a spiral intensity distribution in the near diffraction zone. Note that a similar distribution is formed in the near diffraction zone upon diffraction by a binary helical axicon [28] and a more complex structure, which is the refractive twisted axicon [29]. Moreover, at a distance $10 \lambda$ from the optical element, a compact light spot is formed on the optical axis, which is stored at a considerable distance. The length of the axial light segment increases with the degree of the SPP $q$. A similar diffraction-free distribution can be obtained using an annular diaphragm [30-31]. In addition, it is possible to scale the wavefront [32] if necessary in such applied problems as: optical manipulation, microscopy, and testing optical systems.

It is difficult to give a sufficiently complete theoretical description of the patterns observed in Tables 4 and 5. The geometric-optical approach is not very applicable for elements of such a small size. Integral transformations using the plane element model are applicable with a noticeable error; in addition, they do not allow you to get an analytical solution. But two conclusions can be drawn.

1) Based on the geometric-optic approach, it can be shown that a ring (for linear SPP) or another closed shape (for nonlinear SPP, if certain conditions are met [21]) will have a gap. This can be seen in Tables 4 and 5 for $q=1$.

2) The appearance of a double contour (differentiating property) is explained using the Kirchhoff integral. This phenomenon is most noticeable when the phase jump between 0 and 360 degrees (minus
a multiple of $2\pi$ is equal to $\pi$. For a linear SPP, this will be at a half-integer charge. For a nonlinear SPP, the phase jump $\pi$ can be at any charge, since the charge is determined by the derivative of the function $\psi(x, y)$, and not by the function itself. For $q=1$, the phase jump is $2\pi$, for $q=2$ $12\pi+0.56$ $\pi$, and for $q=3$ $78\pi+0.88$ $\pi$. Therefore, the double contour should be most noticeable when $q=3$. However, this difference is not very noticeable in the pictures in Table 4, since the value of $\sigma$ is small and there are additional spots.

3. Conclusion
This report shows the action of a spiral phase plate in the near diffraction zone, taking into account the three-dimensional structure of the optical element. Based on the results obtained, the action of linear and nonlinear phase plates is compared. In addition, we conducted studies of the SPP action with polarized sources. It was revealed that for linear polarization, a spiral intensity distribution is formed in the immediate vicinity of the optical element. In addition, a compact light spot is formed at a distance of $10\lambda$, which is stored at a considerable distance. The length of the axial light segment increases with increasing degree of the SPP $q$.

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