Scale, Gauge Couplings, Soft Terms and Toy Compactification in M-theory on $S^1/Z_2$

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Abstract

In M-theory on $S^1/Z_2$, we point out that to be consistent, we should keep the scale, gauge couplings and soft terms at next order, and obtain the soft term relations: $M_{1/2} = -A$, $|M_0/M_{1/2}| \leq 1 / \sqrt{3}$ in the standard embedding and $M_{1/2} = -A$ in the non-standard embedding with five branes and $K_{5,n} = 0$. We construct a toy compactification model which includes higher order terms in 4-dimensional Lagrangian in standard embedding, and discuss its scale, gauge couplings, soft terms, and show that the higher order terms do affect the scale, gauge couplings and especially the soft terms if the next order correction was not small. We also construct a toy compactification model in non-standard embedding with five branes and discuss its phenomenology. We argue that one might not push the physical Calabi-Yau manifold’s volume to zero at any point along the eleventh dimension.

PACS: 11.25.Mj; 04.65.+e; 11.30.Pb; 12.60. Jv

Keywords: M-theory; Compactification; Supersymmetry; Scale; Coupling

August 1999

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1 Introduction

M-theory on $S^1/Z_2$ suggested by Horava and Witten [1] is a 11-dimensional Supergravity theory with two boundaries where the two $E_8$ Yang-Mills fields live on respectively. A lot of study on compactification (standard embedding and non-standard embedding) and phenomenology have been done [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41]. And as we know, the key which connects the M-theory on $S^1/Z_2$ to the low energy phenomenology is the SUSY breaking soft terms, because if we knew the soft terms, by running RGE, we can obtain the low energy SUSY particle spectra and discuss their search in the future collider. On the other hand, if we knew the low energy SUSY particle spectra from future collider, we might derive the soft terms by solving RGE [42]. Therefore, we need to understand the SUSY breaking soft terms as well as possible.

The original eleven-dimensional Lagrangian obtained by Horava and Witten [1] is at order of $\kappa^{2/3}$. And Witten’s solution to the compactification of M-theory on $S^1/Z_2$ which have N=1 supersymmetry in 4-dimension just considered the next order expansion of the metric [2]. Moreover, after compactifying the theory on the deformed Calabi-Yau manifold, A. Lukas, B. A. Ovrut and D. Waldram obtained the 4-dimensional Lagrangian which is at order of $\kappa^{4/3}$ [19]. The explicit expansion parameter which is defined in the following section, is $x$. However, the soft terms: $M_{1/2}, M_0^2, A$ which were obtained previously in standard embedding for the simplest compactification [22], include the higher order correction, i.e., when we expand the soft term $M_{1/2}, M_0^2, A$ as polynomial in $x$ for $|x| < 1$, we have the terms which are proportional to $x^n$ where $n > 1$. This is inconsistent to the original calculation, because we did not include higher order correction to the previous Lagrangian, or the Kähler potential, gauge kinetic function and superpotential. In short, the correct soft terms should be at order of $x$, and this is a good approximation only when $x$ is small. In this paper, we calculate the soft terms $M_{1/2}, M_0^2, A$ at order of $x$ in standard embedding, and compare with previous calculation. We find out that $x < 0.2$, we have good approximation, we will have observable deviation if $x$ is large, i.e., $x > 0.5$. Therefore, the phenomenology discussions, like low energy SUSY particle spectra and collider search, should keep in the small $x$ region. In addition, we also discuss the scale and gauge couplings at order $x$, and the detail discussions are similar to those in [14, 33] with small $x$. Furthermore, we obtain the following soft term relations in standard embedding or non-standard embedding without five brane for the simplest compactification:

$$M_{1/2} = - A, \quad |\frac{M_0}{M_{1/2}}| \leq \frac{1}{\sqrt{3}}. \quad (1)$$

Because the original eleven-dimensional Lagrangian is at order of $\kappa^{2/3}$. And Witten’s solution is at next order, it is very difficult to do the calculation which includes higher order correction unless one makes some simplification. In order to
discuss higher order correction and compare the differences of the soft terms, we use
the following ansatz in our toy model: (1) we just consider the Lagrangian which was
obtained by Horava and Witten \cite{1}, (2) we do not consider the higher order expansion
of the deformed metric, (3) we do not consider the higher order correction to the
bulk fields. Because the massive modes’ contributions are supressed by the factor
$\frac{v_p^{1/6}}{\pi \rho_p}$ which is at about 0.1 order and very small when we consider the intermediate
unification \cite{13, 80}, we just consider the massless modes.

In standard embedding, under this ansatz and considering the Calabi-Yau
manifold with Hodge-Betti number $h_{(1,1)} = 1, h_{(2,1)} = 0$, we calculate the higher order
correction to the Lagrangian, or the scale, gauge couplings, Kähler potential, gauge
kinetic function and superpotential. Under $x^n = 0$ limit for $n > 1$, our results are
the same as those obtained previously \cite{14, 19}. In this toy model compactification,
we find out that:

(I) In order to keep $g_{11,11}$ positive, we can not push the physical Calabi-Yau
manifold’s volume in the hidden sector to zero.

(II) $x = 1$ is not the M-theory limit. $x = 3$ or $y = 1$ ( where $y$ is one-third
of $x$ ) is the limit which pushes the physical Calabi-Yau manifold’s volume in the
hidden sector to zero, but, in order to keep $g_{11,11}$ positive, we require that $x < 3/2$ or
$y < 1/2$. And $x = 3/2$ or $y = 1/2$ is considered as M-theory limit in this paper. In
other words, we define the M-theory limit as the limit which keeps the signature of
the metric invariant, i. e., the total high order corrections to the metric are less than
the zeroth order metric in each component of the metric.

(III) We have strong contraints on the gauge coupling $\alpha_H$ and GUT scale $M_H$
in the hidden sector:

\begin{equation}
\frac{1}{27} \alpha_{GUT} < \alpha_H < 27 \alpha_{GUT} .
\end{equation}

\begin{equation}
\frac{1}{\sqrt{3}} M_{GUT} < M_H < \sqrt{3} M_{GUT} .
\end{equation}

(IV) We notice that, if $y$ is small ( $y < 0.2$ ), in large parameter space, the
magnitude of the gaugino mass is larger than that of the scalar mass, and if $y$ is large
( $y > 0.3$ ), in the most of the parameter space, the magnitude of the gaugino mass is
smaller than that of the scalar mass. However, in the previous soft term analysis \cite{22},
the magnitude of the gaugino mass is often larger than that of the scalar mass in the
standard embedding. In addition, we compare the soft term deviations with the two
scenarios discussed in the second paragrah.

Non-standard embedding in M-theory on $S^1/Z_2$ is also an interesting sub-
ject \cite{27, 30, 31}. We can embed the spin connection to the two $E_8$ gauge fields, so,
after compactification, the gauge groups in the observable sector and hidden sector
will be $G^O$ and $G^H$, which are subgroups of $E_8$. And we can include the five branes,
states which are essentially non-perturbative in heterotic string. The presence of the
five branes is very important to the three generation GUT model building because
they introduce more freedom in the anomaly cancellation condition that consistent vacua need to satisfy \[31, 37\]. And introducing five branes will affect the gauge kinetic function, Kähler potential and non-perturbative superpotential by gaugino condensation in the next order because one introduces more moduli whose real parts are the positions of the five-branes. Without five brane, the non-standard embedding’s results are similar to those in the standard embedding. With five branes, we discuss the scale, gauge couplings and soft terms to the next order, and we find that if there are no Kähler potential for the five brane moduli \( K_5 \), we obtain the soft term relation:

\[ M_{1/2} = -A . \]  

This may be interesting in the low energy phenomenology analysis. Furthermore, we consider the toy model compactification and calculate its Kähler potential, gauge kinetic function, superpotential, and discuss its scale, gauge couplings and soft terms. Our results are:

(I) In order to keep \( g_{11,11} \) positive, we can not push the physical Calabi-Yau manifold’s volume in the hidden sector to zero.

(II) M-theory limit is: \( y_{5b} = \frac{1}{2} \) or \( y_{5b} = -1 \), where \( y_{5b} \) is defined in the following section. In order to keep \( g_{11,11} \) positive, we require that \( y_{5b} < \frac{1}{2} \), and in order to keep the signature of metric \( g_{\mu\nu} \) invariant, we require that \( y_{5b} > -1 \). In short, we obtain: \(-1 < y_{5b} < \frac{1}{2}\).

(III) We have strong constraints on the gauge coupling \( \alpha_H \) and GUT scale \( M_H \) in the hidden sector:

\[ \frac{1}{64} \alpha_{GUT} < \alpha_H < 64\alpha_{GUT} , \]  

\[ \frac{1}{2} M_{GUT} < M_H < 2 M_{GUT} . \]  

Of course, we have more freedom in phenomenology discussion if we include five branes, for we introduce the new parameters \( \beta^i \) where \( i=1, N \). Therefore, we do not do the numerical analysis of the soft terms because we just have three soft term parameters: \( M_{1/2}, M_0^2 \), and \( A \), and we can make them as free parameters by varying \( \beta^i \) and \( \epsilon \) which is defined in the following section.

Finally, we argue that, in general, we might not push the physical Calabi-Yau manifold’s volume to zero at any point along the eleventh dimension.

Our conventions are the following. We denote the eleven-dimensional coordinates by \( x^0, \ldots, x^9, x^{11} \) and the corresponding indices by \( I, J, K, \ldots = 0, \ldots, 9, 11 \). The orbifold \( S^1/Z_2 \) is chosen in the \( x^{11} \)-direction, so we assume that \( x^{11} \in [-\pi\rho, \pi\rho] \) with the endpoints identified as \( x^{11} \sim x^{11} + 2\pi\rho \). The \( Z_2 \) symmetry acts as \( x^{11} \rightarrow -x^{11} \). Then, there exist two ten-dimensional hyperplanes, \( M_i^{10} \) with \( i = 1, 2 \), locally specified by the conditions \( x^{11} = 0 \) and \( x^{11} = \pi\rho \), which are fixed under the
action of the $Z_2$ symmetry. When we compactify the theory on a Calabi-Yau three-fold, we will use indices $A, B, C, \ldots = 4, \ldots, 9$ for the Calabi-Yau coordinates, and indices $\mu, \nu \ldots = 0, \ldots, 3$ for the coordinates of the remaining, uncompactified, four-dimensional space. Holomorphic and antiholomorphic coordinates on the Calabi-Yau space will be labeled by $a, b, c, \ldots$ and $\bar{a}, \bar{b}, \bar{c}, \ldots$. In addition, we denote the hyperplane $M_1^{10}$ as the observable sector, and the hyperplane $M_2^{10}$ as the hidden sector.

2 Scale, Gauge Couplings, Soft Terms and Toy Compactification in Standard Embedding

2.1 Scale, Gauge Couplings, Kähler Function and Soft Terms Revisit

First, let us review the gauge couplings, gravitational coupling and the physical eleventh dimension radius in the M-theory [15]. The relevant 11-dimensional Lagrangian is given by [1]

$$L_B = -\frac{1}{2\kappa^2} \int_{M^{11}} d^{11}x \sqrt{g}R - \sum_{i=1,2} \frac{1}{2 \pi (4 \pi \kappa^2)^{2/3}} \int_{M_1^{10}} d^{10}x \sqrt{g} \frac{1}{4} F_{aI}^2 F^{aI}.$$ (7)

In the 11-dimensional metric [4], the gauge couplings and gravitational coupling in 4-dimension are [2, 11, 15]:

$$8\pi G_N^{(4)} = \frac{\kappa^2}{2 \pi \rho_p V_p},$$

$$\alpha_{\text{GUT}} = \frac{1}{2 V_p (1 + x)} (4 \pi \kappa^2)^{2/3},$$

$$[\alpha_H]_W = \frac{1}{2 V_p (1 - x)} (4 \pi \kappa^2)^{2/3},$$

where $x$ is defined by:

$$x = \frac{\rho_p}{V_p^{2/3}} \left( \frac{\kappa}{4 \pi} \right)^{2/3} \int_X \omega \wedge \frac{\text{tr} F \wedge F - \frac{1}{4} \text{tr} R \wedge R}{8 \pi^2},$$

where $\rho_p$, $V_p$ are the physical eleventh dimension radius and Calabi-Yau manifold’s volume (which is defined by the middle point Calabi-Yau manifold’s volume between

\footnote{Because we think 11-dimensional metric is more fundamental than string metric and Einstein frame, our discussion in this paper use 11-dimensional metric.}
the observable sector and the hidden sector) respectively. From above formula, one obtains:

$$x = \frac{\alpha_H \alpha_{GUT}^{-1} - 1}{\alpha_H \alpha_{GUT} + 1}. \quad (12)$$

The GUT scale $M_{GUT}$ and the hidden sector GUT scale $M_H$ when the Calabi-Yau manifold is compactified are:

$$M_{GUT}^{-6} = V_p(1 + x), \quad (13)$$

$$M_H^{-6} = V_p(1 - x), \quad (14)$$

or we can express the $M_H$ as:

$$M_H = \left( \frac{\alpha_H}{\alpha_{GUT}} \right)^{1/6} M_{GUT} = \left( \frac{1 + x}{1 - x} \right)^{1/6} M_{GUT}. \quad (15)$$

Noticing that $M_{11} = \kappa^{-2/9}$, we have

$$M_{11} = \left[ 2(4\pi)^{-2/3} \alpha_{GUT} \right]^{-1/6} M_{GUT}. \quad (16)$$

And the physical scale of the eleventh dimension in the eleven-dimensional metric is:

$$[\pi p_p]^{-1} = \frac{8\pi}{1 + x} \left( 2\alpha_{GUT} \right)^{-3/2} \frac{M_{GUT}^2}{M_{Pl}^2}, \quad (17)$$

where $M_{Pl} = 2.4 \times 10^{18}$ GeV. From the constraints that $M_{GUT}$ and $M_H$ are smaller than the scale of $M_{11}$, one obtains:

$$\alpha_{GUT} \leq \frac{(4\pi)^{2/3}}{2}; \quad \alpha_H \leq \frac{(4\pi)^{2/3}}{2}, \quad (18)$$

or

$$\alpha_{GUT} \leq 2.7; \quad \alpha_H \leq 2.7. \quad (19)$$

For the standard embedding, the upper bound on $x$ is $0.97$ ($x < 0.97$), for $\alpha_{GUT} = \frac{1}{25}$.

Second, let us review the Kähler potential, gauge kinetic function, superpotential and soft terms in the simplest compactification of M-theory on $S^1/Z_2$. The Kähler potential, gauge kinetic function and superpotential are [16, 19]:

$$K = \hat{K} + \bar{K}|C|^2, \quad (20)$$

$$\hat{K} = -\ln [S + \bar{S}] - 3 \ln [T + \bar{T}]. \quad (21)$$
where $S$, $T$ and $C$ are dilaton, moduli and matter fields respectively. $\alpha$ is the next order correction constant which is related to the Calabi-Yau manifold and it is:

$$\alpha = \frac{x(S + \bar{S})}{T + T}.$$  
(26)

The non-perturbative superpotential due to the gaugino condensation is [21, 43]:

$$W_{np} = h \exp(-\frac{8\pi^2}{C_2(G^H)}(S - \alpha T)),$$  
(27)

where the group in the hidden sector is $G^H$ and $C_2(G^H)$ is the quadratic Casimir of $G^H$. And in this case, $G^H$ is $E_8$ and $C_2(G^H) = 30$.

With the standard formulae [13, 22], one can easily obtain the following soft terms [13, 22]:

$$M_{1/2} = \frac{\sqrt{3}C_0M_{3/2}}{1 + x}(\sin \theta e^{-i\theta_S} + \frac{x}{\sqrt{3}} \cos \theta e^{-i\theta_T}),$$  
(28)

$$M_0^2 = V_0 + M_{3/2}^2 - \frac{3C_0M_{3/2}^2}{(3 + x)^2}(x(6 + x) \sin^2 \theta + (3 + 2x) \cos^2 \theta - 2\sqrt{3}x \cos(\theta_S - \theta_T) \sin \theta \cos \theta),$$  
(29)

$$A = -\frac{\sqrt{3}C_0M_{3/2}}{(3 + x)}((3 - 2x) \sin \theta e^{-i\theta_S} + \sqrt{3}x \cos \theta e^{-i\theta_T}),$$  
(30)

where $M_{3/2}$ is the gravitino mass,

$$F^S = \sqrt{3}M_{3/2}C_0(S + \bar{S}) \sin \theta e^{-i\theta_S},$$  
(31)

$$F^T = M_{3/2}C_0(T + \bar{T}) \cos \theta e^{-i\theta_T},$$  
(32)
\[ C_0^2 = 1 + \frac{V_0}{3M_{3/2}^2}, \tag{33} \]

and \( V_0 \) for the tree level vacuum density.

Generically, the dynamics of the hidden sector may give rise to both \( < F^S > \) and \( < F^T > \), but one type of F term often dominates. Therefore, we concentrate on the two limiting cases: dilaton dominant SUSY breaking \( (F^T = 0) \) and the moduli dominant SUSY breaking \( (F^S = 0) \) \[44, 45\].

(I) Dilaton dominant SUSY breaking scenario \( (F^T = 0) \). In this case, the soft terms become

\[ M_{1/2} = \frac{\sqrt{3} M_{3/2}}{1 + x}, \tag{34} \]

\[ M_0^2 = M_{3/2}^2 - \frac{3M_{3/2}^2}{(3 + x)^2} x (6 + x), \tag{35} \]

\[ A = -\frac{\sqrt{3} M_{3/2}}{3 + x} (3 - 2x). \tag{36} \]

(II) Moduli dominant SUSY breaking scenario \( (F^S = 0) \). In this case, the soft terms become

\[ M_{1/2} = \frac{x}{1 + x} M_{3/2}, \tag{37} \]

\[ M_0 = \frac{x}{3 + x} M_{3/2}, \tag{38} \]

\[ A = -\frac{3x}{3 + x} M_{3/2}. \tag{39} \]

Therefore, we have

\[ M_0/A = -1/3 ; 3 \geq M_{1/2}/M_0 \geq 2. \tag{40} \]

However, the previous calculation is expanded in \( x \) (small \( x \) is considered) and the Lagrangian, Kähler potential and gauge kinetic function are at order of \( x \). Therefore, to be consistent, we should keep the scale, gauge couplings and soft terms

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3 In the discussion of the dilaton dominant SUSY breaking scenario and moduli dominant SUSY breaking scenario, in order to obtain the simple soft term relations, we set \( V_0 = 0 \), \( C_0 = 1 \) and \( \theta_s = \theta_T = 0 \).
at order of $x$, because there may exist other high order correction ($x^2$ and higher) to the Lagrangian, or Kähler potential and gauge kinetic function. Considering $x^n = 0$ for $n > 1$, the following previous scale and gauge coupling equations need to be changed:

$$\alpha_{\text{GUT}} = \frac{1 - x}{2V_p} (4\pi\kappa^2)^{2/3} , \quad \text{(41)}$$

$$[\alpha_H]_W = \frac{1 + x}{2V_p} (4\pi\kappa^2)^{2/3} , \quad \text{(42)}$$

$$M_H = (1 + x/3) M_{\text{GUT}} , \quad \text{(43)}$$

$$[\pi\rho_p]^{-1} = 8\pi (1 - x) (2\alpha_{\text{GUT}})^{-3/2} \frac{M_{\text{GUT}}^3}{M_{Pl}^2} , \quad \text{(44)}$$

and the next order soft terms are:

$$M_{1/2} = \sqrt{3} C_0 M_{3/2} ((1 - x) \sin \theta e^{-i\theta_S} + \frac{x}{\sqrt{3}} \cos \theta e^{-i\theta_T}) , \quad \text{(45)}$$

$$M_0^2 = V_0 + M_{3/2}^2 - \frac{C_0^2 M_{3/2}^2}{3} (6x \sin^2 \theta + 3 \cos^2 \theta - 2\sqrt{3}x \cos(\theta_S - \theta_T) \sin \theta \cos \theta) , \quad \text{(46)}$$

$$A = -\sqrt{3} C_0 M_{3/2} ((1 - x) \sin \theta e^{-i\theta_S} + \frac{x}{\sqrt{3}} \cos \theta e^{-i\theta_T}) . \quad \text{(47)}$$

Obviously, we have $M_{1/2} = -A$. And we obtain the soft terms in the two limiting case:

(I) Dilaton dominant SUSY breaking scenario ($F^T = 0$):

$$M_{1/2} = \sqrt{3} M_{3/2} (1 - x) , \quad \text{(48)}$$

$$M_0 = (1 - x) M_{3/2} , \quad \text{(49)}$$

$$A = -\sqrt{3} M_{3/2} (1 - x) . \quad \text{(50)}$$

Soft term relations are the same as those obtained in the weakly coupled string with dilaton dominant SUSY breaking: $M_{1/2} = -A = \sqrt{3} M_0$.

(II) Moduli dominant SUSY breaking scenario ($F^S = 0$):
\[ M_{1/2} = x M_{3/2} \, , \]  
\[ M_0 = 0 \, , \]  \hspace{1cm} (51) 
\[ A = -x M_{3/2} \, . \]  \hspace{1cm} (53)

Soft term relations: \( M_{1/2} = -A, M_0 = 0 \) are similar to the no-scale case \([K]\): \( M_{1/2} \neq 0 \) and \( M_0 = A = 0 \), for \( A \) is not very important in the RGE running.

Because in general, we have \( M_{1/2} = -A \). We might need to know the relation between the magnitude of \( M_{1/2} \) and that of \( M_0 \). Taking \( V_0 = 0, C_0 = 1 \), we can express \(|M_{1/2}|^2\) as following:

\[ |M_{1/2}|^2 = 3M_0^2 + M_{2/3}^2 x^2 \left( 3 \sin^2 \theta + \cos^2 \theta - 2\sqrt{3} \cos(\theta_S - \theta_T) \sin \theta \cos \theta \right) \, . \]  \hspace{1cm} (54)

We obtain that

\[ M_{1/2} = -A \, , \, |\frac{M_0}{M_{1/2}}| \leq \frac{1}{\sqrt{3}} \, , \]  \hspace{1cm} (55)

for the last term in eq. (51) is obvious positive, and only when \( \theta_S - \theta_T = 0 \, (\pi) \) and \( \theta = \frac{\pi}{6} \, \text{or} \, \theta = \frac{5\pi}{6} \, (\theta = \frac{11\pi}{6}) \), \( |M_{1/2}| = \sqrt{3}|M_0| \). In fig. 1, choosing \( \theta_S = \theta_T = 0 \), we draw \( \frac{M_0}{M_{1/2}} \) versus \( \theta \) by taking \( x = 0.05, 0.1, 0.2, 0.3 \) which are represented by the solid, dots, dotdash, and dashes lines, respectively. We can see that the deviation from \( \pm \frac{1}{\sqrt{3}} \) is large when \( \theta \) closes to 0 or \( \pi \), or \( x \) is large. The soft terms are different from those obtained in the weakly coupled string where one has \([14]\):

\[ M_{1/2} = -A \, , \, |\frac{M_0}{M_{1/2}}| = \frac{1}{\sqrt{3}} \, . \]  \hspace{1cm} (56)

2.2 Toy Model Compactification and its Phenomenology

If we did not consider the higher order correction to the bulk fields, one can easily write down the most general parametrized Kähler potential, gauge kinetic function and non-perturbative superpotential. For the simplest compactification, they are \([14]\):

\[ K = \hat{K} + \bar{K} |C|^2 \, , \]  \hspace{1cm} (57)
\[ \hat{K} = -\ln [S + \bar{S}] - 3 \ln [T + \bar{T}] \, , \]  \hspace{1cm} (58)
\[
\hat{K} = \left(1 + \sum_{i=1}^{\infty} c_i \left( \frac{\alpha(T + \bar{T})}{S + \bar{S}} \right)^i \right) \left( \frac{3}{T + \bar{T}} \right) |C|^2 ,
\]

(59)

\[
f_{\alpha\beta}^O = S \left(1 + \sum_{i=1}^{\infty} d_i \left( \frac{\alpha T}{S} \right)^i \right) \delta_{\alpha\beta} ,
\]

(60)

\[
f_{\alpha\beta}^H = S \left(1 + \sum_{i=1}^{\infty} d_i \left( \frac{-\alpha T}{S} \right)^i \right) \delta_{\alpha\beta} ,
\]

(61)

\[
W_{np} = h \exp \left( -\frac{8\pi^2}{C_2(G^H)} S \left(1 + \sum_{i=1}^{\infty} d_i \left( \frac{-\alpha T}{S} \right)^i \right) \right) .
\]

(62)

Using standard method \cite{44, 45}, one can easily calculate the soft terms. But, here, one introduce many parameters, which is useless to the low energy phenomenology analysis. Therefore, we construct a toy simple model as an example which contains high order correction.

In fact, in order to consider high order correction to the scale, gauge couplings and soft terms in detail, we need to consider the higher order Lagrangian. However, the original Lagrangian is at order of \(\kappa^2/3\) \cite{1}, and the Witten’s solution to the compactification of M-theory on \(S^1/Z_2\) which have N=1 supersymmetry in 4-dimension just considered the next order expansion of the metric \cite{2}. Therefore, it is really very tough to consider the higher order terms realistically. The ansatz of our toy model is that, we just consider the Lagrangian which was obtained by Horava and Witten, we do not consider the higher order expansion of the metric and higher order correction to the bulk fields. Under this ansatz, we can discuss the higher order correction to the scale, gauge couplings, Kähler potential, gauge kinetic function and superpotential.

The bosonic part of the eleven-dimensional supergravity Lagrangian is given by \cite{1}

\[
L_B = \frac{1}{\kappa^2} \int_{M_{11}} d^{11}x \sqrt{g} \left( -\frac{1}{2} R - \frac{1}{48} G_{IJKL} G^{IJKL} - \frac{\sqrt{2}}{3456} e^{i_{11}i_{12}...i_{11}} C_{11} C_{12} C_{13} G_{14}...G_{18}...G_{11} \right) \\
- \sum_{i=1,2} \frac{1}{2\pi(4\pi\kappa^2)^{\frac{1}{2}}} \int_{M_{10}} d^{10}x \sqrt{g} \frac{1}{4} F_{IJ}^a F^{aIJ} ,
\]

(63)

where \(G_{11IJK} = (\partial_1 C_{IJK} \pm 23 \text{ permutations}) + \frac{\sqrt{2}}{2\kappa^2 \sqrt{2N}} \delta(x^{11}) \omega_{IJK}, \ \lambda^2 = 2\pi(4\pi\kappa^2)^{2/3}.

In this paper, we consider the compactification on the Calabi-Yau manifold with Hodge-Betti numbers \(h_{(1,1)} = 1\) and \(h_{(2,1)} = 0\). For the zeroth-order metric, there are only one dilaton and one modulus controlling the overall size of the Calabi-Yau space and the length of the orbifold interval. We write

\[
ds^2 = g^{(0)}_{\mu\nu} dx^\mu dx^\nu + e^{2\varphi} g^{(0)}_{AB} dx^A dx^B + e^{2\varphi} (dx^{11})^2 ,
\]

(64)
so that the physical Calabi-Yau volume $V_p$ is $e^{6a}V$ and the physical length of the
orbifold interval $\pi \rho_p$ is $e^{c} \pi \rho$. For the next order correction, because the massive
modes are suppressed by the factor $\frac{V^{1/6}}{\pi \rho_p}$ which is at about 0.1 order and very small
when we consider the intermediate unification, we just consider the massless mode. And the deformed metric are [13]:

$$ds^2 = (1 + \beta e^{-4a}(\frac{2x^{11}}{\pi \rho} - 1))g^{(0)}_{\mu \nu}dx^\mu dx^\nu + (1 - \beta e^{-4a}(\frac{2x^{11}}{\pi \rho} - 1))e^{2c}(dx^{11})^2,$$

where

$$\beta = \frac{1}{3} \pi^2 \rho \frac{\kappa}{V^{2/3} (4\pi)^{2/3}} \int_X \omega \wedge \frac{tr F \wedge F - \frac{1}{2} tr R \wedge R}{8\pi^2}.$$ 

(65)

The M-theory limit is $\beta e^{-4a} = \pm \frac{1}{2}$. In order to keep $g_{11,11}$ positive, we require that
$-\frac{1}{2} < \beta e^{-4a} < \frac{1}{2}$, which is enough to keep the physical Calabi-Yau manifold’s volume
non-zero at any point along the eleventh dimension.

Defining

$$C_{AB11} = \frac{1}{6} \chi \omega_{AB} \, , \, C_{\mu \nu 11} = \frac{1}{6} B_{\mu \nu} \, ,$$

(67)

and

$$\partial_{[\mu} B_{\nu \rho]} = \frac{1}{3} e^{-12a} \epsilon_{\mu \nu \rho} \delta_{\partial\sigma} \, , \, \dot{c} = c + 2a \, ,$$

(68)

one can obtain the Lagrangian in 4-dimension to the zeroth order [11, 19]:

$$S_0^1 = \pi \rho V \int_{M^4} \sqrt{-g} \left[ -R - 18 \partial_{\mu} a \partial^\mu a - \frac{3}{2} \partial_{\mu} \dot{c} \partial^\mu \dot{c} - 3e^{-2c} \partial_{\mu} \chi \partial^\mu \chi - e^{-12a} \partial_{\mu} \sigma \partial^\mu \sigma \right],$$

(69)

$$S_0^2 = \pi \rho V \int_{M^4} \sqrt{-g} \left[ -3e^{-\dot{c}} D_{\mu} C D^\mu \bar{C} - \frac{3i}{\sqrt{2}} e^{-2c} \left( \bar{C} D_{\mu} C - C D_{\mu} \bar{C} \right) \partial^\mu \chi - \frac{3k^2}{4} e^{-2c-6a} |d_{pq} C^p C^q|^2 - \frac{3k^2}{32} e^{-2c-6a} (\bar{T}^4 C)^2 - \frac{1}{4} e^{6a} F_{\mu \nu}^O F_{\mu \nu}^O - \frac{1}{4} e^{6a} F_{\mu \nu}^H F_{\mu \nu}^H - \frac{\sqrt{2} \sigma}{4} F_{\mu \nu}^O \tilde{F}_{\mu \nu}^O - \frac{\sqrt{2} \sigma}{4} F_{\mu \nu}^H \tilde{F}_{\mu \nu}^H \right],$$

(70)
\[ S_0^3 = \frac{\pi \rho V}{\kappa^2} \int_{M^4} \sqrt{-g} \left[ \frac{3}{8} e^{-2\xi} (C^2 D_\mu \bar{C} D^\mu \bar{C} + C^2 D_\mu C D^\mu \bar{C} - 2|C|^2 D_\mu C D^\mu \bar{C}) \ight. \\
\left. - \frac{3k^2}{8} e^{-3\xi - 6\alpha} |d_{pqr} C^p C^q C^r|^2 \right] , \tag{71} \]

where \( k = 4\sqrt{2\rho \pi (4\pi / \kappa)^{1/3}} \) and for the gauge fields and gauge matter fields, we rescale them as following \([11, 19]\):

\[ C^p \to \pi \sqrt{2\rho} \left( \frac{4\pi}{\kappa} \right)^{1/3} C^p , \quad A_\mu \to \pi \sqrt{2\rho} \left( \frac{4\pi}{\kappa} \right)^{1/3} A_\mu . \tag{72} \]

In above equations, \( S_0^1 \) is from the bulk Lagrangian, \( S_0^2 \) is from the boundary Lagrangian and \( S_0^3 \) is from the boundary Lagrangian with additional \( \delta(0) \) factor which is not well defined. Considering the higher order correction from the deformed metric to the boundary terms, we obtain:

\[ S^2 = \frac{\pi \rho V}{\kappa^2} \int_{M^4} \sqrt{-g} \left[ -3 e^{-\xi} \Lambda_+ \Lambda_- D_\mu C D^\mu \bar{C} - \frac{3i}{\sqrt{2}} \Lambda_+ \Lambda_- e^{-2\xi} \left( CD_\mu C - CD_\mu \bar{C} \right) \partial^\mu \chi \\
- \frac{3k^2}{4} \Lambda_+ \Lambda_- e^{-2\xi - 6a} |d_{pqr} C^p C^q|^2 - \frac{3k^2}{32} \Lambda_+ \Lambda_- e^{-2\xi - 6a} (\bar{C} T^4 C)^2 \\
- \frac{1}{4} \Lambda_+ e^{6a} F_{\mu\nu}^O F_{\mu\nu}^O - \frac{1}{4} \Lambda_- e^{6a} F_{\mu\nu}^H \bar{F}_{\mu\nu}^H - \frac{\sqrt{2\sigma}}{4} \Lambda_+^3 F_{\mu\nu}^O \bar{F}_{\mu\nu}^O \\
- \frac{\sqrt{2\sigma}}{4} \Lambda_-^3 F_{\mu\nu}^H \bar{F}_{\mu\nu}^H \right] , \tag{73} \]

\[ S^3 = \frac{\pi \rho V}{\kappa^2} \int_{M^4} \sqrt{-g} \left[ \frac{3}{8} \Lambda_+ \Lambda_- e^{-2\xi} (C^2 D_\mu \bar{C} D^\mu \bar{C} + \bar{C}^2 D_\mu C D^\mu \bar{C} - 2|C|^2 D_\mu C D^\mu \bar{C}) \\
- \frac{3k^2}{8} \Lambda_+^2 e^{-3\xi - 6a} |d_{pqr} C^p C^q C^r|^2 \right] \frac{1}{(1 + 2\beta e^{\xi - 4a})^{1/2}} , \tag{74} \]

where

\[ \Lambda_+ = 1 + \beta e^{\xi - 4a} , \quad \Lambda_- = 1 - \beta e^{\xi - 4a} . \tag{75} \]

Because \( S^3 \) is proportional to \( \delta(0) \) in the original Lagrangian and its order is \( \kappa^{4/3} \), we use \( S^2 \) to obtain the Kähler potential, gauge kinetic function and the superpotential in this toy compactification:

\[ K = \hat{K} + \bar{K}|C|^2 , \tag{76} \]

\[ \hat{K} = - \ln [S + \bar{S}] - 3 \ln [T + \bar{T}] , \tag{77} \]

\[ \bar{K} = \frac{3}{T + T} \left( 1 + \frac{\beta(T + T)}{S + S} \right)^2 \left( 1 - \frac{\beta(T + T)}{S + S} \right) , \tag{78} \]

\[ 12 \]
\[ f^{O}_{\alpha\beta} = S \left( 1 + \frac{\beta T}{S} \right)^3 \delta_{\alpha\beta}, \quad (79) \]

\[ f^{H}_{\alpha\beta} = S \left( 1 - \frac{\beta T}{S} \right)^3 \delta_{\alpha\beta}, \quad (80) \]

\[ W = \left( 1 + \frac{\beta T}{S} \right)^{3/2} \left( 1 - \frac{\beta T}{S} \right)^{3/2} k d_{xyz} C^x C^y C^z, \quad (81) \]

\[ W_{np} = h \exp \left( -\frac{8\pi^2}{C_2(G^H)} S \left( 1 - \frac{\beta T}{S} \right)^3 \right). \quad (82) \]

where

\[ S = e^{6a} + i\sqrt{2}\sigma, \quad T = e^\hat{c} + i\sqrt{2}\chi. \quad (83) \]

One can easily prove that, when \( \beta \) is very small, i.e., taking \( \beta^2 = 0 \), one obtains the same results as those obtained previously [16, 19].

Now, we will consider the phenomenology in the toy compactification. First, let us discuss the scale and gauge couplings,

\[ 8\pi G_N^{(4)} = \frac{\kappa^2}{2\pi p_p V_p \delta}, \quad (84) \]

\[ \alpha_{GUT} = \frac{1}{2V_p(1 + y)^3 (4\pi\kappa^2)^{2/3}}, \quad (85) \]

\[ \alpha_H = \frac{1}{2V_p(1 - y)^3 (4\pi\kappa^2)^{2/3}}, \quad (86) \]

where

\[ y = \frac{\beta(T + \bar{T})}{S + \bar{S}} = \frac{\alpha_H^{1/3} \alpha_{GUT}^{-1/3} - 1}{\alpha_H^{1/3} \alpha_{GUT}^{-1/3} + 1}, \quad (87) \]

\[ \delta = \frac{\int_{-1}^{1} \sqrt{1 - 2yx(1 + yx)^2(1 - yx)^3} \, dx}{\int_{-1}^{1} \sqrt{1 - 2yx} \, dx}. \quad (88) \]

By the way, comparing the compactification in last subsection, we have: \( y = \frac{\tilde{y}}{3}, \beta = \frac{\tilde{\beta}}{3}. \)
The GUT scale $M_{GUT}$ and the hidden sector GUT scale $M_H$ when the Calabi-Yau manifold is compactified are:

$$M_{GUT}^{-6} = V_p(1 + y)^3,$$  \hspace{1cm} (89)

$$M_H^{-6} = V_p(1 - y)^3,$$  \hspace{1cm} (90)

or we can express the $M_H$ as:

$$M_H = \left(\frac{1 + y}{1 - y}\right)^{1/2} M_{GUT},$$  \hspace{1cm} (91)

And the physical scale of the eleventh dimension is:

$$[\pi \rho_p]^{-1} = \frac{8\pi \delta}{(1 + y)^3} (2\alpha_{GUT})^{-3/2} \frac{M_{GUT}^3}{M_{Pl}^2}.$$  \hspace{1cm} (93)

In order to keep $g_{11,11}$ positive, we require that $-\frac{1}{2} < y < \frac{1}{2}$. In addition, in fig. 2, we plot the $\delta$ versus $y$, and we find that $\delta$ is about 1. So, $\delta$ will not change previous scale picture \[14, 15, 39\]. And the bounds on $M_H$ and $\alpha_H$ are:

$$\frac{1}{27} \alpha_{GUT} < \alpha_H < 27 \alpha_{GUT},$$  \hspace{1cm} (94)

$$\frac{1}{\sqrt{3}} M_{GUT} < M_H < \sqrt{3} M_{GUT}.$$  \hspace{1cm} (95)

In addition, because $-\frac{1}{2} < y < \frac{1}{2}$, the discussions of the scale and gauge couplings are similar to those in \[14, 15, 39\] with small $x$. So, we will not redo the discussion here again.

Second, let us discuss the soft terms from above Kähler potential and gauge kinetic function. Using standard tree level formulae \[14, 15\], we obtain the following soft terms:

$$M_{1/2} = \frac{\sqrt{3} C_0 M_{3/2}}{1 + y} \left[ \sin \theta (1 - 2y)e^{-i\theta_S} + \sqrt{3} y \cos \theta e^{-i\theta_T} \right],$$  \hspace{1cm} (96)

$$M_0^2 = M_{3/2}^2 + V_0 - \frac{M_{3/2}^2 C_0^2}{(1 - y)^2} \left( 3y(2 - 9y + 3y^3) \sin^2 \theta + (1 - 5y^2 + 2y^3 - 2y^4) \cos^2 \theta \right.$$

$$\left. + 2\sqrt{3} y \sin \theta \cos \theta \cos(\theta_S - \theta_T)(-1 + 6y - y^2) \right),$$  \hspace{1cm} (97)
\[ A = -\frac{\sqrt{3}}{1-y^2} M_{3/2} C_0 (1 - 3y + 8y^2) \sin \theta e^{-i\theta_S} - \frac{3y}{1-y^2} M_{3/2} C_0 (1 - 3y) \cos \theta e^{-i\theta_T}, \] (98)

where

\[ F^S = \sqrt{3} M_{3/2} C_0 (S + \bar{S}) \sin \theta e^{-i\theta_S}, \] (99)

\[ F^T = M_{3/2} C_0 (T + \bar{T}) \cos \theta e^{-i\theta_T}, \] (100)

\[ C_0^2 = 1 + \frac{V_0}{3M_{3/2}^2}, \] (101)

and \( V_0 \) for the tree level vacuum density.

Now, we discuss the numerical results for the soft terms by taking \( V_0 = 0, C_0 = 1, \) and \( \theta_S = \theta_T = 0. \) First, we compare the soft terms. We denote the original soft terms (eqs (28-30)) as scenario O, the soft terms (eqs (45-47)) at next to the leading order which is simple as scenario S, the soft terms (eqs (96-98)) in above toy compactification as scenario T. Choosing \( \theta = \frac{\pi}{4}, \) we draw the soft terms: \( M_{1/2}, M_0, A \) versus \( x \) in fig. 3, fig. 4, and fig. 5 respectively. We notice that, when \( x < 0.2, \) three scenarios agree very well, when \( x > 0.5, \) we can see the obvious differences among them. In addition, we can compare the magnitudes of the soft terms in these three scenarios: \( M_{1/2}^O < M_{1/2}^S < M_{1/2}^T, \) \( M_0^S < M_0^O < M_0^T, \) \( |A|^S < |A|^O < |A|^T. \) Therefore, if we discuss M-theory phenomenology from the soft terms, we should keep in the small \( x \) region in order to be consistent, because we do not know the higher order correction, although in realistic model, \( x \) might be large. Second, we analyze the soft terms in the toy compactification, we draw the soft terms versus \( \theta \) by choosing \( y = 0.15, 0.2, 0.3, 0.4, 0.5 \) in fig. 6, 7, 8, 9, 10, respectively. We notice that, if \( y \) is small ( \( y < 0.2 \) ), in large parameter space, the magnitude of the gaugino mass is larger than that of the scalar mass, if \( y \) is large ( \( y > 0.3 \) ), in the most of the parameter space, the magnitude of the gaugino mass is smaller than that of the scalar mass. However, in previous soft term analysis in standard embedding, the magnitude of the gaugino mass is often larger than that of the scalar mass [22]. We also discuss the case that \( y < 0, \) where we just use \( y = -0.3 \) as an example in fig. 11.

3 Scale, Gauge Couplings, Soft Terms and Toy Compactification in Non-Standard Embedding

3.1 Scale, Gauge Couplings, Kähler Function and Soft Terms Revisit

The non-standard embedding without 5-branes is similar to the standard embedding in section 2.1. In general, non-standard embedding includes 5-brane correction, which
introduces additional moduli fields $Z_n$, whose real parts are the 5-brane positions along the eleventh dimension. The Kähler potential, gauge kinetic function and superpotential are [31]:

$$K = \hat{K} + \bar{K}|C|^2, \quad (102)$$

$$\hat{K} = -\ln [S + \bar{S}] - 3\ln [T + \bar{T}] + K_5, \quad (103)$$

$$\bar{K} = \left(\frac{3}{T + \bar{T}} + \frac{3\epsilon\zeta}{S + \bar{S}}\right)|C|^2, \quad (104)$$

$$f^O_{\alpha\beta} = \left(S + 3\epsilon T(\beta^0 + \sum_{n=1}^N (1 - Z_n)^2\beta^n)\right)\delta_{\alpha\beta}, \quad (105)$$

$$f^H_{\alpha\beta} = \left(S + 3\epsilon T(\beta^{N+1} + \sum_{n=1}^N (Z_n)^2\beta^n)\right)\delta_{\alpha\beta}, \quad (106)$$

$$W = d_{xyz} C^x C^y C^z, \quad (107)$$

$$W_{np} = h \exp\left(-\frac{8\pi^2}{C_2(G^H)}(S + 3\epsilon T(\beta^{N+1} + \sum_{n=1}^N (Z_n)^2\beta^n))\right), \quad (108)$$

where

$$\epsilon = \left(\frac{\kappa}{4\pi}\right)^{2/3} \frac{2\pi^2 \rho}{V^{2/3}}, \quad (109)$$

$$\zeta = \beta^0 + \sum_{n=1}^N \left(1 - \frac{1}{2}(Z_n + \bar{Z}_n)\right)^2 \beta^n, \quad (110)$$

and $K_5$ is the Kähler potential for the moduli fields $Z_n$. In addition, $\beta^0, \beta^{N+1}$ are the instanton number in the observable sector and hidden sector, $\beta^n$ where $n=1, \ldots, N$ is the magnetic charge on each 5-brane. The cohomology constraints on $\beta^i$ is:

$$\sum_{n=0}^{N+1} \beta^i = 0, \quad (111)$$

which means the net charge should be zero.
With assumption: \( <Z_n> = <ReZ_n> \), \( <S> = <\bar{S}> \), \( <T> = <\bar{T}> \), we discuss the scale and gauge couplings first [39]:

\[
8\pi G_N^{(4)} = \frac{\kappa^2}{2\pi \rho_p V_p \delta_{5b}}, \tag{112}
\]

\[
\alpha_{\text{GUT}} = \frac{1}{2V_p(1 + e)} (4\pi \kappa^2)^{2/3}, \tag{113}
\]

\[
\alpha_H = \frac{1}{2V_p(1 + e_h)} (4\pi \kappa^2)^{2/3}, \tag{114}
\]

where

\[
\delta_{5b} = \frac{\int_{-\rho}^{\rho} (1 - 2y_{5b}) dx^{11}}{\int_{-\rho}^{\rho} (1 - y_{5b}) dx^{11}}. \tag{115}
\]

\[
e = \frac{3\epsilon \zeta (T + \bar{T})}{S + S}, \tag{116}
\]

\[
e_h = \frac{3\epsilon (T + \bar{T})}{S + S} (\beta^{N+1} + \sum_{n=1}^{N} \frac{1}{2} (Z_n + \bar{Z}_n))^2 \beta^n), \tag{117}
\]

and where

\[
y_{5b} = \frac{\epsilon (T + \bar{T})}{S + S} \left[ 2 \sum_{m=0}^{n} \beta^m(|ReZ| - ReZ_m) - \sum_{m=0}^{N+1} ((ReZ_m)^2 - 2ReZ_m) \beta^m \right] \tag{118}
\]

in the interval \( ReZ_n \leq |ReZ| \leq ReZ_{n+1} \). And the \( ReZ, ReZ_n \) are defined as:

\[
ReZ = \frac{x^{11}}{\pi \rho}, \quad ReZ_n = \frac{x^{11}}{\pi \rho}, \tag{119}
\]

where \( n = 1, \ldots, N \). \( y_{5b} \) is a function of above \( x \), and in general, \( \delta_{5b} \) will not be 1 in non-standard embedding with five branes. However, \( \delta_{5b} \) may close to 1 for we consider the small next order correction, so, it will not change the previous scale picture [34, 35, 39].

The GUT scale \( M_{\text{GUT}} \) and the hidden sector GUT scale \( M_H \) when the Calabi-Yau manifold is compactified are:

\[
M_{\text{GUT}}^{-6} = V_p(1 + e), \tag{120}
\]

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\[
M_H^{-6} = V_p(1 + e_h),
\]

or we can express the \(M_H\) as:

\[
M_H = \left(\frac{1 + e}{1 + e_h}\right)^{1/6} M_{\text{GUT}},
\]

\[
M_{11} = \left[2(4\pi)^{-2/3} \alpha_{\text{GUT}}\right]^{-1/6} M_{\text{GUT}}.
\]

And the physical scale of the eleventh dimension is:

\[
[\pi \rho_p]^{-1} = \frac{8\pi \delta_{5b}}{1 + e (2\alpha_{\text{GUT}})^{-3/2} M_{3/2}^3 M_{\text{Pl}}^2}.
\]

Second, we can also obtain the following soft terms [38, 39]:

\[
M_{1/2} = \frac{1}{1 + e} \left(\tilde{F}^S + e\tilde{F}^T + e\tilde{F}^n\zeta_n\right),
\]

\[
M_0^2 = V_0 + M_{3/2}^2 - \frac{1}{(3 + e)^2} \left(e(6 + e)|\tilde{F}^S|^2
\right.
\]

\[
+ 3(3 + 2e)|\tilde{F}^T|^2 - 6eRe\tilde{F}^S\tilde{F}^T
\]

\[
+ (e\zeta(3 + e)\zeta_{nn} - e^2\zeta_n\zeta_m)\tilde{F}^n\tilde{F}^m
\]

\[
- 6e\zeta_nRe\tilde{F}^S\tilde{F}^n + 6e\zeta_nRe\tilde{F}^T\tilde{F}^n \right),
\]

\[
A = -\frac{1}{3 + e} \left((3 - 2e)\tilde{F}^S + 3e\tilde{F}^T
\right.
\]

\[
+ (3e\zeta_n - (3 + e)\zeta K_{5,n})\tilde{F}^m \right),
\]

where

\[
V_0 = \left|\frac{F^S}{S + S}\right|^2 + 3\left|\frac{F^T}{T + T}\right|^2 + \tilde{F}^n F^m K_{5,nm} - 3M_{3/2}^2,
\]

and

\[
\tilde{F}^n = \frac{F^n}{\zeta},
\]

\[
\tilde{F}^S = \frac{F^S}{S + S}, \quad \tilde{F}^T = \frac{F^T}{T + T},
\]
and \( K_{5,n} = \partial K_5/\partial Z_n \), \( K_{5,m} = \partial^2 K_5/(\partial Z_n \partial Z_m) \), \( \zeta_n = \partial \zeta/\partial Z_n \), \( \zeta_{nm} = \partial^2 \zeta/(\partial Z_n \partial Z_m) \).

The original calculation is at order of \( \epsilon \). Therefore, to be consistent, we should keep the scale, gauge couplings, and soft terms at order of \( \epsilon \), because there may exist other high order correction ( \( \epsilon^2 \) and higher) to the Lagrangian, or the Kähler potential and gauge kinetic function. Considering \( \epsilon^n = 0 \) for \( n > 1 \), the following scale and gauge coupling equations need to be changed:

\[
\alpha_{\text{GUT}} = \frac{1 - e}{2V_p} \left(4\pi \kappa^2\right)^{2/3},
\]

\[
\alpha_{H} = \frac{1 - e_h}{2V_p} \left(4\pi \kappa^2\right)^{2/3},
\]

\[
M_H = (1 + e/6 - e_h/6)M_{\text{GUT}},
\]

\[
[\pi \rho_p]^{-1} = 8\pi \delta_{5b}(1 - e)(2\alpha_{\text{GUT}})^{-3/2} \frac{M_{\text{GUT}}^3}{M_{\text{Pl}}^2},
\]

and the soft terms at order \( \epsilon \) are:

\[
M_{1/2} = (1 - e)\bar{F}^S + e\bar{F}^T + e\bar{F}^n \zeta_n,
\]

\[
M_0^2 = V_0 + M_{3/2}^2 - \left(\frac{2}{3}e|\bar{F}^S|^2 + |\bar{F}^T|^2 - \frac{2}{3}eRe\bar{F}^S \bar{F}^T + \frac{1}{3}e\zeta_{nm} \bar{F}^m \bar{F}^n - \frac{2}{3}e\zeta_{n} Re\bar{F}^S \bar{F}^n + \frac{2}{3}e\zeta_{n} Re\bar{F}^T \bar{F}^n\right),
\]

\[
A = - \left((1 - e)\bar{F}^S + e\bar{F}^T + (e\zeta_n - \zeta K_{5,n}) \bar{F}^n\right).
\]

Obviously, if \( K_{5,n} = 0 \), we obtain \( M_{1/2} = -A \) in this case, too. For the scale and gauge couplings, because we consider small \( \epsilon \) or the small next order correction, the discussions are similar to those in [13, 14, 39] with small \( x \). And because too many parameters in the soft term expressions, we will not do the numerical analysis here, for we can just let \( M_{1/2}, M_0, A \) as free parameters.

### 3.2 Toy Compactification and its Phenomenology

In this section, we will consider the toy compactification in non-standard embedding. First, we consider the case without five-brane, which is similar to the case in subsection...
2.2. Therefore, we just write down the Kähler potential, gauge kinetic function and superpotential:

$$K = \hat{K} + \tilde{K}^O |C_O|^2 + \tilde{K}^H |C_H|^2 ,$$  \hspace{1cm} (138)

$$\hat{K} = - \ln [S + \bar{S}] - 3 \ln [T + \bar{T}] ,$$  \hspace{1cm} (139)

$$\tilde{K}^O = \frac{3}{T + T} \left( 1 + \frac{\epsilon \beta^0 (T + \bar{T})}{S + S} \right)^2 \left( 1 - \frac{\epsilon \beta^0 (T + \bar{T})}{S + S} \right) ,$$  \hspace{1cm} (140)

$$\tilde{K}^H = \frac{3}{T + T} \left( 1 + \frac{\epsilon \beta^0 (T + \bar{T})}{S + S} \right) \left( 1 - \frac{\epsilon \beta^0 (T + \bar{T})}{S + S} \right)^2 ,$$  \hspace{1cm} (141)

$$f^O_{\alpha \beta} = S \left( 1 + \frac{\epsilon \beta^0 T}{S} \right)^3 \delta_{\alpha \beta} ,$$  \hspace{1cm} (142)

$$Re f^H_{\alpha \beta} = S \left( 1 - \frac{\epsilon \beta^0 T}{S} \right)^3 \delta_{\alpha \beta} ,$$  \hspace{1cm} (143)

$$W_O = \left( 1 + \frac{\epsilon \beta^0 T}{S} \right)^{3/2} \left( 1 - \frac{\epsilon \beta^0 T}{S} \right)^{3/2} k d_{xy} C_{O}^x C_{O}^y C_{O}^z ,$$  \hspace{1cm} (144)

$$W_H = \left( 1 + \frac{\epsilon \beta^0 T}{S} \right)^{3/2} \left( 1 - \frac{\epsilon \beta^0 T}{S} \right)^{3/2} k d_{xy} C_{H}^x C_{H}^y C_{H}^z ,$$  \hspace{1cm} (145)

$$W_{np} = h \exp \left( - \frac{8 \pi^2}{C_2(G^H)} S \left( 1 - \frac{\beta T}{S} \right)^3 \right) .$$  \hspace{1cm} (146)

The soft terms in the observable sector are similar to those in the subsection 2.2. The only difference between here and subsection 2.2 is that in section 2.2, the gauge group in the hidden sector is $E_8$, but here the gauge group in the hidden sector is the subgroup of $E_8$.

Now, we consider the case with five-brane. The next order metric is the following [31]:

$$ds^2 = (1 + y_{5b}) g_{\mu \nu}^{(0)} dx^\mu dx^\nu + (1 - y_{5b}) e^{2\alpha} g_{AB}^{(0)} dx^A dx^B + (1 - 2y_{5b}) e^{2\epsilon} (dx^{11})^2 .$$  \hspace{1cm} (147)
M-theory limit is: \( y_{5b} = \frac{1}{2} \) or \( y_{5b} = -1 \). In order to keep \( g_{11,11} \) positive, we require that \( y_{5b} < \frac{1}{2} \), and in order to keep the signature of metric \( g_{\mu\nu} \) invariant, we require that \( y_{5b} > -1 \). In short, we obtain: \(-1 < y_{5b} < \frac{1}{2}\). The physical Calabi-Yau manifold’s volume is obvious non-zero at any point along the eleventh dimension, and if one defined \( V^\text{min}_p \) and \( V^\text{max}_p \) as the minimum and maximum physical Calabi-Yau manifold’s volume along the eleventh dimension, respectively, one obtains that:

\[
V^\text{max}_p < 64 V^\text{min}_p. \tag{148}
\]

Using same ansatz in section 2.2, we obtain the Kähler potential, gauge kinetic function and the superpotential:

\[
K = \hat{K} + \tilde{K}^O |C_O|^2 + \tilde{K}^H |C_H|^2, \tag{149}
\]

\[
\hat{K} = -\ln [S + \bar{S}] - 3 \ln [T + \bar{T}] + K_5, \tag{150}
\]

\[
\tilde{K}^O = \frac{3}{T + T} \left( 1 + \frac{\epsilon (T + \bar{T})}{S + S} \sum_{m=0}^{N+1} \left( 1 - \frac{1}{2} (Z_m + \bar{Z}_m) \right)^2 \beta^m \right) \left( 1 - \frac{\epsilon (T + \bar{T})}{S + S} \sum_{m=0}^{N+1} \left( 1 - \frac{1}{2} (Z_m + \bar{Z}_m) \right)^2 \beta^m \right), \tag{151}
\]

\[
\tilde{K}^H = \frac{3}{T + T} \left( 1 + \frac{\epsilon (T + \bar{T})}{S + S} \sum_{m=0}^{N+1} \left( \frac{1}{2} (Z_m + \bar{Z}_m) \right)^2 \beta^m \right) \left( 1 - \frac{\epsilon (T + \bar{T})}{S + S} \sum_{m=0}^{N+1} \left( \frac{1}{2} (Z_m + \bar{Z}_m) \right)^2 \beta^m \right), \tag{152}
\]

\[
f_{\alpha\beta}^O = S \left( 1 + \frac{\epsilon T}{S} \sum_{m=0}^{N+1} (1 - Z_m)^2 \beta^m \right) \delta_{\alpha\beta}, \tag{153}
\]

\[
f_{\alpha\beta}^H = S \left( 1 + \frac{\epsilon T}{S} \sum_{m=0}^{N+1} Z_m^2 \beta^m \right) \delta_{\alpha\beta}, \tag{154}
\]

\[
W_{np} = h \exp \left( -\frac{8\pi^2}{C_2(G_H^4)} S \left( 1 + \frac{\epsilon T}{S} \sum_{m=0}^{N+1} Z_m^2 \beta^m \right) \right). \tag{155}
\]

With the assumption \(< Z_n > = < \bar{Z}_n >, < S > = < \bar{S} >, \text{ and } < T > = < \bar{T} >,\) we discuss the scale and gauge couplings first,

\[
8\pi G_N^{(4)} = \frac{k^2}{2\pi \rho_p V_p \delta_{5b}}, \tag{156}
\]
\[ \alpha_{\text{GUT}} = \frac{1}{2V_p(1+e^T)^3} (4\pi\kappa^2)^{2/3}, \quad (157) \]

\[ \alpha_{H} = \frac{1}{2V_p(1+e_h^T)^3} (4\pi\kappa^2)^{2/3}, \quad (158) \]

where

\[ e^T = \frac{\varepsilon(T + \bar{T})}{S + S} \sum_{m=0}^{N+1} \left(1 - \frac{1}{2}(Z_m + \bar{Z}_m)\right)^2 \beta^m, \quad (159) \]

\[ e_h^T = \frac{\varepsilon(T + \bar{T})}{S + S} \sum_{m=0}^{N+1} \left(\frac{1}{2}(Z_m + \bar{Z}_m)\right)^2 \beta^m, \quad (160) \]

\[ \delta_{5b}^T = \frac{\int_0^{\pi \rho_p} \sqrt{1 - 2y_{5b}(1 + y_{5b})^2(1 - y_{5b})^3} dx^{11}}{\int_0^{\pi \rho_p} \sqrt{1 - 2y_{5b}} dx^{11}}. \quad (161) \]

The GUT scale \( M_{\text{GUT}} \) and the hidden sector GUT scale \( M_H \) when the Calabi-Yau manifold is compactified are:

\[ M_{-6}^{-6} = V_p(1+e^T)^3, \quad (162) \]

\[ M_{H}^{-6} = V_p(1+e_h^T)^3, \quad (163) \]

or we can express the \( M_H \) as:

\[ M_H = \left(\frac{1+e_h^T}{1+e_h^T}\right)^{1/2} M_{\text{GUT}}, \quad (164) \]

\[ M_{11} = \left[2(4\pi)^{-2/3} \alpha_{\text{GUT}}\right]^{-1/6} M_{\text{GUT}}. \quad (165) \]

And the physical scale of the eleventh dimension is:

\[ [\pi \rho_p]^{-1} = \frac{8\pi \delta_{5b}^T}{(1+e^T)^3} (2\alpha_{\text{GUT}})^{-3/2} \frac{M_{\text{GUT}}^3}{M_{Pl}^2}. \quad (166) \]

In order to keep \( g_{11,11} \) and \( g_{1,1} \) positive, we require that \(-1 < \bar{y}_{5b} < \frac{1}{2}\). So, we obtain:

\[ -\frac{1}{2} < e^T < 1, \quad -\frac{1}{2} < e_h^T < 1. \quad (167) \]
And the bounds on $M_H$ and $\alpha_H$ are:

$$\frac{1}{64} \alpha_{GUT} < \alpha_H < 64 \alpha_{GUT}, \quad (168)$$

$$\frac{1}{2} M_{GUT} < M_H < 2 M_{GUT}. \quad (169)$$

In addition, the discussions of the scale and gauge couplings are similar to those in [14, 15, 39] with small $x$ if we defined the physical eleventh dimension length as $\pi \rho \delta_{Sb}$. So, we will not redo the discussions here again. Furthermore, the $\delta_{Sb}$ might be at about 1 because of the constraints on $\beta^i$ and the charge distributions.

In addition, we obtain the following normalized soft terms:

$$M_{1/2} = \frac{1}{S + S + \epsilon(T + \bar{T})\Omega} \left( F^S \left( 1 - 2 \frac{\epsilon(T + \bar{T})\Omega}{S + S} \right) + 3 \epsilon F^T \Omega - 3 \epsilon F^n(T + \bar{T})\Omega_n \right), \quad (170)$$

$$M_0^2 = M_{3/2}^2 + V_0^2 - |F^S|^2 \left( \frac{\tilde{K}_{SS}^0}{K^0} - \frac{\tilde{K}_S^0 \tilde{K}_\bar{S}^0}{K_{S}^0 K_{\bar{S}}^0} \right) - |F^T|^2 \left( \frac{\tilde{K}_{TT}^0}{K^0} - \frac{\tilde{K}_T^0 \tilde{K}_{\bar{T}}^0}{K_{T}^0 K_{\bar{T}}^0} \right) - F^n \left( \frac{\tilde{K}_{mn}^0}{K^0} - \frac{\tilde{K}_m^0 \tilde{K}_n^0}{K_{m}^0 K_{n}^0} \right) F^m$$

$$- 2 \left( \frac{\tilde{K}_{ST}^0}{K^0} - \frac{\tilde{K}_S^0 \tilde{K}_T^0}{K_{S}^0 K_{T}^0} \right) \text{Re}(\bar{F}^T F^S) - 2 \left( \frac{\tilde{K}_{Sn}^0}{K^0} - \frac{\tilde{K}_S^0 \tilde{K}_n^0}{K_{S}^0 K_{n}^0} \right) \text{Re}(\bar{F}^n F^S)$$

$$- 2 \left( \frac{\tilde{K}_{Tn}^0}{K^0} - \frac{\tilde{K}_T^0 \tilde{K}_n^0}{K_{T}^0 K_{n}^0} \right) \text{Re}(\bar{F}^n F^T), \quad (171)$$

$$A = F^S \left( \frac{1}{S + S} - \frac{3\tilde{K}_S^0}{K^0} \right) + F^T \left( \frac{3}{T + \bar{T}} - \frac{3\tilde{K}_T^0}{K^0} \right) + F^n \left( K_{5,n} - \frac{3\tilde{K}_S^0}{K^0} \right), \quad (172)$$

where

$$\tilde{K}_S^0 = 3 \left( -\frac{\epsilon \Omega}{(S + S)^2} + 2 \frac{\epsilon^2 (T + \bar{T})\Omega^2}{(S + S)^3} + 3 \frac{\epsilon^3 (T + \bar{T})^2 \Omega^3}{(S + S)^4} \right), \quad (173)$$

$$\tilde{K}_{SS}^0 = 6 \left( \frac{\epsilon \Omega}{(S + S)^2} - 3 \frac{\epsilon^2 (T + \bar{T})\Omega^2}{(S + S)^3} - 6 \frac{\epsilon^3 (T + \bar{T})^2 \Omega^3}{(S + S)^4} \right), \quad (174)$$
\[ \tilde{K}^O_{ST} = 6 \left( \frac{\epsilon^2 \Omega^2}{(S + S)^3} + 3 \frac{\epsilon^3(T + \bar{T})\Omega^3}{(S + S)^4} \right), \]  
(175)

\[ \tilde{K}^O_{S\bar{n}} = 3 \left( \frac{\epsilon \Omega_n}{(S + S)^2} - 4 \frac{\epsilon^2(T + \bar{T})\Omega \Omega_n}{(S + S)^3} - 9 \frac{\epsilon^3(T + \bar{T})^2 \Omega^2 \Omega_n}{(S + S)^4} \right), \]  
(176)

\[ \tilde{K}^O_T = 3 \left( -\frac{1}{(T + \bar{T})^2} - \frac{\epsilon^2 \Omega^2}{(S + S)^2} - 2 \frac{\epsilon^3(T + \bar{T})\Omega^3}{(S + S)^3} \right), \]  
(177)

\[ \tilde{K}^O_{TT} = 6 \left( \frac{1}{(T + \bar{T})^3} - \frac{\epsilon^3 \Omega^3}{(S + S)^3} \right), \]  
(178)

\[ \tilde{K}^O_{T\bar{n}} = 6 \left( \frac{\epsilon^2 \Omega \Omega_n}{(S + S)^2} + 3 \frac{\epsilon^3(T + \bar{T})\Omega^2 \Omega_n}{(S + S)^3} \right), \]  
(179)

\[ \tilde{K}^O_n = 3 \left( -\frac{\epsilon \Omega_n}{S + S} + 2 \frac{\epsilon^2(T + \bar{T})\Omega \Omega_n}{(S + S)^2} + 3 \frac{\epsilon^3(T + \bar{T})^2 \Omega^2 \Omega_n}{(S + S)^3} \right), \]  
(180)

\[ \tilde{K}^O_{n\bar{l}} = 3 \left( \frac{\epsilon \beta^n}{2(S + S)} - \frac{\epsilon^2(T + \bar{T})\Omega \beta^n}{(S + S)^2} - 3 \frac{\epsilon^3(T + \bar{T})^2 \Omega^2 \beta^n}{(S + S)^3} \right) \delta_{nl} \]  
\[ + 6 \left( -\frac{\epsilon^2(T + \bar{T})\Omega \Omega_n \Omega_l}{(S + S)^2} - 3 \frac{\epsilon^3(T + \bar{T})^2 \Omega^2 \Omega_n \Omega_l}{(S + S)^3} \right), \]  
(181)

and

\[ \Omega = \sum_{m=0}^{N+1} \left( 1 - \frac{1}{2} (Z_m + \bar{Z}_m) \right)^2 \beta^m, \]  
(182)

\[ \Omega_n = \left( 1 - \frac{1}{2} (Z_n + \bar{Z}_n) \right) \beta^n. \]  
(183)

Because we introduce the new parameters $\beta^i$ where $i=1, N$ if we included five branes, we have a lot of freedom in phenomenology discussions. We do not do the numerical analysis of the soft terms because we just have three soft term parameters $M_{1/2}, M_0^2,$ and $A$, and we can make them as free parameters by varying $\beta^i$ and $\epsilon$. 

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4 The Physical Calabi-Yau Manifold’s Volume

In above toy compactifications, we notice that the physical Calabi-Yau manifold’s volume at any point along the eleventh dimension cannot be zero from the metric directly. And we can argue that this may be the general result. The next order metric can be written as \[ds^2 = (1 + \frac{\sqrt{2}}{6} B)g_{\mu\nu}^{(0)}dx^\mu dx^\nu + \left(1 - \frac{\sqrt{2}}{3} B\right)g_{AB}^{(0)} + \sqrt{2}B_{AB} \]

\[+ (1 - \frac{\sqrt{2}}{3} B)(dx^{11})^2,\]

(184)

where \(\partial_{[I_1 I_2...I_7]} = \frac{1}{7} \epsilon G_{I_1...I_7}\), and \(B_{ab}\) is defined as following \[19\]:

\(B_{\mu\nu\rho\sigma a\bar{b}} = \epsilon_{\mu\nu\rho\sigma}B_{a\bar{b}},\)

(185)

and \(B = B^{(0)AB}B_{AB}\). Because the massive modes are suppressed, \(B_{AB}\) can be written as linear function of the massless modes \[19\]. So, \(\frac{\sqrt{2}}{3}Bg_{AB}^{(0)}\) and \(\sqrt{2}B_{AB}\) have the same sign. In addition, the magnitude of \(\frac{\sqrt{2}}{3}Bg_{AB}^{(0)}\) is larger than that of \(\sqrt{2}B_{AB}\); for example, in the toy compactification, the magnitude of \(\sqrt{2}B_{AB}\) is half of that of \(\frac{\sqrt{2}}{3}Bg_{AB}^{(0)}\) which can be easily proved at next order in general. Therefore, in order to keep \(g_{11,11}\) positive, we may not push the physical Calabi-Yau manifold’s volume to zero at any point along the eleventh dimension.

5 Conclusion

In M-theory on \(S^1/Z_2\), we point out that to be consistant, we should keep the scale, gauge couplings and soft terms at next order in the standard embedding and non-standard embedding, and obtain the soft term relations \(M_{1/2} = -A\), \(|M_0/M_{1/2}| \leq 1/\sqrt{3}\) in the standard embedding and soft term relation \(M_{1/2} = -A\) in non-standard embedding with five branes and \(K_{5,n} = 0\). Furthermore, we construct a toy compactification model which includes higher order terms in 4-dimensional Lagrangian in standard embedding, and discuss its scale, gauge couplings, soft terms, and explicitly show that the higher order terms do affect the scale, gauge couplings and especially the soft terms if the next order correction was not small. We also construct a toy compactification model in non-standard embedding with five branes, calculate its Kähler potential, gauge kinetic function, and discuss the scale, gauge couplings and soft terms. And M-theory limit gives strong constraints on the gauge coupling \(\alpha_H\) and GUT scale \(M_H\) in the hidden sector in these toy models. Finally, we argue that, in general, we might not push the physical Calabi-Yau manifold’s volume to zero at any point along the eleventh dimension.

The phenomenology consequences from the soft terms obtained in this paper and the multi moduli toy compactification are under investigation.
**Acknowledgments**

This research was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-95ER40896 and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.
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Figure 1: $M_0/M_{1/2}$ versus $\theta$. Solid line, dots line, dotdash, dashes line represent $x=0.05, 0.1, 0.2, 0.3$, respectively.
Figure 2: $\delta$ versus $y$. 
Figure 3: $M_{1/2}$ versus $x$ in the unit of gravitino mass. Solid line, dots line, dashes line represent scenario O, S, T, respectively.
Figure 4: $M_0$ versus $x$ in the unit of gravitino mass. Solid line, dots line, dashes line represent scenario O, S, T, respectively.
Figure 5: $A$ versus $x$ in the unit of gravitino mass. Solid line, dots line, dashes line represent scenario O, S, T, respectively. 33
Figure 6: Soft terms versus angle $\theta$ with $y=0.15$ in the unit of gravitino mass.
Figure 7: Soft terms versus angle $\theta$ with $y=0.2$ in the unit of gravitino mass.
Figure 8: Soft terms versus angle $\theta$ with $y=0.3$ in the unit of gravitino mass.
Figure 9: Soft terms versus angle $\theta$ with $y=0.4$ in the unit of gravitino mass.
Figure 10: Soft terms versus angle $\theta$ with $y = 0.5$ in the unit of gravitino mass.
Figure 11: Soft terms versus angle $\theta$ with $y=-0.3$ in the unit of gravitino mass.