EXTRAORDINARY DIMENSION OF MAPS

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Abstract. We establish a characterization of the extraordinary dimension of perfect maps between metrizable spaces.

1. Introduction

The paper deals with extensional dimension of maps, specially, with the extraordinary dimension introduced recently by Ščepin [10] and studied by the first author in [1]. If $L$ is a CW-complex and $X$ a metrizable space, we write $e\dim X \leq L$ provided $L$ is an absolute extensor for $X$ (in such a case we say that the extensional dimension of $X$ is $\leq L$, see [3], [4]). The extraordinary dimension of $X$ generated by a complex $L$, notation $\dim L X$, is the smallest integer $n$ such that $e\dim X \leq \Sigma^n L$, where $\Sigma^n L$ is the $n$-th iterated suspension of $L$ (by $\Sigma^0 L$ we always denote the complex $L$ itself). If $L$ is the 0-dimensional sphere $S^0$, then $\dim L$ coincides with the covering dimension $\dim$. We also write $\dim L f \leq n$, where $f : X \to Y$ is a given map, provided $\dim L f^{-1}(y) \leq n$ for every $y \in Y$. Next is our main result.

Theorem 1.1. Let $f : X \to Y$ be a $\sigma$-perfect map of metrizable spaces, let $L$ be a CW-complex and $n \geq 1$. Consider the following properties:

1. $\dim L f \leq n$;
2. There exists an $F_\sigma$ subset $A$ of $X$ such that $\dim L A \leq n - 1$ and the restriction map $f|(X \setminus A)$ is of dimension $\dim L f|(X \setminus A) = 0$;
3. There exists a dense and $G_\delta$ subset $\mathcal{G}$ of $C(X, I^n)$ with the source limitation topology such that $\dim_L (f \times g) = 0$ for every $g \in \mathcal{G}$;
3′ There exists a map $g : X \to I^n$ is such that $\dim_L (f \times g) = 0$.

Then (3) $\Rightarrow$ (3′) $\Rightarrow$ (1) and (3′) $\Rightarrow$ (2). Moreover, (1) $\Rightarrow$ (3) provided $Y$ is a $C$-space and $L$ is countable.

Here, $f : X \to Y$ is $\sigma$-perfect if $X$ is the union of countably many closed sets $X_i$ such that $f(X_i) \subset Y$ are closed and the restriction maps $f|X_i$ are perfect.

Theorem 1.1 is inspired by the following result of M. Levin and W. Lewis [7, Theorem 1.8]: If $X$ and $Y$ are metrizable compacta then (3) $\Rightarrow$ (3′) $\Rightarrow$ (1) and
$\Rightarrow (2') \Rightarrow (1)$, where $(2')$ is obtained from our condition $(2)$ by replacing $\dim_L f|(X \setminus A) \leq 0$ with $\dim f|(X \setminus A) \leq 0$. Moreover, the implication $(1) \Rightarrow (3)$ was also established in [7] for a finite-dimensional compactum $Y$ and a countable $CW$-complex $L$.

Therefore, we have the following characterization of extraordinary dimension of perfect maps between metrizable spaces:

**Corollary 1.2.** Let $f : X \to Y$ be a perfect surjection between metrizable spaces with $Y$ being a $C$-space. If $L$ is a countable $CW$-complex, then the following conditions are equivalent:

1. $\dim_L f \leq n$;
2. There exists a dense and $G_\delta$ subset $G$ of $C(X, \mathbb{I}^n)$ with the source limitation topology such that $\dim_L (f \times g) \leq 0$ for every $g \in G$;
3. There exists a map $g : X \to \mathbb{I}^n$ such that $\dim_L (f \times g) \leq 0$.

If, in addition, $X$ is compact, then each of the above three conditions is equivalent to the following one:

4. There exists an $F_\sigma$ set $A \subset X$ such that $\dim_L A \leq n - 1$ and the restriction map $f|(X \setminus A)$ is of dimension $\dim f|(X \setminus A) \leq 0$.

The equivalence of the first three conditions follow from Theorem 1.1. More precisely, by Theorem 1.1 we have the following implications: $(2) \Rightarrow (3) \Rightarrow (1) \Rightarrow (2)$. When $X$ is compact, the result of Levin-Lewis which was mentioned above yields that $(2) \Rightarrow (4) \Rightarrow (1)$. Therefore, combining the last two chains of implications, we can obtain the compact version of Corollary 1.2.

Corollary 1.2 is a parametric version of [1, Theorem 4.9]. For the covering dimension $\dim$ such a characterization was obtained by Pasyukov [9] and Toruńczyk [11] in the realm of finite-dimensional compact metric spaces and extended in [12] to perfect maps between metrizable $C$-spaces. Since the class of $C$-spaces contains the class of finite-dimensional ones as a proper subclass (see [5]), the compact version of Corollary 1.2 is more general than the Levin-Lewis result [7, Theorem 1.8]. It is interesting to know whether all the conditions $(1)$-$(4)$ in Corollary 1.2 remain equivalent without the compactness requirement on $X$ and $Y$.

The source limitation topology on $C(X, M)$, where $(M, d)$ is a metric space, can be described as follows: a subset $U \subset C(X, M)$ is open if for every $g \in U$ there exists a continuous function $\alpha : X \to (0, \infty)$ such that $B(g, \alpha) \subset U$. Here, $B(g, \alpha)$ denotes the set \{ $h \in C(X, M) : d(g(x), h(x)) \leq \alpha(x)$ for each $x \in X$ \}. The source limitation topology doesn’t depend on the metric $d$ if $X$ is paracompact and $C(X, M)$ with this topology has the Baire property provided $(M, d)$ is a complete metric space. Moreover, if $X$ is compact, then the source limitation topology coincides with the uniform convergence topology generated by $d$. 
All function spaces in this paper, if not explicitly stated otherwise, are equipped with the source limitation topology.

2. Some preliminary results

Throughout this section $K$ is a closed and convex subset of a given Banach space $E$ and $f: X \to Y$ a perfect map with $X$ and $Y$ paracompact spaces. Suppose that for every $y \in Y$ we are given a property $\mathcal{P}(y)$ of maps $h: f^{-1}(y) \to K$ and let $\mathcal{P} = \{ \mathcal{P}(y) : y \in Y \}$. By $C_{\mathcal{P}}(X|H,K)$ we denote the set of all bounded maps $g: X \to K$ such that $g|f^{-1}(y)$ has the property $\mathcal{P}(y)$ for every $y \in H$, where $H \subset Y$. We also consider the set-valued map $\psi_{\mathcal{P}}: Y \to 2^{C^*(X,K)}$, defined by the formula $\psi_{\mathcal{P}}(y) = C^*(X,K) \setminus C_{\mathcal{P}}(X|\{y\},K)$, where $C^*(X,K)$ is the space of bounded maps from $X$ into $K$.

Lemma 2.1. Suppose that $\mathcal{P} = \{ \mathcal{P}(y) \}_{y \in Y}$ is a family of properties satisfying the following conditions:

(a) $C_{\mathcal{P}}(X|H,K)$ is open in $C^*(X,K)$ with respect to the source limitation topology for every closed $H \subset Y$;

(b) $g \in C_{\mathcal{P}}(X|\{y\},K)$ implies $g \in C_{\mathcal{P}}(X|U,K)$ for some neighborhood $U$ of $y$ in $Y$.

Then the map $\psi_{\mathcal{P}}$ has a closed graph provided $C^*(X,K)$ is equipped with the uniform convergence topology.

Proof. The proof of this lemma follows the arguments from the proof of [12, Lemma 2.6].

Recall that a closed subset $F$ of the metrizable space $M$ is said to be a $Z_m$-set in $M$, if the set $C(\mathbb{I}^m, M \setminus F)$ is dense in $C(\mathbb{I}^m, M)$ with respect to the uniform convergence topology, where $\mathbb{I}^m$ is the $m$-dimensional cube. If $F$ is a $Z_m$-set in $M$ for every $m \in \mathbb{N}$, we say that $F$ is a $Z$-set in $M$.

Lemma 2.2. Suppose $y \in Y$ and $\mathcal{P}(y)$ satisfy the following condition:

• For every $m \in \mathbb{N}$ the set of all maps $h \in C(\mathbb{I}^m \times f^{-1}(y), K)$ with each $h|\{z \times f^{-1}(y)\}$, $z \in \mathbb{I}^m$, having the property $\mathcal{P}(y)$ (as a map from $f^{-1}(y)$ into $K$) is dense in $C(\mathbb{I}^m \times f^{-1}(y), K)$ with respect to the uniform convergence topology.

Then, for every $\alpha: X \to (0, \infty)$ and $g \in C^*(X,K)$, $\psi_{\mathcal{P}}(y) \cap \overline{B}(g,\alpha)$ is a $Z$-set in $\overline{B}(g,\alpha)$ provided $\overline{B}(g,\alpha)$ is considered as subset of $C^*(X,K)$ equipped with the uniform convergence topology and $\psi_{\mathcal{P}}(y) \subset C^*(X,K)$ is closed.

Proof. See the proof of [12, Lemma 2.8]

Proposition 2.3. Let $Y$ be a $C$-space and $\mathcal{P} = \{ \mathcal{P}(y) \}_{y \in Y}$ such that:

(a) the map $\psi_{\mathcal{P}}$ has a closed graph;
(b) $\psi_p(y) \cap B(g, \alpha)$ is a $Z$-set in $B(g, \alpha)$ for any continuous function $\alpha: X \to (0, \infty)$, $y \in Y$ and $g \in C^*(X, K)$, where $B(g, \alpha)$ is considered as a subspace of $C^*(X, K)$ with the uniform convergence topology.

Then the set $\{g \in C^*(X, K) : g \in C_p(X\{y\}, K) \text{ for every } y \in Y\}$ is dense in $C^*(X, K)$ with respect to the source limitation topology.

Proof. Let $G = \{g \in C^*(X, K) : g \in C_p(X\{y\}, K) \text{ for every } y \in Y\}$. It suffices to show that, for fixed $g_0 \in C^*(X, K)$ and a positive continuous function $\alpha: X \to (0, \infty)$, there exists $g \in B(g_0, \alpha) \cap G$. We equip $C^*(X, K)$ with the uniform convergence topology and consider the constant (and hence, lower semi-continuous) convex-valued map $\phi: Y \to 2^{C^*(X, K)}$, $\phi(y) = B(g_0, \alpha_1)$, where $\alpha_1(x) = \min\{\alpha(x), 1\}$. Because of the conditions (a) and (b), we can apply the selection theorem [6, Theorem 1.1] to obtain a continuous map $h: Y \to C^*(X, K)$ such that $h(y) \in \phi(y) \setminus \psi_p(y)$ for every $y \in Y$. Observe that $h$ is a map from $Y$ into $B(g_0, \alpha_1)$ such that $h(y) \in C_p(X\{y\}, K)$ for every $y \in Y$. Then $g(x) = h(f(x))(x)$, $x \in X$, defines a bounded map $g \in B(g_0, \alpha)$ such that $g|f^{-1}(y) = h(y)|f^{-1}(y)$, $y \in Y$. Therefore, $g \in C_p(X\{y\}, K)$ for all $y \in Y$, i.e., $g \in B(g_0, \alpha) \cap G$.

3. Proof of Theorem 1.1

$(1) \Rightarrow (3)$ Suppose that $L$ is countable and $Y$ is a $C$-space. Let $X_i$ be closed subsets of $X$ such that each $f_i = f|X_i: X_i \to Y_i = f(X_i)$ is a perfect map and $Y_i$ is closed in $Y$. Then all $Y_i$’s are $C$-spaces, and since the restriction maps $\pi_i: C(X, \mathbb{I}^n) \to C(X_i, \mathbb{I}^n)$, $\pi_i(g) = g|X_i$, are open, the proof of this implication is reduced to the case when $f$ is a perfect map. Consequently, we may assume that $f$ is perfect.

By [13, Theorem 1.1] (see also [8]), there exists a map $g$ from $X$ into the Hilbert cube $Q$ such that $f \times g: X \to Y \times Q$ is an embedding. Let $\{W_i\}_{i \in \mathbb{N}}$ be a countable finitely-additive base for $Q$. For every $i$ we choose a sequence of mappings $h_{ij}: W_i \to L$, representing all the homotopy classes of mappings from $W_i$ to $L$ (this is possible because $L$ is a countable CW-complex). Following the notations from Section 2, for fixed $i$, $j$ and $y \in Y$ we say that a map $g \in C(X, \mathbb{I}^n)$ has the property $P_{ij}(y)$ provided the map $h_{ij} \circ q: q^{-1}(W_i) \to L$ can be continuously extended to a map over the set $q^{-1}(W_i) \cup (f^{-1}(y) \cap g^{-1}(t))$ for every $t \in g(f^{-1}(y))$.

Let $P_{ij} = \{P_{ij}(y) : y \in Y\}$ and for every $H \subset Y$ we denote $C_{P_{ij}}(X|H, \mathbb{I}^n)$ by $C_{ij}(X|H, \mathbb{I}^n)$. Hence, $C_{ij}(X|H, \mathbb{I}^n)$ consists of all $g \in C(X, \mathbb{I}^n)$ having the property $P_{ij}(y)$ for every $y \in H$. Let $\psi_{ij}: Y \to 2^{C(X, \mathbb{I}^n)}$ be the set-valued map $\psi_{ij}(y) = C(X, \mathbb{I}^n) \setminus C_{ij}(X\{y\}, \mathbb{I}^n)$.
Lemma 3.1. Let \( g \in C_{ij}(X|\{y\}, \mathbb{P}^n) \). Then, there exists a neighborhood \( U_y \) of \( y \) in \( Y \) and a neighborhood \( V_t \subseteq \mathbb{P}^n \) of each \( t \in g(f^{-1}(y)) \) such that \( h_{ij} \circ q \) can be extended to a map from \( q^{-1}(W_t) \cup (f^{-1}(U_y) \cap g^{-1}(V_t)) \) into \( L \).

Proof. Since \( g \in C_{ij}(X|\{y\}, \mathbb{P}^n) \), \( h_{ij} \circ q \) can be extended to a map from \( q^{-1}(W_t) \cup (f^{-1}(y) \cap g^{-1}(t)) \) into \( L \) for every \( t \in g(f^{-1}(y)) \). Because \( L \) is an absolute neighborhood extensor for \( X \), there exists and open set \( G_t \subseteq X \) containing \( f^{-1}(y) \cap g^{-1}(t) \) and a map \( h_t : q^{-1}(W_t) \cup G_t \to L \) extending \( h_{ij} \circ q \). Using that \( f \times g \) is a closed map, we can find a neighborhood \( U_y \times V_t \) of \( (y,t) \) in \( Y \times \mathbb{P}^n \) such that \( S_t = (f \times g)^{-1}(U_y \times V_t) \subseteq G_t \). Next, choose finitely many points \( t(k), k = 1, 2, \ldots, m \), with \( f^{-1}(y) \subseteq \bigcup_{k=1}^{m} S_{t(k)} \) and a neighborhood \( U_y \) of \( y \) in \( Y \) such that \( U_y \subseteq \bigcap_{k=1}^{m} U_y^{(k)} \) and \( f^{-1}(U_y) \subseteq \bigcup_{k=1}^{m} S_{t(k)} \) (this can be done since \( f \) is perfect). If \( t \in g(f^{-1}(y)) \), then \( t \in V_{t(k)} \) for some \( k \) and \( f^{-1}(U_y) \cap g^{-1}(V_{t(k)}) \subseteq S_{t(k)} \). Since \( S_{t(k)} \subseteq G_{t(k)} \), the map \( h_{t(k)} : S_{t(k)} \subseteq H \) is an extension of \( h_{ij} \circ q \) over the set \( q^{-1}(W_t) \cup (f^{-1}(U_y) \cap g^{-1}(V_{t(k)})) \). \( \square \)

Lemma 3.2. The set \( C_{ij}(X|H, \mathbb{P}^n) \) is open in \( C(X, \mathbb{P}^n) \) for any \( i, j \) and closed \( H \subseteq Y \).

Proof. We follow the proof of [12, Lemma 2.5]. For a fixed \( g_0 \in C_{ij}(X|H, \mathbb{P}^n) \) we are going to find a function \( \alpha : X \to (0, \infty) \) such that \( \overline{B(g_0, \alpha)} \subseteq C_{ij}(X|H, \mathbb{P}^n) \). By Lemma 3.1, for every \( z = (y, t) \in (f \times g_0)((f^{-1}(H))) \) there exists a neighborhood \( U_z \) in \( Y \times \mathbb{P}^n \) such that

1. \( h_{ij} \circ q \) can be extended to a map from \( q^{-1}(W_t) \cup (f \times g_0)^{-1}(U_z) \) into \( L \).

Obviously, \( K = (f \times g_0)((f^{-1}(H))) \) is closed in \( Y \times \mathbb{P}^n \), so there exists open \( G \subseteq Y \times \mathbb{P}^n \) with \( K \subseteq G \subseteq \overline{G} \subseteq U = \bigcup \{ U_z : z \in K \} \cup \{(Y \times \mathbb{P}^n) \setminus \overline{G} \} \) is an open cover of \( Y \times \mathbb{P}^n \). Let \( \gamma \) be an open locally finite cover of \( Y \times \mathbb{P}^n \) such that the family

2. \( \{ St(W, \gamma) : W \in \gamma \} \) refines \( \nu \) and \( St(W, \gamma) \subseteq G \) provided \( W \cap K \neq \emptyset \).

Consider the metric \( \rho = d + d_1 \) on \( Y \times \mathbb{P}^n \), where \( d \) is a metric on \( Y \) and \( d_1 \) the usual metric on \( \mathbb{P}^n \), and define the function \( \alpha : X \to (0, \infty) \) by \( \alpha(x) = 2^{-1} \sup \{ \rho((f \times g_0)(x), (Y \times \mathbb{P}^n) \setminus W) : W \in \gamma \} \). Let show that \( \overline{B(g_0, \alpha)} \subseteq C_{ij}(X|H, \mathbb{P}^n) \). Take \( g \in \overline{B(g_0, \alpha)} \), \( y \in H \) and \( t \in g(f^{-1}(y)) \). Then, \( (y, t) = (f \times g)(x) \) for some \( x \in f^{-1}(y) \). Since \( g \) is \( \alpha \)-close to \( g_0 \), there exists \( W \in \gamma \) such that \( W \cap K \neq \emptyset \) and \( W \) contains both \( (f \times g)(x) \) and \( (f \times g_0)(x) \). It follows from (2) that \( (f \times g)^{-1}(W) \subseteq (f \times g_0)^{-1}(U_z) \) for some \( z \in K \). In particular, \( f^{-1}(y) \cap g^{-1}(t) \subseteq (f \times g_0)^{-1}(U_z) \). Consequently, by (1), \( h_{ij} \circ q \) is extendable to a map from \( q^{-1}(W_t) \cup (f^{-1}(y) \cap g^{-1}(t)) \) into \( L \). Therefore, \( \overline{B(g_0, \alpha)} \subseteq C_{ij}(X|\{y\}, \mathbb{P}^n) \) for every \( y \in H \) which completes the proof. \( \square \)
Because of Lemma 3.1 and Lemma 3.2, we can apply Lemma 2.1 to obtain the following corollary.

**Corollary 3.3.** For any \( i \) and \( j \) the map \( \psi_{ij} \) has a closed graph.

**Lemma 3.4.** Let \( g \in C(X, \mathbb{I}^n) \), \( \alpha : X \to (0, \infty) \) and \( y \in Y \). Then, for any \( i, j \), \( \psi_{ij}(y) \cap \overline{B}(g, \alpha) \) is a \( Z \)-set in \( \overline{B}(g, \alpha) \) provided \( \overline{B}(g, \alpha) \) is considered as a subset of \( C(X, \mathbb{I}^n) \) with the uniform convergence topology.

**Proof.** It follows from [7, Theorem 1.8, (1) ⇒ (3)] that if \( m \in \mathbb{N} \), then all maps \( g : \mathbb{I}^m \times f^{-1}(y) \to \mathbb{I}^n \) such that \( \text{e-dim}((\{z\} \times f^{-1}(y)) \cap g^{-1}(t)) \leq L \) for every \( z \in \mathbb{I}^m \) and \( t \in \mathbb{I}^n \), form a dense subset \( G \) of \( C(\mathbb{I}^m \times f^{-1}(y)) \) with the uniform convergence topology. It is clear that, for every \( g \in G \) and \( z \in \mathbb{I}^m \), the restriction \( g|((\{z\} \times f^{-1}(y)) \), considered as a map on \( f^{-1}(y) \), has the following property: \( h_{ij} \circ q \) can be extended to a map from \( q^{-1}(\overline{W}_i) \cup (f^{-1}(y) \cap g^{-1}(t)) \) into \( L \) for any \( t \in \mathbb{I}^n \). Hence, we can apply Lemma 2.2 to conclude that \( \psi_{ij}(y) \cap \overline{B}(g, \alpha) \) is a \( Z \)-set in \( \overline{B}(g, \alpha) \). \( \square \)

Now, we can finish the proof of this implication. Because of Corollary 3.3 and Lemma 3.4, we can apply Proposition 2.3 to obtain that the set \( C_{ij} = C_{ij}(X|Y, \mathbb{I}^n) \) is dense in \( C(X, \mathbb{I}^n) \) for every \( i, j \). Since, by Lemma 3.2, all \( C_{ij} \) are also open, their intersection \( G \) is dense and \( G \) in \( C(X, \mathbb{I}^n) \). Let show that \( \dim(f \times g) \leq 0 \) for every \( g \in G \), i.e., \( \text{e-dim}(f \times g) \leq L \). We fix \( y \in Y \) and \( t \in \mathbb{I}^n \) and consider the fiber \( (f \times g)^{-1}(y, t) = f^{-1}(y) \cap g^{-1}(t) \). Take a closed set \( A \subset f^{-1}(y) \cap g^{-1}(t) \) and a map \( h : A \to L \). Because the map \( q_y = q|f^{-1}(y) \) is a homomorphism, \( h' = h \circ q_y^{-1} : q(A) \to L \) is well defined. Next, extend \( h' \) to a map from a neighborhood \( W \) of \( q(A) \) (in \( Q \)) into \( L \) and find \( W_i \) with \( q(A) \subset W_i \subset \overline{W}_i \subset W \). Therefore, there exists a map \( h'' : \overline{W}_i \to L \) extending \( h' \). Then \( h'' \) is homotopy equivalent to some \( h_{ij} \), so are \( h'' \circ q \) and \( h_{ij} \circ q \) (considered as maps from \( q^{-1}(\overline{W}_i) \) into \( L \)). Since \( h_{ij} \circ q \) can be extended to a map from \( q^{-1}(\overline{W}_i) \cup (f^{-1}(y) \cap g^{-1}(t)) \) into \( L \), by the Homotopy Extension Theorem, there exists a map \( h : q^{-1}(\overline{W}_i) \cup (f^{-1}(y) \cap g^{-1}(t)) \to L \) extending \( h'' \circ q \). Obviously, \( h|(f^{-1}(y) \cap g^{-1}(t)) \) extends \( h \). Hence, \( \text{e-dim}(f^{-1}(y) \cap g^{-1}(t)) \leq L \).

(3) ⇒ (3′) ⇒ (1) The implication (3) ⇒ (3′) is trivial. It is easily seen that in the proof of (3′) ⇒ (1) we can assume \( f \) is perfect. Let \( g : X \to \mathbb{I}^n \) be such that \( \dim_L(f \times g) \leq 0 \) and \( y \in Y \). Then the restriction \( g|f^{-1}(y) : f^{-1}(y) \to \mathbb{I}^n \) is a perfect map with all of its fibers having extensional dimension \( \text{e-dim} \leq L \). Hence, by [2, Corollary], \( \text{e-dim}f^{-1}(y) \leq \Sigma^n L \), i.e, \( \dim_L f \leq n \).

(3′) ⇒ (2) Because of the countable sum theorem, we can suppose that \( f \) is perfect. We fix a map \( g : X \to \mathbb{I}^n \) such that \( \dim_L(f \times g) \leq 0 \). According to [12, Lemma 4.1], there exists an \( F_\sigma \) subset \( B \subset Y \times \mathbb{I}^n \) such that \( \dim B \leq n - 1 \) and
dim(\{y\} \times \mathbb{I}^n) \setminus B \leq 0\) for every \(y \in Y\). Then, applying again [2, Corollary], we conclude that the set \(A = (f \times g)^{-1}(B)\) is as required.

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