Multilevel recursive model of properties existence constraints in machine learning

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Abstract. We considered an a priori knowledge that establishes how the facts of detection some properties in empirically observed objects affect the presence of others. Taking into account such constraints of the existence of properties in the objects of the domain of interest is necessary for the correct formation of the contexts of the tasks of machine learning in the conditions of incompleteness and inconsistency of the initial data. The paper presents a new multilevel recursive model of such constraints. The model is built on the basis of the point of view on the logical and conceptual origin of restrictions using elementary binary relations on a set of measured properties - incompatibility and conditionality.

1. Introduction

One of the defining entities of machine learning is the training sample. As a rule, it is given in the form of a multi-valued formal context (FC) [1-4]:

\[(G^*, M, V, I)\]

where \(G^*\) - is the set of observed objects; \(|G^*| \geq 1, G^* \subseteq G, G\) - all hypothetically conceivable set of objects of the domain of interest (DI); \(M\) - a priori defined set of measurable properties of the objects, \(|M| \geq 1; V\) - is the set of property values; \(I\) - is the ternary relation between \(G^*, M\) and \(V (I \subseteq G^* \times M \times V)\), defined for all pairs of \(G^* \times M\).

In many problems of machine learning, a multi-valued FC must first be transformed into single-valued \((G, M, I)\) [3-8], where \(I\) is the binary "object-property" correspondence, i.e. a collection of truth estimates \(\|b_{xy}\| \in \{\text{Truth}, \text{False}\}\) of basic semantic judgments (BSJ) about the DI of the form \(b_{xy} = "\text{object x has the property y}"\).

The transformation of a multi-valued FC into a single-valued one reduces to the replacement of the original table values by the truth constants Truth and False:

- the False constant replaces the None values - special measurement results of any property of the object of the DI, when the corresponding measurement procedure reveals either a
"semantic mismatch" with the object under study or the value of the measured property lies outside the dynamic range of the measuring instrument [9, 10];

- the **Truth** constant replaces all values except **None**.

From a general point of view, values equal to the information constant **None** appear in a multi-valued FC as a result of conceptual scaling of properties [11, 12]. Another generalizing circumstance is connected with the fact that the original multi-valued FC reflects the fundamental realities of accumulation of empirical information discussed in [3], predetermining its incompleteness and inconsistency.

Therefore, the transformation of a multi-valued FC into a single-valued FC in the general case involves two steps:

- the measured properties of the original multi-valued FC undergo conceptual scaling. The thus changed multi-valued context is transformed into a single-valued context using truth constants of one or another multi-valued logic, chosen to simulate the incompleteness and inconsistency of the data of a multi-valued FC [3]. The independent truth estimates that arise for each BSJ are combined according to the rules of the chosen multi-valued logic. In the final single-valued FC, the I correspondence "objects-properties" turns out to be "soft" (fuzzy, non-strict, etc. - the specification is determined by the terminology of the applied multi-valued logic - see, for example, [13, 14]);

- in a single-valued FC, the I fuzzy binary relation "object-property" is replaced by its section of the level \( \alpha \), where the truth value \( \alpha \) of the multi-valued logic used indicates a subjectively established confidence threshold to the original data.

The collision of these actions consists in the fact that the standard \( \alpha \)-section procedure does not take into account the dependence of the existence of some properties of objects on the existence of their other properties – properties existence constraints (PEC), which are usually found in the DI, and in any case arise as a result of the conceptual scaling of the properties of a multi-valued FC [15].

It is clear that the correct method of transformation of a multi-valued FC into an single-valued one should be based on an adequate PEC model. It is not enough just to state the existence of a system of measurable properties (SMP) – a set of measured properties with binary relations of incompatibility and conditionality defined on it, as it is done, for example, in [16, 17]. To construct an effective method of a correct \( \alpha \)-section of a fuzzy relation "objects-properties", additional knowledge of the structural organization of the SMP is required. In this sense, the two-level PEC model proposed earlier by the authors [18] reflects only one of the simplest, albeit common in practice situations.

This article describes the general PEC model developed by us, which is based on the concept of recursiveness of the "ontogeny" of PEC. The emergence of SIS is presented as the gradual unfolding of hypothetical ideas about the conceptual structure of the DI. The formation of hypotheses obeys the laws of classical logic. At the same time, it can be expressed in terms of conceptual scaling of measured properties, which is more natural in data analysis and machine learning problems.

2. **A priori conceptual notions about the domain of interest and ways of expanding the corresponding system of hypotheses**

It is obvious that the decision to measure the \( z \) property of objects of the DI is a consequence of the hypothesis about the presence in the DI of objects possessing such a property [10].

Conceptually, this means introducing an a priori assumption that the DI characterizes a set of hypothetical formal concepts that have a \( z \) property in their content. However, for only one of these hypothetical concepts, \( z \) will be a distinctive property [4, 10]. Formally, this concept can be described by a tuple

\[
\{ \ldots, z \}' \times \{ \ldots, z \},
\]

where \( \{ \ldots, z \} \) is the content, \( \{ \ldots, z \}' \times \) is the volume of the concept, \( \{ \ldots \} \) - is the designation of the set of inherited properties from the more general hypothetical formal concepts, \( \{ \} \) - is the Galois operator
of the binary "object-property" relation (I). Next to shorten the entries; we use the abridged formula $(\{z\}', \{z\})$ to denote the concept of interest.

In general, the number of distinctive properties of a hypothetical concept $ism \geq 1$, and in the introduced notation this concept is described by a tuple $(\{z\}', \{z\})$.

where $Z = \{z_1, z_2, ..., z_m\}$, $\{Z\}' = \{z_1, z_2, ..., z_m\}' = \{z\}' = \{z_2\}' = ... = \{z_m\}'$.

It is obvious that each measured property determines itself (the binary relation of conditioning is reflexive [16]), and the distinctive properties of $z_i \in Z$ of the formal concept are mutually conditioned.

Thus, the expansion of the set of measurable properties of $M$ means either the replenishment of the distinctive properties of the available hypothetical concepts of the DI, or the appearance of new similar concepts, i.e. change of a priori conceptual notions about DI. In the PEC aspect, in the first case, only sets of mutually conditioned properties are added. The second case requires special analysis.

In logic, only two possibilities for transforming concepts associated with the expansion of the set of features (properties) of the objects under study are known: division and restriction [19]. Therefore, when researching the genesis of PEC, it is necessary to obtain answers to the following questions:

- How is the division and restriction of a priori put forward hypothetical concepts about the DI practically realized?
- What are the dependencies of the existence of some properties of objects on the existence of their other properties formed as a result of operations of division and restriction?

2.1. Division of a hypothetical concept

The division of the concept is associated with the enumeration of all disjoint parts of its volume on a single base - the characteristic (the measured property), which is modified within the scope of the divisible volume [19].

Let the concept $(\{z\}', \{z\})$ be divisible. According to the introduced notation, this is a hypothetical formal concept with the distinctive property $z$ (the general case where the divisible concept is described by the set $Z$ of distinctive properties is tactically more complex and will be considered later).

And let the property $z$, whose values make up the domain $D_z$ (i.e. $\|z\| \in D_z$), play the role of the basis of division.

The "modification" of a property within the divisible volume $\{z\}'$ can naturally be interpreted as a characteristic change in the values of the property $z$ when passing between some disjunctive parts of the domain $D_z$ or (in terms of conceptual scaling) between disjunctive elements of the coverage of this domain, the number of which is not less than two:

$$D_z = \cup_{i=1}^{n} D_z(z_i),$$

where $z_i, i = 1, ..., n, n \geq 2$ – are the new measured properties that, as a result of performing the conceptual scaling procedure, replace the property $z$ in the set $M$; $D_z(z)$ is the coverage element, some non-empty part of the dynamic range of the procedure for measuring the property $z$, which is formed as the dynamic range of the procedure for measuring the property $z_i$, or the domain of the values of this property, $D_z(z) \cap D_z(z_i) = \emptyset$ for $i \neq j$. Next, such a conceptual scaling of the property, known in the literature as a nominal [11, 12], will be called disjunctive scaling.

Since the measured value of the property $z$ in the case under consideration will belong to one and only one domain $D_z(z_i)$, the new measurable properties $z_i, i = 1, ..., n$, entered into the SMP when dividing the hypothetical concept $(\{z\}', \{z\})$ will be incompatible.

In a priori hypotheses about the DI, the divisible concept $(\{z\}', \{z\})$ is replaced by a set of new concepts $(\{z\}', \{z\}), i = 1, ..., n$, for which $\{z\}' \cap \{z_i\}' = \emptyset$ for $i \neq j$ and $\cup_{i=1}^{n} \{z_i\}' = \{z\}'$. This will lead to an expansion of the generalization relation on the set of hypothetical concepts. The expansion will touch upon new concepts, the direct "ancestors" of the replaced concept and, generally speaking, all the "heirs" of the replaced concept.
2.2. Restriction of the hypothetical concept

The restriction of the concept means the introduction of a new concept, the volume of which is part of
the volume of the original concept, and the content is distinguished by an additional attribute (a new
measurable property), inherent only in that part of the objects conceivable in the original concept that
forms the volume of the new concept introduced [19].

In our notation, the restriction of the hypothetical concept (\{z\}', \{z\}) is the formal concept
(\{z, z_1\}', \{z, z_1\}), where z_1 is the new measurable property added to the set M, and \{z, z_1\}' \subseteq \{z\}'.

According to the definition of the restriction operation, the detection of the property z_1 in the object
of the DI is a reliable indication that this object belongs to the volume \{z\}'. But since all objects in
\{z\}' possess the property z, therefore, the presence of the property z_1 in the object under investigation
determines the presence of the property z.

The restriction of the concept (\{z\}', \{z\}) is realized by conceptual scaling of the property z, which
corresponds to the covering of the domain D_i in the above notations by two elements - the domain D,
its part D_i(z_1) \subseteq D_i. Further, such a conceptual scaling of the property will be referred to as a
refining scaling.

Generally speaking, it is possible to construct an arbitrary number of restrictions of the concept
(\{z\}', \{z\}), i.e. concepts of the form
(\{z, z_1\}', \{z, z_1\}),
where z_i, i = 1, ..., n, n \geq 1 — are the new measurable properties added to the set M,\{z, z_i\}' \subseteq \{z\}'. Note
that, \bigcap_{i=1}^{n} D_\zeta(z_i) = \emptyset, since for any intersection of domains of added properties, a new constraint
property can and must be "constructed".

The restricted concept (\{z\}', \{z\}) is preserved as part of the a priori assumptions about the DI and
becomes generic (generalizing) for all its species restrictions of the form (\{z, z_1\}', \{z, z_1\}), which
complement the hypothetical representations of the DI. The expansion of the generalization relation
will touch, and generally speaking, all the "heirs" of the concept, which has become generic for newly
introduced hypothetical concepts.

2.3. Expansion of a priori conceptual representations and the emergence of a system of measurable
properties

We state that the primary decision to measure in observable objects a certain property z defines the
initial hypothetical concept of the DI. Of course, the number of such initial hypotheses and the proto-
properties that determine them is arbitrary, and the set of corresponding hypothetical concepts can be
replenished.

As we found out, the second way of replenishing this set is the division or restriction of any of the
already existing hypothetical formal concepts. It is obvious that such an extension of the set of a priori
conceptual representations about the DI can be recursively continued. Each new formal concept,
generated as a result of a logical operation of division or restriction, may in turn be subject to division
or restriction according to the rules described above (and only thereon).

These two methods exhaust the strategy for the formation of a priori hypothetical concepts about
the DI that are built in the form of disconnected recursive hierarchies.

On the other hand, we have shown that the tactic here is to add new measurable properties:
• proto-properties;
• or properties that are "constructed" as a result of proper conceptual scaling of the distinctive
properties of already existing hypothetical concepts about the DI.

The latter defines a recursive procedure for replenishing a set of measurable properties. At the same
time, two and only two binary relations are formed on the set of measured properties: incompatibility
and conditionality. These relationships are the properties existence constraints (PEC). The set of
measured properties with the incompatibility and conditionality relations defined on it form a system
of measured properties (SMP).
We call the PEC model the description of the structural organization of a specific SMP. To build such models, it is necessary to identify rules that determine the emergence of the structural organization of SMP. As we shall establish further, these rules are conditioned by the method of replenishing a set of measurable properties in the course of the formation of a priori conceptual notions about the DI.

3. Property existence constraints model
So, our modeling is based on an understanding of the logical-conceptual genesis of PEC. With this approach, PEC is a consequence of the existence of developed hypothetical assumptions about the DI. Hypothetical concepts arise when performing a recursive procedure for replenishing a set of measured properties, so our PEC model will be multilevel and recursive. However, the result of the execution of the recursive procedure will depend on the "pre-history" - the incompatibility and conditionality relations into which the scaled measured property is already included. Since the number of variants of such "inclusion" is finite, for the definitive determination of the PEC model it suffices to consider the rules for changing the structural organization of PEC that arise when disjunctive and refining scaling for each such variant.

4. Patterns of the structural organization of a system of measured properties

4.1. Groups of conjugate properties
Figure 1 introduces a graphical notation of the entities we need.

- measured property
- group of conjugate properties
- single group of conjugate properties
- measured properties subjected to conceptual scaling
- the nesting of one group of conjugate properties into another
- group of mutually conditioned measured properties, or MC-group
- set of incompatible measured properties, or an I-group
- a pair of properties with conditionality arising as a result of the refinement scaling of the measured property (it is on the left), or the C-group
- pair of properties within compatibility
- pair of properties with conditionality (the left determines the right one)
- an example of a mixed group of conjugate properties, or M-group

Figure 1. The main entities of the task of forming a structural organization systems of measured properties and their designation (the «≡» sign is used to indicate the equivalence of structural elements and images).
Among these entities, the central concept is the "group of conjugate properties" (GCP):

- the measured property is a single GCP, the only member of which is self-imposed. As a graphic designation of a single GCP, the property icon itself can be used, and in any case the "loop of self-conditioning" in the diagrams will only be implied;
- GCP form all the distinctive properties of a hypothetical formal concept. The members of such a GCP are mutually conditioned;
- the set of measured properties introduced in M as a result of the disjunctive scaling of the measured property forms a GCP whose members are incompatible;
- GCP form a measurable property subjected to refining scaling, together with the measured property introduced in M as a result of this scaling. In such a GCP, the first of these properties are conditioned by the second.

It is easy to see that such a definition of the GCP determines the nesting/substitution relation on the GCP set. In particular:

- GCP, which includes the proto-property or the set of distinctive properties of a hypothetical formal concept, is not nested in any GCP (figure 2a);
- GCP, which includes some part of the set of distinctive properties of the hypothetical formal concept, is nested in the GCP of all these distinctive properties and replaces in it the specified part of the distinctive properties (figure 2b);
- GCP, including a set of measurable properties introduced in M as a result of disjunctive scaling, is nested in the GCP containing the scaled property and replaces it (figure 2c);
- the GCP, including the measured property subjected to refining scaling, together with the measured property introduced in M as a result of this scaling, is nested in the GCP containing the scaled property and replaces it (figure 2d).

The term "substitution" as applied to the GCP emphasizes that on the GCP set as well as on the set of measured properties are defined the incompatibility and conditionality relations.

**Figure 2.** Nesting/substitution of group of conjugate properties.

4.2. Scaling of proto-property

4.2.1. Disjunctive scaling of proto-property. The structural pattern of conjugation of measured properties (PCP) with this scaling unambiguously follows from the definition of the division operation of a concept whose content is exhausted by proto-properties. In figure 3a, in a single GCP, the proto-property is replaced by a group of \( n \geq 2 \) new incompatible measured properties.

4.2.2. Refining scaling of proto-property. The PSP in this case also follows directly from the definition of the restrict operation and is illustrated in figure 3b.

The PSP in figure 3b corresponds to a typical case where the restriction is made by introducing a new single measurable property that becomes the only distinctive property of the new formal sub-concept. However, in the general case, the resulting sub-concept must be characterized by more than one distinctive property \( \{z_1, z_2, \ldots, z_m\}, \ m > 1 \). These new measured properties are mutually conditioned. Therefore, the PSP corresponding to this case acquires two levels as shown in figure 3c.

Another rule for refining scaling of proto-property is that the original concept can be restricted to the formation of \( n \geq 2 \) sub-concepts. The necessary \( n \) new refinement measurable properties for this
purposes must be incompatible (see subsection 2.2). Therefore, the third PSP also has two levels (figure 3d).

In general, the PSP for the refinement scaling of the proto-property acquires a three-level view, shown in figure 3e.

![Diagram showing structural patterns of conjugation of measured properties in the scaling of proto-property.](image)

**Figure 3.** Structural patterns of conjugation of measured properties in the scaling of proto-property.

4.3. **Scaling of the unconditioned measured property**

Unconditioned measured properties are added, for example, in the disjunctive scaling of proto-properties (figure 3a) or as a result of refining scaling (figure 3b).

4.3.1. **Disjunctive scaling of the unconditioned measured property.** When one of the m properties of the I-group is replaced by n new measured properties, no new GCPs are created. The PSP fixes in this case the expansion of the composition of the existing I-group to \((m - 1) + n\) properties (figure 4).

![Diagram showing structural pattern of conjugation of measured properties at disjunctive scaling of the unconditioned measured property.](image)

**Figure 4.** Structural pattern of conjugation of measured properties at disjunctive scaling of the unconditioned measured property.

Disjunctive scaling of the unconditioned measured property, when it itself is a conditioning property (figure 3b), has actually been considered. The resulting PSP is an easily identifiable part of the structural patterns in figures 3d and 3e.

4.3.2. **Refining scaling of the unconditioned measured property.** The unconditionedness of the scaled measured property means that there are no any restraining existential limitations for refining scaling. Therefore, the PSPs that arise when scaling the unconditioned measured property are equivalent to the patterns that arise at refining scaling of proto-property. Thus, these PSPs are easily identifiable parts of the structural patterns presented in Figures 3b-3e, where an unconditioned measured property from an I-group will appear instead of the proto-property.
4.4. Scaling of the conditioned measured property

Let us, for the time being, consider the case when the property being scaled is not mutually conditioned with other properties. Then it can be argued that the conditionality of the measured property indicates that it has already been subjected to refinement scaling (see figure 3).

4.4.1. Disjunctive scaling of the conditioned measured property. Generally speaking, in this case it is required to replace the property conditioned by the m measurable properties, by the n new measurable properties. This gives rise to a combinatorial number of variants of conjugation of properties that already "refine" the actual divisible hypothetical concept, and new properties that determine the division of this concept. The problem is to construct all such variants in order to select a single, corresponding to the set of certain hypotheses about the studied DI. However, it is not difficult to understand that the desired configuration of properties and dependencies between them can be obtained for another, easily controlled scaling sequence of the measured property under consideration, when disjunctive scaling is first performed, and then a proper refinement scaling of the measured properties replacing the property under consideration.

On this basis, disjunctive scaling of the conditioned measured property can be excluded from the number of SMP formation mechanisms.

4.4.2. Refining scaling of the conditioned measured property. It is natural to proceed from the fact that by the time of such scaling, SMP construction came with the use of the considered PSP (see figures 3b-3e). This completely determines the conditions for the solution of the scaling problem under consideration. For the new measured properties introduced with the refinement of n new measurable properties, only the following alternatives exist:

- the properties will fall into the composition of the distinctive properties of the already existing hypothetical formal concept and will enter the corresponding MC-group (figure 3c);
- properties supplement a set of incompatible conditioning properties and the corresponding I-group (figure 3d), and taking into account the intersections of the value domains for old and new "refining" properties, the composition of the I-group will increase by no less than n units;
- both the first and the second will occur, i.e. the corresponding fragment of the structural pattern shown in Figure 3e is realized. With this, the quantitative composition of the MC- and I-groups will correspond to specific hypothetical ideas about the investigated DI.

4.5. Scaling of the measured property, which is a member of the group of mutually-conditioned properties

MC-groups are a common element of the structural organization of SMP (see figures 2a, 3c, 3e). For example, many machine learning tasks and all tasks of classical data analysis by default proceed from the assumption of the mutual conditioning of properties measured in the training sample objects [1, 2].

The division of the volume or the allocation of a part of the volume of a hypothetical formal concept, whose distinctive features constitute the MC-group, cause very specific problems. Their reason is the potential consistency of the options and scaling results of the various measured properties that are members of the MC-group. Consistency is that parts of the concept volume allocated when it is divided or restricted by scaling one of the properties of the MC-group correspond to quite specific parts of the domain values of another property of this group (and only these parts of the domain of the second property). This means that scaling of the first property automatically leads to the corresponding scaling of the second property and vice versa.

4.5.1. Disjunctive scaling of the measured property, which is a member of the group of mutually conditioned properties. Let the MC-group include m measurable properties. Consider for simplicity the MC-group consisting of three \((m = 3)\) measured properties \(\{x, y, z\}\) (figure 5a). Suppose that disjunctive scaling is subjected to the property \(x\), replaced by the new measurable properties \(x_i, i = 1, 2, \ldots, n, n \geq 2\). Then the lack of consistency for a specific (from the point of view of dividing the
value domain) scaling of the property $x$ will determine the conjugation structure given in figure 5b. The consistency of a particular disjunctive scaling of the $x$ (and $y$) property for $n = 2$ defines PSP with a mixed-group, shown in figure 5c. The consistency of scaling at $n = 3$ has already four PSPs – see figures 5d and 5e (the latter shows one of three similar PSPs, determined by permutations of the "leading" properties $x_1, x_2, x_3$). Etc. for various combinations of $m$ and $n$.

![Figure 5](image.png)

**Figure 5.** Structural patterns of conjugation of measured properties at disjunctive scaling of the measured property, which is a member of the group of mutually-conditioned properties.

4.5.2. Refining scaling of the measured property, which is a member of the group of mutually-conditioned properties. It is easy to show that, due to the transitivity of the conditioning relationship, the addition of a new measurable property with such a scaling will only lead to the inclusion of this property in the considered MC-group. Of course, this assertion is also true for the refinement of scaling with $n$ new measurable properties.

5. Conclusion

The proposed PEC model generalizes formal schemes previously constructed in the authors' works, and is still based on elementary constraints of coexistence of any entities - incompatibility and conditionality (neutrality, is traditionally excluded from limitations, but is implicitly almost always taken into account, our model in this sense is also not is an exception).

In the context of data analysis and machine learning, this narrow nomenclature of elementary constraints on the existence of properties (as the term "properties exist constraints" itself) was first explicitly proposed in [16], where it was revealed as a result of redefinition of the semantic limitations of relational database theory [20]. Moreover, in [16] and then in [17], PEC is considered not as a hypothetical but a reliable initial knowledge of the DI, which is used in the construction of single-valued FCs. Such an approach is in principle known in pattern recognition (see, for example, [21]).

The rationale proposed in our work is not only directly addressed to the fundamental laws of classical logic, but also links PEC with hypothetical assumptions that need to be introduced in the traditional formation of training samples as a result of measuring the objects of the DI.
The PEC model presented in the article has undergone a significant complication. However, this does not entail a revision of the heuristic, previously proposed by the authors to correct the standard \( \alpha \)-section of the fuzzy relation "objects-properties" [18, 22]:

- the choice of confidence threshold to the initial data is arbitrary;
- the corresponding to threshold and, in general, the inadmissible composition of the properties of each object of the desired single-valued FC should be successively shortened by cutting off at each step a property that violates the PEC;
- the cut-off mechanism consists in local tightening of the confidence threshold within each GSP of OSS model, when the main criterion for selecting the measured property for clipping is the minimum tightening of the chosen trust threshold [18, 22].

Nevertheless, the algorithm that implements the heuristic with the new PEC model should be substantially modified. First of all, the task of research and development should become recursive components of the algorithm for processing a new multilevel PEC model.

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