Effects of Inner Halo Angular Momentum on the Peanut/X Shapes of Bars

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Abstract

Cosmological simulations show that dark matter halos surrounding baryonic disks have a wide range of angular momenta, which is measured by the spin parameter ($\lambda$). In this study, we bring out the importance of inner angular momentum (<30 kpc), measured in terms of the halo spin parameter, on the secular evolution of the bar using N-body simulations. We have varied the halo spin parameter $\lambda$ from 0 to 0.1, for corotating (prograde) spinning halos and one counterrotating (retrograde) halo spin ($\lambda = -0.1$) with respect to the disk. We report that as the halo spin increases the buckling is also triggered earlier and is followed by a second buckling phase in high-spin halo models. The timescale for the second buckling is significantly longer than the first buckling. We find that bar strength does not reduce significantly after the buckling in all of our models, which provides new insights about the role of inner halo angular momentum unlike previous studies. Also, the buckled bar can still transfer significant angular momentum to the halo in the secular evolution phase, but it reduces with increasing halo spin. In the secular evolution phase, the bar strength increases and saturates to nearly equal values for all the models irrespective of halo spin and the sense of rotation with respect to the disk. The final boxy/peanut shape is more pronounced (~20%) in high-spin halos having higher angular momentum in the inner region compared to nonrotating halos. We explain our results with angular momentum exchanges between the disk and halo.

Unified Astronomy Thesaurus concepts: Spiral galaxies (1560); Galaxy dark matter halos (1880); Galaxy kinematics (602); N-body simulations (1083); Galaxy structure (622); Galaxy evolution (594)

1. Introduction

Cosmological simulations show that dark matter halos play an important role in galaxy formation and evolution (Springel et al. 2005b; Schaye et al. 2015). Dark matter halos grow gradually from the initial primordial perturbations, given the long-range nature of gravity, and result in the formation of huge potential wells in which baryonic matter condenses (White & Rees 1978). As a result dark matter halos in cosmological simulations come with various properties such as different shapes, sizes, angular momenta, and concentration. Furthermore, using isolated disk galaxy evolution studies it has been shown that the various dark matter properties like concentration and shape affect the morphological evolution of the stellar disk via global instabilities (Athanassoula & Misiriotis 2002; Athanassoula 2003; Athanassoula et al. 2013; Kataria et al. 2022).

The origin of halo angular momentum has been shown to be the result of tidal interactions between neighboring galaxies as well as neighboring filament structures, which causes perturbations in the galaxy potentials; this is famously known as the tidal torque theory (Hoyle 1949; Sciama 1955; Peebles 1969; Doroshkevich 1970; White 1984; Barnes & Efthathiou 1987; Schäfer 2009). Recently it has been shown that merger histories of a given halo can significantly affect its angular momentum (Maller et al. 2002; Vivitska et al. 2002), which is slightly different from what is predicted by the tidal torque theory. The rotation of these halos, which is the proxy of its angular momentum is measured by a parameter called the halo spin. It has been shown that the lognormal distribution of the halo spin peaks around 0.035 (Bullock et al. 2001; Hetznecker & Burkert 2006; Bett et al. 2007; Knebe & Power 2008; Ishiyama et al. 2013; Zjupa & Springel 2017) where spin is defined as

$$\lambda = \frac{J}{\sqrt{2GMR}}. \tag{1}$$

Here $J$ is the magnitude of the specific angular momentum of the halo, $M$ is the mass of the halo within the virial radius and $R$ is the virial radius of the halo. It is well known that disks with global instabilities like bars can transport angular momentum to the surrounding halo during its evolution in isolation (Debattista & Sellwood 2000; Athanassoula 2003; Martinez-Valpuesta et al. 2006; Villa-Vargas et al. 2010; Sellwood 2016; Kataria & Das 2018; Lokas 2019). Therefore the initial angular momentum in the dark matter halo, which is a proxy for halo spin, affects the process of angular momentum transfer from disk to halo.

A recent study by Saha & Naab (2013) shows that dark matter halos with higher spin trigger the onset of bar formation earlier than low-spinning halos, which resonates with previous theoretical studies (Weinberg 1985). Furthermore, Long et al. (2014) show that models with increasing halo spin tend to weaken the bar in the secular evolution phase, which is followed by strong buckling of the bar. These studies clearly show that angular momentum transfer between the halo and disk is significantly affected by varying halo spins, which in turn affects the formation and evolution of bars in the disks. The effect of halo shapes on increasing halo spin has also been studied (Collier et al. 2018). Along with this, the effect of halos counterrotating with respect to the disk has been explored by
Collier et al. (2019), and they show that there is a delay in bar formation with the increasing counterrotating spin of halos. All these studies indicate that there is rich dynamics between spinning halos and disks that certainly depend on the initial angular momentum of the dark matter halo.

In this study we focus on the effect of the inner halo angular momentum, which has a close proximity with the disk, on bar formation and evolution. We measure the angular momentum difference of all of our galaxy models in terms of halo spin, assuming that the spin is correlated with increasing inner halo angular momentum. We have varied the inner halo angular momentum by varying prograde/retrograde orbits in the inner halo. Given the role of halo spin on the evolution of bars, we will also focus on the buckling of bars. It is well known that bars undergo buckling instability during their evolution and just after they gain their peak strength (Combes et al. 1990; Pfenniger & Friedli 1991; Raha et al. 1991; Martinez-Valpuesta et al. 2006; Shen et al. 2010; Collier 2020), which leads to a decrease in bar strength. Previous studies (Long et al. 2014; Collier et al. 2018) show that strong buckling along with increase in halo spin leads to weakening of bars in the secular evolution phase. The next interesting question to answer will be “How does weak and intermediate buckling, which does not affect bar strength significantly, respond to different values of halo spin?” So, in this study we address the following questions: (1) how is buckling strength affected by increasing halo spin?; (2) how does the peanut/X shape of a bar depend on the halo spin?; (3) how is the thickening of bars affected by different mechanisms (Sellwood & Gerhard 2020)?

In this article we construct galaxy models with increasing halo spin ($\lambda \equiv 0-0.1$) that are corotating with respect to the disk, as well as one counterrotating halo model. In these models, the spin of halos is controlled by varying inner halo angular momentum only. We evolve these models to see the effect of halo spin on the evolution of bars, like buckling and the formation of boxy/peanut shapes. In Section 2 we provide the numerical techniques and methods used in this study. We further provide result in Section 3 and discussion is done in Section 4. Finally we summarize and conclude our key results in Section 5.

2. Model Setup

2.1. Initial Galaxy Models

All the initial galaxy models are generated using GalIC code (Yurin & Springel 2014), which are not aimed particularly at modeling the Milky Way, although they are similar systems. The code populates orbits of particles in a target stationary potential and achieves the equilibrium N-body realization of the system by solving the collisionless Boltzmann equations. In all of our initial models we have used $10^6$ dark matter halo particles and $10^7$ disk particles. We also ran the convergence test with the number of particles doubled and found that they show similar results.

The total mass ($M_{\text{total}}$) of all of our galaxy model is equal to $6.38 \times 10^{11} M_{\odot}$. The rotation curve near the Sun radius is equal to 250 km s$^{-1}$. The disks in our galaxy models are locally stable as the value of the Toomre parameter $Q$ is greater than 1. The Toomre factor varies with radius and is given by $Q(R) = \frac{\sigma(R)c(R)}{2\pi G \Sigma(R)}$. Here $\sigma(R)$ is the radial dispersion of disk stars, $c(R)$ is the epicyclic frequency of stars, and $\Sigma(R)$ is the mass surface density of the disk. We have shown the radial variation of various disk properties in Figure 1.

The profile of the dark matter halo, which is spherically symmetric, is Hernquist and given as

$$\rho_h = \frac{M_{\text{halo}}}{2\pi} \frac{a}{r(r+a)^3}$$  \hspace{1cm} (2)

where $a$ is the scale length of the halo component. This scale length is related to the concentration parameter of the Navarro–Frenk–White (NFW) halo by $M_{\text{halo}} = M_{200}$ (Springel et al. 2005a) so that the inner profile of the halo is identical to the NFW halo. Here $a$ and $c$ are related as follows

$$a = \frac{R_{200}}{c} \left[2ln(1 + c) - c - 1/1 + c\right]$$  \hspace{1cm} (3)

where $M_{200}$, $R_{200}$ are the virial mass and virial radius for an NFW halo respectively. $M_{200}$ is equal to $5.68 \times 10^{11} M_{\odot}$ and $R_{200}$ is equal to 140 kpc.

The density profile of the disk component has an exponential distribution in the radial direction and a sech$^2$ profile in the vertical direction.

$$\rho_d = \frac{M_d}{4\pi a_0 h^2} \exp\left(-\frac{R}{R_d}\right) \text{sech}^2\left(\frac{z}{z_0}\right)$$  \hspace{1cm} (4)

where $R_d = 2.9$ kpc and $z_0 = 0.58$ kpc are the radial scale length and vertical scale length respectively. In our models there is no stellar particle beyond the radius $12 R_d$. Total mass in the disk component is equal to $6.38 \times 10^{10} M_{\odot} = 0.1 M_{\text{total}}$.

For nonspinning halos the total angular momentum of the halo is close to zero where total retrograde fraction of orbits is almost equal to total prograde fraction. For conducting this study we have prepared five galaxy models with increasing halo spin where total prograde orbit fractions increases successively. Radial variation of the prograde orbits is shown in Figure 2. In order to increase the spin of the halo we have reversed the direction of retrograde orbits within the very central region (<30 kpc), which results in increased prograde orbit fractions. We have also an extra model with negative halo spin in which halo orbits are retrograde with respect to disk. The dark matter density remains unvaried under this orbit reversal, which is in agreement with Jean’s theorem (Jeans 1919; Binney & Tremaine 2008). These reversals of orbits maintain the equilibrium distribution of dark matter as the solution of the collisionless Boltzmann equation is preserved (Lynden-Bell & Woolley 1960; Weinberg 1985). While reversing the direction of halo orbits we have kept the same parameters of the disk. The details of fraction of retrograde orbits in our increasing spin models are given in Table 1. We can see that halo orbits are generally varied in the central region and result in different spin models. This is because halo-disk interaction mostly happens in the central region.

2.2. Simulation Methods

We ran all the N-body simulation using GADGET-2 code (Springel 2005) for evolving our galaxy models. All the galaxy models are evolved up to 9.78 Gyr. The code makes use of the tree method (Barnes & Hut 1986) and computes gravitational forces among all the particles. The time integration of position and velocity of particles is performed using various types of leapfrog methods. The opening angle for the tree is chosen as $\theta_{\text{cut}} = 0.4$, which has resulted in improved force calculations. The softening length for halo, disk, and bulge components have been chosen as 30, 25, and 10 pc respectively. In order to have...
accurate force calculations, we have used the values of the time step parameter to be $\eta \leq 0.15$ and force accuracy parameter $\leq 0.0005$ in all of the simulations. As a result in all of our models, the total angular momentum of the disk and halo is conserved to within 0.1% of the initial value. Throughout the paper we describe our results in terms of code units. Both GalIC and Gadget-2 code have unit mass equal to $10^{10} M_\odot$ and unit length is 1 kpc.

3. Results

3.1. Evolution of the Bar Strength

There are various ways in which bar strength has been defined in the literature (Combes & Sanders 1981; Athanassoula 2003). In this study we look for stellar contribution to the $m=2$ Fourier mode as a proxy for bar strength.

$$a_2(R) = \sum_{i=1}^{N} m_i \cos(2\phi_i) \quad b_2(R) = \sum_{i=1}^{N} m_i \sin(2\phi_i)$$

where $a_2$ and $b_2$ are calculated for all the disk particles, $m_i$ is mass of $i$th star, and $\phi_i$ is the azimuthal angle. We have defined the bar strength as

$$\frac{A_2}{A_0} = \frac{\sqrt{a_2^2 + b_2^2}}{\sum_{i=1}^{N} m_i}.$$  

(5)

So the bar strength is derived using the cumulative value of the $m=2$ Fourier mode for the whole disk, which also varies with radius.

Bar strength for all of our models has been shown in the top panel of Figure 3. We can clearly see that the bar triggering timescale reduces with the increase in halo spin, which is in agreement with earlier studies (Saha & Naab 2013; Long et al. 2014; Collier et al. 2018). We also find that bar formation in our counterrotating halo (SM100) has been delayed as shown by earlier studies (Collier et al. 2019). As soon as the bar strength has reached its maximum value it begins to decrease, which is more prominent with increasing halo spin models. This is due to the buckling of bar, which we discuss in detail in the next subsection. Furthermore, as the bar secularly evolves, we find that the bar strength increases with time and finally saturates at the end of the simulation. We find that all the increasing spin models show similar bar strengths at the end of simulations. This shows that the secular evolution of bars in higher-spin models are quite different compared to previously reported studies (Long et al. 2014; Collier et al. 2018) where the bar becomes weaker with increasing halo spin. We have
confirmed our result by using different galaxy initial condition generator; namely Agama (Vasiliev 2019). We explain this difference in Section 4.

3.2. Buckling of Bars

As all the bars in our simulations buckle as they evolve, we have measured the time evolution of buckling amplitude, which is given by the following equation (Debattista et al. 2006):

$$ A_{\text{Buckle}} = \frac{\sum_{i=1}^{N} m_i \sigma_i \sin \phi_i}{\sum_{i=1}^{N} m_i} \quad (7) $$

where $z$ is the vertical coordinate, $m_i$ is mass of $i^{th}$ star, $\phi_i$ is the azimuthal angle.

The buckling of bar results in the reduced strength of the bars (Raha et al. 1991; Martinez-Valpuesta & Shlosman 2004). We find that the reduction in bar strength after buckling increases with the increase in halo spin (top panel of Figure 3). As the bar has buckled in all galaxy models as soon as the bar reaches its peak strength, further temporal evolution of bar is defined as the secular evolution phase. We have measured the buckling strength using Equation (7), which captures the bending of bars very effectively. Time evolution of the buckling strength is shown in the middle panel of Figure 3. We find that as the halo spin increases for all prograde halo models, the buckling of bar happens earlier. However, for the counterrotating halo model (SM100) the buckling event is more delayed compared to all prograde spinning halos. These trends are correlated with bar formation timescales as discussed in the previous subsection. We find that the chance of a second buckling event increases with increasing halo spin. The second peak in buckling strength is more pronounced for model S075.

We know from previous studies that buckling also shows signatures in the evolution of $\sigma_z/\sigma_R$ (Lokas 2019). To capture this we have plotted the time evolution for the same in Figure 4. We can clearly see the sudden jump in $\sigma_z/\sigma_R$ during the time of buckling for all the models. We notice a few trends clearly; one is that the size in jump increases with increasing halo spin for all prograde orbits. The second trend we notice is that the timescale of these jumps are well correlated with the peak of the buckling strength as seen in the middle panel of Figure 3. We also notice that the second jump in $\sigma_z/\sigma_R$ tends to be more pronounced with increasing spin, which are seen in S050 and S075 models. We also find that the signature of second buckling in the S075 model (which shows a second jump) happens around 3 Gyr.

### Table 1

| Models Name | Spin ($\lambda$) | Prograde Fraction ($R < 10$ kpc) | Prograde Fraction ($R < 15$ kpc) | Prograde Fraction ($R < 20$ kpc) | Prograde Fraction ($R < 30$ kpc) | Overall Prograde Fraction |
|-------------|------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|--------------------------|
| S000        | 0                | 0.36                             | 0.41                             | 0.43                             | 0.45                             | 0.48                     |
| S025        | 0.025            | 0.50                             | 0.50                             | 0.50                             | 0.50                             | 0.52                     |
| S050        | 0.050            | 0.55                             | 0.53                             | 0.52                             | 0.51                             | 0.54                     |
| S075        | 0.075            | 0.56                             | 0.54                             | 0.53                             | 0.52                             | 0.57                     |
| S100        | 0.1              | 0.73                             | 0.65                             | 0.61                             | 0.59                             | 0.62                     |
| SM100       | −0.10            | 0                                | 0                                | 0.02                             | 0.15                             | 0.29                     |

Note. Column (1) model name; (2) spin of the model; (3) fraction of retrograde orbits in dark matter halo within 10 kpc spherical region; (4) fraction of retrograde orbits in dark matter halo within 15 kpc spherical region; (5) fraction of retrograde orbits in dark matter halo within 20 kpc spherical region; (6) fraction of retrograde orbits in dark matter halo within 30 kpc spherical region; (7) fraction of retrograde orbits in all the dark matter halo particles.

#### 3.3. Face-on and Edge-on Morphology of Bars

As a bar secularly evolves, its bar strength increases (top panel of Figure 3), which is reflected in decrease in pattern speed (Figure 7). We can clearly see that bar strength for increasing spin models are similar at the end of the simulation. Furthermore, we find that bar morphology changes as the halo spin increases. We have plotted the face-on and edge-on distribution of bars at the end of the simulations in Figure 5. We can see clearly that the face-on bar shape changes from rectangular to farfalle with increasing halo spin for all prograde spin models. Apart from this we also notice that all of our bars show ansae (handle) features that are quite prominent. These features indicate that all the bars are evolving secularly by capturing orbits from the disk (Martinez-Valpuesta et al. 2006).

However, for the counterrotating halo case the bar maintains a box shape where the spiral is more prominent compared to all prograde spin cases. To quantify the farfalle shape, we further show the line profile along the bar major axis that is passing through the center of bar, i.e., the $y = 4$ kpc and $y = −4$ kpc lines in Figure 6. We can clearly see from these profiles that the bar shows more prominent peanut/X shapes in higher halo spin models. However, for the counterrotating (retrograde) halo spin model (SM100) we see that bar does not show a peanut shape in the face-on profiles.

Figure 5 also shows the edge-on bars for increasing halo spin models. It is clear from this figure that the peanut/X shape also gets enhanced as we increase the halo spin. This has also been captured in the $A_{\text{BFX}}$ strength, which is shown in the bottom panel of Figure 3. The observed shape of the bulge has been suggested to be supported by certain families of orbits mentioned in Patsis et al. (2002) and Athanassoula (2005). Both of the box shapes of bars in face-on as well as edge-on maps have been suggested to be supported by quasiperiodic orbits in stability islands or sticky orbits around them (Chaves-Velasquez et al. 2017).

#### 3.4. Boxy/Peanut Shape

As a result of the buckling a bar grows vertically (Debattista et al. 2004, 2006; Martinez-Valpuesta et al. 2006; Shen et al. 2010; Athanassoula 2016; Kataria & Das 2018). We quantify the boxy/peanut amplitude of the bar, which is basically the rms of the vertical height of the bar and measured within an annular region of radius ranging from 2 to 8 kpc. We define the
time shows varied nature as the halo spin increases. In all of the spinning halo models, there is a steep increase in $A_{BPX}$ at the time of buckling, which decreases with increase in halo spin. After this steep increase there is a gradual increase in $A_{BPX}$; the slope of this gradual increase increases with spin of the halo, which is quite clear as we move from the S000 to S075 models. We find that the steepest gradual slope for model S075 corroborates with a second buckling event. Furthermore, we observe that $A_{BPX}$ increases more gradually for all the prograde spinning models and the final $A_{BPX}$ shows a monotonic increase in $A_{BPX}$ with halo spin. $A_{BPX}$ is around 20% higher in the highest-spin model (S075) compared to the lowest one (S000). For the counterrotating spin model (SM100) we also find that $A_{BPX}$ increases steeply during buckling and then it increases gradually. It has a much higher slope compared to the slope of gradual $A_{BPX}$ increase in all the prograde spin models.

3.5. Bar Pattern Speed and $R$ Parameter

Pattern speed ($\Omega_p$) of the bar has been measured as the change in phase angle $\phi = \frac{1}{2} \tan^{-1} \left( \frac{b_2}{a_2} \right)$ of the bar with time where $a_2$ and $b_2$ are defined by Equation (5). This is measured using the Fourier modes ($m = 2$) in concentric annular bins of size 1 kpc throughout the disk of the galaxy. In order to measure the phase angle of the bar we have used the Fourier mode of the annular region corresponding to the peak in maximum value of $A_2/A_0$.

The time evolution of bar pattern speed has been shown in Figure 7. We can clearly see that the initial bar pattern speed increases with increase in halo spin. As the bar evolves secularly, the bar pattern speed decreases due to the dynamical friction of the halo (Debattista & Sellwood 2000). We notice that the final bar pattern speed for all the increasing spin models saturate to similar values and are correlated with the saturation in bar strengths (top panel of Figure 3) at the end of the simulations. The bar pattern speed for higher-spin halos reduces sharply in the beginning compared to lower-spin halos, although the trend reverses at the end of the simulations. The retrograde halo spin model (SM100) shows the fastest decrease in pattern speed compared to all prograde halo spin models. This clearly indicates that the dynamical friction by the counterrotating halo is stronger compared to the prograde one.

The $R$ parameter is defined as the ratio of corotation radius ($R_{CR}$) to bar major axis ($R_b$). Corotation radius is measured as the distance of the Lagrange points (L1/L2) from the center of disk. It is well known that in the bar rotating frame the centripetal force is balanced by the gravitational force at the Lagrange points; therefore, the effective force is zero (Debattista & Sellwood 2000). We evaluate the gravitational forces and centripetal forces along the bar major axis within the radial bin of 1 kpc size. We measure the L1/L2 Lagrange point as the radius of bin where both gravitational force and centripetal forces are equal.

Bar lengths have been measured using several methods in the literature. Some of these use the ellipticity and position angle variation along the fitted isophotes (Erwin & Sparke 2003; Erwin 2005; Marinova & Jogee 2007; Zou et al. 2014; Kataria & Das 2019) while other studies (Rosas-Guevara et al. 2020) use the radial variation of bar strength (as defined in Equation (6)) and phase angle of the bar ($\phi$). In this study we have used the average of bar lengths from three techniques, namely, $A_2/A_0$ peak methods, $a_5$, and $a_{min}$ (for more

Figure 3. Top panel shows the bar strength evolution with time; middle panel shows the buckling strength evolution with time (here the plots are shifted by arbitrary value for clear comparison) and bottom panel shows evolution of the boxy/peanut strength of bars in all the spinning models.
details see Appendix A), to measure the length of the bar. We also show the visual overlay of bar lengths measured from various methods on the density contours of the bar (S000 model) in Appendix A.

Table 2 shows the values of bar lengths from various measurement methods, corotation radius, and $R$ parameter for all models. We see that all the bars in increasing halo spin models have roughly similar sizes, while the counterrotating halo models (SM100) have slightly smaller bars. We do not find any correlation between the corotation radius and increasing halo spin. However, we do see that all prograde halos as well the retrograde halo show slow bars ($R > 1.4$). These slow bars are a result of the dynamical friction due to the surrounding dark matter halos, which is also similar irrespective of halo spin. We also find the slowest bar is in the case of the counterrotating halo where the bar has reduced its pattern speed despite being triggered late (Figure 7).

### 3.6. Angular Momentum Transport

As a bar evolves it transfers angular momentum from the disk to the surrounding dark matter halo. We have plotted the evolution of the increase in the angular momentum of the halo within a 20 kpc cylindrical region in Figure 8. We can clearly see that the higher-spin halo gains more angular momentum earlier compared to the lower-spin halos during the bar triggering phase. In this bar triggering phase, the rate of angular momentum change is also higher for high-spin halos. As the bars secularly evolve, we find that the rate of angular momentum transfer to the halo increases for low halo spin cases. The rate of angular momentum transfer in the bar triggering phase as well as in the secular evolution phase explains the pattern speed behavior respectively (Figure 7). We can clearly see that the retrograde spin model (SM100) gains angular momentum with the highest rate and the final gain at the end of the simulation is much larger than all the prograde halo spin models. Furthermore, the highest gain in angular momentum is clearly captured in the faster decrease in pattern speed for the counterrotating case.

### 4. Discussion

We see that the prograde halos with increasing spin trigger bars earlier, as also seen in previous studies (Saha & Naab 2013; Long et al. 2014). While in the secular evolution we find that bars keep on growing irrespective of halo spin, which is different from earlier studies (Long et al. 2014; Collier et al. 2018). We attribute this change to difference in the nature of angular momentum distribution for a given spin of the halo. As we know, the halo spin parameter depends on the total angular momentum of the surrounding dark matter halo as shown in Equation (1). Since the halo is distributed over a large radial scale compared to the disk, the total angular momentum of the halo can correspond to different radial distributions of internal halo angular momenta for a given total spin. In this study we increased the angular momentum in the central region of halos to obtain fast-spinning halos as shown in Table 1, which is
represented in terms of decreasing retrograde orbits in the central region. This is because most of the disk–halo angular momentum interaction happen within the bar region. Collier et al. (2018) has shown that in high halo spin models, bar regrowth after buckling is inhibited by the collusion of two factors, namely, (i) bars becoming weaker and (ii) the inability of halo to absorb angular momentum. In this study we clearly see that the buckling of a bar does not weaken the bar significantly (top panel of Figure 3), which is the case in previous studies (Long et al. 2014; Collier et al. 2018). Therefore strong bars are still torqued down due to dynamical friction of the halos despite the higher halo spin. Therefore bars can still exchange angular momentum and grow with increasing halo spin, although the rate of transfer reduces with increasing spin (Figure 8), which is in agreement with previous studies. We further conduct several tests using initial conditions generated from other initial condition generators i.e., Agama (Vasiliev 2019). We confirm our results with new models that are shown in Appendix B.

We further test our high-spin model (Spin100) by increasing the prograde orbits of dark matter halos to larger radii. We find that the Spin100 galaxy model with all the prograde orbits or no retrograde orbits up to 80 kpc does not show bar growth in secular evolution compared to the model with retrograde fraction distribution as shown in Table 1. Figure 9 clearly shows that the bar formation timescale and peak bar strength are similar for both the models. In the secular evolution bar formation is suppressed in the model with all the prograde orbits of halos within 80 kpc radii, as seen in previous studies (Long et al. 2014; Collier et al. 2018). This is because the bar is no longer able to transport angular momentum to the halo in secular evolution, which is not the case with the model with a lower prograde fraction in the central region. Hence, the importance of inner halo angular momentum (<30 kpc) is highlighted by this study, which does not allow bar weakening in secular evolution, even for the high-spin cases. Given the richness in radial variation of the angular momentum of halos for a given halo spin, we show the importance of the initial condition in another study (S. K. Kataria et al. 2023, in preparation).

Figure 5. Figure shows density projection maps at end of simulations (\(t = 9.78\) Gyr) for all the increasing prograde halo spin models; namely XY, XZ, and YZ density maps. XY density map clearly shows that bar changes its shape from square to farfalle as the halo spin increases. Bottom right map corresponds to counterrotating spinning model.
In our prograde high halo spin models we find that there are three phases of bar thickening for which the slope of $A_{BPX}$ decreases in successive phase. The first phase corresponds to a sharp increase in bar thickening as a result of buckling. The second phase of bar thickening points toward the second buckling event, which is less steeper than the first phase. Finally the third phase where bar thickens at a much slower pace points toward the mechanism of capture or passage of bar orbits by 2:1 vertical resonance.

5. Summary and Conclusion

In this article we have conducted N-body experiments to look into the effect of increasing halo spin on bar secular evolution, which results in the formation of boxy/peanut-shaped bars during their evolution. These increasing spin models correspond to increasing angular momentum in the very central regions ($< 30$ kpc) of the galaxy. We also look at a retrograde spinning halo model, which is counterrotating with respect to the disk. We confirm that bars are triggered earlier in the faster-spinning prograde halos while it is delayed in the counterrotating halos as shown in previous studies (Collier et al. 2018, 2019). We summarize our results below.

1. We find that secularly evolved bars in high-spinning prograde halos remain strong until 9.78 Gyr of evolution. We also report that the mild buckling event soon after bar strength peaks where bar strength does not change for less than 2% during buckling.

2. We find that the face-on shape of the bar changes from rectangular to a pronounced farfalle shape as the spin of halo increases at the end of the secular evolution phase.

3. The boxy/peanut shapes of bars get more pronounced as halo spin increases. Furthermore, we quantify that there is around a 20% increase in boxy/peanut strength ($A_{BPX}$) for the lowest- ($S000$) to highest-spin ($S100$) models in our study.

4. We find that evolving bars with increasing halo spins buckle earlier and have a tendency to undergo a second
buckling event, which is enhanced with increasing halo spin. The second buckling events are captured by buckling strength ($A_{\text{buckle}}$) and $\sigma_5/\sigma_R$ ratio. We also find that the timescale of the second buckling event is larger (order of a Gyr) than the first buckling event (150 Myr).

5. We conclude that all the bars in our simulations are slow for which $R \gg 1.4$. The $R$ parameters for all the simulations do not depend much on halo spin in our simulations.

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**Data Availability**

All the simulated data that is generated during this work will be provided on reasonable request to the corresponding author.

**Appendix A**

**Measurements of the Bar Length**

In this Appendix we show the comparison of various methods used to measure bar length as discussed in Section 3.5. We show only the S000 model though we have conducted the similar analysis for all the models. Figure 10 shows fitted isophotes with increasing semimajor axis using the photutils package (Astropy Collaboration et al. 2018). Then we calculate the bar lengths using various definitions like $a_{\text{min}}$ and $a_{\text{max}}$, which correspond to

![Figure 8](image1.png) Angular momentum gained by halos within 20 kpc cylindrical region for all the spinning halo models. Faster-spinning halos have larger angular momentum in the triggering phase of bar while trend reverses during secular evolution.

![Figure 9](image2.png) This figure shows time evolution of bar strength for the high-spin models (Spin100). This clearly shows dark matter halo having all prograde orbits within 80 kpc radius region suppresses the bar strength in secular evolution phase as seen in previous studies (Long et al. 2014; Collier et al. 2018).

| Models Name | Bar Length ($A_{\text{peak}}$) | Bar Length ($a_{\text{peak}}$) | Bar Length ($a_{\text{min}}$) | Avg. Bar Length | Corotation Radius | $R$ Parameter ($R_{\text{CR}}/R_c$) |
|-------------|-------------------------------|--------------------------------|--------------------------------|-----------------|-------------------|---------------------------------|
| S000        | 7.25                          | 12.40                          | 17.43                          | 12.55           | 26.75             | 2.13                           |
| S025        | 5.5                           | 10.89                          | 16.42                          | 10.94           | 25.75             | 2.35                           |
| S050        | 6                             | 10.38                          | 18.04                          | 11.16           | 25.75             | 2.31                           |
| S075        | 7.25                          | 11.14                          | 18.04                          | 12.14           | 28.25             | 2.33                           |
| S100        | 6.25                          | 10.89                          | 18.04                          | 11.73           | 27.75             | 2.37                           |
| SM100       | 6.25                          | 8.86                           | 12.45                          | 9.19            | 24.25             | 2.63                           |

**Note.** Column (1) model name; (2) bar radius measured using $A_{\text{peak}}/A_0$ peak method; (3) bar radius measured using bar phase angle variation by $5^\circ$ ($a_5$); (4) bar radius measured using minimum ellipticity of isophotes of increasing radii ($a_{\text{min}}$); (5) average bar radius of $A_{\text{peak}}/A_0$, $a_5$, and $a_{\text{min}}$; (6) corotation radius measured at $t = 9.78$ Gyr; (7) $R$ parameter, which is the ratio of corotation radius to bar radius.
maximum and minimum ellipticity as the position angle of isophotes change within 10°.

We further measure the bar strength from radial variation of bar amplitude. We have looked for \( \frac{A_2}{A_0} \) and bar phase angle (\( \phi \)) variation with radius as shown in Figure 11. Here \( \frac{A_2}{A_0} \) is measured in radial bins of 1 kpc size using Equation (6). We measure the bar length as the radius where \( \frac{A_2}{A_0} \) peaks. The bar phase angle (\( \phi \)) is measured using Fourier components of bar strength in radial bins of 1 kpc size. We measure bar length, namely, \( a_5 \) and \( a_{10} \), as the radii where \( \phi \) deviates from constant value by 5° and 10° respectively.

When compared visually (Figure 12) we find that both \( a_{\text{min}} \) and \( a_{\text{max}} \) overestimate the bar length while the \( \frac{A_2}{A_0} \) peak method underestimates the bar length. Bar lengths \( a_5 \) and \( a_{10} \) measured using variation of the bar phase angle provide visually reasonable estimates. In this study we use the bar length that is the average of three methods, namely, \( \frac{A_2}{A_0} \) peak, \( a_5 \), and \( a_{\text{min}} \).

**Appendix B**

**Testing Our Results with Other Galaxy Initial Condition Generator**

We test the robustness of our result by using galaxy models generated from the Agama initial condition generator.
(Vasiliev 2019). Agama realizes N-body equilibrium system in an action-based approach. Our Agama models are having the same density distribution for disk and halo components while kinematics are closer to GalIC models. We also make sure that Agama models have the same fractions of retrograde orbits for a given spin of dark matter halo component as shown in Table 1. Figure 13 shows the comparison of bar strength evolution with time for lowest- (Spin000) and highest-spin (Spin100) models generated using GalIC and Agama initial condition generators. We can see the bar formation timescales are similar for a given spin though the bar strength evolution is not exactly the same given a slight difference in the kinematics. We confirm that the bar forms earlier with increasing halo spin with Agama models also. Further similar models also show secular growth for high-spin models and the bar is not suppressed in the secular evolution phase.

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Figure 13. This figure shows the bar strength time evolution for lowest- and highest-spin models. Solid line models are generated using GalIC initial condition generator while dashed line models are generated using Agama initial condition generators.