Knowledge-aided sparse recovery STAP algorithm with off-grid self-calibration for airborne radar

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Abstract: Space-time adaptive processing (STAP) for airborne radar may cause lattice mismatch during sparse recovery processing, which is the off-grid problem. The off-grid problem may lead to degradation of STAP performance. To cope with this problem, this paper proposes a knowledge-aided sparse recovery STAP algorithm with off-grid self-calibration (AO-SR-STAP). The snapshots are decomposed by sparse processing firstly. The off-grid of spare dictionary is calibrated and dense interferences are removed according to the knowledge of clutter distribution. A standard steering vector set is constructed by the prior knowledge of clutter distribution, which is used to calibrate the off-grid of sparse dictionary. The dense interferences in snapshots are removed with knowledge of clutter distribution. Hence, the clutter information of snapshots is estimated accurately and the target in cell under test can be detected completely. The advantage of this algorithm is that the off-grid of sparse dictionary can be calibrated, and dense interferences are filtered effectively. Simulation experiments verify the effectiveness and robustness of the proposed algorithm.

1. Introduction

Space-time adaptive processing (STAP) for airborne radar needs enough training snapshots which are independent and identically distributed (IID) [1-2]. In last decade, the STAP with sparse recovery (SR-STAP) had been proposed [3-4], which only needs few numbers of training snapshots to estimate the clutter covariance matrix. However, the SR-STAP method cannot overcome the impact of dense interference. Additionally, the sparse dictionary used in the SR-STAP method always has mismatch with the steering vectors of the real clutter, called off-grid problem [5]. Moreover, the training snapshots cannot contain moving target signals (generally called outliers) [3]. Hence, the clutter suppression performance of SR-STAP method is degraded. In order to improve the performance of the STAP algorithm, the knowledge-aided STAP (KA-STAP) has been proposed [6-7]. KA-STAP uses the priori information of clutter distribution to assist the design of filters, which can improve the interference suppression performance of STAP effectively.

To cope with the problem of SR-STAP technology performance degradation caused by off-grid, some effective algorithms have proposed [8-9]. However, these algorithms cannot suppress the dense interferences in the training snapshots, which are difficult to use in practice. Proposes a
knowledge-aided sparse recovery STAP algorithm with off-grid self-calibration. This algorithm can calibrate the mismatch of the sparse dictionary, and suppress the dense interferences in training snapshots. Hence, the clutter covariance matrix can be estimated accurately, and performance of STAP is improved significantly.

1.1. Signal model of STAP for airborne radar
The configuration of side-looking airborne phased array radar is shown in figure 1. The height of platform is \( h \), the speed \( v_a \) of the airborne radar, the spatial cone angle of the airborne radar antenna beam is \( \psi \), the pitch angle is \( \theta \), and the azimuth angle is \( \varphi \). The working array of the front-side view uniform linear array airborne radar is an equidistant linear array, which is composed of \( N \) receiving array elements, and the operating wavelength \( \lambda \) of the airborne radar is twice the array element spacing \( d \). Within each CPI, the airborne radar array receives \( M \) pulses at a fixed pulse frequency \( f_r \). Among them, the number of distance units participating in radar detection is \( L \), so after matching the echo, the \( N \times M \times L \) dimension sample data set is formed. In order to facilitate signal processing, the definition of space-time snapshot is proposed, that is, sampling is performed according to the fast time dimension, that is, the data matrix of each distance unit is arranged and calculated in the direction of the column, and finally \( L NM \times 1 \) can be arranged. dimensional column vector \( x(l) \) (1 \( \leq l \leq L \)) [10].

![Figure 1. Working principle diagram of airborne radar](image)

A space-time snapshot sample may contain one or more components, these components are clutter, interference, moving targets and noise. Radar echo signals generated by the reflection of objects other than moving targets during target detection are called clutter. The sum of the clutter emission points on the ground is regarded as the uncorrelated clutter data in each distance unit, assuming that the number of clutter emission points on the ground is \( N_c \) [11]. The space-time steering vector of the \( i \)th clutter reflection point can be represented by

\[
S = a_s(f_s,i) \otimes a_d(f_d,i)
\]

(1)

Where \( \otimes \) represents the calculation of the Kronecker product, \( a_s(f_s,i) \) is the spatial steering vector for the \( i \)th clutter reflection point, and \( a_d(f_d,i) \) is the temporal steering vector for the \( i \)th clutter reflection point, which is defined Are

\[
a_s(f_s,i) = \left[ 1, \exp\left(j 2 \pi f_s \right), \ldots, \exp\left(j (M-1) 2 \pi f_s \right) \right]^T
\]

(2)

\[
a_d(f_d,i) = \left[ 1, \exp\left(j 2 \pi f_d \right), \ldots, \exp\left(j (N-1) 2 \pi f_d \right) \right]^T
\]

(3)

\((\cdot)^T\) represents transposed matrix. The normalized spatial frequency of the \( i \)th clutter reflection point is \( f_{sa} \) and the normalized Doppler frequency is \( f_{da} \) defined as

\[
f_{sa} = \frac{d \cdot \cos \theta \cdot \sin \varphi}{\lambda}
\]

(4)

\[
f_{da} = \frac{2v_a \cdot \cos \theta \cdot \sin \varphi}{\lambda f_r}
\]

(5)

When the \( l \)th space-time snapshot sample \( x(l) \) contains only clutter and noise, expressed by
\[ x(l) = \sum_{i=1}^{N_c} \varepsilon_i S_i + n_0 \]  

(6)

Where \( \varepsilon_i \) represents the power of the \( i \)th clutter emission point, \( S_i \) is the space-time two-dimensional steering vector of the \( i \)th clutter reflection point, and \( n_0 \) is the noise.

The covariance matrix of clutter and noise estimated from snapshot samples can be written as \(^{10}\)

\[ R = \frac{1}{L} \sum_{l=1}^{L} x_l x_l^H \]  

(7)

The optimal STAP weighting vector can be obtained as \(^{19}\)

\[ W_{opt} = \frac{R^+ S_i}{S_i^H R^{-1} S_i} \]  

(8)

\( S_i \) is the steering vector of the target, \((\cdot)^H \) is the conjugate transpose.

The above method is called sample matrix inverse (SMI) STAP. In the actual environment, the training samples generally do not satisfy the IID assumption. In some complex observation environments, dense interference and outliers may appear in the training samples. Therefore, it is very meaningful to study the STAP method with good performance for non-uniform environments, especially those with dense interference.

2. Algorithm principle

2.1. SR-STAP algorithm

Substituting (6) into (7) yields an estimate of the clutter covariance matrix as

\[ R_c = E\{xx^H\} = E\{\sum_{i=1}^{N_c} \varepsilon_i S_i (f_{d,i}, f_{s,i}) + n_0\} = (\sum_{i=1}^{N_c} \varepsilon_i S_i (f_{d,i}, f_{s,i}) + n_0)^{\ast}\} \]  

(9)

Where \( E\{\cdot\} \) stands for the expected value of a random variable. Usually, it is assumed that the signals of different clutter scattering points are incoherent, namely

\[ E\{\varepsilon_i \varepsilon_j^*\} = E\{\varepsilon_i\} E\{\varepsilon_j^*\} = 0, \forall i, j : i \neq j \]  

(10)

Assuming that the noise is independent of the clutter, then (9) can be rewritten as

\[ R_c = \sum_{i=1}^{N_c} E\{|\varepsilon_i|^2\} S_i (f_{d,i}, f_{s,i}) S_i^H (f_{d,i}, f_{s,i}) + \sigma^2 I \]  

(11)

Where, \( E\{|\varepsilon_i|^2\} \) is the clutter covariance matrix, \( \sigma^2 \) denotes the noise power, \( I \) is the \( NM \times NM \) identity matrix. The ULA airborne radar operating in positive side view mode has the following linear relationship between the normalized Doppler frequency \( f_{d,i} \) of its \( i \)th clutter point and its spatial cone angle \( \psi_i \):

\[ f_{d,i} = \frac{2v_0 \cos \psi_i}{\lambda f_s} \]  

(12)

It can be seen from equation (12) that \( f_{d,i} \) and \( \cos \psi_i \) satisfy the proportional relationship under the conditions determined by \( v_0 \) and \( \lambda \), equation (12) appears as a diagonal line on the two-dimensional space-time spectral plane, and the clutter It is mainly concentrated on this oblique line, called the clutter ridge line, and its three-dimensional observation is used to further verify this characteristic of clutter, as shown in Figure 2. Moreover, research shows that the number of space-time steering vectors contained in the dictionary \( \Phi \) is limited. Generally, the range of the spatial cone angle \( \psi_i \) is \( 0 - \pi \). From equations (4) and (5), \( f_s \) and \( f_d \) take values from \([-0.5, 0.5]\). If \( f_s \) is evenly divided into \( N_s \) points, where \( N_s \) is defined as the number of spatial frequency resolution units, the specific division is \([f_{s,1}, f_{s,2}, ..., f_{s,N_s}]\), \( N_s \) can be expressed as \( N_s = \rho_s N \). In the same way, \( f_d \) is divided into \( N_d \) points evenly, which is divided into \([f_{d,1}, f_{d,2}, ..., f_{d,N_d}]\), and \( N_d \) is defined as the number of time-domain frequency resolution units. The specific expression is \( N_d = \rho_d M \). Both \( \rho_s \) and \( \rho_d \) represent the degree of discretization \(^{3}\), and \( \rho_s, \rho_d > 1 \). In practical applications, the discrete parameters \( \rho_s \) and \( \rho_d \) generally range from 4 to 6, the
purpose is to be able to accurately represent the actual distribution of clutter, but also to reduce the discretization error, because $\rho_s$ and $\rho_d$ have large values, So there is $N_s N_d >> N_M$.

Figure 2. Schematic diagram of the distribution of clutter of the airborne ULA radar in frontal view

The discretized spatial frequency interval and the discretized Doppler frequency interval can be expressed as

$$\Delta f_s = \frac{1}{N_s (N_s - 1)}$$

$$\Delta f_d = \frac{1}{N_d (N_d - 1)}$$

Each grid point corresponds to a space-time steering vector. It is assumed that all clutter scattering blocks fall exactly on the discretized space-time grid points.

Then the snapshot of the $l$th range cell can be expressed as

$$x(l) = \sum_{i=1}^{N_s N_d} \epsilon_i S f_{d,i}, f_{s,i} + n_0 = \Phi \alpha + n_0$$

$$\Phi = \left[ S f_{d,1}, f_{s,1} \right] \cdots \left[ S f_{d,N_s}, f_{s,N_d} \right]$$

$$\alpha = \left[ \epsilon_1, \cdots, \epsilon_{N_s N_d} \right]^T$$

The overcomplete space-time dictionary $\Phi$ denotes the collection of all space-time steering vectors. $\alpha$ is a column vector which is consist of sparse representation coefficients. $\alpha$ can be approximated according to the method of minimizing norm, as

$$\alpha = \arg \min \| \alpha \|_p \text{ subject to } \| x - \Phi \alpha \|_2 < \eta$$

where, $\| \cdot \|_p$ denotes the $l_p$-norm, and $\eta$ is the noise power [12].

2.2. AO-SR-STAP algorithm

Figure 3. Clutter power spectrum with off-grid problem
In the aforementioned SR-STAP method, the space-time-frequency points of the space-time steering vector in the base matrix are artificially specified. However, the angle and Doppler frequency of the real clutter scattering source do not necessarily fall completely on the artificially preset grid. This is the grid point mismatch problem, that is, the off-grid problem [13]. As shown in Figure 3. In addition, the SR-STAP method cannot remove dense interference in the training snapshots, which may cause the calculation of the clutter power spectrum to be incorrect, resulting in incorrect target detection results. Therefore, knowledge-aided methods are used to perform calculations to remove dense interference and can correct off-grid problem.

The specific calculation steps are given as follows:

Step 1: Recording the position of \( e_i \) in \( a \) obtained from the first sparse recovery in the order of the element values from large to small, form a set \( \Omega \).

Step 2: The \( i \)th element in \( \Omega \) is selected in turn, calculated the \( l_i \) norm of the difference between the corresponding base vector \( \Phi(\cdot, \Omega(i)) \) and every column vector \( \theta_k \ (k = 1, 2, ..., K) \) in \( \Psi \) are calculated, Matrix \( \Psi \) represents a steering vector matrix constructed based on prior knowledge of clutter ridges, shown by

\[
\Psi = [\theta_1, \theta_2, \ldots, \theta_K]_{K \times K}
\]

\[
z_k = \| \Phi(\cdot, \Omega(i)) - \theta_k \|_2
\]

\( \theta_k \) represents the ideal space-time steering vector of the \( k \)th clutter reflection point on the ridge line of the clutter obtained in the angle-Doppler plane according to a priori knowledge, where \( k = 1, 2, ..., K \).

Step 3: Set a minimum threshold \( \zeta \), find the smallest difference \( z_{\min} \) among all \( z_k \) values obtained by formula (20), and determine the relationship between \( z_{\min} \) and \( \zeta \):

\[
z_{\min} < \zeta
\]

If formula (21) is satisfied, it means that the current point is located on the clutter ridge, and the current vector is replaced with the steering vector on the clutter ridge to perform clutter ridge correction to eliminate the off-grid problem. Otherwise it means that the point is an outlier, and the corresponding value in the sparse recovery coefficient is set to zero to remove the effect of dense interference.

Obtaining the updated sparse recovery vector \( a' \) and Dictionary \( \Phi' \).

Step 4: The updated sparse recovery vector \( a' \) and the dictionary \( \Phi' \) are used to obtain a clean sample \( x_2 \) after removing dense interferences, and \( x_2 \) is sparse recovered as follows.

\[
x_2 = \Phi' * a'
\]

\[
a_\alpha^* = \arg \min \| a' \| \quad \text{subject to} \quad \| x_2(l) - \Phi' a_\alpha^* \|_2 < \eta
\]

Step 5: The column vector obtained by formula (23) records its position in the order of the element values from large to small, and forms a set \( \Gamma \). The \( l \)th snapshot is re-estimate as

\[
X(l) = \Phi'(\cdot, \Gamma(m)) * a_\alpha^*(\Gamma(1:m), l)
\]

Step 6: The \((l+1)\)th snapshot is processed as step1-step5. The clutter covariance matrix \( R \) is calculated as (25), and the STAP optimal filtering weight \( W \) is calculated according to formula (8). Then the snapshot of the range cell under test is filtered to solve the filtered output \( y \) as (25).

\[
R = \frac{1}{L} \sum_{l=1}^{L} X(l) * X^H(l)
\]

\[
y = W^H X
\]

3. Simulation results and performance analysis

This section analyzes the performance of the AO-SR-STAP algorithm through simulation. The simulation parameter settings are shown in Table 1. In the simulation experiment, 10 snapshot samples were selected for the experiment to estimate the clutter covariance matrix. In the algorithm simulation, \( \rho_c=\rho_f=5 \), the minimum threshold \( \zeta=0.1 \). The
simulation experiment compares the performance of the AO-SR-STAP, SMI-STAP \cite{17}, and SR-STAP \cite{11} algorithms in the presence of off-grid problems and intensive interference performance.

3.1. Clutter space-time power spectrum analysis

The clutter space-time power spectrum estimated by AO-SR-STAP, SMI-STAP and SR-STAP are shown in figure 4 (a)–(c).

Table 1. Parameter of airborne radar system

| Parameter                  | Value     |
|----------------------------|-----------|
| Pulses number              | 12        |
| Elements Number            | 10        |
| Radar wavelength / m       | 0.3       |
| Platform velocity / (m / s)| 300       |
| Platform height / m        | 3000      |
| Pulse repetition frequency / Hz | 4000     |
| Azimuth angle /°           | 0         |
| Signal-to noise-ratio / dB | 0         |
| Clutter-to-noise ratio / dB| 60        |
| Interference-to-clutter ratio/dB | -10     |

Figure 4. Clutter space-time power spectrum

It can be clearly seen from the figure that the clutter power spectrum formed by the AO-SR-STAP algorithm simulation is completely concentrated on the clutter ridge line, which is just on the diagonal of the space-time 2D plane, and there is no grid mismatch problem; The SMI-STAP algorithm has a clutter power spectrum distributed over the entire angle-Doppler plane under 10 sample conditions, indicating that there is an error in the calculation of the clutter power spectrum; the SR-STAP algorithm clutter has an off-grid problem because of It is not well concentrated on the diagonal of the space-time two-dimensional plane, and there is obvious power output at the position of the non-clutter ridge, corresponding to dense interference, indicating that it cannot restrain the dense interference present in the sample. Therefore, compared with several other methods, the AO-SR-STAP algorithm can well suppress the impact of dense interference and solve the off-grid problem of clutter.

3.2. Analysis of clutter suppression performance

The second experiment analyzes the clutter suppression performance of the three algorithms AO-SR-STAP, SMI-STAP and SR-STAP. Figure 5 shows the improvement factor curves of the three STAP algorithms. From the figure, it can be clearly seen that the SMI-STAP and SR-STAP algorithms appear to sag at the Doppler frequency of dense interference, which is caused by this phenomenon. The impact of intensive interference is erroneously estimated by clutter covariance matrix. It can be
clearly seen from the figure that AO-SR-STAP only forms the nulls in the clutter area, and is the deepest. Compared with the other two methods, the AO-SR-STAP algorithm has the strongest ability to suppress clutter. In addition, the AO-SR-STAP algorithm has no nulls outside the clutter area, proving that the algorithm will not be affected by dense interference during the filtering process.

3.3. Moving target output analysis
The third experiment analyzes the target detection ability of the above three algorithms for the moving target in the test snapshots. The simulation selects the test snapshots from 120-220 range cells. There is a target in the 170th, which has the same Doppler Frequency as the dense interference in the training snapshots. The simulation results are shown in figure 6. The simulation contains a moving target signal to be detected, the angle and Doppler frequency of the signal are the same as the parameters of the dense interference of the training sample. It can be seen that the SMI-STAP and SR-STAP algorithms are affected by dense interference and off-grid problems, and cannot correctly detect the moving target signal contained in the test snapshot sample. The AO-SR-STAP algorithm can completely overcome the impact of dense interference and off-grid problems, well suppress clutter, and accurately detect moving target signals.

![Figure 5. Improvement factor curve](image1)

![Figure 6. Target output power](image2)

4. Conclusion
This paper studies the STAP algorithm based on knowledge-aided off-grid sparse recovery. One is to solve the error of clutter power spectrum estimation caused by the off-grid problem caused by SR-STAP in the sparse recovery process; the other solves the problem of moving target detection failure caused by dense interference in the training samples of the airborne radar. AO-SR-STAP uses prior knowledge to perform two sparse recovery processes, calibrates the grid points, and overcomes the effects of dense interference. Simulation experiments show that AO-SR-STAP algorithm can overcome the problem of moving target cancellation caused by dense interference, and its performance is better than the existing SR-STAP algorithm.

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