New Stabilization Method for Delayed Discrete-Time Cohen–Grossberg BAM Neural Networks

ER-YONG CONG¹,³, XIAO HAN³, AND XIAN ZHANG², (Senior Member, IEEE)
¹Department of Mathematics, Harbin University, Harbin 150086, China
²School of Mathematical Science, Heilongjiang University, Harbin 150080, China
³School of Mathematics, Jilin University, Changchun 130012, China

Corresponding authors: Xiao Han (hanx@jlu.edu.cn) and Xian Zhang (xianzhang@ieee.org)

This work was supported by the Natural Science Foundation of Heilongjiang Province under Grant LH2019F030.

ABSTRACT This paper deals with the state feedback stabilization problem of delayed discrete-time Cohen–Grossberg BAM neural networks. By the mathematical induction method, stabilizable conditions are derived to ensure that the resulting closed-loop system is globally exponentially stable, and thereby, the desired state feedback controller is designed. These stabilizable conditions are very simple, which can easily verified by using the standard toolbox software (for example, MATLAB). The proposed approach is directly based on the definition of global exponential stability, and does not involve the construction of any Lyapunov–Krasovskii functional. For a special case, it is theoretical proven that the proposed method is superior to an existing one. Moreover, several illustrative examples are given to validate the success of the derived theoretical results.

INDEX TERMS Discrete-time Cohen–Grossberg BAM neural network, stabilization, global exponential stability.

I. INTRODUCTION

In 1983, Cohen and Grossberg introduced a simplified model of neural networks, named as Cohen–Grossberg Neural Networks (CGNNs) [1], which is a single-layer auto-associative Hebbian correlator. Then the bidirectional associative memory neural networks (BAMNNs) were proposed by Kosko [2], [3], which is a type of recurrent neural networks. BAMNNs generalize the single-layer CGNNs to a two-layer pattern-matched heteroassociative circuits, and comes up with a complete and clear pattern stored in memory from an incomplete or fuzzy pattern.

In hardware implementation of neural networks, time delay is an unavoidable factor during the signal transmission between the neurons. Time delay may lead to some complex dynamical behaviors of the whole network, for example, instability, chaos, periodic, and poor performance [4]–[11]. Therefore, it is great important to determine sufficient conditions for the asymptotic or exponential stability of discrete-time delayed BAMNNs [12]–[23].

A considerable number of outcomes have been investigated regarding the combination of CGNNs and BAMNNs (CGBAMNNs) which have been applied to many areas. The CGBAMNN will be a greater network system, which contains more neurons of interactions, since it considers the interactions between the two neural fields. CGBAMNNs include a number of models from neurobiology and population biology, such as Lotka-Volterra systems, CGNNs and BAMNNs as the special cases, thus they will have more functions in pattern recognition, signal processing, ecological system, parallel computing, associative memory, and combinatorial optimization [24]–[26]. Accordingly, many researchers paid more attention on CGBAMNNs. On the basis of the methods dealing with BAM and CGNNs, in recent years, some experts and scholars have proposed and discussed the stability of equilibrium for CGBAMNNs [26]–[37]. Here, we mention only those related to this paper closely. In [35], Cao and Song investigated several novel sufficient conditions ensuring the existence, uniqueness and global exponential stability of equilibrium by using the analysis method, inequality technique and the properties of M-matrix. Zhou et al. [36] analyzed global exponential stability of equilibrium for a class of CGBAMNNs with delays.
Under the assumptions that the activation functions only satisfy global Lipschitz conditions and the behaved functions only satisfy sign conditions, by applying the linear matrix inequality (LMI) method, degree theory and some inequality technique, a novel LMI-based sufficient condition is established for global exponential stability of the concerned neural networks. The assumptions on the activation functions and behaved functions are more general than ones in [29], [30], [32]. Ali et al. [37] studied the problem of asymptotic stability of neural type CGBAMNNs with discrete and distributed time-varying delays. By constructing a suitable Lyapunov–Krasovskii functional and applying reciprocal convex technique and Jensen’s inequality, delay-dependent sufficient conditions for the asymptotic stability are established.

Due to the complex dynamic behaviors, the stabilization problems on the neural networks based on suitable control technique are important in the both theory and application sense. In some real applications, it is required that the dynamic behavior to converge to a stable equilibrium state. Accordingly, some researchers gave attention to study stabilization criteria under which the state trajectories of closed-loop system can be controlled to approach some periodic orbits or equilibriums and to keep them there then after. In [38], a continuous stabilization controller was designed for stabilizing the states of stochastic uncertain BAM neural networks in finite time. Aouiti et al. [39] dealt with the finite-time and fixed time stabilization problems for a class of high-order BAM neural networks with time varying delay. Chinnathambi et al. [40] designed a state feedback controller to stabilize CGBAMNNs with delays.

However, the stabilization problem for delayed discrete-time CGBAMNNs is still an untreated topic in the existing literature. Motivated by the previous discussions, in this paper we study the stabilization issue for delayed discrete-time CGBAMNNs via state feedback controller. Based on the mathematical induction method proposed in [23], the state feedback controller is designed. We derive several novel global exponential stability criteria for the equilibrium of the resulting close-loop system, which have simple form, and hence they can be easily verified via the standard tool software (e.g., MATLAB). Compared with the existing results, the proposed approach does not require to construct any Lyapunov–Krasovskii functional. It is theoretically proven that the obtained global exponential stability criteria are less conservative than ones in [18].

The organization of this paper is as follows. Problem considered in this paper will be formulated in Section II. The stabilization method for delayed discrete-time CGBAMNNs via state feedback controller will be discussed in Section III. In Section IV, we will theoretically compare the methods proposed in this paper and [18]. Numerical examples are provided in Section V to illustrate the effectiveness of the proposed method. Finally, Section VI gives some concluding remarks.

**Notations.** Suppose \( \mathbb{Z}, \mathbb{R} \) and \( \mathbb{C} \) are sets of all integers, real numbers and complex numbers, respectively. Let \( \mathbb{Z}[a, b] \) be the subset of \( \mathbb{Z} \) consisting of all integers between \( a \) and \( b \), and let \( \mathbb{Z}[a, \infty) = \bigcup_{b>a} \mathbb{Z}[a, b] \). For given positive integers \( p \) and \( q \), let \( \mathbb{R}^{p \times q} \) denote the set of all \( p \times q \) matrices over \( \mathbb{R} \). Set \( \mathbb{R}^p = \mathbb{R}^{p \times 1} \).

For a matrix \( M \in \mathbb{R}^{n \times n} \), let \( \lambda(M) = \{ z \in \mathbb{C} : \det(\mathbb{J}_n - M) = 0 \} \). The spectral abscissa of \( M \) is defined by \( s(M) := \max\{\text{Re}\lambda : \lambda \in \lambda(M)\} \), and the spectral radius of \( M \) is defined by \( \rho(M) := \max\{|\lambda| : \lambda \in \sigma(M)\} \). We say that \( M \) is a Metzler matrix if all off-diagonal elements of \( M \) are nonnegative.\(^1\)

For \( A = [a_{ij}] \in \mathbb{R}^{p \times q} \) and \( B = [b_{ij}] \in \mathbb{R}^{p \times q} \), the matrix \( [a_{ij}b_{ij}] \), denoted by \( A \circ B \), refers to the Hadamard product of \( A \) and \( B \), and the symbol \( A \succeq B \) (or \( B \preceq A \)) means that \( a_{ij} \geq b_{ij} \) for all \( i \in \mathbb{Z}[1, p] \) and \( j \in \mathbb{Z}[1, q] \); in particular, if \( a_{ij} > b_{ij} \) for all \( i \in \mathbb{Z}[1, p] \) and \( j \in \mathbb{Z}[1, q] \), then we write \( A \succ B \) (or \( B \prec A \)) instead of \( A \succeq B \) (or \( B \preceq A \)). Let \( |A| = |[a_{ij}]| \.

Denote by \( \mathbb{R}_{+}^{p \times q} \) and \( \mathbb{R}_{-}^{p \times q} \) the sets of all \( p \times q \) nonnegative and positive matrices, respectively.

**II. PROBLEM FORMULATION**

Consider a class of discrete-time CGBAMNNs with time-varying delays and control inputs, which can be described as:

\[
\begin{align*}
\dot{x}_i(k+1) &= x_i(k) - \alpha_i(x_i(k)) \left[ a_i(x_i(k)) \right. \\
&\quad - \sum_{j=1}^{n} c_{ij}f_j(y_j(k)) \\
&\quad - \sum_{j=1}^{n} e_{ij}g_j(y_j(k-h_{ij}(k))) + J_i \bigg] + U_i(k), \\
&\quad i \in \mathbb{Z}[1, n], \quad k \in \mathbb{Z}[0, \infty), \tag{1a}
\end{align*}
\]

\[
\begin{align*}
\dot{y}_j(k+1) &= y_j(k) - \beta_j(y_j(k)) \left[ b_j(y_j(k)) \right. \\
&\quad - \sum_{i=1}^{n} d_{ji}\tilde{f}_i(x_i(k)) \\
&\quad - \sum_{i=1}^{n} w_{ji}\tilde{g}_i(x_i(k-h_{ji}(k))) + \tilde{J}_j \bigg] + V_j(k), \\
&\quad j \in \mathbb{Z}[1, n], \quad k \in \mathbb{Z}[0, \infty), \tag{1b}
\end{align*}
\]

where the subscripts \( i \) and \( j \) stand for the \( i \)th neuron from the neural field \( F_X \) and the \( j \)th neuron from the neural field \( F_Y \), respectively, \( x_i(k) \) and \( y_j(k) \) are the states, \( U_i(k) \) and \( V_j(k) \) are the control inputs, \( \alpha_i(\cdot) \) and \( \beta_j(\cdot) \) represent the amplification functions, \( a_i(\cdot) \) and \( b_j(\cdot) \) denote appropriately behaved functions, \( c_{ij}, e_{ij}, d_{ji} \) and \( w_{ji} \) are constants which denote the synaptic connection weights, \( f_j(\cdot), g_j(\cdot), \tilde{f}_i(\cdot) \) and \( \tilde{g}_i(\cdot) \) denote the activation functions, \( J_i \) and \( \tilde{J}_j \) denote the external inputs, and \( h_{ij}(k) \) and \( \tau_{ji}(k) \) denote the time-varying delays.

\(^1\)Metzler matrices are important, as they arise in a number of application areas, including linear dynamical systems, differential equations, electrodynamics, population dynamics, economics, etc.
For convenience, let’s make the following assumptions:

**Assumption 1:** The delays \( h_{ij}(k) \) and \( \tau_{ij}(k) \) satisfy \( 0 \leq h_{ij}(k) \leq \hat{h}_{ij} \) and \( 0 \leq \tau_{ij}(k) \leq \tilde{\tau}_{ij} \) for any \( k \in \mathbb{Z}[0, \infty) \), where \( \hat{h}_{ij} \) and \( \tilde{\tau}_{ij} \) are known positive integers.

**Assumption 2:** The functions \( \alpha_i(\cdot) \) and \( \beta_j(\cdot) \) are bounded, and satisfy \( \alpha_j \leq \bar{\alpha}_i \) and \( \beta_j \leq \bar{\beta}_i \) for any \( u \in \mathbb{R} \), where \( \bar{\alpha}_i, \bar{\beta}_i, \beta_j \) and \( \beta_j \) are known positive constants.

**Assumption 3:** The functions \( \alpha_i(\cdot) \) and \( \beta_j(\cdot) \) satisfy

\[
\begin{align*}
\alpha_i(0) &= 0, \quad \nu_i \leq \frac{a_i(s_1) - a_i(s_2)}{s_1 - s_2} \leq \bar{\nu}_i, \\
\beta_j(0) &= 0, \quad \mu_j \leq \frac{b_j(s_1) - b_j(s_2)}{s_1 - s_2} \leq \bar{\mu}_j
\end{align*}
\]

for any \( s_1, s_2 \in \mathbb{R} \) with \( s_1 \neq s_2 \), where \( \nu_i, \bar{\nu}_i, \mu_j \) and \( \bar{\mu}_j \) are known positive constants.

**Assumption 4:** The activation functions \( f_j, g_j, \tilde{f}_i \) and \( \tilde{g}_i \) satisfy

\[
\begin{align*}
f_j(0) &= 0, \quad 0 \leq \frac{f_j(s_1) - f_j(s_2)}{s_1 - s_2} \leq \gamma_j^{(1)}(1), \\
g_j(0) &= 0, \quad 0 \leq \frac{g_j(s_1) - g_j(s_2)}{s_1 - s_2} \leq \gamma_j^{(2)}(1), \\
\tilde{f}_i(0) &= 0, \quad 0 \leq \frac{\tilde{f}_i(s_1) - \tilde{f}_i(s_2)}{s_1 - s_2} \leq \tilde{\gamma}_i^{(1)}(1), \\
\tilde{g}_i(0) &= 0, \quad 0 \leq \frac{\tilde{g}_i(s_1) - \tilde{g}_i(s_2)}{s_1 - s_2} \leq \tilde{\gamma}_i^{(2)}(1)
\end{align*}
\]

for any \( s_1, s_2 \in \mathbb{R} \) with \( s_1 \neq s_2 \), where \( \gamma_j^{(1)}, \gamma_j^{(2)}, \tilde{\gamma}_i^{(1)} \) and \( \tilde{\gamma}_i^{(2)} \) are known positive constants.

In the following we will always assume that \((x^+, y^+) \in \mathbb{R}^n \times \mathbb{R}^n\) is the unique equilibrium of CGBAMNN (1) with \( U_i(k) \equiv 0 \) and \( V_j(k) \equiv 0 \), that is,

\[
\begin{align*}
a_i(x_i^+) - \sum_{j=1}^{n} c_{ij} f_j(y_j^+) - \sum_{j=1}^{n} e_{ij} g_j(y_j^+) + J_i &= 0, \\
b_j(y_j^+) - \sum_{i=1}^{n} d_{ij} \tilde{f}_i(x_i^+) - \sum_{i=1}^{n} w_{ij} \tilde{g}_i(x_i^+) + \tilde{J}_j &= 0
\end{align*}
\]

for any \( i, j \in \mathbb{Z}[1, n] \), where \( x_i^+ \) and \( y_j^+ \) are the \( i \)th components of \( x^+ \) and \( y^+ \), respectively.

Set \( u_i(k) = x_i(k) - x_i^+ \) and \( v_j(k) = y_j(k) - y_j^+ \) for any \( k \in \mathbb{Z}[-\sigma, \infty) \) and \( i, j \in \mathbb{Z}[1, n] \), where \( \sigma = \max \{ \max_{1 \leq i \leq n} \hat{h}_{ij}, \max_{1 \leq i \leq n} \tilde{\tau}_{ij} \} \). Due to (1) and (2), we have

\[
\begin{align*}
u_j(k + 1) &= u_i(k) - \alpha_i(u_i(k)) - \sum_{j=1}^{n} c_{ij} f_j(u_j(k)) - \sum_{j=1}^{n} e_{ij} g_j(u_j(k)) + U_i(k), \\
v_j(k + 1) &= v_j(k) - \beta_j(v_j(k)) - \sum_{i=1}^{n} d_{ij} \tilde{f}_i(u_i(k)) - \sum_{i=1}^{n} w_{ij} \tilde{g}_i(u_i(k)) + V_j(k),
\end{align*}
\]

(3a)

where

\[
\begin{align*}
\gamma_i(x, y) &= \alpha_i(x + x_i^+), \quad \beta_j(x, y) = \beta_j(y + y_j^+), \\
\alpha_i(x, y) &= \alpha_i(x + x_i^+), \quad b_j(y, x) = b_j(y + y_j^+) - b_j(y_i^+), \\
f_j(x, y) &= f_j(x + x_i^+) - f_j(y_i^+), \quad g_j(x, y) = g_j(x + x_i^+) - g_j(y_i^+), \\
f_j(x, y) &= f_j(x + x_i^+) - f_j(y_i^+), \quad \bar{g}_j(x, y) = \bar{g}_j(x + x_i^+) - \bar{g}_j(y_i^+).
\end{align*}
\]

Clearly, \((0, 0, 0)\) is the unique equilibrium of system (3), and \( \alpha_i, \beta_j, a_i, b_j, f_j, g_j, \bar{g}_j \) and \( \tilde{g}_i \) satisfy the following inequalities:

\[
\begin{align*}
\alpha_i(s) &\leq \bar{\alpha}_i, \quad \beta_j(s) \leq \bar{\beta}_i, \quad \forall s \in \mathbb{R}, \\
\nu_i &\leq \bar{\nu}_i, \quad \mu_j \leq \bar{\mu}_j, \quad \forall 0 \neq s \in \mathbb{R}, \\
|f_j(s)| &\leq \gamma_j^{(1)}(1)|s|, \quad |g_j(s)| \leq \gamma_j^{(2)}(1)|s|, \quad \forall s \in \mathbb{R}, \\
|\tilde{f}_i(s)| &\leq \tilde{\gamma}_i^{(1)}(1)|s|, \quad |\tilde{g}_i(s)| \leq \tilde{\gamma}_i^{(2)}(1)|s|, \quad \forall s \in \mathbb{R}.
\end{align*}
\]

(4)

When the state feedback controller

\[
U_i(k) = \eta_i(u_i(k)), \quad V_j(k) = \zeta_j(v_j(k)), \quad i, j \in \mathbb{Z}[1, n]
\]

(5)

is applied to system (3), the resulting closed-loop system is obtained as follows:

\[
\begin{align*}
u_j(k + 1) &= (1 + \eta_i)u_i(k) - \alpha_i(u_i(k)) - \sum_{j=1}^{n} c_{ij} f_j(u_j(k)) - \sum_{j=1}^{n} e_{ij} g_j(u_j(k)), \\
v_j(k + 1) &= (1 + \zeta_j) v_j(k) - \beta_j(v_j(k)) - \sum_{i=1}^{n} d_{ij} \tilde{f}_i(u_i(k)) - \sum_{i=1}^{n} w_{ij} \tilde{g}_i(u_i(k))
\end{align*}
\]

(6a)

Here, \( \eta_i \) and \( \zeta_j \) are control gains to be determined.

The initial functions associated the closed-loop system (6) are given by \( u_i(s) = \phi_i(s) \) and \( v_j(s) = \psi_j(s) \) for any \( i \in \mathbb{Z}[1, n] \) and \( s \in \mathbb{Z}[-\sigma, 0] \). Let \( C(\mathbb{Z}[-\sigma, 0], \mathbb{R}^n) \) be the linear space over \( \mathbb{R} \) consisting of all functions \( \phi: \mathbb{Z}[-\sigma, 0] \to \mathbb{R}^n \). Define the norm \( \| \cdot \| \) on \( \mathbb{R}^n \times \mathbb{R}^n \) by \( \| (a, b) \| = (\| a \|^2 + \| b \|^2)^{1/2} \) for any \( a, b \in \mathbb{R}^n \), and the norm \( \| \cdot \|_{\sigma} \) on \( C(\mathbb{Z}[-\sigma, 0], \mathbb{R}^n) \times C(\mathbb{Z}[-\sigma, 0], \mathbb{R}^n) \) by

\[
\| (\psi, \varphi) \|_{\sigma} = \max \left\{ \max_{s \in \mathbb{Z}[-\sigma, 0]} \| \psi(s) \|_2, \max_{s \in \mathbb{Z}[-\sigma, 0]} \| \varphi(s) \|_2 \right\}
\]
for any $\psi, \varphi \in C(\mathbb{Z}[−σ, 0], \mathbb{R}^n)$, where $\| \cdot \|_2$ is the Euclidean norm on $\mathbb{R}^n$.

**Definition 1** [13]: The zero equilibrium of closed-loop system (6) is said to be globally exponentially stable, if there exist scalars $K > 0$ and $γ > 0$ such that every solution of (6), $(u(k), v(k))$ starting from $\varphi, \psi \in C(\mathbb{Z}[−σ, 0], \mathbb{R}^n)$, satisfies

$$\| (u(k), v(k)) \| \leq K e^{γk} \| (\psi, \varphi) \|_σ, \quad \forall k \in \mathbb{Z}[0, \infty),$$

where

$$u(k) = \text{col}(u_1(k), u_2(k), \ldots, u_n(k)), \quad v(k) = \text{col}(v_1(k), v_2(k), \ldots, v_n(k)).$$

The main goal of this paper is to stabilize exponentially the delayed discrete-time system (3) via the state feedback controller (5), that is, find control gains $η_i$ and $ζ_j$ $(i, j \in \mathbb{Z}[1, n])$ such that the zero equilibrium of closed-loop system (6) is globally exponentially stable.

### III. EXPONENTIAL STABILIZATION

In this section, we will investigate sufficient conditions under which the zero equilibrium of closed-loop system (6) is globally exponentially stable. To describe conveniently our main conclusions, we define:

$$A_γ = \tilde{A} - e^{-γ}I_n, \quad B_γ = A^0 e^{\gamma l_h} \circ |E| \Gamma_2 + |C| \Gamma_1,$$

$$C_γ = B^0 e^{\gamma l_d} \circ |W| \Gamma_2 + |D| \Gamma_1, \quad D_γ = \tilde{B} - e^{-γ}I_n,$$

$$\tilde{A} = \text{diag}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n), \quad \tilde{a}_i = \frac{\tilde{a}_h_1 - \tilde{a}_d_1}{2},$$

$$A^0 = \text{diag}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n), \quad e^{\gamma h} = [e^{\gamma h_{1}}]_{n \times n},$$

$$E = [e_{ij}]_{n \times n}, \quad \Gamma_2 = \text{diag}(\gamma_{1}, \gamma_{2}, \ldots, \gamma_{n}),$$

$$C = [c_{ij}]_{n \times n}, \quad \Gamma_1 = \text{diag}(\gamma^{(1)}_{1}, \gamma^{(2)}_{1}, \ldots, \gamma^{(2)}_{n}),$$

$$B^0 = \text{diag}(\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n), \quad e^{\gamma d} = [e^{\gamma d_h_{1}}]_{n \times n},$$

$$W = [w_{ij}]_{n \times n}, \quad \tilde{\Gamma}_2 = \text{diag}(\tilde{\gamma}^{(2)}_{1}, \tilde{\gamma}^{(2)}_{2}, \ldots, \tilde{\gamma}^{(2)}_{n}),$$

$$D = [d_{ij}]_{n \times n}, \quad \tilde{\Gamma}_1 = \text{diag}(\tilde{\gamma}^{(1)}_{1}, \tilde{\gamma}^{(2)}_{1}, \ldots, \tilde{\gamma}^{(2)}_{n}),$$

$$\tilde{B} = \text{diag}(\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n), \quad \tilde{b}_i = \frac{\tilde{b}_h_1 + \tilde{b}_d_1}{2}.$$

**Theorem 1**: Under Assumptions 1–4, if there exist a scalar $γ > 0$ and vectors $\tilde{u}, \tilde{v} \in \mathbb{R}^n$ such that

$$A_γ \tilde{u} + B_γ \tilde{v} \leq 0, \quad C_γ \tilde{u} + D_γ \tilde{v} \leq 0,$$

then the zero equilibrium of closed-loop system (6) is globally exponentially stable, that is, the delayed discrete-time system (3) can be stabilized exponentially via the state feedback controller

$$U_i(k) = \left(\tilde{a}_{i+\tilde{a}_h_1} - 1\right) u_i(k), \quad i \in \mathbb{Z}[1, n],$$

$$V_j(k) = \left(\tilde{b}_{j+\tilde{b}_h_1} - 1\right) v_j(k), \quad j \in \mathbb{Z}[1, n].$$

**Proof**: Choose $K_1 > 0$ such that

$$K_1 \tilde{u} > \text{col}(1, 1, \ldots, 1), \quad K_1 \tilde{v} > \text{col}(1, 1, \ldots, 1).$$

For any but fixed initial functions $\varphi, \psi \in C(\mathbb{Z}[−σ, 0], \mathbb{R}^n)$ of system (6), define

$$\dot{u}(k) = K_1 \| (\psi, \varphi) \|_σ e^{-γk} \tilde{u}, \quad k \in \mathbb{Z}[−σ, \infty),$$

$$\dot{v}(k) = K_1 \| (\psi, \varphi) \|_σ e^{-γk} \tilde{v}, \quad k \in \mathbb{Z}[−σ, \infty).$$

Now we will show by using the mathematical induction method that

$$| u(k) | \leq | \tilde{u}(k) |, \quad | v(k) | \leq \tilde{v}(k), \quad \forall k \in \mathbb{Z}[−σ, \infty).$$

(12)

Indeed, from the definition of $\| \cdot \|_σ$ and the choice of $K_1$, it is clear that

$$| u(q) | \leq | \tilde{u}(q) |, \quad | v(q) | \leq \tilde{v}(q), \quad \forall q \in \mathbb{Z}[−σ, 0).$$

(13)

Assume that (12) holds for $k \in \mathbb{Z}[−σ, q], \quad q \geq 0$. When $k = q + 1$, it follows from (6a) that, for any $i \in \mathbb{Z}[1, n],$

$$| u_i(q + 1) | = | (1 + η_i) u_i(q) + α_i^u(u_i(q)) e^u_i(q) |$$

$$\leq (1 + η_i) | u_i(q) | - \alpha_i^u(u_i(q)) e^u_i(q) \sum_{j=1}^{n} \left| c_{ij} f_j^u(v_j(q)) - e_{ij} g_j^u(v_j(q) - h_j(q)) \right|$$

$$\leq (1 + η_i) | u_i(q) | - \left| \sum_{j=1}^{n} c_{ij} f_j^u(v_j(q)) \right|$$

$$\leq (1 + η_i - θ_i(u_i(q))) | u_i(q) |$$

$$\leq (1 + η_i - θ_i(u_i(q)) | u_i(q) |$$

$$+ \left| \sum_{j=1}^{n} c_{ij} f_j^u(v_j(q)) \right|$$

$$\leq (1 + η_i - \tilde{u}_i(q)) | u_i(q) |$$

$$\leq (1 + η_i - \tilde{u}_i(q)) | u_i(q) |$$

$$\leq (1 + η_i - \tilde{u}_i(q)) | u_i(q) |$$

where $θ_i(u_i(q)) = α_i(u_i(q)) a_i(u_i(q))/u_i(q)$. This, together with (4), gives that

$$| u_i(q + 1) | \leq \max \left\{ (1 + η_i - \tilde{a}_{i+\tilde{a}_h_1}), (1 + η_i - \tilde{a}_{i+\tilde{a}_h_1}) \right\} | u_i(q) |$$

$$+ \sum_{j=1}^{n} \tilde{a}_i | c_{ij} f_j^{(1)}(v_j(q))$$

$$+ \sum_{j=1}^{n} \tilde{a}_i | c_{ij} f_j^{(2)}(v_j(q) - h_j(q)) |.$$

(14)
where $\hat{u}_i(q)$ and $\hat{v}_i(q)$ are the $i$th components of $\hat{u}(q)$ and $\hat{v}(q)$, respectively.

Note that $\max \{1 + \eta_i - \alpha_i \lambda_j, 1 + \eta_i - \bar{\alpha}_i \lambda_j\}$ arises to the minimal value $\tilde{\alpha}_i := \frac{\bar{\alpha}_i \lambda_j - \eta_i}{2}$ when $\eta_i = \frac{\bar{\alpha}_i \lambda_j + \alpha_i \lambda_j}{2} - 1$. So, the inequality (14) implies

$$|u_i(q+1)| \leq \tilde{\alpha}_i K \|(\psi, \phi)\|_\sigma e^{-\gamma q} \hat{u}_i$$

$$+ \sum_{j=1}^{n} \tilde{\alpha}_i |e_{ij}| \gamma_j^{(1)} K \|(\psi, \phi)\|_\sigma e^{-\gamma q} \hat{v}_j$$

$$+ \sum_{j=1}^{n} \tilde{\alpha}_i |e_{ij}| \gamma_j^{(2)} K \|(\psi, \phi)\|_\sigma e^{-\gamma (q-h_j(q))} \hat{v}_j$$

$$\leq K \|(\psi, \phi)\|_\sigma e^{-\gamma q} \left[ \tilde{\alpha}_i \hat{u}_i + \sum_{j=1}^{n} \tilde{\alpha}_i |e_{ij}| \gamma_j^{(1)} \hat{v}_j \right]$$

$$+ \sum_{j=1}^{n} \tilde{\alpha}_i |e_{ij}| \gamma_j^{(2)} e^{\gamma h_j} \hat{v}_j. \ (15)$$

The combination of (10), (11) and (15) gives

$$|u_i(q+1)| \leq \tilde{\alpha}_i K \|(\psi, \phi)\|_\sigma e^{-\gamma q} (\tilde{\alpha}_i + B_i \tilde{v}) \hat{u}_i$$

Using (7) and (10), we obtain that

$$|u_i(q+1)| \leq K \|(\psi, \phi)\|_\sigma e^{-\gamma (q+1)} \hat{u}_i = \hat{u}(q+1). \ (17)$$

By a process similar to one deriving (17), it is easy to give that

$$|v_i(q+1)| \leq K \|(\psi, \phi)\|_\sigma e^{-\gamma (q+1)} \hat{v} = \hat{v}(q+1).$$

In summary, (12) holds.

It follows from (10), (11) and (12) that

$$\|(u(k), v(k))\| = (\|(u(k))\|_2 + \|(v(k))\|_2)^{1/2} \leq ((\hat{u}_i(k))^2 + ((\hat{v}_i(k))^2)^{1/2}$$

$$= K e^{-\gamma k} \|(\psi, \phi)\|_\sigma \left( \|(\hat{u}_i(k))^2 + ((\hat{v}_i(k))^2)^{1/2} \right)$$

for any $k \in \mathbb{Z}[0,\infty)$. Let $K = K_1(\|(\hat{u}_i(k))\|^2 + ((\hat{v}_i(k))^2)^{1/2})$. Then

$$\|(u(k), v(k))\| \leq K e^{-\gamma k} \|(\psi, \phi)\|_\sigma , \forall k \in \mathbb{Z}[0,\infty).$$

Since $\psi, \phi \in C(\mathbb{Z}, [-\sigma, 0], \mathbb{R}^n)$ is arbitrary, we obtain that the zero equilibrium of closed-loop system (6) is globally exponentially stable. This completes the proof.

**Remark 1:** Theorem 1 gives global exponential stabilization conditions for the zero equilibrium of closed-loop system (6). From its proof, it is seen that the proposed method is applicable to the case that the numbers of neurons in the two neural fields are different.

To present more sufficient conditions, we introduce the following result.

**Lemma 1** [41]: Let $A_0 \in \mathbb{R}^{n \times n}$ be a Metzler matrix and $B_0, C_0, D_0 \in \mathbb{R}^{n \times n}$. Then the following statements (i)–(iii) are equivalent:

(i) $\rho(D_0) < 1$ and $s(A_0 + B_0(D_0 - D_0^{-1}C_0) < 0$.

(ii) $A_0x + B_0y < 0$ and $C_0x + D_0y < y$ for some $x, y \in \mathbb{R}^n$.

(iii) $s(A_0) < 0$ and $\rho(C_0(-A_0)^{-1}B_0 + D_0) < 1$.

Combining Theorem 1 and Lemma 1, one can easily derive the following conclusion.

**Theorem 2:** Under Assumptions 1–4 and the state feedback controller (9), the zero equilibrium of closed-loop system (6) is globally exponentially stable, if one of the following statements (i)–(iii) holds:

(a) $A_0 + B_0(\tilde{C} - 2\rho^2) \tilde{v} < 0$ and $\tilde{C} \bar{u} + (D_0 - D_0^{-1}C_0) \tilde{v} < \tilde{v}$ for some $\gamma > 0$ and $\tilde{u}, \tilde{v} \in \mathbb{R}_{\tilde{v}}^n$.

(b) $\rho(D_0 + I_0) < 1$ and $s(A_0 - B_0D_0^{-1}C_0) < 0$ for some $\gamma > 0$.

(c) $s(A_0) < 0$ and $\rho(D_0 + I_0 - C_0A_0^{-1}B_0) < 1$ for some $\gamma > 0$.

Set $\tilde{B} = A_0^0(\tilde{C}|\tilde{C}) + |\tilde{E}|\tilde{C}$ and $\tilde{C} = B_0^0(D_0^0|D_0^0) + |\tilde{C}|\tilde{C}.$

**Theorem 3:** Under Assumptions 1–4 and the state feedback controller (9), the zero equilibrium of closed-loop system (6) is globally exponentially stable, if one of the following statements (i)–(iii) holds:

(i) $\tilde{A} = \tilde{A}_0 + \tilde{B} < 0$ and $\tilde{C} \bar{u} + \tilde{B} \tilde{v} < \tilde{v}$ for some $\tilde{u}, \tilde{v} \in \mathbb{R}_{\tilde{v}}^n$.

(ii) $\rho(\tilde{B}) < 1$ and $s(\tilde{A} + \tilde{B}(I_0 - \tilde{B})^{-1}\tilde{C}) < 1$.

(iii) $s(\tilde{A}) < 1$ and $\rho(\tilde{B} - \tilde{A}(I_0 - \tilde{A})^{-1}\tilde{B}) < 1$.

**Proof:** It is clear that $\lim \gamma^{-\gamma} A_0 = \tilde{A} - \tilde{I}_n$, $\lim \gamma^{-\gamma} B_0 = \tilde{B}$, $\lim \gamma^{-\gamma} C_0 = \tilde{C}$ and $\lim \gamma^{-\gamma} D_0 = \tilde{B} - \tilde{I}_n$. This, together with Theorem 2, completes the proof.

**Remark 2:** Theorems 2 and 3 give the delay-dependent and -independent global exponential stabilization conditions for the closed-loop system (6), respectively.

**IV. METHOD COMPARISON**

In this section, we will give a theoretical comparison between the methods proposed in this paper and [18]. To this end, we consider a simplified case of the delayed discrete-time CGBAMNN (3):

$$u_i(k+1) = u_i(k) - \alpha_i^* \left( u_i(k) \right)$$

$$- \sum_{j=1}^{n} c_{ij} f_j^*(v_j(k))$$

$$- \sum_{j=1}^{n} e_{ij} f_j^*(v_j(k-h_j(k))). \ (18a)$$
\( v_j(k+1) = v_j(k) - \beta_j^s(v_j(k))\left[\bar{b}_j^s(v_j(k)) - \sum_{i=1}^{n} d_{ij} \tilde{a}_i(u_i(k)) - \sum_{i=1}^{n} w_{ij} \tilde{a}_i(u_i(k) - \tau_j(k)))\right], \quad j \in \mathbb{Z}[1,n], \quad k \in \mathbb{Z}[0, \infty). \) (18b)

The following proposition is lent from [18].

**Proposition 1** [18, Corollary 2]: Under Assumptions 1–4, the zero equilibrium of CGBAMNN (18) is globally exponentially stable, if the following statements (i) and (ii) are satisfied:

(i) \( \tilde{a}_1 \tilde{a}_1 \leq 1 \) and \( \tilde{b}_1 \tilde{b}_1 \leq 1; \)

(ii) \( \alpha \beta > \sum_{i=1}^{n} \tilde{b}_i \tilde{a}_i^{(1)}(|d_{ij}| + |w_{ij}|) \quad \text{for } i \in \mathbb{Z}[1,n], \quad j = 1\).

Let

\[
\dot{A}_y = \dot{A} - e^{-\gamma} I_n, \quad \dot{A} = \text{diag}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n), \quad \tilde{a}_i = \text{max} \{1 - \alpha \tilde{a}_i, |1 - \tilde{a}_i|\}, \quad i \in \mathbb{Z}[1,n], \\
\dot{B}_y = \dot{A}^0(e^{\gamma} \dot{A}^0 + |E| + |C|) \Gamma_1, \\
\dot{C}_y = B^0(e^{\gamma} \dot{B}^0 + |W| + |D|) \Gamma_1, \\
D_y = b - e^{-\gamma} I_n, \quad B = \text{diag}(\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n), \\
\dot{b}_i = \text{max} \{1 - \beta \tilde{b}_i, |1 - \tilde{b}_i|\}, \quad i \in \mathbb{Z}[1,n].
\]

Thus, (ii) of Proposition 1 is equivalent to

\[
(I_n - \dot{A}) \dot{u} > [\Gamma_1(|D| + |W|) \dot{B}^0 \dot{v} = \dot{C}_y \dot{v}, \\
(I_n - \dot{B}) \dot{v} > [\Gamma_1(|C| + |E|) \dot{A}^0 \dot{u} = \dot{B}_y \dot{u},
\]

where \( \dot{u} = \dot{v} = \text{col}(1, 1, \ldots, 1) \). Using Lemma 1, we obtain \( s(\dot{A}) < 1 \) and \( \rho(\dot{B} + \dot{B}_y (I_n - \dot{A})^{-1} \dot{C}_y) < 1 \), or equivalently, \( s(\dot{A}) < 1 \) and \( \rho(\dot{B} + \dot{B}_y (I_n - \dot{A})^{-1} \dot{B}_y) < 1 \) (i.e., (III) of Proposition 3 holds). This, together with Lemma 1, completes the proof.

**Remark 3**: Note that Proposition 3 does not require the condition \( \tilde{a}_1 \tilde{a}_1 \leq 1 \) or \( \tilde{b}_1 \tilde{b}_1 \leq 1 \). This, together with Theorem 4, claims that Proposition 3 is less conservative than Proposition 1, and hence the method proposed in this paper is superior to one in [18]. Moreover, it can be observed from the proof of Theorem 3 that Proposition 2 is less conservative than Proposition 3.

**Remark 4**: In [23], the authors investigated the global exponential stability for discrete-time BAM neural network with variable delay. When \( \alpha \beta^s(u_i(k)) = \beta_j^s(v_j(k)) = 1 \), \( \alpha \beta^s(u_i(k)) = \alpha \beta^s(v_j(k)) = 1 \), the delayed discrete-time CGBAMNN (18) becomes the BAMNN in [23]. Therefore, [23, Theorems 2 and 3] are special cases of Propositions 2 and 3, respectively.

**V. ILLUSTRATIVE EXAMPLES**

In this section, we will present the effectiveness of the proposed method by several numerical examples.

**Example 1**: Consider the one-dimensional CGBAMNN (3), where \( c_{11} = -0.15, a_{11} = -0.14, e_{11} = 0.25, w_{11} = 0.23 \), and

\[
\alpha_1^s(s) = 0.5 + 0.2 \cos(3s), \quad \beta_1^s(s) = 0.6 + 0.2 \sin(2s), \\
\alpha_1^s(s) = 3s, \quad \beta_1^s(s) = 2.9s, \\
f^s(s) = f^s(s) = \text{sin}(s), \\
g^s(s) = g^s(s) = \text{tanh}(s), \quad s \in [0, \infty), \\
h_{11}(k) = \tau_{11}(k) = 6, \quad k \in \mathbb{Z}[0, \infty). 
\]

It is clear that (4) is satisfied when \( \alpha_1 = 0.3, \beta_1 = 0.7, \beta_1 = 0.4, \beta_1 = 0.8, \lambda_1 = 1 = 3, \mu_1 = \mu_2 = 2.9 \text{ and } \Gamma_1 = \Gamma_2 = \Gamma_1 = \Gamma_2 = 1 \). When \( (u_1(k), v_1(k)) \equiv (3.9, 1.1) \text{ or } (1.4, -1.2), \quad s \in \mathbb{Z}[-6, 0], \) Figures 1 and 2 display the time domain behaviors of the state trajectories without controller, which claims that the considered open-loop system is not stable.

Now we design a state feedback controller (5) to stabilize the system under consideration. By employing the function eig of MATLAB, it is easy to verify that (ii) of Theorem 3 holds, and hence the considered system can be exponentially stabilized via the state-feedback controller: \( u_1(k) = 0.5 u_1(k), \quad v_1(k) = 0.74 v_1(k), \quad k \in \mathbb{Z}[0, \infty). \) Notice that the condition (b) in Theorem 3 is delay-independent, the global exponential stability of the resulting closed-loop system is independent of the choice of delays \( h_{11}(k) \) and \( \tau_{11}(k) \). Furthermore, the state responses of the closed-loop system are given in Figures 3 and 4 for different initial
functions, which demonstrates that the state trajectories of the resulting closed-loop system approach to zero and to keep them there. So, the resulting closed-loop system is stable due to the designed state-feedback controller.

Example 2: Consider CGBAMNN (3), where

\[
C = \begin{bmatrix}
-0.05 & 0.04 \\
0.02 & 0.03
\end{bmatrix}, \quad D = \begin{bmatrix}
-0.02 & 0.4 \\
0.03 & 0.04
\end{bmatrix},
\]

\[
E = \begin{bmatrix}
0.04 & 0.04 \\
-0.05 & 0.03
\end{bmatrix}, \quad W = \begin{bmatrix}
0.06 & 0.03 \\
-0.04 & 0.02
\end{bmatrix},
\]

\[
\alpha_i^*(s) = 0.5 + 0.25 \sin(s), \quad \alpha_i^*(s) = 0.5 + 0.25 \cos(s),
\]

\[
\beta_i^*(s) = 0.6 - 0.1 \sin(s), \quad \beta_i^*(s) = 0.6 - 0.1 \cos(s),
\]

\[
a_1(s) = a_2(s) = 3s, \quad b_1(s) = b_2(s) = 2s,
\]

\[
f_j^*(s) = g_j^*(s) = \sin(s),
\]

\[
\tilde{f}_j^*(s) = \tilde{g}_j^*(s) = \frac{1}{2} \sin(s), \quad \forall s \in [0, \infty), \quad i, j \in \mathbb{Z}[1, 2],
\]

\[
h_{ij}(k) = r_{ij} + s_{ji} \sin(k\pi/2),
\]

\[
\tau_{ij}(k) = p_{ij} + q_{ij} \cos(k\pi), \quad k \in \mathbb{Z}[0, \infty), \quad i, j \in \mathbb{Z}[1, 2],
\]

where \( r_{11} = r_{21} = r_{22} = 9, r_{12} = 10, s_{11} = s_{12} = s_{21} = s_{22} = 1, p_{11} = 4, p_{21} = p_{22} = 9, q_{11} = 11, q_{12} = q_{21} = q_{22} = 1 \). Clearly, the condition (4) is satisfied when \( \alpha_1 = \alpha_2 = 0.25, \alpha_1 = \alpha_2 = 0.75, \beta_1 = \beta_2 = 0.5, \hat{\beta}_1 = \hat{\beta}_2 = 0.7, \Gamma_1 = \Gamma_2 = I_2 \) and \( \Gamma_1 = \Gamma_2 = 0.5I_2 \). Furthermore, Assumptions 1 are satisfied by taking \( h_{11} = h_{21} = h_{22} = 10, \hat{h}_{12} = 11, \hat{r}_{11} = 5, \hat{r}_{21} = \hat{r}_{22} = 10 \) and \( \hat{r}_{12} = 12 \).

When \( u(s) \equiv \text{col}(-1.6, 4.1), v(s) \equiv \text{col}(-1.3, -3.8), s \in \mathbb{Z}[-12, 0] \), Figures 5 and 6 display the time domain behaviors of the state trajectories without controller, which claims that the considered open-loop system is not stable.

Now we design a state feedback controller (5) to stabilize the system under consideration. For \( \gamma = 0.022 \), by using the function \( \text{eig} \) of MATLAB, it is easy to verify that Theorem 2(b) holds, and hence the considered system can be exponentially stabilized via the state-feedback controller: \( U(k) = 0.5u(t), V(k) = 0.2v(t), k \in \mathbb{Z}[0, \infty) \). Moreover, if we use \( p_{11} = 5 \) instead of \( p_{11} = 4 \) in this example, then Theorem 2(b) is not true, which implies that Theorem 2 is not available in this case. So, the global exponential stability criterion provided in Theorem 2 is delay-dependent. When \( h_{ij}(k) \) and \( \tau_{ij}(k), (i, j \in \mathbb{Z}[1, 2]) \) are taken as above, the state responses of the closed-loop system are given in...
FIGURE 5. State trajectories of the open-loop system (Example 2).

FIGURE 6. State trajectories of the open-loop system (Example 2).

FIGURE 7. State trajectories of the closed-loop system (Example 2).

FIGURE 8. State trajectories of the closed-loop system (Example 2).

FIGURE 9. State trajectories of the considered CGBAMNN. (Example 3).

FIGURE 10. State trajectories of the considered CGBAMNN. (Example 3).

Example 3: Consider CGBAMNN (18) with \( n = 2 \),
\[
\alpha_1^*(s) = \frac{2 + \sin(s)}{4}, \quad \alpha_2^*(s) = \frac{2 + \cos(s)}{4}, \quad \beta_1^*(s) = \frac{4 - \sin(s)}{6}, \quad \beta_2^*(s) = \frac{4 - \cos(s)}{6}, \quad a_i^*(s) = b_i^*(s) = 2s, \quad f_i^*(s) = \sin(s), \quad \tilde{f}_i^*(s) = \frac{1}{2} \sin(s),
\]
\[
h_{ij}(k) = \tau_{ij}(k) \equiv 6, \quad e_{ij} = d_{ij} = 0, \quad e_{ij}^{-} = -\frac{\sqrt{2}}{16}, \quad w_{ij} = \frac{\sqrt{2}}{4}, \quad i, j = 1, 2, \quad s \in [0, \infty), \quad k \in \mathbb{Z}[0, \infty).
\]

Clearly, the inequalities in (4) are satisfied with \( \alpha_1 = \alpha_2 = 0.25, \quad \bar{\alpha}_1 = \bar{\alpha}_2 = 0.75, \quad \beta_1 = \beta_2 = 0.5, \quad \bar{\beta}_1 = \bar{\beta}_2 = \frac{5}{6}, \quad \lambda_1 = \lambda_2 = \lambda_1 = \lambda_2 = \mu_1 = \mu_2 = \bar{\lambda}_1 = \bar{\lambda}_2 = 2, \quad \gamma_1 = \gamma_2 = 1 \) and \( \bar{\gamma}_1 = \bar{\gamma}_2 = 0.5 \).

By using MATLAB, it is obvious that the (i) of Proposition 1 (i.e., [18, Corollary 2]) is not satisfied, and hence Proposition 1 can not be used to check the global exponential stability of the considered CGBAMNN.

It is easily obtained that \( \bar{a}_1 = \bar{a}_2 = 0.5, \quad \bar{b}_1 = \bar{b}_2 = \frac{5}{6} \).

By employing the function \texttt{eig} of MATLAB, it is easy to verify that (II) of Proposition 3 holds, and hence the zero equilibrium of CGBAMNN under consideration is globally exponentially stable. Furthermore, when \( u(s) \equiv \text{col}(-1.6, 4.1) \) and \( v(s) \equiv \text{col}(-1.3, -3.9) \) for any \( s \in \mathbb{Z}[-6, 0] \), the state responses of the resulting closed-loop system under consideration is given in Figures 9 and 10.
The authors would like to thank the anonymous reviewers for their helpful comments and suggestions which improve greatly the original version of this article.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their helpful comments and suggestions which improve greatly the original version of this article.

REFERENCES

[1] M. A. Cohen and S. Grossberg, “Absolute stability of global pattern formation and parallel memory storage by competitive neural networks,” IEEE Trans. Syst., Man, Cybern., vol. SMC-13, no. 5, pp. 815–826, Sep. 1983.

[2] B. Kosko, “Adaptive bidirectional associative memories,” Appl. Opt., vol. 26, no. 23, p. 4947, Dec. 1987.

[3] B. Kosko, “Bidirectional associative memories,” IEEE Trans. Syst., Man, Cybern., vol. SMC-18, no. 1, pp. 49–60, Jan./Feb. 1988.

[4] P. Shi, Y. Zhang, M. Chadli, and R. K. Agarwal, “Mixed H∞ and passive filtering for discrete fuzzy neural networks with stochastic jumps and time delays,” IEEE Trans. Neural Netw. Learn. Syst., vol. 27, no. 4, pp. 903–909, Apr. 2016.

[5] P. Shi, F. Li, L. Wu, and C.-C. Lim, “Neural network-based passive filtering for delayed neural-type semi-Markovian jump systems,” IEEE Trans. Neural Netw. Learn. Syst., vol. 28, no. 9, pp. 2101–2114, Sep. 2017.

[6] Z.-G. Wu, P. Shi, H. Su, and J. Chu, “Stochastic synchronization of Markovian jump neural networks with time-varying delay using sampled data,” IEEE Trans. Cybern., vol. 43, no. 6, pp. 1796–1806, Dec. 2013.

[7] J. Tao, Z.-G. Wu, H. Su, Y. Wu, and D. Zhang, “Asynchronous and resilient filtering for Markovian jump neural networks subject to extended dissipativity,” IEEE Trans. Cybern., vol. 49, no. 7, pp. 2504–2513, Jul. 2019.

[8] X. Wang and G.-H. Yang, “Fault-tolerant consensus tracking control for linear multiagent systems under switching directed network,” IEEE Trans. Cybern., vol. 50, no. 5, pp. 1921–1930, May 2020.

[9] H. Li, N. Zhao, X. Wang, X. Zhang, and P. Shi, “Necessary and sufficient conditions of exponential stability for delayed linear discrete-time systems,” IEEE Trans. Autom. Control, vol. 64, no. 2, pp. 712–719, Feb. 2019.

[10] K. Shi, J. Wang, Y. Tang, and S. Zhong, “Reliable asynchronous sampled-data filtering of T–S fuzzy uncertain delayed neural networks with stochastic switched topologies,” Fuzzy Sets Syst., vol. 381, pp. 1–25, Feb. 2020.

[11] K. Shi, J. Wang, S. Zhong, X. Zhang, Y. Liu, and J. Cheng, “New reliable nonuniform sampling control for uncertain chaotic neural networks under Markov switching topologies,” Appl. Math. Comput., vol. 347, pp. 169–193, Apr. 2019.

[12] S. Mohamad, “Global exponential stability in continuous-time and discrete-time delayed bidirectional neural networks,” Phys. D, Nonlinear Phenomena, vol. 159, nos. 3–4, pp. 233–251, Nov. 2001.

[13] J. Liang and J. Cao, “Exponential stability of continuous-time and discrete-time bidirectional associative memory networks with delays,” Chaos, Solitons Fractals, vol. 22, no. 4, pp. 773–785, Nov. 2004.

[14] J. Liang, J. Cao, and D. W. C. Ho, “Discrete-time bidirectional associative memory neural networks with variable delays,” Phys. Lett. A, vol. 335, nos. 2–3, pp. 226–234, Feb. 2005.

[15] X.-G. Liu, M. Wu, M.-L. Tang, and X.-B. Liu, “Global exponential stability of discrete-time BAM neural networks with variable delays,” in Proc. IEEE Int. Conf. Control Automat., May 2007, pp. 3139–3143.

[16] X.-G. Liu, M.-L. Tang, R. Martin, and X.-B. Liu, “Discrete-time BAM neural networks with variable delays,” Phys. Lett. A, vol. 367, nos. 4–5, pp. 322–330, Jul. 2007.

[17] Y. Chen, W. Bi, and Y. Wu, “Delay-dependent exponential stability for discrete-time BAM neural networks with time-varying delays,” Discrete Dyn. Nature Soc., vol. 2008, pp. 1–14, Nov. 2008.

[18] X. Lu, “Global exponential stability for discrete-time BAM neural network with variable delay,” in Proc. 6th Int. Symp. Neural Netw. (ISNN), Berlin, Germany: Springer, 2009, pp. 19–29.

[19] R. Zhang, Z. Wang, J. Feng, and Y. Jing, “Delay-dependent exponential stability of discrete-time BAM neural networks with time varying delays,” in Proc. Int. Symp. Neural Netw. Berlin, Germany: Springer, 2009, pp. 440–449.

[20] M. Gao and B. Cui, “Global robust exponential stability of discrete-time interval BAM neural networks with time-varying delays,” Appl. Math. Model., vol. 33, no. 3, pp. 1270–1284, Mar. 2009.

[21] R. Raja and S. M. Anthoni, “Global exponential stability of BAM neural networks with time-varying delays: The discrete-time case,” Commun. Nonlinear Sci. Numer. Simul., vol. 16, no. 2, pp. 613–622, 2011.

[22] Y. Shu, X. Liu, F. Wang, and S. Qiu, “Further results on exponential stability of discrete-time BAM neural networks with time-varying delays,” Math. Methods Appl. Sci., vol. 40, no. 11, pp. 4014–4027, Jul. 2017.

[23] E.-Y. Cong, X. Han, and X. Zhang, “Global exponential stability analysis of discrete-time BAM neural networks with delays: A mathematical induction approach,” Neurocomputing, vol. 379, pp. 227–235, Feb. 2020.

[24] A.-P. Chen and Q.-H. Gu, “Periodic solution to BAM-type Cohen–Grossberg neural network with time-varying delays,” Acta Mathematicae Applicatae Sinica, English Ser., vol. 27, no. 3, pp. 427–442, Jul. 2011.

[25] Z. Zhang, J. Cao, and D. Zhou, “Novel LMI-based condition on global exponential stability of discrete-time Cohen–Grossberg BAM neural networks with time-varying delays,” IEEE Trans. Neural Netw. Learn. Syst., vol. 25, no. 6, pp. 1161–1172, Jun. 2014.

[26] J. Jian and B. Wang, “Global Lagrange stability for neutral-type Cohen–Grossberg BAM neural networks with mixed time-varying delays,” Neurocomputing, vol. 116, pp. 1–25, Oct. 2015.

[27] H. Jiang and J. Cao, “BAM-type Cohen–Grossberg neural networks with time delays,” Math. Comput. Model., vol. 47, nos. 1–2, pp. 92–103, 2008.

[28] X. Li, “Exponential stability of Cohen–Grossberg-type BAM neural networks with time-varying delays via impulsive control,” Neurocomputing, vol. 73, nos. 1–3, pp. 525–530, Dec. 2009.
[29] Q. Zhou and L. Wan, “Impulsive effects on stability of Cohen–Grossberg-type bidirectional associative memory neural networks with delays,” Nonlinear Anal., Real World Appl., vol. 10, no. 4, pp. 2531–2540, Aug. 2009.

[30] Y. Xia, “Impulsive effect on the delayed Cohen–Grossberg-type BAM neural networks,” Neurocomputing, vol. 73, nos. 13–15, pp. 2754–2764, Aug. 2010.

[31] X. Li and X. Fu, “Global asymptotic stability of stochastic Cohen–Grossberg-type BAM neural networks with mixed delays: An LMI approach,” J. Comput. Appl. Math., vol. 235, no. 12, pp. 3385–3394, Apr. 2011.

[32] Z. Zhang, W. Liu, and D. Zhou, “Global asymptotic stability of generalized Cohen–Grossberg BAM neural networks of neutral type delays,” Neural Netw., vol. 25, pp. 94–105, Jan. 2012.

[33] W. Xiong, Y. Shi, and J. Cao, “Stability analysis of two-dimensional neutral-type Cohen–Grossberg BAM neural networks,” Neural Comput. Appl., vol. 28, no. 4, pp. 703–716, Apr. 2017.

[34] K. Subramanian and P. Muthukumar, “Existence, uniqueness, and global asymptotic stability analysis for delayed complex-valued Cohen–Grossberg BAM neural networks,” Neural Comput. Appl., vol. 29, no. 9, pp. 565–584, May 2018.

[35] J. Cao and Q. Song, “Stability in Cohen–Grossberg-type bidirectional associative memory neural networks with time-varying delays,” Nonlinearity, vol. 19, no. 7, pp. 1601–1617, 2006.

[36] D. Zhou, S. Yu, and Z. Zhang, “New LMI-based conditions for global exponential stability to a class of Cohen–Grossberg BAM networks with delays,” Neurocomputing, vol. 121, pp. 512–522, Dec. 2013.

[37] M. S. Ali, S. Saravanan, M. E. Rani, S. Elakkia, J. Cao, A. Alsaeedi, and T. Hayat, “Asymptotic stability of Cohen–Grossberg BAM neutral type neural networks with distributed time varying delays,” Neural Process. Lett., vol. 46, no. 3, pp. 991–1007, 2017.

[38] X. Liu, N. Jiang, J. Cao, S. Wang, and Z. Wang, “Finite-time stochastic stabilization for BAM neural networks with uncertainties,” J. Franklin Inst., vol. 350, no. 8, pp. 2109–2123, Oct. 2013.

[39] C. Aouiti, X. Li, and F. Miaadi, “A new LMI approach to finite and fixed time stabilization of high-order class of BAM neural networks with time-varying delays,” Neurocomputing, vol. 121, pp. 512–522, Dec. 2013.

[40] R. Chinnathambi, F. A. Rihan, and L. Shanmugam, “Stabilization of delayed Cohen–Grossberg BAM neural networks,” Math. Methods Appl. Sci., vol. 41, no. 2, pp. 593–605, 2018.

[41] P. H. A. Ngoc and H. Trinh, “Novel criteria for exponential stability of linear neutral time-varying differential systems,” IEEE Trans. Autom. Control, vol. 61, no. 6, pp. 1590–1594, Jun. 2016.

ER-YONG CONG received the B.S. and M.S. degrees from the School of Mathematical Science, Heilongjiang University, in 2003 and 2009, respectively. Since 2003, he has been working at Harbin University, where he is currently a Lecturer with the Department of Mathematical. His research interests include neural networks and stability analysis of delayed dynamic systems.

XIAO HAN received the Ph.D. degree from the School of Mathematics, Jilin University, in 2007. Since 2007, she has been working at Jilin University, where she is currently a Lecturer with the School of Mathematics. Her current research interests include computational mathematics, numerical solutions of differential equations, and actuarial mathematics.

XIAN ZHANG (Senior Member, IEEE) received the Ph.D. degree in control theory from Queen’s University Belfast, U.K., in 2004. Since 2004, he has been with Heilongjiang University, where he is currently a Professor with the School of Mathematical Science. He has authored more than 100 research articles. His current research interests include neural networks, genetic regulatory networks, mathematical biology, and stability analysis of delayed dynamic systems. He is the Vice President of Mathematical Society of Heilongjiang Province. He has received the Second Class of Science and Technology Awards of Heilongjiang Province, in 2005, and the Three Class of Science and Technology Awards of Heilongjiang Province, in 2015. Since 2006, he has been serving as an Editor of Journal of Natural Science of Heilongjiang University.