Weak sensitivity of three-body \((d,p)\) reactions to \(np\) force models

A. Deltuva

Institute of Theoretical Physics and Astronomy, Vilnius University, Saulėtekio al. 3, LT-10257 Vilnius, Lithuania

(Dated: 3 May, 2018)

Background: Adiabatic distorted-wave approximation (ADWA) study of three-body \((d,p)\) transfer reactions [G.W. Bailey, N.K. Timofeyuk, and J.A. Tostevin, Phys. Rev. Lett. 117, 162502 (2016)] reported strong sensitivity of cross sections to the neutron-proton \((np)\) interaction model when the nucleon-nucleus optical potential is nonlocal.

Purpose: The verification of this unusual finding using more reliable methods is aimed for in the present work.

Methods: A rigorous Faddeev-type three-body scattering theory is applied to the study of \((d,p)\) transfer reactions. The equations for transition operators are solved in the momentum-space partial-wave framework.

Results: Differential cross sections for \(^{26}\text{Al}(d,p)^{27}\text{Al}\) reactions are calculated using nonlocal nuclear optical potentials and a number of realistic \(np\) potentials. Only a weak dependence on the \(np\) force model is observed, typically one order of magnitude lower than in the previous ADWA study. The shape of the angular distribution of the experimental data is well reproduced.

Conclusions: Cross sections of \((d,p)\) transfer reactions calculated using a rigorous three-body method show little sensitivity to the \(np\) interaction model. This indicates a failure of the ADWA in the context of nonlocal potentials. Some evident shortcomings of the ADWA are pointed out.

I. INTRODUCTION

Three-body \((d,p)\) transfer reactions with nucleon-nucleus \((NA)\) nonlocal optical potentials (NLOP) and a number of neutron-proton \((np)\) potentials have been studied in Ref. [1] using the adiabatic distorted-wave approximation (ADWA). The authors of Ref. [1] claimed that NLOP leads to a substantial increase of the sensitivity of low-energy \((d,p)\) cross sections to high-momentum components in the deuteron wave function and \(np\) interaction. This sensitivity manifested itself as a strong dependence of \((d,p)\) cross sections on the \(np\) potential model, even exceeding a factor of 2 in particular cases such as the \(^{26}\text{Al}(d,p)^{27}\text{Al}(\text{g.s.})\) reaction. Such sensitivity contrasts with the usual behavior in the low-energy nuclear physics and calls for further studies using alternative methods. The authors of Ref. [1] suggested that the Weinberg state expansion, the basis of ADWA, should be revisited in the presence of nonlocality. The present work, being an extended version of the pioneering study [2], goes well beyond the suggestion of Ref. [1]. I use a rigorous treatment of three-body reactions with NLOP based on the Faddeev theory [3]. Although much more complicated than ADWA in terms of practical implementation [3], this treatment has the advantage of providing an exact solution of the three-body scattering problem. It does not rely on the Weinberg state expansion, thus, implicitly includes all Weinberg states up to infinity. Furthermore, it includes also the so-called “remnant term”, neglected in ADWA of Ref. [1], all relevant \(np\) waves beside the deuteron wave, and the proper Coulomb force acting between the proton and nucleus, not between the center-of-mass (c.m.) of the deuteron and nucleus as in the initial channel treatment by ADWA.

Section II recalls the three-body Faddeev formalism for transition operators. The results for \(^{26}\text{Al}(d,p)^{27}\text{Al}\) reactions, the study of the \(np\) force model sensitivity, and the comparison with the experimental data are presented in Sec. III. Section IV contains the conclusions and discussion, also pointing out some inadequacies of the ADWA.

II. THEORY

A rigorous three-body treatment of nuclear reactions with NLOP based on the Faddeev equations in the Alt-Grassberger-Sandhas (AGS) form [4] was first implemented in Refs. [4, 5]. At the given three-body energy \(E\) in the c.m. frame the transition operators \(U_{\beta\alpha}(E)\) obey the system of integral equations

\[
U_{\beta\alpha}(E) = \delta_{\beta\alpha} G_0^{-1}(E) + \sum_{\gamma=1}^3 \delta_{\beta\gamma} T_\gamma(E) G_0(E) U_{\gamma\alpha}(E) \tag{1}
\]

with \(\delta_{\beta\alpha} = 1 - \delta_{\beta\alpha}\), the free resolvent \(G_0(E) = (E - H_0 + \alpha)\), and the free Hamiltonian for the relative motion \(H_0\). The two-particle transition matrices in three-particle space

\[
T_\gamma(E) = v_\gamma + v_\gamma G_0(E) T_\gamma(E) \tag{2}
\]

are obtained from the corresponding two-particle potentials \(v_\gamma\). Odd-man-out notation is used to label pairs of particles and two-cluster channels \(\alpha\) with asymptotic states \(|\Phi_\alpha\rangle\), while on-shell matrix elements \(|\Phi_\beta(U_{3\alpha}(E)|\Phi_\alpha\rangle\) yield \(\alpha \to \beta\) reaction amplitudes. The integral equations (1) are solved in the momentum-space partial-wave framework, leading to well-converged results for \(^{26}\text{Al}(d,p)^{27}\text{Al}\) differential cross sections, with the proton-nucleus Coulomb force included via the screening and renormalization method. Further technical details on the solution can be found in Refs. [4, 6].
III. RESULTS

I study $^{26}$Al$(d,p)$$^{27}$Al reactions using a number of realistic high-precision $np$ potentials: Argonne V18 (AV18) \cite{1}, charge-dependent Bonn (CD Bonn) \cite{2}, Reid93 \cite{3}, chiral effective field theory ($\chi$EFT) potentials at next-to-next-to-next-to-leading order (N4LO) \cite{4} with regulators of 0.8 and 1.2 fm, and the AV18 potential softened by the similarity renormalization group (SRG) transformation \cite{11, 12} with the flow parameter $\lambda = 1.8$ fm$^{-1}$. The deuteron $D$-state probability, characterizing the strength of the tensor force and high-momentum components, acquires values in a broad range from 2.53\% (SRG) to 5.76\% (AV18), even broader than in Ref. \cite{1}. To compare with ADWA results of Ref. \cite{1}, I take the same NLOP for $p$-$^{26}$Al and $n$-$^{26}$Al, i.e., the parametrization of Giannini and Ricco \cite{13} without the spin-orbit part. Binding potentials for $^{27}$Al are also chosen as in Ref. \cite{1}, i.e., they have local Woods-Saxon form with the radius $r_0 = 1.25$ fm, diffuseness $a = 0.65$ fm, and spin-orbit strength $V_{so} = 6$ MeV, whereas the strength of the central potential is adjusted to the neutron separation energy $S_n$ for the given state.

The results at the deuteron beam energy $E_d = 12$ MeV for $^{26}$Al$(d,p)$$^{27}$Al differential cross sections with ground and excited final states are presented in Fig. 1. Both in the present work and in Ref. \cite{1} the sensitivity to the $np$ potential reaches its maximum at forward angles, therefore the following comparison refers to the c.m. scattering angle $\Theta_{c.m.} = 0$ deg. The largest sensitivity to the $np$ potential using ADWA in Ref. \cite{1} was found for the transfer to the $^{27}$Al ground state with spin/parity $J^p = 5/2^+$ and $S_n = 13.06$ MeV. At $\Theta_{c.m.} = 0$ deg the spread of ADWA results with realistic $np$ potentials in Ref. \cite{1} is roughly a factor of 2.5, or about $\pm 50\%$ when measured from the central value. In contrast, rigorous Faddeev calculations, as shown in the top panel of Fig. 1 exhibit much weaker sensitivity to the $np$ force model, about $\pm 5\%$ when measured from the respective central value. Furthermore, the magnitude of the differential cross section predicted using the Faddeev framework at $\Theta_{c.m.} = 0$ deg is lower by a factor of 2 (CD Bonn) to 4 (AV18) as compared to ADWA; in the latter case even the shape is quite different.

Results for $(d,p)$ transfer reactions leading to excited states of $^{27}$Al are also presented in Fig. 1 this time using AV18, CD Bonn, and SRG potentials only; predictions of other potentials lie between those of AV18 and SRG and are therefore not shown. In all cases the sensitivity to the $np$ force model is significantly weaker than in ADWA predictions of Ref. \cite{1}. For example, for the reaction leading to the $J^p = 9/2^+$, $S_n = 5.25$ MeV state at $\Theta_{c.m.} = 0$ deg the spread of Faddeev-type results, measured from their central values, is $\pm 1\%$ and $\pm 3\%$ for the angular momentum transfer $L = 0$ and $L = 2$, respectively, while the spread of the corresponding ADWA results is about $\pm 7\%$ and $\pm 25\%$. Furthermore, the magnitude of Faddeev-type predictions for all excited states shown in Fig. 1 is lower than ADWA by a factor of roughly 1.5, while the shape is qualitatively similar. Note that $np$ partial waves other than the deuteron wave $^3S_1 - ^3D_1$ contribute to the cross section up to 10\%; those waves are neglected in ADWA \cite{1}.

Although the main goal of the present work is the study

![FIG. 1. Differential cross sections for $^{26}$Al$(d,p)$$^{27}$Al reactions at $E_d = 12$ MeV as functions of the c.m. scattering angle. The neutron separation energy $S_n$, spin and parity of the final nucleus $J^p$, and the orbital angular momentum transfer $L$ are indicated in each panel. Predictions obtained with different realistic $np$ potentials are compared.](image)
FIG. 2. Differential cross sections for $^{26}$Al$(d,p)^{27}$Al reactions at $E_d = 12$ MeV as functions of the c.m. scattering angle. The excitation energy $E_x$, spin and parity of the final nucleus $J^\pi$, and the contributing orbital angular momentum transfer $L$ are indicated in each panel. CD Bonn predictions, rescaled by factors $X(E_x, L)$ given in the text, are compared with the experimental data from Ref. [14].

of $(d,p)$ cross section sensitivity to the $np$ force model, the comparison of angular distributions with the experimental data is also of some interest, even if the calculations neglect the core excitation that is an important dynamic ingredient. To account roughly for this shortcoming, the theoretical predictions are rescaled by factors $X(E_x, L)$, such that the differential cross section $d\sigma_x/d\Omega$ for the given $^{27}$Al state with the excitation energy $E_x$ becomes

$$d\sigma_x/d\Omega = \sum L X(E_x, L) \frac{d\sigma(E_x, L)}{d\Omega}. \quad (3)$$

I emphasize that $X(E_x, L)$ are not spectroscopic factors (SF), but, given the low reaction energy $E_d = 12$ MeV, may be quite close to the SF as the rigorous study including the core excitation indicates [15]. The $X(E_x, L)$ values, obtained by adjusting the CD Bonn results from Fig. 2 to the experimental data from Ref. [14], are $X(3.004, 0) = 0.59$, $X(7.806, 0) = 0.0105$, $X(7.806, 2) = 0.09$, and $X(7.948, 1) = 0.002$, where the excitation energies are given in MeV. The comparison is presented in Fig. 2. One may conclude that theoretical calculations describe the shape of the experimental data [14] quite well.

IV. DISCUSSION AND CONCLUSIONS

The present study of $(d,p)$ reactions, performed in the rigorous momentum-space three-body framework, indicates low sensitivity to the $np$ potential model, and thereby a failure of the ADWA with NLOP. This may appear quite unexpected, since a decent accuracy of the ADWA, 20% or better [14 17], was found when using local optical potentials. However, given that no sensitivity to the $np$ model was observed for local optical potentials [18], much larger disagreement in the case of NLOP probably indicates the inadequacy of ADWA specifically for treating NLOP. In fact, some inadequacy can easily be seen when confronting the leading-order NLOP effect in ADWA [18] to the rigorous Faddeev scattering framework [3]. This comparison is justified, since the leading-order yields a good approximation of the complete ADWA [18], and therefore shortcomings characteristic to the leading-order ADWA should be valid also in the case of the complete ADWA. The leading-order NLOP effect in ADWA is simulated replacing the energy-independent NLOP in the initial deuteron channel by the equivalent local optical potential (ELOP) [18]. Energy-dependent ELOP has to be evaluated at the energy $E_{loc} = E_d/2 + \Delta E$ where, depending on the underlying $np$ potential, $\Delta E$ typically acquires values from 40 to 70 MeV. E.g., $E_{loc} \approx 50$ MeV (72 MeV) for CD Bonn (AV18) models in the considered reactions at $E_d = 12$ MeV. However, an inspection of the interaction terms on the r.h.s. of the Faddeev equation [14], i.e.,

$$\int \langle \Phi_\beta | \bar{\delta}_\beta \gamma | p'_\gamma q_\gamma \rangle d^3 p'_\gamma \langle p'_\gamma | T_\gamma (E - q^2/2M_\gamma) | p_\gamma \rangle$$

$$\frac{d^3 p_\gamma d^3 q_\gamma}{E + i0 - p^2/2\mu_\gamma - q^2/2M_\gamma} \langle p_\gamma q_\gamma | U_{\gamma \alpha} (E) | \Phi_\alpha \rangle \quad (4)$$

reveals that the energy $E - q^2/2M_\gamma$ of any interacting two-particle subsystem $\gamma$ with the intermediate relative momentum $q_\gamma$ is not fixed but depends on the respective spectator momentum $q_\gamma$ that is an integration variable, formally ranging from zero to infinity; $\mu_\gamma$ and $M_\gamma$ are the corresponding reduced masses. The energy of the nucleon-nucleus subsystem therefore formally acquires values from $-\infty$ to $E$, i.e., its upper limit is $E_{N/A}^{max} = E_d A / (A + 2) - 2.2245$ MeV, amounting to $E_{N/A}^{max} \approx 9$ MeV in the present study. Thus, $E_{loc}$ of the order 50 MeV as required in the ADWA appears to be a very unnatural value from the Faddeev formalism point of view, where the three-body amplitudes depend on fully off-shell nucleon-nucleus transition operators $\langle p'_\gamma | T_\gamma (E - q^2/2M_\gamma) | p_\gamma \rangle$ but with the relative two-body energy below $E_{N/A}^{max}$. Note that there is no such a clear contradiction in the case of ADWA with local potentials that are taken at the energy $E_d/2$. Furthermore,
the ELOP that is used in the leading-order ADWA is on-shell equivalent to the original NLOP only at $E_{N_A} = E_{\text{loc}}$ but deviates from it otherwise. As a consequence, ELOP and NLOP do not provide an equivalent description of the initial scattering state in $d + A$ collisions. Most importantly, the ability of the energy-independent NLOP to describe the nucleon-nucleus subsystem over a broader energy range is lost when using ELOP with fixed parameters [12].

In conclusion, ADWA appears to be inadequate for three-body ($d, p$) reactions with NLOP, where low sensitivity to the $np$ interaction models is found by rigorous Faddeev-type calculations. A similar conclusion is drawn also by the alternative study using the approximate continuum-discretized coupled-channel method [19]. However, more significant sensitivity to nucleon-nucleon force models can be expected in truly ab initio calculations without any use of optical potentials, where even the threshold positions may depend on the force model. Rigorous four-particle calculations of $^2\text{H}(d, p)^3\text{H}$ and $^2\text{H}(d, n)^3\text{He}$ reactions [20] provide an example for such dependence.

ACKNOWLEDGMENTS

I thank E. Epelbaum for providing the codes for \chiEFT potentials. I acknowledge the support by the Alexander von Humboldt Foundation under Grant No. LTU-1185721-HFST-E, and the hospitality of the Ruhr-Universität Bochum where a part of this work was performed.

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