GROWTH OF EARLY SUPERMASSIVE BLACK HOLES AND
THE HIGH-REDSHIFT EDDINGTON RATIO DISTRIBUTION

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ABSTRACT

Using a new large-scale (~0.75 Gpc)^3 hydrodynamic cosmological simulation, we investigate the growth rate of supermassive black holes (BHs) in the early universe (z > 4.75). Remarkably we find a clear peak in the typical Eddington ratio (λ) at BH masses of (4-8) × 10^9 M⊙ (typically in halos of ~7 × 10^{11} to 1 × 10^{12} M⊙, close to their shock heating scale), independent of redshift and indicative that most BH growth occurs in the cold-flow-dominated regime. BH growth is enhanced at high-z and by and large regulated by the cosmological evolution of gas density, with λ scaling simply as (1+z)^3. The peak in λ is caused by the competition between increased gas density available in more massive hosts, and a decrease due to active galactic nucleus feedback that becomes effective above the shock heating halo mass scale and at high BH masses. We show that the distribution of λ among both mass-selected and luminosity-selected samples is approximately lognormal. We combine these findings into a single lognormal fitting formula for the distribution of Eddington ratios as a function of (M_{BH}, z). This formula can be used in analytic and semianalytic models for evolving BH populations, predicting BH masses of observed quasars, and, in conjunction with the observed distribution of Eddington ratios, can be used to constrain the BH mass function.

Key words: black hole physics \-- galaxies: active \-- galaxies: evolution \-- methods: numerical \-- quasars: general

Online-only material: color figures

1. INTRODUCTION

It has been well established that supermassive black holes (BHs) are present in the centers of most galaxies, and that they are correlated with the properties of their hosts (e.g., Tremaine et al. 2002). These correlations provide strong evidence that the growth of a BH and the evolution of its host galaxy directly influence one another, such that BH growth is an important aspect of understanding galactic evolution and vice versa.

In general, the link between BH and galactic evolution is attributed to some form of quasar feedback which can result in the self-regulation of the growth of the BH (e.g., Di Matteo et al. 2005). In this model, we would expect BHs to grow rapidly during their early lifetime (i.e., while at low mass), until some point at which the BH feedback begins to significantly affect its environment, resulting in a noticeable decline in growth rate. This effect has been observed in individual BH histories, but such investigations (see, e.g., Sijacki et al. 2009; Di Matteo et al. 2012) have tended to focus on the largest mass BHs, primarily to explain how BHs could grow rapidly enough to produce the extremely large masses (~10^9 M⊙ by z ~ 6) found in observations by the Sloan Digital Sky Survey (e.g., Fan et al. 2006; Jiang et al. 2009). In this Letter we take advantage of a new, very large simulation to investigate the growth histories of early universe BHs across a wide range of masses, probing both the mean and the distribution of growth rates for BHs across a wide range of masses and luminosities, and provide fits for these distributions.

2. METHOD

In this Letter, we investigate a new 533 h^{-1} Mpc hydrodynamic simulation specifically for high-redshift investigations. The simulation uses the massively parallel cosmological TreePM-SPH code P-GADGET (an updated version of GADGET-2; see Springel 2005) incorporating a multi-phase interstellar medium model with star formation (Springel & Hernquist 2003) and BH accretion and feedback (Springel et al. 2005; Di Matteo et al. 2005). It has a gravitational softening length of 5 h^{-1} kpc (comoving) and mass resolution of 2.8 × 10^8 M⊙ for dark matter and 5.7 × 10^7 M⊙ for gas.

The model for BH creation, accretion, and feedback used in our simulation has been investigated and discussed in Sijacki et al. (2007, 2009), Di Matteo et al. (2008), Colberg & di Matteo (2008), DeGraf et al. (2010), and Degraf et al. (2011), and these authors have found that it does a good job of reproducing the \( M_{BH} - \sigma \) relation, the total BH mass density (Di Matteo et al. 2008), the quasar luminosity function (DeGraf et al. 2010), and the expected BH clustering behavior (Degraf et al. 2011). This simple model thus appears to model the growth, activity, and evolution of supermassive BHs in a cosmological context surprisingly well (though the detailed treatment of the accretion physics is infeasible for cosmological scale simulations). We also note that Booth & Schaye (2009) and Johansson et al. (2009) have adopted a very similar model, and have independently investigated the parameter space of the reference model of Di Matteo et al. (2008), as well as varying some of the underlying prescriptions. In addition, this simulation has previously been used to investigate the growth of the first very massive BHs (Di Matteo et al. 2012), statistical properties of quasars (DeGraf et al. 2011), and large-scale high-resolution imaging (Feng et al. 2011). We provide a general summary of the treatment of BHs here. For a more complete description of the model, see Di Matteo et al. (2008), and for analysis of the overall BH populations produced by this simulation, see DeGraf et al. (2011).

2.1. Black Hole Model

BHs are modeled as collisionless sink particles that form in newly emerging and resolved dark matter halos found by a friends-of-friends group finder run at regular intervals (spaced...
by \( \Delta \log a = \log 1.25 \). Any group above a threshold of \( 7 \times 10^{10} M_\odot \) not already containing a BH is seeded with a sink particle with \( M_{\text{BH,seed}} = 7 \times 10^5 M_\odot \). This seeding prescription is chosen to reasonably match the expected formation of supermassive BHs by gas directly collapsing to BHs with \( M_{\text{BH}} \sim M_{\text{seed}} \) (e.g., Bromm & Loeb 2003; Begelman et al. 2006) or by PopIII stars collapsing to \( \sim 10^2 M_\odot \) BHs at \( z \sim 30 \) (Bromm & Larson 2004; Yoshida et al. 2006) followed by sufficient exponential growth to reach \( M_{\text{seed}} \) by the time the host halo reaches \( \sim 10^{10} M_\odot \). The BHs then accrete gas according to 

\[
M_{\text{BH}} = \left(4\pi G^2 M_{\text{BH}}^2 \rho_{\text{BH}}/c_s^2 + v^2\right)^{3/2},
\]

where \( \rho_{\text{BH}} \) is the local gas density (determined from the gas particles within the BH kernel), \( v \) is the velocity of the BH relative to the surrounding gas, and \( c_s \) is the local sound speed accounting for both hot and cold phases (as in Pelupessy et al. 2007). Note that all calculations of \( M_{\text{BH}} \) and \( \dot{M}_{\text{BH}} \) in both the simulation and our analysis are based on the smooth growth from this equation, and thus do not depend on the swallowing of individual smoothed particle hydrodynamics (SPH) particles. To allow for the initial rapid BH growth necessary to produce sufficiently massive BHs at early times \((\sim 10^9 M_\odot)\) by \( z \sim 6 \) we allow for mildly super-Eddington accretion, but impose a maximum of \( 3 \times \dot{M}_{\text{Edd}} \) to prevent artificially high values.

The BH is assumed to radiate with a bolometric luminosity proportional to the accretion rate, \( L = \eta \dot{M}_{\text{BH}} c^2 \), where the radiative efficiency \( \eta \) is fixed to 0.1 throughout both simulation and analysis, 5% of which is isotropically deposited to the local BH kernel as thermal energy. This feedback efficiency is set to 5% based on galaxy merger simulations such that the normalization of the \( M_{\text{BH}}-\sigma \) relation is reproduced (Di Matteo et al. 2005). Note that some newer models have also begun introducing momentum-driven feedback (e.g., Ostriker et al. 2010; DeBuhr et al. 2012).

In addition, BHs can grow through mergers. In cosmological volumes, it is not possible to directly model the physics of the infalling BHs at the smallest scales, so a subresolution model is used. Since it has been shown that final coalescence in a gas-rich environment will be rapid (e.g., Mayer et al. 2007), BH pairs are merged when their separation falls below the spatial resolution, unless they are rapidly passing by one another (relative velocity greater than half the local sound speed). We note that the merging of the BH particles has minimal effect on \( M_{\text{BH}} \), so this merger criterion should not affect our results.

3. RESULTS

3.1. Typical Black Hole Growth Rates

To quantify the growth rate of BHs, we use the mean Eddington ratio \( \lambda = (M_{\text{BH}}/\dot{M}_{\text{Edd}}) \) calculated for every BH (46,508 between 10^7 and 10^9 M_\odot at \( z = 5 \)) over a finite time interval. Because we have the complete BH growth history, we are able to compute this quantity based solely on the gas accretion, and neglect any mass gained through BH mergers (though the mass gained by mergers is small enough to have a negligible effect on our results). In Figure 1, we plot \( \langle \lambda \rangle/(1+z)^3 \) as a function of \( M_{\text{BH,initial}} \) for several redshift ranges. We plot \( \langle \lambda \rangle/(1+z)^3 \) rather than \( \langle \lambda \rangle \) for two reasons: to show that the dependence of \( \langle \lambda \rangle \) on \( M_{\text{BH}} \) is independent of redshift (at least for \( z \geq 4.75 \)) and to show that \( \langle \lambda \rangle \propto (1+z)^3 \).

Regardless of redshift considered, we find similar behavior for the Eddington ratio with respect to mass: more massive BHs grow faster than lower mass BHs, up to a peak growth rate at \( M_{\text{BH}} \sim (4-8) \times 10^7 M_\odot \) while the BHs above this characteristic mass grow more slowly. Thus BHs grow fastest (relative to their current mass) while at intermediate masses, and slow down at higher masses.

We find this peak in the Eddington ratio to be caused by the change in the local gas density available for fueling BH growth. We plot the evolution in local gas density \( \rho_{\text{BH}} \) for each BH in Figure 1, showing a clear peak at \( \sim 5 \times 10^7 M_\odot \). We note that neither the sound speed nor the BH velocity (the other factors in the calculation of \( M_{\text{BH}} \)) exhibit a peak with respect to \( M_{\text{BH}} \). To show how the gas density evolves, in Figure 2 we show the gas density profiles around BHs below the Eddington ratio peak \( \sim 10^7 M_\odot \) (blue), at the peak \( \sim 5 \times 10^7 M_\odot \) (green), and above the peak \( \sim 4 \times 10^8 M_\odot \) (red), each averaged over 100 BHs. In general we find the gas density profile to grow with \( M_{\text{BH}} \) until \( M_{\text{BH}} \sim 5 \times 10^5 M_\odot \), as expected for BHs found in more massive halos). Above \( 5 \times 10^5 M_\odot \) the gas density away from the BH continues to grow, but the innermost density is suppressed, with the suppression increasing with \( M_{\text{BH}} \) in both magnitude and distance. This suppression of the local gas density is caused by the feedback of the BH, with the stronger feedback of high-mass BHs producing the strongest effect (see Di Matteo et al. 2012 for detailed investigation of feedback among massive BHs). Although the details of the density profile are dependent on our SF model, we expect this direct BH feedback to be found independent of the SF model.

We also show the typical mass of halos hosting a given \( M_{\text{BH}} \) on the top axis of Figure 1, noting that the Eddington ratio peaks at a host halo mass of \( \sim 7 \times 10^{11} \) to \( 1 \times 10^{12} M_\odot \). This very closely matches the critical shock heating scale of \( \sim 6 \times 10^{11} M_\odot \).
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Figure 2. Gas density profiles averaged among 100 black holes with mass \( \sim 10^7 M_\odot \) (blue), \( \sim 5 \times 10^7 M_\odot \) (green), \( \sim 4 \times 10^8 M_\odot \) (red). Dotted line shows the gravitational softening length.

(A color version of this figure is available in the online journal.)

Figure 3. Redshift evolution of the Eddington ratio for black holes with \( M_{\text{BH}} > 10^7 M_\odot \) (shaded region shows 1\( \sigma \) standard deviation in \( \log(\lambda) \)) and \( i\)-band quasar magnitude \( m_i < 21 \) (green line) compared with data from Shen & Kelly (2012) (black asterisks). We also show the evolution in the gas density around BHs for comparison (blue dashed line).

(A color version of this figure is available in the online journal.)

(Dekel & Birnboim 2006; Dekel et al. 2009, and consistent with our simulation), above which infalling gas is shock-heated near the virial radius to the virial temperature of the halo. Dekel & Birnboim (2006) suggest that in these halos active galactic nucleus (AGN) feedback becomes more significant, since the diluted shock-heated gas will be more susceptible to heating and pushing by the central AGN. This would thus produce a suppression in the gas density profile, consistent with the picture described above and the downturn in Figure 1.

In addition to the evolution in \( \lambda \) with \( M_{\text{BH}} \), Figure 1 also shows that \( \lambda \) evolves with redshift as \( \sim (1 + z)^3 \), which is also caused by the evolution in the local gas density. In Figure 3 we show the evolution in \( \langle \log(\lambda) \rangle \) with redshift among \( M_{\text{BH}} > 10^7 M_\odot \) BHs (shaded region, showing 1\( \sigma \) standard deviation). We plot the average gas density at the BH (blue dashed line), showing that the evolution in \( \lambda \) is primarily caused by the evolution in \( \rho_{\text{BH}} \) (recall \( M_{\text{BH}} \propto \rho_{\text{BH}} \)). We also compare these results to observational measurements of Shen & Kelly (2012) (black asterisks), showing that this general redshift evolution is consistent with current observations, and the normalization is approximately consistent if we use a similar magnitude cut on the quasar luminosity (\( i\)-band magnitude \( m_i < 21 \) using quasar SED of Hopkins et al. 2007b; green line).

3.2. Eddington Ratio Distributions

In addition to investigating the mean Eddington ratio, we also study the distribution of \( \lambda \) among comparable BHs. Previous work on the \( \lambda \) distribution has often found roughly lognormal distributions using both observational (Kollmeier et al. 2006; Netzer et al. 2007; Netzer & Trakhtenbrot 2007; Willott et al. 2010; Trakhtenbrot et al. 2011) and phenomenological approaches (Shankar et al. 2011) (though Aird et al. 2012 find \( \lambda \) to follow a power law when selecting by host stellar mass rather than BH mass). However, these observational studies necessarily incorporate several uncertainties such as sample selection and scatter in BH mass estimators whereas our simulation allows us to probe our BH Eddington ratios directly. In Figure 4 we show the distribution of Eddington ratios among BHs selected by \( M_{\text{BH}} \) (black histograms). We find that the distribution produced by our simulation is indeed lognormal, in keeping with
observational findings (Kollmeier et al. 2006; Netzer et al. 2007; Netzer & Trakhtenbrot 2007; Willott et al. 2010; Trakhtenbrot et al. 2011). In particular, we note that the distribution remains lognormal regardless of the mass considered, with Figure 4 showing this holds among BHs that are below, at, and above the peak observed in Figure 1.

Because we find \( \lambda \) to follow a lognormal distribution and the mean of that distribution obeys a well-defined dependence on \( M_{\text{BH}} \) (Figure 1), we are able to provide a general fitting formula for \( P(\lambda|M_{\text{BH}}, z) \), the probability distribution of BH Eddington ratios based on redshift and \( M_{\text{BH}} \):

\[
P(\lambda|M_{\text{BH}}, z) = \frac{1}{\lambda \sigma_m \sqrt{2\pi}} e^{-\frac{(\log(\lambda) - \mu_m)^2}{2\sigma_m^2}},
\]

where \( \mu_m \) and \( \sigma_m \) are the mean and standard deviation of \( \log(\lambda) \), respectively, and are fit by \( \sigma_m \approx 0.39 \) and

\[
\mu_m = (1 + z)^3 A e^\left(\frac{\log(\mu_{\text{BH}})}{\sigma_0} \right)^2/\sigma_0^2,
\]

with \( A \approx 0.0094, M_{\mu} = 5 \times 10^7 M_\odot \), and \( \sigma_0 \approx 0.85 \). In Figure 4, we plot the distribution predicted by Equations (1) and (2) (red curve) compared to the actual distribution, showing that this simple formula is capable of reproducing the distribution of \( \lambda \) for BHs in our simulation across a wide range of masses and redshifts, without requiring knowledge of individual BH environments.

In addition to the distribution for a mass-selected sample, in the top panels of Figure 5 we show the Eddington ratio distribution from our simulation (red histogram) compared to the observed distribution from Kollmeier et al. (2006) (black histogram) for two luminosity-selected samples. We again note that the distribution is described by a roughly lognormal distribution, and that our simulation is approximately consistent with observational results.

Furthermore, by combining \( P(\lambda|M_{\text{BH}}, z) \) with the BH mass function (\( \Phi_{\text{BH}} \)) we can obtain the Eddington ratio distribution for a luminosity-selected sample:

\[
P(\lambda|L_{\text{BH}}, z) = \frac{\Phi_{\text{BH}}(M_{\text{BH}}) P(\lambda|M_{\text{BH}}, z)}{\int_0^\infty \Phi_{\text{BH}}(M_{\text{BH}}) P(\lambda|M_{\text{BH}}, z) d\lambda},
\]

where \( M_{\text{BH}} = (\sigma T L_{\text{BH}}/4\pi G m_p c \lambda) \). In Figure 5 we plot this predicted probability distribution (using our simulation’s mass function) in red, showing \( P(\lambda|L_{\text{BH}}, z) \) is well predicted in this manner. We note that this approach is significant as it provides a potentially powerful tool for constraining the BH mass function using observations of the Eddington ratio distribution. We show this in Figure 5 by plotting \( P(\lambda|L_{\text{BH}}, z) \) based on three different local mass functions: the Shankar et al. (2009) mass function (dashed green), the Shankar et al. (2009) mass function derived from the Hopkins et al. (2007b) luminosity function (dashed blue), and the mass function of Hopkins et al. (2007a) (dashed pink). Because \( P(\lambda|L_{\text{BH}}) \) is sensitive to the slope of \( \Phi_{\text{BH}} \), the distribution of \( \lambda \) at high \( L_{\text{BH}} \) (where the mass function is steepest) varies substantially with the mass function used, suggesting that with the improved statistics from upcoming surveys, we could use the observed \( P(\lambda|L_{\text{BH}}) \) to constrain the slope of the BH mass function at high redshift, even without measurements of the \( M_{\text{BH}} \).

In the bottom panel of Figure 5, we compare our simulation to recent observational results at \( z = 4.8 \) (Trakhtenbrot et al. 2011). Here, we see that despite very limited sample sizes (within the overlapping luminosity range there are 8 objects from our simulation and 22 from Trakhtenbrot et al. 2011) we have excellent agreement between the simulation and observations.

4. CONCLUSIONS

Using a new large-scale simulation, we show that the growth of BHs tends to follow a typical growth pattern. In general, we find that BHs grow more rapidly at higher redshift than comparable ones at lower redshift, characterized by \( \lambda \propto (1+z)^{3} \). This scaling is caused by the redshift evolution in the gas density about the BHs and is comparable to current observational data from Shen & Kelly (2012). We note that our results are based on a model with limited resolution, however, we focus on the long-term trends averaged across large samples thereby minimizing the effect of small-scale variations, particularly among the most massive objects.

The typical Eddington ratio also scales with \( M_{\text{BH}} \) such that \( \lambda \) peaks at \( M_{\text{BH}} \sim (4-8) \times 10^7 M_\odot \) (typically found in halos of \( \sim 7 \times 10^{11} \) to \( 1 \times 10^{12} M_\odot \)). This peak is caused by evolution in the density of the gas at halo centers available to fuel BH growth. In general, more massive BHs are found in more massive halos.
with correspondingly higher gas densities, hence \( \lambda \) grows with \( M_{\text{BH}} \) for low masses. However, above \( M_{\text{BH}} \sim 5 \times 10^7 M_{\odot} \) BH feedback has a sufficiently strong effect on the local environment to suppress the nearby gas density. Thus although these more massive BHs are found in more massive halos with generally higher gas densities, the feedback has significantly lessened the density of the innermost gas where accretion occurs. This suppression of the local gas density is stronger for more massive BHs and causes \( \lambda \) to decrease for \( M_{\text{BH}} \gtrsim 5 \times 10^7 M_{\odot} \).

Although the local environment is important for the accretion rate of individual BHs, we show that the distribution of Eddington ratios follows a roughly lognormal distribution regardless of the BH population considered, consistent with current observational findings. We use this, together with the evolution in \( \langle \lambda \rangle \), to provide a simple fitting formula for the distribution of Eddington ratio with \( (M_{\text{BH}}, z) \). This general formula can be used for predicting the growth/evolution of BH populations in theoretical and semianalytic models (such as the evolution of the BH mass function), for predicting the mass of observed high-redshift quasars, and, in conjunction with upcoming observations of the \( \lambda \)-distribution, to constrain the slope of the high-redshift BH mass function.

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