Analytical models of Ohmic heating and conventional heating in food processing

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Abstract. Ohmic heating is a food processing operation in which an electric current is passed through a food and the electrical resistance of the food causes the electric power to be transformed directly into heat. The heat is not delivered through a surface as in conventional heat exchangers but it is internally generated by Joule effect. Therefore, no temperature gradient is required and it origins quicker and more uniform heating within the food. On the other hand, it is associated with high energy costs and its use is limited to a particular range of food products with an appropriate electrical conductivity. Sterilization of foods by Ohmic heating has gained growing interest in the last few years. The aim of this study is to evaluate the benefits of Ohmic heating with respect to conventional heat exchangers under uniform wall temperature, a condition that is often present in industrial plants. This comparison is carried out by means of analytical models. The two different heating conditions are simulated under typical circumstances for the food industry. Particular attention is paid to the uniformity of the heat treatment and to the heating section length required in the two different conditions.

1. Introduction

Ohmic heating is an innovative thermal processing method for the food industry wherein an alternating current is passed through a food with the specific purpose of increasing its temperature. Heating occurs in the form of internal energy generation and offers significant advantages over conventional food processing methods [1]. In fact, during conventional thermal processing, heat transfer occurs from a heated surface to the inner product by means of conduction and convection phenomena. Traditional heat exchangers are therefore related with problems such as fouling and burning of the product directly in contact with the heat transfer surfaces. On the other hand, in Ohmic heating the electrical energy is dissipated into heat and results in more rapid and uniform heating of the entire mass of the food material. However, it is associated with high energy costs and its use is limited to a particular range of products with an appropriate electrical conductivity.

Excellent investigation on theoretical models for predicting the temperature field distribution, for both conventional heat exchangers and Ohmic heaters, may be found in literature [2-6]. Keys and Crawford [2] report the solution of the thermal entry length problem for a Newtonian fluid flowing in a circular tube with prescribed uniform surface temperature. Sparrow and Siegel [3] proposed an analytical solution for the thermal entrance region in circular ducts with adiabatic wall
and uniform internal heat generation. This particular case was further investigated experimentally by Inman [4], who found good agreement with the theoretical model.

The thermal entrance region for a non-Newtonian fluid in a circular duct with uniform wall temperature and internal heat generation source term that depends on temperature was studied by Faraboschi and Di Federico [5]. Pesso and Piva [6] studied a similar problem, but for Newtonian fluids. They proposed analytical solutions for the case of boundary condition of the third kind and temperature dependant heat generation term.

However no Author compared the two different heating conditions in order to understand the real theoretical advantages of Ohmic heating. In the present work comparisons between performances of the two technologies, simulated under typical circumstances for the food industry, are drawn by means of analytical approach.

2. Analytical Models

In order to compare the quality of the product treatment by Ohmic heating with conventional heat exchangers two models are here presented. These models are based on the analytical solution of the convection transfer equations for the case of Newtonian fluid with constant properties in the laminar flow regime. For the case of conventional heat transfer device, the fixed temperature boundary condition is considered. This is a typical situation that may be encountered in many industrial applications, such as condensers. Finally a model for Ohmic heating with uniform internal heat generation and adiabatic wall is presented.

2.1 Constant surface temperature heat exchanger

The convection problem considered is schematically shown in figure 1. The fluid enters the tube at uniform temperature \( T(r,0)=T_0 \), that is lower than the constant surface temperature \( T_s \).

Assuming a steady fully developed flow and neglecting viscous dissipation, axial conduction and internal thermal energy sources, the energy equation for a constant properties fluid reads as follows:

\[
\frac{u \partial T}{\partial x} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \tag{1}
\]

The considered boundary conditions are:

\[
T = T_0 \quad \text{at } x = 0
\]

\[
\frac{\partial T}{\partial r} \bigg|_{r=0} = 0 \quad \text{for } x > 0 \tag{2}
\]

\[
T(x, r_0) = T_s
\]

To solve the problem it is convenient to rewrite equations (1) and (2) in the dimensionless form substituting the following dimensionless variables:
Where $Pe$ is the Peclet number defined as $Pe=2u_m r_0/\alpha$. The energy equation in the dimensionless form becomes:

$$\begin{align} x^* &= \frac{x}{Pe r_0} \quad \xi = \frac{r}{r_0} \quad \Theta(x^*, \xi) = \frac{T_s - T(x, r)}{T_s - T_0} \quad u^* = \frac{u}{u_m} \tag{3} \end{align}$$

with the following dimensionless boundary conditions

$$\begin{align} \Theta(0, \xi) &= 1 \\
\Theta(x^*, 1) &= 0 \\
\frac{\partial \Theta}{\partial \xi}(x^*, 0) &= 0 \tag{5} \end{align}$$

Equation (4) is a linear homogeneous partial differential equation, which may be solved using the method of separation of variables [2]. Therefore, the solution can be expressed by the product:

$$\Theta(x^*, \xi) = R(\xi) \cdot X(x^*) \tag{6}$$

The method of separation of variables yields an infinite series solution for the temperature field [2]:

$$\Theta(x^*, \xi) = \sum_{n=0}^{\infty} C_n R_n(\xi) \exp(-\lambda_n^2 x^*) \tag{7}$$

In the above solution, $\lambda_n^2$ are the so called eigenvalues and $R_n$ are the corresponding eigenfunctions of the following Sturm Liouville equation:

$$\frac{d^2 R}{d\xi^2} + \frac{1}{\xi} \frac{dR}{d\xi} + \lambda^2 (1 - \xi^2) R = 0 \tag{8}$$

Eigenvalues can be obtained from the following transcendental equation:

$$\text{\texttt{1F1}} \left( \frac{1}{2} - \frac{\lambda}{4}, 1, \lambda \right) = 0 \tag{9}$$

Where $\text{\texttt{1F1}}$ is the confluent Hypergeometric function. The expression of the constant $C_n$ in the series can be obtained exploiting the orthogonality property of the eigenfunctions:

$$C_n = \frac{\int_0^1 R_n(\xi) \xi \left(1 - \xi^2\right) d\xi}{\int_0^1 R_n^2(\xi) \xi \left(1 - \xi^2\right) d\xi} \tag{10}$$

The bulk temperature can be calculated from the following expression:

$$\Theta_b(x^*) = 8 \sum_{n=0}^{\infty} \frac{G_n}{\lambda_n^2} \exp(-\lambda_n^2 x^*) \tag{11}$$

Where constants $G_n = -\frac{c_n}{2} \frac{dR_n(1)}{d\xi}$ are introduced to shorten the above expression.
2.2 Ohmic Heating with uniform internal heat generation

The problem considered is schematically shown in figure 2:

Figure 2. Ohmic heating problem: geometry and boundary conditions.

The geometry consists of a collinear cylindrical Ohmic heater with two electrodes at the pipe ends. Considering a steady fully developed flow and neglecting viscous dissipation, axial conduction, the energy equation becomes:

\[ \rho C_p u \frac{\partial T(x, r)}{\partial x} = \frac{k}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T(x, r)}{\partial r} \right) + q_{\text{gen}} \]  \hspace{1cm} (12)

To simplify the problem, the wall of the Ohmic heater is considered adiabatic and the internal heat generation is assumed uniform. Therefore, the boundary conditions related to equation (12) read:

at \( x = 0 \) \hspace{1cm} T = T_0

for \( x > 0 \)

\[ \left. \frac{\partial T}{\partial r} \right|_{r=0} = 0 \]

\[ \left. \frac{\partial T}{\partial r} \right|_{r=r_0} = 0 \]  \hspace{1cm} (13)

Equations (12) and (13) are then expressed in a dimensionless form, defining the dimensionless temperature as follows:

\[ \Theta(x^*, \xi) = - \frac{T(x, r)}{T_0} \]  \hspace{1cm} (14)

While the axial and the radial dimensionless coordinates as the same used in section 2.1. Substituting the velocity profile and the dimensionless terms the energy equation (12) becomes [6]:

\[ (1 - \xi^2) \frac{\partial \Theta}{\partial x^*} = \frac{1}{\xi} \frac{\partial}{\partial \xi} \left( \frac{\xi}{\xi} \frac{\partial \Theta}{\partial \xi} \right) + S_0 \]  \hspace{1cm} (15)

Where \( S_0 = (q_{\text{gen}} r_0^2) / (k^* (T_a - T_0)) \). The boundary conditions are expressed as follows:

\[ \Theta = \Theta(\xi, 0) = -1 \hspace{1cm} \text{at} \hspace{0.5cm} x^* = 0 \]

\[ \left. \frac{\partial \Theta}{\partial \xi} \right|_{\xi=0} = 0 \hspace{1cm} \text{for} \hspace{0.5cm} x^* > 0 \]

\[ \left. \frac{\partial \Theta}{\partial \xi} \right|_{\xi=1} = 0 \]  \hspace{1cm} (16)
Equation (15) may be solved using the method of separation of variables [6]. This yields to the following dimensionless temperature expression:

$$\Theta(x^*,\xi) = \sum_{i=1}^{\infty} A_i \exp(-\lambda_i x^*) f_i(\xi) + 2S_0 x^* - 1 + \frac{S_0}{4} \left( \xi^2 - \frac{1}{2} \xi^4 - \frac{1}{4} \right)$$  

(17)

Where $f_i(\xi)$ are the Poiseuille functions defined as follows:

$$f_i(\xi) = pe(\xi, \mu_i) = \exp\left(-\mu_i \frac{\xi^2}{2}\right) {}_1F_1(a_i, 1, -\mu_i \xi^2)$$  

(18)

in which:

$$\mu_i = \lambda_i^{1/2} \quad a_i = \frac{1}{2} \left( 1 - \frac{\lambda_i}{2}\right)$$  

(19)

The admissible values of the constant of separation $\lambda_i$ are the root of the transcendental equation:

$$\text{at } \xi = 1 \quad \frac{\partial f_i}{\partial \xi} = 0$$  

(20)

The expression of the constant $A_i$ can be obtained from the orthogonality property of the functions $f_i$:

$$A_i = \frac{\int_0^1 \left(-\frac{S_0}{4} \left( \xi^2 - \frac{1}{2} \xi^4 - \frac{1}{4} \right)\right) f_i(\xi)(1 - \xi^2) \xi \, d\xi}{\int_0^1 f_i^2(\xi)(1 - \xi^2) \xi \, d\xi}$$  

(21)

Finally the dimensionless bulk temperature can be calculated from the expression:

$$\Theta_b(x^*) = 2S_0 x^* - 1$$  

(22)

3 Models Implementation

3.1 Constant surface temperature heat exchanger

Finding the solutions of the transcendental equation (9) is a fundamental step for the implementation of the analytical model. In Matlab® environment, by “fzero” function and Symbolic Math Toolbox, eigenvalues can be efficiently calculated noting that their periodicity is $\lambda_n = \lambda_{n-1} + 4$. In the available literature, Kays and Crawford [2] report only the first five values but in many cases they are not enough for the implementation of accurate models. Table (1) reports the values of the first twenty eigenvalues $\lambda_n$ and of the corresponding constants $C_n$ to provide to the reader a powerful tools for further investigations.
Table 1. Calculated eigenvalues $\lambda_n$ and constants $C_n$.

| $n$ | $\lambda_n$    | $C_n$     | $n$ | $\lambda_n$    | $C_n$     |
|-----|----------------|-----------|-----|----------------|-----------|
| 1   | 2.70436        | 1.47643   | 11  | 42.6677        | 0.23322   |
| 2   | 6.67903        | -0.80612  | 12  | 46.6676        | -0.21969  |
| 3   | 10.6734        | 0.58876   | 13  | 50.6675        | 0.20796   |
| 4   | 14.6711        | -0.47585  | 14  | 54.6674        | -0.19768  |
| 5   | 18.6699        | 0.40502   | 15  | 58.6674        | 0.188586  |
| 6   | 22.6691        | -0.35575  | 16  | 62.6673        | -0.18046  |
| 7   | 26.6687        | 0.31916   | 17  | 66.6673        | 0.17317   |
| 8   | 30.6683        | -0.29073  | 18  | 70.6672        | -0.16657  |
| 9   | 34.6681        | 0.26789   | 19  | 74.6672        | 0.16056   |
| 10  | 38.6679        | -0.24906  | 20  | 78.66714       | -0.15507  |

Finally, for the sake of completeness, in figure 3 the first five eigenfunctions, calculated again with the help of the Symbolic Math Toolbox, are represented graphically from $\xi=0$ to $1$.

![Calculated eigenfunctions $R_n$ for $n=1$ to 5.](image)

With the calculated eigenvalues $\lambda_n$, constants $C_n$, and eigenfunctions $R_n$ it is finally possible to obtain the temperature distribution by equation (7).

3.2 Ohmic Heating with uniform internal heat generation

Finding the solutions of the transcendental equation (20) is a key point for the implementation of the analytical model for Ohmic heating. In Matlab® environment, by “fzero” function and Symbolic Math Toolbox, the eigenvalues are efficiently calculated noting that their periodicity is $\lambda_i \approx (\lambda_{i-1}/2+4)^2$. Twenty values of the eigenvalues $\lambda_i$ and constants $A_i$ are reported in Table 2.

Table 2. Calculated eigenvalues $\lambda_i$ and constants $A_i$.

| $i$ | $\lambda_i$    | $A_i$    | $i$ | $\lambda_i$    | $A_i$    |
|-----|----------------|----------|-----|----------------|----------|
| 1   | 25.679612      | -7.25210 | 11  | 2049.8425      | -0.09934 |
| 2   | 83.861755      | 2.30424  | 12  | 2428.3660      | 0.08414  |
| 3   | 174.16674      | -1.129115| 13  | 2838.8968      | -0.07188 |
| 4   | 296.53630      | 0.67020  | 14  | 3281.4341      | 0.06245  |
| 5   | 450.94719      | -0.443768| 15  | 3755.9772      | -0.05440 |
| 6   | 637.38733      | 0.315644 | 16  | 4262.5254      | 0.04820  |
| 7   | 855.84949      | -0.23589 | 17  | 4801.0783      | -0.04258 |
| 8   | 1106.3289      | 0.18315  | 18  | 5371.6354      | 0.03833  |
| 9   | 1388.8224      | -0.14613 | 19  | 5974.1963      | -0.03421 |
In figure first five Poiseuille functions $f_i$, calculated with the *Symbolic Math Toolbox*, are shown graphically from $\xi=0$ to 1.

![Graph of Poiseuille functions](image)

**Figure 4.** Calculated $f_i$ function for $i=1$ to 5.

With the calculated eigenvalues $\lambda_i$, constants $A_i$, and functions $f_i$ it is finally possible to obtain the temperature distribution by equation (17).

### 4 Models Comparison

In order to understand the advantages of Ohmic heating over conventional technologies, the two models presented in section 2 have to be compared under representative circumstances for the food industry. Attention is paid particular on the uniformity of the heat treatment and on the length of the heating section required in the two different conditions.

Firstly, simulations are performed on the Ohmic model (*ohm*). Flow rates, product properties, inlet and outlet product bulk temperatures are fixed. In this case, the volumetric flow rate is fixed at $1.4 \cdot 10^{-4} \text{ m}^3/\text{s}$, while the diameter and the length of the Ohmic heater are $D=63.5 \text{ mm}$ and $L_{\text{ohm}}=3 \text{ m}$. The inlet and outlet product bulk temperature are fixed at $T_{\text{b, in}}=60^\circ\text{C}$ and $T_{\text{b, out}}=100^\circ\text{C}$. Those values are chosen in order to simulate a typical situation that may be encountered in industrial plants. Temperature profiles at different axial coordinates of the heater are therefore obtained. For each of these sections, the attention is focused on the minimum and maximum temperature values, in order to evaluate the product stratification and its evolution through the heater. Some representative temperature profiles are presented in figure 5. The maximum temperature value is always close to the tube wall while the minimum in the center of the tube section.
Figure 5. (a) Bulk and (b) radial temperature profiles at three selected axial positions for the Ohmic heater.

Then simulations are performed on the conventional heat exchanger model (che) with prescribed surface temperature with the same volumetric flow and the same diameter as in the Ohmic case. The basic idea of the comparison is to have, for the two different models, an equivalent thermal treatment which means having the same bulk temperature at the outlet section and the same maximum temperature reached by the product. From the practical point of view, the surface temperature of the conventional heat exchanger is taken equal to the maximum product temperature of the product in the Ohmic model. The required length of the equivalent conventional heat exchanger is obtained by imposing the same inlet and outlet product bulk temperature. In figure 6 the bulk, surface and temperature profiles for the conventional heat exchanger are reported. The required heat exchanger length for having an equivalent thermal treatment is, in this case, $L_{che}=20.9$ m.

Figure 6. (a) Bulk and (b) radial temperature profiles at three selected axial positions for the constant surface temperature heat exchanger.
Simulations are then made for different flow rates varying from $1 \cdot 10^{-4}$ m$^3$/s to $7 \cdot 10^{-4}$ m$^3$/s. The thermal process was always the same (i.e., same maximum product temperature, same inlet and outlet temperature). In figure 7 it is reported the ratio between the length of the conventional heat exchanger and of the Ohmic heater as a function of the volumetric product flow rate. It is possible to say that the Ohmic heating is always better in terms of heating section lengths; in particular as the flow rate increase this advantage is further amplified.

![Figure 7. Heating section lengths ratio](image)

To evaluate the product stratification through the heater in the two different heating processes it is introduced a dimensionless stratification factor $SF$ defined as:

$$SF = \frac{T_{\text{max}} - T_{\text{min}}}{T_b} \quad (22)$$

Where $T_{\text{max}}, T_{\text{min}}$ are the maximum and minimum temperature found at the surface and center of the tube respectively, while $T_b$ is the corresponding bulk temperature for a particular section of the heater. $SF$ is then obtained at different sections of the heater. In figure 8 stratification factors are reported as a function of dimensionless axial coordinate of the two heaters.
Figure 8. Stratification factor

Simulations again are repeated varying the volumetric flow rate. In figure 9 it is reported the maximum value of the stratification factor as a function of the volumetric flow rate for both the Ohmic and the traditional heating condition.

Figure 9. Maximum stratification factor

The stratification factor is always lower in the Ohmic heating than in the conventional heat exchanger. Therefore, as expected, it assures a more uniform thermal treatment of the product.

Finally, it should be stressed that while in Ohmic heaters there is a direct use of electrical energy, in conventional heat exchangers the constant surface temperature is generally obtained by condensing steam in the external jacket. Therefore, a comparison in term of exergy consumption between these two technologies is undoubtedly very interesting but is not easy because it strongly depends on the nature of the energy available [7]. Further efforts are required to investigate this aspect.
5 Conclusions
Analytical models for predicting the temperature field for fully developed laminar circular-tube flow under two different heating condition have been presented. The first model described a typical heat exchanger with constant wall temperature. The second one describes an Ohmic heater with uniform internal heat generation and adiabatic walls. The eigenvalues and eigenfunctions embedded in the two analytical models were reported.

Temperature profiles at different axial position were given for the two heat exchangers by considering the same operating conditions. The stratification of the product and the heating section lengths required by the two different heating conditions were studied and discussed.

For the condition here investigated, which is representative for the food industry, Ohmic heating always showed much more efficient and uniform heating than traditional heat exchangers.

Finally, it has to be highlighted than the analytical models here presented represent a powerful tool which could be useful in the design of this kind of complex devices, with very limited computational costs.

Acknowledgments
This work was partially supported by the Emilia-Romagna Region (POR-FESR 2014-2020). JBT SpA (Parma, Italy) is gratefully acknowledged for the support.

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