Breather Molecular Complexes in a Passively Mode-Locked Fiber Laser

Junsong Peng, Zihan Zhao, Sonia Boscolo,* Christophe Finot, Srikanth Sugavanam, Dmitry V. Churkin, and Heping Zeng*

Breathing solitons are nonlinear waves in which the energy concentrates in a localized and oscillatory fashion. Similarly to stationary solitons, breathers in dissipative systems can form stable bound states displaying molecule-like dynamics, which are frequently called breather molecules. So far, the experimental observation of optical breather molecules and the real-time detection of their dynamics are limited to diatomic molecules, that is, bound states of only two breathers. In this work, the observation of different types of breather complexes in a mode-locked fiber laser: multibreather molecules, and molecular complexes originating from the binding of two breather-pair molecules or a breather pair molecule and a single breather is reported. The intermolecular temporal separation of the molecular complexes attains several hundreds of picoseconds, which is more than an order of magnitude larger than that of their stationary soliton counterparts and is a signature of long-range interactions. Numerical simulations of the laser model support the experimental findings. Moreover, nonequilibrium dynamics of breathing solitons are also observed, including breather collisions and annihilation. This work opens the possibility of studying the dynamics of many-body systems in which breathers are the elementary constituents.

1. Introduction

The concept of soliton is universal and can be applied to a large class of solitary wave propagation phenomena that are observed in most branches of nonlinear science, ranging from fluid dynamics and biology to plasma physics and photonics.[1] The main feature of solitons is that they propagate for a long time without visible changes. In their original studies solitons were attributed to integrable Hamiltonian systems like the focusing 1D nonlinear Schrödinger equation (NLSE),[2] but lately the initial concept has been extended to nonlinear dissipative systems, in which localized wave packets arise from the balance between dispersion or diffraction and nonlinearity (as the conventional solitons) and between gain and losses.[3] Besides their formation, their mutual interactions such as collisions and even the emergence of stable bound states are currently the subject of intense studies in laser physics by means of real-time ultrafast measurements.[4–7] Such dissipative multisoliton complexes, also termed soliton molecules, result from self-organization phenomena in a laser cavity, and show dynamics similar to matter molecules, such as synthesis and vibration. Through transfer of general spectroscopy concepts to the case of dissipative solitons, resonant excitation and all-optical switching of soliton molecules in response to an external perturbation were demonstrated, thus extending the matter-like soliton molecule analogy.[8] While the soliton-pair molecule, consisting of two bound solitons, has been the most studied multisoliton structure, a recent study showed that two soliton-pair molecules can bind subsequently to form a stable molecular complex.[9]

Apart from stationary solitons, numerous nonlinear systems support breathing dissipative solitons exhibiting a periodic oscillatory behavior. Dissipative breathers were first demonstrated in passive Kerr cavities,[10] and subsequently reported in optical microresonators.[11,12] Dissipative systems whose averaged dynamics are governed by the complex Ginzburg–Landau equation (CGLE) also support breather solutions.[13] In the context of passively mode-locked lasers, which constitute a suitable platform for the study of the properties and dynamics of nonlinear dissipative systems, the existence of regimes of operation in which the laser oscillator generates strongly breathing solitons was recently
are related to the Fermi–Pasta–Ulam recurrence\cite{19,20}—a paraconservative nonlinear systems, breathing dissipative solitons breather explosions\cite{27–31} and rogue wave generation.\cite{32}

mentally studied in mode-locked lasers, including soliton and dynamics of dissipative localized structures have been explored. In fact, overlapping soliton pulses cannot form strong bonds in such systems as the effective interaction potential of the pulses is not a minimum.\cite{24} A potential minimum can be induced by dissipative effects only, thus enabling the formation of multipulse bound states. A number of recent works have reported on the observation of breathing soliton-pair molecules in mode-locked fiber lasers.\cite{15,25,26} It is worth mentioning that besides the formation of breather molecules, various notable dynamics of dissipative localized structures have been experimentally studied in mode-locked lasers, including soliton and breather explosions\cite{27–31} and rogue wave generation.\cite{32}

In this work, by employing an ultrafast fiber laser setup whose output is spectrally and temporally analyzed in real time, we demonstrate multibreather molecules and two types of breather molecular complexes (BMCs). One originates from the stable binding of two basic molecules, each made up of a pair of breathing solitons (2 + 2) BMC, and the other one arises from the binding of a breather-pair molecule and a single breather (2 + 1) BMC. The intermolecular temporal separation of such complexes is more than an order of magnitude larger than that of their stationary soliton counterparts. A recent theoretical study has shown that the interaction of well-separated dissipative breathers by exchanging weakly decaying dispersive waves can lead to the formation of bound states as a result of harmonic synchronization.\cite{33} Such dispersive waves emitted by breathers in the anomalous dispersion propagation regime is the driving mechanism for the formation of the breather complexes that are demonstrated in this work. We also explore the nonequilibrium dynamics of BMCs, including collisions of breathers and annihilation of an elementary breather within a BMC. Numerical simulations of the laser model described by the CGLE confirm our experimental observations.

2. Results

2.1. Experimental Setups

The experimental setup for generating breather complexes is an erbium-doped passively mode-locked fiber laser with anomalous path-averaged cavity dispersion, sketched in Figure 1 (see also Supporting Information for details). Mode-locking relies on the nonlinear polarization evolution technique,\cite{34} which is realized through the inclusion of two fiber polarization controllers (PCs) and a polarization dependent isolator, and allows us to tune the nonlinear transfer function by simply rotating the loops, thereby modifying the linear cavity loss, hence the interactions among pulses. The output signal from the laser is analyzed in real time in both the time and frequency domains. We use the dispersive Fourier transform (DFT) method\cite{35,36} to acquire sub-nanometer resolution spectral measurements at the shot-to-shot level. This simple, yet powerful method, which is today a commonly used tool to obtain spectral dynamics on ultrashort time scales, maps the optical spectrum of the laser output onto a temporal waveform that is directly read out on a real-time oscilloscope. This is achieved by stretching the laser output pulses in a dispersive medium that cumulates a group-velocity dispersion large enough for the pulse propagation to satisfy the far-field condition, hence for the stretched waveform to represent the spectral intensity of the initial pulse waveform. The time-domain dynamics of the laser are characterized by measuring time traces of 1D intensity in real time, I(t), and then constructing from these traces the spatio-temporal intensity evolution I(r, z).\cite{37} The latter reveals both the dynamics over the fast time t and the slow evolution propagation coordinate z, which is measured, in our case, as a number of cavity round trips.
2.2. Experimental Results

The laser works in the stable single-pulse mode-locking regime, characterized by soliton-like pulse shaping, when the pump current is set to 44.9 mA. When the pump current is increased to 46 mA, self-starting mode locking is accompanied by the generation of multiple pulses per cavity round trip. The existence of the stable single-pulse regime in a narrow range of pump powers is due to the fact that the cavity length is greater than the typical length of most mode-locked fiber lasers (less than 10 m). In our experiment, the pump current is fixed at 46 mA and we tune the cavity loss through small rotations of the PCs to generate various kinds of breather complexes. We note that the laser can also sustain regimes of emission of two, three and four bound stationary solitons. Figure 2 shows the dynamics of a breather-quartet molecule (“tetratomic molecule”). The spatio-temporal intensity evolution depicted in Figure 2a reveals large periodic variations in the intensities of the four elementary breathers with a period of ≈1000 round trips. The peak intensities within each period of pulsation change by nearly an order of magnitude (Figure 2b). Concomitantly, the pulse temporal duration also varies, whilst the temporal resolution of our detection system (around 30 ps) does not allow us to capture such changes. The use of a time lens\(^{[38]}\) may potentially relax this constraint. However, this technique may prove difficult to implement when the pulses experience large breathing. In spite of the periodic intensity variations, the pulse temporal separation remains nearly fixed at ≈50 ps over consecutive cavity round trips, indicating a strong bond between the pulses. The fast evolutionary behavior of the pulse temporal intensity entails rapid variations of the optical spectrum, which are beyond the speed of traditional spectral measurement tools such as an optical spectrum analyzer. We employ real-time DFT spectral monitoring to capture the spectral evolution of the pulse quartet over cavity round trips. The DFT-based single-shot spectral measurements are shown in Figure 2c. The spectrum features the typical interference pattern that is present in the spectrum of a soliton molecule,\(^{[4]}\) and the separation between the peaks of spectral intensity matches the pulses spacing in time. The evolution of the spectrum over cavity round trips is periodic, and the spectrum largely widens and narrows within each period, with the broadening (compression) naturally occurring in the vicinity of the position where the pulses reach the highest (lowest) peak intensity. The maximum width of the spectrum exceeds the minimum width by more than eight times (Figure 2d). The energy of the pulse quartet (Figure 2c, white curve), calculated by integration of the power spectral density over the whole wavelength band, evolves over round trips synchronously with the spectrum width and pulse temporal intensity. The calibration and accuracy of the DFT spectral measurements are checked by comparing, in the Supporting Information, the average of...
the measured consecutive single-shot spectra with the averaged spectrum is recorded by an optical spectrum analyzer. To complete the quantitative dynamical picture of the breather-quartet molecule, we resolve in real time the relative phases within the molecule using the DFT-based spectral interferometry method, which relies on computing the Fourier transform of each single-shot DFT spectrum and is now rather commonly used with stationary soliton states.\[9,39\] It is worth to mention that the application of this method to the breather complexes generated in our laser cavity is quite challenging as the first-order single-shot autocorrelation traces are too weak when the breathers are at their intensity minima. The internal phase dynamics shown in the Supporting Information confirm the strong nature of the bond between the breather constituents of the molecule.

A notable phenomenon in the breathing process illustrated in Figure 2 is the periodic appearance of wide Kelly sidebands in the shot-to-shot spectra (see, e.g., Figure 2d). These Kelly sidebands are a manifestation of resonant dispersive waves in the temporal domain, which, in the usual perspective, radiate from solitons when they are perturbed by lumped nonlinear losses and various intracavity components in a round trip.\[40\] The low-intensity background pattern synchronized to the breather pattern that is observable in Figure 2a is formed out of the slowly decaying dispersive waves radiated by the breathers,\[41\] which are the main agent of the breathers long-range interaction.\[33\] These dispersive waves are periodically emitted during hundreds of cavity periods and, as it can be seen from Figure 2c (a magnified version of Figure 2c is provided in the Supporting Information) and Figure 2d, the central wavelength of the spectrum and the wavelengths of the Kelly sidebands oscillate synchronously with the pulse energy over cavity round trips.\[41,42\] This wavelength oscillation can be ascribed to cross-phase modulation of the breathers and the radiated dispersive waves.\[41\] As we can notice from Figure 2a, this periodic wavelength shift is mirrored into a periodic shift of the pulse positions in the time domain. In the false color plot of Figure 2a, the pulsations of the individual pulse intensities (round trip number from \(\approx 300\) to \(900\), for example) are accompanied by a positive (to the right) time shift, while no time shift occurs over the duration of the intensity minima. As a result, the pulses at a given round trip number within a period of oscillation appear shifted in time with respect to the pulses at the same round trip number within the previous oscillation period while having the same group velocity. We note that this fact has no physical origin but, as explained in the Supporting Information, is a direct result of the spatio-temporal intensity representation. It is also noteworthy that the signal after temporal stretching has a rather long duration; for instance, the wavelength span in Figure 2c corresponds to a time window of length \(25\) ns. Therefore, the impact of the artificial shift of the pulse temporal positions from one oscillation period to the next (\(\approx 100\) ps in Figure 2a) on the DFT measurements is negligible. The calculated physical time shift induced by the fiber dispersion over consecutive round trips shows good agreement with the artificial shift that is observed in the spatio-temporal intensity map. Although the Kerr effect can also induce a change in the pulse temporal positions, this effect is negligibly small (Supporting Information).

Another type of a breather complex is observed under different PC settings: a \((2 + 2)\) BMC consisting of two bound breather pairs. The spatio-temporal intensity dynamics shown in Figure 3a reveal the existence of two characteristic time scales, corresponding to the pulse separation within each breather-pair molecule and the separation between the two breather-pair molecules. The ratio between the two characteristic times being approximately one to seven indicates that the two times are plausibly associated with two different bond strengths. The intramolecular pulse separation is nearly the same as that of the breather-quartet molecule.
in Figure 2, but the intermolecular separation comes to \( \approx 350 \) ps, an order of magnitude larger than that of the stationary soliton molecular complexes recently reported.\(^{29}\) Through rotation of the PCs, we can turn to a different \((2 + 2)\) BMC (Figure 3b). Compared to the previous case (Figure 3a), the pulses of the two breather-pair molecules are bound at different time intervals, showing that a BMC can have varied intramolecular separations.

By adjusting the linear cavity loss, we can also decrease the number of breathing solitons and prepare a robust breather triplet. An example is provided in Figure 3c, which shows the spatio-temporal evolution of a BMC consisting of a breather-pair molecule bound to a single breather, where the latter can be regarded as a monoatomic molecule. Though the single breather is far apart from the breather pair by about 500 ps, its intensity evolves synchronously with the breather pair over cavity round trips, indicating a strong intermolecular bond. We have also observed a single breather-pair molecule in the laser (see Supporting Information).

Soliton interaction is one of the most exciting areas of research in nonlinear dynamics. In nearly conservative systems, such as Bose–Einstein condensates of atoms confined to a quasi-dimensional waveguide, soliton collisions may result in annihilation depending on the relative phase between the solitons.\(^{41}\) Soliton annihilation also occurs in dissipative systems.\(^{44}\) In our experiment, we have observed breather collisions and destruction within unstable BMCs by rotating the PCs at fixed pump power. Examples are shown in Figure 4. In Figure 4a, the initial condition is a breather quartet. Breather fusions occur at round-trip numbers of \( \approx 2500 \) and 3500, followed by the splitting of each of the two resulting breathers into two pulses at a round-trip number of \( \approx 5000 \). Subsequently, one of the two breather pairs gradually disappears. This is the first time that splitting of breathing solitons is observed, following the observation of soliton splitting.\(^{25,43,46}\) Figure 4b shows three collision events when the initial condition is a breather triplet. In all cases, the scenario resembles that of an elastic collision, in that the two colliding breathers just pass through each other and they are not affected by the collision: they do not merge and their traces do not change after the collision. This is different from the inelastic breather collisions that were recently observed in passive fibers, which represent an example of a conservative system.\(^{22}\) In that case, indeed, the two breathers merged into a single peak. It is also notable that different types of nonequilibrium dynamics of breather molecular complexes can be observed in our laser setup by fine adjustment of the PC settings. For instance, the time separation between the leading and trailing breather within a complex may increase or decrease monotonously over cavity round trips, a dynamical state that we could refer to as thermal expansion or contraction of a breather molecular complex.

2.3. Numerical Simulations

The periodic shift of the central wavelength of the pulsating soliton spectrum as well as the appearance of wide Kelly sidebands on the spectrum were explained in the frame of a vectorial model for the laser operation, based on coupled extended NLSEs and including the effect of cross-phase modulation.\(^{41}\) Our focus is here on verifying the main features of the observed pulse complexes, that is, their periodic breathing behavior. To this end, we have bypassed a quite complex laser model and carried out numerical simulations of the laser based on the master-equation approach,\(^{47}\) namely, we have used the cubic-quintic CGLE. For the context of a passively mode-locked fiber laser, the cubic-quintic CGLE has the form\(^{48}\)

\[
i\omega \xi + \frac{D}{2} \psi_{rr} + |\psi|^2 \psi + \nu |\psi|^4 \psi = i \delta \phi + i \epsilon |\psi|^2 \psi + i \beta \psi_{rr} + i \mu |\psi|^4 \psi
\]  

(1)

where \( \xi \) is the normalized propagation distance traversed in the cavity, \( r \) is the retarded time, \( \psi \) is the normalized envelope of the field, \( D = -\text{sgn}(\beta) \), \( \beta \) is the path-averaged cavity dispersion, and \( \nu \) corresponds to the saturation of the nonlinear refractive index.
Figure 5. Simulation of a breather-quartet molecule. Evolutions of the a) temporal intensity and c) spectrum found for the set of CGLE parameters: \( \nu = 2, \delta = -0.01, \beta = 0.3, \epsilon = 5, \mu = -0.02 \). The evolution of the energy \( Q(\zeta) = \int d\tau |\psi(\zeta, \tau)|^2 \) is also shown. b) Temporal intensity profiles at representative round-trip numbers of maximal and minimal energies within a period of oscillation. d) Corresponding spectra of maximal and minimal extents.

The dissipative terms are written on the right-hand side of Equation (1), where \( \delta > 0 \) (\( \delta < 0 \)) is the net linear gain (loss) coefficient, the term with \( \epsilon \) represents the nonlinear gain (which arises, e.g., from saturable absorption), \( \beta > 0 \) accounts for spectral filtering, and \( \mu < 0 \) represents the saturation of the nonlinear gain. This equation is one of the simplest models for a passively mode-locked laser with a fast saturable absorber, and can describe various complex nonlinear dynamics such as soliton explosions,[27,28,49] rogue waves,[32] multiple pulsing,[50] switching dynamics,[51] and dissipative soliton resonances[52] among the others.[48,53]

By solving the CGLE numerically for the set of parameters given in the caption of Figure 5, we have found periodic breathing dynamics of a bound pulse quartet, which qualitatively account for the behaviors observed in Figure 2. We have also found numerically the different types of BMCs generated in the experiment (Figure 3), as shown in Figure 6.

3. Conclusion

One of the remarkable properties of dissipative solitons, which are mostly absent in integrable systems, is the ability to form robust multipulse bound states (soliton molecules). An optical cavity constitutes an ideal propagation medium to study multiple soliton interactions, since even very weak interactions can be revealed through the virtually unlimited propagation time.[44,54,55]

While soliton pairs constitute the central soliton molecule case, soliton molecules can exist in various isomers,[56] and a large population of optical solitons can self-assembly into macro-molecules and soliton crystals,[57] and even into highly ordered supramolecular structures through the tailoring of their long-range interactions.[58] Although breathers are fundamentally different from stationary solitons, they appear to exhibit similar collective dynamics. The experimental generation of breather-pair molecules in a fiber laser cavity has been reported on a few recent occasions.[15,25,26] The emphasis of the present article is on pushing the similarity in collective behavior between breathing and stationary solitons further. We have reported on the experimental observation and real-time dynamic characterization of different types of breather complexes in a passively mode-locked fiber laser, including tetramolecular structures, and molecular complexes formed by the binding of two diatomic molecules or a diatomic and a monoatomic molecule. We have also observed breather annihilation within an unstable molecular complex when the complex is in the phase of intensity drop. Since breathing dissipative solitons are fundamental modes of many nonlinear physical systems, it is reasonable to assume that the breather dynamics observed in this work will incentivize the investigation of BMCs in various other systems.

Similarly to the stationary soliton scenario,[58] we believe it will be possible to assemble supramolecular structures of optical breathers in a fiber laser by using intense optoacoustic effects in photonic crystal fibers. So far, the studies of localized wave structures in ultrafast fiber optics are limited to the time and frequency domains, as these structures are generated in single-mode fibers. The generation and propagation of pulses in multimode fiber systems have recently drawn great attention.[59] A multimode fiber laser provides a new degree of freedom in the control of coherent
Figure 6. Simulation of various breather molecular complexes. Evolution of the temporal intensity for a) a \((2 + 2)\) breather molecular complex with equal intramolecular pulse separations. b) a \((2 + 2)\) breather molecular complex with different intramolecular separations, and c) a \((2 + 1)\) breather molecular complex. The CCQGLE parameters are \((\nu = 2, \delta = -0.23, \beta = 0.4, \epsilon = 6, \mu = -0.02)\) for the solution shown in panel (a), while the same set of parameters as that of Figure 5 is used for the solutions shown in panels (b) and (c).

light fields: the space domain. We can therefore anticipate that the spatio-temporal engineering of optical pulses will enable the generation of even more complex breather structures.

Supporting Information
Supporting Information is available from the Wiley Online Library or from the author.

Acknowledgements
The authors acknowledge funding support from the National Key Research and Development Program (2018YFB0407100), the National Natural Science Foundation of China (62022033, 12074122, 11621404, and 11727812), Key Project of Shanghai Education Commission (2017-01-07-00-05-E00021), Science and Technology Innovation Program of Basic Science Foundation of Shanghai (18JC1412000), Shanghai Rising-Star Program, and National Key Laboratory Foundation of China (6142412000). The authors acknowledge funding support from the National Key Research and Development Program (2018YFB0407100), the National Natural Science Foundation of China (62022033, 12074122, 11621404, and 11727812), Key Project of Shanghai Education Commission (2017-01-07-00-05-E00021), Science and Technology Innovation Program of Basic Science Foundation of Shanghai (18JC1412000), Shanghai Rising-Star Program, and National Key Laboratory Foundation of China (6142412000). The authors acknowledge funding support from the National Key Research and Development Program (2018YFB0407100), the National Natural Science Foundation of China (62022033, 12074122, 11621404, and 11727812), Key Project of Shanghai Education Commission (2017-01-07-00-05-E00021), Science and Technology Innovation Program of Basic Science Foundation of Shanghai (18JC1412000), Shanghai Rising-Star Program, and National Key Laboratory Foundation of China (6142412000).

Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
The data used in this study are available from the corresponding authors under reasonable request.

Keywords
breathers, modelocking, ultrafast fiber lasers

Received: March 31, 2020
Revised: January 19, 2021
Published online: June 9, 2021

[1] T. Dauxois, M. Peyrard, *Physics of Solitons*, Cambridge University Press, Cambridge 2006.
[2] N. J. Zabusky, M. D. Kruskal, *Phys. Rev. Lett.* 1965, 15, 240.
[3] N. Akhmediev, A. Ankiewicz, *Dissipative Solitons: From Optics to Biology and Medicine*, Springer, Berlin 2008.
[4] G. Herink, F. Kurtz, B. Jalali, D. R. Solli, C. Ropers, *Science* 2017, 356, 50.
[5] K. Krupa, K. Nithyanandan, U. Andral, P. Tchofo-Dinda, P. Grelu, *Phys. Rev. Lett.* 2017, 118, 243901.
[6] J. Liu, X. Yao, Y. Cui, *Phys. Rev. Lett.* 2018, 121, 023905.
[7] F. Kurtz, C. Ropers, G. Herink, *Nat. Photonics* 2020, 14, 9.
[8] Z. Wang, K. Nithyanandan, A. Coillet, P. Tchofo-Dinda, P. Grelu, *Nat. Commun.* 2019, 10, 830.
[9] F. Leo, L. Gelens, P. Emplit, M. Haelterman, S. Coen, *Opt. Express.* 2013, 21, 9180.
[10] M. Yu, J. K. Jang, Y. Okawachi, A. G. Griffith, K. Luke, S. A. Miller, X. Ji, M. Lipson, A. L. Gaeta, *Nat. Commun.* 2017, 8, 14569.
[11] E. Lucas, M. Karpov, H. Guo, M. L. Gorodetsky, T. J. Kippenberg, *Nat. Commun.* 2017, 8, 736.
[12] R. J. Deissler, H. R. Brand, *Phys. Rev. Lett.* 1994, 72, 478.
[13] W. Chang, J. M. Soto-Crespo, P. Vouzas, N. Akhmediev, *Phys. Rev. E* 2015, 92, 022926.
[14] J. Peng, S. Boscolo, Z. Zhao, H. Zeng, *Sci. Adv.* 2019, 5, 1110.
[15] N. Akhmediev, V. Korneev, *Theor. Math. Phys.* 1996, 69, 1089.
[16] E. A. Kuznetsov, *Akad. Nauk SSSR Dokl.* 1977, 236, 575.
[17] Y. C. Ma, *Stud. Appl. Math.* 1979, 60, 43.
[18] C. Bao, J. A. Jaramillo-Villegas, Y. Xuan, D. E. Leaird, M. Qi, A. M. Weiner, *Phys. Rev. Lett.* 2016, 117, 163901.
[19] A. Mussot, C. Naveau, M. Conforti, A. Kudlinski, F. Copie, P. Szrift-giser, S. Trillo, *Nat. Photonics* 2018, 12, 303.
[20] P. Liao, K. Zou, C. Bao, A. Kordts, M. Karpov, M. H. P. Pfeiffer, L. Zhang, Y. Cao, A. Almaiman, F. Alishashi, A. Mohajerin-Araei, A. Falahpour, M. Tur, T. J. Kippenberg, A. E. Willner, *Conf. on Lasers and Electro-Optics, San Jose, CA*, Optical Society of America, Washington 2018.
[21] B. Frisquet, B. Kibler, G. Millot, *Phys. Rev. X* 2013, 3, 041032.
[22] G. Xu, A. Gelash, A. Chabchoub, V. Zakharov, B. Kibler, *Phys. Rev. Lett.* 2019, 122, 084101.
[23] B. A. Malomed, *Phys. Rev. A* 1991, 44, 6954.
[25] H.-J. Chen, Y.-J. Tan, J.-G. Long, W.-C. Chen, W.-Y. Hong, H. Cui, A.-P. Luo, Z.-C. Luo, W.-C. Xu, Opt. Express 2019, 27, 28507.

[26] X. Wang, J. He, B. Mao, H. Guo, Z. Wang, Y. Yue, Y.-G. Liu, Opt. Express 2019, 27, 28214.

[27] J. M. Soto-Crespo, N. Akhmediev, A. Ankiewicz, Phys. Rev. Lett. 2000, 85, 2937.

[28] J. M. Soto-Crespo, N. Akhmediev, Phys. Rev. Lett. 2002, 88, 073903.

[29] X. Wang, J. He, B. Mao, H. Guo, Z. Wang, Y. Yue, Y.-G. Liu, Opt. Express 2019, 27, 28214.

[30] J. Peng, H. Zeng, Commun. Phys. 2019, 2, 34.

[31] J. M. Soto-Crespo, N. Akhmediev, Phys. Rev. Lett. 2000, 85, 2937.

[32] S. T. Cundiff, J. M. Soto-Crespo, N. Akhmediev, Phys. Rev. Lett. 2002, 88, 073903.

[33] A. F. Runge, N. G. Broderick, M. Erkintalo, Optica 2015, 2, 36.

[34] J. Peng, H. Zeng, Commun. Phys. 2019, 2, 34.

[35] J. Peng, H. Zeng, Phys. Rev. Lett. 2019, 12, 034052.

[36] C. Lecaplain, P. Grelu, J. Soto-Crespo, N. Akhmediev, Phys. Rev. Lett. 2012, 108, 233901.

[37] D. Turaev, A. Vladimirov, S. Zelik, Phys. Rev. Lett. 2012, 108, 263906.

[38] D. Noske, N. Pandit, J. Taylor, Electron. Lett. 1992, 28, 2185.

[39] K. Goda, B. Jalali, Nat. Photonics 2013, 7, 102.

[40] A. Mahjoubfar, D. V. Churkin, S. Barland, N. Broderick, S. K. Turitsyn, B. Jalali, Nat. Photonics 2017, 11, 341.

[41] D. Churkin, S. Sugavanam, N. Tarasov, S. Khorev, S. Smirnov, S. Kibis, S. K. Turitsyn, Nat. Commun. 2015, 6, 7004.

[42] P. Ryczkowski, M. Nørh, C. Billet, J.-M. Merolla, G. Genty, J. M. Dudley, Nat. Photonics 2018, 12, 221.

[43] R. Xia, Y. Luo, P. P. Shum, W. Ni, Y. Liu, H. Q. Lam, Q. Sun, X. Tang, L. Zhao, Opt. Lett. 2020, 45, 1551.

[44] A. Komarov, F. Amrani, A. Dmitriev, K. Komarov, D. Meshcheriakov, F. Sanchez, Phys. Rev. A 2012, 85, 013802.

[45] Y. Du, M. Han, P. Cheng, X. Shu, Opt. Lett. 2019, 44, 4087.

[46] X. Wang, Z. Wang, Y. Liu, R. He, J. Zhao, C. Wang, Y. Yue, J. He, B. Mao, J. Hu, Opt. Lett. 2018, 43, 478.

[47] J. H. V. Nguyen, P. Dyke, D. Luo, B. A. Malomed, R. G. Hulet, Nat. Phys. 2014, 10, 918.

[48] N. Akhmediev, J. M. Soto-Crespo, M. Grapinet, P. Grelu, Opt. Fiber Technol. 2005, 11, 209.

[49] J. Peng, M. Sorokina, S. Sugavanam, N. Tarasov, D. V. Churkin, S. K. Turitsyn, H. Zeng, Commun. Phys. 2018, 1, 20.

[50] X. Liu, M. Pang, Laser Photonics Rev. 2019, 13, 1800333.

[51] H. A. Haus, J. Appl. Phys. 1975, 46, 3049.

[52] P. Grelu, N. Akhmediev, Nat. Photonics 2012, 6, 84.

[53] N. Akhmediev, J. M. Soto-Crespo, Phys. Rev. E 2004, 70, 036613.

[54] N. Akhmediev, A. Ankiewicz, J. Soto-Crespo, Phys. Rev. Lett. 1997, 79, 4047.

[55] C. Bao, W. Chang, C. Yang, N. Akhmediev, S. T. Cundiff, Phys. Rev. Lett. 2015, 115, 253903.

[56] W. Chang, A. Ankiewicz, J. Soto-Crespo, N. Akhmediev, Phys. Rev. A 2008, 78, 023830.

[57] W. Renninger, A. Chong, F. Wise, Phys. Rev. A 2008, 77, 023814.

[58] J. N. Kutz, B. Collings, K. Bergman, W. Knox, IEEE J. Quantum Electron. 1998, 34, 1749.

[59] J. K. Jang, M. Erkintalo, S. G. Murdoch, S. Coen, Nat. Photonics 2013, 7, 657.

[60] H. Leblond, A. Komarov, M. Salhi, A. Haboucha, F. Sanchez, J. Optics 2006, 8, 319.

[61] A. Haboucha, H. Leblond, M. Salhi, A. Komarov, F. Sanchez, Phys. Rev. A 2008, 78, 043806.

[62] W. He, M. Pang, D. H. Yeh, J. Huang, C. R. Menyuk, P. S. J. Russell, Nat. Commun. 2019, 10, 5756.

[63] L. C. Wright, D. N. Christodoulides, F. W. Wise, Science 2017, 358, 94.