Theta Dependence of Meson Masses in the Small Mass Limit of the Massive Schwinger Model

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We present a continuum formulation for $\theta$-vacua in the massive Schwinger model on the light-front, where $\theta$ enters as a background electric field. The effective coupling of the external field is partially screened due to vacuum polarization processes. For small fermion masses and small $\theta$ we calculate the mass of the meson and find agreement with results from bosonization.

I. INTRODUCTION

Light-Front (LF) coordinates are natural coordinates for studying many high energy scattering processes, since such processes are often dominated by correlations along light-like directions \[1,2\]. For recent reviews on this subject, see for example Refs. \[1,3\].

The LF framework is very useful also for studying low-energy, nonperturbative physics such as bound states because field theories quantized on the light-front have the vacuum which appears to be trivial (at least as long as zero-mode degrees of freedom are neglected). But how can the LF trivial vacuum incorporate with non-trivial structures (e.g., spontaneous symmetry breaking) of the vacuum? Presumably all the vacuum structures would be extracted from the zero-mode dynamics, if we could correctly take it into account. It is unfortunately extremely difficult to do it. On the other hand, there exists now plenty of evidence that there is no conflict between trivial vacua on the LF and nontrivial vacua in an equal time formulation \[4\], provided one works and interprets the LF Hamiltonian as an effective Hamiltonian in which zero-modes degrees of freedom are integrated out (as opposed to just ommitted). The point is that it seems that effects of the zero mode dynamics can really be simulated by a set of “counterterms,” though their precise forms and the strengths are not easily determined.

Nevertheless, it is still common lore that such an effective LF Hamiltonian approach is not sufficient to account for topologically nontrivial effects. In particular it is generally assumed that $\theta$ vacua in the LF framework can only be described if the zero-modes of the gauge field are included as dynamical degrees of freedom \[5\].

In order to investigate this issue, we are investigating the massive Schwinger model which is known to have $\theta$ vacua \[6\] and where the properties of mesons are dependent on the value of $\theta$ \[8\]. In the equal time formulation of the model, there are (at least) two complementary approaches to describe the physics of $\theta$ vacua: on the one hand, one can formulate the model on a finite interval with periodic boundary conditions and introduce dynamical zero-mode degrees of freedom for the gauge field \[9\]. In this approach, the parameter $\theta$ appears in a way very similar to the lattice momentum in Bloch waves familiar from solid state physics. On the other hand, one can interpret $\theta$ as an external field, generated by “intergalactic capacitor plates” at infinity \[8\]. In the latter approach, where the 1+1 dimensional world is not a circle but the infinite line, $\theta$ is not a dynamical degree of freedom, but instead appears as an integration constant when solving Poisson’s equation.

LF formulations of $\theta$-vacua in the Schwinger model which put the system in a box with periodic boundary conditions and where the $\theta$ vacuum is constructed as a Bloch state can be found in Refs. \[6,10\] (\(m = 0\)) and \[11\] (\(m \neq 0\)). (See also Ref. \[12\].) One finds that $\theta$ vacua can be understood as zero-mode dynamics and $\theta$ plays the role of a Bloch momentum. In particular, the periodicity of the physics in $\theta$ can be quite easily understood, though it is due to non-trivial dynamics (pair creation). Even though the vacuum and meson equations have been obtained, spectra and wave functions have not been calculated explicitly because the continuum limit of the equations is singular (or ambiguous). It is therefore desirable to have an alternate formulation which allows to perform explicit calculations.

The approach to $\theta$ vacua in the LF formulation which we will pursue in this paper is complementary to the “Bloch wave approach” described above and is in fact very similar to Coleman’s formulation of the problem in an equal time framework: we introduce the $\theta$ parameter as an external field, generated by external charges at infinity and then we study the behavior of mesons un-
under the influence of such an external field. The paper is organized as follows: first we derive the effective interaction term caused by such an external electric field. In the rest of the paper we investigate the influence of this field on mesons by approximating mesons as fermion – anti-fermion pairs. In particular, we study the boundary behavior of meson wave-functions, screening effects and the chiral limit of the model.

II. \( \theta \) VACUA AS EFFECTIVE BACKGROUND FIELDS

Let us start with the Lagrangian of the massive Schwinger model

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \gamma^\mu (i \partial_\mu - e A_\mu) - m \psi. \quad (2.1)
\]

We quantize the model on the light-cone, i.e., by regarding \( x^+ = (x^0 + x^1)/\sqrt{2} \) as the “time.” In the \( A^+ = 0 \) gauge equations of motion can be derived in the usual way. In particular, we find that \( A^- \) satisfies the following Poisson equation,

\[
- \partial^2 A^- = ej^+, \quad (2.2)
\]

where \( j^+ = \sqrt{2} : \bar{\psi} \gamma^0 \psi_R : \), \( \psi = (\psi_R, \psi_L)^T \). For the notation used in this paper, see Ref. [13].

Following Coleman, we introduce the \( \theta \) parameter into the massive Schwinger model as an external background field by including an appropriate integration constant in the solution to Eq. (2.2),

\[
A^- (x^-) = -\frac{e^2}{2} \int_{-\infty}^{\infty} dy^- |x^- - y^-| j^+(y^-) - \frac{e \theta}{2 \pi} x^- , \quad (2.3)
\]

yielding an additional interaction term in the LF Hamiltonian,

\[
\delta P^- = -\frac{ie^2 \theta}{2 \pi} \int_{-\infty}^{\infty} dx^- j^+(x^-) x^- . \quad (2.4)
\]

In momentum space, this additional term appears as the derivative of the current operator at zero momentum, i.e.,

\[
\frac{d}{dq^+} j^+(q^+) \bigg|_{q^+ = 0} , \quad (2.5)
\]

where for \( q^+ > 0 \)

\[
\delta P^- = \frac{ie^2 \theta}{2 \pi} \int_{-\infty}^{\infty} dx^- e^{-iq^+ x^-} j^+(x^-)
\]

\[
= \int_{0}^{\infty} \frac{dk^+}{2\pi \sqrt{k^+ (q^+ + k^+)}} \left[ b^\dagger (k^+ + q^+) b(k^+) - d^\dagger (k^+ + q^+) d(k^+) \right]
\]

\[
+ \int_{0}^{q^+} \frac{dk^+}{2\pi \sqrt{k^+ (q^+ - k^+)}} b^\dagger (k^+ + q^+) d(k^+) - d^\dagger (k^+ - k^+), \quad (2.6)
\]

and analogously for \( q^+ < 0 \). The \( b \) and \( d \) are the usual destruction operators for fermions and anti-fermions respectively and, by definition, \( j^+_{\text{diag}} \) and \( j^+_{\text{pair}} \) are those terms in the current operator which are diagonal (off-diagonal) in Fock space. The contributions of \( j^+_{\text{diag}} \) to \( \delta P^- \) [Eq. (2.3)] are straightforward to evaluate, yielding

\[
\delta P^-_{\text{diag}} = \frac{i e^2 \theta}{2 \pi} \int_{0}^{\infty} dk^+ dp^+ \sqrt{k^+ p^+} d(k^+ - p^+)
\]

\[
\times |b^\dagger (k^+) b(p^+) - d^\dagger (k^+) d(p^+)| , \quad (2.7)
\]

e.g., a derivative coupling with opposite signs for fermions and anti-fermions.

How can \( j^+_{\text{pair}} \) affect the meson state? First observe that the (anti-)fermion must have non-zero momentum \( q^+ \) in order for \( j^+_{\text{pair}} \) to contribute. Because wave functions in the massive Schwinger model vanish when one of the momenta goes to zero, one might think that \( j^+_{\text{pair}} \) with \( q^+ \rightarrow 0 \) (hence \( \delta P^-_{\text{pair}} \)) is unimportant. However, we can show that they do not vanish when the momenta of a fermion and an anti-fermion go to zero simultaneously. The details of the argument will be presented in Ref. [16] and in this letter we will restrict ourselves to analyzing the consequences for the coupling of a meson to a constant background field.

In order to investigate whether this has a nontrivial effect on the matrix elements of \( \frac{d}{dq^+} j^+ \bigg|_{q^+ = 0} \), let us consider the matrix elements of \( j^+ \) for small but nonzero momentum transfer. As an example, compare the two diagrams in Fig. [11] which contribute to the coupling of a fermion to an external charge.
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different perturbative diagrams.

\[ V_a = \frac{e^2}{q^+} \int_0^{q^+} dk^+ \left( \frac{\mu^2}{2k^+} - \frac{\mu^2}{2q^+} \right) \theta(x), \]  

(2.8)

while the one loop diagram [Fig. 1b] yields

\[ V_b = \frac{e^2}{2\pi q^+} \int_0^{q^+} dk^+ \left( \frac{\mu^2}{2k^+} - \frac{\mu^2}{2q^+} \right) \theta(x), \]  

(2.9)

(since we are interested in static external sources, we may
effectively limit the validity of our
diagrams vanishes when \( \delta P^\mu = 0 \).)

As a side remark, we should emphasize that in principle the screening factor \( \theta_{eff} \) itself depends on \( \theta_{eff} \) since it is \( \theta_{eff} \) which enters the LF-Hamiltonian and thus
determines the masses and coupling constants of mesons that appear in the sum in Eq. (2.11). However, we will
neglect this effect here and evaluate the screening factor at \( \theta = 0 \), which will effectively limit the validity of our
results to small values of \( \theta \).

III. THE \( \theta \) PARAMETER IN THE TWO
PARTICLE SECTOR

In order to illustrate the physics of the \( \theta \) parameter, let us now focus our attention to the 2-particle sector of the
massive Schwinger model, which is known to provide an
excellent approximation for the lightest meson \( \bar{\psi} \). After
projecting the effective LF Hamiltonian in the presence of
an external background field onto the two particle sector,
one finds the following equation of motion for mesons

\[
\mu^2 \psi(x) = \frac{m^2}{x(1-x)^2} \psi(x) + \int_0^1 dy \frac{1}{x-y} \frac{d}{dy} \psi(y) 
+ \int_0^1 dy \theta_{eff} \frac{d}{dx} \psi(x),
\]  

(3.1)

where a principal value prescription for the singular integral is implied. Here and in the following we will use
units where \( e^2/\pi = 1 \).

\[^3\text{We were not able to give a rigorous proof of this result, but}
\text{convinced ourselves about the correctness by studying many}
\text{different perturbative diagrams.}\]

\[^4\text{Note that since it is the anomaly which is responsible for}
\text{the nonvanishing contribution of the off-diagonal pieces, it is}
\text{only the U(1) current which is affected by this result in a}
\text{multi-flavor version of the model.}\]
The most dramatic modification compared to the Bergknoff Hamiltonian ($\theta = 0$) is that above Hamiltonian is complex (but still Hermitian). The eigenfunctions satisfy

$$\psi(1 - x) = \pm \psi^*(x),$$  \hspace{1cm} (3.2)

which results from charge conjugation invariance. At the boundary $x = 0, 1$ a self-consistent ansatz shows that $\psi(x)$ vaishes like $x^\beta$ and $(1 - x)^{\beta^*}$ respectively, where $\beta$ is the solution to

$$m^2 - 1 + \pi \beta \cot(\pi \beta) + i \theta_{eff} = 0 \quad (3.3)$$

with $\mathbb{R} \beta \in (0; 1)$. Obviously Eq. (3.3) can be satisfied if and only if one allows $\beta$ to be complex. For $\theta_{eff} = 0$ one recovers 't Hooft’s boundary condition $\pi \beta \cot(\pi \beta) = 1 - m^2$, which has only real solutions $\beta_n$ located at $n < \beta_n < n + 1$. For the end-point behavior only $\beta_0$ is important. In the following we will briefly discuss how $\beta_0$ — the solutions to Eq. (3.3) change as a function of $\theta_{eff}$. Detailed proofs can be found in Ref. [10].

For fixed $m$, as one increases $\theta_{eff}$, the imaginary part of $\beta$ increases until at a certain value $\theta_{eff} = \theta_{crit}$, $\beta$ becomes purely imaginary and the solutions to the Bergknoff equation with $\theta$-term become tachyonic. For $m^2 \geq 1$, one finds $\theta_{crit} = \pi$, but for $0 < m^2 < 1$, $\theta_{crit}$ slowly decreases to zero. In the chiral limit one can see this explicitly using (3.3)

$$\frac{(\pi \beta)^2}{3} - i \beta \theta_{eff} - m^2 = 0 \quad (3.4)$$

and thus

$$\pi \beta = \sqrt{m^2 - \frac{3}{4 \pi^2} \theta_{eff}^2 + i \frac{\sqrt{3}}{2 \pi} \theta_{eff}}, \quad (3.5)$$

yielding

$$\theta_{crit} = \frac{2 \pi}{\sqrt{3}} m. \quad (3.6)$$

**IV. VARIATIONAL CALCULATION IN THE CHIRAL LIMIT**

For $m = 0$, the screening factor (2.11) vanishes and thus $\theta_{eff} = 0$ regardless of the value of $\theta$. Since only $\theta_{eff}$ enters the Hamiltonian for mesons, we thus confirm that meson masses become $\theta$-independent for zero fermion masses.

Much more interesting is the limit of small but non-vanishing fermion masses. In this case numerical calculations of the meson spectrum become very tricky because of the singular behavior of the meson wave functions at the boundary. Furthermore, especially for values of $|\theta_{eff}| \approx \pi$, pair creation (which we are suppressing for simplicity) is expected to become very important.

Given that $\psi$ has to satisfy the boundary conditions $\psi(x) \to x^\beta$ and $(1 - x)^{\beta^*}$ for $x \to 0$ and 1, where $\beta$ is determined from Eq. (3.3), it would be natural to make a variational ansatz of the form

$$\psi(x) = x^\beta (1 - x)^{\beta^*}. \quad (4.1)$$

However, we have not been able to derive analytic expressions for matrix elements of the interaction with this ansatz for general (i.e. complex) values of $\beta$. For $|\beta| \ll 1$, an ansatz which has the same end-point behavior as Eq. (3.3), and which is also [like Eq. (4.1)] nearly constant for intermediate values of $x$ is given by

$$\psi(x) = x^\beta (1 - x)^{1 - \beta} + x^{1 - \beta^*} (1 - x)^{\beta^*}. \quad (4.2)$$

The main advantage of this modified ansatz is that it not only satisfies the right boundary conditions but also leads to analytically calculable matrix elements, using (17)

$$\int_0^1 dy \frac{y^\nu (1 - y)^{-\nu}}{y - x} = \pi \sin(\nu \pi) x^\nu (1 - x)^{-\nu} \quad (4.3)$$

for $0 < x < 1$ and $|\Re \nu| < 1$. Using Eq. (4.3) one thus finds

$$\langle V \psi | H | \psi \rangle = \frac{\pi}{\sin(\pi \beta)} + \pi \cot(\pi \beta) \left[ \frac{\beta}{x} - 1 \right] \left( \frac{x}{1 - x} \right)^\beta \quad (4.4)$$

and hence for the expectation value of the Hamiltonian

$$\langle \psi | H | \psi \rangle = 1 + \frac{m^2}{a} + \frac{a^2 + b^2}{3a} \frac{\pi^2}{\theta_{eff}} \frac{b}{a} + \mathcal{O}(\beta^2) \quad (4.5)$$

where $\beta = a + ib$ with $a$ and $b$ real. This expression is minimized for

$$\beta_{min} = \frac{\sqrt{3}}{\pi} \sqrt{m^2 - \frac{3 \theta_{eff}^2}{4 \pi^2} + i \theta_{eff} \frac{3}{2 \pi^2}}, \quad (4.6)$$

which agrees with Eq. (3.3). Substituting (4.6) into (4.5) yields for the invariant mass of the meson

$$M^2 = \left\langle \frac{\psi | H | \psi}{\langle \psi | \psi \rangle} \right\rangle_{min} = 1 + \frac{2 \pi}{\sqrt{3}} \sqrt{m^2 - \frac{3}{4 \pi^2} \theta_{eff}^2}, \quad (4.7)$$

which is valid up to order $\mathcal{O}(m^2)$ and for $\theta_{eff} < m^2/3\pi$. In order to turn Eq. (17) into a prediction for the $\theta$ dependence of the mass, we need to evaluate the screening factor (2.11). For this purpose, we note that

$$g_0^V = 1 - \mathcal{O}(\beta^2), \quad (4.8)$$

which, by completeness $\sum_n (g_n^V)^2 = 1$, also implies that
\[ \sum_{n \neq 0} (g_n)^2 = \mathcal{O}(\beta^2). \quad (4.9) \]

As a result, up to \( \mathcal{O}(\beta^2) \), screening depends only on the
mass shift of the meson, i.e.
\[ \frac{\theta_{\text{eff}}}{\theta} = M^2 - 1. \quad (4.10) \]

To lowest order in \( \theta \), one thus immediately obtains
\[ \theta_{\text{eff}} = \frac{2\pi m}{\sqrt{3}} + \mathcal{O}(\theta^2), \quad (4.11) \]
yielding
\[ M^2(\theta, m) = 1 + \frac{2\pi m}{\sqrt{3}} \left( 1 - \frac{\theta^2}{2} \right) + \mathcal{O}(m^2) + \mathcal{O}(\theta^4), \quad (4.12) \]
which agrees to this order in \( \theta \) with the result from bosonization\[18\]
\[ M^2(\theta, m)_{\text{bos}} = 1 + 2e^\gamma m \cos(\theta) + \mathcal{O}(m^2) \quad (4.13) \]
within 2 %.

One might suspect that the 2 % difference between
Eq. (4.12) and Eq. (4.13) is just a consequence of using
the “wrong” meson mass in Eq. (2.11). However, this is
not the case, since the screening factor enters Eq. (4.12)
quadratically, and using \( m_0^2 \) from bosonization instead of
from the Bergknoff equation results in a \( \theta \)-dependence in
Eq. (4.12) which is 2 % too small instead of 2 % too
large. Nevertheless, we believe that understanding the 2 %
puzzle in the \( \theta = 0 \) case\[19\] will eventually also help
understand the 2 % deviation of the \( \theta \) dependent term
above.

Much more important than the 2 % deviation is the fact that above
calculation gave the correct result within 2 % — especially since the calculation
without screening corrections would have given a \( \theta \) dependence which
diverges as \( \theta \to 0 \). Only after inclusion of the screening
factor did we obtain the correct \( m \)-dependence, i.e. van-
ishing \( \theta \)-dependence in the chiral limit. We consider this
as a strong support for our method of including screening
effects into the effective background field. Once more, we
should emphasize that above result (4.12) was obtained
without including dynamical zero mode degrees of freedom.

V. SUMMARY

We have studied the \( \theta \) dependence of the meson mass
in the massive Schwinger model. The \( \theta \) dependence is
introduced via a static background electric field. The
essential new ingredient which we introduce is a screened
background field, which arises as an effective coupling
to fermion anti-fermion pairs with vanishing momentum.

In the chiral limit we obtain results which are consistent
with results based on chiral perturbation theory.

While above results have been very reassuring, many
open questions remain, including deriving the effective
Hamiltonian and calculating meson masses for \( \theta \approx \pi \).
Such results would be particularly interesting for investi-
gations of “vacuum periodicity”, which should manifest
itself in the effective Hamiltonian formalism as critical
behavior for \( |\theta| = \pi \). Work in this direction is in progress\[10\].

ACKNOWLEDGMENTS

M. B. was supported in part by the D.O.E. (grant no.
DE-FG03-96ER40965), by TJNAF and by an interna-
tionalization supplementary grant from NMSU. K. H.
was partially supported by Sumitomo Foundation (No.
960517).

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