Proton-neutron mixed-symmetry states and a separable approximation for Skyrme interactions

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Abstract. Starting from an effective Skyrme interaction we present a finite rank separable approach for the QRPA calculations taking into account the effects of the phonon-phonon coupling. As an example the properties of the low-lying quadrupole states in $^{94}$Mo are considered. It is shown that the third $2^+$ state is the mixed-symmetry state.

1. Introduction

One of the successful tools for nuclear structure studies is the quasiparticle random phase approximation (QRPA) with the self-consistent mean-field derived from the Skyrme interaction, see a rather complete list of references on that subject in Ref. [1]. The low-energy spectrum of nuclear excitations provides the most sensitive testing ground for the nuclear structure calculations. In particular, properties of the quadrupole-collective isovector excitations of the valence shell of heavy nuclei, so-called mixed-symmetry states [2, 3], are sensitive to the proton-neutron interaction. It is interesting to study the properties of the mixed-symmetry states in the low-energy spectrum calculated with the Skyrme interaction.

The complexity of calculations taking into account the coupling between one-phonon and more complex states increases rapidly with the size of the configuration space. Making use of the finite rank separable approximation [4, 5] for the residual interaction enables one to perform the QRPA calculations in very large two-quasiparticle spaces. The approach has been generalized to take into account the coupling between the one- and two-phonon components of the wave functions [6]. Here, we use an extension of our approach by taking into account the particle-particle residual interaction [7].

One of the well-known examples is the mixed-symmetry $2^+$ state in $^{94}$Mo. The low-energy spectrum of quadrupole excitations in $^{94}$Mo is extensively studied in many experiments [3, 8, 9]. The experimental efforts have stimulated theoretical analysis based either on the interacting boson model (IBM-2) [8] or on the quasiparticle phonon model (QPM) [9, 10, 11]. In this report we describe briefly our method and present the analysis of the properties of the low-lying quadrupole states in $^{94}$Mo.
2. The method

The starting point of the method is the HF-BCS calculation (see for example [12]) of the ground states, where spherical symmetry is imposed on the quasiparticle wave functions. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian in a harmonic oscillator basis. We work in the quasiparticle representation defined by the canonical Bogoliubov transformation:

$$a_{jm}^+ = u_j \alpha_{jm}^+ + (-1)^{j-m} v_j \alpha_{j-m}$$

(1)

where \( jm \) denote the quantum numbers \( nljm \). As effective interactions, a Skyrme interaction with tensor terms [13] in the particle-hole (p-h) channel and a density dependent zero-range Migdal form. For Skyrme interactions all Landau parameters with \( jm \) when \( l \) > 1 terms in \( V_{pp} \). In this work we study only normal parity states and one can neglect the spin-spin terms since they play a minor role [5]. The two-body Coulomb and spin-orbit residual interactions are also dropped. Therefore we can write the residual interaction in the following form:

$$V_{res}^a(r_1, r_2) = N_0^{-1} \left[ F_{0}^{a}(r_1) + F_{0}^{a}(r_1)(r_1 \cdot \tau_2) \right] \delta(r_1 - r_2),$$

(2)

where \( a \) is the channel index \( a = \{ph, pp\} \); \( \tau_2 \) are the isospin operators, and \( N_0 = 2k_Fm^*/\pi^2h^2 \) with \( k_F \) and \( m^* \) standing for the Fermi momentum and nucleon effective mass. The expressions for \( F_{0}^{ph}, F_{0}^{pp} \) and \( F_{0}^{pp}, F_{0}^{pp} \) can be found in Ref.[14] and in Ref.[7], respectively. The p-h matrix elements and the antisymmetrized p-p matrix elements can be written as the separable form in the angular coordinates [4, 7]. Following the paper [4] the radial integrals are calculated accurately by using the \( N \)-point integration Gauss formula. Thus, the residual interaction can be expressed as the sum of \( N \) separable terms [4, 7].

We introduce the phonon creation operators

$$Q_{\lambda \mu i}^+ = \frac{1}{2} \sum_{jj'} \left( X_{jj'}^{\lambda \mu} A^+ (jj' ; \lambda \mu) - (-1)^{\lambda - \mu} Y_{jj'}^{\lambda \mu} A(jj' ; \lambda - \mu) \right),$$

(3)

$$A^+ (jj' ; \lambda \mu) = \sum_{mm'} \langle jmjm' | \lambda \mu \rangle \alpha_{jm}^+ \alpha_{jm'}^+,$$

(4)

where the index \( \lambda \) denotes total angular momentum and \( \mu \) is its z-projection in the laboratory frame. One assumes that the ground state is the QRPA phonon vacuum | 0 \rangle. We define the one-phonon excited states as \( Q_{\lambda \mu i}^+ | 0 \rangle \). Making use of the linearized equation-of-motion approach one can get the QRPA equations [12]. Solutions of the set of linear equations yield the eigen energies and the amplitudes \( X, Y \) of the one-phonon excited states. Using the separable form of the residual interaction, the QRPA equations can be reduced to the secular equation and the matrix dimensions never exceed \( 6N \times 6N \) independently of the size of the two-quasiparticle configuration space [7].

To take into account the effects of the phonon-phonon coupling in the simplest case one can write the wave functions of excited states as

$$\Psi_J(JM) = \left\{ \sum_i R_i (J\nu) Q_{JM1}^+ \right\} + \sum_{\lambda_1 \mu_1, \lambda_2 \mu_2} \left( P_{\lambda_1 \mu_1}^\lambda (J\nu) \left[ Q_{\lambda_1 \mu_1}^+ Q_{\lambda_2 \mu_2}^+ \right]_{JM} \right) | 0 \rangle$$

(5)
with the normalization condition
\[
\sum_i R_i^2(J\nu) + 2 \sum_{\lambda_1\lambda_2} (P_{\lambda_1\lambda_2}^\nu(J\nu))^2 = 1
\]  \hspace{1cm} (6)

Using the variational principle one obtains the set of linear equations \([6]\) which has the same form as the QPM equations \([10]\), but the single-particle spectrum and the parameters of the residual interaction are calculated with the Skyrme force.

We apply our approach to study characteristics of the low-lying \(2^+\) states in \(^{94}\text{Mo}\). As the parameter set in the particle-hole channel, we use the central Skyrme interaction SLy5 \([15]\) and the same zero-range tensor interaction as in Ref. \([13]\). The strength of the surface-peaked zero-range pairing force is taken equal to -870 MeVfm\(^3\) in connection with the soft cutoff at 10 MeV above the Fermi energies as introduced in Ref. \([7, 16]\). This value of the pairing strength is fitted to reproduce the experimental pairing energies of \(^{94}\text{Mo}\).

3. Low-lying quadrupole states in \(^{94}\text{Mo}\)

First, properties of low-lying quadrupole states in \(^{94}\text{Mo}\) are studied within the one-phonon approximation. There are only two \(2^+\) states below 2.5 MeV, see \([17]\). The \(2^+_1\) state is the collective state which has a typical isoscalar character. The dominant neutron and proton phonon amplitudes \(X, Y\) of the \(2^+_1\) state are in phase. In particular, the energy of the first proton two-quasiparticle pole \(1g_9/2, 1g_9/2\) is greater than the energy of the neutron one \(2d_{5/2}, 2d_{5/2}\) and, for the \(2^+_1\) state, the largest contribution to the norm of the \(2^+_1\) state (64\%) comes from the neutron configuration \(2d_{5/2}, 2d_{5/2}\). The dominant neutron and proton amplitudes of the fairly collective \(2^+_2\) state are in phase opposition. As a consequence, the isovector character of the \(2^+_2\) state is reflected in the noticeable size of the \(B(M1; 2^+_2 \rightarrow 2^+_1)\) value. This analysis within the 1p-1h configuration space can help to identify the mixed-symmetry state, but it is only a rough estimate.

Let us now discuss the extension of the space to one- and two-phonon configurations. To construct the wave functions \((5)\) of the low-lying \(2^+\) states up to 3 MeV we use only the \(2^+\) phonons, and all one- and two-phonon configurations with energies up to 8 MeV are included. The inclusion of high-energy configurations plays a minor role in the calculations \([17]\). The calculated \(2^+\) state energies, the largest contributions of the wave function normalization \((6)\), the transition probabilities \(B(E2; 0^+_g.s. \rightarrow 2^+_1)\), \(B(M1; 2^+_1 \rightarrow 2^+_1)\) and the experimental data \([8]\) are given in table 1. Note that the \(B(M1)\) values have been calculated with the \(g\)-factors of free protons and neutrons. One can see that the first \(2^+\) state is mainly the isoscalar \(|2^+_1\rangle_{QRP\Lambda}\), but the two-phonon contribution is appreciable. The second \(2^+\) state has the dominant two-phonon

### Table 1. Energy, structure of the low-lying quadrupole states and the transition probabilities in \(^{94}\text{Mo}\). Experimental data are from Ref. \([8]\).

| \(\lambda\) \(= 2^+_1\) | Energy (MeV) | Structure | \(B(E2; 0^+_g.s. \rightarrow 2^+_1)\) (\(e^2\text{fm}^4\)) | \(B(M1; 2^+_1 \rightarrow 2^+_1)\) (\(\mu^2\)) |
|---|---|---|---|---|
| \(2^+_1\) | 0.871 | 0.6 | 83\% \(|2^+_1\rangle_{QRP\Lambda}\) | 2030(40) |
| \(2^+_2\) | 1.864 | 1.6 | 60\% \(|2^+_1 \otimes 2^+_1\rangle_{QRP\Lambda}\ | 32(7) | 30 | 0.06(2) | 0.01 |
| \(2^+_3\) | 2.067 | 2.0 | 85\% \(|2^+_2\rangle_{QRP\Lambda}\) | 230(30) | 250 | 0.48(6) | 0.84 |
| \(2^+_4\) | 2.393 | 3.2 | 72\% \(|2^+_1 \otimes 2^+_2\rangle_{QRP\Lambda}\) | 27(8) | 20 | 0.07(2) | 0.01 |
configuration $[2^+_1 \otimes 2^+_1]_{QRPA}$ composed of isoscalar phonons. The third $2^+$ state is dominated by the isovector $[2^+_2]_{QRPA}$ and can be associated with the mixed-symmetry state. It is worth to mention that the mixed-symmetry $2^+$ state in $^{94}$Mo was predicted for the first time in Ref. [8] based on the IBM-2 calculation. Although our approach describes the properties of the low-lying states less accurately than more phenomenological ones [9, 11], the results are still in a reasonable agreement with the experimental data [8, 9].

4. Summary

Starting from an effective Skyrme interaction we present a finite rank separable approach for QRPA calculations taking into account the effects of the phonon-phonon coupling. Choosing as an example the nucleus $^{94}$Mo, the properties of the low-energy spectrum of quadrupole excitations are analyzed. It is found that the third $2^+$ state is the mixed-symmetry state successfully predicted within the IBM-2. The structure of the low-lying $2^+$ states calculated in our approach are close to those that were calculated within the QPM. They are generally in a reasonable agreement with experimental data.

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References

[1] Paar N, Vretenar D, Khan E and Coló G 2007 Rep. Prog. Phys. 70 691
[2] Iachello F 1984 Phys. Rev. Lett. 53 1427
[3] Pietralla N, von Brentano P and Lisetskiy A F 2008 Prog. Part. Nucl. Phys. 60 225
[4] Nguyen Van Giai, Stoyanov Ch and Voronov V V 1998 Phys. Rev. C 57 1204
[5] Severyukhin A P, Stoyanov Ch, Voronov V V and Nguyen Van Giai 2002 Phys. Rev. C 66 034304
[6] Severyukhin A P, Voronov V V and Nguyen Van Giai 2004 Eur. Phys. J. A 22 397
[7] Severyukhin A P, Voronov V V and Nguyen Van Giai 2008 Phys. Rev. C 77 024322
[8] Pietralla N et al 1999 Phys. Rev. Lett. 83 1303
[9] Burda O et al. 2007 Phys. Rev. Lett. 99 092503
[10] Soloviev V G 1992 Theory of Atomic Nuclei: Quasiparticles and Phonons (Bristol and Philadelphia: Institute of Physics)
[11] Lo Iudice N and Stoyanov Ch 2000 Phys. Rev. C 62 047302
[12] Ring P and Schuck P 1980 The Nuclear Many Body Problem (Berlin: Springer)
[13] Coló G, Sagawa H, Fracasso S and Bortignon P F 2007 Phys. Lett. B 646 227
[14] Nguyen Van Giai and Sagawa H 1981 Phys. Lett. B 106 379
[15] Chabanat E, Bonche P, Haensel P, Meyer J and Schaeffer R 1998 Nucl. Phys. A 635 231
[16] Krieger S J et al 1990 Nucl. Phys. A 517 275
[17] Severyukhin A P, Arsenyev N N, Voronov V V, Pietralla N and Nguyen Van Giai in preparation