Extended Axion Electrodynamics: Optical Activity Induced by Nonstationary Dark Matter

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Abstract—We suggest a new self-consistent Einstein-Maxwell-axion model based on the Lagrangian which is linear in the pseudoscalar (axion) field and its four-gradient and includes the four-vector of macroscopic velocity of the axion system as a whole. We consider extended equations of axion electrodynamics and the modified gravity field equations and discuss nonstationary effects in the phenomenon of optical activity induced by axions.

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1. INTRODUCTION

Axions (pseudo-Goldstone bosons) are considered to be Weakly Interacting Massive Particles (WIMPs) appearing as a result of a spontaneous phase transition predicted by Peccei and Quinn [1]. These (hypothetic) particles can play a fundamental role in the formation of Dark Matter (DM), whose contribution into the Universe energy balance is estimated to be about 23% (see, e.g., [2–5]). The axion electrodynamics established by Weinberg and Wilczek [6–8] gives us a new instrument for DM investigation since the model of coupling between photons and the pseudoscalar (axion) four- vector; second order in derivatives; third, the cross-terms in the Lagrangian are linear in the pseudoscalar field and its four-gradient; fourth, the Lagrangian of the model includes the macroscopic velocity four-vector of the system as a whole but does not contain its derivatives.

2. EXTENDED MODEL OF AXION-PHOTON COUPLING

2.1. On the Lagrangian of the Extended Model

The standard Einstein-Maxwell-axion model is based on the Lagrangian formalism with the action functional

\[ S(0) = \int d^4x \sqrt{-g} \mathcal{L}_0, \]

\[ \mathcal{L}(0) = \frac{R+2\Lambda}{\kappa} + \frac{1}{2} F_{mn} F^{mn} + \frac{1}{2} \phi \mathcal{F}_m \mathcal{F}^m + \Psi_0^2 \left[ -\nabla_m \phi \nabla^m \phi + V (\phi^2) \right]. \]

Here, \( g \) is the determinant of the metric tensor \( g_{ik} \), \( \nabla_m \) is the covariant derivative, \( R \) is the Ricci scalar, \( \kappa \equiv 8\pi G/c^4 \) is the Einstein coupling constant, \( \Lambda \) is the cosmological constant. The Maxwell tensor \( F_{mn} \) is given by

\[ F_{mn} \equiv \nabla_m A_n - \nabla_n A_m, \quad \nabla_k F^{*ik} = 0, \]

where \( A_m \) is an electromagnetic potential four-vector; \( \mathcal{F}^{mn} \equiv \frac{1}{2} \mathcal{F}_{mnpq} \mathcal{F}^{pq} \) is the tensor dual to \( \mathcal{F}_{pq} \), \( \mathcal{F}^{mnpq} \equiv \frac{1}{2} \mathcal{E}^{mnpq} \) is the Levi-Civita tensor, \( \mathcal{E}^{mnpq} \) is the absolutely antisymmetric Levi-Civita symbol with \( \mathcal{E}^{0123} = 1 \). It is the third term in the Lagrangian that describes the pseudoscalar-photon interaction [9]. The symbol \( \phi \) stands for a pseudoscalar field, this quantity being dimensionless.
axion field itself, \( \Phi \), is considered to be proportional to this quantity \( \Phi = \Psi_0 \phi \) with a phenomenological constant \( \Psi_0 \). The function \( V(\phi^3) \) describes the potential of the pseudoscalar field.

Now we extend the Lagrangian (2) by terms which are quadratic in the Maxwell tensor \( F_{mn} \), linear in \( \phi \) or in \( \nabla_k \phi \), and contain the normalized four-vector \( U^k \) \((U^kU_k = 1)\). The quantity \( U^k \) describes the macroscopic velocity of the axion system as a whole and may be chosen as the timelike eigenvector of the stress-energy tensor of the pseudoscalar (axion) field. To list all the irreducible scalars which satisfy these requirements, let us recall the important identity

\[
\left( F^i_k F^k_j \right) = \frac{1}{4} g^i_j F^m_n F^*_{mn},
\]

Clearly, all the invariants which we could construct using \( g_{ij}, F^i_k, F^*_{mn}, U^k \) as well as \( \phi \) or \( \nabla_k \phi \), definitely contain at least one convolution of the type \( F^i_k F^k_j \). Thus it is easy to check that due to (4) all the new terms in the extended Lagrangian can be reduced to the invariant

\[
\mathcal{L}_{(\text{int})} = \frac{1}{4} \nu F^m_n F^*_{mn} U^k \nabla_k \phi,
\]

where \( \nu \) is some new coupling constant introduced phenomenologically.

### 2.2. Extension of Axion Electrodyamics

The variation of the action functional containing a sum of the Lagrangians \( \mathcal{L}_{(0)} + \mathcal{L}_{(\text{int})} \) with respect to the four-vector potential \( A_i \) gives the equations of axion electrodynamics

\[
\nabla_k H^k_i = 0.
\]

Here the excitation tensor \( H^k_i \) is given by

\[
H^k_i = F^k_i + F^*_{k i} (\phi + \nu \nabla \phi),
\]

and \( \nabla \phi \) is the convective derivative. Using the linear constitutive equations

\[
H^k_i = C^{ikmn} F_{mn},
\]

we readily obtain that the linear response tensor \( C^{ikmn} \) now takes the form

\[
C^{ikmn} = \frac{1}{2} \left( g^{i m} g^{k n} - g^{i n} g^{k m} \right) + \frac{1}{2} \epsilon^{ikmn} (\phi + \nu \nabla \phi).
\]

This means that the dielectric permittivity and magnetic impermeability tensors of the axion-photon system are the same as in vacuum, i.e.,

\[
\varepsilon^{im} = 2C^{ikmn} U_k U_n = \Delta^{im},
\]

where \( \Delta^{im} = g^{im} - U^i U^m \) is the projector, and

\[
(\mu^{-1})_{pq} = -\frac{1}{2} \eta_{pik} C^{ikmn} \eta_{mnq} = \Delta_{pq},
\]

where \( \eta_{pik} \equiv \epsilon_{pikj} U^j \). The tensor of magneto-electric coefficients

\[
\nu^m = \eta_{pik} C^{ikmn} U^i = -\Delta^m (\phi + \nu \nabla \phi),
\]

describing the optical activity effects (see, e.g., [15]), is now characterized by an additional term linear in \( \nabla \phi \).

### 2.3. Pseudoscalar Field Evolution

Variation of the extended action functional with respect to the pseudoscalar field \( \phi \) gives the equation

\[
\nabla_k \nabla^k \phi + \phi V'(\phi^3) = \frac{1}{4\Psi_0^2} \left[ F^*_{mn} F^{mn} (\nu \theta - 1) + \nu \nabla (F^*_{mn} F^{mn}) \right],
\]

where \( \theta \equiv \nabla_k U^k \) is the expansion scalar of the velocity field, and the prime denotes a derivative of the potential \( V(\phi^3) \) with respect to its argument.

### 2.4. Gravity Field Equations

Modified Einstein’s equations obtained by variation of the extended action functional with respect to the metric \( g^{pq} \) can be presented in the form

\[
R_{pq} - \frac{1}{2} g_{pq} R = \Lambda g_{pq}
\]

\[
+ \kappa \left[ T_{pq}^{(EM)} + T_{pq}^{(A)} + \nu T_{pq}^{(s)} \right].
\]

Here the stress-energy tensor of the electromagnetic field

\[
T_{pq}^{(EM)} = \frac{1}{4} g_{pq} F_{mn} F^{mn} - F_{pm} F^m_q
\]

and the stress-energy tensor of the pure axionic field

\[
T_{pq}^{(A)} = \Psi_0^2 \left\{ \nabla_p \phi \nabla_q \phi - \frac{1}{2} g_{pq} \left[ \nabla_m \phi \nabla^m \phi - V(\phi^3) \right] \right\}
\]

are represented by the well-known expressions. The tensor

\[
T_{pq}^{(s)} = -\frac{1}{8} F^*_{mn} F^{*}_{mn} (U_p \nabla_q \phi + U_q \nabla_p \phi),
\]

describes a basically new source term in the right-hand side of the gravity field equations. Let us mention that the term \( \nu T_{pq}^{(s)} \) is obtained by variation of the
term with the interaction Lagrangian (5) by using the formula
\[ \delta U^i = \frac{1}{2} \delta g^{pq} \left( U_p \delta_q^i + U_q \delta_p^i \right) \]
for a variation of the velocity four-vector (see [16] for details). The standard interaction term \( \frac{1}{2} \sqrt{-g} \phi \times F_{mn} F^*_{mn} = \frac{1}{2} \phi E^{ikmn} F_{ik} F_{mn} \) does not contribute to the stress-energy tensor in the process of variation with respect to the metric. Thus the appearance of the term (17) is a new event in modeling of the gravity field of the photon-axion system.

2.5. An Example of Application

Let us consider the propagation of a test electromagnetic wave coupled to the axionic subsystem of the DM in a spatially homogeneous FLRW-type spacetime with the scale factor \( a(t) \). Let the electromagnetic wave propagate in the direction 0\( x \) and be characterized by the potential four-vector \( A_i = \delta_i^2 A_2(t, x) + \delta_i^3 A_3(t, x) \). The equations of axion electrodynamics can now be reduced to
\[
\begin{align*}
\left[ \frac{\partial^2}{\partial t^2} - \frac{1}{a^2} \frac{\partial^2}{\partial x^2} + H \frac{\partial}{\partial t} \right] A_2 &= -\frac{2\dot{\Theta}}{a} \frac{\partial}{\partial x} A_3, \\
\left[ \frac{\partial^2}{\partial t^2} - \frac{1}{a^2} \frac{\partial^2}{\partial x^2} + H \frac{\partial}{\partial t} \right] A_3 &= \frac{2\dot{\Theta}}{a} \frac{\partial}{\partial x} A_2,
\end{align*}
\]
where \( H(t) = \dot{a}/a \) is the Hubble function and
\[ \Theta(t) = \frac{1}{2} \left( \phi(t) + \nu \dot{\phi}(t) \right) \]
(in the cosmological context we use the units with \( c = 1 \)). Clearly, if \( \nu = 0 \) and \( \dot{\phi} \neq 0 \), the electromagnetic wave cannot keep linear polarization, however, in the case where \( \nu \neq 0 \) and the pseudoscalar field evolves exponentially, \( \phi(t) \propto \exp(-t/\nu) \), it can be possible. In the approximation of short wavelengths \( k \gg H \), a solution of (19), (20) for a circularly polarized wave has the form
\[
\begin{align*}
A_2 &= -A_0 \sin \left[ W - \varphi(t) \right], \\
A_3 &= A_0 \cos [W - \varphi(t)], \\
W &= W(t_0) + k \left[ \int_{t_0}^t \frac{dt'}{a(t')} - x \right], \\
\varphi(t) &= \Theta(t) - \Theta(t_0),
\end{align*}
\]
where \( k \) is a constant reciprocal to the wavelength. On the one hand, the quantity \( \varphi(t) \) is expressed in terms of \( \phi \) and \( \dot{\phi} \) (see (23) and (21)) and describes the rotation angle of the polarization vector of the electromagnetic wave traveling through the axion system; it can be studied in optical experiments. On the other hand, it is well known that in the cosmological context the function \( \dot{\phi} \) can be presented in terms of the DM energy density \( \mathcal{E} \) and pressure \( P \) as follows:
\[ \dot{\phi} = \pm \frac{1}{\Psi_0} \sqrt{\mathcal{E}(t) + P(t)}. \]
Thus the extended axion electrodynamics can be considered as a tool for studying nonstationary effects in the evolution of the axionic DM.

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