Spatial Imagination Development through Planar Section of Cube Buildings in Educational Process

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Abstract

The National Curriculum for Upper Secondary Education in Slovakia (ISCED 3A) specifies the mathematical standards and lesson quantity in Slovak schools. The major topic in ISCED 3A is stereometry, considered as one of the most problematic areas for students by a large majority of mathematics teachers. In general, the results of a large-scale opinion survey indicate that a high level of spatial imagination is necessary to solve stereometrical tasks. Slovak and Czech researchers in this field concur in the conviction that an appropriate level of spatial imagination can be systematically developed through selected types of tasks by using special methods, strategies and suitable forms of visualization. The results of several studies emphasize the need for the introduction of these types of tasks also with respect to the implementation of ICT in the educational process.

In our paper, we analyze in detail the conclusions of Slovak and Czech mathematics experts and university teachers whose research field is focused on the development of spatial and geometric imagination. Based on the findings that the subject area is developed mainly through planar sections of solids, we propose a new inspirational type of tasks on sections of cube buildings of special solids constructed from multiple cubes. We discuss the results of pilot tests in the recommendations, where we analyse various aspects of the educational process in regard to the implementation of appropriate geometry software Cabri 3D in teaching support.

Keywords: stereometry, cube, Cabri 3D, spatial imagination, geometric imagination

1 Introduction

Pursuant to National Curriculum for Upper Secondary Education in Slovakia, stereometry is an ingredient of geometry teaching at Slovak schools which content corresponds to the teaching of the positional and metric properties of spatial objects. For this purpose, special attention is given to the solution of the standard tasks focused...
on planar cuts of solids, specifically cubes. Currently, courses in stereometry are supported by introducing ICT in Slovak schools, especially by using interactive geometric software like Cabri 3D. The visualization made possible through implementation of this software helps to solve tasks that would be difficult for spatial and geometric imagination. The software supports the constructive thinking of students. Its use in a presentation of the solution plays an important role in the educational process because a visual perception of 3D objects allows the investigation of stereometrical problems to be more rational.

2 Methodology of stereometry

A lot of Slovak and Czech researchers working in the field of the theory of mathematics education seem to agree in opinion that for the successful handling of these types of tasks, it is necessary that a student possess a sufficient level of spatial imagination. Šedivý (2006), Molnár (2009), Perný et al. (2010). From this point of view spatial imagination is considered a significant factor, but the researchers diverge in their opinions as to its exact definition. The consensus is that spatial imagination is the ability to perceive space, to imagine objects in different positions; and to characterize shapes, their properties and relationships among their elements. Šarounová (1988), Hejný (1990), Tomková (2013), Chávez, Reys & Jones (2005), Pavlovičová (2012). Geometric imagination is considered a higher form of spatial imagination. It is stated that geometric imagination is the specific ability to work with geometric objects in mind, to deduce geometric shapes of real objects, to represent shapes graphically in different positions and finally, to solve geometric problems of real life in plane and space. Hejný (2004). Geometric imagination is underdeveloped in high school students in Slovakia due to different reasons. Vidermanová (2005), Pomp & Václavíková (2010). It turns out that the implementation of ICT in the teaching process can significantly improve the current situation. Boz (2005), Fulier (2006), Jones (2001), Boytchev et. al. (2007), Hubackova (2011), Žilková (2010), Laborde (2000). According to Hejný (1990) there are two principal ways of methodically teaching stereometry. Students use these ways as suitable approaches to solving the more complicated stereometric tasks. The first approach is based on the experience and knowledge that students have obtained in dealing with planimetric tasks. A stereometric problem is decomposed into particular planar tasks, which are solved in familiar algorithmic ways. A synthesis of these particular solutions gives a complex solution to stereometrical tasks. This approach is considered a standard approach which we called the algorithmic principle. The second methodological approach is positional, called the shaped principle. This idea is based on a perspective view of the problem. The stereometrical task is solved like a compact problem in a space and one can search there for connections among the given elements of solid objects. The shape principle requires a high level of geometric imagination, but solutions based on this principle are usually original and simpler than those obtained with the algorithmic principle. We illustrate some applications of these principles in the solutions of specific stereometric tasks by using software support.

3 Cubes in the teaching of stereometry

It is a convention that space is studied through properties of solids where the cube plays a basic role. The cube can be illustrated in two suitable projections at a blackboard. The first one is presented from left and right projection in the fig. 1. Due to a psychological reason of human perception of space images, people prefer the cube projection in the right look. Several studies conjecture that this preference is significant in the solving of stereometric tasks, but it is not a guarantee of correct solutions (see fig. 1).
Fig. 1. A-B The first sample of the student’s solution is correct although the cube is sketched in the left perspective. The solution on the right is false. The section polygon cannot be a planar figure in this position. The front side of polygon must be identical with the diagonal of face of the cube.

It turns out that students often use an algorithmic approach to solve tasks requiring the planar sections of cubes. The cases when it is necessary to use the shape principle are not exceptions (see fig. 2). Students were asked to draw the cut of cube buildings at points K, L and M. One can see that they used the algorithmic principle. They drew up the cutting by the given points lying in one face of the cube at first. Next, the algorithmic principle is fully reflected in the fact that they solved the task only partially. Other students cut only the first cube and did not draw the planar cutting for the entire block of cubes. The students did not analyse the problem from the perspective view. Moreover, some of the selected samples of the students’ partial solutions are not correct, either.

Fig. 2. A–D Four samples of students’ solutions of the selected task. The solutions are not correct. The students used the algorithmic principle. The last two sketches imply that they have used the theorem about the common intersection point of three planes. All samples were obtained by the small probe in geometry course at CPU in Nitra.

3.1 Solving problems using software Cabri 3D

The solutions for standard tasks on planar sections of different solids are a frequent subject for research on spatial imagination. The benefits of the implementation of ICT are of interest to researchers, too. The most widely used software is Cabri 3D. Vaníček (2005), Heshkowitz et al. (2002). One advantage of the software Cabri 3D is its ability to design cube buildings with a built-in design tool called cube. Tasks of this type assist the development of geometrical imagination and are an important part of the Slovak mathematical curriculum at primary schools. Although these tasks are not preferred in high school mathematical curriculum, we see their didactic advantage. Tasks of this type can be used as non-traditional tasks for the planar cutting of cube buildings. We believe that analysis of the solutions offer teachers a resource in how to diagnose the formal transfer of knowledge.
The teacher has a unique type of the tasks in solutions which are applied to both stereometrical principles. If students use the algorithmic principle, then they can construct vertices of the first hexagonal section on the first cube. These vertices are the starting points for cutting the other cubes. From the point of view of the shape principle, it is clear that the lines must also intersect other edges of the cube building.

Fig. 4. The planar section of a cube building executed by software Cabri 3D. The software also visualizes the result in a clear form which cannot be displayed without the use of technology.
Virtual manipulation of the constructed model of a plane cutting through all the cubes gives students an opportunity to see that the plane intersects several cubes (see Fig. 4). It brings a new look to the solution in accordance with the shaped principle.

4. Discussion

As we noted above, the use of interactive geometry software is very important. In the traditional approach to teaching stereometry, it is not possible to solve such difficult tasks because the technical implementation of their sketch assignment on the blackboard is complicated and has insufficient graphical clarity. From this perspective, the appropriate software must be used. The use of software for the visualisation of geometric objects is another reason. Building on our own observations of the process of teaching stereometry, we say that teaching is marked by formalism. (Hejný & Kuřína, 2001). Formalism can be caused by an insufficient supply of concrete visual images of geometric objects and their possible arrangement in the three-dimensional space. One of the ways to break down this problem is to re-build a new student’s structure of separate models by working with software like Cabri 3D.

Acknowledgements

This paper was supported by grant KEGA MINE Slovak Republic titled Solid Geometry in Arrangement of Math Teachers with Accent of Factor Handling Activity and Application Tasks.

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