A multifractal study of charged secondaries produced in relativistic nucleus–nucleus collisions

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Abstract An analysis of the data obtained in 14.5A GeV/c $^{28}$Si–nucleus interactions has been carried out to study the multiplicity fluctuations by using the method of multifractality. A power-law behaviour is found to exist in this study. Experimental results have been compared with those of FRITIOF-generated events. The dependence of various parameters characterizing multifractality for two groups of events chosen on the basis of grey particle multiplicity is also looked into.

1 Introduction

The multiparticle production process, in hadron–nucleus (hA) and nucleus–nucleus (AA) collisions, has been studied extensively in the past [1–27]. The major goal to study AA collisions is to investigate the properties of quark–gluon plasma (QGP) [28, 29] that is the de-confined state of matter. There are many signatures [30–33] of QGP; one of the techniques to study this is the fluctuations in the particle densities. Fluctuations in individual events/interactions may give rise to peaks or spikes in the phase space domains [34–36]. These may be studied using the method of scaled factorial moments of the multiplicity distributions by dividing them into different pseudo-rapidity, $\eta$, bin sizes [37]. Pseudo-rapidity is defined as:

$$\eta = -\ln \tan \frac{\theta}{2}$$

(1)

where $\theta$ is the space angle of a charged particle track made with the mean primary direction.

To study the self-similarity of multiplicity fluctuations in particle production, Bialas and Peschanski [38] proposed that if we study the variations of factorial moments with decreasing bin sizes, it shows a power-law behaviour, which is referred to as intermittency. The search for a link between intermittency and phase transition leads to the thermodynamic formulation of fractal dimensions of which intermittency is a special case. This has been discussed by various workers [39–42].

A fractal or a self-similar object has the characteristics of satisfying a power-law behaviour which reflects the underlying dynamics [43]. In the present work, the method of multifractal moments, $G_{q}$ [43], has been used to investigate the scaling properties of relativistic AA collisions. The investigation of heavy-ion collisions at relativistic energies may reveal some information about the nature of the interaction mechanism. To understand multiparticle production, the nuclear emulsion technique [44, 45] has been used. The nuclear emulsion is a material that detects charged particles only. It is one of the oldest particle detection techniques. It is compact and has a 4$\pi$ detection capability. The nuclear emulsion consists of various elements like hydrogen (H), carbon (C), nitrogen (N), oxygen (O), silver (Ag) and bromine (Br). When a high energy particle or ion interacts with the nuclei of emulsion, a large number of particles are produced.

2 Mathematical formalism

To study multifractality on event-by-event basis, a given pseudo-rapidity range, $\Delta \eta (= \eta_{\text{max}} - \eta_{\text{min}})$, is divided into $M$ bins (1–15) of width $\delta \eta = \Delta \eta / M$. The $q$th-order multifractal moments, $G_{q}$, are defined [41, 43] as:

$$G_{q} = \sum_{j=1}^{M} p_{j}^{q}$$

(2)

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where summation has been performed over the non-empty bins only, which constitute a fractal set. Here \( p_j = n_j/n_t; n_j \) denote the number of particles in the considered bin and \( n_t \) is the total number of particles in the considered \( \eta \) range in an event which is (0–6) that is \( \eta_{\text{max}} \) is 6 and \( \eta_{\text{min}} \) is zero. When it is averaged over the entire data sample consisting of \( N \) events, it may be expressed as:

\[
\langle G_q \rangle = \left( \frac{1}{N} \right) \sum_{1}^{N} G_q
\]

For the observation of multifractality in the rapidity distribution, mean values of multifractal moments, \( \langle G_q \rangle \), should give rise to a power-law behaviour over a small \( \eta \) range in the following way:

\[
\langle G_q \rangle \propto (\delta \eta)^{T_q}
\]

where \( T_q \) are the mass exponents and may be determined from the observed linear dependence of \( \ln \langle G_q \rangle \) on \( \ln M \) using Eq. 4.

The generalized dimensions \( D_q \) and multifractal spectral function, \( f(\alpha_q) \), may be obtained [43] by applying Legendre transform that is the standard procedure of multifractals [38] from the following equations:

\[
D_q = \frac{T_q}{q - 1}
\]

and

\[
f(\alpha_q) = q\alpha_q - T_q
\]

where \( \alpha_q \) is referred to as the Lipchitz–Holder exponents [46] and is defined as:

\[
\alpha_q = \frac{dT_q}{dq}.
\]

Regarding the investigation of multifractality, a spectrum function is considered as one of the main parameters. For the observance of a multifractal structure, the spectral function is smooth concave downwards with its maximum at \( \alpha_q = 0 \). The left \((q > 0)\) and right \((q < 0)\) wings of the plots of the multifractal function, \( f(\alpha_q) \), indicate the fluctuation in the particle density in the dense and sparse regions of single-particle pseudo-rapidity distribution [47]. The in-homogeneity in the pseudo-rapidity distribution is determined by the width of the distribution. There is non-existence of a sharp peak in the plot of \( f(\alpha_q) \), versus \( \alpha_q \) at \( \alpha_q \) corresponding to \( q = 0 \), which reveals the non-smooth nature of the pseudo-rapidity distribution. One of the basic properties of the fractals that describe the scaling behaviour is the generalized dimensions, \( D_q \). It is found that \( D_q \) decreases with increasing order of the moments; this decreasing pattern is known as multifractal, and on the other hand, if \( D_q \) is constant with \( q \), the pattern is referred to as mono-fractal [48, 49].

### 3 Details of the data

The emulsion stack used in the present study was exposed to a 14.5A GeV/c silicon beam at Alternating Gradient Synchrotron (AGS), Brookhaven National Laboratory (BNL). The method of line scanning was adopted to search the interactions by using M4000 Cooke’s series microscopes with 15 × eyepieces and 20 × objectives. The events/interactions were picked up after leaving 3 mm from the leading edges of the pellicles to avoid any distortion effects. The interactions that were produced 35 μm from the top or bottom surface of the pellicles were excluded from the data.

To avoid any contamination of primary events with secondary interactions, the primaries of all the events were followed back up to the edge of the pellicles. Only those events whose primary remained parallel to the main direction of the beam and which did not show any significant change in their ionization were finally picked up as genuine primary events. The different types of particles appear in the form of tracks and usual emulsion terminology [44, 45] was applied to categorise such tracks. The tracks were classified on the basis of their specific ionization \( g^* (= g/g_0) \), where \( g \) is the ionization of the track and \( g_0 \) is the ionization of the primary. The tracks with \( g^* < 1.4 \), \( 1.4 \leq g^* \leq 10 \), and \( g^* > 10 \) were named as shower, grey, and black tracks/particles, respectively. The number of the shower, grey, and black tracks in an event has been denoted by \( N_s, N_g, \) and \( N_b \), respectively.

The angular measurements were taken by using an oil immersion objective of 100 × magnification. For the calculation of space angle, the projected angle is to be measured first. The tracks of relativistic charged particles in the most forward cone overlap with each other so it becomes difficult to measure the projected angle of such tracks directly, for this purpose coordinate method [49] was applied. In this \( X, Y, \) and \( Z \) coordinates were measured. The star vertex, the point from where different tracks originate, is moved along the \( X \)-axis of the stage of the microscope, and when the track looked separated, their \( X, Y, \) and \( Z \) coordinates were measured and the projected angle was calculated. Thus, after applying the above-discussed criterion, a sample of 555 clean events was selected for the present study.
4 Results and discussion

To compare the experimental results with the Lund model, FRITIOF [50, 51], on multiparticle production and Monte Carlo random number generator (MC-RAND), 5000 events for each were generated with similar characteristics as that of experimental events observed in 14.5A GeV/c 28Si–nucleus interactions. FRITIOF events were generated based on the average value of relativistic charged particles and dispersion of its experimental multiplicity distribution so that we get the average value in the case of simulated data as almost the same as that of the experimental events. However, in the generation of the MC-RAND events, we tried to make sure that the multiplicity distribution of the produced particles should be similar to that of experimental events and there should not be any correlation amongst the produced particles. The pseudo-rapidity distribution for experimental as well as generated events is shown in Fig. 1. The shape of the distribution is of Gaussian type for both the data. One more criterion which we applied is that the mean value and dispersion of the produced particles are comparable to the experimental values. The basic difference between FRITIOF and MC-RAND is that the MC-RAND events are correlation-free, whereas FRITIOF events have the same correlation as that of the experimental events [50, 51].

Before we proceed further for results and discussions, we calculated mean number of relativistic charged particles ($\langle N_s \rangle$) for experimental as well as FRITIOF events. The mean number of relativistic charged particles is calculated by dividing the total number of charged particles in the data sample by the total number of events. Mean pseudo-rapidity is calculated as the sum of pseudo-rapidities of all the relativistic charged particles divided by the total number of relativistic charged particles in the entire data sample. The dispersion of the pseudo-rapidity distribution, $D(\eta)$, is calculated as $D(\eta) = \sqrt{\langle \eta^2 \rangle - \langle \eta \rangle^2}$. We found the mean number of relativistic charged particles, average pseudo-rapidity, and dispersion of the pseudo-rapidity distribution for FRITIOF simulated events come out to be 21.32 ± 0.95, 2.83 ± 0.16, and 1.28 ± 0.03, respectively, which are very close to that of the experimental events. As the average values and the dispersion in the two cases, i.e., experimental and FRITIOF, are found to be almost the same, we can say that the experimental data matches with the data of the FRITIOF model.

The values of $\ln \langle G_q \rangle$ have been plotted as a function of $\ln M$ for experimental and FRITIOF in Fig. 2 for $q = -6, -4, -2, 0, 2, 4$ and 6. A linear increase in the variation of fractal moments with increasing bin size, $M$, is observed for both the data sets. Furthermore, the moments with positive values of $q$ and for negative $q$ values give a linear relationship over a wide range of $\ln M$; thus, the moment shows self-similarity in the mechanism of particle production for the nuclear interactions considered. The plots for the simulated events are in good agreement with that of the experimental events. Similar results have been reported for hA [52] and AA [53] collisions.

The variation of $\ln \langle G_q \rangle$ with $\ln M$ at 14.5A GeV/c 28Si–nucleus interactions for two groups of events, i.e. $N_g \leq 1$ and $N_g \geq 2$, is exhibited in Fig. 3. The multifractal moments are observed to increase linearly with decreasing bin width, $\delta \eta$ for the two groups of interactions considered in the study. Furthermore, it is observed that $G_q$ moments have slightly higher values for the interactions having $N_g \geq 2$ than those of having $N_g \leq 1$.

The mass exponents, $T_q$, are obtained by studying the dependence of $\ln \langle G_q \rangle$ on $\ln M$. For carrying out approximations, only the portions of the curves that show linearity are taken into account to avoid the saturation effect. The values of $T_q$, in 28Si–nucleus collisions obtained for experimental and FRITIOF data are displayed in Fig. 4. The mass exponents, $T_q$, are observed to increase with increasing order of the moments, $q$, for both the data sets.

![Fig. 1 Normalized pseudo-rapidity distributions of relativistic charged particles in 28Si–nucleus interactions. Solid (Experimental) and dotted (FRITIOF) curves are the best fits to the data](image-url)
The dependence of the mass exponents, $T_q$, on $q$ for two groups of events, i.e. $N_g \leq 1$ and $N_g \geq 2$, is given in Fig. 5. From the figure, it may be seen that $T_q$ increases with the increase in the order of the moments, $q$. However, it seems to saturate at higher values of $q$. We find that the dependence of $T_q$ on $q$ is almost similar for the two groups of events.

To study the contribution of the dynamical component of the multifractal moments, experimental results were compared with those of the MC-RAND events. For this purpose, the values of $G_q$ as well as $T_q$ moments for the MC-RAND data which is written
Fig. 5 $T_q$ versus $q$ plots for $^{28}$Si–nucleus interactions having $N_g \leq 1$ and $N_g \geq 2$ groups of experimental events

The variation of $T_q$, $T_q^{\text{stat}}$ and $T_q^{\text{dyn}}$ with $q$ in $^{28}$Si–nucleus collisions is shown in Fig. 6. It may be seen from the figure that $T_q$ increases with increasing $q$, however, the rate of increase in the regions which correspond to positive and negative values of $q$ are quite different. In the regions corresponding to the negative $q$ values, the increase in $T_q$ is relatively more rapid in comparison with that for the region in which $q$ have positive values. This observation is consistent with the predictions of the gluon model [47]. Furthermore, the deviation in the values of $T_q$ from $T_q^{\text{stat}}$ may, therefore, lead to the deviation in the $T_q^{\text{dyn}}$ from $(q - 1)$. It should be mentioned that any deviation of $T_q^{\text{dyn}}$ from $(q - 1)$ will indicate the presence of dynamical contribution to the fluctuations. Therefore, $T_q$ may be considered as a more sensitive measure of dynamical fluctuation than $G_q$ itself. One more observation that may be made is that $T_q^{\text{dyn}}$ coincides with $T_q^{\text{stat}}$ in the mid-region of $q$ values, whereas for the $q$ values in the region ($-2 \geq q \geq 2$), a significant departure is observed.

The variations of the generalized dimensions, $D_q$, with $q$ in $^{28}$Si–nucleus collisions for experimental and FRITIOF data are exhibited in Fig. 7. The generalized dimensions are found to be positive for all orders of the moments, $q$ and demonstrate a decreasing trend with increasing $q$. This behaviour is in excellent agreement with the predictions of the multifractal cascade model [54]. It may also be observed from the figure that the values of $D_q$ are greater than unity for $q \leq -2$, and this result is in agreement with those reported [37] earlier for different projectiles over a wide energy range.

To examine the variation of the generalized dimensions with a certain order of the moment, we have plotted $D_q$ as a function of $q$ in Fig. 8 again for the same two groups of events, i.e. $N_g \leq 1$ and $N_g \geq 2$. From the figure, we note that the generalized dimensions, $D_q$, are almost the same in the case of negative as well as positive values of $q$ for both the groups of events. However, it is also observed that these values are slightly higher for $N_g \geq 2$ group of events for positive $q$ values. One of the reasons for the higher value of $D_q$ in the case of $N_g \geq 2$ may be due to an expected increase in the average multiplicity [55].

Figure 9 shows the dependence of $f(\alpha_q)$ on $\alpha_q$ for the experimental and FRITIOF events. The width of the multifractal spectra for experimental events and for the simulated events is almost the same. This means that the nature of the spectra is same as concave

as $G_q^{\text{stat}}$ and $T_q^{\text{stat}}$ respectively, have also been calculated in the same way as that for the experimental data. Further, the statistical component ($T_q^{\text{stat}}$) and dynamical component ($T_q^{\text{dyn}}$) of the mass exponents are related [9] as:

$$T_q^{\text{dyn}} = T_q - T_q^{\text{stat}} + (q - 1) \quad (8)$$

The variations of $T_q$, $T_q^{\text{stat}}$ and $T_q^{\text{dyn}}$ with the moment order, $q$, are shown in Fig. 6. It may be seen from the figure that $T_q$ increases with increasing $q$, however, the rate of increase in the regions which correspond to positive and negative values of $q$ are quite different. In the regions corresponding to the negative $q$ values, the increase in $T_q$ is relatively more rapid in comparison with that for the region in which $q$ have positive values. This observation is consistent with the predictions of the gluon model [47]. Furthermore, the deviation in the values of $T_q$ from $T_q^{\text{stat}}$ may, therefore, lead to the deviation in the $T_q^{\text{dyn}}$ from $(q - 1)$. It should be mentioned that any deviation of $T_q^{\text{dyn}}$ from $(q - 1)$ will indicate the presence of dynamical contribution to the fluctuations. Therefore, $T_q$ may be considered as a more sensitive measure of dynamical fluctuation than $G_q$ itself. One more observation that may be made is that $T_q^{\text{dyn}}$ coincides with $T_q^{\text{stat}}$ in the mid-region of $q$ values, whereas for the $q$ values in the region ($-2 \geq q \geq 2$), a significant departure is observed.

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Figure 9 shows the dependence of $f(\alpha_q)$ on $\alpha_q$ for the experimental and FRITIOF events. The width of the multifractal spectra for experimental events and for the simulated events is almost the same. This means that the nature of the spectra is same as concave
downwards centred around $\alpha_q$ corresponding to $q = 0$ and have a common tangent at an angle of 45°. This observation agrees fairly well with the predictions of the gluon model [47]. However, $f(\alpha_q)$ has no peak in any of the cases studied, which is an indication of the non-smooth nature of the multiplicity distribution of the particles produced in the interactions considered for the present study. To see the dependence of $f(\alpha_q)$ spectra on $\alpha_q$ for two groups of events, $N_\bar{g} \leq 1$ and $N_\bar{g} \geq 2$ are plotted in Fig. 10. The shape of the spectra is found to be same for both groups of events.

5 Concluding remarks

The conclusions that may be drawn from the present investigation are as follows:

1. A power-law behaviour is observed in the variation of $\ln \langle G_q \rangle$ with $\ln M$. A concave downward trend is found to exist for spectral function, $f(\alpha_q)$. These observations, which are for experimental and FRITIOF events, indicate the presence of multifractality in the mechanism of multiparticle production.
2. The deviation of $T_q^{\text{dyn}}$ from $(q - 1)$ indicates the presence of dynamical contribution to the fluctuations.
3. The decreasing trend in the value of $D_q$ with increasing $q$ confirms the presence of multifractality.
4. The variation of the parameters $T_q$, $D_q$ on $q$ and $f(\alpha_q)$ on $\alpha_q$ is observed to be almost similar for the two groups of events, $N_\bar{g} \leq 1$ and $N_\bar{g} \geq 2$. 

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Fig. 9 The dependence of multifractal spectral function, $f(\alpha_q)$, on $\alpha_q$ in $^{28}\text{Si}$–nucleus collisions.

Fig. 10 The dependence of multifractal spectral function, $f(\alpha_q)$, on $\alpha_q$ in $^{28}\text{Si}$–nucleus collisions for two groups of events, $N_g \leq 1$ and $N_g \geq 2$.

Data Availability Statement This manuscript has associated data in a data repository. [Authors’ comment: This is our results.]

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