Nanotechnological Structure for Observation of Current Induced Contact Potential Difference and Creation of Effective Cooper Pair Mass-Spectroscopy

Todor M. Mishonov
Institute of Solid State Physics, Bulgarian Academy of Sciences,
72 Tzarigradsko Chaussee Blvd., BG-1784 Sofia, Bulgaria

Albert Varonov
Institute of Solid State Physics, Bulgarian Academy of Sciences,
72 Tzarigradsko Chaussee Blvd., BG-1784 Sofia, Bulgaria

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Changes of the electron work-function of a superconductor proportional to the square of the current density \( \Delta \phi = -\beta j^2 \) are known as Bernoulli effect in superconductors or current induced Contact Potential Difference (CPD). The temperature dependent constant \( \beta(T; m^*) \) is parametrized by the effective mass of Cooper pairs \( m^* \). In such a way the study of the Bernoulli effect leads to creation of Cooper pair mass-spectroscopy. In this paper a short review on the Bernoulli effect in superconductors is given and a proposed experimental set-up for its measurement is described in detail. The experiment requires standard electronic equipment and can be implemented in every laboratory related to physics of superconductivity. This experimental set-up for observation of current induced CPD requires nano-technological hybrid superconductor structures with atomically clean interfaces and measurement of nano-volt signals with capacitive coupling to the sample.

I. INTRODUCTION. \( e^* \) AND \( m^* \). A HISTORICAL PERSPECTIVE

Starting from the formula for velocity \( v = (\hbar \nabla \theta - e^* A)/m^* \) for a charged super-fluid with wave-function \( \Psi = \sqrt{n} e^{i\theta} \), F. London derived the well-known relation between the variation of the vector-potential \( \delta A \) and the current density \( j = e^* n \nabla v \). This local approximation \( \delta j(r) = (e^* n/m^*) \delta A(r) \) leads to the well-known formulae for the London penetration depth \( 1/\lambda^2 = e^* n/\pi e^2 m^* \) and the quantization of magnetic flux \( \Phi_0 = 2\pi \hbar e^*/e^* \). Hereafter the notation \( [e] \) is only in Gaussian system, in SI \( [e] \) should be substituted by 1. In the relatively exotic case of rotating superconductor with local velocity of the lattice \( V \) to the potential momentum \( m_0 V \) a term containing the inertial mass of the free charge carriers \( m_0 \) is added and the local expression for the current reads \( j = e^* n (\hbar \nabla \theta - e^* A - m_0 V)/m^* \). This complete result gives the possibility for a precise determination of the inertial mass of an electron \( m_e \) moving in a crystal lattice.

The negligible dissipation and approximate energy conservation of superconductors in the radio-frequency range give good conditions for the applicability of the Bernoulli theorem

\[
\frac{1}{2} m^* v^2 n(T) + \rho_{tot} \varphi = \rho_{tot} \zeta, \quad \rho_{tot} = e^* n(t = 0)
\]

for the constancy of the electro-chemical potential \( \zeta \) in the condition of thermodynamic equilibrium with a constant temperature \( T \), cf. the equations of hydrodynamics of a super-fluid by Landau. If the superconductor is a clean metal with closed trajectories on the Fermi surface \( \rho_{tot} = e(n_c - n_h) \) where \( n_c \) and \( n_h \) are densities of electronic states with positive and negative effective masses.

Let us recall that a voltmeter measures the difference of the electro-chemical potential. That is why for the measurement of current induced contact potential difference

\[
\Delta \varphi = -\frac{1}{2} m^* v^2 n/\rho_{tot} = -\beta j^2, \quad \beta(T) \equiv m^*/2\rho_{tot} C_s(T) = \frac{\lambda^2(T = 0)}{\lambda^2(T)} = \frac{n(T)}{n(T = 0)} \in (0, 1),
\]

where \( \Delta \varphi \equiv \varphi(v) - \varphi(0) \). Introducing a dimensionless parameter \( C_\lambda \) close to the critical temperature we have

\[
C_\lambda \equiv -T_c \frac{d}{dT} \left| \frac{\lambda^2(0)}{\lambda^2(T)} \right|_{T_c}, \quad C_s(T) \approx C_\lambda |\epsilon|, \quad |\epsilon| = \frac{T_c - T}{T_c} \ll 1, \quad T < T_c, \quad C_{s,d} \approx 2.6, \quad C_{s,s} \approx 2.0.
\]

For the experiment which we suggest it is indispensable to have capacitive coupling between the sample and the Bernoulli signal detector electrode. For a short review of the problem see Ref. and references therein. We wish to stress out that the Bernoulli theorem for superconductors is not a result of some dynamic theory but a consequence from thermodynamic consideration even of static super currents.
Yet London\textsuperscript{[1]} pointed out, that superconducting alloys behave like they have a negative surface tension\textsuperscript{[2]} and Shubnikov\textsuperscript{[3]} even started to investigate the magnetization of the vortex phase of now called type-II superconductors\textsuperscript{[10]} Many years later in the framework of the Ginzburg-Landau theory\textsuperscript{[11,12]} an approximate dependence\textsuperscript{[13]} of the surface tension $\sigma$ from the Ginzburg-Landau (GL) parameter $\kappa$ was derived

$$\sigma(T) \approx \frac{8}{3\sqrt{2}} p_B(T) \xi(T) \left(1 - \sqrt{\frac{\kappa}{\kappa_c}}\right), \quad p_B(T) = \frac{B^2_c(T)}{2\mu_0}, \quad \kappa \equiv \frac{\lambda(T)}{\xi(T)} \approx \text{const, } \kappa_c = \sqrt{2}, \quad \lambda(T) = \frac{\lambda(0)}{\sqrt{|\epsilon|}}$$

where $B_c(T)$ is the thermodynamic critical field related to the volume density of the condensation energy $p_B(T)$, and $\lambda(0) = \lambda(0)/\sqrt{C_\lambda(0)}$ parameterized penetration depth for $|\epsilon| \ll 1$. Here we wish to mention other well known results of GL theory

$$\kappa = \frac{2^{3/2}\pi}{\Phi_0} \lambda^2(T) B_c(T), \quad \xi(T) = \frac{\xi(0)}{\sqrt{|\epsilon|}}, \quad \epsilon = \frac{T - T_c}{T_c}, \quad -T_c \frac{dB_2^c}{dT} \bigg|_{T_c} = \frac{\Phi_0}{2\pi\xi^2(0)}, \quad \Delta C = \frac{T_c}{\mu_0} \left(\frac{dB_1}{dT} \bigg|_{T_c}\right)^2,$$

where $B_2^c(T) \approx \Phi_0/2\pi\xi^2(T)$ is the super-cooling (for type-I superconductors) magnetic field, $B_c$ is the thermodynamic magnetic field related to the jump of the heat capacity $\Delta C$ (per unit volume) at the critical temperature $T_c$. The argument of $\xi(0)$ means only GL parametrization, but not $T = 0$.

Unfortunately, theoretical considerations and brilliant physical intuition have never been taken seriously. The charge of the superfluid charge carriers was not determined by the flux quantum $e^* = 2\pi\hbar|\epsilon|/\Phi_0$ an elementary consequence of the integer value of the dimensionless momentum circulation $\oint (m^\ast v + e^* A) \cdot d\mathbf{r}/2\pi\hbar$. However scientific archaeology reveals that for first time idea for electron doublets was proposed soon after discovery of superconductivity in 1914 by Sir J. J. Thompson\textsuperscript{[15]} and later on the analyzing relation by superconductivity and Bose-Einstein condensation by Ogg\textsuperscript{[16]} and Shafroth\textsuperscript{[17]} in such a way it is clear why Onsager pointed out that $\Phi_0$ should correspond to $|e^*| = 2|\epsilon|$ long time before the experimental determination.

All equations in the present work are written in almost system-invariant form; as we pointed out in SI units $|\epsilon|$ must be substituted by one. The research by Shubnikov on the now called Abrikosov vortex phase was even more dramatically interrupted\textsuperscript{[10]} The same can be said for the effective mass of the super-fluid charge carrier $m^\ast$. Analyzing only the temperature dependence of the penetration depth $1/\lambda^2(T) = e^* \rho_{\text{cond}} C_s(T)/\epsilon_0 e^2 m^\ast$ the mass of the Cooper pairs remains undetermined. In all experiments with magnetic field only the ratio $n(T)/m^\ast$ participates and with appropriate re-normalization of the density for $m^\ast$ one can take the mass of the Sun $m_\odot$, cf. Ref.\textsuperscript{[18]} In such a way one significant part of the physics of superconductivity remains undeveloped despite of $10^5$ works published on this topic. In order to measure $m^\ast$ it is necessary to study the electric field effect in superconductors using atomically clean superconductor surfaces, which is the main technological difficulty. If we ignore Bernoulli and London-Hall effect in superconductors we have one parametric re-normalization $n(T) \rightarrow C n(T)$ and simultaneously $m^\ast \rightarrow C m^\ast$, which conserves penetration depth ratio

$$\frac{n(T)}{m^\ast} = \frac{C n(T)}{C m^\ast}, \quad C > 0,$$

but this does not mean that $m^\ast$ is not accessible. The density of super-fluid charge carriers can be determined also by measurement of the drift velocity of the super-fluid by Doppler shift of plasmon resonances in thin films, for example Ref.\textsuperscript{[19]} For clean superconductors half of the Cooper pair mass must be the extrapolated to zero frequency optical mass\textsuperscript{[20]}

The purpose of the present work is to suggest a simple experimental set-up and a method for investigation of the Bernoulli effect in superconductors, which is parameterized by the effective mass of Cooper pairs. In such a way it is possible to continue the first determination of the Cooper pair mass\textsuperscript{[21,22]} begun in the Bell labs and interrupted by the end of these laboratories. In the next section we describe the suggested experiment in details.

II. DESCRIPTION OF THE SET-UP AND NOTATIONS

Thin films with thickness $d_{\text{film}} < \lambda$ grown on an insulator substrate are important objects for investigation in the physics of superconductivity. If through such a film a two dimensional current $j_{2D}$ with an acceptable 5% accuracy flows one can consider that the bulk current density is homogeneous across the thickness of the film $j \approx j_{2D}/d_{\text{film}}$, and precisely this current density is on the interface of the superconductor with protecting insulator layer with thickness $d_{\text{ins}}$. We consider a superconducting film grown on a substrate and protected by a very thin insulator layer. We suppose that the interface is perfect and the surface of the superconductor has the properties of the bulk material.
On the protective insulator layer 4 metal electrodes with axial symmetry have to be evaporated, the material of the alloy is irrelevant. In this Corbino geometry we have 1) One circle with radius \( R_1 \). 2) Then after some narrow gap with width \( w \) we have a ring electrode with internal radius \( r_2 = R_1 + w \) and external radius \( R_2 \). This is repeated two times again with two other electrodes with radii 3) \( r_3 = R_2 + w \) and external radius \( R_3 \), and finally 4) a ring electrode with internal radius \( r_4 = R_3 + w \) and an external electrode with radius \( R_4 = a/2 \) equal to the half side of the square substrate, for definiteness let \( a = 5 \text{ mm} \). All probes are wired and AC input drive voltage \( U_{1,3} \) is applied between electrodes (1) and (3) and the time dependent output Bernoulli signal \( U_{2,4} \) is measured by a capacitive coupling to the superconductor layer. The drive and detector circuits are capacitively disconnected. One dimensional version of this setup is depicted in Fig. 2.

The radii have such proportion that areas of the electrodes are approximately equal

\[
S_1 = \pi R_1^2, \quad S_2 = \pi (R_2^2 - r_2^2), \quad S_3 = \pi (R_3^2 - r_3^2), \quad S_4 = \pi (R_4^2 - r_4^2).
\]

The capacities neglecting the effect of ends are approximately \( C_i = \varepsilon_0 \varepsilon_{\text{ins}} S_i / d_{\text{ins}} \), where \( i = 1, 2, 3, 4 \).

The observable Bernoulli potential is determined by the super-fluid velocity and current density \( j_{\text{surf}} \) at the surface of the superconductor. This current should be much smaller than the critical one \( j_c(T) \). For thin superconductor film we have

\[
\Delta \varphi = -b_{\text{thin}} j_{\text{2D}}, \quad b_{\text{thin}} = \frac{\beta}{d_{\text{film}}^2} = \frac{e^*}{2 m^*} \left( \frac{Z \lambda(0) \lambda(T)}{\varepsilon_0 c^2 d_{\text{film}}} \right)^2 = \frac{m^* Z^2(x)}{2 e^* \rho_{\text{tot}} d_{\text{film}}^2 C_4(T)}, \quad Z(x) = \frac{x}{\tanh x}, \quad x = \frac{d_{\text{film}}}{2 \lambda}.
\]

Here the screening factor \( Z(x < 1) \approx 1 \) describes more precise treatment of the current density \( j \propto \exp(z/\lambda) \) across the thickness of the layer \( |z| < d_{\text{film}}/2 \).

Two dimensional current density \( j_{\text{2D}} = I / 2 \pi r \) is proportional to the total drive current \( I \) between the electrodes (1) and (3). The Bernoulli voltage is just the averaged Bernoulli potential beneath the reference electrode (2)

\[
U = \langle \Delta \varphi \rangle_2 = -B_{\text{thin}} r^2, \quad B_{\text{thin}} = \frac{b_{\text{thin}}}{(2\pi)^2} \left\langle \frac{1}{r^2} \right\rangle / \left\langle \frac{1}{r^2} \right\rangle = \frac{1}{R_2} \frac{1}{r^2} d(\pi r^2) \int_{r_2}^{R_2} d(\pi r^2) = \frac{2 \ln \frac{R_2}{r_2}}{R_2^2 - r_2^2}.
\]
FIG. 2: One dimensional variation of the set-up from Fig. 1. AC current generator in the left creates current $I(t)$ through the superconducting wire – the narrow cylinder between points (1) and (3). The superconducting wire with radius $r$ is surrounded by an insulator (Insulator) and there is a normal metal layer around the insulator (Metal). As a whole we have a coaxial cable with a superconducting wire. The detector circuit measures the AC Bernoulli voltage between probes (2) and (4). Comparison with Fig. 1 shows that formally we have infinite capacities $C_1$, $C_3$ and $C_4$ and there is a common point between the drive and detector circuits. Practically we have a co-axial cable with a superconducting wire and normal metal around it separated by an insulator.

$$V_B = -BI^2/2C_2C_4/L_2,4$$

FIG. 3: An effective circuit for observation of the Bernoulli effect in superconductors. A current generator creates a current $I(t)$, which is input for the two-port network. The output voltage $V_B = -BI^2$ is detected through two capacitors with a total capacitance $C_2C_4/(C_2 + C_4)$. The pre-amplifier and the lock-in voltmeter are shown schematically as a voltmeter.

or finally

$$B_{\text{thin}} = \frac{e^*}{4\pi^2m^*} \ln \frac{R_2}{r_2} \left( \frac{Z\lambda(0)\lambda(T)}{\varepsilon_0c^2d_{\text{film}}} \right)^2 = \frac{m^*}{e^*} \left( \frac{2\pi\rho_{\text{ord}}d_{\text{film}}}{(R_2^2 - r_2^2)} \right)^2 C_s(T).$$

(9)

For thin film $d_{\text{film}} \ll 2\lambda(T)$, $Z \approx 1$. In the first expression it is supposed that we know the experimentally determined temperature dependent penetration depth, while in the second one is supposed that we know the total volume density of the charge carriers and the temperature dependent relative super-fluid concentration $C_s(T)$. Those expressions are simplified at temperatures much lower than the critical one $T \ll T_c$ and very thin layers $d_{\text{film}} \ll \lambda(0)$ when we have $Z = 1 = C_s(0)$. As $|e^*| = 2|e|$ the measurement of the Bernoulli voltage $U_{2,4} = U$, see Fig. 1 determines the Bernoulli coefficient $B_{\text{thin}}$ and the effective Cooper pair mass $m^*$.

In order to clarify the work of the set-up in the next section we will analyze the case of a bulk superconductor.

III. BERNOULLI EFFECT ON THE SURFACE OF A BULK SUPERCONDUCTOR

Now let us describe the work of the set-up when the insulator layer and electrodes are deposited on the atomically clean surface of a bulk superconductor. Imagine a good Bi$_2$Sr$_2$CaCu$_2$O$_8$ crystal cleaved by adhesive tape. In this case the volume density of the current decreases exponentially in the bulk of the superconductor $j(z) \propto \exp(-z/\lambda)$, and the current density on the surface $j_{\text{surf}} = j_{2D}/\lambda$ expressed by the two dimensional current density and the temperature dependent penetration depth. At every two dimensional space point $\rho = (x, y)$ surface current density is proportional to the drive current $I \equiv I_{1,3}$ between electrodes (1) and (3)

$$j(\rho) = I/S_{\text{bulk}}(\rho), \quad S_{\text{bulk}}(\rho) = 2\pi r \lambda.$$  

(10)
For thin $d_{\text{film}} \ll \lambda$ films the geometric factor $S_{\text{thin}} = 2\pi rd_{\text{film}}$ is the area of a thin cylindrical slice. For the bulk sample we have

$$\Delta \varphi = -b_{\text{bulk}} j^2_{2D} = \frac{B^2}{2\mu_0 \rho_{\text{tot}}}, \quad b_{\text{bulk}} \equiv \frac{\beta}{\lambda^2} = \frac{1}{2\epsilon_0 c^2 \rho_{\text{tot}}}, \quad B = \mu_0 j_{2D} \leq B_c(T),$$

(11)

where $B$ is parallel to the surface magnetic field created by the two dimensional current. Averaging again the $1/r$ dependence of the current density under the Bernoulli electrode (2) we obtain in the same manner the Bernoulli constant for the set-up with a bulk sample

$$U = -B_{\text{bulk}} l^2, \quad B_{\text{bulk}} = \left\langle \frac{b(|\mathbf{r}|)}{S^2(|\mathbf{r}|)} \right\rangle = \frac{\ln(R_Z/R_2)}{4\pi^2(R_z^2 - r^2) \varepsilon_0 c^2 \rho_{\text{tot}}},$$

(12)

Let us mention the lack of the temperature dependence in the Bernoulli potential for the bulk sample. One can consider that Bernoulli effect in this case is London Hall effect describing how the magnetic pressure $p_B = B^2/2\mu_0 = \rho_{\text{latt}} \Delta \varphi$ is transmitted to the ion lattice with volume density of charge $\rho_{\text{latt}} = -\rho_{\text{tot}}$. Recalling $\Delta \phi = -\int E_z(z) dz$ we can write the volume density of the force acting on the lattice as

$$f_{\text{latt}} = \rho_{\text{latt}} \mathbf{E} = j \times \mathbf{B} = -\nabla p_B = \nabla \cdot \left( \frac{1}{\mu_0} \left( \mathbf{B} \mathbf{B} - \frac{1}{2} B^2 \mathbf{1} \right) \right),$$

(13)

i.e. the Lorentz force is gradient of the magnetic pressure and divergence of the Maxwell stress tensor. Those 3 vectors are mutually orthogonal $E_z = R_{\text{LH}} B_{\varphi} j_{\varphi}$, where $R_{\text{LH}} = 1/\rho_{\text{tot}}$ is the not yet ever measured constant of the London Hall effect written in SI units. Let us remark the original idea by London: for the surface of a bulk crystal a change of the potential depends on the tangential to the surface magnetic field $\Delta \varphi = R_{\text{LH}} B_{\varphi}^2/2\mu_0$. We would like to add that this Bernoulli potential or also the London Hall effect potential is temperature independent, while in Ref. 25 the temperature dependent density of the superfluid is used in their final Eq. (3.23). The reason for this difference is that in Eq. (1) the kinetic energy is proportional to the superfluid density, while the potential energy includes the total charge density of both the superfluid and normal charge carriers.

In such a way measurement of the Bernoulli voltage $U_{2,3} = U$ leads to direct determination of the total density of charge carriers $\rho_{\text{tot}}$. This important parameter can be substituted in the formula for the Bernoulli coefficient for a thin film Eq. (9) to determine the effective mass of Cooper pairs $m^*$. The comparison gives

$$\frac{B_{\text{thin}}(T)}{B_{\text{thick}}} = \left( \frac{Z(T) \lambda(T)}{d_{\text{film}}} \right)^2,$$

(14)

and we have obtained a new method for determination of penetration depth $\lambda(T)$ in type-II superconductors. In order to obtain the derived formulas in Gaussian units we have to replace $\Phi_0 \rightarrow \Phi_0 / c$ and to substitute $\varepsilon_0 = 1/4\pi$ and $\mu_0 = 4\pi$. In Heaviside-Lorenz system we have to substitute $\varepsilon_0 = 1$, $\mu_0 = 1$ and $c = 1$.

Performing analogous calculations for a superconducting wire with radius $r$, i.e. for the “coaxial cable” set-up shown in Fig. 4, we have for thin $r \ll \lambda$ and thick wire $r \gg \lambda$

$$B_{\text{thin}}^{(\text{wire})} = m^*/2e^* \rho_{\text{tot}}^2 C_s(T)(\pi^2)^2 = R_{\text{LH}} \lambda^2(T)/2c \epsilon_0 c^2 (\pi^2)^2, \quad B_{\text{thin}}^{(\text{wire})}/B_{\text{thick}}^{(\text{wire})} = (2\lambda(T)/r)^2,$$

(15)

The general expression containing Bessel functions can be easily programmed for the experimental data processing.

### IV. Experimental Method

Let apply to electrodes (1) and (3) driving voltage, which is a sum of two sinusoidal, one basic and one modulated with much smaller frequency

$$U_{1,3}(t) = U_a \sin(\Omega t) + U_b \sin(\Omega t) \sin(\omega t) = U_a \sin(\Omega t) + \frac{1}{2} U_b [\cos((\Omega - \omega) t) - \cos((\Omega + \omega) t)], \quad \omega \ll \Omega.$$  

(16)

Then through these electrodes a current

$$I_{1,3}(t) = C_{1,3} d_1 U_{1,3} = I_a \cos(\Omega t) - \frac{1}{2} I_b \sin((\Omega - \omega) t) + \frac{1}{2} I_b \sin((\Omega + \omega) t),$$

(17)

$$I_a \approx \Omega C_{1,3} U_a, \quad I_b \approx \Omega C_{1,3} U_b, \quad C_{1,3} = \frac{C_1 C_3}{C_1 + C_3}.$$  

(18)
flows. This current creates Bernoulli voltage
\[ U_{2,4}(t) = -BI_{1,3}^2 = -\frac{1}{2}BI_aI_b \sin(\omega t) + \ldots, \]
where the high frequency terms with frequencies $2\Omega \pm \omega$ are not written. The usage a low noise pre-amplifier should be followed by a selective resonance amplifier tuned to $f = \omega/2\pi$. In this case if we have monochromatic Bernoulli voltage
\[ U_{2,4}(t) \approx -U_B \sin(\omega t), \quad U_B = \frac{1}{2}BI_aI_b. \tag{19} \]
This AC voltage with amplitude $U_B$ has to be measured as connected in series to a capacitor with capacitance $C_{2,4} \equiv C_2C_4/(C_2 + C_4)$. In short, we need a high-frequency signal with frequency $\Omega$, which is modulated with low frequency $\omega$, and the same low-frequency signal is applied to the lock-in as reference signal.

If the thickness of the insulator $d_{ins}$ is not extremely thin, the corresponding impedance $1/\omega C_{2,4}$ is very high and this creates significant difficulties for the measurement. The Bernoulli signal must be bi-linear with respect to $U_a$ and $U_b$. However the detector circuit can have its own non-linearity. This background can be measured above $T_c$ and is reliably removed by the temperature dependence of the Bernoulli signal. The suggested experiment is actually not difficult. The experimental technique gave the possibility for observation of the Bernoulli signal long time ago; confer the pioneer works by Lewis, Bok and Klein, and Morris and Brown simply the importance of the Bernoulli effect in superconductors was not evaluated at those times. Material science of the superconductors was not on the agenda of the condensed matter physics and the measurement of the Cooper pair mass $m^*$ was not related to the Bernoulli effect.

As we demonstrate precise measurements of the Bernoulli constant for thin film $B_{\text{thin}}(T)$ and bulk sample $B_{\text{bulk}}$ give simultaneously the effective mass of Cooper pairs $m^*$, the total volume charge density of the charge carriers $\rho_{\text{tot}}$, and the temperature dependent penetration depth $\lambda(T)$

\[ \rho_{\text{tot}} = \left\langle \frac{b(r)}{S^2(r)} \right\rangle_2 = \frac{\ln(R_2/r_2)}{4\pi^2(R_2^2 - r_2^2)} \varepsilon_0e^2B_{\text{bulk}}, \quad \lambda(T) = \frac{d_{\text{thin}}}{Z(T)} \sqrt{\frac{B_{\text{thin}}(T)}{B_{\text{thick}}}}, \quad m^* = \frac{e^*\rho_{\text{tot}}\lambda^2(0)}{\varepsilon_0e^2}. \tag{20} \]

Here we have to use some extrapolated to zero temperature value using the smooth temperature function $C_s(T) = \lambda^2(0)/\lambda^2(T)$.

The Bernoulli equation Eq. (1) is applicable for currents much smaller than the maximal current density in Ginsburg-Landau approximation for $|\xi| \ll 1$

\[ j_{\text{max}}(T) = \frac{2}{3\sqrt{3}} \frac{\varepsilon_0e^2h/|e^*|}{\xi(T)^2} \approx \frac{2\sqrt{2}}{3\sqrt{3}} \frac{B_c(T)}{\mu_0\lambda(T)}, \quad B_c(T) \approx \frac{1}{2\sqrt{2\pi}} \frac{\Phi_0}{\xi(T)^2}, \quad B_{\text{cl}}(T) \approx \frac{1}{2\pi} \frac{\Phi_0}{\lambda^2(T)} \ln \varpi, \tag{21} \]
where the numerator in the first expression has dimension velocity times electric charge; in Gaussian system we have to substitute $\mu_0 = 4\pi$ and $B_c(T) = e\mu_0(T)$; in the last expression $\varpi \gg 1$. For $j \ll j_{\text{max}}$ the drift momentum of the Cooper pairs $m^*v = \mu(T)$, This GL result is derived minimizing the volume density of the Gibbs free energy $g = (a(T) + p^2/2m^*)n + bn^2/2$ with respect to the density $n$, where for $T < T_c$ the coefficient $a(T) = -\hbar^2/2m^*\xi^2(T) = a_{\text{cl}}$. The impurity and disorder parameter of the GL coefficients $a_0$ and $b$ was calculated in Ref. [30] the impurity dependence of the Cooper pair mass can be understood even in the framework of Pippard-Landau theory cf. Ref. [19]. If the current density is significant within the GL theory one can easily obtain

\[ \frac{Q}{m^*v(T)/\hbar} \leq 1, \quad p(t) = m^*v(t) = e^* \int_0^t E(T) dt, \quad j = e^*v n(T,v), \quad v(t = 0) = 0, \quad \frac{j}{j_{\text{max}}} = f(Q), \quad f(Q) = \frac{3\sqrt{3}}{2} Q(1 - Q^2) \leq 1, \quad -\frac{\Delta^2}{\beta^2} = g(Q), \quad g(Q) = \frac{1}{1 - Q^2}. \tag{22} \]

Needless to say that in the usual BCS theory the superconducting gap $\Delta(T)$ is just the order parameter which minimizes the density the free energy $g$ in the self-consistent BCS calculations. BCS gap is accessible by tunneling spectroscopy and far-infrared spectroscopy as well. In static GL theory, with static effective wave function $\Psi(r)$ and static vector-potential $A(r)$ only the ratio $|\Psi|^2/m^*c$ can be determined. In order to determine the Cooper pair mass it is necessary to study electrostatic Bernoulli potential or London Hall effect in bulk superconductors. Indeed London analyzed this physical situation 70 years ago and the Bernoulli potential in superconductors has been analyzed in the framework of BCS theory many years ago in general, the methods of the statistical physics allow us to analyze
every situation with many particles. For conventional clean superconductors $m^*$ is just extrapolated to zero frequency optical mass.

The goal of the present work is to point out that Cooper pair mass $m^*$ can be measured in every laboratory involved in the research of superconductivity. In the next section we will give a detailed numerical example which will illustrate the order of the value of the predicted effect.

Here we present only one opportunity to explore electric field effects for revealing fundamental properties of superconductors. We consider that electrostatic excitation of the currents is the best method for thin films and two dimensional superconductors. Another possibility for creation of Cooper pair mass spectroscopy by current induced contact potential difference is to use magnetic field for creation of eddy currents as it is for the standard method for mutual inductance measurements. We believe that this method is better for thick films and bulk crystals. Another obvious possibility is to use permanent magnet driven by piezoelectric crystal oscillations.

V. NUMERICAL EVALUATION OF THE BERNOULLI EFFECT

Quadratic variations of the chemical potential as function of current density was considered by London and Landau. This capacitive impedance can be compensated with an inductance $L$ which gives GL parameterizing of the penetration depth $\lambda(0) = 161 \text{ nm}$ and $\varpi = 62$. For these conditions we have: $|e| = 0.0341$, $\lambda(T) = 873 \text{ nm} \gg d_{\text{thin}}$, $Z \approx 1$, $\xi(0) = 140.8 \text{ nm}$, $C_s = 0.0886$. We calculate also $B_{c1}(T) = 994 \mu T$, $B_s(T) = 59.4 \text{ mT}$, $B_{c2}(T) = 1.660 \text{ T}$, $j_{\text{max}}(T) = 29.49 \text{ GA/m}^2$, $\beta = 1.432 \times 10^{-27} \text{ VA}^{-2} \text{ m}^4$, $B_{\text{thin}}(T) = 399 \text{ pVA}^{-2}$.

Let have a $10 \times 10 \text{ mm}$ sample of subtratum, epitaxial Bi$_2$Sr$_2$CaCu$_2$O$_8$, and thin dielectric layer. On the insulator layer 4 concentric metallic probes are evaporated: a central circle with radius $R_1 = 2.50 \text{ mm}$, and 3 ring electrodes with external radii $R_2 = 3.54 \text{ mm}$, $R_3 = 4.33 \text{ mm}$, and $R_4 = 5.00 \text{ mm}$. The gap between electrodes has width $w = 20 \mu \text{m}$, and the internal radii are $r_2 = R_1 + w$, $r_3 = R_2 + w$ and $r_4 = R_3 + w$; see Fig. 1. So that areas of all plane capacitors are approximately equal and $C_{1.3} = 8.59 \text{ nF}$ and $C_{2.4} = 8.50 \text{ nF}$.

The basic sinusoidal voltage with frequency $\Omega/2\pi = 250 \text{ MHz}$ has amplitude $U_0 = 500 \text{ mV}$ and the modulated with low frequency $\omega/2\pi = 250 \text{ kHz}$ signal has the same amplitude $U_b = 500 \text{ mV}$. For the brake-down electric field of the insulator we take $E_{\text{bd}} = 1 \text{ GV/m}$. The maximal electric field in the insulator layer we estimate as $E_{\text{max}} = (U_a + U_b)/d_{\text{ins}} = 100 \text{ MV/m}$, so that $E_{\text{max}}/E_{\text{bd}} = 0.1$ and maximal charge on the sequential capacitors $C_1$ and $C_3$ is $Q_{1.3} = 8.59 \text{ nC}$.

At high-frequencies $\Omega$ the capacitive impedance between probes (1) and (3) is $Y_{1.3} = 1/(\Omega C_{1.3}) = 74 \text{ m}\Omega$ and the corresponding currents are significant $I_a = U_a/Y_{1.3} = 6.75 \text{ A}$ and $I_b = U_b/Y_{1.3} = 6.75 \text{ A}$ is the same. The maximal two dimensional current density is $j_{2D} = (I_a + I_b)/(2\pi r_2) = 852 \text{ A/m}$, and the bulk one $j = j_{2D}/d_{\text{thin}} = 8.52 \text{ GA/m}^2$; $j/j_{\text{max}}(T) = 0.289$. The super-fluid drift velocity we evaluate at $v = j/e^*n_{\text{oc}}C_s(T) = 204 \text{ m/s}$, which is smaller that de-pairing current at this temperature $v_{\text{dep}}(T) = h/m^*\xi(T) = 822 \text{ m/s}$. The dimensionless momentum is $q = m^*v\xi(T)/h = 0.249$. Let us mention also the extrapolation $v_{\text{dep}}(0) = h/m^*\xi(0) = 4.45 \text{ km/s}$.

One can evaluate now the amplitude of the Bernoulli voltage $U_B = 9.09 \text{ nV}$, which after amplification $A = 10^6$ becomes $U_{\text{ampl}} = AU_B = 9.09 \text{ mV}$. For the calibration of $A = 10^6$ amplifier is convenient to use shot or thermal noise.

The rectified signal is collected on the lock-in capacitors for, say, $\Delta t = 5 \text{ s}$. Evaluating the electric voltage noise of the first operational amplifiers of the pre-amplifier to have spectral density parameter $e_N \times 1.2, \text{nV}/\sqrt{\text{Hz}}$, typical noise voltage can be evaluated as $U_N = e_N\sqrt{\Delta t} = 2.68 \text{ nV}$ we obtain Bernoulli signal to noise ratio $r_{S/N} = 3.39$, which reveals that experiment is doable, and Cooper pair mass in cuprates can be determined using standard electronic equipment.

In the considered conditions the Bernoulli effect operates as a de-modulator and one of the difficulties is that the small nano-Volt signal has to be measured hidden in the capacitive coupling with impedance $Y_{2.4} = 1/(\omega C_{2.4}) = 74.9 \Omega$. This capacitive impedance can be compensated with an inductance $L = 1/\omega^2C_{2.4} = 47.6 \mu \text{H}$. Using general impedance converter this inductance can be tunable. A Python script for calculation of this numerical illustration is given in the appendix.
This example demonstrates also the possibilities for different modification. Indispensable is only $C_2$ capacitor. All other electrodes can be soft gold electrodes directly galvanically touched to the surface of the superconductor. In this case breakdown problem of the insulator layer can be avoided. The central gold circle probe (1) and gold ring probe (3) can be terminals of a coaxial cable. Similar system was used for measurement of penetration depth in thin films\(^{31}\) here we suggest a three-axial version of this device, which can give also the Cooper pair mass.

For Bernoulli effect we have to apply strong current and to measure small electric voltage. One electric field effect in superconductors is in some sense complementary. One can apply strong electric field and to measure small magnetization related to eddy currents. We analyze it in the next section.

VI. ELECTRIC FIELD MODULATION OF THE KINETIC INDUCTANCE, CONDENSATION ENERGY AND THE CONDENSATION ENERGY

The order of the Bernoulli effect in high-$T_c$ cuprates leads to very optimistic evaluation of the difficulties of the sample preparation. However the causes of the several unsuccessful attempts with YBa$_2$Cu$_3$O$_{7-\delta}$ perhaps was the degradation of the surface (CuO$_2$)$_2$ bi-layer by vapor. It is necessary to use carefully prepared interfaces for investigation of other field effects in superconductors. The considered structure can be studied by two coil mutual inductance method, where the mutual inductance is parameterized by the kinetic inductance

$$L(T) = \frac{m^*}{e^2 n_{2D}(T)} = \mu_0 \frac{\lambda^2(T)}{d_{\text{film}}}, \quad n_{2D}(T) = d_{\text{film}} n(T),$$

which is determined by two dimensional density $n_{2D}(T)$ of the superfluid particles. The Bernoulli coefficient $b_{\text{thin}}$ can be expressed by the kinetic inductance $L(T)$

$$\Delta \varphi = -b_{\text{thin}} j_{2D}, \quad b_{\text{thin}}(T) = \frac{L(T)}{2 \rho_{\text{tot}} d_{\text{film}}}, \quad \rho_{\text{tot}} = \frac{L(T)}{2 d_{\text{film}} b_{\text{thin}}(T)},$$

and the ratio gives the temperature independent total charge density of charge carriers $\rho_{\text{tot}}$. For clean crystals this volume density $\rho_{\text{tot}}$ describes the Hall effect in strong magnetic field\(^{19}\).

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![FIG. 4: Set-up for Cooper-pair mass spectroscopy by electrostatic modulation of the kinetic inductance; after Ref. 33](image)

Current through drive coil $I_d$ induces eddy current in the superconductor film which is detected as voltage $U_r$ in detector circuit. Modulating voltage $U_{ms}$ is applied between the superconductor layer and the metal electrode. Additional magnetic moment is created by excess superconducting charge carriers induced by electric induction in this plane capacitor on the superconductor-insulator interface. Several principally different experiments van be performed by small modifications of this set-up.

However the considered structure can be used for study of completely different effect: electric field modulation of the kinetic inductance. At this effect a strong electric field is applied between superconducting an insulator layer which forms a plane capacitor with the surface charge density on the plates $Q = \varepsilon_0 \varepsilon_{\text{ins}} U_{ms} / d_{\text{ins}}$, where $U_{ms}$ is the metal-superconductor voltage and $\varepsilon_{\text{ins}}$ is the relative dielectric constant of the insulator layer. The effective mass of
Cooper pairs can be expressed\(^{21}\) by the ratio of the variations
\[
m^* = -2|e|\text{sgn}(e^*)L(0)L(T)\frac{\delta Q}{\delta L(T)}, \quad C_s(T) = \frac{L(0)}{L(T)}.
\]
Actually this method has been used for first determination of \(m^* \approx 11m_e\) in YBa\(_2\)Cu\(_3\)O\(_{7-\delta}\) (Y:123)\(^{22}\) This experimental set-up is depicted in Fig. 4.

As the quality of the interface is crucial for the suggested experiments the breakthrough perhaps will be triggered by the easily cleaved surface of Bi\(_2\)Sr\(_2\)Ca\(_2\)Cu\(_3\)O\(_8\) (Bi:2212). In short, Bi:2212 crystals good for ARPES (Angular Resolved Photo-Emission Spectroscopy) and Bi:2212 films good for making squids are perfect for the pioneering observation of Bernoulli effect in superconductors.

The Eq. (11) describes the transmission of the magnetic pressure to the lattice. This equation can be used up to the critical thermodynamic field \(B_c(T)\) and this gives a new method for determination of the thermodynamic critical field and condensation energy
\[
\rho_{tot} [\varphi(T) - \varphi(T_c)] = \frac{B_c^2(T)}{2\mu_0}
\]
related to the jump of the heat capacity. The method requires the same capacitive junction to the sample. The superconductor is illuminated by a chopped light in order the surface to become a normal metal. The amplitude of modulation of the contact potential difference is determined by the amplitude the induced charge detected by the electronic circuit. In such a way the density of condensation energy \(B_c^2(T)/2\mu_0\) can be measured by the electric measurements without the use of magnetic field.

VII. DISCUSSION AND CONCLUSION

Perhaps the use of Josephson effect and the formula for the frequency created by a constant voltage \(\omega_j = e^*U/\hbar\) is the most famous electric field effect in superconductors. The electric field could be static but we observe a dynamic effect, roughly speaking because the electric potential has to participate in the gauge invariant equations only by \((i\hbar \partial_t - e^*\varphi)\Psi\). The effective mass of Cooper pairs \(m^*\) is a typical parameter of low frequency dynamics of superconductors, even in the case of static electric field. But in any case the determination of \(m^*\) and \(R_{LH}\) requires electric fields (applied strong electric field or measured small electric voltage). In the framework of purely static Ginzburg-Landau theory with static order parameter \(\psi(\mathbf{r})\) and only space dependent vector-potential \(\mathbf{A}(\mathbf{r})\) the effective mass of Cooper pairs \(m^*\) cannot be determined. This statement is trivial but created significant retention of the development of the physics of superconductivity. Authors of grant proposals and referee reports were afraid that something new will be investigated. From social aspect it is well-known that transparent theoretical ideas slowly receive experimental observation. Flux quantization was predicted by London\(^{23}\) in 1950 but was wad observed 11 years after when \(|e^*| = 2|e|\) was trivialized by the BCS theory. Absolutely analogously already 70 years \(m^*\) and \(R_{LH}\) are still in the agenda of the physics of superconductivity, and without the mass of Cooper pairs \(m^*\) remains Hamlet without the Prince but only with an onnagata in the role of Ofelia.

Electric field effects in superconductors were studied mainly in attempts to get electronic applications, however a lot of fundamental physics can be obtained if we carefully investigate cleaved superconductors or nano-structures with good quality of the interface layer. It is necessary to have no degradation of the superconductivity in the last nanometer layer under the interface. We use understandable notions as the effective mass \(m^*\) and volume charge density \(\rho_{tot}\) just to parameterize the effects. However, the contact potential difference and the change of the superconducting properties by electrostatic charge modulation can be studied experimentally and calculated theoretically directly from the microscopic theory even without the help of the so called phenomenological parameters. Roughly speaking \(m^*\) can be determined by thin films, while for bulk samples one can measure \(\rho_{tot}\) and one can determine penetration depth \(\lambda\) observing only Bernoulli effect in superconductors.

A lot of new physics can be obtained by investigating those well-forgotten effects. The technical applications will come without a significant delay. Let us only note that the Bernoulli effect is quadratic for the frequencies up to the superconducting gap. This quadratic effect can be used for creation of high frequency de-modulators and even TeraHertz lock-in voltimeters.

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\(^{21}\) Electronic address: mishonov@bgphysics.eu
\(^{22}\) Electronic address: varonov@issp.bas.bg
Appendix. The Python script for the numerical example and its results

```python
#!/usr/bin/python
print; print; print; print; print; print
print "---------------------------------------------"

from math import sqrt, atan, log, tanh

# SI notations
mili=10.**(-3)
micro=10.**(-6)
nano=10.**(-9)
anoi=-10.**(3)
Mega=10.**(6)
Giga=10.**(9)

# constants
pi=4.0*atan(1.)
mu0=4.0*pi*10**(-7)
c=299792458
eps0=1.0/(mu0*c**2)
e=1.60217662*10**(-19)
m_e=9.10938356*10**(-31)
m_eff=10.0 # effective mass of Cooper pairs
m_star=m_eff*m_e # mass of Cooper pair
e_star=2*e
hbar=1.055*10**(-34)
Phi0=2.*pi*hbar/e_star

# geometry of the set-up
a=10.*mili
R1=a/4.0
R2=sqrt(2.0)*R1
R3=sqrt(3.0)*R1
R4=sqrt(4.0)*R1
print "R1=",R1," ,R2=",R2," ,R3=",R3," ,R4=",R4
w=20.*micro
r2=R1+w
r3=R2+w
r4=R3+w
S1=pi*R1**2
S2=pi*(R2**2-r2**2)
S3=pi*(R3**2-r3**2)
S4=pi*(R4**2-r4**2)
d_ins=10.*nano # effective thickness 1/epsilon_relative 100 Angstrom insulator film, AlO_x?
print "d_ins=",d_ins

# capacitors
C1=eps0*S1/d_ins
```
C2=\varepsilon_0 S_2/d_{\text{ins}}
C3=\varepsilon_0 S_3/d_{\text{ins}}
C4=\varepsilon_0 S_4/d_{\text{ins}}
C13=C1\times C3/(C1+C3)
C24=C2\times C4/(C2+C4)
print "C13=" ,C13 , " , C24=" ,C24

d=2.6
# material properties parameterized by lengths
lambda0=260.*\text{nano} # penetration depth at zero temperature
\hat{\lambda}_0=\lambda_0/\sqrt{C_d}
x0=2.6*\text{nano} # Ginsburg–Landau extrapolation using $B_c^2$
rho_{\text{tot}}=(\varepsilon_0 c^4 )*(\text{m}_{\text{star}}/\text{e}_{\text{star}})/(\lambda_0^2)
n_{\text{tot}}=\rho_{\text{tot}}/e_{\text{star}}
print "n_{\text{tot}}=" , n_{\text{tot}} , " , \rho_{\text{tot}}=" , \rho_{\text{tot}}

d_{\text{film}}=100. \times \text{nano} # superconducting film
T_c=88.0
Delta_T=3.
print "\Delta_T=" , \Delta_T
t=T_c-\Delta_T
mod_{\text{eps}}=(T_c-T)/T_c # {\text{Fig. Fiory}}
print "\text{mod_{\text{eps}}}" , \text{mod_{\text{eps}}}
C_s=C_d*\text{mod_{\text{eps}}}
print "C_s=" , C_s , " , C_d=" , C_d

\lambda_T=\hat{\lambda}_0/\sqrt{\text{mod_{\text{eps}}}}
\xi_T=\xi_0/\sqrt{\text{mod_{\text{eps}}}}
print "\lambda_T=" , \lambda_T
x=d_{\text{film}}/(2.\times\lambda_T)
print "x=" , x
Z=x/\tanh(x)
print "Z=" , Z
print "\xi_T=" , \xi_T
kappa=\lambda_T/\xi_T
print "Kappa=" , \kappa
B_{c2}=\Phi_0/(2.\times \pi \times \xi_T^2)
B_c=\Phi_0/(2.\times \sqrt{2.})\times \xi_T\times \lambda_T
print "B_{c2}" , B_{c2}
print "B_c=" , B_c
B_{c1}=\Phi_0/(2.\times \text{pi} \times \lambda_T^2)\times \log(10.)
print "B_{c1}" , B_{c1}
j_{\text{max}}=(3.\times \sqrt{3})/(3.\times \sqrt{3})\times B_c/(\mu_0 \lambda_T)
print "j_{\text{max}}=" , j_{\text{max}}
B_{\text{thin}}=(\text{m}_{\text{star}}/\text{e}_{\text{star}})\times \log(R_2/r_2)/((R_2^2-r_2^2)\times C_s\times (2.\times \text{pi} \times d_{\text{film}} \times \rho_{\text{tot}})^2)
beta=(\text{m}_{\text{star}}/\text{e}_{\text{star}})/(2.\times \rho_{\text{tot}}^2\times C_s)
print "!!! \text{beta=" , \beta , \text{ " # according Eq. (9) of version of 4 March; the main result }\text{print "beta=" , \beta \text{ # Eq. (2) of version of 5 March}}\text{Frequency}=250.0 \times \text{Mega}nf=250.0 \times \text{kilo}print "f=" , f , " , \text{Frequency=" , \text{Frequency}}
omega=2.\times \text{pi} \times f
Omega=2.\times \text{pi} \times \text{Frequency}
Y13 = 1./(Omega*C13)  # Imaginary impedance of the drive circuit; small
print "Y13 =", Y13
U13a = .5  # amplitude of the basic high-frequency voltage signal
U13b = .5  # amplitude of the modulated voltage signal
E_max = (U13a+U13b)/d_ins  # electric field across driving sections
E_breakthrough = 1.*Giga
print "E_max =", E_max, "  E_max/E_breakthrough =", E_max/E_breakthrough
Ia = U13a/Y13  # current amplitude of the basic high-frequency signal
Ib = U13b/Y13  # current amplitude of the modulated signal
Q_charge = (U13a+U13b)*C13
print "Ia =", Ia, "  Ib =", Ib, "  Q_charge =", Q_charge
U24 = B_thin*Ia*Ib/2.  # amplitude of the monochromatic Bernoulli voltage
Y24 = 1./(omega*C24)  # impedance of the detector capacitors at low frequency
L24 = 1./(C24*omega**2)  # inductance for resonance detection
print "Y24 =", Y24, "  L24 =", L24, "  omega*L24 =", omega*L24
I24 = U24/Y24  # low frequency demodulated current in detector circuit
Ampl = 10.**6  # amplification of the low noise pre amplifier; EPO construction
U_amp = U24*Ampl  # Bernoulli signal to be measured by loc-in

I24 = U24/Y24
j_2D = (Ia+Ib)/(2*pi*r2)  # maximal 2D current density through the superconducting film
j = j_2D/d_film  # maximal 3D current density through the film
v = j/(e_star*n)  # maximal drift velocity of Cooper pairs
v_dep_T = hbar/(m_star*xi_T)  # maximal depairing velocity at the working temperature
v_max_0 = hbar/(m_star*xi_0)  # maximal depairing velocity at zero temperature; order estimate
Q_momentum = m_star*v*xi_T/hbar  # dimensionless momentum of Cooper pairs in GL units

delta_time = 5.  # lock-in measurements
voltage_noise = 1.2*nano  # 1.2 nano Volts/sqrt(Hz)
U_noise = voltage_noise*sqrt(delta_time)
signal_noise = U24/U_noise
print "U_24 =", U24, "  U_noise =", U_noise, "  signal_noise =", signal_noise

Output:

m_eff = 10.0
R1 = 0.0025  R2 = 0.0035355390593  R3 = 0.00433012701892  R4 = 0.005
d_ins = 1e-08
C13 = 8.59228330638e-09  C24 = 8.50172889227e-09
n_tot = 1.04436343427e+27  rho_tot = 334650935.438
Delta_T = 3.0
mod_eps = 0.0340909090909
C_s = 0.0886363636364  C_d = 2.6
lambda_T = 8.730788767e-07
x = 0.0572535765518
Z = 1.00109241864
xi_T = 1.4081666568e-08
kappa = 62.0173672946
Bc2 = 1.66036649352
Bc = 0.0594737755482
Bc1 = 0.000994015135589
j_max = 29499277152.6
!!! B_thin = 3.99429270961e-10
beta = 1.43193264588e-27
f = 250000.0 , Frequency = 250000000.0
Y13 = 0.0740873807908
E_max = 10000000.0 , E_max/E_breakthrough = 0.1
Ia = 6.74878764323 , Ib = 6.74878764323 , Q_chgarge = 8.59282330638e-09
Y24 = 74.8812130373 , L24 = 4.76708607984e-05 , omega*L24 = 74.8812130373
I24 = 1.21475458408e-10
j_2D = 852.462629601
j/j_max = 0.288977463818
j = 8524626296.01
v = 287.389816813
v_dep_T = 822.449827915
v_max_0 = 4454.40932525
Q_momentum = 0.349431426767
!!! U24 = 9.09622967983e-09 , U_noise = 2.683281573e-09 , signal/noise = 3.38996465051
U_amp = 0.00909622967983