Similar constructive method for solving the model of coalbed methane flow through coalbed methane reservoirs

To cite this article: Bao Xi-Tao et al 2018 J. Geophys. Eng. 15 1783

View the article online for updates and enhancements.
Similar constructive method for solving the model of coalbed methane flow through coalbed methane reservoirs

Bao Xi-Tao$^{1,2}$, Yan Yi-Fei$^{2,4}$, Yang Jiang$^1$, Yan Xiang-Zhen$^2$ and Li Shun-Chu$^3$

$^1$ College of Petroleum Engineering, China University of Petroleum, Qingdao 266580, People’s Republic of China
$^2$ Oil and Gas CAE Technology research Center, China University of Petroleum, Qingdao 266580, People’s Republic of China
$^3$ Institute of Applied Mathematics, Xihua University, Chengdu 610039, People’s Republic of China

E-mail: yanyf163@163.com

Received 6 April 2017, revised 10 February 2018
Accepted for publication 13 March 2018
Published 29 May 2018

Abstract
Coalbed methane (CBM) is produced through desorption, diffusion, and seepage processes. However, pressure during CBM production, especially the seepage process, is a very complicated problem. Methods for solving the pressure models for CBM reservoirs are multidisciplinary and rather sophisticated. In this paper, an elementary and simple method for solving the CBM seepage model is proposed. First, a classical seepage model was built to study CBM seepage at a variable flow rate. After the transformation of pressure into pseudo-pressure and dimensionless treatments, a definite problem for the CBM seepage model was obtained. By Laplace transformation, the definite problem can be solved as a boundary value problem in Laplace space. Then, this boundary value problem was solved by the proposed method which is elementary and simple and its solutions have a structure similar to that of a continued fraction. This explains why this method is called the similar constructive method. With this method, solutions for the model of the pressure response of a CBM reservoir can be obtained. A corresponding program for the method was written, which can help engineers working on CBM reservoirs solve similar problems even without pre-training. In addition to the typical analysis curves for well testing such as the conventional log–log graph of the pressure and pressure differential and semi-log graph, another type of analysis curve, i.e., a log–log graph of the first-order pressure derivative was plotted. On such basis, the diagnosis of the pressure response of a CBM well is more accurate. Our results might be of great significance to the theoretical study of pressure responses of CBM reservoirs. The results of this research could provide great simplicity for software developers to create well testing analysis software and well testing interpreters to interpret CBM well test data.

Keywords: CBM reservoir, seepage, similar constructive method, continued fraction, well testing analysis curve

(Some figures may appear in colour only in the online journal)
Natural gas is a clean energy source and demand for it has been growing rapidly in recent years (Jón and Shapour 2012, Li et al. 2015). Coalbed methane (CBM) is a type of unconventional natural gas with advantages including shallower burial depth, low-cost development, and cleanliness. Along with increasing demand for CBM, exploitation of CBM fields has already progressed across the world. However, corresponding problems occur. A coalbed reservoir, different from a conventional reservoir, is a dual-porosity coal seam composed of matrix with minute pores and a unique cleat system. Cleats are well-developed natural fractures with high adsorption capacity, causing an obstacle to CBM’s desorption and occurrence (Gentzis et al. 2007, Connell et al. 2010). Moreover, the fragility and high stress sensitivity of the coalbed reservoir result in the seepage of CBM in the gas flow channel (Kong 2010). Thus, research on the complex desorption-diffusion and seepage of CBM is urgently needed, and is also a difficult goal.

A dynamic model for CBM reservoirs was firstly proposed as an equilibrium adsorption model with the following assumptions (Pavone and Schwerr 1986, Bumb and McKee 1988, Reeves and Pekot 2001): (1) The coal reservoir is treated as a single-porosity medium. (2) There is a continuous equilibrium between gas adsorbed on the porous wall of a coal reservoir and free gas inside the pores. (3) Adsorbed gas escapes from the porous wall of a coal reservoir and flows into the pore when the pressure inside the pores decreases. However, this equilibrium adsorption model does not consider the process of CBM desorption from the porous wall, so it cannot reflect the real desorption time, nor can it report the characteristic parameters of accumulation and migration. Thus, CBM production predicted by this model is higher than the real production.

For the deep and extensive study of the occurrence and migration mechanisms of CBM, a non-equilibrium sorption model, as a new dynamic model, was proposed (Kolesar et al. 1990, Kong 2010). The underlying assumptions are as follows. A coalbed reservoir is considered a dual-porosity reservoir with a fracture–matrix system. There are well-developed micro pores in coal matrix. CBM is mostly adsorbed on the inner surface of the coal matrix. The fissure medium for the seepage in a coalbed reservoir is composed of face cleats and butt cleats. Hence, the process of CBM desorption and diffusion from micro pores to cracks is considered in this model. Thus, this model can better describe the occurrence and migration process of CBM.

Although the CBM seepage model has been significantly improved, the computation is still daunting. It has been proposed that the model can be described by differential equations (Li 2009, Dong et al. 2013, Bai et al. 2014). The solutions of the model have a structure similar to that of continued fractions. Hence, the theory of a similar structure of a solution was proposed for differential equations and has been applied to solve engineering problems. The effectiveness of this theory has been verified by its successful application in solving problems of oil flowing in various reservoirs including homogeneous reservoirs (Li et al. 2006, Wang et al. 2013), dual-porosity reservoirs (Xu et al. 2006, Bao et al. 2013, Li et al. 2013), fractal reservoirs (Sheng et al. 2013, Xu et al. 2013), etc.

However, the theory of a similar structure of a solution is a kind of one-fold theory of the expression of solution. Moreover, the similar structure of solutions for the CBM seepage model has never been demonstrated. Therefore, this study aims to propose an elementary and simple method to solve the model of CBM seepage at a changing production rate. This particular method, called the similar constructive method, can give accurate solutions whose expressions have a structure similar that of a unified continued fraction.

The method works as follows: (1) Guiding functions are constructed with a group of linearly independent solutions of the basic equation of the mathematical model for CBM seepage. (2) Similar kernel functions are built with coefficients of a homogeneous boundary condition of the mathematical model and guiding functions constructed in step 1. (3) Solutions for the mathematical model are built with coefficients of its inhomogeneous boundary
condition and similar kernel functions built in step 2. (4) A numerical inverse method is applied to the solutions obtained in step 3 to obtain the solutions of the CBM seepage model.

On above basis, the solutions of the CBM seepage model were obtained and they can be expressed as a unified continued fraction. Evidently, the similar constructive method is an elementary and simple method and allows us to solve such a model without daunting computation. Besides, this method is easy to use and can be easily applied to engineering problems.

**Theoretical calculations**

**Model of CBM seepage at a variable production rate**

Fluids undergo desorption, diffusion, and seepage processes in a CBM reservoir. In order to study the seepage process in a CBM reservoir, a practical model with some idealized and simplified assumptions was derived.

In this paper, as the schematic diagram of the CBM seepage model shown in figure 1, the flowing fluid in the CBM reservoir was assumed as a single-phase methane gas. The pressure gradient was the driving force. Compared with seepage flux, the diffusion flux was too low to be considered for the single-phase methane flowing in coal cleats. Thus, CBM flow in coal cleats was assumed as the seepage which satisfies Darcy’s law and only radial flow existed in coal cleats. Because the viscosity, compression factor, and deviation factor of gas are functions of pressure, the equation of gas seepage is nonlinear. Thus, the pressure data can be transformed to pseudo-pressure (Al-Hussainy et al 1966) for the single-phase CBM flow and then the equation of gas seepage becomes linear. Studies have shown that the pressure response during the production of CBM, especially the seepage process, is a very complicated problem. Moreover, for a real CBM reservoir, the lower and upper boundaries can be assumed to be closed. Desorption can be described by the Langmuir isothermal adsorption model and adsorption isotherm was satisfied in the initial state of the system. The diffusion process that methane flows into coal cleats from the surface of the coal matrix and primary pores can be described by Fick’s law. There are two types of diffusion for the methane in a CBM reservoir: pseudo-steady-state diffusion and unsteady-state diffusion. They can be described by Fick’s first law and Fick’s second law, respectively. As a common pressure build-up test, it is assumed that the well has been produced with a constant rate for a long time. But it is quite difficult for the production well to hold a constant rate for a long time, especially for gas wells. Hence, in this paper, we assume that the CBM well has been produced with a multi-rate \( q_m(t) \) for a long time. Then the CBM seepage model can be given as fundamental differential equations which include the equations of coal cleats

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = \frac{\phi p_c \partial p_c}{K \frac{\partial t}{\partial t}} + \frac{p_{sc} T}{K T_c} \frac{\partial V}{\partial t} \tag{1}
\]

and coal matrix with a pseudo-steady state

\[
\frac{\partial V}{\partial t} = \frac{D}{r^2} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) \tag{2}
\]

or unsteady state

\[
\frac{\partial V}{\partial t} = \frac{6D \pi^2}{R^2} (V_{sc} - V). \tag{3}
\]

Conditions include initial condition

\[
p(r, 0) = p_i, \tag{4}
\]

internal boundary condition

\[
p_w = \left[ p - Sr \frac{\partial p}{\partial r} \right]_{r=r_e} \tag{5}
\]

and three types of outer boundary conditions including the infinite outer boundary condition

\[
p(r \to \infty, t) = 0, \tag{7}
\]

circular outer boundary with constant pressure

\[
p(r = R, t) = 0, \tag{8}
\]

and closed circular outer boundary condition

\[
\left. \frac{\partial p^2}{\partial r} \right|_{r=R} = 0. \tag{9}
\]
Pseudo-pressure was introduced into the fundamental differential equations given above and then they could be dealt with a dimensionless method (Jones et al. 1989, Thompson et al. 1993, Yadavalli and Jones 1996).

\[ m = \mu_i Z_i \int \frac{P}{\mu Z} \, dp, \quad m_D = \frac{542.867 K(h(m_i - m))}. \]

\[ r_D = \frac{R}{r_w}, \quad R_D = \frac{R}{r_w}, \quad r_D = \frac{1.013 \times 25Kt}{10^{-4} \Delta r_w^i}. \]

\[ \Lambda = 0.101 \times 325 \phi \mu_c + 55.006 K_i h TZ_D p_w. \]

\[ q_{inD} = \frac{q_m(t)}{q_w} \cdot C_D = \frac{6.283 \times 19 C_i h r_w^2}{2}, \quad \alpha = 1.81798 \times 10^4 \cdot \frac{q_w T_w p_i}{K_i h (m_i + m)(m_i + m_w)}. \]

Further, the dimensionless model can be turned into a boundary value problem for an ordinary differential equation in Laplace space by Laplace transformation (Doetsch 2007). The details are as follows.

The basic equation is

\[ \frac{d^2 \varphi_0}{d r^2} + \frac{1 + \mu_s}{r_D} \frac{d \varphi_0}{d r_D} = f(z) \varphi_{inD}(z). \]

(10)

The internal boundary condition is

\[ C_D \varphi_{inD} = (r_D + C_D z \varphi) \frac{d \varphi_{inD}}{d r_D} \bigg|_{r_D = 1} = \varphi_{inD}(z). \]

(11)

The infinite outer boundary condition is

\[ \varphi_{inD}(r_D \to \infty, z) = 0. \]

(12)

The circular outer boundary condition with constant pressure is

\[ \varphi_{inD}(r_D = R_D, z) = 0. \]

(13)

And the closed circular outer boundary condition is

\[ \frac{d \varphi_{inD}}{d r_D} \bigg|_{r_D = R_D} = 0, \]

(14)

where \( C_D \) is the dimensionless well storage constant, \( S \) is the skin factor, and \( z \) is the Laplace variable.

For the pseudo-steady-state model and unsteady-state model, respectively, the expression \( f(z) \) can be defined as

\[ f(z) = \frac{\omega + \alpha(1 - \omega)}{z^\lambda + 1}. \]

(15)

And

\[ f(z) = \omega z + 1 - \frac{\omega}{\lambda} \alpha [\sqrt{\lambda z} \text{cosh} (\sqrt{\lambda z}) - 1], \]

(16)

where \( \omega \) is storability given by

\[ \omega = 0.103 \times 25 \phi \mu_c, \]

\[ \lambda \] is the inter-porosity flow coefficient defined as

\[ \lambda = 1.013 \times 25 K. \]

And \( \alpha \) is the sticking factor defined as

\[ \alpha = 1.81798 \times 10^4 \frac{q_w T_w p_i}{K_i h (m_i + m)(m_i + m_w)}. \]

(19)

Finally, the general solution of the mathematical model can be derived as follows:

\[ \bar{m}_D = A K_0(\sqrt{f(z)} r_D) + B I_0(\sqrt{f(z)} r_D), \]

(20)

where \( A \) and \( B \) are constants.

Solving the model of CBM seepage by the similar constructive method

Using the similar constructive method proposed in appendix A, the CBM seepage model can be solved as a boundary value problem for an ordinary differential equation in Laplace space. Then, the solution whose expression has a structure similar to that of a unified continued fraction was obtained.

The basic equation (10) is a modified Bessel equation of zeroth order. Its two linearly independent solutions are \( \varphi_{inD}(r_D, z) = I_0(\sqrt{f(z)} r_D) \) and \( \varphi_{inD}(r_D, z) = K_0(\sqrt{f(z)} r_D) \). Its general solution is equation (20).

First, guiding functions were constructed with the two linearly independent solutions of the modified Bessel equation:

\[ \varphi_{\alpha,0}(r_D, l) = \begin{bmatrix} I_0(\sqrt{f(z)} r_D) & K_0(\sqrt{f(z)} r_D) \\ I_0(\sqrt{f(z)} l) & K_0(\sqrt{f(z)} l) \end{bmatrix} \]

\[ = \Psi_{\alpha,0}(l, r_D, \sqrt{f(z)}), \]

(21)

\[ \varphi_{\alpha,1}(r_D, l) = \frac{\partial}{\partial l} \varphi_{\alpha,0}(r_D, l) = -\sqrt{f(z)} \times \Psi_{\alpha,0}(l, r_D, \sqrt{f(z)}), \]

(22)

\[ \varphi_{\alpha,1}(r_D, l) = \frac{\partial}{\partial r_D} \varphi_{\alpha,0}(r_D, l) = \sqrt{f(z)} \times \Psi_{\alpha,0}(l, r_D, \sqrt{f(z)}), \]

(23)

\[ \varphi_{\alpha,1}(r_D, l) = \frac{\partial}{\partial r_D} \varphi_{\alpha,1}(r_D, l) = \frac{\partial}{\partial l} \varphi_{\alpha,1}(r_D, l) \]

\[ = -\frac{\omega}{C_D \mu_c} \Psi_{\alpha,1}(l, r_D, \sqrt{f(z)}). \]

(24)

Second, similar kernel functions were built with coefficients of three homogeneous right boundary conditions and guiding functions constructed in step 1.

For the infinite outer boundary, the similar kernel function can be derived as

\[ \Phi_{l}(r_D, z) = \lim_{R_D \to \infty} \frac{\varphi_{\alpha,0}(r_D, R_D)}{\varphi_{\alpha,1}(l, R_D)} \]

\[ = \frac{K_0(\sqrt{f(z)} r_D)}{\sqrt{f(z)} K_0(\sqrt{f(z)} l)}. \]

(25)
For the circular outer boundary with constant pressure, the similar kernel function can be derived as
\[
\Phi_2(r_D, z) = \frac{\varphi_{0,0}(r_D, R_D)}{\varphi_{1,0}(1, R_D)} = \frac{\Psi_{0,0}(r_D, R_D, \sqrt{f(z)})}{\sqrt{f(z)} \Psi_{1,0}(1, R_D, \sqrt{f(z)})}.
\]  
(26)

For the closed circular outer boundary, the similar kernel function can be derived as
\[
\Phi_3(r_D, z) = \frac{\varphi_{0,1}(r_D, R_D)}{\varphi_{1,1}(1, R_D)} = \frac{\Psi_{0,1}(r_D, R_D, v)}{\sqrt{f(z)} \Psi_{1,1}(1, R_D, \sqrt{f(z)})}.
\]  
(27)

Then, the similar kernel function can be expressed as follows:
\[
\Phi(r_D, z),
\begin{cases} 
\Phi_2(r_D, z), & \text{infinite outer boundary}, \\
\Phi_1(r_D, z), & \text{circular outer boundary with constant pressure}, \\
\Phi_3(r_D, z), & \text{circular and closed outer boundary}.
\end{cases}
\]  
(28)

Third, the solution of the boundary value problem was constructed with coefficients of a nonhomogeneous internal boundary condition including \(C_Dz, 1 + C_DzS\) and \(\overline{q}_{in}(z)\), and the similar kernel function derived from step 2:
\[
\overline{p}(r_D, z) = -\overline{q}_{in}(z) \cdot \frac{1}{C_Dz + \frac{1}{1 + C_DzS - \Phi(1, z)}} \cdot \frac{1}{1 + C_DzS - \Phi(1, z)} \cdot \Phi(r_D, z),
\]  
(29)

where \(r_D\) was set to 1 and the solution of the distribution of the dimensionless bottom-hole pressure in the Laplace space can be obtained as follows:
\[
\overline{p}(r_D, z) = \overline{q}_{in}(z) \cdot \frac{1}{C_Dz + \frac{1}{1 + C_DzS - \Phi(1, z)}} \cdot \frac{1}{1 + C_DzS - \Phi(1, z)}.
\]  
(30)

Equation (30) has a structure similar to that of a continued fraction. Consequently, the mathematical model of CBM seepage was solved using the similar constructive method and the solutions have a structure similar to that of a continued fraction.

Fourth, Laplace inversion (Stehfest 1970) was performed and a program was written for the similar constructive method. When \(\overline{q}_{in}(z) = 1/z\), we could obtain three types of analysis curves for CBM well testing at a constant production rate (discussed in next section). Figure 2 shows a flowchart for solving the CBM seepage model.

**Results and discussion**

By using the above program, we obtained three types of analysis curves for well testing: log–log graph of the pressure and pressure differential, semi-log graph, and log–log graph of the first-order derivative. In addition, the parameters, which are important for solving the model of CBM seepage with the pseudo-steady state, were analyzed.
Three types of boundary conditions \( (C_D = 2, S = 1, \alpha = 8, \omega = 0.1, \lambda = 5e4; R_D = 800 \) for the infinite outer boundary condition, the circular outer boundary with constant pressure and closed case) \)

Figure 3(a) demonstrates that the process of CBM flowing in a coal seam can be divided into six main stages:

Stage 1 (denoted by \( \odot \)): The log values of pressure and pressure derivative increase linearly with the log values of \( t_D/C_D \), indicated by straight line segments with a slope of 1.

Stage 2 (denoted by \( \odot \)): Transition flow occurs in stage 2, which is indicated by the hump in the curve.

Stage 3 (denoted by \( \odot \)): This stage can be called the early radial-flow stage and it reflects the characteristics of early radial flow of CBM near the wellbore.

Stage 4 (denoted by \( \odot \)): This stage can be called cross-flow stage, which is indicated by the dent in the curve. In this stage, CBM adsorbed into the matrix of coal seam escapes and flows out.

Stage 5 (denoted by \( \odot \)): As indicated by the pressure derivative curve, the radial flow characteristics occur for the second time. This stage is called the late radial-flow stage.

Stage 6 (denoted by \( \odot \)): Different pressure responses of the CBM well under three boundary conditions can be observed in this stage. Under the infinite outer boundary condition, the log value of the pressure derivative remains constant as the log value of \( t_D/C_D \) increases. Under the circular outer boundary with constant pressure, the log value of the pressure derivative decreases dramatically as the log value of \( t_D/C_D \) increases. For the closed outer boundary, the log value of the pressure derivative increases dramatically as the log value of \( t_D/C_D \) increases.

According to the well testing data, we can plot a log–log graph of the conventional pressure and the pressure derivative, a semi-log graph of the pressure response, and a log–log graph of the first-order pressure derivative. Compared with the raw data, the first-order of the data has more sensitivity. Therefore, the CBM seepage model features can be more obvious in the log–log graph of the first-order pressure derivative. Then, combining figures 3(b) and (c), we can more accurately evaluate the pressure response of the CBM well.

The effects of the parameters on the well testing curves are analyzed in the following sections.

Effects of a dimensionless wellbore storage factor \( (C_D = [1;2;4;8], S = 1, \alpha = 8, \omega = 0.1, \lambda = 5e4; \) the infinite outer boundary condition) \)

In figure 4(a), with an increase in the dimensionless wellbore storage factor \( (C_D) \), the length of the straight line segment in the curve increases. In figure 4(b), with an increase in \( C_D \), the starting point of the first increasing stage of the pressure derivative moves toward the right and the increasing speed increases. In figure 4(c), there is an initial stationary stage, indicated by the straight line segment with a slope of 0 and the length of this line segment increases with an increase in \( C_D \).

Therefore, the dimensionless wellbore storage factor determines the duration of the wellbore storage effect.

Effects of the skin factor \( (C_D = 2, S = [0;1;2;4], \alpha = 8, \omega = 0.1, \lambda = 5e4; \) the infinite outer boundary condition) \)

In figure 5(a), with the increase in skin factor \( (S) \), the first peak value of the pressure derivative increases. In figure 5(b), following the initial stationary stage, there is a rapidly increasing stage of the pressure derivative and the increasing speed increases with the increase in \( S \). In figure 5(c), there is a short decreasing stage following the initial stationary stage (the straight line segment with a slope of 0). Note that the short decreasing stages under different \( S \) values have the same starting point and ending point. However, the log value of the pressure derivative in this stage increases with an increase in the \( S \) value.

Effects of the sticking factor \( (C_D = 2, S = 1, \alpha = [4;8;16;32], \omega = 0.1, \lambda = 5e4; \) the infinite outer boundary condition) \)

In figure 6(a), with an increase in the sticking factor \( (\alpha) \), the first dent in the pressure derivative curve appears at a smaller \( log(t_D/C_D) \) value, the minimum pressure derivative decreases, and the range of \( log(t_D/C_D) \) within which the CBM is in the early radial-flow stage is shortened. In figure 6(b), there is a slowly increasing stage following the first rapidly increasing stage and the degree of slow-down increases with an increase in \( \alpha \). Following the slowly increasing stage, there is a linearly increasing stage, indicated by the straight line segment in the curve. When the \( log(t_D/C_D) \) value remains unchanged, the pressure derivative increases with an increase in \( \alpha \). In figure 6(c), the degree of the concave in the curve along with the decreasing stage increases with an increase in \( \alpha \).

Effects of the storage ratio \( (C_D = 2, S = 1, \alpha = 8, \omega = [0;1;0.2;0.4;0.8], \lambda = 5e4; \) the infinite outer boundary condition) \)

In figure 7(a), with an increase in the storage ratio \( (\omega) \), the first dent in the pressure derivative curve appears at larger \( log(t_D/C_D) \) values, the minimum pressure derivative decreases, and the range of \( log(t_D/C_D) \) within which CBM is in the early radial-flow stage is extended. In figure 7(b), with an increase in \( \omega \), the pressure decreases and then increases when the \( log(t_D/C_D) \) value remains unchanged. In figure 7(c), the degree of the concave in the curve along with the decreasing period decreases with an increase in \( \omega \).

Effects of the inter-porosity flow coefficient \( (C_D = 2, S = 1, \alpha = 8, \omega = 0.1, \lambda = [5e2;5e3;5e4;5e5]; \) the infinite outer boundary condition) \)

In figure 8(a), with an increase in the inter-porosity flow coefficient \( (\lambda) \), the first dent in the pressure derivative curve appears at larger \( log(t_D/C_D) \) values, the minimum pressure derivative decreases, and the range of \( log(t_D/C_D) \) within which CBM is in early radial-flow stage is extended. In figure 8(b), the first increasing stage lasts longer and the slowly increasing stage following the first increasing stage lasts shorter with an increase in \( \lambda \). In figure 8(c), the dent in the pressure derivative curve appears at larger \( log(t_D/C_D) \) values and the degree of the concave is enhanced with an increase in \( \lambda \).
Figure 3. (a) Log-log graph of the conventional pressure and pressure derivative of a CBM well under three outer boundary conditions. (b) Semi-log graph of the pressure response of a CBM well under three outer boundary conditions. (c) Log-log graph of the first-order pressure derivative of the CBM well under three outer boundary conditions.
Figure 4. (a) Log–log graph of the conventional pressure and pressure derivative of the CBM well as affected by the wellbore storage factor ($C_D$). (b) Semi-log graph of the pressure response of the CBM well as affected by $C_D$. (c) Log–log graph of the first-order pressure derivative of the CBM well as affected by $C_D$. 
Figure 5. (a) Log–log graph of the conventional pressure and pressure derivative of the CBM well as affected by the skin factor ($S$). (b) Semi-log graph of the pressure response of the CBM well as affected by $S$. (c) Log–log graph of the first-order pressure derivative of the CBM well as affected by $S$. 

1791
Figure 6. (a) Log–log graph of the conventional pressure and pressure derivative of the CBM well as affected by the sticking factor ($\alpha$). (b) Semi-log graph of the pressure response of the CBM well as affected by $\alpha$. (c) Log–log graph of the first-order pressure derivative of the CBM well as affected by $\alpha$. 
Figure 7. (a) Log–log graph of the conventional pressure and pressure derivative of the CBM well as affected by the storage ratio ($\omega$). (b) Semi-log graph of the pressure response of the CBM well as affected by $\omega$. (c) Log–log graph of the first-order pressure derivative of the CBM well as affected by $\omega$. 
Figure 8. (a) Log–log graph of the conventional pressure and pressure derivative of the CBM well as affected by the inter-porosity flow coefficient ($\lambda$). (b) Semi-log graph of the pressure response of the CBM well as affected by $\lambda$. (c) Log–log graph of the first-order pressure derivative of the CBM well as affected by $\lambda$. 
Figure 9. (a) Log–log graph of the conventional pressure and pressure derivative of the CBM well as affected by the radius of the closed circular outer boundary ($R_D$). (b) Semi-log graph of the pressure response of the CBM well as affected by $R_D$. (c) Log–log graph of the first-order pressure derivative of the CBM well as affected by $R_D$. 
Effects of the outer boundary radius ($C_D = 2$, $S = 1$, $\alpha = 8$, $\omega = 0.1$, $\lambda = 564$ and $R_D = \{400;600;800;1000\}$; the closed circular outer boundary condition)

In figure 9(a), with an increase in the radius of closed boundary ($R_D$), the range of $\log(t_{15}/C_D)$ within which CBM is in the late radial-flow stage is extended and the boundary effect appears at larger $\log(t_{15}/C_D)$ values. In figure 9(b), the boundary effect also appears at larger $\log(t_{15}/C_D)$ values with an increase in $R_D$. In figure 9(c), the same case happens.

Conclusions

A novel seepage model was developed to study CBM seepage at a variable flow rate. This model can be solved as a boundary value problem for a Bessel equation. In addition, the solutions for this model have a structure similar to that of a continued fraction. On such basis, we proposed a method, called the similar constructive method, to solve the CBM seepage model and a corresponding flowchart was given.

According to the similar constructive method, the solutions can be built with coefficients of the left (internal) boundary condition and a similar kernel function. The similar kernel function can be built with coefficients of the right (outer) boundary condition and guiding functions. Guiding functions can be built with two linear solutions of the basic zero-order Bessel equation. We also wrote a calculation program and optimized it to solve such a mathematical model. Finally, the solutions for the model were obtained and their expressions have a structure similar to that of a continued fraction. Therefore, a complex boundary value problem can be solved with the similar constructive method, which is elementary and simple.

In addition to traditional analysis curves for well testing such as a log–log graph of the conventional pressure and pressure differential and a semi-log graph, another type of analysis curve, i.e., a log–log graph of the first-order pressure derivative, was plotted. On the basis of these three analysis curves, we can more accurately evaluate the pressure response of a CBM well. Furthermore, we analyzed the characteristic parameters affecting the pressure response of a CBM well, and the results might be useful for adjusting parameters during the process of well testing.

In sum, this study provides not only new insights into the seepage theory but also a new method solving such seepage model. This method allows for a simplified calculation process and shorter computation time. Also, by employing the three types of analysis curves for well testing, engineers can easily analyze the testing data of a CBM well. Thus, this study has great significance in promoting the development of a seepage theory and well testing technology.

Acknowledgments

This research was supported by National Natural Science Foundation of China (51374228), Major Project of the National Science and Technology of China (Grant No. 2016ZX05017-003-01HZ2), and the Key Laboratory Project of China National Petroleum Corporation (Grant No. 2016A-3905), Postdoctoral Science Foundation of China (No.2017M612375).

Appendix

Principle of the similar constructive method

The mathematical model of CBM seepage can be solved as a boundary value problem for a modified Bessel equation of zeroth order whose boundary conditions are more common.

A boundary value problem for a modified zero-order Bessel equation was proposed as follows:

$$
\begin{bmatrix}
y'' + \frac{1}{x}y' = M y \\
[ay - (1 + ab)]y'|_{x = \alpha} = e \\
[ey + fy']|_{x = \beta} = 0
\end{bmatrix}
$$

where $A, a, b, c, e$, and $f$ are real numbers. Conditions including $M > 0$, $c = 0$, $e^2 + f^2 = 0$, and $0 < \alpha < \beta$ were satisfied. If the boundary value problem had a unique solution, the solution would be the following expression which has a structure similar to that of a continued fraction:

$$
y(x) = -c \cdot \frac{1}{a + \frac{1}{b - \phi(x)}} \cdot \frac{1}{b - \phi(x)} \cdot \phi(x),
$$

where $\phi(x)$ is defined as a function similar to a kernel function

$$
\phi(x) = \frac{e\varphi_{0,0}(x, \beta) + f\varphi_{0,1}(x, \beta)}{e\varphi_{1,0}(\alpha, \beta) + f\varphi_{1,1}(\alpha, \beta)}
$$

and $\varphi_{ij}(x, \gamma)(i, j = 0, 1)$ is defined as a guiding function

$$
\varphi_{0,0}(x, \beta) = \Psi_{0,0}(\beta, x, \sqrt{M}),
$$

$$
\varphi_{0,1}(x, \beta) = \frac{\partial}{\partial \beta} \varphi_{0,0}(x, \beta) = -\sqrt{M} \Psi_{1,0}(x, \beta, \sqrt{M}),
$$

$$
\varphi_{1,0}(\alpha, \beta) = \sqrt{M} \Psi_{0,0}(\beta, \alpha, \sqrt{M}),
$$

$$
\varphi_{1,1}(\alpha, \beta) = -M \Psi_{1,1}(\beta, \alpha, \sqrt{M}),
$$

where

$$
\Psi_{m,n}(x, y, z) = K_m(xz)I_n(yz) + (-1)^{m-n+1} \times I_m(xz)K_n(yz).
$$

Actually, the basic equation of the boundary value problem is a kind of modified Bessel equation of zeroth order and its two linearly independent solutions are $y_1(x) = \phi_0(\sqrt{M}x)$ and $y_2(x) = \phi_0(\sqrt{M}x)$. The general solution of the basic equation can be expressed as follows:

$$
y(x) = A\phi_0(\sqrt{M}x) + BK_0(\sqrt{M}x).
$$
Guiding functions can be constructed by the two linearly independent solutions as follows:

\[
\varphi_{0,0}(x_1, x_2) = \begin{vmatrix} I_0(\sqrt{M}x_1) & K_0(\sqrt{M}x_1) \\ I_0(\sqrt{M}x_2) & K_0(\sqrt{M}x_2) \end{vmatrix}
= \Psi_{0,0}(x_2, x_1, \sqrt{M}). \tag{A10}
\]

\[
y(x) = -c \cdot \frac{e^\varphi_{0,0}(x, \beta) + f^\varphi_{0,0}(x, \beta)}{-ae^\varphi_{0,0}(\alpha, \beta) + af^\varphi_{0,0}(\alpha, \beta) + (1 + ab)e^\varphi_{1,0}(\alpha, \beta) + (1 + ab)f^\varphi_{1,1}(\alpha, \beta)}. \tag{A19}
\]

By finding the partial and total derivatives of \(\varphi_{0,0}(x_1, x_2)\) and using equation (A8), the other three guiding functions can be obtained:

\[
\varphi_{0,1}(x_1, x_2) = \frac{\partial}{\partial x_2} \varphi_{0,0}(x_1, x_2) = -\sqrt{M} \Psi_{1,0}
\times (x_2, x_1, \sqrt{M}), \tag{A11}
\]

\[
\varphi_{1,0}(x_1, x_2) = \frac{\partial}{\partial x_1} \varphi_{0,0}(x_1, x_2) = \sqrt{M} \Psi_{0,1}
\times (x_2, x_1, \sqrt{M}), \tag{A12}
\]

\[
\varphi_{1,1}(x_1, x_2) = \frac{\partial}{\partial x_1} \varphi_{1,0}(x_1, x_2) = \frac{\partial}{\partial x_2} \varphi_{1,0}(x_1, x_2)
= -M \Psi_{1,0}(x_2, x_1, \sqrt{M}). \tag{A13}
\]

Substituting equation (27) into the boundary value problem under the left boundary condition yields

\[
[aI_0(\sqrt{M} \alpha) - (1 + ab)\sqrt{M} I_1(\sqrt{M} \alpha)]d_1 
+ [aK_0(\sqrt{M} \alpha) + (1 + ab)\sqrt{M} K_1(\sqrt{M} \alpha)]
\times d_2 = c. \tag{A14}
\]

Substituting equation (27) into the boundary value problem under the right boundary condition yields

\[
[eI_0(\sqrt{M} \alpha) + f\sqrt{M} I_1(\sqrt{M} \alpha)]d_1 
+ [eK_0(\sqrt{M} \alpha) - f\sqrt{M} K_1(\sqrt{M} \alpha)]d_2 = 0. \tag{A15}
\]

The determinant of the coefficients in equations (A14) and (A15) can be obtained using equations (A10)–(A13):

\[
\Delta = \begin{vmatrix} aI_0(\sqrt{M} \alpha) - (1 + ab)\sqrt{M} I_1(\sqrt{M} \alpha) & aK_0(\sqrt{M} \alpha) + (1 + ab)\sqrt{M} K_1(\sqrt{M} \alpha) \\ eI_0(\sqrt{M} \alpha) + f\sqrt{M} I_1(\sqrt{M} \alpha) & eK_0(\sqrt{M} \alpha) - f\sqrt{M} K_1(\sqrt{M} \alpha) \end{vmatrix}
= -\sqrt{M}(e + abe - af)[I_1(\sqrt{M} \alpha)K_0(\sqrt{M} \alpha) + I_0(\sqrt{M} \alpha)K_1(\sqrt{M} \alpha)]
- ae^\varphi_{0,0}(\alpha, \beta) - af^\varphi_{0,0}(\alpha, \beta) + (1 + ab)e^\varphi_{1,0}(\alpha, \beta) + (1 + ab)f^\varphi_{1,1}(\alpha, \beta). \tag{A16}
\]

Since the boundary problem has a unique solution (\(\Delta \neq 0\)), according to Cramer’s rule, we can obtain the values of parameters \(A\) and \(B\).

\[
A = \frac{c}{\Delta} [eK_0(\sqrt{M} \alpha) - f\sqrt{M} K_1(\sqrt{M} \alpha)], \tag{A17}
\]

\[
B = -\frac{c}{\Delta} \cdot [eI_0(\sqrt{M} \alpha) + f\sqrt{M} I_1(\sqrt{M} \alpha)]. \tag{A18}
\]

Substituting equations (A16)–(A18) into equation (A9) and making use of equations (A10)–(A13), the solution of the mathematical model can be deduced as follows:

Equation (A19) has a structure similar to that of equation (A2) and contains a similar kernel function.

As shown above, we can obtain the similar kernel function and the solution whose expression has a structure similar to that of equation (A2).

The expression of the solution of a boundary value problem for a modified zero-order Bessel equation has a structure similar to that of a unified continued fraction in product form. The form of the structure was built with the coefficients of the nonhomogeneous left boundary condition and a similar kernel function. The similar kernel function was built with the coefficients of the homogeneous right boundary condition and guiding function. The guiding function was built with two linearly independent solutions of the basic equation of the boundary problem. These are the fundamental principles of the similar constructive method. This method can be used to solve such boundary value problem of a modified Bessel equation of zeroth order.

The general steps for applying the similar constructive method to solve such a boundary value problem are given as follows:

First, guiding functions \((\varphi_{ij}(x_1, x_2), (i, j = 0, 1))\) can be constructed with two linearly independent solutions of the basic equation of the boundary problem (equation (A1)).

Second, the similar kernel function can be built with the coefficients \((e, f)\) of the homogeneous right boundary condition and guiding functions constructed in the first step:

\[
\phi(x) = \frac{e^\varphi_{0,0}(x, \beta) + f^\varphi_{0,0}(x, \beta)}{e^\varphi_{1,0}(x, \beta) + f^\varphi_{1,1}(x, \beta)}. \tag{A20}
\]
Third, the solution of such a mathematical model can be constructed with the coefficients of the inhomogeneous left boundary condition and the similar kernel function built in the second step.

References

Al-Hussainy R, Ramey H J and Crawford P B 1966 The flow of real gases though media J. Pet. Technol. 18 624–36
Bai L X et al 2014 Similar structure algorithm for solving boundary value problem of differential equations Appl. Mech. Mater. 574 665–71
Bao X T, Li S C and Gui D D 2013 Similar constructive method for solving the nonlinear spherical percolation model in dual-porosity media Mater. Eng. Adv. Technol., Adv. Mater. Res. 631 265–71
Bumb A C and McKee C R 1988 Gas-well testing in the presence of desorption for coalbed methane and Devonian shale SPE Form. Eval. 3 179–85
Connell L D, Lu M and Pan Z 2010 An analytical coal permeability model for tri-axial strain and stress conditions Int. J. Coal Geol. 84 103–14
Doetsch G 2007 Introduction to the theory and application of the Laplace transformation IEEE Trans. Syst. Man Cybern. 7 631–2
Dong X X et al 2013 Similar constructing method for solving the boundary value problem of the composite first Weber system Am. J. Appl. Math. Stat. 1 76–82
Genitz T, Deisman N and Chalaturnyk R J 2007 Geomechanical properties and permeability of coals from the foothills and mountain regions of western Canada Int. J. Coal Geol. 69 153–64
Jon S and Shapour V 2012 Unconventional gas resources in the USA AIP Conf. Proc. 1453 301–6
Jones J R, Vo D T and Raghavan R 1989 Interpretation of pressure build-up responses in gas condensate wells SPE Form. Eval. 4 93–101
Kolesar J E, Ertekin T and Obut S T 1990 The unsteady-state nature of sorption and diffusion phenomena in the micropore structure of coal: 1.—Theory and mathematical formulation SPE Form. Eval. 5 81–8
Kong X Y 2010 Advanced Mechanics of Fluid Flow in Porous Media (Hefei, China: Press of University of Science and Technology of China, An’hui) pp 503–16 (in Chinese)
Li Q et al 2015 Composition, origin, and distribution of coalbed methane in the HuaiBei coalfield, China Energy Fuels 29 546–55
Li S C 2009 The similar structure of solutions to the boundary value problem for second-order linear homogeneous differential equation J. Xihua Univ. (Nat. Sci. Edn.) 40 1–90
Li S C, Zheng P S and Zhang Y F 2006 The similar structure of pressure distribution in the homogeneous reservoir Pure Appl. Math. 22 459–63 (in Chinese)
Li W P et al 2013 The similar structure of solution in the spherical shaped matrix of dual-porosity reservoir seepage model Appl. Mech. Mater. 339 737–41
Pavone A M and Schwerr F C 1986 Development of coal gas production simulators mathematical models for well test strategies Final Report under GRI Contract No. 5081-321-0457 (https://osti.gov/biblio/6038504)
Reeves S and Pekot L 2001 Advanced reservoir modeling in desorption-controlled reservoirs SPE Rocky Mountain Petroleum Technology Conf. (Keystone, CO, USA) (Society of Petroleum Engineers) SPE 71090-MS
Sheng C C et al 2013 Similar construction method of solution for solving the mathematical model of fractal reservoir with spherical flow J. Appl. Math. 2013 211–44
Stehfest H 1970 Numerical inversion of Laplace transforms [D5] Commun. ACM 13 47–9
Thompson L G, Niu J G and Reynolds A C 1993 Well testing for gas condensate reservoirs SPE Asia Pacific Oil and Gas Conf. (Singapore, 8-10 February) (Society of Petroleum Engineers) SPE-62920-MS
Wang Y, Bao X T and Li S C 2013 Similar constructive method of solution for percolation model through homogeneous reservoir considering the quadratic pressure gradient J. Convergence Inf. Technol. 8 794–801
Xu C X et al 2006 Similar structure of pressure distribution in the dual-porosity reservoir Drilling Prod. Technol. 29 28–30 (in Chinese)
Xu L et al 2013 The similar structure method for solving the model of fractal dual-porosity reservoir Math. Problems Eng. 2013 1–9
Yadavalli S K and Jones J R 1996 Interpretation of pressure transient data from hydraulically fractured gas condensate SPE Annual Technical Conf. and Exhibition (Denver, CO, USA) (Society of Petroleum Engineers) SPE-36556-MS