Is There a Stringy Description of Self-Dual Supergravity in 2+2 Dimensions?

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ABSTRACT

The well known N=2 string theory describes self-dual gravity, as was shown by Ooguri and Vafa sometime ago. In search of a variant of this theory which would describe self-dual supergravity in 2+2 dimensions, we have constructed two new N=2 strings theories in which the target space is a superspace. Both theories contain massless scalar and spinor fields in their spectrum, and one of them has spacetime supersymmetry. However, we find that the interactions of these fields do not correspond to those of self-dual supergravity. In our construction, we have used the basic (2,2) superspace variables, and considered quadratic constraints in these variables. A more general construction may be needed for a stringy description of self-dual supergravity.

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1. Introduction

Various supergravity theories are known to arise as low energy effective field theory limits of an underlying superstring or super $p$-brane theory. For example, all supergravity theories in $D = 10$ and $D = 11$ are associated with certain superstring or super $p$-brane theories. Supergravity theories can also serve as worldvolume field theories for a suitable super $p$-brane theory, the most celebrated example of this being the spinning string theory.

Of course, not all supergravity theories have been associated so far with superstrings or super $p$-branes. An outstanding example is the self-dual supergravity in 2+2 dimensions [1, 2, 3]. There are a number of reasons why this is a rather important example. For one thing, the dimensional reduction to $1+1$ dimensions can give rise to a large class integrable models. Secondly, it can teach us a great deal about quantum gravity. Furthermore, and perhaps more interestingly, a suitable version of self-dual supergravity in 2+2 dimensions may in principle serve as the worldvolume theory of an extended object propagating in $10+2$ dimensions, as has been suggested recently by Vafa [4]. Further tantalizing hints at the relevance of a worldvolume theory in 2 + 2 dimensions have been put forward recently [5].

Since the well known $N = 2$ string theory has the critical dimension of four, it is natural to examine this theory, or its variants, in search of a stringy description of self-dual supergravity. It turns out that this theory actually describes self-dual gravity in 2+2 dimensions, as was shown by Ooguri and Vafa [6] sometime ago. Interestingly enough, and contrary to what one would naively expect, the fermionic partner of the graviton does not arise in the spectrum, and therefore self-dual supergravity does not emerge [6]. This intriguing result led us to look for a variant of the $N = 2$ string theory where spacetime supersymmetry is kept manifest from the outset, thereby providing a natural framework for finding a stringy description of self-dual supergravity. We have constructed two such variants [7, 8], in which (a) we use the basic variables of the 2 + 2 superspace, and (b) we consider constraints that are quadratic in these variables. Surprisingly enough, we find that neither one of the two models describe the self-dual supergravity, suggesting that we probably need to introduce extra worldsheet variables and/or consider higher order constraints. Nonetheless, we believe that our results may be of interest in their own right, and with that in mind, we shall briefly describe them in this note.

Both of the models mentioned above can be constructed by making use of bilinear combinations of the bosonic coordinates $X^{\alpha \dot{\alpha}}$, fermionic coordinates $\theta^\alpha$, and their conjugate momenta $p_\alpha$, to build the currents of the underlying worldsheet algebras. The indices $\alpha$ and $\dot{\alpha}$ label the two dimensional spinor representations of $SL(2)_R \times SL(2)_L \approx SO(2,2)$. In terms of these variables, it is useful to recall the currents of the small $N = 4$ superconformal algebra, namely

\[
T = -\frac{1}{2} \partial X^{\alpha \dot{\alpha}} \partial X_{\alpha \dot{\alpha}} - p_\alpha \partial \theta^\alpha, \\
G^{\dot{\alpha}} = p_\alpha \partial X^{\alpha \dot{\alpha}}, \quad \tilde{G}^{\dot{\alpha}} = \theta_\alpha \partial X^\alpha_{\dot{\alpha}}, \quad (1)
\]
This is the twisted version of the usual realization, since here the \((p, \theta)\) system has dimension \((1, 0)\). An \(N = 2\) truncation of this algebra is given by \[ T = -\frac{1}{2} \partial X^{\dot{a} \dot{a}} \partial X_{\dot{a} \dot{a}} - p_\alpha \partial \theta^\alpha, \quad G^{\dot{a}} = p_\alpha \partial X^{\alpha \dot{a}}, \quad J = p_\alpha p^\alpha. \] (2)

Naively, this system appears to be non-critical. However, the currents are reducible, and a proper quantization requires the identification of the irreducible subsets. Assuming that

(a) the worldsheet field content is \((p_\alpha, \theta, X^{\alpha \dot{a}})\),

(b) the constraints are quadratic in worldsheet fields,

(c) the constraints are irreducible,

we have found that, there exists three possible \(N = 2\) string theories. One of them is the old model shown by Ooguri and Vafa \cite{6} to describe pure self-dual gravity. We will refer to this model as the “\(n = 0\) model”. The other two models were studied in refs. \cite{7, 8}. One of them, which we will refer to as the “\(n = 1\) model”, has spacetime \(N = 1\) supersymmetry, and the other one, which we will refer to as the “new \(n = 0\) model”, has no spacetime supersymmetry. In what follows, we shall give a very brief description of these models.

2. The \(N = 2\) String Models

2.1. The \(n = 0\) Model

This is the usual \(N = 2\) string which has worldsheet \(N = 2\) supersymmetry, but lacks spacetime supersymmetry. The underlying \(N = 2\) superconformal algebra, in the twisted basis described above, is given by

\[ T = -\frac{1}{2} \partial X^{\dot{a} \dot{a}} \partial X_{\dot{a} \dot{a}} - p_\alpha \partial \theta^\alpha, \quad J = p_\alpha \theta^\alpha, \]
\[ G^1 = \theta_\alpha \partial X^{\alpha 1}, \quad G^2 = p_\alpha \partial X^{\alpha 2}. \] (3)

The striking feature of this model is that the only continuous degree of freedom it describes is that of the self-dual graviton \cite{3}. This model has been studied extensively in the literature. See, for example, refs. \cite{9, 10}, where a BRST analysis of the spectrum is given, and various twists and GSO projections leading to massless bosonic and fermionic vertex operators are considered. We now turn our attention to the remaining two models, which we have constructed in \cite{7, 8}.

2.2. The \(n=1\) Model
This model can covariantly be described by the set of currents given in eq. (2). Notice that all currents have spin two, and that the system is critical. Nonetheless, this set of constraints is reducible. All the relations among the constraints can be described in a concise form by introducing a pair of spin-0 fermionic coordinates $\zeta^\alpha$ on the worldsheet. We can then define

$$\mathcal{P}^\alpha = p^\alpha + \zeta_\dot{\alpha} \partial X^{\alpha\dot{\alpha}} + \zeta_{\dot{\alpha}} \partial \theta^\alpha,$$

in terms of which the currents may be written as $\mathcal{T} = \mathcal{P}_\alpha \mathcal{P}^\alpha$, where

$$\mathcal{T} = J + \zeta_\dot{\alpha} G^{\dot{\alpha}} + \zeta_{\dot{\alpha}} \zeta^{\dot{\alpha}} T.$$

The reducibility relations among the constraints can now be written in the concise form

$$\mathcal{P}_\alpha \mathcal{T} = 0.$$

In fact, the system has infinite order reducibility. This can be easily seen from the form $\mathcal{P}_\alpha \mathcal{T} = 0$ for the reducibility relations, owing to the fact that the functions $\mathcal{P}_\alpha$ are themselves reducible, since $\mathcal{P}_\alpha \mathcal{P}^\alpha$ gives back the constraints $\mathcal{T}$. This infinite order of reducibility implies that a proper BRST treatment requires an infinite number of ghosts for ghosts $\mathcal{P}_\alpha$. The construction of the covariant BRST operator is rather cumbersome problem. Some den progress is made on this problem in [7], however, thanks to the fact that the covariant system is critical.

To have an insight into the physical spectrum of the theory, and its basic interactions, it is sufficient to consider the independent subset of constraints, at the expense of sacrificing manifest target space supersymmetry. For example, we can choose the following set of independent constraints

$$T = -\frac{1}{2} \partial X^{\alpha\dot{\alpha}} \partial X_{\alpha\dot{\alpha}} - p_\alpha \partial \theta^\alpha, \quad G^{1} = -p_\alpha \partial X^{\alpha 1},$$

which in fact generate a subalgebra of the twisted $N = 2$ superconformal algebra. Using eq.(3), we can write the remaining constraints, i.e. the dependent ones, as linear functions of the independent constraints.

The BRST operator for the reducible system $(T, G^{1})$ can be easily constructed. We introduce the anticommuting ghosts $(b, c)$ and the commuting ghosts $(r, s)$ for $T$

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1 In [11], Siegel proposed to build a string theory implementing the set of constraints given by $\{ \partial X^{\alpha\dot{\alpha}} \partial X_{\alpha\dot{\alpha}}, p_\alpha \partial \theta^\alpha, p_\alpha p^\alpha, \partial \theta_\alpha \partial \theta^\alpha, p_\alpha \partial X^{\alpha\dot{\alpha}}, \partial \theta_\alpha \partial X^{\alpha\dot{\alpha}} \}$. However, we have checked that the algebra of these constraints does not close [7]. Actually, this non-closure occurs even at the classical level of Poisson brackets, or single OPE contractions [6].

2 Note that, this situation is very much similar to the case of systems with $\kappa$-symmetry.

3 Although the massless states can be shown to be annihilated by the dependent constraints as well, it turns out that there are massive operators with standard ghost structure which do not seem to be annihilated by them [6]. Establishing the equivalence of the massive spectra of the reducible and the irreducible systems would require the analysis of the full cohomology and interactions, including the physical states with non-standard ghost structure.
and $G^1$ respectively. The commuting ghosts $(r, s)$ are bosonized, i.e. $r = \partial \xi e^{-\phi}$, $s = \eta e^\phi$. In terms of these fields, the BRST operator $Q$ is given by

$$Q = c \left( -\frac{1}{2} \partial X^{\alpha \dot{\alpha}} \partial X_{\alpha \dot{\alpha}} - p_\alpha \partial \theta^\alpha - b \partial c - \frac{1}{2} (\partial \phi)^2 - \frac{3}{2} \partial^2 \phi - \eta \partial \xi \right) + \eta e^\phi p_\alpha \partial X^\alpha e^\phi. \quad (8)$$

The theory has spacetime supersymmetry, generated by

$$q^\alpha = \oint p^\alpha, \quad q^1 = \oint \theta_\alpha \partial X^\alpha, \quad q^2 = \oint \theta_\alpha \partial X^\alpha + b \eta e^\phi. \quad (9)$$

The somewhat unusual ghost terms in $q^2$ are necessary for the generator to anti-commute with the BRST operator. It is straightforward to verify that these supercharges generate the usual $N = 1$ spacetime superalgebra

$$\{q_\alpha, q_\beta\} = 0 = \{q^\dot{\alpha}, q^\dot{\beta}\}, \quad \{q^\alpha, q^\dot{\alpha}\} = P^{\alpha \dot{\alpha}}, \quad (10)$$

where $P^{\alpha \dot{\alpha}} = \oint \partial X^{\alpha \dot{\alpha}}$ is the spacetime translation operator.

Since the zero mode of $\xi$ is not included in the Hilbert space of physical states, there exists a BRST non-trivial picture-changing operator $Z = \{Q, \xi\}$ which can give new BRST non-trivial physical operators when normal ordered with others. Explicitly, it takes the form

$$Z = c \partial \xi + p_\alpha \partial X^\alpha e^\phi. \quad (11)$$

Unlike the picture-changing operator in the usual $N = 1$ NSR superstring, this operator has no inverse.

Let us now consider the physical spectrum with standard ghost structure. There are two massless operators

$$V = c e^{-\phi} e^{ip \cdot X}, \quad \Psi = h_\alpha c e^{-\phi} \theta^\alpha e^{ip \cdot X}, \quad (12)$$

which are physical provided with mass-shell condition $p^{\alpha \dot{\alpha}} p_{\alpha \dot{\alpha}} = 0$ and spinor polarization condition $p^\alpha h_\alpha = 0$. The non-triviality of these operators can be established by the fact that the conjugates of these operators with respect to the following non-vanishing inner product

$$\langle \partial^2 c \partial c c e^{-3\phi} \theta^2 \rangle \quad (13)$$

are also annihilated by the BRST operator. The bosonic operator $V$ and the fermionic operator $\Psi$ form a supermultiplet under the $N = 1$ spacetime supersymmetric transformation. The associated spacetime fields $\phi$ and $\psi_\alpha$ transform as

$$\delta \phi = \epsilon_\alpha \psi^\alpha \quad \delta \psi_\alpha = \epsilon^{\dot{\alpha}} \partial_{\alpha \dot{\alpha}} \phi. \quad (14)$$
We can build only one three-point amplitude among the massless operators, namely
\[ \langle V(z_1) \Psi(z_2) \Psi(z_3) \rangle = c_{23} \]  
(15)
where \( b_{ij} \) is defined by
\[ b_{ij} = h_{(i)}^\alpha h_{(j)}^\alpha. \]  
(16)

From this, we can deduce that the \( V \) operator describes a spacetime scalar whilst the \( \Psi \) operator describes a spacetime chiral spin-\( \frac{1}{2} \) fermion. Note that this is quite different from the case of the \( N = 2 \) string where there is only a massless boson and although it is ostensibly a scalar, it is in fact, as emerges from the study of the three-point amplitudes, a prepotential for self-dual Yang-Mills or gravity.

With the one insertion of the picture-changing operator, we can build a four-point function which vanishes for kinematic reasons [7]:
\[ \langle ZV \Psi \int b \Psi \Psi \rangle = (u b_{12} b_{34} + s b_{13} b_{24}) \frac{\Gamma(-\frac{1}{2} s) \Gamma(-\frac{1}{2} t)}{\Gamma(\frac{1}{2} u)}, \]  
(17)
where \( s, t, \) and \( u \) are the Mandelstam variables and \( h_{(1)}^\alpha = p_{(1)}^\alpha. \) The vanishing of the kinematic term, \( i.e. \ u b_{12} b_{34} + s b_{13} b_{24} = 0, \) is a straightforward consequence of the mass-shell condition of the operators and momentum conservation of the four-point amplitude [9]. It might seem that the vanishing of the this four-point amplitude should be automatically implied by the statistics of the operators since there is an odd number of fermions. However, as shown in [7], the picture-changing operator has spacetime fermionic statistics. In fact, that the four-point amplitude eq.(17) vanishes only on-shell, for kinematic reasons, already implies that the picture changer \( Z \) is a fermion. Thus the picture changing of a physical operator changes its spacetime statistics and hence does not establish the equivalence between the two. On the other hand, since \( Z^2 = (ZZ) \) becomes a spacetime bosonic operator, we can use \( Z^2 \) to identify the physical states with different pictures.

Thus, we have a total of four massless operators, namely \( V, ZV \) and their supersymmetric partners. \( V \) and its superpartner \( \Psi \) have standard ghost structure; \( ZV \) and its superpartner \( Z \Psi \) have non-standard ghost structures.

So far we have discussed the massless physical operators. There are also infinitely many massive states. The tachyonic type massive operators, \( i.e. \) those that become pure exponentials after bosonizing the fermionic fields, are relatively easy to obtain, and they have been discussed at length in [7]. An example of such massive operators is as follows
\[ V_n = c(\partial^\alpha p)^2 \cdots p^2 e^{n \phi} e^{ip \cdot X}, \quad M^2 = (n + 1)(n + 2), \]  
(18)
where \( p^2 = p_\alpha p^\alpha. \) These operators correspond to physical states, provided the mass-shell condition is satisfied. Furthermore, they all have non-standard ghost structures. From these operators, we can build non-vanishing four-point amplitudes, which implies the existence of further massive operators in the physical spectrum.
In summary, we emphasize that the model has \( n = 1 \) supersymmetry in the critical 2 + 2 dimensional spacetime. It describes two massless scalar supermultiplets, in addition to an infinite tower of massive states. Examining their interactions, however, we find that they do not correspond to those of self-dual supergravity.

2.3. The New \( n = 0 \) Model

This model is described by the following set of currents \([8]\)

\[
\begin{align*}
T &= -\frac{1}{2} \partial X^{\alpha \dot{\alpha}} \partial X_{\alpha \dot{\alpha}} - p_\alpha \partial \theta^\alpha, \\
J &= \partial (\theta_\alpha \theta^\alpha), \\
G^i &= p_\alpha \partial X^{\alpha i}, \\
\tilde{G}^i &= \theta_\alpha \partial X^{\alpha i}.
\end{align*}
\] (19)

It is easy to see that the currents \((T, G^i, \tilde{G}^i, J)\) have spins \((2, 2, 1, 1)\). In addition to the standard OPEs of \(T\) with \((T, J, G^i, \tilde{G}^i)\), the only non-vanishing OPE is

\[
J(z) G^i(w) \sim \frac{2 \tilde{G}^i}{(z - w)^2} + \frac{\partial \tilde{G}^i}{(z - w)}.
\] (20)

This algebra is related to the small \( \mathcal{N} = 4 \) superconformal algebra, not directly as a subalgebra, but in the following way. The subset of currents \(T, G^i, \tilde{G}^i\) and \(J\) in (19) form a critical closed algebra. However these currents form a reducible set. To achieve irreducibility, we simply differentiate the current \(J\), thereby obtaining the set of currents given in (19). Note that taking the derivative of \(J\) still gives a primary current with the same anomaly contribution, since \(12s^2 - 12s + 2\) takes the same value for \(s = 0\) and \(s = 1\).

To proceed with the BRST quantisation of the model, we introduce the fermionic ghost fields \((c, b)\) and \((\gamma, \beta)\) for the currents \(T\) and \(J\), and the bosonic ghost fields \((s, r)\) and \((\bar{s}, \bar{r})\) for \(G^i\) and \(\tilde{G}^i\). It is necessary to bosonize the commuting ghosts, by writing \(s = \eta e^\phi\), \(r = \partial \xi e^{-\phi}\), \(\bar{s} = \bar{\eta} e^{\bar{\phi}}\) and \(\bar{r} = \partial \bar{\xi} e^{-\bar{\phi}}\). The BRST operator for the model is then given by \([8]\)

\[
Q = \oint c \left( -\frac{1}{2} \partial X_{\alpha \dot{\alpha}} \partial X^{\alpha \dot{\alpha}} - p_\alpha \partial \theta^\alpha - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} (\partial \bar{\phi})^2 - \frac{3}{2} \partial^2 \phi - \frac{1}{2} \partial^2 \bar{\phi} \right)
\]

\[
- \eta \partial \xi - \bar{\eta} \partial \bar{\xi} - b \partial c - \beta \partial \gamma
\]

\[
+ \eta e^\phi p_\alpha \partial X^{\alpha i} + \bar{\eta} e^{\bar{\phi}} \theta_\alpha \partial X^{\alpha i} + \partial \gamma \left( \frac{1}{2} \theta_\alpha \theta^\alpha - \bar{\xi} \eta e^{\phi - \bar{\phi}} \right).
\] (21)

Since the zero modes of \(\xi\) and \(\bar{\xi}\) do not appear in the BRST operator, there exist BRST non-trivial picture-changing operators \([8]\):

\[
Z_\xi = \{Q, \xi\} = c \partial \xi + e^\phi p_\alpha \partial X^{\alpha i} - \partial \gamma \partial \bar{\xi} e^{\phi - \bar{\phi}},
\]

\[
Z_{\bar{\xi}} = \{Q, \bar{\xi}\} = c \partial \bar{\xi} + e^{\bar{\phi}} \theta_\alpha \partial X^{\alpha i}.
\] (22)
It turns out that these two picture changers are not invertible. Thus, one has the option of including the zero modes of $\xi$ and $\tilde{\xi}$ in the Hilbert space of physical states. This would not be true for a case where the picture changers were invertible. Under these circumstances, the inclusion of the zero modes would mean that all physical states would become trivial, since $|\text{phys}\rangle = Q(\xi Z^{-1}_\xi |\text{phys}\rangle)$. In [8], we chose to exclude the zero modes of $\xi$ and $\tilde{\xi}$ from the Hilbert space. It is interesting to note that in this model the zero mode of the ghost field $\gamma$ for the spin–1 current is also absent in the BRST operator. If one excludes this zero mode from the Hilbert space, one can then introduce the corresponding picture-changing operator $Z_{\tilde{\gamma}} = \{Q, \gamma\} = c\partial \gamma$. In [8], we indeed chose to exclude the zero mode of $\gamma$.

In order to discuss the cohomology of the BRST operator eq.(21), it is convenient first to define an inner product in the Hilbert space. Since the zero modes of the $\xi$, $\tilde{\xi}$ and $\gamma$ are excluded, the inner product is given by

$$\langle \partial^2 c \partial c \theta^\alpha \theta^\alpha e^{-3\phi-\tilde{\phi}} \rangle = 1.$$ (23)

Let us first discuss the spectrum of massless states in the Neveu-Schwarz sector. The simplest such state is given by [8]

$$V = c e^{-\phi-\tilde{\phi}} e^{ip \cdot X}.$$ (24)

As in the case of the $N = 2$ string discussed in [4], since the picture-changing operators are not invertible the massless states in different pictures cannot necessarily all be identified. In fact, the picture changers annihilate the massless operators such as (24) when the momentum $p^\alpha$ is zero. However, massless operators in other pictures still exist at momentum $p^{\alpha 1} = 0$. For example, in the same picture as the physical operator $Z_{\tilde{\xi}} V$ that vanishes at $p^{\alpha 1} = 0$ is a physical operator that is non-vanishing for all on-shell momenta, namely [8]

$$\Psi = h^\alpha c \theta^\alpha e^{-\phi} e^{ip \cdot X},$$ (25)

which is physical provided that $p^{\alpha 1} h_\alpha = 0$ and $p_{\alpha\dot{\alpha}} p^{\alpha 2} = 0$. In fact, $Z_{\tilde{\xi}} V$ is nothing but $\Psi$ with the polarisation condition solved by writing $h^\alpha = p^{\alpha 1}$. However, we can choose instead to solve the polarisation condition by writing $h^\alpha = p^{\alpha 2}$, which is non-vanishing even when $p^{\alpha 1} = 0$. Thus, the operators $V$ and $\Psi$ cannot be identified under picture changing when $p^{\alpha 1} = 0$. In fact when $p^{\alpha 1} = 0$ there is another independent solution for $\Psi$, since the polarisation condition becomes empty in this case. A convenient way to describe the physical states is in terms of $\Psi$ given in eq.(25), with the polarisation condition re-written in the covariant form $p^{\alpha \dot{\alpha}} h_\alpha = 0$, together with a further physical operator which is defined only when $p^{\alpha 1} = 0$. In this description, the physical operator $\Psi$ is defined for all on-shell momenta.

If one adopts the traditional viewpoint that physical operators related by picture changers describe the same physical degree of freedom, one would then interpret the
spectrum as containing a massless operator eq.(24), together with an infinite number of massless operators that are subject to the further constraint $p^\alpha \bar{1} = 0$ on the on-shell momentum $\bar{1}$. This viewpoint is not altogether satisfactory in a case such as ours, where the picture changing operators are not invertible. An alternative, and moreover covariant, viewpoint is that the physical operators in different pictures, such as $V$ and $\Psi$, should be viewed as independent. At first sight one might think that this description leads to an infinite number of massless operators. However, as shown in [8], the interactions of all the physical operators can be effectively described by the interaction of just the two operators $V$ and $\Psi$.

Thus the theory effectively reduces to one with just two massless operators, one a scalar and the other a spinorial bosonic operator.

As for the massive states, an infinite tower of them exist, and they have been discussed in [8]. They all have positive mass, and non-standard ghost structure $5$. A typical such tower is given by [8]

$$ V_n = c (\partial^n p)^2 \ldots p^2 e^{n\phi-(n+2)\phi} \partial^{2n+2} \gamma \ldots \partial \gamma e^{ip\cdot X}, \quad (26) $$

where $n > -1$ and the mass is given by $M^2 = (2n + 2)(2n + 3)$. For subtleties concerning the exclusion of the zero-mode of the $\gamma$ field in the Hilbert space of physical states, and the nature of the picture-changing operators in massless versus massive sector of the theory, we refer the reader to [8].

As for the interactions, there is one three-point interaction between the massless operators, namely [8]

$$ \langle \Psi(z_1, p_{1(1)}) \Psi(z_2, p_{2(2)}) V(z_3, p_{3(3)}) \rangle = h_{1(1)} h_{2(2)} \alpha. \quad (27) $$

Note that this three-point amplitude is manifestly Lorentz invariant. There are also an infinite number of massless physical operators with different pictures in the spectrum, and they can all be expressed in a covariant way. As one steps through the picture numbers, the character of the physical operators alternates between scalar and spinorial. The three-point interactions of all these operators lead only to the one amplitude given by eq.(27). In view of their equivalent interactions, all the scalar operators can be identified and all the spinorial operators can be identified.

The massless spectrum can thus be effectively described by the scalar operator eq.(24) and the spinorial operator eq.(25). All four-point and higher amplitudes vanish.

Although the theory contains an infinite tower of physical operators, the massless sector and its interactions are remarkably simple. In particular, although all the

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4It should be emphasized that the possibility of having $p^\alpha \bar{1} = 0$ while $p^\alpha \bar{2} \neq 0$ is a consequence of our having chosen a real structure on the $(2, 2)$ spacetime $[7, 9]$, rather than the more customary complex structure $\bar{1}$.

5By considering the interactions, one can deduce the existence of an infinite tower of massive states with standard ghost-structure as well $[8]$. 

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massive physical operators break Lorentz invariance, the massless operators and their interactions have manifest spacetime Lorentz invariance. If we associate spacetime fields $\phi$ and $\psi_\alpha$ with the physical operators $V$ and $\Psi$, it follows from the three-point amplitude eq. (27) that we can write the field equations [8]:

$$\partial_\alpha \partial^\alpha \phi = \psi^\alpha \psi_\alpha \quad \partial_\alpha \psi^\alpha = \psi^\alpha \partial_\alpha \phi . \quad (28)$$

We have suppressed Chan-Paton group theory factors that must be introduced in order for the three-point amplitude to be non-vanishing in the open string. It is easy to see even from the kinetic terms in the field equations eq. (28) that there is no associated Lagrangian. Note that there is no undifferentiated $\phi$ field, owing to the fact that the theory is invariant under the transformation $\phi \rightarrow \phi + \text{const}$. It is of interest to obtain the higher-point amplitudes from the field equations eq. (28), which should be zero if they are to reproduce the string interactions.

In summary, the new $n = 0$ model has a massless scalar and fermion, in addition to an infinite tower of massive particles. However, the model lacks spacetime supersymmetry. Moreover, while the massless fields have interesting interactions, for which we can write down the field equations not derivable from a Lagrangian, the model does not seem to describe the interactions of self-dual gravity.

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