Ultralight particle creation during Fresh inflation

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Abstract

I study ultralight particle creation which becomes from the Yukawa interaction between the inflaton and the thermal bath during fresh inflation. Particle creation is important in the first stages of fresh inflation, when the nonequilibrium thermal effects are important. I find that the number density of the ultralight created particles is more important as the scale factor growth rate is more large (i.e., for \( p \) large — \( a \sim t^p \)). Ultralight boson fields created during fresh inflation could be an alternative mechanism to cosmological constant to explain the discrepancy between the observed \( \Omega_m \approx 0.2 \) and \( \Omega_{\text{tot}} \approx 1 \), predicted by inflationary models.

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I. INTRODUCTION

Recently a new model of inflation called fresh inflation was introduced [1]. In this model initially the universe there is not thermalized \( [\rho_r(t = t_0) = 0] \), and begins from an unstable primordial matter field perturbation with energy density nearly \( M_p^4 \) and chaotic initial conditions. Later, it describes a second order phase transition, and the inflaton rolls down towards its minimum energetic configuration. Particles production and heating occur together during the rapid expansion of the universe, so that the radiation energy density grows during fresh inflation (\( \dot{\rho}_r > 0 \)). The Yukawa interaction between the inflaton field and other scalar fields in a thermal bath (\( \delta = \Gamma(\theta)\phi^2 \)), lead to dissipation which is responsible for the slow rolling of the inflaton field. So, the slow-roll conditions are physically justified

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and there are not a requirement of a nearly flat potential in fresh inflation. As the viscosity is large enough at the end of fresh inflation, the inflaton will reach a slow-roll regime due to its dynamics becomes overdamped ($\Gamma \gg H$). This fact also provides thermal equilibrium in the last phase of fresh inflation.

In this work I discuss creation of ultralight scalar fields which underwent in the fresh inflationary scenario [1]. Ultralight scalars have been previously discussed in the context of pseudo-Nambu-Goldstone bosons [2]. The basic idea is that as the inflaton relaxes toward its minimum energy configuration, it will decay into lighter fields, generating an effective viscosity. That this indeed happens in warm inflation [3–5], and also has been demonstrated in detail in Refs. [6].

I consider a spatially globally flat Friedmann-Robertson-Walker (FRW) metric
\[ ds^2 = -dt^2 + a^2(t)dx^2. \] (1)

Here, $a$ is the scale factor of the universe. Furthermore, $H = \dot{a}/a$ is the Hubble parameter. The Lagrangian for a $\phi$-scalar field minimally coupled to gravity, which also interacts with another $\psi$-scalar field by means of a Yukawa interaction,
\[ L = -\sqrt{-g} \left[ \frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) + L_{\text{int}} \right], \] (2)

where $g^{\mu\nu}$ is the metric tensor, $g$ is its determinant and $R$ is the scalar curvature. The interaction Lagrangian is given by $L_{\text{int}} \sim -g^2 \phi^2 \psi^2$, where $\psi$ is a scalar field in the thermal bath. Furthermore, the indices $\mu, \nu$ take the values $0, ..., 3$ and the gravitational constant is $G = M_p^{-2}$ (where $M_p = 1.2 \times 10^{19} \text{ GeV}$ is the Planckian mass).

The dynamics for the spatially homogeneous inflaton field is given by
\[ \ddot{\phi} + (3H + \Gamma) \dot{\phi} + V'(\phi) = 0, \] (3)

where $V'(\phi) \equiv \frac{dV}{d\phi}$, $V(\phi) = |\mathcal{M}^2(0)/2|\dot{\phi}^2 + |\lambda^2/4|\phi^4$ (where $\mathcal{M}^2(0) > 0$ is the squared mass at zero temperature) and $H(\phi) = 4\sqrt{\pi G/[3(4 - 3F)]}\mathcal{M}(0)\phi$. For this model (the reader can see a detailed development of fresh inflation in Ref. [1]), the scale factor of the universe evolves as $a \sim t^{2/(3F)}$, where $F = -2\dot{H}/(3H^2)$ is considered as constant. The term $\Gamma \dot{\phi}$ is added in the scalar field equation of motion (3) to describe the continuous energy transferred from $\phi$ to the radiation field, where $\Gamma(\theta) = g_{\text{eff}}^4/(192\pi)\theta$ ($\theta$ is the temperature) and $\theta(t)$ evolves as
\[ \theta(t) = \frac{192\pi}{g_{\text{eff}}^4 \lambda^2} \left\{ \mathcal{M}^2(0) \lambda^2 t + t^{-1} \left[ \lambda^2 \frac{(9F^2 - 18F + 8)}{(4 - 3F)^2} + \mathcal{M}^2(0) \pi G \frac{(192F^2 - 72F^3 - 96)}{(4 - 3F)^2} \right] \right\}, \] (4)

being $g_{\text{eff}} \approx 100$ the effective number of degrees of freedom of the created particles. The persistent thermal contact during warm inflation is so finely adjusted that the scalar field evolves always in a damped regime.
II. ULTRALIGHT PARTICLE CREATION

The fluctuations of the inflaton field $\delta \phi(\vec{x}, t)$ are given by the equation of motion

$$\ddot{\delta \phi} - \frac{1}{a^2} \nabla^2 \delta \phi + (3H + \Gamma) \dot{\delta \phi} + V''(\phi) \delta \phi = 0.$$  

(5)

Here, the additional second term appears because the fluctuations $\delta \phi$ are spatially inhomogeneous. Since $(a/a_0) = (t/t_0)^p$, the equation for the modes of $\delta \phi = a^{-3/2}e^{-\frac{1}{2} \int \Gamma dt} \chi$, with $p = 2/(3F)$, can be written as a Klein-Gordon equation with a time dependent parameter of mass $\mu^2(t) = \frac{k_0^2}{a^2}$, such that

$$k_0^2 = a^2 \left[ \frac{9}{4} (H + \Gamma/3)^2 + 3 \left( \dot{H} + \dot{\Gamma}/3 \right) - V''(\phi(t)) \right].$$  

(6)

The time-dependent wave number $k_0(t)$ separates the infrared (IR) and ultraviolet (UV) sectors. The redefined field fluctuations can be written as a Fourier expansion

$$\chi(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[ a_k \chi_k(\vec{x}, t) + a_k^\dagger \chi_k^*(\vec{x}, t) \right],$$  

(7)

where $\chi_k(\vec{x}, t) = \xi_k(t)e^{i\vec{k} \cdot \vec{x}}$, $\chi_k^*(\vec{x}, t) = e^{-i\vec{k} \cdot \vec{x}} \xi_k^*(t)$, and $(a_k, a_k^\dagger)$ are respectively the annihilation and creation operators. The equation of motion for $\xi_k(t)$ is

$$\ddot{\xi}_k + \left[ \frac{k^2}{a^2} - \frac{k_0^2}{a^2} \right] \xi_k = 0,$$  

(8)

where $k_0^2$ is given by eq. (6). When inflation starts $\Gamma(t = t_0) \approx 0$. Furthermore the time derivative of the width decay $\dot{\Gamma}$ is nearly constant ($\dot{\Gamma}(t) \approx \mathcal{M}^2(0)$), so that the equation (8) can be approximated to

$$\ddot{\xi}_k^{(0)} + \left[ \frac{k^2 t^{-2p-2}}{a_0^2 t_0^{-2p}} - \frac{9}{4} p^2 - 3p - 3 \right] t^{-2} \xi_k^{(0)} = 0.$$  

(9)

The general solution (for $\nu \neq 0, 1, 2, ...$) for this equation is

$$\xi_k^{(0)}(t) = C_1 \sqrt{\frac{t}{t_0}} \mathcal{J}_\nu \left[ \frac{kt^{1-p}}{a_0 t_0^{-p}(p-1)} \right] + C_2 \sqrt{\frac{t}{t_0}} \mathcal{J}_{-\nu} \left[ \frac{kt^{1-p}}{a_0 t_0^{-p}(p-1)} \right],$$  

(10)

where $\nu = \sqrt{9p^2 - 12p - 11}/2(p-1)$, which tends to $3/2$ as $p \to \infty$ (i.e., $\nu \approx 3/2$ for $p \gg 1$), and ($\mathcal{J}_\nu, \mathcal{J}_{-\nu}$) are the linearly independent Bessel functions. For large values of $p$ the Bessel functions are $\mathcal{J}_{3/2}[x] \approx \sqrt{\frac{2}{\pi x}} \left( \sin[x] - \cos[x] \right)$ and $\mathcal{J}_{-3/2}[x] \approx \sqrt{\frac{2}{\pi x}} \left( \cos[x] + \sin[x] \right)$. We can take the Bunch-Davis vacuum such that $C_1 = \sqrt{\pi/2}$ and $C_2 = -\sqrt{\pi/2}$. The general solution of eq. (8) can be written as $\xi_k(t) = e^{\int g(t) dt} \xi_k^{(0)}$, where $g(t = t_0) = 0$. Notice that $\xi_k^{(0)}$ is the solution for the modes when the interaction is negligible ($\Gamma \propto \theta \approx 0$). The dependence of the Yukawa interaction is in the function $g(t)$, which only takes into account...
the thermal effects. Replacing $\xi_k(t) = e^{\int g dt} \xi_k^{(0)}$ in eq. (8), and taking into account eq. (9), one obtains the differential equation for $g$

$$g^2 + \dot{g} = \frac{3}{2} H \Gamma + \frac{\Gamma^2}{4}, \quad (11)$$

which has the solution (for $g \gg \frac{\xi_k^{(0)}}{\xi_k}$)

$$g(t) \simeq \frac{\mathcal{M} \left[ -\frac{3p}{16}, \frac{1}{4}, \frac{\mathcal{M}^2(0)}{2} t^2 \right] \left[ \frac{\mathcal{M}^6(0)}{8} (\mathcal{M}^2(0) t^2 + (3p - 1)) \right] + \mathcal{M} \left[ \frac{1}{2} \left( 2 - \frac{3p}{2} \right), \frac{1}{4}, \frac{\mathcal{M}^2(0)}{2} t^2 \right] \left[ \frac{3}{8} \mathcal{M}^6(0) (1 - p) \right]}{2 \left[ \frac{\mathcal{M}^6(0)}{8} t \left( \mathcal{M}^2(0) t^2 + (3p - 1) \right) - c_1 \mathcal{W} \left[ \frac{1}{2} \left( 2 - \frac{3p}{2} \right), \frac{1}{4}, \frac{\mathcal{M}^2(0)}{2} t^2 \right] \right] + \mathcal{M} \left[ -\frac{3p}{16}, \frac{1}{4}, \frac{\mathcal{M}^2(0)}{2} t^2 \right] \left[ \frac{\mathcal{M}^6(0)}{2} \right]} + \frac{c_1 \mathcal{W} \left[ -\frac{3p}{16}, \frac{1}{4}, \frac{\mathcal{M}^2(0)}{2} t^2 \right] \left[ \frac{\mathcal{M}^6(0)}{8} (\mathcal{M}^2(0) t^2 + (3p - 1)) \right]}{2 \left[ \frac{\mathcal{M}^6(0)}{8} t \left( \mathcal{M}^2(0) t^2 + (3p - 1) \right) - c_1 \mathcal{W} \left[ \frac{1}{2} \left( 2 - \frac{3p}{2} \right), \frac{1}{4}, \frac{\mathcal{M}^2(0)}{2} t^2 \right] \right] + \mathcal{M} \left[ -\frac{3p}{16}, \frac{1}{4}, \frac{\mathcal{M}^2(0)}{2} t^2 \right] \left[ \frac{\mathcal{M}^6(0)}{2} \right]}, \quad (12)$$

where $c_1$ is such that $g(t = t_0) = 0$, due to initially the thermal effects are neglibible. Hence, the number density of ultralight created particles $n_u$ when inflation ends is

$$n_u = a^{-3} e^{-\mathcal{M}^2(0) t^2} \frac{2 \pi^2}{\int_{k_p}^{k_0} \int_{k_p}^{k_0} \beta_k^2 \beta_k^2} \int_{k_p}^{k_0} \frac{d k}{2} \int_{k_p}^{k_0} \frac{d k}{2}, \quad (13)$$

where $k_p$ is the Planckian wave number and $\beta_k$ is given by

$$\beta_k^2(t) = \frac{\langle \xi_k(t) \rangle^2}{(\xi_k^{(0)})^2} \simeq e^{2 \int g(t) dt}, \quad (14)$$

so that $n_u$ can be written as

$$n_u \sim a^{-3} F^2[\theta(t)] \int_{k_p}^{k_0} \frac{d k}{2} \int_{k_p}^{k_0} \frac{d k}{2}, \quad (15)$$

where $F[\theta(t)] = e^{-\frac{\mathcal{M}^2(0)}{2} t^2} e^{\int g(t) dt}$. Notice that the spectral density $\delta_k^{(n_u)} \sim k^{3/2}$ of the created particles is scale invariant in the range $k_p < k < k_0$. This range defines the infrared sector during inflation.

Fig.1: Temporal evolution of the function $F$ for $p = 2$. 

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Figure 1 shows the function $F[\theta(t)]$ as a function of the time in Planckian unities for $p = 2$, $g(t_0) = 0$ and $\mathcal{M}^2(0) = 10^{-12} \text{G}^{-1}$. Can be seen that after $t \gg 10^7 \text{G}^{1/2}$ the thermal effects are unimportant and $F$ tends to zero. However, for $t < 10^7 \text{G}^{1/2}$ the thermal effects are very important for ultralight particle creation. This period agrees with nonequilibrium thermal effects during the first stage of fresh inflation, which also could be responsible for baryogenesis in fresh inflation [3]. Figure 2 shows the function $F[\theta(t)]$ for $p = 20$. Notice that the peak is very much larger. This implies that the number density of the created particles grows when the rate of expansion of the universe increases.

III. FINAL COMMENTS

Recently, the cosmological constant ($\Lambda$) has come back into vogue. Dynamical estimates of the mass density from galaxy clusters suggest that $\Omega_m = 0.2 \pm 0.1$ for the matter that clusters gravitationaly [4]. However, the inflationary scenario suggests that $\Omega_{tot} = 1$. A cosmological constant is one way to resolve the discrepancy between $\Omega_m$ and $\Omega_{tot}$. In this work I studied the consequences of an ultralight boson field created during fresh inflation scenario, which is relaxing to its vacuum stated and dominating the energy density of the Universe. With this hypothesis, the effective vacuum energy at any epoch will be dominated by the heaviest fields that have not yet relaxed to their vacuum state. So, at late times, such fields must be very light. I deal with a potential without symmetry breaking. I find that the number density of ultralight created particles is important in the first stages of fresh inflation, when the nonequilibrium thermal effects are important. However, at the end of fresh inflation the number density of such created particles goes to zero because the thermal equilibrium is restored ($\Gamma \gg H$). Hence, a low density universe could exist without a $\Omega$ problem making fresh inflation compatible with observations indicating a low value of $\Omega_m$ [4-6]. So, ultralight boson fields created during fresh inflation could be an alternative mechanism to cosmological constant to explain the discrepancy between the observed value $\Omega_m \simeq 0.2$ and $\Omega_{tot} \simeq 1$, predicted by inflationary models of the universe.

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