Increasing granular flow rate with obstructions

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We describe a simple experiment involving spheres rolling down an inclined plane towards a bottleneck and through a gap. Results of the experiment indicate that flow rate can be increased by placing an obstruction at optimal positions near the bottleneck. We use the experiment to develop a computer simulation using the PhysX physics engine. Simulations confirm the experimental results and we state several considerations necessary to obtain a model that agrees well with experiment. We demonstrate that the model exhibits clogging, intermittent and continuous flow, and that it can be used as a tool for further investigations in granular flow.

I. Introduction

When does an obstruction placed near a bottleneck increase the flow rate of discrete objects moving through the bottleneck? Answering this question is of great utility on many scales. For example, the safe evacuation of pedestrians moving through confined environments (train stations, stadiums, concert halls, etc) in emergency situations can be of vital importance. And grain falling through a silo, where efficient flow is necessary for production and clogging is undesirable, is another example.

As stated by Magalhaes et al. in Ref. [1], ‘Granular materials are ubiquitous either in nature or in industrial processes’ and so a fundamental understanding of their motions is of intrinsic interest, both from physics and engineering perspectives. The quantitative study of the flow of granular materials has been performed for many decades [2], yet there is enormous scope for many interesting and creative techniques to be developed: experimental, theoretical and computational. Granular flow around an obstruction placed near a bottleneck is of particular interest as it has many potential applications and this has been investigated by many authors [1,3–7].

Increases in flow rate achieved through placement of an obstruction near a bottleneck have been reported by several investigators [8–10]. Another commonly reported phenomenon is that of ‘clogging’, also described as the formation of arches [4]. There are also many novel investigations of granular flow. For example, Wilson et al. [3] consider granular flow of particles that are completely submerged under water and Lumay et al. [7] investigate the flow of charged particles that repel each other. Clearly, granular flow is a very wide and active area of research.

Several authors have investigated the use of physics engines used in games, an example of which can be found in the work by Carlevaro and Pugnaloni [15]. In this paper we present a real time, 3D computer simulation using the PhysX physics engine. The model can be used in the study of granular flow as it exhibits continuous flow, intermittent flow, clogging, and has several parameters that an investigator can control as the simulation...
is occurring. The computer model is based on an experiment that uses spheres for the particles and three different obstruction shapes. We use experimental data to calibrate and validate our model. The model can then be used to determine how the highest flow rates may be achieved both with regard to obstruction position and to find optimal obstruction shapes.

The paper is organized as follows: we present the model in section II. Then we describe the experiments in two parts. Section III deals with flow rate without obstructions, and section IV investigates flow rate with obstructions. We discuss the limitations in section V, and conclude in section VI.

II. The Model

We use PhysX [12], a freely available real time, 3D physics engine for computations. PhysX is widely used in the games industry as, when necessary, it favors speed and stability of computation over accuracy. Integration of the equations of motion is done using an unconditionally stable, semi implicit Euler scheme yielding algebraic equations that are solved using a progressive Gauss-Seidel algorithm, whilst enforcing the Signorini conditions [16]. The time step can be adjusted for greater accuracy at the cost of speed and hence, real time performance of the engine. For this study, we left the time step at its default value of 0.02 seconds.

PhysX uses the Coulomb model for friction and restitution is a measure of the loss of kinetic energy between colliding particles. Details and further references about the rigid body system and friction model used in PhysX can be found in the work by Tonge [16].

The model incorporates the following parameters which are freely specifiable and chosen to agree with experimental results: friction, restitution, and sphere diameter. In addition, our model allows us to vary the angle of inclination of the plane, the gap width and the distance from the obstruction to the gap. We incorporate imperfections in the sphere diameters by allowing them to vary by up to 5%. The model uses 1551 particles whilst ≈ 1000 particles were used in the experiment. Each simulation is performed 20 times.

Unity [13], a freely available game engine, is used as our programming interface to PhysX. We also use Blender [14], a freely available 3D modeling package for modeling the obstructions.

III. Flow rate without obstructions

The basic set up of the experiment with obstructions is shown in Fig. I(a), where particles are released from above the obstruction. The particles themselves are spheres with diameters: 6.15 mm, 7.75 mm and 11.9 mm, referred to as ‘small’, ‘medium’, and ‘large’ respectively, and are shown in Fig. I(b). Note that without obstructions, the particles are released at the gap as shown in Fig. I(c).

Figure 1: (a) Basic set up of the experiment with an obstruction. (b) Particles are spheres of three different diameters. (c) Experiment of flow rate without obstructions, the particles are released at the gap.

We use an inclined plane that makes an angle of 8° with the horizontal. The plane has retaining walls, walls that form a bottleneck and release barriers as shown in Fig. I(a). We pack a known number of spheres of a specified size, shown without texture in Fig. I(b), onto the plane and release them from rest so that they collide with the bottleneck walls, and flow through the gap. The walls of the bottleneck make an angle of 30° to the retaining walls.

Under gravity, the spheres move down the inclined plane, come into contact with the bottleneck walls and flow through the gap. The number of particles is known and the time taken for all particles to flow through the gap is measured, from which the
flow rate, $J$, is determined. The flow rate is thus defined as the number of particles passing through the gap per second.

The experiment is repeated for various gap widths, with the corresponding flow rates measured. The entire experiment is performed individually for small, medium, and large spheres. All dimensions (particles, plane, bottleneck angles, etc) are known, as are the coefficients of friction and restitution of the particles. As per the experiment, in our model we release the particles at rest from the gap as shown in Fig. 1 (c).

It was found that there is a relatively sharp rise in flow rate as the gap is increased. It is precisely this range of distances where the flow rate transitions from clogging to intermittent to continuous flow rates. The model also exhibits clogging as shown in Fig. 2 (a), and intermittent flow. Therefore, in order to get reliable and reproducible flow rates, we decided to perturb the particles just sufficiently to prevent extended periods of clogging which thereby affect flow rates. As described by Garcimartin et al. [11], the problem of clogging can effectively be eliminated by applying external vibrations.

Figure 2: The model exhibits clogging (a) at the gap and between the obstruction and the bottleneck walls (b), consistent with the experiment.

In a similar way, we use the technique of ‘shaking’. In the case of flow rate without obstructions, the bottleneck walls were shaken as shown in Fig. 3(a) from a maximum of 0.5 mm diminishing to zero over one second, every five seconds. The parameters that define a shake are: shake amplitude, direction of shake, shake duration and time between shakes. These parameters can be set arbitrarily in the model and, whilst other times and distances were tested, the above were found to be just sufficient to yield reproducible results.

We varied the values of coefficient of friction and coefficient of restitution in the model until we got good agreement with experimental results. It was found that varying the coefficient of restitution had little to no effect on flow rate.

We found a coefficient of friction of 0.24 yielded model results that gave good agreement with experimental results. The experimentally measured value of the spheres was found to be 0.28±0.4. The results for each diameter sphere are shown in Fig. 4. The model results agree with the experimental results quite well.

### IV. Flow rate with obstructions

The experiment is repeated with medium spheres (of diameter 7.75 mm). However, this time, obstructions (shown in Fig. 3(c)) are placed at varying distances away from the gap, which is now fixed at a width of 3.3 cm.

It was found that there was a relatively sharp rise in flow rate as an obstruction is moved away...
Small (experiment)  
Medium (experiment)  
Large (experiment)  
Medium (model)  
Large (model)  
Small (model)

Figure 4: Flow rate without obstruction versus gap in dimensionless form. Comparison of experiment with simulation using spheres of three different diameters. $J$ is the flow rate, $g$ is the acceleration due to gravity, $d$ is the particle diameter and $W$ is the gap width.

from the gap. It is precisely this range of distances where the flow rate transitions from clogging to intermittent to continuous flow rates. The model also exhibited clogging as shown in Fig. 2(b), and intermittent flow. Therefore, in order to get reliable and reproducible flow rates, we decided to perturb the particles just sufficiently to prevent extended periods of clogging which thereby affect flow rates.

With the gap set at 3.3 cm, no clogging was observed for spheres of 7.75 mm at the gap. However, with the presence of an obstruction, clogging was observed in areas between the obstruction and the bottleneck walls as shown in Fig. 2(b) and Fig. 3(b). To get consistently measurable flow rates in the model, once again, we decided to perturb the system using the technique of ‘shaking’, described above. This time, we decided to shake the obstruction also shown in Fig. 3(b). The obstruction was shaken by a maximum of 1 mm diminishing to zero over one second, every five seconds. The freely specifiable parameters that define a ‘shake’ are the same as those for shaking the bottleneck walls. Figure 5 shows the maximum flow rates for the three obstructions, where the ‘waiting room’ effect, discussed in Ref. [8], occurs for the cylinder and the V obstruction but not for the A obstruction.

Figure 5: Peak flow rates with obstructions, where the ‘waiting room’ effect is present for the cylinder and the V obstruction but not for the A obstruction.

Figure 6 shows flow rates as measured experimentally and with the model. Both sets of results indicate increases in flow rates for certain ranges of distance of obstruction to the gap. For the model, each graph shows error bars and data that were more than two standard deviations away from the mean were not included in the statistical analysis. This was necessary as, even with the shaking technique described above, the particles did occasionally clog for extended periods of time, thereby affecting the flow rate measurements.

Clearly, the model shows all obstructions im-
Figure 6: Results of flow rate with obstructions versus distance in dimensionless form. \( J \) is the flow rate, \( g \) is the acceleration due to gravity, \( d \) is the particle diameter, and \( D \) is the distance from the obstruction to the gap. The experimental results in (a) show that each obstruction has a range of distances where flow rate is greater than with no obstruction. The numerical results (b-d) show that the model also predicts that each obstruction has a range of distances from the gap where the flow rate is greater than flow rate with no obstruction.

proceed flow rate for a range of distances, and reassuringly, the further the obstruction is moved away from the gap, the closer we get to flow rates without obstruction.

In the case of the cylinder, due to its shape and size relative to the bottleneck, we could only start at distances \( \sim 7 \) cm from the gap. We see that there is a steady rise in flow rate as the cylinder is moved further away from the gap. A maximum flow rate is reached and then a slightly sharper decrease is reached until we get flow rates the same as if no obstruction is present.

In the case of the A obstruction, due to its shape and size relative to the bottleneck, we were able to place the obstruction very close to the gap, which is why distances start at \( \sim 0.4 \) cm from the gap. We see a sharp rise in flow rate as the obstruction is moved away from the gap, and then we see a leveling out of the flow rate over a range of distances from 0.8 cm to 2.8 cm. There is a small peak in this range, but we consider this a statistical fluctuation.

Moving further away, from 2.8 cm to 3.2 cm, we see a sharp decrease in flow rate to values that are actually below flow rates with no obstacle. This is an intriguing phenomenon, showing what we might expect: obstructions decrease flow rates.

The flow rate remains constant to a distance of 3.6 cm and then starts rising. At a distance of 4 cm, the flow rate is approximately equal to that with no obstructions.

In the case of the V obstruction, due to its shape and size relative to the bottleneck, we could only start at distances \( \sim 6 \) cm from the gap. We see that there is a steady rise in flow rate that is slower than that of the A obstruction, as the V obstruction is moved further away from the gap. A maximum flow rate is reached, higher than that with no obstruction, and then a decrease occurs until we get flow rates the same as if no obstruction were present.

V. Limitations of PhysX engine

We observed that the model exhibits continuous flow, intermittent flow, and clogging, all phenomen-
ena that have been identified and observed experimentally. The model was quite sensitive to the absolute sizes of the geometric structures used. This is true for all physics engines, as they have to be ‘tuned’ to the range we would like them to work most accurately in. For example, an engine might be tuned to work best with objects whose sizes are of the order of meters but it will not work for objects at the scale of millimeters. To overcome this limitation, we ran the simulations in the dimensions where the performance of the engine was optimal, and then used dimensional analysis to extrapolate the result to the real experimental conditions.

We found that the particles in the model needed to be perturbed more often than in actual experiments, in order to get consistent flow rates. This can be attributed to a real time optimization employed by PhysX, known as ‘sleeping’. When a particle’s angular and/or linear velocity falls below certain threshold values for more than a few frames, these velocities are set to zero and the particle goes into a ‘sleep’ state, in which no collision detection occurs and hence the particle’s velocity remains at zero. The particle ‘wakes up’ when it is subjected to net forces. The reason for this optimization is that it relieves the processor of having to perform needless computations especially with regard to collision detection, thereby allowing much larger numbers of particles to be present and only performing the necessary computations as required. For this study, we left the threshold ‘sleep velocities’ at their default values.

In spite of these limitations, we can confirm that, as reported in various experiments, it is possible to increase flow rate of discrete particles by placing an obstruction at a suitable location near a gap.

VI. Conclusions

We have developed a model that exhibited reasonable quantitative accuracy in the case of no obstructions, and good qualitative agreement when obstructions were introduced. As in experiments, we showed that the flow exhibits clogging, intermittent flow rates, and continuous flow rates. The model also confirms experimental results that placement of an obstruction near a gap can actually increase flow rate through the gap. Better experimental agreement with obstructions will be possible if the roundness of the obstructions is more accurately modeled.

The model is quite flexible in that many parameters can be changed via user input, and without modifying the actual code. In this way, the model can be used as a convenient tool to suggest further experiments and allows us to investigate different obstruction shapes, both of which may help us gain a greater understanding of granular flow.

We have also described a suite of freely available realtime 3D software, which is mature and has many online resources in the form of documentation and tutorials and is being actively developed, which can be used together to create simulations of interest in the area of granular flow.

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