We study the effect of interactions on the bosonic two-particle quantum walk and its corresponding spatial correlations. The combined effect of interactions and Hanbury-Brown-Twiss interference results in unique spatial correlations which depend on the strength of the interaction, but not on its sign. The results are explained in light of the two-particle spectrum and the physics of attractively and repulsively bound pairs. We experimentally measure the weak interaction limit of these effects in nonlinear photonic lattices. Finally, we discuss an experimental approach to observe the strong interaction limit using single atoms in optical lattices.

Introduction.- Understanding highly correlated many-body systems remains both an experimental and theoretical challenge. While there is a rather good understanding of weakly interacting systems, problems involving strong interactions are in general harder to address.

Recently a new approach to the study of quantum dynamics became experimentally accessible through the study of Quantum Walks (QWs) in lattice potentials. Quantum walks are the quantum counterparts of classical random walks on discrete lattice: A quantum particle is initially placed at a particular site of a lattice and then tunnels to neighboring sites with equal probability amplitude. This basic "step" is repeated, but in contrast to the classical case quantum mechanical interference leads to distinctively different dynamics. For example, in periodic lattices the wavefunction width grows ballistically, while in the classical case the expansion is diffusive.

QWs receive increasing attention due to their relation to various physical and bio-physical processes, and their possible use as a primitive for quantum computation algorithms. Theoretically, QWs were studied for the single particle case. Initial experiments studied the physics of single particles by using either classical waves, single photons, or single atoms. Moving from one to two non-interacting particles it has recently been shown that indistinguishable quantum walkers can develop non-trivial correlations due to Hanbury Brown-Twiss (HBT) interferences. Yet, very little is known on the effect of interactions on the dynamics of the few-body QW. As new systems emerge that can accommodate such experiments, a systematic study of this problem starting at small particle numbers may offer a "bottom up" approach in the general thrive to understand dynamical quantum many-body systems.

In this letter we study the effect of inter-particle interactions on the two-particle quantum walk and the resulting spatial correlation. We consider two bosons, each initially localized on a single lattice site, undergoing a QW simultaneously. We find that the interplay between interactions and quantum two-particle (HBT) interference gives rise to fermion-like spatial correlations between the particles. Interestingly, the correlations depend on the strength of the interaction but not on whether it is attractive or repulsive. We explain the observed correlations by calculating the two-particle spectrum, and interpret our results in light of the physics of attractively and repulsively bound pairs. We then present an experimental observation of the weak interaction limit of these effects in nonlinear photonic lattices, and outline an experimental approach to observe the strong interaction limit using single atoms in optical lattices.

QW of two interacting particles.- We start by calculating the QW of two interacting particles. We consider the one dimensional Bose-Hubbard model:

$$H = -J \sum_{\langle i,m \rangle} a_i^\dagger a_m + \frac{U}{2} \sum_m \hat{n}_m (\hat{n}_m - 1)$$

where $a_i^\dagger$ ($a_m$) is the creation (annihilation) operator for a particle at site $m$, $\hat{n}_m = a_i^\dagger a_m$ is the corresponding number operator, $J$ is the tunneling amplitude between nearest neighbors and $U$ is the on-site interaction energy which can be attractive (negative) or repulsive (positive).

We study the QW of two indistinguishable particles, each initially localized on a single site in a periodic lattice. We consider two different initial conditions: One in which the two particles are localized at adjacent lattice sites $|\psi_{\text{initial}}\rangle = a_i^\dagger a_0^\dagger |0\rangle$, and a second in which the particles are initially placed at the same site, $|\psi_{\text{initial}}\rangle = (a_i^\dagger)^2 |0\rangle$. Our focus lies on the particle-density $n_r(t) = \langle a_i^\dagger a_r \rangle$ and...
Let us now turn to the discussion of interaction effects. Fig. 2b-e show the results for increasing repulsive interaction $U$ for in the case of two bosons initially localized at the same site. The spatial correlations show that as $|U|$ increases the two particles tend to propagate as a pair, while the density distribution becomes localized. Fig. 2g-l shows the results for the case in which the particles are initially places at different sites $|ψ_{initial}| = a_1^†a_0^|0⟩$. Here, the particle density depends only weakly on $U$, but the two-particle correlation undergoes a fundamental change: the spatial bunching effects which occur in the non-interacting case gradually transforms to spatial anti-bunching (Fig. 2k). For large values of the interaction strength $|U|$ the correlation between the two bosons becomes very similar to the correlation exhibited by two non-interacting fermions, prepared in the same initial configuration (compare Fig. 2k and l). The non-interacting fermionic and the interacting bosonic matrices become identical at the limit of $|U| → ∞$, while the density becomes identical to the one for $U = 0$. An interesting result is that in both cases the effect of interactions does not depend on the sign of $U$; it is identical for both attractive and repulsive interactions. We note that for initial conditions in which the two particles are further separated in space interactions also drive the system towards fermion-like correlations, only that now they have a more complicated spatial structure—see [16] for additional experimental and theoretical results.

The two particle spectrum. To understand these results we consider the two-particle spectrum of the system, as shown in Fig. 3 for two particles on a lattice with $M = 29$ sites. Each of the two-particle eigenfunctions can be written as $Ψ(r_1, r_2)$ where $r_1$ and $r_2$ are the positions of the two particles on the lattice. Introducing the center of mass coordinate, $R = (r_1 + r_2)/2$ and the relative coordinate, $r = r_1 - r_2$, we can solve the Schrödinger equation with the ansatz $Ψ(r_1, r_2) = \exp(iKR)ψ_M(r)$, where $K$ is the quasi-momentum of the center of mass motion and $ψ_M(r)$ is the pair wavefunction [15].

For finite interaction strength $U$ the spectrum separates into two bands. The main part of the spectrum, containing $[M(M-1)]/2$ eigenstates, consists of scattering states having low probability at $r = 0$, whose energy is given by the non-interacting part of the Hamil-

---

**Figure 2:** (color online). Two-particle correlations for interacting quantum walkers. Left column: correlations after propagation time $T = 4$, where initially the two particles are placed at the same site, $|ψ_{initial}| = (a_0^†)^20⟩$. (a) For zero interactions, the two-particle correlation shows no interference [10]. (b)-(f) As the interaction is increased the correlations shows the formation of bound pairs, while the density distribution (shown on the right of each plot) becomes increasingly localized. Right column: similar results for an initial condition in which the two particles are placed at adjacent sites $|ψ_{initial}| = a_1^†a_0^|0⟩$. (g) At $U = 0$ the correlations show spatial bunching. (h)-(j) The correlations change as the interaction $U$ is increased, while the density is only weakly affected. (k) At strong interactions the correlation is transformed to spatial anti-bunching, similar to the correlation that would be exhibited by two non-interacting fermions initially placed in the same configuration (l). All results are identical for both attractive and repulsive interactions.

The results for $U = 0$ correspond to the results reported in [10, 11]: when the two bosons start the QW at the same site, after propagation each particle can be found on either side of the site of origin, reflected in the four symmetric peaks in the correlation matrix inset for Fig. 2a. When the particles are initially placed at adjacent sites, HBT interference results in spatial bunching, and the two particles propagate together either to the left side of the distribution (peak at the bottom left corner of the correlation matrix in [2k]) or to the right (top right corner). Other initial conditions, in which the particles are further separated in space result in more complicated correlation patterns [10]. Such initial state and the results of interactions will be discussed in [16].
tonian. The smaller part of the spectrum (M eigenstates) consists of states \( \psi_K^{bs}(r) \) which have a large probability for two particles to occupy the same site, i.e., \( |\psi_K^{bs}(0)|^2 \rightarrow 1 \ (U \rightarrow \infty) \) (see insets in Fig. 3). This mini-band has higher or lower energies than the main part of the spectrum, depending on the sign of the interaction; nevertheless, the spatial probability distribution of the two-particle eigenstates is identical.

Using this picture, it is possible to explain the results in Fig. 2 An initial state in which the two particles occupy the same site with strong attractive or repulsive interaction, will mostly contain two-particle states from the smaller mini-band. As a result, the two particles will remain bound as described by Winkler et al. in [14] (see Fig. 2 g-h). A complementary process happens if the particles initially occupy different sites. This initial condition excites mainly scattering states from the main part of the spectrum. As a result, the particles have low probability to be found at the same site throughout the evolution, and will not show bunching.

Let us now turn to the case of strong interactions \( |U| \gg J \). Our goal is to understand the “fermionization” as observed in the correlator \( \Gamma_{q,r}(t) \) for an initial state in which the particles are found at different sites. We start by noting that by focusing on the scattering states we can describe the Hamiltonian \( \hat{H} \) using hard-core bosons, where doubly occupied sites are eliminated from the Hilbert space. Formally we replace the bosonic with spin-1/2 operators: \( a_m^+ \rightarrow S_m^+ \), \( a_m \rightarrow S_m^- \).

Next, we use a standard mapping from spin-1/2 to fermionic operators \( f_m, f_m^\dagger \) [17]. Let us review the essential steps of this mapping to understand the “fermionic” behavior of \( \Gamma_{q,r}(t) \). Spin-1/2 and fermionic operators share the local property \( (f_m^\dagger)^2 = f_m^2 = (S_m^+)^2 = (S_m^-)^2 = 0 \). However, spins on different sites commute, whereas fermions pick up a minus sign. In the sought mapping one corrects for this via the Jordan-Wigner string \( \exp(i\phi_m) \):

\[
S_m^- = e^{-i\phi_m} f_m, \quad S_m^+ = e^{i\phi_m} f_m^\dagger,
\]

with \( \phi_m = \pi \sum_{l=1}^{m-1} f_l^\dagger f_l \).

It is now straight-forward to check that for \( \Gamma_{q,r}(t) \) the Jordan-Wigner string drops out. Hence, the correlation for hard-core bosons is identical to the ones obtained for non-interacting fermions in accordance with our observation in Fig. 2.

**Experimental results.** - The case of the two-photon quantum walk and the resulting HBT correlations were considered in [10] and observed in [11], in a system of waveguide lattices. This system is described by an equation identical to Eq. 1, only that for single photons interactions are negligible, i.e. \( U = 0 \). Ref. [10] presented also a measurement of correlation for classical, thermal light waves, analogous to the intensity correlations predicted by the original HBT work [12]. That experiment showed that the results for thermal inputs captures some (but not all) aspects of the correlations predicted for the quantum, two-particle case.

The waveguide lattice used in that experiment, described in detail in [3, 10], has been shown to support nonlinear effects for high intensity classical light [18]. In this case the system is described by the Hamiltonian:

\[
H = -J \sum_{(l,m)} \Psi_l^\dagger \Psi_m + \gamma \sum_m |\Psi_m|^4
\]

which is the classical or mean-field limit of Eq. 1. It is therefore reasonable to presume that the classical correlations for nonlinear thermal waves in this system will correspond to the results for two interacting quantum particles. Indeed, In the experiments described below we find that in the limit of weak interactions the measured classical correlation for nonlinear thermal waves are similar to the predicted quantum correlations.

In Fig. 4 we present experimental measurements for intensity correlations obtained using nonlinear thermal waves in \( |\psi_{\text{initial}}\rangle = a_0^\dagger a_0^\dagger |0\rangle \). Detailed numerical results are presented in Fig 4. In all figures we compare the correlation fluctuations: \( \Gamma_{q,r}^F(t) = \langle a_q^\dagger a_r^\dagger a_q a_r \rangle - \frac{1}{2} \langle a_q^\dagger a_q \rangle \cdot \langle a_r^\dagger a_r \rangle = \Gamma_{q,r} \), \( \frac{1}{2}n_q \cdot n_r \) which are a better basis for comparison between the quantum and classical (thermal) case. As the results show, for weak interactions the classical HBT correlations follow the quantum predictions - see Fig. 3 and Fig. 3 a-f. However, as interactions become stronger the two systems diverge - while the quantum system exhibits a switch to fermion-like correlations, the classical system cannot follow, and remains with modified, localized correlations - Fig. 3 i-l.

**Proposed cold atoms experiment.** - Experimentally, the strong interaction limit of effects discussed above can be
The fluctuation in intensity correlations can be directly assessed from measurements in the same manner as in [9], and the two QWs [1, 5]. Using a similar approach, we propose starting with an ensemble of two non-interacting particles initially placed at the same locations. (c) Experimental results for nonlinear thermal waves (d) the predictions of the quantum theory for two interacting particles. Note the similarity between the classical results and the quantum prediction in both cases.

Figure 5: (color online). Simulations comparing the fluctuations for the quantum (top panel) and the classical (bottom) cases, with increasing $|U|$ or nonlinear coefficient $|\gamma|$, correspondingly, for $|\psi_{initial}| = a_1^\dagger a_0^\dagger |0\rangle$. The two sets of results look similar for weak interactions (a-c). However, beyond $|U| = 1.5$ (d-g) the two results diverge - the quantum correlations transform to fermionic-like correlations, while the classical correlations become increasingly localized.

Conclusions.- In this letter we have considered the quantum dynamics of two bosons on a lattice, each initially confined to a single site. Such dynamics with two or more particles can be experimentally explored in systems such as described in ref. [9]. As the number of particles increases the problem will become uncomputable, but may remain experimentally accessible.

Note.- During the final completion of this manuscript we became aware of related work [20].

1. J. Kempe, Contemp. Phys., 44, 307, (2003).
2. G. S. Engel, et al., Nature 446, 782-786 (2007).
3. T. Kitagawa, et al., Phys. Rev. A 82, 033429 (2010).
4. A. M. Childs, Phys. Rev. Lett. 102, 180501 (2009).
5. H. B. Perets, et al., Phys. Rev. Lett. 100, 170506 (2008).
6. A. Schreiber et al., Phys. Rev. Lett. 104, 050502 (2010).
7. M. A. Broome, et al., Phys. Rev. Lett. 104, 153602 (2010).
8. M. Karski, et al., Science 325, 174 (2009).
9. C. Weitenberg, et al., arXiv:1101.2076 (2011).
10. Y. Bromberg et al., Phys. Rev. Lett. 102, 253904 (2009).
11. A. Peruzzo, et al., Science 329, 1500 (2010).
12. R. Hanbury Brown and R.Q. Twiss, Nature 177, 27 (1956).
13. This problem can be considered as the quantum counterpart of classical excluded volume diffusion which is non-trivial already in 1D lattices due to the inability to neglect fluctuations; see e.g. D. ben-Avraham and S. Havlin, Diffusion and Reactions in Fractals and Disordered Systems, chapter 10. (Cambridge University Press, 2000).
14. K. Winkler, et al., Nature 441, 853 (2006).
15. P. F. Maldague, Phys. Rev. B 16, 2436 (1977).
16. See xxxxx for supplementary information.
17. F. Jordan and E. Wigner, Z. Phys. 47, 631 (1928).
18. D. N. Christodoulides, F. Lederer, and Y. Silberberg, Nature 424, 424, 817 (2003); F. Lederer et al., Physics Reports 463, 1 (2008).
19. Y. Bromberg, Y. Lahini, E. Small, and Y. Silberberg, Nat. Photon. 4, 721 (2010).