We perform a model-independent global fit to all germane and updated $b \to s \ell \ell$ ($\ell = e, \mu$) data assuming new physics couplings to be complex. Under the approximation that new physics universally affects muon and electron sectors and that either one or two related operators contribute at a time, we identify scenarios which provide a good fit to the data. It turns out that the favored scenarios remain the same as obtained for the real fit. Further, the magnitude of complex couplings can be as large as that of their real counterparts and these are reflected in the predictions of the direct $CP$ asymmetry, $A_{CP}$, in $B \to (K, K^*) \mu^+ \mu^-$ along with a number of angular $CP$ asymmetries, $A_i$, in $B^0 \to K^{0} \mu^+ \mu^-$ decay. The sensitivities of these observables to various solutions are different in the low and high-$q^2$ bins. We also determine observables which can serve as unique identifier for a particular new physics solution. Moreover, we examine correlations between $A_{CP}$ and several $A_i$ observables. A precise measurement of $A_{CP}$ and $A_i$ observables can not only confirm the existence of additional weak phases but can also enable unique determination of Lorentz structure of possible new physics in $b \to s \mu^+ \mu^-$ transition.

I. INTRODUCTION

One of the key open problems in particle physics is the observed recalcitrant disparity between the amount of matter and antimatter in the Universe. It is expected that the Big Bang explosion would have created matter and antimatter in equal amounts. However, it is still not understood how one type of matter triumphed over another in the early Universe. Sakharov’s conditions provide three necessary ingredients required to create the observed baryon asymmetry $\Delta N_{\text{BA}}$. One of these conditions requires a $CP$ violation which can be generated through a complex phase in the Lagrangian that cannot be reabsorbed through the rephasing of the apposite fields.

The SM of electroweak interactions allows for $CP$ violations owing to a complex phase in the quark mixing matrix. The $3 \times 3$ CKM matrix can be parametrized by three angles and a single complex phase. This single phase of the CKM matrix is the only source of $CP$ violation in the SM. This phase evinces itself in several observables in the decays of $K$ and $B$ mesons. In fact, the BaBar and Belle experiments established the CKM paradigm of $CP$ violation through several measurements of observables in the decays of $B$ mesons. However, unlike parity violation, which is maximal, the observed $CP$ violation is small and cannot account for the observed baryon asymmetry. The amount of predicted baryons in the Universe using the CKM formalism falls several orders of magnitude short of the observed value. Therefore one needs to explore beyond the CKM paradigm of the SM.

The $CP$ violating observables in the decays induced by the quark level transition $b \to s \ell^+ \ell^-$ are particularly important in probing new physics. see for e.g. $[2,11]$. This is because these observables are highly suppressed in the SM, i.e., they are predicted to be less than a percent level in the SM $[2,3]$. Even after including the next-to-leading order QCD corrections and hadronic uncertainties, the $CP$ asymmetries are not expected to transcend $1\%$ $[4,5]$. The $CP$-violating observables can be measured at LHC or at Belle-II provided new physics enhances them to a level of a few percent. Therefore measurement of any $CP$ violating observable in $b \to s \mu^+ \mu^-$ sector will provide a luculent signature of new physics.

The decay mode $b \to s \mu^+ \mu^-$ is already in spotlight for a decade due to the fact that it has provide a number of observables whose measurements are in contention with the predictions of the SM. These include number of observables in $B_s \to \phi \mu^+ \mu^-$ and $B \to K^* \mu^+ \mu^-$ decays which are related only to the muon sector. For e.g., the experimental value of the branching ratio of $B_s \to \phi \mu^+ \mu^-$ decay ostentates tension with the SM at 3.5$\sigma$ level $[12,13]$. The measurement of the optimized angular observable $P_3^\prime$ in $B \to K^* \mu^+ \mu^-$ decay in the (4.0 GeV$^2 \leq q^2 \leq 6.0$ GeV$^2$) bin deviates from the SM prediction at the level of 3$\sigma$ $[14–19]$. The measured value of the branching ratio of the decay $B_s \to \mu^+ \mu^-$ also regaled tension with the SM at 2$\sigma$ level $[20,21]$. However, the CMS collaboration recently updated the measurement of the branching ratio of $B_s \to \mu^+ \mu^-$ using the full Run 2 dataset. This resulted in a new world average of the branching ratio $[26]$, which is now in agreement with its SM prediction $[27,28]$.

The measurement of the ratio $R_{K^*} \equiv \Gamma(B^+ \to K^+ \mu^+ \mu^-)/\Gamma(B^+ \to K^+ e^+ e^-)$ showed a scantiness of 3.1$\sigma$ as compared to the SM value in the (1.1 GeV$^2 \leq q^2 \leq 6.0$ GeV$^2$) bin $[29,31]$. Here $q^2$ is the dilepton invariant mass-squared. The measurements of analogous ratio, $R_{K^*}$, in the (0.045 GeV$^2 \leq q^2 \leq 1.1$ GeV$^2$) and (1.1 GeV$^2 \leq q^2 \leq 6.0$ GeV$^2$) bins also nonconcurred with the SM at level of $\sim 2.5\sigma$ $[32]$. In $[33,35]$, it was shown that these deviations are valid even after including the QED corrections. These contestations, known as lepton flavour universality violation (LFUV) was accredited to new physics in $b \to s \mu^+ \mu^-$ or/and $b \to s e^+ e^-$ decays, i.e.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Parameter & Value \\
\hline
$\Delta N_{\text{BA}}$ & $10^{-10}$ \\
$\theta_{\text{CKM}}$ & $10^{-3}$ \\
$\sin^2 \beta$ & $10^{-4}$ \\
\hline
\end{tabular}
\caption{Observed and predicted values of key parameters in the SM and beyond.}
\end{table}

1 These can also be attributed to under estimation of hadronic uncertainties in the SM such as non factorizable power corrections $[30,39]$. 

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it required non-universal couplings in muon and electron sectors. However, on 20th of December 2022, the LHCb collaboration updated these measurements [40, 41] which are now consistent with the SM predictions. As the updated values of $R_K$ and $R_{K^*}$ are now consistent with their SM predictions, these would force the new physics couplings in electron and muon sectors to be nearly universal in nature.

Apart from these LFU ratios, the LHCb collaboration had also provided measurements of new LFU ratios in $B^0 \to K_{S}^{0} \mu^+ \mu^-$ and $B^+ \to K^{+} \mu^+ \mu^-$ channels [12]. These measurements concur with the SM at $\sim 1.5\sigma$ level. In the recent updates of $R_K$ and $R_{K^*}$, the LHCb collaboration included the experimental systematic effects which were absent in the previous analysis [40, 41]. Due to this, the updated values are now consistent with SM predictions. Therefore it is expected that the measurements of these new LFU observables would also suffer from the same systematic effects.

In order to determine the Lorentz structure of possible new physics that can accommodate the anomalous measurements in $b \to s \ell \ell$ decays, a model independent analysis can be performed using the language of effective field theory [39, 43–57]. Barring a few [48, 51], most of these analyses assume new physics Wilson Coefficients (WCs) to be real. In this work, we allow the new physics WCs to be complex and perform a global analysis of all CP-conserving $b \to s \ell \ell \ (\ell = e, \mu)$ data under the assumption that the beyond SM contributions affect both the muon as well as electron sector equally. Apart from the updated values of the LFU ratios $R_K$ and $R_{K^*}$ by the LHCb collaboration in December 2022, the branching ratios of $B \to X_s \mu^+ \mu^-$, $B^0 \to K^0 \mu^+ \mu^-$, $B^+ \to K^+ \mu^+ \mu^-$, $B^0 \to K^{*0} \mu^+ \mu^-$, $B^+ \to K^{*+} \mu^+ \mu^-$ and $B_s^0 \to \phi \mu^+ \mu^-$ in several $q^2$ bins along with $B(B_s^0 \to \mu^+ \mu^-)$ are included in the fits. Further, we include a number of CP-conserving angular observables in $B^0 \to K^{*0} \mu^+ \mu^-$, $B^+ \to K^{*+} \mu^+ \mu^-$ and $B_s^0 \to \phi \mu^+ \mu^-$ decays. Moreover, we also include a number of observables in decays induced by $b \to s e^+ e^-$ transition. These observables are obtained by averaging over the angular distributions of $B$ and $B$ decays.

We take the most frugal approach by considering only one operator or two related operators at a time. For statistically favoured scenarios, we then obtain predictions for several CP-violating observables. For $B^+ \to K^{+} \mu^+ \mu^-$ decay, we calculate the direct CP asymmetry, $A_{CP}$, whereas for $B^0 \to K^{*0} \mu^+ \mu^-$ decay, a number of angular CP-asymmetries, $A_i$’s, are analyzed along with $A_{CP}$. These are obtained by comparing the angular distributions of the corresponding $B$ and $B$ decays. For favoured new physics solutions, we also study correlations between CP violating angular asymmetries in $B^0 \to K^{*0} \mu^+ \mu^-$ decay and $A_{CP}$ which is expected to be measured with the highest statistical significance amongst all $CP$ asymmetries. These correlations would not only reveal the impact of new physics phase on various quantities but would also help in sequestering between the allowed scenarios.

Plan of the work is as follows. In Sec. II we discuss the methodology adopted in this work. We then provide the fit results. Using the fit results, we calculate the direct $CP$ asymmetry in $B^+ \to K^{+} \mu^+ \mu^-$ in Sec. III. In the following section, we obtain predictions of a number of $CP$-violating angular observables related to $B^0 \to K^{*0} \mu^+ \mu^-$ decay. We also study correlations between $A_{CP}$ and several $CP$-violating angular observables related to $B^0 \to K^{*0} \mu^+ \mu^-$ decay. Finally, we conclude in Sec. IV.

II. A FIT TO ALL $b \to s \ell \ell$ DATA

We start by performing a global fit to all $CP$ conserving data in $b \to s \ell \ell \ (\ell = e, \mu)$ by assuming new physics WCs to be complex. The data includes the updated measurements of LFU ratios $R_K$ and $R_{K^*}$ [40, 41] along with the branching ratio of $B_s \to \mu^+ \mu^-$ [26]. For reasons mentioned in the Introduction, we do not include measurements of new LFU ratios $R_{K^0}$, $R_{K^*}$ in the fit. The fit also includes the updated measurements for several $B_\ell \to \phi \mu^+ \mu^-$ observables [13, 55]. We closely follow the methodology adopted in [54] where the new physics couplings were assumed to be real. In this section, we intend to espy the following:

- The impact of the assumption of the complex coupling on the fit, i.e to spell out the differences between the real and complex fits by making use of the most updated data.

- The upper limit on the allowed parameter space of the new weak phases accredited by the current data. This will enable us to identify various $CP$ violating observables where large enhancement over the SM value is possible.

A. Methodology

We include following $CP$ conserving observables in our fit:

1. LFU ratios: Within the SM, $R_K$ is predicted to be close to unity owing to LFU which is deeply instilled in the symmetry structure of the SM. To be more specific, $R_K^{SM} = 1 \pm 0.01$ [29]. This ratio was first measured in 2014 by the LHCb collaboration [59] in $1.0 \text{GeV}^2 \leq q^2 \leq 6.0 \text{GeV}^2$ bin and updated in 2021 [31]. The measured value of $R_K^{exp} = 0.846^{+0.044}_{-0.031}$ [31] retrogressed from the SM prediction at the level of 3.1$\sigma$. This was considered as an inkling of LFUV. In 2017, the notion of LFUV in $b \to s \ell \ell$ was substantiated by the observation of the ratio $R_{K^*}$ by the LHCb collaboration [32]. This measurement
was performed in two $q^2$ bins. The measured values
\[ R_{K}^{\exp} = \begin{cases} 
0.660^{+0.110}_{-0.070} \pm 0.024, & q^2 \in [0.045, 1.1], \\
0.685^{+0.113}_{-0.060} \pm 0.047, & q^2 \in [1.1, 6.0],
\end{cases} \]  
(1)
detour from the SM predictions \[ 30 \] \[ 60 \] at the level of $\sim 2.5\sigma$. Apart from LHCb, Belle collaboration also measured $R_{K}$ in 0.045 GeV$^2 < q^2 < 1.1$ GeV$^2$, 1.1 GeV$^2 < q^2 < 6.0$ GeV$^2$, and 15.0 GeV$^2 < q^2 < 19.0$ GeV$^2$ bins \[ 61 \].

In 2021, LHCb also provided measurements of new LFU ratios $R_{K}$ and $R_{K^*}$. The measured values are \[ 40 \] \[ 41 \]:
\[ R_{K}^{\exp} = \begin{cases} 
0.994^{+0.080}_{-0.029} \text{(stat)}^{+0.022}_{-0.022} \text{(sysy)}, & q^2 \in [0.1, 1.1], \\
0.949^{+0.041}_{-0.022} \text{(stat)}^{+0.022}_{-0.022} \text{(sysy)}, & q^2 \in [1.1, 6.0],
\end{cases} \]  
(2)
\[ R_{K^*}^{\exp} = \begin{cases} 
0.927^{+0.036}_{-0.082} \text{(stat)}^{+0.027}_{-0.026} \text{(sysy)}, & q^2 \in [0.1, 1.1], \\
1.027^{+0.072}_{-0.066} \text{(stat)}^{+0.027}_{-0.026} \text{(sysy)}, & q^2 \in [1.1, 6.0],
\end{cases} \]  
(3)
It is thus obvious that these values are consistent with their SM predictions. We include these updated measurements in the fit along the Belle measurements of $R_{K^*}$. Further, $R_{K}$ and $R_{K^*}$ measurements are excluded from the fit.

2. Branching ratios: We include the updated world average of the branching ratio of the purely leptonic decay $B_s \to \mu^+ \mu^-$ which is $(3.45 \pm 0.29) \times 10^{-9}$ \[ 20 \]. This average value is in excellent agreement with the SM prediction \[ 27 \] \[ 28 \]. We also consider the branching fractions of inclusive decay modes $B \to X_s \mu^+ \mu^-$ and $B \to X_s e^+ e^-$ \[ 62 \] in the fit in the low and high-$q^2$ bins.

We also enshathe measurements of the differential branching fraction of several semileptonic decays. The recently updated measurements of the differential branching fraction of $B_s \to \phi \mu^+ \mu^-$ by LHCb in various $q^2$ intervals are included in the fit \[ 13 \]. Further, the differential branching ratios of $B^0 \to K_e^0 e^+ e^-$ \[ 63 \] \[ 65 \], $B^+ \to K^+ e^+ e^-$, $B^0 \to K^0 \mu^+ \mu^-$ and $B^+ \to K^+ \mu^+ \mu^-$ \[ 63 \] \[ 65 \] in different $q^2$ bins are encapsulated in the analysis. In $b \to se^+e^-$ sector, we include measurement of the differential branching fraction of $B^+ \to K^+ e^+ e^-$ in $1.0 \leq q^2 \leq 6.0$ GeV$^2$ bin \[ 59 \].

3. Angular Observables: We consider a plenitude of CP conserving $B^0 \to K^{*0} \mu^+ \mu^-$ angular observables in the fit. This entails longitudinal polarisation fraction $F_L$, forward-backward asymmetry $A_{FB}$ and observables $S_1, S_4, S_5, S_7, S_8, S_9$ in various $q^2$ bins, as measured by the LHCb collaboration \[ 16 \]. We also include their experimental correlations. We also encompass the angular observables $F_L, F_1, F_4, F_5, F_6$ and $F_8$ measured by ATLAS \[ 67 \] along with $P_1, P'_1$ measured by CMS \[ 68 \]. Further, the measurements of $F_L$ and $A_{FB}$ by CDF and CMS collaborations are also included \[ 64 \] \[ 65 \] in our analysis.

We then consider full set of angular observables in the decay $B^+ \to K^{*0} \mu^+ \mu^-$ which was measured for the first time by the LHCb collaboration in 2020 \[ 69 \]. The optimized angular observables $P_1 - P_5$ and longitudinal polarisation fraction $F_L$ along with their experimental correlations are included in the fit \[ 69 \]. Finally, we include CP conserving angular observables in $B_s \to \phi \mu^+ \mu^-$ decay mode. There are $F_L, S_3, S_4$ and $S_7$ as measured by the LHCb in 2021. The available experimental correlations are also subsumed in the fit \[ 63 \].

In decays induced by $b \to se^+e^-$ transition, we include the longitudinal polarisation fraction $F_L$ in the decay $B^0 \to K^{*0} e^+ e^-$ in $0.002 \leq q^2 \leq 1.12$ GeV$^2$ bin as measured by the LHCb collaboration \[ 70 \]. Further, we also include $P_4$ and $P'_5$ measured by the Belle collaboration in $0.1 \leq q^2 \leq 4$ GeV$^2$, $1.0 \leq q^2 \leq 6.0$ GeV$^2$ and $14.18 \leq q^2 \leq 19.0$ GeV$^2$ bins \[ 71 \].

In order to identify the Lorentz structure of possible new physics that can account for the discrepancies in $b \to sl\ell$ data, we perform a model independent analysis within the framework of effective field theory. For this we consider new physics in the form of vector and axial-vector operators. The effective Hamiltonian for $b \to sl\ell$ transition is then given by
\[
H_{\text{eff}}(b \to sl\ell) = H^{\text{SM}} + H^{\text{IA}}. \tag{4}
\]
Here the SM effective Hamiltonian can be written as
\[
H^{\text{SM}} = -\frac{4G_F}{\sqrt{2}\pi} V_{ts}^* V_{tb} \left\{ \sum_{i=1}^{6} C_i \mathcal{O}_i + C_8 \mathcal{O}_8 \right\} \\
+ C_7 \frac{e}{16\pi^2} \{ \sigma_{\alpha\beta}(m_s P_L + m_b P_R)b \} F_{\alpha\beta} \\
+ C_9 \frac{\alpha_{\text{em}}}{4\pi} (\pi_{\gamma\alpha} P_L b) (\bar{T}_{\gamma\alpha} \ell) \\
+ C_{10} \frac{\alpha_{\text{em}}}{4\pi} (\pi_{\gamma\alpha} P_L b) (\bar{T}_{\gamma\alpha\gamma\ell} \ell), \tag{5}
\]
where $V_{ij}$ are the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The short-distance contributions are enciphered in the WCs $C_i$ of the four-fermi operators $\mathcal{O}_i$ where the scale-dependence is implicit, i.e. $C_i \equiv \ldots$
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{Wilson Coefficient(s)} & \textbf{Real} & \textbf{Complex} \\
\hline
$C_i = 0$ (SM) & $0$ & $0$ \\
\hline
\end{tabular}
\caption{The best fit values of new WCs in various 1D scenarios. Here $\Delta \chi^2 = \chi^2_{SM} - \chi^2_0$ where $\chi^2_0$ is the $\chi^2$ at the best fit point and $\chi^2_{SM}$ corresponds to the SM which is $\chi^2_{SM} \approx 184$.}
\end{table}

$C_i(\mu)$ and $\mathcal{O}_i \equiv \mathcal{O}_i(\mu)$. The operators $\mathcal{O}_i$ ($i = 1, \ldots, 6, 8$) contribute through the modifications $C_i(\mu) \rightarrow C_i^{\text{eff}}(\mu, q^2)$ and $C_9(\mu) \rightarrow C_9^{\text{eff}}(\mu, q^2)$. The new physics effective Hamiltonian can be written as

$$\mathcal{H}^{\text{VA}} = -\frac{G_F}{\sqrt{2}} V_{tb}^* V_{tb} \left[ C_9^{\text{NP}} (s_{\gamma} \gamma P_L b)(\bar{b} \gamma_\alpha \ell) + C_{10}^{\text{NP}} (s_{\gamma} \gamma P_L b)(\bar{b} \gamma_\alpha \gamma_5 \ell) + C_9^{\text{eff}} (s_{\gamma} \gamma P_R b)(\bar{b} \gamma_\alpha \ell) + C_{10}^{\text{eff}} (s_{\gamma} \gamma P_R b)(\bar{b} \gamma_\alpha \gamma_5 \ell) \right].$$

(6)

Here $C_9^{\text{NP}}$, $C_{10}^{\text{NP}}$, $C_9^{\text{eff}}$ and $C_{10}^{\text{eff}}$ are the new physics WCs which are assumed to be real in the current analysis. Following a penurious approach, we ruminate only those scenarios where either only one new physics operator or two operators whose WCs are linearly related, contributes. We call them as “1D” scenarios. Under this assumption, we perform a $\chi^2$ fit to identify solutions which can accommodate the current $b \rightarrow s\ell\ell$ measurements. The fit is performed using the CERN minimization code MINUIT \cite{MINUIT}. The $\chi^2$ which is a function of new physics WCs is defined as

$$\chi^2(C_i, C_j) = [\mathcal{O}_{\text{th}}(C_i, C_j) - \mathcal{O}_{\text{exp}}]^T C^{-1} [\mathcal{O}_{\text{th}}(C_i, C_j) - \mathcal{O}_{\text{exp}}],$$

(7)

where $\mathcal{O}_{\text{th}}(C_i, C_j)$ are the theoretical predictions of the N=179 observables used in the fit and $\mathcal{O}_{\text{exp}}$ are the corresponding central values of the experimental measurements. The total $N \times N$ covariance matrix is obtained by adding the individual theoretical and experimental covariance matrices. The theoretical predictions of N=179 observables along with the theoretical covariance matrix are evaluated using flavio \cite{flavio} where the observables are preimplemented based on refs. \cite{flavio}. The experimental correlations, $\mathcal{O}_{\text{exp}}$, are admitted for the angular observables in $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ \cite{KKMM}, $B^+ \rightarrow K^{*+} \mu^+ \mu^-$ \cite{Bfmumu} and $B_s \rightarrow \phi \mu^+ \mu^-$. Further, for asymmetric errors, we use the larger error on both sides of the central value.

The $\chi^2$ value in the SM is represented by $\chi^2_{SM}$ whereas $\chi^2_{0}$ represents the value at the the best-fit point in the presence of new physics. We then quantify the goodness of fit by $\Delta \chi^2 \equiv \chi^2_{SM} - \chi^2_{0}$ for each new physics scenario. This means that, under the given assumptions, the largest value of this quantity would represent the best possible new physics scenario to accommodate the entire $b \rightarrow s\ell\ell$ data.

B. Fit results

The fit results are presented in Table I. For comparison, we provide the updated fit results for the real WCs. Using the values of $R_K$ and $R_{K^*}$ along with the measurement of LFV ratios $R_{K^0}$ & $R_{K^{*+}}$ \cite{Kstarr} and older world average of the branching ratio of $B_s \rightarrow \mu^+ \mu^-$ \cite{PDG}, it was well established that for real WCs, the new physics solutions $C_9^{\text{NP}}$ and $C_{10}^{\text{NP}} = -C_{10}$ provided a good fit to the data whereas $C_9^{\text{NP}} = -C_9'$ scenario provided a moderate fit, see for e.g. \cite{flavio}. In \cite{flavio}, it was shown that $C_{10}^{\text{NP}}$ scenario also provided a moderate fit to the data at par with $C_9^{\text{NP}} = -C_9$ solution. It is perspicuous from Table I that the updated fit for real WCs still prefers $C_9^{\text{NP}}$ scenario. The $C_9^{\text{NP}} = -C_9$ solution which provided a moderate fit to the older data now provides a good fit at par with $C_9^{\text{NP}}$ solution. However, the value of $\Delta \chi^2$ for $C_9^{\text{NP}} = -C_10$ scenario falls considerably, $\sim 10$ below $\Delta \chi^2$ for $C_9^{\text{NP}}$ solution. Therefore the $C_9^{\text{NP}} = -C_{10}$ scenario can only provide a moderate fit to the current $b \rightarrow s\ell\ell$ data. The situation appears to be more grim for $C_9^{\text{NP}}$ scenario which fails to provide useful improvement in the value of $\chi^2$ as compared to the $\chi^2_{SM}$. This is mainly due to the fact the current world average of the branching ratio of $B_s \rightarrow \mu^+ \mu^-$ is now in excellent agreement with the SM value.

It is also apparent from Table I that the scenarios that are favoured by assuming new physics WCs to be real, remains the preferred ones even for the complex couplings. The $C_9^{\text{NP}}$ and $C_{10}^{\text{NP}} = -C_9'$ scenarios turn out to be the most viable scenarios to accommodate all $b \rightarrow s\ell\ell$ data whereas the $C_9^{\text{NP}} = -C_{10}$ scenario can only provide a moderate fit to the current data. The $C_{10}^{\text{NP}}$ scenario has the lowest value of $\Delta \chi^2$ in comparison to the other three solutions. Hence we drop this from further consideration in this work. Further, the imaginary part of all WCs are allowed to have values similar to that of their real counterparts. This is evident from the 1$\sigma$ range of complex
FIG. 1. Allowed parameter space for new physics Scenarios $\mathcal{C}_9^{\text{NP}}$ (upper left panel), $\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_1^{\text{NP}}$ (upper right panel) and $\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_9'$ (lower panel).

Therefore, it will be intriguing to see whether some of the $\text{CP}$ violating observables can be enhanced up to the current or planned sensitivity of LHCb or Belle-II.

III. DIRECT $\text{CP}$ ASYMMETRY IN $B^+ \to K^+ \mu^+ \mu^-$

The $\text{CP}$ violation can be classified into two types: the direct $\text{CP}$ violating asymmetries and triple product $\text{CP}$ asymmetries. Assume that there are two contributions, $A_1 \propto e^{i\alpha_1}e^{i\beta_1}$ and $A_2 \propto e^{i\alpha_2}e^{i\beta_2}$ in $b \to s\mu^+\mu^-$ decay. Here $\alpha_{1,2}$ and $\beta_{1,2}$ are weak and strong phases, respectively. It can be easily shown that the direct $\text{CP}$ asymmetries are proportional to $\sin(\alpha_1 - \alpha_2) \sin(\beta_1 - \beta_2)$. This implies that these asymmetries can have non-zero values only if the two interfering amplitudes have a relative weak as well as strong phase. On the other hand, as triple product asymmetries are proportional to $\sin(\alpha_1 - \alpha_2) \cos(\beta_1 - \beta_2)$, a relative weak phase between the amplitudes is sufficient to provide a non-zero value.

For $B \to K\mu^+\mu^-$ decays, we only have direct $\text{CP}$ asymmetry, $A_{CP}$, which is defined as

$$A_{CP} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

where $\Gamma$ and $\bar{\Gamma}$ are the decay rates of $B^+ \to K^+\mu^+\mu^-$ and $\bar{B} \to \bar{K}\mu^+\mu^-$ decays. $\bar{\Gamma}$ is obtained from $\Gamma$ by changing the sign of the weak phases. The sign of strong phases remain unchanged. The theoretical expression of $\Gamma(B \to K\mu^+\mu^-)$ is provided in Appendix A.

Effectively, various contributions to $b \to s\mu^+\mu^-$ decay
TABLE II. Predictions of $A_{\text{CP}}$ in $B^+ \to K^+ \mu^+ \mu^-$ decay (1σ range). Here $A_{\text{CP}}^K \equiv A_{\text{CP}}(B^+ \to K^+ \mu^+ \mu^-)$.

| Wilson Coefficients | $A_{\text{CP}}^K_{|\ell=6|} (%)$ | $A_{\text{CP}}^K_{|\ell=15-19|} (%)$ |
|---------------------|---------------------------------|---------------------------------|
| $C_9$ = 0 (SM)      | $\approx 0$                     | $\approx 0$                     |

1D Scenarios:

| $C_{9}^{NP}$        | $(-0.33, 0.58)$                | $(-3.53, 3.54)$                |
|---------------------|--------------------------------|--------------------------------|
| $C_{9}^{NP} = -C_{10}^N$ | $(-0.52, 0.85)$                | $(-5.24, 5.35)$                |
| $C_{9}^{CP} = -C_{0}^N$ | $(0.12, 0.12)$                 | $(-0.16, -0.16)$               |

The differential distribution of $B \to K^*(\to K\pi)\mu^+\mu^-$ decay can be parametrized in terms of one kinematic and three angular variables. The kinematic variable is $q^2 = (p_B - p_K)^2$, where $p_B$ and $p_K$ are the four-momenta of $B$ and $K^*$ mesons, respectively. The angular variables are usually defined in the rest frame of the vector meson $K^*$. These angles are

- $\theta_K$ the angle between $B$ and $K$ mesons where $K$ meson emerges from the decay of a $K^*$,
- $\theta_\mu$, the angle between $-\mu^-$ and $B$ momenta,
- $\phi$ the angle between $K^*$ decay plane and the plane defined by the $\mu^+ - \mu^-$ momenta.

The four-fold decay distribution can be expounded as \cite{4, 5}

$$\frac{d^4I}{dq^2d\cos \theta_\mu d\cos \theta_K d\phi} = \frac{9}{32\pi} I(q^2, \theta_\mu, \theta_K, \phi), \quad (9)$$

where

$$I(q^2, \theta_\mu, \theta_K, \phi) = I_1^* \sin^2 \theta_K + I_1^* \cos^2 \theta_K + (I_2^* \sin^2 \theta_K + I_2^* \cos^2 \theta_K) \cos 2\theta_\mu + I_3 \sin^2 \theta_K \sin^2 \theta_\mu \cos 2\phi + I_4 \sin 2\theta_K \sin 2\theta_\mu \cos \phi + I_5 \sin 2\theta_K \sin \theta_\mu \cos \phi + I_6^* \sin^2 \theta_K \sin \theta_\mu \sin \phi + I_7 \sin 2\theta_K \sin \theta_\mu \sin \phi + I_8 \sin 2\theta_K \sin 2\theta_\mu \sin \phi + I_9 \sin^2 \theta_K \sin^2 \theta_\mu \sin 2\phi. \quad (10)$$
Wilson Coefficient(s) & $A_{3[1−6]}$ (%) & $A_{4[1−6]}$ (%) & $A_{5[1−6]}$ (%) & $A_{6[1−6]}$ (%) & $A_{7[1−6]}$ (%) & $A_{8[1−6]}$ (%) & $A_{9[1−6]}$ (%) \\
$C_i = 0$ (SM) & $\approx 0$ & $\approx 0$ & $\approx 0$ & $\approx 0$ & $\approx 0$ & $\approx 0$ & $\approx 0$ \\

1D Scenarios: & & & & & & & \\
$C_{9}^{NP} = C_{10}^{NP}$ & $(0.00, 0.01)$ & $(-0.10, 0.15)$ & $(0.04, 0.04)$ & $(-0.08, -0.07)$ & $(0.27, 0.29)$ & $(3.03, 3.45)$ & $(-0.31, 0.36)$ \\
$C_{9}^{NP} = -C_{10}^{NP}$ & $(-0.01, 0.02)$ & $(-0.17, 0.23)$ & $(-0.40, 0.49)$ & $(-0.79, 0.60)$ & $(-9.17, 10.38)$ & $(-4.74, 5.34)$ & $(-0.49, 0.55)$ \\
$C_{9}^{NP} = -C_{9}$ & $(-0.24, 0.23)$ & $(-0.24, 0.28)$ & $(0.04, 0.04)$ & $(-0.09, -0.08)$ & $(0.27, 0.32)$ & $(0.91, 1.04)$ & $(-0.21, 0.15)$ \\

TABLE IV. Prediction of various CP violating angular observables (1σ range) in $B^0 \to K^{*0}\mu^+\mu^-$ in the low-$q^2$ region.

Wilson Coefficient(s) & $A_{3[15−19]}$ (%) & $A_{4[15−19]}$ (%) & $A_{5[15−19]}$ (%) & $A_{6[15−19]}$ (%) & $A_{7[15−19]}$ (%) & $A_{8[15−19]}$ (%) & $A_{9[15−19]}$ (%) \\
$C_i = 0$ (SM) & $\approx 0$ & $\approx 0$ & $\approx 0$ & $\approx 0$ & $\approx 0$ & $\approx 0$ & $\approx 0$ \\

1D Scenarios: & & & & & & & \\
$C_{9}^{NP}$ & $(-0.72, 0.73)$ & $(-1.06, 1.07)$ & $(0.10, 0.11)$ & $(-0.21, -0.18)$ & $(0.01, 0.01)$ & $(-0.13, 0.15)$ & $(-0.11, 0.13)$ \\
$C_{9}^{NP} = -C_{10}^{NP}$ & $(-1.11, 1.10)$ & $(-1.62, 1.61)$ & $(-1.51, 1.58)$ & $(-2.79, 2.65)$ & $(-0.40, 0.45)$ & $(-0.20, 0.24)$ & $(-0.17, 0.20)$ \\
$C_{9}^{NP} = -C_{9}$ & $(-1.91, 1.91)$ & $(-2.13, 2.17)$ & $(0.10, 0.12)$ & $(-0.22, -0.18)$ & $(0.01, 0.01)$ & $(-3.58, 3.56)$ & $(-6.40, 6.33)$ \\

TABLE V. Prediction of various CP violating angular observables (1σ range) in $B^0 \to K^{*0}\mu^+\mu^-$ in the high-$q^2$ region.

The expressions of these twelve angular coefficients $I^{(a)}$ are provided in Appendix B. These coefficients are defined as

\[ I_{1,2,3,4,7}^{(a)} \Rightarrow \overline{I}_{1,2,3,4,7}^{(a)}, \quad I_{5,6,8,9}^{(a)} \Rightarrow -\overline{I}_{5,6,8,9}^{(a)}. \]

Here $\overline{I}_{i}^{(a)}$ are the complex conjugate of $I_{i}^{(a)}$. Therefore, one can define twelve CP averaged angular observables as

\[ S_{i}^{(a)}(q^2) = \frac{I_{i}^{(a)}(q^2) + \overline{I}_{i}^{(a)}(q^2)}{d(\Gamma + \overline{\Gamma})/dq^2}, \]

along with twelve CP asymmetries

\[ A_{i}^{(a)}(q^2) = \frac{I_{i}^{(a)}(q^2) - \overline{I}_{i}^{(a)}(q^2)}{d(\Gamma + \overline{\Gamma})/dq^2}. \]

The CP asymmetry in the dimuon mass spectrum is defined as

\[ A_{CP}(q^2) = \frac{d\Gamma/\overline{dq^2} - d\overline{\Gamma}/dq^2}{d\Gamma/dq^2 + d\overline{\Gamma}/dq^2}, \]

where $d\Gamma/\overline{dq^2}$ can be expressed in terms of angular coefficients as

\[ \frac{d\Gamma}{dq^2} = \frac{3}{4} (2I_{+}^{(a)} + I_{-}^{(a)}) - \frac{1}{4} (2I_{-}^{(a)} + I_{+}^{(a)}). \]

Apart from $A_{CP}$, in this work we consider consider $A_{3,4,5}$, $A_{6}$ and $A_{7,8,9}$ observables. These observables are measured by the LHCb collaboration, however, with large errors [15]. The angular observables $A_{3,4,5}$ and $A_{6}$ are direct CP like asymmetries whereas $A_{7,8,9}$ are triple product CP asymmetries [4]. Therefore $A_{7,8,9}$ observables seem to be more sensitive to new weak phases as compared to the other observables.

First of all, we examine $A_{CP}^{K^*}$. Based on predictions obtained in Table III for various favored scenarios, it is pellucid that none of the solutions can enhance $A_{CP}^{K^*}$ to a level of a percent in the low-$q^2$ bin. Therefore, the current $b \to s\ell\ell$ data suggests that the measurement of $A_{CP}$ in $B^0 \to K^{*0}\mu^+\mu^-$ decay in the low-$q^2$ region would be a hellacious task. On the contrary, in the high-$q^2$ bin, all favored new physics scenarios can ameliorate $A_{CP}^{K^*}$ up to 2-3%. However, as the maximum allowed value of $A_{CP}^{K^*}$ for all solutions are close to each other, one needs to look for additional CP violating observables to discriminate between the allowed solutions.

The predictions of several CP violating angular observables in $B^0 \to K^{*0}\mu^+\mu^-$ in the low and high-$q^2$ regions are exhibited in Table IV and V respectively. From Table IV it is unambivalent that all three allowed solutions predict $A_{3,4,5}$, $A_{6}$ and $A_{7,8,9}$ asymmetries to be less than a percent in the low-$q^2$ bin and hence making their observation an arduous endeavor. However, the prediction of observable $A_7$ provides encouraging sign for $C_{9}^{NP} = -C_{10}^{NP}$ solution for which $A_{7[1−6]}$ can be enhanced up to 10%. For all other solutions, $A_{7[1−6]} < 1\%$. Therefore the measurement of $A_{7[1−6]}$ observable can lead to a unique identification of new physics solution in the form of $C_{9}^{NP} = -C_{10}^{NP}$. The $C_{9}^{NP}$ and $C_{9}^{NP} = -C_{10}^{NP}$ solutions can bolster $A_{8[1−6]}$ at the level of 4-5% whereas for $C_{9}^{NP} = -C_{9}$ scenario, $A_{8[1−6]} \lesssim 1\%$. Therefore measurement of $A_{8[1−6]}$ at the level of few percent would discriminate $C_{9}^{NP} = -C_{9}'$ new
TABLE VI. A plot exhibiting discriminating capabilities of various CP violating observables in $B^+ \rightarrow K^+\mu^+\mu^-$ and $B^0 \rightarrow K^{*0}\mu^+\mu^-$ decays. In each row, we show new physics solutions along with the observables where a meaningful enhancement is allowed. These observables are further classified in three categories on the basis of maximum amount of enhancements, $A^{max}$, allowed by the current data. It is obvious that any observable which is placed in $A^{max} \geq 10\%$ or $A^{max} \approx 5-9\%$ column also appears in the preceding columns. The observables marked in green color are termed as unique identifier of the specific new physics solution (appearing in the same row in which these observables appear), i.e., these observables will not appear in any other row. The observables marked in blue and red colors are degenerate observables in the sense that they appear in the preceding columns. The observables marked in green color are termed as unique identifier of the specific new physics scenario from other two scenarios. As both $C_9^{NP}$ and $C_7^{NP} = -C_9^{NP}$ solutions allow almost similar enhancement in the value of $A_9^{min}$, a discrimination between these two solutions would not be possible through this observable.

We now consider predictions of $A_i$ observables in the high-$q^2$ bin as given in Table VI. The most conspicuous feature of predictions in the low-$q^2$ region was related to the observable $A_7$ which insinuated to be a potential observable to verbalize the signatures of weak phase related to the new physics solution $C_9^{NP} = -C_9^{NP}$. However, unlike in the low-$q^2$ bin, all allowed 1D solutions fail to provide an enhancement in $A_7$ above a percent level in the high-$q^2$ region. The $C_9^{NP} = -C_9^{NP}$ scenario failed to make any noticeable indentations in the low-$q^2$ bin as it was unable to provide any detectable enhancements in any of the considered $A_i$ observables. However, in the high-$q^2$ bin, this solution appears to make a riveting impact as it can enhance $A_8$ and $A_9$ observables up to a level $\sim (4 \pm 6)\%$. All other favored scenarios fail to provide any meaningful enhancement in these observables. This thus implies that the observation of either $A_8$ or $A_9$ asymmetries at the level of a few percent in the high-$q^2$ bin may provide confirmatory evidence in support of the $C_9^{NP} = -C_9^{NP}$ scenario.

The observable $A_5^b$ failed to make any imprint in the low-$q^2$ bin. However in [15-19] bin, this can be enhanced up to $\sim (2 \pm 3)\%$ by $C_9^{NP} = -C_9^{NP}$ scenario. The other two scenarios predict $A_5^b < 1\%$. Therefore the observation of $A_5^b$ in the high-$q^2$ bin at the level of a few percent would provide an unambiguous signature of new physics in the form of $C_9^{NP} = -C_9^{NP}$ solution. None of the 1D solutions in [1-6] bin provided any meaningful enhancements in $A_3, A_4, A_5$ angular observables. On the contrary, all of these observables can be enhanced up to a level of a few percent in the high-$q^2$ bin. The $A_5$ observable can be brought to a percent level by $C_7^{NP} = -C_9^{NP}$ solution. All solutions have the potential to bring $A_5$ observable up to a level of a percent or more. A similar job for $A_6$ in [15-19] observable can be done by $C_9^{NP} = -C_9^{NP}$ and $C_9^{NP} = -C_9^{NP}$ solutions.

Thus we see that an accurate measurement of $A_{CP}$ in the high-$q^2$ bin along with a number of $CP$ violating angular observables would enable unique identification of possible new physics in $b \rightarrow s\ell\ell$ transition. This can be easily understood with the help of summary Tab. VI. In this table, we have listed those observables for which the current allowed solutions can provide meaningful enhancement, say values above 2%. For each new physics solutions, the observables are distributed in different blocks based on their allowed values.

The observables represented in green color are unique identifiers. For e.g, the observable $A_{7,8,1-6}$ appears in all three blocks. This implies that the measurement of $A_{7,8,1-6}$ with any value greater than 2% can provide confirmatory evidence for new physics in the form of $C_9^{NP} = -C_9^{NP}$ solution. Similarly, measurement of $A_5^b$ in (2-5)% range would turn out to be another unique identi-
FIG. 2. The left panel portrays correlations between $A_{CP}$ in $B^0 \rightarrow K^{*0}\mu^+\mu^-$ and $B^+ \rightarrow K^+\mu^+\mu^-$ in the high-$q^2$ region for all favored “1D” solutions. A correlation between $A_{CP}$ in $B^0 \rightarrow K^{*0}\mu^+\mu^-$ in the high-$q^2$ bin and $A_8$ observable in the low-$q^2$ region ($A_8^*$) is depicted in the right panel.

FIG. 3. The left panel portrays correlations between $A_{CP}$ in $B^+ \rightarrow K^+\mu^+\mu^-$ in the high-$q^2$ and $A_7$ observable in the low-$q^2$ region for all favored “1D” solutions. A correlation between $A_{CP}$ in $B^+ \rightarrow K^+\mu^+\mu^-$ in the high-$q^2$ bin and $A_8$ observable in the low-$q^2$ region is depicted in the right panel.

The observables represented by blue and red colors provide signatures of new physics for multiple scenarios. The $A_{CP}^{K^+}$ in the high-$q^2$ bin is allowed to have values in the range of $(2-5)\%$ for all three scenarios. Similarly, $A_{CP}^{K^*}$ in high-$q^2$ and $A_8$ in low-$q^2$ bin can be enhanced by $C_9^{NP}$ as well as $C_9^{NP} = -C_{10}^{NP}$ scenarios. Therefore, a careful scrutiny is required to see whether these observables in combination with others can also provide unique identification. It is apparent from the table that
FIG. 4. These plots reveal correlations between $A_{\text{CP}}$ in $B^0 \to K^{*0} \mu^+ \mu^-$ and several $A_i$ observables for all favored “1D” solutions in the high-$q^2$ bin.
the following combinations can also serve as a useful dis-

- \([A^K_{\text{CP}} - A^K_{\text{NP}}}]\) in high-\(q^2\) region: A simultaneous measure-
  ment of these observables can be used as a good dis-
m

- \(A^s_{[1-6]} - A^K_{\text{CP}}\) (high-\(q^2\)): A simultaneous measure-

We now delve correlations in the high-\(q^2\) region. The interrelations between \(A^s_{\text{CP}}\) and \(A^s_i\) are demonstrated in Fig. 3. Here we do not consider \(A^s_7\) observable as none of the allowed 1D scenarios can enhance it up to a level of a percent. The \((A^K_{\text{CP}} - A^s_4)\) and \((A^K_{\text{CP}} - A^s_4)\) corre-
lations are almost similar. These \(CP\) violating angular observables are anti-correlated with \(A^K_{\text{CP}}\) for the three allowed solutions, i.e., a negative value of \(A^K_{\text{CP}}\) would im-
ply \(A^s_{i,4} > 0\) and vice versa. Further, \(|A^s_{\text{CP}}| \approx 3\%\), which is the maximum allowed value of \(A^s_{\text{CP}}\) with the current data, can lead to \(|A_{3,4}| \approx 2\%\) for \(C_{9NP} = -C'_{9}\) scenario. For \(C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP}\) scenarios, \(A_{8,9}^k_{\text{max}}\) implies \(A_{3,4} \approx 1\%\).

From \((A^K_{\text{CP}} - A^s_{8,9})\) plots, it is obvious that \(A^s_i\) (\(A^s_{8,9}\)) has negative (positive) correlations with \(A^K_{\text{CP}}\). For \(C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP}\) solution. For e.g., a measurement of \(A^K_{\text{CP}}\) with a value \(\approx -3\%\) would lead to an observation of \(A^s_5\) (\(A^s_6\)) with a value \(\approx 1\%\) (\(\approx -2\%\)). Therefore simultaneous measurements of \(A^K_{\text{CP}}\) and \(A^s_{8,9}\) can discriminate \(C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP}\) solution from others. The \((A^K_{\text{CP}} - A^s_5)\) and \((A^K_{\text{CP}} - A^s_9)\) correlations features are almost similar for the three scenarios. Here \(A^s_{8,9}\) have anti-correlations with \(A^K_{\text{CP}}\) for the \(C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP}\) scenario. For this solution, a measurement of \(A^K_{\text{CP}} \approx -3\%\) would imply \(A^s_{8,9} \approx 5\%\).

As of now, we have been emphasizing on the measure-
ments of the angular observables \(A^s_i\) along with \(A^K_{\text{CP}}\)’s for discriminating between the allowed solutions. How-
never, these correlations will also be helpful in discarding or identifying a particular scenario with a precise measure-
ment of \(A^s_i\) even if we only have upper bounds on the \(A^s_i\) observables. For e.g., from \((A^K_{\text{CP}} - A^s_8)\) correlation plot in the high-\(q^2\) region, it is evident that a finite value of \(A^K_{\text{CP}}\) indicates a finite value of \(A^s_i\) for \(C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP}\) scenario. In case, the experimental upper bounds on \(A^s_8\) slips below the value predicted by the correlation plot (on the basis of the measured value of \(A^K_{\text{CP}}\)), the given scenario would be disfavored.

This may also help in the identification of \(C_{9NP}^\text{CP}\) solu-
tion. As evident from the left panel of Fig. 3 for this solution (as well as \(C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP}\), \(A^K_{\text{CP}}\) and \(A^K_{\text{CP}}\) in the high-\(q^2\) have positive correlation, and a larger value in one will imply the same for other observable. Therefore if both of these observables are measured, say with a value \(\gtrsim 2\%\), this can only be due to either \(C_{9NP}^\text{CP}\) or \(C_{9NP}^\text{CP} = -C_{9NP}^\text{CP}\) scenario. This degeneracy can be re-
moved by inspecting correlations of \(A^s_i\) with \(A^s_i\) observ-
ables. For e.g., \((A^K_{\text{CP}} - A^s_6)\) plot predicts \(|A^s_6| \approx 2\%\) for \(C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP} = -C_{9NP}^\text{CP}\) solution corresponding to \(|A^s_{10}| \approx 2\%\) and 0 for \(C_{9NP}^\text{CP}\) scenario. Therefore if the experimental upper bound on \(A^s_6\) observable falls below 2%, such a scenario can only be accommodated by the \(C_{9NP}^\text{CP}\) solution.

On similar lines, the simultaneous measurements of \(A^K_{\text{CP}}\) and \(A^K_{\text{CP}}\) in the high-\(q^2\) bins can be used as a good identi-
fier between the possible NP solutions. In case, \(A^K_{\text{CP}}\) is measured at the level of \(\gtrsim 2\%\) and the upper bounds on \(A^K_{\text{CP}}\) shrinks to less than 2%, it would be
enough to identify $C_{9}^{NP} = -C_{9}'$ solution by disfavoring the other two.

V. CONCLUSIONS

Assuming new physics Wilson coefficients to be complex, we perform a model-independent global fit to all apropos $b \rightarrow s\ell\ell^*$ ($\ell = e, \mu$) data. This include updated measurements of $R_K$ and $R_{K^*}$ by the LHCb collaboration in December 2022 together with the updated measurement of the branching ratio of $B_s \rightarrow \mu^+\mu^-$ by the CMS collaboration and the measurements of several $B_s \rightarrow \phi\mu^+\mu^-$ observables. We work under the assumption that the new physics equally affects both the muon and electron sectors. For comparison, we also update the fits for real couplings under the assumption of universal couplings. Considering only one operator or two related operators at a time, we obtain the following:

- The allowed solutions remain the same as obtained for the real fits, i.e. $C_{9}^{NP}$, $C_{9}^{NP} = -C_{9}^{NP}$ and $C_{9}^{NP} = -C_{9}'$ scenarios still provide a good or moderate fits to the data.

- The $C_{9}^{NP}$ and $C_{9}^{NP} = -C_{9}'$ scenarios now becomes the most preferred one as the $\Delta\chi^2$ for $C_{9}^{NP} = -C_{9}^{NP}$ solution falls by $\sim 10$ below $\Delta\chi^2$ values for the other two scenarios. Therefore the $C_{9}^{NP} = -C_{9}^{NP}$ scenario can only be considered as a moderate solution.

- The $C_{10}^{NP}$ scenario which provided a moderate fit to the data before CMS and December 2022 LHCb updates now fails to provide any significant improvement in the value of $\Delta\chi^2$.

We find that the current data allows complex couplings to exist with an upper bound similar to that of their real counterparts. The effect of such a weak phase can show up in some of the CP asymmetries. For the favoured solutions, we obtain predictions of several CP-violating observables in $B^0 \rightarrow K^{*0}\mu^+\mu^-$ and direct CP asymmetry in $B^+ \rightarrow K^+\mu^+\mu^-$. These asymmetries can be observed at the current or planned experimental facilities provided new physics enhances them up to a level of a few percent. Following are our main observations:

- None of the new physics solutions can enhance $A_{CP}$ in the low-$q^2$ bin at the level of a few percent. Such an enhancement is feasible only in the high-$q^2$ region. This is true for $B^+ \rightarrow K^+\mu^+\mu^-$ as well as $B^0 \rightarrow K^{*0}\mu^+\mu^-$. For $B^+ \rightarrow K^+\mu^+\mu^-$, such an enhancement can be provided by $C_{9}^{NP}$ or $C_{9}^{NP} = -C_{10}^{NP}$ solutions, the enhancement being more pronounced for the later solution. For $B^0 \rightarrow K^{*0}\mu^+\mu^-$ decay, all solutions can serve this purpose.

- All allowed solutions predict $A_{3,4,5,6}$ and $A_9$ asymmetries to be less than a percent level in the low-$q^2$ bin. However, the predictions of observable $A_7$ provides eupheptic sign for $C_{9}^{NP} = -C_{10}^{NP}$ solution as $A_{7[1-6]}$ can be enhanced up to 10%. Therefore $A_{7[1-6]}$ can be termed as an unique identifier for $C_{9}^{NP} = -C_{10}^{NP}$ solution.

- The $C_{9}^{NP}$ and $C_{9}^{NP} = -C_{10}^{NP}$ solutions can bolster $A_{8[1-6]}$ at the level of 4-5%.

- The observation of any of the $A_4$, $A_8$ or $A_9$ observables at a level of a few percent in the high-$q^2$ bin may provide confirmatory evidence in support of the $C_{9}^{NP} = -C_{9}'$ scenario.

- The observable $A_{3,4,5}$ and $A_6$ failed to make any impact in the low-$q^2$ bin. However in [15-19] bin, all of these observables can be enhanced up to a level of a percent or more. A measurement of $A_{7}$ up to (2-3)% level would provide unique identification of $C_{9}^{NP} = -C_{10}^{NP}$ solution.

Finally, we study correlations between $A_{CP}$ and other CP asymmetries. Our findings are as follows:

- A simultaneous measurement of $A_{CP}^{K^0}$ and $A_{CP}^{K^*}$ in the high-$q^2$ bin can be used as a good discriminant for $C_{9}^{NP} = -C_{9}'$ solution. The same can also be achieved by simultaneous measurements of $A_{8[1-6]}$ and $A_{CP}^{K^*}$ in the high-$q^2$ region.

- The $(A_{CP}^{K^*-A_3})$ and $(A_{CP}^{K^*-A_4})$ correlations in the high-$q^2$ bin cannot discriminate between any of the solutions whereas a simultaneous measurement of $A_{CP}^{K^*}$ and $A_5$ (or $A_6$) in high-$q^2$ region can distinguish $C_{9}^{NP} = -C_{10}^{NP}$ from other scenarios. A similar identification for $C_{9}^{NP} = -C_{9}'$ solution can be provided by examining $(A_{CP}^{K^*}-A_{8,9})$ correlations in the high-$q^2$ bin.

- If $A_{CP}$ is precisely measured in the high-$q^2$ region, the new physics solutions can also be identified even if we only have upper bounds on the $A_i$ observables. For some scenarios, a discrimination would be possible only through $(A_{CP}^{K^*}-A_{CP})$ correlations in the high-$q^2$ bin.

Therefore the observation of $A_{CP}$ as well as CP violating angular observables will not only provide an evidence of new physics with complex phase but their accurate measurements would also facilitate the unique identification of possible new physics in the decays induced by the $b \rightarrow s\ell\ell^*$ transition. The direct CP asymmetry can be measured at the LHCb or Belle-II, however the measurements of CP violating angular observables require higher statistics which can be attained at the HL-LHC [83].

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Appendix A: Decay rate of $B \to K \mu^+ \mu^-$ decay

The decay rate of $B \to K \mu^+ \mu^-$ is given by \[84\] \[85\]
\[
\Gamma(B \to K \mu^+ \mu^-) = \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 \left( 2a_{\mu}(q^2) + \frac{2}{3} c_{\mu}(q^2) \right),
\]
where
\[
a_{\mu}(q^2) = E(q^2) \left[ q^2 |F_\parallel|^2 + \frac{\lambda}{4} (|F_V|^2 + |F_A|^2) + 2m_\mu (m_B^2 - m_K^2 + q^2) \text{Re}(F_\parallel F_\perp^*) + 4m_B^2 m_B^2 |F_A|^2 \right],
\]
\[
c_{\mu}(q^2) = -\frac{\lambda}{4} \beta_{\mu}^2 E(q^2) (|F_V|^2 + |F_A|^2) ,
\]
with
\[
E(q^2) = \frac{G_F^2 |V_{tb} V_{ts}^*|^2}{512 \pi^2 m_B^2} \beta_\mu \lambda_K.
\]

The expressions of transversity amplitudes can be found in ref. \[85\]. These amplitudes are written in terms of form-factors \[73\] and non-local contributions related with charm-quark loops \[89\] \[87\]. In the high-$q^2$ region, all form-factors reduce to one soft form-factor \[80\] \[87\]. In the high-$q^2$ region too, symmetry relations among the form factors can be delved with the improved Isgur-Wise relation \[88\].

Appendix B: Angular coefficients in $B^0 \to K^{*0} \mu^+ \mu^-$ decay

The angular coefficients appearing in the four-fold distribution of $B \to K^* \to K \pi \mu^+ \mu^-$ decay can be expressed in terms of transversity amplitudes as \[8\]

\[
I_1 = \frac{(2 + \beta_{\mu}^2)}{4} \left[ |A_\parallel|^2 + |A_\perp|^2 + (L \to R) \right]
+ \frac{4m_B^2}{q^2} \text{Re} \left( A_\parallel^L A_{\parallel R}^* + A_\perp^L A_{\perp R}^* \right),
\]
\[
I_2 = |A_\parallel|^2 + |A_\perp|^2 + \frac{4m_B^2}{q^2} \left[ |A_{\parallel L}|^2 + 2 \text{Re} \left( A_{\parallel R}^L \right) \right],
\]
\[
I_3 = \frac{\beta_{\mu}^2}{2} \left[ |A_\parallel|^2 - |A_\perp|^2 + (L \to R) \right],
I_4 = \frac{\beta_{\mu}^2}{2} \left[ \text{Re}(A_\parallel^L A_{\parallel R}^*) + (L \to R) \right],
\]
\[
I_5 = \sqrt{2} \beta_{\mu} \left[ \text{Re}(A_\parallel^L A_{\parallel R}^*) - (L \to R) \right],
I_6 = 2 \beta_{\mu} \left[ \text{Re}(A_\parallel^L A_{\parallel R}^*) - (L \to R) \right],
I_7 = \sqrt{2} \beta_{\mu} \left[ \text{Im}(A_\parallel^L A_{\parallel R}^*) - (L \to R) \right],
I_8 = \frac{\beta_{\mu}^2}{\sqrt{2}} \left[ \text{Im}(A_\parallel^L A_{\parallel R}^*) + (L \to R) \right],
I_9 = \frac{\beta_{\mu}^2}{\sqrt{2}} \left[ \text{Im}(A_\parallel^L A_{\parallel R}^*) + (L \to R) \right].
\]

Here $\lambda = m_B^4 + m_K^4 + q^4 - 2(m_B^2 m_K^2 + m_B^2 q^2 + m_K^2 q^2)$ and $\beta_{\mu} = \sqrt{1 - 4m_B^2 / q^2}$. In the low-$q^2$ region, all form-factors reduce to one soft form-factor \[80\] \[87\]. In the high-$q^2$ region too, symmetry relations among the form factors can be delved with the improved Isgur-Wise relation \[88\].
determined from lattice computations [97, 98].
