Rainbow Vertex Coloring for Line, Middle, Central, Total Graph of Comb Graph

C. S. Hariramkumar* and N. Parvathi
Faculty of Engineering and Technology, Department of Mathematics, SRM University, Kattankulathur - 603203, Tamil Nadu, India; hariramkumar81@gmail.com, Parvathi.n@ktr.srmuniv.ac.in

Abstract

Objectives: To find the rainbow vertex connection number for Line, Central, Middle and Total graphs of Comb Graph.
Methods/Statistical Analysis: The methods to find the rainbow vertex connection number of any graph G is quite different from other coloring problems. Findings: The rainbow vertex connection number for line graph and middle graph of comb graph is \( \left\lceil \frac{n}{2} \right\rceil \) if \( n \) is even, \( \left\lceil \frac{n}{2} \right\rceil - 1 \) if \( n \) is odd, where \( n \geq 7, 11 \). While finding the achromatic number for any graph G, noting that no two adjacent vertices receive the same color. Application/Improvements: The applications of rainbow vertex connection number are same as rainbow connection number.

Keywords: Central and Total Graph of Comb Graph, Line, Middle, Rainbow Vertex Coloring, Rainbow Vertex Connection Number

1. Introduction

In, make-known the 'Rainbow Connection' in which the ideas are implemented in many applications. Rainbow connection expresses ideas as taking distinct passwords in the intermediate agencies connecting between any two agencies. The Concept “Rainbow Connection” implemented on Information security purposes between any two agencies. This application is to share secured information in any path between any two agencies, provided the information cannot view by the third party, which leads to security purposes. This application are modeled in graph theory as edges can be treated as passwords, adjacent edges are assign same color but in the path wise, no two edges have same color connecting between any two vertices. All graphs are finite, simple and undirected. Any notation or terminology follows from the book. The existence of rainbow vertex connection number was brought up by the article. The definitions of Line Graph and Total Graph of any graph are taken. A 1-regular caterpillar is called a Comb. Shows the other references. In Graph Theory, the connectivity is the fundamental graph-theoretic subject in combinatorial, algorithmic sense. There are many concepts existed due to applications raised and durable for the serving the purpose. Among them, the new and fabricating concept is rainbow connection, which provides strength to connectivity.

2. Main Results

Line Graph of Comb Graph

Theorem 4. Let \( G = L(CG) \) where \( \text{diam} \ (G) \geq 2 \), then \( rvc(G) = \left\lceil \frac{n}{2} \right\rceil \).

Proof: Let \( x \in V(G) \), \( x \) is peripheral vertex in G and \( \text{diam} \ (G) \geq 2 \).

Let \( T(x) = \{v \in V(G) : d(x,v) = i\} \) where \( i \in [1, \text{diam}(G)] \). Let \( U_{T(x)} \) (\( i \) is even) = \( V_1 \) and \( U_{T(x)} \) (\( i \) is odd) = \( V_2 \) and \( V_3 = \{x\} \) where \( x \) is peripheral vertex in \( G \).

Case 1: \( \text{diam} \ (G) \) is even of atleast 2

*Author for correspondence
For \( n = 3 \), Here diam \((G) = 2\) then \( rvc\( (G) = 1\). Now we assume diam \((G) \geq 4\)

Define a coloring \( c: V(G) \to [1, \left\lfloor \frac{n}{2} \right\rfloor] \) to the vertices of \( G \) as follows, since

\[
V_1 = U T_i(x)(1 \leq i \leq \text{diam}(G), \text{where} \ i \text{ is even}), \ V_2 = U T_i(x)(1 \leq i \leq \text{diam}(G)-1) \text{ is odd}.
\]

Assign color 1 to the vertices \( u_{n,1} (k \in \mathbb{N}) \) and also to \( u_2 \), then assigning remaining \( \left\lfloor \frac{n}{2} \right\rfloor - 1 \) distinct colors to the vertices \( u_{2k}(k \geq 2) \) as shown in the Figure 1. Let \( P_1, P_2 \ldots P_n \) denotes paths, not necessarily distinct. Let \( x_1, x_2, \ldots, x_n \) be the internal vertices in the path \( P_n \). Where the vertices are colored with colors 1, 1, 2, …., \( \left\lfloor \frac{n}{2} \right\rfloor \), and \( c(x) \neq c(y) \) where \( x \) and \( y \) are not initial and terminal vertices in the path \( P_1 \). Then the path \( P_1 \) is rainbow vertex connected.

\[
\text{Hence} rvc(G) \leq \left\lfloor \frac{n}{2} \right\rfloor.
\]

Suppose that if we take \( rvc(G) < \left\lfloor \frac{n}{2} \right\rfloor \) colors, then any two internal vertices \( x, y \) in the path of length 2k where \( c(x) = c(y) \), which is not a rainbow path. Therefore \( rvc(G) \geq \left\lfloor \frac{n}{2} \right\rfloor \).

Which is also a rainbow vertex connected with maximum colors chosen. Thus \( rvc(G) = \left\lfloor \frac{n}{2} \right\rfloor \).

Case 2: diam \((G) \) is odd where \( \text{diam}(G) \geq 3 \)

Since \( V_1 = U T_i(x) \) where \( 2 \leq i \leq \text{diam}(G)-1 \), \( i \) is even and \( V_2 = U T_i(x) \) where \( 1 \leq j \leq \text{diam}(G) \), \( j \) is odd Let \( c(u_{2k-1}) = 1 \), \( k \in \mathbb{N} \) and also to \( u_2 \) and assigning remaining \( \left\lfloor \frac{n}{2} \right\rfloor - 1 \) distinct colors to the vertices \( u_{2k}(k \geq 2) \).

Let \( P_1, P_2 \ldots P_n \) denotes paths, not necessarily distinct. Let \( x_1, x_2, \ldots, x_n \) be the internal vertices in the path \( P_n \), one can easily find that \( c(x) \neq c(y) \) where \( x \) and \( y \) are not initial and terminal vertices in the path \( P_n \). The path \( P_n \) is rainbow vertex connected. Now we assume that \( d(u_{i},u_{n-1}) < \text{diam}(G) \), the vertices are colored with colors 1, 1, 2, …., \( \left\lfloor \frac{n}{2} \right\rfloor \), Hence \( rvc(G) \leq \left\lfloor \frac{n}{2} \right\rfloor \).

Suppose that we have a vertex coloring for \( G \) with fewer than \( rvc(G) \) colors, any two internal vertices \( x,y \) in the path of length \( \text{diam}(G) \) have same color, then we do not have a rainbow path. Therefore \( rvc(G) \geq \left\lfloor \frac{n}{2} \right\rfloor \). On the other hand, if we take \( rvc(G) > \left\lfloor \frac{n}{2} \right\rfloor \) colors which is also a rainbow vertex connected with maximum colors chosen. Thus \( rvc(G) = \left\lfloor \frac{n}{2} \right\rfloor \).

![Figure 1. Line graph of comb graph.](image)

### 3. Rainbow Vertex Connection Number for Middle Graph of Comb Graph

**Theorem 5.1:** Let \( G = M[CG_n] \) of even order, where \( n \geq 2 \) then \( rvc(G) = \left\lfloor \frac{n}{2} \right\rfloor \).

**Proof:** Let diam \((G) = k \geq 4 \). Let \( u,v \in V(G) \) be such that \( d_G(u,v) = d \) where \( d_G(u,v) \) represents distance between \( u \) and \( v \).

**Case 1:** \( n = 8k-1, k \geq 1 \)

Define coloring to the vertex set \( V(G) \) as follows : \( c(u_i) = 1, i \in \{1, n\} \) and assigning the remaining \( \left\lfloor \frac{n}{2} \right\rfloor - 1 \) distinct colors to the vertices \( u_{i} \) where \( i \in \{2, 4k-1\} \) and \( c(u_i) = 1 \).

Without loss of generality \( u = v_1 \) and \( v = v_n \) and let \( d_G(u, v) = k \). Let \( P_1, \ldots, P_n \) be paths, not necessarily distinct. Among the paths \( P_1, \ldots, P_n \), \( P_i \) be the diameter path whose internal vertices have distinct colors. Thus \( rvc(G) \leq \left\lfloor \frac{n}{2} \right\rfloor \).

By taking \( rvc(G) \geq \left\lfloor \frac{n}{2} \right\rfloor \) colors. Define a vertex coloring for \( G \) as follows : \( c(v_i) = c(u_i) = 1 \) for \( 1 \leq i \leq 2k \) and \( c(u_{i,1}) = c(u_{i,4}) = 1 \) and \( c(u_{i,1}) = k \), \( k \geq 1 \) and \( c(u_i) = i \), where \( i \neq 1, 4k-1, i \geq 3 \), \( i \) is odd. Since \( P_i \) is the diameter path, where \( c(u_{i,1}) = c(u_{n,1}) \), then the path is not rainbow path. Thus \( rvc(G) \geq \left\lfloor \frac{n}{2} \right\rfloor \).
By taking \( rvc(G) > \left\lfloor \frac{n}{2} \right\rfloor \) colors, then there will be rainbow path with maximum colors chosen.
Thus \( rvc(G) = \left\lfloor \frac{n}{2} \right\rfloor \)

**Case 2:** \( n = 8k+3, k \geq 1 \)

Let \( V_1 = \{v_i \mid i \in [1,2k+1]\} \), \( V_2 = \{u_i \mid i \in [1,2k+1]\} \) and \( V_3 = \{u_i \mid i \in [1,4k+1]\} \).

Define a vertex coloring \( c: V(G) \rightarrow [1, \left\lfloor \frac{n}{2} \right\rfloor] \) as follows.
Let \( c(v_i) = c(u_i) = 1 \) for \( i \in [1,2k+1] \)
Assigning the remaining \( \left\lfloor \frac{n}{2} \right\rfloor - 1 \) distinct colors to the vertices \( u_i \) where \( [1,4k+1] \)
and by letting \( c(u_1^{2k}) = 2k-1, k \geq 1 \) and \( c(u_1) = 2k, k \in \mathbb{N} \)
as shown in the Figure 2.

**Figure 2.** Middle graph of comb graph.

Without loss of generality assume \( u = v_1 \) and \( v = v_n \)
and let \( d_G(u,v) = k \). Let \( P_1, \ldots, P_n \) be paths, not necessarily distinct. Among the paths \( P_i (1 \leq i \leq n) \), let \( P_i \) be the diameter path whose internal vertices have distinct colors. Then there exists a rainbow path. Thus \( rvc(G) \leq \left\lfloor \frac{n}{2} \right\rfloor \)

Suppose by taking \( rvc(G) < \left\lfloor \frac{n}{2} \right\rfloor \) colors, then we have to show that there will be no rainbow vertex connected graph. Define a vertex coloring \( c: V(G) \rightarrow [1, \left\lfloor \frac{n}{2} \right\rfloor] \) as follows. \( c(v_i) = c(u_i) = 1 \) for \( (1 \leq i \leq 2k+1) \)
and \( c(u_i) = i \), where \( i \neq 1,4k+1,i \geq 3,i \text{ is odd} \).

Since \( P_i \) is the diameter path, \( c(u_1) = 1 \) and \( c(u_1) = \frac{n}{2} \) where \( u_1 \) and \( u_1 \)
are not initial and terminal vertices. Thus \( rvc(G) \geq \left\lfloor \frac{n}{2} \right\rfloor \).
Suppose if we taking \( rvc(G) > \), then there will be rainbow path with maximum colors \( \left\lfloor \frac{n}{2} \right\rfloor \) chosen. Thus \( rvc(G) = \left\lfloor \frac{n}{2} \right\rfloor \).

**4. Rainbow Vertex Connection Number for Central Graph of Comb Graph**

**Theorem 6.1** Let \( G \) be a Central Graph of Comb Graph with diameter \( d = 3 \), then \( rvc(G) = 2 \).

**Proof:** For \( n = 7 \), it is obvious that \( rvc(G) = 2 \). For \( n \geq 11 \), we distinguish the following cases according to maximum degree \( \Delta(G) \).

**Case 1:** \( \Delta(G) = 4k+1, k \geq 1 \)

Now we assume that \( n = 8k+3, k \geq 1 \). Let \( G \) be a graph with vertex set \( V(G) = V_1 \cup V_2 \cup V_3 \) where \( V_1 = \{v_i \mid 1 \leq i \leq 2k+1\} \), \( V_2 = \{u_i \mid 1 \leq i \leq 2k+1\} \) and \( V_3 = \{u_i \mid 1 \leq i \leq 4k+1\} \). Define a vertex coloring
\( c: V(G) \rightarrow [1, \left\lfloor \frac{n}{2} \right\rfloor] \) as follows. Assign color 1 to the vertex \( v_i \) and assign color 2 to the remaining vertices of \( V_i \) \( c(v_i) = 1 \) for \( (1 \leq i \leq 2k+1) \)
and \( c(u_1) = 1 \) for \( (1 \leq i \leq 4k+1) \) as shown in the Figure 3. Every path is rainbow vertex connected with this coloring. Thus \( rvc(G) \leq 2 \). To show that \( rvc(G) = 2 \).

First we show that \( rvc(G) \neq 1 \). Suppose if we are coloring the vertices with only one color as follows,
\( c(u_i) = c(v_i) = 1 \) for \( (1 \leq i \leq 2k+1) \)
and \( c(u_1) = 1 \) for \( (1 \leq i \leq 4k+1) \). Let \( P_i \) be a path \( u_1, v_i, v_3 \) \( u_{4k+1} \)
and \( v_3 = u_{4k+1} \), since \( c(v_i) = 1 \), there exists no rainbow path \( P_i \).

Thus \( rvc(G) \neq 1 \). Now we show that \( rvc(G) \geq 2 \). Suppose \( c: V(G) \rightarrow [i] \) where \( i \geq 3 \), then every path is rainbow vertex connected with maximum colors. Thus \( rvc(G) = 2 \).

**Figure 3.** Total graph of comb graph.

**Case 2:** \( \Delta(G) = 4k+3, k \geq 1 \)

Now we assume that \( n = 8k+7, k \geq 1 \). Let \( G \) be a graph with vertex set \( V(G) = V_1 \cup V_2 \cup V_3 \) where \( V_1 = \{v_i \mid 1 \leq i \leq 2k+2\} \), \( V_2 = \{u_i \mid 1 \leq i \leq 2k+2\} \) and \( V_3 = \{u_i \mid 1 \leq i \leq 4k+3\} \). Define a vertex coloring
\( c: V(G) \rightarrow [1, \left\lfloor \frac{n}{2} \right\rfloor] \) as follows. Assign color 1 to the vertex \( v_i \) and assign color 2 to the remaining vertices of \( V_i \) \( c(v_i) = 1 \) for \( (1 \leq i \leq 2k+2) \)
and \( c(u_1) = 1 \) for \( (1 \leq i \leq 4k+3) \) as shown in the Figure 3. Every path is rainbow vertex connected with this coloring. Thus \( rvc(G) \leq 2 \). To show that \( rvc(G) = 2 \).

First we show that \( rvc(G) \neq 1 \). Suppose if we are coloring the vertices with only one color as follows,
\( c(u_i) = c(v_i) = 1 \) for \( (1 \leq i \leq 2k+2) \)
and \( c(u_1) = 1 \) for \( (1 \leq i \leq 4k+3) \). Let \( P_i \) be a path \( u_1, v_i, v_3 \) \( u_{4k+3} \)
and \( v_3 = u_{4k+3} \), since \( c(v_i) = 1 \), there exists no rainbow path \( P_i \).

Thus \( rvc(G) \neq 1 \). Now we show that \( rvc(G) \geq 2 \). Suppose \( c: V(G) \rightarrow [i] \) where \( i \geq 3 \), then every path is rainbow vertex connected with maximum colors. Thus \( rvc(G) = 2 \).
Rainbow Vertex Coloring for Line, Middle, Central, Total Graph of Comb Graph

V₂ =\{uᵢ/1≤i≤2k+2\} and V₃ = \{uᵢ/1≤i≤4k+3\}. Define a vertex coloring \(c:V(G) \to \{1,2\}\) as follows. \(c(v₁)=c(uᵢ)=1\) for every \(i (1≤i≤2k+2)\), \(c(uᵢ) =1\) for \(1≤i≤4k+3\) and assigning color 2 to the remaining vertices of \(V₁\). Every path is rainbow vertex connected with this coloring. Thus \(\text{rvc} (G) ≤2\).

To show that \(\text{rvc} (G) ≥2\). The same argument follows from case 1.

7. Rainbow Vertex Connection Number for Total Graph of Comb Graph

**Theorem 7.1**: Let \(G = T[CGₙ]\) of even order where \(n ≥ 2\), then \(\text{rvc}(G) = \frac{n}{3}\) if \(n=7,11\) and \(\text{rvc}(G) = \frac{n}{3} - 1\) if \(n = 4k+11, k ∈ N\).

**Proof**: Let \(G\) be a graph with vertex set \(V(G) = V₁ \cup V₂ \cup V₃\) where \(V₁ = \{vᵢ ; 1≤i≤n\}\), \(V₂ = \{uᵢ;1≤i≤n\}\) and \(V₃ = \{uᵢ¹; 1≤i≤n\}\). We distinguish the proof according to the order of graph \(G\).

**Case 1**: \(n = 7,11\)

For \(n = 7,11\). Define a vertex coloring \(c:V(G) \to \{1,\ldots,\frac{n}{3}\}\) as follows. \(c(vᵢ)=1\) for \(1≤i≤\frac{n}{3}\) and \(c(uᵢ)=i\) for \(1≤i≤\frac{n}{3}\). Then there exists a rainbow path connecting between any two vertices in \(G\), so \(c\) is a rainbow vertex coloring. Thus \(\text{rvc}(G) ≤ \frac{n}{3}\).

Now, we show that \(\text{rvc}(G) ≥ \frac{n}{3}\). If we have vertex coloring fewer than \(\frac{n}{3}\) colors, then the two vertices in the internal vertices have same color. Let \(P₁,…,Pₙ\) be paths, not necessarily distinct. Let \(P\) be a path of length exactly \(\frac{n}{3}\) diam \((G)\). Let \(u,v ∈ V(G)\), then some two vertices \(u\) and \(v\) in the internal vertices have same color, then there will be no rainbow path. Hence by taking \(\text{rvc}(G) ≥ \frac{n}{3}\), which contradicts the definition of rainbow vertex connected. Hence \(\text{rvc}(G) ≥ \frac{n}{3}\).

If we have vertex coloring more than \(\frac{n}{3}\) colors then there exists a rainbow path connecting between every pair of vertices with maximum colors, which contradicts the definition of rainbow vertex connection number. Thus \(\text{rvc}(G) = \frac{n}{3}\) will be minimum compared to \(\text{rvc}(G) ≥ \frac{n}{3}\).

**Case 2**: Now, we assume that \(n = 4k+11, k ≥1\)

Let us consider the two subcases to show that \(\text{rvc}(G) = \frac{n}{3} - 1\)

Let \(G\) be a graph with vertex set \(V(G) = V₁ \cup V₂ \cup V₃\) where \(V₁ = \{vᵢ ; 1≤i≤2k+2\}\) and \(V₂ = \{uᵢ;1≤i≤2k+2\}\) and \(V₃ = \{uᵢ¹; 1≤i≤4k+3\}\).

Define a vertex coloring \(c : V(G) \to \{1,\ldots,\frac{n}{3}\}\) - 1\) as follows. \(c(vᵢ)=1\) for \(1≤i≤2k+2\) and \(c(uᵢ¹)=1\) for \(1≤i≤4k+3\) and assigning the remaining \(\frac{n}{3}\) colors to the vertices of \(V(G)\), \(V₁ \cup V₃\) as follows, \(c(uᵢ)=i\) for \(1≤i≤2k+2\) as shown in the Figure 4.

Let \(P₁,…,Pₙ\) be a paths, not necessarily distinct. Among the paths \(P₁(1≤i≤n)\), let \(P\) be the path having distance \(d₂(u,v)=\text{diam}(G)\), whose internal vertices have distinct colors. Then every path is rainbow Vertex connected, obviously the graph is rainbow vertex connected graph.

Thus \(\text{rvc}(G) ≤ \frac{n}{3} - 1\). If we coloring the vertices fewer than \(\frac{n}{3} - 1\) colors. Suppose if we coloring the vertices with \(\frac{n}{3} - 2\) colors, then we define the vertex coloring \(c₂ : V(G) \to \frac{n}{3} - 2\) as follows. Assigning the color 1 to the vertices of degree exactly 2 and \(c₂(uᵢ¹)=1\) for \(1≤i≤n\), and \(c₂(uᵢ)=2k-1, k ∈ N\), \(c₂(uᵢ¹)=i\) , \(1≤i≤n-1\).where \(i,n\) are even and \(c(uᵢ)=1\) if \(i=n\). Let \(P\) be the path having distance \(d₂(u,v)=\text{diam}(G)\) where two vertices say \(uᵢ\) and \(uᵢ¹\) exists in the internal vertices in the path \(P\), have same color, then the path is not a rainbow path, then the graph is not rainbow vertex connected graph.

Thus if we have fewer than \(\frac{n}{3} - 1\) colors, then the graph is not rainbow vertex connected graph.
Thus \( \text{rvc}(G) \geq \left\lfloor \frac{n}{3} \right\rfloor - 1 \). If we have vertex coloring \( c \) more than \( \left\lfloor \frac{n}{3} \right\rfloor - 1 \) colors, then there will be rainbow vertex coloring with maximum colors chosen which is contradiction to the definition of rainbow vertex connection number. Thus \( \text{rvc}(G) = \left\lfloor \frac{n}{3} \right\rfloor - 1 \) colors will be minimum, which is not contradiction to the definition of rainbow vertex connection number. Therefore \( \text{rvc}(G) = \left\lfloor \frac{n}{3} \right\rfloor - 1 \).

**Subcase 2.2:** For \( n = 8k+11, k \in \mathbb{N} \)

Let \( G \) be a graph with vertex set \( V(G) = V_1 \cup V_2 \cup V_3 \) where \( V_1 = \{v_i; 1 \leq i \leq 2k+3\} \) and \( V_2 = \{u_i; 1 \leq i \leq 2k+3\} \) and \( V_3 = \{u_i; 1 \leq i \leq 4k+5\} \).

Define a vertex coloring \( c_3 : V(G) \to [1, \left\lfloor \frac{n}{3} \right\rfloor - 1] \) as follows.

\[ c(v_i) = 1 \text{ for } (1 \leq i \leq 2k+3) \text{ and } c(u_i^1) = 1 \text{ for } (1 \leq i \leq 4k+5) \text{ and assigning the remaining colors to the vertices of } V(G) \text{ as follows, } c(u_i) = i \text{ for } (1 \leq i \leq 2k+3). \]

Without loss of generality, let \( u = v_1 \) and \( v = v_n \) and \( P: u-v \) is a diameter path. Assume that \( x_1, \ldots, x_n \) are internal vertices in the diameter path have distinct colors where \( x_1 = u_i^1, x_2 = u_2^1, \ldots , x_n = u_{n-1}^1 \). Clearly the Path \( P \) is a rainbow path. Any path of \( \left\lfloor \frac{n}{3} \right\rfloor - 1 \) length less than \( \text{diam}(G) \) have distinct colors is rainbow vertex connected. Then \( \text{rvc}(G) \leq \left\lfloor \frac{n}{3} \right\rfloor - 1 \).

If we coloring the vertices fewer than \( \left\lfloor \frac{n}{3} \right\rfloor - 1 \) colors, define a vertex coloring \( c : V(G) \to [1, \left\lfloor \frac{n}{3} \right\rfloor - 1] \) as follows. Assigning color 1 to the vertices of degree exactly 2, \( c(u_i^1) = 1 \) where \( i = 4k+5, k \geq 1 \) and \( c(u_i) = i \) \( \forall i \in [1, n-1], \) \( i \) is odd, \( c(u_2) = 1 \) if \( i = n \) and \( c(u_{2k}) = 2k, k \geq 1 \).

Let \( P \) be the path of length exactly \( \text{diam}(G) \), since the vertices \( u_1 \) and \( u_n \) have same color and we note that these two vertices are in the internal vertices of the path \( P \), there exists no rainbow path. Let \( P_1 \) be the another path where the vertices \( u_1 \) and \( u_n \) are in the internal vertices have same color, then there exists no rainbow path. Hence it is not possible to take \( \left\lfloor \frac{n}{3} \right\rfloor - 1 \) fewer than. Thus \( \text{rvc}(G) \geq \left\lfloor \frac{n}{3} \right\rfloor - 1 \).

If we have vertex coloring more than \( \left\lfloor \frac{n}{3} \right\rfloor - 1 \) colors, then every path is rainbow vertex connected with maximum colors chosen which contradicts the definition of rainbow vertex connection number. Thus by choosing \( \text{rvc}(G) = \left\lfloor \frac{n}{3} \right\rfloor - 1 \), colors will be the minimum, which satisfies the definition of rainbow vertex connection number. Thus \( \text{rvc}(G) = \left\lfloor \frac{n}{3} \right\rfloor - 1 \).

**Figure 4. Central graph of comb graph.**

### 8. Conclusion

The difference between rainbow vertex connection number for line graph and middle graph of Bi-star Graph is zero; whereas rainbow vertex connection number for central graph is 2 and total graph of Bi-star graph is same if order of a graph is 7 and 11 and it is same if order of a graph starts with 15 follows arithmetic progression with common difference 4. This result can be extended by introducing Randic index as a method of comparing the Randic index and rainbow vertex connection number of line, middle, central and total graph of Bi-star graph. The applications can be raised with Bi-star Graph can be treated as chemical graph problem as comparing rainbow vertex connection number with chemical and physical properties.

### 9. References

1. Chartrand G, Johns GL, McKeon KA, Zhang P. Rainbow connection in graphs. Math Bohem. 2008; 133(1):1–14.
2. Chartrand C, Zhang P. Chromatic Graph Theory. CRC Press; 2008.
3. Krivelevich M, Yuster R. The rainbow connection of a graph is (at most) reciprocal to its minimum degree. J Graph Theory. 2009; 63(3):185–91.
4. Harary F. Graph theory. Naraosa Publishing House; 2001.
5. Saha A, Sambroni E, Bogerd J, Schulz RW, Gac FL, Lareyre JJ. The cell context influences rainbow trout gonadotropin receptors' selectivity. Indian Journal of Science and Technology. 2011 Aug; 4(S8):1–2.
6. Yano A, Jouanno E, Klopp C, Guiguen Y. Gene expression profiling during gonadal differentiation in rainbow trout (Oncorhynchusmykiss) using a Next Generation Sequencing (NGS) approach. Indian Journal of Science and Technology. 2011 Aug; 4(S8):1–6.
7. Nicol B, Yano A, Jouanno E, Branthonne A, Fostier A, Guiguen Y. Follistatin is expressed along with aromatase in female gonads during sex differentiation in the rainbow trout. Indian Journal of Science and Technology. 2011 Aug; 4(S8):1–2.
8. Valdivia K, Jouanno E, Mouri B, Quillet E, Guyomard R, Volf JN, Galiana-Arnoux D, Cauty C, Fostier A, Guiguen Y. Masculinization in rainbow trout carrying the mal mutation is temperature sensitive. Indian Journal of Science and Technology. 2011 Aug; 4(S8):1–1.
9. Kusakabe M, Takei Y, Luckenbach JA. Relaxin-3 and relaxin/insulin-like family peptide receptor 3 in rainbow trout: Sites of gene expression and changes in messenger RNA levels during spermatogenesis in testes. Indian Journal of Science and Technology. 2011 Aug; 4(S8):1–2.