SCTL: Towards Combining Model Checking and Proof Checking

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Abstract. Model checking and automated theorem proving are two pillars of formal methods. This paper investigates model checking from an automated theorem proving perspective, aiming at combining the expressiveness of automated theorem proving and the complete automaticity of model checking. The focus of this paper is on the verification of the temporal logic properties of Kripke models. The properties are specified by an extended computation tree logic that allows polyadic predicate symbols. The main contributions of this paper are: firstly, the development of a sequent calculus for the extended computation tree logic, taking Kripke models as parameters; secondly, the design of a proof-search algorithm for this calculus and a new automated theorem prover to implement it. The verification process is completely automatic, and produces either a counterexample when the property does not hold, or a certificate when it does. The experimental result compares well to existing tools, and the design choices that lead to the efficiency are discussed.

1 Introduction

Model checking [7] and automated theorem proving [12,17] are two pillars of formal verification methods. They differ by the fact that model checking often uses decidable logics, such as propositional modal logics, while automated theorem proving mostly uses undecidable ones, such as first-order logic. Nevertheless, model checking and automated theorem proving have a lot in common, in particular, both of them are often based on a recursive decomposition of problems, through the application of rules.

This paper investigates model checking from an automated theorem proving perspective, aiming at combining the expressiveness of automated theorem proving and the complete automaticity of model checking. We propose a proof system for Computation Tree Logic (CTL) [9,10] (in the style of sequent calculus), taking
Kripke models as parameters. This calculus is called Sequent Calculus for CTL (SCTL), which suggests in fact a slight extension of CTL with polyadic predicate symbols. The proof search in SCTL coincides with checking the validity of a formula in a Kripke model. Using such a proof system has several advantages. First, it permits to give a certificate for the property when it succeeds. Such a certificate can be verified by an independent proof checker, increasing the credibility in the proved property, and can also be combined with proofs built by other means. Secondly, when the verification of the given property fails, it permits to generate counterexamples as formal proofs instead of sequences of states in traditional model checkers. This allows to explain why the formula does not hold, for instance which formula is not valid in which state.

Different proof systems for modal logic have been proposed (see, for instance, [10,11,13,14,20,21]). When designing such a proof system, one of the main issues is to handle co-inductive modalities, for instance, asserting the existence of an infinite sequence of which all elements satisfy some property. It is tempting to reflect this infinite sequence as an infinite proof and then use the finiteness of the model to prune the search-tree in a proof-search method. Instead, we use the finiteness of the model to keep our proofs finite, like in the usual sequent calculus. This is the purpose of the merge rules of SCTL in Figure 1.

Links between model checking and automated theorem proving have been investigated for long. For instance, in the field of Bounded Model Checking (BMC) [3,23], the model checking problems are translated into the satisfiability problem of Boolean formulae which encode both models and properties, and the satisfiability of the temporal formulae is checked on a set of traces of a given model with some limited length. In some on-the-fly model checking techniques, a collection of proof rules are used to infer when a state in a Kripke structure satisfies a temporal formula [2]. However, these rules are not complete, that is because proof structures for co-inductive properties cannot reduce to axioms. They are designed to help on building a “goal-directed” model checking algorithm, not a standalone proof system.

Our approach is different: we prove SCTL formulae in the sequent calculus directly and automatically. This way, a SCTL formula is provable in the sequent calculus (with a Kripke model \(M\) as parameter) if and only if it is valid in \(M\). Thus, neither models nor properties are needed to be encoded. We show also that SCTL is decidable, and proof search in this calculus always terminates. Another contribution of this paper is an implementation of a proof-search algorithm for SCTL. Instead of translating the CTL formulae to Quantified Boolean formulae (QBFs) [23] or to the format of an existing theorem prover [10], we develop a new automated theorem prover tailored for SCTL (SCTLProV), in programming language OCaml\(^5\). Designing our own system gives us a lot of freedom to optimize it. We illustrate the efficiency of SCTLProV by comparing it with the

\(^5\) http://ocaml.org/
proof checker iProver Modulo\(^\text{6}\) and the three model checkers Verds\(^\text{7}\), NuSMV\(^\text{8}\) and NuXMV\(^\text{9}\), on a benchmark with 2880 test cases. The experimental result shows that SCTLProV compares well to the existing tools.

The efficiency of SCTLProV depends on the following design choices: The first is the implementation of the merge rule. The second is that, unlike traditional symbolic model checkers or bounded model checkers, SCTLProV performs verification by unfolding on demand the transition relation, just like on-the-fly model checking; but unlike traditional on-the-fly model checking where only the transition relation is unfolded on demand, SCTLProV unfolds on demand both the transition relation and the formula during the proof search. The double on-the-fly can avoid visiting unneeded states, thanks to the syntax and the inference rules of SCTL. The third is that, unlike some on-the-fly model checking algorithms [2], our proof search algorithm is written in a continuation-passing style \(^\text{11}\), in order to reduce the stack space. The fourth is that, unlike traditional symbolic model checkers that usually encode the models and the properties into some additional data structures, for instance Binary Decision Diagram (BDD)s or SAT-formulae, SCTLProV verifies the given properties directly on the model, in the sense that we need neither translate the models or properties into some third-party structures, nor translate non-boolean state variables into boolean state ones. Translating non-boolean state variables into boolean state variables may increase the number of state variables, which may cause inefficiency in model checkers that are sensitive to the number of state variables. However, just like traditional symbolic model checkers such as NuSMV, we use BDDs to memorize the visited states, reducing space occupation.

The rest of the paper is organized as follows. In Section 2 we introduce the proof system SCTL. In Section 3 we describe the proof search algorithm and the prover SCTLProV. In Section 4 we first illustrate via examples the applications of SCTLProV, then we compare, over 2880 test cases, SCTLProV with iProver Modulo, Verds, NuSMV and NuXMV, respectively. Conclusions and future works are discussed in Section 5.

2 SCTL

In this section, we present briefly SCTL(\(\mathcal{M}\)), a sequent calculus for CTL, taking Kripke models as parameter. Unlike the usual sequent calculus, where a formula is provable if and only if it is valid in all models, a formula is provable in SCTL(\(\mathcal{M}\)) if and only if it is valid in the model \(\mathcal{M}\). The interested reader is referred to [8] for further details of this section.

2.1 Models and Formulae

\textbf{Definition 1 (Kripke model).} A Kripke model \(\mathcal{M}\) is given by

\[^{6}\text{http://www.ensiie.fr/~guillaume.burel/blackandwhite_iProverModulo.html.en}\]
\[^{7}\text{http://lcs.ios.ac.cn/~zwh/verds/index.html}\]
\[^{8}\text{http://nusmv.fbk.eu/}\]
\[^{9}\text{https://nuxmv.fbk.eu/}\]
Definition 2 (Validity). Let $\mathcal{M}$ be a model and $\phi$ be a closed formula, the validity of a formula $\phi$ in the model $\mathcal{M}$ is defined by induction on $\phi$ as follows.

- a non empty set $S$, whose elements are called states,
- a binary relation $\rightarrow$ defined on $S$, such that for each $s$ in $S$, there exists at least one $s'$ in $S$, such that $s \rightarrow s'$,
- and a family of subsets of $S^n$, where $n$ is a natural number, called relations.

We write $\text{Next}(s)$ for the set $\{s' \mid s \rightarrow s'\}$ which is always finite. A path is a finite or infinite sequence of states $s_0, ..., s_n$ or $s_0, s_1, ...$ such that for each $i$, if $s_i$ is not the last element of the sequence, then $s_{i+1} \in \text{Next}(s_i)$. A path-tree is a finite or infinite tree labelled by states such that for each internal node labelled by a state $s$, the children of this node are labelled by the elements of $\text{Next}(s)$.

Properties of such a model are expressed in a language, tailored for this model, that contains, for each state $s$, a constant, also written $s$; and for each relation $P$, a predicate symbol, also written $P$.

The grammar of $\text{SCTL}(\mathcal{M})$ formulae is displayed below:

$$
\phi := \begin{cases} 
T \mid \bot \mid P(t_1, ..., t_n) \mid \neg P(t_1, ..., t_n) \mid \phi \land \phi \mid \phi \lor \phi \mid \\
AX_x(\phi)(t) \mid EX_x(\phi)(t) \mid AF_x(\phi)(t) \mid EG_x(\phi)(t) \mid \\
AR_{x,y}(\phi_1, \phi_2)(t) \mid EUR_{x,y}(\phi_1, \phi_2)(t) 
\end{cases}
$$

where $x, y$ are variables, and each of $t$ and $t_1, ..., t_n$ is either a constant or a variable.

Note that in this language, modalities are applied to formulae and states, binding variables in these formulae. More explicitly, modalities $AX, EX, AF$, and $EG$ bind the variable $x$ in $\phi$, and modalities $AR$ and $EU$ bind respectively the variable $x$ in $\phi_1$ and $y$ in $\phi_2$. Note also that our predicate symbols may have an arbitrary arity, which constitutes a slight extension of $\text{CTL}$. Finally, note that negation is applied to atomic formulae only, so, as usual, negations must be pushed inside the formulae.

The following abbreviations are used

- $\phi_1 \Rightarrow \phi_2 \equiv \neg \phi_1 \lor \phi_2$,
- $EF_x(\phi)(t) \equiv EU_{x,x}(\top, \phi)(t)$,
- $ER_{x,y}(\phi_1, \phi_2)(t) \equiv EU_{x,y}(\phi_2, ((z/x)\phi_1 \land (z/y)\phi_2))(t) \lor EG_y(\phi_2)(t)$, where $z$ is a variable that occurs neither in $\phi_1$ nor in $\phi_2$,
- $AG_x(\phi)(t) \equiv \neg (EF_x(\neg \phi)(t))$,
- $AU_{x,y}(\phi_1, \phi_2)(t) \equiv \neg (ER_{x,y}(\neg \phi_1, \neg \phi_2)(t))$.

Hereafter, a formula starting with one of the modalities $AX, EX, AF, EF$, $AU$ and $EU$ will be called an inductive formula; and a formula starting with one of the modalities $AR, ER, AG$ and $EG$ will be called a co-inductive formula.
In this section, we consider a fixed finite Kripke model $\mathcal{M}$.

Remark 1. From the definition above, we obtain $\mathcal{M} \models EF_x(\phi)(s)$, if there exists an infinite path $s_0, s_1, ...$ starting from $s$ and a natural number $j$ such that $\mathcal{M} \models (s_j/x)\phi$, etc.

2.2 Proofs

In this section, we consider a fixed finite Kripke model $\mathcal{M}$.

First, consider the formula $AF_x(P(x))(s)$. This formula is valid if there exists a finite tree $T$ whose root is labelled by $s$, such that the children of an internal node labelled by a state $a$ are labelled by the elements of $Next(a)$, and such that all the leaves are in $P$. Such a tree can be called a proof of the formula $AF_x(P(x))(s)$.

Now, consider $AF_x(AF_y(P(x,y))(x))(s)$ that contains nested modalities. To justify the validity of this formula, one needs to provide a tree whose root is labelled by $s$, where at each leaf $a$, the formula $AF_y(P(a,y))(a)$ is valid. And to justify the validity of the formula $AF_y(P(a,y))(a)$, one needs to provide other trees. These hierarchical trees can be formalized with the sequent calculus rules

$$
\frac{\vdash (s/x)\phi \quad \vdash AF_x(\phi)(s)}{\vdash AF_x(\phi)(s_1) \ldots \vdash AF_x(\phi)(s_n)}
$$

Example 1. Consider the model formed with the relation

```
\begin{array}{c}
  \vdots \\
  a & b & c & \cdots \\
  \vdots \\
\end{array}
```
and the set \( P = \{ b, c \} \). A proof of the formula \( AF_x(P(x))(a) \) is

\[
\begin{align*}
\vdash P(b) & \quad \text{atom-R} \\
\vdash AF_x(P(x))(b) & \quad \text{AF-R}_1 \\
\vdash P(c) & \quad \text{atom-R} \\
\vdash AF_x(P(x))(c) & \quad \text{AF-R}_1 \\
\vdash AF_x(P(x))(a) & \quad \text{AF-R}_2
\end{align*}
\]

where besides the rules \( \text{AF-R}_1 \) and \( \text{AF-R}_2 \), we use the rule

\[
\vdash P(s_1, ..., s_n) \quad \text{atom-R} \quad (s_1, ..., s_n) \in P
\]

The case of co-inductive formulae, for instance \( EG_x(P(x))(s) \), is more complex than that of the inductive one, such as \( AF_x(P(x))(s) \). To justify its validity, one needs to provide an infinite sequence, that is an infinite tree with only one branch, such that the root of the tree is labelled by \( s \), the child of a node labelled by a state \( a \) is labelled by an element of \( \text{Next}(a) \), and each node of the tree verifies \( P \). However, as the model is finite, we can always restrict to regular trees and use a finite representation of such trees. This leads us to introduce a rule, called \( \text{EG-merge} \), that permits to prove a sequent of the form \( \vdash EG_x(P(x))(s) \), provided such a sequent already occurs lower in the proof. To make this rule local, we re-introduce hypotheses \( \Gamma \) to record part of the history of the proof. The sequent have therefore the form \( \Gamma \vdash \phi \), with a non empty \( \Gamma \) in this particular case only, and the \( \text{EG-merge} \) rule is then just an instance of the axiom rule, that must be re-introduced in this particular case only.

Note that SCTL needs neither contraction rules nor multiplicative \( \vee \)-R rules, because for each atomic formula \( P \), either \( P \) is provable or \( \neg P \) is. Therefore the sequent \( \vdash \neg P \vee P \) is proved by proving either the sequent \( \vdash \neg P \) or the sequent \( \vdash P \). As we have neither multiplicative \( \vee \)-R rules nor structural rules, if we start with a sequent \( \vdash \phi \), then each sequent in the proof has one formula on the right of \( \vdash \) and none on the left. So, as all sequents have the form \( \vdash \phi \), the left rules and the axiom rule can be dropped as well. In other words, unlike the usual sequent calculus and like Hilbert systems, SCTL is tailored for deduction, not for hypothetical deduction.

The rules of SCTL are depicted in Figure 1.

**Theorem 1 (Soundness and Completeness).** If \( \phi \) is closed, then the sequent \( \vdash \phi \) has a proof if and only if \( M \models \phi \) for the given Kripke model \( M \).

### 3 Implementation

We develop a new automated theorem prover SCTLProV (Figure 2) to implement SCTL in programming language OCaml. SCTLProV reads and interprets an input file containing a description of a Kripke model and a finite number of SCTL formulae. To each formula, it searches for a proof, and outputs a certificate (resp. True) when the verification succeeds, or a counterexample (resp. False) when it does not.
The basic idea of the proof search procedure in SCTLProV is as follows: first we give an order over the inference rules of SCTL with the same conclusion (if any), and for each root under consideration of a Continuation Passing Tree (Definition 3), we give an order over the children of this node. Then, to prove an SCTL sequent \( \Gamma \vdash \phi \), we need to find an inference rule of SCTL such that this sequent matches the conclusion of the rule, and then find successively a proof for each of the premises, according to the given orders. Thus, the proving procedure of sequent \( \Gamma \vdash \phi \) transforms into the proving procedure of all its premises with some specific order. One of the major techniques for the implementation of SCTL is based on the concept of continuation, usually used in compiling and programming [122]. Basically, a continuation is an explicit representation of “the rest of the computation”, which will happen next.
sequent, if the corresponding SCTL result. The sequent to prove, \( \Gamma \), for a given sequent \( \Gamma \), we analyze the form of the formula \( \phi \).

\[
\text{Definition 3 (Continuation Passing Tree). A Continuation Passing Tree (CPT) is a binary tree such that}
\]

- every leaf is labelled by either \( t \) or \( f \), where \( t \) and \( f \) are two different symbols;
- every internal node is labelled by an SCTL sequent.

For each internal node in a CPT, the left subtree is called its \( t \)-continuation, and the right one its \( f \)-continuation. A CPT \( c \) with an SCTL sequent \( \Gamma \vdash \phi \) as its root is often denoted by \( \text{cpt}(\Gamma \vdash \phi, c_1, c_2) \), or visually by

\[
\begin{array}{c}
\Gamma \vdash \phi \\
\overset{c_1}{\triangleright} \overset{c_2}{\triangleright}
\end{array}
\]

where \( c_1 \) is the \( t \)-continuation of \( c \), and \( c_2 \) the \( f \)-continuation.

CPTs are evaluated to \( t \) or \( f \) using the rewrite rules presented in Figure 3 which implement the rules of SCTL. The aim of the rewrite rules is to decide, for a given sequent \( \Gamma \vdash \phi \), if the CPT \( \text{cpt}(\Gamma \vdash \phi, t, f) \) reduces to \( t \) or \( f \). To do so, we analyze the form of the formula \( \phi \). If, for instance, it is \( \Gamma \vdash \phi_1 \land \phi_2 \), we transform, using one of the rewrite rules, the tree \( \text{cpt}(\Gamma \vdash \phi_1 \land \phi_2, t, f) \) into \( \text{cpt}(\Gamma \vdash \phi_1, \text{cpt}(\Gamma \vdash \phi_2, t, f), f) \) expressing that if the attempt to prove \( \Gamma \vdash \phi_1 \) succeeds then we attempt to prove \( \Gamma \vdash \phi_2 \), otherwise it just returns a negative result. The CPT \( \text{cpt}(\Gamma \vdash \phi_1, \text{cpt}(\Gamma \vdash \phi_2, t, f), f) \) is in turn transformed according to the form of \( \phi_1 \). As shown in Figure 3 based on these rewrite rules, given a sequent to prove, SCTLProV can generate a certificate—a proof tree of the sequent, if the corresponding SCTL formula is valid in the Kripke model; or a

| Rule | Description |
|------|-------------|
| \( \text{cpt}(l_1 \vdash c_1, c_2) \rightarrow c_1 \) | CPT evaluation rule |
| \( \text{cpt}(l_1 \vdash \top, c_1, c_2) \rightarrow c_2 \) | CPT evaluation rule |
| \( \text{cpt}(l_1 \vdash P(s_1, ..., s_n), c_1, c_2) \rightarrow c_1 \{s_1, ..., s_n \in \Gamma \} \rightarrow c_2 \{s_1, ..., s_n \notin \Gamma \} \) | CPT evaluation rule |
| \( \text{cpt}(l_1 \vdash \neg P(s_1, ..., s_n), c_1, c_2) \rightarrow c_2 \{s_1, ..., s_n \in \Gamma \} \rightarrow c_1 \{s_1, ..., s_n \notin \Gamma \} \) | CPT evaluation rule |
| \( \text{cpt}(l_1 \vdash \phi_1 \land \phi_2, c_1, c_2) \rightarrow \text{cpt}(l_1 \vdash \phi_1, \text{cpt}(l_1 \vdash \phi_2, c_1, c_2), c_2) \) | CPT evaluation rule |
| \( \text{cpt}(l_1 \vdash \phi_1 \lor \phi_2, c_1, c_2) \rightarrow \text{cpt}(l_1 \vdash \phi_1, \text{cpt}(l_1 \vdash \phi_2, c_1, c_2), c_2) \) | CPT evaluation rule |
| \( \text{cpt}(l_1 \vdash \phi, c_1, c_2) \rightarrow \text{cpt}(l_1 \vdash \phi, c_1, c_2) \) | CPT evaluation rule |

Fig. 3. Rewritings over CPTs.
counterexample—a proof tree of the negation of the sequent, otherwise. The correctness of this algorithm is ensured by the proposition below.

**Proposition 1.** Given a sequent \( \Gamma \vdash \phi \),

- \( \text{cpt}(\Gamma \vdash \phi, t, f) \rightarrow^* t \) if and only if \( \Gamma \vdash \phi \) is provable;
- \( \text{cpt}(\Gamma \vdash \phi, t, f) \rightarrow^* f \) if and only if \( \Gamma \vdash \phi \) is not provable.

Proposition 1 also implies that our proof search algorithm always terminates, in the sense that a \( \text{CPT} \) \( \text{cpt}(\Gamma \vdash \phi, t, f) \) always rewrites to \( t \) or \( f \) in finite steps. The pseudo code of the proof search algorithm is shown in Figure 5.

**Input:** An input file \( f \)
**Output:** A boolean result \( r \), and a proof tree \( \text{proof} \)

**Name:** main

1: Parse the input file \( f \), and obtain the Kripke model \( M \), and a formula \( \phi \) in the system \( \text{SCTL}(M) \);
2: \((r, \text{proof}) \leftarrow \text{proof generation}(\text{cpt}(\Gamma \vdash \phi, t, f))\);
3: return \((r, \text{proof})\);

**Fig. 4.** The main algorithm

**Input:** A \( \text{CPT} \) \( c \)
**Output:** A boolean result \( r \), and a proof tree \( \text{proof} \)

**Name:** \text{proof generation}

1: \( \text{proof}_t, \text{proof}_f \leftarrow \emptyset, \text{mark}_t, \text{mark}_f \leftarrow \emptyset \);
2: if \( c = \text{cpt}(\Gamma \vdash \phi, c_1, c_2) \) then
3: if \( b = \text{true} \) then
4: if \( \exists (c, f) \in \text{mark}_t \) then
5: \( \text{proof}_t \leftarrow \text{proof}_t \cup f \);
6: else
7: end if
8: \( \text{proof}_f \leftarrow \text{proof}_f \cup f \);
9: end if
10: end if
11: \( c \leftarrow c' \); \( \text{comment: one step of CPT rewriting} \)
12: if \( b = \text{true} \) then
13: \( \text{mark}_t \leftarrow \text{mark}_t \cup \{c_1, \{\Gamma \vdash \phi, \text{ss}_t\}\} \);
14: \( \text{mark}_f \leftarrow \text{mark}_f \cup \{c_2, \{\Gamma \vdash \phi, \text{ss}_f\}\} \);
15: \{ \text{comment: ss}_t \text{ is the set of sequents occur in } c' \text{ that can be the hypothesises of } \Gamma \vdash \phi \}
16: \{ \text{while ss}_f \text{ is the set of sequents occur in } c' \text{ that can be hypothesive of the negation of } \Gamma \vdash \phi \} \}
17: end if
18: \( c \leftarrow c' \);
19: goto 2;
20: else
21: if \( c = \text{t} \) then
22: return \((\text{true}, \text{proof}_t)\);
23: else
24: return \((\text{false}, \text{proof}_f)\);
25: end if
26: end if

**Fig. 5.** The proof generation algorithm
Example 2. We illustrate how to use the rewrite rules in Figure 3 by considering the proof in Example 1. There are six steps of rewritings in this example, as shown in Figure 6.

Fig. 6. An example of CPT rewritings

Step 1. At this step, on the left side of 1→, the root of the CPT is \( \vdash AF_x(P(x))(a) \). We need to show whether \( \vdash AF_x(P(x))(a) \) is provable, which is not known at that moment yet. So we have to show first whether \( P(a) \) is provable, and then both \( AF_x(P(x))(b) \) and \( AF_x(P(x))(c) \) are successively provable, corresponding applying the \( \textbf{AF-R}_1 \) rule and the \( \textbf{AF-R}_2 \) rule, respectively. We encode those two steps in a single CPT, which is the one on the right side of 1→.

Step 2. Since the atomic formula \( P(a) \) is not provable, the CPT on the left side of 2→ reduces to its right subtree (f-continuation), which is the CPT on the right side of 2→.
Step 3. Like at step 1, we need to show whether $\text{AF}_x(P(x))(a) \vdash \text{AF}_x(P(x))(b)$ is provable, which is not known at that moment yet. So we encode the left subtre (t-continuation) of the CPT which is on the left side of $\frac{3}{4}$, and, by the $\text{AF-}R_1$ rule and the $\text{AF-}R_2$ rule, the two steps to find successively the proofs of $\vdash P(b)$ and of $\text{AF}_x(P(x))(a), \text{AF}_x(P(x))(b) \vdash \text{AF}_x(P(x))(d)$ into the CPT which is on the right side of $\frac{3}{4}$.

Step 4. Like at step 2, we can judge the atomic formula $P(b)$ is provable immediately. So the CPT on the left side of $\frac{4}{5}$ reduces to its left subtree (t-continuation) which is on the right side of $\frac{4}{5}$.

Step 5. Like at step 1 and 3, we can not judge whether the sequent $\text{AF}_x(P(x))(a) \vdash \text{AF}_x(P(x))(c)$ is provable immediately, so we encode the two steps to find successively the proofs of $\vdash P(c)$ and $\text{AF}_x(P(x))(a), \text{AF}_x(P(x))(b) \vdash \text{AF}_x(P(x))(d)$ into the CPT which is on the right side of $\frac{5}{6}$.

Step 6. Like at step 2 and 4, as the atomic formula $P(c)$ is provable, so the CPT on the left side of $\frac{6}{7}$ reduces to its left subtree (t-continuation) which is $t$. Now, the proof search of $\vdash \text{AF}_x(P(x))(a)$ terminates, and we can judge that this sequent is provable.

**Implementation of merges.** In the proof search of sequents with co-inductive formulae (formulae with modality $\text{EG}$ or $\text{AR}$), the merge rules are used to assert that some property holds on an infinite path of states. For every merge rule, the formulae need to be memorized are with the same modality, the only differences are the states contained in the formulae. Thus, it is sufficient to memorize only the states, not the whole formulae, in the implementation of every merge rules.

Note that the idea of merge is also used in the implementation of the proof search for the sequents having formulae starting with the modality $\text{AF}$ or $\text{EU}$. This idea is helpful to avoid infinite proof searches. For instance, for the proof search of the sequent $\vdash \text{EU}_{x,y}(\phi_1, \phi_2)(s)$, we need to find a finite path of which $\phi_2$ holds at the last state, and $\phi_1$ holds at all other states. To avoid the proof search falling into an infinite path, we need a mechanism to detect infinite paths in finite steps, which is exactly the way that merge works. So, although merge is not needed at the level of syntax, it does need at the level of implementation. Similar scenario happens to $\text{AF}$.

Note also that, in order to avoid visiting the same states repeatedly, we use a global memory to remember the visited states during the proof search of each sequent.

**Comparing with some model checking techniques.** The verification procedure in SCTLProV differs from traditional symbolic model checking. For instance, consider a Kripke model with the initial state $s_0$ and transition relation $T$. To verify whether $M, s_0 \models \text{EF} \phi$ holds or not in traditional symbolic model checker such as NuSMV, first one needs to calculate a least fixed point \( \mu Y. (p \lor \text{EXY}) \), then check whether $s_0 \in \text{lfp} \{ \mu Y. (p \lor \text{EXY}) \}$. Calculating lfp corresponds to unfolding of the relation $T$, where states that are not reachable from $s_0$ may be involved. Unlike in NuSMV, there is no need for SCTLProV to calculate a fixed
point of the transition relation. Instead, unfolding of the transition relation stops as soon as the given property is proved or its negation is proved. Unfolding on demand of the transition relation in a Kripke model is exactly the idea of on-the-fly model checking. However, unlike traditional on-the-fly model checking algorithms such as [2], in which unfolding of transition relations is implemented by calling subroutines, SCTLProV unfolds the transition relations by rewriting on CPTs, i.e., implementing backtracking using continuation-passing style when searching the state space, in order to reduce the stack space.

For traditional BMC tools, where the temporal formulae under proving will be unfolded on a set of traces with limited length once for all, one adopts the way of unfolding the formulae partial lazily. For example, in the bounded model checking problem on proving $M, s_0 \models_{k+1} EF \phi$ holds or not, the formula $EF \phi$ will be unfolded on a trace of length $k + 1$ immediately, which means, the tool need to deal with the bulky formula [3]:

$$[M, EF \phi]_{k+1} := \bigwedge_{i=0}^{k-1} T(s_i, s_{i+1}) \land \bigvee_{j=0}^{k} \phi(s_j)$$

To avoid exploring unnecessary states in $M$, SCTLProV unfolds on demand the transition relation $T$. Thus, to verify $\vdash EF_s(\phi)(s_0)$, SCTLProV unfolds the transition relation $T$ and the formula $EF_s(\phi)(s_0)$ as

$$\text{unfold}(S, EF_s(\phi)(s_i)) := \phi(s_i) \lor ((s_i \notin S) \land T(s_i, s_{i+1}) \land \text{unfold}(S \cup \{s_i\}, EF_s(\phi)(s_{i+1})))$$

where $S$ is a set representing the visited states during the proof search, which is in fact our implementation of the merge rule. Unfolding of the relation $T$ is applied only when the formula (also unfolded on demand) is unfolded.

Similar to traditional CTL symbolic model checkers such NuSMV, SCTLProV also has the ability to use BDDs for memorizing visited states, in order to reduce space occupation during the verification procedure. However, unlike NuSMV that translates models and properties into BDDs before verification, SCTLProV only use BDDs to memorize the visited states. Both approaches involve the translating of non-boolean state variables into boolean ones, which may add extra state variables for a given Kripke model. When a Kripke model contains mostly boolean variables, for instance in hardware model checking, memorizing states using BDDs is efficient to reduce space occupation. However, when a Kripke model contains many non-boolean variables, for instance in software model checking, its memory space can explode. SCTLProV can choose to memorize visited states either use BDDs when the model contains mostly boolean variables, or directly when the model contains many non-boolean variables.

To summarize the above discussions, although our approach does not reduce the complexity of model checking problems, we make efforts not to perform unneeded proof search via an on-the-fly style of unfolding both the transition relations and the formulae (double on-the-fly).
4 Examples and Experimental Evaluation

To illustrate the feasibility and the efficiency of SCTLProV, we first consider two examples to show the applications of SCTLProV, then evaluate a benchmark with 2880 test cases, and compare the experimental results with one proof checker and three model checkers.

4.1 Two Examples

The first example is a mutual exclusion algorithm of two processes (process A and process B) described in [19]. In this algorithm, process A and process B run concurrently, and both can enter a critical section during their running. Mutual Exclusion means that both two processes can not enter the critical section at the same time.

Example 3 (The Mutual Exclusion Problem). Our formulation of this problem is based on a Spin [15] version of this problem[12] where a shared variable $mutex$ is used to remember the number of processes that have entered the critical section. A violation of Mutual Exclusion means that in some state of the program, the value of the shared variable $mutex$ is 2.

Model mutual()
{
    Var {flag : Bool; mutex : (0 .. 2); a : (1 .. 6); b : (1 .. 6);}
    Init {flag := false; mutex := 0; a := 1; b := 1;}
    Transition
    {
        a = 1 && flag = false : {a := 2;};
        a = 2 : {a := 3; flag := true;};
        a = 3 : {a := 4; mutex := mutex + 1;/*A has entered the critical section*/};
        a = 4 : {a := 5; mutex := mutex - 1;/*A has left the critical section*/};
        a = 5 : {a := 6;};
        b = 1 && flag = false : {b := 2;};
        b = 2 : {b := 3; flag := true;};
        b = 3 : {b := 4; mutex := mutex + 1;/*B has entered the critical section*/};
        b = 4 : {b := 5; mutex := mutex - 1;/*B has left the critical section*/};
        b = 5 : {b := 6;};
    }
    Atomic {bug(s) := s(mutex) = 2;}
    Spec{find_bug := EU(x, y, TRUE, bug(y), ini);}
In the input file (Figure 7), variable flag is a signal indicating whether there exists a running process. Variables a and b indicate the program counters of the two processes, respectively. The property need to be checked is that whether both of the two processes are already in the critical section at the the same time. We check this property in SCTLProV using the following command:

```
sctl -output output.out mutual.model
```

The result is as follows:

```
verifying on the model mutual...
find_bug: EU(x,y, TRUE, bug(y), ini)
find_bug is true.
```

The proof tree of the property is output to the file “output.out”.

```
0: |- EU(x,y,TRUE,bug(y),{flag:=false;mutex:=0;a:=1;b:=1}) [4, 1]
  4: {flag:=false;mutex:=0;a:=1;b:=1}
  |- EU(x,y,TRUE,bug(y),{flag:=false;mutex:=0;a:=2;b:=1}) [7, 5]
  7: {flag:=false;mutex:=0;a:=1;b:=1}
  |    (flag:=false;mutex:=0;a:=2;b:=1)
  |- EU(x,y,TRUE,bug(y),{flag:=false;mutex:=0;a:=2;b:=2}) [23, 20]
  23:{flag:=false;mutex:=0;a:=1;b:=1}
  |    (flag:=false;mutex:=0;a:=2;b:=1)
  |    (flag:=false;mutex:=0;a:=2;b:=2)
  |- EU(x,y,TRUE,bug(y),{flag:=true;mutex:=0;a:=3;b:=2}) [27, 24]
  27:{flag:=false;mutex:=0;a:=1;b:=1}
  |    (flag:=false;mutex:=0;a:=2;b:=2)
  |    (flag:=true;mutex:=0;a:=3;b:=2)
  |- EU(x,y,TRUE,bug(y),{flag:=true;mutex:=1;a:=4;b:=2}) [31, 28]
  31:{flag:=false;mutex:=0;a:=1;b:=1}
  |    (flag:=false;mutex:=0;a:=2;b:=2)
  |    (flag:=true;mutex:=0;a:=3;b:=2)
  |    (flag:=true;mutex:=1;a:=4;b:=2)
  |- EU(x,y,TRUE,bug(y),{flag:=true;mutex:=1;a:=4;b:=3}) [35, 32]
  35:{flag:=false;mutex:=0;a:=1;b:=1}
  |    (flag:=false;mutex:=0;a:=2;b:=2)
  |    (flag:=true;mutex:=0;a:=3;b:=2)
  |    (flag:=true;mutex:=1;a:=4;b:=2)
```

According to the output above, we can find that after process $A$ have entered the critical section, process $B$ can also enter the critical section.

A simple solution of the mutual exclusion problem (also described in [19]) would be as follows. Variable $x$ and $y$ are signals to indicate whether process $A$ and $B$ are running; $turn$ is the variable indicating it is who’s turn to enter the critical section.

Model mutual()
{
Var {x:Bool; y:Bool; mutex:(0 .. 2); turn:{#turn_1, #turn_2}; a:(1 .. 7); b:(1 .. 7);}
Init {x := false; y := false; mutex := 0; turn := #turn_1; a := 1; b := 1;}
Transition
{
 a = 1 : {a := 2; x := true;};
 a = 2 : {a := 3; turn := #turn_2;};
 a = 3 && (y = false || turn = #turn_1): {a := 4;};
 a = 4 : {a := 5; mutex := mutex + 1;/*A has entered the critical section*/};
 a = 5 : {a := 6; mutex := mutex - 1;/*A has left the critical section*/};
 a = 6 : {a := 7; x := false;};
 b = 1 : {b := 2; y := true;};
 b = 2 : {b := 3; turn := #turn_1;};
 b = 3 && (x = false || turn = #turn_2): {b := 4;};
 b = 4 : {b := 5; mutex := mutex + 1;/*B has entered the critical section*/};
 b = 5 : {b := 6; mutex := mutex - 1;/*B has left the critical section*/};
 b = 6 : {b := 7; y := false;};
}
Atomic {bug(s) := s(mutex) = 2;}
Spec {find_bug := EU(x, y, TRUE, bug(y), ini);}
}

Fig. 8. The input file “mutual_solution.model” indicating the solution.

The verification result of this model would be as follows.

verifying on the model mutual...
find_bug: EU(x, y, TRUE, bug(y), ini)
find_bug is false.
Example 4 (Search States Directly). Consider a Kripke model which has 100001 states. The states in this model are represented by two kinds of state variables: a boolean state variable $b$, and a non-boolean state variable $i$ whose value ranges from integer 0 to integer 100000. The initial state of the model is \{\(b = true, i = 0\)\}. The transition relation $\rightarrow$ over the states is defined by \{\(b = true, i = 2k\) \(\rightarrow\) \{\(b = false, i = 2k + 1\)\}, \(b = false, i = 2k + 1\) \(\rightarrow\) \{\(b = true, i = 2k + 2\)\}\) with \(0 \leq k < 100000\), and \{\(b = true, 100000\) \(\rightarrow\) \{\(b = true, 100000\)\}\). Thus, the transition ends with an infinite loop on the state \{\(b = true, i = 100000\)\}. The property to be verified is to determine whether the state \{\(b = true, i = 100000\)\} is reachable from the initial state. We describe this model and the property to be verified in the input language for NuSMV (NuXMV), Verds, and SCTLProV, respectively (Figure 9); and verify this given property in the four verification tools.

In order to verify this property, NuSMV, NuXMV and Verds have to encode the state variable $i$ into at least 17 ($2^{16} < 100000 < 2^{17}$) boolean state variables. This can make the verification procedure time and space consuming. While in SCTLProV, we choose to search and remember states directly (not translating non-boolean state variables into boolean ones) in this case. As is shown in Table 1 in this case, SCTLProV performs better than the other three tools.
### 4.2 Experimental Evaluation

We evaluate a benchmark with 2880 test cases, and compare the experimental results with four other verification tools: the Resolution-based theorem prover iProver Modulo [4], the QBF-based bounded model checker Verds [23] version 1.48 (henceforth to be referred to as Verds), the BDD-based model checker NuSMV [6] version 2.6.0 (henceforth to be referred to as NuSMV) and its extension NuXMV [5] version 1.0.0 (henceforth to be referred to as NuXMV). This benchmark is introduced in [23], and is based on two types of boolean programs (the concurrent processes and the concurrent sequential processes) and 24 properties. The initial states and transition relations are randomly generated in all the test cases. We believe that this randomness of the test cases will help us recognize the characteristics of each tool. All these examples and test cases ran on a Linux platform with 3.0GB memory and a 2.93GHz * 4 CPU, and the limit of running time is set to be 20 minutes.

The comparison of SCTLProV with respectively iProver Modulo, Verds, NuSMV and NuXMV is based on three aspects: the number of solvable cases in 20 minutes (Table 2), the running speed (Table 3), and the average running time (Figure 10).

The detailed experimental data are summarized in Tables 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11 in the appendix.

| Programs | iProver Modulo | Verds | NuSMV | NuXMV | SCTLProV |
|----------|----------------|-------|-------|-------|----------|
| Con.Proc | 1222 (84.9%)   | 1277 (88.7%) | 1440 (100%) | 1440 (100%) | 1430 (99.3%) |
| Con.Seq.Proc | 594 (41.3%) | 953 (66.2%) | 1440 (100%) | 1440 (100%) | 1432 (99.4%) |
| Sum      | 1816 (63.1%)   | 2230 (77.4%) | 2880 (100%) | 2880 (100%) | 2862 (99.4%) |

**Table 2.** Solvable cases in five tools.

| Programs | iProver Modulo | Verds | NuSMV | NuXMV | SCTLProV |
|----------|----------------|-------|-------|-------|----------|
| Con.Proc | 1414 (98.2%)   | 1427 (99.1%) | 1327 (92.2%) | 1351 (93.8%) |
| Con.Seq.Proc | 1409 (97.8%) | 1431 (99.4%) | 1414 (98.2%) | 1412 (98.1%) |
| Sum      | 2823 (98.2%)   | 2858 (99.2%) | 2741 (95.2%) | 2763 (95.9%) |

**Table 3.** Test cases where SCTLProV outperforms the other four tools.

**Summary:** The experimental results shows that SCTLProV outperforms the other four tools in over 90 percent of the test cases. The comparison of average verification time shows that, along with the number of state variables increases, the average verification time of SCTLProV and iProver Modulo grows slowly, for

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12 Test cases that are solvable in all of the four tools are considered here.
both Concurrent Processes and Concurrent Sequential Processes. The average verification time of Verds also grows slowly as the number of state variables increases for Concurrent Processes, but not for Concurrent Sequential Processes. Whereas for NuSMV and NuXMV, the average verification time grows faster than SCTLProV, iProver Modulo, and Verds for both Concurrent Processes and Concurrent Sequential Processes. However, we can not say that SCTLProV is better than the other four tools, especially NuSMV and NuXMV. SCTLProV solve 18 (0.63%) test cases less than both NuSMV and NuXMV. All these test cases that not solvable in SCTLProV are with AG properties, where we need to explore all the states in these Kripke models. In this situation, the symbolic state search method used by NuSMV and NuXMV is usually more efficient than the direct state search in SCTLProV. In other kinds of properties, SCTLProV usually explore much less states to verify these kinds of properties. This suggests that SCTLProV may be seen as a new feasible method of solving CTL model checking problems and as complementary to NuSMV and NuXMV, which are two state-of-art CTL model checkers.

5 Conclusion and Future Work

This paper provides the first step towards combining model checking and proof checking. We proposed a parameterized sequent calculus SCTL, and developed a new automated theorem prover SCTLProV from scratch, tailored for this calculus. The particular aspects of SCTLProV are as follows: (1) It performs verification automatically and directly over any given Kripke model. (2) In addition of generating counterexamples when the verification of the given property fails, SCTLProV permits to give a certificate for the property when it succeeds. (3) It performs verification in a continuation-passing style and a double on-the-fly
style, thanks to the syntax and inference rules of SCTL. (4) It uses a global memory remembering visited states for implementing the emerge rules. We illustrate the efficiency and feasibility of SCTLProV by comparing it with the three model checkers Verds, NuSMV and NuXMV, and the proof checker iProver Modulo, on a benchmark with 2880 test cases. The experimental results are reported and quite encouraging: SCTLProV outperforms both Verds and iProver Modulo in nearly all the test cases. Although SCTLProV solves 18 (0.63%) cases less than NuSMV and NuXMV, it performs better in more than 90% of the test cases. This suggests that both SCTLProV and the other tools have their own advantages and may be considered as complementary tools. As a matter of fact, NuSMV and NuXMV perform better than SCTLProV in proving some AG properties, while SCTLProV performs better with other kinds of properties.

Note also that the tool SCTLProV can be seen either as a theorem prover, or a model checker that can produce more information than traditional ones. It permits in fact a direct implementation of CTL semantics.

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A Appendix

The appendix is organized as follows: Section A.1 presents the proofs of all the propositions and theorems in the main paper; Section A.2 shows the details of the experimental data for the given benchmark; Section A.3 presents the general description of the tool SCTLProV, which contains the general introduction of the proving procedure, the specification of the input language, and an illustrative example; Section A.4 shows the experimental data of comparing SCTLProV with the other four tools from another computer.

We present the table of contents in the last page.

A.1 Proofs in the Main Paper

The Soundness and Completeness of SCTL We prove the soundness and completeness of SCTL and some related propositions. The interested reader may refer to [8] for further details of the proofs.

Proposition 2 (Finite to infinite sequences). Let \( s_0, \ldots, s_n \) be a finite sequence of states such that for all \( i \) between 0 and \( n - 1 \), \( s_i \rightarrow s_{i+1} \), and \( s_n = s_p \) for some \( p \) between 0 and \( n - 1 \). Then there exists an infinite sequence of states \( s'_0, s'_1, \ldots \) such that for all \( i \), \( s'_i \rightarrow s'_{i+1} \), and all the \( s'_i \) are among \( s_0, \ldots, s_n \).

Proof. Take the sequence \( s_0, \ldots, s_{p-1}, s_p, \ldots, s_{n-1}, \ldots \)
**Proposition 3 (Finite to infinite trees).** Let $\Phi$ be a set of states and $T$ be a finite tree labeled by states such that, for each internal node $s$, the immediate successors of $s$ are the elements of $\text{Next}(s)$ and each leaf is labeled with a state which is either in $\Phi$ or also a label of a node on the branch from the root of $T$ to this leaf. Then there exists an infinite tree $T'$ labeled by states such that for each internal node $s$ the successors of $s$ are the elements of $\text{Next}(s)$, all the leaves are labeled by elements of $\Phi$, and all the labels of $T'$ are the labels of $T$.

**Proof.** Consider for $T'$ the tree whose root is labeled by the root of $T$ and such that for each node $s$, if $s$ is in $\Phi$, then $s$ is a leaf of $T'$, otherwise the successors of $s$ are the elements of $\text{Next}(s)$. It is easy to check that all the nodes of $T'$ are labeled by labels of $T$.

**Proposition 4 (Infinite to finite Sequences).** Let $s_0, s_1, \ldots$ be an infinite sequence of states such that for all $i$, $s_i \rightarrow s_{i+1}$. Then there exists a finite sequence of states $s_0', \ldots, s_n'$ such that for all $i$ between 0 and $n-1$, $s_i' \rightarrow s_{i+1}'$, $s_n' = s_p'$ for some $p$ between 0 and $n-1$, and all the $s_j'$ are among $s_0, s_1, \ldots$.

**Proof.** As the number of states is finite, there exists $p$ and $n$ such that $p < n$ and $s_p = s_n$. Take the sequence $s_0, \ldots, s_n$.

**Proposition 5 (Infinite to finite trees).** Let $\Phi$ be a set of states and $T$ be an infinite tree labeled by states such that for each internal node $s$ the successors of $s$ are the elements of $\text{Next}(s)$ and each leaf is labeled by a state in $\Phi$. Then, there exists a finite tree labeled by states such that for each internal node $s$ the successors of $s$ are the elements of $\text{Next}(s)$ and each leaf is labeled with a state which is either in $\Phi$ or also a label of a node on the branch from the root of $T$ to this leaf.

**Proof.** As the number of states is finite, on each infinite branch, there exists $p$ and $n$ such that $p < n$ and $s_p = s_n$. Prune the tree at node $s_n$. This tree is finitely branching and each branch is finite, hence, by König’s lemma, it is finite.

**Theorem 2 (Soundness).** Let $\phi$ be a closed formula. If the sequent $\vdash \phi$ has a proof $\pi$, then $\models \phi$.

**Proof.** By induction on the structure of the proof $\pi$.

- If the last rule of $\pi$ is $\text{atom-R}$, then the proved sequent has the form $\vdash P(s_1, \ldots, s_n)$, hence $\models P(s_1, \ldots, s_n)$.
- If the last rule of $\pi$ is $\neg$-$\text{R}$, then the proved sequent has the form $\vdash \neg P(s_1, \ldots, s_n)$, hence $\models \neg P(s_1, \ldots, s_n)$.
- If the last rule of $\pi$ is $\top$-$\text{R}$, the proved sequent has the form $\vdash \top$ and hence $\models \top$.
- If the last rule of $\pi$ is $\land$-$\text{R}$, then the proved sequent has the form $\vdash \phi_1 \land \phi_2$. By induction hypothesis $\models \phi_1$ and $\models \phi_2$, hence $\models \phi_1 \land \phi_2$.
- If the last rule of $\pi$ is $\lor$-$\text{R}_1$ or $\lor$-$\text{R}_2$, then the proved sequent has the form $\vdash \phi_1 \lor \phi_2$. By induction hypothesis $\models \phi_1$ or $\models \phi_2$, hence $\models \phi_1 \lor \phi_2$. 

If the last rule of $\pi$ is $\text{AX-R}$, then the proved sequent has the form $\vdash AX_x(\phi_1)(s)$. By induction hypothesis, for each $s'$ in $\text{Next}(s)$, such that $\vdash (s'/x)\phi_1$, hence $\vdash AX_x(\phi_1)(s)$.

If the last rule of $\pi$ is $\text{EX-R}$, then the proved sequent has the form $\vdash EX_x(\phi_1)(s)$. By induction hypothesis, for each $s'$ in $\text{Next}(s)$, $\vdash (s'/x)\phi_1$, hence $\vdash EX_x(\phi_1)(s)$.

If the last rule of $\pi$ is $\text{AF-R}_1$ or $\text{AF-R}_2$, then the proved sequent has the form $\vdash AF_x(\phi_1)(s)$. We associate a finite tree $[\pi]$ to the proof $\pi$ by induction in the following way.

- If the proof $\pi$ ends with the $\text{AF-R}_1$ rule with a subproof $\rho$ of the sequent $\vdash (s/x)\phi_1$, then the tree contains a single node $s$.
- If the proof $\pi$ ends with the $\text{AF-R}_2$ rule, with subproofs $\pi_1, \ldots, \pi_n$ of the sequent $\vdash AF_x(\phi_1)(s_1), ..., \vdash AF_x(\phi_1)(s_n)$, respectively, then $[\pi]$ is the tree $s([\pi_1], ..., [\pi_n])$.

The tree $[\pi]$ has root $s$; for each internal node $s'$, the children of this node are labeled by elements of $\text{Next}(s')$; and for each leaf $s'$ the sequent $\vdash (s'/x)\phi_1$ has a proof smaller than $\pi$. By induction hypothesis, for each leaf $s'$ of $[\pi]$, $\vdash (s'/x)\phi_1$. Hence $\vdash AF_x(\phi_1)(s)$.

If the last rule of $\pi$ is $\text{EG-R}$, then the proved sequent has the form $\vdash EG_x(\phi_1)(s)$. We associate a finite sequence $[\pi]$ to the proof $\pi$ by induction in the following way.

- If the proof $\pi$ ends with the $\text{EG}$-merge rule, then the sequence contains a single element $s$.
- If the proof $\pi$ ends with the $\text{EG}$-rule, with subproofs $\rho$ and $\pi_1$ of the sequents $\vdash (s/x)\phi_1$ and $\Gamma, EG_x(\phi_1)(s) \vdash EG_x(\phi_1)(s')$, respectively, then $[\pi]$ is the sequence $s[\pi_1]$. The sequent $[\pi] = s_0, s_1, ..., s_n$ is such that $s_0 = s$; for all $i$ between $0$ and $n - 1$, $s_i \rightarrow s_{i+1}$; for all $i$ between $0$ and $n$, the sequent $\vdash (s_i/x)\phi_1$ has a proof smaller than $\pi$; and $s_n$ is equal to $s_p$ for some $p$ between $0$ and $n - 1$. By induction hypothesis, for all $i$, we have $\vdash (s_i/x)\phi_1$. Using Proposition 2 there exists an infinite sequence $s'_0, s'_1, ...$ such that for all $i$, we have $s'_i \rightarrow s'_{i+1}$, and $\vdash (s'_i/x)\phi_1$. Hence, $\vdash EG_x(\phi_1)(s)$.

- If the last rule of $\pi$ is $\text{AR-R}_1$ or $\text{AR-R}_2$, then the proved sequent has the form $\vdash AR_x(\phi_1, \phi_2)(s)$. We associate a finite tree $[\pi]$ to the proof $\pi$ by induction in the following way.

- If the proof $\pi$ ends with the $\text{AR-R}_1$ rule with subproofs $\rho_1$ and $\rho_2$ of the sequents $\vdash (s/x)\phi_1$ and $\vdash (s/x)\phi_2$, respectively, or with the $\text{AR}$-merge rule, then the tree contains a single node $s$.
- If the proof $\pi$ ends with the $\text{AR-R}_2$ rule, with subproofs $\rho, \pi_1, ..., \pi_n$ of the sequents $\vdash (s/y)\phi_2$, $\Gamma, AR_{x,y}(\phi_1, \phi_2)(s) \vdash AR_{x,y}(\phi_1, \phi_2)(s_1), ...,\Gamma, AR_{x,y}(\phi_1, \phi_2)(s) \vdash AR_{x,y}(\phi_1, \phi_2)(s_n)$, respectively, then $[\pi]$ is the tree $s([\pi_1], ..., [\pi_n])$.

The tree $[\pi]$ has root $s$; for each internal node $s'$, the children of this node are labelled by the elements of $\text{Next}(s')$; for each node $s'$ of $[\pi]$, the sequent $\vdash (s'/x)\phi_1$ has a proof smaller than $\pi$; and for each leaf $s'$, either the sequent $\vdash (s'/x)\phi_1$ has a proof smaller than $\pi$, or $s'$ is also a label of a node on the
branch from the root of $|\pi|$ to this leaf. By induction hypothesis, for each node $s'$ of this tree $|= (s'/y)\phi_2$ and for each leaf $s'$, either $|= (s'/x)\phi_1$ or $s'$ is also a label of a node on the branch from the root of $|\pi|$ to this leaf. Using Proposition 3 there exists an infinite tree $T'$ labelled by states such that for each internal node $s$ the successors of $s$ are the elements of Next($s$), for each node $s'$ of $T'$, $|= (s'/y)\phi_2$, and for each leaf $s'$ of $T'$, $|= (s'/x)\phi_1$. Thus, $|= AR_{x,y}(\phi_1, \phi_2)(s)$.

- If the last rule of $\pi$ is EU-R$_1$ or EU-R$_2$, then the proved sequent has the form $\vdash EU_{x,y}(\phi_1, \phi_2)(s)$. We associate a finite sequence $|\pi|$ to the proof $\pi$ by induction in the following way.
  
  - If the proof $\pi$ ends with the EU-R$_1$ rule with a subproof $\rho$ of the sequent $\vdash (s/y)\phi_2$, then the sequence contains a single element $s$.
  
  - If the proof $\pi$ ends with the EU-R$_2$ rule, with subproofs $\rho$ and $\pi_1$ of the sequents $\vdash (s/x)\phi_1$ and $\vdash EU_{x,y}(\phi_1, \phi_2)(s')$, respectively, then $|\pi|$ is the sequence $s[\pi_1]$.

  The sequence $|\pi| = s_0, ..., s_n$ is such that $s_0 = s$; for each $i$ between 0 and $n - 1$, $s_i \rightarrow s_{i+1}$; for each $i$ between 0 and $n - 1$, the sequent $\vdash (s_i/x)\phi_1$ has a proof smaller than $\pi$; and the sequent $\vdash (s_n/y)\phi_2$ has a proof smaller than $\pi$. By induction hypothesis, for each $i$ between 0 and $n - 1$, $|= (s_i/x)\phi_1$ and $|= (s_n/y)\phi_2$. Hence, $|= EU_{x,y}(\phi_1, \phi_2)(s)$.

- The last rule cannot be a merge rule.

**Theorem 3 (Completeness).** Let $\phi$ be a closed formula. If $|= \phi$ then the sequent $\vdash \phi$ is provable.

**Proof.** By induction over the structure of $\phi$.

- If $\phi = P(s_1, ..., s_n)$, then as $|= P(s_1, ..., s_n)$, the sequent $\vdash P(s_1, ..., s_n)$ is provable with the rule atom-R.

- If $\phi = \neg P(s_1, ..., s_n)$, then as $|= \neg P(s_1, ..., s_n)$, the sequent $\vdash \neg P(s_1, ..., s_n)$ is provable with the rule $\neg$-R.

- If $\phi = \top$, then $\vdash \top$ is provable with the rule $\top$-R.

- If $\phi = \bot$, then it is not the case that $|= \bot$.

- If $\phi = \phi_1 \wedge \phi_2$, then as $|= \phi_1 \wedge \phi_2$, $|= \phi_1$ and $|= \phi_2$. By induction hypothesis, the sequents $\vdash \phi_1$ and $\vdash \phi_2$ are provable. Thus the sequent $\vdash \phi_1 \wedge \phi_2$ is provable with the $\wedge$-R rule.

- If $\phi = \phi_1 \vee \phi_2$, as $|= \phi_1 \vee \phi_2$, $|= \phi_1$ or $|= \phi_2$. By induction hypothesis, the sequent $\vdash \phi_1$ or $\vdash \phi_2$ is provable and the sequent $\vdash \phi_1 \vee \phi_2$ is provable with the $\vee$-R$_1$ or $\vee$-R$_2$ rule, respectively.

- If $\phi = AX_x(\phi_1)(s)$, as $|= AX_x(\phi_1)(s)$, for each state $s'$ in Next($s$), we have $|= (s'/x)\phi_1$. By induction hypothesis, for each $s'$ in Next($s$), the sequent $\vdash (s'/x)\phi_1$ is provable. Using these proofs and the AX-R rule, we build a proof of the sequent $\vdash AX_x(\phi_1)(s)$.

- If $\phi = EX_x(\phi_1)(s)$, as $|= EX_x(\phi_1)(s)$, there exists a state $s'$ in Next($s$) such that $|= (s'/x)\phi_1$. By induction hypothesis, the sequent $\vdash (s'/x)\phi_1$ is provable. With this proof and the EX-R rule, we build a proof of the sequent $\vdash EX_x(\phi_1)(s)$. 

If $\phi = AF_x(\phi_1)(s)$, as $\vdash AF_x(\phi_1)(s)$, there exists a finite tree $T$ such that $T$ has root $s$, for each internal node $s'$, the children of this node are labelled by the elements of $\text{Next}(s')$, and for each leaf $s'$, $\vdash (s'/x)\phi_1$. By induction hypothesis, for every leaf $s'$, the sequent $\vdash (s'/x)\phi_1$ is provable. Then, to each subtree $T'$ of $T$, we associate a proof $[T']$ of the sequent $\vdash AF_x(\phi_1)(s')$ where $s'$ is the root of $T'$, by induction, as follows.

- If $T'$ contains a single node $s'$, then the proof $[T]$ is built with the $\text{AF-R}_1$ rule from the proof of $\vdash (s'/x)\phi_1$ given by the induction hypothesis.
- If $T' = s'(T_1, ..., T_n)$, then the proof $[T]$ is built with the $\text{AF-R}_2$ rule from the proofs $[T_1], ..., [T_n]$ of the sequents $\vdash AF_x(\phi_1)(s_1), ..., \vdash AF_x(\phi_1)(s_n)$, respectively, where $s_1, ..., s_n$ are the elements of $\text{Next}(s')$.

This way, the proof $[T]$ is a proof of the sequent $\vdash AF_x(\phi_1)(s')$.

If $\phi = EG_x(\phi_1)(s)$, as $\vdash EG_x(\phi_1)(s)$, there exists a path $s_0, s_1, ..., s_n$ such that $s_0 = s$ and for all $i$, $\vdash (s_i/x)\phi_1$. By induction hypothesis, all the sequents $\vdash (s_i/x)\phi_1$ are provable. Using Proposition 4, there exists a finite sequence $T = s_0, ..., s_n$ such that for all $i, s_i \rightarrow s_{i+1}$, the sequent $\vdash (s_i/x)\phi_1$ is provable and $s_n$ is some $s_p$ for $p < n$. We associate a proof $[s_0, ..., s_n]$ of the sequent $EG_x(\phi_1)(s_0), ..., EG_x(\phi_1)(s_{n-1}) \vdash EG_x(\phi_1)(s_n)$ to each suffix of $T$ by induction as follows.

- The proof $[s_n]$ is built with the $\text{EG-merge}$ rule.
- If $i \leq n - 1$, then the proof $[s_i, ..., s_n]$ is built with the $\text{EG-R}$ rule from the proof of $\vdash (s_i/x)\phi_1$ given by the induction hypothesis and the proof $[s_{i+1}, ..., s_n]$ of the sequent $EG_x(\phi_1)(s_i), ..., EG_x(\phi_1)(s_n) \vdash EG_x(\phi_1)(s_{i+1})$.

This way, the proof $[s_0, ..., s_n]$ is a proof of the sequent $\vdash EG_x(\phi_1)(s_n)$.

If $\phi = AR_{x,y}(\phi_1, \phi_2)(s)$, as $\vdash AR_{x,y}(\phi_1, \phi_2)(s)$, there exists an infinite tree such that the root of this tree is $s$, for each internal node $s'$, the children of this node are labelled by the elements of $\text{Next}(s')$, for each node $s'$, $\vdash (s'/y)\phi_2$ and for each leaf $s'$, $\vdash (s'/x)\phi_1$. By induction hypothesis, for each node $s'$ of the tree, the sequent $\vdash (s'/y)\phi_2$ is provable and for each leaf $s'$ of the tree, the sequent $\vdash (s'/x)\phi_1$ is provable. Using Proposition 5, there exists a finite tree $T$ such that for each internal node $s'$ the successors of $s'$ are the elements of $\text{Next}(s')$, for each node $s'$, the sequent $\vdash (s'/y)\phi_2$ is provable and for each leaf $s'$, either the sequent $\vdash (s'/x)\phi_1$ is provable or $s'$ is also a label of a node on the branch from the root of $T$ to this leaf. Then, to each subtree $T'$ of $T$, we associate a proof $[T']$ of the sequent $AR_{x,y}(\phi_1, \phi_2)(s_1), ..., AR_{x,y}(\phi_1, \phi_2)(s_m) \vdash AR_{x,y}(\phi_1, \phi_2)(s')$ where $s'$ is the root of $T'$ and $s_1, ..., s_m$ is the sequence of nodes in $T$ from the root of $T$ to the root of $T'$.

- If $T'$ contains a single node $s'$, and the sequent $\vdash (s'/x)\phi_1$ is provable then the proof $[T']$ is built with the $\text{AR-R}_1$ rule from the proofs of $\vdash (s'/x)\phi_1$ and $\vdash (s'/y)\phi_2$ given by the induction hypothesis.
- If $T'$ contains a single node $s'$, and $s'$ is among $s_1, ..., s_m$, then the proof $[T']$ is built with the $\text{AR-merge}$ rule.
- If $T' = s'(T_1, ..., T_n)$, then the proof $[T']$ is built with the $\text{AR-R}_2$ rule from the proofs $\vdash (s'/y)\phi_2$ given by the induction hypothesis and the proofs $[T_1], ..., [T_n]$ of the sequents.
Given a sequent \( \Gamma \vdash \phi \), as \( |- EU_{x,y}(\phi_1, \phi_2)(s) \), there exists a finite sequence \( T = s_0, ..., s_n \) such that \( |- (s_n/y)\bar{\phi}_2 \) and for all \( i \) between 0 and \( n - 1 \), \( |- (s_i/x)\bar{\phi}_1 \). By induction hypothesis, the sequent \( |- (s_n/y)\bar{\phi}_2 \) is provable and for all \( i \) between 0 and \( n - 1 \), the sequent \( |- (s_i/x)\bar{\phi}_1 \) is provable. We associate a proof \( |s_1, ..., s_n| \) of the sequent \( |- EU_{x,y}(\phi_1, \phi_2)(s) \) to each suffix of \( T \) by induction as follows.

- The proof \( |s_n| \) is built with the \( EG-R_1 \) rule from the proof of \( |- (s_n/y)\bar{\phi}_2 \) given by the induction hypothesis.
- If \( i \leq n - 1 \), then the proof \( |s_1, ..., s_n| \) is built with the \( EG-R_2 \) rule from the proof of \( |- (s_i/x)\bar{\phi}_1 \) given by the induction hypothesis and the proof \( |s_{i+1}, ..., s_n| \) of the sequent \( |- EU_{x,y}(\phi_1, \phi_2)(s) \).

This way, the proof \( |s_0, ..., s_n| \) is a proof of the sequent \( |- EU_{x,y}(\phi_1, \phi_2)(s) \).

**Correctness of the Proof Search Algorithm** To prove proposition \( \square \) in the main paper, we deal with a more general situation, as characterized as follows.

**Proposition 6.** Given a sequent \( \Gamma \vdash \phi \), for all CPTs \( c_1 \) and \( c_2 \),

1. \( ctp(\Gamma \vdash \phi, c_1, c_2) \leadsto c_1 \) if \( \Gamma \vdash \phi \) is provable,
2. \( ctp(\Gamma \vdash \phi, c_1, c_2) \leadsto c_2 \) if \( \Gamma \vdash \phi \) is not provable.

**Proof.** We only analyse the first case, as the analysis of the second case is symmetric to the first case.

Induction on the structure of proof tree of \( \Gamma \vdash \phi \).

- If \( \phi = \top \) or \( \bot \), trivial.
- If \( \phi = P(s_1, ..., s_n) \) where \( P(s_1, ..., s_n) \) is atomic, then \( ctp(\neg P(s_1, ..., s_n), c_1, c_2) \leadsto c_1 \) if \( \neg P(s_1, s_2, ..., s_n) \) is provable.
- If \( \phi = \neg P(s_1, ..., s_n) \) where \( P(s_1, ..., s_n) \) is atomic, then \( ctp(\neg \neg P(s_1, ..., s_n), c_1, c_2) \leadsto c_1 \) if \( \neg P(s_1, s_2, ..., s_n) \) is provable.
- If \( \phi = \phi_1 \land \phi_2 \), then \( ctp(\phi_1 \land \phi_2, c_1, c_2) \leadsto c_1 \) if \( ctp(\phi_1, \phi_2, c_1, c_2) \leadsto c_1 \) and \( \phi_1 \land \phi_2 \) are provable (by induction hypothesis) if \( \phi_1 \land \phi_2 \) are provable.
- If \( \phi = \phi_1 \lor \phi_2 \), then \( ctp(\phi_1 \lor \phi_2, c_1, c_2) \leadsto c_1 \) if either \( ctp(\phi_1 \lor \phi_2, c_1, c_2) \leadsto c_1 \) or \( ctp(\phi_1, \phi_2, c_1, c_2) \leadsto c_1 \) and \( \phi_1 \lor \phi_2 \) are provable (by induction hypothesis) if \( \phi_1 \lor \phi_2 \) is provable.
- If $\phi = AX_x(\psi)(s)$ and $\{s_1, ..., s_n\} = \text{Next}(s)$, then $\text{cpt}(\Gamma \vdash AX_x(\psi)(s), c_1, c_2) \rightsquigarrow^* c_1$ if $\text{cpt}(\Gamma \vdash AX_x(\psi)(s), c_1, c_2) \rightsquigarrow^* c_1$.

- If $\phi = EX_x(\psi)(s)$ and $\{s_1, ..., s_n\} = \text{Next}(s)$, then $\text{cpt}(\Gamma \vdash EX_x(\psi)(s), c_1, c_2) \rightsquigarrow^* c_1$.

- If $\phi = AF_x(\psi)(s)$ and $\{s_1, ..., s_n\} = \text{Next}(s)$, then $\text{cpt}(\Gamma \vdash AF_x(\psi)(s), c_1, c_2) \rightsquigarrow^* c_1$.

\[ \psi \text{ is finite. So, the second condition holds implies that } \Gamma \vdash \text{cpt}(\text{Next} (s), c_1, c_2) \rightsquigarrow^* \text{cpt}(\psi, c_1, c_2) \text{ is provable.} \]

\[ \text{if the second condition holds, then } \Gamma \vdash \text{cpt}(\text{Next} (s), c_1, c_2) \rightsquigarrow^* c_1 \text{ if } \Gamma \vdash \text{Next} (s, c_1) \text{ is provable (by induction hypothesis)} \]

\[ \text{if } \Gamma \vdash \text{Next} (s, c_1) \text{ is provable, where } 1 \leq i \leq n, \text{ and } c_2 \text{ is either } c_2 \text{ when } i = n \text{ or } c_1 \text{ if } i = n \text{ or } c_2. \]

\[ \text{where } 1 \leq i \leq n \text{ and } I^n = \Gamma, AF_x(\psi)(s). \text{ The first condition holds iff } \Gamma \vdash (s/x)\psi \text{ is provable (by induction hypothesis)}, \]

\[ \text{and if the second condition holds if and only if } \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_1), \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_2), \ldots, \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_n) \text{ are all provable, then both conditions hold if and only if } \Gamma \vdash AF_x(\psi)(s) \text{ is provable. So, it is sufficient to prove here the second condition holds if and only if } \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_1), \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_2), \ldots, \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_n) \text{ are all provable:} \]

\[ \Rightarrow \text{ if the second condition holds, then } \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_1), \ldots, \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_n). \text{ That is because otherwise, if } 1 \leq j \leq n \text{ such that } \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_j) \text{ is the first sequent that is not provable, then there exists an infinite path } s_{j_0}, s_{j_1}, s_{j_2}, \ldots, \text{ and } s_{j_0} = s_1 \text{ such that } \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_j) \text{ is not provable for all } k \geq 0, \text{ then by induction hypothesis,} \]

\[ \text{if the second condition holds, then } \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_1), \ldots, \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_n) \text{ are all provable, that is because otherwise, if } 1 \leq j \leq n \text{ such that } \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_j) \text{ is the first sequent that is not provable, then there exists an infinite path } s_{j_0}, s_{j_1}, s_{j_2}, \ldots, \text{ and } s_{j_0} = s_1 \text{ such that } \Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_j) \text{ is not provable for all } k \geq 0, \text{ then by induction hypothesis,} \]

\[ \Gamma^n = \Gamma, AF_x(\psi)(s_1), \ldots, \Gamma^n = \Gamma, AF_x(\psi)(s_n), \text{ and the shape of } c_1, \ldots, c^n \text{ have no impact on the transformations of CPTs here. Note that such } m \geq 0 \text{ exists because our Kripke model is finite. So, the second condition holds implies that } \Gamma, AF_x(\psi)(s) \vdash \]
The shared variables are initially set to a random value in variables are initially set to 0. For each process, the shared variable $s$ and the Boolean programs are as follows:

Programs with Concurrent Processes.

- if $\forall i \in \{1, 2, \ldots, n\}$, $\Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_i)$ is provable, to prove that the second condition holds, it is sufficient to prove that $\text{cpt}(\Gamma' \vdash AF_x(\psi)(s_j), c_1', c_2') \rightarrow c_1'$ for all $1 \leq j \leq n$ and all $c_1', c_2'$, and that $\Gamma' = \Gamma, AF_x(\psi)(s)$. This is easily proved by induction on the structure of the proof tree of $\Gamma, AF_x(\psi)(s) \vdash AF_x(\psi)(s_j)$.

- if $\phi = EG_x(\psi)(s)$ and $\{s_1, \ldots, s_n\} = \text{Next}(s)$, then $\Gamma \vdash EG_x(\psi)(s)$,

  - if $EG_x(\psi)(s) \notin \Gamma$, then $\text{cpt}(\Gamma \vdash EG_x(\psi)(s), c_1, c_2) \rightarrow c_1$ if

    - $\text{cpt}(\Gamma \vdash EG_x(\psi)(s), c_1, c_2) \rightarrow c_1$
    - $\text{cpt}(\Gamma \vdash EG_x(\psi)(s_1), c_1, \text{cpt}(\Gamma' \vdash EG_x(\psi)(s), c_1, c_2)) \rightarrow c_1$
    - $\text{cpt}(\Gamma \vdash EG_x(\psi)(s), c_1, \text{cpt}(\Gamma' \vdash EG_x(\psi)(s_1), c_1, \text{cpt}(\Gamma' \vdash EG_x(\psi)(s), c_1, c_2)) \rightarrow c_1$
    - $\text{cpt}(\Gamma \vdash EG_x(\psi)(s_1), c_1, \text{cpt}(\Gamma' \vdash EG_x(\psi)(s), c_1, c_2)) \rightarrow c_1$

  - $\text{cpt}(\Gamma \vdash EG_x(\psi)(s), c_1, \text{cpt}(\Gamma' \vdash EG_x(\psi)(s_1), c_1, \text{cpt}(\Gamma' \vdash EG_x(\psi)(s), c_1, c_2)) \rightarrow c_1$

  - $\text{cpt}(\Gamma \vdash EG_x(\psi)(s), c_1, \text{cpt}(\Gamma' \vdash EG_x(\psi)(s_1), c_1, \text{cpt}(\Gamma' \vdash EG_x(\psi)(s), c_1, c_2)) \rightarrow c_1$

if there exists an infinite path $s, s_1, s_{1+1}, \ldots, s_{1+i}, \ldots$ such that for all state $s'$ in this path, $\vdash \psi(s')$ is provable, where $\Gamma' = \Gamma, EG_x(\psi)(s), \Gamma_m = \Gamma', EG_x(\psi)(s_1), \ldots, EG_x(\psi)(s_{1+i}), \ldots, EG_x(\psi)(s_{1+i}) \in \Gamma_m$. By induction hypothesis, this holds iff $\Gamma' \vdash EG_x(\psi)$ is provable.

- if $\phi = AF_x, y(\phi_1, \phi_2)(s)$, as are both co-inductive modalities, the analysis is similar to $AF$.

- if $\phi = EU_x, y(\phi_1, \phi_2)(s)$, as are both inductive modalities, the analysis is similar to $AF$.

### A.2 Benchmark and Experimental Data

**Benchmark** The benchmark in this paper is introduced in [23], where two types of random Boolean programs and 24 properties are considered. The description of the benchmark in [23] is as follows.

**Programs with Concurrent Processes.** The parameters of the first set of random Boolean programs are as follows:

| Parameter | Description |
|-----------|-------------|
| a | number of processes |
| b | number of all variables |
| c | number of shared variables |
| d | number of local variables in a process |

The shared variables are initially set to a random value in $\{0, 1\}$, and the local variables are initially set to 0. For each process, the shared variables and the
local variables are assigned the negation of a variable randomly chosen from these variables. We test different sizes of the programs with 3 processes \((a = 3)\), and let \(b\) vary over the set of values \(\{12, 24, 36\}\), then set \(c = b/2, d = c/a\). Each of the 24 properties is tested on 20 test cases for each value of \(b\).

**Programs with Concurrent Sequential Processes.** The parameters of the second set of random Boolean programs are as follows, in addition to \(a, b, c, d\) of the 24 properties is tested on 20 test cases for each value of \(b\).

For brevity, \(T\) limited to 20 minutes. The experimental results are shown in the following tables.

**Experimental Data** All the test cases in this paper are implemented on a Linux platform with 1.9GB memory, and the run-out time for each case was limited to 20 minutes. The experimental results are shown in the following tables. For brevity, \(T\) denote the number of cases in which the property is true, \(F\) is the

\[
\begin{array}{|c|c|}
\hline
i & number of transitions in a process \\
\hline
p & number of parallel assignments in each transition \\
\hline
\end{array}
\]

For each concurrent sequential process, besides the \(b\) Boolean variables, there is a local variable representing program locations, with \(e\) possible values. The shared variables are initially set to a random value in \(\{0, 1\}\), and the local variables are initially set to 0. For each transition of a process, \(p\) pairs of shared variables and local variables are randomly chosen among the shared variables and the local variables, such that the first element of such a pair is assigned the negation of the second element of the pair. Transitions are numbered from 0 to \(t - 1\), and are executed consecutively, and when the end of the sequence of the transitions is reached, it loops back to the execution of the transition numbered 0. For this type of programs, we test different sizes of the programs with 2 processes \((a = 2)\), and let \(b\) vary over the set of values \(\{12, 16, 20\}\), and then set \(c = b/2, d = c/a, t = c,\) and \(p = 4\). Similarly, each property is tested on 20 test cases for each value of \(b\).

**Properties** The properties are specified by a subset of 24 CTL formulae (which are translated to SCTL formulae in the input files for our tool with the same semantics). These properties involve \(AG\), \(AF\) properties, and more complicated ones specified with different combinations of operators with one or two levels of nesting (with two levels of nesting when \(AX\) or \(EX\) is involved). Properties \(p_{01}\) to \(p_{12}\) are shown below, where \(v_i\) are global variables.

\[
\begin{align*}
p_{01} & : AG(\forall_{i=1}^{t} v_i) \\
p_{02} & : AF(\forall_{i=1}^{t} v_i) \\
p_{03} & : AU(v_1, AU(v_2, \forall_{i=1}^{t} v_i)) \\
p_{04} & : AU(v_1, EU(v_2, \forall_{i=1}^{t} v_i)) \\
p_{05} & : AU(v_1, AR(v_2, \forall_{i=1}^{t} v_i)) \\
p_{06} & : AU(v_1, ER(v_2, \forall_{i=1}^{t} v_i)) \\
p_{07} & : AU(v_1, AU(v_2, \forall_{i=1}^{t} v_i)) \\
p_{08} & : AU(v_1, EU(v_2, \forall_{i=1}^{t} v_i)) \\
p_{09} & : AU(v_1, AR(v_2, \forall_{i=1}^{t} v_i)) \\
p_{10} & : AU(v_1, ER(v_2, \forall_{i=1}^{t} v_i)) \\
p_{11} & : AR(AX v_1, AXAU(v_2, \forall_{i=1}^{t} v_i)) \\
p_{12} & : AR(EX v_1, EXEU(v_2, \forall_{i=1}^{t} v_i))
\end{align*}
\]

The properties of \(p_{13}\) to \(p_{24}\) are simply the variations of \(p_{01}\) to \(p_{12}\) by replacing \(\land\) by \(\lor\) and \(\forall\) by \(\exists\), respectively.

**Experimental Data** All the test cases in this paper are implemented on a Linux platform with 1.9GB memory, and the run-out time for each case was limited to 20 minutes. The experimental results are shown in the following tables. For brevity, \(T\) denote the number of cases in which the property is true, \(F\) is the
number of cases in which the property is false. N is the number of all cases that the property is true or false, solv is the number of of solved cases, and adv the number of test cases in which SCTLProV has advantage in speed.

| Test No. | N | Concurrent Processes | Concurrent Seq. Processes |
|----------|---|----------------------|---------------------------|
| Prop.    | solv/T | solv/F | solv/N | adv/T | adv/F | adv/N |
| P00      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P01      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P02      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P03      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P04      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P05      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P06      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P07      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P08      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P09      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P10      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P11      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P12      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P13      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P14      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P15      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P16      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P17      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P18      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P19      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P20      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P21      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P22      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P23      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P24      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |

Table 4. solvable cases (SCTLProV vs iProver Modulo)

| Test No. | N | Concurrent Processes | Concurrent Seq. Processes |
|----------|---|----------------------|---------------------------|
| Prop.    | solv/T | solv/F | solv/N | adv/T | adv/F | adv/N |
| P00      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P01      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P02      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P03      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P04      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P05      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P06      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P07      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P08      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P09      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P10      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P11      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P12      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P13      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P14      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P15      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P16      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P17      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P18      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P19      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P20      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P21      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P22      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P23      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |
| P24      | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 | 60/60 |

Table 5. speed comparisons (SCTLProV vs iProver Modulo)

A.3 The Tool SctlProV
### General Introduction
We implement the idea of our approach as a verification tool on Kripke models, denote \texttt{SCTLProV}. Typically, we use input files to describe the Kripke model and the specification to be verified, as for \texttt{NuSMV} and \texttt{Verds}. The basic verification procedure of \texttt{SCTLProV} is as follows:

1. Read the input file, check for syntax errors;
2. If there is no syntax error detected, then parse the content of the input file into a finite state model $M$ and an SCTL formula $\phi$;
3. Perform the proof search algorithm for the sequent $\vdash \phi$ in $\mathsf{SCTL}(M)$;
4. Produce the result.

### Table 6. solvable cases (\texttt{SCTLProV} vs \texttt{Verds})

| M file | \texttt{SCTLProV} | \texttt{Verds} | M file | \texttt{SCTLProV} | \texttt{Verds} | M file | \texttt{SCTLProV} | \texttt{Verds} | M file | \texttt{SCTLProV} | \texttt{Verds} | M file | \texttt{SCTLProV} | \texttt{Verds} | M file | \texttt{SCTLProV} | \texttt{Verds} |
|--------|------------------|---------------|--------|------------------|---------------|--------|------------------|---------------|--------|------------------|---------------|--------|------------------|---------------|--------|------------------|---------------|
| P01    | 60               | 0.00          | P03    | 60               | 0.00          | P05    | 60               | 0.00          | P07    | 60               | 0.00          | P09    | 60               | 0.00          | P11    | 60               | 0.00          |
| P02    | 60               | 0.00          | P04    | 60               | 0.00          | P06    | 60               | 0.00          | P08    | 60               | 0.00          | P10    | 60               | 0.00          | P12    | 60               | 0.00          |
| P03    | 60               | 0.00          | P05    | 60               | 0.00          | P07    | 60               | 0.00          | P09    | 60               | 0.00          | P10    | 60               | 0.00          | P12    | 60               | 0.00          |
| P04    | 60               | 0.00          | P05    | 60               | 0.00          | P07    | 60               | 0.00          | P09    | 60               | 0.00          | P10    | 60               | 0.00          | P12    | 60               | 0.00          |
| P06    | 60               | 0.00          | P07    | 60               | 0.00          | P09    | 60               | 0.00          | P10    | 60               | 0.00          | P12    | 60               | 0.00          | P12    | 60               | 0.00          |
| P08    | 60               | 0.00          | P09    | 60               | 0.00          | P10    | 60               | 0.00          | P12    | 60               | 0.00          | P12    | 60               | 0.00          | P12    | 60               | 0.00          |
| P10    | 60               | 0.00          | P12    | 60               | 0.00          | P12    | 60               | 0.00          | P12    | 60               | 0.00          | P12    | 60               | 0.00          | P12    | 60               | 0.00          |

### Table 7. speed comparisons (\texttt{SCTLProV} vs \texttt{Verds})

| M file | \texttt{SCTLProV} | \texttt{Verds} | M file | \texttt{SCTLProV} | \texttt{Verds} | M file | \texttt{SCTLProV} | \texttt{Verds} | M file | \texttt{SCTLProV} | \texttt{Verds} | M file | \texttt{SCTLProV} | \texttt{Verds} | M file | \texttt{SCTLProV} | \texttt{Verds} |
|--------|------------------|---------------|--------|------------------|---------------|--------|------------------|---------------|--------|------------------|---------------|--------|------------------|---------------|--------|------------------|---------------|
| P01    | 60               | 0.00          | P03    | 60               | 0.00          | P05    | 60               | 0.00          | P07    | 60               | 0.00          | P09    | 60               | 0.00          | P11    | 60               | 0.00          |
| P02    | 60               | 0.00          | P04    | 60               | 0.00          | P06    | 60               | 0.00          | P08    | 60               | 0.00          | P10    | 60               | 0.00          | P12    | 60               | 0.00          |
Concurrent Processes

- By extending CTL with polyadic predicate symbols.

- In the finite state model, we use the SCTL formulae, instead of CTL formulae, to specify multiple states. We can also specify this in our language. As for the specification introduced to represent either property of a single state or relations between states, we design our language to be suited to describe these three parts of a finite state model.

- The purpose of the SCTL specification language is to describe a finite state model and the specification to be verified against the model. To define a finite model, one usually needs to define the notion of state, the initial state, and the transitions relation between states. We design our language to be suited to describe these three parts of a finite state model.

### Table 8. Solvable Cases (SCTLProV vs NuSMV)

| Proc | N | SCTLProV | NuSMV
|------|---|----------|----------|
| P12 | 60 | 60/60 | 60/60 |
| P11 | 60 | 60/60 | 60/60 |
| P33 | 60 | 60/60 | 60/60 |
| P32 | 60 | 60/60 | 60/60 |
| P21 | 60 | 60/60 | 60/60 |
| P22 | 60 | 60/60 | 60/60 |

### Table 9. Speed Comparisons (SCTLProV vs NuSMV)

| Proc | N | SCTLProV | NuSMV
|------|---|----------|----------|
| P12 | 60 | 60/60 | 60/60 |
| P11 | 60 | 60/60 | 60/60 |
| P33 | 60 | 60/60 | 60/60 |
| P32 | 60 | 60/60 | 60/60 |
| P21 | 60 | 60/60 | 60/60 |
| P22 | 60 | 60/60 | 60/60 |

**Input Language Specification** The purpose of the SCTL specification language is to describe a finite state model and the specification to be verified against the model. To define a finite model, one usually needs to define the notion of state, the initial state, and the transitions relation between states. We design our language to be suited to describe these three parts of a finite state model. In addition, in the SCTL system, a notion of atomic formulae is introduced to represent either property of a single state or relations between multiple states. We also can specify this in our language. As for the specification of the finite state model, we use the SCTL formulae, instead of CTL formulae, by extending CTL with polyadic predicate symbols.
Lexical Tokens. The content of an input file is a sequence of characters, which will be recognized as a sequence of lexical tokens by the lexical analyzer. Among these tokens, a number is a sequence of digits, an identifier is a sequence of characters beginning with an alphabetic character, and followed by any sequence of characters in the set \{A – Z, a – Z, 0 – 9, \_\}. The keywords are listed below:

true false TRUE FALSE not AX EX AF EG AR EU

Any other tokens are in quotes in the syntax descriptions.
Expressions. Expressions in the language consist of variables, constants, and a collection of operator-connected expressions. The syntax of expressions is as follows.

\[
\begin{align*}
\text{expr} &::= \\
&\quad \text{id} \quad \text{;;variable or symbol} \\
&\quad \text{id} \; \text{"(" expr ")"} \quad \text{;;expression evaluated at a state} \\
&\quad \text{id} \; "." \; \text{id} \quad \text{;;local variable of a module} \\
&\quad \text{number} \quad \text{;;integer constant} \\
&\quad \text{"true"} \quad \text{;;logical constant true} \\
&\quad \text{"false"} \quad \text{;;logical constant false} \\
&\quad \text{"#" iden} \quad \text{;;scalar constant} \\
&\quad \text{"!" expr} \quad \text{;;logical negation} \\
&\quad \text{expr} \; \text{"&&"} \; \text{expr} \quad \text{;;logical and} \\
&\quad \text{expr} \; \text{"||"} \; \text{expr} \quad \text{;;logical or} \\
&\quad \text{"-" expr} \quad \text{;;integer negation} \\
&\quad \text{expr} \; \text{"+"} \; \text{expr} \quad \text{;;integer addition} \\
&\quad \text{expr} \; \text{"-"} \; \text{expr} \quad \text{;;integer subtraction} \\
&\quad \text{expr} \; \text{"*"} \; \text{expr} \quad \text{;;integer multiplication} \\
&\quad \text{expr} \; \text{"="} \; \text{expr} \quad \text{;;expression equivalence} \\
&\quad \text{expr} \; \text{"!="} \; \text{expr} \quad \text{;;expression non-equivalence} \\
&\quad \text{expr} \; \text{"<"} \; \text{expr} \quad \text{;;less than} \\
&\quad \text{expr} \; \text{"<="} \; \text{expr} \quad \text{;;less than or equal} \\
&\quad \text{expr} \; \text{">"} \; \text{expr} \quad \text{;;larger than} \\
&\quad \text{expr} \; \text{">="} \; \text{expr} \quad \text{;;larger than or equal}
\end{align*}
\]

Expressions match the pattern \text{"iden \; \{expr\}" only appear in the definition of atomic formulae, which means the expression inside the parentheses are evaluated only at a specified state. This will be explained later in the atomic formulae definition section. The order of precedence of the operators from low to high is

\begin{align*}
\| & \quad \text{and} \\
\&\& & \quad \text{and} \\
+ & \quad \text{add} \\
- & \quad \text{subtract} \\
* & \quad \text{multiply} \\
= & \quad \text{equal} \\
! = & \quad \text{not equal} \\
< & \quad \text{less than} \\
\leq & \quad \text{less than or equal} \\
> & \quad \text{greater than} \\
\geq & \quad \text{greater than or equal}
\end{align*}

Operators of equal precedence are associate to the left. One can also use parentheses to group expressions.

State Variables Declaration. A state in the finite state model is an assignment of a set of state variables. The state variable declaration begins with a \text{Var} keyword, and between \text{"\{" and \text{"\}"} appears the definition of each variable along with its type. The type of a state variable can be either a Boolean, a subrange of the integer set, a scalar, or a user defined module.
User Defined Symbols. It is often more concise if using one single symbol to rep-
represent complicated or commonly used expressions. The declarations of symbols
begins with a Define keyword and are surrounded by "{" and "}".

Initial State Declaration. The initial assignment of the state variables formed the
initial state of the finite state model. The declaration of the initial assignments
for all state variables begins with a Init keyword and are surrounded by "{" and "}".

Note there are two kinds of assignments for the state variables: the assignment
of a state variable by an expression, and the assignment of a state variable
by an instance of a user defined module. For instance, when the assignment
"p := m(1, true)" appears in the initial state declaration of module m', this
means that the state variable p in module m' is initially assigned by the initial
assignment of the state variables in the module m, instantiated by the given
parameter 1 and true. Suppose x is a state variable for module m, then one can
get the assignment of x in module m' by referring to p.x.
Transition Relation Declaration. The transition relation defines the transitions from one state to another. The declaration of transition relation begins with a Transition keyword, and are surrounded by "{" and "}".

```
trans_decl ::
  "Transition"
  "{"
    expr ":" "+" iden ":=" expr ";" iden ":=" expr ";" ... "}" ";"
  ...
  "}"
```

The transition relation is defined by a set of transition options, where each transition option is formed by an guarded expression and a set of state variable assignments. For instance, the transition option "v1=v2 : {v3 := v5+v6; v7 := v8;}" means that when we compute the next state $s'$ of state $s$, we first evaluate the guarded expression $v1=v2$ at state $s$, and if it evaluates to a truth value, then we assign the value of $v5+v6$ to $v3$, and the value of $v8$ to $v7$ in state $s'$. Both $v5+v6$ and $v8$ are evaluated at state $s$. There maybe more than one transition options defined in the transition declaration, and if more than one guarded expressions are evaluated to true, then it refers to a non-deterministic transition.

Atomic Formulae Declaration. By extending CTL with polyadic predicates, SCTL enables us not only to express properties of one single state, but also relations between more than one states in an atomic formula.

```
atomic_decl :: "Atomic"
  "{"
    iden "(" iden "," iden "," ... ")" ":=" expr ";"
    ...
    "}"
```

For instance, "atom1(s1, s2) := (s1(v1) = s2(v2))" defines an atomic formula $atom1(s1, s2)$ such that this formula is true if and only if the assignment of the state variable $v1$ at state $s1$ equals to the assignment of the state variable $v2$ at state $s2$.

Specification Declaration. The specification of a finite model is characterized by a list of SCTL formulae.

```
spec_decl :: "Spec"
  "{"
    iden ":=" formula ";"
    ...
    "}"
```

and
formula ::= "TRUE" ;;Top
| "FALSE" ;;Bottom
| iden "(" iden "," iden "," ... ")" ;;atomic formula
| "not" formula
| formula ":\" formula
| formula ":/" formula
| "AX" "(" iden "," formula "," iden ")" 
| "EX" "(" iden "," formula "," iden ")" 
| "AF" "(" iden "," formula "," iden ")" 
| "EG" "(" iden "," formula "," iden ")" 
| "AR" "(" iden "," iden "," formula "," formula "," iden ")" 
| "EU" "(" iden "," iden "," formula "," formula "," iden ")" 
| "(" formula ")"

Module Declaration. There are two kinds of module declarations in the language: modules containing the declaration of atomic formulae and specification, called the main modules; and modules do not contain the declaration of atomic formulae and specification, called sub-modules. There is exactly one main modules and may be more than one sub-modules in an input file. The main module is like the main function to an C file, while the sub-modules are like normal C functions. The declaration of a main module starts with a Model keyword, and the declaration of a sub-module starts with a Module keyword.

main_module_decl ::= "Model" iden "(" [iden ":" type "," ] ... ")"

{" 
  var_decl
  [symbol_decl]
  init_decl
  trans_decl
  atomic_decl 
  spec_decl 
  "}"

and

sub_module_decl ::= "Module" iden "(" [iden ":" type "," ] ... ")"

{" 
  var_decl
  [symbol_decl]
  init_decl
  trans_decl 
  "}"

Program Structure. A program is formed by a set of declarations of modules.
program ::
    [sub_module_decl]
    ...

main_module_decl

The declaration of sub-modules is optional in an input file.

An Illustrative Example  We show the general usage of our tool by an illustrative example.

*Example 5 (Use of SCTLProV).* This first example concerns the so-called River Crossing Puzzle problem. The question is how can the farmer bring the wolf, the goat, and the cabbage get across the river? We formalize this problem as a Kripke model $M_r$ and the question as a specification. The initial state $s_{ini}$ of model $M_r$ has four components: the initial position of the farmer, the wolf, the goat, and the cabbage. Every transition from one state to another corresponds to every move of the farmer from one side of the river to another, whether he will carry the wolf, the goat, or the cabbage or not. The specification is that if there exists a state $s_r$ can be reachable from $s_{ini}$, such that in $s_r$, all of them get on the other side of the river. the farmer, the wolf, the goat, and the cabbage have crossed the river. These data compose the input file of SCTLProV below:

```plaintext
Model River_Crossing()
{Var {farmer:Bool; wolf:Bool; goat:Bool; cabbage:Bool;}
Init {farmer:=false; wolf:=false; goat:=false; cabbage:=false;}
Transition
{ farmer=wolf: {wolf:=!wolf;};
  farmer=goat: {goat:=!goat;};
  farmer=cabbage: {cabbage:=!cabbage;};
  true: {farmer:=!farmer;};}
Atomic
(safe(s):=!(s(wolf)=s(goat)&&s(wolf)!=s(farmer))&&!(s(goat)=s(cabbage)&&s(goat)!= s(farmer));
  complete(s):=s(farmer)=true&&s(wolf)=true && s(goat)=true&&s(cabbage)=true;}
Spec
{ find:=EU(x,y,safe(x),complete(y),ini); }
```

Note that in this input file, two atomic formulae: safe(s) and complete(s) are given. safe(s) being true means that, in state $s$, neither the goat nor the cabbage can be eaten; complete(s) being true means that, in state $s$, the farmer, the wolf, the goat, and the cabbage all of them have crossed the river. The identifier “ini” represents the initial state. Suppose the input file with name “river.model”. For checking the specification, we can use the following command:

```bash
./sctl -output output.out river.model
```

and the result will display as below:

```plaintext
verifying on the model River_Crossing...
find: EU(x,y,safe(x),complete(y),ini)
find is true, proof output to "output.out".
```
Moreover, as requested, SCTLProV can output a sequence of sequents as a certificate of the verified property in the output file “output.out”. We refer the reader to Appendix for the certificate of the following proof of the sequent

\[ \vdash EU_{x,y}(\text{safe}(x), \text{complete}(y))(s_0): \]

\[ \Gamma_1 \vdash EU_{x,y}(\text{safe}(x), \text{complete}(y))(s_0) \]

where \( \Gamma_i = \{ EU_{x,y}(\text{safe}(x), \text{complete}(y))(s_1), ..., EU_{x,y}(\text{safe}(x), \text{complete}(y))(s_i) \} \).

Note that, by inspecting all the formulae with modality \( EU \) in the certificate, one can find a solution of the river crossing puzzle:

\[
\begin{align*}
\text{s0:} & \{ \text{farmer:=false; wolf:=false; cabbage:=false; goat:=false;} \\
\text{s1:} & \{ \text{farmer:=true; wolf:=false; cabbage:=false; goat:=true;} \\
\text{s2:} & \{ \text{farmer:=false; wolf:=false; cabbage:=false; goat:=true;} \\
\text{s3:} & \{ \text{farmer:=true; wolf:=false; cabbage:=true; goat:=true;} \\
\text{s4:} & \{ \text{farmer:=false; wolf:=false; cabbage:=true; goat:=true;} \\
\text{s5:} & \{ \text{farmer:=false; wolf:=true; cabbage:=false;} \\
\text{s6:} & \{ \text{farmer:=false; wolf:=true; cabbage:=true;} \\
\text{s7:} & \{ \text{farmer:=true; wolf:=true; cabbage:=false;} \\
\end{align*}
\]

On the other hand, if we alter the input file by defining the initial state as:

\[
\text{Init{farmer := false; wolf := true; goat := true; cabbage := false;}}
\]

and then verify the same property with the same command, we get the output result:

verifying on the model River_Crossing...

\[ \text{find: EU(x,y,safe(x),complete(y),ini)} \]

\[ \text{find is false, counterexample output to "output.out".} \]

An counterexample is then output to the file "output.out":

\[
\begin{align*}
0: & \vdash EU(z,y,(\text{safe}(x)),(\text{okay}(y)), (\text{farmer:=false; wolf:=true; goat:=true; cabbage:=false})) [2, 1] \\
2: & \vdash \text{okay((farmer:=false; wolf:=true; goat:=true; cabbage:=false)) []} \\
1: & \vdash \text{safe((farmer:=false; wolf:=true; goat:=true; cabbage:=false)) []}
\end{align*}
\]

This counterexample indicates that \( \vdash EU_{x,y}(\text{safe}(x), \text{okay}(y))(s_{\text{ini}}) \) does not hold, when \( s_{\text{ini}} \) is represented by

\{\text{farmer:=false; wolf:=true; goat:=true; cabbage:=false}\}

as neither \( \vdash \text{safe}(s_{\text{ini}}) \) nor \( \vdash \text{okay}(s_{\text{ini}}) \) holds.

A.4 Experimental Data on Another Computer

We also show our experimental data in another computer (Linux platform with 2.53GHz * 4 CPU, and 1.9GB memory).

13 Each output of a sequent starts with a number, as the index of the sequent, and ends with a sequence of numbers corresponding to the indices of its premises. The merge in a sequent is represented by a set of states instead of formulae.
Programs

| Programs   | iProver Modulo | Verds | NuSMV | NuXMV | SCTLProV |
|------------|----------------|-------|-------|-------|----------|
| Sum        | 1419 (98.5%)   | 1440 (100%) | 1440 (100%) | 1440 (100%) | 1440 (100%) |

| Programs   | iProver Modulo | Verds | NuSMV | NuXMV | SCTLProV |
|------------|----------------|-------|-------|-------|----------|
| Sum        | 1432 (99.4%)   | 1440 (100%) | 1440 (100%) | 1440 (100%) | 1440 (100%) |

Table 12. Solvable cases in five tools.

Table 13. Test cases where SCTLProV outperforms the other four tools.

**Fig. 11.** Average verification time, where the $x$ coordinate is the number of state variables, and the $y$ coordinate is the average time used for each test case.