How Large is the Intrinsic Flux Noise of a Magnetic Flux Quantum, of Half a Flux Quantum and of a Vortex-Free Superconductor?

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Abstract

This article addresses the question whether the magnetic flux of stationary vortices or of half flux quanta generated by frustrated superconducting rings is noisy. It is found that the flux noise generated intrinsically by a superconductor is, in good approximation, not enhanced by stationary vortices. Half flux quanta generated by π-rings are characterized by considerably larger noise.

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Standard electronic devices are based on the manipulation of electrodynamic quantities such as electric charge, current, and magnetic fields. It is obvious that under practical conditions, which require contacts to the devices and operation at a finite temperature $T$, all these devices are subject to noise which arises usually from several sources. Thermal noise and shot noise are prime examples. Digital superconducting devices operate by processing quanta or half quanta of magnetic flux. For some devices, in particular for qubits, their noise needs to be vanishingly small. Exploring whether for superconducting devices a fundamental, ultimate noise limit exists, we ask the following questions: Consider a magnetic flux quantum that is held stationary in a hole of a superconductor cooled to $T \ll T_c$ (Fig. 1). The flux of the vortex penetrates a pick-up coil with an area $A_{\text{coil}}$ of a diameter that is much larger than the London penetration depth $\lambda_L$. Is the flux $\Phi$ in $A_{\text{coil}}$ noisy? Or is the flux of a superconducting vortex under practical operation conditions completely noise-free? If it is noise-free, is it possible to realize devices that operate with flux quanta in a noise-free mode? Of course, being quantized, the fluxoid exactly equals $\Phi_0 = \hbar/2e$ at all instants. But does the flux fluctuate?

To tackle this question, we call to mind that in the superconductor the flux line generates a screening current with density $\vec{J}(\vec{r})$ circling the hole on a path with inductance $L$. The length scale of its penetration into the superconductor is given by $\lambda_L$ [1, 2]. Although these Meissner currents are supercurrents, they fluctuate with $\Delta \vec{J}(\vec{r})$. This noise of the screening currents is caused, in the presence of the magnetic field of the vortex, by thermal and quantum fluctuations of the gauge invariant phase, as well as by fluctuations of the density of the condensate coexisting with the noisy quasiparticle system.

Consequently, the magnetic flux penetrating the hole is noisy, expressed by the vector potential noise $\Delta \vec{A}(\vec{r})$. The noisy screening current and the noisy magnetic field give rise to a gradient of the phase of the superconducting order parameter that is given by:

$$h \vec{\nabla} \varphi = \frac{m}{n 2e} \vec{J} + 2e \vec{A},$$

where $m$ is the Cooper pair mass and $\vec{A}$ is the vector potential [2]. The phase $\varphi$ and the number of Cooper pairs $n$ are conjugate variables, and therefore

$$\Delta \varphi \Delta n \gtrsim 1,$$
an uncertainty relation which holds for large $n$ – the error being of the order of $1/n$ ([2, 3]).

Because the typical order of magnitude of $n$ is $10^{23}$ and $\Delta n/n \sim n^{-1/2} \sim 3 \cdot 10^{-12}$, for any loop closed around a vortex, at any instant, the integrated phase gradient has to amount exactly to an integer number of $2\pi$, the precision being of the order of $10^{-12}$.

Is the total flux penetrating $A_{\text{coil}}$ noisy? We start to answer this question by disregarding current fluctuations at the boundary of $A_{\text{coil}}$, having in mind to return to this boundary effect later. Also we postpone discussions of effects induced by intrinsic equilibrium noise of the current taking place in the absence of the vortex. Then, the topological constraint described yields the requirement that despite of the noise of the local current density $\Delta \vec{J}(\vec{r})$, the noise on the total current $\Delta \vec{I}$, and the noise of the magnetic field $\Delta \vec{A}(\vec{r})$, the total flux through the loop has to equal exactly $\hbar/2e$ in a large spectral range. This refers to all loops that comprise the vortex completely. The noise of the current density and the magnetic field does therefore not generate noise in the total flux through the loop. The noise is quenched by the topological constraint resulting from the uniqueness of the phase. This noise quenching occurs by the following microscopic mechanism: if, for example, at position $\vec{r}$ the screening current fluctuates to exceed its average value, the resulting induction changes the path of $\vec{J}(\vec{r})$ such that its inductance is lowered to the value that keeps the induced flux constant – and noise-free. This process specifically reduces the noise of flux quanta, noise arising from other sources is not necessarily affected.

The noise is quenched over a large spectral range. At high frequencies, the range is limited by the gap frequency $\Delta/\hbar$: the condensate does not respond sufficiently to faster fluctuations. At low frequencies, rare events triggered by highly energetic fluctuations occur when the flux line approaches the boundary of $A_{\text{coil}}$ or even crosses it. Such events are heralded by their large magnetic flux changes. Because the time interval between such jumps scales exponentially with the energy barrier for entry of flux lines into the superconductor, these events are rare and leave correspondingly long, noise-free periods. For many applications, such jumps do not jeopardize device performance. Should a quantum jump, the device has to be reset and started afresh.
Thus we conclude that under the assumptions made, the screening current and the phase fluctuations do not cause flux noise for a superconducting vortex. Due to the macroscopic character of the superconducting flux quantum and the topological requirement of the order parameter phase, the noise generated by a vortex has a very low value, even under practical experimental conditions. Therefore, the frequently practiced encoding of data as separate quantum states presents a route to low noise-data processing; the system can quench low-energy noise at every processing step.

It has to be mentioned that a significant difference exists between the flux noise in a weak-link free superconducting loop and of a loop interrupted with weak-links such as Josephson junctions. In the latter case only the sum of the magnetic flux contribution and the phase difference across the junctions is equal to a multiple of $2\pi$. Under these conditions, noise in the phase difference across the junctions usually controls the flux noise of the loop. In case the junctions are in the zero-voltage state, the phase difference noise is caused via the first Josephson relation by critical current fluctuations. If the junctions are in the voltage state, the phase difference noise is associated via the second Josephson relation with the voltage noise. Therefore, as described by the effective electric circuit of the device (including, in particular, resistive elements) several noise sources are coupled to the flux noise. Such important effects have been thoroughly studied [4, 5, 6]. They also take place in loops with odd numbers of $\pi$-junctions, for example, in tricrystal rings [7]. Keeping the phase difference of $\pi$ across the junction leaves almost a half value of magnetic flux quanta for the flux through the ring. Due to noise in the junctions, the flux noise is significantly enhanced as compared to a loop devoid of weak-links. Nevertheless, also in this case the fluxoid is of course exquisitely noise-free. Half flux quanta generated by frustrated $\pi$-loops can provide a highly precise and stable flux bias for quantum interference devices that also can be rapidly switched (Fig. 2) [8], the flux generated by these loops is, however, not free of noise.

In contrast, the results for weak-link free loops seem to suggest the principal possibility to build noise-free superconducting devices that manipulate flux quanta. Is this possible, indeed? Because the flux of a vortex is, to a good approximation, noise-free, this problem boils down to the question whether under practical conditions also a vortex-free supercon-
Does a vortex-free superconductor induce noise currents in a coil with a diameter $\gg \lambda_L$ as shown in Fig. 3? Yes, the superconductor generates flux noise and, associated with the flux noise, also voltage noise. Supercurrents and quasiparticle currents that are activated by thermal or by quantum fluctuations even in the absence of the vortex, generate fluctuating, local magnetic fields. These fields form closed loops. Loops that are closed within $A_{\text{coil}}$ do not alter the flux in $A_{\text{coil}}$. Flux noise is, however, generated by fluctuating loops that straddle the boundary of $A_{\text{coil}}$ (Fig. 3). By this intrinsic process, superconductors induce a small but finite magnetic flux noise at all parts of their surfaces. Therefore, above the surface of a superconductor, a cloud of minute flux and voltage noise is generated.

This flux noise is closely related to the electromagnetic fluctuations present in condensed matter \cite{9}. It is controlled by the temperature-dependent quasiparticle density and phase fluctuations and thus represents a basic property of the material. The flux noise varies as a function of the distance to the surface of the superconductor. We are not aware that this intrinsic flux noise has been determined for Al at, say, 100 mK, to give an example, or that its spectral density has ever been calculated. Nevertheless it is obvious that the noise is smallest for fully gapped, $s$-wave superconductors. The flux noise causes a minute, temperature dependent, attractive contribution to the force that acts between two closely spaced superconductors. In superconducting qubits, at finite temperature this flux noise provides an ultimately small, yet intrinsic source of decoherence that cannot be overcome.

The effects of a vortex on the superconducting order parameter and also the magnetic field of the vortex are screened in the depth of the superconducting ring. Because the intrinsic flux noise is controlled deep inside the superconductor by current fluctuations on the contour of integration, the intrinsic flux noise and the vortex can interact only slightly. There are several possible mechanisms to cause a small interaction. Fluctuation-induced quasiparticles can, for example, interact with the screening current of the flux line, or be subject to Aharonov-Bohm type phase shifts when circling the vortex. Due to these interactions, the contribution of a stationary vortex to the flux noise is finite – minute, but not exactly zero.
In summary, we conclude that intrinsic flux noise generated by a superconductor is stronger for superconductors with gap nodes. A stationary vortex does, in good approximation, not produce additional noise. Its noise is suppressed by the topology and the macroscopic nature of the vortex. The flux of half vortices generated by frustrated $\pi$-rings is characterized by a significantly larger amount of noise.

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Figure Legends

Fig. 1
Sketch of the device configuration. A vortex $\Phi_0$ is pinned stationary by a hole in a superconductor and penetrates a coil (blue) with an area $A_{\text{coil}}$. The Meissner screening currents $\vec{J}(\vec{r})$ are sketched in red.

Fig. 2
Sketch of a device configuration in which a standard SQUID is biased by half a flux quantum $\Phi_0/2$ generated by a frustrated superconducting loop ($\pi$-loop), formed, e.g., by a tricrystal ring.

Fig. 3
Sketch of a sample configuration to illustrate the flux noise intrinsically generated by a superconductor. Phase fluctuations generate loops of magnetic flux ($A, B$) that penetrate a detector coil (blue) with an area $A_{\text{coil}}$. Loops that straddle the boundary of $A_{\text{coil}}$ ($B$) generate magnetic flux noise in the detector coil, loops that are closed well within $A_{\text{coil}}$ ($A$) do not.
