How to Verify Identity in the Continuous Variable Quantum System?

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Abstract: Continuous variable quantum cryptography has developed rapidly in recent decades, but how to verify identity in the continuous variable quantum system is still an urgent issue to be solved. To solve this problem, we propose a continuous variable quantum identification (CV-QI) protocol based on the correlation of two-mode squeezed vacuum state and the continuous variable teleportation. The bidirectional identity verification between two participants of the communication can be achieved by the proposed CV-QI protocol. In order to guarantee the security, we make full use of the decoy state sequences during the whole process of the proposed CV-QI protocol. Besides, we provide the security analyses of the proposed CV-QI protocol, and analyses indicate that the security of the proposed CV-QI protocol is guaranteed.

Keywords: continuous variable quantum cryptography, verify identity, two-mode squeezed vacuum state, decoy state, teleportation, security

1. Introduction

With the continuous development of quantum communications, a number of continuous variable quantum network dialogue protocols$^{[1-5]}$ have been proposed in the past few years. In 1999, Ralph designed a quantum cryptographic scheme in which the secret key information is modulated on the phase and the amplitude of CW light beam.$^{[6]}$ Although Ralph’s scheme is regarded as the first continuous variable quantum key distribution (CV-QKD) protocol, it is essentially still base on the modulation of the discrete Gaussian states, which is different from the CV-QKD protocols$^{[7-9]}$ utilized nowadays. In 2001, Cerf et al. designed a continuous...
key-distribution scheme taking advantage of a pair of conjugate quantum variables, so that both the key and its carrier are continuous. After that, a lot of quantum communication protocols based on CV-QKD have been put forward. For example, Zhou et al. proposed a continuous variable quantum secret sharing (CV-QSS) scheme in 2013. Gong et al. presented a new-type quantum network dialogue protocol based on continuous variable GHZ states to improve the communication efficiency with the perfect utilization of quantum bits greatly in 2018. In 2019, Ghorai et al. established a lower bound on the asymptotic secret key rate of continuous variable quantum key distribution by using discrete modulation of coherent states, which opens the way to establishing the full security of continuous-variable protocols with discrete modulation.

Continuous variable quantum cryptography has developed rapidly in recent decades, but how to verify identity in the continuous variable quantum system is still a pending issue to be solved. In the general quantum system, the quantum identity authentication protocols have been perfected day by day. In 2000, Zeng et al. proposed the first quantum identity verification protocol with the utilization of quantum key distribution (QKD). Since then, a lot of modified quantum identity verification protocols have emerged. Based on quantum teleportation, Ma et al. proposed the first continuous-variable quantum identity authentication protocol by employing the two-mode squeezed vacuum state and the coherent state. Recently, Zhou et al. put forward a semi-quantum authentication protocol based on the teleportation of W state and the correlation of GHZ-like state, which achieves the mutual identity verification between classical Bob and quantum Alice. So it is natural to consider applying the idea of the semi-quantum authentication protocol to a quantum continuous variable system, and we propose a new continuous variable quantum identification protocol based on the correlation and the teleportation of the weak coherent pulses. In the proposed CV-QI protocol, the two quantum communicators can achieve the mutual identity authentication of each other before communication.

In order to guarantee the security, we make full use of the decoy state sequences
during the whole process of the proposed CV-QI protocol. Concurrently, a set of relatively complete security analyses are given to demonstrate the security of the proposed CV-QI protocol.

The structure of this paper is as follows. In Section 2, the basic principles are introduced in detail. In Section 3, the specific steps of the proposed continuous variable quantum identification protocol are stated. In Section 4, the relatively complete security analyses are provided. Besides, a brief conclusion is reached in Section 5.

2. Basic knowledge

2.1 Two-mode squeezed vacuum state

Two-mode squeezed vacuum state is a kind of entanglement resource commonly used in the quantum communication protocols based on continuous variable quantum correlations. Generally, a two-mod squeezed vacuum light field can be prepared by combining two single-mode squeezed vacuum light fields $\hat{a}_{in1}$ and $\hat{a}_{in2}$ with a 50:50 beam splitter (BS)\[31\], as shown in Figure 1. One of these two single-mode squeezed vacuum fields is created by applying a squeeze operator $\hat{S}(\gamma)$ on a vacuum light field, and the other one is generated by the squeeze operator $\hat{S}(-\gamma)$ (where $\gamma$ is the squeezed parameter and assumed to be real). The canonical quantum quadratures of the output modes of the BS $\hat{a}_{out1}$ and $\hat{a}_{out2}$ can be expressed as follows:

$$\hat{X}_{out1} = \frac{1}{\sqrt{2}} e^{\gamma} \hat{X}_{in1} + \frac{1}{\sqrt{2}} e^{-\gamma} \hat{X}_{in2}, \quad (1)$$

$$\hat{P}_{out1} = \frac{1}{\sqrt{2}} e^{\gamma} \hat{P}_{in1} + \frac{1}{\sqrt{2}} e^{-\gamma} \hat{P}_{in2}, \quad (2)$$

$$\hat{X}_{out2} = \frac{1}{\sqrt{2}} e^{\gamma} \hat{X}_{in1} - \frac{1}{\sqrt{2}} e^{-\gamma} \hat{X}_{in2}, \quad (3)$$
\[ \hat{P}_{\text{out}2} = \frac{1}{\sqrt{2}} e^{-\gamma} \hat{P}_{\text{in}1} - \frac{1}{\sqrt{2}} e^{\gamma} \hat{P}_{\text{in}2}, \quad (4) \]

where \( \hat{X}_{\text{in}1}, \hat{P}_{\text{in}1}, \hat{X}_{\text{in}2} \) and \( \hat{P}_{\text{in}2} \) are the canonical quantum quadrature of the modes \( \hat{a}_{\text{in}1} \) and \( \hat{a}_{\text{in}2} \), the expectations of which follow the Gaussian probability distribution.

It is found that there is an EPR correlation between the two output modes \( \hat{a}_{\text{out}1} \) and \( \hat{a}_{\text{out}2} \), which can be expressed as follows:

\[ \left\langle \left[ \Delta \left( \hat{X}_{\text{out}1} + \hat{X}_{\text{out}2} \right) \right]^2 \right\rangle = \left\langle \left[ \Delta \left( \hat{P}_{\text{out}1} - \hat{P}_{\text{out}2} \right) \right]^2 \right\rangle = \frac{1}{2} e^{2\gamma}, \quad (5) \]

\[ \left\langle \left[ \Delta \left( \hat{X}_{\text{out}1} - \hat{X}_{\text{out}2} \right) \right]^2 \right\rangle = \left\langle \left[ \Delta \left( \hat{P}_{\text{out}1} + \hat{P}_{\text{out}2} \right) \right]^2 \right\rangle = \frac{1}{2} e^{-2\gamma}, \quad (6) \]

Apparently, this correlation increases with the increase of the squeezing parameter \( \gamma \), and peaks when \( \gamma \to +\infty \), i.e.,

\[ \lim_{\gamma \to +\infty} \hat{X}_{\text{out}1} = \hat{X}_{\text{out}2} \quad \lim_{\gamma \to +\infty} \hat{P}_{\text{out}1} = -\hat{P}_{\text{out}2}, \quad (7) \]

In other words, the output modes \( \hat{a}_{\text{out}1} \) and \( \hat{a}_{\text{out}2} \) at this stage are in a maximal entangled state. Here, we adopt a parameter \( F \) to indicate the entanglement degree, which is defined as\(^{[32]}\)

\[ F = \left\langle \left[ \Delta \left( \hat{X}_{\text{out}1} - k_1 \hat{X}_{\text{out}2} \right) \right]^2 \right\rangle_{\text{min}} \times \left\langle \left[ \Delta \left( \hat{P}_{\text{out}1} + k_2 \hat{P}_{\text{out}2} \right) \right]^2 \right\rangle_{\text{min}}. \quad (8) \]

Obviously, the parameter increases with the increasing of the entanglement degree, and approximates to 0 if the condition \( \gamma \to +\infty \) is satisfied.
2.2 Continuous variable quantum teleportation of a coherent state

Quantum teleportation, may be the most novel quantum communication protocols, enable the transmission of an arbitrary unknown quantum state between two parties separated spatially. By a continuous-variable EPR pair as quantum channel, the teleportation for a coherent state can be realized.\cite{33}

Suppose the sender Alice, wants to transfer a coherent state $|\psi\rangle_i = |\hat{X}_i + i\hat{P}_i\rangle$ to the receiver Bob, where $\hat{X}_i$ and $\hat{P}_i$ are the value of the canonical quantum quadrature of the coherent state. To achieve the goal, Alice firstly prepares a pair of entangled light beams $\hat{a}_{out1}$ and $\hat{a}_{out2}$ in two-mode squeezed vacuum state, and send light beam $\hat{a}_{out2}$ to Bob, as shown in Figure 2. And then she performs a Bell Measurement (BM) on the coherent beam and light beam $\hat{a}_{out}$. The results of the BM, which are in the following form, are sent to Bob through classical channel.

$$
\hat{X}_U = \frac{1}{\sqrt{2}} \left( \hat{X}_i - \hat{X}_{out1} \right) \quad \hat{P}_U = \frac{1}{\sqrt{2}} \left( \hat{P}_i + \hat{P}_{out1} \right).
$$

According to the results of the BM, Bob carries out a unitary transformation $D(\beta)$ on his light beam $\hat{a}_{out2}$, where $\beta = 2^{\hat{X}_U + i\hat{P}_U}$. Therefore, the value of the canonical quantum quadrature of the output state can be transformed into
\[ X_O = \hat{X}_{\text{out}2} + \sqrt{2} \hat{X}_U = \hat{X}_i - (\hat{X}_{\text{out}1} - \hat{X}_{\text{out}2}), \]  
(10)

\[ P_O = \hat{P}_{\text{out}2} + \sqrt{2} \hat{P}_U = \hat{P}_i + (\hat{P}_{\text{out}1} + \hat{P}_{\text{out}2}). \]  
(11)

It is clear that when the squeeze parameter $\gamma \to +\infty$ there will be $X_O = \hat{X}_i$, $P_O = \hat{P}_i$. That means the original input coherent state $|\psi\rangle_i$ can be reconstructed at Bob’s cite.

**Figure 2.** Generation of the two-mode squeezed vacuum state

### 3. Continuous variable quantum identification protocol

Suppose Alice and Bob are two participants and they need to verify the identity of each other. All quantum sources are continuous and the continuous variable
quantum identification protocol can be described as follows in detail. Besides, the proposed continuous variable quantum identification protocol can also be expressed by Figure 3.

Step 1 Alice executes the two-mode squeezed operator $\xi(r)$ on vacuum states $|0\rangle_1$ and $|0\rangle_2$ to obtain the two-mode squeezed vacuum states $a_1$ and $a_2$, where $\xi(r) = \exp\left[ r (a_1^+a_2^+-a_1a_2) \right]$. Then Alice randomly selects some time slots $T_1 = \{t_i^1 | i = 0, 1, 2, \ldots, n\}$ to compute the parameter $F^{A}_{12}$ that measures the entanglement degree\cite{33} between $a_1$ and $a_2$.

Step 2 Alice chooses two unitary operations $D\left(\alpha_u = \sqrt{2}x_u + i\sqrt{3}p_u \right)$ and $D\left(\alpha_u = \sqrt{3}x_u + i\sqrt{5}p_u \right)$ at random and performs them on $|0\rangle_{a1}$ and $|0\rangle_{a2}$ to gain the continuous decoy states. Subsequently, Alice randomly arranges these decoy states to generate a new sequence $a_d$ with length $L$. After that, Alice produces continuous decoy states which are the same as states in $a_d$ and randomly selects some time slots $T_2 = \{t_i^2 | i = 0, 1, 2, \ldots, n\}$ to insert these decoy states into sequence $a_2$ in the order of the decoy states in sequence $a_d$ to obtain the mixed state sequence $\hat{a}_2$.

Step 3 Alice executes the two-mode squeezed operator $\xi(r)$ on vacuum states $|0\rangle_3$ and $|0\rangle_4$ to obtain the two-mode squeezed vacuum states $a_3$ and $a_4$, then Alice randomly selects some time slots $T_3 = \{t_i^3 | i = 0, 1, 2, \ldots, n\}$ to calculate the value of entanglement degree parameter $F^{A}_{34}$ between $a_3$ and $a_4$.

Step 4 Alice sends $a_4$ to Bob. When Alice confirms Bob has received $a_4$, she announces the time slot $T_3$ while Bob measures the amplitude or phase of $a_4$ and calculates entanglement degree parameter $F^{B}_{34}$ in time slot $T_3$. Then Alice and Bob compare $F^{A}_{34}$ with $F^{B}_{34}$. If $F^{A}_{34} \to +\infty$, $F^{B}_{34} \to +\infty$ and $F^{A}_{34} = F^{B}_{34}$, Alice sends $a_2$.
to Bob; otherwise, they announce this identification is invalid.

Step 5 Alice measures \( a_d \) and \( a_3 \) with Bell bases, Bob implements the unitary operation \( D(\beta = \sqrt{2}x_u + i\sqrt{2}p_u) \) on \( a_d \) to reconstruct \( a^B_d \).

Step 6 Alice announces time slot \( T_2 \) and Bob compares the decoy states inserted in \( a'_d \) during time slot \( T_2 \) with the reconstructed decoy state sequence \( a^B_d \).

If the decoy states inserted in \( a'_d \) during time slot \( T_2 \) are the same as that in \( a^B_d \), Bob announces that he successfully verifies the identity of Alice and they continue to the next step; otherwise, Alice and Bob terminate this identification.

Step 7 Bob picks out all the decoy states inserted in \( a'_d \) to obtain \( a_2 \), Alice announces time slot \( T_1 \). Bob measures the amplitude or the phase of \( a_2 \) and calculates the entanglement degree parameter \( F_{12}^B \) in the time slot \( T_1 \). Then Bob tells Alice the result of \( F_{12}^B \) by the classical channel and Alice compares \( F_{12}^A \) and \( F_{12}^B \). If \( F_{12}^A = F_{12}^B \), Alice announces that the identity of Bob is legal; otherwise, Alice announces that the identity of Bob is illegal.

4. Security analyses

According to the methods proposed by Zhou et al.,\(^{[16]}\) the security analyses are provided in this section.

4.1 Security analysis based on the beam splitter

Assume Eve is the eavesdropper, and her beam splitter attack may have three steps. First, Eve intercepts \( a_4 \) and lets it pass a beam splitter whose transmittance is \( \eta_1 \) to generate \( a_{4}^E \) and \( a_{4}^{00} \), and then she sends \( a_{4}^E \) that replaces \( a_4 \) to Bob while she remains \( a_{4}^{00} \); Second, Eve intercepts \( a'_2 \) and lets it pass a beam splitter with transmittance \( \eta_2 \) to generate \( a_{2}^{E} \) and \( a_{2}^{10} \), and then she sends \( a_{2}^{E} \) that replaces \( a'_2 \) to Bob while remains \( a_{2}^{10} \); Finally, Eve attempts to obtain the useful message by
measuring $a_E^{00}$ and $a_E^{10}$, respectively. The whole beam splitter attack of Eve is shown in Figure 4.

![Beam splitter attack of Eve](image)

**Figure 4.** Beam splitter attack of Eve

After the operation of Eve on $a_4$,

$$a_E^E = \sqrt{\eta} a_4 + \sqrt{1 - \eta} a_{E1},$$

$$a_E^{00} = \sqrt{\eta} a_{E1} + \sqrt{1 - \eta} a_4.$$ (12) (13)

Bob checks the security of the quantum channel with Alice by comparing the entanglement degree parameter when he received $a_E^E$ instead of $a_4$, however, which cannot find the eavesdropping of Eve since Eve did not change the entanglement characteristics of the squeezed state. After unitary operation $D(\beta = \sqrt{2}x_u + i\sqrt{2}p_u)$, sequence $a_d^B$ that Bob reconstructed is

$$a_d^B = a_4^E + \sqrt{2} (x_u + i p_u).$$ (14)

According to the definition of the conditional variance $V_{A:B}$, $V_{A:B}^{00} = \langle x_d \rangle^2 - \frac{\langle x \cdot x_d \rangle}{\langle x^2 \rangle}$ in the proposed continuous variable quantum identification protocol. Concurrently, conditional variances $V_{A:B}$ and $V_{E:B}$ meet the relationships:

$$V_{A:B}^x \cdot V_{E:B}^p \geq \frac{1}{4}, \quad V_{A:B}^p \cdot V_{E:B}^x \geq \frac{1}{4}. \quad (15)$$
According to Equation (14), it is not difficult to compute that

\[ x_d^B = x_E^x + \sqrt{\eta_1} (x_d - x_3) = \sqrt{\eta_1} x_4 + \sqrt{1 - \eta_1} x_E^{E_1} + \sqrt{\eta_1} (x_d - x_3), \]  

(16)

\[ \left( V_{\Delta B}^\rho \right)_{\min} = \frac{1}{4} (1 - \eta_1) + \frac{\eta_1}{4V_d^x} + \frac{\eta_1}{2} e^{-2r}, \]  

(17)

where \( V_d^x \) represents the variance of the \( x \) component of \( a_d \). Recalling Equation (15), one can obtain

\[ V_{E,B}^x \geq \frac{1}{16 \left( V_{\Delta B}^\rho \right)_{\min}} = \frac{1}{4} \left( 1 - \eta_1 + \frac{\eta_1}{4V_d^x} + 2\eta_1 e^{-2r} \right). \]  

(18)

Similarly,

\[ a_2^E = \sqrt{\eta_2} a_2^E + \sqrt{1 - \eta_2} a_{E_2}, \]  

(19)

\[ a_{10}^E = \sqrt{\eta_2} a_2^E + \sqrt{1 - \eta_2} a_2^E, \]  

(20)

and

\[ \left( V_{\Delta B}^\rho \right)_{\min} = \frac{1}{4} (1 - \eta_2) + \frac{\eta_2}{4V_2^x} + \frac{\eta_2}{2} e^{-2r}. \]  

(21)

Recalling Equation (12)-(21), it is easy to calculate that

\[ \Delta I \geq -\frac{1}{2} \log_2 \left( \frac{1}{1 - \eta_1 + \frac{\eta_1}{V_d} + 2\eta_1 e^{-2r}} \left( 1 - \eta_1 + \eta V_{E_1} + 2\eta_1 e^{-2r} \right) \right) \times \left( 1 - \eta_2 + \frac{\eta_2}{V_2} + 2\eta_2 e^{-2r} \right) \left( 1 - \eta_2 + \eta V_{E_2} + 2\eta_2 e^{-2r} \right). \]  

(22)

where \( \Delta I \) denotes the information transmission rate. The relationship between \( \Delta I \) and other parameters in Equation (22) can also be expressed in Figure 5.

In Figure 5, the X, Y, and Z axes represent parameters \( \eta_1 \), \( \eta_2 \), and \( \Delta I \), respectively, and the curves in colors black, cyan-blue, red and blue indicate the values of \( \Delta I \) when entanglement degree \( r = 10 \), \( r = 1 \), \( r = 0.01 \), and \( r = 0 \), respectively. According to Figure 5, it is clear that the mutual information between Alice and Bob increases as the entanglement degree \( r \) increases. It is worth noting that \( \Delta I > 0 \) if \( r > 0 \), in other words, the proposed continuous variable quantum identification protocol can effectively resist the beam splitter attack of Eve.
4.2 Security analysis based on entanglement degree

Parameter $F$ is given by He,\cite{32} and $F \to 0$ when $r \to +\infty$ while $F \to +\infty$ when $r \to 0$. Obviously, once the entanglement degree of the two-mode squeezed vacuum state is damaged, $r$ will decrease but $F$ will increase rapidly. Generally speaking, $F$ decreases as the entanglement degree increases.

According to the analysis in Sec. 4.1, the entanglement degree of the two-mode squeezed vacuum state remains unchanged, so parameter $F$ will not change and the part that relies on the entanglement degree in the proposed continuous variable quantum identification protocol is secure.

4.3 Security analysis under the joint eavesdropping

Some analyses show that Eve may steal most information if her eavesdropping operation is considered to be a Gaussian operation, which is called “Gaussian eavesdropping optimality theorem”\cite{34-36}.

During the whole proposed continuous variable quantum identification protocol, Alice has three sequences and Bob has two sequences. Therefore, the inequality expressed in Equation (20) can be obtained by the subadditivity and the strong subadditivity of von Neumann entropy. The schematic diagram of the strong subadditivity about Alice’s and Bob’s modes is shown in Figure 6.
\[ S(a_3, a_d : E) = S(a_3, a_d) - S(a_3, a_d | E) \]
\[ = S(a_3, a_d) - \left[ S(a_d | a_3 E) + S(a_3 | a_d E) + S(a_3 : a_d | E) \right], (23) \]
\[ \leq S(a_3) - S(a_3 | A_i B_i E) + S(a_d) - S(a_d | A_i B_i E) \]

where \( A_i \) and \( B_j \) denote the purifications, and \( S(\rho) \) is the von Neumann entropy of state \( \rho \).

**Figure 6.** Schematic of the strong subadditivity

After Alice measures sequences \( a_3 \) and \( a_d \) with Bell bases, one can gain the relation written as Equation (24) in conjunction with Equation (23).

\[ S(a_3, a_d : E) \leq S(a_3 : E_1) + S(a_d : E_2), \]  

(24)

where \( E_1 = A_i B_i E \) and \( E_2 = A_2 B_i E \). Remarkably, there is a Gaussian state \( \rho_{AB}^G \) that also has covariance matrix \( \gamma_{AB} \), and

\[ S(a : E) \leq S(a : E)_G, \]  

(25)

\[ S(a : E)_G = S(E)_G - S(E | a)_G = S(\rho_{AB})_G - S(\rho_{AB | a})_G, \]  

(26)

where \( a \) includes \( a_1 \), \( a_3 \) and \( a_d \). Thus, the secure code rate is

\[ R = I(A : B) - \left[ S(\rho_{AB})_G - S(\rho_{AB | a})_G \right], \]  

(27)

where \( I(A : B) \) is the mutual information between Alice and Bob. However, if the information of Alice and Bob does not meet the Gaussian distribution, the secure code rate is given in Ref. [34], i.e.,

\[ R = I(A : B) - S(a : E) = S(a | E) - H(a | b), \]  

(28)
where $b$ includes $a_2$ and $a_4$, $H$ denotes the Shannon entropy. Recalling Equation (23)-(28), it is easy to calculate that

$$R(\rho_{ab}) \geq R(\rho_{ab}^c). \quad (29)$$

Therefore, the upper bound of the information that Eve steals is computed easily and the security of the proposed continuous variable quantum identification protocol under the joint eavesdropping is guaranteed.

### 4.4 Security analysis under the coherent eavesdropping

The specific theorems and lemma for the proofs of the security of continuous variable quantum protocols are provided.\cite{37} Since every state in the proposed continuous variable quantum identification protocol is prepared, received and measured independently, the security of the proposed continuous variable quantum identification protocol under the coherent eavesdropping is guaranteed according to the analyses of Renner et al.\cite{37}

### 4.5 Non-ideality of the light source

The preparation of quantum state is a crucial part of quantum protocols, but light source noise will affect it. Fortunately, Guo et al. proved that the prepare-and-measurement (P&M) scheme and the entanglement-based (E-B) scheme are equivalent,\cite{38} which can more accurately reflect the impact of light source noise on both two communication parties and the eavesdropper. Subsequently, Usenko et al. created one new scheme to model the light source noise by the beam splitter and the EPR pair.\cite{39} In 2011, Guo et al. designed one unitary transformation model whose versatility is stronger.\cite{40} Therefore, the issue that the non-ideality of the light source influences the security of the proposed continuous variable quantum identification protocol can be solved by the above schemes.\cite{38-40}

### 4.6 Non-ideality of the detector

To detect the information loaded on the quantum state, the homodyne detection is employed in the proposed continuous variable quantum identification protocol. However, the use of the homodyne detection may affect the efficiency and security of the proposed continuous variable quantum identification protocol, worth mentioning,
this issue has been solved by Fossier et al. in 2009.\[41\] Fossier et al pointed that amplifying the quantum state with the phase sensitive amplifier before the homodyne detection can reduce the influence of the non-ideality of the detector, which has been successfully verified by Zhang et al.\[42\] The schematic of the phase sensitive amplifier is shown in Figure 7.

\[\text{Figure 7. Schematic of the phase sensitive amplifier}\]

Apparently, the phase sensitive amplifier is also applicable to the proposed continuous variable quantum identification protocol to guarantee the security and the efficiency under the homodyne detection.

4.7 Finite-size analysis

Compared with infinite-size continuous variable quantum schemes, the secure code rate of finite-size continuous variable quantum protocol is restricted and the transmission distance of states is short. The secure code rate under finite-size is\[43\]

\[R_F = \frac{n}{N} \left[ \delta I(x : y) - S_{\text{cPE}}(y : E) - \Delta(n) \right],\]

where \(N\) is the total amount of data exchanged between Alice and Bob, \(n\) is useful data, \(S_{\text{cPE}}\) is the condition entropy under finite-size, \(x\) and \(y\) are the data that Alice and Bob measured, respectively, \(E\) is the quantum states of Eve, \(\delta\) is the coordination efficiency of the data coordination phase and it can be written as\[44\]

\[\delta = \frac{H(y) - \text{leak}_{\text{EC}}}{I(x : y)},\]

where \(\text{leak}_{\text{EC}}\) is the amount of information exposed between Alice and Bob to
complete data coordination. Parameter $\Delta(n)$ is related to security of confidentiality enhancement and it can be expressed by

$$
\Delta(n) \equiv (2 \dim H_X + 3) \sqrt{\frac{\log_2 \left( \frac{2}{\bar{\varepsilon}} \right)}{n}} + \frac{2}{n} \log_2 \frac{1}{\varepsilon PA},
$$

where $H_X$ is the Hilbert space of the variable $X$, $\bar{\varepsilon}$ is a smoothing parameter, $\varepsilon$ is a smoothing parameter, $\varepsilon PA$ is the failure probability of the confidentiality enhancement process, $\bar{\varepsilon}$ and $\varepsilon PA$ can be specified as any number as small as possible.

Obviously, the secure code rate under finite-size can be calculated easily combining Equation (27)-(29). Therefore, the security under finite-size of the proposed continuous variable quantum identification protocol can be guaranteed.

5. Conclusion

Based on the correlation and the teleportation of two-mode squeezed vacuum states, a new continuous variable quantum identification protocol achieving the mutual identity authentication between two participants is proposed, in which decoy states and the entanglement of the two-mode squeezed vacuum state are utilized to improve the reliability and security. Concurrently, the relatively complete security analyses are built, in which security analyses based on the beam splitter and the entanglement degree, and security analyses under the joint eavesdropping and the coherent eavesdropping provide proof of the theoretical security, while the non-ideality of the light source, the non-ideality of the detector, and the finite-size analysis provide proof of the experimental security. Remarkably, these analyses show that the proposed continuous variable quantum identification protocol is secure in theory and experiment.

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