The Exposure-Background Duality in the Searches of Neutrinoless Double Beta Decay

M.K. Singh,¹,² H.T. Wong,¹ L. Singh,¹,³ V. Sharma,¹,² V. Singh,¹,²,³ and Q. Yue⁴

¹ Institute of Physics, Academia Sinica, Taipei 11529
² Department of Physics, Institute of Science, Banaras Hindu University, Varanasi 221005
³ Department of Physics, School of Physical and Chemical Sciences, Central University of South Bihar, Gaya 824236
⁴ Department of Engineering Physics, Tsinghua University, Beijing 100084

(Dated: January 28, 2020)

Tremendous efforts are required to scale the summit of observing neutrinoless double beta decay ($0\nu\beta\beta$). This article quantitatively explores the interplay between exposure (target mass×data taking time ) and background levels in $0\nu\beta\beta$ experiments. In particular, background reduction can substantially alleviate the necessity of unrealistic large exposure as the normal mass hierarchy (NH) is probed. The non-degenerate (ND)-NH can be covered with an exposure of $O(100)$ ton-year, which is only an order of magnitude larger than those planned for next generation projects – provided that the background could be reduced by $O(10^{-6})$ relative to the current best levels. It follows that background suppression will be playing increasingly important and investment-effective, if not determining, roles in future $0\nu\beta\beta$ experiments with sensitivity goals of approaching and covering ND-NH.

PACS numbers: 14.60.Pq, 23.40.-s, 02.50.-r.
Keywords: Neutrino Mass and Mixing, Double Beta Decay, Statistics.

I. INTRODUCTION

The nature of the neutrinos [1], and in particular whether they are Majorana or Dirac particles, is an important problem in particle physics, the answer to which will have profound implications to the searches and formulation of physics beyond Standard Model and the Grand Unified Theories. Neutrinoless double beta decay ($0\nu\beta\beta$) is the most sensitive experimental probe to address this question [2]. Observation of $0\nu\beta\beta$ implies: (i) that neutrinos are Majorana particles, and (ii) lepton number violation. Since several decades, there are intense activities world-wide committed to the experimental searches of $0\nu\beta\beta$.

Neutrino oscillation experiments [1,3] are producing increasingly precise information on the mass differences and mixings among the three neutrino mass eigenstates. The latest data imply slight preferences of the “Normal Hierarchy” (NH) over the “Inverted Hierarchy” (IH) in the structures of the neutrino mass eigenstates [4]. In parallel, cosmology data [5] provide stringent upper bounds on the total mass of the neutrinos, with good prospects on an actual measurement in the future. Together, a picture emerges providing a glimpse on the parameter space where positive observations of $0\nu\beta\beta$ may reside. Experimental studies are expected to require significant efforts and resources – especially so if NH is confirmed. Detailed quantitative studies on the optimal strategies “to scale this summit” with finite resources would be highly necessary.

The current work addresses one aspect of this issue. We studied the required exposures of $0\nu\beta\beta$-projects versus the expected background $B_0$ before the experiments are performed. The notations and formulation are described in Section I[1] The effects on the “discovery potentials” with varying $B_0$ and the implied experimental strategies, are discussed in Section II[1,15] The connections with the current landscape in neutrino physics are made in Section III[2] via the choice of a particular model on the evaluation of nuclear matrix elements. Various aspects on the interplay between exposure and background in $0\nu\beta\beta$ experiments are discussed in Section III[1,16] Background typically includes two generic components each having different energy dependence – the ambient background and the irreducible intrinsic background from cosmogenic radioactivity and two-neutrino double beta decay ($2\nu\beta\beta$). Only the combined background is considered in this work, while on-going research efforts are attending the different roles of the two components. In particular, the constraints imposed by the $2\nu\beta\beta$ background to detector resolution are discussed in Section III[2]

II. FORMULATIONS AND NOTATIONS

A. Double Beta Decay

The process $0\nu\beta\beta$ in candidate nucleus $A_{\beta\beta}$ refers to the decay

$$\sum_2^N A_{\beta\beta} \rightarrow \sum_{Z+2}^{N-2} A + 2e^- . \quad (1)$$

The experimental signature is distinctive. The summed kinetic energy of the two emitted electrons corresponds to a peak at the transition Q-value ($Q_{\beta\beta}$), which is known and unique for each $A_{\beta\beta}$.

The width of the $0\nu\beta\beta$-peak (denoted by $\Delta$ in %) characterizes the energy resolution of the detector, and is de-
fined — a natural choice and also following convention in the literature for clarity — as the ratio of full-width-half-maximum (FWHM, denoted by \(w_{\mu}\)) to the total measurable energy \(Q_{\beta\beta}\), such that \(w_{\mu}=\Delta Q_{\beta\beta}\).

Various beyond-standard-model processes invoking lepton-number-violation can give rise to \(0\nu\beta\beta\). In the case of the “mass mechanism” where \(0\nu\beta\beta\) is driven by the Majorana neutrino mass, the \(0\nu\beta\beta\) half-life (\(T_{1/2}^{0\nu}\)) can be expressed by \([2, 7]\)

\[
\frac{1}{T_{1/2}^{0\nu}} = G_{A}^{0\nu} \left| m_{\beta\beta} \right|^{2} m_{e} \right|^2 \tag{2}
\]

where \(m_{e}\) is the electron mass, \(G_{A}^{0\nu}\) is the effective axial vector coupling \([8]\), \(m_{\beta\beta}\) is a known phase space factor \([9]\) due to kinematics, \(\left| m_{\beta\beta} \right|\) is the nuclear physics matrix element \([10]\), while \(\langle m_{\beta\beta} \rangle\) is the effective Majorana neutrino mass term which depends on neutrino masses (\(m_{i}\) for eigenstate \(\nu_{i}\)) and mixings (\(U_{ei}\) for the component of \(\nu_{i}\) in \(e_{\mu}\)):

\[
\langle m_{\beta\beta} \rangle = \left| U_{e1}^{2} m_{1} + U_{e2}^{2} m_{2} e^{i\alpha} + U_{e3}^{2} m_{3} e^{i\beta} \right| \tag{3}
\]

where \(\alpha\) and \(\beta\) are the Majorana phases.

The measurable half-life \(T_{1/2}^{0\nu}\) from an experiment which observes \(N_{0\nu}^{obs}\)-counts of \(0\nu\beta\beta\)-events in time \(t_{DAQ}\) in a “Region-of-Interest” (RoI) at an efficiency of \(\varepsilon_{RoI}\) can be expressed as

\[
T_{1/2}^{0\nu} = \ln 2 \cdot N(A_{\beta\beta}) \cdot t_{DAQ} \cdot \left[ \frac{\varepsilon_{RoI}}{N_{0\nu}^{obs}} \right] \tag{4}
\]

where \(N(A_{\beta\beta})\) is the number of \(A_{\beta\beta}\) atoms being probed.

For simplicity in discussions and to allow the results to be easily convertible to different configurations — while capturing the essence of the physics, results in this article are derived in the special “ideal” case where the target is made up of completely enriched \(A_{\beta\beta}\) isotopes. That is, the isotopic abundance (IA) is 100%. In additional, the various experimental efficiency factors are all unity (\(\varepsilon_{exp}=100\%\)). Accordingly, Eq. \(4\) becomes

\[
T_{1/2}^{0\nu} = \ln 2 \cdot \frac{N_{A}}{M(A_{\beta\beta})} \cdot \Sigma \cdot \left[ \frac{\varepsilon_{RoI}}{N_{0\nu}^{obs}} \right] \tag{5}
\]

where \(N_{A}\) is the Avogadro Number, \(M(A_{\beta\beta})\) is the molar mass of \(A_{\beta\beta}\), and \(\Sigma\) denotes the combined exposure (mass\(\times\)\(t_{DAQ}\)) expressed in units of ton-year (ton-yr) at \(A_{\beta\beta}\) at IA=100% and \(\varepsilon_{exp}=100\%\). Effects due to these parameter choices and other assumptions will be discussed in Section \([11]\) where conversion relations to those for realistic experiments are given.

The expression of Eq. \(5\) applies to experiments with counting analysis. More sophisticated statistical methods are usually adopted to extract full information from a given data set. These typically exploit the energy spectral shapes, which are known for the signal and are predictable with uncertainties for the background. However, in the conceptual-design and sensitivity-projection stage of experiments, the simplified and intuitive approach of Eq. \(5\) will suffice, especially so in the low count rate Poisson statistics regime which is of particular interest in this article.

Combining the theoretical and experimental descriptions of \(T_{1/2}^{0\nu}\) from, respectively, Eqs. \([2, 7]\) gives:

\[
\left| M_{0\nu} \right|^{2} = \frac{1}{\langle m_{\beta\beta} \rangle^{2}} \left[ \frac{1}{\Sigma} \sum N_{0\nu}^{obs} \right] \tag{6}
\]

is called “specific phase space” in the literature \([7]\).

\[\text{B. Discovery Potential}\]

In our context, \(B_{0}\) is expected background counts within the RoI around \(Q_{\beta\beta}\). This can, in principle, be predicted with good accuracies prior to the experiments. The sensitivity goals of experiments are typically expressed in the literature \([11]\] as: “Discovery Potential at 3\(\sigma\) with 50% probability” (\(P_{50}\)) and “upper limits at 90% confidence level” which characterize possible positive and negative outcomes, respectively. We focus on \(P_{50}\) in this work, for the reason that next-generation \(0\nu\beta\beta\) experiments should be designed to have the maximum reach of discovery, rather than setting limits.

Poisson statistics is necessary to handle low background and rare signal processes. The dependence of the required average signal (\(S_{0}\)) versus \(B_{0}\) under \(P_{50}\) and other discovery potential criteria are depicted in Figure \([1]\). For a given real and positive \(B_{0}\) as input and using \(P_{50}\) as illustration, the Poisson distribution \(P(\mu; \mu)\) is constructed with mean \(\mu=B_{0}\). The observed count \(N_{0\nu}^{obs}\) is evaluated as the smallest integer which satisfies

\[
\sum_{i=0}^{N_{0\nu}^{obs}} P(i; B_{0}) \geq (1 - 0.00135) \tag{7}
\]

where 0.00135 is the fraction of a Gaussian distribution in the interval \([+3\sigma, \infty]\). This is the minimal observed event integer number with \(\geq3\sigma\) significance over a predicted average background \(B_{0}\). The output \(S_{0}\) is the minimal signal strength corresponding to the case where the average total event \((B_{0}+S_{0})=N_{0\nu}^{obs}\) with \(\geq50\%\) probability. This is evaluated as the minimum value which satisfies another Poisson distribution under the condition:

\[
\sum_{i=0}^{N_{0\nu}^{obs}} P(i; [B_{0}+S_{0}]) \geq 0.5 \cdot \tag{8}
\]

It can be inferred from Figure \([1]\) that the “background-free” level with \(P_{50}\) criteria corresponds to a background of \(B_{0}<10^{-3}\) and a reference-point signal of \(S_{0}=S_{ref}=0.69\).

The ratios of \(S_{0}\) relative to \(S_{ref}\) are depicted in Figure \([1b]\). It can be seen that one would require factors of
in Eq. 2, relative to the values of various segments.

The origin of the negative slopes in various segments. The ground events are indistinguishable experimentally. The steps in Figures 1a&b. In addition, signal and background events are indistinguishable experimentally. The $P_{\text{ref}}$ criteria is applied to (B$_0$+S$_0$) versus B$_0$, while the S$_0$ dependence on B$_0$ is shown in Figures 1a&b. This is the origin of the negative slopes in various segments.

While the predicted average background B$_0$ can be continuous and real numbers, only integer counts can be observed in an experiment. This gives rise to the relations being inequalities in Eqs. 7&8 and consequently the steps in Figures 1a&b. In addition, signal and background events are indistinguishable experimentally. The $P_{\text{ref}}$ criteria is applied to (B$_0$+S$_0$) versus B$_0$, while the S$_0$ dependence on B$_0$ is shown in Figures 1a&b. This is the origin of the negative slopes in various segments.

C. Background Index

The theme of this work is to study the interplay between required exposure and background in 0νββ experiments to meet certain ($m_{\beta\beta}$) target sensitivities.

In realistic experiments, it is more instructive to characterize background with respect to exposure and the RoI energy range, such that the relevant parameter is the “Background Index” (BI) defined as:

$$BI \equiv \frac{B_0(\text{RoI})}{\Sigma}$$

which is the background within the RoI (chosen to be $\equiv w_{\nu\nu}$, following convention) per 1 ton-year of exposure, with dimension [counts/(w$_{\nu\nu}$-ton-yr)]. Background levels expressed in BI are universally applicable to compare sensitivities of varying $A_{\beta\beta}$ in different experiments.

D. Conversion to Realistic Configurations

As explained in Section II A, the (BI, $\Sigma$) results presented in this article correspond to the ideal case where IA=100% and $\varepsilon_{\text{expt}}=100%$. In addition, while the range of $g_\Lambda \in [0.6, 1.27]$ is generally considered possible [7,8], the “unquenched” free nucleon value of $g_\Lambda=1.27$ is adopted.

The required exposure ($\Sigma'$) in realistic experiments would be larger and can be readily converted from the $\Sigma$-values via

$$\Sigma' \approx \Sigma \cdot \frac{1}{IA} \cdot \frac{1}{\varepsilon_{\text{expt}}} \cdot W_\Sigma(g_\Lambda)$$

where $W_\Sigma(g_\Lambda)$ is the weight factor for $\Sigma$ due to the $g_\Lambda$-dependence [10,12] of $T^{\beta\beta}_{\text{ref}}$ in Eq. 2 relative to the values at $g_\Lambda=1.27$. It is depicted in Figure 3 for the case of $^{76}$Ge. The finite band width as a function of $g_\Lambda$ is the consequence of the spread in $|M^{0\nu}|^2$ predictions [10,12]. The specific case where $|M^{0\nu}|^2$ is independent of $g_\Lambda$ implies $\Sigma \propto g_\Lambda^{-4}$ and is denoted by the dotted line.

The background index defined relative to $\Sigma'$ for realistic configurations can accordingly be expressed as

$$BI'(\Sigma') \approx BI \left[ \frac{\Sigma'}{\Sigma} \right]$$

such that $\Sigma' > \Sigma$ and $BI' < BI$. Realistic experiments naturally imply larger exposure and more stringent background requirements.

E. Neutrino Physics Connections

Results from neutrino oscillation experiments [1,3] indicate that the $m_i$ of the three active $\nu_i$ have structures corresponding to either IH or NH. The values of $\langle m_{\beta\beta} \rangle$ are constrained and depend on the absolute neutrino mass scale, and are typically expressed in terms of the...
The best-fit and other diagonal lines correspond to the match-
cidental cancellation which leads to very small 
ran phases (\langle m_{\beta\beta} \rangle) and listed in Table I shows that the vanishing values of \langle m_{\beta\beta} \rangle are disfavored.

The current generation of oscillation experiments may reveal Nature’s choice between the two hierarchy options. In particular, there is an emerging preference of NH over IH [3]. Moreover, the combined cosmology data may provide a measurement on the sum of \langle m_{\beta\beta} \rangle in 0\nu\beta\beta searches will be further constrained.

Extracting neutrino mass information via Eq. 2 from the experimentally measured \Sigma_{\nu\nu} requires knowledge of \langle M^{0\nu} \rangle and \langle g_{A} \rangle. There are different schemes to calculate \langle M^{0\nu} \rangle for different A_{\beta\beta} [10]. Deviations among their results are the main contributors to the theoretical uncertainties. Another source of uncertainties is the values of \langle g_{A} \rangle, which may differ between a free nucleon and complex nuclei [8].

Studies of Ref. [7] suggest that, in the case where 0\nu\beta\beta is driven by the neutrino mass mechanism, there exists an inverse correlation between \langle M^{0\nu} \rangle and \langle g_{A} \rangle. To derive numerical results which would shed qualitative insights without involving excessive discussions on the choice of \langle M^{0\nu} \rangle, we assume that this correlation is quantitatively valid.

We follow Ref. [7] in adopting the geometric means of the realistic ranges for the various \langle M^{0\nu} \rangle in different isotopes. The data points can be parametrized by straight lines at given \langle m_{\beta\beta} \rangle, as depicted in Figure 2. That is, \langle M^{0\nu} \rangle (g_{A} H^{0\nu}) is a constant at fixed \langle m_{\beta\beta} \rangle independent of A_{\beta\beta}. The displayed \langle m_{\beta\beta} \rangle values in Figure 2 correspond to 0\nu\beta\beta decay rates of [N_{\nu\nu}^{0\nu} (\Sigma)=1/ton-yr at \langle g_{A} \rangle=1.27 and full efficiency. The best-fit at this decay rate corresponds to \langle m_{\beta\beta} \rangle=35 \times 10^{-3} eV.

Following Eq. 4 this model leads to a simplifying consequence that

\[ 1 \times \left( \frac{\epsilon_{\nu\nu}^{\text{Ref}}}{N_{\nu\nu}^{0\nu_{\text{obs}}} \Sigma} \right) \propto \frac{1}{\langle m_{\beta\beta} \rangle^2} \]  

at IA=100% and \epsilon_{\nu\nu}^{\text{exp}}=100%, which is universally applicable to all A_{\beta\beta}. The proportional constant can be derived via the best-fit values of Figure 2.

Given a background B_{0} as input, the required S_{0} to establish signal under P_{50}^{\beta\beta} can be derived via Figure 1. This is related to the mean of N_{\nu\nu}^{0\nu_{\text{obs}}} at known \epsilon_{\nu\nu}^{\text{Ref}}. Neutrino physics provides constraints on \langle m_{\beta\beta} \rangle with several scales-of-interest given in Table I. The output values of \Sigma and BI can be derived with Eqs. [2,9] respectively.

The \Sigma-values thus inferred in what follows could be interpreted with the typical uncertainties of a “factor of two, both directions” (that is, within a factor of [0.5, 2.0]...
as $B$ is 32% longer, and smaller (such that for $g_A=1.27$ in the case of $^{76}\text{Ge}$, the finite band width is the consequence of the spread in $|M^{0\nu}|^2$ predictions $|M^{0\nu}|^2$ $\propto \Sigma^2$). The specific case where $|M^{0\nu}|^2$ is independent of $g_A$ such that $\Sigma\propto|g_A|^{-4}$ is denoted by the dotted line.

of the nominal values) to match our current understanding of $|M^{0\nu}|^2$.

III. SENSITIVITY DEPENDENCE

It is well-known, following Eqs. 2-5, that the sensitivity to $\{m_{\beta\beta}\}$ is proportional to $\Sigma^2$ as $B_0\rightarrow0$ and to $\Sigma^4$ at large $B_0$. We further investigate the $B_0$-dependence quantitatively and in the context of the preferred IH and NH ranges with the model of Ref. [7]. The specific $\langle m_{\beta\beta}\rangle$-values of Table II — $(\langle m_{\beta\beta}\rangle,\langle m_{\beta\beta}\rangle_{95\%})$ for both IH and NH — serve to provide reference scales.

A. Required Exposure and Background

The variations of $\langle m_{\beta\beta}\rangle$ versus $B_0$ with different $\Sigma$ at $\text{RoI}=w_{\nu}\sigma_{\nu}$ (such that $\varepsilon_{\text{RoI}}\gtrsim 76\%$) under the criteria of $P^{90}_{50}$ are depicted in Figure 4 with the IH and NH bands superimposed. The matching $T_{1/2}^{0\nu}$ for $^{76}\text{Ge}$ is illustrated. The equivalent half-life sensitivities for other isotopes $A_{\beta\beta}$ can be derived via

$$\left[T_{1/2}^{0\nu}\right]_{A_{\beta\beta}} = \left[T_{1/2}^{0\nu}\right]_{^{76}\text{Ge}} \left(\frac{76}{A_{\beta\beta}}\right).$$

The figure depicts how the same exposure can be used to probe longer $T_{1/2}^{0\nu}$ and smaller $\langle m_{\beta\beta}\rangle$ with decreasing background.

The dependence of $\langle m_{\beta\beta}\rangle$ sensitivities to $B_{\nu}$ is depicted in Figure 5. Taking $\text{RoI}=w_{\beta\sigma}$ is obviously not the optimal choice when the expected background $B_0\rightarrow0$. An alternative choice for low $B_0$ is $\text{RoI}=w_{\beta\sigma}$ covering $\pm 3\sigma$ of $Q_{\beta\beta}$, such that $\varepsilon_{\text{RoI}}\approx 100\%$. Both schemes are illustrated in Figure 5. The choice of $\text{RoI}=w_{\beta\sigma}$ at $B_0\rightarrow0$ would expectedly give better sensitivity by a factor of $\varepsilon_{\text{RoI}}(w_{\beta\sigma})=0.76$, such that the covered $T_{1/2}^{0\nu}$ is 32% longer, or the required $\Sigma$ is 24% less.

The required exposure to probe $\langle m_{\beta\beta}\rangle_{95\%}$ and $\langle m_{\beta\beta}\rangle_{-}$ with both $\text{RoI}$ selections in both IH and NH are depicted in Figure 6. Superimposed as a blue contour is the
TABLE II: Required $\Sigma$ to cover $\langle m_{\beta\beta}^{\text{IH(NH)}}\rangle$ and $\langle m_{\beta\beta}^{\text{IH(NH)}}\rangle$ at different background scenarios in descending order of intensity. The BI-values follow from Eq. 9

| Scenario                        | Background [counts/($w_{\nu}$-ton-yr)] | Required $\Sigma$ (ton-yr) To Cover |
|--------------------------------|----------------------------------------|-------------------------------------|
| Best Published [13]             |                                        | $\langle m_{\beta\beta}^{\text{IH}}\rangle_{95\%}$ | $\langle m_{\beta\beta}^{\text{IH}}\rangle_{95\%}$ | $\langle m_{\beta\beta}^{\text{NH}}\rangle_{95\%}$ | $\langle m_{\beta\beta}^{\text{NH}}\rangle_{95\%}$ |
| Next Generation [13]            |                                        | 27  | 110  | $4.4\times10^4$ | $11\times10^6$ |
| Projected [13]                  |                                        | 6.1 | 19   | $4.7\times10^3$ | $0.97\times10^6$ |
| Benchmark                       |                                         | 7.3 | –    | –               | –               |
| [1 count/($w_{\nu}$-$\Sigma$)] |                                         | 3.1 | –    | $4.6\times10^2$ | –               |
| -IH :{                          |                                         | –   | 15   | –               | –               |
| -NH :{                          |                                         | –   | 330  | –               | –               |
| “Background-Free”               |                                         | –   | 37   | –               | –               |

TABLE III: The range of (S$_0$, B$_0$) to qualify a positive signal to cover $\langle m_{\beta\beta} \rangle_{-}$ for both IH and NH under P$_{95\%}^\nu$, given the observed number of events in RoI = 0-$\nu$/$\beta$-signals and background are combined but indistinguishable at event-by-event level. The smaller $\Sigma$ values among the alternatives of RoI=$w_{\nu}$ or $w_{\beta}$ are selected. The sixth column shows the required BI which is universal to all $A_{\beta\beta}$. Last column lists the required background specifically for $^{76}\text{Ge}$ normalized to $(\text{keV-ton-yr})^{-1}$, and the conversion to other isotopes is referred to Eq. [13]. The $N_{obs}=1$ row corresponds to the background-free conditions. The BI-values follow from Eq. [9]

| Counts Within RoI | Optimal RoI | Universal Exposure | Background/(keV-ton-yr) |
|------------------|-------------|--------------------|-------------------------|
| $N_{obs}^{\nu}$  | $S_0$       | $B_0$              | $\Sigma$ $(\text{ton-yr})$ [counts/($w_{\nu}$-ton-yr)] | $\Delta = 0.12\%$ |
| 1 $\geq 0.69$ $\leq 1.3\times10^{-3}$ | $w_{3\sigma}$ | 1.7 | $\leq 3.1\times10^{-4}$ | $\leq 1.2\times10^{-3}$ |
| 2 $\geq 1.6$ $\leq 5.2\times10^{-2}$ | $w_{3\sigma}$ | 4.0 | $\leq 5.2\times10^{-3}$ | $\leq 2.1\times10^{-3}$ |
| 3 $\geq 2.5$ $\leq 0.21$ | $w_{3\sigma}$ | 6.0 | $\leq 1.4\times10^{-2}$ | $\leq 5.6\times10^{-3}$ |
| 4 $\geq 3.2$ $\leq 0.45$ | $w_{1/2}$ | 10 | $\leq 4.3\times10^{-2}$ | $\leq 1.8\times10^{-2}$ |
| 5 $\geq 3.9$ $\leq 0.77$ | $w_{1/2}$ | 13 | $\leq 6.1\times10^{-2}$ | $\leq 2.4\times10^{-2}$ |
| 10 $\geq 6.6$ $\leq 3.1$ | $w_{1/2}$ | 21 | $\leq 0.14$ | $\leq 6.0\times10^{-2}$ |
| 1 $\geq 0.69$ $\leq 1.3\times10^{-4}$ | $w_{3\sigma}$ | 5.5 | $\leq 0.96\times10^{-6}$ | $\leq 0.38\times10^{-6}$ |
| 2 $\geq 1.6$ $\leq 5.2\times10^{-2}$ | $w_{3\sigma}$ | 1.3 | $\leq 1.6\times10^{-5}$ | $\leq 6.4\times10^{-6}$ |
| 3 $\geq 2.5$ $\leq 0.21$ | $w_{3\sigma}$ | 2.0 | $\leq 4.0\times10^{-5}$ | $\leq 1.7\times10^{-5}$ |
| 4 $\geq 3.2$ $\leq 0.45$ | $w_{1/2}$ | 3.4 | $\leq 1.3\times10^{-4}$ | $\leq 5.4\times10^{-5}$ |
| 5 $\geq 3.9$ $\leq 0.77$ | $w_{1/2}$ | 4.2 | $\leq 1.8\times10^{-4}$ | $\leq 7.5\times10^{-5}$ |
| 10 $\geq 6.6$ $\leq 3.1$ | $w_{1/2}$ | 7.0 | $\leq 4.4\times10^{-4}$ | $\leq 1.8\times10^{-4}$ |

“benchmark” background level at 1 count/($w_{\nu}$-$\Sigma$) where the first background event would occur at a given exposure. The benchmark level also represents the transition in the effectiveness of probing ($m_{\beta\beta}$) with increasing exposure. The shaded regions correspond to the preferred hardware specification space for future 0-$\nu$/$\beta$ experiments – where the exposure should be sufficient to cover at least $\langle m_{\beta\beta} \rangle_{95\%}^{\text{IH(NH)}}$, and there would be less than one background event per $w_{\nu}$ over the full exposure.

The required exposures under various background conditions are summarized in Table [I]. The best published background level is $1.0_{-0.6}^{+0.4}$ counts/(keV-ton-yr) or BI$\sim3$ counts/($w_{\nu}$-ton-yr) from the GERDA experiment on $^{76}\text{Ge}$ [13]. For simplicity, the “best” current background is taken to be BI$\equiv B_0=1$ count/($w_{\nu}$-ton-yr) in what follows. This background would correspond to $\Sigma_{\text{HI(NH)}}=110$ ton-yr (11 Mton-yr) to cover $\langle m_{\beta\beta} \rangle_{-}^{\text{IH(NH)}}$. Such a large required exposure is inefficient and unrealistic, so that the background should be signifi-
cantly reduced to allow the quest to advance. The target exposure is $\Sigma=10$ ton-yr for the next generation $0\nu\beta\beta$ projects to cover IH with ton-scale detector target \cite{11}. Following Figure 5, this exposure would require $\text{BI}<0.21, 0.033)$ counts/(w$_{\nu},\text{-ton-yr})$ to cover $(m_{\beta\beta})_{95\%}$, $(m_{\beta\beta})^{IH}$. This matches the background specifications of $\text{BI}=O(0.1)$ counts/(w$_{\nu},\text{-ton-yr})$.

The background-free ($\text{BI}_{\text{min}}$) — equivalently, minimal-exposure ($\Sigma_{\text{min}}$) — condition is where one single observed event can establish the signal at the $P_{50}^{m_{\beta\beta}}$-criteria. Their values at the benchmark $(m_{\beta\beta})$’s are given in Table I.

The choice of $(m_{\beta\beta})$ to define $\Sigma$ is a conservative one. Since $(m_{\beta\beta})_{95\%}(m_{\beta\beta})$ from Table I the minimum exposure $\Sigma_{\text{min}}$ corresponding to $(m_{\beta\beta})_{95\%}$ is reduced relative to that for $(m_{\beta\beta})$ by a fraction given as $f_{95\%}$.

The variations of $(\text{BI}_{\text{min}},\Sigma_{\text{min}})$ with $(m_{\beta\beta})$ are depicted in Figure 7. As shown by the black dots and also listed in Table I, $\Sigma_{\text{min}}=(0.83,1.7)$ ton-yr at $\text{BI}_{\text{min}}\leq(6.3\times10^{-4}, 3.1\times10^{-5})$ counts/(w$_{\nu},\text{-ton-yr})$ are required to cover $(m_{\beta\beta})_{95\%}$, $(m_{\beta\beta})^{IH}$. The corresponding requirements for NH are $\Sigma_{\text{min}}=(37,550)$ ton-yr at $\text{BI}_{\text{min}}\leq(1.4\times10^{-5}, 0.96\times10^{-6})$ counts/(w$_{\nu},\text{-ton-yr})$. The required $\text{BI}_{\text{min}}$ from $(m_{\beta\beta})$ to $(m_{\beta\beta})_{95\%}$ is reduced by $f_{95\%}=0.49(0.068)$ for IH(NH).

Alternatively, the $\Sigma=10$ ton-yr target exposure of next-generation projects can probe $(m_{\beta\beta})>(5.8\times10^{-3})$ eV, approaching $(m_{\beta\beta})_{\text{NH}}=(4.3\times10^{-3})$ eV, when the background-free condition $\text{BI}_{\text{min}}\leq5.1\times10^{-5}$ counts/(w$_{\nu},\text{-ton-yr})$ is achieved.

The interplay between fractional reduction of BI and $\Sigma$ relative to $\Sigma_{\text{IH}}$ and $\Sigma_{\text{IH}}$ to cover $(m_{\beta\beta})_{95\%}$ and $(m_{\beta\beta})$ in IH(NH) is depicted in Figure 8. Background-free conditions require additional BI-suppression by factors of $3.1\times10^{-4}(0.96\times10^{-6})$, to cover $(m_{\beta\beta})_\text{IH}$ in which cases $\Sigma$ can be reduced by factors of $0.016(5\times10^{-5})$. The shaded regions match those of Figure 1 in displaying the preferred hardware specification space.

The impact of background suppression to the required exposure is increasingly enhanced as smaller values of $(m_{\beta\beta})$ are probed. This is illustrated in Figures 3(b) which display the reduction fraction in $\Sigma$ relative to $\Sigma_{\text{IH}}$ at different background levels. For instance, the
The suppression of BI from 1 to $10^{-3}$ counts/(w,σ-ton-yr) will contribute to the reduction of $\Sigma$ from (27, 110) ton-yr to (1.1, 4.1) ton-yr and from $\Sigma=(44, 11000)$ kton-yr to (0.17, 13) kton-yr to cover $(m_{\beta\beta})_{95\%}^{NIH}$ and $(m_{\beta\beta})_{95\%}^{NH}$, respectively.

In realistic experiments, signals and background are indistinguishable at the event-by-event level. The expected average background $B_0$ and the observed event counts (an integer) in the RoI are the known quantities. They can be used to assess whether a signal is “established” under certain criteria like $P_{50}$. Listed in Table III are the required ranges on $(S_0, B_0)$ to qualify positive signals given the number of observed events. The first row corresponds to the background-free condition, in which one single event is sufficient to establish a signal. Accordingly, the (BI, Σ) values match the entries in the last rows of Table II and are displayed in Figure 7.

Results of Table III apply generically to all $A_{\beta\beta}$ except those for the last column when background is expressed in “/(keV-ton-yr)” unit. The values are specific for $^{76}$Ge, where the best published $\Delta(76\text{Ge})=0.12\%$ of the MJD-experiment [15] is adopted as input. The background requirements for other $A_{\beta\beta}$ can be derived via:

$$\frac{[\text{Background}/(\text{keV-ton-yr})](A_{\beta\beta})}{[\text{Background}/(\text{keV-ton-yr})](76\text{Ge})} = \frac{\Delta(76\text{Ge})}{\Delta(A_{\beta\beta})} \left[ \frac{Q_{\beta\beta}(76\text{Ge})}{Q_{\beta\beta}(A_{\beta\beta})} \right]. \quad (14)$$

B. Limiting Irreducible Background

It is instructive and important to quantify the interplay between various irreducible background channels to the required exposure. In particular, one such irreducible background is the Standard Model-allowed $2\nu\beta\beta$

$$N_Z A_{\beta\beta} \rightarrow \frac{N-2}{Z+2} A + 2e^- + 2\nu_e. \quad (15)$$

The contamination levels to $0\nu\beta\beta$ at the $Q_{\beta\beta}$-associated RoI depend on its half-life ($T^{\nu}_{1/2}$) and the detector resolution. A worse resolution (larger $\Delta$) implies a larger RoI range to search for $0\nu\beta\beta$ signals, and therefore a higher probability of having background events from the $2\nu\beta\beta$ spectral tail.
TABLE IV: The required $\Delta$ for selected $A_{\beta\beta}$, listed in descending order of their measured $T_{1/2}^{2\nu} [2, 13]$, such that the $2\nu\beta\beta$ background within RoI=$w_{\nu\tau}$ would contribute less than the levels specified by the benchmark and background-free conditions to cover $\langle m_{\beta\beta}^{\text{IH(NH)}} \rangle$ and $\langle m_{\beta\beta}^{-\text{IH(NH)}} \rangle$. The BI-values follow from Eq. 9.

| $A_{\beta\beta}$ | $Q_{\beta\beta}$ | $T_{1/2}^{2\nu}$ (yr) | Best $^f$ | $\Delta(\%)$ | Required $\Delta$ (%) |
|-----------------|-----------------|---------------------|----------|--------------|------------------|
| $^{136}\text{Xe}$ | 2.458           | $2.2\times10^{21}$  | 2.7      | $\leq2.18$   | $\leq2.15$       | $\leq1.15$       |
| $^{76}\text{Ge}$ | 2.039           | $1.9\times10^{21}$  | 0.12     | $\leq1.99$   | $\leq1.04$       | $\leq0.99$       |
| $^{130}\text{Te}$ | 2.528           | $8.2\times10^{20}$  | 0.31     | $\leq1.89$   | $\leq0.99$       | $\leq0.99$       |
| $^{82}\text{Se}$ | 2.998           | $9.2\times10^{19}$  | 8.1      | $\leq1.15$   | $\leq0.61$       | $\leq0.61$       |
| $^{48}\text{Ca}$ | 4.268           | $6.4\times10^{19}$  | 6.8      | $\leq0.93$   | $\leq0.49$       | $\leq0.49$       |
| $^{116}\text{Cd}$ | 2.814           | $2.7\times10^{19}$  | 8.4      | $\leq0.99$   | $\leq0.52$       | $\leq0.52$       |
| $^{90}\text{Zr}$ | 3.350           | $2.4\times10^{19}$  | 7.7      | $\leq0.91$   | $\leq0.48$       | $\leq0.48$       |
| $^{150}\text{Nd}$ | 3.371           | $9.3\times10^{18}$  | 7.6      | $\leq0.84$   | $\leq0.45$       | $\leq0.45$       |
| $^{100}\text{Mo}$ | 3.034           | $6.9\times10^{18}$  | 8.0      | $\leq0.76$   | $\leq0.40$       | $\leq0.40$       |

$^f$ Best achieved $w_{\nu\tau}$ resolution at $Q_{\beta\beta}$ from past and ongoing $0\nu\beta\beta$ experiments $[2, 15, 16]$, not including detector R&D programs and future projects.

Depicted in Figure 9 are variations of the required $\Delta$ with $\langle m_{\beta\beta} \rangle$ such that $2\nu\beta\beta$ background within RoI=$w_{\nu\tau}$ would contribute less than the BI-values specified by the benchmark and background-free conditions. The finite width of the band is a consequence of the spread of measured $T_{1/2}^{2\nu}$ [2, 13]. Faster $2\nu\beta\beta$ rates typically require better detector resolution to define smaller RoI. The relative locations for different $A_{\beta\beta}$ within the bands are depicted in the inset.

Listed in Table IV are the required ranges of $\Delta$ to cover $\langle m_{\beta\beta}^{\text{IH(NH)}} \rangle$ and $\langle m_{\beta\beta}^{-\text{IH(NH)}} \rangle$. In particular, the required resolutions to cover $\langle m_{\beta\beta}^{-\text{IH(NH)}} \rangle$ for IH and NH under background-free conditions are $\Delta \leq (0.3 - 0.9)\%$ and $\Delta \leq (0.1 - 0.4)\%$, respectively. The best achieved $\Delta$ for past and ongoing $0\nu\beta\beta$ experiments [2, 15, 16] are included in Table IV and depicted in the right vertical axis of Figure 9 for comparison. In particular, the best published $\Delta (^{76}\text{Ge})=0.12\%$ [19] corresponds to an irreducible $2\nu\beta\beta$ background contribution of $\text{BI} \leq 6 \times 10^{-10}$ count/(ton-yr). This provides a comfortable margin relative to that which satisfies the background-free conditions for $\langle m_{\beta\beta}^{-\text{IH(NH)}} \rangle$ at $\text{BI} \leq 0.96 \times 10^{-6}$ count/(ton-yr).

IV. SUMMARY AND PROSPECTS

As current neutrino oscillation experiments reveal a preference of NH, the strategy of scaling the summit of $0\nu\beta\beta$ should take this genuine possibility into account.

This work studies the relation between the two main factors in improving experimental sensitivities: $(\text{BI}, \Sigma)$. We recall that the presented results are derived with certain input parameter choice: $\text{IA}=100\%$, $\text{expt}=100\%$ and $g_A=1.27$, and that $0\nu\beta\beta$ is driven by the Majorana neutrino mass terms via the mass mechanism while the Signal-to-Background analysis is based on counting experiments without exploiting the spectral shape information at this stage.

Advancing towards ND-NH to cover $\langle m_{\beta\beta}^{\text{NH}} \rangle$ will require large and costly exposure. An unrealistic $O(10)$ Mton-yr enriched target mass is necessary at the current best achieved background level $\text{BI} \sim 1$ count/(ton-yr). Reduction of BI will be play- ing increasingly significant, if not determining, roles in shaping future $0\nu\beta\beta$ projects.

For instance, following Table IV background-free conditions for $\langle m_{\beta\beta}^{\text{NH}} \rangle$ correspond to additional background suppression from the current best $\text{BI} \sim 1$ count/(ton-yr) and benchmark [1 count/(ton-yr)] levels by factors of $(0.96 \times 10^{-6})$ and $(4.4 \times 10^{-3})$, respectively. This would reduce the required $\Sigma$ from 11 Mton-yr and 4600 ton-yr, respectively, to 550 ton-yr. The corresponding minimal-exposure to cover $\langle m_{\beta\beta}^{\text{NH}} \rangle$ is $\Sigma_{\text{min}} \sim 37$ ton-yr, which is only a modest factor beyond the goals of next-generation experiments [14]. The pursuit of background towards $\text{BI} \sim O(10^{-6})$ count/(ton-yr) to probe ND-NH, while challenging, is highly investment-effective, as it
is equivalent to reduction of $\Sigma$ by $O(10)$ Mton-yr and $O(1)$ kton-yr relative to those required for the current best and benchmark background levels, respectively.

This article serves to quantify the merits of background reduction in $0\nu\beta\beta$ experiments, but does not attempt to address the experimental issues on how to realize the feat and how to demonstrate that the suppression factors are achieved when experiments are constructed. We project that the continuous intense efforts and ingenuities from the experimentalists world-wide, with motivations reinforced by the increasing equivalent “market” values, will be able to meet the challenges.

Boosting $\Sigma$ involves mostly in the accumulation of enriched $A_{\beta\beta}$ isotopes and turning these into operating detectors. These processes are confined to relatively few locations and small communities of expertise. The room of development which may overcome the known hurdles is limited. Suppression of the $0\nu\beta\beta$ experimental background, on the other hand, would be the tasks of mobilizing and coordinating the efforts of a large pool of expertise. It is related to the advances in diverse disciplines from novel materials to chemistry processing to trace measurement techniques. Research programs on many subjects requiring low-background techniques may contribute to and benefit from the advances. There would be strong potentials of technological breakthroughs and innovative ideas as the sensitivity goals are pursued. Signal efficiencies are also increasingly costly as sensitivities advance towards ND-NH. For instance, at $\Sigma_{\text{min}}=550$ ton-yr to cover $\langle m_{\beta\beta}\rangle^{\text{NH}}$, a high 90% efficiency to certain selection criterion corresponds to discarding data of $O(10)$ ton-yr strength — already an order of magnitude larger than the combined exposure of all $0\nu\beta\beta$ experiments. It follows that background suppression would preferably be attended at the root level — that radioactive contaminations are suppressed to start with, rather than relying on special signatures and software selection algorithms to identify them.

The next generation of $0\nu\beta\beta$ experiments would cover $\langle m_{\beta\beta}\rangle^{\text{IH}}$. In addition, they should be able to explore the strategies and demonstrate sufficient margins to advance towards $\langle m_{\beta\beta}\rangle^{\text{NH}}$. A significant merit would be to have no irreducible background before reaching the $\mathcal{B}\sim O(10^{-6})$ counts)$/w_{\nu_{\beta\beta}}$-ton-yr) background-free configuration. The detector requirements to achieve this for $2\nu\beta\beta$ are summarized in Figure[9] and Table[LV]. Detailed studies of this background as well as other channels like those due to residual cosmogenic radioactivity and long-lived radioactive isotopes are themes of our on-going research efforts.

V. ACKNOWLEDGEMENT

This work is supported by the Academia Sinica Principal Investigator Award AS-IA-106-M02, contracts 104-2112-M-259-004-MY3 and 107-2119-M-001-028-MY3 from the Ministry of Science and Technology, Taiwan, and 2017-ECP2 from the National Center of Theoretical Sciences, Taiwan.

[1] M. Tanabashi et al., Particle Data Group, Phys. Rev. D 98, 030001 (2018), and in particular, Review 14, P. 251 by K. Nakamura and S.T. Petcov, and references therein.

[2] S. Dell’Oro et al., Advances High Energy Phys. 2016, 2162659 (2016). L. Cardani, SciPost Phys. Proc. 1, 024 (2019), and references therein; M.J. Dolinski, A.W.P. Poon and W. Rodejohann, Ann, Annu. Rev. Nucl. Part. Sci. 69, 219 (2019).

[3] S. Bilenky, Nucl. Phys. B 908, 2 (2016).

[4] T. Nalayya and R.K. Plunkett, New. J. Phys. 18, 015009 (2016); F. Simpson et al., J. Cosmo. Astropart. Phys. 06, 029 (2017); I. Eseban et al., J. High Energy Phys. 01, 106 (2019); M.G. Aartsen et al., arXiv:1902.07771 (2019).

[5] M. Tanabashi et al., Particle Data Group, Phys. Rev. D 98, 030001 (2018), and in particular, Review 25, P. 390 by J. Lesgourgues and L. Verde, and references therein; N. Aghanim et al., arXiv:1807.06209v1 (2018).

[6] A. Dueck et al., Phys. Rev. D 83, 113010 (2011).

[7] R.G.H. Robertson, Mod. Phys. Lett. A 28, 1350021 (2013).

[8] J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 043415 (2013); S. Dell’Oro, S. Marcocci and F. Vissani, Phys. Rev. D 90, 033005 (2014).

[9] J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).

[10] J. Engel et al., Rep. Prog. Phys. 80, 046301 (2017).

[11] G. Benato, Eur. Phys. J. C 75, 563 (2015); A. Caldwell et al., Phys. Rev. D 96, 073001 (2017); M. Agostini, G. Benato and J.A. Detwiler, Phys. Rev. D 96, 053001 (2017); A. Di Iura and D. Meloni, Nucl. Phys. B 921, 829 (2017).

[12] V. A. Rodin et al., Nucl. Phys. A 766, 107 (2006); J. Barea et al., Phys. Rev. C 91, 034304 (2015).

[13] M. Agostini et al., Nature 544, 47 (2017); M. Agostini et al., Phys. Rev. Lett. 120, 123503 (2018); M. Agostini et al., Science 365, 1445 (2019).

[14] G. Wang et al., arXiv:1504.03599 (2015); N. Abgrall et al., AIP Conf. Proc. 1894, 020027 (2017); J. B. Albert et al., Phys. Rev. C 97, 065503 (2018).

[15] C.E. Aalseth et al., Phys. Rev. Lett. 120, 132502 (2018).

[16] G. Anton et al., Phys. Rev. Lett. 123, 161802 (2019).