On the Origin and Nature of Dark Matter

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Abstract

It is discussed how the ideas of entropy and the second law of thermodynamics, conceived long ago during the nineteenth century, underly why cosmological dark matter exists and originated in the first three years of the universe in the form of primordial black holes, a very large number of which have many solar masses including up to the supermassive black holes at the centres of galaxies. Certain upper bounds on dark astrophysical objects with many solar masses based on analysis of the CMB spectrum and published in the literature are criticised. For completeness we discuss WIMPs and axions which are leading particle theory candidates for the constituents of dark matter. The PIMBHs (Primordial Intermediate Mass Black Holes) with many solar masses should be readily detectable in microlensing experiments which search the Magallenic Clouds and measure light curves with durations of from one year up to several years.

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Introduction

We assume Newton’s universal law for the gravitational force \( F \) between every two point particles

\[
F = \left( \frac{GM_1M_2}{R^2} \right)
\]

where \( G \) is a constant, \( M_1, M_2 \) are the two masses and \( R \) is the separation, to be valid at the scale of galaxies and clusters of galaxies. Then there is overwhelming observational evidence \([1,2]\) for the existence of dark matter which does not radiate electromagnetically but which, by assumption, interacts gravitationally according to Eq. (1). The mass of such dark matter in the Milky Way is five or six times the mass of the luminous matter which does radiate electromagnetically.

There is no doubt that the dark matter exists astrophysically and cosmologically but there is no reliable evidence for dark matter in terrestrial experiments. In the present article we shall first address (Section 1) the reason why dark matter exists by using thermodynamic and entropic arguments especially the second law of thermodynamics. The use of entropy may be unfamiliar to particle theorists because one cannot define the entropy for a single particle. Nevertheless, the reason why dark matter exists pre-dates quantum field theory. We next address (Section 2) the nature of the dark matter and will first briefly discuss within particle theory WIMPs and Axions as candidates for dark matter constituents. These are alternatives to the notion that galactic dark matter is comprised of Primordial Intermediate-Mass Black Holes (PIMBHs). Primordial means that they were formed in the first three years of the expanding universe compared to the first three minutes for luminous matter.

For the present discussion, the PIMBHs of special interest, being the most susceptible to observational discovery by microlensing in the near future, have masses in the narrow range

\[
25M_\odot \leq M_{PIMBH} \leq 2,500M_\odot
\]

which is a range accessible to the microlensing experiments with light curves less than ten years in duration for lensing of stars in the Magellanic Clouds. Masses above the upper end in Eq. (2) up to \( 10^6 M_\odot \) are permitted in the Milky Way halo where there is a mass cut-off caused by disk stability \([3]\) for MACHOS away from the galactic centre. Near to the centre of gravity of the Milky Way is the supermassive black hole SagA* with

\[
M_{SagA*} \simeq 4 \times 10^6 M_\odot
\]

and elsewhere in the visible universe there are primordial black holes with masses anywhere up to \( 10^{12} M_\odot \) which include all the supermassive black holes at galactic centres.

Black holes mean the electrically neutral objects predicted by general relativity of Kerr type \([4]\), specified completely by mass and spin. Having discussed both the why (Section 1) and what (Section 2) of dark matter, we end with a Discussion section.
1 On the Origin of Dark Matter

The reason that dark matter exists is based on the thermodynamic concepts of entropy and the second law of thermodynamics. The present section will contain introductory discussions about entropy which was a major accomplishment of nineteenth century theoretical physics. Physicists studying particle theory normally use the language of quantum field theory and can be skeptical about the usefulness of entropy.

Dark matter should be regarded as an astrophysical phenomenon and its appearance in galaxies and clusters of galaxies the result of dynamical evolution in the early universe of extremely large numbers of particles. This leads naturally to the employment of the methods of statistical mechanics and thermodynamics. To understand the cosmological dark matter, we must therefore study assiduously Boltzmann rather than Maxwell.

For the expansion of the visible universe, we are dealing with $\sim 10^{80}$ particles, far more than Avagadro’s number $6.023 \times 10^{23}$ molecules per mole and therefore a statistical treatment should give reliable predictions. Application of entropy to the visible universe is secure because, assuming a finite universe, it is a thermodynamical isolated system since no heat enters or leaves and the visible universe can be regarded as if surrounded by a perfect thermal insulator.

The universe is not as straightforward as, but not so much more complicated than, a box with a mole of ideal gas and elastic collisions between molecules and between molecules and the walls, where with $6.023 \times 10^{23}$ molecules we can derive at low density extremely accurate thermodynamic laws such as the ideal gas law. For the universe, it is a time-dependent system and the exactly accurate dynamics would require solving Boltzmann’s transport equations for all the particles, so we must instead use thermodynamic arguments.

The kinetic theory of gases is a misleadingly simple case where precise macroscopic properties of a box of ideal gas are related to the microscopic properties of the molecules. There are thermodynamic variables $P, V, T$ whose physical significance in terms of experimental measurement is clear. $S$, the entropy, is a state function.

The second half of the 19th century was when entropy was introduced into physics. In the 1850s and culminating in 1865, Clausius thought carefully about Carnot’s earlier cyclic model for a steam engine and it is Clausius who deserves the credit for introducing both entropy and the second law of thermodynamics. Quite a pair of accomplishments! But the connection of entropy to microscopic physics was first made by Boltzmann in 1872.

The French physicist who was the father of thermodynamics and whose work started the intellectual path towards entropy was Sadi Carnot (1796-1832). In his 1824 book S. Carnot began a new field of research, thermodynamics, and his Carnot Cycle was what later led Clausius in 1865 to the idea of entropy. The Carnot Cycle is a simple model which mimics the operation of a steam engine.

Rudolf Clausius (1822-1888), a German physicist and mathematician, may justly be regarded as the father of entropy. He was initially inspired by the Carnot Cycle which requires that
where $T_H, T_L$ are the absolute temperatures of the hot and cold heat reservoirs and $Q_H, Q_L$ the heat absorbed and emitted respectively. In the presence of irreversible processes in a variant of the Carnot Cycle one would, instead of the equality in Eq.(4), have an inequality $(Q_H/T_H) < (Q_L/T_L)$ which gives rise to the second law. This led Clausius to a definition [9] for incremental entropy as the exact differential

$$dS = \left( \frac{\delta Q}{T} \right)$$

near to thermal equilibrium and thence to the second law of thermodynamics $dS \geq 0$. We emphasise that Eq.(5) is appropriate only near to thermal equilibrium because, for example, a thermally insulated box of ideal gas with an unlikely initial condition, e.g. all the molecules in one corner, will rapidly increase its entropy to approach thermal equilibrium despite the fact that $\delta Q \equiv 0$. Clausius denoted entropy by $S$ in honour of Sadi Carnot. The early universe is never near to equilibrium so that Eq.(5) does not apply: $\delta Q = 0$ but $dS > 0$.

Clausius enunciated two laws as follows:
1. The energy of the universe is a constant.
2. The entropy of the universe tends to a maximum.

This succinct statement of the second law is perfect for use in our Discussion section.

For a closed, isolated homogeneous system in which all processes in a cycle are reversible the closed loop integral vanishes:

$$\oint dS = \oint \frac{\delta Q}{T} = 0$$

which implies that the line integral is independent of the path and hence that the increment $dS$ is uniquely defined as an exact differential at least proximate to thermal equilibrium. Therefore we have a sensible thermodynamic state function $S$ whose partial differentiations with respect to the thermodynamic variables permit an expression for relative entropies in an ideal gas.

Kinetic theory shows how the $P, V, T$ thermodynamic variables can be related to the average motions of the molecules using statistical mechanics. The question following Clausius’s work was how to relate the state function $S$ to microscopic variables? Ludwig Boltzmann (1844-1906) was the physicist who solved this problem. He had no experimental evidence for molecules; this had to wait thirty more years until the explanation of Brownian motion made in 1905 by Einstein [5] and Smoluchowski [6].

Boltzmann was in many ways a tragic figure. Few people were convinced of the reality of atoms and molecules before the last year of his life. Further, his statistical, hence inexact, second law of thermodynamics was strongly criticised by Maxwell (1831-1879) who, although he believed in atoms and molecules, never accepted Boltzmann’s 1872 idea of an inexact law of physics.
Boltzmann appreciated that his law was so unlikely to be violated that it might as well have been exact. Another severe criticism came from the distinguished French mathematician Henri Poincaré (1854-1912) who proved a rigorous recurrence theorem [7] which states that all systems must return eventually to their original state. Boltzmann understood that the time scale involved in Poincaré recurrence is far too long to be physically relevant. In any case, Boltzmann’s lack of recognition in the physics and mathematics communities may have contributed in 1906 to his suicide at the early age of 62.

Boltzmann defined as microstates all the possible arrangements of microscopic variables corresponding to a given fixed set of macroscopic or thermodynamic variables. Let \( p_i \) be the probability that the system is in the \( i \)th microstate. Then introducing the constant \( k \) with the same units as \( S \) he represented \( S \) as

\[ S = -k \sum_i p_i \ln p_i \]

He made the ergodic hypothesis that all the \( p_i \) are equal

\[ p_i = \frac{1}{\Omega} \quad \text{for all} \quad i \]

whereupon

\[ S = k \ln \Omega \]

where \( \Omega \) is the total number of microstates. Eq. (9) is one of the most celebrated equations in all of physics.

For an ideal gas, the maximisation of entropy \( S \) means that in the state of thermal equilibrium there is the maximum uncertainty in the molecular motions. We can equivalently say that there is the greatest disorder in thermal equilibrium, and hence that entropy is a measure of disorder.

The H theorem (1872) [10] of Boltzmann is central to the physics although even now in 2018 we are told it cannot be rigorously proved mathematically because, at least far away from equilibrium, it is unknown whether solutions of the Boltzmann transport equation have sufficient analytic smoothness. Nevertheless, the H theorem shows how starting from reversible microscopic mechanics, one can arrive at non-reversible, in a statistical sense, macroscopic dynamics. It explicates the second law of thermodynamics that the entropy of an isolated system cannot decrease. Later in this paper, we shall argue that the early visible universe can be regarded as such an isolated system.

The Boltzmann transport equation is a general requirement for \( f(q,p,t) \) which is the distribution of molecules with position \( q \) and momentum \( p \) at time \( t \) and is written as follows:

\[
\frac{\partial f(q,p,t)}{\partial t} + \left( \frac{p}{m} \right) \frac{\partial f(q,p,t)}{\partial q} + \mathbf{F} \cdot \frac{\partial f(q,p,t)}{\partial p} = \left( \frac{\partial f}{\partial t} \right)_{\text{coll}}
\]

where the RHS is

\[
\left( \frac{\partial f}{\partial t} \right) = \bar{R} - R
\]
in which \( R \ dt \ dq \ dp \) is the number of collisions from time \( t \) to \( (t + dt) \) with initial position \( q \) to \( (q + dq) \) and initial momentum \( p \) to \( (p + dp) \) and \( \bar{R} \ dt \ dq \ dp \) is the number of collisions from time \( t \) to \( (t + dt) \) with final position \( q \) to \( (q + dq) \) and final momentum \( p \) to \( (p + dp) \).

Taking only \( 2 \rightarrow 2 \) elastic collisions into account

\[
R(q, p_1) = \int dp_2 dp_1' dp_2' \mathcal{P}_{p_1 \rightarrow p_2'} f(q, p_1) f(q, p_2) 
\]

(12)

while for the final state

\[
\bar{R}(q, p_1) = \int dp_2 dp_1' dp_2' \mathcal{P}_{p_1 \rightarrow p_2'} f(q, p_1') f(q, p_2') 
\]

(13)

In Eqs. (12) and (13), \( \mathcal{P}_{p_1 \rightarrow p_2', p_2' \rightarrow p_2} \) is the probability density for going from initial state \( p_1, p_2 \) to final state \( p_1', p_2' \) in time \( dt \).

Time-reversal symmetry for the microscopic scattering requires that

\[
\mathcal{P}_{p_1 \rightarrow p_1', p_2 \rightarrow p_2'} = \mathcal{P}_{p_1', p_2' \rightarrow p_1, p_2} 
\]

(14)

and therefore we may rewrite Eq. (11) as

\[
\left( \frac{\partial f}{\partial t} \right) = \int dp_2 dp_1' dp_2' \mathcal{P}_{p_1 \rightarrow p_2'} \left[ f(p_1') f(p_2') - f(p_1) f(p_2) \right] \left[ 1 + \log f(p_1) \right] 
\]

(15)

To identify entropy \( S \) microscopically, presumably inspired by the monotonic increase of the logarithm function, Boltzmann considered what he called the H function \( (S = -H) \) defined by

\[
H(t) = \int dp f(p, t) \log f(p, t) 
\]

(16)

The time derivative of \( H(t) \) is

\[
\left( \frac{dH(t)}{dt} \right) = \int dp \frac{\partial}{\partial t} \left[ f(p, t) \log f(p, t) \right] 
\]

\[
= \int dp \left( \frac{\partial f(p, t)}{\partial t} \right) \left[ 1 + \log f(p, t) \right] 
\]

(17)

Substitution of Eq. (15) into Eq. (17) gives

\[
\left( \frac{dH(t)}{dt} \right) = \int dp_1 dp_2 dp_1' dp_2' \mathcal{P}_{p_1 \rightarrow p_1', p_2 \rightarrow p_2'} \left[ f(p_1') f(p_2') - f(p_1) f(p_2) \right] \left[ 1 + \log f(p_1) \right] 
\]

(18)

Because of the symmetry in Eq. (18), we may freely replace \( f(p_1) \) by \( f(p_2) \) in the logarithm, and add the result to Eq. (18) to obtain
\[
2 \left( \frac{dH(t)}{dt} \right) = \int dP_1 dP_2 dP'_1 dP'_2 P_{P_1,P_2 \rightarrow P'_1,P'_2} \left[ f(P'_1) f(P'_2) - f(P_1) f(P_2) \right] \left[ 1 + \log f(P_1) f(P_2) \right]
\]

(19)

Now we exploit the time-reversal-invariance of \( P \) in Eq(14) to arrive at the fascinating formula which is at the heart of Boltzmann’s derivation:

\[
\left( \frac{dH(t)}{dt} \right) = -\frac{1}{4} \int dP_1 dP_2 dP'_1 dP'_2 P_{P_1,P_2 \rightarrow P'_1,P'_2} \left[ f(P'_1) f(P'_2) - f(P_1) f(P_2) \right] \times \left[ \log f(P_1) f(P_2) - \log f(P'_1) f(P'_2) \right]
\]

(20)

Eq. (20) has a RHS which is negative semidefinite because \((A - B)(\log A - \log B) \geq 0\) and therefore

\[
\left( \frac{dH(t)}{dt} \right) \leq 0
\]

(21)

or, reverting to entropy \( S(t) = -H(t) \), we have arrived at a microscopic derivation of the second law that with a very large number of molecules \( S \) cannot decrease.

The paper by Boltzmann in which he proved the H theorem has been studied and criticised probably as much as any physics paper. One interesting critique [11] is by Von Neumann (1903-1957), available in translation [12].

The reason why the H theorem of Boltzmann is far more powerful than the infinitesimal definition of \( dS \) by Clausius is that it proves that \( dS \geq 0 \) for non-equilibrium systems assuming only the Boltzmann transport equation and the ergodic hypothesis,

What is clear about \( S(t) \) for a box of ideal gas is that with thermal equilibrium Eq(21) becomes an equality and that \( S(t) \) is a maximum. From the definition of \( S \) in Eq.(9) this implies that the number of microstates corresponding to the thermally equilibrated system is the highest, and that therefore the molecular motion is the most disordered.

The H theorem encapsulates this edifice of 19th-century knowledge sufficiently to progress with some confidence from a box of ideal gas to the more interesting case of the early universe.

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To discuss cosmology we begin from Einstein’s equations [13][14] of general relativity

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}.
\]

(22)

We adopt the FLRW metric [15][18] which reflects homogeneity and isotropy
\[ ds^2 = dt^2 - a(t)^2 \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \]

and substituting Eq. (23) into Eq. (22) gives *inter alia* the Friedmann expansion equation \[15\]

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_{TOT} \] (24)

in which the total density \( \rho_{TOT} \) has three important components

\[ \rho_{TOT} = \rho_m + \rho_\gamma + \rho_\Lambda \] (25)

corresponding to matter, radiation and dark energy respectively.

Can the visible universe be regarded a thermodynamically isolated system? The answer is a categoric yes. No heat ever enters or leaves and it can be considered as if its surface were covered by a perfect thermal insulator. It contains a very large number of particles, much bigger than the number of molecules, \( 6.023 \times 10^{23} \), per mole of ideal gas.

We assume that the universe is finite and that the visible universe (VU) is a 2-sphere characterised by one scale, \( R_{VU}(t) \), which is its co-moving radius, monotonically increasing as if from \( R(t = 0) = 0 \) at the Big Bang to \( R_{VU}(t_0) \) the present time \( t_0 \sim 13.8 \text{Gy} \) when

\[ R_{VU}(t_0 = 13.8 \text{Gy}) \sim 44 \text{Gly} \] (26)

is a reasonable value. There is a subtlety after the expansion starts accelerating at time \( t = t_{DE} \sim 9.8 \text{Gy} \) when the dark energy becomes dominant. The universe then acquires an extroverse, a 2-sphere of radius \( R_{ext}(t) \) which is larger than the visible universe for \( t > t_{DE} \); the present values are

\[ R_{ext}(t_0) \sim 52 \text{Gly} > R_{VU}(t_0) \sim 44 \text{Gly} \] (27)

This implies that the present extroverse has a volume 65% larger than the visible universe so that if, say, the VU contains 100 billion galaxies, a further 65 billion galaxies have already exited the VU but it is reasonable to assume that, despite their lack of observability, those extra galaxies have similar dark matter and supermassive black holes to the ones of the visible universe. The accelerated expansion is thus unimportant to our analysis of the dark matter which was formed in the first three years when \( 0 \leq t \leq 3 \text{y} \).

To set the scene, let us make an inventory of entropies of the known objects inside the visible universe, excluding dark matter and dark energy.

We use dimensionless entropy \( S/k \).

- Luminous matter (Baryons) \( \sim 10^{80} \)
- Cosmic Microwave Background \( \sim 10^{88} \)
- Relic neutrinos \( \sim 10^{88} \)
- Supermassive black holes \( \sim 10^{103} \)
For the supermassive black holes, we have made our estimate based on a number $10^{11}$ of galaxies in the visible universe, an average SMBH mass of $10^7 M_\odot$, and the black hole entropy formula

$$S_{BH}(M = \eta M_\odot) \sim 10^{78} \eta^2 \quad (28)$$

These entropies reveal the staggering fact that of all the present entropy in the visible universe a fraction $(1 - 10^{-15})$ is contained in the SMBHs. Equally striking is that, from Eq. (28), one SMBH with mass $10^7 M_\odot$ contains entropy $10^{92}$ which is 10,000 times the entropy of all the CMB permeating the entire visible universe.

This overwhelming domination of entropy by black holes reflects the fact that by far the most efficient way of concentrating entropy is by black holes. This supports the idea that, for example, the dark matter inside clusters of galaxies, imaged by gravitational lensing, exists because in the early universe Nature created very large numbers of primordial black holes to satisfy the second law of thermodynamics.

Although the PBH formation in the early universe is a dynamical question which, to be rigorous, would require solution of Boltzmann’s transport equation, Eq. (10), for $\sim 10^{80}$ particles which is far beyond the capability of any computer, nevertheless we can specify the eras during which the PBHs were formed. The mass of a PBH is determined by the horizon size to be

$$M_{PBH} \simeq 10^5 M_\odot \left( \frac{t}{1 \text{sec}} \right) \quad (29)$$

The MACHO Collaboration [20] found that MACHOs up to mass $25 M_\odot$ could account for no more that 10% (or 20% at a stretch) of the dark matter. We shall assume that the MACHOs they did discover included PBHs. In any case, the other 90% of the dark matter PBHs must be heavier than $25 M_\odot$ which, according to Eq. (29), requires

$$t \geq t_1 \equiv 0.00025 \text{sec} \quad (30)$$

For MACHO searched by a microlensing experiment which targets the Magellanic clouds, the duration ($\tau$) of the light curves for full-width at half maximum is given by

$$\tau \simeq 0.2 \text{year} \left( \frac{M_{MACHO}}{M_\odot} \right)^{\frac{1}{2}} \quad (31)$$

and so if we find it impracticable to wait for more than 10 years to measure a light curve we require $M_{PBH} \leq 2500 M_\odot$ and according to Eq. (29) such a PBH is formed before $t_2$ given by

$$t_2 \equiv 0.025 \text{sec} \quad (32)$$

Let us define PIMBHs as residing in the mass range

$$25 M_\odot \leq M_{PIMBH} \leq 10^6 M_\odot \quad (33)$$
then PIMBHs were formed before a time $t_3$ which is
\begin{equation}
  t_3 = 10 \text{sec}
\end{equation}

Let us define supermassive black holes (SMBHs) to be those black holes in the mass range
\begin{equation}
  10^6 M_\odot \leq M_{SMBH} \leq 10^{12} M_\odot
\end{equation}

so that SMPBHs are formed \cite{21} before $t_4$ given by
\begin{equation}
  t_4 = 10^7 \text{seconds} \simeq 3 \text{years}
\end{equation}

To summarise, all of the cosmological dark matter and the supermassive black holes are formed during the first three years. Normalising the FLRW scale factor $a(t)$ in Eq.\,(23) by $a(t_0) \equiv 1$ at the present time the values of $a(t)$ and the corresponding values for the radius $R_{VU}(t)$ of the visible universe are:
\begin{align*}
  t_1 &= 0.00025s \quad a(t_1) = 2.7 \times 10^{-12} \quad R_{VU}(t_1) = 0.14 \text{ ly.} \\
  t_2 &= 0.025s \quad a(t_2) = 2.7 \times 10^{-11} \quad R_{VU}(t_2) = 1.4 \text{ ly.} \\
  t_3 &= 10s \quad a(t_3) = 5.4 \times 10^{-10} \quad R_{VU}(t_3) = 28 \text{ ly.} \\
  t_4 &= 10^7s \quad a(t_4) = 5.4 \times 10^{-7} \quad R_{VU}(t_1) = 28 \text{ kly.}
\end{align*}

The intermediate-mass PIMBHs are formed between cosmic times $t_1$ and $t_3$, with the most readily detectable made between $t_1$ and $t_2$. The supermassive black holes then appeared between times $t_3 \sim 10s$ and $t_4 \sim 3y$. All of the dark matter was formed during the first three years after the Big Bang, just as all the luminous matter was formed during the first three minutes.
2 On the Nature of Dark Matter

In this Section, we shall first discuss WIMPs (subsection 2.1) and axions (subsection 2.2) which are the two most likely candidates, in that order, for the constituents of cosmological dark matter which arise from extensions of the standard model of particle phenomenology. We shall then return to primordial black holes (subsection 2.3) whose motivation is based instead on entropy as discussed in Section 1.

2.1 WIMPs

By Weakly Interacting Massive Particle (WIMP) is generally meant an unidentified elementary particle with mass in the range between 10 GeV and 1000 GeV and with scattering cross section with nucleons (N) satisfying, according to the latest unsuccessful WIMP direct searches,

$$\sigma_{WIMP-N} < 10^{-46}\text{cm}^2$$

(37)

which is roughly comparable to the characteristic strength of the known weak interaction.

The WIMP particle must be electrically neutral and be stable or have an extremely long lifetime, longer than the age of the universe. In model-building, this stability may be achieved by an *ad hoc* discrete symmetry, for example a $Z_2$ symmetry under which all the standard model particles are even and others are odd. If the discrete symmetry is unbroken, the lightest odd state must be stable and therefore a candidate for a dark matter.

By far the most popular WIMP example comes from electroweak supersymmetry where a discrete R symmetry has the value $R=+1$ for the standard model particles and $R=-1$ for all the sparticles. Such an R parity is less *ad hoc*, being essential to prevent too-fast proton decay. The lightest $R=-1$ particle is stable and, if not a gravitino which has the problem of too-slow decay in the early universe, it is the neutralino, a linear combination of zino, bino and higgsino. The neutralino provides an attractive dark matter candidate.

It is worth briefly recalling the history of electroweak supersymmetry. The standard model [22,25] was in place by 1971 and its biggest theoretical problem was that, unlike QED with only logarithmic divergences, the scalar sector of the standard model generates quadratic divergences which, unless cancelled within a quiver-type construction [26], destabilise the mass of the BEH boson. When supersymmetric field theories were invented [27,28] in 1974, they provided a mathematical solution of the quadratic divergence problem and immediately became popular. Even more so in 1983 when the neutralino was identified [29] as a dark matter candidate and more so again in 1991 when it was pointed out [30] that grand unification apparently works better with hypothetical supersymmetric partners included.
2.1.1 Direct detection

With all of this support for supersymmetry, it is natural to take seriously the dark matter candidate which such a theory predicts. It is a WIMP with mass typically 10-1000GeV and experiencing weak interactions which would suggest a detectable scattering cross-section from nuclei in direct detection experiments.

Some of the detectors for WIMPs have been built using liquid xenon [31][32]. These have produced the strongest upper limits on the existence of WIMPs such as the cross-section quoted above in Eq.(37).

When a WIMP passes through a detector, it can interact with a nucleus which will recoil. The idea is to detect the small energy which is transferred. Experiments may have 1,000 kgs up to 10,000 kgs of detector. Such an experiment needs knowledge of the WIMP-nucleus cross-section and the distribution of WIMPs in the galactic halo.

The WIMP-nucleus interaction can be spin-independent (SI) or spin-dependent (SD). For SI, the nucleons add to a $A^2$ coherence factor. If the WIMP is heavier than the nucleus this becomes $A^4$, so heavy nuclei are the best targets. This is why germanium ($A = 74$) is a popular choice because $74^4 \simeq 3 \times 10^7$. For SD, the WIMP interacts only with the total spin of the nucleus, and the factor $A^2$ is lost. It needs a nucleus with a nonzero spin. In the case of the CDMS experiment, for example, it uses [34][36] a mixture of Ge-74 (spinless) and Ge-73 (spin=$\frac{9}{2}$) in order to be sensitive to both SI and SD scattering of WIMPs.

Concerning the astrophysics, the count rates are highest for slow WIMPs and go to zero for WIMP velocity $540\text{km/s}$ which is the escape velocity from the halo. One needs also to consider the density profile of the halo which is assumed to follow the NFW profile [37][38] obtained from numerical simulations. This peaks at the galactic centre and the density falls, $\rho \sim r^{-2}$, for large $r$. This density may be lumpy because the Milky Way was formed in part by mergers. The Sun is 24,000ly from SagA* and is moving at $\sim 250\text{km/s}$ around the core. The average density of WIMPs is $0.4\text{GeV/cm}^3$ and their relative velocity is what determines the flux. An annual modulation is expected due to the Earth’s orbit around the Sun.

Background noise for direct detection can arise from cosmic rays and solar flares so the detectors are placed deep underground. For example, the Homestake Mine in South Dakota is almost one mile deep. Nevertheless, even there radioactivity of the rock, due to e.g. naturally occurring radon, must be taken into account.

A large detector for WIMPs, the LZ Dark Matter Experiment using seven tons of liquid xenon, is planned for SURF (Sanford Underground Research Facility) in South Dakota [39].

2.1.2 Production

Another way of finding the WIMPs is to look for production in pairs at a particle collider such as the Large Hadron Collider (LHC) which is presently the highest-energy machine in the world with proton-proton collisions at 13 TeV centre-of-mass energy.
Dark matter itself is not detected in a production experiment like LHC. Instead one looks for an apparent violation of energy conservation. If WIMPs were produced at the LHC, the signature would be high-transverse-momentum jets which are easily detected. Pair production of WIMPs should be associated by 1, 2 or more such jets.

If WIMPs are produced and detected, it will still need astrophysical evidence that it is the dark matter.

2.1.3 Indirect detection

A third method to search for WIMPs is to seek astrophysical signals of WiMP annihilation products. Many WIMPs are their own antiparticles and annihilate among themselves into a variety of lighter particles. The end states include $e^+$, $\gamma$ and $\nu$. These are sought by detectors on satellites in space, strings of phototubes embedded in ice at the South Pole, and other techniques.

The strongest signals come from regions most abundant in WIMPs, viz the centres of the Earth and Sun, the galactic centre and dwarf galaxies near the Milky Way. It needs a WIMP density high enough for them to collide and annihilate. WIMP annihilations in the early universe are important in order that the relic density of dark matter can agree with that observed at the present time.

Now the average WIMP density is so low that, in general, they never collide. The only places they will appreciably annihilate is in regions of especially high WIMP overdensity. The Milky Way has a higher density than the universe’s average.

WIMPs traveling through the Earth have a probability of about one in ten billion to hit a nucleus and lose sufficient energy to be captured and pulled to the Earth’s centre of gravity. There they start annihilating. The $e^+$’s cannot escape the Earth’s core because of electromagnetic forces, but the neutrino products of WIMP annihilation can. The failure to observe these in surface neutrino detectors provides a useful upper limit on the WIMP density at the centre of the Earth.

A similar analysis applies mutatis mutandis, to the centre of the Sun. Searches for $\nu$’s from the centres of the Earth and Sun are continuing.

We can go to larger distances from the Earth whereupon a good candidate for WIMP concentration is the Milky Way core at a distance of 24,000 ly. Computer simulations of the dark matter galactic profile suggest a higher density there. The situation in the vicinity of SagA* is not straightforward because (i) merging of black holes into the supermassive black hole could have knocked WIMPs away, and (ii) competing astrophysical processes might be difficult to disentangle.

Dwarf galaxies can produce cleaner signals for WIMPs. Inside the Milky Way are many substructures, including dwarf galaxies which are between $10^{-6}$ and $10^{-3}$ of the mass of the galaxy. There are at least twenty dwarf galaxies inside the Milky Way with exceptionally high ratio of dark matter to luminous matter, and these may provide the best environments for WIMPs.
The intricate decay chains for the WIMP annihilations into standard model particles can be quite complex, but can be calculated with the most certainty for the special case of the MSSM neutralino. Generally the total mass of the final products may add to about ten percent of the original progenitor WIMP mass.

There is a worldwide search for the products of WIMP annihilation using, e.g., satellites and the IceCube detector [40–42] at the South Pole. Excesses of $e^+$'s and $\gamma$'s could provide us with the smoking gun for such indirect detection.

2.2 Axions

The axion particle is now believed, if it exists, to lie in the mass range

$$10^{-6}\text{eV} < M < 10^{-3}\text{eV}$$

The lagrangian originally proposed for Quantum Chromodynamics (QCD) was of the simple form, analogous to Quantum Electrodynamics (QED),

$$L_{QCD} = -\frac{1}{4} G_{\alpha}^{\mu\nu} G_{\alpha}^{\mu\nu} - \frac{1}{2} \sum_{i,a} \bar{q}_i, a \gamma^\mu D_{\mu}^a q_i, b$$

summed over the six quark flavors. The simplicity of Eq.(39) was only temporary and became more complicated in 1975 by the discovery of instantons which dictated that an additional term be allowed in the QCD lagrangian

$$\Delta L_{QCD} = \frac{\Theta}{64\pi^2} G_{\alpha}^{\mu\nu} \tilde{G}_{\alpha}^{\mu\nu}$$

must be added where $\tilde{G}_{\mu\nu}$ is the dual of $G_{\mu\nu}$. Although this extra term is an exact derivative, it cannot be discarded as a surface term because there is now a topologically nontrivial QCD vacuum with an infinite number of different values of the spacetime integral over Eq.(4) all of which correspond to $G_{\mu\nu} = 0$. Normalized as in Eq.(4), the spacetime integral of this term must be an integer, and an instanton configuration changes this integer, or Pontryagin number, by unity.

When the quark masses are complex, an instanton changes not only $\Theta$ but also the phase of the quark mass matrix $M_{\text{quark}}$ and the full phase to be considered is

$$\bar{\Theta} = \Theta + \text{arg}det||M_{\text{quark}}||$$

The additional term, Eq.(4), violates P and CP, and contributes to the neutron electric dipole moment whose upper limit provides a constraint

$$\bar{\Theta} < 10^{-9}$$

which fine-tuning is the strong CP problem.
The hypothetical axion particle then arises from a method to resolve Eq.(6), based on the Peccei-Quinn mechanism which introduces a new global $U(1)_{PQ}$ symmetry which allows the vacuum to relax to $\Theta = 0$. Because this $U(1)_{PQ}$ symmetry is spontaneously broken, it gives rise to a light pseudoscalar axion $[43, 44]$ with mass in the range $100keV < M < 1MeV$.

An axion in this mass range was excluded experimentally but then the theory was modified $[45–48]$ to one with an invisible axion where the $U(1)_{PQ}$ symmetry is broken at a much higher scale $f_a$ and the coupling of the axion correspondingly suppressed. Nevertheless, experiments to detect such so-called invisible axions were proposed, firstly using resonant microwave cavities then using other techniques discussed in sections 2.2.2 and 2.2.3.

2.2.1 Resonant Microwave Cavities

The first method to detect the invisible axions was suggested by Sikivie $[49]$ in 1983. The idea is that the dark matter axions will move through a microwave cavity in a strong magnetic field and be resonantly converted to photons. The very weak coupling of the axion is compensated by their large number, typically $\sim 10^{14} cm^{-3}$ if axions form all the dark matter.

The microwave signal will be almost monochromatic at a frequency corresponding to the axion mass, broadened upward because of the axion’s virial distribution. The expected velocity is $\sim 10^{-3}c$ which leads to a spread in energy $\delta E/E \sim 10^{-6}$.

The lagrangian for the axion coupling is

$$L = \left(\frac{\alpha g_{a\gamma\gamma}}{2\pi f_a}\right)aE.B. \tag{43}$$

The resonant modes which couple to axions are transverse magnetic (TM) modes. The predicted power from axion $\rightarrow$ photon conversion is $[50]$

$$P_a = \left(\frac{\alpha g_{a\gamma\gamma}}{2\pi f_a}\right)^2 VB^2 \rho_A CM_A^{-1}\min(Q_L, Q_a) \tag{44}$$

where $v$ is the cavity volume, $B$ the magnetic field, $\rho_a$ is the axion volume, $C$ is a form factor characterising overlap of a specific TM mode, $Q_L$ is the loaded quality factor and $Q_a$ is the axion quality factor.

The mass range for dark matter axions centres around $1\mu eV$ to $1meV$, although extensions of this mass range in both directions are being studied. The converted photon frequencies are in the range MHz to THz. Experiments have been designed to look at the lower frequencies in this range but to maintain a high quality factor only a few kHz can be scanned at a time.

The scan rate is determined by the time it takes a possible axion signal to be above the cavity’s intrinsic noise, according to the radiometer equation
\[ SNR = \frac{P_a}{P_N} \sqrt{Bt} = \frac{P_a}{kT} \sqrt{\frac{t}{B}} \]  

(45)

with \( P_a \) the power generated by axion-photon conversion, \( P_N = kBT \) is the cavity noise power, \( B \) is the signal bandwidth, \( t \) is the integration time, \( k \) is Boltzmann’s constant and \( T \) is the temperature.

From Eq.(45), a tiny signal power can be amplified by increasing \( P_a \propto VB^2 \), increasing the time \( t \) or minimising \( T \). Most of the research is directed to lowering the intrinsic noise. The earliest such experiments were carried out at Brookhaven National Laboratory [51], then at the University of Florida [52].

The ADMX (Axion Dark Matter eXperiment) at LLNL is one example [53] of a second-generation experiment. ADMX uses a 8.5 Tesla superconducting magnet 110cm in length and a 200 liter stainless steel microwave cavity plated with ultra-pure copper. An adjustable antenna is put through the top cavity plate and its signal is boosted by extremely low noise cryogenic amplifiers which are the most important limiting factor on axion sensitivity.

More recent cavity designs are discussed in [54].

### 2.2.2 Axion Helioscopes

Axions produced in the Solar core free-stream out to be detected on Earth when they convert into low-energy X-rays as they pass through a strong magnetic field. The flux of axions produced in the Sun should have a thermal spectrum with a mean energy of a few keV, and the integrated flux art the Earth is expected [55] to be \( \sim 10^{11} \text{cm}^{-2} \text{s}^{-1} \).

Consider a solar axion passing through a magnetic field \( B \) with length \( L \) then its probability \( P \) of conversion into a photon is given by [50]

\[ P = \frac\left(\frac{\alpha g_{a\gamma\gamma}BL}{4\pi f_a}\right)^2 2L^2 \frac{1 - \cos(qL)}{(qL)^2} \]  

(46)

in which \( g_{a\gamma\gamma} \) is the axion-photon coupling and \( q \) is the axion-photon momentum difference given by \( q = m_a^2/2E \) where \( E \) is the photon energy. Maximum conversion of axions to photons occurs when their fields stay in phase over the length \( L \) of the magnet. This requires [56] that \( qL < \pi \). When the axion mass is small, \( q \rightarrow 0 \) and the axion→photon conversion is greatest. More massive axions tend to go out of phase, but there is a method to compensate by adding a buffer gas which imparts an effective mass to the photon. Different axion masses can then be tuned by varying the gas pressure.

The first axion helioscope was built [57] at BNL (Brookhaven) in 1992 but the limits it obtained were far outside the expected parameters of invisible axion theories. Follow up experiments at the University of Tokyo [58] in 2007-08 obtained more stringent limits such as \( g_{a\gamma\gamma} < (5.6 - 13.4) \times 10^{-10} \text{GeV}^{-1} \) for axions on the mass range \( 0.84 \text{eV} < m_a < 1 \text{eV} \).
Upgraded experiments have been constructed at CERN and the University of Tokyo. At CERN, the experiment [59] is called CAST (= CERN Axion Solar Telescope). CAST uses an LHC magnet of length \( L = 9.3 \text{m} \) and magnetic field \( B = 9 \text{ Tesla} \). It tracks the Sun for 90 minutes a day using a rail system and the double magnet bore permits four X-ray detectors, one at each end of each bore. CAST has achieved a limit of \( g_{a\gamma\gamma}/f_a < 7.6 \times 10^{-8} \text{GeV}^{-1} \). \(^3\)He and \(^4\)He buffer gases are being used to extend the searched region of axion mass.

More sensitive than CAST is TASTE (=Troisk Axion Solar Telescope Experiment) [60].

### 2.2.3 Laser Methods

One might expect that the unique coherent states of photons in lasers can form a detection method for axions, and indeed we shall discuss two of the possibilities for scattering off the laser photons, designated \( \gamma_{\text{LAS}} \).

There will be a transverse magnetic field which itself creates virtual photons, designated \( \gamma^* \). The idea then is to create axions by the scattering process

\[
\gamma_{\text{LAS}} + \gamma^* \rightarrow a \tag{47}
\]

We may look for disappearance of polarised laser photons as they convert into axions by a magneto-optical effect of the vacuum. This arises from a term

\[
aE.B \tag{48}
\]

which is an anomalous coupling.

Such an axion search has been carried out by a Rochester-Brookhaven- Fermilab-Trieste (RBFT) group [61] which at one time found a preliminary positive signal [62] that led to theories of vacuum dichroism in which the two circular polarisations are differentiated. The polarisation of the laser beam can be examined. The original suggestion of a discovery was not supported by further data, but it has inspired a strong group of searchers for axion-like-particles at the DESY Laboratory.

As a second example for axion searches using lasers we briefly discuss what has been called, dramatically, ”light shining through walls” as suggested in [63]. Polarised laser photons pass through a magnetic field with \( E || B \) and axions when produced pass through an absorber (the wall) and are reconverted to axions on the other side [64].

The probability for a photon to convert into an axion in the axion-source region is

\[
\mathcal{P}_{\gamma \rightarrow a} \propto \frac{1}{4} \left( \frac{\alpha g_{\gamma\gamma}}{2\pi f_a} BL \right)^2 \frac{1 - \cos(qL)}{(qL)^2} \tag{49}
\]

and the probability for the axion to reconvert to an observable photon is the same as Eq.(49) and so the total probability for detection of photon\(\rightarrow\)axion\(\rightarrow\)photon is [65].
There is a maximum detectable axion mass because of the oscillation length becoming shorter than the magnetic field length. The RBFT group, already mentioned, found an upper limit on the axion-photon coupling of \( g_{a\gamma\gamma} < 6.7 \times 10^{-7} \text{GeV}^{-1} \) for axions with mass \( m_a < 1 \text{meV} \).

It has been shown that this "light shining though walls" experiment can be resonantly enhanced by encompassing both the production and reconversion magnets in matched Fabry-Perot optical resonators.

Laser induced fluorescence in rare-earth doped materials is being pursued.

2.3 Primordial black holes

The idea that primordial black holes (PBHs) might be formed in the early universe was first proposed by Novikov and Zeldovich in the Soviet Union in 1967. In 1974, the same idea occurred independently in the West to Carr and Hawking. A year later in 1975 Chapline was the first to suggest that the dark matter could be made from PBHs, \( DM = PBHs \), which was a prescient idea. At that time, PBHs were believed to be orders of magnitude lighter than the Sun, the most popular particle theory dark matter candidates, WIMPs and axions, had not yet been invented, and microlensing experiments were unknown.

Forty years later, in 2015, we proposed in Frampton instead the idea, \( DM = PIMBHs \), where the \( PIMBHs \) are many times the mass of the Sun in the intermediate-mass (IM) region between stellar mass and supermassive black holes. At the time, we were unaware of Chapline's work.

Important and influential were the data obtained by the MACHO Collaboration including examples of microlensing light curves for lens masses up to almost 25\( M_\odot \); it is entirely possible that if that experiment had continued beyond 1999, the dark matter PIMBHs could have been discovered although the possible additional time was limited because the Mount Stromlo Observatory was destroyed in the Canberra bushfire of 2003.

There were also the inventions of the WIMP and axion particles where the former became by far the most popular candidate for dark matter. One realization expressed in the 2015 paper was that the failure of the LHC to confirm the presence of electroweak supersymmetry at the same time weakened the motivation for the WIMP and therefore made more likely an astrophysical solution. It was mentioned that the most promising test was by microlensing and this was stressed further in a joint paper, Chapline and Frampton in 2016.

Several other senior physicists have thought about entropy of the universe or about black holes as dark matter. Six examples are Carr, Garcia-Bellido, Jacobson, Linde, Rees and Verlinde. We apologise to any author not mentioned.
Massive Compact Halo Objects (MACHOs) are commonly defined by the notion of compact objects used in astrophysics as the end products of stellar evolution when the nuclear fuel has been expended. They are usually defined to include white dwarfs, neutron stars, black holes, brown dwarfs and unassociated planets, all equally hard to detect because none of them emit significant electromagnetic radiation.

This narrow definition implies, however, that MACHOs are composed of baryonic matter which is too restrictive in the special case of black holes. It is here posited that black holes of arbitrarily high mass up to $10^{12} M_\odot$ can be produced primordially. Nevertheless the acronym MACHO still nicely applies to dark matter PIMBHs which are massive, compact, and in the halo.

Unlike the axion and WIMP elementary particles invented within the framework of quantum field theory which would have a definite mass, the black holes arising as classical solutions of Einstein’s equations have a range of masses. The lightest PBH which has survived for the age of the universe has a lower mass limit $M_{PBH} > 10^{-18} M_\odot \approx 10^{36}$ TeV, thirty-six orders of magnitude heavier than the WIMP with $10 \text{GeV} \leq M_{WIMP} \leq 1,000 \text{ GeV}$. This lower limit on the PBH mass comes from the lifetime formula derivable from Hawking radiation [96].

Because of observational constraints, the dark matter constituents must generally be another twenty orders of magnitude more massive than the lower limit in Eq.(9). We assert that most dark matter black holes are in the mass range between twenty-five and a trillion times the solar mass. The designation intermediate-mass for PIMBHs is appropriate for $25 M_\odot \leq M_{PIMBH} \leq 10^6 M_\odot$ because they lie in mass above stellar-mass black holes with $M_{BH} \leq 25 M_\odot$ and below the supermassive black holes with $M_{BH} \geq 10^6 M_\odot$ which reside in galactic cores.

We shall discuss three methods which can be used to search for PIMBHs: Wide binaries, CMB distortion and Microlensing in subsections 2.3.1, 2.3.2 and 2.3.3 respectively.

### 2.3.1 Wide binaries

There exist in the Milky Way pairs of stars which are gravitationally bound binaries with a separation more than 0.1pc. These wide binaries retain their original orbital parameters unless compelled to change them by gravitational influences, for example, due to nearby PIMBHs. Because of their very low binding energy, wide binaries are particularly sensitive to gravitational perturbations and can be used to place an upper limit on, or to detect, PIMBHs.

The history of employing this ingenious technique is regretfully checkered. In 2004 a fatally strong constraint was claimed by an Ohio State University group [97] in a paper entitled *End of the MACHO Era*, where stellar and higher mass constituents of dark matter were totally excluded.

Five years later in 2009, however, another group this time from Cambridge University [98] reanalyzed the available data on wide binaries and reached an opposite conclusion. They questioned whether any rigorous constraint on MACHOs could yet be claimed, especially as one of the important binaries in the earlier sample had been misidentified.
In 2014, the most recent publication on wide binaries appeared [99] which claims that, after all, some bound on MACHOs can be claimed, so this approach is still very much alive.

Because of the checkered history, however, it seems wisest to proceed with caution in reaching any categoric conclusions from wide binaries, but to acknowledge that they represent a potentially useful source both of constraints on, and the possible discovery of, dark matter PIMBHs.

### 2.3.2 Distortion of the CMB

This approach hinges on the phenomenon of accretion of gas onto the PIMBHs. The X-rays emitted by such accretion of gas are downgraded in frequency by cosmic expansion and by Thomson scattering becoming microwaves which distort the CMB, especially with regard to its very-precisely-measured black-body spectrum.

One early and detailed calculation of this effect by Ricotti, Ostriker and Mack (ROM) [100] has been very influential. ROM employed a specific model for the accretion, the Bondi model, and carried through the computation all the way up to the point of comparison with data from FIRAS on CMB spectral distortions, where FIRAS was a detector attached to the COBE satellite. ROM concluded that MACHOs with many solar masses could provide no more than a tiny fraction $\sim 10^{-4}$ of the dark matter.

The implication of ROM was that dark matter constituents with many solar masses were excluded unless the ROM calculation was in error by four orders of magnitude. Surprisingly the latter was the correct conclusion, as confirmed in 2016 by Ostriker [101]. There were grounds for suspecting this to be the case from observations of the X-rays from the supermassive black hole at the centre of galaxy M87 which were at least four orders of magnitude below the prediction by the Bondi model that assumes spherical symmetry and radial inflow which are questionable assumptions.

More recent papers have made exclusion plots of the allowed fraction of dark matter versus MACHO mass but their limits are sometimes far too severe for the same reason as for ROM. To mention one well-known analysis, Ali-Haïmoud and Kamionkowski [102] obtain upper limits which, while somewhat softer than ROM, remain suspiciously strong because, like ROM, they employ quasi-spherical accretion.

A recent claim by Zumalacarregui and Seljak [103] that absence of lensing by supernovae provides a contradiction to dark matter comprised of many-solar-mass constituents has been criticised in a reanalysis by Garcia-Bellido, Clesse and Fleury [104].

### 2.3.3 Microlensing

Microlensing is the most direct experimental method and has the big advantage that it has successfully found examples of MACHOs. The MACHO Collaboration used a method which had been proposed by Paczynski [105] where the amplification of a distant source by an intermediate gravitational lens is observed. Unbeknownst to Paczynski, the microlensing equations had been
Table 1: Microlensing duration $\hat{t}$ for the case of $n$ PIMBHs per halo with PIMBH mass = $\eta M_\odot$, halo mass = $10^{12} M_\odot$ and universe mass = $10^{23} M_\odot$

| $n$ / Halo | $M = \eta M_\odot$ | Halo Entropy $(S_{\text{halo}}/k)$ | Universe Entropy $(S_U/k)$ | Duration $\hat{t}$ (years) |
|------------|---------------------|-------------------------------|---------------------------|--------------------------|
| $4 \times 10^{10}$ | 25 | $2.5 \times 10^{91}$ | $2.5 \times 10^{101}$ | 1 |
| $10^{10}$ | 100 | $10^{94}$ | $10^{102}$ | 2 |
| $4 \times 10^8$ | 2500 | $2.5 \times 10^{92}$ | $2.5 \times 10^{103}$ | 10 |

calculated much earlier by Einstein \cite{106} who did not publish because he thought such measurements were impracticable. However, Einstein was overly pessimistic because the MACHO Collaboration discovered several striking microlensing light curves.

The method certainly worked well \cite{20} for $M < 25 M_\odot$ and so should work equally well for $M > 25 M_\odot$ provided one can devise a suitable algorithm and computer program to scan enough sources. The longevity of a given lensing event is proportional to the square root of the lensing mass and numerically is given by ($t$ is longevity)

$$t \simeq 0.2 \text{yr} \left( \frac{M_{\text{MACHO}}}{1 M_\odot} \right)^{1/2}$$  \hspace{1cm} (51)

where a transit velocity 200km/s is assumed for the lensing object.

The MACHO Collaboration investigated lensing events with longevities ranging between about two hours and one year. From Eq.(51) this corresponds to MACHO masses between approximately $10^{-6} M_\odot$ and $25 M_\odot$.

The total number and masses of objects discovered by the MACHO Collaboration could not account for all the dark matter known to exist in the Milky Way. At most 10% could be explained.

What is being suggested is that the other 90% of the dark matter in the Milky Way is in the form of MACHOs which are more massive than those detected by the MACHO Collaboration, and which almost certainly could be detected by a straightforward extension of their techniques. In particular, the expected microlensing events have a duration ranging from one year ($25 M_\odot$) to ten years ($2,500 M_\odot$), which is the practical limit for a feasible experiment.

We have simplified the visible universe, without losing anything important by regarding it as containing exactly $10^{11}$ galaxies, each with mass (dominantly dark matter) of exactly $10^{12} M_\odot$. The first three columns of Table 1 consider one halo of dark matter. To a first approximation, we can temporarily ignore the normal matter. The fourth column gives the additive entropy of the universe for well separated halos and the fifth column gives the corresponding microlensing event longevity in years. For a black hole with mass $M_{BH} = \eta M_\odot$, the dimensionless entropy is $S_{BH}/k \sim 10^{77} \eta^2$. 

20
We note that the entries in the fourth column of Table 1 are of the same order of magnitude as the value $S_{\text{SMBH}}/k \sim 10^{103}$ quoted in Section 1 for supermassive black holes. In a study \cite{107} made in 2009 entropies of the universe as high as $S_U/k \sim 10^{106}$ (a million googols) were found for PIMBHs with mass $M_{\text{PIMBH}} = 10^5 M_\odot$. According to Eq.\ (51) the microlensing light curves would then last several decades which seems impracticable except that a more sophisticated data analysis, in the future, might permit identification using only a fraction of the light curve.

For a given total halo mass, $M_{\text{Halo}} = 10^{12} M_\odot$, a smaller number of heavier black holes gives higher entropy because $S_{\text{BH}} \propto M_{\text{BH}}^2$. Such arguments using the concept of the entropy of the universe \cite{108} have for long been suggestive of more black holes than the stellar and supermassive black holes already identified.

The LIGO discovery \cite{109} of gravitational waves from black hole mergers offers some support for our dark matter theory but it is premature to take this support too seriously \cite{110}.

**Discussion**

In this article we have discussed three possibilities for the solution of the dark matter problem although one of the three, the first one we discussed, that involving the PIMBHs, is the most different. Unlike the others, it does not need to assume any new physics beyond the standard model of particle theory. Instead, it relies on Einstein’s equations of general relativity, their black holes solutions and especially the idea of entropy as developed in the nineteenth century.

The other two possibilities are more similar to each other as they both assume new physics beyond the standard model. One (WIMP) assumes a supersymmetric extension of the electroweak sector to ameliorate the scalar quadratic divergence problem and the other (axion) assumes an extension of the strong interaction QCD sector to solve the strong CP problem. The WIMP and the axion particles are predicted in quite different mass ranges and are the subjects therefore of quite different ongoing experiments. Certainly all such experiments are well worth pursuing.

By assumption, the WIMP experiences weak interactions so it can be searched for by direct detection of collisions with nuclei in the laboratory, or by direct production in colliders like the LHC. Indirect detection of WIMPs can be by searching for the products of WIMP annihilation such as gamma rays and neutrinos in nearby galaxies and clusters of galaxies. The WIMP is expected in the mass range 10 GeV to 1 TeV.

The axion first appeared as a particle postulated to resolve the strong CP problem of QCD. The original axion was ruled out by experiment but was replaced in the theory by a very-weakly-coupled very light “invisible” axion. This was initially thought to be undetectable until it was pointed out that one way to detect such axions in the dark matter halo is by using cold resonant cavities with a magnetic field in which dark halo axions are converted into photons. The preferred mass for axions is in the range 1 µeV to 1 meV.

The other solution for dark matter we discussed is one which requires no new physics but uses old physics dating from even before the birth of quantum field theory. In that theory, dark matter
exists because Nature tried to maximise entropy in the early universe. This was accomplished by producing Kerr black holes which concentrate entropy many orders of magnitude more efficiently than anything else. The masses of the most readily detectable primordial intermediate-mass black holes (PIMBHs) are in the range from $25M_\odot$ to $2500M_\odot$. It will be interesting to learn whether these PIMBHs show up in the microlensing observations within the not-too-distant future.

But there is a more fundamental, and we believe decisive, distinction between the three dark matter solutions being discussed. It seems to us equally as important to understand why dark matter exists, as it is to understand what it is. Let us take as representative masses of the three candidates: WIMP : 100 GeV ; Axion : 1 $\mu$eV ; PIMBH : 100$M_\odot$. Given that the total mass of dark matter in the visible universe is $\sim 10^{23}M_\odot$, the choice is between $10^{78}$ WIMPs, $10^{95}$ axions or $10^{21}$ PIMBHs. For the first two cases there is no reason why dark matter exists.

For the case of PIMBHs, however, there is a clear reason why so many were formed during the first three years after the Big Bang. That reason is entropy. Entropy thus explains why about one quarter of the energy of the universe is in the form of PIMBHs, a prediction which is soon readily testable.

We have argued on the basis of Boltzmann’s H theorem that entropy will increase in an isolated out-of-equilibrium system as happens when the large numbers of PBHs are formed in the first three years after the Big Bang. There is, however, an apparent paradox because although the curvature of spacetime, the gravitation, is undergoing strong fluctuations and inhomogeneities, at the same time [21] the electromagnetic photon-electron-proton plasma is in perfect thermal equilibrium. One way out is simply to say that the gravitational and electromagnetic sectors are decoupled, but this apparent paradox merits further study.

Nevertheless, we do know that black holes are by a very wide margin the best concentrators of entropy. Therefore, the reason dark matter exists is as foreseen over 150 years ago by the great physicist Rudolf Clausius in a useful short statement of the second law of thermodynamics, viz. the entropy of the universe tends to a maximum.

Acknowledgements

We thank S. Altmann for useful discussions.

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