Random Design Variations of Hollow-Core Anti-Resonant Fibers: A Monte-Carlo Study

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Abstract—Hollow-core anti-resonant fibers (HC-ARFs) have earned great attention in the fiber optics community due to their remarkable light-guiding properties and broad application spectrum. Particularly nested HC-ARFs have recently reached competitiveness to standard single-mode fibers (SMFs) in theory and even outperform them in certain categories. Key to their success is a precisely fine-tuned geometry, which inherently leaves optical characteristics highly susceptible to minimal structural deviations. When fabricating fibers, these come into play and manifest themselves in various imperfections to the geometry, ultimately worsening the fiber performance. In this article, for the first time to the best of our knowledge, these imperfections are statistically modeled and analyzed on their impact on the propagation loss in a Monte-Carlo fashion and simulated. Randomly varying outer and nested tube wall thicknesses as well as random tube angle offsets are considered. It is observed, that the loss increase caused by angular offsets dominates over varying tube thicknesses by approximately one order of magnitude for FM and two orders of magnitude for HOM propagation at a wavelength of 1.55 μm. Moreover, the higher-order-mode-extinction-ratio (HOMER) is proportional to the intensity of structural variations, indicating an increase in the 'single-modeness' of a fabricated fiber. Furthermore, a bend condition worsens the loss contribution of both effects applied jointly dramatically to a value of +50% at a bend radius of 4 cm compared to +7% for a straight fiber. We believe that our thorough investigations on the random structural perturbations of HC-ARFs will aid in fully exploiting to predict the performance of realistic HC-ARFs after fabrication.

Index Terms—Hollow-core fiber, random fiber perturbations, single-mode fiber, fabrication tolerance, Monte-Carlo analysis.

I. INTRODUCTION

Since their first demonstration in 2011 [1], negative curvature hollow-core anti-resonant fibers (HC-ARFs) proved themselves to be an outstanding optical medium due to their superior and unique optical properties [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19]. HC-ARFs offer low latency, low non-linearity, extremely low power overlap with silica glass which allows high damage threshold and reduced material attenuation, and low anomalous dispersion over a wide transmission range that can not be attained in silica-based solid-core fibers [3]. Such remarkable optical properties of HC-ARFs find numerous applications, including high power delivery [20], [21], [22], [23], gas-based nonlinear optics [24], [25], [26], [27], [28], [29], [30], extreme UV light generation [31], [32], non linear microendoscopy [33], mid-IR transmission [34], [35], optofluidic [36], and terahertz applications [37], [38], [39]. Recently, HC-ARFs have also been used in short-reach data transmission [5], [6], next-generation scientific instruments, low-noise applications [40], quantum state transmission [41], polarization purity [42], and polarization control [43], [44].

Unlike solid-core fibers, the HC-ARF guides light in the air-core based on an unusual and extraordinary guiding mechanism, namely a combination of anti-resonant reflection and inhibited coupling (IC) [3] between the core-guided modes and the continuum of modes of the cladding [45]. The IC guiding mechanism of hollow-core fibers was first proposed by Couny et al. [46], which can be explained by the high degree of the transverse-field mismatch between the core and cladding modes (CMs). In HC-ARFs, the IC between the core-mode and CMs can be enhanced dramatically by using a negative curvature core contour [9], carefully engineering the cladding structure, and choosing a proper number of cladding tubes [47], [48]. Recently, it has been reported that the number of tubes in the cladding structure plays one of the critical factors in significantly reducing the propagation loss and maintaining effectively single-mode operation [47], [48]. The enhanced IC guiding mechanism in HC-ARFs offers a much wider transmission window, a low dispersion, and a lower loss compared to hollow-core photonic bandgap fibers (HC-PBGFs), in which light guides based on the photonic bandgap effect [49], [50].

The loss and optical performance of several HC-ARF architectures based on a “negative curvature” core surround have been studied over the past few years [1], [2], [3], [5], [6], [7], [8], [9], [10], [11], [47], [51]. Most of the earlier HC-ARF designs rely on single circular anti-resonant tubes having a different number of tubes in the cladding [1], [10], [13]. The single-ring HC-ARF design exhibits relatively high propagation loss and is highly sensitive to a small bend radius; the core-modes can be strongly coupled with the cladding modes upon a small bend radius [3]. The propagation loss and bend loss can be
significantly improved by introducing nested tubes inside the outer tubes [2], [3]. Adding nested tubes drastically enhances the IC coupling between the core and cladding modes and thus significantly reduces the propagation loss and macro bend loss [2], [3]. It is worth mentioning that the earlier proposed fiber designs used either eight [2] or six tubes [3] in the cladding. In 2019, Habib et al. first proposed that the number of cladding tubes plays a crucial role in designing wide-band and ultra-low loss fibers with effectively single-mode operation [47]. In [47], the authors thoroughly studied the impact of different numbers of cladding tubes on the propagation loss, transmission window, and single-mode properties. The five-tube HC-ARFs shows a remarkably wide transmission window, low loss, and effectively single-mode operation compared to other numbers of cladding tubes, later experimentally confirmed in [52]. Recently, some pioneering works on nested HC-ARF designs have been reported with remarkably low propagation loss. For example, a propagation loss of 0.28 dB/km from 1510 nm to 1600 nm was demonstrated with a six-tube nested HC-ARF design [15]. Moreover, a five-tube HC-ARF has been reported with a loss of 0.22 dB/km at 1300 nm and 1625 nm [52]. More recently, a double-nested five-tube HC-ARF has been reported with a record low loss of 0.174 dB/km in the C-band [53]. The above mentioned fiber designs and loss performances are without any doubt outstanding achievements; however, the loss values are \(\approx 25\%\) higher compared to single-mode fibers (SMFs) [54]. Hope has recently been resurged by a numerical study that it is possible to attain propagation loss values close to or even less than SMFs by realizing the nested tubes slightly anisotropic [48], [55]. A propagation loss of \(\approx 0.11\) dB/km was demonstrated. A few studies demonstrate the fabrication tolerances in non-silica [56] and silica-based HC-ARF [48]. However, in all previous studies, the effect of random structural perturbations to the fiber geometry, such as random variations in the tube wall thicknesses and random tube gap separations, were not considered. These variations are typically seen in the fiber fabrication process and might impact the overall loss and single-mode propagation performance, and therefore, these effects can not be avoided. Pioneering work in this topic recently published by Petry et al., [57] targets a particular subcase of random geometry variations in the nested cladding elements. The main focus of this study is on inner tubes that are misaligned and anisotropically deformed. Although the results offer initial understanding of the transmission behaviour of practical hollow-core fibers under perturbations, there is still a lack of fundamental knowledge regarding the effect of the most commonly occurring geometric parameters post-production.

Fig. 1. Random design perturbations analyzed in this article illustrated for a five-tube HC-ARF. (a) Ideal (symmetric) nested HC-ARF design with five circular tubes; (b) allowing individual outer and nested tube thicknesses, \(t_o\) and \(t_n\), respectively; (c) allowing individual angle offsets for tubes; (d) geometry features of (b) and (c) combined concurrently. The thickness of the tubes drawn is exaggerated and not to scale for improved visibility. The design parameters are similar to [5], [6].

The article is organized as follows: Section II reviews the typical nested HC-ARF geometry and introduces two types of random design variations it suffers from after fabrication. Section III optimizes those geometries for the lowest propagation loss and studies its susceptibility to the imperfections introduced prior by performing numerous simulations in a Monte-Carlo fashion. In the case of a straight fiber configuration, we take into account the propagation of both single modes and higher-order modes. However, when we introduce a bend condition or change the operational wavelength, our main emphasis is on the single-mode propagation. Moreover, an outlook on further studies is given. Section IV summarizes this article.

II. INTRODUCTION TO RANDOM DESIGN VARIATIONS

The different types of random design variations analyzed in this work are displayed in Fig. 1. For the purpose of illustration, these effects are demonstrated for a five-tube nested HC-ARF geometry since this structure is of primary focus in our random design analysis. However, the techniques introduced here are equally applicable for any other number of tubes. The ideal geometry of a five-tube HC-ARF is shown in Fig. 1(a). In this work, a core diameter of \(D_c = 35\) µm, similar to [48], is selected. The outer diameter \(D\) and nested tube diameter \(d\) depend on the core diameter, number of cladding tubes \(N\), and the tube gap separation \(q\), which is determined in the optimization part in subsection III-A. The nested tube ratio \(d/D\) is kept fixed to 0.5 since this ratio effectively provides single-mode operation [47], [48]. Lastly, the tube wall thickness, usually in the range of...
multiple 100 nm’s is also determined in the optimization step due to its strong impact on the propagation loss. The ideal structure is characterized by constant fiber parameters. However, when fabricating HC-ARFs with sub-μm geometry features in practice, this assumption no longer holds [5]. With many imperfections coming into play, two distinct imperfections are identified to considerably impact the geometry, which are thoroughly addressed in this work. For example, maintaining the same wall thickness for both tubes is typically challenging when fabricating < 400 nm thick tubes [6]. The wall thicknesses of the cladding tubes are assumed to deviate up to ±5% from the target thickness. Also, recent studies show that the mean nested tube thickness $t_n$ varies around ±10% from the outer tube thickness $t_c$ [6], [48], [52]. Tube thickness variations typically arise from tube inflation inconsistencies in the manufacturing process. This applies to both outer and inner tubes and, hence, inherently affects the nested tube radius $d/D$. Our geometry modelling approach reflects this behavior in a logical manner, resulting in slight deviations of $d/D$ from its target value. Moreover, not only do the tubes’ thicknesses vary, but also the absolute tube position itself is displayed in Fig. 1(c). This manifests itself in an individual offset to the tube angles, denoted by $\alpha_1$ ... $\alpha_5$, each introducing a tube rotation around the geometrical center of the fiber. The angular deviation of the tubes from their original axes is illustrated by light-blue shaded areas, with dotted lines representing the original axes. Another way to look at this phenomenon is from the perspective of varying gap separations between adjacent tubes. Of course, those quantities depend on each other and can be converted interchangeably, which is shown in (1.1).

The gap separation $g_{12}$ between adjacent tubes one and two depends on the core diameter $D_c$, outer-tube diameter $D$ [48], outer-tube thickness $t$ (assuming equal thicknesses), and the difference of the respective angle offsets $\Delta \alpha = \alpha_2 - \alpha_1$:

$$g_{12} = \sqrt{2 \left( \frac{D_c}{2} + \frac{D}{2} + t \right)^2 \left[ 1 - \cos \left( \frac{2\pi}{N} \Delta \alpha \right) \right] - D - 2t}$$

(1.1)

By geometrical dependency, the outer tube diameter $D$ can also be calculated from different parameters, namely the number of tubes $N$ and the mean gap separation $g_{\text{mean}}$ according to [58]

$$D = \frac{D_c \sin \left( \frac{\pi}{N} \right) - g_{\text{mean}}}{1 - \sin \left( \frac{\pi}{N} \right)}.$$  

(1.2)

Due to advantages in the handling of angle offsets, e.g., the statistical independence of $\alpha_1$ to $\alpha_5$ (contrary to $g_1$ to $g_5$) and the intuitive connection to the manufacturing quality metrics, the angle offset perspective is pursued throughout this article. Conversion methods, e.g., from angle offset standard deviation to gap separation standard deviation, are left subject to further research. The influence of both effects (wall thickness variations and angle offsets) on the propagation loss is investigated independently as shown in Fig. 1(b) and (c). Next, the impact on the propagation loss is studied considering both effects simultaneously as shown in Fig. 1(d).

To support the effects described prior, additional degrees of freedom are implemented into the simulation model, namely individual nested and outer tube thicknesses $t_{n,1}$ ... $t_{n,5}$ and $t_{o,5}$, respectively, as well as individual tube angle offsets $\alpha_1$ ... $\alpha_5$. In the course of this article, statistical properties are incorporated based on practically feasible geometrical characteristics.

### III. Numerical Results and Discussion

All simulations are performed using finite-element analysis based on COMSOL software in combination with MATLAB-Livelink package. To accurately calculate the modal properties of the fiber, the simulation environment is setup as follows: A perfectly matched-layer (PML) with a 8-layer-deep cylindrical mapped mesh is placed outside the fiber to mimic a non-reflective, perfectly absorbing infinite domain. Mesh element sizes of $\lambda/6$ and $\lambda/4$ in the silica and air regions, respectively, are configured similarly to [3], [47], [48]. To speed up the simulation, mode-searching is performed by an analytical approximation of a capillary model [59]: $n_{\text{guess}} = \sqrt{1 - \left( \frac{U_{mn} \lambda^2}{2 \pi R_c} \right)^2}$, where, $R_c$ is the core radius, $\lambda$ is the wavelength, and $U_{mn}$ is the nth zero of the nth-order Bessel function of the first kind. However, since spatial random structural variations are introduced across the HC-ARF geometry, symmetry inherent under ideal conditions is voided. This makes the well-used practice of exploiting this symmetry by solving only half of the structure using proper boundary conditions no longer feasible. Therefore, the whole domain of the fiber is solved, which leads to an increase in simulation time and memory consumption. The main focus of this article is the propagation loss, which is the primary characteristic of HC-ARF that receives particular attention. Hence, various forms of losses are taken into account to estimate the propagation loss, including confinement/leakage loss, surface scattering loss, and macro bend loss (if applicable). The surface scattering loss is calculated by multiplying the $F$-factor, which represents the normalized electric field intensity at the silica-air boundaries [60] with a calibration factor $\eta$ [3]. $F$ is calculated according to [61]:

$$F = \sqrt{\frac{c}{\mu_0} \int \int E_E^2 + H_H^2 \, ds}$$

with electric field vector $\vec{E}$, magnetic field vector $\vec{H}$, and nature constants $c$ and $\mu_0$. By further exploiting that $F$ scales with $\lambda^2$ and surface scattering loss having a $\lambda^{-1.24}$ dependency [62], wavelength-dependent estimates are implemented by [3]:

$$\alpha_{\text{SSL}} \text{[dB/km]} = \eta F \left( \frac{\lambda}{\lambda_0} \right)^{-3}$$

(2.1)

with calibration factor $\eta = 300$ at $\lambda_0 = 1.55$ μm. Because of the insignificant power overlap with silica walls < $10^{-4}$, effective material loss (EML) is neglected and therefore not included in the calculations.

#### A. Geometry Optimization

A point of reference is required to analyze and compare the effects of random structural variations on the propagation loss to the principle HC-ARF geometry as described in Section II. This point of reference is chosen as the lowest loss exhibited by
the so-called ideal geometry. To identify those points, a propagation loss optimization for five-, six-, seven-, and eight-tube HC-ARFs is performed by utilizing an exhaustive search over the ideal geometry. To identify those points, a propagation loss optimization for five-, six-, seven-, and eight-tube HC-ARFs is performed by utilizing an exhaustive search over the ideal geometry. To identify those points, a propagation loss optimization for five-, six-, seven-, and eight-tube HC-ARFs is performed by utilizing an exhaustive search over the ideal geometry.

Fig. 2. Calculated propagation loss of LP$_{01}$-like FM as a function of the tube thickness $t$ and gap separation $g$ for nested HC-ARFs with an ideal geometry in a (5) five-, (6) six-, (7) seven- and (8) eight-tube configuration. The 2D surface plot is generated using a sweep quantization of $\Delta t = 1$ nm and $\Delta g = 0.25$ μm with linear interpolation. Additional design parameters: Core diameter $D_c = 35$ μm; equal outer and nested tube thickness, $t_o = t_n$; nested/outer tube diameter ratio $d/D = 0.5$. All simulations are performed at wavelength $\lambda = 1.55$ μm.

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the susceptibility of critical fiber characteristics for fundamental mode (FM) and higher-order-mode (HOM) propagation to random structural variations. As mentioned in Section II, fabricated fibers are subject to structural imperfections introduced by the manufacturing process. The outer and nested tubes hereby represent a great susceptibility to process variation, whereas two effects, namely a deviation of the tubes’ thicknesses and the tubes’ positions itself are believed to pose the major contribution [48].

Inference on the optical properties of perturbed HC-ARF geometries is developed by applying a Monte-Carlo technique in which sufficiently large sets of perturbed fiber geometries are sampled and numerically evaluated. Since an infinite amount of geometries is inherent in this continuous sample space, the sample size is determined based on result convergence and feasibility of simulation time. Since the simulation objectives differ slightly throughout this article, additional information is provided in the respective subsections.

In order to represent the process of random perturbations in our calculations, the structural variations are assumed to be distributed in a zero-mean Gaussian fashion, essentially meaning that small offsets are more common than big offsets. Special care must be taken when applying the offsets to the tube angles. To avoid collisions of two adjacent tubes with large, contrary angle offsets, the angle offset distributions are symmetrically truncated at a cut-off angle of $\alpha_{\text{cut-off}} = 4.5^\circ$. Strictly speaking, trimming the tails of a normal distribution decreases the resulting standard deviation of the sampled values since out-of-range values get replaced by smaller, in-range values. In this article, it is always referred to the original standard deviation. To increase the degree of realism of the model, a general 10% increase in the mean thickness of the nested tubes relative to the outer tubes is assumed, which is observed for fabricated fibers due to the manufacturing process. This increase is also reflected in the nested tube thickness standard deviation. However, one could also assume a decreased nested tube thickness [52].

In the first step, the susceptibility of the propagation loss to the effects of random tube thickness variations and random tube angle offsets is investigated. Those effects are both applied independently and simultaneously. Moreover, both effects are also evaluated on HOM propagation by restricting the analysis to the LP$_{11}$-mode. Hence, the standard deviation of the outer tube thicknesses is scanned from 0 nm to 20 nm [nested: $+10\%$] with a step-size of 1 nm, and of the tube angle offsets from $0^\circ$ to $2^\circ$ with a step-size of 0.2°. For every of these combinations, sample groups of 30 random geometries are generated and evaluated for propagation loss, whereas the fundamental-mode and best propagating HOM are separated into two datasets. This results in a sample size of $30 \times 11 \times 21 = 7161$ ($\#_{\text{samples}} \times \#_{\alpha} \times \#_{\text{th}}$).

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Finally, data interpolation can easily be achieved by evaluating and averaging the final row- and column-wise regressions for any intermediate points necessary. The data shown in Fig. 3(a) and (b) is post-processed by utilizing three iterations and interpolating by a factor of 32, effectively eliminating any noise. Algorithm validation is performed by checking the bias between the original data and the post-processed (non-interpolated) data for convergence to sufficiently small values as a function of the sample size. For fundamental mode, the bias, defined as the average quadratic error, converges to 1.88%, 0.88%, 0.55%, and 0.34% for 5, 10, 20, and ultimately 30 samples per combination, which is over one magnitude smaller than practically relevant loss increase values. Therefore, this algorithm is considered to be appropriate.

For the fundamental mode, the loss increases with either standard deviation rising and reaches a maximum point of +7% at the maximum joint random structural deviation (see Fig. 3(a)). It can be seen from Fig. 3(a) that the effect of random tube angle offsets has a much more pronounced impact on the loss than the varying tube thicknesses. The contour lines evolve more vertical with a rising angle offset intensity, confirming this observation and indicating a generally weak impact of the random tube thickness effect. Interestingly, the loss contribution of the angle offset effect is fairly low in the left half of the figure and rapidly picks up in the right half. This is important since it essentially means that angle offset standard deviations up until \( \approx 1^\circ \), paired with a moderate tube thickness variation, do not result in any considerable loss increase \( > 1\% \), but multiply their impact if this threshold is passed. Circling back to the optimization of the ideal structure, this minor impact is consistent with the general insensitivity of the transmission loss of the five-tube structure to both considered geometrical parameters, as shown in Fig. 2a in Section III-A.

The HOM loss increase shows an even more extreme domination of the angle offset effect over the tube thickness effect as seen in Fig. 3(b), with a maximum joint loss increase of 71%. However, its impact rises nearly perfectly linear in contrast to FM loss, as indicated by the intercept points of the 20%, 40%, and 60% loss contour lines with the x-axis at \( \sigma_\alpha = 0.6^\circ, 1.2^\circ, \) and \( 1.8^\circ \), respectively. The higher-order-mode extinction ratio (HOMER), which is defined as the ratio between minimum HOM propagation loss and FM propagation loss, indicating the single-modeness of the fiber, is displayed in Fig. 3(c) as a function of both imperfection standard deviations. HOMER rises about 63% from 1331 to 2175 in the studied range. It can be seen that HOMER practically solely depends on the angle offset standard deviation and increases approximately proportionally with it. In other words, HOMs are more susceptible to random structural variations than FM. This increased effective single-modeness of perturbed fibers can be interpreted as one of the only advantages of random structural variations when single-mode propagation is desired. In summary, the five-tube nested HC-ARF geometry exhibits negligible susceptibility for random tube thickness variations in the analyzed range and provides a stable FM propagation loss for a moderate random tube angle offset effect.
C. Wavelength Dependency of the Imperfections’ Impact

In this subsection, the same effects, i.e., random tube wall thicknesses as well as random tube angle offsets are analyzed as a function of the wavelength. On the one hand, the goal is to learn how the fiber with realistic fabrication perturbations performs over a broader spectrum. On the other hand, it is of great interest for fiber manufacturers to reduce the loss contribution of the dominating random effect according to the aimed operating wavelength. Therefore, three geometry groups with either a random tube thickness imperfection of $\sigma_t = 20 \, \mu m$, a random angle offset imperfection of $\sigma_\alpha = 2^\circ$, and a simultaneous combination of both are generated. These geometry sets are evaluated over a wavelength range of $1 \, \mu m$ to $2 \, \mu m$ using a quantization of $\Delta \lambda = 0.05 \, \mu m$ and evaluated using the respective medians of each group as a function of the wavelength. This results in a sample size of $3 \times 50 \times 31 = 4650 \, (\# \text{groups} \times \# \text{samples} \times \# \lambda)$. The wavelength-dependent loss increase of each group is displayed in % with respect to the ideal geometry in Fig. 4. The ideal geometry’s absolute loss (corresponding to the right y-axis) is displayed in black in dB/km and exhibits the lowest propagation loss over the whole spectrum with a minimum value of $0.15 \, dB/km$ at $\lambda = 1.36 \, \mu m$. As expected, both independently applied random effects exhibit a slightly positive loss increase with the joint configuration forming an upper envelope. Interestingly, the dominating effect switches in the analyzed range at $\lambda_{cross} = 1.3 \, \mu m$. For wavelengths $< \lambda_{cross}$ the random tube thickness imperfection contributes nearly $100\%$ of the loss increase. On the contrary, the random angle offset effect poses the main loss contribution in the range between $1.3 \, \mu m$ to $1.65 \, \mu m$, which is consistent with the previous observations. For higher $\lambda$ values, the shares of both effects balances out to approximately $2:1$ for the angle offset effect vs. the random tube thickness effect. It is worth noting that the absolute loss (and loss increase) of the fiber category with combined imperfections exhibits a global minimum loss of $0.16 \, dB/km$ at the crossing point, which indicates an optimal operation point from the perspective of resilience to random structural perturbations. Lastly, a loss bump around $\lambda = 1.45 \, \mu m$ for the fiber with random angle offsets can be observed, which also impacts the “combined”-perturbed fiber. We could not identify a distinct reason for this effect in our simulation, however, measurement campaigns of similar realistic fibers [52], [63] reported a similar behavior. While for the first reference this effect is attributed to water vapor inside the fiber, we believe that this could be amplified by random angular tube offsets or even fully caused by this effect. In conclusion, the simulations show that the shares of the loss contribution and the relative loss increases are highly dependent on the operating wavelength in the light spectrum.

D. Impact of Random Design Variations Under a Bend Condition

In this subsection, the susceptibility of the fiber propagation loss to the previously described random imperfections while bending the fiber at a constant radius of $5 \, cm$ at first and allowing multiple bend radii afterwards is thoroughly analyzed and discussed. We employed the formula provided in [64], [65] to determine the amount of bend loss. As before, both effects are studied independently and jointly. Similarly to subsection III-B, the impact of different tube thickness standard deviations as well as multiple angle offset standard deviations is investigated in the range of $2 \, nm$ to $20 \, nm$ [nested +10\%] and $0.2^\circ$ to $2^\circ$, respectively, with sample groups of size 100. This results in a sample size of $3 \times 100 \times 10 = 3000 \, (\# \text{groups} \times \# \text{samples} \times \# \lambda)$. A box plot, grouped by the generating standard deviations, is displayed in Fig. 5. The independently calculated results are visualized in (a) and (c), and the joint analysis, for which both imperfections are applied simultaneously with the chosen standard deviations, is shown in (e). The median and mean values of the individual sample groups are indicated by white and green markers, respectively. The results are given in relative loss increase to the ideal, bent structure ($R_{bend} = 5 \, cm$), which exhibits a propagation loss of $5.47 \, dB/km$ at $\lambda = 1.55 \, \mu m$. All three graphs show a clear dependence of the loss increase on the standard deviation. The median and mean values are assumed to rise monotonically in the investigated range, whereas the median rises to $8\%$ and $17\%$ for the random tube thickness and angle offset effects, respectively. The maximum median value of $\approx 50\%$ for the joint case indicates a mutual amplification in loss increase when both effects are applied concurrently. Estimations of the probability density functions (PDF) for the highlighted sample groups are provided in the corresponding graphs in Fig. 5(b), (d), and (f) on the right-hand side. As noted in subsection III-B, the individual geometries exhibit quite a spread of loss values ranging from a multiple of the median loss increase to slightly lower than the propagation loss of the ideal (bent) geometry. At this point, the preference of the median
over the mean is emphasized once again, since a classification into production-relevant confidence intervals, which are based on probabilities, is given in a natural way by the median. Particularly in the case of a one-sided distortion as observed in Fig. 5, the mean value is only of limited meaningfulness. To further visualize this spread, the loss increase values corresponding to the mean value is only of limited meaningfulness. To further visualize this spread, the loss increase values corresponding to the mean value is only of limited meaningfulness. To further visualize this spread, the loss increase values corresponding to the mean value is only of limited meaningfulness.

Fig. 5. Calculated relative transmission loss increase with respect to the ideal structure for imperfect 5-tube nested HC-ARC geometries as a function of the population standard dev. at a constant bend radius of 5 cm, visualized as a box plot. (a) Thickness standard dev. scanned from 2 nm to 20 nm [nested +10%]; (c) angle offset standard dev. scanned from 0.2° to 2°; (e) imperfections from (a,c) applied simultaneously. Right-sided graphs display estimated probability density functions (PDF) for corresponding highlighted sample groups regarding thickness (b), angle offset (d), and combined (f) structural imperfections. The individual sample groups’ median and mean loss increases are indicated by white and green markers, respectively. Individual loss values in the highlighted sample groups of (a) and (c) are plotted over the sample standard dev. in the background. A sample size of $3 \times 100 \times 10 = 30000$ ($\#_{\text{groups}} \times \#_{\text{samples}} \times \#_{\text{th.\&\alpha}}$) is utilized. Additional design parameters: Core diameter $D_c = 35 \mu m$; nested/outer tube diameter ratio $d/D = 0.5$. Mean outer [nested] tube thickness $\sigma_{t_o} = 393 nm$ [$\sigma_{t_n} = 1.1 \cdot \sigma_{t_o} = 432 nm$]; mean angle offset $\Delta \alpha = 0°$ leading to a mean tube gap separation $\sigma = 5.25 \mu m$. The 2D surface plot is generated using a sweep quantization of $\Delta r = 0.5 cm$ and $\Delta \sigma_{t_n} = 2 nm$, $\sigma_{\alpha} = 0.2°$ with a sample size of $20 \times 17 \times 11 = 3740$ ($\#_{\text{samples}} \times \#_{\text{bend-r.}} \times \#_{\text{th.\&\alpha}}$). Standard dev. of nested tube thickness is 10% increased with respect to outer tubes, $\sigma_{t_n} = 1.1 \cdot \sigma_{t_o}$. The mode-field profiles at the top of the figure showcase the electric field intensity for various bend radii for one perturbed HC-ARC geometry sample. Additional design parameters: Core diameter $D_c = 35 \mu m$; nested/outer tube diameter ratio $d/D = 0.5$. All simulations are performed at wavelength $\lambda = 1.55 \mu m$.

As described in the simulations before, bending a five-tube HC-ARC to a radius of 5 cm increases the absolute propagation loss from 0.17 dB/km to 5.47 dB/km. It is also shown that bending amplifies the impact of random structural imperfections on the loss. This raises the question of how this dynamic behaves across various bend radii and whether random imperfections need to be taken into account for minimal bend radii. In order to address this question, a 2D sweep over practical bend radii in the range of 4 cm to 12 cm is carried out while applying a joint imperfection of random tube thicknesses and tube angle offsets with standard deviations ranging from $\sigma_{t_n} = 0 nm$ to 20 nm and $\sigma_{\alpha} = 0°$ to $2°$, respectively. A sample group size of 20 has proven to be sufficient. This results in a sample size of $20 \times 17 \times 11 = 3740$ ($\#_{\text{samples}} \times \#_{\text{bend-r.}} \times \#_{\text{th.\&\alpha}}$).
loss in dB/km as well as the column-wise median loss increase with respect to the $\sigma_{t,\alpha} = 0$ point. Logarithmic background color-scaling with yellow and dark-blue colors denoting high and low loss, respectively, is used because of the high dynamic range of three orders of magnitude. The resulting median loss ranges between 0.28 dB/km to 140 dB/km, with high losses occurring at very low bend radii. The column-wise median loss increase, denoted by grey-colored dots, shows a strong dependency on the bend radius. It ranges from negligibly small values for high bend radii up to 80% for $R_{bend} = 4$ cm. However, it is emphasized that the computed median values are not smoothed because of the logarithmic-x and polynomial-y behavior and therefore fluctuate due to a limited sample size. A linear estimate for the loss increase between $\sigma_{t,\alpha} = 0$ and $\sigma_{t[\alpha]} = 20$ nm [2°] as a function of the bend radius is displayed by a red dotted line. It can be seen that the relative loss contribution due to random imperfections does indeed increase with the reciprocal of the bend radius and can lead to a significant worsening of the propagation loss up to $\approx 50\%$ at $R_{bend} = 4$ cm. As a rule of thumb, one can assume that the loss contribution due to random perturbations increases by approximately 3% per centimeter less bend radius in the studied range. Exemplary mode-field profiles of an imperfect sample geometry as a function of the bend radius are shown at the top of Fig. 6.

E. Further Imperfections and Outlook

The imperfections considered in this article are non-doubly the most visually pronounced ones; however, other imperfections can be observed, which seek further analysis. One of them is an angular offset of the nested tubes, independent of their corresponding surrounding tube. This creates the effect of “rolling” nested tubes inside their surrounding tube within the range of a few degrees. Also, a randomly distributed non-circular, anisotropic shape of the cladding structure can be investigated. The authors suggest deploying machine learning techniques to predict the expected fiber characteristics as an alternative to the classical finite-element approach. These investigations are left for further research.

IV. Conclusion and Future Work

In summary, the impact of random structural variations on the propagation loss of nested hollow-core anti-resonant fibers have been investigated using finite-element modeling and Monte-Carlo simulations. A five-tube geometry is selected and optimized with respect to the tube wall thickness and gap separation to serve as an ideal reference point. The random structural effects considered in this article are varying outer and nested tube wall thicknesses as well as tube angle offsets, whereas the second one is identified to dominate in the majority of tested operating conditions. Analyses of single- and multi-mode propagation loss as a function of the imperfection intensity at a fixed wavelength of 1.55 μm show that random angle offsets have a much more pronounced impact on the loss than random tube thicknesses by a factor of four, leading to an approximate combined loss increase of 7% and 71% for single- and multi-mode propagation, respectively. However, the share is very wavelength-dependent and even reaches a point where the dominant contribution flips to the random tube thickness effect. An interesting phenomenon worth noting is that the fiber with both imperfections applied simultaneously has its lowest (absolute) propagation loss at this flipping point. Focusing only on the fundamental mode, the impact of random structural variations is amplified when a bend condition is applied. The loss contribution by random effects rises with the reciprocal of the bend radius and has an approximately seven-fold impact at $R_b = 5$ cm compared to no bend condition. Overall, a significant worsening of the propagation loss of about 50% due to random structural effects can be expected at a bend radius of 4 cm. Besides utilizing a newly adapted model [19] for calculating the surface scattering loss, follow-up works should consider modeling realistic dependencies between the nested tube ratio $d/D$ and the tube thicknesses due to inflation inconsistencies in the fabrication process.

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