Experimental Quantum Communication Enhancement by Superposing Trajectories

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Traditional quantum communication assumes that particles are transmitted from a sender to a receiver in well-defined trajectories. In spite of this, trajectories can be used as a quantum control to govern the order of different noisy communication channels, and such a control has been shown to enable the transmission of information even when standard quantum communication protocols fail. This result has motivated further investigations on the role of the superposition of trajectories in enhancing communication, which revealed that the use of quantum-control of parallel communication channels, or of channels in series with quantum-controlled operations can also lead to communication advantages. Building upon these findings, here we experimentally compare different ways in which two trajectories through a pair of noisy channels can be superposed. We observe that, within the framework of quantum interferometry, the use of channels in series with quantum-controlled operations generally yields the largest advantages. Our results contribute to clarify the nature of these advantages in experimental quantum-optical scenarios, and showcase the benefit of an extension of the quantum communication paradigm in which both the information exchanged and the trajectory of the information carriers are quantum.

I. INTRODUCTION

The ability to establish secure communication linkages is of paramount importance in any information technology. Quantum cryptography protocols [1, 2] achieve this in a stunning way, enabling a sender and receiver to communicate securely even in the presence of an eavesdropper with unlimited computational power. The crucial ingredient for this feat is the availability of reliable transmission lines for quantum particles. In this framework, any noisy processes affecting the transmission are attributed to the presence of an eavesdropper, and when the noise exceeds a given threshold, the security of the communication is considered compromised. For this reason, the mitigation of any noise arising from faulty transmission lines is an integral part of the efforts to enable secure communication.

All traditional quantum cryptography protocols encode information in quantum states, yet they treat the propagation of information carriers as classical. Nevertheless, the information carriers can propagate along non-classical trajectories, experiencing a coherent superposition of alternative quantum evolutions [3, 4]. Taking advantage of this fact, Gisin et al. [5] realized that quantum superpositions of trajectories can be harnessed to reduce the noise induced by a pair of noisy communication channels. Therein, it was shown that when the quantum information carriers are sent through two noisy channels in a quantum superposition of trajectories, interference between the two resulting noisy processes can sometimes lead to partial cancellation of the noise via post-selection.

Recently, interest in this discovery has been revived by studies emerging from quantum foundations. In particular, it was shown that the superposition of trajectories can generate setups where the order of different channels is in a quantum superposition. These setups produce the same output as a higher-order operation called the quantum switch [6, 7], which takes two quantum channels as input and combines them in a quantum-controlled order. The quantum switch is an instance of a causally-indefinite process; such processes are currently the target of wide-ranging research both for fundamental reasons [8, 10], and for their potential to provide advantages in...
quantum computation \cite{7,11,17}, quantum communication complexity \cite{15,18,20}, and quantum metrology \cite{20}. Moreover, the particular class of causally-indefinite processes based on the superposition of alternative orders can be probed via current experimental technologies, as has been recently done by encoding information in various degrees of freedom of single photons \cite{17,21,27}.

It was further proposed \cite{28,35} and, in some cases, experimentally shown \cite{24,67}, that the quantum switch can also reduce noise in classical and quantum communication. These results stimulated an exploration of quantum communication networks in a coherent superposition of alternative configurations, including quantum-control of parallel noisy channels \cite{8,40}, and channels in series with quantum-controlled operations \cite{41}.

There are many different ways one can imagine superposing the trajectory of a quantum particle through a set of communication channels, each requiring different resources and achieving different advantages. On the one hand, this suggests the need for a thorough information-theoretic understanding of the resources in play, and a unified description of such protocols. One such approach is presented in Refs. \cite{31,39}. On the other hand, this can be viewed as an experimental task, wherein one wishes to classify and quantify implementation-based resources required for the realization of various types of superpositions and their corresponding advantages.

We take the experimental approach here, focusing on three different types of superpositions of trajectories which have been identified in the literature, namely, quantum-control of parallel channels (Fig. 1a), channels in series with quantum-controlled operations (Fig. 1b), and quantum-control of channel order (Fig. 1c). The common resource for the noise reduction in all the three schemes considered is the establishment of a coupling between the trajectories of the information carriers and the degree of freedom on which the noise acts. We experimentally apply these three schemes to various noise models. This enables us to examine the utility and trade-offs of these different types of superpositions in the goal of communicating through a pair of noisy channels.

In particular, in order to perform a comparative analysis of the performance of the three types of superpositions, we measure the coherent information (which is a lower bound for the quantum channel capacity) in the presence of XY, bit-flip, phase-flip and BB84-channels. We show that, within the paradigm of quantum interferometry, the use of channels in series with quantum-controlled operations generally outperforms or equals the other schemes in all the noise models which we consider. While here we study the three schemes individually in order to focus on the source of the coupling between the trajectory and the degree of freedom on which the noise acts, one could of course also combine the different types of superpositions (and, for instance, insert quantum-controlled operations also in the other two schemes), yielding different—potentially larger—advantages from those presented here.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{Combining two channels in a superposition of trajectories. A sender and a receiver communicate under the restriction that the information carrier must cross at least one noisy region. a) Quantum-Control of Parallel Channels. A quantum particle is placed in a quantum superposition of two trajectories, each branch containing a single noisy channel. b) Channels in Series with Quantum-Controlled Operations. Each of the branches of the superposition passes through the noisy channels in the same order, but different unitary operations are applied locally in each branch. c) Quantum-Control of Channel Order. The information carriers are routed through the two channels in different orders. This setup can achieve a genuinely indefinite order of the two channels. d) Classical Trajectories. Throughout this article, we will compare the three quantum superpositions of channels above to classical trajectories. In this regard, if one has access to classical-like trajectories only, one can send the photon through one or the another noisy regions with probabilities $q$ and $1-q$.}
\end{figure}
The rest of this paper is structured as follows. Section II introduces the three different architectures for the quantum superpositions of trajectories through two noisy channels, and summarizes their performance when applied to a simple noise model. Section III reviews the key figures of merit that we use to quantify the performance of our experimental quantum channels, i.e., the quantum capacity and the coherent information. Section IV outlines our experiments, and Section V presents the corresponding results. Finally, Section VI concludes.

II. QUANTUM SUPERPOSITIONS OF TRAJECTORIES

For simplicity, we will focus on two communication channels and two trajectories, as this already captures the key features of the general idea. All experiments hereafter discussed were performed using single photons, where the trajectory is naturally defined by the photon’s path. Quantum information is initially encoded in one of the internal degrees of freedom of the particle; in our case, in its polarization. Then, using linear optical elements, it is relatively easy to place a photon in a superposition of trajectories [21-24]. We will further consider, as does related work, that the noise acts only on the internal degree of freedom (DOF).

To introduce the basic idea, we will start by considering a particular noise model, which was studied for quantum-controlled orders in [30]. Given some single-qubit input state $\rho$ encoded in the internal DOF, the noisy process $\mathcal{C}$ either applies a Pauli-$X$ or -$Y$ operation to the internal state with equal probability:

$$C_{\text{EB}}(\rho) = \frac{1}{2}X \rho X + \frac{1}{2}Y \rho Y.$$  \hfill (1)

If the input to this process is a pure state $\psi_I = \alpha |0\rangle_I + \beta |1\rangle_I$ (where the subscript $I$ denotes the internal DOF), the output is, in general, a mixed state, with all coherence in the computational basis extinguished:

$$C_{\text{EB}}(\psi_I) = \begin{pmatrix} |\beta|^2 & 0 \\ 0 & |\alpha|^2 \end{pmatrix}.$$  \hfill (2)

This channel is an example of a so-called ‘entanglement-breaking’ (EB) channel, and, as such, it cannot be used to transmit any quantum information. One might, of course, still employ it to transmit classical information in the computational basis.

In a standard quantum communication scenario with a single trajectory, information, which is taken to be encoded in an internal DOF of an information carrier, must often propagate through multiple channels. Depending on the physical implementation, the channels can be linked together in different manners. With two channels and classical-like trajectories, the channels can either be put in series, or in a classical mixture of the two (depicted in Fig. [44])—more complex combinations can also be realized, but they all perform strictly worse than a classical mixture. If two copies of the channel of Eq. (1) are put in series, the result is a maximally-dephasing channel $\mathcal{C}(\rho) = \frac{1}{2} \rho + \frac{1}{2} Z \rho Z$, where $Z$ is the Pauli-Z matrix. This also destroys all coherence in the computational basis, and cannot transmit any quantum information. Similarly, placing two of these channels in a classical mixture will not allow the transmission of any quantum information.

In a typical single-trajectory quantum communication scenario, it can be shown that, if each channel is unable to transmit quantum information (i.e., its quantum capacity—to be defined later—is zero), then any combinations of the two channels should also result in a zero capacity channel. This is known as a bottleneck inequality [42]. In the following subsections, we will show that this is not the case when the trajectories are superposed in a quantum fashion. Thus, the bottleneck inequality does not directly apply to communication scenarios with quantum trajectories [29-41].

A. Quantum-Control of Parallel Channels

The first layout that we consider uses a quantum superposition of configurations where two channels are placed in parallel, and their use is controlled by a quantum system, as illustrated in Fig. [44]. This was originally introduced for error filtration [5], and it was more recently reviewed in the general framework of communication through superposed channels in Refs. [35-37] and [38-39]. In this scheme, different independent noisy channels are placed in each branch of the superposition. In Ref. [5], it was shown that by performing a measurement on the trajectory in a suitable basis, and then post-selecting, one can non-deterministically filter out errors in the communication channel. We will now consider an initial pure state encoded in the internal DOF $\psi_I = \alpha |0\rangle_I + \beta |1\rangle_I$, independent noisy channels realised by applying a Pauli-$X$ and -$Y$ with equal probabilities—as described previously and resulting in Eq. (1)—and two trajectories in an equal superposition $|\pm\rangle_T = (|0\rangle_T \pm |1\rangle_T)/\sqrt{2}$ (where $T$ refers to the trajectory DOF). It is then straightforward to calculate the output (the full calculation is presented in Section VII A), and to observe that performing a measurement on the trajectory DOF in the $\{|+\rangle_T,|\pm\rangle_T\}$ basis, and finding $|\pm\rangle_T = (|0\rangle_T - |1\rangle_T)/\sqrt{2}$ leaves the internal DOF in the pure state:

$$\beta |0\rangle_I - \alpha |1\rangle_I,$$  \hfill (3)

which can be unitarily rotated back to $\psi_I$. On the other hand, when the trajectory state is found to be $|+\rangle_T$, the output state is partially mixed:

$$\frac{|\beta|^2}{\alpha^2|\beta|^2 + \beta^2|\alpha|^2} |\beta\rangle_I - i \alpha |1\rangle_I.$$  \hfill (4)

This output state has a reduced purity, but it still maintains some coherence. Although this is not necessar-
ily the optimal measurement strategy or the best noise model to showcase this scheme, it illustrates that a quantum-controlled superposition of noisy channels allows some coherence to reach the receiver. Hence, the sender and the receiver can communicate some quantum information. We will quantify the amount of quantum information precisely in Section III. Communication advantages in this case have been attributed to the ability to quantum coherently control which channel to use [33].

This type of architecture is relatively easy to imagine deploying in practice. Most modern quantum communication takes place via optical fibers. As is often the case, these fibers can be noisy, resulting in a reduced ability to transmit information. Since a photon can easily be sent through a superposition of two (or more) fibers, the use of such parallel architectures could already improve security in existing communication networks.

B. Channels in Series with Quantum-Controlled Operations

At first glance, placing two noisy communication channels in series may not seem to be a promising solution to mitigate the noise they produce. However, by letting them be traversed by two trajectories in a superposition and by allowing different operations in each branch of the superposition, a significant noise reduction can in fact be accomplished. This channel architecture is presented in Fig. 1b. In each branch, the channels 1 and 2 are placed in the same order, and different unitary operations may be inserted. (Such unitary operations are labeled as $U_1$, $U_2$ and $U_3$ in Fig. 1b). In principle, however, more operations could be inserted along the trajectories). This scheme was originally presented in Ref. [31], where it was referred to as a ‘superposition of direct pure processes’.

Let us now consider the action of the superposition of trajectories in series with the noise model of Eq. (1), setting, following the notation of Fig. 1b), $U_1 = Y$, $U_2 = I$, $U_3 = I$ (I being the identity operator). We will again consider the initial state of the system to be $|\psi_I\rangle_T|+\rangle_T$. This time, we will imagine performing a measurement in the computational basis on the qubit stored in the internal DOF. As we show in Section VII A, finding the internal qubit in $|0\rangle_I$ projects the trajectory state into $|\psi\rangle_T$, while finding it in $|1\rangle_I$ projects the trajectory state into $X|\psi\rangle_T$. Hence, this superposition of trajectories perfectly filters out the noise arising from the noisy channels.

It is easy to imagine the implementation of this scheme in a real-world scenario. The two paths (e.g., optical fibers) are simply sent through a few noisy transmission channels in series. (For the scheme to work, the action of each noisy channel must be correlated along the different paths.) Since the two paths are physically distinct, the different unitary operations can easily be applied in each branch of the superposition independently. Such operations can be performed with simple linear optical elements, or even directly using calibrated optical fibers, which always implement some unitary polarization rotation. In Ref. [31] it was also pointed out that, by superposing more than two trajectories, one can perfectly compensate for any arbitrary noise.

C. Quantum-Control of Channel Order

The original source of inspiration for this architecture is the quantum switch [7], a higher-order operation which takes quantum gates and applies them in a quantum superposition of alternative orders. Within quantum-interferometry, a quantum-optical switch exploiting superposition of trajectories in flat space-time has been proposed [33,45], and experimentally demonstrated [17,21-24]. For two quantum operations, this is a quantum process in which a particle is placed in a superposition of two paths, each of which is routed through the two quantum operations in alternative orders (see Fig. 1b)). This scheme features all the necessary requirements for an advantage in quantum information processing over standard channels [25,37], and it can be provably characterized as a causally-indefinite process [22,24,46].

Applying the quantum switch to two copies of the channel in Eq. (1), one finds that the output state is [30]

$$\frac{1}{2} |\psi\rangle_1 \langle \psi| \otimes |+\rangle_T + |+\rangle_1 \langle +| \otimes Z |\psi\rangle_1 \langle \psi| Z \otimes |−\rangle_T \langle −|_T. \quad (5)$$

Analogously to the previous two examples, we will now measure the trajectory in the $\{|+\rangle_T, |−\rangle_T\}$ basis. If the outcome is $|+\rangle_T$, the state has been transmitted perfectly, whereas if one finds $|−\rangle_T$, a simple phase correction is required to exactly restore the initial state.

The resources required to implement the quantum-optical switch in the laboratory are relatively minimal, it simply requires linear optical elements to route the photon through the two noisy channels in a superposition of their orders. However, in order to be effective, this layout requires the action of the two noisy channels on the photon to be suitably correlated both in space and time (as in Fig. 3 of Ref. [40]). Instead, in standard communication networks [50], the noisy regions are usually localized in space and fixed in time. Any such network would thus require the photon to travel back toward the sender to enter the second channel, and this scheme requires this return trip to occur without traversing any further noisy region (which could happen if the two channels introduce noise in the direction from the sender to the receiver, but not vice versa).

D. Comparison

In all of the three schemes above, a particle is placed in a quantum superposition of two trajectories which are...
then routed through various devices and noisy communication channels. All three methods result in a coupling of the internal state to the state of the trajectory, and the two trajectories must maintain coherence in order to show a communication advantage. (Practically, this is required since the trajectory is measured in a superposition basis.)

This coupling to the trajectory DOF is a necessary requirement to achieve any advantages. In the parallel and indefinite order layouts the channels themselves or the routing through their different orders give rise to the coupling, whereas in the series scheme this coupling is aided by the quantum-controlled operations. In light of this, it has been proposed that the quantum-controlled operations used in the superpositions of channels in series (Fig. 1b) should be considered as additional resources (referred to as ‘encoding’, ‘decoding’ and ‘repeaters’ in Refs. 31, 39), as they can couple the internal DOF to the trajectory independently of the choice of noisy channels. However, these operations do not require any additional resources beyond the transmission lines themselves (for example, polarization rotations can arise from the mere twisting of optical fibers, and are effectively unavoidable), which are the same experimental resources used for the other two schemes.

It was also noted that the number of noisy channels traversed by the particle in each branch of the superposition differs between the three schemes 31, 39, 50, 51: the quantum-control of parallel channels contains only one channel in each interferometer arm, whereas the other two schemes contain two channels per arm. Clearly, when the information carrier traverses more noisy channels, the overall noise is higher. However, although the quantum-control of channel order needs at least two channels to create the required coupling between the trajectory and the internal DOF, it is still able to overcome the additional noise caused by the multiple noisy channels. On the other hand, when the channels are used in series with quantum-controlled operations, no specific number of noisy channels is actually required as the coupling arises from quantum-controlled operations and not from the channels themselves. Hence, this scheme can be implemented with any number of channels. It is, therefore, also easy to imagine adding quantum-controlled operations to either of the other two schemes, which could result in further communication advantages. (As mentioned above, we do not study such combinations here in order to focus on the physics of the individual schemes.)

Next, we will quantify the amount of quantum information which can be transmitted using these various schemes individually for different noise models. Overall, we find that, for all the types of noise considered, the use of channels in series with quantum-controlled operations exceeds or equals the performance of the quantum-control of parallel channels and quantum-control of channel order.

III. QUANTIFYING CHANNEL PERFORMANCE

In order to rigorously compare the ability of the different schemes to transmit quantum information, an experimentally accessible figure of merit is necessary. The quantum capacity \(Q(C)\) of a channel \(C\) is the number of qubits that are transmitted for each use of that channel 52, 53. In general, this is a rather complex function that can be difficult to even theoretically assess. However, it is lower bounded by 52:

\[
Q(C) \geq \max_{\rho_{AB}} I_c(C, \rho_{AB}),
\]

where \(I_c\) is the coherent information 54 of the channel with respect to \(\rho_{AB}\), which is defined as

\[
I_c(C, \rho_{AB}) := S(\rho_B') - S(\rho_{AB}'),
\]

where \(\rho_{AB}\) is a bipartite state, \(\rho_{AB}' := (I_A \otimes C)(\rho_{AB})\) is the output state obtained by applying channel \(C\) on system \(B\), \(\rho_B' := \text{Tr}_A[\rho_{AB}']\) is its marginal state, and \(S(\rho) := -\text{Tr}[\rho \log \rho]\) is the von Neumann entropy.

Let us briefly consider a few simple examples, assuming a 2-qubit system, with \(\rho_{AB}\) set to a maximally-entangled pure state. If the channel is unitary (i.e., noiseless), then the initially pure state remains pure after the application of the channel, thus \(S(C(\rho_{AB})) = 0\). However, since \(\rho_{AB}\) is maximally entangled, tracing out the subsystem \(A\) will leave the subsystem \(B\) in a maximally mixed one-qubit state with entropy \(S(C(\rho_B')) = 1\). Therefore the coherent information of a unitary channel, with respect to a maximally entangled probe state is 1. If, on the other hand, the channel induces decoherence, the entangled probe state will become mixed, and the second term will increase: \(S(C(\rho_{AB}')) > 0\). Because the first term cannot be larger than 1, as decoherence is induced the coherent information decreases. \(I_c(C, \rho_{AB})\) is often maximized when \(\rho_{AB}\) is a maximally-entangled state (this was proven to be the case for the quantum switch and a specific noisy model in 29). In any case, the evaluation of \(I_c\) for any arbitrary state sets a lower bound for the quantum capacity of the channel. Throughout the rest of this manuscript, when we refer to the coherent information, we do so with reference to a maximally-entangled Bell state.

One could consider estimating \(I_c(C, \rho_{AB})\) directly by probing the channel with an entangled state 55. In this case, however, the trade-off is that this state will be more prone to errors in the preparation phase (and such errors are to be considered in addition to all the others already mentioned). Thus, our experimental approach will be to first perform quantum process tomography on the superposition of communication channels. With the resulting estimate of the experimental channels, we will then be able to compute \(I_c\) with ideal maximally-entangled states, and will use this metric to quantify the performance of the various schemes.
In our experiment, we study single-qubit channels acting on the polarization DOF, and equal superpositions of trajectories. Since, as we have seen previously, the coherence between the two trajectories is crucial, both the internal DOF and the trajectory must be fully characterized. In general, this requires two-qubit process tomography on the path (trajectory) qubit and the polarization qubit. Notice that the sender only ever encodes information in the polarization DOF, whereas the receiver must measure both the trajectory and the polarization DOFs. Hence, this is effectively a 1-to-2 qubit channel. Because of this, performing full two-qubit process tomography provides more information than is strictly required.

IV. EXPERIMENT

Implementing Noisy Channels—In our experiment, we encode and transmit information in the internal polarization DOF. We induce noise on this DOF using liquid crystal waveplates (LCWP), which can rapidly implement different polarization rotations to effectively decohere the polarization state in a precise and controllable manner. The LCWP retardance can be changed between 0 rad and 2π rad in approximately 100 ms by varying the applied voltage (see Section VII B for more details). Using these fast LCWPs we can change the operations on-the-fly to actively decohere the photon’s polarization, in contrast to previous experiments wherein decoherence was achieved by averaging the results during the data analysis. Nevertheless, the two methods yield the same results, so we will make use of both techniques interchangeably. Physically, the noise models we study can be understood as randomly applying one of four operations (Z, X, Y, or Z) to each photon. The probability of each operation to occur defines the noise channel.

Specifically, we implement four different noisy channels. The first is a generalization of the entanglement-breaking channel C_{EB}(ρ) discussed above. However, in this general case, the X and Y operations are applied with probability 1 − p and p, respectively (one recovers the C_{EB}(ρ) for p = 1/2):

\[ C^p_{XY}(ρ) = (1 − p)XρX + pYρY. \]  

We also study the well-known bit-flip (BF) C_{BF}(ρ) and phase-flip (PF) C_{PF}(ρ) (or dephasing) channels:

\[ C_{BF}(ρ) = (1 − p)ρ + pXρX, \]  
\[ C_{PF}(ρ) = (1 − p)ρ + pZρZ, \]

respectively. Finally we study a depolarizing channel C_{BB84}(ρ), known as the BB84-channel:

\[ C^p_{BB84}(ρ) = (1 − p)^2ρ + (1 − p)pXρX + (1 − p)pZρZ + p^2YρY. \]

For the BB84-channel, when the noise probability is p = 0.5 the channel is completely depolarizing, mapping any input to the maximally-mixed state.

To realize a single channel we use two LCWPs. The first LCWP’s optic axis is set to 0°, and can thus implement either Z or the identity operation by setting the retardance to π rad or 0 rad, respectively. The second LCWP’s optic axis is set to 22.5° to execute X or the identity operation, again by setting the retardance to π rad or 0 rad, respectively. When the first LCWP performs Z and the second X, the net result is Y (up to a phase). Hence, with these two LCWPs we are able to carry out all four required unitary operations, and switch between them in about 100 ms.

In light of this, a straightforward implementation would be to generate a random number from some defined distribution before a photon enters the channel, and then to set the operations accordingly. However, the net result is the same if we allow several photons to pass through the channel for each coin flip, as this allows us to increase the single-photon count rate well above the switching speed of the LCWPs. In our experiment, we employ two different methods for the data acquisition. In the first, we change the applied operation every second. Since our photon rate (detected at the output, after the experiment) is of the order of 3000 Hz, this means that approximately 3000 subsequent heralded photons experience the same unitary operation (see Section VII C for more details). Our Monte Carlo simulations show that, with these numbers, 100 seconds (and 100 different operations) per measurement setting are sufficient to achieve a process fidelity (i.e., the fidelity to achieve the desired noisy channel) above 99% (for details, see Section VII D). In order to ensure an optimal implementation while maintaining a reasonable duration of the data-taking procedure, we used 1000 different internal configurations for our experiment, resulting in a fidelity of 99.98% per channel. In the second technique, we simply take data for each input state and each measurement setting with the LCWPs set to implement a fixed unitary operation. We then weight the data from these different configurations according to the probability distribution of the desired noise model. (This method was also demonstrated in.)

Creating Superpositions of Trajectories—As shown in Fig. 2, we experimentally create different superpositions of trajectories by placing single photons in an equal quantum superposition of paths using a 50/50 beam splitter. The single photons are generated with a standard type-II down-conversion source described in Fig. 2 and in Section VII E. These two paths (trajectories) are then routed through a series of LCWPs, which implement different noisy channels, in a parallel configuration (Panel a), in series (Panel b), or in a quantum superposition of the two alternative orders (Panel c).

All three set-ups are realised through Mach-Zehnder
A) Quantum-Control of Parallel Channels.

The interferometers in the first case (Fig. 2a)), one channel is placed in each interferometer arm. In the second case (Fig. 2b)), the channels are arranged in series in both arms of the interferometer, and additional operations are performed before each channel through waveplates. Finally, the third scheme (Fig. 2c)) is accomplished using a folded Mach-Zehnder interferometer where the two channels appear in alternating order in each of the interferometer’s arms. The setup presented in Fig. 2d) represents a possible realization of a quantum-optical switch wherein the system qubit is encoded in the polarization DOF, and the control qubit in the path DOF. Other encodings for this type of process have been proposed [14, 58] and experimentally demonstrated [24, 26].

Regardless of the detailed implementation, all proposals to implement a quantum-optical switch use one DOF to route a photon through channels in different orders, while the channels act on some other DOF.

In order to perform quantum process tomography to extract the coherent information, we must prepare a tomographically-complete set of input states, and measure in a tomographically-complete number of different bases. In brief, we use waveplates before the first beamsplitter to prepare the state of the polarization qubit in a tomographically-complete set of input states, and measure in all bases (i.e., $|0⟩_i, |1⟩_i, |+⟩_i, |−⟩_i, |R⟩_i$, or $|L⟩_i$) (where $|R⟩_i = (|0⟩ - i|1⟩)/\sqrt{2}$, $|L⟩_i = (|0⟩ + i|1⟩)/\sqrt{2}$), and waveplates and polarizing beamsplitters after the second beamsplitter to measure in all bases $|0⟩, |1⟩, |+⟩, |−⟩$, and

FIG. 2: Experimental Setup. a) Quantum-Control of Parallel Channels. After their polarization is set via a half waveplate (HWP) and a quarter waveplate (QWP), single photons are injected into a Mach-Zehnder interferometer. One noisy channel is placed into each arm of the interferometer, and each channel is realized through two liquid crystal waveplates (LCWP), the first positioned at 0° (to implement $I$ or $Z$ by changing the retardance), the second at 22.5° ($I$ or $X$). By means of a piezo-electric trombone delay line, the photon interfering on the second beamsplitter of the interferometer can be projected onto the bases $\{|+,+\rangle, |-,+\rangle\}$ or $\{|R⟩, |L⟩\}$ of the trajectory. Finally, the photons’ polarization is measured through a sequence of QWP, HWP and a polarizing beamsplitter. b) Channels in Series with Quantum-Controlled Operations. As in the previous scheme, the photons are prepared in polarization via QWP and HWP and injected into a Mach-Zehnder interferometer. In this case, the two noisy channels are placed in the two superposed branches in series with the same order. Also in this case, the channels are realized through LCWP’s. Furthermore, before each noisy channel, additional unitary operations are realized through sequences of QWP, HWP and QWP (before the first channel, the QWP, HWP and QWP are placed in one branch of the trajectory only, whereas between the two channels the waveplates are in both branches, since we only implement cases where $U_2 = U_3$). The rest of the setup is the same as in the previous case. c) Quantum-Control of Channel Order. The preparation and measurement of the photons in polarization happens as in the previous schemes, as well as the realization of the noisy channels, and the projection of the trajectory DOF. In this case, however, the Mach-Zehnder interferometer is folded into two loops so that the photon can travel through the two channels in the two alternative orders in each arm of the interferometer. d) Heralded single-photon source. We generate photon pairs using a type-II spontaneous-parametric-down-conversion source. One photon is directly detected with an avalanche photodiode (upper arm), whereas the other is coupled into an optical fiber and sent to one of the setups a), b), or c). The interferometers in setups a), b), and c) all contain two compensation HWP at the beginning and at the end of the reflected arm, so as to compensate for the phase shifts due to the reflection from the beamsplitter.
V. RESULTS

XY-Channel—Below, we present our results for the three combinations of the noisy channels described in equations (8)–(10). We will first consider two copies of the XY-channel (Eq. (8)). In Section II, we observed that when $p = 0.5$ both the channels in series with quantum-controlled operations and the quantum-control of channel order (with $U_1 = Y$, $U_2 = U_3 = I$) are able to transmit quantum information perfectly. Below, we observe that such a perfect ‘activation’ (in our case, the term refers to a combination of two noisy channels which enables one to communicate through such a combination with less noise than either individually) is verified for all values of $p$. In fact, the purple and brown lines in Fig. 3 show the coherent information for two XY-channels combined in series and in indefinite order, respectively. For both of these situations, the theoretical coherent information is equal to 1 for all $p$, meaning that one qubit per use can be transferred. In the same plot, our experimental data are presented as squares (for the quantum-control of parallel channels), circles (for the channels in series with quantum-controlled operations) and crosses (for the quantum-control of channel order) with matching colors. The dominant source of the statistical errors is the uncertainty in determining the initial states for the process tomography. (In fact, the input states were prepared and characterized at the output of the source, but they were then sent to each experiment via 3m long optical fibers, which introduced additional noise.) This uncertainty leads to the error bars on all the data sets presented in Figs. 3–5 (see Section VII C for more details on error estimation). Nevertheless, all plots display a good agreement between experiment and theory. As expected, the experimentally measured coherent information is slightly lower than that predicted theoretically. This offset is mainly due to the following systematic errors: i. the imperfect visibility when the two trajectories are recombined on the second beamsplitter, ii. phase drifts which can occur during the experimental runs, and iii. slight calibration errors in the LCWPs implementing the channels and the waveplates used for state preparation and measurement. These systematic effects are not included in the calculation of our experimental errors. Full details of the measurement procedure, including photon count rates and measurement times, as well as the statistical and systematic errors affecting the data are presented in Section VII C.

The orange data set reported in Fig. 3 corresponds to the coherent information when the two XY-channels are used in a quantum-controlled superposition. In Section II, we illustrated that, when $p = 0.5$, the output still displays a partial dependence on the input state. However, calculating the coherent information reveals that this is not sufficient to transmit a single qubit per use (i.e., the coherent information is less than 1). Nevertheless, the orange curve indicates that quantum information can still be transmitted, although not at the maximum rate.

The turquoise curve in Fig. 3 represents the coherent information of a single trajectory traversing a sin-
Coherent Information

we consider two different types of noisy channels $C_1, C_2$, (Eq. (9b)). Note that, contrary to the other cases, here $C_3$ (Eq. (9a)), and channel 2 with the phase-flip channel responds to replacing channel 1 with the bit-flip channel individually. (Referring to Fig. 1, this corresponds to replacing channel 1 with the bit-flip channel (Eq. 9a), and channel 2 with the phase-flip channel (Eq. 9b)). Note that, contrary to the other cases, here we consider two different types of noisy channels $C_1, C_2$, rather than two copies of the same channel). In light of this, Ref. [11] pointed out that this idea can also be applied when the noisy channels are placed in series, provided that one allows quantum-controlled operations before and between them, and that this trick allows one to transmit quantum information perfectly (when $U_1 = Y$, $U_2 = U_3 = H$, where $H$ is the Hadamard operation).

We experimentally confirm the predictions of Refs. [29, 31] in Fig. 4. There, we see that, regardless of the noise strength, the channels in series with quantum-controlled operations can, in theory, perfectly transmit quantum information (i.e., the purple line is equal to 1). Our experimental data (purple circles) confirm this, although they do show a slight offset due to the systematic errors discussed above. In this case, the quantum-control of channel order (brown curve for theory, and crosses for experiment) does not work as well. Nonetheless, we do find that for a range of $p$ it outperforms the single use value $1 - H(p)$. For this choice of noisy channels, the quantum-control of parallel channels (orange curve) can transmit more information than their quantum-controlled order. For a large range of $p$, it is larger than the value achievable through the quantum-control of channel order, and the slight theoretical advantage of this latter over the quantum-control of parallel channels for large enough values of $p$ is not observable in our experimental data.

For a fair comparison, we mention that changing the quantum-controlled operations $U_i$ depending on the type of noise could be regarded as an additional resource. In fact, setting the optimal quantum-controlled operations requires one to characterize the noise prior to using the channels. In Section VII G, we compare the performance of the channels in series with quantum-controlled operations for the same unitaries that we use for the $XY$- and BB84-channels (namely, for $U_1 = Y$, $U_2 = U_3 = I$). There, we observe that setting $U_2 = U_3 = I$ results in a performance that is comparable to that of the quantum-control of channel order, and which still outperforms the single-use capacity. In doing so, the quantum-controlled operations remain fixed in this configuration independently of the type of noise.

**BB84-Channel**— As a final example, we consider two copies of the depolarizing BB84-channel (Eq. 10). These results are shown in Fig. 5. Also in this case, the channels in series with quantum-controlled operations (this time with $U_1 = Y$, $U_2 = U_3 = I$), shown in purple, achieves the largest enhancement. While with only two trajectories it is not possible to perfectly transmit quantum information through these noisy channels, Ref. [11] showed that with additional trajectories any type of noise can be perfectly corrected with the quantum superposition of channels in series. The quantum-control of channel order in this case outperforms both the single-use coherent information ($1 - 2H(p)$) and the coherent information of the quantum-control of parallel channels.

These three examples show that, depending on the type of noise, different superpositions of channels can
lead to the ability to transmit different amounts of quantum information. In all the cases we investigated here (wherein the schemes are used individually), even in the presence of experimental imperfections, using the channels in series with quantum-controlled operations appears to be the best candidate to evade the effects of the noise.

VI. DISCUSSION

In this work, we experimentally explored how the degradation of quantum information due to its propagation through noisy channels can be mitigated, and in several cases fully suppressed. This was achieved by sending quantum information carriers through a pair of noisy channels in various superpositions of trajectories. In particular, we studied three types of schemes: the quantum-control of parallel noisy channels, channels in series with quantum-controlled operations, and the quantum-control of channel order.

All of these schemes bear much in common with error filtration [29]. More recently, this has been refined in a number of theoretical works [31, 38, 39, 41, 51, 59], tied into the concept of indefinite causal orders. While enhanced communication based on an indefinite causal order has been experimentally demonstrated [25, 37], an experimental study comparing different superpositions of trajectories in presence of various types of noise has been lacking. Our work is aimed at bridging this gap, by suggesting common ground based on the experimental resources that each of the analysed schemes require.

Our results suggest that, in most quantum-optical communication scenarios, creating a superposition of trajectories through channels in series with quantum-controlled operations should lead to the largest noise reduction. One can easily imagine characterizing the error introduced in various communication channels, and from there setting the unitary operations accordingly. Moreover, Ref. [41] illustrated that these types of schemes can be extended to superpositions of more than two trajectories to achieve complete error cancellation for any type of noise. We have shown experimentally that with only two trajectories it is already possible to completely cancel (after accounting for experimental errors) all the noise arising from two out of the three types of noisy channels we considered. Furthermore, the quantum-controlled operations could also be introduced in the other two schemes and, potentially, they could match the performance of the layout with channels in series.

The large experimental communication enhancements presented here highlight the practical relevance of extending the quantum communication paradigm to scenarios in which not only the information carriers, but also the trajectories along which they propagate are quantum. We expect that the relative ease of implementation of these schemes will enable them to be readily put into practice for the noise-reduction of real-world long-distance quantum communication applications.

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VII. METHODS

A. Case Study: Activation of the EB-Channels in the three Layouts

In this section, we briefly evaluate the output state of the three superposition techniques for the noisy channel described by Eq. (1). We carry out our study by interpreting the action of the two channels as follows. Each channel randomly applies either $X$ or $Y$ with probability 1/2. Hence, the ‘internal configuration’ of the superposition can be understood as either $i$, both channels 1 and 2 implement $X$, $ii$. channel 1 implements $X$, while channel 2 implements $Y$, $iii$. channel 1 implements $Y$, whereas channel 2 implements $X$, or $iv$. both channels 1 and 2 implement $Y$. The final output state will then be a mixture of the output states in these four configurations, each with probability 1/4.

Throughout this section, we will assume that the input state is $|ψ⟩_1 |+⟩_T$, where $|ψ⟩ = α |0⟩ + β |1⟩$, and $I$ ($T$) labels the internal (trajectory) DOF.

Quantum-Control of Parallel Channels— In this scheme, one noisy channel is placed in each trajectory. The action of the two channels in parallel can be interpreted as creating a mixture of the following four unnormalised states at the output:

\[
\begin{align*}
X |ψ⟩_1 |0⟩_T + X |ψ⟩_1 |1⟩_T \\
X |ψ⟩_1 |0⟩_T + Y |ψ⟩_1 |1⟩_T \\
Y |ψ⟩_1 |0⟩_T + X |ψ⟩_1 |1⟩_T \\
Y |ψ⟩_1 |0⟩_T + Y |ψ⟩_1 |1⟩_T ,
\end{align*}
\]

(11a) (11b) (11c) (11d)

where the four states correspond to internal configurations of $X - X$, $X - Y$, $Y - X$, and $Y - Y$, for channel 1 and channel 2, respectively. These states can be rewritten (up to further normalisation) as:

\[
\begin{align*}
X |ψ⟩_1 |+⟩_T \\
(X |ψ⟩_1 + Y |ψ⟩_1 ) |+⟩_T + (X |ψ⟩_1 - Y |ψ⟩_1 ) |−⟩_T \\
(X |ψ⟩_1 + Y |ψ⟩_1 ) |+⟩_T - (X |ψ⟩_1 - Y |ψ⟩_1 ) |−⟩_T \\
Y |ψ⟩_1 |+⟩_T .
\end{align*}
\]

(12a) (12b) (12c) (12d)

Now, measuring the trajectory DOF in the $|+⟩_T, |−⟩_T$ basis and obtaining $|−⟩_T$ (which happens with probability 1/4) projects the internal DOF into $X |ψ⟩_1 - Y |ψ⟩_1$, which can be rewritten (after renormalisation and up to an irrelevant global phase) as in Eq. (3). This is a pure state, which implies that some ability to transmit quantum information has been restored in post-selection. If, on the other hand, one obtains the result $|+⟩_T$ (with probability 3/4), it is straightforward to show that the internal DOF is projected in the mixed state described by Eq. (4).

In the recent papers on superpositions of trajectories [38], it was shown that the output of a quantum-controlled superposition of two channels depends on additional parameters related to the physical realisation of the channels (‘transformation matrices’ in [38] and ‘vacuum amplitudes’ in [39]). In our scheme, these additional parameters reduce to the relative phase between the vacuum and the single-photon subspace of the unitary operations (e.g., the Pauli-$X$ and -$Y$ from above, with transformation matrix $T = (X + Y)/2$). More precisely, the vacuum extension of a qubit unitary $U$ is $U' = e^{iφ} |\text{vacuum}\rangle |\text{vacuum}⟩ + U$, where $U$ acts in the single-photon subspace. In the calculation above, the phase is implicitly set to zero, which is in agreement with our experiment.

Channels in Series with Quantum-Controlled Operations— Let us now consider the action of the superposition of trajectories in series with the quantum-controlled operations (Fig. 11), with $U_1 = Y$, $U_2 = I$, $U_3 = I$. In this case, the input state is transformed into $(|ψ⟩_1 |0⟩_T + Y |ψ⟩_1 |1⟩_T)/\sqrt{2}$ before interacting with the noisy channels.

Again, we can compute the four effective unnormalised states which arise from the different internal configurations of the noisy channels:

\[
\begin{align*}
XX |ψ⟩_1 |0⟩_T + XXY |ψ⟩_1 |1⟩_T \\
YX |ψ⟩_1 |0⟩_T + YXY |ψ⟩_1 |1⟩_T \\
XY |ψ⟩_1 |0⟩_T + XYY |ψ⟩_1 |1⟩_T \\
YY |ψ⟩_1 |0⟩_T + YYY |ψ⟩_1 |1⟩_T .
\end{align*}
\]

(13a) (13b) (13c) (13d)

The order of the above states refers to internal configurations $X - X$, $X - Y$, $Y - X$, and $Y - Y$. These states can be rewritten (up to phases) as:

\[
\begin{align*}
|0⟩_1 |ψ'⟩_T + i |1⟩_1 X |ψ'⟩_T \\
|0⟩_1 |ψ'⟩_T - i |1⟩_1 X |ψ'⟩_T \\
|0⟩_1 |ψ'⟩_T + i |1⟩_1 X |ψ'⟩_T \\
|0⟩_1 |ψ'⟩_T + i |1⟩_1 X |ψ'⟩_T ,
\end{align*}
\]

(14a) (14b) (14c) (14d)

where $|ψ'⟩ = α |0⟩ - iβ |1⟩$. As a result, we see that measuring the internal DOF in the computational basis $\{|0⟩_1, |1⟩_1\}$ projects the trajectory into either $|ψ'⟩_T$ or $X |ψ'⟩_T$, upon obtaining outcomes $|0⟩_1$ or $|1⟩_1$, respectively (each with equal probabilities). Both of these states can be unitarily corrected, allowing one to achieve perfect quantum information transfer through these channels. (Even though here, for simplicity, we restricted ourselves to the case $p = 1/2$, the same reasoning applies whatever the mixing probability $p$ in the definition of the channels, Eq. (8)).
Liquid Crystal Variable Retarders. Their beam deviation is estimated to 2 arc min, their reflectance (per surface) is 0.5%, and their surface quality is 40-20 scratch and dig.

C. Data Acquisition and Error Estimation

Below, we briefly outline the details of the data acquisition and the error estimation in our experiment.

As discussed above, we followed two methods to experimentally construct the noisy channels. In the first, we realized the noise within each channel by generating random numbers in the range [0, 1]. Based on this number, and on the type of noise we wanted to realize (Eqs. (8)–(10)), we assigned a unitary operation from the set \{Z, X, Y, Z\}. In order to ensure a high fidelity of the noise channel (> 99%), we repeated this procedure 1000 times, measuring each configuration for 1s, and integrating the data taking procedure over these 1000 runs. In the second method, we measured all the possible combinations of unitary operations between the two noisy channels 1 and 2, and we then created the desired noise during our data analysis, following the procedure proposed in Ref. [25]. The first method was used to create the noisy channels in the indefinite order channel layout, while the second method was used for all other layouts. We did not observe any significant difference in the performance of the two methods (provided that we applied enough random unitary operations, see Section III D). However, the first method required several days of measurement, the second less than an hour. Since full QPT was not required for the indefinite order arrangement, we only used the first method for these data, and used the second method for all of the remaining channel configurations.

We collect \( \approx 23000 \) entangled photon pairs per second directly from our source. Of these pairs, we selected only one separable polarization component (i.e., \(|H, V\rangle\)), halving the count rate. Finally, the photons were sent through optical fibers to the different experiments. Because of experimental imperfections due to the non-zero reflectivity of the various optical elements, the non-ideal fiber coupling, and the optical fiber’s losses (the distance to travel in optical fiber between the source and the various experiments is about 3m), approximately 3000 photons per second were detected at the end of the experiment.

Finally, because of the long measurement times (particularly, in the case of the physical implementation of the noise in the channels), we observed phase drifts in the two arms of the interferometer. In order to correct these drifts, so as to ensure that we always prepared and projected the desired path qubit states, we actively stabilized the interferometer by means of the delay line controlled by a piezo-actuator. We measured and reset the phase every 20 minutes (which, according to our tests, ensured phase drifts below 1%). Given these count rates,
it follows that we measured about $3 \times 10^6$ photons for each internal configuration in case of physical implementation of the noisy channels, and about 3000 counts for each internal configuration in case of implementation of the noisy channels during data analysis.

The aforementioned imperfections in the path qubit had various consequences in our experiment. First, the phase drift on this qubit caused an uncertainty on the input state, since the phase of the path qubit can fluctuate over time. Moreover, if the phase drifts during the experiment, the purity of the input state can be reduced. In light of this, and of the high number of accumulated counts, the main statistical error in our experiment was related to the input state used for QPT. Therefore, to calculate all our experimental error bars, we determined the input state as follows. We performed quantum tomography of the quantum state directly on the path and polarization qubits in absence of ‘internal operations’ (i.e., setting to identity all the optical components meant to implement the noisy channels later on). We then observed the variation of this state as a function of time, and used this variation to analyze our data for a ‘worst-case’ state, a ‘best-case’ state, and a ‘most-likely’ state. This spread in the input states led to the error bars and the data-points presented in all of our experimental data.

The second main cause of experimental errors in the path qubit arises from the imperfect visibility of the Mach-Zehnder interferometers used to measure it. This reduced visibility, $\approx 0.93 - 0.96$ (depending on the amount of optical elements in each interferometer arm) essentially corresponds to a slight loss of coherence in the two trajectories and, therefore, to a decrease in the ability of the trajectories to restore quantum information. This leads to a systematic offset in our data. Likewise, a slight miscalibration of our waveplates and liquid crystal waveplates may have occurred, this would lead to further systematic errors. These systematic errors have not been included in our error bar calculation. We note, in particular, that a considerably reduced visibility of the interferometer may constitute the greatest experimental challenge in the application of our techniques to real-world quantum communication.

D. Fidelity of Channel Implementation

As described in the main text, we implemented the noisy channels in two different ways. In this section, we will discuss the first method, wherein we randomly apply either a Pauli-X, -Y, -Z or the identity operation for one second of our data acquisition time. The probability of each operation is given by the type of noisy channel we wish to implement (i.e., by one of Eqs. [8]–[10]). The natural question is how many operations must we average over to ensure a faithful implementation of the noisy channels.

To answer this, we used Monte Carlo simulations to study the average ‘process fidelity’ $F_{av}$ as a function of the number of applied operations. We computed the average process fidelity (defined in Ref. [20]) by i. randomly generating 10000 single qubit states from the Haar measure, ii. computing the ideal output state $\rho_{id}$ using Eqs. [8]–[10], iii. simulating the output by applying $N$ randomly chosen operations $\rho_{sim}$, and then iv. computing the average fidelity between $\rho_{id}$ and $\rho_{sim}$ for all $N$ input states. Already for $N = 25$, the average process fidelity is larger than 0.99. As an example, a plot of the average ‘process infidelity’ $(1 - F_{av})$ for the BB84-channel with $p = 0.5$ is shown in Fig. 7 (For the infidelity, a value of 0 indicates a perfect implementation.) We chose this as a representative example since the BB84-channel takes a slightly longer time to converge than all the others (this is because the BB84-process randomly applies one of the 4 operations, while all the others only choose among 2 operations). So, the case shown in Fig. 7 represents the worst case among all the ones studied. Nevertheless, even such a channel converges to the ideal noisy channel quite rapidly with $N$. Finally, since we always implement two channels simultaneously, we apply 1000 different operations, which is far beyond this limit.

E. Single-Photon Source

A CW laser centered at 392nm emits the pump beam for a source producing single photons through a process of type-II spontaneous-parametric-down-conversion. The pump beam traverses a focusing lens with $f = 12.5$cm, and then reaches, at the proper distance, a 3mm-thick beta-barium borate (BBO) crystal. Within the crystal, single photons are generated at a wavelength centred at 784nm. To compensate for the spatial and tempo-
eral walk-off of the resulting single photon pairs, they are sent each through a BBO crystal of 1.5mm thickness. They are finally filtered in polarization through a long-pass filter, and a bandpass filter centered at 785nm with a full-weight-half-maximum of 10nm. The photon pairs rate is 23000/s with a pump power of 85mW.

F. Quantum Process Tomography

Our experimental measurements consist, in general, of performing two-qubit quantum process tomography (QPT) on a path and a polarization qubit. Basically, QPT requires two steps, i. preparing the system in a tomographically-complete set of states before the process, and ii. measuring the system in a complete basis set after the process. For the polarization qubit, this is relatively straightforward. In fact, in all of three superposition methods outlined in Fig. 2, the photons enter the experiment in a single path. At this point a QWP and a HWP are inserted, which allow us to prepare any single-qubit polarization state. After this, the path qubit is prepared by a 50/50 beamsplitter in a quantum superposition of two paths. After the noisy channels, the paths are recombined by another 50/50 beamsplitter. On each of the output paths we place a QWP, a HWP and a polarizing beamsplitter to implement the polarization measurements. Although they are physically different elements, we ensure that the waveplates in each output arm are always set to the same angle, and hence perform the same measurement.

Even though, in all of the communication schemes presented in the main text, the trajectory is simply initialized in an equal superposition (i.e., the path qubit starts out in $|+\rangle_T$, one must prepare this qubit in a complete set of states in order to perform QPT. In order to change the input state of the path qubit between $|+\rangle_T$, $|R\rangle_T = (|0\rangle_T - |1\rangle_T)/\sqrt{2}$, and $|L\rangle_T = (|0\rangle_T + |1\rangle_T)/\sqrt{2}$, we set the relative phase between the two trajectories after the first beamsplitter using a delay stage mounted on a calibrated piezo-actuator. We can also easily prepare $|0\rangle_T$ and $|1\rangle_T$ by blocking either path. Analogously, we measure the path qubit in two different ways. To measure in $\{|+\rangle_T, |−\rangle_T\}$, or $\{|R\rangle_T, |L\rangle_T\}$, we suitably set the relative phase between the two paths before recombining them at the second beamsplitter. We use the same delay stage to both set the phase of the path state, and to measure it in $\{|+\rangle_T, |−\rangle_T\}$, or $\{|R\rangle_T, |L\rangle_T\}$. This can be done by adding the required phase for state preparation and subtracting the phase for state measurement. Such a phase is then converted into a path delay and sent to the trajectory measurement results is $|+\rangle_T$, whereas $|−\rangle_T$ is the effective process when the trajectory measurement outcome is $|−\rangle_T$.

Next, we compute the action of the one-qubit $\chi$-matrix on a maximally-entangled Bell state, to evaluate the two values of the coherent information $I_c(+) < I_c(−)$. Fixing the state of the path qubit in this manner results in the coherent information of the effective one-to-two-qubit channel. We carry out this method based on full QPT for the cases of quantum-control of parallel channels and channels in series with quantum-controlled operations, but for the quantum-control of channel order we can make a simplification to lower bound the coherent information which saves significant measurement time. For these data, we only prepare the path state $|+\rangle_T$, and then measure it in the $\{|+\rangle_T, |−\rangle_T\}$ basis, as described above. With these measurements, we lower bound the coherent information in our channels as follows. We first reconstruct two single-qubit $\chi$-matrices for the target systems, $\chi(+) |−\rangle_T$, $\chi(−) |−\rangle_T$, using single-qubit process tomography on the polarization qubit. In particular, $\chi(+) |−\rangle_T$ is the single-qubit effective process that the information qubit experiences when the trajectory measurement results is $|+\rangle_T$, whereas $\chi(−) |−\rangle_T$ is the effective process when the trajectory measurement outcome is $|−\rangle_T$.

In general, $I_c^{LB}$ sets a lower bound on $I_c$ because of the data processing inequality for coherent information. Furthermore, in absence of additional errors, it can be shown that $I_c^{LB} = I_c$ in the case of the quantum switch.

G. Fixing the Quantum-Controlled Operations Independently of the Noise

In some cases, for instance in a rapidly-varying noise environment, it may be impossible to estimate the type of noise and adapt the quantum-controlled operations accordingly. In these situations, one would need to fix such
Coherent Information

FIG. 8: Experimental BF- and PF-noise data for sub-optimal quantum-controlled operations. The trend of the scheme featuring the channels in series with quantum-controlled operations (Series w/ QC-ops.) performs worse than the quantum-control of channel order (QC-order) for all $p \geq 0.67$, but better than the quantum-control of parallel channels (QC-//-channels) for $p \geq 0.84$. The experimental data for the sub-optimal choice of Series w/ QC-ops. is in good agreement with the expected trend.

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