The work research of two-layer combined reinforced elements in pure bending

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Abstract. The experimental-theoretical studies’ results on the work of two-layer combined reinforced elements with pure bending are presented. The full-scale tests of single-layer and two-layer elements with combined reinforcement showed that the vermiculite-concrete layer increases stiffness and crack resistance by 15–20%. The proposed numerical method for calculating the bilayer elements on bending forces provides high convergence (95%) with the full-scale experiments’ results.

Introduction

One of the ways to increase the reinforced cement structures’ fire resistance is to apply the heat-protective coatings using porous aggregates such as vermiculite and perlite [1-3]. They can be applied to the building structures in the factory and at the construction site [4-5]. The study of the reinforced cement structures’ fire-retardant and structural layers joint work under various stress-strain states is an urgent task.

To study the work of two-layer reinforced cement elements, as well as to analyze the joint work of fire-retardant and structural concrete layers, 4 series of samples (Figure 1) 100 × 400 mm in size, differing in the manufacturing method, the thickness of the fire-retardant layer from vermiculite concrete, the quantity and location steel woven nets, as well as the presence of bar reinforcement were tested for pure bending. The bending test also evaluated the effect of the vermiculite-concrete layer on the strength, stiffness and crack resistance of the bent elements.

The samples with dimensions of 400 × 100 mm for pure bending tests were cut out from the plates with a size of 2000 × 500 mm, which were made on a laboratory setup by vibroforming. The molding of the vermiculite-concrete layer was carried out from below or from above the reinforced fine-grained concrete layer. The mixing zone between the layers was 2 - 3 mm.

For the reinforced fine-grained concrete layer, sand with a particle size modulus of 2.37 and a maximum grain size of 5.0 mm was used. Fine concrete mix was applied in the following composition C:S = 1:2,5 c W/C = 0,4 (R_{comp} = 55 MPa, R_{prod} = 6,5 MPa).
A woven mesh was used to reinforce the samples No. 8 with a wire diameter of 0.7 mm according to GOST 3826–82 and rod reinforcement of class A240 with a diameter of 6 mm.

For vermiculite-concrete layer, expanded vermiculite of the Kovdor deposit with a maximum grain size of 5.0 mm, composition C: vermiculite = 1:3 (by volume), $W/C = 1,25$ ($\rho = 640$ $\text{kg/m}^3$, $R_{\text{comp}} = 2,3$ $\text{MPa}$, $R_{\text{prod}} = 1,3$ $\text{MPa}$).

The samples’ loading was of a short-term nature and was carried out through a special device with piece loads. To monitor the deformations, displacements and the process of crack formation, the following were used: strain gauges with a base of 50 mm; movement type indicators; MPB microscope - 2 with 24x magnification, 0.05 mm scale interval.

The samples were tested for bending at the age of one month. The test was carried out according to the scheme of pure bending in the flat position with the location of the vermiculite-concrete layer in the stretched zone, the span between the supports was 300 mm (Figure 2). Three samples were tested in each series.

**Figure 1.** Reinforcement schemes for the single-layer (1, 3) and two-layer samples with a vermiculite-concrete layer (2, 4)

**Figure 2.** Scheme for testing the series samples No. 2 with pure bending
During the samples’ testing using the microscope, the cracks in the single-layer reinforced cement samples were not detected, however, using the strain gauges, relative deformations at the time preceding failure were recorded, equal to $35 \cdot 10^{-5}$.

When testing the two-layer samples, the first cracks were discovered in the vermiculite-concrete layer with an opening width of 0.01 - 0.02 mm at bending moments of 5 - 6 kN·cm. The maximum crack opening width at the moment preceding the fracture was 0.03 - 0.05 mm. The number of cracks varied from 4 to 7. Some cracks grew along the height of the vermiculite-concrete layer to 10 - 15 mm by the time preceding the fracture. No cracks were found in the stretched face of the reinforced cement layer in bilayer specimens.

Under loads ($M = 5 - 6$ kN·cm), the strains measured in the two-layer samples in the reinforced fine-grained concrete layer’s transition zone to the vermiculite-concrete layer were $10 - 15 \cdot 10^{-5}$. This indicates that the vermiculite-concrete layer significantly increases the stiffness of the single-layer reinforced cement samples.

It should be noted that during the two-layer reinforced cement samples’ tests, there was no violation of the layers’ adhesion, delamination of the vermiculite-concrete layer from the reinforced cement.

Thus, until the cracks’ formation with an opening width of 0.05 - 0.1 mm in the single-layer reinforced cement samples, an additional fire-retardant layer significantly increases their crack resistance.

The test samples’ results and the nature of the cracks’ formation are shown in Figure 3. and Figure 4.

![Figure 3. Deformation of the stretched face (average for the pure bending zone) of the samples](image-url)
To study the operation of single-layer and two-layer combined reinforced samples with pure bending, a deformation model of the cross section was used [6-7]. This method is based on the use of deformation diagrams describing the nonlinear operation of concrete and reinforcement and some law of the relative deformations’ distribution over the element cross section (the flat section hypothesis was used [8]). As the materials’ deformation diagrams, the dependence “stress - relative deformation” of a curvilinear form, constructed according to the Sargin’s formula [9-10], was used.

In contrast to the method of ultimate efforts, the deformation model allows a more complete analysis of the elements’ stress-strain state [7].

The use of the deformation model requires a numerical method for solving the nonlinear problem of calculating normal sections. To solve the nonlinear problem of calculating the normal sections, the following method is proposed.

In deriving the resolving equation, we use the plane sections’ hypothesis. The total strain is represented as the sum of the axial strain $\varepsilon_o$ and deformation is caused by a change in curvature:

$$\varepsilon = \varepsilon_o - z\chi,$$

where $\chi = 1/\rho$ – is the radius of the sample’s bending axis curvature.

Coordinate origin $z$ is taken on the midline of the beam. Stresses in concrete are determined by the formula:

$$\sigma_s(z) = E_s(z)\varepsilon = E_s(z)(\varepsilon_o - z\chi).$$

Armature stresses $i$-th of the spaced layer $z_{s,i}$ from the midline, is determined from the conditions of its joint work with concrete:

$$\sigma_{s,i} = E_{s,i}\varepsilon_o = E_{s,i}(\varepsilon_o - z_{s,i}\chi).$$

The axial force arising in concrete is written as:

$$N_s = b \int_{-h/2}^{h/2} \sigma_s(z) dz = b \int_{-h/2}^{h/2} E_s(z)(\varepsilon_o - z\chi) dz = \varepsilon_o b \int_{-h/2}^{h/2} E_s(z) dz - \chi b \int_{-h/2}^{h/2} \frac{b}{2} \cdot \frac{z}{\chi} dz;$$

The axial force arising in the reinforcement is calculated as follows:

$$N_s = \sum_{i=1}^{n} \sigma_{s,i} A_{s,i} = \sum_{i=1}^{n} E_{s,i} A_{s,i}(\varepsilon_o - z_{s,i}\chi).$$
where \( n \) is the number of reinforcing layers.

Value \( \varepsilon_0 \) is determined from the condition that the total axial force is equal to zero:

\[
N = N_s + N_b = 0.
\]

Substituting (3.4) and (3.5) into (3.6), we obtain:

\[
N = N_s + N_b = EA_{red} \varepsilon_0 - ES_{red} \chi = 0 \rightarrow \varepsilon_0 = \frac{ES_{red} \chi}{EA_{red}},
\]

where

\[
EA_{red} = b \int_{-h/2}^{h/2} E_b(z) \, dz + \sum_{i=1}^{n} E_{s,i} A_{s,i}.
\]

\[
ES_{red} = b \int_{-h/2}^{h/2} E_b(z) \, dz + \sum_{i=1}^{n} E_{s,i} A_{s,i} z_{s,i}.
\]

The bending moment perceived by concrete is defined as:

\[
M_b = b \int_{-h/2}^{h/2} \sigma_b(z) \, dz - \int_{-h/2}^{h/2} \varepsilon_0 b \int_{-h/2}^{h/2} E_b(z) \, dz - \chi b \int_{-h/2}^{h/2} E_b(z) z^2 \, dz.
\]

The bending moment perceived by the reinforcement is calculated as follows:

\[
M_s = \sum_{i=1}^{n} \sigma_{s,i} A_{s,i} \left( \varepsilon_0 - z_{s,i} \chi \right).
\]

The total bending moment perceived by the cross section is written as:

\[
M = M_s + M_b = EI_{red} \chi - ES_{red} \varepsilon_0 = \chi \left( EI_{red} - \frac{(ES_{red})^2}{EA_{red}} \right),
\]

where

\[
EI_{red} = b \int_{-h/2}^{h/2} E_b(z) \, dz + \sum_{i=1}^{n} E_{s,i} A_{s,i} z_{s,i}^2.
\]

The quantities \( EA_{red}, \, EI_{red}, \, ES_{red} \) represent the reduced stiffnesses of the cross section, which are determined by numerical integration according to the trapezoid formula.

From (3.15) the quantity \( \chi \) is determined by the formula:

\[
\chi = \frac{M}{EI_{red} - \frac{(ES_{red})^2}{EA_{red}}}. \quad (11)
\]

The SSS calculation is performed by the step method. The increase in bending moment is made in small portions \( \Delta M \). At each step, the increment of curvature is calculated \( \Delta \chi \) according to the formula

\[
\Delta \chi = \Delta M \left( EI_{red} - \frac{(ES_{red})^2}{EA_{red}} \right),
\]

then the increment of axial strain is determined by the formula

\[
\Delta \varepsilon_0 = ES_{red} \Delta \chi / EA_{red}.
\]

Further, stress and strain increments are determined and the
tangential modulus of concrete elasticity and reinforcement is adjusted. Then the reduced stiffness of the cross section is corrected.

As an equation describing the relationship between stresses and deformations of concrete, the Sargin’s formula is used, written for the stretched zone in the form:

\[
\frac{\sigma}{R_0} = \frac{k\eta - \eta^2}{1+(k-2)\eta},
\]

where \( R \) – is concrete strength, \( \eta = \varepsilon / \varepsilon_R \), \( \varepsilon_R \) – define the deformation at the top of the diagram, \( k = E_0\varepsilon_R / R \), \( E_0 \) – define the initial modulus of concrete elasticity.

For the compressed zone (\( \sigma < 0 \)) the equation (12) is written as:

\[
\frac{\sigma}{R_0} = \frac{k\eta + \eta^2}{1-(k-2)\eta}.
\]

The tangent modulus for the stretched zone is calculated as:

\[
E_{\text{tan}} = \frac{R_0}{\varepsilon_R} \frac{(k-2\eta)(1+(k-2)\eta)-(k\eta - \eta^2)(k-2)}{(1+(k-2)\eta)^2}
\]

For the compressed zone, the tangent module formula takes the form:

\[
E_{\text{tan}} = -\frac{R_0}{\varepsilon_R} \frac{(k+2\eta)(1-(k-2)\eta)+(k\eta + \eta^2)(k-2)}{(1-(k-2)\eta)^2}
\]

For reinforcement, the Prandtl diagram is used, i.e.:

\[
E_{s,\text{acc}} = \begin{cases} E_s, & |\sigma_s| \leq R_s; \\ 0, & |\sigma_s| > R_s. \end{cases}
\]

The results of the numerical experiments obtained by the calculation method presented above are shown in Figure 5.
Figure 5. The calculated dependence of the bending moment in the cross section of the samples on the relative deformation of the stretched face

Summary
Thus, the full-scale and numerical research methods have shown that the vermiculite-concrete layer increases the stiffness and crack resistance of the thin-walled structures with combined reinforcement. The proposed numerical calculation method provides good agreement with the experimental studies on the reinforced cement samples.

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