On the Gaussian Many-to-One X Channel

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Abstract

In this paper, the Gaussian many-to-one X channel, which is a special case of general multiuser X channel, is studied. In the Gaussian many-to-one X channel, communication links exist between all transmitters and one of the receivers, along with a communication link between each transmitter and its corresponding receiver. As per the X channel assumption, transmission of messages is allowed on all the links of the channel. This communication model is different from the corresponding many-to-one interference channel (IC). Transmission strategies which involve using Gaussian codebooks and treating interference from a subset of transmitters as noise are formulated for the above channel. Sum-rate is used as the criterion of optimality for evaluating the strategies. Initially, a 3-transmitter many-to-one X channel is considered and three transmission strategies are analyzed. The first two strategies are shown to achieve sum-rate capacity under certain channel conditions. For the third strategy, a sum-rate outer bound is derived and the gap between the outer bound and the achieved rate is characterized. These results are later extended to the $K$-transmitter case. Next, a region in which the many-to-one X channel can be operated as a many-to-one IC without loss of sum-rate is identified. Further, in the above region, it is shown that using Gaussian codebooks and treating interference as noise achieves a rate point that is within one bit per user from the sum-rate capacity. Subsequently, some implications of the above results to the Gaussian many-to-one IC are discussed. Transmission strategies for the many-to-one IC are formulated and channel conditions under which the strategies achieve sum-rate capacity are obtained. A region where the sum-rate capacity can be characterized to within one bit per user is also identified. Finally, the regions where the derived channel conditions are satisfied for each strategy are illustrated for a 3-transmitter many-to-one X channel and the corresponding many-to-one IC.

\textit{keywords:} Many-to-one interference channel, interference channel, X channel, sum capacity.

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I. Introduction

The interference network is a multi-terminal communication network introduced by Carleial [1], consisting of $M$ transmitters and $N$ receivers, where each transmitter has an independent message for each of the $2^N - 1$ possible non-empty subsets of the receivers. The multiple access channel (MAC), broadcast channel, interference channel (IC), and X channel (XC) are all special cases of the interference network.

In the two-transmitter interference channel, each transmitter communicates an independent message to its corresponding receiver, while the cross channels constitute interference at the receivers. The interference channel has been studied extensively in literature. Although the capacity region of the IC is unknown, several inner and outer bounds for the capacity region and sum-rate capacity have been derived in [2]–[4]. In [5]–[7], sum-rate capacity of the IC is characterized in the low-interference regime: a regime where using Gaussian inputs and treating interference as noise is optimal.

By allowing messages on all the links of the IC, we obtain the X channel, i.e., both transmitters have an independent message for each receiver, for a total of four messages in the system. In this sense, the X channel is a generalization of the IC. The best known achievable region is due to Koyluoglu, Shahmohammadi, and El Gamal [8]. This rate region when specialized to the IC was shown to reduce to the Han–Kobayashi rate region [2], which is the best known achievable region for the IC. The sum-rate capacity result for the Gaussian interference channel in the low-interference regime was extended to the Gaussian X channel in [9].

The many-to-one X channel is a special case of a $K$-transmitter XC, and can be described as an X channel with “many-to-one” connectivity. In the many-to-one channel model, communication links exist between all transmitters and one of the receivers, say receiver $k$, $k \in \{1, \ldots, K\}$, along with a communication link between each transmitter $Tx \ i$, and its corresponding receiver $Rx \ i$, $i \neq k$. As per the X channel model assumption, transmission of messages is assumed on all the links of the channel. The system model for the $K$-transmitter many-to-one XC is shown in Fig. [1] where we have assumed communication links between all transmitters and receiver 1. Thus, each transmitter $Tx \ i$, excluding the first has two independent messages, one for its corresponding receiver $Rx \ i$, and the other to receiver 1 for a total of $2K - 1$ messages in the system. This model has not been studied before.
The many-to-one interference channel is a special case of the many-to-one XC, where each transmitter \((Tx \, i)\) is only interested in communicating with its corresponding receiver \((Rx \, i)\), i.e., each transmitter has only one message. The many-to-one IC is studied in [7], [10]–[12]. In [7], [10], sum-rate capacity of the many-to-one IC is characterized in the low-interference regime. In [11], the capacity region is characterized to within a constant number of bits. The generalized degrees of freedom of the channel is obtained in [11], [12].

We study the more general many-to-one X channel with messages on all the links. Such a channel could prove useful in the analysis of half-duplex relay networks. See [13] for examples of such networks used in optimization of unicast information flow in multistage decode-and-forward relay networks.

The many-to-one XC can also occur as a communication model in cellular downlink. Consider the illustration in Fig. 2, where user 1 is at the cell edge and receives transmissions from the nearby base stations (BS) along with BS 1, while BS 2 and BS 3 simultaneously communicate with users 2 and 3, respectively. In order to improve the system throughput, all three BSs can communicate independent messages to user 1, provided the channel conditions are conducive. The reverse links of this model for uplink transmission form the one-to-many X channel studied.
in [14].

Allowing messages on the cross links leads to some interesting scenarios. Each transmitter excluding the first, can now make a choice, either transmit to its own corresponding receiver, or transmit to receiver 1, or both. Instead of finding outer and inner bounds to the capacity region of the many-to-one XC, we focus on practical transmission scenarios. We define the transmission strategies for this channel as follows.

**Definition 1:** In strategy $\mathcal{M}_k$, transmitter 1 along with $k - 1$ other transmitters form a MAC at receiver 1, while interference caused by the rest of the transmitters is treated as noise, $k = 1, 2, \ldots, K$. All transmitters use Gaussian codebooks.

In Table I, we list all possible strategies as per the above definition for $K = 3$. Thus, in strategy $\mathcal{M}1$, interference caused by transmitters 2 and 3 at receiver 1 is treated as noise, while in strategy $\mathcal{M}3$, receiver 1 does not experience any interference.

The analysis of specific transmission strategies is also motivated by applications to small cell networks. Small cells encompassing femtocells, picocells, and microcells, are used by mobile service providers to increase network capacity and/or extend the service coverage area. Consider the illustration in Fig. 3 where some femto-BSs along with their corresponding users within a small coverage area co-exist in a macro cell consisting of macro users served by the macro BS. To increase the service reliability and throughput, the users can either communicate with the

![Diagram of three base stations (BS 1, BS 2, BS 3) and users (U1, U2, U3) showing applicability of many-to-one X channel in cellular downlink.](image-url)
femto-BS or with the macro-BS. This communication model also results in the many-to-one X channel.

Small cells are seen as an effective means to achieve 3G data off-loading, and many mobile service providers consider small cells as a vital element for managing LTE Advanced spectrum more efficiently compared to using just macrocells. It is in this context that the knowledge of the optimality of different transmission strategies that the users can employ becomes valuable. Femto, pico and micro cells are also used to motivate a slightly similar channel model studied in [15], where a MAC generates interference for a single user uplink transmission. We note that the many-to-one IC was also motivated by considering a similar scenario where multiple short-range peer-to-peer communications create interference for a long-range receiver [11], [12].

We use a 3-transmitter many-to-one XC to evaluate the different strategies. The sum-rate at all the receivers is used as the criterion for optimality. In general, we use genie-aided bounding techniques to derive the sum-rate capacity results in this paper. Specifically, for certain strategies we make use of the concepts of useful genie and smart genie introduced in [7]. A genie is said to be useful if it results in a genie-aided channel whose sum-rate capacity is achieved by Gaussian inputs, while a smart genie is one which does not increase the sum-rate when Gaussian inputs are used [7]. In [7], the genie-aided bounding technique is used to identify the regime under which all the interference can be treated as noise. In our work, we use this technique for scenarios

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| No. | Strategy |
|-----|----------|
| M1  | All transmitters transmit to their corresponding receivers and interference at receiver 1 is treated as noise. |
| M2  | Transmitter 1 and either transmitter 2 or transmitter 3 form a MAC at receiver 1, while the interference from the other transmitter is treated as noise. |
| M3  | All transmitters form a MAC at receiver 1. |

TABLE I
Transmission strategies for a many-to-one XC with 3 transmitters
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where interference from a *subset* of transmitters is treated as noise. We show that strategies $\mathcal{M}1$ and $\mathcal{M}2$ achieve sum-rate capacity under certain channel conditions. For strategy $\mathcal{M}3$, we characterize the gap between the achievable sum-rate of the strategy and a sum-rate outer bound. Later, we extend these results to the $K$-transmitter case.

Next, we identify a region in which the many-to-one XC can be operated as a many-to-one IC without loss of sum-rate and show that using Gaussian codebooks and treating interference as noise achieves a rate point that is within one bit per user from the sum-rate capacity. In the last part of the paper, we observe some implications of the above results for the many-to-one IC. Firstly, we note that strategies similar to the ones defined above can be considered for the many-to-one IC as well. These involve a combination of partial interference cancellation and treating the rest of the interference as noise. We derive the sum-rate optimality of these strategies under certain channel conditions. Secondly, we identify a region for the many-to-one IC where the sum-rate capacity can be characterized to within one bit per user.

The rest of this paper is organized as follows. The system model is presented in Section
In Section III we consider the 3-transmitter many-to-one XC and analyze the different strategies defined earlier. These results are extended to the $K$-transmitter case in Section IV. Some implications of the above results for the Gaussian many-to-one IC are discussed in Section V. Numerical results and illustrations regarding the optimality of the strategies are presented in Section VI. Conclusions are presented in Section VII.

II. System Model

As shown in Fig. 1, the many-to-one XC with $K$ transmitters is described by the following input-output equations

$$
y_1 = h_{11} \tilde{x}_1 + \sum_{j=2}^{K} h_{1j} \tilde{x}_j + n_1 \tag{1}
$$

$$
y_i = h_{ii} \tilde{x}_i + n_i, \quad i = 2, 3, \ldots, K, \tag{2}
$$

where $\tilde{x}_t$ is the transmitted symbol by transmitter $t$, $h_{rt}$ denotes the complex channel gain from transmitter $t$ to receiver $r$, and $n_r$ is the additive complex Gaussian noise at receiver $r$. $h_{ii}$, $i = 2, \ldots, K$, are the direct channels, while $h_{1i}$ are the cross channels. The additive noise $n_r$ is a circularly symmetric complex Gaussian (CSCG) random variable with unit variance, i.e., $n_r \sim \mathcal{CN}(0, 1)$, $r = 1, 2, \ldots, K$.

A. Many-to-one X channel with 3 transmitters in standard form

In order to analyze the strategies, we consider the many-to-one XC with 3 transmitters since the 2 transmitter case results in the Z channel. The Z channel is obtained from the many-to-one XC by retaining only the first two transmitters and removing the rest. In this way, the many-to-one XC can be considered as one possible generalization of the Z channel. The Z channel has been studied in [18], [19].

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1We use the following notation: lowercase letters for scalars, boldface lowercase letters for vectors, and calligraphic letters for sets. $\bar{\cdot}$ denotes complex conjugation, $\cdot^H$ denotes the Hermitian operation, trace($\cdot$) denotes the trace operation, and $\mathbb{E}\{\cdot\}$ denotes the expectation operation.
The many-to-one XC channel with 3 transmitters can be written in standard form (See Fig. 4), i.e.,

\[ y_1 = x_1 + ax_2 + bx_3 + n_1 \]  \hspace{1cm} (3)
\[ y_2 = x_2 + n_2 \]  \hspace{1cm} (4)
\[ y_3 = x_3 + n_3, \]  \hspace{1cm} (5)

where we have used \( a = h_{12}/h_{22} \), \( b = h_{13}/h_{33} \), \( x_i = h_{ii}\tilde{x}_i \), and \( P_i = |h_{ii}|^2\tilde{P}_i \) are the new power constraints [1].

As shown in Fig. 4, the 3-transmitter many-to-one XC has five independent messages, \( W_{11}, W_{12}, W_{13}, W_{22} \) and \( W_{33} \), where \( W_{ij} \) is the message transmitted from transmitter \( j \) to receiver \( i \).

Our motivation for considering the 3-transmitter many-to-one IC first, instead of directly analyzing the \( K \)-transmitter case stems from three perspectives: (i) ease of presentation, (ii) understanding the proof techniques without cumbersome notational details, (iii) better visualization of the regions where the strategies are optimal (as seen in the numerical results presented in Section VI).

III. ANALYSIS OF DIFFERENT STRATEGIES FOR THE 3-TRANSMITTER MANY-TO-ONE XC

We introduce some terminology useful in deriving the results in this section. Let \( y_i^n \) denote the vector of received symbols of length \( n \) at receiver \( i \). Let \( x_i^n \) denote the \( n \) length vector of
transmitted symbols at transmitter $i$. By Fano’s inequality, we have
\[ H(W_i | y_i^n) \leq n\epsilon_n, \quad i = 1, 2, 3 \]
\[ H(W_{1j} | y_1^n) \leq n\epsilon_n, \quad j = 2, 3, \quad (6) \]

where $\epsilon_n \to 0$ as $n \to \infty$.

Before we proceed to analyze the various strategies, we provide a restatement of Lemma 5 in [7], in a form that is easier to apply to the many-to-one X channel. We make use of the following lemma to bound the sum-rate of the many-to-one XC in some cases.

**Lemma 1:** Let $w^n_i$ be a complex sequence with average power constraint trace $\mathbb{E}(w^n_i w^n_{iH}) \leq nP_i$. Let $n^n_i$ be a complex random vector with components that are distributed as independent $CN(0, 1)$ random variables. Assume that $w^n_i$ are independent of each other and also independent of $n^n_i$. Let $w_{iG} \sim CN(0, P_i)$. For some complex constants $c_i$, we have
\[
\sum_{i=1}^K h(w^n_i + n^n_i) - h\left(\sum_{i=1}^K c_i w^n_i + n^n_i\right) \leq n \sum_{i=1}^K h(w_{iG} + n_i) - nh\left(\sum_{i=1}^K c_i w_{iG} + n_1\right),
\]
when $\sum_{i=1}^K |c_i|^2 \leq 1$ and equality is achieved if $w^n_i = w_{iG}^n$, where $w_{iG}^n$ denotes a complex random vector with components that are i.i.d $CN(0, P_i)$.

**Proof:** Let $t^n_i = c_i(w^n_i + n^n_i)$. The left-hand side of (7) can now be written as
\[
\sum_{i=1}^K h(t^n_i) - h\left(\sum_{i=1}^K t^n_i + \tilde{n}_1^n\right) + n \sum_{i=1}^K \log |c_i|^2,
\]
where $\tilde{n}_1^n$ is a complex random vector with components that are i.i.d $CN(0, 1 - \sum_{i=1}^K |c_i|^2)$. The final result follows by applying Lemma 5 in [7], i.e.,
\[
\sum_{i=1}^K h(t^n_i) - h\left(\sum_{i=1}^K t^n_i + \tilde{n}_1^n\right) \leq n \sum_{i=1}^K h(t_{iG}) - nh\left(\sum_{i=1}^K t_{iG} + \tilde{n}_1\right),
\]
where $t_{iG} = c_i(w_{iG} + n_i)$ and equality is achieved if $w^n_i = w_{iG}^n$. Since the variance of $\tilde{n}_1$ cannot be negative, we have the condition $\sum_{i=1}^K |c_i|^2 \leq 1$.

\[\square\]

### A. Optimality of Strategy $\mathcal{M}1$

Here, we are interested in a region where the transmitters use Gaussian inputs to communicate with their respective receivers and interference at receiver 1 is treated as noise. This is usually referred to as the low-interference or the noisy-interference regime in the interference channel literature. We characterize the low-interference sum-rate capacity in the following theorem.
Theorem 1: The sum-rate capacity is achieved by transmitting on the direct channels and treating interference as noise when

\[ |a|^2 + |b|^2 \leq 1, \]  

(8)

and is given by

\[ S = \log \left( 1 + \frac{P_1}{1 + |a|^2 P_2 + |b|^2 P_3} \right) + \log(1 + P_2) + \log(1 + P_3). \]  

(9)

Proof: If \(|b|^2 \leq 1\), we have \(I(W_{13}; y_3^n) \geq I(W_{13}; y_1^n | x_1^n, x_2^n)\). Therefore,

\[ H(W_{13} | y_3^n) \leq H(W_{13} | y_1^n, x_1^n, x_2^n) \leq H(W_{13} | y_1^n) \leq n\epsilon_n, \]  

(10)

where (a) follows from the fact that removing conditioning does not reduce the conditional entropy. Thus, we conclude that \(W_{13}\) is decodable at receiver 3 when \(|b|^2 \leq 1\). Note that in this case

\[ h(x_3^n | y_3^n) = H(W_{13}, W_{33} | y_3^n) = H(W_{13} | y_3^n) + H(W_{33} | y_3^n, W_{13}) \leq H(W_{33} | y_3^n) + H(W_{13} | y_3^n) \leq 2n\epsilon_n, \]  

(11)

where (II) follows from (6) and (10).

Similarly, it can be shown that when \(|a|^2 \leq 1\), \(W_{12}\) is decodable at receiver 2, i.e.,

\[ h(x_2^n | y_2^n) \leq 2n\epsilon_n. \]  

(12)

This means that we can set \(W_{12} = W_{13} = \phi\) (without loss of sum-rate). Thus, we have shown that the presence of cross messages does not improve the sum-rate when \(|a|^2 \leq 1, |b|^2 \leq 1\).
Now, assume \(|a|^2 \leq 1\) and \(|b|^2 \leq 1\). The sum-rate can be bounded as follows:

\[
\begin{align*}
nS & \leq H(W_{11}) + H(W_{12}, W_{22}) + H(W_{13}, W_{33}) \\
& = h(x_1^n) + h(x_2^n) + h(x_3^n) \\
& = I(x_1^n; y_1^n) + I(x_2^n; y_2^n) + I(x_3^n; y_3^n) + \sum_{i=1}^{3} h(x_i^n | y_i^n) \\
& \leq h(y_1^n) - h(ax_2^n + bx_3^n + n_1^n) + h(x_2^n + n_2^n) - h(x_3^n + n_3^n) - h(x_3^n) + 5\epsilon_n \\
& \leq nh(y_{1G}) + nh(x_{2G} + n_2) + nh(x_{3G} + n_3) \\
& - nh(ax_{2G} + bx_{3G} + n_1) - nh(n_2) - nh(n_3) + 5\epsilon_n \\
& = nI(x_{1G}; y_{1G}) + nI(x_{2G}; y_{2G}) + nI(x_{3G}; y_{3G}) + 5\epsilon_n,
\end{align*}
\]

where \(x_{iG} \sim \mathcal{CN}(0, P_i)\), \(y_{iG}\) denotes \(y_i\), with \(x_j = x_{jG}, \forall i, j\), (b) follows from (6), (11) and (12), and in (c), we have used Lemma 1 to bound the term \(h(x_2^n + n_2^n) + h(x_3^n + n_3^n) - h(ax_2^n + bx_3^n + n_1^n)\), under the condition \(|a|^2 + |b|^2 \leq 1\). As \(n \to \infty\), \(\epsilon_n \to 0\), and we have

\[
S \leq \log \left(1 + \frac{P_1}{1 + |a|^2 P_2 + |b|^2 P_3} \right) + \sum_{i=2}^{3} \log(1 + P_i).
\]

This sum-rate bound can be achieved using strategy \(M1\). \qed

**Remark 1:** Theorem 1 was proved for the many-to-one interference channel in [7, Theorem 4] using genie aided bounding techniques. The low-interference regime for the discrete memoryless many-to-one interference channels is proved in [10]. We also note that the result in [7] is a special case of a more general result in [16, Theorem 3], where the sum-rate capacity of a \(K\)-user Gaussian interference channel is characterized in the low-interference regime.

### B. Optimality of Strategy \(M2\)

Here, we ask the following question: are there channel conditions such that the sum-rate capacity is achieved by a two-user MAC at receiver 1 formed by transmitter 1 and either transmitter 2 or transmitter 3, while the interference from the other transmitter is treated as noise? Observe that the other transmitter forms a point-to-point channel and is a source of interference for the two-user MAC. We characterize the sum-rate capacity in the following theorem.

**Theorem 2:** The sum-rate capacity is achieved by the two-user MAC formed by transmitter 1 and either transmitter 2 or transmitter 3 to receiver 1, for the following channel conditions, respectively.
(i) \(|a| \geq \frac{1 + |b|^2 P_3}{\sqrt{1 - |b|^2}}, \quad |b| \leq 1\)

(ii) \(|b| \geq \frac{1 + |a|^2 P_2}{\sqrt{1 - |a|^2}}, \quad |a| \leq 1\).

**Proof:** We prove statement (i) below. This represents the case where transmitters 1 and 2 form a MAC at receiver 1 while interference from transmitter 3 is treated as noise. The proof for the second statement follows along similar lines.

We use genie-aided bounding techniques to derive the optimality of strategy \(M_2\). Specifically, we use the concept of *useful genie* and *smart genie* introduced in [7] to obtain the sum-rate capacity for strategy \(M_2\). Let a genie provide the following side information to receiver 1:

\[ s_1 = x_1 + a x_2 + \eta z_1, \tag{13} \]

where \(z_1 \sim \mathcal{CN}(0, 1)\) and \(\eta\) is a positive real number. We allow \(z_1\) to be correlated to \(n_1\) with correlation coefficient \(\rho\).

A genie is said to be useful if it results in a genie-aided channel whose sum-rate capacity is achieved by Gaussian inputs, i.e., the sum-rate capacity of the genie-aided channel equals

\[ I(x_1 G, x_2 G; y_1 G, s_1 G) + I(x_3 G; y_3 G), \]

where \(x_i G \sim \mathcal{CN}(0, P_i)\), \(y_i G, s_1 G\) are \(y_i\) and \(s_1\) with \(x_j = x_j G, \forall i, j\).

**Lemma 2:** (Useful Genie) The sum-rate capacity of the genie-aided channel with side information (13) given to receiver 1 is achieved by using Gaussian inputs and by treating interference from transmitter 3 as noise at receiver 1, if the following conditions hold:

\[ \eta^2 \leq |a|^2, \quad |b|^2 \leq 1 - |\rho|^2, \tag{14} \]

and the sum-rate of the genie-aided channel is bounded as

\[ S \leq I(x_1 G, x_2 G; y_1 G, s_1 G) + I(x_3 G; y_3 G). \tag{15} \]

**Proof:** The sum-rate of the genie-aided channel can be bounded as

\[
\begin{align*}
    n S & \leq H(W_{11}, W_{12}, W_{22}) + H(W_{13}, W_{33}) \\
    & = I(W_{11}, W_{12}, W_{22}; y_1^n, s_1^n) + H(W_{11} | y_1^n, s_1^n) + H(W_{12} | y_1^n, s_1^n, x_1^n) \\
    & \quad + H(W_{22} | y_1^n, s_1^n, x_1^n, W_{12}) + I(W_{13}, W_{33}; y_3^n) + H(W_{13} | y_3^n) + H(W_{33} | y_3^n, W_{13}) \\
    & \leq I(x_1^n, x_2^n, y_1^n, s_1^n) + H(x_1^n | y_1^n) + H(W_{12} | y_1^n) \\
    & \quad + H(W_{22} | s_1^n, x_1^n) + I(x_3^n; y_3^n) + H(W_{13} | y_3^n) + H(W_{33} | y_3^n), \tag{16}
\end{align*}
\]
where \((a)\) follows from the fact that removing conditioning cannot reduce the conditional differential entropy.

We bound the term \(H(W_{22} | s^n_1, x^n_1)\). If \(\eta^2 \leq |a|^2\), then we have \(I(W_{22} ; s^n_1 | x^n_1) \geq I(W_{22} ; y^n_2)\). Thus,

\[
H(W_{22} | s^n_1, x^n_1) \leq H(W_{22} | y^n_2) \leq n\epsilon_n. 
\tag{17}
\]

Note that the term \(H(W_{13} | y^n_3)\) is bounded as \((10)\) when \(|b|^2 \leq 1\). Using \((6), (10), \text{and } (17)\) in \((16)\), we have

\[
nS \leq I(x^n_1, x^n_2; y^n_1, s^n_1) + I(x^n_3; y^n_3) + 5n\epsilon_n \\
= I(x^n_1, x^n_2; y^n_1) + I(x^n_3; y^n_3) + 5n\epsilon_n \\
= h(s^n_1) - h(s^n_1 | x^n_1, x^n_2) + h(y^n_1 | s^n_1) \\
- h(y^n_1 | s^n_1, x^n_1, x^n_2) + h(y^n_3 | x^n_3) + 5n\epsilon_n \\
= h(s^n_1) - h(\eta z^n_1) + h(y^n_1 | s^n_1) - h(b x^n_3 + n^n_1 | z^n_1) + h(y^n_3) - h(n^n_3) + 5n\epsilon_n \\
\leq nh(s^n_{1G}) - nh(\eta z^n_1) + nh(y^n_{1G} | s^n_{1G}) \\
- h(b x^n_3 + n^n_1) + h(x^n_3 + n^n_3) - nh(n^n_3) + 5n\epsilon_n \\
\leq nh(s^n_{1G}) - nh(\eta z^n_1) + nh(y^n_{1G} | s^n_{1G}) \\
+ nh(x^n_{3G} + n^n_3) - nh(b x^n_{3G} + n^n_1) - nh(n^n_3) + 5n\epsilon_n \\
= n I(x^n_{1G}, x^n_{2G}; y^n_{1G}, s^n_{1G}) + n I(x^n_{3G}; y^n_{3G}) + 5n\epsilon_n,
\]

where \(\tilde{n}_1 \sim \mathcal{CN}(0, 1 - |\rho|^2)\), \((b)\) follows since Gaussian inputs maximize differential entropy for a given covariance constraint and from the application of Lemmas 1 and 6 in \([7]\), \((c)\) follows from applying Lemma 1 in \([6]\) (which is a special case of the extremal inequality considered in \([17]\)) to the term \(h(x^n_3 + n^n_3) - h(b x^n_3 + n^n_3)\), and using the condition \(|b|^2 \leq 1 - |\rho|^2\).

Next, we show that the genie is smart. A smart genie is one which does not improve the sum-rate when Gaussian inputs are used, i.e., \(I(x^n_{1G}, x^n_{2G}; y^n_{1G}) = I(x^n_{1G}, x^n_{2G}; y^n_{1G})\).

**Lemma 3:** (Smart Genie) If Gaussian inputs are used, and interference is treated as noise, then, under the condition

\[
\eta \rho = 1 + |b|^2 P_3, 
\tag{18}
\]
the genie does not increase the sum rate, i.e.,

$$I(x_{1G}, x_{2G}; y_{1G}, s_{1G}) = I(x_{1G}, x_{2G}; y_{1G}).$$  \hspace{1cm} (19)

**Proof:** Note that

$$I(x_{1G}, x_{2G}; y_{1G}, s_{1G}) = I(x_{1G}, x_{2G}; y_{1G}) + I(x_{1G}, x_{2G}; s_{1G} | y_{1G}).$$

The second term on the right hand side can be expanded as

$$I(x_{1G}; s_{1G} | y_{1G}) + I(x_{2G}; s_{1G} | y_{1G}, x_{1G}).$$

Consider

$$I(x_{1G}; s_{1G} | y_{1G}) = I(x_{1G}; x_{1G} + a x_{2G} + \eta z_{1} | x_{1G} + a x_{2G} + b x_{3G} + n_{1}).$$

From Lemma 8 in [7], if $x, n, z$ are Gaussian with $x$ being independent of the two zero-mean random variables $n, z$, then $I(x; x + z | x + n) = 0$, iff $E(z \bar{n}) = E(|n|^{2})$, where $\bar{n}$ denotes the complex conjugate of $n$. Thus, $I(x_{1G}; s_{1G} | y_{1G})$ becomes zero if $|a|^{2}P_{2} + \eta \rho = 1 + |a|^{2}P_{2} + |b|^{2}P_{3}$ which reduces to (18). Now, consider

$$I(x_{2G}; s_{1G} | y_{1G}, x_{1G}) = I(x_{2G}; a x_{2G} + \eta z_{1} | a x_{2G} + b x_{3G} + n_{1}) \overset{(d)}{=} 0.$$

where $(d)$ follows from [7, Lemma 8] and (18). \hfill $\square$

Combining conditions (14) and (18), we have

$$|a| \geq \frac{1 + |b|^{2}P_{3}}{\rho} ; \quad |b| \leq \sqrt{1 - |\rho|^{2}}. \hspace{1cm} (20)$$

For a fixed value of $b$, we have the constraint $|\rho| \leq \sqrt{1 - |b|^{2}}$. Note that choosing $\rho = \sqrt{1 - |b|^{2}}$ results in the best bound for $a$. Thus, (20) can be rewritten as statement (i) in Theorem 2 \hfill $\square$

**C. Gap from optimality of Strategy $\mathcal{M}_3$**

In strategy $\mathcal{M}_3$, all transmitters form a MAC at receiver 1. We derive a sum-rate outer bound to the many-to-one XC and characterize the gap between the outer bound and the achievable sum-rate of strategy $\mathcal{M}_3$. 
Theorem 3: When all transmitters transmit to receiver 1, if
\[ |a| \geq \frac{1 + |b|^2 P_3}{|\rho|} \quad \text{and} \quad |b| \geq 1, \] (21)
then the gap between the sum-rate outer bound and the sum-rate of strategy \( \mathcal{M}_3 \) is given by
\[ \log \left( \frac{1 - (1 + |b|^2 P_3) |\rho|^2}{1 - |\rho|^2} \right), \] (22)
where \( \rho \) denotes a constant with \( |\rho| \in [0, 1] \).

Proof: We use genie-aided techniques to derive the sum-rate outer bound. Let a genie provide the side information given in (13) to receiver 1. We prove below that the genie is useful.

Lemma 4: (Useful Genie) The sum-rate capacity of the genie-aided channel with side information (13) given to receiver 1 is achieved by using Gaussian inputs when all transmitters transmit to receiver 1, if the following conditions hold:
\[ \eta^2 \leq |a|^2, \quad |b|^2 \geq 1, \] (23)
and the sum-rate of the genie-aided channel is bounded as
\[ S \leq I(x_{1G}, x_{2G}, x_{3G}; y_{1G}, s_{1G}). \] (24)

Proof: The sum-rate \( S \) of the genie-aided channel is bounded as
\[
nS \leq H(W_{11}, W_{12}, W_{13}, W_{22}, W_{33}) \\
= I(x_1^n, x_2^n, x_3^n; y_1^n, s_1^n) + H(W_{11} | y_1^n, s_1^n) + H(W_{12} | y_1^n, s_1^n, x_1^n) \\
+ H(W_{22} | y_1^n, s_1^n, x_1^n, W_{12}) + H(W_{13} | y_1^n, s_1^n, x_1^n, x_2^n) + H(W_{33} | y_1^n, s_1^n, x_1^n, x_2^n, W_{13}) \\
\leq I(x_1^n, x_2^n, x_3^n; y_1^n, s_1^n) + H(W_{11} | y_1^n) + H(W_{12} | y_1^n) + H(W_{22} | s_1^n, x_1^n) + H(W_{13} | y_1^n) \\
+ H(W_{33} | y_1^n, x_1^n, x_2^n). \] (25)

We bound the term \( H(W_{33} | y_1^n, x_1^n, x_2^n) \). If \( |b|^2 \geq 1 \), then \( I(W_{33}; y_1^n | x_1^n, x_2^n) \geq I(W_{33}; y_3^n) \). Therefore,
\[
H(W_{33} | y_1^n, x_1^n, x_2^n) \leq H(W_{33} | y_3^n) \leq n\epsilon_n. \] (26)
Note that the term $H(W_{22} \mid s_1^n, x_1^n)$ is again bounded as in (17) if $\eta^2 \leq |a|^2$.

Using (6), (17) and (26) in (25), we have

\[
\begin{aligned}
 nS & \leq I(x^n_1, x^n_2, x^n_3; y^n_1, s^n_1) + 5n\epsilon_n \\
 & = I(x^n_1, x^n_2, x^n_3; y^n_1) + I(x^n_1, x^n_2, x^n_3; s^n_1 \mid y^n_1) + 5n\epsilon_n \\
 & \leq nI(x_{1G}, x_{2G}, x_{3G}; y_{1G}) + h(s^n_1 \mid y^n_1) - h(s^n_1 \mid x^n_1, x^n_2, x^n_3) + 5\epsilon_n \\
 & \leq nI(x_{1G}, x_{2G}, x_{3G}; y_{1G}) + nh(s_{1G} \mid y_{1G}) - nh(\eta z_1 \mid n_1) + 5\epsilon_n \\
 & = nI(x_{1G}, x_{2G}, x_{3G}; y_{1G}, s_{1G}) + 5\epsilon_n,
\end{aligned}
\]

where (a) follows from the optimality of Gaussian inputs for Gaussian MAC, (b) follows from Lemma 1 in [7]. Here, $y_{1G}$ denotes $y_1$ with $x_i$ being Gaussian distributed, i.e., $y_{1G} = x_{1G} + ax_{2G} + bx_{3G} + n_1$. As $n \to \infty$, $\epsilon_n \to 0$ and we get the desired bound.

Unlike in the case of strategy $\mathcal{M}2$, here the genie does in fact increase the sum-rate and hence is not smart. However, we can choose the parameters $\rho$ and $\eta$ to get a good sum-rate outer bound as follows. Consider

\[
I(x_{1G}, x_{2G}, x_{3G}; y_{1G}, s_{1G}) = I(x_{1G}, x_{2G}, x_{3G}; y_{1G}) + I(x_{1G}, x_{2G}, x_{3G}; s_{1G} \mid y_{1G}).
\]

The second term on the right hand side can be expanded as

\[
I(x_{1G}, x_{2G}; s_{1G} \mid y_{1G}) + I(x_{3G}; s_{1G} \mid y_{1G}, x_{1G}, x_{2G}). \quad (27)
\]

In the proof of Lemma 3, we showed that by choosing $\eta \rho = 1 + |b|^2P_3$, we can make $I(x_{1G}, x_{2G}; s_{1G} \mid y_{1G}) = 0$. Now, consider

\[
I(x_{3G}; s_{1G} \mid y_{1G}, x_{1G}, x_{2G}) = I(x_{3G}; \eta z_1 \mid bx_{3G} + n_1) \\
= h(\eta z_1 \mid bx_{3G} + n_1) - h(\eta z_1 \mid n_1) \\
\stackrel{(c)}{=} h(\eta z_1 \mid bx_{3G} + n_1) - h(\eta z_1) \\
= \log \left( \frac{\eta^2(1 + |b|^2P_3) - \eta^2|\rho|^2}{(1 + |b|^2P_3)\eta^2(1 - |\rho|^2)} \right) \\
= \log \left( \frac{1 - (1 + |b|^2P_3)^{-1}|\rho|^2}{1 - |\rho|^2} \right), \quad (28)
\]

where $z_1 \sim \mathcal{CN}(0, 1 - |\rho|^2)$ and (c) follows from [7, Lemma 6]. Note that (28) represents the gap between the sum-rate outer bound and the sum-rate of strategy $\mathcal{M}3$. Combining condition (23) with $\eta \rho = 1 + |b|^2P_3$, we get (21).
Due to the underlying symmetry in the MAC at receiver 1, a result corresponding to Theorem 3 with the channel coefficients $a$, $b$ and power levels $P_2$, $P_3$ interchanged is also true and further can be proved along similar lines. The results of this section are succinctly summarized in Table II.

### D. Recovering known results for the Z channel

We specialize the results in this section to the Z channel. The Z channel is obtained from the many-to-one X channel by retaining only the first two transmitters and removing the rest \[18\], \[19\]. In the many-to-one XC with 3 transmitters shown in Fig. 4, this is equivalent to setting $b = 0$, and considering the outputs at the first two receivers alone. In this case, Theorem 1 reduces to the channel condition $|a|^2 \leq 1$, which is identical to that obtained in \[18\] for the low-interference regime. Theorem 2 reduces to the condition $|a|^2 \geq 1$, which is same as that obtained in \[19\] for the MAC sum-rate at receiver 1 to be the sum-rate capacity of the Z channel.
IV. EXTENSION TO THE $K$-TRANSMITTER MANY-TO-ONE X CHANNEL

The many-to-one XC with $K$ transmitters can be written in standard form (See Fig. 5), i.e.,

\[ y_1 = x_1 + \sum_{j=2}^{K} h_j x_j + n_1 \]
\[ y_i = x_i + n_i, \quad i = 2, 3, \ldots, K, \]

where as before, we have used $h_j = h_{ij} / h_{jj}$, $x_i = h_{ii} \tilde{x}_i$, and $P_i = |h_{ii}|^2 \tilde{P}_i$ are the new power constraints [1].

As shown in Fig. 5, the $K$-transmitter many-to-one XC has $2K - 1$ independent messages, i.e., \{\text{\textit{W}}_{11}, \text{\textit{W}}_{12}, \text{\textit{W}}_{22}, \text{\textit{W}}_{13}, \text{\textit{W}}_{33}, \ldots, \text{\textit{W}}_{1K}, \text{\textit{W}}_{KK}\}, \text{where} \text{\textit{W}}_{ij} \text{is the message transmitted from transmitter} \text{\textit{j}} \text{to receiver} \text{i}. \text{By Fano’s inequality, we have}

\[ H(W_{ii} | y_i^n) \leq n \epsilon_n, \quad i = 1, \ldots, K, \]
\[ H(W_{1j} | y_1^n) \leq n \epsilon_n, \quad j = 2, \ldots, K, \]

where $\epsilon_n \to 0$ as $n \to \infty$.

Since the results for the $K$-transmitter many-to-one XC follow more or less along similar lines as the 3-transmitter case, we state the results along with a brief outline of the proof for each strategy, with additional details provided in places where the proofs differ.
A. Conditions for the sum-rate optimality of strategies $\mathcal{M}1$, $\mathcal{M}2$ and $\mathcal{M}3$

The optimality of strategy $\mathcal{M}1$ follows using similar arguments as in Theorem 1, under the condition $\sum_{i=2}^{K} |h_i|^2 \leq 1$. This condition arises from the use of Lemma 1, as in inequality (c) of Theorem 1. To avoid repeating the details, we omit the proof.

Next, we consider the optimality of strategy $\mathcal{M}2$. Here, we are interested in a region where the sum-rate capacity is achieved by a two-user MAC at receiver 1 formed by transmitter 1 and transmitter $k$, $k = 2, \ldots, K$, while the interference from the other transmitters is treated as noise. We characterize the sum-rate capacity in the following theorem.

**Theorem 4:** The sum-rate capacity is achieved by the two-user MAC formed by transmitter 1 and transmitter $k$ to receiver 1, for the following channel conditions

$$
|h_k| \geq 1 + \frac{\sum_{j=2, j \neq k}^{K} |h_j|^2 P_j}{\sqrt{1 - \sum_{j=2, j \neq k}^{K} |h_j|^2}},
$$

$$
\sum_{j=2, j \neq k}^{K} |h_j|^2 \leq 1.
$$

**(Proof):** Let a genie provide the following side information to receiver 1:

$$s_k = x_1 + h_k x_k + \eta_k z_k,$$

where $z_k \sim CN(0, 1)$ and $\eta_k$ is a positive real number. We allow $z_k$ to be correlated to $n_1$ with correlation coefficient $\rho_k$.

**Lemma 5:** (Useful Genie) The sum-rate capacity of the genie-aided channel with side information (33) given to receiver 1 is achieved by using Gaussian inputs and by treating interference as noise at receiver 1, if the following conditions hold:

$$
\eta_k^2 \leq |h_k|^2, \quad \sum_{j=2, j \neq k}^{K} |h_j|^2 \leq 1 - |\rho_k|^2.
$$

**(Proof):** Using techniques similar to (16)–(18) in Theorem 4, the sum-rate of the genie-aided
channel can be shown to be bounded as

\[
\begin{align*}
nS & \leq I(\mathbf{x}_1^n; \mathbf{y}_1^n, \mathbf{s}_k^n) + \sum_{j=2, j \neq k}^K I(\mathbf{x}_j^n; \mathbf{y}_j^n) + (2K - 1)n\epsilon_n \\
& = I(\mathbf{x}_1^n; \mathbf{y}_1^n, \mathbf{s}_k^n) + I(\mathbf{x}_1^n; \mathbf{y}_1^n | \mathbf{s}_k^n) + \sum_{j=2, j \neq k}^K I(\mathbf{x}_j^n; \mathbf{y}_j^n) + (2K - 1)n\epsilon_n \\
& = h(\mathbf{s}_k^n) - h(\mathbf{s}_k^n | \mathbf{x}_1^n, \mathbf{x}_k^n) + h(\mathbf{y}_1^n | \mathbf{s}_k^n) - h(\mathbf{y}_1^n | \mathbf{x}_1^n, \mathbf{x}_k^n) \\
& \quad + \sum_{j=2, j \neq k}^K [h(\mathbf{y}_j^n) - h(\mathbf{y}_j^n | \mathbf{x}_j^n)] + (2K - 1)n\epsilon_n \\
& = h(\mathbf{s}_k^n) - h(\eta_k \mathbf{z}_n^k) + h(\mathbf{y}_1^n | \mathbf{s}_k^n) - h\left( \sum_{j=2, j \neq k}^K h_j \mathbf{x}_j^n + \mathbf{n}_1^n \right) \\
& \quad + \sum_{j=2, j \neq k}^K [h(\mathbf{y}_j^n) - h(\mathbf{n}_j^n)] + (2K - 1)n\epsilon_n \\
& \leq nh(s_{kG}) - nh(\eta_k \mathbf{z}_k) + nh(y_{1G} | s_{kG}) - h\left( \sum_{j=2, j \neq k}^K h_j \mathbf{x}_j^n + \tilde{\mathbf{n}}_1^n \right) \\
& \quad + \sum_{j=2, j \neq k}^K [h(\mathbf{x}_j^n + \mathbf{n}_j^n) - h(n_j)] + (2K - 1)\epsilon_n \\
& \leq nh(s_{kG}) - nh(\eta_k \mathbf{z}_k) + nh(y_{1G} | s_{kG}) + \sum_{j=2, j \neq k}^K nh(x_{jG} + n_j) \\
& \quad - nh\left( \sum_{j=2, j \neq k}^K h_j x_{jG} + \tilde{\mathbf{n}}_1 \right) - \sum_{j=2, j \neq k}^K nh(n_j) + (2K - 1)\epsilon_n \\
& = nI(x_{1G}; x_{kG}; y_{1G}, s_{kG}) + \sum_{j=2, j \neq k}^K nI(x_{jG}; y_{jG}) + (2K - 1)\epsilon_n, \quad (35)
\end{align*}
\]

where \( \tilde{\mathbf{n}}_1 \sim \mathcal{CN}(0, 1 - |\rho_k|^2) \), (b) follows since Gaussian inputs maximize differential entropy for a given covariance constraint and from the application of Lemma 1 and Lemma 6 in [7], (c) follows from applying Lemma 1 to the term \( \sum_{j=2, j \neq k}^K h(\mathbf{x}_j^n + \mathbf{n}_j^n) - h(\sum_{j=2, j \neq k}^K h_j \mathbf{x}_j^n + \tilde{\mathbf{n}}_1^n) \), and using the condition \( \sum_{j=2, j \neq k}^K |h_j|^2 \leq 1 - |\rho_k|^2 \). \( \square \)

Using similar arguments as in Lemma 3, the genie is smart if

\[
\eta_k \rho_k = 1 + \sum_{j=2, j \neq k}^K |h_j|^2 P_j, \quad (36)
\]
which ensures that the genie does not increase the sum rate, i.e., \( I(x_{1G}, x_{kG}; y_{1G}, s_{kG}) = I(x_{1G}, x_{kG}; y_{1G}) \). As before, the conditions (34) and (36) can be combined to get (32).

In strategy \( \mathcal{M}3 \), three transmitters form a MAC at receiver 1, i.e., transmitter 1 along with two other transmitters form a MAC at receiver 1, and interference from the rest of the transmitters is treated as noise. We derive a sum-rate outer bound to the \( K \)-transmitter many-to-one XC and characterize the gap between the outer bound and the achievable sum-rate of strategy \( \mathcal{M}3 \).

**Theorem 5:** When transmitter 1, transmitter \( k \) and transmitter \( l \) form a MAC at receiver 1, with \( (k, l) \in (2, \ldots, K) \), \( k \neq l \), if

\[
|h_k| \geq \frac{1 + \sum_{j=2, j\neq k}^{K} |h_j|^2 P_j}{|\rho_k|},
\]

\[
|h_l|^2 \geq 1, \quad \sum_{j=2, j\neq k,l}^{K} |h_j|^2 \leq 1,
\]

then the gap between the sum-rate outer bound and the sum-rate of strategy \( \mathcal{M}3 \) is given by

\[
\log \left( \frac{1 - (1 + \sum_{j=2, j\neq k}^{K} |h_j|^2 P_j)^{-1} |\rho_k|^2}{1 - |\rho_k|^2} \right),
\]

where \( \rho_k \) denotes a constant with \( |\rho_k| \in [0, 1] \).

**Proof:** Let a genie provide side information (33) to receiver 1. Using techniques similar to Lemma 4, it can be shown that the genie is useful, if

\[
\eta_k^2 \leq |h_k|^2, \quad |h_l|^2 \geq 1, \quad \sum_{j=2, j\neq k,l}^{K} |h_j|^2 \leq 1 - |\rho_k|^2,
\]

and the sum-rate of the genie-aided channel is bounded as

\[
S \leq I(x_{1G}, x_{kG}, x_{lG}; y_{1G}, s_{kG}) + \sum_{j=2, j\neq k,l}^{K} I(x_{jG}; y_{1G}).
\]

Note that \( I(x_{1G}, x_{kG}, x_{lG}; s_{kG} ; y_{1G}) \) denotes the gap from the sum-rate optimality of strategy \( \mathcal{M}3 \). From the smartness condition for strategy \( \mathcal{M}2 \), it follows that \( I(x_{1G}, x_{kG} ; s_{kG} ; y_{1G}) = 0 \) if (36) is true. The remaining term \( I(x_{lG} ; s_{kG} ; y_{1G}, x_{1G}, x_{kG}) \) can be bounded similar to (28) and results in the expression given in (38). Combining (36) and (39), we get (37). □

The characterization of the optimality of strategies where more than three transmitters form a MAC at receiver 1 can theoretically be obtained using similar techniques as in Theorem 5. However, we defer this to a future work as the characterization of the gap from the outer bound is decidedly more complicated.
B. A region in which the many-to-one XC can be operated as a many-to-one IC

We identify a region in which the many-to-one XC can be operated as a many-to-one IC without loss of sum-rate. To accomplish this, we need to show that the absence of cross messages does not lead to a decrease in the sum-rate. We have the following result.

**Theorem 6:** The many-to-one XC with $K$ transmitters can be operated as a many-to-one IC without loss of sum-rate in the following sub-region

$$|h_i|^2 \leq 1, \quad i = 2, \ldots, K. \quad (41)$$

**Proof:** Using arguments similar to (10)–(11), it can be shown that if $|h_k|^2 \leq 1, k = 2, \ldots, K$, $W_{1k}$ is decodable at receiver $k$ and

$$h(x^n_k | y^n_k) \leq 2n\epsilon_n. \quad (42)$$

Let $|h_i|^2 \leq 1, i = 2, \ldots, K$. The sum-rate can be bounded as follows:

$$nS = H(W_{11}) + \sum_{k=2}^{K} H(W_{1k}, W_{kk})$$

$$= h(x^n_1) + \sum_{k=1}^{K} h(x^n_k)$$

$$= \sum_{k=1}^{K} I(x^n_k ; y^n_k) + \sum_{k=1}^{K} h(x^n_k | y^n_k)$$

$$\leq \sum_{k=1}^{K} I(x^n_k ; y^n_k) + (2K - 1)n\epsilon_n, \quad (43)$$

where (43) follows from (31) and (42). From (43), it is clear that we can set $W_{1k} = \phi, k = 2, \ldots, K$ (without loss of sum-rate). Thus, we have shown that the absence of cross messages does not diminish the sum-rate when $|h_i|^2 \leq 1, i = 2, \ldots, K$. We note that (43) is in fact the sum-rate of the corresponding $K$-transmitter many-to-one IC.

C. Conditions for sum-rate of strategy of $\mathcal{M}_1$ to be within one bit per user from sum-rate capacity

In the following theorem, we show that in sub-region (41), strategy $\mathcal{M}_1$, i.e., using Gaussian codebooks and treating interference as noise, can achieve a sum-rate to within one bit per user of the sum-rate capacity of the Gaussian many-to-one XC.
**Theorem 7:** In sub-region (41), the rate point achieved by strategy $\mathcal{M}1$, i.e., using Gaussian codebooks and treating interference as noise is within one bit per user from the sum-rate capacity of Gaussian many-to-one XC.

**Proof:** Assume $|h_i|^2 \leq 1$, $i = 2, \ldots, K$, i.e., sub-region (41) is true. Let a genie provide the following side-information to receiver $i$, $i = 2, \ldots, K - 1$

$$s_i = \sum_{j=i}^{K} h_j x_j + n_1.$$  \hfill (44)

Using Theorem 6, receiver $i$ is able to decode $x_i^n$ in sub-region (41), with or without the genie signals. Hence, the sum-rate of the genie-aided channel is bounded as follows:

$$nS \leq I(x^n_1; y^n_1) + \sum_{i=2}^{K-1} I(x^n_i; y^n_i, s^n_i) + I(x^n_K; y^n_K) + (2K - 1)n\epsilon_n$$  \hfill (45)

$$= h(y^n_1) - h(y^n_1 | x^n_1) + \sum_{i=2}^{K-1} \left[ I(x^n_i; s^n_i) + I(x^n_i; y^n_i | s^n_i) \right] + h(y^n_K)$$

$$- h(y^n_K | x^n_K) + (2K - 1)n\epsilon_n$$

$$= h(y^n_1) - h(y^n_1 | x^n_1) + \sum_{i=2}^{K-1} \left[ h(s^n_i) - h(s^n_i | x^n_i) + h(y^n_i | s^n_i) - h(y^n_i | s^n_i, x^n_i) \right] + h(y^n_K)$$

$$- h(y^n_K | x^n_K) + (2K - 1)n\epsilon_n.$$  \hfill (46)

Using the definition of the genie signals in (44), we note that the following are true

$$h(y^n_1 | x^n_1) = h(s^n_2)$$

$$h(s^n_k | x^n_k) = h(s^n_{k+1}), \quad k = 2, \ldots, K - 2.$$  \hfill (47)

Using (47) in (46), we have

$$nS \leq h(y^n_1) - h(s^n_{K-1} | x^n_{K-1}) + \sum_{i=2}^{K-1} \left[ h(y^n_i | s^n_i) - h(n^n_i | s^n_i, x^n_i) \right]$$

$$+ h(x^n_K + n^n_K) - h(n^n_K) + (2K - 1)n\epsilon_n$$

$$\leq nh(y_{1G}) - h(h_K x^n_K + n^n_K) + \sum_{i=2}^{K-1} n \left[ h(y_{1G} | s_{iG}) - h(n_i) \right]$$

$$+ h(x^n_K + n^n_K) - nh(n_K) + (2K - 1)n\epsilon_n$$

$$\leq nh(y_{1G}) + \sum_{i=2}^{K-1} n \left[ h(y_{1G} | s_{iG}) - h(n_i) \right] + nh(x^n_{KG} + n_K)$$

$$- h(h_K x^n_{KG} + n_K) - nh(n_K) + (2K - 1)n\epsilon_n.$$  \hfill (48)
where \(x_{iG} \sim CN(0, P_i)\), \(y_{iG}\) denotes \(y_i\) with \(x_j = x_{jG}\), \(\forall i, j\), (a) follows from Lemma 1 in [7] and the fact that Gaussian inputs maximize the differential entropy for a given covariance constraint, (b) follows from applying Lemma 1 in [6] to the term \(h(x_K^n + n_K) - h(h_K x_K^n + n_1^n)\), and using the condition \(|h_K|^2 \leq 1\). Let \(t_i\) denote the following quantity
\[
t_i = 1 + \sum_{j=i}^{K} |h_j|^2 P_j.
\]
Using (49), we rewrite (48) as
\[
nS \leq n \log \pi e(t_2 + P_1) + n \sum_{i=2}^{K-1} \log \left[ \frac{(1 + P_i) t_i - |h_i|^2 P_i^2}{t_i} \right] + n \log \pi e(1 + P_K) - n \log \pi e(t_K) - n \log \pi e + (2K - 1)n \epsilon_n = \log \left( 1 + \frac{P_1}{t_2} \right) + n \sum_{i=2}^{K-1} \log \left[ \frac{(1 + P_i) t_i - |h_i|^2 P_i^2}{t_{i+1}} \right] + n \log(1 + P_K) + (2K - 1)n \epsilon_n.
\]

The achievable sum-rate of a scheme that employs Gaussian codebooks and treats interference as noise is given by
\[
S_{ach} = \log \left( 1 + \frac{P_1}{1 + \sum_{j=2}^{K} |h_j|^2 P_j} \right) + \sum_{i=2}^{K} \log(1 + P_i) = \log \left( 1 + \frac{P_1}{t_2} \right) + \sum_{i=2}^{K} \log(1 + P_i).
\]

Subtracting (51) from (50), the gap \(\delta\) between the genie-aided outer bound and the achievable sum-rate is given by
\[
\delta = \sum_{i=2}^{K-1} \log \left[ \frac{(1 + P_i) t_i - |h_i|^2 P_i^2}{t_{i+1}(1 + P_i)} \right] + (2K - 1)\epsilon_n
\]
\[
= \sum_{i=2}^{K-1} \log \left[ \frac{(1 + P_i)(|h_i|^2 P_i + t_{i+1}) - |h_i|^2 P_i^2}{t_{i+1}(1 + P_i)} \right] + (2K - 1)\epsilon_n
\]
\[
= \sum_{i=2}^{K-1} \log \left[ 1 + \frac{|h_i|^2 P_i}{t_{i+1}(1 + P_i)} \right] + (2K - 1)\epsilon_n
\]
\[
\leq (c) K - 2 + (2K - 1)\epsilon_n,
\]
where we have used \(|h_i|^2 P_i \leq (1 + P_i)\) and \(t_{i+1} \geq 1\) to write (c). As \(n \to \infty\), \(\epsilon_n \to 0\) and therefore \(\delta/K \leq 1\). We note that if \(K = 3\), \(\delta \leq 1\), which implies that the total gap is within one bit.  
\[\square\]
V. GAUSSIAN MANY-TO-ONE INTERFERENCE CHANNEL

In this section, we observe some implications of the above results for the Gaussian many-to-one IC. The system model for Gaussian many-to-one IC written in standard form is same as that of the many-to-one XC shown in Fig. 5, with the exception that the cross messages are now absent, i.e., $W_{1j} = \phi, j = 2, \ldots, K$. From Fano’s inequality, we have

$$H(W_{ii} | y^u_i) = h(x^u_i | y^u_i) \leq n\epsilon_n,$$  (54)

Note that in the Gaussian many-to-one IC, all transmitters excluding the first cause interference for the reception of the intended signal at receiver 1. Transmission strategies can similarly be defined for the Gaussian many-to-one IC and lead to characterization of sum-rate capacity in some sub-regions. The strategies naturally involve a combination of decoding a part of the interference and treating the rest of the interference as noise. This leads to the following definition.

**Definition 2:** In Strategy $MI_k$, interference resulting from transmissions from $k - 1$ transmitters is decoded and canceled at receiver 1, while the rest of the interference from other transmitters is treated as noise, $k \in \{1, \ldots, K\}$.

Thus, strategy $MI_1$ refers to the case where interference from all transmitters is treated as noise, and results in the low-interference regime. Strategy $MI_K$ refers to the case where interference from all transmitters is decoded and canceled at receiver 1.

A. Conditions for the sum-rate optimality of strategy $MI_k$

We use sum-rate as the criterion of optimality for evaluating the strategies. In the $K$-transmitter Gaussian many-to-one XC studied in Section IV-A, we characterized the sum-rate optimality of strategies $M1, M2$ and also characterized the gap from the optimality of strategy $M3$. However, in the Gaussian many-to-one IC, we characterize the sum-rate optimality of all strategies, $MI_1$ to $MI_K$. Without loss of generality, we assume that strategy $MI_k$ refers to decoding interference from transmitters 2 through $k$, while interference from transmitters $k + 1$ through $K$ is treated as noise. The result for the general case where interference from any subset of transmitters of cardinality $k - 1$ is decoded can be obtained from a reordering of the transmitters without any loss in sum-rate.

Let $Q$ denote the set of integers $\{2, 3, \ldots, k\}$. Let $\pi^Q$ denote any permutation of the set $Q$ with $\pi^Q(i)$ denoting the $i$th element of the permutation. We have the following result on the
sum-rate optimality of strategy $\mathcal{M}Tk$, $k \in \{1, \ldots, K\}$.

**Theorem 8:** For a many-to-one IC with $K$ transmitters satisfying the following channel conditions

$$|h_{\pi Q(i)}|^2 \geq 1 + P_1 + \sum_{j \in \pi Q, j > i} |h_{\pi Q(j)}|^2 P_{\pi Q(j)} + \sum_{j = k+1}^{K} |h_j|^2 P_j, \quad i = 1, \ldots, k - 1,$$

(55)

$$\sum_{j = k+1}^{K} |h_j|^2 \leq 1,$$

(56)

for some permutation $\pi Q$, decoding interference from transmitters 2 to $k$ and treating interference from the rest of the transmitters as noise achieves the sum-rate capacity, and is given by

$$S \leq \log \left( 1 + \frac{P_1}{1 + \sum_{j = k+1}^{K} |h_j|^2 P_j} \right) + \sum_{i = 2}^{K} \log(1 + P_i).$$

Proof: First, we prove the converse. Let a genie provide the following genie signals to receiver 1

$$s_1 = (x_2, x_3, x_4, \ldots, x_k).$$

The sum-rate of the genie-aided channel is given by

$$nS = \sum_{i = 1}^{K} H(W_i) = \sum_{i = 1}^{K} h(x^n_i)$$

$$= I(x^n_1; y^n_1, s^n_1) + \sum_{i = 2}^{K} I(x^n_i; y^n_i) + h(x^n_1 | y^n_1, s^n_1) + \sum_{i = 2}^{K} h(x^n_i | y^n_i)$$

(a)$$\leq I(x^n_1; s^n_1) + I(x^n_1; y^n_1 | s^n_1) + \sum_{i = 2}^{K} I(x^n_i; y^n_i) + \sum_{i = 1}^{K} h(x^n_i | y^n_i)$$

(b)$$\leq I(x^n_1; y^n_1 | s^n_1) + \sum_{i = 2}^{K} I(x^n_i; y^n_i) + nK\epsilon_n$$

$$= h(y^n_1 | s^n_1) - h(y^n_1 | x^n_1, s^n_1) + \sum_{i = 2}^{K} [h(y^n_i) - h(y^n_i | x^n_i)] + nK\epsilon_n$$

$$= h(x^n_1 + \sum_{j = k+1}^{K} h_j x^n_j + n_1^n) - h\left( \sum_{j = k+1}^{K} h_j x^n_j + n_1^n \right) + \sum_{i = 2}^{K} [h(y^n_i) - h(y^n_i | x^n_i)] + nK\epsilon_n$$

(c)$$\leq nh\left( x_{1G} + \sum_{j = k+1}^{K} h_j x_{jG} + n_1^n \right) - h\left( \sum_{j = k+1}^{K} h_j x^n_j + n_1^n \right) + \sum_{i = 2}^{K} nh(y_{iG})$$
\[ + \sum_{i=k+1}^{K} h(x_i^n + n_i^n) - \sum_{i=2}^{K} n h(n_i) + nK \epsilon_n \]

\[ \leq n h \left( x_{1G} + \sum_{j=k+1}^{K} h_j x_{jG} + n_{1G} \right) + \sum_{i=2}^{K} n h(y_{iG}) - n h \left( \sum_{j=k+1}^{K} h_j x_{jG} + n_{1G} \right) - \sum_{i=2}^{K} n h(n_i) + nK \epsilon_n \]

\[ = n I(x_{1G} ; y_{1G}, s_{1G}) + \sum_{i=2}^{K} n I(x_{iG} ; y_{iG}) + nK \epsilon_n \]

\[ = n \log \left( 1 + \frac{P_1}{1 + \sum_{j=k+1}^{K} |h_j|^2 P_j} \right) + \sum_{i=2}^{K} n \log(1 + P_i) + nK \epsilon_n, \quad (57) \]

where (a) follows from the fact that removing conditioning cannot reduce the conditional differential entropy, (b) follows from (54) and the independence of \( s_i^n \) and \( x_i^n \), (c) follows since Gaussian inputs maximize differential entropy for given covariance constraints, and (d) follows from the application of Lemma 1 to bound the term \( \sum_{i=k+1}^{K} h(x_i^n + n_i^n) - h \left( \sum_{j=k+1}^{K} h_j x_j^n + n_j^n \right) \), under the condition \( \sum_{j=k+1}^{K} |h_j|^2 \leq 1 \).

For achievability, note that the sum-rate outer bound in (57) can be achieved by using Gaussian inputs, decoding and canceling interference from transmitters 2 to \( k \) and treating interference from transmitters \( k+1 \) to \( K \) as noise. Assume Gaussian inputs are used at each transmitter, i.e., \( x_i = x_{iG}, i = 1, \ldots, K \). The order in which the signals from transmitters 2 to \( k \) are decoded at receiver 1 determines the channel conditions that must be satisfied for achievability. Here, we use \( \pi^Q \) to denote the decoding order at receiver 1, with \( \pi^Q(i) \) decoded and canceled out before decoding \( \pi^Q(j) \) for \( i < j \).

For ease of presentation, we use \( \pi^Q = \{2, 3, \ldots, k\} \) with no permutation, i.e., \( x_{2G} \) is decoded and cancelled out before decoding \( x_{3G} \) and so on.

Notice that,

\[ I(x_{2G} ; y_{1G}) = I \left( x_{2G} ; x_{2G} + \frac{x_1 + \sum_{j=3}^{K} h_j x_{jG} + n_1}{h_2} \right) \geq I(x_{2G} ; y_{2G}), \]

if \( |h_2|^2 \geq 1 + P_1 + \sum_{j=3}^{K} |h_j|^2 P_j \). Similarly, for some \( 2 < l \leq k \), we have

\[ I(x_{lG} ; y_{1G} | x_{2G}, \ldots, x_{(l-1)G}) = I \left( x_{lG} ; x_{lG} + \frac{x_1 + \sum_{j=l+1}^{K} h_j x_{jG} + n_1}{h_l} \right) \geq I(x_{lG} ; y_{lG}), \]
if \(|h_i|^2 \geq 1 + P_1 + \sum_{j=i+1}^{K} |h_j|^2 P_j\). Combining the above channel conditions, we have

\[
|h_i|^2 \geq 1 + P_1 + \sum_{j=i+1}^{K} |h_j|^2 P_j, \quad i = 2, \ldots, k. \tag{58}
\]

Thus, (55) represents the above condition for a random permutation of \(Q\) and (56) is needed to prove the sum-rate outer bound in (57). This completes the proof of the theorem. \(\square\)

**B. Conditions for sum-rate of strategy of \(\mathcal{M}1\) to be within one bit per user from sum-rate capacity**

Here, we obtain a region for the Gaussian many-to-one IC, where the sum-rate capacity can be characterized to within one bit per user. In Theorem 6 we showed that in sub-region (41), the Gaussian many-to-one XC can be operated as a Gaussian many-to-one IC without loss of sum-rate. Further, in Theorem 7 we showed that in the above sub-region, the sum-rate of strategy \(\mathcal{M}1\) is within one bit per user from the sum-rate capacity. Notice that strategy \(\mathcal{M}1\) for the Gaussian many-to-one XC, which involves using Gaussian codebooks and treating interference as noise, corresponds to strategy \(\mathcal{MI}1\) in many-to-one IC. Since the sum-rate capacity of the Gaussian many-to-one XC forms an outer bound on the sum-rate capacity of Gaussian many-to-one IC, we conclude that strategy \(\mathcal{MI}1\) is within one bit per user from the sum-rate capacity of Gaussian many-to-one IC in sub-region (41).

In the following theorem, we show that strategy \(\mathcal{MI}1\) achieves a rate point that is within one bit per user from the sum-rate capacity of Gaussian many-to-one IC in a region that is much larger than sub-region (41). Let \(S\) denote the set of integers \(S = \{2, 3, \ldots, K\}\). Let \(\pi^S\) denote any permutation of the elements of the set \(S\), with \(\pi^S(k)\) denoting the \(k\)th element of the permutation.

**Theorem 9:** The rate point achieved by using Gaussian codebooks and treating interference as noise is within one bit per user from the sum-rate capacity of Gaussian many-to-one IC in the following sub-regions

\[
|h_{\pi^S(i)}|^2 \leq \left(1 + \frac{1}{P_{\pi^S(i)}}\right) \sum_{j = \pi^S(i+1)}^{K-1} |h_j|^2 P_j, \quad i = 1, \ldots, K - 2,
\]

\[
|h_{\pi^S(K-1)}|^2 \leq 1. \tag{59}
\]
Proof: Without loss of generality, we assume $\pi^S = S$, i.e., no permutation of the elements of the set $S$ is assumed. Thus, $\pi^S(1) = 2$, $\pi^S(2) = 3$ and so on till $\pi^S(K - 1) = K$.

Let a genie provide the side-information given in (44) to receiver $i$, $i = 2, \ldots, K - 1$. The sum-rate of the genie-aided channel is bounded as

$$nS = \sum_{i=1}^{K} H(W_{ii}) = \sum_{i=1}^{K} h(x^n_i)$$

$$= I(x^n_1; y^n_1) + \sum_{i=2}^{K-1} I(x^n_i; y^n_i, s^n_i) + I(x^n_K; y^n_K) + h(x^n_1 | y^n_1) + \sum_{i=2}^{K-1} h(x^n_i | y^n_i, s^n_i)$$

$$+ h(x^n_K | y^n_K)$$

$$\leq I(x^n_1; y^n_1) + \sum_{i=2}^{K-1} I(x^n_i; y^n_i, s^n_i) + I(x^n_K; y^n_K) + \sum_{i=1}^{K} h(x^n_i | y^n_i)$$

$$\leq I(x^n_1; y^n_1) + \sum_{i=2}^{K-1} I(x^n_i; y^n_i, s^n_i) + I(x^n_K; y^n_K) + nK\epsilon_n,$$  (60)

where $(a)$ follows from removing conditioning cannot not reduce the conditional differential entropy, and $(b)$ follows from (54). We recognize that (60) is similar to (45). Notice that the constraint $|h_i|^2 \leq 1$, needed to write (45) for the many-to-one XC is not required in the case of many-to-one IC.

By following essentially the same set of steps as in the Theorem 7 and letting $\delta'$ denote the gap between the genie-aided outer bound and the achievable sum-rate for the many-to-one IC, it follows that $\delta'$ is bounded by (52) if $|h_K|^2 \leq 1$. Note that the condition $|h_K|^2 \leq 1$ is required to write the inequality (48) in Theorem 7.

Using (52), we conclude that for a gap of one bit per user, if $|h_i|^2 P_i \leq t_{i+1}(1 + P_i)$, along with $|h_K|^2 \leq 1$, then $\delta' \leq (K - 2) + K\epsilon_n \Rightarrow \delta'/K \leq 1$. We again note that for $K = 3$, $\delta' \leq 1$, implying that a total gap of one bit is obtained from the sum-rate capacity. The above conditions can be rewritten as

$$|h_i|^2 \leq \left(1 + \frac{1}{P_i}\right) \sum_{j=i+1}^{K} |h_j|^2 P_j, \quad i = 2, \ldots, K - 1,$$

$$|h_K|^2 \leq 1.$$

Note that the above region is much larger than sub-region (41), i.e., $|h_i|^2 \leq 1, i = 2, \ldots, K$, obtained for the many-to-one XC in Theorem 7. We illustrate the above region for $K = 3$ in Fig. 9.
The general case for any permutation $\pi^S$ of $S$ can be proved by giving the following genie signal to receiver $\pi^S(i)$, $i = 1, \ldots, K - 2$

$$s_{\pi^S(i)} = \sum_{j = \pi^S(i)} h_j x_j + n_1,$$

and following the steps given above.

**Remark 2:** In [11], inner and outer bounds to the capacity region of the Gaussian many-to-one IC are presented. The inner bound is based on an achievable scheme which uses lattice codes for alignment of interfering signals at receiver 1. The outer bound is proved by giving an appropriately chosen side information to receiver 1. It is shown that the gap between the inner and outer bounds is approximately $5K \log K$ bits per user with $K + 1$ users in the system.

In Theorem 9, we have strengthened the above result for the sub-region in (59), by showing that using Gaussian codebooks and treating interference as noise is within 1 bit per user of the sum-rate capacity of the many-to-one IC.

**VI. Numerical results**

In this section, we illustrate the regions where the derived channel conditions are satisfied for each strategy. For ease of presentation, we consider the 3-transmitter many-to-one XC for evaluating the strategies.

First, we numerically analyze the sum-rate outer bound for the optimality of strategy $M3$, given in Theorem 3. Let the gap between the sum-rate outer bound and the achievable sum-rate of strategy $M3$ given in (28) be denoted by $\Delta$. Using (28) and solving for $\rho$ in terms of $\Delta$, we get

$$|\rho|^2 \leq \frac{2\Delta - 1}{2\Delta - 1/(1 + |b|^2 P_3)}.$$  \hspace{1cm} (61)

In Fig. 6, we plot $|\rho|^2$ as a function of $\Delta$ for different values of $P_3$ for fixed value of $|b| = 1.5$. It can be observed that $|\rho|^2$ is a monotonically increasing function of $\Delta$. Thus, to obtain a lower gap from the outer bound, a lower value of $|\rho|^2$ must be chosen. This in turn makes the sub-region in (21) smaller. This relationship is explored further in the next two plots.

In Fig. 7 and Fig. 8, we plot the sub region in (21) for the sum-rate optimality of strategy $M3$ as a graph in the $|a| - |b|$ plane for various values of $\Delta$, along with the sub-regions in
Fig. 6. Variation of $|\rho|^2$ as a function of the gap $\Delta$ in bits. $|b| = 1.5$.

Table II for strategies $\mathcal{M}1$ and $\mathcal{M}2$. We assume $P_1 = P_2 = P_3 = 0$ dB. As mentioned above, the sub-region in (21) shrinks for increasing values of $\Delta$.

In Fig. 9, we plot the characterization of sum-rate capacity for the Gaussian many-to-one IC obtained in Theorem 9 for a 3-transmitter many-to-one IC. Also plotted are the channel conditions determined in Theorem 8 for strategies $\mathcal{M}I_1$, $\mathcal{M}I_2$, and $\mathcal{M}I3$ to achieve sum-rate capacity. For $K = 3$, and using same notation as in many-to-one XC with $a = h_2$, $b = h_3$, sub-region (59) becomes

(i) $|a|^2 \leq (1 + |b|^2P_3) \left(1 + \frac{1}{P_2}\right)$; $|b|^2 \leq 1$

(ii) $|b|^2 \leq (1 + |a|^2P_2) \left(1 + \frac{1}{P_3}\right)$; $|a|^2 \leq 1$.

The above region is illustrated in the figure for $P_1 = P_2 = P_3 = 3$ dB. As mentioned earlier, for $K = 3$, the total gap between the sum-rate of strategy $\mathcal{M}I1$ and the sum-rate capacity of the 3-transmitter many-to-one IC is less than one bit. Thus, as long as the channel coefficients lie within this region, the sum-rate capacity can be characterized to within one bit. The channel conditions in (55) and (56) in Theorem 8 for $K = 3$ are summarized in Table III. The sum-rate capacity in the low-interference regime, i.e., strategy $\mathcal{M}I1$ was proved in [7].
VII. CONCLUSIONS

We considered the Gaussian many-to-one X channel with messages on all the links. We formulated different transmission strategies and obtained sufficient channel conditions under which the strategies were either optimal or within a gap from an outer bound. In the process, sum-rate capacity was characterized in some sub-regions of the many-to-one X channel. Subsequently, we identified a region in which the many-to-one X channel can be operated as a many-to-one interference channel without loss of sum-rate and further showed that in this region, the sum-rate capacity can be characterized to within one bit per user. We next formulated transmission strategies for the Gaussian many-to-one interference channel and obtained channel conditions under which the strategies achieved sum-rate capacity. We also identified a region where sum-
rate capacity can be characterized to within one bit per user. This region is larger than the region implied by the one bit per user result for the Gaussian many-to-one X channel.

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| Strategy | Channel conditions |
|----------|-------------------|
| $\mathcal{M}I_1$ | $|a|^2 + |b|^2 \leq 1$ |
| $\mathcal{M}I_2$ | (i) $|a|^2 \geq 1 + P_1 + |b|^2 P_3$, $|b|^2 \leq 1$  
(ii) $|b|^2 \geq 1 + P_1 + |a|^2 P_2$, $|a|^2 \leq 1$ |
| $\mathcal{M}I_3$ | (i) $|a|^2 \geq 1 + P_1 + |b|^2 P_3$, $|b|^2 \geq 1 + P_1$  
(ii) $|b|^2 \geq 1 + P_1 + |a|^2 P_2$, $|a|^2 \geq 1 + P_1$ |

**TABLE III**

**SUM-RATE CAPACITY RESULTS FOR A 3-TRANSMITTER MANY-TO-ONE IC IN THEOREM 8**

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Fig. 9. A plot of the channel conditions in Theorem 8 (summarized in Table III) and Theorem 9 for a 3-transmitter many-to-one IC. $P_1 = P_2 = P_3 = 3$ dB.

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