Predictions from type II see-saw mechanism in $SU(5)$

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We propose a simple, testable, $SU(5)$ model within the context of the type II neutrino see-saw mechanism. It is based on requiring renormalizability, the absence of any other matter fields besides those already present in the Standard Model and consistency with all experimental data. These “minimal” requirements, together with group-theoretical considerations, uniquely determine the model and lead to interesting implications. The model predicts correlation between a light $SU(2)$ triplet boson responsible for the type II see-saw mechanism and observable proton decay signatures. It also allows for an enhanced production of doubly charged Higgs particles through the $WW$ fusion process due to a built-in custodial symmetry. This could also have profound impact on the explicit realization of electroweak symmetry breaking. The model also predicts the existence of a light scalar that transforms as a colour octet and electroweak doublet, with interesting phenomenological consequences.

I. INTRODUCTION

Grand unified theories have been used for a long time as a very elegant framework of physics beyond the Standard Model (SM). While they are tightly constrained by limits on the proton decay lifetime and by the requirement of gauge coupling unification, grand unified models are typically quite complicated and hard to test. The existence of non-zero neutrino masses and mixings has brought new experimental evidence for physics beyond the Standard Model. Understanding the neutrino properties in grand unified theories comes rather naturally through the see-saw mechanism, where integrating out large masses leads to the appearance of small masses. There are three types of see-saw models that can provide an understanding of the neutrino phenomenology:

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type I see-saw models require the existence of SM singlets that have Dirac Yukawa couplings to SM leptons; type II see-saw models use an $SU(2)$ scalar triplet with Majorana type couplings to SM leptons; type III models couple a fermionic $SU(2)$ triplet to SM leptons though a Dirac Yukawa type coupling. While naturally explaining many observed features, the main challenge when building grand unified models and specific types of embedding the see-saw mechanism within these is to find the means for testing the validity of such models. Naturally, a lot of effort has been directed towards building “minimal” models, where “minimal” has been understood in many different ways, but always with the goal of introducing a small number of unknowns in order to keep the theory predictive and testable. In this paper we discuss a highly constrained, testable $SU(5)$ model and its possible consequences. It is based on a small set of “minimal” requirements: renormalizability, the absence of any other matter fields besides those already present in the Standard Model and consistency with all present experimental data. Together with group theoretical considerations, these requirements uniquely determine the model and its implications.

We introduce the model in section II and discuss how it addresses the required symmetry breaking, generation of fermion masses and mixing angles. The model has built-in a type II mechanism for neutrino mass generation and it contains a number of interesting scalars, including two electroweak doublets and two electroweak triplets.

In section III we address in detail the issue of proton decay due to gauge boson and scalar exchange, including partial widths, experimental constraints and flavour dependence.

In section IV we discuss gauge coupling unification. We show how the requirement of unification implies the existence of light scalars, as well as observable proton decay signatures.

In section V we start the discussion of the phenomenological implications of the model. One of the very interesting features of the model is that, while having a complicated Higgs structure and many potentially light scalars, it can preserve custodial symmetry at tree level, such that electroweak constraints can be greatly relaxed compared to simple generic models. We discuss some of the possible collider signatures of the model, in particular the phenomenology of a doubly charged scalar. This has been extensively studied in various contexts and its dominant production mechanism is usually the Drell-Yan process. We emphasize here that in our model it is possible to have a large parameter space where $WW$ fusion becomes the dominant production channel. We also discuss correlations between potential collider signatures and proton decay observations.

In section VI we discuss the potential implications of a colour octet, electroweak doublet, which appears in our model and is rather light.
In section VII we present a comparison of our model with other SU(5) grand unified models. We present our conclusions in section VIII.

II. SCALAR SECTOR: SYMMETRY BREAKING, FERMION MASS GENERATION

As previously mentioned, the SU(5) model we investigate is the simplest possible realization that satisfies the following requirements: renormalizability, the absence of other matter fields besides the ones that have already been observed experimentally and a viable phenomenology. We should thus be able to generate both the breaking of SU(5) to the SM group as well as the SM symmetry breaking, all fermion masses and their mixing angles and gauge coupling unification, as well as be consistent with proton decay and other experimental constraints.

The scalars present in the model are determined by the above requirements. Let us start by specifying the representations responsible for the fermion mass generation. The $i$th generation of the SM matter fields resides in $10_i$ and $\overline{5}_i [1]$. To be specific $10_i = (1, 1, 1)_i \oplus (\overline{3}, 1, -2/3)_i \oplus (3, 2, 1/6)_i = (e_i^C, u_i^C, Q_i)$ and $\overline{5}_i = (1, 2, -1/2)_i \oplus (\overline{3}, 1, 1/3)_i = (L_i, d_i^C)$, where $Q = (u \quad d)^T$, $L = (\nu \quad e)^T$ and $i = 1, 2, 3$. In order to generate mass through a renormalizable operator, a Higgs representation must have a component that is both electrically neutral and SU(3) colour singlet and be in the tensor product of the appropriate matter field representations. For the up quarks one needs either a 5 or 45 of Higgs since the up quark mass originates from the contraction of $10_i$ and $10_j$: $10 \times 10 = \overline{5} \oplus 45 \oplus \overline{5}$. Only 5 and 45 dimensional representations have a component that is both electrically neutral and colour singlet that could thus obtain a phenomenologically allowed vacuum expectation value (VEV). The down quark and charged lepton masses originate from the contraction of $10_i$ with $\overline{5}_j$: $10 \times \overline{5} = 5 \oplus 45$. This time both the 5 and 45 of Higgs are needed to obtain phenomenologically allowed masses. Neutrinos on the other hand reside in $\overline{5}_i$. Their Majorana mass originates from the symmetric contraction of $\overline{5}_i$ with $\overline{5}_j$. Recall $\overline{5} \times \overline{5} = 10 \oplus \overline{15}$, where $\overline{15}$ ($10$) is a (anti)symmetric representation. Hence, to generate the Majorana neutrino masses at the tree level, one must use a 15 of Higgs which happens to have a neutral component as part of a $Y = 2$ SU(2) triplet. This is an SU(5) implementation of the so-called type II see-saw mechanism [2,3,4]. In addition to 5, 15 and 45 dimensional scalar representations one also needs a 24 of Higgs in order to break the SU(5) symmetry. These representations decompose under the SM as $5 = (\Psi_D, \Psi_F) = (1, 2, 1/2) \oplus (3, 1, -1/3)$, $15 = (\Phi_a, \Phi_b, \Phi_c) = (1, 3, 1) \oplus (3, 2, 1/6) \oplus (6, 1, -2/3)$, $24 = (\Sigma_8, \Sigma_3, \Sigma_{(3,2)}, \Sigma_{(3,3)}, \Sigma_{24}) =$
(8, 1, 0) ⊕ (1, 3, 0) ⊕ (3, 2, −5/6) ⊕ (3, 2, 5/6) ⊕ (1, 1, 0), \[45 = (\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7) = (8, 2, 1/2) ⊕ (6, 1, −1/3) ⊕ (3, 3, −1/3) ⊕ (3, 2, −7/6) ⊕ (3, 1, −1/3) ⊕ (3, 1, 4/3) ⊕ (1, 2, 1/2).

This decomposition will be useful when we discuss gauge coupling unification and proton decay. This completes the specification of the Higgs sector that is uniquely determined by group-theoretical considerations once our requirements are imposed. In what follows we will always assume that all terms allowed by the gauge symmetry are present in the Lagrangian density and specify them only when necessary.

A beautiful feature of \(SU(5)\) is that the phenomenologically allowed symmetry breaking chain is unique, i.e, \(SU(5) \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)_{\text{em}}\). The grand unified symmetry is broken down to the SM by the VEV of the SM singlet \(\Sigma_{24}\) in the \(24\) of Higgs:

\[\langle 24 \rangle = v_{24}/\sqrt{30} \text{diag}(2, 2, 2, −3, −3)\]. The symmetry breaking takes place at the so-called GUT scale \(M_{\text{GUT}}\) where the SM gauge couplings unify into \(g_{\text{GUT}}\). At this stage proton decay mediating gauge bosons \(X\) (\(Y\)) absorb the \((\Sigma_{3,2})\) degrees of freedom and become massive. Their masses are \((M_X \cong M_Y \equiv M_V = M_{\text{GUT}} = \sqrt{5/12}v_{24}\)g\). Due to this particular feature one is able to make accurate statements with regard to proton decay signatures via gauge mediation. The electroweak symmetry of the SM is subsequently broken by the VEVs of \(SU(2)\) doublets \(\Psi_D\) and \(\Delta_7\) as well as the VEVs of \(SU(2)\) triplets \(\Phi_a\) and \(\Sigma_3\). The first two are the sources of the charged fermion masses while the third one is the generator of neutrino masses. The VEV of \(\Sigma_3\), on the other hand, affects masses of \(X\), \(Y\), \(W\) and \(Z\) gauge bosons. We will see that the fields that participate in electroweak symmetry breaking should be light in order to have a phenomenologically viable model.

Fermion masses follow from the Yukawa potential:

\[V = (Y_1)_{ij} (10^{\alpha\beta})_i (\bar{\Phi}_a)_j \Phi^*_{\alpha\beta} + (Y_2)_{ij} (10^{\alpha\beta})_i (\bar{\Phi}_b)_j 45^{\alpha\beta}_{\alpha\beta} + (Y_3)_{ij} (\bar{\Phi}_a)_i (\bar{\Phi}_b)_j 15^{\alpha\beta}_{\alpha\beta} + \epsilon_{\alpha\beta\gamma\delta\epsilon} \left[(Y_4)_{ij} (10^{\alpha\beta})_i (10^{\gamma\delta})_j 5' + (Y_5)_{ij} (10^{\alpha\beta})_i (10^{\gamma\delta})_j 45^{\alpha\beta}_{\gamma\delta}\right], \quad i, j = 1, 2, 3, (1)\]

where Greek indices are contracted in the \(SU(5)\) space. The mass matrices, in an obvious notation, are

\[M_D = (Y_1^T v_5^* + 2 Y_2^T v_{45}^*) / \sqrt{2}, \quad (2a)\]

\[M_E = (Y_1 v_5^* - 6 Y_2 v_{45}^*) / \sqrt{2}, \quad (2b)\]

\[M_N = Y_3 v_{15}, \quad (2c)\]

\[M_U = \left[4 (Y_4^T + Y_4) v_5 - 8 (Y_5^T - Y_5) v_{45}\right] / \sqrt{2}, \quad (2d)\]
where \( \langle 5 \rangle = v_5 / \sqrt{2} \), \( \langle 45 \rangle_{15}^{25} = \langle 45 \rangle_{35}^{35} = v_{45} / \sqrt{2} \) and \( \langle 15 \rangle = v_{15} \). Y_1, Y_2, Y_4 and Y_5 are arbitrary \( 3 \times 3 \) Yukawa matrices, while Y_3 represents a symmetric \( 3 \times 3 \) matrix. The factor of 3 difference between the second terms in Eqs. (2a) and (2b) is the so-called Georgi-Jarlskog \([5]\) factor. Its origin is due to the fact that 45 satisfies the following conditions: \( (45)_{\beta \alpha}^{\alpha \beta} = -(45)_{\beta \alpha}^{\beta \alpha} \), \( \sum_{\alpha=1}^{5} (45)_{\alpha \beta}^{\alpha \beta} = 0 \). Hence, one has \( \sum_{i=1}^{3} \langle 45 \rangle_{i5}^{45} = -\langle 45 \rangle_{45}^{45} \). The fermion mass eigenstate basis is defined through the following transformations: \( U^T C M U = M_{U}^{\text{diag}} \), \( D^T C M D = M_{D}^{\text{diag}} \), \( E^T C M E = M_{E}^{\text{diag}} \) and \( N^T C M N = M_{N}^{\text{diag}} \). \( M_{U,D,E,N}^{\text{diag}} \) represent diagonal matrices with real eigenvalues.

Eqs. (2a) and (2b) imply \( M_E^T = (-3) M_D \) if a 5 (45) dimensional Higgs representation is present. In other words, the one Higgs doublet scenario predicts \( m_\tau / m_b = m_\mu / m_s = m_e / m_d \) at the GUT scale, which is in conflict with experimental observations. This is why both the 5 and 45 of Higgs are needed. Eq. (2d), on the other hand, shows that 5 (45) induces a symmetric (antisymmetric) part in \( M_U \). This is very important for the discussion of the flavour dependence of the proton decay signatures. Only if \( M_U \) is a purely symmetric matrix does this dependence disappear in some of the decay channels \([6]\). It is thus clear that in any realistic model the \( SU(5) \) symmetry cannot insure even a partial absence of the flavour dependence in the proton decay signatures \([6,7,8,9]\). We will address this issue in more detail in section III.

Although the Higgs sector looks rather cumbersome, it is the simplest one that yields satisfactory phenomenology while preserving the matter content of the SM. At this point it seems difficult for the model to have any firm and testable predictions unless some additional assumptions are imposed. Fortunately, as we soon demonstrate, the model does predict experimentally observable proton decay. It also predicts that some of the scalars have to be light enough to be of experimental interest in order for unification to take place. Here we refer to \( \Psi_D, \Phi_a, \Sigma_3, \Delta_1 \) and \( \Delta_7 \). If some of these fields are not very light they would jeopardize proton stability and hence rule out the model. In addition, there is a clear correlation between a light \( \Phi_a \) and the proton decay signatures that could allow unambiguous determination of the underlying mechanism of the neutrino mass generation. Recall, the VEV of \( \Phi_a \) generates massive neutrinos. We thus turn to the discussion of proton decay signatures and constraints.
III. PROTON DECAY

Our main predictions rely strongly on consistent application of the current experimental bounds on the partial proton decay lifetimes that constrain the mass spectrum of the model.

The vector gauge boson \( d = 6 \) operators contributing to the decay of the proton in the \( SU(5) \) framework are well-known \[10, 11, 12, 13, 14\]:

\[
\mathcal{O}_1 = k^2 \epsilon_{ijk} \epsilon_{\alpha\beta} \overline{u}_{ia}^{C} \gamma^{\mu} Q_{jaa} \overline{e}_{b}^{\gamma} \gamma_{\mu} Q_{k\beta b,} \tag{3a}
\]

\[
\mathcal{O}_2 = k^2 \epsilon_{ijk} \epsilon_{\alpha\beta} \overline{u}_{ia}^{C} \gamma^{\mu} Q_{jaa} \overline{d}_{kb}^{C} \gamma_{\mu} L_{i\beta b}. \tag{3b}
\]

\( i, j \) and \( k \) are the colour indices, \( a \) and \( b \) are the family indices, \( \alpha, \beta = 1, 2 \) and \( k^2 = 2\pi\alpha_{GUT}M_{(X,Y)}^{-2} \).

The effective operators for decay channels take the following form in the physical basis \[6\]:

\[
\mathcal{O}(e_{\alpha}^{C}, d_{\beta}) = k^2 \left[V_{1}^{11}V_{2}^{\alpha \beta} + (V_{1}V_{UD})^{1\beta}(V_{2}V_{UD}^{\dagger})^{\alpha 1}\right] \epsilon_{ijk} \overline{u}_{i}^{C} \gamma^{\mu} u_{j}^{C} \overline{e}_{k}^{C} \gamma_{\mu} d_{k\beta}, \tag{4a}
\]

\[
\mathcal{O}(e_{\alpha}, d_{\beta}) = k^2 V_{1}^{11}V_{3}^{\beta \alpha} \epsilon_{ijk} \overline{u}_{i}^{C} \gamma^{\mu} u_{j}^{C} \overline{d}_{k}^{C} \gamma_{\mu} e_{k}, \tag{4b}
\]

\[
\mathcal{O}(\nu_{l}, d_{\alpha}, d_{\beta}) = k^2 (V_{1}V_{UD})^{10}(V_{3}V_{EN})^{\beta \alpha} \epsilon_{ijk} \overline{u}_{i}^{C} \gamma^{\mu} d_{j\alpha} \overline{d}_{k}^{C} \gamma_{\mu} \nu_{l}, \quad l = 1, 2, 3. \tag{4c}
\]

\( V_{1} = U_{C}^{\dagger} U, V_{2} = E_{C}^{\dagger} D, V_{3} = D_{L}^{\dagger} E, V_{UD} = U^{\dagger} D = K_{1}V_{CKM} K_{2} \) and \( V_{EN} = E^\dagger N = K_{3}V_{PMNS} \) are unitary mixing matrices. \( K_{1,3} \) and \( K_{2} \) are diagonal matrices containing three and two phases, respectively. \( V_{CKM} \) (\( V_{PMNS} \)) is the usual Cabibbo-Kobayashi-Maskawa (Pontecorvo-Maki-Nakagawa-Sakata) matrix that describes the mixing angles and phases of quarks (leptons).

In what follows we will focus our attention on the proton decay into either a \( \pi \) or \( K \) meson and charged antilepton. For a discussion that treats decays into antineutrinos see \[15\]. The widths for the decays into charged antileptons are:

\[
\Gamma(p \to \pi^{0} e_{\beta}^{+}) = \frac{C(p, \pi)}{2} A_{1}^{2} \left[A_{S,R}^{2} \left|V_{1}^{11}V_{3}^{1\beta}\right|^{2} + A_{S,L}^{2} \left|V_{1}^{11}V_{2}^{\beta 1} + (V_{1}V_{UD})^{11}(V_{2}V_{UD}^{\dagger})^{\beta 1}\right|^{2}\right],
\]

\[
\Gamma(p \to K^{0} e_{\beta}^{+}) = C(p, K) A_{2}^{2} \left[A_{S,R}^{2} \left|V_{1}^{11}V_{3}^{2\beta}\right|^{2} + A_{S,L}^{2} \left|V_{1}^{11}V_{2}^{\beta 2} + (V_{1}V_{UD})^{12}(V_{2}V_{UD}^{\dagger})^{\beta 2}\right|^{2}\right],
\]

where

\[
C(a, b) = \frac{(m_{a}^{2} - m_{b}^{2})^{2}}{8\pi m_{a}^{3} f_{\pi}^{2}} A_{L}^{2} |\alpha|^{2} k^{4}. \tag{5}\]

The relevant \( A_{i} \) factors are: \( A_{1} = 1 + D + F \) and \( A_{2} = 1 + \frac{m_{\pi}}{m_{B}}(D - F) \) \[15\]. To generate numerical results we use \( m_{\pi} = 938.3 \text{ MeV}, D = 0.81, F = 0.44, m_{B} = 1150 \text{ MeV}, f_{\pi} = 139 \text{ MeV}, A_{L} = 1.25, |V_{ud}| = 0.97377, |V_{ub}| = 3.96 \times 10^{-3}, \) and \( \alpha = 0.015 \text{ GeV}^{2} \) \[16\]. Here, \( \alpha \)
is the so-called matrix element. In addition one needs to evaluate the leading-log renormalization of the operators $\mathcal{O}(e_c, d_\beta)$ and $\mathcal{O}(e_\alpha, d_\beta^c)$ from the GUT scale to $M_Z$ which is described by the coefficients $A_{SL}$ and $A_{SR}$ respectively. (The QCD running below $M_Z$ is captured by the coefficient $A_L$.) These coefficients are $[13, 17, 18]$: 

$$
A_{SL(R)} = \prod_{i=1,2,3} \prod_{I \leq M_Z \leq M_{GUT}} \frac{\alpha_i(M_{I+1})}{\alpha_i(M_I)} \prod_{O} \frac{\gamma_{L(R)}(O)}{\prod_{O} \gamma_{L(R)}(O)} , \quad \gamma_{L(R)} = (23(11)/20, 9/4, 2). \tag{6}
$$

$b_i^J$ are the usual $\beta$-function coefficients due to particle $J$ of mass $M_J$ and $\alpha_i(M_I)$ are the gauge coupling constants at the scale $M_I$. We use the following experimental values at $M_Z$ in the $\overline{MS}$ scheme $[19]$: $\alpha_3 = 0.1176 \pm 0.0020$, $\alpha^{-1} = 127.906 \pm 0.019$ and $\sin^2 \theta_W = 0.23122 \pm 0.00015$.

To predict the partial lifetimes of the proton for these decay channels we still need to know $k$, $V_1^{ib}$, $V_2$ and $V_3$. In addition there are two diagonal matrices containing CP violating phases, $K_1$ and $K_2$. Therefore it is impossible to test a general $SU(5)$ scenario through the decay of the proton unless we specify both the full flavour structure and mass spectrum of the GUT model. What is then usually assumed for the flavour structure, in order to extract a conservative limit on the GUT scale, is that $U_C = U$, $D_C = E$ and $E_C = D$. Under these assumptions the dominant proton decay mode is $p \rightarrow \pi^0 e^+$ and the theoretical prediction for this channel comes out to be $\tau_{\text{theo.}} = 3.1 \times 10^{33} (M_{GUT}/10^{16} \text{GeV})^4 \alpha_{GUT}^{-2} (\alpha/0.015 \text{ GeV}^3)(A_{SR}^2 + 3.8 A_{SL}^2)^{-1}$ years. The current experimental limit on the partial lifetime $\tau_{\text{exp.}} > 4.4 \times 10^{33}$ years $[20]$ thus translates into the following bound on $M_{GUT}$: $M_{GUT} > 2.6 \times 10^{16} \sqrt{\alpha_{GUT}} \text{ GeV}$ where we take $A_{SL} = A_{SR} = 2.5$.

Of course, if both the particle content and mass spectrum of the model are known it is possible to evaluate $A_{SL}$ and $A_{SR}$ more accurately.

For the proton decay through scalar exchange—for example via $\Psi_T$—the relevant couplings are Yukawa couplings of the first and second generation that are expected to be of the order of $\mathcal{O}(10^{-6} - 10^{-4})$. We thus get the relevant scale at which scalar exchange becomes dominant by replacing $\alpha_{GUT}$ with $Y^2$. This in turn yields a lower bound on the phenomenologically allowed scalar mass to be around $10^{12}$ GeV.

Finally, let us for completeness discuss the flavour dependence of the experimental bound on $M_{GUT}$. We will assume for the sake of argument the following flavour scenario $[9]$: $(V_1 V_{UD})^{1\alpha} = 0$ and $V_2^{\alpha \beta} = V_3^{\alpha \beta} = 0$ ($\alpha = 1$ or $\beta = 1$). It is easy to see from Eq. $[4c]$ that there will be no decays into antineutrinos while the only surviving channel with an antilepton in the final state is actually $p \rightarrow K^0 \mu^+$. We get $\Gamma(p \rightarrow K^0 \mu^+) = C(p, K) A_2^2 [A_{SR}^2 + A_{SL}^2] |V_{ub}|^2$ which is more then
six orders of magnitude smaller than the decay width when \( U_C = U, D_C = E \) and \( E_C = D \). Both flavour scenarios are a priori possible.

In order to show that our model predicts observable proton decay signatures and prefers if not predicts certain light scalars including the SU(2) triplets with \( Y = 0 \) and \( Y = 2 \), we need to address the issue of gauge coupling unification with these results in mind. We will explicitly assume the flavour scenario where \( U_C \approx U, D_C \approx E \) and \( E_C \approx D \) in what follows.

### IV. GAUGE COUPLING UNIFICATION

The behavior of the gauge couplings between the electroweak and the GUT scale is described by three renormalization group equations—one for each gauge coupling of the SM \( \alpha_i (i = 1, 2, 3) \). If we impose unification and accordingly eliminate the unified coupling constant \( \alpha_{GUT} \), we are left with only two relevant equations [21]. These are:

\[
\begin{align*}
\frac{B_{23}}{B_{12}} &= \frac{5 \sin^2 \theta_W - \alpha/\alpha_3}{8 \left( 3/8 - \sin^2 \theta_W \right)} = 0.716 \pm 0.005, \\
\ln \frac{M_{GUT}}{M_Z} &= \frac{16\pi}{5\alpha} \frac{3/8 - \sin^2 \theta_W}{B_{12}} = \frac{184.9 \pm 0.2}{B_{12}},
\end{align*}
\]

where the right-hand sides reflect the latest experimental measurements of the SM parameters [19].

In view of the fact that we are interested in proton decay signatures of our model, Eq. (7b) is especially interesting. Namely, for a given minimal \( B_{12} \) value of a specific model, it is possible to obtain associated upper bound on \( M_{GUT} \), which is a crucial ingredient for accurate proton decay predictions. The \( B_{ij} \) coefficients on the other hand depend on the specific particle spectrum. More precisely, \( B_{ij} = B_i - B_j \), where \( B_i \) coefficients are given by:

\[
B_i = \sum_I b_{iI} r_I, \quad r_I = \frac{\ln M_{GUT}/M_I}{\ln M_{GUT}/M_Z}, \quad (0 \leq r_I \leq 1).
\]

The SM content with \( n \) light Higgs doublet fields has \( B_1 = 40/10 + n/10, B_2 = -20/6 + n/6 \) and \( B_3 = -7 \). Hence the SM case (\( n = 1 \)) yields \( B_{23}^{SM}/B_{12}^{SM} = 0.53 \). Clearly, additional particles with masses below the GUT scale are required for successful unification. In addition to satisfying Eq. (7a), any potentially realistic grand unified scenario must generate large enough GUT scale in order to satisfy the proton decay constraints. The careful analysis of the \( X \) and \( Y \) mediated proton decay from the previous section implies a lower bound on the GUT scale in SU(5) to be \( M_{GUT} > 4-5 \times 10^{15} \) GeV. Again, we have assumed that \( U_C \approx U, D_C \approx E \) and \( E_C \approx D \).
Our first aim is to show that successful unification implies proton decay signatures that are within the reach of the future proton decay experiments regardless of the exact mass spectrum of the scalars in our model. Our discussion will also imply that some of the scalars—those that do not mediate proton decay—are always very light in order to have phenomenologically acceptable proton decay widths. We thus start by looking at the impact of light scalars $I$ with negative contributions toward the $B_{12}$ coefficient (i.e., $\Delta b_{12}^I = b_1^I - b_2^I < 0$) on unification, since only those fields can raise the GUT scale. We show that, if the vector gauge boson mediated proton decay is suppressed beyond the experimentally established limit, then the proton decay due to scalar exchange is experimentally accessible and vice versa.

The multiplets with negative contribution to $B_{12}$ are $\Phi_D$, $\Sigma_3$, $\Delta_1$, $\Delta_3$, $\Delta_7$, $\Phi_a$ and $\Phi_b$. We have underlined them for convenience in Table I where we list all the $\Delta b_{ij} = b_i - b_j$ contributions. $\Delta_3$ and $\Phi_b$ cannot be arbitrarily light in order to avoid existing experimental limits on partial proton decay lifetimes. The fact that a $\Delta_3$ exchange could contribute to proton decay has been recently pointed out [22]. Other fields that mediate proton decay but have positive $B_{12}$ contributions are $\Psi_T$, $\Delta_5$ and $\Delta_6$. We have placed a line over them in Table I for convenience. All these scalars should have masses of the order of $10^{12}$ GeV or higher unless some special arrangements take place in the Yukawa sector that would suppress their contributions to proton decay. The important point is that if these scalar fields are as light as $10^{12}$ GeV their proton decay signatures would be at their present experimentally established limits. With that in mind we now determine an upper bound on the GUT scale.

We take the fields that do not mediate proton decay and set their masses to $M_Z$, i.e., $r(\Psi_D, \Sigma_3, \Delta_1, \Delta_7, \Phi_a) = 1$, to get $B_{12} = (110/15 - 1/15 - 15/15)$. The first two contributions are the usual SM contributions to $B_{12}$ while the net effect of all other scalar multiplets on $B_{12}$ is rather small. In fact, this yields via Eq. (7b) that $M_{GUT} \simeq 6 \times 10^{14}$ GeV. This is clearly below the lower bound on $M_{GUT}$ as inferred from the experimentally measured limits on proton decay lifetime. If we now take into account that $\Delta_3$ and $\Phi_b$ could only be as light as $10^{12}$ GeV, which roughly translates into $r(\Delta_3, \Phi_b) \leq 1/3$, we obtain $M_{GUT} \leq 3 \times 10^{16}$ GeV which should be considered as a conservative upper bound on the GUT scale in our model at one-loop. If any of the fields with negative contributions to $B_{12}$ is actually heavier than what we have assumed, then the GUT scale would accordingly go down in proportion to the corresponding $b_{12}$ contribution. In particular, if we want to suppress proton decay rates due to the scalar exchange by taking $\Delta_3$ and $\Phi_b$ masses to be significantly above $10^{12}$ GeV, we would significantly enhance decay rates due to the vector
gauge boson exchange.

### Table I: The scalar $B_{ij}$ coefficient contributions.

| $\Psi_D$ | $\Psi_T$ | $\Sigma_8$ | $\Sigma_3$ | $\Delta_1$ | $\Delta_2$ | $\Delta_3$ | $\Delta_4$ | $\Delta_5$ | $\Delta_7$ | $\Phi_a$ | $\Phi_b$ | $\Phi_c$ |
|----------|----------|------------|------------|-------------|-------------|-------------|-------------|-------------|-------------|-----------|-----------|-----------|
| $\Delta b_{23}$ | $\frac{1}{6}$ | $\frac{-1}{6}$ | $\frac{-3}{6}$ | $\frac{2}{6}$ | $\frac{-4}{6}$ | $\frac{-5}{6}$ | $\frac{1}{6}$ | $\frac{-1}{6}$ | $\frac{1}{6}$ | $\frac{4}{6}$ | $\frac{1}{6}$ | $\frac{-5}{6}$ |
| $\Delta b_{12}$ | $\frac{-1}{15}$ | $\frac{1}{15}$ | $0$ | $\frac{-5}{15}$ | $\frac{-8}{15}$ | $\frac{2}{15}$ | $\frac{-27}{15}$ | $\frac{17}{15}$ | $\frac{16}{15}$ | $\frac{-1}{15}$ | $\frac{-1}{15}$ | $\frac{-7}{15}$ | $\frac{8}{15}$ |

These are obviously good news as far as the testability of the model is concerned. The model certainly implies that proton decay should take place within the experimentally accessible range [23, 24] regardless of the exact scalar mass spectrum. In addition, some of the scalars with negative $B_{12}$ contributions that do not mediate proton decay are rather light. Here, in particular, we refer to $\Psi_D$, $\Sigma_3$, $\Delta_1$, $\Delta_7$ and $\Phi_a$. $\Psi_D$ and $\Delta_7$ are SU(2) doublets, $\Sigma_3$ and $\Phi_a$ are SU(2) triplets while $\Delta_1$ transforms as a doublet of SU(2) and octet of SU(3). Each of these fields is interesting in its own right, especially from the point of view of accelerator physics, which is an exciting prospect.

We have so far neglected the fact that Eqs. (7a) and (7b) should be solved simultaneously. Let us do that within the following scenario. Let us (i) fix the mass of $\Phi_a$ to 300 GeV which will certainly be within the reach of accelerator experiments, (ii) impose $M_{\Sigma_8} \geq 10^5$ GeV as required by nucleosynthesis considerations (see for example discussion in Ref. [25] and references therein), (iii) set $M_{\Delta_3} = M_{\Phi_b} = 10^{12}$ GeV and (iv) vary all other fields in the model within their allowed range in order to maximize $M_{GUT}$ via Eqs. (7b) and Eq. (7a). This simple exercise yields $M_{GUT} \leq 1.4 \times 10^{16}$ GeV. This is in a good agreement with our previous analysis. This value is obtained when $\alpha_{GUT}^{-1} = 29.4$, $M_{\Sigma_3} = M_Z$, $M_{\Sigma_8} = 10^5$ GeV, $M_{\Delta_1} = M_Z$, $M_{\Delta_2} = 2 \times 10^{10}$ GeV and $M_{\Delta_7} = M_Z$. All other fields are at the GUT scale. In this case the predicted proton lifetime for $p \rightarrow \pi^0 e^+$ due to gauge mediation is a factor of 51 above the current experimental limit while the proton lifetime due to scalar mediation is at the present limit.

If the scalar exchange induced proton decay is suppressed then the vector boson exchange contributions is experimentally accessible. To illustrate that we set $M_{\Delta_3} = M_{\Phi_b} = 10^{13}$ GeV and keep $\Phi_a$ again at 300 GeV to obtain $M_{GUT} \leq 5.2 \times 10^{15}$ GeV. The predicted proton lifetime through the gauge boson mediation is then exactly at the current experimental limit.

Note that in both cases we set some of the fields at the $M_Z$ scale, which is likely not realistic. In other words, the upper bound on the GUT scale we discuss here is very conservative. With this...
in mind we turn to the important question of testing this model in collider experiments.

V. ELECTROWEAK SYMMETRY BREAKING SECTOR

Let us start with the discussion of the electroweak symmetry breaking sector of the model. It comprises two $SU(2)$ doublets—$\Psi_D$ and $\Delta_7$—as well as two $SU(2)$ triplets—$\Phi_a$ and $\Sigma_3$. Interestingly enough, this particular Higgs content can preserve the custodial symmetry of the SM at tree level and thus accommodate precision electroweak constraints. In fact, it corresponds to the content of the models that have been tailor-made to accomplish just that [26, 27, 28]. Here we have an example where the same kind of setup could naturally emerge within a well-motivated GUT framework.

Since our primary concern is the possibility to test the underlying see-saw mechanism, we observe that one of the consequences of this custodial symmetry could be that the couplings of the see-saw triplet $\Phi_a$ to the gauge bosons are much larger than expected from the standard limits set by electroweak precision measurements.

Testing the electroweak symmetry breaking sector will be very challenging due to its complexity. However, one nice feature is that $\Phi_a$ contains a doubly charged Higgs boson $\Phi_a^{\pm\pm}$ that does not mix with any other Higgs field in the model. This makes the analysis of its experimental signatures relatively model independent. With this in mind we limit our discussion of accelerator signatures of light scalar particles mainly to the $\Phi_a^{\pm\pm}$ production and subsequent decay.

There are a number of well-motivated models that all have potentially light $Y = 2$ triplet(s). These are primarily the left-right symmetry models [29, 30], little Higgs models [31, 32, 33], and certain type II see-saw extensions of the SM [34]. Due to this and the fact that $\Phi_a^{\pm\pm}$ does not mix with other Higgs fields there exists a large body of work on the doubly charged Higgs signatures in current [35, 36, 37] and future colliders [36, 38, 39, 40, 41]. We accordingly point out only those salient features that could make our model different from other models.

The dominant production of $\Phi_a^{\pm\pm}$s at the Tevatron and LHC is either through the Drell-Yan (DY) $\Phi_a^{++}\Phi_a^{--}$ pair production or $WW$ fusion into a single doubly charged component of $\Phi_a$. $WW$ fusion is proportional to the triplet VEV which is primarily bounded from above by the electroweak precision measurements due to its impact on the so-called $\rho$ parameter. This bound is around 2 GeV within the framework of the SM extended with a $Y = 2$ triplet only. In our model however, both $Y = 0$ and $Y = 2$ triplets get VEVs in addition to the two Higgs doublets, i.e.,
\[ \langle 5 \rangle = v_5 / \sqrt{2}, \langle 45 \rangle_1^{15} = \langle 45 \rangle_2^{25} = \langle 45 \rangle_3^{35} = v_{45} / \sqrt{2}, \langle \Sigma_3 \rangle = v' \text{ and } \langle 15 \rangle = v_{15}. \] If we take all these VEVs into account the net tree-level contribution to the \( \rho \) parameter is

\[ \rho = \frac{v_5^2 + v_{45}^2 + 4v_{15}^2 + 4v'^2}{v_5^2 + v_{45}^2 + 8v_{15}^2}. \] (9)

The \( W \) mass is given as

\[ M_W = g^2 / 4(v_5^2 + v_{45}^2 + 4v_{15}^2 + 4v'^2), \]

where \( \alpha_2 = g^2 / (4\pi) \cdot \sin^2 \theta_W \), on the other hand, is not affected by additional VEVs at all. It is easy to see that \( \rho \approx 1 \) naturally at the tree level as long as \( v_L \approx v' \), regardless of their absolute value. In fact, \( v_L(\approx v') \) could be as large as 80 GeV as far as the \( \rho \) parameter and perturbativity of the top Yukawa constraints are concerned at the tree level. This is possible in any \( SU(5) \) scenario with [42] or without supersymmetry [43, 44, 45] that implements the type II see-saw mechanism, as well as in the corresponding \( SO(10) \) models.

There are thus two distinct regions in the parameter space of our model in terms of \( v_L \) values. If \( v_L \approx v' \approx v_5 \approx v_{45} \) then the \( WW \) fusion into a doubly charged component of \( \Phi_a \) would overcome DY production of the \( \Phi_a^{++} \Phi_a^{--} \) pair and its subsequent decay could primarily be into a \( WW \) pair instead of a pair of charged leptons. The \( WW \) pair would eventually decay into a pair of charged leptons and pair of neutrinos 10\% of the time that would then enable the detection of \( \Phi_a^{++} \) at the LHC. The crucial point is that this process has a rather small SM background. Analysis based on the ATLAS simulation shows the possibility to detect \( \Phi_a^{++} \) as heavy as 1 TeV if \( v_L \sim 29 \) GeV [39] at LHC. If on the other hand \( v_L \approx v' \ll v_5 \approx v_{45} \), then the DY production would dominate and subsequent \( \Phi_a^{++} \Phi_a^{--} \) decay into charged leptons would constitute a clean signal. The most recent analysis put the LHC reach at around 700 GeV in the \( l^\pm l^\pm \) channel [40, 41].

In our model it is also possible to correlate \( \Phi_a^{++} \) detection with the expected proton decay signatures. The main difference between the two distinct regions in parameter space is in the strength of the Majorana neutrino Yukawa couplings in \( Y_3 \). In the first case these would be extremely small and would not allow the mapping of the neutrino mass matrix through the decay of \( \Phi_a^{++} \) into a pair of charged leptons: \( \Gamma(\Phi_a^{++} \rightarrow l_i^+ l_j^+) \sim |(Y_3)_{ij}|^2 \). The second case is more promising in that respect since the relevant Yukawa couplings could be sufficiently large. In addition, the branching ratios could shed light on the particular realization of the mass hierarchy in the neutrino sector, For example, the normal hierarchy scenario implies \( BR(\Phi_a^{++} \rightarrow l_i^+ l_j^+) \approx 1/3 \) for \( i, j = 2, 3 \).

The best current limits on the \( \Phi_a^{\pm\pm} \) mass come from searches performed at the Tevatron. The lower bound on \( \Phi_a^{\pm\pm} \) comes out to be around 130 GeV assuming exclusive same-sign dilepton decays [35]. This bound however is derived by explicitly assuming negligible \( v_L \). In case of
inclusive searches for dilepton events there exists some excess of events in a recently published analysis [37].

To summarize, the main difference between our model and the majority of models that incorporate a $Y = 2$ triplet only lies in (i) the possibility to have its couplings to gauge bosons significantly enhanced and (ii) the ability to correlate proton decay with the triplet detection.

VI. COLOUR OCTET

Gauge coupling unification constraints impose a firm upper bound on $M_{\Delta_1}$ in our model. We find that $M_{\Delta_1} < 250$ TeV holds for any successful unification scenario. This is yet another important prediction of our model. With that in mind we should stress that $\Delta_1$, being an $SU(3)$ octet that has doublet like couplings to matter, is phenomenologically very interesting. Its experimental signatures and relevant limits on its couplings to matter have been recently discussed within the context of minimal flavour violation [46, 47]. In that context it is assumed that its couplings to matter and the corresponding mass matrices of the matter fields are proportional to each other in the mass eigenstate basis. The phenomenology in that scenario, the relevant constraints on the octet couplings and mass as well as a recent analysis of its potential production at LHC can be found in Refs. [48, 49].

In our case however the couplings of the octet to the matter fields make only one part of the linear combination that enters the relevant mass matrices as shown in Eqs. (2). Thus, they cannot be brought to diagonal form through the same bi-unitary transformations that define the matter field mass eigenstate basis. Clearly, the strength of the exchange of neutral components of $\Delta_1$ will be constrained due to the tree-level contributions towards $F^0 - \bar{F}^0$ mixing processes $(F = K, B, D)$. For example, using the vacuum saturation approximation for the hadronic matrix element [50], we find a new contribution towards $\epsilon_K$ coming from the $\Delta_1$ exchange to be

$$\epsilon_K \simeq \frac{\sqrt{2} f_K^2 M_K B_K}{9 \Delta M_K M_{\Delta_1}^2} \text{Im} \left[ 4(D^T Y_2 D_C)^*_{21}(D^T Y_2 D_C)_{21} \right].$$  (10)

Using $B_K = 0.75$, $\Delta M_K \simeq 3.48 \times 10^{-12}$ MeV, $f_K \simeq 160$ MeV, $M_K \simeq 498$ MeV and requiring that $\Delta_1$ exchange contributes to $\epsilon_K$ an amount less than the experimental value of that quantity ($|\epsilon_K| = 2.23 \times 10^{-3}$ [19]) gives the following limit

$$M_{\Delta_1}^2 > 2 \times 10^{14} \text{Im} \left[ 4(D^T Y_2 D_C)^*_{21}(D^T Y_2 D_C)_{21} \right] \text{GeV}^2.$$  (11)
Our discussion has made it clear that at least some of the entries of $Y_2$ have to be non-zero in order to correct equality of down quark to charged lepton ratios. So, if and when the mass of $\Delta_1$ is determined, we would have a handle on the strength of Yukawa couplings of the 45 of Higgs using constrains such as the one from the $K$ sector. Notice that, unlike a flavour changing neutral current generating doublets of $SU(2)$ that are singlets of $SU(3)$ that can also contribute to the processes such as $\mu-e$ conversion and/or $\mu \rightarrow e\gamma$, our octet is very selective since it couples only to quarks.

VII. $SU(5)$ MODEL COMPARISONS

It might come as a surprise that our model with so many Higgs multiplets does so well in terms of its potential accelerator and proton decay signatures. To better show the origin and quality of its predictive power we compare our model with other possible extensions of the original Georgi-Glashow (GG) $SU(5)$ scenario.

A. $SU(5)$ model with type I see-saw mechanism

We start with the $SU(5)$ model that extends the GG model with a 45 dimensional Higgs representation and at least two right-handed neutrinos, singlets of $SU(5)$, in order to give neutrinos their mass. That setup has fewer fields that can influence gauge coupling unification than our model. It is in fact already ruled out experimentally unless there exists some suppression of $d=6$ proton decay operators due to scalar exchange [22]. This runs against conclusions reached in previous analysis [21, 51].

The only relevant degrees of freedom in that model, as far as the upper bound on the GUT scale is concerned, are $\Sigma_3, \Delta_1, \Delta_3$ and $\Delta_7$. See Table I for relevant $b_{ij}$ coefficients. We can thus plot the lines of constant $M_{\Delta_1}$ and $M_{\Delta_3}$ in the $M_{GUT}-M_{\Sigma_3}$ plane using Eqs. (7a) and (7b) for a fixed mass of $\Delta_7$. In other words, we can show all viable particle mass spectra of the theory that yield gauge coupling unification. Recall, $\Delta_7$ is the usual $SU(2)$ doublet that resides in 45. We set $M_{\Delta_7} = M_Z$ in order to get the most conservative upper bound on $M_{GUT}$.

The available parameter space of the theory is shown in Fig. I. Again, any given point that is not excluded in Fig. I represents a particle spectrum that yields exact one-loop unification. A dashed line corresponds to a lower phenomenological bound on the GUT scale as given by the proton decay constraints. Clearly, the GUT scale is also bounded from above at around $10^{45.9}$ GeV.
due to the constraint $M_{\Delta_1} \geq M_Z$. Although unification does take place, proton decay constraints are not all successfully satisfied. In particular, the mass of $\Delta_3$, i.e. the scalar that mediates proton exchange, is below the experimentally inferred bound of $10^{12}$ GeV in the otherwise allowed region. Simply put, the simplest $SU(5)$ model with type I see-saw is already experimentally ruled out under rather reasonable assumptions about its flavour structure. We also estimate from Fig. 1 that the partial proton decay lifetime due to the vector gauge boson mediation is at most a factor of 10 away from its present bound. So, even if the scalar contributions are assumed to be suppressed, this model would be ruled out by the next generation of proton decay experiments. In addition, it is clear that the mass of $\Delta_1$ is bounded from above by the proton constraints to be less than $10^4$ GeV.

It is now clear why our model fairs so well. The only additional fields that affect the GUT scale in our case are $\Phi_a$ and $\Phi_b$. The former has very small $B_{12}$ contribution and thus makes
no significant disturbance of the unification picture we present in Fig. [1] The latter, which does have a potentially significant $B_{12}$ contribution, cannot contribute too much due to the existing lower bound on its mass that originates from proton decay constraints. If combined, both of these contributions have just enough strength to satisfy all phenomenological constraints and allow $M_{\Delta_3}$ at or above $10^{12}$ GeV. Clearly, even after the 15 of Higgs is taken into account, the masses of $\Sigma_3$, $\Delta_1$ and $\Delta_7$ must still be very low, of the order of the electroweak scale. That is exactly what we have observed in our previous discussion of the model with both 15 and 45.

### B. $SU(5)$ with the hybrid—type I + type III—see-saw mechanism

Another model we would like to compare our model with is a recent extension of the GG model that realizes hybrid see-saw of the type I and type III nature [25]. (For detailed studies of that model see Refs. [52, 53].) This model extends the GG model with only one extra adjoint representation of fermions $24_F = (\Omega_8, \Omega_3, \Omega_{(3,2)}, \Omega_{(3,2)}, \Omega_{24}) = (8, 1, 0) \oplus (1, 3, 0) \oplus (3, 2, -5/6) \oplus (\bar{3}, 2, 5/6) \oplus (1, 1, 0)$ and predicts a very light $SU(2)$ fermionic triplet with $Y = 0$. We first compare these two models on general grounds in terms of their predictions for the GUT scale. $B_{ij}$ contributions of $24_F$ components are four times larger than those of corresponding $\Sigma$ components. Hence, in this particular case $B_{12}^{\min} = 22/3 - 1/15 - 5/15 - 20/15$ where the third (fourth) term is due to the $\Sigma_3$ ($\Omega_3$) contribution. This should be compared with $B_{12}^{\min} = 22/3 - 1/15 - 5/15 - 8/15 - 1/15 - 1/15 - 27/15 r_{\Delta_3} - 7/15 r_{\Phi_a}$, where $r_{\Phi_a}, r_{\Delta_3} \leq 1/3$ due to phenomenological constraints. We thus obtain comparable values for $B_{12}^{\min}$ in both cases. So even though there are only two degrees of freedom that can minimize $B_{12}$ in the model with $24_F$, their impact on the running of gauge couplings and hence proton decay predictions equals the impact of all the fields in our scenario. The important difference, of course, is that this model is based on higher-dimensional operators while our model is renormalizable. If the idea of hybrid see-saw is implemented within the simplest renormalizable scenario its predictive power is significantly compromised [54].

Since we are interested in the possibility to test the underlying mechanism for neutrino mass generation within the grand unified framework, we assume that the relevant scale for the fields that generate neutrino mass in both models is 300 GeV, i.e., $M_{\Omega_3} = M_{\Phi_a} = 300$ GeV, and compare them after we obtain the upper bound on the GUT scale. The result of this simple numerical comparison is summarized in Table [II]
TABLE II: Comparison between our model with type II see-saw mechanism and the hybrid scenario where both type I and type III see-saw mechanisms are used. We assume $M_{\Omega_4} = M_{\Phi_a} = 300$ GeV and maximize the GUT scale. In both cases $M_{\Sigma_3} = M_Z$ and $M_{\Psi_T} = M_{\text{GUT}}$.

| MODEL            | $A_{SR}$ | $A_{SL}$ | $(M_{\text{GUT}}/10^{16}$ GeV) | $\alpha_{GUT}^{-1}$ | $\tau_{d=6}$ gauge/τ_{exp.} | $\tau_{d=6}$ scalar/τ_{exp.} |
|------------------|----------|----------|-------------------------------|---------------------|-----------------------------|-----------------------------|
| Doršner-Mocioiu  | 2.8      | 3.0      | 1.4                           | 29.4                | 51                          | 1                           |
| Bajc-Senjanović  | 2.5      | 2.7      | 1.5                           | 37.6                | 150                         | 15000                       |

It is evident from Table II that our model insures correlations between the direct detection of the field responsible for the neutrino mass generation and observable proton decay signatures. That possibility is less likely in the model with the hybrid see-saw implementation.

VIII. CONCLUSIONS

We have investigated a well-motivated $SU(5)$ model which implements a type II see-saw mechanism for neutrino mass generation. The model is uniquely determined by requiring renormalizability, the lack of any additional matter fields besides those already observed, gauge coupling unification and a viable phenomenology.

We have shown it is possible to test the underlying mechanism for neutrino mass generation through accelerator signatures and correlations with observable proton decay. The model predicts that all fields that can participate in electroweak symmetry breaking are light. Due to a built-in custodial symmetry, the constraints from precision electroweak measurements are relaxed compared to standard general analysis and our model allows a possible enhancement of the couplings of the $Y = 2$ $SU(2)$ triplet to gauge bosons. This sort of setup can work in any $SU(5)$ theory with type II see-saw neutrino mass generation. The doubly charged Higgs present in the model offers promising opportunities for collider searches. In addition our model predicts a very light $SU(2)$ doublet that transforms as an octet of $SU(3)$, with interesting phenomenological consequences.

We have also shown that the proton decay signal is within reach of the next generation of experiments and it is correlated with the possible collider signatures of the electroweak scalars. We have also compared our model with the $SU(5)$ models that implement (i) the type I see-saw mechanism and (ii) the so-called hybrid scenario that combines the type I and type III see-saw. We came to the conclusion that the minimal $SU(5)$ theory with type I see-saw is already excluded by experimental
limits on partial proton decay lifetimes. Our model also gives more promising signatures than the hybrid scenario.

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