Semileptonic $B_q \to D_q^* l \nu$ transitions in QCD

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The form factors relevant to $B_q \to D_q^*(J^P = 1^-)l \nu$ ($q = s, d, u$) decays are calculated in the framework of the three point QCD sum rules approach. The heavy quark effective theory prediction of the form factors are obtained. The total decay width and branching ratio for these decays are also evaluated using the $q^2$ dependencies of these form factors.

Semileptonic pseudoscalar $B_q$ decays are crucial tools to restrict the Standard Model (SM) parameters and search for new physics beyond the SM. These decays provide possibility to calculate the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, leptonic decay constants as well as the origin of the CP violation.

$B_q \to D_q^* l \nu$ decays occur via $b \to c$ transition and form factors are central objects in studying of these decays. For the calculation of these form factors, we use the QCD sum rules method.

**Sum rules for the $B_q \to D_q^* l \nu$ transition form factors**

The $B_q \to D_q^*$ transitions occur via the $b \to c$ transition at the quark level. At this level, the matrix element for this transition is given by:

$$M_q = \frac{G_F}{\sqrt{2}} V_{cb} \bar{\tau} \gamma_\mu (1 - \gamma_5) l \tau \gamma_\mu (1 - \gamma_5) b.$$  \hspace{1cm} (1)

To derive the matrix elements for $B_q \to D_q^* l \nu$ decays, it is necessary to sandwich Eq. (1) between initial and final meson states. The amplitude of the $B_q \to D_q^* l \nu$ decays can be written as follows:

$$M = \frac{G_F}{\sqrt{2}} V_{cb} \bar{\tau} \gamma_\mu (1 - \gamma_5) l < D_q^* (p', \epsilon) | \bar{\tau} \gamma_\mu (1 - \gamma_5) b | B_q (p) >.$$  \hspace{1cm} (2)

Considering Lorentz and parity invariances, the matrix element $< D_q^* (p', \epsilon) | \bar{\tau} \gamma_\mu (1 - \gamma_5) b | B_q (p) >$ appearing in Eq. (2) can be parameterized in terms of the form factors below:

$$< D_q^* (p', \epsilon) | \bar{\tau} \gamma_\mu b | B_q (p) > = i \frac{f_V (q^2)}{(m_{B_q} + m_{D_q^*})} \epsilon_{\mu \rho \beta} \epsilon^{* \nu} p^\rho p^\beta,$$  \hspace{1cm} (3)

$$< D_q^* (p', \epsilon) | \bar{\tau} \gamma_\mu \gamma_5 b | B_q (p) > = i \left[ f_0 (q^2) (m_{B_q} + m_{D_q^*}) \epsilon^{* \mu} (\epsilon^{*} p) q_\mu - \frac{f_- (q^2)}{(m_{B_q} + m_{D_q^*})} (\epsilon^{*} p) q_\mu \right],$$  \hspace{1cm} (4)

where $f_V (q^2)$, $f_0 (q^2)$, $f_+ (q^2)$ and $f_- (q^2)$ are the transition form factors and $P_\mu = (p + p')_\mu$, $q_\mu = (p - p')_\mu$. In order to calculate these form factors, the QCD sum rules method is applied. Initially the following correlator is considered:

$$\Pi_{\nu \mu} (p^2, p'^2, q^2) = i^2 \int d^4 x d^4 y e^{-ipx} e^{ip'y}$$

$$\times < 0 | T[J_\nu D^*_\epsilon (y)] J^{A \nu}_\mu (0) J_{B_q} (x) | 0 >,$$  \hspace{1cm} (5)

where $J_{\nu D^*_\epsilon} (y) = \bar{\tau} \gamma_\nu \epsilon$ and $J_{B_q} (x) = \bar{\tau} \gamma_5 q$ are the interpolating currents of $D_q^*$ and $B_q$ mesons, respectively and $J^{A \nu}_\mu = \bar{\tau} \gamma_\nu \gamma_5 b$ and $J^{A} = \bar{\tau} \gamma_5 b$ are vector and axial vector transition currents.

From the QCD (theoretical) sides, $\Pi_{\nu \mu} (p^2, p'^2, q^2)$ can also be calculated by the help of OPE and the double dispersion representation for the coefficients of corresponding Lorentz structures as:

$$\Pi^{per} = - \frac{1}{(2\pi)^2} \int ds' \int ds \frac{\rho_0 (s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{subt. terms.}$$  \hspace{1cm} (6)

The spectral densities $\rho_0 (s, s', q^2)$ can be calculated from the usual Feynman integral with the help of Cutkosky rules which implies that all quarks are real. After calculations for the corresponding spectral densities the following expressions are obtained:

$$\rho_V (s, s', q^2) = 4N_c I_0 (s, s', q^2)$$

$$[ (m_q - m_e) A + (m_e - m_q) B - m_q ],$$

$$\rho_0 (s, s', q^2) = - 2N_c I_0 (s, s', q^2)$$

$$[ 2m_q^2 - 2m_{B_q}^2 (m_e + m_b)$$

$$+ m_q (q^2 + s + s' - 2m_b m_e) + [q^2 (m_b - m_q)$$

$$+ s(3m_q - 2m_e - m_b) + s' (m_q - m_b)] A$$

$$+ [q^2 (m_e - m_q) + s (m_q - m_e)$$

$$+ s' (3m_q - 2m_b - m_e)] B + 4(m_b - m_e) C ],$$

$$\rho_+ (s, s', q^2) = 2N_c I_0 (s, s', q^2)$$

$$[ m_q + (3m_q - m_b) A$$

$$+ (m_q - m_e) B$$

$$+ 2(m_q + m_b) D + 2(m_q - m_b) E ],$$

$$\rho_- (s, s', q^2) = 2N_c I_0 (s, s', q^2)$$

$$[ -m_q + (m_q + m_b) A$$
\[ I_0(s, s', q'^2) = \frac{1}{4\lambda(s, s', q'^2)}, \]
\[ \lambda(a, b, c) = a^2 + b^2 + c^2 - 2ac - 2bc - 2ab, \] (8)

where

\[
\begin{align*}
\Delta &= m_b^2 - m_q^2 - s, \\
\Delta' &= m_c^2 - m_q^2 - s', \\
ν &= s + s' - q'^2, \\
A &= \frac{1}{\lambda(s, s', q'^2)}(\Delta' u - 2\Delta s), \\
B &= \frac{1}{\lambda(s, s', q'^2)}(\Delta u - 2\Delta' s), \\
C &= \frac{1}{2\lambda(s, s', q'^2)}[6\Delta\Delta' s'u + \Delta^2(2ss' + u^2)] - 2\Delta u + m_q^2(-4ss' + u^2)), \\
D &= \frac{1}{\lambda(s, s', q'^2)}[6\Delta\Delta' s'u + \Delta^2(2ss' + u^2)] \\
&= \frac{1}{\lambda(s, s', q'^2)}[6\Delta\Delta' s'u + \Delta^2(2ss' + u^2)] \\
&= \frac{1}{\lambda(s, s', q'^2)}[6\Delta\Delta' s'u + \Delta^2(2ss' + u^2)] \\
&= \frac{1}{\lambda(s, s', q'^2)}[6\Delta\Delta' s'u + \Delta^2(2ss' + u^2)] \
\end{align*}
\] (9)

The subscripts \( V \) and \( \pm \) correspond to the coefficients of the structures proportional to \( iε_{μναβ}p^αp^β \cdot g_{νμ} \) and \( \frac{1}{2}(p_0p_ν + p_νp_0) \), respectively. In Eq. (7) \( N_c = 3 \) is the number of colors.

The contribution of power corrections, i.e., the contributions of operators with dimensions \( d = 3, 4 \) and 5, are given in Ref. [11].

By equating the phenomenological expression and the OPE expression, and applying double Borel transformations with respect to the variables \( p^2 \) and \( p'^2 (p^2 \rightarrow M^2, p'^2 \rightarrow M'^2) \) in order to suppress the contributions of higher states and continuum, the QCD sum rules for the form factors \( f_V, f_0, f_+, \) and \( f_- \) are obtained:

\[
\begin{align*}
f_i(q'^2) &= \frac{\kappa}{f_B} f_{B_s} f_{B_s} e^{M^2_{q}/M^2_{s} + M^2_{q}/M^2_{s}} \\
&\times \left[ \frac{1}{(2π)^2} \int_{(m_s + m_s)^2}^{s_0} ds' \int_{f(s')}^{s_0} ds\rho(s, s', q'^2)e^{-q'^2/M^2_{q}/M^2_{s}} \right] \\
&= B(f_i^{(3)} + f_i^{(4)} + f_i^{(5)})\}, \\
\end{align*}
\] (10)

where \( i = V,0 \) and \( 0 \) and \( \pm \), and \( B \) denotes the double Borel transformation operator and \( η = \frac{m_{B_s} + m_{D^*_s}}{2} \) for \( i = V, 0 \) and \( \pm \), and \( η = \frac{m_{D^*_s}}{m_{B_s} + m_{D^*_s}} \) for \( i = V, 0 \) and \( \pm \). Here \( ϵ = +1 \) for \( i = \pm \) and \( ϵ = -1 \) for \( i = 0 \) and \( V \). In Eq. (10), in order to subtract the contributions of the higher states and the continuum, the quark-hadron duality assumption is used.

Next, we present the infinite heavy quark mass limit of the form factors for \( B_q \rightarrow D^*_qν \) transitions. In HQET, the following procedure are used (see [2, 3, 4]). First, we use the following parametrization:

\[
y = νν' = \frac{m_{B}^2 + m_{D^*_q}^2 - q'^2}{2m_B m_{D^*_q}} \] (11)

where \( ν \) and \( ν' \) are the four-velocities of the initial and final meson states, respectively and \( y = 1 \) is called zero recoil limit. Next, we try to find the \( y \) dependent expressions of the form factors by taking \( m_B \rightarrow ∞ \), \( m_c = m_B/2 \), where \( z \) is given by \( √y = y + y'^2 - 1 \) and setting the mass of light quarks to zero. In this limit the Borel parameters take the form \( M^2 = 2T_1 m_B \) and \( M'^2 = 2T_2 m_c \) where \( T_1 \) and \( T_2 \) are the new Borel parameters.

The new continuum thresholds \( ν_0 \), and \( ν'_0 \) take the following forms in this limit

\[
ν_0 = \frac{s_0 - m_B^2}{m_B}, \quad ν'_0 = \frac{s'_0 - m_c^2}{m_c}, \] (12)

and the new integration variables are defined as:

\[
ν = \frac{s - m_B^2}{m_B}, \quad ν' = \frac{s' - m_c^2}{m_c}. \] (13)

The leptonic decay constants are rescaled:

\[
\hat{f}_{B} = \frac{\sqrt{m_B} f_{B}}{m_B}, \quad \hat{f}_{D^*_q} = \frac{\sqrt{m_c} f_{D^*_q}}{m_c}. \] (14)

After the standard calculations, we obtain the \( y \)-dependent expressions of the form factors as follows:

\[
f_V = \frac{(1 + \sqrt{z})}{48\hat{f}_{D^*_q} \hat{f}_{B}} e^{z_{1/4}} \left\{ \frac{3}{2} \right\}_{\pi^2(y + 1)/\sqrt{y^2 - 1}} \] (15)

\[
f_0 = \frac{(1 + \sqrt{z})}{16\hat{f}_{D^*_q} \hat{f}_{B}} e^{z_{1/4}} \left\{ \frac{3}{2} \right\}_{\pi^2(y + 1)/\sqrt{y^2 - 1}} \]
\[ \begin{align*}
\theta(2y\nu\nu' - \nu^2 - \nu'^2) + \frac{<\bar{q}q>}{3} \\
\left[ \left( \frac{1}{2} + \frac{1}{2z} + \frac{1}{\sqrt{z}} \right) \left( 16 - m_0^2 \frac{(1)}{T_1^2} + \frac{1}{T_1^2} \right) \\
- m_0^2 \left( \frac{1}{2T_1^2} + \frac{1}{2T_2^2} + \frac{1}{3T_1T_2} \left( 1 + \frac{1}{\sqrt{z}} + \frac{1}{z} \right) \right) \right] \},
\end{align*} \]

where \( \Lambda = m_{B_s} - m_b \) and \( \bar{\Lambda} = m_{D_s} - m_c \).

**Numerical analysis** This section is devoted to the numerical analysis for the form factors \( f_V(q^2), f_0(q^2), f_+(q^2) \) and \( f_-(q^2) \). The threshold parameters \( s_0 \) and \( s_0' \) are determined from the two-point QCD sum rules: \( s_0 = (35 \pm 2) \text{ GeV}^2 \) and \( s_0' = (6 - 8) \text{ GeV}^2 \). The Borel parameters \( M_1^2 \) and \( M_2^2 \) are not physical quantities, hence form factors should not depend on them. Reliable regions for Borel parameters are \( 10 \text{ GeV}^2 < M_1^2 < 25 \text{ GeV}^2 \) and \( 4 \text{ GeV}^2 < M_2^2 < 10 \text{ GeV}^2 \).

To determine the decay width of \( B_q \to D_s^*\ell\nu \), the \( q^2 \) dependence of the form factors \( f_V(q^2), f_0(q^2), f_+(q^2) \) and \( f_-(q^2) \) in the whole physical region \( m_0^2 \leq q^2 \leq (m_{B_s} - m_{D_s})^2 \) are needed. The value of the form factors at \( q^2 = 0 \) are given in Table I.

Figs. 1, 2, 3 and 4 show the dependence of the form factors \( f_V(q^2), f_0(q^2), f_+(q^2) \) and \( f_-(q^2) \) on \( q^2 \). To find the extrapolation of the form factors, we choose the following fit function.

\[ f_i(q^2) = \frac{a}{(q^2 - m_0^2)} + \frac{b}{(q^2 - m_{f_{it}}^2)}. \]

### Table I: The value of the form factors at \( q^2 = 0 \).

| \( f_V \)             | \( f_0 \)             | \( f_+ \)             | \( f_- \)             |
|-----------------------|-----------------------|-----------------------|-----------------------|
| \( f_V(0) \)         | \( f_0(0) \)         | \( f_+(0) \)         | \( f_-(0) \)         |
| 0.36 \pm 0.08        | 0.47 \pm 0.13        | 0.46 \pm 0.13        | 0.24 \pm 0.05        |
| 0.17 \pm 0.03        | 0.24 \pm 0.05        | 0.24 \pm 0.05        | 0.14 \pm 0.025      |
| 0.11 \pm 0.02        | 0.14 \pm 0.025      | 0.13 \pm 0.025      | 0.13 \pm 0.025      |
| -0.13 \pm 0.03       | -0.16 \pm 0.04      | -0.15 \pm 0.04      |                      |

### Table II: Parameters appearing in the fit function for form factors of the \( B_s \to D_s^*(2112)\ell\nu \) at \( M_1^2 = 19 \text{ GeV}^2 \), \( M_2^2 = 5 \text{ GeV}^2 \).

The values for \( a \), \( b \), and \( m_{f_{it}}^2 \) are given in Table II for example for \( s \) case.

| \( a \)    | \( b \)    | \( m_{f_{it}}^2 \) |
|-----------|-----------|-------------------|
| \( f_V \) | \( f_0 \) | \( f_+ \)  |
| 55.03     | -54.30    | 23.18             |
| 1.43      | -4.32     | 18.80             |
| 1.14      | -2.57     | 14.88             |
| -2.80     | 3.43      | 14.60             |

The numerical values of the above mentioned ratios and a comparison of our results with the predictions of [2] which presents the application of the subleading Isgur-Wise form factors for \( B \to D^*\ell\nu \) are shown in Table III. Note that the values in this Table are obtained with \( T_1 = T_2 = 2 \text{ GeV} \) correspond to \( M_1^2 = 19 \text{ GeV}^2 \) and \( M_2^2 = 5 \text{ GeV}^2 \).

The next step is to calculate the differential decay width in terms of the form factors (see Ref. [1]). The branching ratios are obtained as:

\[ B(B_s \to D_s^*(2112)\ell\nu) = (1.89 - 6.61) \times 10^{-2}, \]
\[ B(B_d \to D_s^*(2010)\ell\nu) = (4.36 - 8.94) \times 10^{-2}, \]
\[ B(B_u \to D_s^*(2007)\ell\nu) = (4.57 - 9.12) \times 10^{-2}. \]
TABLE III: The values for the $R_i$ and comparison of $R_{1,2}$ values with the predictions of [9].

| y | 1 (zero recoil) | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |
|---|----------------|-----|-----|-----|-----|-----|
| $q^2$ (GeV$^2$) | 10.69 | 8.57 | 6.45 | 4.33 | 2.20 | 0.08 |
| $R_1$ | 1.34 | 1.31 | 1.25 | 1.19 | 1.10 | 0.95 |
| $R_2$ | 0.80 | 0.99 | 1.10 | 1.22 | 1.30 | 1.41 |
| $R_3$ | -0.80 | -0.79 | -0.80 | -0.81 | -0.80 | -0.80 |
| $R_4$ | 0.50 | 0.64 | 0.77 | 0.94 | 1.20 | 1.46 |
| $R_5$ | -0.50 | -0.51 | -0.56 | -0.62 | -0.71 | -0.89 |
| $R_6$ | -0.80 | -0.67 | -0.64 | -0.61 | -0.55 | -0.53 |
| $R_1$ [9] | 1.31 | 1.30 | 1.29 | 1.28 | 1.27 | 1.26 |
| $R_2$ [9] | 0.90 | 0.90 | 0.91 | 0.92 | 0.92 | 0.93 |

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