On the position of a heavy Higgs pole

Adrian Ghinculov and Thomas Binoth

Albert–Ludwigs–Universität Freiburg, Fakultät für Physik
Hermann–Herder Str.3, D-79104 Freiburg, Germany

Abstract

Higher loop calculations in the Higgs sector of the standard model at the Higgs mass scale have shown that perturbation theory diverges very badly at about 1 TeV in the on–shell renormalization scheme. The prediction of the position of the Higgs pole in the complex s plane becomes unreliable. We show that in the pole renormalization scheme this appears to have much better convergence properties, while showing good agreement with the on–shell scheme over the validity range of the latter. This suggests that the pole scheme should be preferable for phenomenological studies of heavy Higgs bosons.

We discuss whether this behaviour can be the result of a certain relation between the on–shell mass and the pole mass at the nonperturbative level.
On the position of a heavy Higgs pole

Adrian Ghinculov and Thomas Binoth

Albert–Ludwigs–Universität Freiburg, Fakultät für Physik,
Hermann–Herder Str.3, D-79104 Freiburg, Germany

Abstract

Higher loop calculations in the Higgs sector of the standard model at the Higgs mass scale have shown that perturbation theory diverges very badly at about 1 TeV in the on–shell renormalization scheme. The prediction of the position of the Higgs pole in the complex s plane becomes unreliable. We show that in the pole renormalization scheme this appears to have much better convergence properties, while showing good agreement with the on–shell scheme over the validity range of the latter. This suggests that the pole scheme should be preferable for phenomenological studies of heavy Higgs bosons.

We discuss whether this behaviour can be the result of a certain relation between the on–shell mass and the pole mass at the nonperturbative level.

1 Introduction

The divergent nature of the perturbation theory can be particularly disturbing especially when the coupling constant is large and the divergent behaviour sets in at low order in the loop expansion. Apart from the fundamental problem of disentangling the physical information out of a divergent perturbative expansion, on the phenomenological side one may find out that one is unable to make quantitative predictions of sufficient accuracy because the first few radiative corrections are large, after which the divergent behaviour sets in.

In the hope of elucidating the problem of the electroweak symmetry breaking mechanism, the LHC will be able to search for a Higgs boson up to masses of the order of 1 TeV. A number of higher order calculations of processes at the Higgs resonance became available recently. They indicate that for such heavy Higgs bosons the higher order radiative corrections become indeed large [1]. In the on–shell renormalization scheme, if the Higgs mass is larger than 930 GeV, the two–loop correction to the Higgs decay into vector bosons exceeds the one–loop correction [2, 3]. For the Higgs decay into fermions, this happens at about 1.1 TeV [4, 5]. Similar conclusions are valid for the $gg \rightarrow H \rightarrow ZZ$ process [6], which will be tested at the LHC, and for
the \( f \bar{f} \rightarrow H \rightarrow f' \bar{f}' \) and \( f \bar{f} \rightarrow H \rightarrow ZZ \) scatterings \([6, 7]\), which were proposed as a production mechanism for Higgs bosons at a possible muon collider.

If the one-and two-loop corrections which are available are not accidentally very small or very large, and are indicative for the divergent behaviour of the perturbation series, then one must conclude that beyond these limits – of the order of 900—1000 GeV, depending on the process – the perturbation theory is totally unreliable in the on-shell scheme. Even for lower masses, the theoretical uncertainty due to the unknown higher order corrections may be substantial. This view is supported by calculations involving resummations of higher order logarithmic contributions \([10]\), where by examining the scheme dependence of the results one can estimate the size of the unknown higher order corrections.

How to recover a physical prediction out of a divergent and not Borel summable perturbative expansion is still an open question. Considerable progress has been made in understanding the large order behaviour of perturbation theory, and this may lead to recipes for summing up the perturbative expansion. As an example, the commensurate scale relations among effective charges show promise of dealing with the factorial behaviour associated with the infrared renormalon structure of QCD \([11]\). The justification is that this factorial growth is anyway related to the unknown behaviour of the beta function in a region where the perturbative solution is unreliable.

As it happens, worsening the divergent behaviour of a perturbative expansion is a much easier task. One way of doing this is suggested by ref. \([12]\). Suppose one starts with a perturbative expansion in a given renormalization scheme, with an expansion parameter \( \lambda \). Suppose that this series converges well, up to a high enough loop order, for an expansion parameter \( \lambda \) smaller than a critical value \( \lambda_c \). In this scheme the physics corresponding to values of \( \lambda \) smaller than \( \lambda_c \) are described well, with controllable accuracy. In order to worsen the divergency of the perturbation series, one can define a new renormalization scheme by choosing a new expansion parameter \( \bar{\lambda} \), so that the exact, nonperturbative relation between the two expansion parameters, \( \bar{\lambda} = \bar{\lambda}(\lambda) \), has a cut starting at \( \lambda = \lambda' \). \( \lambda' \) can be chosen to be conveniently smaller than \( \lambda_c \). Of course, then the power expansion in terms of the new expansion parameter \( \bar{\lambda} \) will converge satisfactorily only over a smaller onset of physics than the perturbation theory in the original scheme. For instance, one can arrange things so that the renormalization conditions in the new scheme have a solution only for a limited onset of physics, corresponding to \( \lambda < \lambda' \).

Of course, this also may happen accidentally, if one performs a calculation in an inconvenient renormalization scheme. In ref. \([12]\) it was argued that under certain assumptions the on–shell renormalization conditions for a boson propagator may have no solution for a range of physics for which the pole mass and width are well defined. To show this, a model was considered which allows an all–order, nonperturbative solution.

If the exact solution is unknown, and one only knows a few orders in perturbation
theory, as is the case with the standard model Higgs sector, it is more difficult to establish unambiguously whether such a mechanism is indeed present. For instance, perturbation theory in the on–shell scheme only gives the Higgs propagator as an expansion in the on–shell Higgs mass $m_H$, and one cannot directly decide whether the divergent behaviour which one observes is mainly due to a strong selfinteraction of the Higgs field or if there is a limit of the values of the on–shell mass $m_H$ beyond which the on–shell renormalization conditions have no solution. If that was the case, larger values of $m_H$ would be unphysical, however stronger selfcouplings than this limit may be possible, and may be appropriately described in the pole renormalization scheme.

In the following section we review the existing knowledge on the position of a heavy Higgs pole coming from perturbation theory in the on–shell scheme. We then derive the corresponding result in the pole renormalization scheme, and show that it converges much better. We analyze the nonperturbative $1/N$ expansion of an $O(N)$ model. We find no branching point in the $m_H = m_H(M)$ relation at leading order which could explain the observed divergency pattern, and speculate on the possibility that such a branching point may be induced at $O(1/N)$.

2 The Higgs pole in the on–shell scheme

We are interested in effects related to a heavy Higgs boson, so we adopt the framework proposed in ref. [13] for calculating leading effects in the Higgs mass. This reduces to considering the Higgs–Goldstone Lagrangian of the standard model in Landau gauge.

The Higgs propagator receives quantum corrections which lead to a momentum dependent self–energy. In the on–shell scheme, the Higgs propagator reads:

$$P(s) = \frac{i}{s - m_H^2 + \Sigma(s)} ,$$  

and the on–shell mass is defined by the renormalization condition:

$$\Re \left( iP^{-1}(m_H^2) \right) = 0 .$$  

The quartic coupling $\lambda$ is related at all orders to the on–shell mass $m_H$ by the relation $\lambda = G_F/(2\sqrt{2}\pi^2) m_H^2 \equiv c m_H^2$, were $G_F = 1.16637 \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant.

The self–energy $\Sigma(s)$ is given in perturbation theory by a power series with the expansion parameter $m_H$. Its real part has no constant or linear terms in $s$, since these are absorbed by the mass and wave function renormalization.

The propagator in eq. 1 has a complex pole at an energy $s_P$ which is given by the equation $P^{-1}(s_P) = 0$. The pole mass and width are then defined by $s_P = (M - i\Gamma/2)^2$. Clearly, the on–shell mass $m_H$ and the pole mass $M$ are not the same.

In the on–shell scheme, one can express $M$ and $\Gamma$ as power series in the expansion parameter $m_H$. For calculating the location of the Higgs pole one has to solve the
equation \( P^{-1}(s_P) = 0 \) in the second Riemann sheet of the complex \( s \) plane. It is easy to do this at the one–loop order, where the analytical expression of the self–energy is trivial. We are interested in including the two– and three–loop contributions as well, and for some of these contributions only on–shell results have been calculated so far. Nevertheless, one can solve the pole equation in the complex plane up to the desired order if enough on–shell information is available.

To do this, it is useful to double expand the self–energy in the coupling constant \( \lambda \) and in the energy distance from the on–shell mass \( s - m_H^2 \):

\[
\Sigma(s) = A + (s - m_H^2) B + (s - m_H^2)^2 C + \ldots
\]

\[
A = i \Im \Sigma(m_H^2) = i m_H^2 \left( a_1 \lambda + a_2 \lambda^2 + a_3 \lambda^3 + a_4 \lambda^4 + \ldots \right)
\]

\[
B = i \Im \Sigma'(m_H^2) = i \left( b_1 \lambda + b_2 \lambda^2 + b_3 \lambda^3 + \ldots \right)
\]

\[
C = \Sigma''(m_H^2)/2 = m_H^{-2} \left( c_1 \lambda + c_2 \lambda^2 + \ldots \right)
\]

Here, \( a_j, b_j \) are real valued, whereas \( c_j \) are complex in general. The corrections to the Higgs decay width are known at two–loop order, so \( A \) is known with three–loop precision. \( B \) is known to two–loop, and \( C \) is known to vanish at one–loop.

The terms in the expansion which we need for finding a consistent solution of the pole equation at three–loop level are \([2]–[8]\):

\[
a_1 = 3\pi/8, \quad a_2 = a_1 \cdot 0.350119, \quad a_3 = a_1 \cdot (0.97103 + 0.000476)
\]

\[
b_1 = 0, \quad b_2 = 1.002245, \quad c_1 = 0.2181005
\]

Indeed, one can convince oneself by solving directly the pole equation up to order \( \lambda^4 \) that its solution reads:

\[
s_P = m_H^2 \left[ 1 - i(a_1 \lambda + a_2 \lambda^2 + a_3 \lambda^3) - (a_1 b_2 - a_1^2 c_1) \lambda^3 \right] + \mathcal{O}(\lambda^4)
\]

and therefore the higher order unknown coefficients \( a_4, b_3 \) and \( c_2 \) are not needed for solving the pole equation consistently with three–loop accuracy.

One further obtains:

\[
\sqrt{s_P} = m_H - i \frac{c^2}{2} a_1 m_H^3 + \frac{c^2}{8} (a_1^2 - 4i a_2) m_H^5
\]

\[
+ \frac{c^3}{16} \left[ (4a_1 a_2 - 8a_1 b_2 + 8a_1^2 c_1) + i(a_1^3 - 8a_3) \right] m_H^7,
\]

so that the pole mass and width read:
Figure 1: The real and the imaginary parts of the position of the Higgs pole at LO, NLO and NNLO in the on–shell perturbative expansion. In fig. a), the 1/N result practically coincides with all the other curves up to about $m_H = 872$ GeV. Its behaviour is shown in detail in fig. 3. For comparison, the [2/1], [1/2] (a) and the [1/1] (b) Padé approximants of the perturbation series are also shown, as well as the LO 1/N result. The Padé approximants [2/1], [1/2] of fig. (a) practically coincide for this range of $m_H$.

The above relation between the pole and the on–shell masses agrees with the result derived in ref. [9].

We plot eqns. 7 in fig. 1. For comparison, we show also the [2/1], [1/2] (a) and the [1/1] (b) Padé approximants of eqns. 7, as well as the leading order of the nonperturbative 1/N expansion, which will be discussed in the following. The two–loop correction to the pole width equals the one–loop correction at about 1
TeV, similar to the results of ref. [2, 3]. The corrections to the pole mass appear to be somehow smaller. The [2/1] and [1/2] Padé approximants of fig. 1 a) do not improve much the agreement of perturbation theory with the $1/N$ expansion for large couplings; the [1/1] approximant of fig. 1 b) shows qualitatively a saturation of the mass similar to the $1/N$ result, but the numerical discrepancy remains considerable even for masses as low as 600—650 GeV.

The position of the pole in the on–shell scheme, which is given by eq. 6, is shown in fig. 2, along with the $1/N$ expansion and the pole renormalization scheme result, which will be derived in the following. We have marked in fig. 2 the point beyond which the NNLO correction to eq. 6 exceeds the NLO correction in absolute value. This corresponds to an on–shell mass of about 980 GeV, but the perturbative series may be untrustworthy long before. Of course, other measures of the degree of divergency of the series are possible – for instance the point beyond which the distance between the NNLO and the NLO results is larger than that between the NLO and the LO results, for a given pole mass. This happens for a mass larger than about 710 GeV.

### 3 The Higgs pole in the pole scheme

In the pole renormalization scheme, the Higgs propagator reads:

$$P(s) = \frac{i}{s - M^2 + \Sigma(s)} \quad (8)$$

The coupling of the theory is parameterized by the pole mass $M$, which is defined by the condition:

$$P^{-1}((M - \frac{1}{2}\Gamma)^2) = 0 \quad (9)$$

The Higgs width is then expressed as an expansion in $M$.

We cannot solve directly the renormalization conditions at three–loop order because the analytical continuation of the self–energy in the second Riemann sheet is not available. Nevertheless, the relation between the observables $M$ and $\Gamma$ is independent of some intermediary renormalization scheme in which they are calculated. In the on–shell scheme, $M$ and $\Gamma$ are given by eqns. 7, so one can invert the $m_H$ power series of $M$ to obtain $m_H$ as a power expansion in $M$, and substitute in the second line of eqns. 7, for obtaining the following pole renormalization scheme result:

$$\Gamma = M \left[ a_1 c M^2 + c^2 a_2 M^4 + c^3 (a_3 - a_3^1/2) M^6 \right] \quad (10)$$

This relation is plotted in fig. 2, along with the on–shell result and the prediction of the leading order $1/N$ expansion. The interesting point is that the pole scheme expansion appears to converge much better than the on–shell scheme. In eq. 10, the
NLO and the NNLO corrections become equal for \( M = 1.74 \) TeV. At the same time, the NNLO predictions in the on–shell and the pole schemes agree very well over the energy range where the former is supposed to be a good approximation. However, the \( 1/N \) expansion deviates considerably from the perturbative solution already at 600—650 GeV, where perturbation theory should provide a reliable result.

If the size of the NLO and NNLO corrections in the pole scheme is representative for the divergency of the perturbative series, this behaviour suggests that the pole scheme is a better framework for describing a heavy Higgs for phenomenological purposes.

## 4 Beyond perturbation theory

A number of recipes exist in QCD for finding an appropriate renormalization scheme, in which the perturbative results ought to convergence better. The accuracy of the calculation can be spoiled by choosing a very different renormalization scale because of the presence of large logarithms in higher orders.

The perturbative expansion at the Higgs energy scale appears to converge much better in the pole renormalization scheme than in the on–shell scheme, but no large logarithms can be made responsible for this. In this section we address the question whether this feature may be due to the existence at the nonperturbative level of a certain relation between the on–shell and the pole masses.

The convergence pattern which is observed in the on–shell versus the pole scheme can be understood if one assumes that there is a nonperturbative relation between \( m_H \) and \( M \), \( m_H = m_H(M) \), which has a branching point at a value \( M_c \) of the order of 1 TeV. Of course, this would induce a strong divergent behaviour in the on–shell scheme near the singularity, even if there the Higgs field would not be truly strong selfinteracting. Beyond that value, the on–shell mass ceases to be a good parameterization of the Higgs selfcoupling. Nevertheless, the quartic coupling of the Higgs field may still be not very strong, and the physics at energies comparable to the Higgs mass may be described appropriately in the pole scheme.

The authors of ref. [12] considered a model of a W boson coupled to a large number of light fermions, which is an exactly solvable model, and which displays such a relation between the on–shell and the pole masses. It is more difficult to establish whether a similar scenario is indeed present in the standard model. To gain further insight, one has to go beyond the standard perturbation theory. One promising approach is the nonperturbative \( 1/N \) expansion of an \( O(N) \) sigma model. We anticipate that the leading order solution of the \( 1/N \) expansion does not display the type of relation between \( m_H \) and \( M \) we are looking for. Nevertheless, such a mechanism may be generated at next–to–leading order.

For fixing the notations, we consider an \( O(N) \) sigma model with the Lagrangian:
Figure 2: The position of the Higgs pole in the complex $\sqrt{s}$ plane. We show the on–shell scheme (OS) result of eq. 6, the pole scheme (PS) result of eq. 10, and the leading order $1/N$ expansion result of eqns. 14. The points beyond which the NNLO correction is larger than the NLO correction are marked for the pole scheme (top) and for the on–shell scheme (bottom).
\[ \mathcal{L} = \frac{1}{2} (\partial_{\mu} \Phi) (\partial^{\mu} \Phi) + \frac{1}{2} \mu^2 \Phi^2 - \frac{\lambda}{N} \Phi^4. \]  

(11)

One performs the calculation as an expansion in $1/N$, keeps only the leading order, and sets in the end $N = 4$ and the vacuum expectation value of the Higgs field $v$ to 246 GeV. We merely quote the result for the Higgs propagator, without repeating this well–known summation of Goldstone loops [14]:

\[ P(s) = i \left( s - \frac{2 \lambda v^2}{1 - \frac{\lambda}{4\pi}(\log(s/\mu^2) - i\pi)} \right)^{-1} \]  

(12)

In this expression, the divergency of the bubble diagrams was absorbed in the renormalization of the coupling constant $\lambda$, and one still has the freedom to perform a finite renormalization. Of course, the relation between the Higgs mass and width is independent of the actual renormalization scheme.

Following Einhorn [14], we will define the coupling constant at the energy scale of the Higgs pole, $\mu^2 = |s_P|$, which leads to a convenient parameterization of the results. Therefore, with the definitions:

\[ s_P = \left( M - \frac{i}{2} \Gamma \right)^2 = \mu^2 e^{-2i\theta} \]

\[ x = \tan(\theta), \quad x \in (0, 1) \]

(13)

one obtains the following parameterization:

\[ M = \frac{4\pi v}{\sqrt{\pi} + 2 \arctan(x)} \frac{\sqrt{x}}{1 + x^2} \]

\[ \Gamma = \frac{8\pi v}{\sqrt{\pi} + 2 \arctan(x)} x \frac{\sqrt{x}}{1 + x^2} \]

\[ \lambda = \frac{8\pi^2}{\pi + 2 \arctan(x)} x \]

\[ \mu^2 = \frac{16\pi^2 v^2}{\pi + 2 \arctan(x)} x \frac{\sqrt{x}}{1 + x^2} \]

(14)

For each value of $x$, which measures the coupling strength of the Higgs field, the on–shell Higgs mass $m_H$ is given by the following transcendental equation:

\[ m_H^2 \left\{ \left[ 1 - \frac{\lambda}{4\pi^2} \log(\frac{m_H^2}{\mu^2}) \right]^2 + \left( \frac{\lambda}{4\pi} \right)^2 \right\} = 2v^2 \lambda \left[ 1 - \frac{\lambda}{4\pi^2} \log(\frac{m_H^2}{\mu^2}) \right], \]  

(15)
Figure 3: The nonperturbative relation between the Higgs pole mass $M$ and the on–shell mass $m_H$ which results from the $1/N$ expansion at leading order. No singularity exists in this relation at this order, for real values of $M$ in the allowed range under the saturation point.

with $\lambda$ and $\mu^2$ given by eqns. 14.

At low values of the coupling, the pole mass and the on–shell mass have practically the same value, but start to deviate for larger couplings. As the coupling $x$ increases, the pole mass saturates and then starts to decrease, while the width continues to grow. The pole mass $M$ reaches its maximum of 867 GeV for $x = 0.501$. The on–shell mass has a similar behaviour. Its maximum of 872 GeV is reached for a slightly larger value of the coupling, $x = 0.515$.

The relation between the pole and the on–shell masses is shown in fig. 3. It shows that in this approximation both masses are practically equally good parameterizations of physics. They reach their maxima at nearly the same values of the coupling, and for any value of the coupling $x \in (0, 1)$ both on–shell and pole renormalization conditions have solutions. No branching point is present in the exact relation between $m_H$ and $M$ for real $M$ smaller than the saturation value, at leading order in the $1/N$ expansion.

The important difference between the leading order $1/N$ solution of the Higgs sector and the model of ref. [12], which exhibits the pathological behaviour of the on–shell mass we are looking for, is that the self–energy of the latter contains linear terms in $s$, which are absent in eq. 12. This is related to the absence of a contribution to the Higgs wave function renormalization at leading order in the $O(N)$ model. However, contributions of this type are present at next–to–leading order in the $1/N$ expansion.

It is difficult to find out if the linear terms which should appear at NLO in the $1/N$ expansion are large enough to induce the branching point in the $m_H = m_H(M)$
relation we are looking for, without actually performing a NLO calculation of the Higgs propagator. This is unfortunately a very challenging task.

Still, one can make a guess about the size of higher order contributions in the $1/N$ expansion by comparing the leading order with the perturbative result. As one can see in fig. 2, the perturbative results and the LO $1/N$ result differ considerably already for a Higgs mass of the order of 600 GeV. At these values of the Higgs mass perturbation theory appears to be well under control. The convergence in both on–shell and pole schemes is good, and the two schemes agree very well at NNLO. In fact, the NNLO perturbative results in the pole and on–shell scheme agree well up to about 900 GeV. Unless one takes the view that there is something fundamentally wrong with either the $1/N$ expansion, or perturbation theory, or both, this discrepancy suggests that the $O(1/N)$ corrections are numerically rather substantial.

5 Conclusions

Perturbation theory at the Higgs energy scale diverges very badly at about 1 TeV in the on–shell scheme. In particular, the two–loop correction to the Higgs width exceeds the one–loop correction if the on–shell Higgs mass is larger than 930 GeV. For a Higgs boson in this mass range the prediction of the position of the Higgs pole is rather unreliable.

We show that the pole renormalization scheme has much better convergence properties. In this scheme, the two–loop corrections to the width become as large as the one–loop ones only at 1.74 GeV. This suggests that the pole renormalization scheme is preferable for describing a heavy Higgs boson in phenomenological studies of heavy Higgs production at future colliders.

This choice of renormalization scheme cannot be justified in the same way one chooses the renormalization scale in QCD for resuming large logarithms in higher orders. However, this different behaviour with respect to the convergence range of the on–shell scheme versus the pole scheme may be the result of the existence of a relation between $m_H$ and $M$ at the nonperturbative level. The observed convergence properties are consistent with the assumption that the function $m_H = m_H(M)$ has a singularity at an energy of the order of 1 TeV, and beyond this critical value the on–shell mass is ill–defined.

We examine the leading order nonperturbative solution of the $O(N)$ model in the $1/N$ expansion. No branching point is present in the $m_H = m_H(M)$ function in this approximation for real values of $M$. The rather large deviation of the leading order $1/N$ expansion from the perturbative result in the range where the latter is expected to be accurate leaves room for substantial $O(1/N)$ corrections. At next–to–leading order in the $1/N$ expansion, the Higgs self–energy $\Sigma(s)$ is expected to acquire linear terms in $s$, similarly to the model studied in ref. [12], which exhibits a branching point. Whether the $O(1/N)$ corrections induce indeed a branching point in the $m_H = m_H(M)$ relation at about 1 TeV, is an open question.
Acknowledgements

We are indebted to Scott Willenbrock, Jochum van der Bij, and George Jikia for useful discussions. One of us (A. G.) is grateful to Stanley Brodsky for very interesting discussions, and would also like to thank the theory department of Brookhaven National Laboratory for its hospitality, and the US Department of Energy (DOE) for support. T. B. gratefully acknowledges the hospitality of CERN, where part of the work was done. The work of A. G. was supported by the Deutsche Forschungsgemeinschaft (DFG).

References

[1] A. Ghinculov, *Freiburg-THEP 96/14 (1996)*, [hep-ph/9607455](http://arxiv.org/abs/hep-ph/9607455) and references therein.

[2] A. Ghinculov, *Nucl. Phys. B455* (1995) 21.

[3] A. Frink, B.A. Kniehl, D. Kreimer, K. Riesselmann, *Phys. Rev. D54* (1996) 4548.

[4] A. Ghinculov, *Phys. Lett. B337* (1994) 137; (E) B346 (1995) 426.

[5] L. Durand, B.A. Kniehl and K. Riesselmann, *Phys. Rev. D51* (1995) 5007; *Phys. Rev. Lett. 72* (1994) 2534; (E) *Phys. Rev. Lett. 74* (1995) 1699.

[6] A. Ghinculov and J.J. van der Bij, *Nucl. Phys. B482* (1996) 59.

[7] A. Ghinculov and J.J. van der Bij, *Nucl. Phys. B436* (1995) 30.

[8] V. Borodulin and G. Jikia, *Freiburg-THEP 96/19 (1996)*, [hep-ph/9609447](http://arxiv.org/abs/hep-ph/9609447)

[9] S. Willenbrock and G. Valencia, *Phys. Lett. B247* (1990) 341.

[10] U. Nierste, K. Riesselmann, *Phys. Rev. D53* (1996) 6638.

[11] S.J. Brodsky and H.J. Lu, *SLAC-PUB-7098 (1996)*, [hep-ph/9601301](http://arxiv.org/abs/hep-ph/9601301)

[12] W. Beenakker, G.J. van Oldenborgh, J. Hoogland, R. Kleiss, *Phys. Lett. B376* (1996) 136.

[13] W.J. Marciano and S.S.D. Willenbrock, *Phys. Rev. D37* (1988) 2509.

[14] M.B. Einhorn, *Nucl. Phys. B246* (1984) 75 and references therein.