Non-Poisson queue with normal logistic distribution (case study in Semarang automatic toll gate)

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Abstract. Queue theory is a method that can be used to analyze the performance of a service system. This study aims to apply queue theory for analysis system performance’s size at the Gayamsari Toll Gate by using data obtained through observation. Then the analysis step starts with checking the steady state assumption, goodness of fit tests to data, determines the queue model, and calculates the system performance’s size. Based on analysis with Sigma Magic software, the results of the study obtained queue model with normal distribution on the number of arrivals and logistic distribution on the number of services data. From this model, it can be used to calculate the service system performance’s size at the Gayamsari Toll Gate. The result of this calculation include estimates of the waiting time and the number of vehicles in the system or queue. It can be concluded that the service system at the Gayamsari Toll Gate is in good condition because the waiting time is less than half minute.

1. Introduction
To improve the quality of its services, in 2017 PT. Jasa Marga Tbk has begun implementing a full non-cash payment system at all toll gates it manages. With the implementation of the system, PT Jasa Marga Tbk has replaced all existing regular toll booths to all Automatic Toll Gates (GTO). This results in toll road users having to make non-cash payments using the toll card as a means of payment. According to [4], when a vehicle stops to make a toll payment, at the same time the transaction processing system at the toll booth makes the payment process by reducing the value of money on the toll card and transferring it to the operator's account.

Changes in the payment system at the toll gate have indeed occurred several times where there are certainly advantages and disadvantages. The benefits obtained by implementing a non-cash payment system are to speed up transaction time, reduce the amount of cash that must be handled, increase the accuracy of the transaction, and increase the efficiency of the number of toll officers. However, after conducting research at the Gayamsari Toll Gate, the fact is that several times there are still long lines of vehicles at the toll gate. This is generally caused by a number of customers who come in for example, such as having a toll card balance, not having a toll card, being confused to attach a toll card, and having a toll card that is no longer active. From some of these problems often arise long lines especially when the number of vehicle arrivals is high and the problems that occur have not been quickly resolved properly.

Therefore, to find out how the system performance at the Gayamsari Toll Gate after the implementation of the full non-cash payment system, a vehicle queue study was conducted. The study was conducted to obtain data on the number of arrivals, the number of services, the time between arrivals, and the time of vehicle service at the Gayamsari Toll Gate. From the existing data, a measurement of the performance of the service system will be performed at the Gayamsari Toll Gate using the Matlab GUI. So we get an estimate of the waiting time and the number of vehicles that are in the system and the queue. The reason for choosing the Gayamsari Toll Gate as a research object is
that the Gayamsari Toll Gate is one of the busy toll gates in the Semarang area with a total of 5 GTO service facilities. In addition, the toll gates often arise long lines of vehicles even though a full non-cash payment system has been implemented.

2. Literature review

2.1. Queue theory
According to [2], queue theory is developed to obtain a model that is used to predict the pattern of a queue system. A queue system can be described by the arrival of customers to get service, waiting to be served if not immediately served, and leaving service facilities after being served. In this case what is meant by the customer is not always human. But customers in this line can be vehicles waiting to get service at toll booths. According to [8], the difference between the number of customer requests for the ability of service facilities will cause two effects, namely the emergence of queues and the emergence of service facility unemployment.

2.2. The Characteristic of queue
According to [3], there are six important factors that are closely related to the line of queues, namely the distribution of arrivals, distribution of service time, service facilities, queue discipline, size of the queue, and the source of the call.

2.3. Steady state
According to [9], the purpose of the analysis of the queue situation is to develop measures of system performance so that it can be used as material for real system evaluation. Condition conditions are met if the average number of vehicles coming does not exceed the average number of vehicles that have been serviced or can be written as:

\[
\rho = \frac{\lambda}{c\mu} < 1
\]

According to [2], if the steady state conditions are not met so that the average number of vehicles that come to the system exceeds the average number of vehicles that have been served. This can cause a line of queues to continue to form where the queue will become longer and grow longer over time, except at certain points vehicles are not allowed to enter.

2.4. Poisson process
According to [2], it is generally assumed that in the queue process the level of arrival and level of customer service follows the Poisson distribution. The fundamental characteristic of the Poisson distribution is that the mean is the same as the variance or can be donated as \( \lambda \). According to [7], several assumptions for the Poisson process are:
1. Independent
2. Homogeneity in time
3. Regularity
Therefore, data on the number of arrivals and the number of customer services are tested beforehand whether to follow the Poisson distribution or not. This is because in general the queue process is assumed to follow the Poisson process and the Poisson distribution is one of the distributions contained in the Poisson process.

2.5. Kolmogorov Smirnov test
According to [1], the distribution match test is used to evaluate to what extent a model is able to approach the real situation it describes. One of the distribution compatibility tests is the Kolmogorov Smirnov test, assuming the data consists of free observations \( X_1, X_2, \ldots, X_n \), which is a random sample of size \( n \) from a distribution function that is as yet unknown and it was donated as \( F(x) \). The Kolmogorov Smirnov test steps are as follows:

Hypothesis
\[
H_0 : F(x) = F_0(x)
\]
\[
H_1 : F(x) \neq F_0(x)
\]

Test statistics is
\[
D = \text{Sup} |F(x) - F_0(x)|
\]
with : \( F(x) \): The cumulative opportunity function calculated from sample data.  
\( F_0(x) \): The cumulative opportunity function of the hypothesized distribution.  
Criteria of test value is reject \( H_0 \) in level of significant \( \alpha \) if the \( D > D_{\text{table}} \) or \( p\)-value < \( \alpha \).

2.6. Queue model

There are several models in the queue system, including:  
1. The (M/M/c):(GD/\infty/\infty) queue model

According [9], in the model (M/M/c):(GD/\infty/\infty), if the number of customers in the system is \( n \) equal to or greater than \( c \) then the combined departure rate of the facility is \( c \). If \( n \) is less than \( c \), then the rate of service is \( n \mu \). Then, the probability \( n \) customers can be written as follows:

\[
P_n = \begin{cases} 
\left( \frac{\rho^n}{n!} \right) p_0, & 0 \leq n \leq c \\
\left( \frac{\rho^n}{c^{n-c}c!} \right) p_0, & n > c 
\end{cases}
\]

\( p_0 = \left\{ \sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!(\frac{c}{c-n})} \right\}^{-1}, \quad \frac{\rho}{c} < 1
\]

The formula for looking up performance measures in the queue model (M/M/c):(GD/\infty/\infty) as follows:

\[
L_q = \frac{\rho^{c+1}}{(c-1)! (c-\rho)^2} p_0
\]

\[
L_s = L_q + \rho
\]

\[
W_q = \frac{L_q}{\lambda}
\]

\[
W_s = W_q + \frac{1}{\mu}
\]

\( c \) is the number of service facilities.; \( p_0 \) is probability value for 0 customers; \( \rho \) is comparison between the average number of arrivals and the average number of customer services; \( \lambda \) is average number of customer arrivals; \( \mu \): average number of customers that have been served.

2. The (G/G/c):(GD/\infty/\infty) queue model

In this model there are differences in the calculation of \( L_q \) where the \( L_q \) calculation on this model is based on the model (M/M/c):(GD/\infty/\infty). As for the calculation of \( L_s \), \( W_q \), and \( W_s \) have the same formula as the model (M/M/c):(GD/\infty/\infty). According to [2], the formula for finding performance measures in the model (G/G/c):(GD/\infty/\infty) as follow as:

\[
L_q = \frac{\rho^{c+1}}{(c-1)! (c-\rho)^2} p_0
\]

\[
L_s = \frac{L_q}{\lambda}
\]

\[
W_q = \frac{L_q}{\mu}
\]

\[
W_s = W_q + \frac{1}{\mu}
\]

2.7. Normal Distribution

According to [6], the random variable \( X \) with a density function:

\[
f(x) = \frac{1}{\sqrt{2\pi \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}, \quad -\infty < x < \infty
\]

is normal distribution with parameter \( \mu \) where \( -\infty < \mu < \infty \) and \( \sigma > 0 \).

The mean and variance of Normal distribution as follow as:

\[
E(X) = \mu
\]
\[ \text{Var}(X) = \sigma^2 \]

2.8. **Logistic Distribution**

According to [5], a Logistic distribution is one continuous distribution that has two parameters and is defined for all real numbers. The donated for logistic distribution is \( X \sim \text{Logistic}(\mu, \sigma), \sigma > 0 \). For \( \mu \) is located parameter and \( \sigma \) is scale parameter. The density function of logistic distribution as follow as:

\[
f(x) = \frac{e^{(x-\mu)/\sigma}}{\sigma(1+e^{-(x-\mu)/\sigma})^2}; \quad -\infty < x < \infty
\]

The mean and variance of Logistic distribution as follow as:

\[
E(X) = \mu \\
V(X) = \frac{\sigma^2 \pi^2}{3}
\]

3. **Methods**

The type of data used in this study is primary data obtained through direct observation of the 5 GTOs at the Gayamsari Toll Gate. The study was conducted at the Gayamsari Toll Gate for five days, namely on Saturday, Sunday, Monday, Wednesday, and Thursday from December 22\(^{nd}\) until December 27\(^{th}\) for 10 hours, starting at 07.00 am-05.00 pm. The data used in this study are data on the time between vehicle arrivals, vehicle service times, the number of vehicle arrivals, and the number of vehicle services at the Gayamsari Toll Gate. The steps of analysis data as follow as:

1. Input the data of research
2. The data obtained must meet steady state conditions. If the steady state conditions have not been met then it is recommended to change the time interval so that the steady state conditions can be met.
3. Conduct Poisson distribution test to find out whether the data of the number of arrivals and the number of services are Poisson distributed or not. The distribution match test used is the Kolmogorov-Smirnov test. If the hypothesis is accepted, it can be concluded that the data is Poisson distributed, if the hypothesis is rejected then the data is considered to follow a general distribution.
4. If the data is General distribution, the specifications of the General distribution will be determined through Sigma Magic software. Next, do a fit test of the distribution again to ensure the correctness of the distribution obtained through Sigma Magic.
5. Determine the queue model
6. Determine system performance measures which include \( L_q, L_s, W_q \).
7. Draw conclusions from the results of an analysis of services at the Gayamsari Toll Gate.

4. **Results and discussion**

Table 1 vehicle services produce the same value that is 86 vehicles in 5 minute intervals. The data on the number of arrivals and the number of vehicle services is calculated from the number of vehicles that come and the number of vehicles that have been served in accordance with the selected time interval, namely 5 minute intervals. Actually the selection of time intervals in the queue theory is free as long as it has to be homogeneous. For example, a 5 minute time interval is selected, so the number of arrivals and the number of vehicle services is calculated for every 5 minute time interval. The reason the researchers chose the 5 minute interval was that the calculation value could still be detected if a manual distribution match test was performed on the data. It showed that the average number of arrivals and the number of services was 85,665 and 85,661.

| Table 1. Descriptive analysis of the data |
|-------------------------------------------|
| Variables                          | Max | Min | Average | Variance |
|-------------------------------------------|
| The number of arrival                  | 131 | 38  | 85,665  | 232,43   |
| The number of services                  | 130 | 33  | 85,6617 | 234,388  |
4.1. Steady-State condition
The data of research resulted
\[ \lambda = 85,665 \text{ vehicles in every 5 minute} \]
\[ \mu = 85,6617 \text{ vehicles in every 5 minute} \]
\[ \rho = \frac{\lambda}{c \mu} = \frac{85,665}{5 \times 85,6617} = 0.2000077 <1 \]
can be concluded that the condition of steady state is fulfilled.

4.2. Poisson distribution test

| Table 2. Poisson distribution test |
|-------------------------------|----------------|
| D-value | 0.133 | 0.136 |
| p-value | 0.000 | 0.000 |

Based on Table 2 can be concluded that reject H0 so the number of arrval and the number of services not Poisson distribution. Or we can say that they have general distribution.

4.3. Determine the model
Because the distribution of data on the number of arrivals and the number of services is a General distribution, the model is obtained: (G/G/5):(GD/\infty /\infty ). Then because the data is General distribution, the specifications of the General distribution can be determined through Sigma Magic software. As for the Sigma Magic software, the following results are obtained:

| Table 3. Distribution of the data |
|---------------------------------|----------------|
| Variables                        | Distribution  | P-value   | Conclusion |
| The number of arrival            | Normal        | 0.426     | Ho Accept  |
| The number of services           | Logistic      | 0.273     | Ho Accept  |

From Table 3, H0 was received in the data on the number of arrivals and data on the number of services because the P-value> \( \alpha \) (0.05) so that it can be concluded that the number of arrivals is normally distributed, while the number of services is Logistics.

So based on the results of the distribution test on the data of the number of arrivals and the number of vehicle services above, the queue model is obtained (Norm/Logistic/5):(GD/\infty /\infty ). The model states that the Gayamsari Toll Gate has a queue model with the number of arrivals of vehicles with Normal distribution, the number of service vehicles with Logistics distribution, the number of service facilities of 5 GTO with first come first served (FIFO) rules, the number of vehicles coming in is not limited, and the source of the call is not unlimited limited.

4.4. System performance measure

| Table 4. System performance measure in Gayamsari Toll Gate |
|----------------------------------------------------------|
| System performance                       | Value       |
| c                                        | 5           |
| \( \lambda \)                              | 85,665      |
| \( \mu \)                                  | 85,6617     |
| \( Vt' \)                                  | 0,300162    |
| \( Vt \)                                   | 0,032334    |
| \( Ls \)                                   | 2,16887     |
| \( Lq \)                                   | 1,16883     |
| \( Ws \)                                   | 0,0253181   |
6

Note:
1. Number of service facilities is 5 GTO.
2. The average number of vehicle arrivals (λ) is 85,665 vehicles per 5 minutes. This means that within 5 minutes there were 86 vehicles coming at the Gayamsari Toll Gate.
3. The average number of vehicle services (μ) is 85.6617 vehicles per 5 minutes. This means that within 5 minutes there were 86 vehicles that had been completed serviced at the Gayamsari Toll Gate.
4. The time variance between vehicle arrivals is 0.300162 minutes.
5. The time variant of vehicle service is 0.032334 minutes.
6. The estimated number of vehicles in the system (Ls) is 2.16887 vehicles. This means that the total number of vehicles that are being serviced and that are waiting in line is 2 vehicles per minute.
7. The estimated number of vehicles in line (Lq) is 1.16883 vehicles. This means that the number of vehicles that are waiting in line is 1 vehicle per minute.
8. The estimated waiting time in the system (Ws) is 0.0253181 minutes. This means that the waiting time for a vehicle from entering the queue until the service has been completed is 0.0253181 minutes.
9. The estimated waiting time in line (Wq) is 0.0136442 minutes. This means that the waiting time for one vehicle while in the queue is 0.0136442 minutes.
10. The probability that a service facility (GTO) is unemployed is 0.367802. This indicates that the service system is quite busy because the chance of GTO is unemployed at 36.78%.

In general, the ideal waiting time for vehicles ranging from queuing to finish servicing is about half a minute or 30 seconds where the value is obtained from estimates when conducted research directly at the Gayamsari Toll Gate. Based on the results of the GUI calculation of the System Performance Size, it can be concluded that the provision of 5 GTO at the Gayamsari Toll Gate is already in good condition. This is because the vehicle waiting time is short and the number of vehicles that are queuing is also small so that customers do not have to wait long to be able to make toll payments. Then the probability of unemployed GTO is 36.78%. This indicates that the provision of 5 GTOs at the Gayamsari Toll Gate has been effective because GTO is quite busy and the level of utilization of GTO in serving vehicles is high.

5. Conclusion
Based on the analysis and discussion obtained in the previous chapter, it can be concluded that the right queue model to describe the condition of the service system at the Gayamsari Toll Gate is a queue model with Normal distribution on the number of arrivals and distribution of Logistic data on the number of vehicle services. Then from the results of GUI Size System Performance, it can be concluded that overall service at the Gayamsari Toll Gate is already in good condition. This is because the vehicle waiting time is short and the number of vehicles that are queuing is also small so that customers do not have to wait long to be able to make toll payments.

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