Evidence for Large N Phase Transition in $\mathcal{N} = 4$ Super Yang-Mills Theory at Finite Temperature

Miao Li

Enrico Fermi Institute
University of Chicago
5640 Ellis Avenue, Chicago, IL 60637, USA

The AdS/CFT correspondence provides valuable constraints on the possible exact form of various physical quantities in the $\mathcal{N} = 4$ super Yang-Mills theory in the large N limit. We examine the free energy as the expansions in a small as well as in a large 't Hooft parameter $\lambda$. We argue that it is impossible to smoothly extrapolate from the weak coupling regime to the strong coupling regime, thus there must exist a large N phase transition in $\lambda$ at a finite temperature. We also argue that there is no world-sheet instanton in the background of the Euclidean anti-de Sitter black hole.
The AdS/CFT correspondence conjectured by Maldacena [1] has enabled us to study, for instance, $\mathcal{N} = 4$ super Yang-Mills theory [2,3] as well as nonsupersymmetric Yang-Mills theories [4] in the large $N$ and the large ‘t Hooft parameter $\lambda = g^2_{YM}N$ limit. Many qualitative verifications of this conjecture, and in some cases quantitative predictions have been obtained.

One generic feature of those quantitative predictions is that physical quantities as functions of $\lambda$ in the large $N$ limit typically have $\lambda = \infty$ as a branch point. Examples are anomalous scaling dimensions [2,3], the rectangular Wilson loops [4] and the leading correction to the free energy [8]. For a finite theory such as the $\mathcal{N} = 4$ SYM, it was argued that $\lambda = 0$ should be a regular point order by order in the large $N$ expansion [9]. As in the old matrix models [13], one hopes that a physical quantity in the large $N$ limit should have a Taylor expansion in $\lambda$, and the convergent radius is finite and probably universal. This radius is largely determined by the exponential growth of the number of the planar Feynman diagrams in the perturbation theory [14,15]. An ansatz for the Wilson loop is proposed in [6] and according to this ansatz $\lambda_c = -2\pi^2$ is a singular point. (This can be modified by a factor of a rational number.) It is actually the second branch point as required by the fact that $\lambda = \infty$ is a branch point. Thus we expect that $2\pi^2$ or something similar is the universal convergent radius.

We generalize that analysis to the free energy in this note. We show that an attempt to extrapolate from the weak coupling regime to the strong coupling regime runs into contradiction. Therefore there exists a phase transition at a positive critical value of $\lambda$. We also discuss the possibility of the world-sheet instantons in the Euclidean anti-de Sitter black hole background.

1. Large and small $\lambda$ expansions

In a perturbative calculation of the free energy, the growth of the number of Feynman diagrams should be the same as in calculating a zero temperature physical quantity. The only modification to be made is to replace the usual propagators by the thermal propagators. This modification makes the perturbative behavior of a finite temperature quantity drastically different from its zero temperature counterpart. The major cause is that an electric mass as well as a mass for scalars are generated at one-loop level. This mass scale is proportional to $\sqrt{\lambda T}$. Thus, the perturbative expansion of a physical quantity is no longer
analytic in \( \lambda \), but in \( \sqrt{\lambda} \). For instance, the free energy, beyond the two loop result proportional to \( \lambda \), will have a term \( \lambda^{3/2} \) due to the summation of the so-called ring diagrams [11]. Each ring diagram of a bosonic field is infrared divergent, but the summation is finite. The nonanalytic terms generated this way can not be cancelled by exploiting supersymmetry, since there is no such infrared divergent diagram for a fermionic field, due to the fact that a mass for a fermion is already generated at the tree level due to the anti-periodic boundary condition. Thus, we expect the perturbation series of the free energy take the following schematic form

\[
F = \sum_{n=0}^{\infty} \lambda^n + \sum_{n=0}^{\infty} \lambda^{\frac{3}{2}+n},
\]

powers of larger half integers can be generated, for instance, by higher order terms in the electric mass. Note that we have assumed terms such as \( \lambda^2 \ln \lambda \) occurring in QCD are absent here. Such terms, if exist, can be generated in our scheme presented in the next section.

The free energy of the large N SYM was calculated in [7] in the \( \lambda = 0 \) limit, and found to be

\[
F = -\frac{\pi^2}{6} N^2 V_3 T^4,
\]

where \( V_3 \) is the three volume. However the black hole thermodynamics predicts

\[
F = -\frac{\pi^2}{8} N^2 V_3 T^4.
\]

The mismatch is a factor 3/4. The black hole result is the result for \( \lambda = \infty \), thus there is no contradiction here. If the exact formula is \( F = -\pi^2/6N^2V_3T^4F(\lambda) \), then \( F(0) = 1 \) and \( F(\infty) = 3/4 \). A two-loop result is not known yet. On general grounds, one expects to have

\[
F_{\text{two-loop}} = a \frac{\lambda}{\pi^2},
\]

where \( a \) is a rational number.

Using the stringy correction to the effective action [11,12], it was found that in the large \( \lambda \) limit the next to leading order correction is [8]

\[
\frac{45}{32} \zeta(3) (2\lambda)^{-3/2}.
\]

It has a positive sign. We rewrite, for reason becomes obvious shortly,

\[
F(\lambda) = 3/4 + G(\lambda),
\]
then $G(\lambda)$ behaves as $\lambda^{-3/2}$ in the large $\lambda$ limit, and its value at $\lambda = 0$ is $1/4$. As discussed above, we expect that it has a Taylor expansion for small $\lambda$, corresponding to the usual perturbative expansion. For large $\lambda$, we expect to have, schematically

$$G(\lambda) = \sum_{n \geq 0} \lambda^{-\frac{3}{2} - n} + \sum_{n \geq 0} \lambda^{-2-n}, \quad (1.5)$$

where the first sum comes from corrections of form $(\alpha')^{3+2n}$, and the second sum comes from corrections of form $(\alpha')^{4+2n}$. Throughout this paper $n$ will be used to denote a natural number, except otherwise explicitly stated.

One of the ways to see the origin the coefficient $\zeta(3)$ of the term $\lambda^{-3/2}$ is through an inspection of the Virasoro-Shapiro amplitude \cite{12}

$$A = \frac{1}{stu} \frac{\Gamma(1 - \alpha's)\Gamma(1 - \alpha't)\Gamma(1 - \alpha'u)}{\Gamma(1 + \alpha's)\Gamma(1 + \alpha't)\Gamma(1 + \alpha'u)}. \quad (1.6)$$

Expanding the above formula in $\alpha'$ is equivalent to expanding it in terms of the Mandelstam variables. This gets translated into the effective action as expansion with number of derivatives. At the order $(\alpha')^3$, the coefficient $d^3 \ln \Gamma(x = 1)/dx^3$ appears, which is the origin of the factor $\zeta(3)$. At the order $(\alpha')^{n+3}$, many such derivatives will appear, and the highest one is just $d^{n+3} \ln \Gamma(x = 1)/dx^{n+3} = (-1)^{n+3} \Gamma(n + 3) \zeta(n + 3)$. Thus, we expect that the coefficient of the term $\lambda^{-n/2-3/2}$ which corresponds to the order $(\alpha')^{n+3}$ take the form

$$\zeta(n + 3) + \zeta(3)\zeta(n) + \ldots, \quad (1.7)$$

where the general term is a product of zeta’s whose arguments sum to $n + 3$, and each argument is an odd integer.

For a zero temperature physical quantity, such as the interaction strength of a heavy quark-anti-quark pair, there are two branch points on the complex $\lambda$ plane, one is located at infinity, another is $\lambda_c < 0$. New features appear for a finite temperature quantity, even if we assume that there is no phase transition, so that the quantity is described by the same function for both small $\lambda$ and large $\lambda$. We see from the above analysis, that there are already two branch points, one is infinity, another is $\lambda = 0$. The latter is new, due to infrared divergences at a finite temperature. Now that it is perfectly possible that there exists a branch point at a positive $\lambda$, thus signaling a phase transition. There is no need to connect the branch point at $\lambda = 0$ and the one at $\lambda = \infty$, if there is a phase transition, since the free energy is by no means governed by the same function on the whole complex plane.
2. Failure of interpolation

Assuming that $G(\lambda)$ is a smooth function on the whole positive axis, then the following Mellin transform

$$G^*(s) = \int_0^\infty G(\lambda)\lambda^{s-1}d\lambda$$

(2.1)

exists for $0 < \Re s < 3/2$, owing to the decay behavior $G(\lambda) \sim \lambda^{-3/2}$ for large $\lambda$. Note that the function $F(\lambda)$ does not have a Mellin transform. Given $G^*(s)$ defined for a range of $s$, it can be analytically extended to the whole complex plane of $s$. Since $G(\lambda)$ is assumed to be a smooth function on the positive axis, $G^*(s)$ is analytic everywhere except at poles. Since we start with the assumption that there is no phase transition, therefore the second singularity of the perturbative $G(\lambda)$ is assumed to be negative.

$G(\lambda)$ is determined by $G^*(s)$ through the inverse Mellin transform

$$G(\lambda) = \frac{1}{2\pi i} \int_C ds(2\lambda)^{-s}G^*(s),$$

(2.2)

where the contour $C$ is a straight line with a constant $\Re s$ satisfying $0 < \Re s < 3/2$, we also rescaled $G^*(s)$ by a factor $2^{-s}$. According to the discussion in the last section, $G^*(s)$ must have poles at all integers and half integers with three exceptions $s = \pm1/2$, $s = 1$. For a small $\lambda$, the contour is closed on the left half plane, picking up poles at negative integers and negative half integers. Since the small $\lambda$ series is convergent for a sufficient small $\lambda$, thus the function $G^*(s)$ must grow at most as an exponential function for large negative $\Re s$. On the other hand, there is no such constraint for a large positive $\Re s$. Thus, for a large enough $\lambda$, the contour can not be closed on the right half plane. However, one can shift the contour rightward, thus gradually picks up negative powers in $\lambda$, and eventually the expansion could be only an asymptotic expansion.

Thus, $G^*(s)$ must contain a factor $(s^2 - 1/4)(s - 1)/\sin(2\pi s)$. This factor contains all the desired poles. As pointed out earlier, at a positive power, the residue has the leading term proportional to $\zeta(2s)$. This factor appears for $s = n + 3/2$. However, for a large negative $\Re s$, this function grows faster than an exponential function. This can be seen using the following functional relation

$$\zeta(2s) = 2^{2s}\pi^{2s-1}\Gamma(1 - 2s)\zeta(1 - 2s)\sin(\pi s).$$

(2.3)
One can not get rid of the factor $\Gamma(1 - 2s)$ by simply multiplying $\zeta(2s)$ by $1/\Gamma(1 - 2s)$, since this factor has zeros at positive integers and half integers. Without loss of generality, a factor $\Gamma(a)/\Gamma(a - 2s)$ can be included, where $a$ is not an integer. Thus one can write

$$G^*(s) = \frac{(s^2 - 1/4)(s - 1)}{\sin(2\pi s)} \left( \frac{\Gamma(a)\zeta(2s)}{\Gamma(a - 2s)} f(s) + \ldots \right), \quad (2.4)$$

where $f(s)$ is not specified yet. Now, $\zeta(2s)$ vanishes if $s$ is a negative integer, thus all the poles of the form $s = -n < 0$ are canceled. We shall argue later that the poles at $s = -n - 1/2$ should not be canceled by zeros in $f(s)$.

For a fixed $\Re s$ and large $\Im s$, we have

$$\left| \frac{1}{\sin(2\pi s)\Gamma(a - 2s)} \right| \sim e^{-\pi|\Im s|},$$

where we have ignored a power factor. The factor $|\zeta(2s)|$ is bounded on the contour $C$. Thus, for the integral (2.2) with the integrand given in (2.4) to make sense, $|f(s)|$ can grow no faster than the exponential $\exp(\pi|\Im s|)$ for large $|\Im s|$. Also, in order to close the contour on the left half plane to reproduce the perturbation series, $|f(s)|$ should not grow faster than an exponential function for a large negative $\Re s$.

Now the inverse Mellin transform dictates an asymptotic expansion for a large $\lambda$:

$$G(\lambda) = -\frac{\sin(\pi a)\Gamma(a)}{16\pi^2} \sum_{n=0}^{\infty} (n+1)(n+2)(n+4)\Gamma(n+4-a)f(n/2+3/2) \zeta(n+3)(2\lambda)^{-n/2-3/2} + \ldots, \quad (2.5)$$

where we have the desired factor $\zeta(n+3)$. For $n = 0$, the other factors should comprise to give $45/32$.

$\zeta(2s)$ vanishes if $s$ is a negative integer, as seen in (2.3). It has a simple pole at $s = 1/2$. This is a undesired feature, thus $f(1/2) = 0$ in order not to have a term $\lambda^{-1/2}$ in the large $\lambda$ expansion. Although $\zeta(2s)$ does not give rise to positive integral powers, it gives rise to positive half-integral powers as well as to the constant term

$$G(\lambda) = -\frac{1}{16\pi}f(0) + \sum_{n=1}^{\infty} (-1)^{n+1}(n^2 + n)(2n+3)\frac{\zeta(2n+2)f(-n-1/2)}{4\pi^2}$$

$$\frac{\Gamma(a)\Gamma(2n+2)}{\Gamma(2n+1+a)} \left( \frac{\lambda}{2\pi^2} \right)^{n+1/2}. \quad (2.6)$$

This series is convergent, since $f(-n-1/2)$ by definition does not grow faster than an exponential function. The general perturbation theory indicates that all the $\pi$ factor is
what already included in \((\lambda/2\pi^2)^{n+1/2}\), and the zeta function \(\zeta(2n+2)\) is \(\pi^{2n+2}\) multiplied by a rational number, thus \(f(-n-1/2) \sim \pi^{-2n}\). If so, since for large \(n\), \(\zeta(2n+2) \to 1\), the convergent radius does not contain a factor \(\pi^2\). Indeed, if \(f(-n-1/2)\) has the same sign for sufficiently large \(n\), the singular point \(\lambda_c\) will be negative. Notice that the constant term must be a rational number \((1/4)\), this requires \(f(0) \sim \pi\), and this is consistent with the pattern \(f(-n-1/2) \sim \pi^{-2n}\).

However, the coefficients in \((2.5)\) must be rational, except for the factor \(\zeta(n+3)\). It is 45/32 at \(n = 0\) as computed in [8]. It is fairly easy to see from the strategy of that calculation that one will get a rational number at the order \((\alpha')^{n+3}\), for each factor \(\zeta(n+3)\), \(C\zeta(n+2)\) etc. Now the factor \(\sin(\pi a)\Gamma(a)\Gamma(n+4-a)\) in \((2.5)\) is a rational number times \(\pi\). Thus, \(f((n+3)/2)\) must be a rational number times \(\pi\) to cancel the factor \(\pi^2\) in the denominator in \((2.5)\). The only way to solve this problem is to use

\[ f(s) = \pi^{2s+1}h_1(s) + \pi h_2(s), \]  

\[(2.7)\]

such that \(h_1(s)\) has zeros at \(s = (n+3)/2\), while \(h_2(s)\) has zeros at \(s = -(n+3)/2\). Both \(h_i(s)\) must be bounded by exponential functions for a general \(s\). It is quite easy to construct functions with the these properties.

It appears therefore that it is possible to interpolate between the small \(\lambda\) expansion and the large \(\lambda\) expansion. Now we prove that the form as given in \((2.7)\) makes the contour integral in \((2.2)\) diverge. We need to show that \(|h_1(s)|\) grows as fast as \(\exp(2\pi|\Im s|)\) for large \(|\Im s|\). To see this, consider the contrary, that \(h_1(s)\) grows more slowly such that the following integral

\[ \frac{1}{2\pi i} \int_C ds \lambda^{-s} \frac{(s^2 - 1/4)(s-1)h_1(s)}{\sin(2\pi s)}, \]  

\[(2.8)\]

is well-defined, where the contour \(C\) is the same as in \((2.2)\). Since \(h_1(s)\) is bounded by an exponential function for a large negative \(\Re s\), the above integral results in a small \(\lambda\) expansion with a negative singularity in \(\lambda\). This means that the above integral can be extended for all positive \(\lambda\), and \(\lambda = \infty\) should be no more than a regular singularity such as a branch point. On other hand, there is no large \(\lambda\) expansion resulting from integral \((2.5)\), since the integrand has no poles to the right of the contour \(C\). This implies \(\lambda = \infty\) must be an essential singularity, namely the integral for a large \(\lambda\) must be smaller than any negative power of \(\lambda\). This is a contradiction. We conclude therefore that the integral \((2.8)\) is not well-defined, and \(|h_1(s)|\) grows no more slowly than \(\exp(2\pi|\Im s|)\), so the original integral \((2.2)\) is not well-defined too.
To complete our proof of the impossibility of an interpolation, we need to argue that the form in (2.4) is logically unavoidable, namely, the large $\lambda$ expansion (2.5) necessarily implies the small $\lambda$ expansion (2.6). One might think, for instance, to have poles located on the the right half-plane only. In this case there is no corresponding small $\lambda$ expansion, namely closing the contour formally on the left half-plane will result in null answer. This indicates that $\lambda = 0$ is an essential singularity, contradicting our assumption that it is just a branch point. For example, one might start with

$$G^*(s) = \Gamma(3/2 - s)\zeta(2s)f(s),$$

where the first factor has poles located at $s = (n+3)/2$. This results in a large $\lambda$ expansion

$$G(\lambda) = \sum_{n \geq 0} \frac{(-1)^n}{n!} \zeta(2n + 3)f(n + 3/2)(2\lambda)^{-n-3/2}.$$

If $f$ behaves as a power function asymptotically, then the above sum is convergent for all $\lambda$, and $G(\lambda)$ is dominated by a factor $\exp(-a/\lambda)$ for a small $\lambda$, and apparently this implies that $\lambda = 0$ is an essential singularity. To remove this factor, one need to require, for instance, $f(n + 3/2) \sim n!$. Such a factor will introduce poles on the left-half plane, and these poles must be the ones exhibited in (2.4).

Having shown that it is impossible to extrapolate from the small coupling regime to the large coupling regime, there is no need to discuss how to generate terms $\lambda^n$ in the perturbation series. Actually, if we start with a function such as $\zeta(3)\zeta(2s - 3)$, we will be able to generate the positive integral powers. The above discussion can be repeated, and we will find again the same type of contradiction. We also tried to introduce poles at $s = -n$ in $f(s)$ in (2.4). It turned out at $\lambda^n$ a factor $\zeta(2n + 1)$ appear. This is not permissible. In particular, a factor $\zeta(3)$ should not appear in the two loop calculation.

We have assumed terms such as $\lambda^2 \ln \lambda$ are absent. These terms can be included into our consideration by introducing double poles in $G^*(s)$.

3. Absence of world-sheet instantons

Although we have provided a strong argument for the existence of a large $N$ phase transition somewhere in the positive axis of $\lambda$, we are still interested in how the general large $\lambda$ expansion series looks like. For instance, we would like to ask whether this expansion is an asymptotic one with the general coefficient growing as $n!$. The answer is
likely negative. One way to see this is through a more careful execution of the type of investigation presented in sect. 2 above \(\text{[1,7]}\). Here we provide another piece of evidence for this, the absence of the world-sheet instantons.

Unlike in the flat background, the free energy starts to receive contributions from the sphere in the black hole background. The reason for this is quite simple, that the Euclidean time circle is contractible in the black hole background, so the sphere can be wound around this circle. Thus, the leading contribution to the free energy is proportional to \(1/g_s^2 \sim N^2\).

If the general coefficient of the term \(\lambda^{-(n+3)/2}\) in the free energy goes as \(n!\), we would expect that the origin of this behavior is the existence of a spheric world-sheet instanton. For instance, a term \(\exp(-c\sqrt{\lambda})\) might be generated by a spheric minimal surface.

The topology of the Euclidean anti-de Sitter black hole is \(S^5 \times R^3 \times D\), where the 5 sphere of constant curvature is irrelevant for our question. The factor \(R^3\) corresponds to the spatial volume of D3-branes, and the infinite disk \(D\) is parametrized by the radial coordinate of the anti-de Sitter space and the Euclidean time circle. The metric on \(R^3 \times D\) reads

\[
ds^2 = \frac{1}{R^2U^2}(U^4 - U_T^4)dt^2 + R^2U^2(U^4 - U_T^4)^{-1}dU^2 + \frac{U^2}{R^2}ds_3^2, \tag{3.1}
\]

where \(R^2 = \sqrt{2\lambda}\), and \(ds_3^2\) is the Euclidean metric on \(R^3\). We have set \(\alpha' = 1\). Using the following coordinates transformation

\[
U^4 = U_T^4(1 + b^2), \quad t = \frac{R^2}{2U_T}\psi, \tag{3.2}
\]

the metric is put into a simpler form

\[
ds^2 = \frac{R^2}{4} \left( \frac{db^2}{1 + b^2} + \frac{b^2d\psi^2}{\sqrt{1 + b^2}} + \sqrt{1 + b^2}ds_3^2 \right), \tag{3.3}
\]

where we have rescaled the metric on \(R^3\). It becomes clear that the circumference of \(\psi\) is \(2\pi\) in order to get rid of a conical singularity at \(b = 0\). Let \(b = \sinh \rho\), the metric reads

\[
ds^2 = \frac{R^2}{4} \left( d\rho^2 + \sinh \rho \tanh \rho d\psi^2 + \cosh \rho ds_3^2 \right). \tag{3.4}
\]

Due to the trivial topology of \(R^3\), the only reasonable possibility for a world-sheet instanton is to embed the sphere into the coordinates \((\rho, \psi)\). The space of this part looks like a cigar near the tip \(\rho = 0\), since \(\sinh \rho \tanh \rho = \rho^2 - \rho^4/6 + \ldots\). But the actual radius of the circle gets ever larger with large \(\rho\). Mapping the sphere onto the tip of the cigar, the
sphere must be at least doubly folded. Intuitively, there can be no minimal area embedding, due to the fact that \( \sinh \rho \tanh \rho \) is an increasing function. One can rule out this possible minimal area embedding by simply considering the most symmetric embedding. Let the world-sheet metric be \( (dr^2 + r^2 d\phi^2)/(1 + r^2)^2 \). Without loss generality, assume half of the sphere \( r = (0, 1) \) is embedded into the tip of the cigar, and the other half is folded back. Let \( \psi = \phi \). The boundary conditions are \( \rho(r = 0) = 0, \partial_r \rho(r = 1) = 0 \). The latter condition is to ensure the embedding be smooth. The world sheet action is

\[
S = c(2\lambda)^{1/2} \int r dr \left( (\rho')^2 + \sinh \rho \tanh \rho \right),
\]

where \( c \) is a numerical constant. Use \( r = \exp(-R) \), the action becomes

\[
S = c(2\lambda)^{1/2} \int dR \left( (\rho')^2 + \sinh \rho \tanh \rho \right).
\]

The general solution to the minimal area problem is given by

\[
\rho' = \pm \sqrt{C + \sinh \rho \tanh \rho},
\]

where \( C \) is an integration constant. We can see that the boundary condition can not be satisfied by a nontrivial solution. The condition \( \rho(r = 0) = 0 \) becomes \( \rho(R = \infty) = 0 \). This is possible only when \( C = 0 \). Thus the solution is

\[
\rho' = \pm \sqrt{\sinh \rho \tanh \rho}.
\]

The other condition, \( \rho'(R = 0) = 0 \) can never be satisfied by the above solution unless \( \rho = 0 \). But this is a trivial solution: The whole sphere is mapped to the tip \( \rho = 0 \). The same conclusion can be drawn using the Nambu-Goto action.

We also checked that the corrected metric at the order \( (\alpha')^3 \) as given in [8] does not allow the simplest type of world-sheet instanton as discussed above.

4. Conclusion

The conclusion of this note is quite discouraging, that there exists a phase transition in the large N finite temperature theory of \( \mathcal{N} = 4 \) SYM. Note that this phase transition is different from that discussed in [4], there the existence of a phase transition has to do
with the finite volume. Here the phase transition exists for any finite temperature, although there is evidence that there is no such phase transition at zero temperature \[3\]. The continuum theory of SYM must be well-defined in the strong coupling regime, and Maldacena’s conjecture provides an efficient way to control the theory at zero temperature in this regime. A finite temperature theory in the strong coupling regime must be well-defined too, and hopefully Maldacena’s conjecture can be extended to this case, as advocated in \[4\]. If so, the string theory on AdS space must exhibit the same phase transition at a finite temperature. The transition occurs when the size of AdS is comparable to the string scale, while the string coupling is kept very small.

Our result casts doubt on the program of studying the large N pure Yang-Mills theory using Maldacena’s conjecture. It is true that the strong coupling regime shares many qualitative features with the weak coupling regime \[4,16,17,18\], such as confinement, magnetic screening. Nevertheless to gain any quantitative control one must study string theory in the background of the anti-de Sitter black hole with a small size \(\lambda\). Some preliminary evidence for disagreement between the two regimes was already noticed in \[19\].

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References

[1] J. Maldacena, hep-th/9711200.
[2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, hep-th/9802109.
[3] E. Witten, hep-th/9802150.
[4] E. Witten, hep-th/9803131.
[5] J. Maldacena, hep-th/9803002; S.-J. Rey and Yee, hep-th/9803001.
[6] M. R. Douglas and M. Li, to appear.
[7] S. S. Gubser, I. R. Klebanov and A. W. Peet, hep-th/9602135.
[8] S. S. Gubser, I. R. Klebanov and A. A. Tseytlin, hep-th/9805156.
[9] T. Banks and M. B. Green, hep-th/9804170.
[10] J. I. Kapusta, “Finite temperature field theory”, Cambridge University Press (1989).
[11] M. T. Grisaru and D. Zanon, Phys. Lett. B177 (1986) 347; M. D. Freeman, C. N. Pope, M. F. Sohnius and K. S. Stelle, Phys. Lett. B178 (1986) 199; Q.-H. Park and D. Zanon, Phys. Rev. D35 (1987) 4038.
[12] D. Gross and E. Witten, Nucl. Phys. B277 (1986) 1.
[13] E. Brezin and S. R. Wadia, “The large N expansion in quantum field theory and statistical physics: from spin system to two-dimensional gravity”, World Scientific (1993).
[14] J. Koplik, A. Neveu and S. Nussinov, Nucl. Phys. B123 (1977); W. T. Tuttle, Can. J. Math. 14 (1962) 21.
[15] G. ’t Hooft, in “Progress in gauge field theory”, eds. G. ’t Hooft et al. Plenum Press (1984).
[16] A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, hep-th/9803137; S.-J. Rey, S. Theisen and J.-T Yee, hep-th/9803135.
[17] M. Li, hep-th/9803252; hep-th/9804175.
[18] D. J. Gross and H. Ooguri, hep-th/9805129.
[19] J. Greensite and P. Olesen, hep-th/9806235.