Topological force and torque in spin-orbit coupling systems

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Abstract – The topological force and torque are investigated in systems with spin-orbit coupling. It is demonstrated that the topological force and torque appears as a pure relativistic quantum effect in an electromagnetic field. The origin of both topological force and torque is the \textit{Zitterbewegung} effect. Considering nonlinear behaviors of spin-orbit coupling, we address possible novel phenomena driven by the topological forces.

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Recently, the spin-orbit coupling has become an interesting topic due to the spin Hall effect [1–5]. It provides an efficient route to generate and control quantum spin state electrically. Generally speaking, the spin-orbit coupling arises as a relativistic quantum effect from the Dirac equation, and describes the interaction of the electron spin, momentum and electromagnetic field. In a system with spin-orbit coupling, the semiclassical equations of the electron were studied recently, and some novel effects have been found [6–16]. However, in systems with spin-orbit coupling, there remain some questions to be answered. As well known, the non-relativistic approximation of the Dirac equation can be obtained from the Foldy-Wouthuysen transformation. In the transformation, there exists a gauge potential in momentum space [16,17], so one question is, what is the position operator in semiclassical equations? Since the position operator is the space-time parameter, its definition is significant. If we are incapable of defining it correctly, we could not have a complete understanding of spin-orbit coupling.

In this paper, we investigate the gauge field of the position operator in momentum space, and its related effects. Based on the dynamic continuity equation, we derive the quantum force and torque which contain two parts. The conventional part has the same form as in classic electrodynamics, and the topological part originates from the spin-orbit coupling and other terms from relativistic quantum correction. We notice that the topological part has a close relation to the \textit{Zitterbewegung} effect. For a two-dimensional system in a magnetic field, we propose that the topological force and torque can reveal more complicated phenomena.

\textbf{Topological velocity.} – The Dirac equation of the electron with the wave function $\Psi = (\varphi, \chi)^T$ reads $i\hbar \frac{\partial}{\partial t} \Psi = H \Psi$, with the Hamiltonian $H = c\alpha \cdot \pi + \beta mc^2 + V(x)$, where $\pi = P - \frac{e}{c} A$, $\alpha$ and $\beta$ are the $4 \times 4$ Dirac matrices, $V(x) = e\phi$ is a scalar potential, and $A$ is a vector potential for a magnetic field, $B = \nabla \times A$. $m$ and $e$ are the electron mass and charge, and $c$ is the speed of light. To reveal the spin-orbit coupling in Dirac equation, we perform the Foldy-Wouthuysen transformation $\Psi = U(\pi) \Psi$, where $U(\pi) = e^{iS}$, is a unitary transformation [18,19]. Choosing $S = -i(\beta \alpha \cdot \pi/2mc)$, and substituting it into the Dirac equation, we obtain the transformed Hamiltonian

$$H' = \beta mc^2 + e^{iS} \alpha \cdot \pi e^{-iS} + e^{iS} V(i\hbar \partial_p) e^{-iS} - e^{iS} i\hbar \partial_t e^{-iS},$$

(1)

The scalar potential becomes $V(D)$ with the covariant derivative defined by $D = i\hbar \partial_p + A$, with the pure gauge potential $A = ihU(\pi)^\dagger \partial_p U(\pi)$. By defining the covariant derivative, we have the position operator $X = i\hbar \partial_p + A$. The gauge potential $A$ here is trivial. Neglecting the interband transition, and considering the adiabatic approximation, we find the non-trivial gauge potential

$$A = \frac{\hbar(\pi \times \Sigma)}{4m^2c^2},$$

(2)

where $\Sigma = 1 \otimes \sigma$. 

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By expanding the Hamiltonian $H'$, and taking $S' = \hbar c \mathbf{\alpha} \cdot \mathbf{E}/4m^2c^3$ to do another transformation $\Psi'' = e^{iS'} \Psi'$, we get the non-relativistic Hamiltonian

$$H_{sch} = \frac{\pi^2}{2m} + e\phi - \frac{e\hbar}{2mc} \mathbf{\sigma} \cdot \mathbf{B} - \frac{e^2\hbar^2}{8m^2c^2} \nabla \cdot \mathbf{E} - \frac{\hbar^2}{8m^2c^2} \mathbf{\sigma} \cdot (\mathbf{E} \times \mathbf{\pi} - \mathbf{\pi} \times \mathbf{E}) + \frac{e^2\hbar^2}{4m^2c^2} \mathbf{\sigma} \cdot \mathbf{B},$$

(3)

where the first two terms refer to kinetic energy and scalar potential, the third and the sixth terms are the Zeeman energy and its relativistic correction, the fourth term is a Darwin term that refers to the correction of the scalar potential, and the fifth term is the so-called spin-orbit coupling term. With this Hamiltonian, the Schrödinger equation with wave function $\psi$ can be rewritten as

$$i\hbar \frac{\partial}{\partial t} \psi = H_{sch} \psi,$$

where $\psi = (1 + (\mathbf{\sigma} \cdot \mathbf{\pi})^2/8m^2c^2) \varphi$ is the wave function $\varphi$ which has performed unitary transformations. After the unitary transformations, the wave function $\chi \sim O(1/c^2)$, so that we can neglect it. Based on this fact, we can regard the wave function $\psi$ as having undergone the normalization. In the Schrödinger equation of $\psi$, the position operator has been redefined by a covariant derivative in the momentum space $\mathbf{X} = \mathbf{D} = i\hbar \partial_{\mathbf{p}} + \mathbf{A}$, with the gauge potential $\mathbf{A} = \hbar (\mathbf{\pi} \times \mathbf{\sigma})/4m^2c^2$.

We now turn our attention to the gauge potential induced by transformation $U(\mathbf{p})$. In our process, we consider the adiabatic approximation, so it is straightforward to interpret the gauge potential $\mathbf{A}$ as the non-Abelian Berry gauge potential, and it is Berry connection in momentum space, the Berry phase $\theta = \oint_{\mathbf{p}} \mathbf{A}$, where $C$ is the loop in the momentum space. The Berry gauge potential describes the non-trivial geometry of the fiber bundle of the Hamiltonian eigenvectors over the phase space, it describes the influence of negative-energy states in positive-energy space.

Using $\mathbf{A} = \hbar (\mathbf{\pi} \times \mathbf{\sigma})/4m^2c^2$ and the relation $F_{ij} = \partial_{p_i} \mathbf{A}_j - \partial_{p_j} \mathbf{A}_i = (i/\hbar)[\mathbf{A}_i, \mathbf{A}_j]$, we get the non-Abelian Berry gauge curvature $F_{ij} = -\epsilon_{ijk} \hbar (\mathbf{\sigma}_k/2m^2c^2 - (\mathbf{\sigma} \cdot \mathbf{\pi})/8m^2c^2)$. Let $\lambda = -\frac{e^2}{2}\mathbf{\sigma}_k$, $\pi \equiv \mathbf{P}$, introducing a dual vector $F_k = \epsilon_{ijk} F_{ij}$, then the gauge field can be rewritten as $F_k = (\lambda/m^2c^2)(1 - \mathbf{P}^2/4m^2c^2)$, where the first term presents a monopole the radial length at the rest momentum $mc$ with its strength given by $\frac{\hbar}{2} \mathbf{\sigma}_k$ in the momentum space, the second term is the high order of correction. If we neglect the second term in $F_k$, then

$$F_k = \lambda/(mc)^2.$$

(4)

We note that this field only has two values $\pm \frac{\hbar}{2}$ in the momentum space, so the difference between this field and the ordinary field $F_k = \lambda / \mathbf{P}^2$ [20], is that this field is uniform. Obviously, this gauge field is a consequence of non-relativistic approximation $\mathbf{P} \ll mc$. In classical dynamics, the rest energy $mc^2$ does not affect the motion of particles in the non-relativistic limit. However, in quantum mechanics, the rest momentum $mc$ induces a gauge field in the momentum space, then it will influence the motion of particles.

We notice that this gauge field can induce topological effects. Noting that after the unitary transformation, the covariant position operator of the Dirac equation is defined by $\mathbf{X}$, and the commutator of covariant operator is not trivial: $[x_i, x_j] = i\hbar F_{ij}$. This relation is characteristic of the unitary transformation, it is this relation that induces the gauge field in the non-relativistic approximation. Defining $\mathbf{x} = i\hbar \partial_{\mathbf{p}}$ as the canonical position operator, then $\mathbf{X} = x + A$, and the semiclassical equation of motion is $\dot{\mathbf{X}} = \dot{x} + \mathbf{A} \times \mathbf{F}$, with $F_{ik} = F_{ki}/2$. Apparently, the last term of the equation above is a topological term, and results from the non-trivial commutator of the covariant position operator. It acts as the Lorentz force in the momentum space.

By using the Hamiltonian (3), we define the velocity $\mathbf{X} = \dot{x} + (e\hbar/4m^2c^2)(\mathbf{E} + (\mathbf{\pi} \times \mathbf{B}) \times \mathbf{\sigma})$, where $\dot{x} = \pi/m + (e\hbar/4m^2c^2)\mathbf{E} \times \mathbf{\sigma}$ is the conventional velocity. The second term in $\mathbf{X}$ actually is a topological term, it is caused by the gauge field $F_k = \lambda/(mc)^2$, and we regard it as a topological velocity. Introducing the electron intrinsic magnetic moment $\hat{\mathbf{m}} = (e\hbar/2mc)\mathbf{\sigma}$, we can rewrite the velocity as

$$\dot{\mathbf{v}} = \dot{\mathbf{v}} + \frac{(\mathbf{E} \times \hat{\mathbf{m}})/2c}{m},$$

(5)

where $\dot{\mathbf{v}} = \dot{\mathbf{x}}$ is the conventional velocity operator which corresponds to position operator $\mathbf{x}$, $\mathbf{E}' = \mathbf{E} + (\mathbf{\pi}/mc) \times \mathbf{B}$ is the total electric field which affects an electron in its local coordinate frame. If we remind that the gauge field is induced by the interference between the negative and positive-energy states, this topological term in eq. (5) is contributed by the rest energy under the influence of negative states. Comparing with two velocities, we find that the terms related with $\mathbf{E}$ in eq. (5) are twice those of the conventional velocity. Besides, our definition adds a contribution of the magnetic field. One can easily find that these differences between the two velocities is caused by the gauge field in momentum space.

Topological force and torque. – This is very interesting in the topological force which acts on the electron in an electromagnetic field. Let us start from the dynamic continuity equation. The dynamic continuity equation represents the change form of a physical quantity in space-time, it always requires the existence of a source such as a force or a torque.

Denote by $\rho^M = \psi^\dagger \rho^M \psi$ the momentum density, where $\rho^M = mc\mathbf{v}$. Using $\psi^\dagger \rho^M \psi = \psi^\dagger \rho^M \psi + \psi^\dagger \rho^M \psi$ and $i\hbar \dot{\psi} = H_{sch} \psi$, then we arrive at

$$\dot{\rho^M} + \nabla \cdot \mathbf{J}^M = f,$$

(6)

where $\mathbf{J}^M = \psi^\dagger \mathbf{J}^M \psi = (1/2) \psi^\dagger \{ \rho^M, \mathbf{v} \} \psi$, and $\mathbf{v}$ is the conventional velocity. The force is given by $f = (1/\hbar c) \times [\rho^M, H_{sch}] = f_{con} + f_{top}$, where $f_{con}$ is the conventional force which corresponds to the position operator $\mathbf{x}$, and $f_{top}$ is the topological force. The conventional force can be written as $f_{con} = \rho^C \mathbf{E} + (\mathbf{C} \times \mathbf{B})/c + (1 + (\mathbf{\pi}^2/2m))/(mc^2) \mathbf{\nabla} \cdot \mathbf{E} \times (\mathbf{\pi} \times \mathbf{B})$, where
\[ \rho^C = e\psi^\dagger \psi \quad \text{and} \quad J^C = e\psi^\dagger \hat{\psi} \psi \] are the charge and the current densities for the spin-aligned electrons, and \( P \) is defined by \( P = \psi^\dagger \hat{P} \psi = \psi^\dagger (\frac{\pi}{2mc} \times \hat{m}) \psi \), it correlates with the electrical polarization induced by the spin-orbit coupling. The total magnetic field \( \mathbf{B}' = \mathbf{B} - (\pi/2mc) \times \mathbf{E} \) presents an electron experiences in its local coordinate frame. Meanwhile, considering the topological velocity, the topological force can be found to be

\[ f_{\text{top}} = \frac{e}{2mc^2} \mathbf{E}' \times (\hat{\mathbf{m}} \times \mathbf{B}'). \] (7)

It is clear that the topological force is an additional term to the conventional force, so that we can divide the total force into two parts: \( f = f_{\text{cla}} + f_{\text{so}} \), where \( f_{\text{cla}} \) is the force produced by the spin-orbit coupling. The force \( f_{\text{so}} \) can be written as

\[ f_{\text{so}} = \rho^C \mathbf{E} + (\mathbf{J}^C \times \mathbf{B})/c + \left(1 + \frac{\pi^2/2m}{mc^2}\right) (\nabla \mathbf{B}) \cdot \hat{\mathbf{m}} + (\nabla \mathbf{E}) \cdot \mathbf{P}; \] (8)

it is well known that the classical forces are Lorentz, “Stern-Gerlach”, and electric dipole forces. All the forces above have corresponding forces in classical dynamics, so we combine them to a classical force \( f_{\text{cla}} \).

The force \( f_{\text{so}} \) can be written as \( f_{\text{so}} = (e/2mc^2) \mathbf{E}' \times (\hat{\mathbf{m}} \times \mathbf{B}') \), where \( \mathbf{E}' = \mathbf{E} + (\pi/2mc) \times \mathbf{B} \), this force is of order \( 1/c^2 \), for it is contributed by the spin-orbit term in Hamiltonian (3) and the gauge potential. To explore the physical meaning, we divide this force into two parts,

\[ f_{\text{so}} = e\psi^\dagger \left( \frac{\mathbf{E}' \times \hat{\mathbf{m}}}{mc^2} \right) \times \mathbf{B}' \psi + e\psi^\dagger \left( \frac{\mathbf{B}' \times \mathbf{E}'}{mc^2} \right) \times \hat{\mathbf{m}} \psi, \] (9)

where the first term indicates the electron experiencing an electrical field \( \mathbf{E}' = [(\mathbf{E}' \times \hat{\mathbf{m}})/mc^2] \times \mathbf{B}' \), and it is an electric force. One can find that the second term of \( f_{\text{so}} \) is a pure quantum effect, since \( (\mathbf{E} \times \mathbf{B}')/mc^2 \) is dimensionless, this term is not related with any classical quantity, it is a pure quantum force, as shown in fig. 1. It is a new force.

When the magnetic field vanishes, the force \( f_{\text{so}} \) reduces to \( f_{\text{so}} = -\psi^\dagger (e/2mc^2)(\hat{\mathbf{m}} \times \mathbf{E})(\frac{\pi}{mc} \times \mathbf{E}) \psi \). This force originates from the second term of \( f_{\text{so}} \), so it is a pure quantum force, it can be used to explain the Zitterbewegung of the electron in the spin-orbit coupling system [7,8]. The direction of this force is perpendicular to the electrical field, and to the velocity and the spin polarization direction, so it is a transverse force. This force is relevant to the generation of the charge Hall effect driven by the spin current [8].

When the electrical field vanishes, the force \( f_{\text{so}} \) becomes \( f_{\text{so}} = e\psi^\dagger (e/2mc^2)(\hat{\mathbf{m}} \times \mathbf{E})(\frac{\pi}{mc} \times \mathbf{E}) \psi \), this force also originates from the second term of \( f_{\text{so}} \), and it is also a pure quantum force. It is a new force. The direction of this force parallels the magnetic field, and is perpendicular to the velocity and the spin polarization direction, so this force also is a transverse force. In fact, this force has duality with \( f_{\text{cla}} \), so it also can be known from the Zitterbewegung effect.

We now turn to the angular-momentum continuity equation. Denoting \( \rho^{AM} \) as the total angular-momentum density, then \( \rho^{AM} = \rho^O + \rho^S \), where the orbit momentum density \( \rho^O = \psi^\dagger \mathbf{X} \times \mathbf{P} \psi \), the spin density \( \rho^S = (h/2\psi)^\dagger \mathbf{\sigma} \psi \). Similar to the momentum continuity equation, we arrive at

\[ \rho^{AM} + \nabla \cdot \mathbf{J}^{AM} = \mathbf{T}. \] (10)

The angular-momentum current is defined by \( \mathbf{J}^{AM} = (1/2)\psi^\dagger \left( \mathbf{X} \times \mathbf{J}^{AM} + (h/2)\mathbf{\sigma} \times \tilde{\mathbf{v}} \right) \psi \), where the first term is the orbit angular-momentum current, and the second term is the spin current \( \mathbf{J}^{S} \). The total torque is given by \( \mathbf{T} = (1/ih)[\rho^{AM}, \mathbf{H}_{\text{seq}}] = \mathbf{T}_{\text{con}} + \mathbf{T}_{\text{top}} \), where \( \mathbf{T}_{\text{con}} \) is the conventional torque which corresponds to the position operator \( \mathbf{x} \), \( \mathbf{T}_{\text{top}} \) is the topological torque. The conventional torque can be read \( \mathbf{T}_{\text{con}} = \psi^\dagger \mathbf{x} \times \tilde{\mathbf{f}} \psi + \mathbf{m} \times \mathbf{B} \), where \( \tilde{\mathbf{f}} \) is an operator with \( \tilde{\mathbf{f}} = \psi^\dagger \tilde{\mathbf{f}} \psi \). Meanwhile, considering the topological velocity, the topological torque reads

\[ \mathbf{T}_{\text{top}} = \psi^\dagger \mathbf{A} \times \tilde{\mathbf{f}} \psi + \psi^\dagger \mathbf{x} \times \tilde{\mathbf{f}}_{\text{top}} \psi, \] (11)

where the first term is related to the gauge potential, and the second term is contributed by the topological force, so both terms can be known from the Zitterbewegung effect.

Of course, the total torque can be divided into the orbit torque \( \mathbf{T}^{O} \) and the spin torque \( \mathbf{T}^{S} \). Combining the conventional and the topological torque, the orbit torque has the form \( \mathbf{T}^{O} = \psi^\dagger \mathbf{X} \times \tilde{\mathbf{f}} \psi \). Obviously, \( \mathbf{T}^{O} \) has a corresponding torque in classical dynamics. The spin torque is given by \( \mathbf{T}^{S} = \mathbf{m} \times \mathbf{B} - \psi^\dagger \mathbf{m} \times (\frac{\pi}{mc} \times \mathbf{E}) \psi \). The first term is contributed by the magnetic field. The second term is the influence of electric field \( \mathbf{E} \), which can induce a magnetic field in the electron’s own coordinate frame, where \( 1/2 \) is the Thomas precession factor. These two terms can be found in the Bargman-Michel-Telegdi equation which describes the precession of the electron’s spin [21].

Now we can write the spin continuity equation as \( \rho^S + \nabla \cdot \mathbf{J}^{S} = \mathbf{T}^{S} \), where the spin density is defined by \( \rho^S = (h/2)^\psi \tilde{\mathbf{f}} \psi \), the spin torque is defined by \( \mathbf{T}^{S} \). The spin current is \( \mathbf{J}^{S} = (h/4)\psi^\dagger \left( \mathbf{\sigma} \times \tilde{\mathbf{v}} \right) \psi \). \( \tilde{\mathbf{v}} \) is the conventional velocity operator.
Nonlinear spin–orbit coupling. – We apply the previous results to a two-dimensional system under a weak magnetic field $\mathbf{B} = (0, 0, B)$, with $A = (0, x B, 0)$. The Hamiltonian reads $H_{2D} = \pi^2/2m - \mu_B \sigma \cdot \mathbf{B} + (eF/2) \times (\mathbf{E} \times \sigma) \cdot \pi + e \mathbf{E} \cdot \mathbf{z}$, where $\mu_B$ is Bohr magnetic momentum, $F = (h/2)/(m \epsilon)^2$ is the gauge field, the electrical field $\mathbf{E} = (E, 0, 0)$. The topological velocity $v_\theta = h k_y/m - \mu_B F \sigma \cdot k_y - eF \sigma \cdot \mathbf{E} \cdot \mathbf{z}$, where $k_y$ is the wave vector. The spin current operators read $J^S_\sigma = (h^2/2m)\sigma \cdot k_y - (h/2) \times \mu_B F \sigma \cdot \mathbf{E} \cdot \mathbf{z}$, where the superscript presents the spin direction, the subscript denotes the direction of the electron motion. It is found that the topological term here only concerns with momentum. In this case, the trajectories read $X = x + h F k_y \sigma_z / 2$, $Y = y$. The spin current $J^S_\pi = \sigma_{zH} E$, where the spin Hall coefficient $\sigma_{zH} = -(e/4\pi) (\delta - \delta_c / 4 - \epsilon / 2)$, where $\delta = eB\ell_\pi / c$, $e = \hbar k^2_F$ are dimensionless parameters, $k_F$ is the Fermi momentum. It is clear that the second term in the spin Hall coefficient $\sigma_{zH}$ is contributed by the gauge field.

For spin current $J^S_\sigma$, there is a pure quantum force perpendicular to the magnetic field, it is given by $f^\sigma_{\mu} = \mu_B (e F / m c^2)^3 \psi^\dagger \sigma \cdot \psi^\dagger B$. We can find that this force is proportional to the magnetic field. The directions of the spin polarization, the current and the force are perpendicular to each other. This force originates from the Zitterbewegung effect.

If we construct a spin-orbit coupling Hamiltonian as follows:

$$H = \frac{\pi^2}{2m} - \mu_B \sigma \cdot \mathbf{B} + \eta(\pi \times \theta) \cdot \nu,$$

where $\eta = (\hbar e / 4 m^3 c^3) B$ is a spin-orbit coupling parameter, $\nu$ is a unit direction which is along the $z$ direction, and $\theta = \theta(\pi, \sigma) = \pi \times \sigma$ is a vector presenting spin-orbit coupling types, we can also obtain the same velocity of the two-dimensional system above (the system has eliminated the scalar potential). This indicates that the Hamiltonian $H_{2D}$, which takes the gauge field into account, is equivalent to a system with more complex spin-orbit coupling. This equivalence shows that the gauge field can present a more complicated spin-orbit coupling in the spin-orbit coupling system, such as $\pi \times \sigma$, with $\theta = \pi \times \sigma$ or $(\pi \cdot \sigma) \mathbf{n}$, where $\mathbf{n}$ is a unit direction. In fact, Hamiltonian (11) is a generalization of Rashba model in which $\theta(\pi, \sigma) = \sigma [22]$.

In the usual spin-orbit coupling system, the types of spin-orbit coupling are $\pi \times \sigma$ and $\pi \cdot \sigma$. They will exist as long as the spin-orbit coupling potential is not weak. However, these two types only present the first level of spin-orbit coupling, the types which we consider presenting the complicated level of spin-orbit coupling, such as $\pi \times \theta$, can also exist in a spin-orbit coupling system. Comparing with the first level, the complicated level of spin-orbit coupling is the coupling of the orbit and the first level. The first level is proportional to the orbit parameter $\pi$, and the complicated level is proportional to the square of the orbit parameter, so one can find that the complicated level reflects the non-linear character of spin-orbit coupling system. In fact, there are not only the linear spin-orbit coupling, but also the non-linear spin-orbit coupling in a spin-orbit coupling system. The first level of spin-orbit coupling such as $\pi \times \sigma$ or $\pi \cdot \sigma$ is the linear spin-orbit coupling, the complicated level such as $\pi \times \theta$ reflects the non-linear spin-orbit coupling. As a consequence, we can say that the complicated level presents the non-linear character of a spin-orbit coupling system.

The significance of the topological force and torque lies in the fact that they provide a complete picture of describing the motion of a non-relativistic electron in a spin-orbit coupling system. It is found that the topological force appears as a pure quantum effect when both electrical and magnetic fields exist in the system, while the topological torque originates from the topological force and a gauge potential. Through these force and torque, the Zitterbewegung effect can be understood deeply, more complicated phenomena such as the nonlinear spin-orbit coupling also can be revealed. This implies that the spin-orbit coupling can produce novel effects.

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REFERENCES

[1] Hirsch J. E., Phys. Rev. Lett., 83 (1999) 1834.
[2] Murakami S., Nagaosa N. and Zhang S. C., Science, 301 (2003) 1348.
[3] Sinova J., Culcer D., Niu Q., Sinitsyn N. A., Jungwirth T. and MacDonald A. H., Phys. Rev. Lett., 92 (2004) 126603.
[4] Shen S. Q., Ma M., Xie X. C. and Zhang F. C., Phys. Rev. Lett., 92 (2004) 256603.
[5] Entin-Wohlman O., Aharony A., Galperin Y. M., Kozub V. I. and Vinokur V., Phys. Rev. Lett., 95 (2005) 086603.
[6] Culcer D., Sinova J., Sinitsyn N. A., Jungwirth T., MacDonald A. H. and Niu Q., Phys. Rev. Lett., 93 (2004) 046602.
[7] Jiang Z. F., Li R. D., Zhang Shou-Cheng and Liu W. M., Phys. Rev. B, 72 (2005) 045201.
[8] Shen S. Q., Phys. Rev. Lett., 95 (2005) 187203.
[9] Nikolic B. K., Zarlo L. P. and Welack S., Phys. Rev. B, 72 (2005) 075335.
[10] Bernevig B. A. and Zhang S. C., Phys. Rev. Lett., 95 (2005) 016801.
[11] Wang Y., Xia K., Su Z. B. and Ma Z. S., *Phys. Rev. Lett.*, 96 (2006) 066601.
[12] Jin P. Q., Li Y. Q. and Zhang F. C., *J. Phys. A*, 39 (2006) 7115.
[13] Rashba E. I., *Phys. Rev. B*, 68 (2004) 241315; 70 (2004) 161201.
[14] Sun Q. F. and Xie X. C., *Phys. Rev. B*, 72 (2005) 245305.
[15] Bolte J. and Keppeler S., *Ann. Phys. (N.Y.*), 274 (1999) 125; Spohn H., *Ann. Phys. (N.Y.*), 282 (2000) 420.
[16] Bliokh K. Yu., *Europhys. Lett.*, 72 (2005) 7.
[17] Bérard A. and Mohrbach H., hep-th/0404165.
[18] Foldy L. L. and Wouthuysen S. A., *Phys. Rev.*, 78 (1950) 29.
[19] Bjorken J. and Drell S., *Relativistic Quantum Mechanics* (McGraw-Hill) 1964.
[20] Berry M. V., *Proc. R. Soc. London, Ser. A*, 392 (1984) 45.
[21] Bargmann V., Michel L. and Telegdi V. L., *Phys. Rev. Lett.*, 2 (1959) 435.
[22] Bychkov Y. A. and Rashba E. I., *J. Phys. C*, 17 (1984) 6039.