A Method of the Voronoi Seabed Sedimentary Domain Boundary Reconstruction

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Abstract. Seabed sediment point data is the data basis for expressing the sediment elements. The determination of the extent of the seabed sediment type is the data base of the geometric model and the seabed sediment thematic chart. For the existing point element selection method cannot meet the application requirements. This paper uses the weighted idea to different data density and complex marine environment. Taking into account the trend of the sediment size and charting compensation effect, the influence weight of the point-like bottom data is analyzed, and the range of influence is reasonably determined. It provides a certain reference for the production of seabed sediment thematic chart and has high use value and broad application prospects.

1. Introduction
In the digital chart, due to the loading limit, the sediment data is less and the distribution is irregular [1]. The sediment survey data is limited by the marine meteorological environment, the seabed topographic environment and the offshore survey environment [2]. In some sea areas, the sediment detection cannot be performed completely according to the survey line, and the dot data is irregularly arranged and the distribution density of the station is inconsistent [3][4]. In order to obtain higher density and larger data volume, the multi-source bottom point data of the same sea area can be combined to form a new data set. There are overlapping points or ineffective points with very close distances, which leads to limitations such as large differences in distribution density and large data redundancy. Based on the theory of digital charting, spatial analysis of seabed sediment and the representation method of chart elements, this paper designs the thematic layer of seabed sediment according to the characteristics of sediment elements, and puts forward some innovative practices on the generation and representation of marine sediment elements.

2. Weight analysis of the influence range of seabed sediments

2.1. Seabed sediment group generation
After the Voronoi diagram is generated by the indirect method, the Voronoi polygons with the same sediment type are first combined to obtain the seabed sediment distribution domain, and the points in the same sedimentary distribution domain form a point group. The process of polygon merging is
actually the process of deleting the common edges for adjacent and identical underlying type regions [5]. As shown in Figure 1.

![Seabed sediment Voronoi diagram](image1)

![Seabed sediment distribution boundary](image2)

**Figure.1 Seabed sediment distribution domain**

### 2.2. Boundary point acquisition

The selection of boundary points needs to determine a certain point group distribution range. Considering the importance of boundary points, it is necessary to obtain boundary point processing within the same point group. The basic idea of obtaining point group boundary points is that the convex hull algorithm obtains the boundary to obtain [6]. As shown in Figure 2, the specific steps are as follows:

1. Generating a convex hull from a point set element;
2. Extracting the convex hull boundary;
3. Extract the points that make up the boundary;
4. Constructing a line with the point of the boundary.

![Convex hull algorithm](image3)

**Figure.2 Schematic diagram of obtaining boundary points**

### 2.3. Boundary contraction vector and relative weight

#### 2.3.1. Boundary contraction vector

Boundary contraction vector is the part vector of the vector in the direction of the vertical bisector of the corresponding boundary. Represented by $l_{ij}$, represents the contraction vector between points $a_i$ and $a_j$, and its length $|l_{ij}|$ satisfies the formula (1)[7].

$$|l_{ij}| = |R_i| \times |\cos(R_i, a_i a_j)|$$

#### 2.3.2. Relative weight

The boundary is extended by the direction of the contraction vector, and the degree of the change can define the relative weight $w_{ij}$. The weight is only applied between $a_i$ and $a_j$, the size of relative weight is positively correlated with the length of the contraction vector, which satisfies the formula (2). The function $f$ is a normalization function, and the weight must be normalized to a certain interval in the specific reconstruction calculation [8].
\[ w_{ij} = f(|l| + 1) \]  

Figure 3 shows the calculation of the \( a_0 \) contraction vector, \( \text{---} \) represents \( a_0 \) and \( a_1 \) represent their corresponding Voronoi boundaries. The contraction vector \( l_{01} \) is the projection of \( R_0 \) on \( a_0a_1 \), 
\[ |l_{01}| = |R| \times |\cos \alpha_{01}|. \]

3. Voronoi boundary reconstruction

3.1. Voronoi reconstruction boundary equation

3.1.1. Voronoi algorithm principle.

The process of indirectly generating a Voronoi polygon using the Delaunay triangulation is shown in Figure 4. In the figure, \( o \) is the midpoint of the corresponding line segment. For the line segment \( a_0a_1 \), the vertical bisector intersects the triangle edge with \( d_{01} \), and so on, the polygon enclosed by the vertical bisector of each triangle of the \( a_0 \) connection is the Voronoi polygon \( S_0 \).

![Figure 4 Voronoi Polygon Principle](image)

3.1.2. Voronoi boundary reconstruction equation.

The relative weight \( w_{ij} \) determined according to the boundary contraction vector can reflect the "contention" ability between \( a_i \) and \( a_j \). When the relative weights are equal, the intersection \( d_{ij} \) of the Voronoi boundary and the corresponding triangle edge is the midpoint of the triangle edge. The boundary contraction direction is the same or opposite to the \( a_i a_j \) direction, so the "contention" of the two points on the Voronoi boundary is equivalent to the competition for the intersection point \( d_{ij} \), and
the position of $d_{ij}$ can be adjusted according to the relative weights of the two. The adjusted intersection $d_{ij}$ satisfies the formula (3).

$$\frac{a_id_{ij}}{a_jd_{ij}} = \frac{1}{w_{ij}}$$

(3)

Let the points $a_i(x_i, y_i)$ and $a_j(x_j, y_j)$ correspond to the local weighting improved polygon boundary line is $L_{ij}$, the equation of the line is: $(y - y_i)(y_j - y_i) = (x - x_i)(x_j - x_i)$. Equation (3) is substituted into the equation of the line, and the coordinates of the intersection $d_{ij}$ after local improvement are:

$$d_{ij}\left(\frac{1}{1+w_{ij}}(x_j-x_i)+x_i, \frac{1}{1+w_{ij}}(y_j-y_i)+y_i\right)$$

(4)

According to the coordinates of the Voronoi boundary and the straight line and the improved intersection point $d$, the linear equation corresponding to the Voronoi reconstruction boundary between $a_i$ and $a_j$ satisfies the formula (5).

$$y = -\frac{x_i-x_j}{y_i-y_j}\left(x - \frac{1}{1+w_{ij}}(x_j-x_i)+x_i\right) + \frac{1}{1+w_{ij}}(y_j-y_i)+y_i$$

(5)

3.2. Maintaining the boundary reconstruction of Voronoi characteristics

3.2.1. Double distribution domain boundary.

According to the reconstruction boundary $L_{ij}$, $L_i$ is sorted in the order of generation, and the first and last boundaries $L_0$ and $L_n$ are set as the frame boundary. The adjacent two line segments intersect to obtain the intersection point $q_j$, and $q_j(x_i, y_i)$ is the intersection of $L_i$ and $L_{i+1}$. Connect $q_j$ in turn to reconstruct the Voronoi boundary.

In order to maintain the characteristics of the convex polygon of the Voronoi polygon, $q(i)$ needs to be further processed. As shown in Figure 5, since $\angle q_jq_{i+1}q_{i+2} < 90^\circ$, $q_i$ and $q_{i+1}$ are invalid points, which need to be processed. The specific implementation steps are as follows:

1. If $i < n - 1$, it is determined whether $(x_j - x_{i+1})(x_{i+2} - x_{i+1}) + (y_j - y_{i+1})(y_{i+2} - y_{i+1})$ is greater than or equal to 0 (judgment the angle is an obtuse angle); if yes, proceed to step 2; if not, proceed to step 3; If $i = n - 1$, go to step 4;
2. Calculate the intersection $(a, b)$ of $L_i$ and $L_{i+1}$, let $x_i = x_{i+1} = a$, $y_i = y_{i+1} = b$, i++, return to step (1);
3. i++, return to step (1);
4. Delete the points with the same coordinates in the point set $\{q_i\}$, connect the points in the point set in turn, and reconstruct the Voronoi boundary.
3.2.2. Multi-distribution domain boundary processing.

Voronoi boundary reconstruction of multi-distribution domain is completed on the basis of dual distribution domain. As shown in Figure.6, if the area A has 0-k total k+1 intersection points, the starting point of the area distribution field C is \( q_{k+1} \). To maintain the boundary properties of Voronoi, let \( x_{k+1} = x_i, \ y_{k+1} = y_j \), and update the corresponding boundary equation.

4. Experiment and analysis

Two sets of distribution domain reconstruction experiments were performed on the double-distribution domain boundary and the multi-distribution domain boundary respectively, and the normalization interval of relative weight was \( [1, 3] \). Specific experimental data are shown in Table 1.

| Experimental data | Number of distribution domains | Number of boundary segment | Relative weight distribution | Weight normalization interval |
|-------------------|--------------------------------|---------------------------|-----------------------------|-------------------------------|
| Experimental data 1 | 2 | 19 | \([1.48, 2.77]\) | \([1, 3]\) |
| Experimental data 2 | 3 | 26 | \([0.3, 6.1]\) | \([1, 3]\) |

As shown in Figure.6, the direction of the arrow is the overall direction of the trend vector of the region, and the direction of contraction of the relative weight to the boundary of the Voronoi distribution domain is substantially consistent with the direction of the overall trend vector of the region. The closer the vertical direction of the reconstructed boundary is to the opposite direction of the trend vector, the more severe the boundary shrinks.
5. Conclusion

The Voronoi seabed sedimentary distribution domain boundary reconstruction method can effectively represent the change trend of the substrate property. The reconstructed Voronoi distribution domain boundary can reflect the trend vector direction and size at the boundary point and prove with the overall trend vector of the region, in a certain extent, it can reflect the sediment transport direction and transport capacity of sediments in the distribution domain. This paper perfects the algorithm and representation method of seabed sediment elements, and makes a diversified expression of the Seabed sediment, more scientifically organizes the seabed sediment data, and enriches the application field of the seabed sediment thematic chart.

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