Scheduling in Multi-hop Wireless Networks with Priorities

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Abstract—In this paper we consider prioritized maximal scheduling in multi-hop wireless networks, where the scheduler chooses a maximal independent set greedily according to a sequence specified by certain priorities. We show that if the probability distributions of the priorities are properly chosen, we can achieve the optimal (maximum) stability region using an i.i.d random priority assignment process, for any set of arrival processes that satisfy Law of Large Numbers. The pre-computation of the priorities is, in general, NP-hard, but there exists polynomial time approximation scheme (PTAS) to achieve any fraction of the optimal stability region. We next focus on the simple case of static priority and specify a greedy priority assignment algorithm, which can achieve the same fraction of the optimal stability region as the state of art result for Longest Queue First (LQF) schedulers. We also show that this algorithm can be easily adapted to satisfy delay constraints in the large deviations regime, and therefore, supports Quality of Service (QoS) for each link.

Keywords: Maximal scheduling, wireless networks, stability, delay, priority, large deviations.

I. INTRODUCTION

Efficient scheduling algorithms in wireless networks have been the subject of intensive research in the past few years. A fundamental issue is that the optimal scheduling algorithm, which can achieve a super set stability region of any scheduler, involves solving a Maximum Weighted Independent Set (MWIS) problem [1], which is, in general, NP-Hard [2]. This phenomenon is exaggerated by the requirement that the MWIS problem has to be solved in every time slot, which renders it impossible to be implemented in applications due to the high computing resource consumptions. As an alternative, low complexity suboptimal schedulers with guaranteed efficiency ratio are needed, among which, maximal scheduling has been the focus of recent research. Similar to the maximal matching in the switch scheduling literature [10], maximal scheduling is a low complexity algorithm in wireless networks with a constant efficiency ratio which is inversely proportional to the maximum number of independent links in a link’s neighborhood [3], [5]. Not only is it sound theoretically, maximal scheduling is also well supported in practice by efficient distributed algorithms with constant overheads (e.g. see [4]).

Furthermore, it has been observed that the performance guarantee for maximal schedulers can be improved significantly by considering specific maximal schedulers, since the class of maximal schedulers is quite broad. Recently, LQF scheduling has been shown to yield a much larger stability region than the worst case maximal scheduling. During scheduling, an LQF scheduler produces a maximal independent set greedily following a sequence according to the queue length order of each link, from the longest to the shortest. It is shown that in certain networks, where the topology satisfies the so called “local-pooling” condition [8], LQF is optimal. For general networks, the performance of LQF scheduling can be bounded by its “local-pooling factor” [9], which is a function of the network topology and interference model. In the worst case, LQF scheduling can guarantee an efficiency ratio between 1/6 and 1/3 in geometric networks with the K-hop interference model.

Promising as it is, presently there are still some open issues for the LQF scheduling. First of all, the stability region of LQF is not well characterized, i.e., it is unclear whether a network is stable or not under an arrival process whose rate is outside the stability region of the the worst case maximal scheduler. Moreover, unlike maximal matching for switch scheduling [10], where a centralized controller is available, distributed LQF scheduling in wireless networks is hard to implement, and is subject to performance degradation due to asynchronous queue length information updates. Finally, it is hard to adapt LQF scheduling to support QoS in wireless multi-hop networks, due to the intractability of its analysis, which allows its performance to be understood only in certain networks with sufficient symmetries [11].

Realizing the limits of LQF scheduling, we try to improve the performance of maximal scheduling from a different perspective. Note that in LQF scheduling, in each time slot n, the scheduler picks the links following a sequence according to queue lengths. We generalize this to prioritized maximal scheduling, where in each time slot the scheduler chooses a maximal schedule following a sequence specified by a priority vector p(n), which is not necessarily the same as the queue length order. The priority process p(n) is, in general, a random process, and includes the priorities generated by the max-weight scheduling [1] and LQF scheduling as special cases. Therefore, it can achieve the optimal stability region, if the priorities are properly chosen. In this paper, we show...
that a simple i.i.d random process \( p(n) \) suffices to achieve the optimal stability region. Furthermore, the distribution of \( p(n) \) can be computed, or approximated by a PTAS, if the arrival rate value provided as of each link is available. In applications, the arrival rates can be obtained from either online estimation or the parameters by upper layer services, such as digital voice or encoded video. Since arrival rates parameters and pre-computation of the priorities are only needed when the network topology changes, this combined priority pre-computation and maximal scheduling approach is suitable for slowly changing wireless multi-hop networks.

Next we consider a special class of prioritized maximal schedulers, where \( p(n) \) is constant over time. In principle, this is simpler than the LQF scheduler, where dynamic priorities are used, and its performance is not influenced by asynchronous queue length updates. Interestingly, we show that if the priorities are properly chosen, we can achieve the same efficiency ratio as LQF scheduling. Moreover, we give a specific lower bound characterization of the stability region of prioritized maximal scheduling with a fixed \( p \). Further, we provide an algorithm that can successfully compute a stabilizing priority \( p \) as long as the arrival rate vector is inside the lower bound region of any fixed priority scheduler. Therefore, it is easy to check the stability of the prioritized maximal scheduler for any arrival process that is within a certain guaranteed fraction of the optimal stability region.

Finally we consider the delay constrained scheduling problem in wireless networks. With a major class of services being real time services, which are sensitive to congestion in the network causing buffer overflows, the issue of determining and guaranteeing delay in wireless networks is of equal, if not more, importance compared to the issue of stability. It is desirable to implement a scheduling algorithm such that the queue overflow probability for each link is below a certain small threshold. In this paper we try to solve this problem by using maximal scheduling with constant priority. We first analyze delay guarantees of the worst case maximal scheduler, as well as those of the prioritized maximal scheduler in the large deviations regime, and formulate an upper bound on the queue overflow probability. By exploring the similarity between the stability constrained and the delay constrained scheduling problems, we then propose a greedy priority assignment algorithm, which is adapted from the priority assignment algorithm designed for the stability case, with the guarantee that it will generate a satisfying priority \( p \) as long as the delay constraint can be achieved by some priority.

The paper is organized as follows. Section II introduces the system model. Section III describes the prioritized maximal scheduler with random priorities. We consider the constant priority case in Section IV and adapt it to support delay constraints in Section V. Section VI concludes this paper.

### II. System Model

#### A. Network Model

We model the topology of the network as a directed graph \( G = (V, E) \), where \( V \) is the set of user nodes, and \( E \) is the set of communication links. A link \( i = (u, v) \in E \) only if node \( v \) is in the transmission range of node \( u \). The interference is modeled by an undirected interference graph \( G_c = (V_c, E_c) \), where \( V_c \) is the set of links, and \( E_c \) is the set of pairwise conflicts. Two links \((i, j) \in E_c \) if and only if they are not allowed to transmit together. For example, in the primary interference model, the only constraint is that a user node can not transmit and receive simultaneously. Therefore, two links \((i, j) \in E_c \) if and only if they share a common node, in which case the scheduling problem is reduced to a matching problem in \( G \). This model arises naturally in switch scheduling, and is suitable for wireless networks using Bluetooth or FH-CDMA physical layers [3]. If secondary interference is considered, one model is the \( K \)-hop interference model [2], which requires that two links within \( K \) hops can not transmit at the same time (Note that the 802.11 DCF (Distributed Coordination Function) corresponds to \( K = 2 \)). See Fig. 1 for an illustration of the network topology and interference graph for a sample network consisting of 6 links.

For each link \( i \), define its neighbor set as \( N_i = \{ j : (i, j) \in E_c \} \). Thus, the transmission of link \( i \) is successful if and only if no link in \( N_i \) is transmitting. Denote the interference degree of link \( i \) as \( \Delta_i \), which is the cardinality of the maximum independent set in the subgraph formed by \( \{i\} \cup N_i \). It has been shown that \( \Delta = \max_{i \in V_c} \Delta_i \) is related to the efficiency ratio of maximal scheduling [5]. To analyze the performance ratio of the prioritized maximal scheduling, we need the following definitions. Denote a node removal sequence as \( r = (i_1, i_2, \ldots, i_{|V_i|}) \), which is a permutation of \( \{1, 2, \ldots, |V_c|\} \), and consider removing the nodes from \( G_c \) according to the sequence \( r \). Thus, \( i_k \) denotes the index of the \( k \)-th node that is removed. If a certain node \( i \) is removed in some step \( k \) (i.e., \( i_k = i \)), define \( d_i^{(r)} \) as the interference degree of link \( i \) when it gets removed in the subgraph at that step (i.e., without nodes \( i_1, i_2, \ldots, i_{k-1} \)). Define

\[
\delta = \min_{r \in \Pi} \max_{i \in V_c} d_i^{(r)},
\]

where \( \Pi \) is the set of \( n! \) removal (permutation) sequences.

It has been shown that \( 1/\delta \) is a lower bound on the local pooling factor for a network [9], and hence, the lower bound

![Fig. 1](image-url)
on the efficiency ratio of the LQF scheduling. We will show later that $\delta$ is also a lower bound on the efficiency ratio of maximal scheduling with constant priority.

### B. Traffic Model

We assume the network is time synchronized, and that the transmitter of each link $i$ is associated with an external arrival process $A_i(n)$, which is the cumulative number of packet arrivals during the first $n$ time slots. We further assume that the arrivals happen at the end of each time slot, and in each time slot, the packet arrivals are bounded above by a constant. The arrival processes are also subject to the Strong Law of Large Numbers (SLLN), i.e.,

$$\lim_{n \to \infty} A_i(n)/n = a_i \quad \text{for all } i \in V_c,$$

with probability 1 (w.p.1), where the constant $a_i$ is the arrival rate for link $i$. Note that these assumptions on the arrival process are quite mild, as the arrivals can be dependent over time slots and also among different links. As a consequence, in this paper we only focus on single hop traffic, since it is straightforward to generalize these results to the multi-hop case (since departures from one queue may very well be arrivals to another queue, in our mild assumption).

### C. Scheduling Model

In each time slot, a scheduler $\pi$ chooses an independent set for transmission. In this paper, we are interested in maximal scheduling, where a scheduled set of links has the property that no other link can be added without violating the interference constraints. Denote the family of maximal independent sets as $\mathcal{M}$. We will also treat a maximal independent set $m \in \mathcal{M}$ as a column vector such that $m_i = 1$ if $i \in m$, and $m_i = 0$ otherwise, as long as there is no confusion. The queueing dynamics at each link can be expressed as

$$Q_i(n) = Q_i(0) + A_i(n) - D_i(n)$$

where $Q_i(n)$ is the queue length at link $i$ at the end of time slot $n$ and $D_i(n)$ is the cumulative departures during the first $n$ time slots. Clearly, $D_i(n)$ must ensure that $Q_i(n) \geq 0$.

For prioritized maximal schedulers, during scheduling in time slot $n$, an independent set is produced, as described above, guided by a priority vector $p(n)$, where $p_i(n)$ represents the priority of link $i$. We assume that link $i$ has higher priority than link $j$ if and only if $p_i(n) < p_j(n)$ and that all priorities are distinct. Thus, the scheduler chooses depatures following a sequence specified by $p$, from the highest to the lowest, and schedules link $i$ for departure if it has a nonempty queue and no higher priority neighbor has already been scheduled.

The throughput of a scheduler $\pi$ is represented by its stability region $\mathcal{A}_s$, which is the set of stable arrival rates under $\pi$. We define stability to be rate stability, i.e.,

$$\lim_{n \to \infty} A_i(n)/n = \lim_{n \to \infty} D_i(n)/n = a_i \quad \text{w.p.1}$$

(3) for all $i \in V_c$. It has been shown that the max-weight scheduler in [1] can achieve the maximum stability region, $\mathcal{A}_{\text{max}}$, which is the interior of the convex hull of the maximal independent sets in $G_c$ (we do not consider the boundary points in this paper). We begin our study of prioritized maximal schedulers by considering the stability region of prioritized maximal schedulers with random priorities, in the next section.

### III. Scheduling with Random Priorities

In this section, we analyze the stability region of prioritized maximal schedulers when the priority vectors $(p(n))$ form a random process. First of all, as a subclass of maximal schedulers, any prioritized maximal scheduler can support the following region [5]:

$$\mathcal{A}_{\text{min}} = \{a: a_i + \sum_{j \in N_i} a_j < 1, 1 \leq i \leq |V_c|\}. \quad (4)$$

In order to obtain a better bound, we need to explore the impact of the priority vectors. In fact, the stability region of a priority scheduler is closely related to the choice of priorities. For example, consider a star shaped interference graph, with center link 1 and $n-1$ outer links. If we assign the priorities such that the center link 1 always has the lowest priority, then for any arrival rate $a_1 + \sum_{j=2}^n a_j > 1$, we can find an arrival process which makes link 1 unstable. Therefore, the stability region coincides with $\mathcal{A}_{\text{min}}$. As another example, consider the following choice of priorities: in each time slot, we first solve the MWIS problem

$$m^*(n) = \arg\max_{m \in \mathcal{M}} Q(n)^T m.$$  \quad (5)

Then, we assign all links with $m^*_i(n) = 1$ the highest priorities (the order does not matter) while the links with $m^*_i(n) = 0$ are assigned the remaining (lowest) priorities (the order does not matter). Since $m^*(n)$ is an independent set, all links in the independent set have the local highest priority, i.e., the highest priority in that specific neighborhood. After scheduling, the scheduled links maximize the weighted sum $\mathbf{m}$, and thus $\mathcal{A}_{\text{max}}$ in this example, can be achieved [1]. However, the computation of priorities is prohibitive, since $\mathcal{A}_{\text{max}}$ requires solving an NP-hard problem in every time slot. In fact, a much simpler prioritized scheduler can also achieve $\mathcal{A}_{\text{max}}$, by utilizing only an i.i.d random process $(p(n))$.

We need the following lemma.

**Lemma 1.** For any arrival rate vector $a$ and an i.i.d random process $(p(n))$ of priority vectors, if there exists a $S = (S_1, S_2, \ldots, S_n)$ where set $S_i \subseteq N_i$, such that

$$a_i + \sum_{j \in N_i \setminus S_i} a_j < \Pr(p_i < p_j, \forall j \in S_i), \quad (6)$$

$a$ is stable under the maximal scheduler with process $(p(n))$.

**Proof:** See in Appendix.

Thus, this lemma provide a lower bound on the stability region achievable by $(p(n))$. The following is the main theorem of this section. It uses Lemma 1 to show that any $a \in \mathcal{A}_{\text{max}}$ can be stabilized by a properly chosen $(p(n))$.

**Theorem 1.** In an arbitrary network with $n$ nodes, for any $a \in \mathcal{A}_{\text{max}}$, there exists a stabilizing prioritized maximal scheduler with an i.i.d process $(p(n))$. Furthermore, the support of $(p(n))$ consists of at most $n+1$ elements.

**Proof:** Since $a$ is in an open set $\mathcal{A}_{\text{max}}$, there exists an $\epsilon > 0$ such that $a + \epsilon e \in \mathcal{A}_{\text{max}}$, where $e = (1, 1, \ldots, 1)^T$. According
to the Carathéodory theorem, \(a + \epsilon e\) can be represented as a convex combination of at most \(n + 1\) maximal independent sets, i.e.,

\[
a + \epsilon e = \sum_{k=1}^{n+1} \theta_k m^{(k)}, \quad m^{(k)} \in \mathcal{M} \tag{7}
\]

where \(\theta \geq 0\) and \(e^T \theta = 1\). We associate a priority vector \(p^{(k)}\) with each \(m^{(k)}\) such that if \(m^{(k)}_i = 1\), \(i\) has the highest priority, and other priorities are arbitrary. During the scheduling in time slot \(n\), the scheduler chooses \(p^{(k)}\) with probability \(\theta_k\). Therefore, we have

\[
\Pr(p_i < p_j, \forall j \in N_i) \geq \sum_{k: m^{(k)}_i = 1} \theta_k = \sum_k \theta_k m^{(k)}_i = a_i + \epsilon
\]

and the stability follows from Lemma 1 by setting \(S_i = N_i\) for all \(1 \leq i \leq n\)\noindent ■

Intuitively, Theorem 1 shows that the traditional coloring approach, which is used for scheduling in networks with constant traffic, is still applicable in the presence of stochastic packet dynamics. Furthermore, the hardness remains almost the same, since (7) is equivalent to a coloring problem. Note (7) is only executed during the pre-computation phase. Thus, compared to the max-weight scheduler, which essentially solves an NP-hard problem in every time slot, the burden on the prioritized maximal scheduler during the scheduling phase is significantly relieved. Finally, we show that there exists PTAS for the pre-computation of the priorities.

**Theorem 2:** Given a network and interference model, if there exists a PTAS for the MAXCUT problem, then there exists a PTAS for computing the stabilizing priorities.

**Proof:** Suppose we have \(a + \epsilon e \in (1 - \epsilon) A_{\text{max}}\) for some \(\epsilon' > 0\). The priorities can be obtained by solving the following optimization:

\[
\min_{x, s} \quad e^T x \\
\text{subject to} \quad Mx \succeq a + \epsilon e, x, s \geq 0 \tag{8}
\]

where \(M\) is the matrix whose columns correspond to all \(m \in \mathcal{M}\). This problem can be solved by binary search over \(t\), where in each step we assume \(e^T x = t\) and solve the following

\[
\max_{x, s} \quad s \\
\text{subject to} \quad \frac{(Mx)_i}{a_i + \epsilon'} \geq s, \quad 1 \leq i \leq |V_c| \\
e^T x = t, x, s \geq 0. \quad (9)
\]

By strong duality, this can be solved by the dual problem

\[
\min_{\lambda} \quad f(\lambda) \\
\text{subject to} \quad \lambda \succeq 0, e^T \lambda = 1 \tag{10}
\]

where \(f(\lambda)\) is the optimal value of the following problem

\[
\max_{x} \quad \sum_{i=1}^{n} \lambda_i \frac{(Mx)_i}{a_i + \epsilon'} \quad 1 \leq i \leq |V_c| \\
\text{subject to} \quad e^T x = t, x, s \geq 0. \quad (11)
\]

It has been shown that an \(\epsilon\)-approximate solution for (9) can be obtained by solving \(\Theta(\epsilon)\)-approximate solutions for (11) \(O(|V_c| \epsilon^{-2} + \log |V_c|)\) times, see [16] for an approximation algorithm using logarithmic potential reduction. The only thing remains is to check that (11) is a MWIS problem. Note that (11) is a linear program, with a simplex constraint, and therefore, the optimal value is attained at a vertex, i.e.,

\[
f(\lambda) = \max_{m \in \mathcal{M}} w(\lambda)^T m \quad (12)
\]

where \(w_i(\lambda) = \frac{\lambda_k}{a_i + \epsilon}\), showing that \(f(\lambda)\) finds a MWIS. Thus, the theorem holds.

For most network and interference models, there exists a PTAS for the MWIS problem, see [7] for an example using the graph partitioning technique in geometric graphs. Note that one can also achieve similar performance by using a PTAS for the max-weight scheduler during the scheduling [12]. Compared to their approach, our method has the advantage that the scheduling phase has low complexity, which is independent of approximation ratio, since the approximation algorithm is only executed during the priorities pre-computation phase.

**IV. Scheduling with Constant Priority**

In this section we consider a special class of prioritized schedulers, where \(p(n) = p\) is a constant vector, i.e., the priority for each link is fixed. This is certainly easier to implement, since it does not require a global coordination variable, specifying the priority vector applying in each time slot. We will focus on both of the priority pre-computation and the performance guarantees.

**A. Priority Assignment**

In order to search for the optimal priority, we need to analyze the associated stability region for an arbitrary priority vector \(p\). By setting \(S_i = \{j \in N_i : p_i < p_j\}\) for each link \(i \in V_c\) and apply Lemma 1, we get a lower bound

\[
a_i + \sum_{j \in N_i} a_j 1(p_i - p_j) < 1 \quad \forall i \in V_c \tag{13}
\]

where \(1(\cdot)\) is the indicator function

\[
1(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad \tag{14}
\]

Denote this stability region as \(A_p\) and the stability region achieved by any constant priority as \(A = \bigcup_p A_p\). We are interested in computing a stabilizing priority for a given \(a \in A\). We will first propose the algorithm, and then show its optimality. See also in [6]. Note that the proof below is new, compared to [6], and so is the condition that \(A \subseteq \mathcal{A}'\).

**Algorithm 1 Stable-Priority \((G, c, a)\)**

1. **for** \(k = 1\) to \(|V_c|\) **do**
2. \(s \leftarrow \arg\min_{v \in V_c} \{a_i + \sum_{j \in N_i} a_j\};\)
3. \(p_s \leftarrow n + 1 - k;\)
4. Remove \(s\) from \(V_c\) and its incident edges from \(E_c;\)
5. **end** for
6. **return** \(p\)
In each step, the algorithm chooses node $s$, the center node of a neighborhood, and assign it the next lowest priority.

The following lemma is the key to the proof.

**Lemma 2:** (Alternate definition of $A$) For the $A$ defined above, we have $A = A'$ where $A'$ is any set of rate vectors in $\mathbb{R}_+^n$ that satisfies the following conditions:

1) (Coordinate-convex) If $a \in A'$ and $a' \preceq a$, then $a' \in A'$.

2) (Worst-case stable) For any nonzero $a \in A'$, there exists $1 \leq i \leq n$ such that $a_i > 0$ and $a_i + \sum_{j \in N_i} a_j < 1$.

**Proof:** Suppose $a \in A$, then there exists $p$ such that $a \in A_p$. Thus, property (1) holds from the fact that $A_p$ is a polytope, and thus contains any $a' \preceq a$. For this priority $p$, denote $i$ as the lowest priority link with positive arrival rate. Then, $S_i = \phi$, so that $\phi$ is the same as property (2). Therefore we have $A \subseteq A'$.

Suppose $a^{(1)} \in A'$. From property (2) we have

$$a_{i_1}^{(1)} + \sum_{j \in N_{i_1}} a_{j}^{(1)} < 1$$

for some $i_1$. Assign $p_{i_1} = |V_c|$, and form a new rate vector $a^{(2)}$ from $a^{(1)}$ by setting $a_{i_1}^{(1)} = 0$ (which induces a reduced graph). From property (1), $a^{(2)}$ is also in $A$, and from property (2) there exists $i_2 \neq i_1$ such that $a_{i_2}^{(2)} > 0$ and

$$a_{i_2}^{(2)} + \sum_{j \in N_{i_2}} a_{j}^{(2)} < 1.$$ 

We then set $p_{i_2} = |V_c| - 1$ and repeat the similar procedure until the highest priority, i.e., priority (1) is assigned. Thus, a $p$ vector is obtained. We claim that $a \in A_p$, due to the fact that

$$a_{i_k} + \sum_{j \in N_{i_k}} a_{j} 1(p_i - p_j) = a_{i_k}^{(k)} + \sum_{j \in N_{i_k}} a_{j}^{(k)} < 1$$

for all $1 \leq k \leq |V_c|$. Therefore we have $A' \subseteq A$, and the lemma holds.

From the above proof, it is straightforward to prove the following theorem:

**Theorem 3:** If $a \in A$, Stable-Priority will return a $p$ such that $a \in A_p$.

**B. Performance Guarantee**

We next analyze the performance of the maximal scheduling with the constant priority $p$ generated by Stable-Priority. This can be measured by the efficiency ratio $\gamma$, which is defined as

$$\gamma = \sup \{ \sigma : \sigma A_{\text{max}} \in A \}.$$ 

To gain insight, we first consider the efficiency ratio of greedy graph vertex coloring, where a sequence of vertices are removed first and the colors are assigned in the reverse order. In fact, if the arrival processes have equal rate, and can be approximated by constant fluids, then the scheduling problem is reduced to a graph coloring problem, where the rate is inversely proportional to the number of colors used. Denote the removal sequence in a greedy coloring algorithm as $r = (i_1, i_2, \ldots, i_n)$. When the nodes are colored in the reverse order, we need at most $d^{(r)} + 1 = \max_k d_k^{(r)} + 1$ colors, where $d_k^{(r)}$ is the degree of node $i_k$ when it gets removed. Suppose that the above maximum is achieved at node $i_k$. When $i_k$ gets removed, its neighborhood needs at least $\lceil d_k^{(r)}/\delta(k+1) \rceil$ colors (in any optimal coloring), and at most $d^{(r)} + 1$ colors in the sequential coloring. Therefore the efficiency ratio for this choice of sequence is lower bounded by $1/\delta(r)$. In the worst case, we have the efficiency ratio of $1/\Delta = 1/\max_{r \in \Pi} \delta(r)$, which corresponds to the efficiency ratio of the worst case maximal scheduler [5]. On the other hand, if we choose the sequence properly, we can achieve $1/\delta = 1/\min_{r \in \Pi} \delta(r)$. In the following we show that the efficiency ratio $1/\delta$ can, indeed, be achieved by $A$.

**Theorem 4:** $\frac{1}{\delta} A_{\text{max}} \subseteq A$.

**Proof:** Denote the removal sequence which achieves $\delta$ as $r = (i_1, i_2, \ldots, i_{|V_c|})$. For any $a \in A$, define a sequence of arrival rate vectors as follows:

$$a^{(1)} = a,$n
$$a^{(k)} = a^{(k-1)} - a_{i_k}^{(k-1)} e_{i_k} \quad 2 \leq k \leq |V_c|,$$ 

where $e_{i_k}$ is the vector that is all-zero except for an 1 in the $i_k$th entry. In other words, $a^{(k)}$ is obtained from $a^{(k-1)}$ by setting the $i_k$th entry to zero. We have for any $k$

$$a_{i_k}^{(k)} + \sum_{j \in N_{i_k}} a_{j}^{(k)} < \delta$$

due to the fact that at most $\delta$ links’s in $i_k$’s neighborhood when it gets removed can transmit in each time slot. Now for the priority assignment such that $p_{i_k} = n+1-k$ for $1 \leq k \leq n$, we have

$$a_{i_k} + \sum_{j \in N_{i_k}} a_{j} 1(p_{i_k} - p_j) = a_{i_k}^{(k)} + \sum_{j \in N_{i_k}} a_{j}^{(k)} < \delta$$

for all $i_k$. Thus we have $\frac{1}{\delta} a \in A_p$ and the claim holds.

Interestingly, the efficiency ratio of $1/\delta$ has been shown to also be a lower bound on the local pooling factor [9] for an interference graph, which is the efficiency ratio promised by LQF scheduling. This is no coincidence, since in the special case of constant fluid arrivals with equal rate, both belong to the family of greedy coloring algorithms, and hence have similar performance guarantees.

We next apply this result to certain networks and interference models to obtain examples of worst case guarantees.

**1) K-hop Interference:**

We first consider the K-hop interference model, which can be used for a large class of networks. For instance, the ubiquitous IEEE 802.11 DCF is usually modeled as a K-hop interference model with $K = 2$, due to the RTS-CTS message exchanges. In the K-hop interference model, two links $(i,j) \in E_c$ if and only if the distance between one node in link $i$ (transmitter or receiver) and one node in link $j$ (transmitter of receiver) is less than a threshold $K r$, where $r$ is the transmission range of a node. Summarizing the result in [9], we have the following corollary.

**Corollary 1:** In a geometric graph with $K$-hop interference model, the prioritized maximal scheduling with constant priority can achieve an efficiency ratio between $1/6$ and $1/3$. 

2) PHY-Graph:

We next consider the PHY-Graph [17], which is a more realistic interference model explicitly incorporating the physical layer parameters, i.e., the signal-to-interference-plus-noise ratio (SINR). In the PHY-Graph model, two links \((i, j) \in E_c\) if and only if either of the following is true: 1) the distance between the transmitter of link \(i\) and the receiver of link \(j\) is less than \(c_{ij}\), or 2) the distance between the transmitter of link \(j\) and the receiver of link \(i\) is less than \(c_{ji}\), where \(c = (SNR_t)^{\frac{1}{2}}\) is a function of the SINR threshold \(SNR_t\) and path loss exponent \(\kappa\), and \(l_i, l_j\) are the link lengths of \(i, j\), respectively. From their coloring bound, which is essentially an upper bound on \(\delta\), we have the following corollary.

Corollary 2: In a geometric graph with PHY-Graph interference model, for fixed \(\kappa\), the efficiency ratio of prioritized maximal scheduling with constant priority is a nondecreasing function of \(SNR_t\). Particularly, when the \(SNR_t\) is sufficiently high, the efficiency ratio is bounded above by \(1/7\).

V. DELAY-Aware Scheduling

In this section we try to adapt the prioritized maximal scheduling to support QoS constraints, in the form of an upper bound on queue-overflow probability. Specifically, we assume that the system constraint on the queue over flow probability for link \(i\) is

\[
\Pr(Q_i(0) > B_i) \leq \epsilon
\]

where \(Q_i(0)\) is the stationary queue length and \(B_i\) is buffer capacity. For large \(B_i\), we can assume that the buffer capacity if infinite and approximate the queue overflow problem as the following: calculate \(\theta^*_i\), where

\[
\theta^*_i = \lim \inf_{B_i \to \infty} -\frac{1}{B_i} \log \Pr(Q_i(0) > B_i) \geq \epsilon'.
\]

Here \(\epsilon = \exp(-B_i\epsilon')\) while \(\theta^*_i\) represents the delay exponent of \(Q_i\) in the large deviations regime. In order to measure the delay performance of a scheduler \(\pi\) in the large deviations regime, using a similar notation to the stability region \(A_\pi\), we define the delay region

\[
\Theta_\pi = \{ \theta \in \mathbb{R}^n_+ : \theta \preceq \theta^* \}
\]

where \(\theta^*\) is defined as Eqn. (17), i.e., the set of guaranteed delay exponents under \(\pi\). In contrast to the stability region, where any the arrival process satisfying SLLN applies, the delay region is quite sensitive to the arrival process model, especially the burstiness of the process. Therefore, we have to first re-define the arrival process model before analyzing the delay region of maximal schedulers.

A. Assumptions

We need to slightly modify the arrival process model, so as to apply the Large Deviations Principle (LDP). We now assume that the arrival processes are independent among the links. For each link \(i\), we will follow the model in [14]. Specifically, we have the following assumptions:

A1: (The Gärtner-Ellis theorem applies)

For each link \(i\), the log moment generation function

\[
\Lambda_i(\theta) = \lim_{n \to \infty} \frac{1}{n} \log \mathbb{E}(e^{\theta A_i(n)})
\]

exists for all \(\theta\), from which we can get the rate function via the Legendre transform

\[
\Lambda^*_i(\mu) = \sup_{\theta} (\theta \mu - \Lambda_i(\theta)).
\]

A2: (The sample path LDP applies)

For all \(s \in \mathbb{N}, \epsilon_1, \epsilon_2 > 0\) and for every scalars \(b_0, b_1, \ldots, b_{s-1}\), there exists \(N > 0\), such that for all \(n > N\) and all \(1 = k_0 \leq k_1 \leq \ldots \leq k_s = n\),

\[
P_i \geq e^{-n\epsilon_2 + \sum_{j=0}^{s-1} (b_{j+1} - b_j)\Lambda^*_i(b_j)}
\]

where \(P_i\) is the probability of the following event:

\[
\{|A_i(k_{j+1}) - A_i(k_j) - (k_j - k_{j+1})b_j| \leq \epsilon_1 n, 0 \leq j \leq s\}
\]

Intuitively, this is the event that the arrival process is constrained to lie within a tube around \(s\) linear segments of slopes \(b_0, b_1, \ldots, b_{s-1}\), respectively.

A3: (Convex dual analog of the sample path LDP)

For all \(s \in \mathbb{N}\), there exists \(N > 0\) and a function \(g(\cdot)\) with \(0 < g(x) < \infty\) when \(x > 0\), such that for all \(n \geq N\) and all \(1 = k_0 \leq k_1 \leq \ldots \leq k_s = n\),

\[
\mathbb{E}(\exp(\theta^T Z)) \leq \exp\left(\sum_{j=1}^{s} (k_j - k_{j-1})\Lambda(\theta_j) + g(\theta_j)\right)
\]

where \(\theta = (\theta_1, \theta_2, \ldots, \theta_s)\) and \(Z = (A_i(k_0), A_i(k_2) - A_i(k_1), \ldots, A_i(k_s) - A_i(k_{s-1}))\).

It should be noted that the processes satisfying the above three properties form a broad class, which includes most common models for bursty traffic in realistic networks, such as renewal and Markov-modulated processes. For detailed discussions, see [13].

B. Worst Case Maximal Scheduling

We first consider the delay region of an arbitrary maximal scheduler. In the following, we analyze the delay exponent for a fixed link \(i\). Denote the original network with a chosen arbitrary maximal scheduler as system \(S\). Since it is analytically intractable to give an exact characterization of the delay region, we want to create a system \(S'\) such that 1) it is relatively easier to analyze the delay region in \(S'\) and that 2) the delay region of \(i\) in \(S'\) is a lower bound of that in \(S\). Note that due to maximal scheduling, when link \(i\) has a nonempty queue, we can guarantee that, in each such time slot, there is at least one departure in \(i\)’s neighborhood (which includes \(i\)). Therefore an obvious dominant system for \(S\) is a server with service capacity \(1\), which is fed by one queue of queue length equal to the sum of the queues in \(\{i\} \cup N_i\). We can lower bound the delay exponent of link \(i\) in \(S\) using the following bound based on \(S'\):

\[
\Pr(Q_i(0) \geq x) \leq \Pr(Q'_i(0) + \sum_{j \in N_i} Q_j(0) \geq x),
\]
where the RHS corresponds to the single queue in $S'$ (equal to the sum of queues in $S$) and can be calculated using standard large deviations techniques. However, empirical results show that this bound is quite loose, especially in the cases where some neighbors in $N_i$ are likely to grow large, whereas $Q_i$ is not. To get a better lower bound, we need to analyze the queue length dynamics of link $i$ alone. Since we are analyzing the worst case maximal scheduler, we assume a prioritized maximal scheduler where link $i$ has the global lowest priority and the other links in $N_i$ have the global highest priority, so that $i$ cannot transmit if any link in $N_i$ has nonempty queue. Based on this, we create a system $S'$ consisting of a clique formed by the nodes in $\{i\} \cup N_i$, and $i$ is assigned the lowest priority. The priorities of the other links are arbitrary. We will show that $S'$ is a dominant system over $S$ in the following lemma. Intuitively, we are assuming the worst case correlations among the neighbors in $N_i$, such that the available time slot for link $i$ is minimized. This, together with the scheduler which assigns link $i$ the lowest priority, can be shown to dominate the original system $S$ with any scheduler. We will prove this domination in the following lemma.

**Lemma 3**: For any $x \geq 0$ and $n \geq 0$, if $Q_i(0) = Q'_i(0)$, we have

$$
\Pr(Q_i(n) \geq x) \leq \Pr(Q'_i(n) \geq x)
$$

(21)

where $Q_i(n)$, and $Q'_i(n)$ are the queue lengths of link $i$ at time slot $n$ in $S$ and $S'$, respectively.

**Proof**: We will show that in an arbitrary sample path $\omega$ with the same arrival processes and initial queue lengths for both systems, we have $Q_i(n, \omega) \leq Q'_i(n, \omega)$ for all $n \geq 0$. We assume both systems use the first-in-first-out policy, and denote the arrival time and departure time of the $k$th packet at link $i$ in $S$ as $t_k$ and $s_k$, respectively. Similarly for $S'$ we use $t'_k$ and $s'_k$. From the queueing equations in both systems, we have

$$
Q_i(n) = Q_i(0) + \sum_{k=1}^{\infty} 1_{[0, n]}(t_k) - \sum_{k=1}^{\infty} 1_{[0, n]}(s_k)
$$

$$
Q'_i(n) = Q'_i(0) + \sum_{k=1}^{\infty} 1_{[0, n]}(t'_k) - \sum_{k=1}^{\infty} 1_{[0, n]}(s'_k),
$$

where $1_S(x)$ is the indicator function that $x \in S$. Note that for any sample path $\omega$, $Q_i(0, \omega) = Q'_i(0, \omega)$ and $t_k(\omega) = t'_k(\omega)$ due to the assumptions. Thus to show that $Q_i(n) \leq Q'_i(n)$ w.p.1 for all $n \geq 0$, it is sufficient to show that $s_k(\omega) \leq s'_k(\omega)$ for all $k \geq 1$ and $\omega$. For simplicity we will drop the index $\omega$ in the following and proof by contradiction.

We will use induction on $k$. Suppose that this is not true for the first packet, i.e., $s_1 > s'_1$. In $S'$, due to maximal scheduling, each time slot from $t'_1 + 1$ to $s'_1 - 1$ is occupied by the neighbors of $i$ (which is why the packet in $i$ has not been served yet). Note that we always have

$$
D_i(n) + \sum_{j \in N_i} D_j(n) \geq D'_i(n) + \sum_{j \in N_i} D'_j(n),
$$

(22)

for any $n \geq 1$ since both systems are work-conserving and the links in $S'$ have edges between them (they form a clique), even when $S$ does not. Thus from $s_1 > s'_1$, we have $D_i(s'_1 - 1) = 0$ and

$$
\sum_{j \in N_i} D_j(s'_1 - 1) \geq \sum_{j \in N_i} D'_j(s'_1 - 1),
$$

following Eqn. (22). Therefore, we have

$$
0 = \sum_{j \in N_i} Q'_j(s'_1 - 1) = \sum_{j \in N_i} A_j(s'_1 - 1) - \sum_{j \in N_i} D'_j(s'_1 - 1) \geq \sum_{j \in N_i} A_j(s'_1 - 1) - \sum_{j \in N_i} D_j(s'_1 - 1) = \sum_{j \in N_i} Q_j(s'_1 - 1)
$$

(23)

Equality (a) is explained as follows. Since $i$ transmits in $S'$ at time $s'_1$, no other neighbor is contending for that slot (neighbors have higher priority). It must be that all the neighbors have empty queues in the previous slot. The inequality (b) is explained as follows. Since $i$ does not transmit in time slot $s'_1$ in $S$ (we know it transmits at $s_1 > s'_1$), that must be due to some transmitting neighbor occupying that slot. Since arrivals occur at the end of each slot, that neighbor must have a nonempty queue in the previous slot $s'_1 - 1$. Since this is a contradiction, we have proved the case for $k = 1$.

Now suppose it is true for up to $k - 1$ packets. For the $k$th packet, any time slot between $t'_k + 1$ to $s'_k - 1$ is occupied by the neighbors in $S'$. If $s_k > s'_k$, we have $s_k - 1 \leq s'_k < s_k$, and

$$
D_i(s'_k - 1) = D'_i(s'_k - 1) = k - 1
$$

(24)

Similar to (23), we get

$$
0 = \sum_{j \in N_i} Q'_j(s'_k - 1) = \sum_{j \in N_i} A'_j(s'_k - 1) - \sum_{j \in N_i} D'_j(s'_k - 1) \geq \sum_{j \in N_i} A_j(s'_k - 1) - \sum_{j \in N_i} D_j(s'_k - 1) = \sum_{j \in N_i} Q_j(s'_k - 1) > 0,
$$

i.e., a contradiction. Thus the lemma holds.

Having shown the the dominance of $S'$, we next analyze the delay performance of link $i$ in $S'$ in the following lemma.

**Lemma 4**: For link $i$ in system $S'$, we have

$$
\lim_{x \to \infty} \frac{1}{x} \log \Pr(Q_i(t) \geq x) = \theta_i^*
$$

(25)

where $\theta_i^*$ is the largest root of the following equation

$$
\Lambda_i(\theta) + \inf_{0 \leq u \leq \theta} \left[ \sum_{j \in N_i} A_j(u) - u \right] = 0
$$

(26)

**Proof**: We construct a system $S''$ equivalent to $S'$, consisting of two nodes, where node 1 corresponds to link $i$ and node 2 corresponds to the links in $N_i$. Thus, the queue of node 2 is the sum of the queues of link $i$’s neighbors. We assume that the arrival process to node 2 is equal to the sum arrivals in $N_i$, and that there is an edge between 1 and 2. During the scheduling, we assign higher priority to node 2. Note that from link $i$’s perspective, $S'$ and $S''$ yield the same queueing dynamics.
For a 2-node system, according to [14], the delay exponent \( \theta^* \) for queue 1 is given by the largest root of the equation

\[
\Lambda_1(\theta) + \inf_{0 \leq u \leq \theta} [\Lambda_2(u) - u] = 0. \tag{27}
\]

Thus the lemma holds from the fact that the arrival process of node 2 is \( A_2(n) = \sum_{i \in N_1} A_i(n) \).

From the above discussions, we have the following theorem:

**Theorem 5:** In a network where the arrival processes satisfy A1-A3, we have \( \Theta_{\min} \subseteq \Theta_\pi \) for any maximal scheduler \( \pi \), where \( \Theta_{\min} = \{ \theta \in \mathbb{R}_{++}^n : \theta < \theta^* \} \) and \( \theta^*_i \) is the largest root of the following equation

\[
\Lambda_i(\theta) + \inf_{0 \leq u \leq \theta} \left[ \sum_{j \in N_i} \Lambda_j(u) \mathbf{1}(p_i - p_j - u) \right] = 0. \tag{28}
\]

**Proof:** Note that the neighbors in \( N_i \) with lower priorities than link \( i \) are invisible to link \( i \). Therefore, we create a dominating system \( S' \) consisting of a clique formed by link \( i \) and its neighbors that have higher priorities than link \( i \). Using a similar argument, one can show that Lemma 3 holds. Therefore, Lemma 5 holds, following Lemma 4.

Similarly to \( \mathcal{A} \), we denote \( \Theta = \cup_p \Theta_p \), i.e., the delay region guaranteed by any constant priority scheduler. We are interested in computing a proper priority \( p \), when given a QoS constraint in the form of the vector \( \theta \). We will show this can be solved by an algorithm adapted from Stable-Priority, under the assumption that \( \theta \in \Theta \), by showing that \( \Theta \) has a structure similar as \( \mathcal{A} \).

**Lemma 6:** \( \Theta' = \Theta' \), where \( \Theta' \) is any delay region in \( \mathbb{R}_+^n \) that satisfies the following conditions:

1) \( \Theta' \) is coordinate-convex;
2) For any nonzero \( \theta \in \Theta' \), there exists \( 1 \leq i \leq n \) such that \( \theta_i > 0 \) and \( \theta_i \) is less than the largest root of the following equation

\[
\Lambda_i(\theta) + \inf_{0 \leq u \leq \theta} \left[ \sum_{j \in N_i} \Lambda_j(u) \mathbf{1}(\theta_j) - u \right] = 0, \tag{29}
\]

where \( \mathbf{1}(\cdot) \) is the indicator function in (14).

**Proof:** Suppose \( \theta \in \Theta \). Then, \( \theta \in \Theta_p \) for some \( p \). Therefore property (1) holds following the fact that \( \Theta_p \) is coordinate-convex. We can assume that \( p \) is such that the links with zero delay exponents have the lowest priorities. Next, consider the lowest priority link \( i \) with \( \theta_i > 0 \) according to \( p \). We have \( \theta_i < \theta^*_i \) where \( \theta^*_i \) is the largest root of the following equation

\[
\Lambda_i(\theta) + \inf_{0 \leq u \leq \theta_i} \left[ \sum_{j \in N_i} \Lambda_j(u) \mathbf{1}(p_i - p_j - u) \right] = 0. \tag{30}
\]

Thus property (2) holds from the fact that \( \mathbf{1}(p_i - p_j) = \mathbf{1}(\theta_j) \) for all \( j \in N_i \). Hence we have \( \Theta \subseteq \Theta' \).

Now suppose \( \theta^{(1)} \in \Theta' \). We first assign the lowest priorities to any link \( i \) such that \( \theta_i^{(1)} = 0 \). According to property (2), we can find link \( i_1 \) such that \( 0 < \theta_i^{(1)} < \theta_i^{(1)*} \) where \( \theta_i^{(1)*} \) is described in Eqn. (22). We assign \( i_1 \) the current lowest priority available and set \( \theta_i^{(1)*} = 0 \) to get a new vector \( \theta^{(2)} \in \Theta \). Repeat the above process until all the links are assigned priorities. Denoting the resulting priority vector as \( p \), it is easy to check that \( \theta^{(1)} \in \Theta_p \). Hence \( \Theta' \subseteq \Theta \). Therefore the lemma holds.

Based on the above lemma, we can construct a priority assignment algorithm, such that whenever \( \theta \in \Theta \), the algorithm will output a satisfying priority. This is the delay-analogue of Stable-Priority algorithm, which was constructed using Lemma 2.

**Algorithm 2** Delay-Priority \((G_c, \theta)\)

1: for \( k = 1 \) to \( n \) do
2: for \( i \in V_c \) do
3: Compute the largest root \( \theta^*_i \) of the equation:
\[
\Lambda_i(\theta) + \inf_{0 \leq u \leq \theta} \left[ \sum_{j \in N_i} \Lambda_j(u) \mathbf{1}(\theta_j) - u \right] = 0
\]
4: end for
5: \( s \leftarrow \min \{ i \in V_c : \theta_i < \theta^*_i \} \)
6: \( p_s \leftarrow n + 1 - k \)
7: \( \theta_s \leftarrow 0 \)
8: end for
9: return \( p \)

We conclude with the following theorem, which is the delay-analogue of Theorem 3.

**Theorem 6:** If \( \theta \in \Theta \), Delay-Priority will generate a priority \( p \) such that the \( \theta \in \Theta_p \).

VI. CONCLUSION

This paper considered the prioritized maximal scheduling problem in multi-hop networks for arbitrary correlated arrival processes. We first considered the random priority case and showed that one can achieve the optimal stability region with i.i.d priorities. Then we focused on the constant priority case and proposed a priority assignment algorithm, which, combined with maximal scheduling, can achieve an efficiency ratio, which is the same as that achieved by the state of art LQF scheduling. We also analyzed the delay performance of maximal schedulers in the large deviations regime, assuming independent arrival processes, and proposed a delay-aware prioritized maximal scheduling algorithm.

APPENDIX

**Proof of Lemma 1:** Due to space limit, we only give an outline of the proof, for a detailed discussion about fluid limits please refer to [5] and [15].
1) **The existence of fluid limits:** Note that the support of functions $A_i(n), D_i(n)$ and $Q_i(n)$ is $\mathbb{N}$, we extend it to $\mathbb{R}+$ using linear interpolation, which results in continuous functions (e.g., $A_i(t), t \in \mathbb{R}$.). For any sample path $\omega$, define a family of functions as

$$f^r(t, \omega) = \frac{f(rt, \omega)}{r} \quad (31)$$

where $f(\cdot)$ could be $A_i(\cdot), D_i(\cdot)$ or $Q_i(\cdot)$. Since both the arrivals and the departures in each time slot are bounded, and by linearity in (2), the functions defined in (31) are Lipschitz continuous with the same Lipschitz constant (irrespective of $r$), and hence equi-continuous on any compact interval. Since they are also uniformly bounded on $[0, t]$, according to Arzelà-Ascoli theorem, in any $[0, t]$, there exists a subsequence $r_{nk}$ and continuous functions $\hat{A}_i(\cdot), \hat{D}_i(\cdot)$ and $\hat{Q}_i(\cdot)$, such that (uniform convergence)

$$\lim_{k \to \infty} \sup_{t \in [0,t]} |f^r_{nk}(t, \omega) - f^r_{nk}(t, \omega)| = 0$$

where $f(\cdot)$ could be $A(\cdot), D(\cdot)$ or $Q(\cdot)$. Any $(\hat{A}_i(\cdot), \hat{D}_i(\cdot), \hat{Q}_i(\cdot))$ satisfying the above is defined as a fluid limit.

2) **Properties of fluid limits:** For any fluid limit, we have $\hat{A}_i(t) = a_i t$ w.p.1, since $A_i(n)$ satisfies SLLN, and

$$\hat{Q}_i(t) = \hat{Q}_i(0) + a_i t - \hat{D}_i(t) \quad (32)$$

for all $i \in V_c$. The network is stable if, for any fluid limit with $\hat{Q}_i(0) = 0, \forall i$, we have $\hat{Q}_i(t) = 0$ for all $i$ and all $t > 0$.

3) **Stability proof of Lemma 1:** Define the Lyapunov function

$$L_i(t) = Q_i(t) + \sum_{j \in N_i/S_i} f_j(t) \quad (33)$$

To prove the stability, it is sufficient to prove that if $\bar{L}_i(0) = 0$ for all $i$, we have $\bar{L}_i(t) = 0$ for all $i$ and $t > 0$. Suppose that this is not true, then there exists $i \in V_c$ and $t > 0$ such that

$$\bar{L}_i(t) = \max_{\tau \in [0,t]} \bar{L}_i(\tau) \quad (34)$$

$$\bar{Q}_i(t) = x > 0 \quad (35)$$

for details see the proof for (25) and (26) in [5]. Since $\bar{Q}_i(t)$ is a uniformly continuous function, there is a $t' < t$ such that $\bar{Q}_i(\tau) \geq x/2$ for $\tau \in [t', t]$. Note that since $\bar{Q}_i(t)$ is a fluid limit, there is a subsequence $r_{nk}$ such that

$$\bar{Q}_i(\tau) = \lim_{k \to \infty} \frac{Q_i(r_{nk}\tau)}{r_{nk}} \geq \frac{x}{2} \quad (36)$$

for every $\tau \in [t', t]$. Particularly, for large enough $k$, we have

$$\bar{Q}_i(r_{nk}\tau) \geq \frac{x}{4} r_{nk} \geq 1 \quad \forall \tau \in [t', t] \quad (37)$$

i.e., the queue of link $i$ is nonempty during $[r_{nk} t', r_{nk} t]$. Therefore we have

$$L_i(r_{nk} t) - L_i(r_{nk} t') = \sum_{j \in \{i\} \cup (N_i/S_i)} [A_j(r_{nk} t) - A_j(r_{nk} t')] - \sum_{j \in \{i\} \cup (N_i/S_i)} [D_j(r_{nk} t) - D_j(r_{nk} t')]$$

Due to the randomized priority scheduling, since queue $i$ is non-empty, if $i$ has higher priority than any link in $S_i$ in a certain slot, we can guarantee one departure in $\{i\} \cup (N_i/S_i)$. Since the priorities are i.i.d across time slots, by SLLN

$$\lim_{k \to \infty} \frac{\sum_{i \in \{i\} \cup (N_i/S_i)} [D_j(r_{nk} t) - D_j(r_{nk} t')]}{r_{nk}}$$

$$\geq (t - t') \Pr(\text{queue } i \text{ has local highest priority})$$

$$= \Pr(p_i < p_j, \forall j \in S_i)(t - t').$$

Note that if,

$$\lim_{k \to \infty} \frac{\sum_{i \in \{i\} \cup (N_i/S_i)} [A_j(r_{nk} t) - A_j(r_{nk} t')]}{r_{nk}}$$

$$= (a_i + \sum_{j \in N_i/S_i} a_j)(t - t') \quad (SLLN)$$

$$< \Pr(p_i < p_j, \forall j \in S_i)(t - t')$$

then we have

$$\bar{L}_i(t) - \bar{L}_i(t') = \lim_{k \to \infty} \bar{L}_i^{r_{nk}}(t) - \bar{L}_i^{r_{nk}}(t') < 0$$

which contradicts Eqn. (34). Therefore the stability of the network is proved and the lemma holds.

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