Geometry of the monopole clusters at different scales *

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We present results of measurements of various geometrical characteristics of the monopole clusters in the maximally Abelian projection of SU(2) lattice gauge theory. We observe scaling for the observables tested. Short clusters correspond to random walks at small scale but have long-range correlations at the hadronic scale. On the other hand, the percolating cluster at the hadronic scale does not correspond to a random walk.

1. INTRODUCTION

The monopoles of the maximal Abelian projection of SU(2) gluodynamics seem to be adequate dynamical variables for the description of confinement mechanism (for a review and references see, e.g.\textsuperscript{[1]} and further investigation of their properties is important. The monopoles are observed as trajectories closed on the dual lattice. Moreover, these trajectories form two types of clusters, the percolating cluster and finite ones, for a review and references see, e.g.\textsuperscript{[2]}. In particular, the percolating cluster, spreading over the whole of the lattice is responsible for the confinement.

Although phenomenology of the monopoles is quite rich, there is no understanding of the nature of the monopoles on the fundamental level. Thus, further accumulation of the data on the monopoles and tests of various models seems to be the best strategy available now. Here we report on observation of scaling properties of various geometrical characteristics of the monopole clusters. We perform calculations on various lattices at different values of $\beta$, the number of gauge fields configurations is given below in Table 2. Further details can be found in\textsuperscript{[3,4]}.

In Sect.\textsuperscript{[2]} we discuss the scaling behavior of the various characteristics of the percolating monopole cluster. The geometrical properties of finite clusters are presented in Sect.\textsuperscript{[3]} The density of clusters is discussed in Sect\textsuperscript{[4]} Conclusions are given in Sect.\textsuperscript{[5]}

2. PERCOLATING CLUSTER GEOMETRY

2.1. Segments and Crossings

The percolating cluster consists of segments (that is, trajectories between crossings) and crossings. The natural parameters to be measured are the average length of the segment, $\langle l_{\text{segm}} \rangle$ and the average Euclidean distance between the end points of the segments, $\langle d \rangle$. The data on $\langle l_{\text{segm}} \rangle$ we obtained demonstrate scaling\textsuperscript{[3,4]}:

$$\langle l_{\text{segm}} \rangle \approx 1.60 \text{ fm} .$$

The value of $\langle d \rangle$ exhibits stronger variation with the lattice spacing $a$ and can be fitted as:

$$\langle d \rangle \approx (0.3 + (a/\text{fm} - 0.11)) \text{ fm} ,$$

for the values of $a$ tested, 0.06 fm $< a < 0.16$ fm.

Scaling of the average segment length implies scaling of the number of the cluster self-crossings per unit physical length of the monopole trajectory, $N_{\text{cross}}/l_{\text{perc}}$. In Fig.\textsuperscript{[1]} the number of crossing points per unit length of the monopole trajectory is shown by filled circles. According to direct
measurements \[3,4\]: \[ N_{cross}/l_{perc} \approx 0.3 \text{ fm}^{-1} \]. This number changes if we exclude the closed loops of finite length (in lattice units) connected to the percolating cluster. The number of crossings reduces and is compatible with zero in the continuum limit if we exclude the loops of the length up to 8 (or up to larger length), see Fig. 1.

2.2. Angular Correlations

On a hypercubic lattice one can measure three different angular correlations between tangents to the monopole trajectory. Namely, two links connected by \( l \) cluster links may have the same, the opposite and neither the same nor the opposite ("the other") directions. We normalize these probabilities to unity for the random walk case. It was reported in \[3,4\] that the probability \( P_{\text{other}} \) is equal to unity within error bars for \( l \) larger than several lattice spacings while the probability \( P_{\text{same}} \) converges exponentially to unity:

\[
P_{\text{same}} - 1 = A_s e^{-\mu_s l a}.
\]

By definition the sum of all three probabilities is equal to 3. This means that the difference \( P_{\text{opposit}} - 1 \) falls off exponentially with the same (up to statistical errors) decay parameter and amplitude:

\[
P_{\text{opposit}} - 1 = -A_o e^{-\mu_o l a}.
\]

\[
\mu_s \approx \mu_o, \quad A_s \approx A_o.
\]

As it is seen from the Table 1 this is indeed true for \( \mu_s \) and \( \mu_o \), which are also independent of \( \beta \) within the errors, and thus correspond to the continuum limit.

Table 1

| \( \beta \) | \( \mu_s \) (MeV) | \( \mu_o \) (MeV) |
|---|---|---|
| 2.45 | 290(60) | 290(20) |
| 2.50 | 290(20) | 250(20) |
| 2.55 | 271(15) | 271(15) |
| 2.60 | 273(12) | 277(15) |

3. FINITE CLUSTERS GEOMETRY

A set of finite clusters is characterized by its spectrum, an average number of clusters as the function of their length, \( l \). First measurements of the spectrum were reported for one value of \( \beta \) in ref. \[2\] and it was found that the spectrum has \( 1/l^3 \) form. Our data confirm this behavior for all the values of \( a \) considered, see Table 2. As for the cluster radius, we observe:

\[
\langle r \rangle \sim \sqrt{l \cdot a}.
\]

For interpretation of the data on finite clusters see \[5,4\].

4. DENSITY OF CLUSTERS

The standard definition of the densities of the percolating and the finite clusters is:

\[
< l_{\text{perc}} > \equiv 4 \rho_{\text{perc}} a^4 N_{\text{sites}},
\]

\[
< l_{\text{fin}} > \equiv 4 \rho_{\text{fin}} a^4 N_{\text{sites}},
\]

where \( < l_{\text{perc}} > \) and \( < l_{\text{fin}} > \) is the average length of the corresponding clusters, \( N_{\text{sites}} \) is the number of the lattice sites.

In Fig 2 we show the monopole density \( \rho_{\text{perc}} \) as a function of the lattice spacing \( a \). Our results are in agreement with those of ref. \[6\] but obtained with higher statistics. The dependence on \( a \) is
Table 2
The fit of the length spectrum of the finite clusters by the function $1/l^\alpha$, for various values of $\beta$ on the lattices $L^4$; $N_{\text{conf}}$ is the number of considered gauge fields configurations at given $\beta$ and $L$.

| $\beta$  | 2.30 | 2.35 | 2.40 | 2.40 | 2.40 | 2.45 | 2.50 | 2.55 | 2.60 |
|----------|------|------|------|------|------|------|------|------|------|
| $L$      | 16   | 16   | 16   | 24   | 32   | 24   | 24   | 28   | 28   |
| $N_{\text{conf}}$ | 100  | 100  | 300  | 137  | 35   | 20   | 50   | 40   | 50   |
| $\alpha$ | 3.12(4) | 3.10(4) | 2.98(2) | 2.95(2) | 2.97(2) | 2.91(3) | 3.02(3) | 3.06(3) | 3.11(4) |

Figure 2. Density of the finite clusters $\rho_{\text{fin}}$ and percolating clusters $\rho_{\text{perc}}$; solid and dashed lines, are fits by a constant and function (10) respectively.

rather weak and the fit of the data by a constant for $\beta > 2.35$ gives:

$$\rho_{\text{perc}} = 7.70(8) \text{ fm}^{-3} .$$

The density of the finite clusters can be perfectly fitted (dashed curve on Fig.2) as

$$\rho_{\text{fin}} = C_1 + \frac{C_2}{a},$$

where $C_1 = -6.1(5) \text{ fm}^{-3} , C_2 = 1.55(4) \text{ fm}^{-2}$. The negative value of the constant $C_1$ means that the fit is not valid for large (unphysical) values of the lattice spacing.

5. CONCLUSIONS

The observed scaling behavior of geometrical characteristics of the monopole clusters supports a conjecture that the monopoles might correspond to gauge invariant objects. Most remarkable, the scaling behaviour holds for small $a$. With implication that the monopole size can be very small, see also [7]. Moreover, the $1/a$ divergence of the finite clusters density means that these clusters are associated with two dimensional surfaces embedded into the four dimensional space [4]. This relation between monopoles and surfaces is in a sense very close to the relation between P-vortices and monopoles observed in ref. [8], see also discussion in the talk presented by S.N. Syritsyn at this conference [9].

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