Quantum Key Distribution and Chaos Bandwidth Effects on Impact Security of Quantum Communications

Mahdi H. Al Hasani*1, Kais A. M. Al Naimee1, 2
1Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq
2Istituto Nazionale di Ottica- CNR, Largo E Fermi 6, 50125 Firenze, Italy

Abstract

The influence of bias current on the bandwidth of chaotic signals in semiconductor lasers by optical feedback has been studied experimentally and numerically. The measured data reveal that the bandwidth increase when the system becomes chaotic and this chaotic signal has a broadband spectrum so it can be used as a carrier for the quantum key. Mixing chaotic signal and quantum key make a very small change in chaotic bandwidth that does not affect the security of data transmitted.

Keywords: Bandwidth, Chaos, Quantum key distribution, optical communications

Introduction

Semiconductor lasers are devices of great importance for optical communication systems due to their small size, high efficiency, and high speed for direct modulation [1, 2]. Semiconductor lasers with a large modulation bandwidth or high speed for direct modulation have important applications in both analog and digital optical communications [3, 4]. There is much enthusiasm for the advancement of fast semiconductor lasers, for very high data rate digital transmission [5, 6]. Semiconductor lasers show chaos under various physical conditions such as optical feedback (OFB) or optoelectronic feedback (OEFB) [7, 8]. Optical and optoelectronic feedback in a semiconductor laser has been utilized for impacting the properties of a laser, for example, the linewidth, the wavelength of emanation, the threshold current; and furthermore to destabilize a laser and force it to enter complex chaotic dynamics [9,10]. Chaotic oscillations of semiconductor lasers have attracted extensive consideration attributable to their critical applications in secure communications [11]. The chaotic emission must have a high transmission capacity; an extensive data transfer capacity is required to guarantee encryption at a high bit rate in secure communications [12]. Chaos and its applications in secure communications have been a noteworthy concentration in recent researches [13- 15]. A chaotic

*Email: mahdialhassany@yahoo.com
signal produced by a laser has a high frequency and expansive broadband advantages [16]. Chaotic variations in semiconductor lasers have a wide range spectrum and the achievable maximum frequency is typically bigger than the relaxation oscillation frequency of the solitary lasers. In this manner, we can transmit a signal which contains higher frequency components than the relaxation oscillation [17]. Chaos complexity plays a key role as a criterion for the security of the procedure, which is ensured by the way that a semiconductor laser with feedback has large degrees of freedom [12].

A high rate of data transmission is one of the key specialized properties of secure data communications. The transmitting rate of data signals is an element of the chaotic carrier bandwidth, so the improvement of the chaotic carrier bandwidth is major to achieve high information rate transmission in chaotic laser secure communications [16, 18].

The chaotic carriers can be used to improve the security of quantum cryptography, the target of quantum cryptography also known as Quantum Key Distribution (QKD) is to take advantage of the properties of quantum optics in so as to secretly share a random bit sequence between two parties. Once the key has been exchanged, the transmitter uses this key to encrypt the message and send it to the receiver over a public channel [19, 20]. In QKD the security is ensured by the laws of quantum mechanics, and in this manner is extremely high [21]. QKD systems use quantum states, for example, polarization, to encode data on single photons [22]. The key is sent over a quantum channel. Any information obtained by an unauthorized third party about the exchanged key lead to transmission errors occur because of the quantum mechanical nature of photons [20]. To recognize these errors, legitimate users compare a sample of the shared bits. If errors exist in the verification process, the users conclude that the system is hacked [23].

In this research, the effect of bias current on chaotic bandwidth has been studied, and the effects of the quantum key on chaotic bandwidth when combined together are also studied.

**Experimental method**

The optical chaotic signal is experimentally produce using semiconductor laser with an optical feedback, our experimental setup has been described previously [24]. The chaotic signal is also generated numerically using the two rate equation of Lang and Kobayashi model that describes semiconductor laser with optical feedback [25]. In this paper the quantum key is produced by using BB84 protocol, the experimental setup of BB84 protocol is illustrated in Figure-1.

![Figure 1-Quantum Cryptography Setup with the Bases + (0° and 90°) and x (-45° and 45°).](image_url)

This setup is utilized to make a quantum key which it comprises two parts, the first part is the sending unit “Alice” consists of a single photon source which is polarized horizontally and a $\frac{\lambda}{2}$ plate which it turns the polarization of the incident light by double the physical rotation angle of the wave
plate (polarization rotator). The second part is the receiving unit “Bob” consists of polarization rotator, Polarizing Beam-splitter (PBS) and two detectors. The polarizing beam-splitter cube reflects the vertically polarized (90°) component of the incident light, while passing the horizontally polarized (0°) component. The two detectors are connected to the electronic and they detect the incoming photon from Alice. Since polarization rotator has two bases + and x, Alice has to make two random decisions for key generating. Bob also sets his polarization rotator to differentiate between + and x bases. Accordingly, Bob only needs the settings 0° and 45°. If Bob select the + basis and Alice sends in the + basis, Bob obtains an unambiguous result; this applies correspondingly if both choose the x basis. But what if Bob chooses a different basis than Alice; the result is that 45° polarized light will be sent to the beam-splitter. For a continuous beam, half is transmitted and half is reflected. However, assuming that only one photon is sent, only one of the two detectors can respond. The detector that responds is then left to chance. If the two bases do not match, Bob will nevertheless measure a signal on one of the two detectors. The probability of detect the photon on one of the two detectors is 50% respectively.

During this work, the quantum key is also created numerically by simulating the various stages of BB84 protocol. In this stage several considerations are taken into account, the source is assumed to be perfectly single photon source. Alice will randomly choose her polarization bases and bits, Bob also choose his bases randomly and the quantum channel is assumed to be free space (the noise and effect of quantum channel are neglected).

**Results and Discussion**

In this research, the dynamical system changes from periodic state to a chaotic state by increasing the bias current of the system and we will see the bandwidth for each state. Figure-2 numerically shows the influence of bias current on the bandwidth of the dynamical system. It shows that the bandwidth increase when the system become chaotic at bias current 0.41$I_{th}$, from the figure we notice that the bandwidth of chaotic carrier proportional to the bandwidth of semiconductor laser, since the bandwidth of chaos is corresponding to the laser’s relaxation oscillation frequency and this frequency is reliant on the square root of the difference between the bias current and the threshold current.

Figure-3 illustrates the experimental result in the experimental work the periodic or regular region could not obtain only the chaotic region is obtained since in optical feedback the system is very sensitive to optical phase variations and is difficult to get the regular state in the laboratories[26, 27]. This broadband chaotic carrier useful in transfer a wide band of frequencies, the wider (or broader) the bandwidth means that greater data carrying capacity. Broader band of chaos will carry multiple signals like speech, music, video, etc.

![Figure 2-Influence of bias current on bandwidth (numerical).](image-url)
The chaotic signal will be used as a carrier to transmitting quantum key (Q.key); this key is generated by BB84 protocol with length 60 bits and mixed with the chaotic signal by the mixer. To mix quantum key with chaos the key is converted from binary bits to pulse shape as illustrated in Figures-(4, 5).

Figure 3-Influence of intensity on bandwidth (experimental).

Figure 4-Numerical Q.key in pulse shape.

Figure 5-Experimental Q.key in pulse shape.
After converting encrypted message to the strain of pulses, the train of pulses of the encrypted message and chaotic signal are brought together. Figures-(6 and 7) illustrate numerical and experimental chaotic signals mixed with the encrypted message, these figures show that the encrypted message is successfully hidden within chaotic signals with a highly masking degree. Mixing of quantum cryptography and chaos creates a very secure system.

**Figure 6**- Numerical mixed chaotic signal and encrypted message (a) time series, (b) Power spectra of the chaotic signal with Q.key.

**Figure 7**- Experimental mixed chaotic signal and encrypted message (a) time series, (b) Power spectra of the chaotic signal with Q.key.

The effect of the quantum key on chaotic bandwidth is shown in Tables-(1, 2). From these tables, both numerical and experimental values of chaotic bandwidth before and after mixing with quantum key change very small and this change is almost unnoticeable.

**Table 1**- Numerical chaotic bandwidth with and without quantum key

| Bias current (a.u) | Bandwidth (MHz) |
|-------------------|-----------------|
| Without Q. Key    | With Q. Key     |
| 0.44              | 517.041         | 517.894         |
| 0.5               | 507.949         | 509.268         |
| 0.56              | 518.04          | 519.153         |
| 0.59              | 514.796         | 514.984         |
Table 2-Experimental chaotic bandwidth with and without quantum key

| Bias current (nW/m²) | Bandwidth (GHz) | Without Q. Key | With Q. Key |
|---------------------|-----------------|----------------|-------------|
| 100                 | 1.58453         | 1.5855         |
| 180                 | 1.59559         | 1.59518        |
| 220                 | 1.52478         | 1.52418        |
| 260                 | 1.58412         | 1.58249        |

Figures-(8 and 9) illustrate numerically and experimentally a comparison of bandwidth before and after mixing with the quantum key. From Figure-8 we notice that when the system is regular the bandwidth with the quantum key is different from the bandwidth of system without the quantum key while in the chaotic region the bandwidth of the system is almost identical in the two cases. The same behavior is shown experimentally in Figure-9, this means the bandwidth of chaotic region does not affect with quantum key and the chaotic carrier can be used to hide and transfer the quantum key with high efficiency. Broadband nature of chaotic carrier is a very important parameter for chaos optical communication, because it determined the maximum bit rate that could be transmitted by chaos.

**Figure 8**-Numerical comparison of bandwidth with Q. key and without Q. key.

**Figure 9**-Experimental comparison of bandwidth with Q. key and without Q. key.
Conclusions
In conclusions, the bandwidth of the dynamical system relies upon bias current of the semiconductor laser, this bandwidth is greatly incremented when the system turns to the chaotic regime by increasing the bias current of the system. The chaotic signal has broadband that allows high data carrying the capacity transmission. The quantum key can be included by a chaotic signal with a high degree of masking since it has a broad spectrum. It is shown that this quantum key does not affect the chaotic bandwidth so it can be carried with a chaotic signal with high efficiency.

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