Vacuum fine tuning and empirical estimations of masses of the top-quark and Higgs boson

Abstract

The fine-tuning principles are analyzed in search for predictions of top-quark and Higgs-boson masses. The modification of Veltman condition based on the compensation between fermion and boson vacuum energies within the Standard Model multiplets is proposed. It is supplied with the stability under rescaling and with the requirement of minimum for the physical v. e. v. for the Higgs field (zero anomalous dimension). Their joint solution for top-quark and Higgs-boson couplings exists for the cutoff $\Lambda \approx 2.3 \cdot 10^{13}\, GeV$ that yields the low-energy values $m_t = 151 \pm 4\, GeV$; $m_H = 195 \pm 7\, GeV$.

1 Introduction

The standard Model (SM) describes the strong and electroweak particle interactions with a good precision in a whole range of energies which have been available in experiments [1]. Still few open problems are well known in SM to be resolved in order to justify all the principles which the Standard Model is based on and to determine eventually all its phenomenological parameters. In particular the detection of top-quark and of scalar Higgs particle is wanted in the nearest future [1, 2]. Respectively the estimations for their masses have invoked a lot of efforts to understand possible extensions of SM [2, 3] where an underlying dynamics leads to the formation of scalar particles [4, 5].

Meantime there exist few phenomenological principles within the minimal SM which make it possible to find relations between top-quark

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and Higgs-boson masses and are weakly dependent of the details of a fundamental theory underlying the SM. These principles are coming from the assumption that the SM is actually an effective theory applicable consistently for low energies. Accordingly its coupling constants and dimensional parameters absorb all the influence of high-energy degrees of freedom and of new heavy particles as well. Of course, the form of effective action of Green-Wilson type generally depends on the preparation procedure but the consistent effective action coinciding with the SM action is supposedly that one which is minimally sensitive to very high energies. Still the memory of the high-energy dynamics responsible for the parameter formation exists just giving the relations between dimensional parameters and certain coupling constants. The latter statements allow to formulate following phenomenological principles which could be fulfilled in the quantum SM giving various predictions for the masses of heavy particles.

1. The strong fine tuning for the Higgs field parameters (v.e.v and its mass) that consists in the cancellation of large radiative contributions quadratic in ultraviolet scales bounding the particle spectra in the effective theory (Veltman condition).

2. The weak fine tuning that provides the cancellation of logarithmic cutoff dependence in certain coupling constants and simulates thereby the quasi-fixed ultraviolet behavior.

Different implementations of both strong and weak fine-tuning we shortly survey in Sect.2.

3. We propose also the strong fine tuning for vacuum energies that provides the cancellation of large divergencies quartic in ultraviolet scales which might effect drastically in formation of cosmological constant. Of course, a disbalance in vacuum energies for an effective theory may happen to be compensated by those ones from virtual high-energy components. However we suppose here that the consistent preparation of an effective model can
provide the essential decoupling of low-energy world from very high energies and therefore for an appropriate choice of ultraviolet scales the huge vacuum energies do not naturally appear [11]. This requirement of vacuum adaptation leads as we shall see to a modification of Veltman condition.

The vacuum-energy fine tuning when combined with others leads to predictions for the top-quark and Higgs-boson masses within the range of validity of the Standard Model. In Sect.3 we examine the compatibility of above principles and find the corresponding estimations for t-quark and Higgs-boson masses.

4. The possible reduction of the SM coupling constants [12, 13] and the related stability of the renormalization-group(RG) flow for the Higgs self-coupling and/or for its coupling to t-quark (quasi-fixed infrared point [14]) at relatively low energies.

5. The bounds on acceptable values of t-quark and H-boson based on the vacuum stability in scalar sector [15] and on the triviality of scalar models in four dimensions [16, 17]. These very interesting schemes are beyond of the scope of the present paper though they deserve our attention when combined with the fine-tuning rules. As well the investigation of compatibility of fine tuning rules with scenarios of composite Higgs-particle [18] is postponed to a further research.

2 Strong and weak fine-tuning for Higgs fields

1. It is well known that the scalar sector in the Weinberg-Salam theory contains the quadratic divergences in the tadpole diagrams and in the scalar self-energy. In early 80th the rule of cancellation for quadratic divergences [7] was proposed in the electroweak sector of the SM. This cancellation occurs if the fermion and boson loops are tuned due to specific values of coupling constants. To one loop level
it was found that the condition:
\[
(2M_W^2 + M_Z^2 + m_H^2) = \frac{4}{3} \sum_{\text{flavors, colors}} m_f^2
\]  
(1)
removes quadratic divergences both from the Higgs-field v.e.v. and
the Higgs boson self-energy. From (1) it is easy to see that if the
t-quark is the only heavy fermion, \(m_t \geq 70\, \text{GeV}\) bound should hold.

2. The idea of stability for Higgs v. e. v. against the cutoff variation
was extended to the weak fine-tuning in \([10]\). At the one-loop level
the finiteness of radiative corrections for Higgs v.e.v. is declared and
the scale independence of the cutoff \(\Lambda\) is imposed. Actually, it means
the separate cancellation of quadratic and logarithmic divergencies at
one loop.
\[
4m_t^2 = M_H^2 + M_Z^2 + 2M_W^2; \\
4m_t^4 = \frac{1}{2} M_H^4 + M_Z^4 + 2M_W^4.
\]  
(2)
The mass predictions are: \(m_t = 120\, \text{GeV}; M_H = 190\, \text{GeV}\). If one takes
the value \(\sin^2\Theta_W \approx 0.25\) these conditions are compatible with can-
cellation of logarithmic divergencies in \(e^+e^-H\) vertex \([9]\)(see below).
But it is worth to notice, that the cancellation of logarithmic divergencies for Higgs v.e.v can be useful at the 1-loop level only if two-loop quadratic divergencies are smaller than the 1-loop logarithmic ones. But for the values of \(\Lambda \geq 10\, \text{TeV}\) logarithmically divergent part in v.e.v is less than \(10^{-3}\) in comparison with 1-loop quadratic divergencies (the factor \(\frac{v^2}{\Lambda^2} \ln \frac{\Lambda^2}{v^2}\)) while two-loop quadratic corrections have the
factor \(10^{-2}\) \((1/16\pi^2)\). It means that even at smaller energies two-loop contributions cannot be neglected (we shall illustrate this fact more carefully further on). One should also bear in mind that for energies less or equal than 1 TeV the absolute value of the logarithmic contributions to v.e.v is rather small in comparison to the bare v. e. v. and
that in the SM there are quite a few of logarithmic divergencies which
survive after this cancellation (see below).

3. The further development of above tuning was outlined by R.Dec-
ker and J. Pesteau [8]: their idea (in 1980) was of the lepton mass finiteness (based on the assumption that leptons are not composite particles even beyond the SM). Their one-loop calculation deals with logarithmic fine-tuning including the logarithms from Higgs tadpoles and leads to the following relation:

\[ 2M_W^4 + M_Z^4 + \frac{1}{2}m_H^4 + 2M_W^2 \sin^2 \theta_W \cdot m_H^2 = \frac{4}{3} \sum_{\text{flavors, colors}} m_f^4. \]  

(3)

This relation yields the lower mass bound for the t-quark \( m_t \geq 78 \, \text{GeV} \), while combined with the \( \rho \)-parameter restriction \( m_t \leq 200 \, \text{GeV} \) yields also the upper bound \( M_H \leq 350 \, \text{GeV} \) (in 1980 it was \( m_t \leq 295 \, \text{GeV} \), \( M_H < 450 \, \text{GeV} \) respectively).

The contribution of the Higgs-boson exchange to the fermion self-energy is neglected in (3) since all the leptons are light compared with \( M_W, Z \) and thereby the self-energy of t-quark (and of other heavy quarks) remains large. This is the reason why the lepton self-energy only has been declared to be stabilized. On the other hand in the approach [8] the quadratic divergences of tadpoles were not taken into account (in their usage of dimensional regularization the \( d=4 \) poles are equivalent to logarithmic divergences only). However the equation (3) is incompatible with (4), when one cannot avoid the problem of quadratic divergences ignored in (3).

Another application of this scheme has been made recently providing cancellation in the neutrino self-energy and leading to the equation:

\[ 2M_W^4 + M_Z^4 + \frac{1}{2}m_H^4 + \frac{m_e^2 - m_\nu_e^2}{2}m_H^2 = \frac{4}{3} \sum_{\text{flavors, colors}} m_f^4. \]  

(4)

For the neutrino mass the quadratic and logarithmic divergences can be cancelled simultaneously leading to the prediction: \( m_t = 121 \, \text{GeV} \), \( M_H = 194 \, \text{GeV} \).

Still one can dispute with the very idea to compensate large logarithmic corrections coming from tadpoles non-running under renormalization and from self-energy diagrams contributing into the anomalous
dimension of the mass. The former ones provide the finiteness of the basic EW scale when being cancelled in the Higgs-field v. e. v. (see (2)). If it takes place then one can easily check that neither (3) nor (4) are not fulfilled simultaneously.

4. The weak fine tuning has been applied to the $e^+ e^- H$ vertex in [9] as the cancellation of logarithmic divergences therein. This vertex is not afflicted with quadratic divergences and therefore it is safe in the weak fine tuning. The equation different of (3), (4) has been obtained,

$$m_t^2 = \frac{5}{2} M_Z^2 - M_W^2$$  \hspace{1cm} (5)

which is well compatible with (1), and yields the following mass predictions for $m_t = 120 GeV$, $M_H = 190 GeV$.

5. Few papers were in search of certain unknown symmetries within or beyond the SM which would provide the systematic cancellation of the quadratic divergencies. If this symmetry were local then quadratic divergencies would cancel at each order of the perturbation theory. The special continuation of the dimensional regularization method for two-loop quadratic divergencies aside dim = 4 was adopted in [19]. However, methods used in [19]-[22] treat poles in $d \neq 4$ at different positions for a particular number of loops and bring different results depending on the recipe of contituation in dimensions. The dimensional reduction yields the one-loop Veltman condition, whereas the dimensional regularization replaces it by another equation, whose physical meaning is obscure (see the discussion in [20]). Furthermore the separate cancellation of one and two-loop condition happens to be compatible only if the QCD coupling is actually turned off. The possibility of three-loop cancellation (which is necessary under such a treatment) remains an open question [21].

6. Another interesting discovery had been brought by computing of the RG-flow for coupling $\lambda$ (the 1-loop equation for $\lambda$ is nonlinear
and cannot be solved exactly):

\[ 16\pi^2 \frac{\partial \lambda}{\partial \tau} = 12 \left( \lambda^2 + (g_t^2 - A) \lambda - g_t^4 + B \right) \]

(6)

\[ A = \frac{1}{4} \left( 3g^2 + g'^2 \right) ; \quad B = \frac{1}{16} g'^4 + \frac{1}{8} g^2 g'^2 + \frac{3}{16} g^2 \]

It was found \([3, 14]\) that \(\lambda(\tau)\) tends to the Hill’s quasi-fixed point for \(\lambda (\partial \lambda / \partial \tau = 0)\) in the wide intermediate energy region for any boundary conditions at high energies. For their composite H-boson scenario this yields \(m_t \approx 240 GeV, \quad M_H \approx 250 GeV\), but the above property of eq.(6) should be taken into account for any low-energy predictions in the SM.

Thus we shall follow the preparation way for a low-energy effective action based on a momentum cutoff.

3 Vacuum-energy fine tuning and predictions for t-quark and H-boson masses

Let us consider the SM as a low-energy limit of a more fundamental theory and suppose that the only heavy fermion, t-quark is involved in its dynamics within the selected energy range. Respectively we neglect the masses of all lighter fermions. When there is no expected supersymmetry below Grand-Unification scales we apply different scales for the design of SM-effective action for bosons \(\Lambda_B \ll \Lambda_{comp}\) and for fermions \(\Lambda_F \ll \Lambda_{comp}\). Among bosons the universal scale is introduced in order not to induce the explicit breaking of a Grand Unification symmetry below a scale of compositeness \(\Lambda_c\). As well the unique scale for fermions ensures the horizontal symmetry in the ultraviolet region.

We require for the SM the suppression of very large contributions (leading divergencies) into dimensional physical parameters that is equivalent to the absence of their strong scale dependence. The latter means in addition to the strong fine-tuning that the cancellation of
contributions into vacuum energy should take place, i.e., first of all the contributions which are quartic in cutoffs.

\[ T_{\mu\nu} \sim g_{\mu\nu} E_{\text{vac}} \approx 0; \quad 4N_F \Lambda_F^4 = (3N_B + N_S) \Lambda_B^4; \]

\[ \alpha^2 = \frac{\Lambda_b^4}{\Lambda_F^4} = \frac{4N_F}{3N_B + N_S} = 2.4 \quad (7) \]

where \( N_F = 24 \) is a number of flavor and color fermion degrees of freedom for three generations, \( N_B = 12, N_S = 4 \) are numbers of flavor and color degrees of freedom for vector and scalar bosons respectively.

The strong fine-tuning condition in this case reads at the one-loop level

\[ 4m_t^2 = \alpha(2M_W^2 + M_Z^2 + m_H^2) \quad (8) \]

Taking into account the effects of all loops one comes to the integral Veltman condition:

\[ \int_{v_0}^{\Lambda_F} \frac{4g_t^2d^4k}{k^2 + m_t^2} = \int_{v_0}^{\Lambda_B} d^4k \left\{ \frac{g^2}{k^2 + M_W^2} + \frac{g^2}{2k^2 + \lambda} \right\} + \frac{\lambda}{k^2 + M_Z^2} \quad (9) \]

where the conventional denotations for the electroweak \( g, g' \), Higgs-quartic, \( \lambda \) and the Yukawa \( t \)-quark, \( g_t \) coupling constants are used. When integrating by parts one can conclude that the leading contribution is the modified Veltman condition at the scale \( \Lambda \). The latter is supplemented in the next-to-one-loop approximation with its renorm-derivative (having small combinatorial factor) and so on. At one-loop level of accuracy, all the renorm-derivatives except for the first one are zero. Demanding the weak dependence of \( \lambda \) one has to impose both the modified Veltman condition and its renorm-derivative.

\[ D \equiv 16\pi^2 \frac{\partial}{\partial \tau}; \quad \tau = \ln \frac{\Lambda}{v_0} \]

\[ \begin{cases} f \equiv 4g_t^2 - 2\alpha(\lambda + A) = 0 \\ Df = 0 \end{cases} \quad (10) \]
The explicit form of the second stability condition is:

\[
D_f = 8g_i^2 \left[ \frac{9}{2}g_i^2 - 8g_3^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2 \right] \\
-24\alpha \left[ \lambda^2 + (g_i^2 - A)\lambda + B - g_i^4 \right] \\
-2\alpha(-19g^4 + 41\frac{1}{3}g'^4)/4
\]  
(11)

where in Eqs.(10), (11) the denotations are borrowed from [14]:

\[
A \equiv \frac{3}{4}g^2 + \frac{1}{4}g'^2; \quad B = \frac{1}{16}g'^4 + \frac{1}{8}g^2g'^2 + \frac{3}{16}g^2
\]  
(12)

In order to calculate the solution of Eqs.(10) let us introduce the following variables:

\[
x_A \equiv g_i^2/A; \quad y_A \equiv \lambda/A; \quad z_A \equiv g_3^2/A
\]  
(13)

Evidently,

\[
y_A = 2\frac{2}{\alpha}x_A - 1.
\]  
(14)

Then the equation for the \(t\)-quark Yukawa coupling constant reads,

\[
k_1x_A^2 + k_2x_A + C = 0,
\]  
(15)

where the coefficients are:

\[
k_1 = 36 - 24\alpha(4/\alpha^2 + 2/\alpha - 1) \approx -36.787;
\]  
(it does not depend on energy scale)

\[
k_2 = -64z_A - 24 - \frac{16g^2}{3A} + 24\alpha\left(\frac{6}{\alpha} + 1\right)
\]

\[
C = -24\alpha(1 + \frac{B}{A^2}) - \frac{2\alpha}{A^2}(-19g^4 + \frac{41}{3}g'^2)/4
\]  
(16)

Numerically the existence of a solution is very sensitive to both the value of \(\alpha \approx 1.55\) and the value of the strong coupling constant \(\alpha_3 = g_3^2/4\pi\). In what follows the updated averaged value of \(\alpha_3\) is taken from [23] as \(\alpha_3(M_Z) = 0.118 \pm 0.007\).

In Table 1 one can find the estimations for masses of \(t\)-quark and \(H\)-boson for different cutoffs in preparation of low-energy effective action for SM.
Table 1. Predictions from modified Veltman condition and its renorm-derivative.

| $\Lambda$, GeV | $10^{15}$ | $10^{14}$ | $2.3 \cdot 10^{13}$ |
|----------------|----------|----------|-------------------|
| $\ln \Lambda$  | 34.54    | 32.24    | 30.63             |
| $\alpha_3(\Lambda)$ | 0.0233  | 0.0248   | 0.0261            |
| $\alpha_2(\Lambda)$ | 0.0222  | 0.0228   | 0.0231            |
| $\alpha_1(\Lambda)$ | 0.0138  | 0.0135   | 0.01335           |
| $m_t(\Lambda)$, GeV | 105     | 99       | 79               |
| $M_H(\Lambda)$, GeV | 114     | 98       | 22               |
| $m_t(100 \text{ GeV})$ | 170     | 164      | 146              |
| $M_H(100 \text{ GeV})$ | 202     | 196      | 177              |

The low energy values of $m_H$ are evaluated with help of the IR quasi-fixed point for the Higgs self-coupling \[14\] which has been established at one-loop level.

The above stability conditions ensure the strong fine-tuning both to two-loop level and numerically. So one may consider the Higgs-field v.e.v. as a fundamental scale of SM. Still the physical parameter $v$ related to the above v.e.v. appears in the renormalized lagrangian in place of $\langle H \rangle \approx v_0$ and differs from it due to the wave function renormalization. The latter difference arises from non-vacuum diagrams and creates the anomalous dimension for the renormalized value of $v$.

The renormalization-group equation then deals with the RG flow generated by

$$\gamma_v = \left( \frac{3}{4} (3g^2 + g'^2) - 3g_t^2 \right) v(M).$$  \hspace{1cm} (17)

The natural supplement for the set of stability conditions of Veltman type might be the requirement to have zero anomalous dimension for $v$ that in turn corresponds to the true minimum for the spontaneous symmetry-breaking effects \[10\]. Luckily it happens to be compatible with modified Veltman conditions. Below on we display the joint solution of equations $\gamma_v = 0; \ f = 0$ for a wide range of cutoffs.
Table 2. Predictions from the modified Veltman condition and the absence of anomalous dimension.

One can compare Tabs. 1 and 2 and find the overlapping for $\Lambda \sim 2.5 \cdot 10^{13} GeV$ that gives in turn

$$m_t(100 GeV) \approx 155 GeV; \quad m_H(100 GeV) \approx 187 GeV$$

. On the other hand the usage of one-loop approach when deriving low-energy values brings a theoretical error that can be estimated
by averaging of discrepancy between modified Veltman condition and Hill’s condition as follows:

\[
\begin{align*}
    m_t &= 151 \pm 4 \text{GeV} \\
    m_H &= 195 \pm 7 \text{GeV}
\end{align*}
\] (18)

These predictions are within the accepted range for above masses found by overlapping of different experimental and theoretical bounds [2].

4 Conclusions and extensions

We have shown that the modification of Veltman condition caused by the strong fine-tuning of vacuum energies makes it possible to define the effective SM with a finite cutoff with fundamental EW scale and related particle masses essentially less than the cutoff. It has not been available in the original formulation (for \( \alpha = 1 \) there is no solution [21]). Furthermore it is consistent also at the two-loop level and compatible with the absence of anomalous dimension for the electroweak scale. The corresponding mass predictions are: \( m_t \approx 151 \pm 4 \text{GeV} \); \( M_H \approx 195 \pm 7 \text{GeV} \). The value of the cutoff contains an uncertainty connected with the experimental error in \( \alpha_S \), the latter one causes the following error bar for cutoff, \( 5 \cdot 10^{12} \text{GeV} < \Lambda_F < 5 \cdot 10^{14} \text{GeV} \). However, the masses can be defined more accurately through the absence of the anomalous dimension and the RG flow.

It may be interesting to combine the above conditions with other principles following from the infrared analysis. In particular we pay attention to the embedding the vacuum fine tuning into models with composite scalars following the discussion in [18]. It definitely will bring the intermediate scale where the above fine-tuning relations are necessary conditions to start the renormalization-group flow down to lower energies for all dimensional parameters with the RG equations of the conventional SM.
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