Using vibrational analysis to investigate the batted-ball performance of baseball bats

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Abstract

Hollow nonwood baseball bats have performance advantages over wooden bats. The hollow barrel bats act as a spring or trampoline during the bat-ball collision. The trampoline effect can occur at different natural frequencies known as hoop modes. To examine the effect of the hoop mode on the bat-ball collision, modal analysis and performance measurements were performed on a sample of hollow nonwood baseball bats. A nonlinear mass spring model was created to show the effect of hoop frequency has on performance. It has been shown that by decreasing the hoop frequency of the baseball bat from that of the new bat the performance of the baseball bat increases.

Keywords: Baseball; Batted Ball Speed; Ball Exit Speed Ratio; Batted Ball COR; Vibrations; Modal Analysis; Hoop Frequency

1. Introduction

Hollow nonwood bats were introduced into the game of baseball in the 1970s as a cost-effective alternative to wood bats. Being comprised of aluminium, the hollow bats are more durable than their wood counterparts, allowing for teams to buy fewer bats for the season, and thereby reducing the cost to teams for bats. Nonwood bats are also able to have their properties, such as barrel diameter, wall thickness, swing weights (MOI, moment of inertia), engineered to produce a better performing baseball bat than wood. Previous studies have shown that hollow bats outperform wood bats [1, 2]. However, the subtleties of why one nonwood bat outperforms another nonwood bat of the same length, weight and MOI are not always obvious.

During the collision between the baseball and the bat, a large amount of energy is lost when the ball deforms and compresses around the barrel of the bat. In a wood bat, the barrel of the bat is essentially rigid, such that all of the deformation associated with the bat-ball collision occurs in the baseball. However, in hollow nonwood bats, the barrel can flex during the collision. Thus, in comparison to a wood bat/ball collision, the baseball deforms less when impacting a “flexible” hollow barrel. Thus, less energy is dissipated by the ball due to the reduced ball deformation, and the energy used to compress the bat barrel can be returned to the baseball as the barrel rebounds. This phenomenon of how the barrel flexes, or breaths in and out, is known as the “trampoline effect”.

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The rate or how fast the trampoline effect occurs during the collision can be related to the mass and stiffness of the barrel of the bat, and therefore, one of the natural frequencies of the barrel of the bat. The natural frequency in which the bat trampoline effect occurs is known as the hoop mode of the barrel of the baseball bat.

In this research, experimental modal analysis is used to obtain the hoop-mode frequencies of hollow nonwood (aluminium and composite) baseball bats. Impact testing is performed on a large sample of nonwood bats to develop a database of hoop-frequencies and bending-frequency performances for a wide range of collegiate and high school baseball bats. The batted-ball performance characteristics of the bats are measured using a high-speed air cannon. The measurements are then used to build a database of modal and performance data to explore the relationship between hoop frequency and batted-ball performance for nonwood bats. The different performance metrics, e.g. Ball Exit Speed Ratio (BESR), Batted Ball Speed (BBS), and Bat-Ball COR (BBCOR), are investigated to see their correlation between the performance metrics and the bat’s modal behaviours.

2. Baseball Bat Vibration Modes

Hollow baseball bats exhibit two major vibration modes when excited by a ball impact. These two modes, bending and hoop, are shown in Fig 1. The bending mode of vibration is felt by a baseball player as a stinging sensation in the hands. The bending modes are excited when the baseball is not hit on the “sweet spot”. The relationship observed shows the greater the vibration response in the bat the greater the energy loss during the bat-ball collision.

The hoop mode of the baseball bat occurs in the barrel of hollow nonwood bats. The “pinging” sound heard when the ball strikes the bat is a result of the hoop modes of the baseball bat, also known as the bell modes. Fig. 1 shows the mode shapes for a baseball bat and an average frequency at which the oscillations occur.

3. Nonlinear Mass Spring Model

A nonlinear spring-mass model as developed by Russell was modified to simulate the collision between the baseball and the baseball bat to simulate the trampoline effect of the barrel. The nonlinear spring-mass model was originally developed to describe the collision between golf balls and golf ball club head [3]. This model was also used to describe the collision between softball and softball bat barrels [4]. Because of the nonlinear mechanical behaviour of the materials used to make the baseball, a nonlinear system is needed to describe the behaviour of the baseball during the collision with the baseball bat. Fig. 2 depicts the model of the nonlinear system. The differential equations of motion described in the model are stated as

\[
m_1\ddot{x}_1 = -k_1(x_1 - x_2)x_1 - x_2^a - c_1(\dot{x}_1 - \dot{x}_2)x_1 - x_2^b
\]

\[
m_2\ddot{x}_2 = -k_2x_2 - c_2\dot{x}_2 + k_1(x_1 - x_2)x_1 - x_2^a + c_1(\dot{x}_1 - \dot{x}_2)x_1 - x_2^b
\]

where the mass of the ball and effective mass of the barrel are \(m_1 = 0.145\) kg and \(m_2 = 0.09\) kg, respectively. The stiffness of the ball is \(k = 60 \times 10^6\) N/m and the dampening of the ball \(c_1 = 5000\) N-s/m. The dampening of the barrel
of the baseball bat was $c_2 = 100 \text{ N-s/m}$. The $a$ and $b$ exponents in Eqns. 1 and 2 are used to characterize the nonlinear behaviour of the ball. These nonlinear terms were determined by computing the CCOR (Cylindrical Coefficient of Restitution) values for the ball model. First, a rigid solid cylinder was used to simulate the spring stiffness and mass of the baseball bat. Then different combinations of $a$ and $b$ were iterated for each inbound velocity of the test. The CCOR values were calculated for five different velocities and then compared to experimental data collected using a dynamic stiffness machine. The dynamic stiffness machine utilizes a high-speed camera and light gates to measure velocity and uses load cells to measure force. The values of $a$ and $b$ that gave the best correlation between the model and the experimental data for a CCOR test were determined to be 0.6495 and 0.5202, respectively. Fig. 3 shows the calculated values compared to the measured values of CCOR.

![Fig. 2. Mass-spring model of baseball and baseball bat collision](image1)

![Fig. 3. Comparison of the analytical model results with experimental CCOR data.](image2)

With the values of the constants known for the nonlinear spring-mass model, Eqns. 1 and 2 can be numerically solved for the maximum rebound velocity of the baseball as a function of the hoop frequency. The CCOR values were computed and normalized to values for a ball impacting a rigid bat to show the increase in performance for hollow bats. Fig. 4 shows the normalized collision efficiency versus hoop frequency plot for a baseball bat barrel. Based on Fig. 4, the optimal hoop frequency for a baseball bat is approximately 1250Hz. Baseball bats that are close to this hoop frequency will have more efficient collisions with the baseball than a baseball bat that falls either below or above this frequency.

![Fig. 4. Normalized Collision Efficiency Results of Mass Spring Model](image3)
4. Methods

The methods used to obtain the performance characteristics of baseball bats as well as the vibrational properties are described in this section. Several different performance metrics were investigated.

4.1 Modal Testing

Modal testing was performed on a large sample of nonwood bats comprised of aluminium and composite barrels. Bats were hung by Bungee cords to approximate a free-free condition. Two accelerometers were placed near the end of the barrel and 90° from one another. An impact hammer was used to excite the natural frequencies. A Dactron signal conditioner and Pro-Photon software were used to collect data from a force gage and the two accelerometers to compute the frequency response function (FRF) of a baseball bat. The hoop frequency was determined when the FRF from each of the two accelerometers overlaid at peak amplitudes on the FRF.

4.2 Performance Metrics

The performances of the baseball bats were determined using a high speed air cannon setup in accordance to NCAA BESR certification protocol [5]. The baseball bat was loaded into a clamp six inches (15.24 cm) from the knob of the bat. Balls were fired at the stationary baseball bat that was free to rotate about the six-inch (15.24-cm) location. The inbound and rebound velocities of the baseball were measured using a series of three light gates. Three performance metrics were investigated for comparing with hoop frequency. These performance metrics are Ball Exit Speed Ratio (BESR), Batted-Ball Speed (BBS) and Bat-Ball COR (BBCOR).

The Ball Exit Speed Ratio equation is stated as:

\[
BESR = \frac{V_{\text{rebound}}}{V_{\text{inbound}}} - \delta v + 0.5
\]

where: \(\delta v = 136 \, \text{mph} - V_{\text{contact}}\) \((\delta v = 60.8 \, \text{m/s} - V_{\text{contact}})\)

and where \(V_{\text{rebound}}\) is the velocity of the baseball coming off the bat in the BESR test, and \(V_{\text{inbound}}\) is the inbound velocity of the baseball in the BESR test. The bat velocity at which the baseball makes contact with the barrel of the bat is \(V_{\text{contact}}\). This velocity is based upon the impact location on the barrel of the bat.

The batted-ball equation is stated as

\[
BBS = V_{\text{pitch}} \left( \frac{V_{\text{rebound}}}{V_{\text{inbound}}} + V_{\text{bat}} \left( \frac{V_{\text{rebound}}}{V_{\text{inbound}}} + 1 \right) \right)
\]

where \(V_{\text{pitch}}\) is the velocity of the pitch of the baseball and \(V_{\text{bat}}\) is the velocity of the bat at the impact location based upon a swing speed model [2].

Bat-Ball COR was the last performance metric investigated. The equation for BBCOR is

\[
BBCOR = \left( \frac{V_{\text{rebound}}}{V_{\text{inbound}}} \right) (1 + k) + k
\]

where:

\[
k = \frac{m_{\text{ball}} x^2}{I}
\]

and where \(x\) is the distance between the pivot location and the impact location on the barrel. The variable \(m_{\text{ball}}\) is the mass of the baseball used during the test.
5. Results

Several baseball-bat performances were measured along with the respective hoop frequencies of the bats. The baseball bats were of lengths ranging from 31 to 34 inches (78.7 to 86.4 cm) and compliant with the NCAA -3 drop rule for weight (length (in inches) - weight (in ounces) ≥ -3). The set of bats consisted of both aluminium and composite barrel baseball bats. The total range of hoop frequencies recorded was from 1400 to 2800 Hz. The hoop frequencies of the composite-barrel baseball bats span a much wider frequency range than aluminium bats. The wider range for composite bats gives manufacturers more flexibility over the resulting mechanical properties of the bat in comparison to aluminium. The first hoop frequencies versus first bending mode data were plotted for the aluminium and composite barrel bats. Fig. 5 shows the hoop frequency range for the composite bats is much wider than that of the aluminium barrel bats.

![Graph showing hoop frequency vs. first bending frequency with Aluminium Baseball Barrels and Composite Baseball Barrels plotted.](image)

**Fig. 5.** First hoop frequency vs. first bending frequency to show wide range of hoop frequencies for composite bats compared to aluminium bats.

The hoop frequency range was divided into 50-Hz increments where the performance metric was averaged over a range of ± 25 Hz. Fig. 6 shows a plot of hoop frequency versus performance metrics, i.e. BESR, BBCOR and BBS.

Fig. 6 shows a correlation between hoop frequency and all performance metrics. As the hoop frequency of the barrel of the baseball bat increases, the performance decreases. These results correlate well to the nonlinear baseball bat-ball model as previously discussed. The data points within the frequency range of 2800 to 3000 Hz are each only one bat. Had there been more bats within this frequency range, it is thought they would also follow the same trend. Obtaining baseball bats with frequencies less than 1400 Hz is difficult because it is believed that the durability of the baseball bats is significantly less for bats in this frequency range. Therefore, manufacturers are unlikely to produce bats that are not capable of withstanding repeated impacts.
6. Conclusion

For nonwood baseball bats, the trampoline effect of the barrel of the bat can cause the performance of the baseball bat to outperform wood baseball bats. A nonlinear spring-mass model simulating the bat ball collision of a hollow baseball bat showed there is an ideal range for the hoop frequency of the barrel of the baseball bat to maximize the collision efficiency. Testing of various baseball bats showed the hoop frequency of the baseball does have an effect on the performance. Performance metrics of BESR, BBS, and BBCOR showed similar correlation to the efficiency plot: decreasing the hoop frequency will cause an increase in the performance of the baseball bat. Hoop frequency can be a useful predictor of estimating the performance of nonwood hollow baseball bats.

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