Future Measurements of Transversity

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Abstract: A review of envisaged future quark transversity measurements is presented.

1 Introduction

A quark of a given flavor in the nucleon is characterized by three twist-two quark distributions: the number density distribution $q(x)$, the helicity distribution $\Delta q(x)$, and the experimentally unknown transversity distribution $\delta q(x)$, which characterizes the distribution of the quark’s transverse spin in a transversely polarized nucleon. For non-relativistic quarks $\delta q(x) = \Delta q(x)$ can be expected, but generally both distribution functions are independent. Soffer’s inequality for each quark flavor, $2|\delta q(x, Q^2)| \leq q(x, Q^2) + \Delta q(x, Q^2)$, restricts possible values of the transversity distributions.

The transversity distribution was first discussed by Ralston and Soper \cite{ralston-soper} in doubly transverse polarized Drell-Yan (DY) scattering. Its measurement is one of the main goals of the spin program at RHIC. The transversity distributions $\delta q(x)$ are not accessible in inclusive DIS, because they are chiral-odd and only occur in combinations with other chiral-odd objects. In semi-inclusive DIS of unpolarized leptons off transversely polarized nucleons several methods have been proposed to access $\delta q(x)$ via specific single target-spin asymmetries.

2 Measurement of $\delta q(x)$ in pp Collisions

An evaluation of the DY asymmetry $A_{TT} \sim \sum e_\gamma^2 \delta q_i(x) \delta \bar{q}_i(x)$ was carried out \cite{ralston-soper} by assuming the saturation of Soffer’s inequality for the transversity distributions. The maximum possible asymmetry at RHIC energies was estimated to be $A_{TT} = 1\div2\%$ (see fig.\textsuperscript{1b}). At smaller energies, e.g. for a possible fixed-target experiment HERA-$N$ \cite{hera-n} ($\sqrt{s} \approx 40$ GeV), the asymmetry is expected to be higher.

A better sensitivity to $\delta q(x)$ is expected in a measurement of two-meson correlations with the nucleon’s transverse spin. The interference effect between the $s$- and $p$-waves of the two-meson system allows the quark’s polarization information to be carried through $\vec{k}_+ \times \vec{k}_- \cdot \vec{S}_\perp$ \cite{corr-interf}. Here, $\vec{k}_+$ and $\vec{k}_-$ are the meson momenta, and $\vec{S}_\perp$ is the proton spin vector. The corresponding asymmetry depends on the unknown chiral-odd interference quark fragmentation function (FF), $\delta \hat{q}_i(z)$. The function $\delta \hat{q}_i(z)$ has a theoretical upper bound and could be measured in $e^+e^- \to (\pi^+\pi^-X)(\pi^+\pi^-X)$. To estimate a possible level of the asymmetry at RHIC energies two assumptions were made \cite{corr-interf}: i) $\delta q(x, Q^2)$ saturates Soffer’s inequality, and ii) $\delta \hat{q}_i(z)$ saturates its upper bound. This approach produces the maximally possible asymmetry. The projections of the asymmetry measurement with PHENIX at RHIC \cite{phenix} are shown in fig.\textsuperscript{1b}.

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Figure 1: Projections for measurements with PHENIX ($\sqrt{s} = 200$ GeV) of a) Drell-Yan asymmetry $A_{TT}$ ($L = 320$ pb$^{-1}$) [3], and b) two-pion asymmetry ($L = 32$ pb$^{-1}$) [6].

3 Measurements of $\delta q(x)$ in SIDIS

In semi-inclusive deep inelastic lepton scattering (SIDIS) off transversely polarized nucleons there exist several methods to access transversity distributions. One of them, namely twist-3 pion production [7], uses longitudinally polarized leptons and a double spin asymmetry is measured. The other methods do not require a polarized beam, they rely on polarimetry of the scattered transversely polarized quark: i) measurement of the transverse polarization of $\Lambda$’s in the current fragmentation region [8]; ii) observation of a correlation between the nucleon’s transverse spin vector and the normal to the two-meson plane [4]; iii) observation of the Collins effect in quark fragmentation through the measurement of pion single target-spin asymmetries [9]. HERMES data [10] indicate that the polarized FF $H_{1L}^{q\perp}(z)$, responsible for the Collins effect, is quite sizeable.

3.1 Future Measurements at HERMES

The expected statistics for running at HERMES ($E = 27.5$ GeV) with a transversely polarized proton target will consist of about seven millions reconstructed DIS events. As average beam and target polarizations $P_B = 50\%$ and $P_T = 75\%$, respectively, are used for the analysis. DIS events are defined as those satisfying the following set of kinematic cuts: $Q^2 > 1$ GeV$^2$, $W > 2$ GeV, $0.02 < x < 0.7$, $y < 0.85$. The following cuts were assumed for the kinematic variables of the pion: $x_F > 0$, $z > 0.1$, $P_{h\perp} > 0.05$ GeV.

The approximation $\delta q(x) = \Delta q(x)$ could be used for the evaluation of the below given projections in view of the relatively low $Q^2$-values at HERMES.

Twist-3 Pion Production. An effect of the transversity distributions in the spin-dependent cross-section of pion production in DIS of longitudinally polarized leptons on a transversely polarized nucleon target can appear only at the twist-3 level, when $\delta q(x)$ contributes through coupling with the chiral-odd twist-3 FF $\hat{e}(z)$ [7]. A simple relation between $\hat{e}(z)$ and the unpolarized FF $D(z)$ has been predicted in the chiral quark model [11] $\hat{e}(z) = zD(z)\frac{m_q}{M} \approx \frac{1}{3}zD(z)$, where $m_q$ is the constituent quark mass. In the particular case of forming the asymmetry for
Figure 2: Projections for a measurement at HERMES of a) twist-3 asymmetry for \( \pi^+ + \pi^- \) production (dash-dotted curve — \( \delta q(x) = 0 \), dashed curve — saturation of the Soffer’s inequality, solid curve — \( \delta q(x) = \Delta q(x) \)), b) two-pion asymmetry.

the sum of \( \pi^+ \) and \( \pi^- \) production on a proton target:

\[
A(x, y, \phi) = \cos \phi \cdot \frac{2M x}{\sqrt{Q^2}} \cdot \frac{2y\sqrt{1-y}}{1+(1-y)^2} \cdot \frac{g_T(x) + h_1(x)/3x - (1-y/2)g_1(x)}{F_1(x)},
\]

where \( h_1(x) = \frac{1}{2} \sum_i e_i^2 \delta q_i(x) \), \( g_T(x) = g_1(x) + g_2(x) \), and \( \phi \) is the azimuthal angle between the lepton scattering plane and the spin plane. The projections for a measurement of this twist-3 pion asymmetry at HERMES are shown in fig. 2a, where the asymmetry \( \tilde{A}(x, y) = P_B \cdot P_T \cdot A(x, y, \phi) / \cos \phi \) is shown for convenience.

Two-Meson Correlations with Transverse Spin. As has been noted above, the asymmetry in this case depends on the unknown chiral-odd interference quark FF \( \delta \hat{q}_I(z) \), \( A_{\perp \parallel} \sim \sum_a e_a^2 \delta q_a(x) \delta \hat{q}_I^a(z) \).

A maximally possible asymmetry with respect to the interference FF can be obtained with its upper bound. For \( \pi^+ \pi^- \) pair production at a proton target the maximal asymmetry takes the form:

\[
A_{\text{max}} = -\frac{\pi}{\sqrt{32}} \cos \phi \sin (\delta_0 - \delta_1) \frac{\delta u_\parallel(x) - \frac{1}{4} \delta d_\parallel(x)}{D_{nn}\left(u(x) + \bar{u}(x)\right) + \frac{1}{4}(d(x) + \bar{d}(x))},
\]

where \( \cos \phi = \vec{k}_+ \times \vec{k}_- \cdot \vec{S}_\perp / |\vec{k}_+ \times \vec{k}_-||\vec{S}_\perp| \); \( D_{nn} \) is the transverse polarization transfer coefficient, and \( \delta_{0,1} = \delta_{0,1}(m^2) \) are strong interaction \( \pi \pi \) phase shifts. The asymmetry has been calculated in two regions of the two-pion mass to avoid averaging to zero due to the factor \( \sin (\delta_0 - \delta_1) \). The projections for its measurement at HERMES are shown in fig. 2b in terms of \( \tilde{A}_{\text{max}} = P_T \cdot A_{\text{max}} / \cos \phi \).
Collins effect. In the case of an unpolarized beam and a transversely polarized target the following weighted asymmetry [12] provides access to the quark transversity distribution via the Collins effect:

\[
A_T(x, y, z) \equiv \frac{\int d\phi^f \int d^2 P_{h\perp} \frac{|P_{h\perp}|}{2M_h} \sin(\phi^f + \phi^f_h) \left( d\sigma^\uparrow \uparrow - d\sigma^\downarrow \downarrow \right)}{\int d\phi^f \int d^2 P_{h\perp} \left( d\sigma^\uparrow \uparrow + d\sigma^\downarrow \downarrow \right)},
\]  

(3)

where \(P_{h\perp}\) is the pion’s transverse momentum and the azimuthal angles are defined in the transverse space giving the orientation of the lepton plane (\(\phi^f\)) and the orientation of the hadron plane (\(\phi^f_h = \phi_h - \phi^f\)) or spin vector (\(\phi^s = \phi_s - \phi^f\)) with respect to the lepton plane. The asymmetry (3) can be estimated from

\[
A_T(x, y, z) = P_T \cdot D_{nn} \cdot \frac{\sum_q e_q^2 \delta q(x) H_{1}^{(1)}(z)}{\sum_q e_q^2 q(x) D_{1}^{u}(z)}.
\]  

(4)

The assumption of \(u\)-quark dominance was used to calculate the expected asymmetry \(A_{\pi^+}^T(x)\) [14]. In this case the asymmetry for a proton target reduces to

\[
A_{\pi^+}^T(x, y, z) = P_T \cdot D_{nn} \cdot \frac{\delta u(x)}{u(x)} \cdot \frac{H_{1}^{(1)}(z)}{D_{1}^{u}(z)}.
\]  

(5)

The approach of Ref. [12] is adopted to estimate \(H_{1}^{(1)}(z)/D_{1}^{u}(z)\).

The factorized form of expression (4) with respect to \(x\) and \(z\) allows the simultaneous reconstruction of the shape for both unknown functions \(\delta u(x)\) and \(H_{1}^{(1)}(z)/D_{1}^{u}(z)\) if measurements of the asymmetry are done in \((x, z)\) bins, while the relative normalization cannot be fixed without a further assumption. The differences between \(\delta q(x)\) and \(\Delta q(x)\) are smallest in the region of intermediate and large values of \(x\) [14]. Hence the assumption \(\delta q(x_0) = \Delta q(x_0)\) at \(x_0 = 0.25\) was made to resolve the normalization ambiguity. The projections for a measurement of \(A_{\pi^+}^T(x), \delta u(x),\) and \(H_{1}^{(1)}(z)/D_{1}^{u}(z)\) at HERMES are shown in fig. 3.

\[\begin{array}{ccc}
\begin{array}{c}
\text{a) } A_{\pi^+}^T(x) \\
\text{b) } \delta u(x) \\
\text{c) } H_{1}^{(1)}(z)/D_{1}^{u}(z)
\end{array}
\end{array}\]

Figure 3: a) The weighted asymmetry \(A_{\pi^+}^T(x)\) in different intervals of \(z\); b) the transversity distribution \(\delta u(x)\), and c) the ratio of the fragmentation functions \(H_{1}^{(1)}(z)/D_{1}^{u}(z)\) as it would be measured by HERMES. The asterisk in b) shows the normalization point. The hatched bands in b) and c) show projected systematic uncertainties due to the normalization and the \(u\)-quark dominance assumptions.
3.2 The Experiment TESLA-N

The basic TESLA-N idea is to use one of the arms of the presently planned $e^+e^-$ collider TESLA at DESY to accomplish collisions of longitudinally polarized electrons ($E = 250$ GeV, $P_B = 90\%$) with a polarized solid-state fixed target. Presently, the target materials NH$_3$ ($P_T = 0.8$, $f = 0.176$) and $^6$LiD ($P_T = 0.3$, $f = 0.44$) appear as the best choices to study electron scattering off polarized protons and deuterons, respectively.

The physics projections presented below are based on an integrated luminosity of 100 fb$^{-1}$. This represents a conservative estimate for one year of data taking.

Measurements of single target-spin asymmetries due to the Collins effect in the production of positive and negative pions on proton and deuteron targets ($A_{\pi^+,\pi^-}^{p,d}$) allow, under reasonable assumptions, the simultaneous reconstruction of the shapes of the unknown functions $\delta q(x, Q^2)$ and $H_1^{\perp(1)}(z)/D_1(z)$. Again the relative normalization cannot be fixed without an independent measurement or a further assumption. This ambiguity can be resolved by relating $\delta q(x)$ to $\Delta q(x)$ at small values of $Q^2$. Following ref. [14], the normalization ambiguity is resolved by assuming $\delta u(x_0, Q^2_0) = \Delta u(x_0, Q^2_0)$ at $x_0 = 0.25$.

The projections for the measurement of $\delta u_v(x, Q^2)$ at TESLA-N are shown in fig. 4. A broad range of $0.003 < x < 0.7$ can be accessed in conjunction with $1 < Q^2 < 100$ GeV$^2$. A simultaneous reconstruction of the quark transversity distributions $\delta d_v$, $\delta u$, and $\delta d$ is attained with a somewhat lower accuracy. Projections for the accuracies of a measurement of the $u$- and $d$-quark tensor charges are ±0.01 and ±0.02 at the scale of 1 GeV$^2$, respectively. At the same time, precise values would be measured for the ratio of polarized and unpolarized favored quark fragmentation functions $H_1^{\perp(1)}(z)/D_1(z)$, assumed to be flavor-independent in this analysis.

![Figure 4: The transversity distribution $\delta u_v(x, Q^2)$ as it would be measured at TESLA-N. The curves show the LO $Q^2$-evolution of $\delta u_v$ obtained with a fit to the simulated asymmetries.](image-url)
4 Summary

A measurement of the transversity distributions in polarized proton-proton collisions at RHIC will be possible if the chiral-odd interference fragmentation function, $\delta \hat{q}$, is not heavily suppressed relative to its theoretical upper bound.

The HERMES experiment using a transversely polarized proton target will be capable in a few years to measure simultaneously and with good statistical precision the $u$-quark transversity distribution $\delta u(x)$ and the ratio of the fragmentation functions $H_{1}^{(1)u}(z)/D_{u}^{u}(z)$.

A measurement of the quark transversity distributions as a function of $x$ and $Q^2$ with good statistical accuracy requires a new high luminosity and high energy polarized lepton-nucleon experiment. The physics potential of the TESLA-N project is demonstrated by showing that an accurate measurement of the $x$- and $Q^2$-dependence of the transversity quark distributions would be possible.

References

[1] J. Ralston, D.E. Soper, Nucl. Phys. B152 (1979) 109.
[2] O. Martin et al., Phys. Rev. D60 (1999) 117502.
[3] V.A. Korotkov and W.-D. Nowak, Nucl. Phys. A622 (1997) 78c.
[4] R.L. Jaffe, X. Jin, J. Tang, Phys. Rev. Lett. 80 (1998) 1166.
[5] J. Tang, MIT-CTP-2769, 1998, hep-ph/9807560.
[6] M. Grosse Perdekamp, DIS-2000, Liverpool, 25-30 April 2000; Workshop Future Transversity Measurements, BNL, 18-20 September 2000.
[7] R.L. Jaffe, X.J. Ji, Phys. Rev. Lett. 71 (1993) 2547.
[8] R.L. Jaffe, Phys. Rev. D54 (1996) 6581.
[9] J.C. Collins, Nucl. Phys. B396 (1993) 161;
    J.C. Collins, S.F. Heppelmann, G.A. Ladinsky, Nucl. Phys. B420 (1994) 565.
[10] HERMES Coll., A. Airapetian et al., Phys. Rev. Lett. 84 (2000) 4047.
[11] X. Ji, Z.-K. Zhu, hep-ph/9402303, 1994.
[12] A.M. Kotzinian, P.J. Mulders, Phys.Lett. B406 (1997) 373.
[13] http://www.ifh.de/hermes/future/ and F. Ellinghaus, these Proceedings.
[14] V.A. Korotkov, W.-D. Nowak, K.A. Oganessyan, DESY 99-176, hep-ph/0002268, subm. to EPJ C.