BRANE GAS COSMOLOGY AND LOITERING

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In Brane Gas Cosmology (BGC) the initial state of the universe is taken
to be small, dense and hot, with all fundamental degrees of freedom near
thermal equilibrium. This starting point is in close analogy with the Standard
Big Bang (SBB) model. In the simplest example, the topology of the universe
is assumed to be toroidal in all nine spatial dimensions and is filled with a
gas of $p$-branes. The dynamics of winding modes allow, at most, three spatial
dimensions to become large, providing a possible explanation to the origin
of our macroscopic (3+1)-dimensional universe. Specific solutions are found
within the model that exhibit loitering, i.e. the universe experiences a short
phase of slow contraction during which the Hubble radius grows larger than
the physical extent of the universe. This phase is studied by combining the
dilaton gravity background equations of motion with equations that determine
the annihilation of string winding modes into string loops. Loitering provides
a solution to the brane problem (generalised domain wall problem) in BGC
and the horizon problem of the SBB scenario. In BGC the initial singularity
problem of the SBB scenario is solved, without relying on an inflationary phase
due to the presence of the T-duality symmetry in the theory.

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1 Introduction and Motivation

One of the most significant dilemmas in string theory is the dimensionality problem. A consistent formulation of superstring theory requires the universe to be (9+1)-dimensional but empirical evidence demonstrates that the universe is (3 + 1)-dimensional.

The typical resolution to this apparent problem is to hypothesise that six of the spatial dimensions are curled up on a near Planckian sized manifold, and are therefore difficult to detect in the low energy world that we live in. But if this is the case, the question naturally arises, why is there a difference in size and structure between our large 3-dimensional universe and the 6-dimensional compact space? What physical laws demand that spacetime be split in such an unusual way? Since we are assuming superstring theory is the correct theory to describe the physical universe, the answer to these questions must come from within the theory itself.

Although the dimensionality problem is a very severe problem from a cosmological viewpoint, it is rarely addressed. For example, brane world cosmological scenarios derived from string theories typically impose the identification of our universe with a 3-brane. * All current models fail to explain why our universe is a $d$-brane of spatial dimension $d = 3$, opposed to any other value of $d$, and furthermore fail to explain why our universe is this particular 3-brane opposed to any other 3-brane which may appear in the theory. Due to the current unnatural construction of such models, it seems likely that they will inevitably require some form of the anthropic principle in order to address the dimensionality problem.

The dimensionality problem is not unique to string theory, however; it is an equally challenging problem for cosmology. A truly complete cosmological model (if it is possible to obtain such a thing), whether derived from $M$-theory, quantum gravity or any other theory, should necessarily explain why we live in (3 + 1)-dimensions.

Because this conundrum is an integral part of both superstring theory and cosmology, it seems likely that only an amalgamation of the two will be capable of producing a satisfactory solution. After all, if one is going to evolve from a 9-dimensional space to a 3-dimensional space, one is going to require dynamics, and the dynamics of our universe are governed by cosmology.

In this talk, we consider an approach to string cosmology which addresses the dimensionality problem and which is in close analogy with the usual starting point of standard big-bang cosmology. This scenario is called “Brane Gas Cosmology” (BGC) †. In BGC, the universe starts out small, dense, hot and with all fundamental degrees of freedom near thermal equilibrium. † For simplicity, the background spatial geometry is as-

*For an elementary introduction to brane world constructions see, e.g. [1].
†Note that mathematically it is not possible for the universe to be in thermal equilibrium as the FRW cosmological model does not possess a time-like Killing vector. However, it is true that the universe has been very nearly in thermal equilibrium. Obviously, the departures from equilibrium make things interesting!
sumed to be toroidal, and the universe is filled with a hot gas of \( p \)-branes, the fundamental objects appearing in string theories.

The motivations for this cosmological scenario, are the problems of the standard Big Bang model (such as the presence of an initial singularity), the problems of string theory (such as the dimensionality problem), and are cosmological. By cosmological we mean that we wish to stay in close contact with the standard Big Bang model, and therefore maintain the initial conditions of a hot, small and dense universe. Some other attempts to incorporate \( M \)-theory into cosmology, such as the existing formulations of brane world scenarios, are motivated from particle physics and make little connection with what we know about the origins of the universe. They also suffer from the dimensionality problem mentioned above. Why do the extra dimensions have the topologies they do? Why should a 3-brane be favored over any other \( p \)-brane for our universe, and why should we live on one particular 3-three brane versus another?

Here we present the simplest (and hence, least realistic) model of BGC. The universe is assumed to be toroidal, and the brane gas is constructed from Type IIA superstring theory. More realistic models with brane gases constructed from various branches of the \( M \)-theory moduli space, and with compactifications on manifolds of non-trivial homology are considered in [4].

The branes may wrap around the cycles of the torus (winding modes), they can have a center-of-mass motion along the cycles (momentum modes) or they may simply fluctuate in the bulk space (oscillatory modes). By symmetry, we assume equal numbers of winding and anti-winding modes. As the universe expands, the winding modes become heavy and halt the expansion [5]. Spatial dimensions can only dynamically decompactify if the winding modes can disappear, and this is only possible (for string winding modes) in \( 3 + 1 \) dimensions [6]. Thus, BGC may provide an explanation for the observed number of large spatial dimensions. However, causality demands that at least one winding mode per Hubble volume will be left behind, leading to the \textit{brane problem} for BGC [2], a problem analogous to the domain wall problem of standard cosmology.

There is a simple solution to the brane problem: the winding modes will halt the expansion of the spatial sections, and lead to a phase of slight contraction \( (loitering [7, 8]) \) during which the Hubble radius becomes larger than the spatial sections and hence all remaining winding modes can annihilate in the large \( 3 + 1 \) dimensions. We supplement the equations for the dilaton gravity background of BGC [3, 9] by equations which describe the annihilation of string winding modes into string loops. Solutions exist in which the winding modes cause the universe to contract for a short time and enter a loitering phase. During the phase of contraction the number density of the remaining winding modes increases and the winding and anti-winding modes begin to annihilate. The winding branes appear as solitons (analogous to cosmic strings) in the bulk space. The annihilation of winding and antiwinding modes (analogous to cosmic string intersections) leads to the production of string loops which has the same equation of state as cold matter [3].

Brane Gas Cosmology is a simple nonsingular model which addresses some of the
problems of the SBB scenario, simultaneously providing a dynamic resolution to the dimensionality problem of string theory.

The organisation of this talk is as follows. We begin with a brief review of the Brane Gas model in Section 2. Our concrete starting point is presented, followed by a derivation of the equation of state for a gas of branes and an analysis of the background dynamics. This is followed (in Section 3) by a discussion of the dilaton gravity equations in the presence of a brane gas, of the benefits of loitering, and of attractor solutions. In Section 4 we supplement the system of equations with equations which describe the annihilation of string winding modes into string loops, and based on this we provide a detailed analysis of a loitering solution. We use both numerical and analytic methods to study the solutions. We conclude, in Section 5, with a brief summary and a few conjectures concerning supersymmetry breaking, the effective breaking of T-duality, and dilaton mass generation in the late universe.

2 Brane Gas Cosmology

Over the past decade it has become clear that fundamental strings are not the only fundamental degrees of freedom in string theory. D-branes are also part of the spectrum of fundamental states. In the Brane Gas scenario we explore some possible effects of D-branes on the early universe. The original model of [2] is based on two key assumptions: firstly that the initial state of the Universe corresponded to a dense, hot gas in which all degrees of freedom were in thermal equilibrium, and secondly that the topology of the background space admits one-cycles. Note that even in spaces without one-cycles in the compactified space there are cases where only three spatial dimensions can become large [4].

The first main point of the scenario in [6], is that T-duality will lead to an equivalence of the physics if the radius of the background torus changes (in string units) from \( R \) to \( \frac{1}{R} \). This corresponds to an interchange of momentum and winding modes. Thus, \( R \) becoming small is equivalent to \( R \) tending to infinity. Neither limit corresponds to a singularity for string matter. For example, the temperature \( T \) obeys

\[ T\left(\frac{1}{R}\right) = T(R). \]

Thus, in string cosmology the big bang singularity can be avoided. The second point suggested in [6] was that string winding modes would prevent more than three spatial dimensions from becoming large. String winding modes cannot annihilate in more than three spatial dimensions (by a simple classical dimension counting argument). In the context of dilaton cosmology, a gas of string winding modes (which has an equation of state \( \tilde{p} = -(1/d)\rho \), where \( \tilde{p} \) and \( \rho \) denote pressure and energy density, respectively, and \( d \) is the number of spatial dimensions) will lead to a confining potential in the equation of motion for \( \lambda = \log(a) \), where \( a(t) \) is the scale factor of the universe [6]. Note that
this is not the result which is obtained in a pure metric background obeying the Einstein equations. The dynamics of classical strings in higher dimensional expanding backgrounds was studied numerically in [11], confirming the conclusions of [6].

However, it is now clear that string theory has a much richer set of fundamental degrees of freedom, consisting - in addition to fundamental strings - of D-branes [11] of various dimensionalities. The five previously known consistent perturbative string theories are now known to be connected by a web of dualities [12], and are believed to represent different corners of moduli space of a yet unknown theory called M-theory. Which branes arise in the effective string theory description depends on the particular point in moduli space.

The question we address in [2] is whether the inclusion of the new fundamental degrees of freedom will change the main cosmological implications of string theory suggested in [6], namely the avoidance of the initial cosmological singularity, and the singling out of 3 as the maximal number of large spatial dimensions, in the context of an initial state which is assumed to be hot, dense and small, and in a background geometry which admits string winding modes.

Our concrete starting point is 11-dimensional M-theory compactified on S¹ to yield 10-dimensional Type IIA string theory. The resulting low energy effective theory is supersymmetrized dilaton gravity. As fundamental states, M-theory admits the graviton, 2-branes and 5-branes. After compactification, this leads to 0-branes, 1-branes, 2-branes, 4-branes, 5-branes, 6-branes and 8-branes as the fundamental extended objects of the 10-dimensional theory. The dilaton represents the radius of the compactified S¹. We are in a region of moduli space in which the string coupling constant $g_s$ is smaller than 1.

The details of the compactification will not be discussed here, however we will briefly mention the origins of the above objects from the fundamental eleven-dimensional, M-theory perspective. The 0-branes of the IIA theory are the BPS states of nonvanishing $p_{10}$. In M-theory these are the states of the massless graviton multiplet. The 1-brane of the IIA theory is the fundamental IIA string which is obtained by wrapping the M-theory supermembrane around the S¹. The 2-brane is just the transverse M2-brane. The 4-branes are wrapped M5-branes. The 5-brane of the IIA theory is a solution carrying magnetic NS-NS charge and is an M5-brane that is transverse to the eleventh dimension. The 6-brane field strength is dual to that of the 0-brane, and is a KK magnetic monopole. The 8-brane may be viewed as a source for the dilaton field [13].

We assume that all spatial dimensions are toroidal (radius $R$), and that the universe starts out small, dense, hot, and in thermal equilibrium. Thus, the universe will contain a gas of all branes appearing in the spectrum of the theory. Note that this starting point is in close analogy with the hot big bang picture in standard cosmology, but very different from brane-world scenarios in which the existence of a particular set of branes is postulated from the outset without much justification from the point of view of cosmology.

There have been several interesting studies of the cosmology of brane gases. Maggiore and Riotto [14] (see also [13]) studied the phase diagram of brane gases motivated by
\(M\)-theory as a function of the string coupling constant and of the Hubble expansion rate (as a measure of space-time curvature) and discovered regions of the phase diagram in which brane gases determine the dynamics, and regions where the effective action is no longer well described by a ten-dimensional supergravity action. Given our assumptions, we are in a region in moduli space in which the ten-dimensional effective description of the physics remains true to curvature scales larger than that given by the string scale. In this paper, we consider the time evolution of the system through phase space starting from some well-defined initial conditions. We will argue that as a consequence of T-duality, curvature scales where the ten-dimensional description breaks down are never reached.

In another interesting paper, Park et al. [16] take a starting point very close to our own, a hot dense gas of branes. However, they did not consider the winding and oscillatory modes of the branes.

In the following section we will study the equation of state of the brane gases for all values of their spatial dimension \(p\). We will separately analyze the contributions of winding and non-winding modes (the latter treated perturbatively). The results will be used as source terms for the equations of motion of the background dilaton gravity fields, following the approach of [5]. We find that the winding modes of any \(p\)-brane lead to a confining force which prevents the expansion of the spatial dimensions, and that the branes with the largest value of \(p\) give the largest contribution to the energy of the gas in the phase in which the scale factor is increasing.

In Section 3 we argue that the main conclusions of the scenario proposed in [4] are unchanged: T-duality eliminates the cosmological singularity, and winding modes only allow three dimensions of space to become large. We point out a potential problem (the brane problem) of cosmologies based on theories which admit branes in their spectrum of fundamental states. This problem is similar to the well-known domain wall problem [17] in cosmological models based on quantum field theory. It is pointed out that a phase of loitering (see e.g. [7]) yields a natural solution of this problem, and it is shown that the background equations of motion may well yield a loitering stage during the early evolution of the universe. Some limitations of this model and avenues for future research are discussed in the final section.

### 3 Equation of State of Brane Gases

As mentioned in the Introduction, our starting point is Type IIA string theory on a 9-dimensional toroidal background space (with the time direction being infinite), resulting from the compactification of \(M\)-theory on \(S^1\). The overall spatial manifold has topology \(\mathcal{M}^{10}_{\text{IIA}} = S^1 \times T^9\). The fundamental degrees of freedom are the fields of the bulk background (resulting from the graviton in \(M\)-theory), 1-branes, 2-branes, 4-branes, 5-branes, 6-branes and 8-branes.
The low-energy bulk effective action is given by

\[ S_{\text{bulk}} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} e^{-2\phi} \left[ R + 4G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha} \right], \]  

where \( G \) is the determinant of the background metric \( G_{\mu\nu} \), \( \phi \) is the dilaton, \( H \) denotes the field strength corresponding to the bulk antisymmetric tensor field \( B_{\mu\nu} \), and \( \kappa \) is determined by the 10-dimensional Newton constant in the usual way.

The total action is the sum of the above bulk action and the action of all branes present. The action of an individual brane with spatial dimension \( p \) has the Dirac-Born-Infeld form

\[ S_p = T_p \int d^{p+1}\xi e^{-\phi} \sqrt{-\det(g_{mn} + b_{mn} + 2\pi\alpha' F_{mn})} \]  

where \( T_p \) is the tension of the brane, \( g_{mn} \) is the induced metric on the brane, \( b_{mn} \) is the induced antisymmetric tensor field, and \( F_{mn} \) the field strength tensor of gauge fields \( A_m \) living on the brane. The total action is the sum of the bulk action (2) and the sum of all of the brane actions (3), each coupled as a delta function source (a delta function in the directions transverse to the brane) to the 10-dimensional action.

The induced metric on the brane \( g_{mn} \), with indices \( m, n, \ldots \) denoting space-time dimensions parallel to the brane, is determined by the background metric \( G_{\mu\nu} \) and by scalar fields \( \phi_i \) (not to be confused with the dilaton \( \phi \)) living on the brane (with indices \( i, j, \ldots \) denoting dimensions transverse to the brane) which describe the fluctuations of the brane in the transverse directions:

\[ g_{mn} = G_{mn} + G_{ij} \partial_m \phi_i \partial_n \phi_j + G_{in} \partial_m \phi_i. \]  

The induced antisymmetric tensor field is

\[ b_{mn} = B_{mn} + B_{ij} \partial_m \phi_i \partial_n \phi_j + B_{i[n} \partial_{m]} \phi_i. \]  

In addition,

\[ F_{mn} = \partial_{[m} A_{n]} \].

In the string frame, the fundamental string has tension

\[ T_f = (2\pi\alpha')^{-1}, \]

whereas the brane tensions for various values of \( p \) are given by

\[ T_p = \frac{\pi}{g_s} \left( 4\pi^2\alpha' \right)^{-(p+1)/2}, \]  

where \( \alpha' \sim l_{st}^2 \) is given by the string length scale \( l_{st} \) and \( g_s \) is the string coupling constant. Note that all of these branes have positive tension.

In the following, we compute the equation of state of the brane gases for a general value of \( p \). For our considerations, the most important modes are the winding modes. If
the background space is $T^9$, a $p$-brane can wrap around any set of $p$ toroidal directions. The modes corresponding to these winding modes by T-duality are the momentum modes corresponding to center of mass motion of the brane. The next most important modes for our considerations are the modes corresponding to fluctuations of the brane in transverse directions. These modes are, in the low-energy limit, described by the brane scalar fields $\phi_i$. In addition, there are bulk matter fields and brane matter fields.

Since we are mainly interested in the effects of a gas of brane winding modes and transverse fluctuations on the evolution of a spatially homogeneous universe, we will neglect the antisymmetric tensor field $B_{\mu\nu}$. We will use conformal time $\eta$ and take the background metric to be given by

$$G_{\mu\nu} = a(\eta)^2 \text{diag}(-1,1,\ldots,1),$$

where $a(\eta)$ is the cosmological scale factor.

If the transverse fluctuations of the brane are small (in the sense that the first term on the right hand side of (4) dominates) and the gauge fields on the brane are small, then the brane action can be expanded as follows:

$$S_{\text{brane}} = T_p \int d^{p+1}\zeta a(\eta)^{p+1} e^{-\phi} \times$$

$$e^{\frac{1}{2} \text{tr} \log (1 + \partial_m \phi_i \partial_n \phi_i + a(\eta)^{-2} \alpha' F_{mn})}$$

$$= T_p \int d^{p+1}\zeta a(\eta)^{p+1} e^{-\phi} \times$$

$$(1 + \frac{1}{2} (\partial_m \phi_i)^2 - \pi^2 \alpha'^2 a^{-4} F_{mn} F^{mn}).$$

The first term in the parentheses in the last line corresponds to the brane winding modes, the second term to the transverse fluctuations, and the third term to brane matter. We see that, in the low energy limit, the transverse fluctuations of the brane are described by a free scalar field action, and the longitudinal fluctuations are given by a Yang-Mills theory. The induced equation of state has pressure $p \geq 0$.

The above result extends to the case of large brane field and brane position fluctuations. It can be shown [18] that large gauge field fluctuations on the brane give rise to the same equation of state as momentum modes ($E \sim 1/R$) and are thus also described by pressure $p \geq 0$. In the high energy limit of closely packed branes, the system of transverse brane fluctuations is described by a strongly interacting scalar field theory [19] which also corresponds to pressure $p \geq 0$.

We will now consider a gas of branes and determine the equations of state corresponding to the various modes. The procedure involves taking averages of the contributions of all of the branes to the energy-momentum tensor, analogous to what is usually done in homogeneous cosmology generated by a gas of particles.

Let us first focus on the winding modes. From (10) it immediately follows that the winding modes of a $p$-brane give rise to the following equation of state:

$$\tilde{p} = w_p \rho \quad \text{with} \quad w_p = \frac{-P}{d}$$

$$\text{(11)}$$
where \( d \) is the number of spatial dimensions (9 in our case), and where \( \tilde{p} \) and \( \rho \) stand for the pressure and energy density, respectively.

Since both the fluctuations of the branes and brane matter are given by free scalar fields and gauge fields living on the brane (which can be viewed as particles in the transverse directions extended in brane directions), the corresponding equation of state is that of "ordinary" matter with
\[
\tilde{p} = w\rho \quad \text{with} \quad 0 \leq w \leq 1. \tag{12}
\]

Thus, in the absence of a scalar field sector living on the brane, the energy will not increase as the spatial dimensions expand, in contrast to the energy in the winding modes which evolves according to (as can again be seen immediately from (11))
\[
E_p(a) \sim T_p a(\eta)^p, \tag{13}
\]

where the proportionality constant depends on the number of branes. Note that the winding modes of a fundamental string have the same equation of state as that of the winding modes of a 1-brane, and the oscillatory and momentum modes of the string obey the equation of state (12).

In the context of a hot, dense initial state, the assumption that the brane fluctuations are small will eventually break down. Higher order terms in the expansion of the brane action will become important. One interesting effect of these terms is that they will lead to a decrease in the tension of the branes [14]. This will occur when the typical energy scale of the system approaches the string scale. At that point, the state of the system will be dominated by a gas of branes.

The background equations of motion are [5, 9]
\[
- d\dot{\lambda}^2 + \dot{\varphi}^2 = e^{\varphi} E \tag{14}
\]
\[
\dot{\lambda} - \dot{\varphi} \dot{\lambda} = \frac{1}{2} e^{\varphi} P \tag{15}
\]
\[
\ddot{\varphi} - d\dot{\lambda}^2 = \frac{1}{2} e^{\varphi} E, \tag{16}
\]

where \( E \) and \( P \) denote the total energy and pressure, respectively,
\[
\lambda(t) = log(a(t)), \tag{17}
\]

and \( \varphi \) is a shifted dilaton field which absorbs the space volume factor
\[
\varphi = 2\phi - d\lambda. \tag{18}
\]

In our context, the matter sources \( E \) and \( P \) obtain contributions from all components of the brane gas:
\[
E = \sum_p E^w_p + E^{nw}
\]
\[
P = \sum_p w_p E^w_p + wE^{nw}, \tag{19}
\]
where the superscripts $w$ and $nw$ stand for the winding modes and the non-winding modes, respectively. The contributions of the non-winding modes of all branes have been combined into one term. The constants $w_p$ and $w$ are given by (11) and (12). Each $E_p^w$ is the sum of the energies of all of the brane windings with fixed $p$.

4 Brane Gases in the Early Universe

The first important conclusion of [6] was that in the approach to string cosmology based on considering string gases in the early universe, the initial cosmological (Big Bang) singularity can be avoided. The question we will now address is whether this conclusion remains true in the presence of branes with $p > 1$ in the spectrum of fundamental states.

The two crucial facts leading to the conclusions of [6] were T-duality and the fact that in the micro-canonical ensemble the winding modes lead to positive specific heat, leading to limiting Hagedorn temperature. Both of these facts extend to systems with branes. First, as is obvious, each brane sector by itself preserves T-duality. Secondly, it was shown in [20] that if two or more Hagedorn systems thermally interact, and at least one of them (let us say System 1) has limiting Hagedorn temperature, then at temperatures close to the Hagedorn temperature of System 1, most energy flows into that system, and the joint system therefore also has limiting Hagedorn behavior. Hence, as the universe contracts, the T-duality fixed point $R = 1$ is reached at a temperature $T$ smaller than the Hagedorn temperature, and as the background space contracts further, the temperature starts to decrease according to (1). There is no physical singularity as $R$ approaches 0.

Let us now turn to the dynamical de-compactification mechanism of 3 spatial dimensions suggested in [6]. We assume that the universe starts out hot, small and in thermal equilibrium, with all spatial dimensions equal (near the self-dual point $R = 1$). In this case, in addition to the momentum and oscillatory modes, winding modes of all $p$-branes will be excited. By symmetry, it is reasonable to assume that all of the net winding numbers cancel, i.e. that there are an equal number of winding and anti-winding modes.

Let us assume that the universe starts expanding symmetrically in all directions. As $\lambda$ increases, the total energy in the winding modes increases according to (13), the contribution of the modes from the branes with the largest value of $p$ growing fastest. In exact thermal equilibrium energy would flow from the winding modes into non-winding modes. However, this only can occur if the rate of interactions of the winding modes is larger than the Hubble expansion rate.

Generalising the argument of [6], from a classical brane point of view it follows (by considering the probability that the world-volumes of two $p$-branes in space-time intersect) that the winding modes of $p$-branes can interact in at most $2p+1$ large spatial dimensions.

\*\*The Hagedorn temperature is not reached at finite energy density [20].

\*$^\text{Large compared to the string scale.}$
Thus, in $d = 9$ spatial dimensions, there are no obstacles to the disappearance of $p = 8$, $p = 6$, $p = 5$ and $p = 4$ winding modes, whereas the lower dimension brane winding modes will allow a hierarchy of dimensions to become large. Since for volumes large compared to the string volume the energy of the branes with the largest value of $p$ is greatest, the 2-branes will have an important effect first. They will only allow 5 spatial dimensions to become large. Within this distinguished $T^5$, the 1-brane winding modes will only allow a $T^3$ subspace to become large. Thus, it appears that the mechanism proposed in [6] will also apply if the Hilbert space of states includes fundamental branes with $p > 1$.

To what extent can these classical arguments be trusted? It was shown in [21] that the microscopic width of a string increases logarithmically as the energy with which one probes the string. In our cosmological context, we are restricted to energy densities lower than the typical string density, and thus the effective width of the strings is of string scale [21]. Similar conclusions will presumably apply to branes of higher dimensionality. However, no definite results are known since a rigorous quantization scheme for higher dimensional branes is lacking.

The cosmological scenario we have in mind now looks as follows: The universe starts out near the self-dual point as a hot, dense gas of branes, strings and particles. The universe begins to expand in all spatial directions as described by the background equations of motion (14 - 16). As space expands and cools (and the brane tension therefore increases), the branes will eventually fall out of thermal equilibrium. The branes with the largest value of $p$ will do this first. Space can only expand further if the winding modes can annihilate. This follows immediately from the background equations of motion (15). If the equation of state is dominated by winding modes (which it would be if the universe where to continue expanding), then (with the help of (11) and (13)) it follows that the right hand side of that equation acts as if it comes from a confining potential

$$V_{eff}(\lambda) = \beta_p e^{e^{e^{\lambda}}}$$

where $\beta_p$ is a positive constant which depends on the brane tension.

The unwinding of $p$-branes poses no problem for $p = 8$, $p = 6$, $p = 5$ and $p = 4$. The corresponding brane winding modes will disappear first. However, the $p = 2$ branes will then only allow 5 spatial dimensions to expand further (which 5 is determined by thermal fluctuations). In this distinguished $T^5$, the one-branes and fundamental strings will then only allow a $T^3$ subspace to expand. Thus, there will be a hierarchy of sizes of compact dimensions. In particular, there will be 2 extra spatial dimensions which are larger than the remaining ones. A careful counting of dimensions leads to the resulting manifold

$$M_{IIA}^{10} = S^1 \times T^4 \times T^2 \times T^3,$$

where the $S^1$ comes from the original compactification of $M$-theory and the hierarchy of tori are generated by the self-annihilation of $p = 2$ and $p = 1$ branes as described above.

*For a discussion of the microphysics of brane winding mode annihilation see e.g. [22] and references therein.
From equation (21) it appears that the universe may have undergone a phase during which physics was described by an effective six-dimensional theory. It is tempting to draw a relation between this theory and the scenario of [23, 24] however, since the only scale in the theory is the string scale it seems unlikely that the extra dimensions are large enough to solve the hierarchy problem. This will be studied in a future publication.

Note that even when winding mode annihilation is possible by dimension counting, causality imposes an obstruction. There will be at least 1 winding mode per Hubble volume remaining (see e.g. [25, 26]). In our four-dimensional space-time, all branes with \( p \geq 2 \) will look like domain walls. This leads to the well known domain wall problem [17] for cosmology since one wall per Hubble volume today will overclose the universe if the tension of the brane is larger than the electroweak scale. Due to their large tension at low temperatures, even one-branes will overclose the universe.

There are two ways to overcome this domain wall problem. The first is to invoke cosmic inflation [27] at some stage after the branes have fallen out of equilibrium but before they come to dominate the energy of the universe. The scenario would be as follows: initially, the winding modes dominate the energy density and determine the dynamics of space-time. Once they fall out of equilibrium, most will annihilate and the energy in the brane winding modes will become subdominant. The spatial dimensions in which unwinding occurs will expand. It is at this stage that the ordinary field degrees of freedom in the theory must lead to inflation, before the remnant winding modes (one per Hubble volume) become dominant.

In our context, however, there is another and possibly more appealing alternative - loitering [7]. If at some stage in the universe the Hubble radius becomes larger than the spatial extent of the universe, there is no causal obstruction for all winding modes to annihilate and disappear. More precisely, the “causal horizon”, meaning the distance which light can travel during the time when \( H^{-1} \) is larger than the spatial extent.

5 A Loitering Universe

Within the context of dilaton gravity, cosmological solutions which exhibit a loitering phase appear rather naturally due to the presence of winding modes. The background
equations of motion (equations (14) to (16)) simplify by letting $l = \dot{\lambda}$ and $f = \dot{\phi}$:

\begin{align}
\dot{l} &= \frac{pl^2}{2} + lf - \frac{pf^2}{2d}, \\
\dot{f} &= \frac{dl^2}{2} + \frac{f^2}{2}.
\end{align}

(22) 

(23)

Notice that for positive energy density $E$, equation (14) implies that $\dot{\phi}$ will never change sign. We are interested in studying the initial conditions with $\dot{\phi} < 0$. If $\dot{\phi} \neq 0$ the boosting effect of the dilaton on $\lambda$ will invalidate the adiabatic approximation used in the derivation of the EOM. Furthermore, growing $\phi$ together with expanding $\lambda$ implies the growth of the effective coupling $\exp \phi$ in contradiction with a weak coupling assumption [5]. We will therefore consider solutions to the background EOM with initial conditions corresponding to an expanding universe $\dot{\lambda} > 0$ with $\dot{\phi} < 0$.

For a fixed value of $p$, the phase space of solutions is two-dimensional and is spanned by $l = \dot{\lambda}$ and $f = \dot{\phi}$. If we start in the energetically allowed (positive $E$) part

\begin{equation}
|l| < \frac{1}{\sqrt{d}} |f|
\end{equation}

of the upper left quadrant of phase space with $l > 0$ and $f < 0$ corresponding to expanding solutions with $\dot{\phi} < 0$, then the solutions are driven towards $l = 0$ at a finite value of $f$ (see Fig. 1). More precisely, there are three special lines in phase space with

\begin{equation}
\frac{\dot{l}}{\dot{f}} = \frac{l}{f},
\end{equation}

(25)

which correspond to straight line trajectories through the origin. They are

\begin{equation}
\frac{l}{f} = \pm \frac{1}{\sqrt{d}} \frac{p}{d}.
\end{equation}

(26)

Solutions in the energetically allowed part of the upper left quadrant are repelled by the special line $l/f = -1/\sqrt{d}$ and approach $l = 0$. For $l \to 0$, both $\dot{l}$ and $\dot{f}$ remain finite

\begin{equation}
\dot{l} \simeq \frac{-p}{2d} f^2, \quad \dot{f} \simeq \frac{f^2}{2}.
\end{equation}

(27)

Hence, the trajectories cross the $l = 0$ axis at some time $t_1$. This means that the expansion of space stops and the universe begins to contract. The crossing time $t_1$ is the first candidate for a loitering point.

Note that the dynamics of the initial collapsing phase are very different from the time reverse of the initial expanding phase. In fact, as the special line $l/f = p/d$ is approached, $\dot{l}$ changes sign again, and the trajectory approaches the static solution $(l, f) = (0, 0)$. - also
implying a fixed value for the dilaton. At late times, the time evolution along the above mentioned special line corresponds to contraction with a Hubble rate whose absolute value is decreasing,

$$H(t) = -\frac{1}{|H^{-1}(t_0)| + \beta(t - t_0)}, \quad (28)$$

(where $t_0$ is some starting time along the special line) with

$$\beta = \frac{p}{2} + \frac{d}{2p}. \quad (29)$$

Thus, the evolution slows down and loitering is reached, even if the time evolution at $t_1$ is too rapid for loitering to occur then. Note that these considerations assume that the winding modes are not decaying. If they decay too quickly, this could obviously prevent loitering. The above provides an accurate description of the early universe, before winding modes have self-annihilated. We will examine the case of loop production and the late time evolution of the universe in Section 6. Given our cosmological starting point there is no horizon problem since space was initially of string size. However, two of the other problems of standard cosmology which the inflationary scenario successfully addresses, namely the flatness and the formation of structure problem, persist in our scenario. Note, in particular, that it seems necessary to have something like inflation of the large spatial dimensions in order to produce a universe which is larger than the known Hubble radius. It is of great interest to explore the possibility of finding a solution of these problems in the context of string theory (e.g. along the lines of the string-driven inflationary models of [28] - [31]).

### 6 Unwinding and Loop Production

We now wish to extend the analysis presented in Section 5 in order to study the late time behaviour of the universe, i.e. to include the effects of winding mode annihilation and loop production.

Recall that after the winding modes have annihilated, a three-dimensional subspace will grow large. In what follows we will therefore take $d = 3$. The strings in the theory are the last branes to unwind which implies that at late times we should consider the case of $p = 1$. When the winding strings self-annihilate they create loops in the $(3 + 1)$-dimensional universe.

We now set up the equations describing the unwinding and corresponding loop production. They are analogous to the corresponding equations for cosmic strings in an expanding universe. First, note that the energy density $\rho_w$ in winding strings can be expressed in terms of the string tension $\mu$ and the number $\tilde{\nu}(t)$ of winding modes per “Hubble” volume $t^3$ as

$$\rho_w(t) = \mu \tilde{\nu}(t) t^{-2}. \quad (30)$$
Figure 1: Phase space trajectories of the solutions of the background equations (14 - 16) for the values $p = 2$ and $d = 9$. The energetically allowed region lies near the $l = 0$ axis between the special lines $a$ and $c$, which are the lines given by $l/f = \pm 1/\sqrt{d}$. The trajectory followed in the scenario investigated in this paper starts out in the upper left quadrant close to the special line $c$ (corresponding to an expanding background), crosses the $l = 0$ axis at some finite value of $f$ (at this point entering a contracting phase), and then approaches the loitering point $(l, f) = (0, 0)$ along the phase space line $b$ which corresponds to $l/f = p/d$. 
Since loops are produced by the intersection of two winding strings, the rate of loop production is proportional to $\tilde{\nu}^2$:

$$\frac{dn(t)}{dt} = c\tilde{\nu}(t)^2 t^{-4},$$  \hspace{1cm} (31)

where $n(t)$ is the number density of loops and $c$ is a proportionality constant expected to be of the order 1. The energy density in the winding modes decreases both due to the expansion of space and due to the decay into loops:

$$\frac{d\rho_w(t)}{dt} + 2l \rho_w(t) = -c' \mu t \frac{dn(t)}{dt} = -cc' \mu \tilde{\nu}(t)^2 t^{-3},$$  \hspace{1cm} (32)

where $c'$ is a constant which relates the mean radius $R = c' t$ of a string loop to its length $\ell$. Without loop production ($c = 0$), the energy density $\rho_w$ redshifts corresponding to the equation of state $p = -\frac{1}{3} \rho$. This explains the coefficient of the Hubble damping term in (32).

Inserting equation (31) into the energy conservation equation (32), we obtain an equation for $\tilde{\nu}(t)$:

$$\frac{d\tilde{\nu}(t)}{dt} = 2\tilde{\nu}(t^{-1} - 1) - cc' t^{-1} \tilde{\nu}^2.$$  \hspace{1cm} (33)

In addition to $\rho_w(t)$, we will also require information about the energy density in loops, $\rho_l(t)$. The energy density in loops obeys the conservation equation

$$\frac{d\rho_l(t)}{dt} + 3l \rho_l(t) = cc' \mu \tilde{\nu}(t)^2 t^{-3}.$$  \hspace{1cm} (34)

As in equation (32), the second term on the left hand side of the equation represents the decrease in the density due to Hubble expansion, with the coefficient reflecting the equation of state $p = 0$ of a gas of static loops, and the term on the right hand side representing the energy transfer from winding modes to loops. Without loop production, $\rho_l(t)$ would scale as

$$\rho_l(t) = g(t) e^{-3(\lambda(t) - \lambda_0)},$$  \hspace{1cm} (35)

with $g(t)$ constant. Here $\lambda_0 = \lambda(t_0)$, where $t_0$ is some initial time, and $g(t)$ is a function which obeys the equation

$$\frac{dg(t)}{dt} = cc' \mu t^{-3} \tilde{\nu}^2 e^{3(\lambda(t) - \lambda_0)}.$$  \hspace{1cm} (36)

Using the expressions for $\rho_w(t)$ and $\rho_l(t)$ as sources for the energy density $E$ and pressure $P$ in equations (14) to (16) we can obtain background equations analogous to equations (22) and (23):

$$\dot{l} = lf + \frac{1}{2} l^2 - \frac{1}{6} f^2 + \frac{1}{6} g e^{\varphi + 3\lambda_0},$$ \hspace{1cm} (37)

$$\dot{f} = \frac{1}{2} f^2 + \frac{3}{2} l^2.$$ \hspace{1cm} (38)

**For more on strings in an expanding universe see, e.g. [32].**
Figure 2: A solution of the background equations (37) and (38) including the effects of loop production. This depicts a typical solution which starts in the energetically allowed region of the phase space. The solution crosses the $l = 0$ axis at some finite value of $f$ (at which point the universe enters a contracting phase), and then crosses the $l = 0$ line a second time when the winding modes have fully annihilated. At this point the universe begins to expand, is matter dominated and the dilaton is assumed to become massive.

(Recall that $p = 1$ and $d = 3$ in the above equations.)

Equations (33), (36), (37) and (38), along with the equations $l = \dot{\lambda}$ and $f = \dot{\varphi}$ provide six, first-order, differential equations which fully describe the universe during the process of loop production. Note that the $c = 0$ case corresponds to no loop production and the background equations reduce to the previous equations (22) and (23). These provide us with the initial conditions required for a numerical analysis.

Figure 2 demonstrates the behaviour of a typical numerical solution to the EOM having initial conditions in the energetically allowed region of the phase space and accounting for the effects of loop production. Recall that when no loops are produced (see Fig. 1) the solutions of interest cross over the $l = 0$ line only once and approach the origin of the phase space as $t \to \infty$. When the decay of the winding modes is taken into account the solutions are pushed back over the $l = 0$ line as in Fig. 2.

In more detail, the dynamics of our loitering solution are depicted in Figures 3, 4, 5 and 6. Figure 3 shows the time evolution of the Hubble expansion rate $H = \dot{l}$. Note that since we have set Newton’s constant $G = 1$ in our background equations, time is measured in Planck time units.

By comparing the value of $a(t)$ from Fig. 4, we see that

$$H^{-1}(t) \gg a(t)$$

during the loitering phase. Keeping in mind that the initial spatial size of the tori is Planck scale, it follows immediately from equation (39) (and from the time duration of the loitering phase) that loitering lasts sufficiently long to allow causal communication over the entire spatial section. This is reflected in Fig. 5 which shows that the winding
Figure 3: The time evolution of $H = l$. The loitering phase begins when $l(t)$ crosses the $l = 0$ line for the first time and ends when $l(t)$ crosses back over the $l = 0$ line.

Figure 4: The time evolution of the scale factor $a$. By comparing this plot with Fig. (8) we see that the loitering phase lasts long enough to allow all winding modes to self-annihilate in the large three-dimensional universe.
Figure 5: Time evolution of $\tilde{\nu}$. Initially, $\tilde{\nu}$ increases as the universe contracts. The winding modes begin to self-annihilate ($\tilde{\nu}$ decreases) and eventually vanish ($\tilde{\nu} \to 0$).

Figure 6: Time evolution of $g$. Note that $g$ goes to a constant when $\tilde{\nu}$ goes to zero.

modes completely annihilate by the end of the loitering phase, after which $g(t)$ tends to a constant (Fig. 6).

Our modified picture is as follows: the universe begins to expand until the winding modes become too massive and force the expansion to stop and contraction to begin. This corresponds to the solution in Fig. 1 crossing over the $l = 0$ axis and signals the beginning of the loitering phase. The loitering phase ends when the solution is pushed back over the $l = 0$ line due to the decay of the winding modes. From this point on the universe begins to expand again.

Soon after the solutions cross the $l = 0$ line for the second time, our equations become singular. At the exact moment that the winding modes vanish ($\tilde{\nu}(t) \to 0$), all the fields $l, f, \lambda$ and $\varphi$ in our equations of motion blow up. This singularity does not concern us however since it can be eliminated by the introduction of a simple potential $V(\phi)$ used to freeze the dilaton at the moment loop production is exhausted. The massless dilaton does not appear in nature and therefore such a potential is required in any string-theory-motivated cosmological model at late times. The precise mechanism responsible for dilaton mass generation is unknown, although it is often suspected that this mechanism
will coincide with the breaking of supersymmetry. From this analysis we are lead to the
conjecture that dilaton mass generation may coincide with the elimination of winding
modes. We will comment more on this below.

For the time being, let us assume that the dilaton has frozen at the value $C_{\phi} = \phi(t_{\text{freeze}})$ and therefore $\dot{\phi} = \ddot{\phi} = 0$. We will also assume that this occurs when the winding modes have vanished, $\tilde{\nu}(t) \to 0$ and hence the number of loops has reached a constant so that $g(t) \to C_g$. Now the EOM simplify greatly. By fixing the dilaton in
equations (37) and (38) we can derive an equation for the scale factor (after shifting back
to the true dilaton $\phi$):

$$\dot{a} - C_{\gamma} a^{-\frac{1}{2}} = 0,$$

where $C_{\gamma}$ is a constant given by

$$C_{\gamma} = \sqrt{\frac{C_2}{12}} e^{C_\phi + \frac{1}{2} \lambda_0}.$$  

The most general solution to equation (40) is

$$a(t) = \left( \frac{3C_{\gamma}}{2} \right)^{\frac{2}{3}} (t^2 - 2Ct + C^2)^{\frac{1}{4}},$$

where $C$ is an integration constant. For the value $C = 0$ or for large values of $t$ (late
times), the scale factor grows as

$$a(t) \sim t^{\frac{4}{3}},$$

which is exactly the correct behaviour for a matter dominated universe.

Our interpretation is that the winding modes look like solitons in the (3+1)-dimensional
universe. The self-annihilation of these winding modes corresponds to the creation of mat-
ter in the universe and the scale factor evolves appropriately. In our equations, the loops
are modelled as static. In reality, the loops will oscillate and decay by emitting (mostly)
gravitational radiation, thus producing a radiation dominated universe.

Let us return to the issue of SUSY breaking and dilaton mass generation. One thing
which appears inevitable within the context of this model is the spontaneous “breaking” of
T-duality in the large four-dimensional universe. This is most easily understood once all of
the winding modes have self-annihilated since it is impossible to create new ones. It would
cost too much energy for a brane to wrap around the large dimensions. Thus, the state of
the system is not symmetric under T-duality, and in the absence of string winding modes
and for fixed dilaton, our background equations reduce to those of Einstein’s General
Relativity which does not exhibit the $R \leftrightarrow 1/R$ symmetry of string theory.

It is also interesting to note that there seems to be a relation between the amount of
supersymmetry in a theory and the presence of T-duality. Using a specific example in [33],
Aspinwall and Plesser show that T-duality can be broken by nonperturbative effects in
string coupling. Furthermore, a holonomy argument is given to show that T-dualities should only be expected when large amounts of supersymmetry are present. It seems likely that the dynamics in the BGC scenario will cause SUSY to break. This result may be in agreement with the possibility of dilaton mediated SUSY breaking occurring simultaneously with the breaking of T-duality.

Considering the above evidence we are inclined to hypothesize about the possible relations between supersymmetry breaking and the breaking of T-duality, as well as dilaton mass generation and the vanishing of winding modes.

7 Conclusions and Speculations

In this talk we have introduced the model of Brane Gas Cosmology presented in [2]. The simplest starting point of BGC is $M$-theory compactified on $S^1$ which gives ten-dimensional, Type IIA string theory. Brane gases constructed from other branches of the $M$-theory moduli space are considered in [4]. In the toy model presented here, we assume toroidal topology in all nine, spatial dimensions. The initial conditions for the universe include a small, hot, dense gas of the $p$-branes in the theory. These fundamental degrees of freedom are assumed to be in thermal equilibrium. T-duality ensures that the initial singularity of the SBB model is not present in this scenario.

We compute the equation of state for the brane gas system and the background equations of motion. We study the solutions which initiate in the energetically allowed region of the phase space. The universe expands until winding modes force the expansion to stop and a phase of slow contraction (loitering) to begin. Loitering provides a solution to the brane problem introduced in [2]. It also provides solutions to the horizon problem of the SBB model without relying on an inflationary phase.

The counting argument of [2] demonstrates that winding modes will allow a hierarchy of dimensions in the $T^9$ to grow large. When the string winding modes self-annihilate we are left with a large $T^3$ subspace, simultaneously explaining the origin of our $(3 + 1)$-dimensional universe and solving the dimensionality problem of string theory.

Branes wrapped around the cycles of the torus appear as solitonic objects in the early universe. They are topological defects (domain walls for $p \geq 2$). When the winding modes and anti-winding modes self-annihilate, matter is produced and the universe begins to expand again. We hypothesize that winding mode annihilation may correspond to dilaton mass generation. We also believe there may be a relation between SUSY breaking and the breaking of T-duality, although we cannot provide any direct evidence for this. Once winding states have vanished, we cannot map momentum modes into winding modes via T-duality. The effective “breaking” of T-duality in the large universe requires further study and may be of interest to finite temperature string theorists.

Particle phenomenology demands compactification on manifolds with non-trivial holonomy (e.g. Calabi-Yau three-folds) if the four-dimensional low energy effective theory is
to have $\mathcal{N} = 1$ supersymmetry. Although we have only examined the trivial toroidal compactification here, more realistic compactifications are considered in [4].

Brane Gas Cosmology provides an alternative method of incorporating string and $M$-theory into cosmology to popular “brane world” scenarios. In our opinion, BGC has the advantage over such models in that its foundations are analogous to those of the Standard Big-Bang model. In the BGC model the universe starts out small, hot and dense, with no initial singularity.

All the current versions of brane world scenarios embedded in string theories rely on the compactification of the extra dimensions by hand. In our opinion this is a considerable problem which is often overlooked. A dynamical mechanism in BGC leads naturally to four large space-time dimensions.
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References

[1] D. A. Easson, “The interface of cosmology with string and M(illennium) theory,” to appear in Int. J. Mod. Phys. A (2001) [hep-th/0003086].

[2] S. Alexander, R. H. Brandenberger and D. Easson, “Brane gases in the early universe,” Phys. Rev. D 62, 103509 (2000) [arXiv:hep-th/0005212].

[3] R. Brandenberger, D. A. Easson and D. Kimberly, “Loitering phase in brane gas cosmology,” arXiv:hep-th/0109163.

[4] D. A. Easson, “Brane gases on K3 and Calabi-Yau manifolds,” arXiv:hep-th/0110225.

[5] A. A. Tseytlin and C. Vafa, “Elements of string cosmology,” Nucl. Phys. B 372, 443 (1992) [hep-th/9109048].

[6] R. Brandenberger and C. Vafa, “Superstrings In The Early Universe,” Nucl. Phys. B 316, 391 (1989).

[7] V. Sahni, H. Feldman and A. Stebbins, “Loitering universe,” PRINT-91-0070 (CITA,TORONTO).

[8] H. A. Feldman and A. E. Evrard, “Structure in a loitering universe,” Int. J. Mod. Phys. D 2, 113 (1993) [astro-ph/9212002].

[9] G. Veneziano, “Scale factor duality for classical and quantum strings,” Phys. Lett. B 265, 287 (1991).

[10] M. Sakellariadou, “Numerical Experiments in String Cosmology,” Nucl. Phys. B 468, 319 (1996) [hep-th/9511075].

[11] J. Polchinski, “TASI lectures on D-branes,” hep-th/9611050.

[12] E. Witten, “Bound States Of Strings And p-Branes,” Nucl. Phys. B 460, 335 (1996) [hep-th/9510135].
[13] J. Polchinski, “String theory. Vol. 2: Superstring theory and beyond,” *Cambridge, UK: Univ. Pr. (1998)* 531 p.

[14] M. Maggiore and A. Riotto, “D-branes and cosmology,” Nucl. Phys. B **548**, 427 (1999) [hep-th/9811089].

[15] A. Riotto, “D-branes, string cosmology and large extra dimensions,” Phys. Rev. D **61**, 123506 (2000) [hep-ph/9904485].

[16] C. Park, S. Sin and S. Lee, “The cosmology with the Dp-brane gas,” Phys. Rev. D **61**, 083514 (2000) [hep-th/9911117].

[17] Y. B. Zeldovich, I. Y. Kobzarev and L. B. Okun, “Cosmological Consequences Of The Spontaneous Breakdown Of Discrete Symmetry,” Zh. Eksp. Teor. Fiz. **67**, 3 (1974) [Sov. Phys. JETP **40**, 1 (1974)].

[18] A. Hashimoto and W. I. Taylor, “Fluctuation spectra of tilted and intersecting D-branes from the Born-Infeld action,” Nucl. Phys. B **503**, 193 (1997) [hep-th/9703217].

[19] P. K. Townsend, “Brane theory solitons,” [hep-th/0004039].

[20] S. A. Abel, J. L. Barbon, I. I. Kogan and E. Rabinovici, “String thermodynamics in D-brane backgrounds,” JHEP **9904**, 015 (1999) [hep-th/9902058].

[21] M. Karliner, I. Klebanov and L. Susskind, “Size And Shape Of Strings,” Int. J. Mod. Phys. A **3**, 1981 (1988).

[22] A. Sen, “Non-BPS states and branes in string theory,” [hep-th/9904207].

[23] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, “The hierarchy problem and new dimensions at a millimeter,” Phys. Lett. B **429**, 263 (1998) [hep-ph/9803315].

[24] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, “New dimensions at a millimeter to a Fermi and superstrings at a TeV,” Phys. Lett. B **436**, 257 (1998) [hep-ph/9804398].

[25] T. W. Kibble, “Topology Of Cosmic Domains And Strings,” J. Phys. AA **9**, 1387 (1976).

[26] R. H. Brandenberger, “Topological defects and structure formation,” Int. J. Mod. Phys. A **9**, 2117 (1994) [astro-ph/9310041].

[27] A. H. Guth, “The Inflationary Universe: A Possible Solution To The Horizon And Flatness Problems,” Phys. Rev. D **23**, 347 (1981).
[28] C. P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh and R. J. Zhang, “The inflationary brane-antibrane universe,” JHEP 0107, 047 (2001) [arXiv:hep-th/0105204].

[29] S. H. Alexander, “Inflation from D - anti-D brane annihilation,” arXiv:hep-th/0105032.

[30] S. A. Abel, K. Freese and I. I. Kogan, “Hagedorn inflation of D-branes,” JHEP 0101, 039 (2001) [arXiv:hep-th/0005028].

[31] N. Turok, “String Driven Inflation,” Phys. Rev. Lett. 60, 549 (1988).

[32] A. Vilenkin and E. P. S. Shellard, Cosmic Strings and other Topological Defects, Cambridge University Press, 1994.

[33] P. S. Aspinwall and M. R. Plesser, “T-duality can fail,” JHEP 9908, 001 (1999) [hep-th/9905036].