Reducing manufacturing costs: the dynamic tolerance system

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Abstract: Can tolerance limits in production be extended without violating the specifications of design and development? Adherence to tolerances is crucial for the technical quality of a manufactured good. Fit functions of modules often involve highly precise tolerances, incurring high costs. To avoid or at least reduce these costs, this paper develops a new approach to tolerances, the Dynamic Tolerance System. This approach allows the manufacturer to expand the tolerance limits of single parts of an assembly group and enables the production of assembly groups that are subject to specific fit functions. Thereby, the Dynamic Tolerance System does not intervene in the design process, does not require expensive machines or error-prone control-loops, and does not use complex assembly procedures. Rather, it relies on modern measurement systems and the infrastructure already developed and deployed within the framework of Industry 4.0 to extend tolerance limits in production without violating the designer’s specifications.

Subjects: Production Systems; Manufacturing & Processing; Intelligent Systems

Keywords: Tolerances; industry 4.0; high precision; cyber physical system; smart production

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PUBLIC INTEREST STATEMENT
The present paper develops a new approach to manufacturing components at high levels of precision, i.e., within tight tolerance limits. Such tight tolerances guarantee the functionality of products. However, they are often costly to manufacture as they, e.g., require additional machining steps or the use of highly precise machines. The Dynamic Tolerance System (DTS) approach developed here addresses this issue. It aims at manufacturing closely tolerated components of an assembly group as cost-effectively as possible. In essence, the components produced are automatically adapted to each other during manufacturing. This allows individual components’ tolerance zones to be increased significantly, which reduces costs. At the same time, the functionality of the assembly and thus its quality are maintained. Another advantage of the DTS is that it does not negatively influence designers. On the contrary, it unburdens them by enabling them to factor in highly precise components despite resource constraints.
1. Introduction
Quality ensures that “the customer comes back and not the product”. To manufacture high-quality products with great reliability, a high degree of geometric accuracy is required. Tight tolerances guarantee this accuracy. However, they incur high manufacturing costs, which may diminish companies’ competitiveness. Also, tight tolerances are difficult to achieve due to various sources of error influencing the tools used, such as geometric and thermal errors (see, e.g. (Guo, Liu, Li, & Hong, 2015) (Liu, Li, Wang, & Tan, 2011)).

All contributions to date that attempt to overcome this challenge of narrow tolerances and high production costs optimize tools, machines, or process parameters via the Design of Experiments (DoE) method (Siebertz, van Bebber, & Hochkirchen, 2017). While this approach is very successful, it cannot factor numerous external influences into the quality models or optimization tools, such as unforeseen vibrations, temperature fluctuations, tool wear, and irregularities in the components within every batch.

Considering these shortcomings, this paper will outline a new approach to dealing with precise tolerances in order to improve common multistage production lines, the “Dynamic Tolerance System” (DTS). The DTS uses optical measuring systems and smart algorithms in practice. In this way, it is able to reduce the need for production process optimization. Additionally, it reduces conformity costs by extending the tolerance limits of single parts without violating the specifications of design and development. Therefore, this new approach will lower production costs, reduce the rejection rate and ensure the reliability as well as the functionality of the product throughout its entire life cycle.

In short, the advantages of the DTS are:

- No additional effort (tolerance management) by the designer is required.
- The designer can use very tight tolerances without having to consider the production costs.
- There is no risk that the fit function will not work, as the DTS focuses on the fit function(s). The reliability of the product is ensured.
- By enlarging the tolerance limits of the counterparts, production costs can be reduced significantly.
- The rejection rate can be reduced and the quality of production increased.

The paper will proceed as follows: in the first part, it will give an introduction to the concept of tolerances and their use. Thereafter, it will discuss the costs and difficulties associated with achieving highly precise tolerances. The third part will critically reflect upon the state of the art of handling precise tolerances. Subsequently, the requirements for this new approach will be explained. A fifth part outlines the concept of the new approach. Lastly, a conclusion will be drawn concerning the strength and limits of this approach.

2. Tolerances
DIN ISO 286–1:2010 (DIN ISO 286-1, 2010–02)) describes the basis of tolerances, deviations and fits in great detail. It represents “the internationally accepted code system for tolerances on linear size”. DIN ISO 286–1:2010 specifies geometrical products (geometrical product specification, GPS). According to this standard, the need for limits and fits for machined components was brought about mainly by the requirement to be able to interchange mass-produced parts and by the inherent inaccuracy of manufacturing methods. This was coupled with the fact that “exactness” of size was found to be unnecessary for most features of a particular component to function. Rather, it is sufficient to manufacture a given component so that its size lies within two permissible limits, i.e., a tolerance. The tolerance thus defines the variation in size acceptable in manufacturing while ensuring the interchangeability of the individual components and the functional fit requirement, the so-called “fit function”, which guarantees functionality between two parts.
As well as providing this definition, DIN ISO 286–1:2010 also establishes a certain terminology summarized in the following table and illustrated by Figure 1:

**Terminology (DIN ISO 286–1, 2010–02):**

- **Nominal size:** size of a feature of perfect form as defined by the drawing specification
- **Actual size:** size of the associated integral feature
- **Limits of size:** extreme permissible sizes of a feature of size
- **Upper limit of size:** largest permissible size of a feature of size (ULS)
- **Lower limit of size:** smallest permissible size of a feature of size (LLS)
- **Limit deviation:** upper limit deviation or lower limit deviation from nominal size

**Standard tolerance IT:** any tolerance belonging to the ISO code system for tolerances on linear sizes

3. Costs and difficulties associated with manufacturing tight tolerances

There is an increasing demand for more accurate production processes conforming to a growing number of boundary conditions. This need for increased production precision and therefore for smaller dimensional deviations incurs a significant rise in production costs (Yeo, Ngoi, & Chen, 1996) (Abdel-Malek & Asadathorn, 1994) (Muthu, Dhanalakshmi, & Sankaranarayanasamy, 2009) (Hsieh, 2006). In Figure 2, Milberg illustrates an estimation of the costs of production versus ISO-grades.

In his estimate, Milberg assumes that the accuracy of a component does not have a linear, but rather an exponential effect on the production costs (see also Yeo et al., 1996) (Sfantsikopoulos, 1990) (Hsieh, 2006) (Bohn, 1998) (Muthu et al., 2009); 13). The production costs thus double with each ISO-grade. Additional costs arise as the machines (for example, high precision linear axes or clamping devices) need to become more accurate over a long time period. Also, tight tolerances are difficult to achieve because environmental effects such as temperature or humidity can have negative effects on the precision of production.Compensating such negative effects can be very cost-intensive and error-prone. Additionally, in order to ensure manufacturing precision, frequent maintenance intervals are required. However, these increase production costs (because production must cease during maintenance, and the costs of spare parts, as well as the costs of the maintenance operative must be met). To prolong intervals between maintenance, higher quality components can be used, but these also incur higher costs.

However, tight tolerances are sometimes unavoidable, especially when it comes to tolerance chains. Tolerance chains occur when modules are assembled from parts that have to fit together. The overall tolerance of a tolerance chain can be calculated by summing all individual tolerances. This means that if a module requires a specific overall tolerance, the tolerances of each individual
component have to be added up to equal this overall tolerance. Thus, as shown in Figure 3, the required dimensional precision of each part increases with an increasing number of parts in the tolerance chain. This is important, given that assemblies and components are becoming increasingly complex across industries. The accuracy requirements are therefore continuously increasing; (Dietrich & Schulze, 2017) (Liebl, 2018).

4. State of the art of handling precise tolerances
The problem of ever tighter tolerances has been clear for a long time. In the past decades, many approaches have therefore been developed in order to manufacture the required increasingly precise components at low production costs. In the following, the existing approaches are divided into three categories:

(1) Design-based approaches
(2) Machine-based approaches
(3) Assembly-based approaches
4.1. Design-based approaches

To date, multiple models have attempted to deal with the problem of tight tolerances. Tolerance Stack Analysis (calculating the effect of the accumulated variation that is allowed in specific tolerances) with worst-case assumptions and Root Sum Squared (RSS) are common methods to study the variability of the range of the tolerances (see, e.g. (Chase & Parkinson, 1991) (Scholz, 1995); 18).

RSS is a statistical method which assumes a normal distribution of tolerances across all components. It describes the variation of dimensions with the help of two parameters: μ, which is the mean, and σ, which is the standard deviation. The standard deviation of the combined parts (σ_{tot}) is fully described with:

$$\sigma_{tot} = \sqrt{\sum_{i=0}^{n} \sigma_i^2}$$

where n is the number of parts in a combination and σ_{i} is the standard deviation of part i.

Those basic approaches have been extended by more modern methods. In 1991, Richard W. Johnson introduced a method based on RSS and worst-case analysis in a patent. It specifies an overall tolerance and assigns variable tolerances to each component and its features. The method requires three so-called “tolerance points” for each tolerance to be defined: tolerance point one is the point where the costs cannot be significantly reduced by decreasing the precision. The second tolerance point marks the instance where the costs start to increase significantly when tolerances are reduced, and the third tolerance point marks the point where a further increase of precision is not obtainable (Johnson, n.d.).

Other design-based methods originated from comparisons of 3D-CAD (Computer Aided Design) models with measured 3D-Part models. They are currently used to model complex datasets. These tools include Generalized Additive Models (GAM) that replace the linear regression form of the covariates with a sum of smooth functions (Hastie, & Tibshirani, 1986). GAM can be applied to any likelihood regression model and have proven useful for uncovering non-linear effects in the covariates. Another tool is the non-recursive Multivariate Adaptive Regression Splines (MARS; 21). It is a flexible approach to a regression analysis and, as opposed to recursive partitioning, the method produces continuous models with continuous derivates. Lastly, the Minimax Probability Machine Regression (MPMR; 22) allows a direct estimation of a lower probability bound. Zhang and Xie (Zhang & Xie, 2014) developed another design-based approach. This optimizes assembly tolerances by determining several cost functions for machining, obtaining a total cost function, and spreading the tolerances according to it.

A computer-based approach by Tornquist and Jarvinen (Tornquist & Jarvinen, 2009) uses a database with a hierarchic structure for all parts to assign and optimize the tolerances. Sato and Masayoshi’s (Yuichi Sato & Masayoshi Hashami, 2003) Monte-Carlo-population with all dimensional and assembly information about the part in the assembly group. It conducts a backwards analysis of the acceptable tolerances and generates an individual assembly. The method works with a recursive if-iteration.

A fruitful method to tackle high dimensional variation in the production process might also be the tolerance method developed by Hernández and Tutsch (Hernández & Tutsch, 2013), which is based on the observation of a subset of units. A more practical concept is the response surface function. It can be used to generate deviation models of the individual component vertices. The result is used to predict deviations for specific cases so that they can be manufactured with optimized tool paths, which leads to minimized inaccuracy (Behera et al., 2015).
An approach to avoid trial and error runs in production planning is to use the “model of manufactured parts”, which verifies the ability to machine a series of parts with a given process setup within a certain manufacturing tolerance. Following Bui et al. (Bui, Villeneuve, & Sergent, 2013), the method can be used to confirm process plans in relation to functional tolerances or to determine achievable functional tolerances by means of experimental data as input variables.

A new approach to assign tolerances in the design process is an algorithm developed by Ransing et al. (Ransing, Batbooti, Giannetti, & Ransing, 2016). It determines and addresses risks as well as opportunities of tolerance models by detecting not only the percentage of defects per batch but by penalizing undesired and unexplained quality variations among batches.

Xu and Keyser (Xu & Keyser, 2016) use high dimension ellipsoids to represent tolerance zones. This leads to a tolerance space that is not limited by the number of degrees of freedom. They also provide geometrical and statistical explanations as well as the analytical structures for automatic tolerance allocation.

Another interesting idea is to base tolerance selection on mechanical requirements. This means that the consideration of process and product properties as well as features creates tolerances based on the scheme that the inventors developed (Paul, Drake, Richardson, Dale, & van Wyk, 2004).

4.2. Machine-based approaches
In addition to these design-based approaches, manufacturing processes can be optimized in order to produce parts with precise geometric tolerances at low cost. For this, constructive, maintenance-technical or control-technical measures can be taken.

Response-surface approaches are often used to optimize production processes. The first such approach was developed in 1951 by Box et al. (Box & Wilson, 1951). To date, all types of manufacturing systems are optimized by technical control systems. An example is demonstrated by Fein et al. (Fein et al., 2016).

Special machine components can also be used to optimize process stability and the precision of the manufacturing processes. Some examples, like special tool coatings or special frame materials, of this can be found in the literature (Kalveram, 2005) (Bach, MÖhwald, Laarmann, & Wenz, 2006) (Schneider, Belashchenko, Dratwinski, Siegmann, & Zagorski, 2006) (Nebeling, 2015) (Fu et al., 2016).

The optimization of machine parameters is also essential for the optimization of manufacturing processes in order to produce tighter tolerances. Statistical design of experiments (DoE) is often used for this purpose (Siebertz et al., 2017) (Selvarajan, Sathiya Narayanan, Jayapaul, & Manohar, 2016).

4.3. Assembly-based approaches
In order to produce assemblies with tightly tolerated components, there are also some methods used within the assembly process. Inter alia, the group exchange method sorts the manufactured components into dimensional classes. Components within these dimensional classes are then matched to each other in a next step (Lotter & Wiendahl, 2013) (Szyminski, 2013).

Another assembly-based approach entails providing a certain component of an assembly with a so-called fit addition. This fit addition or allowance can, for example, take the form of a washer. However, in order to calculate the needed size of the fit addition, the components of the assembly must first be measured (Spur, Feldmann, & Schöppner, 2013) (Hoenow & Meißner, 2016). Wing-boxes constitute a practical example for such an application of a fit addition. Their rib feet must come into contact with the skin of the wings. If they do not, this can incur a significant loss of structural strength. This problem is currently tackled by applying a liquid or solid shim (which means having to purchase and assemble extra parts, which results in higher production costs). To avoid this, Chouvion et al., (Chouvion, Popov, Ratchev, Mason, & Summers, 2011) proposed the use
of various automation techniques for a shimless assembly. The method reduces the tolerances of
the manufacturing process and therefore facilitates seamless manufacturing, but requires a large
amount of data and therefore sensors to collect these data. Muelaner et al. (Muelaner, Martin, &
Maropoulos, 2013) propose an architecture for such a manufacturing process using integrated
digital metrology systems to improve cost efficiency.

Manohar et al. (Manohar et al., 2018) show that the sensor data acquired by automated systems
can be used to predict 99% of the shim gaps using machine learning with historical data. The
resulting algorithm uses only 3% of the laser scan points usually required for this task, and thus
potentially accelerates the manufacturing process and makes it more cost-effective.

Another method is the so-called setting method. The last component of an assembly is designed
in such a way that it is adjustable or self-adjusting (Lotter & Wiendahl, 2013) (Hoenow & Meißner,
2016). Examples include the setting by a thread, the number of cup springs in a cup spring package
or the total resistance of an electrical circuit using a potentiometer.

To ensure interchangeability, the total permissible tolerance must not be exceeded—even when
components are manufactured at their respective tolerance limits. However, it is very unlikely that
only components manufactured at tolerance limits will be paired (worst-case). Statistical tolerance
is based on the assumption that a so-called worst-case pairing is virtually non-existent in reality
(Jorden & Schütte, 2014) (Haberhauer, 2011).

4.4. Conclusion
The present section has outlined design-, machine- and assembly-based approaches to enable the
increasingly precise manufacturing of components at a low cost. In the following, the three
categories of solutions will now be evaluated based on the following criteria: the interchangeability
of components, potential cost savings, risks associated with the approach, and the required design
effort. Each criterion will be rated from 1 to 5, where 1 denotes poor fulfilment and 5 denotes
excellent fulfilment of the criterion.

Components manufactured using design-based approaches are fully interchangeable
(Interchangeability = 5). However, these methods can only be applied by the designer, which sig-
nificantly increases the design effort and thus also the costs (Design effort = 1, Cost Saving = 3). Since
the methods are often based on very complex algorithms, an error-free application cannot be
assumed (Risk = 3).

Machine-based approaches guarantee 100% interchangeability, as the manufacturing processes
are extremely precise and the components can be manufactured within the desired tolerance
limits (Interchangeability = 5). This means that there is no additional work for the designer (Design
effort = 5). However, the complex optimization algorithms are partially error-prone, which is why
a low residual risk remains (Risk = 4). Thus, machine-based approaches are often very costly, and
the purchase of such precise manufacturing technologies often does not pay off economically
(Cost saving = 1).

In order to implement most assembly-based approaches, both the measurement technology
and the logistics and infrastructure have to be expanded. In addition, further steps in the assembly
process are required. This is reflected in low-cost savings (Cost saving = 2). Some assembly-based
approaches also require that assemblies must be adjustable. This incurs additional effort for the
construction (Design effort = 3). In addition, the assembly procedures cannot always guarantee
the interchangeability of components (Interchangeability = 2). Due to the many manual interven-
tions (and thus the risk of random errors), a loss of the fit functions is also likely (Risk = 4).

The following table summarizes this evaluation of the approaches based on the four criteria.
As stated in the introduction, the DTS is neither a machine-, nor an assembly- or design-based approach. In order to be able to compare it with the existing approaches, the following two sections will first introduce and outline this new approach. In the conclusion, the DTS will then be compared with the existing approaches based on the four evaluation criteria applied above. It will be shown that implementing the DTS allows companies to manufacture parts of assembly groups with tight tolerances at very low costs. Thereby, it does not entail any extra risks or require additional effort by the designer. However, the interchangeability of parts is limited.

5. A new approach to handling precise tolerances

This section explains the “Dynamic Tolerance System” (DTS) approach, a new method to produce assembly groups with high tolerances. This approach uses the advantages of cyber-physical systems as well as current improvements of optical measurement systems. In this way, it can reduce the production costs of components with tight tolerances without intervening in the design process. However, it is important to mention that this approach can only be implemented for modules in which no individual components are exchanged.

Figure 4 illustrates the main concept of this idea:

In an initial step, “Production Part I”, the first characteristics of the fit function are produced, whereby they must meet the defined tolerances. In the next step, “Measurement Part I”, the characteristics of the produced part are measured. Based on these measurements, a micro-computer, using smart algorithms, calculates the new tolerance limit as well as the new nominal size for the assembly group’s counterpart. This information is then transferred to the next process step, which produces this second part. Standard tolerances, which are specified in the design process, are thereby complied with but the costs of the production process can potentially be more than halved (see below). It can also be expected that, using this method, the functionality and the reliability of the fit function will increase.

To illustrate the model, an example of a calculation will now be given, using the case of a simple shaft-hub connection.

Example of a simple shaft-hub-connection:

Typical-standardized tolerances are often used in shaft-hub-connections. Usually, designers use the DIN ISO 286–1:2010—ISO code system for tolerances of linear sizes for shaft-hub-connections. This standard defines three main fits (DIN EN ISO 286–1, 1990–11):

(1) Clearance fit—provides a clearance between the hole and the shaft
Interference fit——provides an interference between the hole and the shaft

Transition fit——provides either a clearance or interference between the hole and the shaft

The example illustrated in Figure 5 shows a typical interference fit:

In the following, it will be shown how the DTS can work using the example of a shaft-hub-connection: A generic algorithm will first be developed. The algorithm will then be adapted for different shaft-hub-connections. Thereby, besides the tolerance gained, the paper will also address the potential reductions in manufacturing costs that can be achieved by implementing the DTS.

Development of the generic algorithm:

To apply the new DTS, the conditions of the fit function first needs to be defined. The conditions for the corresponding algorithm to guarantee the fit function are:

(1) The fit function is only guaranteed if the minimum and maximum distances are complied with.
(2) The tolerance gain must not negatively affect the mechanical and the geometrical properties of each part.

Hence, the maximum distance between a shaft and a hub can be calculated as follows:

\[ \text{Max}A = \frac{\text{ULS}_2 - \text{LLS}_1}{C} \]  

Analogously, the minimum distance can be calculated as follows:

\[ \text{Min}A = \frac{\text{LLS}_2 - \text{ULS}_1}{C} \]  

After manufacturing the first part (in this example the hub), its exact diameter can be measured (MP means measurement point).

\[ MP_1 \in [\text{LLS}_1 : \text{ULS}_1] \]  

To take the measurement inaccuracy \( M_j \) into account, the following values with a maximal and a minimal possible measurement value will be calculated:

\[ MU_{U1} = MP_1 - M_j \]  
\[ MU_{O1} = MP_1 + M_j \]

Hence, using the data generated above, the new manufacturing dimension (C) and the new tolerances (T) for the second part (in this example the shaft) can be calculated:
Whereby:

\[ \text{ULS}_{2\text{new}} = \mu_{U1} + \text{Max}A \]  \hspace{1cm} (9)

\[ \text{LLS}_{2\text{new}} = \mu_{O1} + \text{Min}A \]  \hspace{1cm} (10)

**Examples of different shaft-hub-connections and the achieved cost reductions:**

**Interference fit:**

12H7/r6 has a small oversize which corresponds to an interference fit. Joining is only possible using force.

A drilling hole with a diameter of 12H7 is defined from 12.000 mm to 12.018 mm and a shaft with the diameter of 12r6 is defined from 12.023 mm to 12.034 mm. Thus, the shaft has a tolerance of 11 µm.

Applying the generic algorithm, the following calculating steps are necessary: Using equations (2) and (3):

\[ \text{Max}A = 12.034\text{mm} - 12.000\text{mm} = 0.034\text{mm} \]

\[ \text{Min}A = 12.023\text{mm} - 12.018\text{mm} = 0.005\text{mm} \]

By way of example, the hole for a shaft-hub-connection has been drilled and the real drilling hole diameter was measured at MP\(_1\) = 12.005 mm. The measurement uncertainty of the measuring instrument is +-0.004 mm. Using equations (5) and (6):

\[ \mu_{U1} = 12.005\text{mm} - 0.004\text{mm} = 12.001\text{mm} \]

\[ \mu_{O1} = 12.005\text{mm} + 0.004\text{mm} = 12.009\text{mm} \]

In the next step, the new limits of the shaft’s tolerance can be calculated by using equations (9) and (10):

\[ \text{ULS}_{2\text{new}} = 12.001\text{mm} + 0.034\text{mm} = 12.035\text{mm} \]

\[ \text{LLS}_{2\text{new}} = 12.009\text{mm} + 0.005\text{mm} = 12.014\text{mm} \]

Whereby the new manufacturing dimension (C) and the new tolerance (T) is (compare equations (7) and (8)):

\[ C_{2\text{new}} = \frac{12.035\text{mm} + 12.014\text{mm}}{2} = 12.025\text{mm} \]

\[ T_{2\text{new}} = 12.035\text{mm} - 12.014\text{mm} = 0.021\text{mm} \]

This demonstrates the benefits of applying the DTS: The tolerance of 11 µm initially defined by the design corresponds to a tolerance class of IT6. Using the DTS, the tolerance of the shaft can be increased to 21 µm. This corresponds to a tolerance class of IT7. According to Milberg (Milberg, 1992) (see also Figure 5 above), a tolerance class of IT6 corresponds to a factor of 2.1 of the relative manufacturing costs, whereas the tolerance class of IT7 corresponds to a factor of 1.6.

Assuming that the production of the shaft costs 5.00 € per piece when manufactured according to tolerance class IT6, the following cost reduction (CR) can be calculated:
Clearence fit:

12F8/h9 has a medium clearance. Joining is possible without any effort.

A drilling hole with a diameter of 12F8 is defined from 12.016 mm to 12.043 mm and a shaft with the diameter of 12r6 is defined from 11.957 mm to 12.000 mm. Thus, the shaft has a tolerance of 43 µm.

Again, using the generic algorithm developed above, equations (2) and (3) are:

\[
\begin{align*}
Max A &= 12.000 \text{ mm} - 12.016 \text{ mm} = -0.016 \text{ mm} \\
Min A &= 11.957 \text{ mm} - 12.043 \text{ mm} = -0.086 \text{ mm}
\end{align*}
\]

In this example, let us assume that the hole of the shaft-hub-connection has been drilled and the real drilling hole diameter was measured at MP \( = 12.020 \text{ mm} \). The measurement uncertainty of the measuring instrument is \( \pm 0.004 \text{ mm} \). Using equations (5) and (6):

\[
\begin{align*}
Mu_{h1} &= 12.020 \text{ mm} - 0.004 \text{ mm} = 12.016 \text{ mm} \\
Mu_{o1} &= 12.020 \text{ mm} + 0.004 \text{ mm} = 12.024 \text{ mm}
\end{align*}
\]

Now, the new limits of the shaft’s tolerance can be calculated again using equations (9) and (10):

\[
\begin{align*}
UL_{S2}^{\text{new}} &= 12.016 \text{ mm} - 0.016 \text{ mm} = 12.000 \text{ mm} \\
LL_{S2}^{\text{new}} &= 12.024 \text{ mm} - 0.086 \text{ mm} = 11.938 \text{ mm}
\end{align*}
\]

Whereby the new manufacturing dimension (C) and the new tolerance (T) is (compare equations (7) and (8)):

\[
\begin{align*}
C_{S2}^{\text{new}} &= \frac{12.000 \text{ mm} + 11.938 \text{ mm}}{2} = 11.969 \text{ mm} \\
T_{S2}^{\text{new}} &= 12.000 \text{ mm} - 12.938 \text{ mm} = 0.062 \text{ mm}
\end{align*}
\]

Again, this demonstrates the benefits of applying the DTS: The tolerance of 43 µm initially defined by the design corresponds to a tolerance class of IT9. Using the DTS, the tolerance of the shaft can be increased to 62 µm. This is equivalent to a tolerance class of IT10. According to Milberg (see above), this corresponds to the factors 1.2 and 1.1 of the relative manufacturing costs, respectively. If the production of the shaft costs 5.00 € per piece when manufactured according to tolerance class IT9, the following cost reduction is thus achievable:

\[
\begin{align*}
CR &= 5.00 \frac{\epsilon}{\text{pcs}} \left( 1 - \frac{1.1}{1.2} \right) = 0.42 \frac{\epsilon}{\text{pcs}}
\end{align*}
\]

Transition fit:

12H7/k6 has clearance and interference. Joining is possible with low force or manual force.

A drilling hole with a diameter of 12H7 is defined from 12.000 mm to 12.018 mm and a shaft with the diameter of 12r6 is defined from 12.001 mm to 12.012 mm. Thus, the shaft has a tolerance of 11 µm.

Using the generic algorithm for shaft-hub-connections, equations (2) and (3) are:
\[ \text{Max} A = 12.012 \text{mm} - 12.000 \text{mm} = 0.012 \text{mm} \]
\[ \text{Min} A = 12.001 \text{mm} - 12.018 \text{mm} = -0.017 \text{mm} \]

Assuming that the hole has been drilled, the real drilling hole diameter equalled \( MP_1 = 12.016 \text{ mm} \) and the measurement uncertainty of the measuring instrument was \( +0.004 \text{ mm} \), equations (5) and (6) are:
\[ MU_{U1} = 12.016 \text{mm} - 0.004 \text{mm} = 12.012 \text{mm} \]
\[ MU_{O1} = 12.016 \text{mm} + 0.004 \text{mm} = 12.020 \text{mm} \]

Using equations (9) and (10), the new limits of the shaft’s tolerance are:
\[ ULS_{2_{\text{new}}} = 12.012 \text{mm} + 0.012 \text{mm} = 12.024 \text{mm} \]
\[ LLS_{2_{\text{new}}} = 12.020 \text{mm} - 0.017 \text{mm} = 12.003 \text{mm} \]

Whereby the new manufacturing dimension (C) and the new tolerance (T) is (compare equations (7) and (8)):
\[ C_{2_{\text{new}}} = \frac{12.024 \text{mm} + 12.003 \text{mm}}{2} = 12.014 \text{mm} \]
\[ T_{2_{\text{new}}} = 12.024 \text{mm} - 12.003 \text{mm} = 0.021 \text{mm} \]

Again, the DTS can achieve significant cost reductions. The tolerance of 11 \( \mu \text{m} \) can be increased to 21 \( \mu \text{m} \); the tolerance class of IT6 is then replaced by a tolerance class of IT7. This corresponds to a factor of 1.6 rather than 2.1 of the relative manufacturing costs. Assuming that the shaft’s production costs equalled 5.00 € per piece when manufactured according to tolerance class IT6, the potential cost reduction is:
\[ CR = 5.00 \text{€} \text{pcs} \left( 1 - \frac{1.6}{2.1} \right) = 1.19 \text{€} \text{pcs} \]

The different examples of shaft-hub-connections have shown that production costs can be reduced significantly by implementing the DTS. In the examples given, production costs were lowered by 1.19€ (interference and transition example) or 0.42 € (clearance example) per module.

However, in order to implement the DTS, initial investments in a measurement system and in the production infrastructure are necessary. The payback period of the implementation of the DTS in the given examples thus depends on the following variables:

1. Necessary investment costs (depending on the existing machinery)
2. Number of units produced
3. Type of fit of the shaft-hub-connection

6. Requirements for a “dynamic tolerance system”

The new approach to handling precise tolerances presented here relies on the current realization of the fourth industrial revolution or “Industry 4.0” (Kagermann, Lukas, & Wolfgang, 2011). This new paradigm is a prerequisite for the implementation of the model outlined above. Specifically, Industry 4.0 implements Cyber-Physical Systems (CPS). CPS is “integrations of computation with physical processes. Embedded computers and networks monitor and control the physical processes, usually with feedback loops where physical processes affect computations and vice versa” (Lee, 2008). CPS is able to automate full production processes on a large scale. According to the “Verein Deutscher Ingenieure” (The Association of German Engineers, VDI) (VDI/VDE Gesellschaft (Hrsg.), 2013), CPS work as follows: all production data are acquired (for example, via sensors) and
then processed with algorithms that connect and interpret the data and use the output to make decisions. A continuous exchange of information (for example, about products, processes and production status) between all participants within the value chain is required, so within CPS, manufacturing machines are all interlinked. This is crucial for the DTS, as it requires the measurement system to be able to communicate with a computer which calculates parameters that then have to be transferred to the next production step.

Moreover, Industry 4.0 is implementing changes in control commands necessary for the DTS. Currently, control commands cannot be communicated and processed “bottom-up”. As a result, information cannot be passed on to the controller automatically and without manual configuration. However, in the course of the fourth industrial revolution, a bottom-up control logic is being developed (Vogel-Heuser, Bauernhansl, & Hompel, 2017).

A last change important for the DTS introduced by Industry 4.0 is the traceability of individual parts and products within the manufacturing process. Following the guidelines of the VDI, single parts can be tracked and their measured dimensions can be recorded (Borgmeier, Grohmann, & Gross, 2017). This is mandatory to accomplish the DTS: the data recorded on trackable individual parts can be used to recalculate the new nominal size with new tolerances of that part’s counterpart. This information then has to be communicated to the machine which produces the counterpart.

The rise of Industry 4.0 thus creates exactly those framework conditions that enable and are necessary for a cost-effective implementation of the new DTS approach presented in this paper. The surveillance monitoring of each part already built into the production process makes it easy to adapt the originally planned process and to integrate another measurement system that can determine the real part dimensions.

Besides Industry 4.0, developments and trends in production measurement technology are also of great importance for this work. In particular, the current trend towards a 100% inspection as well as the increasing degree of integration of production measurement technology are important prerequisites for the DTS, as at least one part has to be measured. In addition, the feedback of acquired measurement data plays an increasingly important role for measurement technology, which is in turn important for the DTS, as the measured value constitutes the input parameter (Bettenhausen, Schmitt, & Berthold, 2011) (Artischewski, 2014) (Imkamp et al., 2016).

The third change in product development and manufacturing that is important for the DTS is the so-called modularization of products. Because of this modularization, repairs are often no longer conducted by replacing individual parts. Rather, they are mainly conducted at the module level (Rieg & Steinhilper, 2018) (Göpfert, 1998; Kober, 2018). This is significant for the DTS, as the interchangeability of individual parts is associated with additional expenses when the approach is implemented. However, this drawback is mitigated by the fact that the ongoing modularization is currently reducing the importance of and need for such interchangeability.

7. Conclusion
Existing approaches to manufacturing components with precise tolerances and thus guaranteeing product fit functions are expensive. To overcome this problem, the present paper proposes a new approach to handling such precise tolerances, the “Dynamic Tolerance System” DTS. The approach is an addition to the production process. It enables the production of very tight tolerance fits at a lower cost.

Drawing on the example of a shaft-hub-connection, the paper develops a simple and adjustable mathematical model that is able to recalculate the tolerance of a second part as a function of the previous manufacturing step. By implementing the DTS, the actual-required tolerances for each component can be machined without loss of quality but with reduced manufacturing costs. The
DTS should also improve the reliability of the fit function and thus the quality of the product while holding costs constant. Finally, the DTS may enable an optimization of both production costs and quality.

Importantly, the DTS does not negatively influence the design process. On the contrary, the designer may profit from the approach due to its enabling extremely precise production of parts.

The present paper has also illustrated that the infrastructure necessary to implement the DTS already exists. In fact, the time is ripe for this new approach given the concept of Industry 4.0, fast and highly precise measurement systems, communicating machines, smart components that know “who” they are, and the increasing practice of replacing modules instead of single parts.

However, much research and testing remain to be done pertaining to this new approach. In particular, from a theoretical perspective, a specific algorithm has to be developed for every case (e.g., linear dimension chains). Moreover, from the technical point of view, pilot projects need to be conducted to implement the approach in real-world production environments. These may serve to verify its anticipated advantages, including a decisive cost reduction and an increased reliability of the fit function.

The approach is subject to several limitations. In particular, the required degree of maturity concerning the networking of machines and smart work-pieces has not yet been reached by many companies. A lack of standards often leads to great difficulties in this respect (Hofmann & Oettmaier, 2017) (Schröder, 2016). Therefore, implementing the DTS would require investments in new measurements systems as well as a suitable infrastructure (Industrie 4.0). A further limitation of the DTS is that enabling the interchangeability of individual parts requires increased expenditure. In the case of a repair, the production data of the assembly must be available to enable the exchange of an individual part. Alternatively, the paired components must be measured again in order to determine the adapted geometry data of the component to be replaced. However, the paper has also shown that this drawback might be mitigated by the current modularization of products: this trend increasingly questions the need to be able to exchange individual parts given the increasingly common replacement of entire modules in the case of repairs.

Table 2 expands Table 1 (see above) to compare the DTS to existing approaches which attempt to enable the achievement of tight tolerances at low costs. In the table, 1 denotes poor fulfilment and 5 denotes excellent fulfilment of the criterion.

The table shows that the DTS can only achieve an interchangeability of individual parts with increased effort (Interchangeability = 2). Also, the necessary measurement technology and infrastructure incur additional costs but enlarging the tolerance limits puts this into perspective (Cost saving = 4). However, since the implementation of functionality is the goal of the DTS, the risk of defective assemblies is very low (Risk = 5). Also, there is no interference in the design process (Design effort = 5). The designer can thus fully concentrate on the functionality of the assembly group.

### Table 1. Evaluation of the approaches

|                     | Interchangeability | Cost saving | Risk | Design effort | ∑  |
|---------------------|--------------------|-------------|------|---------------|----|
| Design-based approaches | 5                  | 3           | 3    | 1             | 12 |
| Machine-based approaches   | 5                  | 1           | 4    | 5             | 15 |
| Assembly-based approaches     | 2                  | 2           | 4    | 3             | 11 |
In sum, the DTS is able to manufacture components at very tight tolerances for a lower cost than achievable using other existing approaches. By increasing the tolerance of single parts, enormous cost savings can be achieved in manufacturing. Thereby, it is to expected that implementing the DTS will become highly economical in the future due to the ongoing fourth Industrial Revolution (Industry 4.0), current progress in production measurement technology, and the increasing modularization of products.

**Table 2. Comparison of the approaches**

|                       | Interchangeability | Cost saving | Risk | Design effort | ∑   |
|-----------------------|--------------------|-------------|------|---------------|-----|
| Design-based approaches| 5                  | 3           | 3    | 1             | 12  |
| Machine-based approaches| 5                 | 1           | 4    | 5             | 15  |
| Assembly-based approaches| 2                 | 2           | 4    | 3             | 11  |
| DTS                   | 2                  | 4           | 5    | 5             | 16  |

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