1. INTRODUCTION

The nature of dark energy (DE) or geometrical dark energy (GDE) is one of the intrinsic queries of modern cosmology that we are still looking for. According to the analyses of the high quality observational data, the present accelerating phase of the universe is quite well described in the framework of the general relativity together with a cosmological constant – the so called $\Lambda$CDM model. However, due to many theoretical and observational shortcomings associated with the $\Lambda$CDM cosmology, searches for alternative descriptions have been necessary. Apart from the well known cosmological constant/fine tuning and cosmic coincidence problems affecting the $\Lambda$CDM scenario, recent observations indicate that the CMB measurements of some key cosmological parameters within this minimal $\Lambda$CDM scenario do not match with the values measured by other cosmological probes. Specifically, the very well known $H_0$ tension between Planck and the estimate obtained by the SH0ES collaboration $^1$, or the $S_8$ tensions with the cosmic shear measurements KiDS-450 $^2$ or DES $^3$, or CFHTLenS $^4$. Furthermore, when a curvature is considered into the cosmic picture $^5$, all these tensions are exacerbated revealing a possible crisis for the cosmology. Thus, in order to circumvent these problems, several alternative cosmological models have been introduced in the literature aiming to solve or alleviate such tensions in an effective way.

For a possible solution or alleviation of the $H_0$ tension, one can see an incomplete list of works $^{11-60}$ and for $S_8$ one can look at the following works $^{45, 54, 56, 61-64}$. In this article we consider two metastable DE models introduced recently by Shafieloo et al. $^{40}$ (also see $^{41}$). In these models, DE has a decaying nature into other components. The most notable point in such models is that the decay of DE depends only on its intrinsic nature and not on the expansion of the universe. Thus, it is expected that metastable DE models could explore some inherent nature of the dark sector, specially the DE. Our observational constraints on the metastable DE models should be considered stringent for the following reasons: (i) we have considered the cosmological perturbations for the models, an indispensable tool to understand the large scale structure of the universe, (ii) we have included the final Planck 2018 data $^{65-67}$. A quick observation from our analyses is that the metastable DE models are able to alleviate the $H_0$ tension. The article is organized in the following way. In section 2 we present the gravitational equations in a Friedmann-Lemaître-Robertson-Walker (FLRW) universe and the metastable DE models that we wish to study in this work. In section 3 we discuss the observational data and the methodology applied to constrain the models. Then we discuss the results of our analyses in section 4. Finally, in section 5 we close our work with a brief summary of all the findings.

2. METASTABLE DARK ENERGY MODELS

In this section we review two metastable DE models introduced recently in $^{40-41}$. As usual we assume the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) geometry characterized by the line el-
emt $ds^2 = -dt^2 + a^2(t) [dx^2 + dy^2 + dz^2]$, where $a(t)$ (hereafter $a$) is the expansion scale factor of the universe. The gravitational sector is assumed to be described by the Einstein's field equations: 

\[ 3H^2 = \frac{8\pi G}{3} \sum_i \rho_i, \]  

\[ 2\dot{H} + 3H^2 = -4\pi G \sum_i p_i, \]  

where an overhead dot denotes the derivative with respect to the cosmic time and $H \equiv \dot{a}/a$ is the Hubble rate of the FLRW universe. Let us now introduce the metastable DE models that we wish to investigate in this article.

### 2.1. Model I

The first metastable DE model that we aim to study follows the evolution law [40][41]:

\[ \dot{\rho}_x = -\Gamma \rho_x, \]  

where $\rho_x$, as already mentioned, denotes the energy density of DE and $\Gamma$ is a constant which could be either positive or negative and its dimension is the same of the Hubble rate, $H$, of the FLRW universe. Note that $\Gamma = 0$ implies that $\rho_x = \text{constant}$ featuring the cosmological constant. For $\Gamma > 0$, DE density has a decaying character while for $\Gamma < 0$, DE density is increasing. Note further that other cosmic fluids, namely baryons, radiation and cold dark matter follow the usual conservation equation, that means, $\dot{\rho}_i + 3H(p_i + \rho_i) = 0$, where $i = \{b, r, c\}$.

Now, if we focus on the evolution of DE as given in eqn. [3], that means, $\rho_x + \Gamma \rho_x = 0$, one could quickly find its equivalent structure by comparing it with the standard evolution of DE

\[ \dot{\rho}_x + 3H(1 + w_x)\rho_x = 0, \]  

which naturally introduces a dynamical equation of state of DE, $w_x = p_x/\rho_x$. Thus, comparing [3] and [4], one could determine, $w_x = -1 + \Gamma/\dot{H}/H_0$, where we introduce $H_0$, i.e. the present value of $H$. In other words, $\Gamma$ will give us an estimate of the deviation of the dark energy equation of state from the cosmological constant.

Let us now proceed with the evolution of this model at the level of perturbations. This is an useful addition in this work because the present two metastable DE models have not been investigated considering their large scale behaviour. We consider the perturbed FLRW metric and synchronous gauge for evaluating the equations, see [68] for more details. The perturbations equations in this gauge can be calculated to be

\[ \delta_x' = -(1 + w_x) \left( \theta_x + \frac{h'}{2} \right) - 3H(c_{sx}^2 - w_x) \left[ \delta_x + 3H(1 + w_x)\frac{\theta_x}{k^2} \right] - 3Hw_x \frac{\theta_x}{k^2}, \]  

\[ \theta_x' = -H(1 - 3c_{sx}^2) \theta_x + \frac{c_{sx}^2}{1 + w_x} k^2 \delta_x', \]  

\[ \delta_c' = \left( \theta_c + \frac{h'}{2} \right), \]  

\[ \theta_c' = -H \theta_c, \]  

In order to understand better the Model I we have investigated the temperature anisotropy in the CMB spectra and matter power spectra for various numerical values of the dimensionless parameter $\Gamma/H_0$. In Fig. 1 we have shown the corresponding plots. In particular, we show the CMB TT spectra in the left panel and matter power spectra in the right one. One can clearly see that even if we increase the value of $\Gamma/H_0$, there is no significant changes in the spectra. In fact, a very mild deviation is present in the low multipoles of the CMB spectra.

### 2.2. Model II

We now introduce the second metastable DE model in this work as following [40][41]:

\[ \dot{\rho}_x' + 3H(1 + w_x)\rho_x = 0, \]  

where $\rho_x$, as already mentioned, denotes the energy density of DE and $\Gamma$ is a constant which could be either positive or negative and its dimension is the same of the Hubble rate, $H$, of the FLRW universe. Note that $\Gamma = 0$ implies that $\rho_x = \text{constant}$ featuring the cosmological constant. For $\Gamma > 0$, DE density has a decaying character while for $\Gamma < 0$, DE density is increasing. Note further that other cosmic fluids, namely baryons, radiation and cold dark matter follow the usual conservation equation, that means, $\dot{\rho}_i + 3H(p_i + \rho_i) = 0$, where $i = \{b, r, c\}$.

Now, if we focus on the evolution of DE as given in eqn. [3], that means, $\rho_x + \Gamma \rho_x = 0$, one could quickly find its equivalent structure by comparing it with the standard evolution of DE

\[ \dot{\rho}_x + 3H(1 + w_x)\rho_x = 0, \]  

which naturally introduces a dynamical equation of state of DE, $w_x = p_x/\rho_x$. Thus, comparing [3] and [4], one could determine, $w_x = -1 + \Gamma/\dot{H}/H_0$, where we introduce $H_0$, i.e. the present value of $H$. In other words, $\Gamma$ will give us an estimate of the deviation of the dark energy equation of state from the cosmological constant.

Let us now proceed with the evolution of this model at the level of perturbations. This is an useful addition in this work because the present two metastable DE models have not been investigated considering their large scale behaviour. We consider the perturbed FLRW metric and synchronous gauge for evaluating the equations, see [68] for more details. The perturbations equations in this gauge can be calculated to be

\[ \delta_x' = -(1 + w_x) \left( \theta_x + \frac{h'}{2} \right) - 3H(c_{sx}^2 - w_x) \left[ \delta_x + 3H(1 + w_x)\frac{\theta_x}{k^2} \right] - 3Hw_x \frac{\theta_x}{k^2}, \]  

\[ \theta_x' = -H(1 - 3c_{sx}^2) \theta_x + \frac{c_{sx}^2}{1 + w_x} k^2 \delta_x', \]  

\[ \delta_c' = \left( \theta_c + \frac{h'}{2} \right), \]  

\[ \theta_c' = -H \theta_c, \]  

In order to understand better the Model I we have investigated the temperature anisotropy in the CMB spectra and matter power spectra for various numerical values of the dimensionless parameter $\Gamma/H_0$. In Fig. 1 we have shown the corresponding plots. In particular, we show the CMB TT spectra in the left panel and matter power spectra in the right one. One can clearly see that even if we increase the value of $\Gamma/H_0$, there is no significant changes in the spectra. In fact, a very mild deviation is present in the low multipoles of the CMB spectra.
where $\Gamma$ is a constant having the same dimension as that of the Hubble constant, hence, $\Gamma/H_0$ is the dimensionless quantity which one can estimate using the observational data. One can see that the above model is actually an interacting scenario (see [69–95, 97–115]) in which pressureless DM is interacting with vacuum non-gravitationally where $\Gamma$ is the coupling parameter of this interacting scenario. As usual the interaction rate does not depend on any parameter related to the expansion of the universe, and this is the basic nature of the metastable DE models. The sign of $\Gamma$ determines the flow of energy between the dark two sectors. For $\Gamma > 0$, DE decays into DM while for $\Gamma < 0$, the situation is reversed, that means energy flows from DM to DE. We consider a general picture allowing $\Gamma$ to take both positive and negative values, with $\Gamma = 0$ recovering the non-interacting $\Lambda$CDM cosmology.

Let us now consider the perturbations equations for this model. We have again considered the perturbed FLRW metric [68] and the synchronous gauge. Within this formalism, one can write down the perturbations equations of the above model as,

$$\dot{\rho}_x = -\Gamma \rho_x, \quad \dot{\rho}_c + 3H \rho_c = \Gamma \rho_x$$

$$\delta_c = -\frac{1}{2} \left( \theta_c + \frac{h'}{2} \right) + Q \frac{\delta_c}{\rho_c} = -\frac{h'}{2} - \frac{a \Gamma \rho_x}{\rho_c} \delta_c$$

$$\theta'_c = -H \theta_c,$$

where $\theta_c = 0$. In a similar way, for Model II we have shown the temperature anisotropy in the CMB spectra and matter power spectra for various numerical values of the dimensionless parameter $\Gamma/H_0$ in Fig. 2. Specifically, the left panel of Fig. 2 shows the CMB TT power spectra and the right panel of Fig. 2 shows the matter power spectra.
The features of the spectra are quite different compared to the Model I. As one can see from the CMB TT power spectra, a mild change in the dimensionless coupling parameter $\Gamma/H_0$ will produce an observable change there. In fact, for negative values of $\Gamma/H_0$ (DM decaying into DE), the amplitude of the first acoustic peak in the CMB TT spectra decreases. The opposite scenario holds when the energy flow takes place from DE to DM ($\Gamma > 0$). Similar effects are observed in the matter power spectra, but in this case when $\Gamma/H_0$ increases, the amplitude of the matter power spectrum is more suppressed.

3. OBSERVATIONAL DATA AND METHODOLOGY

This section is devoted to describe the observational datasets, statistical techniques and the priors imposed on various free parameters related to the aforementioned metastable dark energy models, namely, Model I and Model II. In what follows we describe the observational datasets first:

- **Planck2018**: The latest cosmic microwave background (CMB) measurements from final 2018 Planck legacy release [65][67] have been adopted.
- **BAO**: Measurements of the BAO data from different astronomical missions [116][118] have been used.
- **DES**: The first-year measurements of the Dark Energy Survey (DES) experiment [5][6][119], as adopted by the Planck collaboration in [65] have been analyzed.
- **R19**: The recent measurement of the Hubble constant from a reanalysis of the Hubble Space Telescope data using Cepheids as calibrators, giving $H_0 = 74.03 \pm 1.42$ km/s/Mpc at 68% CL [1] has been considered. It is important to comment that this $H_0$ value is in tension at 4.4σ with the Planck’s estimation within the $\Lambda$CDM cosmological set-up.

To constrain the metastable DE scenarios we use our modified version of the publicly available markov chain monte carlo package CosmoMC [120][121], an excellent cosmological code having a fine convergence diagnostic by Gelman-Rubin [122]. This code includes the support for Planck 2018 likelihood [65][67]. The models we are considering have one extra free parameter, $\Gamma$, compared to the flat $\Lambda$CDM model (six-parameters). Therefore the parameter space of the models is:

$$\mathcal{P}_1 \equiv \left\{ \Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \log[10^{10} A_s], \Gamma/H_0 \right\} ,$$

(13)

where $\Omega_b h^2$, $\Omega_c h^2$, are the physical density for baryons and cold dark matter, respectively; $\theta_{MC}$ denotes the ratio of the sound horizon to the angular diameter distance.

| Parameter          | Prior (Model I)         | Prior (Model II)         |
|--------------------|------------------------|-------------------------|
| $\Omega_b h^2$     | [0.005, 0.1]           | [0.005, 0.1]            |
| $\Omega_c h^2$     | [0.01, 0.99]           | [0.01, 0.99]            |
| $\tau$             | [0.01, 0.8]            | [0.01, 0.8]             |
| $n_s$              | [0.5, 1.5]             | [0.5, 1.5]              |
| $\log[10^{10} A_s]$| [2.4, 4]              | [2.4, 4]                |
| $100\theta_{MC}$   | [0.5, 10]             | [0.5, 10]               |
| $\Gamma/H_0$       | [-1, 1]                | [-1, 0.7]               |

TABLE I: We show the flat priors on the free parameters of both metastable DE models for the statistical simulations.

$\tau$ refers to the reionization optical depth; $n_s$ denotes the scalar spectral index; $A_s$ being the amplitude of the primordial scalar power spectrum; and $\Gamma/H_0$ being the free parameter of the metastable models normalized to the Hubble constant value. For the statistical analyses, we have imposed flat priors (see Table I) on the above free parameters.

4. RESULTS AND ANALYSES

In this section we describe the main observational constraints extracted from both the metastable DE scenarios considering the set of observational datasets and their combinations shown in section 3 In what follows we discuss the constraints.

4.1. Model I

The results of the observational constraints for this model have been summarized in Table II and in Fig. 3. In Fig. 3 selecting some of the key parameters of this model we show their one-dimensional posterior distributions and the 2D joint contours at 68% and 95% CL.

We start with the results of Model I from Planck2018 alone dataset which are shown in the second column of Table II. We see that the dimensionless parameter $\Gamma/H_0$ quantifying the decay of DE is slightly larger than zero at more than 1σ, and completely unconstrained at 95% CL for Planck2018 alone. And this parameter is correlated with most of the key parameters of this model. The fact that the $\Gamma/H_0$ is mostly unconstrained from the CMB alone, can be easily deduced by looking at Fig. 1 where changing $\Gamma/H_0$ we do not observe any differences. In Fig. 3 we can see a strong positive correlation with the Hubble constant, $H_0$, that is shifting its estimate towards
larger values ($H_0 = 69.3^{+5.9}_{-3.5}$, 68% CL, Planck2018) compared to what we obtained in a $\Lambda$CDM model with the same data [65], by relaxing significantly its error bars, and thus, alleviating the $H_0$ tension with R19 within one standard deviation. Thanks to the geometrical degeneracy present in the CMB data between $H_0$ and $\Omega_{m0}$, we find additionally that this model prefers a lower value of the matter density. As we can see from Fig. 4, a strong anti-correlation is present between $\Gamma/H_0$ and $\Omega_{m0}$.

When the BAO data are added to Planck2018, we can now bound $\Gamma/H_0$ at 95% CL, as we can see in the third column of Table II and in the 3D scattered plot of Fig. 4. This is due to the strong constraining power the BAO data have on $\Omega_{m0}$ that strongly anti-correlates with $\Gamma/H_0$, as

Thus, this scenario alleviates the larger values ($H_0 = 70.0^{+3.6}_{-2.2}$, 95% CL, Planck2018+BAO) compared to what we obtained in a $\Lambda$CDM model with the same data [65], by relaxing significantly its error bars, and thus, alleviating the $H_0$ tension with R19 within one standard deviation. Thanks to the geometrical degeneracy present in the CMB data between $H_0$ and $\Omega_{m0}$, we find additionally that this model prefers a lower value of the matter density. As we can see from Fig. 4, a strong anti-correlation is present between $\Gamma/H_0$ and $\Omega_{m0}$.

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| Parameters | Planck2018 | Planck2018+BAO | Planck2018+DES | Planck2018+R19 |
|-----------|------------|----------------|---------------|--------------|
| $\Omega_{m0}$ | $0.303^{+0.026}_{-0.053}$ | $0.306^{+0.014}_{-0.016}$ | $0.263^{+0.012}_{-0.027}$ | $0.263^{+0.009}_{-0.011}$ |
| $H_0$ | $69.3^{+5.9}_{-3.5}$ | $68.3^{+1.7}_{-3.4}$ | $73.6^{+3.7}_{-6.9}$ | $73.8^{+1.4}_{-2.5}$ |
| $\chi^2$ | $2771.046$ | $2779.456$ | $3293.906$ | $2771.620$ |
A strong anti-correlation between $\Gamma/H_0$ and $\Omega_m$, and a positive correlation between $\Gamma/H_0$ and $H_0$ are present. For Planck alone, upper left panel, $\Gamma/H_0$ is unconstrained, while the addition of external datasets to Planck 2018 helps in constraining this parameter.

![3D scattered plots at 95% c.l. in the plane $\Gamma/H_0$ vs $\Omega_m$, coloured by the Hubble constant value $H_0$ for Model I.](image)

**TABLE III:** Summary of the observational constraints at 68% and 95% CL on the cosmological scenario driven by the metastable DE scenario, *Model II*, using different observational datasets.

| Parameters               | Planck2018 | Planck2018+BAO | Planck2018+DES | Planck2018+R19 |
|--------------------------|------------|----------------|----------------|----------------|
| $\Omega_m h^2$           | 0.064 $^{+0.022}_{-0.022}$ | $0.091^{+0.034+0.051}_{-0.023+0.056}$ | $0.0998^{+0.0071+0.015}_{-0.0077-0.014}$ | $<0.050$ $<0.099$ |
| $\Omega_b h^2$           | 0.02231 $^{+0.0015+0.00030}_{-0.0015-0.00031}$ | $0.02233^{+0.0014+0.00028}_{-0.0015-0.00028}$ | $0.02237^{+0.0015+0.00029}_{-0.0016-0.00028}$ | $0.02236^{+0.0016+0.00030}_{-0.0016-0.00028}$ |
| $100\theta_{MC}$         | 1.0444 $^{+0.031+0.0049}_{-0.0331-0.0049}$ | $1.0425^{+0.012+0.0037}_{-0.0022-0.0032}$ | $1.04183^{+0.0049+0.00095}_{-0.0049+0.00091}$ | $1.0461^{+0.0031+0.0039}_{-0.0031-0.0039}$ |
| $\tau$                   | 0.054 $^{+0.0076+0.016}_{-0.0077-0.015}$ | $0.052^{+0.0076+0.016}_{-0.0076-0.016}$ | $0.055^{+0.0077+0.016}_{-0.0076-0.016}$ | $0.055^{+0.0074+0.014}_{-0.0074-0.015}$ |
| $n_s$                    | 0.9724 $^{+0.040+0.0082}_{-0.0042-0.0081}$ | $0.9736^{+0.0039+0.0079}_{-0.0040-0.0079}$ | $0.9739^{+0.0041+0.0081}_{-0.0040-0.0083}$ | $0.9740^{+0.0041+0.0083}_{-0.0041-0.0082}$ |
| $\ln(10^{10}A_s)$       | 3.055 $^{+0.014+0.0033}_{-0.016-0.0033}$ | $3.056^{+0.015+0.0032}_{-0.016-0.0032}$ | $3.056^{+0.015+0.0033}_{-0.017-0.0032}$ | $3.056^{+0.015+0.0032}_{-0.015-0.0030}$ |
| $\Gamma/H_0$             | $< -0.39 < 0.19$ | $-0.29^{+0.30+0.54}_{-0.28-0.53}$ | $-0.21^{+0.082+0.17}_{-0.090-0.17}$ | $-0.21^{+0.082+0.17}_{-0.090-0.17}$ |
| $\Omega_m$               | 0.18 $^{+0.07+0.19}_{-0.13-0.16}$ | $0.242^{+0.073+0.13}_{-0.063-0.14}$ | $0.261^{+0.017+0.038}_{-0.019-0.034}$ | $0.12^{+0.034+0.140}_{-0.084-0.098}$ |
| $H_0$                    | 70.3 $^{+3.3+4.9}_{-2.0-4.9}$ | $69.0^{+4.4+3.1}_{-1.8-3.0}$ | $68.6^{+0.54+1.1}_{-0.54-1.1}$ | $72^{+0.92+1.27}_{-1.0-3.4}$ |
| $\chi^2$                 | 2771.716 | 2780.014 | 3295.094 | 2773.360 |

Table of the number of events in each bin for the $\Gamma/H_0$ and $\Omega_m$ constraints.

We then combine CMB from Planck2018 with DES data because they are no more in tension in this model. As we can see in [3] that the addition of DES data to...
Planck in the plane $\sigma_8 - \Omega_{m0}$ does not shift significantly the 2D contours, as expected when two datasets are in tension with each other. The results of Planck2018+DES combination are summarized in the fourth column of Table [I]. We see that in this case we have an important lower limit on $\Gamma / H_0$, that is larger than zero (i.e. a cosmological constant model), at about 2 standard deviations. This means we have an evidence of increasing DE for this combination of data. This strong constraint is due to the important bound we have on $\Omega_{m0}$, that takes a lower value in this case giving $\Omega_{m0} = 0.263^{+0.012}_{-0.027}$ (68% CL, Planck2018+DES). Thanks to the three-parameter correlation shown in Fig. 4, we find that $H_0$ goes up having its mean value very close to R19 together with large error bars. Thus, within 68% CL, clearly the tension in $H_0$ is released with this model and combination of data.

Finally, since the tension between the Planck2018 and R19 [1] measurements is solved for this model scenario, we can safely add R19 to Planck2018. The results for the combined dataset Planck2018+R19 are shown in the last column of Table [I]. We find a very strong indication of decaying DE with $\Gamma / H_0 > 0$ at more than 2$\sigma$. In fact we have $\Gamma / H_0 > 0.53$ at 95% CL for Planck2018+R19. The constraints we have for this combination of data in Model I are fully consistent with the Planck2018+DES bounds as well, showing a resolution of the tension with the cosmic shear data at the same time.

For a better understanding on the constraints on $H_0$ for different combinations, in Fig. 5 we present all of them in a whisker plot diagram, where additionally, we show the constraints on $H_0$ from Planck2018 (the vertical sky-blue band) [65] and the local estimation (the vertical grey band) from R19 [1].

4.2. Model II

The results of the observational constraints for this model are shown in Table [III] and in Fig. 6. In Fig. 6 selecting some of the key parameters of this model we show their one-dimensional posterior distributions and the 2-dimensional joint contours at 68% and 95% CL.

For Planck2018 alone we find an indication of a $\Gamma / H_0$ different from zero at more than 1$\sigma$. In fact, we have the upper limit $\Gamma / H_0 < -0.39$ at 68% CL. This clearly shows that the transfer of energy from DM to DE is preferred by Planck2018 data. However, at 2$\sigma$, $\Gamma = 0$ is back in agreement with the data. On the other hand, from Fig. 6 we find a strong anti-correlation between $H_0$ and $\Gamma / H_0$, thus, as long as $\Gamma / H_0$ decreases, $H_0$ should increase. This fact is reflected by the Hubble constant constraint $H_0 = 70.3^{+3.3}_{-2.9}$ (68% CL), which clearly shows that the tension on $H_0$ between Planck2018 and R19 is solved within 2 standard deviation. Moreover, for this model, because of the flow of energy from DM to DE, we find a lower estimation of cold dark matter ($\Omega_{m0} = 0.18^{+0.07}_{-0.13}$ at 68% CL) than its estimation within the $\Lambda$CDM model as obtained by Planck2018 in [65]. This is clearly expected.
for the geometrical degeneracy present in the CMB data: if we have less dark matter, we see a shift of the acoustic peaks and we need a larger $H_0$ value to have them back in the original position.

When BAO data are added to Planck2018, thanks to the robust constraint BAO data give on the matter density $\Omega_m$, we find that $\Omega_m$ slightly increases with respect to the Planck2018 alone case ($\Omega_m = 0.242^{+0.079}_{-0.063}$ at 68% CL), but it is still lower than the Planck2018 value in a $\Lambda$CDM model [65]. Because of the positive correlation present between $\Omega_m$ and $\Gamma/H_0$, as we can see in Figs. 7 and 6 we find that $\Gamma/H_0$ is back in agreement with zero within one standard deviation. This means that $\Gamma/H_0$, i.e., the rate of energy transfer between the dark sectors, is in agreement with the expected value in the minimal $\Lambda$CDM model. Finally, because of the very well known anti-correlation between $\Omega_m$ and $H_0$, we see that the Hubble constant shifts towards lower value compared to its estimation from Planck2018 alone, and moreover, its error bars are significantly decreased. Thus, the tension on $H_0$ slightly increases at 2.5σ, but of course in this model scenario, this is always less than the 4.4σ tension between Planck2018 [65] and the SH0ES collaboration [1] within the minimal $\Lambda$CDM scenario. Moreover, because of the extraction method, the BAO data are not completely reliable in fitting extended DE models, as already pointed out in [55].

Now we consider the next two datasets Planck2018+DES and Planck2018+R19. In both cases since the tension between the datasets (Planck2018, DES) and (Planck2018, R19) is solved in this scenario, we can safely combine them. The results for Planck2018+DES and Planck2018+R19 are shown in the last two columns of Table III. For Planck2018+DES we see a really strong bound on $\Gamma/H_0$, that is lower than zero at more than 2σ and very well constrained. Since it is slightly less negative than Planck2018 and Planck2018+BAO, for the three parameter correlation we see in Fig. 7 we will have a slightly larger values of $\Omega_m$ and smaller of $H_0$ with respect to the previous cases. For this reason the Hubble constant tension with R19 is restored in this scenario at about 3.6σ. Finally, for Planck+R19 we have a very strong upper limit on $\Gamma/H_0$, that is less than zero at several standard deviations.
deviations. That means essentially we have an increasing DE scenario for this metastable DE model. Concerning \( \Omega_m \) estimations, similarly to the previous cases, the matter density again decreases.

Finally, we refer to Fig. 5 showing the whisker plot of \( H_0 \) at 68% CL with its measurements by different observational data. The whisker plot in Fig. 5 clearly shows how the tension on \( H_0 \) is alleviated for most of the data combination, with the exception of Planck2018+DES. In summary, within this metastable DE scenario, the energy density of DE is increasing, as reported by the observational data preferring a negative value for \( \Gamma/H_0 \).

5. SUMMARY AND CONCLUDING REMARKS

In this work we investigate two metastable DE models by considering their evolution at the level of linear perturbations and constrain their parameter space in light of the latest observations. The consideration of perturbations equations is the essence of this work since the perturbations equations, specifically for the first metastable DE model have not been studied in earlier works. Concerning the observational data, we use the CMB measurements from final Planck 2018 release [66, 67], BAO [116–118], DES [5, 6, 119] and a measurement of \( H_0 \) from SH0ES collaboration (R19) [1]. In particular, we consider Planck2018, Planck2018+BAO, Planck2018+DES and Planck2018+R19. The inclusion of additional data to CMB is used to break the degeneracies between the parameters. In the last two cases, the combination of Planck2018 to either DES or R19 is possible since the tension between these datasets are solved within these models.

For the first metastable DE model \( \sigma \), we have summarized the results in Table I and in Figs. 4 and 5. The results show that for all the observational data \( \Gamma/H_0 > 0 \) is suggested, which indicates that DE has a decaying nature within this context. While we mention that for Planck2018 alone, \( \Gamma/H_0 \) remains positive at about 68% CL, such an evidence becomes stronger for Planck2018+DES and Planck2018+R19 datasets. However, for Planck2018+BAO, \( \Gamma = 0 \) is in agreement with the data within 68% CL. Additionally, we find that

FIG. 7: 3D scattered plots at 95% CL in the plane \( \Gamma/H_0 \) vs \( \Omega_m \), coloured by the Hubble constant value \( H_0 \) for Model II. On the contrary of the Model I, a strong positive correlation between \( \Gamma/H_0 \) and \( \Omega_m \), and a negative correlation between \( \Gamma/H_0 \) and \( H_0 \) are present.
within this model, the tension on $H_0$ is mostly solved. To be precise, we notice that for Planck2018 data alone, Planck2018+DES and Planck2018+R19, the tension on $H_0$ is significantly alleviated within $1\sigma$. However, for Planck2018+BAO, the tension on $H_0$ is just reduced at $2.6\sigma$ (see Fig. 5 for a better understanding).

The results of the second metastable DE model are shown in Table II and Fig. 6. From the results, one can clearly conclude that, within this model scenario, $\Gamma/H_0 < 0$ is preferred for all the data combination, with the exception of Planck2018+BAO where $\Gamma = 0$ is in agreement with the data within 68% CL. So, for most of the observational data, an increasing of the DE density is favored. The tension on $H_0$ is alleviated for Planck2018 within $2\sigma$, however for Planck2018+BAO is weakened at $2.5\sigma$ and for Planck2018+R19 is completely restored.

In summary, based on the observational results, we see that the recent observations support an indication of non-zero $\Gamma/H_0$, and hence, they are in support of metastable DE models. Moreover, metastable DE models with just an additional extra free parameter $\Gamma/H_0$ can solve quite efficiently the Hubble constant tension.

Last but not least, we would like to emphasize that the choice of the metastable DE models is not unique. Since the nature of DE is not purely understood, thus, there is no reason to exclude other metastable DE models. For instance, some alternatives to the exponential choice of model I can be considered. In a similar way, one could also generalize model II by considering other functional forms. Although model II describes an interacting scenario and similar choices are available in the literature, however, the exact functional form of the interaction rate is not yet revealed. Hence, we believe that metastable DE models must gain a considerable attention in the cosmological community due to the fact that within such models, the extrinsic properties of the universe do not come into the picture, only the intrinsic nature of DE plays the master role.

6. ACKNOWLEDGMENTS

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