Fluctuations and isentropes near the chiral critical endpoint

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Isentropic trajectories crossing the chiral phase transition near the critical endpoint (CEP) are studied for two light quark flavors. The calculations are performed within an effective chiral model with quark-meson interactions, belonging to the same universality class as QCD. We confront mean-field thermodynamics with the functional renormalization group approach, where fluctuations are properly taken into account. We establish a connection between modifications of the isentropic trajectories found in mean-field calculations at the crossover transition near the CEP and the order of the phase transition in the chiral limit. Furthermore, the isentropes obtained with the renormalization group are completely smooth at the crossover transition and do not in any way reflect the proximity of the CEP. In particular, our results do not show the recently conjectured focusing of isentropes from the crossover region towards the critical endpoint.

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1. INTRODUCTION

The determination of the phase structure and the critical behavior of strongly interacting matter is one of the major goals addressed theoretically in studies of QCD at finite temperature T and quark chemical potential μ and experimentally in ultrarelativistic nuclear collisions. One of the fundamental predictions of QCD, is the existence of a boundary line in the (T, μ)-plane that separates the confined, chirally broken hadronic phase from the deconfined chirally symmetric quark-gluon plasma (QGP) [1].

The existence of such a phase boundary was recently established by first-principle calculations in Lattice Gauge Theory (LGT) [2]. For vanishing μ and for two massless flavors, the chiral transition in the presence of an axial anomaly was argued [3] to be second-order and in the universality class of the O(4)-spin model. For finite quark masses, due to the explicit breaking of the chiral symmetry, the second-order transition is most likely replaced by a rapid crossover. Moreover, arguments based on effective models [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] indicate that at large μ and moderate temperatures the transition along the phase boundary is first-order.

The different nature of the transition at high and low μ suggests that the QCD phase diagram exhibits at least one critical endpoint (CEP), where the first-order phase transition line ends in a second-order critical point, followed by a crossover region [14, 15, 16, 17]. It is expected that the static critical properties of the second-order chiral endpoint are governed by the universality class of the 3D Ising model [18]. The universal properties of the phase diagram have recently been studied extensively in an effective theory within the functional renormalization group (FRG) approach, e.g. [5, 14, 20, 21].

Although there is still no direct proof for the existence of the CEP in the QCD phase diagram, many phenomenological models predict such a point. However, its location is still strongly model dependent [14]. Lattice QCD simulations also provide indications for a possible CEP in the QCD phase diagram [2].

The critical behavior and the position of the CEP can be identified by means of observables sensitive to the singular part of the free energy. Of particular interest in this context are observables which reflect the fluctuations of conserved charges [14, 15, 16, 17]. For systems in thermodynamic equilibrium, both baryon number and charge density fluctuations diverge at the CEP [14], independently of the value of the quark mass. However, a similar divergence is expected also at a first-order phase transition in non-equilibrium situations, where the spinodal instabilities are reached [24].

The hydrodynamic expansion of an ideal fluid follows trajectories of constant entropy, the so called isentropes. Due to baryon-number conservation, such trajectories correspond to contours of constant entropy per baryon s/n in the temperature–chemical potential plane. In Ref. [27] it was pointed out that an expanding system, which follows an isentropic trajectory crossing a first-order phase transition between the quark-gluon plasma and the hadronic phase, is focused towards the CEP, if s/n, at a given point (μ, T) on the phase boundary, is larger in the quark-gluon plasma. This implies that a larger range of initial conditions will end up in the vicinity of the CEP.

Recently it was argued that the CEP acts as an attractor for isentropic trajectories also on the crossover side, i.e., for values of the chemical potential μ smaller
An increase of the bulk viscosity near the CEP [33] is suggested for a broad range of critical phenomena [5, 19, 20, 38]. Nonperturbative effects are properly accounted for. The flow equation for a chiral effective theory

The quark-meson model is a low-energy effective theory, which incorporates the chiral symmetry of QCD. For two quark flavors and SU(3)_c color symmetry the model Lagrangian reads

\[ L = \bar{\psi} \left[ i \partial_t - g \left( \sigma + i \gamma_5 \vec{\pi} \cdot \vec{\pi} \right) \right] \psi + \frac{1}{2} \left( \partial_\mu \phi \right)^2 - U(\sigma, \vec{\pi}), \]  

where \( \phi = (\sigma, \vec{\pi}) \) is the O(4)-representation for the isoscalar \( \sigma \)- and the isovector \( \vec{\pi} \)-mesons. The two-flavor quark field \( \psi \) couples to mesons via the flavor-blind Yukawa coupling constant \( g \). The purely mesonic potential is given by

\[ U(\sigma, \vec{\pi}) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} (\phi^2)^2 - h\sigma, \]  

where \( \lambda > 0 \) is the mesonic self-coupling. For a negative \( m^2 \), the vacuum exhibits spontaneous chiral symmetry breaking, where the chiral \( SU(2)_L \times SU(2)_R \) symmetry of the Lagrangian is broken down to the \( SU(2) \) vector symmetry. At high temperatures and/or densities the spontaneously broken chiral symmetry is restored. The external field \( h \), which is related to the current quark masses, breaks the chiral symmetry explicitly.

In the chiral quark-meson model, explicit gluonic degrees of freedom are missing. The effect of gluons is to a certain extent implicitly included in the coupling constants. However, recent extensions of the quark-meson model in this context (e.g. by including the Polyakov loop) are possible [11] and currently under investigation.

2.1. FRG equation for the effective potential

In this section we present the flow equation for the thermodynamic potential \( \Omega(T, \mu) \) in the FRG approach.
Within this scheme, we then compute the thermodynamic potential as well as the entropy and baryon number densities near the chiral phase transition, including the effects of fluctuations. The FRG method yields the average effective action \( \Gamma_k \) at the momentum scale \( k \). This scale is introduced in the flow equation via regulator functions, which act as mass terms in the propagators. The effect of the regulators is to suppress the propagation of particles with momenta smaller than \( k \). In the infrared limit (\( k = 0 \)) fluctuations at all wavelengths have been integrated out and \( \Gamma_{k=0} \) is the full effective action.

The FRG flow equation smoothly interpolates the physics between the ultraviolet (UV) \( \Lambda \) and the IR scale \( \Lambda_0 \). For the quark-meson model it contains two terms (\( t = \ln(k/\Lambda) \))

\[
\partial_t \Gamma_k[\Phi, \Psi] = \frac{1}{2} \text{Tr} \left[ \partial_t R_{B,k} \left( \Gamma_{B,k}^{(2)}[\Phi, \Psi] + R_{B,k} \right)^{-1} \right]
- \text{Tr} \left[ \partial_t R_{F,k} \left( \Gamma_{F,k}^{(2)}[\Phi, \Psi] + R_{F,k} \right)^{-1} \right].
\]

The first term in Eq. (2.3) represents the bosonic flow with the regulator \( R_{B,k} \), while the second part stands for the fermionic contribution with the regulator \( R_{F,k} \). The bosonic and fermionic fields are denoted by \( \Phi \) and \( \Psi \), respectively. The full bosonic (fermionic) inverse propagator includes the term \( \Gamma_{B,F,k}^{(2)} \), the second functional derivative of \( \Gamma_k \) with respect to the corresponding fields \( \Phi \) or \( \Psi \).

As initial condition for the flow equation (2.3) one chooses a bare effective action at the UV scale \( \Lambda \). Consequently, at the momentum scale \( \Lambda \), \( \Gamma_k \) coincides with the classical action \( S \), i.e., \( \Gamma_{k=\Lambda} \equiv S \). During the evolution, the initial bare action is renormalized and finally, at \( k = 0 \), corresponds to the full effective action \( \Gamma_{k=0} = \Gamma \).

### 2.2. Leading order derivative expansion

The functional flow equation (2.3) is exact and is equivalent to an infinite tower of coupled partial differential equations for \( n \)-point functions (\( n \geq 2 \)). In order to solve this equation, a suitable approximation scheme is required.

In the leading order (LO) derivative expansion the FRG flow equation reduces to an ordinary non-linear differential equation \( \frac{d}{dt} \Gamma_{B,F}^{(2)} \). This can be solved either directly, for the effective potential on a momentum-space grid or by expanding it in powers of the order parameter around a minimum. In this letter we use both methods and compare the results. For the quark-meson model, in four Euclidean space-time dimensions, the LO derivative expansion yields \[ 19, 43 \]

\[
\Gamma_k[\psi, \bar{\psi}, \phi] = \int d^4x \left[ \bar{\psi} \left( \frac{\partial}{\partial x} + g(\sigma + i\tau \cdot \vec{\gamma}_5) \right) \psi + \frac{1}{2} (\partial_\mu \phi^2) + \hat{U}_k(\rho) \right],
\]

where \( \hat{U}_k(\rho) \) is the scale-dependent effective potential, cf. Eq. (2.4). In this expression we have introduced a symmetric field variable \( \rho \) defined by \( \rho = \frac{1}{2} \bar{\sigma}^2 = \frac{1}{2} \left( \sigma^2 + \bar{\sigma}^2 \right) \). For a uniform system it is convenient to deal with the thermodynamic potential density \( \Omega_k \equiv (T/V) \Gamma_k = U_k(\rho = \rho_0, k) \), where the scale-dependent minimum of the potential is labelled by \( \rho_0, k \). In this exploratory work, we neglect the scale evolution of the Yukawa coupling \( g \). This approximation does not significantly affect the critical properties \( 13, 43, 44 \).

In thermal equilibrium the extension of the flow equation to finite \( T \) and \( \mu \) is done within the Matsubara formalism: the integration over \( \rho_0 \) is replaced by a summation over the corresponding Matsubara frequencies: \( p_0 \rightarrow 2n\pi T \) for bosons and \( p_0 \rightarrow (2n+1)\pi T \) for fermions. A finite chemical potential is introduced in the fermionic part of the Lagrangian by shifting the derivative with respect to the Euclidean time \( \tau \), \( \partial_\tau \rightarrow \partial_\tau - \mu \). We assume \( SU_f(2) \)-symmetry and set \( \mu = \mu_u = \mu_d \).

We employ the optimized regulators \( 15, 40 \) for fermions

\[
R_{F,k}(q) = (\frac{\bar{q}}{\bar{q}} + i\mu \gamma_0) \left( \frac{\bar{q}^2 + k^2}{\bar{q}^2 + q^2} - 1 \right) \theta(k^2 - q^2)(2.5)
\]

and bosons

\[
R_{B,k}(q) = (k^2 - q^2) \theta(k^2 - q^2),
\]

where \( \bar{q}_0 = q_0 + i\mu \). With these, the momentum integration and Matsubara summation in the RG flow equation can be done analytically, resulting in the following flow equation for the thermodynamic potential density \( \Omega_k \)

\[
\frac{d}{dt} \Omega_k(T, \mu; \rho_0, k) = \frac{k^4}{12\pi^2} \left[ \frac{1}{E_x} \left( \frac{1}{E_x} + 2n_B(E_x) \right) + 1 + 2n_B(E_x) \right] - 2\nu_q \theta \left[ 1 + \frac{n_F(E_q) - n_F(E_q)}{E_q} \right].
\]

In Eq. (2.7) \( n_B(x) = [e^{x/T} - 1]^{-1} \) denotes the Bose-Einstein and \( n_F(x) = [e^{(\pi^2 x)/T} + 1]^{-1} \) the Fermi-Dirac distribution functions for bosons, quarks and antiquarks, respectively. The single-particle energies of pions, sigmas and quarks are given by \( E_{\pi,\sigma,q} = \sqrt{k^2 + M_{\pi,\sigma,q}^2} \) where the masses are scale dependent: \( M_{\pi,\sigma,q}^2 = \tilde{\Omega}_k, M_{\pi,\sigma}^2 = \bar{\Omega}_k + 2\rho_0 k \tilde{\Omega}_k \) and \( M_{\sigma}^2 = 2g^2 \rho_0 k \). The prime on the potential denotes the derivative of \( \tilde{\Omega}_k = \tilde{\Omega} + \hbar 2 \sqrt{\rho} \) with respect to \( \rho \) evaluated at the minimum \( \rho_0, k \). The quark degeneracy factor is \( \nu_q = 2N_f N_c = 12 \).

Although the RG flow equation (2.7) looks rather innocuous, it is quite powerful due to non-linearity implied by the self-consistent determination of the single-particle energies. This scheme very efficiently accounts for long-range fluctuations and nonperturbative dynamics near the chiral phase transition \( 41, 43 \).
2.3. Solving the FRG flow equation

In order to solve the flow equation (2.7), we employ two distinct methods. The first one is the grid method, where the potential is discretized on a one-dimensional \( \rho_i \) grid. This leads to a set of coupled flow equations for the scale-dependent potential \( \Omega_k(T, \mu; \rho_i) \) at each grid point \( \rho_i \). Using the bare potential as initial condition at a given UV cutoff \( \Lambda \), the scale evolution of the thermodynamic potential is obtained by finding the minimum with respect to variations of the field \( \rho \) at each scale. For details concerning the numerical implementation we refer to Ref. \[47\].

The second method is based on a Taylor-expansion, up to a maximum power \( N \), of \( \Omega_k \) around the scale-dependent running minimum \( \rho_{0,k} = \sigma_{0,k}^2/2 \)

\[
\Omega_k(T, \mu; \rho) = \sum_{m=0}^{N} \frac{a_{m,k}}{m!} (\rho - \rho_{0,k})^m. \tag{2.8}
\]

The coefficients \( a_{m,k} \) are functions of the scale \( k \) and of the temperature and chemical potential.

On the one hand, the expansion method yields the potential only in a limited range around the minimum, due to the finite convergence radius of the Taylor expansion. It is therefore difficult to describe a first-order phase transition, where two local potential minima have to be considered. On the other hand, the advantage of the Taylor expansion is its simplicity; only \( N+1 \) coupled differential equations have to be solved.

The minimum of the thermodynamic potential is determined by the stationarity condition

\[
\frac{d\Omega_k}{d\sigma} \bigg|_{\sigma=\sigma_{0,k}} = \frac{d\Omega_k}{d\sigma} \bigg|_{\sigma=\sigma_{0,k}} - \hat{h} = 0. \tag{2.9}
\]

In the Taylor expansion scheme this yields a relation between the coupling \( a_1 \) and the expectation value of the scalar field \( \sigma_0 \)

\[
a_{1,k} = \hat{h}/\sigma_{0,k}. \tag{2.10}
\]

We truncate the expansion up to the third order, \( N = 3 \), and obtain the following set of flow equations:

\[
\partial_k a_{0,k} = \frac{h}{\sqrt{2\rho_{0,k}}} \partial_k \rho_{0,k} + \partial_k \Omega_k, \tag{2.11}
\]

\[
\partial_k \rho_{0,k} = \frac{h}{\sqrt{2\rho_{0,k}}} \partial_k \Omega_k \tag{2.12}
\]

\[
\partial_k a_{2,k} = a_{3,k} \partial_k \rho_{0,k} + \partial_k \Omega''_k, \tag{2.13}
\]

\[
\partial_k a_{3,k} = \partial_k \Omega'''_k. \tag{2.14}
\]

The meson masses are then given by

\[
M_{\pi,k}^2 = \frac{h}{\sqrt{2\rho_{0,k}}} \quad M_{\sigma,k}^2 = \frac{h}{\sqrt{2\rho_{0,k}}} + 2 \rho_{0,k} a_{2,k}, \tag{2.15}
\]

and the quark mass by \( M_{q,k}^2 = 2g^2 \rho_{0,k} \).

The flow equations (2.11), (2.14) are solved numerically starting from the initial conditions for the coefficients \( a_{i,k} \) \( (i = 0, 1, 2, 3) \) at the ultraviolet scale \( \Lambda \). The model parameters of the Lagrangian (2.11) are chosen such that at the scale \( k = 0 \), the chiral symmetry is spontaneously broken in vacuum with a pion mass of \( M_\pi = 138 \text{ MeV} \) and a finite expectation value of the sigma field \( \langle \sigma \rangle = \sigma_0 \), which is identified with the pion decay constant \( f_\pi = 93 \text{ MeV} \). This, together with our choice for the sigma mass in vacuum \( M_\sigma = 670 \text{ MeV} \) and the constituent quark mass \( M_q = 335 \text{ MeV} \), are obtained with \( h = 1.771 \times 10^6 \text{ MeV}^3 \), \( g = 3.6 \), \( \Lambda = 950 \text{ MeV} \), \( a_{1,\Lambda} = (582\text{MeV})^2 \), \( a_{2,\Lambda} = 35.2 \) and \( a_{3,\Lambda} = 0 \). The corresponding values for the parameters of the Lagrangian (2.1) are \( \lambda = a_{2,\Lambda}/2 \) and \( m^2 = a_{1,\Lambda} - \lambda \sigma_0^2/\Lambda \), where \( \sigma_0 = h/a_{1,\Lambda} \simeq 5.2 \text{ MeV} \) is the starting value for the scalar condensate.

2.4. Isentropic thermodynamics

The solution of the flow equation (2.7) yields the pressure \( p(T, \mu) = -\Omega_k(T, \mu; \rho_{0,k}) \big|_{k=0} \) as a function of \( T \) and \( \mu \). The relevant observables, the entropy density \( s(T, \mu) \) and the quark-number density \( n(T, \mu) \) are obtained from the pressure as derivatives with respect to temperature and quark chemical potential

\[
s = \frac{\partial p(T, \mu)}{\partial T} = -\frac{\partial [a_{0,k} - h\sigma_{0,k}]}{\partial T} \bigg|_{k=0}, \tag{2.16}
\]

\[
n = \frac{\partial p(T, \mu)}{\partial \mu} = -\frac{\partial [a_{0,k} - h\sigma_{0,k}]}{\partial \mu} \bigg|_{k=0}. \tag{2.17}
\]

The isentropic trajectories in the \((T, \mu)\)-plane are then obtained as contours of constant entropy per baryon \( s/n \).

3. MEAN-FIELD APPROXIMATION

In order to examine the influence of fluctuations on the thermodynamics, in particular on the isentropic trajectories, we compare the FRG results with those obtained in the mean-field (MF) approximation, where quantum and thermal fluctuations are neglected.

The partition function of the quark-meson model can be formulated as a path integral over meson and quark/antiquark fields in Euclidean space-time. In the MF approximation, the meson fields in the action are replaced by their expectation values. The integration over the quark/antiquark fields yields the fermionic determinant. The resulting thermodynamic potential of the chiral quark–meson model is of the form \[ \tilde{\Omega} \[48\].

\[
\Omega(T, \mu; \langle \sigma \rangle, \langle \vec{\pi} \rangle) = \Omega_{qqq}(T, \mu; \langle \sigma \rangle) + U(\langle \sigma \rangle, \langle \vec{\pi} \rangle) \tag{3.1}
\]

with the meson potential \( U \) of Eq. (2.2). The quark/anti-
The quark contribution is given by

\[ \Omega_{qq}(T, \mu; \langle \sigma \rangle) = \nu_q T \int \frac{d^3k}{(2\pi)^3} \left\{ \ln \left(1 - n^-(E_q)\right) + \ln \left(1 - n^+(E_q)\right) - \frac{E_q}{T} \right\} , \quad (3.2) \]

where the last term is the divergent vacuum contribution. As will be shown below, this term influences the shape of the isentropic trajectories near the phase boundary.

In the MF approximation, the expectation value \( \langle \sigma \rangle \) is determined by the corresponding classical equation of motion, the gap equation. This is obtained by minimizing the thermodynamic potential in the \( \sigma \)-direction

\[ \frac{\partial \Omega}{\partial \sigma} \bigg|_{\text{min}} = 0 . \quad (3.3) \]

The solution of the gap equation determines the \( T \)- and \( \mu \)-dependence of the chiral order parameter \( \langle \sigma \rangle(T, \mu) \) and the constituent quark mass \( M_q = g \langle \sigma \rangle \). The expectation values of the pion fields \( \langle \vec{\pi} \rangle \) vanish. We have chosen the model parameters in the MF analysis so as to reproduce the same vacuum physics as in the FRG approach.

In Fig. 1 and 2 we show the isentropes obtained in the mean-field approximation with and without the vacuum contribution. The divergence of the vacuum term is regularized by a momentum cutoff \( \Lambda = 583 \text{ MeV} \). We note that the position of the CEP is shifted to smaller temperature and larger values of the chemical potential, namely from \((T_c, \mu_c) = (130, 147)\) to \((51, 316) \text{ MeV}\), when the vacuum contribution is included. Furthermore, the crossover temperature at \( \mu = 0 \) is increased from approximately 150 to almost 210 MeV, while the critical chemical potential at \( T = 0 \) is changed by less than 10%.

Our results also show that the kink in the isentropic trajectories found at the phase boundary in the calculation, where the vacuum contribution is neglected, is to a large extent an artifact of the approximation. The thermal part of \( (3.2) \) yields a term of the form \( M^4_q \log M^2_q \). With such a term in the effective potential, the phase transition in the chiral limit is first order for all densities, from \( \mu = 0 \) and \( T \approx 200 \text{ MeV} \) to \( T = 0 \) and \( \mu \approx 300 \text{ MeV} \) \cite{49}, as recently found in Ref. \cite{5}. Due to the non-zero latent heat, the isentropic trajectories are discontinuous at the phase boundary in the \( T - \mu \) plane. When a sufficiently strong explicit symmetry breaking term is introduced, the first order transition is smoothed into a crossover and the discontinuity in the isentropes shows up as kinks at the phase boundary. However, when the vacuum term is included, the logarithmic term is cancelled and the transition in the chiral limit is second order in flavor \( SU(2) \). Consequently, in the chiral limit the isentropes are continuous and when the explicit symmetry breaking term is turned on the isentropic trajectories remain smooth, as seen in Fig. 2.

These considerations offer a plausible explanation for the different behavior of the isentropes near the phase

\[ \text{FIG. 1: Isentropes computed in the mean-field approximation to the quark-meson model, neglecting the vacuum term in Eq. (3.2). The values of } s/n \text{ are indicated at each isentropic trajectory. The chiral phase boundary, composed of a crossover and a first-order transition, is indicated by a broken and a full line, respectively. The bullet on the phase boundary indicates the position of the CEP in the MF approximation to this model.} \]

\[ \text{FIG. 2: The same as Fig. 1 but including the vacuum term in Eq. (3.2).} \]
boundary in the flavor SU(2) versions of the quark-meson and Nambu–Jona-Lasinio (NJL) models found in Ref. [48]. In the quark-meson model the vacuum term was dropped, while in the NJL model it must be included, since the model otherwise does not yield a state with spontaneously broken chiral symmetry. Consequently, one expects a kink at the transition in the quark-meson model and smooth isentropes in the NJL model, in agreement with the results of Ref. [48]. We conclude that the strong kink structure in the crossover region, which is obtained by neglecting the quark vacuum loop, is unphysical.

In two recent papers, the isentropes have been computed in effective chiral models including the coupling to the Polyakov loop in the mean-field approximation [50, 51]. Also in this case, the singular term cancels between the thermal and vacuum contributions. Hence, one expects a kink in the quark-meson model with Polyakov loop and smooth trajectories in the Polyakov–NJL (PNJL) model. This is confirmed on a qualitative level by the calculation of Ref. [50] and similar trajectories have been obtained in the flavor SU(3) version of the PNJL model in Ref. [51]. The cause of a less pronounced structure found in the isentropes obtained in the PNJL model as the CEP is approached, remains unclear. A possible connection with the first order transition in the chiral limit of the flavor SU(2) model [52] is uncertain, since similar trajectories are obtained in the two flavor PNJL model, where in the same limit the transition is second order [51].

At the first-order transition to the broken symmetry phase, the isentropes are deflected to larger µ and smaller T, away from the CEP, because the entropy per baryon in the co-existence region is, in chiral effective models lacking gluonic degrees of freedom, smaller in the symmetric phase. This may be different if the chiral and deconfinement transitions coincide at finite µ, since in QCD the gluons, are expected to contribute a major part of the entropy per baryon. However, if the so called quarkyonic phase [53] is realized, the isentropes at the first-order transition from hadronic to quarkyonic matter may be similar to those found in the quark-meson model.

4. ISENTROPIC TRAJECTORIES IN THE RG APPROACH

As indicated in the introduction, the influence of fluctuations on the isentropic trajectories at the chiral transition and in particular at the critical endpoint of QCD is of high current interest [26, 28, 29]. We address this problem by comparing the isentropic trajectories obtained using the FRG approach with those obtained in the MF approximation in the preceding section.

Using the thermodynamic potentials obtained in the FRG approach, we determine the phase diagram of the model [54]. The crossover line corresponds to a maximum of the chiral susceptibility. In Fig. [5] we show the resulting phase diagram together with the corresponding isentropes. As in the MF approach, the phase diagram has a generic structure and exhibits a critical endpoint, which separates the first-order phase transition from the rapid crossover. At the CEP, the transition is of second-order and belongs to the universality class of the Ising model in three dimensions.

A comparison with the MF results shows that the FRG phase diagram is very similar to that obtained in the mean-field calculation with the vacuum term included. In particular, the position of the critical endpoint, the crossover temperature at µ = 0 and the critical chemical potential at T = 0 are almost identical. Thus, for the quark-meson model, the effect of fluctuations on the phase diagram are fairly small, once the fermion vacuum term is taken into account. With our choice of model parameters the FRG approach yields approximately (Tc, µc) = (14, 328) MeV within the grid method (FRGgrid), while using the Taylor expansion method (FRGTaylor) we find (Tc, µc) = (64, 336) MeV, as shown in Fig. [6]. Thus, the location of the CEP is not only model dependent [14], but also depends on the method the dynamics is implemented in the renormalization group flow equation. The two approaches differ in the treatment of higher order, irrelevant operators. This does not affect universal quantities, like critical exponents, but it can affect non-universal quantities, like the location of the critical endpoint.

A shift of the CEP to larger values of the chemical potential and smaller temperatures was also found in the proper-time renormalization group approach relative to the mean-field solution in Ref. [5]. In view of our results, it is likely that a major part of this shift is due to the fermion vacuum term, which was not included in the mean-field approximation.

We also show the isentropic trajectories in the (T, µ)-plane, obtained in the FRGTaylor and in the FRGgrid approaches. The isentropes show a smooth behavior everywhere, and agree qualitatively with the mean-field trajectories shown in Fig. [2] i.e., those obtained with the vacuum term. It is reassuring that the FRGTaylor and the FRGgrid methods lead to a very similar isentropic trajectories. Some deviations at large µ, seen in Fig. [6] indicate limitations of the Taylor expansion method due to the truncation of the series [2, 8] at third order. Recent LGT results [55] on the isentropic trajectories near the QCD crossover line show qualitatively a very similar smooth behavior as obtained in our model with the FRG approach and shown in Fig. [6] and [2].

A comparison of the FRG and MF results shows no qualitative change of the isentropic trajectories in the vicinity of the CEP. In particular, the isentropes remain smooth also when the effect of long-wavelength fluctuations is consistently included and show no sign of focusing towards the CEP. This is clearly illustrated in Fig. [1] which shows a blow-up of the FRG isentropes in the vicinity of the CEP. Since the quark-meson model is in the same universality class as QCD, our results do
not support the universality of the focusing phenomenon conjectured in Refs. [26, 28]. We stress that the RG treatment of fluctuations employed here, reproduces the $Z(2)$ universal scaling of the relevant physical observables at the CEP.

The fact that the focusing effect is not universal can be understood in general terms. The point is that the entropy and the baryon density are both obtained as first derivatives of the thermodynamic potential $\Omega$, which remain finite at the CEP, since only second- and higher-order derivatives diverge. Consequently, the singular part of the entropy per baryon does not diverge at the CEP and hence is not guaranteed to dominate over the regular background contribution. It follows that the isentropic trajectories are not universal, since they depend on the relative strength of the universal singular part and the non-universal background. In other words, the characteristic shape of the isentropes in the vicinity of the CEP can vary from model to model, even though they belong to the same universality class. The model constructed in Refs. [26, 28] yields focusing of the isentropes towards the CEP because the singular part of the thermodynamic potential is chosen by hand to be very large. In chiral models, where the critical region around the CEP and around the $O(4)$ transition line is small [5, 21], and consequently the relative strength of the singular part of $\Omega$ is small, it is unlikely that the focusing effect of the isentropic trajectories reported in [26] can be observed.

For completeness we note that the fact that this effect is not universal does not exclude the possibility that in QCD the relative strength of the singular part is large. If this is the case, focusing may then potentially be relevant for nucleus-nucleus collision experiments [28, 29]. However, then a detailed study of the equilibration of long-range fluctuations in an expanding system, along the lines of Ref. [17] is needed in order to decide whether the isentropic trajectories are relevant or not, as discussed in the introduction.

## 5. SUMMARY

The isentropic trajectories (contours of fixed entropy per baryon) in the QCD phase diagram describe possible paths of the hydrodynamic evolution of a thermal medium created in nucleus-nucleus collisions. We investigated the behavior of the isentropic trajectories within the chiral quark-meson model for two-quark flavors. The thermodynamics was formulated using functional renormalization group (FRG) techniques and the results were compared with two variants of the mean-field (MF) approximation, one neglecting and the other one including the fermion vacuum term.

Our studies of the isentropic trajectories near the chiral phase transition were motivated by recent findings that the chiral critical endpoint (CEP) acts as an attractor for the isentropes, leading to a focusing towards the CEP [20]. It was argued that the focusing effect would have
important phenomenological consequences for nucleus-nucleus collisions \cite{24-29}. A comparison of the MF and the FRG results for the isentropes show that the kink structure in the transition region, observed in some MF calculations, is washed out when the fermion vacuum contribution is properly included. Furthermore, in spite of the fact that the entropy density and the baryon-number density rise rapidly near the crossover transition and also as the CEP is approached, the isentropes around the CEP remain very smooth. These results raise doubts concerning the focusing of isentropes and its phenomenological consequences. Although, as already mentioned, the isentropic trajectories of our model differ from those of QCD, the singular part of the thermodynamic potential near the CEP is universal. Our results show that in the quark-meson model the singular contribution to the entropy per baryon near the CEP is subdominant and hence that the conjectured focusing of the isentropic trajectories towards the CEP is not universal. However, the possibility still remains that in some systems of the same universality class focusing may appear, if accidentally close to the CEP the singular term dominates. Thus, we conclude that the existence of such an effect in QCD matter is unlikely but not completely excluded.

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