Techniques for Continuous Touch-Based Authentication Modeling

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ABSTRACT

The field of touch-based authentication has been rapidly developing over the last decade, creating a fragmented and difficult-to-navigate area for researchers and application developers alike due to the variety of methods investigated. In this study, we perform a systematic literature analysis of 30 studies on the techniques used for feature extraction, classification, and aggregation in touch-based authentication systems as well as the performance metrics reported by each study. Based on our findings, we design a set of experiments to compare the performance of the most frequently used techniques in the field under clearly defined conditions. In addition, we introduce three new techniques for touch-based authentication: an expanded feature set (consisting of 149 unique features), a multi-algorithm ensemble-based classifier, and a Recurrent Neural Network based stacking aggregation method. The comparison includes 14 feature sets, 11 classifiers, and 5 aggregation methods. In total, 219 model configurations are examined and we show that our novel techniques outperform the current state-of-the-art in each category. The results are also validated across three different publicly available datasets. Finally, we discuss the findings of our investigation with the aim of making the field more understandable and accessible for researchers and practitioners.

1 INTRODUCTION

Over the past two decades, the world has become increasingly reliant on mobile devices, such as smartphones and tablets, for professional, social, and leisure activities. It is estimated that 81% of the world population is in possession of a smartphone today, including children and the elderly [1]. Mobile devices have grown to deliver services far beyond what can be considered trivial and mundane and now are used for activities that involve the processing and storage of sensitive and private information.

Almost all mobile devices offer basic mechanisms for authentication - proving ownership and identity, to prevent unauthorized people from accessing private data. Thus far, the prevalent methods for authentication used by mainstream smartphones include pins, pattern unlocking, face recognition, and fingerprint matching. However, such methods have been shown to not always be reliable as they can be inconvenient to use [15] or prone to various attacks [9, 24, 42].

More recently, the scientific community has introduced continuous mobile authentication systems which ensure user identity throughout the whole session. Such systems can make use of the unique way people type on their phone [25], the sensors of their smartphones [50], by continuously recording video of their faces [18] and other behavioral characteristics. Touch-based authentication methods, specifically, make use of the way people interact with the screen of their smartphones. These methods rely on assessing whether the interaction patterns of the person authenticating match the general behavior of the original owner. Touch-based authentication offers significant advantages over traditional methods, as it is relying on dynamic and inherent properties of users rather than static attributes and memorized sequences. Touch-based authentication systems can be used as a second-factor authentication in sensitive applications such as banking and finance, where suspicious behavior can be flagged and additional checks requested for certain transactions. The technology can also be used to detect whenever a malicious user gets a hold of an unlocked phone. The system, then, would detect the difference in behavior and request an additional authentication method to verify identity. However, despite almost a decade of research into this field and positive sentiment from the community for such technology [43], touch-based authentication methods still lack widespread adoption and integration into our daily devices. This issue can be attributed to the way studies are evaluated and the resulting overestimation of performance [26]. However, contributions in this area are also severely fragmented, primarily due to a lack of a common framework to compare and evaluate models against a well-defined benchmark to accurately assess what can be considered state-of-the-art. In order to make impactful contributions and improve techniques for touch-based authentication, it is imperative to clarify the landscape and provide methods to reason about model performance.

To this end, in this paper, we aim to answer the following research questions:

(1) RQ1: What are the current techniques for performing touch-based authentication? Which features, classifiers, aggregation methods, and metrics are used and how can we derive a theory for grouping them into common categories?

(2) RQ2: How can we establish the best-performing techniques despite the variety in models and evaluation datasets?

(3) RQ3: Which techniques are most important in building robust and well-performing touch-based authentication methods and how can we use this to improve upon the state-of-the-art in the field?

To answer our research questions, we make use of a two-fold approach. First, we conduct a systematic literature review to establish a broad understanding of existing methods for touch-based authentication. We then extract features, classifiers, aggregations, and metrics used for each study and categorize feature extraction and aggregation methods. Second, to determine and improve on state-of-the-art models, we evaluate a carefully selected range of models along common parameters using three open-source datasets, allowing us to understand which are the best-performing methods. We use this to propose a set of techniques that outperform...
the current state-of-the-art and evaluate them against the current best-performing methods.

In this paper we make the following contributions:

- Through a systematic literature review we analyzed 30 papers, extracted 149 features, and categorized touch-based authentication methods.
- We performed a comparison of 219 model configurations across 3 datasets, allowing us to determine the best-performing features, classifiers, and aggregation methods in the field.
- We proposed a novel set of accumulated features, an ensemble-based classification model, and a Recurrent Neural Network (RNN) stacking-based aggregation method, all of which outperform the current state-of-the-art.

2 BACKGROUND

Continuous mobile authentication systems passively verify that a user enrolled in a device is the one persisting on it. This is done by comparing new patterns of interaction with the legitimate ones of the enrolled user. When a significant mismatch is detected, the system can block the malicious user and notify the owner of the device. For instance, this can be useful when a pin has been stolen or an unauthorized user gets access to an unlocked phone. Then, when the unauthorized user starts using the phone, they will be stopped by the continuous authentication system as the pattern of usage deviates from the owner of the smartphone.

There are many ways in which continuous authentication on mobile devices can be performed, including keystrokes [29], taps [65], multi-touch gestures [51], sensors [16, 50], freeform gestures [60], active vibration signal [59] and active gesture methods [12]. However, the focus of this paper is on touch-dynamics - horizontal and vertical displacements on touch-capacitive displays done using a single finger which are called strokes. These are derived from the coordinates and pressure points at contact while interacting with the screen. Some touch-based authentication systems augment these strokes with additional data such as sensor information from the accelerometer and gyroscope. However, the focus of our study is on stroke-based systems.

The lifecycle of a continuous touch-based authentication system is illustrated in Figure 1. The data collection step could be the experimental setup for a study or in the case of a deployed system it could be the enrollment phase where individual templates of behavior are created. The feature extraction step in touch-based authentication aims at obtaining unique information from touchscreen interactive sessions with the smartphone which can be used to differentiate between users of the system. The classifier step relies on models to make a decision about the legitimacy of a particular swipe based on enrollment patterns. These are typically machine learning algorithms that are trained on the features extracted in the previous steps. Furthermore, a single swipe may not provide enough distinguishing information for an acceptable authentication performance. For this purpose, some systems perform aggregation of successive swipes to improve system performance. In the final step, a variety of metrics could be used to capture and report the success of the biometric system.

2.1 Related Work

The first touch-based mobile authentication systems were proposed in the early 2010s inspired by previous work on the use of mouse movement patterns for authentication on desktop computers [28] and other continuous authentication developments. Several studies [17, 21, 41, 54] survey the historical development of touch-based authentication, reporting on the progress, remaining challenges, and performance of models in the field. However, such surveys do not consider the quantitative performance differences between feature extraction, classification and aggregation methods under fair conditions, thus lacking best-practice recommendations for researchers and practitioners. Other works [8, 48] do perform a comparison between a limited number of classification methods. Nevertheless, our study differentiates itself by its magnitude in terms of classifiers examined and investigation of numerous feature extraction and aggregation methods. Furthermore, we use the accumulated knowledge to propose methods that perform better in each step of the system’s lifecycle.
2.2 Datasets

There are dozens of studies that design and implement their own experiments for data collection as shown in [26]. However, the vast majority do not share the resulting dataset upon publication. We present 9 publicly available touch-based authentication datasets in Table 1. We use the following criteria derived from [26] to select the datasets applicable to our investigation. Naturally, the data in question needs to be accessible at the time of requesting it. That is not always the case as some of the servers hosting the data have gone down since paper publication. The dataset itself should contain a group of users using the same smartphone model. The users should have performed at least 2 separate sessions of the experimental tasks. Furthermore, each swipe needs to contain information about its (X, Y) coordinates as well as touch area and pressure values. These are required for the majority of feature extraction approaches. The only publicly available datasets which we consider usable under these conditions are Touchalytics [23], BioIdent [7] and CEP [26]. We focus on these three datasets in the rest of this study and describe the reason for not using the other datasets in the "Notes" column in Table 1. For each of the three datasets, we select the largest subset of users who use the same phone model and perform 2 or more sessions.

3 TECHNIQUES FOR TOUCH-BASED AUTHENTICATION

In this section, we perform a systematic literature review of papers proposing systems for touch-based authentication. We quantify the prevalence of techniques for feature extraction, classification and aggregation in touch-based authentication systems as well as the methods for measuring model performance. Furthermore, we group the approaches into semantically similar categories in order to consolidate the understanding of the field.

3.1 Methods

The objective of the literature review is to understand the methods used in each of the core components of the continuous touch-based authentication lifecycle (Figure 1) so that we can next re-evaluate them in a common benchmark. For our systematic literature review we relied on PRISMA [36] to guide the search strategy to identify articles that proposed and evaluated touch-based authentication models. The search was limited to English language and peer-reviewed published articles. We exclusively made use of the Google Scholar database and used the following search terms: ((touch-based OR touchscreen) AND (authentication OR biometric*)) OR touch dynamics OR touch biometrics OR touch authentication OR continuous touch.

Articles were included if the methods focus on touch-based authentication and made use of machine-learning-based models. We did include articles that, in touch-dynamics-based features, also make use of other features, such as ones based on accelerometer and gyroscope data. However, in our performance evaluation, we do not make use of the additional features as such data is not available in all the publicly available datasets we consider in our study, making comparison difficult. We then implemented an ancestry approach with the articles meeting the inclusion and exclusion criteria. Our keyword-based search identified a total of 685 articles. Following the screening step, we were left with 103 articles. Screening involved the inspection of the title and description of papers for eligibility, as well as removing any duplicates. After our complete eligibility criteria were applied, we included 30 articles in the final review. For each article, we manually tabulated the features, classification methods, and aggregation methods, as well as the metrics used to evaluate performance.

3.2 Findings

The results of our survey are summarized in Table 2. We organize our findings into four sections, each encapsulating the results for a step of the touch-based authentication lifecycle - features, classifiers, aggregations, and metrics (reporting of results). We deliberately do not include the performance reported by each study due to the variety of metrics and datasets used, making the exact data meaningless to compare across the studies.

3.2.1 Features. We found that we can broadly categorize features according to three classes:

- **Swipe-based**: These features are based on data derived from individual swipes. Typically, the features are generated by examining a list of (X, Y, pressure, area) points that form a complete swipe. Examples of such features include the starting X or Y position, the length of the swipe trajectory, the average pressure of the swipe, etc.

- **Image-based**: These methods are based on generating an image that represents the swipe on a 2D plane. The images are then fed into image processing pipelines for texture and shape extraction [6] or to compute a difference score between images [64].

- **Session-based**: These methods are based on the properties of whole sessions, rather than a single or small group of swipe-based features. Examples of such features include the number of swipes per session, the average time duration of swipes per session, the average time duration between swipes per session, etc.

Most studies make use of **swipe-based** features (80%). 13% make use of **session-based** features and 7% of **image-based** ones. The prevalence of **swipe-based** features can be explained by the high computational cost associated with image processing and the long feature accumulation period of **session-based** methods, during which the device is left unprotected.

In total, 16 (67%) of the 24 **swipe-based** based studies, defined their features and extraction methods sufficiently to be reproducible. Another 4 (17%) of the **swipe-based** studies have feature sets that can be only partially reproduced as a number of features do not have clear and non-ambiguous descriptions. For instance, one of the articles has a feature described as "the angle of moving during swiping" [56], without detailing how and which angle is calculated. For further 4 (17%) of the **swipe-based** studies, we could not infer the individual features used for authentication due to broad category definitions rather than specific feature descriptions or the information not being provided by the authors at all.

In total we identified 149 **swipe-based** features from all papers. The list of features can be seen in Appendix B. The average number of features per paper is 24 where the largest number of **swipe-based**
In total, we found that a large proportion of the studies (77%) perform the optional aggregation step in their analysis respectively. The maximum number of classifiers included in a single study is 12 [30]. In total, 12 (44%) of the classifiers appear only once across all studies.

3.2.3 Aggregation. In total, we found that a large proportion of the studies (77%) perform the optional aggregation step in their touch-based authentication systems. There are multiple ways of approaching the processing of a group of swipes to extract optimal performance. We found that we can broadly categorize the aggregation methods into the following four classes:

- **Mean**: The average or median value of the scores of each swipe returned by the classifier.
- **Vote**: The most common binary prediction (legitimate user or attacker) for each swipe decided by the classifier.
- **Feed**: In this approach, all of the stroke features are fed into the model at once and a single prediction is obtained. For instance, if there are 10 features with a window size of 5 (i.e. group of five consecutive swipes), we input all 50 features into the model at once.
- **Trust**: There is a large variation in this category but the general idea is to make use of a statistical formula that outputs a score as new swipes are considered. The score is updated by rewarding positive predictions and penalizing negative predictions proportionally to the individual classifier predictions. An instance of such aggregation methods is the dynamic trust model [37], which is tailored to continuous authentication biometric systems. This specific implementation has been used in [38] and [61].

We present the prevalence of each of these methods in Figure 2. The Mean aggregation approach is the most frequently used one (20%). The Vote, Feed, Trust methods are used in 6%, 13% and 10% of the studies respectively. As mentioned, 23% of the studies do not use aggregation at all, and a further 27% use solutions that do not fall into the categories described above. For instance, the systems using session-based features are making decisions based on a large aggregation of swipes but cannot be included in any of the other categories we describe.

3.2.4 Metrics. Depending on the needs of a particular system, there are a variety of metrics that can be used to measure the performance of a model for touch-based authentication. These include FAR (False Acceptance Rate), FRR (False Rejection Rate), EER (Equal Error Rate), Accuracy, ROC curve (Receiver Operating Characteristic), and others. Statistics for the prevalence of these metrics in the field can be found in Figure 2. The variety of metrics shown illustrates the difficulty in comparing results reported in touch-based authentication studies. In this study, we aim to ease this comparison by reporting on differences in approaches when they are examined under the same conditions and by reporting the results using the same metrics. The reporting of results and impact of metric choices for

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Table 1: Publicly available touch-dynamics datasets. ● - currently accessible without additional processes, ○ - can be accessed through email or special process, ⊗ - link or instructions currently not working. Links accessed on 20 January 2022. The “Usable” column denotes the largest group of users with the same phone model, at least two sessions, and data for coordinates, pressure, and area.

| Dataset          | Publication Year | Total Users (Usable) | Sessions | Accessible | Notes                                      |
|------------------|------------------|----------------------|----------|------------|--------------------------------------------|
| Touchalytics [23]| 2012             | 41 (15)              | 3        | ●          | -                                           |
| WVW [48]         | 2013             | 190 (0)              | 2        | ○          | Data currently not accessible              |
| TCPA [58]        | 2014             | 32 (0)               | 1        | ●          | Participants conduct only a single session |
| UMIDAA-02 [33]   | 2016             | 48 (0)               | 11-429   | ●          | Touch area values unavailable              |
| Biodent [7]      | 2016             | 71 (26)              | 1-4      | ●          |                                            |
| TGA [53]         | 2019             | 31 (0)               | 8        | ●          |                                            |
| Brainrun [40]    | 2020             | 2344 (0)             | 1-1105   | ●          | Touch area and pressure values unavailable |
| HuMIdb [4, 5]    | 2020             | 600 (0)              | 1-5      | ●          | Each session contains only a single swipe  |
| UMDAA-02 [33]    | 2022             | 470 (64)             | 1-30     | ●          |                                            |

1. http://www.mariofrank.net/touchalytics/
2. http://www2.latech.edu/ phoha/BTAS-2013.htm
3. http://www.cudroid.com/urmajesty
4. https://umdada2.github.io/
5. https://ms.sapientia.ro/ manyi/bioident.html
6. http://zasyed.com/jss18dataset.html
7. https://ora.ox.ac.uk/objects/uuid:5f1abaa7-52a4-430b-9208-128d9f1832fd
8. http://www2.latech.edu/ phoha/BTAS-2013.htm
9. https://zenodo.org/record/2598135
10. https://github.com/BiDAlab/HuMIdb
11. https://ora.ox.ac.uk/objects/uuid:5f1abaa7-52a4-430b-9208-128d9f1832fd

Features identified in a single paper is 59 [56], and the smallest is 5 [31].
Table 2: Techniques in touch-based authentication studies. Classifier and metric abbreviations are given in Appendix A. The following symbols are used in the table: ? - unclear, • - we can completely reproduce the features described in the paper and can compare it with other feature sets, ○ - we can reproduce part of the features described in the study but cannot compare it with other feature sets, ○ - features are not described well enough to be reproduced.

| Study (Year) | Feature Extraction (Count) | Feature Reprod. | Classifiers | Metrics | Aggregation |
|--------------|--------------------------|----------------|-------------|---------|-------------|
| [19] (2012)  | Swipe-based (53)         | ○             | DT, RF, BN  | FAR, FRR | Vote        |
| [23] (2012)  | Swipe-based (31)         | •             | kNN, SVM    | EER     | Mean        |
| [32] (2013)  | Swipe-based (13)         | •             | SVM         | ACC     | Feed        |
| [10] (2013)  | Swipe-based (?)          | ○             | OC-SVM, SVM | FAR, FRR, ACC | Trust |
| [48] (2013)  | Swipe-based (28)         | •             | LR, SVM, RF, NB, NN, kNN, BN, SM, Euclidian, DT | EER | Feed |
| [12] (2014)  | Swipe-based (?)          | ○             | kNN         | ACC     | Feed        |
| [35] (2014)  | Session-based (8)        | —             | IF, SVM, NB, BN, RF | ACC | Feed |
| [63, 64] (2014) | Image-based — Proprietary | — | SVM         | ACC, EER | Other |
| [49] (2015)  | Swipe-based (58)         | •             | kNN, SVM, NN, RF | ROC, FRR, FAR | Feed |
| [62] (2015)  | Swipe-based (27)         | •             | SVM, KSR, KDTGR | EER | Mean |
| [31] (2016)  | Session-based (5)        | —             | kNN, RF, SVM | ACC | Trust |
| [38] (2015)  | Swipe-based (15)         | •             | NN, CPANN   | ANGA, ANIA | Trust |
| [39] (2015)  | Swipe-based (5)          | •             | StrOUD      | ROC, EER | N/A |
| [51] (2016)  | Image-based              | —             | SVM, DT, RF, NB | ACC | Other |
| [66] (2017)  | Session-based (59)       | ○             | SVM, RF, DT | AUC | N/A |
| [30] (2018)  | Session-based (5)        | —             | AB, NB, kNN, LDA, LR, NN, RF, SVM, OC-SVM, LOF, IF, EE | FAR, FRR, HTER, AUC | N/A |
| [34] (2018)  | Session-based (21)       | —             | DT, NB, Kstar, RBFN, NN, PSO-RBFN | FAR, FRR, AER | Other |
| [22] (2018)  | Swipe-based (8)          | •             | IF          | ANGA, ANIA | Trust |
| [55] (2019)  | Swipe-based (28)         | •             | NN          | ACC, EER | N/A |
| [53] (2019)  | Swipe-based (18)         | •             | NB, NN, RC, RF, BN, DT | ACC | N/A |
| [46] (2019)  | Swipe-based (18)         | •             | IF, OC-SVM  | ACC | Trust |
| [3] (2020)   | Swipe-based (?)          | ○             | NN          | FAR, FRR, EER | N/A |
| [44] (2021)  | Swipe-based (12)         | •             | NN          | ACC, AUC, FRR, FAR | Mean |
| [27] (2021)  | Swipe-based (30)         | •             | OC-SVM, kNN, NN, DT, RF, NB | ACC, FAR, FRR, EER, ROC | N/A |

Authentication systems has been discussed thoroughly by Sugrim et al. [52].

In this paper, we report our results using the EER metric. The EER is the point at which the FAR and FRR are equal on the ROC curve. The ROC curve is obtained by varying the threshold for acceptance into the biometric system. Therefore, there is a value of the threshold which corresponds to the EER. While some systems might benefit from choosing thresholds for optimizing better FAR or FRR, we believe EER is the most representative of the general performance of a system. This is also supported by the related work [14, 26, 52]. In particular, [26] show that when comparing two touch-based authentication models, the performance differences between them on the ROC curve are largely consistent with the difference at the EER point.
Figure 2: The prevalence of classifiers, aggregation methods, and performance metrics in touch-based authentication studies. The "Other" category means the particular methods have been used less than 3 times in the case of Classification and Metrics and less than 2 times in the case of Aggregation.

4 PERFORMANCE EVALUATION

In this section, we evaluate touch-based authentication techniques, determine which are the best-performing ones, and introduce a new state-of-the-art feature set, classifier, and aggregation method.

4.1 Methods

The objective of this performance evaluation is to determine the best-performing existing feature sets, classifiers, and aggregation methods. Furthermore, we aim to identify a set of novel techniques and compare them to the current state-of-the-art. Finally, the study aims to understand whether the results obtained are valid across multiple publicly available datasets. To this end, we examine how each classifier performs on different feature sets and then compare aggregation methods independently.

Throughout our study, we follow the recommendations from [26] for fair evaluation of touch-based authentication systems. We create the following model for each user and record the mean EER of all users at the end. We select users which have performed at least two sessions and use the same phone model. At first, we split the data of a target user, selecting their first 80% of sessions for positive training data and the remaining 20% for positive testing data. We split the rest of the users into independent training and testing groups at random. The users in each group never overlap. The negative data for training or testing is then obtained by selecting a swipe at random while cycling through the respective group of users until the number of negative training or testing swipes is equal to the positive one. The combined training set is then used to train a binary model and the testing set for evaluating the performance of the model. This whole process is repeated 10 times for each experiment and we report the mean of the results from each repetition. At each of these iterations, we randomly select the training and testing user groups. The one-class classifiers employ the same process, however, the negative training data is not used.

The SVM, RF, NB, kNN, DT, OC-SVM, LR, and IF classifiers we investigated were implemented using the popular scikit-learn [11] machine learning library. The Neural Networks were implemented using TensorFlow [2] and the Keras [13] API. The Bayesian Network implementation was done on the WEKA [57] machine learning library using a Python wrapper. The implementation details of each classification algorithm were left as close to the default as possible. Where we had to make decisions (e.g., in the case of kNN and Neural Networks), we looked at the related work and performed preliminary experiments to decide on the hyperparameters. The final parameters for each classifier are given below:

- Support Vector Machine (SVM) - RBF kernel with a 'scale' coefficient and probability estimations enabled.
- Random Forests (RF) - 100 estimators, max depth of 20, and probability estimations enabled.
- Neural Network (NN) - feed-forward with three hidden layers of 150, 150, and 75 with a 'ReLU' activation function. The output layer has a 'Sigmoid' activation function which outputs a probability of a match between 0 and 1. Batch-normalization is applied at each layer and a 0.3 dropout between the hidden layers. The optimizer is 'Adam' with a 'binary cross-entropy' loss function. The network is trained with a batch size of 20 over 50 epochs.
- Naive Bayes (NB) - gaussian naive bayes implementation.
- k Nearest Neighbors (kNN) - number of neighbors - 18.
- Decision Trees (DT) - gini criterion and no maximum depth.
- Bayesian Network (BN) - K2 algorithm for structure learning and Simple Estimator for predictions.
- One-Class - Support Vector Machine (OC-SVM) - RBF kernel with a 'scale' coefficient.
- Logistic Regression (LR) - LBFGS solver with L2 penalty and maximum iterations of 1000.
- Isolation Forest (IF) - 100 estimators.

4.2 Comparison

In order to compare the performance of selected feature sets, we reproduced the 16 feature sets marked as "Feature Reproducible" in Table 2. Four of the studies [38, 45, 55, 62] implement the exact
Table 3: Reproducible feature sets used in the performance comparison. The additional (non-touch-based) features were used in the final proposed model by the paper. However, we do not re-implement them due to the lack of such data in all of our datasets.

| Study                  | Year of Proposal | Touch Features Count | Additional Features |
|------------------------|------------------|----------------------|---------------------|
| Frank et al. [23]      | 2013             | 30                   | ✗                   |
| Li et al. [32]         | 2013             | 14                   | ✗                   |
| Serwadda et al. [48]   | 2013             | 28                   | ✗                   |
| Xu et al. [58]         | 2014             | 37                   | ✗                   |
| Murmuria et al. [39]   | 2015             | 5                    | (sensors, power) ✓  |
| Antal et al. [7]       | 2015             | 15                   | ✗                   |
| Mahbub et al. [33]     | 2016             | 24                   | ✗                   |
| Shen et al. [49]       | 2016             | 58                   | ✗                   |
| Filippov et al. [22]   | 2018             | 11                   | ✗                   |
| Syed et al. [53]       | 2019             | 18                   | ✗                   |
| Rocha et al. [44]      | 2021             | 12                   | ✗                   |
| Incel et al. [27]      | 2021             | 30                   | (sensors) ✓         |

same group of features as other ones in the set, leaving us with 12 unique and complete feature sets. More details for each feature set are given in Table 3. Some of the studies enhance their touch-based features with auxiliary data, such as ones coming from the accelerometer or gyroscope, however, we do not reproduce these features due to the lack of such data across all datasets.

We compared the 9 most frequently used classifiers in touch-based authentication studies as shown in Section 3.2.2 across all of the 12 feature sets and report their performance in EER. We also compared all four aggregation techniques described in Section 3.2.3 to highlight the best-performing method. In addition, we include the analysis of the Median of scores as an alternative to the Mean approach. The aggregation window we chose in this set of experiments is 5 based on the availability of data and the diminishing returns of larger window sizes as shown in the related literature [23, 26]. For this comparison, we use the novel classifier and feature set described below.

We also introduce 3 novel techniques which we have not identified in other touch-based authentication studies. We include these in our final analysis:

4.2.1 Novel feature set. We compiled a new feature set (ALL) by implementing all swipe-based features from our literature review. These are derived from the X, Y, pressure, and area values of a swipe as described in Appendix B.

In addition to this, we utilized a feature selection algorithm which reduces the total number of features from the dataset. The goal of such approaches is to ensure better computational performance and overall results. For instance, this can be achieved by pruning features that contribute little to the output of the classifier or even have a negative effect on it. The feature selection algorithm we use is Analysis of Variance (ANOVA) using the F-value between features. In order to ensure the method generalizes well, we used the three datasets (CEP [26], Biodent [7] and Touchalytics [25]). We first selected n number of features for each dataset using ANOVA. Then, we only kept features that are sampled in at least two of the three datasets. We experimented with sizes of 50, 75, 100, and 125 for the parameter n. In our preliminary results, we established that n = 125 is the best-performing one in our case and we use it for the rest of this study. However, we highlight that in this case, the general method for feature selection is more important for further research or industry applications rather than the individual features we chose.

4.2.2 Novel classifier. We propose an ensemble method (ENS) for classification based on a combination of results from other classifiers. Ensemble methods are a well-known strategy used to combine multiple machine learning models which produce a result better than the outcome of each individual classifier. This is due to the fact that on some examples, some classifiers might perform poorly but on average models will agree on the correct decision. The algorithm we use outputs a final score by averaging out the probabilities from the predictions of the best-performing individual classifiers. We performed preliminary experiments with three different combinations of classifiers of sizes 3 (SVM, RF, NN), 5 (SVM, RF, NN, kNN, LR), and 7 (SVM, RF, NN, kNN, LR, NB, DT) and found that the best-performing one in our case is the one consisting of SVM, Random Forest, and Neural Network. Similar to the novel feature set selection, the specific group of classifiers that we chose is less of interest than the proposed method itself.

4.2.3 Novel aggregation method. We introduce a Recurrent Neural Network (RNN) stacking algorithm which takes as an input a list of scores and outputs a final probability based on them. Stacking involves training a model on the outputs of other models in order to produce a final result, much like other aggregation methods described in Section 3.2.3. In addition, we hypothesized that the sequential nature of the RNN processing would work well with the temporal nature of swipes in touch dynamics. As such our RNN model consists of a single hidden Long Short-Term Memory (LSTM) layer of size 20 with a ‘tanh’ activation function and one ‘sigmoid’ output layer. We use an LSTM layer in order to avoid the ‘vanishing gradient problem’ associated with traditional RNNs where long-term gradients tend to go to 0 (vanish) or explode and go to infinity. Similar to the NN described in Section 3.2.2, we use the ‘Adam’ optimizer with a binary cross-entropy as a loss function.

4.3 Results

The results from the feature set and classifier comparisons can be found in Table 4. On average, the best-performing feature set is the one generated from the ANOVA method which we proposed with an average of 17.05% EER across all classifiers. Furthermore, using ALL identified swipe-based features resulted in a similar performance. The lowest EER from the studies we re-implemented was [58] with an average of 17.48% EER over all the machine learning algorithms examined. We attribute the low performance of some feature sets such as [44] (28.02%) to the small number of features included.
Table 4: Performance of classifiers applied to different feature sets on the CEP dataset. No aggregation methods are used and the results are reported in EER (%). The average of each row and column is given.

| Features | SVM  | RF   | NN   | NB  | BN  | KNN | DT  | LR  | OC-SVM | IF  | ENS |
|----------|------|------|------|-----|-----|-----|-----|-----|--------|-----|-----|
| [23]     | 14.15| 13.75| 13.48| 21.30| 18.67| 16.76| 22.41| 18.06| 25.80  | 26.22| 12.86| 18.50|
| [32]     | 15.09| 14.64| 14.60| 21.77| 18.51| 17.48| 23.37| 20.50| 24.59  | 26.74| 13.91| 19.20|
| [48]     | 14.10| 14.56| 13.57| 20.54| 18.07| 16.26| 22.88| 17.84| 23.97  | 25.39| 13.10| 18.21|
| [58]     | 13.50| 13.40| 13.22| 19.94| 16.12| 16.12| 22.98| 17.79| 23.75  | 23.90| 12.46| 17.48|
| [49]     | 15.79| 15.36| 14.97| 23.17| 20.46| 19.23| 24.00| 19.01| 27.46  | 29.19| 14.36| 20.27|
| [7]      | 14.00| 14.39| 13.95| 20.79| 18.30| 16.34| 22.88| 19.02| 23.66  | 24.41| 13.32| 18.28|
| [39]     | 20.43| 18.78| 19.60| 24.45| 22.16| 21.70| 25.62| 25.41| 26.58  | 26.13| 18.62| 22.68|
| [33]     | 15.83| 15.69| 15.28| 22.55| 19.72| 18.07| 24.23| 20.43| 27.27  | 27.45| 14.60| 20.10|
| [22]     | 15.17| 15.88| 15.22| 21.15| 19.71| 16.86| 23.78| 21.51| 23.17  | 24.29| 14.67| 19.22|
| [53]     | 16.71| 15.71| 16.00| 23.27| 19.07| 18.99| 23.83| 21.78| 27.68  | 27.09| 15.13| 20.48|
| [44]     | 25.54| 24.74| 24.70| 29.99| 26.39| 26.50| 31.07| 28.69| 33.06  | 33.48| 24.02| 28.02|
| [27]     | 13.68| 14.25| 13.52| 21.19| 19.99| 16.52| 22.72| 17.71| 24.31  | 25.62| 12.79| 18.37|
| All      | 12.77| 12.78| 12.45| 21.67| 16.21| 15.86| 21.71| 15.88| 23.93  | 25.03| 11.70| 17.27|
| ANOVA    | 12.57| 13.00| 12.36| 19.84| 16.37| 15.63| 21.94| 15.81| 23.45  | 24.96| 11.67| 17.05|

| Mean     | 6.35 |
| Median   | 6.47 |
| Vote     | 10.59|
| Feed     | 7.07 |
| Trust    | 6.55 |
| Stacking | 6.28 |

However, the final model of the study uses additional features from sensor data which could result in much better overall performance.

In terms of classifiers, on average, our ensemble method was the best-performing with an average of 14.51% EER across all feature sets. The three individual classifiers in the ensemble (SVM, RF, and NN) also form a well-performing group with 15.67%, 15.49%, and 15.20% respectively. The one-class classifiers (OC-SVM and IF) produced the worst results in our experiments. Overall the best-performing model consisted of our proposed ANOVA feature set and ensemble classifier with an overall performance of 11.67% EER for a single swipe.

However, touch-based systems should not operate on a single swipe as it can be insufficient for authentication. Aggregation methods, based on multiple consecutive swipes result in improved system performance. The results from the aggregation experiment can be found in Table 5. The proposed RNN stacking algorithm was the best-performing method in the aggregation comparison with 6.27% EER. The Mean (6.35%), Median (6.47%), Trust (6.55%) and Feed (7.07%) methods resulted in very similar performance. The worst performing aggregation method in our experiments was Vote with 10.59% EER. Nevertheless, all of the aggregation methods perform better than using a single swipe to make authentication decisions.

4.3.1 Dataset comparison. In order to determine the generality of our results, we replicated our experiments across three datasets - CEP [26], Biodent [7] and Touchalytics [23]. When examining each category (features, classifiers, and aggregations) we choose the best-performing methods determined in our previous experiments and keep them constant (e.g., for all classifiers comparisons, we use the best-performing feature set).

The results of our comparison across all three datasets can be found in Figure 3. Our ANOVA-based feature set and the ensemble classifier consistently outperform the other methods across all three datasets. However, the RNN stacking algorithm was the third-best-performing on the Biodent and Touchalytics datasets. The Trust and Feed aggregation methods were the best-performing on these datasets by a margin of 0.52% and 1.78% EER respectively.

Xu et al. [58] and Frank et al. [23] are other consistently well-performing feature sets with 37 and 31 features respectively. Similarly, the SVM, RF, and NN classifiers provide stable performance across all the datasets examined.

5 DISCUSSION

This study serves as an overview and improvement over the techniques for feature extraction, classification, and aggregation used in touch-based authentication studies. The experiments conducted in Section 4 suggest that when performance is the only concern for a touch-based authentication system, the optimal model would make use of our proposed feature set, ensemble classifier, and RNN stacking aggregation which achieves an EER of 6.28% using a window of 5 swipes on the large CEP dataset. However, the lowest EER we achieved using this model is 4.80% by increasing the aggregation window to 16 strokes.
Techniques for Continuous Touch-Based Authentication Modeling

The relative performance benefits of each individual technique are shown in Table 6. We highlight the next best and the median performing techniques while featuring the difference in EER with the novel ones proposed in this paper. While the improvements might be perceived as marginal compared to the second-best methods, they are quite significant compared to the median, particularly in the feature extraction and classification cases. These results highlight the importance of fair comparison between models which can be helpful for decision-making in the broader touch-based authentication community.

It is worth noting that the results in our experiments might not match the results originally reported by a particular study, sometimes by a large margin. For instance, the EER we obtain using the Touchalytics [23] feature set on their dataset is multiple times higher than the one attained in the original study. This is due to the fair evaluation practices we follow as described in Section 4.1. Substantially less (16) of the original 41 users in the dataset fit into our criteria and were used in our evaluation. Many of them had done only 2 sessions, resulting in training and testing data skew closer to 50%/50% rather than the target of 80%/20%. Furthermore, we report the mean EER, while the original study reports the median.

The results we obtained, however, do not guarantee that the techniques we propose are best suited for all datasets and applications. For instance, our models minimize Equal Error Rates but might require more resources leading to computational power needs and time to execute. To this end, our analysis examines a large number of techniques and these can be used as alternatives for each stage of the touch-authentication lifecycle.

We have established that the best-performing model in terms of EER consists of our ANOVA feature set, ensemble classifier (with SVM, RF, and NN), and a stacking LSTM-based with an EER of 6.28%. However, computational performance could limit the possibility of using this model in practice, particularly due to the mobile environment it is intended for. In this case, we recommend the use of less sophisticated architecture to be used which can still deliver similar

### Figure 3: Performance of (a) feature sets, (b) classifiers, and (c) aggregation methods across CEP, Bioident and Touchalytics touch-based authentication datasets.

| Feature Set | CEP | Bioident | Touchalytics |
|-------------|-----|----------|--------------|
| ANOVA       |     |          |              |
| Xu et al    |     |          |              |
| Incel et al |     |          |              |
| Frank et al |     |          |              |
| Serwadda et al | |     |              |
| Mahbub et al |     |          |              |
| Filippov et al | |     |              |
| Syed et al  |     |          |              |
| Murmuria et al | |     |              |
| Rocha et al |     |          |              |

| Classifier | CEP | Bioident | Touchalytics |
|------------|-----|----------|--------------|
| ENS        |     |          |              |
| SVM        |     |          |              |
| NN         |     |          |              |
| RF         |     |          |              |
| kNN        |     |          |              |
| LR         |     |          |              |
| DT         |     |          |              |

| Aggregation Method | CEP | Bioident | Touchalytics |
|--------------------|-----|----------|--------------|
| Stacking           |     |          |              |
| Mean               |     |          |              |
| Median             |     |          |              |
| Trust              |     |          |              |
| Feed               |     |          |              |
| Vote               |     |          |              |
Table 6: Difference between the novel techniques proposed in this paper and the next best and median methods in touch-based authentication literature. The differences in EER are reported in percentage points (%). The best performing feature sets, classifiers and aggregation methods are ANOVA, Ensembl and Stacking.

|                | CEP          | Biodent      | Touchalytics |
|----------------|--------------|--------------|--------------|
| **Features**   | Xu et al. (+0.79) | Shen et al. (+1.22) | Li et al. (+0.21) |
| **Classification** | NN (+0.69) | SVM (+0.76) | RF (+0.79) |
| **Aggregation** | Mean (+0.07) | Median (-1.03) | Feed (-1.78) |
| **Mean**       | +1.94        | +2.08        | +1.62        |
| **Aggregation** | +4.14        | +1.44        | +3.71        |
| **Touchalytics** | +0.27        | +0.08        | +0.06        |

results but at a more cost-efficient computational performance. Furthermore, we found that there is some variation between the results we obtained on each dataset. For this reason, selecting consistently well-performing models might be preferred for some applications. While EER is a good measure for the overall performance of biometric systems, in continuous authentication the focus can be on guaranteeing a low False Negative Rate to ensure adequate usability of the system. However, that is application-specific and requires further examination which is beyond the scope of this paper.

Even though we ultimately performed the comparison on all three datasets, we believe the results on the CEP dataset are the most representative. That is due to the larger size in terms of users, sessions performed, and the length of each session.

5.1 Limitations

There are several limitations to our experimental approach and results. Firstly, the implementation details of some features, classifiers and aggregation methods might not be perfectly reproduced from related work, despite our best effort. Furthermore, the categories we have grouped techniques in might be quite broad with many internal differences between studies. For instance, implementing a generic trust model algorithm will not necessarily represent the nuances of all models falling under this category. Similarly, Neural Network implementations may vary between papers and differ from our architecture, and optimizing the hyperparameters of models might lead to better overall performance. We believe that one-class classifiers in particular can achieve better results by fine-tuning the parameters.

Furthermore, some of the classification algorithms and feature sets that are not as prevalent in the field or are not reproducible might outperform the more popular methods we examine. Finally, the fact that the methods we examine are mostly consistent throughout the three datasets is encouraging, however, application to other touch-based authentication datasets might result in much different behavior.

6 CONCLUSION

In this paper, we performed a comprehensive review of the approaches for feature extraction, classification, and aggregation in the field. We investigated the prevalence of each technique in the relevant literature and categorized the feature extraction and aggregation methods. Furthermore, we presented and described a set of 149 unique features extracted from related work and identified 9 publicly available datasets for touch-based authentication. We benchmarked the performance of the most common feature sets, classifiers, and aggregation methods in the field with a set of experiments consisting of a total of 219 model configurations. We introduced a novel feature set, ensemble-based classifier, and an RNN-based stacking aggregation method that outperform the state-of-the-art by 0.79%, 0.69%, and 0.07% EER respectively. Finally, we concluded that our findings are largely similar across multiple datasets and provided a discussion of our results, including the limitation of the study.

REFERENCES

[1] [in d .]. Ericsson Mobility Report 2021. https://www.ericsson.com/en/reports-and-papers/mobility-report/reports/november-2021. Accessed 20 January 2022.
[2] Martin Abadi, Ashish Agarwal, Paul Barham, Eugene Brevdo, Zhifeng Chen, Craig Citro, Greg S. Corrado, Andy Davis, et al. 2015. TensorFlow: Large-Scale Machine Learning on Heterogeneous Systems. https://www.tensorflow.org/. Software available from tensorflow.org.
[3] Mohammed Abuhamad, Tamer Abuhamad, and DaeHun Nyang. 2020. AUTOson: Deep-Learning-Based Implicit Continuous Authentication Using Smartphone Sensors. IEEE Internet of Things Journal 7, 6 (2020), 5088–5092. https://doi.org/10.1109/JIOT.2020.2975779
[4] Alejandro Acien, Aythami Morales, Julian ÑÛre, Rubén Vera-Rodriguez, and Ivan Bartolome. 2020. Be-CAPTCHA: Detecting Human Behavior in Smartphone Interaction using Multiple Inbuilt Sensors. CoRR abs/2002.00918 (2020). arXiv:2002.00918 https://arxiv.org/abs/2002.00918
[5] Alejandro Acien, Aythami Morales, Julian ÑÛre, Rubén Vera-Rodriguez, and Oscar Delgado-Mohatar. 2020. Be-CAPTCHA: Bot Detection in Smartphone Interaction using Touchscreen Biometrics and Mobile Sensors. CoRR abs/2005.13655 (2020). arXiv:2005.13655 https://arxiv.org/abs/2005.13655
[6] Jamal Ahmad, Muhammad Sajjad, Ziahoor Jan, Irfan Mehmood, Seungmin Rho, and Sung Wook Baik. 2017. Analysis of interaction trace maps for active authentication on smart devices. Multimedia Tools and Applications 76, 3 (01 Feb 2017), 4069–4087. https://doi.org/10.1007/s11042-016-3450-y
[7] Margit Antal, Zsofia Bork, and LaszlÓ Zsolt SzabÓ. 2015. Information revealed from scrolling interactions on mobile devices. Pattern Recognition Letters 56 (2015), 7–13. https://doi.org/10.1016/j.patrec.2015.01.011
[8] Margit Antal and LaszlÓ Zsolt SzabÓ. 2016. Biometric Authentication Based on Touchscreen Swipe Patterns. Procedia Technology 22 (2016), 862 – 869. https://doi.org/10.1016/j.protcy.2016.01.061 9th International Conference Interdisciplinary in Engineering, INTER-ENG 2015, 8-9 October 2015, Targu Mures, Romania.
[9] Adam J. Aviv, Katherine Gibson, Evan Mospop, Matt Blaze, and Jonathan M. Smith. 2010. Smudge Attacks on Smartphone Touch Screens. In Proceedings of the 4th USENIX Conference on Offensive Technologies (Washington, DC) (WOOT’10). USENIX Association, USA, 1–7.
[10] Cheng Bo, Lan Zhang, Xiang-Yang Li, Quyuan Huang, and Yu Wang. 2013. SilentSense: Silent User Identification via Touch and Movement Behavioral Biometrics. In Proceedings of the 19th Annual International Conference on Mobile Computing & Networking (Miami, Florida, USA) (MobiCom 13) Association for Computing Machinery, New York, NY, USA, 187–190. https://doi.org/10.1145/250423.2504572
[11] Lars Buttinck, Gilles Louppe, Mathieu Blondel, Fabian Pedregosa, Andreas Mueller, Olivier Grisel, Vlad Niculae, Peter Prettenhofer, Alexandre Gramfort,
IEEE Transactions on Information Forensics and Security 11, 3 (2016), 498–513. https://doi.org/10.1109/TIFS.2015.2502528

[50] Zdrėka Sitová, Jaroslav Sedrná, Qing Yang, Ge Peng, Gang Zhou, Paolo Gasti, and Kieran S. Balugani. 2016. HMOC: New Behavioral Biometric Features for Continuous Authentication of Smartphone Users. IEEE Transactions on Information Forensics and Security 11, 5 (2016), 877–892. https://doi.org/10.1109/TIFS.2015.2506542

[51] Yunpeng Song, Zhongmin Cai, and Zhui-Li Zhang. 2017. Multi-touch Authentication Using Hand Geometry and Behavioral Information. In 2017 IEEE Symposium on Security and Privacy (SP). 357–372. https://doi.org/10.1109/SP.2017.54

[52] Shridatt Sagrin, Can Liu, Meghan McLean, and Janne Linnqvist. [n.d.]. Robust Performance Metrics for Authentication Systems. Network and Distributed Systems Security Symposium (NDSS) Symposium 2019 (n.d.). https://doi.org/10.14722/ndss.2019.23531

[53] Zahid Syed, Jordan Helnmeck, Sean Banerjee, and Bojan Cukic. 2019. Touch: User Verification on Smartphones via Tapping Behaviors. In Proceedings of the 26th Annual International Conference on Mobile Computing and Networking (MobiCom ’20). Association for Computing Machinery, New York, NY, USA, 3722–3735. https://doi.org/10.1145/3372224.3380244

[54] Ian H. Witten, Eibe Frank, Mark A. Hall, Christopher J. Pal, and Martin A. Hall. 2011. Practical machine learning tools and techniques. In DATA MINING, Vol. 2. 4.

[55] Hui Xu, Yangfan Zhou, and Michael R. Lyu. 2014. Towards Continuous and Passive Authentication via Touch Biometrics: An Experimental Study on Smartphones. In 10th Symposium On Usable Privacy and Security (SOUPS 2014). USENIX Association, Menlo Park, CA, 187–198. https://www.usenix.org/conference/soups2014/proceedings/presentation/xu

[56] Yuan, Xiaolong, Yiqun Li, and Miao Li. 2020. TouchPass: Towards Behavior-Irrelevant on-Touch User Authentication on Smartphones Leverageig Vibrations. In Proceedings of the 25th Annual International Conference on Mobile Computing and Networking (MobiCom ’20). Association for Computing Machinery, New York, NY, USA, Article 24, 13 pages. https://doi.org/10.1145/3372224.3380901

[57] Yulong Yang, Graedagh D. Clark, Janne Linnqvist, and Antti Oulasvirta. 2016. Free-Form Gesture Authentication in the Wild. Association for Computing Machinery, New York, NY, USA, 3722–3735. https://doi.org/10.1145/2858036.2858270

[58] Yalang Yang, Bin Guo, Zhiwen Yu, Zhihui Chen, and Xiaolong Yang. 2019. BehaveSense: Continuous authentication for security-sensitive mobile apps using behavioral biometrics. Ad Hoc Networks 84 (2019), 9–18. https://doi.org/10.1016/j.adhoc.2018.09.015

[59] Heng Zhang, Vishal M. Patel, Mohammed Fathy, and Rama Chellappa. 2015. Touch Gesture-Based Active User Authentication Using Dictionaries. In 2015 IEEE Winter Conference on Applications of Computer Vision. 207–214. https://doi.org/10.1109/WACV.2015.35

[60] Xin Zhao, Tao Feng, and Weidong Shi. 2013. Continuous mobile authentication using a novel Graphic Touch Gesture Feature. In 2013 IEEE Sixth International Conference on Biotics: Theory, Applications and Systems (BTAS). 1–6. https://doi.org/10.1109/BTAS.2013.6712747

[61] Xi Zhao, Tao Feng, Weidong Shi, and Ioannis A. Kakadiaris. 2014. Mobile User Authentication Using Statistical Touch Dynamics Images. IEEE Transactions on Information Forensics and Security 9, 11 (2014), 1780–1789. https://doi.org/10.1109/TIFS.2014.2350916

[62] Nan Zheng, Kun Bai, Hai Huang, and Haining Wang. 2014. You Are How You Touch: User Verification on Smartphones via Tapping Behaviors. In 2014 IEEE 22nd International Conference on Network Protocols. 221–232. https://doi.org/10.1109/INCP.2014.43

A ABBREVIATIONS

A.1 Classifiers

AB - AdaBoost

BN - Bayesian Network

CPANN - Counter Propagation Artificial Neural Network

DT - Decision Tree

EE - Elliptic Envelope

ENS - Ensemble

GB - Gradient Boosting

HMM - Hidden Markov Models

IF - Isolation Forest

KDTGR - Kernel Dictionary-based Touch Gesture Recognition

KSRC - Kernel Sparse Representation-based Classification

LOF - Local Outlier Factor

LR - Logistic Regression

NB - Naive Bayes

NN - Neural Networks

OC-SVM - OneClass Support Vector Machine

PSO-RBFN - Particle Swarm Optimization Radial Basis Function Network

RC - Random Committee

RF - Random Forest

SM - Scaled Manhattan

 SVM - Support Vector Machine

StrOUd - Strangeness based Outlier Detection

kNN - k Nearest Neighbors

A.2 Metrics

ACC - Accuracy

ANGA - Average Number of Genuine Actions

ANIA - Average Number of Impostor Actions

AUC - Area Under Curve

FAR - False Acceptance Rate

FRR - False Rejection Rate

HTER - Half Total Error Rate

ROC - Receiver Operating Characteristic

B ALL FEATURES
Table 7: Geometric features found in related work. Perc. stands for percentile and Std. Dev. for standard deviation. Full details about each of the features can be found in the corresponding papers. Note that [45, 55, 62] use the same features as [23] and [38] uses the same as [7] except they omit the mid-stroke pressure.

| Feature Studies | Feature | Studies |
|-----------------|---------|---------|
| 1-2. Start X,Y [7, 23, 32, 49, 58] | 43. Std. Dev. acceleration [48, 49] |
| [22, 27, 33, 53] | | |
| 3-4. Stop X,Y [7, 23, 33, 49, 58] | 44-47. First Quartile pressure, area, velocity, acceleration [48] |
| [22, 27, 53] | | |
| 5. Stroke duration [7, 23, 32, 48, 49, 58] | 48-51. Third Quartile pressure, area, velocity, acceleration [48] |
| [22, 27, 33, 39, 53] | | |
| 6. End-to-end distance [7, 23, 39, 48, 58] | 52-55. Extreme point 1,2 - X,Y [48] |
| [22, 27, 33, 53] | | |
| 7. Mid-stroke pressure [7, 23, 27, 48, 49, 53] | 56. Last 2 points tangent [48] |
| [22, 27, 28, 48] | | |
| 8. Mid-stroke area [7, 23, 27, 48] | 57. Velocity at first point [58] |
| [23, 32, 48, 49, 58] | | |
| 9. Length of Trajectory [7, 22, 27, 53] | 58-60. Area, Pressure, Velocity at last point [58] |
| 10. Inter-stroke time [23, 33, 53] | 61. Last moving direction [58] |
| 11. Mean Resultant Length [7, 23, 27, 33] | 62. Average points distance [44, 49, 58] |
| 12. Median acceleration at first 5 points [23, 27, 33] | 63. Std. Dev. points distance [49, 58] |
| 13. Median velocity at last 3 points [23, 27, 33] | 64-68. LDP X, Y, Area, Pressure, Velocity [58, 61] |
| 14. Average velocity [23, 48, 49, 58] | 69-71. Start to LDP Latency, Length, Direction [58] |
| [7, 22, 27, 53] | | |
| 15. Up/Down/Left/Right [7, 23, 33] | 72-74. LDP to Stop Latency, Length, Direction [58] |
| 16. Direction of direct line [7, 23, 27, 39, 53, 58] | 75. Ratio distance to LDP Length [58] |
| 17. Average direction [23] | 76. Total displacement length [49] |
| 18. Ratio of direct distance to trajectory length [23, 27, 33, 53, 58] | 77. Ratio of displacement and trajectory length [49] |
| 19. 20% perc. velocity [23, 27, 33, 53] | 78-81. Median, IQR, Skewness, Kurtosis of distance [49] |
| 20. 50% perc. velocity [23, 27, 33, 48, 49, 53] | 82-86. Avg, Std. Dev, IQR, Skewness, Kurtosis of deviation [49] |
| 21. 80% perc. velocity [23, 27, 33, 53] | 87-92. Avg, Median, Std Dev, IQR, Skewness, Kurtosis of pairwise angles [49] |
| 22. 20% perc. acceleration [23, 27, 33] | 93-98. Avg, Median, Std. Dev, IQR, Skewness, Kurtosis of phase-angles [49] |
| 23. 50% perc. acceleration [23, 27, 33, 48, 49] | 99. Displacement to duration ratio [49] |
| 24. 80% perc. acceleration [23, 27, 33] | 100-102. IQR, Skewness, Kurtosis of velocities [49] |
| 25. 20% perc. deviation [23, 33] | 103-108. Avg, Median, Std. Dev., IQR, Skewness, Kurtosis of angular-velocities [49] |
| 26. 50% perc. deviation [23, 33, 49] | 109-111. IQR, Skewness, Kurtosis of accelerations [49] |
| 27. 80% perc. deviation [23, 33] | 112-114. IQR, Skewness, Kurtosis of pressures [49] |
| 28. Largest deviation [7, 23, 33] | 115-116. Min, Max pressure [44, 56] |
| 29. Pressure at first point [32, 58] | 117-118. Min, Max area [44, 56] |
| 30. Area at first point [32, 58] | 119-120. Min, Max velocity [56, 61] |
| 31. First moving direction [32, 58] | 121-124. Min, Max, Mean of pressure changes [56] |
| 32. Average moving direction [7, 27, 32, 33] | 125-128. Min, Max, Mean of area changes [56] |
| 33. Average moving curvature [32] | 129-130. X, Y at max velocity [61] |
| 34. Average curvature distance [32] | 131-132. X, Y at min velocity [61] |
| 35. Average pressure [32, 39, 48, 49, 58] | 133-135. Quadratic fit pressure x2, x, n [44] |
| 36. Average touch area [22, 32, 39, 44, 48, 58] | 136-138. Min, Max, Avg time duration between points [44] |
| 37. Max-area portion [32] | 139-140. Max deviation of mean X, Y [27] |
| 38. Min-pressure portion [32] | 141-142. 20% perc. deviation of mean X, Y [27] |
| 39. Average acceleration [48, 49] | 143-144. Median deviation of mean X, Y [27] |
| 40. Std. Dev. pressure [48, 49, 58] | 145-146. 80% perc. deviation of mean X, Y [27] |
| 41. Std. Dev. area [48, 58] | 147-148. Direction vector X, Y [22] |
| 42. Std. Dev. velocity [33, 48, 49] | 149. Horizontal/Vertical flag [53] |