Interaction of the branes in the Presence of the Background Fields: the Dynamical Non-intersecting Perpendicular Wrapped-Fractional Configuration

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Abstract

We shall obtain the interaction of the D$p_1$- and D$p_2$-branes in the toroidal-orbifold spacetime $T^n \times \mathbb{R}^{1,d-n-5} \times \mathbb{C}^2/\mathbb{Z}_2$. The configuration of the branes is: non-intersecting, perpendicular, moving-rotating, wrapped-fractional with background fields. For this, we calculate the bosonic boundary state corresponding to a dynamical fractional-wrapped Dp-brane in presence of the Kalb-Ramond field, a $U(1)$ gauge potential and an open string tachyon field. The long-range behavior of the interaction amplitude will be extracted.

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1 Introduction

In development of string theory, a crucial role is played by D-branes \[1\]. An adequate tool for describing D-branes and their interactions is the boundary state formalism \[2\]-\[20\]. In this method a D-brane appears as a source (sink) for emitting (absorbing) all closed string states. All properties of a D-brane are prominently encoded on a boundary state, hence, the interaction between two D-branes is acquired by the overlap of their corresponding boundary states via the closed string propagator. Thus, the boundary state approach for various setups of the stationary and dynamical D-branes with internal background fields in the compact and non-compact spacetimes has been applied \[9\]-\[15\]. In fact, among these configurations the systems with fractional D-branes exhibit some appealing and marvelous behaviors \[16\]-\[23\].

Therefore, in one hand we have the fractional branes which appear in the various parts of the M- and string theories. For example, propagating fractional branes provide a simple and explicit starting point for the definition of Matrix theory \[24, 25\]. Besides, they have been used for demonstrating the gauge/gravity correspondence \[21\]. On the other hand, we have the wrapped branes which have a widespread application in string theory. The fractional and wrapped branes motivated us to study a special setup of the fractional-wrapped branes. In our definition these are the wrapped branes which live in the fixed points of the non-compact orbifold \(\mathbb{C}^2/\mathbb{Z}_2\).

In this paper we apply the boundary state formalism to calculate the interaction amplitude between two non-intersecting perpendicular fractional-wrapped D-branes in the framework of the bosonic string theory. As a result, we shall demonstrate that intersections of the worldsheet of the exchanged closed string with the worldvolumes of the branes create the interaction. However, the background spacetime partially is compact on a torus, and has the topological structure

\[
T^n \times \mathbb{R}^{1,d-n-5} \times \mathbb{C}^2/\mathbb{Z}_2, \quad n \in \{0, 1, \ldots, d-5\}.
\]

We shall consider an arbitrary torus of the set \(\{T^n | n = 0, 1, \ldots, d-5\}\). In addition, we introduce the antisymmetric tensor \(B_{\mu\nu}\), the \(U(1)\) gauge potentials and open string tachyon fields on the worldvolumes of the branes. The branes are dynamical, i.e. they are rotating and are moving within their volumes. In this setup there are various parameters which the strength of the interaction can be adjusted by them. We shall show that the
long-range force, extracted from the interaction amplitude, has a damping nature.

Note that the complete form of the theory includes both the twisted and untwisted sectors under the $\mathbb{Z}_2$-group. Thus, the total interaction amplitude is sum of the amplitudes of the twisted and untwisted sectors. However, since for the various setups the untwisted sector has been extremely studied we shall only concentrate on the twisted sector of our setup.

This paper is organized as follows. In Sec. 2, we obtain the boundary state associated with a dynamical fractional-wrapped $Dp$-brane with the above-mentioned internal background fields. In Sec. 3, the interaction amplitude of two non-intersecting perpendicular $Dp_1$- and $Dp_2$-branes will be calculated. In Sec. 3.1, behavior of the amplitude concerning the large distances of the branes will be investigated. Section 4 is devoted to the conclusions.

2 The $Dp$-brane boundary state

Our starting point is to consider a fractional $Dp$-brane which lives in the $d$-dimensional spacetime, including a toroidal portion $T^n$, and an orbifold part $\mathbb{C}^2/\mathbb{Z}_2$ where the $\mathbb{Z}_2$ group acts on the coordinates $\{x^a | a = d - 4, d - 3, d - 2, d - 1\}$. The orbifold is noncompact, so its fixed points are located at the hyperplane $x^a = 0$. The $Dp$-brane is stuck at these fixed points. In the $d$-dimensional orbifoldized spacetime the brane can possesses the dimensions $p \leq d - 5$.

We begin with the following sigma-model action for closed string

$$S = -\frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \left( \sqrt{-g} g^{ab} G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \epsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \right) + \frac{1}{2\pi\alpha'} \int_{\partial\Sigma} d\sigma \left( A_\alpha \partial_e X^\alpha + \omega_{\alpha\beta} J^\alpha_{\tau\beta} + T^2(X) \right). \quad (2.1)$$

This action contains the antisymmetric field $B_{\mu\nu}$, the tachyon field $T^2(X)$ and the $U(1)$ gauge potential $A_\alpha(X)$. Since the states of the tachyon and gauge fields belong to the open string spectrum they intrinsically adhere to the brane, and hence their corresponding fields appear in the boundary action. $g^{ab}$ and $G_{\mu\nu}$ are the metrics of the worldsheet and the $d$-dimensional spacetime, respectively. $\Sigma$ is the worldsheet of the emitted closed string, and $\partial\Sigma$ is its boundary. The set $\{x^a\}$ specifies the directions along the $Dp$-brane worldvolume.
Here we assume the background fields $G_{\mu \nu}$ and $B_{\mu \nu}$ to be constant, and we apply $T^2 = \frac{1}{2} U_{\alpha \beta} X^\alpha X^\beta$ as the profile of the tachyon field with the constant symmetric matrix $U$. For the $U(1)$ gauge potential we use the trusty gauge $A_\alpha = -\frac{1}{2} F_{\alpha \beta} X^\beta$ where the field strength is constant. The constant antisymmetric angular velocity $\omega_{\alpha \beta}$ represents the linear motion and rotation of the brane, and $J_{r}^{\alpha \beta} = X^\alpha \partial_r X^\beta - X^\beta \partial_r X^\alpha$ specifies the angular momentum density. Therefore, the dynamics of the brane is inside its volume. In fact, the internal fields draw some specific alignments in the brane, and hence in the brane worldvolume there is a broken Lorentz symmetry. This implies that this dynamics is imaginable.

Vanishing of the variation of this action with respect to $X^\mu(\sigma, \tau)$ gives the equation of motion and the following boundary state equations

$$\left( Q_{\alpha \beta} \partial_\tau X^\beta + F_{\alpha \beta} \partial_\tau X^\beta + U_{\alpha \beta} X^\beta + B_{\alpha i} \partial_\sigma X^i + B_{\alpha a} \partial_\sigma X^a \right)_{\tau=0} |B_x\rangle = 0,$$

$$(X^i - y^i)_{\tau=0} |B_x\rangle = 0 ,$$

$$(X^a - y^a)_{\tau=0} |B_x\rangle = 0 ,$$

(2.2)

where the total field strength possesses the definition $F_{\alpha \beta} = B_{\alpha \beta} - F_{\alpha \beta}$, and we defined $Q_{\alpha \beta} = \eta_{\alpha \beta} + 4 \omega_{\alpha \beta}$. The coordinates $\{x^i\}$ refer to the non-orbifold directions, perpendicular to the brane worldvolume. The parameters $\{y^i\}$ specify the location of the brane. Due to the location of the brane at the fixed points of the orbifold we have $\{y^a = 0 | a = d - 4, \ldots, d - 1 \}$.

The mode expansion of the closed string coordinates along the non-orbifoldy directions $x^\alpha$ and $x^i$ is

$$X_\lambda(\sigma, \tau) = x^\lambda + 2 \alpha' p^\lambda \tau + 2 L^\lambda \sigma + \frac{i}{2} \sqrt{2 \alpha'} \sum_{m \neq 0} \frac{1}{m} \left( \alpha_m^\lambda e^{-2im(\tau-\sigma)} + \tilde{\alpha}_m^\lambda e^{-2im(\tau+\sigma)} \right), \quad \lambda \in \{\alpha, i\},$$

(2.3)

where $L^\lambda$ is zero for the non-compact directions and $L^\lambda = N^\lambda R^\lambda$ for the compact directions. $R^\lambda$ is the radius of compactification and $N^\lambda$ is the winding number of the closed string around the compact direction $x^\lambda$. Note that the extra dimensions, which are required by string theory, are compact so small that they cannot be observed. Hence, we introduced the toroidal compactification on some spatial directions. However, the closed string coordinates along the orbifoldy directions have the following solution

$$X^a(\sigma, \tau) = \frac{i}{2} \sqrt{2 \alpha'} \sum_{r \in \mathbb{Z} + 1/2} \frac{1}{r} \left( \alpha_r^a e^{-2ir(\tau-\sigma)} + \tilde{\alpha}_r^a e^{-2ir(\tau+\sigma)} \right),$$

(2.4)
Using the above mode expansions we acquire

\begin{align}
(2\alpha' Q_{\alpha\beta} p^\beta + 2\mathcal{F}_{\alpha\beta} L^\beta + U_{\alpha\beta} x^\beta) |B\rangle^{(0)} &= 0 , \\
U_{\alpha\beta} L^\beta |B\rangle^{(0)} &= 0 , \\
(x^i - y^i) |B\rangle^{(0)} &= 0 , \\
L^i |B\rangle^{(0)} &= 0 ,
\end{align}

(2.5)

for the zero-mode portion of Eqs. (2.2), and

\begin{align}
&\left[ \left( Q_{\alpha\beta} - \mathcal{F}_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta} \right) \alpha^\beta_m + \left( Q_{\alpha\beta} + \mathcal{F}_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta} \right) \tilde{\alpha}^\beta_{-m} \right] |B\rangle^{(osc)} = 0 , \\
(\alpha^i_m - \tilde{\alpha}^i_{-m}) |B\rangle^{(osc)} &= 0 , \\
(\alpha^a_r - \tilde{\alpha}^a_{-r}) |B\rangle^{(osc)} &= 0 ,
\end{align}

(2.6)

for the oscillating part of Eqs. (2.2). The second equation of Eqs. (2.5) implies that for an invertible matrix $U_{\alpha\beta}$ we receive $\ell^\alpha = 0$, where $\ell^\alpha$ is eigenvalue of $L^\alpha$. Thus, the tachyon field wonderfully prevents the closed strings from winding around the compact directions of the brane. For a non-invertible matrix $U_{\alpha\beta}$ the closed strings are permitted to possess such windings. However, for the next purposes we assume that $U_{\alpha\beta}$ to be invertible. The last equation of Eqs. (2.5) reveals that if the direction $x^i$ is compact we obtain $\ell^i = 0$, consequently, the closed strings cannot wind around the non-orbifoldy perpendicular directions.

The first equation of Eqs. (2.5) induces the relation

\begin{equation}
p^\alpha = -\frac{1}{2\alpha'} \left[ (Q^{-1} U)^{\alpha}_{\beta} x^\beta + 2(Q^{-1} F)^{\alpha}_{\beta} \ell^\beta \right].
\end{equation}

That is, the momentum of an emitted closed string, along the brane worldvolume, depends on its center of mass position, the rotation and motion of the brane, the background fields and its winding numbers around the compact directions of the brane. This momentum clarifies that a peculiar potential drastically acts on the emitted closed strings.

Using the coherent state method, the oscillating part of the boundary state takes the form

\begin{equation}
|B\rangle^{(osc)} = \prod_{n=1}^{\infty} [\det M_{(n)}]^{-1} \exp \left[ -\sum_{m=1}^{\infty} \left( \frac{1}{m} \alpha^\lambda_{-m} S_{(m)}^{\lambda\lambda'} \tilde{\alpha}^{\lambda'}_{-m} \right) \right] \\
\times \exp \left[ -\sum_{r=1/2}^{\infty} \left( \frac{1}{r} \alpha^a_r \tilde{\alpha}^a_{-r} \right) \right] |0\rangle_\alpha \otimes |0\rangle_{\tilde{\alpha}} ,
\end{equation}

(2.8)
where $\lambda, \lambda' \in \{\alpha, i\}$. The matrix $S_{(m)}$ is given by

$$S_{(m)}^{\lambda \lambda'} = \left((M_{(m)}^{-1}N_{(m)})_{\alpha \beta}, -\delta_{ij}\right),$$

$$M_{(m)\alpha \beta} = Q_{\alpha \beta} - F_{\alpha \beta} + \frac{i}{2m} U_{\alpha \beta},$$

$$N_{(m)\alpha \beta} = Q_{\alpha \beta} + F_{\alpha \beta} - \frac{i}{2m} U_{\alpha \beta}. \quad (2.9)$$

Advent of the normalization factor is anticipated by the disk partition function $[26]-[28]$. In fact, the coherent state method gives the boundary state (2.8) under the constraint $S_{(m)}S_{(-m)}^T = 1$. This equation introduces some relations among the parameters $\{\omega_{\alpha \beta}, U_{\alpha \beta}, F_{\alpha \beta}\}$, and hence reduces the number of independent parameters.

The boundary state associated with the zero modes possesses the following solution

$$|B\rangle^{(0)} = \frac{T_p}{2 \sqrt{\det(U/2)}} \int_{-\infty}^{\infty} \exp\left[\frac{i}{2} \alpha' \left(U^{-1} Q + Q^T U^{-1}\right)_{\alpha \beta} p^\alpha p^\beta + i(U^{-1} F)_{\alpha \beta} \ell^\alpha p^\beta\right]$$

$$\times \prod_{\alpha=0}^p \left[|p^\alpha\rangle dp^\alpha\right] \otimes \prod_{i=p+1}^{d-5} \left[\delta\left(x^i - y^i\right) |p^i_L = p^i_R = 0\rangle\right]. \quad (2.10)$$

The total boundary state is

$$|B\rangle = |B\rangle^{(osc)} \otimes |B\rangle^{(0)} \otimes |B_{gh}\rangle,$$

where $|B_{gh}\rangle$ is the known boundary state corresponding to the conformal ghost fields

$$|B_{gh}\rangle = \exp\left[\sum_{m=1}^{\infty} (c_{-m}\tilde{b}_{-m} - b_{-m}\tilde{c}_{-m})\right] \frac{\epsilon_0 + \tilde{\epsilon}_0}{2} |q = 1\rangle |\tilde{q} = 1\rangle. \quad (2.11)$$

This boundary state is independent of the orbifold projection, brane dynamics, compactification and the background fields.

3 The D-branes interaction

For showing the importance of the D-branes interactions and connection of this subject with the main problems of physics, for example, one can say that such interactions in the brane-world have been proposed as origin of the inflation $[29, 30]$. In addition, Big-Bang has been actually created by the collision of two D-branes $[31]$. These interactions also have been considered in the early universe for describing the radiation-dominated era. Besides, since the gravity is a force that can penetrate and interact across the branes,
interaction of these hypersurfaces produce the added gravity inside our D3-brane, i.e. our world [32, 33]. Furthermore, the dark matter somehow can be a result of the gravitational interaction between the branes [34]. Finally, the branes interactions also shed light on the gauge/gravity correspondence [21].

Among the parallel, intersecting, and non-intersecting configurations of the D-branes the first and the second setups have been vastly investigated, while the third one has not attracted a considerable attention of the researchers. In fact, this configuration has some particular and remarkable properties. For example, for a system of two non-intersecting D-branes the emitted closed strings from one of the branes usually do not hit to the second brane. Therefore, a zero or very weak interaction is expected. For a system of the non-intersecting perpendicular branes we shall demonstrate that this expectation does not occur. However, some other properties concerning this configuration will be extracted.

In general, let \( d_{or} \) be the dimension of the orbifoldy directions, and \( d_I \) be the number of those directions which are perpendicular to both branes. For having a setup of completely perpendicular branes, i.e. branes without any common spatial direction, there is the following restriction

\[
d = 1 + p_1 + p_2 + d_I + d_{or}.
\]

For the setups with non-intersecting perpendicular branes we should have \( d_I \geq 1 \), i.e.

\[
p_1 + p_2 \leq d - d_{or} - 2. \quad (3.1)
\]

Now we consider the following embedding for the non-intersecting perpendicular \( Dp_1 \)-and \( Dp_2 \)-branes

|       | \( x^0 \) | \( x^1, \ldots, x^{p_1} \) | \( x^{p_1+1}, \ldots, x^{p_1+p_2} \) | \( x^{p_1+p_2+1}, \ldots, x^{d-5} \) | \( x^{d-4}, \ldots, x^{d-1} \) |
|-------|--------|-------------------|-------------------|-------------------|-------------------|
| \( Dp_1 \) | \( \times \) | \( \times \ldots \times \) | | | |
| \( Dp_2 \) | \( \times \) | | \( \times \ldots \times \) | | |

Therefore, for our setup with four orbifoldy directions Eq. (3.1) leads to the condition \( p_1 + p_2 \leq d - 6 \).
The interaction between two D-branes can be conveniently computed by the overlap of the two boundary states, corresponding to the two D-branes, via the closed string propagator. That is, \( \mathcal{A} = \langle B_1 \vert D \vert B_2 \rangle \) where the closed string propagator \( D \) is defined by

\[
D = 2\alpha' \int_0^\infty dt \ e^{-tH_{\text{closed}}},
\]

and the closed string Hamiltonian is given by

\[
H_{\text{closed}} = H_{\text{ghost}} + \alpha' p^\lambda p_\lambda + 2 \left( \sum_{n=1}^{\infty} (\alpha_n^{\lambda} \alpha_n^{\lambda} + \tilde{\alpha}_n^{\lambda} \tilde{\alpha}_n^{\lambda}) + \sum_{r=1/2}^{\infty} (\alpha_r^{a} \alpha_r^{a} + \tilde{\alpha}_r^{a} \tilde{\alpha}_r^{a}) \right) - \frac{d-6}{6}.
\]

(3.2)

The change of the zero-point energy of the Hamiltonian is due to the orbifold projection. After lengthy calculations the interaction amplitude finds the following form

\[
\mathcal{A} = \sqrt{2\pi T_1 T_2} \bigg/ \alpha' \sqrt{\det (U_{1/2}) \det (U_{2/2})} \prod_{n=1}^{\infty} \left[ \det M_{(n)1} M_{(n)2} \right]^{-1} \Theta^3 \left( \frac{y_1^{I_n} - y_2^{I_n}}{i\alpha' t} \right) \exp \left( -\frac{1}{4\alpha' t} \sum_{I_n} (y_1^{I_n} - y_2^{I_n})^2 \right) \frac{\exp \left[ \frac{1}{2} \left( \chi^\dagger G^{-1} \chi + \rho^\dagger R^{-1} \rho \right) + \xi \right]}{\sqrt{\eta \det G \det R}} \prod_{n=1}^{\infty} \left[ \det[1 - S_{(n)1} S_{(n)2} e^{-4nt}]^{-1} (1 - e^{-4nt})^2 (1 - e^{-2(2n-1)t})^{-4} \right],
\]

(3.3)

where

\[
\eta \equiv 2\alpha' + i\alpha' \left[ Q_1^T U_1^{-1} + U_1^{-1} Q_1 - Q_2^T U_2^{-1} - U_2^{-1} Q_2 \right]_{00},
\]

and \( V_0 \) is the common worldvolume of the branes, i.e. the length of the time direction. We decomposed the directions \( \{x^I = p_1 + p_2 + 1, \ldots, d-5\} \), which are perpendicular to both branes, into the compact and non-compact subsets, i.e.

\[
\{I\} = \{I_n\} \cup \{I_c\}.
\]

d_{I_n} \text{ is the number of the noncompact directions } \{x^{I_n}\}. \text{ The other variables have the}
following definitions

\[
G_{\alpha'_1 \beta'_1} = 2t' \delta_{\alpha'_1 \beta'_1} + i \alpha' (Q^T U_1^{-1} + U_1^{-1} Q_1)_{\alpha'_1 \beta'_1} \\
- 2 \alpha' (Q_1^T U_1^{-1} + U_1^{-1} Q_1)_{\alpha'_1 0} (Q_1^T U_1^{-1} + U_1^{-1} Q_1)_{\beta'_1 0},
\]

\[
R_{\alpha'_2 \beta'_2} = 2t' \delta_{\alpha'_2 \beta'_2} - i \alpha' (Q_2^T U_2^{-1} + U_2^{-1} Q_2)_{\alpha'_2 \beta'_2} \\
- 2 \alpha' (Q_2^T U_2^{-1} + U_2^{-1} Q_2)_{\alpha'_2 0} (Q_2^T U_2^{-1} + U_2^{-1} Q_2)_{\beta'_2 0},
\]

\[
\chi_{\beta'_1} = -2 (U_1^{-1} F_1)_{\alpha'_1} \ell_{\alpha'_1} - y_{\beta'_1} - 4 \alpha' (U_1^{-1} F_1)_{\alpha'_1 0} \ell_{\alpha'_1} (Q_1^T U_1^{-1} + U_1^{-1} Q_1)_{\beta'_1 0} \\
+ 4 \alpha' (U_2^{-1} F_2)_{\alpha'_2 0} \ell_{\alpha'_2} (Q_1^T U_1^{-1} + U_1^{-1} Q_1)_{\beta'_2 0},
\]

\[
\rho_{\beta'_2} = -2 (U_2^{-1} F_2)_{\alpha'_2} \ell_{\alpha'_2} - y_{\beta'_2} + 4 \alpha' (U_2^{-1} F_1)_{\alpha'_1 0} \ell_{\alpha'_1} (U_1^{-1} Q_2 + Q_2^T U_2^{-1})_{\beta'_2 0} \\
- 4 \alpha' (U_2^{-1} F_2)_{\alpha'_2 0} \ell_{\alpha'_2} (U_1^{-1} Q_2 + Q_2^T U_2^{-1})_{\beta'_2 0},
\]

\[
\xi = -\frac{1}{\eta} \left[ 2 (F_1 U_1^{-1})_{\alpha'_1 0} \ell_{\alpha'_1} (F_1 U_1^{-1})_{\beta'_1 0} \ell_{\beta'_1} + 2 (U_2^{-1} F_2)_{\alpha'_2 0} \ell_{\alpha'_2} (U_2^{-1} F_2)_{\beta'_2 0} \ell_{\beta'_2} \\
+ 4 (F_1 U_1^{-1})_{\alpha'_1 0} \ell_{\alpha'_1} (U_2^{-1} F_2)_{\beta'_2 0} \ell_{\beta'_2} \right].
\]

(3.4)

In these variables the primed indices represent the spatial directions of the branes, e.g. for the Dp1-brane there is \( \{ x^{\alpha_1} \} = \{ x^0 \} \cup \{ x^{\alpha'_1} | \alpha'_1 = 1, 2, \ldots, p_1 \} \).

We see that the common worldvolume of the branes, i.e. \( V_0 \), appeared in the amplitude. For the non-intersecting perpendicular branes \( V_0 \) is the length of the time direction, which is nonzero and hence there is an interaction. This elaborates that only a portion of the interaction is the effect of “emitting closed strings from volume of one brane and absorbing them by the volume of the other brane”. The main part of the interaction is due to the fact that the worldsheets of the exchanged closed strings intersect the worldvolumes of both branes.

The amplitude (3.3) implies that the strength of the interaction is exponentially damped by the square distance of the branes. In the fourth line the determinant part is contribution of the oscillators of the non-orbifoldy directions, the factor \( \prod_{n=1}^{\infty} (1 - e^{-4nt})^2 \) is the ghosts contribution, and \( \prod_{n=1}^{\infty} (1 - e^{-2(2n-1)t})^{-4} \) comes from the oscillators of the orbifoldy directions. Note that Eq. (3.3) has the symmetry \( A^1 |_{1+2} = A \), which is induced by the formula \( A = \langle B_1 | D | B_2 \rangle \).

For acquiring the interaction amplitude in the non-compact orbifoldized spacetime, in Eqs. (3.3) and (3.4) we should replace \( I_n \rightarrow I, \ d_{I_n} \rightarrow d_I = d - p_1 - p_2 - 5, \ \Theta_3 \rightarrow 1, \ \ell_{\alpha'_1} \rightarrow 0 \) and \( \ell_{\alpha'_2} \rightarrow 0 \).
3.1 Interaction of the distant branes

Behavior of the interaction amplitude for the distant branes represents the long-range force of the theory. That is, after an enough long time the massless closed string states possess a dominant contribution on the interaction, while the contributions of all massive states, except the tachyon state, are drastically damped. Therefore, in the dimension $d = 26$, we introduce the limit $t \to \infty$ on the oscillating part of the general amplitude, i.e. on the last line of Eq. (3.3). Since in this dimension the states of the metric, antisymmetric tensor and dilaton are massless they have zero winding and zero momentum numbers. Thus, we receive

$$
A_{\text{long-range}} = \frac{\sqrt{2\pi T_{p_1} T_{p_2} \alpha' V_0}}{2(2\pi)^{d-p_1-p_2-5}} \prod_{n=1}^{\infty} \left[ \det M^{(n)_1}_{M} \det M^{(n)_2} \right]^{-1} \frac{\sqrt{\det (U_1/2) \det (U_2/2)}}{\sqrt{\det (U_1/2) \det (U_2/2)}}
$$

$$
\times \int_0^\infty dt \left\{ \left( \frac{\pi}{\alpha' t} \right)^{d_{4\infty}} \exp \left( -\frac{1}{4\alpha' t} \sum_{I_{\alpha}} (y_{I\alpha}^1 - y_{I\alpha}^2)^2 \right) \right\}
$$

$$
\times \prod_{I_{\alpha}} \Theta_5 \left( \frac{y_{I\alpha}^1 - y_{I\alpha}^2}{2\pi R_{I\alpha}} \right)^{i\alpha' t} \exp \left[ \frac{1}{2} \left( y_{I\alpha}^2 G^{-1} y_{I\alpha}^2 + y_{I\alpha}^T R^{-1} y_{I\alpha} \right) \right]^{-1} \sqrt{\eta \det G \det R}
$$

$$
\times \lim_{t \to \infty} \left\{ e^{3t} + Tr(S_{(n=1)_1}^\dagger S_{(n=1)_2}) e^{-t} \right\}.
$$

We observe that the contribution of the massless states, i.e. the gravitation, dilaton and Kalb-Ramond, vanishes. Therefore, the orbifold projection extremely eliminates the long-range force. Precisely, this is an effect of the constant term of the closed string Hamiltonian (3.2) which is imposed by the orbifold projection. Note that the untwisted sector of the theory possesses the long-range force. Hence, the total amplitude which comes from the both twisted and untwisted sectors contains a non-vanishing long-range force. However, according to the negative mass squared of the tachyon, the divergent part of Eq. (3.5) elaborates exchange of the tachyonic state. Due to the orbifold projection this divergence is unlike the conventional case.

4 Conclusions and summary

We obtained the boundary state corresponding to a dynamical fractional-wrapped Dp-brane in the following backgrounds: the Kalb-Ramond field, a $U(1)$ gauge potential and
a tachyon field. The brane lives in an orbifold-toroidal spacetime $T^n \times \mathbb{R}^{1,d-n-5} \times \mathbb{C}^2/\mathbb{Z}_2$. We observe that presence of the tachyon field prominently affects the winding of the closed strings around the wrapped directions of the brane.

The emitted closed string acquires a momentum along the brane worldvolume, which is unlike the conventional case. This momentum depends on the position of the closed string and its winding numbers around the compact directions of the brane. This peculiar result demonstrates that the toroidal compactification, background fields, linear and angular motions of the brane, impose a potential on the emitted closed strings.

The amplitude of the interaction for two non-intersecting perpendicular fractional-wrapped $Dp_1$- and $Dp_2$-branes, in the foregoing setup, was calculated. Since the amplitude is nonzero we conclude that the worldvolumes of the branes and the worldsheets of the exchanged closed string play the main role for the interaction. Precisely, for a system of non-intersecting branes the worldsheet of any exchanged closed string intersects the worldvolumes of both branes, and hence interaction takes place. However, the various parameters, i.e., the elements of the Kalb-Ramond and tachyon matrices, the angular and linear velocities of the branes, the dimensions of the spacetime and branes, the orbifoldy directions, the closed string winding and momentum numbers, the coordinates of the branes location, and the radii of compactification, give a general feature to the interaction amplitude. The strength of the interaction can be accurately adjusted by these parameters.

From the total amplitude the interaction concerning the exchange of the massless states and tachyon state was extracted. In the 26-dimensional spacetime the contribution of the massless states vanishes. This is an unconventional and marvelous effect which was imposed by the orbifold projection. In the dimension $d = 32$ we receive a long-range force. In this dimension the divergence part of this interaction, due to the tachyon exchange, finds the conventional form $e^{4t} \to \infty$. Generally, for a given dimension of the orbifold one can manipulate the spacetime dimension to obtain a large distance amplitude of the branes: in the damping form, in the usual form and or in the divergent form.

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