MULTIPARTICLE PRODUCTION AND PERCOLATION OF STRINGS\textsuperscript{a}

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It is shown that the multiplicity distribution associated to self-shadowing events satisfies an universal equation in terms of the minimum bias distribution. The number of elementary collisions, basic in the structure of any multiple scattering, is reduced if a collective effect like percolation of strings takes place. The main consequences of the percolation are briefly discussed.

1 Introduction

Most of the exciting and forthcoming data on particle production on hadron-hadron and nucleus-nucleus collisions can be understood by conventional physics. This does not mean lack of excitement. Indeed, in this paper we show first as an example a surprising general law on multiplicity distributions associated to events which are shadowed by themselves, obtained using nothing but unitarity \textsuperscript{1}.

Also, some data of the forthcoming experiments in the Relativistic Heavy Ion Collider (RHIC) and in the Large Hadron Collider (LHC) can give us many surprises related to new physics. As an example of this, some effects of the percolation of strings\textsuperscript{2,3}, that can be detected already at RHIC, are discussed in this paper.

2 Multiplicity Distributions Associated to Events Shadowed by Themselves.

A multiple collision can be seen as a superposition of many elementary collisions, and it is worth to ask for the propagation of properties existing at the level of elementary collisions. In this way, for example, it is well known the $A-$behavior of the hadron-nucleus cross sections for events which are shadowed.

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by themselves. Indeed, for these events, denoted by C-events, the resulting $h-A$ event is of type C only and only if at least one of the elementary collisions if of type C. It is said, the algebra

$$
C \times \text{no } C = C \\
\text{no } C \times \text{no } C = \text{no } C \\
C \times C = C
$$
is satisfied

From the inelastic cross section

$$
\sigma_{hA}^{hA} = 1 - (1 - \sigma_T(b))^A = \sum_{n=1}^{A} (\sigma_T(b))^n (1 - \sigma_T(b))^{A-n}
$$

retaining in the sum

$$
(\sigma T(b))^n \approx \sum_{i=0}^{n} \binom{n}{i} \sigma_C^i \sigma_{\text{no } C}^{n-i} T(b)^n
$$

the terms with $i \geq 1$, the C-cross section is obtained:

$$
\sigma_{hA}^{hA} = 1 - (1 - \sigma_C T(b))^A
$$

If $\alpha_C$ is small, $\sigma_{hA}^{hA}$ behaves as $A \sigma_C$, otherwise it behaves as $A^{2/3} \sigma_C$.

Let us next consider the particle distribution associated to a rare event C (Drell-Yan pairs, annihilation cross section, particles of a given type produced in a rapidity interval, slow nucleons, high $p_T$ particle, etc.)

If $\alpha_C$ is the probability of event C to occur in an elementary collision and $N(\nu)$ is the number of events with $\nu$ elementary collisions in a nucleus-nucleus collision, the number of events where event C occurs is given by

$$
N_C(\nu) = \sum_{i=1}^{\nu} \binom{\nu}{i} \alpha_C^i (1 - \alpha_C)^{\nu-i} N(\nu)
$$

If event C is rare ($\alpha_C$ small), we can retain the leading term in $\alpha_C$ and

$$
N_C(\nu) \simeq \alpha_C \nu N(\nu)
$$

The total number of C-events and total number of events are:
\[ \sum N_C(\nu) = \alpha C \langle \nu \rangle N \]
\[ \sum N(\nu) = N \]  

Therefore
\[ P_C(\nu) = \frac{\nu}{\langle \nu \rangle} P(\nu) \]  

The multiplicity distribution on produced particles \( P(n) \) is given by
\[ P(n) = \sum_{\nu=1}^{n_1 + \ldots + n_\nu = n} \varphi(\nu) p(n_1)p(n_2) \ldots p(n_\nu) \]  

and the different moments of the multiplicity distribution can be expressed in terms of the different moments of the distribution on the number of elementary collisions and of the distribution on the particles produced in the elementary collision. In particular:
\[ \langle n \rangle = \langle \nu \rangle \bar{n} \]
\[ \frac{D^2}{\langle n \rangle^2} = \frac{\langle \nu^2 \rangle - \langle \nu \rangle^2}{\langle \nu \rangle^2} + \frac{1}{\langle \nu \rangle} \frac{d^2}{\bar{n}^2} \]  

\( \frac{\rho^2}{\langle n \rangle} \) increases with the complexity of the systems involved, from 0.09 in \( e^+e^- \) to 0.25-0.3 in \( pp \) collisions and to 0.8-1 in \( AB \) collisions. In general, the main contribution to the different normalized moments of the total multiplicity distribution is given by the normalized moments of the distribution on the number of elementary collisions, in such a way that in (7) \( \nu \) can be exchanged by \( n \) or \( E_T \), the transverse energy (experimentally \( n \) depends linearly on \( E_T \)):
\[ P_C(n) = \frac{n}{\langle n \rangle} P(n), \quad P_C(E_T) = \frac{E_T}{\langle E_T \rangle} P(E_T) \]  

Relations (10) are universal, independent of \( \alpha_C \). In figure 1 our prediction for Drell-Yan pairs according to (10) is compared with the NA38, SU data. Similar good agreements are obtained for many other cases like high \( p_T \) particles produced in \( \alpha \alpha \) collisions, \( W^\pm \) and \( Z^0 \) production in \( pp \) collisions (see figure 2) or \( pp \) annihilations, as can be seen in reference 1.
2.1 $J/\Psi$ Suppression

In the case of $J/\Psi$ production, final state destructive absorption will eliminate the linearity dependence of $N_C(\nu)$ on $\nu$ by making the effective number of collisions where event C appears smaller. This means that instead of equation (6) we have

$$N_C(\nu) = \alpha_C \nu^\varepsilon N(\nu), \varepsilon < 1$$ (11)

and

$$P_C(E_T) = \frac{E_T^\varepsilon}{\langle E_T^\varepsilon \rangle} P(E_T)$$ (12)

In figure 3 it is seen that the $E_T$ distribution associated to $J/\Psi$ in $SU$ collisions can be described by formula (12) with $\varepsilon = 0.7$.

Notice that from (11) and (12)

$$N_{J/\Psi}(E_T)/N_{DY}(E_T) \sim 1/E_T^{1-\varepsilon}$$ (13)

This ratio decreases with $E_T$ and the curvature, the second derivative, is always positive. In all computations of $J/\Psi$ absorption, including destruction
by comovers, the tendency for a large $E_T$ saturation occurs, which implies positive curvature.

There is, however, another possibility for changing the $\nu$ linearity of equation (6): in a Quark Gluon Plasma (QGP), the $J/\Psi$ and $\Psi'$ formation will be prevented. Now $\alpha_C$ is a function of $\nu$ vanishing for large $\nu$. If the transition is very sharp:

$$\alpha_{J/\Psi}(\nu) = \alpha_{J/\Psi} \quad \nu \leq \nu^*$$
$$\alpha_{J/\Psi}(\nu) = 0 \quad \nu > \nu^*$$

or making a more reasonable approximation

$$\alpha_{J/\Psi}(\nu) = \alpha_{J/\Psi} \exp(-\nu^2/\nu^*^2)$$

Now

$$N_{J/\Psi}(E_T)/N_{DY}(E_T) \sim \exp(-E_T^2/E_T^*^2)$$
in the curvature. At first sight, the NA50 experimental data in $PbPb$ presents a change in the curvature for central events. However, it has been shown that it can be described without any change using formula (12) with $\varepsilon = 0.6$. This value is obtained previously from the low $E_T$ associated probability.

Coming back to our basic relation (16) we can mention another possibility for distinguish absorption or comovers effects from QGP formation. In the first case $P_J(\Psi(E_T)) \geq P(E_T)$ when $E_T \to \infty$, while if plasma is formed $P_J(\Psi(E_T)) < P(E_T)$. The $SU$ data are consistent with the first inequality.

Independently, whether or not, QGP is formed, it is worth to ask for some reason to distinguish central $PbPb$ collisions at SPS energies from the rest of collisions. The answer is percolation of strings.

3 Percolation of Strings

In many models of hadronic collisions, color strings are exchanged between projectile and target. The number of strings grows with the energy and with the number of nucleons of the participating nuclei. When the density of strings becomes high, the string color fields begin to overlap and eventually individual string may form a new string which has a higher color charge at its ends corresponding to the $SU(3)$ sum of the color charges located at the ends of the
original strings. As a result, the higher color means a larger string tension and heavy flavor is produced more efficiently. Also, as the effective number of strings decreases, the mean multiplicity is less than in the case of independent string fragmentation.

In impact parameter space these strings are seen as circles inside the total interaction area. As the number of strings increases, more strings overlap. Above a critical density of strings percolation occurs, so that paths of overlapping circles are formed through the whole collision area, see figure 4. Along these paths the medium behaves like a color conductor. The percolation is a second order phase transition and give rise to a non-thermalized QGP on a nuclear scale. The percolation threshold $\eta_c$ is related to the critical density of circles $n_c$ by the expression

$$\eta_c = \pi r^2 n_c$$

(17)

where $\pi r^2$ is the transverse surface of an string.

$\eta_c$ has been computed using Monte-Carlo simulation, direct-connectedness expansions and other methods. All the results are in the range $\eta_c = 1.12 - 1.18$. Taking $r = 0.2\, fm$ which reproduces the $\bar{\Lambda}$ enhancement observed in $AB$ collisions, it is obtained

$$n_c = 8.9 - 9.3\, \text{strings/fm}^2$$

(18)

In table 1 the number of strings exchanged for central $pp$, $SS$, $FeAir$, $SU$ and $PbPb$ collisions is shown together with their densities. It is seen that at SPS energies only the density reached in central $PbPb$ collisions is above the
Table 1: Number of strings (upper numbers) and their densities (fm$^{-2}$) (lower numbers) in central pp, SS, SU and PbPb collisions at SPS, RHIC and LHC energies.

| $\sqrt{s}$ (AGeV) | Collisions |
|-------------------|------------|
|                   | pp | SS | FeAir | SU | PbPb |
| 19.4              | 4.2 | 123| 89    | 268| 1145 |
|                   | 1.3 | 3.5| 4.42  | 7.6 | 9.5  |
| 200               | 7.2 | 215| 144   | 382| 1703 |
|                   | 1.6 | 6.1| 7.16  | 10.9| 14.4 |
| 5500              | 13.1| 380| 255   | 645| 3071 |
|                   | 2.0 | 10.9| 12.67 | 18.3| 25.6 |

critical density. At RHIC, for central AgAg collisions, the density of strings is 9, just at the critical value. At LHC energies, SS central collisions are already over the critical density. Let us mention that for central FeAir collisions, the critical density is reached at LHC energies. In the laboratory frame, this means that between $10^{17}$ eV and $10^{18}$ eV the percolation of strings occurs in central FeAir collisions. Several experiments of cosmic rays have pointed out a change on the behavior of the atmospheric cosmic ray cascade around the energies mentioned above. The maximum depth of the cascade becomes larger indicating that the energy of the primary is dissipated slower. This is what it was expected if percolation occurs. The multiplicity of FeAir is suppressed and therefore the depth of the cascade increases. The usual belief is that a change of the chemical composition of the primary occurs, and as the energy increases in the range $10^{16}$ eV to $10^{19}$ eV there are less Fe and more protons in the primary cosmic ray. Instead of this we pointed out the possibility that the hadronic phase transition takes place without requiring any ad-hoc change of the chemical composition of the cosmic ray.

Let us point out that if percolation occurs, the energy-momentum of the large cluster of strings is the sum of the energy-momentum of each individual string, and therefore when this object fragments it will produce particles with very large longitudinal momentum, outside of the nucleon-nucleon kinematical limits. This cumulative and spectacular effect could be checked by Brahms detector at RHIC. The percolation in this way behaves like a powerful accelerator.

In 2-dimensional percolation it is known that the fraction $\phi$ of the total area occupied by strings is determined by the formula

$$\phi = 1 - \exp(-\eta)$$

In the vicinity of the phase transition it is satisfied the scaling law for the
number of clusters with \( n \) strings\(^2\)

\[
\langle \nu_n \rangle = n^{-\tau} F(n^{\sigma} (\eta - \eta_c)) \quad |\eta - \eta_c| \ll 1 \quad n \gg 1
\]

(20)

where the critical indices are \( \tau = 187/91 \) and \( \sigma = 36/91 \). The function \( F(z) \) is finite at \( z = 0 \) and falls off exponentially for \( |z| \to \infty \). The equation (20) is of limited value, since near \( \eta = \eta_c \) the bulk of the contribution is still supplied by low values of \( n \), for which (20) is not valid. However from (1) one can finds non-analytical parts of other quantities of interest at the transition point. In particular, one finds a singular part of the total number of clusters \( M = \sum \nu_n \) as

\[
\Delta \langle M \rangle = c |\eta - \eta_c|^{8/3}
\]

(21)

This singularity is quite weak: not only \( \langle M \rangle \) itself but also its two first derivatives in \( \eta \) stay continuous at \( \eta = \eta_c \) and only the third blows up as \( |\eta - \eta_c|^{-1/3} \). This singularity implies also a change on the behavior of the multiplicity. The strength of this singularity would depend on the dependence\(^2\) of the multiplicity of a cluster on the number of strings. Reasonable assumptions, imply that the second derivative of the multiplicity blows up.

4 Conclusions

It is well known that the structure of multiple scattering and the shadowing can explain many unexpected features of experimental data. Here, as an example of this, it is shown that the self-shadowing events have an universal associated multiplicity distribution independently of the detailed type of events, as far as their cross section is small.

Although most of the experimental data of present and may be future heavy ion collisions can be explained by conventional physics, it is sure that there will be surprises and excitement related to the forthcoming RHIC and LHC. As an example, we discuss the possibility of percolation of strings. If this occurs, exotic phenomena should appear like the production of many very fast particle with longitudinal momentum much larger that the permitted by the nucleon-nucleon kinematics or a dumping on the expected multiplicities. It is pointed out, that the physics of cosmic-ray and the new accelerators begin to overlap. In this sense, the observed change in the depth of maximum atmospheric shower could be related to the percolation of strings formed in iron-air collisions.

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References

1. J. Dias de Deus, C. Pajares and C.A. Salgado, Phys. Lett. B409 474-478 (1997); B408 417-421 (1997); B407 335-340 (1997).
2. N. Armesto, M.A. Braun, E.G. Ferreiro and C. Pajares, Phys. Rev. Lett. 77 3736-3738 (1996).
3. M. Braun, C. Pajares and J. Ranft, Santiago preprint US-FT/24-97 (1997).
4. R. Blakenbecler, A. Capella, J. Tran Tan Van, C. Pajares and A.V. Ramallo, Phys. Lett. B107 106-108 (1981).
5. C. Pajares and A.V. Ramallo, Phys. Rev. D31 2800-2812 (1985).
6. C. Charlot (NA38 Collaboration), Proc XXV Rencontres de Moriond, Ed. Frontieres (1990)
7. T. Matsui and H. Satz, Phys. Lett. B178 416 (1986).
8. M. Gonin (NA50 Collaboration), Nucl. Phys. A610 404c (1996).
9. J. Dias de Deus and J. Seixas, hep-ph/9803406.
10. N.S. Amelin, M.A. Braun and C. Pajares, Phys. Lett. B306 312 (1993).
11. H. Sorger, M. Berenguer, H. Stocker and W. Greiner, Phys. Lett. B289 6 (1992).
12. M. Nardi and H. Satz, Bielefeld preprint BI-TP 98/10 (hep-ph/9805247).
13. C. Pajares, D. Sousa and R. Vázquez, U. Santiago preprint US-FT/15-98 (hep-ph/9805475).
14. M.B. Isichenko Rev. Mod. Phys. 64 961 (1992).
15. D. Stauffer and A. Aharony, Introduction to Percolation Theory, Ed. Taylor and Francis (1984).