The Role of Dissipation in the Two–Color Second–Order Coherence Function for Light Sources

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Abstract. We discuss the effect of the dissipation and pump rates in the two-photon spectrum (2PS) for different light sources. In this paper we show the second-order coherence function for the different frequencies given of the systems, we show how this measure reveals more information about the light source, and we show how it changes with dissipation and pump rates. We emphasize the importance of modelling system as an open quantum system for a more realistic description of the spectrum $S(\omega)$ and 2PS of field light.

1. Introduction

The study of light sources with well-defined emission statistics and coherence properties has always been of scientific interest due to its possible technological and fundamental applications. Research has focused on thermal light sources [1, 2], quantum emissions [3] and coherent emissions [4, 5], among others. The photon statistics and coherence properties are the main tools for discussing the nonclassical properties of light fields. This can be described in the framework of Glauber’s theory of coherence function [6]. We distinguish between the various orders of coherence, according to the number of conditions satisfied. The first-order coherence define the emission spectrum which, gives us the intensity of photons emitted by the system $S(\omega)$ at a certain frequency in the steady state $\hat{\rho}_{ss}$:

$$S(\omega) = \frac{1}{2\pi} \int_{0}^{\infty} d\tau e^{i\omega\tau} \langle \hat{a}^\dagger \hat{a}(\tau) \rangle,$$

where $\hat{a}$ ($\hat{a}^\dagger$) is the annihilation (creation) field operator, $\tau$ is the delay time with $\tau = 0$ the time in which the system reaches the steady state and the expected values $\langle \ldots \rangle$ are calculated in $\hat{\rho}_{ss}$. The quantum theory of optical second-order coherence was developed by Glauber [6] in an attempt to understand the Hanbury Brown & Twiss (HBT) experiment [7], and both are cornerstones in the research of quantum light. The second-order coherence function $g^{(2)}(\tau)$ is defined by [8]:

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger \hat{a}^\dagger(\tau) \hat{a}(\tau) \hat{a} \rangle}{\langle \hat{a}^\dagger(\tau) \hat{a}(\tau) \rangle \langle \hat{a}^\dagger \hat{a} \rangle}.$$  

Where again the expected values are calculated at the steady state. At zero delay time $\tau = 0$, the second-order coherence function $g^{(2)}(0)$, for some quantum states, is able to take values...
that are excluded in the classical optical coherence theory in which holds the inequalities \(g_{\text{classic}}(0) \geq g_{\text{classic}}^{(2)}(\tau) \geq 1\). In the quantum case we can have:

\[
g^{(2)}(0) \begin{cases} > 1 & \text{bunching} \\ = 1 & \text{coherent} \\ < 1 & \text{antibunching} \end{cases}
\]

Therefore the antibunching regime is a strong signature of non–classical light.

A generalization of the coherence theory is a measure of the coherence simultaneously in times and frequencies [9]. Mathematically, this amounts to adding exponential decays in the Fourier transform of the time–autocorrelation. To first-order, the output field allows to compute the time-dependent power spectrum of emission as the density of output photons with frequency \(\omega_1\) at time \(T_1\), i.e.,

\[
S^{(1)}_{\Gamma_1}(\omega_1, T_1) = \frac{\Gamma_1}{2\pi} \int_{-\infty}^{T_1} dt_1' dt_1'' e^{-\frac{\Gamma_1}{2}(T_1-t_1')} e^{-\frac{\Gamma_1}{2}(T_1-t_1'')} \langle \hat{a}(t_1')\hat{a}(t_1'') \rangle ,
\]

where \(\Gamma_1\) is interpreted as the detector’s line width. This so-called physical spectrum reduces to the Wiener–Khinchin theorem in the steady state and in the limit \(\Gamma_1 \to 0\) [9].

The generalization at second-order is the so-called \textit{two–color second–order coherence function}, which measure the intensity correlation by two detectors with delay time \(\tau\) at frequencies \(\omega_1\) and \(\omega_2\) [8, 10]. These photon correlations combining both their frequency and time information are now routinely measured in the laboratory. These experiments have proven extremely powerful in characterising quantum systems such as a resonantly driven emitter [11], the strong coupling of light and matter [12], to perform quantum state tomography [13], to monitor heralded single photon sources [14] or to access spectral diffusion of single emitters [15]. This correlator can be defined as:

\[
S^{(2)}_{\Gamma_1, \Gamma_2}(\omega_1, T_1; \omega_2, T_2) = \frac{\Gamma_1 \Gamma_2}{(2\pi)^2} \int_{-\infty}^{T_1} dt_1' dt_1'' e^{-\frac{\Gamma_1}{2}(T_1-t_1')} e^{-\frac{\Gamma_1}{2}(T_1-t_1'')} \times \int_{-\infty}^{T_2} dt_2' dt_2'' e^{-\frac{\Gamma_2}{2}(T_2-t_2')} e^{-\frac{\Gamma_2}{2}(T_2-t_2'')} \langle \hat{T}_+ [\hat{a}(t_1')\hat{a}(t_2'')] \hat{T}_- [\hat{a}(t_1)\hat{a}(t_2)] \rangle ,
\]

where we have defined \(\hat{T}_+ \) (\(\hat{T}_-\)) to order the operators in a product with the latest time to the far left (far right) [8]. Normalising this expression yields the sought two–color second–order coherence function

\[
g^{(2)}_{\Gamma_1, \Gamma_2}(\omega_1, T_1; \omega_2, T_2) = \frac{S^{(2)}_{\Gamma_1, \Gamma_2}(\omega_1, T_1; \omega_2, T_2)}{S^{(1)}_{\Gamma_1}(\omega_1, T_1) S^{(1)}_{\Gamma_2}(\omega_2, T_2)} .
\]

This is an increasingly popular experimental observable by inserting filters in the arms of a standard HTB setup [3]. We compute efficiently and easily \(g^{(2)}_{\Gamma_1, \Gamma_2}(\omega_1, T_1; \omega_2, T_2)\) through the coupling of two ‘sensors’ to the open quantum system of interest, using a new theory of photodetection described in [16], through the Lindblad Master Equation (LME) [17]

\[
\frac{d}{dt} \hat{\rho}(t) = -i \left[ \hat{H}_{\text{system}} + \hat{H}_{\text{sensor}}, \hat{\rho}(t) \right] + \left( \frac{\kappa}{2} \hat{c} \hat{c}^\dagger + \frac{P}{2} \hat{\eta} \hat{\eta}^\dagger + \sum_{i=1}^{2} \frac{\Gamma_i}{2} \hat{c}_i \right) \hat{\rho}(t);
\]

\[
\hat{H}_{\text{sensor}} = \sum_{i=1}^{2} \left[ \omega_i \xi_i^\dagger \xi_i + \varepsilon_i (\hat{a}^\dagger \xi_i + \xi_i^\dagger \hat{a}) \right] ; \quad \hat{n}_i = \xi_i^\dagger \xi_i ;
\]

\[
\mathcal{L}_c \hat{\rho}(t) = 2\varepsilon \hat{\rho}(t) \hat{c}^\dagger - \hat{c}^\dagger \hat{\rho}(t) \hat{c} - \hat{\rho}(t) \hat{c}^\dagger \hat{c} .
\]
where $\hat{H}_{\text{system}}$ describes the system with $\kappa(P)$ the dissipation(pumping) rate. $\hat{H}_{\text{sensor}}$ is the coupling between the system and sensors represented by two-level operators $\hat{\zeta}_i$ with $i = 1, 2$, with $\varepsilon_i$ the coupling strength between sensors and system. The sensors are two level systems characterised by transition frequencies $\omega_i$, and decay rates $\Gamma_i$. The two–color second–order coherence function is given by [16]:

$$g^{(2)}_{1,2}(\omega_1, T_1; \omega_2, T_2) = \lim_{\epsilon_1, \epsilon_2 \to 0} \frac{\langle \hat{n}_1(T_1) \hat{n}_2(T_2) \rangle}{\langle \hat{n}_1(T_1) \rangle \langle \hat{n}_2(T_2) \rangle}. \quad (8)$$

The simultaneous measure of the two–color second–order coherence function—$T_1 = T_2 = 0$, define the two-photon spectrum (2PS), which describe the statistic of the light sources in function of the frequency. In this work we compute the 2PS for the different systems, under the theory of the generation of light states.

2. Generation of the light sources

In this section we briefly review the generation theory of some relevant quantum states of light: coherent, thermal and photon number states. We focus in the dissipation effects on the second-order coherence function. We consider the following three cases that model optical cavities for the generation of light states:

$$\begin{align*}
\hat{H}_{\text{th}} &= \omega_a \hat{a}^\dagger \hat{a} & \kappa = 0.1 & P = 0.01, \quad (9) \\
\hat{H}_{\text{coh}} &= \omega_a \hat{a}^\dagger \hat{a} + \omega_a (\hat{a}^\dagger + \hat{a}) & \kappa = 0.1 & P = 0.01, \quad (10) \\
\hat{H}_{\text{SPS}} &= \omega_a \hat{a}^\dagger \hat{a} + \omega_a (\hat{a}^\dagger + \hat{a}) - \omega_0 (\hat{a}^\dagger \hat{a}^2 + \hat{a}^2 \hat{a}^\dagger) & \kappa = 0.1 & P = 0, \quad (11)
\end{align*}$$

where $\omega_a$ is the frequency of the free field. Their respective quantum algebra is given by the bosonic commutation rule ($[\hat{a}, \hat{a}^\dagger] = 1$). The first case (9), is the single mode electromagnetic field Hamiltonian. The steady state of (9) under a LME evolution (7) is a mixed thermal state. Physically is the incoherent superposition of many uncorrelated sources. Note that to generate a thermal state, it is always necessary to interact with the cavity walls ($\kappa \neq 0$ and $P \neq 0$). The second one (10), is the Hamiltonian to generate a coherent state by displacing the vacuum state $|0\rangle$ [18]. To generate a coherent state of light we need an externally forced-driven harmonic oscilator, for example a classical oscillating current that is resonantly coupled to the electromagnetic field in a coherent process ($\kappa = 0$ and $P = 0$). But real systems are not perfectly isolated and therefore we take dissipation effects into account: $\kappa \neq 0$ and $P \neq 0$. Finally, the third case (11), is the spontaneous parametric down conversion (SPDC) effective Hamiltonian to produce a single photon source (SPS) [19]. As in the previous model, we simulate a more realistic SPDC source introducing a dissipative rate.

3. Results and discussions

The second-order coherence function at zero delay $g^{(2)}(0)$, can be calculated analytically in the above systems restricting ourselves to the steady state and the $N = 6$ photon manifold. For thermal and SPS sources we have $g^{(2)}(0)|_{\text{th}} = 2$–bunching and $g^{(2)}(0)|_{\text{SPS}} = 0$–antibunching, respectively. In the coherent source we obtain:

$$g^{(2)}(0)|_{\text{coh}} = 2 - \frac{16(P - \kappa)^2}{(P(P - \kappa)^2 + 4\kappa)^2}, \quad (12)$$

where we have taken $\omega_a = 1$. This expression reveals several interesting features: (i) When $P \to 0$ and $\kappa \to 0$, then $g^{(2)}(0) \to 1$–coherent, (ii) if $P = \kappa$, or $P \to \infty$ ($\kappa \to \infty$), then the system is bunching $g^{(2)}(0) \to 2$, and (iii) there exist values for which the emission is coherent.
Figure 1. Steady state second-order coherence function in the time domain $g^{(2)}(\tau)$ for the different sources: $\hat{H}_{\text{coh}}$ (blue line), $\hat{H}_{\text{th}}$ (green line) and $\hat{H}_{\text{SPS}}$ (purple line). Exact theoretical states: Coherent state $|\alpha|^2 = 1$, $\kappa = 0$, $P = 0$ (blue dashed line), thermal state $\bar{n} = 0.11$, $\kappa = 0.1$, $P = 0.01$ (green dashed line) and Fock state $|1\rangle$, $\kappa = 0.15$, $P = 0.06$ (purple dashed).

Figure 2. $g^{(2)}(0)$ for the coherent source (see (12)). In the white region the system is coherent, in the red region is bunched and blue region is antibunched. Note that the system can not has $g^{(2)}(0) > 2$.

with $P \neq 0$ and $\kappa \neq 0$.

We have solved numerically (9, 10 and 11) under the dynamic given by (7) in the steady state limit, on a Hilbert space truncated to $N_{\text{max}} = 20$ photons that ensure convergence. We used different decay $\kappa$ and pump $P$ rates, in units of $\omega_a$. In the figure 1 we show the $g^{(2)}(\tau)$ behaviour for the different sources and compared with the exact theoretical states. Unsurprisingly, the thermal source $\hat{H}_{\text{th}}$ has a $g^{(2)}(\tau)$ that coincides exactly with the theoretical thermal state. The SPS $\hat{H}_{\text{SPS}}$ produces antibunching emission and its second-order coherence function oscillates with an envelope given by the theoretical single-photon state. These oscillations are due to the decay is smaller than the fundamental frequency of the system, but for some delay times the emission is bunched. In the case of the coherent source $g^{(2)}(\tau)$ oscillates around the theoretical value $g^{(2)}(\tau) = 1$. The $g^{(2)}(0)$ of the coherent source $\hat{H}_{\text{coh}}$ is described by (12), and it depends on the decay and incoherent pump rates. In the figure 2, we show that the source is able to have bunched or coherent emission controlling the dissipation and pump rates. The antibunching region is an effect of the truncation in the solution. In figure 3 we have studied $g^{(2)}(\tau)$ for $\hat{H}_{\text{coh}}$ with different decay and pump parameters. For $P < \kappa$ the second-order coherence function oscillates around 1 and the detection is bunched, for $P > \kappa$ the second-order coherence function is damped and the detection can be coherent or bunched. This behaviour resides in the limited range of validity of (12).

We have studied the impact of the dissipation in the 2PS using the sensing method (see (8)) for all three sources. The results are presented in figure 4. The spectrum and the 2PS for the thermal source is showed in the panel (a). Its spectrum corresponds to a Lorentzian lineshape centred in one. The detection in the 2PS is bunched for all frequency pairs correlated. The 2PS of the $\hat{H}_{\text{th}}$ is structureless and evidences than the emission process is indistinguishable in frequency [16]. The panel (b) shows the spectrum and 2PS for a coherent source with $\kappa = 0.1$ and $P = 0.01$. The spectrum shows two peaks centred in $\omega = 0$ and $\omega = 1$, and the 2PS
Figure 3. Coherent source $g^{(2)}(\tau)$ for different dissipation and pump regimes. Purple line: $P = 0.67$ and $\kappa = 1$; blue line: $P = 0.01$ and $\kappa = 0.1$; green line: $P = 0.1$ and $\kappa = 0.069$. In the inset the zero delay value are shown.

Figure 4. Spectrum (top panel) and 2PS (bottom panel) for the different sources. $\hat{H}_{th}$ in (a), $\hat{H}_{coh}$ in (b), and $\hat{H}_{SPS}$ in (c). The figure colour code is in a logarithmic scale with: blue $<\log(1)$–antibunching, white $=\log(1)$–coherent and red $>\log(2)$–bunching. In the top panel the values of $g^{(2)}(0)$ are shown.

shows bunching and antibunching in the emission. Panel (c) shows the spectrum and the 2PS when we have the SPS. Its spectrum exhibits a Mollow triplet [20], and the 2PS exhibits a richer and complex pattern around the peaks. The correlations of the central peak with its side-peaks produce antibunched detection, which corresponds to distinguishable emissions occurring in different paths. The correlation between the side-peaks produce bunched detection due to the indistinguishable frequencies of emissions with opposite signs. The correlation of the peaks with itself can be indistinguishable bunching if the system has a cascade emission (central peak) or it is antibunching (side peaks).

The figure 5 shows the spectrum and the 2PS for the coherent source $\hat{H}_{coh}$ at different values of $g^{(2)}(0)$. The spectrum exhibits a narrow peak centred in $\omega = 0$ and a lateral Lorentzian lineshape centred in $\omega = 1$, with variable width that changes with pumping and dissipation rates. The 2PS shows than the coherent source has two frequency modes relevant in the cascade emission process at $\omega = 0$ and $\omega = 1$ and their correlations can be distinguishable or indistinguishable.

In the figure 5(a) we show the highly coherent emission regimen, i.e., $g^{(2)}(0) \approx 1$. In this case the correlation at $\omega = 0$ is distinguishable in frequency and the correlation at $\omega = 1$ is indistinguishable. In this regime, the spectrum is composed by more than one frequency mode and the 2PS structure is more complex. The detection filtering around $\omega = 1$ is highly bunched.
Figure 5. Spectrum (top panel) and 2PS (bottom panel) for the coherent source $\hat{H}_{coh}$. Different regimens in $g^{(2)}(0)$ are shown. Colour code same as in figure 4.

The opposite situation occurs when $g^{(2)}(0) \approx 2$ –see figure 5(c). In this case the correlation at $\omega = 0$ is indistinguishable and at $\omega = 1$ is distinguishable in frequency. The spectrum widens and the 2PS structure disappears, leaving an antibunching region non-localised. Far from the coherent condition (see figure 5(b)), the coherent source produces antibunched detection filtering distinguishable emissions around $\omega = 1$.

4. Conclusions
In this work, the effects of the dissipation and pump rates on the second order coherence function for three models of quantum light sources were studied. We have identified a new condition $\kappa \neq 0$ and $P \neq 0$ where the coherent source $\hat{H}_{coh}$ has incoherent emission. We have simulated, for different parameter regimes the steady state under an open quantum systems dynamic, and calculated the spectrum, the second-order coherence function and the 2PS. Our results emphasize the importance of modelling the quantum source as an open quantum system for a more realistic description of the $S(\omega)$ and 2PS of light field. The values of the dissipation $\kappa$ and pumping $P$ rates can be used as control parameters to modify the spectral or correlation properties of the sources. Finally, we have showed the importance that has the filtering of the emitted light over the bunching or antibunching properties, particularly in the case of single photon and coherent sources.

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