Performance Analysis of Linear Quadratic Regulator Method in Microalga Growth with Control of Light Intensity

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Abstract. Microalgae is an alternative energy source that considered capable of answering fuel oil scarcity problem. The growth of microalgae plays an important role in biomass production. One of the factors that influence the growth of microalgae is the control of light intensity so that it is necessary to adjust the intensity of the incoming light to produce maximum biomass. Control the intensity of light on the microalga growth can be modelled in the form of differential equations and can be seen as a system. The control method chosen in this study was a linear quadratic regulator (LQR), as well as a comparison with the previous method, namely Pontryagin minimum principle (PMP). The simulation results show that at LQR an increase in the amount of biomass reaches 91.77% from initial condition. While the PMP has doubled from its initial condition.

1. Introduction

One of which done several countries to increase energy independence and avoid national energy crisis is to prepare potential biofuels (biomass) potential, which derived from microalgae biomass materials. Microalgae is the most efficient plants in capturing, utilizing solar energy, and $CO_2$ for photosynthesis [1]. Indonesia has very abundant biodiversity, including the microalgae biodiversity. Microalgae is generally microscopic microorganisms (diameters between 3-30 μm) which included in algae class and live as colonies and single cells in all fresh and marine waters. The morphology of microalgae is unicellular or multicellular but there is no clear division of organ functions in the component cells. That is what distinguishes microalgae from higher plants. Microalgae is low-level plants that have the potential to produce biomass raw materials. Based on several studies, microalgae have a very large ability to produce biomass of approximately 60% of dry weight. Related to the production of microalgae, the process of making this biomass begins with the cultivation of microalgae, harvesting, and drying. The results of the development of microalgae in one-ton tubs in aquaculture media can only produce 2-3% of the biomass. The results obtained are very far from the potential of microalgae itself. So, to get the maximum biomass production, the calculation of the control principle is needed. A lot of biomass results are influenced by light and the level of nutrient dilution which will later be used as photosynthetic material to produce biomass [2].
The microalgae growth is discussed in many studies [3-10]. In this study, literature used derived from the research Rizky Nur Ardiansyah [11] entitled "Optimization of Microalgae Growth with Arrangement of Light Intensity by Pontryagin Minimum Principle". The optimal control carried out with control of light intensity so as to produce more biomass. Optimal control is needed in accordance with the chosen model. Therefore, in this study linear quadratic regulator (LQR) method was used, then a comparison was made between Linear Quadratic Regulator (LQR) method and Pontryagin Minimum Principle (PMP) from previous work [11]. Then the simulation is done using MATLAB software. From the simulation results, a comparison of biomass results between methods will be made.

2. Basic Theory

This section explains the theory used in this research to obtain the results of system analysis and optimal control of the system

2.1 The Mathematical Model

The mathematical model of microalgae growth that used is the Droop model that has been developed [12] to:

\[
\frac{ds}{dt} = DS_{in} - DS - \bar{\rho}\left(\frac{s}{s+K_S}\right)\left(\frac{q_l-q_0}{q_1-q_0}\right)X
\]

\[
\frac{dq}{dt} = \rho\left(\frac{s}{s+K_S}\right)\left(\frac{q_l-q_0}{q_1-q_0}\right) - \mu\left(\frac{l}{l+K_L}\right)(1 - \frac{q_0}{q})Q
\]

\[
\frac{dx}{dt} = \mu\left(\frac{l}{l+K_L}\right)\left(1 - \frac{q_0}{q}\right)X - rX - DX
\]

with:

- \(Q(t)\) : Cell quota
- \(X(t)\) : Biomass concentration
- \(S(t)\) : Nutrient concentration
- \(S_{in}(t)\) : The concentration of external nutrients that enter microalgae
- \(D\) : Dilution rate
- \(\mu(q)\) : Growth of microalgae
- \(p(s)\) : Absorption of biomass
- \(\bar{\rho}\) : Maximum growth in microalgae (internal nitrogen quota is limited)
- \(\rho\) : Maximum absorption of biomass nutrition
- \(K_S\) : Half saturation constant
- \(Q_0\) : Minimum cell quota (below this level there is no microalgae growth)
- \(r\) : Respiration.
- \(l\) : Light.
- \(K_L\) : Half-saturation light coefficient.

| Table 1 The values of the parameters used |
|-----------------------------------------|
| Parameter     | Value                       |
|---------------|-----------------------------|
| \(S_{in}\)    | 1 mgN/L                    |
| \(D\)         | 0.4 gN gC^{-1} day^{-1}    |
| \(l\)         | 1.2 \(\mu\) mol quanta. m^{-2}s^{-1} |
| \(\bar{\rho}\) | 0.073 gN gC^{-1} day^{-1}  |
| \(\mu\)       | 1.7 day^{-1}               |
| \(r\)         | 0.07 day^{-1}              |
| \(K_S\)       | 0.0012 gN/m^3              |
| \(K_L\)       | 20 \(\mu\) mol quanta. m^{-2}s^{-1} |
| \(Q_l\)       | 0.25 gN C^{-1}             |
| \(Q_0\)       | 0.05 gN C^{-1}             |
Source: Bernard, O., Grognard, Frederic., and Masci, P. “Microalgal biomass surface productivity optimization based on a photobioreactor model” [12].

2.2 Optimal Control Design
This section discusses the design of LQR controls for a system of mathematical models of microalgae growth. LQR is a control system that consists of a system and gains feedback system with cost function is obtained by:

\[ J = \frac{1}{2} x^T P_c x + \frac{1}{2} \int_{t_0}^{t_f} (x^T Q_c x + u^T R_c u) dt \]  

(4)

LQR which has been described is the design for a control system with an infinite time span where \( t_f \) is the final time of control and \( Q_c \) is a symmetrical matrix, semi-definite is positive and \( R_c \) is a symmetrical matrix, positive definite. The next step is to find the feedback gain regulator. The law of control of a system can be written:

\[ u_c = -K_c x \]  

(5)

with \( K_c \) is the regulator's gain feedback value obtained from:

\[ K_c = R_c^{-1} B^T P_c \]  

(6)

with \( P_c \) is found with Riccati's equation settlement:

\[ A^T P_c + P_c A - P_c B R_c^{-1} B^T P_c + Q_c = 0 \]  

(7)

In completing the Riccati's equation above, the value of \( Q_c \) and \( R_c \) is needed first. \( Q_c \) is a positive matrix of symmetry and semi-definite system variables. \( R_c \) is a matrix of weight values that enter asymmetrical and definite positive systems. The values of \( Q_c \) and \( R_c \) are obtained by trial and error. The following are the steps in determining the \( K_c \) gain [13]:
1. Determine the value of \( R_c \) and \( Q_c \).
2. The \( R_c \) value is selected first and the determination of \( Q_c \).
3. Look for the value of \( P_c \) with the MATLAB toolbox.
4. Look for the gain value \( K_c \).
5. System simulation with simulink MATLAB.
6. Characteristic analysis to obtain the optimal system, namely the fastest stable time and maximum results.

3. Results and Discussion
This section discusses system analysis, equilibrium point and stability, controllable and observable analysis. After that, optimal control obtained using Linear Quadratic Regulator (LQR) and simulated using MATLAB software.

3.1 Equilibrium and stability point
Based on definition of equilibrium point [14], the equilibrium point obtained by equating zero states in equations (1), (2), and (3), namely \( \dot{s} = 0, \dot{q} = 0, \dot{x} = 0 \).

Obtained:

\[ \bar{E}_1 = \left( 1, \frac{(1+a_1)(a_2a_6)+a_4a_5a_8}{(1+a_1)(a_2a_6)+a_4a_8}, 0 \right) \]  

(8)

and

\[ \bar{E}_2 = (\bar{s}_2, \bar{q}_2, \bar{x}_2) \]  

(9)
which,

\[
\bar{S}_2 = \frac{a_1(a_2)^2(a_6)^2a_7 - a_1a_2a_6(a_7)^2 - a_1a_2a_6a_7}{(a_4a_6)(a_2a_3a_6a_7 - a_2a_3a_6)(a_2a_6a_7 - 1)} + \frac{a_1(a_2)^2(a_6)^2a_7 - a_1a_2a_6a_7 - a_1a_2a_6}{a_2a_6(a_2a_6a_7 - 2) + a_2a_6(a_2a_6a_7 - 1)}
\]

\[
\bar{q}_2 = \frac{a_1(a_2)^2(a_6)^2a_7 + a_1a_2a_6a_7 + a_1(a_2)^2(a_6)^2 - a_1a_2a_6a_7 - a_1a_2a_6}{a_2a_6(a_2a_6a_7 - 2) + a_2a_6(a_2a_6a_7 - 1)}
\]

\[
\bar{x}_2 = \frac{a_2a_6a_7 - 1}{a_2a_6a_7 - 1}
\]

Then, linearized the model and obtained Jacobi matrix is:

\[
J = \begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & 0 \\
0 & a_{32} & a_{33}
\end{pmatrix}
\]

(10)

which,

\[
a_{11} = -1 - \frac{sx_4(a_5 - q)}{(s+a_1)^2} + \frac{sx_4(a_5 - q)}{(s+a_1)^2}
\]

\[
a_{12} = \frac{sx_4}{s+a_1}
\]

\[
a_{13} = \frac{sa_4(q - a_5)}{s+a_1}
\]

\[
a_{21} = \frac{sa_4a_6(a_5 - q)}{(s+a_1)^2}
\]

\[
a_{22} = -s/a_1
\]

\[
a_{23} = a_2a_6
\]

\[
a_{33} = a_2a_6(1 - \frac{1}{q}) - a_7 - 1
\]

Then the system stability is analysed around equilibrium points \(\bar{E}_1\) and \(\bar{E}_2\). For equilibrium point \(\bar{E}_1\), obtained:

\[
\lambda_1 = -1
\]

\[
\lambda_2 = -\frac{a_4a_6}{a_6} - a_2a_6 = -1.152
\]

\[
\lambda_3 = a_2a_6(1 - \frac{(1+a_1)(a_2a_6 + a_4a_6)}{(1+a_1)(a_2a_6 + a_4a_6)}) - a_7 - 1
\]

\[
= -0.9923
\]

For equilibrium point \(\bar{E}_2\), Routh Hurwitz criteria are used and based on calculations are stable.

### 3.2 Controllable and observable analysis

The microalgae growth model is said to be controlled if the matrix \(M_c = \begin{pmatrix} B & AB \end{pmatrix} \begin{pmatrix} A & B \\ B & 0 \end{pmatrix}\) has a rank equal to \(n\) [14].

which,

\[
\tilde{A} = \begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & 0 \\
0 & a_{32} & a_{33}
\end{pmatrix}
\]
and
\[
\bar{B} = \begin{pmatrix}
-a_2(q-1) \\
ax(1-\frac{1}{q}) \\
0
\end{pmatrix} = \begin{pmatrix}
0 \\
b_2 \\
b_3
\end{pmatrix}
\]

For matrix \(\bar{AB}\) obtained:
\[
\bar{AB} = \begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & 0 \\
0 & a_{32} & a_{33}
\end{pmatrix} \begin{pmatrix}
0 \\
b_2 \\
b_3
\end{pmatrix}
= \begin{pmatrix}
a_{12}b_2 + a_{13}b_3 \\
a_{22}b_2 \\
a_{32}b_2 + a_{33}b_3
\end{pmatrix} = \begin{pmatrix}
c_1 \\
c_2 \\
c_3
\end{pmatrix}
\]

For matrix \(\bar{A^2}\) obtained:
\[
\bar{A^2B} = \begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & 0 \\
0 & a_{32} & a_{33}
\end{pmatrix} \begin{pmatrix}
c_1 \\
c_2 \\
c_3
\end{pmatrix}
= \begin{pmatrix}
a_{11}c_1 + a_{12}c_2 + a_{13}c_3 \\
a_{21}c_1 + a_{22}c_2 \\
a_{32}c_1 + a_{33}c_3
\end{pmatrix} = \begin{pmatrix}
d_1 \\
d_2 \\
d_3
\end{pmatrix}
\]

the \(M_c\) control matrix is formed into:
\[
M_c = \begin{pmatrix}
0 & c_1 & d_1 \\
b_2 & c_2 & d_2 \\
b_3 & c_3 & d_3
\end{pmatrix}
\]

then the calculation obtained rank \(M_c = 3\). Thus the microalgae growth model is controlled.

Then, the observation analysis can be done by forming the \(M_o\) observation matrix, namely:
\[
M_o = \begin{pmatrix}
C & CA \\
CA & CA^2
\end{pmatrix}
\]

which,
\[
\bar{A} = \begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & 0 \\
0 & a_{32} & a_{33}
\end{pmatrix}
\]

and
\[
\bar{C} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

For Matrix \(CA\) obtained:
\[
CA = \begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & 0 \\
0 & a_{32} & a_{33}
\end{pmatrix}
\]

Then Matrix \(CA^2\) obtained:
obtained observability matrix $M_o$:

$$
M_o = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & 0 \\
a_{11}a_{21} + a_{12}a_{22} & a_{12}a_{21} + a_{13}a_{32} & a_{11}a_{13} + a_{13}a_{33} \\
a_{21}a_{32} & a_{22}a_{32} + a_{32}a_{33} & a_{23}^2
\end{pmatrix}
$$

(12)

then the calculation obtained rank $M_o = 3$. Thus the microalgae growth model is observable.

### 3.3 Simulation Results Before Controlled

In this section simulated on the microalgae growth model with initial conditions given before optimal control.

![Figure 1. Simulation of microalgae growth models before controlled](image)

Based on Fig.1 can be seen the simulation results in 10 days with conditions of initial nutrient concentration is 20, the initial cell quota concentration is 10, and the initial concentration of microalgae is 1.825. The graph shows that the concentration of nutrients continues to decline from first day to seventh day. The cell quota concentration also decreased from the first day to the fourth day. Because nutrition and cell quota have decreased, this condition causes the biomass concentration to decrease continuously from the first day to the fourth day.
From the figure above, there is always a decrease in the amount of microalgae concentration and continuously decreases until the concentration runs out. Increasing the number of microalgae without control depends on the initial administration of nutrient concentration, and cell quota.

3.4 Simulation Results After Controlled
Before performing optimal control on the microalgae growth model, look for \( Q_c \) and \( R_c \) matrix first using MATLAB software by trial and error so that \( Q_c \) and \( R_c \) matrix obtained which make the system optimal.

In the initial conditions \((s_0, q_0, x_0) = (20,10,1.825)\) taken:

\[
Q_c = \begin{pmatrix}
0.35 & 0 & 0 \\
0 & 0.99 & 0 \\
0 & 0 & 0.99
\end{pmatrix}
\]

and

\[
R_c = (1)
\]

Then, from equation (7) the value of \( P_c \) obtained using Riccati’s equation with MATLAB, obtained:

\[
P_c = \begin{pmatrix}
0.1744 & 0.0218 & 0.1143 \\
0.0218 & 0.0458 & 0.1144 \\
0.1143 & 0.1144 & 0.6508
\end{pmatrix}
\]

So that from equation (6), value of \( K_c \) is found, obtained:

\[
K_c = \begin{pmatrix}
-0.0349 & -0.9515 & 0.1675
\end{pmatrix}
\]

The following are the simulation results after optimal control is performed.

![Figure 2. Changes in the number of microalgae concentrations](image)

In Fig. 2 can be seen that the number of microalgae concentrations has increased by 3,4999 after optimal control, then decreased to 2,416 until fifth day and stable concentration until tenth day. In other words, the number of microalgae concentrations increased by 91.77% from the initial amount.
Figure 3. Changes in nutrients concentrations.

In Fig. 3 can be observed that the concentration of nutrients continues to decrease as well as the concentration of uncontrolled nutrients until sixth day reaches 5.199, after which the nutrient concentration is stable until the tenth day with different initial conditions.

Figure 4. Change in cell quota concentration
Based on Fig. 4 shows that after control, the cell quota concentration always decreases as well as the cell quota concentration without control, it can be seen that the cell quota concentration decreases near 0.2355.

Then the simulation results are displayed for the level of light intensity.

![Figure 5](image)

**Figure 5.** Simulation of the light intensity level

Fig. 5 shows that the optimization value of the light intensity level on first day is 9.9071. Then it decreases continuously until it reaches 0.05 on the second day until the tenth day. This shows that the control of light intensity can produce biomass almost twice that of initial biomass. This means that the control of light intensity can produce more biomass than without control.

Performance comparison of microalgae growth between LQR and PMP from previous work:

a. LQR method can increase biomass 91.77% of the initial amount. While the PMP has doubled from its initial condition.

b. LQR, optimal light intensity occurs at 9.9071. While PMP occurs at 0.18.

4. Conclusions

Based on the analysis of simulation results presented in the previous section, a number of things can be summarized as follows:

- The optimal control of light intensity from microalgae growth occurred at 9.9071.
- Control of light intensity level can produce more biomass than without control. This means that light intensity has important role in microalgae growth so as to produce more biomass.
- To get optimal results in increasing the growth of microalgae using the LQR method, we get a matrix of the weight of the state $Q_c$, the control weight matrix $R_c$, and matrix $K_c$ respectively.
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