The $\eta N$ S-wave scattering length in one resonance model

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We show that the S-wave $\eta N$ scattering length can be extracted in a model independent way within the scope of the multichannel model, but with the restricting assumption that only one resonance is included per partial wave. One has only to use the information on $\pi N$ elastic S-wave T-matrix at the $\eta$ production threshold, and near threshold $\pi^- p \to \eta n$ total cross section. The results are independent of the particular parametrization of the elastic T-matrix and of the number of channels. For the described model the unitarity requires that the number of channels must be at least three. These assumptions are more general then the assumptions of the existing single resonance models up to now used for extracting the $\eta N$ S-wave scattering length. The agreement of our approach with former estimates is anticipated and confirmed for all cases for which the input data agree with the commonly accepted values.
1 Introduction

In 1985 Bhalerao and Liu [1] have constructed a coupled channel isobar model for the $\pi N \rightarrow \pi N$, $\pi N \rightarrow \eta N$ and $\eta N \rightarrow \eta N$ T-matrices with $\pi N$, $\eta N$ and $\pi \Delta$ ($\pi \pi N$) as isobars. A single resonance separable interaction model for $S_{11}$, $P_{11}$, $P_{33}$ and $D_{13}$ partial waves has been used. They have used only $\pi N$ elastic scattering data as a constraint while their prediction for the $\eta$ production cross section has been compared with, at that time the most recent data [2]. Their conclusion has been that the S-wave $\eta N$ interaction is attractive, and they have extracted for the S-wave scattering length the value of $a_{\eta N} = (0.27 + i 0.22)$ fm.

Wilkin [4] based his calculation on an S-wave threshold enhancement calculation, used the $\eta$ total cross section near threshold to fix the imaginary part of the T-matrix and obtained the real part by fitting the $\pi^- p \rightarrow \eta n$ production cross section up to the center of mass momentum in the $\eta n$ system of 1.2 (fm$^{-1}$). He quotes the value of $a_{\eta N} = (0.55 \pm 0.20 + i 0.30)$ fm.

Abaev and Nefkens [5] have also used a form of a S-wave single resonance model, adjusted the resonance parameters to reproduce the $\pi^- p \rightarrow \eta n$ production channel to the best of their ability and extracted the S-wave scattering length as: $a_{\eta N} = (0.62 + i 0.30)$ fm.

Arima et al [3] have studied the nature of the S-wave resonances $S_{11}(1535)$ and $S_{11}(1650)$ concerning their couplings with the $\eta N$ channel using the two quark-model wave functions with pure intrinsic spin states for the isobars. The dynamical coupling of the isobars to $\pi N$ and $\eta N$ channels are described by the meson-quark coupling. In addition to analyzing the agreement of the model with the $\pi N$ elastic and $\eta$ production data they have obtained the S-wave scattering length $a_{\eta N} = (0.98 + i 0.37)$ fm.

In this article we show that the agreement of the listed results with the exception of Arima et. al. [3] is to be expected as the knowledge of $\pi N$ elastic T-matrix, originating from any of a world collection of PWA [6, 7, 8, 9] and the threshold value of the $\pi^- p \rightarrow \eta n$ cross section are sufficient to calculate the $\eta N$ S-wave scattering length value in any, single resonance model and all cited models indeed do assume using only one resonance for the dominant $S_{11}$ partial wave. Variations among the obtained $\eta N$ S-wave scattering length values can be completely attributed to the
somewhat different assessment of the input data.

2 Formalism

For the convenience of the reader we give the collection of the essential, but well known formulae.

The model of using only one resonance per partial wave is the simplest, manifestly unitary, way to construct the multichannel T-matrix for any choice of connected two body processes. For any number of two-body channels, T-matrix is given by

$$T_{\alpha}(W) = (T_{ab,\alpha}(W)) \equiv \begin{pmatrix} T_{11,\alpha}(W) & T_{12,\alpha}(W) & T_{NN,\alpha}(W) \\ T_{21,\alpha}(W) & T_{22,\alpha}(W) & T_{2N,\alpha}(W) \\ \vdots & \vdots & \vdots \\ T_{N1,\alpha}(W) & T_{N2,\alpha}(W) & T_{NN,\alpha}(W) \end{pmatrix}$$  \hspace{1cm} (1)

where $W$ is total center of mass energy, $a, b = 1, \ldots, N$ denote two body channels involving one nucleon, e.g.:

$$T_{11} = T_{\pi N \pi N} \equiv T_{\pi \pi}$$
$$T_{22} = T_{\eta N \eta N} \equiv T_{\eta \eta}$$
$$T_{12} = T_{\pi N \eta N} \equiv T_{\pi \eta}$$

... ... ...

and $\alpha$ denotes all additional quantum numbers (isospin, angular momentum, ...). From here on, we restrict our analysis to only one resonance (no background) per partial wave. With that restriction the following relation among matrix elements can be written:

$$T_{ab}(W) = y_a(W)y_b(W)T_0(W)$$
$$\sum_{a=1}^{N} y_a(W)^2 = 1$$  \hspace{1cm} (3)

$$T_0(W) \equiv \sum_{a=1}^{N} T_{aa}(W)$$
where $y_a(W)$ are real quantities and are related to resonance branching ratios $x_a$ and for Breit-Wigner parametrisation $x_a \equiv y_a(M^2)$. Eq. (3) gives relations for diagonal elements:

$$\frac{T_{aa}}{T_{bb}} = P_{ab} \quad P_{ab} \text{ is real positive number}$$  \hfill (4)

and for non diagonal ones:

$$T_{ab} = \pm \sqrt{T_{aa} T_{bb}}$$ \hfill (5)

what means the proportionality between any two complex T-matrix elements of the multichannel T-matrix. This fact does not depend on any particular parametrization of the T-matrix elements either in the case of Breit-Wigner parametrization or in the K-matrix approach. It only depends on the single resonance character of the model. We define the unitary multichannel S-matrix:

$$S_\alpha(W) = 1 + 2i T_\alpha(W)$$

The partial wave scattering amplitudes are defined as:

$$f_{ab,\alpha}(W) = \frac{T_{ab,\alpha}(W)}{\sqrt{p_a p_b}} \quad a, b = 1, 2, ... N$$ \hfill (7)

where $p_a$ is the center of mass momentum of particles in channel $a$

$$p_a = \frac{[W^2 - (M_a + m_a)]^2 [W^2 - (M_a - m_a)^2]}{2W} \quad a, b = 1, 2, ... N$$ \hfill (8)

and $M_a, m_a$ are the corresponding masses.

The total cross sections is given by

$$\sigma_{a\rightarrow b,\alpha}^{\text{tot}}(W) = 4\pi \frac{|T_{ab,\alpha}(W)|^2}{p_a^2}$$

and using Eq. (5) we get:

$$\sigma_{a\rightarrow b,\alpha}^{\text{tot}}(W) = 4\pi \frac{|T_{aa,\alpha}(W)| |T_{bb,\alpha}(W)|}{p_a^2}.$$ \hfill (9)

The total cross section summed over all additional quantum numbers $\alpha$ is given by:

$$\sigma_{a\rightarrow b}^{\text{tot}}(W) = \sum_\alpha \sigma_{a\rightarrow b,\alpha}^{\text{tot}}(W).$$ \hfill (10)
To obtain the scattering length does not depend on the details of the model used, only the unitarity and the single resonance character of the model are required. The S matrix for the angular momentum \( l \) is written in the following way:

\[
S_{aa,l}(W) = \frac{1 + i k_{a,l}(p_a)}{1 - i k_{a,l}(p_a)}; \quad k_{a,l}(p_a) = \tan \delta_{a,l}(p_a),
\]

(11)

\( p_a \) is the channel \( a \) center of mass momentum, \( l \) is the angular momentum quantum number, and \( \delta_{a,l} \) is the channel \( a \) partial wave phase shift. For the diagonal S-matrix elements we apply the low energy expansion\(^*\):

\[
p_a^{2l+1} \cot \delta_{a,l}(p_a) = \frac{1}{a_{a,l}} + \frac{r_{a,l}}{2} p_a^2 + O(p_a^4)
\]

which defines the channel \( a \) scattering length and effective range \( a_{a,l} \) and \( r_{a,l} \), respectively.

In the terms of the scattering length and the effective range the scattering amplitude \( f_{aa,l}(p_a) \) is given by

\[
f_{aa,l}(p_a) \approx \frac{a_{a,l} p_a^{2l}}{1 - i a_{a,l} p_a^{2l+1} - \frac{1}{2} a_{a,l} r_{a,l} p_a^2}
\]

(13)

So the expression for the scattering length \( a_{a,l} \) is given as:

\[
a_{a,l} = \lim_{p_a \to 0} \frac{f_{aa,l}(p_a)}{p_a^{2l+1} f_{aa,l}(p_a)} = \frac{T_{aa,l}(W_a)}{p_a^{2l+1}}
\]

(14)

and because of the threshold behavior \( f_{aa,l} \sim p_a^{2l} \), the scattering lengths can be approximated by

\[
a_{a,l} \approx \frac{f_{aa,l}(p_a)}{p_a^{2l}} = \frac{T_{aa,l}(W_a)}{p_a^{2l+1}}
\]

(15)

for the very small \( p_a \) i.e. for the energy \( W_a \) very close to the threshold of channel \( a \). Namely for the S-wave we have

\[
a_{a,0} \approx f_{aa,0}(p_a) = \frac{T_{aa,0}(W_a)}{p_a}
\]

(16)

This expression for the S-wave scattering length \( a_{a,0} \) is used through this article.

\(^*\)Different textbooks take opposite sign for the scattering length, see [10] vs [11]. Here we follow Ref. [10].
If we apply the single resonance formulae \((4, 5)\) for any of the single resonance models, the scattering length for the channel \(a\) is proportional to any T-matrix element at threshold \(W_a\):
\[
\frac{a_{a,a}}{T_{b,b,a}(W_a)} = r_{ab,a} \quad r_{ab,a} \text{ is a real positive number,} \tag{17}
\]
\[
\frac{a_{a,a}}{T_{b,c,a}(W_a)} = r_{abc,a} \quad r_{abc,a} \text{ is a real number.} \tag{18}
\]
When only one channel is opened, the scattering length is a real quantity. Upon opening inelastic channels, the scattering length becomes complex and its imaginary part is related to the total cross sections via the optical theorem. For the multichannel case \([10, 11]\) we have:
\[
\text{Im } f_{aa}(p_a, \vartheta = 0) = \frac{p_a}{4\pi} \sum_x \sigma_{a\rightarrow x}^{\text{tot}}(W) \tag{19}
\]
where \(x\) denotes all opened channels. The optical theorem \((19)\) is automatically satisfied if S-matrix is a unitary matrix.

If we know one of the S-wave diagonal matrix elements \(T_{aa,0}\) at energy \(W_b\) where the channel \(b\) opens \((b \neq a)\) and \(\sigma_{a\rightarrow b}^{\text{tot}}/p_b\) near threshold \(W_b\), where S-wave dominates, and using Eqs. \((10, 16)\) we get
\[
a_{b,0} = \frac{1}{4\pi} \frac{T_{aa,0}(W_b)}{|T_{aa,0}(W_b)|^2} \frac{\sigma_{a\rightarrow b,0}^{\text{tot}}(W_b)}{p_b} \approx \frac{1}{4\pi} \frac{T_{aa,0}(W_b)}{|T_{aa,0}(W_b)|^2} \frac{\sigma_{a\rightarrow b}^{\text{tot}}(W_b)}{p_b}. \tag{20}
\]
This value depends only on the fact that a single resonance model is used. A particular parametrization or a number of channels does not play any importance.

We restrict our analysis to the \(I = 1/2 \pi N\) scattering with inelastic channels \(\eta N, \pi \Delta, (\pi \pi)_{S\text{-wave}} N\), etc. The optical theorem \((19)\) for the \(\eta N \rightarrow \eta N\) scattering amplitude near \(\eta N\) threshold with the assumption of the S-wave domination gives:
\[
\text{Im } f_{\eta \eta,0}(p_\eta, \vartheta = 0) \overset{p_\eta \rightarrow 0}{=} \frac{p_\eta}{4\pi} \left( \frac{\sigma_{\eta N \rightarrow \pi N}^{\text{tot}}(W) + \sigma_{\eta N \rightarrow \pi \Delta}^{\text{tot}}(W)}{\sigma_{\eta N \rightarrow (\pi \pi)_{S\text{-wave}} N}^{\text{tot}}(W) + \ldots} \right) \tag{21}
\]
Using Eq. \((16)\), isospin algebra and the detailed balance we get the lower bound for the imaginary part of the \(\eta N\) S-wave scattering length:
\[
\text{Im } a_{\eta N,0} \geq \frac{3p_\eta^2 \sigma_{\eta N \rightarrow \eta N}^{\text{tot}}(W_\eta)}{8\pi p_\eta} \tag{22}
\]
where the $p_\pi$ is the c.m. momentum of the particles in the \( \pi N \) channel at \( \eta N \) threshold \( W_\eta \). Using the experimental value of the \( \eta \) production total cross section near \( \eta N \) threshold \( W_\eta \):

\[
\frac{\sigma_{\pi^-p\rightarrow\eta n}^{\text{tot}}(W_\eta)}{p_\eta} = (21.2 \pm 1.8) \mu\text{b/MeV} \tag{23}
\]

taken from Ref. \[12\], we obtain the constraint on the imaginary part of the \( \eta N \) scattering length based exclusively on the optical theorem:

\[
Im\ a_{\eta N,0} \geq (0.24 \pm 0.02) \text{ fm}. \tag{24}
\]

Knowing \( \pi N \) elastic T matrix and \( \pi^- p \rightarrow \eta N \) near \( \eta N \) threshold total cross section Eq. (20) and some isospin algebra gives:

\[
a_{\eta N,0} \approx \frac{3}{2} \frac{p_\pi^2}{4\pi} \frac{T_{\pi\pi}(W_\eta)}{|T_{\pi\pi}(W_\eta)|^2} \frac{\sigma_{\pi^-p\rightarrow\eta n}^{\text{tot}}(W_\eta)}{p_\eta}. \tag{25}
\]

The result is independent on the details of the model, on the T-matrix parametrization, the number of channels, which, because of unitarity, can not be lower then three, etc.

There are several \( \pi N \) elastic phase shift analyses \[6, 7, 8, 9\]. The \( \pi^- p \rightarrow \eta n \) experimental total cross section is as well quite well known \[12\]. Using the different \( \pi N \) elastic phase shift analyses and \( \pi^- p \rightarrow \eta n \) total cross section from Eq. (23) as input we get the distribution of values for the \( \eta N \) scattering length, for the single resonance model, and analyze its sensitivity to the input.

\section{3 Numerical results and conclusions}

In Tables 1–3 and Fig. 1 we show the allowed spread in the \( \eta N \) S-wave scattering length within the framework of any single resonance model, based on the afore described, model independent approach. Numerical values for the \( \eta N \) S-wave scattering length, given in Table 1, were obtained using the Eq. (23) and the \( \pi N \) elastic T-matrices near \( \eta N \) threshold \( (W = 1487 \text{ MeV}, \ p_\pi = 432 \text{ MeV/c}) \) from various obtainable phase shift analyses \[6, 8, 9\] and the \( \eta \) production total cross section from Eq. (23) \[12\]. If we introduce the uncertainty of 0.01 for real and imaginary
part of the input T matrix and the uncertainty of 1.8 \(\mu\text{b}/\text{MeV}\) for the \(\eta\) production total cross section the resulting uncertainty for the \(\eta N\) scattering length is of the order of 0.03 fm for real and for imaginary part. Main contribution to the total uncertainty is coming from the uncertainty of the \(\eta\) production cross section. The contribution from the T-matrix uncertainty is \(\sim 5\%\). In Table 2 we give the results for the \(\eta N\) scattering length given elsewhere together with the input: \(\pi N\) elastic T-matrices and the near threshold \(\eta\) production total cross section when possible. A compilation of all results is shown in Fig. 1. Open circles represent the \(\eta N\) S-wave scattering length values obtained on the basis of \(\pi N\) elastic T-matrices of references \([1, 4, 5, 6]\) and the \(\eta\) production total cross section given in Eq. (23). Full normal and inverse triangles represent the original and modified scattering length values of Ref. \([1]\), and the full square gives the value given by Ref. \([7]\). The full circles represent the value of Ref. \([4]\). Open square represent the value obtained in our full model \([13]\) restricted to the single resonance only and an open normal and inverse triangles represent the values of our full, multiresonance model without any restrictions whatsoever using three or four poles in the \(P_{11}\) partial wave. The full star represents the value of Ref. \([3]\), but let us mention that this value is not obtained within the framework of the single resonance model.

The Eqs. (17, 18) can be used as a check if some particular model is a single resonance one, or there is some influence of higher resonances and/or background terms. If we specify Eqs. (17, 18) for all possible variations of our model \([13]\) we get:

\[
r_{\pi \eta,0} = \frac{a_{\eta N,0}}{T_{\pi,0}(W_\eta)}.
\] (26)

For single resonance models the ratio

\[
R = \frac{\text{Im} \, r_{\pi \eta,0}}{\text{Re} \, r_{\pi \eta,0}}
\] (27)

must be zero. This ratio is shown in Table 3 for all cases where both \(\eta N\) S-wave scattering length and \(\pi N\) elastic T matrix at \(\eta N\) threshold are available and well known. As we can see this ratio is very close to zero for any single resonance model. Certain deviations from the exact zero occur because some T matrices are just read

* The \(\text{Im} \, r_{\pi \eta,0}\) is of course 0, but for the reason of comparison, the scaling factor \(\text{Re} \, r_{\pi \eta,0}\) is introduced.
off the figures in different publications because they have not been available in the numerical form. For the values originating from Table 1 this ratio is of course exactly zero.

Results are very indicative:

As it has been shown in [14], the $\eta N$ S-wave scattering length obtained in the framework of the full model, when other resonances and the background term in particular are explicitly introduced in the unitary way is significantly more attractive then any of the predictions obtained within the limits of the single resonance models. All single resonance models with good $\pi N$ elastic T-matrix at threshold lie within the limits which are obtained using realistic $\eta N$ production cross section, and the $\pi N$ elastic T-matrix. The solution of Arima et al. [3] is much closer to our final value, and as their model is not of single resonance nature, and henceforth it is not in the contradiction with the suggested analyses. However, the prediction of Ref. [5] we consider as non-reliable because the threshold value of the $\pi N$ elastic T-matrix differs significantly from worldwide accepted values.

Conclusion:

The simple, model independent mechanism for extracting the $\eta N$ S-wave scattering length from the near threshold values of the $\pi N$ elastic T-matrices and the $\eta$ production total cross section exists if only one resonance is used per partial wave. All reported values, based on the single S-wave resonance assumption agree with our model independent results. The simple criterion is offered which, on the bases of the input $\pi N$ elastic S-wave T-matrix and the obtained $\eta N$ scattering length, gives the estimate of the importance of non-single resonance ingredients for the presented model. The realistic $\eta N$ S-wave scattering length based on the full, three coupled channel, multiresonance and manifestly unitary model [13] is, and should be very different from any single resonance prediction, because the coupled channel process can not be reliably described using such a simplified model. The analysis of [14] gives the new prediction for the $\eta N$ S-wave scattering length with much more attraction than previously reported.
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Table 1: The $\eta N$ S-wave scattering length within the framework of any single resonance model, using $\pi N$ elastic T matrix at $\eta N$ threshold from different phase shift analyses [6, 7, 8, 9] and $\eta$ production total cross section [12] from Eq. (23) as the input. The $\pi N$ T matrix is given in the first column, and resulting $\eta N$ S-wave scattering length in the second one.

| Solution   | $T_{\pi N \pi N}(W_\eta)$         | $a_{\eta N}$ [fm]       |
|------------|-----------------------------------|-------------------------|
| CMB        | $(0.376 + i0.439)$                 | $(0.269 + i0.315)$      |
| KH80       | $(0.332 + i0.393)$                 | $(0.301 + i0.356)$      |
| KA84       | $(0.390 + i0.374)$                 | $(0.320 + i0.307)$      |
| FA84       | $(0.400 + i0.339)$                 | $(0.348 + i0.296)$      |
| CV90       | $(0.412 + i0.328)$                 | $(0.356 + i0.283)$      |
| KV90       | $(0.418 + i0.334)$                 | $(0.350 + i0.279)$      |
| SM90       | $(0.407 + i0.303)$                 | $(0.379 + i0.282)$      |
| FA93       | $(0.403 + i0.344)$                 | $(0.344 + i0.293)$      |
| WI94       | $(0.408 + i0.339)$                 | $(0.348 + i0.289)$      |
| SP95       | $(0.399 + i0.338)$                 | $(0.350 + i0.296)$      |
## Table 2: Results of other analyses for the $\eta N$ S-wave scattering length.

The scattering length is given in first column, and for single resonance models for which Eq. (25) is valid, the $\pi N$ elastic T matrix at $\eta N$ threshold and corresponding $\eta$ production total cross section near threshold are given in last two columns.

| Model                      | $a_{\eta N}$  | $T_{\pi\pi}(1487$ MeV) | $\sigma_{\pi^-N\to\eta N}(W_{\eta})/p_{\eta}$ |
|----------------------------|----------------|-------------------------|-----------------------------------------------|
| Bhalerao-Liu [1]           | (0.27 + i0.22) | (0.38 + i0.31)*         | 15.0                                          |
|                            | (0.28 + i0.19) | (0.37 + i0.25)*         | 13.4                                          |
| "modified" Bhalerao-Liu [14]| (0.38 + i0.31) | (0.38 + i0.31)*         | 21.2                                          |
|                            | (0.44 + i0.30) | (0.37 + i0.25)*         | 21.2                                          |
| Abaev and Nefkens [5]      | (0.62 + i0.30) | (0.298 + i0.145)        | 20.2                                          |
| Our SR model [14]          | (0.404 + i0.343)| (0.345 + i0.293)        | 21.2                                          |
| Arima et al. [3]           | (0.98 + i0.37) | –                       | –                                             |
| Wilkin [4]                 | (0.55 + i0.30) | –                       | –                                             |
| Our full model [14]        | (0.886 + i0.274)| –                       | –                                             |
|                            | (0.876 + i0.274)| –                       | –                                             |

* data are read of the graphs
Table 3: The test of the single resonance character of different models. The $\eta N$ S-wave scattering length is given in first column, and the $\pi N$ elastic T matrix at $\eta N$ threshold in second one. The ratio defined in Eq. (27) is given in the last column. For the single resonance models this ratio must be close to zero. Deviations from zero can be used as a measure for the importance of the background terms and/or higher resonances.

| Model                    | $a_{\eta N}$ [fm] | $T_{\pi N}(1487 \text{ MeV})$ | $R$    |
|--------------------------|-------------------|-------------------------------|--------|
| Bhalerao-Liu [1]         | (0.27 + $i0.22$)  | (0.38 + $i0.31$)*             | −0.001 |
|                          | (0.28 + $i0.19$)  | (0.37 + $i0.25$)*             | 0.002  |
| Abaev and Nefkens [5]    | (0.62 + $i0.30$)  | (0.298 + $i0.145$)            | 0.002  |
| Our SR model [14]        | (0.404 + $i0.343$) | (0.345 + $i0.293$)            | 0.000  |
| Arima et al. [3]         | (0.98 + $i0.37$)  | (0.47 + $i0.37$)*             | −0.316 |
| Our SRBG [14]            | (0.691 + $i0.174$) | (0.381 + $i0.392$)            | −0.618 |
| Our CSRBG [14]           | (0.968 + $i0.281$) | (0.413 + $i0.317$)            | −0.391 |
| Our full model [14]      | (0.886 + $i0.274$) | (0.373 + $i0.331$)            | −0.453 |
|                          | (0.876 + $i0.274$) | (0.375 + $i0.330$)            | −0.446 |

* data are read of the graphs
Figure Caption

Fig. 1. $\eta N$ S-wave scattering length.
Open circles represent the $\eta N$ S-wave scattering length values obtained on the basis of $\pi N$ elastic T-matrices of references [6, 7, 8, 9] and the $\eta$ production total cross section given in Eq. (23). Full normal and inverse triangles represent the original and modified scattering length values of Ref. [1], and the full square gives the value given by Ref. [3]. The full circles represent the value of Ref. [4]. Open square represent the value obtained in our full model [13] restricted to the single resonance only and an open normal and inverse triangles represent the values of our full, multiresonance model with three and four resonances in the $P_{11}$ partial wave, respectively. The full star represents the value of Ref. [3].
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