A Method for Local Clock’s Error With Kalman Filter Based on Mahalanobis Distance

Xinghua Chi1*, Zhaohan Zhu1, Zhouyang Dong1, Fenghui Xu1 and Jian Xu

132023 Troops, Dalian, Liaoning Province, 116000, China
*Corresponding author’s e-mail: 1259475019@qq.com

Abstract. In order to improve the punctuality of local clock, it needs to use a proper filter to weaken the influence caused by noise in raw clock’s error data. In this paper, An improved Kalman Filter based on Mahalanobis distance is proposed, and the process are shown in details. The final test shows the effectiveness of the improved method in estimation of local clock’s error with gross error.

1. Introduction
Due to the time system of GNSS is easy to be disturbed, the time synchronization information is noisy. In the adverse environment of occlusion, deception and so on, GNSS signal is easy to be lost, which leads to serious degradation of timing performance. Kalman Filter is usually used to process clock difference data of timing. This paper introduces how the local clock punctuality system works and gives the local clock error mode[1-5]. Then, the principle and applicable conditions of the online estimation method of local clock error based on Kalman Filter are analyzed[5-9]. Finally, combined with the characteristics of clock data, an improved Kalman Filter based on Mahalanobis distance is proposed and the simulation test is carried out.

2. Influence of gross error on clock error estimation performance
The downloaded IGS post-precise ephemeris were used as the source of simulation data to verify the results. Data segments for approximately 18 days (51,840 in total) from the 1st day of week 1678 to the 4th day of week 1680 of satellite PRN18 with a data sample interval of 30s.
In clock error data, the distribution of clock error is mainly determined by the quantity and amplitude. Therefore, two different cases are verified respectively. One has the same amplitude and different quantity, and the other has different amplitude and the same quantity. We adopted the parameters that amplitude as 80ns, gross error as 0.2%(104), 0.6%(312) and 1.8%(923) of all data respectively. The results are shown in figure 1(a). Then, we adopted the parameters that gross error as 0.6%, amplitude as 40ns, 160ns and 320ns respectively. The results are shown in figure 1(b). The error statistics of the test results are shown in table 1. In the case of fixed gross error amplitude, the larger the number of data with gross data, the greater the error of Kalman Filtering results. Compared with results of Kalman Filtering, which do not contain gross error in the table 1, the results appear to be biased. The results are similar when the number of gross errors is fixed and the amplitude of gross errors is increased. It can be seen from the test that the gross errors reduce the performance of Kalman Filtering to a certain extent, so it is necessary to deal with the gross errors.

### Table 1. Kalman Filtering results with different gross errors

| Method                        | Mean   | Minimum | Maximum | Mean square error |
|-------------------------------|--------|---------|---------|------------------|
| **Amplitude of gross error: 80ns** |        |         |         |                  |
| Quantity of gross error: 0.2% | 0.16   | -1.7    | 2.8     | 0.55             |
| Quantity of gross error: 0.6% | 0.46   | -1.4    | 2.8     | 0.7              |
| Quantity of gross error: 1.8% | 1.3    | -0.51   | 3.2     | 1.5              |
| **Quantity of gross error: 0.6%** |        |         |         |                  |
| Amplitude of gross error: 40ns | 0.23   | -1.6    | 1.9     | 0.58             |
| Amplitude of gross error: 160ns | 0.91 | -0.88   | 5.9     | 1.1              |
| Amplitude of gross error: 320ns | 1.8    | -0.85   | 12      | 1.9              |
| No error                      | Kalman | -1.8    | 1.7     | 0.52             |

3. **Kalman Filter clock error estimation based on Mahalanobis distance**

For Kalman Filter, if the hypothesis that the observed noise conforms to the Gauss distribution is true, there is no gross error in the observation and the actual observation is as follows[9-13]:
\[ \rho(y_k) = N \left( y_k; \hat{y}_k|k-1, P_{\hat{y}_k|k-1} \right) \]

\[ = \frac{\exp \left( -\frac{1}{2}(y_k - \hat{y}_k|k-1)^T (P_{\hat{y}_k|k-1})^{-1} (y_k - \hat{y}_k|k-1) \right)}{\sqrt{(2\pi)^m |P_{\hat{y}_k|k-1}|}} \]

Where, \( y_k \) is the actual observation and \( \hat{y}_k|k-1 \) is the one-step predicted value respectively; \( P_{\hat{y}_k|k-1} \) is the covariance matrix of \( y_k \) and \( \hat{y}_k|k-1 \); \( |P_{\hat{y}_k|k-1}| \) is the determinant of matrix \( P_{\hat{y}_k|k-1} \). If the assumption that the observed noise conforms to the Gauss distribution is not true, or gross error is existed, then the above equation will no longer be true.

In other words, if we hold that the above equation is invalid, then we can correspondingly hold that the observed noise does not conform to the Gauss distribution or the observation is gross error. The following statistics are defined as follows:

\[ \gamma_k = \left( y_k - \hat{y}_k|k-1 \right)^T \left( P_{\hat{y}_k|k-1} \right)^{-1} \left( y_k - \hat{y}_k|k-1 \right) \]

(2)

The above statistic is the square of Mahalanobis distance. The Mahalanobis distance is defined as follows:

\[ d_k = \sqrt{\left( y_k - \hat{y}_k|k-1 \right)^T \left( P_{\hat{y}_k|k-1} \right)^{-1} \left( y_k - \hat{y}_k|k-1 \right)} \]

(3)

The Mahalanobis distance is a normalized dimensionless distance, representing the normalized distance between an observation \( y_k \) and its mean \( \hat{y}_k|k-1 \).

For a given level of significance, such as \( 1 - \alpha \), where \( \alpha \) is generally selected as a small magnitude, such as 0.05, the corresponding upper quartile of \( \alpha \) is \( \chi_{\alpha} \). When the hypothesis is true, the definitions are as follows:

\[ P_r(y_k - \chi_{\alpha}) < \alpha \]

(4)

Where, \( P_r(.) \) is the probability of a random event.

When a gross error is detected, the work need to be done is to suppress the impact of the gross error on parameter estimation. Notice that the relative information in the observation that the information minus the prediction of the observation, is actually contained in the innovation vector, and the variance of the innovation vector measures the magnitude of the relative information.

In order to suppress the influence of gross error observation, a natural choice is to increase the covariance of the innovation vector and reduce its weight in parameter estimation. The scaling factor \( \kappa \) is introduced to expand the covariance of the innovation vector. The formula is as follows:

\[ \bar{P}_{e_k|k} = \kappa P_{e_k|k} = \kappa P_{\hat{y}_k|k-1} \tilde{y}_k|k-1 \]

(5)

When:

\[ \tilde{y}_k = \left( y_k - \hat{y}_k|k-1 \right)^T \left( P_{\hat{y}_k|k-1} \right)^{-1} \left( y_k - \hat{y}_k|k-1 \right) = \chi_{\alpha} \]

(6)

Then:

\[ \kappa_k = \begin{cases} 1 & \text{if } \tilde{y}_k \leq \chi_{\alpha} \\ \frac{y_k}{\chi_{\alpha}} & \text{else} \end{cases} \]

(7)

Thus, the gross error suppression and filter estimation are completed. The processing flow is shown in figure 2.
4. Simulation verification

The tests adopted were divided into three groups according to the addition of different types of noise, namely, the cases where only the white noise data were added (the noise variance was 250ns), the cases where only the gross error was added (the amplitude of the gross error was about 5 times of the white noise mean variance value, a total of 104, which was about 0.2% of the total data), and the cases where both kinds of noise were added.

Firstly, the detection performance of the algorithm is tested. In the cases that only the gross error and two kinds of noise exist, 104 and 94 gross errors were detected respectively, and the detected gross error results are shown in figure 3. The result graph is not given because there is no gross error detected in the case of only white noise existed. In the case of only gross error existed, the leakage rate and error rate of detected gross error are both 0. However, in the case of both two kinds of noise existed, some gross error data are not detected successfully under the influence of white noise, and the leakage rate is about 10%. It is clear that white noise will affect the gross error detection performance of the new algorithm to some extent. In general, the new algorithm can effectively detect the added gross error data.

Secondly, the filtering performance of the algorithm is verified. In the case of only adding white noise, the new algorithm does not detect the gross error, and the processing result is the same as the traditional Kalman Filtering result. The test results of only the gross error and the existence of both two kinds of noise are shown in figure 4, and the statistics of error results are shown in table 2.

It can be seen from figure 4 and table 2 that white noise affects the estimation performance of the new algorithm under certain conditions, resulting in some data being detected as white noise by mistake. Moreover, the error rate of detection has also increased compared with only gross error exited. However, because its error rate of detection can be controlled basically (within 1%), it has little influence on the final result. Compared with the traditional Kalman Filtering method, the improved
method can effectively suppress the maximum and minimum amplitude of error. For example, in the case of only gross error exited, the maximum error value is reduced from 11ns to 6.4ns. At the same time, the improved method increases the filtering accuracy by about 5% under the influence of the bias error caused by reducing the error.

![Figure 4. Schematic diagram of Kalman Filtering results based on Mahalanobis distance](image)

![Table 2. Error statistics of Kalman Filter test based on Mahalanobis distance](table)

### Table 2. Error statistics of Kalman Filter test based on Mahalanobis distance

| Method       | Mean   | Minimum | Maximum | Mean square error |
|--------------|--------|---------|---------|------------------|
| Only gross error       | Kalman | 0.17    | -1.9    | 11               | 0.56       |
| Only white noise       | Kalman Mahalanobis | 0.10    | -1.8    | 6.4              | 0.54       |
|                      | Kalman | -0.25   | -11     | 8.7              | 0.89       |
|                      | Kalman Mahalanobis | -0.25   | -11     | 8.7              | 0.89       |
| Both                  | Kalman | 0.7     | -16     | 22               | 1.3        |
|                      | Kalman Mahalanobis | 0.63    | -15     | 19               | 1.2        |
| No error              | Kalman | 0.01    | -1.8    | 1.7              | 0.52       |

5. Conclusion
This paper analyzes the principle and applicable conditions of the local clock error online estimation method based on Kalman Filter, and points out the shortcomings of this method. Considering the characteristics of clock error data, a clock error online estimation method based on Kalman Filtering and Mahalanobis distance is proposed. The results of simulation test show that the improved method can eliminate the filtering influence caused by gross error and improve the accuracy of filtering results.

Acknowledgments
This work was supported by the National Science Foundation under Grant 61071006.

References
[1] Yu, H., Hao, J., Liu, W., etc. (2016) A real-time anomaly monitoring algorithm for satellite clock. Geomatics and Information Science of Wuhan University, 41(1): 106-110.
[2] Zhao, W. (2015) Key technology of high precision frequency synthesis and transmission. University of Chinese Academy of Sciences, Beijing.
[3] Rao, Y., Bai, Y. (2015) A Method based on first-order difference for type-judging and processing of outliers. Journal of Time and Frequency, 38(4): 227-234.

[4] Lan, L. (2016) GPS satellite clock error predication model. Guilin University of Technology, Guilin.

[5] Mi, H., Xie, J., Song, Z., etc. (2014) An overview for BeiDou on-board time & frequency system development history. Journal of Navigation and Positioning, 2: 1-5.

[6] Li, B., Cheng, H. (2012) Design and implementation of time maintenance test platform. Journal of Astronautic Metrology and Measurement, 32(1): 9-12.

[7] Ma, Y. (2011) Research of self-adaptive hold technique of oven controlled crystal oscillator. Xidian University, Xian.

[8] Zhai, J. (2013) Analytical investigation on time-keeping system and two way satellite time and frequency transfer. China Jiliang University, Hangzhou.

[9] Lu, J. Gao, Y., Lin, S., etc. (2013) Study of method of maintaining consistency between primary and backup clock in time/frequency system. Journal of Time and Frequency, 36(4): 222-228.

[10] Lu, J. (2013) Research on primary and backup clock synchronization. University of Chinese Academy of Sciences, Beijing.

[11] Teng, Y., Shi, Y., Zheng, Z. (2011) Research on GPS receiver positioning algorithm under bad conditions. Chinese Journal of Scientific Instrument, 32(8): 1879-1884.

[12] Teng, Y. (2011) Study on the techniques of data processing in GPS receiver. University of Electronic Science and Technology of China, Chengdu.

[13] Li, Z., Li, Z., Lu, Y. (2011) Time correction estimation of single reference station based on Kalman Filtering. Computer Engineering, 37(11): 251-253.