Design of a direction finder based on the output of parallel hydroacoustic surveillance system

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Abstract. The problem of synthesizing a direction finding algorithm based on a sample composed of the output signals of a limited number of channels of a scanning sonar system is considered. The problem is formulated as a search for the maximum of the likelihood function by its lattice representation. Options for using the optimal and quadratic interpolation are considered. The errors of displacement and loss of potential accuracy are estimated depending on sample dimension, angular step of scanning, and on the amount of the target movement within the measurement interval. The advantage of the proposed approach is shown in terms of its potential accuracy compared to the traditional approach to constructing direction finding paths based on Guanella’s technique.

Introduction
Finding the direction to the sources of underwater signals has traditionally been one of the basic problems solved by means of hydroacoustic tools along with the problems of signal detection and classification. Historically, in the pre-digital era, this problem was solved by creating a special automatic tracking channel for each signal source, while the scanning system (SS) solved the problem of detection by forming a fan of channels that covered the entire scan sector but had a radically lower efficiency due to the fact that, for certain technical reasons, scanning was sequential. The architecture of the tracking channel was based on Guanella scheme [1], using the channels formed on the antenna array halves and oriented towards the direction finding object. These days, such a solution seems to be an anachronism, firstly, due to the loss of accuracy typical of the originally non-optimal Guanella scheme, and secondly, due to the unnecessary duplication of signal processing when solving the problems of target detection and direction finding separately. Indeed, assuming the SS calculates the values of the likelihood function in the directions of expected signal arrival, the direction finding problem is reduced to finding the maximum of the likelihood function by its lattice representation.

The main specific feature of this approach application to the underwater direction listening is that the problem of finding the direction to extremely weak, threshold signals is especially important. In this case, the signal adaptive processing can be reduced to limiting the power of coherent interference at the preliminary stage of signal processing [2], which makes it possible to consider the problem of weak signal direction finding without taking into account the presence of local interference. Moreover, it is the underwater sound signals, for the processing of which it is important to take into account the amount of the target angular movement in the estimation time interval relative to the directions of the SS channels. The latter is of particular relevance in the case of AUVs which are prone to rolling and...
which can also operate in close vicinity to the signal source. Finally, in underwater acoustic applications, the receiving channels of a direction finding system are usually used for some tasks (such as underwater surveillance [3], positioning [4], communication [5], etc.) which are not related to direction finding, but require a receiving channel to be formed to provide complex spectra from the direction to the signal source, which is determined taking into account pitch/roll of the vehicle, as well as the current estimate of the angle of signal arrival and its derivatives.

The aim of this work is to formulate the principles of construction and to specify the implementation parameters of an almost optimal direction finder which forms its receiving channels oriented in an arbitrary direction of measurement, using the outputs of the SS channels that are formed with some angular step in the sector overlapping this direction.

1. Channel formation algorithm

The structure of an optimal discriminator [6, 7] includes, as an initial signal processing operation, the calculation of complex spectra from the output of the sum and difference channels \( z(\omega_n) \) and \( z_\Delta(\omega_n) \), generated at a fixed frequency of spectral analysis \( \omega_n \) according to the expressions

\[
    z(\mathcal{G}) = \hat{V}(\mathcal{G})^\ast \hat{X} \quad \text{and} \quad z_\Delta(\mathcal{G}) = \hat{V}_\Delta(\mathcal{G})^\ast \hat{X},
\]

where \( \hat{V}(\mathcal{G}) = \exp(-j \cdot \omega_n \cdot \bar{r}(\mathcal{G})) \) is the vector of the weights of the sum channel formation, \( \bar{r} \) is the vector of delays, determined by the path difference of the signal arriving at the antenna from the direction of channel formation \( \mathcal{G} \), with the dimension of the number of hydrophones \( L \); + is Hermitian conjugation sign; \( \hat{V}_\Delta(\mathcal{G}) = \delta V(\mathcal{G}) / \partial \mathcal{G} \).

We will confine ourselves to considering the case of direction finding for a narrowband signal. Therefore, the designation of the circular frequency \( \omega_n \) is omitted here and in the expressions below.

Considering that

\[
    \hat{V}_\Delta(\mathcal{G}) \approx \frac{(\hat{V}(\mathcal{G} + \frac{\Delta}{2}) - \hat{V}(\mathcal{G} - \frac{\Delta}{2}))}{\Delta}
\]

with a sufficiently small increment of the bearing \( \Delta \), the task is reduced to the formation of only sum channels in arbitrary directions \( \mathcal{G} \) and \( \mathcal{G} \pm \frac{\Delta}{2} \) by weigh summation of the outputs of the SS channels formed in fixed directions; we will call them basic directions. Let us consider only the case of using no more than four basic directions arranged with a scanning step, namely

\[
    \mathcal{G}_0 - \Delta_0, \mathcal{G}_0, \mathcal{G}_0 + \Delta_0, \mathcal{G}_0 + 2 \Delta_0,
\]

where \( \mathcal{G}_0 \) is a basic direction closest to the signal arrival direction on the left.

The problem of interpolation consists in obtaining an estimate \( z(\mathcal{G}) \) in the form

\[
    z(\mathcal{G}) = \hat{A}(\mathcal{G})^\ast W^\ast \hat{X}, \quad (1)
\]

where \( \hat{A}(\mathcal{G}) \) is the vector of interpolation coefficients,

\[
    W = [\hat{V}(\mathcal{G}_0 - \Delta_0), \hat{V}(\mathcal{G}_0), \hat{V}(\mathcal{G}_0 + \Delta_0), \hat{V}(\mathcal{G}_0 + 2 \Delta_0)]
\]

is the matrix of the weights of the SS channels formation, and the product \( W \hat{A}(\mathcal{G}) \) is the vector of weight coefficients corresponding to the channel formed by interpolation (1).

The vector

\[
    \hat{V}_{ ERR} = \hat{V} - W \hat{A}(\mathcal{G})
\]

is the vector of the error of weight coefficients when forming the channels.

The value of the weight coefficients of the channels formation \( \hat{A}(\mathcal{G}) \), optimal by the criterion of the minimum root-mean-square error, \( P = \hat{V}_{ ERR}^\ast \hat{V}_{ ERR} = (\hat{V}(\mathcal{G}) - W \hat{A}(\mathcal{G}))^\ast (\hat{V}(\mathcal{G}) - W \hat{A}(\mathcal{G})) - \Delta \rightarrow \min \), has the form:
\[
\hat{A}(\theta) = \left( W^* W \right)^{-1} W^* \tilde{V}(\theta).
\] (3)

The accuracy of the weight coefficients interpolation depends on the ratio of the directivity pattern (DP) width at the level of 3 dB \(\Delta_{0.7}\) and the angular step of the basic channels formation \(\Delta_\vartheta\).

2. Bias errors

Figures 1 and 2 show the values of the bias errors of the channel formation weight coefficients in modulus and phase, calculated in accordance with (2) in the options of using the optimal interpolation (3) with the dimension 4 (a), dimension 3 (b), and quadratic interpolation (c). The results are given for the case of a linear equidistant antenna, when \(L=64\), and \(\Delta_\vartheta / \Delta_{0.7}\) is 0.8 for optimal interpolation and 0.6 for quadratic interpolation. When forming the channels, the phase center is located in the antenna’s center of gravity, which is essential when using quadratic interpolation, the coefficients of which are real. Separate curves in Figs 1 and 2 correspond to different values of the difference \(\vartheta - \vartheta_0\).

Fig.1. Bias of the weight coefficients’ moduli during channel formation using optimal interpolation of dimension 4 (a), dimension 3 (b), and quadratic interpolation (c).

Fig.2. Bias of weight coefficients’ phase during channel formation using optimal interpolation of dimension 4 (a), dimension 3 (b), and quadratic interpolation (c).

As can be seen from Figs. 1 and 2, in the case of quadratic interpolation, the coefficients are downshifted in amplitude and monotone in phase, which leads to the bias errors both in the transmission coefficient and bearing. At the same time, in the case of optimal interpolation, the deviation of the weight coefficients in amplitude and in phase is sign-variable. As a result, in the
process of weight summation when forming the scanning channels, the errors of the coefficients are averaged over the aperture, which provides the advantage of optimal interpolation.

Fig. 3. RMS error in weight coefficients formation depending on the direction of the channel formation using the optimal interpolation of the dimension 4 (a), dimension 3 (b), and quadratic interpolation (c).

Fig. 4. Bias error during direction finding using the optimal interpolation of dimension 4 (a), dimension 3 (b), and quadratic interpolation (c).

Fig. 5. Bias of formed transmission channel in modulus when using the optimal interpolation of dimension 4 (a), dimension 3 (b), and quadratic interpolation (c).
Figures 3, 4 and 5 show the values of the RMS error in the weight coefficients formation (Fig. 3), as well as the shift errors in the bearing (Fig. 4) and the modulus of the transmission coefficient (Fig. 5), depending on the direction of channel formation for different values of the ratio of the scan step to the DP width $\Delta_\vartheta / \Delta_{0.7}$. Figures 3–5 demonstrate the advantage of optimal interpolation over quadratic interpolation, which consists in the possibility to achieve equal or smaller errors with a larger step of scanning. In particular, in Fig. 3, the same ten percent level of the maximum RMS error of the weights is achieved at the ratio $\Delta_\vartheta / \Delta_{0.7} = 0.4$ in the case of quadratic interpolation, $\Delta_\vartheta / \Delta_{0.7} = 0.7$ with the optimal one using the same three channels as the quadratic one, and $\Delta_\vartheta / \Delta_{0.7} = 0.8$ when using four channels. Moreover, as can be seen from Figs. 4 and 5, the maximum values of the bias of the bearing estimates in the scanning step fractions for the same $\Delta_\vartheta / \Delta_{0.7}$ in the same algorithms are 0.005, 0.012 and 0.025, and the maximum bias of the estimates of the transmission coefficient modulus for the formed channel is 0.57%, 0.9% and 2% respectively. As has been noted above, with the same level of the RMS error of the weight coefficients, the gain with optimal interpolation is achieved due to averaging the error over the aperture.

As can be seen from Figs. 4 and 5, the decrease in the bias error due to the decrease in the scan step during direction finding within the considered range of the parameter values is exponential. For instance, every time the scan step decreases by one tenth of $\Delta_\vartheta / \Delta_{0.7}$, the error reduces approximately 2.5–3 times with optimal interpolation in four channels, 2 times with optimal interpolation in three channels, and 1.3 times with quadratic interpolation.

The upper limit of the permissible magnitude of the scanning step, at which it is still possible to use interpolation in the channels formation, is determined by approximation to the value corresponding to the angle between the directions to the DP maximum and to the first zero of the DP. If the scanning step is equal to this value, the signal processing should be implemented in the space of frequency-wavenumber samples with a dramatic increase in the number of the scanning channels used [8].

It should be noted that, if necessary, the bias errors in direction finding can be calculated and compensated.

3. Potential Accuracy

The expression for calculating the lower bound of the variance $\sigma^2$ of the bearing estimates at some particular frequency of the spectral analysis has the form

$$
\sigma^2 = \left[ \operatorname{tr}( \Theta \frac{\partial}{\partial \vartheta} \Theta^{-1} \frac{\partial}{\partial \vartheta} \Theta^{-1} ) \right],
$$

where $\Theta = W_0^* \left( s \mathbf{V}(\vartheta) \mathbf{V}(\vartheta)^* + n \mathbf{I} \right) W_0$ is the correlation matrix of signals by the output of the direction finder formed channels; $W_0$ is the rectangular matrix composed of the weight vectors of the direction finder channels formation, obtained by interpolation:

$$
W_0 = \left[ \mathbf{W}\mathbf{A}(\vartheta), (\mathbf{W}\mathbf{A}(\vartheta + \Delta/2) - \mathbf{W}\mathbf{A}(\vartheta - \Delta/2))/\Delta \right];
$$

$s, n$ are the power of the measured signal and noise (in this work, we assume that $s = 0.1$ (n/L); $\mathbf{I}$ is the identity matrix. When calculating the potential accuracy of direction finding by the output of antenna elements, $W_0$ is a unit matrix of dimension $L$.

If, when constructing a direction finder, time averaging is used for the estimation of signals correlation in the sum and difference channels, then the value of change in the angle of signal arrival during the time of averaging should be taken into account when forming the matrix $\Theta$:

$$
\Theta = W_0^* \left( s \frac{1}{\Delta} \sum_{\vartheta=1}^{\Delta} \mathbf{V}(\vartheta) \mathbf{V}(\vartheta)^* + n \mathbf{I} \right) W_0,
$$
where \( \vartheta = \vartheta + \vartheta \left( r - \frac{R}{2} \right) / R \), \( \vartheta \) is angular movement of the signal in the averaging interval, \( R \) is the number of spectral analysis intervals in the averaging interval (it is assumed that \( \vartheta \) / \( R \) << \( \vartheta \)).

Figure 6 shows the graphs of potential accuracy of a stationary signal direction finding versus the angle of its arrival in the cases of optimal interpolation and four channels of the SS (a), quadratic interpolation (b), and quadratic interpolation in the case of angular movement of the signal by a half of the scanning step within the measurement interval (c) and (d). The values are normalized to the potential accuracy of direction finding by the output of the antenna elements in a stationary situation (Fig. 6a, 6b and 6c), and to the potential accuracy calculated taking into account the angular movement of the signal (Fig. 6d).

It follows from Fig. 6 that the greatest loss of potential accuracy during direction finding by the SS output occurs when the direction of signal arrival is close to the direction of one of the SS channels. In the same case, as can be seen from Figs. 3–5, the bias error is minimal. This phenomenon is explained by the fact that the signal level in the basic channels used for interpolation and not coinciding with the direction of signal arrival turns out to be lower to the extent that the DP level decreases. This leads to weaker signal-to-noise ratio in the interpolated difference channel and, accordingly, to an increase in bearing fluctuations.

Figure 6 (a) and (b) also shows that, although the loss of accuracy in the case of optimal interpolation proves to be 2–4 times less than with quadratic interpolation, the overall level of losses in both cases is insignificant, i.e. by about an order of magnitude less than the 15% losses that are originally typical of the Guanella scheme traditionally used for direction finding.

Based on the above, we can conclude that an interpolation direction finder with the scanning step values of approximately 0.8 \( \Delta_{0.7} \) with optimal interpolation and approximately 0.5 \( \Delta_{0.7} \) with quadratic one makes it possible to achieve efficiency that practically does not differ from the optimal one in a situation where the angular movement of signal during the estimation process is negligible.

In a situation where the angle of signal arrival changes by half of the scanning step during the estimation, the effects of losses from the use of the interpolation procedure are combined with the effect of the signal energy distribution in the angular movement sector. Figure 6c illustrates the fact that the angular movement of the signal has a dramatic effect on the direction finding performance. In this case, if we compare Fig. 6d and Fig. 6b, it is evident that the loss of accuracy due to the direction finder operation based on the SS output remains practically the same, both in the stationary case and in the case of the signal angular movement.
In more detail, the effect of changing angle of the signal arrival on the accuracy of direction finding can be judged from Fig. 7 which shows the graphs of the maximum (based on the arrival angle) values of the standard deviation corresponding to the potential accuracy of direction finding using quadratic interpolation, depending on the value of the signal arrival angle change within the estimation interval. The values are normalized to the potential accuracy of direction finding by the output of the antenna elements in a stationary situation in Fig. 7a, and to the potential accuracy calculated taking into account the angular movement of the signal in Fig. 7b.

As can be seen from Fig. 7, the dependence of losses in the direction finding accuracy, caused by the signal angular movement within the considered limits can be characterized as quadratic. The accuracy loss to a level corresponding to a direction finder designed according to the Guanella scheme approximately corresponds to the angular movement $\Delta_\gamma = 0.5\Delta_{0.7}$, which should be selected as the maximum permissible value. It can be seen in Fig. 7b that with the scanning step being about $0.5–0.6$ of $\Delta_{0.7}$, the loss in direction finding accuracy due to the use of the interpolation procedure is negligible, provided that the angular movement of the signal within the measurement interval $\Delta_\gamma$ does not exceed $0.7–0.8$ of $\Delta_{0.7}$. When this value is exceeded, the direction finding error grows exponentially.

Conclusions

1. A direction finder which practically does not differ from the optimal one in terms of performance, can be constructed based on the SS output with the use of optimal interpolation and a with a scanning step not exceeding 0.8 of the DP width at a level of 3 dB at the upper frequency of the operating range.
2. Quadratic interpolation, which is fundamentally simpler in implementation than the optimal one, provides a similar result with a scanning step of about $0.4–0.5$ of the DP width at the upper frequency of the operating range, provided that the phase center of the channels formation is located in the region of the center of gravity of the antenna working spot.
3. Good predictability of bias errors for the considered options of channels formation allows, makes it possible, if necessary, to compensate for these errors to practically insignificant values.
4. The magnitude of the change in the direction of the measured signal arrival relative to the SS channels in the estimation interval should not exceed half the width of the DP at a level of 3 dB at the upper frequency of the operating range.

5. When using more complex antennas compared to the one considered, it may be necessary to reduce the given values of the parameters of the algorithms.

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