Identification of eccentricity for disc milling cutter of indexable two sided inserts

Gensheng Li¹, Chao Xian² and Hongmin Xin³

Abstract
Tool eccentricity has a significant impact on machining quality, accuracy, and operation status of machine tool. It is difficult to accurately identify tool eccentricity. In this paper, the mathematical models of instantaneous undeformed cutting thickness and cutting force considering tool eccentricity are determined by theoretical method. Based on the model, the identification method for eccentricity parameters is proposed, and the eccentricity parameters of disc milling cutter is identified. According to the identified parameters, the cutting force is verified. The results show that most of the values of measured cutting forces are greater than the predicted ones considering tool eccentricity. In the future, it is necessary to establish a new cutting force model considering both tool eccentricity and tool wear.

Keywords
Tool eccentricity, cutting force, models, eccentricity parameters, identification method

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Introduction
Due to the manufacturing and installation errors of the tool-spindle clamping system, the geometric axis of the tool often does not coincide with the spindle rotation axis, which is called tool eccentricity or runout. A large number of studies show that the influence of tool eccentricity on the cutting process is more obvious than the geometric manufacturing error of the tool. Tool eccentricity, runout will directly affect the instantaneous undeformed cutting thickness, and then affect the machining accuracy of parts. At the same time, the influence of tool eccentricity on surface roughness and tool life should not be ignored. Therefore, it is very important to analyze the tool runout and establish a milling force model including tool eccentricity runout. In recent years, the identification methods of tool eccentricity emerge in endlessly. The existing identification methods can be roughly divided into two types: analytical identification method and optimization identification method.

Kline and Devor¹ modify the feed rate of each tooth by using the tool eccentricity, and establishes a simplified calculation model of instantaneous cutting thickness, and analyzes the influence of tool eccentricity on milling force. Wang and Huang,² Wang and Zheng,³ and Liang and Wang⁴ present a method to estimate cutter axis offset in ball end milling. They establish the expression for the total milling force in the consideration of cutter offset, and find that cutter offset

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parameters can be expressed Fourier series coefficients for the offset related force component, and then cutter offset parameters can be directly calculated from the measured force component. Liu et al.\(^5\) also propose that the peak value of milling force is a discrete sine function of cutting edge number, and its mean value is the sum of nominal force without eccentricity and fluctuating force caused by eccentricity. Based on this finding, a data fitting method for estimating eccentricity angle and eccentricity is proposed. Hekman and Liang\(^6\) propose a method for on-line identification of runout parameter, which is in view of a recursive formula of time-dependent Fourier transform of cutting forces and the mechanistic model considering cutter runout. Seethaler and Yellowley\(^7\) present two methods to estimate the tooth runout in milling process. One use torque to determine the tooth runout; the other use in plane component to determine the tooth runout. Liu et al.\(^8\) consider that the cutting force of end milling cutter is divided into nominal force and eccentric force. The nominal force and eccentric force are expanded by trigonometric series of periodic signals, and the expressions of harmonic amplitude and phase are obtained. A method of estimating runout parameters by using harmonic frequency of spindle and tooth of cutting force is proposed. First, the rotation center of the spindle is determined by dial indicator and index plate, then the difference between the effective cutting radii of each cutter tooth is measured and calculated, then the analytic equations are established according to the difference between the two groups of radii, and finally the parameters of cutter eccentricity are solved by numerical method.\(^9\) Firstly, the surface is milled with small cutting depth and low rotation speed, and the width of micro grooves at two different positions on the surface is extracted by digital image processing, and then the analytical equations are established according to the two sets of width values. Finally, the geometric parameters of tool eccentricity are solved by numerical solution.\(^10\) Zhang et al.\(^11\) establish a method to solve the cutter radial runout parameters from the cutting force model which the cutting coefficients are constant. This method successfully avoids the numerical oscillation from the differential model and eliminates the influence of ill posed problems in calibration. Jing et al.\(^12\) develop an approach to model and simulate runout parameters of tool-holder-spindle in micro milling using CCD. Sun and Guo\(^13\) develop a new approach to determine the instantaneous cutting forces in 5-axis milling processes in the presence of radial cutter runout taking into consideration tool motion analysis. The undeformed chip thickness model is presented, and cutting coefficients are identified, and identification of runout parameters based on the measured cutting forces is given. Li et al.\(^14\) established the micro milling force model by spatial analytic geometry. In this model, the tool runout is identified by only one parameter, that is, the center distance perpendicular to the feed direction. Therefore, it can be easily calibrated by calculating the ratio of forces corresponding to different cutting edges.

Zhang et al.\(^15\) develop an approach for obtaining the tool runout parameters based on measuring displacement and solving analytical equations related to the process parameters. Ranganath and Sutherland\(^16\) propose a cutter runout model that includes the effects of cutter grind, parallel axis offset, and cutter tilt in peripheral milling process. The parallel axis runout parameters can be obtained by solving the minimum sum of the square errors between the radii of the cutting edges established by the model and the radii of the measured cutting edges. Krüger and Denkena\(^17\) find that cutting energy per cutting edge is proportional to the chip volume per cutting edge and their standardized values are equal. The standardized cutting energy is obtained by measuring the cutting energy, and the standardized volume of material removal is obtained by theoretical calculation in the predefined cutting parameters and cutting conditions. Then, the runout parameters can be successfully identified by solving minimum value of the difference between standardized cutting energy and standardized volume by Nelder-Mead simplex algorithm. Ko and Cho\(^18\) propose the runout parameter can be determined by selecting the value which minimizes standard deviation value of the cutting force coefficient at the given instantaneous uncut chip thickness in the cutting force model in every revolution of the cutter. Wan et al.\(^19–21\) hold that cutting forces are divided into a nominal component independent of the runout and a perturbation component induced by the runout. Then and the runout parameter is selected as such one that generates minimum of the squared difference between the simulated and measured cutting forces at all sampled instants. Liu et al.\(^22\) use multi-objective optimization and make that the residual error of each harmonic of milling force frequency domain signal is minimized, and the optimal solution of milling force coefficient and eccentricity parameter is obtained by iteration. Liu et al.\(^23\) obtained the resultant force waveform of micro milling by fast Fourier transform, took the combination of resultant force waveform and cutting parameters as input, took the runout as objective, and identified the tool runout parameters by neural network method. Zhang et al.\(^24\) measured the eccentric displacement of the cutting tool and the actual radius difference of each cutting tooth by the laser displacement sensor, and then took the square sum of the difference between the actual radius difference and the
theoretical radius difference for each cutting tooth as the objective to carry out iterative calculation to obtain the eccentric angle.

It can be seen from the above analysis that the analytical identification method mainly expresses the cutting force or cutting force coefficients as Fourier series, and then obtains the eccentricity parameters by relevant derivation and experiment, so the calculation of the identification method is more complex. The optimization identification method takes the square of the difference between the measured value and the predicted value or others as objective, and then establishes the optimization model. The eccentricity parameters can be obtained by optimization. This method needs many iterations and the calculation is not simple enough. Xin et al.\textsuperscript{25–27} and Zhang et al.\textsuperscript{28} proposed a new method to remove the large margin of blisks channel by disc milling cutter instead of ball-end milling cutter or plunge milling cutter. Zhang et al.\textsuperscript{28} presented a theoretical force model for the milling method, but the cutting force coefficients are difficult to identify. In this paper, the cutting force model for single tooth cutting considering cutter eccentricity in disc milling process is given. The cutting force is decomposed into nominal force and eccentric force caused by eccentricity. Based on this, the identification method of eccentric parameters is proposed.

**Experimental method and materials**

**Experimental equipment and materials**

**Milling machine.** For the machining of blisk contour, the plunge milling + side milling process has a long processing cycle, low efficiency, and high cost. Northwest Polytechnic University proposed a new process, which changed the plunge milling + side milling process into the disc milling + plunge milling + side milling process, and developed an efficient and powerful composite CNC milling machine for aero engine blisk, as shown in Figure 1. The equipment technology integrates the three processes of disc milling, plunge milling, and side milling into a special equipment. Through one-time positioning and clamping, disc milling + plunge milling + side milling process can be realized. The grooving and expanding the groove of blisk are realized by disc milling and plunge milling, and the blade profile is finished and cleaned by side milling, as shown in Figure 2. The cutting test shows that the disc milling efficiency is three to four times higher than that of plunge milling.

Workpiece. GH4169 is a common material for blisk manufacturing, which has good fatigue performance, high strength, and good thermal stability. The material used in this test is GH4169, which is solid solution treated after forging. The size of workpiece is $265 \text{ mm} \times 165 \text{ mm} \times 23.5 \text{ mm}$.

**Milling cutter.** The milling cutter used for the experiment is a disc milling cutter of indexable two sided inserts, which consists of a cutter disc and inserts connected by screws. There are 40 inserts, 20 left inserts and 20 right inserts, as shown in Figure 3. The maximum radius of the milling cutter is 420 mm, and the maximum milling width is 10 mm.

![Figure 1. High-efficiency and powerful composite CNC milling machine.](image)

![Figure 2. Composite milling process.](image)
The angle between the cutting edge of inserts and the horizontal plane is 2.2°, as shown in Figure 4. So when the milling cutter is machining the workpiece, it is an oblique cutting process.

The insert material is carbide with physical coating on the surface and the coating material is TiAlCrN. Figure 5 displays the geometry of the insert and the thickness is 4.3 mm. The positioning and clamping mode of inserts and cutter disc is shown in Figure 6. The milling cutter parameters are shown in Table 1.

**Measurement of cutting force**

In this experiment, cutting process is carried out without the use of cutting fluid and in a symmetrical way, that is, the workpiece is clamped in the symmetrical direction of the axial trajectory of the milling cutter. Cutting forces are used for tool eccentricity

| Number of teeth | Diameter (mm) | Thickness (mm) | Rake angle (°) | Flute length (mm) | Helix angle (°) |
|-----------------|--------------|---------------|---------------|------------------|----------------|
| 40              | 420          | 10            | 8             | 6                | ± 2.2          |
identification and the Kistler 9255B three-direction dynamometer is used to measure the cutting force. As shown in Figure 7, the dynamometer is fixed on the workbench, and the workpiece is fixed on the dynamometer. The experimental principle is shown in Figure 8. When the disc milling cutter cuts the workpiece, the cutting force is applied to the dynamometer through the workpiece. The dynamometer converts the force signal into electrical signal, and then the electrical signal is processed by the amplifier and the acquisition instrument, and then transmitted to the computer. The computer can collect the complete cutting force signal.

Determination of cutting parameters

The cutting parameters of disc milling are mainly composed of spindle speed and feed speed, and the cutting depth is constant and the value of cutting depth is 10 mm. The maximum cutting diameter of disc milling cutter is 420 mm, and the cutting force is relatively large, so the spindle speed is generally less than 100 r/min and the spindle speed is set to 50 r/min. Because the cutting depth is constant, when the feed speed exceeds a certain value, the cutting force will increase rapidly, resulting in inserts damage. Therefore, in order to avoid inserts damage, when the feed speed unit is r/min and the feed speed unit is mm/min, the value of feed speed must be less than the spindle speed, so the feed speed should be less than 50 mm/min and the feed speed is set to 25 mm/min. This combination of cutting parameters can achieve high efficiency, stability, and long tool life. The levels of the cutting parameters are displayed in Table 2.

Instantaneous undeformed cutting thickness model under eccentricity

Figure 9 shows the schematic diagram of eccentric milling. O' is the geometric center and O is the center of rotation. The length of line OO' is the eccentricity ρ. The insert with the largest milling thickness is the first insert, and the other inserts are named as the second insert, the third insert, ..., and the Nth insert in the reverse direction of the spindle rotation. E is the equivalent point of the cutting edge of the ith insert, and the angle θi between O'E and OO' is the eccentric angle of the cutting edge of the ith insert. EO is the radius of rotation and EO' is the nominal radius.

As the angle of tooth sweep is less than the angle between teeth, the cutting process is single tooth cutting, that is, only one insert is cutting the workpiece at any one time in the cutting process. Therefore, the instantaneous undeformed cutting thickness of the ith insert under eccentricity is to be

\[ h(\phi_i) = f_i \sin \phi_i + R_i - R_{i-1}, \quad \phi_i \in [0, \beta] \]  

The eccentric angle of milling cutter is the angle between any insert and eccentric direction. According to this definition, the angle \( \theta_i \) between the first insert and the eccentric direction OO’ is the eccentric angle. It can be seen from Figure 9 that the expression of the eccentric angle for the ith tooth is to be

\[ \theta_i = \theta_1 + \frac{2\pi}{N} i, \quad i \in [0, 1, 2, ..., N-1] \]  

\[ R_i = R + \rho \cos \theta_i, \quad i \in [0, 1, 2, ..., N-1] \]

By substituting equations (2) and (3) into equation (1), it is to be
Force model for single tooth cutting under eccentricity

Figure 10 show the geometric relationship. The axis of the disc milling cutter is parallel to the X axis; the feed direction is consistent with Y axis of machine tool. Because the thickness of workpiece is much smaller than the diameter of milling cutter, the milling process is symmetrical milling, so the radial cutting force is equal to the Y-direction cutting force; the tangential force is equal to the Z-direction cutting force; the axial force is equal to the X-direction cutting force, which is approximately equal to zero. It can be seen from the figure that the immersion angle is $\theta_1 = 6.41^\circ$.

Since the inserts are new and there has never been cut before the experiment, the wear is not considered. In the cutting process, the radial cutting force and tangential cutting force for the $i$th insert can be expressed as:

$$h(\phi_i) = f_i \sin \phi_i + 2 \rho \sin \frac{\pi}{N} \sin \left( \left( \theta_1 - \frac{N + 1}{N} \pi \right) + \frac{2 \pi}{N} i \right),$$

$$\phi_i \in [0, \beta], i \in [0, 1, 2, ..., N-1]$$

$$\phi_i \in [0, \beta], i \in [0, 1, 2, ..., N-1]$$

(4)

By substituting equation (4) into equation (5), they are to be

$$F_{r,i}(\phi_i) = C_{r,i}(\psi) + A_{r,i} \sin \left( \phi + \frac{2 \pi}{N} i \right)$$

(6)

where

$$C_{r,i}(\phi_i) = k_{r,i} b f_i \sin \phi_i$$

(7)

$$A_{r,i} = 2 k_{r,i} b \rho_{r,i} f_i \sin \frac{\pi}{N}$$

(8)

$$\varphi = \theta_1 - \frac{N + 1}{N} \pi$$

(9)

It can be seen from equation (6) that force model for single tooth cutting under eccentricity is a function of spindle rotation angle and insert number, which is composed of two parts. The first part equation (7) is the force of single tooth milling with no eccentricity, and the second part equation (8) is related to eccentricity $\rho_{r,i}$, that is, milling force enhancement caused by eccentricity.

Identification for eccentricity parameters and cutting force coefficients

It can be considered that the spindle rotation angle $\phi_i$ of each insert corresponding to the average milling force is equal. Let $\phi_i$ at this point be the definite value $\psi$, it is to be

$$F_{r,i}(\phi_i) = C_{r,i}(\psi) + A_{r,i} \sin \left( \varphi + \frac{2 \pi}{N} i \right)$$

(10)

where

$$C_{r,i}(\psi) = k_{r,i} b f_i \sin \psi$$

(11)

$$A_{r,i} = 2 k_{r,i} b \rho_{r,i} f_i \sin \frac{\pi}{N}$$

(12)

$$\rho_{r,i} = \frac{A_{r,i}}{2 C_{r,i}(\psi) \sin \frac{\pi}{N}}$$

(13)
It can be considered that the cutting rotation angle $\psi$ corresponding to the average value of the milling force for each insert is approximately equal to the half of the immersion angle, that is $\psi = \beta / 2$.

According to equations (10)–(13), the eccentricity and eccentric angle can be identified according to the following steps:

1. Doing experiment, the average value of cutting force for each insert is measured.
2. Using equation (10) to fit the average value of milling force for each insert, the estimated values of $\varphi_i$, $C_{rr}(\psi)$ and $A_{rr}$ are obtained.
3. The values of eccentric angle $\theta_1$ and eccentricity $\rho$ are estimated by equations (9) and (13).

In order to identify the cutting force of each rotation for disc milling cutter, a right insert is removed in the cutting process. The cutting force signals after filtering high frequency parts are shown in the Figure 11. It can be seen from the Figure 11 that when the cutter is not cutting the workpiece, the cutting force is not zero, which is caused by the vibration of the machine tool.

As described in section “Milling cutter”, the disc milling cutter of indexable two sided inserts includes 40 inserts, 20 left inserts and 20 right inserts. When the milling cutter cuts the workpiece, each insert will cause a cutting force signal. So there are 20 cutting force signals for left inserts and 20 cutting force signals for right inserts. The average values of radial force, tangential force and cutting force for each insert can be obtained from Figure 11, as shown in Table 3.

According to section “Identification for eccentricity parameters and cutting force coefficient” and fitting equation (10) with MATLAB, fitting curve of $F_r$ is displayed in Figure 12 and the expression of $F_r$ for the right inserts is obtained as follows:

$$ F_{rr}(\phi_i) = 470.5 + 31.73 \sin \left(1.707 + \frac{\pi i}{10}\right) \tag{14} $$

![Figure 11. Cutting force signal.](image)

| No. | Right insert | Light insert |
|-----|--------------|--------------|
| $F_{rr}$ | $F_{tr}$ | $F_{rl}$ | $F_{tl}$ |
| 1 | 507.9 | 302.3 | 618.6 | 331.9 |
| 2 | 456.7 | 350.8 | 561.6 | 418.9 |
| 3 | 485.1 | 269.7 | 658.6 | 320.8 |
| 4 | 485.7 | 301.5 | 595.2 | 367.9 |
| 5 | 488.7 | 291.8 | 632.7 | 339.6 |
| 6 | 442.3 | 490.5 | 561.8 | 573.1 |
| 7 | 407.9 | 274 | 500.4 | 291.2 |
| 8 | 464.1 | 348.8 | 597.8 | 407.7 |
| 9 | 396.2 | 360.5 | 497.5 | 443.2 |
| 10 | 517.3 | 338 | 721.2 | 464 |
| 11 | 464.8 | 351.8 | 699.1 | 430.1 |
| 12 | 436.5 | 204.8 | 616.1 | 197.6 |
| 13 | 461.1 | 367.3 | 642.4 | 477 |
| 14 | 530 | 343.3 | 804 | 328.9 |
| 15 | 498.5 | 300.1 | 620.1 | 366.5 |
| 16 | 500 | 278.6 | 633.1 | 338.2 |
| 17 | 385.4 | 224.3 | 478.4 | 252.4 |
| 18 | 576.9 | 321.8 | 758.8 | 389.9 |
| 19 | 385.4 | 224.3 | 478.4 | 252.4 |
| 20 | 576.9 | 321.8 | 758.8 | 389.9 |

![Table 3. The average values of radial force, tangential force, and cutting force.](image)
Fitting curve of $F_t$ is displayed in Figure 13 and the expression of $F_t$ for the right inserts is obtained as follows:

$$F_t(\phi_i) = 615.3 + 54.65 \sin \left(15.05 + \frac{\pi i}{10}\right)$$  \hspace{1cm} (15)$$

Fitting curve of $F_t$ is displayed in Figure 14 and the expression of $F_t$ for the left inserts is obtained as follows:

$$F_t(\phi_i) = 308 + 27.44 \sin \left(4.533 + \frac{\pi i}{10}\right)$$  \hspace{1cm} (16)$$

It can be seen from Figures 12–15 that the most deviation rates between the theoretical value and the actual value are less than 30%. It is concluded that the fitting degree is high.

Theoretical feed per tooth is to be
According to section “Identification for eccentricity parameters and cutting force coefficient,” the eccentricity parameter of the inserts is listed in Table 4.

It can be seen from Table 4 that the eccentricity determined by different force expressions has little difference, which means that the accuracy of parameters is high.

In the cutting force expression, compared with the eccentric part, the part with no eccentricity part is larger and the fitting accuracy is higher. Therefore, the cutting force coefficient is calculated by equation (11) rather than equation (12). According to equations (14)–(17), the value of $C_{r,t}(\psi)$ can be obtained by comparing with equation (11), while the values of $b$, $f_t$ and $\sin\psi$ are known, then the values of cutting force coefficients can be obtained. The cutting force coefficients are shown in Table 5.

### Experimental verification

The parameters of experimental verification are shown in Table 6.

The eccentricity parameters and cutting force coefficients have been obtained by the above methods. The theoretical values of cutting forces corresponding to the cutting parameters in Table 6 can be obtained by equation (10). The measured values can be obtained by experiment. The comparison between predicted values and measured values is shown in Figures 16 and 17.

As can be seen from Figures 16 and 17, most of the predicted values are smaller than the measured values, which is due to the fact that the cutting force model does not take into account inserts wear, which can lead to excessive cutting force. A few of the predicted values are higher than the measured values, which is due to the fact that the manufacturing and installation errors of milling disc and inserts.

### Table 4. Identification results of eccentricity parameters.

| The expression of the source | $\theta$ (°) | $\rho$ (mm) |
|-----------------------------|--------------|-------------|
| $F_{rr}$                    | 97.80        | 0.00030     |
| $F_{tr}$                    | 142.30       | 0.00040     |
| $F_{rl}$                    | 259.72       | 0.00040     |
| $F_{tl}$                    | 266.82       | 0.00055     |

### Table 5. Results of cutting force coefficients.

| $k_{rr}$ | 84155.16 |
|----------|----------|
| $k_{tr}$ | 110054.56|
| $k_{rl}$ | 55089.88 |
| $k_{sl}$ | 65624.93 |

### Table 6. The cutting parameters level of experimental verification (mm).

| $n$ | 50 |
|-----|----|
| $f$ | 25 |
| $a_p$ | 10 |
Discussion and conclusions

Cutter eccentricity will reduce the machining quality and accuracy of parts and cutter life. The model of instantaneous undeformed cutting thickness and cutting force model for single tooth cutting considering cutter eccentricity are obtained through theoretical analysis. The cutting force is decomposed into nominal force without eccentricity and eccentric force caused by eccentricity in the model. The identification method for eccentricity and eccentric angle is given. Based on this method, the eccentricity parameters and cutting force coefficients for disc milling cutter are identified. It is verified and it is shown that most of the measured values for cutting forces are larger than the predicted values that is calculated by the cutting force model equation (10), which is because the cutting force model does not consider inserts wear. Compared with the method of literature in the introduction, the approach presented in this paper does not need complex calculation such as Fourier series decomposition or optimization iteration, and only needs one cutting test to obtain the eccentricity parameters by measuring the cutting force. However, the manufacturing and installation errors of the milling disc and inserts are inevitable, and the inserts will inevitably be worn, which will lead to a certain deviations between the measured values and the real values for cutting force, and then cause a certain deviations between the identified values and the real values for eccentric parameters. In the future, it is necessary to establish a new cutting force model considering both tool eccentricity and tool wear.

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**Appendix**

**Notation**

- $\phi_i$: Spindle rotation angle for the $i$th insert, rad
- $f_i$: Theoretical feed per tooth, mm/tooth
- $R$: Nominal radius, mm
- $R_i$: The actual rotation radius of the $i$th insert, mm
- $\beta$: Immersion angle of milling cutter, rad
- $h(\phi_i)$: Instantaneous undeform cutting thickness, mm
- $\theta_i$: Eccentric angle for the $i$th insert, rad
- $N$: Number of teeth
- $\rho$: Eccentricity, mm
- $b$: Flute length, mm
- $k_r$: Radial cutting force coefficient
- $k_t$: Tangential cutting force coefficient
- $a_p$: Cutting depth, mm
- $F_{i,r}$: Radial force of right inserts, N
- $F_{i,t}$: Tangential force of right inserts, N
- $F_{L,r}$: Radial force of left inserts, N
- $F_{L,t}$: Tangential force of left inserts, N
- $F_i$: Radial force acting on the cutter, N
- $F_{i,t}$: Tangential force acting on the cutter, N
- $n$: Spindle speed, r/min
- $f$: Feed speed, mm/min
- $k_{rr}$: Radial cutting force coefficient of right inserts, N/mm
- $k_{rt}$: Tangential cutting force coefficient of right inserts, N/mm
- $k_{tl}$: Radial cutting force coefficient of left inserts, N/mm
- $k_{tt}$: Tangential cutting force coefficient of left inserts, N/mm