Abstract: This paper describes an analysis of the effects of adjusting the intensity of filter backwash to the water temperature. The consequences of the lack of such adjustment for the life of filter beds, the amount of water used for backwashing, the amount of energy used for backwashing and the quality of the first filtrate are presented. In order to determine the losses and profits resulting from controlling the intensity of backwash water depending on its temperature, an analysis was carried out at a water treatment plant in southern Poland. Laboratory measurements were used to determine the granulation and specific gravity of sand grains filling the filtration beds. On the basis of measurements on a semi-technical scale, the magnitudes of filter bed expansion were determined for average monthly wash water temperatures. They were first calculated from the Richardson–Zaki equation, using different formulae for the value of the exponent of the power in this equation. Due to significant differences in the density and shape of grains covered with a permanent deposit after several years of filter operation, a satisfactory match between the formulae known from the literature and the results of expansion measurements was not obtained. Therefore, a new formula for the bed expansion was developed based on the Richardson–Zaki equation. A good fit of this formula to the experimental results was obtained. Monthly average values of water temperature were compiled, and on this basis the required amount of backwash water and energy was computed. The computations were made for 25% of fluidized bed expansion. Possible energy and water savings were estimated, as well as further gains from keeping the required expansion of the porous bed constant regardless of the wash water temperature.

Keywords: bed expansion; fluidization; water filtration

1. Introduction

Backwashing of rapid filters used in water treatment plants was traditionally carried out in the USA without the use of compressed air [1–4]. Since flushing with water alone is not very effective [5–8], it was necessary to use various procedures to support this effectiveness. However, for a long time in Europe, backwashing started with vigorous mixing of filter bed grains with compressed air, during which sudden changes in pressure gradient occurred and grains rubbed against each other [9–11]. First, however, the raw water inflow is closed and the water table in the filter chambers is allowed to drop to a height only a few centimeters above the top surface of the filter bed. As a result, the energy supplied to mix the grains of the filter bed is not dissipated in the water above this bed and the distances between grains remain small during “air washing”, which favors frequent collisions leading to deposit abrasion. The most effective is the air-water flushing [12,13], during which the compressed air supply is closing and the backwash water supply is opening simultaneously. The compressed air supply has to be completely closed before the water table above the filter bed approaches the edge of the overflow. This is necessary to prevent the loss of part of the grain size of the filter bed during backwashing. Afterward, backwashing is carried out with water only. In this final stage of washing,
the grains do not collide vigorously with each other and there is no effective removal of the sediment retained on the surface of the grains, but rather only the displacement of the water suspension that has formed during the two previous washing processes [14,15]. Backwashing is carried out with clean water, and the inlet to the filter is only closed when the backwash water flowing out is clean. In spite of the clarity of the water outflowing from a filter after backwashing, some of the removable sediment and durable deposits remain on the surface of the filtration bed grains, which have adhered so strongly to the grain surface that they are not only not removed during subsequent backwashing but are even difficult to remove partially during treatment of these grains with hydrochloric acid or a concentrated solution of hypochlorous acid [16]. Therefore, such treatments are not applied in practice. After closing the backwash water inflow, the fluidized grains inevitably fall to the bottom of the chamber and rub against each other [17–19]. This causes some of the removable sediment to be stripped off into the water filling the filter bed, as well as partially into the wash water remaining between the surface of the fixed filter bed and the edges of the overflow troughs [17,18]. When the filter is put back into operation, this water suspension causes poor quality in the first filtrate [15,17] and contributes up to 60% to 90% of all solid phase particles that enter the filtrate during the full filtration cycle. Several methods are used to counteract this unfavorable phenomenon [11,18,20]. Thus, it can be concluded that backwashing of rapid filters requires the consumption of treated water and energy and, moreover, it contributes to the deterioration of the quality of the first filtrate and, if carried out with too high intensity, may cause loss of grains from the filtration beds. In addition, it sometimes even contributes to a building catastrophe, resulting in the need for a general reparation of some filters. This last issue is not the subject of this article. Instead, attention will be focused here on all other problems mentioned.

The expansion of the filter bed at the same backwash intensity is higher in winter when the water viscosity is higher and lower in summer. These differences are greater the finer the grains in the bed are and the lower the density of the mineral of which they are composed. Thus, using the same backwash intensity throughout the year may lead to excessive energy expenditure and additional consumption of water and the danger of losing grains of the filtration bed in winter, or to keeping too small expansion of the bed in summer and thus to obtaining only partial washing out of contaminants contained in the bed. The selection of the right backwash intensity depending on the average temperature of the water in the individual months of the year is important both for water and energy demand and for the quality of drinking water. In addition, reducing the amount of heavily contaminated wash water reduces the costs associated with the treatment of these waters and the treatment of the resulting sludge [21–24]. Proper adjusting of the backwash intensity is important for preventing media grain loss [25,26].

The aim of the research was to develop and experimentally verify a computational algorithm allowing us to select the backwash intensity of rapid filters and granular activated carbon (GAC) filters, guaranteeing minimum water demand during fluidization and minimum energy consumption while maintaining the required level of bed expansion, guaranteeing an appropriately high degree of bed purification and at the same time not causing the loss of fine grains. The developed algorithm seeks to select parameters for the backwash intensity of filter in particular months of the year. It was assumed that the algorithm would be a tool to quickly estimate the annual gains for the filter station resulting from adjusting the backwash intensity. An analysis of one of the larger drinking water filter stations in southern Poland was performed to answer the question of how much reduction in water and energy demand could be expected by adjusting the backwash intensity of filter to the average water temperatures in the different months of the year in the climate prevailing in Poland.
2. Materials and Methods

2.1. Calculation Algorithm

Amirtharajah [7] and Kawamura [27,28] and later Sakkas and Lekkas [29] derived by means of analytical methods the equation for the backwash gradient, which is equivalent to the Camp–Stein equation for the mixing gradient. The gradients developed by Amirtharajah [7] and Kawamura [27,28] result in similar optimal porosity of fluidized filter bed [30], while using the approach of Sakkas and Lekkas [29] gives a slightly higher value [31]. However, many years of practice result in applying lower expansion of fluidized filter bed. Hence, in the algorithm adopted in the calculations, it was assumed that the expansion $p$ defined by Equation (1) should be 25%.

$$ p = \frac{V - V_0}{V_0} $$  \hspace{1cm} (1)

where:
- $p$—expansion of the bed during fluidization,
- $V$—volume of the filter bed during fluidization,
- $V_0$—bed volume before fluidization.

For an expansion value $p$ of 25%, a bed porosity $\varepsilon$ was calculated from Equation (1) and the mass balance Equation (2):

$$ V \cdot (1 - \varepsilon) = V_0 \cdot (1 - \varepsilon_0) $$  \hspace{1cm} (2)

where:
- $V$—bed volume during fluidization,
- $\varepsilon_0$—known bed porosity before backwash,
- $\varepsilon$—bed porosity during fluidization.

It is known that the expansion of the filter bed depends on the temperature of the backwash water, especially for granular activated carbon beds and for anthracite and sand beds of fine grain size [32,33]. The finer the grain size and the lower the grain density, the lower the free sedimentation velocity $v_s$ of the filter grains and the more laminar character of their fall, and thus the greater the influence of the dynamic viscosity coefficient $\mu$ on this sedimentation. The $\mu$ value strongly depends on the water temperature $T$. For eighty years, the relationship between the free sedimentation velocity $v_s$ and the intensity of filter bed washing $v$ and the porosity $\varepsilon$ ($v$) has been traditionally described by the Richardson–Zaki Equation (3) [34]:

$$ \frac{v}{v_s} = \varepsilon^n $$  \hspace{1cm} (3)

where:
- $v$—apparent velocity of the fluid in the fluidization zone,
- $v_s$—free sedimentation velocity of individual grains of the bed,
- $n$—power exponent.

In recent years, there have been attempts to describe the expansion of a fluidized filter bed [35] which are not based on Equation (3). However, the determination of the free sedimentation velocity of grains depending on the nature of the movement can be determined from the equations summarized in Table 1 without the knowledge of parameters that are difficult to determine, which makes Equation (3) of great practical importance. From Equation (3), it is possible to calculate the desired values of backwash intensity for the already known porosity $\varepsilon$ of the bed during fluidization, providing that the values of the free sedimentation velocity $v_s$ and the exponent “$n$” are determined. Equation (3) is physically meaningful only for backwash intensities exceeding the minimum fluidization velocity and not exceeding the value of $v_s$. Both the free sedimentation velocity $v_s$ and the exponent “$n$” in Equation (3) depend in the general case on the Reynolds number $Re$. This relationship is more pronounced for fine grains and for grains of low-density material such
as granular activated carbon. Free sedimentation velocities $v_s$ should be determined from the finest grain fractions up to $d_{90}$. It is commonly assumed that preferably 90% and no less than 80% of grains by weight should be fluidized during backwashing by water alone. The expansion is calculated for each fraction separately and then a weighted average is determined taking into account the mass contribution of the individual fractions. This way of calculating the total expansion is a commonly used simplification, although the suspended grains mix with each other during backwash [36]. In monograph [37], several equations for the value of the exponent “$n$” in the Richardson–Zaki Equation (3) were compiled, in the general case depending on the material and grain size and the value of the Reynolds number $Re_s$ for free sedimentation of grains. The free sedimentation velocity $v_s$ of single grains depends on the fluid flow regime. It was calculated from the equations summarized in Table 1 [38]. In Table 1, the sedimentation velocities $v_s$ were calculated using the equations valid for the ranges of the Reynolds and the Archimedes numbers, defined by Equations (4) and (5).

$$Re_s = \frac{d \cdot v_s}{\nu}$$  \hspace{1cm} (4)

$$Ar = \frac{g \cdot d^3 \cdot (\rho_g - \rho_w)}{\nu^2 \cdot \rho_w}$$  \hspace{1cm} (5)

where:

$Re_s$—Reynolds number for free sedimentation of grains,

$Ar$—Archimedes number,

$d$—equivalent diameter of the grain,

$\rho_g$—the density of the grain,

$\rho_w$—density of water,

$\nu$—kinematic viscosity,

$g$—acceleration of gravity.

Table 1. Equations describing the free sedimentation velocity $v_s$ of the single solid phase grain in water as a function of $Re_s$ and $Ar$ [38].

| Laminar Sedimentation | Sedimentation in Transition | Charakter of Flow | Turbulent Sedimentation |
|------------------------|-----------------------------|------------------|-------------------------|
| $10^{-4} < Re_s < 2$  | $2 < Re_s < 500$            | $500 < Re_s < 2 \cdot 10^5$ |
| $1.8 \cdot 10^{-3} < Ar < 36$ | $36 < Ar < 8.3 \cdot 10^4$ | $8.3 \cdot 10^4 < Ar < 1.3 \cdot 10^{10}$ |
| $\lambda = 24/Re_s$  | $\lambda = 18.5/Re_s^{0.6}$ | $\lambda = 0.44$ |
| $Re_s = 0.056 \cdot Ar$ | $Re_s = 0.152 \cdot Ar^{0.715}$ | $Re_s = 1.74 \cdot Ar^{0.5}$ |
| $v_s = (0.056 \cdot Ar) \cdot v/d$ | $v_s = (0.152 \cdot Ar^{0.715}) \cdot v/d$ | $v_s = (1.74 \cdot Ar^{0.5}) \cdot v/d$ |
| $v_s = 0.056 \cdot (g \cdot d^2 / \nu) \cdot (\rho_g / \rho_w)$ | $v_s = 0.152 \cdot (d^{1.145 / 0.430}) \cdot [g \cdot (\rho_g / \rho_w)]^{0.715}$ | $v_s = 1.74 \cdot d^{0.5} \cdot [g \cdot (\rho_g / \rho_w)]^{0.5}$ |

where $\lambda$—friction factor.

For the backwash intensities $v$ determined from Equation (3), the amount of backwashing water was calculated according to Equation (6)

$$W_i = t_p \cdot v_i \cdot A$$  \hspace{1cm} (6)

where:

$W_i$—volume of water used for a single backwash of one filter in the ‘$i$-th’ month of the year,

$v_i$—intensity of bed washing in the “$i$-th” month of the year (depending on temperature),

$A$—the area of a single filter bed,

$t_p$—backwash time with water only, increased by 50% of the water and air backwash time when simultaneously closing the air inlet and opening the water inlet.
The amount of water used to backwash the entire filter station throughout the year can be determined from Equation (7).

\[ W = FN \cdot W_i \cdot f \cdot 365 \] (7)

where:
- \( W \) — volume of water used for backwash of all filters in the station during the year,
- \( FN \) — number of filters in the station,
- \( f \) — average backwashing frequency of one filter per day.

In compiling the energy gains, it was also necessary to first count the energy consumed in fluidizing the filter bed during the backwash with water alone. The hydraulic gradient \( H/L \) was determined from Ergun’s Equation (8), in which the first expression describes the laminar linear component of the flow resistance through the porous bed, as in the Kozeny–Carman equation, while the second term describes the non-linear hydraulic gradient component as a function of backwash intensity \( v \).

\[
\frac{h}{L} = 4.167 \cdot \mu \cdot \left(1 - \varepsilon_0 \right)^2 \cdot \left(\frac{6}{\psi d}\right) v^2 + 0.292 \cdot \left(1 - \varepsilon_0 \right) \cdot \left(\frac{6}{\psi d}\right) \varepsilon_0^3 \cdot \left(6 \cdot \psi \cdot d \right) v^2 \] (8)

where:
- \( h \) — pressure height loss,
- \( L \) — thickness of the filter bed,
- \( \mu \) — dynamic viscosity,
- \( \psi \) — spherical coefficient.

During backwashing of filter media by water alone, the resistance to the flow is almost linear in a function of backwash intensity, until the minimum fluidization velocity is reached. Thereafter, a disturbance occurs before the minimum fluidization velocity is reached and then the flow through the fluidized bed produces an almost constant hydraulic resistance. Increasing the backwash intensity increases the bed expansion, but hardly changes the magnitude of flow resistance, which can be explained by the balance of forces acting on the individual grains. For any backwash intensity, the gravity force minus the Archimedes buoyancy force is equal to the hydrodynamic force. The latter force does not change with backwash intensity, which is possible due to the increase in bed expansion. The amount of pressure loss \( h_{\text{min}} \) caused by the flow through the fluidized bed can be calculated from Equation (9). It describes the equilibrium of forces at the moment of lifting all grains of the bed for a backwash intensity equal to the minimum fluidization velocity.

\[ h_{\text{min}} = L \cdot (1 - \varepsilon_0) \cdot (\rho_\ell - \rho_w) / \rho_w \] (9)

Excluding the small variations in pump-motor efficiency resulting from speed adjustments, the power consumed by the pump just to maintain fluidization during backwash with water alone is therefore proportional to the intensity of the backwash. The energy dissipation to overcome the flow through the pipelines and underdrain must also be added, which requires individual calculations for each filter station. Laminar flow in this case is beyond the scope of interest. The flow in the transient zone is more similar to turbulent flows for larger economic diameters of pipes, especially considering that these pipes are operating under higher economic flow velocities.

The annual amount of energy \( E \) used to backwash all filters in the station can be determined from Equation (10):

\[ E = N_p \cdot t_p \cdot FN \cdot f \cdot 365 \] (10)

where:
- \( E \) — energy,
\( N_p \) — power at pump shaft during filter flushing.

\[
N_p = \frac{Q_p \cdot H_p \cdot \rho_w \cdot g}{\eta_p}
\]

(11)

where:

- \( Q_p \) — pump capacity during backwash,
- \( H_p \) — pump head during backwash,
- \( \eta_p \) — pump efficiency.

The pump capacity \( Q_p \) during the backwashing by water alone is calculated from Equation (12) in which \( W_i \) is defined by Equation (6).

\[
Q_p = \frac{W_i}{t_p}
\]

(12)

The pump head during backwash can be calculated using Equation (13)

\[
H = H_s + H_b + H_d + H_p
\]

(13)

where:

- \( H_s \) — static head,
- \( H_b \) — drop of pressure height on the filter bed \((H_b \geq h_{\text{min}})\),
- \( H_d \) — drop of pressure height on underdrain,
- \( H_p \) — drop of pressure height on pipeline.

2.2. Replacements of the Filter Media during Plant Operation

Over a longer period of time, abrasion of the grains occurs during air and water backwashing as well as rounding of the grain edges, and thus a change in the sphericity \( \psi \). The sphericity \( \psi \) is defined as the ratio of the sphere surface area having the same volume as the grain to the real grain surface area, and ranges from 0.98 to 0.78 for natural mineral grains and may be even lower than 0.70 for artificially crushed grains [39]. For sand grains, the sphericity coefficient depends on the geological origin of the sand. Fluvial sands have more rounded shapes than those of glacial origin. According to a study in South Africa [40], only 0.3% of the weight of the activated carbon filter media was lost through abrasion, while 19% was lost during transport and almost 20% during thermal regeneration. Therefore, abrasion should not change the grain size classification of minerals from which filter beds are made. However, the formation of a biofilm by bacteria and the sticking together of grains, followed by the carrying away of low-density agglomerates formed in this way, is a common occurrence. The same applies to the loss of grains as a result of selecting too high backwash intensity for winter conditions. Moreover, with time, the grains of the filtration bed become partially covered with non-removable sediment, which changes the density of grains to such an extent that it may significantly influence the amount of bed expansion during washing [31]. Therefore, it is advisable to check periodically the filter grain size, or to measure on a technical scale the expansion for a known backwash water temperature.

2.3. Economy of the Filter Backwash System

When comparing the backwash system of water rapid filters with and without temperature-dependent intensity control, a choice must first be made as to how to determine the backwash intensity. If you choose the intensity of backwash for summer conditions, you take the risk of losing some of the grains in winter. If, on the other hand, the backwash intensity is set for winter conditions, there is no risk of losing some of the grains in summer, but at high backwash water temperatures, the expansion is too small, so the backwash is carried out incorrectly, which is likely to affect the quality of the filtrate. Paradoxically, setting backwash for winter conditions can also indirectly lead to grain loss.
through insufficient bed expansion, which leads to an excessive biofilm sticking to the grains. There are many possibilities to set backwash parameters for conditions intermediate between winter and summer. For system comparison, it is assumed here that backwash is actually carried out year-round at the expansion set for summer conditions. A case study was then carried out in which the backwash intensity was adjusted to the average monthly temperatures.

2.4. Rapid Filter Station and Granular Activated Carbon Filters at the Rudawa River Water Treatment Plant

The analyses were carried out at one of four water treatment plants in the city of Krakow in southern Poland. The plant draws water from the Rudawa River. The maximum capacity of the station is 55,000 m$^3$/d, and the current production varies between 22,000 m$^3$/d and 28,000 m$^3$/d. The raw water in the first stage is preliminarily oxidized with sodium permanganate, then it is coagulated in the rapid mixing chamber and flocculated in the slow mixing chambers. After flocculation, the water flows into horizontal settling tanks where floc sedimentation takes place. The water thus prepared goes first to rapid filters and then to filters with granular activated carbon. The filter station is currently equipped with 10 filter chambers filled with quartz sand and 8 chambers filled with granulated activated carbon. The height of the sand beds is 100 cm and the height of the granular activated carbon beds is 220 cm. The surface area of each of the sand filters as well as the filters filled with activated carbon is 30 m$^2$. The capacity of one sand filter varies between 150 and 200 m$^3$/h and the contact time in the GAC filter is 18 min. The sand filters are washed at an average frequency of once a day except on days with heavy rainfall. Backwash is carried out in air, water-air and water systems in sequence. First, the bed is agitated with air and then, while simultaneously closing the air inflow and opening the backwash water inflow, the backwash is conducted in the water-air system. The process ends with backwash with water only. GAC filters are only backwashed once a week not because of clogged pores, but because of the bacteriology of the treated water.

A specially designed optical device the Raven Environmental Products, Saint Louis, Missouri, USA SID-10200 Sludge Interface Detector was used to measure the expansion of the filter bed in the Rudawa River water treatment plant. The device allows the height of the bed to be determined during fluidization from a higher level.

Laboratory tests were carried out on a column (Figure 1) with a square cross-section of 20.5 cm $\times$ 20.5 cm. It was assumed that the column cross-section is large enough that the wall effect accompanying fluidization should not significantly affect the experimental results. In addition, a uniform distribution of backwash flow during fluidization was maintained across the cross-section of the column by using perforated plates in the subfilter chamber and a stainless-steel wedge wire grating with triangular cross-section and narrow slots of 0.25 mm, spaced parallel every two millimeters.

The column was filled with sand taken from one of the rapid filters at the Rudawa River water treatment plant under analysis. Two sand samples were taken independently from the filter, dried in the laboratory, sieved, and weighed, and then two independent sieving curves were produced based on the measurements taken. As the curves are very close to each other, the sand samples taken were considered to be representative for this filter. The sand samples were mixed together and the sieving curve calculated after mixing at known weight ratios. The representative sieving curve is presented in Figure 2.
Figure 1. Scheme of laboratory set-up (1—tank with raw water and pump, 2—heater, 3—thermometer, 4—overflow tank, 5—column filled with filter media, 6—flowmeter).

Figure 2. Sieving curve.

The height of the column was 2.0 m. The height of the unfluidized sand bed was equal to 42 cm. Backwashing of the sand bed took place from a much higher placed small tank with an overflow into which water was pumped from a large lower tank where tap water was heated to the required temperature. Water flowing through the overflow in the upper tank was returned to the lower tank. The flow was measured using two flow meters in series.
3. Results
3.1. The Exponential “n” in Richardson–Zaki Equation

The analysis was conducted for the filter station at the Rudawa River water treatment plant. The tests were carried out directly at the station and at the stand described in the previous paragraph and built in the laboratory of the Cracow University of Technology.

The average monthly raw water temperatures at the Rudawa water treatment plant are shown in Figure 3. The plant is supplied with river water through retention tanks with a retention time of 7 days, which results in an equalization of the daily raw water temperature entering the station.

![Figure 3. Monthly averages water temperatures in four consecutive years at the station plant and averages of these averages.](image)

Measurements were carried out to determine the grain density of the filter bed in the Rudawa water treatment plant (WTP). The results show that the density of sand grains in the studied bed together with a layer of sediment permanently bound on their surface was 2087 kg/m$^3$ and was significantly lower than the density of different quartz varieties, which usually ranges between 2648 and 2651 kg/m$^3$. Such low density values of the grains meant that they were covered by a large amount of permanently bound sediment. The biofilm consists of approximately 90% extracellular polymeric substances and 10% bacteria. Its density is close to that of water, such this adhesion reduces the density of the sand covered by biofilm. Most of the equations proposed in the literature for calculating the “n” value of the power exponent appearing in the Richardson–Zaki Equation (3) are applied to grains of silica sand or other materials, but do not cover a significant amount of permanently bound sediment. It was therefore necessary to develop an equation that would describe the expansion of grains with such low density and very strongly curved edges. To calculate the exponent “n” in Equation (3), Function (14) was developed by fitting the coefficients in this equation using the method of least squares applied to the results of expansion measurements.
Figure 4 shows measured values and the results of the bed expansion calculations. Calculations were performed using the equations in Table 1 and Equations (1)–(5) and Equation (14) fitted to the experimental measurements for temperatures of 13, 19, and 25 °C. The coefficients of determination $R^2$ for the filter bed expansion determined, considering Equation (14) and the measured values, were 0.982 for 13 °C, 0.962 for 19 °C and 0.950 for 25 °C, respectively.

$$n = 3.32R e^{-0.012}$$  \hspace{1cm} (14)

Figure 4. Measured and computed values of sand bed expansion $p(\%)$ as a function of flushing intensity for flushing water temperatures of 13 °C, 19 °C, 25 °C together with trend lines.

To smooth the results of the calculations shown in Figure 4, trend lines were drawn for them together with the equations summarized in Table 2. High values of $R^2$ with a relatively high number of measurements, and therefore a high number of degrees of freedom, indicate a good fit of the trend lines.

Table 2. Trend lines for filter bed expansion as a function of flushing intensity.

| Temperature | Approximation Equation            | $R^2$  | Equation |
|-------------|----------------------------------|--------|----------|
| 13 °C       | $p(\%) = 1029.4 \, v^2 + 12.7 \, v - 0.0178$ | 0.9966 | (15)     |
| 19 °C       | $p(\%) = 1112.4 \, v^2 + 5.7406 \, v - 0.0039$ | 0.9817 | (16)     |
| 25 °C       | $p(\%) = 1379.6 \, v^2 - 2.3157 \, v + 0.006$ | 0.9805 | (17)     |

Based on the proposed algorithm, considering the equations in Table 1, Equations (1)–(5) and the newly developed Equation (14) fitted to the experimentally measured points in Figure 4, expansion curves for temperatures of 3, 5, 10, 13, 19, 25, and 28 °C were determined and are shown in Figure 5.
3.2. Viscosity Coefficient

The dynamic viscosity of water varies with temperature according to Equation (18) [41], in which $\mu(T)$ and $\mu(T_0)$ are the dynamic viscosities corresponding to the instantaneous temperature $T$ and the reference temperature $T_0$, respectively.

$$\frac{\mu(T)}{\mu(T_0)} = \frac{1 + 0.0337 \cdot T + 0.00022 \cdot T^2}{1 + 0.0337 \cdot T_0 + 0.00022 \cdot T_0^2}$$

(18)

where:

$T$—temperature in °C,

$T_0$—reference temperature in °C.

For backwash below the minimum fluidization velocity, the hydraulic resistance of the flow depends almost linearly on the backwash intensity. However, once the sand is fluidized, the flow is no longer laminar linear, as the current lines are not fixed in time and the local flow velocities at the grain–water interface are different from zero. For granular activated carbons in work [42], expansion was described for different temperatures of washing water using the results of measurements carried out for a reference temperature. It was assumed that the expansion is identical for the same product of the dynamic viscosity raised to the power of “z” and washing intensity $v$, thus for the same product $v \cdot \mu$. Equation (18) shows that $\mu(T) = \mu(T_0)/(1 + 0.0337 \cdot T_0 + 0.00022 \cdot T_0^2)/(1 + 0.0337 \cdot T + 0.00022 \cdot T^2)$. For the reference temperature $T_0 = 13$ °C, the value $\mu(t_0) = 1.2028 \text{ mPa} \cdot \text{s}$. Taking the expansion curve $\mu(t_0 = 13$ °C) as a starting point, the trend line $v = v(p)$ for backwash intensity was first calculated. Measurements and the calculations are technically irrelevant for $p$ expansions below 6%. To obtain a better fit of the trend line, these data were not considered. After omitting the measurements for $p < 6\%$, a very good fit was obtained with a high $R^2$ value for the approximation $v(p)$ with a power function. This function was used to calculate $v(p)$ for $T_0 = 13$ °C, for $p$ values the same as those recorded in measurements taken for $T = 19$ °C and 25 °C. The equation $v(T) = v(T_0) \cdot [\mu(T_0)/\mu(T)]^z$ was then used for the same expansions to predict calculated $v(p)$ values for 19 and 25 °C. The results of calculations and measurements are compared in Figure 6 for $z = 1$. The calculated $v(p)$ values are close to the measurements even for $z = 1$. However, it was decided that measurements of $p$ made for only three close temperatures (one of which is the reference temperature) were not sufficient to evaluate the applicability of the equation $v(T) = v(T_0) \cdot [\mu(T_0)/\mu(T)]^z$. Another widely accepted prediction method $p(v, T)$ was used to calculate the potential for water and energy savings during the backwashing of filters at the Rudawa water station (south of Poland).
Figure 6. Result of fitting calculated values of bed expansion to measured values for temperatures of 13 °C and 25 °C.

3.3. Calculations for an Expansion of 25%

Since the 25% expansion lies in the middle of the recommended expansion range of the filter bed, calculations of the expected water and energy savings were carried out for this value of expansion. For this purpose, from the equations in Table 2 and from Equation (14), the values of the backwash intensity $v(T)$ at which 25% expansion will occur were calculated for temperatures of 3, 5, 10, and 28 °C. The results are quite accurately approximated by the linear Equation (19), which is presented in Figure 7. In Equation (19), the temperature $T$ is expressed in °C and backwash intensity $v$ in m/h.

$$v = 0.5928 \cdot T + 32.095$$  \hspace{1cm} (19)

Figure 7. Intensity of rapid filter backwashing for 25% expansion, as a function of temperature.
3.4. Analysis of Annual Water Savings in a Rapid Filter Plant

Based on the developed mathematical algorithm, the annual quantity of water used for the backwashing of rapid filters was determined for the analyzed filter station in the water treatment plant, once with the assumption that the intensity of backwash was maintained at the same level throughout the year, and the second time that it was adjusted to the average monthly temperature. In the case of the adjustment of the backwash intensity, it was assumed that it was adjusted to the average monthly temperature so as to maintain a constant expansion of 25% throughout the year. In the case of the unadjusted backwash, it was assumed that the backwash intensity was the same throughout the year and that it guaranteed an expansion of 25% in the warmest month. Table 3 shows the results of the water consumption calculations for the backwashing with intensity adjustment. Table 4 shows the results of the calculations of water consumption for backwashing without monthly adjustment of intensity. Finally, Table 5 compares the monthly and annual results and the water savings resulting from adjusting the backwash intensity. These gains amount to about 854,100 m³.

Table 3. Amount of water used for the backwash of rapid filters with monthly adjustment of backwash intensity.

| Month   | Average Temperature (°C) | Backwash Intensity (m/h) | Water Consumption Per One Backwash of One Filter (m³) | Monthly Water Consumption of One Filter Backwash (m³) | Monthly Water Consumption of All Filters (m³) |
|---------|---------------------------|---------------------------|-----------------------------------------------|--------------------------------|---------------------------------------------|
| January | 7.63                      | 37.5                      | 187.5                                         | 5812.5                                      | 58,125                                      |
| February| 6.84                      | 36.2                      | 181                                           | 5249                                        | 52,490                                      |
| March   | 8.40                      | 39                        | 195                                           | 6045                                        | 60,450                                      |
| April   | 12.04                     | 42                        | 210                                           | 6300                                        | 63,000                                      |
| May     | 15.19                     | 43.6                      | 218                                           | 6758                                        | 67,580                                      |
| June    | 18.01                     | 44.8                      | 224                                           | 6720                                        | 67,200                                      |
| July    | 19.20                     | 45.7                      | 228.5                                         | 7083.5                                      | 70,835                                      |
| August  | 18.96                     | 45.2                      | 226                                           | 7006                                        | 70,060                                      |
| September | 16.24                    | 43.9                      | 219.5                                         | 6585                                        | 65,850                                      |
| October | 14.12                     | 43                        | 215                                           | 6665                                        | 66,650                                      |
| November| 10.89                     | 41.2                      | 206                                           | 6180                                        | 61,800                                      |
| December| 8.86                      | 39.5                      | 197.5                                         | 6122.5                                      | 61,225                                      |

Yearly backwash water consumption of all filters (m³) 765,265

Table 4. Amount of water used for backwash of rapid filters without monthly backwash intensity adjustment.

| Month   | Backwash Intensity (m/h) | Water Consumption Per One Backwash of One Filter (m³) | Monthly Water Consumption of One Filter Backwash (m³) | Monthly Water Consumption of All Filters (m³) |
|---------|---------------------------|-----------------------------------------------|--------------------------------|---------------------------------------------|
| January | 45.7                      | 228.5                                         | 7083.5                                      | 70,835                                      |
| February| 45.7                      | 228.5                                         | 6626.5                                      | 66,265                                      |
| March   | 45.7                      | 228.5                                         | 7083.5                                      | 70,835                                      |
| April   | 45.7                      | 228.5                                         | 6855                                        | 68,550                                      |
| May     | 45.7                      | 228.5                                         | 7083.5                                      | 70,835                                      |
| June    | 45.7                      | 228.5                                         | 6855                                        | 68,550                                      |
| July    | 45.7                      | 228.5                                         | 7083.5                                      | 70,835                                      |
| August  | 45.7                      | 228.5                                         | 7083.5                                      | 70,835                                      |
| September | 45.7                     | 228.5                                         | 6855                                        | 68,550                                      |
| October | 45.7                      | 228.5                                         | 7083.5                                      | 70,835                                      |
| November| 45.7                      | 228.5                                         | 6855                                        | 68,550                                      |
| December| 45.7                      | 228.5                                         | 7083.5                                      | 70,835                                      |

Yearly backwash water consumption of all filters (m³) 836,310
Table 5. Water savings for backwash of rapid filters with monthly adjustment of backwash intensity vs. backwash at constant intensity throughout the year.

| Month    | Monthly Water Consumption Without Seasonal Backwash Intensity Adjustment (m³) | Monthly Water Consumption with Seasonal Backwash Intensity Adjustment (m³) | Backwash Water Savings (m³) | Backwash Water Savings (%) | Water Savings in Relation to Total Station Water Production (%) |
|----------|--------------------------------------------------------------------------------|--------------------------------------------------------------------------|----------------------------|-----------------------------|---------------------------------------------------------------|
| January  | 70,835                                                                         | 58,125                                                                   | 12,710                     | 17.94                       | 1.64                                                          |
| February | 66,265                                                                         | 52,490                                                                   | 13,775                     | 20.79                       | 1.90                                                          |
| March    | 70,835                                                                         | 60,450                                                                   | 10,385                     | 14.66                       | 1.38                                                          |
| April    | 68,550                                                                         | 63,000                                                                   | 5550                       | 8.10                        | 0.72                                                          |
| May      | 70,835                                                                         | 67,580                                                                   | 3255                       | 4.60                        | 0.43                                                          |
| June     | 68,550                                                                         | 67,200                                                                   | 1350                       | 1.97                        | 0.18                                                          |
| July     | 70,835                                                                         | 70,835                                                                   | 0                          | 0.00                        | 0.00                                                          |
| August   | 70,835                                                                         | 70,060                                                                   | 775                        | 1.09                        | 0.10                                                          |
| September| 68,550                                                                         | 65,850                                                                   | 2700                       | 3.94                        | 0.36                                                          |
| October  | 70,835                                                                         | 66,650                                                                   | 4185                       | 5.91                        | 0.54                                                          |
| November | 68,550                                                                         | 61,800                                                                   | 6750                       | 9.85                        | 0.90                                                          |
| December | 70,835                                                                         | 61,225                                                                   | 9610                       | 13.57                       | 1.24                                                          |
| **Total**| **836,310**                                                                    | **765,265**                                                              | **71,045**                 | **8.50**                    | **0.78**                                                     |

From Tables 3–8, it can be concluded that the greatest water and energy savings occur in winter when water temperatures are at their lowest. Comparisons were made for the case of adjusting the backwash intensity once to the summer water temperature (July) and then to the average water temperature in each month. Hence, in July, the backwash intensity is identical in both cases, so there are no water or energy savings in this month.

Table 6. Energy consumption for backwashing of rapid filters without monthly adjustment of the backwash intensity to water temperature.

| Month    | Backwash Intensity (m/h) | Pump Flowrate (m³/h) | Discharge Static Head (m) | Pressure Drop on the Filter bed (m) | Pressure Drop on Underdrain (m) | Power at Pump Shaft (kW) | Time of All Rapid Filters Water Backwash during the Month (h) | Energy for Filter Water Backwash (kWh) |
|----------|--------------------------|----------------------|--------------------------|-------------------------------------|--------------------------------|--------------------------|---------------------------------------------------------------|---------------------------------------|
| January  | 45.70                    | 1371                 | 3                        | 4.60                                | 1.45                          | 52.00                    | 51.7                                                          | 2686.90                               |
| February | 45.70                    | 1371                 | 3                        | 4.51                                | 1.42                          | 51.32                    | 48.3                                                          | 2480.54                               |
| March    | 45.70                    | 1371                 | 3                        | 4.69                                | 1.48                          | 52.68                    | 51.7                                                          | 2721.85                               |
| April    | 45.70                    | 1371                 | 3                        | 5.13                                | 1.60                          | 55.90                    | 50.0                                                          | 2795.03                               |
| May      | 45.70                    | 1371                 | 3                        | 5.53                                | 1.71                          | 58.78                    | 51.7                                                          | 3036.85                               |
| June     | 45.70                    | 1371                 | 3                        | 5.90                                | 1.80                          | 61.42                    | 50.0                                                          | 3071.01                               |
| July     | 45.70                    | 1371                 | 3                        | 6.06                                | 1.84                          | 62.54                    | 51.7                                                          | 3231.49                               |
| August   | 45.70                    | 1371                 | 3                        | 6.03                                | 1.83                          | 62.32                    | 51.7                                                          | 3219.92                               |
| September| 45.70                    | 1371                 | 3                        | 5.67                                | 1.74                          | 59.75                    | 50.0                                                          | 2987.66                               |
| October  | 45.70                    | 1371                 | 3                        | 5.39                                | 1.67                          | 57.79                    | 51.7                                                          | 2986.07                               |
| November | 45.70                    | 1371                 | 3                        | 4.99                                | 1.56                          | 54.87                    | 50.0                                                          | 2743.57                               |
| December | 45.70                    | 1371                 | 3                        | 4.75                                | 1.49                          | 53.08                    | 51.7                                                          | 2742.47                               |

Yearly energy consumption for all filters backwash (kWh): 34,703.35
Table 7. Energy consumption for backwash of rapid filters with monthly adjustment of backwashing intensity.

| Month | Backwash Intensity (m/h) | Pump Flowrate (m³/h) | Discharge Static Head (m) | Pressure Drop on the Filter Bed (m) | Pressure Drop on Under-drain (m) | Power at Pump Shaft (kW) | Time of All Rapid Filters Water Backwash during the Month (h) | Energy for Filter Water Backwash (kWh) |
|-------|--------------------------|----------------------|--------------------------|-----------------------------------|---------------------------------|--------------------------|-------------------------------------------------------------|---------------------------------------|
| January | 37.5 | 1125 | 3 | 3.69 | 1.45 | 38.37 | 51.7 | 1982.67 |
| February | 36.2 | 1086 | 3 | 3.47 | 1.42 | 35.95 | 48.3 | 1737.48 |
| March | 39 | 1170 | 3 | 3.93 | 1.48 | 41.22 | 51.7 | 2129.94 |
| April | 42 | 1260 | 3 | 4.67 | 1.60 | 48.95 | 50.0 | 2447.55 |
| May | 43.6 | 1308 | 3 | 5.25 | 1.71 | 54.54 | 51.7 | 2818.11 |
| June | 44.8 | 1344 | 3 | 5.77 | 1.80 | 59.49 | 50.0 | 2974.67 |
| July | 45.7 | 1371 | 3 | 6.06 | 1.84 | 62.54 | 51.7 | 3231.49 |
| August | 45.2 | 1356 | 3 | 5.95 | 1.83 | 61.23 | 51.7 | 3163.50 |
| September | 43.9 | 1317 | 3 | 5.42 | 1.74 | 56.05 | 50.0 | 2802.33 |
| October | 43 | 1290 | 3 | 5.04 | 1.67 | 52.48 | 51.7 | 2711.60 |
| November | 41.2 | 1236 | 3 | 4.45 | 1.56 | 46.65 | 50.0 | 2332.68 |
| December | 39.5 | 1185 | 3 | 4.03 | 1.49 | 42.34 | 51.7 | 2187.55 |

Yearly energy consumption for all filters backwash (kWh) 30,519.58

Table 8. Energy savings on backwash rapid filters with monthly adjustment of backwash intensity versus backwash at constant intensity throughout the year.

| Month | Energy Consumption without Monthly Backwash Intensity Adjustment (kWh) | Energy Consumption with Monthly Backwash Intensity Adjustment (kWh) | Energy Savings (kWh) | Energy Savings (%) |
|-------|---------------------------------------------------------------------|-------------------------------------------------------------------|---------------------|-------------------|
| January | 2686.90 | 1982.67 | 704.23 | 26.21 |
| February | 2480.54 | 1737.48 | 743.06 | 29.96 |
| March | 2721.85 | 2129.94 | 591.90 | 21.75 |
| April | 2795.03 | 2447.55 | 347.48 | 12.43 |
| May | 3036.85 | 2818.11 | 218.74 | 7.20 |
| June | 3071.01 | 2974.67 | 96.34 | 3.14 |
| July | 3231.49 | 3231.49 | 0.00 | 0.00 |
| August | 3219.92 | 3163.50 | 56.42 | 1.75 |
| September | 2987.66 | 2802.33 | 185.32 | 6.20 |
| October | 2986.07 | 2711.60 | 274.48 | 9.19 |
| November | 2743.57 | 2332.68 | 410.89 | 14.98 |
| December | 2742.47 | 2187.55 | 554.92 | 20.23 |
| Total | 34,703.35 | 30,519.58 | 4183.78 | 12.06 |

3.5. Analysis of Annual Energy Savings in a Rapid Filter Plant

The energy savings resulting from adjusting the backwash intensity of the filter plant on a monthly basis were also analyzed. The differences in energy consumed were mainly due to the different amounts of water required for backwash, and to a lesser extent the different bed resistances during fluidization. Bed resistance to flow during fluidization was determined from Equation (9), and power and energy from Equations (10)–(11). The results of calculations for filters washed with the same intensity are summarized in Table 6,
with the intensity adjusted monthly to temperature in Table 7, and a comparison of both calculations in Table 8.

3.6. Granulated Activated Carbon

In the water treatment plant for which an analysis of the benefits of adjusting the backwash intensity to the water temperature was carried out, all of the water after the sand filters is diverted, without ozone treatment, to eight filters with GAC which are in urgent need of replacement. A decision on the type of granulated activated carbon to be used has not yet been taken. Four types of new GAC produced by Chemviron Carbon were selected, for which equations previously developed by one of the co-authors of this article were used [42]. Equations (20)–(23) for backwash intensity as a function of water temperature and GAC bed expansion are presented in Table 9. Based on these equations and monthly average temperatures, the backwash intensity required to achieve bed expansion of 25% for the four selected GAC types during the year was calculated and is shown in Figure 8. Based on the backwash intensities thus determined, the volume of backwash water expected to be used each month of the year for all the GAC filters was calculated for the conditions at the Rudawa River water treatment plant and is shown in Figure 9. As in the case of rapid sand filters, the monthly and annual water consumption for backwashing GAC filters without and with backwash intensity control was calculated. Calculations of water savings achieved by using backwash intensity control of the filters were carried out for the four selected GAC types and are presented in Tables 10 and 11. Tables 10 and 11 compare the backwash water savings from adjusting the backwash intensity of the GAC filters first to the summer water temperature in July and then to the average water temperatures in each month. Hence, the backwash intensity in July was identical in both cases, so there were no water savings in that month. As expected, the highest savings were achieved in winter, when the required expansion could be achieved at a much lower intensity than the adjusted intensity for the summer season (July).

Table 9. Approximation equations for backwash intensity as a function of water temperature \( t \) and bed expansion \( p \) taken from [42].

| Name of Carbon | Formula for New GAC Bed Backwash Intensity |
|----------------|------------------------------------------|
| Filtrasorb 100 | \( v(p, T) = \left\{ \begin{array}{l} 13.9 + 2.025 \cdot (p - 5)^{0.746} + 0.002 + 0.001 \cdot (p - 5)^{0.963} \cdot (10 - T)^2 \\ 1 + 0.0337 \cdot T + 0.00022 \cdot T^2 + 0.00022 \cdot T^2 \cdot (10 - T)^2 \end{array} \right. \) (20) |
| Filtrasorb 200 | \( v(p, T) = \left\{ \begin{array}{l} 8.4 + 1.143 \cdot (p - 5)^{0.792} + 0.003 + 0.001 \cdot (p - 5)^{0.839} \cdot (10 - T)^2 \\ 1 + 0.0337 \cdot T + 0.00022 \cdot T^2 + 0.00022 \cdot T^2 \cdot (10 - T)^2 \end{array} \right. \) (21) |
| Filtrasorb 400 | \( v(p, T) = \left\{ \begin{array}{l} 7.64 + 0.802 \cdot (p - 5)^{0.834} + 0.0017 + 0.0015 \cdot (p - 5)^{0.696} \cdot (10 - T)^2 \\ 1 + 0.0337 \cdot T + 0.00022 \cdot T^2 + 0.00022 \cdot T^2 \cdot (10 - T)^2 \end{array} \right. \) (22) |
| Filtrasorb TL820| \( v(p, T) = \left\{ \begin{array}{l} 14.74 + 1.858 \cdot (p - 5)^{0.758} + 0.134 + 0.023 \cdot (p - 5)^{0.908} \cdot (10 - T)^2 \\ 1 + 0.0337 \cdot T + 0.00022 \cdot T^2 + 0.00022 \cdot T^2 \cdot (10 - T)^2 \end{array} \right. \) (23) |
Figure 8. Backwash intensities of different GAC types at 25% expansion and average monthly water temperatures.

Figure 9. Volume of water used for backwashing 8 GAC filters at Rudawa River water reatment plant with monthly adjustment of backwash intensity.
Table 10. Backwash water savings of 8 GAC filters with monthly adjustment of backwash intensity versus backwash at constant intensity throughout the year in cubic meters.

| Month   | FS100  | FS200  | FS400  | FSTL820 |
|---------|--------|--------|--------|---------|
| January | 1186.86| 1098.29| 708.57 | 938.86  |
| February| 1257.71| 1169.14| 761.71 | 992.00  |
| March   | 1098.29| 1027.43| 655.43 | 868.00  |
| April   | 726.29 | 690.86 | 442.86 | 566.86  |
| May     | 407.43 | 389.71 | 248.00 | 318.86  |
| June    | 106.29 | 124.00 | 70.86  | 88.57   |
| July    | 0.00   | 0.00   | 0.00   | 0.00    |
| August  | 17.71  | 35.43  | 17.71  | 17.71   |
| September| 301.14| 301.14 | 194.86 | 230.29  |
| October | 513.71 | 496.00 | 318.86 | 407.43  |
| November| 850.29 | 797.14 | 513.71 | 673.14  |
| December| 1045.14| 974.29 | 637.71 | 832.57  |
| Total   | 7510.86| 7103.43| 4570.29| 5934.29 |

Table 11. Backwash water savings of 8 GAC filters with monthly adjustment of backwash intensity versus backwash at constant intensity throughout the year in percent.

| Month    | FS100 | FS200 | FS400 | FSTL820 |
|----------|-------|-------|-------|---------|
| January  | 17.59 | 25.41 | 19.42 | 14.36   |
| February | 18.64 | 27.05 | 20.87 | 15.18   |
| March    | 16.27 | 23.77 | 17.96 | 13.28   |
| April    | 10.76 | 15.98 | 12.14 | 8.67    |
| May      | 6.04  | 9.02  | 6.80  | 4.88    |
| June     | 1.57  | 2.87  | 1.94  | 1.36    |
| July     | 0.00  | 0.00  | 0.00  | 0.00    |
| August   | 0.26  | 0.82  | 0.49  | 0.27    |
| September| 4.46  | 6.97  | 5.34  | 3.52    |
| October  | 7.61  | 11.48 | 8.74  | 6.23    |
| November | 12.60 | 18.44 | 14.08 | 10.30   |
| December | 15.49 | 22.54 | 17.48 | 12.74   |
| Total    | 9.27  | 13.70 | 10.44 | 7.57    |

4. Discussion

It is well known that porous media bed expansion depends on water temperature, at least for fine grains and for GAC. However, most filter plants do not have a backwash intensity control system adjusting the backwash to the current temperature. There are several reasons for this situation. One of them is that it has not been reported how much water and energy can be saved. The study presented here gives an estimation of the saving opportunities. These possibilities are underestimated because they do not take into account the savings in the backwash water treatment or the pressure losses in the backwash water supply lines. Surprisingly significant changes over time can occur to filter media, including not only the changes in shape and size, but also the overall density of the grains, taking into account impurities permanently adhered to the grains surface. A new formula (14) for the exponential “$n$” of the Richardson–Zaki equation was developed here. This equation is
valid for the sand from the filter plant studied here, which has been in operation for many years. It would be interesting to see if this formula is also correct for other sand media used for filtration purposes in surface water treatment plants. In the future, more possibilities for water and energy saving should be analyzed and the results obtained should be applied.

Analyses carried out in this article for a medium-sized water treatment plant located in the northern part of Europe have shown almost 10% water savings per year for backwashing of rapid filters and also GAC filters through the use of backwash intensity control. The amount of water saved in this way was 836,310 m$^3$ per year using rapid filters and, depending on the type of GAC, from 4570 to 7511 m$^3$.

Although the analysis of energy savings due to backwash intensity regulation showed a gain of more than 10% in the year, relatively small benefits are expected, since a relatively low amount of energy is consumed for filter backwash.

In addition to adjusting the backwash intensity to the water temperature, other actions can be taken to save water and energy. When using anthracite-sand beds, and multi-layer beds in general, the aim is usually to lay down a coarse layer of material with a lower density on the bed surface. The pores in this layer are large, leading to a significantly longer filter cycles. A granular activated carbon (GAC) bed can be used simultaneously as a mechanical filter and sorbent, which also leads to a significant reduction in the amount of backwash water. A thin layer of GAC is also used as the top layer of multi-layer water rapid filters. GAC filters upstream of sand filters are increasingly used in Japan. The reason for this is the large number of microorganisms inhabiting the carbon dust from filter beds filled with granular active carbon. These microorganisms are extremely resistant to disinfection because they are partially hidden in the largest pores of the dust. Moreover, the most popular disinfectants based on chlorine turn out to be ineffective in these cases. Although water savings for backwashing the filtration-adsorption system in the order GAC-sand filters are obviously not the main reason for using sand filters after GAC, these savings are evident. In the traditional sand filter-ozonation-GAC systems, granular activated carbon is usually washed weekly due to the bacteriology of the treated water, and the fine-grained sand filter bed becomes clogged quickly, filtering the water suspension of high concentration flowing from the settling tanks. On the other hand, in the GAC-sand system, granulated activated carbon does not clog so quickly due to the large size of the pores and the sand filter can be backwashed rarely due to the low concentration of the inflowing suspension. The choice of the filter operation system also has a great impact on saving water and energy. Many years of research, summarized in [43], have shown that it is difficult to decide on the basis of numerous experiments which filtrate is better, that coming from the operation of identical filters with a constant or with a declining variable filtration rate [44]. The performed numerical calculations [43,45,46] based on a simple capillary model (unit bed element) concluded with the suggestion that the filtrate quality from an identical rapid filter operated in a variable declining rate (VDR) system was slightly better, when the filter cycles ended after obtaining the same head pressure losses of flow through the bed. However, according to the calculations, a slightly better filtrate was obtained in the system with a constant filtration rate, when the filter cycles were of the same length [43]. First, theoretically [47], and then experimentally [48], it has been shown that in the case of filter plants controlled by variable declining rate operation systems, in comparison with constant rate systems, longer filter cycles, and thus saving water for backwashing, are achieved in the VDRF operation systems if the ratio of the hydraulic load of the most recently backwashed filter to the average hydraulic load of the all filters in the plant is higher than 1.2. This VDR system can be used for the time-varying turbidity of raw water [49,50], but it requires adjustment when changing the water temperature and it is troublesome to use this system in large stations due to small fluctuations in the water table above the filters, which makes it difficult to determine the appropriate time of subsequent backwashing in stations [51]. Thus, financial benefits do not always determine the adopted technical solutions. Other energy and water saving possibilities
such as appropriate internal pipe lining [52,53] or leakage protection [54] inside the filter station are of marginal importance.

5. Conclusions
- An algorithm for selecting backwash intensity for rapid filters and GAC filters in a water treatment plant was successfully developed and applied to an example.
- An algorithm has been developed for determining water and energy savings due to the use of backwash intensity control for rapid and GAC filters in the water treatment plant in individual months of the year.
- A new formula was developed to determine the exponent “n” in the Richardson–Zaki equation for selecting appropriate backwash intensity.
- It was found that the temperature dependence of backwash intensity at rational 25% expansion is almost linear.
- An analysis of a municipal water treatment plant located in northern Europe with a current average capacity of about 25,000 m$^3$/d showed a potential annual saving of about 71,045 m$^3$ of water used in backwashing the sand filter beds, representing about 8.5% of the total volume of water used for this purpose and about 0.78% of the total production of the water treatment plant.
- Potential water saving during backwashing of GAC filters with monthly intensity adjustment ranged from 4570 to 7511 m$^3$, depending on bed type. This represents from 7.57 to 13.7 per cent of the water used for this purpose.
- Although the analysis of energy saving due to backwash intensity regulation showed a gain of 13.6% of total annual energy used for this purpose, relatively small energy benefits are expected as relatively little energy is used to backwash rapid and GAC filters.

Author Contributions: Conceptualization, M.Z. and W.D.; methodology, W.D. and M.Z.; software, M.Z. and W.D.; validation, M.Z. and W.D.; formal analysis, M.Z. and W.D.; investigation, M.Z and W.D.; resources, M.Z and W.D.; data curation, M.Z. and W.D.; writing—original draft preparation, M.Z. and W.D.; writing—review and editing, W.D. and M.Z.; visualization, M.Z. and W.D.; project administration, M.Z. and W.D.; funding acquisition, M.Z. and W.D. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Waterworks Kraków.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Acknowledgments: The authors are grateful to the postgraduate students Monika Plata and Paweł Guzdek for their assistance in collecting sand samples throughout the whole depth of the filters as well as for sieving and drying the sand and predicting its density.

Conflicts of Interest: The authors declare no conflict of interest. The funder had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.
Nomenclature

\( A \)  \hspace{1em} \text{area of a single filter surface:}

\( Ar \)  \hspace{1em} \text{Archimedes number,}

\( d \)  \hspace{1em} \text{equivalent diameter of a grain,}

\( E \)  \hspace{1em} \text{energy,}

\( f \)  \hspace{1em} \text{average backwashing frequency of one filter per day,}

\( FN \)  \hspace{1em} \text{number of filters in the plant,}

\( g \)  \hspace{1em} \text{acceleration due to gravity,}

\( h \)  \hspace{1em} \text{pressure height loss,}

\( h_{\text{min}} \)  \hspace{1em} \text{pressure height loss necessary to fluidize filter media,}

\( H \)  \hspace{1em} \text{pump head during backwash,}

\( H_s \)  \hspace{1em} \text{static head,}

\( H_b \)  \hspace{1em} \text{pressure height drop on filter media,}

\( H_d \)  \hspace{1em} \text{pressure height drop on underdrain,}

\( H_p \)  \hspace{1em} \text{drop of pressure height on pipeline,}

\( L \)  \hspace{1em} \text{thickness of filter bed,}

\( n \)  \hspace{1em} \text{power exponent from the Richardson–Zaki Equation (3),}

\( N_p \)  \hspace{1em} \text{power at pump shaft during filter flushing,}

\( p \)  \hspace{1em} \text{expansion of the bed during fluidization,}

\( Q_p \)  \hspace{1em} \text{pump capacity,}

\( Re_s \)  \hspace{1em} \text{Reynolds number for free sedimentation of grains,}

\( T \)  \hspace{1em} \text{temperature in °C,}

\( T_o \)  \hspace{1em} \text{reference temperature in °C,}

\( t_p \)  \hspace{1em} \text{backwash time by water,}

\( V \)  \hspace{1em} \text{volume of the filter bed during fluidization,}

\( V_o \)  \hspace{1em} \text{bed volume before fluidization,}

\( v \)  \hspace{1em} \text{apparent velocity of the fluid in the fluidization zone,}

\( v_i \)  \hspace{1em} \text{intensity of water backwash for filter } i,\hspace{1em}

\( v_s \)  \hspace{1em} \text{free sedimentation velocity of individual grains in water,}

\( W \)  \hspace{1em} \text{volume of water used for backwash of all filters in the station during the year,}

\( W_i \)  \hspace{1em} \text{volume of water used for a single backwash,}

\( \epsilon_o \)  \hspace{1em} \text{known bed porosity before backwash,}

\( \epsilon \)  \hspace{1em} \text{bed porosity during fluidization,}

\( \rho_g \)  \hspace{1em} \text{the density of the grain,}

\( \rho_w \)  \hspace{1em} \text{density of water,}

\( \mu \)  \hspace{1em} \text{dynamic viscosity,}

\( \psi \)  \hspace{1em} \text{sphericity,}

\( \eta_p \)  \hspace{1em} \text{pump efficiency,}


References

1. Amirtharajah, A. Some theoretical and conceptual views of filtration. J. Am. Water Work. Assoc. 1988, 80, 34–36. [CrossRef]
2. Cleasby, J.L.; Baumann, E.R. Selection of Optimum Rates for Sand Filters; Iowa State University Bull.: Ames, IA, USA, 1969.
3. Montgomery, J.M. Water Treatment: Principles and Design; John Willey & Sons, Inc.: New York, NY, USA, 1985.
4. Fair, G.M.; Geyer, J.C.; Okun, D.A. Water and Wastewater Engineering; John Willey & Sons, Inc.: New York, NY, USA, 1968.
5. Snowball, M. Reducing backwash with air scouring. Filtr. Sep. 2006, 43, 39–40. [CrossRef]
6. Logsdon, G.S. Effective management and operation of coagulation and filtration. Water Air Soil Pollut. 2000, 123, 159. [CrossRef]
7. Amirtharajah, A. Optimum backwashing of sand filters. J. Environ. Eng. Div. Proc. ASCE 1978, 104, 917–932. [CrossRef]
8. Amirtharajah, A. Optimum backwashing of filters with air scour: A review. Water Res. 1993, 27, 195–211. [CrossRef]
9. Sholji, I.; Sholji, I.; Johnson, F.A. 1987 Comparison of backwash models for granular media. J. Environ. Eng. ASCE 1967, 113, 532–549. [CrossRef]
10. Amirtharajah, A. Fundamentals and theory of air scour. J. Environ. Eng. ASCE 1984, 110, 573–590. [CrossRef]
11. Amirtharajah, A. The interface between filtration and backwashing. Water Res. 1985, 19, 581–588. [CrossRef]
12. Stevenson, D.G. Process conditions for the backwashing of filters with simultaneous air and water. Water Res. 1995, 29, 2594–2597. [CrossRef]
13. Fitzpatrick, C.S.B. Media properties and their effect on filter performance and backwashing. Water Sci. Technol. 1993, 27, 213–221. [CrossRef]
14. Hemmings, D.G.; Fitzpatrick, C.S.B. Pressure signal analysis of combined water and air backwash of rapid gravity filters. *Water Res.* 1997, 31, 356–361. [CrossRef]
15. Amburgey, J.E. Improving Filtration for Removal of Cryptosporidium Oocysts and Particles from Drinking Water. Ph.D. Thesis, Georgia Institute of Technology, Atlanta, GA, USA, 2002.
16. Clements, M. Changes in the Mechanical Behaviour of Filter Media Due to Biological Growth. Ph.D. Thesis, Rand Afrikaans University, Johannesburg, South Africa, 2004; p. 150.
17. Amirtharajah, A.; Wetstein, D.P. Initial degradation of effluent quality during filtration. *J. Am. Water Work. Assoc.* 1980, 72, 518–524. [CrossRef]
18. Amburgey, J.E.; Brouckaert, B.M. Practical and theoretical guidelines for implementing the extended terminal subfluidization wash (ETSW) backwashing procedure. *J. Water Supply Res. Technol. AQUA* 2005, 54, 319–337. [CrossRef]
19. Amburgey, J.E.; Amirtharajah, A. Strategic filter backwashing techniques and resulting particle passage. *J. Environ. Eng. ASCE* 2005, 4, 535–547. [CrossRef]
20. Amburgey, J.E. Optimization of the extended terminal sub fluidization wash (ETSW) filter backwashing procedure. *Water Res.* 2005, 39, 314–330. [CrossRef]
21. Fitzpatrick, C.S.; Gregory, J. Coagulation and Filtration. In *Handbook of Water and Wastewater Microbiology*; Mara, D., Horan, N., Eds.; School of Civil Engineering, University of Leeds: Leeds, UK, 2003; pp. 433–656.
22. United States Environmental Protection Agency. *Filter Backwash Recycling Rule, Technical Guidance Manual*; EPA 816-R-02-014; Office of Ground Water and Drinking Water: Washington, DC, USA, 2002; 172p.
23. Skolubovich, Y.; Voytov, E.; Skolubovich, A.; Iluina, L. Cleaning and reusing backwash water of water treatment plants. In Proceedings of the IOP Conference Series: Earth and Environmental Science, Bandung, Indonesia, 22–23 March 2017; Volume 90, p. 012033. [CrossRef]
24. Dąbrowski, W.; Korczak, P. Filter plants operation in respect to backwashing. *Wydaw. Politech. Krak.* 2008, 184. (In Polish)
25. Ferrara, A.P. Controlling bed losses of granular activated carbon through proper filter operation. *J. Am. Water Work. Assoc.* 1975, 67, 535–544. [CrossRef]
26. Kawamura, S.; Najm, N.N.; Gramith, K. Modifying a backwash through to reduce media loss. *J. Am. Water Work. Assoc.* 1980, 45–60. [CrossRef]
27. Kawamura, S. Design and operation of high-rate filters—part 1. *J. Am. Water Work. Assoc.* 1975, 67, 535–544. [CrossRef]
28. Kawamura, S. Design and operation of high-rate filters—part 2. *J. Am. Water Work. Assoc.* 1975, 67, 653–662. [CrossRef]
29. Sakkas, N.D.; Lekkas, T.D. Hydrodynamic characteristics of the solid-liquid fluidized bed developing during the backwash of filter media. *Environ. Technol. Lett.* 1989, 10, 151–156. [CrossRef]
30. Turan, M. Velocity gradient in filter backwashing. *J. Environ. Eng.* 1992, 118, 776–790. [CrossRef]
31. Brouckaert, B.M. Hydrodynamic Detachment of Deposited Particles in Fluidized Bed Filter Backwashing. Ph.D. Thesis, Georgia Institute of Technology, Atlanta, GA, USA, July 2004; 359p.
32. Dąbrowski, W.; Plata, M. Changes of sand density impact on water filter backwashing. *Electron. J. Pol. Agric. Univ. Civ. Eng.* 2007, 10, 1.
33. Clements, M. Granular activated carbon management at a water treatment plant. Master’s Thesis, Civil Engineering, Rand Afrikaans University, Johannesburg, South Africa, 2002; p. 172.
34. Grabarczyk, C. *Hydromechanics of Water Filtration*; WNT: Worcester, MA, USA, 2010; 400p. (In Polish)
35. Siwiec, T. The shericity of grains of filtration beds applied for water treatment on examples of selected minerals. *Electron. J. Pol. Agric. Univ. Civ. Eng.* 2007, 10, 1.
36. Grabarczyk, C. *Hydromechanics of Water Filtration*; WNT: Worcester, MA, USA, 2010; 400p. (In Polish)
37. Grabczyk, C. *Hydromechanics of Water Filtration*; WNT: Worcester, MA, USA, 2010; 400p. (In Polish)
38. Siwiec, T. The shericity of grains of filtration beds applied for water treatment on examples of selected minerals. *Electron. J. Pol. Agric. Univ. Civ. Eng.* 2007, 10, 1.
39. Clements, M. Granular activated carbon management at a water treatment plant. Master’s Thesis, Civil Engineering, Rand Afrikaans University, Johannesburg, South Africa, 2002; p. 172.
40. Kovacs, G. *Seepage Hydraulics*; Akademiai Kiado: Budapest, Hungary, 1981; 730p.
41. Grabarczyk, C. *Hydromechanics of Water Filtration*; WNT: Worcester, MA, USA, 2010; 400p. (In Polish)
42. Siwiec, T. The shericity of grains of filtration beds applied for water treatment on examples of selected minerals. *Electron. J. Pol. Agric. Univ. Civ. Eng.* 2007, 10, 1.
43. Mackie, R.I.; Zielina, M.; Dąbrowski, W. Filterate quality from different filter operations. *Acta Hydrochim. Hydrobiol.* 2003, 31, 1–10. [CrossRef]
44. Dąbrowski, W. The progression of flow rates in Variable Declining Rate Filter systems. *Acta Hydrochim. Hydrobiol.* 2006, 34, 442–452. [CrossRef]
45. Macki, R.I.; Dąbrowski, W.; Zielina, M. Numerical study of a rational rule for the operation of variable declining rate filters in response to changes in raw water quality. *Environ. Prot. Eng.* 2003, 29, 45–51.
46. Macki, R.I.; Dąbrowski, W.; Zielina, M. Numerical experiments into optimisation of VDR Filters. *Environ. Prot. Eng.* 2007, 4, 27–39.
47. Dąbrowski, W. Rational Operation of Variable Declining Rate Filters. *Environ. Prot. Eng.* 2011, 37, 35–53.
48. Zielina, M.; Dąbrowski, W.; Mackie, R.I. Laboratory and numerical experiments into efficient management of VDR filter plants. *Environ. Prot. Eng.* **2015**, *41*, 101–120. [CrossRef]

49. Dąbrowski, W.; Mackie, I. Influence of temperature on the performance of variable declining rate filters for drinking water. *Arch. Hydro-Eng. Environ. Mech.* **1994**, *41*, 37–51.

50. Zielina, M.; Dąbrowski, W. Impact of raw water quality on operation of variable declining rate filter plants. *Environ. Prot. Eng.* **2011**, *2*, 133–140.

51. Dąbrowski, W. Should variable declining rate filters be operated as one large or separate plants? *Desalination Water Treat.* **2020**, *185*, 105–110. [CrossRef]

52. Bielski, A.; Zielina, M.; Młyńska, A. Analysis of heavy metals leaching from internal pipe cement coating into potable water. *J. Clean. Prod.* **2020**, *265*, 121425. [CrossRef]

53. Dąbrowski, W.; Li, F. Mortar lining as a protective layer for ductile iron pipes. *Int. J. Civ. Eng.* **2021**, *2*, 133–140.

54. Tchorzewska-Cieślak, B.; Pietrucha-Urbanik, K.; Eid, M. Functional safety concept to support hazard assessment and risk management in Water-Supply Systems. *Energies* **2021**, *4*, 947. [CrossRef]