Spontaneous CP Phases
and Flavour Changing Neutral Currents
in the Left-Right Symmetric Model

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Abstract
We study the behaviour of the flavour changing neutral currents in the
Left-Right Symmetric Model related to the presence of spontaneous CP
phases. To do this, we explore four cases corresponding to combinations
of maximal and no CP violation in both the lepton and quark sector.
We find that we can constrain the flavour changing neutral currents to
the experimental limit, by adjusting the CP-violating phase of the quark
sector, opening the possibility to obtain a large CP violation in the lepton
sector as well as new Higgs bosons at the electroweak scale.

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1 Introduction

The Left-Right Symmetric Model (LRSM) was developed looking for a natural origin for parity violation. Introducing a new group of symmetries, $SU(2)_R$, we can have a new “weak” interaction with the same coupling constant of the weak interaction of the Standard Model (SM), but acting only on the right-handed particles. The resulting model is very interesting because it introduces new gauge bosons which we can search for in accelerator experiments. The experimental constraints on the masses of the three new gauge bosons ($W^+_R, W^-_R, Z^0_R$) tell us that they have to be at least of $715 \text{ GeV}$ for the charged bosons, and $564 \text{ GeV}$ for the neutral one. If we have only one bidoublet of Higgs bosons in order to improve the Higgs mechanism in this kind of models with left- and right-handed particles organized in doublets, we will get the same masses for the left-weak and the right-weak bosons with equal electric charge. The usual way to avoid this problem is by introducing new Higgs bosons in the model. The most popular method to do so in the LRSM is through two Higgs triplets each one acting on one type of particles depending on its chirality. With these two triplets, we are able to explain the small masses of the left-handed neutrinos by the so-called see-saw mechanism, whilst giving experimentally compatible heavy masses to the right-handed neutrinos.

New interesting things appear: The SM is based on the group of symmetries $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, where the values that the hypercharge quantum number $Y$ takes for each particle do not have any physical meaning. When we extend the SM to the LRSM, we have the group of symmetries $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, where the hypercharge quantum number $Y$ now becomes $B - L$, the difference between the baryonic number $B$ and the leptonic number $L$.

Another interesting thing in the LRSM is the possibility of explaining the observed $CP$ violation. The SM is able to explain it because there are three families of particles. With three families of particles it is possible to absorb all the complex phases arising from the Yukawa sector of the Lagrangian except for one. This remaining phase appears in the Cabibbo-Kobayashi-Maskawa (CKM) matrix and it finally can explain the observed $CP$ violation in the kaon system. In the LRSM we can impose a global $CP$ symmetry on the complete lagrangian, in order to avoid explicit complex phases in the Yukawa couplings, and obtain them spontaneously through the vacuum expectation values arising from the symmetry breaking mechanism. Two spontaneous $CP$ phases appear, $\alpha$ and $\theta$, which may be allocated in the $CKM$ matrix and in the analogous matrix for the lepton sector respectively. Once again, this is really
interesting because it opens the possibility of having \( CP \) violation in the lepton sector too, which is not possible in the \( SM \).

However, everything is not perfect in the \( LRSM \). Due to the presence of a bidoublet of Higgs fields, Flavour Changing Neutral Currents (\( FCNC \)) appear in the model involving the neutral scalar bosons. The experimental constraints tell us that if the \( FCNC \) exist, they must be suppressed enough so that the experiments will not be sensitive to them. The \( SM \) does not present any \( FCNC \) because it has only one doublet of Higgs fields. One possible way to avoid the \( FCNC \) in the \( LRSM \), without doing any fine-tuning on the coupling constants, is to have really heavy masses for the scalar bosons which mediate the \( FCNC \). However this is not always possible and it depends on the values of the parameters of the scalar potential as well as on the values that the spontaneous \( CP \) phases take.

The aim of this article is to study the relationship between the amount of the \( FCNC \) and the values taken for the spontaneous \( CP \) phases \( \alpha \) and \( \theta \). To do this, we explore four cases in which we have values of 0 or \( \pi/2 \) for the two \( CP \) phases. Two of these cases (what we are going to call \( I \) and \( IV \)) were already studied in the literature, and the other two (\( II \) and \( III \)) are novel. As we shall see later, \( \alpha \) is associated with the quark sector and \( \theta \) with the lepton sector. Therefore, we study four combinations of maximal and no \( CP \) violation in both sectors. What we find is that, in order to suppress the \( FCNC \) in the \( LRSM \), we have to adjust only the \( CP \)-violating phase of the quark sector. So, we may have any value for the other \( CP \)-violating phase, implying the possibility to obtain a large \( CP \) violation in the lepton sector, which is of great importance for current and future experiments. In addition, we find other scalar particles at the electroweak scale, different from the Higgs boson of the \( SM \), which are important in the phenomenology of the model, and that, analogously to the case of \( CP \) violation in the lepton sector, are the focus of most current and future experiments.

The outline of the paper is the following: In Section 2 we present the main features of the \( LRSM \). In Section 3 we show how the \( FCNC \) arise in the \( LRSM \). In Section 4 we present the four cases described above together with the minimization conditions taken for the scalar potential. We analyze their relevance and viability with respect to the \( FCNC \). Finally in Section 5 we conclude. In Appendix A we show the complete mass matrices for the scalar particles. In Appendix B we present how to get the minimum possible order of magnitude for the masses of the flavour changing scalar bosons, by means of experimental constraints from neutrinos.
2 The model

The LRSM is based on the group of symmetries $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes C \otimes P$, where the discrete parity ($P$) symmetry stands for the same coupling constant $g$ for the $SU(2)_L$ and the $SU(2)_R$ groups. Additionally, the discrete charge-parity ($CP$) symmetry assures that there is no explicit $CP$ violation. Therefore we have to search for it in a spontaneous way.

According to the left-right symmetry, quarks and leptons are placed in left- and right-handed doublets:

\[
\begin{align*}
\Psi_{iL} &= \left( \begin{array}{c} \nu_i \\
 e_i \end{array} \right)_L \equiv (2,1,-1), \quad \Psi_{iR} = \left( \begin{array}{c} \nu_i \\
 e_i \end{array} \right)_R \equiv (1,2,-1), \\
Q_{iL} &= \left( \begin{array}{c} u_i \\
 d_i \end{array} \right)_L \equiv (2,1,1), \quad Q_{iR} = \left( \begin{array}{c} u_i \\
 d_i \end{array} \right)_R \equiv (1,2,1),
\end{align*}
\]

where $i = 1, 2, 3$ is the generation index, and the representation with respect to the gauge group is explicitly given.

The gauge bosons consist of two triplets:

\[
\begin{align*}
W_{\mu L} &= \left( \begin{array}{c} W^{+}_{\mu} \\
 Z^{0}_{\mu} \\
 W^{-}_{\mu} \end{array} \right)_L \equiv (3,1,0), \quad W_{\mu R} = \left( \begin{array}{c} W^{+}_{\mu} \\
 Z^{0}_{\mu} \\
 W^{-}_{\mu} \end{array} \right)_R \equiv (1,3,0),
\end{align*}
\]

and one singlet:

\[
B_{\mu} = B^{0}_{\mu} \equiv (1,1,0).
\]

As both quarks and leptons are placed in doublets, we need a bidoublet of scalar bosons to implement the symmetry breaking mechanism:

\[
\Phi = \left( \begin{array}{cc}
\phi^{0}_1 & \phi^{+}_1 \\
\phi^{-}_2 & \phi^{0}_2
\end{array} \right) \equiv (2,2,0).
\]

However, this bidoublet leads to the same masses for the left-weak and the right-weak bosons with equal electric charge. To avoid this problem, we have to extend the Higgs sector by introducing two triplets:

\[
\Delta_L = \left( \begin{array}{cc}
\delta^{+}_L & \delta^{++}_L \\
\delta^{0}_L & -\frac{\delta^{+}_L}{\sqrt{2}}
\end{array} \right) \equiv (3,1,2), \quad \Delta_R = \left( \begin{array}{cc}
\delta^{+}_R & \delta^{++}_R \\
\delta^{0}_R & -\frac{\delta^{+}_R}{\sqrt{2}}
\end{array} \right) \equiv (1,3,2),
\]

3
which have the additional interesting feature of implementing the see-saw mechanism. This mechanism reproduces the observed light masses of the left-handed neutrinos, whilst giving experimentally compatible heavy masses to the right-handed ones. [6]-[9]

The symmetry breaking pattern of the bidoublet and the triplets is given by:

\[
\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k_1 e^{i\alpha_1} & 0 \\ 0 & k_2 e^{i\alpha_2} \end{pmatrix}, \\
\langle \Delta_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_L} & 0 \end{pmatrix}, \\
\langle \Delta_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R e^{i\theta_R} & 0 \end{pmatrix},
\]

(2.6)

where \(k_1, k_2, v_L, v_R, \alpha_1, \alpha_2, \theta_L, \text{and } \theta_R\) are real numbers. There are some constraints on the values that the vacuum expectation values \(k_1, k_2, v_L, \text{and } v_R\) may take: \(v_L\) must be much smaller than \(k_1\) and \(k_2\) to keep the well known experimental condition \(M_W^2/M_Z^2 \simeq \cos^2 \theta_W\). In addition, \(v_R\) must be at least \(2.7 \times 10^7\) GeV to give really heavy masses to the right-weak bosons \(W_R^+, W_R^-, \text{and } Z_{R,0}^0\), and to fit the experimental constraints coming from neutrinos. [5, 6, 9]

Under unitary transformations of the fermionic fields, the scalar fields transform according to the relations:

\[
\Phi \longrightarrow U_L \Phi U_R^+, \\
\Delta_L \longrightarrow U_L \Delta_L U_L^+, \\
\Delta_R \longrightarrow U_R \Delta_R U_R^+,
\]

(2.7)

where we can absorbe some of the phases of the scalar fields by defining:

\[
U_L = \begin{pmatrix} e^{i\gamma_L} & 0 \\ 0 & e^{-i\gamma_L} \end{pmatrix}, \\
U_R = \begin{pmatrix} e^{i\gamma_R} & 0 \\ 0 & e^{-i\gamma_R} \end{pmatrix}, \\
\gamma_L = \frac{\theta_L}{2}, \\
\gamma_R = \gamma_L - \alpha_2.
\]

(2.8)

\[3k^2 = k_1^2 + k_2^2 \simeq (246 \text{ GeV})^2.\]

\[4\text{See Appendix B.}\]
By means of these definitions, we are left with two genuine phases which we call \( \alpha \) and \( \theta \):

\[
\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k_1 e^{i\alpha} & 0 \\ 0 & k_2 \end{pmatrix},
\]

\[
\langle \Delta_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix},
\]

\[
\langle \Delta_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R e^{i\theta} & 0 \end{pmatrix}.
\]  

(2.9)

The Lagrangian is \( CP \) invariant by definition, so the only sources of \( CP \) violation in the \( LRSM \) are these two phases, obtained spontaneously.

The Yukawa terms in the Lagrangian are given by:

\[
- \mathcal{L}_Y^l = \sum_{a,b} h_{ab}^l \overline{\Psi}_a L \Phi \Psi_b R + \tilde{h}_{ab}^l \overline{\Psi}_a L \tilde{\Phi} \Psi_b R + i f_{ab} [\overline{\Psi}_a^T C \tau_2 \Delta_L \Psi_b L + (L \leftrightarrow R)] + h.c.,
\]

(2.10)

for the leptons, and by:

\[
- \mathcal{L}_Y^q = \sum_{a,b} h_{ab}^q \overline{Q}_a L \Phi Q_b R + \tilde{h}_{ab}^q \overline{Q}_a L \tilde{\Phi} Q_b R + h.c.,
\]

(2.11)

for the quarks, where \( h_{ab}^l, \tilde{h}_{ab}^l, \) and \( f \) are the Yukawa coupling matrices, \( \tilde{\Phi} = \tau_2 \Phi^* \tau_2 \), \( C \) is the Dirac's charge-conjugation matrix, and \( a, b \) label different generations. Looking at the above expressions, we can see that the spontaneous \( CP \) phase \( \theta \) enters only in the lepton sector whilst the spontaneous \( CP \) phase \( \alpha \) enters in both the lepton and the quark sector. Therefore, the phase in the \( CKM \) matrix that is responsible for the observed \( CP \) violation in the quark sector, is function of \( \alpha \) but not of \( \theta \). Then, once we have adjusted \( \alpha \) to be consistent with the experimental data, the amount of \( CP \) violation in the lepton sector will be determined only by \( \theta \). These facts lead us to relate the spontaneous \( CP \) phase \( \alpha \) with the quark sector, and the spontaneous \( CP \) phase \( \theta \) with the lepton sector, although \( \alpha \) will be responsible of a possible \( CP \) violation in the lepton sector too.

The complete Lagrangian must be invariant under the transformations:

\[
\Psi_L \leftrightarrow \Psi_R, \quad Q_L \leftrightarrow Q_R, \\
\Delta_L \leftrightarrow \Delta_R, \quad \Phi \leftrightarrow \Phi^*,
\]

(2.12)
due to the left-right symmetry requirements.

Thus, the most general scalar potential can be written as:

\[V = V_\Phi + V_\Delta + V_{\Phi\Delta},\]

with

\[V_\Phi = -\mu_1^2 Tr (\Phi^\dagger \Phi) - \mu_2^2 \left[ Tr (\tilde{\Phi} \Phi^\dagger) + Tr (\tilde{\Phi}^\dagger \Phi) \right] + \lambda_1 \left[ Tr (\Phi \Phi^\dagger) \right]^2 + \lambda_2 \left[ Tr (\tilde{\Phi} \Phi^\dagger) \right]^2 + \lambda_3 \left[ Tr (\tilde{\Phi} \Phi^\dagger) Tr (\tilde{\Phi}^\dagger \Phi) \right] + \lambda_4 \left\{ Tr (\Phi^\dagger \Phi) \left[ Tr (\tilde{\Phi} \Phi^\dagger) + Tr (\tilde{\Phi}^\dagger \Phi) \right] \right\},\]

\[V_\Delta = -\mu_3^2 \left[ Tr (\Delta_L \Delta_L^\dagger) + Tr (\Delta_R \Delta_R^\dagger) \right] + \rho_1 \left\{ \left[ Tr (\Delta_L \Delta_L^\dagger) \right]^2 + \left[ Tr (\Delta_R \Delta_R^\dagger) \right]^2 \right\} + \rho_2 \left[ Tr (\Delta_L \Delta_L) Tr (\Delta_L^\dagger \Delta_L^\dagger) + Tr (\Delta_R \Delta_R) Tr (\Delta_R^\dagger \Delta_R^\dagger) \right] + \rho_3 \left[ Tr (\Delta_L \Delta_L^\dagger) Tr (\Delta_R \Delta_R^\dagger) \right] + \rho_4 \left[ Tr (\Delta_L \Delta_L) Tr (\Delta_R^\dagger \Delta_R^\dagger) + Tr (\Delta_L^\dagger \Delta_L^\dagger) Tr (\Delta_R \Delta_R) \right],\]

\[V_{\Phi\Delta} = \alpha_1 \left\{ Tr (\Phi^\dagger \Phi) \left[ Tr (\Delta_L \Delta_L^\dagger) + Tr (\Delta_R \Delta_R^\dagger) \right] \right\} + \alpha_2 \left[ Tr (\tilde{\Phi} \Phi) Tr (\Delta_R \Delta_R^\dagger) + Tr (\tilde{\Phi}^\dagger \Phi) Tr (\Delta_L \Delta_L^\dagger) \right] + Tr (\tilde{\Phi} \Phi^\dagger) Tr (\tilde{\Phi} \Phi^\dagger) + Tr (\tilde{\Phi} \Phi) Tr (\tilde{\Phi}^\dagger \Phi) Tr (\Delta_L \Delta_L^\dagger) \right\} + \alpha_3 \left[ Tr (\Phi \Phi^\dagger \Delta_L \Delta_L^\dagger) + Tr (\Phi^\dagger \Phi \Delta_R \Delta_R^\dagger) \right] + \beta_1 \left[ Tr (\Phi \Delta_R \Phi^\dagger \Delta_L^\dagger) + Tr (\Phi^\dagger \Delta_L \Phi \Delta_R^\dagger) \right] + \beta_2 \left[ Tr (\tilde{\Phi} \Delta_R \Phi^\dagger \Delta_L^\dagger) + Tr (\tilde{\Phi} \Delta_L \Phi \Delta_R^\dagger) \right] + \beta_3 \left[ Tr (\Phi \Delta_R \tilde{\Phi} \Phi^\dagger \Delta_L^\dagger) + Tr (\Phi^\dagger \Delta_L \Phi \Delta_R^\dagger) \right],\]
where we have written out each term completely to display the full parity symmetry. Note that, as a consequence of the discrete left-right symmetry, all the terms in the potential are self-conjugate. Therefore, all the parameters must be real, avoiding any explicit source of CP violation.

3 The FCNC

We are going to concentrate on the Yukawa terms for the quark sector, eq. (2.11):

\[-L_Y = \sum_{a,b} h^q_{ab} \overline{Q_aL} \Phi Q_bR + \tilde{h}^q_{ab} \overline{Q_aL} \tilde{\Phi} Q_bR + h.c. \quad (3.1)\]

In this Lagrangian $Q$ denotes the flavour eigenstates. Introducing the vacuum expectation values, eq. (2.9), into the Yukawa terms, we obtain the following mass matrices for the up and down quarks:

\[M^u_{ab} = \frac{1}{\sqrt{2}} \left( h^q_{ab} k_1 e^{i\alpha} + \tilde{h}^q_{ab} k_2 \right), \]
\[M^d_{ab} = \frac{1}{\sqrt{2}} \left( h^q_{ab} k_2 + \tilde{h}^q_{ab} k_1 e^{-i\alpha} \right). \quad (3.2)\]

To diagonalize these mass matrices, we have to rotate the flavour eigenstates into the mass eigenstates, which we are going to call $Q^0$:

\[Q^u_L = U_L Q^0_L, \quad Q^u_R = U_R Q^0_R, \]
\[Q^d_L = V_L Q^0_L, \quad Q^d_R = V_R Q^0_R. \quad (3.3)\]

In this way, we can write the non diagonal mass matrices in terms of the diagonal ones as:

\[M^u = U_L M^u_{\text{diag}} U_R^\dagger, \]
\[M^d = V_L M^d_{\text{diag}} V_R^\dagger. \quad (3.4)\]

For $k_1^2 \neq k_2^2$ and $k_\pm^2 \equiv k_1^2 \pm k_2^2$ we can invert equations (3.4) to solve for $h^q$ and $\tilde{h}^q$ in terms of the diagonal matrices for the up and down quarks:

\[5\text{The parameters } \alpha_1 \text{ and } \alpha_2 \text{ in the scalar potential are different to the phases of } \Phi \text{ in eq. (2.3).} \]
\[ h^q = \frac{\sqrt{2}}{k_-^2} \left( k_1 e^{-i\alpha} U_L M_{\text{diag}}^u U_R^\dagger - k_2 V_L M_{\text{diag}}^d V_R^\dagger \right), \]
\[ \tilde{h}^q = \frac{\sqrt{2}}{k_-^2} \left( -k_2 U_L M_{\text{diag}}^u U_R^\dagger + k_1 e^{i\alpha} V_L M_{\text{diag}}^d V_R^\dagger \right). \quad (3.5) \]

Due to the left-right transformation, eq. (2.12), \( h^q \) and \( \tilde{h}^q \) must be hermitian. Then, we can define the CKM matrices for the LRSM as:

\[ K_L = U_L^\dagger V_L, \]
\[ K_R = U_R^\dagger V_R, \quad (3.6) \]

which are related through the relation:

\[ K = K_L = K_R^* \quad (3.7) \]

We can now write the general interaction term for the quark mass eigenstates with the neutral \( \phi \)-type Higgs fields:

\[ \frac{\sqrt{2}}{k_-} u_L^0 \left[ M_{\text{diag}}^u (k_1 e^{-i\alpha} \phi_1^0 - k_2 \phi_2^0) + K_L M_{\text{diag}}^d K_R^\dagger (-k_2 \phi_1^0 + k_1 e^{i\alpha} \phi_2^0) \right] u_R^0 \]
\[ + \frac{\sqrt{2}}{k_-} d_L^0 \left[ M_{\text{diag}}^d (k_1 e^{i\alpha} \phi_1^0 - k_2 \phi_2^0) + K_L M_{\text{diag}}^u K_R^\dagger (-k_2 \phi_1^0 + k_1 e^{-i\alpha} \phi_2^0) \right] d_R^0. \quad (3.8) \]

Defining two new orthogonal neutral fields: \[ \phi_1^0 = \frac{1}{|k_+|} \left( -k_2 \phi_1^0 + k_1 e^{i\alpha} \phi_2^0 \right), \]
\[ \phi_2^0 = \frac{1}{|k_+|} \left( k_1 e^{-i\alpha} \phi_1^0 + k_2 \phi_2^0 \right), \quad (3.9) \]

with inverse transformations:

\[ \phi_1^0 = \frac{1}{|k_+|} \left( -k_2 \phi_1^0 + k_1 e^{i\alpha} \phi_2^0 \right), \]
\[ \phi_2^0 = \frac{1}{|k_+|} \left( k_1 e^{i\alpha} \phi_1^0 + k_2 \phi_2^0 \right), \quad (3.10) \]
it is possible to write the general interaction term as:

\[
\frac{\sqrt{2}}{k^2} d_L^0 \left[ \phi_0^0 \frac{k^2}{|k_+|} M^u_{\text{diag}} + \phi_+^0 \left( -2k_1 e^{-i\alpha} k_2 \frac{M^u_{\text{diag}} + |k_+| K_L M^d_{\text{diag}} K_R^+}{|k_+|} \right) \right] u_R^0
\]

\[
+ \frac{\sqrt{2}}{k^2} d_L^0 \left[ \phi_{-0}^0 \frac{k^2}{|k_+|} M^d_{\text{diag}} + \phi_{+0}^0 \left( -2k_1 e^{i\alpha} k_2 \frac{M^d_{\text{diag}} + |k_+| K_L M^u_{\text{diag}} K_R^+}{|k_+|} \right) \right] d_R^0,
\]

where we can see clearly that there are FCNC in the LRSM associated with the \( \phi_+^0 \) boson.

One possible way to avoid these FCNC, without performing any fine-tuning on the coupling constants or the vacuum expectation values, is to give a really heavy mass to the \( \phi_+^0 \) boson. In the next section, we are going to investigate four cases in which we have different values for the spontaneous CP phases \( \alpha \) and \( \theta \). The idea is to search for a model with a really heavy mass for \( \phi_+^0 \), for example, of the order of \( v_R \) \( (10^7 \text{ GeV}) \). If we are able to find it, we will have a model with FCNC supressed enough to be consistent with the experimental constraints, and with the additional feature of having a spontaneous origin for the observed CP violation.

### 4 Spontaneous CP phases and FCNC

In Section 2 we noticed that the spontaneous CP phase \( \alpha \) is related directly with the amount of CP violation in the quark sector, whilst \( \theta \) is related to the lepton sector. In this section, we are going to investigate the effects on the mass spectrum of the scalar sector, and therefore, on the FCNC, of giving maximal CP violation in both the quark and the lepton sector \((\alpha = \pi/2, \ \theta = \pi/2)\), maximal CP violation in the quark sector and no CP violation in the lepton sector \((\alpha = \pi/2, \ \theta = 0)\), no CP violation in the quark sector and maximal CP violation in the lepton sector \((\alpha = 0, \ \theta = \pi/2)\), and no CP violation in both the quark and the lepton sector \((\alpha = 0, \ \theta = 0)\). The idea is to find what restrictions on the spontaneous CP phases are needed to obtain an experimentally consistent LRSM. To do that we need the components of the scalar mass matrices presented in Appendix A. The four cases defined above will be called cases I, II, III, and IV, respectively.
4.1 Case I: $\alpha = \pi/2$ and $\theta = \pi/2$

In this case, we have maximal $CP$ violation in both the quark and the lepton sector. The minimization conditions arising from the scalar potential, eq. (2.13), are the following:

\[ \beta_2 = \frac{\beta_3 k_2^2}{k_1^2}, \]
\[ \rho_1 = \frac{\rho_3}{2} + \beta_1 \frac{k_1 k_2}{2v_L v_R}, \]
\[ \lambda_2 = \frac{\lambda_3}{2} - \alpha_3 \frac{v_L^2 + v_R^2}{8 (k_3^2 - k_1^2)} + \beta_1 \frac{v_L v_R}{k_1 k_2}, \]
\[ \mu_1^2 = -2 (2\lambda_2 - \lambda_3) k_2^2 + \frac{\alpha_1}{2} \left( v_L^2 + v_R^2 \right) + \lambda_1 \left( k_1^2 + k_2^2 \right) + \beta_1 \frac{k_2}{2k_1} v_L v_R, \]
\[ \mu_2^2 = \frac{\lambda_4}{2} (k_1^2 + k_2^2) + \frac{\alpha_2}{2} \left( v_L^2 + v_R^2 \right) + \beta_2 \frac{k_1}{2k_2} v_L v_R, \]
\[ \mu_3^2 = \frac{1}{2} \left[ \alpha_1 \left( k_1^2 + k_2^2 \right) + \alpha_3 k_2^2 + 2 \rho_1 \left( v_L^2 + v_R^2 \right) \right]. \quad (4.1) \]

Introducing these minimization conditions into the neutral scalar mass matrix, eq. (A.1), and going to the basis \{\phi_+^r, \phi_+^l, \delta_+^R, \delta_+^L, \phi_-^r, \phi_-^l, \delta_-^R, \delta_-^L\} through the general rotation matrix:

\[
R = \frac{1}{|k_+|} \begin{pmatrix}
  k_1 \cos \alpha & k_2 & 0 & 0 & k_1 \sin \alpha & 0 & 0 & 0 \\
  -k_2 & k_1 \cos \alpha & 0 & 0 & k_1 \sin \alpha & 0 & 0 & 0 \\
  0 & 0 & |k_+| & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & |k_+| & 0 & 0 & 0 & 0 \\
  -k_1 \sin \alpha & 0 & 0 & 0 & k_1 \cos \alpha & -k_2 & 0 & 0 \\
  0 & k_1 \sin \alpha & 0 & 0 & -k_2 & -k_1 \cos \alpha & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & |k_+| & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & |k_+| 
\end{pmatrix}
\]

(4.2)

we find the following neutral scalar mass matrix where we only have put the leading terms represented by generic symbols, and used the fact that $v_L v_R \sim k^2$, which results from avoiding fine-tuning of most parameters of the scalar potential.\footnote{There is no way to avoid fine-tuning the parameters of the scalar potential. We have chosen to fine-tune the $\mu^2$ elements which permits to have fine-tunings on the fewest parameters of the scalar potential.}

10
\[
M^2 = \begin{pmatrix}
\alpha R^2 & (\lambda + \beta)k^2 & 0 & \beta k R & \beta k^2 & \alpha R & \alpha k R & 0 \\
(\lambda + \beta)k^2 & \alpha R^2 & 0 & \beta k R & \beta k^2 & \alpha R & \alpha k R & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta k R & \beta k^2 & 0 & \beta k^2 & \beta k^2 & \alpha R & \alpha k R & 0 \\
\beta k^2 & \beta k^2 & 0 & \beta k^2 & \beta k^2 & \alpha R & \alpha k R & 0 \\
\alpha k R & \alpha k R & 0 & (\rho_3 + \beta)k^2 & \alpha R & \alpha k R & 0 & 0 \\
\rho_3 & \beta k^2 & 0 & \beta k^2 & \beta k^2 & \alpha R & \alpha k R & 0 \\
\beta k R & \beta k^2 & 0 & \beta k^2 & \beta k^2 & \alpha R & \alpha k R & 0 \end{pmatrix}
\] 

Defining two new orthogonal fields:

\[
\phi_+ = \frac{1}{|k_+|}(-k_2 \phi_1 + k_1 \phi_2), \\
\phi_- = \frac{1}{|k_+|}(k_1 \phi_1 + k_2 \phi_2),
\]

and their corresponding rotation matrix \( R_+ \):

\[
R_+ = \frac{1}{|k_+|} \begin{pmatrix}
k_1 & k_2 & 0 & 0 \\
-k_2 & k_1 & 0 & 0 \\
0 & 0 & |k_+| & 0 \\
0 & 0 & 0 & |k_+| \end{pmatrix},
\]

we can obtain the singly and doubly charged mass matrices in the basis \( \{\phi_+, \phi_-, \delta_R^+, \delta_L^+\} \) and \( \{\delta_R^{++}, \delta_L^{++}\} \) respectively:

\[
M^{2+} = \begin{pmatrix}
\alpha R^2 & (1 - i)\alpha R & (1 + i)\alpha k R & (1 - i)\beta k R \\
(1 + i)\alpha R & \alpha R^2 & (1 - i)\alpha k R & (1 - i)\beta k R \\
(1 - i)\alpha k R & (1 + i)\alpha k R & k^2 & (1 + i)\beta k^2 \\
(1 + i)\beta k R & (1 + i)\beta k R & (1 - i)\beta k^2 & \beta k^2 \end{pmatrix},
\]

\[
M^{2++} = \begin{pmatrix}
\rho_3 & [(1 + i)\beta + i\rho]k^2 \\
[(1 - i)\beta - i\rho]k^2 & \beta k^2 \end{pmatrix}.
\]

We performed a detailed numerical analysis where we chose an order of magnitude for \( \nu_R \) of \( 10^7 GeV \), according to the experimental constraints on neutrinos.\footnote{See Appendix B.}
and a value of 0.7 for the free dimensionless parameters of the potential. What we found is that, in this model with maximal $CP$ violation in both the quark and the lepton sector, the neutral scalar boson $\phi_0^0$ which contains a significant admixture of $\phi_i^+$ is light enough to allow for large $FCNC$. Therefore, this kind of model is experimentally unacceptable. Tables 1, 2, and 3, show the normalized components of the mass eigenstates and the corresponding order of magnitude for the masses. Some previous conclusions about the state of this case were presented in [4]. Since there were a lot of typos there, these conclusions were erroneous. One of the authors of [4] noted this mistake in [7].

### 4.2 Case $II$: $\alpha = \pi/2$ and $\theta = 0$

In this case, we have maximal $CP$ violation in the quark sector and no $CP$ violation in the lepton sector. The minimization conditions arising from the scalar potential, eq. (2.13), are the following:

\[
\begin{align*}
\beta_2 &= \frac{1}{k_1^2} \left[ \beta_3 k_2^2 - (2 \rho_1 - \rho_3) v_L v_R \right], \\
\lambda_2 &= \frac{\lambda_3}{2} - \frac{1}{4 (k_2^2 - k_1^2)} \left[ \frac{\alpha_3}{2} (v_L^2 + v_R^2) + (\beta_2 + \beta_3) v_L v_R \right], \\
\mu_1^2 &= -2 (2 \lambda_2 - \lambda_3) k_2^2 + \frac{\alpha_1}{2} (v_L^2 + v_R^2) + \lambda_1 (k_1^2 + k_2^2) - \beta_2 v_L v_R, \\
\mu_2^2 &= \frac{\lambda_1}{2} (k_1^2 + k_2^2) + \frac{\alpha_2}{2} (v_L^2 + v_R^2) + \frac{\beta_1}{4} v_L v_R, \\
\mu_3^2 &= \frac{1}{2} \left[ \alpha_1 (k_1^2 + k_2^2) + \alpha_3 k_2^2 + 2 \rho_1 (v_L^2 + v_R^2) \right].
\end{align*}
\]

(4.8)

Introducing these minimization conditions into the neutral scalar mass matrix, eq. (A.1), and going to the basis \{ $\phi_r^-, \phi_r^+, \delta_R^r, \delta_L^r, \phi_i^+, \phi_i^-, \delta_R^i, \delta_L^i$ \} and \{ $\phi_+^r, \phi_+^i, \delta_R^+, \delta_L^+$ \} through the rotation matrices $R$ and $R_+$ respectively, we find the following neutral, singly charged, and doubly charged scalar mass matrices:

\[
M^2 = \begin{pmatrix}
M_{B11}^2 & M_{B12}^2 \\
M_{B12}^T & M_{B22}^2
\end{pmatrix},
\]

(4.9)

\[^8\text{Of the order of } k = 246 \text{ GeV.}\]
\[ M_{B11}^2 = \begin{pmatrix} \alpha v_R^2 & (\lambda + \beta) k^2 & akv_R & (2\rho_1 - \rho_3)kv_R \\ (\lambda + \beta) k^2 & \alpha v_R^2 & akv_R & \beta kv_R \\ akv_R & akv_R & 2\rho_1 v_R^2 & (2\rho_1 + \rho_3) k^2/2 \\ (2\rho_1 - \rho_3)kv_R & \beta kv_R & (2\rho_1 + \rho_3) k^2/2 & (\rho_3 - 2\rho_1)v_R^2/2 \end{pmatrix}, \] (4.10)

\[ M_{B12}^2 = \begin{pmatrix} \beta k^2 & \alpha v_R^2 & 0 & \beta kv_R \\ [\beta + \rho_3 - 2\rho_1]k^2 & \beta k^2 & 0 & [\beta + \rho_3 - 2\rho_1]kv_R \\ 0 & \alpha kv_R & 0 & \beta k^2 \\ \beta kv_R & [\beta + \rho_3 - 2\rho_1]kv_R & \beta k^2 & 0 \end{pmatrix}, \] (4.11)

\[ M_{B22}^2 = \begin{pmatrix} 2(\rho_3 - 2\rho_1)k^2 & \beta k^2 & 0 & (\rho_3 - 2\rho_1)kv_R \\ \beta k^2 & \alpha v_R^2 & 0 & \beta kv_R \\ 0 & 0 & 0 & (2\rho_1 - \rho_3) k^2/2 \\ (\rho_3 - 2\rho_1)kv_R & \beta kv_R & (2\rho_1 - \rho_3) k^2/2 & (\rho_3 - 2\rho_1)v_R^2/2 \end{pmatrix}, \] (4.12)

\[ M^{2+} = \begin{pmatrix} M_{B11}^{2+} & M_{B12}^{2+} \\ M_{B12}^{2+} & M_{B22}^{2+} \end{pmatrix}, \] (4.13)

\[ M_{B11}^{2+} = \begin{pmatrix} \alpha v_R^2 & -i\alpha v_R^2 \\ i\alpha v_R^2 & \alpha v_R^2 \end{pmatrix}, \] (4.14)

\[ M_{B12}^{2+} = \begin{pmatrix} (1 - i)akv_R & i\left[(\rho_3 - 2\rho_1)/\sqrt{2}\right] - (1 - i)\beta kv_R \\ (1 + i)akv_R & [(1 + i)\beta - i\left((\rho_3 - 2\rho_1)/\sqrt{2}\right)]kv_R \end{pmatrix}, \] (4.15)
\[
M_{22}^{2+} = \begin{pmatrix}
\alpha k^2 & [i(2\rho_1 - \rho_3)/2]k^2 \\
-i[(2\rho_1 - \rho_3)/2] + (1 - i)\beta k^2 & 2(\rho_3 - 2\rho_1)v_R^2
\end{pmatrix}
\]

(4.16)

\[
M^{2++} = \begin{pmatrix}
\rho v_R^2 & [(1 + i)\beta + \rho + (\rho_3 - 2\rho_1)/2]k^2 \\
(1 - i)\beta + \rho + (\rho_3 - 2\rho_1)/2 & (\rho_3 - 2\rho_1)v_R^2/2
\end{pmatrix}
\]

(4.17)

In our numerical analysis we also found a neutral scalar boson \(\phi_0^i\) containing a significant admixture of \(\phi^i_+\), which tells us that this model is also unacceptable. What we can conclude is that, independently of the values that the spontaneous \(CP\) phase \(\theta\) takes, we will not be able to obtain an experimentally consistent \(LRSM\) if the amount of \(CP\) violation in the quark sector is maximal. Tables 4, 5, and 6, show the normalized components of the mass eigenstates and the corresponding order of magnitude for the masses.

4.3 Case III: \(\alpha = 0\) and \(\theta = \pi/2\)

In this case, we have no \(CP\) violation in the quark sector and maximal \(CP\) violation in the lepton sector. The minimization conditions arising from the scalar potential, eq. (2.13), are the following:

\[
\rho_1 = \frac{\rho_3}{2},
\]

\[
\beta_2 = \left(\frac{1}{k_1^2}\right)(\beta_1 k_1 k_2 + \beta_3 k_2^2),
\]

\[
\mu_1^2 = \lambda_1 (k_1^2 + k_2^2) + 2\lambda_4 k_1 k_2 + \frac{\alpha_1 (k_1^2 - k_2^2) - \alpha_3 k_2^2}{2(k_1^2 - k_2^2)}(v_L^2 + v_R^2),
\]

\[
\mu_2^2 = (2\lambda_2 + \lambda_3) k_1 k_2 + \frac{\lambda_4}{2}(k_1^2 + k_2^2) + \frac{2\alpha_2 (k_1^2 - k_2^2) + \alpha_3 k_1 k_2}{4(k_1^2 - k_2^2)}(v_L^2 + v_R^2),
\]

\[
\mu_3^2 = \frac{1}{2}\left[\alpha_1 (k_1^2 + k_2^2) + 4\alpha_2 k_1 k_2 + \alpha_3 k_2^2 + 2\rho_1 (v_L^2 + v_R^2)\right].
\]

(4.18)

The neutral, singly charged, and doubly charged scalar mass matrices in the new basis are:
When we performed our numerical analysis in this kind of model, we found that every neutral scalar eigenstate containing a significant admixture of $\phi_0^+$, real or imaginary, has a mass of the order of $\nu_R$. Since the minimum value that $\nu_R$ can take is $2.7 \times 10^7$ GeV, this is a model with FCNC that are consistent with the experimental constraints. It is interesting to note that, in addition to the field analogous of the Higgs boson of the SM, two other neutral scalar particles, one singly charged scalar particle, and one doubly charged scalar particle appear with masses at the electroweak scale. There is no experimental constraint to this order of magnitude for the masses of the new scalar particles. [2, 20] This is very interesting because the search for new scalar particles is the focus of most current and future experiments. [19] Any experimental evidence about this issue could give us some light about the validity and viability of this class of models with left-right symmetries. Tables 7, 8, and 9, show the normalized components of the mass eigenstates and the corresponding order of magnitude for the masses.

---

9These non-SM neutral scalar bosons contain a negligible admixture of $\phi_0^+$, so their contributions to the FCNC are suppressed.
4.4 Case IV: $\alpha = 0$ and $\theta = 0$

In this case, we have no $CP$ violation in both the quark and the lepton sector. The minimization conditions arising from the scalar potential, eq. (2.13), are the following:

$$\beta_2 = \frac{1}{k_1^2} (-\beta_1 k_1 k_2 - \beta_3 k_2^2 + (2\rho_1 - \rho_3) v_L v_R),$$
$$\mu_1^2 = \lambda_1 (k_1^2 + k_2^2) + 2\lambda_4 k_1 k_2$$
$$+ \frac{1}{2(k_1^2 - k_2^2)} [2(\beta_2 k_1 - \beta_3 k_2^2) v_L v_R + [\alpha_1 (k_1^2 - k_2^2) - \alpha_3 k_2^2] (v_L^2 + v_R^2)],$$
$$\mu_2^2 = (2\lambda_2 + \lambda_3) k_1 k_2 + \frac{\lambda_4}{2} (k_1^2 + k_2^2)$$
$$+ \frac{1}{4(k_1^2 - k_2^2)} [2(\beta_1 k_1^2 - k_2^2 - 2k_1 k_2 (\beta_2 - \beta_3)) v_L v_R$$
$$+ [2\alpha_2 (k_1^2 - k_2^2) + \alpha_3 k_1 k_2] (v_L^2 + v_R^2)],$$
$$\mu_3^2 = \frac{1}{2} [\alpha_1 (k_1^2 + k_2^2) + 4\alpha_2 k_1 k_2 + \alpha_3 k_2^2 + 2\rho_1 (v_L^2 + v_R^2)]. \tag{4.22}$$

The neutral, singly charged, and doubly charged scalar mass matrices in the new basis are:

$$M^2 = \begin{pmatrix} M^2_{B11} & 0 \\ 0 & M^2_{B22} \end{pmatrix}, \tag{4.23}$$

$$M^2_{B11} = \begin{pmatrix} \lambda k_1^2 & \lambda k_2^2 & \alpha k v_R & (2\rho_1 - \rho_3) k v_R \\ \lambda k_1^2 & \alpha k v_R & \alpha k v_R & \beta k v_R \\ \alpha k v_R & \alpha k v_R & 2\rho_1 v_R^2 & (2\rho_1 + \rho_3) k^2 / 2 \\ (2\rho_1 - \rho_3) k v_R & \beta k v_R & (2\rho_1 + \rho_3) k^2 / 2 & (\rho_3 - 2\rho_1) v_R^2 / 2 \end{pmatrix}. \tag{4.24}$$
Finally, in this last case our numerical analysis leads us to the same conclusions already obtained by Deshpande et. al. \[6\] In this model we find that all the non-SM Higgs bosons have a mass of the order of \(v_R\), avoiding any large FCNC. This model is exactly equal to the SM in the limit in which \(v_R\) goes to the infinity. If we compare the scalar mass spectrum of the cases III and IV, we find that the value which \(\theta\) takes only affects the order of the masses of the scalar particles and not the amount of the FCNC. Thus, we can conclude from the two last cases that, independently of the value that the spontaneous CP phase \(\theta\) takes, we will have an experimentally consistent LRSM if there is no CP violation in the quark sector. To avoid an explicit origin for the CP violation in the quark sector, we have to adjust \(\alpha\) to be small enough so as not to change the main features and results found, and to lead to the correct experimental value for the CKM phase of the SM. Effectively, to obtain the correct value for the CKM phase of the SM, we need a value close to zero for the spontaneous CP phase \(\alpha\). \[14\] Tables 10, 11, and 12, show the normalized components of the mass eigenstates and the corresponding order of magnitude for the masses.
5 Conclusions

In this paper, we have presented a detailed analysis of the relationship between the FCNC and the two spontaneous CP phases present in the LRSM with one doublet and two triplets of scalar bosons. Such quantity of scalar bosons is necessary in order to give phenomenologically acceptable masses to the right-weak bosons, and to implement the see-saw mechanism. Each spontaneous CP phase is related with one sector of matter: the quark or the leptonic one. Different combinations of maximal and null values between the two spontaneous CP phases lead us to four different cases corresponding to maximal and/or no CP violation in the quark and the lepton sector. What we found is that, each one of these cases leads to a different mass spectrum for the scalar bosons, allowing or avoiding FCNC.

The main result of this paper is that the only way to suppress the FCNC is to adjust close to zero the spontaneous CP phase associated with the quark sector, as in the cases III and IV. Then, the amount of CP violation in the lepton sector does not depend on the theoretical restrictions studied in this paper and, therefore, a possible large CKM-like phase for the lepton sector will only be restricted by elementary-process constraints. Additionally, in the case III, we have the possibility to observe new scalar bosons at the electroweak scale, which is a source of theoretical inspiration and many experimental works. The other non-SM particles, and their corresponding phenomenology, are at the scale of $10^7$ GeV, which is well beyond the reach of the next generation of accelerators. This result comes from restrictions arising from neutrino masses. From the cases I and II we can see that the FCNC are present if the CP violation in the quark sector is maximal. Therefore, this kind of models are experimentally unacceptable. We notice that our results were obtained in a general framework, which has let us study the four cases presented above. The results obtained in the case IV agree with those previously found in [6], which implies the consistence of our results for the other three cases.

The LRSM is then viable; it gives a natural explanation to the origin of the parity ($P$) violation and the charge-parity ($CP$) violation too. It does not have large FCNC that enter in conflict with the experimental data, and gives us a rich phenomenological world of CP violation in the lepton sector and new Higgs particles, neutral, singly charged, and doubly charged, at the electroweak scale.
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A Mass matrices for the scalar particles

In this appendix we show the mass matrices for the neutral, singly charged, and doubly charged scalar particles. We present the results before substituting the minimization conditions. Although these mass matrices have already been given in [4], there are lots of typos there, which led to erroneous conclusions. One of the authors of [4] noted this mistake in [7].

A.1 Neutral scalar mass matrix

The components of the symmetric neutral scalar mass matrix in the \{\phi_i^r, \phi_i^l, \delta_i^r, \delta_i^l, \phi_i^i, \phi_i^s, \delta_i^R, \delta_i^L\} basis are:

\[
M_{11}^R = -\mu_1^2 + \lambda_1 \left[ k_1^2 \left( 2 \cos^2 \alpha + 1 \right) + k_3^2 \right] + 2 \left( 2 \lambda_2 + \lambda_3 \right) k_2^3 + 6 \lambda_4 k_1 k_2 \cos \alpha, \\
+ \frac{1}{2} \alpha_1 \left( v_L^2 + v_R^2 \right) + \beta_2 v_L v_R \cos \theta,
\]

\[
M_{12}^R = -2 \mu_2^2 + 2 \left( \lambda_1 + 4 \lambda_2 + 2 \lambda_3 \right) k_1 k_2 \cos \alpha + \lambda_4 \left[ k_1^2 \left( 2 \cos^2 \alpha + 1 \right) + 3 k_2^2 \right] \\
+ \alpha_2 \left( v_L^2 + v_R^2 \right) + \frac{1}{2} \beta_1 v_L v_R \cos \theta,
\]

\[
M_{13}^R = \alpha_1 k_1 v_R \cos \alpha \cos \theta + 2 \alpha_2 k_2 v_R \cos \theta + \frac{1}{2} v_L \left( \beta_1 k_2 + 2 \beta_2 k_1 \cos \alpha \right),
\]

\[
M_{14}^R = \alpha_1 k_1 v_L \cos \alpha + 2 \alpha_2 k_2 v_L + \frac{1}{2} v_R \left( \beta_1 k_2 \cos \theta + 2 \beta_2 k_1 \cos \left( \theta - \alpha \right) \right),
\]

\[
M_{15}^R = \lambda_1 k_1^2 \sin 2 \alpha + 2 \lambda_4 k_1 k_2 \sin \alpha + \beta_2 v_L v_R \sin \theta,
\]

\[
M_{16}^R = -8 \lambda_2 k_1 k_2 \sin \alpha - \lambda_4 k_1^2 \sin 2 \alpha - \frac{1}{2} \beta_1 v_L v_R \sin \theta,
\]

\[
M_{17}^R = \alpha_1 k_1 v_R \cos \alpha \sin \theta + 2 \alpha_2 k_2 v_R \sin \theta + \beta_2 k_1 v_L \sin \alpha,
\]

\[
M_{18}^R = \frac{1}{2} v_R \left( \beta_1 k_2 \sin \theta + 2 \beta_2 k_1 \sin \left( \theta - \alpha \right) \right),
\]

\[
M_{22}^R = -\mu_1^2 + \lambda_1 \left( k_1^2 + 3 k_2^2 \right) + 2 \left( 2 \lambda_2 \cos 2 \alpha + \lambda_3 \right) k_3^2 + 6 \lambda_4 k_1 k_2 \cos \alpha \\
+ \frac{1}{2} \left( \alpha_1 + \alpha_3 \right) \left( v_L^2 + v_R^2 \right) + \beta_3 v_L v_R \cos \theta,
\]
\[ M_{23}^2 = \alpha_1 k_2 v_R \cos \theta + 2 \alpha_2 k_1 v_R \cos \alpha \cos \theta + \alpha_3 k_2 v_R \cos \theta + \frac{1}{2} v_L \left( \beta_1 k_1 \cos \alpha + 2 \beta_3 k_2 \right), \]

\[ M_{24}^2 = (\alpha_1 + \alpha_3) k_2 v_L + 2 \alpha_2 k_1 v_L \cos \alpha + \frac{1}{2} v_R \left( \beta_1 k_1 \cos (\theta - \alpha) + 2 \beta_3 k_2 \cos \theta \right), \]

\[ M_{25}^2 = 2 (\lambda_1 - 4 \lambda_2 + 2 \lambda_3) k_1 k_2 \sin \alpha + \lambda_4 k_1^2 \sin 2 \alpha + \frac{1}{2} \beta_1 v_L v_R \sin \theta, \]

\[ M_{26}^2 = -4 \lambda_2 k_1^2 \sin 2 \alpha - 2 \lambda_1 k_1 k_2 \sin \alpha - \beta_2 v_L v_R \sin \theta, \]

\[ M_{27}^2 = \alpha_1 k_2 v_R \sin \theta + 2 \alpha_2 k_1 v_R \cos \alpha \sin \theta + \alpha_3 k_2 v_R \sin \theta + \frac{1}{2} \beta_1 k_1 v_L \sin \alpha, \]

\[ M_{28}^2 = \frac{1}{2} v_R (\beta_1 k_1 \sin (\theta - \alpha) + 2 \beta_3 k_2 \sin \theta), \]

\[ M_{33}^2 = -\mu_3^2 + \frac{1}{2} \alpha_1 (k_1^2 + k_2^2) + 2 \alpha_2 k_1 k_2 \cos \alpha + \frac{1}{2} \alpha_3 k_2^2 + \rho_1 v_R^2 \left( 2 \cos^2 \theta + 1 \right) + \frac{1}{2} \beta_3 v_L^2, \]

\[ M_{34}^2 = \frac{1}{2} \beta_1 k_1 k_2 \cos \alpha + \frac{1}{2} \beta_2 k_1^2 \cos 2 \alpha + \frac{1}{2} \beta_3 k_2^2 + \rho_3 v_L v_R \cos \theta, \]

\[ M_{35}^2 = \alpha_1 k_1 v_R \sin \alpha \cos \theta - \beta_2 k_1 v_L \sin \alpha, \]

\[ M_{36}^2 = -2 \alpha_2 k_1 v_R \sin \alpha \cos \theta + \frac{1}{2} \beta_1 k_1 v_L \sin \alpha, \]

\[ M_{37}^2 = \rho_1 v_R^2 \sin 2 \theta, \]

\[ M_{38}^2 = -\frac{1}{2} \beta_1 k_1 k_2 \sin \alpha - \frac{1}{2} \beta_2 k_1^2 \sin 2 \alpha, \]

\[ M_{44}^2 = -\mu_3^2 + \frac{1}{2} \alpha_1 (k_1^2 + k_2^2) + 2 \alpha_2 k_1 k_2 \cos \alpha + \frac{1}{2} \alpha_3 k_2^2 + 3 \rho_1 v_L^2 + \frac{1}{2} \beta_3 v_R^2, \]

\[ M_{45}^2 = \alpha_1 k_1 v_L \sin \alpha + \frac{1}{2} \beta_1 k_2 v_R \sin \theta + \beta_2 k_1 v_R \sin (\theta - \alpha), \]

\[ M_{46}^2 = -2 \alpha_2 k_1 v_L \sin \alpha - \frac{1}{2} \beta_1 k_1 v_R \sin (\theta - \alpha) - \beta_3 k_2 v_R \sin \theta, \]

\[ M_{47}^2 = \frac{1}{2} \beta_1 k_1 k_2 \sin \alpha + \frac{1}{2} \beta_2 k_1^2 \sin 2 \alpha + \rho_3 v_L v_R \sin \theta, \]

\[ M_{48}^2 = 0, \]

\[ M_{55}^2 = -\mu_3^2 + \lambda_1 \left[ k_1^2 \left( 2 \sin^2 \alpha + 1 \right) + k_2^2 \right] + 2 (\lambda_1 - 2 \lambda_2 + \lambda_3) k_1^2 \cos \alpha + \frac{1}{8} \alpha_1 (v_L^2 + v_R^2) - \beta_2 v_L v_R \cos \theta, \]

\[ M_{56}^2 = 2 \mu_4^2 - 8 \lambda_2 k_1 k_2 \cos \alpha - \lambda_4 \left[ k_1^2 \left( 2 \sin^2 \alpha + 1 \right) + k_2^2 \right] - \alpha_2 (v_L^2 + v_R^2) + \frac{1}{2} \beta_1 v_L v_R \cos \theta, \]

\[ M_{57}^2 = \alpha_1 k_1 v_R \sin \alpha \sin \theta + \frac{1}{2} \beta_1 k_2 v_L + \beta_2 k_1 v_L \cos \alpha, \]

\[ M_{58}^2 = -\frac{1}{2} \beta_1 k_2 v_R \cos \theta - \beta_2 k_1 v_R \cos (\theta - \alpha), \]

\[ M_{66}^2 = -\mu_3^2 + \lambda_1 \left( k_1^2 + k_2^2 \right) + 2 (\lambda_1 - 2 \lambda_2 \cos 2 \alpha + \lambda_3) k_1^2 + 2 \lambda_4 k_1 k_2 \cos \alpha + \frac{1}{2} \left( \alpha_1 + \alpha_3 \right) (v_L^2 + v_R^2) - \beta_3 v_L v_R \cos \theta, \]
\[ M_{07}^2 = -2\alpha_2 k_1 v_R \sin \theta - \frac{1}{2} \beta_1 k_1 v_L \cos \alpha - \beta_3 k_2 v_L, \]
\[ M_{08}^2 = \frac{1}{2} \beta_3 k_1 v_R \cos (\theta - \alpha) + \beta_3 k_2 v_R \cos \theta, \]
\[ M_{17}^2 = -\mu_3^2 + \frac{1}{2} \alpha_1 (k_1^2 + k_2^2) + 2\alpha_2 k_1 k_2 \cos \alpha + \frac{1}{2} \alpha_3 k_2^2 + \rho_1 v_L^2 \left( 2 \sin^2 \theta + 1 \right) + \frac{1}{2} \rho_3 v_R^2, \]
\[ M_{18}^2 = \frac{1}{2} \beta_1 k_1 k_2 \cos \alpha + \frac{1}{2} \beta_2 k_1^2 \cos 2\alpha + \frac{1}{2} \beta_3 k_2^2, \]
\[ M_{27}^2 = -\mu_3^2 + \frac{1}{2} \alpha_1 (k_1^2 + k_2^2) + 2\alpha_2 k_1 k_2 \cos \alpha + \frac{1}{2} \alpha_3 k_2^2 + \rho_1 v_L^2 + \frac{1}{2} \rho_3 v_R^2. \]

(A. 1)

### A.2 Singly charged scalar mass matrix

The components of the hermitian singly charged scalar mass matrix in the \{ \phi^+_1, \phi^+_2, \delta^+_R, \delta^+_L \} basis are:

\[ M_{11}^{\pm 2} = -\mu_1^2 + \lambda_1 (k_1^2 + k_2^2) + 2\lambda_4 k_1 k_2 \cos \alpha + \frac{1}{2} \alpha_1 (v_L^2 + v_R^2) + \frac{1}{2} \alpha_3 v_R^2, \]
\[ M_{12}^{\pm 2} = 2\mu_2^2 + 2 (2\lambda_2 e^{i\alpha} + \lambda_3 e^{-i\alpha}) k_1 k_2 - \lambda_4 (k_1^2 + k_2^2) - \alpha_2 (v_L^2 + v_R^2), \]
\[ M_{13}^{\pm 2} = \frac{1}{2\sqrt{2}} \alpha_3 k_1 v_R e^{i(\theta - \alpha)} - \frac{1}{2\sqrt{2}} v_L (\beta_1 k_2 + 2\beta_2 k_1 e^{i\alpha}), \]
\[ M_{14}^{\pm 2} = \frac{1}{2\sqrt{2}} \alpha_3 k_2 v_L + \frac{1}{2\sqrt{2}} v_R (\beta_1 k_1 e^{i(\theta - \alpha)} + 2\beta_3 k_2 e^{i\alpha}), \]
\[ M_{22}^{\pm 2} = -\mu_3^2 + \lambda_1 (k_1^2 + k_2^2) + 2\lambda_4 k_1 k_2 \cos \alpha + \frac{1}{2} \alpha_1 (v_L^2 + v_R^2) + \frac{1}{2} \alpha_3 v_R^2, \]
\[ M_{23}^{\pm 2} = \frac{1}{2\sqrt{2}} \alpha_3 k_2 v_R e^{i\alpha} + \frac{1}{2\sqrt{2}} v_L (\beta_1 k_1 e^{i\alpha} + 2\beta_3 k_2), \]
\[ M_{24}^{\pm 2} = \frac{1}{2\sqrt{2}} \alpha_3 k_1 v_L e^{i\alpha} - \frac{1}{2\sqrt{2}} v_R (\beta_1 k_2 e^{i\alpha} + 2\beta_2 k_1 e^{i(\theta - \alpha)}), \]
\[ M_{33}^{\pm 2} = -\mu_3^2 + \frac{1}{2} \alpha_1 (k_1^2 + k_2^2) + 2\alpha_2 k_1 k_2 \cos \alpha + \frac{1}{4} \alpha_3 (k_1^2 + k_2^2) + \rho_1 v_L^2 + \frac{1}{2} \rho_3 v_R^2, \]
\[ M_{34}^{\pm 2} = \frac{1}{4} \beta_1 (k_1^2 + k_2^2) + \frac{1}{2} (\beta_2 e^{-i\alpha} + \beta_3 e^{i\alpha}) k_1 k_2, \]
\[ M_{44}^{\pm 2} = -\mu_3^2 + \frac{1}{2} \alpha_1 (k_1^2 + k_2^2) + 2\alpha_2 k_1 k_2 \cos \alpha + \frac{1}{4} \alpha_3 (k_1^2 + k_2^2) + \rho_1 v_L^2 + \frac{1}{2} \rho_3 v_R^2. \]

(A. 2)
A.3 Doubly charged scalar mass matrix

The components of the hermitian doubly charged scalar mass matrix in the \( \{ \delta_R^{++}, \delta_L^{++} \} \) basis are:

\[
M_{11}^{++2} = -\mu_3^2 + \frac{1}{2} \alpha_1 (k_1^2 + k_2^2) + 2 \alpha_2 k_1 k_2 \cos \alpha + \frac{1}{2} \alpha_3 k_1^2 + (\rho_1 + 2 \rho_2) \nu_R^2 + \frac{1}{2} \rho_3 \nu_L^2,
\]

\[
M_{12}^{++2} = 2 \rho_4 \nu_L \nu_R e^{i \theta} + \frac{1}{2} \beta_1 k_1 k_2 e^{i \alpha} + \frac{1}{2} \beta_2 k_2^2 + \frac{1}{2} \beta_3 k_1^2 e^{2i \alpha},
\]

\[
M_{22}^{++2} = -\mu_3^2 + \frac{1}{2} \alpha_1 (k_1^2 + k_2^2) + 2 \alpha_2 k_1 k_2 \cos \alpha + \frac{1}{2} \alpha_3 k_1^2 + (\rho_1 + 2 \rho_2) \nu_L^2 + \frac{1}{2} \rho_3 \nu_R^2.
\]  

(B. 3)

B \( \nu_R \) from neutrino physics

In this appendix we are going to show how to obtain an order of magnitude for \( \nu_R \) from the experimental restrictions on neutrino masses.

The Yukawa terms in the Lagrangian for the lepton sector are given by:

\[
- \mathcal{L}_Y = \sum_{a,b} h_{ab} \Psi_a L \Phi \Psi_b R + \tilde{h}_{ab} \Psi_a L \tilde{\Phi} \Psi_b R + i f_{ab} \left[ \Psi_a^T \tau_2 \Delta_L \Psi_b L + (L \leftrightarrow R) \right] + h.c.,
\]

where, as in the quark Yukawa case, \( h^l \) and \( \tilde{h}^l \) must be hermitian. For convenience, we will work with a single generation, and ignore the spontaneous \( CP \) phases. Introducing the vacuum expectation values into eq. (B. 1), we obtain the following mass terms:

\[
\frac{1}{\sqrt{2}} \left[ \left( h^l k_1 + \tilde{h}^l k_2 \right) \nu_L \nu_R + \left( h^l k_2 + \tilde{h}^l k_1 \right) \nu_R e_R + f \left( \nu_R \nu_R^c + \nu_L \nu_L^c \right) \right] + h.c.,
\]

(B. 2)

Neutrino mass terms derive both from the \( h^l \) and \( \tilde{h}^l \) terms, which lead to a Dirac mass, and from the \( f \) term, which leads to a Majorana mass. Defining, as usual, \( \psi^c \equiv C \left( \psi \right)^T \), it is convenient to employ the self-conjugate spinors:

\[
\nu = \frac{1}{\sqrt{2}} \left( \nu_L + \nu_R^c \right), \quad N = \frac{1}{\sqrt{2}} \left( \nu_R + \nu_L^c \right).
\]

(B. 3)

Thus, the neutrino mass terms can be written as:
\[
\begin{pmatrix}
\nu \\
N
\end{pmatrix}
\begin{pmatrix}
\sqrt{2}f v_L & h_D k_+ \\
h_D k_+ & \sqrt{2}f v_R
\end{pmatrix}
\begin{pmatrix}
\nu \\
N
\end{pmatrix},
\]  

(B. 4)

where for simplicity we have defined:

\[
h_D = \frac{1}{\sqrt{2}} \frac{h^l k_1 + \tilde{h}^l k_2}{k_+}.
\]  

(B. 5)

Given the phenomenological condition \( v_L \ll k_1, k_2 \ll v_R, \nu \) and \( N \) are approximate mass eigenstates with masses:

\[
m_N \approx \sqrt{2}f v_R, 
\]  

(B. 6)

\[
m_\nu \approx \sqrt{2} \left[ f v_L - \frac{h_D^2 k_+^2}{2f v_R} \right].
\]  

(B. 7)

Additionally, the electron mass is given by:

\[
m_e = \frac{1}{\sqrt{2}} \left( h^l k_2 + \tilde{h}^l k_1 \right) = h_D^e k_+,
\]  

(B. 8) 

with:

\[
h_D^e = \frac{1}{\sqrt{2}} \frac{h^l k_2 + \tilde{h}^l k_1}{k_+}.
\]  

(B. 9)

Normally, we expect \( h_D \) and \( h_D^e \) to be similar in size. Then, substituting eq. (B. 4) and eq. (B. 5) into eq. (B. 7) and taking into account that \( k_+^2 \approx v_L v_R \), we arrive at the following expression for \( v_R \) in terms of \( k_+ \) and the masses of \( \nu, N \), and \( e \):

\[
v_R^2 \approx k_+^2 \frac{m_N^2}{m_\nu m_N + m_e^2}.
\]  

(B. 10)

We can see from this expression that the minimum value that \( v_R \) can take is determined by the lower bound on the mass of \( N \) and the upper bound on the mass of \( \nu \).

Taking the following central values from [2]:

\[
m_e = 5.11 \times 10^{-4} \text{ GeV},
\]  

\[
m_\nu < 3 \times 10^{-9} \text{ GeV},
\]  

\[
m_N > 73.5 \text{ GeV},
\]  

(B. 11)
we arrive at the lower bound for $v_R$: \( v_R > 2.7 \times 10^7 \text{ GeV}. \) (B. 12)

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Table 1. Case I: Normalized components of the neutral mass eigenstates and orders of magnitude for the masses. The most general way to express the scalar mass eigenstates (first column of the table) is through a linear combination of the flavour eigenstates (first row of the table). The values in the table mean the corresponding weight in the linear combination. We have approximated these values to two significative digits. In the last column we present the order of magnitude (O.M.) for the masses of the eigenstates of the first column. This applies for all the other tables.

|       | $\phi_+^-$ | $\phi_+^+$ | $\delta_R^+$ | $\delta_L^+$ | $\delta_R^+$ | $\delta_L^+$ | O.M. Masses |
|-------|-------------|-------------|---------------|---------------|---------------|---------------|-------------|
| $\phi_A$ | 0.50       | 0           | 0             | 0             | 0.50          | 0.71          | $v_R$       |
| $\phi_B$ | -0.50      | 0           | 0             | 0             | -0.50         | 0.71          | $v_R$       |
| $\phi_C$ | 0          | 0.71       | 0             | 0.50          | 0             | 0             | $v_R$       |
| $\phi_D$ | 0          | 0          | -0.71         | 0             | 0             | 0.71          | $v_R$       |
| $\phi_E$ | 0          | 0.71       | -0.50         | 0             | 0             | 0             | $v_R$       |
| $\phi_F$ | 0          | 0          | 0             | 0.58          | 0.58          | 0             | $k$         |
| $\phi_G$ | -0.41      | 0          | 0             | -0.82         | 0.41          | 0             | 0           |
| $\phi_H$ | 0          | 0          | -1.00         | 0             | 0             | 0             | 0           |

Table 2. Case I: Normalized components of the singly charged mass eigenstates and orders of magnitude for the masses.

|       | $\phi_+^+$ | $\delta_R^+$ | $\delta_L^+$ | O.M. Masses |
|-------|------------|---------------|---------------|-------------|
| $\phi_A$ | $-0.71i$  | 0.71          | 0             | $v_R$       |
| $\phi_B$ | 0         | 0             | 0             | $v_R$       |
| $\phi_C$ | 0.71       | -0.71i       | 0             | 0           |
| $\phi_D$ | 0         | 1.00          | 0             | 0           |

Table 3. Case I: Normalized components of the doubly charged mass eigenstates and orders of magnitude for the masses.

|       | $\delta_R^{++}$ | $\delta_L^{++}$ | O.M. Masses |
|-------|-----------------|-----------------|-------------|
| $\phi_A$ | 0.71            | 0.32 - 0.63i    | $v_R$       |
| $\phi_B$ | -0.32 - 0.63i  | 0.71            | $v_R$       |
Table 4. Case II: Normalized components of the neutral mass eigenstates and orders of magnitude for the masses.

|   | $\phi^\pm_-$ | $\phi^\pm_+$ | $\delta^+_R$ | $\delta^+_L$ | $\phi^\pm_-$ | $\delta^+_R$ | $\delta^+_L$ | O.M. Masses |
|---|--------------|--------------|--------------|--------------|--------------|--------------|--------------|-------------|
| A | 0.50         | 0            | 0.71         | 0            | 0.50         | 0            | 0            | $v_R$       |
| B | -0.50        | 0            | 0.71         | 0            | -0.50        | 0            | 0            | $v_R$       |
| C | 0            | 1.00         | 0            | 0            | 0            | 0            | 0            | $v_R$       |
| D | 0            | 0            | -0.81        | 0            | 0            | 0            | 0.58         | $v_R$       |
| E | 0            | 0            | -0.58        | 0            | 0            | 0            | -0.81        | $v_R$       |

Table 5. Case II: Normalized components of the singly charged mass eigenstates and orders of magnitude for the masses.

|   | $\delta^{++}_R$ | $\delta^{++}_L$ | O.M. Masses |
|---|----------------|----------------|-------------|
| A | 0.71           | 0.71i          | $v_R$       |
| B | 0              | 0              | $v_R$       |
| C | 0.71i          | 0.71           | $v_R$       |

Table 6. Case II: Normalized components of the doubly charged mass eigenstates and orders of magnitude for the masses.

|   | $\delta^{++}_R$ | $\delta^{++}_L$ | O.M. Masses |
|---|----------------|----------------|-------------|
| A | 1.00           | 0              | $v_R$       |
| B | 0              | 1.00           | $v_R$       |
Table 7. Case III: Normalized components of the neutral mass eigenstates and orders of magnitude for the masses.

|       | $\phi^0_-$ | $\phi^0_+$ | $\delta^0_R$ | $\delta^0_L$ | $\phi^0_-$ | $\delta^0_R$ | $\delta^0_L$ | O.M. Masses |
|-------|-------------|-------------|---------------|---------------|-------------|---------------|---------------|-------------|
| $\phi_A^0$ | 0           | 0.71        | 0             | 0             | 0           | 0.71          | 0             | $v_R$       |
| $\phi_P^0$ | 0           | 0           | 0             | 0             | 1.00        | 0             | 0             | $v_R$       |
| $\phi_C^0$ | 0           | -0.71       | 0             | 0             | 0           | 0.71          | 0             | $v_R$       |
| $\phi_D^0$ | 0.50        | 0           | 0             | 0.63          | -0.50       | 0             | 0             | -0.32       |
| $\phi_E^0$ | 0           | 0           | 0             | 0.45          | 0           | 0             | 0             | 0.89        |
| $\phi_F^0$ | -0.71       | 0           | 0             | 0             | -0.71       | 0             | 0             | $k$         |
| $\phi_G^0$ | 0.50        | 0           | 0             | -0.63         | -0.50       | 0             | 0             | 0.32        |
| $\phi_H^0$ | 0           | 0           | 1.00          | 0             | 0           | 0             | 0             | 0           |

Table 8. Case III: Normalized components of the singly charged mass eigenstates and orders of magnitude for the masses.

|       | $\phi^+^+$ | $\phi^+^-$ | $\delta^+^+_R$ | $\delta^+^-_L$ | O.M. Masses |
|-------|------------|------------|-----------------|-----------------|-------------|
| $\phi_A^+$ | 1.00      | 0          | 0               | 0               | $v_R$       |
| $\phi_P^+$ | 0         | 0          | 0.71            | 0.71            | $k$         |
| $\phi_C^+$ | 0         | 1.00       | 0               | 0               | $k$         |
| $\phi_D^+$ | 0         | 0          | -0.51           | 0.86            | 0           |

Table 9. Case III: Normalized components of the doubly charged mass eigenstates and orders of magnitude for the masses.
Table 10. Case IV: Normalized components of the neutral mass eigenstates and orders of magnitude for the masses.

|        | φ⁺ | φ⁺ | δ⁺ | δ⁺ | φ⁺ | φ⁺ | δ⁺ | δ⁺ | O.M. Masses |
|--------|----|----|----|----|----|----|----|----|-------------|
| φ⁺<sub>A</sub> | 0  | 0  | -1.00 | 0  | 0  | 0  | 0  | 0  | v<sub>R</sub> |
| φ⁺<sub>B</sub> | 0  | 0  | 0  | 0  | 0  | 1.00 | 0  | 0  | v<sub>R</sub> |
| φ⁺<sub>C</sub> | 0  | 1.00 | 0  | 0  | 0  | 0  | 0  | 0  | v<sub>R</sub> |
| φ⁺<sub>D</sub> | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1.00 | v<sub>R</sub> |
| φ⁺<sub>E</sub> | 0  | 0  | 0  | 1.00 | 0  | 0  | 0  | 0  | v<sub>R</sub> |
| φ⁺<sub>F</sub> | 1.00 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | k           |
| φ⁺<sub>G</sub> | 0  | 0  | 0  | 0  | -0.71 | 0  | -0.71 | 0  | 0           |
| φ⁺<sub>H</sub> | 0  | 0  | 0  | 0  | 0.71 | 0  | -0.71 | 0  | 0           |

Table 11. Case IV: Normalized components of the singly charged mass eigenstates and mass orders or magnitude.

|        | φ⁺ | φ⁺ | δ⁺ | δ⁺ | O.M. Masses |
|--------|----|----|----|----|-------------|
| φ⁺<sub>A</sub> | -1.00 | 0  | 0  | 0  | v<sub>R</sub> |
| φ⁺<sub>B</sub> | 0  | 0  | 0  | -1.00 | v<sub>R</sub> |
| φ⁺<sub>C</sub> | 0  | 0.71 | 0.71 | 0  | 0           |
| φ⁺<sub>D</sub> | 0  | 0.71 | -0.71 | 0  | 0           |
| φ⁺<sub>E</sub> | 0  | 0  | 0  | 0  | 0           |

Table 12. Case IV: Normalized components of the doubly charged mass eigenstates and orders of magnitude for the masses.

|        | φ⁺⁺ | φ⁺⁺ | δ⁺⁺ | δ⁺⁺ | O.M. Masses |
|--------|-----|-----|-----|-----|-------------|
| φ⁺⁺<sub>A</sub> | 1.00 | 0  | v<sub>R</sub> |
| φ⁺⁺<sub>B</sub> | 0  | 1.00 | v<sub>R</sub> |