Theory of Striped Hall Ferromagnets

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Abstract

We study spin and charge striped states at the half-filled high Landau level in the zero Zeeman energy limit using a Hartree-Fock approximation. It is shown that a ferromagnetic striped Hall state is more stable than the antiferromagnetic striped state or charge striped state. We calculate the collective excitations using the single mode approximation.

Key words: Charge density wave, spin density wave, striped state, collective excitation, quantum Hall system

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Recently, striped states have been proposed for the anisotropic states in the quantum Hall system at several filling factors. At half-filled high Landau levels, highly anisotropic states are observed[1,2] and explained by the anisotropic charge density wave or charge striped state.[3,4] Furthermore at even integer fillings, highly anisotropic states have been observed in a quantum well system.[5] A kind of the striped state or domain wall related to spin or pseudospin is a candidate of the anisotropic state.[6,7] Considering the spin and pseudospin degree of freedom, a very rich structure has been predicted theoretically.[8,9,10]

In this paper we consider a possibility of spin and charge striped states in an ideal 2D electron system at half-filled high Landau level in the zero Zeeman energy limit. Using the Hartree-Fock approximation, it is shown that a ferromagnetic striped Hall state has a lower energy than the antiferromagnetic striped state or charge striped state.

We calculate the collective excitations using the single mode approximation. There are two kinds of the excitations, namely the phonon and spin wave due to the spontaneous breakdown of the translational symmetry and spin rotational symmetry, respectively. It is shown that the dispersion of the spin wave has a weaker anisotropy than the phonon dispersion.

Let us consider the 2D electron system in a perpendicular strong magnetic field. The kinetic energy is quenched in the Landau level space and neglected in this paper. Then we consider only the following interaction Hamiltonian

\[ H_{\text{int}} = \frac{1}{2} \int \frac{d^2k}{(2\pi)^2} \rho(k)V(k)\rho(-k) \] (1)

where \( V(k) = 2\pi q^2/k \) and \( \rho \) is the density operator. The Zeeman energy term is neglected. We use the unit \( h = c = 1 \) and set \( a = \sqrt{2\pi/eB} = 1 \). In the von Neumann lattice formalism[11], the density operator \( \int d^2xe^{ik\cdot x} \sum_\alpha \psi_\alpha^\dagger(x)\psi_\alpha(x) \) is written in the projected l th Landau level as
\[\rho(k) = f_l(k) \sum_{\alpha} \int_{BZ} \frac{d^2p}{(2\pi)^2} b^\dagger_{\alpha}(p)b_{\alpha}(p - k)\]
\[\times e^{-\frac{i}{\hbar}k_y(p_y - k_y)}, \quad (2)\]

where \(f_l(k) = e^{-k^2/8\sigma}L_l(k^2/4\pi)\), \(\hat{k} = (r k_x, k_y/r)\), \(\alpha = \uparrow, \downarrow\), and \(r\) is an asymmetric parameter of the unit cell of von Neumann lattices. In the mean field theory of the striped state, the parameter \(r\) becomes the period of the stripe. The spin density operators projected to the \(l\) th Landau level are defined by
\[s_i(k) = f_l(k) \sum_{\alpha} \int_{BZ} \frac{d^2p}{(2\pi)^2} b^\dagger_{\alpha}(p)\sigma^i_{\alpha\beta}(p) b_{\beta}(p - k)\]
\[\times e^{-\frac{i}{\hbar}k_y(p_y - k_y)}, \quad (3)\]

where \(\sigma_i\) is the Pauli matrix and \(i = 1, 2, 3\). Ladder operators are defined by \(s_{\pm} = s_1 \pm i s_2\).

In the Hartree-Fock approximation, we consider the following mean field
\[\langle b^\dagger_{\beta}(p)b_{\alpha}(p')\rangle \approx U_{\alpha\beta}(p)\]
\[\times \sum_N (2\pi)^2 \delta(p - p' + 2\pi N)e^{i\phi(p,N)}, \quad (4)\]

where \(U_{\alpha\beta}(p)\) is periodic function in the magnetic Brillouin zone and \(\phi(p,N) = \pi(N_x + N_y) - N_y p_x\). Using the Pauli matrix, \(U_{\alpha\beta}(p)\) is written as
\[U_{\alpha\beta}(p) = U_0(p)\frac{\delta_{\alpha\beta}}{2} + \sum_i U_i(p)\frac{\sigma^i_{\alpha\beta}}{2}. \quad (5)\]

\(U_0\) and \(U_i\) are distributions of the number density and spin density in the momentum space, respectively. Using the mean field, Hartree-Fock Hamiltonian reads
\[H_{HF} = \sum_{\alpha\beta} \int_{BZ} \frac{d^2p}{(2\pi)^2} b^\dagger_{\alpha\beta}(p)\epsilon_{\alpha\beta}(p)b_{\alpha\beta}(p)\]
\[-\frac{1}{2} \sum_{\alpha\beta} \int_{BZ} \frac{d^2p}{(2\pi)^2} \epsilon_{\alpha\beta}(p)U_{\alpha\beta}(p)(2\pi)^2 \delta(p = 0). \quad (6)\]

The one-particle energy \(\epsilon_{\alpha\beta}(p)\) is given by
\[\epsilon_{\alpha\beta}(p) = \int_{BZ} \frac{d^2p'}{(2\pi)^2} [\delta_{\alpha\beta}\nu(p - p')\text{tr}(U(p')) - \nu(p - p')U_{\alpha\beta}(p')]\]. \quad (7)

The Hartree potential \(\nu_l\) and Fock potential \(\nu_F\) are given by
\[\nu_l(p) = \sum_N v_l(2\pi N)e^{-iN_x p_x + iN_y p_y}, \quad \nu_F(p) = \sum_N v_l(p + 2\pi N), \quad \text{where} \quad v_l(p) = f_l(p)V(p) \text{and} \]
\[\tilde{p} = (p_x/r, r p_y)\]. In the followings we present three self-consistent Hartree-Fock state at \(\nu = 2l + 1/2\).

(a) **Ferromagnetic striped state**: First we consider the ferromagnetic striped state, in which the spin up state is filled periodically in \(x\) direction and spin down state is completely empty. The period of the stripe is given by parameter \(r\). The corresponding Fermi sea is a stripe extending in \(p_x\) direction.\([12,13]\) See Fig. 1 (a). The corresponding mean field is \(U_0(p) = \theta(\pi/2 - |p_y|), \quad U_i(p) = \theta(\pi/2 - |p_p|)\). Using this mean field, the one-particle energy is obtained as
\[\epsilon_{\alpha\beta}(p) = \begin{pmatrix} \epsilon_l(p_y) & 0 \\ 0 & \epsilon_l(p_y) \end{pmatrix}, \quad (8)\]

where
\[ \epsilon_\downarrow(p_y) = \int_{-\pi/2}^{\pi/2} \frac{dp_y'}{2\pi} \int_{-\pi}^{\pi} \frac{dp_y}{2\pi} \nuHF(p - p'), \]
\[ \epsilon_\uparrow(p_y) = 0, \] (9)
where \( \nuHF = \nuH - \nuF \). The energy per particle \( E_f(r) \) is given by
\[ E_f(r) = \int_{-\pi/2}^{\pi/2} \frac{dp_y}{2\pi} \epsilon_\uparrow(p_y) \] (10)

(b) Antiferromagnetic striped state: Next we consider an antiferromagnetic striped state in which the spin and charge density is periodic in \( x \) direction and uniform in \( y \) direction. The corresponding Fermi sea is two stripes extending in \( p_x \) direction. See Fig. 1 (b).

Then the mean field for the antiferromagnetic striped state is given by \( U_0(p) = \theta(\pi/4 - |p_y|) + \theta(|p_y| - 3\pi/4), U_i(p) = \theta(\pi/4 - |p_y|) - \theta(|p_y| - 3\pi/4) \). Using this mean field, the one-particle energy is obtained as
\[ \epsilon_{\alpha\beta}(p) = \begin{pmatrix} \epsilon_\uparrow(p_y) & 0 \\ 0 & \epsilon_\downarrow(p_y) \end{pmatrix}, \] (11)
where
\[ \epsilon_\uparrow(p_y) = \left( \int_{-\pi}^{\pi} + \int_{3\pi/4}^{\pi/4} \right) \frac{dp_y'}{2\pi} \frac{dp_y}{2\pi} \nuHF(p - p') \]
\[ + \int_{-\pi/4}^{\pi/4} \frac{dp_y'}{2\pi} \frac{dp_y}{2\pi} \nuHF(p - p'), \]
\[ \epsilon_\downarrow(p_y) = \epsilon_\uparrow(p_y - \pi). \] (12)

The energy per particle \( E_a(r) \) is given by
\[ E_a(r) = \int_{-\pi/4}^{\pi/4} \frac{dp_y}{2\pi} \epsilon_\uparrow(p_y) \]
\[ + \left( \int_{-\pi}^{\pi} + \int_{3\pi/4}^{\pi/4} \right) \frac{dp_y}{2\pi} \epsilon_\downarrow(p_y). \] (13)

(c) Charge striped state: Finally we consider the charge striped state in which the spin up states and down states are filled periodically in \( x \) direction and uniformly in \( y \) direction. In this state, translation symmetry in \( x \) direction is broken. The corresponding Fermi sea is a stripe extending in \( p_x \) direction. See Fig. 1 (c).

Then the mean field for the spin striped state is given by \( U_0(p) = 2\theta(\pi/4 - |p_y|), U_i(p) = 0 \). Using this mean field, the one-particle energy is obtained as
\[ \epsilon_{\alpha\beta}(p) = \begin{pmatrix} \epsilon_\uparrow(p_y) & 0 \\ 0 & \epsilon_\downarrow(p_y) \end{pmatrix}, \] (14)

Let us compare the Hartree-Fock energy obtained in the previous calculation for three striped states. The energy at \( \nu = 4 + 1/2 \) is plotted in Fig. 2 as a function of the period of the stripe \( r \). Note that the antiferromagnetic and charge striped states have a same energy. As seen in the Figure, the ferromagnetic striped state has the lower energy than the other states for all period. We call this stable ferromagnetic striped state the striped Hall ferromagnet.

We consider the low-lying excitations due to the spontaneous symmetry breaking in the striped Hall ferromagnet. We approximate the low-lying excitation states for the charge and spin excitation as follows,
\[ |k, \text{charge}⟩ = ρ(k)|0⟩, |k, \text{spin}⟩ = s_−(k)|0⟩, \] (17)
where the state \( |0⟩ \) is the striped Hall ferromagnet state obtained in the Hartree-Fock approximation. Note that \( s_+(k)|0⟩ = 0 \). These excitations correspond to the NG modes of the spontaneous breaking of the translation and spin rotation symmetry. We call these excitations phonon and spin wave, respectively.

The excitation energies for these collective mode are given by
\[ \begin{align*}
\epsilon_\uparrow(p_y) &= \int_{-\pi/4}^{\pi/4} \frac{dp_y'}{2\pi} \epsilon_\uparrow(p_y) + \epsilon_\downarrow(p_y) \]
\[ = \int_{-\pi/4}^{\pi/4} \frac{dp_y'}{2\pi} \epsilon_\uparrow(p_y) + \epsilon_\downarrow(p_y), \] (16)

where
for small $k$ can be seen that $\Delta_{\text{phonon}}$ is highly anisotropic [13]. On the other hand, it can be seen in this figure, the spectrum for the spin wave has $\Delta_{\text{spin}}$ and calculated using the following commutation relations:

$$\Delta_{\text{phonon}}(k) = \frac{\langle k, \text{charge}|(H_{\text{int}} - E_0)|k, \text{charge} \rangle}{\langle k, \text{charge}|k, \text{charge} \rangle}$$

$$\Delta_{\text{spin}}(k) = \frac{\langle k, \text{spin}|(H_{\text{int}} - E_0)|k, \text{spin} \rangle}{\langle k, \text{spin}|k, \text{spin} \rangle},$$

where $E_0$ is the ground state energy.

Under the assumption that the ground state is given by the Hartree-Fock state for the striped Hall ferromagnet, Eq. (18) is written by using the double commutation as

$$\Delta_{\text{phonon}}(k) = \frac{\langle 0|\rho(-k)|H_{\text{int}}\rho(k)|0 \rangle}{2\langle 0|\rho(-k)\rho(k)|0 \rangle},$$

$$\Delta_{\text{spin}}(k) = \frac{\langle 0|s_+(-k)|H_{\text{int}}s_-(k)|0 \rangle}{\langle 0|s_+(-k)s_-(k)|0 \rangle},$$

and calculated using the following commutation relations

$$[\rho_+(k), \rho_+(k')] = -2i \sin \left(\frac{k \times k'}{4\pi}\right) \rho_+(k + k'),$$

$$[s_+\pm(k), \rho_+(k')] = -2i \sin \left(\frac{k \times k'}{4\pi}\right) s_\pm(k + k'),$$

$$[s_+(k), s_-(k')] = -i \sin \left(\frac{k \times k'}{4\pi}\right) \rho_+(k + k') + 2\cos \left(\frac{k \times k'}{4\pi}\right) s_\pm(k + k'),$$

and $[s_\pm(k), s_\pm(k')] = 0$, where $\rho_+(k) = \rho(k)/f_i(k)$ and $s_\pm(k) = s_i(k)/f_i(k)$.

The results for $\nu = 4 + 1/2$ are shown in Fig. 3. As seen in this figure, the spectrum for the spin wave has a weaker anisotropy than the phonon spectrum. We can see $\Delta_{\text{phonon}}(k_z, 0) = 0$ and the spectrum for the phonon is highly anisotropic [13]. On the other hand, it can be seen that $\Delta_{\text{spin}}(k_z, 0) \propto k_z^2$, $\Delta_{\text{spin}}(0, k_y) \propto k_y^2$, for small $k$.

In summary we have calculated the Hartree-Fock energy of spin and charge striped states at half-filled high Landau level in the zero Zeeman energy limit. We have shown that a ferromagnetic striped Hall state has a lower energy than the antiferromagnetic striped state or charge striped state. Furthermore, excitation spectra for low-lying excitations, phonon and spin wave, have been obtained in the single mode approximation.

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