Numerical Study of Contrast Degradation of BEC Interferometers and Countermeasures

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Abstract. Despite promising features, the BEC-based atomic interferometer needs to overcome momentum recoil caused by the internal atom-atom interaction so as to serve for high precision measurements. We visualize the momentum recoil using the Husimi distribution function and study the BEC’s dynamics in phase space. In so doing, we discuss possible countermeasures for contrast degradation in the manner of ref.[1] as well as hitherto unexpected interesting dynamical features.

1. Introduction
The Bose-Einstein condensates is believed to have promising capacities for precision inertial measurements owing to their high brilliance and high coherence. Proof-of-principle experiments have been indeed demonstrated for BEC-based atom interferometer schemes in recent years\cite{2,3,4,5}. Crucial atom optical elements such as mirrors, beam-splitters, and optical lattice gratings have been also studied as part of coherent matter-wave manipulation of cold atoms (see \cite{6} and references therein for non-condensates). Exploring various applications of the BEC-based atom interferometer is obviously very important. However, the atom-atom interaction among $10^4$ or even a greater number of atoms causes some hitches like degradation of the interference fringe contrast on account of dephasing as well as decoherence in the course of collisions. It is thus also important to explore remedial measures with a view to applications in precision inertial measurement.

The fringe contrast is dictated by both the amplitude and relative phase of the interfering components. Thus, its study motivates a representation of the dynamics in terms of the phase and amplitude in an appropriate phase space. In quantum electronics, three alternative quasi-probability distributions\cite{7}, namely the Glauber-Sudarshan P-function, the Wigner function, and the Husimi Q-function have been studied and applied for analysis. These functions serve as a tool for monitoring the phase and amplitude information, thus also provide insight into the phase space dynamics of the split BEC components, and into the operation of the atom interferometer. Of the three, the positive-definite Husimi function is what we employ for analysis.
in the present paper, namely

\[ P_\alpha(z, p; t) = \frac{1}{2\pi\hbar} \left| \int_{-\infty}^{\infty} e^{-\frac{(z-z_0)^2}{2\hbar^2} - i p q / \hbar} \Psi(q, t) \, dq \right|^2, \]

(1)
defined at an arbitrary instance \( t \) where \( \Psi(q, t) \) represents the solution of the time-dependent Gross-Pitaevskii equation (2) below, and \( \alpha \) sets the phase-space resolution as appropriate for each specific case. We shall describe our theoretical approach, and then discuss the momentum recoil in phase space. We examine two alternative countermeasures, namely momentum-shift and time-delay separately. An interesting quantum feature due to some local oscillations in the colliding BEC components will be mentioned at the end as a preview of the ongoing investigation.

2. Theoretical Approach

As for the equation of motion, we use the Gross-Pitaevskii equation, namely the mean field approximation to numerically study the dynamics of a condensate split by the Bragg diffraction in a 1D Mach-Zehnder-type atom interferometer. It must be thus understood that major features of the coherent components are presumed to be largely retained during the process despite possible inelastic transitions left out here. We have

\[ i\hbar \frac{\partial \Psi(z, t)}{\partial t} = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + f_1(t) \times \frac{1}{2} m \omega_z^2 z^2 + f_2(t) \times \hbar \Omega \cos(k z - \omega t) + g_{1d} N |\Psi|^2 \right) \Psi(z, t), \]

(2)
where \( g_{1d} = -2\hbar^2 / ma_{1d} \) is the effective coupling strength deduced from the 3D s-wave scattering length[8]. Here the energy-momentum conservation reads \( k = k_1 + k_2 \approx 2k \) and the detuning \( \omega = \omega_2 - \omega_1 \). This equation is applied to the specific case of the 1-dimensional \(^{87}\text{Rb} \) atom, presuming a 1D trap and the Bragg diffraction laser field under an actual experimental condition. Here \( f_1(t) \) and \( f_2(t) \) give the time profile of the externally controlled fields. A typical time diagram for the \( \pi/2 - \pi - \pi/2 \) pulse sequence is shown for the case of trap-off in Fig. 1 with the spatial evolution of the split BEC components in parallel. Here the trap is applied for confining the atoms and setting up the initial BEC, but is turned off once the interferometer sequence begins. The signal is picked up by measuring the population of either of the two momentum states \(|0\hbar k \rangle \) or \(|2\hbar k \rangle \) after merging them by the second \( \pi/2 \) pulse where \( \hbar k \) represents the momentum of a single photon. The contrast is the dynamic range of the signal, namely \( \nu = I_{\text{max}} - I_{\text{min}} \) where \( I \) represents the signal amplitude. It should be noted that in other types of interferometer such as in the Horikoshi-Nakagawa experiment, the trap field is exploited for its isochronism and as a mirror \((\pi\text{-pulse})\) in order to reduce contrast degradation.

3. Contrast Degradation and Compensation

3.1. Exceptional Contrast Recovery in the case of Trap-On \((f_1(t) \equiv 1)\)

Let us first show in Fig. 2, the contrast of an \( \text{imperfectly} \) split BEC in the case of trap-on \((f_1(t) \equiv 1)\), “imperfectly” in the sense that the \( \pi \) pulse is applied while the split components still overlap spatially. What happens during this short period is sufficiently rich and proves instructive for our purpose. Let us also note that this corresponds to the actual time span used in earlier proof-of-principle experiments. The parameter of importance here is \( T = T_2 + T_1 = 2T_1 \). This specific case shows an unexpected recovery of contrast for \( N = 8000 \), \( i.e. \) for stronger non-linearity, after the decline to nearly zero. A somewhat weaker rise is seen near the shoulder in the case of \( N = 3000 \) too. Let us examine the phase space dynamics that accounts for this puzzling phenomenon. Fig. 3 shows what happens for \( N = 8000 \) shortly before the third pulse is applied by formally applying the \( \pi/2 \)-pulse unitary matrix to the \(|0\hbar k \rangle \) and \(|2\hbar k \rangle \) components. The corresponding time \( t \) is marked in Fig. 2 by a circle on the local maximum.
of the contrast. Fig. 4 on the other hand corresponds to the local minimum marked by another circle. As a function of $T$, there is spatial shift in the relative position coordinates of the components, allowing them to eventually acquire a larger amplitude overlap in phase space, the trap counterbalancing the recoil (see Fig. 2 for the two values of $N$). This feature of contrast recovery manifests itself as a linear protrusion in Fig. 3 of reference [9]. The relative phase of the two components plays a key role in defining finer structures, a fact to be brought attention to for the trap-on case below, i.e. $f_1(t) \equiv 0$ at $t > 0$.

Figure 1. Position vs time for 1D Mach-Zehnder interferometer (above). Time sequence for the trap $f_1(t)$ and the laser pulse sequence $f_2(t)$ for the trap-off case (below).

3.2. Countermeasures in the case of Trap-Off ($f_1(t) \equiv 0$, $t > 0$)
Compensating for the momentum recoil by shifting the two components in phase space can counteract the contrast degradation to some extent. One way to shift in phase space is to impart momentum difference $\Delta k = \vec{k}_2 - \vec{k}_1$ (see Eq. (2) for $\vec{k}_1$ and $\vec{k}_2$). The time-delay approach is another remedial measure but is somewhat nontrivial for the trap-on case[1] and requires an additional consideration. Here we present the case of trap-off. The $\Delta k$ scheme leads to a considerable improvement as in Fig. 5, and the $\Delta T$ scheme to some improvement as in Fig. 6. In principle, one may apply the $\Delta T$ and $\Delta k$ schemes in combination to achieve an even better amplitude overlap of the BEC components.

Figure 2. Contrast vs $T$ for an imperfectly split initial stage with trap on. Strong contrast recovery for $N = 8000$ and a small hump around the shoulder for $N = 3000$. Here $T = T_1 + T_2 = 2T_1$.

Figure 3. Husimi distribution shown with contours near local maximum at $T \simeq 0.4$ marked by circle in Fig. 2. Observe relatively large overlap.

Figure 4. Husimi distribution shown with contours near local minimum at $T \simeq 0.2$ marked by circle in Fig. 2. Observe relatively small overlap.
Let us now consider the relative phase between the split components. Treating once again the time period where the BEC is imperfectly split, we map the contrast using color coding in Fig. 7 as a function of $T$ and $N$, the latter being the number of particles. Consider a section at some fixed $T$. Then as a function of $N$, we notice the contrast declines rapidly due to the increase in momentum recoil. However, there appear residual oscillations in contrast, namely an interference fringe with respect to $N$. The Husimi distribution function (not shown) indicates that each time the contrast attains a local minimum, there emerges a unit of circulation in phase space resembling a vortex in a rotating 2D BEC. The local circulations pertain to local oscillations in coordinate space so that an increase in vorticity by unity corresponds to a creation of vibrational quantum. A further investigation of this phenomenon is currently under way.

4. Conclusions
We studied the effect of momentum recoil of the BEC components that takes place at the moment of splitting as well as merging. The contrast degradation caused by such recoil can be remedied at least partially by the time-delay and/or the momentum shift scheme(s). BEC-based atomic interferometers are still at their infancy in the sense that they have not been applied for high precision measurements in real systems. There are many technical issues to be cleared both theoretically and experimentally.

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