A nonlinear mechanism of charge qubit decoherence in a lossy cavity: the quasi normal mode approach

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In the viewpoint of quasi normal modes, we describe a novel decoherence mechanism of charge qubit of Josephson Junctions (JJ) in a lossy micro-cavity, which can appear in the realistic experiment for quantum computation based on JJ qubit. We show that the nonlinear coupling of a charge qubit to quantum cavity field can result in an additional dissipation of resonant mode due to its effective interaction between those non-resonant modes and a resonant mode, which is induced by the charge qubit itself. We calculate the characterized time of the novel decoherence by making use of the system plus bath method.

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I. INTRODUCTION

The superposition principle is most basically governing quantum world. It is also the foundation of quantum information technology. The ideal coherent superposition state can only be preserved in quantum world isolated from external influences. However, the influence of the surrounding environment can never be blocked off completely. Now the coherence is an essential requirement for quantum information and the decoherence will result in errors to reduce the power of quantum computation and quantum communication or even destroys it. The quantum decoherence has become the biggest obstacle to its implementation. In practice, we need a qubit with long decoherence time and a longer lifetime medium to implement quantum logic gate. To overcome quantum decoherence, one should know its dynamic details theoretically and experimentally in various physical systems including all kinds of qubits.

Recently in solid state based quantum computation, Josephson Junction (JJ) qubits: charge qubit, flux qubit or their hybridizations, demonstrate their large potential as a candidate of scalable quantum computation. On the one hand, the Rabi oscillation in a Cooper-pair box (charge qubit) and the entanglement between a flux qubit and a superconducting quantum interference device (SQUID) are realized experimentally. Up to now, the decoherence time of JJ qubit has been the order of 5μs. On the other hand, to implement quantum computation, one should integrate many qubits to form a quantum coherent network. To this end, a longer lifetime medium is required to transfer quantum information among these qubits in the network. Some investigations have shown that quantized field in microwave cavity might be a good candidate, whose lifetime is of order 1ms. For this purpose, the integration of JJ qubit and cavity QED has become a focus in exploring the JJ qubit based quantum computing.

In spite of these exciting advances, the relatively short coherent time is still a problem in implementing the JJ based quantum computing in large scale and the mechanism of decoherence of JJ qubit is not very clear, especially in the presence of external field. The fluctuation of background charge is a well-known source of quantum decoherence, but it is not the unique one. For a real dc-SQUID, the fluctuations of gate voltage and magnetic flux produced by the screening current circulating around the dc-SQUID may also cause additional quantum decoherence in the charge qubit-cavity system. To bring out more clearly the physical mechanism of nonlinear decoherence described in this paper, we can avoid the effect of screening current in the physical case of the dc-SQUID. In this treatment, cavity modes in a lossy cavity are divided into a single resonant mode and other non-resonant multi-modes. Due to the effective interaction between those non-resonant modes and the resonant one causes an additional dissipation of the resonant mode. This is just the novel mechanism of quantum decoherence for the charge qubits in a lossy cavity.

The paper is organized in the following sections. With the quasi normal mode approach, the model in Sec. II describes a charge qubit interacting with a lossy cavity. In Sec. III, we demonstrate how the nonlinear coupling...
of a charge qubit to cavity field induces the effective interaction between resonant mode and non-resonant ones. In Sec. IV, we find that the nonlinear coupling leads to energy dissipation of resonant mode of cavity field which is linked to quantum decoherence of a charge qubit in a lossy cavity in Sec. V.

II. MODEL: CHARGE QUBIT COUPLED TO THE LOSSY CAVITY

In this paper, we consider a single mode quantum field in a lossy cavity with frequency $\omega \sim 300GHz$ (typically in the microwave domain) and quality factor $Q \sim 10^6$ coupled to a charge qubit, in which the charging energy $E_C \sim 122\mu eV$ and the Josephson coupling energy $E_J \sim 34\mu eV$ [14]. The charge qubit considered in this paper is a dc-SQUID consisting of two identical Josephson junctions enclosed by a superconducting loop. It is located in a lossy cavity which is produced by a semitransparent mirror. The similar case is discussed for quantum dissipation of semiconductor exciton in a lossy cavity [16]. In this paper, we can describe the magnetic field in a lossy cavity with the quasi mode approach [17, 18, 19, 20]. And we can divide cavity modes into two sets of normal modes with different frequencies. Similarly to the central frequency of the resonant mode of cavity $\omega = 0$. Quasi-spin operators corresponding to single resonant mode of frequency $\omega$ and non-resonant modes of frequencies $\omega_j, V (V_j)$ the electromagnetic mode volume corresponding to resonant mode and non-resonant mode respectively and $L$ the length of the cavity. The constant $M_j$ in Eq.(1) is proportional to a Lorentzian [19]

$$M_j = \frac{\lambda^2}{\sqrt{\left(\omega_j - \omega\right)^2 + \left(\omega\right)^2}}$$

where $\gamma$ is the decay rate of a quasi mode of cavity, $\lambda$ the bandwidth associated with the cavity wall transparency and $\omega$ the central frequency of the resonant mode of cavity. It is obvious that $M_j$ will acquire the maximum value when the frequency of non-resonant mode $\omega_j$ is very close to the central frequency of the resonant mode $\omega$. In our investigation, the lossy cavity contained only one quasi mode.

The Hamiltonian for a charge qubit (dc-SQUID) can be written as in Ref. [21],

$$H = 4E_C \left(n_g - \frac{1}{2}\right) \sigma_z - E_J \cos \left(\frac{\Phi}{\Phi_0}\right) \sigma_x.$$ (3)

Where $E_C$ is the charging energy and $E_J$ the Josephson coupling energy, $E_C >> E_J$ for a charge qubit. $\Phi$ is the magnetic flux generated by controlled classical magnetic field and quantum cavity field and $\Phi_0 = h/2e$ the flux quanta. As a control parameter, the dimensionless gate charge $n_g = C_g V_g/2e$ is determined by the gate voltage $V_g$ applied on the gate capacitance $C_g$. Quasi-spin operators

$$\sigma_z = |0\rangle_q \langle 0|_q - |1\rangle_q \langle 1|_q, \quad \sigma_x = |0\rangle_q \langle 1|_q + |1\rangle_q \langle 0|_q$$

FIG. 1: Schematic of the charge qubit-cavity system. Superconducting microwave cavity with parameter, $R = 2.55mm$, $L = 0.5cm$.
are defined in the charge qubit basis ($\ket{0}_q$ and $\ket{1}_q$).

In Fig. the dc-SQUID is located at the position of the antinode of standing wave field in cavity, i.e., $z = L/2$. Then corresponding to the field decomposition $B_y = B_r + B_{nr}$, the magnetic flux $\Phi_y = \Phi_r + \Phi_{nr}$ enclosed by the dc-SQUID is explicitly given by

$$\Phi_r = -iS\left(\frac{\hbar \omega}{\varepsilon_0 V_c^2}\right)^{\frac{1}{2}}(a - a^\dagger),$$

$$\Phi_{nr} = -iS \sum_j M_j \left(\frac{\hbar \omega_j}{\varepsilon_0 V_{c}^2}\right)^{\frac{1}{2}} \sin \left(-\frac{\omega_j L}{c/2}\right) \left(a_j - a_j^\dagger\right).$$

In a straightforward way, we derive the Hamiltonian of the qubit-cavity system from Eq. (3),

$$H = 4E_C \left(n_q - \frac{1}{2}\right) \sigma_z - E_J \cos(\phi_x + \phi_q) \sigma_x + \hbar \omega a^\dagger a + \sum_j \hbar \omega_j a_j^\dagger a_j. \quad (4)$$

Where $\phi_x$ and $\phi_q$ are phase and “phase operator” generated by the flux $\Phi_x$ and $\Phi_q$ respectively,

$$\phi_x = \frac{\pi \Phi_x}{\Phi_0},$$

$$\phi_q = -i \phi_0 (a - a^\dagger) - i \sum_j \phi_j \left(a_j - a_j^\dagger\right)$$

and

$$\phi_0 = \frac{\pi S}{\Phi_0} \left(\frac{\hbar \omega}{\varepsilon_0 V_c^2}\right)^{\frac{1}{2}},$$

$$\phi_j = M_j \frac{\pi S}{\Phi_0} \left(\frac{\hbar \omega_j}{\varepsilon_0 V_{c}^2}\right)^{\frac{1}{2}} \sin \left(-\frac{\omega_j L}{c/2}\right).$$

In the above discussions to achieve the simplified model we have ignored the effect of the screening current. However, if we are interested in the effect of the screening current, the inductance of the loop of dc-SQUID is not zero, the screening current will induce the additional decoherence. In presence of the screening current, we can not neglect the difference between the practical magnetic flux $\Phi$ threading the dc-SQUID and the external magnetic flux $\Phi_x$. It can be determined by the following equation

$$\phi = \phi_x + \frac{\pi}{2} \beta_m \sin \phi$$

where $\beta_m = 2LL_c/\Phi_0$ is usually called screening parameter and $\phi = \pi \Phi/\Phi_0$ and $\phi_x = \pi \Phi_x/\Phi_0$. This equation simply shows that the total flux $\Phi$ is the sum of the external flux and the induced flux determined by $\Phi_x$. If the screening parameter $\beta_m$ is small enough, we can approximately solve this equation in a single value domain with the technique of perturbation recursion up to the second order

$$\phi = \phi_x + \frac{\pi}{2} \beta_m \sin \phi_x + \left(\frac{\pi}{2} \beta_m\right)^2 \sin \phi_x \cos \phi_x. \quad (5)$$

When $\beta_m \pi/2 << 1$, i.e., $\beta_m << 2/\pi$, we can ignore the effect of the screening current. But the additional nonlinear terms containing $(\phi_x)^2$ at least should also induce the additional nonlinear interaction between the resonant mode and the non-resonant ones. This is the further result of the decoherence of charge qubit in the lossy cavity. However, to clearly demonstrate the physics of the central results of our paper, we suppose $\beta_m << 2/\pi$ and ignore the effect of the screening current. Therefore we do not give much details for this problem.

### III. MODE INTERACTION INDUCED BY NONLINEAR COUPLING TO CAVITY

As shown in Fig. two spherical mirrors form microwave cavity containing a single mode standing wave field. And an external classical magnetic field is also injected into the cavity. In this paper, the geometry of cavity is described by the parameters: the curvature radius $R = 2.55$mm, the width between two mirrors $L = 0.5$cm. By some straightforward calculations, we get that the cavity field $B = (\hbar \omega / \varepsilon_0 V_c^2)^2 = 7.52 \times 10^{-11}$ (Tesla) and $\phi_0 = \pi \Phi_0 / \Phi_0 = 1.14 \times 10^{-5}$. In a low photon number cavity, we find that $\phi_q << \phi_c$, thus there is only a weak polynomial nonlinearity in Eq. (4).

To simplify the Hamiltonian in Eq. (3), we expand $\cos(\phi_x + \phi_q)$ in terms of small quantity $\phi_q$ up to the second order,

$$\cos(\phi_x + \phi_q) = \left(1 - \frac{1}{2} \phi_q^2\right) \cos \phi_x - \phi_q \sin \phi_x. \quad (6)$$

Obviously we can know that the second order term $\phi_q^2 = (\phi_x + \phi_{nr})^2$ includes the term $\phi_x \phi_{nr}$ which results in the nonlinear coupling between resonant mode and non-resonant modes of cavity field, on which the results of this paper is based. The first order term $\phi_q$ is linearly dependent of $\phi_x$ and $\phi_{nr}$, which can not lead to the coupling between resonant mode and non-resonant ones. Therefore the nonlinear coupling in terms of $\phi_q^2 \sim \phi_x \phi_{nr}$ will induce energy dissipation and quantum decoherence of the charge qubit in a lossy cavity simultaneously.

To clearly demonstrate the effect of quantum decoherence of a charge qubit in a lossy cavity we tune the gate voltage $V_g$ such that $n_q = 1/2$ to eliminate the effect of background charge fluctuation up to the linear order. Then the effective Hamiltonian corresponding to a standard quantum measurement model reads

$$H = H^{(0)} \langle 0 \rangle \langle 0 \rangle + H^{(1)} \langle 1 \rangle \langle 1 \rangle, \quad (7)$$

which is diagonal with respect to eigenstates of quasi spin operator $\sigma_z$, $\ket{0} = \ket{0}_q + \ket{1}_q$ and $\ket{1} = \ket{0}_q - \ket{1}_q$.

As seen in Eq. (7), the second order term $\phi_q^2 \sim \phi_x \phi_{nr}$ results in the interaction $\sim \left(a_j - a_j^\dagger\right) \left(a - a^\dagger\right)$ between single resonant mode and other non-resonant ones, while the first order term $\phi_q$ results in the forced terms $\left(a - a^\dagger\right)$.
and \((a_j - a_j^\dagger)\) in the above Hamiltonian. With the rotating wave approximation (RWA), we can drop down the terms of \(a^2 (a_j^2)\) and \(a_j a (a_j^\dagger a_j^\dagger)\) in \(\cos(\phi_c + \phi_q)\) and get an effective Hamiltonian
\[
H^{(k)} = H_s^{(k)} + H_I^{(k)} + H_B^{(k)} + N^{(k)}
\]
where
\[
H_s^{(k)} = \hbar \Omega^{(k)} a_j^\dagger a_j - i \xi^{(k)} (a_j - a_j^\dagger),
\]
\[
H_I^{(k)} = \sum_j g_j^{(k)} \left( a_j a_j^\dagger + a_j^\dagger a_j \right),
\]
\[
H_B^{(k)} = \sum_j \hbar \omega_j a_j^\dagger a_j - i \sum_j \xi_j^{(k)} (a_j - a_j^\dagger)
\]
and the parameters in the above equation can be explicitly expressed as
\[
N^{(k)} = \frac{(-1)^k}{\hbar} \left( \phi_0^2 E_j \cos \phi_c - E_j \cos \phi_c \right),
\]
\[
\Omega^{(k)} = \omega + \frac{(-1)^k}{\hbar} \phi_0^2 E_j \cos \phi_c,
\]
\[
g_j^{(k)} = \frac{(-1)^k}{\hbar} \phi_0 \phi_0 E_j \cos \phi_c,
\]
\[
\xi^{(k)} = \frac{(-1)^k}{\hbar} \phi_0 \phi_0 E_j \sin \phi_c,
\]
\[
\xi_j^{(k)} = \frac{(-1)^k}{\hbar} \phi_0 \phi_0 E_j \sin \phi_c
\]
for \(k = 0, 1\). Here \(H_s^{(k)}\) describes a system with a forced oscillator of frequency \(\Omega^{(k)}\), \(H_B^{(k)}\) describes a bath of many forced oscillators of frequency \(\omega_j s\), and \(H_I^{(k)}\) describes the coupling of the resonant mode to non-resonant modes. The coupling constant \(g_j^{(k)}\) owns a Lorentz type factor, i.e.,
\[
g_j^{(k)} \sim \frac{1}{\sqrt{\left(\omega_j - \omega\right)^2 + \left(\frac{\omega}{2}\right)^2}}.
\]
It means that the resonant mode of cavity field dominates the strength of the interaction mostly.

**IV. QUANTUM DISSIPATION OF RESONANT MODE OF CAVITY FIELD**

In this section, we study quantum dissipation of the resonant mode of cavity field. In each component of the Hamiltonian in Eq. [5], \(H^{(k)}\) can result in quantum dissipation of the resonant mode. To solve dynamic equation governed by the effective Hamiltonian \(H^{(k)}\), we rewrite the above Hamiltonian into the new form
\[
H^{(k)} = \hbar \Omega^{(k)} b_j^\dagger b_j + \sum_j \hbar \omega_j b_j^\dagger b_j + \sum_j g_j^{(k)} \left( b_j^\dagger b_j + b_j b_j^\dagger \right) + \varphi_k
\]
by defining a new set of bosonic operators \(b (b_j^\dagger)\) and \(b_j (b_j^\dagger)\), which are the displacements of operators \(a\) and \(a_j\),
\[
b = a + \lambda,
\]
\[
b_j = a_j + \lambda_j.
\]
Here, \(\varphi_k\) is the constant, and \(\lambda\) and \(\lambda_j\) are dependent of the forced terms \((a - a^\dagger)\) and \((a - a^\dagger)\) of the effective Hamiltonian \(H^{(k)}\) in Eq. [8].

For any coherent state \(|\alpha\rangle_a|\alpha\rangle_{a_j}\) defined with respect to annihilation operators \(a\) and \(a_j\), we get coherent state \(|\alpha\rangle_b|\alpha\rangle_{a_j}\) defined with respect to annihilation operators \(b\) and \(b_j\),
\[
|\alpha\rangle_b = e^{-\lambda a^\dagger} |\alpha - \lambda\rangle_a,
\]
\[
|\alpha\rangle_{a_j} = e^{-\lambda a_j^\dagger} |\alpha - \lambda_j\rangle_{a_j}.
\]

Obviously we can see that the effective Hamiltonian \(H^{(k)}\) in Eq. [10] describes a typical dissipative system of a single-mode boson soaked in a bath of many bosons (we have studied its wave function structure in details [23]). The well-known solutions of Heisenberg equations for the Hamiltonian \(H^{(k)}\) is given in Ref. [23],
\[
b^{(k)} (t) = u^{(k)} (t) b_j (0) + \sum_j v_j^{(k)} (t) b_j (0),
\]
\[
b_j^{(k)} (t) = e^{-i\omega_j t} b_j (0) + u_j^{(k)} (t) b_j (0) + \sum_s v_{j,s}^{(k)} (t) b_j (0).
\]
Where \(b^{(k)} (t)\) and \(b_j^{(k)} (t)\) represent the time evolution of operators \(b\) and \(b_j\). And we also get the solutions of Heisenberg equations for \(a^{(k)} (t)\) and \(a_j^{(k)} (t)\) representing the time evolution of operator \(a\) and \(a_j\) driven by the Hamiltonian \(H^{(k)}\) as
\[
a^{(k)} (t) = b^{(k)} (t) - \lambda,
\]
\[
a_j^{(k)} (t) = b_j^{(k)} (t) - \lambda_j,
\]
where
\[
u^{(k)} (t) = - g_j^{(k)} e^{-i(\Omega^{(k)} + \Delta \Omega^{(k)}) t},
\]
\[
u_{j}^{(k)} (t) = - g_j^{(k)} e^{-i\omega_j t} \left( 1 - e^{-i(\Omega^{(k)} + \Delta \Omega^{(k)}) - \omega_j} t e^{-i\Omega^{(k)} t} \right),
\]
\[
u_{j,s}^{(k)} (t) = - g_{j,s}^{(k)} e^{-i\omega_j t} \left( 1 - e^{-i(\Omega^{(k)} + \Delta \Omega^{(k)} - \omega_j)} \lambda, e^{-i\Omega^{(k)} t} + \Lambda \right),
\]
\[
\Lambda = t, \text{ when } s = j,
\]
\[
\Lambda = \frac{e^{-i(\omega_j - \omega_j) t} - 1}{\omega_s - \omega_j}, \text{ when } s \neq j.
\]
where $\Delta \Omega^{(k)}$ is frequency shift of $\Omega^{(k)}$ corresponding to two different Hamiltonians $H^{(k)}$. In general, $\Delta \Omega^{(k)}$ can be absorbed into $\Omega^{(k)}$, i.e., $\Omega^{(k)} \sim \Omega^{(k)} + \Delta \Omega^{(k)}$. If the forced terms of the Hamiltonian in Eq. (8) are absent, we will get that $a^{(k)}(t) = b^{(k)}(t)$ and $a^{(k)}_j(t) = b^{(k)}_j(t)$.

Here we calculate the time evolution of the mean photon number of the resonant mode of cavity field corresponding to two different Hamiltonians $H^{(k)}$. When we assume that the initial state of all modes of cavity field is prepared in Fock state $|n, \{n_j\}_a = |n\rangle_a \otimes |\{n_j\}_a$, the mean photon number of resonant mode corresponding to $H_k$ is calculated as

$$n^{(k)}(t) = \langle a^{(k)\dagger}(t) a^{(k)}(t) \rangle = e^{-\gamma t} n + F(\lambda, \{\lambda_j\}, \{n_j\}, t)$$

(15)

where the time dependent constant $F(\lambda, \lambda_j, n_j, t)$ is dependent of $\lambda, \lambda_j$ and $n_j$ and the time dependent term $a \exp (-\gamma t)$ characterizing the quantum dissipation induced by non-resonant modes. Thus we can know that the time evolution of mean number of the resonant mode $n^{(k)}(t)$ is the sum of two parts: (1) quantum dissipation $a \exp (-\gamma t)$ induced by non-resonant modes; (2) the constant $F(\lambda, \{\lambda_j\}, \{n_j\}, t)$ generated by the first order term $\phi_q$ and mean number of non-resonant modes $n_j$.

Through some simple calculations, we find that the constant $F(\lambda, \{\lambda_j\}, \{n_j\}, t)$ will approach zero when the forced terms vanish and the initial state of the bath (non-resonant modes) in vacuum state $|\{n_j\} = |\{0\}_j\rangle$. Then we get the same results of Ref. 24 that the time evolution of mean photon number of resonant mode is

$$n^{(k)}(t) = n e^{-\gamma t}.$$

(16)

It means that vacuum fluctuation of non-resonant modes leads to quantum dissipation of resonant mode when the forced terms of operators $a$ and $a_j$ are absent. In the other words, the nonlinear coupling directly causes quantum dissipation of the resonant mode in a lossy cavity.

In contrast to the model of single boson interacting with a bath of many bosons, the constant $F(\lambda, \lambda_j, n_j, t)$ provides the different effect that the mean number of resonant mode of cavity field doesn’t approach zero when time $t \rightarrow \infty$.

V. DECOHERENCE INDUCED BY THE DISSIPATION OF RESONANT MODE

Technically the process of quantum decoherence is described by the time evolution of the reduced density matrix of the coupled qubit-cavity system. To analyze it, we can calculate reduced density matrix for the time evolution of the charge qubit. The pure decoherence process means that the off diagonal elements of the reduced density matrix of the qubit vanish, while the diagonal elements remain unchanged in an ideal case.

Now if the initial state of cavity field is in coherent state, i.e., the resonant mode of cavity field is in a coherent state $|\alpha\rangle$ and other non-resonant modes of cavity field in coherent state $|\{\alpha_j\}_a\rangle$, the initial state of the total qubit-cavity system can be written as

$$|\Psi(0)\rangle = (C_0 |0\rangle + C_1 |1\rangle) \otimes |\alpha, \{\alpha_j\}_a\rangle.$$

Then we can easily get the time evolution of the wave function for the qubit-cavity system

$$|\Psi(t)\rangle = U(t) |\Psi(0)\rangle = C_0 |0\rangle \otimes |\varphi^{(0)}(t)\rangle + C_1 |1\rangle \otimes |\varphi^{(1)}(t)\rangle (17)$$

where $U^{(k)}(t) = \exp (-i H^{(k)} t)$ are evolution operators for the effective Hamiltonians $H^{(k)}$ and

$$|\varphi^{(k)}(t)\rangle = U^{(k)}(t) |\alpha, \{\alpha_j\}_a\rangle.$$

(18)

Then we can obtain time evolution of density matrix for the qubit-cavity system

$$\rho(t) = |\psi(t)\rangle \langle \psi(t)|$$

(19)

and calculate the reduced density matrix of the qubit

$$\rho_s(t) = C^*_0 C_0 |0\rangle \otimes |\varphi^{(0)}(t)\rangle + C^*_1 C_1 |1\rangle \otimes |\varphi^{(1)}(t)\rangle + h.c.$$

(20)

As a measure of the coherence of quantum system, the decoherence factor of charge qubit can be calculated as

$$D(t) = \left| \left\langle \varphi^{(1)}(t) | \varphi^{(0)}(t) \right\rangle \right|.$$  (21)

For any coherent state, we have

$$|\alpha\rangle_b = \exp \left[ a^\dagger b(0) - a^* b(0) \right] |0\rangle_b.$$

Obviously we can see that

$$U^{(k)}(t) |0, \{0\}_j\rangle_b = 0.$$

Formally, we can define

$$O^{(k)}(t) = U^{(k)}(t) O(0) U^{(k)\dagger}(t)$$

for any operator $O^{(k)}$ corresponding to $H^{(k)}$ and get the time dependent equation

$$\frac{dO^{(k)}(t)}{dt} = i \left[ O^{(k)}(t), H_k \right].$$

(22)

So we have $A^{(k)}(t) = U^{(k)}(t) b(0) U^{(k)\dagger}(t)$ and $A^{(k)}_j(t) = U^{(k)}(t) b_j(0) U^{(k)\dagger}(t).$ By substituting $-i$ into Heisenberg equation with $i$, we can get the solutions of Eq. (22)

$$A^{(k)}(t) = u^{(k)*}_j(t) b_j(0) + \sum_j v^{(k)*}_j(t) b_j(0),$$

$$A^{(k)}_j(t) = e^{i\omega_j t} b_j(0) + u^{(k)*}_j(t) b_j(0) + \sum s v^{(k)*}_{j,s}(t) b_s(0).$$
To demonstrate the effect of quantum dissipation of resonant mode induced by non-resonant modes on decoherence of charge qubit, here we do not consider the effect of the forced terms and set $\lambda = 0$ and $\lambda_j = 0$.

In the above section, we have known that the vacuum fluctuation induced by the non-resonant modes can result in quantum dissipation of resonant mode. So we assume that the initial state of non-resonant modes is in vacuum state $|\{0\}_b\rangle = |\{0\}_a\rangle$ and initial state of resonant mode coherent state $|\alpha\rangle_b = |\alpha\rangle_a$. And time evolution of decoherence factor of the charge qubit is

$$D(t) = e^{-\frac{\langle|\alpha|^2\rangle}{2} \left( |u^{(1)}(t)|^2 + \sum_j |v^{(1)}_j(t)|^2 \right)}.$$ (23)

**FIG. 2:** Decoherence factor $D(t)$ in Eq. (23) as the function of the time $t$ with the value of $\alpha = 2$.

With commutation relation $[A^{(k)}(t), A^{(k)*}(t)] = 1$, we have

$$\left|u^{(k)}(t)\right|^2 + \sum_j \left|v^{(k)}_j(t)\right|^2 = 1.$$ (24)

According to Eq. (18), we have

$$|\varphi_k(t)\rangle = |\alpha u^{(k)}(t)\rangle_b \otimes \Pi_j |\alpha v^{(k)}_j(t)\rangle_{b_j}.\tag{25}$$

And the decoherence factor in Eq. (23) becomes

$$D(t) = e^{-\left|\alpha\right|^2 \left( |u^{(1)}(t)|^2 + \sum_j |v^{(1)}_j(t)|^2 \right)} \times e^{\frac{\left|\alpha\right|^2}{2} \sum_j (v^{(1)*}_j(t)v^{(0)}_j(t) + u^{(1)*}_j(t)u^{(0)}_j(t))}.$$ (26)

From the above results in Eq. (19) and Eq. (15), we know that the term $v^{(1)*}_j(t)v^{(0)}_j(t)$ is proportional to $\phi_0^j \phi_0^j \sim \phi_0^j$ which is a 4th order term of nonlinear expansion in Eq. (19). Then we can omit it in the calculation of the decoherence factor and get

$$D(t) = e^{-\left|\alpha\right|^2 \left(1 - e^{-\gamma t} \cos[(\Delta \Omega(1) + \Delta \Omega(0) - \Delta \Omega(0))t]\right)}.$$ (27)

**VI. CONCLUSIONS**

Before concluding this paper, we would like to remark the influences of the fluctuations of gate charge $n_0$ around 1/2. We notice that the classical fluctuation of gate voltage is not the unique source of decoherence. The most recent investigations have demonstrated that 1/f noise due to the background charge fluctuation, which also plays an important role in the decoherence of charge qubit [14, 26, 27]. Maybe there also exists some unknown source of decoherence. For simplicity we only deal with the decoherence of charge qubit by considering that such fluctuations can indeed occur in a real dc-SQUID, but it may be ignored so as to bring out more clearly the novel mechanism. Our investigation only emphasizes the role that nonlinear coupling plays in the decoherence of charge qubit in some case.

In this paper, we have discovered the novel phenomenon of quantum decoherence of charge qubit in a lossy cavity, where we adopt the quasi mode approach to deal with a lossy cavity. We find that the nonlinear coupling between the charge qubit and the cavity field can induce the interaction between resonant mode and non-resonant modes of cavity field. Based on this observation, we achieve a nice model for this novel decoherence mechanism that a forced oscillator (resonant mode) interacts with a bath of many forced oscillators (non-resonant modes). The decoherence factor is calculated to demonstrate an oscillating decay of quantum coherence of a charge qubit in the lossy cavity. In addition, we have shown that vacuum fluctuation provided by these non-resonant modes can cause the quantum dissipation of resonant mode. Consequently, the quantum dissipation of resonant mode directly results in quantum decoherence of charge qubit. These analysis describe the novel source of quantum decoherence for a charge qubit in the lossy cavity.

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