Odd-frequency Cooper pairs near the surfaces of superfluid $^3$He-B

S Higashitani$^1$, S Matsuo$^1$, Y Nagato$^2$ and K Nagai$^1$

$^1$ Graduate School of Integrated Arts and Sciences, Hiroshima University, Kagamiyama 1-7-1, Higashi-Hiroshima 739-8521, Japan
$^2$ Information Media Center, Hiroshima University, Kagamiyama 1-4-2, Higashi-Hiroshima 739-8511, Japan
E-mail: seiji@minerva.ias.hiroshima-u.ac.jp

Abstract. We discuss the odd-frequency pair amplitudes generated near the surfaces of superfluid $^3$He-B using the quasiclassical Green’s function. We demonstrate that even-frequency and odd-frequency pairs coexist near the surfaces of superfluid $^3$He-B. In addition, we show that the odd-frequency pair amplitude has quite similar energy dependence to the surface density of states, which has characteristic low-energy structure reflecting the formation of the Andreev bound states. In the low energy limit the both coincide exactly with each other.

1. Introduction

It is well established that the Cooper pairs with spin-triplet $p$-wave symmetry are formed in superfluid $^3$He. One of the remarkable characteristics of the anisotropic superfluid is that the order parameters and the quasiparticle states are strongly modified near the surfaces [1–3]. The anisotropic order parameters vary in space over several coherence lengths from the surfaces. The Andreev bound states are generated near the surfaces, causing a characteristic low energy structure in the surface density of states below the bulk gap. Such low energy excitations in the $p$-wave superfluid have been observed clearly by transverse acoustic impedance experiments on superfluid $^3$He-B [4–7]. Similar surface effects are expected also in unconventional superconductors.

We discuss here the surface effect in superfluid $^3$He-B from the aspect of the formation of the odd-frequency Cooper pairs. It has been pointed out in recent years that the odd-frequency pairs arise as a consequence of spatial inhomogeneity, irrespective of the presence of the pairing interaction responsible for the odd-frequency pairing. A variety of inhomogeneous superconducting and superfluid systems has been discussed in this context [8–13]. An intriguing example is a dirty normal metal/spin-triplet superconductor junction. In this system, a novel type of the proximity effect occurs [10]: the proximity-induced superconductivity in the dirty normal layer is dominated by the odd-frequency spin-triplet $s$-wave pairs. There can in general coexist even-frequency and odd-frequency pairs near the interface owing to broken translational symmetry. This admixture of the Cooper pairs is the origin of the anomalous proximity effect [13]. We shall show that the odd-frequency pairs with various orbital symmetries appear near the surfaces of superfluid $^3$He-B. We mention also on a close relation between the odd-frequency pairs and the Andreev bound states.
2. Quasiclassical theory

We use the quasiclassical Green’s function to study the surface effect in superfluid $^3$He. We consider the B phase of superfluid $^3$He occupying half-infinite space $z > 0$ bounded by a flat surface at $z = 0$. The surface may be rough enough to scatter the $^3$He-quasiparticles diffusively. To incorporate the diffusive scattering effect into the quasiclassical theory, we employ the random scattering matrix model [3, 14] and parameterize the boundary condition using a specularity parameter $S$ ($0 \leq S \leq 1$; $S = 1$ corresponds to the specular surface and $S \to 0$ to the diffusive limit where the $^3$He-quasiparticles are scattered isotropically).

The quasiclassical Green’s function $\hat{g}$ is a $4 \times 4$ matrix in the Nambu space and has the following matrix structure:

$$
\hat{g} = \begin{pmatrix}
  g(\hat{p}, \epsilon, z) & f(\hat{p}, \epsilon, z) \\
  f(-\hat{p}, -\epsilon^*, z)^* & g(-\hat{p}, -\epsilon^*, z)^*
\end{pmatrix},
$$

(1)

$$
g(\hat{p}, \epsilon, z) = g(\hat{p}, \epsilon^*, z)^T,
$$

(2)

$$
f(\hat{p}, \epsilon, z) = -f(-\hat{p}, -\epsilon, z)^T,
$$

(3)

where $g$ and $f$ are $2 \times 2$ matrices in spin space, $\hat{p}$ is a unit vector specifying the direction of the Fermi momentum, and $\epsilon$ is a complex energy variable. The submatrix $g$ carries information on the angle-resolved local density of states,

$$
n(\hat{p}, E, z) = \text{Im} \left[ \frac{1}{2} \text{Tr} g(\hat{p}, E + i0^+, z) \right],
$$

(4)

while $f$ corresponds to the pair amplitude defined as a $2 \times 2$ spin-space matrix. One can decompose $f$ into the spin-singlet and spin-triplet parts, $f = (f_0 + f \cdot \sigma) i\sigma_2$. In the system under consideration, the spin-singlet pair amplitude $f_0$ vanishes and we have only to consider the spin-triplet one $f$.

The spatial dependence of $\hat{g}$ is governed by the Eilenberger equation

$$
iv_F \hat{p}_z \partial_z \hat{g} = \begin{pmatrix}
  \epsilon & \Delta(\hat{p}, z) \\
  -\Delta(\hat{p}, z)^T & -\epsilon
\end{pmatrix},
$$

(5)

where $v_F$ is the Fermi velocity and $\Delta(\hat{p}, z) = d(\hat{p}, z) \cdot \sigma i\sigma_2$ is a $2 \times 2$ matrix of the spin-triplet $p$-wave gap function. The $d$-vector in the half-infinite superfluid $^3$He-B has the form

$$
d(\hat{p}, z) = (\Delta_0(z)\hat{p}_x, \Delta_0(z)\hat{p}_y, \Delta_1(z)\hat{p}_z).
$$

(6)
Here $\Delta_0(z)$ and $\Delta_1(z)$ are spatially dependent gap functions, both of which tend for $z \to \infty$ asymptotically to a constant $\Delta_B$, the bulk gap of superfluid $^3$He-B. In fig. 1 we show the spatial dependence of the self-consistent gap functions $\Delta_{0,1}(z)$ calculated by using the random scattering matrix model [3, 14] for the boundary condition at the surface.

3. Surface odd-frequency pairs
The pair amplitude $f$ has in general even-frequency and odd-frequency components,

$$f_{\text{EF}} = \frac{1}{2} \left[ f(\hat{p}, \epsilon, z) \pm f(\hat{p}, -\epsilon, z) \right]$$

$$= \frac{1}{2} \left[ f(\hat{p}, \epsilon, z) \mp f(-\hat{p}, \epsilon, z) \right].$$

The second equality follows from eq. (3). The self-consistent solution of $f$ in the present system can be written as

$$f(\hat{p}, \epsilon, z) = (f_{||}(\hat{p}_z, \epsilon, z) \cos \phi, f_{||}(\hat{p}_z, \epsilon, z) \sin \phi, f_{\perp}(\hat{p}_z, \epsilon, z)),$$

where $\phi = \arctan(\hat{p}_y/\hat{p}_x)$ is the azimuthal angle of $\hat{p}$.

Although the gap function is purely $p$-wave, the pair amplitudes with various orbital symmetries appear near the surfaces of superfluid $^3$He-B. To demonstrate that, we plot in fig. 2 the magnitude of the several partial-wave components of the retarded pair amplitude $f_{\perp}(\hat{p}_z, E + i0^+, z)$ at $E = 0.5\Delta_B$ as a function of $z/\xi_0$, where $\xi_0 = v_F/2\pi T_c$ is the coherence length. The even (odd) partial-wave components of $f_{\perp}$ have the odd (even)-frequency symmetry. When the surface is specular ($S = 1$), $f_{\perp}$ satisfies the boundary condition $f_{\perp}(\hat{p}_z, \epsilon, 0) = f_{\perp}(-\hat{p}_z, \epsilon, 0)$. It follows from this reflection symmetry and eq. (8) that $f_{\perp}$ at the specular surface has no even-frequency component and is dominated by the odd-frequency pairs. Even for the rough surface, the pair amplitudes with various orbital symmetries are generated near the surface [see fig. 2(b)]. These results are typical examples of the manifestation of the odd-frequency pairs in inhomogeneous systems.
The surface odd-frequency pair amplitude has characteristic frequency dependence reflecting the formation of the Andreev bound states. In fig. 3, the magnitude of $\text{Re} f^{\text{OF}}(\hat{p}, E + i0^+, 0)$ is plotted as a function of $E/\Delta B$ and the result is compared with the surface density of states, $n(\hat{p}, E, 0)$. The two quantities show very similar $E$-dependence. In the limit $E \to 0$, the both coincide exactly with each other; this property can be shown analytically to hold not only at the surface but also at arbitrary positions. The finite midgap density of states can therefore be interpreted as the manifestation of the odd-frequency pair amplitude.

Acknowledgments
We would like to thank Y. Tanaka and Y. Asano for helpful discussions. This work is supported in part by a Grant-in-Aid for Scientific Research (No. 21540365) and the “Topological Quantum Phenomena” (No. 22103003) KAKENHI on Innovative Areas from MEXT of Japan.

References
[1] Ambegaokar V, de Gennes P G and Rainer D 1974 Phys. Rev. A 9 2676
[2] Buchholtz L J and Zwicky R G 1981 Phys. Rev. B 23 5788
[3] Nagato Y, Yamamoto M and Nagai K 1998 J. Low Temp. Phys. 110 1135
[4] Aoki Y et al. 2005 Phys. Rev. Lett. 95 075301
[5] Murakawa S et al. 2009 Phys. Rev. Lett. 103 155301
[6] Murakawa S 2011 J. Phys. Soc. Jpn. 80 013602
[7] Nagai K, Nagato Y, Yamamoto M, Higashitani S 2008 J. Phys. Soc. Jpn. 77 111003
[8] Bergeret F S, Volkov A F and Efetov K B 2001 Phys. Rev. Lett. 86 4096
[9] Bergeret F S, Volkov A F and Efetov K B 2005 Rev. Mod. Phys. 77 1321
[10] Tanaka Y and Golubov A A 2007 Phys. Rev. Lett. 98 037003
[11] Tanaka Y, Tanuma Y and Golubov A A 2007 Phys. Rev. B 76 054522
[12] Golubov A A, Tanaka Y, Asano Y and Tanuma Y 2009 J. Phys.: Condens. Matter 21 164208
[13] Higashitani S, Nagato Y and Nagai K 2009 J. Low Temp. Phys. 155 83
[14] Nagato Y, Higashitani S, Yamada K and Nagai K 1996 J. Low Temp. Phys. 103 1