Estimation of the admissibility of the destructive testing method

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Abstract. In reliability theory, there is often a need to establish a relationship between parameters non-observed simultaneously. An example of such parameters is the operating time of the same product in different modes, because it’s impossible to experience the same product before destruction in several modes. In carrying out such destructive testing, the following question often arises: how to determine the operating time of a product in some specific, set mode, knowing the time of operation of this product in a different mode. By "mode" in this case a certain condition types for testing construction products are implied, for example, various climatic conditions, loads affecting the building construction, as well as other external influences in which the work of the product is possible. In practice, this problem is reduced to the investigation of the truth or falsity of the hypothesis that the operating time of the product in the first and second modes ($\varepsilon_1$ and $\varepsilon_2$) depends on some function ($\varphi$), chosen on the basis of a qualitative analysis of the problem. Since in real conditions when determining the reliability of products you often deal with samples of small size, in this paper we investigated the admissibility of applying the technique for calculating the reliability of building structures and products for small samples. The simulation results are shown in tables and are graphically depicted.

1. Introduction
In engineering research on reliability, there is often a need to establish a relationship between parameters non-observed simultaneously [1,17]. An example of such parameters is the operating time of the same product in different modes, because it’s impossible to experience the same product before destruction in several modes [2,13,14]. In carrying out such destructive testing, the following question often arises: how to determine the operating time of a product in some specific, set mode, knowing the time of operation of this product in a different mode [15,16,18]. By "mode" in this case a certain condition types for testing construction products are implied, for example, various climatic conditions, loads affecting the building construction, as well as other external influences in which the work of the product is possible [19,20].

The research objective is to improve the methods for calculating the reliability of building structures and products in case of small samples using modern mathematical simulation methods based on the theory of stress testing.

2. Methods
In practice [15, 16] this problem is reduced to the study of the truth or falsity of the following hypothesis:
\[ \xi(\varepsilon_1) = \varphi \xi(\varepsilon_2) \]  
(1)

where \( \xi(\varepsilon_1) \), \( \xi(\varepsilon_2) \) are the operating time of the product in the modes \( \varepsilon_1 \) and \( \varepsilon_2 \);

\( \varphi \) is a function chosen on the basis of a qualitative analysis of the problem [6, 7]. For example, if \( \varphi \) is a linear function, the hypothesis (1) takes the following form

\[ \xi(\varepsilon_1) = C(\varepsilon_1, \varepsilon_2) \xi(\varepsilon_2) \]  
(2)

Let’s consider one of the methods of statistical verification of the hypothesis (2) contained in the works [1, 3, 11, 12]. We choose two sets of \( m \) and \( n \) studied products in a random manner. A batch of \( n \) products is tested in the variable mode \( \tilde{\varepsilon} \) with a random switching torque \( \tau: \tilde{\varepsilon}(t, \tau) = \{ \varepsilon_1, 0 \leq t < \tau \} \) \( \{ \varepsilon_2, \tau \leq t < \infty \} \).

The distribution law \( H(t) = P(\tau < t) \) can be arbitrary with a probability density \( h(t) > 0 \) for all \( t > 0 \), i.e.

\[ H(t) = \int_0^t h(\xi) d\xi, h(t) > 0, t > 0. \]

In practice, the switching torque \( \tau \) can be determined in the following way. We number all \( n \) products of the batch in a random manner. From the tables [4] we choose \( n \) random numbers \( \mu_1, \mu_2, ..., \mu_n \), distributed according to the uniform law. The mode switching torque for the \( i \)th element is calculated by the formula \( \tau_i = H^{-1}(\mu_i), i = 1, 2, ..., n \), where \( H^{-1} \) is the inverse function of \( H \).

Initially, the product under number \( i \) is tested for a time \( \tau_i \) in the \( \varepsilon_1 \) mode, then, if it does not fail, in the \( \varepsilon_2 \) mode. As a result of testing for each product, we obtain the values \( \theta_i(\varepsilon_1), \theta_i(\varepsilon_2), i = 1, 2, ..., n \), i.e. the operating time of the product in the \( \varepsilon_1 \) and \( \varepsilon_2 \) modes. The verification of relation (2) is based on the following fact [8, 9]. Let the random variable \( \eta_c \) be distributed according to the law \( \mathcal{F}_c(t, \tilde{\varepsilon}) = P(\eta_c < t) \), and \( \mathcal{F}(t, \varepsilon_1) \) - is the distribution of the failure instants \( \xi(\varepsilon_1) \) in the \( \varepsilon_1 \) mode. Then, if

\[ \mathcal{F}(t, \varepsilon_1) = \mathcal{F}_c(t, \tilde{\varepsilon}), t \geq 0 \]  
(3)

the random variables \( \xi(\varepsilon_1), \xi(\varepsilon_2) \) are connected by the relation (2). If (3) is not satisfied, (2) is also not the case. We check the relation (3) using the Smirnov criteria. As it’s known, it is applicable in case when \( \mathcal{F}(t, \varepsilon_1) \) and \( \mathcal{F}_c(t, \tilde{\varepsilon}) \) are estimated from the results of tests of different samples.

Values

\[ \eta_c = \theta_i(\varepsilon_1) + c\theta_i(\varepsilon_2), i = 1, 2, ..., n \]  
(4)

represent the realizations of the random variable \( \eta_c \). The distribution estimation \( \mathcal{F}_c(t, \tilde{\varepsilon}) \) will be

\[ \mathcal{F}_0 = \frac{d(t)}{n} \]  
(5)

where \( d(t) \) - the number of values \( \eta_c \) from (4), less than \( t \). The second set of products is tested in the \( \varepsilon_1 \) mode, and we determine for all \( m \) products instants of their failures \( \lambda_1(\varepsilon_1), \lambda_2(\varepsilon_1), ..., \lambda_m(\varepsilon_1) \).

Then we set

\[ \mathcal{F}(t, \varepsilon_1) = \frac{\ell(t)}{m} \]  
(6)

where \( \ell(t) \) – the number of values \( \lambda_i(\varepsilon_1), i = 1, 2, ..., m \) less than \( t \).

We calculate the deviation

\[ D_c = \sup_i |\mathcal{F}_0(t, \tilde{\varepsilon}) - \mathcal{F}(t, \varepsilon_1)| = \sup_i |\frac{d(t)}{n} - \frac{\ell(t)}{m}| \]  
(7)

According to the Smirnov criterion, the hypothesis (2) is considered non-contradictory to testing data, if \( D \leq D_\beta \), where \( \beta \) is the level of significance.

3. Results and discussion

In this paper, the results of computer simulation of the testing method procedure described above [5] are presented. Also investigated the admissibility of using this method for reliable determination of the coefficient \( C(\varepsilon_1, \varepsilon_2) \) for small values of \( m \) and \( n \). In practice, you often deal with samples of small size, in addition, for large values of \( m \) and \( n \) other effective methods for performing destructive tests were developed [14], therefore the values of \( m \) and \( n \) were chosen from 2 to 20 for the simulation. The sample volumes were assumed to be equal, i.e. in what follows \( m=\bar{n} \). As a law of distribution of the failure instants \( \lambda(\varepsilon_1) \) a uniform distribution on the interval \((0,1)\) was chosen, which is not an essential
restriction, since any continuous distribution can be reduced to a uniform distribution by means of the transformation \( t = F(x) \), where \( F(x) \) is the distribution function of the continuous distribution law. In practice, the switching torques \( \tau \) are usually chosen in accordance with the law of distribution of the failure instants \( \lambda(\varepsilon_1) \). Therefore, the switching torques \( \tau \) were also assumed to be distributed uniformly regarding \((0,1)\). Because for the purposes of this paper, the absolute value of the coefficient \( C(\varepsilon_1, \varepsilon_2) \) from (2) plays no role, but only the relative error in determining \( C(\varepsilon_1, \varepsilon_2) \) is of interest, \( \eta \) from formula (4) was also assumed to be uniformly distributed regarding \((0,1)\). This corresponds to the fact that the coefficient \( C(\varepsilon_1, \varepsilon_2) \) in (2) must be equal to 1.

For each of the considered \( n \), three independent realizations of the random variables \( \xi_i, \tau_i, \lambda_i, i = 1,2,...,n \) were simulated, estimates of the distributions \( F(\xi, \varepsilon), F(t, \varepsilon) \) were formed from the formulas (5), (6) for the obtained values of \( \xi, \tau, \lambda \), and the deviation \( D_c \) was calculated from the formula (7) with \( C=0.25; 0.5; 0.75; 0.9; 1; 1.1; 1.25; 1.5; 2; 2.5; 3; 5; 10 \). For each \( n \) independent model realizations were obtained.

It is obvious that for a fixed \( n \) the deviations \( D(c) \) can take the values of \( 0, \frac{1}{n}, ..., \frac{n-1}{n}, 1 \). In each of the 1000 realizations, we have obtained its own set of \( D(c) \). Tables 1-6 show the results of 1000 model realizations for all considered \( n \). As generalizing characteristics for the values of \( D_1(c), D_2(c), ..., D_{1000}(c) \) we used estimates of mathematical expectation \( \overline{m_c} \) and variance \( \overline{\sigma_c^2} \) of the set of obtained deviations \( D_j(c), j = 1,2,...,1000 \), computed from the known [5] formulas \( \overline{m_c} = \frac{1}{1000} \sum_{i=1}^{1000} D_i(c), \overline{\sigma_c^2} = \frac{1}{1000} \sum_{i=1}^{1000} [D_i(c) - \overline{m_c}]^2 \).

The results of mathematical simulation are given in Tables 1-6, and are also conditionally shown in the form of a graph in Figure 1. The \( C(\varepsilon_1, \varepsilon_2) \) values, most preferable for a given sample volume \((n)\), are connected by a solid line on the graph.

| Table 1. Table of simulation results for \( n = m = 2 \) |
|-----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                 | \( \overline{m} \) | \( \overline{\sigma^2} \) |
| \( D \)         | 0.25        | 0.50        | 0.75        | 0.90        | 1.00        | 1.10        | 1.25        | 1.50        | 2.00        | 2.50        | 3.00        | 5.00        | 10.0        |
| 2\(^{-1}\)      | 0.25        | 0.50        | 0.75        | 0.90        | 1.00        | 1.10        | 1.25        | 1.50        | 2.00        | 2.50        | 3.00        | 5.00        | 10.0        |
| 1               | 345         | 345         | 407         | 376         | 376         | 345         | 376         | 314         | 314         | 344         | 375         |             |             |

| Table 2. Table of simulation results for \( n = m = 3 \) |
|-----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                 | \( \overline{m} \) | \( \overline{\sigma^2} \) |
| \( D \)         | 0.25        | 0.50        | 0.75        | 0.90        | 1.00        | 1.10        | 1.25        | 1.50        | 2.00        | 2.50        | 3.00        | 5.00        | 10.0        |
| 3\(^{-1}\)      | 0.25        | 0.50        | 0.75        | 0.90        | 1.00        | 1.10        | 1.25        | 1.50        | 2.00        | 2.50        | 3.00        | 5.00        | 10.0        |
| 2/3             | 593         | 625         | 611         | 518         | 503         | 503         | 471         | 470         | 439         | 408         | 517         | 627         | 611         |
| 1               | 157         | 109         | 109         | 94          | 78          | 78          | 78          | 78          | 93          | 93          | 109         | 109         | 141         |

| \( \overline{m} \) | 636         | 614         | 610         | 569         | 553         | 553         | 542         | 542         | 542         | 531         | 578         | 615         | 631         |
| \( \overline{\sigma^2} \) | 044         | 039         | 040         | 044         | 042         | 042         | 043         | 043         | 047         | 047         | 046         | 039         | 042         |
### Table 3. Table of simulation results for \( n = m = 4 \)

| \( D \) | 0.25 | 0.50 | 0.75 | 0.90 | 1.00 | 1.10 | 1.25 | 1.50 | 2.00 | 2.50 | 3.00 | 5.00 | 10.0 |
|------|------|------|------|------|------|------|------|------|------|------|------|------|-------|
| 1/4  | 62   | 62   | 0    | 62   | 62   | 62   | 62   | 62   | 62   | 187  | 187  | 63   | 63   |
| 2/4  | 437  | 500  | 563  | 564  | 564  | 502  | 502  | 627  | 502  | 377  | 564  | 439  |
| 3/4  | 438  | 438  | 437  | 374  | 374  | 374  | 436  | 436  | 311  | 11   | 436  | 373  | 436  |
| 1    | 63   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 62   |

\( \bar{m} \) | 626  | 594  | 609  | 578  | 578  | 593  | 593  | 562  | 531  | 562  | 578  | 624  |

\( \sigma^2 \) | 0.31 | 0.22 | 0.15 | 0.21 | 0.21 | 0.22 | 0.22 | 0.19 | 0.30 | 0.35 | 0.21 | 0.31 |

### Table 4. Table of simulation results for \( n = m = 6 \)

| \( D \) | 0.25 | 0.50 | 0.75 | 0.90 | 1.00 | 1.10 | 1.25 | 1.50 | 2.00 | 2.50 | 3.00 | 5.00 | 10.0 |
|------|------|------|------|------|------|------|------|------|------|------|------|------|-------|
| 1/6  | 0    | 0    | 31   | 31   | 0    | 0    | 0    | 0    | 0    | 32   | 94   | 0    | 0    |
| 2/6  | 188  | 218  | 220  | 377  | 376  | 376  | 407  | 439  | 376  | 314  | 376  | 156  | 156  |
| 3/6  | 344  | 408  | 562  | 436  | 468  | 468  | 406  | 374  | 405  | 499  | 438  | 564  | 439  |
| 4/6  | 406  | 343  | 156  | 156  | 156  | 156  | 187  | 187  | 93   | 186  | 249  | 343  |
| 5/6  | 62   | 31   | 31   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 31   | 31   |

\( \bar{m} \) | 557  | 531  | 489  | 453  | 463  | 463  | 463  | 458  | 458  | 432  | 468  | 526  | 557  |

\( \sigma^2 \) | 0.20 | 0.18 | 0.17 | 0.16 | 0.13 | 0.13 | 0.15 | 0.16 | 0.17 | 0.17 | 0.15 | 0.14 | 0.22 |

### Table 5. Table of simulation results for \( n = m = 10 \)

| \( D \) | 0.25 | 0.50 | 0.75 | 0.90 | 1.00 | 1.10 | 1.25 | 1.50 | 2.00 | 2.50 | 3.00 | 5.00 | 10.0 |
|------|------|------|------|------|------|------|------|------|------|------|------|------|-------|
| 2/10 | 0    | 95   | 31   | 93   | 62   | 62   | 93   | 94   | 126  | 158  | 63   | 0    | 0    |
| 3/10 | 156  | 344  | 376  | 563  | 563  | 469  | 406  | 437  | 374  | 310  | 186  | 124  |
| 4/10 | 438  | 311  | 468  | 250  | 281  | 343  | 438  | 344  | 280  | 343  | 374  | 439  | 375  |
| 5/10 | 249  | 156  | 125  | 94   | 94   | 126  | 63   | 125  | 220  | 189  | 158  | 156  | 251  |
| 6/10 | 94   | 62   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 93   | 157  | 123  |
| 7/10 | 63   | 32   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 62   | 62   |
| 8/10 | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 63   |

\( \bar{m} \) | 447  | 384  | 369  | 335  | 341  | 353  | 347  | 350  | 359  | 356  | 391  | 447  | 482  |

\( \sigma^2 \) | 0.11 | 0.14 | 0.05 | 0.06 | 0.06 | 0.06 | 0.07 | 0.09 | 0.09 | 0.11 | 0.13 | 0.18 |
Table 6. Table of simulation results for \( n = m = 20 \)

| D   | 0.25 | 0.50 | 0.75 | 0.90 | 1.00 | 1.10 | 1.25 | 1.50 | 2.00 | 2.50 | 3.00 | 5.00 | 10.0 |
|-----|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 3/20| 0    | 63   | 0    | 0    | 0    | 63   | 63   | 0    | 0    | 0    | 0    | 0    | 0    |
| 4/20| 188  | 188  | 63   | 375  | 437  | 563  | 251  | 251  | 63   | 0    | 0    | 0    | 0    |
| 5/20| 63   | 250  | 374  | 187  | 188  | 0    | 249  | 125  | 251  | 314  | 63   | 0    | 0    |
| 6/20| 187  | 62   | 188  | 188  | 125  | 187  | 62   | 249  | 312  | 188  | 313  | 0    | 0    |
| 7/20| 63   | 250  | 212  | 187  | 187  | 375  | 124  | 124  | 248  | 188  | 314  | 189  |      |
| 8/20| 250  | 62   | 0    | 63   | 63   | 0    | 188  | 62   | 0    | 186  | 187  | 312  |      |
| 9/20| 0    | 62   | 63   | 0    | 0    | 0    | 0    | 0    | 188  | 250  | 188  | 249  | 62   |
| 10/20| 62  | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 62   | 125  | 187  |      |
| 11/20| 124| 63   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 125  | 0    |      |
| 12/20| 63  | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 188  |      |
| 13/20| 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 62   |

\( \bar{m} \) 369 303 300 269 263 259 272 284 322 334 365 428 465
\( \sigma^2 \) 016 010 004 004 005 005 005 006 006 006 005 005 010

Figure 1. Dependence of \( C(\varepsilon_1, \varepsilon_2) \) on the sample volume \( (n) \).

The tables obtained from the simulation results allow one to estimate the error in determining the coefficient \( C(\varepsilon_1, \varepsilon_2) \) for different product samples. Thus, the application of the proposed modified technique on the basis of the original Kartashov method [1,10] will allow obtaining more accurate results of the product reliability evaluation for small samples.

4. Conclusion
As shown by the analysis of Tables 1-6 and the graph (Figure 1), for \( n,m \geq 10 \) the error in determining the coefficient \( C(\varepsilon_1, \varepsilon_2) \) under these simulation conditions does not exceed 10%, which is quite acceptable for destructive tests. The application of the proposed method, taking into account the evaluation of these errors, makes it possible, on the basis of a small sample, to select the optimal operating time of products in a variable mode.
References

[1] Kartashov G D 1977 Fundamentals of the stress testing theory Moscow Znanie 1 52.
[2] Chiganova N 2016 Reliability theory application for building structures reliability determination MATEC Web of Conferences 86 https://doi.org/10.1051/matecconf/20168602009
[3] Gnedenko B G, Belyaev Yu K, Slov’ev A D 2012 Mathematical methods in the reliability theory Moscow Nauka 2 582
[4] Bol’shev L N, Smirnov N V 1983 Tables of mathematical statistics Moscow Nauka 1 2017
[5] Ryzhik I M, Gradshtein I S 1963 Tables of integrals, sums, series, and products Moscow Fizmatgiz 4 1100
[6] Kartashov G D 1972 Experiments with unobserved simultaneously parameters in the stress testing theory All-Russian scientific research institute for standardization and certification in the branch of mechanical engineering collection book 1 41-43
[7] Epstein B 1960 Estimation from Life Test Data Technometrics 2 167-183
[8] Van der Varden B L 1962 Mathematical statistics Moscow Foreign Literature Publishing House 1 435
[9] Loeve M 1962 Probability Theory Moscow Foreign Literature Publishing House 1 719
[10] Kartashov G D 1973 Establishing a connection between non-simultaneously measured quality characteristics Moscow Elektronnaya tehnika 7 886
[11] Kendall M J, Stewart A 1979 Statistical inferences and connections Moscow Fizmatgiz 3 900
[12] Kobzar A I 2006 Applied mathematical statistics (For engineers and scientists) Moscow Fizmatgiz 1 816
[13] Ushakov I A 2008 Course in the system reliability theory Moscow Drofa 1 239
[14] Kartashov G D 1972 Evaluation of reliability of products operating in variable mode Moscow Tehnicheskaya kibernetika 4 226
[15] Tarasov D V, Tarasov R V, Makarova L V, Ermishina Ya A 2015 Improvement of quality control of construction products Sovremennye problemy nauki i obrazovaniya 1-1
[16] Makarova L V, Tarasov R V, Rezevich K S 2015 Competitiveness assessment of construction products Sovremennye problemy nauki i obrazovaniya 1
[17] Nelson W 2004 Accelerated Testing Statistical Models, Test Plans and Data Analysis New Jersey: John Wiley & Sons 601
[18] Wasserman L 2006 All of Nonparametric Statistics N.Y.: Springer Science+Business Media 272
[19] Gamiz M L, Kulasekera K B, Limnios N, Lindqvist P 2011 Applied Nonparametric Statistics in Reliability London: Springer-Verlag 227
[20] Sheshkin D J 2000 Handbook of parametric and nonparametric statistical procedures Boca Raton: Chapman & Hall/CRC 1002