Connection between the elastic $G_{E_p}/G_{M_p}$ and $P \to \Delta$ form factors.

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It is suggested that the falloff in $Q^2$ of the $P \to \Delta$ magnetic form factor $G_M^*$ is related to the recently observed falloff of the elastic electric form factor $G_{E_p}/G_{M_p}$. Calculation is carried out in the framework of a GPD model whose parameters are determined by fitting the elastic form factors $F_{1p}$ and $F_{2p}$ and isospin invariance. When applied to the $P \to \Delta$ transition with no additional parameters, the shape of $G_{M}^*$ is found to exhibit the requisite falloff with $Q^2$.

The $P \to \Delta(1232)$ form factor $G_M^*$ exhibits a more rapid decrease with respect to $Q^2$ than is typically observed in other baryons $^1_2$, such as $G_{M_p}$ in elastic scattering from a proton, or $A_{1/2}$ in the transition $P \to S_{11}(1535)$. A recent Jefferson Lab (JLab) measurement $^3$ finds that the ratio $G_{E_p}/G_{M_p}$ for elastic scattering falls with $Q^2$ more rapidly than previously expected. This has given rise to much theoretical activity $^4_5$ to attempt to understand the underlying physics. In this note it is suggested that this behavior in $G_{E_p}/G_{M_p}$ is related to that of $G_M^*$.

As a basis it is assumed that the form factor is dominated by soft mechanisms, and a GPD-handbag approach $^6_7_8$ is utilized. Form factors are the zero'th moments of the GPDs with skewedness $\xi = 0$. For elastic scattering, the Dirac and Pauli form factors are given by

$$F_1(t) = \int_{-1}^{1} \sum_q H^q(x, \xi, t) dx$$

$$F_2(t) = \int_{-1}^{1} \sum_q E^q(x, \xi, t) dx$$

where $q$ signifies flavors. In the following, with $\xi=0$, for brevity the GPD’s are denoted $H^q(x, t) \equiv H^q(x, 0, t)$, and $E^q(x, t) \equiv E^q(x, 0, t)$.

Resonance transition form factors access components of the GPDs which are not accessed in elastic scattering. The $N \to \Delta$ form factors are related to isovector components of the GPDs $^9_{10}$:

$$2G_M^* = \int_{-1}^{1} \sum_q H^q_M(x, t) dx$$

$$2G_E^* = \int_{-1}^{1} \sum_q H^q_E(x, t) dx$$

$$2G_C^* = \int_{-1}^{1} \sum_q H^q_C(x, t) dx$$

where $G_M^*$, $G_E^*$ and $G_C^*$ are magnetic, electric and Coulomb transition form factors $^11$, and $H_M^q$, $H_E^q$, and $H_C^q$ are isovector GPDs, which can be related to elastic GPDs in the large $N_c$ chiral limit through isospin rotations. Analogous relationships can be obtained for the $N \to S_{11}$ and other transitions. Here, the connection between GPDs involved in the elastic and $N \to \Delta$ form factors is explored to obtain the connection between the $Q^2$ dependence of the $G_{E_p}$ and $G_M^*$.

In refs. $^6_{10}$ it is noted that, in the large $N_c$ limit, assuming chiral and isospin symmetry the GPDs for the $P \to \Delta(1232)$ transition are expected to be isovector components of the elastic GPDs, given by

$$H_M^{(IV)} = \frac{2}{\sqrt{3}} E_M^{(IV)} = \frac{2}{\sqrt{3}} (E_u - E_d, 1)$$

$E_u$ and $E_d$ are the GPD’s for the $u$ and $d$ quarks respectively. Thus the $P \to \Delta$ form factor should be obtainable by analysis of the Pauli form factor $F_{2p}$ (eq.2). The Dirac and Pauli form factors, $F_{1p}$ and $F_{2p}$, are related to the measured Sachs form factors $G_{M_p}$ and $G_{E_p}$ by

$$F_{1p}(Q^2) = \frac{1}{\tau + 1} (\tau G_{M_p}(Q^2) + G_{E_p}(Q^2))$$

$$F_{2p}(Q^2) = \frac{1}{\kappa(\tau + 1)} (G_{M_p}(Q^2) - G_{E_p}(Q^2))$$

with $\tau = Q^2/4M_p$. To obtain $E_u$ and $E_d$, needed for eq. 6 the available data for $G_{M_p}$ and the recent JLab data $^9$ on $G_{E_p}/G_{M_p}$ were fit, as reported in ref. $^{12}$, using a parameterization of the GPDs such as in $^{13}_{14}_{15}_{16}$.

The specific functional form for $H_P(x, t)$ and $E_P(x, t)$ is a Gaussian plus small power law shape in $-t (\equiv Q^2)$ to account for the high the measured form factors at very high $Q^2$.

$$H_P(x, t) = f(x) \exp(\bar{x}t/4x\lambda_H) + \cdots$$

$$E_P(x, t) = k(x) \exp(\bar{x}t/4x\lambda_E) + \cdots,$$

in which $\bar{x} \equiv 1 - x$, and $\cdots$ indicates the addition of small power components added in ref. $^{12}$ to account for

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higher $Q^2$ contributions to the form factors. The conditions at $Q^2=0$ are $H(x,0) = e_u f_u(x) + e_d f_d(x)$ and 
$E(x,0) = k_u(x) + k_d(x)$. Here, $f_u(x)$ and $f_d(x)$ are 
the $u$ and $d$ valence quark distribution functions mea-
sured in DIS. The functions $k_u(x)$ and $k_d(x)$ are not 
obtainable from DIS. Following ref. [15] the form used 
was $k_q(x) \propto \sqrt{1-x} f^q(x)$, with normalization obtained 
employing isospin symmetry, and by requiring the proton 
and neutron form factors to have their known values near 
$Q^2=0$, that is $F_{1p}(0) = 1$, $F_{2p}(0) = 1.79$, $F_{1n}(0) = 0$, $F_{2n}(0) = -1.91$. This gives

$$F_{2u}(0) \equiv \kappa_u = \int k_u(x) dx = 1.67$$

$$F_{2d}(0) \equiv \kappa_d = \int k_d(x) dx = -2.03$$

and

$$F_{1u}(0) = \int e_u f_u(x) dx = 2/3$$

$$F_{1d}(0) = \int e_d f_d(x) dx = 1/3$$

Adequate fits to $G_{Mp}$ and $G_{Ep}/G_{Mp}$, or equivalently $F_{1p}$ 
and $F_{2p}/F_{1p}$, were obtained with $\lambda_H = 0.76$ GeV/c and 
$\lambda_E = 0.67$ GeV/c. The results are shown in figs. [1] and [2].

The resulting $E^p_{1}$ and $E^p_{2}$ were inserted into eq. 4 to 
 obtain an estimate for $G^*_M$. At $Q^2=0$ one gets $G^*_M(0) = 2.14$, which is somewhat lower than the experimental 
value of $G^*_M(0) \sim 3$. Such a disagreement is not surpris-
ing [4] [10], given the very approximate nature of eq. 4. 
The obtained $G^*_M$ was overall renormalized to take this 
ratio into account, and the result is shown in fig. 5.

The similar shapes of the curves in figs. 2 and 3 can be 
ascribed to their connection via eq. 6. This can be under-
stood by the observation that $F_2$ is nearly all isovector 
spin-flip, as is the $G^*_M$. However, the difference in the 
mass of the $\Delta(1232)$ and the nucleon, which is a mea-
sure of the SU3 symmetry breaking, and the fact that 
$F_1$ also has an isovector component would make the ob-
served non-negligible differences in the normalization not 

![Figure 1](image1.png)

**Fig. 1:** Dirac form factor $F_{1p}(Q^2)$ relative to the dipole 
$G_D = 1/(1 + Q^2/0.71)^2$. The data are extracted using the 
recent JLab. data [3] for $G_{Ep}/G_{Mp}$, and a recent reevaluation 
[20] of SLAC data of $G_{Mp}$ [15] [10]. The curve is the result of the fit as discussed in the text.

![Figure 2](image2.png)

**Fig. 2:** The Pauli form factor $F_2/1.79F_D$ relative to the 
dipole $F_D = 1/(1 + Q^2/0.71)^2$. The data are extracted using 
the recent JLab. data [3] for $F_{2p}/F_{1p}$, multiplied by the fit 
curve for $F_{2p}/F_{1p}$ shown in fig. 1. The curve is the result of the simultaneous fit to the $G_{Ep}/G_{Mp}$ and $G_{Mp}$ data as discussed 
in the text and fig. 1.

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![Figure 3](image3.png)

**Fig. 3:** The $N \rightarrow \Delta$ magnetic form factor $G^*_M(Q^2)/3Q_D$ 
relative to the dipole $G_D = 1/(1 + Q^2/0.71)^2$. 
The data are a compendium of world data by ref. [21]. The curve is the result of the procedures discussed in the text.

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