Abstract. Pupils from 5th, 7th and 9th grades compared the weight of a small standard mass and a big bag hanging in a pulley at different positions in equilibrium. In all the three age groups the majority (about 70%) of the pupils stated that the lower hanging bag is heavier. Less than 15% of the pupils in the 7th and 9th grades understood that the bodies must have the same weight. Pupils’ reasoning was classified into four categories that were interpreted as mental models according to Vosniadou’s framework theory. Only about 5% of the seventh graders and 10% of the ninth graders seem to grasp the scientific model Motion that leads to the concept of gravity.

Keywords: conceptual change, weight, mental model, physics education, secondary school.

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Introduction

According to the constructivist view of learning the student is seen as an active builder of his/her knowledge. In order for learning to take place it is necessary for the student to connect the new knowledge with the existing knowledge structure. Constructivism underlines the view of knowledge as a form of mental representation, a construction of the human mind (Matthews, 1998; Shapiro, 1994). In current studies cognitive constructions are referred to as mental models according to Vosniadou’s framework theory. Only about 5% of the seventh graders and 10% of the ninth graders seem to grasp the scientific model Motion that leads to the concept of gravity.

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Vosniadou (1994) has put forward a theoretical framework to explain the nature of conceptual change that takes place in the learning of physics. She and her co-workers (Vosniadou & Ioannides, 1998; Vosniadou, Ioannides, Dimitrakopoulou, & Papademetriou, 2001) have also been interested in the development of the concept of force when students are exposed to teaching of science. Vosniadou argues
that already early on, children have certain constraints or entrenched presuppositions about the behaviour of physical objects. These are organised in a framework theory of naïve physics, which is not available to conscious awareness and hypothesis testing. A framework theory includes the basic ontological and epistemological presuppositions that define the concept of the physical object. Children also construct specific theories to explain a limited range of phenomena (like the movement of objects). Specific theories consist of beliefs that give rise to mental models, under the constraints and the presuppositions of the framework theory. Beliefs are generated through observation and/or through information presented by the culture under the constraints of the framework theory. Framework and specific theories provide the basis for generating situation specific representations of mental models, during problem solving situations.

Mental models are dynamic and generative representations that can be manipulated mentally to provide causal explanations of phenomena and to make predictions. Even when constructed on the spot, mental models contain many permanent features because they are constrained by underlying framework and specific theories. Formation of mental model is a cognitive process in which external information like observation changes into an internal model that later affects how new information will be interpreted and embraced.

Two different kinds of mental models can be found before the accepted scientific model is reached. Before children have had any teaching of science they have spontaneously constructed initial mental models. There is a relatively small number of mental model types out of which specific, context sensitive, situational mental model types are constructed. With development and learning children’s initial representations undergo either spontaneous or instructionally-based changes. The former are results of enriched observations in the cultural context or of other kinds of cultural learning (such as language learning) whereas the latter are results from specific science instructions. Children synthesise teacher’s scientific explanation with aspects of their initial conception. They construct such synthetic models to combine the information they receive from teaching with certain presuppositions and beliefs supported by their everyday experience.

The present study set out to explore how pupils in 5th to 9th grade (average age 11 to 15, respectively) would understand forces in equilibrium. The main aim was to find out what aspects of force the teacher should introduce before treating the topic systematically. This topic was chosen on the basis of considerable experience of teaching physics to secondary students and assessing their responses to basic problems concerning forces. Some students seem to identify the pull exerted by the earth with the heaviness of an object tending to make it to go down. Champagne, Klofker & Anderson (1980) applied a problem based on Atwood’s machine (non-stretching string passing over a flywheel with objects hanging on the ends) in their test to measure college physics students’ preconceptions about motion. According to their results about four students in five believed that being “lower (closer to the earth) implies heavier” for objects suspended on the Atwood’s machine. Gunstone & White (1981) carried out a more definite research on the same topic with a bicycle wheel mounted as a pulley. They found that 27% of first year university physics students reasoned the lower hanging block of wood to be heavier than the higher hanging bucket of sand (of the same mass). Some of these students drew even inappropriate analogies to seesaws and beam balances. In Mohapatra’s & Bhattacharyya’s (1989) pencil-and-paper test about 60% of the 9th grade pupils stated that the downward force on the lower hanging body was more than that on the higher hanging body even though it was mentioned in the question that the two bodies were of equal mass. The researchers concluded after the interview that this misinterpretation was because the pupils associate the concept of weight and not moment of force with balancing in a physical balance and then they apply this idea to the case of the pulley.

Bao and Redish (2003) argue that different learning contexts may lead to two different mental processes: activation of existing knowledge or creation of new alternative conceptual understanding. The dominant process depends on the familiarity of the content. Earlier fixed conceptions hinder a new way of thinking. When people meet a new phenomenon they generally try to apply old and familiar ideas to explain it. In this study we wanted to look at how existing
and familiar explanations are used when pupils try to find a proper explanation for a phenomenon that enables them to try several alternative explanations.

Thus the research problem in this study was how pupils’ conception of weight changes from fifth to ninth grade when two bodies are hanging in a pulley in balance.

Methodology of research

Task and participants

During a science lesson pupils were asked to compare the weight of a small standard mass and a clearly bigger bag hanging in a pulley (Fig. 1). Before the test, a flywheel was fixed on its axis and turned around in both directions so that the pupils could see that it was moving freely. This was done to ensure that all pupils would understand what a pulley is. The question was given to pupils from 5th, 7th and 9th grades (average ages 11 to 15). During the test the pupils were not allowed to discuss with each other whereas it was stressed that they should give reasons for their answers. One of the researchers carried out the test in the primary schools (four schools, five different teachers). In the lower secondary schools (six teaching groups in both grades) the physics teachers carried out the test during lessons following written instructions.

Table 1. The number of the pupils taking part in the test.

| Grade/Average age | Boys | Girls | Totals |
|-------------------|------|-------|--------|
| 5th/11            | 51   | 46    | 97     |
| 7th/13            | 49   | 49    | 98     |
| 9th/15            | 51   | 58    | 109    |
| **Totals**        | **151** | **153** | **304** |

Figure 1. A standard mass and a bag are hanging in a pulley. The pulley can move freely and the string is very light. What can you say about the weight of the standard mass A and the bag B compared to each other? Give reasons for your thinking.

Data analysis

The pupils’ answers were first classified into four main categories:
I. Standard mass is heavier,
II. Bag is heavier,
III. Bag and standard mass weigh the same.
IV. Rejected

If the answer contained no reasoning it was rejected and thus not taken into account in the categories I to III. The answers in the three first main categories were further divided into subcategories according to the reasoning, Categorisation of the reasons was carried out together by both researchers. After negotiations the common alternatives were agreed. If a pupil gave more than one reason the answer was placed according to the reason that gives the higher subcategory in the categorisation. The following five subcategories are the same in the three main categories and placed in hierarchical order according to the abstraction level. The pupils' reasoning in a certain subcategory differs slightly according to the main category. Examples of the pupils' answers are given in each case.

1. **Motion.** The scientifically correct argument is based on the movement of the bag, the standard mass or the flywheel. This idea includes some notion of the idea of effects of gravity i.e. of the force concept.

   The bag goes upwards and the standard mass downwards. (I) The bag has moved downwards. (II) The bag and the standard mass stay at their positions. (III)

2. **Position.** The argument is based on the positions of the bag and the standard mass. Some pupils gave also a quantitative estimate of the weights.

   The standard mass hangs higher. (I) The bag hangs lower. (II) The standard mass pulls the bag to the same level. (III) The bag weighs four times as much as the standard mass. (III)

3. **Appearance.** The pupils pay attention to the concrete appearance of the bag and the standard mass.

   The standard mass looks heavy. (I) The bag is larger. (II)

4. **Material.** The pupils give concrete properties to the bag and the standard mass.

   The standard mass is of metal and the bag is of plastic. (I)

   The bag contains something. (II)

5. **No argument or confusing idea.** In most cases the pupils only stated their thought about the weights of the bag and of the standard mass compared to each other.

   The bag is lighter than the standard mass. (I) The bag is heavier than the standard mass. (II) Because the bag is larger than the standard mass gravitation effects more on it and therefore it is lower than the standard mass even though it weighs the same. (III)

**Results of Research**

Results are presented in three sections. In the first section the distribution of the students' answers to the main categories is given. The second section presents the distribution of the students' reasoning into the subcategories in the three main categories. In the third section the development of the mental models represented by the subcategories is introduced.

**Distribution of the pupils' answers in the main categories**

As seen in Figure 2 in all the three age groups the majority of the pupils stated that the lower hanging bag is heavier (main category II). The 5th grade pupils considered that the standard mass is heavier than the bag almost twice as frequently as the older pupils (main category I). The 7th and 9th graders had the correct idea that the standard mass and the bag are of equal weight more frequently than the 5th graders (main category III). The amount of rejected answers was fairly small in all the grades (main category IV) indicating that this question was meaningful to the pupils. According to the chi-square test the results of the 7th and 9th graders do not differ significantly whereas the results of the 5th graders differ significantly ($p < 0.05$) both from those of the 7th graders ($\chi^2(3) = 11.12$) and those of the 9th graders ($\chi^2(3) = 8.91$).
PUPILS’ MENTAL MODELS OF A PULLEY IN BALANCE
(P. 26–34)

Figure 2. Distribution of the pupils’ answers in the main categories. I Standard mass is heavier; II Bag is heavier; III Bag and standard mass weigh the same; IV Rejected. The amount of the pupils: 5th grade: 97, 7th grade: 98, 9th grade: 104.

Distribution of the pupils’ arguments in the subcategories

The subcategory 2 Position contains the most general argument used by the pupils in all the three grades (see Figure 3). The corresponding argument is based on the position of the lower hanging bag resembling the beam of the balance that sinks on the side of the heavier load. (We presume that all the pupils are familiar with the picture of an old-fashion two armed beam balance even though the balances in the shops nowadays mainly have a digital reading.) Also the (altogether five) answers, in which the pupils tried to estimate how much heavier the bag is than the standard mass, are included in this subcategory. The total distribution of the answers according to the grades is as follows: 5th grade 40%; 7th grade 36%; 9th grade 43%. The Position reasoning is mostly used in main group II BAG IS HEAVIER. In some cases the conclusions STANDARD MASS IS HEAVIER (main group I) as well as THE BAG AND THE STANDARD MASS WEIGH THE SAME (main group III) are also supported with this kind of reasoning (in main group I by 10 pupils; in main group III by 4 pupils).

Figure 3. Distribution of the pupils’ arguments in the subcategories. 1. Motion; 2. Position; 3. Appearance; 4. Material; 5. No argument or confusing idea.

In the subcategory 1 Motion the argument that the forces acting on the bag and the standard mass are equal when the system is not in motion and thus the background idea that the string pulls both the bag and the standard mass upward with the same force as the Earth (gravitation) is pulling them downward is beginning to develop in the minds of the pupils (5th grade 7%; 7th grade 12%; 9th grade 27%). This reasoning is used in all main categories but least in main category I (only four pupils). It is also interesting to notice that 9th graders are using it more in main category II than in main category III.

The arguments combined in the subcategory Material are also used in all the main categories.
but their number declines steadily from 5th grade to 9th grade (5th grade 13%; 7th grade 8%; 9th grade 5%). In grades 5 and 7 the pupils pay more attention to the size (appearance) of the bag and the standard mass (about 20%) than in grade 9 (about 9%). This difference is significant (p<0.05). Furthermore, the reasoning of subcategory Appearance is not used at all in main category III and only two 5th graders are using it in main category I.

The number of the 9th graders and the 7th graders differ significantly (p<0.05) in subcategory No argument or confusing idea. In the main category II containing most of the pupils' proposals the 7th graders' answers differ significantly (p<0.05) both from the 5th graders' and 9th graders' answers.

When all the subcategories in the main category I are compared statistically the results of the 5th graders and the 7th graders do not differ significantly ($\chi^2(4) = 3.54$) whereas the results of the 9th graders differ significantly $p < 0.01$ ($\chi^2(4) = 14.99$) from those of the 7th graders and significantly $p < 0.001$ from those of the 5th graders ($\chi^2(4) = 18.85$). The comparison of the main categories indicates that the 7th graders choose the main categories in the similar way as the 9th graders. The comparison of the subcategories shows that the 7th graders' reasoning, however, is closer to the reasoning of the 5th graders than that of the 9th graders.

Pupils' mental models

As shown in Figure 2 the number of pupils' answers classified in the main category I STANDARD MASS IS HEAVIER drops almost to the half from the 5th grade to the 7th and the 9th grade. Correspondingly, a clear increase is noticeable in the number of the answers classified to the main category III BAG AND STANDARD MASS WEIGH THE SAME whereas the number of answers in the most general choice, the main category II BAG IS HEAVIER, stays more or less the same. We argue that these results are due to the changes in pupils' reasoning, and that they can be interpreted as changes in the development of pupils' mental models of the physical world.

Figure 4. Pupils' mental models of a pulley in balance. The subcategories that consist less than 5% of the answers are left out to make the figure clearer.

In the main category I STANDARD MASS IS HEAVIER about 8% of the 5th grade pupils base their argument on the material of the standard mass or the bag: The standard mass is of metal. The bag looks like a teabag. The bag is of plastic. The bag is empty. The numbers in this subcategory Material decrease to 2% in grade 7 and to 1% in grade 9 when pupils get older and start their studies in physics and chemistry. This subcategory can thus be interpreted as an initial model. Pupils have constructed it on the
basis of their earlier experiences with different objects they have seen and carried (see Vosniadou and Ioannides, 1998). They pay attention to the intensive properties like what the bodies may contain.

The subcategory **Appearance** exists only in the main category II BAG IS HEAVIER if the few odd answers are left out. About a fifth of the pupils in grades 5 and 7 support their conclusion with the size of the bag: **The bag is heavier because it is larger**. The number of these arguments drops below 10% in grade 9. Subcategory **Appearance** can be regarded as a synthetic model because pupils have included into the initial model (**Material**) aspects that they have studied in science lessons (mass and volume). However, this model is not instructionally based but formed spontaneously. In the history of science also the old Greeks thought that the weight of an object depends on the size and form of the object (Jammer, 1961). According to one of the ontological presuppositions in Vosniadou's framework theory of naïve physics (Vosniadou, 1994, Vosniadou & Ioannides, 1998) children regard the objects especially according to what they look like.

About a third of the pupils in each grade - the subcategory **Position** - base their conclusion that the bag is heavier on the position of the bag: **"The bag is heavier as it hangs lower"**. This is interpreted as that the pupils make the connection to the behaviour of a balance. One 9th grader even expressed the idea in words: **"In this situation the pulley is acting as a balance"**. Also Mohapatra & Bhattachaayya (1989) quote a pupil's explanation in the interview after answering the corresponding question in the questionnaire: **"take the example of a physical balance; the pan which has the heavier weight goes down."** Even when the interviewer tries to point out that **"this is not a physical balance"** the pupil insists: **"one can think this to be same type"**. The idea of balance contains the same notion as students' interpretation of a falling body: the heaviness of the object tends to make it go down to the ground, Aristotle's natural resting place (Minstrell, 1982, Champagne et al., 1980). This interpretation is also confirmed by the experience that pulling heavy objects down is easier than pulling them up (Gunstone and White, 1981). Subcategory **Position** is thus a synthetic model created by the pupils who have not yet understood the idea of gravity but who try to transfer the idea of balance to the functioning of the pulley. They interpret the information they have received according to their own "common sense" i.e. according to the presuppositions and restrictions of their own framework theory. This kind of conception is formed slowly in the cultural context.

In the main category I STANDARD MASS IS HEAVIER 6% of the 9th graders are classified to the subcategory **Position**, which may indicate that these pupils do not properly understand the functioning of a balance. There is, however, another possible explanation. Pupils' conviction that the standard mass is heavier hinders them to pay attention to the proper behaviour of the balance. This conviction may be due to the concrete models that were dominant earlier like **Material** (the standard mass is of metal) or **Appearance** (the bigger the heavier) or to the experience that the higher an object is the bigger effect it causes when it falls down.

In the 7th and 9th grades some of the pupils start to pay attention to the movement of the pulley. But they combine this with the balance idea and therefore they tend to end up with a wrong conclusion: **The bag weighs more because it is moving downward and the standard mass is moving upward**. Only 5% of the pupils in the 7th grade and 9% of the pupils in the 9th grade have formed the correct idea of equilibrium: **They are of equal weight otherwise the heavier one would fall down**.

Conclusions and Implications

There seems to be a spontaneous development in pupils' mental models about the behaviour of a pulley in balance starting from the concrete model **Material** up to the scientific model **Motion**. The initial mental model **Material** is formed early and pupils have applied it successfully in many situations. It contains the belief that metallic objects are heavy and those of soft material are light and thus it leads to an intuitive conclusion that the standard mass is heavier. About 8% of the 5th graders are still thinking like this (see Figure 4). However, a few 7th and 9th graders wrote that **Standard mass is heavier. Heavier hangs higher and therefore the lighter bag is in balance with the standard mass**. This reasoning can be interpreted so that the stable mental model **Material** and the consequent belief that the metallic standard


mass is heavier led these pupils to apply an intuitive rule that a heavier object hangs higher (Stavy & Tirosh, 1996). These pupils realized that the system is in balance but the use of the initial mental model hindered them to focus on the fact that the objects were not in motion.

Everyday common experience that the weight of an object depends on its size (volume) joined with the general causal reasoning (Andersson, 1986) or the intuitive rule the bigger – the heavier (Stavy & Tirosh, 1996) led some pupils to conclude that the big bag is heavier and thus apply the spontaneous model Appearance (Figure 4). However, only 10% of the ninth graders are anymore using it.

The next conceptual change takes place when some of the pupils perceive to connect the object to its position. This leads to the dominant Position model that is applied in understanding of the behaviour of a beam balance. It reinforces the wrong conclusion – the big bag is heavier – made spontaneously on the basis of Appearance model coming first into mind. Position model has also been supported by physics teaching at the lower secondary school especially in the connection of measurements with beam balances and investigations of the laws of moments.

Some pupils connect, however, the functioning of the pulley, which the teacher showed at the beginning of the test by rotating the easily moving flywheel with the Motion model. According to Vosniadou & Ioannides (1998) it is a synthetic model because it is a product of physics teaching. Here again the wrong conclusion based on the Position model hinders many pupils to draw the right inference and realize that there is another explanation.

Only a few of the pupils understood that the bag and the standard mass had the same weight. For this they had first of all to realise that in the case of the pulley in balance neither the size nor the position of the hanging bodies had any effect on their weight. As Arons (1990, p. 71) stresses it is important to help pupils to understand that the force we feel when we hold an object is not the weight of the object but the contact force the object exerts on us. The weight of the object is the gravitational force exerted by the earth on the object.

As shown above pupils have several misconceptions:
- the denser the material is the heavier the object is [Material model];
- the bigger the object the heavier it is [Appearance model];
- the lower the object hangs the heavier it is [Position model].

With the aid of the pulley the teacher can demonstrate that these conceptions are not valid. Instead when pupils start to pay attention to the motion of the objects in the pulley they may realize that the weight of the object is related to the concept of gravity [Motion model].

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Resumen

БЛОК В РАВНОВЕСИИ: УМСТВЕННЫЕ МОДЕЛИ УЧАЩИХСЯ

Олави Гаккарайнен, Майя Агтии

Настоящее исследование имело целью установить, как учащиеся 5-9 классов (в возрасте 11-15 лет, соответственно) понимают равновесие сил. В письменном тесте ученикам предлагалось сравнить вес маленькой гири и мешка явно большего размера, висящих на блоке в различных положениях в равновесии, и дать обоснованные ответы. До проведения теста ученикам был продемонстрирован маховик, закреплённый на своей оси и поворачиваемый в обоих направлениях так, чтобы ученики могли наблюдать его свободное вращение.

Сначала ответы учеников были объединены в четыре главные группы: I - тяжелее гиря; II - тяжелее мешок; III - гиря и мешок весят одинаково; IV - забракованные ответы (необоснованные ответы отклонялись и не включались в категории I-III). Во всех трёх возрастных группах большинство учеников (около 70%) утверждали, что свисающий ниже мешок тяжелее. Менее 15% учеников в 7 и 9 классах поняли, что эти тела должны иметь равный вес. Затем ответы в каждой из трёх первых групп разделили на пять подгрупп в зависимости от данного обоснования, в порядке убывания уровня абстракции.

"Движение": научно корректные доводы, основанные на том, что ни мешок, ни гиря не движутся (5 класс - 7%; 7 класс - 12%; 9 класс - 27%). "Положение": доводы, основанные на взаимном положении мешка и гири (5 класс - 40%; 7 класс - 36%; 9 класс - 43%). "Внешний вид": ученики обратили внимание на внешний вид мешка и гири (5 класс - 21%; 7 класс - 19%; 9 класс - 8%). "Материал": ученики указали конкретные свойства мешка и гири (5 класс - 13%; 7 класс - 8%; 9 класс - 5%). "Отсутствие доводов как таковых или их запутанность": во многих случаях ученики лишь выражали своё мнение о весе мешка по сравнению с весом гири (5 класс - 12%; 7 класс - 20%; 9 класс - 7%). Далее обоснования ответов учеников интерпретировались в рамках теории Восняду как умственные модели, демонстрирующие частичную прогрессию. Модели "Внешний вид" и "Положение" преобладали в 5 и 7 классах, модели "Движение", ведущую к идее силы тяжести.

Ключевые слова: умственные модели, средняя школа, обучение физики, масса.

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