Resistence of fractal structures to natural and man-made influences

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Abstract. The theory of fractals is applicable for creating the real objects and for constructing the independent elements of the framework or the whole structure of the building. It’s supposed that this type of building has reliability and stability to natural and man-made influences. Innovative program was developed to research the fractal forms. The object of the study was a three-dimensional Mandelbrot fractal. New terminology has been developed and applied to three-dimensional fractals of even and odd powers. External structures of various powers of three-dimensional fractal were modeled and researched. The resistance analysis of fractal structures was conducted by using a power coefficient of the fractal’s exostructure. It’s pointed, that fractal structure is most rationally to produce by 3D printing. The 3D model of the fractal was printed from ABS plastic and can be used as a frame of the unique building.

1. Introduction
The safety of human life is the main aspect of special attention in all industries, including construction. Unique high-rise buildings are characterized by increased sensitivity to natural and man-made impacts. This parameter highlights skyscrapers among all types of building structures. Reliability and survivability of the system are providing the solution to the problem of object security. An assumption is made that a unique high-rise fractal-formed building will have increased values of object survivability.

Fractals are expressed not in the primary geometric forms, but in the algorithms and sets of the mathematical operations [1-3]. Algorithms are transformed into geometric forms using different methods of IT for visualization of the fractals. The fractal geometry of Mandelbrot studies nonsmooth, rough, foamy objects as well as objects capped with pores, cracks and holes. The possible field of application of the theory of fractals in civil engineering is quite extensive [4-9]. The theory of fractals is applicable for creating the real objects and for constructing the independent elements of the framework or the whole structure of the building.

The object of the study is a three-dimensional Mandelbrot fractal. Assuming that any plane figure is the orthogonal projection of some volume object, one may consider the existence of three-dimensional fractal shapes. The volumetric fractal was used as the study object instead of repeated variations of the two-dimensional fractal because the structure Mandelbulb can be perceived as a natural object. Three-dimensional fractals are common in human’s lives, their forms are complete, smooth and understandable, at the same time the forms are in constant. This proves that the fractal tends to chaos, which is limited by the mathematical equations.

The model of the structure of a three-dimensional fractal allows to conduct research of the different shapes. The fractal, as an object of unusual complex geometry, is most rationally to produce by 3D printing. This technology is used to solve the task of modeling fractal structures and the possibility of
introducing them into fractal architecture in the future.

The objectives of the research included the following:

1. To model various external structures of a three-dimensional fractal.
2. To create and to adopt appropriate terminology.
3. To study resistance of the fractal structure to natural and man-made impacts with using the power coefficient of the fractal's exostructure.
4. To create the fractal structure with using 3D printing technology.

2. Methods

Innovative program «External structure of the fractal set» was developed to create an external structure of the fractal [10]. The purpose of the algorithm is to determine the points of the fractal surface and then create a finite element model consists of shells. Coordinates of points are determined by checking the belonging them to the surface of the fractal after number of iterations was given. Verification of the belonging of points is carried out in spherical coordinates by changing cyclically at the beginning the distance from the center (r), then the horizontal angle (φ), then the vertical angle (θ). If the current point is outside the surface, then the previous one belonged to it.

The calculation of a large number of points with using small steps of angles makes it possible to obtain a more detailed and smooth surface of the shell. This increases the accuracy of the calculation, but it has a strong effect on the duration of points generation. Therefore, it matters to set correctly the initial steps for r, φ and θ to obtain the better finite element model as a result. The coordinates of the points and the sets of finite elements form the external structure of the fractal set, which have a rational distribution of the density of the points along the surface to create the most optimal grid of elements. All data is written to a text file.

Fractal structure was modeled in the program developed by the authors, with export of topological data to the SCAD software. External surface of the structure is approximated by isoparametric triangular finite elements with six degrees of freedom in the node. The input data for the calculation were the physical and mechanical characteristics of titanium applied to the elements of the fractal structure. The initial radius of the sphere for the evolution of the fractal structure is 50 m.

To analyze the fractal structure the authors adopted the following terminology, which is valid for structures with power n>3. Power of fractal or quasi-volume is an analogue of the concept of local dimension of a fractal set. The models of structures of fractal were developed with fixed number of initial points of the first iteration surface for all exploring powers.

Determination of stress-strain state of exostructure was made by SCAD software with using finite element method. The fractal power was varied in the series of calculations. In the numerical experiment of the exostructure shaping was investigated the even powers of fractals from 6 to 44. All models had the same number of nodes 253270, the triangular finite elements 505680, the power of system of linear equations 1517040. The analysis of exostructure can be conducted by using the power coefficient of the fractal's exostructure. This power coefficient shows the ratio of the work of external forces to the total external loads.

3. Results and discussion

The exostructure is the external structure of the three-dimension set. The exostructure consist of exobase, surface of f-quarks and crown (Figure1).
A quark is a type of elementary particle and a fundamental constituent of matter in physical theory. On the base of this analogy, authors introduce the notion of f-quark as a unit of the exostructure. F-quarks are classified in depending on their form in the structure of fractal:

- quasielliptic f-quarks;
- quasishperical f-quarks.

The exobase is the bottom support part of the fractal exostructure. The reference points of exobase are the reference points of quasielliptic f-quarks, which form funnel at the center by joining. Increasing power leads to the exobase becomes more detailed and occupies a smaller volume of the fractal exostructure.

The Surface of f-quarks is the middle part of the fractal exostructure and consist of quasishperical f-quarks which have homogeneous surface distribution.

The Crown is the top part of the fractal exostructure and consist of quasielliptic f-quarks. An increasing power leads to the crown becomes more detailed and occupies a smaller volume of the fractal exostructure.

A study compared of influence even and odd powers on shaping of the exostructure.

For power 4, the exostructure is asymmetric and heterogeneous with respect vertical axis. Evolution of the surface of f-quarks occurs with considerable fluctuations by volume. The boundary of the transition of the exobase to the surface of f-quarks is implicit, at the same time the crown occupies ¼ of the exostructure (Figure 2a). For power 5, the exostructure is symmetrical with respect vertical and horizontal axes. The crown of the exostructure repeats the exobase completely, the quasielliptic f-quarks form funnel in the both cases. F-quarks have a homogeneous distribution by surface (Figure 2b).
The exostructure of the power 8 fractal has a pronounced exobase consisting of seven supporting quasielliptic f-quarks, which form funnel (Figure 2c). The evolution of the fractal occurs relative to the central vertical axis. Each level of surface of f-quarks consist of the seven quasispherical f-quarks. Specialty of the even power of fractal is its crown, which consist of the quasielliptic f-quarks forming the top. Total number of the f-quarks is 56, seven on eight levels of the structure. The exostructure of the odd power 9 fractal is absolutely symmetrical; it means that the shape of the crown coincides with the shape of the exobase (Figure 2d). They are formed by the quasielliptic f-quarks, which form funnel. Total number of f-quarks is 64, eight on eight levels of the structure.

Dependence of the formation of structures can be clearly seen when considering fractal powers more than 8. The results of shaping low power exostructures are significantly different from each other. Currently research of fractal forms indicates that in structures of odd power more than 9, crown and exobase are absolutely identical. The funnel of the crown simplifies the shape of the structure; therefore, odd power is not considered in the future.

In the numerical experiment of the fractal structures was investigated the even powers of fractals from 6 to 44. The structure of the power 6 is not symmetric, the crown is strongly pronounced, also the exobase occupies the third part of the exostructure volume. The maximum radius of the location of the surface points reaches 60.7 m.

Considering further exostructures, we note that the crown and exobase become more detailed with the increasing powers, but occupy a smaller volume of the fractal exostructure in constant to surface of f-quarks (Figure 3). Increasing power leads to more intensive distribution of f-quarks at the surface, the maximum radius tends to the original radius 50 m.

![Figure 3. Fractal structures: a) power 14; b) power 24; c) power 44.](image)

The fractal structure is considered as the base for the development of design solutions for a unique high-rise building. Therefore, the main task is to control the stress-strain state of load-bearing structures in order to ensure safety. This study allows to earlier identify the possibility of a building collapse under the influence of man-made or natural loads. The analysis of fractal structure was conducted by using the power coefficient of the fractal's exostructure.
Figure 4 shows a diagram of the change the power coefficient of the fractal's exostructure. For the complete compare of the external structure of fractal set was researched the spherical surface with the same input data. Power coefficient of the sphere conditionally corresponds to a fractal power 0 in a diagram.

For structure power 6, the power coefficient has extreme value 14.04. Then the curve of the diagram decreases nonlinearly and reach the minimum value 2.23, which one belong to powers 14 and 16. This means the possible advantage of the selected exostructures. After power 16 the diagram increases and reaches a new extreme value 6.78, which one belongs to power 24. Analysis of the research results shows that the changing of power coefficient has an undulating character, with every new extreme value less than the previous. Fluctuations of the values in the end of the diagram are not as significant as at the beginning of its developing. The sphere has value of the power coefficient nearly 2.52, which is more than the minimum power coefficient value 2.23 of the fractal exostructure power 14.

Analysis of the stress-strain state of fractal structures allows to establish that the structures of power from 12 to 18 has the best resistant to natural and man-made impacts. General principles of object shaping expand the practical recommendation and provide to consider in construction fractal structures of power 8 to 20.

The fractal theory is applicable to the creation of real objects, to design of elements of the building frame or the whole structure [11]. Predefined form is not a fixed or unchanged, but has a growth principle in accordance with a certain law. Fractal architecture captures the modification of parameters within the framework of self-similar natural structures or their shaping environment. Innovative 3D printing technology is used to solve the task of modeling fractal structures and the possibility of introducing them into fractal architecture.

The development of 3D printing allows creating objects that are different in geometry, colors, properties and material [12-16]. The fractal, as an object of unusual complex geometry, is most rationally to produce by 3D printing from ABS plastic.

Creation of 3D model of a fractal structure for printing has differences from modeling a fractal for visualization. It is necessary in preparing an object for 3D printing to have a closed geometry of the object without any overlapping and intersections of polygons in each element.

Figure 5 shows the three-dimensional fractal power 8 produced by 3D printing from ABS plastic. For creation of the fractal model was used design model executed in the program "External structure of the fractal set".
Figure 5. Model of the three-dimensional fractal power 8.

The 3D printer software analyzes the model, splits it into layers with a thickness of 0.15 mm before sending fractal structure to print. The inner space of the fractal is filled with honeycomb structure. The parameters of the filling are set in the 3D printer software, in this case 10%.

The fractal model consists of two parts connected by cylindrical rods with chamfers at the ends. As a result, we obtain an exact model of a three-dimensional fractal set with a significant drawback - a seam in the middle. Such defect can be avoided by several methods. The main one is creating an additional support for the model, which in the future will be manually removed and then the surface of the fractal will be processed. Another method requires expensive equipment and printing a model of two materials. The first material is standard plastic (ABS, PLA, PC, ASA, etc.), from which the outer structure of the fractal set will be made, the second material - plastic, water-soluble (PVA), from which additional support will be made. Consequently, the entire fractal will be printed as one object with high level of accuracy and quality.

4. Conclusions

The study of the high-rise building life cycle with respect to natural and man-made impacts is one of the main tasks of modern design of buildings and structures. Fractal geometry allows to develop a unique object which is characterized by reliability and survivability at all stages of the life cycle.

During this research authors developed a terminology for describing the three-dimensional Mandelbrot fractal. This allowed developing a fundamentally new procedure of modeling three-dimensional fractal and realizing the resistance analyses of fractal structures to natural and man-made influences.

The following tasks were solved:

1. External structures of various powers of three-dimensional fractal were modeled and researched. SCAD software allows to visualize external structure of different shapes of fractal. Fractals of even and odd powers from 4 to 44 were considered in the article.
2. The developed terminology allows to analyze and compare various powers of three-dimensional fractal. This simplifies further work and expands the possibilities for further study of the Mandelbrot fractal. Results of numerical modeling allow to develop practical recommendations for clarifying the design parameters of the unique object being designed.
3. The power coefficient of the fractal's exostructure allows to study the work of the structure. As a result, the power of the exostructure 14 is the most efficient structure. Research and assessment of the consequences of possible scenarios of natural and man-made impacts allows to analyze the work of structures. This approach will ensure the safe operation of the building throughout its life cycle.
4. The 3D model of the fractal was printed from ABS plastic. This model of power 8 of the fractal can be considered as a conceptual form of a building.

References
[1] Lymar T Yu, Matrova T S and Staroverova N Yu 2014 Fractal search algorithm in relational databases Bulletin of the South Ural State University Series Computational Mathematics and Software Engineering 3 61
[2] M Wilkinson et al 2012 J. Phys. A: Math. Theor. 45 415102
[3] Yu Ryzhikova et al 2018 J. Phys.: Conf. Ser. 1141 012059
[4] D D Khamidulina and S A Nekrasova 2018 IOP Conf. Ser.: Mater. Sci. Eng. 451 012026
[5] Kravchenko G M and Pudanova L I 2019 IOP Conf. Ser.: Mater. Sci. Eng. 698 022017
[6] S Feranie et al 2019 J. Phys.: Conf. Ser. 1280 022056
[7] Shanshan J., Jinxí Z., Baoshan H. Fractal analysis of effect of air void on freeze–thaw resistance of concrete. Construction and Building Materials. 2013 47 126–130.
[8] Rayneau-Kirkhope D, Mao Y and Farr R 2012 Ultralight fractal structures from hollow tubes Phys. Rev. Lett. 109 204301
[9] Optimization of fractal space frames under gentle compressive load 2013 (Phys. Rev.) E 87 063204
[10] Kravchenko G M, Vasi’ev S E and Pudanova L I 2017 External structure of the fractal set, The state registration patent for ECM N 2017615949 26 05.2017
[11] Kravchenko G. Modeling the External Structure of a Fractals. 2017 IOP Conference Series: Earth and Environmental Science, 90 012100
[12] Miss Prachi M. 2013 3D printing making the digital real International Journal of Engineering Sciences & Research Technology 2 1822-1825
[13] Zhiguo L., Ying C and Fanjie J. 2014 Analysis of the related concept of 3D printing building material Tianjin Construction Science and Technology 24 8-12
[14] Wittbrodt B T, Glover A G, Laureto J, Anzalone G C, Oppliger D, Irwin J L and Pearce J M 2013 Life-cycle economic analysis of distributed manufacturing with open-source 3-D printers Mechatronics 23 713-726
[15] Lipson H and Kurman M 2013 Fabricated: The new world of 3D printing (John Wiley & Sons)
[16] Das S, Bourell D L and Babu S S 2016 Metallic materials for 3D printing Mrs Bulletin 41 729-741