2-LOCAL AUTOMORPHISMS ON FINITE-DIMENSIONAL LIE ALGEBRAS

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ABSTRACT. We prove that every 2-local automorphism on a finite-dimensional semi-simple Lie algebra $\mathcal{L}$ over an algebraically closed field of characteristic zero is an automorphism. We also show that each finite-dimensional nilpotent Lie algebra $\mathcal{L}$ with $\dim \mathcal{L} \geq 2$ admits a 2-local automorphism which is not an automorphism.

Keywords: Semi-simple Lie algebra, nilpotent Lie algebra, automorphism, 2-local automorphism.

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1. INTRODUCTION

In last decades a series of papers have been devoted to mappings which are close to automorphism and derivation of associative algebras (especially of operator algebras and C*-algebras). Namely, the problems of description of so called local automorphisms (respectively, local derivations) and 2-local automorphisms (respectively, 2-local derivations) has been considered. Later similar problems were extended for non associative algebras, in particular, for Lie algebras case. The present paper is devoted to study of 2-local automorphisms of finite dimensional Lie algebras.

Let $\mathcal{A}$ be an associative algebra. Recall that a linear mapping $\Phi$ of $\mathcal{A}$ into itself is called a local automorphism (respectively, a local derivation) if for every $x \in \mathcal{A}$ there exists an automorphism (respectively, a derivation) $\Phi_x$ of $\mathcal{A}$, depending on $x$, such that $\Phi_x(x) = \Phi(x)$. These notions were introduced and investigated independently by Kadison [11] and Larson and Sourour [13]. Later, in 1997, P. Šemrl [14] introduced the concepts of 2-local automorphisms and 2-local derivations. A map $\Phi : \mathcal{A} \to \mathcal{A}$ (not linear in general) is called a 2-local automorphism (respectively, a 2-local derivation) if for every $x, y \in \mathcal{A}$, there exists an automorphism (respectively, a derivation) $\Phi_{x,y} : \mathcal{A} \to \mathcal{A}$ (depending on $x, y$) such that $\Phi_{x,y}(x) = \Phi(x)$, $\Phi_{x,y}(y) = \Phi(y)$. In [14], P. Šemrl described 2-local derivations and 2-local automorphisms on the algebra $B(H)$ of all bounded linear operators on the infinite-dimensional separable Hilbert space $H$. Namely, he has proved that every 2-local automorphism (respectively, 2-local derivation) on $B(H)$ is an automorphism (respectively, a derivation). A similar result for finite-dimensional case appeared later in [12]. Further, in [1], a new techniques was introduced to prove the same result for an arbitrary Hilbert space $H$ (no separability is assumed).

Afterwards the above considerations give arise to similar questions in von Neumann algebras framework. First positive results have been obtained in [2] and [3] finite and semi-finite von Neumann algebras respectively, by showing that all 2-local derivations on these algebras
are derivations. Finally, in [4], the same result was obtained for purely infinite von Neumann algebras. This completed the solution of the above problem for arbitrary von Neumann algebras.

It is natural to study corresponding analogues of these problems for automorphisms or derivations of non-associative algebras. We shall consider the case of finite-dimensional Lie algebras.

Let $\mathcal{L}$ be a Lie algebra. A derivation (respectively, an automorphism) $\Phi$ of a Lie algebra $\mathcal{L}$ is a linear (respectively, an invertible linear) map $\Phi : \mathcal{L} \to \mathcal{L}$ which satisfies the condition $\Phi([x, y]) = [\Phi(x), y] + [x, \Phi(y)]$ (respectively, $\Phi([x, y]) = [\Phi(x), \Phi(y)]$) for all $x, y \in \mathcal{L}$. The set of all automorphisms of a Lie algebra $\mathcal{L}$ is denoted by $\text{Aut}\mathcal{L}$.

The notions of a local derivation (respectively, a local automorphism) and a 2-local derivation (respectively, a 2-local automorphism) for Lie algebras are defined as above, similar to the associative case. Every derivation (respectively, automorphism) of a Lie algebra $\mathcal{L}$ is a local derivation (respectively, local automorphism) and a 2-local derivation (respectively, 2-local automorphism). For a given Lie algebra $\mathcal{L}$, the main problem concerning these notions is to prove that they automatically become a derivation (respectively, an automorphism) or to give examples of local and 2-local derivations or automorphisms of $\mathcal{L}$, which are not derivations or automorphisms, respectively.

For a finite-dimensional semi-simple Lie algebra $\mathcal{L}$ over an algebraically closed field of characteristic zero, the derivations and automorphisms of $\mathcal{L}$ are completely described in [9].

Recently in [6] we proved that every local derivation on semi-simple Lie algebras are derivations and gave examples of nilpotent finite-dimensional Lie algebras with local derivations which are not derivations.

Earlier in [5] the authors have proved that every 2-local derivation on a semi-simple Lie algebra $\mathcal{L}$ is a derivation, and showed that each finite-dimension nilpotent Lie algebra, with dimension larger than two, admits a 2-local derivation which is not a derivation.

In [8], Chen and Wang initiated study of 2-local automorphisms of finite-dimensional Lie algebras. They have proved that if $\mathcal{L}$ is a simple Lie algebra which belongs to one of the types $A_l$ ($l \geq 1$), $D_l$ ($l \geq 4$), or $E_k$ ($k = 6, 7, 8$) over an algebraically closed field of characteristic zero, then every 2-local automorphism of $\mathcal{L}$, is an automorphism.

In the present paper we generalize the result of [8] and prove that every 2-local automorphisms of finite-dimensional semi-simple Lie algebras over an algebraically closed field of characteristic zero is an automorphism. Moreover, we show that each finite-dimensional nilpotent Lie algebra admits a 2-local automorphism which is not an automorphism.

2. 2-LOCAL AUTOMORPHISMS ON FINITE-DIMENSIONAL SEMI-SIMPLE LIE ALGEBRAS

In this paper the notations concerning Lie algebras mainly follow [9].

All algebras and vector spaces are considered over an algebraically closed field $\mathbb{F}$ of characteristic zero.

The main result of this section is given as follows.
Theorem 2.1. Let $\mathcal{L}$ be an arbitrary finite-dimensional semi-simple Lie algebra over an algebraically closed field $\mathbb{F}$ of characteristic zero. Then any (not necessarily linear) 2-local automorphism $T : \mathcal{L} \to \mathcal{L}$ is an automorphism.

Let $\mathcal{L}$ be a Lie algebra. The center of $\mathcal{L}$ is denoted by $Z(\mathcal{L})$:

$$Z(\mathcal{L}) = \{ x \in \mathcal{L} : [x, y] = 0, \forall y \in \mathcal{L} \}.$$  

A Lie algebra $\mathcal{L}$ is said to be solvable if $\mathcal{L}^{(k)} = \{ 0 \}$ for some integer $k$, where $\mathcal{L}^{(0)} = \mathcal{L}$, $\mathcal{L}^{(k)} = [\mathcal{L}^{(k-1)}, \mathcal{L}^{(k-1)}]$, $k \geq 1$. Any Lie algebra $\mathcal{L}$ contains a unique maximal solvable ideal, called the radical of $\mathcal{L}$ and denoted by $\text{Rad}\mathcal{L}$. A non trivial Lie algebra $\mathcal{L}$ is called semi-simple if and only if its Killing form is non degenerate, i.e. $\langle x, y \rangle = 0$ for all $x, y \in \mathcal{L}$ implies that $x = 0$.

A Cartan subalgebra $\mathcal{H}$ of a semi-simple Lie algebra $\mathcal{L}$ is a nilpotent subalgebra which coincides with its centralizer: $C(\mathcal{H}) = \{ x \in \mathcal{L} : [x, h] = 0, \forall h \in \mathcal{H} \} = \mathcal{H}$.

A Cartan subalgebra $\mathcal{H}$ of a finite-dimensional semi-simple Lie algebra $\mathcal{L}$ is abelian, i.e. $[x, y] = 0$ for all $x, y \in \mathcal{H}$.

From now on in this section let $\mathcal{L}$ be a finite-dimensional simple Lie algebra of the rank $l$, and let $\mathcal{H}$ be a fixed standard Cartan subalgebra of $\mathcal{L}$ with the corresponding root system $R \subset \mathcal{H}$. Denote by $\Delta = \{ \alpha_1, \alpha_2, \ldots, \alpha_l \}$ a fixed base of $R$, and by $R^+$ the set of corresponding positive root system of $\mathcal{L}$ relative to $R$. For $\alpha \in R$, let

$$L_\alpha = \{ x \in \mathcal{L} : [h, x] = \alpha(h)x, \forall h \in \mathcal{H} \}$$

be the one-dimensional root space relative to $\alpha$. For each $\alpha \in R^+$, let $e_\alpha$ be a non-zero element of $L_\alpha$, then there is a unique element $e_{-\alpha} \in L_{-\alpha}$ such that the linear span of the elements $e_\alpha$, $e_{-\alpha}$, and $h_\alpha = [e_\alpha, e_{-\alpha}]$ is a three-dimensional simple subalgebra of $\mathcal{L}$.
isomorphic to the simple Lie algebra \( \mathfrak{sl}(2, \mathbb{F}) \). The set \( \{h_\alpha, e_\alpha, e_{-\alpha} : \alpha \in R^+\} \) forms a basis of \( \mathcal{L} \).

By [7, Lemma 2.2], there exists an element \( d \in \mathcal{H} \) such that \( \alpha(d) \neq \beta(d) \) for every \( \alpha, \beta \in R, \alpha \neq \beta \). Such elements \( d \) are called strongly regular elements of \( \mathcal{L} \). Again by [7, Lemma 2.2], every strongly regular element is a regular semi-simple element, i.e.

\[ \{x \in \mathcal{L} : [d, x] = 0\} = \mathcal{H}. \]

Choose a fixed strongly regular element \( d \in \mathcal{H} \). Also set

\[ (2.1) \quad q = \sum_{\alpha \in R} e_\alpha. \]

**Lemma 2.2.** Let \( \Phi \) be an automorphism on \( \mathcal{L} \) such that \( \Phi(d) = d \). Then

(i) For any \( h \in \mathcal{H}, \Phi(h) = h; \)

(ii) For any \( \alpha \in R, \) there exists a nonzero \( c_\alpha \in \mathbb{F} \) such that \( \Phi(e_\alpha) = c_\alpha e_\alpha. \)

**Proof.** For \( \alpha \in R \) we have

\[ [d, \Phi(e_\alpha)] = [\Phi(d), \Phi(e_\alpha)] = \Phi([d, e_\alpha]) = \Phi(\alpha(d)e_\alpha) = \alpha(d)\Phi(e_\alpha), \]

i.e.

\[ (2.2) \quad [d, \Phi(e_\alpha)] = \alpha(d)\Phi(e_\alpha). \]

Let us represent the element \( \Phi(e_\alpha) \) in the form

\[ \Phi(e_\alpha) = h_\alpha + \sum_{\beta \in R} c_\beta e_\beta, \]

where \( h_\alpha \in \mathcal{H}, c_\beta \in \mathbb{F}. \) Multiplying both sides of the last equality by \( \alpha(d) \) and taking into account (2.2), we have

\[ (2.3) \quad \sum_{\beta \in R} c_\beta \alpha(d)e_\beta = \alpha(d)h_\alpha + \sum_{\beta \in R} c_\beta \alpha(d)e_\beta. \]

By properties of strongly regular elements, \( \alpha(d) \neq 0, \) and \( \beta(d) \neq \alpha(d) \) for any \( \beta \neq \alpha. \) Thus by the equality (2.3), \( h_\alpha = 0 \) and \( c_\beta = 0 \) for any \( \beta \neq \alpha, \beta \in R \). Therefore \( \Phi(e_\alpha) = c_\alpha e_\alpha. \)

Since \( \Phi \) is a bijection, then \( \Phi(e_\alpha) \neq 0, \) and so \( c_\alpha \neq 0. \) Thus (ii) holds.

Take an arbitrary element \( h \in \mathcal{H}. \) We have

\[ [d, \Phi(h)] = [\Phi(d), \Phi(h)] = \Phi([d, h]) = \Phi(0) = 0. \]

Since \( d \) is a regularly semi-simple element, it follows that \( \Phi(h) \in \mathcal{H}. \)

For every \( \alpha \in R \) we have

\[ [\Phi(h), \Phi(e_\alpha)] = \Phi([h, e_\alpha]) = \Phi(\alpha(h)e_\alpha) = \alpha(h)\Phi(e_\alpha) = \alpha(h)c_\alpha e_\alpha. \]

i.e.

\[ [\Phi(h), \Phi(e_\alpha)] = \alpha(h)c_\alpha e_\alpha. \]

On the other hand, taking into account \( \Phi(h) \in \mathcal{H}, \) we have

\[ [\Phi(h), \Phi(e_\alpha)] = [\Phi(h), c_\alpha e_\alpha] = c_\alpha \alpha(h)\Phi(h)e_\alpha. \]
Thus \( c_\alpha \alpha(h) = c_\alpha \alpha(\Phi(h)) \), which implies that \( \alpha(h) = \alpha(\Phi(h)) \) for any \( \alpha \in R \). Therefore \( \Phi(h) = h \), i.e., (i) holds. The proof is complete.

The following assertion has been proved in [8, Lemma 3.3] for the case of simple algebras of types \( A_l \) \((l \geq 1)\), \( D_l \) \((l \geq 4)\), \( E_k \) \((k = 6, 7, 8)\). From Lemma 2.2 we get the following generalization for an arbitrary semi-simple Lie algebra.

**Lemma 2.3.** Let \( T \) be a 2-local automorphism \( L \) such that \( T(d) = d \). Then

1. \( T(h) = h \) for all \( h \in \mathcal{H} \);
2. for every \( \alpha \in R \), there exists a nonzero \( c_\alpha \in \mathbb{F} \) such that \( T(e_\alpha) = c_\alpha e_\alpha \).

**Proof.** For \( h \in \mathcal{H} \) take an automorphism \( \Phi_{h,d} \) such that

\[ \Phi_{h,d}(h) = T(h) \text{ and } \Phi_{h,d}(d) = T(d). \]

Since \( \Phi_{h,d}(d) = T(d) = d \), Lemma 2.2 implies that \( \Phi_{h,d}(h) = h \), and therefore \( T(h) = h \).

By a similar way we can check (2). The proof is complete.

**Lemma 2.4.** Let \( T \) be a 2-local automorphism on a finite-dimensional semi-simple Lie algebra \( L \) such that \( T(d) = d \). Then \( T \) is linear.

**Proof.** Firstly we show that

\[ \langle T(x), T(y) \rangle = \langle x, y \rangle \]

for all \( x, y \in L \). Indeed, taking into account \( \text{Aut}(L) \)-invariance of the Killing form we obtain that

\[ \langle T(x), T(y) \rangle = \langle \Phi_{x,y}(x), \Phi_{x,y}(y) \rangle = \langle x, y \rangle. \]

Now let \( x, y, z \in L \) be arbitrary elements. Using equality (2.4) we obtain that

\[ \langle T(x+y), T(z) \rangle = \langle x+y, z \rangle = \langle x, z \rangle + \langle y, z \rangle = \langle T(x), T(z) \rangle + \langle T(y), T(z) \rangle = \langle T(x)+T(y), T(z) \rangle, \]

i.e.

\[ \langle T(x+y), T(z) \rangle = \langle T(x)+T(y), T(z) \rangle. \]

or

\[ \langle T(x+y) - T(x) - T(y), T(z) \rangle = 0. \]

Putting in the last equality \( z = h \in \mathcal{H} \) and \( z = e_\alpha \), where \( \alpha \in R \), by Lemma 2.3 we obtain that

\[ \langle T(x+y) - T(x) - T(y), h \rangle = 0 \]

and

\[ \langle T(x+y) - T(x) - T(y), e_\alpha \rangle = 0 \]

for all \( h \in \mathcal{H} \) and \( \alpha \in R \). Taking into account bilinearity of the Killing form we have that

\[ \langle T(x+y) - T(x) - T(y), w \rangle = 0 \]

for all \( w \in L \). Since the Killing form \( \langle \cdot, \cdot \rangle \) is non-degenerate, the last equality implies that \( T(x+y) = T(x)+T(y) \) for all \( x, y \in L \).
Finally,
\[ T(\lambda x) = \Phi_{\lambda x,x}(\lambda x) = \lambda \Phi_{\lambda x,x}(x) = \lambda T(x). \]
So, \( T \) is linear. The proof is complete. \( \square \)

**Proof of Theorem 2.1.** Let \( T : \mathcal{L} \to \mathcal{L} \) be a 2-local automorphism and suppose that \( d \in \mathcal{H} \) is a strongly regular element and \( q \) is the element defined by (2.1). Take an automorphism \( \Phi_{d,q} \) on \( \mathcal{L} \) (depending on \( d \) and \( q \)) such that
\[ \Phi_{d,q}(d) = T(d) \quad \text{and} \quad \Phi_{d,q}(q) = T(q). \]
Set \( T_1 = \Phi_{d,q}^{-1} \circ T \). Then \( T_1 \) is a 2-automorphism such that
\[ T_1(d) = d \quad \text{and} \quad T_1(q) = q. \]
By Lemma 2.4, \( T_1 \) is linear.
Taking into account Lemma 2.3 we have
\[ T_1(q) = T_1 \left( \sum_{\alpha \in R} e_\alpha \right) = \sum_{\alpha \in R} T_1(e_\alpha) = \sum_{\alpha \in R} c_\alpha e_\alpha. \]
On the other hand
\[ T_1(q) = q = \sum_{\alpha \in R} e_\alpha. \]
Comparing the last two equalities we obtain that \( c_\alpha = 1 \) for all \( \alpha \in R \). So,
\[ T_1(h) = h \quad \text{and} \quad T_1(e_\alpha) = e_\alpha \]
for all \( h \in \mathcal{H} \) and \( \alpha \in R \). Finally, in view of the linearity of \( T_1 \), the last two equalities imply that \( T_1(x) = x \) for all \( x \in \mathcal{L} \). Thus \( T = \Phi_{d,q} \) is an automorphism. The proof is complete. \( \square \)

3. 2-LOCAL DERIVATIONS ON NILPOTENT LIE ALGEBRAS

In this section we give examples of 2-local automorphisms on nilpotent Lie algebras which are not automorphisms.

A Lie algebra \( \mathcal{L} \) is called **nilpotent** if \( \mathcal{L}^k = \{0\} \) for some \( k \in \mathbb{N} \), where \( \mathcal{L}^0 = \mathcal{L} \), \( \mathcal{L}^k = [\mathcal{L}^{k-1}, \mathcal{L}] \), \( k \geq 1 \).

Let \( D \) be a nilpotent derivation on \( \mathcal{L} \), i.e. \( D^n = 0 \) for some \( n \in \mathbb{N} \). Then
\[ \Phi(x) = (\exp D)(x) = x + D(x) + \frac{D^2(x)}{2!} + \cdots + \frac{D^{n-1}(x)}{(n-1)!} \]
is an automorphism (see, e.g. [10, P. 9]).

**Theorem 3.1.** Let \( \mathcal{L} \) be a \( n \)-dimensional Lie algebra with \( n \geq 2 \). Suppose that
(i) \( \dim[\mathcal{L}, \mathcal{L}] \leq n - 2; \)
(ii) the intersection \( Z(\mathcal{L}) \cap [\mathcal{L}, \mathcal{L}] \) is non trivial.

Then \( \mathcal{L} \) admits a 2-local automorphism which is not an automorphism.
**Proof.** Let us consider a decomposition of $\mathcal{L}$ in the following form

$$\mathcal{L} = [\mathcal{L}, \mathcal{L}] \oplus V.$$  

Due to the assumption $\dim[\mathcal{L}, \mathcal{L}] \leq n - 2$, we have $\dim V = k \geq 2$. Let $\{e_1, \ldots, e_k\}$ be a basis of $V$ and let $f$ be a homogeneous non additive function on $\mathbb{F}^2$.

According to the assumptions (ii) of the theorem there exists a non zero central element in $[\mathcal{L}, \mathcal{L}]$. Let us fix it as $z \in Z(\mathcal{L})$. By the proof of Theorem 3.1 from [5] it follows that an operator $T$ on $\mathcal{L}$ defined as

$$T(x) = f(\lambda_1, \lambda_2)z, \quad \text{for} \quad x = x_1 + \sum_{i=1}^{k} \lambda_i e_i \in \mathcal{L},$$

where $\lambda_i \in \mathbb{F}, i = 1, \ldots, k, x_1 \in [\mathcal{L}, \mathcal{L}]$, is a 2-local derivation which is not a derivation.

Set

$$\Delta(x) = x + T(x).$$

Let us show that $\Delta$ is a 2-local automorphism. Again by the proof of Theorem 3.1 from [5] it follows that for every pair $x, y \in \mathcal{L}$ there exist $a, b \in \mathbb{F}$ such that the operator $D$ defined as

$$D(x) = (a\lambda_1 + b\lambda_2)z, \quad \text{for} \quad x = x_1 + \sum_{i=1}^{k} \lambda_i e_i \in \mathcal{L},$$

is a derivation and

$$T(x) = D(x), \quad T(y) = D(y).$$

Since $D(\mathcal{L}) = \{\lambda z : \lambda \in \mathbb{F}\}$ and $z \in Z(\mathcal{L}) \cap [\mathcal{L}, \mathcal{L}]$, it follows that $D$ is a nilpotent derivation of order 2, i.e. $D^2 = 0$. Therefore,

$$\Phi(x) = (\exp D)(x) = x + D(x)$$

is an automorphism. Since

$$\Delta(x) = \Phi(x), \quad \Delta(y) = \Phi(y),$$

$\Delta$ is a 2-local automorphism, as required. The proof is complete. \hfill $\square$

Let $\mathcal{L}$ be an $n$-dimensional nilpotent Lie algebra with $n \geq 2$. Then $[\mathcal{L}, \mathcal{L}] \neq \mathcal{L}$, otherwise, $\mathcal{L}^k = \mathcal{L} \neq \{0\}$ for all $k \geq 1$.

Suppose that $\dim[\mathcal{L}, \mathcal{L}] = n - 1$. Then direct computations show that

$$\{0\} \neq \mathcal{L}^1 = [\mathcal{L}, \mathcal{L}] = [[\mathcal{L}, \mathcal{L}], \mathcal{L}] = \mathcal{L}^2,$$

and therefore $\mathcal{L}^k = \mathcal{L}^{k+1} \neq \{0\}$ for all $k \in \mathbb{N}$. This contradicts the nilpotency of $\mathcal{L}$. So $\dim[\mathcal{L}, \mathcal{L}] \leq n - 2$.

Note that for any nilpotent Lie algebra $Z(\mathcal{L}) \cap [\mathcal{L}, \mathcal{L}] \neq \{0\}$, because $\{0\} \neq \mathcal{L}^{k-1} \subseteq Z(\mathcal{L}) \cap [\mathcal{L}, \mathcal{L}]$, where $\mathcal{L}^{k-1} \neq \{0\}$ and $\mathcal{L}^k = \{0\}$. So, Theorem 3.1 implies the following result.

**Theorem 3.2.** Let $\mathcal{L}$ be a finite-dimensional nilpotent Lie algebra with $\dim \mathcal{L} \geq 2$. Then $\mathcal{L}$ admits a 2-local automorphism which is not an automorphism.

The corollary below is an immediate consequence of Theorem 3.2.
Corollary 3.3. Let $\mathcal{L}$ be a finite-dimensional abelian Lie algebra with $\dim \mathcal{L} \geq 2$. Then $\mathcal{L}$ admits a 2-local automorphism which is not an automorphism.

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