Thermal analysis of flat roof systems with regards to their thermal insulation and exterior surface emissivity coefficient

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Abstract. In this study, a numerical analysis of the thermal performance of a flat roof system is presented. The dynamic thermal behaviour of this system is assessed with regards to the effects produced by an insulation layer and the thermal emissivity of the exterior surface exposed to the outdoor environment thermal conditions, such as the ambient air temperature and radiation (short-wave and long-wave). Analysed building components correspond to insulated and non-insulated flat roof assemblies formed by reinforced concrete. For their exterior surface, a varying value of its thermal emissivity is assumed. In this context, the aim of this investigation is to assess the dynamic thermal behaviour of multi-layered roof configurations in terms of thermal flux through their construction; this allows us to determine the overall heat exchange in a daily basis, in the aspect of the above-mentioned parameters. In order to analyse the thermal response of the flat roof located in the Greek region, during the cooling period, a FEM model is introduced (one-dimensional model). Results underline the significance to consider the thickness of the thermal insulation layer, as well as the effect of various emissivity coefficients on heat transfer processes.

Keywords: flat roof, thermal analysis, heat flux, insulation thickness, emissivity coefficient

Nomenclature

\( A \) surface area \([m^2]\)

\( d \) layer thickness \([m]\)

\( \lambda \) thermal conductivity, heat transfer coefficient due to conduction \([W/(m \cdot K)]\)

\( \rho \) bulk density \([kg/m^3]\)

\( C_p \) specific heat capacity at constant pressure \([J/(kg \cdot K)]\)

\( \rho \cdot C_p \) volumetric heat capacity \([J/(m^3 \cdot K)]\)

\( R \) thermal resistance \([K/W]\)

\( C \) thermal capacitance \([J/K]\)

\( T_{out} \) ambient-air temperature \([K]\)

\( T_{in} \) indoor space temperature \([K]\)

\( T_{sur} \) surface temperature \([K]\)

\( T_{se} \) external surface temperature \([K]\)
1. Introduction

One of the most critical challenges that all modern societies have to face is the high energy demand. It is a real fact that in the European region the building sector accounts about 40% of the total energy consumption. It is therefore obvious that particular attention should be paid in order to minimize energy consumption in buildings providing at the same time health, safety and thermal comfort to its residents.

Several studies dealing with the above-mentioned problem have been conducted throughout the years. Stazi et al. [1] analysed the effect of different building envelope techniques on energy consumption, comfort levels and other environmental aspects, considering the Mediterranean climate with a hot and dry summer. In the same direction, Kontoleon and Bikas [2] underlined the importance of the solar absorptivity of an opaque wall on the thermal inertia parameters, such as time lag and decrement factor by employing a dynamic thermal-network model. In another study, Zbigniew Zapalowicz [3] investigated the influence of the irradiance and ambient temperature on the roof coating temperature. Furthermore, he calculated the heat flux through the roof into the interior of the building by exposing that a proper material selection for the roof coating can significantly decrease the heat flux into the environment just below the roof. Recently, thermal inertia parameters were also investigated in terms of heat flux transferred from the external to the internal side of walls by considering the variation of wall solar absorptivity and orientation, air conditioning regime, locality and presence of shortwave thermal excitation on the internal side of the wall [4].

In this study, a numerical analysis of the thermal performance of a flat roof system is presented. The dynamic thermal behaviour of this system is assessed with regards to the thermal emissivity of the exterior surface exposed to the outdoor environmental conditions. These conditions correspond to the ambient air temperature, as well as to short-wave and long-wave radiation heat exchanges. The mathematical formulation defining the physical processes taking place is given in the second section of this study. Section three provides an overview of the modelling of a flat roof configuration exposed to specific external and internal environmental conditions (during the cooling period in the northern region of Greece). It is essential to mention that the analysed building components correspond to insulated and non-insulated reinforced concrete slabs with a varying thermal emissivity coefficient of their exterior surface. In this section, the FEM program (COMSOL Multiphysics) is also presented. Results obtained from the thermal analysis are shown in section four. As it is seen, the effect of emissivity on both exterior and interior surface temperatures, as well as on daily heat exchanges between the interior surface and the indoor environment is significant. Its impact on the thermal response of a flat roof is highly affected by the existence of the thermal insulation layer. Finally, section five concludes the results occurred from this study.

2. Thermal response of horizontal building elements in the aspect of their emissivity

An essential part as regards the design of a building envelope is the determination of the dynamic thermal response of its exterior surfaces. In this context, horizontal surfaces affect significantly the
overall heat fluxes through building envelopes. As it is well known, the building envelope is the physical barrier between the outdoor and indoor environment. Accordingly, the one-dimensional heat flow through building assemblies relies on the geometrical characteristics and the thermophysical properties of material layers \((d, \lambda, \rho \text{ and } C_p)\); the aforementioned aspects underline the thermal resistance \(R\) and thermal capacity \(C\) of a building system. The mathematical formulation of \(R\) and \(C\) is given below [5]:

\[
R = \frac{d}{\lambda \cdot A} \tag{1}
\]

\[
C = \rho \cdot C_p \cdot d \cdot A \tag{2}
\]

These aspects affect the heat transfer mechanisms through multi-layered building components in terms of temperature fluctuations in the time domain.

A common approach to analyse and assess heat propagation via a building component is to adopt the sol-air temperature concept for the outdoor environment. More specifically, the sol-air temperature acting on a flat roof assembly corresponds to an equivalent forcing function that takes into account: (a) the ambient air temperature variations \(T_{out}\), (b) the impact of incident solar radiation \(Q_{sol}\), while considering the solar absorptivity \(\alpha_{sol}\) of a studied surface and (c) the heat exchanges with long-wave radiation between the studied horizontal surface \(T_{sur}\) and the sky-dome \(T_{sky}\) as a function of its emissivity \(\varepsilon\). For the aims of this investigation a mathematical expression proposed by the Chartered Institution of Building Services Engineers (CIBSE) [6] is applied:

\[
T_{sol-air} = T_{out} + \frac{\alpha_{sol} \cdot Q_{sol}}{h_e} + \varepsilon \cdot \sigma \cdot \left(\frac{T_{sky}^4 - T_{sur}^4}{h_e}\right) \tag{3}
\]

Figure 1. Schematic representation of the considered thermal model for the flat roof system.

As shown in equation (3), the thermal emissivity of the exterior surface affects critically heat flows through the roof configuration. In this context, the thermal emissivity describes a material’s ability to emit or release the thermal energy which has absorbed. A perfect radiator –known as a ‘black body’– emits the entire amount of absorbed energy, while common materials emit less energy than a black body at the same temperature. The emissivity of different materials depends on their chemical properties as well as the nature of their final surface. Typical building materials may have an emissivity coefficient value ranging from 0.60 to 0.90. In many studies, it is assumed – as a simplification of the modelling process- that the emissivity has little impact on the thermal response of the building components. However, in some cases, the role of the long-wave radiation heat exchanges seems to be more important. For instance, when very high temperatures appear in the outer surface of a roof, a high emissivity coating may help to discharge the absorbed heat and finally achieve a more comfortable indoor environment. In this context, it would be interesting to investigate the extent to
which higher values of the emissivity coefficient can improve the thermal performance of a roof configuration during the cooling period.

3. Thermal model

3.1. Analysed roof assemblies
The assemblies under investigation correspond to horizontal reinforced concrete slabs that may incorporate a thermal insulation layer in their exterior surface as shown in figure 2. The reinforced concrete slabs have a constant thickness of \( d_{\text{slab}} = 0.20 \text{ m} \), while they may be uninsulated or have an insulation layer of thickness \( d_{\text{ins}} = 0.10 \text{ m} \). The insulation material considered for the building components is XPS (rigid extruded polystyrene foam board insulation). It is also assumed that the outer surface of the flat roof configuration has a coating of negligible thickness. Moreover, the outdoor absorption coefficient of the coating is taken equal to \( \alpha_{\text{sol}} = 0.50 \). The thermal response of the roof configurations was investigated for different values of the thermal emissivity coefficient of the outer coating, \( \varepsilon = 0, \varepsilon = 0.50, \varepsilon = 0.95 \). The geometrical characteristics and the thermophysical properties of building materials applied to the thermal model are listed in table 1.

![Figure 2. Investigated roof configuration including a concrete slab, a thermal insulation layer and a coating of negligible thickness with solar absorptivity \( \alpha \) and thermal emissivity \( \varepsilon \).](image)

Table 1. Geometrical characteristics and thermophysical properties of building materials.

| Building material       | Layer thickness \( d \) [m] | Thermal conductivity \( \lambda \) [W/(m·K)] | Bulk density \( \rho \) [kg/m\(^3\)] | Specific heat capacity \( C_p \) [J/(kg·K)] |
|------------------------|-----------------------------|---------------------------------------------|-------------------------------------|---------------------------------------------|
| Reinforced concrete    | 0.20                        | 2.500                                       | 2,400                               | 1,000                                       |
| Extruded polystyrene   | 0.10                        | 0.035                                       | 40                                  | 1,450                                       |

3.2. Environmental conditions
The consideration of precise meteorological information is decisive in order to analyse accurately the thermal performance of the assumed building components. The current thermal analysis is carried out for a typical day during the summer period in the mild Mediterranean region by considering Thessaloniki climatic conditions (northern Greece at latitude 40° 38’ N). The cooling period corresponds to the high levels of temperatures and sunshine, which in turn affect significantly the heat flows through building envelopes. It is possible to regulate the evolution of heat by means of the sol-air concept \( T_{\text{sol-air}} \) by equation (3). The values of the (a) daily ambient-air temperature variations \( T_{\text{out}} \), (b) incident solar radiation variations \( Q_{\text{sol}} \) and (c) sky temperature variations \( T_{\text{sky}} \) are specified in figure 3(a)-(b). These data [7] are illustrative of the considered conditions that take place in the investigated area. Finally, the indoor temperature setting \( T_{\text{in}} \) is assumed to be constant in the time domain and its value is taken equal to 25°C.
3.3. Transient analysis

For the aims of this study, a thermal analysis has been carried out by using the COMSOL Multiphysics software. A one-dimensional thermal model has been adopted, that makes provision for the occurring heat transfer mechanisms, in time-steps of $\Delta t = 300$ s (5 minutes). The thermal analysis is carried out for 7 days, the first 6 of which are pre-conducted to eliminate the initial effect (stabilize the system to a steady periodic state). It is also worth mentioning that the implemented solver uses the Finite Element Method to resolve this dynamic problem (heat transfer processes); thus, the analytical equations used are the following [8]:

- Conduction through the solid sections of the flat roof structure.

$$\rho \cdot C_p \frac{dT}{dt} + \rho \cdot C_p \cdot u \cdot \nabla T = \nabla \cdot (\lambda \cdot \nabla T) + Q \quad (4)$$

- Combined convective and short-wave radiative heat flux at both the internal and external boundaries of the examined assembly.

$$-n \cdot (-\lambda \cdot \nabla T) = h_i \cdot (T_{in} - T_{st}) \quad \text{and} \quad -n \cdot (-\lambda \cdot \nabla T) = h_e \cdot (T_{eq} - T_{se}) \quad (5)$$

- Long-wave radiative heat exchanges between the exterior surface of the flat roof and the sky.

$$-n \cdot (-\lambda \cdot \nabla T) = \varepsilon \cdot \sigma \cdot (T_{sky}^4 - T_{se}^4) \quad (6)$$

In order to define the outside boundary conditions, the variable $T_{eq}$ is used. As it can be seen in equation (7), this variable is an equivalent temperature excitation taking into account the ambient air temperature variations $T_{out}$ and the impact of incident short-wave solar radiation $Q_{sol}$, while considering the solar absorptivity $a_{sol}$ of the studied surface. Using equations (5) and (6) to describe the outside boundary conditions, the sol-air temperature concept described in section 2 can be compatibly adopted with the standards set by the software.

$$T_{eq} = T_{out} + \frac{a_{sol} \cdot Q_{sol}}{h_e} \quad (7)$$

The results of the study are summarized by means the following indices that quantify the flat roof thermal performance on a daily basis:

- the range of temperatures occurring at the outer side of the flat roof $\Delta T_{se} = T_{se,max} - T_{se,min}$;
- the difference between the indoor temperature (fixed at 25 °C) and the mean temperature of the exterior surface $\Delta T'_{se} = T_{se,mean} - T_{in}$;
- the range of temperatures occurring at the inner side of the flat roof $\Delta T_{si} = T_{si,max} - T_{si,min}$;
- the difference between the indoor temperature (fixed at 25 °C) and the mean temperature of the interior surface $\Delta T'_{si} = T_{si,mean} - T_{in}$;
- the daily heat exchange $E$ from the interior surface of the assumed flat roof configurations to the indoor environment $E = h_i \sum (T_{si} - T_{in}) \Delta t$. 

![Figure 3](image-url)
4. Results and discussion

4.1. External surface temperature variations $T_{se}$

The graphs below provide information about changes in the temperature of the external surface $T_{se}$ of a flat roof during a typical day in the cooling period, by taking into account different values of the thermal emissivity coefficient, $\epsilon = 0$, $\epsilon = 0.50$, $\epsilon = 0.95$. In figures 4(b) and 4(d), the temperature variations $\Delta T_{se}$ and $\Delta T'_{se}$ are presented (blue and green vertical bars of column graph, respectively).

As can be seen from the graphs in figure 4(a)-(d), the increase of the value of the thermal emissivity coefficient of the outer coating $\epsilon$ causes a significant decrease in the temperatures occurring at the exterior surface of the roof $T_{se}$, as well as to the temperature variations $\Delta T_{se}$ and $\Delta T'_{se}$.

For a non-insulated roof ($d_{ins} = 0$) having a thermal emissivity coefficient of $\epsilon = 0$ the maximum temperature noted is $T_{se,max} = 41.72 \, ^\circ C$, while for $\epsilon = 0.95$ the corresponding value is equal to $T_{se,max} = 37.08 \, ^\circ C$, representing a reduction of 4.64 °C. In a similar way, the maximum temperatures appearing in the outer surface of an insulated roof ($d_{ins} = 10$ cm) are $T_{se,max} = 57.11 \, ^\circ C$ and $T_{se,max} = 45.62 \, ^\circ C$ for $\epsilon = 0$ and $\epsilon = 0.95$, respectively. This corresponds to a decrease of 11.49 °C.

It is essential to underline that the occurrence of the insulation layer intensifies the temperature variations between the indoor and outdoor environment. In the case of the insulated roof higher temperatures occur in the external surface and so a high thermal emissivity outer coating can be largely instrumental to the thermal discharging of the roof.

Figure 4. Impact of the outer coating’s thermal emissivity $\epsilon$ on the external surface temperature $T_{se}$ during a typical day in the cooling period and on temperature variations $\Delta T_{se}$ / $\Delta T'_{se}$ of a flat roof either (a), (b) non-insulated $d_{ins} = 0$ cm or (c), (d) having an insulation layer of $d_{ins} = 10$cm.
4.2. Internal surface temperature variations $T_{si}$

Figures 5(a)-(d) illustrate the changes in the temperature of the internal surface of a flat roof system, taking into account different values of the thermal emissivity coefficient, $\varepsilon = 0, 0.50, 0.95$.

The increase of the thermal emissivity value causes a slight decrease in the temperatures appearing in the internal surface of the roof $T_{si}$, as well as in the temperature variations $\Delta T_{si}$ and $\Delta T'_{si}$ (blue and green rectangular bars of column graph, respectively).

For a non-insulated roof ($d_{ins} = 0$) with a thermal emissivity coefficient $\varepsilon = 0$, the maximum temperature appearing is $T_{si,max} = 33.03$ °C, while for $\varepsilon = 0.95$ it decreases to $T_{si,max} = 30.57$ °C, representing a reduction of 2.46 °C. Evidently, the effect of the thermal emissivity affects substantially heat fluxes due to long-wave radiation and the ensuing temperature variations at the interior surface of the analysed flat roof configurations.

In the case of an insulated roof ($d_{ins} = 10$ cm), the maximum temperatures noted are $T_{si,max} = 25.47$ °C and $T_{si,max} = 25.25$ °C for $\varepsilon = 0$ and $\varepsilon = 0.95$, respectively. This corresponds to a 0.22 °C decrease.

At last, as it regards to the indoor temperature swing it is mostly affected by the changes of the thermal emissivity value in the case of the non-insulated roof.

![Figure 5](image-url)
4.3. Daily heat exchanges between the interior surface and the indoor environment

Figures 6(a) and 6(b) illustrate the thermal energy $E$ that is transferred from the internal surface of a flat roof to the indoor environment on a daily basis, taking into account different values of the thermal emissivity coefficient of the roof’s outer coating. In general, an increase of the thermal emissivity value causes an almost linear decline to the thermal energy, while the energy transferred in the case of a non-insulated roof is an order of magnitude higher than that of an insulated one. More specifically, as it regards to the non-insulated roofs the thermal energy transferred from the internal roof surface to the indoor environment is $E = 1.027\ \text{kWh/m}^2\text{/day}$, $E = 0.729\ \text{kWh/m}^2\text{/day}$ and $E = 0.524\ \text{kWh/m}^2\text{/day}$ for $\varepsilon = 0$, $\varepsilon = 0.50$ and $\varepsilon = 0.95$, respectively. For the case of an insulated roof the thermal energy is equal to $E = 0.083\ \text{kWh/m}^2\text{/day}$, $E = 0.055\ \text{kWh/m}^2\text{/day}$, $E = 0.038\ \text{kWh/m}^2\text{/day}$, for $\varepsilon = 0$, $\varepsilon = 0.50$ and $\varepsilon = 0.95$, respectively. In case of both a non-insulated and an insulated roof, when the thermal emissivity value increases from $\varepsilon = 0.50$ to $\varepsilon = 0.95$, there is an approximately 30% decrease of the thermal energy transferred from internal roof surface to the indoor environment. Thus, the thermal emissivity of the outer coating affects significantly the heat flux into the indoor environment just below the roof.

![Figure 6](image_url)

**Figure 6.** Thermal energy $E$ transferred from the internal surface of the roof to the indoor environment on a daily basis for different values of the thermal emissivity $\varepsilon$ of the outer coating of (a) an uninsulated roof $d_{\text{ins}} = 0$ cm and (b) a roof with an insulation layer of $d_{\text{ins}} = 10$ cm.

5. Conclusions

The present study investigated the effect of thermal emissivity on the thermal performance of a flat roof. The influence of thermal emissivity was analyzed in terms of the temperature variations on both internal and external surface of the roof, as well as the thermal energy transferred from the interior surface of the roof to the indoor environment. The thermal analysis was carried out for roof formations consisting of reinforced concrete with either an XPS insulation layer in their external side or no insulation at all and an outer coating of negligible thickness with a fixed solar absorptivity coefficient and varying thermal emissivity. Simulation results have been obtained by employing a dynamic thermal-network model that has been analyzed in discrete time steps, using the nodal approach.

It has been shown that thermal emissivity, which affects the exterior forcing function (sol-air temperature), has a significant effect on the temperature variations of the roof’s surfaces. The increase of thermal emissivity coefficient of the outer coating leads to a reduction of both internal and external surface temperatures. Analogously, the same decreasing trend was highlighted on the outdoor and indoor temperature swing. The existence of the insulation layer proved to intensify the temperature variations between the indoor and outdoor environment. More specifically, comparing the case of a
non-insulated roof with an insulated one, under the same thermal excitation, has shown that the later has higher temperatures in its external and lower temperatures in its internal surface. As a consequence, a high thermal emissivity outer coating established on an insulated roof can lead to a substantial decrease of the external surface temperatures. Finally, results underlined the significance of the thermal emissivity of the outer coating as a mean to mitigate the heat transfer from the internal surface of the roof to the indoor environment during the cooling period.

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