Nonperturbative definition of closed string theory via open string field theory

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Abstract

In typical examples of the AdS/CFT correspondence, the world-sheet theory with holes in the presence of D-branes is assumed to be equivalent in a low-energy limit to a world-sheet theory without holes for a different background such as $AdS_5 \times S^5$. In the case of the bosonic string, we claim under the assumption of this equivalence that open string field theory on $N$ coincident D-branes can be used to provide a nonperturbative definition of closed string theory based on the fact that the $1/N$ expansion of correlation functions of gauge-invariant operators reproduces the world-sheet theory with holes where the moduli space of Riemann surfaces is precisely covered.
1 Introduction

One of the most important problems in theoretical physics is to formulate quantum gravity in a consistent manner. While the quantization of general relativity in the framework of quantum field theory turned out to be difficult, it was found that string theory consistently describes on-shell scattering amplitudes involving gravitons. However, string theory only provides a perturbative definition of such on-shell scattering amplitudes with respect to the string coupling constant.

One possible approach to a nonperturbative formulation of string theory would be to introduce a spacetime field for each oscillation mode of the string and construct an action of those spacetime fields. The resulting theory in this approach is called string field theory. Since gravitons are described as states of the closed string, a natural approach to quantum gravity would be to construct closed string field theory. For closed bosonic string field theory, the gauge invariance of the classical action turned out to be anomalous and we need quantum corrections to the action at each loop order to recover the gauge invariance [1]. The existence of such closed string field theory is useful when we handle phenomena such as vacuum shift and mass renormalization in the perturbative string theory as reviewed in [2]. However, formulating closed string field theory at the quantum level nonperturbatively by the path integral does not seem to be promising because of these quantum corrections to the action. As the origin of the quantum corrections is related to the decomposition of the moduli space of Riemann surfaces, we do not expect any improvement of the situation in the generalization to closed superstring field theory.

Then how can we formulate string theory nonperturbatively? The typical origin of the string perturbation theory is the $1/N$ expansion of gauge theories with $N \times N$ matrix degrees of freedom [3]. Following this remarkable insight of ’t Hooft, the long history of research on string theory indicates that string theory can be defined nonperturbatively in terms of such gauge theory.

In this paper we claim that open string field theory instead of closed string field theory can play a role of such gauge theory and can be used to provide a nonperturbative definition of closed string theory. In the rest of this paper we present five questions and their answers which will lead us to this claim. The first question we ask is what kind of closed string theory we should consider.
2 Five questions

2.1 What kind of closed string theory should we consider?

While the $1/N$ expansion in the gauge theories has a structure of the genus expansion in string theory, we do not see the smooth world-sheet picture in Feynman diagrams of matrix fields written in the double-line notation. One attempt to generate the smooth world-sheet picture was to take the double scaling limit of matrix models [4, 5, 6]. This successfully defines string theory nonperturbatively, but it worked out only for low spacetime dimensions where physical degrees of freedom of gravitons are absent.

Then the conjecture called the AdS/CFT correspondence [7] was put forward, and it can be regarded as providing a nonperturbative definition of closed string theory in terms of a quantum field theory without containing gravity. Type IIB superstring theory on $AdS_5 \times S^5$, for example, is conjectured to be defined nonperturbatively by $\mathcal{N} = 4 U(N)$ super Yang-Mills theory in four dimensions, and the string coupling constant of type IIB superstring theory on $AdS_5 \times S^5$ is given by $1/N$ in accord with the idea by 't Hooft. To understand how closed string theory appears from a theory without gravity in this conjecture, let us recall the standard explanation of the AdS/CFT correspondence following section 3.1 of the review [8].

Consider type IIB superstring theory on a flat spacetime in ten dimensions with $N$ coincident D3-branes. In the low-energy region where the energy of the system is much lower than the string scale $1/\sqrt{\alpha'}$, closed strings and open strings are decoupled. Then closed string theory becomes a free theory in ten dimensions and open string theory becomes $\mathcal{N} = 4 U(N)$ super Yang-Mills theory in four dimensions.

Next consider type IIB superstring theory on the three-brane solution of supergravity. Because of the redshift factor, an object brought closer and closer to the three-brane appears to have lower and lower energy for the observer at infinity. In the same low-energy limit, excitations propagating in ten dimensions and excitations in the near horizon region are decoupled, and we have a free theory in ten dimensions and type IIB superstring theory on $AdS_5 \times S^5$, which is the near horizon geometry of the three-brane solution. We are then led to the conjecture that $\mathcal{N} = 4 U(N)$ super Yang-Mills theory in four dimensions is the same as type IIB superstring theory on $AdS_5 \times S^5$.

As can be seen from this explanation, the AdS/CFT correspondence tells us that the world-sheet theory with holes in the presence of D-branes is equivalent to a different world-sheet theory on a curved background in the low-energy limit. This equivalence was shown in the context of the large $N$ duality of the topological string [9] by Ooguri and Vafa [10]. For developments in the superstring, see, for example, [11] [12] [13]. While establishing this equivalence is a crucial ingredient for proving the AdS/CFT correspondence, we assume this equivalence in this paper and we instead concentrate on two other aspects. The first aspect is to see that the world-sheet
theory with holes is a consistent perturbation theory which can be interpreted as a theory of closed strings. The second aspect is how the world-sheet theory with holes can be reproduced by a theory from the open string sector.

Let us begin with the question of whether the world-sheet theory with holes can be interpreted as a consistent perturbation theory of closed strings. Compared with the discussion in the topological string, we have to be more careful in the physical string theory. First of all, the moduli space of Riemann surfaces with holes must be covered for consistency. In the moduli space there are regions where two boundaries are close, and such a region corresponds to propagation of an open string. So we may think that open strings are necessary for unitarity, and this should be a theory of closed strings and open strings. We claim that this is not necessarily the case in the context relevant for the AdS/CFT correspondence. First, in this context we focus on a sector which is analogous to the gauge-invariant sector of gauge theory. When we consider correlation functions of gauge-invariant operators in gauge theory, we never see poles from fields which are not gauge invariant. Second, correlation functions of one closed string vertex operator and one open string vertex operator on the disk are generically nonvanishing so that the closed string propagation and the open string propagation are mixed when the interaction is turned on. On-shell states in the interacting theory need to be identified by diagonalizing such propagators and the long propagation of an open string does not necessarily generates an on-shell pole. While the world-sheet contains holes, such on-shell states can be regarded as closed string states just as we regard Wilson loops as closed strings in gauge theory. Feynman diagrams of gauge theory in the double-line notation are similar to the world-sheet with holes, but one important difference is that the world-sheet with holes should be associated with Riemann surfaces, and the covering of the moduli space of Riemann surfaces is crucial for consistency. Note that the number of holes is irrelevant for the consistent world-sheet picture as long as the moduli space of Riemann surfaces is covered. This should be contrasted with the world-sheet picture which appears in the double scaling limit of the matrix models.

When we regard the world-sheet theory with holes as closed string theory, what corresponds to the string coupling constant? Let us consider the theory on $N$ coincident D-branes, and we organize the Feynman diagrams in terms the ’t Hooft coupling constant as usual. Then the coupling constant of closed string theory is given by $1/N$. Assuming the equivalence of this world-sheet theory with holes to a world-sheet theory without holes for a different background, we expect that this theory contains gravity in the low energy, and this is the perturbation theory that we want to reproduce by a theory without gravity. Let us now present our answer

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1 The situation is analogous to the mass renormalization of closed string theory reviewed in [2]. On-shell states in the interacting theory are generically different from those in the free theory.

2 In the hexagon approach based on the integrability [14], the world-sheet picture appears even for the weak-coupling region, and our viewpoint might provide an explanation of this feature.
to the first question.

**Question 1**

What kind of closed string theory should we consider?

**Answer**

We should consider the world-sheet theory with holes which can be interpreted as a consistent perturbation theory of closed strings and which we expect to contain gravity.

It would be difficult to see this world-sheet picture directly in $\mathcal{N} = 4$ super Yang-Mills theory because it is the theory after taking the low-energy limit. Before taking the low-energy limit, the dynamics on the D-branes is described by *open string field theory*, and gauge invariance of open string field theory is closely related to the world-sheet picture. Now the second question we ask is what quantities we should consider in open string field theory.

### 2.2 What quantities should we consider in open string field theory?

In the context of the AdS/CFT correspondence, we consider correlation functions of gauge-invariant operators in $\mathcal{N} = 4$ super Yang-Mills theory. In string field theory, it is in general difficult to construct gauge-invariant operators. It is in fact an important feature of string field theory and it is part of the reason that the interacting string field theory is believed to be unique up to field redefinition given a free theory. In open bosonic string field theory with the cubic interaction in terms of the star product [15], however, there are a class of gauge-invariant operators and we can define a gauge-invariant operator for each on-shell closed string vertex operator [16, 17]. These gauge-invariant operators have been mainly discussed in the context of the classical theory. They were evaluated for a classical solution to extract information on the boundary conformal field theory corresponding to the classical solution [18, 19]. In that context they were called gauge-invariant observables, gauge-invariant overlaps, Ellwood invariants, and so on. We call them gauge-invariant operators as we consider them in the quantum context.

The action of open bosonic string field theory [15] is given by

$$S = -\frac{1}{2} \langle \Psi, Q \Psi \rangle - \frac{1}{3} \langle \Psi, \Psi \ast \Psi \rangle,$$

where $\Psi$ is the open string field of ghost number 1, $Q$ is the BRST operator, $\langle A, B \rangle$ is the BPZ product of $A$ and $B$, and $A \ast B$ is the star product of $A$ and $B$. Before we explain the
definition of the gauge-invariant operators, it will be useful to recall the definitions of the BPZ inner product and the star product.\footnote{More detailed explanations can be found in the review\cite{20}, where only the basic knowledge of conformal field theory in Chapter 2 of the textbook by Polchinski\cite{21} is assumed.}

The BPZ inner product for a pair of states $A_1$ and $A_2$ is defined by the following two-point correlation function on the upper half-plane:

$$\langle A_1, A_2 \rangle = \langle h_1 \circ A_1(0) h_2 \circ A_2(0) \rangle_{UHP}, \tag{2.2}$$

where $A_1(0)$ and $A_2(0)$ are the operators corresponding to the states $A_1$ and $A_2$, respectively, in the state-operator correspondence. Here and in what follows we denote the operator mapped from $\mathcal{O}(\xi)$ in the local coordinate $\xi$ under a conformal transformation $f(\xi)$ by $f \circ \mathcal{O}(\xi)$. The conformal transformations $h_1(\xi)$ and $h_2(\xi)$ are given by

$$h_1(\xi) = \tan\left(\arctan \xi - \frac{\pi}{4}\right) = \frac{\xi - 1}{\xi + 1}, \quad h_2(\xi) = \tan\left(\arctan \xi + \frac{\pi}{4}\right) = -\frac{\xi + 1}{\xi - 1}. \tag{2.3}$$

See figure 1 for illustration of this definition.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{bpz-inner-product.png}
\caption{The definition of the BPZ inner product.}
\end{figure}

The star product is defined by the following three-point correlation function on the upper half-plane:

$$\langle A_1, A_2 \ast A_3 \rangle = \langle f_1 \circ A_1(0) f_2 \circ A_2(0) f_3 \circ A_3(0) \rangle_{UHP}, \tag{2.4}$$

where $A_1(0)$, $A_2(0)$, and $A_3(0)$ are the operators corresponding to the states $A_2$, $A_2$, and $A_3$, respectively, in the state-operator correspondence and the conformal transformations $f_1(\xi)$,
$f_2(\xi)$, and $f_3(\xi)$ are given by

\begin{align*}
  f_1(\xi) &= \tan \left[ \frac{2}{3} \left( \arctan \xi - \frac{\pi}{2} \right) \right], \\
  f_2(\xi) &= \tan \left( \frac{2}{3} \arctan \xi \right), \\
  f_3(\xi) &= \tan \left[ \frac{2}{3} \left( \arctan \xi + \frac{\pi}{2} \right) \right].
\end{align*}

(2.5)

See figure 2 for illustration of this definition.

![Figure 2: The definition of the star product.](image)

Let us finally explain the definition of the gauge-invariant operator. The gauge-invariant operator $\mathcal{A}_V[\Psi]$ for an on-shell closed string vertex operator $V$ of ghost number 2 is defined by the following correlation function on the upper half-plane:

\[ \mathcal{A}_V[\Psi] = \langle f_{\text{mid}} \circ V(0) f_I \circ \Psi(0) \rangle_{\text{UHP}}, \]

(2.6)

where $\Psi(0)$ is the operator corresponding to the state $\Psi$ in the state-operator correspondence. The conformal transformation $f_I(\xi)$ associated with the identity string field is given by

\[ f_I(\xi) = \tan \left( 2 \arctan \xi \right) = \frac{2\xi}{1 - \xi^2}, \]

(2.7)

and $f_{\text{mid}}(\xi)$ is the translation to the open-string midpoint:

\[ f_{\text{mid}}(\xi) = \xi + i. \]

(2.8)

See figure 3 for illustration of this definition.

These gauge-invariant operators have an interesting origin in open-closed string field theory. A one-parameter family of formulations for open-closed bosonic string field theory were
constructed in [22], and it was observed that in a singular limit the action reduces to that of the cubic open bosonic string field theory with an additional vertex which couples one off-shell open string field and one on-shell closed string field:

\[ S = -\frac{1}{2} \langle \Psi, Q\Psi \rangle - \frac{1}{3} \langle \Psi, \Psi * \Psi \rangle + \langle J(\Phi), \Psi \rangle, \] (2.9)

where \( \Phi \) is the on-shell closed string field,

\[ Q\Phi = 0, \] (2.10)

and \( J(B) \) is a map from a closed string field \( B \) to an open string field. The vertex \( \langle J(B), A \rangle \) for an open string field \( A \) and a closed string field \( B \) is defined by

\[ \langle J(B), A \rangle = \langle f_{mid} \circ B(0) f_I \circ A(0) \rangle_{UHP}, \] (2.11)

where \( A(0) \) and \( B(0) \) are the operators corresponding to the states \( A \) and \( B \), respectively, in the state-operator correspondence. The Grassmann parity of \( J(B) \) is the same as that of \( B \) mod 2. Two important properties associated with \( J(\Phi) \) are as follows:

\[ QJ(\Phi) = 0, \quad J(\Phi) * A = A * J(\Phi), \] (2.12)

where \( A \) is an arbitrary open string field. The kinetic term of the closed string field is absent so that the resulting theory is no longer open-closed string field theory. It is open string field
theory, and the coupling of the on-shell closed string field and the off-shell open string field can be regarded as source terms for the gauge-invariant operators:

$$\langle J(\Phi), \Psi \rangle = \sum_{\alpha} G_{\alpha} A_{\mathcal{V}_\alpha}[\Psi],$$

(2.13)

where the collective label $\alpha$ generically contains both continuous and discrete variables and the summation over $\alpha$ should be understood to include integrals for continuous variables. The source $G_{\alpha}$ for $A_{\mathcal{V}_\alpha}[\Psi]$ is related to $\Phi$ via the expansion

$$\Phi = \sum_{\alpha} G_{\alpha} \Phi_{\alpha},$$

(2.14)

where $\Phi_{\alpha}$ is the state corresponding to $\mathcal{V}_\alpha$ in the state-operator correspondence.

An important consequence from this relation of the gauge-invariant operators and open-closed string field theory is that Feynman diagrams for correlation functions of the gauge-invariant operators are given by Riemann surfaces containing holes with bulk punctures and the moduli space of such Riemann surfaces is covered. Note that open bosonic string field theory with the cubic vertex in terms of the star product plays a distinguished role in this context.

Let us now consider the theory on $N$ coincident D-branes. If we evaluate correlation functions of the gauge-invariant operators in the $1/N$ expansion, by construction it reproduces the perturbation theory we mentioned in §2.1. Let us present our answer to the second question.

**Question 2**

What quantities should we consider in open string field theory?

**Answer**

We should consider correlation functions of the gauge-invariant operators. The moduli space of Riemann surfaces associated with Feynman diagrams is covered, and the $1/N$ expansion can be interpreted as a closed string perturbation theory.

In open string field theory we can also consider dynamics of open strings in addition to the gauge-invariant operators. This may be related to the discussion on non-singlet sectors in the matrix models [23]. While this is an interesting direction to explore, we concentrate on correlation functions of the gauge-invariant operators in this paper.

The action (2.9) generates the complete set of Riemann surfaces containing an arbitrary number of holes with an arbitrary number of bulk punctures as Feynman diagrams for correlation functions of the gauge-invariant operators, but such Riemann surfaces contain at least one
hole and contributions from Riemann surfaces without any holes are missing. In the context of on-shell scattering amplitudes, they are necessary for factorization. Then our third question is what we lose in the missing Feynman diagrams.

2.3 What do we lose in the missing Feynman diagrams?

Let us again recall the explanation of the AdS/CFT correspondence. Both in the description with D-branes and in the description with the three-brane solution of supergravity, there are two decoupled sectors in the low-energy limit. One of them is a free theory in ten dimensions from the closed string sector, and we identified the two descriptions of the other sector. What we want is of course the interacting sector. In the low-energy limit, the missing contributions correspond to those of a free closed string theory, which we want to discard. Therefore our answer to the third question is as follows.

Question 3

What do we lose in the missing Feynman diagrams?

Answer

Nothing in the low-energy limit!

Previously there were some attempts to reproduce closed string theory without holes in the world-sheet from correlation functions of the gauge-invariant operators, for example, using tachyon condensation. While these attempts are interesting, we emphasize that our approach is different and we are not trying to reproduce closed string theory on a flat spacetime.

Note that after taking the low-energy limit the quantities we are considering are no longer on-shell scattering amplitudes. There may be a physical interpretation about correlation functions of the gauge-invariant operators before taking the low-energy limit, but we have not figured it out.

In the low-energy limit, gauge-invariant operators with vertex operators for massive closed string states will not play an important role. We emphasize that they are massive closed string states in the presence of D-branes, and massive closed string states in $AdS_5 \times S^5$ arise from the massless sector in the asymptotically flat spacetime by taking the low-energy limit. Under the assumption of the equivalence between the world-sheet theory with holes and a world-sheet theory without holes for a different background, the $1/N$ expansion of correlation functions of the gauge-invariant operators in the low-energy limit should incorporate interactions in terms of massive closed string states in the different background such as $AdS_5 \times S^5$.

To summarize, we claim that the evaluation of correlation functions of the gauge-invariant operators in the $1/N$ expansion can be interpreted as a closed string perturbation theory in
the low-energy limit. Therefore, if open string field theory for finite $N$ is a consistent quantum theory, it provides a nonperturbative definition of closed string theory. Now the fourth question we ask is whether open string field theory is a consistent quantum theory.

### 2.4 Is open string field theory a consistent quantum theory?

In general, we do not expect open bosonic string field theory to be a consistent quantum theory because of the presence of tachyons in the open string channel and in the closed string channel. In the topological string or in the noncritical string, however, there are backgrounds without tachyons and it would be interesting to consider the quantum theory for gauge-invariant operators of open bosonic string field theory. For example, three-dimensional Chern-Simons gauge theory can be formulated as open string field theory on topological A-branes [24]. The duality in the B-model topological string theory is also discussed recently [25], and it would be interesting to consider open string field theory in this context. In the case of the noncritical string, it was shown by Gaiotto and Rastelli that the Kontsevich model [26] can be realized as open string field theory [27]. Another interesting arena is the recent discussion on D-instanton contributions in two-dimensional string theory [28, 29, 30, 31, 32].

On the other hand, open superstring field theory can be a consistent quantum theory. When we quantize open superstring field theory, we know that both the Neveu-Schwarz sector and the Ramond sector are necessary for consistency. While the action of open superstring field theory involving the Ramond sector had not been constructed for many years, this problem was recently overcome and we now have several formulations of open superstring field theory which are complete at the classical level [33, 34, 35, 36]. We consider that the formulations of open superstring field theory need to be developed further and it is an important question to address whether or not open superstring field theory is consistent as a quantum theory, but at the same time we consider that we are in a position to discuss how we use open superstring field theory to understand the mechanism which realizes the AdS/CFT correspondence. Let us now present our answer to the fourth question.

**Question 4**

Is open string field theory a consistent quantum theory?

**Answer**

Open bosonic string field theory for the topological string or the noncritical string can be a consistent quantum theory. Open superstring field theory can also be a consistent quantum theory, which motivates us to extend our discussion to the superstring.
In our perspective gauge invariance of open string field theory at the quantum level is crucially important. For open bosonic string field theory, the cubic theory in terms of the star product is just one gauge-invariant formulation and there are other formulations which are also gauge invariant at the classical level. At the quantum level, however, the cubic theory in terms of the star product seems to play a distinguished role. For open superstring field theory, as we mentioned before, there are several formulations at the classical level, but there might be a distinguished theory at the quantum level.

At any rate, we should seriously think about quantization of open string field theory. Our fifth question is then whether we can make sense of the path integral of open string fields.

2.5 Can we make sense of the path integral of open string fields?

As we mentioned in the introduction, we do not consider the path integral of closed string field theory to be promising for a nonperturbative definition of closed string theory. On the other hand, there will be a better chance of making sense of the path integral of open string field theory. However, it may be still difficult because the open string field contains infinite component fields.

Actually, we define closed string theory by taking the low-energy limit of open string field theory so that we can in principle integrate out massive fields of open string field theory following the approach developed by Sen [37]. In general, the resulting theory in terms of massless fields will be very complicated. If we can identify a theory in the same universality class, however, we can use it to define closed string theory nonperturbatively.

In the case of D3-branes, for example, the resulting theory will be equivalent to $\mathcal{N} = 4$ super Yang-Mills theory in the low-energy limit. As we mentioned before, it is difficult to see the world-sheet picture in $\mathcal{N} = 4$ super Yang-Mills theory, but we can keep track of the relation to the world-sheet in the theory after integrating out massive fields. This can be a promising way for proving the AdS/CFT correspondence. Our answer to the fifth question is as follows.

Question 5

Can we make sense of the path integral of open string fields?

Answer

While there is a possibility of making sense of the path integral of open string fields, we can also consider integrating out massive fields to obtain a theory in terms of massless fields. If we can identify a theory in the same universality class, we can use it to define closed string theory nonperturbatively.
One puzzling feature in our approach is that the gauge-invariant operator is a linear functional of the open string field and apparently it does not look like operators which couple to closed strings such as the energy-momentum tensor. The resolution of this puzzle is also related to taking the low-energy limit. In the process of integrating out massive fields, couplings of the closed string and multiple open string fields are generated, and the gauge-invariant operators in terms of massless fields will resemble single-trace operators of $U(N)$ gauge theories in the low-energy limit \cite{38,39}. We expect that the world-sheet calculations of the energy-momentum tensor of noncommutative gauge theory in \cite{40} or of the BFSS matrix model \cite{41} in \cite{42} are reproduced by taking the low-energy limit in our approach.

We believe that there are several advantages in our approach. First, we define correlation functions of the gauge-invariant operators before taking the low-energy limit, and this can provide a well-defined setting to discuss the correspondence after taking the limit such as the AdS/CFT dictionary between correlation functions on the boundary and supergravity calculations in the bulk \cite{43,44}. Second, we have a relation between gauge-invariant operators and closed string states from the beginning, and this can be an advantage, although it might be difficult to keep track of the relation after taking the low-energy limit. Third, our discussion can be applied to any background by taking an appropriate limit discussed in \cite{45,46,47}. In particular, our discussion does not directly rely on conformal symmetry in the limit or on supersymmetry.

3 Discussion

We want to have a consistent formulation of quantum gravity. For this purpose we want to define closed string theory nonperturbatively. Instead of the ordinary world-sheet theory of the closed string, we consider the world-sheet theory with holes in this paper. Our expectation that this perturbation theory contains gravity is based on the assumption that this theory is equivalent to a world-sheet theory without holes for a different background. This is a crucial assumption, and one approach to the proof is to consider particular examples. The most promising example would be the world-sheet theory with holes where the boundary conditions for D3-branes are imposed, and this theory is believed to be equivalent to the world-sheet theory for the $AdS_5 \times S^5$ background in the low-energy limit. Another possible approach would be to use open-closed string field theory. We can formally integrate out the open string field to show that the resulting theory has the same algebraic structure as closed string field theory, although it is subtle to integrate out massless fields from the open string field. Furthermore, the algebraic structure of closed string field theory strongly indicates that it contains gravity, but we do not know what background the resulting theory describes in general, and we do not even know whether the background has a geometric interpretation.
The action (2.9) for $N$ coincident D-branes generates Feynman diagrams for the world-sheet theory with holes where the moduli space of Riemann surfaces is precisely covered, and this implies that the $1/N$ expansion of correlation functions of the gauge-invariant operators reproduces the perturbation theory which we expect to contain gravity. This is the main observation of this paper. The proof of the covering of the moduli space in [22] is truly remarkable, and the challenge is to extend the proof to the supermoduli space of super-Riemann surfaces. Such an extension would establish that open superstring field theory is a consistent quantum theory, and we hope that such open superstring field theory can be used to provide a nonperturbative formulation of quantum gravity based on the scenario described in this paper.

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