Double emulsion production in glass capillary microfluidic device: Parametric investigation of droplet generation behaviour

Seyed Ali Nabavi, Goran T. Vladisavljević, Sai Gu, Ekanem E. Ekanem

Offshore, Process and Energy Engineering Department, Cranfield University, Cranfield MK43 0AL, United Kingdom
Department of Chemical Engineering, Loughborough University, Loughborough LE11 3TU, United Kingdom

HIGHLIGHTS

- Compound droplet formation in glass capillary devices was parametrically studied.
- Good agreement was achieved between experiments and VOF–CSF numerical model.
- Model reproduced well dripping, narrowing jetting, widening jetting and transitions.
- Effects of fluid properties, flow rates and geometry on droplet size were explored.
- Results are applicable for encapsulation of CO₂ solvents in core/shell microcapsules.

GRAPHICAL ABSTRACT

ABSTRACT

A three-phase axisymmetric numerical model based on Volume of Fluid–Continuum Surface Force (VOF–CSF) model was developed to perform parametric analysis of compound droplet production in three-phase glass capillary devices that combine co-flow and countercurrent flow focusing. The model predicted successfully generation of core–shell and multi-cored double emulsion droplets in dripping and jetting (narrowing and widening) regime and was used to investigate the effects of phase flow rates, fluid properties, and geometry on the size, morphology, and production rate of droplets. As the outer fluid flow rate increased, the size of compound droplets was reduced until a dripping-to-jetting transition occurred. By increasing the middle fluid flow rate, the size of compound droplets increased, which led to a widening jetting regime. The jetting was suppressed by increasing the orifice size in the collection capillary or increasing the interfacial tension at the outer interface up to 0.06 N/m. The experimental and simulation results can be used to encapsulate CO₂ solvents within gas-permeable microcapsules.

1. Introduction

Double water-in-oil-in-water emulsions are three-phase dispersions composed of inner aqueous droplets dispersed in larger oil droplets, which are themselves dispersed in another aqueous phase (Utada et al., 2005). The presence of intermediate (middle) fluid as a protective shell or semipermeable barrier which separates the inner aqueous phase from the outer one, makes double
emulsions suitable in a wide range of applications including food industry (Edris and Bergstæhl, 2001; Muschiolik, 2007), cosmetics (Gallarate et al., 1999), controlled delivery (Kim and Park, 2004; Yamaguchi et al., 2002), and encapsulation (Aines et al., 2013; Martinez et al., 2012). Microcapsules for encapsulation of CO₂ solvents within a CO₂ permeable polymer are a promising alternative to conventional liquid CO₂ capture materials such as monoethanolamine (Aines et al., 2013). The most challenging step in the production of tailor-made CO₂ solvent capsules is to fabricate monodispersed core/shell template droplets with controllable shell thickness. The thinner the shell and the higher the surface area of the innermost droplet, the higher the permeability to CO₂ and the CO₂ loading capacity, but the mechanical stability of capsules over multiple absorption–desorption cycles is also important. In order to achieve active control over the capsule production, understanding the underlying physics behind double emulsion formation in microfluidic devices is obligatory.

Conventional emulsification devices such as high-shear homogenizers and colloid mill (King and Keswani, 1994; Ma and Hsu, 1996) are based on high shear mixing of immiscible liquids. These devices, however, suffer from poor droplet size reproducibility and are not suitable for generation of core/shell droplets. Recently, microfluidic emulsification devices have attracted much attention due to their unprecedented level of control over droplet size and morphology. The most common microfluidic strategies for drop generation are flow focusing (Chen et al., 2009; Pannacci et al., 2008; Seo et al., 2007), T-junction (Nissako et al., 2005; Okushima et al., 2004) and co-flowing (Cramer et al., 2004; Herrada et al., 2010; Perro et al., 2011). Utada et al. (2005, 2007) developed a glass capillary device by combining co-flow and flow focusing in coaxial glass capillaries. In microfluidic devices, droplets can be produced in two main regimes, dripping and jetting, depending on the balance between interfacial, viscous, inertial and gravity forces (Lagus and Edd, 2013). The experimental studies of drop generation in glass capillary devices were focused on the effects of fluid flow rates and geometry on the size and morphology of compound droplets (Chang and Su, 2008; Chang et al., 2009; Chen et al., 2012; Chu et al., 2007; Erb et al., 2011; Lee and Weitz, 2008; Martinez et al., 2012; Shirk et al., 2013) and dripping-to-jetting transition (Utada et al., 2005).

Although considerable experimental research has been devoted to double emulsion formation in microfluidic devices, there are only a few numerical parametric studies on microfluidic compound droplet production (Herrada et al., 2010; Park and Anderson, 2012; Radev and Tchavdarov, 1988; Suryo et al., 2006; Vu et al., 2013; Zhou et al., 2006). Zhou et al. (2006) have studied the formation of compound droplets in a planar (2D) flow focusing microfluidic device using diffuse-interface method. However, in their setup all inlet streams are delivered from the same direction, while in 3D glass capillary device, the outer fluid flows counter-currently to the inner and middle fluid. Park and Anderson (2012) developed a two dimensional axisymmetric numerical model based on a ternary diffuse-interface method to predict dripping-to-jetting transition in glass capillary device and the effect of outer fluid flow rate on the size of double emulsion droplets. However, the model was not validated and a simplified setup was assumed with all inlet streams flowing co-currently. Vu et al. (2013) have used the front-tracking finite difference method to study the effect of surface tension, inlet velocities and radius ratio on the size of compound droplets formed in an axisymmetric co-flow device under jetting and dripping regime. However, the effect of viscosities and densities of middle and outer fluid was not investigated and again, a co-flowing outer fluid was considered.

There are many approaches used for simulation of free surface flow, which can be classified into interface tracking methods, such as immersed boundary method (Shin and Juric, 2002; Tryggvason et al., 2001), finite element method (Notz et al., 2001; Wilkes et al., 1999), boundary integral method (Cristini et al., 2001, 1998), and interface capturing methods including Level set method (Osher and Fedkiw, 2001), constrained-interpolation-profile method (Yabe et al., 2001), and volume of fluid method (VOF) (Hirt and Nichols, 1981). The VOF method coupled with CSF (continuum surface force) (Brackbill et al., 1992) approach was found to be very successful in predicting droplet formation and break up (Chen et al., 2013b; Vladislavjevic et al., 2014; Zhang, 1999), droplet deformation (Chen et al., 2013a) and dripping to jetting transitions (Chen et al., 2013b; Herrada et al., 2008).

To the best of our knowledge, this is the first numerical simulation study of double emulsion formation in an axisymmetric three-phase glass capillary device developed by Utada et al. (2005). We have used a two-dimensional axisymmetric VOF approach (Hirt and Nichols, 1981) coupled with continuum surface force (Brackbill et al., 1992) to simulate generation of core/shell droplets encapsulating CO₂ solvents. Overall 13 parameters (8 physical parameters of the fluids, 3 fluid flow rates and 2 geometrical parameters of the device) have been systematically varied to investigate their effects on the droplet generation behaviour.

2. Governing equations and numerical method

2.1. Governing equations

The governing equations are those for the conservation of mass (continuity), Eq. (1), and momentum, Eq. (2),

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0$$  \hspace{1cm} (1)

$$\frac{\partial (\rho \mathbf{U})}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) = -\nabla P + \nabla \cdot \left( \mu (\nabla \mathbf{U} + \nabla \mathbf{U}^T) \right) + F_b$$  \hspace{1cm} (2)

where $\mathbf{U}$ is the velocity vector, $P$ is the pressure, and $t$, $\mu$ and $\rho$ are the time, dynamic viscosity and density, respectively. A source term is denoted by $F_b$ and includes two body forces, the gravitational force and interfacial tension force, $F_o$. Since the length-scale is in the order of micro-scale, the gravitational acceleration is negligible and $F_b = F_o$.

In the VOF model, a momentum equation, Eq. (2), was solved for all phases, and tracking of the interface was achieved by solving a continuity equation for volume fraction, $f$, of one or more phases. Eq. (3).

$$\frac{\partial f}{\partial t} + \nabla \cdot (f \mathbf{U}) = 0$$  \hspace{1cm} (3)

Eq. (3) explains the advection of fluids through the cells to gather the required information for reconstruction of the interface. The portion of each cell filled with fluids is determined by volume fraction in each cell where

\begin{align*}
  f &= 0 \quad \text{the cell is filled with fluid} \, 1 \\
  0 &< f < 1 \quad \text{the interface of fluids exists in the cell} \\
  f &= 1 \quad \text{the cell is filled with fluid} \, 2
\end{align*}

The dynamic viscosity and density in the momentum equation, Eq. (2) were calculated as follows:

$$\mu = f_1 \mu_1 + \left(1-f_1\right)\mu_2,$$  \hspace{1cm} (5)

$$\rho = f_1 \rho_1 + \left(1-f_1\right)\rho_2,$$  \hspace{1cm} (6)

The subscripts 1 and 2 present the first and second existing phases in the cell when the cell is filled with two phases. Continuum surface force (CSF) method was utilized to calculate the interfacial surface force term at the free surface in Eq. (2),
where

\[ F_\sigma = \sigma \kappa \nabla f \]  
\[ \kappa = \nabla \cdot \hat{n} \]  
\[ \hat{n} = \frac{\nabla f}{|\nabla f|} \]

(7) (8) (9)

where \( \sigma \) is the interfacial tension and \( \kappa \) is the local curvature of the interface calculated as follows:

\[ \nabla \cdot \hat{n} \]

(8)

where \( \hat{n} \) is the unit normal defined as

\[ \hat{n} = \frac{\nabla f}{|\nabla f|} \]

2.2. Numerical method

The governing equations were solved by utilizing an unsteady pressure-based segregated algorithm established in finite-volume based commercial software Ansys® Fluent v. 14.0. The discretized moment equation was approximated using both second order upwind and QUICK scheme in order to get the most accurate solution. Since there was no discrepancy between the solutions obtained by the two schemes, second order upwind scheme was chosen to carry on the simulations. The interpolation of pressure term on faces and avoids interpolation errors. The pressure-based segregated algorithm established in this work.

A summary of the boundary conditions applied in this work.

| Boundary     | Type       |
|--------------|------------|
| Inlets       | Mass flow inlet |
| Outlet       | Pressure outlet |
| Symmetry axis| Axis       |
| Wall         | No-slip wall |

Table 2

The properties of the fluids used for experimental double emulsion preparation.

| Phase       | Density (kg/m³) | Viscosity (mPa s) |
|-------------|-----------------|-------------------|
| Inner phase | 1012            | 1.24              |
| Middle phase| 940             | 10.37             |
| Outer phase | 1107            | 7.91              |

and capillary numbers of inner, \( \text{Ca}_1 \), middle, \( \text{Ca}_2 \) and outer phase, \( \text{Ca}_3 \)

\[ \text{We}_1 = \frac{\rho u_1^2 D_N}{\sigma_1} \]
\[ \text{We}_2 = \frac{\rho u_3^2 D_{\text{jet}}}{\sigma_2} \]
\[ \text{Ca}_1 = \frac{\mu_1 u_1}{\sigma_1} \]
\[ \text{Ca}_2 = \frac{\mu_2 u_2}{\sigma_2} \]
\[ \text{Ca}_3 = \frac{\mu_3 u_3}{\sigma_2} \]

where \( \rho \), \( \sigma \), \( \mu \), and \( u \) are density, interfacial tension, viscosity, and average velocity, respectively, \( D_N \) is a diameter, and subscripts 1, 2, 3, jet, and N stand for inner, middle and outer phase, compound jet, and nozzle, respectively. The values of \( u_1 \), \( u_2 \), \( u_3 \), and \( D_{\text{jet}} \) were directly measured in the vicinity of exit orifice from the numerical data, while \( u_1 \) was calculated as \( u_1 = 4Q_1/(\pi D_N^2) \). Since the difference in density between the inner and middle phase is very small, the outer fluid density is used for estimation of \( \text{We}_2 \).

3. Experimental method

3.1. Material

The inner phase was composed of 95 wt% deionized water and 5 wt% glycerol (Fisher scientific, UK). The middle phase was a mixture of 2 wt% xiameter® rsn-0744 resin (UNIVAR, UK) and 98 wt% polydimethyl siloxane (PDMS) fluid (Dow Corning 200/10c S fluid, VWR, UK). The outer phase was a mixture composed of 58 wt% deionized water, 40 wt% glycerol and 2 wt% polyvinyl alcohol (PVA, \( M_w = 13,000-23,000 \) g mol⁻¹, 87–89% hydrolyzed, Sigma-Aldrich, UK). Prior to each use, the outer phase was kept in a quiescent state for 10 min to allow entrained air bubbles to rise to the surface. Table 2 lists the density and viscosity of all phases, measured using a pycnometer and capillary viscometer, respectively. The interfacial tension at the inner and outer interface of 22.6 and 7.3 mN/m, respectively, was measured using a Kruss DSA-100 pendant drop tensiometer.

3.2. Device fabrication

Round capillaries were supplied from Intracel (Royston, UK) with an inner diameter of 0.58 mm, an outer diameter of 1 mm, and a length of 150 mm. A Flaming/Brown micropipette puller (P-97, Sutter Instrument Co., Linton Instrumentation, Norfolk, UK) was used to break each round capillary into two identical parts, each with a tapered end. The pulled capillary was mounted onto a microforge microscope (Narishige MF–830) to observe the shape of the tip while it was sandpapered to the final size. The tip of both
capillaries was then treated with octadecyltrimethoxysilane and 2-[methoxy(polyethyleneoxy)propyl]trimethoxysilane to make the glass surface hydrophobic and hydrophilic, respectively.

The injection capillary was inserted into a square capillary (AIT Glass, Rockaway, US) with an inner width of 1 mm. The two capillaries were axially centred on a microscope slide and fix in a position using two-part epoxy glue (5-Minute Epoxy® Devcon). The collection tube was then inserted into the square capillary, placed at a desirable distance from the injection capillary, axially centred and fix to the microscope slide. Three hypodermic needles with polypropylene hubs (BD Precisionglide® 20 G, Sigma-Aldrich, UK) were attached to the capillaries to introduce the three phases to the device (Fig. 2). A needle hub with a single groove was used to deliver the inner phase to the opening of the injection capillary. Needle hubs with two grooves were placed at the intersections of the injection and collection capillary with the square capillary and used to introduce the middle and outer phase, respectively. The device was left 4–5 h to allow the epoxy to fully cure.

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.ces.2015.03.004.

3.3. Generation of double emulsions

Three gas-tight glass syringes were filled with the three phases, mounted on syringe pumps (Harvard Apparatus model 11 Elite) and delivered through polyethylene medical tubing with 0.86 mm I.D. and 1.52 mm O.D. Once the compound jet emerged in the collection tube, double emulsion droplets were started to form (Fig. 2). Droplet formation was monitored using an inverted microscope (XDS-3, GX Microscopes, UK) and Phantom V9.0 high-speed camera interfaced to a PC computer. The pictures were taken at a speed of 3000 frames per second and the videos were processed using ImageJ software.

4. Results and discussions

Validation of numerical model

The qualitative validations of the numerical model with conducted experiments are presented in Fig. 3. It is obvious that the CFD model is capable of predicting morphological changes of compound droplets during formation in the collection capillary, including formation of satellite droplets. Due to fast droplet formation, the model does not account for the adsorption of surfactant molecules at the interfaces, which leads to the coalescence of inner droplets in numerical simulations, behaviour not observed in experiments (Fig. 3). Since the droplet formation time was negligible compared to the time needed for the surfactants to reach the equilibrium saturation, the interfacial tension was considered time-independent.

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4.1. Experimental results

The effect of phase flow rates on the inner droplet diameter, $D_1$, the outer droplet diameter, $D_2$, and the shell thickness, $t_s$, is presented in Fig. 4. Solid and dashed lines in Fig. 4a–c are theoretical predictions based on observed droplet generation frequencies and flow rate settings. As can be seen in Fig. 4c, $D_1$ can be varied by tuning the flow rate ratio $Q_1/Q_2$, where an increase in $Q_1/Q_2$ leads to a significant increase in $D_1$, while $D_2$ remains almost constant, which results in thinning the shell. In contrast, an increase in $Q_2/Q_1$ increases $D_2$ and reduces $D_1$ and consequently, the shell becomes thicker, as shown in Fig. 4b. An increase in $Q_3$ considerably reduces both $D_2$ and $D_1$ while $t_s$ remains nearly constant (Fig. 4a). Fig. 4d

![Fig. 2. A fabricated three-phase glass capillary device with $D_{Orif} = 255 \, \mu m$ (Up). A region marked by the red dashed line is magnified in the lower figure. The direction of inner, middle and outer fluid is shown by the blue, green and red arrow, respectively. Supplementary movie 1.](http://dx.doi.org/10.1016/j.ces.2015.03.004)

![Fig. 3. A qualitative comparison of the numerical (right) and experimental (left) drop generation behaviour. All scale bars are 300 \, \mu m. Supplementary movie 2.](http://dx.doi.org/10.1016/j.ces.2015.03.004)
4.2. Parametric numerical study of compound droplet formations

In this section parametric numerical investigation of compound droplet formation in glass capillary microfluidic devices is presented. The chosen liquid phase properties are within a range typically found for encapsulation of conventional CO₂ solvents such as potassium carbonate and MEA (monoethanolamine) within polymerizable liquid polymers (Aines et al., 2013). The encapsulation of 30 wt% aqueous MEA solution within a UV-curing liquid polymer adhesive (NOA 61) is simulated in Figs. 5, 7–9.

4.2.1. Increasing $Q_3$ reduces both $D_1$ and $D_2$

The effect of outer fluid flow rate, $Q_3$, on droplet formation is illustrated in Fig. 5. In the dripping regime ($Q_3/Q_{sum} < 5.17$), compound droplets are formed very close to the entry of the collection capillary ($l_2/D_{ref} > 1.2$, Fig. 5f) and the size of both inner and outer droplets is significantly reduced with increasing $Q_3$ (Fig. 5a, b), which is coupled with an increase in both droplet generation frequencies (Fig. 5g). $Q_3$ has a higher impact on $D_2$ than $D_1$, which leads to the reduction in shell thickness, $t_s$. A short-term increase in the size of inner and outer drops at $Q_3/Q_{sum} > 7$ in Fig. 5e was due to transition from dripping to narrowing jetting regime, but both $D_1$ and $D_2$ resumed its downward trend at $Q_3/Q_{sum} > 7.18$. A temporary increase in the drop size can be explained by the variations of $C_3$ and $W_2$ presented in Fig. 5h. At the transition point, although there is a considerable decrease in $D_{ref}$, the steep jump in $W_2$ implies a sudden increase in the velocity of compound jet, while there is a constant increase in $C_3$. Therefore, the velocity gradient at the outer interface decreases and thus, the shearing imposed by the outer phase on the compound jet also decreases, which results in an increase in droplet formation time of both drops, $1/f_1$ and $1/f_2$ (Fig. 5g).

Since $Q_{sum} = const$, an increase in droplet formation times results in larger $D_1$ and $D_2$ values.

The transition to jetting regime leads to a dramatic increase in break up lengths (Fig. 5c, d), with $l_2/D_{ref}$ reaching 8.7 at $Q_3/Q_{sum} = 26$ (Fig. 5f). The size of inner and compound droplets continue to decrease in the jetting regime so that the diameter of the compound droplet is less than one half of the orifice diameter at $Q_3/Q_{sum} = 26$. A reduction in droplet size as a result of increase in the outer fluid flow rate can be explained by an increase in the viscous stress exerted by the outer fluid to compound jet, which reduces the drop formation time and thus, increases droplet generation frequencies $f_1$ and $f_2$ to almost 200 Hz at $Q_3/Q_{sum} = 26$ (Fig. 5g). Since $f_1=f_2$ at all flow rates shown in Fig. 5, each compound droplet contains a single internal drop. A shear stress at the outer interface, $τ_{2,3}$, is as follows (see the Supplemental material S1):

$$τ_{2,3} = \frac{32\mu_3 Q_3}{\pi} \frac{\alpha}{D_{ref}^3(\alpha^2 - 1)^2}$$  \hspace{1cm} (11)

where $\alpha$ is given by Eq. (S.12). The variation of $τ_{2,3}$ with $Q_3$ calculated from Eq. (11) is shown in Fig. 6. An increase in $Q_3$ leads to an increase in $τ_{2,3}$, but the effect is less pronounced at $Q_3/Q_{sum} > 20$. 

![Fig. 4. The size of inner and outer droplets and the shell thickness as a function of phase flow rates: (a) $Q_1=2\text{ ml/h}, Q_2=4\text{ ml/h}$; (b) $Q_1=1.5\text{ ml/h}, Q_3=25\text{ ml/h}$; (c) $Q_2=3\text{ ml/h}, Q_1=12\text{ ml/h}$ and (d) Comparison of experimental data with theoretical predictions. MC in the legend stands for mass conservation.](image-url)
droplets adjacent to the tip of the injection tube. At Ca3 negligible compared to interfacial forces leading to the formation of and accordingly, the shell becomes thicker (Fig. 7e). An increase in There is an abrupt increase in causing the transition from dripping to narrowing jetting regime. viscous and interfacial forces are of the same order of magnitude of the compound jet. At Ca3 results in higher velocity gradient at the inner interface and thus increases the shear force exerted on the inner jet by the middle fluid. Therefore, the drop generation frequency of inner droplets increases (Fig. 7g), leading to the formation of smaller inner droplets. According to the mass balance equation: \( f_1 = 6Q_1/(D_1^3) \). Since Q1 is a constant, f1 varies inversely as the cube of D1. An increase in f2 in Fig. 7g is a consequence of increase in Q2. An increase in Q2 results in higher We2 values and longer break-up lengths, L1 and L2, as shown in Fig. 7h and f respectively. A widening shape of the compound jet is a consequence of higher velocity of the middle fluid compared to that of the outer fluid. Therefore, a drag force at the outer interface tends to slow down and widen the compound jet. As Q2 increases, a difference in velocity between the middle and outer fluid increases, leading to the formation of increasingly wider and longer compound jet. Since in widening jetting regime, the momentum force of compound jet is responsible for jet breakup, higher Q2 values result in a faster droplet production and higher f2 values, as shown in Fig. 7g. Based on the fluid properties used in the simulations, the successful core–shell droplet generation is at 0.58 < Q2/Q1 < 46.7. For Q2/Q1 < 0.58, the shell of compound jet is so narrow that the inner fluid ruptures the compound jet and the droplet formation fails. For Q2/Q1 > 46.7, the compound jet diameter exceeds the orifice diameter, Dout and wets the collection tube leading to failure in droplet formation.

4.2.3. Increasing Q1 increases both D1 and D2
Fig. 8 demonstrates the effect of innermost fluid flow rate, Q1, on the drop formation behaviour. The drop formation in the dripping regime occurs over the Q1/Q2 range from 0.13 to 0.66, corresponding to 0.013 < We1 < 0.61 (Fig. 8a–c). An increase in Q1 leads to an increase in both inertial and viscous forces exerted by the inner fluid, which results in an increase in both We1 and Ca1 (Fig. 8h). Since there is no considerable change in L1 and L2 over the range of 0.13 < Q1/Q2 < 0.66 (Fig. 8g), an increase in the momentum of inner fluid results in bigger inner droplet. In the dripping regime Q1 does not have any noticeable effect on the shear stress at the outer interface and thus, D2 increases only slightly with increase in D2 with Q2 may be attributed to the high inertial of the middle fluid which leads to faster introduction of middle fluid to the droplet before drop detachment. An increase in Q2 results in higher velocity at the inner interface and thus increases the shear force exerted on the inner jet by the middle fluid. Therefore, the drop generation frequency of inner droplets increases (Fig. 7g), leading to the formation of smaller inner droplets. According to the mass balance equation: \( f_1 = 6Q_1/(D_1^3) \). Since Q1 is a constant, f1 varies inversely as the cube of D1. An increase in f2 in Fig. 7g is a consequence of increase in Q2. An increase in Q2 results in higher We2 values and longer break-up lengths, L1 and L2, as shown in Fig. 7h and f respectively. A widening shape of the compound jet is a consequence of higher velocity of the middle fluid compared to that of the outer fluid. Therefore, a drag force at the outer interface tends to slow down and widen the compound jet. As Q2 increases, a difference in velocity between the middle and outer fluid increases, leading to the formation of increasingly wider and longer compound jet. Since in widening jetting regime, the momentum force of compound jet is responsible for jet breakup, higher Q2 values result in a faster droplet production and higher f2 values, as shown in Fig. 7g. Based on the fluid properties used in the simulations, the successful core–shell droplet generation is at 0.58 < Q2/Q1 < 46.7. For Q2/Q1 < 0.58, the shell of compound jet is so narrow that the inner fluid ruptures the compound jet and the droplet formation fails. For Q2/Q1 > 46.7, the compound jet diameter exceeds the orifice diameter, Dout and wets the collection tube leading to failure in droplet formation.

4.2.2. Increasing Q2 reduces D1 while increasing D2
An increase in Q2 results in higher D2 values, while D1 is reduced and accordingly, the shell becomes thicker (Fig. 7e). An increase in Q2 results in higher D2 values, while D1 is reduced and accordingly, the shell becomes thicker (Fig. 7e). An increase in Q2 increases the middle fluid velocity and correspondingly the compound jet inertia.

Fig. 5. The variation of flow field with Q2. (a) Q2/Qsum=1.44; (b) Q2/Qsum=5.17; (c) Q2/Qsum=11.48 and (d) Q2/Qsum=17.22. Constant parameters: \( \rho_1 = 1180 \text{ kg/m}^3 \), \( \mu_1 = 1200 \text{ kg/m}^3 \), \( \rho_2 = 0.0396 \text{ Pa s} \), \( \mu_2 = 0.0648 \text{ Pa s} \), \( \rho_3 = 0.0472 \text{ Pa s} \), \( \sigma_{12} = 0.000574 \text{ N/m} \), \( \sigma_{23} = 0.0137 \text{ N/m} \), \( Q_1 = 0.12 \text{ ml/h} \), \( Q_2 = 0.92 \text{ ml/h} \), Dout = 300 \text{ \mu m} \).

Fig. 6. The shear stress at the outer interface at the orifice of the collection capillary as a function of Q3. Constant parameters: \( \rho_1 = 1180 \text{ kg/m}^3 \), \( \mu_1 = 1200 \text{ kg/m}^3 \), \( \rho_2 = 0.0396 \text{ Pa s} \), \( \mu_2 = 0.0648 \text{ Pa s} \), \( \rho_3 = 0.0472 \text{ Pa s} \), \( \sigma_{12} = 0.000574 \text{ N/m} \), \( \sigma_{23} = 0.0137 \text{ N/m} \), \( Q_1 = 0.12 \text{ ml/h} \), Q2 = 0.92 ml/h, Dout = 300 \text{ \mu m} \).
Q\(_1\) (Fig. 8e). Since the size of inner drop increases with increasing Q\(_1\) and the size of outer drop remains nearly constant, the shell becomes thinner. As Q\(_1\)/Q\(_2\) exceeds a critical value of 0.66, any further increase in Q\(_1\) results in developing a high speed jet of the inner fluid leading to a burst of the middle shell and failure of drop formation (Fig. 8d). In addition, for Q\(_1\)/Q\(_2\) < 0.032, the core–shell formation is periodically interrupted by the formation of shell droplet without any encapsulated inner droplet. A failure in drop formation when Q\(_1\) exceeds a certain critical value was also reported earlier (Chang and Su, 2008).

The influence of Q\(_1\) on drop formation in jetting regime is presented in Fig. 9. As shown in Fig. 9e, D\(_1\) follows the same trend as in dripping regime, but D\(_2\) increases more significantly, albeit at a slower pace than D\(_1\), so that the shell thickness is reducing. At very low Q\(_1\)/Q\(_2\) value of 0.0248, where We\(_1\) ≈ 0.006, inner droplets are formed in a narrowing jetting regime (due to the high Q\(_2\) value of 2.46 ml/h), while the outer droplets are formed in a widening mode, resulting in the formation of large compound drops with very small inner drops (Fig. 9a). Formation of inner and outer drops in different regimes results in a considerable difference between L\(_1\) and L\(_2\), which leads to the generation of multi-cored droplets. As Q\(_1\) increases, the inner drop formation switches from narrowing to widening mode, resulting in virtually the same value of L\(_1\) and L\(_2\) and formation of core/shell droplets. A high speed jet was observed at Q\(_1\)/Q\(_2\) > 0.62 and We\(_1\) > 3.83 (Fig. 9d), similar to that shown in Fig. 8d. However, in this case the shell rupture occurs after droplet pinch off, further downstream.
g). Within this range, as fore, there is an increasing tendency to pull back the forming inner jetting regime (Section 4.3.1), where further widening has been observed in droplet generation during transition from dripping to dripping transition mode. The same trend in opposite way have also been reported by Lagus and Edd (2013). Higher values of \( \sigma_{23} \) suppress the inner jet formation by pulling back the compound jet and switching the droplet formation to dripping regime, as shown in Fig. 10a–d. Consequently, both break-up lengths, \( L_1 \) and \( L_2 \), decrease with increasing \( \sigma_{23} \) (Fig. 10f), which is consistent with the findings of Herrada et al. (2010). Two different behaviours have been observed on the graphs in Fig. 10e and g. At \( \sigma_{23} \approx 0.005–0.06 \) \( \text{N/m} \), an increase in \( \sigma_{23} \) considerably delays the pinch off events and reduces both \( f_1 \) and \( f_2 \), resulting in an increase in \( D_1 \) and \( D_2 \). An increase in the size of the outer drop is more significant than that of the inner drop causing \( t_1 \) to grow.

A different type of behaviour was observed at high interfacial tensions at the outer interface of 0.060–0.095 \( \text{N/m} \) (Fig. 10e and g). Within this range, as \( \sigma_{23} \) increases both inner and outer droplets shrink causing the drop generation frequency to increase in order to maintain constant volume flow rates, \( Q_1 \) and \( Q_2 \). The reason for this behaviour could be droplet formation in jetting to dripping transition mode. The same trend in opposite way may have been observed in droplet generation during transition from dripping to narrowing jetting regime (Section 4.3.1), where further increase in \( Q_3 \) led to a sudden growth in both \( D_1 \) and \( D_2 \).

4.2.5. Increasing \( \sigma_{12} \) suppresses inner fluid jetting

The effect of interfacial tension at the inner interface, \( \sigma_{12} \), on the drop formation is shown in Fig. 11. Starting from \( \sigma_{12} = 0.00574 \) \( \text{N/m} \), where both inner and outer droplets are formed in jetting regime, increasing \( \sigma_{12} \) causes a reduction in \( C_a_1 \) (Fig. 11h), which implies higher interfacial force compared to shear force of inner jet. Therefore, there is an increasing tendency to pull back the forming inner jet towards the tip of the nozzle. At \( \sigma_{12} = 0.01 \) \( \text{N/m} \) (Fig. 11a), the inner and middle phase jets are of similar length (\( L_1 \approx L_2 \)). As \( \sigma_{12} \) increases to 0.015 \( \text{N/m} \), \( L_2 \) remains fairly constant while the inner jet shrinks due to increased interfacial tension force at the inner interface which causes development of capillary instability on the inner jet. Consequently the inner jet pinches off faster, leading to smaller \( D_1 \) values (Fig. 11e and g). Indeed, for 0.005 < \( \sigma_{12} < 0.03 \) \( \text{N/m} \), both \( L_1 \) and \( L_2 \) decreases with increasing \( \sigma_{12} \) (Fig. 11a–c). This effect is more pronounced for the inner fluid, causing the difference between \( L_1 \) and \( L_2 \) to increase, which leads to the formation of multi-cored compound droplets (Fig. 11b).

In the \( \sigma_{12} \) range of 0.015–0.03 \( \text{N/m} \), the formed inner droplets, which have the diameter comparable to that of the compound jet, move downstream inside the compound jet, which results in the appearance of capillary-like waves on the outer interface. Therefore, the outer interface becomes more unstable and consequently the compound jet breaks closer to the nozzle exit, meaning a reduction in \( D_2 \) (Fig. 11c). The further increase in \( \sigma_{12} \) over the range from 0.04 to 0.06 \( \text{N/m} \) leads to reduction in both \( D_1 \) and \( D_2 \), where \( D_1 \) is considerably smaller than the compound jet diameter. Therefore, the capillary waves previously formed on the outer surface (Fig. 11c) due to large inner droplet are suppressed (Fig. 11d), and the compound jet pinch off is delayed leading to an increase in \( D_2 \). It should be mentioned that the droplet formation at high \( \sigma_{12} \) values is very chaotic leading to formation of multi-cored droplets along with generation of large satellite droplets (Fig. 11d).

4.2.6. Increasing \( \mu_3 \) increases both \( D_2 \) and \( D_1 \)

The effect of outer fluid viscosity \( \mu_3 \), on the production of compound drops is shown in Fig. 12. According to Fig. 12a, an increase in \( \mu_3 \) leads to a linear increase in \( C_a_3 \) due to higher viscous force acting on the outer interface, which results in stretching both the inner phase and middle phase jet (Fig. 12f). The higher level of shearing exerted by outer fluid accelerates the droplet formation, which results in higher droplet formation frequencies \( f_1 \) and \( f_2 \), and smaller drop diameters, \( D_1 \) and \( D_2 \) (Fig. 12e and g). As can be seen in Fig. 12a,b, the variations of \( D_1 \) and \( D_2 \) with \( \mu_3 \) are strongly correlated, leading to almost constant shell thickness.

4.2.7. Increasing \( \mu_2 \) increases both \( L_2 \) and \( L_1 \)

Fig. 13 shows the effect of middle fluid viscosity, \( \mu_2 \) on droplet production. As \( \mu_2 \) increases, there is an increasing tendency for

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**Fig. 9.** The variation of flow field with \( Q_2 \) in jetting mode: (a) \( Q_1/Q_2 = 0.0248 \); (b) \( Q_1/Q_2 = 0.112 \); (c) \( Q_1/Q_2 = 0.372 \) and (d) \( Q_1/Q_2 = 0.744 \). Constant parameters: \( \rho_1 = 1180 \text{ kg/m}^3 \), \( \rho_2 = 1170 \text{ kg/m}^3 \), \( \rho_3 = 1200 \text{ kg/m}^3 \), \( \mu_1 = 0.0396 \text{ Pa s} \), \( \mu_2 = 0.0648 \text{ Pa s} \), \( \mu_3 = 0.0482 \text{ Pa s} \), \( \sigma_1 = 0.00574 \text{ N/m} \), \( \sigma_2 = 0.0137 \text{ N/m} \), \( Q_1 = 2.46 \text{ ml/h} \), \( Q_3 = 6 \text{ ml/h} \), \( D_{\text{out}} = 200 \mu\text{m} \). The keys are the same as in Fig. 5.
transition from dripping to jetting (Fig. 13a–f). In the current numerical model, the effect of adsorption of surfactants onto the interfaces was not considered, so that droplet coalescence occurs, as shown in Fig. 13a–c. As shown in Fig. 13j, the capillary number of middle phase \( \text{Ca}_2 \) increases with increasing \( \mu_2 \), since \( \text{Ca}_2 \mu_2 \) at constant \( \mu_1 \) and \( \sigma_{23} \). The increased \( \mu_2 \) leads to an increase in viscous stress on inner and outer fluids from middle fluid, causing both liquid jets to stretch in the downstream direction. As shown in Fig. 11g, the transition from dripping to jetting caused by an increase in \( \mu_2 \) from 0.002 to 0.005 Pa s results in a steep increase in both \( D_2 \) and \( D_4 \) and drop formation time. However, there is no considerable difference in drop formation time and accordingly drop size before and after dripping-to-jetting transition. This fact was pointed out by Suryo et al. (2006) who stated that the droplet size and formation time are insensitive to middle phase viscosity in a certain range of viscosity ratios, \( \mu_3/\mu_2 \). The production frequency of compound droplets in the dripping regime was relatively high, \( f_2 = 120 \text{ s}^{-1} \), compared to the one in jetting mode, \( f_2 = 20 \text{ s}^{-1} \). It can be explained by the fact that at very low \( \mu_2 \) (in dripping regime), due to large viscosity \( \mu_3 = \mu_2 \) the required force to pinch off the compound jet and make compound droplets is much smaller than that at high \( \mu_2 \) values and the compound droplet pinch off accelerates.

4.2.8. Reducing \( \mu_1 \) leads to formation of inner droplets in dripping regime

As illustrated in Fig. 14, \( D_2 \) is almost unaffected by the innermost fluid viscosity, \( \mu_1 \), due to negligible effect of \( \mu_1 \) on the shear stress at the outer interface. However, there is a steep increase in \( D_1 \) as \( \mu_1 \) increases from 0.001 to 0.019 Pa s, followed by a sharp drop in \( f_1 \) from nearly 600 s\(^{-1}\) to 60 s\(^{-1}\). At \( \mu_1 = 0.001 \text{ Pa s} \), very small inner droplets are formed in the vicinity of nozzle exit and there is a large difference in drop generation frequency between
According to (Chu et al., 2007), when the predicted value of $L_2$ increases in shell, the break-up of compound jet facilitates, resulting in shorter consequently decreasing the number of inner droplets trapped by from the compound jet and results in larger sequential formation of inner droplets delays the drop pinch off between two integers, $N$. Inner droplets are engulfed by an outer droplet and number of internal droplets can be predicted from the relation: $N_f = \frac{N_1 f_1}{f_2}$. An increase in $\mu_1$ to 0.0396 Pa s at $L_2$ beyond $\mu_1 = 0.0396$ Pa s results in an increase in $C_a$, corresponding to higher viscous forces of the inner fluid and a slight increase in both $L_1$ and $L_2$. However, there is no significant change in $f_1$ and $f_2$ and therefore $D_1$ and $D_2$.

### 4.2.9. Fluid densities do not have any impact on flow field

The effect of fluid density on droplet formation is shown in Fig. 15. No difference in flow field was observed as the density of the outer fluid was varied from 600 to 1800 kg/m$^3$. It could be explained by the fact that due to low fluid velocity and low density ratio, the convection term of the Navier–Stokes equation which includes the effect of density is negligible. The same behaviour was found when the density of inner and middle fluid was modified.
4.2.11. Increasing L changes the droplet formation pattern at low flow rates

The effect of distance between the injection and collection capillary, L, on the drop formation is shown in Fig. 17 and Table S.3. As L increases at high flow rates (Fig. 17a,b), D2 decreases while D1 is almost unaffected, leading to a decrease in the shell thickness (Table S.3). The increased distance L causes the compound jet to slow down before it enters the collection tube. Therefore, a magnitude of the inertial force acting on both jets due to smaller velocities of the inner and outer interface, the time interval between two consecutive breakup events increase from 13 to 15 ms causing an increase in D1 and D2 by 4–8% (Table S.2, jetting).

Although L does not have a considerable impact on drop generation behaviour at high flow rates, it dramatically affects the drop generation pattern at low flow rates (Fig. 17c,d). In Fig. 17c, the droplets are formed in the widening jetting regime due to very low velocity of the outer phase, which is surpassed by the middle phase velocity. An increase in L considerably reduces the middle fluid velocity at the orifice of the collection tube. Since in this case the middle phase flow rate is relatively low, the viscous force originating from the middle phase is not sufficiently high to elongate the inner fluid and the pinch off happens at a very small distance from the exit orifice. Also, the generation frequency of the outer drops is much higher than that of the inner drops and this non-uniformity in drop generation frequency leads to alternating generation of middle phase droplets and compound droplets. Therefore, in this case the optimum value of L/D of changes between 0.3 and 0.83.

5. Conclusion

An in-depth parametric study of compound droplet formation in a three-phase glass capillary device was performed by developing a VOF-CSF numerical model. The model successfully predicted the experimental data and was used to investigate the effect of fluid flow
droplets was non-synchronised, leading to alternating generation of compound and single droplets. As the interfacial tension at the outer interface increased from 0.005 to 0.06 N/m, the size of compound droplets increased and the jetting regime was highly suppressed. On the other hand, an increase of interfacial tension at the inner interface suppressed jetting of the inner fluid only and led to the formation of multi-cored droplets. The number of inner droplets in multi-cored droplets was consistent with the ratio of generation frequency of inner and outer droplets. As the viscosity of the middle phase increased, jet break up lengths increased until a transition from dripping to jetting occurred. As the viscosity of inner phase increased, the size of inner droplets increased and the number of inner droplets encapsulated within each outer droplet was reduced, but no impact on the size of outer droplets was observed. Due to small droplet sizes and low density ratios, no difference in droplet formation was observed at different flow rates.

The size of both inner and outer droplets was reduced as the outer fluid flow rate, \( Q_2 \), or the viscosity of the outer fluid, \( \mu_2 \), was increased, which led to the transition from dripping to narrowing jetting regime at certain critical value of \( Q_3 \) and \( \mu_3 \). An increase in the middle fluid flow rate, \( Q_3 \), reduced the size of inner drops and increased the size of outer drops, which led to a transition to widening jetting regime at the critical \( Q_2 \) value. As the inner fluid flow rate, \( Q_1 \), increased in dripping regime, considerably larger inner droplets were observed but the size of outer droplets remained nearly constant. The jetting regime was suppressed by increasing the size of the orifice in the collection capillary. When the distance between the injection and collection capillary was too large, at low fluid flow rates the generation of inner and outer droplets was non-synchronised, leading to alternating generation of compound and single droplets. As the interfacial tension at the outer interface increased from 0.005 to 0.06 N/m, the size of compound droplets increased and the jetting regime was highly suppressed. On the other hand, an increase of interfacial tension at the inner interface suppressed jetting of the inner fluid only and led to the formation of multi-cored droplets. The number of inner droplets in multi-cored droplets was consistent with the ratio of generation frequency of inner and outer droplets. As the viscosity of the middle phase increased, jet break up lengths increased until a transition from dripping to jetting occurred. As the viscosity of inner phase increased, the size of inner droplets increased and the number of inner droplets encapsulated within each outer droplet was reduced, but no impact on the size of outer droplets was observed. Due to small droplet sizes and low density ratios, no difference in droplet formation was observed at different fluid densities.

Although the developed model accurately predicted generation of compound droplets in glass capillary devices, due to the presence of two interfaces it was computationally very expensive and time consuming. In the future study, a grid adaption method will be coupled to the current CFD model to refine the meshes along the interface, so as to reduce the size of meshes.

**Fig. 16.** The variation of flow field with \( D_{\text{out}} \) at high flow rates. (a) \( D_{\text{out}} = 150 \mu m \) and (b) \( D_{\text{out}} = 300 \mu m \). Constant parameters: \( \rho_1 = 1180 \text{ kg/m}^3, \rho_2 = 1170 \text{ kg/m}^3, \rho_3 = 1200 \text{ kg/m}^3, \mu_1 = 0.0396 \text{ Pa.s}, \mu_2 = 0.06482 \text{ Pa.s}, \mu_3 = 0.0482 \text{ Pa.s}, \sigma_{12} = 0.00574 \text{ N/m}, \sigma_{23} = 0.0137 \text{ N/m}, Q_1 = 0.27 \text{ ml/h}, Q_2 = 2.46 \text{ ml/h}, Q_3 = 15 \text{ ml/h}, D_N = 25 \mu m \). The variation of flow field with \( D_{\text{out}} \) at low flow rates: (c) \( D_{\text{out}} = 150 \mu m \); (d) \( D_{\text{out}} = 300 \mu m \). Constant parameters: \( \rho_1 = 1180 \text{ kg/m}^3, \rho_2 = 1170 \text{ kg/m}^3, \rho_3 = 1200 \text{ kg/m}^3, \mu_1 = 0.0396 \text{ Pa.s}, \mu_2 = 0.06482 \text{ Pa.s}, \mu_3 = 0.0482 \text{ Pa.s}, \sigma_{12} = 0.00574 \text{ N/m}, \sigma_{23} = 0.0137 \text{ N/m}, Q_1 = 0.12 \text{ ml/h}, Q_2 = 0.92 \text{ ml/h}, Q_3 = 2.4 \text{ ml/h}, D_N = 25 \mu m \). The keys are the same as in Fig. 5.

**Fig. 17.** The variation of flow field with \( L' \) at high flow rates (\( Q_1 = 0.27 \text{ ml/h}, Q_2 = 2.46 \text{ ml/h}, Q_3 = 15 \text{ ml/h} \)): (a) \( L' = 90 \mu m \); (b) \( L' = 250 \mu m \). The variation of flow field with \( L' \) at low flow rates (\( Q_1 = 0.12 \text{ ml/h}, Q_2 = 0.92 \text{ ml/h}, Q_3 = 2.4 \text{ ml/h} \)): (c) \( L' = 90 \mu m \) and (d) \( L' = 250 \mu m \). Constant parameters: \( \rho_1 = 1180 \text{ kg/m}^3, \rho_2 = 1170 \text{ kg/m}^3, \rho_3 = 1200 \text{ kg/m}^3, \mu_1 = 0.0396 \text{ Pa.s}, \mu_2 = 0.06482 \text{ Pa.s}, \mu_3 = 0.0482 \text{ Pa.s}, \sigma_{12} = 0.00574 \text{ N/m}, \sigma_{23} = 0.0137 \text{ N/m}, Q_1 = 0.12 \text{ ml/h}, Q_2 = 0.92 \text{ ml/h}, Q_3 = 2.4 \text{ ml/h}, D_N = 25 \mu m \). The keys are the same as in Fig. 5.
A | surface area, m²
---|---
Ca | capillary number, dimensionless
Co | Courant number, dimensionless
D | droplet diameter, m
Dₙ | injection nozzle diameter, m
D₉₅ | exit orifice diameter, m
E | integral constant, Pa m
E₁₂ | integral constant, m/s
E₂₁ | integral constant, m/s
f | volume fraction-droplet generation frequency, 1/s
F₉ | body force, N
F₀ | interface force, N
L | break up length, m
L' | distance between collection tube and injection nozzle, m
n | unit normal, dimensionless
P | pressure, Pa
Q | volume flow rate, m³/s
r | radial direction, m
R | radius, m
S | spreading coefficient, N/m
σ | interfacial tension, N/m
U | mean velocity, m/s
We | Weber number, dimensionless
x | axial direction, m
α | radius ratio, dimensionless
κ | curvature of interface, 1/m
μ | dynamic viscosity, kg/m s
ρ | density, kg/m³
σ | interfacial tension, N/m
θ | contact angle, dimensionless

Subscripts
1 | inner phase
2 | middle phase
3 | outer phase
1,2 | interface between inner and middle phases
2,3 | interface between middle and outer phases
jet | jet
Orif | orifice
sum | Q₁ + Q₂
w | wall

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Appendix A. Supporting information

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