QCD Phenomenology
based on a Chiral Effective Lagrangian

Tetsuo Hatsuda and Teiji Kunihiro*

Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan
*Faculty of Science and Technology, Ryukoku University, Seta, Otsu-city
520-21, Japan

Abstract

We review the Nambu-Jona-Lasinio (NJL) approach to the dynamical breaking of chiral symmetry in Quantum Chromodynamics (QCD). After a general overview of the non-perturbative aspects of QCD, we introduce the NJL model as a low-energy effective theory of QCD. The collective nature of hadrons and the constituent quark model are treated in a unified way. Various aspects of QCD related to the dynamical and explicit breaking of chiral symmetry and the axial anomaly can be well described. The subjects treated in Part I include the vacuum structure of QCD, mass spectra and coupling constants of hadrons, flavor mixing in mesons, the violation of the OZI rule in baryons, and the validity of the chiral perturbation in QCD. A subtle interplay between the axial anomaly and the current-quark masses is shown to play important roles, and a realistic evaluation of the strangeness and heavy quark contents of hadrons is given. Also the problem of elusive scalar mesons is studied in detail. For a pedagogical reason, we first present an account of basic ingredients and detailed technical aspects of the NJL model using simple versions of it.

In Part II, the NJL model is applied to the system at finite temperature ($T$) and density ($\rho$) relevant to the early universe, interior of the neutron stars and the ultra-relativistic heavy ion collisions. After a brief introduction of the field theory at finite temperature, phenomena associated with the restoration of chiral symmetry in the medium are examined. The subjects treated here include the quark condensates in the medium, meson properties at finite $T$ ($\rho$) and their experimental implications. A special attention is paid to fluctuation phenomena near the critical temperature, i.e., possible existence of soft modes in the scalar channel and a jump of the quark-number susceptibility in the vector channel.

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Part I

1 Overview

1.1 Introduction

The dynamics of mesons and baryons are now believed to be described by the theory of quarks and gluons, i.e., the quantum chromodynamics (QCD) \[1\]. The final goal of the study of QCD is to classify all the hadronic phenomena in terms of the quark-gluon degrees of freedom. For very high energy processes such as the deep inelastic lepton-hadron scattering (DIS), this program has been achieving remarkable success, which is due to the asymptotic-free nature of QCD; the perturbation theory works in the high energy regime \[2\]. On the other hand, at low energies comparable to the low lying hadron masses (\(\sim 1\text{ GeV}\)), QCD shows non-perturbative behaviors such as the confinement of quarks and gluons and the dynamical breaking of chiral symmetry, which make the analytic study of QCD very difficult.

Nevertheless, with the accumulated experimental information, we have now a good number of empirical facts at low energies which are waiting to be explained in a unified manner, and thereby give us hints for making a unified picture of the non-perturbative nature of QCD. Some examples are:

(i) **Non-perturbative vacuum structure** \[3\]: Unlike the QED case, the QCD vacuum has non-perturbative condensations of quarks and gluons. This fact is extracted from the studies of the current algebra, QCD sum rules and the lattice QCD. However, we do not yet know the essential physical mechanism of such condensations.

(ii) **Existence of the Nambu-Goldstone (NG) bosons and dynamical breaking of chiral symmetry (DBCS)** \[4\]: The existence of the light pseudo-scalar mesons (pion, kaon and \(\eta\)) suggests that they are the NG-bosons associated with the dynamical breaking of \(SU_L(3) \otimes SU_R(3)\) chiral symmetry down to \(SU_V(3)\). The pair condensation \(\langle \bar{q}q \rangle\) in the vacuum deduced from the current algebra relations also supports this picture.

(iii) **\(U_A(1)\) anomaly** \[5, 6\]: The heavy pseudo-scalar particle \(\eta'\) and also the anomalous decay \(\eta \rightarrow 3\pi\) suggest that the \(U_A(1)\) quark-current is not conserved even partially. This explicit breaking of \(U_A(1)\) symmetry in the quantum level has a close connection to a topological nature of the QCD ground state.

(iv) **Explicit \(SU_V(3)\) breaking**: The hadron mass differences in the same \(SU_V(3)\) multiplet look as if being governed by the linear breaking of the \(SU_V(3)\) symmetry which is summarized as the Gell-Mann-Okubo (GMO) mass formula \[7\]. Although such \(SU_V(3)\) breaking is dictated with the current quark masses in the QCD Lagrangian \((m_{u,d} \sim O(10)\text{ MeV for } u\text{ and } d\text{ quarks and } m_s \sim O(200)\text{ MeV for the strange quark})\), the relation of the quark mass differences and those of the hadron masses is not simple.

(v) **Success of the constituent quark model**: Low lying mesons and baryons except for NG-bosons are well described as systems composed of massive \(u\), \(d\) and \(s\) quarks with \(M_{u,d} \sim 300\text{ MeV and } M_s \sim 500\text{MeV}\). These masses are determined so as to reproduce the magnetic moments of the octet baryons. In this constituent quark picture, the baryon mass splittings are governed by the \(SU_V(3)\) breaking due to the quark mass difference \((M_s \neq M_u)\), the Fermi-Breit type spin-spin interaction between the constituent quarks and the orbital excitations of the quarks inside the baryons \[8\]. The GMO mass formula and its generalization are obtained successfully in this picture:
One of the remarkable examples is the $\Sigma - \Lambda$ mass splitting which is due to $M_s \neq M_u$ and the spin-spin interaction.

(vi) **OZI rule in hadrons**: The flavor changing processes are suppressed in most of the mesons (such as the $J/\Psi$ decay into light hadrons, small $\omega - \phi$ mixing etc) which is summarized as the Okubo-Zweig-Iizuka (OZI) rule \[9\]. There are, however, several exceptions: A large flavor mixing in the iso-singlet NG bosons ($\eta$ and $\eta'$) is known from the 2-photon decays \[10\], for instance. The non-negligible contents of strangeness and heavy quarks in the nucleon are suggested from the analysis of the pion-nucleon sigma term ($\Sigma_{\pi N}$) \[11\], the spin-dependent structure function in the deep inelastic scattering \[12\] and the heavy quark production in high energy proton-nucleus reactions \[13\].

Each empirical fact has so far been studied separately; a physical picture to unify them has not been presented yet. The lattice simulation of QCD, which is supposed to be the most fundamental approach for such non-perturbative problems, is unfortunately not mature enough to give us a clear physical idea.

One should also note that it is an interesting subject to study how these aspects of QCD change at finite temperature ($T$) and finite baryon density ($\rho$). Such a study is quite relevant to the physics of the ultra-relativistic heavy ion collisions, the early universe and the neutron stars \[14\].

In this report, we try to give a qualitative description of the important aspects of QCD such as (i)-(vi) and the hot/dense QCD in a unified way with the use of an effective theory of QCD at low energies. Our Lagrangian takes a form of the Nambu-Jona-Lasinio (NJL) type \[15\] and has three independent ingredients, i.e., DBCS, $U_A(1)$ anomaly and the explicit symmetry breaking due to the current quark masses. Various empirical aspects of QCD are then described as a mutual interplay among the three ingredients. Furthermore, the simultaneous description of the ground state and the collective excitations in the NJL model allows us to study the change of hadron properties in hot/dense medium in a self-consistent way.

We will introduce a NJL-type 4-fermion interaction between the massless quarks with $U_L(3) \otimes U_R(3)$ chiral invariance to describe DBCS. The interaction has a strong attractive force between quark and anti-quark in the $J^P=0^+$ channel. This induces the instability of the Fock vacuum of massless quarks to realize the non-perturbative ground state with $\bar{q}q$ condensation, which is quite similar to the BCS mechanism of the superconductivity \[16\]. Due to the pair condensation, the original symmetry of the theory is broken down to $U_V(3)$ and the dynamical quark mass $M$ is generated. The mass $M$ is nothing but a quantity which should be identified with the constituent quark mass in the constituent quark model. Actually, the NJL model predicts $M$ around 300 (500) MeV for $u, d$ quarks ($s$ quark). Thus the notion of chiral symmetry and the success of the non-relativistic quark model is reconciled. After DBCS, there still remains residual interactions between the constituent quarks, which give rise to collective excitations, i.e., mesons in the new vacuum. In the pseudo-scalar channel, the strong residual force makes the massless NG bosons, while in the scalar channel, scalar bosons (the chiral partners of NG bosons) with the mass $2M$ are formed. The simple relation for the mass-ratio $m_{NG} : M : m_{scalar}=0:1:2$ is called the Nambu-relation \[15, 17\]. Up to this point, there is no flavor mixing among different flavors if only the 4-fermion interactions are adopted (decoupling of flavors).

As a next ingredient, we will introduce the current quark masses, which immediately causes mass splittings among the mesons (and baryons) with different flavors. In particular, $uu$, $dd$ and $ss$ are preferred as the mass eigenstates due to a combined effect of the quark-masses and the decoupling of flavors in the interaction. Thus the
realistic composition \( \pi^0 \sim u\bar{u} - d\bar{d}, \eta \sim u\bar{u} + d\bar{d} - 2s\bar{s} \) and \( \eta' \sim u\bar{u} + d\bar{d} + s\bar{s} \) are not yet realized at this level, while other particles are sitting almost in the right places.

To describe baryons, we will utilize a constituent quark model with the quark mass identified with \( M \) introduced above. The resulting masses are in excellent agreement with the empirical values, and hence satisfy the celebrated GMO mass formula and the equal-mass spacing rule. Nevertheless, the strange quark mass \( m_s \sim O(200) \) MeV turns out to be too large as an expansion parameter in QCD in contrast with \( m_{u,d} \). In other words, the success of the GMO mass-formula does not imply the validity of chiral perturbation (expansion by \( m_s \)). This observation resolves the wide-spread confusion about the magnitudes of the OZI violation in baryons. (See Chapter 3).

Finally, a term simulating the \( U_A(1) \) anomaly, which we shall call Kobayashi-Maskawa'-t Hooft (KMT) term \([18, 19]\), is introduced. The KMT term is a 6-fermion interaction written in a determinantal form and gives rise to a mixing between different flavors. On the contrary, the effect of large \( m_s \) tends to suppress the mixing of \((u, d)\) quarks with \( s \) quark, as noted above. Thus various observables related to the flavor mixing are the results of an interplay between the two effects. For example, in the pseudo-scalar channel, the KMT term dominates over the effect of \( m_s \). This causes a large flavor mixing among iso-singlet NG bosons towards the right composition \((\pi^0 \sim u\bar{u} - d\bar{d}, \eta \sim u\bar{u} + d\bar{d} - 2s\bar{s} \) and \( \eta' \sim u\bar{u} + d\bar{d} + s\bar{s} \)). Simultaneously, the \( \eta' \) mass is pushed up by the KMT term, hence the resolution of the \( U_A(1) \) problem.

In the scalar channel, the situation is found opposite and the \( s \) and \((u,d)\) do not mix so much. Nevertheless the small mixing still remains and shows up in the \( ss \) content of the nucleon. By fixing the strength of the KMT term in the \( \eta - \eta' \) sector, one can thus calculate non-valence components of baryons, at least those in the scalar channel. Furthermore, by combining the results with the heavy quark mass expansion in QCD, one can evaluate heavy-quark contents of mesons and baryons which are relevant to the heavy-quark production experiments.

It is in order here to mention the significance of the scalar meson \( \sigma \) (chiral partner of the pion) in QCD. A model-independent consequence of DBCS is the existence of the pion and its chiral partner \((\sigma)\): The former is the phase fluctuation of the order parameter \( \bar{q}q \) while the latter is the amplitude fluctuation of \( \bar{q}q \). The importance of \( \sigma \) has been overlooked so far since the existence of such a resonance is obscure in the \( \pi - \pi \) scattering phase shift in the \( I = J = 0 \) channel below 1 GeV \([10]\). On the other hand, light \( \sigma \) below 1 GeV is suggested theoretically \([23]\). The quark version of the NJL model gives \( m_\sigma \simeq 2M \simeq 700 \) MeV \([20]\) as a result of the Nambu-relation, and the mended symmetry by Weinberg predicts \( m_\sigma \simeq m_\rho \simeq 770 \) MeV \([10]\). Even the lattice data (taking into account only the connected diagram though) shows rather low-mass scalar meson \([17]\).

The apparent contradiction between the theory and experiment can be resolved, if the \( \sigma \rightarrow 2\pi \) decay width is so large and comparable to \( m_\sigma \). This picture is again qualitatively supported by the NJL model (and also in the linear \( \sigma \) model) where the decay width is calculable without introducing free parameters.

Now once the temperature of the system is raised, the \( \bar{q}q \) pair condensation tends to be broken by the thermal fluctuation. In the NJL model in the chiral limit \((m = 0)\), the condensation disappears above \( T_c \sim (150 - 200) \) MeV, which indicates that the restoration of chiral symmetry occurs above \( T_c \). Associated with this phase transition, collective excitations, in particular, the phase fluctuation (pion) and the amplitude fluctuation (\( \sigma \)) change their properties. As \( T \rightarrow T_c \), \( \sigma \) tends to degenerate with \( \pi \); thus \( m_\sigma \) decreases and so does its width due to the depletion of the phase space.
This means that the \( \sigma \)-meson, which is a broad resonance in the vacuum, becomes a good elementary excitation with a small width near \( T_c \) \([27, 30]\). Another interesting observation is that, even above \( T_c \), there may exist a large fluctuation of the order parameter with the quantum numbers of \( \pi \) and \( \sigma \) \([27, 30]\). The modes are analogous to those corresponding to the fluctuation of the order parameter in the superconductor above \( T_c \), which is responsible for an anomalous increase of the current conductivity (Maki-Aslamazov-Larkin mechanism) \([18]\). Because of their small mass and width, they should be as good elementary excitations as quarks and gluons above \( T_c \). Recent lattice QCD simulations of the hadronic modes above \( T_c \) seem to support this picture \([49]\).

The restoration of chiral symmetry under the strong background field (either electromagnetic field or color field) is also an interesting problem. Since an extensive review on this subject has been published \([50]\), we will not pursue this problem in this report.

Our effective Lagrangian (the NJL model) embodies essential ingredients of QCD at low energies with a few parameters and is workable but admittedly not equivalent with the whole QCD dynamics. An advantage of such approach lies in the fact that one can easily test physical ideas by rather simple calculations and also get possible phenomenological consequences of the basic ingredients of QCD. Our intention is not to give a precise quantitative description of the data within a few \% level. Instead, we are interested in developing physical ideas toward getting a unified picture and in calculating observables which would be hard to do in other approaches. In this sense, our effective theory written with quark fields is an intermediate theory between the QCD Lagrangian and the low energy chiral Lagrangians written in terms of only meson fields with infinite number of coupling constants. Once one gets qualitative ideas from our theory, one can check them by new laboratory experiments or numerical experiments (lattice QCD).

In the following sections of this chapter, we will recapitulate the basic aspect of QCD and also the rules to construct effective Lagrangians at low energies. In Chapter 2, we will give rather detailed description of the treatment of the Lagrangian in simplest cases. In Chapter 3, the model is applied to the world with three flavors (up, down and strangeness), in which the properties of the mesons are examined with an attention to the interplay of the three ingredients above. In Chapter 4, baryons are examined in our model, and the relation to the constituent quark model are made clear. These are Part I of this report. Part II deals with the systems at finite temperature. In Chapter 5, we give a general introduction of Part II and a brief account of field theory at finite temperature. In Chapter 6, we discuss changes of hadron properties associated with the chiral restoration. In Chapter 7, we explore possible existence of color-singlet collective modes above \( T_c \) as a precursor of DBCS. Chapter 8 is on another aspect of high \( T \) phase, i.e., the hadronic correlation in the vector channel and the quark-number susceptibility. Chapter 9 is devoted to a brief summary and perspectives.

1.2 Basic aspects of QCD

The classical QCD Lagrangian having local color \( SU(3) \) symmetry is written as

\[
\mathcal{L}_{QCD}^{cl} = \bar{q}(i\gamma_\mu D^\mu - m_q)q - \frac{1}{4} F^{a}_{\mu\nu} F^{a}_{\mu\nu},
\]

where \( q \) denotes the quark field with three colors and \( N_f \) flavors \( q=(u, d, s, \cdots) \), \( m_q \) is a mass matrix for current quarks \( m_q=\text{diag.}(m_u, m_d, m_s, \cdots) \), \( D_\mu(\equiv \partial_\mu - igt^a A^a_\mu) \) is
a covariant derivative with colored gauge field $A^a_\mu$ ($a=1 \sim 8$), $g$ the strong coupling constant and $t^a$ the $SU(3)$-color Gell-Mann matrix ($[t^a, t^b] = i f_{abc} t^c$, $\text{tr}(t^a t^b) = \delta^{ab}/2$). The field strength $F^a_{\mu\nu}$ is defined as $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$.

If one tries to carry out the perturbation theory in terms of $g$ in the quantum level, one has to introduce the gauge fixing term and an associated ghost term obtained from the Faddeev-Popov procedure. Furthermore, because of the regularization/renormalization procedure, a mass scale $\mu$ (renormalization scale) comes into the game.

The $\mu$-dependence of the renormalized strong coupling constant is governed by the $\beta$-function

$$\mu \frac{d}{d\mu} g(\mu) = \beta(g) \ . \quad (1.2)$$

As far as $g$ is small enough, $\beta$ is calculable in the perturbation theory

$$\beta(g) = -\frac{\beta_0}{(4\pi)^2} g^3 - \frac{\beta_1}{(4\pi)^4} g^5 + \cdots \ , \quad (1.3)$$

with

$$\beta_0 = 11 - \frac{2}{3} N_f \ , \ \beta_1 = 102 - \frac{38}{3} N_f \ , \quad (1.4)$$

where $N_f$ denotes the number of active flavors. Thus one arrives at the effective coupling constant or running coupling

$$\alpha_s(\mu) = \frac{g^2(\mu)}{4\pi} = \frac{12\pi}{(33 - 2N_f) \ln(\mu^2/\Lambda_{QCD}^2)} \left[1 - \frac{6(153 - 19N_f) \ln(\mu^2/\Lambda_{QCD}^2))}{(33 - 2N_f)^2 \ln(\mu^2/\Lambda_{QCD}^2)} \right] + \cdots \ . \quad (1.5)$$

Here $\Lambda_{QCD}$ is a scale parameter characterizing the change of $\alpha_s$ as a function of $\mu$. It depends on the subtraction scheme and the number of active flavors. Analyses of the various high energy processes show

$$\Lambda^{(3)}_{MS} = (290 \pm 30) \text{MeV},$$
$$\Lambda^{(4)}_{MS} = (220 \pm 90) \text{MeV},$$
$$\Lambda^{(5)}_{MS} = (140 \pm 60) \text{MeV}, \quad (1.6)$$

where the $\overline{MS}$ scheme is used and superscripts indicate the number of active flavors. Eq.(1.5) tells us that the running coupling decreases logarithmically as $\mu$ increases and the perturbation theory works well for large $\mu$. This is called the asymptotic freedom. Correspondingly, the running mass obeys the relation

$$\mu \frac{d}{d\mu} m(\mu) = -\gamma_m(g) m(\mu) \ , \quad (1.7)$$

1This does not necessarily mean that the expansion by $g$ is convergent for large $\mu$. Instead, the expansion is at most asymptotic. Large order behaviors of the expansion is reviewed in [55].
with the mass anomalous dimension

\[ \gamma_m = 2 \left( \frac{\alpha_s}{\pi} \right) + \left( \frac{101}{12} - \frac{5}{18} N_f \right) \left( \frac{\alpha_s}{\pi} \right)^2 + \cdots . \] (1.8)

The light quark masses determined from the hadron mass splittings and the QCD sum rules read

\[ m_u(1\text{GeV}) = (5.1 \pm 0.9)\text{MeV}, \quad m_d(1\text{GeV}) = (9.0 \pm 1.6)\text{MeV}, \quad (1.9) \]

from ref. [58] and

\[ m_u(1\text{GeV}) = (5.6 \pm 1.1)\text{MeV}, \quad m_d(1\text{GeV}) = (9.9 \pm 1.1)\text{MeV}, \quad (1.10) \]

\[ m_s(1\text{GeV}) = (161 \pm 28)\text{MeV}, \quad m_u(1\text{GeV}) = (5.6 \pm 1.1)\text{MeV}, \quad m_d(1\text{GeV}) = (9.9 \pm 1.1)\text{MeV}, \quad (1.10) \]

from ref. [59]. Within the perturbation theory, \( \mu \) should be chosen so that the higher order terms in the expansion are effectively suppressed. Thus \( \mu \) should be a typical scale of the system. For example, in the deep inelastic lepton-hadron scattering (DIS), the typical scale is \( Q^2 = -q^2 \) with \( q^2 \) being the four-momentum transfer to hadrons. For the systems at very high temperature (\( T \)) or at high baryon-number density, \( \mu \) will be identified with \( T \) or the fermi-momentum \( p_F \) of the system [60, 61]. This is the reason why we can expect the weakly interacting system of quarks and gluons in the high temperature/density plasma (quark-gluon plasma) [14]. The DIS processes have been investigated in great detail in the past two decades and the violation of the Bjorken scaling predicted by QCD was confirmed [4]. As for the quark-gluon plasma (QGP), the future projects on the ultra-relativistic heavy ion collisions at BNL (RHIC project) and at CERN (LHC project) will shed light on its nature.

Now, if \( \mu \) is comparable to \( \Lambda_{\overline{MS}} \), the perturbation theory does not work anymore and the non-perturbative phenomena play dominant roles, as noted in the previous section. Extensive lattice QCD studies [62], although they are still confined in a small lattice volume, confirm that the low-energy dynamics is really dominated by the non-perturbative configuration of the self-interacting gluon field. Unfortunately, because of the limited computer time and power, one does not yet reach a truly reliable QCD simulation including the virtual \( q - \bar{q} \) pair of light quarks. Furthermore, it is rather difficult to extract essential physics only from the direct numerical simulations. (Several attempts have been recently started to single out the dominant gluonic fluctuations on the lattice [63].)

### 1.3 Vacuum structure of QCD

Because of the non-perturbative interactions among quarks and gluons, the ground state of QCD has a non-trivial structure. This structure also affects the properties of the elementary excitations on the ground state, i.e., mesons and baryons.

The presence of the non-zero vacuum expectation values of certain operators is one of the evidences of such non-perturbative vacuum structure. For example, the current algebra relation (Gell-Mann-Oakes-Renner relation) [14] tells us

\[ f_\pi^2 m_{\pi^\pm}^2 \simeq -\hat{m} \langle \bar{u}u + \bar{d}d \rangle \] (1.11)

where \( \hat{m} = \frac{m_u + m_d}{2} \) is an averaged mass of \( u \) and \( d \) quarks. By taking the known pion decay constant \( f_\pi = 93 \text{ MeV} \) and \( \hat{m}(1\text{GeV}) = (7 \pm 2) \text{ MeV} \), one gets

\[ \langle \bar{u}u \rangle \simeq \langle \bar{d}d \rangle \simeq -[(225 \pm 25)\text{MeV}]^3 \quad \text{at} \quad \mu^2 = 1\text{GeV}. \] (1.12)
This means that the QCD ground state has condensation of quark and anti-quark pairs, which is, as noted in a previous section, quite analogous to the ground state of the BCS type superconductor where there is a condensation of the electron Cooper pairs $\langle \psi^\uparrow \psi^\downarrow \rangle \neq 0$ [4].

The analyses based on the QCD sum rules [65] for the heavy as well as light quark systems show that the gluons also condense in the vacuum [65, 66]

$$\langle \alpha_s F_{\mu\nu}^a F^a_{\mu\nu} \rangle = (350 \pm 30 \text{ MeV})^4 .$$

The non-zero value of the gluon condensate is also suggested by the numerical simulations on the lattice [67, 68].

There are of course lots of other scalar operators which have vacuum expectation values. One of such operators with physical significance is the four-quark operator

$$\alpha_s \langle (\bar{u}u)^2 \rangle = (1.8 - 3.8) \cdot 10^{-4} \text{ GeV}^6 ,$$

which is essential to determine the masses of $\rho$ and $\omega$ mesons in the QCD sum rules [65, 66].

Besides the fact that $\bar{q}q$ and $F_{\mu\nu}^a F^a_{\mu\nu}$ are the low dimensional operators, they have a close connection with the symmetry properties of the classical QCD Lagrangian $L_{QCD}$. Let’s summarize here the several symmetries of $L_{QCD}$.

(i) **Global chiral symmetry**: Under the transformation

$$q_L \rightarrow e^{i\lambda^a \alpha^a} q_L, \quad q_R \rightarrow e^{i\lambda^a \beta^a} q_R,$$

with $\lambda^0 = \frac{1}{\sqrt{2N_f}}$, $\lambda^i = i^i/2$ ($i = 1 \sim N_f^2 - 1$), $L_{QCD}$ has $U_L(N_f) \otimes U_R(N_f)$ symmetry in the chiral limit ($m_q = 0$).

(ii) **Dilatational symmetry**: Under the scale transformation

$$q(x) \rightarrow e^{3/2} q(e^{-1}x), \quad A^a_{\mu}(x) \rightarrow e A^a_{\mu}(e^{-1}x) ,$$

as well as $x_\mu \rightarrow e^{-1}x_\mu$, $L_{QCD}$ is invariant except for the quark mass term.

In the quantum level, some of these symmetries are broken by a quantum effect (anomaly) even in the chiral limit. The corresponding conservation laws are now modified as

$$\partial_\mu (\bar{q}_\gamma^a \lambda^a q) = i \sum_{i,j} \bar{q}_i (m_i - m_j) \lambda^a q_j \quad (a = 0 \sim N_f^2 - 1)$$

$$\partial_\mu (\bar{q}_\gamma^a \gamma_5 \lambda^a q) = i \sum_{i,j} \bar{q}_i (m_i + m_j) \gamma_5 \lambda^a q_j \quad (a = 1 \sim N_f^2 - 1)$$

$$\partial_\mu (\bar{q}_\gamma^a \gamma_5 q) = i \sum_i \bar{q}_i 2m_i \gamma_5 q_i + 2N_f \frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}^a_{\mu\nu}$$

$$\partial_\mu D^\mu = \Theta^\mu = (1 + \gamma_m) \sum_i \bar{q}_i m_i q_i + \frac{\beta}{2g} F_{\mu\nu}^a F^a_{\mu\nu} ,$$

Here the index $i, j$ stand for the flavor, $\tilde{F}^a_{\lambda\rho} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu}^a$ and $\Theta_{\mu\nu}$ is the energy momentum tensor of QCD. The third and last equation show the **axial anomaly** [69] and the **trace anomaly** [70], respectively.
Now the existence of $\langle \bar{q}q \rangle \neq 0$ implies that the $SU_L(N_f) \otimes SU_R(N_f)$ symmetry is broken down to $SU_V(N_f)$ i.e., the vacuum breaks the global chiral symmetry $Q^a_5 \parallel 0 \neq |0\rangle$.

Here we note an interesting physical consequence of the trace anomaly: Taking the vacuum expectation value of the last equation of eq. (1.17), one can see that QCD vacuum has a smaller energy density than the perturbative vacuum $\epsilon_{\text{vac}} - \epsilon_{\text{pert.}} = \langle \Theta_{\mu}^{\mu} \rangle \approx -\frac{7}{8} \langle \alpha_s \pi F_{\mu\nu} F_{\mu\nu}^a \rangle < 0$, (1.18)

where a small contribution from the quark condensates is neglected for simplicity, and $N_f=6$. Here the last inequality is due to eq.(1.13).

1.4 Dynamical quark mass and chiral symmetry breaking

The success of the constituent quark model implies that many of hadron properties can be described by a simple assumption that massive quarks with $M_{u,d}(M_s) \sim 300(500)$ MeV interact rather weakly inside hadrons. Such large masses of the constituent quarks are certainly not the same objects as the current-quark masses in the QCD Lagrangian.

In fact, $\langle \bar{q}q \rangle \neq 0$, which we saw in the previous section, suggests the existence of the "dynamical mass" in the quark propagator. The quark condensate in QCD is given by the trace of the full quark propagator $S_F$:

$$\langle \bar{q}q \rangle = -i \lim_{y \to x^+} \text{Tr} S_F(x,y) .$$

(1.19)

Since $\bar{q}q$ is a gauge invariant quantity, one can take any gauge to evaluate $S_F(x,y)$ and it will have a general form in the momentum space as

$$S_F(p) = \frac{A(p^2)}{p \cdot \gamma - B(p^2)} .$$

(1.20)

At least, within the perturbation theory in the chiral limit, there is no mixing between left handed quarks and right handed quarks. Therefore $B(p^2)=0$ and the quark condensate never takes a non-zero value just because of a simple trace identity $\text{Tr} \gamma_\mu = 0$. On the other hand, $B(p^2)$ can be generated in a non-perturbative way, as shown originally by Nambu and Jona-Lasinio in somewhat different context and also by several authors in the approximated version of QCD such as the ladder QCD approach. In the NJL model,

$$B(p^2) = M ,$$

(1.21)

which is a result of the self-consistent equation (the Schwinger-Dyson equation) with the contact 4-fermi interaction (See Fig.1.1). In the ladder QCD, $B(p^2)$ has a momentum dependence which is obtained by the infinite sum of the rainbow diagram (See Fig.1.2). In both cases, the non-perturbative generation of the scalar self-energy (or the dynamical mass) is intimately related to the generation of the non-zero quark condensate in the vacuum.

Fig. 1.1, Fig. 1.2
Here one might wonder the validity of the approximate form of $B(p^2)$ such as 
(1.21) which gives rise to a pole of $S_F(p^2)$. (The QCD vacuum is confining, therefore 
there should be no such pole in the exact quark propagator.) In fact, the generation 
of the dynamical mass is not the end of the story: There will be a long-range force 
together with massive quarks to confine them and also there will be a short range spin-spin 
interaction between massive quarks. The former will modify the low momentum part 
of the propagator to kill the on-shell propagation of free quarks. This is actually the 
whole idea of the constituent quark model supplemented by the notion of the dynamical 
symmetry breaking;

$$H_{QCD} = H_{\text{massless quarks}} + H_{\text{massless gluons}} + V_{\text{int}}$$

(1.22)

$$\simeq H_{\text{massive quarks}} + v_{\text{int}},$$

where $v_{\text{int}}$ is the interaction between massive quarks and is expected to be much weaker 
than the interaction between bare massless quarks, $v_{\text{int}} << V_{\text{int}}$ \[72\]. In short, the 
massive quarks, which are generated by the chiral symmetry breaking, make a good 
basis to represent the QCD Hamiltonian at low energies.

Another important observation of the above picture is that the conserved quantum 
numbers (such as charge, $SU_V$(3) flavor quantum numbers etc) are the same for current 
quarks and the constituent quarks. This is essentially due to the non-renormalization 
of the partially conserved currents \[73\]:

$$\langle \bar{q} \gamma_\mu \lambda^a q \rangle_{\text{low } \mu^2} = \langle \bar{q} \gamma_\mu \lambda^a q \rangle_{\text{high } \mu^2} \quad (a = 0 \sim 8)$$

(1.23)

$$\langle \bar{q} \gamma_\mu \gamma_5 \lambda^a q \rangle_{\text{low } \mu^2} = \langle \bar{q} \gamma_\mu \gamma_5 \lambda^a q \rangle_{\text{high } \mu^2} \quad (a = 1 \sim 8).$$

The currents normalized at low $\mu^2$ (corresponding to the currents for constituent 
quarks) are the same as those normalized at high $\mu^2$ (corresponding to the currents for 
current quarks). Because of this property, almost all the counting rules and the classifi-
cation based on the $SU_V$(3) symmetry works for massive quarks as well as massless 
quarks, which is again one of the basic advantages of the quark model. One of the 
well-known exception of this rule is the flavor-singlet axial charge of the constituent 
quark $g_0^A$. Since this quantity is scale dependent due to the axial anomaly, $g_A^0$ of a 
current quark is different from that of the constituent quarks.

The constituent quark picture (1.22) has some similarity with the idea of the Landau 
theory of the Fermi liquid \[74\]. In the Landau theory, the basic properties of the Fermi 
liquid at low temperatures can be accounted for in terms of weakly interacting quasi-
particles; they have the same quantum numbers as the bare particles but may have 
different physical properties by wearing clothes of the interaction adiabatically. Our 
constituent quarks (current quarks) in QCD at low energies are analogous to the quasi-
particles (bare particles) in the Landau theory.

### 1.5 Effective theories of QCD

The basic idea of the effective theories is found in many areas of physics: Suppose that 
we are treating the system with many degrees of freedom. If we are only interested in 
the dynamics of a few or small degrees of freedom in the total system, we will try to 
derive an effective theory only for the relevant degrees of freedom by eliminating the 
irrelevant degrees of freedom \[75\]. The way to eliminate irrelevant variables depends 
on the system. In many cases, it can be done only in an approximate manner and in
some cases, one does not even have any idea how to do it. Unfortunately, low energy QCD is in the last category at the present time. In this case, one has to write down the effective Lagrangian based on some general constraints such as the symmetry properties. In effective theories developed by Weinberg [76] and later by Gasser-Leutwyler [77], infinite numbers of coupling constants are allowed to describe the nature. Our approach is different from theirs and may be categorized as an intermediate step between QCD and such approaches as we will see below.

Our basic clues to write down the low energy effective theory of QCD are the following:

(i) **Effective quark theory**: we would like to formulate an effective theory which can be a basis for the constituent quark model. Therefore, the theory is written in terms of quark variables without explicit gluons.

(ii) **Chiral symmetry**: the theory should preserve the same global chiral symmetry as the QCD Lagrangian does, and should also develop the dynamical breaking of the symmetry.

(iii) **Low energy degrees of freedom**: we are interested in the low energy properties of the quark dynamics where the energy scale is smaller than some cutoff scale $\Lambda \simeq 1$ GeV. The short distant dynamics above $\Lambda$ to be dictated by the perturbative QCD will be treated as a small perturbation.

Thus the Lagrangian of the effective theory is generally written as

$$L_{\text{eff}}(x) = \sum_n c_n O_n(x) \left( \frac{1}{\Lambda} \right)^{\text{dim}O_n - 4}$$

where $O_n$ are the local chiral-invariant operators written with the quark fields and $c_n$ are the $c$-number dimensionless coupling constants. The theory is defined at the scale below $\Lambda$, thus the internal loop momenta of the theory should be cutoff at $\Lambda$. The effect of the higher dimensional operators are suppressed with the powers of $p^2/\Lambda^2$ where $p^2$ is the typical momentum of the system one treats. Once we truncates the above series and determine the finite number of coupling constants with suitable physical inputs, one can use the theory to calculate some other quantities. Note here that the cutoff scale $\Lambda$ will depends on the number of terms one takes in the above expansion.

### 1.6 Nambu-Jona-Lasinio model

A simplest effective theory satisfying the requirement in the previous section is the Nambu-Jona-Lasinio (NJL) model. Originally, the model was introduced to describe the pion as a bound state of the nucleon and the anti-nucleon [15]. We will replace their nucleon field by the quark field and adopt only the lowest dimensional operators as $O_n$. Then $L_{\text{eff}}$ for $N_f = 3$ reads

$$L_{\text{NJL}} = \bar{q}(i\gamma \cdot \partial - m_q)q + \sum_{a=0}^8 g_s \left[ (\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2 \right] + \cdots,$$

where $\cdots$ denotes all the possible 4-fermi interactions with global $U_L(3) \otimes U_R(3)$ symmetry.

Now in the quark level, we already know that $U_A(1)$ symmetry is broken by the axial anomaly. Since we do not have the explicit gluon field, we have to take into
account this effect in the tree level of our quark Lagrangian. The lowest dimensional operator which preserves \( SU_L(3) \otimes SU_R(3) \) but breaks \( U_A(1) \) is

\[
\mathcal{L}_{KMT} = g_D \det \left[ \bar{q}_i (1 - \gamma_5) q_j + \text{h.c.} \right].
\]  

This is a dimension 9 term introduced in [18, 19]. Eq.(1.25) and (1.26) are the basic effective Lagrangian [37]-[43] which we are going to examine in detail in this article.

A main drawback of the NJL model is a lack of confinement. However, as we have discussed before around eq.(1.22), the net effects \( v_{\text{int}} \) of confinement as well as the short range interactions may be treated as a small perturbation. This idea was first discussed by Goldman-Haymaker [72]. In our model, DBCS is described by the extended NJL model including the KMT term, hence our basic picture may be represented as

\[
\mathcal{L}_{\text{QCD}} \simeq \mathcal{L}_{\text{NJL}} + \mathcal{L}_{\text{KMT}} + \epsilon (\mathcal{L}_{\text{conf}} + \mathcal{L}_{\text{OGE}}). 
\]

The effective quark theory for \( N_f=3 \) with \( U_A(1) \) breaking term has a long history of research. The \( U_A(1) \) breaking term in the quark level eq.(1.26) was first written down by Kobayashi and Maskawa on 1970 to describe the mixing property of \( \eta - \eta' \) and also the heavy \( \eta' \) mass. After the discovery of the topological configuration in QCD, i.e., instantons, ’t Hooft derived an effective interaction for quarks under the instanton background which turned out to be the same structure as the Kobayashi-Maskawa’s except for an interaction with an explicit color \( SU(3) \) matrix. Thus we call (1.26) the KMT-term. Schechter and his collaborators applied the 4-fermi interaction + KMT-term to describe the nonet mesons within the long wave length approximation.

More serious studies based on the random-phase approximation or the Bethe-Salpeter equation have been carried out by Kunihiro and Hatsuda, by Bernard, Jaffe and Meissner and by Kohyama, Kubodera and Takizawa. Alkofer and Reinhardt developed a useful path-integral bosonization procedure for the Lagrangian with 6-fermi interactions. (see also the later extensive studies).

The 4-fermion NJL model had been also studied as a model for extended hadrons. The pioneering works by Eguchi and Sugawara and Kleinert initiated the studies of such theories in the mid 70’s. It was later shown that the NJL model reproduces and relates the basic hadronic parameters remarkably well by Ebert and Volkov, Volkov, and Hatsuda and Kunihiro. A realistic application of the model to the phenomena at finite \( T \) medium were initiated by Hatsuda and Kunihiro and later the systems at finite density and at finite external field have been studied.

The relation of the NJL model to the non-linear chiral model due to Skyrme, Wess and Zumino, and Witten (SWZW) has been also clarified by the works of Dhar and Wadia and Ebert and Reinhardt. The higher derivative terms of the chiral Lagrangian compiled by Gasser and Leutwyler were successfully reproduced by simple quark-loop integrals and the Wess-Zumino term is derived as an imaginary part of the effective action coming also from the quark loops. (See also ref. [94]).

Applications of the NJL model to the baryons are still under active studies. In this report, we will take a simplest constituent-quark picture where the residual interactions are assumed to be small (see Chapter 3 for details). Other approaches in the NJL

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2The \( N_f=2 \) version of \( \mathcal{L}_{KMT} \) or its non-local version has been also studied by several authors.

3In ref.[21], the NJL model was used as the Lagrangian describing the quark dynamics in the interior of the chiral bag.
model include the diquark-quark model (see e.g. \[97\]), soliton model (see e.g. \[98\]) and Faddeev approach (see e.g. \[99\]).

It should be also remarked that, in the strong-coupling lattice QCD, the effective quark theory similar to the NJL model is obtained \[100\]. This strong-coupling effective theory has been also applied to the hot and dense medium \[101\]. There are also some attempts to derive the NJL model from the continuum QCD; see e.g. ref. \[102\].

### 1.7 Relation to the other approaches

In this subsection, we will give a brief comment on the relation between the NJL approach and the other approaches for low energy hadrons.

Because of the dynamical symmetry breaking caused by the 4-fermi interaction, the following things happen simultaneously in the NJL model; non-zero vacuum condensate, generation of the dynamical quark mass, the creation of the massless Nambu-Goldstone boson (\(\pi\)) and the creation of the chiral partner (\(\sigma\)). In the NJL model, only the quarks are the basic degrees of freedom and \(\pi\) and \(\sigma\) are \(q\bar{q}\) bound states.

In the path-integral bosonization of the NJL model, by introducing the auxiliary fields \(\pi^a \sim \bar{q_i} \gamma_5 \lambda^a q\) and \(\sigma^a \sim \bar{q_i} \lambda^a q\), the partition function can be written as an integral over \(q, \bar{q}, \pi\) and \(\sigma\). If one further freezes the \(\sigma^a\) degrees of freedom, one arrives at an effective theory of quarks and pions in the non-linear liariization. We will call this effective theory as ”chiral quark model” (\(\chi\)QM) which has been studied by many authors independently of the development of the NJL model \[103\].

One can further integrate out the quark field explicitly or approximately and arrive at an effective theory written only in terms of the chiral fields and its derivatives \[94, 95, 104\]. The final form is nothing but the SWZW type chiral Lagrangian, which contains infinite number of derivative terms and also summarizes the anomalies in QCD.

There is one advantageous point of the NJL derivation of these effective Lagrangians. One can calculate all the coupling constants between quarks and mesons appearing in the chiral quark models and all the coupling constants in the chiral Lagrangian in the SWZW Lagrangian; they are controlled only by a few parameters in the NJL model.\footnote{This is analogous to the Gor’kov’s derivation \[105\] of the Ginzburg-Landau theory of the superconductivity \[106\].} This fact in turn gives a crucial test of the NJL model since some of the coupling constants in the chiral Lagrangian are known experimentally. We will come to this point later and we just quote here that within the 10-20% level, the NJL theory agrees to all the empirical facts, thus quite successful even as a quantitative theory for hadrons.

Here it is worth-while to mention the complementary roles of the non-perturbative dynamics at low energies and the perturbative QCD at high energies. Let’s write down the dispersion relation for the two point function of the bilinear composite operators such as \(\bar{q} \Gamma q\) with \(\Gamma\) being some gamma matrix:

\[
\text{Re}\Pi(q^2) = \frac{P}{\pi} \int_0^{\infty} \frac{\text{Im}\Pi(s)}{q^2 - s} ds + \text{(subtractions)} .
\]

Since the low energy dynamics below \(\Lambda\) is assumed to be dominated by \(\mathcal{L}_{NJL}\) and the higher energy part is dominated by the perturbative QCD (PQCD), \(\text{Im}\Pi\) has a hybrid form

\[
\text{Im}\Pi(s) = \theta(\Lambda^2 - s)\text{Im}\Pi_{NJL}(s) + \theta(s - \Lambda^2)\text{Im}\Pi_{PQCD} .
\]
Thus, as is evident from the dispersion relation, \( \text{Re}\Pi(q^2) \) is a superposition of the high energy and low energy contribution. The cutoff \( \Lambda \) has a meaning of the starting point of the continuum threshold of the spectral function or approximately the position of the second resonances of the mesons. It is interesting that this 4-momentum cutoff \( \Lambda \) turns out to be around 1.4 GeV which is quite consistent with the analysis of the QCD sum rules and also the locations of the second excited states of the low lying mesons.

The relative importance of NJL and PQCD depends on the region of \( q^2 \) which one is looking at. For low \( q^2 \) below the cutoff, \( \text{Re}\Pi \) is dominated by the NJL part (i.e. hadronic poles) and the PQCD part gives only a correction of \( O(q^2/\Lambda^2) \). On the other hand, for \( q^2 \to -\infty \), \( \text{Re}\Pi \) is dominated by the PQCD with power corrections coming from the NJL part of \( O(\Lambda^2/q^2) \). In the latter case, the operator product expansion works for \( \text{Re}\Pi \) and the power corrections are written with the various vacuum condensates. In this sense, the NJL model can even give an approximate evaluation of such non-perturbative condensates. The original work of Sakurai on the finite energy sum rule for vector mesons is implicitly based on this picture [107].

### 1.8 Related review articles

Since there have appeared several review articles on the NJL model recently, we will list them up here for the readers. One of the earliest review article is [44] by one of the present authors. This covers the development before 1989, some parts of which have overlap with this article. Reviews with some emphasis on the di-quark picture of the nucleon and the nuclear applications include [97] and [108]. [109] pays attention to the meson spectra, and the electro-magnetic and weak interactions of hadrons in the NJL model. [50] puts emphasis on the NJL model with strong external fields. The extensive applications of the model to baryons can be seen in [98]. These reviews including this article are complementary with each other.

Since the number of papers published in this field is enormous and still developing, it is beyond our ability to search and list up all the publications. We apologize to our colleagues if their important contributions are omitted in this review.
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