CP violation in $B \to PP$ in the SM with SU(3) symmetry

Han-Kuei Fu, Xiao-Gang He, Yu-Kuo Hsiao, and Jian-Qing Shi
Physics Department, National Taiwan University, Taipei, Taiwan, R.O.C

Abstract

In this paper we study CP violation in $B \to PP$ decays in the Standard Model using SU(3) flavor symmetry. With SU(3) symmetry only seven hadronic parameters are needed to describe $B \to PP$ decays in the SM when annihilation contributions are neglected. The relevant hadronic parameters can be determined using known experimental data from $B \to \pi\pi$ and $B \to K\pi$. We predict branching ratios and CP asymmetries for some of the unmeasured $B \to PP$ decays. Some of the CP asymmetries can be large and measured at B factories. The effects of annihilation contributions can also be studied using present experimental data. Inclusion of annihilation contributions introduces six more hadronic parameters. We find that annihilation contributions are in general small, but can have significant effects on CP asymmetries and some $B_s \to PP$. 

1
I. INTRODUCTION

In this paper we study rare charmless hadronic $B \rightarrow PP$ decays using SU(3) flavor symmetry in the Standard Model (SM). Here $P$ is one of the SU(3) octet pseudoscalar mesons. SU(3) analysis for rare charmless B decays have been studied by many groups and have obtained several interesting results, such as relations between different decay branching ratios, and ways to constrain and/or to determine the phase $\gamma$ [1–6]. SU(3) symmetry for $B \rightarrow PP$ decays is expected to be a good approximation because the energies released in these decays are larger than the hadronization scale. Test of SU(3) symmetry has been shown to be possible by using relations predicted and also using some $B_s \rightarrow PP$ decays in an electroweak model independent way [4]. Here we will take SU(3) symmetry as our working hypothesis. We will also study how SU(3) breaking effects affect the results.

Recently it has been shown that if enough $B \rightarrow PP$ decay branching ratios can be measured, in the framework of SU(3) symmetry, the associated hadronic parameters and their CP conserving final state interaction (FSI) phases, can be systematically studied [7]. The CP violating phase $\gamma$ in the KM matrix can also be constrained. Comparison of the phase $\gamma$ constrained this way with other constraints, the consistence of the SM can be checked. Once the hadronic parameters are determined, CP asymmetries in these decays can be predicted. We will carry out an analysis using the most recent data on rare charmless $B \rightarrow PP$ decays to determine hadronic parameters, to predict several other decay branching ratios and CP asymmetries in $B \rightarrow PP$ decays.

We start with a few comments on the determination of the CP violating phase $\gamma$ using information from $\epsilon_K$ in $K^0 - \bar{K}^0$ mixing, $B - \bar{B}$ mixings and $|V_{ub}/V_{cb}|$. Very stringent constraint on the CP violating phase $\gamma$ [7–9] can be obtained by using experimental information on various KM matrix elements [7,9]. Some of the most stringent constraint come from CP violating parameter $\epsilon_K$, $|V_{ub}/V_{cb}|$, $\Delta m_{B_s}$. The recently measured $\sin(2\beta)$ also provide important information. Although $B_s - \bar{B}_s$ mixing has not been measured, one can still use information on the upper bound on $\Delta m_{B_s}$ to constrain the phase. One of the method to
include $B_s - B_s$ mixing information, in a global $\chi^2$ fit of $\gamma$, is to use the amplitude method \[10\]. Using the input numerical values of the parameters in these processes as in Ref. \[7,9\], and the new averaged value of $\sin(2\beta) = 0.78 \pm 0.08$ \[11–14\], we obtain the best fit value of $\gamma$ to be $59^\circ$. The 68% C.L. and 95% C.L. allowed regions, in the Wolfenstein parameters $\rho$ and $\eta$ plane, are shown for this case in Figure 1.

The usage of upper bound on $\Delta m_{B_s}$ to constrain $\gamma$ is not without controversy because the result depends on how the bound is included. To avoid uncertainties due to this, we propose to fit the value for $\gamma$ without the use of $\Delta M_{B_s}$ bound. As a by product one can obtain a prediction for the range of $\Delta m_{B_s}$. Carrying out an $\chi^2$ analysis, we find that the best values with 68% C.L. errors for $\gamma$ and $\Delta M_{B_s}$, and their 95% C.L. ranges are given by

$$\gamma = 59^{+26}_{-16}^\circ, \quad 39^\circ \sim 94^\circ, \quad 95\% \text{ C.L. range;}$$

$$\Delta M_{B_s} = 17.9^{+4.4}_{-3.3}, \quad 10.8 < \Delta m_{B_s} < 26.1, \quad 95\% \text{ C.L. range.} \quad (1)$$

In Figure 1, we also show the allowed region (the dashed lines) in $\rho$ and $\eta$ plane. Since experiments constrain $\Delta m_{B_s}$ to be larger than $14.9$ ps$^{-1}$ \[15\] at the 95% C.L., the range for $\Delta m_{B_s}$ should be taken to be $14.9 \sim 26.1$ ps$^{-1}$ at the 95% C.L.. This prediction is consistent with the prediction of $\Delta m_{B_s} = 29^{+16}_{-11}$ ps$^{-1}$ \[16\] from measurement of $\Delta \Gamma_s$ and lattice calculation of $\Delta \Gamma_s/\Delta m_{B_s}$. The predicted range of $\Delta m_{B_s}$ can be measured at future
hadron colliders, such as LHCb, HERAb and BTeV. This will provide an important test for the SM.

The value of $\gamma$ obtained above will serve as a reference value for comparison when we study $B \to PP$ decays. We will make use of the values obtained in two ways. We will first study the consistency of the value obtained here and the one to be obtained from $B \to PP$ decays. The other way is to fix $\gamma$ at its best fit value determined above and to use experimental data on $B \to PP$ decays to fix hadronic parameters using SU(3) symmetry, and to predict other unmeasured branching ratios and CP asymmetries.

In Section II, we will briefly review the SU(3) parameterizations for $B \to PP$ decay amplitudes in the SM, and to study the consistence of $\gamma$ by comparing the constraint discussed earlier and that from $B \to PP$ decays. In Section III we study SU(3) hadronic parameters, the branching ratios and CP asymmetries for $B \to PP$ decays. In Section IV, we study the effects of annihilation amplitudes on $B \to PP$ decays. In Section V, we discuss some of the implications from our studies and draw conclusions.

II. SU(3) HADRONIC PARAMETERS AND THE PHASE $\gamma$

In SM the decay amplitudes for $B \to PP$ can be written as

$$ A(B \to PP) = \langle PP | H_{eff}^q | B \rangle = \frac{G_F}{\sqrt{2}} [V_{ub} V_{uq}^* T(q) + V_{ub} V_{tq}^* P(q)], \quad (2) $$

where $B = (B_u, B_d, B_s) = (B^-, B^0, B^{0*})$. $T(q)$ contains contributions from the tree operators as well as penguin operators due to charm and up quark loop corrections to the matrix elements, while $P(q)$ contains contributions purely from penguin due to top and charm quarks in loops.

SU(3) flavor symmetry can relate different $B \to PP$ decays. Therefore, knowing some of the branching ratios, other branching ratios and associated CP violating rate asymmetries can be predicted. As far as the SU(3) structure is concerned, the effective Hamiltonian contains 3, 6, and $\bar{15}$ which define three types of SU(3) invariant amplitudes. We use the
notations in Ref. [7]. In Table I we list $B \rightarrow PP$ decays in terms of the SU(3) invariant amplitudes.

| TABLE I. SU(3) decay amplitudes for $B \rightarrow PP$ decays. |
|---------------------------------------------------------------|
| $\Delta S = 0$                                               |
| $T_{\pi^- N^5}(d) = \frac{8}{\sqrt{2}} C^{T}_{15}$             |
| $T_{\pi^- N^6}(d) = \frac{2}{\sqrt{6}} (C^{T}_{3} - C^{T}_{6} + 3A^{T}_{15} + 3C^{T}_{15})$, |
| $T_{\pi^- K^0}(d) = C^{T}_{3} - C^{T}_{6} + 3A^{T}_{15} - C^{T}_{15}$, |
| $T_{\pi^+ K^+}(d) = 2A^{T}_{3} + C^{T}_{3} + C^{T}_{6} + A^{T}_{15} + 3C^{T}_{15}$, |
| $T_{\pi^0 K^0}(d) = \frac{1}{\sqrt{2}} (2A^{T}_{3} + C^{T}_{3} + C^{T}_{6} + A^{T}_{15} - 5C^{T}_{15})$, |
| $T_{\pi^0 K^0}(d) = \frac{1}{\sqrt{3}} (-C^{T}_{3} + C^{T}_{6} + 5A^{T}_{15} + C^{T}_{15})$, |
| $T_{\pi^0 K^0}(d) = \frac{1}{\sqrt{2}} (2A^{T}_{3} + \frac{1}{3} C^{T}_{3} - C^{T}_{6} - A^{T}_{15} + C^{T}_{15})$, |
| $T_{\pi^0 K^0}(d) = \frac{1}{\sqrt{2}} (C^{T}_{3} + C^{T}_{6} - A^{T}_{15} - 3C^{T}_{15})$, |
| $T_{\pi^0 K^0}(d) = -\frac{1}{\sqrt{2}} (C^{T}_{3} + C^{T}_{6} - A^{T}_{15} - 5C^{T}_{15})$, |
| $T_{\pi^0 K^0}(d) = -\frac{1}{\sqrt{2}} (C^{T}_{3} + C^{T}_{6} - A^{T}_{15} - 5C^{T}_{15})$, |

| $\Delta S = -1$                                               |
| $T_{\pi^- K^0}(s) = C^{T}_{3} - C^{T}_{6} + 3A^{T}_{15} - C^{T}_{15}$, |
| $T_{\pi^- K^0}(s) = \frac{1}{\sqrt{2}} (C^{T}_{3} - C^{T}_{6} + 3A^{T}_{15} + 7C^{T}_{15})$, |
| $T_{\pi^+ K^+}(s) = \frac{1}{\sqrt{6}} (-C^{T}_{3} + C^{T}_{6} - 3A^{T}_{15} + 9C^{T}_{15})$, |
| $T_{\pi^0 K^0}(s) = \frac{1}{\sqrt{2}} (C^{T}_{3} + C^{T}_{6} - A^{T}_{15} - 5C^{T}_{15})$, |
| $T_{\pi^0 K^0}(s) = \frac{1}{\sqrt{2}} (C^{T}_{3} + C^{T}_{6} - A^{T}_{15} - 5C^{T}_{15})$, |
| $T_{\pi^0 K^0}(s) = \frac{1}{\sqrt{2}} (C^{T}_{3} + C^{T}_{6} - A^{T}_{15} - 5C^{T}_{15})$, |
| $T_{\pi^0 K^0}(s) = \frac{1}{\sqrt{2}} (C^{T}_{3} + C^{T}_{6} - A^{T}_{15} - 5C^{T}_{15})$, |
| $T_{\pi^0 K^0}(s) = \frac{1}{\sqrt{2}} (C^{T}_{3} + C^{T}_{6} - A^{T}_{15} - 2C^{T}_{15})$, |

In general there are both tree and penguin amplitudes $C^{T,P}_{3,6,15}$, $A^{T,P}_{3,6,15}$. $C_6$ and $A_6$ always appear as $C_6 - A_6$ and we take this combination to be $C_6$. The amplitudes $A_i$ are referred as annihilation amplitudes. In total there are 10 complex hadronic parameters (20 real parameters with one of them to be an overall unphysical phase). However simplification can be made because the following relations in the SM,

$$C^P_6 = -\frac{3}{2} \frac{c^{tc}_{10} - c^{tc}_{10c}}{c_1 - c_2 - 3(c^{uc}_{10c} - c^{uc}_{10c})/2}C^T_6 \approx -0.013C^T_6,$$

$$C^P_{15} (A^P_{15}) = -\frac{3}{2} \frac{c^{tc}_{10} + c^{tc}_{10c}}{c_1 + c_2 - 3(c^{uc}_{10c} + c^{uc}_{10c})/2}C^T_{15} (A^T_{15}) \approx +0.015C^T_{15} (A^T_{15}).$$

Here we have used the Wilson coefficients obtained in Ref. [17]. With the above relations, there are less independent parameters which we choose them to be, $C^{T,P}_{3} (A^{T,P}_{3})$, $C^{T}_6$, and
Using the fact that an overall phase can be removed without loss of generality, we will set $C^P_3$ to be real, there are in fact only 13 real independent parameters for $B \rightarrow PP$ in the SM,

$$C^P_3, \quad C^T_3 e^{i\delta_3}, \quad C^T_6 e^{i\delta_6}, \quad C^T_{15} e^{i\delta_{15}}, \quad A^P_3 e^{i\delta_{A_3}}, \quad A^P_6 e^{i\delta_{A_6}}, \quad A^P_{15} e^{i\delta_{A_{15}}}.$$ 

Further the amplitudes $A_i$ correspond to annihilation contributions and are expected to be small. In this section, we neglect these amplitudes. In this case there are only 7 independent hadronic parameters

$$C^P_3, \quad C^T_3 e^{i\delta_3}, \quad C^T_6 e^{i\delta_6}, \quad C^T_{15} e^{i\delta_{15}}.$$ 

(4) The phases in the above are defined in such a way that all $C_i^{T,P}$ are real positive numbers. We will discuss how the annihilation contributions affect the decays in Section IV.

SU(3) may not be an exact symmetry for $B \rightarrow PP$. The amplitudes $C_i$ for $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$ will be different if SU(3) is broken. At present it is not possible to calculate the breaking effects. To have some idea about the size of the SU(3) breaking effects, we work with the factorization estimate. To leading order the relation between the amplitudes for $B \rightarrow \pi\pi$ decays $C_i(\pi\pi)$ and the amplitudes for $B \rightarrow K\pi$ decays $C_i(K\pi)$ can be parameterized as $C_i(K\pi) = rC_i(\pi\pi)$, and $r$ is approximately given by $r \approx f_K/f_\pi = 1.22$.

Here we have assumed that the SU(3) breaking effects in $f_i$ and $F_0^{B\rightarrow i}$ are similar in magnitude, that is, $f_K/f_\pi \approx F_0^{B\rightarrow K}/F_0^{B\rightarrow \pi}$. Using the above to represent SU(3) breaking effect, we can obtain another set of fitting results. Compared with $B \rightarrow K\pi$, there is also SU(3) breaking effect in $B_s \rightarrow K\pi$ proportional to $F^{B_s\rightarrow K}/F_B^{B\rightarrow K}$ or $F^{B_s\rightarrow \pi}/F_B^{B\rightarrow \pi}$. We will take them to be approximately 1. There are different ways to determine the hadronic parameters $C_i$ and $\delta_i$. A consistent and systematic way of carrying out such an analysis is to perform a $\chi^2$ analysis by taking into account all experimental data on $B \rightarrow PP$. We will use this method to obtain the hadronic parameters and also the CP violating phase $\gamma$.

In Table[1] we list present available experimental data on $B \rightarrow PP$ decays. In general the errors for the experimental data in Table[1] are correlated. Due to the lack of knowledge of the error correlation from experiments, in our analysis, for simplicity, we take them to be
uncorrelated and assume the errors obey Gaussian distribution taking the larger one between $\sigma_+$ and $\sigma_-$ to be on the conservative side. When combining from different measurements, we take the weighted average. For the data which only presented as upper bounds, we assume them to obey Gaussian distribution and taking the error $\sigma$ accordingly.

We will carry out our $\chi^2$ analysis with the KM matrix elements $V_{us} = \lambda$, $V_{cb} = A\lambda^2$, $V_{ub} = |V_{ub}|e^{-i\gamma}$ fixed by [8] $\lambda = 0.2196$, $A = 0.835$ and $|V_{ub}| = 0.09|V_{cb}|$ and take $\gamma$ to be a free parameter to be determined in this section. The total parameters to be determined are the 7 hadronic parameters in Eq. (4) and $\gamma$.

In Figure 2, we show the $\chi^2$ as a function of the phase $\gamma$. From the figure we see that for the case with exact SU(3) symmetry $\gamma$ between $20^\circ \sim 160^\circ$, the $\chi^2$ is reasonably small and allowed at the one sigma level. Although there are minimal points in the curve, they are not deep enough to single out one point with high significance. $\gamma$ around $60^\circ$ is certainly allowed. There is no inconsistency between the allowed range of $\gamma$ obtained here and that in the previous section. For the case with broken SU(3) symmetry, the region with $\gamma$ around $110^\circ$ is not favored. But $\gamma$ around $60^\circ$ is still allowed at 90% C.L.. Accurate experimental data in the near future will provide us with more information.

![FIG. 2. $\chi^2$ vs. $\gamma$ phase without annihilation terms. The solid line is for the case with exact SU(3) symmetry, and the dotted line is for the case with SU(3) breaking described in the text.](image-url)
TABLE II. The branching ratios for $B \to PP$ in units of $10^{-6}$.

| Branching ratio and CP asymmetries | Cleo       | Belle      | Babar      | Averaged Value |
|-----------------------------------|------------|------------|------------|----------------|
| $Br(B_u \to \pi^-\pi^0)$          | $5.6^{+2.6}_{-2.3} \pm 1.7$ | $7.0 \pm 2.2 \pm 0.8$ | $4.1^{+1.1+0.8}_{-1.0-0.7}$ | $4.9 \pm 1.1$ |
| $Br(B_u \to K^-K^0)$              | $< 5.1(90\%C.L.)$ | $< 3.8(90\%C.L.)$ | $< 1.3(90\%C.L.)$ | $0 \pm 0.8$ |
| $Br(B_d \to \pi^+\pi^-)$         | $4.3^{+1.6}_{-1.4} \pm 0.5$ | $5.1 \pm 1.1 \pm 0.4$ | $5.4 \pm 0.7 \pm 0.4$ | $5.2 \pm 0.6$ |
| $Br(B_d \to \pi^0\pi^0)$         | $2.2^{+1.7+0.7}_{-1.3-0.7}$ | $2.9 \pm 1.5 \pm 0.6$ | $0.9^{+0.9+0.8}_{-0.7-0.6}$ | $1.7 \pm 0.9$ |
| $Br(B_d \to K^-K^+)$              | $< 1.9(90\%C.L.)$ | $< 0.5(90\%C.L.)$ | $< 1.1(90\%C.L.)$ | $0 \pm 0.3$ |
| $Br(B_d \to \bar{K}^0K^0)$       | $1.8^{+1.8}_{-1.2} \pm 1.8$ | $< 13(90\%C.L.)$ | $< 7.3(90\%C.L.)$ | $1.8 \pm 2.5$ |
| $Br(B_u \to \pi^-\bar{K}^0)$     | $18.2^{+4.6}_{-4.0} \pm 1.6$ | $18.8 \pm 3.0 \pm 1.5$ | $17.5^{+1.8}_{-1.7} \pm 1.3$ | $17.9 \pm 1.7$ |
| $Br(B_u \to \pi^0K^-)$            | $11.6^{+3.0+1.4}_{-2.7-1.3}$ | $12.5 \pm 2.4 \pm 1.2$ | $11.1^{+1.3}_{-1.2} \pm 1.0$ | $11.5 \pm 1.3$ |
| $Br(B_d \to \pi^+K^-)$            | $17.2^{+2.5}_{-2.4} \pm 1.2$ | $21.8 \pm 1.8 \pm 1.5$ | $17.8 \pm 1.1 \pm 0.8$ | $18.6 \pm 1.1$ |
| $Br(B_d \to \pi^0\bar{K}^0)$     | $14.6^{+5.9+2.4}_{-5.1-3.3}$ | $7.7 \pm 3.2 \pm 1.6$ | $8.2^{+3.1}_{-2.7} \pm 1.2$ | $8.8 \pm 2.3$ |
| $A_{CP}(B_u \to \pi^-\pi^0)$     | $0.31 \pm 0.31 \pm 0.05$ | $-0.02^{+0.27}_{-0.26} \pm 0.10$ | $0.13 \pm 0.21$ |
| $A_{CP}(B_d \to \pi^+\pi^-)$     | $0.94^{+0.25}_{-0.31} \pm 0.09$ | $-0.02 \pm 0.29 \pm 0.07$ | $0.42 \pm 0.22$ |
| $A_{CP}(B_u \to \pi^-\bar{K}^0)$ | $0.18 \pm 0.24$ | $0.46 \pm 0.15 \pm 0.02$ | $-0.17 \pm 0.10 \pm 0.02$ | $0.04 \pm 0.08$ |
| $A_{CP}(B_u \to \pi^0K^-)$        | $-0.29 \pm 0.23$ | $-0.04 \pm 0.19 \pm 0.03$ | $0.00 \pm 0.11 \pm 0.02$ | $-0.05 \pm 0.09$ |
| $A_{CP}(B_d \to \pi^+K^-)$        | $-0.04 \pm 0.16$ | $-0.06 \pm 0.08 \pm 0.01$ | $-0.05 \pm 0.06 \pm 0.01$ | $-0.05 \pm 0.05$ |
TABLE III. The best fit values and their 1σ errors of the hadronic parameters using all data in Table II with annihilation terms set to be zero and γ = 59°.

|         | SU(3) exact | SU(3) break |
|---------|-------------|-------------|
|         | central value | error range   | central value | error range   |
| $C_3^P$ | 0.138       | 0.003        | 0.113         | 0.003         |
| $C_3^T$ | 0.248       | 0.111        | 0.245         | 0.074         |
| $C_{15}^T$ | 0.155 | 0.112        | 0.142         | 0.092         |
| $C_{15}^T$ | 0.142 | 0.014        | 0.145         | 0.014         |
| $\delta_3$ | 38.10⁰  | 29.69⁰       | 34.74⁰       | 23.51⁰       |
| $\delta_6$ | 83.17⁰  | 35.97⁰       | 71.59⁰       | 29.42⁰       |
| $\delta_{15}$ | 4.78⁰  | 17.84⁰       | 3.36⁰        | 15.31⁰       |

III. BRANCHING RATIOS AND CP ASYMMETRIES FOR $B \to PP$

In the previous section we have seen that the CP violating phase γ determined using data from $B \to PP$ and from $\epsilon_K$, $B - \bar{B}$ mixing and $|V_{ub}/V_{cb}|$ are not in conflict. One may want to combine these two to predict a combined best fit value for γ. At present the fit from the first section for γ has a much better error range. The combined fit will give a value for γ similar to the one in the previous section [7]. In this section we will use the best fit value of γ = 59° from the first section as a known value to study in more details about $B \to PP$ decays.

The best fit values for the hadronic parameters are given in Table III. The magnitudes of $C_i$ are the same order of magnitude as the factorization predictions [7]. The CP conserving phases $\delta_i$, which can not be calculated in factorization approximation, can be determined from the $\chi^2$ analysis performed here. We see from Table III that these CP conserving phases can be large.

Using the above determined hadronic parameters, one can easily obtain the branching
ratios and CP asymmetries for $B \to PP$. We use the following definition for the CP violating rate asymmetry,

$$A_{CP} = \frac{\Gamma(B_i \to PP) - \Gamma(\bar{B}_i \to \bar{P}\bar{P})}{\Gamma(B_i \to PP) + \Gamma(\bar{B}_i \to \bar{P}\bar{P})}. \quad (5)$$

In general $P$ can be any one of the SU(3) pseudoscalar octet mesons, $\pi$, $K$ and $\eta_8$. Here we will limit our study to $P = \pi, K$ to avoid complications associated with $\eta_1$ and $\eta_8$ mixings. In this case there are total 16 decay modes. Among them the decay amplitudes for $B_d \to K^-K^+$, $B_s \to \pi^-\pi^+, \pi^0\pi^0$ only receive annihilation contributions. Since we have neglected annihilation contributions they would have vanishing branching ratios. At present none of them have been measured experimentally. The present bound on $B_d \to K^-K^+$ is consistent with this prediction.

In Tables IV and V we show the results for the branching ratios and CP asymmetries for the other 13 decays. We see that the best fit values of the branching ratios for the ones have experimental measurements are similar and agree with each other within error bars. We also predict the branching ratios for $B_s \to K^+\pi^-, K^0\pi^0, K^-K^+, K^0\bar{K}^0$ decays. These decay modes are predicted to be large and can be measured at hadron colliders, such as, CDF, HERAb and LHCb. The SM and SU(3) flavor symmetry can be tested.

SU(3) symmetry predicts some of the CP asymmetries to be equal. From Table I we obtain

$$A_{CP}(B_d \to \bar{K}^0K^0) = A_{CP}(B_u \to K^-K^0),$$
$$A_{CP}(B_d \to \pi^+\pi^-) = A_{CP}(B_s \to K^+\pi^-),$$
$$A_{CP}(B_d \to \pi^0\pi^0) = A_{CP}(B_s \to K^0\pi^0),$$
$$A_{CP}(B_d \to \pi^+K^-) = A_{CP}(B_s \to K^+K^-),$$
$$A_{CP}(B_u \to \pi^-\bar{K}^0) = A_{CP}(B_s \to K^0\bar{K}^0). \quad (6)$$

When SU(3) is broken, in general these relations may no longer hold. However in the special pattern of the SU(3) breaking we are dealing with the above relations still hold.
Experimental measurements of CP asymmetries for these modes can provide important test for the SU(3) flavor symmetry.

In the SU(3) limit there are also some relations between rate differences defined as,
\[ \Delta(B_i \to PP) = \Gamma(B_i \to PP) - \Gamma(\bar{B}_i \to \bar{P}\bar{P}) \],
between \( \Delta S = 0 \) and \( \Delta S = -1 \) modes due to a unique feature of the SM in the KM matrix element that \[ 21 \]
\[ \text{Im}(V_{ub}V_{ud}^*V_{tb}^*V_{td}) = -\text{Im}(V_{ub}V_{us}^*V_{ts}^*V_{ts}) \].

We find \[ 4 \]
\[ \Delta(B_d \to \pi^+\pi^-) = -\Delta(B_d \to \pi^+K^-), \]
\[ \Delta(B_d \to \pi^0\pi^0) = -\Delta(B_d \to \pi^0\bar{K}^0), \]
\[ \Delta(B_d \to \bar{K}^0K^0) = -\Delta(B_u \to \pi^-\bar{K}^0). \] (7)

These rate difference relations can also provide important information.

The best fit values for \( A_{CP} \) can be large with several of them reaching 30\%, such as the asymmetries for \( B_d \to \pi^+\pi^- , \pi^0\pi^0 \), and \( B_s \to K^+\pi^- , K^0\pi^0 \). \( B_d \to \pi^+\pi^- \) provides the best chance to measure CP asymmetry. The fact that the size of \( A_{CP} \) for these modes are large, can be easily understood from the following. Using the above relations, one would obtain
\[ A_{CP}(B_d \to \pi^+\pi^-) = A_{CP}(B_s \to K^+\pi^-) = -A_{CP}(B_d \to \pi^+K^-) \frac{Br(B_d \to \pi^+K^-)}{Br(B_d \to \pi^+\pi^-)} , \]
\[ A_{CP}(B_d \to \pi^0\pi^0) = A_{CP}(B_s \to K^0\pi^0) = -A_{CP}(B_d \to \pi^0\bar{K}^0) \frac{Br(B_d \to \pi^0\bar{K}^0)}{Br(B_d \to \pi^0\pi^0)}. \] (8)

In all the above cases the ratio of the branching ratios are larger than one, a small \( A_{CP} \) of the decay modes on the right hand side can induce a large \( A_{CP} \) for the decay modes on the left hand side. The situation with SU(3) breaking case is also similar.

The cases for \( B_u \to \pi^-\bar{K}^0 , B_d \to K^0\bar{K}^0 \) and \( B_s \to K^0\bar{K}^0 , B_u \to K^-K^0 \) are particularly interesting. In the factorization approximation, the tree amplitude for these modes are almost zero. In terms of the SU(3) amplitudes that implies \( \Delta C = C^T_3 - C^T_6 - \frac{C^T_{15}}{15} \) is close to zero. CP asymmetries are predicted to be very small. \( \Delta C \) is small, however, does not follow from SU(3) symmetry. Rescattering effect may make it significantly deviate from zero. From Table \[ \text{[1]} \] we can see that the best fit value for \( \Delta C = 0.035 - 0.013 i \) for exact \( 0.011 - 0.004 i \) for break is small, but
TABLE IV. The prediction of the branching ratio without annihilation terms and $\gamma=59^\circ$.

| Branching ratio | SU(3) Exact          | SU(3) break          |
|-----------------|----------------------|----------------------|
|                 | central value | Error( Max , Min )   | central value | Error( Max , Min )   |
| $B_u \to \pi^-\pi^0$ | 5.3          | ( 6.3 , 4.2 )       | 5.4          | ( 6.5 , 4.4 )       |
| $B_u \to K^-K^0$   | 0.7           | ( 1.1 , 0.6 )       | 1.1          | ( 1.4 , 1.0 )       |
| $B_d \to \pi^+\pi^-$ | 5.1          | ( 5.7 , 4.5 )       | 5.0          | ( 5.6 , 4.4 )       |
| $B_d \to \pi^0\pi^0$ | 1.3          | ( 2.1 , 0.7 )       | 1.1          | ( 1.9 , 0.6 )       |
| $B_d \to \bar{K}^0K^0$ | 0.7          | ( 1.0 , 0.6 )       | 1.0          | ( 1.3 , 0.9 )       |
| $B_u \to \pi^-\bar{K}^0$ | 19.1        | ( 20.3 , 18.0 )    | 19.2        | ( 20.3 , 18.1 )    |
| $B_u \to \pi^0K^-$   | 10.5         | ( 11.1 , 10.0 )    | 10.8        | ( 11.3 , 10.3 )    |
| $B_d \to \pi^+K^-$   | 18.5         | ( 19.5 , 17.6 )    | 18.4        | ( 19.3 , 17.5 )    |
| $B_d \to \pi^0\bar{K}^0$ | 8.6          | ( 9.0 , 8.1 )      | 8.3         | ( 8.7 , 7.9 )      |
| $B_s \to K^+\pi^-$   | 4.8           | ( 5.3 , 4.2 )      | 7.0         | ( 7.8 , 6.2 )      |
| $B_s \to K^0\pi^0$   | 1.2           | ( 2.0 , 0.7 )      | 1.6         | ( 2.7 , 0.8 )      |
| $B_s \to K^+K^-$   | 17.4         | ( 18.3 , 16.5 )    | 26.6        | ( 27.9 , 25.3 )    |
| $B_s \to K^0\bar{K}^0$ | 16.8        | ( 17.9 , 15.8 )    | 25.9        | ( 27.4 , 24.5 )    |

within errors it can be away from zero. Translating this into CP violating rate asymmetries for $B_u \to \bar{K}^0\pi^-$ and $B_s \to K^0\bar{K}^0$, we see that the best fit value is small, but non-zero asymmetries can not be ruled out. This can lead to large asymmetries for $B_u \to K^-K^0$ and $B_d \to K^0\bar{K}^0$ within the error bars, as can be seen from Table V experiments.

We note that the CP asymmetry for $B_u \to \pi^-\pi^0$ is zero in Table V resulting from SU(3) (or isospin) symmetry. In principle it should have a small asymmetry due to the different short distance strong and electroweak penguins, but it is negligibly small and have been neglected.

At present no CP asymmetry in $B \to PP$ has been measured. To see how sensitive the
TABLE V. The prediction of the CP asymmetry without annihilation terms and $\gamma=59^\circ$

| Asymmetry        | SU(3) Exact |                      | SU(3) break |                      |
|------------------|-------------|----------------------|-------------|----------------------|
|                  | Central value | Error (Max,Min)     | Central value | Error (Max,Min)     |
| $B_u \to \pi^- \pi^0$ | 0.00        | ( 0.00 , 0.00 )     | 0.00        | ( 0.00 , 0.00 )     |
| $B_d \to \pi^+ \pi^-$ | 0.32        | ( 0.46 , 0.18 )     | 0.24        | ( 0.35 , 0.12 )     |
| $B_u \to \pi^- \bar{K}^0$ | 0.00        | ( 0.05 , -0.04 )    | 0.00        | ( 0.04 , -0.03 )    |
| $B_u \to \pi^0 K^-$ | -0.01       | ( 0.06 , -0.10 )    | -0.01       | ( 0.06 , -0.10 )    |
| $B_d \to \pi^+ K^-$ | -0.09       | ( -0.05 , -0.13 )   | -0.10       | ( -0.05 , -0.14 )   |
| $B_u \to K^- K^0$ | -0.09       | ( 0.85 , -0.91 )    | -0.03       | ( 0.74 , -0.78 )    |
| $B_d \to \pi^0 \pi^0$ | 0.37        | ( 0.64 , -0.58 )    | 0.32        | ( 0.56 , -0.38 )    |
| $B_d \to \bar{K}^0 K^0$ | -0.09       | ( 0.85 , -0.91 )    | -0.03       | ( 0.74 , -0.78 )    |
| $B_d \to \pi^0 \bar{K}^0$ | -0.06       | ( 0.06 , -0.13 )    | -0.07       | ( 0.06 , -0.15 )    |
| $B_s \to K^+ \pi^-$ | 0.32        | ( 0.46 , 0.18 )     | 0.24        | ( 0.35 , 0.12 )     |
| $B_s \to K^0 \pi^0$ | 0.37        | ( 0.64 , -0.58 )    | 0.32        | ( 0.56 , -0.38 )    |
| $B_s \to K^+ K^-$ | -0.09       | ( -0.05 , -0.13 )   | -0.10       | ( -0.05 , -0.14 )   |
| $B_s \to K^0 \bar{K}^0$ | 0.00        | ( 0.05 , -0.04 )    | 0.00        | ( 0.04 , -0.03 )    |
TABLE VI. The best fit values and their error ranges for the hadronic parameters without annihilation terms and $\gamma = 59^\circ$ using data on branching ratios and CP asymmetry on $B_d \rightarrow K^+\pi^-$.  

| SU(3) exact | SU(3) break |
|-------------|-------------|
|             | central value | error range | central value | error range |
| $C_i^p$     | 0.139        | 0.003       | 0.114        | 0.003       |
| $C_i^T$     | 0.280        | 0.112       | 0.271        | 0.074       |
| $C_i^T$     | 0.176        | 0.147       | 0.182        | 0.103       |
| $C_i^T$     | 0.141        | 0.014       | 0.143        | 0.014       |
| $\delta_3$  | 29.54$^0$    | 29.86$^0$   | 27.92$^0$    | 19.86$^0$   |
| $\delta_6$  | 75.15$^0$    | 32.15$^0$   | 65.18$^0$    | 22.51$^0$   |
| $\delta_{15}$ | $-13.33^0$  | 21.71$^0$   | $-15.50^0$   | 19.03$^0$   |

bounds on CP asymmetries in Table I affect the analysis, we carried out an analysis using mostly branching ratio information. If we do not use any CP violating data, we find that the branching ratios are not affected very much. However, in this case there is a degeneracy in identifying particle and anti-particle branching ratios. This implies that one can only determine the size of the asymmetries but not the signs. To determine the sign, one should use at least one CP asymmetry data point to lift the degeneracy. For this purpose we select one CP asymmetry data point, the asymmetry for $B_d \rightarrow \pi^+K^-$, for which all experimental measurements have similar central values although there is still a large error bar to establish the measurement. We list the results in Tables VI, VII and VIII. From the Table VI, we see that the size of the hadronic parameters $C_i$ are not affected very much, but the CP conserving phase $\delta_i$ can vary quite a lot, especially for $\delta_{15}$. In terms of the branching ratios and CP asymmetries we find that branching ratios are similar, but CP asymmetries can be quite different which can be seen from Tables VII and VIII. The differences are largely caused by the differences in $\delta_i$. It is therefore very important to have good CP asymmetry
TABLE VII. The prediction of the branching ratio without annihilation terms and $\gamma=59^\circ$ using

measurement which not only provide information for CP violation but also information for

detailed dynamics of hadronic physics.
TABLE VIII. The prediction of the CP asymmetry without annihilation terms and $\gamma=59^\circ$ using data on branching ratios and CP asymmetry from $B_d \to K^+\pi^-$. 

| Asymmetry     | SU(3) Exact       | SU(3) break       |
|---------------|-------------------|-------------------|
|               | Central value     | Error (Max,Min)   | Central value | Error (Max,Min) |
| $B_u \to \pi^-\pi^0$ | 0.00 ( 0.00 , 0.00 ) | 0.00 ( 0.00 , 0.00 ) |
| $B_d \to \pi^+\pi^-$ | 0.21 ( 0.38 , 0.03 ) | 0.15 ( 0.27 , 0.03 ) |
| $B_u \to \pi^-\bar{K}^0$ | 0.00 ( 0.05 , -0.05 ) | 0.00 ( 0.04 , -0.04 ) |
| $B_u \to \pi^0K^-$ | 0.06 ( 0.17 , -0.2 ) | 0.09 ( 0.18 , -0.07 ) |
| $B_d \to \pi^+K^-$ | -0.06 ( -0.01 , -0.11 ) | -0.06 ( -0.01 , -0.11 ) |
| $B_u \to K^-\bar{K}^0$ | -0.01 ( 0.94 , -0.94 ) | 0.00 ( 0.79 , -0.79 ) |
| $B_d \to \pi^0\pi^0$ | 0.60 ( 0.77 , -0.85 ) | 0.53 ( 0.67 , -0.17 ) |
| $B_d \to \bar{K}^0K^0$ | -0.01 ( 0.94 , -0.94 ) | 0.00 ( 0.79 , -0.79 ) |
| $B_d \to \pi^0\bar{K}^0$ | -0.12 ( 0.15 , -0.18 ) | -0.15 ( 0.02 , -0.22 ) |
| $B_s \to K^+\pi^-$ | 0.21 ( 0.38 , 0.03 ) | 0.15 ( 0.27 , 0.03 ) |
| $B_s \to K^0\pi^0$ | 0.60 ( 0.77 , -0.85 ) | 0.53 ( 0.67 , -0.17 ) |
| $B_s \to K^+K^-$ | -0.06 ( -0.01 , -0.11 ) | -0.06 ( -0.01 , -0.11 ) |
| $B_s \to K^0\bar{K}^0$ | 0.00 ( 0.05 , -0.05 ) | 0.00 ( 0.04 , -0.04 ) |
IV. EFFECTS OF ANNIHILATION CONTRIBUTIONS

In the analyses of the previous sections we have neglected annihilation contributions to \( B \to PP \) decays. In this section we study the effects of the annihilation terms on \( B \to PP \) decays. The inclusion of annihilation contributions introduce 6 more hadronic parameters. They are

\[
A^T_3 e^{i \delta^T_3}, A^P_3 e^{i \delta^P_3}, A^T_{15} e^{i \delta^T_{15}},
\]

(9)

In total we would have 13 parameters. From Table II we see that there are 15 experimental data points. In principle, the 13 hadronic parameters under consideration can be determined. In Tables IX, X and XI we show the results on the hadronic parameters, \( B \to PP \) branching ratios and CP asymmetries.

From Table IX we see that the size of the best fit annihilation parameters \( A_i \) are small compared with the non-annihilation terms \( C_{3,15} \). This confirms the conjecture that annihilation contributions are small. The allowed ranges are, however, large and therefore cannot rule out the possibility of having significant annihilation contributions. We have to wait improved experiments to obtain more precise information. We note that \( A_i \) actually have similar size as \( C_6 \).

The branching ratios for \( B_d \to K^- K^+ \), \( B_s \to \pi^+ \pi^- \) and \( B_s \to \pi^0 \pi^0 \) which only receive contribution from annihilation are not vanishing any more. The branching ratios are expected to be small. From Table IX, we indeed find that these branching ratios are among the small ones.

It is interesting to note that although the annihilation amplitudes are small, in certain decay modes, such as \( B_s \to K^+ K^- \) and \( B_s \to K^0 \bar{K}^0 \), the effects on the branching ratios can be significant. This is because that although \( A^P_3 \) is small compared with \( C_{3,15} \), and is comparable with \( C_6^T \), but enhanced by a KM factor \( |V_{tb} V_{ts}^*/V_{ub} V_{us}^*| \). These modes provide good places to study the annihilation contributions. It can be seen that SU(3) breaking effects are also large in these decays. From Table XI, we also see that CP violation can be
TABLE IX. The best fit values and their errors for the hadronic parameters with annihilation terms and $\gamma = 59^{\circ}$.

|          | SU(3) exact | SU(3) break |
|----------|-------------|-------------|
|          | central value | error range | central value | error range |
| $C_{3}^{P}$ | 0.138       | 0.004       | 0.113         | 0.003       |
| $C_{3}^{T}$ | 0.208       | 0.181       | 0.225         | 0.112       |
| $C_{6}^{T}$ | 0.043       | 0.206       | 0.095         | 0.163       |
| $C_{15}^{T}$ | 0.141       | 0.014       | 0.143         | 0.014       |
| $\delta_{3}$ | 31.65$^{0}$ | 57.7$^{0}$  | 26.42$^{0}$   | 35.94$^{0}$ |
| $\delta_{0}$ | 97.74$^{0}$ | 147.97$^{0}$ | 81.90$^{0}$   | 52.38$^{0}$ |
| $\delta_{15}$ | 8.54$^{0}$  | 21.67$^{0}$ | 4.58$^{0}$    | 16.75$^{0}$ |
| $A_{3}^{P}$ | 0.025       | 0.042       | 0.017         | 0.022       |
| $A_{3}^{T}$ | 0.061       | 0.143       | 0.039         | 0.082       |
| $A_{15}^{T}$ | 0.036       | 0.075       | 0.023         | 0.043       |
| $\delta_{A_{3}^{P}}$ | $-13.78^{0}$ | 89.52$^{0}$ | $-42.27^{0}$ | 98.46$^{0}$ |
| $\delta_{A_{3}^{T}}$ | 73.46$^{0}$ | 107.18$^{0}$ | 54.88$^{0}$ | 113.5$^{0}$ |
| $\delta_{A_{15}^{T}}$ | $-131.12^{0}$ | 180.51$^{0}$ | $-175.94^{0}$ | 197.56$^{0}$ |
TABLE X. The prediction of the branching ratios with annihilation terms and $\gamma=59^\circ$

| Branching ratio | SU(3) Exact | SU(3) break |
|-----------------|-------------|-------------|
|                 | central value | Error( Max, Min ) | central value | Error( Max, Min ) |
| $B_u \to \pi^-\pi^0$ | 5.2 | ( 6.2, 4.1 ) | 5.3 | ( 6.4, 4.3 ) |
| $B_u \to K^-K^0$ | 0.7 | ( 1.0, 0.6 ) | 1.0 | ( 1.3, 0.9 ) |
| $B_d \to \pi^+\pi^-$ | 5.2 | ( 5.8, 4.5 ) | 5.1 | ( 5.7, 4.5 ) |
| $B_d \to \pi^0\pi^0$ | 1.4 | ( 2.2, 0.7 ) | 1.2 | ( 2.0, 0.6 ) |
| $B_d \to K^-K^+$ | 0.1 | ( 0.4, 0.0 ) | 0.1 | ( 0.4, 0.0 ) |
| $B_d \to \bar{K}^0K^0$ | 1.9 | ( 4.4, 0.4 ) | 2.2 | ( 4.5, 0.8 ) |
| $B_u \to \pi^-\bar{K}^0$ | 18.9 | ( 20.2, 17.5 ) | 18.8 | ( 20.0, 17.6 ) |
| $B_u \to \pi^0K^-$ | 10.3 | ( 11.1, 9.6 ) | 10.6 | ( 11.2, 10.0 ) |
| $B_d \to \pi^+K^-$ | 18.6 | ( 19.7, 17.6 ) | 18.6 | ( 19.5, 17.6 ) |
| $B_d \to \pi^0\bar{K}^0$ | 8.6 | ( 9.6, 8.1 ) | 8.4 | ( 8.9, 7.9 ) |
| $B_s \to K^+\pi^-$ | 4.1 | ( 6.7, 2.6 ) | 6.4 | ( 8.8, 4.7 ) |
| $B_s \to K^0\pi^0$ | 1.1 | ( 2.2, 0.4 ) | 1.4 | ( 2.6, 0.6 ) |
| $B_s \to \pi^+\pi^-$ | 2.3 | ( 9.5, 0.0 ) | 1.1 | ( 4.3, 0.0 ) |
| $B_s \to \pi^0\pi^0$ | 1.1 | ( 4.7, 0.0 ) | 0.5 | ( 2.2, 0.0 ) |
| $B_s \to K^+K^-$ | 31.8 | ( 51.9, 7.1 ) | 41.3 | ( 66.6, 14.8 ) |
| $B_s \to K^0\bar{K}^0$ | 30.9 | ( 50.7, 6.5 ) | 39.6 | ( 65.0, 13.5 ) |
| Asymmetry        | SU(3) Exact |                      | SU(3) break |                      |
|------------------|-------------|----------------------|-------------|----------------------|
|                  | Central value | Error (Max,Min)    | Central value | Error (Max,Min)    |
| $B_u \rightarrow \pi^-\pi^0$ | 0.00 | ( 0.00 , 0.00 ) | 0.00 | ( 0.00 , 0.00 ) |
| $B_d \rightarrow \pi^+\pi^-$ | 0.41 | ( 0.61 , 0.20 ) | 0.37 | ( 0.52 , 0.18 ) |
| $B_u \rightarrow \pi^-K^0$ | 0.01 | ( 0.05 , -0.04 ) | 0.00 | ( 0.04 , -0.03 ) |
| $B_u \rightarrow \pi^0K^-$ | -0.03 | ( 0.05 , -0.14 ) | -0.02 | ( 0.06 , -0.12 ) |
| $B_d \rightarrow \pi^+K^-$ | -0.06 | ( -0.01 , -0.11 ) | -0.07 | ( -0.02 , -0.12 ) |
| $B_u \rightarrow K^-K^0$ | -0.24 | ( 0.82 , -0.96 ) | -0.08 | ( 0.73 , -0.81 ) |
| $B_d \rightarrow \pi^0\pi^0$ | 0.19 | ( 0.72 , -0.99 ) | 0.19 | ( 0.62 , -0.86 ) |
| $B_d \rightarrow K^-K^+$ | 0.83 | ( 1.00 , -1.00 ) | 0.77 | ( 1.00 , -1.00 ) |
| $B_d \rightarrow \bar{K}^0K^0$ | 0.78 | ( 1.00 , -1.00 ) | 0.40 | ( 1.00 , -1.00 ) |
| $B_d \rightarrow \pi^0K^0$ | -0.02 | ( 0.13 , -0.12 ) | -0.04 | ( 0.10 , -0.13 ) |
| $B_s \rightarrow K^+\pi^-$ | 0.26 | ( 0.54 , 0.03 ) | 0.20 | ( 0.36 , 0.06 ) |
| $B_s \rightarrow K^0\pi^0$ | 0.12 | ( 0.68 , -0.86 ) | 0.21 | ( 0.62 , -0.64 ) |
| $B_s \rightarrow \pi^+\pi^-$ | -0.04 | ( 1.00 , -1.00 ) | -0.04 | ( 1.00 , -1.00 ) |
| $B_s \rightarrow \pi^0\pi^0$ | -0.04 | ( 1.00 , -1.00 ) | -0.04 | ( 1.00 , -1.00 ) |
| $B_s \rightarrow K^+K^-$ | -0.06 | ( -0.02 , -0.24 ) | -0.10 | ( -0.04 , -0.23 ) |
| $B_s \rightarrow K^0\bar{K}^0$ | -0.05 | ( 0.18 , -0.22 ) | -0.02 | ( 0.11 , -0.14 ) |
affected significantly. CP asymmetries in $B_d \to K^0\bar{K}^0$ can be more than 50% with a not too small branching ratio.

V. DISCUSSIONS AND CONCLUSIONS

We have studied branching ratios and CP violating rate asymmetries in $B \to PP$ decays in the Standard Model using SU(3) flavor symmetry. In the SM when annihilation contributions are neglected only seven hadronic parameters are needed to describe $B \to PP$ decays, six more hadronic parameters are needed to include the annihilation contribution. We have shown that present experimental data on these decays can be used to systematically determine hadronic parameters, in particular the CP conserving FSI phases.

Although great efforts have been made to understand the dynamics of low energy strong interactions to calculate theoretically the decay amplitudes and the CP conserving FSI phases for $B \to PP$ decays, such as factorization approximation with improvement from QCD corrections \cite{22}. It is still far away from being able to predict with high confidence level the amplitudes. Still factorization calculations may provide some ideal about the order of magnitude. We have numerically studied the predictions of factorization approximation for the size of the SU(3) invariant amplitudes. We found that the size of the hadronic amplitudes obtained in this paper are in the same order of magnitudes as those from factorization calculations \cite{4}, but the FSI phases, which can not be reliably calculated in factorization approximation, can be very different and large. We also found that the annihilation contributions are generally small, but can have significant effects on some decays, such as $B_s \to K^+K^-, K^0\bar{K}^0$.

We attempted to study SU(3) breaking effects in $B \to PP$ decays by assuming a simple pattern for the breaking effects. We found that although the general features are not changed very much, in certain decays the effects can be large, such as the branching ratios for $B_s \to K^+K^-, K^0\bar{K}^0$. Therefore these modes can be good modes to study SU(3) breaking effects.
We predicted branching ratios for several $B_s \rightarrow PP$ decays. These decay branching ratios can be measured at future hadron colliders. The SM and SU(3) flavor symmetry can be tested.

At present CP violating rate asymmetries in $B \rightarrow PP$ have not been measured. The use of SU(3) flavor symmetry can also provide important information on CP violation in the Standard Model. Using the best fit values for the hadronic parameters, we also obtained CP violating rate asymmetries for $B \rightarrow PP$ decays. We found that some of the asymmetries can be large and within the reach of $B$ factories. CP asymmetry in $B_d \rightarrow \pi^+\pi^-$ can be as large as 30% and even larger ones for $B_d \rightarrow KK$. It can be expected that with more accurate experimental measurements, the study of CP violating rate asymmetries can provide crucial information about dynamics for $B$ decays in the Standard Model.
REFERENCES

[1] M. Savage and M. Wise, Phys. Rev. D39, 3346(1989); ibid D40, Erratum, 3127(1989); 
X.-G. He, Eur. Phys. J. C9, 443(1999); N. G. Deshpande, X.-G. He, and J.-Q. Shi, 
Phys. Rev. D62, 034018(2000).

[2] M. Gronau et al., Phys. Rev. D50, 4529 (1994); D52, 6356 (1995); ibid, 6374 (1995); 
A.S. Dighe, M. Gronau and J. Rosner, Phys. Rev. Lett. 79, 4333 (1997); L.L. Chau et 
al., Phys. Rev. D43, 2176 (1991); D. Zeppendfeld, Z. Phys. C8, 77(1981).

[3] M. Gronau and D. London, Phys. Rev. Lett. 65, 3381(1990); M. Gronau and J. Rosner, 
Phys. Rev. D57, 6843(1998); ibid, D61, 073008(2000); Phys. Lett. B482, 71(2000), M. 
Gronau, D. Pirjol and T.-M. Yan, Phys. Rev. D60, 034021(1999).

[4] X.-G. He, J.-Y. Leou and C.-Y. Wu, Phys. Rev. D62, 114015(2000); N. G. Deshpande 
and X.-G. He, Phys. Rev. Lett. 75, 1703(1995).

[5] Y.-F. Zhou et al., Phys. Rev. D63, 054011(2001); M. Bargiotti et al., e-print hep- 
ph/0204029.

[6] R. Fleischer and T. Mannel, Phys. Rev. D57, 2752(1998); M. Neubert and J. Ros- 
ner, Phys. Lett. B441, 403(1998); M. Neubert and J. Rosner, Phys. Rev. Lett. 81, 
5076(1998); X.-G. He, C.-L. Hsueh and J.-Q. Shi, Phys. Rev. Lett. 84, 18(2000); M. 
Gronau and J. Rosner, Phys. Rev. D57, 6843(1998); N.G. Deshpande and X.-G. He, 
Phys. Rev. Lett. 75, 3064(1995).

[7] X.-G. He et al., Phys.Rev. D64 034002(2001).

[8] Particle Data Group, Eur. Phys. J. C15 1(2000).

[9] S. Mele, Phys.Rev. D59 113011(1999); A. Ali, D. London, Eur.Phys.J. C9 687(1999); 
F. Parodi, P. Roudeau and A. Stocchi, Nuovo. Cim. A112, 833(1999); F. Caravaglias 
et al., e-print hep-ph/0002171.
[10] H. G. Moser and A. Roussani, Nucl. Inst. Meth. A384, 491(1997).

[11] T. Affolder et al. (CDF Collaboration), Phys. Rev. D61, 072205(2000).

[12] The ALEPH Collaboration, Phys. Lett. 492, 259(2000).

[13] The Babar Collaboration, e-print hep-ex/0203007.

[14] The Belle Collaboration, e-print hep-ex/0205020.

[15] http://lepbosc.web.cern.ch/LEPBOSC, CKM workshop February 2002, CERN, Geneva.

[16] G. Boix, e-print hep-ex/0104048.

[17] N. Deshpande and X.-G. He, Phys. Lett. B336, 471(1994).

[18] D. Cronin-Hennessy, et al. (CLEO Collaboration), Phys. Rev. Lett. 85, 515(2000); D.M. Asner, et al. (CLEO Collaboration), Phys. Rev. D65, 031103(2002).

[19] M.Z. Wang (Belle Collaboration), talk presented at Lake Louise Winter Institute 2002 on Fundamental Interactions, Lake Louise, Alberta, Canada, February (2002).

[20] B. Aubert, et al. (Babar Collaboration), Phys. Rev. Lett. 87, 151802 (2001). J. Olsen and M. Bona (Babar Collaboration), talks presented at American Physical Society’s 2002 Meeting of The Division of Particles and Field, Williamsburg, Virginia, USA, May 24 - 28 (2002).

[21] C. Jarlskog, Phys. Rev. Lett. 55, 1093 (1985); Z. Phys. C29, 491 (1985); O.W. Greenberh, Phys, Rev. D 32, 1841 (1985); D.-D. Wu, Phys, Rev. D 33,860 (1986).

[22] M. Beneke et al., Phys. Rev. Lett. 83, 1914 1999; Nucl. Phys. B591, 313 (2000); Y.-Y. Keum, H.-n. Li and I. Sanda, Phys. Lett. B504, 6(2001); Phys. Rev. D63, 054008(2001).