The association between conceptual understanding and reasoning ability in mathematics: An analysis of DNR-based instruction models

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Abstract. Mathematical reasoning is a tool used to understand mathematical concepts. Mathematical understanding is the basic ability in learning mathematics which is a mental construction process that connects, makes generalizations and synthesizes internal and external representations of mathematical objects. It is suspected that these two abilities have a close relationship. Based on these assumptions, this study aimed to describe and reveal if there was an association between mathematical understanding and mathematical reasoning of college students who were taught using the Duality Necessity Repeated reasoning (DNR)-Based Instruction in the Basic Mathematics Course. 107 prospective primary education students participated in the study. Data was collected using a test consisting of four problems related to two-variable linear equations. Association analysis was assessed using Spearman test. The result showed that there was a strong and positive association between the ability of mathematical concept understanding and mathematical reasoning. This implies that college students’ mathematical reasoning abilities are strongly influenced by their understanding of mathematical concepts.

1. Introduction

Mathematical understanding is an ability to understand the basic nature of mathematical knowledge [1]. One of the basic natures of mathematics is having abstract objects. Understanding abstract objects of mathematics cannot be done just by memorizing. As Idris stated [2] that mathematical understanding is not only remembering mathematical concepts or procedures, but it requires deep understanding. The definition of further mathematical understanding is put forward by Wiggins & Mc Tighe [3] as a mental construction and abstraction; being able to communicate, solving problems and demonstrating reasoning [4]; being able to prove relationships and the representation of concepts in a situation or problem solving [5]. This corresponds to Harel's remark that mathematics is ways of thinking (WoT) and ways of understanding (WoU); duality that has a reciprocal relationship where both of them are mental actions such as interpreting, summing up, proving, explaining, generalizing and so on [6]. Therefore, a person is said to have the ability of mathematical understanding when he or
she is capable of performing the mental acts as mentioned earlier. This is in line with Hoosain’s opinion [7] that mathematical understanding includes mathematical facts and mathematical proofs.

Mathematical reasoning is the ability to think logically. It has something to do with making connection between concepts and situations and the ability to make conclusions. The study of mathematical reasoning has been extensively researched. For example, English’s [8] study found that mathematical reasoning is imaginative, and Russel’s [9] study stated that reasoning is a tool of understanding abstraction. Research findings on mathematical concept understanding and reasoning have also been widely practiced. For example, a research conducted by Hoosain [7] recommended educators to set students’ understanding as their goal in teaching mathematics. A theoretical study conducted by Kieran [10] suggested that mathematical concept understanding is sustainable and continuous. Hence, it can be concluded that there are some stages involved in achieving mathematical concept understanding.

Previous research on pre-service teachers’ mathematical ability show that there are at least five weaknesses possessed by students: (1) lack of prerequisite knowledge, (2) lack of understanding of the basic concepts of mathematics, (3) lack of focus in learning, (4) inability to identify problems, and (5) poor logical reasoning ability [11]. This is similar with a study conducted by Fuadi, Johar and Munzir [12] involving a number of eight grade students which was aimed to determine the improvement of mathematical concept understanding and reasoning after applying contextual learning model. The result shows that students who are taught by contextual learning model perform better than those who are taught with conventional learning. From some of these studies, there has been no research that aims to see the association between the ability of mathematical concept understanding and mathematical reasoning using the DNR-Based Instruction. Therefore this is what seems to be the novelty of this research. For that reason, it is necessary to develop an instrument to measure students’ mathematical concepts understanding and mathematical reasoning.

The DNR-Based Instruction model was developed by Harel [13] who suggested that the goal of mathematics learning is to help students constructing ways of thinking and ways of understanding [6]. DNR-Based Instruction consists of Duality, Necessity and repeated Reasoning, which are also the principles in Harel learning [14]. The duality principle is the fundamental concept of DNR. The principle of necessity is that what teacher wants to convey is what they need. This is called intellectual need [15]. This highly supportive model characteristic is what this research conducted.

2. Method
This study aimed to describe students’ mathematical concept understanding and mathematical reasoning ability and to determine whether there is an association between the conceptual understanding and mathematical reasoning of students who were taught with DNR - Based Instruction learning. Then, an instrument test related to mathematical conceptual understanding indicators and mathematical reasoning indicators was designed.

The indicators of mathematical conceptual understanding included being able to relate a concept to another concept, represent a mathematical situation in various ways, determine better representations. The indicators of mathematical reasoning included being able to perform calculations based on mathematical formula and draw general conclusions based on the mathematical process/concepts involved. This instrument test was developed based on validation process by four experts. The experts were two lecturers of mathematics education, an evaluation lecturer, and a linguistic lecturer. This instrument had been tested on 35 students at a university in Ternate, Indonesia.

The participants of this study were 107 students at the University of Khairun Ternate, Indonesia, who were enrolled in the Basic Mathematics Course. The scores of the students’ conceptual understanding and mathematical reasoning were obtained from the test given at the end of the lesson. The test consisted of four problems as presented in table 1.
### Tabel 1. Problem description.

| No | Problems                                                                                                                                 |
|----|----------------------------------------------------------------------------------------------------------------------------------------|
| 1  | Mr. Amir bought five recreational park tickets for two adults and three children for Rp105,000.00. Meanwhile Pak Iksan bought three tickets for adults and five tickets for children for Rp165,000.00. State this situation in the most appropriate form you can think of to determine the price of each recreational park ticket. |
| 2  | A convenience store sells two types of rice. Rice type I and rice type II. The price of rice type I is Rp10,000.00 and the price of rice type II is Rp13,000.00. The sales on that day were Rp2,935,000.00 and the amount of rice sold was 250 liters.  
   a. Which type of rice sells better? Explain your answer.  
   b. Suppose that the amount of rice sold was 370 liters and the sales were Rp4,450,000. How many litters was each type of the rice sold?  
   c. What is your conclusions based on these two problems? |
| 3  | The circumference of an isosceles triangle is 20 cm. If the length of both feet added by 3 cm and the length of the base is two times the original length, then the circumference is 34 cm. Represent your answers using more than two ways to calculate the lengths of all three sides of the isosceles triangle. |
| 4  | There is a rectangular object with a circumference of 22 cm. If the length is changed into three times the original length and the width twice the original width, then the circumference of the object becomes 58 cm. Find the area of the rectangle and explain how you get your answer. |

Descriptive analysis was used to obtain the description of the students’ ability of mathematical concept understanding and mathematical reasoning, and to predict the association between the two variables using Spearman's rho test. The interpretation of the Spearman association coefficient is presented in Table 2.

### Table 2. Interpretation of Spearman’s association coefficients.

| Association coefficient | Interpretation               |
|-------------------------|------------------------------|
| 0.00 to 0.20            | There is almost no association |
| 0.21 to 0.40            | Weak association             |
| 0.41 to 0.60            | Moderate association         |
| 0.61 to 0.80            | High association             |
| 0.81 to 1.00            | Perfect association          |

### 3. Result and discussion

In general, the average scores of the students' mathematical concept understanding and mathematical reasoning were 78.055 and 72.98, respectively. This data showed that the students’ mathematical concept understanding was higher than their mathematical reasoning. This result is in accordance with Kieran’s study [10] which stated that understanding of mathematical concepts is the basis of mathematical mastery and the foundation for learning more advanced mathematical concepts. Students’ existing knowledge plays an important role in connecting new ideas and information [16].

Before conducting statistical tests to determine whether there was an association between the two capabilities, data normality was tested using Kolmogorov Smirnov Z test. Normality test result showed that one of the data was not normally distributed. Therefore the association test used Spearman non-parametric test instead. The hypotheses were:
H_0: There is no association between the ability of mathematical concept understanding and mathematical reasoning ability.

H_1: There is an association between the ability of mathematical concept understanding and mathematical reasoning ability.

The testing criterion based on Spearman association test of the hypothesis was that if the significance value was less than the degree of \( \alpha = 0.05 \) then H_1 would be accepted. Otherwise H_0 would be accepted.

The results of Spearman’s association test between the ability of mathematical concept understanding and mathematical reasoning are shown in Table 3.

**Table 3.** Association between students’ mathematical understanding and mathematical reasoning.

| Spearman’s rho | mathematical concept understanding | Mathematical reasoning |
|----------------|-----------------------------------|------------------------|
| Spearman’s rho | Correlation coefficient 1.000     | 0.801**               |
| Sig. (2-tailed)| .                                  | 1.000                  |
| N              | 107                                | 107                    |
| *mathematical reasoning* | Correlation coefficient 0.801** | 1.000                  |
| Sig. (2-tailed)| 0.000                             | .                      |
| N              | 107                                | 107                    |

Based on Table 3, it could be seen that the correlation coefficient based on Spearman test between the ability of mathematical concept understanding and mathematical reasoning was 0.801, and the value of significance (p value) was 0.000. Therefore, based on the criteria of the association, it could be interpreted that the ability to understand mathematical concepts and mathematical reasoning were highly and positively correlated. This meant that the ability to understand mathematical concepts and mathematical reasoning had a positive relationship with high closeness. In other words, it could be interpreted that the higher ability mathematical concept understanding was, the higher mathematical reasoning. Students with high understanding of mathematical concepts tended to have high reasoning abilities. Thus it could be said that learning with DNR-Based Instructions developed the ability to understand mathematical concepts and mathematical reasoning. This finding is in line with Harel’s research [13] that the DNR-Based Instructions learning model helped students to devise ways of thinking and improved their understanding of mathematics, thus ensuring a positive impact on their mathematical concept understanding and reasoning.

### 4. Conclusion

Based on the findings, it could be concluded that the students’ mathematical concepts understanding ability was higher than their mathematical reasoning ability. There was a strong and positive association between the ability to understand mathematical concepts and mathematical reasoning.

**Acknowledgments**

Thank you to Yulyanti Harisman and Dr. Karman La Nani, M.Si who have helped to review this article.

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