A PEPS is given by a graph and a set of linear maps $P$.

```
|   |   |
|---|---|
| 0 | 0 |
```

A PEPS is efficiently created by a quantum computer.

```
|   |   |
|---|---|
| 0 | 0 |
```

PEPS = Postselection!

### Complexity crash course

Let $f : [0,1]^n \rightarrow [0,1]$ be an efficiently computable boolean function. Define $x \mapsto f(x) \in \{0,1\}$. Defining tasks for complexity classes:

- **NP**: Is $x \in S$?
- **PP**: Is $x \in S$? (A function class.)
- **QMA**: All problems where each yes instance $x$ has a quantum proof $\rho_+$ and each no instance $x$ has a quantum proof $\rho_-$.

#### PEPS and Postselection

A PEPS is given by a graph and a set of linear maps $P$.

```
|   |   |
|---|---|
| 0 | 0 |
```

Every state created by postselection is a PEPS!

```
|   |   |
|---|---|
| 0 | 0 |
```

### The computational complexity of PEPS

**N. Schuch**, M. M. Wolf, F. Verstraete, and J. I. Cirac

1. Max-Planck-Institut für Quantenoptik, Garching, Germany
2. Institute for Quantum Information, Caltech, Pasadena, USA
3. Fakultät für Physik, Universität Wien, Wien, Austria

#### Abstract

The Projected Entangled Pair States (PEPS) formalism which underlies DMRG has proven very successful for the description of complex many-body systems. While in one dimension, PEPS can be simulated efficiently, the situation in two and more dimensions is less clear. In this work, we determine the power of creating PEPS as well as the complexity of classically simulating PEPS. We also consider ground states of gapped Hamiltonians and show how they can be approximated by PEPS. Finally, we prove that the hardness of creating ground states is lower than for arbitrary PEPS. The central tool for our proofs is a new duality between PEPS and postselection.

**Our questions:**
- What is the power of creating PEPS?
- What is the complexity of simulating PEPS?
- How can PEPS approximate ground states?
- What is the power of creating ground states?

#### The power of creating PEPS

What is the power of a “PEPS oracle”, i.e. a black box which can create any PEPS?

```
|   |   |
|---|---|
| 0 | 0 |
```

PEPS = Postselection!

#### The power of a PEPS oracle equals PP.

Using PostBQP=PP [3]:

```
|   |   |
|---|---|
| 0 | 0 |
```

---

### The complexity of simulating PEPS

#### PEPS–postselection duality:

compute expectation value

contract tensor network

```
|   |   |
|---|---|
| 0 | 0 |
```

#### The complexity of simulating PEPS & the complexity of contracting tensor networks is both in PP.

**Problem:** Use techniques from simulation of normal $q$-circuits:

1. Rewrite with only Toffoli & Hadamard.
2. The amplitude $a_{xw}$ of an output state $\omega$ is the sum of the transition probabilities over all intermediate configurations $C = (c_0, \ldots, c_{n-1})$:
   
   $a_{xw} = \sum_{c_0, \ldots, c_{n-1}} p(c_0) p(c_1 | c_0) \ldots p(c_{n-1} | c_{n-2}) p(xw | c_{n-1})$

3. The probability for output state $\omega$ is $p(\omega) = \sum_{xw} a_{xw}$.

4. $a_{xw} (\rho_s, \omega) \rightarrow$ the probabilities after postselection.  e.g. $p = \sum_{xw} a_{xw}$, can be computed in PP.

#### Proof:

- [1] F. Verstraete and J. I. Cirac, cond-mat/0407066
- [2] F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 200601 (2006); quant-ph/0507035.
- [3] S. Aaronson, Proc. R. Soc. Lond. A 461, 2473 (2005); quant-ph/0412187.
- [4] M.B. Hastings, Phys. Rev. B 73, 085115 (2006); cond-mat/0508054.

#### References:

---

### PEPS and Postselection

**What are PEPS?**

A PEPS is given by a graph and a set of linear maps $P$.

```
|   |   |
|---|---|
| 0 | 0 |
```

### Motivation

```
|   |   |
|---|---|
| 0 | 0 |
```

#### Postselection = Cooling!

```
|   |   |
|---|---|
| 0 | 0 |
```

The ground state of any polynomially gapped Hamiltonian can be approximated by the boundary of a PEPS with one extra dimension.

---

### The complexity of simulating PEPS

```
|   |   |
|---|---|
| 0 | 0 |
```

#### In general, it is as hard as general PEPS, i.e. PP!

**Why?**

- Take PP-hard problem encoded in a PEPS.
- Add random small $\epsilon$ - PEPS becomes injective, i.e. it is the unique ground state of a local Hamiltonian.
- Use oracle to get ground state – the original PEPS.
- Read out the result: PP problem solved!

Note: the energy gap is exponentially vanishing!

#### For gapped Hamiltonians, this is at most QMA!

**Solution!**

```
|   |   |
|---|---|
| 0 | 0 |
```

Q. Computer postprocessing

#### Problem!

```
|   |   |
|---|---|
| 0 | 0 |
```

Q. Computer postprocessing (as before)

#### No

```
|   |   |
|---|---|
| 0 | 0 |
```

Q. Computer postprocessing (as before)

#### Yes

```
|   |   |
|---|---|
| 0 | 0 |
```

Q. Computer postprocessing (as before)

#### Only for yes instances, the proof will be accepted!

...and thus (probably) less hard than arbitrary PEPS!