The surface temperature of free evaporating drops

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Abstract. Complex experimental and theoretical investigation of heat and mass transfer processes was performed at evaporation of free liquid drops. For theoretical calculation the emission-diffusion model was proposed. This allowed taking into account the characteristics of evaporation of small droplets, for which heat and mass transfer processes are not described in the conventional diffusion model. The calculation results of evaporation of droplets of different sizes were compared using two models: the conventional diffusion and emission-diffusion models. To verify the proposed physical model, the evaporation of droplets suspended on a polypropylene fiber was experimentally investigated. The form of droplets in the evaporation process was determined using microphotography. The temperature was measured on the surfaces of evaporating drops using infrared thermography. The experimental results have showed good agreement with the numerical data for the time of evaporation and the temperature of evaporating drops.

Introduction
Interest in the study of evaporation of liquid droplets is associated with a variety of practical applications, ranging from fuel burning and intensification of heat and mass transfer processes to formation of nanostructures [1-3]. The study of the evaporation of liquid droplets is a classical fundamental problem, and it is the subject of a large number of works [4-7]. However in the processes associated with droplet evaporation, there are still many fundamental issues that need deeper study. At evaporation the droplet size continuously decreases, which leads to the necessity of solving this problem in non-stationary conditions. The droplet size is a significant parameter that determines the influence of certain factors (surface tension, gravity, the free path of vapor molecules, etc.) on the process of drop evaporation. A physical model of drop evaporation should be chosen depending on the most significant factors.

There are a number of physical models describing the behavior of evaporating liquid droplets that have been proposed and are currently used [10-13]. It should be noted that the majority of studies use the diffusion approach of Maxwell for calculations of droplets evaporation. The diffusion model describes well the evaporation of large drops, but has limitations in the description of the droplets of small size. In addition to the diffusive approach, some researchers propose gas-kinetic models to describe evaporation [8, 14]. Thus, in [8] it is demonstrated that consideration of the kinetic approach to small drops evaporation leads to an increase in the droplets evaporation time in comparison with purely diffusion mechanism of evaporation.

Experimental investigations of droplets evaporation widely apply a technique of drops suspended on a thin fiber, which allows obtaining a good approximation to the conditions of free drop evaporation [5].
The experiments usually pay main attention to determining the varying geometry of the drops using photo and video equipment [1, 12, 13]. The temperature of drops in these studies is measured using contact methods. Drops suspended on a thermocouple have become a frequent practice allowing controlling the temperature change in the droplets in the evaporation process. For example, Watanabe et al. [12] investigated the evaporation of drops, suspended on a thin thermocouple, to study combustion of isolated droplets of liquid fuel. Gan et al. [1] conducted experiments on the evaporation of drops (suspended on thin K-type thermocouple) of different fuel mixtures with the addition of nanoparticles under natural and forced convection. A significant problem in such studies is heat supply to the droplet through the thermocouple due to high thermal conductivity of the thermocouple material, which leads to a significant error of the experiment.

Yang and Wong [13] studied the evaporation of drops, suspending them on the end of a thin quartz fiber to minimize the heat input to the drop. Similar experiments at elevated pressures and temperatures were carried out by Ghassemi et al. [15]. They found that at low ambient temperatures, the thermal conductivity of the fiber mostly influenced the early stage of evaporation, and at high temperatures the influence extended to the whole process. K. Han et al. [13] considered the effect of heat conduction of the thermocouple and the quartz fiber on the evaporation of suspended single droplets under different ambient temperatures. The results showed that the influence of quartz fiber on the evaporation of drops was substantially less than the one of the thermocouple. Nevertheless, there was an increase in evaporation rates, associated with the additional supply of heat. Thus, measuring the drop temperature with thermocouples implies significant errors due to the supply of heat through the thermocouple. In addition, such measurements are local in nature and do not provide information about the temperature distribution on the drop surface.

In recent years, temperature measurements on evaporating drop surfaces have been widely done using the infrared thermography technique [17-19]. The advantages of this method are contactlessness, high sensitivity and high-speed performance.

The aim of this work is a comparative analysis of the calculation results of free drops evaporation according to the diffusion model and the emission-diffusion model for different diameters of spherical water drops. To verify the proposed physical model the calculated changes in temperatures of suspended droplets and times of their evaporation were compared with the experimental results, obtained using the contactless infrared thermography and microphotographing.

1. Description of the computational model

For the calculations the emission-diffusion model of liquid droplets evaporation was used [8, 20]. The model is based on a dynamic equilibrium between the vapor flow, obtained in the framework of the gas-kinetic approach, and the diffusion flow. This approach, in contrast to the purely diffusional one, allows more accurately taking into account the characteristics of evaporation of small and large drops in the final stages of evaporation.

A conventional Maxwell diffusion model of droplets evaporation suggests that the concentration of vapor molecules in the vicinity of the drop surface is always equal to saturated vapor concentration at the temperature of the ambient gaseous medium. Obviously, this assumption is highly simplifying and is not always realizable. First, the surface emission of vapor permanently prevails over the diffusion flow, which entrains vapor into the ambient medium. Second, the assumption does not take into account the drop temperature and its dynamics. The drop temperature at the beginning of evaporation decreases rapidly and then for a certain time period keeps an approximately constant value that is lower than in the medium. In this case, the concentration of vapor molecules near the surface is determined not only by the ambient temperature.

The mentioned contradiction in a conventional diffusion model of drops evaporation can be surmounted involving the gas-kinetic description of the vapor flow from the droplet surface. At that, the flow on the surface must be equal to the diffusion vapor flow at a large distance from the droplet. The actual diffusion flow of vapor starts forming at a distance of several lengths of free path of vapor
molecules from the surface. So the equality of the flows can be achieved by introducing some intermediate vapor layer. The emission of vapor molecules can be defined in the framework of Hertz–Knudsen approach. In this case, the mass flow of vapor \( J_{HK} \) emitted by the droplet surface to the medium may be written as follows [8, 21]:

\[
J_{HK} = \frac{\gamma S}{\sqrt{2\pi R_v}} \frac{P_s(T_d)}{\sqrt{T}} \cdot (1 - \phi),
\]

where \( S \) is the surface area of the droplet, \( T_d \) is the temperature of the droplet, \( \gamma \) is the coefficient of accommodation, \( R_v \) is the gas constant for vapor, \( P_s \) is the saturated vapor pressure, and \( \phi \) is the relative humidity of the intermediate vapor layer near the drop surface. For calculations we used the traditionally accepted value of accommodation coefficient for water vapor \( \gamma = 0.04 \) [8, 22].

The diffusion flow of the vapor mass \( J_D \) at a distance from a spherical droplet taking into account the Stefan flow [5] may be expressed:

\[
J_D = -2\pi Dc_\mu \mu R \cdot Sh \cdot \ln \left( \frac{1 - \phi}{1 - \phi_c} \right),
\]

where \( D \) is the molecular diffusion coefficient of vapor in the medium, \( \mu \) is the molar mass of vapor, \( c_\mu \) is the molar concentration of the vapor-gas mixture, \( R \) is the universal gas constant, \( \phi_c \) is the relative humidity of the medium at a great distance from drops, \( T_a \) is the ambient air temperature. The equality between the vapor flows \( J_{HK} \) and \( J_D \) allows determining \( \phi \) for the intermediate layer. To calculate the temperature drop let us write the equation of change in internal drop energy, at that supposing that the inhomogeneity of the temperature field inside the drop is small, and the drop has a temperature \( T_d \). In this case, the change in the internal drop energy at its temperature change may be expressed:

\[
dE = mc \cdot dT_d,
\]

where \( m \) is the mass of the drop, and \( c \) is the specific heat capacity of fluid. The internal energy of the drop changes due to heat exchange with the medium and its evaporation. The total heat flux during heat exchange with the medium is denoted by \( Q_1 \), and the heat absorption at evaporation, by \( Q_2 \). Heat flux \( Q_1 \) includes conductive, free-convective, and radiant heat fluxes:

\[
Q_1 = \alpha S \cdot (T_a - T_d) + \varepsilon \sigma S \cdot (T_{a}^4 - T_d^4),
\]

where \( \alpha \) is the heat transfer coefficient taking into account conduction and free convection, \( \varepsilon \) is the emissivity factor of the drop surface, and \( \sigma \) is the Stefan-Boltzmann constant. The heat transfer coefficient is determined as follows:

\[
\alpha = \frac{Nu \cdot 2R}{\lambda_a},
\]

where \( \lambda_a \) is the heat conductivity of the ambient medium.

\( Nu \) and \( Sh \) are numbers taking into account conductive and convective heat and mass transfer [16]:

\[
Nu = 2 + 0.60Pr^{1/3}Gr^{1/4}, \quad Sh = 2 + 0.60Pr^{1/3}Gr^{1/4}
\]

where \( Gr \) is the Grashof number, determined by the droplet diameter \( d \):

\[
Gr = \frac{gd^4(T_d - T_a)}{v^2T_d}
\]
Pr is the Prandtl number, taken to be \( Pr = 0.7 \).
The value \( Q_2 \) can be determined taking into account the enthalpy of the vapor, heated to a temperature of the medium:

\[
Q_2 = L \cdot (1 + B) \frac{dm}{dt},
\]

where \( L \) is the specific heat of vaporization, taken at the ambient temperature \( T_a \), the number \( B \) is

\[
B = \frac{c \cdot (T_a - T_d)}{L},
\]

In the end, the equation of change in internal drop energy is:

\[
m c \frac{dT_a}{dt} = 2 \pi \lambda_a R \cdot Nu \cdot (T_a - T_d) + L \cdot (1 + B) \frac{dm}{dt} + 4 \pi R^2 \varepsilon \sigma \cdot (T_a^4 - T_d^4),
\]

In addition to the equation of change in internal energy (10) the calculated system of equations includes the equation of change in the drop mass (11) and the equations of equality of the mass vapor flow of Hertz-Knudsen and the diffusion flow (12).

\[
\frac{dm}{dt} = -\frac{\gamma S}{\sqrt{2 \pi R_p}} \cdot \frac{P_s(T_d)}{\sqrt{T_d}} \cdot (1 - \phi),
\]

\[
\frac{\gamma S}{\sqrt{2 \pi R_p}} \cdot \frac{P_s(T_d)}{\sqrt{T_d}} \cdot (1 - \phi) = -2 \pi D_{c,\mu} R \cdot S h \cdot \ln \left( \frac{1 - \phi}{1 - \phi} \right),
\]

The differential equations (10), (11) were solved numerically by the method of Runge-Kutta-Merson of the fifth order with correction of the calculation step in each iteration. The value of \( \phi \) was determined from the nonlinear equation (12) using the Newton method. Based on the developed algorithm for solving the system of equations we implemented a computer program for calculating the evaporation of free liquid drops with the use of emission–diffusion model.

2. Analysis of calculation results

(a)

(b)

Figure 1. The temperature change of the large droplets at evaporation: (a) according to the emission-diffusion model, (b) according to the diffusion model.
Comparative calculations of evaporation of free spherical water drops of different diameters were performed with the use of both emission-diffusion model (EDM) and the conventional diffusion model (DM). All calculations were carried out at air temperature $t_a = 21^\circ C$, relative humidity $\varphi = 50\%$ and initial droplet temperature $t = 21^\circ C$. The calculation results of temperature change in large droplets with an initial diameter of 1mm, 2mm and 3 mm are shown in figure 1.

In the article, the conventional value, relative to which the drops are divided into small and large, is the initial diameter of about 25 microns. The choice of this value is due to the following. Diffusion model well describes the evaporation of drops at small Knudsen numbers, when the free path length of water molecules in the air is by 2-3 orders of magnitude smaller than the initial diameter of the drop. With decreasing diameter, this condition is violated and the diffusion model ceases to meet the necessary requirements. The present investigation and [8] show that in the studied range of temperatures and relative humidity it is at this diameter when the deviation of the drop evaporation time, determined by diffusion and emission-diffusion models, starts.

According to the results of calculations using the EDM (figure 1a) for the large droplets at the initial stage there was a sharp temperature decrease to that exceeding the wet bulb temperature $t_{WB}$ by 1$^\circ$ - 1.5$^\circ$C. After the initial cooling there was a long time period of relatively stable drop temperature. At the final stage of droplets evaporation there was a sharp increase of their temperature to ambient one. Calculations of the evaporation time of large drops on DM (figure 1b) gave the close results to the calculations according to EDM. However, in contrast to them, at the final stage of evaporation the drop temperature did not increase to $t_a$. In the process of evaporation there was a gradual decrease of the drop temperature to the wet bulb temperature.

In contrast to the evaporation of large drops there were significant differences between the results of calculations according to EDM and DM for small drops. Figure 2 shows the results of calculations of temperature change of evaporating drops with an initial diameter of 5 µm, 10 µm and 20 µm.

![Figure 2](image)

**Figure 2.** Temperature changes in small drops at evaporation: (a) according to emission-diffusion model, (b) according to diffusion model.

The calculation of the evaporation of drops was carried out using the two compared models under the same conditions. In all cases it was assumed that the drops have the same initial temperature. The drops were taken to be thermally uniform. The temperature field uniformity was assessed by Biot number $Bi$. For the conditions $Bi = 0.09 – 0.13$, when $Bi<1$, the drops could be considered thermally uniform. Fig.2a shows the dependences of changes in the temperature of drops with micron dimensions in the evaporation process. It follows from the figure that the minimum temperature of drops cooling was 1-2 $^\circ$C higher than the wet bulb temperature $t_{WB}$. After rapid cooling at the initial stage of evaporation, the temperature of drops gradually increased to ambient temperature. The
minimum temperature of the drop cooling decreased with the increase in the initial diameter and approached the wet bulb temperature. The results of calculations in Fig.2b were obtained under the same conditions as for 2a, but using the diffusion model. It follows from the figure that at initial stage of evaporation the drops are cooled to the wet bulb temperature and keep it until their complete evaporation. Comparing Fig.2.a and Fig.2b it may be inferred that the emission-diffusion model gives a longer evaporation time than the diffusion model. The difference for the drop with $d_0 = 5 \, \mu m$ was more than 2 times.

3. Testing the computational model
For testing the proposed computational model a series of experimental studies of evaporation of water droplets of different diameters was performed. The experiments were conducted to measure the time of droplets evaporation and their temperature. Evaporation of free drops was simulated experimentally using suspended drops. Mono-fibers of different diameters, made of polypropylene with low thermal conductivity, were used as the suspension. Such experimental statement allowed minimizing the effect of the fiber and obtaining a good approximation to the conditions of free drops evaporation. The experiments were conducted at a constant temperature $t_a = 27.5 \, ^\circ C$ and humidity $\varphi = 27\%$. Absolute error of measurement of the ambient relative humidity did not exceed $\pm 3\%$, and of ambient temperature $\pm 0.2 \, ^\circ C$. Absolute measurement error for drop evaporation time did not exceed 2 seconds. Initial droplet diameters were measured by digital microphotography method with an accuracy of $\pm 10 \, \mu m$. In the evaporation process, the temperature distribution on the drop surface was determined using infrared thermography [23, 24]. Temperature measurement was conducted with 5 seconds interval. In the experiments, we used the television camera NEC TH 7102WV, which operates in the spectral range $8\div14 \, \mu m$ and has a relative temperature accuracy $0.2 \, ^\circ C$. The measurements were carried out with the use of micro lens TH 71-377. As a result, the average temperatures of the surfaces of the evaporating water droplets with different diameters were obtained. Figure 3 presents the comparison of the obtained experimental data with the calculations results on the EDM.

![Figure 3](image1)

**Figure 3.** Comparison of the results of calculations and experiments on measuring temperature of the evaporating suspended droplets; calculations: 1- 0.99 mm, 2- 1.47 mm, 3- 2.0 mm; experiments: 4- 0.99 mm, 5- 1.47 mm, 6- 2.0 mm.

![Figure 4](image2)

**Figure 4.** Comparison of results of calculations and experiments at the time of evaporation of the suspended drops. 1–without convection and supply of heat through the fiber; 2– with convection and without supply of heat through the fiber; 3– with convection and supply of heat through the fiber.

From figure 3 it follows that the calculation results are in good agreement with the experimental data at the initial stage of drop evaporation with a sharp decrease of temperature and at the stage with a
slightly changed temperature of drops, exceeding $t_{WB}$ by about 1.5°C. At the final stage of drop evaporation both the calculated and experimental data showed an increase in the drop temperature. However, the calculation shows a more dramatic increase in the temperature at the final stage and a longer total time of drop evaporation compared to the experiment. This was probably due to edge effects and deviation from the spherical shape of a drop at the end of evaporation.

In the process of drop evaporation microphotography was performed, which allowed recording the change of the drop shape and determining its geometrical dimensions. Using the proposed EDM the evaporation time of droplets of different diameter was calculated. Figure 4 shows the comparison of calculation results with experimental data.

The results of calculations demonstrate good agreement with experimental data taking into account free-convective heat exchange of the droplet with ambient air and additional supply of heat flux to the droplet along the polypropylene fiber.

**Conclusions**

The authors of the work have proposed and tested (in calculations of free spherical water droplets) the emission-diffusion model of evaporation, based on a dynamic equilibrium between the vapor flow, obtained in the kinetic approach of Hertz-Knudsen, and the diffusion flow of Maxwell. The proposed model allows calculating evaporation of droplets of various sizes.

The results for the time of evaporation of large water drops calculated using the emission-diffusion model are in good agreement with the results of calculations on the conventional diffusion model. For small water droplets, the results of calculations on the emission-diffusion model have given significantly longer evaporation times and higher minimum temperatures of droplets at evaporation than those obtained on the diffusion model.

Experiments with evaporation of the suspended drops have showed that the given free-convective heat transfer of drops and additional heat flux to the drops over the suspended fibers, the calculation results for the droplet evaporation times performed on the emission-diffusion model and their temperature during evaporation well coincided with experimental results.

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