The uncertainty in Galactic parameters

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1 INTRODUCTION

The fundamental parameters that define the Solar position and velocity within the Galaxy remain uncertain to a remarkable degree. The major remaining uncertainty is that in the distance from the Sun to the Galactic centre, \( R_0 \). The most recent results from studies of stellar orbits in the Galactic centre (Ghez et al. 2008, Gillessen et al. 2009) give values of \( R_0 = (8.4 \pm 0.4) \) kpc and \( R_0 = (8.33 \pm 0.35) \) kpc respectively. These can be compared to the earlier estimate in the review by Reid (1993) of \( R_0 = (8.0 \pm 0.5) \) kpc.

The total velocity of the Sun about the Galactic centre is the sum of the velocity of the local standard of rest (LSR), \( v_0 \), and the peculiar motion of the Sun with respect to the LSR in the same direction, \( V_\odot \). This total velocity can be determined using the apparent proper motion of Sgr A* and \( \mu_A^* \), since it is expected to be moving with a peculiar velocity less than \( \sim 1 \) km s\(^{-1} \) at the Galactic Centre. This constraint on the peculiar motion of Sgr A* is justified by the observation that the velocity of Sgr A* perpendicular to the plane is consistent with zero, with uncertainties \( \sim 1 \) km s\(^{-1} \) (Reid & Brunthaler 2004), and because this motion is thought to be due to stochastic forces from discrete interactions with individual stars, so the velocity in the plane should be similar to that perpendicular to it (Chatterjee, Hernquist, & Loeb 2002). Reid & Brunthaler (2004) found \( \mu_A^* = (6.379 \pm 0.024) \) mas yr\(^{-1} \), which corresponds to \( (v_0 + V_\odot)/R_0 = (30.2 \pm 0.2) \) km s\(^{-1} \) kpc\(^{-1} \), or a velocity about the Galactic centre (using the Ghez et al. result for \( R_0 \)) of \( v_0 + V_\odot = (252 \pm 11) \) km s\(^{-1} \), where by far the dominant uncertainty comes from the value of \( R_0 \).

Analysis of the GD-1 stellar stream (Koposov, Rix, & Hogg 2004) has recently been used to suggest a significantly lower value, \( v_0 = 221^{+16}_{-20} \) km s\(^{-1} \).

The velocity of the Sun with respect to the local standard of rest, \( v_\odot \), is often assumed to be well known and constrained to within \( \sim 0.5 \) km s\(^{-1} \) because of the analysis of the dynamics of nearby stars conducted by Dehnen & Binney (1998, henceforth DB98), and again more recently, using identical techniques, by Aumer & Binney (2009). DB98 found

\[
\begin{align*}
\mathbf{v}_\odot & \equiv (U_\odot, V_\odot, W_\odot) \\
& = (10.00 \pm 0.36, 5.25 \pm 0.62, 7.17 \pm 0.38) \text{ km s}^{-1},
\end{align*}
\]

which has been widely accepted and used. However Binney (2009, henceforth B09) suggests that the value for \( V_\odot \) determined in these papers may be an underestimate by \( \sim 6 \) km s\(^{-1} \). This is because the analysis by DB98 uses Stromberg’s equation, which is derived under the assump-
tion that the Galactic potential is axisymmetric, and extrapolates to zero velocity dispersion. In practice, the Galactic potential is not axisymmetric, and the smaller the velocity dispersion of a population, the more it is affected by departures from axisymmetry. B09 uses a more global approach, ensuring that more emphasis is placed on stellar populations with high velocity dispersions, which one would expect to be less affected by the non-axisymmetry of the Galactic potential.

Reid et al. (2009) brought together observations of masers seen in high mass star-forming regions (hmsfrs) in the Milky Way. They used simple statistical tools in an effort to determine the values of $R_0$ and $v_0$, using the DB98 value of $v_\odot$, initially under the assumption that the hmsfrs were moving on circular orbits in a flat rotation curve, for which they offer best-fitting parameters $R_0 = (8.24 \pm 0.55)$ kpc and $v_0 = (265 \pm 26)$ km s$^{-1}$, but with a high $\chi^2$ value.

In addition they considered a model in which the hmsfrs were moving with a characteristic velocity with respect to their local circular velocity. They found that they achieved a significantly improved fit to their data using this model with a peculiar velocity $\sim 15$ km s$^{-1}$ in the opposite direction to rotation. This fit yielded $R_0 = (8.40 \pm 0.36)$ kpc and $v_0 = (254 \pm 16)$ km s$^{-1}$. They briefly considered a model in which the value of $v_\odot$ was allowed to vary, finding an acceptable fit with $v_\odot = (9, 20, 10)$ km s$^{-1}$. However, believing the DB98 result to be “well determined” they did not pursue the matter further.

In this paper we re-examine the data described in Reid et al. (2009), and conduct a likelihood analysis for various models of the velocity distribution of the maser sources. This enables us to exploit these data more thoroughly. We also consider the implications of the results from B09 on the interpretation of these data. In Section 2 we explain what the data consist of, describe our models and our statistical technique; in Section 3 we give the raw results and examine their significance. We discuss the implications of these results in Section 4.

2 METHODS

2.1 The data

These data, as given in Table 1 of Reid et al. (2009), consist of measurements for 18 masers of: Galactic coordinates $(l_i, b_i)$, which we can assume to be exact; parallaxes $\pi_i$; proper motions $\mu_{l,i}$ and $\mu_{b,i}$; line-of-sight velocities $v_{l,\text{LSR},i}$. For each quantity Reid et al. give an error and we assume that this error together with the measured value of the quantity defines a Gaussian probability distribution for the true value of the quantity. For the proper motions $\mu_{l}$ and $\mu_{b}$ we shall use the more conventional notation $\mu_{l0}$ and $\mu_{b0}$. The line-of-sight velocity $v_{l,\text{LSR}}$ is relative to an obsolete estimate of the LSR. Fortunately the underlying heliocentric line-of-sight velocity $v_r$ can be recovered from $v_{l,\text{LSR}}$ without impact on the associated uncertainty (Reid et al. 2004, Appendix).

$$-v_r(R)e_\phi,$$  \hspace{1cm} (2)

where $v_r(R) > 0$ and the minus sign reflects the fact that the Galaxy rotates clockwise in our coordinate system. We explore three forms for $v_r(R)$

- A flat rotation curve, $v_r(R) = v_0$, with $v_0$ being a (positive) free parameter.
- A power-law rotation curve $v_r(R) = v_0(R/R_0)\alpha$, with $v_0$, $R_0$ and $\alpha$ being free parameters.
- A rotation curve corresponding to that given by the Galactic potential Model I in §2.7 of Binney & Tremaine (2008, henceforth GDI), linearly scaled to variable values of $R_0$ and $v_0$.

In each case, we take the parameters of $v_r(R)$ to have uniform prior probability distributions. This choice ensures that we determine what these data tell us, without prejudice. The velocity of a maser can be expected to differ from the local circular speed. We separate this difference into a random component and (in some cases) two systematic components. Like Reid et al. (2009), we consider the possibility that the velocity of hmsfrs has a systematic offset from the circular velocity, $v_{\text{SFR}}$. We also consider the possibility that the expansion of the shell within which a maser occurs will

2.2 Our models

The motion of both the Sun and the masers is dominated by circular motion around the Galactic centre with velocity $-
u_r(R)e_\phi$, where $\nu_r(R) > 0$ and the minus sign reflects the fact that the Galaxy rotates clockwise in our coordinate system. We explore three forms for $\nu_r(R)$

- A flat rotation curve, $\nu_r(R) = \nu_0$, with $\nu_0$ being a (positive) free parameter.
- A power-law rotation curve $\nu_r(R) = \nu_0(R/R_0)^\alpha$, with $\nu_0$, $R_0$ and $\alpha$ being free parameters.
- A rotation curve corresponding to that given by the Galactic potential Model I in §2.7 of Binney & Tremaine (2008, henceforth GDI), linearly scaled to variable values of $R_0$ and $\nu_0$.
displace the maser’s velocity from that of the star, and if we are biased towards seeing masers on the near side of that shell, we will see a bias in radial velocity. We represent this as an average peculiar motion \(v_m \mathbf{e}_s\), where \(\mathbf{e}_s\) is the unit vector from the Sun to the maser. We assume that the random component of the maser velocities has a Gaussian distribution of dispersion \(\Delta_v\). In total we take the probability distribution of the velocities of masers to be
\[
p(v) \, \mathrm{d}^3v = \frac{1}{(2\pi \Delta_v^2)^{3/2}} \exp\left(-\frac{|v - \mathbf{v}|^2}{2\Delta_v^2}\right),
\]  
where
\[
\mathbf{v} = -v_c(R)\mathbf{e}_\odot - v_{\text{SFR}} + v_m \mathbf{e}_s.
\]
Notice that positive values of \(v_{\odot, \text{SFR}}\) imply that the masers will lead Galactic rotation, and that motion of the masers towards us will be reflected in negative values of \(v_m\). In most cases we fix both \(v_{\text{SFR}} = 0\) and \(v_m = 0\). The various velocities are illustrated in Fig. 1.

We sometimes take the Sun’s motion with respect to the LSR, \(\mathbf{v}_\odot\), to be specified, and sometimes we fit it to the data.

For given values of \(l_i, b_i\), the heliocentric distance \(s_i\), and \(R_0\), we deduce the probability distributions in proper motion and line-of-sight velocity given the above probability distribution of the velocity (eq. 3). Since our focus is on the Galaxy’s parameters, we marginalise over the \(s_i\). We do this under the assumption that the probability distribution of the \(s_i\) is that implied by the assumption of Gaussian errors in the parallaxes.

2.3 Statistical analysis

2.3.1 Likelihood function

In order to determine the best-fitting model parameters, we maximise the likelihood function. This is
\[
\mathcal{L} (\boldsymbol{\theta}) \propto \prod_i \int \mathrm{d}s_i \, p(\text{data} | \boldsymbol{\theta}),
\]
where \(p(\text{data} | \boldsymbol{\theta})\) is the conditional probability of the \(i\)th observation, given the model with parameters
\[
\boldsymbol{\theta} \equiv (v_0, R_0, \alpha, v_\odot, \Delta_v, s_i, \ldots).
\]
For each distance \(s_i\), the model provides a multivariate Gaussian probability distribution in \(v_{\odot,i}, \mu_{\alpha,i}\) and \(\mu_{\Delta_v,i}\). The data also provide a multivariate Gaussian distribution for the underlying values of these variables. We obtain \(p(\text{data} | \boldsymbol{\theta})\) by integrating the product of these two Gaussian distributions over all three variables.

The likelihood function defines the a posteriori probability distribution of the model parameters \(\boldsymbol{\theta}\). We use a Metropolis algorithm (Metropolis et al. 1953) to identify the peak in this probability distribution, and to characterise its width around the most probable model. The Metropolis algorithm is a Markov Chain Monte Carlo method for drawing a representative sample from a probability distribution, such as the likelihood function. We start with some choice for the parameters \(\boldsymbol{\theta}\), and calculate the associated likelihood \(\mathcal{L}(\boldsymbol{\theta})\). We then

(i) choose a trial parameter set \(\boldsymbol{\theta}’\) by moving from \(\boldsymbol{\theta}\) in all directions in parameter space, by an amount chosen at random (from a Gaussian distribution);
(ii) determine \(\mathcal{L}(\boldsymbol{\theta}’)\);
(iii) choose a random variable \(r\) from a uniform distribution in the range [0,1];
(iv) if \(\mathcal{L}(\boldsymbol{\theta}’)/\mathcal{L}(\boldsymbol{\theta}) > r\), accept the trial parameter set, and set \(\boldsymbol{\theta} = \boldsymbol{\theta}’\). Otherwise do not accept it.
(v) Return to step (i).

The first few values of \(\boldsymbol{\theta}\) are ignored as “burn-in”, which helps to remove the dependence on the initial value of \(\boldsymbol{\theta}\). We repeat the procedure until the chain of \(\boldsymbol{\theta}\) values constitutes a fair sample of the probability distribution – we establish that the burn-in period is sufficiently long by comparing chains that have different starting \(\boldsymbol{\theta}\) (e.g. Gelman & Rubin 1992).

2.3.2 Bayesian evidence

In addition to comparing models that differ only in the values taken by a given set of parameters, we have to assess the value of adding a parameter to a model. Adding a parameter is guaranteed to increase the maximum likelihood achievable for given data, but is that increase statistically significant or just the consequence of an enhanced ability to fit noise? The Bayesian methodology for making such assessments is now well established and described by Heavens (2009), for example.

One calculates the “evidence” for the model under different priors for whatever parameters are either fixed a priori or varied. The evidence is the total probability of the data after integrating over all parameters:
\[
p(\text{data} | \text{Model}) = \int \mathrm{d}^n\theta \, p(\text{data} | \boldsymbol{\theta}, \text{Model}) \, p(\boldsymbol{\theta} | \text{Model}).
\]
Here \(p(\boldsymbol{\theta} | \text{Model})\) is the prior on the parameters. For a parameter \(\theta_i\) that is varied, we take the prior to be uniform within a range of width \(\Delta \theta_i\), that is large enough to encompass any plausible value of \(\theta_i\), (so \(p(\theta_i | \text{Model}) \, d\theta_i = d\theta_i / \Delta \theta_i\), over this range). The priors on parameters that are always varied play little role because we are interested in the ratio of the evidence when a parameter \(\theta_i\) is fixed to when it is varied. When \(\theta_i\) is fixed, its prior is a delta function at the chosen value, so the ratio of the evidences when \(\theta_i\) is fixed to when it is varied is
\[
\frac{p(\text{data} | \text{Model}, \theta_i \text{ fixed})}{p(\text{data} | \text{Model})} = \frac{\int \mathrm{d}^{n-1}\theta \, p(\text{data} | \theta, \text{Model}) \, \Delta \theta_i}{\int \mathrm{d}^n\theta \, p(\text{data} | \theta, \text{Model})},
\]
where upper integral excludes \(\theta_i\) and the lower one includes it.

By Bayes theorem the ratio of the a posteriori probabilities of the model when \(\theta_i\) is fixed to when is varied is the ratio of the corresponding evidences times the ratios of the prior on \(\theta_i\) being fixed to that on it being variable. We take this second factor to be unity, so the a posteriori probabilities of the models is simply the ratio of the evidences.

The integration over the space of parameters that is required to calculate the evidence is exceedingly costly if done by brute force. We approximate it by assuming that in the vicinity of its peak, \(p(\text{data} | \boldsymbol{\theta}, \text{model})\) can be approximated by a Gaussian \(p \propto \exp(-\mathbf{r}^T \cdot \mathbf{K} \cdot \mathbf{r})\), where \(\mathbf{K}\) is a matrix whose eigenvectors and eigenvalues can be estimated from

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so here we focus on what can be inferred about $v_\odot$ given $v_{\text{SFR}} = 0$. In view of the degeneracy between $v_\odot$ and $v_{\text{SFR}}$ we do not report results obtained when both $v_\odot$ and $v_{\text{SFR}}$ are varied.

When $v_{\text{SFR}} = 0$, the peak likelihood is higher when $v_\odot$ is fixed to the B09 value than when it is fixed to the DB98 value. To assess the significance of this increase in likelihood, we calculate the ratio of the evidences for the two models as described in Section 2.3.2

$$\frac{p(\text{DB98}|\text{data})}{p(\text{B09}|\text{data})} \simeq \begin{cases} 2 \times 10^{-4} & \text{for } \alpha = 0 \\ 1 \times 10^{-4} & \text{for } \alpha \text{ variable} \\ 4 \times 10^{-4} & \text{GDII} \end{cases}$$

(9)

Thus regardless of the adopted rotation curve, the data strongly favour upward revision of $V_\odot$ from 5.2 km s$^{-1}$ to 11 km s$^{-1}$.

When $v_\odot$ is a free parameter, the peak likelihood of the B09 model is surpassed at yet larger values of $V_\odot$. To determine whether the increase in likelihood that occurs when $v_\odot$ is set free from the B09 value, we again calculate the relevant ratio of the evidences. Since the two models now differ in whether $v_\odot$ is fixed or free, the priors on the components of $v_\odot$ now become relevant (cf eq. 3). We have adopted $\Delta U_\odot = \Delta V_\odot = \Delta W_\odot = 100$ km s$^{-1}$. With these values we have

$$\frac{p(\text{B09}|\text{data})}{p(v_\odot \text{ free}|\text{data})} \simeq \begin{cases} 0.6 & \text{for } \alpha = 0 \\ 0.01 & \text{GDII} \end{cases}$$

(10)

Therefore under these assumptions the increase in likelihood attained on setting $v_\odot$ free from the B09 value is not statistically significant. The ratio of evidences for the case when $v_\odot$ is set to the DB98 value or is set free is

$$\frac{p(\text{DB98}|\text{data})}{p(v_\odot \text{ free}|\text{data})} \simeq \begin{cases} 8 \times 10^{-4} & \text{for } \alpha = 0 \\ 7 \times 10^{-5} & \text{for } \alpha \text{ variable} \\ 0.01 & \text{GDII} \end{cases}$$

(11)

Therefore, even with a generous choice of $\Delta U_\odot$, etc., the data reject the possibility that the Sun has the DB98 value of $v_\odot$. Reducing the widths $\Delta U_\odot$, etc., of the priors on $v_\odot$ would strengthen the case against the DB98 value of $v_\odot$.

The choice of $\Delta U_\odot \sim 100$ km s$^{-1}$ is reasonably generous, and it is sensible to ask what value of $\Delta U_\odot$, etc., would bring $p(\text{B09}|\text{data})/p(v_\odot \text{ free}|\text{data})$ down to unity. In the case where the GDII rotation curve is used, it would require a reduction to a value $\Delta U_\odot \sim 30$ km s$^{-1}$ — this is smaller than is reasonable given that the value for $V_\odot$ found when it is allowed to vary is already $\sim 15$ km s$^{-1}$ greater than the canonical DB98 value.

It is worth noting that the case for setting $v_\odot$ free from either the DB98 value of the B09 value is weakest when the best-motivated rotation curve is adopted — the GDII curve. The lowest values of both $v_\odot$ and $R_0$ are found for the models with the DB98 value of $v_\odot$, and $v_{\text{SFR}}$ set to zero. Both $v_\odot$ and $R_0$ take larger values for models with the B09 value of $v_\odot$, and larger still for models with either $v_\odot$ or $v_{\text{SFR}}$ allowed to vary. The lowest values of $v_\odot$ and $R_0$ for the models in which the peculiar velocities are allowed to vary are 232 km s$^{-1}$ and 7.7 kpc respectively.

Fig. 3 suggests a reason why the value of $v_\odot$ or $v_{\text{SFR}}$ in the model has such a large impact on the best fitting values of $v_\odot$ and $R_0$. It shows the residual velocities of the

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**Figure 2.** Plot showing contours of the Likelihood function (marginalised over $\Delta \nu$) for a model with the GDII rotation curve, and the DB98 $v_\odot$. There is clearly a strong correlation between the values found for $v_\odot$ and $R_0$. Contours are drawn at likelihood differing from the maximum likelihood $\Delta \mathcal{L} = 0.25, 0.5, 1, 2$. The Metropolis algorithm yields a set of points in parameter space which sample $p(\text{data}|\text{Model})$. Principal component analysis of this sample yields the eigenvectors and eigenvalues of $K$. **3 RESULTS**

We first investigate models in which the masers are at rest with respect to their host stars (i.e., $v_\text{m} = 0$).

Table 1 gives the peak likelihoods of our models, as well as the best-fitting parameters and the corresponding uncertainties. The best-fitting value of $v_\text{c}$ varies in the range $(200 - 279)$ km s$^{-1}$, and that of $R_0$ between 6.7 and 8.9 kpc. As Fig. 2 illustrates, $v_\text{c}$ and $R_0$ are strongly correlated with the result that $v_\text{c}/R_0$ is confined to the relatively narrow range $29.9 - 31.6$ km s$^{-1}$ kpc$^{-1}$.

When $v_\odot$ is a free parameter, the best-fitting values of $U_\odot$ and $W_\odot$ are close to the DB98 values, and essentially independent of the form of the rotation curve, whereas $V_\odot$ varies in the range 16.5 - 19.5 km s$^{-1}$. Similarly, when $v_\odot$ is fixed at the DB98 value and $v_{\text{SFR}}$ is a free parameter, only $v_\text{SFR}$ takes a value that is far removed from that which one would naively expect — it moves in the range $-11.0$ to $-14.8$ km s$^{-1}$. Thus the data suggest either that the Sun is circulating significantly faster than the circular speed, or that the HMSFR has an appreciable rotational lag.

The fits are almost perfectly degenerate between $v_\odot$ and $v_{\text{SFR}}$: they constrain only the difference between these velocities. However, we shall argue in Section 4 that significantly non-zero values of $v_{\text{SFR}}$ are physically implausible.
Table 1. Log likelihoods for best-fitting models with flat rotation curves (\(\alpha = 0\)) and power law rotation curves (\(\alpha \neq 0\)) and rotation curves taken from GDII. The first 9 likelihoods are for models in which the sources are assumed to have no typical velocity – for the first 3 the value of \(v_\odot\) is that found in DB98; the next 3 are for models in which \(v_\odot\) is assumed to be that suggested by B09; and the next 3 are for models in which \(v_\odot\) is allowed to vary freely; the final 3 are for cases in which \(v_\odot\) is taken to be the DB98 value, and the mean source peculiar motion with respect to their local standard of rest, \(v_{\text{SFR}}\), is allowed to vary freely.

| \(v_0\) (km s\(^{-1}\)) | \(R_0\) (kpc) | \(\alpha\) | \(U_\odot\) (km s\(^{-1}\)) | \(V_\odot\) (km s\(^{-1}\)) | \(W_\odot\) (km s\(^{-1}\)) | \(\Delta v\) (km s\(^{-1}\)) | \(v_0/R_0\) (km s\(^{-1}\)kpc\(^{-1}\)) | log(\(\mathcal{L}\)) |
|--------------------------|-------------|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-------------|
| DB98                    |             |        |                 |                 |                 |                 |                 |             |
| 200 ± 20                | 6.7 ± 0.5   | 0      | 10.0            | 5.2             | 7.2             | 10.0 ± 1.3      | 30.1 ± 1.7      | 26.8        |
| 209 ± 26                | 6.9 ± 0.7   | 0.10 ± 0.16 | 10.0             | 5.2             | 7.2             | 10.0 ± 1.3      | 30.4 ± 1.7      | 27.1        |
| 218 ± 20                | 7.1 ± 0.5   | GDH    | 10.0            | 5.2             | 7.2             | 9.3 ± 1.2       | 30.8 ± 1.6      | 30.7        |
| B09                     |             |        |                 |                 |                 |                 |                 |             |
| 215 ± 19                | 7.0 ± 0.5   | 0      | 10.0            | 11.0            | 7.2             | 8.1 ± 1.1       | 30.5 ± 1.5      | 35.7        |
| 228 ± 24                | 7.4 ± 0.6   | 0.15 ± 0.13 | 10.0             | 11.0            | 7.2             | 8.0 ± 1.1       | 30.8 ± 1.5      | 36.5        |
| 235 ± 19                | 7.6 ± 0.5   | GDH    | 10.0            | 11.0            | 7.2             | 7.6 ± 1.0       | 31.1 ± 1.5      | 38.9        |
| \(v_\odot\) free       |             |        |                 |                 |                 |                 |                 |             |
| 232 ± 24                | 7.7 ± 0.6   | 0      | 8.1 ± 2.9       | 18.6 ± 2.4      | 9.7 ± 2.0       | 7.1 ± 1.0       | 30.0 ± 1.8      | 42.5        |
| 258 ± 32                | 8.6 ± 0.9   | 0.27 ± 0.13 | 8.1 ± 2.8       | 19.5 ± 2.5      | 10.1 ± 2.0      | 7.1 ± 1.0       | 29.9 ± 1.7      | 45.4        |
| 246 ± 24                | 8.1 ± 0.6   | GDH    | 8.3 ± 2.8       | 16.5 ± 2.4      | 9.9 ± 2.0       | 7.0 ± 1.0       | 30.3 ± 1.8      | 43.5        |

| \(v_{\text{SFR}}\) free |     |        |                 |                 |                 |                 |                 |             |
| 241 ± 24                | 7.7 ± 0.6   | 0      | 3.3 ± 2.8       | -12.9 ± 2.4     | 2.5 ± 2.0       | 7.0 ± 1.0       | 31.1 ± 1.7      | 42.9        |
| 279 ± 33                | 8.9 ± 0.9   | 0.25 ± 0.12 | 2.0 ± 2.8       | -14.8 ± 2.5     | 3.0 ± 2.0       | 6.6 ± 1.0       | 31.5 ± 1.6      | 45.5        |
| 259 ± 23                | 8.2 ± 0.6   | GDH    | 2.5 ± 2.8       | -11.0 ± 2.4     | 2.8 ± 2.0       | 7.1 ± 1.0       | 31.6 ± 1.7      | 43.8        |

| \(v_0\) (km s\(^{-1}\)) | \(R_0\) (kpc) | \(U_\odot\) | \(V_\odot\) | \(W_\odot\) | \(v_{\text{SFR}}\) | \(v_{\text{SFR}}\) | \(\Delta v\) | \(v_0/R_0\) (km s\(^{-1}\)kpc\(^{-1}\)) | log(\(\mathcal{L}\)) |
|--------------------------|-------------|-----------|-----------|-----------|-----------------|-----------------|-----------------|-----------------|-------------|
| 233 ± 20                | 7.3 ± 0.5   | 10.0      | 5.2       | 7.2       | -8.1 ± 2.7      | 8.0 ± 1.1       | 31.6 ± 1.5      | 36.2        |
| 247 ± 19                | 7.8 ± 0.4   | 10.0      | 11.0      | 7.2       | -6.2 ± 2.4      | 6.8 ± 0.9       | 31.4 ± 1.4      | 43.1        |
| 259 ± 25                | 8.2 ± 0.5   | 10.7 ± 3.3 | 15.0 ± 2.7 | 10.0 ± 2.0 | -4.8 ± 2.9      | 6.5 ± 1.0       | 31.4 ± 1.9      | 46.0        |

Table 2. Similar to Table 1. This shows the best-fitting parameters (and corresponding log likelihoods) for models with a GDII rotation curve, and with the value \(v_{\text{SFR}}\) allowed to vary.

3.1 Maser line-of-sight velocities

Finally we consider models in which we allow for a systematic difference between the measured line-of-sight velocity of a maser and the line-of-sight velocity of the exciting star – such a difference would arise if masing occurred on the near side of an expanding shell around the star. We have done this for all the rotation curves described in this paper, but the results are sufficiently similar to one another that we only present (in Table 2) the results for the GDII rotation curve.

We again compare the different models by calculating the ratios of their evidences, using the prior for \(v_{\text{SFR}}\) uniform in a range of width \(\Delta v_{\text{SFR}} = 20\) km s\(^{-1}\). We find

\[
p(v_{\text{SFR}} = 0 | \text{data}) \approx \begin{cases} 0.01 & \text{for DB98} \\ 0.06 & \text{B09} \\ 0.3 & v_\odot \text{ free} \\ 0.3 & v_{\text{SFR}} \text{ free} \end{cases}
\]

so in each case the data support adding the extra free parameter.

If we accept this extra parameter in our models, we have to reconsider the evidence for a change in \(v_\odot\) (or \(v_{\text{SFR}}\)). The relevant ratios of evidences are:

\[
p(\text{DB98, } v_{\text{SFR}} \text{ free} | \text{data}) \approx 0.001, \\
p(\text{B09, } v_{\text{SFR}} \text{ free} | \text{data}) \approx 0.3, \\
p(\text{DB98, } v_{\text{SFR}} \text{ free} | \text{data}) \approx 220.
\]

So the B09 value of \(v_\odot\) is still favoured. Moreover, when \(v_\odot\) is taken to be free, the likelihood still peaks at an even larger value of \(V\) than that of B09.

We note that the sources providing the strongest evidence that a non-zero value of \(v_{\text{SFR}}\) is needed are those ringed in the the bottom right plot of Fig. 4 all associated with the
Figure 3. Residual velocities left after the best-fitting model velocities are subtracted, for models with GDII rotation curves and $v_\odot$ taking the DB98 value (top-left); the B09 value (top-right); or taken to be a free parameter (bottom-left). We also plot the residual velocities left after subtracting the best fitting model velocities for a model with the B09 $v_\odot$ value, and $v_m$ taken to be a free parameter (bottom-right). Open squares correspond to the objects for which the likelihood improves most significantly between the best-fitting DB98 model and the best-fitting free Solar velocity one ($\Delta(\log L_i) > 1$), and crosses to all other sources. The Sun is represented by a solid circle. The solid lined ellipses around each point are $1\sigma$ measurement error ellipses, where the velocity error perpendicular to the line of sight in each case is due to a combination of the uncertainty in the proper motion and in the parallax. The uncertainty in the parallax also means that the residual velocities shown here are not an ideal illustration of the difference between the model and the data, because the peculiar motions can only be given for a chosen position (in this case the position corresponding to the quoted parallax). However this does provide a useful guide, especially for sources relatively close to the Sun, which have small position uncertainties. The dashed ellipse in the bottom-right plot is drawn around the three objects for which the likelihood improves most significantly when $v_m$ is taken to be a free parameter ($\Delta(\log L_i) > 1$ for B09 $v_\odot$).

Perseus spiral arm \cite{Reid et al. 2009}. If we exclude these sources, the best fitting value of $v_m$ is $\sim -3$ km s$^{-1}$ for any assumption about $v_\odot$ we consider. This is approximately within the uncertainty on $v_m$, and if we considered only this subset of the data in equation (12), the evidence would not support adding the extra free parameter $v_m$ (though it should be noted that it is still a radial velocity towards the Sun). It is, therefore, possible that what we have modelled as an offset in the radial velocities of all observations is actually primarily due to a large peculiar velocity of the objects.
These results, and in particular the large variation in $v_0$ and $R_0$ depending on the other model parameters, makes it impossible to constrain tightly either $v_0$ or $R_0$ from these data – the smallest best-fitting values of these parameters are, respectively, 40 per cent and 33 per cent smaller than the largest best-fitting values. The choice of rotation curve has a significant impact, and we do not investigate all possible rotation curves. It is worth noting that our two most favoured models in Table 1 (according to Bayesian evidence, so somewhat dependent on our choice of priors) have best-fitting values of $v_0$ that differ by 30 km s$^{-1}$ and of $R_0$ that differ by 1 kpc from one another (the models are those with B09 $v_⊙ +$ GD rotation curve and free $v_⊙ +$ power-law rotation curve, respectively).

However, for all but one of our models the best fitting $v_0/R_0$ lie in the narrow range $(29.8 - 31.5)$ km s$^{-1}$ kpc$^{-1}$, with typical uncertainties $\sim 1.5$ km s$^{-1}$ kpc$^{-1}$. This corresponds to best fitting $(v_0 + V_⊙)/R_0$ in the range $(30.9 - 32.5)$ km s$^{-1}$ kpc$^{-1}$ (again, depending on the model), with similar uncertainties. These values are slightly larger than the value of $(v_0 + V_⊙)/R_0 = 30.2 \pm 0.2$ km s$^{-1}$ kpc$^{-1}$ found from the proper motion of Sgr A* (Reid & Brunthaler 2004), but consistent to within (less than) twice the uncertainties on our values.

If we allow for a bias in the maser radial velocities with respect to the Sun by allowing the parameter $v_0$ to take a non-zero value in equation (1), this does improve the fit. However we have seen that this is primarily due to a group of maser sources in the Perseus spiral arm, so the cause may be a large peculiar motion in that part of the Galaxy. Even if the bias in measured radial velocity is real, it does not affect the result: (i) that in different models these data are fit best by very different values of $v_0$ and $R_0$; (ii) that the DB98 $v_0$ is rejected by these data, and (iii) that the B09 $v_⊙$ is favoured over making $v_0$ a free parameter.

Reid et al. (2009) focused on the idea that these data show that the HMFsRs are orbiting the Galaxy with a velocity that lags the circular speed by $-v_{φ,SFR} \sim 15$ km s$^{-1}$. Our analysis of similar models shows that the data are fit better by a slightly smaller offset of $(11 - 14.8)$ km s$^{-1}$ to the circular speed. We have also shown that an equivalent fit to the data can be obtained by instead increasing to $\sim 17$ km s$^{-1}$ the amount $V_0$ by which the Sun is assumed to circulate faster than the circular speed. In fact, the sources that provide the strongest statistical support for $v_{φ,SFR} \sim -15$ km s$^{-1}$ are found near the Sun (Fig. 3), so a change in $v_⊙$ has a very similar effect to a change in $v_{SFR}$. More maser data at different Galactic azimuths could break this degeneracy, as well as reducing the correlation between the values of $v_0$ and $R_0$ seen in Fig. 2.

If the proposal of Reid et al. (2009) that $v_{φ,SFR} \sim -15$ km s$^{-1}$ were correct, the HMFsRs would all have to be close to apocentre. Since the lifetime of an HMFsR is short compared to a typical epicycle period and spiral structure must play a significant role in their formation, it is not inherently implausible that the maser stars are all close to apocentre. However, two arguments make it unlikely that the maser stars are on orbits as eccentric as is implied by a 15 km s$^{-1}$ offset from circular motion at apocentre. First, non-axisymmetric structure in the Galaxy’s potential modulates the tangential velocity of gas by only $\sim 7$ km s$^{-1}$ (e.g. Binney & Merrifield 1998 §9.2.3), so the HMFsRs would be moving with a peculiar velocity significantly higher than the gas from which they formed. Second, by GDII (equation 3.100), a quarter of an epicycle period later the $U$ velocities of these stars would be $(2\Omega/\kappa)15$ km s$^{-1} \simeq 22$ km s$^{-1}$, where $\Omega$ and $\kappa$ are the circular and radial frequencies at the star’s location. Hence the radial velocity dispersion of a population of such stars would be at least $\sim 22/\sqrt{2} \simeq 15$ km s$^{-1}$. The velocity dispersion of stars observed locally increases with age on account of heating by irregularities in the Galaxy’s gravitational field (e.g. GDII §10.4.1) and the velocity dispersion of the bluer stars in the Hipparcos catalogue is $\sim 10$ km s$^{-1}$ (Aumer & Binnemans 2007) rather than 15 km s$^{-1}$. Thus the conjecture of Reid et al. (2009) not only requires the maser stars to be confined to apocentre but also requires them to have more eccentric orbits than the generality of young stars. Recognising this problem, Reid et al. suggested that the orbits of these stars became more circular early in their lives. However, any random scattering process spreads stars more widely in phase space and therefore increase the mean eccentricity of the masering stars. Only a dissipative process could increase the phase-space density of these stars by moving them to more circular orbits, and no such process is known.

If the B09 value of $V_0$ is correct, the lag to circular motion required to optimise the fit to the data is so small ($\sim 5$ km s$^{-1}$) that the above objections to the proposal that HMFsRs systematically lag rotation become moot. However, the formalism of Bayesian inference says that when the B09 velocity of the LSR is accepted, there is no convincing evidence for a systematic lag of the HMFsR.

Throughout this paper we have proceeded under the assumption that the velocity of the LSR is the same thing as the circular velocity at $R_0$. It is worth noting that this is not necessarily the case. The LSR is defined to be the velocity of a closed orbit as it passes the current Solar position, which will only be a circular orbit if the Galactic potential is axisymmetric. Therefore the velocity of the LSR may be offset from the circular velocity curve. However (much like the possibility of a systematic lag of the HMFsR) we should note that if we accept the B09 value of $V_0$, there is no convincing evidence for this offset.

We conclude that these data provide a compelling case for revising $V_0$ upward. Our Bayesian analysis (equation 10) supports the revision upwards of $V_0$ to 11 km s$^{-1}$ suggested by B09 on the basis of modelling predominantly old disc stars with relatively large random velocities. It should, however, be recognised that an even better fit to these data comes from somewhat larger upward revisions to $V_0 \sim 16 - 20$ km s$^{-1}$.

5 CONCLUSIONS

We have reanalysed observations of HMFsRs reported in Reid et al. (2009) using a maximum-likelihood approach to
exploit fully the data, in an effort to determine the values of $R_0$, the Galactocentric radius of the Sun, and $v_0$, the circular velocity of the LSR. We have found that the best-fitting values, considered separately, are strongly dependent on the Galaxy model used to interpret the data, but that the ratio $v_0/R_0$ is consistently found to lie in the range $29.8 - 31.5$ km s$^{-1}$ kpc$^{-1}$.

We have also used these data to explore the value of the Sun’s peculiar velocity $v_\odot$, in light of the recent argument of B09 that the canonical DB98 value is incorrect. We find that these data support the conclusion that $V_\odot$ is significantly higher than the DB98 value. By a small but significant margin, the data prefer models with the B09 revision of $V_\odot$ to $\sim 11$ km s$^{-1}$ over models in which $V_\odot$ is left as a free parameter (equation 10). The best-fitting models have $U_\odot$ and $W_\odot$ near to the DB98 values, and the even larger value of $V_\odot \sim 16 - 20$ km s$^{-1}$. Reid et al. (2009) suggested that HMSFRs significantly lag circular rotation. We have investigated the possibility that the HMSFRs have a typical peculiar velocity $v_{SFR}$ and find that the models only constrain the velocity difference $v_{SFR} - v_\odot$. We have argued that models in which the HMSFRs have large peculiar velocities in the opposite direction to Galactic rotation are neither needed nor plausible.

This work, in conjunction with B09, casts severe doubt on the accuracy of the widely used DB98 value for $V_\odot$. This must be a concern for anyone interested in the dynamics within the Milky Way because velocities are inevitably measured with respect to the Sun. Both observations of more masers and the development of a detailed dynamical model of the Galaxy’s spiral structure would contribute to establishing more securely what the true value of $V_\odot$ is.

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After this paper was accepted for publication, Rygl et al. (2009) presented similar observations of maser sources. Incorporating these observations in our analysis does not materially affect our conclusions.

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