Off-shell Behavior of the $\pi - \eta$ Mixing Amplitude

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We extend a recent calculation of the momentum dependence of the $\rho - \omega$ mixing amplitude to the pseudoscalar sector. The $\pi - \eta$ mixing amplitude is calculated in a hadronic model where the mixing is driven by the neutron-proton mass difference. Closed-form analytic expressions are presented in terms of a few nucleon-meson parameters. The observed momentum dependence of the mixing amplitude is strong enough as to question earlier calculations of charge-symmetry-breaking observables based on the on-shell assumption. The momentum dependence of the $\pi - \eta$ amplitude is, however, practically identical to the one recently predicted for $\rho - \omega$ mixing. Hence, in this model, the ratio of pseudoscalar to vector mixing amplitudes is, to a good approximation, a constant solely determined from nucleon-meson coupling constants. Furthermore, by selecting these parameters in accordance with charge-symmetry-conserving data and SU(3)-flavor symmetry, we reproduce the momentum dependence of the $\pi - \eta$ mixing amplitude predicted from chiral perturbation theory. Alternatively, one can use chiral-perturbation-theory results to set stringent limits on the value of the $NN\eta$ coupling constant.

I. INTRODUCTION

Most theoretical efforts devoted to the understanding of meson mixing in charge-symmetry-violating (CSV) observables start from a nucleon-nucleon ($NN$) interaction constructed from coupling constants previously determined from empirical two-nucleon data. In addition, meson mixing amplitudes are obtained, either, from experiment (in the case of $\rho - \omega$ mixing) or are inferred (for $\pi - \eta$ mixing) from the mass splittings of the SU(3) octet of pseudoscalar mesons \[1,2\]. These values for the mixing amplitudes, however, reflect physics relevant to the timelike region. Nevertheless, these on-shell values are still used in constructing the CSV component of the $NN$ interaction which requires, instead, information about meson mixing in the spacelike region.

In a recent publication, Goldman, Henderson, and Thomas have questioned the previously accepted procedure of using the on-shell value for the $\rho - \omega$ mixing amplitude in calculating CSV observables \[3\]. Two calculations that address the momentum dependence of the $\rho - \omega$ mixing amplitude have recently been completed. In the first calculation, Goldman, Henderson, and Thomas, have estimated the momentum dependence of the mixing
amplitude in a model where the mixing was generated by quark-antiquark (\(q\bar{q}\)) loops and thus driven by the up-down quark mass difference [3].

More recently, Piekarewicz and Williams have also tested the on-shell prescription in a model in which the mixing was, in contrast, generated by nucleon-antinucleon (\(NN\)) loops and hence driven by the neutron-proton mass difference [4]. Closed-form analytic expressions for the mixing amplitude were presented in terms of a few nucleon-meson parameters. An important assumption of their model is that the coupling of mesons to \(NN\) loops is determined by the underlying theory and, therefore, ultimately constrained by empirical two-nucleon (NN) data. Using standard values for these couplings they reported a value for the \(\rho - \omega\) mixing amplitude at the on-shell point in agreement with experiment.

In spite of some obvious differences between the two models, both calculations predicted a substantial momentum dependence for the \(\rho - \omega\) mixing amplitude and suggested that the presence of a node in the \(NN\) potential around \(r \sim 0.9\) fm should suppress the \(\rho - \omega\) contribution to the CSV potential [3-4]. To lend support to these claims, Maltman and Goldman have tested (in a calculation of \(\pi - \eta\) mixing) the assumptions underlying the quark-loop model by using constraints imposed from chiral perturbation theory [5,6]. They concluded that the quark-loop model of Ref. [3] should provide a reliable estimate of the momentum dependence of the (off-diagonal) \(\rho - \omega\) mixing amplitude at least in the region of applicability of chiral perturbation theory.

These findings suggest that previous calculations of CSV observables based on the on-shell assumption are suspect and should be re-examined. For example, \(\rho - \omega\) mixing is believed to account for half of the size of the difference between the neutron and proton analyzing power measured in elastic neutron-proton scattering at IUCF [7-9]. Furthermore, \(\rho - \omega\) mixing seems to also play an important role in explaining the binding-energy difference between mirror nuclei (Nolen-Schiffer anomaly) [10] as well as in accounting for the difference in \(NN\) scattering lengths [1,11,12].

However, it might be difficult to reexamine these results by looking at \(\rho - \omega\) mixing in isolation. Although \(\rho - \omega\) mixing has been recognized as an important source of charge symmetry violation, its role in describing observables is far from preeminent. Indeed, most CSV observables arise from a, sometimes delicate, sum of many different contributions [13]. Furthermore, some of these additional contributions, specifically \(\pi - \eta\) mixing, have been calculated by employing the now suspect on-shell approximation. It is therefore natural, in view of the substantial momentum dependence displayed by the \(\rho - \omega\) mixing amplitude, to extend our calculations to the pseudoscalar sector.

Pseudoscalar mixing has traditionally been regarded as marginally important in understanding CSV observables. For example, \(\pi - \eta\) mixing does not contribute to the neutron-proton analyzing power difference. Furthermore, \(\pi - \eta\) mixing is believed to generate a small contribution (of the order of 10%) to the difference in \(NN\) scattering lengths as well as to the Nolen-Schiffer anomaly [11-12]. This should be contrasted with vector (\(\rho - \omega\)) mixing which, until very recently, was believed to play a crucial role in explaining all three CSV observables. One should realize, however, that once the on-shell prescription is relaxed and the full momentum dependence of the mixing amplitudes is incorporated, it is not clear what the final outcome of the calculations will be.

In this work we extend our previous hadronic calculation of vector mixing to the pseudoscalar sector. We assume that the \(\pi - \eta\) mixing amplitude is generated by \(NN\) loops and
thus driven by the neutron-proton mass difference. In Sec. II we present the formalism and evaluate the mixing amplitude analytically. As in the case of vector ($\rho - \omega$) mixing we obtain a strongly momentum-dependent $\pi - \eta$ mixing amplitude. In spite of the substantial momentum dependence displayed by both mixing amplitudes we find an almost constant ratio of pseudoscalar to vector amplitudes. In Sec. III we show that by employing accepted values for the nucleon-meson parameters we are able to reproduce the momentum dependence of the $\pi - \eta$ mixing amplitude predicted from a recent calculation using chiral perturbation theory ($\chi$PT) [5]. Finally, our conclusions and suggestions for future work are presented in Sec. IV.

II. FORMALISM

The starting point for our calculation is a hadronic model in which the $NN$ interaction is mediated by several meson exchanges. For the purpose of this calculation it is sufficient to specify the coupling of nucleons to the two lightest pseudoscalar mesons. For the coupling of nucleons to the neutral pion we will assume a pseudoscalar representation

$$L_{NN\pi} = -ig_{\pi}\bar{\psi}\gamma^5\vec{\tau}\cdot\vec{\pi}.$$ (1)

Although there are reasons to believe that a pseudovector coupling might be more appropriate in calculating low-energy observables [14,15], we will show, at least for a mixing amplitude calculated to one loop, that the two representations yield an identical momentum dependence. Because of the special role played by the pion in the $NN$ interaction, the value of the $NN\pi$ coupling constant is very well known. In contrast, the $NN\eta$ coupling is not well determined by one boson exchange (OBE) models of the $NN$ interaction. In particular, the full Bonn potential yields excellent fits to two-nucleon data without introducing a pseudoscalar-isoscalar meson [16,17]. The $NN\eta$ coupling constant is, instead, determined from SU(3)-flavor symmetry and is conventionally assumed to vary in the range $g_{\eta}^2/4\pi \sim (0.50 - 1.00)$ [12,13]. In the present work we use a pseudoscalar representation for the $NN\eta$ coupling

$$L_{NN\eta} = -ig_{\eta}\bar{\psi}\gamma^5\psi\eta,$$ (2)

and adopt the value of Ref. [2] for the $NN\eta$ coupling constant (see Table I).

Given the interaction Lagrangian, one can then proceed to calculate the contribution from $\pi - \eta$ mixing to the $NN$ potential [1,3],

$$V_{\pi\eta}^{NN}(q) = -\frac{g_{\pi}g_{\eta}\langle\pi|H|\eta\rangle}{(q^2 - m_\pi^2)(q^2 - m_\eta^2)}\gamma^5(1)\gamma^5(2)\left[\tau_3(1) + \tau_3(2)\right].$$ (3)

In the present work we employ the same hadronic model previously used in calculating $\rho - \omega$ mixing to evaluate the momentum dependence of the $\pi - \eta$ mixing amplitude. Most of the formalism has been presented before and we include some of the details here just for completeness [3].

The $\pi - \eta$ mixing amplitude can be evaluated to leading (one-loop) order

$$\langle\pi|H|\eta\rangle_{ps} = -g_{\pi}g_{\eta}\Pi_{ps}(q^2),$$ (4)
in terms of the pion-eta mixing self-energy

\[ i \Pi_{\text{ps}}(q^2) = \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \gamma^5 \tau_z G(k + q) \gamma^5 G(k) \right], \tag{5} \]

with an assumed pseudoscalar \(NN\pi\) coupling. The isospin trace can be evaluated by writing isoscalar and isovector components of the nucleon propagator

\[ G(k) = \frac{1}{2} G_p(k)(1 + \tau_z) + \frac{1}{2} G_n(k)(1 - \tau_z) \equiv G_0(k) + G_1(k) \tau_z \tag{6} \]

in terms of individual proton and neutron contributions

\[ G_p(k) = \frac{k + M_p}{k^2 - M_p^2 + i\epsilon}, \quad G_n(k) = \frac{k + M_n}{k^2 - M_n^2 + i\epsilon}. \tag{7} \]

After performing the isospin trace one obtains a \(\pi-\eta\) mixing amplitude driven by the difference between proton and neutron loops:

\[ \Pi_{\text{ps}}(q^2) = \Pi_{\text{ps}}^{(p)}(q^2) - \Pi_{\text{ps}}^{(n)}(q^2). \tag{8} \]

As is common place in field-theoretical models, most of the integrals appearing in calculating vacuum correction to tree-level amplitudes are divergent and must be renormalized. To isolate the singularities one first regularizes all integrals and then removes the divergences by inserting appropriate counterterm contributions. For example, one can isolate the singularities in the proton contribution to the \(\pi-\eta\) mixing amplitude by using dimensional regularization \[19\],

\[ \Pi_{\text{ps}}^{(p)}(q^2) = -\frac{1}{4\pi^2} \left[ \Gamma(\epsilon) \left( M_p^2 - \frac{q^2}{2} \right) + \frac{q^2}{3} \right. \]

\[ - \int_0^1 dx \left( M_p^2 - 3x(1 - x)q^2 \right) \ln \left( \frac{M_p^2 - x(1 - x)q^2}{\Lambda^2} \right) \right]. \tag{9} \]

Here \(\Lambda\) is an arbitrary renormalization scale and \(\Gamma(\epsilon) = (\epsilon^{-1} - \gamma + \cdots)\) is the gamma function evaluated in the limit of \(\epsilon \to 0\), and \(\gamma\) is the Euler-Mascheroni constant. Having isolated the singularities one can render the integral finite by appropriate counterterm subtractions. Notice, however, that in contrast to the \(\rho-\omega\) mixing amplitude, the singularities can not be removed by simply subtracting the corresponding neutron contribution \[1\] [note that the singularity is now proportional to the mass term; see Eq. (1)]. Nevertheless, all divergences can be eliminated by appropriate counterterm subtractions \[14,15\]. The precise value for the (infinite) coefficients must be specified by imposing appropriate renormalization conditions. Since the proton-neutron subtraction removes the \(q^2\) singularity a single counterterm is sufficient to render the amplitude finite. This counterterm is chosen in such a way that the \(\pi-\eta\) mixing amplitude reproduces the “experimental” value at the on-shell \(\eta\)-meson point \[1,2\]:

\[ \langle \pi | H | \eta \rangle_{\text{ps}} \bigg|_{q^2 = m_\eta^2} = -4200 \text{ MeV}^2. \tag{10} \]
Hence, after performing the appropriate subtractions one obtains a finite $\pi - \eta$ mixing amplitude that can be written as

$$\Pi_{ps}(q^2) \equiv \Pi_{ps}(q^2 = 0) + \Pi_{pv}(q^2),$$

(11)

where

$$\Pi_{pv}(q^2) = -\frac{q^2}{8\pi^2} \int_0^1 dx \ln \left[ \frac{M_p^2 - x(1-x)q^2}{M_n^2 - x(1-x)q^2} \right],$$

(12)

and where the constant $\Pi_{ps}(q^2 = 0)$ is adjusted so that the renormalization condition [Eq. (10)] is fulfilled. The full momentum dependence of the $\pi - \eta$ mixing amplitude is, therefore, contained in the term $\Pi_{pv}(q^2)$. This expression can be evaluated in closed form and reduces, to leading order in the neutron-proton mass difference, to the following simple form:

$$\frac{\Pi_{pv}(q^2)}{M^2} = \frac{1}{\pi^2} \frac{\Delta M}{M} \left\{ \begin{array}{ll}
\frac{1}{\xi} \tan^{-1} \left( \frac{1}{\xi} \right), & \text{for } 0 < q^2 < 4M^2; \\
\frac{1}{2\xi} \ln \left( \frac{\xi - 1}{\xi + 1} \right), & \text{otherwise,}
\end{array} \right.$$  

(13)

where

$$M = \frac{1}{2}(M_n + M_p), \quad \Delta M = (M_n - M_p), \quad \text{and} \quad \xi = \left| 1 - \frac{4M^2}{q^2} \right|^{1/2}. $$  

(14)

We have appended the pseudovector (pv) subscript to the above expression because it is, in fact, the result that one would have obtained if a pseudovector (as opposed to a pseudoscalar) $NN\pi$ coupling had been adopted. Hence, pseudoscalar and pseudovector representations generate the same momentum dependence for the $\pi - \eta$ mixing amplitude. The equivalence (at least to one loop) between pseudoscalar and pseudovector representations is not difficult to understand. For on-shell nucleons the equivalence of the two representations is well known. Since the imaginary part of vacuum polarization is related to the decay of a pseudoscalar meson into (on-shell) $N\bar{N}$ pairs, both representations yield identical imaginary parts [15]. Moreover, the polarization is an analytic function of $q^2$. Hence, the real part of the polarization can be written in terms of a (subtracted) dispersion integral involving only the imaginary part. Consequently, both representations should (and do) yield, up to a constant, the same one-loop mixing amplitude.

As we will show later, vacuum corrections generate a substantial momentum dependence for the $\pi - \eta$ mixing amplitude (see Fig. 1). By itself, this should be sufficient reason to question earlier results based on the on-shell assumption. However, one should first test the reliability of the model. For example, could it be possible to calculate other observables (e.g., ratio of mixing amplitudes) that might be less sensitive to the model assumptions? Can one compare these hadronic results with other calculations (e.g., chiral perturbation theory) that, while having a limited ($q^2$) range of applicability, might be perceived as having a stronger theoretical underpinning? (See Ref. [5] for a comparison of quark-loop model results to predictions from chiral perturbation theory). In what follows we will try to answer some of these questions.
In a recent work on $\rho - \omega$ mixing we showed that the mixing amplitude calculated in a hadronic model was given by

$$\langle \rho | H | \omega \rangle = g_\rho g_\omega q^2 \left[ \Pi_{vv}(q^2) + C_\rho \Pi_{vt}(q^2) \right],$$

(15)

where $g_\rho(g_\omega)$ is the $NN\rho(\omega)$ coupling constant, $C_\rho \equiv f_\rho/g_\rho$ is the ratio of tensor to vector $NN\rho$ coupling (see Table I), and $\Pi_{vv}$ and $\Pi_{vt}$ are the transverse components of the vector-vector and vector-tensor polarizations respectively [4].

The existence of a nonzero $\rho - \omega$ mixing amplitude is, of course, very well established. The mixing has been experimentally observed in the measurement of the pion form factor at the $\omega$-meson point [13,18,20]. In contrast, no direct experimental measurement exists for $\pi - \eta$ mixing. The magnitude of the mixing must, therefore, be inferred from the mass splitting of the pseudoscalar octet [1,2]. However, the theoretical procedure employed in extracting the value of the $\pi - \eta$ mixing amplitude is now the subject of some controversy [5]. Thus, it is safe to assume that the on-shell value of the $\pi - \eta$ mixing amplitude is not well known much less its full momentum dependence.

The momentum dependence of the $\pi - \eta$ mixing amplitude, however, can be constrained, at least within the present hadronic model, from knowledge of the off-shell behavior of the corresponding vector amplitude. Indeed, the pseudoscalar and vector mixing amplitudes are intimately related in the model. The origin for this relation is the following identity between vacuum polarization amplitudes:

$$\Pi_{pv}(q^2) = -q^2 \Pi_{vt}(q^2).$$

(16)

This, in turn, translates into the following relation between the ratio of mixing amplitudes

$$\frac{\langle \pi | H | \eta \rangle_{pv}}{\langle \rho | H | \omega \rangle} = \left( \frac{g_\pi g_\eta}{f_\rho g_\omega} \right) \left[ \frac{1}{1 + g_\rho \Pi_{vv}/f_\rho \Pi_{vt}} \right],$$

(17)

where $\langle \pi | H | \eta \rangle_{pv}$ is the $\pi - \eta$ mixing amplitude minus its value at $q^2 = 0$. Because of the similar momentum dependence of $\Pi_{vv}$ and $\Pi_{vt}$, the above relation indicates that the ratio of mixing amplitudes is essentially constant. We believe that such relations between mixing amplitudes might be useful in the study of meson mixing, particularly, if they prove to be less sensitive to the assumptions of the model (notice that the weak momentum dependence of the ratio will be preserved even after the inclusion of form factors; see Fig. 2).

### III. RESULTS

The momentum dependence of the $\pi - \eta$ mixing amplitude is shown in Fig. 1. Two sets of calculations are displayed. One set (solid and dashed lines) uses a value of $g_\eta^2/4\pi = 0.5$ for the $NN\eta$ coupling constant [1]. The other set (dot-dashed and dotted lines) uses, instead, the more recent value of $g_\eta^2/4\pi = 0.9$ [2]. Furthermore, the solid and dot-dashed lines show results for the mixing amplitude using point nucleon-meson couplings at all values of $q^2$. In contrast, the dashed and dotted lines show results for a mixing amplitude modified by the introduction of form factors in the spacelike region. These form factors are introduced by modifying the point coupling in the following way:
\[ g_\pi \to g_\pi(q^2) \equiv g_\pi \left(1 - q^2/\Lambda_\pi^2\right)^{-1} ; \quad g_\eta \to g_\eta(q^2) \equiv g_\eta \left(1 - q^2/\Lambda_\eta^2\right)^{-1} , \]

with the numerical values for the cutoffs determined from empirical two-nucleon data (see Table I). Also shown (using data-like symbols) are recent results obtained by Maltman using chiral perturbation theory \[4\]. The size of the “error bars” is meant to represent the spread in the results induced by the uncertainty in the theoretical determination of the electromagnetic contribution to the mass-squared splitting in the kaon system. Although the overall normalization of the hadronic result was obtained by imposing appropriate renormalization conditions, the fact that the observed momentum dependence agrees with the predictions from chiral perturbation theory is a significant result. This finding, added to the success of the hadronic model in reproducing the value of the \(\rho - \omega\) mixing amplitude at the on-shell point \[4\], gives us confidence in extending the model beyond the region of applicability of chiral perturbation theory.

Perhaps the most uncertain parameter in our calculations is the value of the \(NN\eta\) coupling constant (the remaining three parameters, \(g_\pi, M_p,\) and \(M_n\), are very well known). Hence, one could attempt to use results from \(\chi PT\) to set some limits on the hadronic \(NN\eta\) coupling constant. In fact, the constraints imposed by \(\chi PT\) on \(g_\eta\) are tighter than Fig. 1 might suggest. This can be observed by expanding our analytic results for the \(\pi - \eta\) mixing amplitude [Eq. (13)] to leading order in \(q^2/4M^2\), i.e.,

\[ \langle \pi | H | \eta \rangle_{ps} \simeq - \left[a_0 + a_1 q^2/m_\eta^2\right] , \]

where

\[ a_0 = g_\pi g_\eta \Pi_{ps}(q^2 = 0) ; \quad a_1 = \frac{g_\pi g_\eta}{4\pi^2} \left(\frac{\Delta M}{M}\right) m_\eta^2 . \]

The value for the constant \(\Pi_{ps}(q^2 = 0)\) (and hence \(a_0\)) has been chosen to satisfy the on-shell renormalization condition [see Eq. (14)]. The value for the slope, on the other hand, in conjunction with the following limits set by chiral perturbation theory \[5\]

\[ 280 \text{ MeV}^2 \leq a_1 \leq 360 \text{ MeV}^2 , \]

can be used to constrain the value of the \(NN\eta\) coupling constant to the range

\[ 0.32 \leq \frac{g_\eta^2}{4\pi} \leq 0.53 . \]

These values are somehow smaller, but still consistent, with the limits \(g_\eta^2/4\pi \sim (0.5 - 1.0)\) inferred from \(SU(3)\)-flavor symmetry \[4,13\,18\]. Furthermore, these results support the assertion of Ref. \[2\] that a value of \(g_\eta^2/4\pi \sim 0.9\) does, indeed, provide an upper limit on the importance of pseudoscalar mixing and that values as large as \(g_\eta^2/4\pi \sim 4\), as seems to be required by static approximations to OBE models of the NN interaction \[16,17\], are not realistic.

The ratio of the pseudoscalar to the vector mixing amplitude [Eq. (17)] is plotted in Fig. 2 with (dashed line) and without (solid line) form factor corrections. This result indicates that in spite of the substantial momentum dependence displayed by both amplitudes, their ratio is essentially constant over the entire range of momentum shown in the figure. Notice that
this result holds even in the presence of form factor corrections (variations are less than 10%). Furthermore, because the tensor to vector $NN\rho$ coupling is large ($C_\rho = 6.1$), and because $\Pi_{vv} \sim \Pi_{vt}$, the ratio of pseudoscalar to vector mixing amplitudes can be approximated by a simple ratio of nucleon-meson coupling constants

$$\frac{\langle \pi | H | \eta \rangle_{ps}}{\langle \rho | H | \omega \rangle} \simeq \left( \frac{g_\pi g_\eta}{g_\rho g_\omega} \right).$$  \hspace{1cm} (21)

Finally, in Fig. 3 we present the contribution from $\pi - \eta$ mixing to the singlet ($^1S_0$) component of the NN interaction in the nonrelativistic limit. This is given, for example, in the on-shell limit and with no form factor corrections by

$$V_{^1S_0}^{\pi\eta}(q) = -\frac{g_\pi g_\eta \langle \pi | H | \eta \rangle_{ps}}{(q^2 + m_\pi^2)(q^2 + m_\eta^2)} \left( \frac{q^2}{4M^2} \right).$$  \hspace{1cm} (22)

This result is indicated by the solid line. The dashed line shows results obtained using also on-shell mixing but with the point coupling modified by form factor corrections according to Eq. (18). Finally, the dot-dashed line shows the effect of off-shell mixing and form factors on the potential. Although off-shell mixing reduces the overall strength of the potential, the size of the suppression is comparable to the reduction observed in going from point couplings to form factors. Consequently, these changes are not as dramatic as the ones observed in the analogous calculation of $\rho - \omega$ mixing [4]. There, in addition to a strong suppression, off-shell mixing generated a NN potential opposite in sign (over the entire spacelike region) relative to the conventional on-shell contribution.

**IV. CONCLUSIONS**

We have calculated $\pi - \eta$ mixing in a hadronic model where the mixing was generated by $N\bar{N}$ loops and thus driven by the neutron-proton mass difference. We have presented closed-form analytic expressions for the mixing amplitude in terms of a few meson-nucleon parameters. By adopting values for these parameters inferred from SU(3)-flavor symmetry or from CSC two-nucleon data, we obtained a momentum dependence for the mixing amplitude in agreement with recent results from chiral perturbation theory. Alternatively, $\chi$PT results were used to constrain the value of the hadronic $NN\eta$ coupling constant to the range $0.32 \leq g_\eta^2/4\pi \leq 0.53$. These values were slightly smaller, albeit still consistent, with the limits inferred from SU(3)-flavor symmetry.

We have, also, related the off-shell behavior of the $\pi - \eta$ and $\rho - \omega$ mixing amplitudes. In the present model the momentum dependence of the pseudoscalar and vector amplitudes are not independent. In fact, we found the ratio of pseudoscalar to vector mixing amplitudes to be practically constant and determined essentially from meson-nucleon coupling constants. Since this result was only slightly modified by form factor corrections, we believe that the ratio might be less sensitive to the particular assumptions of the model.

To gauge the effects from off-shell mixing, we calculated the contribution from $\pi - \eta$ mixing to the singlet component of the NN potential. We have found that the changes generated from off-shell mixing were comparable to those observed when the point coupling was modified by vertex corrections. Although off-shell mixing lead to a suppression of the
NN potential relative to the on-shell value, the changes were nowhere as dramatic as those observed for $\rho - \omega$ mixing [3,4].

These off-shell corrections to the NN potential should, however, be taken with caution until additional momentum-dependent corrections are investigated. For example, we have assumed that $NN$ loops generate a nonzero meson-mixing amplitude. These $NN$ loops, however, are also responsible for the vacuum dressing of the (unmixed) $\pi$ and $\eta$ propagators. These additional dressing should be studied for consistency but might not be of practical importance.

In order to justify this last point we must go back the recurring, yet little understood, topic of vertex corrections in hadronic field theories. In principle, the calculation of (diagonal) self-energy corrections to the $\pi$ and $\eta$ propagators could parallel the calculation of off-diagonal (e.g., $\pi - \eta$ or $\rho - \omega$) mixing. In contrast to off-diagonal mixing, however, diagonal self-energy corrections involve the sum, as opposed to the difference, of proton and neutron loops. Thus, no small parameter (like $\Delta M/M$ in the case of off-diagonal mixing) emerges in the calculation of the unmixed propagators. Consequently, one must use Dyson’s equation to sum the lowest order self-energy correction to infinite order. However, this procedure modifies the analytic structure of the propagator and, because the structure of the $NN$—meson vertex is typically neglected, generates an unphysical ghost pole at spacelike momenta [21,22].

Recently, however, Allendes and Serot have computed the lowest order self-energy correction to the $\omega$-meson propagator by including a $NN\omega$ vertex approximated by its on-shell form [23]. Solving Dyson’s equation with this vertex-corrected self-energy resulted in an $\omega$-meson propagator with no ghost poles and finite at large spacelike momenta. Moreover, no significant changes in the low momentum region ($-q^2/M^2 < 1$) were seen between the free and dressed propagators. This last result is the only justification we have, so far, in presenting the NN potential of Fig. 3. How will vertex corrections modify the pseudoscalar propagators and how sensitive are these corrections to the off-shell behavior of the vertex are important open questions that must be address before a clear picture of the role of meson mixing on CSV observables will emerge.

ACKNOWLEDGMENTS

This research was supported by the Florida State University Supercomputer Computations Research Institute and U.S. Department of Energy contracts DE-FC05-85ER250000, DE-FG05-92ER40750.
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FIGURES

FIG. 1. The \( \pi - \eta \) mixing amplitude as a function of \( q^2 \) with (dashed and dotted lines) and without (solid and dot-dashed lines) the inclusion of form factors in the spacelike region. The experimental-like symbols show results from a calculation by Maltman using chiral perturbation theory \[5\].

FIG. 2. The ratio of pseudoscalar \( (\pi - \eta) \) to vector \( (\rho - \omega) \) mixing amplitudes as a function of \( q^2 \) with (dashed line) and without (solid line) the inclusion of form factors in the spacelike region. Calculations were done with the parameters of Table \[\text{I}\].

FIG. 3. The contribution from \( \pi - \eta \) mixing to the NN potential as a function of \( q^2 \) using the off-shell value for the mixing amplitude (dot-dashed line), and the on-shell value with (dashed line) and without (solid line) the inclusion of on-shell form factors at the external nucleon legs. Calculations were done with the parameters of Table \[\text{I}\].
TABLE I. Meson masses, coupling constants, tensor-to-vector ratio and cutoff parameters. (see Table 4 of Ref. [17] and Eq. (31) of Ref. [2]).

| Meson | Mass (MeV) | $g^2/4\pi$ | $C = f/g$ | $\Lambda$ (MeV) |
|-------|-----------|------------|-----------|------------------|
| $\pi$ | 138       | 14.1       | —         | 1300             |
| $\eta$ | 549       | 0.90       | —         | 1500             |
| $\rho$ | 770       | 0.41       | 6.1       | 1400             |
| $\omega$ | 783       | 10.6       | 0.0       | 1500             |