Abstract

When the anti-periodic boundary condition is imposed for a bulk field in extradimensional theories, independently of the background metric, the lightest component in the anti-periodic field becomes stable and hence a good candidate for the dark matter in the effective 4D theory due to the remaining accidental discrete symmetry. Noting that in the gauge-Higgs unification scenario, introduction of anti-periodic fermions is well-motivated by a phenomenological reason, we investigate dark matter physics in the scenario. As an example, we consider a five-dimensional SO(5) × U(1)X gauge-Higgs unification model compactified on the $S^1/Z_2$ with the warped metric. Due to the structure of the gauge-Higgs unification, interactions between the dark matter particle and the Standard Model particles are largely controlled by the gauge symmetry, and hence the model has a strong predictive power for the dark matter physics. Evaluating the dark matter relic abundance, we identify a parameter region consistent with the current observations. Furthermore, we calculate the elastic scattering cross section between the dark matter particle and nucleon and find that a part of the parameter region is already excluded by the current experimental results for the direct dark matter search and most of the region will be explored in future experiments.
1 Introduction

The standard model (SM) of particle physics successfully explains almost of all the experimental results around the electroweak scale. Nevertheless, the SM suffers from several problems and this fact strongly motivates us to explore physics beyond the SM. One of them is the so-called hierarchy problem originating from the ultraviolet sensitivity of the SM Higgs doublet mass, and another one is the absence of candidates for the dark matter particle. In this paper we propose an extra-dimensional scenario which can provide a possible solution to these two problems.

Among many models proposed to solve the hierarchy problem, we concentrate on the gauge-Higgs unification scenario [1, 2]. In this scenario, the SM Higgs doublet field is identified with an extra-dimensional component of the gauge field in higher-dimensional gauge theories where the extra spacial dimensions are compactified to realize four-dimensional effective theory at low energies. The higher-dimensional gauge symmetry protects the Higgs doublet mass from ultraviolet divergences [2, 3], and hence the hierarchy problem can be solved. In the context of the gauge-Higgs unification scenario, many models have been considered in both the flat [4]-[7] and the warped [8] background geometries [9]-[13]. However, the latter problem has not been investigated in this scenario, except for a few literatures [14, 15, 16], and in this paper, we propose a dark matter candidate which can be naturally incorporated in the gauge-Higgs unification scenario.

In the next section, we show a simple way to introduce a candidate for the dark matter particle in general higher-dimensional models. In a sharp contrast with the usual Kaluza-Klein (KK) dark matter in the universal extradimension scenario [17], our procedure is independent of the background space-time metric. In section 3, we apply this to the gauge-Higgs unification scenario and show that a dark matter candidate as a weakly-interacting-massive-particle (WIMP) emerges. For our explicit analysis, we consider a gauge-Higgs unification model based on the gauge group SO(5)×U(1)_X in five-dimensional warped background metric with the fifth dimension compactified on the S^1/Z_2 orbifold. In section 4, we evaluate the relic abundance of the dark matter particle and its detection rates in the direct dark matter detection experiments. Section 5 is devoted to summary.

2 A new candidate for the dark matter

A stable and electric charge neutral WIMP is a suitable candidate for the dark matter. In general, a certain symmetry (parity) is necessary to ensure the stability of a dark matter
particle. Such a symmetry can be imposed by hand in some models or it can be accidentally realized such as the KK parity \cite{17}. The KK parity is actually an interesting possibility for introducing a dark matter candidate in higher-dimensional models. However, we need to elaborate a model in order to realize the KK parity in general warped background geometry \cite{18}. In a simple setup, the KK parity is explicitly broken by a warped background metric and the KK dark matter is no longer stable \cite{19}. So, here is an interesting question: Is it possible in extradimensional models to introduce a stable particle independently of the background space-time metric, without imposing any symmetries by hand? In the following we address our positive answer to this question. In fact, when we impose the anti-periodic (AP) boundary condition on bulk fields, the lightest AP field turns out to be stable.

In models with the toroidal compactification, no matter what further orbifoldings are, the Lagrangian $L$ should be invariant under a discrete shift of the coordinate of the compactified direction,

$$L(x, y + 2\pi R) = L(x, y),$$

(1)

where $x$ and $y$ denote the non-compact four dimensional coordinate and the compact fifth-dimensional one with a radius $R$, respectively. When we introduce some fields which have the AP boundary condition as

$$\Phi(x, y + 2\pi R) = -\Phi(x, y),$$

(2)

these fields never appear alone but always do in pairs in the Lagrangian, since the Lagrangian must be periodic. Thus, there exists an accidental $Z_2$ parity, under which the AP (periodic) fields transform as odd (even) fields. This concludes that the lightest AP field is stable and can be a good candidate for the dark matter if it is colorless and electric-charge neutral.

In this way, a dark matter candidate can be generally incorporated as the lightest AP field in higher-dimensional models. However, except for providing the dark matter candidate, there may be no strong motivation for introducing such AP fields. In fact, AP fields often plays a crucial role in the gauge-Higgs unification scenario to make a model phenomenologically viable, and therefore a dark matter candidate is simultaneously introduced in such a model.

### 3 Gauge-Higgs Dark Matter

We show a model of the gauge-Higgs unification, which naturally has a dark matter candidate. The dark matter particle originates from an AP field which is introduced in a model

\footnote{Similarly to the KK parity, Lagrangian on the boundaries must be restricted to respect the $Z_2$ parity.}
for a phenomenological reason as will be discussed below.

We know well that, it is difficult, in simple gauge-Higgs unification models with the flat metric, to give a realistic top quark mass and a Higgs boson mass above the current experimental lower bound. This difficulty originates from the fact that effective Higgs potential in the gauge-Higgs unification model results in the Wilson line phase of order one. When we consider the gauge-Higgs unification scenario in the warped metric of the extra dimension, this problem can be solved because of the effect of the warped metric, although the Wilson line phase of order one is obtained from effective Higgs potential. However, as is claimed in Ref. [12], a small Wilson line phase is again required in order for the scenario to be consistent with the electroweak precision measurements. Therefore, it is an important issue in the gauge-Higgs unification scenario how to naturally obtain a small Wilson line phase.

A simple way is to introduce AP fermions in a model. It has been shown in Ref. [5] that a small Wilson line phase is actually obtained by introducing AP fermions. This is the motivation we mentioned above. An AP fermion, once introduced, not only reduces unwanted new particle effects to the precisely measured SM parameters but also provides a dark matter candidate as its lightest electric-charge neutral component. We call the dark matter candidate in the AP fermion “gauge-Higgs dark matter” in this paper. The interactions between the dark matter and the Higgs field is largely controlled by the gauge symmetry, since the Higgs field is a part of the gauge field in the gauge-Higgs unification scenario. This fact leads to a strong predictive power of the model for the dark matter phenomenology.

### 3.1 A model

Here we explicitly examine a 5D gauge-Higgs unification model with a dark matter particle. The model is based on the gauge symmetry $SO(5) \times U(1)_X$ compactified on the simplest orbifold $S^1/Z_2$ with the warped metric

$$ds^2 = G_{MN} dx^M dx^N = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2,$$

where $M = 0, 1, 2, 3, 5$, $\mu = 0, 1, 2, 3$, $\sigma(y) = k|y|$ at $-\pi R \leq y \leq \pi R$, $\sigma(y) = \sigma(y + 2\pi R)$, and $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1, -1)$ is the 4D flat metric. We define the warp factor $a = \exp(-\pi k R)$ and as a reference value, we set the curvature $k$ and the radius $R$ to give the warp factor $a = 10^{-15}$.

The bulk $SO(5)$ gauge symmetry is broken down to $SO(4) \simeq SU(2)_L \times SU(2)_R$ by the boundary conditions [20]. Concretely, the gauge field and its 5th component transform

---

2 In Ref. [14], with a similar purpose, a similar $Z_2$ symmetry is imposed but by hand.
around the two fixed points $y_0 = 0$ and $y_L = \pi R$ as

$$A_\mu(x, y_i - y) = P_i A_\mu(x, y_i + y) P_i^\dagger,$$

$$A_5(x, y_i - y) = -P_i A_5(x, y_i + y) P_i^\dagger,$$  \hspace{1cm} (4)

under the $Z_2$ parity, where $P_0 = P_L = \text{diag.}(-1, -1, -1, -1, +1)$ for the five-by-five anti-symmetric matrix representation of the generators acting on the vector representation, 5.

As for the remaining $\text{SO}(4) \times U(1)_X$ gauge symmetry, the $\text{SU}(2)_R \times U(1)_X$ is assumed to be broken down to the hypercharge symmetry $U(1)_Y$ by a VEV of an elementary Higgs field put on the $y = 0$ orbifold fixed point. Now the remaining gauge symmetry is the same as the SM, where there exists the zero-mode of $A_5$ which is identified as the SM Higgs doublet (possessing the right quantum numbers). When the zero mode of $A_5$ develops a non-trivial VEV, the $\text{SO}(4)$ symmetry is broken down to $\text{SO}(3) \simeq \text{SU}(2)_D$ which is the diagonal part of $\text{SU}(2)_L \times \text{SU}(2)_R \simeq \text{SO}(4)$. Taking the boundary Higgs VEV into account, the electromagnetic $U(1)_{\text{EM}}$ is left with unbroken. Thanks to the custodial symmetry which is violated only at the $y = 0$ fixed point, that is, a superheavy energy scale, the correction to the $\rho$-parameter is naturally suppressed [12]. This allows the KK scale as low as a few TeV without any contradictions against current experiments.

The components of gauge field are explicitly written as

$$A_M = \begin{pmatrix}
0 & A_3^V & -A_2^V & A_1^A & A_1^H & A_1^V \\
0 & A_1^V & A_2^A & A_2^H & A_2^V \\
0 & A_3^A & A_3^H & A_3^V & A_3^A \\
0 & A_4^H & A_4^V & A_4^A & A_4^H \\
0 & 0 & 0 & 0 & 0
\end{pmatrix},$$  \hspace{1cm} (6)

where

$$A_i^V = \frac{1}{\sqrt{2}} (A_i^L \pm A_i^R), \hspace{1cm} (i = 1, 2, 3),$$  \hspace{1cm} (7)

$$A_i^F = \frac{1}{\sqrt{2}} (A_i^F + iA_i^F), \hspace{1cm} (F = V, A, H).$$  \hspace{1cm} (8)

The zero-modes of $A_5$ exist on $A_H$ and its VEV can be rotated into only $(A_1^H)_5$ component by the $\text{SO}(4)$ symmetry, by which the Wilson line phase $\theta_W$ is defined as

$$W \equiv e^{i \theta_W} = P \exp \left(-ig \int_{-\pi R}^{\pi R} dy \, G_{55}^5 (A_1^H)_5 \right),$$  \hspace{1cm} (9)

Note that introducing the elementary Higgs field at the $y = 0$ orbifold fixed point has no contradiction against the motivation of the gauge-Higgs unification scenario since the mass of the Higgs fields and their VEVs are of the order of the Planck scale. In this case, they decouple from TeV scale physics.
where $P$ denotes the path ordered integral. For vanishing $\theta_W$, the SM gauge bosons are included in $A_L$ and $B_X$ (which is the gauge boson of the $U(1)_X$ symmetry), while the $A_H$ component is mixed into the mass eigenstates of weak bosons for non-vanishing $\theta_W$.

We do not specify the fermion sector of the model but just assume it works well, since this sector is not strongly limited by the gauge symmetry and has a lot of model-dependent degrees of freedom. Thus, in our following analysis we leave the Higgs boson mass $m_h$ and the Wilson line phase $\theta_W$ as free parameters, which should be calculated through the loop induced effective potential [21]-[23] once the fermion sector of the model is completely fixed.

Let us now consider an AP fermion, $\psi$, as a $5_0$-multiplet under $\text{SO}(5) \times \text{U}(1)_X$, in which the dark matter particle is contained. A parity odd bulk mass parameter $c$ of this multiplet is involved as an additional parameter [24]. The wave function profile along the compactified direction is written by the Bessel functions with the index $\alpha = |\gamma_5 c + 1/2|$ [24] and the localization of the bulk fermion is controlled by the bulk mass parameter. We choose the boundary conditions of this multiplet so that the singlet component of the $\text{SO}(4)$ is lighter than the vector one for small $\theta_W$ with $c > 0$.

After the electroweak symmetry breaking, the forth and fifth components are mixed with each other through the non-vanishing Wilson line phase in $(4, 5)$ component, while the first, second and third ones are not. The combinations of forth and fifth components make up two mass eigenstates: The lighter one is nothing but the dark matter particle, $\psi_{\text{DM}}$, and we denote the heavier state as $\psi_S$. The first, second and third components denoted as $\psi_i$ ($i = 1, 2, 3$) have nothing to do with the electroweak symmetry breaking, and thus degenerate up to small radiative corrections. They are heavier than $\psi_S$. Note that only dark matter particles themselves have no couplings with the weak gauge bosons, which are linear combinations of $A_V$, $A_A$ and $A_H$ (and $B_X$), and the couplings between the dark matter particle and the weak gauge bosons are always associated with the transition from/to the heavier partners, $\psi_i$. On the other hand, both types of couplings exist among the dark matter particle and the Higgs boson. At the energy scale below the 1st KK mode mass, the effective Lagrangian is expressed as

$$
\mathcal{L}_{\text{DM}}^{4D} = \sum_{i=1,2,3,5,\text{DM}} \bar{\psi}_i (i\partial - m_a) \psi_i + y_{\text{DM}} \bar{\psi}_{\text{DM}} H \psi_{\text{DM}}
+ \bar{\psi}_S H (y_S + y_P \gamma_5) \psi_{\text{DM}} + \bar{\psi}_{\text{DM}} H (y_S - y_P \gamma_5) \psi_S
+ \sum_{i=1,2,3} \bar{\psi}_i W_i (g_i^V + g_i^A \gamma_5) \psi_{\text{DM}} + \bar{\psi}_{\text{DM}} W_i (g_i^V + g_i^A \gamma_5) \psi_i,
$$

(10)

where we denote $Z$ as $W^3$, and set $g_i^h = g_2^h$ due to the remaining $U(1)_{\text{EM}}$ symmetry.

Once we fix the free parameters $\theta_W$ and $c$ (also the warp factor), we can solve the bulk
equations of motion for $A_M$ and $\psi$ (see for example Ref. [11]) and obtain the mass spectra of all the states and effective couplings in Eq. (10) among AP fields, the gauge bosons and the Higgs boson, independently of the Higgs boson mass $m_h$ (which is another free parameter of the model as mentioned above). Using calculated spectra and the effective couplings, we investigate phenomenology of the gauge-Higgs dark matter in the next section. Since we have only three parameters (or four if we count also the warp factor), the model has a strong predictive power.

### 3.2 Constraints

Before investigating the gauge-Higgs dark matter phenomenology, we examine an experimental constraint on the Wilson line phase $\theta_W$.

In Ref. [12], it is claimed that $\theta_W$ should be smaller than 0.3 or the KK gauge boson mass larger than 3 TeV in order to be consistent with the electroweak precision measurements. Using the relation between $m_W$ and $m_{K K} \equiv \pi ka$ (see for example Ref. [11]),

$$m_W \simeq \frac{\theta_W}{\sqrt{\ln(a^{-1})}} \frac{m_{K K}}{\pi}, \quad (11)$$

and the formula for the first KK gauge boson mass $m_1 = 0.78m_{K K}$, the latter constraint is translated as $\theta_W \lesssim 0.4$.

According to these bounds, we restrict our analysis in the range of a small Wilson line phase, namely, $\theta_W \leq \pi/10$. We expect that AP fields not only provide the dark matter particle but also is helpful to realize such small value of $\theta_W$.

### 4 Phenomenology of gauge-Higgs dark matter

Now we are in a position to investigate the gauge-Higgs dark matter phenomenology. We first estimate the relic abundance of the dark matter and identify the allowed region of the model parameter space which predicts the dark matter relic density consistent with the current cosmological observations. Furthermore, we calculate the cross section of the elastic scattering between the dark matter particle and nucleon to show implications of the gauge-Higgs dark matter scenario for the current and future direct dark matter detection experiments.

---

4 Constraints in the case with the flat metric is discussed in Ref. [25].
4.1 Relic abundance

In the early universe, the gauge-Higgs dark matter is in thermal equilibrium through the interactions with the SM particles. According to the expansion of the universe, temperature of the universe goes down and the dark matter eventually decouples from thermal plasma of the SM particles in its non-relativistic regime. The thermal relic abundance of the dark matter can be evaluated by solving the Boltzmann equation,

\[
\frac{dY}{dx} = - s \langle \sigma v \rangle \frac{1}{xH} \left( 1 - \frac{x}{3} \frac{d \log g^*}{dx} \right) (Y^2 - Y_{EQ}^2),
\]

where \( x = m_{DM}/T \), \( \langle \sigma v \rangle \) is the thermal averaged product of the dark matter annihilation cross section \( \langle \sigma \rangle \) and the relative velocity of annihilating dark matter particles \( \langle v \rangle \), \( Y(\equiv n/s) \) is the yield defined as the ratio of the dark matter number density \( n \) to the entropy density \( s \) of the universe, and the Hubble parameter \( H \) is described as \( H = \sqrt{(8\pi/3)G_N \rho} \) with the Newton’s gravitational constant \( G_N = 6.708 \times 10^{39} \text{ GeV}^{-2} \) and the energy density of the universe \( \rho \). The explicit formulas for the number density of the dark matter particle, the energy density, and the entropy density are given, in the Maxwell-Boltzmann approximation, by

\[
n = \frac{g_{DM}}{2\pi^2} K_2(x) m^3 x, \quad \rho = \frac{\pi^2}{30} g_* T^4, \quad s = \frac{2\pi^2}{45} g^* s T^3,
\]

where \( K_2 \) is the modified Bessel function of the second kind, \( g_{DM} = 4 \) is the spin degrees of freedom for the gauge-Higgs dark matter, and \( g_* \) \( (g^*) \) is the effective massless degrees of freedom in the energy (entropy) density, respectively.

In non-relativistic limit, the annihilation cross section can be expanded with respect to a small relative velocity as

\[
\sigma v = \sigma_0 + \frac{1}{4} \sigma_1 v^2 + \cdots,
\]

where \( v \simeq 2\sqrt{1 - 4m^2/s} \) in the center-of-mass frame of annihilating dark matter particles. The first term corresponds to the dark matter annihilations via \( S \)-wave, while the second is contributed by the \( S \)- and \( P \)-wave processes. In the Maxwell-Boltzmann approximation, the thermal average of the annihilation cross section is evaluated as

\[
\langle \sigma v \rangle \equiv \frac{1}{8 x^4 K_2(x)^2} \int_{4x^2}^\infty ds \sqrt{s}(s - 4x^2) K_1(\sqrt{s}) \sigma_{ann}
\]

\[
= \sigma_0 + \frac{3}{2} \sigma_1 x^{-1} + \cdots,
\]

where a unit \( T = 1 \) is used in the first line. There are several dark matter annihilation modes in both \( S \)-wave and \( P \)-wave processes (see Eq. (10)), such as \( \bar{\psi}_{DM} \psi_{DM} \to W^+W^-, ZZ, HH \).
through $\psi_i$, $\psi_S$ and $\psi_{DM}$ exchanges in the $t$-channel and $\bar{\psi}_{DM}\psi_{DM} \rightarrow f\bar{f}, W^+W^-, ZZ, HH$ through the Higgs boson exchange in the $s$-channel, where $f$ stands for quarks and leptons.

Once the model parameters, $\theta_W$, $c$ and $m_h$, are fixed, magnitudes of $\sigma_0$ and $\sigma_1$ are calculated.

With a given annihilation cross section, the Boltzmann equation can be numerically solved. The relic density of the dark matter is obtained as $\Omega_{DM}h^2 = m_{DM}s_0 Y(\infty)/(\rho_c/h^2)$ with $s_0 = 2889$ cm$^{-3}$ and $\rho_c/h^2 = 1.054 \times 10^{-5}$ GeV cm$^{-3}$. Here, we use an approximate formula \[26\] for the solution of the Boltzmann equation:

$$\Omega_{DM}h^2 = 8.766 \times 10^{-11}(\text{GeV}^{-2}) \left( \frac{T_0}{2.75\text{K}} \right)^3 \frac{x_f}{\sqrt{g^*(T_f)}} \left( \frac{1}{2} \sigma_0 + \frac{3}{8} \sigma_1 x_f^{-1} \right)^{1},$$

where $x_f = m_{DM}/T_f$ is the freeze-out temperature normalized by the dark matter mass, and $T_0 = 2.725$ K is the present temperature of the universe. The freeze-out temperature is approximately determined by \[26\]

$$\sqrt{\frac{\pi}{45G_N}} \frac{45g_{DM}}{8\pi} \frac{\pi^{1/2}e^{-x_f}}{g^*(T_f) x_f^{1/2} g^*(T_f)} m_{DM} \left( \frac{1}{2} \sigma_0 + \frac{3}{4} \sigma_1 x_f^{-1} \right) \delta(\delta + 2) = 1.$$  \hspace{1cm} (18)

Here the parameter $\delta$ defines $T_f$ through a relation between the yield $Y$ and its value in thermal equilibrium, $Y - Y_{EQ} = \delta Y_{EQ}$, whose value is chosen so as to keep this approximation good. We set $\delta = 1.5$ according to Ref. \[26\]. In these approximations, we include the factor $1/2$ due to the Dirac nature of the gauge-Higgs dark matter (see the discussion below Eq. (2.16) of Ref. \[26\]).

Let us now compare the resultant dark matter relic density for various $\theta_W$ and $c$ with the observed value \[27\]:

$$\Omega_{DM}h^2 = 0.1143 \pm 0.0034.$$ \hspace{1cm} (19)

The result is depicted in Figure 1, where the Higgs boson mass is set as $m_h = 120$ GeV. The regions consistent with the observations are indicated by red, while too little (much) abundances are obtained in the region above the red line on the upper-left (in the other regions). There are two allowed regions: One is the very narrow region in upper-right, where the right relic abundance is achieved by the enhancement of the annihilation cross section through the $s$-channel Higgs boson resonance, so that the dark matter mass is $m_{DM} \simeq m_h/2 = 60$ GeV there. The other one appears in upper-left with the dark matter mass around a few TeV, where dark matter particles can efficiently annihilate into the weak gauge bosons and the Higgs bosons through the processes with heavy fermions in the $t$-channel.
Figure 1: *The relic abundance*: The relic abundances consistent with the observations are obtained in the two red regions. In the upper-left corner outside the red region, the relic abundance is predicted to be too little, while over-abundance of the dark matter relic density is obtained in the other region. The contours corresponding to fixed dark matter masses are also shown. Here, the Higgs mass has been taken to be $m_h = 120$ GeV.

### 4.2 Direct detection

Next we investigate the implication of the gauge-Higgs dark matter for the direct detection experiments [28]. A variety of experiments are underway to directly detect dark matter particles through their elastic scatterings off nuclei. The most stringent limits on the (spin-independent) elastic scattering cross section have been reported by the recent XENON10 [29] and CDMS II [30] experiments: $\sigma_{el}(\text{cm}^2) \lesssim 7 \times 10^{-44} - 5 \times 10^{-43}$, for a dark matter mass of $100 \text{ GeV} \lesssim m_{DM} \lesssim 1 \text{ TeV}$. Since the gauge-Higgs dark matter particle can scatter off a nucleon through processes mediated by the Higgs boson in the $t$-channel, a parameter region of our model is constrained by this current experimental bound.

The elastic scattering cross section between the dark matter and nucleon mediated by the Higgs boson is given as

$$\sigma_{el}(DM + N \rightarrow DM + N) = \frac{y_{DM}^2 m_N^2 m_{DM}^2}{\pi v_h^2 m_h^4 (m_{DM} + m_N)^2} |f_N|^2,$$

where $m_N = 0.931 \text{eV}$ is the nucleon mass [31], and $v_h = 246 \text{GeV}$ is the VEV of the Higgs.
expected search limits by future experiments are also shown. The contributions from the light quarks to the hadron matrix element is evaluated by lattice QCD. In our analysis, we use the conservative value, $f_N$ is defined as

$$f_N = \langle N \mid \sigma_q m_q \bar{q}q - \frac{\alpha_s}{4\pi} G_{\mu\nu} G^{\mu\nu} \rangle N \rangle = m_N \left( \frac{2}{9} f_{T_G} + f_{T_u} + f_{T_d} + f_{T_s} \right),$$

where $q$ represents light quarks ($u, d,$ and $s$) and $G_{\mu\nu}$ is the gluon field strength. Contributions from the light quarks to the hadron matrix element is evaluated by lattice QCD simulations.

$$f_{T_u} + f_{T_d} \simeq 0.056, \quad f_{T_s} < 0.038,$$

while the contribution by gluon $f_{T_G}$ is determined from the trace anomaly condition:

$$f_{T_G} + f_{T_u} + f_{T_d} + f_{T_s} = 1.$$  

In our analysis, we use the conservative value, $f_{T_s} = 0$.

For various values of parameters, $\theta_W$ and $c$, with $m_h = 120$ GeV fixed, we evaluate the elastic scattering cross sections between the dark matter particle and nucleon. The result is shown in figure 2. The parameter sets in red regions lead to the appropriate dark

Figure 2: The direct detection: The red regions correspond to parameter sets that predict the right abundance. The parameter sets with $\theta_W > \pi/10$ are indicated in gray, and those excluded by the current bound from the direct detection experiments are in brown. The expected search limits by future experiments are also shown.
matter abundances. The gray region corresponds to $\theta_W > \pi/10$, which we do not consider as discussed in section 3.2. The already excluded region from XENON10 [29] and CDMS II [30] experiments is shown in brown, by which a part of the red region with $m_{DM} = 2 - 3$ TeV is excluded. Here, we naively extrapolate the exclusion limit beyond 1 TeV, although the experimental bounds shown in the original papers are depicted in the range $m_{DM} \leq 1$ TeV [31]. The other three lines indicate expected future limits by XMASS [35], SCDMS [34] and , XENON100 [33] respectively from above to below. The allowed region with the dark matter mass around TeV is fully covered by the future experiments. On the other hand, most of the narrow region consistent with the observed dark matter abundance is out side of the reach of the future experiments.

5 Summary

In extradimensional theories, the AP boundary condition for a bulk fermion can be imposed in general. We show that the lightest mode of the AP fields can be stable and hence become a candidate for the dark matter in the effective 4D theory due to the remaining accidental discrete symmetry. This mechanism works even with general non-flat metric, in contrast to the KK parity which does not work in a simple warped model.

Although we can introduce the AP fields in various phenomenological extradimensional models, they are usually not so strongly motivated except for providing the dark matter particle. In contrast, it is worth noting that in the gauge-Higgs unification scenario, AP fields often play a crucial role to realize a phenomenologically viable model. Thus, we examine the possibility of the dark matter in the gauge-Higgs unification scenario. We find that due to the structure of the gauge-Higgs unification, the interactions of the dark matter particle with the SM particles, especially with the Higgs boson, are largely controlled by the gauge symmetry and the model has a strong predictive power for the dark matter phenomenology. Because of this feature, we call this scenario as the gauge-Higgs dark matter scenario.

We have investigated this scenario based on a five-dimensional $SO(5) \times U(1)_X$ gauge-Higgs unification model compactified on the warped metric as an example. This model is favorable because it contains the bulk custodial symmetry and thus a few TeV KK scale can be consistent with the electroweak precision measurement. We have evaluated the relic abundance of the dark matter particle and identified the parameter region of the model to be

5 We would like to thank Yoshitaka Itow for his advise on the current experimental bounds for the dark matter mass beyond 1 TeV.
consistent with the observed dark matter relic density. We have found two allowed regions: One is a quite narrow region where the right dark matter relic density is achieved by the dark matter annihilation through the Higgs boson resonance, so that the dark matter mass is close to a half of the Higgs boson mass. In the other region, the dark matter annihilation process is efficient and the dark matter particle with a few TeV mass is consistent with the observations. Furthermore, we have calculated the cross section of the elastic scattering between the dark matter particle and nucleon and shown the implication of the gauge-Higgs dark matter scenario for the current and future direct dark matter detection experiments. It turns out that the region with a few TeV dark matter mass is partly excluded by the current experiments and the whole region can be explored by future experiments. On the other hand, most of the narrow region is outside of the experimental reach.

Acknowledgments

This work is supported in part by a Grant-in-Aid for Science Research from the Ministry of Education, Culture, Sports, Science and Technology, Japan (Nos. 16540258 and 17740146 for N. H., No. 21740174 for S. M. and No. 18740170 for N. O.), and by the Japan Society for the Promotion of Science (T.Y.).

References

[1] N. S. Manton, Nucl. Phys. B 158 (1979), 141.
  D. B. Fairlie, J. of Phys. G 5 (1979), L55; Phys. Lett. B 82 (1979), 97.

[2] Y. Hosotani, Phys. Lett. B 126 (1983), 309; Ann. of Phys. 190 (1989), 233; Phys. Lett. B 129 (1983), 193; Phys. Rev. D 29 (1984), 731.

[3] N. V. Krasnikov, Phys. Lett. B 273 (1991), 246.
  H. Hatanaka, T. Inami and C. S. Lim, Mod. Phys. Lett. A 13 (1998), 2601.
  G. R. Dvali, S. Randjbar-Daemi and R. Tabbash, Phys. Rev. D 65 (2002), 064021.
  N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 513 (2001), 232.
  I. Antoniadis, K. Benakli and M. Quiros, New J. Phys. 3 (2001), 20.

[4] C. Csaki, C. Grojean and H. Murayama, Phys. Rev. D 67 (2003), 085012.
  G. Burdman and Y. Nomura, Nucl. Phys. B 656 (2003), 3.
  N. Haba and Y. Shimizu, Phys. Rev. D 67 (2003), 095001 [Errata; 69 (2004), 059902].
  I. Gogoladze, Y. Mimura and S. Nandi, Phys. Lett. B 560 (2003), 204; ibid. 562 (2003),
K. w. Choi, N. y. Haba, K. S. Jeong, K. i. Okumura, Y. Shimizu and M. Yamaguchi, J. High Energy Phys. 02 (2004), 037.
G. Cacciapaglia, C. Csaki and S. C. Park, J. High Energy Phys. 03 (2006), 099.
C. A. Scrucca, M. Serone and L. Silvestrini, Nucl. Phys. B 669 (2003), 128.
C. A. Scrucca, M. Serone, L. Silvestrini and A. Wulzer, J. High Energy Phys. 02 (2004), 049.
G. Panico, M. Serone and A. Wulzer, Nucl. Phys. B 739 (2006), 186.
C. A. Scrucca, M. Serone and L. Silvestrini, Nucl. Phys. B 669 (2003), 128.
C. S. Lim and N. Maru, Phys. Lett. B 653 (2007), 320.

[5] N. Haba, Y. Hosotani, Y. Kawamura and T. Yamashita, Phys. Rev. D 70 (2004), 015010.
N. Haba and T. Yamashita, J. High Energy Phys. 04 (2004), 016.
N. Haba, K. Takenaga and T. Yamashita, Phys. Lett. B 615 (2005), 247.
N. Haba, S. Matsumoto, N. Okada and T. Yamashita, JHEP 0602 (2006) 073.

[6] Y. Hosotani, S. Noda and K. Takenaga, Phys. Rev. D 69 (2004), 125014; Phys. Lett. B 607 (2005), 276.

[7] K. Hasegawa, C. S. Lim and N. Maru, Phys. Lett. B 604 (2004), 133.

[8] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999), 3370.

[9] R. Contino, Y. Nomura and A. Pomarol, Nucl. Phys. B 671 (2003), 148.
K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719 (2005), 165.
A. D. Medina, N. R. Shah and C. E. M. Wagner, Phys. Rev. D 76 (2007), 095010.

[10] Y. Hosotani and M. Mabe, Phys. Lett. B 615 (2005), 257.
Y. Hosotani, S. Noda, Y. Sakamura and S. Shimasaki, Phys. Rev. D 73 (2006), 096006.
Y. Sakamura and Y. Hosotani, Phys. Lett. B 645 (2007), 442.
Y. Sakamura, Phys. Rev. D 76 (2007), 065002.

[11] Y. Hosotani and Y. Sakamura, Prog. Theor. Phys. 118 (2007), 935.

[12] K. Agashe and R. Contino, Nucl. Phys. B 742 (2006) 59.
M. S. Carena, E. Ponton, J. Santiago and C. E. M. Wagner, Nucl. Phys. B 759 (2006) 202. Phys. Rev. D 76 (2007) 035006.
[13] G. F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, J. High Energy Phys. 06 (2007), 045.
A. Falkowski, S. Pokorski and J. P. Roberts, J. High Energy Phys. 12 (2007), 063.

[14] M. Regis, M. Serone and P. Ullio, JHEP 0703 (2007) 084.
G. Panico, E. Ponton, J. Santiago and M. Serone, Phys. Rev. D 77 (2008) 115012.

[15] M. Carena, A. D. Medina, N. R. Shah and C. E. M. Wagner, Phys. Rev. D 79 (2009) 096010.

[16] Y. Hosotani, P. Ko and M. Tanaka, Phys. Lett. B 680 (2009) 179.

[17] T. Appelquist, H. C. Cheng and B. A. Dobrescu, Phys. Rev. D 64 (2001) 035002.

[18] K. Agashe, A. Falkowski, I. Low and G. Servant, JHEP 0804 (2008) 027.

[19] N. Okada and T. Yamada, arXiv:0905.2801 [hep-ph], to be published in Phys. Rev. D.

[20] Y. Kawamura, Prog. Theor. Phys. 103 (2000) 613; ibid 105 (2001) 691; ibid 105 (2001) 999.

[21] M. Kubo, C. S. Lim and H. Yamashita, Mod. Phys. Lett. A 17 (2002), 2249.
A. Delgado, A. Pomarol and M. Quiros, Phys. Rev. D 60 (1999), 095008.
K. Takenaga, Phys. Lett. B 570 (2003), 244.
A. Aranda and J. L. Diaz-Cruz, Phys. Lett. B 633 (2006), 591.
N. Haba and T. Yamashita, JHEP 0402 (2004) 059.
N. Haba, K. Takenaga and T. Yamashita, Phys. Rev. D 71 (2005) 025006.
K. Kojima, K. Takenaga and T. Yamashita, Phys. Rev. D 77 (2008) 075004.

[22] N. Maru and T. Yamashita, Nucl. Phys. B 754 (2006) 127.
Y. Hosotani, N. Maru, K. Takenaga and T. Yamashita, Prog. Theor. Phys. 118 (2007) 1053.

[23] K. y. Oda and A. Weiler, Phys. Lett. B 606 (2005), 408.
N. Haba, S. Matsumoto, N. Okada and T. Yamashita, Prog. Theor. Phys. 120 (2008) 77.

[24] T. Gherghetta and A. Pomarol, Nucl. Phys. B 586 (2000) 141.

[25] Y. Adachi, C. S. Lim and N. Maru, arXiv:0904.1695 [hep-ph]; arXiv:0905.1022 [hep-ph].
[26] P. Gondolo and G. Gelmini, Nucl. Phys. B 360 (1991) 145.

[27] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 180 (2009) 330.

[28] For a review, see, for example, G. Jungman, M. Kamionkowski and K. Griest, Phys. Rept. 267 (1996) 195.

[29] J. Angle et al. [XENON Collaboration], Phys. Rev. Lett. 100 (2008) 021303.

[30] Z. Ahmed et al. [CDMS Collaboration], Phys. Rev. Lett. 102 (2009) 011301.

[31] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667 (2008) 1.

[32] H. Ohki et al., Phys. Rev. D 78 (2008) 054502.

[33] XENON Collaboration, http://www-sk.icrr.u-tokyo.ac.jp/xmass/index-e.html

[34] SCDMS Collaboration, http://xenon.astro.columbia.edu/

[35] XMASS Collaboration, http://cdms.berkeley.edu/