Neutrino emission due to Cooper pairing of protons in cooling neutron stars: Collective effects

L. B. Leinson*

Departamento de Física Teórica, Universidad de Valencia
46100 Burjassot (Valencia), Spain.

Abstract

The process of neutrino-pair radiation due to formation and breaking of Cooper pairs of protons in superconducting cores of neutron stars is considered with taking into account of the electromagnetic coupling of protons to ambient electrons. It is shown that plasma polarization strongly modifies the effective vector weak current of protons. Collective response of ambient electrons to the proton quantum transition contributes coherently to the complete interaction with the neutrino field and enhances the rate of neutrino-pair production by two orders of magnitude.

*On leave from: Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation RAS
142092 Troitsk, Moscow Region, Russia
Except for the early stages of evolution, temperatures inside neutron stars are lower than the critical temperatures for proton superconductivity $T_{cp}$ and/or neutron superfluidity $T_{cn}$. Conventionally, neutron superfluidity as well as proton superconductivity, exponentially reduce the heat capacity of neutron matter and the rates of neutrino production from nucleon reactions in the core of a neutron star [1], [2], [3], [4]. When $T < T_{cp}, T_{cn}$, nuclear matter in the stellar core consists of two components. One part of nucleons forms a condensate of Cooper pairs, the other one represents unpaired quasi-particles. Both processes, Cooper-pair formation and pair-breaking coexist in statistical equilibrium and result in additional neutrino-pair emission from the neutron star. This mechanism of energy loss from superfluid neutron stars was first proposed and evaluated for neutron singlet-state superfluidity by Flowers et al. [6], but it has been included into cooling simulations only recently [7], [8], [9]. It was shown, that the phase transition of neutrons in a superfluid state does not decelerate the process of cooling, but, on the contrary, neutrino emission due to formation and breaking of Cooper pairs accelerates the neutron star cooling.

Cooper pairing of protons take place likely in $^1S_0$ state [5]. Since the total spin of the Cooper pair in the $^1S_0$ state is zero, the axial-vector contribution of the weak neutral current vanishes. Contribution to $\nu \bar{\nu}$ emissivity comes only from the vector weak interaction. Therefore, neutrino emission produced by proton pairing is conventionally assumed to be strongly suppressed due to the numerical smallness of the weak vector coupling constant of protons [10]. Such inference is made on the basis of calculations which ignored electromagnetic correlations among the charged particles in a QED plasma. Actually, protons in the plasma are coupled to ambient electrons via the electromagnetic field. By undergoing a quantum transition to the paired state, protons polarize the medium, thus inducing the motion of electrons inside the Debye sphere around them. The electron weak current associated to this motion generates neutrinos coherently with the weak current of protons, because the wavelength $\lambda$ of radiated neutrino pairs is much larger than the electron Debye screening distance $D_e$ (typically, $D_e^2/\lambda^2 \lesssim 10^{-2}$). In the present article we study an effective weak current of protons with taking into account the collective contribution of ambient electrons,
and estimate the neutrino-pair emissivity due to formation and breaking of proton Cooper pairs.

In what follows we use the system of units \( \hbar = c = 1 \) and the Boltzmann constant \( k_B = 1 \). The low-energy Lagrangian of proton interaction with the neutrino field in vacuum can be written in a point-like approach

\[
\mathcal{L}_{\text{vac}} = -\frac{G_F}{\sqrt{2}} \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu j^\mu, \tag{1}
\]

where \( G_F \) is the Fermi coupling constant, and the proton weak current is of the form

\[
j^\mu = \bar{\psi} \gamma^\mu (C_V - C_A \gamma_5) \psi \tag{2}
\]

Here \( \psi \) stands for the proton field; \( C_V \) and \( C_A \) are the vector and axial-vector coupling constants of the proton, respectively. Reserving the capital letter notations \( C_V \) and \( C_A \) for proton coupling constants, we will use, at the same time, small letters, \( c_V \), for electron coupling constants with

\[
c_V = \frac{1}{2} + 2 \sin^2 \theta_W \simeq 0.96 \quad \text{for electron neutrinos, and}
\]

\[
c'_V = -\frac{1}{2} + 2 \sin^2 \theta_W \simeq -0.04 \quad \text{for muon and tau neutrinos; } \theta_W \text{ is the Weinberg angle.}
\]

To describe the above-mentioned collective effects, one has to include in the matrix element of the reaction two Feynman diagrams shown in Fig. 1. The sum of these diagrams represents the effective weak coupling of in-medium protons with the neutrino field. The matrix element of this interaction should be calculated between exact initial and final states of two proton quasi-particles undergoing quantum transition to the paired state. The first diagram in Fig. 1 represents the proton weak interaction in vacuum, and the second one is the contribution of ambient electrons. Some explanations are necessary here. At the first sight, the second diagram contains additionally a small factor \( e^2 = 1/137 \). However, the fine structure constant enters the matrix element in a combination with the electron number density, known as the Debye screening distance. Therefore, the parameter of the problem is \( k^2 D_e^2 \), where \( k \) is the momentum carried out by the neutrino-pair. As will be shown, in the case \( k^2 D_e^2 \lesssim 1 \) the second diagram gives an important contribution and should be necessarily included in the matrix element of the reaction.
The electron loop in the second diagram can be expressed in terms of polarization tensors of the electron gas, defined in the one-loop approximation

$$\Pi^{\mu \rho}(K) = 4\pi i e^2 \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \gamma^\mu \hat{G}(p) \gamma^\rho \hat{G}(p + K),$$

(3)

$$\Pi_5^{\mu \rho}(K) = 4\pi i e^2 \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \gamma^\mu \hat{G}(p) \gamma^\rho \gamma_5 \hat{G}(p + K).$$

(4)

Here $\hat{G}(p)$ is the in-medium electron Green’s function, and $K = (\omega, \mathbf{k})$ is the four-momentum transfer. Thus, the in-medium effective weak current of protons is of the following form

$$J^\mu = \bar{\psi} \gamma^\mu (C_V - C_A \gamma_5) \psi - \frac{1}{4\pi} \left( \bar{\psi} \gamma^\lambda \psi \right) D_{\lambda \rho} \left( c_V \Pi^{\rho \mu} - c_A \Pi_5^{\rho \mu} \right),$$

(5)

where $D_{\lambda \rho}(K)$ is the photon propagator in the medium. An extra factor $(4\pi)^{-1}$ appears here because the factor $4\pi$ is traditionally included in the definition of polarization tensors. The minus sign in front of this factor is due to the fact that the second diagram includes a proton charge $e > 0$ in the left-hand electromagnetic vertex of the virtual photon, and an electron charge $-e$ in the right-hand vertex. The corresponding $e^2$ factor which appears accordingly to the second diagram, is also included in the definition of polarization tensors.

To write the in-medium photon propagator one has to take into account that the superconductive electric current of protons and the normal electric current of electrons coexist, participating in electromagnetic oscillations of the medium. The total Lagrangian density of the complex field $\Psi$ of proton Cooper pairs, and electrons interacting with the electromagnetic field $A_\mu$ has the form

$$L = |(\partial_\mu + 2ieA_\mu) \Psi|^2 + M_{Cp}^2 \Psi^2 - \lambda |\Psi|^4 - \frac{1}{16\pi} F_{\mu \nu}^2 - j_\mu^e A_\mu + L_0^e$$

(6)

where $\lambda$ is a constant for proton-proton interaction resulting in the proton pairing, and $M_{Cp} \simeq 2M_p^*$ is the mass of the Cooper pair consisting of two protons of effective mass $M_p^*$; $F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the tensor of electromagnetic field; $j_\mu^e$ is the electron current, and $L_0^e$ is the Lagrangian density of free electrons. As a result of spontaneous breaking of the symmetry of the ground state of the system, the vacuum expectation value of the Cooper
pair field is nonzero $\langle |\Psi|^2 \rangle = \Psi_0^2$. Since the ground state of the system corresponds to zero spin and zero total momentum of the Cooper pair the vacuum expectation value is connected with the number density of Cooper pairs $N_s$ by the relation $N_s = 2E\Psi_0^2$, where $E \approx 2M_p^*$ is the energy of the Cooper pair corresponding to zero total momentum, and $N_s$ is one-half the number density of paired protons $N_p$. Thus,

$$\Psi_0^2 = \frac{N_p}{8M_p^*} \quad (7)$$

We represent the field of the Cooper condensate in the form

$$\Psi = \Psi_0 (1 + \rho) \exp(-2ie\phi) \quad (8)$$

where $\rho$ and $\phi$ are arbitrary real functions of the coordinates and time, and substitute in the Lagrangian density (6) for the fields $\rho$ and $\phi$. Taking into account the following identity

$$|(\partial_\mu + 2ieA_\mu) \Psi|^2 = \Psi_0^2 (\partial_\mu \rho)^2 + 4e^2\Psi_0^2 (A_\mu - \partial_\mu \phi)^2 (1 + \rho)^2 \quad (9)$$

we can make the gauge transformation $A'_\mu = A_\mu - \partial_\mu \phi$. Since the quantity $F_{\mu\nu} = F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu$ as well as the electron current $j'_{\mu} = j_{\mu}^e$ are gauge-invariant we obtain

$$L = \psi_0^2 (\partial_\mu \rho)^2 + 4e^2\Psi_0^2 A'_\mu (1 + \rho)^2 + M_{Cp}^2 |\psi|^2 - \lambda |\psi|^4 - \frac{1}{16\pi} F_{\mu\nu}^2 - j_{\mu}^e A'^{\mu} + L_e \quad (10)$$

As a result, the Goldstone field $\phi$ is absorbed by the gauge transformation. Considering this and taking variations of the Lagrangian with respect to the field $A'^\mu$, in the linear approximation we obtain the equation

$$\partial_\nu \partial^{\nu} A'_\mu + 32\pi e^2\Psi_0^2 A'_\mu = 4\pi j_{\mu}^e, \quad \partial^{\mu} A'_\mu = 0 \quad (11)$$

In the absence of the electron current ($j_{\mu}^e = 0$), this equation would describe the eigen photon modes of mass $m_\gamma = \sqrt{32\pi e^2\Psi_0^2} = \sqrt{4\pi N_p e^2 / M_p^*}$. This is the well-known Higgs effect. However, the induced electron current should also be taken into account. In the momentum representation $4\pi j_{\mu}^e = -\Pi_{\mu \nu} A'^{\nu}$, and the Eq.(11) takes the form

$$K^2 A'_\mu - m_\gamma^2 A'_\mu - \Pi_{\mu \nu} (K) A'^{\nu} = 0, \quad K^{\mu} A'_\mu = 0 \quad (12)$$
To specify the components of the polarization tensor, we select a basis constructed from the following orthogonal four-vectors

$$h^\mu \equiv (\omega, k)/\sqrt{K^2}, \quad l^\mu \equiv (k, \omega n)/\sqrt{K^2},$$

where the space-like unit vector $n = k/k$, $k = |k|$ is directed along the electromagnetic wave vector $k$. Thus, the longitudinal basis tensor can be chosen as $L^\rho\mu = -l^\rho l^\mu$. The transverse (with respect to $k$) components of $\Pi^{\rho\mu}$ have a tensor structure proportional to the tensor $T^{\rho\mu} \equiv (g^{\rho\mu} - h^\rho h^\mu + l^\rho l^\mu)$, where $g^{\rho\mu} = \text{diag}(1, -1, -1, -1)$ is the signature tensor. In this basis, the polarization tensor has the following form

$$\Pi^{\rho\mu} (K) = \pi_l (K) L^{\rho\mu} + \pi_t (K) T^{\rho\mu}, \quad (13)$$

In the case of a strongly degenerate ultrarelativistic electron plasma ($v_F \simeq 1$), the longitudinal and transverse polarization functions are

$$\pi_l = \frac{1}{D_e^2} \left( 1 - \frac{\omega^2}{k^2} \right) \left( 1 - \frac{\omega}{2k} \ln \frac{\omega + k}{\omega - k} \right), \quad (14)$$

$$\pi_t = \frac{3}{2} \frac{\omega^2}{\omega_{pe}} \left( 1 + \left( \frac{\omega^2}{k^2} - 1 \right) \left( 1 - \frac{\omega}{2k} \ln \frac{\omega + k}{\omega - k} \right) \right). \quad (15)$$

The electron plasma frequency and the Debye screening distance are defined as

$$\omega_{pe}^2 = \frac{4}{3\pi} e^2 \mu_e = \frac{4\pi n_e e^2}{\mu_e}, \quad \frac{1}{D_e^2} = 3\omega_{pe}^2, \quad (16)$$

with $\mu_e$ and $n_e$ being, respectively, the chemical potential and the number density of electrons. The axial polarization tensor is given by

$$\Pi_5^{\rho\mu} (K) = \pi_A (K) i\hbar A e^{\rho\mu\lambda\sigma} 0, \quad \pi_A (K) = \frac{\sqrt{K^2}}{2\mu_e} \pi_l (K). \quad (17)$$

The total energy of neutrino-pairs is $\omega \sim \Delta \ll \mu_e$. When $\omega \geq k$, polarization functions differ in the order of magnitude, namely, $\pi_A \sim \pi_{l,t} \Delta/\mu_e$. Therefore, the axial polarization of the medium can be neglected.

---

1 Our Eq. (14) differs from Eq. (A39) of the Ref. [11] by an extra factor $(\omega^2/k^2 - 1)$ because our basis $l^\mu, h^\mu$ is different from that used by Braaten and Segel. All components of the complete tensor (13) identically coincide with that obtained in [11] for the degenerate ultrarelativistic case.
In the absence of the electron current, the propagator of a massive photon is of the form

\[ D_{0\mu\nu} = \frac{4\pi}{K^2 - m_\gamma^2} \left( g_{\mu\nu} - \frac{K^\mu K^\nu}{m_\gamma^2} \right) \] (18)

The in-medium photon propagator, taking into account the induced electron current, can be found with the aid of the Dyson’s equation. The solution is:

\[ D_{\lambda\rho}(K) = \frac{4\pi}{K^2 - m_\gamma^2 - \pi_l} L_{\lambda\rho} + \frac{4\pi}{K^2 - m_\gamma^2 - \pi_t} T_{\lambda\rho} - \frac{4\pi}{m_\gamma^2} h_\lambda h_\rho. \] (19)

Due to conservation and gauge invariance of the electron current \( h_\lambda \Pi^{\lambda\mu} = \Pi^{\lambda\mu} h_\mu = 0 \), the last term of Eq.(19) does not contribute to the effective weak current of a proton in the medium.

Insertion of (13) and (19) into (5) with the above approximation yields the effective weak current of a proton in the medium.

\[ J^\mu = \bar{\psi} \gamma^\mu (C_V - C_A \gamma_5) \psi - c_V \left( \bar{\psi} \gamma_\lambda \psi \right) \left( \frac{\pi_l L^{\lambda\mu}}{\omega^2 - k^2 - m_\gamma^2 - \pi_l} + \frac{\pi_t T^{\lambda\mu}}{\omega^2 - k^2 - m_\gamma^2 - \pi_t} \right) \] (20)

The poles of this expression give the eigen photon modes in the medium. In the case of \( K^2 > 0 \) the eigen mode frequency \( \omega(k) \) is larger than the plasma frequency of electrons. At temperatures \( T < T_c \ll \omega_{pe} \), the number of such excited oscillations in the medium is exponentially suppressed, therefore contributions from the poles can be neglected.

The contributions of longitudinal and transverse virtual photons to the effective weak current (20) are proportional to the following factors

\[ F_l(k, \omega) \equiv \frac{\pi_l}{\omega^2 - k^2 - m_\gamma^2 - \pi_l} = -\frac{\varphi(x)}{D_\epsilon^2 \omega^2 (1 - x^2) - m_\gamma^2 D_\epsilon^2 + \varphi(x)} \] (21)

\[ F_t(k, \omega) \equiv \frac{\pi_t}{\omega^2 - k^2 - m_\gamma^2 - \pi_t} = \frac{1 + \varphi(x)}{2D_\epsilon^2 \omega^2 (1 - x^2) - m_\gamma^2 D_\epsilon^2 - (1 + \varphi(x))} \] (22)

with \( x = k/\omega \) and

\[ \varphi(x) = x^{-2} \left( 1 - x^2 \right) \left( 1 - \frac{1}{2x} \ln \frac{1 + x}{1 - x} \right), \quad m_\gamma^2 D_\epsilon^2 = \frac{\mu_e}{3M_p^*} \] (23)
The latter equality takes into account local neutrality $N_p = n_e$ of the medium.

The neutrino wave-length is much larger than the Debye screening distance, therefore, $D_e^2 \omega^2 (1 - x^2) \lesssim 4 \Delta^2 D_e^2 (1 - x^2) \ll \varphi(x)$. Neglecting this small term we obtain

$$F_l(x) = -\frac{\varphi(x)}{\varphi(x) - m_e^2 D_e^2}, \quad F_t(x) = -\frac{1 + \varphi(x)}{1 + \varphi(x) + m_e^2 D_e^2}. \quad (24)$$

Due to the electron contribution the effective proton interaction with the neutrino field has a complicated form. According to Eq.(20), the vector part of the effective proton weak current is substantially modified by polarization of the medium. Assuming the proton effective mass to be $M_p^* \simeq 0.8 M_p$, functions $F_l(x), F_t(x)$ are plotted in the Fig. 2 for typical values of the electron chemical potential $\mu_e \simeq 100$ MeV and $\mu_e \simeq 200$ MeV.

For the moment, we shall estimate the contribution of the medium polarization to the neutrino emissivity caused by formation and breaking of proton Cooper pairs and leave the complete calculation for a future work. To roughly estimate the contribution of collective effects, we take advantage of the fact that $F_l(x)$ and $F_t(x)$ are slowly varying functions, and replace them in Eq.(20) by a constant value $-0.8$. With this simplification we obtain

$$J^\mu \sim \bar{\psi} \gamma^\mu (C_V - C_A \gamma_5) \psi + 0.8 c_V \left( \bar{\psi} \gamma_\lambda \psi \right) \left( L^\lambda + T^\lambda \right). \quad (25)$$

Taking into account the identity $L^\lambda + T^\lambda \equiv g^\lambda - h^\lambda h^\mu$, and gauge invariance of the electromagnetic current of protons $\left( \bar{\psi} \gamma_\lambda \psi \right)_{fi} K^\lambda = 0$, we can reduce the effective proton weak current (25) as follows

$$J^\mu \sim \bar{\psi} \gamma^\mu (C_V + 0.8 c_V - C_A \gamma_5) \psi. \quad (26)$$

Thus, the effective vector weak coupling of in-medium protons is of the order of $C_V + 0.8 c_V$, in contrast to $C_V = \frac{1}{2} - 2 \sin^2 \theta_W \simeq 0.04$ for bare protons in vacuum. If protons interact with electron neutrinos then $C_V + 0.8 c_V \simeq 0.8$, while for muon and tau neutrinos $C_V + 0.8 c_V \simeq 0.008$. The total spin of a Cooper pair in the $^1S_0$ state is zero therefore, the contribution to

---

2 Strictly speaking, this equality is valid only at zero temperature, when all protons are paired.
the $\nu\bar{\nu}$ emissivity was proportional to $C_V^2$. Taking into account the collective effects, one has to replace $C_V$ by $C_V + 0.8c_V$ for electron neutrinos and by $C_V + 0.8c_V'$ for muon and tau neutrinos. Thus, the $\nu\bar{\nu}$ emissivity is enhanced about two orders of magnitude:

$$\frac{Q}{Q_0} \sim \frac{(C_V + 0.8c_V)^2 + 2(C_V + 0.8c_V')^2}{3C_V^2} \sim 133.$$  \hspace{1cm} (27)

This ratio demonstrates the large importance of collective processes in the core of a superconducting neutron star. As $C_V \ll c_V$, the leading contribution to $Q$ comes from the weak current of electrons which is electromagnetically induced by the quantum transition of the initial proton.

The neutrino emissivity due to Cooper pairing of protons, as well as that for neutron pairing is proportional to the Fermi momentum of the relevant particles, i.e. it only weakly ($\sim n_{p,n}^{1/3}$) depends explicitly on the partial number density. Besides that, both the superfluid and the superconducting energy gaps, calculated theoretically, sensitively depend on the model of strong interactions and vary in the range from ten keV to some MeV \cite{13}, \cite{14}, \cite{15}, \cite{16}. Thus, though the number density of protons is much smaller than that of neutrons, their partial contribution to the total energy losses is very sensitive to the ratio between the energy gaps of neutrons and protons. It can be dominant at some cooling stage, depending on temperature, $T_{cn}$, $T_{cp}$, and density. Therefore, the neutrino radiation, caused by a singlet-pairing of protons, should be taken into consideration in the scenario of cooling of superconducting neutron stars at the same level, as other basic processes of neutrino production.

We have shown in the present paper that the effective vector weak current of in-medium protons is strongly modified by ambient electrons. Their weak current, electromagnetically induced by the proton quantum transition, enhances neutrino production due to Cooper pairing of protons by two orders of magnitude. An analogous conclusion can be made about neutrino-pair production due to Cooper pairing of $\Xi^-$ hyperons. Though the vector constant of the weak coupling for $\Xi^-$ hyperons is small relatively to those for neutrons and $\Sigma^\pm$, the
interaction of $\Xi^-$ with neutrinos is enhanced in the medium by ambient electrons.

ACKNOWLEDGMENTS

This work was supported in part by Spanish Grant DGES PB97- 1432, and the Russian Foundation for Fundamental Research Grant 00-02-16271.
REFERENCES

[1] S. Tsuruta, V. Canuto, J. Lodenquai, M. Ruderman, *ApJ*, **176** (1972) 739.

[2] D. Page, J. H. Applegate, *ApJ (Lett.)*, **394** (1992) L17.

[3] D. Page, *ApJ (Lett.)*, **428** (1994) 250.

[4] K. P. Levenfish, D. G. Yakovlev, *Astron. Lett.*, **22** (1996) 47.

[5] T. Takatsuka, R. Tamagaki, *Progr. Theor. Phys. Suppl.*, **112** (1993) 27.

[6] E. Flowers, M. Ruderman, P. Sutherland, *ApJ*, **205** (1976) 541.

[7] C. Shaab, D. Voskresensky, A. D. Sedrakian, F. Weber, M. K. Weigel, *A&A*, **321** (1997) 591.

[8] D. Page, In: Many Faces of Neutron Stars (eds. R. Buccheri, J. van Peredijs, M. A. Alpar). *Dordrecht: Kluver* 1998, p. 538.

[9] D. G. Yakovlev, A. D. Kaminker, K. P. Levenfish, In: Neutron Stars and Pulsars (ed. N. Shibazaki et al.), Tokio: *Universal Akademy Press* 1998, p. 195.

[10] D. G. Yakovlev, A. D. Kaminker, K. P. Levenfish, *A&A*, **343** (1999) 650.

[11] E. Braaten and D. Segel, *Phys.Rev.*, **D48** (1993) 1478.

[12] G. G. Raffelt, ”Stars as laboratories for fundamental physics: the astrophysics of neutrinos, axions, and other weakly interacting particles”, *The University of Chicago Press*, 1996.

[13] R. Tamagaki, *Prog. Theor. Phys.*, **44** (1970) 905.

[14] L. Amundsen, E. Ostgaard, *Nucl. Phys.*, **A422** (1985) 163.

[15] M. Baldo, J. Cugnon, A. Lejeune, U. Lombardo, *Nucl. Phys.*, **A536** (1992) 349.

[16] T. Takatsuka, R. Tamagaki, *Prog. Theor. Phys.*, **112** (1993) 27.
Fig 1. Feynman graphs contributing to the effective proton interaction with a neutrino field in the medium. The first diagram presents the vacuum-like weak interaction. The second diagram, with the electron loop, describes the interaction via an intermediate virtual photon, shown by the dashed line.
Fig 2. Functions $F_l(x)$, $F_t(x)$ are plotted for two cases of the electron chemical potential $\mu_e \simeq 100 \text{ MeV}$ (dashed lines) and $\mu_e \simeq 200 \text{ MeV}$ (solid lines). The proton effective mass $M_p^* = 0.8M_p$. 