NNLL Threshold Resummation for Top-Pair and Single-Top Production

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Abstract—I discuss threshold resummation at NNLL accuracy in the standard moment-space approach in perturbative QCD for top-pair and single-top production. For top quark pair production I present new approximate NNLO results for the total cross section and for the top quark transverse momentum and rapidity distributions at 8 TeV LHC energy. I discuss the accuracy of the soft-gluon approximation and show that the NLO and NNLO approximate results from resummation are practically indistinguishable from exact NLO and partial NNLO results. For single top production I present new approximate NNLO results for the total cross sections in all three channels at the LHC and also for the top quark transverse momentum distributions in $t$-channel production and in top-quark associated production with a $W$ boson. For both $t\bar{t}$ and single-top production the agreement of theoretical results with LHC and Tevatron data is excellent.

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1. INTRODUCTION

Threshold resummation is important for increasing the theoretical accuracy of top quark total and differential cross sections at hadron colliders. The top quark is the heaviest known elementary particle and its study is of unique importance both theoretically and in experiments at the LHC and previously at the Tevatron. The top quark cross section is well-known to receive large contributions from soft-gluon emission near partonic threshold. Resummation of soft-gluon corrections at next-to-leading-logarithm (NLL) accuracy [1, 2] and beyond depends on the color structure of the hard scattering.

Soft-gluon resummation was developed recently at next-to-next-to-leading-logarithm (NNLL) accuracy in various approaches for $t\bar{t}$ differential [3–6] and total [3–5, 7, 8] cross sections. It is very important to note that although all these approaches are formally NNLL, they are quite different from each other both theoretically and numerically, and “NNLL” can mean different things since the variables used for the threshold logarithms, the formalism employed, and the practical choices made in its implementation vary widely. Differences between the approaches include performing the resummation for the double-differential cross section in single-particle-inclusive (1PI) [4–6] and/or pair-invariant-mass (PIM) [3] kinematics versus doing the resummation solely for the total cross section using production threshold [7, 8]; using moment-space resummation in perturbative QCD [4, 6, 7] versus using Soft-Collinear Effective Theory (SCET) [3, 5, 8]; choices for the analytical and numerical implementation of the expressions and for damping factors away from threshold; and keeping subleading terms of various origins. A review explaining in detail many of the differences can be found in [9]. In the double-differential approach the resummation is sensitive to the kinematical invariants of the partonic process; this sensitivity is lost in the total-cross-section-only approach where the production threshold (also known as absolute threshold) that is used is simply a special case of the more general partonic threshold employed in 1PI and PIM differential kinematics (see discussion in [9]). We note that the formalism presented here is the only calculation using the moment-space perturbative-QCD NNLL resummation for the double-differential cross section in 1PI kinematics, $d\sigma/dp_T dY$, to calculate approximate NNLO total cross sections and transverse momentum, $p_T$, and rapidity, $Y$, distributions for both top-pair production [4, 6] and single-top production [10–12].

In the next section we briefly describe our resummation formalism. In Section 3 we present results for top pair production, including the total cross section, the top quark transverse momentum distribution, and the top quark rapidity distribution. New results for 8 TeV LHC energy are presented and compared with results at 7 TeV. A discussion of the excellent accuracy of the soft-gluon approximation at NLO and NNLO is also presented. In Section 4 we discuss new results for single top production in the $t,s$, and $tW$ channels. New results for the total cross sections at 8 TeV LHC energy are shown together with (updated) results at 7 TeV. Furthermore new theoretical calculations and results for the top (and antitop) transverse momentum distributions in $t$-channel production are presented at LHC and Tevatron energies. A new calculation for the top quark transverse momentum distribution in $tW$ production at the LHC is also presented. We conclude in Section 5.
2. RESUMMATION AT NNLL

We begin with a brief description of our formalism for NNLL resummation of soft-gluon corrections. We consider partonic processes of the form

\[ f_1(p_1) + f_2(p_2) \rightarrow t(p) + X, \]  

where \( f_1 \) and \( f_2 \) represent partons (quarks or gluons), \( t \) represents the top quark, and \( X \) represents additional final-state particles. The partonic cross section explicitly involves the kinematical invariants \( s = (p_1 + p_2)^2 \), \( t_1 = (p_1 - p)^2 - m_t^2 \), \( u_1 = (p_2 - p)^2 - m_t^2 \), with \( m_t \) the top quark mass, as well as factorization scale \( \mu_F \) and the renormalization scale \( \mu_R \). The physical cross section is in principle independent of \( \mu_F \) and \( \mu_R \), but a dependence appears when we truncate the infinite perturbative series at finite order.

Near partonic threshold for the production of the top-quark final state, the cross section receives logarithmic contributions that arise from incomplete cancellations between virtual terms and terms from soft-gluon emission. These contributions are of the form \[ \ln(s_4/m_t^2/s_4) \] where \( s_4 = s + t_1 + u_1 \) measures distance from partonic threshold. Soft-gluon resummation depends critically on the color structure of the partonic process as well as its kinematics.

The resummation of threshold logarithms is performed in moment space, it is based on the factorization properties of the cross section, and it employs the eikonal approximation for describing the emission of soft gluons from partons in the hard scattering. By taking moments, logarithms of \( s_4 \) produce powers of \( \ln N \), with \( N \) the moment variable. The resummation was first performed at NLL accuracy in [1, 2] and at NNLL accuracy in [4].

We factorize the moments of the partonic cross section in dimensional regularization as in [1, 2]

\[ \widehat{c}_{\mu} \rightarrow tX(N, \epsilon) = \psi_{\mu} \psi_{\mu} (N, \mu, \epsilon) \psi_{\mu} \psi_{\mu} (N, \mu, \epsilon) \]
\[ \times \mathcal{H}_{\mu}^{\mu_1 \rightarrow tX} (\alpha_s(\mu_R)) \mathcal{S}_{\mu}^{L_I \rightarrow tX} (\frac{m_t}{\mu_R}, \alpha_s(\mu_R)) \]
\[ \times \prod_j J_j (N, \mu, \epsilon) + \mathcal{O}(1/N), \]  

where \( \psi \) are center-of-mass distributions for the incoming partons, \( H_\mu \) is the \( N \)-independent hard matrix in the space of color exchanges (with color indices \( I, L \)), \( S_{\mu} \) is the soft matrix, and \( J \) are functions for massless partons in the final state. More details about the definitions of these functions and the construction of the eikonal cross section can be found in [2].

The hard-scattering matrix involves contributions from the amplitude of the process and the complex conjugate of the amplitude, \( H_\mu = h^{\mu_1}_l h^\mu_r \). The soft function \( S_{\mu} \) represents the emission of noncollinear soft gluons from the partons in the scattering. The color tensors of the hard scattering connect together the eikonal lines to which soft gluons couple (see Fig. 1).

The \( N \)-dependence of the soft matrix \( S_{\mu} \) can be resummed by renormalization group analysis. \( S_{\mu} \) satisfies the renormalization group equation [1, 2]

\[ \left( \mu \frac{\partial}{\partial \mu} + \beta(g_s) \frac{\partial}{\partial g_s} \right) S_{\mu} = - (\Gamma^\mu_1 L_\mu S_{\mu} - S_{\mu} (\Gamma_{S})_{\mu} \mu \), \]

where \( \beta \) is the QCD beta function and \( g_s = 4\pi \alpha_s \). \( \Gamma_{S} \) is the soft anomalous dimension matrix and it is calculated in the eikonal approximation by explicit renormalization of the soft function.

The exponentiation of logarithms of \( N \) in the functions \( \psi \) and \( J \) in the factorized cross section, together with the solution of the renormalization group Eq. (2.3), provide us with the complete expression for the resummed (double-differential) partonic cross section in moment space

\[ \hat{\sigma}_{\text{res}}(N) = \exp \left[ \sum_j E_{a}^{f_j}(N_j) \right] \exp \left[ \sum_j E_{a}^{f}(N^j) \right] \]
\[ \times \exp \left[ \sum_j \int d\mu_{a} \Gamma_{f_j}^{\mu} (\alpha_s(\mu), N) \right] \text{Tr} \left[ H_{f_j}^{f_j-\rightarrow tX} (\alpha_s(\mu)) \right] \]
\[ \times \exp \left[ \int d\mu_{\mu} \Gamma_{s}^{\mu} (\alpha_s(\mu)) \right] \delta^{f_j-\rightarrow tX} (\alpha_s) \]
\[ \times \exp \left[ \int d\mu_{\mu} \Gamma_{s}^{\mu} (\alpha_s) \right] \delta^{f_j-\rightarrow tX} (\alpha_s) \]  

Fig. 1. The soft-gluon matrix \( S_{\mu} \). The eikonal lines connect with color tensors \( c_L \) and \( c_R \).
The first and second exponents in Eq. (2.4) control collinear and soft gluon emission \([13, 14]\) from incoming and outgoing partons, respectively, while the third exponent controls the factorization scale dependence of the cross section. Explicit expressions for these exponents can be found in \([15]\). The evolution of the soft gluon function is controlled by the soft anomalous dimension matrix \(\Gamma_S\) via the solution to Eq. (2.3).

The ultraviolet poles in loop diagrams involving eikonal lines play a direct role in the renormalization group evolution equations that are used in the calculation of the soft anomalous dimensions \([1, 2, 16]\). The soft anomalous dimension matrices in our formalism have been calculated at two loops for the partonic processes in top-pair production in \([4, 16]\) and for single-top production in \([10–12]\).

### 3. TOP-PAIR PRODUCTION

The threshold resummation formalism discussed here has been used to calculate top-pair production at the Tevatron and the LHC \([4, 6]\). We expand the NNLL resummed cross section to NNLO. We add the NNLO soft-gluon corrections from this expansion to the exact NLO \([17–19]\) expressions and denote the result as approximate NNLO. Here we present new results for the current 8 TeV LHC energy for the total cross section and the top quark transverse momentum and rapidity distributions. The factorization and renormalization scales are set equal to each other and denoted by \(\mu\). Throughout we use the MSTW2008 \([20]\) NNLO parton distribution functions (pdf).

We begin by examining the validity and numerical accuracy of the threshold soft-gluon approximation by comparing exact NLO and approximate NLO results (with \(\mu = m_t\)) for the total \(t\bar{t}\) cross section and the top quark \(p_T\) distribution. The comparison is shown in the left two plots of Fig. 2. The top left plot displays the exact and approximate NLO corrections, i.e. \(O(\alpha_s)\) corrections, for the total top-pair cross section at 7 and 8 TeV LHC energy. We note that the approximation is excellent with only around 2 to 3% difference between approximate and exact corrections. The approximate results include only \(q\bar{q}\) and \(gg\) channels (for which resummation is performed) while the exact results also include the \(qg\) and \(g\bar{g}\) channels (the contribution from the latter two channels is quite small). In fact if only the sum of the \(q\bar{q}\) and \(gg\) channels are included in the exact result the difference between approximate and exact corrections is only 1 to 2%. This very good agreement between exact and approximate NLO results is true not only for the total cross section but also for differential distributions. The bottom left plot of Fig. 2 shows that the exact NLO transverse

![Fig. 2. Comparison of exact and approximate NLO results for the total top-pair cross section and top-quark \(p_T\) distribution (left); NNLO approximate top-pair total cross section compared with data at the LHC (right).](image-url)
momentum distribution of the top quark is again almost identical to the approximate NLO result (the results are for 8 TeV LHC energy and \( m_t = 173 \text{ GeV} \)). We note that there are some choices to be made in the analytical structure and numerical implementation of the calculation, all strictly equivalent at partonic threshold but not entirely equivalent away from it, and different choices can give somewhat different results. The agreement between exact and approximate results shown here proves the validity, accuracy, and importance of our approach and the appropriateness of our choices, and it is the motivation for studying higher-order soft-gluon corrections in the same approach and with the same choices. As we will discuss shortly, recent partial NNLO results are also in excellent agreement with the approximate NNLO results from our formalism, thus further proving the advantages and accuracy of our approach.

In the right plot of Fig. 2 we show the NNLO approximate total top-pair production cross section at the LHC at 8 TeV energy, and for comparison also at 7 TeV energy, as a function of top quark mass. Here we have set \( \mu = m_t \) for our central results. The uncertainties from scale variation and from the pdf are added in quadrature. The results at 7 TeV are compared with ATLAS and CMS data [21, 22]. The 8 TeV results are compared with recent CMS data [23]. Excellent agreement is found between theory and experiment for both energies.

The new result for the current 8 TeV LHC energy with a top quark mass \( m_t = 173 \text{ GeV} \) and using MSTW2008 NNLO pdf is

\[
\sigma_{\tilde{t}\tilde{t}}^{\text{NNLAppox}} (m_t = 173 \text{ GeV}, 8 \text{ TeV}) = 234^{+10}_{-7} \pm 12 \text{ pb},
\]

where the first uncertainty is from scale variation \( m_t/2 < \mu < 2m_t \), and the second is from the pdf at 90% C.L. We note that independent variation of \( \mu_F \) and \( \mu_R \) does not increase the scale uncertainty at LHC energies. At 7 TeV the corresponding result is \( 163^{+7}_{-5} \pm 9 \text{ pb} \).

At the Tevatron the total top-pair cross section is \( 7.08^{+0.20}_{-0.24} \pm 0.31 \text{ pb} \) [4], in very good agreement with recent results from CDF [24] and D0 [25]. Independent variation of \( \mu_F \) and \( \mu_R \) changes the upper scale error at the Tevatron from \( +0.00 \) to \( +0.20 \). A lot of theoretical work by many groups over the last few years (see [9] for complete references) has made possible the calculation of exact NNLO corrections. Recently there have been numerical results of the exact NNLO contribution from the fermionic channels to the total top-pair cross section [26]. Since the \( q\bar{q} \) channel is dominant at the Tevatron this is supposed to be a good approximation to the still unknown complete NNLO at that collider. We note that our approximate NNLO results at the Tevatron are virtually indistinguishable from the results in [26] (the difference between \( 7.08^{+0.20}_{-0.24} \) versus \( 7.07^{+0.20}_{-0.31} \) pb [26] for \( m_t = 173 \text{ GeV} \) is at the per mille level with very similar scale uncertainty from independent \( \mu_F \) and \( \mu_R \) variation; in fact the scale dependence of our result is slightly smaller than that of [26]), indicating that future complete NNLO results will make very little, if any, practical difference for Tevatron and probably LHC measurements of the \( \tilde{t}\tilde{t} \) cross section. We also emphasize that the comparison between exact and approximate results from resummation in [26] is quite distinct from ours since different resummation formalisms are used.

The fact that corrections beyond the soft-gluon approximation make very little difference was actually inferred from the study in [27]. The calculation there was done in both 1PI and PIM kinematics and it was shown that when terms beyond NLL are included, the difference between the results is reduced and vanishes at partonic threshold. This indicated that further terms would make very little difference, and thus the fact that the complete NNLL terms and now the exact NNLO terms make very little difference was largely expected. This was also discussed in [28] and [4]. The more recent calculations at NNLL and NNLO prove the correctness of our earlier arguments and the robustness of our results.

No NNLO differential distributions are known at present but given the agreement of the exact and approximate NLO distributions, the agreement between the approximate and partially exact NNLO total cross sections, and the lessons learned from the comparison of approximate NLO and NNLO results calculated in 1PI and PIM kinematics, we expect that approximate NNLO should be sufficient for all practical purposes and exact NNLO results will make no significant difference to the distributions.

We also note that once the complete NNLO results are known for either the total cross section or differential distributions, the next natural step will be to add the approximate NNNLO corrections. In fact, a first study of those was performed in [29] for the \( q\bar{q} \) channel at the Tevatron and it was found that the NNNLO corrections are small although not insignificant.

A major strength of our double-differential calculation is that it can be used to calculate differential distributions. Of particular significance is the study of transverse momentum and rapidity distributions. In the left plot of Fig. 3 we show the top-quark transverse momentum distribution, \( d\sigma/dp_T \), at the LHC. New results for 8 TeV LHC energy are plotted and compared with results at 7 TeV. The approximate NNLO top quark \( p_T \) distributions are shown with a top quark mass \( m_t = 173 \text{ GeV} \) and the scale choices \( \mu = m_t \) and \( \mu = m_T \), where the transverse mass \( m_T \) is defined by \( m_T = (p_T^2 + m_t^2)^{1/2} \). In the right plot of Fig. 3 we show the \( K \)-factors, i.e. the ratios of NNLO approxi-
mate/NLO results for the top-quark $p_T$ distribution at 8 TeV LHC energy with scale choices $\mu = m_t$ and $\mu = m_T$. We observe that the $K$-factor for $\mu = m_t$ rises significantly at large $p_T$ as contrasted to that with $\mu = m_T$. The choice of the best scale is still an open question and it will be interesting to see LHC data at very high $p_T$.

In our perturbative expansion at approximate NNLO we have logarithms of $\mu_F/m_t$ and $\mu_R/m_t$ in plus-distribution terms as well as in $\delta(s_\alpha)$ terms, so $\mu = m_t$ is a natural scale choice for our results.

In the left plot of Fig. 4 we present the normalized top-quark $p_T$ distribution, $(1/\sigma) d\sigma/dp_T$, at 7 TeV LHC energy and compare it with recent data from CMS in the dilepton channel. We note the very good description of the CMS data by the theoretical predictions. Similar agreement is also found with CMS $l + \text{jets}$ data [30] as shown in [31].

In the right plot of Fig. 4 we show new results for the top quark rapidity distribution, $d\sigma/dY$, at the LHC at 8 TeV energy and, for comparison, at 7 TeV as well. The approximate NNLO top quark rapidity distributions are shown with a top quark mass $m_t = 173$ GeV and the scale choice $\mu = m_t$. The NNLO soft gluon corrections enhance the distribution without a significant change in shape relative to NLO. Our results are also in excellent agreement with CMS data [30] as shown in [31].

We also note that results for $p_T$ distributions and for rapidity distributions and the forward-backward
Table 1. Results for single-top, single-antitop, and combined approximate NNLO cross sections at 7 TeV LHC energy with $m_t = 173$ GeV in pb

| Channel | $\sigma(t)$, pb | $\sigma(\bar{t})$, pb | $\sigma(t) + \sigma(\bar{t})$, pb |
|---------|----------------|----------------------|-----------------------------|
| 7 TeV   | 43.0$^{+1.6}_{-0.2}$ ± 0.8 | 22.9 ± 0.5$^{+0.7}_{-0.9}$ | 65.9$^{+2.1+1.5}_{-0.7-1.7}$ |
| $t$-channel | 3.14 ± 0.06$^{+0.12}_{-0.10}$ | 1.42 ± 0.01$^{+0.06}_{-0.07}$ | 4.56 ± 0.07$^{+0.18}_{-0.17}$ |
| $s$-channel | 7.8 ± 0.2$^{+0.5}_{-0.6}$ | 7.8 ± 0.2$^{+0.5}_{-0.6}$ | 15.6 ± 0.4 ± 1.1 |

Table 2. Results for single-top, single-antitop, and combined approximate NNLO cross sections at 8 TeV LHC energy with $m_t = 173$ GeV in pb

| Channel | $\sigma(t)$, pb | $\sigma(\bar{t})$, pb | $\sigma(t) + \sigma(\bar{t})$, pb |
|---------|----------------|----------------------|-----------------------------|
| 8 TeV   | 56.4$^{+2.1}_{-0.3}$ ± 1.1 | 30.7 ± 0.7$^{+0.9}_{-1.1}$ | 87.2$^{+2.8+2.0}_{-1.0-2.2}$ |
| $t$-channel | 3.79 ± 0.07 ± 0.13 | 1.76 ± 0.01 ± 0.08 | 5.55 ± 0.08 ± 0.21 |
| $s$-channel | 11.1 ± 0.3 ± 0.7 | 11.1 ± 0.3 ± 0.7 | 22.2 ± 0.6 ± 1.4 |
In Fig. 6 we show new results for the top quark transverse momentum distribution in $t$-channel single top production at the LHC and the Tevatron (left) and the antitop quark transverse momentum distribution in $t$-channel production at the LHC (right).

**5. CONCLUSIONS**

The threshold resummation of soft-gluon contributions provides a powerful method to accurately calculate top quark total cross sections and differential distributions for both top-pair and single-top production. Resummation is performed at NNLL accuracy for the double-differential cross section and used to provide approximate NNLO results for total and differential cross sections. We have shown the applicability, validity, and accuracy of the approach by explicit comparisons to complete NLO and partial NNLO calculations. The comparisons show that currently available partial exact NNLO corrections are virtually indistinguishable from the NNLO approximations in our formalism (which is distinct from other resummation formalisms) both in the central result and in the theoretical uncertainty, and they thus indicate that future complete NNLO corrections will very likely have a negligible impact on the existing results from our formalism. New state-of-the-art results for top-pair and single-top total cross sections and transverse momentum and rapidity distributions are presented at LHC and Tevatron energies, with particular attention to
the 8 TeV LHC energy. All theoretical results, for both 
\( t\bar{t} \) and single-top production, and for both total cross sections and differential distributions, are in excellent agreement with recent LHC and Tevatron data.

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