The top quark polarization beyond the Standard Model in electron-positron annihilation

Electron-positron annihilation into a pair of top quarks is considered at the energy of future collider CLIC. Polarization components of the top quark are calculated with the $\gamma t\bar{t}$ and $Zt\bar{t}$ interactions which follow from Lagrangian of effective field theory of the Standard Model. The polarization vector as a function of the scattering angle is calculated and averaged components of polarization are analyzed as functions of anomalous coupling constants. Extrema of these observables (maxima, minima and saddle points) are studied as functions of the $e^+e^-$ energy.

**Key words:** Top quark, effective field theory, quark polarization, anomalous couplings

1. Introduction

The top quark was discovered in 1994 at the Tevatron. Due to its enormous mass $m_t = 173.0 \pm 0.4$ GeV (direct measurements) the top quark is a short-lived particle so it can be detected by studying the products of the decay $t \rightarrow W^+b \rightarrow \ell^+\nu_\ell b$. The top quark is important in various theories of “new physics” which is not described by the Standard Model (SM). The useful and detailed information about particle properties can be obtained from polarization experiments. In $pp$ collisions such measurements have been performed at the LHC at $\sqrt{s} = 7$ TeV [2,3] and $\sqrt{s} = 8$ TeV [4].

The top quark precision measurements at the $e^+e^-$ colliders are expected to be cleaner than those at the $pp$ colliders because interaction of electrons with the photon and $Z$-boson is well understood. Future experiments at the $e^+e^-$ colliders are supposed to be highly sensitive to physics beyond the Standard Model (BSM) [5,6]. And the important direction in the progress of studying physics of the top quark is polarization measurements at the electron-positron colliders.

The present paper is oriented on future TeV scale linear accelerator – Compact Linear Collider (CLIC). It is expected that CLIC will improve measurements and limits on some physical parameters which have been obtained so far at the LHC and the Tevatron (see, e.g., [7,8]). CLIC is foreseen to operate with center-of-mass energies $\sqrt{s} = 380$ GeV, 1.5 TeV, and 3 TeV and with corresponding total luminosities 500 fb$^{-1}$, 1.5 ab$^{-1}$, and 3 ab$^{-1}$, respectively [9].

In this work we study polarization of the top quark produced in the electron-positron annihilation, $e^+e^- \rightarrow t\bar{t}$. We calculate the top quark polarization components in framework of effective field theory (EFT) Lagrangian of the SM which satisfies $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry. This
Lagrangian contains the usual renormalizable SM Lagrangian $\mathcal{L}_{SM}^{(4)}$ and terms of dimension $d \geq 5$ which account for physics BSM. Terms of dimension six generate anomalous contributions to the photon-quark and Z-boson-quark interaction vertices which are similar in structure to radiative corrections in the SM.

In Section 2 we consider the Lagrangian with BSM coupling constants $\kappa$ and $\kappa_z$ which are directly related to the anomalous magnetic dipole moment (AMM) and anomalous weak-magnetic dipole moment (AWMM), respectively. These couplings are expressed through the complex Wilson coefficients with values constrained by the up-to-date fit in Ref. 10 to experimental data from the Tevatron and the LHC Run 1 and Run 2. The authors 10 analyzed cross sections at energies up to 13 TeV for the single and pair top production, and presented bounds on Wilson coefficients for the dimension-6 operators. Slightly different bounds have been suggested in Refs. 11, 12 which used the electroweak data and $B$-meson decays. In our work the Wilson coefficients from 11, 12 are used for obtaining bounds on couplings $\kappa$ and $\kappa_z$.

In Section 3 the polarization of the top quark in the reaction $e^+ e^- \to t \bar{t}$ at the center-of-mass energy $\sqrt{s} = 380$ GeV is calculated. Dependencies of the quark polarization on the anomalous couplings $\kappa$ and $\kappa_z$ are analyzed. The cross section for unpolarized quarks and the averaged over scattering angle components of polarization are studied as functions of couplings $\kappa$ and $\kappa_z$. The extreme points (maxima, minima and saddle points) of the corresponding two-dimensional surfaces are found and analyzed. In Section 4 we present conclusions, and Appendices A and B contain some details of formalism.

2. Lagrangian of $\gamma f \bar{f}$ and $Z f \bar{f}$ interaction

We consider the process of electron-positron annihilation into a pair of quarks

$$e^-(k) + e^+(k') \to (\gamma^*, Z^*) \to t(p) + \bar{t}(p'),$$

where $e^-(e^+)$ denotes an electron (positron) and $t(\bar{t})$ describes top quark (antiquark) with the mass $m_t$. Here $p (p')$ is the top quark (antiquark) 4-momentum and $k (k')$ is the electron (positron) 4-momentum.

We assume that the Lagrangian describing interaction of the photon and Z-boson with the quark includes the SM contribution and terms BSM which are related to new physics. For the photon the Lagrangian is

$$\mathcal{L}_{ff\gamma} = e f \left( Q f \gamma^\mu A_\mu + \frac{1}{4 m_f} \sigma^{\mu\nu} F_{\mu\nu}(\kappa + i \kappa_5) \right) f,$$

(2)

and for the Z-boson the Lagrangian reads

$$\mathcal{L}_{f\bar{f}Z} = \frac{g}{2 \cos \theta_w} f \left( \gamma^\mu Z_\mu (v_f - a_f \gamma_5) \right.$$

$$+ \frac{1}{4 m_f} \sigma^{\mu\nu} Z_{\mu\nu}(\kappa_z + i \kappa_5) \left.) f. \right(3)$$

Here $f = (\ell, q)$ stands for the fermion field, $A^\mu$ and $Z^\mu$ are fields of the photon and Z-boson,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu,$$

(4)

and $g = e/\sin \theta_w$ with $\theta_w$ denoting the weak mixing angle.

In these equations $Q_f$ is electric charge of the fermion in units of the positron charge $e = \sqrt{4\pi \alpha_{em}}$, $\alpha_{em}$ is the fine structure constant, $v_f, a_f$ are the vector- and axial-vector couplings

$$v_f = T^f_3 - 2 Q_f \sin^2 \theta_w, a_f = T^f_3,$$

(5)

with $T^f_3$ being the 3d component of the weak isospin. In particular, $v_e = -\frac{1}{2} + 2 \sin^2 \theta_w$, $a_e = -\frac{1}{2}$ for an electron, and $v_\ell = \frac{1}{2} - \frac{1}{2} \sin^2 \theta_w$, $a_\ell = \frac{1}{2}$ for a top quark.

The terms in Eqs. (2) and (3) proportional to $\kappa$ and $\kappa_z$ determine the CP even interaction, while those proportional to $\tilde{\kappa}$ and $\tilde{\kappa}_z$ determine the CP odd interaction.

The BSM terms in Eqs. (2) and (3) for $f = q$ arise in framework of EFT. These terms in the EFT Lagrangian appear after integration over heavy degrees of freedom connected with new physics. The EFT Lagrangian has the structure 13, 14

$$\mathcal{L}_{eff} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_i C_i^{(5)} O_i^{(5)} + \frac{1}{\Lambda^2} \sum_i C_i^{(6)} O_i^{(6)} + \ldots,$$

(6)

where $\Lambda$ is the scale of new physics, gauge-invariant operators $O_i^{(d)}$ for $d \geq 5$ are expressed through the fields of the SM, and $C_i^{(d)}$ are Wilson coefficients.
where index ‘3’ means the 3rd quark generation, \( \bar{\phi} \equiv i \tau_2 \phi^* \), \( \phi, \bar{\phi} \) are the SM Higgs doublet, and strength tensors of the gauge fields are defined as

\[
B_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \\
W^a_{\mu \nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g^{abc} W^b_\mu W^c_\nu,
\]

where \( a, b, c = 1, 2, 3 \).

Eqs. (11), after replacing \( \langle \bar{\phi} \rangle \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \), give rise to the couplings \( \kappa, \kappa \) and \( \kappa_z, \kappa_{\bar{z}} \) in Eqs. (2) and (3).

For the electromagnetic interaction (2) one finds from (7) the \( \gamma t \bar{t} \) couplings

\[
\kappa = \sqrt{2} m_t \frac{1}{m_z} c_w s_w \text{Re} (s_w \bar{C}^{33}_{uW} + c_w \bar{C}^{33}_{uB\phi}), \\
\tilde{\kappa} = \sqrt{2} m_t \frac{1}{m_z} c_w s_w \text{Im} (s_w \bar{C}^{33}_{uW} + c_w \bar{C}^{33}_{uB\phi}),
\]

which are related to the anomalous magnetic and electric dipole moments of the top quark, respectively. Hereafter \( c_w \equiv \cos \theta_w, \ s_w \equiv \sin \theta_w, \ m_z \) is the mass of the Z boson and we defined

\[
\bar{C}^{33}_{uW} = \frac{v^2}{\Lambda^2} C^{33}_{uW}, \\
\bar{C}^{33}_{uB\phi} = \frac{v^2}{\Lambda^2} C^{33}_{uB\phi},
\]

where \( v = 246 \) GeV is the vacuum value of the scalar Higgs field and the scale parameter \( \Lambda \) is taken 1 TeV.

For the Z-boson interaction (3) one finds from (7) the relations for the \( Z t \bar{t} \) couplings

\[
\kappa_z = 2 \sqrt{2} m_t \frac{1}{m_z} c_w \text{Re} (s_w \bar{C}^{33}_{uW} - c_w \bar{C}^{33}_{uB\phi}), \\
\tilde{\kappa}_z = 2 \sqrt{2} m_t \frac{1}{m_z} c_w \text{Im} (C_u \bar{C}^{33}_{uW} - c_w \bar{C}^{33}_{uB\phi}),
\]

which are related to the anomalous weak-magnetic and weak-electric dipole moments, respectively.

All the information on the anomalous \( \gamma t \bar{t} \) and \( Z t \bar{t} \) interactions is contained in two complex coefficients \( \bar{C}^{33}_{uW} \) and \( \bar{C}^{33}_{uB\phi} \). The bound on the coefficient \( \bar{C}^{33}_{uB\phi} \) comes from experiments on the decay of \( B \) mesons, \( B \to X_s \gamma \), in particular, on the measured branching ratio and \( CP \) asymmetry. The coefficient \( \bar{C}^{33}_{uW} \) is constrained by the LHC data at 7 and 8 TeV on the single top-quark production and \( W \) polarization in the top-quark decay. According to (12), the present bounds on the real values of the coefficients, in our definition (14), are

\[
\bar{C}^{33}_{uW} = (-0.661, 0.030), \\
\bar{C}^{33}_{uB\phi} = (-0.363, 0.054).
\]

The imaginary part of the coefficients is not constrained from the data and is chosen zero in our calculations. Correspondingly the couplings \( \kappa \) and \( \kappa_z \) are absent.

Substituting the bounds in (12) in Eqs. (9), (11) one obtains the bounds on the coupling constants of the BSM physics:

\[
\kappa = (-2.21, 0.40), \\
\kappa_z = (-0.43, 1.08).
\]

In the following sections the values (13) will be used.

In the considered process \( e^+ e^- \to t \bar{t} \) the invariant energy squared \( s \geq 4m_t^2 \), and the threshold is far from the photon pole \( (s = 0) \) and Z-boson pole \( (s = m_Z^2) \). Therefore it is appropriate to introduce the \( q^2 \) dependence of the couplings \( \kappa \) and \( \kappa_z \) as follows

\[
\kappa(s) = Q_t F_2^\gamma(s), \\
\kappa_z(s) = v_t F_2^Z(s),
\]

where \( F_2^\gamma, F_2^Z(s) \) are some form-factors.

The coupling \( \kappa(0) \) is directly related to the anomalous part of the quark magnetic moment \( \mu_t \), via the relations (13)

\[
\kappa(0) = Q_t F_2^\gamma(0) = Q_t a_t, \\
\mu_t = \frac{e Q_t}{2m_t} (1 + a_t).
\]

In the SM, for a point-like Dirac particle without radiative corrections, \( a_t \) and \( \kappa(0) \) are equal to zero. Modification arises due to radiative corrections and can also stem from new physics. The radiative corrections to \( a_t \) have been evaluated to one loop in the EW theory (19, 21) and two loops in QCD (22, 24). Here we cite the results of Ref. (25) for radiative corrections to couplings \( \kappa(0) \) and \( \kappa_z(0) \) which are evaluated to the two loops in QCD and to the lowest order
in electroweak couplings
\[ \kappa(0)_{rc} = Q_t F^2_\gamma(0)_{rc} = 2.0 \times 10^{-2}, \]
\[ \kappa_z(0)_{rc} = v_t F^2_{Z\gamma}(0)_{rc} = 5.75 \times 10^{-3}, \] (16)
where
\[ F^2(0)_{rc} = F^2_Z(0)_{rc} = F_2(0)_{rc} = 3.0 \times 10^{-2}. \] (17)
Here ‘rc’ is abbreviation for ‘radiative corrections’ and \( F_2(s)_{rc} \) is the form-factor defined in \([26]\). The values in (16) are calculated \([25]\) with the strong coupling \( \alpha_s(\mu) = 0.108 \) at the renormalization scale \( \mu = m_t \).
The QED correction to \( \kappa(0) \) in the leading order, \( \kappa(0)_{rc,\text{QED}} = Q_t^2 \alpha_{em}/(2\pi) \), turns out to be of the same order as the three loop correction in QCD and is much less than the value in Eqs. (16).

The bounds in Eqs. (13) are much wider than the values in (16). It will be important to obtain the more precise constraints on \( \kappa(s) \) and \( \kappa_z(s) \) from high precision measurements on \( e^+e^- \) colliders.

3. Top quark polarization and discussion of results
In the “tree level” approximation this process is described by two diagrams with the photon and Z-boson exchanges in the s-channel (see Fig. 1).

![Fig. 1. Tree-level diagrams for the process \( e^- e^+ \to (\gamma^*, Z^*) \to t\bar{t} \).](image)

Since outgoing quarks are polarized we use the covariant fermion density matrix:
\[
u(p, m) \bar{u}(p, m) = \frac{1}{2} (\not{\!p} + m)(1 + \gamma^5 \not{\!a}),
\]
\[
u(p, m) \bar{v}(p, m) = \frac{1}{2} (\not{\!p} - m)(1 + \gamma^5 \not{\!a}),
\] (18)
where \( u, \bar{u} \) are the Dirac spinors for particle, \( v, \bar{v} \) – for anti-particle. In Eqs. (18) \( \not{\!a} \equiv \gamma_\mu a^\mu \), where \( a^\mu \) is the 4-vector of fermion polarization. In the rest frame of fermion, \( p = (m_f, 0) \), this vector can be expressed through the 3-vector \( \xi \) of the fermion double average spin
\[ a = \left( 0, \vec{\xi} \right), \quad a \cdot p = 0. \] (19)
Similarly the polarization vector of the antiquark, \( a' \), can be defined.

Further the coordinate system is chosen in the same way as in Ref. \([21]\) and is shown in Fig. 2. The quarks momenta are directed along the OZ axis and the lepton momenta lie in the XOZ plane, \( \theta \) is the angle between the electron and the quark momenta.

![Fig. 2. Coordinate system.](image)

In the center-of-mass system the 4-momenta of electron and positron are
\[ k = (E, E \sin \theta, 0, E \cos \theta), \]
\[ k' = (E, -E \sin \theta, 0, -E \cos \theta), \] (20)
and the momenta and polarization vectors of quarks and antiquarks are
\[ p = (E, 0, 0, V E), \]
\[ p' = (E, 0, 0, -V E), \]
\[ a = (\gamma V \xi_x, \xi_x, \xi_y, \gamma \xi_z), \]
\[ a' = (-\gamma V \xi'_x, \xi'_x, \xi'_y, \gamma \xi'_z), \] (21)
where \( E = \sqrt{s}/2 \) is the energy of the lepton or quark, \( V \) – velocity of the top quark, \( \gamma = E/m_t = 1/\sqrt{1-V^2} \) is the Lorentz factor. Eqs. (21) are obtained by using Lorentz transformation from the rest frame of the quarks.

For polarized quarks the cross section is a function of the polarization components \( \xi_i, \xi'_j \) \( (i, j = (x, y, z)) \) and has the form
\[ \frac{d\sigma}{d\Omega}(\theta, \vec{\xi}, \vec{\xi'}) = \frac{3 \alpha_{em}^2(s) V}{16 E^2} N \left( 1 + P_i \xi_i + P'_j \xi'_j + Q_{ij} \xi'_i \xi'_j \right), \] (22)
where \( N \) is related to the cross section for unpolarized quarks as
\[ \frac{3 \alpha_{em}^2(s) V}{16 E^2} N = \frac{1}{4} \frac{d\sigma_0}{d\Omega}(\theta). \] (23)
The factor 3 accounts for the quark colors, $d\Omega$ is the solid angle for the top quark, $\alpha_{em}(s)$ is the energy-dependent fine structure constant, $\vec{P}$ and $\vec{P'}$ are the quark and anti-quark polarizations, respectively, which arise in the process of the quark production.

Here we do not consider correlations of polarizations described by the term $\sim Q_{ij}$, and neglect the last term in Eq. (22). Therefore

$$\frac{d\sigma}{d\Omega}(\theta, \xi, \xi') = \frac{1}{4} \frac{d\sigma_0}{d\Omega}(\theta) \left( 1 + P_t \xi_t + P'_t \xi'_t \right).$$

(24)

The components of the top-quark polarization $P_i(\kappa, \kappa_z)$ are shown on Fig. 3. Calculations are performed for the invariant energy $\sqrt{s} = 380$ GeV and are obtained using the bounds (13) for the couplings $\kappa$ and $\kappa_z$. Besides we use numerical values of physical constants in Table 1.

Table 1. Numerical values of the physical constants

| Constant  | Value       | Constant  | Value       |
|-----------|-------------|-----------|-------------|
| $m_t$     | 173.0 GeV   | $a_e$     | -0.50123    |
| $m_z$     | 91.1876(21) GeV | $a_t$     | 0.5         |
| $\Gamma_z$ | 2.4952(23) GeV  | $v_e$     | -0.03783    |
| $\sin^2\theta_w$ | 0.23155(4)     | $v_t$     | 0.191       |

The solid curves show calculations in the SM, in which the coupling constants in $[9]$ and $[11]$ are equal to zero. The other curves show calculations with the BSM coupling constants $\kappa$ (the plots on the left) and $\kappa_z$ (the plots on the right) which are constrained in Eqs. (13).

Firstly, we note that the component $P_y$ of the polarization vector, which is perpendicular to the reaction plane, is very small in the SM and in the considered extension of the SM. This component is generated at tree level from the $\gamma - Z$ interference and is proportional to the imaginary part of the $Z$-boson propagator. $P_y$ is smaller by several orders of magnitude than the other components of the polarization and takes values of the order $10^{-4} - 10^{-3}$.

Secondly, the transverse to the quark momentum component, $P_x$, is sensitive to the coupling $\kappa$, and especially to the coupling $\kappa_z$ (see upper plots in Fig. 3). Varying the coupling $\kappa_z$ can change considerably the polarization $P_x$.

Thirdly, the longitudinal component of the polarization vector $P_z$ is sizable in the SM, and it is also sensitive to the BSM couplings. It is interesting to note that the polarization $P_z$ reaches value +1 for $\kappa_z = 0.566$ and $\kappa = 0$ at the forward angle, correspondingly the transverse polarization vanishes at this angle.

Based on Fig. 3 one can see that the terms BSM, which are included in the effective Lagrangian (1), make quite a large contribution to the polarization of the top quark.

In order to demonstrate the simultaneous effect of both couplings $\kappa$ and $\kappa_z$ on the polarization components we perform averaging over the scattering angle in the following way

$$\left\langle P_i \right\rangle = \frac{\left\langle N P_i \right\rangle}{\left\langle N \right\rangle},$$

(25)

for $i = (x, y, z)$. Here we define the average value of any quantity dependent on the scattering angle as

$$\left\langle A \right\rangle = \frac{1}{2} \int_0^\pi A(\theta) \sin \theta \, d\theta.$$

(26)

The polarization components averaged in this way for the energy $\sqrt{s} = 380$ GeV are shown on Fig. 4. As can be seen from Fig. 4, the surfaces $\left\langle P_i \right\rangle$ as functions of $\kappa$ and $\kappa_z$ have extreme points: maxima, minima and saddle points. The averaged transverse component $\left\langle P_y \right\rangle$ has a saddle point at the values $\kappa = -0.609$, $\kappa_z = -0.177$. The perpendicular to the scattering plane component $\left\langle P_y \right\rangle$ and the longitudinal component $\left\langle P_z \right\rangle$ have the sharp minimum and also maximum. The component $\left\langle P_y \right\rangle$ gives little information being extremely small and not significant from experimental point of view. The minimal value of $\left\langle P_z \right\rangle$ is $-0.7843$ which appears at $\kappa = -0.6237$ and $\kappa_z = -0.014$, while the maximum value, 0.6523, appears at $\kappa = -0.623$ and $\kappa_z = -0.377$.

Let us analyze the maximum of $\left\langle N \right\rangle^{-1}$ proportional to the inverse cross section for the unpolarized quarks, $\left\langle N \right\rangle^{-1} = 12 \alpha_{em}(s)^2 \pi V s^{-1} \sigma_0^{-1}$. The surface $\left\langle N \right\rangle^{-1}$ has a distinct maximum at $\kappa = -0.624$, $\kappa_z = -0.179$ with the value $\left\langle N \right\rangle^{-1}_{max} = 102.32$. If we calculate the ratio $\frac{\kappa_z(\kappa)}{\kappa_z(\kappa)} = 3.48$, then it is seen that this ratio is the same as the ratio of the couplings $\kappa$ and $\kappa_z$ in the SM with radiative corrections (see Eqs. 14)

$$\frac{\kappa_z(\kappa)}{\kappa_z(\kappa)} = \frac{Q_t}{V_t} \approx 3.48.$$

(27)

Note that the ratio (27) is valid for any upper quark $u, c, t$. For the lower quarks $d, s, b$ the corresponding
Fig. 3. Components of the top quark polarization as functions of the scattering angle $\theta$ for a center-of-mass energy $\sqrt{s} = 380$ GeV. Left panel shows polarization components for several values of the BSM coupling $\kappa$ while taking $\kappa_z = 0$, and the right panel shows polarization components for several values of the BSM coupling $\kappa_z$ while taking $\kappa = 0$.

The location of the maximum of the surface $\langle N \rangle^{-1}$ changes with changing the energy $\sqrt{s}$. We can make an assumption, albeit without proof, that the true ratio is very different, $Q_{v_q}/Q_{\bar{u}_q} = 0.96$, due to the different electric charges and weak isospin values.
values of the anomalous couplings $\kappa(s)$ and $\kappa_z(s)$ correspond to the position of the maximum of $\langle N \rangle^{-1}$, or minimum of the cross section for unpolarized quarks. Then this allows one to find variation of form-factors in Eq. (28) with the energy. The photon and Z-boson form-factors are equal to each other and have the form

$$F_2(s) = - \left( 1 + \frac{s - 4m_t^2}{12m_t^2} \right)^{-1}.$$  \hfill (28)

This dependence is shown in Fig. 4. As can be seen from Eq. (28), the absolute value of the form-factor is equal to 1 at the reaction threshold $s = 4m_t^2$, and with increasing invariant energy the form-factor decreases.

4. Conclusions

The process of electron-positron annihilation into a pair of top quarks is considered in the conditions of future electron-positron colliders. We chose the invariant energy $\sqrt{s} = 380$ GeV which corresponds to the planned conditions of the first run of CLIC. The components of the top-quark polarization are calculated in the SM and BSM in framework of effective field theory in which additional $\gamma t\bar{t}$ and $Zt\bar{t}$ couplings are generated from the dimension-6 operators. For the values of couplings $\kappa$ and $\kappa_z$, describing new physics, we use the existing experimental bounds from Refs. [11, 12] which are obtained from analysis of data on $B$-meson decays $B \to X_s \gamma$ and experiments at the LHC and the Tevatron on the top-quark production and decays.

The dependence of the quark polarization components on the scattering angle is calculated for various values of $\kappa$ and $\kappa_z$. Our calculation shows that the sensitivity of the single-spin observables to the BSM couplings is sizable.
In Eqs. (A4), (A5) and BSM where \( q \) and BSM are studied as functions of \( \kappa \) and \( \kappa_z \). The extreme points (maxima, minima and saddle points) of the corresponding two-dimensional surfaces are found and analyzed. Under an assumption that the true values of the couplings \( \kappa \) and \( \kappa_z \) may correspond to position of the minimum of the cross section for unpolarized top quarks, we obtained the energy dependence of the form-factors in the \( \gamma t\bar{t} \) and \( Zt\bar{t} \) vertices.

In real experiments the top-quark polarizations can be determined from measurement of angular distributions of the decay products of the top quark \( (t \rightarrow W^+ b \rightarrow \ell^+ \nu \ell b) \). We are planning to address this aspect in future and also to extend calculations to the case of polarized electron and positron beams.

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### Appendix A: Matrix element of \( e^+e^- \rightarrow t\bar{t} \) in the tree-level approximation

The matrix element can be written as a sum of the SM and BSM terms

\[
M = M^{SM} + M^{BSM}.
\]  

(A1)

For the process with the photon exchange the matrix elements are in the SM

\[
i M^{SM}_\gamma = -ie^2 Q_t \frac{g_{\mu\nu}}{q^2} \bar{u}(p)\gamma^\mu v(p') \bar{v}(k')\gamma^\nu u(k),
\]

(A2)

and BSM

\[
i M^{BSM}_\gamma = e^2 \frac{g_{\mu\nu}}{q^2} \bar{u}(p) (\kappa \frac{\sigma^\mu\lambda}{2m_t} q_\lambda) v(p') \bar{v}(k')\gamma^\nu u(k),
\]

(A3)

where \( q = k + k' = p + p' \), \( \sigma^\mu\nu = \frac{i}{2} (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) \), \( g_{\mu\nu} \) is metric tensor with the standard signature \((+,-,-,-)\), \( u(k) \), \( v(k') \) are the Dirac spinors for electron and positron, and \( u(p) \), \( v(p') \) are the spinors for the quark and antiquark. The invariant energy squared is \( q^2 \equiv s = 4E^2 \).

For the process with the Z-boson exchange matrix elements are in the SM

\[
i M^{SM}_Z = i \frac{g^2}{4c_w} \frac{(g_{\mu\nu} - \frac{2g_{\mu\nu}}{m_Z^2})}{q^2 - m_Z^2 + i m_Z \Gamma_Z} \bar{u}(p)\gamma^\mu (v_{\ell} - a_5 \gamma^5)v(p') \bar{v}(k')\gamma^\nu (v_e - a_5 \gamma^5) u(k),
\]

(A4)

and BSM

\[
i M^{BSM}_Z = -\frac{g^2}{4c_w} \frac{(g_{\mu\nu} - \frac{2g_{\mu\nu}}{m_Z^2})}{q^2 - m_Z^2 + i m_Z \Gamma_Z} \bar{u}(p) (\kappa_z \frac{\sigma^\mu\lambda}{2m_t} q_\lambda) v(p') \bar{v}(k')\gamma^\nu (v_e - a_5 \gamma^5) u(k).
\]

(A5)

In Eqs. (A4), (A5) \( m_z \) and \( \Gamma_z \) are the mass and the width of the Z boson, respectively.
Appendix B: Averaging the polarization components over the scattering angle

In this Appendix we present the expressions for the averaged components of the top quark polarization in Eq. (25). These components are shown in Fig. 4. The obtained expressions are functions of the coupling constants $\kappa$ and $\kappa_z$.

The dimensionless constant $\langle N \rangle$, independent of the top quark polarization, is

$$
\langle N \rangle = \frac{1}{48E^2m_t^2c_w^4s_w^4(\Gamma_2^2m_z^2 + (m_z^2 - 4E^2)^2)}
\left[ c_w^2s_w^2(\Gamma_2^2m_z^2 + (m_z^2 - 4E^2)^2)
\times \left( 3m_t^4(\kappa + 2Q_t)^2 + m_t^2E^2(4Q_t^2(V^2 + 3) + 12\kappa Q_t(V^2 + 3) + 3\kappa^2(V^2 + 6)) + \kappa^2(5V^2 + 3)E^4 \right)
+ E^6 \left( a_z^2 + v_z^2 \right) \left( 8m_t^2(2V^2a_t^2 + v_t(6\kappa - (V^2 - 3)v_t)) + \kappa^2 \left( 21m_z^2 + (5V^2 + 3)E^2 \right) \right)
- 2E^4v_t c_w^2 s_w^2 \left( 4E^2 - m_z^2 \right) \left( 8m_t^2 v_t(3\kappa - Q_t(V^2 - 3)) + \kappa^2 \left( 3m_t^2(7\kappa + 8Q_t) + \kappa(5V^2 + 3)E^2 \right) \right) \right].
$$

(B1)

The average components of the top quark polarization vector read:

$$
\langle P_x \rangle = -\frac{1}{\langle N \rangle} \frac{\pi E a_t}{8m_t c_w^2 s_w^2(\Gamma_2^2m_z^2 + (m_z^2 - 4E^2)^2)}
\left[ c_w^2s_w^2(m_z^2 - 4E^2) \left( v_t(m_t^2(\kappa + 2Q_t) + \kappa E^2) + \kappa_z(m_t^2 Q_t + E^2(2\kappa + Q_t)) \right) + 2E^2 v_t v(E^2(m_t^2 + \kappa_z E^2)) \right],
$$

(B2)

$$
\langle P_y \rangle = \frac{1}{\langle N \rangle} \frac{\pi V E T_z a_t a_r m_z}{8m_t c_w^2 s_w^2(\Gamma_2^2m_z^2 + (m_z^2 - 4E^2)^2)}
\right],
$$

(B3)

$$
\langle P_z \rangle = -\frac{1}{\langle N \rangle} \frac{2VE^2 a_t}{3c_w^4 s_w^4(\Gamma_2^2m_z^2 + (m_z^2 - 4E^2)^2)}
\left[ E^2 a_z^2(v_t + \kappa_z) + v_t \left( c_w^2 s_w^2(m_z^2 - 4E^2)(\kappa + Q_t) \right)
\left( E^2 v_t(v_t + \kappa_z) \right) \right].
$$

(B4)

As can be seen from the equations above, the factor $\langle N \rangle$ contains terms linear and quadratic in the coupling constants $\kappa$ and $\kappa_z$, and the components of the polarization are complicated functions of these constants.

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