STEADY ON MHD HEAT AND MASS TRANSFER FLOW OF AN INCLINED POROUS PLATE IN THE PRESENCE OF RADIATION AND CHEMICAL REACTION

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Abstract: In the present paper steady on MHD heat and mass transfer flow of a viscous incompressible fluid flow through a porous medium in the presence of radiation, heat source and chemical reaction effects are considered. The set of non linear differential equations can be solved analytically. Effects of various parameters like magnetic parameter, permeability parameter, Grashof number, modified Grashof number, radiation parameter, heat source parameter, Prandtl number, Schmidt number, chemical reaction and an angle of inclination on the velocity, temperature and concentration profiles as well as the skin friction, Nusselt number and Sherwood number are discussed qualitatively and shown graphically.

Keywords: MHD, heat and mass transfer, inclined plate, heat source and porous medium.

1. Introduction:
Many transport processes occurring both in nature and in industries involve fluid flows with the combined heat and mass transfer. Such flows are driven by the multiple buoyancy effects arising from the density variations caused by the variations in temperature as well as species concentrations. Natural convection processes involving the combined mechanisms are also encountered in many natural processes, such as drying, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, evaporation at the surface of a water body, energy transfer in a wet cooling tower and flow in a desert cooler, heat and mass transfer occur simultaneously. In the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Chemical reactions can be modeled as either homogeneous or heterogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. A homogeneous reaction is one that occurs uniformly throughout a given phase. The species generation in a homogeneous reaction is the same as internal source of heat generation. On the other hand, a heterogeneous reaction takes place in a restricted area or within the boundary of a phase. In bio-medical application, the drug permeation through human glands, flow of fluids in lungs, blood vessels, arteries and so on. Haldar and Shit [1] presented the thermal radiation and Hall Effect on MHD heat and mass transfer flow over an inclined permeable stretching sheet. Bhuvaneswari et al. [2] presented an exact analysis of radiation convective flow heat and mass transfer over an inclined plate in a porous medium. Raptis and Kafousias [3] studied the magnetohydrodynamic free convective flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Sudheer Babu and Satyanarayana [4] studied the effects of the chemical reaction and radiation absorption on free convective flow through porous medium with variable suction in the presence of uniform magnetic field. Chen [5] presented the analysis to study natural convection flow over a permeable inclined surface with variable wall replete with examples of heat transfer in the laminar flow temperature and
concentration. Barik and Dash [6] investigated unsteady magnetohydrodynamic flow past an inclined porous heated plate. Reddy et al. [7] investigated mass transfer and radiation effects of unsteady MHD free convective fluid flow embedded in a porous medium with heat generation/absorption. Patil and Kulkarni [8] studied the effects of chemical reaction on free convective flow of a polar fluid through porous medium in the presence of internal heat generation. Ganesan et al. [9] presented the study of natural convection flow of inclined plate. H. Kumar [10] considered the MHD heat and mass transfer flow over an isothermal inclined plate at constant concentration gradient. Reddy et al. [11] have discussed the effect of thermal radiation on an unsteady magnetohydrodynamic flow past an inclined porous heated plate in the presence of chemical reaction with heat and mass transfer by using a closed analytical form method. Shit and Haldar [12] studied MHD flow, heat and mass transfer over an inclined permeable stretching sheet with thermal radiation and Hall Effect. Tak et al. [13] investigated the interaction of radiation with free convection in Darcian porous media by taking into account the Soret and Dufour’s effects. Abdullah [14] found an analytical solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, dufour-soret effect and Hall Effect using homotopy analysis method. Kim [15] has analysed the unsteady MHD convection flow of polar fluids past a vertical moving porous plate in a porous medium. Muthucumaraswamy and Vijayalakshmi [16] investigated MHD and chemical reaction on flow past an impulsively started semi-infinite vertical plate with thermal radiation. Mamta Goyal, and Annu Banshiwal, [17] studied MHD heat and mass transfer flow over a semi infinite inclined plate at constant concentration gradient embedded in a porous medium with heat source.

In the present study MHD heat and mass transfer flow of a viscous incompressible fluid flow through a porous medium in the presence of radiation, heat source and chemical reaction effects are considered. Using a perturbation technique, the solution of the set of differential equations which are solved analytically.

2. Mathematical Formulation:
Consider a steady, two-dimensional laminar flow of an incompressible, viscous, electrically conducting fluid past a semi-infinite inclined porous plate embedded in a porous medium and subjected to a uniform magnetic field $B_0$ normal to the direction of flow. The physical configuration and coordinate system of problem is shown in Fig. 1. The x-axis is taken along the inclined plate and the y-axis normal to the plate.

The magnetic Reynolds number are assumed to be very small so that the induced magnetic field is considered to be negligible compared to the applied magnetic field. The governing equations of mass, momentum, energy and concentration for steady flow with Boussinesq’s approximation are as follows

\[
\frac{\partial u}{\partial y} = 0, \quad (1)
\]

\[
\nu \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + g B_T \cos \alpha (T - T_\infty) + g B_c \cos \alpha (c - c_\infty) - \frac{\sigma B_0^2 u}{\rho} - \frac{v}{k} u, \quad (2)
\]

\[
\nu \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p} (T - T_\infty), \quad (3)
\]

\[
\nu \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} - K_c (c - c_\infty), \quad (4)
\]

where $u$ and $v$ are corresponding velocity component along and perpendicular to the surface, $\nu$ is the kinematic viscosity, $g$ is the acceleration due to gravity, $B_T$ is the coefficient of volume expansion for the heat transfer and $B_c$ is the volumetric coefficient of concentration expansion, $k$ is the permeability of porous medium, $\alpha$ is the angle of inclination, $\sigma$ is the fluid electrical conductivity, $T$ is the fluid
temperature, \( T_\infty \) is the far field temperature, \( \propto \) is the thermal conductivity, \( Q \) heat source parameter, 
\( \rho \) is the density of the fluid, \( C_p \) is specific heat at constant pressure, \( c \) is the species concentration, \( c_\infty \) is the far field concentration, \( D \) is the chemical molecular diffusivity, with boundary conditions

\[
\begin{align*}
\frac{\partial u}{\partial y} &= 0, v = v_\infty, T = T_\infty, -D \frac{\partial c}{\partial y} = m_0 \text{ at } y = 0, \\
u \to 0, T \to T_\infty, c \to c_\infty \text{ as } y \to \infty.
\end{align*}
\]

(5)

Figure 1: Physical configuration of the problem

The equation of continuity (1) gives

\[ \nu = -v_\infty. \]

(6)

Introducing the following non-dimensional parameters in equations (2), (3) and (4)

\[
Y = \frac{u}{u_\infty}, U = \frac{u}{u_\infty}, \theta = \frac{T - T_\infty}{T_\infty - T_\infty}, \phi = \frac{C - C_\infty}{(m_0 \nu / \nu D)},
\]

(7)

From the above dimensionless variables, we have

\[
u = U u_\infty, T = \theta(T - T_\infty) + T_\infty
\]

and

\[ C = \phi(m_0 \nu / \nu D) + C_\infty. \]

Now we substitute the values of above derivatives in equations (2), (3) and (4). By simplifying these equations, we obtain the following ordinary differential equations in terms of dimensionless variables

\[
\frac{d^2 U}{dY^2} + \frac{dU}{dY} + Gr \theta + Gm C - (M + \frac{1}{K}) = 0,
\]

(8)

\[
\frac{d^2 \theta}{dY^2} + Pr \frac{d\theta}{dY} + Pr S \theta = 0,
\]

(9)
\[
\frac{d^2 \phi}{dY^2} + Sc \frac{d\phi}{dY} - Kr \phi = 0
\]  

(10)

where

Grashof number \( Gr = \frac{gB^2(T_w - T_\infty)\nu}{u_w^2 v_w^2}\), Modified Grashof number \( Gm = \frac{gB^2 m_w v^2}{u_w^3 v_w^2 D}\),

Magnetic field number \( M = \frac{B^2 v \sigma}{\nu^2 \rho}\), Permeability parameter \( K = \frac{\nu^2}{\nu^2 k}\),

Prandtl number \( Pr = \frac{\rho \nu C_p}{k} = \frac{v}{\alpha T}\), Heat source parameter \( S = \frac{Qv}{\rho C_p \nu^2 w}\), Schmidt number \( Sc = \frac{v}{D}\). 

The boundary condition (5) turns into

\[U = 0, \theta = 1, \frac{\partial C}{\partial Y} = -1, \text{ at } Y = 0,\]

\[U \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0, \text{ as } Y \rightarrow \infty\]

Solution of equations (8) to (10) with boundary condition (11) is as follows

\[U = A_3 e^{-m_3 y} + A_1 e^{-m_1 y} + A_2 e^{-m_2 y}, \theta = e^{-m_2 y}, C = \frac{1}{m_1} e^{-m_1 y}\]

3. Results and Discussion

In order to get the effects of magnetic parameter, permeability parameter, Grashof number, Modified Grashof number, angle of inclination, chemical reaction, Schmidt number, Prandtl number and heat source parameter on the velocity, temperature and concentration profiles are shown in Fig.2-14. For numerical calculation of \( M=1, K=0.5, Gr=5, Gm=2, \alpha = \pi /6, Kr=0.5, Sc=0.60, Pr = 0.71 \) and \( Q=0.5\). Fig.2 presents the velocity profiles for different values of the magnetic parameter. It is observed that the magnetic parameter increases, the velocity profile decreases this is because an increase in applied magnetic field strength causes greater interaction between the fluid motion and magnetic field and hence an increase in Lorentz force , since this force opposes the buoyancy force velocity will be decreased. 

Fig.3 illustrates the effect of the velocity profile for various values of the permeability parameter (K). The velocity increases with an increase in permeability parameter K. It can clearly seen that as permeability parameter increases, the velocity increases because when permeability increase then flow resistance decreases which leads to increase the velocity. 

Fig.4 depicts the velocity profiles for different values of Grashof number (Gr). It is observed that the Grashof number \( Gr \) increases the velocity field increases. This is because of the increasing the strength of the flow and the ratio of the thermal buoyancy force to the viscous hydrodynamic force. In Fig.5 exhibits the velocity profiles for different values of the modified Grashof number (Gm). It is noticed that an approximate the ratio of buoyancy to viscous force acting on fluid. As expected, it is observed that there is a rise in the velocity due to the enhancement of species buoyancy force.
Fig. 6 represents the velocity profiles for different values of angle of inclination (\(\alpha\)). It is observed that the \(\alpha\) increases, the velocity decreases, it is fact the inclination decreases automatically fluid velocity increases.

Figs 7 and 8 represent the velocity and concentration profiles for different values of chemical reaction parameter (Kr). It is noticed that the chemical reaction increases in both the cases of velocity and concentration decreases due to the fact that increase in chemical reaction parameter Kr gives rise to an increase in viscosity of fluid which means velocity boundary layer thickness decreases. Also due to increase in values of Kr, the concentration of fluid particles near the plate drops, which results in decreasing the effect of mass buoyancy forces and thus decrease the fluid velocity. Due to increase in Kr, the constituents from higher concentration zone (adherent to the plate) moves towards the species in lower concentration zone (free stream) results of which decreases the concentration boundary layer thickness, thus decreasing the values of concentration.

Figs. 9 and 10 shows that the effect of velocity and concentration profiles for different values of Schmidt number (Sc). It is observed that the Schmidt number increases, the velocity and concentration profiles both decreases because Schmidt number is a dimensionless number defined as the ratio of momentum diffusivity and mass diffusivity, and is used to characterize the fluid flows in which there are simultaneous momentum and mass diffusion convection processes.

Figs. 11 and 12 represents the velocity and temperature profiles for different values of Prandtl number (Pr). It is observed that the velocity and temperature experience reduction for increasing value of Prandtl number.

Fig.13 and Fig.14 depict the velocity and temperature profile for different values of the heat source parameter (S). It is noticed that an increase in the heat source parameter S the velocity increases but both trends are observed in the temperature profiles.
Fig.6. Velocity profiles for different values of angle of inclination ($\alpha$).

Fig.7. Velocity profiles for different values of chemical reaction parameter ($Kr$).

Fig.8. Concentration profiles for different values of chemical reaction parameter ($Kr$).

Fig.9. Velocity profiles for different values of Schmidt number ($Sc$).

Fig.10. Concentration profiles for different values of Schmidt number ($Sc$).

Fig.11. Velocity profiles for different values of Prandtl number ($Pr$).
Fig. 12. Temperature profiles for different values of Prandtl number (Pr).

Fig. 13. Velocity profiles for different values of heat source parameter (S).

Fig. 14. Temperature profiles for different values of heat source parameter (S).

Table I

| M     | K     | Gr   | Gm   | Pr   | S    | Sc   | Kr   | Sf   | Nu   | Sh   |
|-------|-------|------|------|------|------|------|------|------|------|------|
| 1.0000| 1.0000| 1.0000| 0.5000| 0.7100| 0.1000| 0.6000| 0.5000| -0.8204| 0.5896| 1.0000|
| 3.0000| 1.0000| 1.0000| 0.5000| 0.7100| 0.1000| 0.6000| 0.5000| -0.6282| 0.5896| 1.0000|
| 5.0000| 1.0000| 1.0000| 0.5000| 0.7100| 0.1000| 0.6000| 0.5000| -0.5311| 0.5896| 1.0000|
| 1.0000| 1.5000| 1.0000| 0.5000| 0.7100| 0.1000| 0.6000| 0.5000| -0.8758| 0.5896| 1.0000|
| 1.0000| 2.0000| 1.0000| 0.5000| 0.7100| 0.1000| 0.6000| 0.5000| -0.9085| 0.5896| 1.0000|
| 1.0000| 2.5000| 1.0000| 0.5000| 0.7100| 0.1000| 0.6000| 0.5000| -0.9303| 0.5896| 1.0000|
| 1.0000| 1.0000| 1.0000| 0.5000| 1.0000| 0.1000| 0.6000| 0.5000| -0.7809| 0.8873| 1.0000|
| 1.0000| 1.0000| 1.0000| 0.5000| 3.0000| 0.1000| 0.6000| 0.5000| 0.5245| 2.8964| 1.0000|
| 1.0000| 1.0000| 1.0000| 0.5000| 5.0000| 0.1000| 0.6000| 0.5000| 0.7630| 4.8979| 1.0000|
| 1.0000| 1.0000| 1.0000| 0.5000| 1.0000| 0.1000| 0.6000| 0.5000| -1.0008| 0.5896| 1.0000|
| 1.0000| 1.0000| 1.0000| 0.5000| 0.7100| 0.1000| 0.6000| 0.5000| -0.9539| 0.5896| 1.0000|
| 1.0000| 1.0000| 1.0000| 0.5000| 0.7100| 0.1000| 0.6000| 0.5000| -0.8204| 0.5896| 1.0000|
4. Conclusions:

- The velocity profile increases with the increase of thermal Grashof number (Gr), modified Grashof number (Gm) and permeability parameter (K) of the medium and decreases with the increase of magnetic field parameter (M), chemical reaction parameter (Kr), Schmidt number (Sc), heat source parameter (S), Prandtl number (Pr) and inclination angle ($\alpha$).
- The temperature of the fluid decreases with the increase of heat source parameter (S) and Prandtl number (Pr).
- The concentration profiles decreases with the increase of chemical reaction parameter (Kr) and Schmidt number (Sc).
- Skin friction increases with an increase in Schmidt number (Sc), Prandtl number (Pr) and magnetic field parameter (M) and it shows the reverse effect in case of chemical reaction parameter (Kr) and permeability parameter (K).
- The rate of heat transfer i.e. Nusselt number (Nu) increases with an increase of Prandtl number (Pr) and there is no effect on Shear wood number (Sh).

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