Primordial gravastar from inflation

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Abstract

The dS bubbles can nucleate spontaneously during inflation, and will be stretched by the cosmological expansion to astrophysical scale. We report on a novel phenomenon that such a bubble might develop into a gravastar (an ultra-compact object with a dS core) after inflation, which witnessed the occurrence of inflation and would survive till today. It is pointed out that if a primordial gravastar was involved in one of the LIGO/Virgo gravitational wave (GW) events, the post-merger object could be a gravastar that will eventually collapse into a black hole. As a result, the late-time GW ringdown waveform will exhibit a series of “echoes” with intervals increasing with time.

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The recent direct detections of gravitational waves (GWs) made by the LIGO Scientific and the Virgo Collaborations have opened a new window to probe the strong gravity physics. The detections are usually attributed to coalescences of binary black holes (BHs) [1] or neutron stars [2]. However, it is still significant to ask if they could be explained by coalescences of other ultra-compact objects [3]. Particularly, it has been showed in Refs.[4][5] that if the post-merger object is a horizonless exotic compact object, such as a gravitational vacuum star (GVS) called gravastar [6][7], the GW ringdown waveform will consist of the primary signal, which is almost identical to that of a ringing-down BH, and a series of “echoes”. As the echoes encode informations of new physics or quantum gravity [8], the corresponding signals have been searched in the GW data by many research groups [9][10][11][12].

Actually, like the primordial BHs [13][14][15][16][17], the GVSs might also be primordial, and could play distinct roles in cosmological evolution. It has been pointed out in Ref.[18] that the light primordial GVSs could be a perfect candidate for cold dark matter (CDM). Moreover, the interaction between the GVS DM with baryons can help to cool the baryons at the cosmic dawn, and so explain the EDGES’s recent observation [19][20]. However, if they exist, how can the primordial GVSs come into being? It is still an open question.

The spacetime of a GVS can be described by the Schwarzschild metric, except the Schwarzschild horizon and the region inside are replaced by a dS core [6]. Nonetheless, the physical origin of the dS core is unclear, see e.g.[3]. On the other hand, the dS bubbles can spontaneously nucleate during inflation [21], and be stretched by the cosmological expansion to astrophysical scales [22][23]. A natural idea would be that the precursor of a GVS might be just such a dS bubble. In this report, we will explore the possibility that a primordial GVS-like object developed from a dS bubble that nucleated during inflation. We will also discuss its implications for the LIGO/Virgo events.

Generally, inflation is driven by a scalar field slow rolling along its effective potential \( V_{\text{inf}} \). In a Landscape [24][25], nucleations of bubbles with low-energy \( (V_b < V_{\text{inf}}) \) vacua occur inevitably. During inflation, the bubble radius \( r_* \) will be stretched by the expansion of universe, so that after the inflation ends, \( r_* \) may be far larger than \( 1/H_{\text{inf}} \). Dependent on the nucleation rate \( \lambda \) per Hubble volume (= \( 1/H_{\text{inf}}^4 \)), the number density of bubble \( dn = \lambda dr_*/(r_* + H_{\text{inf}}^{-1})^4 \) can have a special distribution [22].

When the inflation ends, the inflaton energy \( 0 < \rho_{\text{inf}} \simeq V_{\text{inf}} \) outside bubbles will release into radiation. The bubble wall moves highly relativistic related to matter, but will become
at rest with respect to matter in a very short time\cite{23}. Once the bubble wall is at rest with respect to the Hubble flow, it will be surrounded by an infinitesimal layer devoid of matter with a negligible vacuum energy, see also \cite{26}. In this case, the metric is

\[ ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \]

(1)

with

\[ f(r) = \begin{cases} 
1 - \frac{2M}{r} & \text{for } r > r_*, \\
1 - H_b^2 r^2 & \text{for } r < r_*,
\end{cases} \]

(2)

which is Schwarzschild outside the bubble wall \((r > r_*)\), and is de Sitter inside the bubble \((r < r_*)\). We assumed the bubble wall is located at \(r = r_*\), \(H_b^2 = 8\pi V_b/3\) and \(G = 1\).

The equation of motion for the bubble wall \(r_*\) is given by \cite{27}

\[ \beta_D - \beta_S = H_\sigma r_*, \]

(3)

with

\[ |\beta_D| = \left(1 + \dot{r}_*^2 - H_b^2 r_*^2\right)^{1/2}, \quad |\beta_S| = \left(1 + \dot{r}_*^2 - \frac{2M}{r_*}\right)^{1/2}, \]

(4)

which is equivalent to

\[ \dot{r}_*^2 + V(r_*) = 0, \]

(5)

where

\[ V(r_*) = 1 - \left(1 - \frac{H_b^2}{H_\sigma^2}\right)\frac{M}{r_*^3} - \frac{M^2}{H_b^2 r_*^4} - \left(H_\sigma^2/4 + H_b^2/2 + \frac{H_b^4}{4H_b^2}\right) r_*^2, \]

(6)

\(H_\sigma = 4\pi\sigma\) with \(\sigma\) being the bubble wall tension, and \(\dot{r} = dr/d\tau\) with \(\tau\) being the proper time of the bubble wall. It has been showed in Ref.\cite{27} that if \(\sigma\) is constant, \(r_*\) will either decrease to 0 after reaching its maximal value, or simply grow without bound, see the insert in Fig. 1.

As a result, a bubble will either stop expanding and collapse to a Schwarzschild singularity, or it will develop into a baby universe that initially connects to the parent universe (the exterior of the bubble) through a wormhole and will detach as the wormhole pinches off eventually. In both cases, the primordial BHs will form \cite{23}, see also \cite{28} \cite{29}.

Generally, if the bubble wall consists of the fluids with a state equation \(w\), one will have \(\sigma = \sigma_0/r_*^{2(1+w)}\) with \(\sigma_0\) being a constant, see e.g.\cite{30} \cite{31} for relevant physics. The scalar field bubble wall corresponds to the fluids with \(w = -1\), and hence \(\sigma = \sigma_0\). In Ref.\cite{6}, the shin shell (or wall) with the stiff fluid \((w = 1)\) has been adopted for GVS. In the following,
we will consider such a scenario that after inflation the bubble wall will go through a rapid phase transition (PT) at $t_0$, and after the PT, the bubble wall will consist of the stiff fluid, so that the wall tension $\sigma = \sigma_0(r_0/\tilde{r}_*)^4$, where $r_0$ is the radius of bubble wall at $t \simeq t_0$. Defining $x = 4\pi\sigma_0\tilde{r}_*^4$, we have

$$\left(\frac{d\tilde{r}_*}{d\tau}\right)^2 + V_{\text{eff}}(\tilde{r}_*) = 0,$$

where

$$V_{\text{eff}}(\tilde{r}_*) = 1 - \left(1 - \tilde{H}_b^2\tilde{r}_*^8\right)\frac{\dot{\mathcal{M}}}{\tilde{r}_*} - \dot{\mathcal{M}}^2\tilde{r}_*^4 - \left(\frac{1}{4\tilde{r}_*^8} + \frac{\tilde{H}_b^2}{2} + \frac{\tilde{H}_b^2}{4}\tilde{r}_*^8\right)\tilde{r}_*^2$$

with the dimensionless parameters $\mathcal{M} = x\mathcal{M}$, $\tilde{r}_* = r_*x$ and $\tilde{H}_b = H_b/x$. Different from the case of constant $\sigma$, depending on the values of $\dot{\mathcal{M}}$ and $\tilde{H}_b^2$, we may have a metastable static solution for the bubble wall. Numerically solving Eqs. $V_{\text{eff}}(\tilde{r}_*) = 0$, $\partial V_{\text{eff}}/\partial \tilde{r}_* = 0$ and $\partial^2 V_{\text{eff}}/\partial \tilde{r}_*^2 = 0$, we find $\dot{\mathcal{M}} = 0.55$, $\tilde{H}_b = 0.4$ and $\tilde{r}_* \simeq 1.2$, which correspond to the static point ‘A’ shown in Fig.1. When the bubble wall expands and arrives at the ‘A’ point, it will be at rest for a long time. In this case, the asymptotic radius is

$$\frac{r_*}{\mathcal{M}} = \frac{\tilde{r}_*}{\mathcal{M}} \sim 2.18,$$

which implies the bubble is slightly larger than the Schwarzschild radius, and lies inside the light-ring (or photosphere) of the corresponding Schwarzschild mass ($r = 3\mathcal{M}$). Also, we have the bubble radius $r_* = (\tilde{r}_*\tilde{H}_b)/H_b \simeq 0.5/H_b < 1/H_b$. Therefore, a primordial GVS-like object comes into being. The existence of the light-ring indicates that the GVS looks like a BH to a distance observer, and belongs to the ultra-compact objects [32]. The
conformal diagram of the primordial GVS developing from a dS bubble is shown in Fig. 2. \( \beta_S = \tilde{M} \tilde{r}_*^2 - 1/(2 \tilde{r}_*^2) - \tilde{H}_b^2 \tilde{r}_*^5/2 > 0 \) suggests that the polar angle in Schwarzschild space coordinates is increasing [22].

![Conformal diagram of the spacetime outside the bubble.](image)

FIG. 2: The conformal diagram of the spacetime outside the bubble. The Schwarzschild metric has been matched on a matter-dominated FRW universe. The red line shows the trajectory of the bubble wall, which expands initially and will approach to a constant after the wall went through a PT. The blue line shows the location of the light-ring (the photosphere) of the corresponding Schwarzschild mass. Notice that the interior of the bubble (region on the left hand side of the red line) should be replaced by the dS spacetime.

The mass \( M_{PGVS} = \tilde{M}/(x) \) of the primordial GVS is

\[
M_{PGVS} = \frac{\tilde{M} \tilde{H}_b}{H_b} \sim \frac{M_P^2}{H_b} \simeq \left( \frac{1 \text{Gev}}{\Lambda_b} \right)^2 M_\odot,
\]

\( \text{Eq. (10)} \)

where \( H_b \simeq \sqrt{V_b/M_P} = \Lambda_b^2/M_P \) is used. This indicates that for \( \Lambda_b \sim 1 \text{Gev} \), we will have the primordial GVS with \( M_{PGVS} \sim 1 M_\odot \) (the radius \( r_{PGVS} \sim 10^3 m < 1/H_b \)). However, \( M_{PGVS} \) may also be estimated straightly by using Eq.(4),

\[
M = \left( \frac{H_b^2 - H_\sigma^2}{2} \right) r_*^3 + H_\sigma r_*^2 \sqrt{1 + \dot{r}_*^2 - H_b^2 \dot{r}_*^2}.
\]

\( \text{Eq. (11)} \)

When the bubble wall arrives at the ‘A’ point, \( \dot{r}_A = 0 \), so for \( H_\sigma \ll H_b \) and \( r_A \simeq 0.5/H_b \), we will get Eq.(10) approximately. Interestingly, \( M_{PGVS} \) given by Eq.(10) actually corresponds to the critical mass defined in Ref.[23].

In this scenario, for the different values of \( \sigma_0 \), the PT of bubble wall must occur at the different time \( t_0 \), and so does the bubble radius \( r_0 \). Combining \( \tilde{H}_b = H_b/x \), \( \tilde{r}_* = r_* x \) and \( x = 4\pi \sigma_0 \tilde{r}_0^4 \), we have

\[
r_0 \sim \left( \frac{M_P^2}{H_b^3 \sigma_0} \right)^{1/4} \sim \left( \frac{M_P \Lambda_b^2}{\sigma_0} \right)^{1/4} (1/H_b).
\]

\( \text{Eq. (12)} \)
To have PGVSs form after the PT, $r_0$ should satisfy $r_0 < r_{*A} < 1/H_b$, which requires $\Lambda_b^2 M_P < \sigma_0$ (equivalently $H_b < H_{\sigma_0}$) and can be implemented easily. To get (10), we require $H_{\sigma_A} \lesssim H_b$, which is not conflicted with $H_b < H_{\sigma_0}$, since $\sigma_A = \sigma_0 (r_0/r_{*A})^4 < \sigma_0$. Given Eqs.(10) and (12), we plot the dependence of $\mathcal{M}_{PGVS}$ on $r_0$ for the fixed $\sigma_0 = M_3^2$ in Fig.3.

According to (10), $\mathcal{M}_{PGVS}$ will be fixed by $H_b$ (or $\Lambda_b$), if the primordial GVSs could form. If the inflation really occurred in a Landscape with different dS vacua [24], then the bubbles with different $\Lambda_b$ will nucleate inevitably, and one could expect that the primordial GVSs can have a multi-peak mass spectrum. For $\Lambda_b \simeq M_P$, we have $\mathcal{M}_{PGVS} \sim 10^{-5} g$, which is consistent with that of Planck-mass relic of evaporating BHs, as proposed in Ref.[33]. However, $\Lambda_b$ should satisfy $\Lambda_b < \Lambda_{inf} \simeq (H_{inf}^2 M_P^2)^{1/4} \ll M_P$, therefore we actually have e.g. $\mathcal{M}_{PGVS} > 10 g$ for $\Lambda_{inf} \sim 10^{16}$Gev. It is interesting that the light primordial GVSs with $\mathcal{M}_{PGVS} > 10^{15} g \sim 10^{-18} M_\odot$ could constitute all of cold DM and be perfect candidates for the DM, as the GVSs do not evaporate or evaporate much slower than BHs [18]. The scenario proposed in this report actually provides a mechanism responsible for such primordial GVSs.

![FIG. 3: The mass $\mathcal{M}_{PGVS}$ of primordial GVS with respect to $M_\sigma = \sigma^{1/3}$ and $r_0/(1/H_b)$.](image)

In seminal Ref.[6] proposed GVS (see also [7]), it was speculated that the dS interior of GVSs, as well as the shell with stiff fluid, could be produced by some PT during the gravitational collapse [34][35][36]. However, we showed that the dS region of GVSs might be none other than the interior of dS bubble nucleated during inflation. If so, some of GVSs would be primordial, and hence could be a potential probe to the physics of inflation.

A primordial GVS can absorb radiation and dust so that it eventually collapses into a BH. Nonetheless, it is possible that some of the primordial GVSs would survive till today. According to Eq. (10), if $\Lambda_b \sim 0.1$Gev, we could have primordial GVSs with $\mathcal{M}_{PGVS} \sim 10 - 100M_\odot$, which could be responsible for the LIGO/Virgo events in a similar way as
primordial BHs (see [37] for a review on the primordial BHs). If such a GVS merges with a BH, we might have a post-merger GVS with a larger mass $M > M_{\text{PGVS}}$. The addition of mass alters the shape of $V_{\text{eff}}(\tilde{r}_*)$ in (8), so that $V_{\text{eff}}(\tilde{r}_*) < 0$ at $\tilde{r}_* A$. Assuming $\Delta M = M - M_{\text{PGVS}} < M_{\text{PGVS}}$, at $\tilde{r}_* A$ we have

$$\frac{d\tilde{r}_*/x}{d\tau} = -\sqrt{|V_{\text{eff}}(\tilde{r}_*)|} = -\left[\left(1 - \tilde{H}_b^2 \tilde{r}_*^8\right)\frac{\Delta \tilde{M}}{\tilde{r}_*} + 2\tilde{M}_{\text{PGVS}}\tilde{r}_*^4 \Delta \tilde{M}\right]^{1/2} \approx -\left(1.4 \frac{\Delta M}{M_{\text{PGVS}}}\right)^{1/2} < 0,$$

where we have used Eq.(8) and $\Delta \tilde{M}/\tilde{M} = \Delta M/M$, as well as $V_{\text{eff}}(\tilde{r}_* A) = 0$ for $M = M_{\text{PGVS}}$. Thus the shell of post-merger GVS (the wall of bubble) will acquire a velocity, which is inward (or initially outward but will reverse shortly), and the GVS will eventually collapse into a BH.

It is interesting that a post-merger GVS will bring a distinct GW echo signal. As the Schwarzschild horizon is replaced by a surface that reflects GWs, the reflection will result in a series of echoes in the ringdown waveform [4][5]. While it is collapsing, the post-merger GVS has the reflection surface move towards the would-be Schwarzschild horizon. As a result, the intervals of successive echoes will gradually increase with time [38]. Considering that when the $i$th reflection occurred, the reflection surface has just moved to $r_i(t)$, we can obtain that after the ringdown primary signal, the $n$th echo will appear approximately at $t_n^\text{echo} - t_{\text{merge}} = 2\sum_{i=1}^{n} \Delta t_i$ with

$$\Delta t_i \approx \int_{r_i}^{r_{i-1}} \frac{dr}{v \cdot f(r)} = \int_{r_i}^{3M} \frac{dr}{f(r)},$$

where we assumed the shift velocity $v = d\tilde{r}/d\tau \simeq \text{const}$ of the reflection surface for simplicity. According to Eq. (14), defining $\ell_i = r_i - 2M$, we have $\ell_i/M \simeq (\ell_{i-1}/M)\frac{1}{\ell_0}$ ($\ell_0 \approx 0.18M$). We plot $(t - t_{\text{merge}})/M$ with respect to the shift velocity $v$ of the shell in Fig.4, which suggests that at a short time-segment, if we would like to see more than one echo, $v$ should satisfy $v \lesssim 0.95$. Together with Eq.(13), we have $\Delta \tilde{M}/\tilde{M}_{\text{PGVS}} \lesssim 0.6$. Taking the LIGO/Virgo GW event, GW170608 ($12^{+7}_{-2}M_\odot, 7^{+2}_{-2}M_\odot$) [39], as a special example of primordial GVS/BH coalescence, we have $\Delta M/M_{\text{PGVS}} \simeq 0.6$ and $v \simeq 0.9$. Combining it with (14), we will see that the first two echoes will appear approximately at $t_{1,2}^{\text{echo}} - t_{\text{merge}} \simeq 5\text{ms}, 50\text{ms}$, respectively. Currently, the signal-to-noise ratio of the echo signals is too low to detect echo at enough
confidence level, however, as argued in Ref.[3], the proposed Einstein Telescope has the potential of identifying such signals.

\[ \text{FIG. 4: The time at which the } n\text{th echo appears approximately with respect to the shift velocity } v \text{ of the shell.} \]

In summary, we report on a novel phenomenon that the GVSs might come into being through the nucleation of bubbles during inflation. The primordial GVSs not only witnessed the occurrence of inflation, but also can survive till today and play distinct roles in cosmological evolution. It is pointed out that if such primordial GVSs were involved in LIGO/Virgo GW events, the late-time GW ringdown waveform will exhibit a series of echoes, whose intervals increasing with time. Such echo signals could be detectable with the higher-sensitivity GWs detectors. In addition, the inspiral stage could also be used to detect the primordial GVSs, e.g.[3][40][41].

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