Detailed study of ELAIS N1 field with the uGMRT - II. Source Properties and Spectral Variation Of Foreground Power Spectrum from 300-500 MHz Observations

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ABSTRACT
Understanding the low-frequency radio sky in depth is necessary to subtract foregrounds in order to detect the redshifted 21 cm signal of neutral hydrogen from the Cosmic Dawn, Epoch of Reionization (EoR) and post-reionization era. In this second paper of the series, we present the upgraded Giant Metrewave Radio Telescope (uGMRT) observation of the ELAIS N1 field made at 300-500 MHz. The image covers an area of \( \sim 1.8 \text{ deg}^2 \) and has a central background rms noise of \( \sim 15 \mu \text{Jy beam}^{-1} \). We present a radio source catalog containing 2528 sources (with flux densities \( > 100 \mu \text{Jy} \)) and normalized source counts derived from that. The detailed comparison of detected sources with previous radio observations is shown. We discuss flux scale accuracy, positional offsets, spectral index distribution and correction factors in source counts. The normalized source counts are in agreement with previous observations of the same field, as well as model source counts from the Square Kilometre Array Design Study (SKADS) simulation. This work demonstrates the improved capabilities of the uGMRT.

Key words: Cosmology – diffuse emission - interferometric - surveys - galaxies

1 INTRODUCTION
In the structure formation history of the Universe, after the epoch of recombination at \( z \approx 1100 \), the Universe was completely neutral and consisted mostly of neutral hydrogen (HI). In the absence of any radiating sources, the Universe entered to an era known as the cosmic ‘Dark Ages’. The formation of first stars and galaxies inaugurated another phase of the Universe, the so-called ‘Cosmic Dawn’ (CD) era spanning the redshift range \( 30 > z > 12 \). The high-energy photon emanating from the first stars and quasars began to heat and ionize HI in the surrounding Inter-Galactic Medium (IGM) and forced the Universe to go through a patchy phase transition from being fully neutral to completely ionized. This epoch is marked as ‘Epoch of Reionization’ (EoR) spanning the redshift range \( 12 > z > 6 \) (For more details see: Madau 1997; Furlanetto, Oh, & Briggs 2006; Pritchard & Loeb 2012; Barkana 2016; Dayal & Ferrara 2018).
The redshifted 21 cm line signal generated by the hyperfine transition of neutral hydrogen atom (HI) in the IGM is an excellent probe of the early Universe at $z > 6$ (Field 1958; Madau 1997; Furlanetto, Oh, & Briggs 2006; Pritchard & Loeb 2012). Several low-frequency radio interferometers such as the Donald C. Backer Precision Array to Probe the Epoch of Reionization (PAPER, Parsons et al. 2014), the low-frequency Array (LOFAR, van Haarlem et al. 2013), the Murchison Wide-field Array (MWA, Li et al. 2018) and the Hydrogen Epoch of Reionization Array (HERA, Neben et al. 2016; DeBoer et al. 2017), are trying to measure fluctuations in the cosmological 21 cm signal by means of power spectra. Upcoming instruments such as the Square Kilometre Array (SKA) will have enough sensitivity to resolve physical scales down to 5-10 Mpc (comoving) in the sky plane and corresponding physical scale along line of sight at $z \sim 6$–10 allowing for tomographic imaging of the 21 cm signal (Koopmans et al. 2015; Mondal, Bharadwaj, & Datta 2018; Mondal et al. 2019).

In addition to these, measuring brightness temperature fluctuations of 21 cm signal in the post EoR era ($z < 6$) is a promising tool to study the large scale structure of the Universe in three dimensions. This novel technique is widely known as HI intensity mapping. HI is a biased tracer of dark matter density field. Hence, measurement of the post EoR power spectrum can be used to study large scale structure formation, Baryon Acoustic Oscillation (BAO), neutrino mass, source clustering, etc (Bharadwaj, Nath, & Sethi 2001a; Bharadwaj & Pandey 2003; Bharadwaj & Ali 2005; Wiytse & Loeb 2008; Visbal, Loeb, & Wiytse 2009; Bull et al. 2015; Santos et al. 2015). There are some instruments such as BAOBAB (Pober et al. 2013a), BINGO (Battye et al. 2012), CHIME (Bandura et al. 2014), the Tianlai project (Chen et al. 2016), HIRAX (Newburgh et al. 2016), SKA1-MID (Bull et al. 2015), OWFA (Subrahmanya, Manoharan, & Chengalur 2017) trying to measure Baryon Acoustic Oscillations (BAO) over a redshift range $z \sim 0.5$–2.5, which can be used as a standard ruler to constrain the Dark Energy equation of state.

The main challenge to detect the cosmological HI signal, common to all of these experiments, is the strong contamination of systematic effects (ionospheric distortion, telescope response, calibration, etc) and bright foregrounds (Galactic and extragalactic) (Datta, Bhattacharjee, & Carilli 2009). Foreground sources include diffuse Galactic synchrotron emission from our Galaxy (DGSE) (Shaver et al. 1999), free-free emission from Galactic and extragalactic sources (Cooray & Furlanetto 2004), faint radio-loud quasars (Di Matteo et al. 2002), synchrotron emission from low-redshift Galaxy clusters (Di Matteo, Ciardhi, & Miniati 2004), extragalactic point sources, etc. Typically, foregrounds are 4-5 orders of magnitude stronger than the redshifted HI signal (Zaldarriaga, Furlanetto, & Hernquist 2004; Bharadwaj & Ali 2005; Jelić et al. 2008; Bernardi et al. 2009; Jelić et al. 2010; Zaroubi et al. 2012; Chapman et al. 2015). There are several different ways to deal with foregrounds, but all the methods rely on the fact that foreground sources to have a smooth spectral shape. However, the redshifted HI 21 cm signal has spectral structure (Pritchard & Loeb 2012). In fact, this difference in spectral properties between the strong foreground and faint 21 cm signal can be used favourably for the detection of the cosmological signal (Datta, Bowman, & Carilli 2010). Hence, the accuracy in the knowledge of the spectral “smoothness” of the foreground becomes critical. Our current study makes an attempt to constrain the spectral behaviour of the foregrounds near the redshifted 21 cm signal frequencies. The three main techniques proposed to overcome foreground contamination are foreground avoidance, foreground removal and foreground suppression. Instead of isotropic 1D power-spectrum, $P(k)$, of HI brightness temperature fluctuation, cylindrical (2D) power spectrum, $P(k_\ell, k_\perp)$ is a useful diagnostic in terms of foreground avoidance. Spectral smoothness of foregrounds confines the majority of foreground power to low $k_\perp$ modes, resulting in “Foreground Wedge”. In foreground avoidance technique, the EoR signal is searched for outside this wedge, in so called “EoR Window” (Datta, Bowman, & Carilli 2010; Parsons et al. 2012; Vedantham, Udaya Shankar, & Subrahmanyan 2012; Pober et al. 2013; Thyagarajan et al. 2013; Dillon et al. 2015). However, error in calibration of chromatic instrument and insufficient knowledge on wedge boundary can leak foreground power into the wedge and consequently detection of 21 cm signal becomes challenging even inside the EoR window. Foregrounds can be modelled very precisely and subtracted from the data set. Also, without any modelling a component analysis method can be used to mitigate foregrounds (For detail see: Chapman et al. 2012, 2013). The foregrounds can be suppressed by weighting foregrounds dominated modes appropriately (Liu & Tegmark 2011).

The power spectrum of DGSE is generally modelled as a power law both as a function of frequency and angular scale (Santos, Cooray, & Knox 2005; Datta, Choudhury, & Bharadwaj 2007): 

$$P(\nu) = A(\frac{\nu}{\nu_0})^{-\alpha}$$

where $\nu$ is the power law index of the angular power spectrum (APS) of DGSE and $\alpha$ is the mean spectral index. There are several observational constraint on angular fluctuation in DGSE (Ali, Bharadwaj, & Chengalur 2008; Jacobelli et al. 2013; Bernardi et al. 2009; Ghosh et al. 2012; Jacobelli et al. 2013; Choudhuri et al. 2017; Chakraborty et al. 2019). Motivated by power law nature of synchrotron emission, spectral evolution of fluctuation in DGSE is also modelled as a power law. Basic principle of foreground subtraction technique is to fit this simple power law model for DGSE along frequency axis for each pixel of a data cube and subtract it from the data. However, constraint on spectral variation of foreground power spectrum based on observation is necessary to model DGSE in a most precise manner. As, any error in foreground subtraction can remove the whole 21 cm signal from the data set, accurate modelling is crucial (see Sec. 7).

In this aspect, deep understanding of low-frequency radio sky is important to model extragalactic and Galactic foregrounds. We need prior knowledge in Spectral Energy Distribution (SED), clustering and evolutionary properties of extragalactic sources in radio band to achieve accurate modelling of foregrounds (Jelić et al. 2008, 2010; Prandoni 2018). Spatial distribution of sources, in general, is assumed to be Poissonian or a very simple clustering with a power law feature (Ali, Bharadwaj, & Chengalur 2008; Jelić et al. 2010; Trott et al. 2016). Source counts are also assumed to follow a single power law (Intema et al. 2017; Hurley-Walker et al. 2020).
et al. 2017; Franzen et al. 2019). But, several studies show a deviation from this single power law model at sub-mJy to μJy flux densities (Williams et al. 2016; Prandoni 2018; Hale et al. 2019). Any error in modelling of foregrounds can be harmful in 21 cm signal detection.

In addition, differential source counts can give constraint on the nature of extragalactic sources. We have good understanding of source population at high flux densities for 1.4 GHz observations. Source counts below 1 mJy is a subject of much debate. Several low-frequency deep survey found a flattening in normalized source counts around 1 mJy (Windhorst et al. 1985; Williams et al. 2016; Prandoni et al. 2018; Hale et al. 2019). This suggests a increase in population of Star Forming Galaxies (SFG), radio-quiet AGN at low flux densities (Jackson & Wall 1999; Prandoni et al. 2018). Also, detection of SFGs and AGNs through their radio emission at low-frequency together with their redshift information will help us to understand astrophysical properties of these sources such as luminosity, size of the source, cosmic-ray electron population, etc (Simpson 2017; Norris 2017). Our knowledge of low-frequency sky is poor compared with that of > 1.4 GHz and as a consequence empirical constraint on low-frequency source count is limited. This is mainly because of the fact that reaching high SNR at low-frequency is extremely challenging.

So, deep survey at low-frequency is important not only for modelling foregrounds to detect cosmological 21 cm signal, but it is equally useful to understand astrophysical properties of extragalactic sources. In our first paper of this series (Chakraborty, et al. 2019), we have shown the angular power spectrum of Galactic and extragalactic foregrounds. The spatial behavior of DGSE in this field has been quantified. We have also shown, the effect of direction-dependent and direction-independent calibration in estimation of angular power spectrum of DGSE in this field using 32 MHz (GSB: GMRT Software Backend) bandwidth data of the ELAIS N1 field. In this second paper of the series, we present the detailed study of the ELAIS N1 field with wide bandwidth data (300-500 MHz) using uGMRT. ELAIS N1 field has been previously studied at other frequencies (Ciliegi 1999; Garn et al. 2008; Sirothia et al. 2009; Jelić et al. 2014; Taylor & Jagannathan 2016). The NVSS (Condon et al. 1998) and FIRST (Becker, White, & Helfand 1995) surveys both cover the ELAIS N1 region, but only to relatively shallow 5σ limits of 2.25 and 0.75 mJy respectively.

Although, this field has been studied in different frequencies, the information at low-frequency with high resolution is still lacking. We have observed the ELAIS N1 field at Band-3 (300 - 500 MHz) using uGMRT in GTAC (GMRT Time Allocation Committee) cycle-32 for 25 hours to get a Band-3 (300 - 500 MHz) using uGMRT in GTAC (GMRT Software Backend) bandwidth data of the ELAIS N1 field. The observation spanned 13 hours. The ELAIS N1 field is at high Galactic latitudes (l = 86.95°, b = +44.48°). During the GTAC cycle 32, ELAIS N1 field was up at night time and the observation was carried out at night for all four days. The observation spanned a frequency range of 300-500 MHz, i.e, bandwidth (BW) is 200 MHz. The whole band was divided into 8192 channels, resulting in frequency resolution of 24 KHz. The integration time used was 2s. This high time and frequency resolution was helpful in identifying and flagging RFI (see Sec. 3.1). The uv-coverage of ELAIS N1 field, using uGMRT, can be seen in the left panel of Fig. 1. We have a densely filled core of the uv-plane. The large bandwidth fills the uv-plane radially and long observational time fills the plane azimuthally using Earth’s rotational speed. However, gaps in the uv-plane limits the quality of the final image. In the right panel of Fig. 1, we show the relative number of baseline distribution at different multipole (ℓ = U/2π) to demonstrate the sensitivity of the uGMRT at different angular scale in the estimation of the angular power spectrum (see Sec. 7).

We have observed a flux calibrator 3C286 in the beginning and in the middle of each observing session. We have also observed 3C48 at the end since 3C286 was not up during the last scan of the observation. We have observed a Phase calibrator J1549+506 near the target field every 25 minutes in between scans on the target field. The total on-source data after exclusion of calibrators scans is ~ 13 hours. The observation summary along with the correction factors are described in Table 1.

3 DATA REDUCTION

In this section we describe the RFI mitigation, calibration and imaging procedures in detail which result in high fidelity image of the ELAIS N1.

3.1 Flagging

Low-frequency radio observation with uGMRT is affected gravely by man-made radio transmitters. In general, this spurious signal, which is several orders of magnitude
stronger than the weak astronomical signal of interest, is known as radio-frequency interference (RFI). The effect of RFI is particularly strong at frequencies below 600 MHz in uGMRT observation. RFI can be of many forms but in most cases it is localized in frequency domain or it persists for a short time interval (For more detail see Offringa et al. 2010A).

There are numerous techniques available to mitigate RFI from the post-correlation radio data. The widely used method is to identify RFI in high time and frequency resolution and flagged them from the total data set. Also, this unfavourable signal may completely corrupt some baselines or any particular antenna. It is necessary to remove those baselines and any bad antenna before calibration and imaging. We have taken the data with high time and frequency resolution (24 KHz and 2s) to flag RFI in most efficient way without loosing much of the signal. This results in high data volume (4 TB) for further processing. We have applied AOFLAGGER (Offringa, van de Gronde, & Roerdink 2012) to the high resolution data set. It detects anomaly in time-frequency domain per baseline per polarization and flagged them. For more details regarding working methodology of AOFLAGGER please see: Offringa et al. (2010A), Offringa et al. (2010B), Offringa, van de Gronde, & Roerdink (2012).

### 3.2 Calibration

After flagging in high time-frequency resolution we have averaged the data down to 2048 channels and 8s integration time. We have calibrated four night’s observations separately but in a consistent manner. The calibration is done with exactly same parameters for different night’s data sets. Then during CLEANing we use all the calibrated visibilities to make a continuum image. We have not done any polarization calibration as of now. This defers to later work. The data reduction is done using a CASA based pipeline. Here we briefly describe the steps of our data reduction procedures.

We have first run Hanning-smoothing to reduce the Gibbs ringing across frequency channels. Then we have flagged 5% of total number of channels (2.5% on each side) from the edge of the bandpass using flagging mode: quack in CASA. We have used Perley & Butler (2013) to set the model of our standard primary calibrators 3C286 and 3C48. After setting the flux density model of primary calibrator we follow the traditional direction-independent calibration technique. First we have done a initial delay, gain and bandpass calibration to look for remaining bad data. For gain calibration we have used short solution interval of 16s. Then we have applied this initial calibration to the primary calibrators and run RFLAG (an automated flagging routine in CASA) on the calibrated data to remove RFI. We have done this initial calibration followed by flagging with RFLAG for
Figure 2. The above uGMRT image is zoomed-in total intensity image of ELIAS N1 at 400MHz (bandwidth 200MHz). The Central off-source noise is \(\sim 15\, \mu\text{Jy beam}^{-1}\). The image covers a central area of \(\sim 1.2\, \text{deg}^2\). This illustrates that a large number of weak sources are detected due to high signal-to-noise ratio achieved here.

3.3 Imaging

After calibration we have used the \texttt{CASA} task \texttt{CLEAN} to make a combined continuum image with 4 days data sets. The FoV at 400 MHz of uGMRT is large \((1.15^\circ \times 1.15^\circ)\). Hence, we have taken 256 \(w\)-projection planes to correct for non-coplanar nature of the array. To account for the large bandwidth and spectral structure of the sources present in the field, we have used MS-MFS algorithm (Rau & Cornwell 2011) in \texttt{CASA} and choose \(nterms = 2\). We choose Briggs robust parameter = -1, which gives nearly uniform weighting. This particular choice of robust parameter produces a near-Gaussian central PSF and suppresses the broad wings. We made a large image of size \(3^\circ \times 3^\circ\) to include bright sources lying outside the FoV. Modelling these sources during \texttt{CLEAN}ing is important, otherwise large sidelobes of those will distort the image within the primary beam area.
3.4 Self-Calibration

After getting the first image, we have performed 4 rounds of phase only self-calibration. The solution interval of self-calibration loops are 4min, 2min, 1min and 30sec. After 4 rounds of self-calibration and imaging loop, the signal-to-noise ratio (SNR) saturates and we got the final image. The off-source rms noise achieved near the centre of the FoV is \( \sim 15 \ \mu \text{Jy beam}^{-1} \). The size of the synthesized beam is \( 4.6'' \times 4.3'' \) and the position angle (pa) of the beam is \(-34.2'\). The central zoomed-in part of the image is shown in Fig. 2.

We have corrected for the beam model of uGMRT to measure real sky fluxes. The primary beam of uGMRT is usually modelled as eighth order polynomial. This fitted polynomial is given by:

\[
1 + \left( \frac{a}{10''} \right)^2 + \left( \frac{b}{10''} \right)^4 + \left( \frac{c}{10''} \right)^6 + \left( \frac{d}{10''} \right)^8
\]

where, \(x\) is in terms of separation from pointing position in arc-minutes times the frequency in GHz and \(a, b, c, d\) are the coefficients. For Band-3 (250-500 MHz), the values are: \(a = -2.939, b = 33.312, c = -16.659, d = 3.066\). We use these primary beam parameters provided by uGMRT staff to make a primary beam corrected map of the ELAIS N1 field. We have imposed a cut at 20\% of the peak of the primary beam response. Fig. 3 shows the 1.5" × 1.5" image after primary beam correction.

4 SOURCE CATALOG

We have assembled a source catalog using P\textsubscript{v}BDSF (Mohan & Rafferty 2015) to characterize sources present in this field. Along with source catalog P\textsubscript{v}BDSF produces the rms and residual map. The rms map shows the variation of noise across the whole field whereas residual map gives the image of the field with all the modelled sources are subtracted. The left panel of Fig. 4 shows rms map of the field. It is clear from the image that rms is varying across the FoV with an increased value near the bright sources and at the edge of the FoV. In the right panel of the Fig. 4, the area and the corresponding percentage of the image that has a noise value less than a given value is shown. We have used the primary beam-corrected image to extract the sources with corrected flux values. P\textsubscript{v}BDSF calculates varying rms map across the FoV using a sliding box window \( \text{rms}_{\text{box}} = (180, 50) \) (i.e a box size of 180 pixels in every 50 pixels). Signal to noise ratio is generally high near bright artifacts. First, we have identified those bright regions whose peak amplitude are higher than adaptive threshold of 150 \( \sigma \), where \( \sigma \) is the clipped rms across the entire map. Then to avoid counting artefacts as real sources, we have used a small \( \text{rms}_{\text{box}} = (35, 7) \) around those bright regions. P\textsubscript{v}BDSF identifies islands of contiguous emission over a pixel threshold and then fit multiple Gaussian to each island. We have imposed a threshold of 3\( \sigma_{\text{rms}} \) to detect islands and pixel threshold of 6\( \sigma_{\text{rms}} \) for source detection.

The PSF may vary across the FoV due to ionospheric fluctuation in low-frequency observation. So, at any position in the image the actual PSF is different from the restoring beam by a certain amount. To address this issue, we have calculated variation of PSF using P\textsubscript{v}BDSF with \texttt{psf\_vary\_do = True}\(^2\). It selects a list of sources which are likely to be unresolved ("S" flagged sources from PYBDM output) and with high SNR (>10\( \sigma \)). The number of unresolved sources used are 468. Then using Voronoi tessellation the whole map is tessellated into tiles around those bright sources and with in which PSF shape has been calculated. The spatial variation of PSF is then interpolated across the whole image and the effects of PSF variation are corrected for.

P\textsubscript{v}BDSF groups nearby Islands within an island into sources. The total flux is obtained by summing the fluxes of grouped Gaussians. The uncertainty in total flux is calculated by summing the Gaussian uncertainties in quadrature. The source position is set to be its centroid and source size is measured using moment analysis with the knowledge of the image restoring beam. We have also checked the residual rms map and Gaussian model image after fitting to exclude any false detection or a artefacts. A total of 41 sources has been identified as spurious sources by visual inspection. They are mainly side-lobes of few bright sources (artefacts), at the edge of the FoV, detected as real sources by P\textsubscript{v}BDSF. They are residing within 2\( '' \) of those bright sources and also do not have any counterpart in high frequency catalog. We have removed those sources from the final catalog.

The number of beams per source is used to determine whether the image is confused or not. If the average number of pixels between two sources are less than 25 then the image is assumed to be confused. Here we got 540 pixels corresponding to nearly 10 beams per source. This ensures that it is not confusion limited. The confusion noise limit for this observation is \( \sim 2.0 \mu \text{Jy beam}^{-1} \) (Condon, et al. 2012).

We have compiled 2528 sources within 20\% of uGMRT primary beam at 400 MHz with flux densities greater than 100 \( \mu \text{Jy} \) (>6\( \sigma \)). A sample of the catalog shown in Table 2. The selection of extended and unresolved sources is discussed below.

4.1 Classification of sources

Classification of sources as resolved and point-like is complicated based on P\textsubscript{v}BDSF derived source properties. This is mainly due to time and bandwidth smearing, which artificially extend the sources in the image plane. The error in calibration and varying noise are also responsible to scatter the ratio of integrated flux density (\( S_{\text{int}} \)) to peak flux density (\( S_{\text{peak}} \)). As a consequence of that, we can not simply classify the sources as extended or resolved based on the requirement of \( (S_{\text{int}}/S_{\text{peak}}) > 1 \). In fig. 5, we have plotted \( (S_{\text{int}}/S_{\text{peak}}) \) as a function of \( (S_{\text{peak}}/\sigma) \), where \( \sigma \) is the local rms. The distribution is skewed at low SNR.

The consequence of bandwidth and time average smearing is the reduction of peak flux densities of the sources whereas the integrated flux density remains same. As a result the ratio of total to peak flux density is not equals to one for originally unresolved sources. The magnitude of this effect depends on the radial distance from the pointing centre, channel width (frequency resolution) and integration time

\(^2\) For more details on different parameters please see: https://www.astron.nl/citt/pybdsf/
Figure 3. Primary beam corrected image of ELIAS N1 at 400MHz. The image extends over an area of ∼ 1.8 deg$^2$. The off source rms at the center is ∼ 15 µJy beam$^{-1}$ and beam size is 4.6′′ × 4.3′′.

Table 2. Sample of uGMRT 400 MHz source catalog of ELAIS N1 field.

| Id | RA (deg) | E_RA (arcsec) | DEC (deg) | E_DEC (arcsec) | Total_flux (mJy) | Peak_flux (mJy beam$^{-1}$) | Major (arcsec) | Minor (arcsec) | PA (degree) | rms (mJy beam$^{-1}$) |
|----|---------|--------------|----------|----------------|-----------------|--------------------------|---------------|--------------|------------|------------------|
| 0  | 243.79  | 0.33         | 54.58    | 0.27           | 1.29            | 0.68                     | 6.8           | 5.7          | 65.20      | 0.06             |
| 1  | 243.77  | 0.95         | 54.40    | 0.58           | 2.37            | 0.33                     | 15.0          | 9.2          | 74.08      | 0.05             |
| 3  | 243.78  | 0.31         | 54.64    | 0.38           | 0.79            | 0.47                     | 6.7           | 4.8          | 35.81      | 0.06             |

Notes: The columns of the final catalog (fits format) include source id, positions, error in positions, flux densities, peak flux densities, sizes, position angle and local rms noise.
Figure 4. Left image is showing the local rms noise measured in the final map. Local noise is high near the bright sources and at the edge of FoV. Right: Cumulative area of the final map with a rms noise level below the given value.

Figure 5. The ratio of integrated to peak flux density \( \frac{S_{\text{int}}}{S_{\text{peak}}} \) as a function of signal-to-noise ratio \( \frac{S_{\text{peak}}}{\sigma_L} \) of sources. Extended sources are shown in red and point-like sources in green.

(time resolution). We have theoretically estimated the combined effect of bandwidth and time smearing and found that measured peak flux density is 97% of the expected value (see Condon et al. 1998) at the maximum distance (45′) from the phase centre. Hence, smearing is not an issue in our case.

We have used the method described in Franzen et al. (2015, 2019) for identification of resolved sources. The requirement for a source to be extended at the 3σ level is, following Franzen et al. (2015):

\[
\ln \left( \frac{S_{\text{int}}}{S_{\text{peak}}} \right) > 3 \sqrt{\frac{(\sigma_S)^2}{S_{\text{int}}} + \left( \frac{\sigma_{S_{\text{peak}}}}{S_{\text{peak}}} \right)^2}
\]  

where \( \sigma_S \) and \( \sigma_{S_{\text{peak}}} \) are the uncertainties on integrated flux density \( S_{\text{int}} \) and peak flux density \( S_{\text{peak}} \) respectively. We have found 401 (the red circles) resolved sources based on the above requirements and 2127 sources have been classified as unresolved or point-like sources at this frequency of observation with uGMRT (see Fig. 5).

5 COMPARISON WITH OTHER RADIO CATALOGS

In this section, we present detailed comparison with other radio catalogs having overlapping regions. We have compared our catalog with NVSS and FIRST all-sky survey and GMRT observation of the ELAIS N1 field at 325 MHz by Sirothia et al. (2009) and at 610 MHz by Garn et al. (2008). Given the uncertainty in uGMRT beam model (primary beam) and ionospheric fluctuations at low-frequency, it is essential to cross check with previous radio catalogs. The multi-frequency information available in literature for this field allow us to quantify any systematic offsets in flux density and position of sources. Counterpart of our catalog sources are identified using a 5′ search radius for all previous catalogs. Individual catalog has a flux density limit \( S_{\text{limit}} \) based on completeness and sensitivity of that particular observation. We have scaled that flux density limit of different catalogs to 400 MHz using a spectral index of \( \alpha = -0.8 \) \( (S \propto \nu^\alpha) \). Hence, different flux density limits correspond to a flux cut at 400 MHz \( (S_{\text{cut,400MHz}}) \). For comparison with other catalogs, we have used only those sources whose flux density at 400 MHz is greater than the flux cutoff. Details of different survey parameters are mentioned in Table 3.

5.1 Flux density offset

Uncertainties in the flux density scale (e.g Perley & Butler 2013) and uGMRT beam (primary beam) model can cause for systematic offsets in flux density. We have compared uGMRT flux densities with GMRT observation of ELAIS N1 at 610 MHz by Garn et al. (2008). We follow the same selection criteria as described in Williams et al. (2016). We have
The median value of flux ratios together with the 16th and 84th percentiles. We have plotted RA and DEC of these sources in the Fig. 6, where the colorbar shows the flux density ratio. We have not found any significant variation across the image. In the central part of the map, the ratio is close to one for a significant number of sources.

To cross validate this result with other radio observations, we have performed a similar comparison with NVSS (Condon et al. 1998), FIRST catalog (Becker, White, & Helfand 1995) and also with 325 MHz GMRT observation (Sirothia et al. 2009). Here again we restrict our requirements of sample selection as described above. We have determined the flux density ratio for these samples after proper scaling. The median value of flux ratios together with the errors are 0.95±0.2, 1.06±0.4, 1.09±0.2 for NVSS, FIRST and 325 MHz GMRT catalog respectively.

In Fig. 7, we have shown the flux densities of selected sources at 400 MHz uGMRT observation as a function of flux densities of counterparts in other catalog. We have not found any significant deviation in flux density ratios for these different catalog comparisons. The median value is also close to 1 for all cases. We can say that the systematic effect is negligible here and we opt for no correction in the flux density due to systematic offsets.

### 5.1.1 Flux scale accuracy

To check the overall reliability of the flux scale and to account for the uncertainties in spectral index, we have compared a small number of sources which are detected at higher frequency (FIRST 1.4 GHz) as well as in lower frequency (uGMRT 325 MHz; Sirothia et al. 2009) maps. Here again we restrict our choice to compact, high signal-to-noise and well isolated sources. We have properly scaled the flux densities measured at 400 MHz using uGMRT with other radio catalogs at different frequencies 325 MHz GMRT (green), 610 MHz GMRT (magenta), NVSS (blue), FIRST (red). The black dashed line corresponds to $S_{\text{uGMRT}}/S_{\text{other}} = 1$.

![Figure 7. Comparison of total flux density of compact sources measured at 400 MHz using uGMRT with other radio catalogs at different frequencies 325 MHz GMRT (green), 610 MHz GMRT (magenta), NVSS (blue), FIRST (red). The black dashed line corresponds to $S_{\text{uGMRT}}/S_{\text{other}} = 1$.](image)

### Table 3. Details of previous radio catalogs. The frequency of observation, resolution, flux limit of a survey and flux cut at 400 MHz corresponding to flux limit (assuming $\alpha = -0.8$) are mentioned in different columns.

| Catalog          | Frequency (MHz) | Resolution (arcsec) | $S_{\text{lim}}^{\text{uGMRT}}$ (mJy) | $S_{\text{lim}}^{\text{400MHz}}$ (mJy) |
|------------------|-----------------|---------------------|--------------------------------------|--------------------------------------|
| uGMRT            | 400 MHz         | 4.6′′                | 0.10                                 | 0.10                                 |
| NVSS             | 1.4 GHz         | 45′′                 | 2.5                                  | 6.8                                  |
| FIRST            | 1.4 GHz         | 5.4′′                 | 1.0                                  | 2.7                                  |
| GMRT             | 610 MHz         | 6′′                   | 0.27                                 | 0.37                                 |
| GMRT             | 325 MHz         | 9′′                   | 0.26                                 | 0.22                                 |

† $S_{\text{lim}}$ is the flux density limit of the corresponding catalog.

![Figure 6. Map of the ratios of integrated flux densities for high signal-to-noise, compact and isolated uGMRT 400 MHz sources with respect to GMRT 610 MHz sources. The colorscale is showing the flux density ratio.](image)
to Perley & Butler (2013) to put them in the same flux scale. Then, for this sample of sources, we have first calculated the spectral index by comparing FIRST and 325 MHz flux densities and then predicted the uGMRT flux density at 400 MHz. The mean value of spectral index for these sources is \( \alpha = -0.77 \). In Fig. 8, we have shown predicted to measured flux density ratio as a function of uGMRT flux densities. The mean flux density ratio is 1.02 with standard deviation of 0.17. We can conclude that the corrected uGMRT flux density is consistent with Perley & Butler (2013) scale.

5.2 Positional accuracy

We have not done any direction-dependent calibration for this wide bandwidth uGMRT observation of the ELAIS N1 field. Phase only self-calibration can reduce fluctuation in phase but only near the apparent bright sources. There are residual phase errors after final calibration procedure, causing uncertainty in the source positions. Ionospheric fluctuation at low-frequency also induces positional offsets. Here we have assessed the astrometric accuracy of our catalog by comparing source positions with 1.4 GHz FIRST catalog (Becker, White, & Helfand 1995; Thyagarajan et al. 2011). Due to high frequency, FIRST catalog has better resolution (5.4") and also ionospheric fluctuation is comparatively small. The positional accuracy of FIRST catalog is better than 1" (Becker, White, & Helfand 1995).

Again we have selected a sample of small, isolated and compact sources following the criteria described in Sec. 5.1. This gives us a sample consists of 135 sources for comparison. We have measured the offset in right ascension (RA) and declination (DEC) of these sources as (following Williams et al. 2016):

\[
\begin{align*}
\delta_{\text{RA}} &= R_{\text{uGMRT}} - R_{\text{FIRST}} \\
\delta_{\text{DEC}} &= D_{\text{uGMRT}} - D_{\text{FIRST}}
\end{align*}
\]

The median offset in RA and DEC are 0.28" and 0.56". There is no systematic variation of positional offset (\( \delta_{\text{RA}} \) and \( \delta_{\text{DEC}} \)) across the FoV. We have done similar analysis with GMRT 610 MHz catalog (Garn et al. 2008). Here the mean offset in RA and DEC are 0.06" and 1". The offset in DEC is slightly higher in this case. But ionosphere is more unstable at 610 MHz GMRT observation and also no direction-dependent calibration has been performed for this observation (Garn et al. 2008). In Fig. 9, the histogram of offset in RA and DEC for both catalogs is shown. Given the pixel size of 1.5" of uGMRT image, these offsets are negligible.

We made a correction in the final catalog for uGMRT source positions with a constant offset, i.e., \( \delta_{\text{RA}} = 0.28" \) and \( \delta_{\text{DEC}} = 0.56" \) (based on FIRST catalog offsets).

5.3 Spectral index distribution

Characterization of spectral properties of sources in ELAIS N1 field is done by comparing flux densities with previous high frequency radio catalogs. For comparison, we have used 610 MHz GMRT observation of the same field (Garn et al. 2008) and FIRST (1.4 GHz) and NVSS (1.4 GHz) catalogs. We follow the same source selection procedure as in Sec. 5.1. The sample includes compact, isolated and high SNR sources, whose flux density values are above the flux limit of corresponding catalogs. The number of sources used to estimate spectral index distribution for different catalog comparison are: 44 (NVSS), 135 (FIRST) and 80 (GMRT, 610 MHz).

We have assumed a synchrotron power-law distribution with single spectral index, i.e., \( S_\nu \propto \nu^\alpha \), where \( \alpha \) is the spectral index. We have compared the flux density of matched sources between two catalogs and then estimate the \( \alpha \) value. In Fig. 10, we have shown the histogram of \( \alpha \) for the sources in our catalog matched to other three different catalogs.
The median spectral indices with errors from 16th and 84th percentile for different catalogs are: $-0.81^{+0.28}_{-0.32}$ (1.4 GHz FIRST), $-0.70^{+0.31}_{-0.24}$ (1.4 GHz NVSS) and $-0.68^{+0.36}_{-0.52}$ (610 MHz GMRT). Garn et al. (2008) have reported a spectral index value of -0.7 by comparing flux densities with FIRST catalog. Sirothia et al. (2009) measured a median spectral index of -0.83 after matching sources with 1.4 GHz FIRST catalog. They have also reported a more steeper median value of spectral index -1.28 after comparing flux densities with 610 MHz GMRT catalog of Garn et al. (2008). Here, the median value of $\alpha$ estimated after comparing with different radio catalogs is close to -0.7, which is in agreement with previous measurements. A detailed study of spectral index of sources using multi-frequency data as well as analysis of in band uGMRT spectral indices is deferred to future work.

6 SOURCE COUNTS

We have estimated the differential source counts based on wide-band flux densities from P$_v$BDSF catalog output. At low-frequency, distribution of sources as a function of flux density is important to understand population of different radio galaxies. We know from simulation (Wilman et al. 2008) as well as from different observations that star forming galaxies (SFGs) and the radio quiet quasars (RQQ) are most dominant populations at faint flux densities. But, there are very few observational constraints on source population at sub mJy level, mainly below 0.5 mJy. Characterization of the spatial and spectral nature of the foreground sources down to $\mu$Jy level flux density is crucial for telescopes like LOFAR, MWA, HERA and SKA in order to detect the faint cosmological HI 21 cm signal.

Here, we have measured the differential source counts at 400 MHz down to 120 $\mu$Jy ($>5\sigma$). But, direct quantification of source counts based on P$_v$BDSF output does not contemplate true extragalactic source distribution, specially at low frequencies and at faint end of flux density bins. We need to correct for incompleteness, false detection rate, Eddington bias, resolution biases as well as visibility area effects. These correction factors are described in detail below.

6.1 False detection rate

False detection rate defines as the number of spurious sources detected by the source finding package (P$_v$BDSF) as real ones due to noise spikes and artifacts. If the distribution of noise in the image is symmetric about zero, i.e, positive noise spikes have equivalent negative spikes in the image, then number of falsely detected (spurious) sources would be identical to the number of “negative” sources in the inverted (or, negative) image. In order to quantify this, we run P$_v$BDSF on the inverted (negative) image with exactly same parameters as used in our source finding algorithm (Sec. 4). We have detected a total of 243 sources with negative peaks less than $-5\sigma$.

To correct the flux density bins for FDR, we have binned the number of negative sources detected in the inverted image in 20 logarithmic bins and compared this with the positive sources detected in the original image. Fraction of real sources in each bin is defined as (Hale et al. 2019) -

\[ f_{\text{real},i} = \frac{N_{\text{catalog},i} - N_{\text{inv},i}}{N_{\text{catalog},i}}. \]  

where $N_{\text{inv},i}$ and $N_{\text{catalog},i}$ are the number of detected sources in $i^{th}$ flux density bin for inverted and original image respectively. The correction factor due to false detection is shown in Fig. 11. The errors in FDR are calculated using Poissonian errors. We have multiplied this fraction (Eqn. 5) to the number of sources detected in each flux density bin in the original catalog.

6.2 Completeness

A source catalog constructed using P$_v$BDSF output is not complete. There are some factors which can cause for under-estimation as well as over-estimation of the source counts.
that it can not be detected above the noise. Although these
source, the peak flux density may be significantly reduced
Table 4.

| S (mJy) | $S_c$ (mJy) | N | $S^2$dN/dS (Jy$^{-1}$sr$^{-1}$) | FDR | Completeness | Corrected $S^2$dN/dS (Jy$^{-1}$sr$^{-1}$) |
|---------|-------------|---|-------------------------------|-----|-------------|-------------------------------------|
| 0.120-0.191 | 0.166 | 218 | 6.635 ± 0.183 | 0.876 ± 0.004 | 3.44$^{+0.32}_{-0.33}$ | 20.01 ± 0.55 |
| 0.191-0.303 | 0.244 | 688 | 18.269 ± 0.391 | 0.951 ± 0.001 | 1.10$^{+0.05}_{-0.08}$ | 19.14 ± 0.41 |
| 0.303-0.482 | 0.380 | 701 | 27.627 ± 0.665 | 0.942 ± 0.001 | 0.80$^{+0.07}_{-0.08}$ | 20.95 ± 0.50 |
| 0.482-0.766 | 0.603 | 381 | 27.542 ± 0.938 | 0.913 ± 0.002 | 0.82$^{+0.07}_{-0.06}$ | 20.79 ± 0.71 |
| 0.766-1.218 | 0.943 | 191 | 25.957 ± 1.263 | 0.832 ± 0.006 | 0.87$^{+0.07}_{-0.09}$ | 18.85 ± 0.92 |
| 1.218-1.935 | 1.511 | 112 | 30.882 ± 1.971 | 0.750 ± 0.011 | 0.83$^{+0.07}_{-0.07}$ | 19.36 ± 1.24 |
| 1.935-3.076 | 2.344 | 68 | 35.184 ± 2.887 | 0.765 ± 0.013 | 0.82$^{+0.07}_{-0.10}$ | 22.31 ± 1.83 |
| 3.076-4.889 | 3.956 | 50 | 60.132 ± 5.760 | 0.700 ± 0.019 | 0.94$^{+0.11}_{-0.17}$ | 39.69 ± 3.80 |
| 4.889-7.772 | 5.850 | 31 | 62.327 ± 7.585 | 0.710 ± 0.024 | 0.93$^{+0.12}_{-0.11}$ | 41.55 ± 0.85 |
| 7.772-12.353 | 10.058 | 28 | 137.190 ± 17.573 | 0.893 ± 0.010 | 1.02$^{+0.17}_{-0.12}$ | 125.32 ± 16.05 |
| 12.353-19.635 | 15.092 | 22 | 186.968 ± 27.023 | 0.955 ± 0.005 | 1.09$^{+0.17}_{-0.11}$ | 194.80 ± 28.15 |
| 19.635-31.209 | 24.877 | 13 | 242.412 ± 45.584 | 0.923 ± 0.010 | 1.12$^{+0.22}_{-0.17}$ | 252.61 ± 47.50 |
| 31.209-49.607 | 37.404 | 7 | 227.605 ± 58.330 | 0.857 ± 0.026 | 1.38$^{+0.38}_{-0.25}$ | 270.35 ± 69.28 |
| 49.607-78.849 | 61.337 | 4 | 281.753 ± 95.524 | 1.00 | 1.36$^{+0.36}_{-0.07}$ | 384.87 ± 130.48 |
| 78.849-125.330 | 85.496 | 1 | 101.645 ± 68.924 | 1.00 | 1.32$^{+0.47}_{-0.05}$ | 134.88 ± 91.46 |
| 125.330-199.212 | 138.951 | 1 | 215.329 ± 146.014 | 1.00 | 1.38$^{+0.38}_{-0.27}$ | 298.87 ± 202.66 |
| 199.212 - 316.645 | 250.260 | 2 | 1179.466 ± 565.546 | 1.00 | 1.00$^{+0.07}_{-0.05}$ | 1179.46 ± 565.54 |
| 316.645 - 503.305 | 372.677 | 2 | 2008.035 ± 962.850 | 1.00 | 1.00$^{+0.05}_{-0.02}$ | 2208.83 ± 1059.13 |
| 503.305 - 800.0 | 798.478 | 1 | 4244.269 ± 2878.116 | 1.00 | 1.00$^{+0.08}_{-0.07}$ | 4244.26 ± 2878.12 |

Notes: This table includes the flux density bins, central of flux density bin, the raw counts, normalized source counts, False Detection Rate (FDR), completeness and corrected normalized source counts.

This makes the catalog incomplete. To quantify those, we carried out simulation in the image plane. Incompleteness means given a flux density limit, we are still unable to detect sources above that limit due to varying noise in the image. This results in underestimation of source counts near the flux density detection limit. Eddington bias causes noise to redistribute low flux density sources in higher fluxes. Due to steep source counts at low flux density bins, this bias is significant near the detection limit. As a consequence, there may be boost in source counts in the faintest bins. Magnitude of this boost is governed by signal-to-noise ratio and source count slope.

Resolution bias signifies that the detection probability of a resolved source is less than point-like sources in our peak flux density selection during P$\nu$BDSF run. For a extended source, the peak flux density may be significantly reduced that it can not be detected above the noise. Although these extended sources have same integrated flux density as the unresolved ones, we are unable to detect them and hence resolution bias reduces our source counts.

We have quantified these biases by injecting 1000 sources into our primary beam corrected image (not the residual rms map as in Williams et al. 2016). Out of these, 100 sources (10%) are extended, i.e., sizes are greater than beam size. We scatter the sources randomly in the image plane. The flux densities of simulated sources are drawn randomly from a power law distribution (dN/dS $\propto S^{-1.6}$; Intema et al. 2011; Williams et al. 2013) between 80 $\mu$Jy to 1 Jy. We have created 100 such simulations. These simulations inherently take into account the confusion of sources and visibility area of sources at different flux density bins (Hale et al. 2019; Franzen et al. 2019; Williams et al. 2016). For each simulated image, we have extracted sources using P$\nu$BDSF following the same criteria as described in Sec. 4.
There were sources prior to our simulation in the original image. We already have the source catalog corresponding to the original image. Now after detection of sources from the simulated image, we have subtracted the original sources from the post-simulation source counts (which consists of injected sources and original sources). We have binned these sources in 20 logarithmic bins in flux density. The correction factor then calculated as (following Hale et al. 2019) -

\[
\text{Correction}_{i} = \frac{N_{\text{injected},i}}{N_{\text{recovered},i}}
\]

here, \(\text{Correction}_{i}\) is the completeness correction factor in the \(i^{th}\) flux density bin. \(N_{\text{injected},i}\) is the number of injected sources and \(N_{\text{recovered},i}\) is the number of sources recovered after subtracting original pre-simulation sources in the \(i^{th}\) bin. This method of quantifying completeness already takes into account the resolution bias as well as the Eddington bias (Hale et al. 2019). The completeness correction factor is shown in Fig. 11. We are quoting the median value of 100 simulations for each flux density bins as a correction factor and the associated errors are from 16th and 84th percentiles.

### 6.3 Differential Source count

We have estimated the Euclidian-normalized differential source counts from the source list generated by PyBDSF. We have corrected the source counts for FDR and completeness. The correction factors are multiplicative to the original source counts. We have also corrected for effective area for different flux density bins over which a source can be detected. This is due to the fact that the noise is varying significantly across the image (see Fig. 4). Hence, faint sources can not be detected over the full image. So, we have found out the fraction of area (\(f\)) over which a source with a given flux density can be detected (its visibility area) and weighted the source counts by the reciprocal of that fraction (Windhorst et al. 1985). The normalized source counts can be seen in Fig. 12. We have binned the sources in 20 logarithmic bins in flux density down to 120 \(\mu \text{Jy}\). This is the deepest source counts at this low-frequency. The error in source count for each bin is Poisson errors. The source counts and associated errors are given in table 4. We have compared this source counts with 325 MHz (Sirothia et al. 2009) and 610 MHz (Garn et al. 2008) GMRT source counts of ELAIS N1 field after scaling to 400 MHz assuming a spectral index of -0.8. These source counts are in agreement with our findings.

We have also compared our source counts with \(S^3\) - SKADS simulation by Wilman et al. (2008). We have taken 1.4 GHz flux densities of \(S^3\) simulation and scaled it to 400 MHz using \(\alpha = -0.8\). SKADS-simulation uses different multi-frequency observation to model luminosity function, clustering of sources, classification of different sources, etc and...
gives a synthetic radio catalog (see Wilman et al. 2008 and the references therein). We have shown the source counts of SFG, RQQ and AGN and all sources (combination of all) in Fig. 12. It is observed that the source population of RQQ and SFG’s are increased at low flux densities and give rise to flattening in the total source counts below 1 mJy. We have also found a similar feature in the normalized source counts signifies the increased population of SFG and RQQ at low flux density bins. Our observed counts is consistent with this simulated model. However, our observed counts is little higher than $3^3$ simulation in the flux density range 10 mJy to 100 mJy. The exact reason behind this excess is unknown. However, the models used to generate the simulated catalog in SKADS are based on high frequency data available in literature (Wilman et al. 2008; Williams et al. 2016). So, some deviation may also be plausible.

We have found that completeness correction is most dominant effect in low flux density bins whereas FDR correction is not large at these flux densities. Another possible error can be induced by incorrect primary beam model of uGMRT. The primary beam pattern may change due to antenna movement in azimuth-elevation while tracking the target field across the sky. Also, there can be errors in the estimation of the primary beam pattern from relevant data. In order to understand the effect of these errors/deviations in the primary beam pattern, we have considered about 10% error around the best-fitted values of the four parameters of the primary beam model of uGMRT at Band-3 (see Eqn. 2). We have estimated the normalized dN/dS curve with the errors in the four parameter values. Our results show no significant deviation from the normalized dN/dS obtained with best-fitted values of the four beam parameters. Hence, we can conclude that this curve is robust against any beam errors within 10%.

7 SPECTRAL EVOLUTION OF DGSE POWER SPECTRA

After removal of point sources from the observed data set, DGSE is still higher than the HI signal by orders of magnitudes. The smooth spectral behavior of foregrounds holds promise to extract the faint cosmological signal amidst these bright foregrounds. But extracting the signal requires knowledge of spatial as well as spectral nature of foregrounds. Here we have quantified how amplitude of angular power spectrum of DGSE is evolving as a function of frequency.

DGSE is generally modelled as a power law in both angular scale and frequency (see Eqn. 1). This is an empirical model of foregrounds. Several previous observations have measured the APS of DGSE for different fields and measured the value of the power law index ($\beta$) lies between [1.5 to 3.0] (Ali, Bharadwaj, & Chenguang 2008; Iacobelli et al. 2013; Bernardi et al. 2009; Ghosh et al. 2012; Iacobelli et al. 2013; Choudhuri et al. 2017). La Porta et al. (2008) has studied 408 MHz Haslam map (Haslam et al. 1982) and 1420 MHz map created by Reich & Reich (1988) after combining Northern and Southern sky survey. They have measured the APS of Galactic synchrotron emission ($C_\ell$) for different Galactic latitudes. Then using the mean APS at two frequencies (408 MHz and 1420 MHz) they have calculated mean spectral index ($\alpha$) using the relation (La Porta et al. 2008):

$$\langle C_\ell(\nu_1) \rangle = \langle C_\ell(\nu_2) \rangle \left(\frac{\nu_1}{\nu_2}\right)^{-2\alpha} \quad (7)$$

(Note that: La Porta et al. 2008 has used $\alpha$ as the power law index of APS and $\beta$ as the mean spectral index. So, our notation is just opposite to them)

The obtained mean spectral index lies between [2.9 to 3.2] for different Galactic latitudes. Using this mean spectral index, they have extrapolated $C_\ell$ to 23 GHz and check the consistency of synchrotron APS with the WMAP observation of foregrounds at 23 GHz. They have derived the mean spectral index by comparing amplitude of APS at two frequencies and extrapolate this to higher frequency. Different astrophysical components contribute in a different manner to the APS of foregrounds at different frequencies. As a consequence of that, fluctuation in DGSE can also vary as a function of frequency. Hence, estimating $\alpha$ based on only two discrete frequency samples may overlook the detail intricacies of synchrotron power spectrum as a function of frequency.

We have used the wide bandwidth (200 MHz) data of ELAIS N1 field to find out the spectral behavior of fluctuation in DGSE, i.e, spectral evolution of $\alpha$. First, we have subtracted the point source model (generated during CLEANING) from the calibrated data set by using $UVSUB$ in CASA. The residual data (after subtraction) mainly consists of DGSE and residual point sources below the noise level. Then, we have used the Tapered Gridded Estimator (TGE) (Choudhuri et al. 2014, 2016) to quantify the APS of DGSE from the residual visibility data set. TGE uses visibility correlation after gridding the calibrated data on a regular grid and subtracts the positive noise bias (by not including self-correlation of visibilities) to give unbiased estimate of the angular power spectrum ($C_\ell$) (For more details please see: Choudhuri et al. 2014, 2016).

We have shown in Chakraborty, et al. (2019) that with large tapering of FoV, we can estimate the angular power spectrum of diffuse radiation even without direction-dependent calibration. We can neglect the undesired effects of bright sources at large distance from the centre of the FoV in the estimation of $C_\ell$, by using the higher tapering of sky response for direction-independent calibration in comparison with direction-dependent calibration (Chakraborty, et al. 2019). Here, since we have not done any direction-dependent ionospheric calibration, we have used the same tapering parameter (f = 0.5) as used in Chakraborty, et al. (2019) for direction-independent calibration. This ensures that the estimation of $C_\ell$ is not be affected by direction-dependent calibration effects.

We have divided the residual visibility data of whole bandwidth (200 MHz) into 25 chunks of 8 MHz band. For each chunk of 8 MHz band, we have run TGE to estimate the angular power spectrum (2D). This gives us $C_\ell$ (APS) of DGSE at the central frequency of the $i^{th}$ chunk. Due to flagging and sparse uv-coverage, we are able to estimate APS of DGSE for only 13 chunks of residual visibilities. Then for each chunk we have found a $\ell$ range where $C_\ell$ shows a steep power law behavior, which is characteristics of DGSE (see Fig. 13). We inferred that DGSE is dominant for that $\ell$ range, beyond which residual point sources begins to dominate over DGSE. For that particular angular multipole ($\ell$)
Figure 13. The estimated angular power spectrum ($C_\ell$) with 1−σ error bar (green curve) as a function of angular multipole $\ell$ for 13 sub-bands. The vertical dashed lines (in maroon) shows $\ell$ range to fit a power law model and the black dashed line shows the best-fitting $C_\ell^M = A_\ell^{\beta}$. The value of angular power law index $\beta$ is mentioned in each plot. In the last panel, we also show the angular scale corresponding to the $\ell$ range probed here.
range we fit a power law of the form:

\[ C_{\ell}^i = A_i \ell^{-\beta_i}, \]  

where \( A_i \) and \( \beta_i \) are the amplitude and power law index of APS for the \( i \)th chunk. We have normalized the APS of all 13 chunks at \( \ell_0 = 1200 \), i.e., \( C_{\ell=1200}^i = A_i \). We have checked with other values of \( \ell_0 \), but our findings are consistent. The value of \( \beta \) for all 13 sub-bands lies between [1.8 to 3]. All the plots of \( C_{\ell}^i \) as a function of \( \ell \) for all 13 sub-bands are presented in the Fig. 13.

The values of \( C_{\ell=1200} \) at the central frequency of 13 sub-bands (\( \nu_0 \)) is being plotted in Fig. 14. We have also plotted the measured value of the amplitude of DGSE power spectrum at 325 MHz (Chakraborty, et al. 2019) in magenta. The spectral variation of APS for DGSE or the Multi-Frequency Angular Power Spectrum (MFAPS) of the DGSE is modelled as \( C_\ell(\nu) \propto \nu^{-2\alpha} \). Here, we have also fitted a power law in frequency to the whole frequency range given as:

\[ C_{\ell=1200}(\nu) = A \nu^{-2\alpha} \]

The value of \( \alpha \) for whole frequency range is \( 2.9 \pm 0.21 \). The reduced \( \chi^2 \) (\( \chi^2 R \)) value for this fit is 1.6. We have shown the fitted curve (black) in Fig. 14.

Previously, de Oliveira-Costa et al. (2008) presented a global sky model (GSM) of diffuse radio background using different total power large-area radio surveys between 10 MHz and 94 GHz. In their model (GSM), spectral index of diffuse emission at 150 MHz is \( \sim 2.6 \) above the Galactic plane. EDGES team has measured the spectral index of diffuse radio emission using all-sky averaged data. They have reported a mean spectral index at high Galactic latitudes to be \( 2.52 \pm 0.04 \) between frequency range 150 - 408 MHz (Rogers & Bowman 2008). Mozdzen, Bowman, Monsalve & Rogers (2017) have found a spectral index nearly \( 2.62 \pm 0.02 \) in frequency range 90-190 MHz using EDGES high-band system. Recently, using EDGES low-band system, Mozdzen, Mahesh, Monsalve, Rogers & Bowman (2019) measured a spectral index lies between \([2.54-2.59]\) in frequency range 50-100 MHz. We have estimated the MFAPS of DGSE power spectrum for the first time with a wide-band radio interferometric observation. Our findings for ELAIS N1 field is consistent with previous total power observations.

Since the reduced \( \chi^2 \) value for the single spectral index fit is high, we explored the possibility of a broken power law fit to the data as well with a break at 405 MHz (\( \nu_{\text{break}} \)). The broken power law model is given by:

\[ C_{\ell=1200}(\nu) = \begin{cases} A \left( \frac{\nu}{\nu_{\text{break}}} \right)^{-2\alpha_1}, & \text{for } \nu < \nu_{\text{break}} \\ A \left( \frac{\nu}{\nu_{\text{break}}} \right)^{-2\alpha_2}, & \text{for } \nu > \nu_{\text{break}} \end{cases} \]  

The best fitted values of spectral index for this case is \( \alpha_1 = \ldots \)

---

**Figure 14.** Angular power spectrum of DGSE normalized at \( l = 1200 \) as a function of frequency. The magenta triangle is the measured power spectrum of DGSE at 325 MHz (Chakraborty, et al. 2019). The observed values are consistent with the previous measurement.
2.1 ± 0.2 and \( \alpha_2 = 4.8 \pm 0.4 \). The value of reduced \( \chi^2 \) is 0.3 for this broken power law fitting.

From the above two attempts to fit the MFAPS data with a broken or single power law, none of the models can be ruled out. The error bars in the MFAPS data makes it difficult to distinguish between both the models. Hence, a single spectral index of the MFAPS of DGSE cannot be ruled out. This is consistent with the findings so far with other radio telescopes and other parts of the sky.

For the broken power law model, a break in the power law around 405 MHz suggests that there is a suppression of power above \( f_{\text{break}} = 405 \text{ MHz} \) and is due to “synchrotron age”. The observed value of spectral index above the break (\( \alpha_2 \)) is in between the JP (Jaffe-Perola) and the KP (Kardashev-Pacholczyk) model (Myers & Spangler 1985; Carilli, Perley, Dreher & Leahy 1991). The corresponding “synchrotron age” of the plasma is 80 Myr (using Eqn.1 of Carilli, Perley, Dreher & Leahy 1991). assuming average magnetic field \( B = 10 \mu \text{G} \). A deeper analysis of spectral variation of the MFAPS requires more sensitive and much wider bandwidth data which is outside the scope of this paper.

### 8 CONCLUSION

In this paper, we have shown deep observation of the ELAIS N1 field with the uGMRT at 300 - 500 MHz spanning a sky coverage of \( \sim 1.8 \) deg\(^2\). The field lies at high galactic latitude (\( b = +44.48^\circ \)) due to which it helps us to study extragalactic sources. Here we present the image of ELAIS N1 field and the catalog extracted from that image. The final image reaches a rms depth of \( \sim 15 \text{ µJy beam}^{-1} \) near the phase centre. The catalog presented here contains 2528 sources.

We have discussed in detail the comparison of our catalog with previous radio catalogs at other frequencies. We have found that flux scale is nearly consistent with other observations and the estimated ratio of flux densities of selected sample of sources when compared with other catalogs are close to 1. We have also checked for astrometry after comparing with high frequency catalogs. The positional offset typically constrained within \( \sim 0.5'' \). This ensures the good agreement of positional information of radio sources with other radio catalogs. A well constrained positional accuracy is needed for identification of sources in optical catalogs which in turn helps us to study spectroscopic property of those sources. We have not shown cross matching with multi frequency data (other than radio) available for this field here. This defers to later work. Finally, we have estimated spectral indices after comparing flux densities with other low and high frequency radio catalogs covering the ELAIS N1 field. We have found a median spectral index of \( \sim -0.7 \) after comparing with 1.4 GHz NVSS and FIRST catalog and with low-frequency GMRT observations of the ELAIS N1 field (610 MHz GMRT). A detailed investigation of spectral index using other frequency band data is deferred to later work.

We also present the Euclidian-normalized source counts below 1 mJy. This flattening corresponds to increase in population of SFGs and radio-quiet AGNs.

Finally, we have quantified the fluctuations in DGSE in this field and found out its evolution as a function of frequency. In general, DGSE is modelled as a simple power law both in angular and frequency domain. Although there is a hint of a broken power law in the MFAPS of DGSE, the sensitivity of the current observation prevents us from ruling out the single power law fit. Hence, more sensitive observations using much wider bandwidth data is required to infer conclusively.

It should be noted that foreground modeling is critical for redshifted 21 cm signal experiments. Any errors in modeling the foregrounds can affect the detection of redshifted HI 21 cm signal. This study of spectral variation of the DGSE will facilitate to create more sensitive spectral and spatial models of the foreground, in particular the DGSE. This study also helps us to understand the foreground properties in this field and will be helpful for next generation telescopes such as the LOFAR, PAPER, HERA, SKA, which are trying to detect the 21 cm signal from the EoR and post-EoR epoch.

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### REFERENCES

Alam S., Albareti F. D., Allende Prieto C., et al., 2015, ApJS, 219, 12
Ali S. S., Bharadwaj S., Chengalur J. N., 2008, MNRAS, 385, 2166
Barkana R., 2016, PhR, 645, 1
Battye R. A., Brown M. L., Browne I. W. A., et al., 2012, arXiv, arXiv:1209.1041
Bandura K., Addison G. E., Amiri M., et al., 2014, SPIE, 9145, 914522
Baars J. W. M., Genzel R., Pauliny-Toth I. I. K., Witzel A., 1977, A&A, 61, 99
Becker R. H., White R. L., Helfand D. J., 1995, ApJ, 450, 559
Becker R. H., Fan X. G., White R. L., et al., 2001, AJ, 122, 2850
Bernardi G., de Bruyn A. G., Brentjens M. A., et al., 2009, A&A, 500, 965
Bernardi G., Zwart J. T. L., Price D., et al., 2016, MNRAS, 461, 2847
Bharadwaj S., Sethi S. K., 2001, JApA, 22, 293
Bharadwaj S., Nath B. B., Sethi S. K., 2001, JApA, 22, 21
Bharadwaj S., Nath B. B., Sethi S. K., 2002, IAUS, 199, 108
Bharadwaj S., Pandey S. K., 2003, JApA, 24, 23
Bharadwaj S., Ali S. S., 2005, MNRAS, 356, 1519
Bowman J. D., Morales M. F., Hewitt J. N., 2009, ApJ, 695, 183
Shaver P. A., Windhorst R. A., Madau P., de Bruyn A. G., 1999, A&A, 345, 380
Simpson C., 2017, RSOS, 4, 170522
Sokolowski M., Tremblay S. E., Wayth R. B., et al., 2015, PASA, 32, e004
Swarup G., Ananthakrishnan S., Kapahi V. K., Rao A. P., Subrahmanya C. R., Kulkarni V. K., 1991, CuSc, 60, 95
Sirothia S. K., Dennefeld M., Saikia D. J., Dole H., Ricquebourg F., Roland J., 2009, ASPC, 407, 27
Singh S., Subrahmanyan R., Shanlark N. U., Rao M. S., Girish B. S., Raghu Nathan A., Somash S., Srivani K. S., 2018, ExA, 45, 269
Shimwell T. W., Tasse C., Hardcastle M. J., et al., 2019, A&A, 622, A1
Subrahmanya C. R., Manoharan P. K., Chengalur J. N., 2017, JApA, 38, 10
Taylor A. R., Jagannathan P., 2016, MNRAS, 459, L36
Thyagarajan N., Helfand D. J., White R. L., Becker R. H., 2011, ApJ, 742, 49
Thyagarajan N., Udaya Shankar N., Subrahmanyan R., et al., 2013, ApJ, 776, 6
Trott C. M., Pindor B., Procopio P., et al., 2016, ApJ, 818, 139
van Haarlem M. P., Wise M. W., Gunst A. W., et al., 2013, A&A, 556, A2
van Haarlem M. P., Wise M. W., Gunst A. W., et al., 2013, A&A, 556, A2
Vedantham H., Udaya Shankar N., Subrahmanyan R., 2012, ApJ, 745, 176
Visbal E., Loeb A., Wyithe S., 2009, JCAP, 10, 030
Windhorst R. A., Miley G. K., Owen F. N., Kron R. G., Koo D. C., 1985, ApJ, 289, 494
Wilman R. J., Miller L., Jarvis M. J., et al., 2008, MNRAS, 388, 1335
Williams W. L., Intema H. T., Röttgering H. J. A., 2013, A&A, 549, A55
Williams W. L., van Weeren R. J., Röttgering H. J. A., et al., 2016, MNRAS, 460, 2385
Wyithe J. S. B., Loeb A., 2008, MNRAS, 383, 606
Zaroubi S., de Bruyn A. G., Harker G., et al., 2012, MNRAS, 425, 2964
Zaldarriaga M., Furlanetto S. R., Hernquist L., 2004, ApJ, 608, 622

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