Atomic Kapitza-Dirac effect with quadrupole transitions

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Abstract

Interactions between atoms and light fields are usually described in the electric-dipole approximation. We show that electric-quadrupole terms are important in the Kapitza-Dirac arrangement for light gratings on resonance with a quadrupole atomic transition. We derive the diffraction patterns, which in some cases are experimentally verifiable with the same techniques used with dipole transitions.

Keywords: Diffraction by light gratings; Atomic quadrupole effects

1 Introduction

The interaction between atomic systems and light fields is usually described in the electric-dipole approximation. However, there are some scenarios where one must go beyond that approximation. In this paper we want analyze a situation where the electric-quadrupole term becomes important in the Kapitza-Dirac arrangement.

In the Kapitza-Dirac effect a beam of atoms or electrons is diffracted by a standing light wave, usually a laser [1, 2, 3, 4, 5]. Recently, the theory of the effect has been extended to two-particle systems [6, 7]. In the case of atoms, one chooses the wavelength of the optical grating in the proximity of an atomic transition, in order to enhance the strength of the interaction. Most times this is a dipole transition, and the dipole transition matrix element is the only relevant in the problem.

An interesting situation emerges when, instead, the optical wavelength is close to a quadrupole transition. Then the quadrupole transition matrix element becomes dominant. A similar situation was considered in [8] for the trapping of cold atoms in optical lattices.

We shall analyze in detail the scenario in order to estimate when the quadrupole-type diffraction is experimentally verifiable. We shall evaluate the diffraction
2 Quadrupole transitions

As signaled in the Introduction, if we choose the wavelength of the standing light wave close to a quadrupole transition, the quadrupole term will be dominant. This is the case when the light detuning with respect to the atomic frequency couples a s-type ground atomic state with an excited d-type one, instead of a p-type one as it is the case in dipole transitions. In Calcium, for instance, the lowest excitation is from this type.

In order to analyze this scenario we must consider the interaction ruling the transition. We closely follow the presentation in [8]. A beam of atoms interacts with a linearly polarized laser. We denote by $x$ the propagation direction of light and by $z$ the oscillation one. The relevant interaction in the problem is given by the potential

$$V = \frac{e}{mc} A(X, x, t) \hat{p}_z$$

with $m$ the mass of the electron, $\hat{p}_z$ the z-component of its momentum operator, $X$ the coordinate of the center of mass of the atomic system and $x$ the relative one [8]. Being essentially the Kapitza-Dirac effect an one-dimensional problem, we restrict our considerations to that longitudinal variable (parallel to the grating). The transversal or perpendicular variables are important determining the duration of the interaction, but do not play any additional relevant role in the diffraction pattern.

The electromagnetic potential at the position of the electron suffering the transition in the atom can be expressed as $A(x_e, t) = A(X + x, t)$. In the dipole approximation the dependence on the relative coordinate can be neglected and we have $A(x_e, t) \approx A(X, t)$. In contrast, when the quadrupole term is taken into account we must consider the next order in the multipole expansion:

$$A(x_e, t) \approx A(X, t) + x \left( \frac{\partial A}{\partial X} \right) (X, t)$$

If we choose, as usual, for the standing light $A(X) = A_0 \cos(k_L X) \cos(\omega_L t)$ with $k_L$ and $\omega_L$ the wave vector and frequency of the laser beam, the contribution of the quadrupole term will have the form $-A_0 k_L \sin(k_L X) \cos(\omega_L t)$.

With this expansion the interaction potential decomposes as

$$V = \frac{e}{mc} A(X, t) \hat{p}_z + \frac{e}{mc} x \left( \frac{\partial A}{\partial X} \right) (X, t) \hat{p}_z = V_D + V_Q$$

patterns, which are similar to those derived in the dipole approximation, but with different values of the parameters. If we consider a two-mode standing light wave containing simultaneously the dipole- and quadrupole-resonant frequencies we can observe a richer behaviour.
The optical potential \([9, 8]\) associated with this interaction is

\[
V_{\text{op}} = \frac{|\langle e|V|g \rangle|^2}{\hbar \Delta}
\]  

with \(e\) and \(g\) the excited and ground atomic states, and \(\Delta\) the detuning of this transition frequency with the laser one.

Oscillating the light variables with a periodicity much smaller than the time scale of the atomic center of mass we must average over that periodicity, \(\tilde{V}_{\text{op}} = T_{L}^{-1} \int_{0}^{T_{L}} V_{\text{op}} dt\), with \(T_{L} = 2\pi/\omega_{L}\). The quadratic dependence on \(V\) ensures that the optical potential does not average to zero.

The first type of contribution in \(V\) is dominant for dipole transitions and gives the usual dipole term

\[
\tilde{V}_{\text{op}}^{D}(X) = \frac{\hbar |\Omega_{D}|^2}{4\Delta} \cos^2(k_{L}X)
\]  

with the Rabi frequency \(\Omega_{D} = (eA_{0}/\hbar c m) < e|\hat{p}_{z}|g >\). Similarly, for quadrupole transitions the potential reduces to the quadrupole term

\[
\tilde{V}_{\text{op}}^{Q}(X) = \frac{\hbar |\Omega_{Q}|^2}{4\Delta} \sin^2(k_{L}X)
\]  

where now the Rabi frequency is \(\Omega_{Q} = (eA_{0}k_{L}/\hbar c m) < e|x\hat{p}_{z}|g >\). Note the additional presence of the factor \(k_{L}\) with respect to the dipole case. The explicit evaluation of the matrix elements for some atoms can be found in \([8]\).

In summary, we can write the potentials generated by the standing wave light as

\[
\tilde{V}_{\text{op}}^{D}(X) = V_{D}^{0} \cos^2(k_{L}X)
\]  

when the light grating frequency is close to a dipole transition, whereas for a quadrupole one we have

\[
\tilde{V}_{\text{op}}^{Q}(X) = V_{Q}^{0} \sin^2(k_{L}X)
\]  

As signaled before, the intensity of the potentials, \(V_{D}^{0}\) and \(V_{Q}^{0}\), is determined by the Rabi frequencies.

### 3 Diffraction patterns

The knowledge of the interaction potential allows one to evaluate the diffraction patterns. We shall consider two situations. The first one is the interaction of the atomic beam with a laser only having one mode, close to the quadrupole transition. The second one corresponds to a laser with two modes, one close to the dipole transition and the other to the quadrupole one. On the other hand, as it is well-known, there are two regimes in the Kapitza-Dirac effect, diffraction for thin light gratings and Bragg’s scattering for thick ones. Then for each situation we must consider separately the two regimes.
3.1 Patterns for a single-mode laser

We consider first diffraction. We assume that the momentum of the atoms is large compared to that of the photons. Then the kinetic energy remains approximately constant and may be neglected. We can use the Raman-Nath approximation. In addition to the Raman-Nath approximation two conditions must be fulfilled to derive the diffraction pattern in the usual way [9]. On the one side, the rate of spontaneous emission must be small during the interaction. Mathematically, this condition reads $\Delta \gg \Gamma$, where $\Delta$ is the detuning and $\Gamma$ the decaying rate. On the other hand, the evolution must be adiabatic, $\Delta > 1/\tau$, with $\tau$ the interaction time.

If the atom is initially in the state $\exp(ik_0X)$, the final one is

$$e^{iV_0^Q(X)\tau/\hbar} e^{ik_0X} = e^{iV_0^D\tau/2\hbar} \sum_{n=\infty}^{\infty} i^n J_n \left( \frac{V_0^Q\tau}{2\hbar} \right) e^{i(2nk_L+k_0)X}$$

where we have used the formula $\exp(i\xi \cos \phi) = \sum_{n=\infty}^{\infty} i^n J_n(\xi) \exp(in\phi)$ with $J_n$ the n-th order Bessel function.

Thus, up to irrelevant global phases, we have the same pattern obtained for dipole-type transitions. In an intuitive picture, the contributions to each peak correspond to even multi-photon interchange processes. The first one, $n = 1$, represents one-photon absorption followed by one stimulated emission at the same frequency ($k_0 \rightarrow k_0 + 2k_L$). Of course, $n = 0$, corresponds to no photon interchange. The only important difference is that the Bessel functions depend on $V_0^Q$ instead of $V_0^D$.

Next, we consider if these patterns can be studied experimentally. We give an approximate evaluation of their intensity, based on the values presented in [8]. Typical values of $V\tau/\hbar$ for the observation of Kapitza-Dirac diffraction are of the order of unity [2]. Some of the experiments demonstrating the effect have been carried out with Na atoms with the values $\tau \approx (1/14) \times 10^{-6}s$ and $V/\hbar \approx 18 \times 10^6 s^{-1}$, that is, we must consider potentials of the order of $V \approx 10^{-8} eV$. On the other hand, for the quadrupole-type decaying rate we have $\Gamma \approx 10^3 s^{-1}$; using a detuning $\Delta \approx 1 GHz$ (the detuning used in [8] in the numerical simulations) we guarantee adiabatic evolution and the absence of spontaneous emission. The relation between the maximum potential induced by a light grating in a frequency close to the quadrupole transition for Na atoms and the laser intensity has been evaluated in [8]. Using these data we have that the above values of the potential can be reached with an intensity of $10^8$ to $10^9 W/m^2$, not too different from those used in the dipole case. The same experimental procedure used in that case seems to be adequate for the verification of our proposal.

In the Ca case a potential of $10^{-8} eV$ can be reached with intensities close to $10^8 W/m^2$, for a detuning of $70 kHz$ [8]. Taking as in the Na case an interaction time of $10^{-7}s$ we need, in order to have an adiabatic evolution, a detuning of
\( \Delta \approx 10^8 s^{-1} \). Note that with this choice the rate of spontaneous emission is negligible, because for the Ca case we have \( \Gamma \approx 300 s^{-1} \). Assuming a relation of the type \( V \sim I/\Delta \) (as it is true in the perturbative case [8]), we would need an intensity \( I \approx 10^{11} W/m^2 \) for \( \Delta \approx 10^8 s^{-1} \), that is, four orders of magnitude above the usual values in this type of experiments. For these intensities, one generates the standing wave using counter-propagating laser pulses, whose duration is only about 10ns [4], shorter than the interaction time, making useless the usual schemes for our system. We conclude that the case of Na atoms can be tested with the standard techniques, being the Ca case much more demanding.

In Bragg’s scattering, a simple calculation (see, for instance [2], replacing the cos-type function by a sin-type one) shows that the transmission and scattering amplitudes are the same of the dipole approximation with the change of \( V_D^0 \) by \( V_Q^0 \). In particular, the transmission and scattering probabilities are

\[
P_{tra} = \cos^2\left(\frac{V_Q^0}{4\hbar}\right); \quad P_{sca} = \sin^2\left(\frac{V_Q^0}{4\hbar}\right)
\]

For real experiments with Na (and dipole transitions) the interaction times are of the order of \( 10^{-5} s \) and the potentials of \( 10^{-10} eV \) [2]. The detuning is again of the order of 1GHz. For the Ca and Na atoms these values can be reached with laser intensities of, respectively, \( 10^8 \) and \( 10^7 W/m^2 \) [8]. Although these intensities are lower than in the case of diffraction, the demonstration of Bragg’s scattering is in general more demanding, because now the focusing of the laser and the coherence properties of thick standing waves are more difficult to implement [5].

### 3.2 Patterns for a two-mode laser

Now, we consider a two-mode laser. One of the modes has a frequency close to the quadrupole transition, and the other to the dipole one. We consider, for simplicity, the case of the Na atom which, as seen before, seems to be accessible to the experimental scrutiny.

First of all, we need the potential for the light-atom interaction. In the case of two laser modes inducing dipole transitions the potential can be approximately expressed as the sum of the two optical potentials associated with the modes (see, for instance, a short discussion in [10]). In the case of two modes related to dipole and quadrupole transitions we expect a similar behavior, specially taking into account that the crossed effects between the two transitions can be neglected. Then we can use a two-mode light potential of the type

\[
V_{TM} \approx V_D^0 \cos^2(k_D x) + V_Q^0 \sin^2(k_Q x)
\]

In general, \( V_D^0 \) and \( V_Q^0 \) can be different, although we choose them of the same order of magnitude. We denote by \( k_D \) and \( k_Q \) the wave vectors of the
laser modes associated with the dipole and quadrupole transitions. The wave function after the interaction is

\[ e^{i\frac{V_T M(X)\tau}{\hbar}} e^{i\frac{V_0^D \tau}{2\hbar}} e^{i\frac{V_0^Q \tau}{2\hbar}} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} i^{n+m} \times \]

\[ J_n \left( \frac{V_0^D \tau}{2\hbar} \right) J_m \left( \frac{V_0^Q \tau}{2\hbar} \right) e^{i(2nk_D+2mk_Q+\kappa_0)X} \]

(12)

Figure 1: Probability of detection of atoms in the lower diffraction orders versus the dimensionless parameter \( w = \frac{V_0^D \tau}{\hbar} \). The black and red curves represent respectively the single- and two-mode cases. The continuous, dashed and dotted curves correspond to the values \( n = 0 \) (and \( m = 0 \) for two modes), \( n = 1 \) (and \( m = 0 \) for two modes), and \( n = 0 \) and \( m = 1 \) (only for two modes). We use for the intensity of the potentials the values \( V_0^D = 1 \) and \( V_0^Q = 0.8 \).

Several interesting properties easily follow from this expression. The probability for the central or no-diffraction peak, \( n = m = 0 \), is \( J_0^2 \left( \frac{V_0^D \tau}{2\hbar} \right) J_0^2 \left( \frac{V_0^Q \tau}{2\hbar} \right) \) that differs from the single-mode probability for this case. Having two channels of diffraction, the contributions of both must be present. For any other diffraction order the number of peaks doubles. For instance, for the first order we have the peaks \( n = \pm 1 \) (\( m = 0 \)) and \( m = \pm 1 \) (\( n = 0 \)), which take place for
different momentum interchanges, \( \pm 2h k_D \) and \( \pm 2h k_Q \). The amplitude of, for instance, the peak \( n = 1 \ (m = 0) \) is \( J_1^2(V_0^D \tau/2\hbar)J_0^2(V_0^Q \tau/2\hbar) \). It implies again a modification with respect to the single mode case: the detection probability in that channel is modulated by the coefficient \( J_0^2(V_0^Q \tau/2\hbar) \), associated with the presence of a second channel.

We represent the amplitudes of the non-diffraction and first diffraction peaks in Fig. 1. The detection probabilities are always smaller in the two-mode arrangement.

Finally, we consider Bragg’s scattering in the two-mode scenario. In order to have Bragg’s scattering, the modulus of the longitudinal wave vector of the incident particle must be equal (or very close) to one of the wave vectors of the laser modes, that is, \( k_0 = \pm k_D \) or \( k_0 = \pm k_Q \). Being \( k_D \neq k_Q \) we can only have a type of Bragg-scattering. At variance with the diffraction regime, we do not have modifications of the scattering patterns in the two-mode case.

4 Conclusions

We have analyzed in this paper the atomic Kapitza-Dirac effect with quadrupole transitions, instead of the dipole ones usually considered in the literature. The effect is, in principle, accessible to experimental scrutiny. For some atoms (for instance, Na) the diffraction patterns associated with quadrupole atomic transitions are observable with experimental parameters (laser intensity,...) close to those associated with dipole transitions.

In addition to the demonstration of the existence of observable quadrupole effects in this regime of the matter-light interaction our scheme could be interesting in some other aspects. Our proposal would be the first example of a diffraction arrangement driven by a quadrupole force. From a more practical point of view, the diffraction of atoms can be used to measure the transition quadrupole matrix elements, by fitting the experimental data to the distributions \( |J_n(V_0^Q \tau/2\hbar)|^2 \). The values of the matrix elements are of interest to validate calculational methods.

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