ON THE MECHANISM OF HAWKING RADIATION

V.A. Berezin, A.M. Boyarsky† and A.Yu. Neronov‡

† Institute for Nuclear Research of the Russian Academy of Sciences
60-th October Anniversary Prosp., 7a, 117312, Moscow, Russia

‡ Theoretische Physik, Universität München, D-80333, München, Germany

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In our previous papers the first step was made to the construction of a global wave function on the configuration space of a self-gravitating shell. The asymptotic behaviour of analytical wave functions at the infinities was analyzed. As a result, a discrete mass spectrum of a quantum black hole and a discrete spectrum for the Hawking radiation were found. In the present paper we study a global quasiclassical solution inside and outside the horizon. The result is rather unexpected: for a quasiclassical solution with two waves of equal amplitudes under the horizon we obtain, in the outer region of the black hole, ingoing and outgoing waves with the amplitudes $Z_{\text{in}}$ and $Z_{\text{out}}$ such that $Z_{\text{in}}/Z_{\text{out}} = \exp(-\delta A/(4m_{\text{pl}}^2))$ where $A$ is the black hole horizon area. This result exactly coincides with the main result of the Hartle and Hawking consideration [21], from which one can derive the value of the black hole temperature and entropy.

1. Introduction

Conventional quantum field theory is a set of quantum mechanics for $N$-particle states and transitions between them. Its starting point is a Fock space spanned by the states of (approximately) free particles with different values of energies and momenta. Having at first a quantum mechanical description of these particles, we construct their Hilbert space and then take into account the processes of creation and annihilation, thus secondary quantizing the theory. But it is now well known that $N$-particle states at the Planckian scale could be rather different from free particle states. Indeed, there is no free particle state for particles with trans-planckian energies moving in different directions. The Compton wavelength in this case is smaller than the gravitational radius of the particle. Such particles will inevitably form black holes (see e.g. [1]).

So it is natural, when dealing with quantum field theory at the Planckian scale, first to try to construct a Hilbert space containing $N$ gravitating particles before second quantization.

At first sight, this $N$-particle quantum mechanics is introduced only as a technical tool regularizing quantum field theory at the Planckian scale. This could be done in different ways. In particular, there could be different models of quantum mechanics of gravitating particles. But fortunately there is a language for studying the physical content of these models at already the quantum-mechanical level. Namely, since the “black hole states” should be present in Hilbert space, this first-quantized quantum mechanics should be suitable for describing such physical phenomena as the Hawking radiation, mass spectrum, black hole entropy. Analyzing certain predictions about these phenomena (like the Hawking radiation spectrum or the black hole entropy formula) given by different quantum mechanics, one could choose among them. But in gravitational models we have some special property: the gravitational field is not just the field responsible for the gravitational interaction between particles, but it also determines the global structure of the spacetime. Usually the space-time itself is the configuration space for particle dynamics. So, in general, when we take into account the gravitational interaction, the structure of the “configuration space” becomes dependent on the initial conditions, as could be observed by analyzing classical solutions. Hence it is impossible to define a configuration space for gravitating particles naively. On the quantum-mechanical level another problem arises. In quantum mechanics we are supposed to have a superposition of different classical states. It means that in the case of a self-gravitating system we are forced to work in terms of superposition of different space-time geometries. There have been plenty of attempts to describe this picture but it still remains a puzzle how to construct any field theory on such “quantum space-time”.

This problem is the most important if we want to construct a field theory taking into account the gravitational interaction at the Planckian scale. As a matter of fact, it has very little to do with the problem of scattering, creation and annihilation of gravitons. Being present in field theory, the problem of “quantum space-time” arises already at the quantum-mechanical level, i.e. for systems with a finite number of degrees
of freedom. Thus we can try to solve it in this simplest case when it is easier to obtain a correct definition of the configuration space.

How can we construct a quantum mechanics of gravitating particles without facing the problems of creation and annihilation of gravitons? A possible approach to this problem is to take into account only some global degrees of freedom of the gravitational field relevant for the dynamics of gravitating particles. These degrees of freedom are topological in the absence of sources.

It is well known that gravitational theories contain topological degrees of freedom. For example, 2+1 gravity is pure topological and equivalent to the Chern-Simons theory with the gauge group $ISO(2,1)$. Spherically symmetric 3+1 gravity without matter is also topological due to the Birkhoff uniqueness theorem.

On the other hand, in some complicated cases, when the theory is not topological, one can look for its topological sectors. If we are interested only in some particular observables, it could be enough to study quantum mechanics of these global degrees of freedom. For example, we could separate the dynamics of global degrees of freedom from the dynamics in volume by extracting some important surface terms from the action functional thus obtaining a field theory on the boundary surface.

The corresponding field theories in such topological models should have a finite number of physical degrees of freedom. Thus the quantum-mechanical phase space is present in the problem from the very beginning. Then one has to add some sources and the proper Hamiltonian which results in the transition from topological to physical theory, as could be easily seen, for example, in the 2D Yang-Mills case (cf. § and refs. therein). As a result, one has a quantum-mechanical problem (it is often integrable) describing the dynamics of these topological degrees of freedom. Mathematically this finite-dimensional system is obtained by Hamiltonian reduction.

Let us try to apply this approach to gravitating particles. To construct a quantum mechanics for such a topological field theory with a particle we need first to clarify what is the "classical phase space". The latter can be defined as the space of all classical solutions modulo gauge transformations. For the case of self-gravitating particles in $2+1$ dimensions this approach was used in [2].

In what follows (as in our previous papers) we will study maybe the simplest gravitational model of the type described above — spherically-symmetric gravity with a self-gravitating thin dust shell (see [11][10][14]).

In this model we see among the classical solutions space-time manifolds with different geometries. In Fig. 1 the geometry of the complete Schwarzschild space-time is shown. It contains two isometric regions with two singularities (future and past) at $R = 0$ and two infinities in the left and right asymptotically flat regions $R_+$ and $R_-$. In Fig. 2 different types of the corresponding Carter-Penrose diagrams are presented. In the “black hole case” (a) the turning point of the shell lies in the $R_+$-region and at this point it can be seen by an observer at the right infinity. In the “wormhole case” (b) the turning point of the shell is on the opposite side of the Einstein-Rosen bridge in the $R_-$-region and could not be seen from $R_+$-infinity. In the case of unbounded motion (c) the shell starts from the infinity $R = \infty$ in the $R_+$-region and collapses to $R = 0$ forming a singularity.

All these classical configurations should be present in the finite-dimensional phase space of the gravitating shell.

Due to the high symmetry it is possible in this model to fulfil the above reduction explicitly. In the case of spherically symmetric gravity without matter such a reduction was made by K. Kuchař ([12]). The resulting “topological” gauge-invariant degree of freedom is a variable $m$ (and the corresponding momentum) which is defined by the boundary conditions at the infinity and is nothing but the Schwarzschild mass measured by an observer at infinity. If a thin dust shell is included as a source, we have (after reduction) another “physical” degree of freedom, describing the shell motion. The variable $m$ enters into the resulting equation for the gravitating shell and can be formally considered as a parameter. There is another parameter, the bare mass of the shell $M$. In classical mechanics, cases (a) and (b) are realized for bounded motion of the shell when $M/m < 2$ and $M/m > 2$, respectively.

There were several attempts to construct both classical and quantum mechanics for such a system. For example, in [15] the reduced phase space for a self-gravitating shell was constructed as a set of initial data for the black hole case (a) only. In Ref. [16] the local wave function of a self-gravitating null shell was found to describe the effects of back reaction in non-thermal corrections to the spectrum of the Hawking radiation. But in quantum mechanics the wave function should be defined over the whole configuration space and it is of crucial importance to construct a global picture for the dynamics of the system in hand. Some important results, such as the quantization conditions, can be obtained only from global properties of the wave function. So it is necessary in our case to construct a global configuration space taking into account all classical solutions, in particular, the “wormhole” case (b)
as well as the “black hole” case (a) for any value of the ratio \( M/m \). But for each particular value of this ratio the part of the global configuration space representing classical solutions with the opposite sign of \((M/m - 2)\) should be classically forbidden in an effective quantum mechanics of the self-gravitating shell. For example, in the black hole case \((M/m < 2)\) the part of the configuration space representing wormhole classical solutions is classically forbidden.

This is the first qualitative difference of the self-gravitating shells motion from test particle (shell) motion on a fixed Kruskal background when the shell can move in all parts of the Carter-Penrose diagram irrespective of the value of \( M/m \). The appearance of an additional classically forbidden region in the configuration space, where the wave function should exponentially decrease, results in a new quantization condition for the parameters \( M \) and \( m \) \((10, 14)\). This effect (whose physical consequences are discussed in \((14)\) and will be discussed below) can be illustrated by the following simple quantum-mechanical example. Let us consider the following radial Schrödinger equation:

\[
\frac{d^2 R(r)}{dr^2} + \frac{2dR(r)}{r \; dr} + \frac{2m}{\hbar^2} \left( E - A \frac{r^2}{r^2} + B \right) R(r) = 0.
\]

Then, let us suppose that the asymptotic behaviour of the wave function at negative infinity \( r \rightarrow -\infty \) along the real line is also important for some physical reasons (for example, the true configurations variable is the area \( s = r^2 \), and the classical configuration space is a positive semi-axis \( s > 0 \)). In this case it is easy to see from the exact solution (see \((14)\)) that this new requirement, together with usual ones at \( r \rightarrow \infty \) and \( r \rightarrow 0 \), gives not only a quantization condition for the parameter \( E \), but also one more quantization condition, so that the parameter \( A \) is quantized as well.

In \((10)\) the formalism was constructed to describe global properties of the configuration space and globally-defined quantum mechanics for the case of a self-gravitating shell. In this case, after reduction, using the Kuchař gauge-invariant variables, we are left with the only nontrivial equation describing the dynamics of the shell. Formally this dynamics is one-dimensional. The variable \( \bar{R} \) which describes the position of the shell is gauge-invariant and has the meaning of the shell radius. But to parameterize the whole configuration space it is not enough to have \( \bar{R} \) varying from 0 to \( \infty \). This can be easily seen from the observation that it is impossible to distinguish black-hole-type classical solutions from the wormhole-type solutions in terms of the variable \( R \) (see Fig. 2). This variable covers the configuration space twice. Fortunately there is a way to avoid this difficulty. The equation which governs the dynamics of the shell is an equation in finite differences. The shift of the argument of the wave function in this equation occurs along the imaginary axis, and this means that the equation is actually defined not on the real line, but over some complex manifold. The equation for the shell dynamics contains the square root of the “Schwarzschild factors” \( \sqrt{1 - \frac{2m}{r}} \). So the natural complex manifold for the equation is the Riemannian surface \( SF \) on which the coefficients of the equation are analytical functions. This Riemannian surface is just a two-(real)dimensional sphere obtained after gluing two complex planes along the sides of the cuts made on each plane along the interval between the branching points of the coefficients on the real line. The configuration space for our self-gravitating shell is the real section of \( SF \). This configuration space properly represents different classical solutions for the self-gravitating shell (for a detailed analysis see \((14)\) and \((18)\)). All main results from our models are due to the non-trivial structure of the configuration space. This real section \( \text{Im}(\rho) = 0 \) of the Riemannian surface \( SF \) covers the real line twice, as is shown in Fig. 8. It consists of the \( V_+ \), \( T_+ \) and \( R_+ \) intervals. The sign of the branching function is taken to be + on the intervals \( V_+ \), \( R_+ \) and - on the intervals \( T_- \), \( R_- \) and \( (F_{in}F_{out})^{1/2} = \pm i \sqrt{|F_{in}F_{out}|} \) on \( T_\pm \) respectively.

In \((10)\) \((13)\) the first step to the construction of a global wave function on such a complicated configuration space was made. The asymptotic behaviour of the analytical wave functions at the infinities in \( R_+ \) and \( R_- \) regions was analyzed. As a result, a discrete mass spectrum of bound states and a discrete spectrum for infinite motion of the system were found. Analyzing these two spectra, the Bekenstein-Mukhanov mass spectrum for black holes \((17)\) was obtained.

In the present paper we study a global quasiclassical solution for the \( V_+ \), \( T_+ \) and \( R_+ \) regions. The result is rather unexpected — for a quasiclassical solution with two waves of equal amplitudes under the horizon we obtain, after analytical continuation in the \( R_+ \) region, ingoing and outgoing waves with the amplitudes \( Z_{in} \) and \( Z_{out} \). Namely,

\[
Z_{in}^2/Z_{out}^2 = \exp\{-\delta A/(4m^2_{pl})\}
\]

where \( A \) is the black hole horizon area. This exactly coincides with the main result of the Hartle and Hawking consideration \((21)\) from which one can derive the values of the black hole temperature and entropy.

2. Quantum mechanics of self-gravitating massless particles

As was shown in \((10)\) \((14)\), the radial relativistic Schrödinger equation for the massless self-gravitating null-dust shell has the form

\[
\Psi(S + i\zeta) + \Psi(S - i\zeta) = \frac{F_{in} + F_{out}}{\sqrt{F_{in}F_{out}}} \Psi.
\]
Here
\[ S = R^2/(4G^2m^2) \]  
(3)
is a dimensionless variable which measures the area of the shell (\( G \) is the gravitational constant and \( m = m_{\text{out}} \) is the Schwarzschild mass of the black hole as seen by an observer at infinity). The dimensionless shift parameter is
\[ \zeta = m_{\text{pl}}^2/(2m^2); \quad m_{\text{pl}} = \sqrt{\hbar/c/G} \]  
(4)
\((m_{\text{pl}} \text{ is the Planck mass}). The functions \( F_{\text{in, out}} \) are just the coefficients of the Schwarzschild metric inside and outside the shell:
\[ F_{\text{out}} = 1 - 1/\sqrt{S}, \quad F_{\text{in}} = 1 - \mu/\sqrt{S} \]  
(5)
\((\mu = m_{\text{in}}/m_{\text{out}} \text{ is the quotient of the Schwarzschild masses inside and outside the shell}). If we suppose that the energy of the null shell
\[ \epsilon = m_{\text{out}} - m_{\text{in}} \]  
(6)
is much smaller than the black hole mass \( m_{\text{out}} \), then
\[ m_{\text{in}} \approx m_{\text{out}} = m; \quad \mu \approx 1 - \epsilon/m. \]  
(7)
So in the limit of test particles (when the back reaction is not taken into account) we can expand all the quantities in powers of the small parameter \( \epsilon/m \).

Another limiting situation is when the shift parameter is small compared to a characteristic scale on which the wave function varies significantly, then one can approximate the shifted wave function \( \Psi(\zeta) \) by its Taylor expansion near \( S \), so that Eq. (2) takes the form
\[ -\zeta^2\Psi''(S) + 2\Psi(S) = \frac{F_{\text{in}} + F_{\text{out}}}{\sqrt{F_{\text{in}}F_{\text{out}}}}\Psi(S), \]  
(8)
which is just the usual Schrödinger equation of nonrelativistic quantum mechanics. This is natural because the limit \( \zeta \to 0 \) (or \( m_{\text{pl}} \to 0 \)) is either nonrelativistic, or classical, or the limit of weak gravitational field. Thus we see that the coefficient in the right-hand side of Eq. (2) plays the role of a potential term for the motion of the null shell in a gravitational field.

One important note is that the “truncated” equation (5) is certainly not valid in the vicinity of the in- and out- horizons — they are singular points of Eq. (2) and we cannot assume that \( \Psi \) varies slowly there.

The shift of the argument of the wave function in Eq. (2) is along the imaginary axis, this means that the equation is actually defined not on the real line, but over some complex manifold. The natural complex manifold for Eq. (2) is the Riemannian surface \( S_F \) of the branching function
\[ (F_{\text{in}}F_{\text{out}})^{1/2} = \frac{(\rho - 1)(\rho - \mu)^{1/2}}{\rho} \]  
(9)
\((\rho = \sqrt{S}) \) on which the coefficients of the equation are analytical functions. This Riemannian surface \( S_F \) is just a two-(real)-dimensional sphere obtained after gluing two complex planes along the sides of the cuts made on each of them along the interval \( \rho \in (\mu, 1) \) of the real line. The configuration space for our self-gravitating shell is the real section of \( S_F \). This configuration space represents properly a different classical solution for a self-gravitating shell (for a detailed analysis see [13] and [15]). All main results in our models are due to a non-trivial structure of the configuration space. This real section \( \text{Im}(\rho) = 0 \) of the Riemannian surface \( S_F \) covers the real line twice, as is shown in Fig. 3. It consists of the intervals \( V_\pm, T_\pm \) and \( R_\pm \). The sign of the branching function (9) is taken to be + on the intervals \( V_+, R_+, \) — on intervals \( T_-, R_- \), and \((F_{\text{in}}F_{\text{out}})^{1/2} = \pm i\sqrt{|F_{\text{in}}F_{\text{out}}|} \) on \( T_\pm \), respectively.

### 3. Quasiclassical wave function

In our previous papers the main attention was devoted to the behaviour of the wave function at the infinities in the \( R_+ \) and \( R_- \) regions. Here we study the properties of the quasiclassical wave function near the horizons \( s = 1 \) and \( s = \mu \). Consider the quasiclassical solutions of Eq. (2) in the form
\[ \Psi = \exp \left\{ \frac{i\Omega(S)}{\zeta} \right\} (\phi_0 + \zeta\phi_1 + \ldots). \]  
(10)
Substituting (10) into (2), one gets in the zero order in \( \zeta \) the Hamilton-Jacobi equation for \( P_S = \partial\Omega/\partial S \):
\[ \cosh\{P_S\} = \frac{F_{\text{in}} + F_{\text{out}}}{2\sqrt{F_{\text{in}}F_{\text{out}}}} \]  
(11)
whence it follows
\[ P_S = \pm \ln \left( \frac{F_{\text{out}}}{F_{\text{in}}} \right)^{1/2} = \pm \ln \left( \frac{\sqrt{S} - 1}{\sqrt{S} - \mu} \right)^{1/2}. \]  
(12)
The \( \pm \) signs correspond to expanding and collapsing trajectories of the null shell.

The points \( S = S_0 \) where \( P_S = 0 \) are the turning points of the shell classical motion. From (12) it is easy to see that there are no such points for the null-shell motion. But the points \( S = 1 \) and \( S = \mu^2 \) (the points where the apparent horizons of the internal and external Schwarzschild metrics are situated) are singular points of Eq. (2). The quasiclassical anzatz (10) is not a good approximation for solving Eq. (2) near these points.

In the region \( S \in (\mu^2, 1) \) between the horizons the momentum \( P_S \) (12) has an imaginary part, so this is an analogue of the “classically forbidden” region if we use the analogy with nonrelativistic quantum mechan-
ics provided by the form \( S \) of Eq. (2)\(^2\). But it should be noted that this analogy is not direct because this region is not entirely forbidden classically: of course, we have in this region a trajectory of a particle falling to the black hole singularity. The true origin of this “special region” is that it is indeed classically forbidden for a particle trajectory going outside the black hole. In some sense, the origin of this region is quite different from that of the region situated between \( S = \mu^2 \) and \( S = 0 \). The result is that we have only an ingoing (or outgoing) wave in \( T_\pm \) regions, respectively. The wave in the opposite direction is enormously damped near the horizon relative to the “correct” quasiclassical waves in each regions. This quasiclassical picture reproduces the classical behaviour of the shell.

If the energy \( \epsilon \) of the shell is small as compared to the mass of black hole, then \( \mu \) is close to 1 and the region situated between \( S = \mu^2 \) and \( S = 1 \) is very narrow and the contribution of the damped waves is not negligible. In what follows we will try to take this contribution into account and to determine its physical meaning.

### 3.1. States inside and outside the horizon

Another important feature in the present consideration which differs from the situation described in Eq. (11), when \( \mu \) was equal to zero, is the appearance of the region \( V_\pm \). In this region we also have two real solution of Eq. (11) and thus two different quasiclassical waves. But the nature of this region is quite different from that of the region \( R_\pm \). We should stress that the coordinate \( R \) under the horizon (including the region \( V_\pm \)) is actually a time coordinate, and the quasiclassical wave function

\[
    \Psi \sim \exp \left\{ \frac{i}{\zeta} \int_S P_S d\tilde{S} \right\} 
\]

with \( P_S > 0 \) represents a wave moving forward in time, while the solution with \( P_S < 0 \) represents a wave moving backward in time.

The classical particle trajectory under the horizon which starts near \( R = 0 \), propagates backward in time up to the horizon, then is reflected from it and then propagates forward in time back to the singularity \( R = 0 \).

According to the usual interpretation of waves propagating backward in time we might treat them as antiparticles which propagate forward in time, but with the opposite sign of energy. This is clear from Eq. (11). The energy of the particle is \( \epsilon = \delta m = m_{\text{out}} - m_{\text{in}} \). If we take the solution of (11) with the minus sign before the logarithm in the r.h.s., we can write

\[
    P_S = -\ln \frac{F_{\text{in}}}{F_{\text{out}}} = +\ln \frac{F_{\text{out}}}{F_{\text{in}}} = +\ln \frac{\tilde{F}_{\text{in}}}{F_{\text{out}}} 
\]

where \( \tilde{F}_{\text{in}} = F_{\text{out}} \) and \( \tilde{F}_{\text{out}} = F_{\text{in}} \). This means that \( n_{\text{out}} = m_{\text{in}} \), \( \tilde{m}_{\text{in}} = m_{\text{out}} \) and \( \tilde{E} = -E \) — instead of treating part of the trajectory as a trajectory of a particle of energy \( E \) propagating back in time, we can treat as that of a particle of energy \(-E\) propagating forward in time. Each solution describes the situation when either a particle or an antiparticle eventually falls into the singularity because there is a probability flow directed to \( R = 0 \). Now we have to make the first step in the construction of the global quasiclassical solution in the configuration space: we should glue the waves in the \( V_\pm \) region with the waves in the \( R_\pm \) region. This may be done as usual by analytical continuation through the complexified configuration space. We will not consider below the continuation of the solutions to the \( R_\pm \) and \( V_\pm \) regions, so the above continuation will actually be made through the ordinary complex plane.

The integral in (13) can be calculated explicitly in the case in question and takes the form

\[
    \int_S P_S d\tilde{S} = \int x \ln \frac{x - 1}{x - \mu} dx \\
    = \frac{(x - a)^2}{2} \left[ \ln(x - a) - \frac{1}{2} \right] \\
    + a(x - a) \ln(x - a) - 1 \right|_{a=1}^{a=\mu} 
\]

(here \( x = \sqrt{S} \)).

We can continue this expression analytically from \( V \) to the \( R \) region along the contour situated far from the branching points \( x = 1 \) and \( x = \mu \). The only result of such a continuation is the appearance of the additional terms \( i \pi \) in each logarithm:

\[
    \frac{(x - a)^2}{2} \left( \ln(x - a) - \frac{1}{2} + \pi i \right) 
\]
As a result, we obtain that the quasiclassical wave function acquires, after the analytical continuation, an additional factor in its amplitude

\[
\Psi = A \exp \left\{ \int_S P_S d\tilde{S} \right\}.
\]

The second solution acquires the reversed factor

\[
\Psi = A \exp \left\{ -\int_S P_S d\tilde{S} \right\}.
\]

So for a state with both waves having equal amplitudes in the \( V_+ \) region we have in the \( R_+ \) region outgoing and ingoing waves with amplitudes related to each other as in \( (16), (17) \). The validity of the above consideration depends now on the following important problem to be analyzed. The quasiclassical ansatz \( (13) \) is not a good approximation for the true wave function not only at the points \( S = 1 \) and \( S = \mu^2 \) — the branching points of the momentum \( P_S \) — but also at the so-called Stokes lines (see e.g. \( [22] \)). These lines are solutions of the equation

\[
\text{Im } \int P_S d\tilde{S} = 0.
\]

To take seriously the above analytical continuation, we must be sure that the path in the complex plane along which we continue our quasiclassical solution does not intersect Stokes lines. Otherwise we can lose some important part of the quasiclassical solution which is exponentially small compared with the wave \( (13) \) before the Stokes line but becomes large after the intersection. But fortunately in our case it can be easily seen (analytically as well as numerically) that we can reach the region \( (1, \infty) \) from \( (0, \mu^2) \) through the complex plane without intersecting the Stokes lines.

### 4. Hawking radiation spectrum

Eq. (2) is a field equation for first-quantized self-gravitating massless particles in the field of a black hole. (We suppose that it is this equation that must replace the radial Klein-Gordon equation for the \( s \)-modes of a scalar field if we want to take into account its back reaction onto the gravitational field of the black hole.)

We suppose that the vacuum state of second quantized theory inside the horizon consists of zero-mode oscillations of particles with different energies. The natural property of the vacuum would be that all the possible zero modes with different energies \( \epsilon \) are present with the same amplitude. A particle-antiparticle pair with the energy \( \epsilon \) falling into the singularity is presented in our model as a solution which consists, under the horizon, of both quasiclassical waves (forward and backward with respect to the variable \( R \) which is time-like in the \( V \)-region) with equal amplitudes.

Now from the previous section we know that such a state under the horizon gives us the ingoing and outgoing waves with the amplitudes \( Z_\text{in} \) and \( Z_\text{out} \),

\[
P = \frac{Z_\text{in}}{Z_\text{out}^2} = \exp \left\{ -2\pi \frac{1 - \mu^2}{\zeta} \right\}.
\]

Let us look at the last formula in more details. We must recall the definition of \( \zeta \) and rewrite \( (19) \) in a more convenient form:

\[
P = \exp \left\{ -\frac{2\pi^2 m^2_{\text{out}} (m^2_{\text{out}} - m^2_{\text{in}})}{m^2_{\text{pl}}^2} \right\} = \exp \left\{ -\frac{4\pi^2 \delta R^2}{m^2_{\text{pl}}^2} \right\}.
\]

Introducing the area of the horizon \( A \), we obtain finally

\[
P = \exp \left\{ -\frac{1}{4} \frac{\delta A}{m^2_{\text{pl}}} \right\}.
\]

This result precisely coincides with the main result of Hartle and Hawking in \([21]\). Following their line of reasoning, we can treat this probability distribution as the Gibbs distribution

\[
P = \exp \{-\delta m/T\}.
\]

We see that it follows from the comparison of the two distributions that the correct mass formula for the Schwarzschild black hole is valid:

\[
\delta m = T \delta A/4.
\]

Thus we have arrived at the conclusion that our first-quantized model for a self-gravitating particle describes such an important phenomenon of black-hole physics as the Hawking radiation.

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