$w_d = -1$ in interacting quintessence model

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Abstract
A model consisting of quintessence scalar field interacting with cold dark matter is considered. Conditions required to reach $w_d = -1$ are discussed. It is shown that depending on the potential considered for the quintessence, reaching the phantom divide line puts some constraints on the interaction between dark energy and dark matter. This also may determine the ratio of dark matter to dark energy density at $w_d = -1$.

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1 Introduction
Dark energy model is a candidate to explain the present accelerated expansion of the universe [1]. In this scenario 70% of the universe is assumed to be permeated by a smooth energy component with negative pressure, dubbed as dark energy. A simple model introduced to describe dark energy is a scalar dynamical field with a suitable potential [2]. In this model an exact solution for Friedmann equations is only accessible for some kinds of potentials [3].

Based on recent astrophysical data, it seems that the dark energy component has an equation of state parameter, $w_d \simeq -1$, in the present epoch [4]. Therefore to study the dynamical behavior of dark energy in the present era, instead of trying to find an exact solution, one can restrict himself to the region $w_d \simeq -1$, where equations can be solved approximately.

In a noninteracting dark energy model, it is expected that density of dark matter ($\rho_m$), in the present epoch, be very less than the density of dark energy component ($\rho_d$). This lies on the fact that the equation of state (EoS) parameter of dark energy, $w_d$, is less than $-\frac{1}{3}$, therefore it redshifts more slowly than the (dark) matter component. But the ratio of dark matter
to dark energy, \( r \), is of order unity in the present epoch: \( r \sim 3/7 \), this is known as the coincidence problem [5]. This forces us to consider interaction between dark energy and matter which allows energy exchange between these components [6].

In this paper we assume that the universe is filled with a scalar field, whose EoS satisfies \(-1 \leq w_d < -\frac{1}{3}\) (dubbed as quintessence), and (cold) dark matter with mutual non-gravitational interaction. As we have mentioned obtaining an exact solution for the Friedman equations in the presence of interactions even for simple potentials is not straightforward. Hence using some natural conditions which must be obeyed by the quintessence field in the vicinity of \( w_d \approx -1 \), we seek the required conditions (posed on the interactions and scalar potentials) allowing the system to reach at \( w_d = -1 \) without requirement to obtain an analytic and an exact solution for the problem. So using this method one can show that if a proposed quintessence model with a specific potential and interaction permits reaching to \( w_d \approx -1 \) in the present era or not.

Besides, conditions on the interaction term and the potential which may be functions of energy densities may pose some natural conditions on the ratio of dark matter to dark energy density showing whether the occurrence of coincidence problem near the era where \( w_d = -1 \) is occurred is only a coincidence.

At the end, using the interaction \( Q = H(\lambda_m \rho_m + \lambda_d \rho_d) \) and via Taylor series expansion we illustrate and emphasize our results.

We use units \( G = k_B = c = 1 \) throughout the paper.

### 2 Quintessence model and \( w_d = -1 \)

The spatially flat Friedmann Robertson walker space time is described by the metric

\[
ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2),
\]

where \( a(t) \) is the scale factor. We assume that this universe is filled by dark energy and (cold) dark matter with densities \( \rho_d \) and \( \rho_m \) respectively. Dark energy component is a scalar field, \( \phi \), with potential \( V(\phi) \). Energy density, \( \rho_d > 0 \), and pressure, \( P_d < 0 \), of dark energy are given by

\[
\rho_d = \frac{\dot{\phi}^2}{2} + V(\phi),
\]

\[
P_d = \frac{\dot{\phi}^2}{2} - V(\phi).
\]

The EoS parameter of dark energy, given by

\[
w_d = \frac{P_d}{\rho_d} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)},
\]
satisfies $-1 \leq w_d < -\frac{1}{3}$. The scalar field with this EoS parameter is dubbed quintessence. The inequality $w_d < -\frac{1}{3}$ is necessary for accelerating the expansion of the universe. The Hubble parameter, $H = \frac{\dot{a}}{a}$, satisfies Friedmann equations

$$H^2 = \frac{8\pi}{3} \rho, \quad \dot{H} = -4\pi(P + \rho),$$

where $\rho$ and $P$ are the total energy density and pressure of the universe: $\rho = \rho_d + \rho_m$, $P = P_d$. We consider an interaction between dark matter and dark energy components:

$$\dot{\rho}_d + 3H(P_d + \rho_d) = -Q \quad \rho'_m + 3H\rho_m = Q. \quad (5)$$

By substituting (2) in the above equation we find the evolution equation for the quintessence field

$$\dot{\phi}(\ddot{\phi} + 3H\dot{\phi} + V'(\phi)) = -Q. \quad (6)$$

The ratio of dark matter to dark energy, $r = \frac{\rho_m}{\rho_d}$, satisfies

$$\dot{r} = 3Hrw_d + \frac{Q}{\rho_d}(1 + r). \quad (7)$$

Using $w_d < -\frac{1}{3}$ and the above equation one can show that in noninteracting quintessence model: $\dot{r} < -Hr$, and therefore $r \to 0$ eventually, in contrast to the recent data which assess $r \sim \mathcal{O}(1)$. We can also obtain an expression for time evolution of $r$ in terms of $\Omega_d$ defined by $\Omega_d = \frac{\rho_d}{\rho_c}$ (where $\rho_c = \frac{3H^2}{8\pi}$ is the critical density):

$$\dot{r} = -\frac{\Omega_d}{\Omega_d^2}. \quad (8)$$

By comparing (7) and (8) we arrive at

$$w_d = \frac{\Omega_d}{3H\Omega_d(1 - \Omega_d)} - \frac{Q}{3H\rho_d(1 - \Omega_d)}. \quad (9)$$

In the absence of interaction we have $\dot{\Omega}_d > H(1 - \Omega_d)$, which implies that $\Omega_d$ is an increasing function of time. In contrast, $\Omega_m$, defined by $\Omega_m = \frac{\rho_m}{\rho_c}$, is a decreasing function in this case. Hence the introduction of $Q$ in the above equation, via exchanging energy, depending on the form of $Q$ may prevent $r = \frac{\rho_m}{\rho_d}$ to go to zero, and can be a remedy for the coincidence problem.

Eq. (9) is a general equation for dark energy models. In the scalar field quintessence model we can also make use of

$$2V(\phi) = (1 - w_d)\rho_d \quad \dot{\phi}^2 = (1 + w_d)\rho_d. \quad (10)$$
to obtain
\[(1 - w_d)\dot{\rho}_d = 2V'(\phi)\dot{\phi}, \quad (11)\]
and subsequently
\[\pm \sqrt{(1 + w_d)\rho_d V'(\phi)} = \frac{1}{2}(-\dot{w}_d\rho_d + (1 - w_d)\dot{\rho}_d). \quad (12)\]

\(+(-)\) corresponds to \(\dot{\phi} \geq (\leq)0\). By substituting
\[\dot{\rho}_d = \frac{3H^2}{8\pi} \left(-3H\Omega_d(1 + w_d\Omega_d) + \dot{\Omega}_d\right), \quad (13)\]
which can be verified by taking the time derivative of \(\rho_d = \Omega_d\rho_c\), into \((12)\), and by making use of \((9)\), we arrive at
\[\dot{w}_d = \mp 2V'(\phi)\sqrt{\frac{1 + w_d}{\rho_d}} - 3H(1 - w_d^2) - \frac{Q}{\rho_d}(1 - w_d). \quad (14)\]

We use the above equation to study the behavior of the model in the vicinity of the time, \(t = t_0\), when \(w_d(t_0) = -1\). The equation of state parameter of the quintessence is equal to or greater than \(-1\), \(w_d \geq -1\), therefore at \(t_0\) we must have \(\dot{w}_d(t_0) = 0\). Otherwise there will be a neighborhood of \(t_0\), in which \(w_d < -1\). Hence a necessary condition to reach at \(w_d(t_0) = -1\) in the quintessence model is \(\dot{w}_d = 0\) at \(t = t_0\). Even when \(t_0 \to \infty\), this is asymptotically valid. This lies on the fact if \(w_d \to -1\) when \(t \to \infty\), we must also have \(\frac{dw_d}{dt}(\in \mathbb{R}) = 0\). For \(\lim_{w_d \to -1} \frac{V'(\phi)}{V(\phi)} \sqrt{(1 + w_d)} = 0\) (e.g. for bounded \(\frac{V'(\phi)}{V(\phi)}\) at \(t = t_0\) \(14\) reduces to
\[\frac{Q(t_0)}{\rho_d(t_0)} = 0. \quad (15)\]

E.g. if one takes the interaction term such as \(Q = \lambda \rho_m\rho_d\) (\(\lambda > 0\)), as in the present epoch \(r \sim \mathcal{O}(1)\), we have \(\frac{Q}{\rho_d} \neq 0\) and it is clear that \(w_d = -1\) cannot occur in the present era.

Note that because of the presence of additional terms in \((6)\) with probable singular behavior at the limit \(w_d \to -1\), \(Q(t_0) = 0\) (at \(w_d = -1\)) may not be derived directly from \((6)\) unless \(\lim_{w_d \to -1} \frac{V'(\phi)}{V(\phi)} \sqrt{(1 + w_d)} = 0\).

Beside the above natural condition, \(\ddot{w}_d(t_0) \geq 0\) is also a necessary condition to reach at \(w_d(t_0) = -1\). This is due to the fact that \(w_d = -1\) may only be the global minimum of \(w_d(t)\). Note that even if \(w_d = -1\) occurs at \(t_0 \to \infty\), \(\lim_{t \to \infty} \ddot{w}_d(t) = 0\) must be satisfied (provided \(\ddot{w}_d \in \mathbb{R}\)). Now we must explain this natural condition in terms of the interaction term and the potential (inconsistency of the potential and the considered interaction with
this condition indicates that the model is not able to reach at \( w_d = -1 \). To do so by getting another time derivative of eq. (14), at \( t = t_0 \) we obtain

\[
\ddot{w}_d(t_0) = -2 \frac{d}{dt} \left( \frac{Q}{\rho_d} \right) + 2 \frac{d}{dt} \left( \frac{V'(\phi) \sqrt{1 + w_d}}{V(\phi)} \right). \tag{16}
\]

Note that the right hand side must be evaluated at \( t = t_0 \). By putting (14) into (16) we get

\[
\ddot{w}_d(t_0) = -2 \frac{d}{dt} \left( \frac{Q}{\rho_d} \right) + 2 \frac{V'(\phi)}{V(\phi)} \frac{Q}{\rho_d \sqrt{1 + w_d}} \tag{17}
\]

But at \( t = t_0 \)

\[
\frac{d}{dt} \left( \frac{Q}{\rho_d} \right) = \frac{\dot{Q}}{\rho_d} + \left( \frac{Q}{\rho_d} \right)^2 = \frac{\dot{Q}}{\rho_d} \tag{18}
\]

holds, therefore \( \ddot{w}_d(t_0) > 0 \) requires that at \( t = t_0 \):

\[
- \dot{Q}(t_0) + (V'(\phi))^2 \pm \frac{V'(\phi)}{\sqrt{V(\phi)}} \frac{Q(t_0)}{\sqrt{1 + w_d}} \geq 0. \tag{19}
\]

For interactions of the form \( Q(\rho_m, \rho_d) \), at \( t = t_0 \) we have \( \dot{Q}(t_0) = -3H \rho_m \frac{\partial Q}{\partial \rho_m} \) and the equation (19) at \( t = t_0 \) reduces to

\[
- 3H \rho_m \frac{\partial Q}{\partial \rho_m} + (V'(\phi))^2 \pm \frac{V'(\phi)}{\sqrt{V(\phi)}} \frac{Q}{\sqrt{1 + w_d}} \geq 0. \tag{20}
\]

Note that the form used in (17) for the third term in the right hand side is suitable only when \( Q \) can be expressed in terms of \( 1 + w_d \), (e.g. for interactions containing positive power of \( \dot{\phi} \) which following (10) can be written in terms of \( 1 + w_d \) like the interaction considered for the inflaton during the reheating process: \( Q \propto P_d + \rho_d = \dot{\phi}^2 \)). In general, at \( t = t_0 \), using l’Hôpital’s rule, it is also possible to write (17) in the form

\[
\ddot{w}_d(t_0) = -2 \frac{d}{dt} \left( \frac{Q}{\rho_d} \right) \pm \frac{V'(\phi)}{\sqrt{V(\phi)}} \sqrt{2\ddot{w}_d(t_0)} \tag{21}
\]

For intermediate time (we mean \( t_0 \rightarrow \infty \)), and for \( t < t_0 \), we have \( \dot{w}_d < 0 \) and for \( t > t_0 \), \( \dot{w}_d > 0 \) holds. \( \{+(-)\} \) corresponds to the case where \( \dot{\phi} \leq (\geq) 0 \) when \( t \leq t_0 \). (21) has real roots (for \( \ddot{w}_d \)) provided that at \( t = t_0 \)

\[
4\dot{Q}(t_0) \leq (V'(\phi))^2. \tag{22}
\]
The above inequality can be viewed as a constraint on the parameters of the model. In general, if the first nonzero derivative of \( w_d \) at \( t = t_0 \) is of order \( n \), at \( t = t_0 \) we have

\[
w_d^{(n)}(t_0) = -2 \left( \frac{Q}{\rho_d} \right)^{(n-1)} + 2 \left( \frac{V'(\phi)}{\sqrt{\rho_d}}(\sqrt{1 + w_d(t_0)}) \right)^{(n-1)}.
\] (23)

In this case, evenness of \( n \) and \( w_d^{(n)}(t_0) > 0 \), together with \( \dot{w}_d(t_0) = 0 \) are sufficient conditions for \( w_d \) to have a global minimum at \( t_0 \). The generalization of (21) is then

\[
w_d^{(n)}(t_0) = -2 \left( \frac{Q}{\rho_d} \right)^{(n-1)} + 2 \left( \frac{(n-1)!}{(n-2)!} \right) \left( \frac{V'(\phi)}{\sqrt{\rho_d}} \right)^{(n-1)} \sqrt{\frac{w_d^{(n)}(t_0)}{n}}.
\] (24)

To derive the above equation we have assumed that \( \left( \frac{V'(\phi)}{\sqrt{\rho_d}} \right) \) and its time derivatives up to order \( (\frac{n}{2} - 1) \) are continuous and bounded at \( t = t_0 \). If \( w_d \) tends asymptotically to \(-1\), all of derivatives of \( w_d \) may be zero in this limit. In this situation \( t_0 \) is the point of infinite flatness and for infinitely differentiable \( w_d \), can only occur at infinity, \( t_0 \to \infty \).

In the following, to elucidate our results, as an example, we consider the interaction \( Q = H(\lambda_d \rho_m + \lambda_m \rho_d) \).

Following (15), we deduce that in order to reach at \( w_d = -1 \), we must have

\[
r(t_0) = -\frac{\lambda_d}{\lambda_m}.
\] (26)

This equation determines the ratio of dark matter to dark energy in terms of the parameter of interaction at \( t = t_0 \). As we consider \( r \) as continuous function of comoving time, we also expect that this ratio is approximately given by \( r \approx -\frac{\lambda_d}{\lambda_m} \) in the vicinity of \( t_0 \) where \( w_d(t_0) = -1 \) occurs.

As a result, near \( w_d = -1 \), the value of \( r(t_0) \) is specified by the constant parameters of the interaction. If the present value of \( w_d \) is believed to be \( w_d \approx -1 \) [3], based on astrophysical data [4], we can get \(-\frac{\lambda_d}{\lambda_m} \simeq 3/7 \). In this method we cannot assess \( \lambda_d \) and \( \lambda_m \) separately. Note that for interactions which do not satisfy (26) (e.g. models with \( \frac{\lambda_d}{\lambda_m} > 0 \)), \( w_d = -1 \) is not accessible.

To investigate the condition (22), we note that at \( w_d = -1 \)

\[
\dot{Q}(t_0) = 8\pi \lambda_d(1 + r)\rho_d^2.
\] (27)

Hence the inequality (22) becomes

\[
32\pi \lambda_d(1 + r)\rho_d^2(t_0) \leq (V'(\phi))^2.
\] (28)
Note that the above constraint depends on the form of the potential of the quintessence field. We shall discuss this issue shortly after some remarks.

The conditions like (15) and (22), are only necessary conditions to reach \( w_d = -1 \). Indeed we have used the fact that if in an interacting quintessence model \( w_d = -1 \) is achieved, then equations like (15) and (22) must be hold. But we did not prove that \( w_d = -1 \) is allowed in the model. In fact describing the exact form of \( w_d \) (to see whether \( w_d = -1 \) is accessible) requires solving the equation (6) with one of the Friedmann equations in (4), which in the presence of interaction (even in its absence), as we mentioned in the introduction, is not straightforward for a general potential. Despite this, to study more about the behavior of the system near \( w_d = -1 \), we can restrict ourselves to the neighborhood of \( t = t_0 \) where the Hubble parameter is presumed to be differentiable and consider the interaction (25). For a differentiable (at least in an open interval containing \( t = t_0 \)) Hubble parameter, in the vicinity of \( t_0 \) (an open set containing \( t_0 \)) following [8] we use the following Taylor expansion

\[
H = h_0 + h_1 (t - t_0)^\beta + \mathcal{O}(t - t_0)^{\beta+1}, \quad \beta \geq 1. \tag{29}
\]

In this open set we consider the following expressions (Taylor expansion):

\[
w_d = -1 + w_0(t - t_0)^{\alpha} + \mathcal{O}(t - t_0)^{\alpha+1}
\]
\[
r = r_0 + r_1 (t - t_0)^{\gamma} + \mathcal{O}(t - t_0)^{\gamma+1}. \tag{30}
\]

The equation of state parameter of the system is related to \( w_d \) through the relation \( w = w_d \Omega_d \). We have also \( w = -1 - \frac{2\dot{H}}{3H^2} \), therefore near \( w_d = -1 \), \( w = -\Omega_d \) and \( \dot{H} \neq 0 \), and \( \beta \) in (29) begins with \( \beta = 1 \). We write the equation (7) as

\[
\dot{r} = 3rH \left[ w_d + \frac{1}{3} \left( \frac{r + 1}{r} \right) (\lambda_d + r \lambda_m) \right] \tag{31}
\]

Putting (29) and (30) into (31) gives: \( \gamma = 1 \) and \( r_1 = -3r_0 h_0 \). To elucidate our results we must specify the potential. Here we consider the quadratic and the exponential potentials. For the quadratic potential

\[
V(\phi) = \frac{1}{2} m^2 \phi^2, \tag{32}
\]

by substituting (29) and (30) in (14) we arrive at

\[
\pm 2m \sqrt{1 - w_d^2} = w_d + 3H(1 - w_d^2) + H(\lambda_d + \lambda_m r)(1 - w_d). \tag{33}
\]

Comparing the coefficients of the expressions with the same power of \( t \) in both sides of (33) forces us to take \( \alpha = 2 \) and \( r_0 = -\frac{\lambda_d}{\lambda_m} \), in accordance with (15). We also obtain the equation

\[
\pm 2m \sqrt{2w_0} = 2w_0 + 2\lambda_m r_1 h_0 = 2w_0 - 6\lambda_d h_0^2 \tag{34}
\]
in agreement with the previous result \((21)\). The above equation has real roots provided that

\[
8\pi\lambda_d(1 - \frac{\lambda_d}{\lambda_m})\phi^2(t_0) \leq 1,
\]

which is the same as \((28)\). This poses a condition on the value of the quintessence field at \(w_d = -1\). In terms of total energy density this inequality may be written as

\[
16\pi\lambda_d\rho(t_0) \leq m^2.
\]

For the exponential potential

\[ V = v_0 \exp(\lambda \phi), \quad v_0 > 0, \]

\((37)\) reduces to

\[
\pm \lambda (1 - w_d)\sqrt{\rho_d(1 + w_d)} = w_d + 3H(1 - w_d^2) + H(\lambda_d + \lambda_m r)(1 - w_d).
\]

Again, by substituting \((29)\) and \((30)\) in \((38)\), and by comparing the coefficients of the same power of \(t\) in both sides we arrive at: \(\alpha = 2, \quad r_0 = -\frac{\lambda}{\lambda_m}\) and

\[
\pm \lambda \sqrt{\rho_d(t_0)w_0} = w_0 + 3\lambda_d h_0^2,
\]

in agreement with \((15)\) and \((21)\). The necessary condition to have real roots for \((39)\) is then

\[
32\pi\lambda_d(1 + r_0) < \lambda^2
\]

By taking \(r_0 = -\frac{\lambda}{\lambda_m} \approx 3/7\) as the present estimated value, we obtain

\[
\frac{320\pi}{7}\lambda_d \leq \lambda^2,
\]

which shows that access to \(w_d = -1\) is not feasible for models with arbitrary \(\lambda\) and \(\lambda_d\).

3 Conclusion

In this paper we considered a spatially flat FRW universe composed of dark matter and dark energy components. The dark energy was assumed to be a quintessence scalar field interacting with dark matter (see \((2)\) and \((5)\)). A general expression for time derivative of EoS parameter of dark energy was derived (see \((14)\)), upon which we discussed some necessary conditions and relation between the interaction term and the potential of the quintessence to reach at \(w_d = -1\) (see \((15)\) and \((22)\)). We also examine our results by approximation method based on series expansion near the time when \(w_d = -1\).
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