A Novel Control-Oriented Cell Transmission Model Including Service Stations on Highways

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Abstract—In this paper, we propose a novel model that describes how the traffic evolution on a highway stretch is affected by the presence of a service station. The presented model enhances the classical Cell Transmission Model (CTM) dynamics by adding the dynamics associated with the service stations, where the vehicles may stop before merging back in the main stream. We name it Cell Transmission Model with service station (CTM-s). We discuss its flexibility in describing different complex scenarios where multiple stations are characterized by different drivers’ average stopping times corresponding to different services. The model has been developed to help designing control strategies aimed to decrease traffic congestion. Thus, we discuss how classical control schemes can interact with the proposed CTM-s. Finally, we demonstrate the proposed model through numerical simulations and assess the effects of service stations on traffic evolution, which appear to be beneficial especially for relatively short congested periods.

I. INTRODUCTION

In recent years, mobility is becoming a central issue in many countries and some alarming statistics show a growing need for change. For example, some studies show that the cost of congestion for the EU is no less than 267 billion € per year. Moreover, an inefficient transportation system affects not only the citizens’ well-being, but also the environment, since traffic jams heavily increase the emission of CO₂ [2].

The classical solution to the traffic demand management problem is to increase roads capacity, or to build alternative routes. Although this solution produces tangible benefits [3], policymakers and researchers are exploring alternative interventions that may be faster and cheaper to implement. These solutions heavily rely on mathematical models to be able to assess a priori their feasibility and impact.

In traffic systems, the introduction of traffic models dates back to the 50s, with publication of the Lighthill-Whitham-Richards (LWR) model [4], which is a macroscopic model based on the equation of vehicles conservation representing traffic dynamics in an aggregate way. An alternative to macroscopic models is given by microscopic or mesoscopic traffic models [5, Sec. 2.3.1]. The firsts explicitly capture drivers’ individual behaviors, while the seconds also account for car-following and vehicles interaction phenomena. Microscopic or mesoscopic traffic models are not convenient when the goal is to design traffic control strategies due to their computational complexity, definitely much higher than that of macroscopic models. One of the most renowned macroscopic discrete traffic models is the so-called CTM, developed in the 90s for highway traffic in road stretches and networks [6]. Several variations of such a model has been developed throughout the years to address its limitations, e.g., see [7].

The role of these models has been of paramount importance in the development of active traffic demand management mechanisms. Such interventions may be designed to incentivize positive driver behaviors, and have their roots in behavioral economics and psychology [8]. Recently, inspired by the so-called “valley filling” objective in smart grids (see e.g. [9], [10]) and ramp metering control in highways, the authors of [11], [12] have proposed a monetary incentive based policy for plug-in hybrid and electric vehicles to alleviate traffic congestion. Another approach is to intervene by imposing some physical constraints or penalizing undesired actions. This category includes ramp metering, traffic lights and tolls control. The interested reader is referred to [13], [14] and the references therein for the details.

In this paper, we propose a novel highway traffic model that includes the dynamics of any service station along the highway. The presented model enhances the classical CTM dynamics by modelling the presence of service stations where the vehicles may stop and then merge back in the main stream. The resulting Cell Transmission Model with service station (CTM-s) is a macroscopic highway traffic model capable of describing the dynamical effect of service stations on highway traffic. For example, the CTM-s can model a single service station that provides different services (refueling, ancillary services, charging for electric vehicles), or multiple service stations. We discuss possible control schemes that leverage the CTM-s to perform active traffic demand management. For each of these schemes, we discuss how it can be implemented and which would be the control actions put in place.

II. THE CTM WITH SERVICE STATIONS

We model a generic highway stretch in which there may be on- and off-ramps that allow the vehicles to exit and merge into the main stream, respectively. Furthermore, we assume the presence of at least one multi-purpose service station where a fraction of the drivers in the main stream stops to refuel or use ancillary services such as the restaurant.
or restroom, see [1, Fig. 1]. These vehicles obey different dynamics from those simply entering or exiting the highway: after a certain amount of time spent at the service station, they merge back in the main stream. This creates a coupling between the two flows that has to be carefully modelled.

In the remainder of the section, we use the “classical” formulation of the CTM as cornerstone to build the proposed CTM-\$s\$.

A. Model variables

Each time interval \([kT,(k+1)T)\) of length \(T\) is denoted by an integer \(k \in \mathbb{N}\). The highway stretch is modeled as a chain of \(N\) consecutive cells and the vehicles in each cell \(i \in \mathcal{N} := \{1, \ldots, N\}\) are assumed to move with constant speed \(v_i(k)\). Two adjacent cells are connected via an interface, where the vehicles can \(i\) proceed to the next cell; \(ii\) exit the main stream via an on-ramp or by stopping at a service station; \(iii\) merge into the main stream of the next cell via an on-ramp or by exiting a service station. We assume the presence of \(M\) service stations. Each station \(p \in \mathcal{M} \subseteq \mathcal{N} \times \mathcal{N}\) is located between any two cells \(i, j \in \mathcal{N}\), and \(|\mathcal{M}| = M\). Station \(p\) correspond to the ordered couple \((i, j) \in \mathcal{M}\), where \(i \in \mathcal{N}\) represents the cell from which the vehicles may access the service station, while \(j \in \mathcal{N}\) denotes the one in which they merge back. Notice that multiple stations can be represented by the same couple, since stations can share enter and exit points, see Figure 1(a).

We denote the set of all the service stations having access point at \(i\) by \(\mathcal{E}_{i}^{m} = \{ p \in \mathcal{M} \mid \exists j \in \mathcal{N} \text{ s.t. } p = (i, j) \}\). Those that merge back into cell \(i\) by \(\mathcal{E}_{i}^{out}\) and note that \(\sum_{i \in \mathcal{N}} |\mathcal{E}_{i}^{in}| = \sum_{i \in \mathcal{N}} |\mathcal{E}_{i}^{out}| = M\).

Following [5, Sec. 3.3], we briefly introduce the variables used in the proposed CTM-\$s\$ for a generic cell \(i \in \mathcal{N}\) and service station \(p = (i, j)\), during an interval \(k \in \mathbb{N}\). A set of fixed parameter is associated to each cell \(i\):

- \(L_i\) [km]: cell length;
- \(\tau_i\) [km/h]: free-flow speed;
- \(w_i\) [km/h]: congestion wave speed;
- \(\rho_i^{\text{max}}\) [veh/h]: maximum cell capacity;
- \(\rho_i^{\text{max}}\) [veh/km]: maximum jam density;
- \(r_i^{\text{max}}\) [veh/h]: maximum capacity of the on-ramp exiting a service station.

The variables used to describe the dynamics (during the time interval \(k \in \mathbb{N}\)) are:

- \(\rho_i(k)\) [veh/km]: traffic density of cell \(i\);
- \(\Phi_i(k)\) [veh/h]: total flow entering (exit)ing cell \(i\);
- \(\phi_i(k)\) [veh/h]: flow entering cell \(i\) from \(i-1\);
- \(r_i(k)\) [veh/h]: flow of vehicles merging into the main stream via an on-ramp;
- \(r_i^{\text{out}}(k)\) [veh/h]: flow of vehicles merging into the main stream from the service stations in \(\mathcal{E}_{i}^{out}\);
- \(s_i(k)\) [veh/h]: flow of vehicles leaving the main stream via an off-ramp;
- \(s_i^{\text{out}}(k)\) [veh/h]: flow of vehicles leaving the main stream to enter the service stations in \(\mathcal{E}_{i}^{in}\);
- \(\beta_i(k)\): split ratio associated with the off-ramp;
- \(\beta_i^{\text{out}}(k)\): split ratio associated with the service stations in \(\mathcal{E}_{i}^{in}\);
- \(\ell_{i}(k)\) [veh/h]: number of vehicles at the service station \((i, j)\);
- \(\ell_{i}(k)\) (or \(\ell_{p}\)) [veh]: number of vehicles queuing at the service station \((i, j)\) due to the impossibility of merging back in the main stream during previous time intervals;
- \(\delta_{i}(k)\) (or \(\delta_{p}\)): the average number of time intervals spent at the service station \((i, j)\) by the drivers. It is assumed constant for the sake of simplicity, but in general may be time varying and we discuss this aspect in Section III;
- \(p_i\): relative priority of all the flows entering in cell \(i\) in case of congestion. By definition, this vector has to belong to a simplex, i.e., \(1^T p_i = 1\) where \(1\) is a vector of all \(1\) of the correct dimension. The component of \(p_i\) associated with the main stream is denoted by \(p_i^{\text{main}} > 0\). For simplicity, we assume this vector constant over time and discuss this in Section III.

In the remainder, given a variable \(x\), we denote its components associated with \(q \in \mathcal{M}\) by \([x]_q\).

In general, at the interface between cells \(i\) and \(i+1\) there can be the access and exit of multiple service stations. Consequently, \(\beta_{i}^{\text{out}} \in \mathbb{R}^{\mathcal{E}_{i}^{out}}\), where \([\beta_{i}^{\text{out}}]_q\) represents the split ratio of vehicles entering the \(q\)-th station in \(\mathcal{E}_{i}^{in}\). Following a similar reasoning, we attain that \(r_i^{\text{max}}\) [veh/h], \(r_i^{\text{out}}\) [veh/h] and \(\ell_{i}(k)\), \(\delta_{i}(k), \ell_{i}(k)\) \(\in \mathbb{R}^{\mathcal{E}_{i}^{in}}\). Finally, to ease the notation, we assume that for a cell \(j\) there cannot be in-flows deriving from both an on-ramp and the exiting of a service station. The role of this assumption will appear clear in Section II-B.

To clarify the role of the variables defined above, we present two possible configurations schematized in Figure 1.

**Example 1 (Single multi-modal service station):** In Figure 1(a), we represent the case of single service station.
where drivers can stop for different reasons, e.g., refueling, ancillary services, or charging a plug-in electric vehicles.

These different types of services are described via $M = 3$ service stations that share the same entry and exit points, i.e., cells $i$ and $j$, respectively. So, $|E_i| = |E_j| = |E_i \cap E_j| = 3$. This scheme allows us to use different split ratios $[\beta_i^p]_q, q \in M$, for the three different types of services, and different $[\delta_i^p]_q, q \in M$, describing the average time spent at the corresponding service station.

**Example 2 (Multiple service stations):** In Figure 1(b), we represent a generic configuration in which there are $M = 3$ service stations, i.e., $M = \{m_1, m_2, m_3\}$, where $m_1 = (i, j - 1)$, $m_2 = (i, j)$, $m_3 = (i + 1, j)$. Here, $m_1, m_2$ share the same entry point, so $[\beta_i^p]_q \in \mathbb{R}^2$ since $E_i = \{m_1, m_2\}$, while $\beta_{i+1}^p \in \mathbb{R}^2$. Similarly, $m_2$ and $m_3$ merge back in the same cell $j$, so $E_j = \{m_2, m_3\}$, and thus $[\beta_j^p]_q \in \mathbb{R}^2$.

Conversely, $|E_i \cap E_j| = |E_i \cap E_j| = |E_i \cap E_j| = 1$ so $\ell_p, \delta_p \in \mathbb{R}$ for all $p \in M$.

**B. CTM-s dynamics**

We are now ready to introduce the dynamics of the proposed CTM-s. The evolution of the demand $s_i$ of cell $i \in N$ is governed by

$$r_i(k + 1) = r_i(k) + T \frac{D}{L} \left( \Phi_i^+(k) - \Phi_i^-(k) \right),$$

where the inflow and outflow are defined respectively as

$$\Phi_i^-(k) := \phi_{i+1}(k) + s_i(k) + s_i^+(k)$$

$$\Phi_i^+(k) := \phi_i(k) + r_i(k) + 1 \Phi_i^-(k),$$

and $1 \in \mathbb{R}[E_i]$. As in [5, Sec. 3.3.1], the flow $s_i(k)$ is a fraction $\beta_i$ of the total flow exiting the cell, so $s_i(k) = \frac{\beta_i(k)}{1 - \beta_i}(k + 1 \Phi_i^-(k))$. Likewise, the total flow entering the service station is defined as

$$s_i^+(k) = \sum_{q \in E_i^+} [\beta_i^q](k)_{\phi_{i+1}} = \sum_{q \in E_i^+} [\beta_i^q](k)_{\Phi_i^-(k)}.$$

The only constraint that has to be satisfied by the split ratios is $0 \leq \beta_i(k) \leq 1$. Notice that we do not explicitly model the supply of the service station since, if needed, can be incorporated by imposing dynamic constraints on $\beta_i^p(k)$.

The vehicles entering a service station $q = (i, j)$ during $k$ linger there for $\delta_q$ time intervals before trying to merge back. The number of vehicles at the service station evolves as

$$\ell_q(k + 1) = \ell_q(k) + T \left[ s_q^-(k) - [r_q^+](k) \right],$$

where $s_q^-(k) := [\beta_q^p](k)_{\Phi_q^-}(k)$ and $q \in E_i \cap E_j$. Loosely speaking, $s_q^-(k) \in \mathbb{R}[E_i \cap E_j]$ comprises the fractions of $s_q^-(k)$ in (3) entering the service station $q$.

After $\delta_q$ time intervals, the vehicles attempt to exit the service station to merge back into the main stream. However, if this flow exceeds the supply of the receiving cell, then some vehicles remain at the service station and wait for merging back during the next intervals, thus creating a queue.

To keep track of these vehicles, we introduce the state variable $e_q(k)$, whose dynamics evolve as follows

$$e_q(k + 1) = e_q(k) + T \left[ s_q^-(k) - [r_q^+](k) \right].$$

With a slight abuse of notation, we can compactly define the flow of vehicles that attempts to exit during $k$ as $s_q^-(k) - \delta_q$. If the flow $s_q^-(k) - \delta_q > e_q(k)$ exceeds the capacity of the on-ramp connected to cell $j$, then only part of it is able to merge back. Specifically, the demand of the ramp exiting the service station $(i, j)$ and connecting to cell $j$ reads as

$$D_q^j(k) = \min \left(\frac{e_q(k)}{T}, r_q^\text{max} \right).$$

**Remark 1 (Number of variables):** A drawback of the current formulation of the proposed CTM-s is the necessity of $\delta_q$ variables to track the evolution of $\ell_q$ over time. This might be overcome by rephrasing the dynamics of the service station via a stochastic variable that obeys a Poisson distribution. We leave the investigation of this alternative formulation to future works.

We denote the demand of the on-ramps of cell $j$ during $k$ by $D_j^\text{ramp}(k)$, on the other hand, similarly to the classical CTM, the demand of cell $i$ and the supply of cell $j$ are respectively

$$D_j(k) = \min \left(1 - \beta_i(k), 1 \Phi_i^-(k)\right) \Psi_i \rho_i(k), q_i^\text{max},$$

$$S_j(k) = \min \left(w_j(k) \rho_j^\text{max}(k) - \rho_j(k), q_j^\text{max} \right).$$

We define next the relation between the demand/supply of cells and ramps and the flows of vehicles that transit from one to another. Let us discuss individually the free-flow and the congested case for cell $j$. We only focus on the case in which there is an in-flow deriving from the exit of a service station, i.e., $|E_j| > 0$, and refer the reader to [5, Eq. 3.33-3.34] for the case in which the flow is due to an on-ramp.

1) **Free-flow case:** This is the simplest scenario arising if $D_j(k) + \sum_{p \in E_j^\text{out}} D_p^j(k) \leq S_j(k)$.

Then, all the vehicles are able to enter cell $j$ during $k$, and thus the flows read as

$$\phi_j(k) = D_j(k)$$

$$[r_q^+](k) = \frac{e_q(k)}{T}, \forall q \in E_j^\text{out}.$$

2) **Congested case:** The total demand exceeds the supply of cell $j$, i.e., $D_j(k) + \sum_{p \in E_j^\text{out}} D_p^j(k) > S_j(k)$. This might occur for different reasons, so we discuss them separately.

- **If** $D_j(k) > p_i^\text{max} S_j(k)$ and $\sum_{p \in E_j^\text{out}} D_p^j(k) \leq (1 - p_i^\text{max}) S_j(k)$, then the complete flow from the stations can enter the cell $j$, so the flows read as

$$\phi_j(k) = S_j(k) - \sum_{p \in E_j^\text{out}} D_p^j(k)$$

$$[r_q^+](k) = D_q^j(k), \forall q \in E_j^\text{out}.$$
If $D_{j-1}(k) \leq p_j^{ms}S_j(k)$ and $\sum_{p \in E_j}^D D_p(k) > (1 - p_j^{ms})S_j(k)$, then the whole flow from the previous cell can enter cell $j$, thus

$$\phi_j(k) = D_{j-1}(k).$$

(10)

The remainder of the supply is split among the ramps exiting the service station, i.e., $\sum_{p \in E_j}^D D_p(k) = S_j(k) - D_{j-1}(k)$. Finding the exact expression for the flow of each ramp must be done with an iterative procedure. First, we split the ramps of the stations $E_j^{out}$ in two subsets that are $1\overline{E}$ and $1E$ (1 denotes the procedure’s iteration). The former is composed by those ramps $p \in E_j^{out}$ satisfying

$$D^s_p > \frac{S_j(k) - D_{j-1}(k)}{|E_j^{out}|},$$

while $1E = E_j^{out} \setminus 1\overline{E}$. From this, we conclude that the complete flow of the ramps in $1E$ manages to enter cell $j$ during $k$, thus

$$[r^s_p]_q = D^s_p(k), \forall p \in 1E.$$  

(11)

Next, we define $2\overline{E}$ as those $p \in 2\overline{E}$ that satisfy

$$D^s_p > \frac{S_j(k) - D_{j-1}(k) - \sum_{p \in 1\overline{E}} D_p(k)}{|1\overline{E}|},$$

and $2E = 1\overline{E} \setminus 2\overline{E}$. Also in this case we obtain that

$$[r^s_p]_q = D^s_p(k), \forall q \in 2E.$$  

(12)

By repeating this procedure, we reach an iteration $y$ such that $y\overline{E} = \emptyset$. Therefore, the flow from the ramps in $y\overline{E}$ cannot enter completely in cell $j$ and thus the remaining supply

$$\hat{S}_j(k) := S_j(k) - D_{j-1}(k) - \sum_{t=1}^y \sum_{p \in 1\overline{E}} D_p(k)$$

is divided among them based on their priority

$$[r^s_p]_q = \frac{[p]_q}{\sum_{p \in y\overline{E}} [p]_q} \hat{S}_j(k),$$  

(13)

for all $q \in y\overline{E}$.

If $D_{j-1}(k) > p_j^{ms}S_j(k)$ and $\sum_{q \in E_j^{out}}^D D_q(k) > (1 - p_j^{ms})S_j(k)$, then

$$\phi_j(k) = p_j^{ms}S_j(k).$$

(14)

Therefore, the resulting flows read as

$$[r^s_p]_q = D^s_p(k), \forall p \in \bigcup_{t=1}^y t\overline{E},$$

$$\hat{S}_j(k) := (1 - p_j)S_j(k) - \sum_{t=1}^y \sum_{q \in t\overline{E}} D^s_q,$$

$$[r^s_p]_q = \frac{[p]_q}{\sum_{p \in y\overline{E}} [p]_q} \hat{S}_j(k), \forall q \in y\overline{E}.$$ 

This concludes the formulation of the CTM-$s$, in fact all the dynamics associated with the variables introduced in the previous section have been defined. The formulation above, even though it might seem convoluted, boils down to very simple and intuitive equations when it is applied to specific cases like the ones depicted in Figure 1.

III. OVERVIEW OF POSSIBLE CONTROL STRATEGIES

This section proposes a high-level discussion on how the proposed CTM-$s$ can be utilized for designing novel traffic control schemes.

1) Control of the flow exiting the service station: A natural idea is to introduce a ramp-metering mechanism to modulate the flow of vehicles merging back into the main stream, i.e., $r^s$. This can be implemented introducing a control signal $r^s_q(k)$ in (6) for every $q \in M$. Then, similarly to [5, Eq. 3.39], the demand becomes

$$D^s_q(k) = \min \left( \frac{\delta_q - \delta_q(k) + e_q(k)}{T}, r^s_{qc}(k), r^s_{max} \right).$$

The design of $r^s_{qc}(k)$ can be performed applying ramp-metering control methods such as ALINEA [15], where the local control influences the exit flow based solely on the traffic conditions of the highway. Alternatively, one can take direct advantage of the knowledge of the CTM-$s$ dynamics by introducing an MPC-based or event-triggered control scheme, such as those discussed in [13, Sec. 5.3 and 5.4].

The intuition behind these schemes is that limiting the flow of vehicles merging back in the main stream during rush hours decreases the overall traffic congestion. Overall, these controls might temporarily increase $e_q(k)$.

2) Incentive based traffic control: the control over the entering flow cannot be performed in a direct way. In fact, the drivers passing by the entrance of the service stations can decide whether to stop or not. Thus, the value of $\beta(k)$ arises from the individuals’ decisions.

Game theory can be a suitable tool to analyze such phenomena since there is a rich literature that studies how a game can model and influence decision-making processes [16], [17]. In the game, the payoff would be associated with the current and future traffic conditions estimated via the CTM-$s$. Then, the decision can be influenced via incentives. The game’s outcome during $k$ would define the value of $\beta(k)$. A similar approach can also be used to influence $\delta(k)$. The nature of these incentives may be diverse and varies from a discounted energy price for the charging of electric vehicles [18] to several benefits for using the ancillary
services. The effectiveness of using rewards to reinforce a desirable behavior is supported by a large volume of empirical evidence [19] and the proposed CTM-\$s can prove to be an important tool to shift from static incentives to dynamic ones.

3) Optimal service station positioning: the proposed CTM-\$s has high flexibility and can easily describe many service stations configurations, by manipulating $\beta$, $\delta$ and $r^\*$. This feature can be exploited to create computationally tractable optimization problems to encompass the optimal configuration and positioning of service stations along a long and complex highway. This would improve current studies that rely directly on micro-simulators that require the use of algorithms that cannot guarantee the solutions’ optimality, e.g., the genetic algorithm applied to SUMO.

IV. SIMULATIONS

In this section, we analyse the CTM-\$s in the case of single and multiple service stations and, in particular, we study the effect that the model’s parameters have on the overall traffic congestion. We consider a highway stretch divided in $N = 9$ cells. The associated parameters are reported in [1, Tab. I]. We assume that there are no on- and off-ramps. The Total Travel Time (TTT) for the highway stretch in free flow is 2.15 min. The additional travel time during the time interval $k$ due to congestions can be computed as

$$\Delta(k) := \sum_{i \in N} \frac{L_i}{v_i(k)} - \frac{L_i}{\pi_1}, \quad (15)$$

where $v_i(k)$ is the actual velocity in cell $i$ during $k$. In the following, we denote by $\Delta_0$ the quantity in (15) obtained when there is no service station. During peak congestion we have $\max(\Delta_0(k)) = 56$ s, which corresponds to an increment of the TTT of 41.5%. The value of $\Delta$ is then a good indicator of the overall traffic congestion. Notice that a reduction of $\Delta$ implies, in turn, that the TTT of the vehicles decreases.

We perform the simulations over a time horizon of 3h and consider a time interval of $T = 10$ s, so $k \in [0, 1080]$. To better study the model’s features, we consider a simplified piece-wise linear flow entering the first cell that is defined for all $k$ as

$$\phi_1(k) := \max(500, -7.04|k - 540| + 2400).$$

We design $\phi_1$ to resemble the flow appearing during a typical morning rush hour in the considered highway stretch (a scaled version of $\phi_1$ is depicted in Figure 3). In this configuration, the period of high vehicles inflow lasts 1.5h.

A. Single service station

First, we assume the presence of a single service station between cells 2 and 4 and explore the effects that it has on traffic congestion for different values of $\beta^*$ and $\delta^*$. As performance index we use

$$\pi := \frac{\max_k(\Delta_0(k)) - \max_k(\Delta(k))}{\max_k(\Delta_0(k))},$$

which denotes the percentage of peak congestion reduction. If $\pi$ value is 1, then the introduction of the service stations completely eliminate the traffic congestion, while if it is 0 there is no improvement. Here, the priority of the main stream is set to $p^m_4 = 0.97$.

In Figure 2, it can be noticed that an increment in $\beta^*$ has a greater effect on the congestion than an increment in $\delta$. In fact, even for $\delta(2,4) = 5$ min, we achieve $\pi = 0.64$ for $\beta^* = 0.15$ and $\pi = 0.30$ for $\beta^* = 0.06$. These correspond to $\max(\Delta(k))$ being equal to 17 s and 39 s, respectively. As expected, the longer the drivers stop at the service station the higher the effect is on the traffic. In fact, if $\delta(2,4) = 40$ min, then $\pi = 0.97$ for $\beta^* = 0.15$ and $\pi = 0.54$ for $\beta^* = 0.06$.

Notice that a high $\beta^*$ might lead to an undesirable number of drivers waiting for merging back into the main stream. This issue can be amplified or mitigated by varying the priority $p_4$. To study this phenomena, we examine evolution of $e_4(k)$ over time for different values of $p^m_4$. We explore this scenario in Figure 3, where we plot the value of $e_4(k)$ in the case in which the priority $p_4$ varies from 0.95 to 0.99 and we choose $\delta(2,4) = 15$ min and $\beta^* = 0.05$. The maximum value of $e_4$ is always reached at $\delta(2,4)$ from the peak of $\phi_1$. The maximum number of vehicles simultaneously waiting for merging is $e_4 = 11$ when $p_4 = 0.99$ while only $e_4 = 1$ if $p_4 = 0.95$. A higher priority usually leads to a reduction of $\Delta$, since during congested periods the flow of vehicles entering the service station is bigger than the one exiting it.
This positive effect may be overshadowed by higher queues at the service stations.

### B. Multiple-purpose service station

Next, we consider the case of a multi-purpose service station, as depicted in Figure 1.a, placed between cells 2 and 4. The drivers can choose among three different services leading to distinct time periods spent at the station, so \( \beta^s \in \mathbb{R}^3 \) and \( \delta \in \mathbb{R}^3 \). In the following, we use \( p_4 = [0.97 \ 0.1 \ 0.1 \ 0.1]^\top \) where \( p_4^{\text{ms}} = 0.97 \).

Firstly, we assume that the service station offers three different services which generally require the user to stop for an average period of 5, 15 and 30 min, respectively. We assume that the service requiring more time is used less often than the other two, i.e., the associated \( \beta^s \) is smaller.

In Figure 4, we show the effect that this service station has on the traffic congestion in the case of different entering flows. We simulate three scenarios where the total flow entering the service station \( 1^\top \beta^s \) increases of 5\% every time, going from 0.05 to 0.15. We obtain that \( \pi = 31.3 \) for \( 1^\top \beta^s = 0.05 \), \( \pi = 51.5 \) for \( 1^\top \beta^s = 0.10 \), and \( \pi = 77.1 \) for \( 1^\top \beta^s = 0.15 \). These values are similar to the ones that we would obtain in the case of a single service station with \( \beta^s = 1^\top \beta^s \). They show a steep reduction of the time spent in the traffic congestion due to the presence of the service stations.

The opposite scenario that is a case in which the inflow is fixed and the time spent at the service station increases is omitted for space limitations and the interested reader can find it in [1, Sec. IV.B].

### V. Conclusion and outlooks

The CTM-\( s \) model is a novel macroscopic traffic model based on the CTM. It is particularly suited to describe the traffic on highways (or simple routes) in which there are service stations that can affect the dynamics. The flexibility of the model allows to easily describe several different scenarios like multi-modal service stations in which drivers stop for different reasons. Interestingly, the model shows that the introduction of a service station can reduce traffic congestion, in the case of a short traffic congestion, and it highlights that the number of vehicles stopping is more relevant than the time spent at the service station by the drivers. The dynamics of the model have been developed to be easily interconnected with many classical control schemes used in the literature to perform traffic management.

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