Rotating Black Holes on Codimension-2 Branes

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It has recently been demonstrated that certain types of non-tensional stress-energy can live on tensional codimension-2 branes, including gravitational shockwaves and small Schwarzschild black holes. In this note we generalize the earlier Schwarzschild results, and construct the exact gravitational fields of small rotating black holes on a codimension-2 brane. We focus on the phenomenologically interesting case of a three-brane embedded in a spacetime with two compactified extra dimensions. For a nonzero tension on the brane, we verify that these solutions also show the “lightning rod” effect found in the Schwarzschild solutions, the net effect of which is to rescale the fundamental Planck mass. This allows for larger black hole parameters, such as the event horizon, angular momentum, and lifetime than would be naively expected for a tensionless brane. It is also found that a black hole with angular momentum pointing purely along the brane directions has a smaller horizon angular velocity than the corresponding tensionless case, while a hole with bulk components of angular momentum has a larger angular velocity.

I. SCHWARZSCHILD BLACK HOLES ON CODIMENSION-2 BRANES

It has long been known that the fundamental scale of gravity could be much lower than the four-dimensional Planck scale, $M_P \sim 10^{19}$ GeV in theories with extra dimensions [1]-[4]. This realization has generated considerable interest in the possibility that microscopic black holes can form at energies that will be achievable at the forthcoming Large Hadron Collider (LHC). If the fundamental gravitational scale, $M_*$, is of the order of a TeV, then the LHC may be able to create these black holes through a gravitational lensing process [5] at the rate of approximately one per second [6], which then quickly decay through the emission of Hawking radiation. This would provide a very interesting arena in which one can study quantum gravity effects and has been investigated extensively in the literature, for example in [6]-[15]. One particularly interesting avenue along which one can study small black holes is to put them on codimension-2 branes in which a 3-brane floats in a 6D bulk, for example.

Codimension-2 branes have been the topic of much interest recently [16]-[24] because of their remarkable properties. In general, putting a brane with tension in a bulk spacetime curves the brane as well as the bulk, making the system typically very difficult to analyze. However, on codimension-2 branes the vacuum energy on the brane can be “offloaded” into the bulk cusping it into a cone centered on the brane, leaving the brane flat. This considerably simplifies the task of finding solutions. For example, exact solutions have been found in [25] for a relativistic particle. In the relativistic limit the equations of motion of the particle are exactly described by linear equations and yield a gravitational shockwave, generalizing the 4D Aichelburg-Sexl solutions [26]. These shockwave solutions can be viewed as a six-dimensional black hole on the brane boosted to a relativistic speed, since the gravitational field lines are flattened completely transverse to the direction of motion due to Lorentz contraction. The exact Schwarzschild black hole solution on a tensional codimension-2 brane is, to our knowledge, the first example of an exact black hole on a 3-brane, which we now review.

The black hole is constructed in analogy with the method of Aryal, Ford, and Vilenkin (AFV) [28], which describes a black hole threaded by a straight cosmic string of mass per unit length $\mu$. The net effect of the cosmic string is to induce a conical deficit in the surrounding spacetime, which remains locally flat, but has a conical singularity along the string. The AFV solution is constructed by starting with the four-dimensional Schwarzschild solution. The cosmic string is then threaded along an axis of symmetry of the black hole (which is trivial in the Schwarzschild case). One accounts for its presence by cutting out a wedge from the polar angle around the symmetry axis, the size of which is proportional to $\mu$, and then identifying the edges of the cut, rescaling the polar angle, $\phi \rightarrow b\phi$, where $b = 1 - 4\mu$ [30]. This gives the conical topology since the polar angle doesn’t run around a full $2\pi$ radians.

A small black hole on a tensional 3-brane is constructed along the same lines, where by “small” we mean that the hole has a horizon size smaller than the compactification scale of the extra dimensions, making it a true six-dimensional object. Here we follow the construction given in [29]. We start with the six-dimensional Schwarzschild solution given by

$$ds^2_6 = -\left(1 - \left(\frac{r_0}{r}\right)^3\right)dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^3} + r^2d\Omega_4,$$  \hspace{1cm} (1)

where $r_0$ is the event horizon which depends on the mass of the black hole, and $d\Omega_4$ is the metric of a 4D unit sphere. To include the brane, we choose a three-dimensional symmetry hypersurface along which we thread the brane (generalizing the axis of symmetry in the AFV solution). Then, including the tension on the
brane tells us that we should cut out a wedge from the polar angle that runs around that hypersurface, the size of the wedge depending on the tension, $\lambda$.

The method discussed above is simple in the so-called uniform coordinates, obtained from Eq. (1) by the coordinate transformation

$$r = \mathcal{R} \left( 1 + \frac{1}{4} \left( \frac{r}{\mathcal{R}} \right)^3 \right)^{\frac{2}{3}},$$

in terms of which Eq. (1) becomes

$$ds^2_6 = - \left( \frac{4R^3 - \lambda^2}{4R^3 + \lambda^2} \right) dt^2 + \left( 1 + \frac{1}{4} \left( \frac{r}{\mathcal{R}} \right)^3 \right)^{\frac{2}{3}} \left( d\mathcal{R}^2 + dR^2 + d\phi^2 + B^2 d\rho^2 + r^2 d\phi^2 \right).$$

We can write the metric on the unit sphere in terms of polar coordinates, $R^2 = \hat{x}^2 + \rho^2$, where $\hat{x}$ denotes the coordinates along the brane, and $\rho$ measures distance in the bulk. Then $dR^2 + R^2 d\Omega_4 = d\hat{x}^2 + d\rho^2 + \rho^2 d\phi^2$. Finally, to include the brane we cut out a wedge in the $\rho, \phi$ plane of angular opening $\lambda/M_6$, then rescale $\phi \rightarrow B\phi$, where $29$

$$B = 1 - \frac{\lambda}{2\pi M_6^4}. \quad (4)$$

This gives the exact solution for a Schwarzschild black hole on a tensional 3-brane

$$ds^2_6 = - \left( 1 - \left( \frac{m}{r} \right)^3 \right) dt^2 + \left( 1 + \frac{1}{4} \left( \frac{r}{\mathcal{R}} \right)^3 \right)^{\frac{2}{3}} \left( d\hat{x}^2 + d\rho^2 + B^2 d\rho^2 + B^2 d\phi^2 \right). \quad (5)$$

While the above construction was performed in uniform coordinates to facilitate easy comparison with the AFV solution, one can easily express the metric in a Schwarzschild form. In this case one starts with Eq. (1) and writes $d\Omega_4 = d\Omega_4 + B^2 \prod_{k=1}^6 \sin^2(\theta_k) d\psi^2$, which gives $14$

$$ds^2_6 = - \left( 1 - \left( \frac{m}{r} \right)^3 \right) dt^2 + \left( 1 + \frac{1}{4} \left( \frac{r}{\mathcal{R}} \right)^3 \right)^{\frac{2}{3}} \left[ \frac{dr^2}{1 - \left( \frac{r}{\mathcal{R}} \right)^3} + r^2 \left\{ d\theta^2 + \sin^2 \theta \left[ d\phi^2 + \sin^2 \phi \left( d\chi^2 + B^2 \sin^2 \chi d\psi^2 \right) \right] \right\} \right], \quad (6)$$

which is particularly simple, and one can check that it still satisfies Einstein’s equations in the bulk. It’s easy to see that Eq. (5) can be obtained from Eq. (1) simply by rescaling the angle $\psi \rightarrow B\psi$. It is further clear that $\psi$ has associated with it a Killing vector, $\partial_\psi$, which immediately suggests that $\psi$ is the angle to rescale. This will be useful when we extend this construction to the rotating black hole solutions.

The ADM mass of the black hole is $29$

$$m = 2M_6^4 r_0^3 \int d\Omega_4. \quad (7)$$

Performing the integral in the tensionless case yields the area of the 4-sphere. However, when we include a nonzero $\lambda$, the full integration is over a sphere with a deficit angle, and so the area element depends on the tension, $d\Omega_4 \rightarrow B d\Omega_4$. The integral in Eq. (1) gives a factor of $2\pi^2 B$, and so

$$m = 4\pi^2 BM_6^4 r_0^3,$$

from which we can read off the horizon size

$$r_0 = \left( \frac{m}{4\pi^2 BM_6^4} \right)^{\frac{1}{3}} = \frac{1}{B^{1/3} r_s}, \quad (8)$$

where $r_s$ is the usual 6D Schwarzschild radius in the zero tension limit.

Eq. (6) shows a very interesting feature. Since $B = 1 - \frac{\lambda}{2\pi M_6^4}$ we see that the horizon size can be much larger than naively expected on the basis of the black hole’s mass alone. In particular, the horizon size can become very large for near-critical branes where $\lambda \rightarrow 2\pi M_6^4$. Of course, the horizon size cannot grow too large, as the fundamental description of the hole as a 6D object begins to fail as the horizon size approaches scales comparable to the compactification radius of the extra dimensions. After that, the hole will look approximately four-dimensional, where the 4D Planck mass is determined in the usual way using Gauss’s law, $M_4^2 = M_6^4 \times \text{Vol}(y^6) \sim (BL^2)M_6^4$, where $L$ is the compactification radius. Thus, the 4D Planck scale already includes the effects of a non-zero tension.

Qualitatively, the reason for this enhancement of the horizon is clear: the extra dimensions are bent into a teardrop, changing gravitational coupling goes $1/M_6^4 \rightarrow 1/M_6^4 \chi$, we see that the effective coupling is larger for a nonzero tension than that set by simply the fundamental Planck scale, $M_6$. This explains why the horizon size can be larger than expected – gravity appears stronger. We will also verify this lightning rod effect when we extend the black hole solutions to include spin, to which we turn now.

$^1$ Notice that the same argument holds for Eq. (5), where $\partial_\psi$ is the Killing vector, suggesting that we rescale $\phi$ in that form of the metric.
II. ROTATING BLACK HOLES ON CODIMENSION-2 BRANES

The metric for a spinning black hole in six dimensions is considerably more complicated than the simple Schwarzschild solution given in Eq. (11), and can be expressed in Boyer-Lindquist coordinates as in (37). Here we differ slightly from (37) and take the opposite sign for the two angular momentum parameters, \( a_i \rightarrow -a_i \) to be more consistent with other literature. Note, also, that \( r \) is not the usual flat-space radial coordinate, but is defined instead by the elliptical coordinate relation

\[
\frac{r^2}{a_1^2} + \frac{(y^1)^2}{a_2^2} + \frac{(x^2)^2}{a_3^2} = 1,
\]

where \( x^i \) are the brane coordinates, and \( y^a \) are the bulk coordinates. With this the metric is

\[
ds_6^2 = -dt^2 + r^2 d\alpha^2 + \frac{\Pi F}{1 - \mu r} dr^2 + \sum_{i=1}^2 \left[ (r^2 + a_i^2)(d\mu_i^2 + \mu_i^2 d\phi_i^2) + \frac{\mu_i}{r \Pi} (dt - a_i \mu_i d\phi_i)^2 \right].
\]

Here \( \mu \) is proportional to the mass of the black hole, and

\[
\Pi = \prod_{i=1}^2 (r^2 + a_i^2),
\]

Eq. (9) is written in a compact form, but in order to extend the spinning black hole solutions to include a nonzero tension, it will be more convenient to expand the sum in Eq. (9), rewriting it in the following way:

\[
ds_6^2 = -(1 - \frac{\mu r}{\Pi F}) dt^2 - \frac{2\mu r}{\Pi F} (a_1 \mu_1^2 d\phi_1 + a_2 \mu_2^2 d\phi_2) dt + \frac{F}{1 - \mu r} dr^2 + r^2 \left( d\alpha^2 + d\mu_1^2 + d\mu_2^2 + \mu_1^2 d\phi_1^2 + \mu_2^2 d\phi_2^2 \right)
\]

\[
+ a_1^2 \left[ d\mu_1^2 + \mu_1^2 \left( 1 + \frac{\mu r}{\Pi F} \mu_1^2 \right) d\phi_1^2 \right] + a_2^2 \left[ d\mu_2^2 + \mu_2^2 \left( 1 + \frac{\mu r}{\Pi F} \mu_2^2 \right) d\phi_2^2 \right] + 2a_1a_2 \frac{\mu r}{\Pi F} \mu_1^2 \mu_2^2 d\phi_1 d\phi_2.
\]

To include the brane we proceed as before and choose a symmetry hypersurface and thread the brane along it. Because we have two angular momentum parameters we now have a choice of two hypersurfaces along which we can thread the brane. We can choose the normal to the plane swept out by either \( \phi_1 \) or \( \phi_2 \). Once again, it’s clear that the Killing vectors, \( \partial_{\phi_1}, \partial_{\phi_2} \) suggest the angles that we should rescale. The general metric, Eq. (13), is symmetric under the simultaneous interchange \( a_1 \leftrightarrow a_2 \) and \( \phi_1 \leftrightarrow \phi_2 \). So without loss of generality, let us choose \( \phi_2 \) as defining the angle about the symmetry hypersurface. Then, rescaling \( \phi_2 \rightarrow B \phi_2 \) as in Eq. (10) yields our metric

\[
ds_6^2 = -(1 - \frac{\mu r}{\Pi F}) dt^2 - \frac{2\mu r}{\Pi F} (a_1 \mu_1^2 d\phi_1 + a_2 B \mu_2^2 d\phi_2) dt + \frac{F}{1 - \mu r} dr^2 + r^2 \left( d\alpha^2 + d\mu_1^2 + d\mu_2^2 + \mu_1^2 d\phi_1^2 + B^2 \mu_2^2 d\phi_2^2 \right)
\]

\[
+ a_1^2 \left[ d\mu_1^2 + \mu_1^2 \left( 1 + \frac{\mu r}{\Pi F} \mu_1^2 \right) d\phi_1^2 \right] + a_2^2 \left[ d\mu_2^2 + B^2 \mu_2^2 \left( 1 + \frac{\mu r}{\Pi F} \mu_2^2 \right) d\phi_2^2 \right] + 2a_1a_2 \frac{\mu r}{\Pi F} \mu_1^2 \mu_2^2 d\phi_1 d\phi_2.
\]

Eq. (14) has an interesting feature. The obvious choice for orienting the symmetry hypersurface would be the plane defined by the axis of rotation of the black hole. This would be the only choice in the analogous construction of a spinning black hole threaded by a cosmic string. However, in Eq. (14) we have two choices of angle through which the hole can rotate. We have chosen to rescale \( \phi_2 \rightarrow B \phi_2 \), orienting the brane along the normal to \( \phi_2 \), but we have not specified the rotation axis. Choosing the rotation axis to be \( \phi_2 \), by setting \( a_1 \equiv 0 \), say, corresponds to cutting the wedge out of the rotation axis such that the hole would not complete a full revolution through \( 2\pi \) radians, but only through \( 2\pi B \) radians. The hole would be spinning orthogonal to the brane with its angular momentum pointing along the brane. Choosing \( \phi_1 \) instead cuts the wedge from an axis orthogonal to the direction of spin, such that the hole still completes a full \( 2\pi \) radian revolution. The hole would be spinning on the brane with its angular momentum orthogonal to the brane.

The two direction cosines, \( \mu_i \), and the \( \alpha \) coordinates are subject to the constraint

\[
\mu_1^2 + \mu_2^2 + \alpha^2 = 1,
\]

which could be satisfied by expressing \( \mu_i \) and \( \alpha \) in terms of angles on a unit 2-sphere, for example, but we’ll leave the expressions general for now. In general a \( D \)-dimensional Kerr solution has \( \left[ \frac{D-1}{2} \right] \) angular momentum parameters, where \( |x| \) denotes the integer part of \( x \). Hence, Eq. (9) has two parameters, \( a_i \). Unlike in four dimensions, for six or more dimensions black holes with a fixed mass can have an arbitrarily large angular momentum (37).
III. A SPECIAL CASE – A SINGLE ROTATION PARAMETER

It is already clear that the tension on the brane can have an interesting effect on the black hole properties through the lightning rod effect. We will see that this effect will persist to other aspects of the black hole such as the temperature and the lifetime. The metric in Eq. (14) is general, as are all the expressions derived from it so far and we could continue with the most general case. In this section, however, let us focus on a particular case of phenomenological interest that will elucidate the effects of the tension in as simple and direct a way as possible.

We suppose that the black hole is formed through the collision of two particles on a delta function brane. If the collision occurs with a nonzero impact parameter, $b$, then the hole will have angular momentum. Since the particles are constrained to live on the brane, the hole will spin on the brane and the initial angular momentum will have a single value, directed along the brane. By conservation of angular momentum, the black hole will subsequently have a single nonzero rotation parameter, $a$. We can determine a simpler form for the metric in this limit starting with Eq. (14) and setting $a_2 = 0$ and letting $a_1 = a$, which is required by the combination of our choice of rescaling $\phi_2 \rightarrow B\phi_2$, and also having the angular momentum point along the brane. Then making the substitutions $\phi_1 = \phi$, $\phi_2 = \psi$, and

$$\mu_1 = \sin \theta, \quad \mu_2 = \cos \theta \sin \chi, \quad \alpha = \cos \theta \cos \chi,$$

we find

$$ds_6^2 = -(1 - \frac{\mu}{r^3}) dt^2 - \frac{2\mu}{r^3} \sin^2 \theta dtd\phi + \frac{\mu^2}{r^3 + \mu^2} d\psi^2$$

$$+ \rho^2 d\theta^2 + \sin^2 \theta \left( r^2 + a^2 + \frac{\mu^2}{\rho^2} \sin^2 \theta \right) d\phi^2 + r^2 \cos^2 \theta \left( d\chi^2 + B^2 \sin^2 \chi d\psi^2 \right).$$

where $\rho^2 = r^2 + a^2 \cos^2 \theta$. Let us now analyze the parameters associated with Eq. (18). Eq. (16) still gives $\mu$, while Eq. (17) reduces to

$$J = \frac{1}{2} Ma.$$

The particles have a center of mass energy $\sqrt{s} = M_i$, which is fixed by the accelerator, with an impact parameter $b$. Then, the initial angular momentum will be $J_i = bM_i/2$ in the center of mass frame. The black hole is formed when the particles pass within a distance of each other smaller than the event horizon of the black hole which depends on its mass and also its angular moment-
tum. This sets an upper limit on the impact parameter of $b \leq 2r_H(M,J)$ [13], and so therefore an upper limit on the angular momentum (for a fixed mass). After the collision, a black hole of mass $M$ and angular momentum $J = bM/2$ is formed. Comparison with Eq. (19) shows that $a = b$, and so the angular momentum parameter is really just a measure of the impact parameter (for a black hole formed in this way). Plugging in for $a \rightarrow b_{\text{max}}$ in Eq. (19) we find

$$J_{\text{max}} = \frac{Mb_{\text{max}}}{2} = Mr_H.$$

The horizon occurs where $r^3_H + r_H a^2 = \mu$, which we
can write as

\[ r_H^3 \left( 1 + \frac{a^2}{r_H^2} \right) \equiv r_H^3 \left( 1 + \frac{a_\star^2}{r_H^2} \right) = \mu, \]

where \( a_\star \equiv a/r_H = 2J/J_{\text{max}} \). Because \( 0 \leq a_\star \leq 2 \), we have \( \frac{1}{\mu} \leq r_H^2 \leq \mu \), so \( a_\star \) makes very little difference and we can take the above expression as defining the event horizon of the spinning black hole rather than solving the cubic equation. Then, using Eq. (16),

\[ r_H = \left( \frac{M}{4\pi^2(1 + a_\star^2)M_{6\,\text{eff}}} \right)^{\frac{1}{3}}. \tag{21} \]

Note the presence of the effective Planck scale in Eq. (21), showing the lightning rod effect. It’s immediately clear that the angular momentum, Eq. (20), can be larger than would be naively expected \(( \sim B^{-1/3}J_{\text{max}} \), where \( J_{\text{max}} \) is the tensionless case \). It is also clear that the ergosphere will be similarly enhanced.

Given that for a black hole of a fixed mass the horizon can be larger for a tensional brane than for one without tension, one would also expect the temperature of the black hole to decrease as a function of mass since large black holes have a lower temperature than small ones. The temperature can be determined by Wick rotating the time coordinate in the usual way and is given by

\[ T_H = \frac{3 + a_\star^2}{4\pi r_H(1 + a_\star^2)}, \tag{22} \]

which is, indeed, smaller than the zero tension limit. Eq. (22) has an associated entropy

\[ S_{BH} = \frac{M}{4T_H} \frac{3 + a_\star^2}{1 + a_\star^2}, \tag{23} \]

and is also larger than one would naively expect.

Following their production, the black holes evaporate first shedding their hair during a “balding” phase. The holes then enter a spin-down phase during which they shed their angular momentum, after which the hole is described by the Schwarzschild metric and decays through the emission of Hawking radiation, radiating mainly into brane modes \[10\]. Beyond this one needs a full theory of quantum gravity to exactly describe the subsequent final decay. Because of the tension, the temperature is smaller and the angular momentum is higher than the “braneless” case, and so one would expect the lifetime of the black hole to be increased. The lifetime of a six-dimensional black hole can be estimated from thermodynamics as \( dE/dt \sim dM/dt \sim \text{(Area)} \times T_H^3 \), giving \( \tau \sim 1/M_{6\,\text{eff}}(M/M_{6\,\text{eff}})^{5/3} \) after integrating and using Eqs. (21) and (22). So one should expect that \( \tau \approx B^{-8/3}\tau_0 \), where \( \tau_0 \) is the tensionless case. Thus, the black holes should linger a while longer after production. Finally the angular velocity of the horizon is given by

\[ \Omega_H = \frac{a_\star}{r_H(1 + a_\star^2)}, \tag{24} \]

and is found by looking at null geodesics in the \( \phi \) direction. The velocity is smaller than the zero tension limit since the horizon is larger.

Eq. (24) has been determined for a black hole with angular momentum along the brane. When the angular momentum has only bulk components, the hole could be rotating through the bulk angle from which the deficit angle had been cut. The hole would be described by Eq. (18), but with \( \phi \to B\phi \) instead of rescaling \( \psi \). In this case, the angular velocity would read

\[ \Omega_H = \frac{a_\star}{Br_H(1 + a_\star^2)}, \tag{25} \]

which is larger than the tensionless case \(( \Omega_H \sim B^{-2/3}\Omega_{H0} \), after including the factor of \( B^{-1/3} \) in \( r_H \), and where \( \Omega_{H0} \) is the tensionless case \). The increase is due to the fact that the hole is now only completing a rotation through \( 2\pi B \leq 2\pi \) radians, as discussed above, and so the velocity should be higher for a fixed angular momentum. This would be an additional amplification due to the tension beyond the rescaling of the Planck mass. This is in contrast to Eq. (24) where the angular velocity is reduced due to the tension.

The black holes with their angular momentum pointing along the brane will lose this momentum through the Penrose process (superradiance). This process enhances the emission of higher-spin particles such that graviton emission could be the dominant effect \[38\], though other brane fields still participate. After the hole has spun down and enters the Hawking phase of its evaporation, the emission of gravitons into the bulk could possibly cause the hole enough recoil to leave the brane \[39\] - \[41\], overcoming the small perturbation restoring force due to the tension. However, for codimension-2 branes this effect is likely to be small \[42\].

Although the presence of the brane can give additional effects as described above, it is clear that the main effect of the tension really is, in every case so far considered, to simply rescale \( M_4^2 \to M_{6\,\text{eff}} \). This can have useful experimental consequences. For example, since the cross section for production of a black hole is \( \sigma \approx \pi r_H^2 \), one would expect an increase in the cross section due to the tension on the brane \[14\]. The black holes would be easier to make, and so one should find an increase in the production rate. This effect, combined with the increase in the lifetime described above should be useful in the experimental study of TeV-scale black holes at the LHC, if nature has chosen to behave in this way (see \[14\] for a discussion of the effects of a nonzero tension on the evaporation of a Schwarzschild black hole).

In fact, one can see that the tension on the brane may be able help in the production of black holes in another way. The LHC will be able to create TeV-scale black holes, and it is usually assumed that the fundamental scale of gravity needs to be in this range for this process to occur. However, due to the lightning rod effect, one might need only \( M_{6\,\text{eff}} \) to be in the TeV range. Since \( M_{6\,\text{eff}} = B^{1/4}M_6 \) one can see that for near-critical branes
one might have a value for $M_6$ higher than a TeV and still create black holes! Hence, the fundamental scale of gravity could potentially be higher than the experimentally accessible range, but we still have a possibility of exploring quantum gravity if the tension is near critical.

IV. CONCLUSIONS

In general, braneworld systems can be very difficult to analyze. Tension on the brane can curve the brane, as well as the bulk. However, codimension-2 branes allow the tension to be off-loaded into the bulk, leaving the brane flat which can considerably simplify the analysis. In this note we have benefited significantly from this simplification to describe the exact solution for a black hole spinning on a codimension-2 brane. This solution shows the same lightning rod behavior as the Schwarzschild solution of [29]. This behavior rescales the fundamental Planck mass, amplifying various properties of the black hole such as the event horizon, the angular momentum, and the lifetime. The temperature of the black hole is correspondingly smaller. The combination of all these effects may prove to be useful in the experimental study of TeV scale black holes since the holes should be easier to produce and live longer than the corresponding tensionless case.

While the main effect of the tension on the brane is to rescale the Planck mass, it can also have additional effects. As we have seen in Eq. 28, the angular velocity of a black hole with bulk angular momentum can be amplified to a value larger than the corresponding tensionless case. This effect is not present in a black hole with purely brane angular momentum components, and is an amplification beyond simply the usual lightning rod effect which seeks to decrease the angular velocity.

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