Influence of Heat Transfer on MHD Oscillatory Flow for Eyring-Powell Fluid with Variable Viscosity Through a Porous Medium

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Abstract

The aim of this paper is the effect of heat transfer on the oscillating flow of the hydrodynamics of magnetizing Eyring-Power fluid with variable viscosity through a porous medium for two kinds of geometries "Poiseuille flow and Couette flow". We used "perturbation technique" to obtain a clear formula for fluid motion. The results that obtained are illustrated by graphs.

Keywords: Eyring-Powell fluid, MHD, Oscillatory flow, Variable viscosity, Porous medium.

1. Introduction

The concept of porous media is used in many fields of applied science and engineering: mechanics (acoustics, geo mechanics, soil mechanics, rock mechanics), engineering (petroleum engineering and construction engineering), geosciences (hydrography and petroleum geology), Geophysics, biophysics, materials science, etc..

We review some of the research that touched on the movement of the Powell-Eyring fluid in some channels. S. A. Gaffar, V. R. Prasad, E. K. Reddy (1), studies analyze the nonlinear, non-isothermal, magnetohydrodynamic (MHD) free convection boundary layer flow, heat and mass transfer of non-Newtonian Eyring-Powell fluid from a vertical surface in a non-Darcy, homogenous porous medium, isotropic, in the presence of on slip currents and hall currents and on slip currents. S. O. Alharbi, A. Dawar, Z. Shah, W. Khan, M. Idrees, S. Islam and I. Khan (2), studies briefly examined the entropy generation in magnetohydrodynamic (MHD) Eyring-Powell fluid over an unsteady oscillating porous stretching sheet. M. Khan, M. Irfan, W.A.Khan, L. Ahmad (3), studies scrutinize the steady three-dimensional magnetohydrodynamics (MHD) flow of Powell-Eyring nanofluid with convective and the nanoparticles mass flux conditions. Additionally. H. A. Ogunseye, H. Mondal, P. Sibanda, H. Mamбли (4), they checked the flow and heat transfer in the flow of the first nano-eric across the expansion surface using nano-conductivity viscosity models. W. Ibrahim and B. Hindebu (5), this study analyzed the MHD boundary layer flow of Eyring-Powell nanofluid past stretching cylinder with Cattaneo-Christov heat flux model. Wissam Sadiq K., Dheia G.S. Al-Khafajy (7), they studied the influence of heat transfer on magnetohydrodynamics (MHD) for the oscillatory flow of Williamson fluid with variable viscosity model through a porous medium channel. Dheia G.S. Al-Khafajy (8), studied the effect of heat-transfer on MHD oscillatory flow of Jeffrey fluid with variable viscosity model.
through a porous medium. Dheia G.S. Al-Khafajy (9), study the radiation and mass transfer effects on MHD oscillatory flow for Jeffery fluid with variable viscosity through porous channel in the presence of chemical reaction. Al-Khatib and Wilson (10), the development of Poiseuille flow of the yield stress fluid was discussed. Soundalgekar and Bhat (11), have investigated the MHD oscillatory flow of a Newtonian fluid in a Channel with heat transfer. M. Vidhya, N. Niranjana, A. Govindarajan (12) discussed the effect of two-dimensional unstable thermal flow during the movement of a dusty non-pressure liquid. This study aimed to analyze the mathematical model of the effects of heat transfer on the oscillating flow of the hydrodynamics of magnetizing Eyring-Power fluid with variable viscosity through a porous medium for two types of engineering conditions "Poiseuille flow and Couette flow".

This research consists of six sections in the first section, which is the introduction, research on this topic shows. The second section includes the form of the flow channel with the formulation of the governing equations with boundaries conditions and the formula of the Eyring-Powell fluid equation. In the third section, we note the dimensionless transformations which helps us formulate the governing equations in a way that helps in solving them. Section four includes problem-solving and finding the formula for variable viscosity and velocity for two types of engineering conditions "Poiseuille flow and Couette flow". In the fifth and sixth sections, we discuss the results that we obtained through the illustrated graphs and review the most important observations that we reached.

2. Mathematical Formulation

consider the flow of an Eyring-Powell fluid in a porous medium of width $h$ under the effects of the electrically applied magnetic field and radioactive heat transfer as depicted in Fig 1. Supposed that the fluid has very small electromagnetic force produced and the electrical conductivity is small. We are considering Cartesian coordinate system such that, $(u(y), 0, 0)$ is a velocity vector in which is the $x$-component of velocity and $y$ is perpendicular to the $x$-axis.

![Fig 1 Channel format: (i) Poiseuille flow and (ii) Couette flow](image)

The basic equations governing for Eyring – Powell fluid are given by:

The continuity equation is given by:

$$\rho \left( \frac{\partial u}{\partial x} + \frac{\partial \bar{u}}{\partial y} \right) = 0.$$  

(1)

The momentum equations are:

In the $x$-direction: 

$$\rho \left( \frac{\partial u}{\partial x} + \frac{v}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial (\rho \beta)}{\partial y} + \rho g (T - T_0) - \sigma B_0^2 \bar{u} - \frac{\mu(T)}{k}(\bar{u}).$$  

(2)

In the $y$-direction: 

$$\rho \left( \frac{\partial v}{\partial y} + \frac{u}{\partial x} \right) = -\frac{\partial p}{\partial y} + \frac{\partial (\rho \beta)}{\partial x} + \frac{\partial (\rho g (T - T_0))}{\partial y} - \frac{\mu(T)}{k}.$$  

(3)

The temperature equation:

$$\rho \frac{\partial T}{\partial \bar{x}} = \frac{k}{C_p} \frac{\partial^2 T}{\partial \bar{y}^2} - \frac{1}{C_p} \frac{\partial q}{\partial \bar{y}}.$$  

(4)

where $\bar{u}$ is the axial velocity, $\rho$ is the density of the fluid, $p$ is the pressure, $\alpha$ is the electrical conductivity, $B_0$ is the strength of the magnetic field, $\beta$ is the acceleration due to gravity, $T$ is temperature, $C_p$ is specific heat at constant pressure, $q$ is the radiation heat flux, $\mu(T)$ fluid
viscosity dependent on temperature and \( R \) is thermal conductivity.

The corresponding boundary conditions are given below:

\[
\begin{align*}
\text{for Poiseuille flow:} \\
\text{velocity:} & \quad u = 0 \text{ at } \bar{y} = 0, \\
\text{temperature:} & \quad T = T_0 \text{ at } \bar{y} = 0, \\
\text{for Couette flow:} \\
\text{velocity:} & \quad u = U_0 \text{ at } \bar{y} = 0, \\
\text{temperature:} & \quad T = T_1 \text{ at } \bar{y} = 0.
\end{align*}
\]

(5)

The radioactive heat flux [6] is given by:

\[
\frac{\partial q}{\partial y} = 4\eta^2(T_0 - T).
\]

(6)

where \( \eta \) is the radiation absorption.

The fundamental equation for Eyring – Powell fluid given by:

\[
\begin{align*}
S &= -\overline{\mathbf{p}} : \mathbf{I} + \tau, \\
\bar{\tau} &= \mu(T)\nabla \bar{\nabla} + \frac{1}{B_1 A_1} \text{sinh}^{-1} \left( \frac{1}{\bar{\tau}} \right),
\end{align*}
\]

where \( \overline{\mathbf{p}} \) is the pressure, \( \mathbf{I} \) is the unit tensor, \( \bar{\tau} \) is the extra stress tensor, \( \mu(T) \) is the variable shear rate viscosity and \( \nabla \bar{\nabla} \) is the velocity gradient. We can write the component of extra stress tensor according as follows:

\[
\bar{\tau}_{12} = \bar{\tau}_{21} = 0, \quad \bar{\tau}_{12} = \left( \mu(\theta)\mu_0 + \frac{1}{B_1 A_1} \right) \frac{\partial u}{\partial y} - \frac{1}{6B_1 (A_1)^2} \left( \frac{\partial u}{\partial y} \right)^3.
\]

(7)

3. Method of Solution

The governing equations for non-dimensional conditions are:

\[
\begin{align*}
\chi &= \frac{\bar{y}}{h}, \\
\mu &= \frac{\bar{y}}{h}, \\
\rho &= \frac{\bar{y}}{\mu_0}, \\
\bar{p} &= \frac{\bar{y}}{\bar{p}} \frac{\partial \bar{p}}{\partial \bar{p}}, \\
\bar{t} &= \frac{\bar{y}}{\bar{t}}, \\
\bar{r} &= \frac{\bar{y}}{\bar{r}}, \\
\bar{T} &= \frac{\bar{y}}{\bar{T}}, \\
\bar{P} &= \frac{\bar{y}}{\bar{P}}, \\
A &= \frac{\bar{y}}{\bar{A}}, \\
\Re &= \frac{\bar{y}}{\bar{R}}, \\
\Da &= \frac{\bar{y}}{\bar{D}}, \\
\Gr &= \frac{\bar{y}}{\bar{Gr}}, \\
W &= \frac{\bar{y}}{\bar{W}}, \\
\theta &= \frac{\bar{y}}{\bar{T}}, \\
\theta &= \bar{T}.
\end{align*}
\]

(8)

where \( "\mu" \) is the mean flow velocity, \( "\mu_0" \) is Darcy number, \( "\Re" \) is Reynolds number, \( "\Gr" \) is magnetic parameter, \( "\Pe" \) is the Peclet number, \( "\Gr" \) is the radiation parameter,\( "\Gr" \) is Thermal Grashof number and \( "\Gr" \) is the radiation parameter.

Substituting equations (6) - (8) into equations (1) - (5), we have the following of non-dimensional equations:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\Re \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \Gr \theta - \left( M^2 + \frac{\mu(\theta)}{\Da} \right) u, \\
\frac{\partial p}{\partial x} &= 0, \\
\Pe \frac{\partial \theta}{\partial x} &= \frac{\partial^2 \theta}{\partial y^2} + (N^2) \theta, \\
\tau_{12} &= \left( \mu(\theta) + W \right) \frac{\partial u}{\partial y} - A \left( \frac{\partial u}{\partial y} \right)^3.
\end{align*}
\]

(9)

(10)

(11)

(12)

(13)

With the boundary conditions:

\[
\begin{align*}
u(0) &= 0, \quad u(1) = 0 \quad \text{(for Poiseuille flow)} \\
\theta(0) &= 0, \quad \theta(1) = 1 \quad \text{(for Couette flow)}
\end{align*}
\]

(14)

(15)

(16)

After simplifying the equation resulting from compensating the equation (13) into equation (10), we have:

\[
\Re \frac{\partial u}{\partial x} + \Gr \theta - 3A \left( \frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + \left( \mu(\theta) + W \right) \frac{\partial^2 u}{\partial y^2} - \left( M^2 + \frac{\mu(\theta)}{\Da} \right) u
\]

(17)

4. Solution of the Problem
4.1. Solution of the Heat Equations
Using the separating variables method, assuming \( \vartheta(y,t) = e^{i\omega t} \vartheta_0(y) \) for heat equation (12), where \( \omega \) is the frequency of oscillation with the boundary condition (16).

The heat equation solution
\[
\vartheta(y,t) = \csc(2) \sin(2y) e^{i\omega t}
\]
where \( Z = \sqrt{N^2 - i\omega Pe} \).

4.2. Solution of the Motion Equation
To solve the motion equation for two types flow “Poiseuille flow and Couette flow”. Let
\[
\frac{dp}{dt} = -\lambda e^{i\omega t}, \quad u(y,t) = f(y)e^{i\omega t}
\]  
where \( \lambda \) is a real constant and \( \omega \) is the frequency of the oscillation.

The Reynold's model and variation of viscosity with temperature are defined as:
\[
\mu(\vartheta) = e^{-\alpha\vartheta}.
\]

By using the Maclaurin series, we get:
\[
\mu(\vartheta) = 1 - \alpha\vartheta \quad \alpha << 1.
\]

In this case, the viscosity is fixed at \( \alpha = 0 \), by substituting Eq. (19) and Eq. (21) into Eq. (17), we get:
\[
Re \frac{\partial}{\partial y} \left(f(y)e^{i\omega t}\right) = \lambda e^{i\omega t} + (1 - \alpha \vartheta) \frac{\partial^2}{\partial y^2} \left(f(y)e^{i\omega t}\right) + W \frac{\partial^2}{\partial y^2} \left(f(y)e^{i\omega t}\right) - 3A \left( \frac{\partial^2}{\partial y^2} \left(f(y)e^{i\omega t}\right) \right)^2 \quad \text{Re} \frac{\partial^2}{\partial y^2} \left(f(y)e^{i\omega t}\right)
\]
\[
3A \left( \frac{\partial^2}{\partial y^2} \left(f(y)e^{i\omega t}\right) \right)^2 + Gr \vartheta_0(y)e^{i\omega t} - \left( M^2 + \frac{(1 - \alpha \vartheta)}{Da} \right) \left(f(y)e^{i\omega t}\right). \quad (22)
\]

We assume a small value of \( A \), equation (22) is a non-linear differential equation and it is hard to find an exact solution, so will be used the perturbation technique to find the problem solution, as follows:
\[
f = f_0 + Af_1 + O(A^2). \quad (23)
\]

Substituting equation (23) into equation (22) with boundary conditions, then equating the like powers of \( A \), we have:
\[
Re i\omega (f_0 + Af_1) = \lambda + (1 - \alpha \vartheta) \frac{\partial^2}{\partial y^2} (f_0 + Af_1) + W \frac{\partial^2}{\partial y^2} (f_0 + Af_1) - 3A \left( \frac{\partial^2}{\partial y^2} (f_0 + Af_1) \right)^2 e^{2i\omega t} + Gr \vartheta_0 - M^2(f_0 + Af_1) - \frac{(1 - \alpha \vartheta)}{Da} (f_0 + Af_1). \quad (24)
\]

4.2.1 Poiseuille flow
The solution of the equation (24) for Poiseuille flow by using boundary condition (14).

**I - Zeros-order system \( (A^0) \)**
\[
Re i\omega f_0 = \lambda + Gr \vartheta_0 + (1 - \alpha \vartheta) \frac{\partial^2 f_0}{\partial y^2} + W \frac{\partial^2 f_0}{\partial y^2} - \left( M^2 + \frac{(1 - \alpha \vartheta)}{Da} \right) f_0. \quad (25)
\]

The associated boundary conditions are:
\[
f_0(0) = f_0(1) = 0.
\]

**II - First-order system \( (A^1) \)**
\[
Re i\omega f_1 = (1 - \alpha \vartheta) \frac{\partial^2 f_1}{\partial y^2} + W \frac{\partial^2 f_1}{\partial y^2} - 3e^{2i\omega t} \left( \frac{\partial^2 f_0}{\partial y^2} \right) \left( \frac{\partial f_0}{\partial y} \right)^2 - \left( M^2 + \frac{(1 - \alpha \vartheta)}{Da} \right) f_1. \quad (26)
\]

The associated boundary conditions are:
\[
f_1(0) = f_1(1) = 0.
\]

Equations (25) and (26) have been found by the expansion in terms of \( A \). We give some physical meaning to the problem by considering that when \( \alpha \) is small and we use the perturbations series with parameters \( \alpha \). We substitute for \( f_j \) (for \( j = 0,1 \)) by expansion
and equating the coefficients of like powers in \( \alpha \), then the following set of equations are obtained.

**A. Approximation of Solution for**

By substituting equation (27) into equation (25), we get

\[
Re \ i \omega (f_{00} + \alpha f_{01}) = \lambda + Gr \theta_0 + (1 - \alpha \theta) \frac{\partial^2}{\partial y^2} (f_{00} + \alpha f_{01}) + W \frac{\partial^2}{\partial y^2} (f_{00} + \alpha f_{01}) - (M^2 + \frac{1-\alpha \theta}{\partial \alpha}) (f_{00} + \alpha f_{01})
\]

Equating the coefficients of like powers in \( \alpha \), we obtain:

- **Zeros-Order System** \((\alpha^0)\)

\[
\frac{\partial^2 f_{00}}{\partial y^2} - \left( \frac{\omega Re + M^2 + 1}{1 + W} \right) f_{00} = - \left( \frac{\lambda + Gr \theta_0}{1 + W} \right).
\]

The associated boundary conditions are: \( f_{00}(0) = f_{00}(1) = 0 \)

- **First-Order System** \((\alpha^1)\)

\[
\frac{\partial^2 f_{01}}{\partial y^2} - \left( \frac{\omega Re + M^2 + 1}{1 + W} \right) f_{01} = \left( \frac{1}{1 + W} \right) \left( \frac{\partial}{\partial y} \frac{\partial f_{00}}{\partial y} - \frac{\partial}{\partial \alpha} f_{00} \right).
\]

The associated boundary conditions are: \( f_{01}(0) = f_{01}(1) = 0 \)

The perturbation solutions of the equations (28) and (29), with boundary conditions, are given \( f_0 = f_{00} + \alpha f_{01} \).

**B. Approximation of Solution for**

By substituting for given by the expression equations (27) into equations (26), we get:

\[
Re \ i \omega (f_{10} + \alpha f_{11}) = (1 - \alpha \theta) \frac{\partial^2}{\partial y^2} (f_{10} + \alpha f_{11}) + W \frac{\partial^2}{\partial y^2} (f_{10} + \alpha f_{11}) - 3e^{2i \omega t} \left( \frac{\partial^2}{\partial y^2} (f_{00} + \alpha f_{01}) \right) \left( \frac{\partial}{\partial y} (f_{00} + \alpha f_{01}) \right)^2 - \left( M^2 + \frac{1-\alpha \theta}{\partial \alpha} \right) (f_{10} + \alpha f_{11}).
\]

Equating the coefficients of like powers in \( \alpha \), we obtain:

- **Zeros-Order System** \((\alpha^0)\)

\[
\frac{\partial^2 f_{10}}{\partial y^2} - \left( \frac{\omega Re + M^2 + 1}{1 + W} \right) f_{10} = \frac{3}{1 + W} e^{2i \omega t} \left( \frac{\partial^2 f_{00}}{\partial y^2} \right) \left( \frac{\partial f_{00}}{\partial y} \right)^2.
\]

The associated boundary conditions are: \( f_{10}(0) = f_{10}(1) = 0 \)

- **First-Order System** \((\alpha^1)\)

\[
\frac{\partial^2 f_{11}}{\partial y^2} - \left( \frac{\omega Re + M^2 + 1}{1 + W} \right) f_{11} = \frac{1}{1 + W} \left( \frac{\partial^2 f_{10}}{\partial y^2} + 6e^{2i \omega t} \left( \frac{\partial^2 f_{10}}{\partial y^2} \right) \left( \frac{\partial f_{10}}{\partial y} \right) \left( \frac{\partial f_{10}}{\partial y} \right) \right) + 3e^{2i \omega t} \left( \frac{\partial^2 f_{10}}{\partial y^2} \right) \left( \frac{\partial f_{10}}{\partial y} \right)^2 - \left( \frac{\partial}{\partial \alpha} \right) f_{10}.
\]
The associated boundary conditions are: \( f_{11}(0) = f_{11}(1) = 0 \).

The perturbation solutions of the equations (30) and (31), with boundary conditions, give is:

\[ f_i = f_{i0} + \alpha f_{i1} \]

Finally, the perturbation solutions up to second term for \( f \) are given by \( f = f_{0} + \alpha f_{1} \). We did not write the solution because it is very long.

### 4.2.2 Couette flow

The upper flake is locomotion and the lower flake is fixed with the velocity \( U_0 \), boundary conditions for the Couette flow problem as problem defined \( u(0) = 0, \ u(1) = U_0 \).

By the same previous method that we used to solve the equation and on the terms of the Poiseuille flow equation (17). The solution has been calculated by the perturbation technique and the results have been discussed during graphs.

### 5. Results and Discussion

We discuss the influence of heat transfer on MHD oscillatory flow for Eyring – Powell fluid through a porous medium with variable viscosity for two types of engineering conditions "Poiseuille flow and Couette flow" by using the graphical illustrations. We provide numerical assessments of analytical results and some of the graphically significant results are presented in figures (2-13). We used the (MATHEMATICA-12) program to find numerical results and illustrations. The velocity profile of the Poiseuille flow is shown in figures (2-7). Figure 2 shows the velocity profile \( u \) decreases with the increasing \( Da \) and \( \omega \). Figure 3 illustrates the influence of \( P \) and \( \alpha \) on the velocity profiles function \( u \) vs. \( y \). It is found that the velocity increasing with the increasing \( P \) while the velocity increases with the increasing \( \alpha \). In figure 4 the velocity profile \( u \) increases with the increasing of \( \lambda \) and \( Re \), respectively, while the velocity increasing with the increasing the parameters \( M \) and \( N \) in figure 5. Figure 6 illustrates the influence of \( A \) and \( W \) on the velocity profiles function \( u \) vs. \( y \). It is found that the increasing of \( A \) the velocity decreases and the velocity decreases with the increasing of \( W \). We found that the velocity decreases with the increasing of \( Gr \) and \( \omega \) in figures 7. The velocity profile of Couette flow is shown in figures (8-13). It is found that the velocity increases with increasing of the parameters \( \alpha \), \( Pe \), \( Re \), \( \lambda \), \( M \) and \( N \) respectively, while the velocity decreases with the increasing \( Da \), \( \omega \), \( Gr \), \( W \) and \( A \).

**Fig 2:** "Poiseuille flow" Velocity profile for \( Da \) and \( \omega \) with \( A = 0.1, M = 1, Re = 2, W = 0.5, \lambda = 1, Pe = 0.7, N = 2, Gr = 1, t = 0.5, \alpha = 0.1 \.
Fig 3: "Poiseuille flow" Velocity profile for $Pe$ and $\alpha$ with $\omega = 1, A = 0.1, M = 1, Re = 2, W = 0.5, \lambda = 1, Da = 0.8, N = 2, Gr = 1, t = 0.5$.

Fig 4: "Poiseuille flow" Velocity profile for $\lambda$ and $Re$ with $\omega = 1, A = 0.1, M = 1, W = 0.5, Pe = 0.7, Da = 0.8, N = 2, Gr = 1, t = 0.5, $\alpha = 0.1$.

Fig 5: "Poiseuille flow" Velocity profile for $M$ and $N$ with $\omega = 1, A = 0.1, Re = 2, W = 0.5, \lambda = 1, Pe = 0.7, Da = 0.8, Gr = 1, t = 0.5, $\alpha = 0.1$. 
Fig 6: "Poiseuille flow" Velocity profile for $A$ and $W$ with $\omega = 1, M = 1, Re = 2, \lambda = 1, Pe = 0.7, Da = 0.8, N = 2, Gr = 1, t = 0.5, \alpha = 0.1.$

Fig 7: "Poiseuille flow" Velocity profile for $Gr$ and $\omega$ with $A = 0.1, M = 1, Re = 2, W = 0.5, \lambda = 1, Pe = 0.7, Da = 0.8, N = 2, t = 0.5, \alpha = 0.1.$

Fig 8: "Couette flow" Velocity profile for $Da$ and $\omega$ with $A = 0.1, M = 1, Re = 2, W = 0.5, \lambda = 1, Pe = 0.7, N = 2, Gr = 1, t = 0.5, \alpha = 0.1.$

Fig 9: "Couette flow" Velocity profile for $Pe$ and $\alpha$ with $\omega = 1, A = 0.1, M = 1, Re = 2, W = 0.5, \lambda = 1, Da = 0.8, N = 2, Gr = 1, t = 0.5.$
Fig 10: "Couette flow" Velocity profile for $\lambda$ and $Re$ with $\omega = 1$, $A = 0.1$, $M = 1$, $W = 0.5$, $Pe = 0.7$, $Da = 0.8$, $N = 2$, $Gr = 1$, $t = 0.5$, $s = 0.1$.

Fig 11: "Couette flow" Velocity profile for $M$ and $N$ with $\omega = 1$, $A = 0.1$, $Re = 2$, $W = 0.5$, $\lambda = 1$, $Pe = 0.7$, $Da = 0.8$, $Gr = 1$, $t = 0.5$, $s = 0.1$.

Fig 12: "Couette flow" Velocity profile for $A$ and $W$ with $\omega = 1$, $M = 1$, $Re = 2$, $\lambda = 1$, $Pe = 0.7$, $Da = 0.8$, $N=2$, $Gr = 1$, $t = 0.5$, $s = 0.1$. 
Fig 13: "Couette flow" Velocity profile for $\frac{Gr}{Re}$ and $\omega$ with $A=0.1$, $M=1$, $Re=2$, $W=0.5$, $\lambda=1$, $Pe=0.7$, $Da=0.8$, $N=2$, $t=0.5$, $\alpha=0.1$.

6. Concluding Remarks

We discuss the influence of heat transfer on MHD oscillatory flow for Eyring-Powell fluid with variable viscosity through a porous medium. Using the perturbation technique we found the velocity. We used different values to find the results of pertinent parameters, namely Darcy number, Peclet number, Grashof number, magnetic parameter, the radiation parameter, the Schmidt number, the Soret number, the heat generation parameter frequency of the oscillation and Reynold number. we noted that in two types of flow "Poiseuille and Couette" the velocity increases with increasing of the parameters $\omega$, $Pe$, $Re$, $Gr$, $N$ and $M$ respectively, while the velocity decreases with the increasing $Da$, $\omega$, $Gr$, $W$ and $A$.

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