Modal optimization approach for composite aeroelastic wing model based on a derivative-free BFGS method

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Abstract. The optimization problem of aeroelastic wing model may lead to highly coupled nonlinear equations whose Jacobian matrix is hard to be calculated, followed by adjusting the variable domain due to design variable importance in terms of their sensitivity. To overcome the problem, an optimization function for the modal frequency is established. And a suitable line search approach is combined with the BFGS method to improve its efficiency. Some numerical results are reported to show efficiency of the proposed method. An integrated modal optimization approach based on the BFGS is developed for the model design of a composite aeroelastic wing model. The stiffness of the key beams of the wing model is designed to be the optimization variables, bending frequency and torsional frequency are designed to be the optimization objective. Then the modal of CHN-T1 is optimized, it is shown that the method proposed is suitable for the optimization of composite aeroelastic wing model with a high efficiency.

1. Introduction

In order to obtain the flutter characteristics of the whole aircraft or components, wind tunnel flutter test is a necessary step in the process of aircraft development, and flutter test model is an important part of flutter test [1]. Flutter model is a dynamic similar model which satisfies the similar aerodynamic force, stiffness and mass of the object. It is designed and processed according to a certain similar scale. At present, the commonly used forced models mainly include metal model and composite material model [3]. Especially in the scale wind tunnel model manufacturing of large aircraft, due to the corresponding reduction of structural stiffness, traditional metal materials cannot meet the requirements of geometry, stiffness and quality. Composite material model is made of resin-based fiber reinforced composites, which has the characteristics of low modulus. Compared with metal model, composite material model has higher similarity, and has become an important flutter model [4, 5]. After the flutter model is processed, the modal parameters of the model need to be obtained by modal test (generally ground vibration test). The purpose is to verify whether the model meets the design requirements. Generally, the difference between the main modal frequencies and the design frequencies of the model is less than 5%, and the modal modes are basically the same.

In order to complete the design and optimization of the composite wind tunnel model, it is necessary to establish a direct mathematical model of the geometry and material parameters of the model and the dynamic performance of the model. Especially in the reverse calculation, because of the huge amount of calculation, high performance optimization algorithm is required. At present, optimization algorithms
have been widely used in aerodynamic tailoring [7], aeroelasticity [8], structural design [9], including

criterion method, mathematical programming method and intelligent algorithm. Sensitivity optimization

method is first used in composite wind tunnel model. Sensitivity optimization method has the

characteristics of local convergence, and can get the local optimal solution. In order to solve its local

convergence and find the global optimal solution, the genetic/sensitivity hybrid optimization method

was proposed by Yang Chao et al. and used in the design of side-merged structural model [10]. Genetic

algorithm is a method of searching the optimal solution by simulating natural evolution process,

including selection, crossover, mutation and so on. In order to obtain the global optimal solution, a large

number of populations need to be generated, which brings huge computational load, and is difficult to

apply to the rapid optimization of multi-design variables flutter model. With the gradual maturity of the

fast calculation method of BFGS (Brayden-Fletcher-Goldfarb-Shanno) with global convergence

characteristics and its successful application in optimization methods [11], this paper studies the

frequency optimization method of flutter test model based on the fast optimization method of BFGS

method, which has both high efficiency and high precision.

2. Modal optimization method based on Jacobin-Free BFGS

The modal optimization problem of wind tunnel flutter test model can be reduced to the extreme problem

of non-linear equations. The BFGS algorithm with global convergence is used to find the minimum

point of multivariate function, and its theory has gradually matured. Based on BFGS, the objective

function of wing frequency optimization with influence coefficient is constructed.

\[
\min_{x} A(x)
\]

\[
A(x) = \sqrt{\sum_{i=1}^{n} c_i^2 (x_i - x_{i0})^2}
\]

where  \( A(x) \) is the objective function,  \( x_i \) is the variable to be optimized,  \( x_{i0} \) is the experimental value. It

is worth mentioning that different optimization parameters have different importance [12]. Therefore,

this paper uses variable weight coefficient  \( c_i \) to consider this design sensitivity problem.

In the optimization method, the traditional Newton method and Newton iteration method can be used

to solve the system of nonlinear equations. However, because of the complexity of structure and

frequency calculation when considering frequency optimization, the Jacobin matrix of the system of

non-linear equations is difficult to display. Compared with Newton's method and Newton's iteration

method, quasi-Newton's method is more competent for its calculation work [13]. In the construction of

iteration scheme, linearization equation is used instead of non-linear calculation.

\[
L_k(x) = A_k x + b_k = 0
\]

Among them,  \( k \) is the k-step iteration and  \( A_k \) is the full-rank Jacobin matrix of the system of non-

linear equations (2). Because of the need to constantly update the calculation of  \( A_k \) and  \( b_k \) in the

iterative calculation, it increases the amount of calculation, especially when  \( A_k \) is difficult to calculate

or does not exist. If the values of  \( A(x) \) at some points are known,  \( A_k \) and  \( b_k \) can be determined

by interpolation, and a class of methods without computing Jacobin matrix can be obtained. Brayden

algorithm has become one of the famous quasi-Newton methods because of its small computational

complexity and stable algorithm. According to the interpolation properties of the secant method, the

inverse matrix iteration scheme of Jacobin matrix is constructed by two different independent variables

\( x^j \) and  \( x^{j+1} \).
$B_{k+1} = B_k + \Theta_k \left( \frac{-B_k s_k y_k^T B_k + y_k^T y_k}{s_k^T B_k s_k} \right)$

(4)

$s_k = x_{k+1} - x_k$, $y_k = F(x_k) - F(x_{k+1})$.

Braydens correction formula is obtained. The BFGS method can avoid solving the complex Jacobin matrix of the optimization objective function, but it has local convergence. In order to construct a BFGS method with global convergence, this paper introduces a derivative-free linear search method based on approximate normal gradient, and achieves global convergence by retrieving $x_{k+1}$ in each iteration step [14]

$$\|A(x_k + ad_k)\| - \|A(x_k)\| \leq \sigma \|A(x_k + ad_k) - A(x_k)\|$$

(5)

In the above formula, $\sigma \in (0,1)$, \{\$\varepsilon_k\$\} is a sequence of positive real numbers satisfied \(\sum_{k=0}^{\infty} \varepsilon_k < \varepsilon < \infty\).

Following is the frequency optimization flow of BFGS method with linear search.

Step 1: Set the wing stiffness distribution as the optimization parameter, and take the frequency as the monitoring variable to construct the optimization objective function (1).

Step 2: Given the initial value of optimization parameters $x_0$ and the sum of convergence coefficients $\varepsilon_1$ and $\varepsilon_2$, initialize the initial estimation matrix $B_0$ and calculate $F(x_0)$.

Step 3: If $\|A(x_k)\| \leq \delta$ turn to step 6, otherwise calculate $d_k$ according to the following formula:

$$d_k = a_k d = -a_k B_k A(x_k), a_k = 1.$$

Step 4: If formula (5) satisfies, then $x_{k+1} = x_k + a_k d_k$; otherwise, $a_k = \rho a_k$ return to step 3.

Step 5: Setup $s_k = x_{k+1} - x_k$, $y_k = A(x_{k+1}) - A(x_k), \theta_k = 1$.

Step 6: Calculate formula (5), if $\det(B_{k+1}) \neq 0$, assign $k = k + 1$ and return to step 2, otherwise assign $\theta_k = \eta \theta_k$ and return to step 5.

Step 7: Output solution $x$ and optimization frequency.

3. BFGS Method Validation

The BFGS algorithm with global convergence property developed in this paper is suitable for the optimization of non-linear equations. To verify its effectiveness, 17 sets of non-linear equations in reference [14] are calculated and compared with the original results as shown in Table 1.

In this paper, the parameters are set as follows:

$$\sigma = 0.5, \rho = 0.2, \varepsilon_k = 0.5/k^2, \eta = 0.8, \delta = 10^{-10}, x_0 = (0,0,...0)^T, B_0 = I$$

The computational results show that even for some equations which do not satisfy the uniform non-singular assumption, the convergence results are good, and the convergence mode of $A(x_k)$ is smaller than the original calculation results. Table 2 further compares the BKT, NI and total calculation steps (TS) of the 17 equations mentioned above. Compared with the original calculation results, 8 groups of BKT steps are smaller, 14 groups of NI are smaller and 13 groups of TS are smaller. The method developed in this paper has higher efficiency.
Table 1. Comparisons of iteration steps calculated in this paper with reference [14], I: The calculating method in this paper, II: The calculating method in reference [14].

| No. | BKT | I       | II       |
|-----|-----|---------|---------|
| 1(1)|     | 0       | 2       |
|     | NI  | 28      | 6       |
|     | NORM| 6.18648974E-010 | 3.69436467E-0012 |
|     | BKT | 22      | 24      |
| 2(3)|     |         |         |
|     | NI  | 19      | 17      |
|     | NORM| 4.77143839E-10 | 2.38951784E-0012 |
|     | BKT | 2       | 1       |
| 3(2)|     |         |         |
|     | NI  | 27      | 31      |
|     | NORM| 2.05218333E-10 | 3.24256509E-0011 |
|     | BKT | 0       | 0       |
| 4(3)|     |         |         |
|     | NI  | 3       | 2       |
|     | NORM| 8.94069263E-11 | 8.94069726E-0011 |
|     | BKT | 6       | 7       |
| 5(3)|     |         |         |
|     | NI  | 14      | 12      |
|     | NORM| 2.77388197E-11 | 2.28055025E-0014 |
|     | BKT | 889     | 1651    |
| 6(6)|     |         |         |
|     | NI  | 211     | 333     |
|     | NORM| 2.81484198E-10 | 9.81004324E-0012 |
|     | BKT | 17      | 12      |
| 7(3)|     |         |         |
|     | NI  | 19      | 18      |
|     | NORM| 1.06859940E-10 | 1.20985694E-13 |
|     | BKT | 40      | 29      |
| 8(4)|     |         |         |
|     | NI  | 27      | 23      |
|     | NORM| 4.13369440E-10 | 3.52400505E-0011 |
|     | BKT | 13      | 11      |
| 9(2)|     |         |         |
|     | NI  | 19      | 14      |
|     | NORM| 5.83911549E-12 | 4.82542583E-0011 |
|     | BKT | 4       | 4       |
| 10(3)|    |         |         |
|     | NI  | 14      | 12      |
|     | NORM| 1.62014066E-11 | 1.07978781E-0012 |
|     | BKT | 12      | 3       |
| 11(2)|    |         |         |
|     | NI  | 17      | 11      |
|     | NORM| 1.50045571E-10 | 2.21079707E-0012 |
|     | BKT | 915     | 199     |
| 12(2)|    |         |         |
|     | NI  | 327     | 114     |
|     | NORM| 3.77680670E-11 | 4.07477480E-0011 |
|     | BKT | 9       | 9       |
| 13(2)|    |         |         |
|     | NI  | 16      | 17      |
|     | NORM| 3.06880163E-10 | 2.44920164E-0013 |
|     | BKT | 2       | 2       |
| 14(2)|    |         |         |
|     | NI  | 10      | 9       |
|     | NORM| 1.25580131E-10 | 2.59750844E-0011 |
|     | BKT | 22      | 11      |
| 15(3)|    |         |         |
|     | NI  | 22      | 14      |
|     | NORM| 2.38350824E-10 | 2.08273690E-0012 |
|     | BKT | 8       | 9       |
| 16(2)|    |         |         |
|     | NI  | 15      | 12      |
|     | NORM| 1.54408415E-11 | 8.79997619E-0012 |
|     | BKT | 15      | 9       |
| 17(3)|    |         |         |
|     | NI  | 15      | 12      |
|     | NORM| 4.21043151E-10 | 2.89976083E-0011 |
Table 2. Comparisons of iteration steps calculated in this paper with reference [14]

| Step number | Smaller | Equal | Larger |
|-------------|---------|-------|--------|
| BKT         | 8       | 4     | 5      |
| NI          | 14      | 0     | 3      |
| TS          | 13      | 1     | 3      |

4. Finite element modal optimization

The model structure of CHN-T1 is beam structure, which consists of upper skin, lower skin, front skin, rear skin, wall and wing rib. Along the span, the wing can be divided into mid-wing and outer-wing. Based on the finite element method, the model of composite skin, long beam and wing rib is established by using the composite plate element. The solid element is used to establish the supporting structure, foam and other models, and the weight block is used for counterweight. The complex finite element model [15] of the wind tunnel model of the composite material is established, as shown in Figure 1.

Figure 1. CHN-T1 finite element model

The modal analysis of the wing is carried out by using the modal analysis method in Nastran software, and the first five order arrays are obtained. The first, second and third order modes are respectively the first, second and third bend modes of the wing, the fourth mode is the first bend in the wing surface, and the fifth mode is the first twist mode of the wing. The modals of each order are shown in Figure 2. Considering that wing bending and torsion are the main influencing modes in wing flutter analysis, the third-order mode of in-plane bending is neglected, and the first, second, fourth and fifth modes are selected as optimization objects to carry out modal optimization. The stiffness of the front and rear walls, which are critical to complete and torsion, is selected as the optimization variable, and the modal frequency optimization is carried out by using the optimization method in this paper under the condition that the quality is not changed.
Fig. 3 shows the convergence process of each order of modal frequencies in the process of uniform weight optimization. From the graph, it can be seen that each order of modal frequencies fluctuates at 20 steps and converges after 50 steps. After convergence, the first frequency is optimized from 11.1Hz to 12.3Hz, the second frequency from 34.7Hz to 38.5Hz, the fourth frequency from 85.1Hz to 39.4Hz, and the fifth frequency from 110.7Hz to 112.5Hz.

In the frequency optimization of the uniform weight coefficient, the bend is in the first mode, and the test frequency is 12.0Hz. However, the optimized value is 12.3, and the relative error is 2.5%. The reason is that the sensitivity of the first-order mode in the optimization function is too low because of the lower frequency value. In order to improve the accuracy of first-order modal optimization, based on the importance of sensitivity of each variable in the optimization objective, the first-order modal weight coefficient is set to 10.0, the second-order modal weight coefficient is set to 3.0, and the fifth-order modal weight coefficient is set to 1.0. In this paper, the variable coefficient optimization method is used to optimize the modal frequencies. Formula (2) is used to calculate the optimization residuals of the modal frequencies. Formula (2) is used to calculate the optimization residuals of the modal frequencies of each order. In the unified weight coefficient, the optimization method presented in this paper fluctuates greatly in the first 30 steps, but after more than 50 steps, the whole convergence enters a stable stage. In the variable weight coefficient, the residual value is basically stable before 40 steps. Therefore, the optimization method adopted in this paper can quickly realize the modal optimization of the wing.

Table 3 shows the comparison between the test frequency and the optimized frequency. Among the unified weights, the largest error is the second-order bending mode with a relative error of 2.5%; the smallest error is the first-order torsional mode with a relative error of 0.1%; and the optimal bending frequency with a relative error of 1.8%. The optimization results show that the modal frequencies of each order are in good agreement with the experimental frequencies. In variable weight optimization,
the maximum error occurs in three bends of -1.6%. The errors of bending, second bending and torsion are 0.42%, 0.52% and 0.44%, respectively. The relative errors of all modes, which are critical to flutter, are less than 1%. The results show that the frequency errors of the first bend and the second bend decrease obviously, while the torsion frequency errors increase slightly. The weight coefficient improves the influence of residual function on low frequency modal and reduces the influence on high frequency modal. The optimization method in this paper is suitable for the modal optimization of flutter wind tunnel test model.

| Mode | Test frequency/Hz | Unified optimization frequency/Hz | Relative error | Variable weight optimization frequency/Hz | Relative error |
|------|-------------------|----------------------------------|----------------|-------------------------------------------|----------------|
| 1    | 12.0              | 12.3                             | 2.5%           | 12.05                                     | 0.42%          |
| 2    | 38.50             | 39.4                             | 2.3%           | 38.7                                      | 0.52%          |
| 4    | 95.40             | 94.9                             | 0.5%           | 93.8                                      | -1.6%          |
| 5    | 112.50            | 112.6                            | 0.1%           | 113.0                                     | 0.44%          |

5. Conclusion

Aiming at the problems of implicit optimization function and incalculable Jacobin matrix, the modal optimization method of composite wind tunnel flutter test model was established by BFGS method. The sensitivity of design variables was considered by variable weight coefficient method. The modal optimization of CHN-T1 model was studied by this method. The results show that:

(1) Combining the BFGS method with the linear search method, an efficient method for calculating the system of nonlinear equations with global convergence is developed; the variable weight coefficient method is used to consider the different sensitivity of each optimization frequency design; the above BFGS method and the variable weight coefficient method are applied to frequency optimization, and a modal optimization method for wind tunnel flutter test model is developed.

(2) Frequency optimization of CHN-T1 wind tunnel flutter test model is carried out by using the method developed in this paper. Compared with the test results, the maximum relative error of each order modal frequency which has a key impact on flutter is less than 1%. The method in this paper not only guarantees the efficiency of optimization calculation, but also obtains the modal frequency of composite wind tunnel flutter test model accurately and reliably.

(3) The optimization residuals and modal frequencies of each order can converge after 50 steps of iteration, which shows that this method can quickly and efficiently complete the modal optimization of the composite wind tunnel flutter test model.

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