Measurement-based quantum computation
and undecidable logic

Abstract We establish a connection between measurement-based quantum computation and the field of mathematical logic. We show that the computational power of an important class of quantum states called graph states, representing resources for measurement-based quantum computation, is reflected in the expressive power of (classical) formal logic languages defined on the underlying mathematical graphs. In particular, we show that for all graph state resources which can yield a computational speed-up with respect to classical computation, the underlying graphs—describing the quantum correlations of the states—are associated with undecidable logic theories. Here undecidability is to be interpreted in a sense similar to Gödel’s incompleteness results, meaning that there exist propositions, expressible in the above classical formal logic, which cannot be proven or disproven.

Keywords Quantum information theory · Quantum computation · Logic

1 Introduction

Quantum computers are devices that use quantum mechanics for enhanced ways of information processing \( \mathbb{N} \). Indeed, it is known that problems such as integer
factoring can be performed significantly faster on a quantum computer than on any known classical device [2]. Despite these exciting perspectives, the questions:

“What are the essential resources that give quantum computers their computational power”

and

“Are quantum computers fundamentally more powerful than classical devices?”

remain to date largely unanswered.

The existence of several models for quantum computation, each based on different concepts, indicates that there may not be a straightforward answer to these difficult questions. The new paradigm of measurement-based, or one-way quantum computation [12], [13] has lead to novel perspectives in these respects. The introduction of this model established that certain many-qubit quantum states, such as the 2D cluster states [14], exhibit the remarkable property that universal quantum computation can be achieved by simply individually measuring the qubits of the system in a specific order and basis, and by classical processing of the measurement results. The initial state of the system then serves as the resource for the entire computation which is (in part) consumed in the process. This is in sharp contrast to the quantum circuit model, where computations are realized via unitary evolution. Within the measurement-based paradigm for quantum computation, fundamental questions regarding the speed-up of quantum with respect to classical computation can be formulated and investigated in an alternative, and in several cases much more concise, way. In particular, the introductory questions of this paper can be restated as

“Which resource states for measurement-based quantum computation (MQC) yield a computational speed-up over classical computers?”

This question will be addressed in the present article.

As entanglement can only decrease in a one-way computation, the enhanced computational power of such a quantum computer (beyond a classical Turing machine) must originate in the entanglement structure of its resource state. Owing to this insight, a series of papers have recently been devoted to investigating which types of entanglement needs to be present in any resource state which achieves the desired enhanced computational power [3] [4] [5] [6] [7] [8] [9] [10] [11]. In this paper we establish a new necessary condition for resource states to yield a computational speed-up with respect to classical computers. The crucial point of this result is that the present criterion is entirely different in nature with respect to previously established requirements—in particular, it will not be stated in terms of entanglement. In the following we will focus on resource states belonging to the rich class of graph states, which are generalizations of the 2D cluster states and which play an important role in several applications in quantum information theory (e.g., one-way quantum computation, quantum error-correction, multipartite entanglement theory, communication schemes; see [15] for a review). A graph state on \( n \) qubits is defined by means of a mathematical graph on \( n \) vertices, which completely encodes the correlations in the system. Our main result will be a connection between the possibility of obtaining a computational speed-up w.r.t. classical computation by performing MQC on graph state resources, and certain properties of the associated graphs in relation with mathematical logic theory—more particularly, the decidability of logic theories.
As is well known since Gödel’s incompleteness theorems [16], every formal system that is sufficiently interesting (or rich) contains statements which can neither be proved to be true nor to be false within the axiomatic framework of the system. Gödel’s incompleteness theorem not only applies to formal systems relating to natural numbers (cf. Peano arithmetic) but also to graphs. Indeed, many interesting graph properties can be expressed within a (classical) formal language, denoted by $\mathcal{L}$ (the exact definition of this language is stated below). Examples of such properties are planarity or 2-colorability of graphs. In this paper we will show that the computational power of graph states—as resources for measurement-based quantum computation—is reflected in the expressive power of the formal language $\mathcal{L}$ defined on the underlying graphs, which encode the set of quantum correlations in the system. In particular, the following result will be obtained.

**Theorem.** Graph state resources for measurement-based quantum computation can only yield a computational speed-up over classical computers if the formal language $\mathcal{L}$ defined on the underlying graphs is undecidable.

Here undecidability is to be interpreted in a sense similar to Gödel, meaning that there exist propositions, expressible in the logic $\mathcal{L}$, which cannot be proven or disproven.

The Theorem provides a necessary condition to assess the computational power of graph state resources by considering the underlying graphs, which, by the very definition of graph states, render a classical encoding of the quantum correlations in the states. The concept of undecidability is to be regarded as a notion of complexity of the graphs—and hence of the correlations in the system—stated independently of any quantitative (entanglement) measure.

This paper aims at connecting two quite different fields of research, namely quantum computation and mathematical logic. Therefore, in the following we will give a brief review of the basic concepts of measurement-based (one-way) quantum computation, as well as logic theories on graphs. This will allow us to reformulate the main theorem in a precise manner. The proof of the Theorem is obtained by combining our previous results regarding entanglement width and MQC on graph states [11], and a recent graph theoretic result [17]. We will conclude the paper with an interpretation of these results.

### 2 Graph states and measurement-based quantum computation

We will be concerned with a particular class of multi-party quantum states, called *graph states* [15], which are generalizations of the 2D cluster states. A graph state $|G\rangle$ on $m$ qubits is the joint fixed point (i.e., an eigenvector with eigenvalue 1) of $m$ commuting correlation operators

$$K_a := \sigma_x^{(a)} \bigotimes_{b \in N(a)} \sigma_z^{(b)},$$

where $\sigma_x$ and $\sigma_z$ are Pauli matrices, and the upper indices denote on which qubit system these operators act. Moreover, $N(a)$ denotes the set of neighbors of qubit $a$ in the graph $G$. Thus, a system in a graph state has $\langle K_a \rangle = 1$ for every $a$. For
example, a 2D cluster state is obtained if the underlying graph is a \( k \times k \) square lattice \( C_{k \times k} \) (thus \( m = k^2 \)).

The family of 2D cluster states is known to be a *universal* resource for measurement based quantum computation, in that any unitary operation can efficiently be simulated by performing measurements on a 2D cluster state of appropriate dimensions \([12]\). As graph states are generally highly entangled, and as they can efficiently be prepared by applying a suitable poly-sized quantum circuit to a product input state, they form natural candidates to serve as resources for MQC. We envisage a situation where an (infinitely large) family of graph states

\[
\Psi = \{|G_1\rangle, |G_2\rangle, \ldots \}
\]

with growing system size is considered, and where local measurements can be performed on arbitrary members of \( \Psi \), thus allowing to implement quantum computations of arbitrary length. For example, \( \Psi \) could be the family of 2D cluster states, \( \Psi = \{C_{k \times k} : k = 1, 2, \ldots \} \).

In this setting we are interested in resources \( \Psi \) for MQC which yield a computational speed-up over classical devices. Given a family of states \( \Psi \), we will say that efficient classical simulation of MQC on \( \Psi \) is possible, if for every state \( |G_i\rangle \in \Psi \) it is possible to simulate every LOCC protocol (short for *local operations and classical communication*) on a classical computer with overhead \( \text{poly}(m_i) \), where \( m_i \) denotes the number of qubits on which the state \( |G_i\rangle \) is defined \([11]\). Evidently, resources which allow a computational speed-up over classical computers do not allow efficient classical simulation of MQC.

In the following we will focus on the graphs associated to families of graph states, and the formal languages defined on them. Note that by definition \([1]\) the graph \( G \) is an encoding of the correlations present in the corresponding graph state. Therefore, any property of these graphs reflects a property of the corresponding states, and, more particularly, the correlations in these states.

3 Graphs and logic

Next we define some basic notions of logic theory which are necessary to state our main results concisely below. We refer to Refs. \([18]\) \([19]\) for an extensive treatment. We also emphasize that we will favor clarity of the exposition over mathematical rigor.

In fundamental aspects of graph theory one is interested in formal approaches to formulate graph properties such as 2-colorability, connectedness, planarity, etc. This formalization is obtained by defining a logical calculus in which such graph properties can be expressed. Roughly speaking, a logic on a graph \( G \) corresponds to a set of rules which determine the basic constituents with which statements regarding \( G \) can be constructed. Such formalization in terms of logic allows, in principle, artificial devices to mechanically prove or disprove, for a given graph or set of graphs, properties expressible in this logic.

The simplest logic is *first-order logic*, which is obtained by allowing formulas containing the following elementary components:

- Quantifications \( \exists x \) and \( \forall x \) over vertices \( x \) of the graph (i.e., “There exists a vertex \( x \) such that \([\ldots]\)”, or “For all vertices \( x \) it holds that \([\ldots]\)”) and connectives \( \land, \lor, \) and \( \neg \), i.e., the logical “AND”, “OR” and “NOT”.
Moreover, it is allowed to express whether two vertices are adjacent in the graph or not. This is formally achieved by introducing the symbol \( \text{edge} \), which is defined by

\[
\text{edge}(a, b) = \begin{cases} 
\text{True} & \text{if } \{a, b\} \text{ is an edge} \\
\text{False} & \text{otherwise,}
\end{cases}
\]  

(3)

for every pair of vertices \( a, b \) of the graph.

A simple example of a first-order logic formula on a graph \( G \) is “There exist vertices \( x, y, \) and \( z \) such that \( x \) is connected to \( y \) and \( y \) is connected to \( z \)” or, more formally,

\[
\exists x \exists y \exists z \text{ edge}(x, y) \land \text{edge}(y, z). \tag{4}
\]

It turns out that first-order logic is often not rich enough to express interesting graph properties. One therefore extends first-order logic by allowing more elementary symbols. In particular, one may supplement first-order logic with the following elements:

- Next to variables \( x, y, z, \ldots \) denoting vertices of the graph, one also allows set variables \( X, Y, Z, \ldots \) (indicated by capital letters), which denote subsets of vertices.
- Furthermore, one adds quantifications \( \forall X, \exists X \) over such sets.
- Finally, one introduces elementary formulas of the form \( x \in X \), which allow one to express that a vertex \( x \) belongs to a certain subset \( X \).

The logical calculus which is thus obtained is called monadic second-order logic, or MS logic in short. MS logic is strictly more expressive than first-order logic, i.e., there are problems which can be expressed with MS logic which cannot be expressed using only first-order logic. An example of an MS formula on a graph is

\[
\exists X \exists Y \{ \forall z (z \in X \lor z \in Y) \land \\
\forall z \forall z' \neg \text{edge}(z, z') \lor \neg(z, z' \in X \lor z, z' \in Y) \}.
\]

This formula expresses that the graph can properly be colored with 2 colors (i.e., the vertices can partitioned in two classes such that no two adjacent vertices are in the same class). Many interesting graph properties can be expressed in MS logic, among which there are several NP-hard problems (such as e.g. 3-colorability), indicating that MS logic on graphs has a considerable expressive power.

A slight extension of MS logic is obtained by including atomic formulas of the form \( \text{Even}(X) \), indicating that the set \( X \) has even cardinality. In this way one obtains MS logic with the additional possibility to count modulo two, denoted by \( C_2 \text{MS logic} \), which will be our topic of interest, i.e., it corresponds to the logic \( \mathcal{L'} \) as denoted above. \( C_2 \text{MS logic} \) is an interesting extension of MS logic which is moreover physically interesting in that it e.g. allows to express whether two graph states are local unitary equivalent.
4 Decidability of logic theories

Next we introduce the fundamental notion of \textit{decidability} of logic theories. We again refer to [18] [19] for details. Let $\mathcal{G} = \{G_1, G_2, \ldots\}$ be a (finite or infinite) family of graphs, and let $\mathcal{L}$ denote $C_2$MS logic. The $\mathcal{L}$-theory of $\mathcal{G}$ is defined to be the collection of all formulas $\varphi$, expressed in the logic $\mathcal{L}$, which are satisfied (or “True”) for all graphs in the family $\mathcal{G}$. The set $\mathcal{G}$ is said to have a \textit{decidable} $\mathcal{L}$-theory if for every formula $\varphi$ expressed in the logic $\mathcal{L}$, it is possible to decide (in finite time) whether or not $\varphi$ belongs to the $\mathcal{L}$-theory of $\mathcal{G}$. The set $\mathcal{G}$ is said to have an \textit{undecidable} $\mathcal{L}$-theory if it does not have a decidable $\mathcal{L}$-theory.

For example, the formula $\varphi$ could correspond to graph planarity, 2-colorability, etc. Then the question is asked whether all graphs in a given family of graphs $\mathcal{G}$ are planar, 2-colorable, etc. If every possible such question, that is, every formula expressible in the language $\mathcal{L}$, can be answered in finite time, then this family is said to have a decidable $\mathcal{L}$-theory. The decidability or undecidability of a logic theory $\mathcal{L}$ on a set $\mathcal{G}$ is a reflection of both the expressive power of the logic $\mathcal{L}$—“How many properties can be expressed in the logic $\mathcal{L}$?”—and the complexity (regarded in a colloquial sense) of the family $\mathcal{G}$—“How rich is the structure of the graphs in $\mathcal{G}$?”.

Before giving examples of (un)decidable MS theories, is important to make the following two remarks. First, decidability of a logic theory is not concerned with the \textit{efficiency} with which problems can be solved—one only asks whether it is \textit{in principle} possible to verify whether a given formula $\varphi$ is true, where one does not care about e.g. the computational complexity of a possible verification algorithm. Second, note that any first-order or MS theory is decidable on \textit{finite} families of graphs $\mathcal{G} = \{G_1, G_2, \ldots, G_N\}$. This is simply because, in this finite regime, any formula can be verified by an exhaustive enumeration of cases. Thus, decidability is only relevant when infinite structures are considered.

Let us now give some important examples. First, let $\mathcal{G}_{\text{bin}}$ be the set of all binary tree graphs, which are regarded as so-called incidence structures. A milestone result was obtained by Rabin, who proved that $\mathcal{G}_{\text{bin}}$ has a decidable $C_2$MS theory [21]. This result has many important implications in graph theory and computer science. Further, let $\mathcal{G}_{\text{2D}}$ be the set of all 2D $(k \times k)$ lattice graphs. Then $\mathcal{G}_{\text{2D}}$ has an \textit{undecidable} $C_2$MS theory [22]. As final examples, let $\mathcal{G}_{\text{tri}}$ and $\mathcal{G}_{\text{hex}}$ be the sets of all triangular lattice graphs and hexagonal lattice graphs, respectively, regarded as adjacency structures. Then also $\mathcal{G}_{\text{tri}}$ and $\mathcal{G}_{\text{hex}}$ have \textit{undecidable} $C_2$MS theories [23].

5 Main results

Keeping in mind that the graph states corresponding to the 2D rectangular, hexagonal and triangular lattices have been shown to be universal resources for MQC [10], the above examples already suggest a connection between the computational power of a family of graph states as a resource for MQC, and the $C_2$MS logic defined on the underlying graphs. This connection will now be fully established, as we are now in a position to precisely state and prove the main result of this paper. Let $\mathcal{G} = \{G_1, G_2, \ldots\}$ be an (infinitely large) family of graphs and let $\Psi(\mathcal{G})$
be the associated family of graph states. The main Theorem can then precisely be formulated as follows.

**Theorem.** If a family of graphs $\mathcal{G}$ has a decidable $C_2MS$ logic theory then MQC performed on the graph state resource $\Psi(\mathcal{G})$ can classically be simulated efficiently.

Thus, this results states that any family of graphs with a decidable $C_2MS$ logic theory cannot give rise to a graph state resource for MQC which yields a computational speed-up as compared to classical computers.

The proof of the Theorem is in fact quickly obtained by invoking previous results of the present authors and a highly nontrivial result from graph theory. The proof has two main ingredients (i) and (ii):

(i) Courcelle and Oum [17] proved that every class of graphs $\mathcal{G}$ which exhibits a divergence with respect to a graph invariant called rank-width (we refer to Ref. [19] for definitions) must have an undecidable $C_2MS$ theory.

(ii) In previous work [11], the present and other authors proved that every family of resource states $\Psi(\mathcal{G})$ where $\mathcal{G}$ has a bounded rank-width, allows an efficient simulation of MQC.

Combining (i) and (ii) then yields the proof of the Theorem.

Next we elaborate on the above proof strategy. Let us first introduce the notion “rank-width”. The rank-width $\text{rwd}(G)$ [19] of a graph $G$ is a parameter which measures how well a graph can be approximated by means of certain “tree-like” structures. Graphs with small rank-width include, e.g., a one-dimensional chain with open or closed boundary conditions, or a 2-dimensional “stripe”, which is a $d \times n$ square lattice where $n$ may be arbitrarily large, but where $d$ is held fixed. Graphs of large rank-width include e.g. 2-dimensional $n \times n$ lattices (for growing $n$), or lattices of higher dimensions. For completeness, we give here the definition of the rank-width, which is quite technical [the reader who is not interested in these mathematical details may skip to the next paragraph]. Let $G$ be a graph with vertex set $V = \{1, \ldots, n\}$ and edge set $E$. Let $\Gamma$ be the $n \times n$ adjacency matrix of $G$, i.e. one has $\Gamma_{ab} = 1$ if $\{a, b\} \in E$ and $\Gamma_{ab} = 0$ otherwise. For every bipartition $(A, B)$ of the vertex set $V$, define $\Gamma(A, B)$ to be the $|A| \times |B|$ submatrix of $\Gamma$ defined by

$$\Gamma(A, B) := (\Gamma_{ab})_{a \in A, b \in B}. \tag{5}$$

Let $T$ be a subcubic tree, which is a tree such that every vertex has exactly 1 or 3 incident edges (see Fig. 1a). The vertices which are incident with exactly one edge are called the leaves of the tree. For a given fixed graph $G$, we will be interested in the collection of all possible subcubic trees $T$ with exactly $n$ leaves $V := \{1, \ldots, n\}$, which are identified with the $n$ vertices of $G$. Letting $e = \{i, j\}$ be an arbitrary edge of such a tree $T$, we denote by $T \setminus e$ the graph obtained by deleting the edge $e$ from $T$. The graph $T \setminus e$ then consists of exactly two connected components, which naturally induce a bipartition $(A_f, B_f)$ of the set $V$ (see Fig. 1a).
The rank-width of the graph \( G \) is now defined by the following optimization problem:

\[
\text{rwd}(G) = \min_T \max_{e \in T} \text{rank}_2 \Gamma(A_e^T, B_e^T).
\] (6)

Here the minimization is taken over all subcubic trees \( T \) with \( n \) leaves, which are identified with the \( n \) vertices in the graph. Moreover, \( \text{rank}_2 \Gamma(A, B) \) denotes the rank of the matrix \( \Gamma(A, B) \) when arithmetic is performed modulo 2.

One notices that the construction involving the subcubic trees is designed to single out a specific class of bipartitions \( (A_e^T, B_e^T) \) of the vertex set of \( G \), over which the min-max optimization problem is performed. For a given tree \( T \) one considers the maximum, over all edges \( e \) in \( T \), of the quantity \( \text{rank}_2 \Gamma(A_e^T, B_e^T) \); then the minimum, over all subcubic trees \( T \), of such maxima is computed. As an example, it can be shown that the rank-width of a 1D chain is equal to 1 (independent of the length of the chain), whereas an \( n \times n \) square lattice has rank-with of \( O(n) \) and thus increases with the size of the lattice.

While the rank-width is indeed a rather involved mathematical concept, it turns out to be crucial for the current investigation. In particular, the following highly nontrivial result by Courcelle and Oum is of particular interest:

Every class of graphs \( \mathcal{G} \) with an unbounded rank-width must have an undecidable \( C_2 MS \) theory.

This result connects the notion of rank-width of a class of graphs with the decidability of the logic theory of this class. We will use this result to prove the Theorem by juxtaposing it to a result obtained by the present and other authors, which relates the rank-width of a (family of) graph(s) with the computational power of the associated (family of) graph state(s). For, it was proved in [11] that:

Consider a family of graphs \( \mathcal{G} \) with a bounded rank-width. Then the graph state resource \( \Psi(\mathcal{G}) \) allows an efficient classical simulation of MQC.

The combination of this result with the one obtained by Courcelle and Oum, then immediately yields the proof of the Theorem.

Finally, we conclude this paragraph by remarking that the Theorem represents a sufficient condition for a resource \( \Psi(\mathcal{G}) \) to be simulatable, but not a necessary
one. For, there exist graph state resources with an unbounded rank-width—and therefore an undecidable $C_2MS$ logic theory—for which MQC is nevertheless simulatable; examples of such resources are given by the so-called “toric code states” [24], or graph states with logarithmically growing rank-width [11].

6 Discussion

In the Theorem the desired connection between measurement based quantum computation on graph states and mathematical logic theories on the underlying graphs is fully obtained. It is the authors’ opinion that the present results should be regarded as conceptual results, aimed at establishing a connection between seemingly remote areas of research, rather than yielding direct practical applications. We are aware that assessing whether a family of graphs has a decidable $C_2MS$ theory is a formidable task, and that logic theory itself is a dynamic area of research with difficult outstanding problems [25]. Therefore the present results are not likely to e.g. directly provide new examples of states on which MQC can be simulated efficiently. Nevertheless, we believe that our findings present a new, and possibly deep, perspective towards understanding the central issue of what the computational power of quantum computers with respect to classical devices is. The present connection to logic theory offers an entirely new view on “how complex” states need to be in order for them to possibly provide computational speed-ups, next to more standard considerations regarding entanglement. While the Theorem in fact follows from previously obtained results regarding MQC and entanglement width of graph states [10] [11], the resulting logic criteria are entirely different in nature.

Finally, one might be inclined to relate ($C_2MS$) logic formulas defined on graphs to the content of the quantum computations (i.e., measurement patterns) implemented on the corresponding graph states. As far as the authors are aware, there does not seem to be a direct relation between the classical logic defined on graphs and the quantum measurements—that might be associated to a quantum logic—on the corresponding graph states. Thus far it seems that the classical logic theories, and the issue of their (un)decidability, are related to assessing the complexity of the graphs, and hence of the (correlations in the) graph states, with no direct correspondence to quantum algorithms. However, it would be very interesting to investigate this issue in more detail, and we leave this as an open problem.

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