Mathematical modeling of milling machines for processing turbine blades

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Abstract. A special place in the theory of shaping is occupied by the method of compiling mathematical models of technological processes for obtaining surfaces of parts using cutting. The mathematical model includes a description of the shape of the cutting tool and the law of its movement relative to the workpiece. In this case, several reference systems are used, and the entire calculation is carried out taking into account the complex movement of bodies. The matrix expression of the laws of motion of bodies can be the basis for creating a model of the forming process. In many cases, the envelope surface can serve as a model of the surface treated with a cutting tool. From these positions, the envelope theory is an integral part of the theory of shaping. The mathematical description of the envelope depends on the type of task for the surface family. The mathematical model of the technological process of forming includes a description of the shape of the producing surface and the law of its movement relative to the workpiece, as well as obtaining the equation of the processed surface as the envelope of the family of producing surfaces.

1. Introduction

The sequence of creating a mathematical model is as follows: first, the equation of the generating surface in the initial position is written in a stationary system. Then the transition is made to a system that rotates with the workpiece. In this mobile system, the envelope of the family of producing surfaces is defined, which serves as a model of the processed surface. In many problems, you need to know the surface for a fairly long distance. In such problems, replacing a function describing a surface with its approximate expression as a power series segment may lead to significant errors or be unacceptable at all. In these cases, numerical methods are used to obtain the surface. Mathematical modeling allows you to take into account the shortcomings and complexities of the forming process described above. The structure of elements of the design model of a variable radius milling cutter contains the following elements: the producing surface of the blade, the cutter, the i-th cutting edge, the front, back surface, and the transition matrix.

2. Methodology

2.1. Modeling of surfaces of the pen blades

The basis for the design of the milling cutter is the modeling of the producing surface of the milling cutter, which can be performed using the geometric theory of surface formation. To ensure smoothness and the desired speed of the cutter relative to the blade, it is necessary to ensure the maximum length
of the arc of the cutter surface; it can be controlled using two parameters: the offset of the cutter axis relative to the blade and the ratio of the rotation angles of the workpiece and the tool.

The first step is to model the surface to be processed. The process of modeling a cutter for processing the outer surface is almost identical to modeling a cutter for processing the inner surface. The difference is in the direction of rotation: the rotation of the tool relative to the part when milling the inner surface occurs along the way. The problem was solved in a flat coordinate system, then its solutions were reflected in a three-dimensional coordinate system. Therefore, for the case of blade sections, the sections of the producing surface of the tool that processes the part are expressed using envelopes. The tool surface equation depends on the type of surface to be processed and how it is set. Since the surface to be processed is characterized by three blade cross-sections with specified coordinates, the spline adjustment method was used using the specified coordinates. Spline-elastic flexible line. If you specify a sequence of points through which the curve must pass, then you can use a spline described by a polynomial of degree and named Hermite. However, it is advisable to use a polynomial given by the Lagrange or Newton formula that provides continuity:

$$y_0 = a_0 \cdot x^m + a_1 \cdot x^{m-1} + a_2 \cdot x^{m-2} + \cdots + a_{m-1} \cdot x^1 + a_m$$

(1)

In this case, we used an implementation of least squares approximation that includes an array $x$. We transform the equations obtained for three sections into an array of conditional $y$ for the function that defines the surface of the part. The principle is the same, but in volume, using a family of curves. Expressing the obtained equations in the array $y$, we obtain the equation of the section of the surface to be processed: $r_0(x, z)$.

2.2 Modeling of the mill’s producing surface

In order to get the equation of the producing surface, we transform the equation of the surface of the part into the coordinate system of the tool. The equation of the tool surface is expressed in terms of the parameters $x$ and $z$, and in General look like this:

$$\vec{r}_f(\varphi, \varphi', x, z) = A_0^{-1}(\varphi, \varphi') \cdot \vec{r}_0(x, z)$$

(2)

where $\vec{r}_0(x, z)$ - equation of the surface to be processed; - matrix for setting the axis of rotation of the cutter:

$$A_3(a, b, \varphi, h, \varphi') = A^3(a) \cdot A^2(b) \cdot A^6(\pi) \cdot A^6(\varphi) \cdot A^7(h) \cdot A^5(\varphi')$$

(3)

where $A^3(a) = \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ - matrix that takes into account the offset along the axis $X$;

$$A^2(b) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ - matrix that takes into account the offset along the axis $Y$;

$$A^6(\pi) = \begin{pmatrix} \cos(\phi) & \sin(\phi) & 0 & 0 \\ \sin(\phi) & \cos(\phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ - rotation matrix that takes into account the initial angle rotation of the blade relative to the coordinate system;
– blade rotation matrix around the axis Z;

– matrix that takes into account the offset along the axis Y – distance between the workpiece and tool axes;

– tool rotation matrix around the Z axis by angle φ';

\[ A^5(\varphi) = \begin{bmatrix} \cos(\varphi) & \sin(\varphi) & 0 & 0 \\ \sin(\varphi) & \cos(\varphi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ A^2(h) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & h \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ A^6(\varphi') = \begin{bmatrix} \cos(\varphi') & \sin(\varphi') & 0 & 0 \\ \sin(\varphi') & \cos(\varphi') & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ x, y – \text{coordinates of the blade cross-section profile point that determine the center of rotation of the workpiece; at the first stages, they are taken arbitrarily; } \varphi – \text{ angle of rotation of the blade; } h – \text{ the distance between the axes of rotation of the workpiece and the blade; - the angle of rotation of the tool.} \]

On the parameters it is necessary to impose links; so we set specific values for parameters: \( x, y, y' \). On the angles of rotation of the workpiece and tool impose a connection in the form of a transfer ratio:

\[ \varphi = C \cdot \varphi' \] (4)

where \( C \) – a constant that expresses the ratio between the rotation angles of the tool and the workpiece.

Corners \( \varphi \) and \( \varphi' \) they are connected by a gear ratio that determines the dependence of the rotation speeds of the tool and the workpiece. The relationship is built in such a way that for one complete rotation of the tool around its axis, the part makes a rocking motion, making a rotation at a certain angle. The solution to calculating the maximum and minimum angles is the result where the determinant of the envelope matrix of a family of surfaces is equal to 0.

The producing surface is given by the radius vector \( \overrightarrow{r} \), an arbitrary point on the tool surface, described using the envelope matrix of the surface family.

On the parameters let's impose an envelope relation, which we define by solving with respect to the parameter \( \varphi \) equations,

\[ \tau_{f1} = \tau_{f}(\varphi'(|B| = 0)); \] (5)

where \( B \) – matrix of partial derivatives vector \( \overrightarrow{r}(x, \varphi') \) on the parameters \( x, \varphi' \):

\[ B = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial \varphi'} & \frac{\partial y}{\partial \varphi'} \end{bmatrix} = 0. \] (6)

Since in General the regular surfaces of the family are given by the equations:

\[ x = x(u, \varphi'); \] (7)
\[ y = y(u, \varphi'). \]  

(8)

Thus, the vector \( \overrightarrow{r'}(x, \varphi, \varphi') \) brought to view \( \overrightarrow{r'}(x, \varphi) \).

By solving the equation and converting the radius vector \( \overrightarrow{r'} \) in the coordinate system of the cutter, we get the final equation of the section of the producing surface of the cutter.

For the first section of the cutter:

\[
\overrightarrow{r'}(x, z) = (-\sin(\varphi' + \varphi') \cdot \overrightarrow{r_0}(x, z) - \sin(\varphi') \cdot b + b \cdot \sin(\varphi' + \varphi') \cdot \overrightarrow{r_0}(x, z) - \cos(\varphi') \cdot b + b \cdot \cos(\varphi' + \varphi'), z, 1)
\]

(9)

Creating an array of the obtained equations for three sections; based on them, it becomes possible to perform modeling in a three-dimensional coordinate system by adding a parameter \( z \) – length of the working part of the cutter.

The tool surface, which is analogous to the Lagrange curve, is built using the family of curves. The radius vector of such a surface is described by the formula:

\[
\overrightarrow{r}_f(x, i) = \sum_{i=1}^{n} L_i(t, z) \overrightarrow{r_f}(u, x),
\]

(10)

where \( L_i \) – the coefficients of Lagrange in \( t_i = j \):

\[
L_i(t, z) = \frac{\prod_{j=0, j\neq i}^{n} (z - j)}{\prod_{j=0, j\neq i}^{n} (t - j)}
\]

(11)

Each - line of such a surface is a Lagrange curve. As a result, their family creates the producing surface of the cutter.

2.3 Modeling of the cutting edges of the cutter, front and back surfaces

An important step in the process of designing a shaped cutter with a variable radius is the modeling of the cutting elements of the tool. The type of cutting edge equation depends on the type of front surface (flat, helical) and the value of the angle of inclination of the flat front surface \( \varphi \). The final solution is not the equation of the cutting edge, but an array of coordinates, since the solution was not presented in analytical form, but in numerical form. For the studied case of a curved front surface, the cutting edge is the result of the intersection of the producing surface with a plane passing through the axis of rotation of the tool and located at a certain angle \( \theta \) relative to the axis \( X \). The resulting edge does not have a tilt angle \( \omega \) and it belongs to the same plane of the radial section of the cutter, but it is not straight [1-3].
The equation of the front surface of the cutting edge can be obtained by solving the equation with respect to the parameters \( \theta \) and \( z \) vector \( \mathbf{r}_f \). To express this equation, follow these steps:

We express the equation of the front surface using the installation vector:

\[
\mathbf{r}_{ni} = A_{ni} \cdot A_{ust} \cdot e^4,
\]

where \( \mathbf{r}_{ni} \) – equation of the plane of the front surface of the I-th cutting edge in its own coordinate system; \( e^4 = [0 \ 0 \ 0 \ 1]^T \) – unit vector of zero length; \( A_{ni} \) – matrix for setting the front surface plane:

\[
A_{ni} = A^6(\gamma) \cdot A^2(b) \cdot e^4,
\]

where \( A^2(b) \) – motion matrix relative to the coordinate \( y \); \( A^6(\gamma) \) – rotation matrix that takes into account the front angle of the cutting edge; \( \theta_i \) – parameter that determines the angular position of the cutting edge; \( \gamma \) – front corner; \( A_{ust} \) – the matrix for setting the front surface plane, which can be defined as (Fig. 1)

\[
A_{ust}(\theta, z) = A \left( \mathbf{k}_{kf}, \mathbf{j}_{kf}, \mathbf{r}_{kf}(\theta_{jk}, z) \right).
\]

where \( A \) – function that determines the position of axes represented in the mill coordinate system; \( \mathbf{k}_{kf} \) – vector that defines the position of the axis \( Z_{x_f} \) represented in the milling cutter coordinate system

\[
\mathbf{k}_{kf} = [0 \ 0 \ 1 \ 0]^T,
\]

\( \mathbf{j}_{kf} \) – vector that defines the position of the axis \( Y_{z_f} \) represented in the milling cutter coordinate system

\[
\mathbf{j}_{kf} = \mathbf{r}_{fjk}(\theta_{jk}, z) \bigg|_{x_f j_f, y_f j_f}.
\]
vector that defines the position of the origin of the back surface coordinate system in the mill coordinate system

\[ \overline{t}_{fjk}(\theta_{jk}, z) = \overline{t}_{fj}(x, z) \big|_{x=f(\theta_{jk})} \]  \hspace{1cm} (17)

where \( \theta_{i} \) – parameter that determines the angular position of the cutting edge;

Also for the back surface:

\[ \overline{r}_{3i} = A_{3di} \cdot A_{up} \cdot e^{4} \]  \hspace{1cm} (18)

where \( \overline{r}_{3i} \) – equation of the plane of the front surface of the \( i \)-th cutting edge in its own coordinate system;

\( A_{3di} \) – matrix installation plane of the rear surface

\[ A_{3} = A_{2i} A^{6}(\alpha) \cdot A^{2}(b) \]  \hspace{1cm} (19)

\( A^{6}(\alpha) \) – rotation matrix that takes into account the rear angle of the cutting edge;

\( A_{2i} \) – matrix for setting the back surface plane at \( \alpha = 0 \);

\( \alpha \) – back corner.
Figure 3. Installation diagram the plane of the front surface.

Based on the obtained solutions, it becomes possible to model the normal to the front and back surfaces. Due to the fact that the straight cutting edge with $\omega = 0$ easier and cheaper to manufacture in the future, we will build the model for this option.

2.4 Modeling of the cutting surface formed by the cutting edges of the cutter

In order to be able to perform the cutting process by the chosen method, the surface to be processed must have a curvature. Based on the equations of cutting edges shaped cutters and model forming system, construct the equation of the surface described by the cutting edge of the $i$–th tooth of the cutter in the machining process:

$$\overline{QR}_i(t, z, t) = A^0(t) \cdot \overline{r}_x(\theta_{j, i}, z).$$  \hspace{1cm} (20)

where $A^0(t)$ – matrix of the transition from the coordinate system of the workpiece to the coordinate system that produces the milling surface during milling:

$$A^0(t) = A^1(a) \cdot A^2(h) \cdot A^6(\pi) \cdot A^6(\varphi(t)) \cdot A^2(h) \cdot A^4(\varphi'(t));$$  \hspace{1cm} (21)

where $\varphi(t)$ – parameter of rotation of the mill relative to the workpiece

$$\varphi(t) = \frac{\pi \cdot N_o \cdot t}{30},$$  \hspace{1cm} (22)

$N_o$ – the rotation of the tool, rpm; $t$ – time, s.

2.5 Method for calculating the estimated indicators of forming processes

The error value of the processed blade profile $\Delta_r$ measured on the axis $Y$ it is one of the main indicators of the quality of these mills. This value $\Delta_r$ define how

$$\Delta_r = \max(\Delta_{r_i}), i = 1, 3$$  \hspace{1cm} (23)

where $\Delta_{r_i}$ – the value of the profile error formed by the $i$-th ($i+1$) tooth of the covering cutter
\[
\Delta_{ri} = [\overline{Q}_{ri}(z, t_i) - Y_j]_{z=0} \quad (i, \overline{Q}_{ri}(t_j))
\]

where \( \overline{Q}_{ri}(z, t_i) \) – recording the cutting surface of the \( i \)-th cutting edge; \( t_i \) – the time at which the \( i \)-th tooth forms \( \Delta_{ri} \); \( Y_j \) – the point on the nominal surface of the part corresponding to the point of the cutting surface \([\overline{Q}_{ri}(z, t_i)]_{z=0} \); \( N_{sub} \) – number of mill teeth.

Figure 4 Schemes for determining the value during processing: a) internal surface; b) external surface.

\[
\begin{align*}
\dot{i} \cdot \overline{Q}_{ri}(t_i) &= \dot{i} \cdot \overline{Q}_{ri+1}(t_{i+1}) \\
\dot{j} \cdot \overline{Q}_{ri}(t_j) &= \dot{j} \cdot \overline{Q}_{ri+1}(t_{j+1})
\end{align*}
\]

Thus, based on the obtained formulas, we can calculate the estimated parameters of the model of the projected cutter.

3. Conclusion
As a result, the equations of the cutting edges of the cutter and the equations of the producing surface were obtained. Parametric equations of the back and front surfaces of the cutting teeth were constructed. It was also proposed to introduce corrections to reduce the amount of profile distortion after overshooting in the embedding motion parameter [4-5].
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