Estimation of Geodetic Parameters with VLBI Data of Last 5 Years

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Abstract  The meaning to research the potential of VLBI for geodetic applications is summarized. And the observation models and their related parameters of geodetic interest are investigated. Then, the principle and method of using the random model in VLBI data processing are investigated. With the world wide VLBI data from 2000-2004, the conditions to compute the parameters of geodetic interest are introduced, and so are the computing methods and processes. And the computed results of the parameters of geodetic interest are analyzed.

Keywords  VLBI; observation model; parameters of geodetic interest; random model; computation and analysis

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Introduction

Very-long-baseline interferometry (VLBI) has turn out to be a revolution to geodesy, geodynamics and astrometry since its appearance in 1967. For its high precision and resolution, many missions such as geodetic positioning, the conjunction of reference frame, the monitoring of Earth rotation and polar motion, the estimaton crustal movement, the mapping of galaxy etc. have achieved an unprecedented accuracy. VLBI contains enormous potential for “providing information of geodesy, geodynamics and astrometry”, so it is researched worthwhile. However, the former studies indicate that the instability of hydrogen atom clock and the disturbance of atmosphere have become the main obstacles to improve the accuracy. Least-square method and least-square collocation are used in the traditional VLBI software in which the clock biases and some atmosphere disturbances are approximated by piecewise linear polynomial, the division of the time session and the selection of polynomial order need to be adjusted according to the fact, which may affect the efficiency. Hence it is difficult to improve the accuracy.

Herring, et al.\textsuperscript{[1]} have proposed a new method that use Kalman filtering method to establish the stochastic model of clock parameters and atmosphere parameters, which can authentically reflect the geodetic values as well as the stochastic processes values of geodynamics and celestial parameters estimation. This method was recognized by many scholars and subsequently was admitted by IERS in 1994. Today much softwares such as SOLVK, GEODYN use this method.

Wuhan University has taken part in the combination pilot project (CPP) of IERS and has become one of the research centers. The global observed data of GPS, SLR, DORIS and VLBI are available for us. We have programmed the PANDA-SVLBI software,
which can run on PC. Besides, we use real VLBI observation data to debug the software, compute and analyse the results.

1 VLBI observation model and corresponding geodetic parameters

As shown in Fig.1, the geometrical relationship between the time delay ($\tau$) and delay rate ($\dot{\tau}$) caused by telescope $A$ and telescope $B$ on the ground in conventional inertial celestial coordinate system (CIS) can be described as [2]:

$$\tau = -X_g \cdot X_G / c$$  \hspace{1cm} (1)

$$\dot{\tau} = -(X_g \cdot X_G + X_g \cdot \dot{X}_G) / c$$  \hspace{1cm} (2)

Fig.1  VLBI geometry

The baseline vector from ground station $A$ to $B$ is:

$$X_B = X_B - X_A$$  \hspace{1cm} (3)

The CIS coordinates of a ground observation station $G$ can be obtained from its conventional terrestrial coordinates System (CTS) $X_G$ with the three well-known conversion steps: the precession ($P$), the nutation ($N$) and the earth rotation ($\text{UT1} - \text{UTC}$, $x_p$, $y_p$).

With the clock biases and the clock drift, the observations of VLBI can be described as the implicit function of the following parameters.

$$\tau = \tau(X_G, X_G, \text{UT1} - \text{UTC}, x_p, y_p, N, P, \Delta C_0, \Delta C_1)$$

$$\dot{\tau} = \dot{\tau}(X_G, X_G, \text{UT1} - \text{UTC}, x_p, y_p, N, P, \Delta C_1)$$  \hspace{1cm} (4)

The following is the geodetic parameters included in above expression.

1) Geodetic parameters: the coordinates of ground observation station $X_G$.  
2) Astrometry parameters: the position of radio sources $X_\nu$, express in right ascension and declination $\alpha_\nu, \delta_\nu$.  
3) Ties between CIS and CTS (geodynamics parameters): earth rotation parameters (EOP) $x_p, y_p$, $\text{UT1} - \text{UTC}$; nutation parameters, nutation in longitude ($d\psi$) and nutation in obliquity ($d\epsilon$).  
4) Solid tide parameters: the Love numbers ($h_2, l_2$), which tell how the earth gravity field changes due to its solid tide.  
5) Superabundant parameters: clock biases and clock drift $\Delta C_0, \Delta C_1$; atmosphere zenith delay $\tau_z$.

The precession parameter can be calculated by some formulas, it is not listed in above parameters [3].

2 Principle of VLBI computation using stochastic model

Traditional VLBI data process software uses least-square method and least-square collocation, in which most stochastic biases such as clock biases and atmosphere disturbances are approximated by piecewise linear polynomial. The division of the time session and the selection of polynomial order need to be decided manually, which may affect the efficiency of the data process. And it is also difficult to improve the accuracy. The instability of hydrogen atom clock and the disturbance of atmosphere have become the main obstacles to improve the accuracy.

Using the Kalman filtering method and VLBI observations to compute the geodetic parameters, stochastic model will take the place of polynomial parameter models, which was previously used in least-square method. Following is the description.

Assumed that vector $l$ is the difference between the theoretical value computed by PANDA-SVLBI software and the observations, so after linearization the observation equations can be described as:

$$v_t = A_t x_t - l_t$$  \hspace{1cm} (5)

where $x_t$ is the correction of the apriori values of parameter vector; $A_t$ is the partial derivative matrix; $v_t$ is random noise vector; $t$ denotes the epoch.

The state transition equation denoting the dynamics characteristic of the parameters can be described as:

$$x_{rel} = S_t x_t - w_t$$  \hspace{1cm} (6)
where \( x_{t+1} \) is the parameter corrections vector at the epoch of \( t+1 \); \( S_t \) is the state transition matrix at epoch of \( t \), it can be used to prognosticate the state at epoch of \( t+1 \); \( w_t \) is the stochastic disturbance vector between \( t \) and \( t+1 \), it may affect the state. As for non-stochastic parameter \( w=0 \), it comprises the position of ground VLBI station, the coordinates of radio sources and the EOP parameters, while the stochastic parameter comprise the stochastic process components of the clock and the atmosphere disturbance.

Eqs.(5) and (6) describe the course of linear dynamics, as for Kalman filtering, we can assume that:

\[
\begin{align*}
\langle v_t \rangle &= 0, \quad \langle w_t \rangle = 0 \\
\langle v_t w_{t+j} \rangle &= 0, \quad \langle v_t x_{t+j} \rangle = 0 \\
\langle v_{t+j} \rangle &= 0, \quad j \neq 0; \quad \langle w_t w_{t+j} \rangle = 0, \quad j \neq 0 \\
\langle x_{t+j} \rangle &= 0, \quad j > 0
\end{align*}
\]

< > denotes the expectation of the variables. We define \( t+j \) as the time, which is different from the time \( t \), that means the measurements are independent of stochastic motion and the stochastic disturbances are also independent of each other at different time.

Kalman filtering estimation should follow the sequence of observation, using the state at the epoch of \( t \) to predict the observations at the time of \( t+1 \), and then to modify the predicted value to get the estimated values at the time of \( t+1 \). That means, to predict

\[
\hat{x}_{t+1} = S_t \hat{x}_t + w_t
\]

To modify

\[
\hat{x}_{t+1} = \hat{x}_{t+1} + K (y_{t+1} - A_{t+1} x_{t+1})
\]

\[
C_{t+1} = C_{t+1} - K A_{t+1} C_{t+1}
\]

\( C_t \) is the covariance matrix, \( K \) denotes the gain of Kalman filtering, it can be calculated as:

\[
K = C_{t+1} (A_{t+1} w_{t+1} + A_{t+1} C_{t+1} A_{t+1}^T)^{-1}
\]

Eqs.(8) to (12) is the whole course of process that Kalman filtering does in processing observations. If the computation at epoch of \( t+1 \) finishes, the variables at the epoch of \( t+2 \) will replace the variables of \( t+1 \), the variables at the epoch of \( t+1 \) will replace the variables of \( t \), and the computation continues till all the observations to be comprised. At the beginning of the Kalman filtering, transcendental parameters \( X_0 \) and their covariance matrix \( C_0 \) have to be given, this course is called forward Kalman filtering. When all the observations have been used for computation, the estimated values of non-stochastic parameters can be solved out. There are also backward Kalman filtering methods, just the time is backward.

3 Main procedure of VLBI stochastic model computation

3.1 Establish clock and atmosphere stochastic model

At the beginning, a suitable stochastic process has to be chosen for reflecting the characteristic of the clock and the atmosphere in Kalman filtering, the research results by Herring and other scholars indicate that the change of station clock can be considered as the addition of random walk and the integrated random walk, while the change of atmosphere is just the course of random walk. As for a VLBI experiment, the corresponding parameter of clock and atmosphere model should be given. Alan standard deviation \( \sigma_{clk} \) and sample intervals \( \tau_{atk} \) reflect the characteristic of stochastic parameter, while the atmosphere stochastic parameter is the standard deviation of atmosphere zenith delay \( \sigma_{atm} \) in a certain time interval \( \tau_{atm} \).

All of these parameters are included in a control file in the software, which also contains other parameters, for example the edit multiplier, error constant, selecting the parameters, inputting transcendental precision, selecting data cutting rate and so on. These parameters can be chosen by the operator in order to get a precise solution.

3.2 Compute statistic characteristic parameters of stochastic model

In order to calculate the covariance \( W_t \) caused by the stochastic disturbance in the process of Kalman filtering, the power spectral density (PSD)of the random walk and the integrated random walk of the clock ,as well as the random walk of the atmosphere
is the statistic characteristic parameters needed in data process.

The PSD of the random walk of the clock is:

\[ \Phi_{\text{clk}}(\tau) = \sigma_{\text{clk}}^2 \tau_{\text{clk}} / 2 \] (13)

The PSD of the integrated random walk of the clock is:

\[ \Phi_{\text{iclk}}(\tau) = 3\sigma_{\text{iclk}}^2 / \tau_{\text{iclk}}^3 \] (14)

The PSD of the atmosphere random walk is:

\[ \Phi_{\text{atm}}(\tau) = \sigma_{\text{atm}}^2 / \tau_{\text{atm}} \] (15)

### 3.3 Compute parameters and predicted and modified values of covariance matrix at epoch of \( t+1 \)

According to Eq.(8), state transition matrix \( S_t \) is necessary for computing the predicted value \( \hat{X}_{t+1} \), but in fact it is not indispensable. The parameters can be divided into two parts according to whether they vary with time. For those parameters which do not vary with time (station coordinates, radio sources), to take the modified values at the time of \( t \) as the predicted values at the epoch of \( t+1 \). For those parameters which vary with time (clock parameter, atmosphere zenith delay), just to consider the linear change between \( t \) and \( t+1 \) in the computation, that means to use the modified values at epoch of \( t \) add the product of change rate and the time interval as the predicted value at epoch of \( t+1 \). The predicted covariance matrix can be calculated in similar way. It is not difficult to compute the modified values according to Eqs.(10) to (12). When all the computation above finish, it is not difficult to compute the modified values according to Eqs.(10) to (12).

### 4 Computation and analysis

#### 4.1 Software

We have programmed PANDA-SVLBI software to compute the geodetic parameters with VLBI observations, and we have debugged the program, and have processed the observations, and have analyzed the results.

#### 4.2 Parameter selection

The linear polynomial parameters model of the clock and the atmosphere delay in least-square method is replaced by stochastic model when to compute the geodetic parameters with Kalman filtering and VLBI observations.

Non-stochastic parameters comprise the position of VLBI station, the coordinates of the radio resources and EOP parameters.

Known parameters comprises the solid tide parameters, ocean load tide correction, atmosphere correction and precession parameters.

#### 4.3 Process method selection and comparison

1) Method 1. The coordinates of the radio source is fixed, other parameters are the unknown parameters to be solved, such as VLBI station coordinates, station clock bias and drift, atmosphere zenith delay and the ties between CIS and CTS. Due to the influence of VLBI systematic error, the precision of the baseline process just reach on the level of decimeter. The magnitude of difference between the ties between CIS and CTS and its annually reported values by IERS almost equals to the parameters themselves, or even greater.

2) Method 2. The coordinates of the radio resource and the VLBI station are fixed; other parameters are the unknown parameters to be solved, such as station clock bias and drift, atmosphere zenith delay and the ties between CIS and CTS. The result of this method is encouraging. The following research is based on this method, and our main analysis will focus on the precision of the ties between CIS and CTS.

#### 4.4 Precise computation and analysis from 2000 to 2004

Till now, we have computed and analysed the VLBI data from 2000 to 2004, the time interval of the computation is 7 d. The mean precision of the five parameters \( x_p, y_p, UT1, d\psi, d\epsilon \) in 2003 are 0.077 mas, 0.069 mas, 3.198 μs, 0.155 mas, 0.062 mas respectively, which has averagely improved for 52%, 30%, 40%, 20%, 25% comparing with 10 a ago (0.11 mas, 0.09 mas, 0.005 ms, 0.19 mas, 0.08mas)[4].
4.5 External precision from 2000 to 2004

In order to examine the reliability of using PANDA-SVLBI software, we also have computed the ties between CIS and CTS with the data from 2000 to 2004 and compared the results with the IERS annually report (2004). The two serials are perfectly consistent, most of the differences in \( x_p \) and \( y_p \) are less than 0.3 mas. The absolute-mean value of polar motion difference is 0.30 mas, UT1 is 0.03 ms, \( d\psi \) is 0.23 mas, \( d\epsilon \) is 0.09 mas, \( d\psi \) and \( d\epsilon \) is 0.16 mas on average.

5 Conclusions

The PANDA-SVLBI software can get a precisely solution of ties between CIS and CTS with the ground stations data under the condition that the coordinates of the ground stations and the radio sources are fixed. But if only fix the coordinate of the radio sources, although our software is able to compute, the precision of baseline vector and ties between CIS and CTS is not ideal, this method still needs further to be researched. Besides, when we process the data of recent years, we find some new stations and radio sources data has already existed in the observations, but the coordinates and rates of those newly built stations are still unavailable, the newly added radio sources have not been listed in the radio source almanac yet, which will influence the precision of the parameter estimation to some extent and still need to be compensated. And this researchment will be very valuable to study space VLBI and deep space detection\(^{[5,6]}\).

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