$p - p'$ Branes in PP-wave Background

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ABSTRACT

We present several supergravity solutions corresponding to both $Dp$, as well as $Dp - Dp'$ systems, in NS-NS and R-R PP-wave background originating from $AdS_3 \times S^3 \times R^4$. The $Dp$ brane solutions, $p = 1, ..., 5$ are fully localized, whereas $Dp - Dp'$ are localized along common transverse directions. We also discuss the supersymmetry properties of these solutions and the worldsheet construction for the $p - p'$ system.

August 2002

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1 Introduction

PP-waves \([1, 14]\) are known to be an interesting class of supersymmetric solutions of type IIB supergravity, with wide applications to gauge theories. For these applications, such solutions are considered in the “Penrose” limit of strings in \(AdS_p \times S^p\) background \([4]\). In several cases, PP-waves are also known to be maximally supersymmetric solutions of supergravities in various dimensions \([17–19]\), and are known to give rise to solvable string theories from worldsheet point of view as well. The particular case of \(AdS_5 \times S^5\) is of special interest, with applications to \(N = 4, D = 4\) gauge theories in the limit of large conformal dimensions and R-charges \([6]\). Interestingly, the \(D\)-branes of string theories also have an appropriate representation in such gauge theories, in terms of operators corresponding to ‘giant gravitons’ and ‘defects’ \([14, 15]\).

In this paper, we continue the search for explicit \(D\)-brane supergravity solutions in string theories in PP-wave background. Our study will be mainly concentrated to the NS-NS and R-R PP-waves arising out of \(AdS_3 \times S^3\) geometry \([10]\). We, however, also present branes in other PP-waves such as the ones in ‘little’ string theories etc.. Explicit supergravity solution of \(D\)-branes, along with open string spectrum, has been studied in \([20–26]\). Our new result includes a set of solutions corresponding to the brane systems of the type: \(Dp - Dp'\) in both IIA and IIB theories, as well as explicit worldsheet construction in these cases. We would like to mention that \(Dp\)-branes from worldsheet point of view have already been obtained in NS-NS PP-wave background earlier \([35]\). Our result gives realization of such \(D\)-branes from the supergravity point of view. We also find it interesting to note that, unlike the case of \(AdS_5 \times S^5\) PP-wave, in our case we are able to obtain several ‘localized’ \(D\)-brane solutions. \(Dp\)-branes in R-R PP-wave background are obtained by applying a set of \(S\) and \(T\)-duality transformations on the brane solutions in NS-NS background. Such background PP-wave configurations were already discussed in the context of \(D7\)-branes in \([21]\). The worldsheet construction of these branes then follows from the results in \([21]\), and will also be discussed below. We, however, point out that there is a crucial difference between the solutions presented in this paper and those in \([21]\). In the present case, only light-cone directions of the PP-wave are along the branes, the remaining four are always transverse to them. On the other hand, for the solutions in \([21]\) the directions transverse to the brane are flat.

The rest of the paper is organised as follows. In section-2, new (supersymmetric) \(Dp\) as well as \(Dp - Dp'\) branes are presented in both NS-NS and R-R PP-wave backgrounds. Supersymmetry properties of these branes have been examined in detail in section-3 and it is shown that they preserve some amount of unbroken supersymmetry. Open string constructions of branes is discussed in section-4. Section-5 is devoted to the branes in ‘little string theory’ background. We conclude in section-6 with some general remarks.
2 Supergravity Solutions

2.1 $D$-Branes in NS-NS PP-wave background

We now start by writing down the $D$-string solutions in the PP-wave background originating from the Penrose limit of NS-NS $AdS_3 \times S^3 \times R^4$ \cite{[10]}. The supergravity solution of a system of $N$ $D$-strings in such a background is given by:

$$\begin{align*}
  ds^2 &= f_1^{-\frac{1}{2}} (2dx^+dx^- - \mu^2 \sum_{i=1}^{4} x_i^2 (dx^+)^2) + f_1^\frac{1}{8} \sum_{a=1}^{8} (dx^a)^2, \\
  e^{2\phi} &= f_1, \quad H_{+12} = H_{+34} = 2\mu, \\
  F_{+-a} &= \partial_a f_1^{-1}, \quad f_1 = 1 + \frac{N g_s l_s^6}{r^6},
\end{align*}$$

with $f_1$ satisfying the Green function equation in the 8-dimensional transverse space. We have explicitly verified that the solution presented in (2.1) satisfy the type IIB field equations (see e.g. \cite{[27, 28]}). One notices that in this case, constant NS-NS 3-forms along the PP-wave direction are required precisely to cancel the $\mu$-dependent part of $R_{++}$ equation of motion.

Starting from the $D$-string solution in eqn. (2.1), one can write down all the $Dp$-brane solutions ($p = 1, ..., 5$) in NS-NS PP-wave background by applying successive T-dualities along $x_5, ..., x_8$. As is known, this procedure also involves smearing of the brane along these directions. For example, a $D3$-brane solution has a form:

$$\begin{align*}
  ds^2 &= f_3^{-\frac{1}{2}} (2dx^+dx^- - \mu^2 \sum_{i=1}^{4} x_i^2 (dx^+)^2 + (dx_5)^2 + (dx_6)^2) \\
  &\quad + f_3^\frac{1}{2} \sum_{a=1,4,7,8} (dx_a)^2, \\
  H_{+12} &= H_{+34} = 2\mu, \\
  F_{+-56a} &= \partial_a f_3^{-1}, \quad e^{2\phi} = 1,
\end{align*}$$

with $f_3$ being the harmonic function in the transverse space of the $D3$-brane.

Now we present the supergravity solution of intersecting ($Dp$ – $Dp'$)-brane system in PP-wave background. These solutions are described as ‘branes lying within branes’. In particular for $D1 – D5$ case, the solution is given by:

$$\begin{align*}
  ds^2 &= (f_1 f_5)^{-\frac{1}{2}} (2dx^+dx^- - \mu^2 \sum_{i=1}^{4} x_i^2 (dx^+)^2) + \left(\frac{f_1}{f_3}\right)^\frac{1}{2} \sum_{a=5}^{8} (dx^a)^2
\end{align*}$$
\[ + \left( f_1 f_5 \right) \frac{1}{2} \sum_{i=1}^{4} (dx_i)^2, \]

\[ e^{2\phi} = \frac{f_1}{f_5}, \]

\[ H_{+12} = H_{+34} = 2\mu, \]

\[ F_{+a} = \partial_a f_1^{-1}, \quad F_{mnp} = \epsilon_{mnp} \partial_0 f_5, \quad (2.3) \]

with \( f_1 \) and \( f_5 \) satisfying the Green function equations for \( D1 \) and \( D5 \)-branes respectively. Eqn. (2.3) provides one of the main results of our paper and shows that, as in the flat space, intersecting brane solutions are possible in the PP-wave background as well. We have once again checked that the solution presented above do satisfy type IIB field equations of motion.

One can now apply T-duality transformations to generate more intersecting brane solutions starting from the one given in eqn. (2.3) \[30, 31\]. Note that the directions \( x^5, \ldots, x^8 \) are transverse to the \( D \)-string in eqn. (2.3), whereas they lie along the longitudinal directions of \( D5 \). As a result, one can easily obtain solutions of the type \( D2 - D4 \) as well as \( D3 - D3' \) in this PP-wave background. These solutions will give a PP-wave generalization of the intersecting solutions given in \[32\]. We however skip the details of this analysis.

### 2.2 \( p - p' \) Branes in R-R PP-Wave Background

In this section, we will present the \( Dp \) as well as \( (Dp - Dp') \)-branes in R-R PP-wave of \( AdS_3 \times S^3 \times R^4 \). These backgrounds can be obtained from the solutions given in the last subsection by applying \( S \) and \( T \)-duality transformations in several steps. For example, from the \( D3 \)-brane solution in NS-NS PP-wave background (2.2), one gets a \( D3 \)-brane in RR PP-wave background under \( S \)-duality transformation. Now applying \( T \)-duality along the directions \( (x^5, x^6) \), we can generate a \( D \)-string solution. On the other hand, by applying \( T \)-duality along two transverse directions, \( (x^7, x^8) \), of the \( D3 \)-brane, one gets a \( D5 \)-brane lying along \( (x^+, x^-, x^5, \ldots, x^8) \) directions. Supergravity solution of a system of \( N \) \( D \)-strings is then given explicitly by:

\[ ds^2 = \frac{1}{2} (2dx^+dx^- - \mu^2 \sum_{i=1}^{4} x_i^2 (dx^+)^2 + f_1^{-\frac{1}{2}} \sum_{a=1}^{8} (dx_a)^2) \]

\[ e^{2\phi} = f_1, \quad F_{+1256} = F_{+3456} = 2\mu, \]

\[ F_{+-a} = \partial_a f_1^{-1}, \quad f_1 = 1 + \frac{Ng_s l_s^6}{r^6}, \quad (2.4) \]
with $f_1$ satisfying the Green function in 8-dimensional transverse space. One notices that the solution has a constant 5-form field strength.

The supergravity solution of $D5$-brane is given by:

$$ds^2 = f_5^{-\frac{4}{5}}(2dx^+dx^- - \mu^2 \sum_{i=1}^{4} x_i^2(dx^i)^2) + \sum_{a=5}^{8} (dx^a)^2 + f_5^2 \sum_{i=1}^{4} (dx^i)^2$$

$$e^{2\phi} = f_5^{-1}, \quad F_{+1256} = F_{+3456} = F_{+1278} = F_{+3478} = 2\mu,$$

$$F_{mnp} = \epsilon_{mnpq} \partial_q f_5, \quad f_5 = 1 + \frac{Ng_s l_s^2}{r^2}, \quad e^{2\phi} = f_5^{-1} f_5^2, \quad F_{+1256} = F_{+3456} = F_{+1278} = F_{+3478} = 2\mu,$$

with $f_5$ satisfying the Green function in the transverse directions $(x^1, ..., x^4)$. Now we will present the $(Dp-Dp')$-brane solutions in RR PP-wave background. In particular, to write down the supergravity solution of a $(D1-D5)$ system, we made an ansatz which combines the $D$-string of eqn. (2.4) and $D5$-brane given in eqn. (2.5). The final configuration is as follows:

$$ds^2 = (f_1 f_5)^{-\frac{1}{5}}(2dx^+dx^- - \mu^2 \sum_{i=1}^{4} x_i^2(dx^i)^2) + \left(\frac{f_1}{f_5}\right)^{\frac{4}{5}} \sum_{a=5}^{8} (dx^a)^2$$

$$+ (f_1 f_5)^{\frac{4}{5}} \sum_{i=1}^{4} (dx_i)^2,$$

$$e^{2\phi} = \frac{f_1}{f_5},$$

$$F_{+1256} = F_{+3456} = F_{+1278} = F_{+3478} = 2\mu,$$

$$F_{+-i} = \partial_i f_1^{-1}, \quad F_{mnp} = \epsilon_{mnpq} \partial_q f_5, \quad (2.6)$$

with $f_1$ and $f_5$ being the Green function in the common transverse space. One can check that the solution presented above do satisfy the type IIB field equations. Once again, more $p-p'$ branes can be obtained from the $D1-D5$ solution in (2.6) by applying $T$-dualities.

One may also attempt to find a $(D1-D5)$ solution by taking a decoupling limit, followed by the Penrose scaling, of the solution presented in [23] in a similar way as the $D5$-brane solution in [21]. The starting solution along which one would take the Penrose limit is as follows:

$$ds^2 = \frac{1}{(H_1' H_5')^{1/2}} \left[ r^2 (-dt^2 + dx^2) + \left(\frac{H_1'}{H_5'}\right)^{1/2} \frac{R_1^2}{r^2} dr^2 \right]$$

$$+ \left(\frac{H_1'}{H_5'}\right)^{1/2} (d\psi^2 + \sin^2 \psi d\Omega_5^2) + (H_1' H_5')^{1/2} (dy^2 + y^2 d\Omega_3^2), \quad (2.7)$$
where
\[ H_1 = 1 + \frac{R_1^2}{x^2}, \quad H_5 = 1 + \frac{R_5^2}{x^2}, \quad H'_1 = 1 + \frac{R'_1}{y^2}, \quad H'_5 = 1 + \frac{R'_5^2}{y^2}. \] (2.8)

One however notices that different terms in the metric above come with different powers of \( H'_1 \), leading to difficulty in choosing a ‘null geodesic’ to define an appropriate Penrose limit and find brane solutions.

3 Supersymmetry Analysis

3.1 NS-NS PP-wave

In this section we will present the supersymmetry of the solutions described earlier in section-(2.1). The supersymmetry variation of dilatino and gravitino fields of type IIB supergravity in ten dimension, in string frame, is given by \[23, 33, 34\]:

\[
\delta \lambda_{\pm} = \frac{1}{2} (\Gamma^\mu \partial_\mu \phi + \frac{1}{12} \Gamma^{\mu \nu \rho} H_{\mu \nu \rho}) \epsilon_{\pm} + \frac{1}{2} \epsilon^\phi (\pm \Gamma^M F^{(1)}_M + \frac{1}{12} \Gamma^{\mu \rho} F^{(3)}_{\mu \rho}) \epsilon_\mp,
\] (3.1)

\[
\delta \Psi_{\mu}^\pm = \left[ \partial_\mu + \frac{1}{4} \left( w_{\mu \hat{a} \hat{b}} \mp \frac{1}{2} H_{\mu \hat{a} \hat{b}} \right) \Gamma^{\hat{a} \hat{b}} \right] \epsilon_{\pm} + \frac{1}{8} e^\phi \left[ \mp \Gamma^\mu F^{(1)}_{\mu} - \frac{1}{3!} \Gamma^{\mu \nu \rho} F^{(3)}_{\mu \nu \rho} \mp \frac{1}{2 \cdot 5!} \Gamma^{\mu \nu \rho \sigma \delta} F^{(5)}_{\mu \nu \rho \sigma \delta} \right] \Gamma_\mu \epsilon_\mp,
\] (3.2)

where we have used \((\mu, \nu, \rho)\) to describe the ten dimensional space-time indices, and hat’s represent the corresponding tangent space indices. Solving the above two equations for the solution describing a D-string as given in eqn. (2.1), we get several conditions. First, the dilatino variation gives:

\[
\Gamma^{\hat{a}} \epsilon_\pm - \Gamma^{\hat{1} \hat{2}} \epsilon_\mp = 0,
\] (3.3)

\[
(\Gamma^{\hat{1} \hat{2}} + \Gamma^{\hat{3} \hat{4}}) \epsilon_\mp = 0.
\] (3.4)

In fact, both the conditions (3.3) and (3.4), are required for satisfying dilatino variation condition. Gravitino variation gives the following conditions on the spinors:

\[
\delta \psi_{\pm}^\pm \equiv \partial_\pm \epsilon_{\pm} \mp \frac{\mu}{2} f_1^{-\frac{1}{2}} (\Gamma^{\hat{1} \hat{2}} + \Gamma^{\hat{3} \hat{4}}) \Gamma^\pm \epsilon_\mp = 0, \quad \delta \psi_{\mp} \equiv \partial_- \epsilon_\pm = 0,
\]

\[
\delta \psi_{\pm}^a \equiv \partial_a \epsilon_\pm = -\frac{1}{8} f_{1a} \epsilon_\pm, \quad \delta \psi_{\mp}^i \equiv \partial_i \epsilon_\pm = -\frac{1}{8} f_{1i} \epsilon_\pm.
\] (3.5)
In writing the above set of equations, we have also imposed a necessary condition:

\[ \Gamma^\dagger \epsilon_\pm = 0, \]  

(3.6)
in addition to (3.3). Further, by using

\[ (1 - \Gamma^{12\tilde{3}4})\epsilon_\pm = 0, \]  

(3.7)
all the supersymmetry conditions are solved by spinors: \( \epsilon_\pm = \exp(-\frac{1}{8}ln(f_1)f^0_\pm) \), with \( f^0_\pm \) being a constant spinor. D-string solution in eqn. (2.1) therefore preserves 1/8 supersymmetry. All other \( Dp \)-branes \((p = 1, \ldots, 5)\), obtained by applying T-dualities as discussed above, will also preserve same amount of supersymmetry.

Next, we will analyze the supersymmetry properties of the intersecting branes. We will concentrate on the \((D1 - D5)\)-case explicitly. The dilatino variation gives the following conditions on the spinors:

\[ \Gamma^i \epsilon_\pm - \Gamma^i\tilde{i} \epsilon_\mp = 0, \]  

(3.8)
\[ \Gamma^i \epsilon_\pm + \frac{1}{3!} \epsilon_{ij\tilde{k}} \Gamma^{j\tilde{k}} \epsilon_\mp = 0, \]  

(3.9)
\[ (\Gamma^{\dagger i\dagger 2} + \Gamma^{\dagger i\tilde{3}\tilde{4}}) \epsilon_\mp = 0. \]  

(3.10)
One needs to impose all the three conditions, specified above for the dilatino variation to vanish. On the other hand, the gravitino variation gives:

\[ \delta \psi_\pm \equiv \partial_+ \epsilon_\pm \mp \frac{\mu}{2} (f_1f_5)^{-\frac{1}{2}} (\Gamma^{12} + \Gamma^{34}) \Gamma^\dagger \epsilon_\pm = 0, \quad \delta \psi_\pm \equiv \partial_- \epsilon_\pm = 0, \]  

\[ \delta \psi_i^\pm \equiv \partial_i \epsilon_\pm = -\frac{1}{8} \left[ \frac{f_{1,a}}{f_1} \epsilon_\pm + \frac{f_{5,a}}{f_5} \right] \epsilon_\pm, \quad \delta \psi_a^\pm \equiv \partial_a \epsilon_\pm = 0. \]  

(3.11)
In writing down the above gravitino variations we have once again made use of the projection \( \Gamma^\dagger \epsilon_\pm = 0 \). The above set of equations can be solved by imposing:

\[ (1 - \Gamma^{12\tilde{3}4})\epsilon_\pm = 0, \]  

(3.12)
in addition to (3.8) and the solution is given as: \( \epsilon_\pm = \exp(-\frac{1}{8}ln(f_1f_5))\epsilon^0_\pm \). One therefore has 1/8 supersymmetry for the \((D1 - D5)\) solution presented in eqn. (2.3).
3.2 R-R PP-wave

Now, we will present the supersymmetry of the $Dp$ as well as $Dp - Dp'$ branes in R-R PP-wave background given in section-(2.2). First we will discuss the supersymmetry of the $D$-string in eqn. (2.4). The dilatino variation (3.1), gives:

$$\Gamma^\hat{a} \epsilon_{\pm} - \Gamma^{\hat{i} \hat{a}} \epsilon_{\mp} = 0.$$  \hfill (3.13)

Gravitino variation gives the following conditions on the spinors:

$$\delta \psi^\pm_+ \equiv \partial_+ \epsilon_{\pm} + \frac{\mu}{8} f^{-\frac{1}{2}} f_1 \left( (\Gamma^{\hat{i} \hat{2} \hat{5} \hat{6}} + \Gamma^{\hat{i} \hat{3} \hat{4} \hat{5} \hat{6}}) + (\Gamma^{\hat{i} \hat{2} \hat{7} \hat{8}} + \Gamma^{\hat{i} \hat{3} \hat{4} \hat{7} \hat{8}}) \right) \Gamma^\hat{-} \epsilon_{\mp} = 0,$$

$$\delta \psi^\pm_- \equiv \partial_- \epsilon_{\mp} = 0,$$

$$\delta \psi^\pm_a \equiv \partial_a \epsilon_{\pm} = -\frac{1}{8} f_{1,i} \epsilon_{\pm}, \quad \delta \psi^\pm_i \equiv \partial_i \epsilon_{\pm} = -\frac{1}{8} f_{1,a} \epsilon_{\pm},$$  \hfill (3.14)

where we have once again used a necessary condition: $\Gamma^{\hat{i} \hat{a}} \epsilon_{\pm} = 0$, in addition to (3.13). Further, by using the condition:

$$(1 - \Gamma^{\hat{1} \hat{2} \hat{3} \hat{4}}) = 0,$$  \hfill (3.15)

all the supersymmetry conditions are satisfied, thus preserving $1/8$ unbroken supersymmetry.

Next, we present the supersymmetry property of the $D1 - D5$ solution written in eqn. (2.6). The dilatino variation (3.1), gives the following conditions on spinors:

$$\Gamma^{\hat{i}} \epsilon_{\pm} - \Gamma^{\hat{i} \hat{a}} \epsilon_{\mp} = 0,$$  \hfill (3.16)

$$\Gamma^{\hat{i}} \epsilon_{\pm} + \frac{1}{3!} \epsilon_{ijkl} \Gamma^{\hat{j} \hat{k} \hat{l}} \epsilon_{\mp} = 0.$$  \hfill (3.17)

On the other hand, the gravitino variation (1.2), gives the following conditions:

$$\delta \psi^\pm_+ \equiv \partial_+ \epsilon_{\pm} + \frac{\mu}{8} (f_1 f_5)^{-\frac{1}{2}} \left( (\Gamma^{\hat{i} \hat{2} \hat{5} \hat{6}} + \Gamma^{\hat{i} \hat{3} \hat{4} \hat{5} \hat{6}}) + (\Gamma^{\hat{i} \hat{2} \hat{7} \hat{8}} + \Gamma^{\hat{i} \hat{3} \hat{4} \hat{7} \hat{8}}) \right) \Gamma^\hat{+} \epsilon_{\mp} = 0,$$

$$\delta \psi^\pm_- \equiv \partial_- \epsilon_{\mp} = 0, \quad \delta \psi^\pm_i \equiv \partial_i \epsilon_{\pm} = 0,$$

$$\delta \psi^\pm_a \equiv \partial_a \epsilon_{\pm} = -\frac{1}{8} \left[ f_{1,i} - f_{5,i} \right] \epsilon_{\mp},$$  \hfill (3.18)

where we have once again used a necessary condition: $\Gamma^{\hat{i} \hat{a}} \epsilon_{\pm} = 0$ along with the ones in (3.10) and (3.17). The above set of equations can be solved by imposing further:

$$(1 - \Gamma^{\hat{1} \hat{2} \hat{3} \hat{4}}) \epsilon_{\pm} = 0.$$  \hfill (3.19)

One therefore has $1/8$ supersymmetry for the $D1 - D5$ system described in eqn. (2.6) as well.
4 Worldsheet Construction of $p - p'$ Branes

4.1 NS-NS PP-wave

In this section, we will discuss the $(D1 - D5)$-brane system, constructed earlier in the paper, from the point of view of first quantized string theory in Green-Schwarz formalism, in light-cone gauge. In the present case, in flat directions $x_{\alpha}(\alpha = 5, \ldots, 8)$, we have the Dirichlet boundary condition at one end and Neumann boundary condition at the other end of the open string. Along $x_i$ $(i = 1, \ldots, 4)$ directions, one has the usual Dirichlet boundary condition. The relevant classical action to be studied in our case (after imposing the light-cone gauge conditions on fermions and bosons [10]) is as follows [35]:

$$L = L_b + L_f,$$ (4.1)

where

$$L_b = \partial_+ u \partial_- v - m^2 x_i^2 + \partial_+ x_i \partial_- x_i + \partial_+ x_{\alpha} \partial_- x_{\alpha}$$

$$+ \mu \sum_{(i,j) = (1,2),(3,4)} x^i (\partial_+ u \partial_- x^j - \partial_- u \partial_+ x^j),$$ (4.2)

$$L_f = i S_R (\partial_+ - m M) S_R + i S_L (\partial_- + m M) S_L,$$ (4.3)

with

$$m \equiv \alpha' p^u \mu = 2 \alpha' p_u \mu,$$ (4.4)

$$M = -\frac{1}{2} (\gamma^{12} + \gamma^{34}).$$ (4.5)

Eight component real spinors $(S_L, S_R)$ have been obtained from 16-component Majorana-Weyl spinors in the left and the right sector after solving the light-cone gauge conditions.

The equations of motion and boundary conditions for bosons $x^i, x^\alpha$ in our case are as follows:

$$\partial_+ \partial_- x_{i_1} + m^2 x_{i_1} - m \epsilon^{i_1 j_1} (\partial_- x^{j_1} - \partial_+ x^{j_1}) = 0, \quad \partial_+ \partial_- x_{\alpha} = 0,$$ (4.6)

$$\partial_\tau x^\alpha\big|_{\sigma=0} = \partial_\tau x^\alpha\big|_{\sigma=\pi} = 0, \quad x^i\big|_{\sigma=0,\pi} = \text{constant}. \quad (4.7)$$
The solutions to the bosonic equations of motion, with the boundary conditions specified above, is given by (defining \( X^{\hat{1}} = \frac{1}{\sqrt{2}}(x^1 + ix^2) \) and \( X^{\hat{2}} = \frac{1}{\sqrt{2}}(x^3 + ix^4) \)):

\[
X^{\alpha}(\sigma, \tau) = i \sum_{r \in \mathbb{Z}} \frac{1}{r} \alpha^{\alpha}_{r} e^{-ir\tau} \cos r\sigma, \quad (\alpha = 5, \ldots, 8),
\]

(4.8)

\[
X^{\hat{i}}(\sigma, \tau) = e^{-2im\sigma} \left[ x_0 + (x_1 e^{2im\sigma} - x_0) \frac{\sigma}{\pi} + i \sum_{n \neq 0} \frac{1}{n} \alpha^{i}_{n} e^{-in\tau} \sin n\sigma \right].
\]

(4.9)

To consider the equations of motion of fermions, we note that the matrix \( M \) evidently breaks the \( SO(8) \) symmetry further, and thereby splits the fermions in the \( 8 \rightarrow 4 + 4 \) way: \( S_L \rightarrow (\tilde{S}_L, \hat{S}_L), S_R \rightarrow (\tilde{S}_R, \hat{S}_R) \):

\[
\gamma^{1234} \begin{pmatrix} \tilde{S}_{L,R} \\ \hat{S}_{L,R} \end{pmatrix} = \begin{pmatrix} -\tilde{S}_{L,R} \\ \hat{S}_{L,R} \end{pmatrix}.
\]

(4.10)

In this connection, one also introduces \( 4 \times 4 \) matrices \( \Lambda \) and \( \Sigma \):

\[
\gamma^{12} \begin{pmatrix} \tilde{S}_{L,R} \\ \hat{S}_{L,R} \end{pmatrix} = - \begin{pmatrix} \Lambda \tilde{S}_{L,R} \\ \Sigma \hat{S}_{L,R} \end{pmatrix},
\]

(4.11)

with \( \Lambda^2 = \Sigma^2 = -1 \). \( \Lambda \) and \( \Sigma \) in the above equation are \( 4 \times 4 \) antisymmetric matrices with eigenvalues \( \pm i \). Using these notations, one has:

\[
M \begin{pmatrix} \tilde{S}_{L,R} \\ \hat{S}_{L,R} \end{pmatrix} = \begin{pmatrix} \Lambda \tilde{S}_{L,R} \\ 0 \end{pmatrix}.
\]

(4.12)

The equations of motion written in terms of \( (\tilde{S}_L, \hat{S}_L) \) and \( (\tilde{S}_R, \hat{S}_R) \) are then of the form:

\[
\partial_+(e^{2m\tau} \tilde{S}_R) = 0, \quad \partial_-(e^{-2m\tau} \tilde{S}_L) = 0,
\]

(4.13)

\[
\partial_+ \tilde{S}_R = 0, \quad \partial_- \tilde{S}_L = 0.
\]

(4.14)

Now we will write down the boundary conditions for the fermions in the mixed sector. As the equations of motion and the boundary condition for the components \( \tilde{S}_{L,R} \) are identical to the ones in flat space, we only concentrate on finding explicit solution for \( \tilde{S}_{L,R} \) below. Following [10, 21, 33], one can write down the boundary conditions for the fermions as:

\[
\tilde{S}_L|_{\sigma=0} = -\tilde{S}_R|_{\sigma=0},
\]

(4.15)
\[
\tilde{S}_L|_{\sigma=\pi} = \tilde{S}_R|_{\sigma=\pi},
\]
\[
\tilde{S}_L|_{\sigma=0,\pi} = \tilde{S}_R|_{\sigma=0,\pi}.
\]

The solution for $\tilde{S}_{L,R}$ equations of motion (4.13), with the above boundary condition, can be read from [35], and has the following form:

\[
\tilde{S}_L = -e^{-2m\sigma}\Lambda \sum_{r \in (z+\frac{1}{2})} s_r e^{-ir(r+\sigma)},
\]
\[
\tilde{S}_R = e^{-2m\sigma}\Lambda \sum_{r \in (z+\frac{1}{2})} s_r e^{-ir(r-\sigma)}.
\]

The canonical quantization conditions as well as the worldsheet hamiltonian for the $D1-D5$ system discussed above can also be written in a straightforward manner following the procedure in [35]. We skip these details.

### 4.2 R-R PP-wave

Now, we present the worldsheet analysis of the $(D1-D5)$-system discussed earlier in section-(2.2). This can be done by realizing that the PP-wave background for these solutions is given by a $T$-dual configuration of the ones presented in [10, 21]. More explicitly, in the worldsheet action in the present case:

\[
L = L_B + L_F,
\]

where

\[
L_B = \partial_+ u \partial_- v - m^2 x_i^2 + \partial_+ x_i \partial_- x_i + \partial_+ x_\alpha \partial_- x_\alpha,
\]
\[
L_F = i S_R \partial_+ S_R + i S_L \partial_+ S_L - 2im S_L M S_R,
\]

with

\[
m \equiv \alpha' p^\mu \mu = 2\alpha' p^\mu \mu,
\]
\[
M = -\frac{1}{2} (\gamma^{12} + \gamma^{34})\gamma^{56},
\]

the terms involving fermions are easily seen to be related to the ones in [10, 21] through $T$-dualities along $x^5$ and $x^6$. This in fact leads to the relation:

\[
S'_R = \gamma^{56} S_R,
\]

and reproduces the original action in [10]. The mode expansion for fermions as well as canonical quantization conditions can therefore be also written down in a straightforward manner. We end this section by pointing out that, since the $D$-brane solutions found in this paper are preserving less than $1/2$ supersymmetry, some of the restrictions on the brane directions, imposed using zero mode considerations [20] do not directly apply above.
5 Branes in ‘Little String Theory’

5.1 Supergravity Backgrounds

In this section, we discuss the branes in the Penrose limit of ‘little string theory’ (LST). PP-waves of non-local theories have been discussed recently in the literature \[36–38\]. Among them, ‘little string theory’ arises on the world volume of NS5-brane when a decoupling limit, \(g_s \to 0\) with fixed \(\alpha'\), is taken \[39, 40\]. To construct our solution, we start with the NS5-brane solution given by the metric and dilaton:

\[
ds^2 = -dt^2 + dy_5^2 + H(r)(dr^2 + r^2 d\Omega_3^2),
\]
\[e^{2\phi} = g_s^2 H(r),\]  \hspace{1cm} (5.1)

with \(H(r) = 1 + \frac{N_l^2}{r^2}\). The near horizon limit of the above solution is the linear dilaton geometry, which in the string frame is given by,

\[
ds^2 = Nl_s^2 \left(-dt^2 + \cos^2 \theta d\psi^2 + d\theta^2 + \sin^2 \theta d\phi^2 + \frac{dr^2}{r^2}\right) + dy_5^2,\]  \hspace{1cm} (5.2)

with \(t = \sqrt{Nl_s} \tilde{t}\). PP-wave background of LST is then found by applying Penrose limit to (5.2). We however consider the case after applying S-duality on eqn. (5.2). Applying S-duality transformation and then taking Penrose limit (as described in \[39\]), the background solution is given by:

\[
ds^2 = -4 dx^+ dx^- - \mu^2 z^2 dx^+ dx^+ + (d\bar{z})^2 + dx^2 + dy_5^2
\]
\[e^{2\phi} = \text{Const},\]
\[F_{+12} = C\mu,\]  \hspace{1cm} (5.3)

where \(F_{+12}\) is the 3-form field strength in the \(\bar{z}\)-plane. Now we proceed to analyze the existence and stability of branes in this background. The supergravity solution for a system of D5-branes in this background is given by:

\[
ds^2 = f^{-\frac{1}{2}} \left(-4 dx^+ dx^- - \mu^2 z^2(dx^+)^2 + \sum_{i=1}^{2}(dz_i)^2 + \sum_{p=1}^{2}(dy_p)^2\right)
\]
\[+ f^\frac{1}{2} \sum_{a=1}^{4}(dx_a)^2,\]
\[e^{2\phi} = f^{-1}, \quad F_{+12} = 2\mu,\]
\[ F_{mnp} = \epsilon_{mnpq} \partial_r f, \quad f = 1 + \frac{N g_s l_s^2}{r^2}. \quad (5.4) \]

One notices that the background has only one constant 3-form field strength \( (F_{+12}) \). We have once again verified that the solution presented above satisfies type IIB field equations.

One can then write down (by applying S-duality on (5.4)) the NS5-brane in a PP-wave background of the ‘little string theory’ \[ 36 \] as:

\[ ds^2 = -4 dx^+ dx^- - \mu^2 z^2 (dx^+)^2 + \sum_{i=1}^{2} (dz_i)^2 + \sum_{p=1}^{2} (dy_p)^2 + f \sum_{a=1}^{4} (dx_a)^2 \]

\[ e^{2\Phi} = f, \quad H_{+12} = 2\mu, \]

\[ H_{mnp} = \epsilon_{mnpq} \partial_r f, \quad (5.5) \]

with \( H \)’s being the NS-sector 3-form field strengths.

We will now consider the dilatino and gravitino variation of the solution presented in eqn. (5.4) to study the supersymmetry properties. The dilatino variation gives equations:

\[ \Gamma^a \epsilon_+ + \frac{1}{3!} \epsilon_{a\hat{b}\hat{c}\hat{d}} \Gamma^{\hat{b}\hat{c}\hat{d}} \epsilon_+ = 0, \quad (5.6) \]

\[ \Gamma^{+\hat{1}\hat{2}} \epsilon_+ = 0. \quad (5.7) \]

The gravitino variation leads to the equations:

\[ \delta \Psi^\pm_+ \equiv \partial_+ \epsilon_+ + \frac{\mu^2 z^2}{2} \Gamma^{+\hat{1}\hat{2}} \epsilon_+ + \frac{1}{16} \mu^2 z^2 \hat{f} \hat{a} \hat{b} \hat{c} \hat{d} \Gamma^{+\hat{a}\hat{b}\hat{c}\hat{d}} \epsilon_+ - \frac{\mu}{4} \Gamma^{+\hat{1}\hat{2}} \Gamma^- \epsilon_+ = 0, \quad (5.8) \]

\[ \delta \Psi^-_+ \equiv \partial_- \epsilon_+ = 0, \quad (5.9) \]

\[ \delta \Psi^\pm_i \equiv \partial_i \epsilon_+ - \frac{\mu}{4} \Gamma^{+\hat{1}\hat{2}} \delta_{\hat{a}} \Gamma^{\hat{a}} \epsilon_+ = 0. \quad (5.10) \]

\[ \delta \Psi^\pm_p \equiv \partial_p \epsilon_+ - \frac{\mu}{4} \Gamma^{+\hat{1}\hat{2}} \delta_{\hat{p}} \Gamma^{\hat{p}} \epsilon_+ = 0. \quad (5.11) \]

\[ \delta \Psi^\pm_a \equiv \partial_a \epsilon_+ + \frac{f a}{f} \epsilon_+ - \frac{\mu}{4} f \Gamma^{+\hat{1}\hat{2}} \delta_{\hat{a}} \Gamma^{\hat{a}} \epsilon_+ = 0, \quad (5.12) \]
In writing these set of equations we have used the condition (5.6). Imposing the condition $\Gamma^{+}\epsilon = 0$, we further reduce them to:

$$\partial_{\pm}\epsilon_{\pm} - \frac{\mu}{4}\Gamma^{\pm i\pm\Gamma_{-}}\epsilon = 0,$$

(5.13)

$$\partial_{-}\epsilon_{\pm} = 0, \quad \partial_{i}\epsilon_{\pm} = 0, \quad \partial_{p}\epsilon_{\pm} = 0, \quad \partial_{a}\epsilon_{\pm} = -\frac{1}{8}\frac{f}{f}\epsilon_{\pm},$$

(5.14)

Since eqns. (5.13) and (5.14) are integrable ones, hence in this case we get 1/4 supersymmetry. It will also be nice to give a worldsheet construction for such D-branes.

6 Conclusion

In this paper we have presented several supersymmetric $Dp$ and $Dp - Dp'$-brane configurations in PP-wave background and analyzed their supersymmetry properties. We have also presented $(D1 - D5)$-brane construction from the point of view of massive Green-Schwarz formalism in the light cone gauge in NS-NS and R-R PP-wave of $AdS_{3} \times S^{3} \times R^{4}$. It will be interesting to study the gauge theory duals of the branes presented in this paper by using operators such as ‘defects’. One could possibly also look at the black hole physics using the $(D1 - D5)$ system presented here in an attempt to understand their properties.

Acknowledgement

We thank Rashmi R. Nayak, Koushik Ray and Sanjay for useful discussions.

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