Semiclassical Quantum Computation Solutions to the Count to Infinity Problem: A Brief Discussion

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(Dated: May 26, 2003)

In this paper we briefly define distance vector routing algorithms, their advantages and possible drawbacks. On these possible drawbacks, currently widely used methods split horizon and poisoned reverse are defined and compared. The count to infinity problem is specified and it is classified to be a halting problem and a proposition stating that entangled states used in quantum computation can be used to handle this problem is examined. Several solutions to this problem by using entangled states are proposed and a very brief introduction to entangled states is presented.

I. INTRODUCTION

Distance vector routing is a popular dynamic routing algorithm which is used in many applications due to its simplicity and ease of implementation. Although it is not a technically superior algorithm, its availability even before it was standardized made it the most common algorithm used by now. Despite such advantages of distance vector routing algorithm which was firstly proposed by Ford and Fulkerson [1, 2], the algorithm has a very critical problem embedded in it, which arises when one of the nodes in the network goes down (or is isolated from the network). Since distance vector routing simply depends on routing table exchange of a node with its neighbors, the other nodes neighboring the node that has just gone down still think that their other neighbors have a better path leading to that isolated node, which in turn starts an endless exchange of data between the nodes, and it is well known as count to infinity problem.

A good standard in distance routing protocols is called RIP (Routing Information Protocol) [3] and it solves count to infinity problem by adding more check actions and limitations to the system, these methods are called split horizon and poisoned reverse. Split horizon together with poisoned reverse solves loops in the network up to and including two gateways and if more than two gateways are in a loop the problem is not eliminated. Poisoned reverse loads a special meaning to the infinite distance metric (16 for RIP) and updates other nodes’ routing tables accordingly to avoid looping so that the the neighboring nodes do get infinite metric entry into its corresponding table to immediately prevent a loop. But this poisoned reverse has a serious problem that it limits the bandwidth of the system since the packets that prevent the loop get bigger and bigger as the network neighborhood enlarges. For this reason, RIP is suggested to be implemented in networks no more than 15 hops (i.e., 15 gateways connecting asynchronous networks to each other is an example). Therefore, the major ways of preventing count to infinity problem lead to more complications and changes in the protocol, which adds overheads in using time and space sources (i.e., more delays and less bandwidth with no improvement in performance but some stabilization). Moreover, these preventing algorithms are only applicable when the network size is small, which is due to the the time considerations in loop detection. To propose a solution to the count to infinity problem with minimal loss in time and space, we will approach the system as a cause for the count to infinity problem and show that classical computers are not capable of solving this problem which in turn will need entanglement as a solution for the problem. Next section will show that count to infinity problem is a very hard problem (in view of algorithmic complexity considerations) to be completely solved by classical computers if the case is imminent in a certain algorithm.

II. REDUCTION OF COUNTING TO INFINITY PROBLEM TO THE HALTING PROBLEM

The halting problem is one of the oldest unsolvable problems of computation theory. It stems from Hilbert’s Entscheidungs problem (decision problem) which asks whether there is an algorithm for solution of any problem given. The halting problem is Turing’s answer to this question and it is a well known issue that, unfortunately, all problems do not have feasible algorithms for their solution. The halting problem can be proved by the following discussion [4]. Define a Turing machine $M_x$ which halts upon input of a specific number $x$ so that each such Turing machine $M$ is related with a Turing number $x$. And define the halting function $h(x)$ as

$$h(x) = \begin{cases} 0, & \text{if machine with Turing number } x \text{ does not halt upon input of } x \\ 1, & \text{if machine with Turing number } x \text{ halts upon input of } x \end{cases}$$
Therefore the halting problem can be stated as follows: Does the machine with number \( x \) halt upon input of the same number \( x \), or equivalently is there an algorithm to evaluate \( h(x) \)? Now suppose that we have such an algorithm denoted by \( \text{HALT}(x) \) that evaluates the function \( h(x) \). Then define the function \( \text{TURING}(x) \) with the pseudocode

\[
\text{TURING}(x): \\
y = \text{HALT}(x) \\
\text{if } y = 0 \text{ then halt} \\
\text{else loop forever} \\
\text{end if}
\]

Since we assumed \( \text{HALT()} \) is a valid program \( \text{TURING()} \) is also a valid program and for the halting function, \( h(t) = 1 \) if and only if \( \text{TURING()} \) halts on input of \( t \) (Note that if \( h(t) = 1 \), Turing machine ends operation giving a result and the algorithm in this way automatically ends). But from the program, \( \text{TURING()} \) halts on input of \( t \) if and only if \( h(t) = 0 \). Thus \( h(t) = 1 \) if and only if \( h(t) = 0 \), which is a contradiction and there is no algorithm \( \text{HALT}(x) \) for evaluating \( h(x) \).

Count to infinity problem can also be defined in the class of problems that have the characteristics of halting problem and to show this it is enough to show a function derived from the count to infinity problem, which models the function \( h(x) \) above. Our \( h(x) \) for the count to infinity problem can be given as

\[
h(x) = \begin{cases} 
0, & \text{if there is not a well defined route to a just isolated node with acceptable amount of metric used} \\
1, & \text{if there is a well defined route to a just isolated node with some optimum finite amount of metric used}
\end{cases}
\]

Here the problem only covers the nodes that are on the path that is not available just after the destination node is isolated. If there were an algorithm to determine a finite cost path we would then have \( h(x) = 1 \) after some acceptable amount of time and the system will halt with a specific route ending the program while \( \text{TURING()} \) procedure is in a loop. Vice versa, if there were an algorithm that can evaluate \( h(x) = 0 \), \( \text{TURING()} \) procedure would stop and a decision would be definitely made. But for the count to infinity problem, both of those do not concur each other’s decision as in the halting problem and in fact this problem is an open ended issue in computer science that implies no current solution with classical computers. When one attempts to solve this problem based on a classical computer architecture, he must either completely change the algorithm or take some precautions not to let it happen, which in turn adds significant overheads and limitations to the system. Split horizon with poisoned reverse is such a precaution that still has problems but works well in small networks to some extent.

For these reasons we will propose a new feature coming from quantum theory, the entangled states to solve this problem for larger networks, which change the hardware by using a phenomenon that has no classical counterpart in physics. The main aim of this change in hardware is to design a feasible system that extends to distant networks with minimum overhead, to significantly reduce the time and space complexity of the system which is practically applicable by the users of the network.

### III. QUANTUM COMPUTATION BASICS AND ENTANGLEMENT

Quantum computation is a theory based on quantum theory and today’s theory of computation. Below are some mathematical and physical tools that are used in this study for defining entangled states:

1) A qubit (equivalent of a bit in today’s computers) is defined as

\[
|\Psi\rangle = a|0\rangle + b|1\rangle
\]

where \(|0\rangle \) and \(|1\rangle \) are orthonormal bases for this system and \( a \) and \( b \) are complex numbers satisfying the equation \(|a|^2 + |b|^2 = 1\) where \(|a|^2\) and \(|b|^2\) are the probabilities for the \(|\Psi\rangle\) for evaluating to 0 or 1 upon measurement of that qubit. As it is seen from eq. \( 1\) a qubit is the superposition of both being 0 or 1 as a value before it is measured by a measurement device.

2) Any state \(|\Psi\rangle\) has a hermitian conjugate \(<\Psi|\) whose inner product is given as \(<\Psi|\Psi\rangle = 1\) when normalized. Note that \(<0|0\rangle = 1\rangle = 1\) and \(<0|1\rangle = 0|0\rangle = 0\) and

\[
|\Psi\rangle = a|0\rangle + b|1\rangle \Rightarrow <\Psi| = a^* <0| + b^* <1|
\]

and

\[
<\Psi|\Psi\rangle = |a|^2 + |b|^2 = 1
\]

3) Define a projection operator \( P \) for the bases \(|0\rangle\) and \(|1\rangle\) which has the properties

\[
P_0 = |0\rangle <0| \\
P_1 = |1\rangle <1| \\
P_0|0\rangle = |0\rangle \\
P_1|0\rangle = 0 \\
P_1|1\rangle = 0 \\
P_0 + P_1 = I
\]

\[
<0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

\[
<0| = \begin{pmatrix} 1 & 0 \end{pmatrix} \\
<1| = \begin{pmatrix} 0 & 1 \end{pmatrix}
\]
Note that projection operators have two eigenvalues which are either 0 or 1.

4) For any operator $A$ the expected value of that operation on a state is given as $\langle \Psi | A | \Psi \rangle = a \in \mathbb{R}$ where $a$ is the expected value of a measurement by a device.

5) Tensor product $\otimes$ is used to define an ensemble of two or more states (or qubits) with the following properties.

- For an ensemble of two states
  $$|\Psi > \otimes |\varphi > = a|0 > \otimes |0 > + b|0 > \otimes |1 > + c|1 > \otimes |0 > + d|1 > \otimes |1 >$$
  or shortly
  $$|\Psi \varphi > = a|00 > + b|01 > + c|10 > + d|11 >$$
  where $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$

- For an operator $A$ that is to be applied on $|\Psi >$ and operator $B$ to be applied on $|\varphi >$ we have
  $$(A \otimes B)(|\Psi > \otimes |\varphi >) = A|\Psi > \otimes B|\varphi >$$
  = $a|\Psi > \otimes b|\varphi >$

And this property applies for the expected value also.

After defining some mathematical tools we can develop the concept of entanglement on these. Entanglement was first proposed by Einstein, Podolsky and Rosen as a paradox that implies incompleteness of Quantum Theory. Entangled states are special types of states that are defined by only one state function even if there are more than one state. As an example, one can separate the ensemble $a|01 > + b|00 > = |0 > \otimes (a|1 > + b|0 >)$ and separately apply operators on each state but on an entangled state such as $\frac{1}{\sqrt{2}}(|00 > + |11 >)$, there is no way to separate these states. This immediately results in the conclusion that if two quantum states are entangled, and if one of them is measured by applying an operator, say $A$ (or more explicitly the operator $A \otimes I$), the result of a measurement by the same operator would be known without measurement even if they are many light years apart from each other. This situation, in turn, might seem to mean violating rules of relativity stating that information cannot be propagated faster than light and there is a paradox according to Einstein, Podolsky and Rosen. This paradox was resolved by Bell in 1960’s that these states really appear in nature and there is no violation of relativity since one should send the way of measuring one state if the other states are supposed to give the same information.

Later in 1990’s EPR states proved themselves to be very valuable, one can use them to create some type of quantum parallelism such that algorithms that are proved to be exponential in time in classical computers were evaluated in polynomial time in a quantum computer. Moreover entangled states enable to teleport a state from one place to another by simply evolving one entangled state in the sender’s side so that receiver’s state also evolves giving out the teleported state as a result.

Now, we will define the entangled states and the operators we will use in our discussion of networks. We will use the projection operators $P_0$ and $P_1$ and the entangled state $|\Psi > = \frac{1}{\sqrt{2}}(|00 > + |11 >)$. If we did the measurement on the sender’s qubit we will have

$$(P_0 \otimes I)|\Psi > = \frac{1}{\sqrt{2}}|0 > \otimes |0 > = |\varphi >$$

as a result of measurement on one entangled state. An expected value of $\frac{1}{2}$ will be the output as a result.

And the measurement on the other entangled state after this evolution gives

$$(I \otimes P_0)|\varphi > = \frac{1}{\sqrt{2}}|0 > \otimes |0 >,$$

the same result. Note that if $P_1$ is applied on $|\varphi >$ the result would be 0.

Therefore, we will fix the following convention. Assume that two nodes $A$ and $B$ in a network have corresponding entangled pairs with them. If the sender $A$ detects that it is completely isolated from the network it does a measurement on its qubit by the operator $P_0$ outputting an expected value of $\frac{1}{2}$ as a result of his measurement. When the receiver $B$ sees that no information is coming via classical channels it does an operation of $P_1$ on its entangled state. If the node $A$ is isolated node $B$ will get a 0 as a result, if $A$ is still connected to the network it does no operation on the system and $B$ will get a result of $\frac{1}{2}$ from the measurement. This can be also stated in terms of the base states that in this configuration you either get the two-bit result 00 from measurement of this two qubit system where the measurement result is nonzero or you get nothing which means a zero as output.

IV. APPLICATION OF ENTANGLEMENT TO THE COUNTING TO INFINITY PROBLEM

Entanglement may be applied in many different ways to a classical network. We will now study these applications, and then compare them with each other. During these comparisons, we will assume that the entangled states can be transported via a quantum channel so that nodes can share them.

One application may be implementing entangled states (there may be a lot of pairs shared between two nodes) between neighboring nodes of a selected node $A$ and the distant gateways. When that node $A$ goes down, neighboring node learns it and then sets its entangled state accordingly and the distant gateway periodically measures its entangled state. Since count to infinity procedure has already begun near the node $A$, this gateway
is aware of the situation and it may guarantee that the problem does not pass through it to outer networks and stays in a limited area so that if split horizon with poisoned reverse is still active the resolution of the problem becomes faster. This can also help the distance vector routing to be used in much larger networks since count to infinity problem is guaranteed to be limited in a smaller area if it occurs and split horizon with poisoned reverse can handle it up to some acceptable level. Here the infinity will be represented by the number of hops to reach the distant gateway that knows the situation at node A plus 1. This system requires periodical transportation of entangled pairs since the distant gateway periodically consumes the states it has by measuring them to learn if node A has detected any problems around itself. This scenario is so simple but even in this case the complexity of informing a distant node about a change in topology becomes in $O(1)$ time. Variants of this type of informing a change in topology may be continuously applied among nodes but this also adds an overhead of qubit exchange on the network. However, this exchange does not significantly affect the performance since we only send 0 or 1 data and qubits are independent of each other due to the fact that each of them is discretely entangled and this makes the quantum channel error tolerant, i.e. if one or two qubits are mistakenly measured on the way, they become useless but this does not affect the other qubits and this is not a significant source of unreliability.

Another application that is more sophisticated is the exchange of entangled qubits generated at a node while exchanging the distance vector routing data between neighbors. When one node updates its table, it also gets corresponding entangled qubits generated by its neighbors. This procedure can be explained on a simple network lined up on a straight line which has the morphology

$$A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow \cdots$$

In this system each node exchanges entangled bits for each entry of the distance vector routing table. This means that when a node gets data from a neighbor and decides that neighbor has the shortest path to some other node, it also gets entangled qubits generated by that neighboring node. When it needs entry exchange next time, it also measures the entangled bits it has for detecting any change in morphology if the neighboring node that has the other qubit of the entangled pair already detected it. Therefore, for the above network system each node exchanges entangled qubits for each of its entries and in each exchange time these qubits are measured. If node A goes down, it immediately measures its qubit with an appropriate operator and when node B requests data for routing table exchange, it gets no answer from node A and it immediately measures its entangled qubit corresponding to this entry and learns that node A is not reachable anymore. Then node B looks for another route for reaching node A and it detects that node C has a route to node A without detecting that node C’s route passed through it. Node B still remembers the result of its measurement state for entry A and when it exchanges entry data between node C it also sends some entangled qubits for the corresponding entry of node A to node C. And since node B knows from its former measurement that node A has gone down it immediately measures entangled qubits it has for entry A to state that node A has gone down. Note that at this time a loop has just begun and count to infinity problem has just showed itself, i.e. it occurred in the system. After a set of exchanges between neighboring nodes for the entry of node A, the node C again looks for its neighbors for a path to node A and before it gets entry data from node B, it measures the entangled bits it has for entry A and learns that node B says that node A is blocked on its side and any path data coming from node A for path B is not reliable. Therefore node C then learns that node A is not reachable via node B and looks for other neighbor for alternative paths to node A. In this way, during each exchange each node first measures its entangled qubits shared with its neighbor for an entry in routing table and according to this data it trusts in the metric its neighbor informs it for another node. Thus the system quickly and asynchronously (i.e., no periodical measurement of qubits, they are measured just before each exchange of routing data and generated and exchanged while exchanging the routing data) collapses to a stable system with its new network topology. The protocol defined above can be applied on more complex networks than the above example and it prevents the count to infinity problem in an efficient way when it occurs.

V. CONCLUSIONS

In this paper we briefly defined the distance vector routing algorithm characteristics and examined possible causes and results of count to infinity problem. We investigated the advantages, disadvantages and limitations of the mostly used methods to avoid count to infinity problem or recover from it. These methods are split horizon and poisoned reverse which can be used together also. The reduction of count to infinity problem to halting problem let us examine the problem from the point of computation complexities and a decision problem and we proposed to change the hardware to solve the problem in order to have a more robust network algorithm without significantly increasing the complexity of the distance vector routing algorithm. For this reason a statistical and nondeterministic theory of computation, the quantum computation theory is used to develop such algorithms. The novel states called entangled states enable the network node to communicate with themselves without depending on the network connectivity in the case of network topology change, which is possible if a measurement protocol of states is established. Moreover the time complexity of learning any change (or equivalently, making a measurement on an entangled state) between any node with any amount of distance between them is...
$O(1)$ in the case of a network topology change, which significantly increases the size and quality of service of the network on which distance vector routing can be applied.

[1] R. E. Bellman, *Dynamic Programming.* (Princeton University Press., 1957).
[2] L. R. Ford and D. R. Fulkerson, *Flows in Networks* (Princeton University Press, Princeton, 1962), ISBN 0-691-07962-5.
[3] C. Hedrick, *Request for comments: 1058*, can be found from web page www.ietf.org/rfc (1988).
[4] M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).
[5] P. W. Shor, SIAM Journal on Computing 26, 1484 (1997).
[6] D. Deutsch, in *Proc. Royal Society, A400, 97* (1985).