Ranking the market efficiency of the Philippine stock exchange industry sectors using the multifractal detrended fluctuation analysis

J M V Antenorcruz, M C F Manzano and R C Batac
Physics Department, De La Salle University, 2401 Taft Avenue, 1004 Manila, Philippines
jude_maria_antenorcruz@dlsu.edu.ph

Abstract. In this study, the market efficiency of the Philippine stock exchange industry sectors was ranked through the multifractal detrended fluctuation analysis wherein the generalized Hurst exponents for each sector were obtained. The difference between the minimum and maximum generalized Hurst exponents for each sector was used to measure market efficiency. For detrending polynomials of degree one, the following efficiency ranking was obtained, from most efficient to least efficient: properties, mining and oil, holding firms, industrial, finance and services. The results imply that since the services sector was found to be the least efficient sector, then investors do not react quickly to information entering the market, thus reflecting on the sector’s index. Conversely, the results also imply that the properties sector is the most efficient industry in the Philippine stock exchange. The multifractal detrended fluctuation analysis was also conducted for quadratic and cubic detrending polynomials, and different efficiency rankings were obtained, it was found that services and properties were still the most inefficient and efficient sectors, respectively. This study has provided additional evidence on the link between multifractality and market efficiency, and it has also been found that such a link still exists when considering industries under a country’s stock exchange.

1. Introduction
Studies have shown that there is a causal link between a country’s stock market development and its economic growth [1-3]. One way to promote stock market development is through the improvement of stock market efficiency, which is the extent in which prices of stocks would fully reflect all information about the market. One of the first formulations of efficient markets is the Efficient Market Hypothesis or the EMH. The EMH models pricing processes of assets as martingales, and this formulation eliminates the dependence of market efficiency on assumptions made by traditional asset-pricing models such as the random walk and the geometric Brownian motion [4]. Under the EMH, the pricing processes of assets are assumed to be efficient, in which the best guess of the next price of an asset can only be contained in the set of all past observations of the price and that current changes in the price of an asset is uncorrelated with previous price changes. The assumptions of the EMH do not consider the investment horizons of investors in the market. Realistically, some investors prefer long-term investments, whereas some investors prefer to take advantage of the short-term movements in the prices of assets. Thus, investors react to information entering the market depending on their investment horizons. If investment horizons are considered, then this implies that not all markets are perfectly efficient. In fact, studies have shown that empirical evidence is not consistent with the
assumptions of the EMH [5]. Thus, an alternative to the EMH was formulated, which is the Fractal Market Hypothesis or the FMH which considers varying investment horizons in markets [6,7]. Pricing processes under the FMH are modelled as multifractal series. A representative mathematical model of the FMH is the Multifractal Model of Asset Returns or the MMAR [8]. Under the MMAR, the memory of a financial time series is characterized by the Hurst exponent, which is defined, for a complete set of horizons $\tau$:

$$
H = \lim_{\tau \to \infty} \frac{\ln RS_H(\tau)}{\ln \tau}
$$

(1)

Where $H \in (0,1)$ and $RS_H$ is known as Hurst’s range scale statistic, which is defined for a time series $x(t)$ as:

$$
RS_H(\tau) = \frac{1}{c_2^{0.5}} \left[ \text{Max}_{1 \leq t \leq T} \sum_{i=1}^{\tau} \{x(t) - m_i\} - \text{Min}_{1 \leq t \leq T} \sum_{i=1}^{\tau} \{x(t) - m_i\} \right] \geq 0
$$

(2)

With a variance:

$$
c_2 = \frac{1}{T} \sum_{t=1}^{T} [x(t) - m_i]^2
$$

(3)

And mean:

$$
m_i = \frac{1}{T} \sum_{t=1}^{T} x(t)
$$

(4)

The series is long-range correlated for $H > 0.5$, for $H < 0.5$ it is anti-correlated and for $H = 0.5$ it is uncorrelated. A generalization of $H$ is called the generalized Hurst exponent $H_q = H(q)$ which is related to standard multifractal analysis [9]. One way to obtain the generalized Hurst exponents for a time series is through the Multifractal Detrended Fluctuation Analysis or the MF-DFA [9]. The MF-DFA has been used in several studies in which stock market efficiency was quantified. [10-13] In these studies, the difference between the minimum and the maximum generalized Hurst exponents for a certain composite index was interpreted as market efficiency:

$$
\Delta H_q = H(q_{\text{max}}) - H(q_{\text{min}})
$$

(5)

Most studies that used the MF-DFA to quantify market efficiency considered composite indices of countries. No studies have been done which used the same methodology on a more microscopic scale of stock markets, such as groups of individual firms like blue chip companies. Furthermore, studies conducted in the Philippines involving market efficiency have not used the MF-DFA or any multifractal approach [14]. In this study, the efficiency of the Philippine stock exchange industry sectors was measured and ranked using equation (5), in which the generalized Hurst exponents were obtained through the MF-DFA. The data used in this study are logarithmic returns obtained from daily closing prices of the finance, holding firms, industrial, mining and oil, properties and services industry sectors of the Philippine stock exchange from August 4, 2008 to August 3, 2018 which contains 2439 observations of daily closing prices in each sector. This range of data was chosen in order to consider the effects of long-term investment horizons of investors in the Philippine stock exchange.

2. Methodology

This study used the MF-DFA to obtain the generalized Hurst exponents for each sector, and it has the following steps [9]:

(i) Determine the profile $Y(i)$ where $x_i$ is the series of length $N$ and $\langle x \rangle$ is the mean of the series:

$$
Y(i) = \sum_{i=1}^{N} x_i - \langle x \rangle
$$

(6)
(ii) Divide the profile $Y(i)$ into $N = \lfloor N/s \rfloor$ non-overlapping segments of equivalent length $s$. The series of length $N$ does not need to be a multiple of the time scale $s$, thus a short part at the end of the profile will remain. To consider this short part of the series, $2N_s$ segments are considered in total, starting from the other end of the series.

(iii) The variance is then calculated for each segment by averaging over all data points $i$ in the $v^{th}$ segment, for $v = 1, \ldots, 2N_s$ to include the short part of the series, since the series of length $N$ is usually not a multiple of the time scale $s$. Here, $p_v(i)$ is the local trend in the $v^{th}$ segment of the profile which is determined through a least-squares fit of the data. The fitting polynomial $p_v(i)$ can be of order $m$ where $m = 1, 2, \ldots$, and trends of order $m$ are eliminated from the profile. In this study, the fitting polynomial $p_v(i)$ is called the detrending polynomial.

\[
F^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} [Y[N-(v-N_s)s+i]-p_v(i)]^2
\]

(iv) The MF-DFA fluctuation function is then obtained by averaging $F^2(v, s)$ over all segments, and this function increases for large values of $s$ as a power law:

\[
F_q(s) = \left( \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(v, s)]^{q/2} \right)^{1/q} \sim s^{H(q)}
\]

For $q = 0$, the fluctuation function is:

\[
F_0(s) = \exp \left( \frac{1}{4N_s} \sum_{v=1}^{2N_s} \ln[F^2(v, s)] \right) \sim s^{H(0)}
\]

(v) The natural logarithm of the fluctuation function increases linearly with the natural logarithm of the time scales $s$:

\[
\ln[F_q(s)] = \beta_q + H(q) \ln(s)
\]

The slope $H(q)$ is called the generalized Hurst exponent, which can be obtained through least-squares. The steps of the MF-DFA outlined above are repeated for different values of $q$ and scales $s$. The parameters $q \in [-20, 20]$ and $s \in \{24, 480\}$ with a step of $\Delta s = 4$ were used, following a set of guidelines in choosing these parameters in conducting the MF-DFA for financial time series [15]. A pre-processing algorithm for the MF-DFA [15] was also employed, to determine the best detrending polynomial of order $m$, and it was found that detrending polynomials of order $m = 1$ is most suitable to use for all industry sectors. This study has also considered quadratic and cubic detrending polynomials, to compare any differences in the results and any possible implications of using different orders of the detrending polynomial. Once the generalized Hurst exponents were obtained for each industry sector, equation (5) was used as a measure of market efficiency, wherein $H(q_{min}) = H(-20)$ and $H(q_{max}) = H(20)$.

3. Results and Discussion

The MF-DFA was conducted on the finance, holding firms, industrial, mining and oil, properties and services industry sectors of the Philippine stock exchange. As mentioned in the previous section,
detrending polynomials of orders $m = 1, 2, 3$ were considered and the appropriate parameters were used. The generalized Hurst exponents of all industry sectors were obtained from the MF-DFA.

Figure 1. The generalized Hurst exponents, $H(q)$, against $q$ for all industry sectors, after conducting the MF-DFA for $m = 1$ (a), $m = 2$ (b) and $m = 3$ (c). The top three least multifractal sectors have light blue markers whereas the top three most multifractal sectors dark blue markers.

Figure 1 shows the generalized Hurst exponents obtained from the MF-DFA using detrending polynomials of order $m = 1, m = 2$ and $m = 3$, respectively. A higher variability in $H(q)$ corresponds to a higher degree of multifractality. Graphically, it can be observed that there is only a slight difference in the generalized Hurst exponents for all sectors when detrending polynomial orders $m = 1$ and $m = 2$ are used in the MF-DFA. However, when detrending polynomial order $m = 3$ is used in the MF-DFA, the obtained generalized Hurst exponents differ greatly, relative to the generalized Hurst exponents obtained when detrending polynomial orders $m = 1$ and $m = 2$ are used in the MF-DFA. The difference between the minimum and maximum generalized Hurst exponents for each industry sector was then obtained through equation (5). These are shown in table 1 below.

Table 1. Each industry sector’s $\Delta H$ for detrending polynomial orders $m = 1, 2, 3$.

| Detrending polynomial order | Finance $\Delta H$ | Holding firms $\Delta H$ | Industrial $\Delta H$ | Mining and oil $\Delta H$ | Properties $\Delta H$ | Services $\Delta H$ |
|----------------------------|------------------|-----------------|------------------|------------------|------------------|------------------|
| $m = 1$                    | 0.2641           | 0.1973         | 0.2299           | 0.1774           | 0.1662           | 0.4284           |
| $m = 2$                    | 0.2529           | 0.2243         | 0.2781           | 0.2025           | 0.1725           | 0.4644           |
| $m = 3$                    | 0.3717           | 0.3636         | 0.5039           | 0.5235           | 0.3138           | 0.5289           |

The calculated values for $\Delta H$ in table 1 are consistent with what is shown in figure 1, since the $\Delta H$ values in all sectors for $m = 1$ and $m = 2$, when compared, only have slight differences, whereas the $\Delta H$ values for $m = 3$ are much larger. The quantity $\Delta H$ can be interpreted as the market efficiency of an industry sector, since, for a monofractally uncorrelated time series such as a random walk process or a martingale, the generalized Hurst exponents should not vary [9], and therefore $\Delta H$ should
be equal to zero. In this study, we have found that all values for $\Delta H$ are non-zero for all sectors, thus all sectors are inefficient. An efficiency ranking was then constructed, wherein smaller values of $\Delta H$ correspond to higher market efficiency and larger values of $\Delta H$ correspond to lower market efficiency. Thus, for detrending polynomials of order $m = 1$, the efficiency ranking is as follows, from most efficient to least efficient: properties, mining and oil, holding firms, industrial, finance and services. For detrending polynomials of order $m = 2$, we have: properties, mining and oil, holding firms, finance, industrial, and services. Finally, for detrending polynomials of order $m = 3$ we have: properties, holding firms, finance, industrial, mining and oil and services. Different efficiency rankings may be obtained for higher detrending polynomial orders, however, the best order $m$ to use in this study is that of $m = 1$, as determined by the pre-processing algorithm for the MF-DFA [14]. Thus, the results obtained by using detrending polynomials of order $m = 1$ gives the most accurate efficiency ranking of the industry sectors.

4. Conclusion
The efficiency ranking obtained through the MF-DFA shows that the most efficient sector in the Philippine stock exchange is the properties sector, whereas the least efficient sector is the services sector, as shown in the previous section. It can be observed that even when cubic detrending polynomials were used, these two sectors are still the most efficient and least efficient sectors, respectively. To explain why the services sector was found to be the least efficient sector in the Philippine stock exchange, we recall that market efficiency is a concept which deals with information entering the market. In a perfectly efficient market, investors should react quickly to information entering the market and this would reflect on the prices of assets in the market. Now, since it was found that the services sector is the least efficient industry sector, investors in this sector must have long-term investment horizons, which means they would not react quickly to information entering the market, thus contributing to the sector’s inefficiency. This finding is consistent with the fact that the Philippine economy’s services sector is known to be one of the country’s largest industries. It follows therefore, that the efficiency of the properties sector can also be explained by the short-term investment horizons of most investors in this sector. Due to these findings, this study has provided additional support for the fractal market hypothesis, since a link between multifractality and market efficiency was still found for a stock market’s individual sectors. We note, however, that the efficiency ranking for the mining and oil, finance, holding firms and industrial sectors were found to be different for all detrending polynomial orders used, thus, it is recommended that future studies further investigate these sectors. It is also recommended that future studies investigate if evidence for the fractal market hypothesis can still be found for the case of individual stocks, and if possible, construct a model that provides a clear analytical link between multifractality and market efficiency.

References
[1] Enisan A A and Olufisayo A O J. Econ. Bus. 32 316–24
[2] Marques L M, Fuihhas J A and Marques A C 2013 Econ. Model. 32 316–24
[3] Pan L and Mishra V Econ. Model. 68 661–73
[4] Malkiel B G and Fama E F 1970 J. Finance 25 383–417
[5] Mantegna R N and Stanley H E 2016 An introduction to econophysics: correlations and complexity in finance (Cambridge, UK: Cambridge University Press) chapter 2
[6] Peters E E 1994 Fractal market analysis: applying chaos theory to investment and economies (New York: Wiley) chapter 3
[7] Bierman H 1997 J. Portfolio Manag. 23 51–55
[8] Mandelbrot B B, Fisher A and Calvet L 1997 A multifractal model of asset returns (New Haven, CT: Cowles Foundation for Research in Economics at Yale University)
[9] Kantelhardt J W, Zschiegner S A, Koscienly-Bunde E, Halvin S, Bunde A and Stanley H 2002 Physica A 316 87-114
[10] Zunino L, Tabak B, Figliola A, Prez D, Garavaglia M and Rosso O 2008 Physica A 387 6558-6566
[11] Yang L, Zhu Y and Wang Y 2016 Physica A 18 271-276
[12] Yuan Y, Zhuang X and Jin X 2009 *Physica A* **388** 2189-2197
[13] Rizvi S A R, Dewandaru G, Bacha O I and Masih M 2014 *Physica A* **407** 86-99
[14] Aquino R Q 2006 *Appl. Econ. Lett.* **13** 463-470
[15] Thompson J R and Wilson J R 2016 *Math. Comput. Simul.* **126** 63-88