Bounds for Energy Efficient Survivable IP over WDM Networks with Network Coding

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Abstract—In this work we establish analytic bounds for the energy efficiency of the 1+1 survivable IP over WDM networks using network coding. The analytic bounds are shown to be in close agreement with our previously reported results. They provide verification of the MILP and heuristics proposed previously and an efficient, compact means of evaluating the network results, and allow the performance of large networks to be determined easily.

Index Terms—Energy Efficiency; IP/WDM; MILP model; Analytic Bounds; Network coding.

I. INTRODUCTION

AFTER the introduction of network coding for the first time in [1], the contributions of network coding (NC) to various networking domains have accelerated, demonstrating the potential it has to improve the networking throughput. The work in optical networks, however, has been incomparable to its wireless counterpart due to the multicast nature of the wireless medium that is not inherent in the optical medium. The ability of network coding to reduce the overall traffic in the network, and therefore improve the network throughput, provides a motivation for using network coding to achieve energy efficiency by requiring less operating resources than the conventional approach. The benefits of introducing network coding (NC) in optical networks to improve robustness and efficiency has been reported in [2], [3], and [4]. In our previous work [5] and [6] we studied the energy efficiency gained by implementing network coding in non-bypass and bypass core networks by performing an XOR (Exclusive OR) operation on the bidirectional flows of unicast connections. Network coding elevates the traditional functionality of network nodes to incorporate coding operations on traffic flows and hence, by mixing signals at specific nodes rather than duplicating the signals end to end, more efficient network resource utilization can be achieved.

In [7] the authors provided a 1 + N network coding protection scheme, and through integer linear programmes and simulation they showed that significant cost savings over the 1 + 1 approach can be achieved. Network coding was proposed in [8] and [9] as a technique to improve protection in 1 + N protection schemes that employ p-cycles. The p-cycles are used to protect multiple bidirectional link-disjoint connections, which are also link disjoint from the p-cycle links. In [10] network coding is used to provide protection against node failures by reducing the problem to a problem of multiple link failures as a consequence of the node failure. In [11] it is shown that for networks with multiple subdomains, network coding can be used to enable the network to survive any node or link failure in each subdomain. The study of 1 + 1 protection schemes with network coding was reported in [12] through an integer non linear programme. This study is limited however, to equal traffic demands between different sources, provides results that are considerably lower than those achievable through network coding, and constrains the network coding only to nodes with nodal degree greater than or equals to 3. Our work is different in that it focuses on the widely implemented 1+1 protection scheme where it provides optimal and thorough solutions to protection with network coding focusing on improving the energy efficiency of the network. As far as our knowledge goes, no practical implementation is found yet in the industry, but we are optimistic with the considerable resource savings that network coding can achieve, the implementation will follow.

Energy efficiency in IP over WDM network has attracted a considerable attention from the research community driven by economic and environmental impact. The exponential growth of data intensive applications and the increasing number of Internet connected devices necessitates a shift in the way the network is designed and operated. As a global effort to tackle the energy consumption challenge in ICT, the GreenTouch consortium of leading expert in Industry and academia was formed in 2010 with the goal of achieving a 1000x energy efficiency improvement in 2020 compared to 2010 levels. The GreenTouch results for the core network are reported in [20]. A good survey of some of the techniques for energy efficiency in core networks can be found in [13], [14]. We used MILP models and heuristics in our previous work to improve energy efficiency in IP over WDM networks considering renewable energy sources [15], studying core networks with data centers [16], optimising physical topology [17], reducing traffic through distributed clouds [18], optimum design for future high definition TV [19], optimal P2P content distribution [20] and virtual network embedding [21]. We introduced network coding for energy efficient IP over WDM networks in [5] and [6], by encoding bidirectional flows using an XOR operation, and presented a thorough study of the use of network coding to improve the energy efficiency in core networks in unicast settings [22].

In our previous work [23] we proposed and designed a 1+1 protection scheme for core networks with network coding and optimized the network coding allocation and network operation using a mixed integer linear program and heuristics, providing encouraging results for energy efficiency improvement of up to 37% compared to the conventional 1+1 protection scheme. In this work we complement our study and analysis by deriving for the first time analytical bounds and close form...
expressions for the network coding as well as the conventional 1+1 protection scheme which verify the MILP and heuristic results in [23] and enable the performance of large networks to be determined easily. We also study large network sizes that are highly complex using the MILP approach, as well as provide a detailed study on the special full mesh and ring topologies.

The remainder of this paper is organized as follows: Section II provides the analytical bounds for the conventional and network coded core networks with 1+1 protection. In Section III we derive the bounds for regular topologies. Finally the paper is concluded in Section IV.

II. 1 + 1 PROTECTION WITH NETWORK CODING

Consider the network coding scheme where an example is shown in Fig. 1, representing a comparison between the conventional (Fig. 1a) and the network coded (Fig. 1b) 1+1 protection scheme in an arbitrary topology. Consider the two demands (3, 11) and (2, 11) originating from node 3 and 2 respectively and sharing the same destination. With the conventional 1+1 protection scheme (Fig. 1a) both demands use a single wavelength (\(\lambda_1\)) for the working path and each working path traverses three links. Since they share the 4 links of their protection paths, they are forced to use different wavelengths, (\(\lambda_1\)) for the demand (2, 11) and (\(\lambda_2\)) for demand (3,11), using a total of 16 wavelengths in the network, considering all links and paths.

Our proposed network coding scheme is shown in (Fig. 1b), where the protection paths use the same wavelength (\(\lambda_1\)) after being encoded at node 1, and later decoded at the destination node (i.e. 11). The benefits of using such a scheme are in reducing the total number of distinct wavelength used in the network, where in this case only the wavelength (\(\lambda_1\)) is used, as well as reducing the total number of wavelengths in the network: in this example from 16 to 12. A reduction of 25% in total resources, and 40% in protection resources (6 wavelengths instead of 10).

The resource savings and therefore the power consumption reduction in this scheme depends on the network topology, the location and number of network coding enabled nodes as well as the nature of demands. In our previous paper [23] we studied the scheme and determined the optimum allocation using a MILP optimization model followed by 5 heuristics. The real time and most optimal version of the heuristics is called the Optimal search heuristic (OSH). The heuristic reduces the size of the problem by dividing the set of encodable paths into clusters and searching for optimal coding operations on these clusters rather than the whole network. The working and protection paths are found using the Suurbelle algorithm. Four other suboptimal but faster heuristics are found by limiting the search for a suitable encoding pair. For each pair of demands there exists 4 paths in total: a working path and protection path for each. This produces 4 combinations for encoding, and therefore 4 possible specific heuristics, we call them w-w, p-p, w-p and p-w where the letters w and p designate the working and protection path respectively. The heuristics assume a distributed approach to determine the encoding decision. The network state can be communicated using the conventional routing protocol mechanism to exchange network state. For higher order codes and coding more than two paths, a centralized control may have more value. It would be interesting to consider the SDN and virtualization ideas presented in [24] and [25] for optical networks generally and for protection and failure recovery specifically as a future area of investigation.

In this work we complement our work by developing closed form expressions and analytic bounds for the power consumptions as a function of the hop count, the network size and the demand volume. We also study regular topologies and study scenarios that proved too complex for the MILP approach such as large network sizes.

III. ANALYTIC BOUNDS

The power consumption of the network is calculated as the sum of the power consumption of individual components. This approach is used by the research community and adopted by the GreenTouch consortium [26]. To simplify the analytical formulas, we consider the network devices components to have a linear power profile where the power consumption of a device is proportional to the traffic served. Other power profiles exist and have been investigated in our previous work [15], such as the on-off, cubic and log10 profiles.

The total power consumption of the survivable optical networks with network coding for this scheme is given by

\[
P = \left(\frac{P_p + P_t}{B}\right) \sum_{d \in D} \sum_{m,n} V^d \left(x_{mn}^d + y_{mn}^d\right) - \left(\frac{P_p + P_t}{B}\right) \sum_{d_1,d_2} \sum_{m,n} \min(V^{d_1}, V^{d_2}) \frac{d_{1,m}^d d_{2,m}^d}{2},
\]

where the first term is the total power consumption of the network operating without network coding, and the second term is the power consumption reduction achieved by network coding. \(P_p, P_t\) are the power consumption of a router port and a transponder in Watts, \(B\) is the capacity of a wavelength in
Gbps, $V^d$ is the volume of demand $d$ in Gbps, $D$ is the set of demands. The variable $x^d_{m,n}$ is a Binary variable, $x^d_{m,n} = 1$ if the working path of demand $d$ is routed over link $(m,n)$, and $x^d_{m,n} = 0$ otherwise. The variable $y^d_{m,n}$ is the equivalent of $x^d_{m,n}$ for protection paths. $\beta^d_{mn}$ is a binary variable, $\beta^d_{mn} = 1$ if demand $d$ is encoded with demand $d_2$ on link $(m,n)$, and $\beta^d_{mn} = 0$ otherwise. Note that the values of power consumption ($p_0$ and $p_t$) count for the OEO conversion at all nodes. The routing from source to destination uses the optical non-bypass approach where all intermediate nodes have OEO conversion. The power consumption contributions of the XOR operations, and of the EDFAs have not been included as their associated power consumption is low and to simplify the expressions. According to the GreenTouch core network energy efficiency study [26], the EDFAs power consumption constitutes a small portion of the overall power consumption compared to the routers and transponders power consumption. For the 2010 values the EDFAs consume 5% of the overall power consumption, while for the predicted business as usual values for 2020 they consume less than 2% at port speeds of 400Gbps.

Let the expression given in (1) be divided into its two summable components, which we refer to as $P_1$ and $P_2$ (i.e. $P = P_1 - P_2$), such that

$$P_1 = \left( \frac{p_p + p_t}{B} \right) \left\{ \sum_{d \in D} \sum_{m,n} V^d (x^d_{m,n} + y^d_{m,n}) \right\}, \quad (2)$$

$$P_2 = \left( \frac{p_p + p_t}{B} \right) \left\{ \sum_{d_1, d_2 \in D, m,n} \min(V^{d_1}, V^{d_2}) \left( \frac{\beta^{d_1}_{d_2}}{2} \right) \right\}. \quad (3)$$

The value $P_1$ represents the power consumption of the baseline conventional 1+1 protection approach, while $P_2$ is the reduction as a result of using network coding. Expression (2) can be rewritten as

$$P_1 = \left( \frac{p_p + p_t}{B} \right) \left\{ \sum_{d \in D} V^d \sum_{m,n} (x^d_{m,n} + y^d_{m,n}) \right\}. \quad (4)$$

Given the fact that the sum of the hop count of the working and protection paths of a given demand is always greater than or equal to twice the minimum hop count $h^d_{min}$ of the path serving it, that is

$$\sum_{m,n} (x^d_{m,n} + y^d_{m,n}) \geq 2h^d_{min}, \quad (5)$$

therefore $P_1$ can be written as

$$P_1 \geq \left( \frac{p_p + p_t}{B} \right) \left\{ \sum_{d \in D} 2V^d h^d_{min} \right\}. \quad (6)$$

Assuming that all demands are routed through the minimum hop count path of the network, i.e. $h^d_{min} = h_{min}, \forall d \in D$, we then have

$$P_1 \geq \left( \frac{p_p + p_t}{B} \right) h_{min} \left\{ \sum_{d \in D} 2V^d \right\}, \quad (7)$$

which gives

$$P_1 \geq 2\left( \frac{p_p + p_t}{B} \right) N(N - 1) V h_{min}. \quad (8)$$

Expression (8) represents a lower bound on the power consumption of the first component of the total power consumption of the network coded case as a result of routing traffic flows in the working and the protection paths, without the network coding component. It also represents the lower bound on the power consumption of the conventional case, which we refer to as $P_0$, where

$$P_0 \geq 2\left( \frac{p_p + p_t}{B} \right) N(N - 1) V h_{min}. \quad (9)$$

The upper bound of $P_2$ is found by starting from the fact that the minimum volume of two demands is never greater than their average, that is

$$\min(V^{d_1}, V^{d_2}) \leq \frac{V^{d_1} + V^{d_2}}{2}, \quad (10)$$

then (3) becomes

$$P_2 \leq \left( \frac{p_p + p_t}{B} \right) \left\{ \sum_{d_1, d_2 \in D, m,n} V^{d_1} + V^{d_2} \left( \frac{\beta^{d_1}_{d_2}}{2} \right) \right\}. \quad (11)$$

The expression $\min(V^{d_1}, V^{d_2})$ has its highest value when the maximum traffic is equal to the minimum traffic, therefore the equality in (10) is met when $V^{d_1} = V^{d_2} = V^{d_1, d_2}$. In this case (12) becomes

$$P_2 \leq \left( \frac{p_p + p_t}{B} \right) \left\{ \sum_{d_1, d_2 \in D, m,n} V^{d_1, d_2} \left( \frac{\beta^{d_1}_{d_2}}{2} \right) \right\}. \quad (13)$$

The expression $\sum_{m,n} \beta^{d_1}_{d_2}$ represents the number of shared links between the demand pair $(d_1, d_2)$. We refer to this value as $h^d_{d_2}$, where

$$h^d_{d_2} = \sum_{m,n} \beta^{d_1}_{d_2}. \quad (14)$$

Therefore equation (13) becomes:

$$P_2 \leq \left( \frac{p_p + p_t}{B} \right) \left\{ \sum_{d_1, d_2} V^{d_1, d_2} h^d_{d_2} \right\}. \quad (15)$$

Considering the lower bound of the component $P_1$ in (6) and the upper bound of the component $P_2$ in (15), we can get a lower bound on the total power consumption by combining the two, since $P = P_1 - P_2$, minimising $P$ is achieved by minimising $P_1$ and maximising $P_2$. The total power is then lower bounded by the following

$$P \geq \left( \frac{p_p + p_t}{B} \right) \left\{ 2 \sum_{d \in D} V^d h_{min} - \frac{1}{2} \sum_{d_1, d_2} V^{d_1, d_2} h^d_{d_2} \right\}. \quad (16)$$

The result in (16) analytically confirms our heuristic in (23) that can provide close to optimal solution. The heuristic produces a good solution by employing the following principles
• Select the minimum number of hops for the working and
  protection paths (minimising the first term of (16))
• Encode a demand with another demand that has the highest
  number of shared hops and closest demand volume
  (maximising the second term of (16))
• More weight is given to finding minimal hop paths than
  searching for better encoding pair (from the equation, the
  weight ratio of the first to second terms is 4:1)
• Three heuristics can be conceived. The first finds encod-
  able pairs by searching only for the highest link sharing,
  the second searches for the demand with the closest traffic
  volume, and a better heuristic searches for the highest
  sharing and closest traffic volume, at the expense of
  increased complexity. The first heuristic approaches the
  performance of the third the smaller the traffic variation
  becomes.

The bound (16) can be reduced by setting \( V^d = V^{d,d_2} \), which gives

\[
P \geq \left( \frac{p_p + p_t}{B} \right) \sum_{d \in D} V^d \left( 2h_{d_{\text{min}}}^d - \frac{1}{2} \sum_{d_2} h_{d_2}^d \right),
\]

(17)

Since each demand is constrained to be encoded with a maximum of a single other demand only, which is expressed as

\[
\sum_{d_2 \in D} h_{d_2}^d \leq 1.
\]

(18)

Therefore we let the value \( \hat{h}^d = \sum_{d_2} h_{d_2}^d \) represent the amount of shared links (hops) between demand \( d \) and the demand it is encoded with. Equation (17) can then be reduced to

\[
P \geq \left( \frac{p_p + p_t}{B} \right) \left( \sum_{d \in D} V^d \left( 2h_{d_{\text{min}}}^d - \hat{h}^d \right) \right),
\]

(19)

which is equal to

\[
P \geq 2\left( \frac{p_p + p_t}{B} \right) \left( \sum_{d \in D} V^d \left( h_{d_{\text{min}}}^d - \frac{\hat{h}^d}{4} \right) \right),
\]

(20)

If we define the variable \( \hat{h}^d \) as the characteristic hop count for demand \( d \), such that

\[
\hat{h}^d = h_{d_{\text{min}}}^d - \frac{\hat{h}^d}{4},
\]

(21)

then the lower bound of the total power becomes

\[
P \geq 2\left( \frac{p_p + p_t}{B} \right) \left( \sum_{d \in D} V^d \hat{h}^d \right).
\]

(22)

Using Chebyshev Sum Inequality, i.e.

\[
\frac{1}{n} \sum_{k=1}^{n} a_k b_k \geq \left( \frac{1}{n} \sum_{k=1}^{n} a_k \right) \left( \frac{1}{n} \sum_{k=1}^{n} b_k \right),
\]

(23)

then, (22) can be written as

\[
P \geq 2\left( \frac{p_p + p_t}{B} \right) \left( \frac{1}{N(N-1)} \sum_{d \in D} V^d \sum_{d \in D} \hat{h}^d \right),
\]

(24)

which gives

\[
P \geq \frac{2(p_p + p_t)}{B} V \sum_{d \in D} \hat{h}^d,
\]

(25)

where \( V = \sum_{d \in D} V^d \) is the average demand volume. Defining \( \hat{h} = \sum_{d \in D} \hat{h}^d \) as the average characteristic hop count, then

\[
P \geq \frac{2(p_p + p_t)}{B} N(N-1) \hat{h}
\]

(26)

The lower bound given in (26) bears resemblance to the lower bound of the conventional case in (9), where the minimum hop count of the conventional case \( h_{d_{\text{min}}} \) is replaced by the characteristic minimum hop count of the network coding case \( \hat{h} \).

**IV. Regular topologies**

In the previous work [23] we established that the star and the line topologies exhibits no network coding benefits as the concept of protection does not apply. Here we develop formulas and bounds for the full mesh and ring topologies for the case where protection paths are encoded together, and study the impact of the network size on the performance of network coding.

**A. Full mesh topology**

The total power consumption under conventional protection is given by

\[
P_0 = \left( \frac{p_p + p_t}{B} \right) \left( \sum_{d \in D} \sum_{m,n} V^d (x_{mn}^d + y_{mn}^d) \right)
\]

(27)

For the full mesh topology, the optimal paths are the direct path (a single hop) for the working path, and a path with an intermediate node for the protection path (2 hops). This means \( \sum_{m,n} x_{mn}^d = 1 \) and \( \sum_{m,n} y_{mn}^d = 2, \forall d \in D \). Therefore

\[
P_0 = \left( \frac{p_p + p_t}{B} \right) \left( \sum_{d \in D} 3V^d \right)
\]

(28)

which can be written as

\[
P_0 = 3\left( \frac{p_p + p_t}{B} \right) VN(N-1).
\]

(29)

For the network coded approach the network power consumption is given by

\[
P = P_0 - \left( \frac{p_p + p_t}{B} \right) \sum_{d_1,d_2,m,n} \min(V_{d_1},V_{d_2}) \frac{\beta_{d_1,d_2}}{2},
\]

(30)

which can be reduced to the following, given that equal traffic demands that produces the highest savings

\[
P = P_0 - \left( \frac{p_p + p_t}{B} \right) V \sum_{d_1,d_2,m,n} \beta_{d_1,d_2}.
\]

(31)

\[
P = P_0 - \left( \frac{p_p + p_t}{B} \right) V \sum_{d_1,d_2,m,n} \beta_{d_1,d_2}.
\]

(32)

Since the number of encodable pairs in each cluster in the encodable graph depends on the total number of nodes in the network, the total number of encoded nodes depends on
the network size. This is illustrated in Fig. 3 for full mesh topologies of size 4 (clusters of size 3), 5 and 6 nodes respectively. If the network has an even number of nodes, then each cluster in the encoded graph will have an odd number of demands (i.e. each receiving node in the network will have demands from \(N-1\) nodes, \(N\) is even and hence each cluster has an odd number of demands). With an odd number of demands, one demand cannot be paired and hence cannot be network coded and is therefore transmitted using conventional router ports and transponders. This leads to a higher power consumption compared to a network with an odd number of nodes. In the latter case (network with an odd number of nodes) each cluster has an even number of demands, therefore all demands can be encoded leading to higher power savings. As such the odd and even cases have to be treated separately. For the full mesh topology with an odd number of nodes (e.g. 5 nodes network, 4 cluster nodes), any two encodable demands have a single hop shared between them (recall the working path for the full mesh is a single hop, and the protection path is 2 hops), therefore

\[
\sum_{d_1,d_2,m,n} d_{mn} = N(N-1).
\]  

Therefore the total power consumption for the network coded case is

\[
P = \left(\frac{p_p + p_t}{B}\right) N(N - 1) \frac{5V}{2}.
\]  

Therefore the total savings is given by

\[
\phi_{\text{odd}} = 1 - \frac{(p_p + p_t)N(N - 1)\frac{5V}{2}}{3(p_p + p_t)VN(N - 1)} = 0.166
\]  

which means the savings are upper bounded by a value of 16.67%.

For the full mesh topology that has an even number of nodes, each cluster will have an odd number of encodable demands, which means that a single encodable node (demand) will not be encoded due to the pairing of all other demands, making the number of encodable demands \(N-2\), in each of the \(N\) clusters. This fact makes the power savings for the even case less than the power savings of the odd case in (35). With \(N\) clusters, and \(N-2\) encodable demands in each cluster, the total number of shared hops is given by

\[
\sum_{d_1,d_2,m,n} d_{mn} = N(N-2).
\]  

The total power consumption of the even case of the full mesh topology under network coding becomes

\[
P = 3\left(\frac{p_p + p_t}{B}\right) VN(N - 1) - \left(\frac{p_p + p_t}{B}\right) V\frac{N(N - 2)}{2},
\]  

which gives

\[
P = \left(\frac{p_p + p_t}{B}\right) VN \left(\frac{5N - 4}{2}\right).
\]  

Therefore, the power saving is given by

\[
\phi_{\text{even}} = 1 - \frac{(p_p + p_t)VN\left(\frac{5N - 4}{2}\right)}{3(p_p + p_t)VN(N - 1)}.
\]  

which leads to

\[
\phi_{\text{even}} = \frac{N-2}{6(N-1)}.
\]  

From equations (35) and (40), we can see that the power consumption fluctuates between the upper value (i.e. 16.67%) when the number of nodes is odd, and the value given by equation (40) with an even number of nodes. These fluctuations, however, decrease as the number of nodes grows making the network power consumption converge to 16.67% for any number of nodes. This decrease in fluctuations follows the inverse of the number of nodes and is given by

\[
e(N) = \frac{1}{6} - \frac{N-2}{6(N-1)} = \frac{1}{6(N-1)},
\]  

and for a very large number of nodes

\[
\lim_{N \to \infty} e(N) = \lim_{N \to \infty} \frac{1}{6(N-1)} = 0.
\]  

Fig. 5 Shows a comparison between the power consumption between the MILP, analytical and the OSH heuristic for the 5 nodes full mesh topology and compares them with the conventional MILP scenario. It clearly shows that the analytical results matches the MILP results. It also shows a linear dependency between the power consumption and the average demand volume, as can be seen from equation (34) when the number of nodes \(N\) is fixed for a given network. This slope of the curve is given as \((\frac{p_p + p_t}{2}N(N - 1))\). We show in Fig. 4 the power savings of full mesh topologies with a number of nodes ranging from 3 nodes up to 15 nodes. The concept of multi path protection and therefore the concept of network coded protection doesn’t apply to networks with number of nodes below 3. It is obvious that encoding both working flows together produces no savings as both working flows use the direct link between each node in the network which is not shared with the direct link of a working path of another demand. It also shows that the Optimal Search Heuristic (OSH) [23] is superior, while the form of the heuristic that encodes protection paths together produces optimal savings at even network sizes. The savings of the optimal heuristic jumps increasing and decreasing as the network size changes between odd and even number of nodes, agreeing with the analytical formulas, but overall converges to the maximum possible savings value (i.e. 16.67%).

B. Ring topology

The power consumption of the conventional 1+1 protection of the ring is given as

\[
P_0 = \left(\frac{p_p + p_t}{B}\right) \left(\sum_{d \in D, m,n} V^d(x_{mn}^{d} + y_{mn}^{d})\right).
\]  

The total count of working hops for the odd number of nodes is given as

\[
h_w = \sum_{d \in D, m,n} x_{mn}^{d} = 2N \left(1 + 2 + ... + \left(\frac{N - 1}{2}\right)\right) = \frac{(N-1)N(N+1)}{4}.
\]
Since each working path of length \( k \) has a protection path of length \( N - k \) in the other direction, this makes the total number of protection hops for the case of odd number of nodes

\[
h_p = \sum_{d \in D} \sum_{m,n} y_{mn}^d
\]

\[
= 2N \left( N - 1 + N - 2 + \ldots + N - \frac{N-1}{2} \right)
\]

\[
= N(N-1)(\frac{3N-1}{4}), \quad (45)
\]

and the total number of hops of both working and protection paths for the odd number of nodes is given as

\[
\sum_{d \in D} \sum_{m,n} (x_{mn}^d + y_{mn}^d)
\]

\[
= \frac{(N-1)N(N+1)}{4} + N(N-1)(\frac{3N-1}{4})
\]

\[
= N^3 - N^2. \quad (46)
\]

For the case of even number of nodes, the number of working hops is

\[
h_w = \sum_{d \in D} \sum_{m,n} x_{mn}^d
\]

\[
= 2N \left( 1 + 2 + \ldots + (\frac{N-2}{2}) \right) + N \frac{N}{2}
\]

\[
= \frac{N^3}{4}, \quad (47)
\]

and the number of protection hops is given by

\[
h_p = \sum_{d \in D} \sum_{m,n} y_{mn}^d
\]

\[
= 2N \left( N - 1 + N - 2 + \ldots + N - \frac{N-2}{2} \right) + N \frac{N}{2}
\]

\[
= \frac{N^2(3N-4)}{4} \quad (48)
\]

and the total number of hops of both working and protection

\[
h_{wp} = 2N \left( \frac{N-2}{2} - 1 - 2 - \ldots - \frac{N-2}{2} \right) + \frac{N^2}{2}
\]

\[
= \frac{N^2(3N-4)}{4} \quad (49)
\]
paths for the even number of nodes is given as

$$\sum_{d=0}^{\infty} \sum_{m,n} (x_m^d + y_m^d) = \frac{N^3}{4} + \frac{N^2(3N - 4)}{4} = N^3 - N^2$$

(50)

This expression is for the conventional case and is the same for rings of odd and even number of nodes (i.e. (56) is the same as (50)).

1) Rings with odd size: We start with the case of a ring with odd number of nodes, as shown in Fig. 5. The figure shows a ring with 11 and 13 nodes where all nodes send to node 11 and 13, respectively. To maximise the number of shared links, protection paths are encoded together so the longest protection path is encoded with the second longest protection path and so on, leading to a number of shared hops that is equal to the number of hops of the shorter protection path. This is shown in Fig. 5, where we pair the source nodes of demands that can be encoded. Fig. 5 shows that the demands (1, 11) and (2, 11) are encoded together, where demand (1, 11) has a protection path with a length of 10 hops and demand (2, 11) has a protection path of 9 hops, leading to 9 shared hops. The same principle applies between demands (3, 11) and (4, 11) leading to 7 shared hops, which is equal to the length of the protection path of demand (4, 11). The same applies to demands [(10, 11), (9, 11)] and [(8, 11), (7, 11)]. As node 5, and node 6 do not share a protection path because they send their protection signals in opposite directions, they do not get encoded together.

The second example of a 13 nodes ring, shows that all nodes can find another node to be paired with. Therefore, compared to the 11 nodes ring, better savings are achieved. As a result, the power savings obtained under network coding go up and down as the number of nodes in the odd ring changes between the odd number where $\frac{N - 1}{2} \mod 2 = 1$, classified as odd-1, and the odd number where $\frac{N - 1}{2} \mod 2 = 0$, which is classified as odd-2. For example, when $N = 11$, we have $\frac{11 - 1}{2} \mod 2 = 1$ meaning 11 nodes belong to group 1 (i.e. odd-1), and when $N = 13$, we have $\frac{13 - 1}{2} \mod 2 = 0$ meaning a ring with 13 nodes belongs to group 2 (i.e. odd-2).

The second example of a 13 nodes ring, shows that all nodes can find another node to be paired with. Therefore, compared to the 11 nodes ring, better savings are achieved. As a result, the power savings obtained under network coding go up and down as the number of nodes in the odd ring changes between the odd number where $\frac{N - 1}{2} \mod 2 = 1$, classified as odd-1, and the odd number where $\frac{N - 1}{2} \mod 2 = 0$, which is classified as odd-2. For example, when $N = 11$, we have $\frac{11 - 1}{2} \mod 2 = 1$ meaning 11 nodes belong to group 1 (i.e. odd-1), and when $N = 13$, we have $\frac{13 - 1}{2} \mod 2 = 0$ meaning a ring with 13 nodes belongs to group 2 (i.e. odd-2).

The total power saving for this case is represented by

$$\phi_{odd1} = \frac{N(N - 3)(3N - 1)}{8(N^3 - N^2)}.$$  

(54)

$$\lim_{N \to \infty} \phi_{odd1} = \lim_{N \to \infty} \frac{N(N - 3)(3N - 1)}{8(N^3 - N^2)} = \frac{3}{8} = 37.5\%.$$  

(55)

For the second odd group, represented by Fig. 5b, the total number of encodable demands in each half is given by $\frac{N - 1}{2}$ as only the destination node is not selected. This gives

$$h_t(odd2) = 2N \left[ (N-2)+(N-4)+\ldots+(N-\frac{N-1}{2}) \right],$$  

(56)

which gives

$$h_t(odd2) = 2N \left[ N \cdot \frac{N - 1}{4} - 2 - \frac{N - 1}{2} \right].$$  

(57)

which can be written as

$$h_t(odd2) = 2N \left[ N \cdot \frac{N - 1}{4} - 2 \sum_{k=1}^{\frac{N-1}{2}} k \right] = \frac{3}{8} N(N - 1)^2.$$  

(58)
This makes the total power saving
\[
\phi_{\text{odd2}} = \frac{3}{8} N(N - 1)^2, \tag{59}
\]
\[
\lim_{N \to \infty} \phi_{\text{odd2}} = \lim_{N \to \infty} \frac{3}{8} N(N - 1)^2 = \frac{3}{8} = 37.5\%. \tag{60}
\]

2) Rings with even sizes: Here we also face the same distinction between two sets of even ring sizes, where rings of size (4, 8, 12, ...) will be in a different group (i.e. even-1 and have a different expression compared to the group (i.e. even-2) containing the other ring sizes (6, 10, 14, ...). This is illustrated in Fig. 6. In both cases, the destination node and another demand source node (i.e. node 5 in Fig. 6a, and node 7 in Fig. 6b) are not paired. A ring with an even size \(N\) is classified into its appropriate group, and hence its bound, by checking if \(\frac{N-2}{2}\) mod 2 = 1, if so it belongs to the even-1 group, and it belongs to even-2 when \(\frac{N-2}{2}\) mod 2 = 0. For example, when \(N = 12\), \(\frac{12-2}{2}\) mod 2 = 1 meaning 12 nodes belong to group 1, and when \(N = 14\), \(\frac{14-2}{2}\) mod 2 = 0 meaning a ring with 14 nodes belong to group 2.

![Fig. 6: Encoding pairs for two rings with different even number of nodes](image)

We start by the even-1 group represented by Fig. 6a. The total number of encoded pairs in the ring is given by \(N(2 \cdot \frac{N-4}{2} + \frac{N}{2})\), where the number of encoded pairs on one side is \(\frac{N-4}{2}\) which is deduced by removing four nodes (i.e. 12, 5, 6 and 7) to maintain the symmetry needed for the whole expression, while the \(\frac{N}{2}\) accounts for the shared hop count of the encoded pair (node 6 and node 7). Therefore the total number of shared hops is given by

\[
h_t(\text{even1}) = N \left(2 \left[ N \cdot \frac{N-4}{4} - 4 - \frac{N-4}{2} \right] \right) + \frac{N}{2}, \tag{61}
\]

The additional \(\frac{N}{2}\) term inside the brackets is the shared hop count as a result of encoding between node 6 and node 7 in Fig. 6a, which equals

\[
h_t(\text{even1}) = N \left(2\left[ N \cdot \frac{N-4}{4} - 2 \sum_{k=1}^{\frac{N-4}{2}} k \right] + \frac{N}{2} \right), \tag{62}
\]

which gives

\[
h_t(\text{even1}) = N^2 \left[ \frac{1}{2} + \frac{3(N-4)}{4} \right]. \tag{63}
\]

The savings for the even-1 ring is

\[
\phi_{\text{even1}} = \frac{1}{2} N^2 \left(\frac{3N-2}{8} \right), \tag{64}
\]

therefore

\[
\lim_{N \to \infty} \phi_{\text{even1}} = \lim_{N \to \infty} \frac{3}{8} = 37.5\%. \tag{65}
\]

For the even-2 ring, the same approach applies, just by removing two nodes (destination node and central node), and it becomes completely symmetrical, having a number of encodable demands on each side of the destination node given by \(\frac{N-2}{2}\). Therefore giving the following total number of shared hops

\[
h_t(\text{even2}) = N \left(2 \left[ N \cdot \frac{N-2}{4} - 4 - 4 - \frac{N-2}{2} \right] \right), \tag{66}
\]

which can be written as

\[
h_t(\text{even2}) = N \left(2 \left[ N \cdot \frac{N-2}{4} - 2 \sum_{k=1}^{\frac{N-2}{2}} k \right] \right), \tag{67}
\]

so the savings of the even-2 ring is

\[
\phi_{\text{even2}} = \frac{1}{2} N(3N-2), \tag{68}
\]

therefore

\[
\lim_{N \to \infty} \phi_{\text{even2}} = \lim_{N \to \infty} \frac{1}{2} N(3N-2) = 37.5\%. \tag{69}
\]

Fig. 7 Shows a comparison between the power consumption between the MILP, analytical and the OSH heuristic for the 5 nodes ring topology and compares them with the conventional MILP scenario, demonstrating an exact match between the analytic and MILP results.

We evaluate the impact of the ring size by showing the power savings of ring topologies ranging from 3 nodes up to 15 nodes as shown in Fig. 8. The figure shows that the analytical formulas developed for the 2 cases of the even number of nodes and the other two cases of the odd number of nodes matches exactly the results of the heuristic, all together converging to the highest possible savings of 37.5% as the number of nodes grows. The figure also shows that the other heuristics (i.e. w-w, w-p, and p-w), have comparable savings around 15% that are far inferior to the OSH heuristic and the p-p heuristic.

The figure also shows that the difference between the values of the analytical formulas for the odd-1 and odd-2 case are higher than the difference of the analytical value between even-1 and even-2 cases. This can be explained with the aid of Fig.
Fig. 7: Power consumption of the 5 node ring topology with equal demands using the MILP, Heuristic and analytical bound (5) and Fig. (6), where in the case of an even number of nodes, 2 nodes get left out each time for both even cases, while for the odd case, one node gets left out in one case and three at the other. This also explains why the first odd group has the highest savings where only 1 node gets left out (all nodes are encoded). It also shows that the power savings of the OSH heuristic are higher than the heuristic p-p in the odd-2 case (e.g. size 7, 11 and 15), because in this case not all nodes are encodable and the heuristic tries all possible combinations while the heuristic p-p chooses only protection paths.

Fig. 8: The power savings of the ring topology under different network sizes.

V. CONCLUSION

In this work we developed analytical bounds and closed form expressions for energy efficient survivable IP over WDM networks that use network coding, encoding the protection paths of demands using simple XOR operations. The analytical bounds were developed also for the conventional 1+1 protection scheme without network coding as a function of the average demand volume, the network size and the minimum hop count. We introduced a new concept, the characteristic hop count, and showed that the power consumption of the network coded case is a function of this characteristic hop count alongside the average traffic volume and the network size. We also studied regular topologies with emphasis on the full mesh and the ring topologies, providing a study on large network sizes, proving that the mesh, and ring topologies to exhibit savings that approach asymptotically 16.7% and 37.5% respectively. We also provided a closed form expression of the total number of hops as a function of network size for the full mesh and ring topologies. The implementation of network coding in this work uses the same algorithms of routing and path allocation algorithms used in the conventional approach, making the application to existing network a matter of a small incremental addition, as using a simple xor operation is much simpler compared to the highly complex existing techniques such as forward error correction. An interesting direction of further study however is to analyses the additional efficiency gained against the higher complexity incurred by using much higher order network coding techniques. The impact of a centralized control and management using SDN as opposed to the distributed control and the contrast between them regarding the power consumption at different types of codes is also a future direction with significant value.

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