The effect of spot size on quality factor of Laser Doppler Velocimeter

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Abstract. Quality factor is an important evaluation index of Laser Doppler Velocimeter (LDV). In order to study the effect of spot size on quality factor, the effects of signal light intensity and transit time on the quality factor are discussed theoretically. In addition, the distribution of quality factor of LDV was measured and examined. Theoretical analysis and experimental results demonstrate that: Due to the symmetry of Gaussian beam, the distribution of quality factor is symmetric. Furthermore, the light intensity of Doppler signal at the waist spot of Gaussian beam is the largest while the transit time of moving particles diminishes due to the decrease of spot diameter, causing spectrum leakage. Under the influence of both Doppler signal intensity and transit time, the distribution of quality factor appears concave phenomenon.

1. Introduction

In 1964, Yeh and Cummins confirmed that the velocity of a fluid could be obtained via utilizing the technique of laser Doppler frequency shift[1]. Based on it, Laser Doppler Velocimeter (LDV) can obtain the moving velocity of the carrier through gauging the interference signal of reference together with signal light scattered by the moving surface. On account of the complicated characteristics of the scattering surface, the signal-to-noise ratio of the Doppler signal will be reduced, causing signal loss named signal shedding. Under this condition, the scattered light contains no Doppler frequency information, thus the results obtained by any algorithm are invalid[2].

In order to evaluate the effectiveness of Doppler signal, quality factor $Q$ is defined as the ratio of the peak spectrum in the signal spectrum to the root mean square of all other points[3], as shown in Eq. (1), where $P_k$ is the spectrum peak value of Doppler signal, $P_i$ is the spectral value of other points, and $N$ is the number of different frequencies contained in the Doppler signal. In order to realize the validity of Doppler signal, the threshold value of quality factor $Q_0$ is firstly specified in the measuring process. If the quality factor of LDV $Q$ is below the threshold value $Q_0$, the signal is considered to be invalid. Otherwise, the signal is judged to be effective. Therefore, the value of quality factor reflects the

$$Q = \frac{P_k}{\sqrt{\sum_{i=0}^{k-1} P_i^2 + \sum_{i=k+1}^{N-2} P_i^2 + \frac{N}{2-1}}}$$ (1)
quality of the Doppler signal, and it is of great significance to understand the distribution characteristics of the quality factor to improve the performance of LDV.

In order to investigate the effect of spot size on quality factors of LDV, firstly, this paper analyzes the influence of the change of spot size on the quality factors theoretically. Afterwards, the distribution of the quality factor at different measuring distances is measured through experiments. Theoretical analysis and experimental results show that the quality factors are symmetrically distributed, and there is a concave phenomenon at the position of the waist spot.

2. Theory

2.1. Effect of spot size on quality factor when only considering Doppler signal strength

The relationship between laser Doppler signal strength $I$ and optical path structure parameters is \cite{4}:

$$ I = \sqrt{E_i(t)} = \frac{aqD}{d} \sqrt{\frac{\pi k_2}{2k_1}} $$ (2)

where $q$ is the average light intensity on the observation surface, $a$ is a constant related to the reference light intensity, $\sqrt{E_i(t)}$ is the root mean square of the output current of the photodetector, $k_1, k_2, k_3$ are constants, $d$ is the spot diameter on the scattering surface (assuming it is circular), and $D$ is the diameter of the photosensitive surface of the photodetector.

As derived from Eq. (2), shrinking the diameter of the spot is able to enhance the intensity of Doppler signal. According to the linearity of Fourier transform, the transform of $I$ is proportional to $I$. With an escalating $I$, the spectrum peak value of Doppler signal $P_k$ will increase, while $P_i$ ($i \neq k$) is principally the spectrum value of noise and independent of the signal light intensity. Therefore, the increment of $P_i$ ($i \neq k$) is less than $P_k$. According to Eq. (1), the value of quality factor will simultaneously ascend. From the above analysis we can obtain:

$$ Q = \frac{k_3}{d} $$ (3)

where $k_3$ is the scale factor. Therefore, when other factors are not considered, the quality factor will increase as the spot size decreases. The relationship between $Q$ and $d$ is shown in Fig.1. As the measurement point is approaching the waist spot, $Q$ performs an upward trend. Nevertheless, when the measurement point is deviating from the waist spot, the Doppler signal strength is weakened due to the excessive light spot, causing the escalation of $Q$. As the spot size approaches zero, the quality factor approaches infinity. On account of the symmetry of the Gaussian beam, the relationship between spot size and quality factor is also symmetrical.

![Fig.1. The effect of spot size on quality factor when transit time is constant](image)
2.2. Effect of spot size on quality factor when only considering transit time

The time that a moving particle passes through the spot area is called the transit time. As shown in Fig.2, as the spot of Gaussian beam decreases, the transit time of moving particles decreases. The ideal Doppler signal is a cosine function, as shown in Fig.3 (a). But the genuine Doppler signal is composed of the scattered signal fragments of multiple mobile particles (without considering the noise). The lights scattered by dissimilar particles are at the identical frequency, however, in different phases. Assume that the total signal length is $T$, the signal length produced by a single moving particle is $t_0$. It can be obtained that:

$$t_0 = \frac{d}{v}$$

when the measured velocity is fixed, the smaller the spot is, the shorter the signal length of a single moving particle is. The number of signal segments $M$ can be associated with spot size $d$ by:

$$M = \frac{T}{t_0} = \frac{Tv}{d}$$

when the sampling frequency and sampling numbers are fixed, the total length of the signal $T$ is constant. As derived from Eq. (4) and Eq. (5), it is necessary for LDV to gather more signal of the mobile particles with the purpose of obtaining equivalent amount of sampling points under the circumstance of a shrinking spot diameter of the Gaussian beam. Subsequently, the Doppler signal comprises more but shorter segments, as illustrated in Fig.3 (b).

$$\mathcal{X}[x(t)] = \sum_{n=1}^{M-1} \int_{t_n}^{t_{n+1}} \cos(\omega_n t + r_n \pi) \exp(-i\omega t) dt + \int_{t_{M-1}}^{t_0} \cos(\omega_0 t + r_0 \pi) \exp(-i\omega t) dt$$

![Fig.2. Moving particles pass through a Gaussian beam](image)

![Fig.3. Doppler signal (a) The ideal Doppler signal (M=1); (b) The genuine Doppler signal (M=10)](image)
where \( r_n \) is a random number. As the quantity of segments \( M \) in Doppler signal increases, the amplitude of the high-frequency component gradually amplifies, together with an escalating amount of breakpoints. In compliance with energy conservation principle, the amplitude at the peak is smaller than that in the ideal signal spectrum, creating the spectrum leakage (see Fig.4). On the other hand, with the increase of \( M \), the similarity between the practical and the theoretical Doppler signal progressively decreases. As a result, the peak frequency is approximate to the perfective situation, however, not identical.

Fig.4. Spectrum of actual Doppler signal (a) Spectrum of ideal Doppler signal \((M=1)\); (b) Spectrum of actual Doppler signal \((M=10)\)

From Fig.4, the spectrum leakage diminishes the amplitude of the peak spectrum \( P_k \), while the spectrum amplitude of other points \( P_i \) \((i \neq k)\) increases. Concurrently, as derived from Eq. (1), the quality factor will decrease. According to Eq. (1), Eq. (5) and Eq. (6), the relationship between the quality factor \( Q \) and the spot size can be obtained by simulation as shown in the blue scatter diagram in Fig.5. As the spot size approaches zero, the quality factor approaches zero.

Fig.5. The effect of spot size on quality factor when Doppler signal strength is constant

Through curve fitting, the relationship between quality factor and spot size can be obtained as (See the red curve in Fig.5):

\[
Q = s_1d^8 + s_2d^7 + s_3d^6 + s_4d^5 + s_5d^4 + s_6d^3 + s_7d^2 + s_8d + s_9
\]

(7)

where \( s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9 \) are constants related to the scattering characteristics of moving particles. On the basis of the interpretation above, under the equivalent spot brightness, shrinking the spot diameter will cause the descent of transit time of moving particles, which will lead to a decline in quality factor.
2.3. Effect of spot size on quality factor

The radius of Gaussian beam together with the distance between waist spot and measuring point are associated by \(^{(5)}\):

\[
\omega(z) = \omega_0 \sqrt{1 + \left(\frac{z}{f}\right)^2}
\]  

(8)

where \(f\) is characterized as \(f = \frac{\pi \omega_0^2}{\lambda}\), \(\lambda\) is the wavelength of laser beam, \(\omega(z)\) is the radius of Gaussian beam, \(\omega_0\) is the waist spot of Gaussian beam, and \(z\) is the distance between waist spot and measuring point. According to \(d = 2\omega(z)\) substituting Eq. (8) into Eq. (3) and Eq. (7) respectively, it can be obtained:

\[
Q = \frac{k}{\omega_0} \sqrt{1 + \left(\frac{z}{f}\right)^2}
\]  

(9)

\[
Q = s_1 \left[1 + \left(\frac{z}{f}\right)^2\right] + s_2 \left[1 + \left(\frac{z}{f}\right)^2\right]^2 + s_3 \left[1 + \left(\frac{z}{f}\right)^2\right]^3 + s_4 \left[1 + \left(\frac{z}{f}\right)^2\right]^4 + s_5 \left[1 + \left(\frac{z}{f}\right)^2\right]^5 + s_6 \left[1 + \left(\frac{z}{f}\right)^2\right]^6 + s_7 \left[1 + \left(\frac{z}{f}\right)^2\right]^7 + s_8 \left[1 + \left(\frac{z}{f}\right)^2\right]^8 + s_9 \left[1 + \left(\frac{z}{f}\right)^2\right]^9
\]  

(10)

where \(s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9\) are constants related to the scattering characteristics of moving particles. Take the derivative of Eq. (9) and Eq. (10), respectively, which represents the slope of Eq. (9) and Eq. (10), and the relationship between the absolute value of the slope of both and \(z\) is shown in Fig.6.

Fig.6. Relationship between the absolute value of the slop and \(z\)
Based on Eq. (9) and Eq. (10), we can sketch the relationship between $Q$ and $z$ as Fig. 7, where green part represents Gaussian light and $O$ is the waist point. We can see that the radius of the spot decreases as the measuring point approaches the position of waist spot. The shrink of the spot diameter of the Gaussian beam increases the intensity of signal light, resulting in an escalation in the quality factor. Meanwhile, the shrink simultaneously reduces the transit time of the moving particles, causing spectral leakage which generates a decline in the quality factor.

As depicted in Fig.6, In the first place, the absolute value of the slope of Eq. (9) is greater than the absolute value of slope of the Eq. (10), which means the influence of shrinking the spot on the signal light intensity is more significant than that caused by spectral leakage (before point $A$ in Fig.6 and Fig.7), the quality factor gradually increases at this stage. As the light spot dwindles, when the measuring point locates between $A$ and $O$, the absolute value of slope of the Eq. (10) is greater than the absolute value of the slope of Eq. (9), that is the impact of the spectral leakage will exceed the effect of the signal light intensity on the quality factor, which will consequently be declined. Therefore, quality factor attains the maximum value at point $A$. When the measuring point transfers to the position of the waist spot (point $O$ in Fig.6 and Fig.7), the absolute value of the slope of Eq. (10) and Eq. (9) are equal, the affection is identical to that of signal light intensity on quality factor, which attains the minimum value. In obedience to the symmetry of Gaussian beam, the alteration of spot diameter on the other side of the waist spot has the equivalent impact on the quality factor. Accordingly, the quality factors are also distributed symmetrically. Furthermore, there is a concave phenomenon at the position of the waist spot. The minimum value of the quality factor appears at the waist spot (point $O$ in Fig.6 and Fig.7), while the maximum values emerge at both sides of the waist spot (point $A$ and point $B$ in Fig.6 and Fig.7).

3. Experiment and Discussion

In order to eliminate the experimental contingency, a beam transformation system composed of lenses $F_1$ and $F_2$ were used, where $F_1$ is an electrically tunable lens\cite{6}. The focal length of $F_1$ can be changed by control the driving current of $F_1$. And the distribution of quality factor in different Gaussian beam structures is studied by changing the focal length of $F_1$. As sketched in Fig.8, the output beam of LDV is still Gaussian beam after travelling through the lenses. The chopper with fixed angular speed is employed as the velocity source. For the parameter: $L$ is the range from the measuring point to $F_2$. 

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Fig.7. Relationship between $Q$ and $z$ under the condition that the Doppler signal strength is constant (red curve) or that the transit time is constant (blue curve).
The distribution of $Q$ under different focal length of $F_1$ are shown in Fig.9, where $x$ axis is the distance from the measuring point to the lens $F_2$; $y$ axis is the focal length of $F_1$, $z$ axis is the value of $Q$. Under the circumstance of different focal length of $F_1$, the value of $Q$ gradually escalates with the increase of the measuring distance. However, when the separation reaches the position near the waist spot, the value proceeds to decrease; it attains the minimum value at the waist spot, after which the value increases to a certain figure after passing the waist spot; finally, it progressively dwindles. The overall distribution of $Q$ appears concave. It can be seen from Fig.8 that the relation between $Q$ and $L$ is consistent with the trend analyzed in section 2, so the theoretical analysis is in line with reality.

4. Conclusion

The article discusses the influence of light intensity and transit time on the quality factor of LDV, and analyzes the relationship between the spot size of Gaussian beam and the quality factor through simulation. Theoretical analysis and experimental results show that under the dual influence of signal light intensity and transit time, the quality factor is symmetrically distributed about the waist spot, and the concave phenomenon appears at the location of the waist spot. Therefore, in the actual application process, it is necessary to comprehensively consider the effect of the spot size on the signal light intensity and the transit time, so as to design a higher performance LDV.

Acknowledgement

The authors gratefully acknowledge the support from the fund of the Major basic autonomous research projects of college of Advanced Interdisciplinary Studies, National University of Defense Technology, Changsha, China under Grant No. ZDJC19-12.

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