Cyclotron Resonance in Two-Dimensional Electron Single-Layers and Double-Layers in Tilted Magnetic Field

N Goncharuk, L Smrčka and J Kučera
Institute of Physics ASCR, Cukrovarnická 10, 162 53 Praha 6, Czech Republic

Abstract. The far-infrared absorption in two-dimensional electron layers subject to magnetic field of general orientation was studied theoretically. The Kubo formula was employed to derive diagonal components of the magneto-conductivity tensor of two-dimensional electron single-layers and double-layers. A parabolic quantum well was used to model single-layer systems. Bilayer systems were represented by a pair of tunnel-coupled, strictly two-dimensional quantum wells. Obtained results were compared to experimental data.

1. Introduction

The absorption of far-infrared radiation in two-dimensional electron layers subject to magnetic fields is used to determine the electron cyclotron mass. In a quasi-classical picture of the cyclotron resonance a magnetic field $B_\perp$, perpendicular to the plane of the two-dimensional electron gas (2DEG), drives electrons with the cyclotron frequency $\omega_c$ along trajectories similar to the shape of a Fermi contour, but multiplied by $\hbar/|e|B_\perp$, and rotated by $\pi/2$. The frequency $\omega$ of circularly polarized components of the electromagnetic wave either adds to or subtracts from $\omega_c$. The current $\vec{J}(\omega)$ induced in the layer by the electric field $\vec{E}(\omega)$ dissipates, and the corresponding power $P$ is expressed by

$$ P = \frac{1}{T} \int_0^T \vec{J}(\omega) \cdot \vec{E}(\omega) \, dt = \frac{1}{T} \sum_{\alpha=x,y} \int_0^T \sigma_{\alpha\alpha} E_\alpha^2(\omega) \, dt. $$

In this expression $T = 2\pi/\omega$ is a period of the cyclotron motion and $\sigma_{\alpha\alpha}$ are the diagonal components of the conductivity tensor. The absorption power $P$ reaches maximum at the cyclotron resonance frequency, i.e. when $\omega = \omega_c$. The magnetic field at which the cyclotron resonance occurs, $B_\perp c$, defines the cyclotron effective mass by $m^*_c = |e|B_\perp c/\omega_c$. For circular Fermi contours the cyclotron effective mass and the effective mass are identical, $m^*_c = m^*$. For the case of anisotropic effective mass $m^*_x \neq m^*_y$ (an elliptic Fermi line) $m^*_c = \sqrt{m^*_x m^*_y}$. In a quantum-mechanical picture, the electron states are quantized by the magnetic field $B_\perp$ into Landau levels and transitions between adjacent levels are measured.

Here we present the theoretical study of the cyclotron resonance in symmetric 2D single-layers and double-layers subject to magnetic fields of general orientation. In this case the in-plane cyclotron mass becomes a function of both $B_\parallel$ and $B_\perp$.

A parabolic quantum well with a confining potential $V_{\text{conf}}(z) = m^* \Omega^2 z^2/2$ is used to model single-layer systems. Bilayer systems are represented by a pair of tunnel-coupled, strictly two-dimensional quantum wells, located at the distance $d$. These simple models allow to carry out most calculations analytically and are still able to capture the essence of physics of single-layer and bilayer systems. Results obtained for a single well are compared to experimental data of Takaoka et al [1].
Assuming the Landau gauge of the vector potential, \( \vec{A} = (-B_\perp y + B_\parallel z, 0, 0) \), and a wave function in the form \( \psi \propto e^{ikx} \varphi(y, z) \), the Hamiltonian can be written as

\[
H = \frac{1}{2m^*}(\hbar k_x + eB_\perp y - eB_\parallel z)^2 + \frac{p_y^2}{2m^*} + \frac{p_z^2}{2m^*} + V_{\text{conf}}(z). \tag{2}
\]

Its eigenenergies are Landau levels degenerated in \( k_x \) with a factor \( |e|B_\perp / \hbar \). For \( B_\parallel = 0 \) the variables \( y \) and \( z \) in the corresponding Schrödinger equation can be separated and for two lowest electron subbands denoted by an index \( j = 0, 1 \) we get two fans of Landau levels with the level index \( i = 0, 1, 2, \ldots \).

In tilted fields, the in-plane component \( B_\parallel \) influences strongly the orbital motion of electrons and modifies the structure of Landau levels corresponding to \( B_\parallel = 0 \). In our symmetric systems a part of the Hamiltonian proportional to a product \( \varphi(xz) \) is responsible for mixing of levels from different subbands when their level indices \( i \) differ by \( \pm 1 \). We denote the new modified fans of Landau levels by a subband index \( m = 0, 1 \) and the new levels by an index \( n = 0, 1, 2, \ldots \). Note that in our symmetric systems the eigenfunctions \( \varphi_{m,n} \) of the Hamiltonian are either even or odd:

\[
\varphi_{m,n}(y, z) = (-1)^{m+n} \varphi_{m,n}(-y, -z). \tag{3}
\]

2. Magneto-conductivity tensor

According to (1) only the “in-phase” parts of the diagonal components of the magneto-conductivity tensor are necessary to evaluate the dissipated power. We employed the Kubo formula to derive \( \sigma_{\alpha \alpha} \) of single-layers and double-layers subjected to magnetic fields of general orientation. Since our aim is to study mainly the field-dependence of the energy levels \( E_{m,n} = E_{m,n}(B_\parallel, B_\perp) \) and of the transitions between them, we completely neglect the broadening of levels by scattering of electrons. With this simplifying assumption we get, as a response to the linearly polarized electromagnetic wave \( \tilde{E}(\vec{r}, t) = 2\hbar \cos(\vec{q}r - \omega t) \) in the dipole approximation, the conductivity in the form

\[
\sigma_{\alpha \alpha} = \frac{2\pi e^2}{\omega} \sum_{\mu, \nu} f(E_\nu) f(E_\mu) |\mu \nu| \langle \nu | V | \mu \rangle^2 \left[ \delta(E_\nu - E_\mu + \hbar \omega) - \delta(E_\nu - E_\mu - \hbar \omega) \right]. \tag{4}
\]

Here \( f \) is the Fermi-Dirac distribution function, \( \mu \) or \( \nu \) stands for a Landau level double-index \( m, n \) and \( \alpha \) denotes \( x \) and \( y \). The in-plane components of the conductivity \( \sigma_{xx} \) and \( \sigma_{yy} \) involve the matrix elements of

\[
v_x = -\frac{\hbar}{m^*} \frac{\partial}{\partial x} - \omega_\perp y + \omega_\parallel z, \quad v_y = -\frac{\hbar}{m^*} \frac{\partial}{\partial y}, \tag{5}
\]

the in-plane components of the velocity operator.

The single-layer with a parabolic confining potential represents a particularly simple example which can be solved analytically. In tilted magnetic fields, the cyclotron frequency corresponding to the perpendicular field configuration, \( \omega_\perp = |e|B_\perp / m^* \), and the frequency \( \Omega \) defining the separation of two subbands, are replaced by \( \omega_1 \) and \( \omega_2 \) given by

\[
\omega_{1,2} = \sqrt{\omega_\parallel^2 + (\omega_\perp + \Omega)^2} \mp \sqrt{\omega_\parallel^2 + (\omega_\perp - \Omega)^2} \frac{2}{2}, \tag{6}
\]

where \( \omega_\parallel \) means \( \omega_\parallel = |e|B_\parallel / m^* \).
The components of the conductivity tensor read

\[ E_{m,n} = \hbar \omega_2 (m + \frac{1}{2}) + \hbar \omega_1 (n + \frac{1}{2}) \]  \tag{7} \]

It follows from equations (3) and (5) that only intrasubband transitions, \( \delta m = 0 \), between the neighbouring Landau levels, \( \delta n = \pm 1 \), or intersubband transitions, \( \delta m = \pm 1 \), between the Landau levels with the same index \( n \), \( \delta n = 0 \), are possible. Two different values of the cyclotron effective mass are connected with two processes, \( m_{c,1}^* = |e| B_{\perp} / \omega_1 \) and \( m_{c,2}^* = |e| B_{\perp} / \omega_2 \). Moreover, the conductivity can be written as a linear combination of

\[ \sigma_1 = \frac{\pi e^2}{m^* \omega} \sum_{m,n} \hbar \omega_1 (n + 1) [ f(E_{m,n}) - f(E_{m,n+1}) ] \left[ \delta (\hbar \omega_1 - \hbar \omega) - \delta (\hbar \omega_1 + \hbar \omega) \right] \]  \tag{8} \]

\[ \sigma_2 = \frac{\pi e^2}{m^* \omega} \sum_{m,n} \hbar \omega_2 (m + 1) [ f(E_{m,n}) - f(E_{m+1,n}) ] \left[ \delta (\hbar \omega_2 - \hbar \omega) - \delta (\hbar \omega_2 + \hbar \omega) \right] \]

The components of the conductivity tensor read

\[ \sigma_{\alpha \alpha} = W_{\alpha,1} \sigma_1 + W_{\alpha,2} \sigma_2 \]  \tag{9} \]

where the weight factors \( W_{\alpha,1} \) and \( W_{\alpha,2} \) are different for \( \sigma_{xx} \) and \( \sigma_{yy} \)

\[ W_{x,1} = \frac{\omega_2^2 - \omega_2^1}{\omega_2^0 - \omega_2^1}, \quad W_{x,2} = \frac{\omega_2^0 - \omega_2^1}{\omega_2^0 - \omega_2^1}, \]  \tag{10} \]

\[ W_{y,1} = \frac{\omega_2^0 \Omega_2^2 - \omega_1^2}{\Omega_2^0 \omega_2^0 - \omega_1^2}, \quad W_{y,2} = \frac{\omega_2^0 \omega_2^0 - \Omega_2^2}{\Omega_2^0 \omega_2^0 - \omega_1^2}, \]

and \( \omega_1 \) is given by \( \omega_1^2 = \omega_0^2 + \omega_2^0 \).

The detailed description of the model of the bilayer systems can be found e.g. in [3]. This model results in a pair of coupled differential equations for one-dimensional wave functions \( \varphi_L(y) \) and \( \varphi_R(y) \) localized in the left and right wells:

\[ -\frac{\hbar}{2 m^*} \varphi_L''(y) + \frac{m^* \omega_2^2}{2} \left( y + \frac{B_\parallel d}{B_\perp} \right)^2 \varphi_L(y) + t \varphi_R(y) = E \varphi_L(y), \]  \tag{11} \]

\[ -\frac{\hbar}{2 m^*} \varphi_R''(y) + \frac{m^* \omega_2^2}{2} \left( y - \frac{B_\parallel d}{B_\perp} \right)^2 \varphi_R(y) + t \varphi_L(y) = E \varphi_R(y). \]  \tag{12} \]

Here, the orbit centres in individual wells are shifted due to the in-plane component of the magnetic field and the hopping integral \( t \) describes the coupling of two two-dimensional electron layers.

We solve the coupled equations numerically and use the resulting eigenenergies and eigenfuncions to evaluate the cyclotron effective mass and transition matrix elements. The equation (3) helps to determine whether the Landau level belongs to the fan of a ground or an excited subband.

The above described approach is not appropriate when \( B_\perp \) is very small in comparison with \( B_\parallel \). In such a case the quasi-classical description [3, 4] should be employed. For small \( B_\perp \)-components of magnetic fields it yields the same results as the quantum mechanical approach but it fails when both \( B_\parallel \) and \( B_\perp \) are strong. Then the quantum mechanical approach must be used and the interpretation of cyclotron resonance experiments becomes more complicated as cyclotron effective masses depend on both \( B_\parallel \) and \( B_\perp \).
3. Results and discussion

3.1. Two-dimensional electron single-layers

Within the parabolic-confinement approximation, the 2D electron single-layer system decouples into two harmonic oscillators with eigenfrequencies (6). The confinement frequency $\Omega$ yields the subband separation in perpendicular magnetic field.

In tilted magnetic fields, if $B_\perp$ is small ($\omega_\perp \ll \Omega$), $\omega_1$ is determined mainly by $\omega_\perp$ and the quantum number $n$ plays the role of the Landau band index, while $\omega_2$ reduces to $\Omega$ and the quantum number $m$ represents the subband index. In high perpendicular magnetic fields ($\omega_\perp \gg \Omega$) the roles of both indices are interchanged.

The $B_\perp$-dependence of eigenenergies $E_{0,n}$ and $E_{1,n}$, $n = 0, 1, 2 \ldots$ is shown in figure 1. The in-plane component of magnetic field increases the subband separation and changes the Landau-level structure. In tilted magnetic fields Landau-level mixing arises and not only intrasubband transitions between adjacent Landau levels of a single subband but also intersubband transitions between Landau levels with the same index and from different subbands are allowed. Figure 2 presents the field dependence of two different types of effective cyclotron masses associated with the eigenfrequencies $\omega_1$, $\omega_2$ and of weights of the intrasubband ($W_{x,1}$) and the intersubband ($W_{x,2}$) transitions are governed by the transition matrix elements which, similarly as $m_c^*$ itself, strongly depend on both $B_\parallel$ and $B_\perp$.

If 2D electron systems have only one occupied subband, various electron concentrations can be modelled by changing $\Omega$, the higher the concentration, the narrower the well. We have found that wider parabolic wells are more sensitive to the magnitude of both magnetic field
components than narrower (figure 3) and have shown (figure 4) that our model can reasonably fit the $B_{\parallel}$- and $B_{\perp}$-dependence of experimental data [1].

A convenient way to measure the cyclotron resonance in 2D electron systems is FIR optical magneto-absorption measured by Fourier transform spectrometer. The absorption process is accompanied by electron excitation from an occupied to an unoccupied level whereby the energy difference is an integer multiple of $\hbar\omega_{\perp} = \hbar|e|B_{\perp}/m^*_{c}$. At radiation energies equal to these differences the measured magnetoconductivity exhibits maxima.

The effective cyclotron mass can be calculated from the separation energy of Landau levels corresponding to allowed intrasubband or intersubband transitions. For fixed $B_{\perp}$ and $B_{\parallel}$ Landau levels are equidistant (figure 5) and cyclotron effective masses connected with different transitions are identical (figure 6).

### 3.2. Two-dimensional electron double-layers

The coupling between quantum wells due to electron tunneling through the barrier removes the degeneracy of the energy spectrum and the lowest bound states of individual quantum wells form symmetric and antisymmetric pairs. In the perpendicular magnetic field the separation between symmetric and antisymmetric states is characterised by the interwell hopping energy $2|t|$. Results for 2D electron bilayers were calculated numerically. As follows from the Kubo formula (4), the diagonal components of the magnetoconductivity can be expressed in terms of a sum of dimensionless oscillator strengths $f_{\alpha,1}, f_{\alpha,2}$ [5] where the index $\alpha$ denotes $x$ and $y$ as in the case of single-layer, and the indices 1 and 2 indicate intrasubband and intersubband transitions, respectively. Due to system symmetry, selection rules hold [5] and only transitions with nonzero oscillatory strength are allowed. These include intrasubband transitions $\delta m = 0, \delta n = \pm 1$ and intersubband transitions $\delta m = \pm 1, \delta n = 0$.

In what follows we concentrate on the lowest transitions from $n = 0$ which are more pronounced than all other channels. Figures 7 and 8 show $B_{\perp}$-dependence of eigenenergies calculated for a symmetrical double well structure and of oscillator strengths corresponding
to the strongest permitted transitions in given system.

The Landau levels with high quantum numbers can be described both quasi-

Figure 5. (left) Fan diagram of eigenenergies against the in-plane $B_\parallel$ component of magnetic field at fixed $B_\perp = 0.5T$ corresponding to the energy of subband separation $\hbar \Omega = 4\text{meV}$ for the model of 2D electron single-layer systems.

Figure 6. (right) Cyclotron effective masses $m^*_c$ calculated from the difference between Landau levels of the eigenenergy spectrum (figure 5) for two types of transitions $\delta m = 0$ $\delta n = \pm 1$ and $\delta m = \pm 1$ $\delta n = 0$ as a function of the in-plane magnetic field.

Figure 7. (left) Eigenenergies in the 2D electron bilayer systems as a function of $B_\perp$. Solid curves correspond to tilted magnetic fields with fixed $B_\parallel = 0.5T$ with an anticrossing due to subband-Landau-level coupling $|t| = 2\text{meV}$. Blue lines represent the bonding and red lines the antibonding Landau-level subband. Dotted curves represent the case where $B_\parallel = 0T$.

Figure 8. (right) The $B_\perp$-dependence of oscillator strengths $f_{x,1}$, $f_{x,2}$ of allowed optical transitions between Landau levels in 2D electron double-layer systems corresponding to fixed $B_\parallel = 0.5T$ and $|t| = 2\text{meV}$. Index 1 determines intrasubband transitions between adjacent Landau levels $m = 0, n = 0$ and $m = 0, n = 1$ from bonding subband. Index 2 denotes intersubband transitions between Landau levels $m = 0, n = 0$ from bonding subband and $m = 1, n = 0$ from antibonding subband.
Cyclotron Resonance in Two-Dimensional Electron ...  

Figure 9. (left) Eigenenergies of the 2D electron bilayer systems as a function of $B_\parallel$ at fixed $B_\perp = 0.5$ T and $|t| = 2$ meV. Blue and red lines denote Landau levels from bonding and antibonding subband, respectively.

Figure 10. (right) Cyclotron effective masses $m^*_c$ calculated from energy difference between adjacent Landau levels of the band structure spectrum (figure 9) as a function of the in-plane magnetic field.

classically [3, 4] and quantum-mechanically. Using our quantum-mechanical approximation for the case of slightly tilted field we have calculated spectrum of eigenenergies. With increasing in-plane component of magnetic field due to the process of electron transfer from the higher occupied antibonding subband to the lower bonding subband the separation energy between bonding and antibonding subbands grows until the antibonding subband is completely depopulated. Further growth of $B_\parallel$ leads to the degeneracy of Landau levels from the bonding subband as a result of the process is accompanied by complete suppression of interwell tunneling when electrons move in one of individual wells. Figure 10 shows the field-dependence of cyclotron effective masses calculated from eigenenergy spectrum presented on figure 9. The effective cyclotron masses corresponding to the highest antibonding subband decrease with increasing of the in-plane field, while cyclotron masses corresponding to the lowest bonding subband increase. The positions of singularities of cyclotron masses corresponding to the lowest bonding subband are related to the values of $B_\parallel$ at which coupled double quantum wells move to two decoupled electron layers. As the number of occupied levels depends on the Fermi energy, the increase in the electron concentration shifts the singularity position to the side of larger in-plane fields.

The field-induced electron-layer decoupling process, shown in figure 11, illustrates the aforementioned explanation. Wave function components of individual layers $\phi_L(y)$, $\phi_R(y)$ are shifted by the distance $y_0 = d B_\parallel / 2 B_\perp$ where $d$ is the distance between the potential minima in the left and the right well on the z-axis. The increase of the in-plane field is followed by a separation growth between $\phi_L(y)$ and $\phi_R(y)$. Moreover, with the increase of $B_\parallel$ wave function components of higher odd states lose their nodes and tend to coincide with the nodeless wave function components of lower even states.
4. Conclusion

To summarize, we have performed the theoretical study of the cyclotron resonance in 2D single-layers and double-layers subject to magnetic fields of general orientation to explain existing experimental results. For strong $B_\parallel$ and $B_\perp$, the quantum-mechanical approach must be involved and $m_\parallel$ becomes a function of both components. We have calculated the field-dependence of the energy levels and transitions matrix elements between them. The Kubo formula was employed to derive diagonal components of magneto-conductivity tensor for both systems. The resulting conductivities are linear combinations of two 2D conductivities connected with intrasubband and intersubband transitions. Conclusions obtained for the model of the parabolic quantum well and for the model of coupled double-quantum wells are analogous. All results are obtained in a good qualitative agreement with experimental data.

5. Acknowledgements

This work has been supported by the Grant Agency of the Czech Republic under Grant No 202/01/0754. The experimental curves in figure 4 are presented with kind permission of S. Takaoka.

References

[1] Aikawa H, Takaoka S, Oto K, Murase K, Saku T, Hirayama Y, Shimomura S and Hiyamizu S 2002 Physica E 12 578-580
[2] Hu J and MacDonald A H 1992 Phys. Rev. B 46 12554-12559
[3] Smrčka L, Jungwirth T 1995 J. Phys.: Condens. Matter 7 3721-3732
[4] Takaoka S, Kuriyama A, Oto K, Murase K, Shimomura S, Hiyamizu S, Cukr M, Jungwirth T, Smrčka L 2000 Physica E 6 623-626
[5] Davies J H 1998 The Physics of Low-Dimensional Semiconductors (Cambridge University Press) 316-321