Jump of azimuthal velocity in a creeping two-fluid swirling flow

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Abstract. This work reveals a jump of azimuthal velocity at the interface that can be important for vortex bioreactors. The rotating lid controls the flow of two fluids in a vertical cylindrical container, the other walls of which are stationary. Near the lid, a centrifugal force pushes the upper fluid toward the sidewall, thereby developing its centrifugal meridional circulation. This motion drives the bulk counter-circulation in the lower fluid at a slow rotation. The use of sunflower oil for the upper fluid, alcohol-glycerine solutions for the lower fluid and the assembly of averaged PIV images allow measuring velocity and visualizing pattern even in a creeping motion. This advanced experimental method helps to detect velocity jumps occurring at the interface.

1. Introduction
The development of aerial vortex bioreactors has attracted the attention of researchers to two-fluid swirling flows [1-4]. The propeller controls the flow of air, which in turn controls the flow of water. The air stream transports oxygen to the interface, where oxygen diffuses into water. Water circulation improves the mixing of oxygen with other ingredients. Thus, the aerial vortex bioreactor provides soft and subtle mixing, useful for the growth of biological tissue culture. The flow of water should not be too fast to avoid rupture of the crop under strong shear loads.

A suitable model of a bioreactor is a flow in a sealed vertical container filled with water and air [3]. The rotating lid creates a stream of air and water, while the other walls of the container are stationary. The tissue fraction is small compared to the water fraction and may be neglected in the model while the flow characteristics are being studied. Simple geometry and well-defined boundary conditions and control parameters are convenient for both experimental and numerical studies. Comparison of numerical and experimental results helps understand the paradoxical features and topological changes in characteristic of two-fluid vortex flows.

The experimental study of vortices in air-water flows is difficult because the steady laboratory water flow is so slow (due to the low density and dynamic viscosity of air compared with those of water) that speed measurement becomes problematic. It is no coincidence that the first experiments on the study of rotating two-fluid flows in vertical cylindrical containers were devoted to silicon-oil-water [5], soybean-oil-water [6] and sunflower-oil-water systems [7]. The use of oils for the upper liquid made has revealed (a) the impressive shapes of the interface [5], (b) the destruction of vortices in the upper liquid [6] and (c) the hysteretic growth and destruction of the drainpipe column [7].
Naumov et al. [8] replaced water, as the lower liquid, with a water-glycerin solution with viscosity close to that of oil and more than that of water by two orders of magnitude. This replacement drastically increases the speed of the lower fluid for small Re values.

This experimental method has helped reveal new striking features of vortex flows consisting of two liquids: (a) a radial velocity discontinuity at the interface [8], (b) multiple topological transformations of the lower fluid flow, and (c) the formation of a thin circulation layer serving as a liquid bearing separating the bulk meridional centrifugal circulation of the upper and lower fluids [9].

The purpose of current research is to investigate the flow of two rotating liquids with close kinematic viscosities along the interface and to compare the measured flow characteristics at small Re consistent with the theory of creeping swirling flows. As the lower fluid, we use an alcohol-glycerin solution, whose density is close to the oil density. The advanced technique helps reveal jumps in vortex velocity at the interface (in addition to radial velocity jumps). This modification serves to find the creeping swirling flow near the interface and jumps in vortex velocity at the interface (in addition to radial velocity jumps).

The upper-fluid flow, being directly driven by the rotating lid, is almost independent of the lower flow for large Re [8]. However, for small and moderate Re, the lower-fluid features do affect the upper-fluid motion, especially near the interface.

Current experimental research shows that the measured flow characteristics at small and moderate Re are consistent with the theory of creeping swirling flows [10, 11]. This agreement, being interesting in itself, confirms our experimental results, in particular, the jump in the azimuthal velocity at the interface. The range $0 < \text{Re} < 100$ corresponds to the creeping flow where nonlinear effects are small compared with linear ones [12]. For this range, no visible deformation of the interface occurs.

2. Experimental set-up and technique

To study a two-component flow, we use experimental setup for contactless optical methods, schematically shown in Figure 1. Experiments are performed in a vertical cylindrical container ($R = 45$ mm, $h = 90$ mm) made of optical glass. The motion is driven by the upper disk, which rotates with an angular velocity $\omega$, while the other walls are fixed. The rotation of the disk is set by the stepper motor. A water jacket with a square cross section is employed to correct optical distortion and to maintain a constant temperature.

![Figure 1. The scheme of fluid motion (left) and the experimental setup for measurements in the horizontal cross-section (right).](image_url)
The container is filled with sunflower oil \((o)\) as the top liquid \((h_o = R)\) and with a solution of alcohol-glycerin \((sg)\), whose density is close to the oil density, as the bottom liquid \((h_{sg} = R)\). The indices "o" and "sg" denote "oil" and "spirit-glycerin solution". At a room temperature of 22.6\(^\circ\)C, the fluid densities and kinematic viscosities are \(\rho_o = 914.7\ \text{kg/m}^3\), \(\rho_{sg} = 922\ \text{kg/m}^3\), \(\nu_o = 54.86\ \text{mm}^2/\text{s}\), and \(\nu_{sg} = 7.366\ \text{mm}^2/\text{s}\). The rotation strength is characterized by the Reynolds number \(Re = \omega R^2/\nu_o\).

The velocity fields were measured by Particle Image Velocimetry (PIV). The use of PIV allowed obtaining an instantaneous velocity distribution in the investigated cross section and observing an instantaneous flow pattern within the two-dimensional plane of a light sheet. As an optical source for forming the light sheet we used Nd: YAG pulsed laser POLIS v3.2 with the following characteristics: wavelength of 532 nm, light sheet thickness of 1 mm, the energy pulse power of 120 mJ, and the operation frequency of 2 Hz. Measurements were performed in horizontal cross-section. Images were registered by POLIS camera v1.0 with lenses Nikon AF 28 mm f/2.8D Nikkor.

3. Results

For the following analysis of experimental results, it is instructive to compare them with the Hills theory \([10]\). Figure 2 shows the dependence of the maximum tangential velocity \(V_{\theta_{\max}}\) on the vertical coordinate \(z\) near the interface (located at \(z = 45\ \text{mm}\)) in (I) the case of mono-fluid (oil) and in (II) the case of two fluids. Symbols represent the results of measurements, and the curves show the predictions of the theory \([10]\) for the slow flow of mono-fluid created by the rotation of the end disk in a semi-infinite cylinder.

The agreement of experiment (cross symbols) and theory (dashed curve) in the case of mono-fluid (Fig. 2) indicates that the flow is creeping, in which advection is small compared to diffusion. In the case of two liquids, the continuous dependence (dotted curve in Fig. 2) does not agree with the experiment (square symbols). It is assumed that a jump in the velocity \(V_\theta\) occurs at the interface so that the angular momentum \(\rho r V_\theta\) is continuous. This hypothesis (lower curve in Fig. 2) is consistent with the measurement data.

![Figure 2](image_url)

*Figure 2.* Comparison of experiment (symbols) and theory (lines) for velocity variations in the z-direction in mono and two fluid flow.
Figure 3 shows a comparison of experimental values of normalized azimuthal velocity $V_0$ (crosses) with the Hills theory (line) at $Re = 50$ and 150. The graph in Figure 3 has a logarithmic scale of the ordinate axis. Hills considered a creeping flow in a semi-infinite pipe, driven by rotation of the end disk and found that away from the disk vicinity, the distribution of swirl velocity has the presentation $V_0 = C \exp(\lambda z/R) S(r)$, where $C$ is a constant, $\lambda = 3.83 \times 10^{11}$ and $S(r)$ is the radial profile normalized by its maximal value. Choosing $S(r) = 1$ reduces relation to that for $V_{0\text{max}}$, which is the maximum of $V_0$ at a fixed $z$ value. Next, taking $C = V_{0\text{max}} \exp(-\lambda z/R)$ we find a match with the experimental and theoretical values of $V_{0\text{max}}$ at $z = z_0 = 47$ mm, where $V_{0\text{max}}$ is $V_{0\text{max}}$ at $z = z_0$. This transforms equation to $V_{0\text{max}} = V_{0\text{max}} \exp[\lambda(z-z_0)/R]$. The line equation in Fig. 3 is $V_{0\text{max}} = V_{0\text{max}}(z_0) \exp[3.83(z - z_0)/R]$. As shown in Figure 3, the experimental values at $Re = 50$ are consistent with Hill's theory, and at $Re = 150$, the graphs diverge. This confirms the fact that at $Re > 100$ the flow is not creeping [12].

![Figure 3](image-url)

**Figure 3.** Comparison of experimental values with the Hills theory at $Re =$ (a) 50, (b) 150.

The experimental study [9] found that there is a slip of the radial velocity at the interface of a two-fluid swirling flow. The radial velocities in upper and lower fluid are close at $Re = 50$, while the velocities are very different, even having opposite directions at the near-axis part of the interface for $Re = 100, 200,$ and $300$. In the present work, we studied the distribution of the azimuthal velocity near the interface.
Figure 4. Dependence of \( r \)-profiles of normalized azimuthal velocity \( V_\theta \) at \( \text{Re} = 50 \) (a) and 150 (b). At \( z = 43 \text{ mm} \), the profiles are similar to that in the creeping flow for \( \text{Re} = 50 \) and \( \text{Re} = 150 \). In contrast at \( z = 47 \text{ mm} \), the converging flow of the upper fluid shifts the maximum location of \( V_\theta \) closer to the axis.

Conclusions
In this experimental study of a two-fluid swirling flow, two rotating liquids have close kinematic viscosities. The motion is driven by the rotating lid in a sealed vertical cylindrical container. Slip condition of the azimuthal velocity at the interface is observed. It is assumed that a jump in the velocity \( V_\theta \) occurs at the interface, whereas the angular momentum \( \rho r V_\theta \) is continuous. Other important results of this study are: (a) the measured flow characteristics at small \( \text{Re} \) are consistent with the theory of creeping swirling flows and (b) the velocity jump occurs at any \( \text{Re} \) regardless of the flow types considered.

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References
[1] Ramazanov Yu A, Kislykh V I, Kosyuk I P, Bakuleva N V and Shchurikhina V V 2007 New Aspects of Biotechnology and Medicine ed. A M Egorov 87–91
[2] Liow K Y S, Thouas G, Tan B T, Thompson M C and Hourigan K 2008 IFMBE Proceedings 23 1672
[3] Liow K Y S, Tan B T, Thouas G and Thompson M C 2009 Modern Physics Letters B 23 121
[4] Lo Jacono D, Nazarinia M and Brons M 2009 Phys. Fluids 21 111704
[5] Fujimoto S and Takeda Y 2009 Phys. Rev. E 80 015304
[6] Tsai J C, Tao C Y, Sun Y C, Lai C Y, Huang K H, Juan W T and Huang J R 2015 Phys. Rev. E 92 031002
[7] Naumov I V, Herrada M A, Sharifullin B R and Shtern V N 2018 Phys. Rev. Fluids 3 024701
[8] Naumov I V, Herrada M A, Sharifullin B R and Shtern V N 2018 Phys. Fluids 30 074101
[9] Naumov I V, Glavny V G, Sharifullin B R and Shtern V N 2019 Phys. Rev. Fluids 4 054702
[10] Hills C P 2001 Phys. Fluids 13 2279–86
[11] Shtern V 2012 J. Fluid Mech. 711 667–80
[12] Naumov I V, Sharifullin B R, Kravtcova A Yu and Shtern V N 2020 Exp. Therm. Fluid. Sci. 116 110116