IMPORTANCE OF MERIDIONAL CIRCULATION IN FLUX TRANSPORT DYNAMO: THE POSSIBILITY OF A MAUNDER-LIKE GRAND MINIMUM

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ABSTRACT

Meridional circulation is an important ingredient in flux transport dynamo models. We have studied its importance on the period, the amplitude of the solar cycle, and also in producing Maunder-like grand minima in these models. First, we model the periods of the last 23 sunspot cycles by varying the meridional circulation speed. If the dynamo is in a diffusion-dominated regime, then we find that most of the cycle amplitudes also get modeled up to some extent when we model the periods. Next, we propose that at the beginning of the Maunder minimum the amplitude of meridional circulation dropped to a low value and then after a few years it increased again. Several independent studies also favor this assumption. With this assumption, a diffusion-dominated dynamo is able to reproduce many important features of the Maunder minimum remarkably well. If the dynamo is in a diffusion-dominated regime, then a slower meridional circulation means that the poloidal field gets more time to diffuse during its transport through the convection zone, making the dynamo weaker. This consequence helps to model both the cycle amplitudes and the Maunder-like minima. We, however, fail to reproduce these results if the dynamo is in an advection-dominated regime.

Key words: Sun: activity – Sun: dynamo – sunspots

1. INTRODUCTION

An important aspect of solar activity are the grand minima periods during which the activity level is strongly reduced. The best example of this is the Maunder minimum during 1645–1715 (Eddy 1976; Ribes & Nesme-Ribes 1993). It is important not only to the solar physics community but also to the space weather and Earth climate community as it may be correlated to the Little Ice Age. It is not an artifact of few observations, but a real phenomenon (Hoyt & Schatten 1996). There is evidence that solar-like stars also show Maunder-like minima (Baliunas et al. 1995). The Maunder minimum episode may be divided into two periods: one from 1645 to 1670 and another from 1670 to 1715. During the first period, the sunspot number was almost zero in both the hemispheres, whereas during the second period, a few sunspots appeared in the southern hemisphere. So there was a strong north–south asymmetry of sunspot number in the last phase of the Maunder minimum (Ribes & Nesme-Ribes 1993; Sokoloff & Nesme-Ribes 1994). Another important feature is the sudden initiation, but slow recovery (Usoskin et al. 2000). From the study of the cosmogenic isotope $^{10}\text{Be}$ in polar ice core, Beer et al. (1998) suggested that the usual 11 year period of solar activity (Schwabe cycle) continued during the Maunder minimum, although the overall activity level was weaker. However, from the study of $^{14}\text{C}$ data in tree rings, Miyahara et al. (2004) found the same cyclic behavior but with a period of 13–15 years instead of the regular 11 year period (see also Miyahara et al. 2010).

Flux transport dynamo models developed by several authors (Wang et al. 1991; Choudhuri et al. 1995; Durney 1995; Dikpati & Charbonneau 1999; Kükner et al. 2001; Nandy & Choudhuri 2002; Chatterjee et al. 2004; Guerrero & Muñoz 2004; Muñoz-Jaramillo et al. 2009; Hotta & Yokoyama 2010a) are the most promising models for studying the solar cycle. It explains many important features of the regular solar cycle including 11 year periodicity, equatorial migration of the sunspot belt, poleward migration of large-scale field and parity, etc. It also reproduces many irregular features of the solar cycle (Charbonneau & Dikpati 2000; Choudhuri & Karak 2009; Karak & Choudhuri 2010a, 2010b). The flux transport dynamo model consists of the following processes. A strong toroidal component of the magnetic field is generated in the tachocline due to the strong differential rotation (Parker 1955). When this toroidal magnetic field rises above the stable layer, the magnetic buoyancy acts on it causing it to rise to the surface to form sunspots. These sunspots decay to give rise to the poloidal component of the magnetic field through the Babcock–Leighton mechanism (Babcock 1961; Leighton 1969), which then gets advected (also diffused) to the pole by the meridional circulation. This field ultimately reaches the tachocline where it again gets stretched to give rise to the toroidal field and the cycle completes. In this process, the poloidal field is produced from the decay of the tilted bipolar sunspots. These tilts are due to the Coriolis force acting on the rising flux tubes (D’Silva & Choudhuri 1993). In addition, when these flux tubes rise through the convection zone, they are subjected to the convective buffeting (Longcope & Choudhuri 2002). This gives rise to a scatter in the tilt angles around the mean given by Joy’s law (Wang & Sheeley 1989). We therefore believe that the Babcock–Leighton process is not a deterministic process. As a result, the polar field changes randomly as its production is governed by the random process (Choudhuri et al. 2007). Following this idea, Choudhuri & Karak (2009) have proposed that at the beginning of the Maunder minimum, the polar field fell to a very low value. With this assumption, they have succeeded in reproducing a Maunder minimum using a flux transport dynamo model. Some authors (Choudhuri 1992; Gómez & Mininni 2006; Wilmot-Smith et al. 2005; Brandenburg & Spiegel 2008; Moss et al. 2008; Usoskin et al. 2009) are also able to reproduce Maunder-like grand minima by introducing stochastic fluctuations or nonlinearity in dynamo parameters. By introducing stochastic fluctuations in the flux transport dynamo simulations, Charbonneau et al. (2004) are able to reproduce intermittencies which resemble the Maunder minimum. On the other hand, Beer et al. (1998) proposed the nonlinearity to be the cause of such irregularities of the solar cycle.
Another vital source of irregularity in the flux transport dynamo model is the meridional circulation which plays a major role in transporting the magnetic fields. It determines not only the period but also the amplitude of the solar cycle (Wang et al. 1991; Dikpati & Charbonneau 1999; Hathaway et al. 2003; Yeates et al. 2008). It also varies stochastically with time, giving randomness in the solar activity. Recently several authors (Hathaway 1996; Haber et al. 2002; Basu & Antia 2003; Gizon & Rempel 2008; Hathaway & Rightmire 2010) have reported its value in the upper part of the convection zone and found strong temporal variation of its amplitude. Javaraiah & Ulrich (2006) studied group sunspot data during cycles 12–23 and found a cycle-to-cycle variation of mean meridional motion of sunspot groups (a proxy of the meridional flow). Several other indirect studies (Wang et al. 2002; Hathaway et al. 2003; Georgieva & Kirov 2010) have found the evidences of the fluctuations of the meridional circulation. Moreover, using a low-order model constructed from the sunspot number as the proxy of the toroidal field, Passos & Lopes (2008) suggested that the last 11 magnetic cycles can be modeled using variable meridional circulation speed only. Using a similar model, Passos & Lopes (2009) have also concluded that the stochastic fluctuations in the α-effect cannot trigger a grand minimum, rather they argued that the strong decrease of meridional circulation can do so.

The motivation of this work is to find out and understand the influence of the meridional circulation on both cycle-to-cycle amplitude variation of the solar cycle and producing grand minima. Therefore, first, we try to model the periods of the last 23 sunspot cycles just by varying the amplitude of meridional circulation to see its effect on the amplitude of the solar cycle. We have shown the results from a diffusion-dominated flux transport dynamo model (e.g., Chatterjee et al. 2004). By the term “diffusion-dominated” we mean that the diffusivity of the poloidal field in the whole convection zone is high enough (~10^{12}–10^{13} cm^2 s^{-1}; see also Jiang et al. 2007 and Yeates et al. 2008 for details). However, we discuss the result from an advection-dominated flux transport dynamo model (e.g., Dikpati & Charbonneau 1999) as well. By the term “advection-dominated” we mean that the diffusivity of the poloidal field in the whole convection zone is low (~10^{10}–10^{11} cm^2 s^{-1}). In this model, therefore, the diffusivity is not so important in transporting the fields. In a high-diffusivity model, we have found that most of the cycle amplitudes get modeled up to some extent when we try to model the periods of the last 23 cycles just by varying the amplitude of meridional circulation only. Therefore, we conclude that a major part of the fluctuations of the amplitude of the solar cycle may come from the fluctuations in meridional circulation. However, we do get a completely different result, if the dynamo is in the advection-dominated regime. The physics of getting these two completely different results from these two models can be understood based on the Yeates et al. (2008) study, which we mention in the appropriate place.

Next, we also study the importance of meridional circulation in producing a Maunder-like grand minimum in both the high-diffusivity model and the low-diffusivity model. We propose that at the beginning of the Maunder minimum, the amplitude of meridional circulation dropped to a low value and then after a few years it increased again to the original value. With this assumption, we have checked the possibility of producing (or not producing) a Maunder-like grand minimum in both the models. In the next step, we have also included the fluctuations of polar field to capture the fluctuations in the Babcock–Leighton process along with that of meridional circulation to model such kind of grand minimum.

2. MODEL

The evolution of poloidal and toroidal components of the magnetic field in the flux transport dynamo models is governed, respectively, by the following two equations:

\[
\frac{\partial A}{\partial t} + \frac{1}{s}(\mathbf{v} \cdot \nabla)(sA) = \eta_p \left( \nabla^2 - \frac{1}{s^2} \right) A + \alpha B, \tag{1}
\]

\[
\frac{\partial B}{\partial t} + \frac{1}{r} \left[ \frac{\partial}{\partial r} (rv_B) + \frac{\partial}{\partial \theta} (v_\theta B) \right] = \eta_t \left( \nabla^2 - \frac{1}{s^2} \right) B + \frac{\sigma (B_p, \nabla) + \frac{1}{r} \frac{d \eta_t}{dt} \partial (rB)}{r \frac{d}{dr}}, \tag{2}
\]

where \(A(r, \theta)\) and \(B(r, \theta)\), respectively, correspond to the poloidal and toroidal components and \(s = r \sin \theta\). Here \(\mathbf{v}\) is the velocity of the meridional flow, \(\alpha\) is the coefficient which describes the generation of the poloidal field near the solar surface from the decay of bipolar sunspots, \(\Omega\) is the internal angular velocity of the Sun, and \(\eta_p, \eta_t\) are the turbulent diffusivities for the poloidal and toroidal fields.

To study the evolution of the magnetic fields, we have to solve the above two equations with the given parameters. Recently this problem has been extensively studied by two different groups (Dikpati & Charbonneau 1999 and Chatterjee et al. 2004). Dikpati & Charbonneau (1999; also Dikpati et al. 2004) use a very low value \((10^{10}–10^{11} \text{ cm}^2 \text{ s}^{-1})\) of the diffusivity of the poloidal field in the convection zone, whereas Chatterjee et al. (2004; also Jiang et al. 2007) use a very high value \((10^{12}–10^{13} \text{ cm}^2 \text{ s}^{-1})\). Although both models can explain some important features of the solar cycle (e.g., 11 year periodicity, equatorward propagation of the sunspot belt; Dikpati & Charbonneau 1999; Chatterjee et al. 2004), in many cases, they give completely different results (Chatterjee et al. 2004; Jiang et al. 2007). Therefore, we are curious to see the importance of meridional circulation in modeling both the solar cycle and the Maunder-like grand minimum in these two models separately. To do this, we first use the high-diffusivity model of Chatterjee et al. (2004) with the same profiles of \(\alpha\) coefficient, differential rotation, meridional circulation, and diffusivity as given in the “standard model” of Chatterjee et al. (2004). We do not repeat them here. However, Chatterjee & Choudhuri (2010b) have changed some parameters of this model which are listed in Table 2 of their paper. In the present work, we use these parameters except we take \(\Gamma = 3.025 \times 10^8 \text{ m}\) and the surface value of meridional circulation \(v_\theta = 22 \text{ m s}^{-1}\). With these parameters, the period of the solar cycle comes out to be around 11 years. In all the analyses, only the amplitude of meridional circulation is varied while all other parameters of the model are held fixed. A related question at this stage may arise that any change in the meridional circulation may be accomplished by the change in the differential rotation. Since the present understanding of these two is very primitive and our aim is to understand the effect of the fluctuation of meridional circulation on the solar cycle, we have taken differential rotation to be constant in all analyses.

For the low-diffusivity calculations, if we decrease the diffusivity to a very low value \((10^{10} \text{ cm}^2 \text{ s}^{-1})\) in the previous model, we do not get an oscillatory solution. Therefore, we have
Figure 1. (a) Variation of amplitudes of meridional circulation $v_0$ (in m s$^{-1}$) with time (in yr). Dashed line is calculated from the relation $T = 97.48v_0^{-0.696}$ by putting the periods of the last 23 cycles data. The solid line is the variation of $v_0$ used to match the theoretical periods with the observed periods. (b) Variation of theoretical sunspot number (dashed line) and observed sunspot number (solid line) with time. Both sunspot numbers are smoothed by a Gaussian filter of FWHM $= 1$ year. In addition, the theoretical sunspot number is scaled by a factor to match the observed value. (c) Scatter diagram showing the theoretical and observed periods (in yr). (d) Scatter diagram showing peak theoretical sunspot number and peak observed sunspot number. The linear correlation coefficients and the corresponding significance levels are shown in panels (c) and (d).

3. METHODOLOGY

First, we model the last 23 cycles by fitting the periods with variable meridional circulation in a high-diffusivity model. We have done this in the following way. It is known that the meridional circulation acts as a clock to regulate the period of the solar cycle (Wang et al. 1991; Dikpati & Charbonneau 1999; Hathaway et al. 2003). Stronger meridional circulation gives a shorter period and vice versa. It is found that in the flux transport dynamo model, the period of the solar cycle roughly scales as the inverse of the meridional circulation amplitude. Therefore, we first calculate the dependency of period ($T$) on $v_0$ by running the model at different values of $v_0$ and we find a relation $T = 97.48v_0^{-0.696}$ (when $19 \leq v_0 \leq 30$). Then from the observed periods of the last 23 cycles we can calculate the corresponding $v_0$ from the above relation. The dashed line in Figure 1(a) shows this variation. However, these values of $v_0$ do not reproduce the observed periods properly; therefore, we fit them by trial and error methods. We did not try to match the periods of each cycle accurately, which is a little bit difficult. We change $v_0$ abruptly between two cycles and not during a cycle. In addition, we do not change $v_0$ if the period difference between two successive cycles is less than 5% of the average period. The solid line in Figure 1(a) shows the variation of $v_0$ used to model the periods of the cycles.

Now, we repeat the same analysis in a low-diffusivity model too. In this model, the dependency of the period on $v_0$ is given by Equation (12) of Dikpati & Charbonneau (1999). From the observed periods, we calculate $v_0$ according to this relation and we show this variation by the dashed line in Figure 2(a). Then we fit the periods of the last 23 cycles by trial and error in the same way as we have done earlier.

Next, to reproduce the Maunder minimum, we decrease $v_0$ rapidly to a very low value in both the hemispheres. We have done this in the decaying phase of the last sunspot cycle before the Maunder minimum. Then we keep $v_0$ at a low value for around 1 year and then we again increase it slowly to the usual value but at different rates in two hemispheres. In northern hemisphere, $v_0$ is increased at a slightly lower rate than in the southern hemisphere. Note that we have varied only $v_0$ and no other parameters of the model.

In the next step, we have included the effect of the fluctuations in the Babcock–Leighton process of generating the poloidal field along with the fluctuations of meridional circulation. As the Babcock–Leighton process involves randomness, the poloidal field at the end of a cycle will be different from the average field obtained in the mean field model. Choudhuri et al. (2007) have proposed that the cumulative effect of the fluctuations of the Babcock–Leighton process can be taken into account just by multiplying a random factor $\gamma$ with the poloidal fields above $0.8 R_{\odot}$ at every minimum of the solar cycle. The poloidal fields below $0.8 R_{\odot}$ are left unchanged as it is believed that these are from the earlier cycles. We follow the same algorithm here.
Hence, we decreased the poloidal field by a factor $\gamma$ along with $v_0$. We run the model for different values of $\gamma$ from 0 to 1 at each value of $v_0$ from a very low value to the average value. Then we find out the critical values of $v_0$ and the corresponding $\gamma$ factor for which we get a Maunder-like minimum.

We have also checked whether a strong fluctuation of meridional circulation can produce a Maunder-like grand minimum in the low-diffusivity model. To do this, we have decreased $u_0$ at the same ratio as we have done earlier in the high-diffusivity model. However, here we do not include the effect of fluctuations of the polar field along with the fluctuations of meridional circulation as we have done earlier.

4. RESULTS

4.1. Modeling Last 23 Cycles

First, we discuss the results of fitting the periods of the last 23 cycles by varying $v_0$ in the high-diffusivity model. The results are shown in Figure 1. It may be noted that we have not tried to match the periods exactly. In order to show how good the theoretical periods have been matched with that of the observed periods, we have shown the scatter diagram between them in Figure 1(c) (linear corr. coeff. = 0.90). The solid line in Figure 1(a) shows the variation of $v_0$ required to fit the periods of the last 23 cycles. As this $v_0$ determines the period of the solar cycle, this may be taken as an indicative of how $v_0$ had varied over the last few centuries. It appears from the same figure that during cycles 1–10, the meridional circulation had large variation with a timescale of around 15 years. On the other hand, during cycles 11–22, the meridional circulation had relatively smaller variation with a timescale of around 45 years. With the limited data we have, we cannot say whether the behavior of cycles 1–10 is more typical in the long run or the behavior of cycles 11–22 is more typical. Looking at Figure 1(a), we can only surmise that the meridional circulation probably has long-time variations having large coherence time.

Indeed, Karak & Choudhuri (2010b) have also assumed a longer coherence time of the meridional circulation while studying the Waldmeier effect. So just by fitting the periods of the last 23 cycles data we get some idea about both the amplitude variation and the timescale of meridional circulation over the last few centuries. Next, in Figure 1(b), we show the theoretical sunspot series (eruptions) by the dashed line along with the observed sunspot series by the solid line. The theoretical sunspot series has been multiplied by a factor to match the observed value. It is very interesting to see that the amplitude of the theoretical sunspot cycle has a similar trend as that of the observed sunspot cycle. In order to see the correlation between the amplitudes of peak theoretical sunspot number and that of observed sunspot number, we have shown the scatter diagram between them in Figure 1(d). We have found a significant correlation between these two (having linear Pearson’s corr. coeff. = 0.75 with a significance level of 99.9%). This suggests that a major part of the fluctuations of the amplitude of the solar cycle may come from the fluctuations of meridional circulation. This is a very important result of this analysis.

It is also interesting to see the results from the low-diffusivity model. We show the results of this analysis in Figure 2. The solid line in Figure 2(a) shows the variation of $u_0$ over the last 23 cycles required to fit the periods of the cycles. It may be noted that in this model the value of $u_0$ required to get an 11 year period is $14.5 \text{ m s}^{-1}$. Although the amplitude of meridional circulation at the surface (what in the previous model is $v_0$) is different than $u_0$, this $u_0$ variation in Figure 2(a) is similar to the previous result from the high-diffusivity model. Therefore, the low-diffusivity model also gives a similar result of the amplitude variation and the timescale variation of meridional circulation. Now let us see how the amplitude of the sunspot cycle responds with the amplitude of meridional circulation. In this model, the magnetic energy density of the toroidal field ($B^2$) at latitude $15^\circ$ at the base of the convection zone is a measure of the sunspot number (Dikpati & Charbonneau 1999) which we also have followed.

Figure 2. Same as Figure 1, but from the low-diffusivity model. Here, the dashed line in panel (a) shows the variation of $u_0$ calculated from Equation (12) of Dikpati & Charbonneau (1999) by putting the observed periods of the last 23 cycles.
In Figure 2(b), we have shown the theoretical sunspot number by a dashed line along with the observed sunspot number by a solid line. We see from this figure that the amplitudes of this theoretical sunspot number are not matching with the observed amplitudes. Rather we have found a weak inverse correlation between these two amplitudes (linear corr. coeff. = -0.13).

Now, we explain the physics behind these two completely different results. It is well accepted that the period of the solar cycle is strongly determined by the value of $v_0$. This is true in both the high-diffusivity model and the low-diffusivity model. However, the value of $v_0$ affects the amplitude of the solar cycle differently in these two models. This can be understood on the basis of the Yeates et al. (2008) study in the following way. We have seen that in these models, the toroidal field is generated by the stretching of the poloidal field in the tachocline. The production of this toroidal field is more if the poloidal field remains in the tachocline for a longer time and vice versa. This is exactly observed in the observational data (see Figure 1(C) of Charbonneau & Dikpati 2000). However, this is not the case in the low-diffusivity model. This is because in this model the diffusive decay of the fields is not of much importance. As a result, the slower meridional circulation means that the poloidal field remains in the tachocline for a longer time and therefore it produces more toroidal field, giving rise to a strong cycle. Ultimately, we get stronger amplitudes for longer periods (having lower values of meridional circulation) and vice versa, which is not observed. Therefore, we do not get a correct correlation between the amplitudes of theoretical sunspot number and that of observed sunspot number when only the meridional circulation is varied.

### 4.2. Modeling Maunder Minimum

In Figure 3, we show the theoretical results covering the Maunder minimum episode from the high-diffusivity model. Figure 3(a), shows the maximum amplitude of meridional circulation $v_0$ varied over this period in two hemispheres. The key point to note is that in our calculation we have increased $v_0$ differently in two hemispheres in the recovery phase. In northern hemisphere (shown by the solid line in Figure 3(a)), $v_0$ is increased slightly at a lower rate than southern hemisphere (shown by a dashed line). Consideration of this north–south asymmetry of meridional circulation may be justified. It is unlikely that the meridional circulation decreased by the same amount in both hemispheres. Even if it did, it is very unlikely that it again increased at the same rate. In addition, recent observational studies (Haber et al. 2002, Basu & Antia 2003) of surface meridional circulation give the evidence of hemispheric asymmetry of meridional circulation which can lead to the asymmetry in solar activity (Dikpati et al. 2004). In Figure 3(b), we show the butterfly diagram of sunspot numbers, whereas in Figure 3(c), we show the variation of total sunspot number along with the individual sunspot numbers in two hemispheres (see the
In order to facilitate comparison with observational data, we have taken the beginning of the year to be 1635. Note that in Figures 3(b) and (c), the sunspot number at the beginning of the Maunder minimum suddenly falls to zero value, whereas in the recovery phase, it increases slowly. In addition, the north–south asymmetry of sunspot number observed in the last phase of the Maunder minimum is remarkably reproduced. In order to compare our results with the observational results, the readers may compare our Figure 3(b) with Figure 1(a) of Sokoloff & Nesme-Ribes (1994) and Figure 3(c) with Figure 1 of Usoskin et al. (2000). We also show the energy density of the toroidal field calculated at latitude 15° at the bottom of the convection zone in Figure 3(d). Here, the rapid decrease of the \( v_0 \) to a critical value (10 m \( \text{s}^{-1} \)) from its average value (22 m \( \text{s}^{-1} \)) triggers a Maunder minimum. However, this critical value depends on the value of the diffusivity (and also on the alpha coefficient) as it determines the growth rate (Choudhuri & Karak 2009). Indeed, we do not know the exact value of diffusivity in the convection zone; we only have some idea of its order of magnitude estimate. Therefore, if we take a slightly higher (lower) value of diffusivity, then we can reproduce the Maunder minimum even at a higher (lower) critical value of \( v_0 \).

Figure 4. Theoretically calculated average radial magnetic field (absolute value in arbitrary units) in the solar wind.

Figure 5. Profile of amplitude of meridional circulation which produces the Maunder minimum similar to Figure 3(b). The format is identical to Figure 3(a).

caption of Figure 3(c)).}


to get rise to a large-scale polar field for the new cycle (Stenflo 1972; Wang et al. 1989; Dikpati et al. 2004). At the maximum phase of the solar cycle, many sunspot eruptions go on at the surface and at the same time, these sunspots decay in a short timescale to generate the poloidal field. At this time, if the amplitude of meridional circulation decreases strongly, then the poloidal field, which is being generated at the surface at low latitude, is not able to move toward the pole to cancel the opposite polarity field of the previous cycle; rather, a part of it diffuses away toward the opposite hemisphere to cancel out the opposite polarity field. Therefore, it is not able to generate a large amount of the polar field for the next cycle. Naturally, the decrease of meridional circulation causes a weaker polar field (see also Wang & Sheeley 2003 and Baumann et al. 2004 for details). This weaker polar field gives a weaker toroidal field (Choudhuri et al. 2007). Now in our simulation, the length of a grand minimum has been determined by the time for which the value of meridional circulation remains at a low value and the rate at which meridional circulation is increased after the grand minimum has started. In this way, we are able to reproduce any grand minimum of any length.

We have no knowledge of how meridional circulation varied during the Maunder minimum. In the above calculation, at the beginning of the Maunder minimum, we have decreased \( v_0 \) rapidly to a low value, whereas in the recovery phase, we increased it rather slowly. However, our results are completely independent of the way that we increase \( v_0 \) in the recovery phase. For example, the variation of \( v_0 \) in Figure 5 reproduces an exactly similar Maunder minimum, which we have evinced in Figure 3(b). Here we underscore that if the meridional circulation falls very slowly to a low value, then we do not get the sudden fall of sunspot number at the beginning of the Maunder minimum. However, if we are to believe the observational results of Usoskin et al. (2000), then it says that at the beginning of the Maunder minimum, solar activity decreased abruptly, but built up gradually. This strongly suggests that the toroidal field decreased suddenly below a certain value which triggered the Maunder minimum and then it took sometime to regain its usual value. If this sudden fall of the toroidal field is due to the reduction of meridional circulation, then it probably indicates that the meridional circulation dropped rapidly to a very low value at the beginning. However, it does not indicate anything about the rate of recovery of meridional circulation.

Let us now discuss the physics of getting the Maunder minimum in this model. The meridional circulation affects the generation not only of the toroidal field but also that of the polar field (Wang & Sheeley 2003; Baumann et al. 2004). Poloidal field generated at the surface through the decay of sunspots is advected by meridional circulation (and also diffuses) toward the pole where it cancels the opposite polarity field of the previous cycle; rather, a part of it diffuses away toward the opposite hemisphere to cancel out the opposite polarity field. Therefore, it is not able to generate a large amount of the polar field for the next cycle. Naturally, the decrease of meridional circulation causes a weaker polar field (see also Wang & Sheeley 2003 and Baumann et al. 2004 for details). This weaker polar field gives a weaker toroidal field (Choudhuri et al. 2007). In addition, according to the discussion of the previous section the lower value of meridional circulation means more time for the diffusive decay of the poloidal field during its transport through the convection zone, leading to less generation of the toroidal field in the tachocline. This weak toroidal field is insufficient to produce sunspot eruptions (D’Silva & Choudhuri 1993; Fan et al. 1993; Caligari et al. 1995) and therefore triggers a grand minimum. However,
Choudhuri & Dikpati (1999) have shown that the poloidal field modeling the distribution of a large-scale solar magnetic field, at some point, the toroidal field exceeds the critical value of and therefore the generation of the toroidal field is enhanced. Again, the poloidal field does not get much time to diffuse when the amplitude of meridional circulation is increased. 

Figure 6. Parameter space of amplitude of the meridional circulation \(v_0\) and the poloidal field reduction factor \(\gamma\). The line shows the values of these parameters which give Maunder-like grand minima.

When the amplitude of meridional circulation is increased again, the poloidal field does not get much time to diffuse and therefore the generation of the toroidal field is enhanced. At some point, the toroidal field exceeds the critical value of producing sunspots eruption. In this way, it recovers from the grand minimum state.

The flux tube simulations (Choudhuri 1989; D’Silva & Choudhuri 1993; Fan et al. 1993) suggest that the initial magnetic field inside the flux tube has to be of order \(10^5\) G and the sunspot eruption takes place only when the toroidal field reaches this value. In this model also, the sunspot eruptions take place if the value of the toroidal field exceeds this critical value (Nandy & Choudhuri 2001; Chatterjee et al. 2004). During the Maunder minimum, the toroidal field at the base of the convection zone was below this value and there was no eruption. However, the weak solar cycles continued during this period. Note that in the absence of sunspot eruptions, the usual Babcock–Leighton process does not work. However, the toroidal field advects (and also diffuses) toward the upper part of the convection zone by the upward meridional circulation. In this case, the \(\alpha\) coefficient in our equation acts like a traditional mean field \(\alpha\) (Parker 1955). In modeling the distribution of a large-scale solar magnetic field, Choudhuri & Dikpati (1999) have shown that the poloidal field has two sources—the Babcock–Leighton mechanism and the \(\alpha\)-effect working on a weaker subsurface toroidal field. During the Maunder minimum—when there were no sunspots—the dynamo continued due to the \(\alpha\)-effect operating on the weaker toroidal field. Also, the \(\alpha\)-effects operating in the tachocline (Dikpati & Gilman 2001; Bonanno et al. 2002, and references therein) or the \(\alpha\)-effect arising due to the buoyancy instability at the base of the convection zone (Ferriz-Mas et al. 1994) may play a role in this episode. In this calculation, however, we do not consider these \(\alpha\)-effects.

Now we include the effect of fluctuations of the polar field along with the fluctuations of meridional circulation in this model. We have seen earlier that the strong decrease of \(v_0\) can lead to a grand minimum. However, in that case, its value had to be reduced to a significantly low value, which may be a bit surprising. We have mentioned that the polar field fluctuates randomly as its generation mechanism—the Babcock–Leighton process—involves randomness. Therefore, it is important to include these fluctuations along with the fluctuations of meridional circulation in modeling grand minima. We find that if we decrease the polar field by a factor of \(\gamma\), then even at a moderately lower value of \(v_0\), we are able to reproduce Maunder-like grand minima. We have repeated our calculation at different values of \(v_0\) and \(\gamma\) and found that for each value of \(v_0\) there is a corresponding value of \(\gamma\) which can give a Maunder-like grand minimum. These values are shown by the line in Figure 6. The values lower than these \(\gamma\)’s give grand minima longer than the Maunder minimum (like Spörer minimum), whereas larger values give grand minima shorter than the Maunder minimum. These two regions are indicated in Figure 6. We mention that Choudhuri & Karak (2009) have reproduced the Maunder minimum just by decreasing the poloidal field to 0.2 (actually \(\gamma_N = 0.0\) and \(\gamma_S = 0.4\)) of its original value, keeping \(v_0\) unchanged. However, in the present calculations, we have to reduce \(v_0\) to around 13.5 m s\(^{-1}\) along with the poloidal field reduction to 0.2. This is because in those calculations we had reduced the toroidal field slightly along with the change of the poloidal field to compensate for the overlap between cycles. Here, we have not done this kind of questionable change of toroidal field. Additionally, here the values of \(\alpha\) and \(n_\gamma\) are different than those calculations giving a different dynamo growth rate.

Now let us briefly discuss the results of the low-diffusivity model. In this model, lesser circulation speed means that the poloidal field gets more time to produce the toroidal field in the tachocline. Thus, smaller \(v_0\) gives more toroidal field and finally a stronger cycle. Consequently, if the dynamo is in the advection-dominated regime, it does not produce any grand minimum; rather, it produces stronger cycles. This is exactly what we find in our simulation. The results are shown in Figure 7. In this figure, panel (a) shows the variation of \(u_0\) during the Maunder minimum episode, whereas panel (b) shows the theoretical sunspot number with time. We see that the sunspot number has rather increased when the meridional circulation is decreased rapidly from its
average value (14.5 m s\(^{-1}\)) to a very low value (6.6 m s\(^{-1}\)). However, for one cycle (around 1680–1690 and also around 1725–1735) after two cycles of the strong reduction of \(v_0\), we get a reduction of sunspot number. This is due to the decrease of the polar field during the low value of \(v_0\) as explained earlier. It may be noted that in this model, the decrease of the polar field affects the sunspot cycle of two cycles later (see Figure 9 of Jiang et al. 2007). We may point out that if we increase \(u_0\) to a very high value for several years, then we may get a Maunder-like minimum in this model. However, in this case, the cycle periods will be unrealistically short and no observation during the Maunder minimum validates this.

### 5. CONCLUSION

We have studied the importance of meridional circulation on the period and the amplitude of the solar cycle and also on the Maunder-like grand minimum in two different regimes of a flux transport dynamo model, namely, advection-dominated and diffusion-dominated. First, we have approximately modeled the periods of the last 23 solar cycles by varying \(v_0\) alone. From this study, we get some idea about both the amplitude variation and the timescale of the meridional circulation over the last few centuries. Moreover, we have seen that when we match the periods of these cycles by varying \(v_0\) in the high-diffusivity model, most of the cycle amplitudes also get modeled up to some extent. Therefore, we conclude that a major part of the fluctuations of solar cycle amplitude may come from the meridional circulation fluctuations. In the low-diffusivity model, the cycle amplitudes do not get modeled at all when we repeat the same analysis.

We have also shown the possibility of producing (or not producing) a Maunder-like minimum in two models by varying the meridional circulation speed. To do this, we have assumed that at the beginning of the Maunder minimum the amplitude of meridional circulation largely reduced and then after a few years it increased again. It may be noted that this is not an ad hoc assumption. There are several independent arguments that support our assumption. First, Wang & Sheeley (2003) used the flux transport model to simulate the evolution of the Sun’s magnetic dipole moment, polar fields, and open flux under Maunder minimum conditions and suggested that the poleward surface flow speed was reduced from 20 m s\(^{-1}\) to 10 m s\(^{-1}\) during that time. Second, the flux transport dynamo model predicts an inverse correlation between the meridional circulation speed and the cycle period (Wang et al. 1991; Dikpati & Charbonneau 1999). On the other hand, from the study of \(^{14}\)C data during the Maunder minimum, Miyahara et al. (2004; also Miyahara et al. 2010) reported that the periods of the solar cycle were longer compared with the usual 11 year period. Now, if the flow speed determines the solar cycle period, then it indicates that the meridional circulation speed during the Maunder minimum was lower than the usual value. Third, while simulating Sun’s large-scale magnetic field during cycles 13–22, Wang et al. (2002) have shown that the regular polarity reversal is obtained only if the flow speed is assumed to be correlated with the cycle amplitude (see also Hathaway et al. 2003). On the other hand, Yeates et al. (2008) have shown that the cycle amplitude is positively correlated with the speed of meridional circulation in the diffusion-dominated regime. Now, both Beer et al. (1998) and Miyahara et al. (2004) found that the amplitudes of solar cycle were weaker during the Maunder minimum. Therefore, if flow speed determines the cycle amplitude, then it probably indicates that the meridional circulation during the Maunder minimum was weaker than the usual one. Last, using the low-order dynamo model, Passos & Lopes (2009) suggest that the stochastic fluctuations in the \(\alpha\)-effect cannot trigger a grand minimum; rather, they argued that the decrease in the amplitude of the meridional flow can do so.

We have no idea why the meridional circulation dropped to a very low value. Therefore, our assumption may be a bit questionable with the present understanding. However, this assumption enables us to reproduce most of the important features of the Maunder minimum remarkably well. It may be noted that to produce a Maunder-like minimum in our model, we did not have to decrease \(v_0\) to an unrealistically low value (10 m s\(^{-1}\) is required here).

We emphasize that we can reproduce these results only if the dynamo is in the diffusion-dominated regime. However, recently several independent works on the parity (Chatterjee et al. 2004; Hotta & Yokoyama 2010b), the hemispheric coupling (Chatterjee & Choudhuri 2006; Goel & Choudhuri 2009), the correlation between the polar field at the minimum and the sunspot number in the next cycle (Jiang et al. 2007), the correct value of the polar field (Hotta & Yokoyama 2010a), the correlation between the flow speed and the cycle amplitude (Wang et al. 2002; Hathaway et al. 2003; Yeates et al. 2008), and the Waldmeier effect (Karak & Choudhuri 2010b) suggest that the solar dynamo is working in the diffusion-dominated regime and not in the advection-dominated regime. In addition, from the low-order time-delay dynamo model, Wilmot-Smith et al. (2006) have shown that the irregular activity and the Maunder-like events are more readily excited in the diffusion-dominated regime and not in the advection-dominated regime.

However, we are not pointing out that the decrease of meridional circulation is the “only” possibility to produce grand minima, but the abrupt decrease of the polar field due to fluctuations in the Babcock–Leighton process may be a valid possibility (Choudhuri & Karak 2009). As the measurement of either the meridional circulation or the polar field during the Maunder minimum is unavailable, we do not know which possibility is correct. In this paper, we have adequately shown that a large fluctuation of meridional circulation alone can lead to a grand minimum.

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