The Chiral Limit of Non-compact QED

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Non-compact QED\textsubscript{3} with four-component fermion flavor content $N_f \geq 2$ is studied numerically near the chiral limit to understand its chiral symmetry breaking features. We monitor discretization and finite size effects on the chiral condensate by simulating the model at different values of the gauge coupling on lattices ranging in size from $10^3$ to $50^3$. Our upper bound for the dimensionless condensate $\beta^2 \langle \bar{\Psi} \Psi \rangle$ in the $N_f = 2$ case is $5 \times 10^{-5}$.

1. Introduction

Over the last few years, QED\textsubscript{3} has attracted a lot of attention because of its potential applications to models of high $T_c$ superconductivity. It is believed to be confining and exhibit features such as dynamical mass generation when the number of fermion flavors $N_f$ is smaller than a critical value $N_{fc}$. It is therefore an interesting and challenging model and an ideal laboratory in which to study more complicated gauge field theories.

We are considering the four-component formulation of QED\textsubscript{3} where the Dirac algebra is represented by the $4 \times 4$ matrices $\gamma_0$, $\gamma_1$ and $\gamma_2$. This formulation preserves parity and gives each spinor a global $U(2)$ symmetry generated by $1$, $\gamma_3$, $\gamma_5$ and $i\gamma_3\gamma_5$; the full symmetry is then $U(2N_f)$. If the fermions acquire dynamical mass the $U(2N_f)$ symmetry is broken spontaneously to $U(N_f) \times U(N_f)$ and $2N_f^2$ Goldstone bosons appear in the particle spectrum.

At the present time the issue of spontaneous chiral symmetry breaking in QED\textsubscript{3} is not very well understood. Studies based on Schwinger-Dyson equations (SDEs) using the photon propagator derived from the leading order $1/N_f$ expansion suggested that for $N_f$ less than some critical value $N_{fc}$ the answer is positive with $N_{fc} \approx 3.2$ \cite{1}. Apparently, for $N_f > N_{fc}$, the attractive interaction between a fermion and an antifermion due to photon exchange is overwhelmed by the fermion screening of the theory’s electric charge. More recent studies which treat the vertex consistently in both numerator and denominator of the SDEs have found a value for $N_{fc}$ either in agreement with the original study or slightly higher, with $N_{fc} \approx 4.3$ \cite{2}. Finally an argument based on a thermodynamic inequality has predicted $N_{fc} \leq \frac{3}{2}$ \cite{3}. There have also been numerical attempts to resolve the issue via lattice simulations. The obvious advantage in this case is that one can study any $N_f$ without any assumption concerning the convergence of expansion methods. However, the principal obstruction to a definitive answer has been large finite volume effects resulting from the presence of a massless photon. Numerical studies of the quenched case have shown that chiral symmetry is broken \cite{4}, whereas in the case of simulations with dynamical fermions opinions have divided on whether $N_{fc}$ is finite and $\approx 3$ or whether chiral symmetry is broken for all $N_f$ \cite{5}. Recent studies of the $N_f = 1$ model on small lattices appear in \cite{6}.

In our study we used the Hybrid Monte Carlo algorithm to simulate the non-compact version of lattice QED\textsubscript{3} with staggered fermions. In the continuum it corresponds to the four-component spinor formulation of the model \cite{7}. We implement even-odd partitioning which implies that a single flavor of one-component staggered fermions can be simulated, which corresponds to $N_f = 2$. 



in the continuum limit.

2. Results

In this section we discuss the results of lattice simulations of QED$_3$ with $N_f \geq 2$. In our study we tried to detect and control the various drawbacks of the lattice method: (i) The lattice itself distorts continuum space-time physics considerably unless the lattice spacing $a$ can be chosen small compared to the relevant physical wavelengths in the system. (ii) the size of the lattice $L^3$ must be large compared to the dynamically generated correlations in the system; and (iii) the chiral limit can only be studied by simulating light fermions of mass $m_0$ in lattice units.

Figure 1. Dimensionless condensate $\beta^2\langle \bar{\Psi}\Psi \rangle$ vs. dimensionless bare mass $\beta m_0$ for $N_f = 2, 4, 8, 16$, $\beta = 0.6$ on a $16^3$ lattice.

In Fig.1 we plot the dimensionless chiral condensate $\beta^2\langle \bar{\Psi}\Psi \rangle$ vs. the dimensionless bare mass $\beta m_0$ (where $\beta \equiv \frac{1}{\sigma a}$) for $N_f = 2, 4, 8, 16$. The coupling $\beta = 0.6$ and the lattice volume is $16^3$. As $N_f$ increases the chiral condensate decreases (for $m_0 \geq 0$) because the interaction between the fermion and the antifermion is screened. However, as $m_0 \to 0$ all the curves tend to pass smoothly through the origin. This motivated us to study in more detail the pattern of chiral symmetry breaking at small $N_f$ on larger volumes near the chiral limit.

In order to check whether our lattice data are characteristic of the continuum limit we plot in Fig.2 $\beta^2\langle \bar{\Psi}\Psi \rangle$ vs. $\beta m_0$ for $N_f = 2$ and coupling $\beta = 0.45, 0.60, 0.75, 0.90$. To disentangle the lattice discretization effects from the finite size effects we keep the volume in physical units $(L/\beta)^3$ constant. It can be inferred from the graph that discretization effects are small for $\beta \geq 0.60$, because the data almost fall on the same line within the resolution of our analysis.

In Fig.3 we present our results for the chiral condensate vs. bare mass for $N_f = 2$ at different values of the coupling $\beta$ and constant physical volume $(L/\beta)^3$. We infer that finite size effects become small for $L \geq 24$ and all the lines tend to pass smoothly through the origin. Our analysis of meson masses and susceptibilities in scalar and pseudoscalar channels showed that these quantities suffer from very strong finite size effects and therefore did not allow us to reach such a definitive conclusion.

Next we discuss the results from $N_f = 2$ simulations at $\beta = 0.75$. The $\beta = 0.75$ data set is closer to the continuum limit than the data extracted at $\beta = 0.60$. However, particular care is required because weak coupling data are very sensitive to finite size effects and accurate measurements require simulations on large lattices. In Fig.4 we present the results for the chiral condensate vs. fermion bare mass extracted from simulations with lattice sizes ranging from $10^3$ to $50^3$. These simulations were performed very close to the chiral limit, i.e. with $m_0 \leq 0.005$. We can
see from the figure the finite size effects are under relatively good control and the data tend to pass smoothly through the origin. Therefore, we conclude that for $N_f = 2$, $\beta^2 \langle \bar{\Psi} \Psi \rangle \leq 5 \times 10^{-5}$, which is a strong indication that QED$_3$ may be chirally symmetric for $N_f \geq 2$.

Figure 3. Dimensionless condensate vs. dimensionless bare mass for $N_f = 2$, $\beta = 0.6$ and lattice sizes $8^3, 16^3, 24^3, 32^3, 48^3$.

Figure 4. Condensate vs. bare mass for $N_f = 2$, $\beta = 0.75$ and lattice sizes $10^3, 20^3, 30^3, 40^3$ and $50^3$.

3. Conclusions

In our study of QED$_3$ with $N_f \geq 2$ we attempted to establish whether chiral symmetry is broken or not by studying the behavior of the chiral condensate close to the continuum limit $g \to 0$, on different volumes in order to detect and control finite size effects and near the chiral limit $m_0 \to 0$.

Our upper bound for the condensate in the $N_f = 2$ case is $\beta^2 \langle \bar{\Psi} \Psi \rangle \leq 5 \times 10^{-5}$ and all the lines of $\beta^2 \langle \bar{\Psi} \Psi \rangle$ vs. $\beta m_0$ tend to pass smoothly through the origin which may imply that chiral symmetry is restored for $N_f \geq 2$. We are continuing this study to check if chiral symmetry is broken in the case of $N_f = 1$.

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