Adjustable electronic phantom for volume magnetic susceptibility measurements

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Abstract. The principle is based on idea to create an electronic model (ES) of the magnetized real object consisting of a set of identical small elements, each of the volume of \( \Delta V^{(E)} \). The sources of identical magnetization are represented by inserted miniature magnetization coils supplied by the current \( i_s \), radius \( r_c \) and the number of turns \( N_c \). The ES behaves like the original sample (of the same dimensions and shape) in the magnetization field \( H_o \) with external magnetic susceptibility \( \chi^{(o)} = \pi N_c r_c^2 i_s / (\Delta V^{(E)} H_o) \). The paper deals with the samples of the cylindrical shape inserted to the axial homogeneous magnetic field. The principle validity of given ideas and derived base relations has been proved experimentally using a 2nd-order SQUID gradiometer.

1. Introduction
Applications of the SQUID magnetometric systems in biomedicine [1] have significantly increased interest in the field of measurement of the magnetic susceptibility of the various substances with various shapes. A typical example is the measurement of ferritin content in the human liver [2] where the problem of low value (~ \( 10^{-6} - 10^{-7} \)) (SI) volume magnetic susceptibility measurement is complicated with the problems of the liver signal selection from the signals of the surrounding tissue [3]. These signals are often markedly greater than measured signal. A further typical problem in low magnetic susceptibility measurements is a weak signal response to magnetization field. This requires the use of relatively strong magnetization fields, however, with the consequent tendency to overlap (with intensity of many orders higher) the useful signal in the sensor location. The useful signal low level is connected with the problem of the ambient disturbing magnetic field reduction when measuring in magnetically unshielded environment and with consideration of the measuring system intrinsic noise [4]. The above given is connected with the problem of determination of individual contributions of several sources, with different shape, distributed in space [5].

2. Theoretical analysis
The disturbing magnetic field in magnetically unshielded laboratories in suburban areas and the intrinsic noise of the superconducting quantum magnetometers (SQM) with SQUID sensors has a character of the 1/f noise in the low-frequency band. Therefore on general conditions it is advantageous to avoid the frequency band below ~ 1 Hz and to chose the AC magnetization fields of higher frequencies. In further text the instantaneous values will be used for the signal quantities. For the magnetic moment \( m_s \) (figure 1) of the isotropic sample (with the shape of the rotational ellipsoid)
of the volume of $V_s$, magnetized in the external homogeneous magnetic field with the intensity $H_o$ (SI units), it holds \[ m_s = M_s V_s, \] where $M_s = \chi_{\text{ext}}^{(s)} H_o$, or $M_s = \chi^{(s)} H_{\text{int}}$, (1)

where

- $M_s$ is volume magnetization and for magnetic field $H_{\text{int}}$ in the volume of the sample it holds $H_{\text{int}} = H_o / (1 + N \chi)$,

- $\chi^{(s)}$ is volume magnetic susceptibility of the sample’s material - which is related to the external susceptibility $\chi_{\text{ext}}^{(s)}$ by the expressions

$$
\chi^{(s)} = \chi_{\text{ext}}^{(s)} / (1 - N \chi_{\text{ext}}^{(s)}), \quad \text{or} \quad \chi_{\text{ext}}^{(s)} = \chi^{(s)} / (1 + N \chi^{(s)}),
$$

where $N$ $(0 \leq N \leq 1)$ is the demagnetization factor depending on the sample’s shape.

From these equations for the external volume susceptibility of measured sample it follows

$$
\chi_{\text{ext}}^{(s)} = \frac{m_s}{V_s} H_o^{-1}, \quad \text{and} \quad \chi^{(s)} = \left( \frac{H_o V_s}{m_s} - N \right)^{-1}. \quad (3)
$$

For small susceptibility ($|N \chi^{(s)}| << 1$), the influence of the demagnetization factor $N$ can be neglected and equation (3) becomes

$$
\chi^{(s)} \equiv \frac{m_s}{V_s} H_o^{-1}. \quad \left( |\chi^{(s)}| \leq 10^{-2}, \text{uncertainty } \pm 1 \%ight). \quad (3a)
$$

On preceding assumptions (isotropy of the material, homogeneity of the magnetization field, the shape of rotational ellipsoid), for the magnetic moment volume density $M_s = m_s / V_s$ of measured sample and for the volume density of each its volume element $\Delta V_s$ with the magnetic moment $\Delta m_s$ it holds

$$
\Delta m_s / \Delta V_s = m_s / V_s.
$$

It is possible to achieve such state also with a dimension- and shape-equivalent electronic model (ES) of measured object, if it consists of a set of identical volume elements $\Delta V_s^{(ES)}$, all of them with the same internal generator of the magnetic moment $\Delta m_s^{(ES)}$ (figure 1). This generator is represented by “miniature” current loop with the radius $r_c$ supplied by the current $i_s$. The loop with $N_c$ turns induces the magnetic moment $\Delta m_s^{(ES)} = N_c \pi r_c^2 i_s$ and each element has the volume density of the magnetic moment

$$
\frac{\Delta m_s^{(ES)}}{\Delta V_s^{(ES)}} = \frac{m_s}{V_s}.
$$

(4a)

All current loops are equally oriented; their surfaces are perpendicular to the direction of the considered magnetization field, and they are serially connected. The size of the loops is designed in such a way that they lay inside, in the centres of the volume elements. The better the condition $\Delta V_s^{(ES)} \to 0$ (and concurrently $r_c \to 0$) is fulfilled, the more precise the simulation is. For given parameters, the relation (4a) holds for ES if the supplying current has the value $i_s = i_{so}$

$$
i_{so} = \frac{\Delta V_s^{(ES)}}{\pi N_c r_c^2} M_s.
$$

(5)
For the susceptibility $\chi^{(ES)}$ of such created electronic sample (equation (3)) it holds that

$$
\chi^{(ES)}_{\text{ext}} = \frac{\pi N i_s^2}{\Delta V_s^{(ES)} H_o} \text{ or } \chi^{(ES)}_{\text{ext}} = \frac{\pi N i_s^2 \mu_o}{\Delta V_s^{(ES)} B_o} i_s,
$$

where $B_o = \mu_o H_o$ ($\mu_o = 4\pi \times 10^{-7}$) is the magnetic induction corresponding to intensity of the external magnetization field $H_o$ in vacuum. If the supplying current $i_s = i_o$ (equation (5)), the “electronic sample” exactly imitates the state of real measured sample inserted to the magnetic field $H_o$ and for their magnetic susceptibilities it holds $\chi^{(ES)}_{\text{ext}} = \chi^{(s)}_{\text{ext}}$. The volume susceptibility $\chi^{(s)}$ of the measured sample material is then given by the equation (2). For small susceptibility values, it holds $\chi^{(s)} = \chi^{(s)}_{\text{ext}}$ (equation (3)).

The volume elements (VE) with the volume of $\Delta V_s^{(ES)}$ (figure 1) are arranged in the rings denoted $q = 1,2,...,q_{\text{max}}$ with constant width $s_o$. Their further two dimensions are determined by the circumference $l_o$ of the central circles of the rings with diameters $R_q$ and the layer thickness $h_o$. These parameters are chosen in such a way that the radius $r_s$ of the cylindrical sample, its height $h_s$, and the circumferences $2\pi R_q$ of the central circles are integer multiples of $s_o$, $h_o$, and $l_o$, respectively. Therefore it holds

$$
s_o = r_s / q_{\text{max}}, \quad R_q = s_o(q - 0.5), \quad q = 1,2,...,q_{\text{max}},
$$

where $q_{\text{max}}$ is the total number of the rings (concurrently the number of the greatest ring),

$$
h_o = h_s / w_{\text{max}}, \quad w = 0,1,...,w_{\text{max}} - 1,
$$

where $w_{\text{max}}$ is the total number of layers and $w$ is the number of layer in the direction from the gradiometer ($w_{\text{max}}$ is chosen in such a way to ensure $h_o = s_o$),

$$
l_o = \frac{2\pi R_{q-1}}{p_{q-1}} = \frac{\pi s_o}{p_{q-1}},
$$

where $p_{q-1}$ is the total number of the volume elements for the first ($q = 1$) – smallest ring. When the shape of the volume elements approximates to the cube with the length of the edge $s_o$, the condition $l_o = s_o$ should be fulfilled, which implies that $p_{q-1} = 3$. Then for the number of the volume elements in the rings $q$ and for their volume $\Delta V_s^{(ES)}$ it holds

$$
p_{q} = 6q - 3, \quad \Delta V_s^{(ES)} = \frac{\pi}{3} h_o s_o^3 = \text{const.}
$$

The number of all volume elements in the whole ES volume is

$$
N_{v} = w_{\text{max}} \sum_{q=1}^{w_{\text{max}}} p_{q}. \quad (p_{q} = 3,9,15,...) \quad N_{v} = 3w_{\text{max}},12w_{\text{max}},27w_{\text{max}},48w_{\text{max}}...)
$$

The exact mathematical model for the cylindrical samples with these parameters is given in detail in [5].
3. Experiment – the basic operation

In practical measurement with prepared ES it is advantageous to replace the instantaneous values of electrical and magnetic quantities by corresponding amplitudes $I$, $H_0$, and $B_0$ (or their peak-to-peak values: $I_{pp}$, $H_{pp}$, $B_{pp}$; where $I_{pp}=2I$, ...) of the harmonic signals with frequency $f$.

$$i(t) = I \sin(\omega t), \quad H_0(t) = H_0 \sin(\omega t), \quad B_0(t) = B_0 \sin(\omega t) \quad (\omega = 2\pi f, \quad t - \text{time}, \quad B_0 = \mu_0 H_0). \quad (8)$$

1) In the state when the measured sample is not inserted, the effect of the magnetization field, with the amplitude $B_0$ (or $H_0$) penetrating to the SQG (in measurement in magnetically unshielded environment it is usually the gradiometer (G) of the 2nd order), is compensated by the standard way. After compensation (zero output signal without the sample) the sample is inserted to the measuring place and the amplitude $(s)_{out}$ of SQG output signal is recorded.

2) The magnetization field is turned off and the shape- and volume-identical ES is inserted to the same position towards the gradiometer. The amplitude of the AC current (with frequency $f$) supplied to the current loops of its volume elements $\Delta V^{(ES)}_i$ is adjusted to the value $I = I_{iso}$, where the SQG output signal achieves the same amplitude $U^{(ES)}_{out} = U^{(iso)}_{out}$. This way the state $\chi^{(i)}_{ext} = \chi^{(ES)}_{ext}$ is reached and the susceptibility of measured sample $\chi^{(i)}_{ext}$ is determined by the relations $(6)$ $(i, \Rightarrow I_{iso}, B_0, \Rightarrow B_0, H_0, \Rightarrow H_0)$. 

**Figure 1.** Experimental setup at measurement of magnetic susceptibility of the cylindrical samples in homogeneous magnetization field:

a) cylindrical sample with the 2nd order gradiometer,
b) elementary volume segment of the cylindrical sample.
Often it is advantageous to utilize the electronic sample as a calibration standard which is supplied by current with the amplitude $I = I_{\text{cal}}$ and the induced signal $U_{\text{out}}^{(\text{cal})}$ is read on the SQG output. For the susceptibility of the sample, which induced the output signal $U_{\text{out}}^{(s)}$ in the magnetic field $B_0$, it holds

$$\chi^{(s)} = \frac{K_B U_{\text{out}}^{(s)}}{B_0 U_{\text{out}}^{(\text{cal})}}$$

where the calibration constant

$$K_B = \frac{\pi N r_c^2 \mu_0}{\Delta V_r^{(\text{ES})}} I_{\text{cal}} \ [\text{T}].$$

(9)

For known ES parameters, dimensions of the sensing 2nd order gradiometer (figure 1.: baselength $b$, radius $r_g$ and the loops area $A_l$) and the SQG transfer parameters it is possible theoretically to compute the dependence of the output signal $U_{\text{out}}^{(s)}$ on the current $I$ and on the position of electronic sample. Such computation for cylindrical samples is done in [6]. There are also confronted the measured and theoretically computed dependences of the output signal $U_{\text{out}}^{(s)}$ of the simple ES (radius $r_g = 1.41$ cm, height $h_s = 2.3$ cm, two layers, 8 volume elements in total, calibration susceptibility $\chi_{\text{cal}} = 2.066 \times 10^{-4} I_{\text{cal}}/B_0$ [dimensionless, $\lambda_{pp}, T_{pp}$]) on its distance from the gradiometer.

4. Discussion

For the real samples of the same shape and dimensions, the ES can be used as a standard for the volume magnetic susceptibility with adjustable positive and negative magnetic susceptibility. In [6] a good agreement between the parameters measured by a 2nd order SQUID gradiometer for a prepared cylindrical ES and the parameters theoretically computed using the derived relations has been proved. Using the ES in check measurement of the volume susceptibility of distilled water the result $\chi^{(s)} = -9.09 \times 10^{-9} \pm 5\%$ has been obtained.

5. Conclusions

A method of computation and realization of electronic samples of the magnetic materials is designed. It is based on simulation of constant volume density of the magnetic moment of the real sample by a set of identical current-driven magnetization coils in the volume elements of the electronic sample. The results of experiment were consistent with the theoretical analysis.

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