A Novel Approach to Conduct Resistance Calculation Considering Skin Effect

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Abstract. With the rapid growth of power electronics, increasingly more non-linear loads access the power grid, aggravating harmonic pollution. To calculate harmonic loss, it is necessary to calculate the resistance of a conductor considering skin effect, however, most method base on the current density on a semi-infinite plane. Based on the principle of engineering electromagnetics, this paper derived the distribution of current density on a finite plane, which is also suitable for the transmission line, and therefore developed a more practical and precise enough version of frequency-dependent resistance of current carrying bar or transmission line considering skin effect. In order to make this method more convincing, a corresponding physical experiment was also conducted, proving the feasibility of this method.

1. Introduction

The increasingly more use of non-linear loads has far-reaching consequences on our power grid. Harmonics created by non-linear generates extra loss during electricity transmission. Therefore, subject to thermal stability limits, the capacity of transmission lines is also limited. Some measurement shows that residential areas and industrial enterprises, such as shopping malls, hotels and office buildings, contain current harmonics mostly between 6.25%-26.8% in 380V side, with certain point even reaching 48.8%. In neutral point, harmonic content stays more than 80%, with maximum even up to 215%.[1] Harmonic distortion not only has a serious impact on the safety of power grid but also raises the cost of power transmission.[2]

There are other already existing equation to calculate the resistance of transmission line, one of them is the most commonly used model on practical application, which was put forward by Wakileh G J[3]. The impedance of the transmission line system is given by the following equation:

\[ Z(n) = \sqrt{n} (R + jX) \]  

(1)

According to this equation, \( R_n = \sqrt{n}R_1 \) is considered to be the resistance of nth order harmonic. Although this formula seems to be concise, but the result becomes inaccurate when it comes to the resistance of high order harmonic.

Morgen gave a much more accurate version to calculate the resistance of the harmonic. Morgen ignore the electrical conductibility of the steel core in ACSR [4] (Aluminum Conductors Steel-
Reinforced) and consider ACSR as a conductive tube to calculate harmonic resistance.[5] The equation is shown as below.

\[
\begin{align*}
\beta &= \frac{2t}{d} \\
z &= 8\pi^2r^2f\gamma \\
\gamma &= \frac{1}{A_0R_{dc} \times 10^9} \\
A_0 &= \pi t (d - t)
\end{align*}
\]

(2)

where 
\(t\) = the thickness of the aluminum layer of ACSR,
\(d\) = the diameter of ACSR
\(f\) = the AC frequency
\(R_{dc}\) = the DC resistance per unit length of ACSR.

This calculation method is pretty accurate comparing to the former method, but the process of calculation becomes more tedious and the range of application is limited. Therefore, it is not appropriate to apply to engineering.

Based on the principle of engineering electromagnetics, this paper derived the distribution of current density on a finite plane, which is also suitable for the transmission line, and therefore developed a more practical and precise enough equation to calculate the resistance of current carrying bar or transmission line considering skin effect.

2. The Distribution of Current Density on a Finite One-dimensional Field

Neglecting the influence of displacement current, the electromagnetic field in the conductor can be approximated as the Magnetoquasistatic field, which satisfies the following basic equations [6,7]

\[
\begin{align*}
\nabla \times H &= \gamma E \\
\nabla \cdot B &= 0 \\
\nabla \times E &= -\frac{\partial B}{\partial t} \\
\nabla \cdot D &= 0
\end{align*}
\]

(3)

It can be derived that[8]

\[
\nabla^2 E = \mu_0 \gamma \frac{\partial E}{\partial t}
\]

(4)

and

\[
\nabla^2 H = \mu_0 \gamma \frac{\partial H}{\partial t}
\]

(5)

The corresponding equation in complex the complex form becomes

\[
\begin{align*}
\nabla^2 \hat{E} &= j \omega \mu_0 \gamma \hat{E} = p^2 \hat{E} \\
\nabla^2 \hat{H} &= j \omega \mu_0 \gamma \hat{H} = p^2 \hat{H}
\end{align*}
\]

(6)

where

\[
p = \sqrt{j \omega \mu_0} = \sqrt{\frac{\omega \mu_0}{2}} (1 + j) = \frac{1}{d} (1 + j)
\]
Skin Depth $d = \frac{2}{\omega \mu}$.

Notice that the solution form of this equation is

$$\hat{E}_y = Ae^{-px} + Be^{px}$$  \hspace{1cm} (7)

According to the differential form of Ohm's law, this indicates that the current density in the conductor decreases exponentially with increasing distance to the conductor surface. In other words, the current inside the conductor will concentrate on the surface of the conductor.

Assume a one-dimensional field shown in figure 1.

![Figure 1. A Finite One-dimensional Field.](image)

It is obvious that when $x=0$, $\frac{d\hat{E}_y}{dx} \bigg|_{x=0} = 0$.

Therefore we can draw the conclusion that $A=B$.

If the $E_y=0$ when $x=r$ and $x=-r$, then (7) can be rewritten as

$$\hat{E}_y = E_0 \frac{e^{-px} + e^{px}}{e^{-pr} + e^{pr}}$$  \hspace{1cm} (8)

Noticing that $e^{-pr} = e^{-\frac{r}{d}(x+j)} = e^{-\frac{r}{d}}(\cos \frac{r}{d} - j \sin \frac{r}{d})$, the norm of $|e^{-pr}|$ is $e^{-\frac{r}{d}}$. Generally speaking, the skin depth is much smaller than the radius and $e^{-\frac{r}{d}}$ decreases exponentially, therefore we can ignore this term. Then (8) becomes

$$\hat{E}_y = E_0[e^{-p(x+r)} + e^{p(x-r)}]$$  \hspace{1cm} (9)

Thus

$$\hat{J}_y = \gamma \hat{E}_y = \gamma E_0[e^{-p(x+r)} + e^{p(x-r)}] = J_0[e^{-p(x+r)} + e^{p(x-r)}]$$  \hspace{1cm} (10)

Bring $x=r-d$ into (9), we can find that the electric field intensity at skin depth is approximately $E_0(1 + e^{-\frac{2r}{d}})e^{-1}$. However, by definition, the electric field intensity at skin depth is $E_0e^{-1}$. The cause of this difference is the skin depth is deducted on a infinite plane and we can observe from the expression of the skin depth that it is only related to the frequency of the harmonic, which is strange because it should also be related the radius of the conductor.

### 3. The Resistance Considering Skin Effect

Assume a current carrying bar with unit height, unit width and length of $2r$ shown in figure 2. And its distribution of current density toward $y$-axis is
\[ \dot{J}_y = J_{y0}[e^{-p(x,r)} + e^{p(x,r)}] = \gamma E_{y0}[e^{-p(x,r)} + e^{p(x,r)}] \]  

(11)

**Figure 2.** A Current Carrying Bar with unit height, unit width and length of 2r.  
Noticing that the cube is symmetrical, we can only consider the left hand side or right hand side. This paper takes the left half part.  
According to the differential form of Gauss' law \( \nabla \times \vec{E} = -j\omega \mu \vec{H} \), the following equation can be obtained

\[ \frac{d\vec{E}}{dx} \cdot \vec{e}_z = -j\omega \mu \dot{\vec{H}} \]  

(12)

The complex power flows into the conductor can be calculated by Poynting's theorem\[9\] as following

\[ 2ZI^2 = -\frac{1}{S} \int (\vec{E} \times \dot{\vec{H}}^*) \cdot dS \]  

(13)

Because of the electric field intensity is toward y-axis and the magnetic field intensity is toward z-axis, the complex power flows the x-axis. Therefore, (13) can also be written as

\[ Z = -\frac{1}{I^2} \int \frac{1}{S} \int [\vec{E} \times \dot{\vec{H}}^*] \cdot dS = \frac{1}{I^2} \int_{0}^{1} \int_{0}^{1} \dot{E}_{y0} \dot{H}_{z0} dydz \]  

(14)

According to the Ampère's circuital law, if \( r \) is much bigger than skin depth, the line integral on the left hand side of the surface of current carrying bar on plane xOz is

\[ \oint_{l} H \cdot dl = \oint_{l} H_z \cdot dl = \int_{0}^{1} \dot{H}_{z0}dz = \hat{H}_{z0} - 2H_{z0} \cdot e^{-pr} \approx \dot{H}_{z0} = \dot{I} \]  

(15)

Thus

\[ \dot{I}^* = \int_{0}^{1} \dot{H}_{z0}dz \]  

(16)

And the current flow through left half part is

\[ \dot{I} = \int_{S} \dot{J}_y \cdot dS = \int_{-r}^{0} \dot{J}_y dx = \int_{-r}^{0} \gamma E_{y0}[e^{-p(x,r)} + e^{p(x-r)}] dx \]  

(17)

Because of \( \int_{0}^{1} \dot{E}_{y0} dy = \dot{E}_{y0} \), the impedance should be
\[ Z = R + jX = \frac{j\gamma}{T^2} \dot{E}_{\rho_0} = \frac{\dot{E}_{\rho_0}}{I} = \frac{p(1-e^{-2\mu r})}{\gamma} \] (18)

Using Euler formula, the real part of \( Z \) should be the resistance, which is

\[ R = \frac{1-e^{-\nu_0}(\sin \nu_0 + \cos \nu_0)}{\gamma d} \] (19)

where \( \nu_0 = \frac{2r}{d} = \sqrt{\frac{\omega \mu \gamma}{2}} \) \( 2r = \sqrt{\frac{2n \pi \mu f_1 \mu \gamma}{2}} \);

\( f_1 \) = Fundamental frequency.

Noticing that the current carrying bar could be a small part of transmission line, which will also related to \( d \) and \( \nu \). So we can draw the conclusion that

\[ R_s = \frac{[1-e^{-\nu_0}(\sin \nu_0 + \cos \nu_0)]}{[1-e^{-\nu_1}(\sin \nu_1 + \cos \nu_1)]} \sqrt{nR_1} \] (20)

4. Experiment

An experiment was conducted to prove the formula. The experiment circuit shown in figure 3 explains what kinds of equipment was used during experiment and how they connected.

**Figure 3.** The experimental equipment connection

The wire to be tested was a 4mm iron conductor with the length of 25m and its starting point and terminal were connected to the power analyzer. The programmable power supply can generate three phase sinusoidal voltage from 50Hz to 1100Hz and the setting voltage is 220V. During the experiment, the temperature of the conductor is monitored and guaranteed to remain the same. After testing, calculate the resistance using the formula \( R = \frac{P_1 - P_2}{I^2} \)

where \( P_1 \) = the power measured at the beginning of the wire.

\( P_2 \) = the power measured at the terminal of the wire.

The results can be found in the following table.

**Table 1.** Experimental data.
## The Order of harmonic

| Order | Current (A) | Initial Power (W) | Terminal Power (W) | Power Loss (W) | Resistance (Ohm) | Real ratio | Calculated Ratio | Root n |
|-------|-------------|-------------------|-------------------|---------------|-----------------|------------|-----------------|--------|
| 1     | 8.7546      | 1928              | 1890.8            | 37.2          | 0.485           | 1          | 1               | 1      |
| 2     | 8.6896      | 1910.5            | 1858              | 52.5          | 0.695           | 1.432      | 1.376           | 1.414  |
| 3     | 8.6063      | 1886.6            | 1825.1            | 61.5          | 0.830           | 1.711      | 1.661           | 1.732  |
| 4     | 8.5175      | 1860              | 1791.7            | 68.3          | 0.941           | 1.940      | 1.914           | 2      |
| 5     | 8.4181      | 1829.7            | 1755.9            | 73.8          | 1.041           | 2.146      | 2.141           | 2.236  |
| 6     | 8.3105      | 1796.3            | 1718              | 78.3          | 1.133           | 2.336      | 2.347           | 2.449  |
| 7     | 8.1972      | 1760.9            | 1678.7            | 82.2          | 1.223           | 2.520      | 2.536           | 2.645  |
| 8     | 8.0766      | 1723              | 1637.9            | 85.1          | 1.304           | 2.688      | 2.711           | 2.828  |
| 9     | 7.95        | 1683.5            | 1596.5            | 87            | 1.377           | 2.836      | 2.876           | 3      |
| 10    | 7.8205      | 1642.6            | 1554.4            | 88.2          | 1.442           | 2.971      | 3.031           | 3.162  |
| 11    | 7.6874      | 1600.9            | 1511.9            | 89            | 1.506           | 3.103      | 3.179           | 3.317  |
| 12    | 7.5582      | 1559.4            | 1470.6            | 88.8          | 1.554           | 3.203      | 3.321           | 3.464  |
| 13    | 7.4291      | 1519              | 1430.3            | 88.7          | 1.607           | 3.311      | 3.456           | 3.606  |
| 14    | 7.3045      | 1480.2            | 1392              | 88.2          | 1.653           | 3.406      | 3.587           | 3.742  |
| 15    | 7.184       | 1444              | 1355.9            | 88.1          | 1.707           | 3.517      | 3.713           | 3.873  |
| 16    | 7.0718      | 1410.3            | 1322.2            | 88.1          | 1.762           | 3.629      | 3.834           | 4      |
| 17    | 6.9629      | 1379              | 1291.1            | 87.9          | 1.813           | 3.735      | 3.952           | 4.123  |
| 18    | 6.86        | 1350.1            | 1262.1            | 88            | 1.870           | 3.853      | 4.067           | 4.243  |
| 19    | 6.7609      | 1323.1            | 1234.8            | 88.3          | 1.932           | 3.980      | 4.178           | 4.359  |
| 20    | 6.6672      | 1297.2            | 1209.1            | 88.1          | 1.982           | 4.083      | 4.287           | 4.472  |

In table 1, 'real ratio' means the ratio of n-th order harmonic resistance to fundamental resistance, which is the resistance when the power supply generate 50Hz voltage. 'Calculated ratio' means the ratio of \( R_n \) to \( R_1 \) Calculated by (20). 'Root n' means the ratio of root n to 1, which is the ratio of resistance calculated by (1).

As we can see from table 1, the ratio given by (20) is much more accurate than (1) as the frequency increases and its expression form is much more simple than (2). The error when the frequency is low is because skin depth is not negligible comparing to \( r \). As long as the frequency increases, the skin depth becomes much smaller and the formula is much more accurate.

## 5. Conclusion

Based on the principle of engineering electromagnetics, this paper derived the distribution of current density on a finite plane and therefore developed a more practical and precise enough version of frequency-dependent resistance of current carrying bar or transmission line considering skin effect, which is proved to be consistent with facts.

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