Thermodynamic instabilities in warm and dense asymmetric nuclear matter and in compact stars

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Abstract. We investigate the possible thermodynamic instability in a warm and dense nuclear medium where a phase transition from nucleonic matter to resonance-dominated $\Delta$-matter can take place. Such a phase transition is characterized by both mechanical instability (fluctuations on the baryon density) and by chemical-diffusive instability (fluctuations on the isospin concentration) in asymmetric nuclear matter. Similarly to the liquid-gas phase transition, the nucleonic and the $\Delta$-matter phase have a different isospin density in the mixed phase. In the liquid-gas phase transition, the process of producing a larger neutron excess in the gas phase is referred to as isospin fractionation. A similar effects can occur in the nucleon-$\Delta$ matter phase transition due essentially to a $\Delta^{-}$ excess in the $\Delta$-matter phase in asymmetric nuclear matter. In this context we also discuss the relevance of $\Delta$-isobar degrees of freedom in the bulk properties and in the maximum mass of compact stars.

1. Introduction

One of the very interesting aspects in nuclear astrophysics and in the heavy-ion collisions experiments is a detailed study of the thermodynamical properties of strongly interacting nuclear matter away from the nuclear ground state.

The new accumulating data from x-ray satellites provide important information on the structure and formation of compact stellar objects. Concerning the structure, these data are at first sight difficult to interpret in a unique and selfconsistent theoretical scenario, since some of the observations indicate rather small radii and other observations indicate large values for the mass of the star.

On the other hand, the information coming from experiments with heavy ions in intermediate- and high-energy collisions is that the EOS depends on the energy beam but also sensibly on the electric charge fraction $Z/A$ of the colliding nuclei, especially at not too high temperature\cite{1, 2, 3, 4}. Moreover, the study of nuclear matter with arbitrary electric charge fraction results to be important in radioactive beam experiments and in the physics of compact stars.

In this article, we study a hadronic equation of state (EOS) at finite temperature and density by means of a relativistic mean-field model with the inclusion $\Delta(1232)$-isobars\cite{5, 6, 7} and by requiring the Gibbs conditions on the global conservation of baryon number and net electric charge. Transport model calculations and experimental results indicate that an excited state of baryonic matter is dominated by the $\Delta$ resonance at the energies from the BNL Alternating Gradien Synchrotron (AGS) to RHIC\cite{8}. Moreover, in the framework of the nonlinear Walecka model, it has been predicted that a phase transition from nucleonic matter to $\Delta$-excited nuclear
matter can take place and the occurrence of this transition sensibly depends on the \( \Delta \)-meson coupling constants \([9, 10]\).

The main goal of this paper is to show that, for asymmetric warm and dense nuclear medium, the possible \( \Delta \)-matter phase transition is characterized by mechanical and chemical-diffusive instabilities. Similarly to the liquid-gas phase transition \([11]\), chemical instabilities play a crucial role in the characterization of the phase transition and can imply a very different electric charge fraction \( Z/A \) in the coexisting phases during the phase transition. In this context we also discuss the relevance of \( \Delta \)-isobar degrees of freedom in the bulk properties of compact stars.

2. Phase transition and stability conditions

We are dealing with the study of a multi-component system at finite temperature and density with two conserved charges: baryon number and electric charge. For such a system, the Helmholtz free energy density \( F \) can be written as

\[
F(T, \rho_B, \rho_C) = -P(T, \mu_B, \mu_C) + \mu_B \rho_B + \mu_C \rho_C,
\]

with

\[
\mu_B = \left( \frac{\partial F}{\partial \rho_B} \right)_{T, \rho_C}, \quad \mu_C = \left( \frac{\partial F}{\partial \rho_C} \right)_{T, \rho_B}.
\]

In a system with \( N \) different particles, the particle chemical potentials are expressed as the linear combination of the two independent chemical potentials \( \mu_B \) and \( \mu_C \) and, as a consequence,

\[
\sum_{i=1}^{N} \mu_i \rho_i = \mu_B \rho_B + \mu_C \rho_C.
\]

Assuming the presence of two phases (denoted as \( I \) and \( II \), respectively), the system is stable against the separation in two phases if the free energy of a single phase is lower than the free energy in all two phases configuration. The phase coexistence is given by the Gibbs conditions

\[
\mu_I^B = \mu_I^B, \quad \mu_I^C = \mu_I^C,
\]

\[
P_I(T, \mu_B, \mu_C) = P_{II}(T, \mu_B, \mu_C).
\]

Therefore, at a given baryon density \( \rho_B \) and at a given net electric charge density \( \rho_C = y \rho_B \) (with \( y = Z/A \)), the chemical potentials \( \mu_B \) are \( \mu_C \) are univocally determined. An important feature of this conditions is that, unlike the case of a single conserved charge, the pressure in the mixed phase is not constant and, although the total \( \rho_B \) and \( \rho_C \) are fixed, baryon and charge densities can be different in the two phases. For such a system in thermal equilibrium, the possible phase transition can be characterized by mechanical (fluctuations in the baryon density) and chemical instabilities (fluctuations in the electric charge density). As usual the condition of the mechanical stability implies

\[
\rho_B \left( \frac{\partial P}{\partial \rho_B} \right)_{T, \rho_C} > 0.
\]

By introducing the notation \( \mu_{i,j} = (\partial \mu_i / \partial \rho_j)_{T, P} \) (with \( i, j = B, C \)), the chemical stability for a process at constant \( P \) and \( T \) can be expressed with the following conditions \([10]\)

\[
\rho_B \mu_{B,B} + \rho_C \mu_{C,B} = 0,
\]

\[
\rho_B \mu_{B,C} + \rho_C \mu_{C,C} = 0.
\]

Whenever the above stability conditions are not respected, the system becomes unstable and the phase transition takes place. The coexistence line of a system with one conserved charge becomes in this case a two dimensional surface in \((T, P, y)\) space, enclosing the region where mechanical and diffusive instabilities occur.
By increasing the temperature and the baryon density during the high energy heavy ion collisions \((T \approx 50 \text{ MeV} \text{ and } \rho_B \geq \rho_0)\), a multi-particle system with \(\Delta\)-isobar and pion degrees of freedom may take place.

In analogy with the liquid-gas case, we are going to investigate the existence of a possible phase transition in the nuclear medium by studying the presence of instabilities (mechanical and/or chemical) in the system. The chemical stability condition is satisfied if

\[
\left( \frac{\partial \mu_C}{\partial y} \right)_{T,P} > 0 \quad \text{or} \quad \left\{ \begin{array}{ll}
\left( \frac{\partial \mu_B}{\partial y} \right)_{T,P} < 0, & \text{if } y > 0, \\
\left( \frac{\partial \mu_B}{\partial y} \right)_{T,P} > 0, & \text{if } y < 0.
\end{array} \right. \tag{8}
\]

In the Fig. 1, we report the baryon and electric charge chemical potential isobars as a function of \(y\), at fixed temperature \(T = 50 \text{ MeV}\) and \(x_{\sigma\Delta} \equiv g_{\sigma\Delta}/g_{\sigma N} = 1.3\) (the ratio related to the scalar \(\sigma\) meson-\(\Delta\) coupling constants) in the GM3 parameters set [5].

![Figure 1. Baryon (right panel) and electric charge (left panel) chemical potential isobars as a function of \(y\) at \(T = 50 \text{ MeV}\) and \(x_{\sigma\Delta} = 1.3\). The curves labeled \(a\) through \(g\) have pressure \(P=9,7,6,5,4,3,2 \text{ MeV/fm}^3\), respectively.](image_url)

From the analysis of the above chemical potential isobars, we are able to construct the binodal surface relative to the nucleon-\(\Delta\) matter phase transition. In Fig. 2, we show the binodal section at \(T = 50 \text{ MeV}\) and \(x_{\sigma\Delta} = 1.3\).

The right branch (at lower density) corresponds to the initial phase (I), where the dominant component of the system is given by nucleons. The left branch (II) is related to the final phase at higher densities, where the system is composed primarily by \(\Delta\)-isobar degrees of freedom (\(\Delta\)-dominant phase). In presence of \(\Delta\)-isobars the phase coexistence region results very different from what obtained in the liquid-gas case, in particular it extends up to regions of negative electric charge fraction and the mixed phase region ends in a point of maximum asymmetry with \(y = -1\) (corresponding to a system with almost all \(\Delta^-\)-particles, being antiparticles and pions contribution almost negligible in this regime). We analyze the phase evolution of the system during the isothermal compression from an arbitrary initial point \(A\), indicated in Fig. 2. In this point the system becomes unstable and starts to be energetically favorable the separation into two phases, therefore an infinitesimal \(\Delta\)-dominant phase appears in \(B\), at the same temperature and pressure. Let us observe that, although in \(B\) the electric charge fraction is substantially negative, the relative \(\Delta^-\) abundance must be weighed on the low volume fraction occupied by the phase II near the point \(B\). During the phase transition, each phase evolves towards
a configuration with increasing $y$, in contrast to the liquid-gas case, where each phase evolves through a configuration with a decreasing value of $y$ (with the exception of the gas phase after the maximum asymmetry point).

Finally, we investigate the relevance $\Delta$-isobar degrees of freedom in the bulk properties of compact star in the framework of the same equation of state discussed above but considering $\beta$-stable and electric-charge neutral nuclear matter at $T = 0$.

In Fig. 3, we report the mass-radius relations in absence (no $\Delta$) and in presence of $\Delta$-isobars with different scalar coupling ratios ($x_{\sigma\Delta} = 1.0$ and $x_{\sigma\Delta} = 1.2$). The presence of $\Delta$-isobar degrees of freedom smooths the equation of state and reduces the maximum gravitational mass. On the other hand very compact object with smaller radii can be formed. This matter of fact can be very relevant in the interpretation of recent astrophysical observations [12].

**Figure 2.** Binodal section at $T = 50$ MeV and $x_{\sigma\Delta} = 1.3$.

**Figure 3.** Mass-radius relations in absence (no $\Delta$) and in presence of $\Delta$-isobars with different scalar coupling ratios ($x_{\sigma\Delta} = 1.0$ and $x_{\sigma\Delta} = 1.2$).

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