General properties of exchange rate of national money versus some foreign currencies in Albania

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Abstract. In this work we have considered the study of the exchange rate series for the specific case where the formal financial market is not active. In those situations, we would be interested in the parallelization of the exchange rate with financial indexes for stabilized financial market. We observed that the stationarity of the distribution for some the exchange rate of currencies traded in the country differs significantly. The time dynamics shows the presence of the elements of local critical behavior, but those tendencies attenuate and fade away in an a periodic fashion. Next, we considered and evidenced the correlation distances and dissimilarity between exchange rates of national currencies versus euro and dollar and golden prices. It resulted that two exchange rates do have different distance from golden price taken for references. The correlation distance between the series of the return in different period has evidenced that there is not a regular behavior in this respect.

Key words: exchange rates, correlations, financial mathematics

1. Introduction

The exchange rate of a national currency against another tradable currency depends on many parameters as the purchasing parity and prices level in both countries, the interest rates, inflations, governments’ debt, or stock etc., see references [1] etc. Depending on the specifics of the economies involved in the both sides of the exchange mechanism, other factors may be part of the game also, see [3], [2] and references therein for details. As a result, the time data series of the exchange rates are similar with other financial data. We will consider in this work the exchange rates Albanian currency toward USD and EURO, so the specifics of the country’s economy are relevant. Herein, we will consider the time evolution and dynamics on the data series, the specifics average values, the distribution of the return, the features of the volatility etc. Firstly, we highlight the system’ specifics, which motivate the step by step descriptive analysis and limitation on the qualitative elements. Just to remember in this issue is the fact that Albania has established a formal financial market but practically it is not operational. Next, the country is making effort to be integrated in the (mega) EU economic system and therefore monetary policy are strictly monitored. Note that also the exchange rates are strictly used by authorities as an additional financial instrument in the framework of macro-economic policies. But also, because of such specific circumstances, the study of the Lek/other currency exchange rate series has become a very important and interesting object for researchers form different point of view, see [12], [14], [15], aside of many and parodic research and publication from central Bank of Albania. In the general case, we note that aside various econometric models developed by finance doctrines, successful analyses have been
Presented by mathematicians and physicists in the framework of the econo-physics studies, see [2],[3], [6] or as case study for non-equilibrium statistics and complex systems, see [10], [7] etc. Also, the socioeconomic processes have been considered in the framework of the non-equilibrium statistical mechanics extensions, [4], [5] [6], [17], [18] etc. We will follow the discussion from a descriptive statistical point of view but also some specific behaviour related to the complexity science would be mentioned and highlighted. In this case, exchange rates series under study are considered as a typical complex system, because of the heterogeneity of the factors effecting it, but again, we do not go deeper in the modelling or technical analysis limiting the work in the framework of the empirical-descriptive analysis.

2. Note on the exchange rate properties and the analytic procedures used in this work

Let first briefly introduce some econometric elements related to the exchange rates. In a very simplified model, the basic exchange rate (Q) in the equilibrium is calculated as

\[ Q = \sum_i \beta_i E \frac{P_{f,i}}{P_d} \]  

(1)

where \( E \) is the nominal exchange rate, \( P_{f,i} \) are the foreign purchasing parity or price level and \( P_d \) is the domestic price. The nominal exchange rate (or the official exchange rate) is defined as the number of units in domestic currency that can purchase the unit of foreign currency. Usually, the nominal exchange rate for transition economies is based on the countries policies and is fixed or pegged against a basket of major world currencies, see [2], [1], [3] etc. The parameter \( \beta_i \) in (1) is the weight assigned to each real exchange rate of domestic currency against foreign currency (i) and is given by the relation

\[ \beta_i = \frac{(g_{3006}, g_{3051}, g_{3007}, g_{3002}, g_{3005}, g_{3007})}{(g_{3000}, g_{3010}, g_{3003}, g_{3002}, g_{3005}, g_{3007})} \]  

(2)

where “export” and “import” are calculated in the domestic currency and include all trading activities toward the country (i), [2]. Next, by introducing the tradable (T) and not tradable (NT) equations and taking logarithmic form, the real logarithmic exchange rate is given by the simple linear formula

\[ q = (e + p_{f,i} - p^T) - ((1 - \beta)(p^{NT} - p^T) - (1 - \beta_f)(p^{NT}_{f,i} - p^T_{f,i})) \]  

(3)

wherein we assigned by lower-case the logarithm of the quantities in the equation (1). The fluctuation in the real exchange rates is driven by the real tradable part \((e + p_{f,i} - p^T)\) and the ratio of the domestic to the foreign relative prices of non-tradable and tradable goods [2]. A similar equation is obtained under money approach for nominal exchange rate that appears in (2)

\[ e_t = (m_t - m^T) - a_1(y^T - y^f) - a_2(i_t - i^f) + \epsilon_t \]  

(4)

Here \( m_t \) is money supply, \( i_t \) is the interest rate and \( y \) represent incomes or a GDP measure. More realistic models reveal the fact that even in the most simplified approaches, the exchange rates are affected by a couple of interconnected and competitive factors that impose a very complex volatility and fluctuation of this financial quantity. Therefore, a stochastic term can be added in the empirical formulas, and we assigned it by \( \epsilon_t \). Note that the most relevant quantity in the financial indexes dynamics is the return \( \frac{P_{t+1} - P_t}{P_t} \)

where \( \{P\} \) are the instants values for the financial quantity considered. Therefore, in our case the quantity

\[ r_t = \ln \frac{q_{t+1}}{q_t} \leq \frac{q_{t+1} - q_t}{q_t} \]  

(5)

is the most important indicator in the exchange rate dynamics. It has been shown that the return of the exchange rates exhibits similar dynamics with other financial indexes [7]. In [4] it is argued that the distribution of the returns generally obeys a q-lognormal shape given by function...
\[ \rho(x) = A \frac{1}{x^q} e^{\frac{-|\ln_q x - \mu|^2}{2\sigma^2}} \]  

(6)

where \( \ln_q \) is given by relation \( \ln_q x = \frac{x^{1-q} - 1}{1-q} \), \( A \) is the normalization constant, \( \mu \) and \( \sigma \) are constants that play the role of the mean and standard deviation. In the limit \( q \to 1 \) the classical lognormal is recovered.

Borland, in [5] has proposed a generalized return model where the volatility \( \omega \) follow a q-gaussian distribution given by equation

\[ P = A(1 - (1 - q)\beta \omega^2)^{1/q} \]  

(7)

In this framework we will verify the distribution of the return of exchange rate for the data series considered. The procedure of the fit is based on the nonlinear least squares. Another interesting property for exchange rates, again like the other financial quantities dynamics is the self-organization or the Discrete Scale of Invariance Structure (DSI). The DSI structure is a specific scale invariance where the scaling parameter is discrete that is in the equation \( O(x) = \mu O(ax) \) parameter \( \alpha: [\alpha_1, \alpha_2, \ldots, \alpha_n] \) In [7] the dynamics of prices, exchange rates etc have been expressed by the log-periodic function (LPP) \( y = y_0 + A(t - t_c)^m (1 + B\cos(\omega \log(t - t_c) + \varphi)) \) where \( t_c \) is the critical time, \( y \) is the logarithm of the price, \( \omega \) is the cyclic frequency related to the DSI parameters and \( A, BC \) are constants. Other forms of log-periodic are used subsequently and we will choose in our case the

\[ y = y_0 + A(t - t_c)^m (t - t_c)^m (B\cos(\omega \log(t - t_c) + \varphi) + C\sin(\omega \log(t - t_c) + \varphi)) \]  

(8)

The fitting of forms (4) is based on the genetic algorithm, starting by a linearization procedure called ‘salving’

\[ y = a + bf(t) + cg(t) + dh(t) \]

\[ f(t) = (t - t_c)^m \]

\[ g(t) = (t - t_c)^m \cos(\omega \log(t - t_c) + \varphi) \]

\[ h(t) = (t - t_c)^m \sin(\omega \log(t - t_c) + \varphi) \]  

(9)

In each step of the Genetic Algorithm procedure, we generate \( f(t), g(t), h(t) \) by assigning firstly randomly values \( \{t_c, \omega, \varphi\} \) in a \( R^4 \) subset defined for the system and solve the linear problem by simply calculating \( \{a, b, c, d\} \) form the regression equation

\[ \begin{pmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} f_i \\ \sum_{i=1}^{n} g_i \\ \sum_{i=1}^{n} h_i \end{pmatrix} = \begin{pmatrix} N \sum_{i=1}^{n} f_i \sum_{i=1}^{n} g_i \sum_{i=1}^{n} h_i \\ \sum_{i=1}^{n} f_i \sum_{i=1}^{n} f_i \sum_{i=1}^{n} g_i \sum_{i=1}^{n} h_i \\ \sum_{i=1}^{n} g_i \sum_{i=1}^{n} g_i \sum_{i=1}^{n} g_i \sum_{i=1}^{n} h_i \\ \sum_{i=1}^{n} h_i \sum_{i=1}^{n} h_i \sum_{i=1}^{n} h_i \sum_{i=1}^{n} h_i \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \]  

(10)

so the \( k \) solution of the types \( \{t_c, \omega, \varphi, a, b, c, d\} \) are generated accordingly. Next, after realization the crossover between each two-initial solution say
\[
\{t_{c, \omega, \varphi}\}_{\text{child}} = \frac{\{t_{c, \omega, \varphi}\}_{\text{parent1}} + \{t_{c, \omega, \varphi}\}_{\text{parent2}}}{2}
\]

(11)
a set of the ‘k’ child solutions is realised. By ranking solutions according to the deviance \( R^2 = \text{sum}(y_{\text{data}} - y_{\text{computed}}) \), the k individuals having smallest \( R^2 \) ‘survived’ whereas others are eliminated as by the natural selecting process. Here the first generation is completed and the process reiterate until reaching the conditions \( R^2 < \text{threshold} \). We analysed the LP behaviour in selected time windows to check for possible critical behaviour. Since DSI structure could be potentially overlapped by other effects, we have used an approximative form based on the q-functions also. Finally, we have analysed the fractal and multifractal structure of the series for a detailed understanding of the self-affinities of the data. Again, for the clarity of the reading we remember that the self-affinity or scaling for a process \( X(t;p) \) where \( t \) is the time and \( \{p\} \) are parameters, is expressed by the e relation

\[
X(at, p) \sim a^H X(t, p)
\]

(12)
The self-similarity index \( H \) appearing in (7) is called also the Hurst exponent [15]. The generalised Hurst exponent is introduced successively by using q-order moment statistics

\[
\frac{\langle |X(t+r) - X(t)|^q \rangle}{\langle |X(t)|^q \rangle} \sim \tau^q H(q)
\]

(13)
where \( H(q) \) is called the generalised Hurst exponent, see [17] for a generalised analysis or [19] for methods of application in multi-disciplinary framework. The most used procedure for the calculation of the hurst exponent and other quantities related to them is the Detrended Fluctuation Analysis (DFA) as explained shortly in the reference [11]. If the hurst exponent varies according to the values of the data series, the structure is called multifractal. There is a lot of mathematical, geometrical, and physical properties related to the multifractal behaviour, which we are skipping for now herein. Note that from algorithmic point of view, the calculation of the multifractal properties would be realised via a similar procedure called the Multifractal-DFA. Herein we have used an algorithm provided by [11] and therefore we are not detailing the steps.

3. General properties and measures of the exchange rate Lek/USD/EURO and price of Gold

As we mentioned above, the exchange rate lives in an open system, where the liquidity market is only a part of the whole. In our system, the regulation of the Central Bank in Albania has implied a series of rules and regulations to guarantee the financial and economic stability, therefore the outside borders of the series are strongly effectuated by those policies, see annual publication in [20]. Next, we should mention that Albania has a relatively small economy at around 15 bn Euro GDP, and the its GDP/capita is significantly lower than in the US or European countries. So, from econometric point of view the money streams is expected to flow inward the country effectuating the reduction of the price of Euro or USD etc. (in Lek). In the reference [12] it is stated the flows per capita from the big economy (Europe-E) with higher GDP per capita to the small one with lower GDP per capita (Albania-A) would be estimated by

\[
\frac{\delta m}{\delta N} \sim \ln(\text{GDP}_E(A)) - \ln(\text{GDP}_E(E)) \geq 0
\]

(14)
So, the net inward flow of the foreign currency contributes in the depreciation of the ratio Euro/All. Adding to that we have e net negative flows of the population in the direction \( A \to E \). Another very critical issue is the effect of the informal economy pressure. Considerable amount of money remains out of the regularized financial structures generating interior irregular transfers. The informal economy can absorb and emit irregularly the money that implying distortion of the cash currencies balances. The estimated that the level informality in Albania during 2000-2015 is has been at 35-40% of the GDP.,[12] so the effect would be very significant. As a preliminary conclusion, a formal mathematical expression that take into account all those particular factors seem to be unrealistic in for our observable. Therefore, the study of the dynamics of the exchange rates series Lek/Euro, USD, and other properties as well are proposed herein in the framework of descriptive analysis.
4. The descriptive analyses of the exchange rates series of the period 2008-2021

Herein we will focus our attention on the distributions of the returns (volatilities), the correlation and autocorrelation, and the trend of series for the period 2008-2021. By exploring the properties of function fitted to the empiric distribution, we have identified the nature of the dominant process, the stationarity level of the global state, and also the grade of the complexity. According to [4] the level of stationarity is given by the q-parameter of the q-gaussian fitted in the real data distribution. Note that econometrists usually employ the lognormal functions in similar cases, whereas in reference [5], a thorough analysis based on the econophysics framework has argued that the q-lognormal and q-gaussian are more realistic pdf forms to describe the distributions of the volatilities. Based on those arguments, in the reference [8] we have reported that the best fitted distribution for the densities of the rate of return (volatility) of USD/Lek and EURO/Lek in the period [2000, 2019] have been the q-gaussian shapes. Herein, we tried both forms, the q-lognormal and q-gaussian. By using nonlinear least squares fitting method, we obtained that q-lognormal has fitted very well but it is algorithm-depended and also it destabilises by the bin-width change. In reference [14] it is argued that the more acceptable distribution in those cases should be the one that is more unaffected one toward the bin-width change, and we will refer it as an auxiliary argument in the selection of the most descriptive distribution if the statistics of the goodness of fit are not decisive. Here we have performed the fitting by using the nonlinear least square algorithm, where we chose ‘trust region’ algorithm in the MATLAB procedure. Also, we used Freedman-Djacoic optimization rule for the bin to account for the fat tail distribution evidenced in the preliminary analysis. We observed that the q-Gaussian and q-lognormal are the best candidates as pdf in this case, but q-gaussian was the better alternative on the above criteriums. The empirical distribution and their fitted forms are shown in Figure 1.
Figure 1. Distribution of the volatilities and q-Gaussians fitted to them

The parameter obtained from q-Gaussian fit are 2.1063; 1.4876; 1.6032 and 1.4684 for Euro/Lek, Gold Price, USD/Lek and Pound Lek exchange rate respectively. We observed that the distribution of Euro/Lek volatility (rate of return) belongs to the typical nonstationary distribution and as a physical consequence the respective state is considered in the intensive evolution and dynamics. Based on the theoretical approach [4] and [5], and referring to the previous work in those subjects [12], the overall state of the rate of return belong to the zone of undefined variance, and therefore we conclude that the variance calculated for the time series of rate of return in the period [2008, 2021] is variable, undefined. From the other part, the rate of return for USD/Lek, Pound/Lek and price of gold are characterised by highly nonstationary distribution in the edge of the limit \( q = \frac{5}{3} \) which is the boundary of the stationarity. The USD/Lek case has the distribution that belongs to the zones of the infinite variance \( (q>\frac{5}{3}) \). We note that the incertitude on the assessment for the parameters on the nonlinear fitting calculation is not neglectable, so we assume that \( q \approx \frac{5}{3} \) should be considered that \( q < \frac{5}{3} \) is not verifiable and therefore, all the distributions are considered as highly nonstationary. The nonstationary level of the global rate of return states observed so far suggests us to restrict our study in the descriptive analysis, because modelling and alternative deterministic approach are not efficient in this situation. From this general point of view and constatation, we followed the exploration for the presence of any regularity on the series expressed on their correlations and autocorrelations. In this case, we supposed that series of exchange rates for specific periods could have been correlated or anti-correlated as response result of tactical action of the state agencies on the exchange rates. Therefore, a quite simple end evident behaviour is considered as an important indicator for further analytic discussion. The most interesting time windows are those related to a fiscal year which represent year by year adjustment of the exchange rate strategies. Shorter interval would probability indicate the in-time correction policy.
So, we performed the correlation between sequential series of one year and half year windows. For the euro/lek pair we obtained that correlation may exists between series of exchange rates of 2017, 2018 and 2019. However, there is no correlation for rate of return, meaning that stochastic term imposed by secondary banc activities and also form the free currency’s trade including black market, are typically random and stochastically independent. There is more significant correlation (absolute values greater than 0.5) in the series of the gold prices. Again, no correlation trace for the volatility for the price. We observe that the coefficients of correlation between deferent periods for Lek/USD exchange rate occur densely compared to the euro/Lek. The same behaviour is observed in the series of lek Pound exchanges. Considering that only one or two period’s correlation could be meaningful, we concluded that for two currency’s Pound and USD, there are trace of correlation but in all they are alternated and no strong correlation occurs. For stochastic processes, an interesting parameter is the distance between two series given by formula

\[ d_{x_i,x_j} = \sqrt{2(1 - \rho_{x_i,x_j})} \]  

Where the \( \rho_{ij} \) is the correlation coefficient.

### Table 1. The Pearson correlation coefficient for partial series of longitude half of year.

|      | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 2008 | 1    | 0.2708 | 0.7339 | -0.2091 | 0.6849 | -0.1403 | 0.3012 | -0.169 | 0.0881 | 0.4746 | 0.1651 | 0.4651 | 0.1084 |
| 2009 | 0.2708 | 1    | 0.2433 | 0.0013 | 0.434 | 0.3624 | -0.4048 | -0.6492 | -0.5999 | -0.4174 | -0.7818 | -0.3801 | -0.0736 |
| 2010 | 0.7339 | 0.2433 | 1    | -0.2464 | 0.7303 | -0.3381 | 0.2784 | -0.3227 | 0.037 | 0.5649 | 0.2725 | 0.5826 | 0.0019 |
| 2011 | -0.2091 | 0.0013 | -0.2464 | 1    | 0.1925 | 0.4843 | -0.1807 | 0.5064 | 0.3644 | -0.1259 | -0.188 | -0.1667 | 0.007 |
| 2012 | 0.6849 | 0.434 | 0.7303 | 0.1925 | 1    | 0.1386 | 0.1968 | -0.1302 | 0.1579 | 0.4175 | -0.034 | 0.359 | 0.1072 |
| 2013 | -0.1403 | 0.3624 | -0.3381 | 0.4843 | 0.1386 | 1    | -0.1249 | 0.1468 | -0.0804 | -0.481 | -0.5733 | -0.4592 | -0.0341 |
| 2014 | 0.3012 | -0.4048 | 0.2784 | -0.1807 | 0.1968 | -0.1249 | 1    | 0.2009 | 0.4067 | 0.5154 | 0.5139 | 0.4388 | 0.0636 |
| 2015 | -0.169 | -0.6492 | -0.3227 | 0.5064 | -0.1302 | 0.1468 | 0.2009 | 1    | 0.7729 | 0.2734 | 0.5032 | 0.2626 | 0.2522 |
| 2016 | 0.0881 | -0.5999 | 0.037 | 0.3644 | 0.1579 | -0.0804 | 0.4067 | 0.7729 | 1    | 0.6763 | 0.6844 | 0.5362 | 0.1484 |
| 2017 | 0.4746 | -0.4174 | 0.5649 | -0.1259 | 0.4175 | -0.481 | 0.5154 | 0.2734 | 0.6763 | 1    | 0.7817 | 0.8013 | 0.063 |
| 2018 | 0.1651 | -0.7818 | 0.2725 | -0.188 | -0.034 | -0.5733 | 0.5139 | 0.5032 | 0.6844 | 0.7817 | 1    | 0.7865 | 0.1804 |
| 2019 | 0.4651 | -0.3801 | 0.5826 | -0.1667 | 0.359 | -0.4592 | 0.4388 | 0.2626 | 0.5362 | 0.8013 | 0.7865 | 1    | 0.2437 |
| 2020 | 0.1084 | -0.0736 | 0.0019 | 0.007 | 0.1072 | -0.0341 | 0.0636 | 0.2522 | 0.1484 | 0.063 | 0.1804 | 0.2437 | 1    |

| Sem1 | Sem2 | Sem3 | Sem4 | Sem5 | Sem6 | Sem7 | Sem8 | Sem9 | Sem10 | Sem11 | Sem12 | Sem13 |
|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|
| 0.1479 | 0.2295 | 0.5049 | -0.6235 | -0.5498 | -0.5968 | 0.3045 | 0.4177 | 0.0361 | -0.4418 | -0.2629 | -0.4598 | -0.0848 |
| 0.3934 | -0.2629 | -0.458 | 0.7846 | 0.294 | 0.5326 | 0.8746 | 0.2957 | 0.0761 | 0.2623 | -0.2371 | -0.1302 | -0.5651 |
| 0.4177 | 0.5326 | 0.7846 | -0.5049 | -0.6235 | -0.458 | 0.294 | 0.5326 | 0.8746 | 0.2957 | 0.0761 | 0.2623 | -0.2371 |
| 0.0361 | 0.2957 | 0.0761 | -0.6235 | -0.458 | 0.294 | 0.5326 | 0.8746 | 0.2957 | 0.0761 | 0.2623 | -0.2371 | -0.1302 |
| 0.4177 | 0.5326 | 0.7846 | -0.5049 | -0.6235 | -0.458 | 0.294 | 0.5326 | 0.8746 | 0.2957 | 0.0761 | 0.2623 | -0.2371 |
| 0.4177 | 0.5326 | 0.7846 | -0.5049 | -0.6235 | -0.458 | 0.294 | 0.5326 | 0.8746 | 0.2957 | 0.0761 | 0.2623 | -0.2371 |

It is taken by the report covariance and square roots of product of variances.
It gives on-by one distance on the series terms. There are many metrics as Manhattan distance, Minkovsky distance, Mahalanobis distance etc, but the Euclidian distance is considered herein in this attribute. It does not need specific complicated interpretation and is obtained simply as the numerical value of the cosine between two vectors (series).

\[
\rho_{x_i x_j} = \frac{\text{cov}(x_i, x_j)}{\sqrt{\text{var}(x_i) \cdot \text{var}(x_j)}}
\]  

(16)

We calculated the Euclidian distance for the pairs taken from three time series of the exchange rates and used it to estimate the similarity of their trend over time. So, we obtained that the distances between the gold price and Euro/Dollar to Lek exchange rates has started from are currently oscillation regime at the period 2008-2010, and next follow a period of long value in the 2010-2014, after which has entered a relaxing trend of a distance around 0.3, Figure 2. The distance of Pound/Lek exchange rate started by the same behaviour in the beginning, remain between the first two distances for the period 2010-2014 and next has changed significantly it values to remain in high value around the 0.9. It resulted that within fluctuation term, the exchange rates Euro/Lek and Dollar/Lek and the price of Gold have entered a stage of apparent similarity but keeping also significant changes. As seen in the Figure 2, the distances of the volatilities have apparently smaller Euclidian distance.

5. The multifractal analysis and the long-term trend of the Lek/Euro/USD exchange rate series

By calculating the fractal and multifractal parameters for series of the exchange rates we have analysed the self-similarity properties, the persistence and anti-persistence issues etc. For this procedure we have operated the algorithm [11] based on the MF-DFA technique. When analysing the presence of the discrete scale of invariance structure (DSI) as a special case of the scaling properties for series, we have
performed an ad hoc genetic algorithm, following the procedures discussed in [7] and implemented by an ad hoc routine in [12]. The log periodic function is taken in the form given by equation (8). We do not go in detail to check for critical time, cyclic frequency properties etc., which considered as of high physical importance in [7], because the aim of this work is the evidence of properties from a global a comparative view. The aim of the log-periodic fit is primary to mark the presence of the DSI. In principle we can perform this test by a straightforward procedure of statistical test, but the residuals $R_{LP} = Y_{data} - Y_{LP}$ are basically an Ornstein-Uhlenbeck[8] process which is difficult to be tested.

We have obtained good statistic on the log-periodic fits with real data. We have obtained the LP trend and consequently the critical behaviour in the short terms. In this case we can realize quantitative assessment for LP parameters and DSI structure. From a more global view, we can acknowledge the LP underlying trend as a real dynamical behaviour, much more acceptable than a polynomial one. So, can use the LP trend to detrend series better than polynomial forms used in many applications. The m-exponent of the power in the equation (4) indicates the grade of the growth as we go near the cortical time $t_c$, so it is important indicator of the dynamics of the times series. Herein, it is found to be 0.963793; 0.854559; 0.573736; 0.756802 respectively for Euro/Lek, Gold price, Dollar/Lek and Pound/Lek rate of exchange. The critical time (in trading days after 03 January 2008), are 1416.9; 5126.9; 2064.7; and 4069.3 respectively. The fit is shown in the Figure 3. Next, we propose to use LP fit in long term for detrending analysis in some specific and empirical application.

![Figure 3. Long term LP trend of the series](image-url)
Figure 4. The generalized Hurst exponent for exchange rates in Albania

For a more quantitative approach, as for example in the examination of critical time for the long-term series, we observed that the extended q-LP form proposed in [9] and used also in [12] remains very productive in the sense that the critical time obtained by the fit has reproduced very well the local peaks. Finally, in the analysis of the multifractal properties for the series we have calculated the Hurst, generalized Hurst exponent and the fractal spectrum. So, for all series we observe that the monotonicity of the q-Hurst hold, and it keep decreasing for series Euro/Lek, Gold Prices, Pound/Lek. For series Dollar Lek. Also, we observe that the H-curve shows e near to left-side rotated sigmoid in the interval [-2,2], Figure 4. The increasing region is apparently significant in the interval [-1,1]. Therefore, we associate the normal q-Hurst behaviour with a clear multifractal structure of the series analysed. In this descriptive view, self-affinity, the persistent or intermittent behaviour can be easily analysed on the multifractal analysis framework.

6. Conclusions

The time data series of the exchange rates of national versus two global currencies in Albania shows the common properties of the financial indices on the free financial markets. Each series exhibits specific properties related to the short- and long-term correlation. The overall stats spanned on the period of observation 2008-2021 is nonstationary for each of the series considered. The best fitted distribution is the q-Gaussian. For localised periods there are signs of the correlation of semestral series. The Euclidian distances between each pair fluctuate rapidly in the beginning of the time interval and relaxes to a considerable value in the period 2010-2014. Next the exchange rate of Pound/Lek start increasing the distance form Euro/Lek and the same pattern is meet on the respective volatilities. Series are characterised by discrete scale of invariance and the trend of them is fitted well with a log-periodic form. Series are multifractal, and many properties related to the multifractality can be described by multifractal analysis.
7. References

[1] Betts, C. & Devereux, M. Exchange rate dynamics in a model of pricing-to-market. Journal of International Economics 50 215 –244

[2] Reza Y. Siregar. The Concepts of Equilibrium Exchange Rate: A Survey of Literature. n Extended version of the Report prepared for the 2006-2007 Exchange Rate Policy Evaluation Project of the Independent Evaluation Office (IEO), the International Monetary Fund, Washington, D.C.

[3] Mohsin, S. Khan & Peter, J. Montiel. Real Exchange Rate Dynamics in a Small, Primary-Exporting Country. International Monetary Fund SPapers, Vol. 34, No.

[4] C. Tsallis. Economics and Finance: q-Statistical Stylized Features Galore. Entropy. 2017; 19(9):457. https://doi.org/10.3390/e19090457

[5] Lisa Borland. Phys. Rev. Lett. 89, 098701 – Published 7 August 2002

[6] Mantegna, R. &Stanley, H. (2007). An introduction to econophysics: correlations and complexity in finance. Cambridge University Press New York, NY, USA.

[7] Sornette, D.&Johansen,A.(2001). Significance of Log- periodic Precursors to Financial Crashes. Quantitative Finance, 1: 452.

[8] P Geraskin, D Fantazzini. Everything you always wanted to know about log-periodic power laws for bubble modelling but were afraid to ask. The European Journal of Finance 19 (5), 366-391

[9] D. Prenga, M. Ifti, Int. J. Mod. Phys. CS 16, 1 (2012).

[10] Tsallis, C. (2017) Economics and Finance Features Galore: q-Statistical Stylized. Entropy, 19, 457; doi:10.3390/e19090457.

[11] Espen A. F. Ihlen. Introduction to multifractal detrended fluctuation analysis in Matlab. Front. Physiol., 04 June 2012 | https://doi.org/10.3389/fphys.2012.00141

[12] D. Prenga, S. Kovaçi, E Kushta. An Econo-Physics View on the Historical Dynamics of the Albanian Currency vs. Euro Exchange Rates AUDŒ, Vol. 16, no. 1/2020, pp. 254-267

[13] Banerjee, A & Yakovenko, V. (2010). Universal patterns of inequality. New Journal of Physics 12, 075032.

[14] Cera, Gentijan; Dokle, Eda; Çera, Edmond (2015) : Do the News Affect the EUR/ALL Exchange Rate Volatility?, Economic Review: Journal of Economics and Business, ISSN 1512-8962, University of Tuzla, Faculty of Economics, Tuzla, Vol. 13, Iss. 1, pp. 21-28

[15] Natasha Ahmetaj, Merita Bejtja. Determinants of the Real Equilibrium Exchange Rate in Albania: An Estimation Based on the Co-Integration Approach. ACRN Journal of Finance and Risk Perspectives

[16] Constantino Tsallis. The Nonadditive Entropy Sq and its Applications in Physics and Elsewhere: Some Remarks. Entropy 2011, 13, 1765-1804; doi:10.3390/e13101765

[17] G.P. Pavlos, L.P. Karakatsanis, M.N. Xenakis, A.E.G. Pavlos, A.C. Iliopoulos, D.V. Sarafopoulos.

[18] [Exton, Harold (1983), q-hypergeometric functions and applications, Ellis Horwood Series: Mathematics and its Applications, Chichester: Ellis Horwood Ltd., ISBN 978-0-85312-491-7, MR 0708496

[19] R. Lopes, N. Betrouni. Fractal and multifractal analysis: A review. Medical Image Analysis 13 (2009) 634–649