Some Properties of Lie Algebras

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Opinion

Traditionally, Lie algebras have been used in physics in the context of symmetry groups of dynamical systems, as a powerful tool to study the underlying conservation laws [1,2]. At present, space-time symmetries and symmetries related to degrees of freedom are considered. For instance, non-trivial Heidelberg algebra arises right in the base of the Hamiltonian mechanics. Hamiltonian mechanics describes the state of a dynamic system with 2n variables (n coordinates and n momenta), and the other interesting observable physics quantities are functions of them. Kuranishi [3] proved that for any finite dimensional semisimple Lie algebra over a field F of characteristic zero there exist two elements X, Y ∈ L which generate L. Work on simple Lie algebras of prime characteristic began nearly 75 years ago. Much of this work has concentrated on the case of restricted Lie algebras (also called Lie p-algebras). Robert Zeier and Zoltán Zimborás [4] given a subalgebra h of a compact semisimple Lie algebra g and a finite dimensional, faithful representation θ of h, then h^g if dim com([0 ⊗ 0]/0)=dim com([0 ⊗ 0]). Bai Ruipu, Gao Yansha and Li Zhengheng [5] proved L be a Lie algebra, D be an idempotent derivation. Then the image of D on L is denoted by L=D(L), is an abelian ideal of L, and the kernel of D, is denoted by K=KerD is a subalgebra of L . Zhang Chengcheng, Zhang Qingcheng [6] Let L be a Lie color algebra. Then adL={adx | x ∈ L} is a Lie color subalgebra of End(L), which is said to be the inner derivation algebra, where a Lie color algebra is a G-graded F-vector space L=⊕_{g ∈ G} L_g with the bilinear product [·,·]: L × L → L satisfying some conditions. Recently, David A. Towers [7] proved there are many interesting results concerning the question of what certain intrinsic properties of the maximal subalgebras of a Lie algebra L imply about the structure of L itself. In this paper, we give some properties on subalgebras and semisimple of Lie algebras with others concepts.

Preliminaries

A finite-dimensional Lie algebra is a finite-dimensional vector space L over a field F together with a map [·,·]: L × L → L, with the following properties: (1) [x,y]+[y,x]=0 for all x, y ∈ L and [αx, y]+[x,αy]=α[y,x] for all x, y ∈ L and α ∈ F, (2) [x,y+w]=[x,y]+[x,w] for all x, y, w ∈ L and [x,αy]=[x,y] for all x, y ∈ L and [αx, y]=[x,αy] for all x, y ∈ F, (3) [x, y+z]=[x, y]+[x, z]+[z, y]=0 for all x, y, z ∈ L. A Lie algebra is said to be semi-simple if Rad(L)=0. If L=Fn then g(L) is denoted g(n, F), so we not need to denote sl(n, F). Then trace xy=∑_{i=1}^{n} x_i y_i = ∑_{i=1}^{n} x_i y_i . Thus, we get the trace (xy-yx)=0 for all x, y ∈ sl(n, F), so in particular [x,y] ∈ sl(n, F) for all x,y ∈ sl(n,F), as required.

Proposition 1

Let sl(n, F) be the subset of gl(n, F) consisting of matrices with trace 0. This is an subalgebra of gl(n, F).

Proof: We have the relation sl(n,F) ⊆ gl(n, F), so we not need to prove sl(n,F) is a linear subspace. Let x=(x_{ij}), y=(y_{ij}) ∈ sl(n,F).Then trace xy=∑_{i,j=1}^{n} x_{ij} y_{ij}= ∑_{i,j=1}^{n} x_{ij} y_{ij} . Thus, we get the trace (xy-yx)=0 for all x, y ∈ gl(n, F), so in particular [x,y] ∈ sl(n, F) for all x,y ∈ sl(n,F), as required.

Proposition 2

Let L be Lie algebra and L^* is nilpotent Lie algebra, then

(i) if the Killing form κ(L^*, RadL) is non-degenerate then L is semi simple Lie algebra.

(ii) if the Killing form is κ(x, L^*), then κ is non-degenerate.

Proof: (i) According to our hypothesis, we have L^* is a nilpotent Lie algebra with the Killing form κ(L^*,RadL)=0. Suppose κ is non-degenerate which is implies Rad L=0. Therefore, L is semi simple Lie algebra as required.

(ii) We have we have L^* is a nilpotent Lie algebra with the Killing form defined as κ(x, L^*)=0.Since L^*=0,we get x=0. Therefore, κ is non-degenerate as required.

Proposition 3

If L be associative Lie algebra then it is semi simple.

Proof: According to our hypothesis, L is associative which means satisfying the following relation [L|[L,L]]]=0. Now discuss the behavior of the bracket [L|[L,L]]. Firstly, if [L,L]=0. Then L=[[L,L],[L,L]]=0, which leads to L^*=[[L,L],[L,L]]=0,……, and L=[[[L,L],[L,L]],[[L,L],[L,L]]]=0, which leads to L^*[=0. Thus, L is solvable Lie algebra. In other words, L is semisimple Lie algebra. Secondly, if L=0. Then L=[[[L,L],[L,L]],[[L,L],[L,L]]]=0.

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L′=[LL′]=[L,L,L]=0.

L′′=[LL′′]=[L,[L,L]]]=0,.....,L^n=0. Then L is a nilpotent Lie algebra, where any abelian or nilpotent Lie algebra is solvable Lie algebra, which means L is semi simple Lie algebra as required.

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