The Cosmic Distance Duality Relation with Strong Lensing and Gravitational Waves: An Opacity-free Test

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Abstract

The cosmic distance duality relation (CDDR) is a fundamental rule in cosmological studies. Given the redshift, it relates luminosity distance \( D^L \) with angular diameter distance \( D^A \) through \((1 + z)^2 D^A / D^L \equiv 1\). Many efforts have been made to test CDDR with various observational approaches. However, to the best of our knowledge, those methods are always affected by cosmic opacity, which could violate CDDR owing to the non-conservation of photon number. Such a mechanism is more related to astroparticle physics. In this work, in order to directly study the nature of spacetime, we propose a new strategy to test CDDR, with strong lensing providing \( D^A \) and gravitational waves (GWs) providing \( D^L \). It is known that the propagation of GWs is unaffected by cosmic opacity. We demonstrate that distances from observations of optical lensing are also opacity-free. These two kinds of distance measurements make it possible to test spacetime. Our results show that the constraints on the deviations of CDDR will be very competitive with current techniques.

Key words: distance scale – gravitational waves: strong – gravitational waves

1. Introduction

The development of modern cosmology relies heavily on the measured distance–redshift relation. While the redshift \( z \) of a celestial object is relatively easy to obtain accurately using spectral lines, cosmological distance measurements are very important. In a general spacetime, the direct observational quantities are luminosity distance \( D^L \) and angular diameter distance \( D^A \). Theoretically, if the three conditions

1. the spacetime is described by a metric theory of gravity,
2. photons travel along null geodesics,
3. photon number is conserved,

are satisfied then \((1 + z)^2 D^A / D^L \equiv 1\), which is called the “cosmic distance duality relation” (CDDR; Etherington 1933). Note that Conditions 1 and 2 are related to the nature of spacetime and are more fundamental, while Condition 3 usually corresponds to astrophysical mechanisms or particle physics. Testing the validity of CDDR with observation would either strengthen our current knowledge of the universe or reveal new physics/astrophysical mechanisms (Bassett & Kunz 2004). Various methods have been used to test CDDR.

To perform a test on CDDR, one needs a measurement of luminosity distance plus one of angular diameter distance at the same redshift. For example, the most commonly used combination consists of \( D^L \) data from Type-Ia supernovae (SNe Ia) as standard candles and \( D^A \) data from galaxy clusters (Bernardis et al. 2006; Holanda et al. 2010, 2012; Li et al. 2011; Hu & Wang 2018). It has been conjectured that cosmological dust might dim the observed SNe Ia (Lima et al. 2011). Other dimming mechanisms include extragalactic magnetic fields turning photons into light axions, gravitons, Kaluza–Klein modes associated with extra dimensions or a chameleon field. They are taken together as the cosmic opacity (Avgoustidis et al. 2010; Liao et al. 2015a), which could change the \( D^L \) measurements and lead to violation of CDDR. Meanwhile, \( D^A \) from galaxy clusters is based on observations of the surface brightness in X-rays and the Sunyaev–Zel’dovich effect (Uzan et al. 2004):

\[
D^A \propto \frac{\Delta T_{\text{CMB}}^2}{S_X},
\]

where \( \Delta T_{\text{CMB}} \) is the temperature change when the cosmic microwave background (CMB) radiation passes through the hot intracluster medium (Sunyaev–Zel’dovich effect). \( S_X \) is the X-ray surface brightness of the galaxy cluster. They are both affected by the cosmic opacity since the measurements are quantities of intensity (Li et al. 2013). In other methods, \( D^L \) can come from gamma-ray bursts (GRBs) at high redshifts (Holanda et al. 2017). Likewise, luminosity distances of GRBs also depend on cosmic opacity. \( D^A \) can come from ultra-compact radio sources (Li & Lin 2018) and baryon acoustic oscillations (BAOs; Wu et al. 2015); however, they either suffer from cosmic opacity or assume a \( \Lambda \)CDM model, making the test model-dependent.

If one wants to exclude the impact of cosmic opacity (also the cosmological models), i.e., to model-independently test the spacetime alone, direct opacity-free distance measurements should be used. On the one hand, gravitational waves (GWs) were proposed as standard sirens to give \( D^L \) (Fu et al. 2019; Qi et al. 2019; Yang et al. 2019). There are two benefits: first, the propagation of GWs is unaffected by cosmic opacity; second, they provide direct luminosity distances while SNe Ia in principle provide relative distances. On the other hand, strong lensing by elliptical galaxies has been used to study cosmology (Chae 2003; Oguri 2007; Paraficz & Hjorth 2009, 2010; Cao et al. 2012; Oguri et al. 2012; Chen et al. 2019; Tu et al. 2019; Wong et al. 2019). The ratios of angular diameter distance from strong lensing observations carry information on \( D^A \) (Liao et al. 2016; Yang et al. 2019). For an ideal model, assuming that an elliptical lens galaxy is described by a
singular isothermal sphere (SIS), once the Einstein radius \( R_E \) and the central velocity dispersion \( \sigma_v \) are measured from the separation angle of multiple images of the active galactic nucleus (AGN) and spectroscopy, one can infer the ratio of two angular diameter distances \( D_A^l / D_A^s \approx \theta_E / \sigma_v^2 \), where the subscripts \( l, s \) denote lens and source, respectively. However, realistic lenses plus their environments are more complex (Jiang & Kochanek 2007). A universal simple model such as SIS or its extensions for all lenses can introduce severe systematics (Xia et al. 2017). More observational quantities and detailed analysis are required to model individual lensing systems one by one (Suyu et al. 2017). Besides, when inferring \( D_A^s \), a flat universe has to be assumed (Holanda et al. 2016; Liao et al. 2016; Yang et al. 2019). Furthermore, for a CDDR test, one needs two \( D_A^l \) data at the same redshifts of the lens and source.

Current state-of-the-art lensing programs (for example, the H0LiCOW (Suyu et al. 2017)) are focusing on time delay lenses. With measurements of time delays between AGN images, the central velocity dispersion of the lens, the host galaxy arcs plus lens galaxy imaging, and the mass fluctuation along the line of sight (LOS), a good algorithm with blind analysis to control the systematics can provide the “time delay distance,” which is a combination of three angular diameter distances, \( D_{\text{sys}} = (1 + z_l)D_A^l D_A^s / D_A^l \), and depends primarily on the Hubble constant (Suyu et al. 2017). Furthermore, time delay lenses were recently found to be more powerful for cosmological studies with the capability to measure the angular diameter distances to the lenses \( D_A^l \) (Paraficz & Hjorth 2009; Jee et al. 2015, 2016; Wong et al. 2019; Yildirim et al. 2019).

The angular diameter distances can be used in a CDDR test (Rana et al. 2017), though still in an opacity-dependent way with SNe Ia, whereas we will focus on disentangling the nature of spacetime from cosmic opacity with GWs.

In this work, we show that the measurements of angular diameter distance from strong lensing are unaffected by the cosmic opacity. Combined with standard sirens, they could provide a direct opacity-free test of CDDR. Unlike previous CDDR tests with lenses where only \( R_E \), \( \sigma_v \), and \( \Delta t \) are measured (Liao et al. 2016; Rana et al. 2017), state-of-the-art and planned projects could measure individual lenses much better with high-quality data from multiple aspects.

This paper is organized as follows: Section 2 introduces \( D_A^l \) from strong lensing and we explain why it is opacity-free. Section 3 introduces \( D_A^L \) from GWs. We give the analysis and results in Section 4 and draw a conclusion in Section 5. The flat \( \Lambda \)CDM model with \( \Omega_M = 0.3 \) and \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) is assumed for simulating the lensing data and GW data.

## 2. Angular Diameter Distances from Strong Lensing

Strong lensing by elliptical galaxies has become a powerful tool for studying astrophysics and cosmology (Treu 2010). Systems with time delay measurements can yield a direct measurement of the angular diameter distance to the lens \( D_A^l \) (Jee et al. 2015, 2016).

Note that \( D_A^l \) can be obtained easily with very simple assumptions (Rana et al. 2017). However, we will introduce the determination of \( D_A^l \) on the standard of the H0LiCOW program (Suyu et al. 2017). For illustration, we briefly show how to measure \( D_A^l \) with observations. The time delay distance is given by

\[
(1 + z_l) D_A^l D_A^s = \frac{c \Delta t}{\Delta \phi (\xi_{\text{ens}})},
\]

where \( c \) is the speed of light, \( \Delta t \) is the time delay measured by the light curves, \( \Delta \phi = \sqrt{[(\theta_l - \beta)^2 / 2 - \psi(\theta_l) - (\theta_l - \beta)^2 / 2 + \psi(\theta_l)]} \) is the Fermat potential difference for image angular positions \( \theta_l \) and \( \theta_s \), and \( \beta \) denotes the source position, and \( \psi \) is the two-dimensional lensing potential determined by the surface mass density of the lens \( \kappa \) in units of critical density \( \Sigma_{\text{crit}} = c^2 D_A^l / (4\pi G D_A^s D_A^l) \) through the Poisson equation \( \nabla^2 \psi = 2\kappa \). \( \Delta \phi \) is determined by the lens model parameters \( \xi_{\text{ens}} \), which can be inferred with high-resolution imaging data. Note that the LOS mass structure would also affect the measured time delay distance (Rusu et al. 2017).

Meanwhile, the general form (not limited to the SIS model) of the distance ratio can be expressed as (Birrer et al. 2016, 2019)

\[
D_{ls}^A = \frac{c^2 J (\xi_{\text{ens}}, \xi_{\text{light}}, \beta_{\text{ant}})}{(\sigma_P)^2},
\]

where \( \sigma_P \) is the LOS-projected stellar velocity dispersion of the lens galaxy. It provides extra constraints on the cosmographic inference. \( J \) captures all the model components computed from angles measured on the sky (the imaging) and the stellar orbital anisotropy distribution. It can be written as a function of lens model parameters \( \xi_{\text{ens}} \), the light profile parameters \( \xi_{\text{light}} \), and the anisotropy distribution of the stellar orbits \( \beta_{\text{ant}} \). We refer to Section 4.6 of Birrer et al. (2019) for detailed modeling related to \( J \).

Thus the angular diameter distance to the lens can be given by (Birrer et al. 2016, 2019)

\[
D_A^l = \frac{1}{1 + z_l} \frac{c \Delta t}{\Delta \phi (\xi_{\text{ens}})} \frac{c^2 J (\xi_{\text{ens}}, \xi_{\text{light}}, \beta_{\text{ant}})}{(\sigma_P)^2}.
\]

Note that a full Bayesian analysis considering covariances between quantities should be applied when dealing with real data. For more details of such a process, we refer to Birrer et al. (2019) and Jee et al. (2015).

It is worth noting that for gravitational lensing it is the angle measure that matters, while the intensity measure only contributes to the signal-to-noise ratio (S/N). The cosmic opacity can change the absolute intensity but not the relative intensity, thus not biasing the distance determination. Besides, the velocity dispersion based on spectroscopic measurements is also free of the intensity. Therefore, we point out in this paper that distances measured by gravitational lensing should be independent of cosmic opacity.

To make a forecast of the uncertainties of \( D_A^l \), one needs to repeat the process in Yildirim et al. (2019) for each system, which proved that \( D_A^l \) can be determined with 1.8% precision in the best cases, such as the lens system RXJ1131-1231. In a general case, \( D_A^l \) may be measured with larger uncertainties. However, such simulation is beyond this work because of its complexity. We make several assumptions just to give the expected uncertainty level for \( D_A^l \) determination. Similar to Jee et al. (2016), we assume that each quantity in Equation (4) can be measured with a precision of a few per cent currently and in the near future. For example, the time delays measured from
light curves, the spatially resolved velocity dispersion of the lens galaxy, the LOS mass fluctuation, the Fermat potential, and the parameter \( J \) determined from highly resolved imaging. These uncertainties will result in \( D_L^i \) determinations with errors of a few per cent. We adopt a 5\% uncertainty of \( D_L^i \) as the benchmark, which is also used in Jee et al. (2016) and Linder (2011), and as the aim of the HOliCOW program (Suyu et al. 2017). We also consider a 10\% uncertainty for comparison.

Current surveys, for example the Dark Energy Survey (Treu et al. 2018) and the Hyper Suprime-Cam Survey (HSC; More et al. 2017), and the upcoming Large Synoptic Survey Telescope (LSST; Oguri & Marshall 2010), are bringing us new lensed quasars. To achieve the 5\% precision in \( D_L^i \) determination, the lensing systems should be good enough with high-quality observations. First, the time delays should be measured with an uncertainty of a few per cent. The Time Delay Challenge program showed that only 400 well-measured measurements with 5\% precision. We adopt a 5\% uncertainty of \( D_L^i \) as the benchmark, which is also used in Jee et al. (2016) and Linder (2011), and as the aim of the HOliCOW program (Suyu et al. 2017). We also consider a 10\% uncertainty for comparison.

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For illustration purposes, the strain of a chirp waveform is
\[
 h \propto M^{5/3} \bar{f}^{2/3} D_L^{-1},
\]
where \( M \) is the chirp mass of the binary system, \( \bar{f} \) is the frequency. \( \tilde{f} \propto M^{5/3} \bar{f}^{11/3} \). Therefore, measuring \( h, f, \) and \( \tilde{f} \) from the waveform is sufficient to determine the luminosity distance \( D_L \) (Nissanke et al. 2010).

For cosmological studies, one should also know the redshift of the GW source. However, the GW itself does not carry information on the redshift (unless considering the tidal effect, Messenger & Read 2012). An effective way to obtain the redshift is from the EM counterparts, for example the short gamma-ray burst (SGRB), which is one of the most promising EM counterparts of a BNS. Once it is confirmed, the redshift can be measured from its host galaxy or afterglow.

The next generation of GW detectors, for example the Einstein Telescope (ET; Abernathy et al. 2011), will broaden the accessible volume of the universe by three orders of magnitude, promising tens to hundreds of thousands of detections per year. The detection can reach \( z_{GW} = 5 \) with \( S/N > 8 \). To simulate the mock data, we follow the work by Cai & Yang (2017), Zhao & Wen (2018), and Wang & Wang (2019). The source redshift distribution follows
\[
P(z_{GW}) \propto \frac{4\pi \chi^2(z_{GW}) R(z_{GW})}{H(z_{GW})(1 + z_{GW})},
\]
where \( \chi \) is the comoving distance and
\[
R(z_{GW}) = \begin{cases} 
1 + 2z_{GW}, & z_{GW} \leq 1 \\
3/4, & 1 < z_{GW} < 5 \\
0, & z_{GW} \geq 5.
\end{cases}
\]

Since SGRBs are strongly beamed, only nearly face-on configurations can provide the redshifts, the probability is \( \sim 10^{-3} \), and the ET is supposed to detect \( \sim 10^2 \) signals with accurate redshifts. For a nearly face-on system, the instrumental

**Figure 1.** Selected redshift distribution of the lenses that can provide angular diameter distance measurements with 5\% precision. Fifty-five lenses are expected to be observed from current and upcoming projects.

**Figure 2.** Mock \( D_L^i \) data with an uncertainty level of 5\% from strong lensing.
uncertainty is given by

$$\sigma_{L,\text{inst}}^2 \approx \left\langle \frac{\partial H}{\partial D_L} \frac{\partial H}{\partial D_L'} \right\rangle^{-1},$$

(7)

where the angle bracket denotes the inner product. $H \propto 1/D_L$ is the Fourier transform of the waveform, then $\sigma_{L,\text{inst}} \approx D_L/\rho$, where $\rho$ is the combined S/N, determined by the square root of the inner product of $H$. For detecting a GW signal, $\rho > 8$ is usually taken as the minimum requirement. The uncertainty of the inclination $\iota$ would also affect the S/N; for example, the S/N would be changed by a factor of 2 from $\iota = 0^\circ$ to $\iota = 90^\circ$.

Therefore, we set the instrumental uncertainty of the luminosity distance:

$$\sigma_{L,\text{inst}} \approx \frac{2D_L}{\rho}.$$  

(8)

In addition to the instrumental uncertainty, the weak lensing effect of a large-scale structure is another important systematic, especially for high-redshift sources. Ignoring them would result in biased distance measurements. Following Zhao et al. (2011), $\sigma_{L,\text{inst}}/D_L = 0.05z_{\text{GW}}$ is adopted in our simulation. The total uncertainty on the luminosity distance is given by

$$\sigma_L = \sqrt{(\sigma_{L,\text{inst}}^2 + (\sigma_L')^2).}$$

(9)

To test CDDR, we choose GWs whose redshifts are $z_{\text{GW}} < 1.25$ such that they can match the lens redshifts. In Cai & Yang (2017), the total number is assumed to be 100 or 1000 up to $z_{\text{GW}} = 5$, while Zhao & Wen (2018) assumed 100 detections within $z_{\text{GW}} = 2$. Note that most of the detected sources are at low redshifts. In this work, we take 300 sources within $z_{\text{GW}} < 1.25$. The redshift distribution and the corresponding luminosity distances and uncertainty levels are shown in Figures 3 and 4.

4. Methodology and Results

To test any deviation from CDDR, we parameterize it with $(1 + z)^2D_A/D_L = \eta(z)$. Any $\eta(z) \neq 1$ would challenge the validity of CDDR. Since the test is only applied at low redshift $z < 1.25$ and following the literature (Yang et al. 2013; Wu et al. 2015; Liao et al. 2016; Li & Lin 2018; Holanda et al. 2017; Ruan et al. 2018), we Taylor-expand $\eta(z)$ in two ways: firstly with $z$, $\eta(z) = 1 + \eta_0z$; secondly with the scale factor $a = 1/(1 + z)$, $\eta(z) = 1 + \eta_1z/(1 + z)$.

For a CDDR test, in principle the measured luminosity distance and angular diameter distance should correspond to the same redshift. However, since the two distances are from different systems, their redshifts cannot always be matched perfectly. One way to deal with this is to find the nearest data pair, if the redshift difference is small enough, then they can be taken as having the same redshift. In the literature, one usually take $\Delta z < 0.005$ as the criterion (Li et al. 2011; Yang et al. 2013; Liao et al. 2016; Qi et al. 2019), and simulations showed that this would introduce negligible systematic errors. In this work, we adopt a stricter criterion $\Delta z < 0.003$. This value is chosen such that we will still have enough matched pairs. Figure 5 is from one of the simulations. One can see that with our criterion there will be $\sim 50$ data pairs available.

Since this work aims at predicting a constraint on $\eta(z)$ rather than using realistic data to draw a conclusion, we adopt two random processes to give an unbiased result reflecting an average constraining power. First, we randomly select the redshifts of lensing and GWs; second, for each selected data
set, we distribute different noise realizations to generate the mock data. For each mock data set, we carry out minimization using the “scipy.optimize.minimize” function in Python to find the best fits of $\eta_0$ and $\eta_1$. The statistical quantity used in the minimization process is given by

$$\chi^2 = \sum_i \left[ \frac{D_i^L - D_i^A(1 + z_i)^2\eta(z_i)}{\sigma_{L,i}^2 + \sigma_{A,i}^2(1 + z_i)^4\eta(z_i)^2} \right]^2.$$  \hspace{1cm} (10)

The total error in the denominator consists of errors from GW and strong lensing contributions. Since the typical values are both several per cent across the redshift range, their contributions are comparable.

We take all the best fits from each minimization as the expected distributions of $\eta_0$ and $\eta_1$, and plot their probability distribution functions (PDFs) in Figures 6 and 7, respectively. They stand for the constraining power on the deviation of CDDR. Since the PDFs are approximately Gaussian-like, we calculate the standard deviations as the 1σ uncertainty levels. The numerical results are summarized in Table 1 along with part of the results from current methods for comparison. We also plot the reconstruction of $\eta(z)$ for the two parameterizations with their errors in Figure 8. Therefore, while the CDDR test is opacity-free, our method should be very competitive in constraining the deviation parameters. We emphasize again that we do not attempt to make a constraint on CDDR from the realistic data in this work, but rather propose a new method and discuss its power based on mock data and future observing conditions. Therefore, only the errors matter and the best fits are meaningless.

5. Conclusion and Discussions

We propose an opacity-free test of CDDR with strong lensing and GWs. This work is different from theirs. We use direct $D_A$ measurements from strong lensing, whereas Yang et al. (2019) used measurements of distance ratio (just a dimensionless quantity). Their idea was to extract information on the angular diameter distance from the distance ratios, thus the angular diameter distances are obtained in an indirect manner. However, when using the distance ratio data to infer the distances, one has to assume a flat universe in the FLRW metric such that the distance sum rule can be applied, $D_0 = D_L - (1 + z)L_D/(1 + z_L)$ (Rässänen et al. 2015; Liao et al. 2016; Liao 2019). Therefore, to some degree, Yang et al. (2019) actually tested the flatness of the universe rather than a more profound spacetime nature. Also noted by us is the work by Rana et al. (2017), who used 12 realistic time delay lenses to infer $D_A$, and compared them with $D_L^2$ from SNe Ia. However, their work is related to cosmic opacity, and SNe Ia only give relative conditions above $z = 0$ and that it increases with $z$, i.e., the photon number decreases rather than increases when the light travels through the universe. The luminosity distance will look larger than expected. For mechanisms that violate Conditions 1 and 2, we do not limit the sign of $\eta_0$ and $\eta_1$. Among various theories of gravity that
violating CDDR, one may conjecture that one possibility could be related to non-conserved gravitons.

To give a robust test, while the observational precision is important, the systematic errors in each observation should be carefully dealt with. In current strong lensing techniques, blind analysis is being conducted to control the systematics (Suyu et al. 2017). More astrophysical processes are being understood, for example the time delays caused by microlensing (Tie & Kochanek 2018). For standard sirens, the detector calibration errors, weak lensing effects, and template models are being better understood. With the development of these two aspects, we will be able to test CDDR in an opacity-free way in the near future.

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**Table 1**

Comparisons with Current Opacity-dependent Tests on CDDR

| Data | $\eta_0$ | $\eta_1$ |
|------|----------|----------|
| (A) SNe Ia + galaxy clusters | 0.16$^{+0.56}_{-0.30}$ (E) or 0.02$^{+0.20}_{-0.17}$ (S) | … |
| (B) SNe Ia + BAOs | 0.027 ± 0.064 | 0.039 ± 0.099 |
| (C) SNe Ia + strong lensing ($D_{ls}^i/D_{ls}^s$) | $-0.05^{+0.28}_{-0.31}$ | … |
| (D) SNe Ia + ultracompact radio sources | $-0.06 \pm 0.05$ | $-0.18 \pm 0.16$ |
| (E) SNe Ia + GRBs + strong lensing ($D_{ls}^i/D_{ls}^s$) | 0.00 ± 0.1 | $-0.36^{+0.37}_{-0.42}$ |
| (F) H II galaxies + strong lensing ($D_{ls}^i/D_{ls}^s$) | 0.0147$^{+0.35}_{-0.066}$ | … |
| GWs + strong lensing ($D^s(z)$) (this work with $\sigma_8/D^s = 5\%$) | ±0.026 | ±0.047 |
| GWs + strong lensing ($D^s(z)$) (this work with $\sigma_8/D^s = 10\%$) | ±0.034 | ±0.058 |

Note: A: Yang et al. (2013), with (E) = elliptical, (S) = spherical; B: Wu et al. (2015); C: Liao et al. (2016); D: Li & Lin (2018); E: Holanda et al. (2017); F: Ruan et al. (2018).
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