Spiral Shocks in Young Circumbinary Disks

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Abstract. Numerical simulations of binary star formation suffer from serious mismatch with observations. Equal mass binaries are abundant in numerical simulations while also unequal mass binaries are commonly observed. The discrepancy should be due to errors in the numerical simulations. In this paper we discuss the evaluation of the Coriolis force as a source of errors in the numerical simulations. Simulations of an accreting young binary is often performed in the frame co-rotating with the binary. We demonstrate that the specific angular momentum changes spuriously at a shock front, if it is evaluated either solely with the density and velocity at the cell center. We show that the spurious change is erased out if a half of it is evaluated from the numerical flux on the cell surface. We name this method of evaluating the Coriolis force HH type since a half is evaluated from the numerical mass flux and the other half is from the momentum density in the cell. We prove that simulations conserve the momentum measured in the rest frame only when HH type Coriolis force is adopted. The numerical error is serious around shock waves since the difference between the numerical mass flux and momentum density is large there. The shock waves drive gas accretion through angular momentum transfer and should be simulated accurately.

1. Introduction
A young binary accretes gas from the circumbinary disk and each component star grows in mass by the gas accretion. Thus the accretion is important to the primary and secondary masses at the zero age main sequence. Artymowicz and Lubow [1] studied this stage based on their SPH simulations. Their and most of the simulations thus far show that the secondary accretes more gas. It implies that nearly equal mass binaries should be abundant. However, this theoretical prediction is not consistent with the observations that secondaries have considerably lower masses than primaries in many binaries (see, e.g., Reipurth et al. [2] and the references therein). The discrepancy suggests some unknown numerical errors in our simulations.

A clue to solve the discrepancy can be obtained from direct comparison with observations. Takakuwa et al. [3] have discovered spiral waves in the circumbinary disk of L1551 NE from high resolution imaging observation with ALMA in its cycle 0. The spiral waves are also seen in the simulations and drive accretion through angular momentum transfer. Since the resolution and sensitivity of ALMA have been improved, more detailed comparison will be possible in near future.

In this paper we examine the Coriolis force taken into account in the simulations of an accreting young binary as a source of numerical errors. The simulations thus far have employed
the frame co-rotating with the binary in order to reduce numerical errors due to advection of the circumstellar disks. In compensation, the hydrodynamical equations contain an extra source term, the Coriolis force, and are no more in the conservation form in a strict sense. As shown in the subsequent sections, this can be a source of serious error around strong shock waves. We propose the best numerical recipe to evaluate the Coriolis force in this paper.

This paper is organized as follows. We propose our method of evaluating the Coriolis force after inspecting the hydrodynamical equations in §2. We confirm the validity of our method by comparing types of the methods of evaluating the Coriolis force in §3. Implications of our results are given in §4.

2. Methods
We solve the hydrodynamical equations in the co-rotating frame to simulate accretion onto a young binary. For simplicity, the binary is assumed to rotate in the $z = 0$ plane in the Cartesian coordinates with a constant angular velocity, $\Omega$. The primary and secondary of the binary are assumed to have the masses, $M/(1 + q)$ and $Mq/(1 + q)$, respectively, where $M$ and $q$ denote the total mass and the mass ratio, respectively. We neglect the mass of the accreting gas for simplicity. Then the gravitational potential of the binary is expressed as

$$\Phi = - \frac{GM}{(1 + q) |r - r_1|} - \frac{GMq}{(1 + q) |r - r_2|},$$

$$r_1 = \frac{aq}{1 + q} e_x,$$

$$r_2 = - \frac{a}{1 + q} e_x,$$

$$a = \left( \frac{GM}{\Omega^2} \right)^{1/3},$$

where $G$, $a$, and $e_x$ denote the gravitational constant, binary separation, and the unit vector in the $x$-direction. We use the unit system in which $a$ and $\Omega$ are unity.

In the co-rotating frame the hydrodynamical equations are expressed as

$$\frac{\partial p}{\partial t} + \nabla \cdot (\rho v) = 0,$$

$$\frac{\partial}{\partial t} (\rho v) + \nabla \cdot (\rho vv) + \nabla \left( \rho c_s^2 \right) = - \rho [\nabla \Phi + 2\Omega \times v + \Omega \times (\Omega \times r)],$$

where the symbols, $\rho$, $v$, $r$, $c_s$, and $\Phi$ denote the density, velocity, position, the sound speed, and the gravitational potential, respectively. The gas is assumed to be isothermal for simplicity. The vector, $\Omega = \Omega e_z$, denotes the angular velocity of the co-rotating frame.

In numerical simulations we employ the conservation form of the hydrodynamical equations to ensure the conservation of mass and momentum of the flow. Equation of continuity has no source term, while equation of momentum conservation has some external source term such as gravity. In addition to the gravity, the Coriolis force and centrifugal force appear as source terms, i.e., in the right hand side [cf. Eq. (6)] in the co-rotating frame. We argue that a half of the Coriolis force should be evaluated by the cell center value and the other half from the numerical flux. As shown later, this evaluation ensures momentum conservation in the rest frame.

Our argument is based on the following theoretical consideration. Eq. (6) is derived from equation of momentum conservation in the rest frame,
where \( \mathbf{u} \) denotes the velocity in the rest frame. The \( x \)-component of Eq. (6) is obtained by the inner product of the unit vector in the \( x \) direction and Eq. (7),
\[
\mathbf{e}_x \cdot \left[ \frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) + \nabla \left( \rho c_s^2 \right) \right] = -\rho \mathbf{e}_x \cdot \nabla \Phi.
\]
(8)

It is rewritten as
\[
\frac{\partial}{\partial t} [\rho (\mathbf{e}_x \cdot \mathbf{u})] + \nabla \cdot [\rho \mathbf{u} (\mathbf{e}_x \cdot \mathbf{u})] + (\mathbf{e}_x \cdot \nabla) \left( \rho c_s^2 \right) = -\rho (\mathbf{e}_x \cdot \nabla) \Phi + \Omega \rho (\mathbf{e}_y \cdot \mathbf{u}),
\]
(9)
since the time derivative of the unit vector is expressed
\[
\frac{\partial}{\partial t} \mathbf{e}_x = \Omega \times \mathbf{e}_x = \Omega \mathbf{e}_y.
\]
(10)

Here the symbol, \( \mathbf{e}_y \), denotes the unit vector in the \( y \)-direction. Similarly we obtain
\[
\frac{\partial}{\partial t} [\rho (\mathbf{e}_y \cdot \mathbf{u})] + \nabla \cdot [\rho \mathbf{u} (\mathbf{e}_y \cdot \mathbf{u})] + (\mathbf{e}_y \cdot \nabla) \left( \rho c_s^2 \right) = -\rho (\mathbf{e}_y \cdot \nabla) \Phi - \Omega \rho (\mathbf{e}_x \cdot \mathbf{u}).
\]
(11)

The last term in Eq. (9) and that in Eq (11) yield a half of the Coriolis force.

The rest half of the Coriolis force is derived as follows. The rest frame velocity, \( \mathbf{u} \), is related with the co-rotating frame velocity, \( \mathbf{v} \), via
\[
\mathbf{u} = \mathbf{v} + \Omega \times \mathbf{r}.
\]
(12)

Thus we obtain
\[
\mathbf{e}_x \cdot \mathbf{u} = \mathbf{e}_x \cdot \mathbf{v} - \Omega y = v_x - \Omega y,
\]
\[
\mathbf{e}_y \cdot \mathbf{u} = \mathbf{e}_y \cdot \mathbf{v} + \Omega x = v_y + \Omega x.
\]
(13)

Substituting Eqs. (13) and (14) into Eqs. (9) and (11) we obtain
\[
\frac{\partial}{\partial t} [\rho (v_x - \Omega y)] + \frac{\partial}{\partial x} [\rho v_x (v_x - \Omega y)] + \frac{\partial}{\partial y} [\rho v_y (v_x - \Omega y)] + \frac{\partial}{\partial z} [\rho v_z (v_x - \Omega y)]
+ \frac{\partial}{\partial x} \left( \rho c_s^2 \right) = -\rho \frac{\partial \Phi}{\partial x} + \rho \Omega (v_y + \Omega x),
\]
(15)
\[
\frac{\partial}{\partial t} [\rho (v_y + \Omega x)] + \frac{\partial}{\partial x} [\rho v_x (v_y + \Omega x)] + \frac{\partial}{\partial y} [\rho v_y (v_y + \Omega x)] + \frac{\partial}{\partial z} [\rho v_z (v_y + \Omega x)]
+ \frac{\partial}{\partial y} \left( \rho c_s^2 \right) = -\rho \frac{\partial \Phi}{\partial y} - \rho \Omega (v_x - \Omega x),
\]
(16)

We obtain Eq. (6) by substituting the mass conservation,
\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v_x) + \frac{\partial}{\partial y} (\rho v_y) + \frac{\partial}{\partial z} (\rho v_z) = 0,
\]
(17)

into Eq. (15) and (16). Note that the second half of the Coriolis force is introduced by substituting the mass conservation. It implies that this half of the Coriolis force should be evaluated consistently with the evaluation of the mass conservation. It means that we should use the mass flux across the numerical cell surface to evaluate the Coriolis force. Otherwise the momentum of the flow is not conserved in the rest frame. On the other hand, the first half of
the Coriolis force is due to rotation of the unit vectors in the co-rotating frame. Thus it should be evaluated by using the velocity and density in the numerical cell considered.

We examine three types of methods of evaluating the Coriolis force in order to assess the argument mentioned above.

In the first type, the source term, $2\rho \Omega \times \mathbf{v}$, is evaluated by the density and velocity in the cell. Then the $x$- and $y$-component of the source term due to the Coriolis force is evaluated to be

$$ F_{CC,x}(x_i, y_j, z_k) = 2\Omega \rho(x_i, y_j, z_k) v_y(x_i, y_j, z_k), $$
$$ F_{CC,y}(x_i, y_j, z_k) = -2\Omega \rho(x_i, y_j, z_k) v_x(x_i, y_j, z_k), $$

where $(x_i, y_j, z_k)$ denotes the central position of the numerical cell considered. We call this cell center (CC) type in the following. CC type is likely to be used in most of numerical simulations, although it is not written explicitly in the literature.

In the second type, the source term is evaluated from the mass flux across the cell surface, i.e.,

$$ F_{NF,x}(x_i, y_j, z_k) = \Omega \left[ F_{p,x}(x_i + \Delta x/2, y_j, z_k) + F_{p,x}(x_i, y_j - \Delta y/2, z_k) \right], $$
$$ F_{NF,y}(x_i, y_j, z_k) = -\Omega \left[ F_{p,y}(x_i, y_j + \Delta y/2, z_k) + F_{p,y}(x_i - \Delta x/2, y_j, z_k) \right], $$

where $\Delta x$ and $\Delta y$ denote the cell width in the $x$- and $y$-directions, respectively. The symbols, $F_{p,x}$ and $F_{p,y}$ denote the $x$- and $y$-components of the numerical mass flux used in the equation of mass conservation [Eq. (5)]. We call this numerical flux (NF) type in the following. NF type is employed in Hanawa et al. [4] and Takakuwa et al. [3].

We propose to evaluate the source term by half & half mixture of CC and NF types,

$$ F_{HH,x} = \frac{1}{2} (F_{CC,x} + F_{NF,x}), $$
$$ F_{HH,y} = \frac{1}{2} (F_{CC,y} + F_{NF,y}). $$

This is called half & half (HH) in the following.

We use the numerical simulation code SFUMATO (Matsumoto [5]) to solve the hydrodynamical equations. The temporal and spatial accuracies are set to be of the second order. We use the nested grids of 5 levels in which the coarsest grid covers the region of $|x| \leq 12.8a$, $|y| < 12.8a$, and $|z| < 6.4a$. Finer grids cover smaller spaces around the origin with higher spatial resolution. Each grid contains $128 \times 128 \times 64$ cubic cells.

The fixed boundary is set on the cylindrical surface of $\sqrt{x^2 + y^2} = 12.8a$ and the planes of $|z| = 6.4a$. The specific angular momentum is fixed to be $j_{\text{inf}} = 1.2\Omega a^2$. The radial infall velocity is set to be

$$ v_r = \begin{cases} -\left( \frac{2GM}{\sqrt{x^2 + y^2 + z^2}} - \frac{j_{\text{inf}}^2}{x^2 + y^2} \right)^{1/2}, & (\text{otherwise}) \\ 0, & \frac{2GM}{\sqrt{x^2 + y^2 + z^2}} > \frac{j_{\text{inf}}^2}{x^2 + y^2} \end{cases}. $$

Note that the specific angular momentum is defined in the rest frame. The density is fixed to be $\rho_{\text{inf}} = 1.0$ outside the boundary.

The initial density inside the boundary is set to be uniform at $\rho_0$ and the initial velocity is set to vanish in the rest frame, i.e., $\mathbf{v} = -\mathbf{\Omega} \times \mathbf{r}$. The CFL number is taken to be 0.7 unless otherwise noted.
3. Results
In this section we compare the three types of evaluating the Coriolis force (CC, NF, and HH) and demonstrate how numerical simulations depend on them. First we compare the numerical simulations in which the mass ratio and sound speed are set to be \( q = 0.2 \) and \( c_s = 0.1 \), respectively. The initial density is \( \rho_0 = 0.1 \) inside the boundary and \( \rho_{\text{inf}} = 1.0 \) for the gas infalling from the boundary. The infalling gas has a constant specific angular momentum, \( j_{\text{inf}} = 1.2 \).

Figure 1 shows the density and velocity distributions at \( t = 9.43 \). The top, middle and bottom panels denote the results obtained with CC, NF, and HH types of the Coriolis force. The left panels denote the density on the mid plane \((z = 0)\) by color and the velocity in the co-rotating frame, \( v \), on the mid plane by arrows. The right panels denote the specific angular momentum, \( j_z \), on the midplane by color and the velocity in the rest frame, \( u \), by the arrows. The color bars shown in the right of the panels. Some gas elements have specific angular momentum outside the range shown in the color bar.

In the period \( t < 9.43 \) the infalling gas remain in the outer region of \( \sqrt{x^2 + y^2} > 6 \) on the mid plane. The density and velocity are both symmetric around the \( z \)-axis. The gravity is almost symmetric around the axis since the quadruple moment of the gravity is very small there. Thus the specific angular momentum of each gas element should be conserved. However, it is largely changed from its initial value, \( j_{\text{inf}} = 1.2 \), when either CC type or NF type of the Coriolis force is employed. Note that the gas element rotate clockwise in the ring like region of \( \sqrt{x^2 + y^2} \simeq 8 \) in the top right panel (CC type). The specific angular momentum is larger than the initial value outside the ring. On the other hand, the specific angular momentum is enhanced in the ring of \( \sqrt{x^2 + y^2} \simeq 8 \) and lowered outside the ring when NF type Coriolis force is employed. Both CC and NF types of the Coriolis force induce unphysical change in the specific angular momentum but in the opposite directions each other.

When HH type Coriolis force is employed, the specific angular momentum remains nearly constant although it is slightly enhanced in the ring of \( \sqrt{x^2 + y^2} \simeq 8.5 \). This is reasonable since HH type is an intermediate of CC and NF types. The unphysical changes seen in CC and NF types are canceled out each other in HH type.

The unphysical change in the specific angular momentum may have been missed in simulations thus far. The velocity distributions \((v)\) shown in the left panels of Fig. 1 look quite normal, although the top right and middle right panels show apparently abnormal rotation. This serious error is detectable only when the rest frame velocity is displayed. It also should be noted that the error is emphasized in the specific angular momentum, \( j \). The corresponding error in the rotation velocity is lower since \( \Delta v_{\text{rot}} = \Delta j / \sqrt{x^2 + y^2} \).

The unphysical change in the specific angular momentum is more serious, when the density contrast is higher between the infalling gas \((\rho_{\text{inf}})\) and pre-existing gas \((\rho_0)\). We have confirmed this dependence by lowering the density of pre-existing gas to \( \rho_0 = 0.01 \) and \( 10^{-3} \) for NF type Coriolis force. When the density contrast is higher, a stronger shock wave arises between the pre-existing and newly infalling gases. Remember that the numerical mass flux differs very much from the product of the density and velocity at the cell center. Thus the results depend seriously on the evaluation of the Coriolis force.

Even after 10 revolution of the binary the numerical simulation depend considerably on the evaluation of the Coriolis force. Figure 2 is the same as Figure 1 but for the stage of \( t \simeq 71.73 \). The color scales are modified to cover a wider range in the density and specific angular momentum. The specific angular momentum is unnaturally high in the model obtained with CC type Coriolis force. Hence the circumbinary disk is more extended in the model. In the model obtained with NF type Coriolis force, the specific angular momentum is artificially decreased in the model computed and accordingly more gas is accreted onto the binary.

Figure 3 shows the structure of the circumbinary disk at the stage shown in Figure 2. The
Figure 1. The density and velocity distributions at $t = 9.43$ for the model of $q = 0.2$, $c_s = 0.1$ and $j_{\text{inf}} = 1.2$. Color denote the density on the mid plane ($z = 0$) in the left panels and the specific angular momentum in the right panels. The arrows denote $v$ in the left panels and $u$ in the right panels. CC, NF and HH types Coriolis force is employed in the models shown in the top, middle and bottom panels, respectively.

left, middle and right panels denote the models obtained with CC, NF and HH types Coriolis force, respectively. The upper panels show the cross section in the $xz$ plane ($y = 0$) while the lower panels do the distribution in the $xy$ plane ($z = 0$). The density is denoted by color and the velocity is shown by arrows.
Figure 2. The same as Fig. 1 but for $t \simeq 71.73$ with different color scales.

All the models show a geometrically thin circumbinary disk, S-shaped spiral shock waves, and vertically expanded infalling envelope. However, the difference is not negligibly small. The difference is in part due to the specific angular momentum of infalling gas. The specific angular momentum is higher in the model obtained with CC type Coriolis force (see Fig. 2). Thus the circumbinary disk is extended more in the radial direction in CC model. Some part of the difference is due to spiral shock waves seen in the circumbinary disk. This possibility should be examined more carefully.
Figure 3. The structure of the circumbinary disk is shown for the models shown in Fig. 2. The left, middle, and right panels show the models obtained with CC, NF, and HH type Coriolis force, respectively. The upper panels denote the cross sections in the $xz$ plane while the lower panels do those in the $xy$ plane.

4. Discussions
As shown in the previous section, numerical simulations depend on the method of evaluating Coriolis force. A serious numerical error arises when the Coriolis force evaluated either by CC type or by NF type. On the other hand, the numerical error is much smaller when the Coriolis force is evaluated by HH type, i.e., the average of CC and NF types. We have also found that the structure of circumbinary disk and the S-shaped spiral shock waves are also affected by the methods of evaluating the Coriolis force. Strong spiral waves are also excited near the primary and secondary. They are expected to suffer from similar numerical errors and may be a source of discrepancy.

The numerical error is ascribed to the fact that the Coriolis force appears as a source term in the hydrodynamical equations. In other words, the hydrodynamical equations are not in the conservation form in a strict sense. The source term should vanish when the equations are transformed into those in the rest frame. However, it may not be true when the hydrodynamical equations are discretized. The mass flux and momentum density are evaluated differently in numerical simulations, although both of them are the product of the density and velocity, $\rho v$. The former is evaluated on the cell surface and named numerical flux, while the latter is regarded as a cell average. The numerical flux is intentionally taken to be different from the simple average in modern numerical hydrodynamics. This difference is a key to avoid numerical oscillation around a shock wave. We need to think whether the Coriolis force is proportional to either the mass flux or the momentum density.

In §2 we have demonstrated that the half of the Coriolis force is due to the mass conservation and accordingly proportional to the mass flux. The other half is ascribed to the rotation of unit vector in the co-rotating frame and should be proportional to the momentum density. We have confirmed this theoretical argument in §3.
Our method of evaluating the Coriolis force is simple and easy to implement. Still the advantage is large. The conservation of specific angular momentum is greatly improved. This recipe can be applied to any numerical simulations performed in a rotating frame. It could be applied to rotational frame dragging (Lense-Thirring) effect in general relativistic hydrodynamics since this effect corresponds to rotation of the frame.

We also argue that the rest frame velocity should be examined when the numerical simulation is performed in a co-rotating frame. As demonstrated in Figure 1, a serious numerical error can be hidden and missed easily when the velocity is shown in the co-rotating frame.

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