Improving the speed of initial alignment for marine strapdown inertial navigation systems using heading control signal feedback in extended Kalman filter

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Abstract
In this article, a method was proposed for strapdown inertial navigation systems initial alignment by drawing on the conventional alignment method for stable platform navigation systems. When a vessel is moored, the strapdown inertial navigation system contributes to the disturbing motion. Moreover, the conventional methods of accurate alignment fail to succeed within an acceptable period of time due to the slow convergence of the heading channel in the mooring conditions. In this work, the heading was adjusted using the velocity bias resulting from the component of the angular velocity of the Earth on the east channel on the strapdown inertial navigation systems analytic platform plane to accelerate convergence in the initial alignment of navigation system. To this end, an extended Kalman filter with control signal feedback was used. The heading error was calculated using the north channel residual velocity of the strapdown inertial navigation systems analytic platform plane and was entered into an extended Kalman filter. Simulation and turntable experimental tests were indicative of the ability of the proposed alignment method to increase heading converge speed in mooring conditions.

Keywords
Strapdown inertial navigation system, fine alignment, mooring condition, extended Kalman filter

Date received: 23 October 2018; accepted: 14 November 2019

Introduction
Initial alignment is the first work and crucial phase stage in strapdown inertial navigation systems (SINS). The objective of alignment is to provide a highly accurate initial attitude matrix between the navigation frame and body frame. In other words, the SINS needs to accomplish alignment before it starts the navigation phase. Convergence speed and accuracy associated with initial alignment are two important criteria for INSs, so in many applications we need to do the initial alignment with precise accuracy and rapid speed. Also, under mooring conditions, the SINS used in a ship suffers from the external disturbance components caused by the motions of the sea waves and wind waves, therefore a rapid and accurate alignment of a ship’s SINS is hard to achieve. Therefore, this problem has...
attenuate the effect of external disturbed movements and approaches. SINS under mooring conditions have utilized different methods to meet the denoising requirement and lead to the increase in accuracy. But these FIR filters are designed with very large orders (FIR) digital filters to attenuate disturbing accelerations, and these methods have been employed by Sun et al.22 to cancel the high-frequency sensors noise effects. Li et al.22 employed the cascaded finite impulse response (FIR) filters lead to time-delay problems which are not suitable for real-time implementations. Combination of a Kalman filter and a IIR digital filter to filter the outer disturbances in a real-time way was proposed by Lu et al.27,28 in which the computation time is much shorter than the wavelet methodology. Using of inertial frame-based alignment and the projection of gravity in the inertial frame, a new approach was proposed by Sun et al.29 to calculate the attitude matrix between inertial frame and body frame. In this way, the disturbed movements can attenuate and lead to the change of velocity error in the north. With the north velocity error feedback for azimuth angle correction, the error diminishes over time and the platform coordinate match the navigation coordinate. This approach has been used in SINS for alignment with changes in its structure. However, the basic of this method relies on static conditions and in the case of external conditions, caused by the sea waves will be inaccurate.13

The other method is integration alignment, which estimates misalignment angles based on the modern control theory. For the case of static conditions, this approach can accomplish alignment rapidly and precisely by using modern filtering methods. However, the estimation time is affected and increased by the distribution. Under the mooring conditions, the alignment process undergoes extra disturbed accelerations and angular velocities. Therefore, the vehicle had to wait for the fulfillment for a long alignment time and the alignment duration will be beyond 20 min.23 To overcome this challenge, studies on alignment of SINS under mooring conditions have utilized different approaches.

In some research studies, the prefilters are used to attenuate the effect of external disturbed movements and sensor noise. In the studies by El-Sheimy et al.24 and Nasar and El-Sheimy,25 wavelet de-noising technique is used to cancel the high-frequency sensors noise effects. Li et al.22 employed the cascaded finite impulse response (FIR) filters to attenuate disturbing accelerations, but these FIR filters are designed with very large orders to meet the de-noising requirement and lead to the increasing of the computation burden. In the study by Sun and Sun,26 the infinite impulse response (IIR) digital low-pass filter was used to reduce the influence caused by the disturbed movements on the gyroscope and accelerometer outputs with small filter orders. Nevertheless, all these digital filters lead to time-delay problems which are not suitable for real-time implementations. Combination of a Kalman filter and a IIR digital filter to filter the outer disturbances in a real-time way was proposed by Lu et al.27,28 in which the computation time is much shorter than the wavelet methodology. Using of inertial frame-based alignment and the projection of gravity in the inertial frame, a new approach was proposed by Sun et al.29 to calculate the attitude matrix between inertial frame and body frame. In this way, the disturbed movements can attenuate and lead to the change of velocity error in the north. With the north velocity error feedback for azimuth angle correction, the error diminishes over time and the platform coordinate match the navigation coordinate. This approach has been used in SINS for alignment with changes in its structure. However, the basic of this method relies on static conditions and in the case of external conditions, caused by the sea waves will be inaccurate.13

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both simulation tests are illustrated in the fifth section. Finally, the conclusion is presented in the sixth section.

Coordinate frame definitions

The coordinate frames used in this article are defined as follows:

1. The $n$ frame is the navigation frame with its origin at center of marine vehicles’ mass, the $x_n$ axis points to the north, the $y_n$ axis points to the east, and the $z_n$ axis points to the downward.

2. The $b$ frame is the body frame with its origin at center of marine vehicles’ mass, the $x_b$ axis points to the forward, the $y_b$ axis points to the right of the vehicle, and the $z_b$ axis points to the downward of the vehicle.

Conventional platform alignment method

The velocity error feedback method is used for initial alignment of platform INS. The general idea of this method is to use three control loops to adjust the roll, pitch, and heading initial angles. The pitch and roll control loops, based on the accelerometer output, create commands for stable platform motors to horizontally align the plane. In the closed loop, the output of two horizontal accelerometer channels at the first stage is aligned with the plane. In addition, the commutation is sent to the vertical channel motor through the integration of the north accelerometer based on the image of the Earth’s angular velocity on the eastern channel gyroscope (due to deviation from the east). In principle, this angular velocity of the east channel gyroscope accounts for the tilt in the stable plane. This tilt causes an image of the gravity acceleration on the north accelerometer. Integration of this acceleration is used to correct the heading channel. This rotation of the heading of the plane continues to a point where no further increase on the accelerometer of the northern channel is created. It means that the east gyroscope is located in the east and no longer has any image of the Earth’s angular velocity. Of course, the overall structure of the stable platform has complexities, and here the generalities of performance are expressed. This process can be used in a strapdown structure with a slight change in its functionality. The full description of the platform is provided in the studies of Ben12 and Kouba,34 while Li et al.35 illustrate it in more details. These references are based on classical control.

SINS alignment structure

Conventional SINS alignment algorithms have used error model to estimate and correct SINS attitude errors like tilt and heading error. The error propagation model in navigation systems is extracted from the INS in the geographic frame and is represented in the state-space equations form. Different SINS error models in alignment were proposed.36,37 Using the linearized model of the SINS error and utilizing the zero average speed of the ships as measurement signal, the designed Kalman filter will estimate the attitude error. So, it is possible to correct heading and tilts errors by estimated errors. In the following, we introduced error model with internal feedback loop to improve the speed convergence of the SINS initial alignment.

Navigation error propagation model

The error propagation model used in this article is a linear error model that revolves around the working point with six state variables and is based on the $\varphi$ angle error model. The two states are velocity and angle errors. The state variables are as follows

\[
x = [\delta V_N, \delta V_E, \delta V_D, \varphi_N, \varphi_E, \varphi_D]^T
\]  

The position equations in the navigation frame are as follows

\[
\dot{L} = \frac{V_N}{r_L + h}, \quad \dot{t} = \frac{V_E}{(r_t + h) \cos L}, \quad \dot{h} = -V_D
\]  

After differentiating the above equations, the following relation is obtained using the simplifying assumptions38:

\[
r_L + h = r_t + h = r_L = r_t = r = r_c
\]

\[
\begin{align*}
\delta \dot{L} &= \frac{1}{r} \delta V_N - \frac{V_N}{r^2} \delta h \\
\delta \dot{t} &= \frac{1}{r \cos L} \delta V_E + \frac{V_E}{r \cos L} \delta L - \frac{V_E}{r^2 \cos L} \delta h \\
\delta \dot{h} &= -\delta V_D
\end{align*}
\]  

The velocity equations in the navigation frame are expressed as follows

\[
\dot{V}^N = f^N + g^N - (2 \omega^N_{IE} + \omega^N_{EN}) \times V^N
\]  

After differentiating the above equations, we have

\[
\begin{align*}
\delta \dot{V}^N &= \delta f^N + \delta g^N - (2 \delta \omega^N_{IE} + \delta \omega^N_{EN}) \\
&\times V^N - (2 \omega^N_{IE} + \omega^N_{EN}) \times \delta V^N
\end{align*}
\]  

and35

\[
\begin{align*}
\omega^N_{EN} &= [l \cos L - \dot{L} \sin L] \\
\delta \omega^N_{EN} &= [\delta \dot{L} \cos L - \dot{L} \sin L \delta L - \delta \dot{L} - \dot{L} \sin L - \dot{L} \cos L \delta L] \\
\omega^N_{IE} &= [\omega_{ic} \cos L \ 0 \ - \omega_{ic} \sin L] \\
\delta \omega^N_{IE} &= [-\omega_{ic} \sin L \delta L \ 0 \ - \omega_{ic} \cos L \delta L]
\end{align*}
\]  

The following relations are obtained by expanding equation (5) and neglecting variations of $g$ and elements containing the position error
\[ \delta V_N = B_N + g\varphi_N + \frac{V_D}{r}\delta V_N - 2\left(\frac{V_E}{r \cos L} + \omega_c\right)\sin L\delta V_E + \frac{V_N}{r}\delta V_D \]  
(7)

\[ \delta V_E = B_E - g\varphi_N + \left(\frac{V_E}{r \cos L} + 2\omega_c\right)\sin L\delta V_N + \left(\frac{V_N\tan L + V_D}{r}\right)\delta V_E + \left(\frac{V_E}{r \cos L} + 2\omega_c\right)\cos L\delta V_D \]  
(8)

\[ \delta V_D = B_D + \delta g - \frac{2V_N}{r}\delta V_N - 2\left(\frac{V_E}{r \cos L} + \omega_c\right)\cos L\delta V_E \]  
(9)

where \( B_j \) 's are the maps of the accelerometer errors on the navigation axis

\[ \left[ B_N \ B_E \ B_D \right]^T = C_B^N \delta f^B \]  
(10)

Attitude is represented by the minimum number of parameters, which is three. These three parameters are the rotation vector components that describe the small-angle error from the true navigation frame to the calculated navigation frame on the navigation system computer.\(^{38}\) We have

\[ C_B^N = C_{B_{\text{true}}}^N, \quad \hat{C}_B^N = C_{B_{\text{analytical}}}^N = C_B^P \]  
(11)

where \( P \) is the coordinate that will be used instead of \( N \) on the navigation system computer, and represents the arithmetic coordinate or the analytic stable plane. When the navigation system has an error in calculating the rotational attitude, the rotation vector from \( N \) to \( P \) is shown by \( \phi \). For a small \( \phi \) we have

\[ C_B^P = I - \Phi = I - [\phi \times] \]  
(12)

Besides, since \( \phi \) values are small, they can provide good approximations of Euler angles between the \( P \) and \( N \) frames. As shown in the following, the resulting linearized differential equation for \( \phi \) error vector is expressed as follows

\[ \dot{\varphi} = -\Omega_{IN}\varphi - C_B^N \delta \omega_{IB} + \delta \omega_{IN}^N \]  
(13)

and

\[ \delta \omega_{IN}^N = \delta \omega_{EN}^N + \delta \omega_{IE}^N \]  
(14)

The true navigation coordinate (\( N \)) is considered an unknown coordinate, and thus the related rotation matrix and the velocity vector will have errors. The components of equation (13) after simplification are as follows

\[ \dot{\varphi}_N = \left(\omega_D - \frac{V_E}{r} \tan L\right)\varphi_N + \left(\frac{V_N}{r}\right)\varphi_D + \frac{1}{r}\delta V_E + \omega_D\delta L - D_N \]  
(15)

\[ \dot{\varphi}_E = -\left(\omega_D - \frac{V_E}{r} \tan L\right)\varphi_N + \left(\frac{\omega_N}{r}\right)\varphi_D - \frac{1}{r}\delta V_N - D_E \]  
(16)

\[ \dot{\varphi}_D = -\left(\frac{V_N}{r}\right)\varphi_N - \left(\frac{\omega_N}{r}\right)\varphi_E - \frac{\tan L}{r}\delta V_E \]  
(17)

\[ \omega_N = \omega_{ie} \cos L, \quad \omega_D = -\omega_{ie} \sin L \]  
(18)

where

\[ [D_N \ D_E \ D_D]^T = C_B^N \delta \omega_{IB} \]

In case the navigation system is not moving in relation to the Earth, the error model is as follows

\[ \begin{cases} 
\delta V_N = -f_D\varphi_E + f_E\varphi_D + 2\omega_D\delta V_E \\
\delta V_E = f_D\varphi_N - f_N\varphi_D - 2\omega_D\delta V_N + 2\omega_N\delta V_D \\
\delta V_D = -f_E\varphi_N + f_N\varphi_E - 2\omega_N\delta V_E 
\end{cases} \]  
(19)

The angle error with the control signal feedback (\( c\delta V_N \)) is also calculated as follows

\[ \begin{cases} 
\dot{\varphi}_N = \omega_D\varphi_E + \frac{1}{r}\delta V_E \\
\dot{\varphi}_E = -\omega_D\varphi_N + \omega_N\varphi_D - \frac{1}{r}\delta V_N \\
\dot{\varphi}_D = -\omega_N\varphi_E - \frac{\tan L}{r}\delta V_E + c\delta V_N 
\end{cases} \]  
(20)

where \( \omega_{ie} \) denotes the angular velocity of the Earth relative to the inertial frame, \( L \) stands for latitude, and \( r \) represents the mean radius of the Earth.

Remark 1. The heading angle convergent in equation (20) is very slow, since the Earth angular rate is too slow and the sensor noise is too large. To improve heading angle convergence, we employed a feedback from north velocity error with \( c \). The \( c \) gain should be chosen in such a way that the heading convergence rate improves, but the heading angle noise must not be amplified. In practice, the estimated variance of the heading angle must be set as a big value to depress the sensor noise. If not, its convergent speed will be very low.

Remark 2. In this error model, we assumed that the gyroscopes do not have bias errors, or we accept the bias error of the gyroscope in the final error (due to the inability to observe the bias error of the gyroscope in general, in the problem of marine alignment). Equations (15) to (17) show that any bias in the sensors is manifested as an angle error, and this angle error exists until the bias is corrected, resulting in the error in the analytic stable plane. As a result, the error is mirrored in the accelerometers image and it causes a constant velocity error as shown by equation (19).
Kalman filter with control signal

Assume the system model includes the known control signal expressed as follows

\[ x_k = \Phi_{k-1}x_{k-1} + Cu_k + Gw_{k-1} \]  

where \( u_k \) denotes the known control signal.

The structure of the Kalman filter algorithm in this case is shown in the following

\[ \hat{x}_k = \Phi_{k-1}\hat{x}_{k-1} + Cu_k + K(z_k - H\Phi_{k-1}\hat{x}_{k-1} - HCu_k) \]

Besides, \( K_k \) is calculated via the conventional Kalman filter equation.

Note that the introduction of the known control signal to the system model does not influence the calculation equations for the gain matrix \( K_k \) and only the a priori estimate of the state vector in equation (22) changes as follows

\[ \hat{x}_{k|k-1} = \Phi_{k-1}\hat{x}_{k-1} + Cu_k \]

State-space model

As mentioned in the previous section, the space-state model is expressed via equation (21).38

For \( \Phi \)

\[ \Phi = \begin{bmatrix} 0 & 2\omega_D & 0 & 0 & -f_D & f_E \\ -2\omega_D & 0 & 2\omega_N & f_D & 0 & -f_N \\ 0 & -2\omega_N & 0 & f_E & f_N & 0 \\ 0 & 1/r & 0 & 0 & \omega_D & 0 \\ -1/r & 0 & 0 & -\omega_D & 0 & \omega_N \\ 0 & -tanL/r & 0 & 0 & -\omega_N & 0 \end{bmatrix} \]  

(24)

For \( G \)

\[ G = \begin{bmatrix} C_b^a & 0_{3 \times 3} \\ 0_{3 \times 3} & -C_b^a \end{bmatrix} \]  

(25)

For \( u \) and \( C \)

\[ u = \delta V_N, \quad C = [0 \ 0 \ 0 \ 0 \ c]^T \]  

(26)

The measurements used are the velocity error measurements that are entered into the filter along with the specified covariance matrix \( R \)

\[ y = [\delta V_m]^T \]  

(27)

SINS alignment algorithm

For increasing the rate of convergence to true heading, we combined the extended Kalman filter (EKF) with the improved error model. For this purpose, we utilized a control signal feedback from the north velocity error to change the SINS heading channel damping based on the traditional platform INS structure. For the error model presented in equation (20), the convergence rate of the roll and pitch is much greater than the heading convergence rate when the \( c \) gain is zero. We could improve the heading convergence rate by adjusting the value of the gain \( c \). But it is necessary the north speed variation only depend on the heading changes. So, as gyrocompass alignment method, we create an analytic stable plane in the SINS structure, then north channel velocity residual error is used as the control signal feedback for the heading channel. The SINS structure with the control signal feedback EKF is depicted in Figure 1. In the proposed structure, the control signal feedback is not used for 25 s. Afterward, using the horizontally analytic platform plane, we start using the control signal feedback in EKF. After creating the horizontally analytic platform plane, \( \delta V_n \) is used with \( c \) gain and the feedback in the heading channel EKF. The \( c \) gain can be selected based on the trade-off between convergence speed and heading angle noise. In the course of convergence, the gain is reduced from high gain to low gain to secure good convergence.

Simulation results

In this section, the simulation study shows the capability of the proposed algorithm to correct the initial heading error.
and confirm the performance under the marine mooring condition. Input data (accelerations and angular velocities) that fed to the algorithm were created using the relationships expressed in the next sections.

**Sensor model**

Table 1 presents the parameters of the IMU model used in our simulation.

| Parameters values for IMU error model |
|-----------------------------------------|
| **Fixed bias**                         | 100 µg          |
| **Velocity random walk**               | 0.05 m/s/√h    |
| **Scale factor error**                 | 50 ppm          |
| **Input axis misalignment**            | 10 arcseconds   |

| Parameters values for gyrooscope error model |
|----------------------------------------------|
| **Fixed drift**                             | 0.01 deg/h     |
| **Angular random walk**                    | 0.01 deg/√h    |
| **Scale factor error**                     | 20 ppm          |
| **Input axis misalignment**                | 5 arcseconds    |

**IMU**: inertial measurement unit.

Table 1. Parameters values for IMU error model.

| Parameters values for accelerometer error model |
|-----------------------------------------------|
| **Fixed bias**                               | 100 µg          |
| **Velocity random walk**                    | 0.05 m/s/√h    |
| **Scale factor error**                      | 50 ppm          |
| **Input axis misalignment**                 | 10 arcseconds   |

| Parameters values for gyroscope error model |
|---------------------------------------------|
| **Fixed drift**                             | 0.01 deg/h     |
| **Angular random walk**                    | 0.01 deg/√h    |
| **Input axis misalignment**                | 5 arcseconds    |

In the above equations, $S, m, \text{bias, dfr}, \text{ and rnd}$ represent the scale factor errors, misalignment errors, constant bias, fix drift, and random noise, respectively. Moreover, $a^{\text{acc}}$ and $\omega^{\text{gyr}}$ represent the actual IMU outputs, while $a^T$ and $\omega^T$ denote the true values.

**Sea turbulence model**

To simulate the designed algorithm, there is a need for a model of sea turbulence. Different models have been suggested for this purpose in different papers. In this section, the models in some papers are presented and one of them selected for use in this research. In the study by Scherzinger and Reid,\textsuperscript{37} it is assumed that the ship is in an anchorage and the angles of its heading, pitch, and rolling are changed as follows

$$\psi = 30^\circ + 5^\circ \cos \left( \frac{2\pi}{7} t + \frac{\pi}{3} \right)$$

$$\theta = 7^\circ \cos \left( \frac{2\pi}{5} t + \frac{\pi}{4} \right)$$

$$\phi = 10^\circ \cos \left( \frac{2\pi}{6} t + \frac{\pi}{7} \right)$$

where $\psi$ is the heading angle, $\theta$ is the pitch angle, and $\phi$ is the roll angle. Also, its velocity changes as follows

$$V_D = A_D + \omega_D \cos(\omega_D t + \varphi_D)$$

where $i = x, y, z; A_D = 0.02m, A_D = 0.03m, A_D = 0.3m,$

$\omega_D = 2\pi/T_D, T_D = 7s, T_D = 6s, T_D = 8s,$ and $\varphi_D$ is considered a normal distribution in the interval $[0, 2\pi]$.  

![Figure 2. The heading angle estimation using the usual method (without using the heading error feedback).](image-url)
Perturbation velocities due to high-frequency vibrations are considered as follows

\[ V_{DH_i} = \frac{A_{DH_i}}{2\pi f_{DH_i}} + \cos(2\pi f_{DH_i} t + \varphi_{DH_i}) \]  

(33)

where \( i = x, y, z \), \( A_{DH_x} = 4.2g \), \( A_{DH_y} = 3.8g \), \( A_{DH_z} = 4.0g \), \( f_{DH_x} = 300Hz \), \( f_{DH_y} = 250Hz \), \( f_{DH_z} = 400Hz \), and \( \varphi_{DH_i} \) is considered a normal distribution in the interval \( [0, 2\pi] \).

In the study by Sun and Sun,\textsuperscript{20} it is assumed that the ship is in an anchor and the angles of its heading, pitch, and rolling are changed as follows

\[ \psi = 10' \cos \left( \frac{2\pi}{6} t \right) \]  

\[ \theta = 6' \cos \left( \frac{2\pi}{8} t \right) \]  

\[ \phi = 12' \cos \left( \frac{2\pi}{10} t \right) \]  

(34)

where \( \psi \) is the heading angle, \( \theta \) is the pitch angle, and \( \phi \) is the roll angle. Also, its velocity changes as follows
where $i = x, y, z$ means the north, east, and down in the navigational frame, and $A_x = 0.02m$, $A_y = 0.02m$, $A_z = 0.35m$, $T_x = 7s$, $T_y = 6s$, $T_z = 8s$, and $\varphi_i$ is a normal distribution in the interval $[0 \ 2\pi]$. The authors\textsuperscript{20} also provide a model that is almost the same as in the study by Gao.\textsuperscript{30} In the study by Chang et al.,\textsuperscript{2} only the attitude model is pointed out. Because of the validation and completeness of simulation, the model proposed by Gu et al.\textsuperscript{40} is used in this article for simulations. Of course, to evaluate and compare results with different conditions, the models presented in other references are also used. This is necessary to investigate the comprehensiveness of the algorithm under different perturbation conditions.

For simulation, the following parameters are considered for the filters:
- The initial value of the state $x_0 = 0.0 \times 1$.
- The initial covariance of the state $P_0 = \text{diag}[(0.1)^2 \ (0.1)^2 \ (0.01)^2 \ (0.01)^2 \ (0.01)^2]$ \hfill (36)

The values are represented in m/s and rad/s. The covariance of the process noise $Q = \text{diag}[(0.05)^2 \ (0.05)^2 \ (0.001)^2 \ (0.001)^2 \ (0.001)^2]$ \hfill (37)

The values are represented in m/s and rad/s. The covariance of the measurement noise $R = \text{diag}[(0.01\text{m/s})^2 \ (0.01\text{m/s})^2 \ (0.01\text{m/s})^2]$ \hfill (38)

**Simulation results**

First, the data from the sea oscillation model are used to examine the results with the EKF without the heading measurement feedback error. In this case, we assume an initial heading error of approximately $10^\circ$, and filter functions for 300 s. Figures 2 to 4 illustrate the slow convergence behavior of the conventional method (without heading measurements). As seen in Figure 2, this process is repeated in the following in the presence of sea oscillations using the speed error in the north direction as the simulation error (Figures 5 to 7). In this case, the initial error is assumed to be at least

\[
V_i = A_i + \frac{2\pi}{T_i} \cos \left( \frac{2\pi}{T_i} t + \varphi_i \right) + \text{rand} \quad (35)
\]
As shown in Figure 6, convergence occurred in less than 60 s. It can be concluded from the simulation result that the proposed algorithm significantly increased the convergence speed which is important criteria for INSs. The reason is that the feedback from north velocity error improved error model dynamics.

**Experiment results**

We placed the fiber optic gyroscope (FOG)-IMU-based SINS which is produced by our laboratory on the two-axis turntable that can rotate along its vertical and latitudinal axis under the motor drive. The IMU is using accelerometers with a precision of 100 μg, and gyros with a precision of 0.01 deg/h. The IMU can be seen in Figure 8. A turntable test is conducted to validate the performance of the alignment algorithm presented in this article. We run the turntable to simulate the mooring conditions. The data update rate is 400 Hz and we use 300 s data. The turntable performs the sinusoidal oscillation along its vertical and latitudinal axis under the motor drive. The oscillation period is 8 s and its magnitude is 10°. The initial level misalignment errors are set to be 10°. The filtering parameters for the filters are the same as in the fifth section. The roll, pitch, and heading angles with EKF and without the heading measurement feedback error are shown in Figures 9 to 11.
The roll, pitch, and heading angles with EKF and with the heading measurement feedback error are presented in Figures 12 to 14.

According to experimental tests, the proposed alignment method increased heading convergence under mooring conditions. Also, it can be understood from results that the large heading angle error problem is solved well, therefore the proposed method does not require coarse alignment.

Conclusion

An improved alignment algorithm for marine SINS based on the control signal feedback in EKF was proposed in this article. The major improvement carried out in this work was meant to reduce the time of heading angle convergence in the mooring conditions alignment. In the proposed algorithm, the north velocity error in the analytic stable plane of SINS was used as a control signal damping feedback to heading error in EKF. The heading error is not directly available. However, the propagation error equations imply that after horizontal alignment of SINS analytical plane, the velocity error in the north direction is directly related to the error of the heading angle. This error can be entered into the filter as a control signal, accelerating the heading angle convergence. As regards the simulation and turntable test results, this algorithm significantly improved the convergence rate of the heading angle compared to the conventional methods.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

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