The quest for low-energy supersymmetry and the role of high-energy $e^+e^−$ colliders

F. Zwirner

Theory Division, CERN, Geneva, Switzerland

Abstract

The motivations for low-energy supersymmetry and the main features of the minimal supersymmetric extension of the Standard Model are reviewed. Possible non-minimal models and the issue of gauge coupling unification are also discussed. Theoretical results relevant for supersymmetric particle searches at present and future accelerators are presented, with emphasis on the role of a proposed $e^+e^−$ collider with $\sqrt{s} = 500$ GeV. In particular, recent results on radiative corrections to supersymmetric Higgs boson masses and couplings are summarized, and their implications for experimental searches are discussed in some detail.

Plenary talk at the Workshop on Physics and Experiments with Linear Colliders, Saariselkä, Lapland, Finland, 9–14 September 1991

CERN-TH.6357/91
December 1991

1On leave from INFN, Sezione di Padova, Italy.
1 Introduction

Realistic models of low-energy supersymmetry have been studied for about 15 years, starting with the pioneering works of Fayet [1] and continuing with more and more systematic investigations [2], but there is no decisive experimental evidence yet either in favour of or against this idea. It is then almost a duty for the theoretical speaker on the subject (the experimental aspects are discussed in ref. [3]) to argue in favour of the following two statements:

- Low-energy supersymmetry is, today more than ever, a phenomenologically viable and theoretically motivated extension of the Standard Model.
- High-energy $e^+e^-$ colliders can play a crucial role in testing it experimentally.

With the above two goals in mind, the discussion will be organized as follows. This introduction will end with a brief reminder of the motivations for low-energy supersymmetry. Sect. 2 will introduce the Minimal Supersymmetric extension of the Standard Model (MSSM), and its possible non-minimal alternatives. Plausible theoretical constraints on the MSSM, including the ones coming from gauge coupling unification, will be also discussed. Sects. 3 and 4 will take a closer look at the particle spectrum of the MSSM, thus providing an introduction to the experimental discussion of ref. [3]. The present limits from LEP I and Tevatron and the expected sensitivity of LEP II and LHC/SSC will be reviewed, followed by some theoretical considerations on the potential of a 500 GeV $e^+e^-$ collider (EE500). Sect. 3 will discuss recent results on radiative corrections to Higgs boson masses and couplings, and their implications for experimental searches. Sect. 4 will deal with supersymmetric partners of quarks, leptons, gauge and Higgs bosons. Finally, sect. 5 will contain some concluding remarks.

1.1 Motivations for low-energy supersymmetry

There are many good reasons to believe that supersymmetry [4] and its local version, supergravity [5], could be relevant in a fundamental theory of particle interactions. Symmetries, even when broken, have been very important
in establishing modern particle theory as we know it today: supersymmetry is the most general symmetry of the S-matrix consistent with relativistic quantum field theory \[6\], so it is not inconceivable that Nature might make some use of it. Also, superstrings \[7\] are the present best candidates for a consistent quantum theory unifying gravity with all the other fundamental interactions, and supersymmetry appears to play a very important role for the quantum stability of superstring solutions in four-dimensional space-time. Experimental data, however, tell us that supersymmetry is not realized exactly, and none of the above motivations gives us any insight about the scale of supersymmetry breaking.

The only motivation for low-energy supersymmetry, i.e. supersymmetry effectively broken around the electroweak scale, comes from the naturalness or hierarchy problem \[8\] of the Standard Model (SM), whose formulation will now be sketched. Despite its remarkable phenomenological success \[9\], it is impossible not to regard the SM as an effective low-energy theory, valid up to some energy scale \(\Lambda\), at which it is replaced by some more fundamental theory. Certainly \(\Lambda\) is less than the Planck scale \(M_P \sim 10^{19}\) GeV, since one needs a theory of quantum gravity to describe physics at these energies. However, the study of the Higgs sector of the SM suggests that \(\Lambda\) should rather be close to the Fermi scale, \(G_F^{-1/2} \sim 300\) GeV. The argument goes as follows. Consistency of the SM requires the SM Higgs mass to be less than \(O(1\) TeV\). If one then tries to extend the validity of the SM to energy scales \(\Lambda \gg G_F^{-1/2}\), one is faced with the fact that in the SM there is no symmetry to justify the smallness of the Higgs mass with respect to the (physical) cut-off \(\Lambda\). This is apparent from the fact that in the SM one-loop radiative corrections to the Higgs mass are quadratically divergent. Motivated by this problem, much theoretical effort has been devoted to finding descriptions of electroweak symmetry breaking which modify the SM at scales \(\Lambda \sim G_F^{-1/2}\). Here supersymmetry comes into play because of its improved ultraviolet behaviour with respect to ordinary quantum field theories \[10\], due to cancellations between bosonic and fermionic loop diagrams. If one wants to have a low-energy effective Lagrangian valid up to scales \(\Lambda \gg G_F^{-1/2}\), with one or more elementary scalar fields, kept light without unnatural fine-tunings of parameters, the solution \[11\] is to introduce supersymmetry, effectively broken in the vicinity of the electroweak scale. This does not yet explain why the scale \(M_{\text{SUSY}}\) of supersymmetry breaking is much smaller than \(\Lambda\),
but at least links the Fermi scale $G_F^{-1/2}$ to the supersymmetry-breaking scale $M_{\text{SUSY}}$, and makes the hierarchy $G_F^{-1/2} \sim M_{\text{SUSY}} \ll \Lambda$ stable against radiative corrections.

2 The MSSM

The most economical realization of low-energy supersymmetry is the Minimal Supersymmetric extension of the Standard Model [2], whose defining assumptions are listed below.

1: Minimal gauge group.
   In the MSSM, the gauge group is just $G = SU(3)_C \times SU(2)_L \times U(1)_Y$, as in the SM. Supersymmetry then implies that spin-1 gauge bosons belong to vector superfields, together with their spin-$\frac{1}{2}$ superpartners, the gauginos.

2: Minimal particle content.
   The MSSM contains just three generations of quark and lepton spin-$\frac{1}{2}$ fields, as does the SM, but embedded in chiral superfields together with their spin-0 superpartners, the squarks and the sleptons. In addition, to give masses to all charged fermions and to avoid chiral anomalies, one is forced to introduce two more chiral superfields, containing two complex spin-0 Higgs doublets and their spin-$\frac{1}{2}$ superpartners, the higgsinos.

3: Exact $R$-parity.
   Once the gauge group and the particle content are given, to determine a globally supersymmetric Lagrangian, $\mathcal{L}_{\text{SUSY}}$, one must specify an analytic function of the chiral superfields, the superpotential. To enforce baryon and lepton number conservation in renormalizable interactions, in the MSSM one imposes a discrete, multiplicative symmetry called $R$-parity, defined as
   \begin{equation}
   R = (-1)^{2s + 3B + L},
   \end{equation}
   where $s$ is the spin quantum number. In practice, the $R$-parity assignments are $R = +1$ for all ordinary particles (quarks, leptons, gauge and Higgs bosons), $R = -1$ for their superpartners (squarks, sleptons,
gauginos and higgsinos). The most general superpotential compatible with gauge invariance, renormalizability and $R$-parity is

$$f = h^U Q U^c H_2 + h^D Q D^c H_1 + h^E L E^c H_1 + \mu H_1 H_2,$$

(2)

where $Q, U^c, D^c, L, E^c$ are the chiral superfields containing the left-handed components of ordinary quarks and leptons, $H_1$ and $H_2$ are the two Higgs chiral superfields, and family and group indices have been left implicit for notational simplicity. The first three terms are nothing else than the supersymmetric generalization of the SM Yukawa couplings, whereas the fourth one is a globally supersymmetric Higgs mass term. Exact $R$-parity has very important phenomenological consequences: ($R$-odd) supersymmetric particles are always produced in pairs, their decays always involve an odd number of supersymmetric particles in the final state, and the lightest supersymmetric particle (LSP) is absolutely stable.

4: **Soft supersymmetry breaking.**

The above three assumptions are sufficient to completely determine a globally supersymmetric renormalizable Lagrangian, $\mathcal{L}_{\text{SUSY}}$. The MSSM Lagrangian is obtained by adding to $\mathcal{L}_{\text{SUSY}}$ a collection $\mathcal{L}_{\text{soft}}$ of explicit but soft supersymmetry-breaking terms, which preserve the good ultraviolet properties of supersymmetric theories. In general, $\mathcal{L}_{\text{soft}}$ contains $[12]$ mass terms for scalar fields and gauginos, as well as a restricted set of scalar interaction terms proportional to the corresponding superpotential couplings

$$-\mathcal{L}_{\text{soft}} = \sum_i \tilde{m}_i^2 |\varphi_i|^2 + \frac{1}{2} \sum_A M_A \lambda_A \lambda_A + \left( h^U A^U Q U^c H_2 + h^D A^D Q D^c H_1 + h^E A^E L E^c H_1 + m_3^2 H_1 H_2 + \text{h.c.} \right),$$

(3)

where $\varphi_i (i = H_1, H_2, Q, U^c, D^c, L, E^c)$ denotes the generic spin-0 field, and $\lambda_A (A = 1, 2, 3)$ the generic gaugino field. Observe that, since $A^U, A^D$ and $A^E$ are matrices in generation space, the most general form of $\mathcal{L}_{\text{soft}}$ contains in principle a huge number of free parameters. Moreover, for generic values of these parameters one encounters phenomenological problems with flavour-changing neutral currents $[13]$. 

4
with new sources of CP-violation \[1\] \[15\] and with charge- and colour-breaking vacua.

5: **Unification assumptions.**

All the above problems can be solved at once if one assumes that the running MSSM parameters, defined at the one-loop level and in a mass-independent renormalization scheme, obey a certain number of boundary conditions at some grand-unification scale \(M_U\). First of all, one assumes grand unification of the gauge couplings

\[
g_3(M_U) = g_2(M_U) = g_1(M_U) \equiv g_U, \tag{4}
\]

where \(g_1 = \sqrt{3/5} \cdot g'\) as in most grand-unified models. Furthermore, one assumes that all soft supersymmetry-breaking terms can be parametrized, at the scale \(M_U\), by a universal gaugino mass

\[
M_3(M_U) = M_2(M_U) = M_1(M_U) \equiv m_{1/2}, \tag{5}
\]

a universal scalar mass

\[
\tilde{m}_{H_1}^2(M_U) = \tilde{m}_{H_2}^2(M_U) = \tilde{m}_{Q}^2(M_U) = \ldots = \tilde{m}_{E^c}^2(M_U) \equiv m_0^2, \tag{6}
\]

and a universal trilinear scalar coupling

\[
A^U(M_U) = A^D(M_U) = A^E(M_U) \equiv A, \tag{7}
\]

whereas \(m_3^2\) remains in general an independent parameter. In addition, all possible CP-violating phases besides the Kobayashi-Maskawa one are set to zero at the scale \(M_U\).

### 2.1 Non-minimal alternatives to the MSSM

The above assumptions, which define the MSSM, are plausible but not compulsory. Relaxing them leads to non-minimal supersymmetric extensions of the SM, which typically increase the number of free parameters without a corresponding increase of physical motivation.

\[1\] The phenomenology of CP violation in supersymmetric models has been discussed recently, in connection with high-energy \(e^+e^-\) colliders, in ref. \[14\].
For example, relaxing assumption 1, a low-energy gauge group larger than the SM one could be considered, as is possible in non-minimal grand-unification schemes and in some string compactifications, and as was originally suggested in some models for spontaneous breaking of global supersymmetry. However, the present limits on the masses and mixing of extra gauge bosons are so stringent that such a departure is certainly not motivated by now.

Similarly, there are various possibilities to enlarge the particle content of the MSSM, relaxing assumption 2. One possibility is the introduction of additional chiral superfields with the quantum numbers of exotic states contained in the fundamental 27 representation of $E_6$: under assumption 1, however, these states have naturally superheavy masses and decouple from the low-energy effective theory. A particularly popular variation, which corresponds to the simplest non-minimal model, is constructed by adding a gauge-singlet Higgs superfield $N$ and by requiring purely trilinear superpotential couplings. Without unification assumptions, this model has already two more parameters than the MSSM, but with an assumption analogous to eq. (7) the number of free parameters remains the same as in the MSSM. Folklore arguments in favour of this model are that it avoids the introduction of a supersymmetry-preserving mass parameter $\mu \sim G_F^{-1/2}$, and that the homogeneity properties of its superpotential recall the structure of some superstring effective theories. A closer look, however, shows that these statements should be taken with a grain of salt. First, in the effective low-energy theory with softly broken global supersymmetry, the supersymmetric mass $\mu \sim G_F^{-1/2}$ could well be a remnant of local supersymmetry breaking, if the underlying supergravity theory has a suitable structure of interactions. Moreover, when embedded in a grand-unified theory, the non-minimal model with a singlet Higgs field might develop dangerous instabilities. Also, the trilinear $N^3$ superpotential coupling, which is usually invoked to avoid a massless axion, is typically absent in string models. Phenomenological aspects of the non-minimal model with an extra singlet have been studied recently, in connection with high-energy $e^+e^-$ colliders, in ref. [19], and will not be discussed here.

Assumption 3 is of crucial importance, since relaxing it can drastically modify the phenomenological signatures of supersymmetry. If one does not impose $R$-parity, the most general superpotential compatible with gauge invariance and renormalizability contains, besides the terms of eq. (3), also
the following ones:

\[ \Delta f = \lambda Q D^c L + \lambda' L LE^c + \mu' LH_2 + \lambda'' U^c D^c D^c. \]  

(8)

The first three terms on the right-hand side of eq. (8) obey the selection rule \( \Delta B = 0, |\Delta L| = 1 \), and the last one the selection rule \( \Delta L = 0, |\Delta B| = 1 \). Their simultaneous presence would be phenomenologically unacceptable, since they could induce, for example, fast proton decay mediated by \( \tilde{d} \) squarks. However, imposing discrete symmetries weaker than \( R \)-parity one can allow for some of the terms in eq. (8), and therefore for explicit \( R \)-parity breaking, in a phenomenologically acceptable way \[20\]. Another possibility \[21\] is that \( R \)-parity is spontaneously broken by the VEV of a sneutrino field, but it is by now experimentally ruled out by LEP data. In order to obtain acceptable models with spontaneously broken \( R \)-parity, one would need to introduce several extra fields and parameters. The phenomenology of models with broken \( R \)-parity at high-energy \( e^+ e^- \) colliders has been recently studied in ref. \[22\], and will not be discussed here.

To comment assumption 4, one has to discuss the problem of supersymmetry breaking. Models with spontaneously broken global supersymmetry have to face several phenomenological difficulties, which can be solved only at the price of introducing rather baroque constructions. Present theoretical ideas, however, favour the possibility that supersymmetry is spontaneously broken in the hidden sector of some underlying supergravity (or superstring) model, communicating with the observable sector (the one containing the states of the MSSM) only via gravitational interactions. As for the precise mechanism of spontaneous supersymmetry breaking, there are several suggestions, among which non-perturbative phenomena such as gaugino condensation \[23\] and string constructions such as coordinate-dependent compactifications \[24\], but none of them has yet reached a fully satisfactory formulation. It then appears to be a sensible choice to parametrize supersymmetry breaking in the low-energy effective theory by a collection of soft terms, without strong assumptions on the underlying mechanism for spontaneous supersymmetry breaking.

Besides solving naturally the phenomenological problems connected with flavour-changing neutral currents, new sources of CP violation, charge and colour breaking vacua, and proliferation of free parameters, assumption 5 is strongly suggested by ideas about grand unification and spontaneous breaking of local supersymmetry in a hidden sector; it receives further support by
the present indications on the structure of the low-energy effective supergravity theories of string models. We shall discuss later other phenomenological and theoretical facets of the unification assumptions.

2.2 Supersymmetric grand-unification

Starting from the boundary condition of eq. (4), one can solve the appropriate renormalization group equations (RGE) to obtain the running gauge coupling constants $g_A(Q) \ (A = 1, 2, 3)$ at scales $Q \ll M_U$. At the one-loop level, and assuming that there are no new physics thresholds between $M_U$ and $Q$, one finds [25]

$$
\frac{1}{g_A^2(Q)} = \frac{1}{g_U^2} + \frac{b_A}{8\pi^2} \log \frac{M_U}{Q} \ (A = 1, 2, 3),
$$

(9)

where the one-loop beta-function coefficients $b_A$ depend only on the $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers of the light particle spectrum. In the MSSM

$$
b_3 = -3, \quad b_2 = 1, \quad b_1 = \frac{33}{5},
$$

(10)

whereas in the SM

$$
b_3^0 = -7, \quad b_2^0 = -\frac{19}{6}, \quad b_1^0 = \frac{41}{10}.
$$

(11)

Starting from three input data at the electroweak scale, for example [3]

$$
\alpha_3(m_Z) = 0.118 \pm 0.008, \quad \alpha_{em}^{-1}(m_Z) = 127.9 \pm 0.2, \quad \sin^2 \theta_W(m_Z) = 0.2327 \pm 0.0008,
$$

(12) (13) (14)

where $\alpha_A = g_A^2/(4\pi)$, $\sin^2 \theta_W = g^2/(g^2 + g'^2)$, $\alpha_{em} = \alpha_2 \cdot \sin^2 \theta_W$, and all running parameters are defined in the modified minimal subtraction scheme $\overline{MS}$ [26], one can perform consistency checks of the grand-unification hypothesis in different models.

In the minimal $SU(5)$ model [27], and indeed in any other model where eq. (4) holds and the light-particle content is just that of the SM (with no intermediate mass scales between $m_Z$ and $M_U$), eqs. (3) and (11) are
incompatible with experimental data. This was first realized by noticing that the prediction $M_U \simeq 10^{14-15}$ GeV, obtained by using as inputs eqs. \((12)\) and \((13)\), is incompatible with experimental data on nucleon decay \[28\]. Subsequently, also the prediction $\sin^2 \theta_W \simeq 0.21$ was shown to be in conflict with experimental data \[29\], and this conflict became even more significant \[30\] after the recent LEP precision measurements.

In the MSSM, assuming for simplicity that all supersymmetric particles have masses of order $m_Z$, one obtains \[31\] $M_U \simeq 10^{16}$ GeV (which increases the proton lifetime for gauge-boson-mediated processes beyond the present experimental limits) and $\sin^2 \theta_W \simeq 0.23$. At the time of refs. \[31\], when data were pointing towards a significantly smaller value of $\sin^2 \theta_W$, this was considered by some a potential phenomenological shortcoming of the MSSM. The high degree of compatibility between data and supersymmetric grand unification became manifest \[29\] only later, after improved data on neutrino-nucleon deep inelastic scattering were obtained, and was recently re-emphasized, after the LEP precision measurements, in refs. \[32,30\]. One should not forget, however, that unification of the MSSM is not the only solution which can fit the data of eqs. \((12)\)–\((14)\): for example, non-supersymmetric models with \emph{ad hoc} light exotic particles or intermediate symmetry-breaking scales \[33\] could also do the job. The MSSM, however, stands out as the simplest physically motivated solution.

If one wants to make the comparison between low-energy data and the predictions of specific grand-unified models more precise, there are several factors that should be further taken into account. After the inclusion of higher-loop corrections and threshold effects, eq. \((11)\) is modified as follows

\[
\frac{1}{g^2_A(Q)} = \frac{1}{g^2_U} + \frac{b_A}{8\pi^2} \log \frac{M_U}{Q} + \Delta^{th}_A + \Delta^{l>1}_A \quad (A = 1, 2, 3). \tag{15}
\]

In eq. \((15)\), $\Delta^{th}_A$ represents the so-called \emph{threshold effects}, which arise whenever the RGE are integrated across a particle threshold \[34\], and $\Delta^{l>1}_A$ represents the corrections due to two- and higher-loop contributions to the RGE \[35\]. Both $\Delta^{th}_A$ and $\Delta^{l>1}_A$ are scheme-dependent, so one should be careful to compare data and predictions within the same renormalization scheme. $\Delta^{th}_A$ receives contributions both from thresholds around the electroweak scale (top quark, Higgs boson, and in SUSY-GUTs also the additional particles of the MSSM spectrum), and from thresholds around the grand-unification scale (superheavy gauge and Higgs bosons, and in SUSY-GUTs also their
superpartners). Needless to say, these last threshold effects can be computed only in the framework of a specific grand-unified model, and typically depend on a number of free parameters. Besides the effects of gauge couplings, $\Delta^{\ell_3}_{\ell_1}$ must include also the effects of Yukawa couplings, since, even in the simplest mass-independent renormalization schemes, gauge and Yukawa couplings mix beyond the one-loop order. In minimal $SU(5)$ grand unification, and for sensible values of the top and Higgs masses, all these corrections are small and do not affect substantially the conclusions derived from the naïve one-loop analysis. This is no longer the case, however, for supersymmetric grand unification. First of all, one should notice that the MSSM by itself does not uniquely define a SUSY-GUT, whereas threshold effects and even the proton lifetime (due to a new class of diagrams [36] which can be originated in SUSY-GUTs) become strongly model-dependent. Furthermore, the simplest SUSY-GUT [37], containing only chiral Higgs superfields in the 24, 5 and $\overline{5}$ representations of $SU(5)$, has a severe problem in accounting for the huge mass splitting between the $SU(2)$ doublets and the $SU(3)$ triplets sitting together in the $5$ and $\overline{5}$ Higgs supermultiplets. Threshold effects are typically larger than in ordinary GUTs, because of the much larger number of particles in the spectrum, and in any given model they depend on several unknown parameters. Also two-loop effects of Yukawa couplings can be quantitatively important in SUSY-GUTs, since they depend not only on the top-quark mass, but also on the ratio $\tan \beta = v_2/v_1$ of the VEVs of the two neutral Higgs fields: as will be made clearer below, these effects become large for $m_t \gtrsim 140$ GeV and $\tan \beta \sim 1$, which correspond to a strongly interacting top Yukawa coupling. All these effects have been recently re-evaluated [38] after the enthusiasm created by refs. [30]. The conclusion is that, even imagining a further reduction in the experimental errors of eqs. (12)–(14), it is impossible to claim indirect evidence for supersymmetry and to predict the MSSM spectrum with any significant accuracy. The only safe statement is [32] that, at the level of precision corresponding to the naïve one-loop approximation, there is a remarkable consistency between experimental data and the prediction of supersymmetric grand unification, with the MSSM $R$-odd particles roughly at the electroweak scale.

To conclude the discussion of supersymmetric grand unification, it is worth mentioning how the unification constraints can be applied to the low-energy effective theories of four-dimensional heterotic string models. The basic fact to realize is that the only free parameter of these models is the
string tension, which fixes the unit of measure of the massive string excitations. All the other scales and parameters are related to VEVs of scalar fields, the so-called moduli, corresponding to flat directions of the scalar potential. In particular, there is a relation among the string mass $M_s \sim \alpha'^{-1/2}$, the Planck mass $M_P \sim G_N^{-1/2}$, and the unified string coupling constant $g_{\text{string}}$, which reflects unification with gravity and implies that in any string vacuum one has one more prediction than in ordinary field-theoretical grand unification. In a large class of string models, one can write down an equation of the same form as (15), and compute $g_U$, $M_U$, $\Delta_{\text{th}}$, . . . in terms of the relevant VEVs [39]. In the $\overline{\text{DR}}$ scheme [40], which is the most appropriate for supersymmetric models, one finds $M_U \simeq 0.7 \times g_U \times 10^{18}$ GeV, more than one order of magnitude higher than the naive extrapolations from low-energy data illustrated before. This means that significant threshold effects are needed in order to reconcile string unification with low-energy data: for example, the minimal version of the flipped-$SU(5)$ model [11] is by now ruled out [12]. To get agreement, one needs some more structure in the spectrum, either at the compactification scale or in the form of light exotics [43]. Once the present string calculations will be sufficiently generalized, unification constraints will provide a very important phenomenological test of realistic string models.

2.3 More on the MSSM

It is perhaps useful, at this point of the discussion, to remind the reader of some other phenomenological virtues and theoretical constraints of the MSSM, besides the solution of the ‘technical’ part of the hierarchy problem and the grand unification of gauge couplings.

It was already said that, because of $R$-parity, the LSP is absolutely stable. In most of the otherwise acceptable parameter space, the LSP is neutral and weakly interacting, rarely a sneutrino and typically the lightest, $\tilde{\chi}$, of the neutralinos (the mass eigenstates of the neutral gaugino-higgsino sector). Then the LSP is a natural candidate for cold dark matter [44,45]. In particular, for generic values of parameters one naturally avoids an excessive $\tilde{\chi}$ relic density, but one often obtains cosmologically interesting values for it. This should also be considered an important consistency check of the MSSM, since a coloured or electrically charged LSP would be in conflict with astrophysical observations [15]. Recent analyses of supersymmetric dark matter, taking into account the LEP limits, can be found in ref. [46].
Another remarkable fact to be noticed is that LEP precision measurements of the Z properties put little indirect constraints, via radiative corrections \[47\], on the MSSM parameters. This is not the case, for example, of technicolour and extended technicolour models, which are severely constrained by the recent LEP data \[48,49\]. In the MSSM, the most important effect could be given by additional contributions to the effective $\rho$ parameter coming from the stop-sbottom sector: these can be sizeable only in the case of large mass splittings in the stop-sbottom sector, in which case the upper bound on the top-quark mass, $m_t \lesssim 180$ GeV, obtained in the SM by fitting the electroweak precision data, can be further strengthened. However, deviations from the SM predictions due to loops of supersymmetric particles are typically small for generic values of the parameters.

A further predictive aspect of the MSSM is the possibility of computing low-energy parameters, in particular the soft supersymmetry-breaking masses, in terms of the few parameters assigned as boundary conditions at the unification scale. To do this, it is sufficient to solve the corresponding RGE \[50\], analogous to the ones given above for the gauge couplings. For the gaugino masses, one finds

$$M_A(Q) = \frac{g_A^2(Q)}{g_U} m_{1/2} \quad (A = 1, 2, 3). \quad (16)$$

For the top Yukawa coupling, neglecting mixing and the Yukawa couplings of the remaining fermions, one gets ($t \equiv \log Q$)

$$\frac{dh_t}{dt} = \frac{h_t}{8\pi^2} \left( -\frac{8}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{13}{18} g'^2 + 3 h_t^2 \right). \quad (17)$$

A close look at eq. \[17\] can give us some important information about the top-quark mass in the MSSM. The important thing to realize is that the running top Yukawa coupling has an effective infrared fixed point \[51\], smaller than in the SM case \[52\]. However high the value one assigns to it at the unification scale, $h_t$ evaluated at the electroweak scale never exceeds a certain maximum value $h_t^{\text{max}} \simeq 1$. This implies that, for any given value of $\tan \beta$, there is a corresponding maximum value for the top quark mass. A naive one-loop calculation gives

$$\begin{align*}
\tan \beta : & \quad 1 \quad 2 \quad 4 \quad 8 \quad \infty \\
\ m_t^{\text{max}} (GeV) : & \quad 139 \quad 176 \quad 191 \quad 195 \quad 196. \quad (18)
\end{align*}$$
For the soft supersymmetry-breaking scalar masses, under the same assumptions as above, and considering for the moment the sfermions of the third family, one finds

\[
\frac{d\tilde{m}_i^2}{dt} = \frac{1}{8\pi^2} \left[ - \sum_{A=1,2,3} c_A(i) g_A^2 M_A^2 + c_t(i) h_t^2 F_t \right],
\]

where \( i = H_1, H_2, Q, U^c, D^c, L, E^c, \)

\[ F_t \equiv \tilde{m}_Q^2 + \tilde{m}_{U^c}^2 + \tilde{m}_{H_2}^2 + A_t^2, \]

and the \( c_A(i), c_t(i) \) coefficients are given by

\[
\begin{align*}
  i : & \quad H_1 \quad H_2 \quad Q \quad U^c \quad D^c \quad L \quad E^c \\
  c_3(i) : & \quad 0 \quad 0 \quad \frac{16}{3} \quad \frac{16}{3} \quad \frac{16}{3} \quad 0 \quad 0 \\
  c_2(i) : & \quad 3 \quad 3 \quad 3 \quad 0 \quad 0 \quad 3 \quad 0 \\
  c_1(i) : & \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{15} \quad \frac{1}{15} \quad \frac{4}{15} \quad \frac{3}{5} \quad \frac{12}{5} \\
  c_t(i) : & \quad 0 \quad 3 \quad 1 \quad 2 \quad 0 \quad 0 \quad 0 
\end{align*}
\]

Similar equations can be derived for the remaining soft supersymmetry-breaking parameters and for the superpotential Higgs mass \( \mu \). Also, the inclusion of the complete set of Yukawa couplings, including mixing, is straightforward. In general, the RGEs for superpotential couplings and soft supersymmetry-breaking parameters have to be solved by numerical methods. Analytical solutions can be obtained for the soft squark and slepton masses when the corresponding Yukawa couplings are negligible:

\[
\tilde{m}_i^2 = m_i^2 + m_{1/2}^2 \sum_{A=1}^{3} \frac{c_A(i)}{2b_A} \left( 1 - \frac{1}{F_A^2} \right),
\]

where

\[
F_A = 1 + \frac{b_A}{8\pi^2 g_U^2} \log \frac{M_U}{Q}.
\]

For example, one gets \( \tilde{m}_Q^2, \tilde{m}_{U^c}^2, \tilde{m}_{D^c}^2 \sim m_0^2 + (5 - 8) m_{1/2}^2, \tilde{m}_L^2 \sim m_0^2 + 0.5 m_{1/2}^2, \)

\[ \tilde{m}_{E^c}^2 \sim m_0^2 + 0.15 m_{1/2}^2. \]

It should be stressed that also eqs. (16) and (22), in analogy with eq. (4), are valid up to higher-order corrections and threshold effects, so their accuracy should not be overestimated.

One of the most attractive features of the MSSM is the possibility of describing the spontaneous breaking of the electroweak gauge symmetry as
an effect of radiative corrections \cite{53}, via a generalization of the mechanism discussed first by Coleman and E. Weinberg \cite{54} in the context of the SM. It is remarkable that, starting from universal boundary conditions at the unification scale, it is possible to explain naturally why fields carrying colour or electric charge do not acquire non-vanishing VEVs, whereas the neutral components of the Higgs doublets do. We give here a simplified description of the mechanism in which the physical content is transparent, and we comment later on the importance of a more refined treatment. The starting point is a set of boundary conditions on the independent model parameters at the unification scale $Q = M_U$. One then evolves all the running parameters from the grand-unification scale to a low scale $Q \sim m_Z$, according to the RGE, and considers the renormalization-group-improved tree-level potential

$$V_0(Q) = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 H_2 + \text{h.c.})$$

$$+ \frac{1}{8} g^2 \left( H_2^\dagger \tilde{\sigma} H_2 + H_1^\dagger \tilde{\sigma} H_1 \right)^2 + \frac{1}{8} g'^2 \left( |H_2|^2 - |H_1|^2 \right)^2,$$

where

$$m_1^2 \equiv \tilde{m}_{H_1}^2 + \mu^2, \quad m_2^2 \equiv \tilde{m}_{H_2}^2 + \mu^2,$$

and it is not restrictive to choose a field basis such that $m_3^2 \leq 0$. All masses and coupling constants in $V_0(Q)$ are running parameters, evaluated at the scale $Q$. The minimization of the potential in eq. (24) is straightforward. To generate non-vanishing VEVs $v_1 \equiv \langle H_1^0 \rangle$ and $v_2 \equiv \langle H_2^0 \rangle$, one needs

$$B \equiv m_1^2 m_2^2 - m_3^4 < 0.$$  

In addition, a certain number of conditions have to be satisfied to have a stable minimum with the correct amount of symmetry breaking and with unbroken colour, electric charge, baryon and lepton number: for example, all the running squark and slepton masses have to be positive. A crucial role in the whole process is played by the top Yukawa coupling, which strongly influences the RGE for $\tilde{m}_{H_4}^2$, as should be clear from eqs. (19)–(21). For appropriate boundary conditions, the RGE drive $B < 0$ at scales $Q \sim m_Z$, whereas all the squark and slepton masses remain positive as desired, to give a phenomenologically acceptable breaking of the electroweak symmetry.

The use of $V_0(Q)$ is very practical for a qualitative discussion as the one given above, but it relies on the assumption that, once the leading logarithms
have been included in the running parameters, all the remaining one-loop corrections to the scalar potential can be neglected at the scale $Q \sim m_Z$. However, as shown for example in ref. [32], for a quantitative discussion of gauge symmetry breaking it is necessary to use the full one-loop effective potential, which in the Landau gauge and in the $\overline{DR}$ renormalization scheme [40] is given by

$$V_1(Q) = V_0(Q) + \frac{1}{64\pi^2} \text{Str}\left\{\mathcal{M}^4(Q) \left[\log \frac{\mathcal{M}^2(Q)}{Q^2} - \frac{3}{2}\right]\right\}.$$  (27)$$

In eq. (27), $\text{Str}\{f(M^2) = \sum_i (-1)^{2J_i}(2J_i + 1)f(m_i^2)$ denotes the conventional supertrace, where $m_i^2$ is the field-dependent mass eigenvalue of the $i$-th particle of spin $J_i$, and field-independent terms have been neglected. To give an example, the VEVs determined from $V_0(Q)$ are strongly scale-dependent, whereas the ones determined from $V_1(Q)$ are not, as it should be. Only at a scale $\hat{Q}$, of the order of the stop masses, is the use of $V_0(Q)$ a good approximation. This is a result of the fact that mass-independent renormalization schemes, like $\overline{MS}$ or $\overline{DR}$, do not automatically include decoupling: since the most important contributions to $V_1(Q)$ come from the stop sector, the optimal scale at which to freeze the evolution of the running parameters turns out to be of the order of the stop masses. Another aspect of this effect, with important phenomenological consequences, are the radiative corrections to Higgs boson masses and couplings, which will be discussed in the following section.

To conclude the discussion of radiative symmetry breaking, we show now that in the MSSM, with universal boundary conditions, one expects

$$1 \lesssim \tan \beta \lesssim \frac{m_t}{m_b}.$$  (28)$$

The simplest proof relies on the relation, derived from the minimization of $V_0(Q)$

$$\frac{v_2}{v_1} = \frac{m_1^2 + m_Z^2/2}{m_2^2 + m_Z^2/2}.$$  (29)$$

The boundary conditions at the unification scale is $m_1^2(M_U) = m_Z^2(M_U)$, and, neglecting as before all Yukawa couplings except $h_t$, the RGE for the difference $m_1^2 - m_2^2$ reads

$$\frac{d(m_1^2 - m_2^2)}{dt} = -\frac{3}{8\pi^2} h_t^2 F_t.$$  (30)$$
Imagine now that $\tan \beta < 1$, and observe that the top and bottom masses are given by $m_t^2 = h_t^2 v_t^2$ and $m_b^2 = h_b^2 v_b^2$, respectively. Then $m_t \gg m_b$ implies $h_t \gg h_b$, which makes eq. (30) a good approximation. Solving now eq. (30) at the scale $\hat{Q}$, where the use of $V_0(Q)$ is justified, and observing that it is always $F_t > 0$, one finds $m_1^2 > m_2^2$. But eq. (29) then tells us that $\tan \beta > 1$, in contradiction with the starting assumption. Similarly, including in eq. (30) the contributions of the bottom and $\tau$ Yukawa couplings, one can prove that $\tan \beta \lesssim m_t/m_b$.

3 Higgs bosons

We begin the discussion of the MSSM particle spectrum with the ($R$-even) Higgs boson sector. As explained in the previous section, the MSSM contains two complex Higgs doublets of opposite hypercharge, $H_1 \equiv (H_1^0, H_1^-)$ and $H_2 \equiv (H_2^+, H_2^0)$. After their neutral components develop non-vanishing VEVs, $v_1$ and $v_2$, which can be taken to be real and positive without loss of generality, one is left with five physical degrees of freedom. Three of these are neutral (two CP-even, $h$ and $H$, and one CP-odd, $A$) and two are charged ($H^\pm$). The starting point for a discussion of Higgs-boson masses and couplings in the MSSM is the potential of eq. (24). Besides the minimization conditions, which relate $v_1$ and $v_2$ with the potential parameters, a physical constraint comes from the fact that the combination $(v_1^2 + v_2^2)$, which determines the $W$- and $Z$-boson masses, must reproduce their measured values. Once this constraint is imposed, in the approximation of eq. (24) the MSSM Higgs sector contains only two independent parameters. A convenient choice, which will be adopted here, is to take as independent parameters $m_A$, the physical mass of the CP-odd neutral boson, and $\tan \beta \equiv v_2/v_1$. The parameter $m_A$ is essentially unconstrained, even if naturalness arguments suggest that it should be smaller than $O(500 \text{ GeV})$, whereas for $\tan \beta$ the range permitted in the MSSM is given by formula (28).

In the approximation of eq. (24), the mass matrix of neutral CP-even Higgs bosons reads

$$(\mathcal{M}_R^0)^2 = \left[\begin{array}{cc} \cot \beta & -1 \\ -1 & \tan \beta \end{array}\right] \frac{m_Z^2}{2} + \left[\begin{array}{cc} \tan \beta & -1 \\ -1 & \cot \beta \end{array}\right] \frac{m_A^2}{2} \sin 2\beta$$

(31)
and the charged Higgs mass is given by

\[ m_{H^\pm}^2 = m_W^2 + m_A^2. \]  

From eq. (31), one obtains

\[ m_{h,H}^2 = \frac{1}{2} \left[ m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2m_Z^2\cos^2 2\beta} \right], \]  

and also celebrated inequalities as \( m_W, m_A < m_{H^\pm}, m_h < m_Z < m_H, m_h < m_A < m_H \). Similarly, one can easily compute all the Higgs-boson couplings by observing that the mixing-angle \( \alpha \), required to diagonalize the mass matrix (31), is given by

\[ \cos 2\alpha = -\cos 2\beta \frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2} \left(-\frac{\pi}{2} < \alpha \leq 0\right). \]  

For example, the tree-level couplings of the neutral Higgs bosons are easily obtained from the standard model Higgs couplings if one multiplies them by some appropriate \( \alpha \)- and \( \beta \)-dependent factors [55]. An important consequence of the structure of the Higgs potential (24) is the existence of at least one neutral CP-even Higgs boson, \( h \), weighing less than \( m_Z \). This raised the hope that the crucial experiment on the MSSM Higgs sector could be entirely performed at LEP II (with sufficient centre-of-mass energy, luminosity and b-tagging efficiency), and took some interest away from higher energy colliders. However, it was recently pointed out [56–58] that the Higgs-boson masses are subject to large radiative corrections, associated with the top quark and its \( SU(2) \) and supersymmetric partners. Several papers [62–75] have subsequently investigated various aspects of these corrections and their implications for experimental searches at LEP and LHC-SSC. In the following subsection, we shall summarize the main effects of radiative corrections on Higgs-boson parameters.

Footnote 2: Previous studies [59, 61] either neglected the case of a heavy top quark [59, 60], or concentrated on the violations of the neutral Higgs-mass sum rule [61] without computing corrections to individual Higgs masses.
3.1 Radiative corrections to Higgs boson-masses and couplings

As far as Higgs-boson masses and self-couplings are concerned, a convenient approximate way of parametrizing one-loop radiative corrections is to substitute the tree-level Higgs potential of eq. (24) with the one-loop effective potential of eq. (27), and to identify masses and self-couplings with the appropriate combinations of derivatives evaluated at the minimum. The comparison with explicit diagrammatic calculations \[58,66,70,75\] shows that the effective potential approximation is more than adequate for our purposes. Also, inspection shows that the most important contributions to eq. (27) come from the field-dependent mass matrices of the top and bottom quarks and squarks, whose explicit expressions depend on a number of parameters and can be found in ref. \[68\]. To simplify the discussion, in the following we will take a universal soft supersymmetry-breaking squark mass, \( \tilde{m}_Q^2 = \tilde{m}_{Uc}^2 = \tilde{m}_{Dc}^2 \equiv \tilde{m}_Q^2 \), and we will assume negligible mixing in the stop and sbottom mass matrices, \( A_t = A_b = \mu = 0 \). More complete formulae for arbitrary values of the parameters can be found in refs. \[65,68\], but the qualitative features corresponding to the above parameter choices are representative of a very large region of parameter space. In the case under consideration, and neglecting D-term contributions to the squark masses, the neutral CP-even mass matrix is modified at one loop as follows

\[
M_R^2 = (M^0_R)^2 + \begin{pmatrix} \Delta_1^2 & 0 \\ 0 & \Delta_2^2 \end{pmatrix},
\]

where

\[
\Delta_1^2 = \frac{3g^2m_b^4}{16\pi^2m_W^2\cos^2\beta} \log \frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2},
\]

\[
\Delta_2^2 = \frac{3g^2m_t^4}{16\pi^2m_W^2\sin^2\beta} \log \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}.
\]

From the above expressions one can easily derive the one-loop-corrected eigenvalues \( m_h \) and \( m_H \), as well as the mixing angle \( \alpha \) associated with the one-loop-corrected mass matrix \[35\]. The most striking fact in eqs. \[35\]–\[37\] is that the correction \( \Delta_2^2 \) is proportional to \( (m_t^4/m_W^2) \). This implies that, for \( m_t \) in the range allowed by experimental limits and by eq. \[18\], the tree-level predictions for \( m_h \) and \( m_H \) can be badly violated, and so for the
related inequalities. The other free parameter is \( m_{\tilde{q}} \), but the dependence on it is much milder. In the following, when making numerical examples, we shall always choose the representative value \( m_{\tilde{q}} = 1 \, \text{TeV} \). The reader can easily rescale the displayed results to different values of \( m_{\tilde{q}} \). To illustrate the impact of these results, we display in fig. 1 contours of the maximum allowed value of \( m_h \) (reached for \( m_A \to \infty \)), in the \((m_t, \tan \beta)\) plane. To plot different quantities of physical interest in the \((m_A, \tan \beta)\) plane, which is going to be the stage of the following phenomenological discussion, one needs to fix also the value of \( m_t \). In the following, we shall work with the representative value \( m_t = 140 \, \text{GeV} \), which is near the centre of the presently allowed range. As an example, we show in fig. 2 contours of constant \( m_h \) and \( m_H \) in the \((m_A, \tan \beta)\) plane. One-loop corrections to the charged Higgs mass have also been computed in refs. \[68–71\], and found to be small, at most a few GeV, for generic values of the parameters.

The effective potential method allows also the computation of the leading corrections to the trilinear and quadrilinear Higgs self-couplings. For example, the leading radiative correction to the trilinear \( hAA \) coupling, which plays a major role in the determination of the \( h \) branching ratios, is \[68\]

\[
\lambda_{hAA} = \lambda^0_{hAA} + \Delta \lambda_{hAA},
\]

where

\[
\lambda^0_{hAA} = -\frac{igm_Z}{2 \cos \theta_W} \cos 2\beta \sin(\beta + \alpha),
\]

and, neglecting the bottom Yukawa coupling and the D-term contributions to squark masses

\[
\Delta \lambda_{hAA} = -\frac{igm_Z}{2 \cos \theta_W} \frac{3g^2 \cos^2 \theta_W \cos \alpha \cos^2 \beta}{8\pi^2} \frac{m_t^4}{m_W^4} \log \frac{m_{\tilde{q}}^2 + m_t^2}{m_t^2}.
\]

Similarly, also the other Higgs self-couplings receive large corrections \( O(m_t^4/m_W^4) \).

Finally, one should consider Higgs couplings to vector bosons and fermions. Tree-level couplings to vector bosons are expressed in terms of gauge couplings and of the angles \( \beta \) and \( \alpha \). The most important part of the radiative corrections is taken into account by using one-loop-corrected formulae to determine \( \alpha \) from the input parameters. Other corrections are at most \( O(m_t^2/m_W^2) \) and can be safely neglected for our purposes. Tree-level couplings to fermions are expressed in terms of the fermion masses and of the
angles $\beta$ and $\alpha$. In this case, the leading radiative corrections can be taken into account by using the one-loop-corrected expression for $\alpha$ and running fermion masses, evaluated at the scale $Q$ which characterizes the process under consideration.

### 3.2 The discovery potential of LEP and LHC-SSC

In this section, we briefly summarize the implications of the previous results on MSSM Higgs-boson searches at LEP [64,65,68] and the LHC-SSC [73,74].

As already clear from tree-level analyses [55], the relevant processes for MSSM Higgs boson searches at LEP I are $Z \rightarrow hZ^*$ and $Z \rightarrow hA$, which play a complementary role since their rates are proportional to $\sin^2(\beta - \alpha)$ and $\cos^2(\beta - \alpha)$, respectively. An important effect of radiative corrections [68] is to render possible, for some values of the parameters, the decay $h \rightarrow AA$, which would be kinematically forbidden according to tree-level formulae. Experimental limits which take radiative corrections into account have by now been obtained by the four LEP collaborations [76], using different methods to present and analyse the data, and different ranges of parameters in the evaluation of radiative corrections. The presently excluded region of the $(m_A, \tan \beta)$ plane, for our standard parameter choice, is given in fig. 3 [73], where the solid line corresponds to the exclusion contour given in the first of refs. [76].

The situation in which the impact of radiative corrections is most dramatic is the search for MSSM Higgs bosons at LEP II. At the time when only tree-level formulae were available, there was hope that LEP could completely test the MSSM Higgs sector. According to tree-level formulae, in fact, there should always be a CP-even Higgs boson with mass smaller than $(h)$ or very close to $(H)$ $m_Z$, and significantly coupled to the $Z$ boson. However, as should be clear from the previous section, this result can be completely upset by radiative corrections. A detailed evaluation of the LEP II discovery potential can be made only if crucial theoretical parameters, such as the top-quark mass and the various soft supersymmetry-breaking masses, and experimental parameters, such as the centre-of-mass energy, the luminosity, and the b-tagging efficiency, are specified. Taking for example $\sqrt{s} = 190$ GeV, $m_t \gtrsim 110$ GeV, and our standard values for the soft supersymmetry-breaking parameters, in the region of $\tan \beta$ significantly greater than 1 the associated production of a $Z$ and a CP-even Higgs can be pushed beyond the kinematic-
cal limit. Associated $hA$ production could be a useful complementary signal, but obviously only for $m_h + m_A < \sqrt{s}$. Associated $HA$ production is typically negligible at these energies. To give a measure of the LEP II sensitivity, we plot in fig. 3 contours associated to two benchmark values of the total cross-section $\sigma(e^+e^- \rightarrow hZ, HZ, hA, HA)$. The dashed lines correspond to $\sigma = 0.2\,pb$ at $\sqrt{s} = 175$ GeV, which could be seen as a rather conservative estimate of the LEP II sensitivity. The dash-dotted lines correspond to $\sigma = 0.05\,pb$ at $\sqrt{s} = 190$ GeV, which could be seen as a rather optimistic estimate of the LEP II sensitivity. Of course, one should keep in mind that there is, at least in principle, the possibility of further extending the maximum LEP energy up to values as high as $\sqrt{s} \simeq 230 - 240$ GeV, at the price of introducing more (and more performing) superconducting cavities into the LEP tunnel \cite{77}.

Similar considerations can be made for charged Higgs searches at LEP II with $\sqrt{s} \simeq 190$ GeV. In view of the $\beta^3$ threshold factor in $\sigma(e^+e^- \rightarrow H^+H^-)$, and of the large background from $e^+e^- \rightarrow W^+W^-$, it will be difficult to find the $H^\pm$ at LEP II unless $m_{H^\pm} \lesssim m_W$, and certainly impossible unless $m_{H^\pm} < \sqrt{s}/2$. We also know \cite{68,71} that for generic values of the parameters there are no large negative radiative corrections to the charged Higgs mass formula, eq. (32). Thus there is very little hope of finding the charged Higgs boson of the MSSM at LEP II (or, to put it differently, the discovery of a charged Higgs boson at LEP II would most probably rule out the MSSM).

The next question is then whether the LHC and SSC can explore the full parameter space of the MSSM Higgs bosons. A systematic study of this problem, including radiative corrections, has been recently performed in ref. \cite{73} (see also \cite{74}), following the strategy outlined in ref. \cite{78}. The analysis is complicated by the fact that the $R$-odd particles could play a role both in the production (via loop diagrams) and in the decay (via loop diagrams and as final states) of the MSSM Higgs bosons. For simplicity, one can concentrate on the most conservative case in which all $R$-odd particles are heavy enough not to play any significant role. Still, one has to perform a separate analysis for each $(m_A, \tan \beta)$ point, to include radiative corrections (depending on the parameters of the top-stop-bottom-sbottom system), and to consider Higgs boson decays involving other Higgs bosons. We make here only a few general remarks on the LHC case, for the representative parameter choice $m_t = 140$ GeV, $m_{\tilde{q}} = 1$ TeV, $A_t = A_b = \mu = 0$, sending the reader to ref. \cite{73} for a more complete discussion, and to ref. \cite{74} for a review of recent
simulation work.

Beginning with the neutral states, when $h$ or $H$ are in the intermediate mass range (80–130 GeV) and have approximately SM couplings, the best prospects for detection are offered, as in the SM, by their $\gamma\gamma$ decay mode. In general, however, $\sigma \cdot BR(h, H \rightarrow \gamma\gamma)$ is smaller than for a SM Higgs boson of the same mass. As a rather optimistic estimate of the possible LHC sensitivity, we display, in fig. 4, lines corresponding to $\sigma \cdot BR(h, H \rightarrow \gamma\gamma) \sim 2/5$ of the corresponding value for a SM Higgs of 100 GeV. Only in the region of the $(m_A, \tan \beta)$ plane to the right of the line denoted by ‘a’ (in the case of $h$) and above the line denoted by ‘b’ (in the case of $H$) the $\gamma\gamma$ signal exceeds the chosen reference value. Almost identical considerations can be made for the production of $h$ or $H$, decaying into $\gamma\gamma$, in association with a $W$ boson or with a $t\bar{t}$ pair. When $H$ and $A$ are heavy, in general one cannot rely on the $ZZ \rightarrow 4l^\pm$ ($l = e, \mu$) decay mode, which gives the ‘gold-plated’ Higgs signature in the SM case, since $H$ and $A$ couplings to vector-boson pairs are strongly suppressed: only for small $\tan \beta$ and $2m_Z < m_H < 2m_t$ might the decay mode $H \rightarrow ZZ \rightarrow 4l^\pm$ still be viable despite the suppressed branching ratio. Again, as an estimate of the possible LHC sensitivity, we show in fig. 4, under the line denoted by ‘c’, the region of the $(m_A, \tan \beta)$ plane corresponding to $\sigma \cdot BR(H \rightarrow 4l^\pm) > 10^{-3}$ pb ($l = e, \mu$). For very large values of $\tan \beta$, and moderately large $m_A$, the unsuppressed decays $H, A \rightarrow \tau^+\tau^-$ could give visible signals, in contrast to the SM case. As a very optimistic estimate (especially in the small $m_A$ region!) we show in fig. 4, above the line denoted by ‘d’, the region of the parameter space corresponding to $\sigma \cdot BR(H, A \rightarrow \tau^+\tau^-) > 1$ pb. Finally, in the region of parameter space corresponding to $m_A \lesssim m_Z$, the charged Higgs could be discovered via the decay chain $t \rightarrow bH^+ \rightarrow b\tau^+\nu_\tau$, which competes with the standard channel $t \rightarrow bW^+ \rightarrow bl^\pm\nu_l$ ($l = e, \mu, \tau$). A convenient parameter is the ratio $R \equiv BR(t \rightarrow \tau^+\nu_\tau b)/BR(t \rightarrow \mu^+\nu_\mu b)$. As a very optimistic estimate of the LHC sensitivity, the line of fig. 4 denoted by ‘e’ delimits from the right the region of the $(m_A, \tan \beta)$ plane corresponding to $R > 1.1$. For all processes considered above, similar remarks apply also to the SSC.

In summary, a global look at figs. 3 and 4 shows that there is a high degree of complementarity between the regions of parameter space accessible to LEP II and to the LHC-SSC. However, for our representative choice of parameters, there is a non-negligible region of the $(m_A, \tan \beta)$ plane that is presumably beyond the reach of both LEP II and the LHC-SSC. This
potential problem could be solved, as we said before, by a further increase of the LEP II energy beyond the reference value $\sqrt{s} \lesssim 190$ GeV. Otherwise, this is the ideal case for a 500 GeV (or even less) $e^+e^-$ collider, as we shall see below. Even if in the future a Higgs boson will be found at LEP or the LHC-SSC, with properties compatible with those of a MSSM Higgs boson, it appears difficult to search effectively for all the Higgs states of the MSSM at the above machines. Again, as we shall see below, EE500 could play an important role in investigating the properties of the newly discovered Higgs boson and in looking for the remaining states of the MSSM.

3.3 Production mechanisms at high-energy $e^+e^-$ colliders

We now present, following ref. [80], cross-sections for the main production mechanisms of neutral susy Higgses in $e^+e^-$ collisions at $\sqrt{s} = 500$ GeV, namely:

\[
\begin{align*}
  e^+e^- &\to hZ \quad [\sigma \propto \sin^2(\beta - \alpha)], \\
  e^+e^- &\to Hz \quad [\sigma \propto \cos^2(\beta - \alpha)], \\
  e^+e^- &\to hA \quad [\sigma \propto \cos^2(\beta - \alpha)], \\
  e^+e^- &\to Ha \quad [\sigma \propto \sin^2(\beta - \alpha)], \\
  e^+e^- &\to hv\bar{\nu} \quad [\sigma \propto \sin^2(\beta - \alpha)], \\
  e^+e^- &\to Hv\bar{\nu} \quad [\sigma \propto \cos^2(\beta - \alpha)], \\
  e^+e^- &\to he^+e^- \quad [\sigma \propto \sin^2(\beta - \alpha)], \\
  e^+e^- &\to He^+e^- \quad [\sigma \propto \cos^2(\beta - \alpha)].
\end{align*}
\]

Other production mechanisms of interest are discussed in refs. [80,81], and details about experimental searches can be found in refs. [82,83]. We have included radiative corrections to the masses $m_h, m_H$ and to the mixing angle $\alpha$ for our standard parameter choice. We have neglected loops from the gauge-gaugino-Higgs-higgsino sector, which are known to give corrections smaller than the ones we have included. We have also neglected proper vertex corrections to vector boson-Higgs boson couplings and initial-state radiation.

In discussing our results, it is useful to estimate the cross-section for which we believe that any of the listed processes will be detectable. A cross-section of 0.01 pb will lead to 25 events for an integrated luminosity of 10 fb$^{-1}$ after multiplying by an efficiency of 25%; the latter is a crude estimate of the
impact of detector efficiencies, cuts, and branching ratios to usable decay channels. It will be helpful to keep this benchmark cross-section value in mind as a rough criterion for where in parameter space a particular reaction can be useful.

Fig. 5 shows contours of $\sigma(e^+e^- \rightarrow hZ)$ and $\sigma(e^+e^- \rightarrow HZ)$, respectively, in the $(m_A, \tan \beta)$ plane. Observe that the two processes are truly complementary, in the sense that everywhere in the $(m_A, \tan \beta)$ plane there is a substantial cross-section for at least one of them ($\sigma > 0.01 \text{ pb}$). This should be an excellent starting point for experimental searches. Similar considerations hold for $hA$, $HA$ production, whose cross-sections are shown in fig. 6. As long as one of the two channels is kinematically accessible, the inclusive cross-section is large enough to provide a substantial event rate. Even in this case the two processes are complementary, and together should be able to probe the region of parameter space corresponding to $m_A \lesssim 200$ GeV. We now move to single Higgs production via vector-boson fusion. The cross-sections for $h,H$ production via $WW$ and $ZZ$ fusion are given in figs. 7 and 8, respectively: they have been obtained using exact analytical formulae, rescaled from ref. [84]. Obviously, since the $AWW$ and $AZZ$ vertices are absent at tree level, one cannot get substantial $A$ production with this mechanism for sensible values of the parameters. The $ZZ$ fusion processes are suppressed by an order of magnitude with respect to the $WW$ fusion ones, but could still be useful for experimental searches.

The global picture which emerges from our results is the following. If no neutral Higgs boson is previously discovered, at EE500 one must find at least one neutral susy Higgs, otherwise the MSSM is ruled out. If $m_A$ is not too large, at EE500 there is the possibility of discovering all of the Higgs states of the MSSM via a variety of processes, including charged-Higgs-boson production, which has not been discussed here. In the event that a neutral Higgs boson is discovered previously at LEP or the LHC-SSC, with properties compatible with one of the MSSM Higgs states, EE500 would still be a very useful instrument to investigate in detail the spectroscopy of the Higgs sector, for example to distinguish between the SM, the MSSM and possibly other non-minimal supersymmetric extensions.
4 \textit{R}-odd particles

We now briefly review the \textit{R}-odd spectrum of the MSSM, to introduce the discussion of supersymmetric particle searches at $e^+e^-$ and hadron colliders.

In the spin-0 sector, one has sleptons and squarks, $\tilde{f} \equiv (\tilde{\nu}_L, \tilde{\ell}_L, \tilde{\nu}^c_L \equiv \tilde{\nu}_R, \tilde{u}_L, \tilde{d}_L, \tilde{d}^c_L \equiv \tilde{d}_R)$, with generation indices left implicit as usual. Neglecting intergenerational mixing, their diagonal mass terms are given by

$$m_{\tilde{f}}^2 = \tilde{m}^2 + m_f^2 + m_D^2,$$

(41)

where $\tilde{m}$ is the soft supersymmetry-breaking mass, $m_f$ is the corresponding fermion mass, and

$$m_D^2 = m_Z^2 \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} (Y \sin^2 \theta_W - T_\beta \cos^2 \theta_W).$$

(42)

For the sfermions of the first two generations, $\tilde{f}_L$-$\tilde{f}_R$ mixing is negligible and the soft masses are given by eq. (22), so one can express $m_{\tilde{f}}$ in terms of the basic parameters $m_{1/2}, m_0$ and $\tan \beta$. Notice for example that, neglecting the lepton masses, $SU(2)$ invariance alone requires $m_{\tilde{\nu}}^2 = m_{\tilde{\ell}}^2 - m_W^2 [ (\tan^2 \beta - 1) / (\tan^2 \beta + 1) ]$. For the sfermions of the third generation, the off-diagonal term in the $\tilde{f}_L$-$\tilde{f}_R$ mass matrix

$$m_{\tilde{f}_{LR}}^2 = m_f \times \begin{cases} A_f + \mu \tan \beta & (f = e, d) \\ A_f + \mu / \tan \beta & (f = u) \end{cases},$$

(43)

might be non-negligible, so that the mass eigenstates ($\tilde{f}_1, \tilde{f}_2$) are non-trivial admixtures of the interaction eigenstates ($\tilde{f}_L, \tilde{f}_R$). Also, to compute the soft contributions to the masses in terms of the basic parameters one has to solve numerically the associated RGE.

In the spin-$\frac{1}{2}$ sector, one has the strongly interacting gluinos, $\tilde{g}$, with mass $m_\tilde{g} \equiv M_3$ directly given by eq. (16). In addition, one has the weakly interacting \textit{charginos} and \textit{neutralinos}, i.e. the charged and neutral mass eigenstates corresponding to electroweak gauginos and higgsinos. Charginos ($W^\pm, H^\pm$) mix via the $2 \times 2$ matrix

$$\begin{pmatrix} M_2 & \sqrt{2}m_W \sin \beta \\ \sqrt{2}m_W \cos \beta & \mu \end{pmatrix},$$

(44)
whose mass eigenstates are denoted by \( \tilde{\chi}_{\pm}^k \) \((k = 1, 2)\), and neutralinos \((\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)\) mix via the \(4 \times 4\) matrix

\[
\begin{pmatrix}
M_1 & 0 & -m_Z \cos \beta \sin \theta_W & m_Z \sin \beta \sin \theta_W \\
0 & M_2 & m_Z \cos \beta \cos \theta_W & -m_Z \sin \beta \cos \theta_W \\
-m_Z \cos \beta \sin \theta_W & m_Z \cos \beta \cos \theta_W & 0 & -\mu \\
m_Z \sin \beta \sin \theta_W & -m_Z \sin \beta \cos \theta_W & -\mu & 0
\end{pmatrix},
\]

whose mass eigenstates are denoted by \( \tilde{\chi}_i^0 \) \((i = 1, 2, 3, 4)\). Notice that the lightest neutralino \( \tilde{\chi} \equiv \tilde{\chi}_1^0 \), which in most of the acceptable parameter space is the LSP, is in general a non-trivial admixture of gauginos and higgsinos, and not just a pure photino \( \tilde{\gamma} \equiv \cos \theta_W \tilde{B} + \sin \theta_W \tilde{W}_3 \) as often assumed in phenomenological studies. In the MSSM, all the masses and couplings in the chargino-neutralino sector can be characterized by the three parameters \( m_{1/2}, \mu, \) and \( \tan \beta \).

To give an idea of the structure of the MSSM \( R \)-odd spectrum, we show in figs. 9 and 10 (updated from ref. [85]) contours of some selected sparticle masses in the \((m_0, m_{1/2})\) and in the \((\mu, m_{1/2})\) planes, respectively, for the representative values \( \tan \beta = 2 \) and \( \tan \beta = 10 \).

### 4.1 Searches for sleptons

The most stringent limits on sleptons come from unsuccessful searches for the processes \( Z \to \tilde{l}^+ \tilde{l}^- \) and \( Z \to \tilde{\nu} \tilde{\nu} \) at LEP I. In the mass range of interest, and assuming that \( \tilde{\chi} \) is the LSP, the main decay modes are \( \tilde{l}^\pm \to l^\pm \tilde{\chi} \) and \( \tilde{\nu} \to \nu \tilde{\chi} \). Indirect but powerful information can be extracted from the precise measurements of the total and partial \( Z \) widths. Direct searches are sensitive to charged sleptons only, and look for acoplanar lepton pairs with missing transverse momentum. Experimental details on slepton searches at LEP I can be found in refs. [86,87]. Crudely speaking, one can summarize the present limits by \( m_{\tilde{l}}, m_{\tilde{\nu}} \gtrsim m_Z/2 \). In the future, LEP II will be sensitive to charged sleptons up to \( m_{\tilde{l}} \simeq 80–90 \) GeV, whereas the limits on \( m_{\tilde{\nu}} \) are not expected to improve. At large hadron colliders like the LHC-SSC, slepton searches appear problematic [88], since the Drell-Yan production cross-sections are small and the backgrounds are large. It is then clear that high-energy \( e^+e^- \) colliders can play a very important role in slepton searches, as will be now outlined.
Theoretical aspects of slepton production and decay at EE500 have been recently investigated in ref. [89]. The production mechanisms considered in this study are

\begin{align}
e^+e^- & \to \tilde{e}_L^+\tilde{e}_L^-, \tilde{e}_R^+\tilde{e}_R^-, \tilde{e}_L^+\tilde{e}_R^- \\
e^+e^- & \to \tilde{\mu}_L^+\tilde{\mu}_L^-, \tilde{\mu}_R^+\tilde{\mu}_R^- \\
e^+e^- & \to \tilde{\nu}\tilde{\nu}.
\end{align}

(46)

(47)

(48)

The first two processes in (46) occur via \((\gamma, Z)\) exchange in the s-channel and \(\tilde{\chi}_i^0\) exchange in the t-channel. The last process in (46) receives only t-channel contributions, the two processes in (47) only s-channel contributions. The processes in (48) occur via Z exchange in the s-channel, with \(\tilde{\chi}_k^\pm\) exchange in the t-channel also contributing in the case of \(\tilde{\nu}_e\). In general, then, the production cross-sections depend not only on the slepton masses, but also on the parameters of the chargino-neutralino sector.

As far as decay modes are concerned, one has to take into account the possibility of cascade decays, \(\tilde{l}_i^\pm \to l^\pm \tilde{\chi}_i^{0\neq 1} \to \ldots, \tilde{l}_L^\pm \to \nu\tilde{\chi}_k^\pm \to \ldots, \tilde{\nu} \to \nu\tilde{\chi}_i^{0\neq 1} \to \ldots, \tilde{\nu} \to l^\pm \tilde{\chi}_k^\mp \to \ldots\), in addition to the direct decays \(\tilde{l}_{L,R}^\pm \to l^\pm \tilde{\chi}\), \(\tilde{\nu} \to \nu\tilde{\chi}\). Also the relevant branching ratios depend on the parameters of the chargino-neutralino sector.

A detailed analysis of the whole parameter space will not be attempted here. The most likely case, in view of the theoretical constraints on the MSSM, seems to be the one in which the lightest sleptons are \(\tilde{\ell}_R^\pm (l = e, \mu, \tau)\). In this case one obtains [89] sizeable cross-sections, \(O(10 \text{ fb})\) or more, up to slepton masses of 80–90% of the beam energy, which should allow for a relatively easy detection if the mass difference \((m_{\tilde{\ell}_R} - m_{\tilde{\chi}})\) is not too small [3].

4.2 Searches for squarks and gluinos

Being strongly interacting sparticles, squarks and gluinos are best searched for at hadron colliders. Production cross-sections for \(\tilde{g}\tilde{g}, \tilde{g}\tilde{q}, \tilde{q}\tilde{q}\) pair-production in \(pp\) or \(p\bar{p}\) collisions are relatively model-independent functions of \(m_{\tilde{g}}\) and \(m_{\tilde{q}}\). As far as signatures are concerned, one has to distinguish two main possibilities: if \(m_{\tilde{g}} < m_{\tilde{q}}\), then \(\tilde{g} \to q\bar{q}\) immediately after production, and the final state is determined by \(\tilde{g}\) decays; if \(m_{\tilde{q}} < m_{\tilde{g}}\), then \(\tilde{g} \to \tilde{g}\tilde{q}\) immediately after production, and the final state is determined by \(\tilde{g}\) decays. The first case
is favoured by the theoretical constraints of the MSSM. In old experimental analyses, it was customary to work under a certain set of assumptions: 1) five or six ($\tilde{q}_L, \tilde{q}_R$) mass-degenerate squark flavours; 2) LSP $\equiv \tilde{\chi}_1^0$, with mass negligible with respect to $m_{\tilde{q}}, m_{\tilde{g}}$; 3) the dominant decay modes of squarks and gluinos are the direct ones, $\tilde{g} \rightarrow q\bar{q}\tilde{\gamma}$ if $m_{\tilde{g}} < m_{\tilde{q}}$ and $\tilde{g} \rightarrow q\gamma$ if $m_{\tilde{q}} < m_{\tilde{g}}$. The signals to be looked for are then multijet events with a large amount of missing transverse momentum. To derive reliable limits, however, one has to take into account that the above assumptions are in general incorrect. For example, one can have cascade decays $\tilde{g} \rightarrow q\bar{q}\tilde{\chi}^{\pm}_{i \neq 1}, q\bar{q}_{i \neq 1} \rightarrow ...$ and $\tilde{q} \rightarrow q\tilde{\chi}^0_{i \neq 1}, q\tilde{\chi}^0_{k} \rightarrow ...$. The effects of these cascade decays become more and more important as one moves to higher and higher squark and gluino masses. Taking all this into account, the present limits from the Tevatron collider are roughly $m_{\tilde{q}} \gtrsim 150$ GeV, $m_{\tilde{g}} \gtrsim 135$ GeV [90]. At the LHC and SSC, one should be able to explore squark and gluino masses up to 1 TeV and probably more [91]. In general, therefore, EE500 will not be competitive for squark and gluino searches. Its cleaner environment, however, could be exploited for a detailed study of squark properties if they are discovered at sufficiently low mass. Also, there are special situations which might be difficult to study at large hadron colliders: for example, the case of a stop squark significantly lighter than all the other squarks. The threshold behaviour for stop production in $e^+e^-$ collisions has been recently studied in ref. [92].

4.3 Searches for charginos and neutralinos

The most stringent limits on charginos and neutralinos come [93,86,87] from unsuccessful LEP I searches for the processes $Z \rightarrow \tilde{\chi}\tilde{\chi}$ (contributing to the invisible Z width), $Z \rightarrow \tilde{\chi}^0_{i \neq 1}$ (originating spectacular one-sided events) and $Z \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^-_1, \tilde{\chi}^0_{i \neq 1} \tilde{\chi}^0_{j \neq 1}$ (originating acoplanar leptons or jets accompanied by missing energy). The presently excluded region of the $(\mu, m_{1/2})$ plane is shown, for the two representative values $\tan \beta = 2$ and $\tan \beta = 10$, in fig. 10. As a crude summary, one could say that all states different from $\tilde{\chi}$ have to be heavier than $m_Z/2$, whereas LEP data alone would still allow for arbitrarily light $\tilde{\chi}$. For LEP II searches, the most effective process should be $\tilde{\chi}^+_1 \tilde{\chi}^-_1$ pair production, with $\tilde{\chi}^0_{i \neq 1}$ pair production slightly less effective in probing parameter space because of the smaller cross-section. At large hadron colliders [94], it seems very difficult to improve the LEP II sensitivity significantly, especially if the top quark mass is significantly smaller than 200
GeV, as now favoured by radiative-correction analyses.

Theoretical aspects of chargino and neutralino production and decays at EE500 have been recently studied in ref. [89], and experimental simulations are reported in ref. [3]. In the case of charginos, the most important production diagrams involve the s-channel exchange of (γ, Z) and the t-channel exchange of $\tilde{\nu}_e$. The cross-section then depends not only on the parameters of the chargino-neutralino sector, but also on the sneutrino mass, and there can be significant destructive interference between the two classes of diagrams. As for chargino decays, if the sneutrino is light enough the dominant decay mode is $\tilde{\chi}^\pm_1 \rightarrow l^\pm \tilde{\nu}$, whereas in the case of a heavy sneutrino the dominant decay modes are $\tilde{\chi}^\pm_1 \rightarrow q\bar{q}^{\prime} \tilde{\chi}$, $l^\pm \nu \tilde{\chi}$. Experimental analyses show that at EE500 one can enormously extend the parameter space accessible to LEP II, and reach chargino masses of the order of 80–90 % of the beam energy, provided that the mass difference $m_{\tilde{\chi}^\pm_1} - m_{\tilde{\chi}}$ is not too small: this unfortunate situation could occur in the region of parameter space where $|\mu| < < m_{1/2}$.

5 Conclusions

In summary, in this talk we have argued that the MSSM is a calculable, theoretically motivated and phenomenologically acceptable extension of the SM. Of course, only experiment can tell if low-energy supersymmetry is actually realized in Nature, but, to use the words of one of the speakers at this Workshop, searching for supersymmetry does not look like fishing in a dead sea. For a global view of the present limits and of the discovery potential of future machines, including EE500, it is useful to look again at the most important parameters of the MSSM

$$m_A, \ \ tan \beta, \ \ m_0, \ \ m_{1/2}, \ \ \mu,$$

(49)

which, together with $m_t$, determine the main features of its particle spectrum.

The Higgs sector mainly depends on $(m_A, \ tan \beta)$, but also $m_t$ and (to a lesser extent) the other parameters play a role via the large radiative corrections. As an example, we have considered the case $m_t = 140$ GeV, $m_{\tilde{q}} = 1$ TeV, $A_t = A_b = \mu = 0$, summarized in figs. 3–8. Fig. 3 shows that LEP I, despite its remarkable achievements, has explored only a small part, roughly $m_A \lesssim 45$ GeV, of the natural parameter space for the MSSM Higgs bosons. A much greater sensitivity will be achieved at LEP II, where, for standard
values of the machine parameters ($\sqrt{s} = 190$ GeV, $\int \mathcal{L} dt = 500$ pb$^{-1}$), one should be able to test $m_A \lesssim 80$ GeV, $\tan \beta \lesssim 3$. However, as a result of the large radiative corrections, the rest of the $(m_A, \tan \beta)$ parameter space will not be accessible to LEP II. The LHC-SSC can greatly improve over LEP II, as shown in fig. 4. The most promising experimental signatures are $h, H \rightarrow \gamma\gamma$ (inclusive or in association with $W$ or $t\bar{t}$), $H \rightarrow ZZ \rightarrow 4l^\pm$, $H, A \rightarrow \tau^+\tau^-$, $t \rightarrow bH^+ \rightarrow b\tau^+\nu\tau$. Combined, they might be able to probe the whole $(m_A, \tan \beta)$ plane, with the exception of $m_Z \lesssim m_A \lesssim 200$ GeV, $2 \lesssim \tan \beta \lesssim 10$. For our choice of parameters, then, the overlap between LEP II and the LHC-SSC is likely not to be complete, giving rise to a possible violation of the so-called ‘no-lose theorem’. A further increase of the LEP II energy might save the day. On the other hand, as shown in figs. 5–8, EE500 is guaranteed to observe at least one neutral Higgs boson or to rule out the MSSM. In particular, in the region of the parameter space which is most difficult for the LHC-SSC, EE500 can perform a detailed spectroscopy of the MSSM Higgs sector, observing all its physical states.

As for the ($R$-odd) supersymmetric particles, the situation is summarized in figs. 9 and 10. Again, we can see that the already impressive limits obtained by LEP I and Tevatron have ruled out only a small part of the natural parameter space. LEP II and the upgraded Tevatron will provide higher but still limited sensitivities, corresponding roughly to $m_{\tilde{t}}, m_{\tilde{\chi}^\pm} \lesssim 80–90$ GeV, $m_{\tilde{g}}, m_{\tilde{q}} \lesssim 200$ GeV. The LHC-SSC should definitely cover the rest of the parameter space, via gluino and squark searches up to masses of 1 TeV or higher. However, a comparable sensitivity can be reached by EE500 via chargino and slepton searches up to masses of 200 GeV or even higher. If no signal of supersymmetry is found at the LHC-SSC, EE500 can provide the definitive confirmation that the MSSM is ruled out, in a cleaner environment and in a more model-independent way. In the optimistic case that a signal is found at the LHC-SSC, EE500 would constitute a unique facility for the direct production of weakly interacting supersymmetric particles, which should allow for a detailed spectroscopy of the MSSM.

In conclusion, for what concerns supersymmetry, EE500 is the ideal complement to the LHC-SSC, and the case for it could not be stronger.
Acknowledgements

I am grateful to my collaborators A. Brignole, J. Ellis, Z. Kunszt and G. Ridolfi for their contributions to our common work on supersymmetric Higgs bosons, and to them and F. Pauss for discussions concerning the present paper.
References

1. P. Fayet, Nucl. Phys. B90 (1975) 104, Phys. Lett. 64B (1976) 159 and 69B (1977) 489.

2. For reviews and references, see, e.g.:
   H.-P. Nilles, Phys. Rep. 110 (1984) 1;
   H.E. Haber and G.L. Kane, Phys. Rep. 117 (1985) 75;
   S. Ferrara, ed., ‘Supersymmetry’ (North-Holland, Amsterdam, 1987);
   R. Barbieri, Riv. Nuovo Cimento 11 (1988) 1.

3. J.-F. Grivaz, these Proceedings.

4. Yu.A. Gol’fand and E.P. Likhtman, JETP Lett. 13 (1971) 323;
   D.V. Volkov and V.P. Akulov, Phys. Lett. 46B (1973) 109;
   J. Wess and B. Zumino, Nucl. Phys. B70 (1974) 39.

5. D.Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, Phys. Rev. B13 (1976) 3214;
   S. Deser and B. Zumino, Phys. Lett. 62B (1976) 335.

6. R. Haag, J. Lopuszanski and M. Sohnius, Nucl. Phys. B88 (1975) 257.

7. For reviews and references see, e.g.:
   M.B. Green, J.H. Schwarz and E. Witten, ‘Superstring Theory’ (University Press, Cambridge, 1987);
   B. Schellekens, ed., ‘Superstring construction’ (North-Holland, Amsterdam, 1989).

8. K. Wilson, as quoted in L. Susskind, Phys. Rev. D20 (1979) 2619;
   E. Gildener and S. Weinberg, Phys. Rev. D13 (1976) 3333;
   G. ’t Hooft, in ‘Recent developments in gauge theories’, Cargese Lectures 1979 (Plenum Press, New York, 1980).

9. For reviews and references, see, e.g.:
   J.R. Carter, J. Ellis and T. Hebbeker, Rapporteur’s talks given at the LP-HEP ’91 Conference, Geneva, 1991, to appear in the Proceedings, and references therein.
10. J. Wess and B. Zumino, Phys. Lett. B49 (1974) 52; J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310; S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77 (1974) 413; M.T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. B159 (1979) 429; S. Ferrara, L. Girardello and F. Palumbo, Phys. Rev. D20 (1979) 403.

11. L. Maiani, Proc. Summer School on Particle Physics, Gif-sur-Yvette, 1979 (IN2P3, Paris, 1980), p. 1; M. Veltman, Acta Phys. Polon. B12 (1981) 437; E. Witten, Nucl. Phys. B188 (1981) 513; see also: S. Weinberg, Phys. Lett. 82B (1979) 387.

12. L. Girardello and M.T. Grisaru, Nucl. Phys. B194 (1982) 65.

13. J. Ellis and D.V. Nanopoulos, Phys. Lett. B110 (1982) 44; R. Barbieri and R. Gatto, Phys. Lett. B110 (1982) 211.

14. Y. Kizukuri and N. Oshimo, contribution to the Proceedings of the Workshop ‘\(e^+e^-\) Linear Colliders at 500 GeV: the Physics Potential’, Hamburg, 1991, and references therein.

15. J. Ellis, S. Ferrara and D.V. Nanopoulos, Phys. Lett. B114 (1982) 231; W. Buchmüller and D. Wyler, Phys. Lett. B121 (1983) 321; J. Polchinski and M.B. Wise, Phys. Lett. B125 (1983) 393; F. del Aguila, J.A. Grifols, A. Mendez, D.V. Nanopoulos and M. Srednicki, Phys. Lett. B129 (1983) 77; J.-M. Frère and M.B. Gavela, Phys. Lett. B132 (1983) 107.

16. R.K. Kaul and P. Majumdar, Nucl. Phys. B199 (1982) 36; R. Barbieri, S. Ferrara and C.A. Savoy, Phys. Lett. B119 (1982) 36; H.P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B120 (1983) 346; J.M. Frère, D.R.T. Jones and S. Raby, Nucl. Phys. B222 (1983) 11; J.-P. Derendinger and C. Savoy, Nucl. Phys. B237 (1984) 307; J. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D39 (1989) 844.

17. J. Polchinski and L. Susskind, Phys. Rev. D26 (1982) 3661; J.E. Kim and H.-P. Nilles, Phys. Lett. B138 (1984) 150;
L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D27 (1983) 2359;
G. F. Giudice and A. Masiero, Phys. Lett. 206B (1988) 480;
K. Inoue, M. Kawasaki, M. Yamaguchi and T. Yanagida, preprint TU-373 (1991);
J.E. Kim and H.-P. Nilles, Phys. Lett. B263 (1991) 79;
E.J. Chun, J.E. Kim and H.-P. Nilles, preprint SNUTP-91-25.

18. H.-P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B124 (1983) 337;
   A.B. Lahanas, Phys. Lett. B124 (1983) 341;
   L. Alvarez-Gaumé, J. Polchinski and M.B. Wise, Nucl. Phys. B221 (1983) 495;
   A. Sen, Phys. Rev. D30 (1984) 2608 and D32 (1985) 411.

19. U. Ellwanger and M. Rausch de Traubenberg, contribution to same Proc. as ref. [14], and references therein;
   B.R. Kim, S.K. Oh and A. Stephan, ibid., and references therein.

20. L.J. Hall and M. Suzuki, Nucl. Phys. B231 (1984) 419;
   F. Zwirner, Phys. Lett. B132 (1983) 103.

21. C. Aulakh and R.N. Mohapatra, Phys. Lett. B119 (1983) 136;
   G.G. Ross and J.W.F. Valle, Phys. Lett. B151 (1985) 375;
   J. Ellis, G. Gelmini, C. Jarlskog, G.G. Ross and J.W.F. Valle, Phys. Lett. B150 (1985) 142.

22. H. Dreiner and S. Lola, contribution to same Proc. as ref. [14], and references therein.

23. H.P. Nilles, Phys. Lett. 115B (1982) 193 and Nucl. Phys. B217 (1983) 366;
   S. Ferrara, L. Girardello and H.P. Nilles, Phys. Lett. 125B (1983) 457;
   J.-P. Derendinger, L.E. Ibáñez and H.P. Nilles, Phys. Lett. 155B (1985) 65;
   M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. 156B (1985) 55.

24. J. Scherk and J.H. Schwarz, Phys. Lett. B82 (1979) 60 and Nucl. Phys. B153 (1979) 61;
   R. Rohm, Nucl. Phys. B237 (1984) 553;

34
C. Kounnas and M. Porrati, Nucl. Phys. B310 (1988) 355;
S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, Nucl. Phys. B318 (1989) 75;
C. Kounnas and B. Rostand, Nucl. Phys. B341 (1990) 641;
I. Antoniadis, Phys. Lett. B246 (1990) 377.

25. H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.

26. W.A. Bardeen, A. Buras, D. Duke and T. Muta, Phys. Rev. D18 (1978) 3998.

27. H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438.

28. For reviews and references, see, e.g. :
   G. Costa and F. Zwirner, Riv. Nuovo Cimento 9 (1986) 1.

29. U. Amaldi, A. Böhm, L.S. Durkin, P. Langacker, A.K. Mann, W.J. Marciano, A. Sirlin and H.H. Williams, Phys. Rev. D36 (1987) 1385;
   G. Costa, J. Ellis, G.L. Fogli, D.V. Nanopoulos and F. Zwirner, Nucl. Phys. B297 (1988) 244.

30. J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. B249 (1990) 442 and B260 (1991) 131;
   P. Langacker and M. Luo, Phys. Rev. D44 (1991) 817;
   U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. B260 (1991) 447;
   F. Anselmo, L. Cifarelli, A. Petermann and A. Zichichi, preprint CERN-PPE/91-123.

31. S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D24 (1981) 1681;
   L.E. Ibáñez and G.G. Ross, Phys. Lett. 105B (1981) 439.

32. G. Gamberini, G. Ridolfi and F. Zwirner, Nucl. Phys. B331 (1990) 331.

33. H. Georgi and D.V. Nanopoulos, Nucl. Phys. B159 (1979) 16;
   F. del Aguila and L.E. Ibáñez, Nucl. Phys. B177 (1981) 60;
   L.E. Ibáñez, Nucl. Phys. B181 (1981) 105;
   P. Frampton and S.L. Glashow, Phys. Lett. 131B (1983) 340;
U. Amaldi, W. de Boer, P.H. Frampton, H. Fürstenau and J.T. Liu, preprint CERN-PPE/91-233.

34. S. Weinberg, Phys. Lett. B91 (1980) 51;
   L.J. Hall, Nucl. Phys. B178 (1980) 75;
   T.J. Goldman and D.A. Ross, Nucl. Phys. B171 (1980) 273;
   P. Binétruy and T. Schücker, Nucl. Phys. B178 (1981) 293, 307;
   I. Antoniadis, C. Kounnas and C. Roiesnel, Nucl. Phys. B198 (1982) 317.

35. M.B. Einhorn and D.R.T. Jones, Nucl. Phys. B196 (1982) 475;
   M.E. Machacek and M.T. Vaughn, Nucl. Phys. B236 (1984) 221.

36. S. Weinberg, Phys. Rev. D26 (1982) 287;
   N. Sakai and T. Yanagida, Nucl. Phys. B197 (1982) 533.

37. S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150;
   N. Sakai, Z. Phys. C11 (1982) 153.

38. R. Barbieri and L.J. Hall, preprint LBL-31238 (1991);
   J. Ellis, S. Kelley and D.V. Nanopoulos, preprint CERN-TH.6140/91;
   G.G. Ross and R.G. Roberts, preprint RAL-92-005.

39. V.S. Kaplunovsky, Nucl. Phys. B307 (1988) 145;
   L. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B355 (1991) 649;
   J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, preprint CERN-TH.6004/91, to appear in Nucl. Phys. B;
   G. Lopez-Cardoso and B.A. Ovrut, preprint UPR-0464T (1991);
   J. Louis, preprint SLAC-PUB-5527 (1991);
   I. Antoniadis, K.S. Narain and T. Taylor, Phys. Lett. B267 (1991) 37;
   G. Lopez-Cardoso and B.A. Ovrut, preprint UPR-0481T (1991);
   J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Phys. Lett. B271 (1991) 307.

40. W. Siegel, Phys. Lett. B84 (1979) 193;
   D.M. Capper, D.R.T. Jones and P. van Nieuwenhuizen, Nucl. Phys. B167 (1980) 479;
I. Antoniadis, C. Kounnas and K. Tamvakis, Phys. Lett. B119 (1982) 377.

41. I. Antoniadis, J. Ellis, J. Hagelin and D.V. Nanopoulos, Phys. Lett. B231 (1989) 65, and references therein.

42. I. Antoniadis, J. Ellis, R. Lacaze and D.V. Nanopoulos, Phys. Lett. B268 (1991) 188.

43. I. Antoniadis, J. Ellis, S. Kelley and D.V. Nanopoulos, preprint CERN-TH.6169/91;
   L.E. Ibáñez, D. Lüst and G.G. Ross, Phys. Lett. B272 (1991) 251.

44. S. Weinberg, Phys. Rev. Lett. 50 (1983) 387;
   H. Goldberg, Phys. Rev. Lett. 50 (1983) 1419;
   L.M. Krauss, Nucl. Phys. B227 (1983) 556.

45. J. Ellis, J.S. Hagelin, D.V. Nanopoulos, K.A. Olive and M. Srednicki, Nucl. Phys. B238 (1984) 453.

46. K.A. Olive and M. Srednicki, Nucl. Phys. B355 (1991) 208;
   J. Ellis, D.V. Nanopoulos, L. Roszkowski and D.N. Schramm, Phys. Lett. B245 (1990) 251;
   L. Roszkowski, Phys. Lett. B262 (1991) 59;
   J.L. Lopez, K. Yuan and D.V. Nanopoulos, Phys. Lett. B267 (1991) 219;
   J. Ellis and L. Roszkowski, preprint CERN-TH.6260/91, UM-TH-91-25.

47. T.K. Kuo and N. Nakagawa, Nuovo Cimento Lett. 36 (1983) 560;
   L. Alvarez-Gaumé, J. Polchinski and M.B. Wise, Nucl. Phys. B221 (1983) 495;
   R. Barbieri and L. Maiani, Nucl. Phys. B224 (1983) 32;
   C.S. Lim, T. Inami and N. Sakai, Phys. Rev. D29 (1984) 1488;
   E. Eliasson, Phys. Lett. B147 (1984) 65;
   Z. Hioki, Progr. Theor. Phys. 73 (1985) 1283;
   J.A. Grifols and J. Solà, Phys. Lett. B137 (1984) 257 and Nucl. Phys. B253 (1985) 47;
   R. Barbieri, M. Frigeni, F. Giuliani and H.E. Haber, Nucl. Phys. B341
A. Bilal, J. Ellis and G.L. Fogli, Phys. Lett. B246 (1990) 459;
A. Djouadi, G. Girardi, C. Verzegnassi, W. Hollik and F.M. Renard, Nucl. Phys. B249 (1991) 48;
M. Drees and K. Hagiwara, Phys. Rev. D42 (1990) 1709;
M. Boulware and D. Finnell, Phys. Rev. D44 (1991) 2054;
M. Drees, K. Hagiwara and A. Yamada, preprint DTP/91/34.

48. M.E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964;
M. Golden and L. Randall, Nucl. Phys. B361 (1991) 3;
B. Holdom and J. Terning, Phys. Lett. B247 (1990) 88.

49. G. Altarelli, preprint CERN-TH.6245/91, talk given at the LP-HEP '91 Conference, Geneva, July 1991, to appear in the Proceedings, and references therein.

50. R. Barbieri, S. Ferrara, L. Maiani, F. Palumbo and C.A. Savoy, Phys. Lett. B115 (1982) 212;
K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Progr. Theor. Phys. 68 (1982) 927 and 71 (1984) 413.

51. J. Bagger, S. Dimopoulos and E. Massó, Phys. Rev. Lett. 55 (1985) 920.

52. B. Pendleton and G.G. Ross, Phys. Lett. B98 (1981) 291;
C. Hill, Phys. Rev. D24 (1981) 691.

53. L.E. Ibáñez and G.G. Ross, Phys. Lett. 110B (1982) 215;
K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, as in ref. [50];
L. Alvarez-Gaumé, M. Claudson and M.B. Wise, Nucl. Phys. B207 (1982) 96;
J. Ellis, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. B121 (1983) 123.

54. S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888;
S. Weinberg, Phys. Rev. D7 (1973) 2887.

55. For reviews and references see, e.g.:
J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, ‘The Higgs Hunter’s Guide’ (Addison-Wesley, New York, 1990).
56. Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. Lett. 85 (1991) 1.
57. J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B257 (1991) 83.
58. H.E. Haber and R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815.
59. S.P. Li and M. Sher, Phys. Lett. B140 (1984) 339.
60. J.F. Gunion and A. Turski, Phys. Rev. D39 (1989) 2701 and D40 (1989) 2333.
61. M. Berger, Phys. Rev. D41 (1990) 225.
62. R. Barbieri, M. Frigeni and M. Caravaglios, Phys. Lett. B258 (1991) 167.
63. Y. Okada, M. Yamaguchi and T. Yanagida, Phys. Lett. B262 (1991) 54.
64. R. Barbieri and M. Frigeni, Phys. Lett. B258 (1991) 395.
65. J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B262 (1991) 477.
66. A. Yamada, Phys. Lett. B263 (1991) 233.
67. J.R. Espinosa and M. Quirós, Phys. Lett. B266 (1991) 389.
68. A. Brignole, J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B271 (1991) 123.
69. P.H. Chankowski, S. Pokorski and J. Rosiek, Phys. Lett. B274 (1992) 191.
70. A. Brignole, preprint DFPD/91/TH/28 (1991), to appear in Phys. Lett. B.
71. M. Drees and M.N. Nojiri, preprint KEK-TH-305 (1991).
72. D.M. Pierce, A. Papadopoulos and S. Johnson, preprint LBL-31416, UCB-PTH-91/58.
73. Z. Kunszt and F. Zwirner, preprint CERN-TH.6150/91, ETH-TH/91-7.

74. J.F. Gunion, R. Bork, H.E. Haber and A. Seiden, preprint UCD-91-29, SCIPP-91/34;
    H. Baer, M. Bisset, C. Kao and X. Tata, preprint FSU-HEP-911104, UH-511-732-91;
    J.F. Gunion and L.H. Orr, preprint UCD-91-15;
    V. Barger, M.S. Berger, A.L. Stange and R.J.N. Phillips, preprint MAD-PH-680 (1991).

75. A. Brignole, preprint CERN-TH.6366/92.

76. D. Décamp et al. (ALEPH Collaboration), Phys. Lett. B265 (1991) 475;
    P. Igo-Kemenes (OPAL Collaboration), L. Barone (L3 Collaboration),
    W. Ruhlmann (DELPHI Collaboration), talks given at the LP-HEP ‘91 Conference, Geneva, 1991, to appear in the Proceedings;
    M. Davier, Rapporteur’s talk at the same Conference, and references therein.

77. D. Treille, private communication;
    C. Rubbia, Rapporteur’s talk given at the LP-HEP ’91 Conference, Geneva, 1991, to appear in the Proceedings;
    U. Amaldi, these Proceedings.

78. Z. Kunszt and F. Zwirner, Proc. Large Hadron Collider Workshop,
    Aachen, 1990 (G. Jarlskog and D. Rein, eds.) (CERN 90-10, ECFA 90-133, Geneva, 1990), Vol.II, p. 578.

79. F. Pauss, lectures given in the CERN Academic Training Programme,
    December 1991, and references therein.

80. A. Brignole, J. Ellis, J.F. Gunion, M. Guzzo, F. Olness, G. Ridolfi,
    L. Roszkowski and F. Zwirner, contribution to the Workshop ‘e+e−
    Linear Colliders at 500 GeV: the Physics Potential’, Hamburg, 1991,
    to appear in the Proceedings.

81. H.E. Haber, these Proceedings.
82. P. Janot, contribution to the same Workshop as ref. [80].

83. S. Komamiya, these Proceedings.

84. D.R.T. Jones and S.T. Petcov, Phys. Lett. B84 (1979) 440;
    K. Hikasa, Phys. Lett. B164 (1985) 385;
    R. Cahn, Nucl. Phys. B255 (1985) 341;
    G. Altarelli, B. Mele and F. Pitolli, Nucl. Phys. B287 (1987) 285.

85. G. Ridolfi, G.G. Ross and F. Zwirner, same Proc. as ref. [78], Vol. II, p. 605.

86. D. Décamp et al. (ALEPH Collaboration), Phys. Lett. B236 (1990) 86;
    P. Abreu et al. (DELPHI Collaboration), Phys. Lett. B247 (1990) 157;
    B. Adeva et al. (L3 Collaboration), Phys. Lett. B233 (1989) 530;
    M.Z. Akrawy et al. (OPAL Collaboration), Phys. Lett. B240 (1990) 261.

87. D. Décamp et al. (ALEPH Collaboration), preprint CERN-PPE/91-149, submitted to Physics Reports.

88. F. del Aguila, L. Ametller and M. Quirós, same Proc. as ref. [78], Vol. II, p. 663;
    F. del Aguila and L. Ametller, Phys. Lett. B261 (1991) 326.

89. A. Bartl, W. Majerotto and B. Mösslacher, contribution to the same Workshop as ref. [80].

90. H. Baer, X. Tata and J. Woodside, Phys. Rev. D44 (1991) 207.

91. C. Albajar, C. Fuglesang, S. Hellman, E. Nagy, F. Pauss, G. Polesello and P. Spicas, same Proc. as ref. [78], Vol. II, p. 621;
    H. Baer et al., preprint FSU-HEP-901110, to be published in the Proceedings of the 1990 DPF Summer Study on High Energy Physics, Snowmass, CO, June 25-July 13, 1990.

92. I.I. Bigi, V.S. Fadin and V. Khoze, preprint UND-HEP-91-BIG03.
93. D. Décamp et al. (ALEPH Collaboration), Phys. Lett. B244 (1990) 541;
M.Z. Akrawy et al. (OPAL Collaboration), Phys. Lett. B248 (1990) 211.

94. R. Barbieri, F. Caravaglios, M. Frigeni and M. Mangano, same Proc.
as ref. [78], Vol.II, p. 658; Nucl. Phys. B367 (1991) 28.
Figure captions

Fig.1: Contours of $m_h^{\text{max}}$ (the maximum value of $m_h$, reached for $m_A \to \infty$) in the $(m_t, \tan \beta)$ plane, for $m_{\tilde{q}} = 1$ TeV.

Fig.2: Contours of a) $m_h$ and b) $m_H$, in the $(m_A, \tan \beta)$ plane, for $m_{\tilde{q}} = 1$ TeV and $m_t = 140$ GeV.

Fig.3: Schematic representation of the present LEP I limits and of the future LEP II sensitivity in the $(m_A, \tan \beta)$ plane, for $m_{\tilde{q}} = 1$ TeV and $m_t = 140$ GeV. The solid lines correspond to the present LEP I limits. The dashed lines correspond to $\sigma(e^+e^- \to hZ, HZ, hA, HA) = 0.2$ pb at $\sqrt{s} = 175$ GeV, which could be seen as a rather conservative estimate of the LEP II sensitivity. The dashed-dotted lines correspond to $\sigma(e^+e^- \to hZ, HZ, hA, HA) = 0.05$ pb at $\sqrt{s} = 190$ GeV, which could be seen as a rather optimistic estimate of the LEP II sensitivity.

Fig.4: Pictorial representation of the future LHC sensitivity in the $(m_A, \tan \beta)$ plane, for $m_{\tilde{q}} = 1$ TeV and $m_t = 140$ GeV.

Fig.5: Contours of a) $\sigma(e^+e^- \to hZ)$ and b) $\sigma(e^+e^- \to HZ)$, in the $(m_A, \tan \beta)$ plane, for $\sqrt{s} = 500$ GeV.

Fig.6: Contours of a) $\sigma(e^+e^- \to hA)$ and b) $\sigma(e^+e^- \to HA)$, in the $(m_A, \tan \beta)$ plane, for $\sqrt{s} = 500$ GeV.

Fig.7: Contours of a) $\sigma(e^+e^- \to h\nu\bar{\nu})$ and b) $\sigma(e^+e^- \to H\nu\bar{\nu})$, in the $(m_A, \tan \beta)$ plane, for $\sqrt{s} = 500$ GeV.

Fig.8: Contours of a) $\sigma(e^+e^- \to h\mu^+\mu^-)$ and b) $\sigma(e^+e^- \to H\mu^+\mu^-)$, via ZZ-fusion, in the $(m_A, \tan \beta)$ plane, for $\sqrt{s} = 500$ GeV.

Fig.9: Present limits and future sensitivity in the $(m_0, m_{1/2})$ plane, for the representative values a) $\tan \beta = 2$, b) $\tan \beta = 10$ and using $\mu$-independent constraints. The shaded area is excluded by the present data, whereas the solid lines correspond to the estimated discovery potential of the complete LEP and Tevatron programs. Dashed lines correspond to fixed values of an ‘average’ squark mass, defined by the relation $m_{\tilde{q}} = \sqrt{m_0^2 + 5.5m_{1/2}^2}$. Dotted lines correspond to fixed values of the mass of...
the lightest charged slepton ($\tilde{e}^c$), as given in the text. The values of the gluino mass as given by eq. (16) are also shown.

Fig. 10: Present limits and future sensitivity in the $(\mu, m_{1/2})$ plane, for the representative values a) $\tan \beta = 2$, b) $\tan \beta = 10$ and using $m_0$-independent constraints. The shaded area is excluded by the present data, whereas the solid lines correspond to the estimated discovery potential of the complete LEP and Tevatron programs. Dashed and dotted lines correspond to fixed values of the lightest chargino and neutralino mass, respectively. The values of the gluino mass as given by eq. (16) are also shown.