Propagation of short current pulses in Josephson transition line and ultrafast qubit control

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Abstract. The results of the study of the impact of single-quantum magnetic flux pulses moving along the Josephson transmission lines (JTL) on the dynamics of states of qubits magnetically coupled with such lines are presented. The JTL dynamics was calculated in the frame of resistively shunted junction model with different damping coefficients. This allowed us to find the form of control current pulses (fluxon) acting on a superconducting qubit. Numerical simulations of the simplest logic operations with superconducting qubit due to calculated fluxon impact are presented.

1. Introduction

In recent years, schemes based on Rapid Single Flux Quantum (RSFOQ) logic have been actively studied for their applicability in reading weak magnetic and current signals from nanoscale structures: bolometers, single photon detectors, and superconducting quantum bits [1-3]. One of these schemes uses an extremely “rapid” ballistic method. In this method, single-quantum pulses (fluxons) corresponding to magnetic flux quanta (voltage pulses \( V(t) \)) is as follows: \( \int V(t) dt = \Phi_0, \Phi_0 = h/2e, h \) is the Planck's constant, \( e \) is the elementary charge) are propagating along the Josephson transmission line (JTL), which is magnetically coupled to the measured qubit [4, 5]. The sensitivity of the ballistic detector is limited by “natural” changes in the speed of movement of single-quantum pulses, which can be caused by thermal fluctuations of currents and technological variations in parameters of JTL elements. In this case, the magnetic energy associated with the field that penetrates from the JTL into the qubit, strongly depends on the specific topology on the chip.

In this paper, we investigate the influence of the voltage and current pulses (“real” fluxons) formed in the transmission lines on the evolution of the state of a superconducting qubit [6-10]. The calculation of the JTL with different damping coefficients allowed us to find the shape of fluxon. Based on the solution of the master equation of the qubit, we was shown that current pulses in the JTL with low damping can control the populations of the qubit by changing only the magnetic coupling energy with high fidelity. The influence of quantum noise in a qubit on the state switching processes is studied.

2. Single-quantum pulse in JTL

The main equations for JTL dynamics in the frame of resistively shunted junction model have the
fallowing form (we use here normalization by the critical current $I_c$ of Josephson junctions, connected in parallel with inductances $L_{JTL}$):

\[
i_{\text{pulse}} + i_b - \frac{(\phi_1 - \phi_2)}{I_{JTL}} = \sin(\phi_1) + \phi_1 + \beta_C \phi_1, \quad (1)
\]

\[
i_b + \frac{(\phi_{k-1} - \phi_k)}{I_{JTL}} - \frac{(\phi_k - \phi_{k+1})}{I_{JTL}} = \sin(\phi_k) + \phi_k + \beta_L \phi_k, \quad (2)
\]

\[
i_b + \frac{(\phi_{N-1} - \phi_N)}{I_{JTL}} = \sin(\phi_N) + \phi_N + \beta_L \phi_N, \quad (3)
\]

where $i_{\text{pulse}}$ is the input normalized current pulse, $i_b$ is the normalized bias current, $I_{JTL}$ is the normalized inductance, $\beta_C = \left(\frac{\omega_c}{\omega_p}\right)^2$ is the McCumber parameter which determines the effective normalized capacitance Josephson junction, $\omega_c$ and $\omega_p$ are the characteristic and plasma frequencies respectively, $\Delta \phi/I_{JTL}$ is the current through the inductance, $N$ is the length of JTL in the number of Josephson junctions. It is important for us to emphasize that the parameter $(\beta_C)^{1/2}$ (damping coefficient) also determines the decay rate of plasma oscillations for Josephson contacts, determines the magnitude of the energy dissipation in the system.

**Figure 1.** Josephson transmission line (JTL) with the phase generator, $G$, on the input. Josephson contacts are marked by crosses; qubit to measure/control denoted by $Q$ (for instance, it may be realized as a superconducting loop interrupted by three Josephson junctions) and which is coupled with the JTL by means of mutual inductance $M$.

In this paper we consider the propagation of fluxons in the JTL (see figure 1) for the case, when bias current is less than the critical one, $i_b < 1$. Otherwise, the system switches to a resistive state. At the initial moment, the phase at the first Josephson junction was equal to 0. The current pulse converted the phase to the value $2\pi$. Due to the second junction is inductively connected to the first junction by means of the inductance $I_{JTL}$, there is a “leaking” current equal to $2\pi / I_{JTL}$. If this current is less than the critical value, then it will be “frozen” between the two junctions, otherwise it will transfer the second junction to the state with phase $2\pi$. In this case, the phase difference between the first and second junctions disappears (the current through the inductance is zero), but a similar situation occurs between the second and third junctions. Thus the fluxon propagates in the JTL.

The JTL is magnetically coupled with a superconducting qubit (this is shown schematically in figure 1). So the current impulse in the inductance located next to the qubit creates a magnetic impulse acting on the qubit. This current value $I_L$ as a function of time was found numerically for various parameters of Josephson junctions and the relations between them.

Numerical simulations were performed with the following JTL parameters: $N = 150$, $I_{JTL} = 3$, $i_b = 0.5$, $\beta_C = 1; 10; 100$. The system of equations (1) - (3) was solved using the 4-th order Runge – Kutta – Fellberg method for each value damping $\beta_C$. The current through the inductance in the middle of the JTL was calculated to eliminate the influence of boundary effects. The currents through the inductance normalized to the magnitude of the critical current for different parameters $\beta_C$ (for different values of energy dissipated near qubit) are shown in figure 2.
Figure 2. The currents induced in the JTL, which through a magnetic coupling affect the state of the qubit. Here are the JTL parameters: \( N = 150, l_{JTL} = 3, i_b = 0.5, \beta_c = 1; 10; 100. \)

3. Switching of qubit states by current pulses
The basic dynamical behavior of a superconducting artificial atom driven by magnetic field can be described by the Hamiltonian [10]:

\[
H = \frac{\hbar}{2} \begin{pmatrix}
\omega_q & \varepsilon(t) \\
\varepsilon(t) & -\omega_q
\end{pmatrix}
\]

(4)

where \( \varepsilon(t) \) describes the impact of the current pulses in JTL and \( \omega_q \) is the frequency of the transition between the basis states of a qubit (\( |0\rangle \) and \( |1\rangle \)).

The superconducting qubit is inductively coupled with the JTL, therefore, the shape of the driving pulse arriving at the qubit is determined by the inductance currents \( I_L \) in the JTL (see figure 1), and the amplitude of the external influence (energy) \( A \) depends on the coupling force of the qubit to the JTL.
line (which is defined by inductance $M$), that is:

$$c(t) = A \left( \frac{I_L}{I_C} \right)^2.$$  

(5)

In experimental conditions the interaction of the qubit with reservoir shows considerable effect on the qubit dynamics. These processes are described here by considering the interaction of a qubit with a bosonic reservoir. We calculated the equation for the density operator of the qubit $\rho$ in the Markov approximation [11]:

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [\hat{\rho}, \hat{H}] + \frac{\Gamma_\phi}{2} (\hat{\sigma}_+ \hat{\rho} \hat{\sigma}_- - \hat{\rho}) + \frac{\Gamma_\sigma}{2} (2\hat{\sigma}_+ \hat{\rho} \hat{\sigma}_- - \hat{\sigma}_+ \hat{\rho} \hat{\sigma}_- - \hat{\sigma}_- \hat{\rho} \hat{\sigma}_+ - \hat{\sigma}_- \hat{\rho} \hat{\sigma}_+)$$  

(6)

where the rate $\Gamma_\phi$ characterizes the process of dephasing, and the parameter $\Gamma_\sigma$ is responsible for the rate of energy loss [11]. We assume that at the initial time the qubit is initialized in the ground state.

Numerically solving equation (6) with (4) for each current control pulse characterizing by different damping $\beta_c$ (see figure 2) we have found the amplitude coefficient $A$ of pulses, which implements the process of switching the qubit states from the ground to the excited qubit state. Time dependences of switching probability $W(t)$ are shown in figure 3. Qubit with characteristic frequency $\omega_q = 5$ GHz was chosen to simulate switching states under the influence of fluxon. Note that the moment of switching for the qubit states is mainly determined by the main Lorentz peak of the current pulse (see figure 3). For small values of damping $\beta_c = 1; 10$ of the pulse current fluctuations has a negligible contribution to the behaviour of populations of the qubit levels. However, it can be seen that for large values of the McCumber parameter ($\beta_c = 100$) plasma oscillations of current $I_L$ is a parasitic phenomenon, which leads to significant errors when switching the qubit. Numerical analysis has shown that the best switching of qubit states for control current pulse with relatively small dissipation ($\beta_c = 10$, see figure 2) can be achieved at the amplitude of $A = 0.02$ GHz. However, we note that the switching error is about 20%, which is unacceptable accuracy for single qubit quantum operations. We conducted a further study and found that this type of pulse (figure 3 (c), black curve) can be used however for qubits with smaller frequencies $\omega_q \sim 100$ MHz.

It is well known that the phase relaxation (loss of coherence) is due to the energy transfer from the ground state to the excited one. Dephasing leads to changes in the off-diagonal elements of the density matrix from Eq. (6) only. Since the phase fluctuations are random, then such kind of energy transfer is accompanied by a loss of coherence over time, $T_\phi = \Gamma_\phi^{-1}$, and leads to equilibrium population of levels (see figure 3, red and blue curves). In turn, the energy relaxation affects both the diagonal and off-diagonal elements and it is a stronger criterion for the destruction of the quantum state. Note that we cannot ensure the damping of the population of the upper level without changing coherence, because this would lead to the loss of positive definiteness of the density matrix. Experimental results show the characteristic relaxation times for superconducting qubit of a few dozens of microseconds [12]. However, since the duration of the control pulse is about of several ps it is sufficient to perform thousands of single-qubit operations [8-9]. This fact gives hope for long-term development of quantum computing through the use of the capabilities of superconducting digital logic circuits [1-3, 10].
Figure 3. The time dependencies of the populations for qubit states switching for three different forms of fluxon control pulses, depicted in figure 1. The qubit frequency is $\omega_q = 5$ GHz, fluxon amplitudes are equal (a) $A = 0.15$ GHz, (b) $A = 0.083$ GHz, (c) $A = 0.02$ GHz. Here are the noise parameters in the system $T_s = 100\mu s$: black curve corresponds to $T_p = 100\mu s$, red – $T_p = 10\mu s$, blue – $T_p = 1\mu s$.

4. Conclusion
The propagation of the magnetic flux (fluxon) in superconducting Josephson circuits was studied depending on the McCumber parameter. When the fluxon passes through the Josephson array in the vicinity of the inductively coupled qubit, the local field pulse causes transitions between the levels of the quantum system. The switching processes for qubit states in the field of the fluxon were numerically simulated. The amplitudes of the current pulses that necessary to optimal switching of the
qubit states are found. Thus we have shown the need to use control pulses, specially created in the circuits of energy-efficient upgrades of rapid single-flux-quantum logic [13].

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