Reflection and dips in elastic scattering

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Abstract

We discuss how the reflective scattering would affect elastic scattering in the region of small and moderate values of \(-t\), in particular, we demonstrate that diffractive pattern in the angular distribution will be kept in a modified form.
Diffraction is a fascinating subject in the light of coming experiments at the LHC. The term diffraction was introduced to the hadron and nuclei scattering with use of optical analogy. This adoption was based on the striking similarities observed in hadron and nuclei scattering and light diffraction by absorbing obstacles. The absorption in hadron and nuclei scattering is considered to be a result of opening many inelastic channels at high energies. Angular distribution in this absorptive approach has typical diffraction pattern with prominent forward peak and secondary maxima and minima and can be associated with wave properties of particle scattering.

In the recent paper [1] we considered saturation of the unitarity condition for scattering matrix in hadron collisions at small impact parameters, when scattering acquires reflective nature, i.e. \( S(s, b)|_{b=0} \to -1 \) at \( s \to \infty \). Approach to the full absorption in head-on collisions — the limit \( S(s, b)|_{b=0} \to 0 \) at \( s \to \infty \) — does not follow from unitarity itself and is merely a result of the assumed saturation of the black disk limit. On the other hand, the reflective scattering is a natural interpretation of the unitarity saturation based on the optical concepts in high energy hadron scattering. Such reflective scattering can be traced to the continuous increasing density of the scatterer with energy, i.e. when density goes beyond some critical value relevant for the black disk limit saturation, the scatterer starts to acquire a reflective ability. The concept of reflective scattering itself is quite general, and results from the \( S \)-matrix unitarity saturation related to the necessity to provide the total cross section growth at \( s \to \infty \). This picture predicts that the scattering amplitude at the LHC energies is beyond the black disk limit at small impact parameters. The prediction for the total, elastic and inelastic cross-sections have been discussed in [1].

Reflective scattering should have also consequences for the differential cross-section of elastic scattering, in the region of small and moderate values of \(-t\). In principle, it is not evident that the presence of the reflective scattering would lead to diffractive angular distributions with diffraction peak followed by dips and bumps. It will be shown further on that diffraction pattern will be kept in a slightly modified form in this case.

We start with unitarity condition for the elastic scattering amplitude \( F(s, t) \) which can be written in the form

\[
\text{Im} F(s, t) = H_{el}(s, t) + H_{inel}(s, t),
\]

(1)

where \( H_{el, inel}(s, t) \) are the corresponding elastic and inelastic overlap function introduced by Van Hove [2]. Physical meaning of each term in Eq. (1) is evident from the graphical representation in Fig. 1. The functions \( H_{el, inel}(s, t) \) are related to the functions \( h_{el, inel}(s, b) \) and via the Fourier-Bessel transforms, i.e.

\[
H_{el, inel}(s, t) = \frac{s}{\pi^2} \int_{0}^{\infty} bdb h_{el, inel}(s, b) J_0(b \sqrt{-t}).
\]

(2)
The elastic and inelastic cross–sections can be obtained as follows:

\[
\sigma_{el, inel}(s) \sim \frac{1}{s} H_{el, inel}(s, t = 0).
\]

As it was already noted, the reflective scattering appears naturally in the \(U\)–matrix form of unitarization. In the \(U\)–matrix approach, the \(2 \rightarrow 2\) scattering matrix element in the impact parameter representation is the following linear fractional transform:

\[
S(s, b) = \frac{1 + iU(s, b)}{1 - iU(s, b)}.
\]

\(U(s, b)\) is the generalized reaction matrix, which is considered to be an input dynamical quantity. The relation (4) is one-to-one transform and easily invertible. Inelastic overlap function \(h_{inel}(s, b)\) is connected with \(U(s, b)\) by the relation

\[
h_{inel}(s, b) = \frac{\text{Im} U(s, b)}{|1 - iU(s, b)|^2},
\]

and the only condition to obey unitarity is \(\text{Im} U(s, b) \geq 0\). Elastic overlap function is related to the function \(U(s, b)\) as follows

\[
h_{el}(s, b) = \frac{|U(s, b)|^2}{|1 - iU(s, b)|^2}.
\]

The form of \(U(s, b)\) depends on the particular model assumptions, but for our qualitative purposes it is sufficient that it increases with energy in a power-like way and decreases with impact parameter like a linear exponent or Gaussian. To simplify the qualitative picture, we consider also the function \(U(s, b)\) as a pure imaginary. At sufficiently high energies \((s > s_0)\), the two separate regions of impact parameter distances can be anticipated, namely the outer region of peripheral collisions where the scattering has a typical absorptive origin, i.e. \(S(s, b)|_{b>R(s)} > 0\) and the inner region of central collisions where the scattering has a combined reflective and absorptive origin, \(S(s, b)|_{b<R(s)} < 0\). The transition to the negative values of \(S\) leads to appearance of the real part of the phase shift, i.e. \(\delta_R(s, b)|_{b<R(s)} = \pi/2\) [1]. It should be noted here that the quasi-eikonal form of unitarization allows
smooth transition to the reflective scattering mode also. Unitarity condition can be obeyed then (beyond the inelastic threshold only) for a special class of the eikonal functions [3].

In what follows we discuss the impact parameter profiles of elastic and inelastic overlap functions for the scattering picture with reflection and for the widely accepted absorptive scattering approach in order to make a conclusion on the absence or presence of the dips and bumps in the differential cross-section of elastic scattering, which are associated with the zeroes of \( \text{Im} \hat{F}(s, t) \) (cf. [4] and references therein).

We start with the well known absorptive picture of high energy scattering corresponding to the black disk limit at \( s \rightarrow \infty \). This limit has already been reached in the head-on proton–antiproton collisions at Tevatron, i.e. \( h_{el}(s, b = 0) \approx 0.25 \) [5]. Thus, one can expect that in the framework of absorptive picture, black disk limit will be reached also at \( b \neq 0 \) at higher energies and the impact parameter profiles of \( h_{el}(s, b) \) and \( h_{inel}(s, b) \) will be similar and have a form close to step function (Fig. 2). The elastic and inelastic overlap functions \( \hat{H}_{el}(s, t) \) and

\[
\hat{H}_{inel}(s, t)
\]

will have similar dependence on \(-t\) which can be approximately described as

\[
\hat{H}_{el, inel}(s, t) \sim \frac{R J_1(R \sqrt{-t})}{\sqrt{-t}}. \tag{7}
\]

Zeroes and maxima of the functions \( \hat{H}_{el}(s, t) \) and \( \hat{H}_{inel}(s, t) \) are located at the same values of \(-t\), they will not compensate each other. As a result differential cross-section of elastic scattering will have well known form with dips and bumps

\[
\frac{d\sigma}{dt} \sim \frac{R^2 J_1^2(R \sqrt{-t})}{-t} \tag{8}
\]
when the absorptive scattering picture is realized. In this case
\[ \sigma_{el,inel}(s) \sim R^2(s). \]  
\hspace{1cm} (9)

Of course, presence of real part of the scattering amplitude and/or contributions from helicity flip amplitudes [6] can modify the diffraction picture and bring the uncertainty into the above conclusion, but we can suppose that those effects are not significant at very high energies.

Let us consider now energy evolution of the elastic and inelastic overlap functions in the scattering picture which includes reflective scattering. With conventional parameterizations of the $U$–matrix the inelastic overlap function increases with energies at modest values of $s$. It reaches its maximum value $h_{inel}(s, b = 0) = 1/4$ at some energy $s = s_0$ and beyond this energy the reflective scattering mode appears at small values of $b$. The region of energies and impact parameters corresponding to the reflective scattering mode is determined by the conditions $h_{el}(s, b) > 1/4$ and $h_{inel}(s, b) < 1/4$. The unitarity limit and black disk limit are the same for the inelastic overlap function, but these limits are different for the elastic overlap function. The quantitative analysis of the experimental data [7] gives the threshold value: $\sqrt{s_0} \simeq 2$ TeV. The function $h_{inel}(s, b)$ becomes peripheral when energy increases in the region $s > s_0$. At such energies the inelastic overlap function reaches its maximum at $b = R(s)$ where $R(s) \sim \ln s$. So, in the energy region, which lies beyond the transition threshold, there are two regions in impact parameter space: the central region of reflective scattering combined with absorptive scattering at $b < R(s)$ and the peripheral region of pure absorptive scattering at $b > R(s)$. Typical pattern of the impact parameter profiles for elastic and inelastic overlap functions in the scattering picture with reflection at the LHC energies is depicted on Fig. 3. Elastic and inelastic overlap functions

![Figure 3: Typical qualitative picture of impact parameter profiles of the elastic and inelastic overlap function in the reflective approach at the asymptotically high energies.](image)

$H_{el}(s, t)$ and $H_{inel}(s, t)$ will also have different dependencies on $-t$. They can be
approximately described as following

\[ H_{el}(s, t) \sim \frac{RJ_1(R\sqrt{-t})}{\sqrt{-t}}, \]  

(10)

but

\[ H_{inel}(s, t) \sim RJ_0(R\sqrt{-t}) \]  

(11)

and

\[ \sigma_{el}(s) \sim R^2(s), \quad \sigma_{inel}(s) \sim R(s). \]  

(12)

In another words, \( H_{el}(s, t) \) dominates over \( H_{inel}(s, t) \) at \(-t = 0\), but it is not the case for the scattering in the non-forward directions. In this region these two functions have similar energy dependencies proportional to \( R^{1/2}(s) \) at rather large fixed values of \(-t\). The mean impact parameter values for elastic and inelastic interactions have also similar energy dependencies

\[ \langle b^2 \rangle_{el}(s) \sim R^2(s), \quad \langle b^2 \rangle_{inel}(s) \sim R^2(s), \]  

(13)

but the value of impact parameter averaged over all interactions

\[ \langle b^2 \rangle_{tot}(s) = \frac{\sigma_{el}(s)}{\sigma_{tot}(s)} \langle b^2 \rangle_{el}(s) + \frac{\sigma_{inel}(s)}{\sigma_{tot}(s)} \langle b^2 \rangle_{inel}(s) \]

acquires the main contribution from elastic scattering according to Eq. 12. Therefore, the inelastic intermediate states will give subleading contribution to the slope of diffraction cone \( B(s) \),

\[ B(s) = \frac{d}{dt} \ln \left( \frac{d\sigma}{dt} \right) |_{t=0}, \]

at asymptotical energies. Indeed, since \( B(s) \sim \langle b^2 \rangle_{tot}(s) \), it can be written in the form

\[ B(s) = B_{el}(s) + B_{inel}(s), \]

where \( B_{el}(s) \sim R^2(s) \), while \( B_{inel}(s) \sim R(s) \). It should be noted that both terms \( B_{el}(s) \) and \( B_{inel}(s) \) are proportional to \( R^2(s) \) in case of the absorptive scattering.

Thus, in the reflective scattering behavior of the function \( H_{inel}(s, t) \) is determined by a peripheral impact parameter profile and its \(-t\) dependence is different. Meanwhile, the elastic overlap function \( H_{el}(s, t) \) has similarities with that function in the case of absorptive approach dependence. As a result, zeroes and maxima of the functions \( H_{el}(s, t) \) and \( H_{inel}(s, t) \) will be located at different values of \(-t\) and zeroes and maxima of \( \text{Im}F(s, t) \) will also be located at different position in the cases of absorptive and the reflective scattering. In the case of reflective scattering, dips and maxima will be located in the region of lower values
of $-t$. We would like to note that the presence of reflective scattering enhances the large $-t$ region by factor $\sqrt{-t}$ compared to absorptive scattering. Despite that these two mechanisms lead at the asymptotics to the significant differences in the total, elastic and inelastic cross-section dependencies, their predictions for the differential cross-section of elastic scattering are not so much different at small and moderate values of $-t$.

We do not consider here the specific model predictions for the differential cross-sections; our aim was to demonstrate that diffraction picture with dips and bumps in the differential cross-section will be kept in the case when the reflective scattering dominates.

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