Obtaining of dependence of sustained angular rotor speed of centrifugal pump with hydrostatic bearings

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Abstract. Manned and cargo vehicles in space industry often use low-capacity centrifugal pumps with direct-current motors with hydrostatic bearings. Such pumps have plenty of benefits, but their main drawback is instability of characteristics. It is caused by wide spread of rotor frequency which influences other parameters. This article describes mathematical model of rotor start-up developed and solved in order to determine sustained rotation frequency. Dependence between angular rotation speed of rotor shaft and all basic construction parameters of pump was determined by Runge-Kutta numerical method with varied pitch.

1. Introduction
Operational scheme of low-capacity centrifugal pump is shown on figure 1.

Figure 1. Structure of low-capacity centrifugal pump with opposite scheme and hydrostatic bearings: 1 – Impeller; 2 – Hydrostatic bearing; 3 – Pair of footstep bearing; 4 – Inlet connection; 5 – Outlet connection; 6 – Motor rotor; 7 – Motor stator; 8 – Motor body; 9 – Body of one of pumps of electric pump unit; 10 – Directions of working liquid are shown by dash lines; 11 – Downstream side; 12 – Throttle; 13 – Pocket; 14 – Hole system in the shaft, providing working liquid return to the impeller
Operating principle of the pump is as following: working liquid is directed to the input of the impeller, from its output it is directed to diverter. Liquid comes to throttles of hydrostatic bearings through channels in pump's body. It is important to note that problems concerning design of centrifugal pump rotors are well described in [1-6].

Research concerning methods of hydrodynamic modelling are of most interest [7-18]. However, research concerning rotor's dynamics is described scantily.

2. Mathematical model

Force diagram is shown on figure 2.

Figure 2. Forces and moments applied to the shaft: forces Mg – rotor weight; Pr – radial force emerging in impeller; N – normal force of support reaction; Rs – static reaction of hydrostatic bearing; moments Mim – impeller moment; Mdf – dry friction moment; Mvf – viscous friction moment; Mr – rotor moment.

They are known from pump shaft equation [1]

\[
\begin{align*}
M \cdot \frac{d^2 y}{dt^2} &= 2 \cdot Rs(t) + 2 \cdot N(t) - 2 \cdot Pr(t) - M \cdot g - 2 \cdot Pr(t) \\
J \cdot \frac{d\omega}{dt} &= Mr(t) - 2 \cdot Mim(t) - 2 \cdot Mst(t) - 2 \cdot Mvf(t) 
\end{align*}
\]

(1)

where

\( M \) – rotor mass;
\( J \) – rotor inertia about its axis;
ω — angular rotational speed of pump shaft;
t — time from the moment of pump start-up;
g — gravity acceleration.

Let's write down members of the system (1). Radial force $Pr(t)$ applied to the impeller [1]:

$$Pr(t) = 0,1 \cdot \rho \cdot D_2^3 \cdot b_2 \cdot \eta \cdot \omega(t)^2$$

(2)

where

$H(t)$ — pump head;
$D_2$ — diameter of output from the impeller;
$b_2$ — width of the impeller at the output;
$\rho$ — working liquid density.

Hydrostatic reaction of the bearing [1]:

$$Rc = l \cdot d \cdot \left[ \frac{p_n \cdot \left( \frac{\pi \cdot d_c^2}{4} \right)^2}{\left( \frac{\mu_c \cdot \pi \cdot d_c^2}{4} \right)^2 + \left( \mu_m \cdot 2 \cdot f_z(y) \right)^2} - \frac{p_n \cdot \left( \frac{\pi \cdot d_c^2}{4} \right)^2}{\left( \frac{\mu_c \cdot \pi \cdot d_c^2}{4} \right)^2 + \left( \mu_m \cdot 2 \cdot f_i(y) \right)^2} \right].$$

(3)

Dry friction moment [1]:

$$M_{df} = \begin{cases} \frac{2 \cdot Pr + M \cdot g - 2 \cdot Rs \cdot \kappa \cdot d}{4} & \text{for } y = 0 \\ 0 & \text{for } y > 0 \end{cases}$$

(4)

Viscous friction moment [1]:

$$M_{vf} = \left( \beta_1 + \beta_2 \cdot (y - \delta)^6 \right) \cdot \omega,$$

(5)

where

$$\beta_1 = 6,28 \cdot 10^{-5} N \cdot m$$

$$\beta_2 = 7,15 \cdot 10^{10} N \cdot m^{5}$$

Impeller moment [1]:

$$M_{im}(t) = \frac{\omega(t) \cdot \pi \cdot \mu \cdot R_2^4}{a} + \rho \cdot Q \cdot R_2^2 \cdot \omega$$

(6)

Rotor moment [1]:

$$M_r(t) = K - K_1 \cdot \omega(t),$$

(7)

where

$K$ and $K_1$ — coefficients of moment-mechanical characteristic of the motor.

Viscous friction moment [1]:

$$P_{vf} = 0,4 \cdot V$$

(8)

Let's write down equation system (1) as follows [1]:
\[
\begin{align*}
\frac{dV}{dt} &= \frac{2 \cdot \rho \cdot \omega^2 \cdot R_2^2 \cdot \eta \cdot l \cdot d - 0,2 \cdot \rho \cdot D_2^3 \cdot b_2 \cdot \eta \cdot \omega^2 - M \cdot g}{M} \\
\frac{dy}{dt} &= V \\
\frac{d\omega}{dt} &= \frac{Mr(t) - 2 \cdot Mim(t) - 2 \cdot Mdf(t) - 2 \cdot Mvf(t)}{J}
\end{align*}
\]  

The system (9) is solved in Mathcad, using Runge – Kutta method of 4th order. The following formula is obtained:

\[
\omega_{\text{max}} = A \cdot (-462,56 \cdot \ln K_1 + 1211.4) \cdot (119.05 \cdot K + 174.95) \cdot (-10.498 \cdot a + 569.09) \times
\]

\[
\times (-13.434 \cdot D1 + 587.15) \cdot (-2.7616 \cdot D2^2 - 1.1665 \cdot D2 + 568.78) \times
\]

\[
\times (-0.0335 \rho^3 + 0.4509 \rho^2 - 15.304 \rho + 589.69),
\]

where

A – scale coefficient describing influence of other construction parameters.

Thus, dependence between sustained angular rotor speed and construction parameters of the pump is obtained. It is important to note that construction parameters are given in non-dimensional form and brought to the following values:

- \( K_1 = 0.0015 \) N·m·sec
- \( K = 0.9 \) N·m
- \( a = 0.001 \) m
- \( D_2 = 0.05 \) m
- \( D_1 = D_2/5 \) m
- \( \rho = 1000 \) kg/m³

3. Conclusion

Obtained model allows to determine sustained angular rotor speed of centrifugal pump with hydrostatic bearings. Such model can be useful in different industry segments, in particular, in space systems.

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