α-nucleus potentials, α-decay half-lives, and shell closures for superheavy nuclei

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Systematic α-nucleus folding potentials are used to analyze α-decay half-lives of superheavy nuclei. Preformation factors of about several per cent are found for all nuclei under study. The systematic behavior of the preformation factors and the volume integrals of the potentials allows to predict α-decay energies and half-lives for unknown nuclei. Shell closures can be determined from measured α-decay energies using the discontinuity of the volume integral at shell closures. For the first time a double shell closure is predicted for \( Z_{magic} = 132, N_{magic} = 194, \) and \( A_{magic} = 326 \) from the systematics of folding potentials. The calculated α-decay half-lives remain far below one nanosecond for superheavy nuclei with double shell closure and masses above \( A > 300 \) independent of the precise knowledge of the magic proton and neutron numbers.

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The α-decay of superheavy nuclei has been studied intensively in the last years \cite{1,2,3,4,5,6,7,8,9,10}. In many papers a simple two-body model was applied \cite{11}, and in most papers a potential was derived which was able to fit the measured α-decay half-lives of the analyzed nuclei. However, most of the studies (with the exception of \cite{2}) did not attempt to use these potentials for the description of other experimental quantities like \( e.g. \) α scattering cross sections or \((n,α)\) fusion reaction cross sections.

Therefore, an alternative approach was followed in \cite{12}. Now the simple two-body model has been combined with systematic α-nucleus folding potentials which are able to describe various properties, and the ratio between the calculated half-life \( T_{1/2,α}^{calc} \) and the experimental half-life \( T_{1/2,α}^{exp} \) has been interpreted as preformation factor \( P \) of the α particle in the decaying nucleus. Besides a systematic behavior of the volume integrals of the folding potentials, preformation factors of the order of a few per cent were found for a large number of nuclei \cite{12,13}.

Only for very few light nuclei some levels have been found where a simple two-body model can exactly reproduce the experimental half-lives or widths, \( e.g. \) for \( ^6\text{Li} = ^2\text{H} \otimes α \) \cite{14} or for \( ^8\text{Be} = α \otimes α \) \cite{12,15}. Already for \( ^{20}\text{Ne} = ^{16}\text{O} \otimes α \) the calculated widths are somewhat larger than the experimentally observed ones \cite{10}. Any simple two-body model with a realistic potential must provide half-lives identical or shorter than the experimental value, because the two-body model assumes a pure α cluster wave function by definition, whereas any realistic wave function is given by the sum over many different configurations. Thus, preformations of a few per cent are a quite natural finding for superheavy nuclei.

The following ingredients were used in this paper. The α-nucleus potential was calculated from a double-folding procedure with an effective interaction \cite{12,17,18}. The nuclear densities were taken from \cite{19} for the α particle and derived from the two-parameter Fermi distributions for \( ^{232}\text{Th} \) and \( ^{238}\text{U} \) in \cite{19} with properly scaled radius parameter \( r \sim A^{1/3}_α \). The total potential is given by the sum of the nuclear potential \( V_N(r) \) and the Coulomb potential \( V_C(r) \):

\[
V(r) = V_N(r) + V_C(r) = \lambda V_F(r) + V_C(r)
\]

The Coulomb potential is taken in the usual form of a homogeneously charged sphere where the Coulomb radius \( R_C \) has been chosen identically with the rms radius of the folding potential \( V_F \), and the folding potential \( V_F \) is scaled by a strength parameter \( \lambda \) which is of the order of \( 0.1 - 1.3 \). This leads to volume integrals of about \( J_R \approx 300 \text{ MeV fm}^3 \) for all nuclei under study and is in agreement with systematic α-nucleus potentials derived from elastic scattering \cite{20,21,22,23,24,25,26}. (Note that as usual the negative sign of \( J_R \) is omitted in this work.) Bound state properties of \( ^{212}\text{Po} = ^{208}\text{Pb} \otimes α \) have been analyzed successfully using the same potential \cite{27}. The centrifugal potential has been omitted for \( L = 0 \) decays. The following study is restricted to even-even nuclei because the additional centrifugal barrier may influence the α-decay half-life for decays with \( L \neq 0 \).

The quotations of the volume integral \( J_R \) and the potential strength parameter \( \lambda \) are practically equivalent for this paper. If one wants to compare this work to folding potentials with a different nucleon-nucleon interaction or even to potentials with a different parametrization (\( e.g. \), Woods-Saxon potentials), the volume integral \( J_R \) has to be used. Therefore the following discussion is restricted to volume integrals. Nevertheless, most figures provide both quantities \( J_R \) and \( \lambda \).

The α-decay width \( Γ_α \) is given by the following formulae \cite{11}:

\[
Γ_α = PF \frac{h^2}{4\mu} \exp \left[ -2 \int_{r_2}^{r_3} k(r) dr \right]
\]

with the preformation factor \( P \), the normalization factor \( F \)

\[
F = \int_{r_1}^{r_2} \frac{dr}{k(r)} = 1
\]

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and the wave number $k(r)$

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}} |E - V(r)|.$$  \hspace{1cm} (4)

$\mu$ is the reduced mass and $E$ is the decay energy of the $\alpha$-decay which was taken from the computer files based on the mass table of Ref. [28] or from Table 1 of [1]. The $r_i$ are the classical turning points. For $0^+ \to 0^+$ s-wave decay the inner turning point is at $r_1 = 0$. $r_2$ varies around 9 fm, and $r_3$ varies strongly depending on the energy. The decay width $\Gamma_\alpha$ is related to the half-life by the well-known relation $\Gamma_\alpha = \hbar / 2T_{1/2,\alpha}$. Following Eq. (2), the preformation factor may also be obtained as

$$P = \frac{T_{1/2,\alpha}^{\text{calc}}}{T_{1/2,\alpha}^{\text{exp}}}$$  \hspace{1cm} (5)

where $\Gamma_\alpha$ or $T_{1/2,\alpha}^{\text{calc}}$ are calculated from Eq. (2) with $P = 1$. For completeness, I define the here predicted half-life for unknown nuclei as $T_{1/2,\alpha}^{\text{pre}} = T_{1/2,\alpha}^{\text{exp}} / P$. The potential strength parameter $\lambda$ was adjusted to the energy of the $\alpha$ particle in the $\alpha$ emitter ($A + 4) = A \otimes \alpha$. The number of nodes of the bound state wave function was taken from the Wildermuth condition

$$Q = 2N + L = \sum_{i=1}^{4} (2n_i + l_i) = \sum_{i=1}^{4} q_i$$  \hspace{1cm} (6)

where $Q$ is the number of oscillator quanta, $N$ is the number of nodes and $L$ the relative angular momentum of the $\alpha$-core wave function, and $q_i = 2n_i + l_i$ are the corresponding quantum numbers of the nucleons in the $\alpha$ cluster. I have taken $q = 5$ for $82 < Z, N \leq 126$ and $q = 6$ for $N > 126$ where $Z$ and $N$ are the proton and neutron number of the daughter nucleus. The above definition of $Q$ slightly deviates from the semi-classical Bohr-Sommerfeld quantum number $G$ as mostly used. One finds $G \approx 22.5$ for all nuclei with $Q = 22$.

Various attempts have been made to determine the preformation factors $P$ experimentally or theoretically [29, 30, 31]. The usage of a simple two-body wave function in connection with the Wildermuth condition Eq. (6) is obviously a quite simple approximation for the description of the complex many-body wave function of a superheavy nucleus which was analyzed e.g. in [32, 33, 34, 35, 36]. Nevertheless, the preformation factor defined as ratio $P = T_{1/2,\alpha}^{\text{calc}} / T_{1/2,\alpha}^{\text{exp}}$ in Eq. (5) may be understood as effective preformation factor. The obtained values for $P$ do only show small variations and can thus be used for the prediction of half-lives of unknown superheavy nuclei in a consistent way. A full discussion of preformation factors is beyond the scope of the present paper.

The resulting preformation factors $P$ for even-even nuclei are shown in Fig. 1. An average value of $P \approx 8\%$ is found. Almost all results lie within a bar of uncertainty of a factor of three. This uncertainty is identical to the results of [1, 2]. However, the values for $P$ are much smaller in this work (see discussion above). A table of the results will be given in a forthcoming paper.

There are two different ways in this simple two-body model to obtain larger $\alpha$-decay half-lives $T_{1/2,\alpha}^{\text{calc}}$ and thus larger preformation factors $P$ as derived from Eq. (5). First, very narrow potentials can be used. In this case the attractive nuclear potential becomes negligible in the region of the Coulomb barrier thus effectively increasing the barrier and increasing the $\alpha$-decay half-life. This idea was followed e.g. in [10], and the differences to the systematic folding potential in the present work are illustrated in Fig. 1 of [12]. A very narrow potential as used in [10] is probably not able to describe quantities beside the $\alpha$-decay half-life. Second, a smaller quantum number $(G$ or $Q)$ can be used. This idea was followed in [1]. In this case the attractive nuclear potential is reduced at all radii, thus again effectively increasing the Coulomb barrier and the $\alpha$-decay half-life. Many quantities are mainly sensitive to the tail of the wave functions at large radii outside the nuclear potential which leads to discrete ambiguities for the volume integral $J_R$ of $\alpha$-nucleus potentials (the so-called “family problem”). However, it has been found in the last years that systematic $\alpha$-nucleus folding potentials have volume integrals $J_R$ around 300 MeV fm$^3$ [20, 21, 22, 23, 24, 25, 26] compatible with the quantum number $Q$ used in the present work and incompatible with the smaller $G$ used in [1].

In principle, the application of a semi-classical model is not necessary for the calculation of $\alpha$-decay half-lives or widths. From the potential in Eq. (4) one can directly calculate the wave function and the width of the decaying state. However, in practice this is difficult because of the low energies and extremely small widths of the states. For $^{212}$Po = $^{208}$Pb $\otimes \alpha$ such a fully quantum-mechanical calculation is possible at the limits of numerical stability. Fig. 2 shows the scattering phase shift $\delta_L$ for the $L = 0$ partial wave as a function of energy which is given by $E = E_0 + i \times \Delta E$ with $E_0 = 8.954088523002\text{MeV}$ and $\Delta E = 2 \times 10^{-15}\text{MeV}$. The points are the results of a
translates to quantum-mechanical (semi-classical) calculation and of α-nation of α clei. As an example, one finds for the decay of $^{298}\text{Be}$ with $\alpha = 10.5$ eV for the semi-classical approximation. As pointed out above, the preformation factors are close to unity for $^{8}\text{Be}$ with $P = 100\%$ (65\%) for the quantum-mechanical (semi-classical) calculation and of the order of a few per cent for $^{215}\text{Po}$ with $P = 4.3\%$ (2.9\%). These results confirm the applicability of the semi-classical model within uncertainties of about 30\%. It is interesting to use the systematic folding potentials for the prediction of properties of unknown superheavy nuclei like $\alpha$-decay energies, $\alpha$-decay half-lives, and shell closures above $N, Z = 126$. The basic building block is the smooth behavior of the strength parameter $\lambda$ of the folding potential and the resulting volume integrals $J_R$ (see Fig. 3 and Table I of [12]).

Within one major shell, one finds variations of $J_R$ from about $J_R \approx 335$ MeV fm$^3$ at the lower end of a shell to about $J_R \approx 280$ MeV fm$^3$ at the upper end of a shell. Between neighboring nuclei the variation in $J_R$ is below $\Delta J_R < 5$ MeV fm$^3$. This allows first the determination of $\alpha$-decay energies for up to now unknown nuclei. As an example, one finds for the decay of $^{298}\text{Be} \rightarrow ^{294}\text{Pb}$ a volume integral of $J_R \approx 296$ MeV fm$^3$ corresponding $\lambda = 1.138$. This leads to a decay energy of $E = 12.87$ MeV. The $\alpha$-decay half-life can be estimated using the given energy and an average preformation factor of $P \approx 8\%$ leading to $T^{\text{pre}}_{1/2,\alpha} \approx 8\mu s$. Whereas the uncertainties for the volume integral $J_R$ and the derived $\alpha$-decay energy are small, the uncertainty of the $\alpha$-decay half-life is strong because of the exponential energy dependence. For a potential strength enhanced (reduced) by 2\% one finds the $\alpha$-decay energy $E = 10.70$ MeV ($E = 14.98$ MeV) and $T^{\text{pre}}_{1/2,\alpha} = 0.97$ s ($T^{\text{pre}}_{1/2,\alpha} = 1.6$ ns) again using $P = 8\%$. A variation of the potential strength of $1\%$ corresponds to a variation of the $\alpha$-decay energy of about 1 MeV which is comparable to the uncertainties of mass formulae [27]. As usual, the reliability of such an extrapolation decreases for nuclei with masses far above the heaviest known nuclei. However, the uncertainties for closed-shell nuclei remain small because of the well-defined volume integral $J_R$ for such nuclei which can be studied at the shell closures at $N = 82, Z = 82,$ and $N = 126$.

Shell closures can be seen as discontinuities in the volume integrals, see Figs. 3 and 4. Whereas the variation between neighboring nuclei remains below $\Delta J_R < 5$ MeV fm$^3$, at shell closures one finds a strong increase of $J_R$ which is directly related to the increase of the quantum number $Q$. Because shell closures are not known a priori for superheavy nuclei, Fig. 4 analyzes the volume integrals around the shell closure at $N = 82$ for Xe, Ba, Ce and Nd isotopes. Below $N = 82$, the wave functions are characterized by $Q = 16$ (full black

\[ \delta_L(E) = \arctan \frac{\Gamma}{2(E_R - E)} \]
The $\alpha$-decay half-life of the doubly-magic nucleus with $Z_{\text{magic}} = 132$, $N_{\text{magic}} = 194$, and $A_{\text{magic}} = 326$ can be calculated using the volume integral $J_R = 279.2 \text{MeV fm}^3$ (taken from $^{208}\text{Pb} = ^{204}\text{Hg} \otimes \alpha$). One finds the energy $E = 18.26 \text{MeV}$ and the corresponding half-life $T_{1/2,\alpha}^{\text{calc}} = 1.16 \times 10^{-12} \text{s}$ with $P = 1$. Again using $P = 8\%$, a realistic prediction of the half-life is $T_{1/2,\alpha}^{\text{pre}} = 1.5 \times 10^{-11} \text{s}$. Including the uncertainty of $P$, the half-life remains below $10^{-10} \text{s}$. The uncertainty of the volume integral $J_R$ at closed shells is smaller than 1%. Increasing $J_R$ by 1% reduces the $\alpha$-decay energy by about 1 MeV and increases the $\alpha$-decay half-life by about a factor of 20. In any case, the half-life remains below 1 ns.

The lower limit of $J_R$ has also been applied for the calculation of the half-life of $^{310}\text{Nd} \rightarrow ^{306}\text{Pb}$ with the widely discussed shell closures at $Z = 126$ and $N = 184$ (e.g., [38, 39]); but also other magic numbers have been discussed (e.g., [40]). Here one obtains the $\alpha$-decay energy $E = 18.82 \text{MeV}$ and the corresponding $\alpha$-decay half-life $T_{1/2,\alpha}^{\text{calc}} = 2.1 \times 10^{-14} \text{s}$ ($P = 1$). The realistic prediction using $P = 8\%$ is $T_{1/2,\alpha}^{\text{pre}} = 2.6 \times 10^{-13} \text{s}$, and including all uncertainties the half-life remains far below $10^{-11} \text{s}$. The above calculations indicate clearly that one cannot expect that any superheavy nucleus above $A > 300$ with magic proton and neutron numbers (whatever these numbers are) has a half-life significantly above one nanosecond.

Because of the significant variation of the decay energy $E$ from about 4 MeV to about 12 MeV for known superheavy nuclei and up to about 20 MeV for the predicted yet unknown doubly-magic superheavy nuclei one may expect a correlation between the potential strength parameter $\lambda$ or the volume integral $J_R$ and the decay energy $E$. This relation is analyzed in Fig. 5 for superheavy nuclei and Fig. 6 for nuclei around $N = 82$.

Fig. 5 seems to indicate that larger decay energies $E$ are correlated to smaller volume integrals $J_R$. However, the underlying reason for this energy dependence of $J_R$ is the smooth variation of $J_R$ within a major shell (see above). At very small energies one finds again a small volume integral of $J_R \approx 280 \text{MeV fm}^3$ for $^{208}\text{Pb} = ^{204}\text{Hg} \otimes \alpha$. As can also be seen from Fig. 6 the volume integrals do not depend on the energy $E$: above $N = 82$ one finds $J_R \approx 310 \text{MeV fm}^3$ for bound ($E < 0$) and unbound ($E > 0$) nuclei, and below $N = 82$ one finds $J_R \approx 280 \text{MeV fm}^3$, again for bound and unbound nuclei.

In conclusion, systematic folding potentials can be used for the calculation of $\alpha$-decay half-lives of superheavy nuclei. Additionally, the systematic behavior of the volume integrals allows to predict $\alpha$-decay energies and half-lives of yet unknown nuclei. The magic numbers $Z_{\text{magic}} = 132$, $N_{\text{magic}} = 194$, and $A_{\text{magic}} = 326$ have been derived from the discontinuities of the volume integrals at shell closures. There is strong evidence that $\alpha$-decay half-lives remain far below one nanosecond even for doubly-magic superheavy nuclei above $A > 300$. 

**FIG. 4:** Potential strength parameter $\lambda$ (upper part) and volume integrals $J_R$ (lower part) around the shell closure $N = 82$ for $^{130-136}\text{Xe} \otimes \alpha$ (diamonds), $^{132-134}\text{Ba} \otimes \alpha$ (circles), $^{134-136}\text{Ce} \otimes \alpha$ (triangles), and $^{140-144}\text{Nd} \otimes \alpha$ (squares) isotopes. One can clearly see the discontinuity at the shell closure $N = 82$ (see text).
FIG. 5: Potential strength parameter $\lambda$ (upper part) and volume integrals $J_R$ (lower) versus decay energy $E$ for superheavy nuclei. Known nuclei are shown as full circles. The extrapolated doubly-magic nucleus with $Z_{\text{magic}} = 132$, $N_{\text{magic}} = 194$, and $A_{\text{magic}} = 326$ is shown as open circle. The open squares are nuclei with $Q = 18$ and $Q = 20$ (see also Fig. 4).

FIG. 6: Potential strength parameter $\lambda$ (upper part) and volume integrals $J_R$ (lower) versus decay energy $E$ around the shell closure $N = 82$ for $^{130-136}\text{Xe} \otimes \alpha$ (diamonds), $^{132-142}\text{Ba} \otimes \alpha$ (circles), $^{134-144}\text{Ce} \otimes \alpha$ (triangles), and $^{140-146}\text{Nd} \otimes \alpha$ (squares) isotopes (see text; description of symbols see Fig. 4).

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