Scattering in Soliton Models and Boson Exchange Descriptions

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ABSTRACT

We argue that the description of meson-nucleon dynamics based on the boson-exchange approach, is compatible with the description of the nucleon as a soliton in the nonrelativistic limit. Our arguments are based on an analysis of the meson-soliton form factor and the exact meson-soliton and soliton-soliton scattering amplitudes in the Sine-Gordon model.

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1. Introduction

The idea that the nucleon is a chiral soliton has attracted considerable attention in the past few years [1]. This idea is supported by large $N_c$ counting arguments in QCD [2]. While the exact effective meson dynamics of which the nucleon is a soliton is not known, the generic character and properties of the low-lying baryons can be understood from simple chiral models of the Skyrme type.

The underlying degrees of freedom in the Skyrme model are pions subject to the tenents of chiral symmetry. The model is consistent with current algebra, and produces solitons that resemble ordinary nucleons. The model provides a natural setting for analysing systematically dynamical issues related to meson-nucleon and nucleon-nucleon scattering. This is a distinct advantage over relativistic bag models and nonrelativistic constituent quark models.

The meson-nucleon scattering amplitude can be systematically expanded in powers of $1/\sqrt{N_c}$ around the classical one soliton solution. The order $1/\sqrt{N_c}$ has been investigated by many authors and yields phase shifts that are overall consistent with the data in the higher partial waves [3]. The expected shortcomings of the P-wave amplitudes have been overcome at next-to-leading order [4]. The deficiencies in the S-wave amplitudes are unavoidable [5].

The nucleon-nucleon scattering amplitude has been more of a challenge since the classical two soliton solution is not known. To leading order in $1/\sqrt{N_c}$ a product ansatz has been used to construct the classical two skyrmion potential. The scattering amplitude to leading order follows automatically. The results of this approach compare favorably with the empirically motivated potentials at short and long distances, but fail to reproduce the intermediate range attraction in the scalar channel [6]. The numerical attempts to the scattering problem involve asymptotic skyrmions and do not seem to relate immediately to the nucleon-nucleon problem [7].

Recently two of us [8] have argued that the nucleon-nucleon problem in the context of Skyrme models can be systematically analysed using a double expansion in both the range $e^{-m_\pi r}$ and the coupling constant $1/\sqrt{N_c}$. As a result the ambiguities related to the ansatz dependence are lifted through the pion fluctuations, and attraction is seen in the central potential at the two pion range. Undoubtedly, this is a step forward in the process of describing
unambiguously the two-nucleon problem in the context of soliton models.

The purpose of the preceding discussion was to emphasize the consistent framework offered by realistic soliton models in the description of a variety of hadronic phenomena. It also provides a systematic framework for calculations based on the semiclassical expansion.

To what extent this description is compatible with the conventional boson-exchange approach remains still unclear. In fact, simple arguments based on naive power counting seems to run into difficulties [8]. It is the purpose of this paper to try to clarify some of these issues in the context of completely integrable models. For definiteness we will use the Sine-Gordon model. In this model many exact non perturbative results are known [9,10], in particular an exact S-matrix [11] and all the form factors [12] are available.

The paper is organised as follows: in section 2, we will analyse the meson-nucleon form factor to one-loop order using the semiclassical expansion. The result is shown to agree with the exact result [12] in the weak coupling (non-relativistic) limit in appendix A. In section 3, we discuss the nonrelativistic reduction of the exact meson-soliton scattering amplitude. In section 4, we discuss the exact soliton-soliton scattering amplitude derived by Zamolodchikov and Zamolodchikov. We work out explicitly the pole of the scattering amplitude in both the relativistic and nonrelativistic limit. In section 5, we show how one can model the nonrelativistic scattering amplitudes using simple boson-exchange models. Our conclusions will be summarized in section 6.

2. Meson-Soliton Form Factor

Consider the Sine-Gordon Lagrangian

\[
L = \int dx \left( \frac{1}{2} (\partial_{\mu} \phi)^2 + \frac{m_0^2}{g^2} (\cos(g\phi) - 1) \right)
\]  

(1)

with the classical one soliton solution

\[
\phi_s(x) = \frac{4}{g} \tan^{-1} \left( e^{-m_0 x} \right)
\]  

(2)
where $g$ is a small parameter ($g^2 \ll 8\pi$). The bare soliton mass is $M_s = 8m_0/g^2$, and to leading order, the semiclassical description is justified. Besides the soliton, there are single meson states and breather modes with masses in the range $g^0 \to g^{-2}$

$$m_n = 2M_s \sin\left(\frac{n}{16} g^2\right) \quad n = 1, 2, ... < \frac{8\pi}{g^2}$$  \hspace{1cm} (3)

As a first step towards understanding meson-soliton dynamics we will discuss the structure of the meson-soliton form factor

$$F(Q) = \langle p_2|\phi(0)|p_1 \rangle$$  \hspace{1cm} (4)

where $|p>$ refers to a momentum eigenstate of the soliton, with $Q = p_1 - p_2$. To leading order, the form factor is of order $g^{-1}$. To evaluate (4) we will use the collective coordinate method [13], although other methods are possible. For that, consider

$$\phi(x, t) = \phi_s(x - X(t)) + \xi(x - X(t), t)$$  \hspace{1cm} (5)

where $X(t)$ will be treated as a collective coordinate conjugate to the total momentum of the soliton. In the semiclassical limit, the meson fluctuations in (5) can be ignored and the result for the form factor to leading order reads (Breit-frame : $Q = (0, q)$)

$$F(q) = \int_{-\infty}^{+\infty} e^{iqx} \phi_s(-x) \, dx = -\frac{2i\pi}{g} \frac{1}{q} \sech(\frac{q\pi}{2m_0})$$  \hspace{1cm} (6)

In the time-like region (6) displays a string of odd poles located at $q_n = im_0(2n + 1)$ with $n = 0, 1, ...$. The pole at $q = 0$ is of kinematical origin. It accounts for the topological charge. Indeed, if we recall that the topological (baryon) current is given by $J^\mu = e^{\mu\nu} \partial_\nu \phi$, then $iqF(q)$ accounts for the proper charge at $q = 0$ (here unity). This point is suggestive of a derivative meson-soliton coupling as given by (8) below. The odd poles correspond to the one-meson and breather modes with the expected masses (6). The occurrence
of only odd poles in (3) suggest that the charge conjugation of the original meson field is odd.

The meson-soliton Yukawa coupling $f_0$ is defined by

$$f_0(q^2) = (q^2 + m_0^2) i q F(q)$$

(7)

Its value at the one meson pole is $f_0(-m_0^2) = -8/g$. This suggests that the meson-soliton coupling is strong and of pseudovector nature. For point like solitons, we have

$$\mathcal{L} = -\frac{8}{g} \bar{\psi} \gamma^\mu \gamma_5 \psi \partial_\mu \phi$$

(8)

where $\psi$ is a fermion field of mass $M_s$. Note that in this form, the pseudovector coupling is strong.

To estimate the role of the quantum effects on the structure of the meson-soliton form factor we will evaluate the one-loop correction to (6). For that, we will use the collective quantization method discussed by Tomboulis [13] (and references therein). For the Sine-Gordon model, the renormalised Hamiltonian to order $g^2$ reads

$$H = M_s + H_0 + H_1 + \mathcal{O}(g^2)$$

(9)

where

$$H_0 = \int dx \left( \frac{\pi^2}{2} + \frac{\xi^2}{2} + \frac{m_0^2 \xi^2}{2} (2 \tanh^2(m_0 x) - 1) \right)$$

$$H_1 = \frac{1}{3} g m_0^2 \int dx \xi \left( \frac{\xi^2}{4\pi} \int \frac{dk}{\sqrt{k^2 + 1}} \frac{\sinh(m_0 x)}{\cosh^2(m_0 x)} \right)$$

(10)

In $H_1$ the term linear in the meson coupling is the mass counterterm following normal ordering in the meson sector. To this order, the soliton does not recoil. The meson field is quantized in a box of length $L$,

$$\xi(x, t) = \sum_n \frac{1}{2\omega_n} \left( b_n \psi_n(x) e^{-i\omega_n t} + b_n^+ \psi_n^+(x) e^{i\omega_n t} \right)$$

(11)
with \((\omega_n^2 = m_0^2 + (m_0q_n)^2)\)

\[
\psi_n(x) = \frac{(\tanh(m_0x) - iq_n)e^{im_0q_nx}}{[(1 + q_n)^2L - 2/m_0\tanh(m_0L/2)]^{1/2}}
\]  

(12)

The one-loop correction to the semiclassical expression (8) for the form factor can be obtained using time-independent Rayleigh-Schrodinger perturbation theory. Generically

\[
<p_2|\phi(x)|p_1> = \sum_m \left( \frac{<p_2|H_1|m><m|\phi(x)|p_1>} {E_0 - E_m} + \frac{<p_2|\phi(x)|m><m|H_1|p_1>} {E_0 - E_m} \right)
\]  

(13)

Using the decomposition (5) for the full quantum field together with the quantized meson fluctuations (11-12), we obtain

\[
<p_2|\phi(x)|p_1>_g = \lim_{L \to \infty} \int dz dy e^{in\phi_s(y)} A_L(x, y, z) B_L(y)
\]  

(14)

where the integrands are given by

\[
A_L = \sum_n \frac{1}{2\omega_n^2} \left( \psi_n(x - z)\psi_n^+(y) + \psi_n(y)\psi_n^+(x - z) \right)
\]  

(15)

\[
B_L = \frac{m_0^2g}{4} \left( \sum_n \frac{1}{\omega_n} \psi_n(y)\psi_n^+(y) - \frac{1}{2\pi} \int \frac{dk}{\sqrt{k^2 + 1}} \right)
\]  

(16)

and \(\phi_s\) is the soliton profile (2). In the continuum limit \((L \to \infty)\) the discrete sums can be turned into continuous integrals with the proper weight for the meson density of states around a non-moving soliton (to the order quoted the soliton does not recoil), i.e.

\[
\rho(k) = \frac{m_0L}{2\pi} - \frac{1}{2\pi}\delta'(k)
\]  

(17)

where \(\delta(q_n) = -2\text{arctan}(q_n)\), denotes the phase shift of the box eigenfunctions (12). As a result
\[ B_{\infty} = \frac{m_0^2 g}{8\pi} \int \frac{dk}{(k^2 + 1)^{3/2}} (\tanh^2(m_0 y) - 1) \]  

(18)

Note that the result is finite (as it should) following the cancellation between the divergence in the mode sum and the mass counterterm in (10). This provides an additional check that mass renormalization in the trivial vacuum sector is sufficient to renormalise the model in the non trivial topological sector as well. Similarly,

\[ A_{\infty} = \frac{1}{4\pi m_0} \int dk \frac{e^{im_0 k(x - y - z)}}{(k^2 + 1)^2} (k + itanh(m_0(x - z)))(k - itanh(m_0y)) + \text{c.c.} \]  

(19)

Inserting (18-19) in (14) and performing some algebra, we obtain

\[ <p_2 | \phi(x) | p_1 > = \frac{m_0 g}{4\pi^2} \int_{-\infty}^{\infty} dz e^{iqz} \int_{-\infty}^{\infty} \frac{dk}{(k^2 + 1)^2} \int_{-\infty}^{\infty} dy \sinh(m_0 y) \frac{\cosh^4(m_0 y)}{cosh^4(m_0 y)} G_1(x, z, k, y) \]  

(20)

where

\[ G_1(x, z, k, y) = \left[ \text{th}(m_0(x - z)) \text{th}(m_0y) + k^2 \right] \cos(m_0 k(x - z - y)) - k \left[ \text{th}(m_0(x - z)) - \text{th}(m_0 y) \right] \sin(m_0 k(x - z - y)) \]  

(21)

The integrals are performed in the order indicated, so that no spurious divergence is generated. Using the result

\[ \int_0^\infty dt \frac{\cos(at)}{\text{ch}^\nu(bt)} = \frac{2^{\nu-2}}{b\Gamma(\nu)} \Gamma \left( \frac{\nu}{2} + \frac{ai}{2b} \right) \Gamma \left( \frac{\nu}{2} - \frac{ai}{2b} \right) \]  

(22)

and integrating by parts, gives

\[ <p_2 | \phi(x) | p_1 > = \left( \frac{-ig}{8q} \right) e^{iqx} \text{sech} \left( \frac{q\pi}{2m_0} \right) \left( \frac{q^2}{2m_0^2} \right) \]  

(23)

Combining (6) with (23) yields the form factor to order \( g^2 \)
\[ < p_2 | \phi(0) | p_1 > = - \frac{i2\pi}{q} \frac{\text{sech}(\frac{q\pi}{2m_0})}{2m_0} \left( \frac{1}{g} + \frac{gq^2}{16\pi m_0^2} + \mathcal{O}(g^2) \right) \]  

(24)

While the pole position has not changed, the residue has been modified. The quantum effects are expected to renormalize the bare position of the pole (here meson mass) and affect the strength of the residues. In fact we expect this result to hold true to higher order in perturbation theory around the soliton background and even in higher dimensions.

3. Meson-Soliton Scattering

The exact meson-soliton scattering amplitude is given by [9],

\[ S_1(\theta_\pm) = \frac{\sinh\theta_\pm + i\cos(\gamma/16)}{\sinh\theta_\pm - i\cos(\gamma/16)} \]  

(25)

In the meson-soliton center of mass frame \( p + k = 0 \), the scattering amplitude (25) can be rewritten in momentum space

\[ S_1(k) = \frac{k(\sqrt{m^2 + k^2} - \sqrt{M^2 + k^2}) - imM\cos(\gamma/16)}{k(\sqrt{m^2 + k^2} - \sqrt{M^2 + k^2}) + imM\cos(\gamma/16)} \]  

(26)

In the semiclassical (nonrelativistic) limit \( \gamma \sim g^2, M \sim M_s, m \sim m_0 \) and (26) reduces to

\[ S_1 = 1 + \frac{2im_0k - 2m_0^2}{k^2 + m_0^2} + \mathcal{O}(g^2) \]  

(27)

On the other hand, the scattering process of a meson off a soliton in the semiclassical description corresponds to background field scattering off a static soliton. If \( \psi_k(x) \) designates the continuum scattering wavefunction of the meson, then from (12) we have

\[ \psi_k(x) = \frac{1}{\sqrt{2\pi \omega_k}} k e^{ikx} \left( 1 + \frac{im_0}{k}\tanh(m_0x) \right) \]  

(28)
where \( \omega_k = \sqrt{m_0^2 + k^2}, \ -\infty < k < \infty \). The S-matrix to order \( g^2 \) follows from (28) through the identification

\[
S_p = \frac{\psi_k(+\infty)}{\psi_k(-\infty)}
\]  

(29)

A little algebra shows that this agrees with (27). Here, we point out that with the Yukawa coupling defined as in (7), it was shown in [14-15] that the Born diagrams for meson-soliton scattering give the same result as potential scattering (29). Our result, while in agreement with [14-15], shows that (29) follows directly from the exact scattering amplitude in the semiclassical limit, as it should.

4. Soliton-Soliton Scattering

The exact relativistic S-matrix for the Sine-Gordon model has been derived by Zamolodchikov and Zamolodchikov [11]. The form of the S-matrix follows from factorisation (absence of pair creation), unitarity and crossing. The soliton-soliton S-matrix element reads

\[
P_{12} + p_1'p_2'|p_1p_2> = -p_1'p_2'|p_1p_2>_-(\theta_-)
\]  

(30)

where \( S \) depends only on the relative rapidities \( \theta = \theta_- \)

\[
S(\theta) = \exp \left( i\pi - i \int_0^{\infty} dk \frac{\sin(k\theta)}{k} \frac{\sinh((\pi - \zeta)k/2)}{\cosh(\pi k/2)\sinh(\zeta k/2)} \right)
\]  

(31)

with \( \zeta = \gamma/8 = g^2/(8 - g^2/\pi) \) and \( \theta_\pm = \theta_1 \pm \theta_2 \). The rapidity variables \( \theta_{1,2} \) are related to the momentum variables \( p_{1,2} \) as follows

\[
p_{1,2} = M(\cosh\theta_{1,2}, \sinh\theta_{1,2})
\]  

(32)

Here \( M = 8m/\gamma \) is the renormalized soliton mass and \( m \) the renormalized meson mass.

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1We are using plane-wave normalisations
Alternative representations of (30-31) can be found in Ref. [10]. In the s-channel, the $S$-matrix exhibits a threshold (branch point) at $s = 4M^2$ and bound states (poles) for $0 < s < 4M^2$. As a result, the soliton-soliton amplitude is a meromorphic function of $\theta$ in the strip $0 < \text{Im}\theta < \pi$ following the mapping $s = 4M^2 \cosh^2(\theta/2)$. The edges of the strip correspond to the cuts in the $s$-plane. Crossing symmetry follows from $\theta \to i\pi - \theta$.

To exhibit the singularity structure of (31), it is best to define $\Omega(\theta)$, a regular function in the strip $0 < \text{Im}\theta < 2\pi$ that satisfies

\begin{align}
S(-\theta) &= \Omega(\theta) S(\theta) = \Omega(\theta - i2\pi) \\
\Omega(\theta)\Omega(\theta - i\pi) &= \eta^{-1}(\theta + i\pi/2) \tag{33}
\end{align}

where $\eta(\theta)$ is an even function with simple poles at $\theta_n = i\pi/2 + in\zeta$ with $n \geq 0$ and zeroes at $\theta_n = i3\pi/2 + in\zeta$ with $n \geq 1$. For more details we refer to Smirnov [12]. The auxiliary function $\eta(\theta)$ satisfies the property

\begin{align}
\eta(\theta - i\zeta) &= \eta(\theta) \frac{\cosh((\theta + i\pi/2)/2)}{\cosh((\theta - i\pi/2 - i\zeta)/2)} \tag{35}
\end{align}

In terms of (33-34) we can rewrite the scattering amplitude (31) in the following form

\begin{align}
S(\theta) &= \eta(\theta + i\pi/2) \eta(\theta - i\pi/2) \tag{36}
\end{align}

Using (35) we obtain

\begin{align}
S(\theta) &= S(\theta - i\zeta) \coth(\theta/2) \coth((\theta - i\zeta)/2) \tag{37}
\end{align}

Iterating (37) we easily find $(n \geq 2)$

\begin{align}
S(\theta) &= S(\theta - in\zeta) \coth(\theta/2) H_n^2(\theta) \coth((\theta - in\zeta)/2) \tag{38}
\end{align}

where we have defined
\[ H_n(\theta) = \prod_{k=1}^{n-1} \coth((\theta - i(n-k)\zeta)/2) \]  

(39)

The expression (38) for the scattering amplitude displays a string of simple poles at \( \theta_n = in\zeta = in\gamma/8 \) with residues given by

\[ R_n = -2\coth(\theta_n/2) H_n^2(\theta_n) \]  

(40)

To extract the semiclassical limit it suffices to notice that this limit is the same as the nonrelativistic limit for which the rapidity becomes the ordinary velocity, \( \theta \to v = p/M_s \). With this in mind the \( S \)-matrix in the semiclassical limit (Breit-frame), near the \( \theta = \theta_n \) pole reads

\[ S = \frac{M_s R_n}{q - q_n} \]  

(41)

with the location of the poles and residues given by

\[ q_n = inm_0 \quad M_s R_n = (-1)^{n+1} \frac{in}{(n!)^2} \frac{(16 g^2)^{2n}}{2^n} m_0 \]  

(42)

The nonrelativistic soliton-soliton \( S \)-matrix in the Sine-Gordon model can also be obtained by solving the factorisation equation and demanding unitarity. The result is [11] (in momentum space)

\[ S(q) = \frac{\sh(i\pi\alpha - 8\pi q/\gamma M)\Gamma(-i8q/\gamma M - \alpha)\Gamma(-i8q/\gamma M + \alpha + 1)}{\sh(\pi q/M)\Gamma(-i8q/\gamma M)\Gamma(1 - i8q/\gamma M)} \]  

(43)

The constant \( \alpha \) is not fixed in (43) because crossing symmetry is not required in the nonrelativistic case. We fix it here by demanding that in the weak coupling limit (i.e. \( \gamma \sim g^2 << 1 \)) the poles and residues of (43) match (41-42). A simple calculation gives

\[ \alpha = \frac{16}{g^2} \]  

(44)
The amplitude (43) implies a potential between the solitons of the form
\[ V(r) = \frac{M\gamma^2}{64} \frac{\alpha^2 - \alpha + 3/4}{\sinh^2((m\gamma/16)r)} \] (45)

In the weak coupling limit \((g^2 << 1)\), we get
\[ V(r) = \frac{32m_0}{g^2} \frac{1}{\sinh^2(m_0 r/2)} = 128m_0 \frac{e^{-m_0 r}}{g^2} \sum_{n=0}^{\infty} e^{-n m_0 r}(1 + n) \] (46)
which is suggestive of a one-boson exchange description of the soliton-soliton scattering in the low-momentum regime. This was also indicated by the pole structure of the S-matrix (41-42). In the next section we will investigate how well the postulated effective interaction (8) supports this picture.

5. Boson-Exchange Models

From the meson-soliton form factor analysis of section 2, we have concluded that in the nonrelativistic limit, meson-soliton dynamics suggests using a pseudovector coupling of the form given by (8).

The soliton-soliton scattering amplitude follows from the direct and exchange diagrams shown in Fig. 1,
\[ < p'_1p'_2|(S-1)|p_1p_2 > = (2\pi)^2 \frac{\delta^2(p'_1 + p'_2 - p_1 - p_2)}{\sqrt{2E_12E_22E'_12E'_2}} T(s, t, u) \] (47)
where \(T\) is the pertinent T-matrix amplitude. The restricted character of the kinematics in 1 + 1 dimensions and the mass-shell conditions yield
\[ \delta^2(p'_1 + p'_2 - p_1 - p_2) = \frac{E_1E_2}{|p_1E_2 - p_2E_1|} (\delta(p'_1 - p_1)\delta(p'_2 - p_2) + \delta(p'_1 - p_2)\delta(p'_2 - p_1))(48) \]

Combining (47) with (48) and going to the center of mass frame \(p_1 = -p_2 = p, E_1 = E_2 = E = \sqrt{M^2 + p^2}\) and \(q = p_1 - p_2 = 2p\) give
\[ <p'_1 p'_2|(S - 1)|p_1 p_2 > = (2\pi)^2 \left( \delta(p'_1 - p_1)\delta(p'_2 - p_2) - \delta(p'_1 - p_2)\delta(p'_2 - p_1) \right) \]
\[ \left( -\frac{64 M^2}{g^2 E} \right) \frac{q}{q^2 + m_0^2} \]  

(49)

As expected there is a pole at \( q = im_0 \) (one-meson exchange) with a residue equal to \( n = 1 \)

\[ M_s R_1 = i\frac{256}{g^4} m_0 \]  

(50)

This is in total agreement with the semiclassical result obtained from the exact S-matrix. It is not hard to show that additional Yukawa-couplings corresponding to the breather modes (bound meson states) in the form of vector or pseudovector couplings (depending on their intrinsic charge conjugation) with the appropriate strength, reproduce the scattering amplitude (43). In fact the character of the residues in (42) is suggestive of the following hierarchy of couplings (generically)

\[ \frac{1}{g} \phi \bar{\psi} \psi, \quad \frac{1}{g^2} \phi \phi \bar{\psi} \psi, \quad \frac{1}{g^3} \phi \phi \phi \bar{\psi} \psi \ldots \]  

(51)

They are to be compared with the ones expected in chiral models where \( g \to f_\pi \) and \( \phi \to \pi \) which are reminiscent of pion-nucleon, rho-nucleon, omega-nucleon .... couplings. We conclude, that in the semiclassical limit the conventional boson-exchange approach is compatible with the soliton-soliton scattering description.

6. Conclusions

We have analysed soliton-soliton and meson-soliton scattering amplitudes in the semiclassical limit. The exact results derived by Zamolodchikov and Zamolodchikov are in agreement with conventional descriptions based on boson-exchange models in the semiclassical (nonrelativistic) limit provided that the couplings are chosen appropriately. We have also shown that the
meson-soliton vertex, while in agreement with the exact results of Smirnov in the semiclassical limit, provides important insights on the meson-soliton dynamics to leading order. We expect the present conclusions to carry through to higher space-time dimensions in more realistic models such as the Skyrme model.
Appendix A : Smirnov’s result in the weak coupling limit

The semiclassical description of the meson-soliton form factor and the one-loop correction discussed in section 2, are consistent with the exact result in [11-12]

\[ F(Q) = -\frac{i2\pi e^{-J(\theta_-)}}{g} \frac{q}{\cosh(\theta_+ / 2)} \left( \frac{\cosh(\theta_- / 2)}{\cosh(\theta_1 \cosh(\theta_2))} \right) \cosh(4\pi\theta_- / \gamma) \]  

(52)

where

\[ J(\theta_-) = \int_0^\infty dx \frac{\sin^2(x\theta_-/2)}{x} \sinh(x(\pi - \zeta)/2) \sinh(\pi x/2) \sinh(3x/2) \]  

(53)

The bracket in (52) involves a relativistic kinematic factor (numerator) and energy normalisation factors (denominator). The combination of the bracket and \(1/q\) is relativistically invariant. In the region \(0 < \text{Im}\theta_- < 2\pi\), the function \(J(\theta_-)\) is regular, so that the exact poles of the form factor follow from the position of the poles in \(\cosh^{-1}(4\pi\theta_- / \gamma)\), i.e.

\[ \theta_n = \frac{i\gamma}{8}(2n + 1) \quad n = 0, 1, ... \]  

(54)

in agreement with the poles derived using the semiclassical approximation.

Using the substitution \(\theta \rightarrow v = p/M_s\) in the semiclassical limit, we have

\[ J(\theta_-) = \frac{g^2}{16\pi} \left( \frac{q^2}{m_0^2} \right) + \mathcal{O}(g^4) \]  

(55)

\[ \cosh(\pi\theta_- / 2\zeta) = \cosh(\pi q / 2m_0) + \mathcal{O}(g^4) \]  

(56)

Inserting (55-56) into (52-53) we obtain after little algebra

\[ < p_2 | \phi(0) | p_1 > = -\frac{i2\pi}{q} \text{sech}(\frac{q\pi}{2m_0}) \left( \frac{1}{g} + \frac{gq^2}{16\pi m_0^2} + \mathcal{O}(g^2) \right) \]  

(57)

in agreement with the result (24) derived in section 2 using perturbation theory. This result shows agreement between perturbation theory in the presence of a single soliton background and the exact result to one loop level and extends the lowest order check first done in [9].
References

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