Stability and Direction Control of a Two-Wheeled Robotic Wheelchair Through a Movable Mechanism

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ABSTRACT A two-wheeled robotic wheelchair (TWRW) has a better manoeuvrability than a conventional four-wheeled wheelchair. However, it is not statically stable near the upright posture or a posture desired by the rider, and an active stability controller is required. Stability control becomes more challenging when a TWRW is also required to move in a desired direction. To rely on wheels’ motions to achieve both stability and direction control tend to impose a large burden on the wheels’ driving motors or other types of actuators in terms of their driving torque and power consumption. Various disturbances in the system also affect the performance of the controller. To solve these problems, this paper presents a stability and direction controller based on the motion of a pendulum-like movable mechanism added to assist the wheels to produce control actions. The dynamic model of the TWRW is established through the Euler-Lagrange formulation in which the disturbances caused by model uncertainties and rider’s motion are considered. A robust second-order sliding mode control is then developed for the stability and the direction control of a TWRW. Simulation results are presented to validate the effectiveness of the proposed method.

INDEX TERMS Two-wheeled robotic wheelchair, stability control, direction control, added movable mechanism, second-order sliding mode control.

I. INTRODUCTION

A conventional robotic wheelchair consists of two driving wheels and two passive casters, where the driving wheels move actively for both mobility and stability of the wheelchair, while the passive casters provide a support for the wheelchair’s stability [1]–[3]. A two-wheeled robotic wheelchair (TWRW) without casters can turn on spot, climb small steps, and thus has a better manoeuvrability than a conventional wheelchair [4]. It is also compact in structure and can maneuver in a narrow space [5].

However, a TWRW which can be modelled as a two-wheeled inverted pendulum, is not statically stable as a conventional wheelchair at the upright posture (defined by a pitch angle) and needs an active controller to be stabilized [6]. When a TWRW is also required to follow a path along a desired direction (defined by a yaw angle), to achieve both stability and direction control is more challenging. The TWRW is also subjected to disturbances caused by the unmodelled dynamics, rider’s motion, sensor noises and uneven surface, etc which affect the controller’s performance [7], [8].

Different motion patterns of a TWRW such as turning, going straight or standing still are produced from the relative angular velocity between two driving wheels; this is called differential wheel drive mechanism [4], [9]–[12]. Such a mechanism has a high demand on torque and power consumption from the driving motors. Another approach for stability control is based on the motion of a movable seat or a linearly moving mass slider under the rider’s seat [13], [14]. Though it needs less torque and power consumption for stability control, its operation range is limited, and it tends to cause unwanted disturbances affecting the comfort of the rider.

For a nonlinear system like a TWRW with different driving mechanisms, the common nonlinear controller Computed Torque Control can be applied for stability and direction control [15]. In this controller, the control inputs (torques) are derived from nonlinear state feedback and closed loop tracking errors through the system dynamic model. However, it requires an accurate dynamic model of the system and is not robust against model and external uncertainties [5]. In comparison, sliding mode control (SMC) is more robust against
disturbances and is more computationally efficient [16]–[18]. In this controller, the closed loop tracking errors are forced to be near a predefined surface, called sliding surface, in the state space of the system. For an underactuated systems where the number of inputs is less than the number of controlled outputs, hierarchical sliding mode control (HSMC) can be applied [19]. In HSMC, the system is divided into several subsystems for each of which, a so called layer sliding surface is designed. The main drawback of SMC control is chattering phenomenon which leads to high vibration in the system. This problem can be solved with quasi-sliding mode control (QSMC) where a smooth sigmoid function is used to replace a non-smooth sign function found in a SMC controller [20]. Another effective solution is higher order sliding mode controller like the second order sliding mode controller (SOSMC) [21]. In this controller, a discontinuous integrator is added to the control input to eliminates the chattering phenomenon.

Both stability control and direction control of a TWRW are required in practice, but this issue has not been addressed in the existing research. A main challenging is to achieve the both control targets well when the system is subject to various disturbances and torques and power consumption are constrained by the limits of the wheel motors’ capacities. In this paper, pendulum-like movable mechanism is added to the TWRW to assist the wheels for stability and direction control. The dynamic model of the system is established using Euler-Lagrange formulation. Disturbances from model uncertainties of system and rider’s motion are also considered. A SOSMC is developed for stability and direction control of the TWRW. The proposed approach is shown to be superior conventional methods in terms of the performance of the controller and the control torque and power consumption needed.

The rest of paper is organized as follows. The TWRW is described and its dynamic model is derived in Sect. II. In Sect. III, a SOSMC for stability and direction control of the TWRW is presented. Simulation results to validate the effectiveness of the proposed control approach are presented and discussed in Sect. IV. Conclusions are given in Sect. V.

II. WHEELCHAIR MODELLING

A TWRW consists of two wheels and a seat for the rider which can rotate freely around the wheels axle; the seat and the rider are combined to form a body. A pendulum like movable mechanism is placed under the seat to assist the wheels to control the TWRW. This mechanism consists of a rod and a mass placed at one end of rod. The mass of rod is small and is neglected. Fig. 1 shows a prototype of a TWRW.

Fig. 2 shows the schematic view of the TWRW and the proposed mechanism. The nomenclature can be found in Table. 1. To derive the dynamic model of the TWRW, the Euler-Lagrange formulation is used, [22]

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i
\]  

where \( L = T - U \) is known as Lagrangian and \( T \) and \( U \) are the kinetic and potential energy of the whole system, respectively. The system’s generalized coordinates and their corresponding inputs are denoted by \( q_i \) and \( Q_i \), respectively. The friction forces between joints are not considered. Also, it is assumed that the wheels don’t slip on the ground. In conventional control method, the wheels torques are the control inputs. Therefore, the overall kinetic and potential energy of system can be obtained as

\[
T = T_r + T_l + T_b, \quad U = U_r + U_l + U_b,
\]

where \( T_r, T_l \) and \( T_b \) are the kinetic energy of right and left wheel and body (including rider and seat frame), respectively. Similarly, \( U_r, U_l \) and \( U_b \) are their potential energies. The kinetic energy of right and left wheel can be shown as

\[
T_r = \frac{1}{2} m_w r^2 \dot{\theta}_r^2 + \frac{1}{2} J_w \dot{\theta}_r^2 + \frac{1}{2} J_w \dot{\theta}_r^2
\]

\[
T_l = \frac{1}{2} m_w r^2 \dot{\theta}_l^2 + \frac{1}{2} J_w \dot{\theta}_l^2 + \frac{1}{2} J_w \dot{\theta}_l^2
\]

\[
\dot{\theta}_y \text{ which is the yaw angular velocity can be obtained as}[23]
\]

\[
\dot{\theta}_y = \frac{r}{d} (\dot{\theta}_r - \dot{\theta}_l)
\]

The kinetic energy of body can be obtained as

\[
T_b = \frac{1}{2} m_b (V^2 + \dot{\theta}_r^2 + \dot{\theta}_l^2 + 2 V \dot{\theta}_r \cos \theta_b)
\]

\[
+ \frac{1}{2} (J_b \dot{\theta}_r^2 \sin^2 \theta_b + J_b \dot{\theta}_l^2 \cos^2 \theta_b + J_b \dot{\theta}_b^2)
\]

where \( V \) is the linear velocity of center of wheels axle which is

\[
V = \frac{r}{2} (\dot{\theta}_r + \dot{\theta}_l)
\]

The potential energy of the right and left wheels and the body can be shown as

\[
U_r = U_l = 0, \quad U_b = m_b g l \cos \theta_b.
\]
Applying Equation (1), the dynamic model of TWRW in the conventional method can be derived and presented as [24]

\[ M_c \ddot{q}_c + H_c + G_c = B_c \tau_c \]  

(4)

where \( q_c \) is the generalized coordinates vector that can be shown as

\[ q_c = \begin{bmatrix} \theta_r & \theta_l & \theta_b \end{bmatrix}^T \]

\( M_c \) is the symmetric matrix called the inertia matrix.

\[
M_c = \begin{bmatrix}
M_{c11} & M_{c12} & M_{c13} \\
M_{c21} & M_{c22} & M_{c23} \\
M_{c31} & M_{c32} & M_{c33}
\end{bmatrix}
\]

\( H_c \) is the Centrifugal and Coriolis forces matrix.

\[
H_c = \begin{bmatrix}
H_{c1} & H_{c2} & H_{c3}
\end{bmatrix}^T
\]

\( M_c \) and \( H_c \) components can be found in Appendix A. \( G_c \) is the gravity matrix.

\[
G_c = \begin{bmatrix} 0 & 0 & -m_b g l \sin \theta_b \end{bmatrix}^T
\]

\( B_c \) is the control coefficient matrix.

\[
B_c = \begin{bmatrix} 1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\]

and \( \tau_c \) is the control input vector.

\[
\tau_c = \begin{bmatrix} \tau_r & \tau_l \end{bmatrix}^T
\]

The input power and energy consumption of right and left wheel motors can be obtained as [25]

\[
P_r = \tau_r \dot{\theta}_r, \quad P_l = \tau_l \dot{\theta}_l, \quad E_r = \int P_r dt, \quad E_l = \int P_l dt.
\]
Considering the disturbances like model uncertainties due to the varying mass of the rider and the variations of the body’s center of gravity (CoG) from the motions of the rider, the dynamic model of system should be reformulated as
\[
\dot{\mathbf{M}}_c \ddot{\mathbf{q}}_c + \dot{\mathbf{H}}_c + \dot{\mathbf{G}}_c + \mathbf{D}_c + \mathbf{R}_c = \mathbf{B}_c \tau_c
\] (5)
where \(\mathbf{D}_c\) and \(\mathbf{R}_c\) denotes the disturbances caused by model uncertainties and change of body’s CoG position, respectively. \(\mathbf{M}_c\), \(\mathbf{H}_c\) and \(\mathbf{G}_c\) are the nominal inertia, centrifugal and gravity matrices, respectively and can be shown as
\[
\dot{\mathbf{M}}_c = \mathbf{M}_c - \Delta \mathbf{M}_c, \quad \dot{\mathbf{H}}_c = \mathbf{H}_c - \Delta \mathbf{H}_c, \quad \dot{\mathbf{G}}_c = \mathbf{G}_c - \Delta \mathbf{G}_c
\]
The disturbance caused by the uncertain mass of the body can be shown as
\[
\mathbf{D}_c = \Delta \mathbf{M}_c \ddot{\mathbf{q}}_c + \Delta \mathbf{H}_c + \Delta \mathbf{G}_c
\]
where
\[
\Delta \mathbf{M}_c = \begin{bmatrix}
\Delta M_{11} & \Delta M_{12} & \Delta M_{13} \\
\Delta M_{21} & \Delta M_{22} & \Delta M_{23} \\
\Delta M_{31} & \Delta M_{32} & \Delta M_{33}
\end{bmatrix}
\]
\[
\Delta \mathbf{H}_c = \begin{bmatrix}
\Delta H_1 \\
\Delta H_2 \\
\Delta H_3
\end{bmatrix}
\]
\[
\Delta \mathbf{G}_c = \begin{bmatrix}
0 \\
0 \\
-\Delta m_b l \sin \theta_b
\end{bmatrix}^T
\]
\(\Delta \mathbf{M}_c\) is a symmetric matrix. \(\Delta \mathbf{M}_c\) and \(\Delta \mathbf{H}_c\) components are shown in Appendix A. \(\Delta m_b = m_b - \bar{m}_b\), where \(m_b\) and \(\bar{m}_b\) are the real and nominal values of body’s mass, respectively.

The effect of change of body’s CoG can be shown as
\[
\mathbf{R}_c = \begin{bmatrix} R_1 & R_2 & R_3 \end{bmatrix}^T
\]
The \(\mathbf{R}_c\) elements are shown in Appendix A. From Equation (5), we have
\[
\ddot{\mathbf{q}}_c = \hat{\mathbf{M}}_c^{-1}(-\dot{\mathbf{H}}_c - \dot{\mathbf{G}}_c - \mathbf{D}_c - \mathbf{R}_c + \mathbf{B}_c \tau_c)
\] (6)
Differentiating Equation (2), with respect to time leads to
\[
\ddot{\theta}_y = \frac{r}{d} (\dot{\theta}_r - \dot{\theta}_l)
\] (7)
From Equation (6), and Equation (7), we have
\[
\begin{aligned}
\ddot{\theta}_b &= A_{c_1} + B_{c_1} + \hat{M}_{c_1}^{-1} \tau_r + \hat{M}_{c_2}^{-1} \tau_l, \\
\ddot{\theta}_y &= \frac{r}{d} (A_{c_2} + B_{c_2} + (\hat{M}_{c_1}^{-1} - \hat{M}_{c_2}^{-1}) \tau_r + (\hat{M}_{c_1}^{-1} - \hat{M}_{c_2}^{-1}) \tau_l).
\end{aligned}
\] (8)
The definition of \(A_{c_1}, B_{c_1}, A_{c_2},\) and \(B_{c_2}\) can be found in Appendix A.

Considering the added pendulum like movable mechanism, the overall kinetic and potential energy of TWRW is obtained as
\[
T = T_r + T_l + T_b + T_p, \quad U = U_r + U_l + U_b + U_p,
\]
where \(T_p\) and \(U_p\) are the kinetic and potential energy of the added movable mechanism, respectively. \(T_p\) and \(U_p\) can be presented as
\[
\begin{aligned}
T_p &= \frac{1}{2} m_p[V^2 + b^2 \dot{\theta}_b^2 + 2bV \dot{\theta}_b \cos \theta_b + \dot{\theta}_b^2 \cos \theta_b + l^2 \dot{\theta}_y^2 \sin^2 \theta_b + 2V \dot{\theta}_y \cos \theta_b + 2 \dot{\theta}_y^2 \cos \theta_b + l^2 \dot{\theta}_y^2 \sin^2 \theta_b + 2b \dot{\theta}_y \cos \theta_b + \dot{\theta}_y^2 \cos \theta_b + l^2 \dot{\theta}_y^2 \sin^2 \theta_b + \dot{\theta}_y^2 \sin \theta_b +\theta_b^2 \cos \theta_b + \dot{\theta}_y^2 \sin \theta_b + \theta_b^2 \cos \theta_b]
\end{aligned}
\]
\[
\begin{aligned}
U_p &= m_p(g(b \cos \theta_b) - l' \cos(\theta_b + \theta_b)).
\end{aligned}
\]
By applying Equation (1), the dynamic model of the whole system is as follows,
\[
\mathbf{M}_p \ddot{\mathbf{q}}_p + \mathbf{H}_p + \mathbf{G}_p = \mathbf{B}_p \tau_p
\] (9)
where
\[
\begin{aligned}
\mathbf{q}_p &= \begin{bmatrix} \theta_r \ \theta_l \ \theta_b \ \theta_p \end{bmatrix}^T \\
\mathbf{M}_p &= \begin{bmatrix}
M_{p11} & M_{p12} & M_{p13} & M_{p14} \\
M_{p21} & M_{p22} & M_{p23} & M_{p24} \\
M_{p31} & M_{p32} & M_{p33} & M_{p34} \\
M_{p41} & M_{p42} & M_{p43} & M_{p44}
\end{bmatrix} \\
\mathbf{G}_p &= \begin{bmatrix}
0 & 0 & G_{p3} & G_{p4} \end{bmatrix}^T \\
\mathbf{B}_p &= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}^T, \quad \tau_p = \begin{bmatrix} \tau_r \ \tau_l \ \tau_p \end{bmatrix}^T
\end{aligned}
\]
$\mathbf{M}_p$, $\mathbf{H}_p$, and $\mathbf{G}_p$ components can be found in Appendix B. The input power and energy consumption of added movable mechanism motor can be obtained as [25]

$$P_p = \tau_p \dot{\theta}_p, \quad E_p = \int P_p dt.$$  

Considering model uncertainties and change of body’s CoG position, the dynamic model is rewritten as

$$\mathbf{M}_p \ddot{\mathbf{q}}_p + \dot{\mathbf{H}}_p + \dot{\mathbf{G}}_p + \mathbf{D}_p + \mathbf{R}_p = \mathbf{B}_p \tau_p$$  (10)

where

$$\begin{align*}
\mathbf{D}_p &= \Delta \mathbf{M}_p \ddot{\mathbf{q}}_p + \Delta \mathbf{H}_p + \Delta \mathbf{G}_p \\
\Delta \mathbf{M}_p &= \begin{bmatrix}
\Delta M_{11} & \Delta M_{12} & \Delta M_{13} & 0 \\
\Delta M_{21} & \Delta M_{22} & \Delta M_{23} & 0 \\
\Delta M_{31} & \Delta M_{32} & \Delta M_{33} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \\
\Delta \mathbf{H}_p &= \begin{bmatrix}
\Delta H_1 \\
\Delta H_2 \\
\Delta H_3 \\
0
\end{bmatrix} \\
\Delta \mathbf{G}_p &= \begin{bmatrix}
0 \\
0 \\
-\Delta m_bgl \sin \theta_b \\
0
\end{bmatrix}^T \\
\mathbf{R}_p &= \begin{bmatrix}
R_1 & R_2 & R_3 & 0
\end{bmatrix}^T
\end{align*}$$

$\Delta \mathbf{M}_p$, $\Delta \mathbf{H}_p$ and $\mathbf{R}_p$ elements are similar to the disturbances matrices derived in conventional method and can be found in Appendix A. In proposed method, $\dot{\theta}_b$ and $\dot{\theta}_y$ can be obtained as

$$\begin{align*}
\dot{\theta}_b &= A_{p_1} + B_{p_1} + \hat{M}_{p_1}^{-1} \tau_r + \hat{M}_{p_2}^{-1} \tau_l + \hat{M}_{p_3}^{-1} \tau_p, \\
\dot{\theta}_y &= \frac{r}{d} [A_{p_2} + B_{p_2} + (\hat{M}_{p_1}^{-1} - \hat{M}_{p_2}^{-1}) \tau_r \\
&+ (\hat{M}_{p_2}^{-1} - \hat{M}_{p_3}^{-1}) \tau_l + (\hat{M}_{p_4}^{-1} - \hat{M}_{p_3}^{-1}) \tau_p].
\end{align*}$$  (11)

The definition of $A_{p_1}$, $B_{p_1}$, $A_{p_2}$, and $B_{p_2}$ are presented in Appendix B.

## III. CONTROLLER DESIGN

### A. CONVENTIONAL METHOD

The control objective is to track the desired yaw angle by the TWRW, while the pitch angle remains zero. To control the pitch and yaw angle through SOSMC, the sliding surface vector is defined as

$$\sigma = \begin{bmatrix}
\sigma_1 \\
\sigma_2
\end{bmatrix}^T, \quad \sigma_1 = e_2 + c_1 e_1, \quad \sigma_2 = e_4 + c_2 e_3.$$  (12)

where $\sigma_1$ and $\sigma_2$ are the sliding surfaces defined for pitch and yaw angle control, respectively. $e_1$, $e_2$, $e_3$, and $e_4$ are the tracking errors of pitch angle, pitch angular velocity, yaw angle, and yaw angular velocity, respectively. $c_1$ and $c_2$ are positive design parameters. $e_1 = \theta_b - \hat{\theta}_b$, $e_2 = \dot{\theta}_b - \hat{\dot{\theta}}_b$, $e_3 = \dot{\theta}_y - \hat{\dot{\theta}}_y$, $e_4 = \ddot{\theta}_y - \ddot{\theta}_y$, $\dot{\sigma}_1$ and $\dot{\sigma}_2$ are the desired values of pitch angle, pitch angular velocity, yaw angle, and yaw angular velocity, respectively. From Equation (12), we have

$$\begin{align*}
\dot{\sigma}_1 &= \dot{e}_2 + c_1 \dot{e}_1 = (\dot{\theta}_b - \hat{\theta}_b) + c_1 e_2, \\
\dot{\sigma}_2 &= \dot{e}_4 + c_2 \dot{e}_3 = (\dddot{\theta}_y - \dddot{\theta}_y) + c_2 e_4.
\end{align*}$$  (13)

According to the structure of SOSMC, we have [26]

$$\dot{\alpha}_1 = u_1 + h_1, \quad \dot{\alpha}_2 = u_2 + h_2.$$  (14)

where $u_1$ and $u_2$ are the equivalent control inputs. $h_1$ and $h_2$ are the disturbances. Comparing Equation (8), Equation (13), and Equation (14), for conventional method we have

$$\begin{align*}
u_1 &= A_{c_1} + \hat{M}_{c_1}^{-1} \tau_r + \hat{M}_{c_2}^{-1} \tau_l - \hat{\theta}_b + c_1 e_2, \quad h_1 = B_{c_1}, \\
u_2 &= \frac{r}{d} [A_{c_2} + (\hat{M}_{c_1}^{-1} - \hat{M}_{c_2}^{-1}) \tau_r + (\hat{M}_{c_2}^{-1} - \hat{M}_{c_2}^{-1}) \tau_l] \\
-\dddot{\theta}_y + c_2 e_4, \quad h_2 = \frac{r}{d} B_{c_2}.
\end{align*}$$  (15)

To develop the SOSMC, $K_{m_1}$ and $K_{M_1}$ which are two positive constants are chosen as

$$0 \leq K_{m_1} \leq 1 \leq K_{M_1}.$$  

There exists two positive constants $q_1$ and $U_{M_1}$ which are selected as

$$|h_1| < q_1 U_{M_1}, \quad 0 < q_1 < 1$$

Also, the positive constant value $C_1$ is chosen as

$$|\dot{h}_1| \leq C_1.$$  

Considering the assumptions above, the equivalent control input $u_1$ is defined as

$$\begin{align*}
u_1 &= -\lambda_1 |\sigma_1|^{0.5} \text{sign}(\sigma_1) + v_1, \\
v_2 &= \left\{ \begin{array}{ll}
-u_2, & |u_2| > U_{M_2} \\
-\alpha_2 \text{sign}(\sigma_2), & |u_2| \leq U_{M_2}
\end{array} \right.
\end{align*}$$  (16)

where, $\lambda_1$ and $\alpha_1$ are two positive constants. Selecting $\lambda_1 > \frac{2}{(K_{m_1} \sigma_1 + C_1) K_{M_1}(1 + q_1)}$ and $\alpha_1 > \frac{1}{C_1/K_{m_1}}$, all tracking errors converge to zero in finite time. The stability proof of SOSMC can be found in [27].

Similar to $u_1$, $u_2$ is defined as

$$\begin{align*}
u_2 &= -\lambda_2 |\sigma_2|^{0.5} \text{sign}(\sigma_2) + v_2, \\
v_2 &= \left\{ \begin{array}{ll}
-u_2, & |u_2| > U_{M_2} \\
-\alpha_2 \text{sign}(\sigma_2), & |u_2| \leq U_{M_2}
\end{array} \right.
\end{align*}$$  (17)

From Equation (15) - Equation (17), the input torque of right and left wheels in conventional method can be obtained through

$$\begin{align*}
\begin{bmatrix}
\tau_r \\
\tau_l
\end{bmatrix} &= \begin{bmatrix}
\hat{M}_{c_1}^{-1} \\
\hat{M}_{c_2}^{-1}
\end{bmatrix} \begin{bmatrix}
\hat{M}_{c_1}^{-1} \\
\hat{M}_{c_2}^{-1}
\end{bmatrix}^{-1} \begin{bmatrix}
F_{c_1} \\
F_{c_2}
\end{bmatrix}
\end{align*}$$  (18)

where

$$\begin{align*}
F_{c_1} &= -A_{c_1} + \dddot{\theta}_b - c_1 e_2 - \lambda_1 |\sigma_1|^{0.5} \text{sign}(\sigma_1) + v_1, \\
F_{c_2} &= -A_{c_2} + \dddot{\theta}_y - c_2 e_4 - \lambda_2 |\sigma_2|^{0.5} \text{sign}(\sigma_2) + v_2.
\end{align*}$$
TABLE 2. Physical parameters of the TWRW for simulation.

| Property | \( m_w \) | \( \theta_0 \) | \( \theta_0 \) | \( I_{x}\) | \( I_{y}\) | \( I_{z}\) | \( I_{p}\) | \( I_{p}\) | \( I_{p}\) | \( r \) | \( d \) | \( b \) | \( l \) | \( l' \) |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Value    | 10       | 80       | 30       | 0.32     | 0.64     | 10.03    | 12.40    | 13.39    | 0.26     | 0.39     | 0.35     | 0.37     | 0.5      | 0.25     | 0.6      | 0.42     |
| Unit     | kg       | kg       | kg       | kg\(m^2\) | kg\(m^2\) | kg\(m^2\) | kg\(m^2\) | kg\(m^2\) | kg\(m^2\) | kg\(m^2\) | kg\(m^2\) | m        | m        | m        | m        |

B. PROPOSED METHOD

From Equation (11), Equation (13), and Equation (14), we have

\[
\begin{aligned}
\begin{bmatrix} u_1 \\ h_1 \\ h_2 \\ \tau_p \end{bmatrix} &= \begin{bmatrix} A_{p_1} + \dot{M}_{p31} \tau_r + \dot{M}_{p31} \tau_l + \dot{M}_{p34} \tau_p - \ddot{\theta}_b_0 + c_1 e_2 \\ B_{p_1} \\ \frac{r}{d} (A_{p_2} + (\dot{M}_{p11} - \dot{M}_{p21}) \tau_r) + (\dot{M}_{p12} - \dot{M}_{p22}) \tau_l - \ddot{\theta}_d + c_2 e_4 \\ \frac{r}{d} B_{p_2} \end{bmatrix}, \\
\end{aligned}
\]

where \( \beta > 0 \). From Equation (16) - Equation (20), the input torque of right and left wheels and added mechanism can be obtained as

\[
\begin{aligned}
\begin{bmatrix} \tau_r \\ \tau_l \\ \tau_p \end{bmatrix} &= W^{-1} \begin{bmatrix} F_{p_1} \\ 0 \\ 0 \end{bmatrix}, \\
\end{aligned}
\]

\[
W = \begin{bmatrix} \dot{M}_{p31}^{-1} & \dot{M}_{p32}^{-1} & \dot{M}_{p34}^{-1} \\ (\dot{M}_{p11} - \dot{M}_{p21}) & (\dot{M}_{p12} - \dot{M}_{p22}) & (\dot{M}_{p14} - \dot{M}_{p24}) \end{bmatrix}, \\
F_{p_1} = -A_{p_1} + \ddot{\theta}_b - c_1 e_2 - \lambda_1 | \sigma_1 |^{0.5} \text{sign}(\sigma_1) + v_1, \\
F_{p_2} = -A_{p_2} + \frac{r}{d} (\ddot{\theta}_d - c_2 e_4 - \lambda_2 | \sigma_2 |^{0.5} \text{sign}(\sigma_2) + v_2).
\]

IV. SIMULATION RESULTS

To demonstrate through simulations the superiority of the proposed control method over conventional ones, the physical dimensions of the TWRW are chosen and listed in Table 2.

![Figure 4](image-url)  
(a): Pitch angle

![Figure 4](image-url)  
(b): Pitch angular velocity

The values selected for the control parameters can be found in Table 3. The following initial conditions are assumed:

\[
\begin{aligned}
\theta_{b_0} = \dot{\theta}_{b_0} = \theta_{x_0} = \dot{\theta}_{x_0} = 0. \\
\end{aligned}
\]

The following control objectives are set: \( \theta_{b_d} = 0, \dot{\theta}_{b_d} = 0, \theta_{y_d} = \frac{\pi}{2} \text{rad}, \dot{\theta}_{y_d} = 0. \)

The stability and direction control performances of the TWRW through conventional and the proposed methods are evaluated when the system is subject to the uncertainties of the mass of the body and its CoG which are respectively assumed as

\[
\Delta m_b = 40 \text{ kg}, \quad \begin{aligned}
0 \leq x_b \leq 5 \text{ cm} & \quad 5 \leq t < 15 \\
0 & \quad \text{otherwise}
\end{aligned}
\]

Fig. 4 shows the response of pitch and its angular velocity. The results show that under both controllers, system can keep its stability as the range of pitch angle and its rate is acceptable and after a period it converges to zero. It can be seen in Fig. 5 that the TWRW can reach its desired yaw angle and yaw angular velocity. The variation of pitch and yaw angle and their rates under the conventional and the proposed method are similar. The required input torque of right and left wheels can be seen in Fig. 6.

The results show that the required torque through the proposed method is lower than the conventional control approach. Similarly, the input power of wheels in the proposed approach is much lower than the conventional one (see Fig. 7). Fig. 8 depicts the input torque and power needed by the added mechanism which is lower than those needed by the right and left wheels.
The energy consumption of the motors through the conventional and proposed approaches can be found in Table 4. It can be seen that the energy consumption of right and left wheels in the proposed approach are much lower than that of the conventional controller. The overall energy consumption including that for the added
mechanism is also much lower than the conventional method.

V. CONCLUSION

In this paper, a novel approach is proposed for stability and direction control of a TWRW. A pendulum-like movable mechanism is added to the TWRW to assist the driving wheels to achieve the both control objectives. The Euler-Lagrange formulation is applied to establish the dynamic model of the system and a SOSMC which is robust against disturbances is developed for stability and direction control. The effectiveness of the proposed approach is simulated while considering disturbances caused by uncertainties of inertia parameter of the dynamic model and the rider’s motion. The simulation results demonstrate that in the proposed approach, the desired pitch and yaw angles of the TWRW desired for stability and direction control are achieved, while the input torque and power consumption for the control system are much lower than conventional methods.

APPENDIXES

APPENDIX A

DYNAMIC MODEL ELEMENTS OF THE CONVENTIONAL METHOD

The components of $M_c$ and $H_c$ are as below:

$$M_{c11} = M_{c22} = (m_w + \frac{1}{4}m_b)r^2 + J_w + \frac{r^2}{d^2}[mb^2 \sin^2 \theta_b + 2J_w \sin \theta_b + J_b \cos \theta_b], \quad M_{c12} = \frac{1}{4}mr^2 - \frac{r^2}{d^2}[mb^2 \sin^2 \theta_b + 2J_w \sin \theta_b + J_b \cos \theta_b],$$

$$M_{c13} = M_{c23} = \frac{1}{2}mr l \cos \theta_b, \quad M_{c33} = mb^2 + J_b,$$

$$H_{c1} = \frac{r}{d} \dot{\theta}_b (\sin \theta_b mb^2 \sin \theta_b + J_b - J_b) - \frac{1}{2}mr l \dot{\theta}_b^3 \sin \theta_b,$$

$$H_{c2} = \frac{-r}{d} \dot{\theta}_b \sin 2\theta_b (mb^2 + J_b - J_b) - \frac{1}{2}mr l \dot{\theta}_b^2 \sin \theta_b,$$

$$H_{c3} = \frac{-1}{2} \dot{\theta}_b \sin 2\theta_b (mb^2 + J_b - J_b).$$

$\Delta M_c$, $\Delta H_c$, and $R_c$ elements are

$$\Delta M_{11} = \Delta M_{22} = (\frac{I_1}{4} + \frac{l^2}{d^2} \sin^2 \theta_b) \Delta mb^2 r^2,$$

$$\Delta M_{12} = \frac{1}{4} \frac{l^2}{d^2} \sin^2 \theta_b \Delta mb^2 r^2,$$

$$\Delta M_{13} = \Delta M_{23} = \frac{1}{2} \Delta mb l \cos \theta_b, \quad \Delta M_{33} = \Delta mb^2 l^2,$$

$$\Delta H_1 = \frac{l}{d} \dot{\theta}_b \sin 2\theta_b - \frac{1}{2} \dot{\theta}_b \sin \theta_b \Delta mb l \dot{\theta}_b,$$

$$\Delta H_2 = -\frac{1}{2} \dot{\theta}_b \sin 2\theta_b + \frac{1}{2} \dot{\theta}_b \sin \theta_b \Delta mb l \dot{\theta}_b,$$

$$\Delta H_3 = -\frac{1}{2} \dot{\theta}_b \sin 2\theta_b, \quad R_1 = \frac{r^2}{d^2} [mb \sin \theta_b \cos \theta_b + \sin \theta_b \dot{\theta}_b^2 + \frac{1}{2} \dot{\theta}_b \sin \theta_b \Delta mb l \dot{\theta}_b + \frac{1}{2} \dot{\theta}_b \sin \theta_b \Delta mb l \dot{\theta}_b],$$

$$\Delta H_4 = \frac{1}{2} \dot{\theta}_b \sin 2\theta_b + \frac{1}{2} \dot{\theta}_b \sin \theta_b \Delta mb l \dot{\theta}_b,$$

$$\Delta H_5 = -\frac{1}{2} \dot{\theta}_b \sin 2\theta_b - \frac{1}{2} \dot{\theta}_b \sin \theta_b \Delta mb l \dot{\theta}_b.$$
\[
- \frac{1}{2} m_p r l' \cos(\theta_b + \theta_p), \quad M_{p14} = M_{p24} = - \frac{1}{2} m_p r l' \cos(\theta_b + \theta_p), \quad M_{p31} = m_b l^2 + m_p (b^2 + l^2) + J_b, \quad J_{p1} - 2 m_p b l' \cos \theta_p, \quad M_{p14} = m_p l^2 - m_p b l' \cos \theta_j + J_{p1} - 2 m_p b l' \cos \theta_p,
\]
\[
H_{p1} = \frac{r}{d} \dot{\theta}_b \dot{\theta}_b \sin(2 \theta_b (m_p l^2 + J_b) - J_{b_j}) - \frac{1}{2} r \dot{\theta}_b \sin(\theta_b (m_p l^2 + m_p b) + \frac{1}{2} m_p r l' (\dot{\theta}_b + \dot{\theta}_p)^2 \sin(\theta_b + \theta_p)
\]
\[
+ \frac{r}{d} m_p \dot{\theta}_b (b^2 \dot{\theta}_b \sin(2 \theta_b + \theta_p) \sin(2 \theta_b + 2 \theta_p) - 2 b l' \dot{\theta}_b \sin(2 \theta_b + \theta_p) - 2 l' \dot{\theta}_b \sin(2 \theta_b + \theta_p) + \frac{r}{d} \dot{\theta}_b (\dot{\theta}_p + \dot{\theta}_b) (J_{p1} - J_{p_j}) \sin(2 \theta_b + 2 \theta_p),
\]
\[
H_{p2} = \frac{- \frac{1}{2} \dot{\theta}_b \sin(2 \theta_b (m_p l^2 + J_b) - J_{b_j})}{- \frac{1}{2} \dot{\theta}_b \sin(2 \theta_b (m_p l^2 + m_p b) + \frac{1}{2} m_p r l' (\dot{\theta}_b + \dot{\theta}_p)^2 \sin(\theta_b + \theta_p)
\]
\[
+ \frac{r}{d} m_p \dot{\theta}_b (b^2 \dot{\theta}_b \sin(2 \theta_b + \theta_p) \sin(2 \theta_b + 2 \theta_p) + l^2 \dot{\theta}_b \sin(2 \theta_b + \theta_p) - 2 b l' \dot{\theta}_b \sin(2 \theta_b + \theta_p) + \frac{r}{d} \dot{\theta}_b (\dot{\theta}_p + \dot{\theta}_b) (J_{p1} - J_{p_j}) \sin(2 \theta_b + 2 \theta_p),
\]
\[
G_{p1} = -(m_p l + m_p b) g \sin \theta_b + m_p g l' \sin(\theta_b + \theta_p), \quad G_{p4} = m_p g l' \sin(\theta_b + \theta_p).
\]

The definition of \( A_{p1}, B_{p1}, A_{p2}, \) and \( B_{p2} \) are as below
\[
A_{p1} = - \tilde{M}_{p1}^{-1} (H_{p1} + G_{p1} - \tilde{M}_{p2}^{-1} (H_{p2} + G_{p2}), \quad B_{p1} = - \tilde{M}_{p1}^{-1} (D_{p1} + R_{p1}) - \tilde{M}_{p3}^{-1} (D_{p3} + G_{p4}), \quad A_{p2} = (\tilde{M}_{p2}^{-1} - \tilde{M}_{p1}^{-1}) (H_{p1} + G_{p1})
\]
\[
+ (\tilde{M}_{p2}^{-1} - \tilde{M}_{p1}^{-1}) (H_{p2} + G_{p2}) + (\tilde{M}_{p3}^{-1} - \tilde{M}_{p1}^{-1}) (H_{p3} + G_{p4}), \quad B_{p2} = (\tilde{M}_{p2}^{-1} - \tilde{M}_{p1}^{-1}) (D_{p1} + R_{p1}) + (\tilde{M}_{p2}^{-1} - \tilde{M}_{p1}^{-1}) (D_{p3} + R_{p3}) + (\tilde{M}_{p4}^{-1} - \tilde{M}_{p1}^{-1}) (D_{p4} + R_{p4}).
\]

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REFERENCES

[1] C. De La Cruz, T. F. Bastos, and R. Carelli, “Adaptive motion control law of a robotic wheelchair,” Control Eng. Pract., vol. 19, no. 2, pp. 113–125, Feb. 2011.
[2] S. Oh and Y. Hori, “Disturbance attenuation control for power-assist wheelchair operation on slopes,” IEEE Trans. Control Syst. Technol., vol. 22, no. 3, pp. 828–837, May 2014.
[3] S. Tashiro and T. Murakami, “Power-assist control of an electric wheelchair considering step passage,” in Proc. IEEE/ASME Int. Conf. Adv. Intell. Mechatron., Sep. 2007, pp. 1–6.
[4] A. Dinale, K. Hirata, M. Zoppici, and T. Murakami, “Parameter design of disturbance observer for a robust control of two-wheeled wheelchair system,” J. Intell. Robot. Syst., vol. 77, no. 1, pp. 135–148, Oct. 2014.
[5] Z. Li and C. Yang, “Neural-adaptive output feedback control of a class of transportation vehicles based on wheeled inverted pendulum models,” IEEE Trans. Control Syst. Technol., vol. 20, no. 6, pp. 1583–1591, Nov. 2012.
[6] C.-H. Chiu, “Adaptive fuzzy control strategy for a single-wheel transportation vehicle,” IEEE Access, vol. 7, pp. 113272–113283, 2019.
[7] J. Huang, T. Zhang, Y. Fan, and J.-Q. Sun, “Control of rotary inverted pendulum using model-free backstepping technique,” IEEE Access, vol. 7, pp. 96965–96973, 2019.
[8] R. Cui, J. Guo, and Z. Mao, “Adaptive backstepping control of wheeled inverted pendulums models,” Nonlinear Dyn., vol. 79, no. 1, pp. 501–511, Sep. 2014.
[9] A. Maddahi and A. H. Shamekhi, “Controller design for two-wheeled self-balancing vehicles using feedback linearisation technique,” Int. J. Vehicle Syst. Model. Test., vol. 8, no. 1. pp. 38–54, 2013.
[10] K. Hirata and T. Murakami, “Stability analysis of disturbance observer based controllers for two-wheeled wheelchair systems,” Adv. Robot., vol. 26, no. 7, pp. 467–477, Feb. 2014.
[11] L. Vermeiren, A. Dequidt, T. M. Guerra, H. Rago-Tirmant, and M. Parent, “Modeling, control and experimental verification on a two-wheeled vehicle with free inclination: An urban transportation system,” Control Eng. Pract., vol. 19, no. 7, pp. 744–756, Jul. 2011.
[12] S. Ahmad, N. H. Siddique, and M. O. Tokhi, “A modular fuzzy control approach for two-wheeled self-balancing,” J. Intell. Robot. Syst., vol. 64, no. 3–4, pp. 401–426, Feb. 2011.
[13] J. Huang, F. Ding, T. Fukuda, and T. Matsuno, “Modeling and velocity control for a novel narrow vehicle based on wheeled mobile inverted pendulum,” IEEE Trans. Control Syst. Technol., vol. 21, no. 5, pp. 1607–1617, Sep. 2013.
[14] Y. Sago, Y. Noda, K. Kakihara, and K. Terashima, “Parallel two-wheeled vehicle with underslung vehicle body,” Mech. Eng. J., vol. 1, no. 4, 2014, Art. no. DR0036.
[15] M. Nikpour, L. Huang, A. M. Al-Jumaily, and B. Lotfi, “Stability control of mobile inverted pendulum through an added movable mechanism,” in Proc. 25th Int. Conf. Mechatron. Mach. Vis. Pract. (M2VIP), Nov. 2018, pp. 1–6.
[16] A. Ghaffari, A. Shariati, and A. H. Shamekhi, “A modified dynamical formulation for two-wheeled self-balancing robots,” Nonlinear Dyn., vol. 83, nos. 1–2, pp. 217–230, Aug. 2015.
[17] P. K. W. Abeygunawardhana, M. Defoort, and T. Murakami, “Self-sustaining control of two-wheeled mobile manipulator using sliding mode control,” in Proc. 11th IEEE Int. Workshop Adv. Motion Control (AMC), Mar. 2010, pp. 792–797.
[18] J. Huang, Z.-H. Guan, T. Matsuno, T. Fukuda, and K. Sekiyama, “Sliding-mode velocity control of mobile-wheeled inverted-pendulum systems,” IEEE Trans. Robot., vol. 26, no. 4, pp. 750–758, Aug. 2010.
[19] M. Yue and X. Wei, “Dynamic balance and motion control for wheeled inverted pendulum vehicle via hierarchical sliding mode approach,” Proc. Inst. Mech. Eng., I. J. Syst. Control Eng., vol. 228, no. 6, pp. 351–358, Apr. 2014.
[20] T. Elmokadem, M. Zribi, and K. Youcef-Toumi, “Control for dynamic positioning and way-point tracking of underactuated autonomous underwater vehicles using sliding mode control,” J. Intell. Robot. Syst., vol. 95, nos. 3–4, pp. 1113–1132, Apr. 2018.
[21] R. Guruganesh, B. Bandypadhyay, H. Arya, and G. K. Singh, “Design and hardware implementation of autopilot control laws for MAV using super twisting control,” J. Intell. Robot. Syst., vol. 90, nos. 3–4, pp. 435–471, Nov. 2018.
[22] S. S. Rao and F. F. Yap, Mechanical Vibrations, vol. 4. Upper Saddle River, NJ, USA: Prentice-Hall, 2011.
[23] N. Esmaeili, A. Alfi, and H. Khosravi, “Balancing and trajectory tracking of two-wheeled mobile robot using backstepping sliding mode control: Design and experiments,” *J. Intell. Robotic Syst.*, vol. 87, nos. 3–4, pp. 601–613, Jan. 2017.

[24] M. G. Rodd, “Introduction to robotics: Mechanics and control,” *Automatica*, vol. 23, no. 2, pp. 263–264, Mar. 1987.

[25] D. S. Naidu, *Mechatronics: Electronic Control Systems in Mechanical Engineering*. London, U.K.: Pearson, 2003.

[26] Y. Shitessel, C. Edwards, L. Fridman, and A. Levant, *Sliding Mode Control and Observation*. New York, NY, USA: Springer, 2014.

[27] Z. Zhao, J. Yang, S. Li, Z. Zhang, and L. Guo, “Finite-time super-twisting sliding mode control for Mars entry trajectory tracking,” *J. Franklin Inst.*, vol. 352, no. 11, pp. 5226–5248, Nov. 2015.

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