The nucleon Drell-Hearn-Gerasimov sum rule within a relativistic constituent quark model

F. Cardarelli\(^{(a)}\), B. Pasquini\(^{(b)}\) and S. Simula\(^{(a)}\)

\(^{(a)}\)Istituto Nazionale di Fisica Nucleare, Sezione Sanità, Roma, Italy
\(^{(b)}\)Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, and
Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy

Abstract

The Drell-Hearn-Gerasimov sum rule for the nucleon is investigated within a relativistic constituent quark model formulated on the light-front. The contribution of the \(N - \Delta\) transition is explicitly evaluated using different forms for the baryon wave functions and adopting a one-body relativistic current for the constituent quarks. It is shown that the \(N - \Delta\) contribution to the sum rule is sharply sensitive to the introduction of anomalous magnetic moments for the constituent quarks, at variance with the findings of non-relativistic and relativized quark models. The experimental value of the isovector-iso-vector part of the sum rule is almost totally reproduced by the \(N - \Delta\) contribution, when the values of the quark anomalous magnetic moments are fixed by fitting the experimental nucleon magnetic moments. Our results are almost independent of the adopted form of the baryon wave functions and only slightly sensitive to the violation of the angular condition caused by the use of a one-body current. The calculated average slope of the generalized sum rule around the photon point results to be only slightly negative at variance with recent predictions of relativized quark models.

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The polarized and unpolarized photoabsorption cross sections off hadronic targets are known to be non-trivially constrained by sum rules arising from low-energy theorems [1] and general properties of the Compton scattering amplitude. In particular, the Drell-Hearn-Gerasimov (DHG) sum rule [2] for the forward spin-flip amplitude of the Compton scattering relates the helicity structure of the photoabsorption cross section with the anomalous magnetic moment of the target (i.e., with a ground-state property of the target). Starting from an unsubtracted dispersion relation for the spin-dependent part of the forward Compton amplitude and using low-energy theorems to prescribe the behaviour of the scattering amplitude at low energy transfer, the DHG sum rule in case of the nucleon reads as

\[ I_N = \int_{\omega_{th}}^{\infty} d\omega \frac{\sigma^{(N)}_{1/2}(\omega) - \sigma^{(N)}_{3/2}(\omega)}{\omega} = -2\pi^2 \alpha \frac{\kappa_N^2}{m_N^2} \]  

where \( \sigma^{(N)}_{1/2} \) (\( \sigma^{(N)}_{3/2} \)) is the total cross section for the absorption of a circularly polarized photon with spin parallel (anti-parallel) to the nucleon spin and \( \kappa_N \) (\( m_N \)) is the anomalous magnetic moment (mass) of the nucleon. In Eq. (1) the photon energy \( \omega \) starts from the pion production threshold \( \omega_{th} \equiv m_\pi(1 + m_\pi/2m_N) \) and \( \alpha \simeq 1/137 \) is the fine structure constant.

The DHG sum rule for the nucleon has not yet been tested experimentally, for direct measurements of \( \sigma^{(N)}_{1/2} \) and \( \sigma^{(N)}_{3/2} \) are still lacking. However, it is possible to estimate the difference \( \sigma^{(N)}_{1/2} - \sigma^{(N)}_{3/2} \) using the multipole decomposition of pion photoproduction amplitudes in unpolarized experiments. Such a decomposition is available only in the single-pion production channel, so that above the two-pion production threshold model-dependent assumptions have to be adopted. Nevertheless, the phenomenological analyses, carried out by a number of authors [3, 4, 5, 6], have provided relevant information on the isospin decomposition of the DHG integral into the isovector-isovector (VV), isoscalar-isoscalar (SS) and mixed isoscalar-isovector (SV) terms, which can be compared with the sum rule predictions obtained from the isospin dependence of \( \kappa_N \) (i.e, \( \kappa_N = \kappa_S + \tau_3 \kappa_V \)), viz.

\[ I_N = I_{VV} + I_{SS} + \tau_3 I_{SV} = -\frac{2\pi^2 \alpha}{m_N^2} (\kappa_V^2 + \kappa_S^2 + 2\tau_3 \kappa_S \kappa_V) \]  

(2)

The main outcome of existing phenomenological analyses is that the dominant \( I_{VV} \) contribution is correctly reproduced and the \( I_{SS} \) term turns out to be quite small, but a striking discrepancy for \( I_{SV} \) is found and is still unexplained; moreover, the \( N - \Delta(1232) \) transition almost exhausts the \( I_{VV} \) integral [3, 4, 5]. Several experiments, involving both real and virtual photons, are planned or underway at various labs in order to investigate both the DHG integral and its \( Q^2 \) evolution, the latter being relevant for the understanding of higher-twist contribution to the sum rules on polarized nucleon structure functions (cf. Ref. [7]). Direct measurements of the polarized photoabsorption cross section will be provided by the ELSA, MAMI, LEGS and GRAAL facilities [8, 9] in the next future, while forthcoming high-quality data from TJNAF [10] at low \( Q^2 \) and DESY [11] at higher \( Q^2 \) will be combined with recent measurements performed at SLAC [12] and HERA [13].

\[ \text{bWe mention that, as far as non-resonant background processes are concerned, only pion production channels are taken into account in existing phenomenological analyses.} \]
From the theoretical point of view the DHG integral has been throughout investigated within non-relativistic [14] and relativized [14, 15] versions of the constituent quark (CQ) model. Though one of the major success of the CQ model is the good overall description of nucleon magnetic moments, both its non-relativistic and relativized versions fail in describing the DHG sum rule [16]. The aim of this letter is to investigate the DHG integral for the nucleon within the relativistic CQ model of Refs. [17, 18]. The basic features of this model are: i) the relativistic composition of the CQ spins obtained via the introduction of the (generalized) Melosh rotations (cf. [19]); ii) the possibility of adopting hadron wave functions derived from an effective Hamiltonian able to describe the mass spectroscopy; iii) the use of a relativistic one-body electromagnetic (e.m.) current which includes Dirac and Pauli form factors for the CQ’s. The latter can be fixed by the non-trivial request of reproducing existing experimental data on pion and nucleon elastic form factors (cf. Ref. [17]).

Within the so-called zero-width approximation, the resonance contribution to the DHG integral can simply be written in the following form (cf. Ref. [15])

\[ I_{res}^N = \sum_R \sum_R I_{R/N}^R = \sum R \frac{4\pi}{m_R^2 - m_N^2} \left( |A_{1/2}^R|^2 - |A_{3/2}^R|^2 \right) \]  

where

\[ A_\lambda^R = \sqrt{\frac{2\pi\alpha}{\omega_R m_N}} \langle \psi_R, \lambda_R = \lambda | e_\mu(+) \cdot J_\mu(0) | \psi_N, \lambda_N = \lambda - 1 \rangle \]  

is the helicity amplitude describing the electromagnetic (e.m.) excitation of the nucleon to a resonance of mass \( m_R \) with spin projection \( \lambda = 1/2, 3/2 \). In Eqs. (3-4) \( \omega_R \equiv (m_R^2 - m_N^2)/2m_N \) is the resonance excitation energy, \( e(+1) \) is the photon polarization four-vector with helicity \(+1\) and \( |\psi_R, \lambda_R \rangle (|\psi_N, \lambda_N \rangle) \) is the resonance (nucleon) spinor corresponding to a spin projection \( \lambda_R (\lambda_N) \). In this work we limit ourselves to the contribution of the \( N - \Delta(1232) \) transition to the DHG integral, because of its expected dominance. Four different forms of the \( N \) and \( \Delta(1232) \) wave functions will be adopted. The first one, which will be referred to as model A, is given by the \( N \) and \( \Delta(1232) \) wave functions corresponding to the effective Hamiltonian of Capstick and Isgur (CI) [20], while in model B the effects of the hyperfine terms of the CI interaction are switched off and in model C only the linear confining part of the CI potential is retained. In all these models the mass of \( u \) and \( d \) quark is \( m_u = m_d = m = 0.22 \) GeV. Moreover, a fourth model (D) is considered, based on the gaussian-like ansatz already adopted in Ref. [21]. However, when \( m = 0.22 \) GeV, the results obtained in model D have been found to be quite similar to those of model C. Therefore, in model D we take the values \( m = 0.33 \) GeV in order to check the sensitivity of our calculations to the value of the light CQ mass. The CQ momentum distribution corresponding to models A – D can be found in Ref. [17]; here, it suffices to remind that, thanks to the effects of the hyperfine terms of the CI interaction, the high-momentum tail of the nucleon wave function drastically increases going from model D to model A. Moreover, in each of the models B, C and D the \( N \) and \( \Delta(1232) \) wave functions are the same, while in model A the high-momentum tail in the \( \Delta(1232) \) resonance is suppressed with respect to the nucleon case by the spin-dependent terms of the CI interaction.

Since we are interested also in the evaluation of the slope of the DHG integral at the photon point, we will present the basic formulae relevant for the calculation of the helicity
amplitudes $A^\Delta_{1/2}$ at finite values of the squared four-momentum transfer $Q^2 \equiv -q \cdot q$. As is well known (cf. [22]), the matrix elements of the $N - \Delta(1232)$ transition e.m. current can be cast in the form

$$T^\mu_{\Delta \lambda N} \equiv \langle \psi_{\Delta \lambda} | J^\mu_\nu(0) | \psi_{N \lambda} \rangle = \sqrt{\frac{2}{3}} \langle \psi_{\Delta \lambda} | \left[ G^\Delta_1(Q^2) K^\mu_{1\nu} + G^\Delta_2(Q^2) K^\mu_{2\nu} + G^\Delta_3(Q^2) K^\mu_{3\nu} \right] | \psi_{N \lambda} \rangle$$

(5)

where the form factors $G^\Delta_i(Q^2)$ and the tensors $K^\mu_{ij}$ are defined as in Ref. [22]. Within the light-front formalism, hadron e.m. form factors at space-like values of the four-momentum transfer can be related to the matrix elements of the plus component of the current, $T^+ \equiv T^0 + \hat{n} \cdot \vec{T}$, where $\hat{n}$ defines the spin quantization axis. The standard choice of a reference frame where $q^+ \equiv q^0 + \hat{n} \cdot \vec{q} = 0$ allows to suppress the contribution of the pair creation from the vacuum [23]. The relations between the matrix elements $T^+_{\Delta \lambda N}$ and the form factors $G^\Delta_i(Q^2)$ are

$$T^+_{\frac{1}{2} \frac{3}{2}} = \frac{Q}{\sqrt{3}} \left[ G^\Delta_1(Q^2) + \frac{m_\Delta - m_N}{2} G^\Delta_2(Q^2) \right]$$

$$T^+_{\frac{3}{2} \frac{1}{2}} = -\frac{Q^2}{3} \left[ \frac{G^\Delta_1(Q^2)}{m_\Delta} + \frac{G^\Delta_2(Q^2)}{2} - \frac{m_\Delta - m_N}{m_\Delta} G^\Delta_3(Q^2) \right]$$

$$T^+_{\frac{1}{2} - \frac{1}{2}} = \frac{Q}{3} \left[ \frac{m_N}{m_\Delta} G^\Delta_1(Q^2) - \frac{m_\Delta - m_N}{2} G^\Delta_2(Q^2) - \frac{Q^2}{m_\Delta} G^\Delta_3(Q^2) \right]$$

$$T^+_{\frac{3}{2} - \frac{3}{2}} = -\frac{Q^2}{2\sqrt{3}} G^\Delta_2(Q^2)$$

(6)

The helicity amplitudes $A^\Delta_{1/2}$ and $A^\Delta_{3/2}$ are related to the multipole form factors $G^\Delta_M$ and $G^\Delta_E$ by

$$A^\Delta_{1/2} = \mathcal{N}(Q^2) \frac{1}{2} \left[ G^\Delta_M(Q^2) - 3G^\Delta_E(Q^2) \right]$$

$$A^\Delta_{3/2} = \mathcal{N}(Q^2) \frac{\sqrt{3}}{2} \left[ G^\Delta_M(Q^2) + G^\Delta_E(Q^2) \right]$$

(7)

where $\mathcal{N}(Q^2) = -\frac{m_N}{m_\Delta} \frac{m_N + m_\Delta}{m_\Delta + m_N} \sqrt{\frac{M^2 + Q^2}{(m_\Delta + m_N)^2 + Q^2}} \sqrt{\frac{2\pi a}{\omega N \omega N}}$ and

$$G^\Delta_M(Q^2) = \frac{m_N}{3(m_\Delta + m_N)} \left\{ 2[m_\Delta M + (m_\Delta + m_N)^2 + Q^2] \frac{G^\Delta_1(Q^2)}{m_\Delta} + 2m_\Delta M G^\Delta_2(Q^2) - 2Q^2 G^\Delta_3(Q^2) \right\}$$

$$G^\Delta_E(Q^2) = \frac{m_N}{3(m_\Delta + m_N)} \left\{ 2m_\Delta M \left[ \frac{G^\Delta_1(Q^2)}{m_\Delta} + G^\Delta_2(Q^2) \right] - 2Q^2 G^\Delta_3(Q^2) \right\}$$

(8)

with $M = (m_\Delta^2 - m_N^2 - Q^2)/2m_\Delta$. 

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Following Refs. [7, 8] we approximate the $I^+$ component of the e.m. current by the sum of one-body $CQ$ currents, viz.

$$I^+(0) \simeq \sum_{q=1}^{3} I^+_q(0) = \sum_{q=1}^{3} \left( e_q \gamma^+ f_1^{(q)}(Q^2) + i \kappa_q \frac{\sigma^+ \rho_{q} \sigma}{2m_q} f_2^{(q)}(Q^2) \right)$$

(9)

where $e_q$ ($\kappa_q$) is the $CQ$ charge (anomalous magnetic moment) and $f_1^{(q)}$ is the corresponding Dirac (Pauli) form factor. As already mentioned, Eq. (9) has been found to be non-trivially consistent with existing data on pion and nucleon elastic form factors [17].

Equation (9) clearly shows that for the $N-\Delta(1232)$ transition the number of independent form factors is not equal to one of the matrix elements of $I^+$. If the exact $I^+$ is adopted, the inversion of Eq. (9) is unique, because the matrix elements of $I^+$ are related by the so-called angular condition (cf. [19]). However, the fulfillment of the angular condition requires the presence of (at least) two-body currents in $I^+$; therefore, since we use the one-body approximation (9), a unique determination of the form factors $G^\Delta_m(Q^2)$ is not possible. In the actual calculation for the $N-\Delta(1232)$ transition we have considered three different angular prescriptions: i) all the form factors $G^\Delta_m$ are extracted from the first three equations in (9) (prescription I); ii) $G^\Delta_1$ and $G^\Delta_2$ are taken as in the previous prescription, but $G^\Delta_3$ is directly obtained from the fourth equation in (9) (prescription II); iii) $G^\Delta_2$ and $G^\Delta_3$ are as in the previous prescription, while $G^\Delta_1$ is derived from the first equation in (9) (prescription III). The multipole form factors $G^\Delta_M$ and $G^\Delta_E$ (Eq. (9)) have been calculated in each prescription and for each model ($A - D$) adopted for the baryon wave functions. At the photon point it turns out that the value of $G^\Delta_M(0)$ is only slightly different (within $\simeq 5\%$) in the various prescriptions, while $G^\Delta_E(0)$ is sharply sensitive to the violation of the angular condition. However, the helicity amplitudes (9) have the same sensitivity to the angular prescriptions as $G^\Delta_M(0)$, because $G^\Delta_E(0)$ is much smaller than $G^\Delta_M(0)$. Indeed, for the ratio $E1/M1 \equiv -G^\Delta_E(0) / G^\Delta_M(0)$, in case of the model $A$, we have obtained the values $0.37\%$, $-2.06\%$, $-1.85\%$ for the prescriptions I - III, respectively. In these calculations we have included the effects due to the $D$-wave of the $\Delta(1232)$ wave function, generated by the tensor term of the $CI$ interaction; however, $D$-wave effects are very small (cf. also [18]), so that the value of $E1/M1$ in our $CQ$ model is mainly governed by relativistic effects in the $S$-waves and therefore is strongly prescription dependent. In what follows our results for the $N-\Delta(1232)$ contribution to the $DHG$ integral will be given together with a theoretical uncertainty calculated as the spread of the values obtained adopting the angular prescriptions I - III.

First of all, we have calculated the nucleon magnetic moments, $\mu_{p[n]}$, both including and excluding the contribution arising from the $CQ$ anomalous magnetic moments $\kappa_q$ in Eq. (9). Our results corresponding to the four adopted forms of the nucleon wave function are reported in Table 1. It can be seen that, when $\kappa_q = 0$, the calculated values of $\mu_p$ and $|\mu_n|$ significantly underestimate the experimental data. In our opinion this fact should be traced back to a typical (momentum-dependent) dilution effect arising from the helicity mixing provided by the relativistic composition of the $CQ$ spins (i.e., by Melosh rotations). As a matter of fact, the suppression factor is remarkably sensitive to the high-momentum components of the nucleon wave function (see models $A$ and $C$) and to the values of the $CQ$ masses (see models $C$ and $D$). Note also that in all the models considered the calculated values of $\kappa_p$ and $\kappa_n$ satisfy
the inequality \( \kappa_p > |\kappa_n| \), whereas the experimental data (\( \kappa_{p[n]}^{\exp} = 1.793 \pm 1.913 \)) exhibit the opposite trend, i.e. \( \kappa_{p[n]}^{\exp} < |\kappa_{p[n]}^{\exp}| \), which is the origin of the positive sign of the \( I^\text{SV} \) part of the experimental GDH sum rule. Then, non-vanishing \( \kappa_q \) are considered in Eq. (4) and their values (reported in Table 1) are fixed by requiring the reproduction of the experimental nucleon magnetic moments, \( \mu_{p[n]}^{\exp} = 2.793 \pm 1.913 \). The largest values of \( |\kappa_q| \) are obtained for models \( A \) and \( D \). In all the wave function models the \( SU(2) \)-symmetry constraint, \( \kappa_u = -2 \kappa_d \), is not fulfilled and, moreover, we find \( |\kappa_u| < |\kappa_d| \) at variance with non-relativistic \( CQ \) models where \( |\kappa_u| \approx |\kappa_d| \). We stress that our finding \( |\kappa_u| < |\kappa_d| \) is a direct consequence of the inequality \( \kappa_{p}^{\exp} < |\kappa_{p}^{\exp}| \) and it is obtained both in case of the models \( B \) - \( D \), where the nucleon wave function is fully \( SU(6) \)-symmetric, and in case of model \( A \), which includes a small mixed-symmetry admixture (\( \approx 1.7\% \)) due to the hyperfine terms of the \( CI \) interaction. The sensitivity exhibited by the calculated \( \mu_N \) to the inclusion of the \( CQ \) anomalous magnetic moments is not surprising. Indeed, in the plus component of the current (Eq. (3)) the Dirac term cannot produce any \( CQ \) spin flip, which should therefore be provided by the Melosh rotations. On the contrary, the Pauli term in Eq. (9) flips the \( CQ \) spin. Therefore, an important feature of our relativistic quark model is the crucial role expected (and found) for \( \kappa_q \) in the calculation of magnetic-type observables (see also below).

In Table 2 the results obtained for the contribution of the \( N - \Delta(1232) \) transition to the \( DHG \) integral (\( I_N^{R = \Delta} \) in Eq. (3)) are shown and compared with the combination (\( I_p + I_n \))/2 of the sum rule expected from the calculated nucleon magnetic moments.\(^{[4]} \) It can clearly be seen that: i) the \( N - \Delta(1232) \) transition almost exhausts the expected \( DHG \) sum rule both for \( \kappa_q = 0 \) and \( \kappa_q \neq 0 \); in our opinion this is related to the facts that the \( \Delta(1232) \) resonance is the lowest excited resonance state and the spatial part of its wave function is expected to have the largest overlap with the nucleon one due to the approximate \( SU(6) \) symmetry; ii) the experimental value of (\( I_p + I_n \))/2 is almost totally reproduced only when the effects of the \( CQ \) anomalous magnetic moments are considered and their values are fixed by fitting the experimental \( \mu_{p[n]}^{\exp} \); iii) the calculated \( I_N^{\Delta} \) is sharply sensitive to the effects due to \( \kappa_u \) and \( \kappa_q \) (up to a factor of \( \sim 2 \) in case of model \( A \)), at variance with the findings of non-relativistic and relativized \( CQ \) models \( [4, 6] \); iv) our results with \( \kappa_q \neq 0 \) are almost independent of the adopted form of the baryon wave functions and, in particular, they are quite insensitive to hyperfine interaction effects; v) the sensitivity to the violation of the angular conditions results to be relatively small (within \( \sim 10\% \)), suggesting that the effects of two-body currents, needed for a full Poincaré-covariance of the e.m. current, could be not large at the photon point.

In Fig. 1 ours prediction for the low-\( Q^2 \) evolution of the \( N - \Delta(1232) \) contribution to the (generalized) \( DHG \) integral are shown and compared with the results of the relativized quark model of Ref. \([15]\). As for the form factors \( f_1^{(q)} (Q^2) \) and \( f_2^{(q)} (Q^2) \) appearing in Eq. (9), in case of model \( A \) we have used the \( CQ \) form factors already determined in Ref. \([17]\) from a fit of pion and nucleon elastic data; the procedure of Ref. \([17]\) has been repeated in case of models \( B \) and \( C \) (cf. also Ref. \([18]\)), obtaining again a nice reproduction of the elastic data. From Fig. 1 it can be seen that a slightly negative value of the average slope around the photon point is obtained for all the wave function models considered; this finding is clearly at variance with

\(^{[6]} \)The \( N - \Delta(1232) \) transition contributes equally to \( I_p \) and \( I_n \), so that we compare the \( N - \Delta(1232) \) contribution to the combination (\( I_p + I_n \))/2 of the \( DHG \) sum rule.
the result of Ref. [15] and implies that, as far as the average slope around the photon point is concerned, the role of the $N - \Delta(1232)$ transition in our relativistic approach is less relevant than the one expected from relativized quark models. Moreover, we should mention that recent calculations based on baryon chiral perturbation theory [25] predict a positive average slope; however, these predictions need to be confirmed by higher-order calculations and, at the same time, the inclusion of the background contribution in our calculation is a pre-requisite for a full comparison. Planned experiments at TJNAF [10] are expected to help in unravelling the low-$Q^2$ behaviour of the generalized GDH sum rule.

Before closing, we want to address briefly the questions of the physical meaning of an $CQ$ anomalous magnetic moment and its consequences in the nucleon DHG sum rule. A finite size and an anomalous magnetic moment are a clear manifestation of an internal structure in a particle. If we want to give a physical meaning to the quantity $\kappa_q$ appearing in Eq. (9), we are naturally led to assume the occurrence of inelastic channels at the $CQ$ level, providing a photoabsorption cross section on the $CQ$, $\sigma^{(q)}_\lambda$, whose helicity structure is constrained by the DHG sum rule at the $CQ$ level, viz.

$$I_q = \int_{\tilde{\omega}_{th}}^{\infty} d\omega \frac{\sigma^{(q)}_{1/2}(\omega) - \sigma^{(q)}_{3/2}(\omega)}{\omega} = -2\pi^2 \alpha \frac{\kappa^2_q}{m^2_q}$$

(10)

where $\tilde{\omega}_{th} \equiv m\pi(1 + m\pi/2m_q)$ is the pion production threshold on the $CQ$. The main problem is clearly how to calculate the contribution of $CQ$ inelastic channels to the nucleon DHG integral, or, in other words, how to evaluate the effects of the internal structure of the $CQ$’s on Eq. (9). To this end we will make use of some formal results, obtained in Ref. [24], which we translate for the case of interest here. In case of a non-relativistic $CQ$ e.m. current, the resonance contribution $I^{res}_N$ (Eq. (3)) satisfies the nucleon DHG sum rule only when the quark anomalous magnetic moments are vanishing. When $\kappa_q \neq 0$, an additional term is present and it can be interpreted as resulting simply from the sum of the $CQ$ spin-dependent forward Compton amplitudes. In Ref. [14] the possible presence of a subtraction at infinity in the dispersion integral is suggested in order to compensate the additional term. However, a different viewpoint can be adopted. Real (as well as virtual) photons can couple to a $CQ$ in two ways: i) an elastic coupling, $q\gamma \rightarrow q$, described by the one-body current (3), which at the hadron level generates the transitions to nucleon resonances and gives rise to the resonance contribution $I^{res}_N$; ii) an inelastic coupling (like, e.g., $q\gamma \rightarrow qM$, where $M$ can be any meson), constrained by the conventional DHG sum rule (11) for the $CQ$’s, which corresponds at the hadron level to meson background production and yields a contribution $I^{bkg}_N$ to the nucleon DHG sum rule. Neglecting at the present stage any interference between final hadron states arising from elastic and inelastic $CQ$ couplings (which might be a not too bad approximation for an inclusive quantity like the DHG sum rule), we may write the nucleon DHG integral as

$$I_N \simeq I^{res}_N + I^{bkg}_N$$

(11)

Inspired by the additivity structure found in [24], we may approximate the contribution $I^{bkg}_N$ as

$$I^{bkg}_N \simeq -2\pi^2 \alpha \langle \psi_N, \frac{1}{2} | \sum_{q=1}^{3} \frac{\kappa^2_q}{m^2_q} \sigma^{(q)}_3 | \psi_N, \frac{1}{2} \rangle$$
where the last equality is obtained after considering a $SU(6)$-symmetric nucleon wave function and the quantity $\langle \gamma_M \rangle$ is a (momentum-dependent) dilution factor resulting from the Melosh rotations of the $CQ$ spins, given explicitly by

$$\langle \gamma_M \rangle = \left( \frac{(m + x M_0)^2 - p_\perp^2}{(m + x M_0)^2 + p_\perp^2} \right)$$

where $M_0$ is the free mass operator (cf. Ref. [17]), $x$ the light-front momentum fraction carried by a $CQ$ in the nucleon, $p_\perp^2$ its transverse momentum squared and the notation $\langle \rangle$ stands for the average over the radial nucleon wave function. In case of the models $A - D$ the relativistic factor $\langle \gamma_M \rangle$ results to be 0.48, 0.51, 0.63 and 0.75, respectively. Thanks to the results of Ref. [24], in the non-relativistic limit (where $\langle \gamma_M \rangle = 1$) the right-hand side of Eq. (11), with $I_{bkg}^N$ given by Eq. (12), satisfies exactly the nucleon DHG sum rule without requiring any subtraction of the dispersion integral at infinity when $\kappa_q \neq 0$. From Eq. (12) it follows that the $I_{SV} = (I_p - I_n)/2$ part of the nucleon sum rule may receive a contribution proportional to $(I_u - I_d)/2$; since in all our models $|\kappa_u| < |\kappa_d|$, the sign of such a contribution is always positive. Keeping in mind the caveat that Eq. (12) is an approximation for $I_{bkg}^N$, our results, obtained for the nucleon DHG integrals both including ($\langle \gamma_M \rangle \neq 1$) and excluding ($\langle \gamma_M \rangle = 1$) the relativistic dilution factor (13), are summarized in Table 3. It can clearly be seen that the contribution to $I_{SV}$ arising from $CQ$ inelastic channels (i.e., from background processes) appears to be of the right sign and order of magnitude. Moreover, the model dependence of our results is partially reduced by the inclusion of relativistic effects related to the Melosh rotations of the $CQ$ spins. The striking difference with the findings of phenomenological analyses [3, 4] might be due to the lack in the latter of the contribution of non-resonant background production of mesons other than the pions. We should mention however that our results for $I_{SV}$ (and, to a less extent, those for $I_{VV}$) need to be confirmed after the inclusion of the contributions resulting from nucleon resonances other than the $\Delta(1232)$.

In conclusion, the Drell-Hearn-Gerasimov sum rule for the nucleon has been investigated within a relativistic constituent quark model formulated on the light-front. The contribution of the $N - \Delta(1232)$ transition has been explicitly evaluated using different forms for the baryon wave functions and adopting a one-body relativistic current for the constituent quarks. It has been shown that the $N - \Delta(1232)$ contribution to the $DHG$ sum rule is sharply sensitive to the introduction of anomalous magnetic moments for the constituent quarks, at variance with the findings of non-relativistic and relativized quark models. The experimental value of the isovector-isovector part of the sum rule is almost totally reproduced by the $N - \Delta(1232)$ contribution, when the values of the quark anomalous magnetic moments are fixed by fitting the experimental nucleon magnetic moments. Our results are almost independent of the adopted form of the baryon wave functions and, in particular, they are quite insensitive to hyperfine interaction effects; moreover, the sensitivity to the violation of the angular condition, caused by the use of a one-body current, is found to be relatively small. The calculated average slope of the generalized sum rule around the photon point results to be only slightly negative at variance with recent predictions from relativized quark models. Eventually, we have stressed
that the relevance of the role played by quark anomalous magnetic moments in our relativistic calculations clearly motivates further theoretical investigation concerning the questions of the physical meaning of the constituent quark anomalous magnetic moment and its consequences in the nucleon DHG sum rule.

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Table 1. Values of the nucleon magnetic moments calculated within our models A – D of the baryon wave functions. In model A the effective Hamiltonian of Capstick and Isgur (CI) \cite{20} is considered, while in model B the effects of the hyperfine terms of the CI interaction are switched off and in model C only the linear confining term of the CI potential is retained. In all these models the masses of $u$ and $d$ quarks are: $m_u = m_d = m = 0.22$ GeV. Model D is based on the gaussian-like ansatz of Ref. \cite{21}, but adopting $m = 0.33$ GeV. The rows labelled $\mu_p(\kappa_q = 0)$ and $\mu_p(\kappa_q = 0)$ are the results obtained assuming $\kappa_q = 0$ in Eq. \eqref{9}. The rows labelled $\kappa_u$ and $\kappa_d$ contain the values of the CQ anomalous magnetic moments needed in Eq. \eqref{9} for the reproduction of the experimental nucleon magnetic moments.

| Model | $A$ | $B$ | $C$ | $D$ |
|-------|-----|-----|-----|-----|
| $\mu_p(\kappa_q = 0)$ | 2.28 | 2.44 | 2.74 | 2.42 |
| $\mu_n(\kappa_q = 0)$ | -1.18 | -1.30 | -1.60 | -1.34 |
| $\kappa_u$ | 0.087 | 0.051 | -0.0057 | 0.075 |
| $\kappa_d$ | -0.157 | -0.129 | -0.07 | -0.155 |

Table 2. Values of the $N – \Delta(1232)$ contribution to the nucleon DHG integral (given in $\mu$barn), calculated using the models A – D for the nucleon and $\Delta(1232)$ wave functions. The theoretical results are reported together with the estimate of the uncertainty related to the violation of the angular condition (see text). The columns labelled $\kappa_q \neq 0$ and $\kappa_q = 0$ correspond to the results obtained with and without the contribution arising from the CQ anomalous magnetic moments in Eq. \eqref{9}. The columns labelled $(p + n)/2$ represent the combination $(I_p + I_n)/2$ of the DHG sum rule \eqref{9}, expected from the nucleon magnetic moments calculated within each model; in the case $\kappa_q \neq 0$, the values adopted for $\kappa_u$ and $\kappa_d$ are those reported in Table 1, which allow to reproduce the experimental value of $(I_p + I_n)/2$.

| Model | $\kappa_q = 0$ | $\kappa_q \neq 0$ |
|-------|----------------|------------------|
| $N – \Delta$ | $(p + n)/2$ | $N – \Delta$ | $(p + n)/2$ |
| A | $-107 \pm 9$ | $-96$ | $-204 \pm 18$ | $-219$ |
| B | $-114 \pm 9$ | $-120$ | $-193 \pm 16$ | $-219$ |
| C | $-165 \pm 7$ | $-183$ | $-197 \pm 9$ | $-219$ |
| D | $-119 \pm 8$ | $-129$ | $-197 \pm 15$ | $-219$ |
Table 3. Values of the nucleon DHG integrals \((I_p + I_n)/2\) and \((I_p - I_n)/2\) (given in \(\mu\text{barn}\)), calculated for each model \((A - D)\) of the baryon wave functions using Eq. (11) with \(I^{\text{bkg}}_N\) given by Eq. (12) and \(I^{\text{res}}_N\) (Eq. (3)) including only the \(N - \Delta(1232)\) contribution. The rows labelled \(\langle \gamma_M \rangle \neq 1\) and \(\langle \gamma_M \rangle = 1\) correspond to the calculation of Eq. (12) with and without the relativistic factor (13), respectively. The row labelled DHG contains the values obtained directly from the DHG sum rule (1). The results of the phenomenological analyses of Refs. [3, 4] are also reported.

| Model | \((I_p + I_n)/2\) | \((I_p - I_n)/2\) |
|-------|--------------------|--------------------|
|       | \(\langle \gamma_M \rangle = 1\) | \(\langle \gamma_M \rangle \neq 1\) | \(\langle \gamma_M \rangle = 1\) | \(\langle \gamma_M \rangle \neq 1\) |
| A     | \(-223 \pm 18\)   | \(-213 \pm 18\)   | 16.5  | 8.3  |
| B     | \(-204 \pm 16\)   | \(-198 \pm 16\)   | 13.6  | 6.9  |
| C     | \(-200 \pm 9\)    | \(-199 \pm 9\)    | 4.7   | 2.9  |
| D     | \(-205 \pm 15\)   | \(-203 \pm 15\)   | 7.9   | 5.9  |
| DHG   | \(-219\)          | 14.7               |
| Ref. [3] | \(-222\)         | \(-39\)           |
| Ref. [4] | \(-225\)         | \(-34\)           |

Figure 1. The \(N - \Delta(1232)\) contribution to the generalized nucleon DHG integral versus the squared four-momentum transfer \(Q^2\). The solid, dashed and dotted lines are our predictions calculated within the models \(A - C\), respectively, adopting the prescription III and using in Eq. (9) the CQ form factors determined by fitting pion and nucleon elastic data (see text). The uncertainty due to the different prescriptions \(I - III\) results to be within \(\sim 10\%\). The dot-dashed line is the \(N - \Delta(1232)\) contribution obtained within the relativized quark model of Ref. [15].