Examples of controlling the technological stress state characteristics of layer-by-layer manufactured products in the framework of the accreted solid model

D A Parshin
Laboratory of Modelling in Solid Mechanics, Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences (IPMech RAS), Vernadskogo Ave. 101 Bldg 1, Moscow 119526, Russia
E-mail: parshin@ipmnet.ru

Abstract. A mathematical model of quasi-statics of accreted deformable solids is proposed for the mechanical analysis of technological processes of slow multilayer power winding of a thin sheet onto the outer side surface of a sufficiently rigid circular mould made in advance. By using the analytical dependences obtained in the framework of this model, two problems of different type on the technological control of the characteristics of the formed product stress state developing during the winding process are formulated and analytically solved. In these problems, setting appropriate programs for changing in course of this process the thickness of the wound sheet and the tension force applied to it is used as the controlling impact.

1. Introduction
If some product is formed due to the layer-by-layer addition of material to its surface, then we can talk about a process of additive manufacturing. Often in such processes, adding the material begins with applying its first elementary layer onto the surface of some prepared template. As a rule, in additive processes, precisely because of their specificity, there is a tangible impact on the formed product of various physical, chemical or mechanical factors that cause deformation of this product already during its formation. The deformation is caused by appearance in the being formed product, due to the named effects, of fields of stresses [1] which can be appropriately called technological stresses.

One of the common possible reasons for the appearance and further development of technological stresses in a layer-by-layer made solid — accreted, or growing solid, in the already accepted phrase — is a pre-stressed state in which elementary layers of the new material added are incorporated into the composition of this solid. Having found out how the deformation of the considered solid in a particular additive technological process will depend on these preliminary stresses, one could actively influence the current and the resulting characteristics of the technological stresses distributions acquired by the products obtained in this process, in other words, control these characteristics by proper varying the pre-stresses created in the being added material.

This paper is devoted to modelling such kind of control on the example of a relatively simple additive manufacturing scheme which makes it possible to construct solutions of the corresponding technological control problems in closed analytical forms that greatly facilitate the identification and quantitative analysis of accompanying mechanical effects and practical peculiarities.
2. Problem setting

As an example, the technological process of manufacturing circular cylindrical products as a result of slow winding of a relatively thin sheet onto a certain mould is considered. The rigidity of the mould is considered to be significantly high in comparison with the rigidity of the product itself formed during the winding process. Therefore, in the proposed mathematical model of the process under study, the rigidity of the mould is assumed to be infinitely large. Equivalent mechanical properties of the resulting material layer are assumed to be homogeneous and isotropic. The formed product is assumed to have a very large axial size, so the simulation is carried out in the approximation of plane strain state. The case of small strain is investigated. At the same time, only such variants of the winding process organization are considered in which the effects of dynamics are insignificant. But even in this case the stress-strain state of the gradually formed solid will obviously have to change over time. Thus, we should be talking about a quasi-static formulation of the problem.

It is assumed that the thickness of the wound sheet is small in relation both to the radius of the mould used for winding and to the total thickness of the material layer formed on it as a result. In this case, it is correct to use the model of continuous accretion, in which an axisymmetric cylindrical solid is formed by continuous attaching to this solid outer side surface annular elementary layers of material that are uniform in thickness. Since we consider the case of small strain, the solid outer variable radius \( R \) continuously increasing during the accretion process can be considered as set at each moment of this process due to the sheet winding program implemented in it, which includes the winding speed and the law of changing over time the thickness \( \Delta \text{sheet} \) of the sheet.

The winding speed is considered to be quite low, which is consistent with the accepted quasi-static approximation. But with a low speed of winding and with using for it, as indicated above, a relatively thin sheet there will be a very small rate of growth over time of the thickness of the entire material layer formed as a result of winding. It is shown in [2] that in such situations, taking into account the deformation aftereffect (creeping) in the material used practically does not affect the resulting final distributions of technological stresses in additively formed solids, and therefore, when there is no need to take into account plastic effects, we can limit ourselves to considering purely elastic deformation, which is done in this paper.

It is obvious that in the absence of rheological effects in the deformation response of the material, an arbitrary quantity can be chosen as a time parameter, which changes strictly monotonically over the course of true time. In the mathematical model constructed in this paper, it is natural to choose the quantity \( R \) as such parameter.

According to the above-formulated purpose of the work, preliminary stresses in the newly added turns of the sheet attached to the solid being formed as a result of winding are considered as those mechanical influences that cause the development of technological stresses in the formed product, and consequently of its deformation. These pre-stresses will occur due to the tension of the sheet during its winding. We will assume that the change of the tension force over time is controlled. Provided that the thickness of the wound sheet is also controlled over time, we can consider the program for changing the preliminary circumferential stress in the superimposed material as an arbitrarily set control action.

If \((r, \phi, z)\) is the polar cylindrical coordinate system associated with the rigid cylindrical mould used (\( z \) is the longitudinal coordinate, \( \phi \) is the angular coordinate, and \( r \) is the radial coordinate), then the mentioned pre-stresses change program can be determined by the law \( t_{\phi \phi}^{\text{ini}}(r) \) of change in the preliminary circumferential stress \( t_{\phi \phi}^{\text{ini}} \) depending on the radius \( r \) of the current wound sheet turn. Considering the laws of change in the course of the winding process of the sheet thickness \( \Delta \text{sheet}(r) \) and the sheet linear (per unit length of the formed product in the axial direction) tension force \( F_{\text{sheet}} (r) \) as given, which coincides with the above-made assumption of a possibility to control these parameters during the process, we can write the following formula for calculating the corresponding preliminary circumferential stresses in the sheet being wound onto the solid external surface having the current radius of value \( r \) :
Here we assume that the force of preliminary stretching the wound sheet does not have time to significantly change in a single turn of the sheet, and therefore in the constructed model of continuous accretion we assume that all the annular elementary layers of the added material are stretched in the circumferential direction uniformly over the angular coordinate. We will also assume that:

1) the wound sheet does not experience tension in the axial direction;
2) the wound sheet is not pressed by any external force to the surface of the formed solid;
3) it is possible to neglect the swirling effects of the wound sheet on the formed solid.

The first assumption means that the axial pre-stresses in the material included in the solid are zero:

\[ t_{zz}^{\text{ini}}(r) = 0. \] (2)

The second and third assumptions mean that there are no radial and tangential stresses on that surface of the solid to which the new material is attached. This is equivalent to the fact that the radial and the corresponding shear pre-stresses in the material included in the solid are, as well as axial, zero:

\[ t_{rr}^{\text{ini}}(r) = t_{\phi r}^{\text{ini}}(r) = 0. \] (3)

We are to emphasize that conditions (2) and (3) do not mean that the axial and the radial stresses will continue to remain equal to zero in the vicinity of the points attached to the solid when these points become internal points of this solid in the course of its continued accreting by new material.

We will denote by \( t_{ij} \) the technological stresses that develop in the product under consideration in the process of its additive formation over the “time” \( R \), where the specified abstract indices \( i \) and \( j \) are to be replaced with any of the coordinate indices \( r, \phi, z \).

It is quite obvious that the above assumptions about the force conditions of winding the material sheet can be expressed by the formulation of the following specific initial conditions:

\[ t_{rr}|_{r=R} = 0, \quad t_{r\phi}|_{r=R} = 0, \quad t_{\phi r}|_{r=R} = t_{\phi \phi}^{\text{ini}}(r), \quad t_{zz}|_{r=R} = 0. \] (4)

(note that each point whose position in the solid being formed is characterized by the coordinate \( r \) begins its “life” in the composition of this solid from the moment of “time” \( R = r \)).

With a detailed qualitative analysis of the simulated winding process, it becomes clear that significant circumferential technological stresses must develop in the resulting product. These stresses are caused by the compression of the formed solid by additional stretched turns of the sheet imposed on it. At the same time, as a result, the surface of the used mould on which the being formed layer of material is growing is also compressed. The corresponding compression value, i.e. the contact pressure on the mould from the side of this layer, increases over time.

In some situations, this effect can play a positive role — for example, when the mould used will be a part of the manufactured product and during operating this product, in order to avoid destruction or excessive deformation of the mould, it will be necessary to partially compensate a certain pressure created inside it. As specific examples, we can mention high-pressure vessels or gun barrels which are reinforced by winding pre-stretched material on them. In other situations, the noted qualitative effect can have very negative consequences, causing, for example, a loss of stability of the mould used already during the winding process.

Regardless of the features of mould working, the strength of the additively formed layer of material obtained on it, both during the winding process and after its completion, will be determined by the technological stresses distributions that occur in this layer throughout its entire thickness [3]. It is also worth noting that the nature of interaction of the mould with the material layer formed on it is of
critical importance for the correct solution of various contact and wear-contact problems arising in engineering and technology in connection with manufacturing and use of such type of products [4–6].

A qualitative understanding of the mechanical effects that occur during the winding of a stretched sheet is not sufficient for a satisfactory solution of all the above issues. A quantitative analysis of the regularities of influence of the parameters characterising the additive process under consideration on the nature of development of the resulting technological stresses distributions in the formed product is required. The results of such an analysis should be the construction of specific programs for changing these parameters in the course of the process, which would provide the required characteristics of the resulting distributions. In this paper, such programs are finally constructed for the case of the need to obtain the required law of change over time of the contact stress on the mould used and for the case of the need to obtain the required final distribution of circumferential stresses over the entire thickness of the wound material layer.

It should be noted that the technological processes of solids additive formation by means of winding are implemented in the vast majority of cases by bringing the being formed solid into rotation around its axis and pulling the wound sheet between the generating line of this solid and the sheet feed mechanism. If the rotation is fast enough the formed solid will be exposed to significant inertia forces. It was demonstrated in [7], how such a mechanical impact can affect the process of technological stresses development in the formed solid. Under certain winding conditions this effect may prevail over the effect of pre-stresses created in the material due to the tension of the sheet being laid onto the solid surface. In the present study, as mentioned above, slow winding processes are considered. In such processes the centrifugal forces of inertia of rotation can be neglected. And the above-mentioned refusal to take into account the dynamic effects of deformation of the being formed product implies the organization of sufficiently smooth rotation, that is the rotation in which the angular velocity changes slowly enough. In such processes it is permissible to neglect the tangential forces of inertia.

3. Research methodology

In the studied technological process we are talking about a deformable solid the material composition of which is replenished with new material elements in the course of deformation of this solid. Such solids, being unconventional for mechanics, are usually called, according to the already established terminology, accreted or growing solids. Numerous studies in the field of mechanics of growing solids (see, e.g., [8–16]) clearly show that to correctly describe the deformation of such solids is not enough to use classical mechanics relations, even when considering them in a variable area of space. This is due to the fact that all accreted solids exhibit a special strain kinematics — because of the fundamental absence of any non-stressed configuration owned by such a solid [17].

One of the approaches used for the formulation of boundary problems in the mechanics of accreted solids, which correctly takes into account such solids kinematics, is to formulate the problems in terms of the velocity characteristics of the solid stress-strain state [18]. Since in the in this paper simulated process of accretion the additional material being attached to the being formed solid moves after its adhesion to the surface of this solid already as part of a solid continuum, then at each time moment of the deformation process there will be a smooth velocity field associated with the deformation process of this continuum inside the entire area of space that this continuum currently occupies. So, in the mechanical problem corresponding to the study undertaken in the present work (set in a descriptive form in the previous Section), the components \( \nu_r, \nu_\phi, \nu_z \) of the deformation velocities field (relative to the adopted “time” \( R \)) in the above-introduced polar cylindrical coordinate system coupled with the rigid mould may act as the main desired characteristics — just as the main desired characteristics in problems on the deformation of solids of constant material composition are the components of the displacement field.

In view of assumed plane strain we can write the following restrictions on the velocity components:

\[
\nu_z \equiv 0, \quad \frac{\partial \nu_r}{\partial z} \equiv \frac{\partial \nu_\phi}{\partial z} \equiv 0. \tag{5}
\]
The above-justified axial symmetry of the constructed model gives additionally
\[
\frac{\partial v_r}{\partial \phi} = \frac{\partial v_\phi}{\partial \phi} = 0, \quad (6)
\]
and neglecting the swirling effects of the wound sheet on the being formed solid (see assumption 3 in the previous Section) gives also the following requirement:
\[
v_\phi \equiv 0. \quad (7)
\]
Joint consideration of conditions (5)–(7) leads to the fact that the component \(v_r\) that depends only on the coordinate \(r\) and the “time” \(R\) turns out to be the only non-zero component of the desired deformation velocities field in the growing continuum under consideration. Such a velocity field generates the strain rate tensor with components
\[
\xi_{rr} = \frac{\partial v_r}{\partial r}, \quad \xi_{\phi\phi} = \frac{v_r}{r}. \quad (8)
\]
The remaining components of the strain rate tensor in the coordinate system under usage are equal to zero over the entire additive process considered.

The non-zero components of the strain rate tensor must be related to the rates of change (with respect to the accepted “time” \(R\)) of the stress tensor components. According to the above, we have to do this by means of the defining relations of the elasticity theory:
\[
\frac{\partial t_{rr}}{\partial R} = 2\mu \left( (\chi + 1) \xi_{rr} + \chi \xi_{\phi\phi} \right), \quad \frac{\partial t_{\phi\phi}}{\partial R} = 2\mu \left( (\chi + 1) \xi_{\phi\phi} + \chi \xi_{rr} \right), \quad \frac{\partial t_{zr}}{\partial R} = 2\mu \chi \left( \xi_{rr} + \xi_{\phi\phi} \right). \quad (9)
\]
Here \(\mu\) is the modulus of elasticity of the second kind of the formed solid product, and the material constant \(\chi\) used is expressed only via the Poisson’s ratio \(\nu\) as follows:
\[
\chi = \frac{\nu}{1 - 2\nu}.
\]
The non-diagonal components of the stress tensor in the coordinate system under usage will be zero due to the form of the strain rate tensor adduced above.

If the desired only nontrivial component \(v_r\) of the deformation velocities field will be determined as a function of the “time” \(R\) and the position \(r\) of a point inside the considered accreted solid, as a result of solving the boundary problem to be set, i.e.
\[
v_r = f(R,r), \quad (10)
\]
then the nontrivial components of the stress tensor will be to be found, in accordance with (8) and (9), by the following integration over the “time” \(R\) taking into account initial conditions (4) where the preliminary stresses \(t_{rr}^{ini}\) are to be calculated by formula (1):
\[
t_{rr} = 2\mu \int_r^R \left[ (\chi + 1) \frac{\partial f(R',r)}{\partial r} + \chi f(R',r) \right] dR', \quad (11)
\]
\[
t_{\phi\phi} = t_{\phi\phi}^{ini}(r) + 2\mu \int_r^R \chi \frac{\partial f(R',r)}{\partial r} + (\chi + 1) f(R',r) \right] dR', \quad (12)
\]
\[ t_{rr} = 2\mu Z \int_R r \left[ \frac{\partial f(R',r)}{\partial R'} + \frac{f(R',r)}{r} \right] dR'. \tag{13} \]

4. Boundary problem for deformation velocities

In the entire time-variable area of space instantly occupied by the solid under consideration, the local equation of equilibrium in stresses must be fulfilled which provides (in the absence of distributed mass forces) that the divergence of the stress tensor at each point of this solid is equal to zero. In the component form, we will have

\[ \frac{\partial t_{rr}}{\partial r} + \frac{t_{rr} - t_{\phi\phi}}{r} = 0. \tag{14} \]

Differentiating this equation by the “time” \( R \) we obtain the following differential equation:

\[ \frac{\partial^2 t_{rr}}{\partial R \partial r} + \frac{1}{r} \left( \frac{\partial t_{rr}}{\partial R} - \frac{\partial t_{\phi\phi}}{\partial R} \right) = 0. \]

Substituting relations (9), (8), and (10) into the latter equation we come to the following equation for the unknown function \( f(R,r) \) that determines the deformation velocities field:

\[ \frac{\partial^2 f(R,r)}{\partial r^2} + \frac{1}{r} \frac{\partial f(R,r)}{\partial r} - \frac{f(R,r)}{r^2} = 0. \tag{15} \]

For differential equation (15) it is necessary to set the boundary conditions:

1) on the fixed boundary \( r = R_{\text{mould}} \), where \( R_{\text{mould}} \) is the radius of the surface of the mould used for winding, onto which the starting elementary layer of the material is superimposed;

2) on the moving boundary \( r = R \), onto which the regular elementary layer of the material is attached at the current time instant.

The first boundary condition should express the fact that there is no movement of the points of the material layer being formed by winding which are attached to the surface of the mould, in the used coordinate system during the entire formation process. This means that

\[ f(R,R_{\text{mould}}) \equiv 0. \tag{16} \]

The second boundary condition for the desired function \( f(R,r) \) must be derived from the in Section 2 mentioned force conditions for attaching the material to the being formed solid. This can be done, for example, by calculating the full derivative with respect to \( r \) of the first of conditions (4):

\[ \frac{d}{dr} t_{rr} \bigg|_{R=r} \equiv 0. \tag{17} \]

Since, in accordance with (11), the component of the stress tensor is a function only of \( R \) and \( r \), then taking into account equation (14) as well as the first and the third conditions (4) we should write

\[ \frac{d}{dr} t_{rr} \bigg|_{R=r} = \frac{\partial t_{rr}}{\partial R} \bigg|_{R=r} + \frac{\partial t_{rr}}{\partial r} \bigg|_{R=r} - \frac{t_{rr}}{R} - t_{\phi\phi} \bigg|_{R=r} = \frac{\partial t_{rr}}{\partial R} \bigg|_{R=r} + \frac{t_{\phi\phi}^m(r)}{r}. \]

Thus, from (17) we obtain

\[ \frac{\partial t_{rr}}{\partial R} \bigg|_{R=r} = -\frac{t_{\phi\phi}^m(r)}{r}. \]
Substituting here the first relation (9) together with (8) and (10) and considering the result at \( r = R \), we obtain the required boundary condition on the moving boundary in the following mixed form:

\[
(\chi + 1) \left. \frac{\partial f(R, r)}{\partial r} \right|_{r=R} + \frac{\chi f(R, R)}{R} = -\frac{t^{\text{ini}}_{\phi\phi}(R)}{2\mu R},
\]

where the function \( t^{\text{ini}}_{\phi\phi}(r) \) is known and is given by formula (1).

Now the unknown function \( f(R, r) \) defining the deformation velocities field can be found from the solution of boundary problem (15), (16), (18) with the time parameter \( R \).

5. **Solving the boundary problem. Stresses in the being formed product**

The general solution of homogeneous differential equation (15) has the form

\[
f(R, r) = A(R) r + \frac{B(R)}{r},
\]

where \( A(R) \) and \( B(R) \) are arbitrary functions. By satisfying boundary conditions (16) and (18) we obtain the following system of algebraic equations for determining the functions \( A(R) \) and \( B(R) \):

\[
\begin{align*}
A(R) R_{\text{mould}} + \frac{B(R)}{R_{\text{mould}}} &= 0, \\
(2\chi + 1) A(R) - \frac{B(R)}{R^2} &= -\frac{t^{\text{ini}}_{\phi\phi}(R)}{2\mu R}.
\end{align*}
\]

We solve it and find

\[
A(R) = -\frac{R t^{\text{ini}}_{\phi\phi}(R)}{2\mu \left(2\chi + 1\right) R^2 + R_{\text{mould}}^2}, \quad B(R) = \frac{R_{\text{mould}}^2 R t^{\text{ini}}_{\phi\phi}(R)}{2\mu \left(2\chi + 1\right) R^2 + R_{\text{mould}}^2}.
\]

Adding (19) with (20) to (11)–(13), we find finally

\[
\begin{align*}
t_{rr} &= \left(\frac{R_{\text{mould}}^2}{r^2} + 2\chi + 1\right) \int_r^g \frac{R' t^{\text{ini}}_{\phi\phi}(R')}{(2\chi + 1) R'^2 + R_{\text{mould}}^2} \, dR', \\
t_{\phi\phi} &= t^{\text{ini}}_{\phi\phi}(r) - \frac{R^2_{\text{mould}}}{r^2} \left(\frac{2\chi}{2\chi + 1}\right) \int_r^g \frac{R' t^{\text{ini}}_{\phi\phi}(R')}{(2\chi + 1) R'^2 + R_{\text{mould}}^2} \, dR', \\
t_{zz} &= -2\chi \int_r^g \frac{R' t^{\text{ini}}_{\phi\phi}(R')}{(2\chi + 1) R'^2 + R_{\text{mould}}^2} \, dR'.
\end{align*}
\]

6. **Examples of controlling the characteristics of technological stresses distributions**

Based on results (21)–(23) obtained, it is possible to set and solve a number of problems on controlling the technological stresses \( t_{ij} \) that develop in the additively manufactured product under consideration.

Below we will consider the following problems as examples:

1) The problem on providing a required law of change of contact stress pressing the layer of material being formed in the process of winding to the mould during this process.
2) The problem on obtaining a required final distribution of circumferential technological stresses in the finished product.
In both problems, the law of changing the pre-stresses \( t^{\text{ini}}_{\phi\phi} \) in the wound sheet during the winding process is considered as the controlling impact on the additively formed material layer. In accordance with (1) this impact is determined by the specified programs of change in thickness \( \Delta_{\text{sheet}} \) of the sheet and in linear force \( F_{\text{sheet}} \) of the sheet tension. The relevance of the technological control problems formulated here was discussed in Section 2.

6.1. Solution of the first control problem

The law of change over “time” \( R \) of contact stress \( t_{\text{contact}}(R) \) on the surface of the mould used can be found from formula (21):

\[
t_{\text{contact}}(R) = t_{\phi\phi}\big|_{R=R_{\text{mould}}} = -2(\chi + 1) \int_{R_{\text{mould}}}^{R} \frac{R't^{\text{ini}}_{\phi\phi}(R')}{(2\chi + 1)R'^2 + R_{\text{mould}}^2} \, dR'.
\]

By differentiating (24) with respect to the variable \( R \) we find the desired law of controlled changing the preliminary circumferential stresses \( t^{\text{ini}}_{\phi\phi}(R) \) that provides the required law of change in the contact stress \( t_{\text{contact}}(R) \):

\[
t^{\text{ini}}_{\phi\phi}(R) = \frac{(2\chi + 1)R^2 + R_{\text{mould}}^2}{2(\chi + 1)R} \frac{dt_{\text{contact}}(R)}{dR}.
\]

Analysing result (25) obtained, we can notice, in particular, the following. Since it is true that \( R \geq R_{\text{mould}} > 0 \) for the outer radius \( R \) of the being formed layer, and the material constant \( \chi \) satisfies the inequality \( \chi > -\frac{1}{2} \) for all compressible materials (that is, with \( \nu < \frac{1}{2} \)), then with any stretching (i.e. positive) preliminary stresses \( t^{\text{ini}}_{\phi\phi} \) created in the wound sheet, the contact stress \( t_{\text{contact}} \) will strictly monotonically decrease throughout the entire winding process. And since the contact pressure onto the mould is equal to zero at the initial moment of this process, the contact stress will become negative (i.e. compressive) immediately after this moment and will continue to increase in its absolute value. This means that the mould will be pressed more and more strongly against the being wound layer of material over time.

6.2. Solution of the second control problem

Let it now be necessary to create some prescribed distribution of the final circumferential stresses \( t^{\text{fin}}_{\phi\phi}(r) \) in the finished product. Denote the final outer radius of the product by \( R_{\text{fin}} \). Then, according to formula (22),

\[
t^{\text{fin}}_{\phi\phi}(r) = t_{\phi\phi}\big|_{R=R_{\text{fin}}} = t^{\text{fin}}_{\phi\phi}(r) + \int_{r}^{R_{\text{fin}}} \frac{R't^{\text{ini}}_{\phi\phi}(R')}{(2\chi + 1)R'^2 + R_{\text{mould}}^2} \, dR'.
\]

Written relation (26) between a given function \( t^{\text{fin}}_{\phi\phi}(r) \) and the desired function \( t^{\text{ini}}_{\phi\phi}(r) \) offers a Volterra integral equation of the second kind in the unknown latter function. We can show that the solution of this equation has the form (we do not reproduce here the procedure for constructing this solution due to the limited scope of the article):

\[
t^{\text{ini}}_{\phi\phi}(r) = t^{\text{fin}}_{\phi\phi}(r) + \frac{(2\chi + 1)r^2 - R_{\text{mould}}^2}{(2\chi + 1)r^2 + R_{\text{mould}}^2} \int_{r}^{R_{\text{fin}}} \frac{R't^{\text{fin}}_{\phi\phi}(R')}{r} \, dR'.
\]
Analysing obtained result (27), we can notice, in particular, that the value of the stretching pre-stress $t_{\phi}^{ini}$ that is to be created in any considered elementary layer of the material attached during the winding process onto the surface of the being formed cylindrical product, to obtain the desired final distribution of the circumferential stresses $t_{\phi}^{fin}(r)$, is affected only by the values of the stresses $t_{\phi}^{fin}$ which have to act in the finished product in those of its elementary cylindrical layers that are to be attached after the considered one.

7. Conclusions
In this paper, a quasi-static model of mechanics of accreted deformable solids is proposed to analyse the process of slow multilayer power winding of a thin sheet of material onto the outer side surface of a pre-made sufficiently rigid circular mould. The mathematical formulation of the corresponding evolutionary boundary problem is given. This problem closed solution is constructed. The stress-strain state of the solid produced under the considered conditions is entirely determined by the preliminary stress state of the elementary layers of additional material sequentially attached to it. The latter state depends on the programs used to change the thickness of the wound sheet and the tension force applied to it during the winding process. The analytical dependences obtained in this work allow us to formulate and solve a number of problems on the technological control of the stress characteristics of the material layer being formed, which are acquired in this process, by proper setting these programs. The paper provides examples of solving such problems. The cases of controlling the contact stress on the mould surface from the side of the layer made on it and of controlling the distribution of the final circumferential stresses throughout the entire thickness of the finished layer are considered. Practical conclusions are made.

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