A finite element-based, integrated process model is presented for a three dimensional, coupled analysis of the thermal and metallurgical behavior of the strip occurring on the run-out-table in hot strip rolling. The validity of the proposed model is examined through comparison with thermal measurements. The model's capability of revealing the effect of diverse process parameters is demonstrated through a series of process simulation.

KEY WORDS: finite element; integrated process model; three dimensional; thermal and metallurgical behavior; run-out-table; hot strip rolling.
perature, due to the increase of undercooling. Note that as carbon content was increased, the latent heat due to bainite formation was decreased, while the latent heat due to pearlite formation was increased. The heat capacity and the latent heat thus predicted were used in predicting the strip temperatures.

Information regarding the other material properties such as thermal conductivity and density of each phase, which were also required for the heat transfer analysis, may be found in the reference.15)

2.2. Phase Transformation Model

Modeling of phase transformation that takes place during cooling of steels has attained considerable development through experimental investigations and analytic approaches. On the assumption that a new phase nucleates at the austenite grain boundary and its rates of nucleation and growth are independent of time, the isothermal kinetics of the decomposition of austenite can be characterized by the Avrami16) type equation:

$$ X/X^* = 1 - \exp(-mt^n) $$

(1)

Where $X$ is the fraction transformed, $t$ is the time, and $X^*$ is the thermodynamic equilibrium fraction, which can be determined from the equilibrium phase diagram at a given temperature and a chemical composition. The constant $m$ depends on the temperature and transformation mechanism, while the time exponent $n$ may be assumed to be a constant over the temperature range when a unique transformation mechanism operates. An isothermal transformation behavior can be characterized by determining these kinetic parameters, $m$ and $n$, from the Avrami plot of the transformation curve.

Equation (1) may be extended to describe the non-isothermal transformation behavior of the strip that takes place on ROT using an additivity rule, based on the theory proposed by Scheil.17) In the context of the additivity rule, the non-isothermal transformation kinetics can be described as the summation of a series of the small isothermal steps on the assumption that the phase transformation is an isokinetic reaction. The transformed phase fraction until the $i$th step, $X_i$, is then expressed as follows:

$$ X_i = 1 - \exp\{m(t' + \Delta t)^n\} = \frac{1}{m} \ln \left( 1 - \frac{X_{i-1}}{X_i^*} \right)^{-1/n} $$

(2)

where $t'$ is the equivalent transformation time needed to achieve the fraction of $X_{i-1}$ at the temperature of the $i$th step and $\Delta t$ is the time step.

The values of $m$ and $n$ for an Fe–C–Mn system, which were obtained from a series of non-isothermal continuous cooling experiments in conjunction with an inverse additivity technique,10,11) were shown in Table 1, for various chemical compositions of 0.03–0.87 wt% C and 0.2–
2.1 wt% Mn. The phase transformation of the steel on ROT in this investigation was calculated under the assumption of the original austenite grain size of 20 μm. It is well known that the transformation behavior may be affected by a retained strain, due to plastic deformation prior to phase transformation. However, under the assumption that the time interval between last stand of finishing mill and cooling start position on ROT is long enough for the deformed structure to be recrystallized, the effect of the retained strain on ROT was neglected in this investigation. As illustrated in Fig. 3, phase transformation behaviors predicted on the basis of the kinetic constants given in Table 1 were found to be in good agreement with the measurements.

3. Heat Transfer Model

A 3-D Eulerian FE model was developed for the analysis of the steady-state heat transfer occurring in the strip on ROT. The governing equation for steady-state heat flow in the strip during cooling is given by

$$\rho C_p u_i T_i = (k T_i)_{i} + \dot{Q}$$

where $u_i$ represents the components of the velocity vector, $\rho$, $C_p$, and $k$ are density, heat capacity and thermal conductivity, respectively, and $\dot{Q}$ represents the latent heat discharged during phase transformation. During phase transformation, $\rho$, $C_p$, $k$ and $\dot{Q}$ may be expressed as

$$\rho = \rho_l \sum_{i=1}^{4} \rho_l X_l$$

$$C_p = \sum_{i=1}^{4} C_{p_l} X_l$$

$$k = \sum_{i=1}^{4} k_l X_l$$

$$\dot{Q} = \sum_{i=1}^{4} \Delta H_i \frac{\Delta X_i}{\Delta t}$$

where $l = 1, 2, 3, 4$ denotes ferrite, pearlite, bainite, and austenite phase respectively, $X_i$ is the fraction of each phase, $\Delta H_i$ is the latent heat discharged during (complete) transformation of austenite into each phase, $\Delta X_i/\Delta t$ is the material time derivative of $X_i$. Note that, once the distribution of $X_i$ in the analysis domain is predicted by the phase transformation model, $\Delta X_i/\Delta t$ may easily be calculated by a finite difference scheme.

During ROT cooling the strip may be subject to various thermal boundary conditions, as follows:

$$T = T_{\text{in}}$$

$$k T_i n_i = -h_w (T - T_w)$$

$$k T_i n_i = -\sigma (T^4 - T_e^4) - h_e (T - T_e)$$

$$k T_i n_i = q$$

where $T_{\text{in}}$, $T_w$, $T_e$, $h_w$, $h_e$, $q$ represent the temperature prescribed surface, the surface where convection heat transfer occurs, the surface exposed to the environment, the heat flux prescribed surface, respectively.

A variational equation which can be derived from the above boundary value problem is

$$\int_{\Omega} \left( \rho c u_i (T_i \omega + k T_i \omega - \dot{Q} \omega) d\Omega + \int_{r_i} h_w (T - T_w) \omega d\Gamma \right)$$

$$+ \int_{r_e} (\sigma (T^4 - T_e^4) + h_e (T - T_e)) \omega d\Gamma - \int_{r_e} q \omega d\Gamma = 0$$

where $\omega$ is an arbitrary function. The finite element ap-

### Table 1. $m$ and $n$ values.

| Phase | $\ln (m)$ | $n$ |
|-------|-----------|-----|
| Austenite to Ferrite | -25.049 + (2.031 + 1.873) C + 2.349 (C) 1.29 + 0.867 Mn | 0.8647 1.750 C + 0.0583 Mn |
| Austenite to Pearlite | 17.359 + (5.021 + 2.977) C + 0.716 Mn | 1.5 |
| Austenite to Bainite | -1.604 + (1.95 + 1.22) C + 0.247 Mn | 1.5 |

Note: 1. $T_{\text{ex}}$: the eutectoid temperature, $T_{\text{start}}$: the temperature at which ferrite starts to form.
2. $C$ is the carbon content in % in the remaining austenite, deduced from the mass balance of carbon in the system. Note that $n$ is expressed as a function of $C$, in order to consider soft impingement due to carbon enrichment in the austenite during phase transformation.
3. AGS: the original austenite grain size ($\mu$m).

Fig. 3. Comparison between measured and calculated transformation behavior during continuous cooling of (a) 0.19 wt% C–0.56 wt% Mn and (b) 0.45 wt% C–0.49 wt% Mn steels.
proximation of the variational Eq. (12) involves approximation of \( u, T, \) and \( \omega \) by

\[
u=\sum_{K} N_{K} u_{K}(t) \quad (13)
\]

\[
T=\sum_{K} N_{K} T_{K}(t) \quad (14)
\]

\[
\omega=\sum_{K} N_{K} W_{K} \quad (15)
\]

where \( U_{K}, T_{K} \) and \( W_{K} \) denote \( u, T \) and \( \omega \) evaluated at nodal point \( K \), respectively, and \( N_{K} \) denotes the FE basis function.

In the strip, a large amount of heat is transported to the downstream by convection due to high processing speed. It is well known that when the convection term governs heat transfer more significantly than the diffusion term, solutions based on the standard Galerkin formulation are often corrupted by spurious node to node oscillations. To remove such a numerical instability, a proper modification of the standard Galerkin formulation was necessary. Details regarding such a modification may be found in the reference.\(^{18}\)

### 4. Integrated Process Model

As shown in Fig. 4, an integrated process model for the coupled analysis of the thermal and metallurgical behavior of the strip consisted of the aforementioned three basic models—thermodynamic model, phase transformation model, and steady-state heat transfer model. Note that the assumption of steady-state heat transfer may be justified as long as the process conditions such as finishing mill discharge temperature (FDT) and the strip speed, as well as the cooling pattern remain unaltered while the strip passes through ROT. Also note that the coupled aspect of the thermal and metallurgical behavior, which arises from the fact that the temperature changes and the changes in the fraction of each phase are interdependent due to the latent heat discharged during phase transformation, was rigorously treated by iterative computation of temperature and fraction transformed until convergence.

### 5. Process Conditions, Thermal Boundary Conditions, and Simulation Strategy

Investigated was the thermal and metallurgical behavior of the strip passing through ROT in POSCO no. 2 hot strip mill, Pohang works. FDT, thickness and width of the strip, and the strip speed on ROT for each of the four cases considered were summarized in Table 2, along with the chemical composition of the strip materials. As shown in Fig. 5(a), the ROT consisted of 15 water tanks with nozzles for laminar cooling on the top surface and for spray cooling on the bottom surface.

The ROT zone was divided into 15 sub zones directly under the tank and 14 sub zones between the two adjacent tanks, plus the entry zone (between the last finishing mill and the first tank) and the exit zone (between the last tank and the down coiler). The surface of each sub zone, which was subject to diverse thermal boundary conditions, was divided into the surface exposed to active nozzles, the wet surface (top surface only), the surface exposed to non-active nozzles (under a tank, and between the two tanks), and the surface exposed to air, as shown in Fig. 5(b). The heat transfer coefficients to be applied to such diverse locations were summarized in Table 3.

The computational procedure started with process simulation for the entry zone, followed by process simulation for the adjacent sub zone. The procedure was repeated until process simulation for the exit zone was completed. Figure 6 shows the finite element mesh adopted for process simulation for each sub zone. Since each sub zone was only a fraction of the entire ROT zone in size, the strategy of successive computation had a definitive advantage of substantially reducing the computation time required, compared to employing the entire ROT zone as the analysis domain.

### 6. Results and Discussion

Illustrated in Fig. 7(a) is the variation of the top and bottom surface temperatures of the strip with time, from the instant of emerging from the last mill stand to the instant of arriving at the down coiler. As may be seen from Table 3, the heat transfer coefficient \( h_{U} \) at the top surface of strip is greater than the heat transfer coefficient \( h_{W} \) at the bottom surface, which leads to the top surface temperatures being smaller than the bottom surface temperatures. However, the top surface temperature will be restored quickly after water cooling. Also, it may be seen that the temperature can be significantly underestimated (and consequently, estimation of fraction of each phase may be erroneous) when the discharge of the latent heat due to phase transformation is neglected. Predicted coiling temperatures (CT) were in excel-
Table 3. Heat transfer coefficients at top and bottom surface of the strip.

| Surface | Heat transfer coefficients | $\sigma \cdot T_{\text{em}} \cdot T_{\text{at}}$ |
|---------|---------------------------|------------------------------------------|
| Top     | $h_w = \frac{2186.7}{10^6} \left( \frac{T}{T_m} \right)^{0.7} \left( \frac{q}{U_w} \right)^{0.75}$ (W/mm² °C) | $\sigma = 5.6697 \times 10^{-6}$ (W/mm² °C) |
|         | $h_{\text{wet}} = \frac{200}{10^6} (2420 - 217.5 \times 75) / (T - 95)^{1.5} \left( \frac{N_{\text{act}}}{12} \right)^{1.5}$ (W/mm² °C) | $\epsilon = \frac{T}{1000} \left( \frac{1.125}{1000} - 0.38 \right) + 1.1$ |
|         | $T_{\text{at}} = \frac{T}{1000}$ | $T_{\text{em}} = T_{\text{at}} = 30$ °C |
|         | $T_{\text{at}} = \frac{T}{1000}$ | $T_{\text{em}} = T_{\text{at}} = 30$ °C |
| Bottom  | $h_w = \frac{267}{10^6} \left( \frac{T}{T_m} \right)^{0.7} \left( \frac{q}{U_w} \right)^{0.75}$ (W/mm² °C) | $h_{\text{wet}} = 2.95 \times 10^{-5}$ (W/mm² °C) |

Note: 1. $h_w^{10}$ and $h_w^{10}$ are the heat transfer coefficients of water cooling at top and bottom surface, respectively. $h_{\text{wet}}^{10}$ is the heat transfer coefficient at the wetting zone, and $h_{\text{wet}}^{10}$ is convective heat transfer coefficient of air cooling.

2. $\sigma^{10}$ and $\epsilon^{10}$ are Stefan-Boltzmann constants and emissivity, respectively.

3. Distribution of $B$ values along the width direction is shown in the figure.

Fig. 5. (a) Cooling pattern along ROT. The number in a box represents the number of active nozzles under each tank. Except No. 15 tank, which has 12 nozzles, each tank has 6 nozzles in total. (b) Thermal boundary conditions for each sub zone.
lent agreement with measurements, as shown in Fig. 7(b). The details of the changes in the temperature distributions at a cross section of the strip that may occur on ROT are illustrated in Fig. 8. Note that the temperature will be greater at the bottom surface than at the top surface in this case, indicating the need for increasing the number of the active nozzles for cooling of the bottom surface, to balance the heat flow. Also note that the temperature distributions are asymmetric along the strip width, due to asymmetric distribution of ‘β’ (see Table 3).

Two entirely different cooling patterns—cooling patterns I and II defined in Fig. 5(a), lead to substantially different temperature histories. For a high carbon steel, the histories of the top surface temperature and phase transformation predicted for the two different cooling patterns are shown in Fig. 9. The two cooling patterns show only 10°C difference in the coiling temperature. On the other hand, the phase evolution will be critically affected by the temperature history, resulting in markedly different final fraction of each phase, as shown in Fig. 10. It is predicted that at the coiling stage, the transformed Pearlite phase will reach about 90% for both two cases, however, when cooling pattern I is applied, the bainite phase will be formed in the strip, especially at the top surface where the temperature will be the lowest, while no bainite will be present when cooling pattern II is applied. Considering that the presence of the bainite phase would be detrimental to tensile strength and ductility of the product, cooling pattern II should be preferred. On the contrary, for a low carbon steel, even though the effect of the cooling pattern on the temperature history as well as on the phase evolution is clear, as shown in Fig. 11, the final fraction of each phase will be far less sensitive to the cooling pattern, as shown in Fig. 12. Not that the final fraction of each phase is asymmetric, either along the thickness or along the width of the strip, due to the difference in the heat transfer characteristics between the top and bottom surface, and due to the effect of ‘β’.

Fig. 6. Finite element mesh used for process simulation for each sub zone.

Fig. 7. (a) Variation of the top and bottom surface temperatures with time. The data are for a material particle at the center of the strip. (b) Top surface temperature distributions along the width of the strip at the down coiler (CT), predictions and measurements (case 1 was considered).

Fig. 8. Variation of temperatures (in °C) with time (case 1 was considered).
Fig. 9. (a) Variation of top surface temperature (in °C) with time; variation of phase with time for (b) case 3 and (c) case 4 (a high carbon steel was considered).

Fig. 10. Effect of cooling pattern on final fraction of each phase for a high carbon steel.
Fig. 11. (a) Variation of top surface temperature (in °C) with time; Variation of phase with time for (b) case 1 and (c) case 2. (A low carbon steel was considered).

Fig. 12. Effect of cooling pattern on final fraction of each phase for a low carbon steel.
7. **Concluding Remarks**

A finite element-based, integrated process model was presented for the three dimensional analysis of the thermal and metallurgical behavior of the strip occurring on ROT. The validity of the proposed model, which can rigorously deal with the interdependent nature between the thermal and metallurgical behavior, was discussed through comparison between the predicted temperatures and the measurements. Then, demonstrated was the capability of the model for revealing the effect of strip material and cooling pattern. In concluding, it is to be emphasized that the proposed model may serve as an effective tool for optimizing the diverse process parameters associated with cooling on ROT in hot strip rolling.

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