HOMOLOGY OF THE INTERSECTION SPACE ASSOCIATED TO THE UNIVERSAL IMPLODED CROSS SECTION OF SU(3)

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Abstract. We compute the homology of the middle perversity intersection space associated to the universal imploded cross section of SU(3), and show that it is different from its intersection homology as it is calculated in [8]. Moreover, we compute the homology of intersection spaces associated to the open cone and suspension over a simply connected, smooth, oriented manifold.

1. Introduction

In what follows we are going to compute the homology of the middle perversity intersection space associated to the universal imploded cross section of SU(3), denoted by \((T^*SU(3))_{\text{impl}}\). As shown in [8], \((T^*SU(3))_{\text{impl}}\) is homeomorphic to the open cone \(c^\circ(Y)\) where

\[
Y = \{(z, w) \in \mathbb{C}^3 \times \mathbb{C}^3 \mid z \cdot w = 0, |z|^2 + |w|^2 = 1\}
\]

is a compact Riemannian manifold of \(\dim_{\mathbb{R}}(Y) = 9\). Moreover, the homology groups of \(Y\) are given by:

\[
\tilde{H}_j(Y) = \begin{cases} 
\mathbb{R} & j = 4, 5, 9 \\
0 & \text{otherwise.} 
\end{cases}
\]

First we are going to prove a more general theorem:

Theorem 1.1. Let \(L\) be a simply connected, oriented smooth manifold of dimension \(l\), and let \(X = c^\circ(L)\). Assume that \(\overline{p}\) is an (extended) perversity. Then:

\[
\tilde{H}_j^\overline{p}(X) = \begin{cases} 
\mathbb{R} & 0 < j < l - \overline{p}(l + 1) \\
\tilde{H}_j(L) & \text{otherwise.} 
\end{cases}
\]

Throughout these notes the letter \(m\) denotes the lower middle perversity. The corollary below is a direct consequence of the previous theorem.

Corollary 1.2.

\[
\tilde{H}^m_j((T^*SU(3))_{\text{impl}}) = \begin{cases} 
\mathbb{R} & j = 5, 9 \\
0 & \text{otherwise.} 
\end{cases}
\]

Furthermore we will prove the following theorem related to the suspension over a smooth manifold:
Theorem 1.3. Let $L$ be a smooth, simply connected, oriented manifold of dimension $l$ and let $p$ denote an extended perversity. Then:

$$
\tilde{H}_j^{p}(\text{susp}(L)) = \begin{cases} 
\tilde{H}_{j-1}(L) & 0 < i < l - p(l + 1) \\
\tilde{H}_j(L) \oplus \tilde{H}_{j-1}(L) & j = l - p(l + 1) \\
\tilde{H}_j(L) & \text{otherwise},
\end{cases}
$$

where by $\text{susp}(L)$ we mean the suspension over $L$.

Remark 1.4. The middle perversity intersection homology of $(T^*SU(3))_{\text{impl}}$ is calculated in [8] and it is given by:

$$
I\tilde{H}_m^m((T^*SU(3))_{\text{impl}}) = \begin{cases} 
\mathbb{R} & j = 4 \\
0 & \text{otherwise}.
\end{cases}
$$

Comparing this with the result of Corollary 1.2, we observe that the homology theories $\tilde{H}_m$ and $I\tilde{H}_m$ do not agree on $(T^*SU(3))_{\text{impl}}$. More generally, given any smooth manifold $L$ and perversity function $p$, the perversity $p$ intersection homology groups of $c^\circ(L)$ are given by (page 58, [7]):

$$
I\tilde{H}_j^p(c^\circ(L)) = \begin{cases} 
\tilde{H}_j(L) & j < l - p(l + 1) \\
0 & \text{otherwise}.
\end{cases}
$$

By comparing this with the result of Theorem 1.1, we see that the homology theories $\tilde{H}_m^p$ and $I\tilde{H}_m^p$ usually do not agree on open cones over simply connected, smooth oriented manifolds.

Remark 1.5. When $X$ is a stratified pseudomanifold of dimension $n$ with an isolated singularity, the following formulas are available for $\tilde{H}_m^p(X)$ (page 221, [5]):

$$
\tilde{H}_m^p(X) = \begin{cases} 
\tilde{H}_j(M, \partial M) & j < k \\
\tilde{H}_j(M) & j > k
\end{cases}
$$

where $k := n - 1 - p(n)$ and $\overline{M}$ is the blowup manifold associated to the space $X$ (as defined in section 3). For the dimension $k$ homology, the following diagram with exact row and columns exists:

$$
\begin{array}{ccccccccc}
0 & \rightarrow & \ker(\tilde{H}_k(\overline{M}) \rightarrow \tilde{H}_k(\overline{M}, \partial M)) & \rightarrow & \tilde{H}_k(\overline{M}) & \rightarrow & I\tilde{H}_k(X) & \rightarrow & 0 \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
& & \tilde{H}_k(X) & & & & \im(\tilde{H}_k(M, \partial M) \rightarrow H_{k-1}(\partial \overline{M})) & & 0
\end{array}
$$
Previous remark provides a proof of Theorem 1.1. When \( X = c^\circ(L) \), the blowup manifold \( \overline{M} \) associated to \( X \) is equal to \( L \times [0,1) \) (as explained in Remark 3.3). In particular, notice that

\[
\tilde{H}_*(\overline{M}) = \tilde{H}_*(L),
\]

and

\[
\tilde{H}_*(\overline{M}, \partial\overline{M}) = \tilde{H}_*(\overline{M}/\partial\overline{M}) = \tilde{H}_*(c^\circ(L)) = 0.
\]

In section 4, we give an alternative proof of this theorem.

**Notations and conventions.** Throughout these notes \( p, q \) are considered to be extended perversities, which are just a sequence of integers ([4], page 10). By \( c^\circ(X) \), the open cone over a topological space \( X \), we mean the quotient space

\[
c^\circ(X) = \left( (0,1] \times X \right)/\sim = \left( (1,x) \sim (1,x') \right).
\]

On the other hand, by \( c(X) \) we mean the closed cone over \( X \). The notation \( I^pX \) stands for the perversity \( p \) intersection space associated to \( X \) (as introduced in [2]). The homology theory \( \tilde{H}_i^{I^p}(X) \) is defined by

\[
\tilde{H}_i^{I^p}(X) = \tilde{H}_i(I^pX; \mathbb{R})
\]

where by \( \tilde{H}_*(X) \) we mean the reduced (singular) homology of \( X \). Finally by dimension of a manifold we always mean its real dimension.

## 2. Symplectic Implosion

In this section we fix \( K \) to be a compact connected Lie group and \((M,\omega)\) a Hamiltonian \( K \)-manifold with equivariant momentum mapping \( \Phi : M \to \mathfrak{t}^* \). Moreover, we assume that \( T \) is a maximal torus of \( K \) and \( \mathfrak{t}^*_+ \) is the fundamental Weyl chamber in \( \mathfrak{t}^* \) with respect to a fixed polarization.

When taking a symplectic quotient at a value of the moment map which is not a regular value, the symplectic quotient is not symplectic. The imploded cross-section is designed to repair this so that we may replace the symplectic quotient by a stratified space where each individual stratas have a symplectic structure.

Define a relation \( \sim \) on \( \Phi^{-1}(\mathfrak{t}^*) \) by setting \( m_1 \sim m_2 \) if there exist \( k \in [K_{\Phi(m_1)}, K_{\Phi(m_2)}] \) such that \( km_1 = m_2 \). It turns out that this defines an equivalence relation on \( \Phi^{-1}(\mathfrak{t}^*) \). Indeed, by equivariance of the moment map \( \Phi \), \( m_1 \sim m_2 \) implies that \( K_{\Phi(m_1)} = K_{\Phi(m_2)} \) and therefore this relation is transitive.

**Definition 2.1.** The symplectic implosion \( M \), denoted by \( M_{impl} \) is:

\[
M_{impl} = M/\sim,
\]

equipped with the quotient space topology.

Considering the left action of \( K \) on itself, one can lift this action to a Hamiltonian action on the cotangent bundle \( T^*K \). Now the implosion of this \( K \)-manifold, \( (T^*K)_{impl} \), is called the universal imploded cross section of \( K \). The following theorem explains why this space is called "universal".
Theorem 2.2. ([6], Theorem 4.9) For any Hamiltonian $K$-manifold $M$, there exist an isomorphism

$$M_{impl} \cong (M \times (T^*K)_{impl}) \sslash_0 K,$$

where the symplectic quotients is with respect to the diagonal action of $K$.

As described in Example 6.16 in [6], the universal imploded cross section of $SU(3)$ has a structure of an irreducible affine complex variety which is given by

$$\{(z, w) \in \mathbb{C}^3 \times \mathbb{C}^3 \mid z \cdot w = 0\}.$$

This space has an isolated singularity at $(0, 0)$. It turns out that this space is homeomorphic to the open cone over compact connected Riemannian manifold

$$Y = \{(z, w) \in \mathbb{C}^3 \times \mathbb{C}^3 \mid z \cdot w = 0, |z|^2 + |w|^2 = 1\}.$$

The intersection homology of $(T^*SU(3))_{impl}$ is computed in [8]. This has been done by first computing the homology of $Y$ by a Mayer-Vietoris argument, and then applying the cone-formula for intersection homology (page 58, [7]) on the manifold $Y$.

3. Conifold transition, Blowup manifold and intersection space

Introduced by Banagl in [2], the method of intersection spaces provides an approach for studying Poincaré duality on singular spaces. Given a perversity $\overline{p}$, this approach associates a CW complex $I^pX$ to a certain class of singular spaces $X$. For our purposes, we only need to understand this construction in the case that $X$ is a Thom-Mather pseudomanifold of depth 1 with a trivial link bundle. The definition of Thom-Mather stratified spaces is given with more generality in [1]. The following definition appeared in [3]:

**Definition 3.1.** A depth one pseudomanifold $X$ with singularity $\Sigma$ is a pair $(X, \Sigma)$, where

1. $\Sigma$ is understood to be a closed subspace and a smooth manifold of codimension at least 2.
2. $X \setminus \Sigma$ is a smooth manifold which is dense in $X$.
3. $\Sigma$ possesses control data consisting of a tube $T \subseteq X$ around $\Sigma$ which is an open set in $X$ together with two maps:

$$\pi : T \rightarrow \Sigma$$
$$\rho : T \rightarrow [0, \infty)$$

such that $\pi$ is a continuous retraction and $\rho$ is a continuous distance function such that $\rho^{-1}(0) = \Sigma$. Moreover, it is required that $(\pi, \rho) : T \setminus \Sigma \rightarrow \Sigma \times (0, \infty)$ is a smooth submersion.

**Remark 3.2.** Notice that when $L$ is a smooth manifold, $c^\circ(L)$ is a depth 1 Thom-Mather pseudomanifold, with $v$ (the vertex of the cone) as its singularity. The link bundle of a depth 1 pseudomanifold is defined in Proposition 8.2 in [3]. In the case that $X = c^\circ(L)$, the link bundle is as follows:

$$L \rightarrow v.$$

Carefully following [4] we write down the construction of the conifold transition and the blowup manifold associated to $(X, \Sigma)$. 4
Take a tubular neighborhood $N$ around the singularity $\Sigma$ and fix a diffeomorphism 
\[ \theta : N - \Sigma \cong L \times \Sigma \times (0, 1). \]

Define the blowup manifold to be: 
\[ \overline{M} = (X - \Sigma) \cup_{\theta} (L \times \Sigma \times [0, 1]). \]

Notice that the blowup manifold is a manifold with boundary $\partial \overline{M} = L \times \Sigma$. Define the conifold transition to be: 
\[ CT(X) = \frac{(X - \Sigma) \cup_{\theta} (L \times \Sigma \times [0, 1])}{(z, y, 0) \sim (z, y', 0)} \quad (4) \]
for all $z \in L$, and for all $y, y' \in \Sigma$.

**Remark 3.3.** Following this construction, one can see that when the singularity $\Sigma$ is a point, $CT(X) = \overline{M}$ is a manifold with boundary $L$. In particular, when $X$ is $c^o(L)$ for some smooth manifold $L$, we have 
\[ CT(X) = \overline{M} \cong L \times [0, 1) \]
Next, we will describe how to construct $I^pX$, the perversity $p$ intersection space associated to $X$ when $(X, \Sigma)$ is a depth 1 pseudomanifold with simply connected link and trivial link bundle (as given in [4], page 8).

Let $l := \dim L$ and set $k := l - \overline{p}(l + 1)$. Assume $f : L_{<k} \rightarrow L$ is a stage $k$ Moore approximation of $L$ ([4], Definition 3.1). That is to say, $H_i(L_{<k})$ are 0 for $i \geq k$ and 
\[ f_* : H_i(L_{<k}) \rightarrow H_i(L) \]
is an isomorphism for $i < k$. Define the map $g : L_{<k} \times \Sigma \rightarrow M$ to be the composition: 
\[ L_{<k} \times \Sigma \xrightarrow{f \times \text{id}_\Sigma} L \times \Sigma = \partial \overline{M} \hookrightarrow \overline{M}. \]
The perversity $\overline{p}$ intersection space $I^pX$ is defined to be: 
\[ I^pX = \text{cone}(g) = \overline{M} \cup_{g} c(L_{<k} \times \Sigma). \quad (5) \]

**Remark 3.4.** In the case of $X = c^o(L)$, we have:
\[ I^pX = \frac{L \times [0, 1) \cup c(L_{<k})}{(f(x), 0) \sim (x, 0)} \]
where $f : L_k \rightarrow L$ is a stage $k = l - \overline{p}(l + 1)$ Moore approximation of $L$.

**4. PROOF OF THEOREM 1.1**

Following the Remark 3.3, we have 
\[ I^pX = L \times [0, 1) \cup c(L_{<k})/ \sim \]
where $k = l - \overline{p}(l + 1)$ and the equivalence relation is given by: 
\[ (x, 0) \sim (f(x), 0), \quad \forall x \in L_{<k}. \]
Here $f : L_{<k} \rightarrow L$ is a stage $k$ Moore approximation of $L$. Define two open sets $A$ and $B$ as follows:
\[ A = \frac{L \times [0, 1) \cup L_{<k} \times [0, \frac{1}{2} + \epsilon)}{(f(x), 0) \sim (x, 0)}, \]

\[ B = \frac{L \times [0, 1) \cup L_{<k} \times [\frac{1}{2} + \epsilon, 1)}{(f(x), 0) \sim (x, 0)}. \]

Here $f : L_{<k} \rightarrow L$ is a stage $k$ Moore approximation of $L$. Define two open sets $A$ and $B$ as follows:
\[ A = \frac{L \times [0, 1) \cup L_{<k} \times [0, \frac{1}{2} + \epsilon)}{(f(x), 0) \sim (x, 0)}, \]

\[ B = \frac{L \times [0, 1) \cup L_{<k} \times [\frac{1}{2} + \epsilon, 1)}{(f(x), 0) \sim (x, 0)}. \]
We observe that \( B \) is contractible, as it is the preimage of \([1, 1/2 + \epsilon)\) in \( c(L_{<k}) \) under the cone map sending \((x, y) \in [0, 1] \times L_{<k} \) to \([0, 1]\). Hence it is homeomorphic to the cone on \( L_{<k} \), so it is contractible. The set \( A \) is homotopy equivalent to \( L \), because the identification map \( f \) identifies each point in \( L_{<k} \) to a point in \( L \). Moreover, we observe that \( A \cap B \) deformation retracts to \( L_{<k} \), as \( A \cap B \) is homeomorphic to \( L_{<k} \times (1/2 - \epsilon, 1/2 + \epsilon) \).

Writing the Mayer-Vietoris sequence, we have the following:

Case I: \( j - 1 \geq k \)

In this case, the M-V sequence gives \( 0 \rightarrow \tilde{H}_j(L) \rightarrow \tilde{H}_j(I^p(X)) \rightarrow 0 \) (because \( \tilde{H}_j(L_{<k}) = \tilde{H}_{j-1}(L_{<k}) = 0 \)). Hence \( I^p(X) \cong \tilde{H}_j(L_{<k}) = 0 \).

Case II: \( j = k \)

The M-V sequence gives

\[
\ldots \rightarrow 0 = \tilde{H}_k(L_{<k}) \xrightarrow{f_*} \tilde{H}_k(L) \xrightarrow{\alpha_k} \tilde{H}_k(I^p(X)) \xrightarrow{\beta_k} \\
\ldots \rightarrow \tilde{H}_{k-1}(L_{<k}) \xrightarrow{f_*} \tilde{H}_{k-1}(L) \xrightarrow{\alpha_{k-1}} \tilde{H}_{k-1}(I^p(X)) \xrightarrow{\beta_{k-1}} \\
\ldots \rightarrow \tilde{H}_{k-2}(L_{<k}) \xrightarrow{f_*} \tilde{H}_{k-2}(L) \xrightarrow{\alpha_{k-2}} \tilde{H}_{k-2}(I^p(X)) \xrightarrow{\beta_{k-2}} \ldots
\]

The maps \( f_* : \tilde{H}_j(L_{<k}) \rightarrow \tilde{H}_j(L) \) are isomorphisms for \( j \leq k - 1 \). This implies \( \alpha_j = \beta_j = 0 \) for \( j \leq k - 1 \). Also \( \beta_k = 0 \), so \( \tilde{H}_k(I^p(X)) \cong \tilde{H}_k(L) \).

Case III: \( j < k \)

Since \( f_* : \tilde{H}_j(L_{<k}) \rightarrow \tilde{H}_j(L) \) is an isomorphism for \( j < k \), \( \alpha_j = \beta_j = 0 \) in this range. This implies the M-V sequence gives

\[
0 \rightarrow \tilde{H}_j(I^pX) \rightarrow 0
\]

which implies \( \tilde{H}_j(I^pX) = 0 \) for \( j \leq k - 1 \).

**Remark 4.1.** When \( X = c^p(L) \), \( I^pX \) deformation retracts to \( \text{Cone}(f) \) where by \( \text{Cone}(f) \) we mean the mapping cone of \( f : L_{<k} \rightarrow L \). Therefore Theorem 1.1 gives the homology group of \( \text{Cone}(f) \).

**5. Associated Intersection Space to a Suspension**

Let \( L \) be a smooth, simply connected oriented manifold of dimension \( L \). By \( X := \text{susp}(L) \) we mean the quotient space obtained from \( L \times [-1, 1] \) by collapsing \( L \times \{1\} \) to one point (denote this point by \( v \)), and \( L \times \{-1\} \) to another point (denoted by \( u \)). We observe that \((X, \{u, v\})\) is a depth 1 Thom-Mather pseudomanifold manifold with trivial link bundle

\[
L \times \{u, v\} \xrightarrow{\pi_2} \{u, v\}.
\]

Following the construction given in section 2, we have \( \bar{M} = CT(X) \cong [-1, 1] \times L \).

Fix a perversity \( p \) and set \( k := l - p(l + 1) \), then

\[
I^pX = L \times [-1, 1] \cup_g c(L_k \times \{u, v\}),
\]

where the map \( g \) is defined to be the composition

\[
L_{<k} \times \{u, v\} \rightarrow L \times \{u, v\} \xrightarrow{\cong} L \times \{-1\} \cup L \times \{1\} \hookrightarrow L \times [-1, 1]
\]

where \( f : L_{<k} \rightarrow L \) is a stage \( k \) Moore approximation of \( L \).
6. Proof of Theorem 1.3

Throughout this section, \( X = \text{susp}(L) \) where \( L \) is a smooth manifold satisfying the conditions given in Theorem 1.3. Moreover, we set \( k = l - \bar{p}(l + 1) \).

First we are going to prove the following lemma:

**Lemma 6.1.** For \( i > k \) we have
\[
\tilde{H} I^p_i(X) = \tilde{H}_i(L).
\]

**Proof.** The proof of this lemma is very similar to the proof of Theorem 1.1. Define two open sets \( A \) and \( B \)
\[
A = L \times [-1, 1] \sqcup L_{<k} \times \{u, v\} \times [0, 1/2 + \epsilon]/\sim
\]
where the equivalence relation is given by:
\[
(l, u, 0) \sim (f(l), -1) \quad (l, v, 0) \sim (f(l), 1).
\]

On the other hand,
\[
B = c(L_{<k} \times \{u, v\}) \setminus L_{<k} \times \{u, v\} \times [0, 1/2 - \epsilon)
\]
By similar reasoning as in the proof of Theorem 1.1, we see that \( A \) deformation retracts to \( L \), \( B \) is contractible and \( A \cap B \) is homotopy equivalent to \( L_{<k} \sqcup L_{<k} \).

For \( j > k \) the M-V sequence gives
\[
0 \rightarrow \tilde{H}_j(L) \rightarrow \tilde{H}_j(I^p(X)) \rightarrow 0
\]
as \( \tilde{H}(L_{<k}) \oplus \tilde{H}(L_{<k}) = 0 \) for \( j \geq k \). \( \square \)

**Lemma 6.2.** For \( i < k \)
\[
\tilde{H}_i(I^p X) = \tilde{H}_{i-1}(L).
\]

**Proof.** This time we cover \( I^p X \) with two different open sets \( C, D \) as follows:
\[
C = L \sqcup g c(L_{<k} \times \{u, v\}) \setminus L_{<k} \times \{u\} \times [1/2 - \epsilon, 1/2 + \epsilon],
\]
\[
D = L \sqcup g c(L_{<k} \times \{u, v\}) \setminus L_{<k} \times \{v\} \times [1/2 - \epsilon, 1/2 + \epsilon].
\]
Now we have that \( C \) and \( D \) deformation retract to \( \text{Cone}(f) \). Moreover, \( C \cap D \) is homotopy equivalent to \( L \). For \( i < k \) the M-V sequence with respect to the cover \( \{C, D\} \) gives
\[
0 \rightarrow \tilde{H}_j(I^p X) \rightarrow \tilde{H}_j(L) \rightarrow 0
\]
as \( \tilde{H}_j(\text{Cone}(f)) \oplus \tilde{H}_j(\text{Cone}(f)) = 0 \) for \( j < k \). \( \square \)

In order to complete the proof of Theorem 1.3, we need to calculate \( H_k(I^p X) \). Once again we consider the M-V sequence with respect to the cover \( I^p X = A \cup B \) given in the proof of Lemma 6.1.

\[
\begin{array}{cccccccc}
\cdots \rightarrow 0 = \tilde{H}_k(L_{<k}) \oplus \tilde{H}_k(L_{<k}) & \gamma_k \rightarrow & \tilde{H}_k(L) & \alpha_k \rightarrow & \tilde{H}_k(I^p(X)) & \beta_k \rightarrow \\
\cdots \rightarrow \tilde{H}_{k-1}(L_{<k}) \oplus \tilde{H}_{k-1}(L_{<k}) & \gamma_{k-1} \rightarrow & \tilde{H}_{k-1}(L) & \alpha_{k-1} \rightarrow & \tilde{H}_{k-1}(I^p(X)) & \beta_{k-1} \rightarrow \\ \\
\end{array}
\]

Using this diagram, we observe that \( \alpha_k \) is injective, as the top line of the diagram gives
\[
0 \xrightarrow{\gamma_k} \tilde{H}_k(L) \xrightarrow{\alpha_k} \tilde{H}_k(I^p X) \rightarrow \cdots
\]
The map $\gamma_i$ is given by

$$\gamma_i : \tilde{H}_i(L_{<k}) \oplus \tilde{H}_i(L_{<k}) \to \tilde{H}_i(L), \quad (\omega, \eta) \mapsto f_*(\omega) + f_*(\eta).$$

For $i < k$, $f_* : \tilde{H}_i(L_{<k}) \to \tilde{H}_i(L)$ is an isomorphism. This implies that $\gamma_i$ is a surjective map with

$$\ker(\gamma_i) = \{ (\omega, -\omega) | \omega \in \tilde{H}_i(L_{<k}) \} \cong \tilde{H}_i(L_{<k}) = \tilde{H}_i(L).$$

In particular we get $\ker(\gamma_{k-1}) \cong \tilde{H}_{k-1}(L)$. Now we can calculate $\tilde{H}_k(I^pX)$

$$\tilde{H}_k(I^pX) = \text{Im}(\beta_k) \oplus \ker(\beta_k) = \ker(\gamma_{k-1}) \oplus \text{Im}(\alpha_k) = \tilde{H}_{k-1}(L) \oplus \tilde{H}_k(L).$$

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