Hierarchical Eclipses

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Abstract. The obscuration of a celestial body that covers another one in the background will be called a “hierarchical eclipse”. The most obvious case is that a star or a planet will be hidden from sight by the moon during a lunar eclipse. Four objects of the solar system will line up then. We investigate this phenomenon with respect to the region of visibility and periodicity. There exists a parallax field constraining the chances for observation. A historic account from the Middle Ages is preserved that we analyse from different viewing angles. Furthermore, we provide a list of events from 0 to 4000 AD. From this, it is apparent that Jupiter is most often involved in such spectacles because its orbit inclination is small. High-inclination orbits reduce the probability to have a coincidence of an occultation of that object with a lunar eclipse.

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1 Introduction

Occultations of planets in the solar system by the moon receive particular attention in astronomical almanacs. They occur at semi-regular intervals of a few years’ time and, as such, they are quite common. In the 21st century there are a total of 59 cases for the classical naked-eye planets, 34 of which happen during daytime. Lunar eclipses are more frequent, and for a fixed location one can expect roughly 1 per year on average.

The impressive lunar eclipse of 27 July 2018 took place in the vicinity of Mars. This gave rise to the idea of a combination of both spectacles: a “hierarchical eclipse”. It will be characterised by four bodies placed in a straight line — Sun, Earth, Moon, and a planet. In this paper we put the “double overlaps”, as they would be seen from the sun, to a test. We try to gain insight into the following questions: Did this kind of consecutive covering of celestial objects happen in the historical past? In particular, are there reports about an eclipsed moon eclipsing another planet simultaneously? How often does such a line-up occur? When will the next opportunity be scheduled? Do exist cycles or, at least, accumulations for these hierarchical eclipses?

The analysis is based on an empirical list of events compiled for the years from 0 to +4,000 AD. We used the simulation software packages Guide 9.1 (2017), Cartes du Ciel 4.0 (2017), and Occult 4.6.0 (2018). The next section presents some few accounts from history that match our configuration. In Section 3 we check the tolerance for visibility, and in the subsequent section we discuss the frequency of the occurrences. Within the scope of our investigation, we achieved some instructive results.

For the sake of clarity, we use the following technical terms: The obscuration of a planet will be called “occultation” in order to distinguish it from the “eclipse” of the moon. The instant of disappearance of the planet behind the moon’s disk is labelled “immersion”, while its re-appearance is the “emersion”. The eclipse contacts with the earth’s umbra are assigned U1 to U4, as defined in the astronomical textbooks: begin of the umbral eclipse (U1), begin of totality (U2), end of totality (U3), and end of the declining phase (U4), respectively.

2 Records from History

One historic account about a hierarchical eclipse was passed down by the medieval priest Roger of Hoveden. His lifetime can only be limited to the years between 1174 to 1201 AD. His lifetime can only be limited to the years between 1174 to 1201 AD. In the relevant paragraph of his chronicle he reports about a “bright star” being involved in a peculiar scene with the eclipsed moon. The record for the year 756 AD reads as follows [3]:

On the eighth day before the calends of December (23rd November), the moon, on her fifteenth day, being about her full, appeared to be covered with the colour of blood, and then, the darkness decreasing, she turned to her usual brightness; but, in a wondrous manner, a bright star followed the moon, and passing across her, preceded her when shining, at the same distance at which it had followed her before she was darkened.

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The procedure is not correctly described from the astronomical point of view. The direction of the course is inverted, and the lunar eclipse took place one year earlier, in 755 AD. The “star” was Jupiter who was overtaken by the moon.
Figure 1: Jupiter was occulted by the lunar disk, which suffered an eclipse itself, on 23 November 755 AD. Simulated view from London.

while the latter was darkened in the earth’s shadow (Figure 1). The event goes back more than four centuries prior to Roger’s time. It appears only natural that scribal errors slip in, especially, when one of the copists in the chain of transmission failed to understand the meaning of the line himself. The original record does not seem to be preserved.

Another record, that would be contemporary with Roger’s life, closely missed a similar sight in Syria. The patriarch of the Orthodox Church in Aleppo, Michael Syrus (1126–1199), vividly depicts the nightmare of a solar eclipse on 11 April 1176. Thereafter he continues [4]:

Fifteen days after [the solar eclipse], in this month of Nissan (April) at the decline of Monday, at dusk, there was an eclipse of the Moon in the part of the sky where the eclipse of the Sun had taken place . . .

Of course, it took place in the opposite part of the sky. The eclipse was partial (mag = 0.673) with the Northern rim unobscured. Above the non-eclipsed edge, Jupiter was shining. The cleric must have regarded the bright spot as a usual star and did not mention it. Somewhat further to the South that spot was occulted, e.g. in Kenya and, even more, in South Africa.

Another great occasion to watch a hierarchical eclipse occurred shortly before the telescopic era. On 26 July 1580 the eclipsed moon met even two planets, Saturn and Uranus. An observer in Japan could have seen the disappearance of Saturn, while someone in Northern Australia the same of Uranus. Only on the small Indonesian island of Koror both planets were occulted, though not simultaneously. The partially eclipsed moon ran over Uranus first, and 10 minutes after its emersion, the occultation of Saturn began. The subsequent lunation provided an occultation of both planets at the same time, but without the eclipse then. However, in that year no-one knew about the existence of Uranus, and telescopes were not invented yet.

As regards modern times, the sole chance to have had a glimpse of Saturn being covered during an eclipse was on 14 December 1796. It was visible in East Asia, but any remark about an observation is not known to us.

3 The View from Earth

The occultation of any background object during a lunar eclipse implies its opposition with the sun. As a matter of fact, Mercury and Venus are excluded.

At first, let us imagine a fictitious observer in the center of the earth looking through a vitreous sphere. The shadow of the earth (umbra) has an apparent radius as large as 1.3° at the distance of the moon. The geocentric observer would see the maximum duration when the planet is placed exactly in the ecliptic and the moon traverses the shadow centrally. Then we have the longest totality, and the moon crosses over the planet alongside its diameter, also providing the longest occultation possible. Such a configuration almost never happens in practice. Usually, the moon passes through the shadow at some displacement from the center, and the planet will hide along a cord behind the moon’s face.

For the other extreme, the moon just scratches both the planet and the umbra with its diametrically opposite fringes. On one edge it would cover the shining dot for a moment and, on the other, it touches the umbra on its antipode. This adds an angle of 0.45° to 0.52° to the extent of the terrestrial shadow, depending on the current distance of the moon, since the size of its disk varies between perigee and apogee. For the planet, it gives ≈ 1.8° of tolerance to stay above or below the ecliptic (Figure 2).

In contrast to that geocentric observer, the real viewer has the advantage to move on the surface of the earth. His topocentric position gains a parallax, as it would be seen from the moon (cosine of his geographical latitude). At the
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Figure 3: Region of visibility for the Jupiter occultation of 755 AD.

poles this accounts for another $\approx 15'$. On the whole, we find the parallactic field of view to be 4 times larger than the diameter of the moon. To ensure the hierarchical eclipse to be seen from some spot on earth (topocentric), the planet must stay inside this parallactic circle (geocentric). If the planet is outside, there will be no point on the globe for the hierarchical eclipse.

Figure 2 shows the field of tolerance for Roger’s occultation event. From the geocentric view, four instants are presented:

(a) At 18:40 UT, the circle reached Jupiter. On the surface of the earth, both the planet and the moon were rising in Dakar, West Africa. The planet was standing already in the umbra, and the occultation of the planet (its immersion) could be seen during totality there. However, from the location of London, Jupiter resided in the penumbra, while the moon was still running towards the planet.

(b) At 19:37 UT, the geocentric observer would have seen the planet closer to the darkened face, while in London its immersion occurred. But the totality has already ended at 19:25 UT, such that the occultation started during the decreasing partial phase. Jupiter entered the umbra, while covered, and returned for visibility (emersion) after the lunar eclipse had fully ended, i.e. beyond the U4-contact.

(c) Going even further eastward, the occultation of Jupiter would be observed from another angle, e.g. in Perm in the Ural: immersion as well as emersion took place with Jupiter standing in the penumbra. The terrestrial shadow crept up slowly towards the planet, while the moon was overtaking it. When the moon reached the planet, the eclipse was close to finish (penumbra neglected).

(d) For Beijing it was not until 22:40 UT as the moon caught up with the shining dot, and the whole procedure passed off sequentially instead of simultaneously.

Note that the fictitious observer at the center of the earth would not have seen any occultation at all. It is a pure effect of the extent of the spherical earth that the hierarchical eclipse happened. The region of visibility is shown as the grey-shaded area in Figure 3. East of the yellow line the total eclipse ended (U3-contact), and the red line marks the border of the finish of the partial phase (U4-contact).

A useful insight is that there are no fixed times for immersion and emersion, but they depend on the location of the observer. The occultation occurs at different times in spite of having deployed the same time frame for reference (like the UT). With regard to Earth’s shadow, the planet seems to stay at different positions in the sky. While the eclipse is in progress, the umbra of the earth moves on, too, reducing the deviation for an eastward observer.
Table 1: Eclipsed moon occults a planet as visible for a “good” observing site. Times in [UT].

| Date [AD] | Ecl. U1 – U4 | Magn. | Planet | Bri. | I – E | Best visibility |
|-----------|--------------|-------|--------|------|-------|-----------------|
| 2 Nov 8   | 21:26 – 23:59 | 0.46  | Mars   | -1.8 | 21:58 – 23:02 | W-Brazil        |
| 195 Jul 10| 23:41 – 03:36 | 1.71  | Saturn | 0.6  | 03:35 – 04:42 | Central Pacific |
| 345 Dec 16| 12:28 – 15:50 | 1.33  | Saturn | -0.4 | 15:27 – 16:22 | E-Africa        |
| 400 Dec 17| 17:05 – 20:27 | 1.06  | Jupiter | -2.7 | 19:46 – 21:11 | Seychelles      |
| 412 Nov 4 | 18:22 – 22:06 | 1.60  | Mars   | -1.9 | 20:06 – 21:05 | S-Africa        |
| 458 Nov 6 | 21:16 – 00:11 | 0.80  | Jupiter | -2.8 | 21:10 – 22:16 | Caucasus        |
| 480 Sep 5 | 03:25 – 06:19 | 0.60  | Jupiter | -2.9 | 03:21 – 03:58 | Hudson-Bay      |
| 513 Jun 4 | 08:02 – 11:50 | 1.34  | Jupiter | -2.7 | 07:23 – 08:39 | Bolivia         |
| 524 May 3 | 16:25 – 20:02 | 1.65  | Jupiter | -2.6 | 19:59 – 20:37 | S-Pole          |
| 755 Nov 23| 16:49 – 20:37 | 1.40  | Jupiter | -2.8 | 19:37 – 20:53 | Europe          |
| 771 Feb 4 | 08:29 – 11:32 | 0.93  | Saturn  | 0.3  | 11:27 – 12:25 | Tasmania        |
| 799 Jul 10| 13:47 – 17:30 | 1.56  | Saturn  | -0.3 | 12:44 – 14:29 | Kazakhstan      |
| 821 May 20| 18:30 – 22:14 | 1.41  | Saturn  | 0.6  | 19:59 – 20:37 | Madagascar      |
| 879 Apr 10| 09:29 – 12:52 | 1.36  | Saturn  | 0.1  | 12:00 – 12:43 | S-Pole          |
| 959 Jun 23| 06:37 – 09:44 | 0.94  | Saturn  | 0.1  | 09:42 – 10:30 | Antarctica      |
| 1052 Dec 8| 20:38 – 00:07 | 1.65  | Saturn  | 0.2  | 19:55 – 20:29 | Caribbean       |
| 1176 Apr 25| 17:43 – 20:46 | 0.67  | Jupiter | -2.5 | 20:20 – 20:29 | Indian Ocean    |
| 1234 Mrc 17| 02:06 – 04:51 | 0.65  | Jupiter | -2.5 | 05:23 – 05:43 | Patagonia       |
| 1312 Jun 19| 18:05 – 21:06 | 1.55  | Saturn  | 0.1  | 20:05 – 21:11 | Namibia         |
| 1407 Nov 15| 11:05 – 14:25 | 1.19  | Saturn  | 0.2  | 11:16 – 11:44 | N-Siberia       |
| 1418 Oct 14| 20:15 – 23:53 | 1.12  | Saturn  | 0.2  | 22:40 – 22:48 | N-Canada        |
| 1462 Jun 12| 00:32 – 03:17 | 0.59  | Saturn  | 0.2  | 03:00 – 03:58 | Antarctica      |
| 1531 Apr 1 | 18:05 – 19:24 | 0.11  | Saturn  | 0.1  | 19:01 – 20:03 | S-Africa        |
| 1580 Jul 26| 09:27 – 12:47 | 1.26  | Saturn  | 0.3  | 10:52 – 11:58 | Japan (+ Uranus!) |
| 1591 Dec 30| 02:12 – 05:45 | 1.57  | Saturn  | 0.4  | 11:00 – 11:45 | Alaska          |
| 1796 Dec 14| 13:05 – 15:27 | 0.49  | Saturn  | 0.3  | 14:52 – 15:55 | Siberia         |
| 2344 Jul 26| 10:40 – 12:41 | 1.34  | Saturn  | 0.1  | 12:20 – 13:44 | N-Pacific       |
| 2429 Jun 17| 10:42 – 11:10 | 0.02  | Saturn  | 0.1  | 10:58 – 11:58 | New Zealand     |
| 2488 Apr 26| 07:42 – 11:02 | 1.38  | Mars    | -1.6 | 07:08 – 08:15 | Antarctica      |
| 2829 Jan 11| 01:41 – 05:33 | 1.81  | Saturn  | 0.4  | 05:26 – 06:40 | N-Pacific       |
| 2932 Jun 9 | 22:13 – 23:50 | 0.21  | Jupiter | -2.6 | 22:32 – 23:38 | E-Brazil        |
| 2977 Jan 26| 07:03 – 10:49 | 1.65  | Saturn  | 0.4  | 07:37 – 08:48 | S-Mexico        |
| 2990 May 1 | 23:57 – 01:09 | 0.09  | Saturn  | 0.2  | 00:26 – 01:21 | S-Chile         |
| 3108 Jun 15| 06:26 – 08:57 | 0.44  | Saturn  | 0.0  | 06:17 – 07:47 | French Polynesia|
| 3218 Jul 30| 00:00 – 02:23 | 0.46  | Saturn  | 0.0  | 00:06 – 01:05 | Madagascar      |
| 3229 Jun 28| 16:30 – 19:20 | 0.55  | Saturn  | 0.2  | 18:03 – 18:48 | Madagascar      |
| 3287 May 19| 14:30 – 17:32 | 0.88  | Saturn  | 0.2  | 14:36 – 15:27 | Fr. S-Antarctica|
| 3376 Aug 21| 18:58 – 20:39 | 0.20  | Saturn  | 0.4  | 19:06 – 20:33 | Maldives        |
| 3444 Dec 17| 16:17 – 19:07 | 0.59  | Saturn  | 0.0  | 17:13 – 17:30 | Arctic          |
| 3461 Jul 14| 22:35 – 02:12 | 1.62  | Saturn  | 0.0  | 22:10 – 22:41 | Antarctica      |
| 3584 Jun 6 | 12:52 – 16:32 | 1.16  | Saturn  | 0.2  | 14:26 – 14:58 | Madagascar      |
| 3805 Jan 28| 08:12 – 11:19 | 0.93  | Saturn  | 0.2  | 08:30 – 09:17 | Central Chile   |
| 3815 Dec 29| 12:28 – 15:37 | 0.89  | Saturn  | 0.2  | 13:19 – 13:58 | Svalbard        |
| 3826 Nov 27| 15:22 – 18:25 | 0.65  | Saturn  | 0.2  | 17:40 – 18:36 | Svalbard        |
| 3870 Jul 26| 00:20 – 03:57 | 1.48  | Saturn  | 0.2  | 00:33 – 01:34 | Seychelles      |
| 3881 Jun 25| 08:28 – 12:04 | 1.82  | Saturn  | 0.2  | 09:26 – 10:10 | S-Australia      |

### 4 Frequency of Hierarchical Eclipses

Table 1 lists all hierarchical eclipses we were able to find between 0 and 4,000 AD. We make no claim for completeness. Dates before 1582 are Julian, thereafter Gregorian. Considered are only cases with at least one planetary contact (immersion or emersion) inside the time interval for the eclipse, which is given in column 2: between U1 and U4. The magnitude in column 3 denotes the maximum eclipse. If below 1.0, the eclipse is partial. Columns 4, 5, and 6 give
the name of the planet, its brightness, and the time of immersion (I) and emersion (E). These latter times correspond to a “good” site of visibility in the last column 7. It may not necessarily be an ideal spot, but it would be very close to it. The star (*) in the first column indicates the hierarchical coverage for the fictitious position at the center of the earth.

It is informative to discover that only four (*) of 49 incidents could be seen from the geocentric point. The others are attributed to the extended surface of the earth. That is to say, we observe them, because one is placed at a certain parallax somewhere on the globe. As the case would be, one could witness either the eclipsing hierarchy, or a simple occultation of the planet, or even nothing. The lunar eclipse, though, is visible in the same way for anyone having the moon above the horizon.

Figure 4 shows a histogram of the data for each century. Hierarchical eclipses seem to be irregularly distributed. There are intervals of accumulation, but we live in an extraordinary long interval of shortage. Jupiter was prominent in the first millennium, while Mars is very rarely occulted, in general. A strict periodicity cannot be extracted for any planet, but some repetitions and gaps do catch attention. The reasons for it rest upon the characteristics of the planetary orbits, as Meeus et al. point out: inclination and eccentricity. The period of eclipses, which is governed by the draconitic period of the moon, is also essential.

For obtaining a cycle, three periods need to be considered. The synodic month of the moon has to be an integer, otherwise there is no full moon. The draconitic month has to be half its number, otherwise there will be no eclipse. And, thirdly, the synodic revolution of the planet must be an integer, too, in order to meet the opposition. All periods are incommensurable and carry a minute displacement against the other on the long run. Here we sketch the qualities briefly for each planet, but a more precise mathematical treatment is still pending.

Mars has an orbital inclination of 1.85° to the ecliptic, but it exhibits the largest departure at the extremes, as seen from Earth. There are only two small windows, each of about 50° centered on the nodes with the ecliptic, in which the planet crosses this reference plane within a favourable latitude accessible for a lunar eclipse. So, the hierarchical eclipse can only happen in the days of April/May and from October to the beginning of December. For the other months the ecliptic latitude of Mars will be too high or too low, and, thus, out of reach for the parallactic field.

A secondary obstacle regards the ellipticity of its orbit. The velocity will not be uniform owing to the perihelion and aphelion. Therefore, this gives rise to an advance or retard as compared to a circular orbit. This means that the lunar node could fail to catch the planet at a convenient instant although the conditions are fulfilled. The “period” for a recurrence, if it exists, will possibly be valid for a small piece of its orbit only, unless extremely long time scales are envisioned.

Jupiter displays the smallest deviation from the ecliptic and can be occulted by an eclipsed moon in any season. This planet is most often involved in hierarchical eclipses, as Figure 4 confirms. However, there are episodes of abundances as well as paucities. Two slight periods of 10.9 and 57.9 years flash up, and they would be more prominent, if the orbit was circular. On the other side, these two quasi-periods have to be merged with the advancing difference with respect to the lunar node. The revolution of the lunar node is controlled by the precession of the moon’s plane, and should comply with half its number of the draconitic period which is 18.61 years.

Again, both series above only satisfy the conditions for a small arc of Jupiter’s orbit. If the incident comes about close to perihelion, then the 11-year period can hold for one or two more chances as in 799–810–821. When the
minute differences on subsequent “hits” have accumulated, the series tears off and there are no hierarchical eclipses for several centuries. See [1] for details.

**Saturn** is an intermediate case. The inclination of its orbit is 2.49° but the planet’s outward distance makes the vertical elevation from the ecliptic appear ±2.3° at maximum. For a few weeks in spring and autumn the planet is beyond the threshold for eclipses, and the hierarchy is suspended. An asset is that Saturn changes its position along the orbit quite slowly and stays inside the admissible belt for lunar eclipses for several years. The conditions seem to provide a larger stability, but we confess not having checked this in detail. We were not able to identify a period for Saturn, for all its hierarchical eclipses seem to proceed at random.

**The Moon** itself is liable to the extent of the circular parallactic field. Its elliptical orbit around Earth brings the anomalistic month as another period into consideration. This type rules the exact size of the circle and determines whether or not the planet will be positioned inside or just slightly outside the ring. The circle “pulsates” in the rhythm of the anomalistic month.

For extremely long timescales, the eccentricity of each orbit varies as well. This holds even for the earth itself. Taken all these factors into account, there will hardly be a cycle of a stable nature. Anyway, all periods turn out incommensurable in the Solar System, while the purpose of any search for periods is usually to find an approximation as good as possible.

5 Anti-Transits

In closing, let us change perspectives. If the moon is able to cover a background object, then the sun will do so as well. Taking planets as targets, they will hide behind the solar disk. One may call this an “anti-transit”. As a matter of principle, it is unobservable, and our examination becomes just an academic question.

In order to have this state of affairs hierarchically, the sun needs to be covered, too, i.e. the moon will be the obvious object to trigger a solar eclipse while the planet is anti-transiting. To achieve that, the planet has to be in the superior conjunction and stay close to one of its nodes. The maximum ecliptic latitude allowed is ±15°, since this is the apparent radius of the sun. Mercury and Venus can join our consideration again. The duration of the anti-transit depends on the relative speed between the sun and the planet. The sun traverses its own diameter in about 12 hours, however. Mercury and Venus move faster than the sun at their superior conjunction and may stay longer than a day behind it, if the passage is central.

If you think, it is too weird, you’re wrong. It happens from time to time, most recently at the total eclipse of 30 June 1954 [2]. Jupiter stepped behind the sun at 10 UT that day and remained obscured for the next 17 hours.

The moon entered the playground around midnight (Southern Scandinavia) and caused an eclipse of the sun between 11 and 14 UT. Thus, two celestial objects were deprived from sight for the observer on Earth. The next opportunity is envisaged for 14 May 2105, when Mercury will perform its anti-transit and a partial solar eclipse will have its stage.

6 Conclusions

We presented the rare phenomenon of an “eclipse-occultation” when a planet at opposition is eclipsed by the moon which, in turn, is eclipsed by the earth. The example of Roger de Hoveden showed that Jupiter’s disappearance occurs at different times for different places, though the same time frame is used. This effect is due to the parallax for the observers on the surface of the earth. In contrast to that, the lunar eclipse for all observers occurs at the same instant.

As known since Antiquity, lunar eclipses can be utilised to measure the time difference between two places on Earth, if they are widely spaced in geographical longitude. In fact, this method was employed in old times for localising time zones or synchronising clocks.

Occultations by the moon can be used to determine its position and speed in the sky. Hence, they reveal the secular acceleration that is based on the exchange of angular momentum between Earth and Moon. For an evaluation of those observations and results, the geographical position of the observer is relevant. The method fails to work when documents from various (unknown) cultural regions are compared, because the occultation takes place at different times, even if a common time scale like the “UT” is used. In case of known places of observation, still a correction procedure has to be applied.

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