Tetrahedral symmetry in nuclei: search for its fingerprints in the Actinide and Rare-Earth regions

D Curien, J Dudek and K Mazurek

1 Institut Pluridisciplinaire Hubert Curien, UMR7178, IN2P3-CNRS and Université de Strasbourg
23 rue du Loess BP 28 - 67037 Strasbourg Cedex 2, France

2 Niewodniczanski Institut of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, Pl-31-342 Krakow, Poland and Grand Accélérateur National d'Ions Lourds
Bd Henri Becquerel BP 55027 - 14076 CAEN Cedex 05, France

E-mail: Dominique.Curien@ires.in2p3.fr

Abstract. In a series of recent theory articles, predictions have been formulated suggesting the existence of nuclear states whose mean-field Hamiltonians and thus the implied geometrical shapes, are characterized by tetrahedral symmetry. Following these publications, a series of experiments for the Rare-Earth region have been proposed and performed. In this article, we shortly summarize the theory evolution developed in the original articles and discuss the status of the issue in light of the preliminary results of the ongoing experimental analyses. More recent theoretical results cross checked with extended literature investigations, suggest that the Actinide region might contain the best experimental candidates to investigate the fingerprints of the symmetry. A proposition is made to prove it via lifetime measurements with the so-called "microwave method".

1. Tetrahedral symmetry - theoretical background and experimental signs

The existence of the so-called high-rank symmetries in nuclear structure physics has been predicted in the framework of the realistic mean-field theory with the universal Woods-Saxon potential and confirmed by using Skyrme-Hartree-Fock-Bogolyubov method [1-8]. The term high-rank refers to specific point-groups symmetries: the tetrahedral and octahedral ones. They are the only symmetries leading to four-fold degeneracies of single-nucleonic levels in deformed nuclei: all other symmetries of deformed nuclei studied in nuclear physics lead only to the two-fold Kramer degeneracies. It can be shown that a classical tetrahedral-symmetric surface (pyramid shape) is invariant with respect to 24 symmetry elements such as fixed-angle rotations and plane reflections and that the octahedral-symmetric surface (diamond shape) is invariant with respect to 48 different symmetry elements. The importance of these symmetries has been deduced from the fact that they can lower the total energy of the nucleus by a few MeV [9].

As a critical consequence, the tetrahedral symmetry helps to produce large shell gaps in the single particle spectra because the degenerate levels "shrink" together [1,2,6] leading to specific tetrahedral shell-closures at the tetrahedral magic numbers: 32, 40, 56, 64, 70, 90, 112, and 136. The presence of...
those magic numbers produces low-lying minima in the nuclear potential energy surfaces that correspond to tetrahedral shapes. These "central" tetrahedral magic nuclei are accompanied by a large number of neighbouring nuclei that form tetrahedral islands throughout the Periodic Table in analogy to the presence of islands of spherical nuclei distributed around combinations of 8, 20, 28, 50, 82, 126 spherical magic numbers. The increased degeneracies in the mean-field spectra of deformed nuclei may lead to increased stability of the corresponding nuclear shapes: this may have particularly important implications for the exotic-nuclei projects. The discovery of tetrahedral and/or octahedral symmetries would then represent a critical step forward in nuclear structure physics.

The calculation results show clearly that the tetrahedral symmetry is responsible for strong shell effects in the Rare-Earth \cite{9,10} and Actinide regions. It can be shown that nuclei with the exact tetrahedral symmetry have all multipole moments equal to zero (except for the $Q_{32}$, the "triaxial octupole" one) and in particular all the quadrupole moments $Q_{2} = 0$. This implies in particular the existence of very exotic rotational bands: because the tetrahedron is a non-spherical shape, nuclei with such shapes can rotate. The corresponding rotational sequences will be characterized by the "usual" $E_{i} - I(I+1)$ energy-spin dependence. But unlike in the "traditional" rotational bands, the $E2$-transitions commonly thought to be synonymous with collective rotation, are predicted either to vanish completely or to be very weak since the quadrupole moments in the exact symmetry limit must vanish. As a consequence the first experimental fingerprints that one can think of are the gamma decay $B_{γ}(E2)/B_{out-out}(E1)$ branching ratios and their evolution with spin along the band. Another important point is that the tetrahedral symmetry is modelled, within the lowest multipolarity spherical harmonics of order $\lambda = 3$ (octupole) and therefore the associated rotational states may carry the negative parity in full analogy to other octupole states.

Figure 1 illustrates the results of the macroscopic-microscopic phenomenological method calculations using the Woods-Saxon Hamiltonian with the "universal-compact" set of parameters as in Refs. \cite{6} and \cite{10}. Comparison shows that the minimum at $\alpha_{32} \approx 0.13$ corresponds to $\alpha_{20} = 0$ and thus to the tetrahedral symmetry. If the measurements show that the quadrupole moment of the underlying rotational band is close to zero then the most likely interpretation would be the tetrahedral rather that pure "pear-shape" octupole deformation since (cf. right-hand of the figure) there is no axial-octupole minimum predicted by theory in this case. Of course, the next step will be to look for unambiguous criteria based on the branching ratios calculated with the help of the group theory methods. But the first condition, necessary although not sufficient, is to verify whether the quadrupole deformations in the discussed states are close to zero.

**Figure 1.** Total energy surfaces in function of the tetrahedral deformation $\alpha_{32}$ (left) and axial "pear shape" octupole deformation $\alpha_{30}$ (right) vs. quadrupole deformation $\alpha_{20}$. Observe that according to calculations a tetrahedral minimum at $\alpha_{20} = 0$ is not accompanied by a "pear-shape" minimum (no competition between the two "octupole" type deformations).
Traditionally, the negative parity bands that are not identified as based on particle-hole excitations have often been interpreted in terms of the octupole degrees of freedom, the latter understood as $\alpha_{30}$ i.e. axially symmetric deformation. These effects take the form of either pear-shape static-deformed minima on the potential energy surfaces or as pear-shape vibrational-bands (called octupole-vibrational for short). To be more precise, according to such a traditional interpretation the nucleus remains quadrupole-deformed in its ground-state, whereas its one-phonon negative-parity octupole-excitation gives rise to the associated band-head. The latter collects then the rotational transitions from the rotational-states above. According to this scenario, the presence of the quadrupole-deformed equilibrium implies that the quadrupole moments and thus the implied rotational collective E2 transitions associated with those band heads should be present. However, as pointed out above, in many situations such E2 transitions seem to be clearly missing what puts in doubt this traditional interpretation in terms of the octupole-vibrational bands as always valid. Our approach adds a new interpretation without negating the validity of the possible two traditional interpretations of the collective octupole-mode related bands. More detailed calculations (which will be reported elsewhere) show that in fact, when the neutron number increases, in many actinide nuclei the pear-shape susceptibility sets in, in the otherwise quadrupole-deformed ground-state minima, with an increasing chances for the traditional interpretation in terms of the low-lying octupole $\alpha_{30}$-vibrations. This tendency is accompanied, in the calculations, by a weakening of the tetrahedral symmetry minima through the lowering of the corresponding potential energy barriers (the tetrahedral minima vanish with increasing N while being relatively strong for the lighter isotopes). Detailed calculation results will be published.

In fact the existing literature contains already reports about several collective rotational bands of the above properties in the Rare-Earth and Actinide nuclei in an excellent correspondence to the predictions summarized above. In particular Refs. [10] and [11] suggest that tetrahedral symmetry has probably been seen in experiments in the early 1980’s presenting a kind of curiosity whose interpretation was left out then. These rotational bands were simply interpreted in terms of octupole vibrational structures ignoring the unusual vanishing of the E2 intra-band transitions. This was particularly the case in the Rare-Earth region where for nuclei such as $^{156}$Gd a negative parity rotational band was observed with vanishing E2’s when the spin is decreasing [12]. Historically, this nucleus was the first experimental case that we have investigated. Now it appears to us from the current literature that the most significant cases might be found in the Actinide region and more specifically in some of the Uranium isotopes.

2. Recent experiments in the Rare-Earth region

Our first theoretical results suggest the existence of very stable total-energy minima corresponding to sizeable tetrahedral deformations in several nuclei around Z=62-70 (Sm, Gd, Dy, Er, Yb) with masses A~150-160. In the Gadolinium nuclei the tetrahedral-symmetric states are predicted to lie relatively low above the ground state (between 0.5 - 2.0 MeV approximately). The predicted barriers separating the "new" structures from the "traditional" ones (the quadrupole deformed ground-state minima) are of the order of several MeV, ranging typically from about 1.5 to about 5 MeV or more in various isotopes of the mentioned elements [6]. Following these indications we have started our experimental effort in this region.

2.1. The $^{156}$Gd JYFL-JUROGAM experiment

In the $^{156}$Gd nucleus a low-lying negative-parity side-band with odd-spins have been reported with no transition of the E2 character below spin 9 [12]. This particular nucleus has been studied in over 20 experiments using varying target-projectile combinations and detection systems [13], but the E2 transitions below spin 9 have never been seen. The presence of the corresponding states has been detected exclusively through the inter-band E1 transitions to the ground-state band. No convincing explanation was given to interpret this observation. Ultimately this band, together with its even-spin partner, was considered as an octupole $K^\pi=1^-$ vibrational band.
As mentioned earlier, at the ideal symmetry limit the dipole and the quadrupole moments of a tetrahedral-symmetric nucleus vanish. Then, according to theory arguments based on the pure symmetry assumption, the first allowed collective transition has an E3 character (pure octupole transitions) and consequently there should be no E2 or E1 transitions! But this is very unlikely because the ratio of the transition probability between E1 and E3 transitions is proportional to $10^{12}$. Therefore at low spin any partial symmetry breaking such as the zero-point motion will favor the electromagnetic decay through E1 transitions. At increasing spins the nuclear rotation will necessarily privilege some particular directions of the total spin and because of the Coriolis interactions, the nucleonic alignment is going to gradually impose the triaxial quadrupole polarization of the core. As the result of those polarization effects we expect that some E2 transitions will become possible above a certain critical spin value in qualitative agreement with the clear observation of weak E2 transitions above spin 11.

The nucleus $^{156}$Gd was therefore considered as a very good candidate to test the possibility to use the $B(E2)/B(E1)$ branching ratios as a first fingerprint of the symmetry. This nucleus could be easily produced with a very high fusion-evaporation cross section using an alpha beam that favors a direct feeding of the low spin yrare states of interest. We have run a $\gamma$-coincidence experiment at Jyväskylä (FIN) with the JUROGAM array using the reaction $^{154}$Sm($\alpha$,2n) at a 27.5 MeV bombarding energy that was pre-determined in an OSCAR pilot experiment at IPN-Orsay. A key issue here is that JUROGAM offers the possibility to study $^{156}$Gd with triple $\gamma$-coincidences. Such a study was never performed on this nucleus in the past with an alpha beam, but it is essential in order to disentangle various gamma multiplets present in these nuclei, unambiguously locate the states of interest and be able to accurately measure the branching ratios in order to be sure that the previous results were not due to some experimental limits.

The preliminary results of the Jyväskylä experiment have been reported in Ref. [11]. These results confirm the pattern of vanishing intra-band E2 transitions below spin 11, however new inter-band transitions between the two negative-parity bands were found. They are very important to pin down the presence of some states of the tetrahedral candidate band that were previously seen only trough their E1 decays-out to the ground-state band and therefore attributed to the negative-parity even-spin band using only the argument of the state's excitation energy. We used the new inter-band transitions also to gate from above the states of interest to extract the branching ratios that are reported in Table 1 (below spin 11 the branching ratio of the odd-spin band are given in terms of upper limits whose estimates take into account the experimental resolving power). They are compared with the available data on $^{222}$Th, a nucleus seen in the literature as one of the best "static" octupole deformed nuclei. From this Table we could conclude the following:

- The behaviour of the two negative-parity bands is very different: the even-spin partner has branching ratio one to two orders of magnitude larger than the odd-spin partners that is our candidate band. (Note that in the case of even-spin negative-parity band the inter-band dipole transitions are in fact non-stretched E1 transitions while in the case of the tetrahedral band they are the stretched ones).
- The E2 intra-band transitions in the even-spin band are seen down to spin 4, but only down to spin 11 in the odd-spin band.
- Both bands have decreasing $B(E2)_{in}/B(E1)_{out}$ branching ratios with spin which is not the case of the octupole band of $^{222}$Th.

Consequently the large structural differences seen in the branching ratios between the compared configurations suggest that they may have different origins. Even though there is no justification to claim the existence of the tetrahedral symmetry in $^{156}$Gd, we can conclude from our accurate measurements that the first-excited negative-parity side-band with odd-spin is a serious candidate because the usual explanation in terms of octupole vibration appears less and less possible with the impossibility to explain the vanishing E2 intra-band transitions in a comprehensive way. As we will see in the following, the final word will probably come from the direct evaluation of the quadrupole moment extracted from lifetime measurements.
2.2. Other experiments in the Rare-Earth region

Three other experiments were performed after the JUROGAM measurement. Two of them aimed at attacking the problem of the vanishing E2 transitions in $^{156}$Gd through different methods and nuclear reactions. The third experiment was performed with GAMMASPHERE at ANL to investigate $^{155}$Dy, an N=90 isotope as opposed to $^{156}$Gd which has N=92 neutrons. All these experiments are still under analysis and we will briefly present here only preliminary results.

2.2.1. $^{156}$Gd LNL-GaSp experiment (Legnaro-I). The idea of this experiment [15] was to study the states of interests in $^{156}$Gd by feeding them not from above in term of spin as in the case of the Jyväskylä fusion-evaporation experiment, but from below via Coulomb excitations using a $^{58}$Ni beam. Therefore these states, if fed, could only be fed via multi-E2 ground-state band transitions followed by a single E3 inter-band transition. This was what we observed since the negative-parity states of the odd-spin band were seen up to spin 17. This fact clearly indicates that the octupole degrees of freedom and possibly the ones leading to tetrahedral symmetry are strongly at work in this nucleus. No E2 transitions below spin 11 are seen and the extracted branching ratios are compatible with the ones in the Jyväskylä measurement within the experimental errors, thus enforcing our previous conclusions.

The absolute intensities of the transitions seen in this experiment have been compiled and will be analysed thanks to the GOSIA code in order to extract a value of the quadrupole moment of the tetrahedral-candidate band.

2.2.2. $^{156}$Gd ILL-GAMS experiment (Grenoble-F). This experiment [16] is aiming at the measurement of the lifetime of the 5$^-$ state from the $\gamma$-ray induced Doppler broadening (GRID technique) of the 5$^-$ to 6$^+$ E1 inter-band transition. For that purpose a very intense $\gamma$-beam produced by the $^{155}$Gd(n,$\gamma$)$^{156}$Gd reaction at the centre of the ILL-reactor is focused on the GAMS spectrometer where the Bragg diffraction is studied in the range of the energy of interest. The second goal was to try to observe the "missing" 5$^-$ to 3$^-$ E2 transition. No clear-cut evidence of the existence of this transition is currently seen in the available data. Therefore if we combine the lifetime measurement obtained for the 5$^-$ states together with the lowest intensity value of the resolving power, we are able to extract an upper limit of the quadrupole moment. We end up with a value which is at its maximum estimated limit, close to the one of the ground-state band. If we take this limit as real, this observation would imply both the presence of the quadrupole band-head of the negative-parity band and yet vanishing E2 transitions. This would be another interesting scenario implying, however, the existence of a very strong E1 moment since otherwise it will be difficult to understand why strong quadrupole moments do not result in observation of the implied E2 transitions. We aim now to get a factor of ten in statistics by a dedicated experiment.

2.2.3. $^{156}$Dy ANL-GAMMASPHERE experiment (Argonne-USA). In $^{156}$Dy the same kind of structure as the one studied in $^{156}$Gd has been reported in the literature: a low-lying negative-parity odd-spin band with E2 inter-band transitions seen only down to spin 9 with no intensities reported [17] and this prevents obtaining the branching ratios. GAMMASPHERE with its very high sensitivity was used to observe the decay of the interesting states populated in the $^{148}$Nd(12C,4n) reaction [18]. A very weak
transition between the $9^-$ and the $7^-$ state was observed but even after an extended analysis, no in-band transitions below spin 7 were seen. The extracted $B_{zz}(E2)/B_{nn}(E1)$ branching ratios show the same pattern and magnitude as in $^{156}\text{Gd}$ with a regular decrease down to spin 9. These results clearly prove that the structure seen in $^{156}\text{Gd}$ is not a single accident but is most probably well spread in this region as predicted by the calculations involving tetrahedral symmetry arguments around the $N=90$ magic number. The next step with GAMMASPHERE will be to measure the lifetime in this band with a plunger experiment.

3. The Actinide region

In the actinide region our theoretical results available today predict the existence of stable total-energy minima laying at about 500 keV or more, above the ground-state and corresponding to sizeable tetrahedral deformations. The barriers separating these minima from the ground-state minimum are of the order of 1.0 to 2.5 MeV and the effect is even strengthened by the tetrahedral and octahedral symmetries superposing. (Recall that the tetrahedral group is a sub-group of the octahedral one and the superposition in question still results in the tetrahedral symmetry). It is worth emphasizing that these barriers are comparable with-, or even more sizeable, than those predicted as responsible for the prolate/oblate shape coexistence phenomena already studied experimentally, among others, in the Mercury region. We believe that these predictions signify a stronger stability in yet undiscovered tetrahedral nuclei as compared to already discovered prolate/oblate/spherical shape co-existing minima. The 230–234 even-even Uranium isotopes are good examples of tetrahedral candidates. Figure 2 shows the series of even-even Uranium isotopes from $^{226}\text{U}$ to $^{238}\text{U}$ for the ground-state bands and the first negative-parity bands. It is clear that the behaviour of the $^{230,234}\text{U}$ isotopes corresponds to what we expect for a pure tetrahedral symmetry: no E2’s intra-band transitions and only E1’s inter-band transitions decaying from the negative-parity excited-states to the ground-state band members. To the contrary, on the lightest-mass side of this group we have $^{226}\text{U}$ nucleus that shows a "characteristic zigzag" pattern typical for the static-octupole deformation. Our microscopic calculations suggest that the underlying nature of the negative-parity bands in these actinide nuclei changes when the neutron number increases. At low neutron numbers $N<136$, calculations confirm the earlier interpretation of the band structure in terms of the $Y_{10}$ static-octupole configurations. At heavier isotopes the $Y_{12}$ tetrahedral interpretation is supported by the theory results. For $N>146$, the tetrahedral minimum is not well formed implying low stiffness against octupole vibrations and suggesting the presence of the low-lying octupole vibrational bands as observed in $^{238}\text{U}$ [19].

Let us observe in passing that all the odd-spin negative-parity bands from 230–238 even-even Uranium isotopes have been usually interpreted in the literature as octupole vibrational bands similarly to many bands in the Rare-Earth nuclei. They are characterized by two rotational branches: the lower-lying zero-phonon positive parity ground-state band and the negative parity one-phonon higher-lying band that usually remains shifted upwards in energy by the phonon’s vibrational energy. However it is important to emphasize that the origin of such a shift (as the energy of octupole vibration) cannot be distinguished from the shift caused by the energy difference between the ground-state minimum and the tetrahedral minimum in the case of the new-symmetry interpretation. But the tetrahedral symmetry offers a natural explanation of the disappearing E2 transitions which is not the case when the usual octupole vibrational states are considered.

One important comment concerns the $I^\pi=1^-\pi$ state. This state is a natural band member in the context of octupole vibrational scenario because they are the so-called $K^{\pi}=0^-$ or $1^-$ bands. But there is a particular arbitrariness in operating with $I^\pi=1^-$ states in our context of tetrahedral deformation. These $1^-$ are "very fragile" states in the sense that they may come from numerous and varying structural origins. In particular, in the tetrahedral-candidate bands we have no way to say whether $1^-$ states belong to the bands as long as nobody has seen the E2 transition from $3^-$ to $1^-$. Consequently, unless we measure them we have to relay on theory. And theory says: if the bands are tetrahedral then $1^-$ states certainly do not belong to the tetrahedral symmetry because the states with spins I<2 must not carry the tetrahedral symmetry (they carry most likely the axial symmetry, and this deduction is a
direct mathematical consequence of the properties of the Wigner functions). On the other hand there are good reasons to have \(^1\)\(^-\) states that hang around with close-to-good excitation energies without being neither octupole nor, certainly, tetrahedral. This is particularly seen in the Rare-Earth region: for most of the candidate nuclei if one would fit a parabola to the energies of states \(^3\)\(^-\), \(^5\)\(^-\) and \(^7\)\(^-\) and then see how it extrapolates to spin \(^1\), all possible scenarios are found; in other words, anything may happen and there are also sometimes situation where the lowest \(^1\)\(^-\) states are higher in energy than \(^3\)\(^-\) states!

Figure 2. Partial level schemes of the uranium isotopes from \(^{226}\)U to \(^{238}\)U showing only the ground-state bands and the first negative parity bands. Energy differences between the negative parity states are given whether an E2 transition was observed or not. Observe a characteristic "E1 zigzag pattern" in \(^{226}\)U as discussed in the text. Data are taken from Ref. [19-21].

Consequently, unless we measure them we have to relay on theory. And theory says: if the bands are tetrahedral then \(^1\)\(^-\) states certainly do not belong to the tetrahedral symmetry because the states with spins \(^I<2\) must not carry the tetrahedral symmetry (they carry most likely the axial symmetry, and this deduction is a direct mathematical consequence of the properties of the Wigner functions). On the other hand there are good reasons to have \(^1\)\(^-\) states that hang around with close-to-good excitation energies without being neither octupole nor, certainly, tetrahedral. This is particularly seen in the
Rare-Earth region: for most of the candidate nuclei if one would fit a parabola to the energies of states $3^-$, $5^-$ and $7^-$ and then see how it extrapolates to spin $1^-$, all possible scenarios are found; in other words, anything may happen and there are also sometimes situations where the lowest $1^-$ states are higher in energy than $3^-$ states!

One question remains untouched and fully open: are the E2 intra-band transitions missing because the reduced B(E2) transition probabilities are small or, alternatively, because the B(E1) transition probabilities are very large? The first scenario will be a direct consequence of the tetrahedral symmetry hypothesis whereas the second one will favour the traditional octupole vibration explanation under the condition that the absence of E2 transitions in the spectra is well understood. Therefore it is now for us of the up-most importance to address this clear question by measuring the lifetimes of the candidate-band members and get very accurate limits for the branching ratios. We then will obtain the transition probabilities that we will use to extract the values of the transitional quadrupole moment. This will enable us to compare them with the ground-state band values: if they are much lower in magnitude than the ground-state band values, we will approach the tetrahedral hypothesis as an explanation for these structures.

4. Lifetimes and the microwave method to measure quadrupole moments

In the Actinide region taking into account the possible nuclear reactions and the structure of the candidate-bands, it is impossible to employ the usual methods for the lifetime measurements of the excited states of the 230–234 Uranium isotopes. The states of interest in these nuclei could only be produced via fusion evaporation reactions with proton and alpha particles that create the nuclei almost at rest. This totally prevents the use of Doppler measurement techniques such as RDM (Recoil Distance Method) with a plunger. Another possibility would be to use fast-timing measurements but this method implies the possibility to gate on a gamma transition above the measured states. Once again this is not possible in our case because of the absence of the E2 intra-band transitions. Therefore and to the best of our knowledge, we can not use the gamma-rays for this measurement. That being said the only remaining possibility is to use the conversion electrons with a "new" approach, but in fact based on the quite old method that is described briefly below.

The microwave method was originally developed at the Weizmann Institute in 1960 to measure in-beam lifetimes in the range of few tenths of picoseconds [22]. In the early 60’s the recoil distance methods were not in use because of the absence of good resolution gamma spectrometers such as the Germanium detectors. A dedicated beam line was then built at the CRN-Strasbourg again in the late 60’s to measure the lifetimes in the Rare-Earth region [23]. Basically this method consists of a high-frequency beam chopper driven by a microwave beam sweeping cavity. This cavity produces very short bursts of charged particles impinging on a target. An electromagnetic shutter-like device is used to select the conversion electrons according to their time of emission from the target. That device is another microwave cavity synchronized with the beam chopper. Changing the relative phase between the two cavities will modulate the electron energy in function of time. Therefore a variable time scale is established between the production of the excited nuclear state and its decay. Measuring the conversion electron energy in a beta-ray spectrometer will then give access to the lifetimes of the levels of interest. Combining these measurements with limits of the branching ratios will give upper limits of the quadrupole moments.

5. Conclusions

Our realistic mean-field predictions that take into account both the tetrahedral and the octahedral symmetry have focussed our experimental interest onto two regions of the nuclear chart: the Rare-Earth and the Actinide regions. Specific cases have been selected according to the current literature knowledge that display hints of the symmetry through the absence or the vanishing of E2 transitions in given rotational bands. A series of experiments performed world-wide have brought up preliminary encouraging results in $^{156}$Gd and $^{156}$Dy nuclei. The Actinide region seems to present purer signs of the symmetry with a possible new type of shape coexistence and shape evolution that we might already
We are now looking forward to new lifetime measurements in both regions that will prove, or refute, in a unique way the hypothesis of the existence of the high-rank tetrahedral and octahedral symmetries in nuclear physics.

Acknowledgments

The works reported here are mainly part of the activities of the TetraNuc (Tetrahedral Nuclei) Collaboration involving 18 European and non-European institutions. This informal collaboration aims at searching for the fingerprints of the tetrahedral symmetry throughout the Nuclear Chart. The authors want to warmly thank all the people that have been taken part in these activities or support them and more specifically the Direction of IN2P3, the Direction of the IPN-Orsay and D. Verney, O. Stezowski and Q.T. Doan at IPN-Lyon, J. Gerl at GSI-Darmstadt, R. Julin and P. Jones at JYFL-Jyväskylä, F. Haas, H. Molique, J. Robin, P. Medina and M. Richer at IPHC-Strasbourg, M. Jentschel, B. Lauss and W. Urban at ILL-Grenoble, D. Tonev at INRNE-Sofia, G. de Angelis at LNL-Legnaro, R. J. Singh at IUAC-New Delhi, L. Riedinger at Uni. of Tennessee-Knoxville, D. Hartley at U.S. Naval Academy-Annapolis and last but not least the Direction of ANL-Argonne. Acknowledgement is due also for the support from IN2P3-COPIN collaboration program through the project no. 05-119 and the support under COPIGAL project "Search for the high-rank symmetries in subatomic physics".

References

[1] Li X and Dudek J 1994 Phys. Rev. C94 R1250
[2] Dudek J, Gozdz A, Schunck N, Miskiewicz M 2002 Phys. Rev. Lett. 88 252502
[3] Dudek J, Gozdz A and Rosly D 2001 Acta Phys. Polon. B32 2625
[4] GozdA , Dudek D and Miskiewicz M 2003 Acta Phys. Polon. B34 2123
[5] Dudek J, Gozdz A and Schunck N 2003 Acta Phys. Polon. B34 2491
[6] Schunck N, Dudek J, Gozdz A and Regan P 2004 Phys. Rev. C69 061305 (R)
[7] Schunck N and Dudek J, Int. 2004 Journal of Modern Phys. E13 213
[8] Schunck N, Dudek J and Frauendorf S 2005 Acta. Phys. Polon. B36 1071
[9] Dudek J, Dobaczewski J, Dubray N, Gozdz A, Pangon V, Schunck N 2007 Int.J.Mod.Phys. E16 516
[10] Dudek J, Curien D, Dubray N, Dobaczewski J, Pangon V, Olbratowski P and Schunck N 2006 Phys. Rev. Lett. 97 07250
[11] Doan Q T et al. 2009 Acta. Phys. Polon. B40,725; PhD Thesis 2009 Uni. C. Bernard, Lyon
[12] Kojijn J et al. 1981 Nucl. Phys. A352 191
[13] Reich C W 2003 Nuclear Data Sheets 99 753
[14] Akovali Y A 1996 Nuclear Data Sheets 77 271
[15] Singh R J 2008 LNL Annual Report and 2009 private communication
[16] Jentschel M 2008 private communication
[17] Riley M A et al. 1998 Nucl. Phys. A486 456
[18] Riedinger L 2009 private communication
[19] Ward D et al 1996 Nucl. Phys. A600 88
[20] Zeyen P et al. 1987 Z. Phys. A328 399; Ackermann B et al.1993 Nucl. Phys. A559 61
[21] Greenless P et al. 1998 J. Phys. G: Nucl. Part. Phys. 24 63
[22] Goldring G 1961 NM 11 29
[23] Gerber J et al. 1969 Revue de Physique Appliquée 4 218; Armbruster R et al. 1970 Nucl. Phys. A143 315; Dar Y et al.1971 NIM 97 251 and reference therein.