UNCERTAINTIES IN SUPERNOVA YIELDS. I. ONE-DIMENSIONAL EXPLOSIONS

PATRICK A. YOUNG1,2 AND CHRIS L. FRYER1,3

Received 2006 June 13; accepted 2007 March 7

ABSTRACT

Theoretical nucleosynthetic yields from supernovae are sensitive to both the details of the progenitor star and the explosion calculation. We attempt to comprehensively identify the sources of uncertainties in these yields. In this paper we concentrate on the variations in yields from a single progenitor arising from common 1D methods of approximating a supernova explosion. Subsequent papers will examine 3D effects in the explosion and the progenitor, and trends in mass and composition. For the 1D explosions we find that both elemental and isotopic yields for Si and heavier elements are a sensitive function of explosion energy. Also, piston-driven and thermal bomb–type explosions have different yields for the same explosion energy. Yields derived from 1D explosions are nonunique.

Subject headings: nuclear reactions, nucleosynthesis, abundances — stars: evolution — supernova remnants — supernovae: general

Online material: machine-readable tables

1. INTRODUCTION

The amount of observational data on chemical abundances is growing at an enormous rate. The Sloan Digital Sky Survey has produced spectra for 

\[ >10^5 \]  

galaxies at low redshifts. Information is also becoming available for smaller numbers of galaxies to redshifts with the current generation of large telescopes (Berger et al. 2006). Observations at high to moderate redshift show a gradual build-up in metallicity (Erb et al. 2007), from which conclusions about the star formation histories of galaxies are drawn. Comparison of metallicities between elliptical and spiral galaxies, members of groups and clusters, and abundance gradients within individual galaxies are used to probe their formation, interaction history, and evolution. Large telescopes are providing detailed abundances for Milky Way halo stars and dwarf galaxies and globular clusters in the local group. The dwarfs and globular clusters show a range of nearly a dex in \([Fe/H]\) (Pritzl et al. 2005). Several ultra–metal-poor stars are now known with \([Fe/H]<-4\) (Aoki et al. 2006). Each has unique abundance anomalies that suggest they have been enriched by only one or a few stars.

Chemical evolution over a large fraction of the age of the universe is becoming a quantitative field of study. The conclusions that can be drawn from this data, however, are only as good as our understanding of how chemical elements are produced in stars.

The best currently practical approach to producing theoretical yields would be to take a set of progenitor stars and explode them in one-dimension (1D) with a range of free parameters such as explosion energy or mass cut. Then a linear combination of the models, weighted by an initial mass function (IMF) would be chosen that produces the desired abundance pattern, i.e., solar, and these would then form a table of yields with mass and metallicity. In fact, the common approach is to take a single set of yields without any exploration of parameter space (Garnett 2002). In either scenario these yields are fit, not predicted. The local group results indicate that very different enrichment histories with very different abundance ratios can lead to the same total enrichment in metallicity. Exploring these pathways requires predictive yields. Identifying the star or stars that enriched the ultra–metal-poor halo stars even more clearly requires predictive yields with a unique correspondence to a progenitor.

Unfortunately, it is not presently possible routinely (or perhaps at all) to produce a truly predictive yield for an ensemble of stars. The nucleosynthesis is sensitive to a number of properties of the progenitor and explosion models, both physical and numerical. The structure of progenitors integrated over the evolution changes dramatically if hydrodynamic processes are included (Young et al. 2005). The evolution also depends on rotation and mass loss (i.e., Meynet & Maeder 2000). The dynamics of the progenitor before and during collapse will also leave an imprint on the explosion. Hydrodynamic motions can result in asymmetries in the shell-burning regions of >10% at the onset of collapse (Meakin & Arnett 2006). The integrated effects can be incorporated into a stellar evolution code after an analytic framework is derived from examining simulations, but the late-stage dynamics must be simulated directly with a computationally expensive multidimensional hydrodynamics code. The composition and mass of the progenitor of course also play a role.

The idea that the convection above the proto–neutron star plays an important role in understanding supernova energetics is gradually becoming accepted (Herant et al. 1994; Fryer & Warren 2002; Blondin & Mezzacappa 2006; Burrows et al. 2006). If this is indeed the case, understanding the explosion and obtaining accurate explosion energetics will require three-dimensional (3D) calculations. We are far from modeling all of the physics, including the convection, with enough detail in two dimensions, let alone three, to accurately estimate explosion energies. Two-dimensional (2D) simulations assuming a 90° wedge geometry (Buras et al. 2003) have obtained different results when modeled using a 180° domain (H. Th. Janka 2006, private communication). The convective instabilities depend sensitively on the equation of state, resolution, and, probably, implementation of the gravitational force (Fryer & Kusenko 2006; Fryer 2006). The convective instabilities also depend on the effects of rotation and asymmetries in the collapsing core (Shimaizu et al. 1994; Fryer & Heger 2000; Kotake et al. 2003; Fryer & Warren 2004).

\[ \text{1 Theoretical Astrophysics, Los Alamos National Laboratories, Los Alamos, NM 87545.} \]
\[ \text{2 Steward Observatory, University of Arizona, Tucson, AZ 85721.} \]
\[ \text{3 Physics Department, University of Arizona, Tucson, AZ 85721.} \]
\[ \text{The common standard for yields is Woosley & Weaver (1995). This compilation only presents yields for a single explosion energy, except at progenitor masses of 30--40 } M_{\odot}. \]
In addition to being important for our understanding of the explosive engine, multidimensional simulations are required to understand the outward mixing of heavy elements and ultimately the amount of this material that is ejected in a supernova (Kifonidis et al. 2000; Hungerford et al. 2003b). This effect is enhanced by explosion asymmetries (Nagataki et al. 1998; Kifonidis et al. 2003; Hungerford et al. 2003b, 2005a). Multidimensional studies of the explosion are just now beginning to put in the relevant physics, and these new studies will no doubt bring new surprises in supernovae explosions. Detailed multidimensional models are required both to understand the supernova engine and the resultant explosion, and at this point in time much more work must be done before we can predict either the explosive energies or the yields from supernovae. For massive stars, explosion mechanisms beyond the standard supernova engine (e.g., Woosley 1993; the collapsar model) will also play a role in nucleosynthetic yields.

Clearly, progress in the fields of chemical evolution, stellar populations, and galaxy assembly is not going to be put on hold while all the outstanding issues in supernovae and nucleosynthesis are resolved. We can, however, attempt to quantify the uncertainties arising from various assumptions in the yield calculations. In this series of papers we identify sources of uncertainty from each aspect of the calculations so that we may include error bars in a new generation of integrated yields. In this first paper we examine changes in yields for a single progenitor arising from various methods of performing 1D explosion calculations. Even though we are able to produce a weak explosion of a 23 M⊙ star in 3D, the simulation requires approximately 7 months of computer time on a moderately sized cluster. Doing multidimensional explosions for even a sparse sampling of a population will not be feasible for some time, so we still must rely on 1D models for integrated yields. In the second paper we examine a full 3D collapse and explosion of the same progenitor and compare it to the 1D explosions. Paper III studies trends with changes in progenitor mass, composition and evolution code physics. A final paper compares our initially spherically symmetric 3D explosion with one with realistic progenitor dynamics as initial conditions.

2. PROGENITOR AND EXPLOSION CALCULATIONS

2.1. Progenitor

We use a 23 M⊙ progenitor of Grevesse & Sauval (1998) solar composition, aspects of which we have examined in Young et al. (2005). The model was produced with the TYCHO stellar evolution code (Young & Arnett 2005). The model is nonrotating and includes hydrodynamic mixing processes (Young & Arnett 2005; Young et al. 2005). The inclusion of these processes, which approximate the integrated effect of dynamic stability criteria for convection, entrainment at convective boundaries, and wave-driven mixing, results in significantly larger extents of regions processed by nuclear burning stages. The $\bar{A} > 16$ core is $\sim$50% larger than a model of the same mass using parameterized overshooting calibrated for lower mass stars. Because of the difference in luminosity and time spent on the giant branch and blue loops, there are also differences in the total extent of mass loss. Mass loss uses the prescriptions of Kudritzki et al. (1989) for OB mass loss and Blöcker (1995) for red giant/supergiant mass loss. The final stellar mass is 14.4 M⊙. Figure 1 shows the mean atomic weight ($\bar{A}$, top) and density (bottom) versus enclosed mass. The He-rich material has been either mixed into the hydrogen envelope (40% He by mass) or significantly enriched by carbon and oxygen from He shell burning convection and has a high $\bar{A}$. The latter is a result of hydrodynamic mixing processes included in the evolution and should be expected to differ significantly from standard stellar models. The total core size of heavy elements resembles more closely the 35 M⊙ model of Limongi & Chieffi (2003). The physics involved are explored in Young & Arnett (2005); Young et al. (2005); Meakin & Arnett (2007); and D. Arnett et al. (2007, in preparation). A 177-element network terminating at 74 Ge is used throughout the evolution. The network uses the NOSMOKER rates from Rauscher & Thielemann (2001), weak rates from Langanke & Martinez-Pinedo (2000), and screening from Graboske et al. (1973). Neutrino cooling from plasma processes and the Urca process is included.

2.2. Explosion Calculations

To model collapse and explosion, we use a 1D Lagrangian code developed by Herant et al. (1994). This code includes three-flavor neutrino transport using a flux-limited diffusion calculation and a coupled set of equations of state to model the wide range of densities in the collapse phase (see Herant et al. 1994; Fryer et al. 1999a for details). It includes a 14-element nuclear network (Benz et al. 1998) to follow the energy generation. This code was used to follow the collapse of the star through bounce. For our neutrino driven explosions, we enhanced the neutrino heating by artificially turning up the neutrino energy. For these explosions, we found that either we got a very strong explosion (1052 ergs) or no explosion at all. This was because if the explosion did not occur quickly, we were unable to drive a supernova explosion due to the quick decay in the electron neutrino and antineutrino fluxes. But such a result (either strong or no explosion) can be overcome if we did not increase the neutrino energy throughout the star (including the proto–neutron star core) or if we were adding this heating in a multidimensional calculation. In a calculation that modifies the neutrino transport by assuming convection can redistribute the energy in the neutron star and separately modifies the neutrino transport above the neutron star, one can easily get a range of explosion energies by changing the neutrino opacities (Fröhlich et al. 2006).
To get a range of explosion energies, we opted to remove the neutron star and drive an explosion by one of two ways: by injecting energy in the innermost 15 zones (roughly 0.035 $M_\odot$) or by driving a piston at the innermost zone (outer edge of the proto–neutron star). The duration and magnitude (piston velocity or energy injection) of these artificial explosions were altered to produce the different explosion energies. During energy injection, the proto–neutron star is modeled as a hard surface. We do not include the neutrino flux from the proto–neutron star, but the energy injected by this neutrino flux is minimal compared to our artificial energy injection. Shortly after the end of the energy injection, we turn the hard neutron star surface to an absorbing boundary layer, mimicking the accretion of infalling matter due to neutrino cooling onto the proto–neutron star. In this manner we can model the explosion out to late times, even if there is considerable fallback.

In this paper we focus on the energy-injection model for explosions. We have produced three sets of explosions (each with a range of energy) using the same base model in which the collapse, bounce, and fail of the bounce shock is followed with no energy injection (first 380 ms after start of collapse). The first set of models assumes a fast explosion in which the energy is injected in just 20 ms, producing an explosion just 400 ms after collapse (roughly 150 ms after bounce). By varying the energy injected, we produce a range of explosion energies. We then modeled two additional sets of explosions in which we injected energy into those inner 15 zones for longer times: 200 ms (roughly 330 ms after bounce) and 700 ms (roughly 830 ms after bounce). Of course, to produce the same final explosion energy, the rate of energy input for longer injection times is much lower than of that of our fast explosions.

The 1D explosions are summarized in Table 1. All of our models assume the same initial mass of the compact remnant: $\sim 1.75 M_\odot$, corresponding to a gravitational mass of the neutron star roughly equal to $1.6 M_\odot$. But unless the explosion is extremely energetic, material falling back onto the proto–neutron star always produces a much more massive compact remnant, generally producing a black hole. We expect such massive progenitors to produce black holes. Although it is likely that all stars above $12 M_\odot$ have some fallback (Fryer et al. 1999b), above $20 M_\odot$, we expect the fallback to be considerable. When an explosion is launched, the inner material all has energy (mostly internal) in excess of the escape energy. However, this material deposits much of its energy into the layers above it. Fallback occurs when this material loses so much energy that it eventually becomes bound again and falls back back onto the compact remnant (see Fryer & Kalogera 2001 for details).

Our different explosion methods lead to different amounts of fallback that, while not physical, allow us probe a range of effects that may occur in nature. First, the piston model moves the material outward in radius, reducing the potential well that this matter must climb out to be ejected. In general, this smaller potential well leads to less fallback for a given energy than thermal bomb models. Thielemann et al. (1996, 2002) and F. X. Timmes (2006, private communication) note that there are differences in yields from piston and thermal bomb explosions. Depending on how one works the piston engine, one could prevent any fallback at all, although many piston engines do allow for fallback (Fryer et al. 1999b). Aufderheide et al. (1991, hereafter ABT91) find that pistons and thermal bombs can produce differences in yields on the order of tens of percent, despite imposing a mass cut to get the desired Fe peak yields. Comparing the different simulations using a thermal bomb, we see that the simulations with a longer energy injection also produce less massive remnants. Because we continue to inject energy into the inner cells for longer time-scales, these zones move further out of the potential well and are less likely to fall back.

But the remnant mass is not the only difference caused by the duration of the energy injection. Figure 2 shows the velocity profiles of four of our simulations as a function of enclosed mass when the shock reaches roughly 1.2 $\times 10^9$ cm. Comparing the weak (0.8 foe $^5$) explosions (solid, dotted lines) to their normal (1.5 foe) explosion counterparts, we see the obvious trend: the more energetic explosion has faster velocities. But note that the shorter delays in the 200 ms explosions produces faster velocities than their 700 ms counterparts. The quicker explosions deposit

![FIG. 2.—Velocity of the ejecta as a function of enclosed mass for two 200 ms explosions: 23e-0.2-0.8 (solid line), 23e-0.2-1.5 (dashed line); and two 700 ms explosions: 23e-0.7-0.8 (dotted line); 23e-0.7-1.5 (long-dashed line). The results are compared when all the shocks are at the same spot. This occurs at different times for the different simulations. Although the more energetic explosions tend to have higher velocities, the duration that the energy is injected also changes the velocity profile.]

$^5$ A foe (fifty-one ergs) is a unit denoting $10^{51}$ ergs. The foe has also been termed a “Bethe.”
2.3. Nucleosynthesis Postprocessing

The network in the explosion code terminates at $^{56}$Ni and cannot follow neutron excess so to accurately calculate the yields from these models, we so turn to a postprocess step. Nucleosynthesis postprocessing was performed with the Burn code (Fryer et al. 2006), using a 524-element network terminating at $^{99}$Tc. The network uses the current NOSMOKER rates described in Rauscher & Thielemann (2001), weak rates from Langanke & Martinez-Pinedo (2000), and screening from Graboske et al. (1973). Reverse rates are calculated from detailed balance and allow a smooth transition to a nuclear statistical equilibrium (NSE) solver at $T > 10^{10}$ K. For this work Burn chooses an appropriate time step based on the rate of change of abundances and performs a log-linear interpolation in the thermodynamic trajectory of each zone in the explosion calculation. The code also has available modes for analytic adiabatic trajectories, arbitrary density trajectories coupled with the equation of state solver in TYCHO (Young & Arnett 2005), big bang conditions, and hydrostatic (stellar) burning. Neutrino cooling from plasma processes and the Urca process is calculated. The initial abundances are those of the 177 nuclei in the initial stellar model, and the network machinery is identical to that in TYCHO.

3. YIELDS

Tables 2 and 3 give final yields for the thermal bomb and piston models, respectively. (Both sets have a 20 ms delay.) The tables terminate at $^{74}$Ge. Heavier elements were not tracked during the star’s evolution, so the final yield of those species would not be complete. The extra extent of the tables does allow us to track the neutron-rich iron peak correctly, without nuclei we tabulate building up artificially at a network boundary. In this paper we concentrate on the iron peak and intermediate-mass elements and defer any discussion of $r$- and $s$-process to a later paper. Figures 4–9 show the graphical yields for the thermal bomb models. Figures 10–12 show yields for piston models.

### TABLE 2

| Species | $\text{23e-0.8}$ | $\text{23e-1.1}$ | $\text{23e-1.2}$ | $\text{23e-1.4}$ | $\text{23e-1.5}$ | $\text{23e-2.0}$ |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|
| $^1$H   | 1.46E+00       | 1.46E+00       | 1.46E+00       | 1.46E+00       | 1.46E+00       | 1.46E+00       |
| $^2$H   | 3.86E-16       | 4.46E-16       | 4.39E-16       | 4.78E-16       | 4.76E-16       | 5.96E-16       |
| $^3$H   | 2.39E-23       | 1.58E-16       | 1.58E-16       | 1.58E-16       | 1.57E-16       | 1.55E-16       |
| $^3$He  | 6.60E-05       | 6.60E-05       | 6.60E-05       | 6.60E-05       | 6.60E-05       | 6.60E-05       |
| $^4$He  | 1.13E+00       | 1.13E+00       | 1.13E+00       | 1.13E+00       | 1.13E+00       | 1.13E+00       |
| $^6$Li  | 2.01E-18       | 2.35E-18       | 2.29E-18       | 2.51E-18       | 2.50E-18       | 3.22E-18       |
| $^7$Li  | 2.89E-10       | 2.89E-10       | 2.89E-10       | 2.89E-10       | 2.89E-10       | 2.89E-10       |
| $^7$Be  | 1.28E-10       | 1.28E-10       | 1.28E-10       | 1.28E-10       | 1.28E-10       | 1.28E-10       |
| $^9$Be  | 1.95E-16       | 3.82E-11       | 6.11E-11       | 1.57E-10       | 4.10E-09       | 1.39E-06       |
| $^{9}$Be | 8.88E-19       | 1.01E-18       | 1.03E-18       | 1.12E-18       | 1.14E-18       | 1.27E-18       |

Note.—Table 2 is published in its entirety in the electronic edition of the *Astrophysical Journal*. A portion is shown here for guidance regarding its form and content.
| Species | 23$p$-0.9 | 23$p$-1.2 | 23$p$-1.6 |
|---------|-----------|-----------|-----------|
| $^1$H   | 1.46E+00  | 1.46E+00  | 1.46E+00  |
| $^2$H   | 1.75E-16  | 1.67E-16  | 1.89E-16  |
| $^3$H   | 6.77E-18  | 1.77E-16  | 1.75E-16  |
| $^3$He  | 6.60E-05  | 6.60E-05  | 6.60E-05  |
| $^4$He  | 1.13E+00  | 1.13E+00  | 1.15E+00  |
| $^4$Li  | 1.26E-18  | 4.24E-18  | 7.05E-18  |
| $^7$Li  | 2.89E-10  | 2.89E-10  | 2.89E-10  |
| $^7$Be  | 1.28E-10  | 1.28E-10  | 1.28E-10  |
| $^8$B   | 3.29E-15  | 1.82E-08  | 1.68E-06  |
| $^8$Be  | 8.77E-19  | 2.52E-19  | 1.69E-19  |

Note.—Table 3 is published in its entirety in the electronic edition of the *Astrophysical Journal*. A portion is shown here for guidance regarding its form and content.

**Fig. 4.**—Mass fraction of elemental yields $X/X_\odot$ vs. proton number $Z$ for the 0.8 foe thermal bomb model. Each element is labeled with the most abundant isotope of that element. A clear trend is seen with energy. The 0.8 foe explosion ejects the H, He, and part of the C/O layers. The heavier elements are represented only by their initial abundances in the star. As explosion energy increases the yields of C/O increase, followed by the intermediate-mass elements. Only at 1.5 foe does ejecta reach Si-burning temperatures during the explosion, and produces mostly $^{56}$Fe, characteristic of high-entropy QSE nucleosynthesis. At 2 foe there is ejecta that reaches NSE conditions and produces $^{56}$Ni as the dominant Fe peak isotope.

**Fig. 5.**—Same as Fig. 4, but for the 1.1 foe thermal bomb model.

**Fig. 6.**—Same as Fig. 4, but for the 1.2 foe thermal bomb model.

**Fig. 7.**—Same as Fig. 4, but for the 1.4 foe thermal bomb model.

**Fig. 8.**—Same as Fig. 4, but for the 1.5 foe thermal bomb model.
The thermal bomb models show a clear trend with final explosion energy. This behavior is simple to interpret. Figure 13 indicates the material that escapes for different thermal bomb input energies on a plot of mean atomic weight $\bar{A}$ versus enclosed mass for the progenitor. (This corresponds to the rightmost set of points in Fig. 17, and it ignores, of course, 3D effects and mixing during the explosion.) Figure 14 shows the peak temperature reached by each mass zone ejected for the 0.8, 1.2, 1.5, and 2.0 foe explosions. The explosions with less than 1.4 foe do not reach even oxygen-burning temperatures in the material that escapes as ejecta. Elements heavier than oxygen are represented only by their initial abundances in the star. With increasing explosion energy a larger fraction of the unprocessed material from the original star is gravitationally unbound, and abundances of intermediate-mass elements begin to come up above solar, progressing toward larger $A$ in order of synthesis temperature. In the 0.8 foe explosion only about half of the O-rich mantle escapes. By 1.4 foe Si-rich material makes it out in the explosion. At

---

**Fig. 9.**—Same as Fig. 4, but for the 2.0 foe thermal bomb model.

**Fig. 12.**—Same as Fig. 10, but for the 1.6 foe piston model.

**Fig. 10.**—Mass fraction of elemental yields $X/X_\odot$ vs. proton number $Z$ for the 0.9 foe piston model. Each element is labeled with the most abundant isotope of that element. Trends with energy are similar to the thermal bombs, but offset to lower energies.

**Fig. 11.**—Same as Fig. 10, but for the 1.2 foe piston model.

**Fig. 13.**—Mean atomic mass $\bar{A}$ vs. enclosed mass in the progenitor. Dashed vertical lines indicate the material that escapes for thermal bomb explosions of the indicated input energies, ignoring 3D effects and mixing.
1.5 foe some Si-rich material in the ejecta reaches quasi-statistical equilibrium temperatures ($>3 \times 10^9$ K) and is burned to the Fe peak. In the high-entropy QSE regime the material is dominated by $^{54}$Fe, with a smaller component of $^{56}$Ni. At 2.0 foe some of the ejecta reaches full NSE ($T > 5 \times 10^9$ K) and is dominated by $^{56}$Ni ($0.3 M_\odot$).

This trend is exactly what we already expect with increasing explosion energy. At this stellar mass a range of energies is possible, and we must consider the entire range of yields. The more interesting comparison, however, is between a thermal bomb and a piston explosion of the same final energy. In terms of yields, the piston explosion mimics a higher energy thermal bomb explosion because the piston is more efficient at accelerating material where the explosion is imposed. Figure 15 shows the peak temperature achieved by the ejecta as a function of initial stellar mass coordinate for 1.2 foe thermal bomb and piston explosions. Material from deeper in the star escapes in the piston explosion. Figure 16 shows the velocities versus initial stellar mass coordinate.

1.5 foe some Si-rich material in the ejecta reaches quasi-statistical equilibrium temperatures ($>3 \times 10^9$ K) and is burned to the Fe peak. In the high-entropy QSE regime the material is dominated by $^{54}$Fe, with a smaller component of $^{56}$Ni. At 2.0 foe some of the ejecta reaches full NSE ($T > 5 \times 10^9$ K) and is dominated by $^{56}$Ni ($0.3 M_\odot$).

This trend is exactly what we already expect with increasing explosion energy. At this stellar mass a range of energies is possible, and we must consider the entire range of yields. The more interesting comparison, however, is between a thermal bomb and a piston explosion of the same final energy. In terms of yields, the piston explosion mimics a higher energy thermal bomb explosion because the piston is more efficient at accelerating material where the explosion is imposed. Figure 15 shows the peak temperature achieved by the ejecta as a function of initial stellar mass coordinate for 1.2 foe thermal bomb and piston explosions. Material from deeper in the star escapes in the piston explosion. Figure 16 shows the velocities versus initial stellar mass coordinate.

1.5 foe some Si-rich material in the ejecta reaches quasi-statistical equilibrium temperatures ($>3 \times 10^9$ K) and is burned to the Fe peak. In the high-entropy QSE regime the material is dominated by $^{54}$Fe, with a smaller component of $^{56}$Ni. At 2.0 foe some of the ejecta reaches full NSE ($T > 5 \times 10^9$ K) and is dominated by $^{56}$Ni ($0.3 M_\odot$).

This trend is exactly what we already expect with increasing explosion energy. At this stellar mass a range of energies is possible, and we must consider the entire range of yields. The more interesting comparison, however, is between a thermal bomb and a piston explosion of the same final energy. In terms of yields, the piston explosion mimics a higher energy thermal bomb explosion because the piston is more efficient at accelerating material where the explosion is imposed. Figure 15 shows the peak temperature achieved by the ejecta as a function of initial stellar mass coordinate for 1.2 foe thermal bomb and piston explosions. Material from deeper in the star escapes in the piston explosion. Figure 16 shows the velocities versus initial stellar mass coordinate.

1.5 foe some Si-rich material in the ejecta reaches quasi-statistical equilibrium temperatures ($>3 \times 10^9$ K) and is burned to the Fe peak. In the high-entropy QSE regime the material is dominated by $^{54}$Fe, with a smaller component of $^{56}$Ni. At 2.0 foe some of the ejecta reaches full NSE ($T > 5 \times 10^9$ K) and is dominated by $^{56}$Ni ($0.3 M_\odot$).

This trend is exactly what we already expect with increasing explosion energy. At this stellar mass a range of energies is possible, and we must consider the entire range of yields. The more interesting comparison, however, is between a thermal bomb and a piston explosion of the same final energy. In terms of yields, the piston explosion mimics a higher energy thermal bomb explosion because the piston is more efficient at accelerating material where the explosion is imposed. Figure 15 shows the peak temperature achieved by the ejecta as a function of initial stellar mass coordinate for 1.2 foe thermal bomb and piston explosions. Material from deeper in the star escapes in the piston explosion. Figure 16 shows the velocities versus initial stellar mass coordinate.
for the 1.2 foe piston and thermal bomb at the same time during the explosion. High velocities persist deeper in the star for the piston since it begins as a mass motion, rather than thermal energy that must be transformed into kinetic energy as in the thermal bomb.

Fryer (2006) developed an analytic method to estimate the explosion energy and remnant mass of a collapsing star based on the critical accretion rate of the infalling star onto the convective region. By varying this critical accretion rate, we can determine the dependence of the remnant mass on explosion energy. This curve is shown along with our 1D results in Figure 17. The pairs of blue points represent the total energy injected in the thermal bomb models (rightmost point) and the final kinetic energy of the explosion (left point). It is difficult to estimate the total energy injected by the piston. The analytic estimate predicts lower remnant masses for a given total energy than the thermal bombs. The piston model predicts the lowest remnant masses for a given explosion energy.

There are several differences between the assumptions in the Fryer (2006) analysis and the assumptions in our artificially driven explosions. First, the energy in the analytic estimate is the energy at the base of the explosion, estimated from the total energy stored in the convective region. This is not the total kinetic energy of the shock as it breaks out of the star. Much of this energy goes into unbinding the star. An alternate way to understand this is that the kinetic energy quoted in our artificial explosion is...
just a fraction of the total explosion energy. It is the total explosion energy that is quoted in most core-collapse engine calculations, but it is the final kinetic energy that is quoted in most estimates based on observations. So the primary difference between these results is the difference between the definitions of explosion energy between these two models. It is difficult to estimate the energy injected in the piston model, but we can calculate the energy injected in our thermal bomb calculations. Unfortunately, much of the energy injected in our artificial explosions ends up falling back onto the compact remnant, so the energy we deposit is likely to be higher than our analytic analysis that assumed all of that energy escaped. We see that this is true for all of our simulations (Fig. 17).

Other differences also exist. In the thermal bomb and piston explosions, the engine continues to be powered even after the explosion is launched. In both the analytic estimate and in a neutrino-driven explosion, the explosion energy is limited to what can be stored in the convective region before the launch of the explosion. After this launch neutrinos will not deposit much more energy into a strong explosion. The continued driving that occurs in our artificial explosions leads to less fallback than we would expect from neutrino-driven explosions. In this respect the analytic estimate is more realistic than our 1D simulations.

Over the energy range 0.8–1.5 foe in the thermal bombs the yield of oxygen increases by 70%, neon by 17%, silicon by 530%, and iron+nickel by 290%. We can also compare thermal bomb and piston explosions of the same energy. At 1.2 foe the yields in the piston explosion increase as follows: oxygen by 8%, Ne by 0.5%, Si by 1011%, and Fe+Ni by 324%. The $^{56}$Ni yield increases from essentially zero to $6.6 \times 10^{-3} M_\odot$. Figure 18 shows the yields ($M_\odot$) of carbon, oxygen, silicon, $^{54}$Fe, $^{56}$Ni, and the total iron plus nickel as a function of explosion energy, with thermal bombs in blue and pistons in red. Carbon decreases at higher energies as more ejecta is heated to C-burning temperatures. Oxygen increases as a larger fraction of the O-rich mantle is ejected, then decreases at energies that produce O-burning temperatures. Silicon has a sharp turn-up where presupernova silicon is finally ejected. $^{54}$Fe rises rapidly for energies that produce Si burning ($T > 3 \times 10^9$ K) and then drops at energies high enough for the equilibrium to favor $^{56}$Ni ($T > 5 \times 10^9$ K). Total Fe+Ni, of course, increases rapidly at energies that produce Si-burning temperatures. These trends are the same for both types of explosion but are shifted to lower energies for the piston models. The effect is especially dramatic for the Fe peak.

We also compare thermal bomb explosions with different explosion delays, as explained in § 2.2. For a star this massive it is
reasonable to expect a long delay in light of 3D simulations (Fryer & Young 2006). Figures 19–22 show the bulk element yields for explosions with additional delays of 0.2 s at 0.8 and 1.5 foe, and 0.7 s at 0.8 and 1.5 foe. These can be compared to Figures 4, 8, 10, and 12. The detailed isotopic yields are presented in Table 4. We find that for longer delays the yield of intermediate-mass and Fe peak elements increases because the longer energy input raises more material out of the potential well. However, this material is at systematically lower velocities and thus lower shock temperatures (see §2.2). As a result, the yield of Fe peak material processed at NSE temperatures is lower in the 1.5 foe explosions with longer delays than the 2.0 foe short delay, even though as much or more total material escapes. The $^{56}$Ni yield increases by 2 orders of magnitude for both moderate and long delays. $^{44}$Ti increases by a factor of 5 with 0.2 s additional delay and by another factor of 5 with 0.7 s as more material is heated to Si-burning QSE and escapes with longer delays. These results should be taken as indicative of the range of yields that can be generated with variation of parameters than a real trend. In 3D a longer delay allows more fallback, so this may be the opposite of what nature does.

4. DISCUSSION

This research provides a framework for estimating uncertainties in yields arising from the shortcomings of 1D explosion models. These errors fall roughly into two regimes, as follows.

In the low explosion energy regime the variation in yields arises simply from the amount of material processed by hydrostatic burning that escapes from the star. Ignoring multidimensional effects (the importance of which should not be underestimated), this can be described by a traditional mass cut.

In the high energy explosion regime we must deal with explosive nucleosynthesis as well. In this case the thermodynamic trajectory of the ejecta is important. The velocity evolution of a piston explosion and a thermal bomb of the same energy or of a piston and a thermal bomb with the same mass cut will differ slightly, imposing an accompanying difference in the thermodynamic evolution. Hence any material that undergoes explosive nucleosynthesis will have different abundances. The 1.2 foe piston and 1.5 foe thermal bomb produce the same mass compact remnant. If we examine the elemental yields we see that the effect is small, as the difference in the velocities is small. In individual isotopes, however, the difference can be larger. $^{56}$Ni is a factor of three larger in the piston driven explosion because its production is dominated by the highest temperature material, in which the largest deviation in thermodynamic histories is seen. This changes the Fe yield by $\sim$25%. The evolution would be substantially different for an explosion with a gain region as in the analytic case, and the abundances correspondingly disparate.

A small change in energy/compact remnant mass in this regime results in a much larger difference in the yields than does a similar change in the low-energy regime due to the steep temperature dependence of the burning. In addition, the time delay of the explosion has an effect in this regime. A longer delay in 1D results in more material escaping, but with lower shock temperatures, resulting in a higher intermediate mass to Fe peak ratio. In 3D we would expect more accretion onto the compact object for long delays, resulting in a lower overall ejecta mass. In 3D the longer delays are the result of a large mass accretion rate onto the compact object for a long period. This allows more infalling material to pass through the standing accretion shock before the explosion is launched. This also results in a lower energy explosion because the energy reservoir in a “real” explosion is finite, whereas a delay in a 1D parameterized explosion can be arbitrary for any energy. Thus, the amount of fallback is also greater. Results from 1D delays can therefore not be taken at face value.

We can choose between two parameters in constructing a grid of yields. If we choose to parameterize by compact remnant mass, which makes more sense in terms of yields, we must still take into account the thermodynamic history of the material that escapes. If we parameterize by explosion energy, which has some hope of being partially constrained by observations of explosions, we are left with a very uncertain remnant mass, amount of ejecta, and thermodynamic history.

Fryer & Kalogera (2001) examine how supernova explosion energy varies with mass. They argued that the supernova explosion energy peaks at an initial progenitor mass of $15\,M_\odot$. For $23\,M_\odot$ they predicted an explosion energy of less than $10^{51}$ ergs. While the progenitors we have used in this paper are very different from the progenitors used in Fryer & Kalogera (2001), we may still use the trends in that work to guess where the parameter space will open up for the uncertainties identified here. We find that large explosion energies are required to expel explosively processed material at $23\,M_\odot$ because of the massive core of the star. It is likely that sufficiently large explosion energies only occur if the collapsar engine is driving the explosion. If such stars do have low explosion energies, then our primary uncertainty will be in the O/Ne/Mg to Si/S part of the periodic table, where we may expect changes in O of tens of percent and changes in Si of factors of a few. In this case it is sufficient to examine a range of total ejecta masses to estimate the range of yields, remembering that the energy corresponding to a given remnant mass differs by $\sim$30% between piston and thermal bomb models and is even larger for a neutrino gain region. As our progenitor decreases in mass, explosion energy rises and the amount of energy required to eject explosively processed material declines. By analogy to the models in this work, the mass range where our expected explosion energy crosses into the regime where QSE and NSE processing take place can have uncertainties in Si by a factor of 10 and in Fe peak by factors of several. Useful explosion diagnostics like $^{56}$Ni can vary by orders of magnitude. At higher explosion energies the main uncertainties lie in the Fe peak, with a smaller contribution in the amount of Si-rich material left unburned. The situation is rendered more difficult by the fact that this parameterization relates progenitor mass to explosion energy, which does not have a one-to-one correspondence with remnant/ejecta mass.

| Species | $d0.2\times0.8$ | $d0.2\times1.5$ | $d0.7\times0.8$ | $d0.7\times1.5$ |
|---------|----------------|----------------|----------------|----------------|
| $^1\text{H}$ | 1.47E+00 | 1.47E+00 | 1.47E+00 | 1.47E+00 |
| $^2\text{H}$ | 3.03E–16 | 4.79E–16 | 3.86E–16 | 5.27E–16 |
| $^3\text{H}$ | 1.63E–16 | 1.66E–16 | 1.52E–16 | 1.66E–16 |
| $^3\text{He}$ | 6.63E–05 | 6.63E–05 | 6.63E–05 | 6.63E–05 |
| $^4\text{He}$ | 1.13E+00 | 1.12E+00 | 1.13E+00 | 1.13E+00 |
| $^6\text{Li}$ | 1.45E–18 | 2.45E–18 | 1.79E–18 | 2.53E–18 |
| $^7\text{Li}$ | 2.90E–10 | 2.90E–10 | 2.90E–10 | 2.90E–10 |
| $^7\text{Be}$ | 1.29E–10 | 1.29E–10 | 1.29E–10 | 1.29E–10 |
| $^8\text{Be}$ | 2.48E–10 | 1.56E–07 | 4.96E–11 | 9.19E–08 |
| $^9\text{Be}$ | 8.50E–19 | 1.19E–18 | 7.24E–19 | 9.68E–19 |

Note.—Table 4 is published in its entirety in the electronic edition of the Astrophysical Journal. A portion is shown here for guidance regarding its form and content.
We expect our maximum uncertainties for lower mass, and thus more common, progenitors. For these stars a two-parameter grid in remnant mass and thermodynamic history must be calculated to estimate the uncertainties. Without a good understanding of the supernova mechanism and the remnant to explosion energy relation, these grids and their corresponding error bars must be large. Perhaps more importantly, the solutions for a given abundance pattern are not unique, especially if we measure only elemental abundances. Ignoring variations in progenitors, initial mass functions, and 3D complications, we can independently adjust the explosion energy distribution, the energy/ remnant mass relation, and the thermodynamic history of an explosion to find a given result. To some extent we can constrain our parameter space by attempting to match the solar abundance pattern (which is itself in dispute! [Asplund et al. 2006]), but this gives us very little leverage in making predictions since it introduces further complications of star formation histories and the nonlinearity of enriching intermediate generations of model stars with our own uncertain yields.

It is worth discussing these results in the context of previous studies. We begin with Woosley & Weaver (1995), which provides the current standard for yields and the template from which later compilations of yields are derived. Globally, we notice that varying the explosion energy or the treatment of the explosion can have very large effects not only on individual, low abundance diagnostic isotopes, but also on the bulk yields of common elements. Woosley & Weaver (1995) examine different effective gravitational potentials in the piston (broadly equivalent to changing the final explosion kinetic energy while keeping the same energy input) for stars \( M > 30 M_\odot \). Below this mass limit they explore some small changes in the method of implementing the piston, which do have effects on important low abundance isotopes. However, they do not change the bulk energetics, assuming a final explosion kinetic energy of \( \approx 1.2 \times 10^{51} \) ergs. Woosley & Weaver (1995) do point out that the energies are uncertain, but they do not present any resulting uncertainties in the yields. If anything, the assumption of constant energy looks worse now.

Our still unsatisfactory understanding of the mechanism at least assures us that the explosion energies for stars in the common 10–20 \( M_\odot \) range will vary a great deal. Uncertainties of factors of more than 10 in silicon and the iron peak can occur for stars of masses common enough to be interesting for purposes of population yields, given what we must currently accept as a reasonable range of explosion energies.

More specifically, we can compare our 1.2 foe piston to the Woosley & Weaver (1995) S22A model. Our yield of carbon is almost exactly twice that of S22A. Oxygen is increased by \( \approx 80\% \), Si by 10\%. The S22A Fe peak is nearly a factor of 7 higher than our 23p-1.2, due almost entirely to their much larger yield of \( ^{56}\text{Ni} \). Almost all of these changes can be traced to differences in the progenitor model. Our progenitor includes mass loss and, most importantly, a much more realistic treatment of hydrodynamics and mixing, which results in much larger core sizes for a given mass. This contributes to the \( ^{56}\text{Ni} \) discrepancy as well, but as Woosley & Weaver (1995) point out, this is also sensitive to the details of the piston.

ABT91 find that the temperature profiles for thermal bombs and pistons are nearly identical in most of the stellar mantle, while they begin to diverge increasingly starting at several percent, when \( T \) exceeds \( 2 \times 10^9 \) K (see their Fig. 7). The mass represented by very large temperature differences is small. Since they define the ejected material by imposing a uniform mass cut, the bulk yields for a given final kinetic energy are similar.

The principal difference in our results versus those of ABT91 results from our treatment of the hydrodynamic evolution. We do not impose a mass cut; rather, we let material with insufficient kinetic energy to become unbound fall back onto the compact object. If we return to Figure 15, which compares the peak temperatures in thermal bomb and piston models with a final kinetic energy of 1.2 foe, we see that the temperatures in the ejected material begin to diverge at about \( T > 2 \times 10^9 \) K. Nevertheless, the difference remains small for the material ejected, just as in the ABT91 study. The two models have very different yields, however, because much more material falls back in the thermal bomb case. Indeed, when we compare the 1.2 foe piston and 1.5 foe thermal bomb or 1.6 foe piston and 2.0 foe thermal bomb, which have similar compact remnant masses, we find that the yields are reasonably similar, with the exception of NSE species such as \( ^{56}\text{Ni} \), which we would expect to show the effects of the initial difference in partition of kinetic and thermal energy between the thermal bomb and piston. Given that we are free to specify the location of the piston, its acceleration, velocity, and duration, we could probably produce more similar kinetic energies for a given compact remnant mass, but this ad hoc fine-tuning obscures the amount of variation produced by taking an independent “best guess” in setting up the two explosions to reproduce the observable constraint of ejecta kinetic energy.

The other notable difference is the mass coordinate at which corresponding temperatures occur in each study. ABT91 use a 20 \( M_\odot \) stellar model from Nomoto & Hashimoto (1988). Our progenitor is a 23 \( M_\odot \) model with hydrodynamics that accounts for hydrodynamic mixing processes that produce much larger nuclear processed cores and burning shells. The much more massive \( \chi \geq 16 \) mantle in our model requires much more energy input to produce a successful explosion, so high temperatures extend to higher mass coordinates, as do the Fe core and O and Si shells. This is exactly as we would expect. Once adjusted for the differences in progenitors, the agreement of our results is surprisingly good.

More recent studies corroborate these results. Umeda & Nomoto (2005) examine nucleosynthesis in Population III stars with varying amounts of fallback and mixing. While it is obviously impossible to compare our results directly, since the progenitors are of entirely different mass, metallicity, and input physics, their methodology has some bearing on this study. Their Figure 14 is particularly interesting, showing the allowed range of fallback masses required to derive a Mg/Fe ratio consistent with extremely metal-poor stars. They choose an initial mass cut of 2.44 \( M_\odot \), but their final allowed values are between 10.2 and 13.6 \( M_\odot \). This indicates that an enormous variety of yields can be generated from a single model by choosing the parameters for explosion energy and amount of fallback and mixing. Limongi & Chieffi (2003) examine nucleosynthesis with a variety of explosion parameters and fallback. Their yields are highly variable for Fe-peak and intermediate-mass elements, depending on the energy of the explosion and the amount of fallback. Their range of variation for CNO and Si is smaller than ours mainly because their range of explosion energies does not extend as low as ours. We cannot compare directly to their results because their progenitors do not include a realistic prescription for mixing during the evolution,
and therefore the extent of the various nuclear processed regions is smaller. However, if we examine their 35 M\(_\odot\) progenitor we can make some approximate comparisons since the extent of the oxygen and heavier core is similar. They show a range of a factor of five in 28Si and about an order of magnitude in 32S, 40Ca, and 56Fe. There is little variation of CNO because the minimum explosion energy is 1.3 fo, which ejects all or nearly all of the CNO mantle. We find similar results for explosion energies this high.

Since we are required by computational limitations to rely on 1D explosion for population yields, the best we can hope for in the near future is use a few 3D simulations provide some insight into energy distribution, the energy/rennant mass relation, and amount of mixing to constrain the size of the 1D grid. But recall that the current uncertainties in the explosion mechanism, even in 3D, prevent such results from being the final yields of stars. It is likely that we will have to live with uncertainties in the yields for some time yet, but understanding these uncertainties should help in understanding the current data.

4.1. O-rich Supernova Remnants

An incidental point of interest of this study concerns O-rich supernova remnants. As a class these remnants have very large masses of O-dominated ejecta and comparatively small enrichments in Si/S and Fe elements. In particular, N132D in the Large Magellanic Cloud (LMC) and 1E 0102.2–7219 in the SMC show no evidence for the products of oxygen burning, either hydrostatic or explosive (Blair et al. 2000). In addition, Zampieri et al. (2003) present an analysis of two dim supernovae, SN 1997D and SN 1997br, which have small explosion energies (<1 fo) and very small masses of 56Ni (<10^{-2} M\(_\odot\)). Taken together, these observations are most simply explained by the explosion of a massive star that does not eject material processed by oxygen burning or later stages.

We see from these results that for a 23 M\(_\odot\) progenitor, explosion energies of 1.2 and 1.5 fo for the piston and thermal bomb models, respectively, are needed to produce a significant amount of 56Ni. Small amounts, a few \times 10^{-3} M\(_\odot\), can be explained by 3D effects and mixing, even in weak explosions, but there is a very large discrepancy between the energy necessary for significant 56Ni production and the energies predicted for stars above 20 M\(_\odot\) by Fryer & Kalogera (2001). We thus expect most stars between \sim 20 M\(_\odot\) and the lower mass limit for Wolf-Rayet stars (30 M\(_\odot\)/?) to eject very little Si and Fe peak, which represents a significant population of objects that both explode as dim supernovae and produce O-rich remnants.

This work was carried out in part under the auspices of the National Nuclear Security Administration of the Department of Energy (DOE) at Los Alamos National Laboratory and supported by contract DE-AC52-06NA25396, by a DOE SciDAC grant DE-FC02-01ER41176, an NNSA ASC grant, and a subcontract to the ASCI FLASH Center at the University of Chicago.

References

Aoki, W., et al. 2006, ApJ, 639, 897
Asplund, M., Grevesse, N., & Sauval, A. J. 2005, Nucl. Phys. A, 777, 1
Aufderheide, M. B., Baron, E., & Thielemann, F.-K. 1991, ApJ, 370, 630
Benz, W., Thielemann, F.-K., & Hills, J. G. 1989, ApJ, 342, 986
Berger, E., et al. 2006, ApJ, submitted (astro-ph/0603689)
Blair, W. P., et al. 2000, ApJ, 537, 667
Blöcker, T. 1995, A&A, 297, 727
Blondin, J. M., & Mezzacappa, A. 2006, ApJ, 642, 401
Buras, R., Rampp, M., Janka, H.-Th., & Kifonidis, K. 2003, Phys. Rev. Lett., 90, 1101
Burrows, A., Livne, E., Dessart, L., Ott, C. D., & Murphy, J. 2006, ApJ, 640, 878
Erb, D., et al. 2007, ApJ, 644, 813
Frolich, C., et al. 2006, ApJ, 637, 415
Fryer, C., Benz, W., Herant, M., & Colgate, S. A. 1999a, ApJ, 516, 892
Fryer, C. L. 2006, NewA Rev., 50, 492
Fryer, C. L., Colgate, S. A., & Pinto, P. A. 1999b, ApJ, 511, 885
Fryer, C. L., & Heger, A. 2000, ApJ, 541, 1033
Fryer, C. L., & Kalogera, V. 2001, ApJ, 554, 548
Fryer, C. L., & Kusenko, A. 2006, ApJS, 163, 335
Fryer, C. L., & Warren, M. S. 2002, ApJ, 574, L65
———. 2004, ApJ, 601, 391
Fryer, C. L., & Young, P. A. 2006, ApJ, submitted
Fryer, C. L., Young, P. A., & Hungerford, A. L. 2006, ApJ, 650, 1028
Garnett, D. R. 2002, ApJ, 581, 1019
Graboske, H. C., Dewitt, H. E., Grossman, A. S., & Cooper, M. S. 1973, ApJ, 181, 457
Grevesse, N., & Sauval, A. J. 1998, Space Sci. Rev., 85, 161
Hamuy, M. 2003, ApJ, 582, 905
Herant, M., Benz, W., Hix, W. R., Fryer, C. L., & Colgate, S. A. 1994, ApJ, 435, 339
Hungerford, A. L., Fryer, C. L., & Rockefeller, G. 2003a, ApJ, 635, 487
Hungerford, A. L., Fryer, C. L., & Warren, M. S. 2003b, ApJ, 594, 390
Kifonidis, K., Plewa, T., Janka, H.-Th., & Müller, E. 2000, ApJ, 531, L123
———. 2003, A&A, 408, 621
Kotake, K., Yamada, S., & Sato, K. 2003, ApJ, 595, 304
Kudritzki, R. P., Pauldrach, A., Puls, J., & Abbott, D. C. 1989, A&A, 219, 205
Langanke, K., & Martinez-Pinedo, G. 2000, Nucl. Phys. A, 673, 481
Limongi, M., & Chieffi, A. 2003, ApJ, 592, 404
Meakin, C. A., & Arnett, D. 2007, ApJ, 665, 690
———. 2006, ApJ, 637, L53
Meynet, G., & Maeder, A. 2000, A&A, 361, 101
Nagataki, S., Hashimoto, M., Sato, K., Yamada, S., & Mochizuki, Y., S. 1998, ApJ, 492, L45
Nomoto, K., & Hashimoto, M. 1988, Phys. Rep., 163, 13
Nomoto, K., Umeda, H., Maeda, K., Ohkubo, T., Deng, J., & Mazzali, P. 2003, Nucl. Phys. A, 718, 277
Pritzl, B. J., Venn, K. A., & Irwin, M. 2005, AJ, 130, 2140
Rauscher, T., & Thielemann, F.-K. 2001, At. Data Nucl. Data Tables, 79, 47
Shimizu, T., Yamada, S., & Sato, K. 2003, ApJ, 594, 332
Thielemann, F.-K., Nomoto, K., & Hashimoto, M.-A. 1996, ApJ, 460, 408
Thielemann, F.-K., et al. 2002, Ap&SS, 285, 21
Umeda, H., & Nomoto, K. 2005, ApJ, 619, 427
Woosley, S. E. 1993, ApJ, 405, 273
Woosley, S. E., & Weaver, T. A. 1995, ApJS, 101, 181
Young, P. A., & Arnett, D. 2005, ApJ, 618, 908
Young, P. A., Meakin, C., Arnett, D., & Fryer, C. L. 2005, ApJ, 629, L101
Zampieri, L., Pastorello, A., Turatto, M., Cappellaro, E., Benetti, S., Altavilla, G., Mazzali, P., & Hamuy, M. 2003, MNRAS, 338, 711

This work was carried out in part under the auspices of the National Nuclear Security Administration of the Department of Energy (DOE) at Los Alamos National Laboratory and supported by contract DE-AC52-06NA25396, by a DOE SciDAC grant DE-FC02-01ER41176, an NNSA ASC grant, and a subcontract to the ASCI FLASH Center at the University of Chicago.