Evaluation of wall functions for RANS turbulence models by unsteady lid-driven cavity flow

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Abstract
Turbulence models do not perform well near walls because the Reynolds number in this interesting area is low. The wall functions and the advantage method can be efficiently used to model fluid flow at the near wall region. The Reynolds averaged Navier-Stokes turbulence models which had the developed wall functions in OpenFOAM, the open source CFD software, were evaluated by the turbulent flow. The 3-D lid-driven cavity flow at the turbulence Reynolds number of 10,000 with various spanwise aspect ratio had been referred to validate these turbulence models. The comparison shown that the realizable $k-\varepsilon$ turbulence model with the standard $k$ and $\varepsilon$ wall functions of OpenFOAM was the most powerful model to represent the velocity profile of water flow in the 3-D lid-driven cavities.

Keywords: Wall function, RANS, Turbulence model, Cavity

1. Introduction
The Reynolds averaged Navier-Stokes (RANS) turbulence model obtained by the evaluation of Reynolds stress in the RANS equation [1]. To calculate the Reynolds stress, the transport equations of the turbulent energy were carried out for the computational fluid dynamics (CFD) modeling. Many transport equations models had been used to estimate the Reynolds stress. The $k-\varepsilon$ and $k-\omega$ models are two-equation transport models widely used to simulate turbulence flow, based on flow over immersed bodies and duct flow [2, 3]. The $k-L-\omega$ turbulence model is the three-equation transport model included in the RANS turbulence model which is also used to simulate turbulent flow [4]. The $\overline{v^2} - f$ model otherwise the RANS turbulence model is composed of four transport equations. It met with great success to simulate some kinds of the open channel flow [5-8]. The turbulence models which were reviewed usually simulated the complex flow domains. The fine grids at the rigid wall of complex domain were required to process these turbulence models efficiently. Consequently, the computational costs were consumed importantly to capture the turbulent flow at the near wall region.

The advantage approach which can reduce the computational costs of the turbulent flow simulation wall functions. In this way, the near wall region of complex flow domain will yield coarse grids. Many wall functions were improved and developed for the remedy of this problem. For example, Parente et
al. [9] had been improved $k - \varepsilon$ model and wall function to simulate the atmospheric boundary layer flow. The source term was added to the dissipation rate equation while the rough surface was improved for wall function. The flow around bluff-body was carried out to demonstrate the improved $k - \varepsilon$ model and wall function. Nazif et al. [10] derived an analytic wall function which significantly reduced the computational costs. The Robin-type wall function had been implemented by using two benchmark cases, near-equilibrium channel flow and flow over a backward-facing step. This function obtained computational costs slightly higher than the standard wall function but better kept the DNS results. Chedevergne [11] had developed wall function by including the roughness corrections in the analytical wall function model to investigate the roughness effect on velocity profile. However, the improved wall function adapted only to the $k - \omega$ SST model. Beaugendre and Morency [12] proposed a wall function which was compatible with the penalization method and vortex formulation to simulate fluid flow around complex geometries. The velocity profiles of the Spalart-Allmaras model, RANS turbulence model, with a wall function agreed with the analytic law of the wall solution. In addition to the turbulent flow, Cao et al. [13] improved heat transfer model based on an enhanced thermal wall function. The RNG $k - \varepsilon$ turbulence model of gas flow was utilized in the heat transfer model. The stable and accuracy predictions were achieved by the improved model. Consequently, the wall functions advantage to reduce simulating time by increase of grid size at near wall but still keep an accurate result.

The flow simulation with turbulence model and wall function which were reviewed were generally performed by the commercial CFD software. It was the great barrier to the researchers whom had not had funding for the license software support. This research aimed to carry out turbulent flow simulation using the open source CFD software, Open FOAM. The RANS turbulence model and wall functions were implemented and valid by the experimental data of reference papers [14-16]. The capable RANS turbulence model and wall functions were proposed to advance the flow simulation in the further works.

2. Wall functions derivations

In the process of derivation of wall function, turbulence parameters are given in non-dimensional form as follows:

$$
U^+ = \frac{\bar{U}}{u_\tau}, k^+ = \frac{k}{u_\tau^3}, \varepsilon^+ = \frac{\varepsilon u_\tau}{\bar{U}}, \omega^+ = \frac{\omega u_\tau}{\bar{U}}, \tau^+ = \frac{\tau}{\rho u_\tau^2}, u_+ = \frac{u_\tau}{\rho}, \nonumber
$$

where $\tau$ is the local wall shear stress, $\bar{U}$ is averaged velocity, $u_\tau$ is the friction velocity, $k$ is the turbulent kinetic energy, $\varepsilon$ is the rate of viscous dissipation, $\nu$ is the kinematic viscosity, $\omega$ is the turbulence frequency, $\nu_2$ is the wall-normal stress, and $f$ is the wall-normal fluctuation.

2.1 The $k - \varepsilon$ turbulence model

The standard $k - \varepsilon$ turbulence model composes two transport equations to solve two turbulence quantities, the turbulence kinetic energy and rate of viscous dissipation [17]. In the viscous sub-layer, the wall shear stress is equal to the shear stress of fluid flow which is written as follows:

$$
\tau = \tau_w = \mu \frac{\partial \bar{U}}{\partial y} \nonumber
$$

The velocity on the solid wall is assessed by the no-slip condition, therefore the solutions of two transport equations are written as follows:

$$
k = u_\tau^2 C_{\mu}^{-1/2}, \varepsilon = C_{\mu}^{3/4} k^{3/2} \frac{\nu}{\kappa y} \nonumber
$$

where $\kappa$ is the Karman coefficient which is equal to 0.41, and $C_{\mu}$ is equal to 0.09.

The fluid flow inside the inertial sub-layer, therefore the fluid velocity can be written by following equation.

$$
U^+ = \frac{\bar{U}}{u_\tau} = \frac{1}{k} \ln(Ey^+), y^+ = \frac{y}{\nu} \sqrt{\frac{\tau_w}{\rho}} \nonumber
$$

2
where $E$ roughness coefficient.

The turbulence kinetic energy and rate of viscous dissipation are written by following equations.

$$k = u_2^2 C_{\mu}^{-1/2}, \varepsilon = \frac{u_2^3}{k y}$$ (5)

### 2.2 The $k - \omega$ model

The $k - \omega$ turbulence model is composed the turbulence kinetic energy and turbulence frequency transport equations [18]. In the viscous sub-layer, these transport equations have solution as written by:

$$\omega^+ = \frac{6}{\beta_2(y^+)^2}k^+ = C_\mu (y^+)^{3.23}$$ (6)

In the inertial sub-layer, the diffusion term is assumed to be small when compared to the other two terms of the frequency transport equation. Using $k^+ = ky^+\omega^+$, the turbulence kinetic energy equation can balance to provide the $\omega^+$ and give $k^+$ by the following equations.

$$\omega^+ = \frac{1}{\kappa \sqrt{C_\mu y^+}}, k^+ = \frac{1}{\sqrt{C_\mu}}$$ (7)

### 2.3 The $\overline{u^2} - f$ model

The $\overline{u^2} - f$ is four transport equations in RANS turbulence model [1]. In the viscous sub-layer, the $\varepsilon^+$ and $k^+$ can be analytic, and written as follows:

$$\varepsilon^+ = \frac{2400}{C_{\varepsilon 2}} \left( \frac{1}{(y^+ + C)^4}, k^+ = \frac{2400}{C_{\varepsilon 2}} \left( \frac{1}{(y^+ + C)^4} + \frac{2y^+}{C^3} - \frac{1}{C^2} \right) \right)$$ (8)

The boundary condition of $f^+$ is derived by assuming that $\overline{u^2}^+$ is equal to $C_{v2}(y^+)^4$, therefore the solution of $\overline{u^2} - f$ equations is written as follows:

$$f^+ = \frac{-4(6 - N)(\overline{u^2})^+}{\varepsilon^+ (y^+)^2}$$ (9)

In the inertial sub-layer, the diffusion term is small, therefore the diffusive flux is constant as follows:

$$ky^+ \frac{dy^+}{dy^+} = C_k$$ (10)

Integrating, equation (10) yields the following equation.

$$k^+ = \frac{C_k}{k} \log(y^+) + B_k$$ (11)

In the same way, the $\overline{u^2}^+$, $\varepsilon^+$ and $f^+$ are obtained as follows:

$$\overline{u^2}^+ = \frac{C_{v2}}{k^+} \log(y^+) + B_{v2}, \varepsilon^+ = \frac{1}{ky^+}, f^+ = N \left( \overline{u^2}^+ \right)^4$$ (12)

To fit the numerical solution of $\overline{u^2} - f$ equation, the following values for the constants are given as follows:

$$C_k = -0.416, B_k = 8.366, C_{v2} = 0.193, B_{v2} = -0.940$$ (13)

### 3. Verifications

The lid-driven cavity flow models was developed following the dimension of three rectangular cavities. The cross section of each cavity was equal to the square size of $150 \times 150$ mm$^2$ (BxH) while the length was various to investigate the side wall effect. The spanwise aspect ratio (SAR) composes, the ration between length (L) and width (B), 0.5/1, 1/1, and 3/1, respectively. Figure 1 shows the schematic of the rectangular cavity flow. The fluid of cavity flow was water which had the constant density and kinematic viscosity of 997.05 kg/m$^3$ and 0.89x10$^{-6}$ m$^2$/s, respectively. The water flow over the cavity along the flow, characteristic length, with the constant velocity of 0.05953 m/s, therefore it sturdily occupied the Reynolds number of lid-driven cavity flow at 10,000. The velocity on five sides of the cavity was fixed by the no-slip boundary condition. The pressure boundary condition was the zero gradient on every
The transient lid-driven cavity flow schematic.

Figure 1. The transient lid-driven cavity flow schematic.

side of cavity flow model. The dimensionless wall distance \( (y^+) \) at the wall region was five which were the maximum size that be still in the viscous sub-layer. The grid structure of the lid-driven cavity flow model was possessed by \( y^+ \); therefore, the uniform grids which were divided the rectangular cavities have the cubical size with the dimension of 2.5×2.5×2.5 mm\(^3\). The RANS turbulence model which was used to simulate the transient lid-driven cavity flow composed the \( k - \varepsilon \), \( k - \omega \), \( k - \omega \) SST, \( RNG \) \( k - \varepsilon \), \( Realizable \) \( k - \varepsilon \) and \( \bar{v}^2 - f \) turbulence model. The turbulence parameters on the rigid wall of the lid-driven cavity flow model including \( k, \varepsilon, \omega, \bar{v}^2 \) and \( f \) were defined according to the wall functions based on the viscous sub-layer. The transient flow was estimate by Euler implicit method with the constant time step of 0.01 seconds. Effects of velocity of water flow inside the cavity was assessed by the limited linear differencing scheme which was class of the total variation diminishing (TVD) schemes equation. The iteration for the pressure-velocity coupling problem of flow had been solved by PISO method. All of lid-driven cavity flow cases, the personal computer with the i7-8700K processor and 8 GB DDR RAM memory was intended to calculate the effect of wall functions together with the RANS turbulence model of OpenFOAM.

4. Results and discussion
The simulation results were illustrated on the cross-section plan at the mid length of cavities. To perceive the phenomenon of lid-driven cavity flow, the stream tracer was used to present the eddy flow in cavities. Figure 2 shows the stream tracer in the cross-section plan at the mid length of lid-driven cavity with a SAR of 3/1. The transient flow inside lid-driven cavity had scenario of the secondary eddy forming. It consisted of one main circulation flow and three secondary eddies. The first secondary eddy established at the right bottom corner of cavity, the downstream secondary eddy, by the near boundary flow past a right-side wall and right angle. The upstream secondary eddy occurred at the left bottom corner of cavity by the near boundary flow past bottom wall and right angle, which was the second order secondary eddy. The final secondary eddy was the upper secondary eddy, which formed at the left top of cavity cause of the near boundary flow past just left side wall of cavity. The shear stress and flow direction were the main cause of the separated flow that the secondary eddies were formed.

Figure 2. The stream tracer of flow in the cross-section plan at the middle length of lid-driven cavity for SAR 3/1.
The transient lid-driven cavity flow was investigated in the time period of 150 seconds. The RANS turbulence models could calculate to form the secondary eddies in different time. In the lid-driven cavity with a SAR of 3/1, the downstream eddy by the \( k - \varepsilon \), \( k - \omega \), \( k - \omega \) SST and RNG \( k - \varepsilon \) turbulence model was formed after 35, 35, 35 and 35 seconds, respectively. Subsequently, the upstream eddy by the \( k - \varepsilon \), \( k - \omega \), \( k - \omega \) SST and RNG \( k - \varepsilon \) turbulence model was formed after 45, 45, 50 and 50 seconds, respectively. The downstream and upstream secondary eddies of the \textit{Realizable} \( k - \varepsilon \) and \( \bar{v}^2 - f \) turbulence model occurred almost in the same time (20 seconds). The upper secondary eddy was formed by every RANS turbulence model. The \( k - \varepsilon \), \( k - \omega \), \( k - \omega \) SST, RNG \( k - \varepsilon \), \textit{Realizable} \( k - \varepsilon \) and \( \bar{v}^2 - f \) turbulence models could form the upper secondary eddy after 125, 70, 70, 90, 100 and 90 seconds, respectively. Cause of the proper wall functions, three secondary eddies could calculate by RANS turbulence models and the sluggish forming of upper secondary was observed by the \( k - \varepsilon \) turbulence model. In the near-wall region, the \( k - \omega \) model had good to implement boundary layer. The \( k - \omega \) and \( k - \omega \) SST were incapable preserving the conformation of secondary eddies completely to 150 seconds due to the end-wall effect arrived to the mid plane of cavity faster than other models. Figure 3 shows the stream tracer in the lid-driven cavity with the SAR of 3/1 by the \textit{Realizable} \( k - \varepsilon \) turbulence model. The secondary eddy forming happened at three regions along the transient flow simulation [18]. Consequently, entire RANS turbulence models which used wall functions except the \textit{Realizable} \( k - \varepsilon \) and \( \bar{v}^2 - f \) turbulence models could calculate secondary eddies in order to experiment.

In the lid-driven cavity with a SAR of 1/1, the upper secondary eddy generation was affected by its end walls. The end walls drag the main circulation of water flow. The upper secondary eddy could not be generated by each RANS turbulence model. The \( k - \varepsilon \), \( k - \omega \) SST, \textit{Realizable} \( k - \varepsilon \) and \( \bar{v}^2 - f \) turbulence models could not form the upper secondary eddy while it was happened by the \( k - \omega \) and RNG \( k - \varepsilon \) turbulence models after 70 and 85 seconds, respectively. Even though the \( k - \omega \) and RNG \( k - \varepsilon \) turbulence model could calculate the upper secondary eddy successfully, it was not the complete pattern of the secondary eddies forming. The end walls were close to the mid plane of lid-driven cavity with a SAR of 0.5/1 were more influent than other larger cavity. The last secondary eddy could ever be formed by any RANS turbulence model. It was eliminated caused of the drag effect grew stronger at the middle plane.

All RANS turbulence model which governed all size of lid-driven cavity mostly formed the secondary eddies corresponding the experiment after 60 seconds. Consequently, the normalized mean velocity profiles, \( \bar{U} / U_B \) and \( \bar{V} / U_B \), by the RANS turbulence model in the lid-driven cavity for three SARs at the corresponding time are plotted by graphs as shown in left of figure 4. In the lid-driven cavity with SAR of 3/1, the \textit{Realizable} \( k - \varepsilon \) turbulence model was in the superb agreement with the experimental data and then the \( k - \varepsilon \) turbulence model was the worst agreement. The different value of \( \bar{U} / U_B \) and \( \bar{V} / U_B \) from the experiment just were 0.0356 and 0.0344. The \textit{Realizable} \( k - \varepsilon \) turbulence model was still superb agreement with the experiment in the lid-driven cavity for the SAR of 1/1 while the worst agreement was change to be the \( k - \omega \) turbulence model. It had the different value of \( \bar{U} / U_B \).

![Figure 3](image-url)

**Figure 3.** The transient flow inside lid-driven cavity for SAR of 3/1 by the \textit{Realizable} \( k - \varepsilon \) turbulence model after (a) 40, (b) 60 and (c) 100 seconds.
Figure 4. The comparison of normalized mean $U$- and $V$-velocity profiles inside, and normalized mean $U$-velocity profiles over the bottom wall of lid-driven cavity for SAR of (a) the 3/1, (b) 1/1 and (c) 0.5/1.
and $\bar{V}/U_B$ from the experiment about 0.0180 and 0.0308. In the most end-wall effect as the lid-driven cavity with the SAR of 0.5/1, the Realizable $k - \varepsilon$ turbulence model was the last turbulence model standing for the good agreement with experiment with the different value of $\bar{V}/U_B$ and $\bar{V}/U_B$ from the experiment were 0.0627 and 0.0401, respectively.

In the right of figure 4 shows the comparison graph of lower part of normalized $U$-velocity profiles between experiment and simulation of the lid-driven cavity with the SARs of 0.5/1, 1/1 and 3/1. The comparison showed that the Realizable $k - \varepsilon$ turbulence model had the normalized mean $U$-velocity profile in the good agreement with experiment more than the other models. It was a good end-wall effect sensitivity. This study had been verified the wall functions. The $k$ and $\varepsilon$ wall functions had been used efficiently with the Realizable $k - \varepsilon$ turbulence model even though both wall functions were derived by the $k - \varepsilon$ turbulence model. Therefore, the simulation result will be good agreement with experiment rather than wall functions of this study if it is derived by pertinent model. Further investigation, the mean $U$-velocity profile could be attributed to the effect of the end wall. As the SAR of cavity decreased, the end-wall effect forced a higher peak $U$-velocity at the near-wall region of bottom cavity.

5. Conclusion
The transient lid-driven cavity flow was modelled by the RANS turbulence models of Open FOAM in this study. According to the simulation results, the RANS turbulence models with the viscous sub-layer wall functions at the maximum $y^+$ of 5 could illustrate sequence of secondary eddy forming in different time periods. It was found that four turbulence models concluding the $k - \varepsilon$, $k - \omega$, $k - \omega$ SST and RNG $k - \varepsilon$ turbulence models could form the secondary eddies respectively in accordance with the experiment. Decrease of wall distance from the mid plane increased effect to generation of the upper secondary eddy, and conserved the complete pattern of secondary eddies. Particularly, the end-wall effect increased the peak of normalized mean $U$-velocity at the near-wall region over the bottom wall of cavity. It was found that only the Realizable $k - \varepsilon$ turbulence model with the $k$ and $\varepsilon$ wall functions could show very good agreement with the experiment. These wall functions which were implemented in Open FOAM could represent the normalized mean $U$-velocity profile in the near-wall region efficiently even though they were derivative by the $k - \varepsilon$ turbulence model. The velocity profile at the near-wall region by the Realizable $k - \varepsilon$ turbulence model may be very accurate if the $k$ and $\varepsilon$ wall functions are derived from the pertinent turbulence model in the future work. However, this explored model advantageously obtained accurate results when simulated near wall flow problems.

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