Theoretical justification of impact speeds of working tools on water-melons

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Abstract. The interaction process between the working tool of a collector and a fruit has been considered, as well as the process of hitting water-melons on various surfaces. A diagram of speed distribution when the working tool interacts with a fruit has been presented, as well as that of the area of expansion of plastic deformation after a shock impact on a fruit. The absolute and allowable increment speeds after a shock have been theoretically determined and justified. The parameters of the allowable speeds for variety “Crimson sweet” and for variety “Kholodok” have been theoretically found and the verification of the theoretical analysis was performed through experimental researches on an installation for determining the critical impact speed on a fruit. The comparison of the theoretical and experimental values of the limit speeds confirms the accuracy of the studies.

1. Introduction

Water-melons are considered to be the largest berries in the vegetable world. As any of the agricultural materials, a water-melon has a complex texture, which is represented, from a physical standpoint, by three main components: solid, liquid, and gaseous. The liquid is the base of its mechanically weak pulp, which is covered with a smooth elastic membrane. The form of a melon which occurs most frequently is spherical; from a mathematical standpoint – close to spherical. Such a form is characterized with essential values of surface stresses. Practically, this statement is confirmed by the phenomena of a rind break by the forces of surface tension, which are of explosive nature even when negligible mechanical impacts are applied. A break is accompanied with deep fractures, sometimes down to the fruit center, the fracture lines are of stochastic nature in the length and directions. If we refer to the fruit build, one should note that the liquid therein, which is in the form of a water solution with other elements, sucrose, fructose, etc., constitutes up to 90 % of the whole weight. Thereby, the liquid is in free condition. It appears from this that the liquid, when an external mechanical impact is applied, can be a source of internal breaking forces of shock or wave types. The hydraulic effect is taken as a basis of formation of these forces. In their value, these forces considerably exceed the forces of external impact on a fruit [1].

When a visual observation is carried out, with the practical conclusions being also taken into account, one should note that the majority of the fruits damaged occur due to impact loads. Hence it appears that the basic constituent of the dynamic loads on water-melon fruits is a shock. The theoretical study of this phenomenon enables to determine the basic cinematic and design parameters for water-melon harvesters [2].
To obtain more consistent results, it is however, necessary to study the physical and mechanical properties. The mathematical models to describe the shock process will be more consistent if the theory of study of various media is used for the induction thereof [3].

2. Materials and methods, results of experimental researches and theoretical studies

Thus, the material deformation process, when dynamic loading is applied, should be considered by assimilating it to a certain mechanical model of a fruit.

Thereby, the liquid is encased in the cell membranes, but they are unable to resist even minor mechanical impacts and virtually destroy even after a mild effort. Seeds, however, form part of a water-melon pulp and they are covered with film membranes with air inside. The total volume of such inclusions can reach up to 5% of a fruit weight. Consequently, up to 7-8% in a single pulp. I S Yegorov determined in his studies that the relaxation time after a shock impact on a water-melon fruit reaches (0.01-0.035) s. The relaxation time is a period during which a wave passes from the striker to the rind and comes back [4].

The subsequent experiments have established that the assumption of a shock impact on a fruit is confirmed by the conclusions about the relaxation.

The question of interaction between the working tool and a fruit has been considered by many researchers and the process of hitting water-melon on different surfaces, which takes place in operation of some collecting units, has been also studied in close detail. We have proposed the study of this phenomenon in respect to the collector.

A shock impact takes place in the contact point between the working tool and a motionless fruit. In mechanics, this phenomenon is described with the equation:

\[ mu - mv = \int_0^t F dt = S, \]

where: \( v \) is the speed of the material point before the shock impact and \( u \) is its speed after the shock impact.

In our case, a water-melon, which is influenced by the working tool with certain speed \( v \), is considered to be the material point.

If the surface of the working tool, which is a plank installed under a certain angle, did not rotate, the impact speed in the contact would equal to its motion speed. And, since the working tool rotates with a certain frequency in the collector structure and, alongside this, it moves translationally, there arise relative and transportation velocities, that is, the cumulative value of the impact speed is equal to the vector sum of the speeds:

\[ \vec{v}_a = \vec{v}_r + \vec{v}_c, \]

where: \( \vec{v}_a \) is the vector of the absolute motion speed of the working tool; \( \vec{v}_r \) is the vector of the transportation velocity; \( \vec{v}_c \) is the vector of the relative velocity.

Transportation velocity \( v_c \) equals to collector motion speed \( v_c = v_u \). The relative velocity is the peripheral speed of the working tool point, which is simultaneously the point of contact with the fruit.

Let us denote the correlation between the transportation and relative velocities through coefficient \( \lambda \), \( \lambda = \frac{v_c}{v_r} \). To determine the absolute increment speed after the shock impact, let us consider the diagram in figure 1.
Figure 1. Diagram of speed distribution when working tool interacts with a fruit.

According to mechanics [5]:

When a shock is direct

\[ v_a = \left[ v_r^2 + v_r^2 \right]^{1/2} \]

When a shock is oblique

\[ v_a = \left( v_r^2 + v_r^2 + 2v_r v_r \cos \gamma \right)^{1/2} \quad (3) \]

Let us replace the transportation velocity with its value:

\[ v_a = \left( \lambda^2 v_r^2 + v_r^2 + 2v_r v_r \lambda \cos \gamma \right)^{1/2} = v_r \left( \lambda^2 + 2\lambda \cos \gamma + 1 \right)^{1/2} \]

In this regard the authors denote \( \left( \lambda^2 + 2\lambda \cos \gamma + 1 \right)^{1/2} = \lambda \), then \( v_a = \lambda v_r \). We consider that a fruit rolls over without gliding under the influence of the working tool.

Shock phenomena arise in the contact point. V P Goryachkin noted that any shock phenomena are inextricably linked with the vibrations of elastic bodies [6]. These vibrations are decaying and, in some cases (a minor shock pulse or a high density of the material), even the first shock wave does not come back.

Many researchers characterize the internal mechanics of a shock with two stages: the first one is the expansion of the compression wave and it finishes when the speeds of any internal particles become zero, the second one is the expansion of the tension wave, which finishes when the speeds of any particles are equal to \( v \) (an absolutely elastic shock) [7].

Thus, a body bounces from the striker with speed \( v \), which is equal to the striker motion speed at the end of the second stage where an absolutely elastic shock takes place. The whole energy obtained during the shock is consumed for the recovery of its shape and the value for the resilience is numerically equal to \( W = \frac{mv^2}{2} \), where \( m \) is the body weight, \( v \) is the striker motion speed. The phenomenon of an incompletely elastic shock is characteristic for the most agricultural processes, with a part of energy being lost for the following motion and, according to the expression by V.P. Goryachkin, composing “the dead part of the shock” [6]. The impact energy is thereby consumed for the plastic deformation and resilience. Identical phenomena take place in cucurbit fruits under a mechanical impact [1]. In conformity with the tests, the plastic deformation involves some deformable volume (figure 2)
To solve the problem of impacting a water-melon with an absolutely rigid body, it is necessary to proceed to discrete models [8]. The discrete model for a viscoelastic medium (Kelvin-Feucht medium) represents a successive combination of elastic and plastic elements. The rheological equation is written in the form of:

$$\sigma = E \varepsilon + \mu \frac{d \varepsilon}{dt},$$

where $E$ is the elastic modulus; $\varepsilon$ is the linear strain of the body; $\mu$ is the viscosity coefficient.

The first summand of the equation describes the elastic properties of the body, the second summand describes the plastic ones. In a manner similar to this equation, one can write down the energy equation during a shock impact:

$$A = W_1 + W_2,$$  \hspace{1cm} (4)

where: $W_1$ is the elastic work of deformation,

$W_2$ is the plastic work of deformation.

The value of work in general, which is obtained during a shock impact, can be written with the equation:

$$A = \int F \cos \varphi \, dt,$$  \hspace{1cm} (5)

where $F$ is the sum of the forces, which impact the system; $v$ is the system joint speed; $\varphi$ is the angle between the sum vector of the forces and speeds according to the diagram (figure 1) $\varphi = \gamma = 0.73$ rad.

The value of the elastic work of deformation of a fruit rind is expressed as:

$$\frac{m(v_v + \Delta v)^2}{2} - \frac{mv_v^2}{2} = m \Delta v + \frac{m(\Delta v)^2}{2},$$  \hspace{1cm} (6)

where $v_v$ is the absolute speed of point “M” of the working tool, which is the point of contact with a fruit; $\Delta v$ is the increment velocity due to the resilience.

By substituting expressions (5) and (6) into equation (4) and taking into account that $v_v = \Delta v$, we will obtain the following expression:

$$\frac{1}{0} \int Fv, \, dt = mkv_v, + 0.5mk^2 (v_v)^2 + W_2$$  \hspace{1cm} (7)

For an absolutely elastic shock, we have the expression:

$$\frac{1}{0} \int mav, \, dt = m \int v_dv_v, + 0.5m \int d (v_v)^2$$  \hspace{1cm} (8)

For an elastoplastic (incompletely elastic) shock:

$$\frac{1}{0} \int mav, \, dt = mkv_v^2 + 0.5mk^2 v_v^2 + W_2$$  \hspace{1cm} (9)

$k$ is the numerical coefficient, which is called a coefficient of a body restitution [9]. It is determined experimentally and is numerically equal to the ratio between a body speed after the impact and the speed
of the striker. The coefficient of a body restitution for water-melons equals to \( k = 0.38 \). Let us determine the part of energy consumed for plastic deformation during a shock impact from the equation:

\[
W_2 = \int_0^l p \xi dl \cdot
\]

(10)

where \( p \) is the specific deformation force in a shock destruction, N/mm\(^2\); \( \xi \) – the section area, mm\(^2\); \( l \) – is the expansion depth of the deformation, mm.

According to the theory, plastic expression (10) represents the most general expression of work of deformation [10]. For an accurate mathematical description of body impact processes, the equilibrium equation of an infinitely small medium volume is applied, the solution of which is difficult in most cases. Let us introduce some assumptions to simplify the mathematical expression:

1. Any sections of the body, which were formed before a shock impact, do not change their shape during the shock impact as well;
2. The normal stresses are equal between each other in any cross section points;
3. Due to a negligible friction coefficient on a water-melon rind \( f = 0.2 \), we do not take into account the shear stress on the fruit surface.

On the basis of the law of least resistance, the movement of body points during a shock impact takes place towards the least resistance and in the direction of the shock impact. Hence we conclude that the plastic deformation develops when a fruit is hit in the volume, which is limited by a spherical segment and a cone (figure 2) with the base in the form of a circle with radius \( r \).

Considering that the body volume before and after the shock impact does not change, we will express the square of the base through the volume: \( F = \frac{V}{h} \), where \( V \) is the constant volume of the body part, which is subject to a shock pulse m\(^3\); \( l \) is the deformation depth, m. Then the complete plastic work of deformation is:

\[
W_2 = W_{2,1} + W_{2,2}
\]

(11)

where \( W_{2,1} \) is plastic work of deformation in the volume limited by the spherical segment; \( W_{2,2} \) is plastic work of deformation in the volume limited by the cone.

\[
W_{2,1} = V_s \int_0^l \frac{p \xi dl}{l} = (3r_c - l) \frac{\pi l^2}{3} \int_0^l \frac{dl}{l} ;
\]

(12)

\[
l_y = r - r \cos \cos \xi ;
\]

\[
dl_y = r_y \sin \xi \sin \xi d\xi ;
\]

where \( \xi \) is the half of the angle between the deformation areas.

Let us substitute the expressions \( l_y \) and \( dl_y \) into formula (14) and, by replacing the limits, we will come to the following expression:

\[
W_{2,1} = \pi r_y^3 \cos^3 \xi - 3 \cos \xi + 2 \int_0^\xi \frac{\sin \xi}{1 - \cos \xi} \rho d\xi = \pi r_y^3 \cos^3 \xi - 3 \cos \xi + 2 \ln \left| \frac{1 - \cos \xi}{\cos \xi} \right| + \xi \pi \rho .
\]

(13)

Here \( \rho \) is a variable and it depends on the depth, but minor fluctuations from the average value do not influence the final result. Therefore, we factor \( \rho \) outside the integral sign in the equation.

\[
W_{2,1} = \pi r_y^3 (\cos^3 \xi - 3 \cos \xi + 2 \ln \left| \frac{1 - \cos \xi}{\cos \xi} \right| + \xi \pi \rho .
\]

(14)

We will express the plastic work of deformation in the cone volume respectively:

\[
W_{2,2} = V_s \int_0^l \frac{p \xi dl}{l} = \frac{1}{3} \pi r_c^3 \int_0^l \frac{dl}{l} p ,
\]

\[
l_e = r_e \cos \xi ,
\]

\[
dl_e = -r_e \sin \xi \sin \xi d\xi .
\]

(15)

where \( V_s \) is the rate of development of the plastic deformation in the cone volume m/s; \( r_e \) is the cone radius, m; \( l_e \) is the deformation depth in the cone volume, m.

By substituting values \( l_e \) and \( dl_e \) obtained into expression (15), we have:
\[ W_2 = -p_{cr} \frac{1}{3} \pi r^3 \cos^2 \xi \int \frac{\sin^2 d \xi}{\cos^2 \xi} = \frac{1}{3} p_{cr} \pi r^3 \cos \xi \ln \left| \cos \xi \right| + \frac{1}{3} p_{cr} \pi r^3 \cos \xi \ln \left| \sec \xi \right| \]  

By substituting values \( W_2 \) and \( W_2^* \) in formula (5), we will obtain:

\[ W_2 = p_{cr} \pi r^3 \frac{\cos^3 \xi - 3 \cos \xi + 2}{3} \ln \left| \cos \xi \right| + \frac{1}{3} \pi r^3 \cos \xi \ln \left| \sec \xi \right| \]  

The first summand of the expression obtained is nothing else than the plastic work of deformation consumed for the destruction of a fruit rind, the second one is the energy consumed for the destruction of the pulp. Value \( p_{cr} \) for the rind will be considerably greater than that for the pulp. As we have arranged that the friction on the fruit surface is very low, specific pressure \( p_{cr} \) will equal to the yield stress, i.e. \( p_{cr} = \sigma_y \), where \( \sigma_y \) is the yield stress [10]. These values can be determined on the basis of the equation of the distribution of normal stress on the contact surface in the absence of friction, which was proposed by Tomlenov A.D.

\[ \sigma_u = \sigma_y \left( 1 + \frac{\pi}{2} - \gamma \right) \]  

where \( \sigma_u \) is the normal stress, which arises when a material is plastic, N/m²; \( \gamma \) is the angle between the axis of the sphere being impressed in and the line from the center to the contact point between the sphere and the material surface; \( \sigma_y \) is the shearing yield stress of the material, N/m².

When a sphere with certain diameter \( d_{sp} \) is impressed in for the extreme event – when the sphere is impressed for the maximum diameter, we have:

\[ P = 1.8 \pi r^3 \sigma_y \]

where: \( r_0 \) is the sphere radius;

\[ \sigma_y = \frac{P}{1.8 \pi r^3} \]

The authors substitute value \( \sigma_y \) into formula (17)

\[ W_2 = \frac{\cos^3 \xi - 3 \cos \xi + 2}{3} \sigma_y \pi r^3 \ln \left| \cos \xi \right| + \frac{1}{3} \sigma_y \pi r^3 \ln \left| \sec \xi \right| \]

When substituting the respective numerical values into expression obtained (19), we come to the conclusion that the multiplier of the first fraction \( \cos^3 \xi - 3 \cos \xi + 2 \to 0 \), and therefore, the whole first summand will tend to zero. Then the plastic work of deformation is determined by the formula:

\[ W_2 = \frac{\cos \xi}{3} P \pi r^3 \ln \left| \sec \xi \right| \]

The complete impact energy:

\[ A = \sqrt{mv^2} = m \pi r^2 + 0.5 m k^2 v^2 + \frac{\cos \xi}{3} P \pi r^3 \ln \left| \sec \xi \right| \]

Let us determine the allowable impact speed of the working tool on a fruit from the expression obtained.

\[ v \leq \left[ \frac{P \pi r^3 \cos \xi}{5.4 r^2 \left( \sqrt{m} - km - 0.5 m k^2 \right)} \right]^{\frac{1}{2}} \]

Numerical methods have been used to solve the expression obtained. To that end, a program was made in Mathcad system and, by substituting unknown values (m, \( \xi \)), the allowable speed forces have been determined for water-melon fruits of varieties: “Crimson sweet” and “Kholodok”. The parameters
of the allowable speeds theoretically found for variety Crimson sweet are at most 1.2 m/s, and those for variety “Kholodok” are 1.4 m/s. Such a difference is explained by the strength of the armor, which is of great importance for “Kholodok” water-melons.

The experimental researches to determine and verify the theoretical analysis, which had been carried out earlier, were performed on special equipment set out in fig. 3.

![Figure 3](https://example.com/f3.png)

**Figure 3.** Installation to determine critical speed of impact on a fruit: 1 – object table; 2 – base; 3 – supports; 4 – stands; 5 – fixing arrow; 6 – spring arm-fixture; 7 – computer.

Water-melon fruits were dropped on a special table in the form of a hard metal plate equipped with a sensor, which was connected to a computer. When a fruit fell on the table from a certain fixed height, the sensor transmitted an impulse to the computer. A diagram of an impact pulse referred to the fruit fall time was built according to the values received.

However, the most justified test results from the practical standpoint have been obtained concerning the storage period of water-melon fruits, which were subject to a shock impact. The diagram of the change in the fruit storage period after a shock impact is set out in figure 4.

![Figure 4](https://example.com/f4.png)

**Figure 4.** Diagram of a change in fruit storage period depending on impact speed.

As reflected by a characteristic change in the curves set out in the diagram, they represent a parabolic dependence. It should be noted that the parameters of the allowable speed are 1.0 m/s for variety “Crimson sweet” and 1.3 m/s for variety “Kholodok”.

### 3. Conclusion

On the basis of the theoretical analysis of the interaction between the working tool and a water-melon fruit, it has been established that an elastoplastic deformation develops in a fruit in the contact point after an impact of the working tool. On the basis thereof, the dependences of the expansion of the elastic and plastic deformations have been established. The equations obtained analytically enable to determine the allowable impact speed of the working tool on a fruit, which equals to 1.2 m/s for “Crimson sweet” fruits and 1.4 m/s for “Kholodok” fruits.
The subsequent experimental researches in regard to the fruit storage periods have confirmed the theoretical parameters obtained and, as reflected by the curves, the speed limits for both varieties set out in diagram (fig.4) correlate as follows: “Crimson sweet” – 1.0 m/s, “Kholodok” – 1.3 m/s. The comparison of the theoretical and experimental speed limit values confirms the accuracy of the researches.

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