On integral characteristics of Polar Lows

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Abstract. Polar mesocyclones, also known as polar lows (PLs), are important maritime mesoscale (horizontal diameter up to 1000 km) weather systems at high latitudes, forming to the pole from the polar front. We consider the possible prognostic criteria of the PLs, in particular, the kinematic helicity as a squared characteristic related to the integral vortex formations and kinematic vorticity number. To calculate such characteristics we use reanalysis data and the results of numerical modeling with the WRF-ARW model for the PLs over the Nordic (Norwegian and Barents) seas. For comparison, experimental data are used. Our estimation of the helicity is based on its remarkable property – the connection of integral helicity content in the Ekman layer with the geostrophic wind velocity. Criteria associated with vorticity and helicity manifested through the PL genesis and development quite clearly. This criteria can be used to increase the efficiency and accuracy of complex forecasting techniques.

1. Introduction

Polar mesocyclones are typical for the Polar Regions and have a considerable influence on the local weather. Their influence will vary depending notably on their size and intensity. It is noteworthy that despite their relevance the problem of finding the well-accepted, universal automatic identification method of polar mesocyclones has not been resolved so far [1,2].

According to the European Polar Lows Working Group [3 and references therein]: “The term polar mesocyclone is the generic term for all meso-α and meso-β cyclonic vortices poleward of the main polar front. The term “polar low” should be used only for intense maritime polar mesoscale cyclones with scales up to 1000 km with a near surface wind exceeding 15 m s⁻¹”. Polar lows (PLs) have a relatively short life time of 3–36 h [4] and often occur in the cold air outbreak where relatively cold air is advected out over relatively warm water. PLs are mainly formed [5 and references therein] near the main deep baroclinic zone, namely the polar front or along the ice edge. The average velocity of PL eye is approximately 50 km h⁻¹.

PLs favor strong surface wind, severe weather and heavy precipitation, namely heavy snows, wind gusts, violent oceanic waves, large temperature gradient between the ocean and the atmosphere, that constitute a significant part of the marine weather risk in subpolar waters [6 and references therein].

PLs in the Northern Hemisphere are mainly formed in the Arctic region and the North Atlantic and are able to travel for distances about 100-2000 km during the period of their existence [1,7-11].
According to various estimates [12], a decrease of PLs frequency in this region by 10-15% and a
decrease of their intensity are predicted in the future. At the same time, the area of genesis of stable
and intense PLs in the Atlantic will move closer to Scandinavia that will increase the risk of extreme
climatic events, such as gale-force wind and swells, icing and others mentioned above, on
infrastructure. Such phenomena are important to consider in ensuring transport operations, in the
design, construction and reconstruction of industrial facilities [4]. PLs are characterized by the lack of
forecasting efficiency and by the sudden nature of the occurrence. Therefore the prognostic parameters
and identification criteria of PLs are of great interest.

In keeping with history one of the first studies of PLs was carried out by Harley D.G. in 1960 [13].
PLs have drawn the attention of the world meteorological community since the 1980s and their
systematic study began in Norway [14]. PLs were initially detected in the analysis of weather maps and
weather station data [15] and later captured from cloud features (spiral form or comma-shaped)
observed in satellite imagery [7,16,17]. Recently PLs have been identified in the numerical models by
the use of reanalysis data. Unfortunately there are the sets of meteorological reanalysis data
representing PLs inadequately as Condron et al. [18] have shown for ERA-40 data. Laffineur et al.
[19] suggest ERA-Interim also misses a substantial fraction of polar lows as only 13 of 29 observed
polar lows show an associated minimum in mean sea level pressure. Furthermore, owing to the sparse
synoptic weather station network, which is typical for the international network in Russia, and to lack
of networks over ocean surface, PLs are not explicitly assimilated into the reanalysis. The satellite
images can be used as assimilation data for reanalysis and may highlights a region of intense PLs
[1,7,10] but do not have enough resolution for accurate PLs identification. The short-lived nature of
many of PLs, and their occurrence in remote areas may cause underestimation of PLs quantity, size
and intensity. In recent years the numerical simulation methods become very popular. Additional
using of the spectral nudging technique [20] to enforce a given large-scale onto the simulations leads
to improve of PLs detecting technique.

In [21] a detailed information on traditional methods of PLs identification such as the
pressure/geopotential height fields and the (geostrophic) vorticity fields are given. However, these
traditional methods fail to obtain sizes of PLs properly in some flows [22]. The former methods
provide inconsistent results in case of a strong background flow or of a few PLs happen to occur close
together. These methods favors larger systems and may exclude weaker PLs [23]. The problem of the
latter methods is that vorticity alone cannot distinguish between sheared and curved flow. Therefore
Schielicke et al. [21] offer kinematic methods (or \( Q \)-methods) namely the kinematic vorticity number
that can distinguish between rotation and deformation of the flow and have already been successfully
applied in the studies on tropical cyclone [24]. The similarity between tropical cyclones and PLs was
described in [14,25]. Kinematic vorticity number is one of the integral parameters using here in our
investigation.

The paper is structured as follows. In the next section we describe the selection of
diagnostic/prognostic criteria used. After introducing the data and methods (section 3), the results are
shown and discussed in sections 4. The conclusions are presented in section 5.

2. Selection of the integral diagnostic/prognostic criteria of PLs

A variety of integral atmospheric parameters describes dangerous convective events such as PLs,
tropical cyclones, storms etc. The most common parameters are the follows [26]:

- Convective available potential energy (CAPE) – an amount of buoyancy energy available to
  produce vertical acceleration of air particle.
- Convective inhibition (CIN) – a numerical measure that indicates the amount of energy
  preventing an air parcel from rising from the surface to the level of free convection.
- Storm-relative environmental helicity or storm-relative helicity and Storm-relative helicity
  (an alternative definition).

Storm-relative environmental helicity is defined as
$SPEH = \int_0^h (v - c) \cdot \left( k \times \frac{\partial v}{\partial z} \right) dz$ \cite{27};

where $v$ – horizontal component of the wind velocity, $c$ – the velocity of the cyclone, $h$ is chosen between 1 and 3 km, $k$ – the unit vector directed vertically, $\frac{\partial v}{\partial z}$ – the wind velocity shear;

whereas Storm-relative helicity (an alternative definition):

\[ H = (v - v_{\text{mean}}) \frac{\partial u}{\partial z} - (u - u_{\text{mean}}) \frac{\partial v}{\partial z} \] \cite{28};

where $u, v$ – wind velocity components in the cyclone, $u_{\text{mean}}, v_{\text{mean}}$ – velocity components of the cyclone center.

However, these parameters have only diagnostic meaning, there is no prognostic capacity in them. Furthermore, their fundamental dependency on the information of upper-level troposphere with rare aerological network and on the parameterization of convection, clouds, microphysics exists. The latter frequently becomes a problem for numerical simulation of observed synoptic situation and is a subject of separate work.

Other existing parameters are inconvenient for PLs forecasting either.

- Helicity index:

\[ S = \frac{8\pi}{3} \int_0^r v^3 dr \] \cite{29};

where $S$ – helicity index (m$^4$s$^{-3}$), $v$ – tangentials component of the wind velocity (m$s^{-1}$), $r$ – radius (m).

The index include the integral of the cube of tangential velocity and basically is the rate of helicity destruction in the surface layer proportional to the power produced by dynamic pressure force. Therefore this parameter is complicated for calculation and is ill-suited to operational forecast.

- A criterion for formation of an intense atmospheric vortex:

\[ \left( \frac{2ht}{hN} \right)^{1/2} \geq 1 \] \cite{25};

where time $t$ would be more than enough to reach the altitude $h$, $h(t) = N^{-1} \left( 2ht \right)^{1/2}$ – the height of the penetrative convection layer, $b$– buoyancy flux at the lower boundary of convective layer, $N$ –Brunt-Vaisala frequency. The criterion is necessary, but not sufficient condition for intense vortex formation. It is integral criterion that expressed through: the parameters of thermodynamic nonequilibrium state between ocean and atmosphere, the temperature, humidity, steady-state stability, development time. Such a criterion require much information collected and processed beforehand, great resources and much time.

We have assumed, that rather effective, simple and fast way to predict PLs is to use the helicity as a squared characteristic related to integral vortex formations and the kinematic vorticity number. We have selected the integral helicity averaged over pre-selected area as the former criterion (see Section 3 for more details):

\[ H_{\text{int}} = \frac{1}{2} \left( u_G^2 + v_G^2 \right) \] \hspace{1cm} (1)

where $u_G, v_G$ – geostrophic wind velocity components in the cyclone.

The latter criterion is the kinematic vorticity number \cite{21}:

\[ W_k = \frac{\| \zeta \|}{\| S \|} = \sqrt{\frac{\zeta^2}{D^2_h + \text{Def}^2 + \text{Def}^2}}, \] \hspace{1cm} (2)
where $\zeta$ is the vertical component of the vorticity vector, $\|S\|$ – Euclidean tensor norm of the strain-rate tensor, $\|\Omega\|$ – Euclidean tensor norm of the vorticity tensor, $D_h$ – horizontal divergence, $Def$ – stretching deformation, $Def'$ – shearing deformation.

In this case, when predicting it is necessary to know only the wind velocity field, complex and time consuming calculations are not required, which is a great advantage for an effective operational forecast.

3. Data and method

3.1. Case study

A PLs formed over the Nordic (Norwegian and Barents) seas in the period between 29 and 31 March 2013 [also described in 30]. A MODIS satellite images of the PLs at 12 UTC on 29 March, 11 UTC on 30 March and 12 UTC on 31 March are presented in figure 1. In the satellite image, the PLs features spiral form or comma-shaped clouds, particularly on 30 March, and a relatively well-defined centre. The PLs formed under the influence of the negative pressure anomaly and the secondary baroclinic zone, stationary front or front of occlusion.

Here we use geopotential height fields at the different levels (500, 700, 850, 975 hPa, the results were consistent with each other, therefore here we will show only the last ones as more illustrative examples) and the velocity fields of the ECMWF reanalysis data and of the results of numerical modeling in the WRF-ARW model.

3.2. Numerical simulation model

To study a synoptic situation, we selected an open research nonhydrostatic mesoscale atmospheric WRF-ARW model (version 3.7.1) – Weather Research and Forecasting [31]. Currently, the open WRF model is one of the most universal and well-functioning open systems of atmospheric simulation. WRF-ARW model grid and parameterizations are shown in table 1.
Table 1. WRF-ARW model grid and parameterizations.

| Parameter                        | Value / Name of parameterization               |
|----------------------------------|------------------------------------------------|
| Run time                         | 01.03.2013 00 UTC-31.03.2013 18 UTC            |
| Number of domains                | 2                                              |
| Map projection                   | Polar                                          |
| Grid distance                    | 10 000 m (10 km) / 3333 m (3,333 km)            |
| Full south-north dimension       | 327/109                                        |
| Full east-west dimension         | 207/90                                         |
| Full vertical dimension          | 50                                             |
| Time step                        | 60 sec                                         |
| Longwave Radiation               | CAM/CAM                                        |
| Surface Layer                    | Monin-Obukhov                                  |
| Land Surface Model               | Noah                                           |
| Planetary Boundary Layer         | Mellor-Jamada-Janjic scheme                    |

3.3. Helicity estimation

Our estimation of the helicity is based upon the connection of integral helicity content in the Ekman layer with the geostrophic wind velocity.

The helicity is defined as the scalar product of velocity and vorticity –

\[
H = v \cdot \text{rot}(v).
\]

The Ekman flow is:

\[
u = u_g \left( 1 - \exp\left(-\frac{z}{h}\right)\cos\frac{z}{h} \right) - v_g \exp\left(-\frac{z}{h}\right)\sin\frac{z}{h};
\]

\[
v = v_g \left( 1 - \exp\left(-\frac{z}{h}\right)\cos\frac{z}{h} \right) + u_g \exp\left(-\frac{z}{h}\right)\sin\frac{z}{h}.
\]

Here \( h = \left(\frac{v}{\Omega}\right) \) – the Ekman scale, \( u_g, v_g \) – the geostrophic wind velocity components in a free atmosphere. The vertical components of the vorticity in these cases can be neglected. In that case the helicity (Ekman flow helicity [32, 33]) is:

\[
H = -u \frac{\partial v}{\partial z} + v \frac{\partial u}{\partial z}.
\]  

(3)

The integral helicity [25] is:

\[
H_{int} = \int_0^z H dz = \frac{1}{2} \left( u_g^2 + v_g^2 \right).
\]

And the helicity density:

\[
H_{den} = \frac{1}{2HGT} \left( u_g^2 + v_g^2 \right).
\]  

(4)

Here \( HGT \) – geopotential height.

The good correlation between the integral helicity and the half-sum of the squared wind velocity component in slightly unstable or neutral stratification conditions [34] allows us to use the geostrophic wind data to facilitate the building of the regional and global helicity fields.
For the moving with polar low square area the result was qualitatively similar to the ones for the stationary square area. Accordingly, we will present the results of numerical simulation with moving square area and the reanalysis data treatment where stationary squares occurred.

Qualitatively behavior of the square-averaged integral helicity curve and of the square averaged helicity density curve correlate well (see figure 2a). In this regard, only one will be shown here:

$$ H_{av} = \overline{H_S} = \frac{1}{S} \sum_{S} H_{int} $$

the square-average of the integral helicity, where $S$– pre-selected area marked by the purple rectangular in figure 3. As will be seen below, even the results averaged over large area remain their practical significance as regard to its prognostic meaning for PLs. Furthermore, the numerical simulation findings of the square-averaged integral helicity quite consistent with the ECMWF reanalysis data concerning their prognostic meaning for these PLs (figure 2b), thereby the results of ECMWF will be demonstrated hereafter.

**Figure 2.** (a)Time behavior of the square average integral helicity (black line) and helicity density (blue) estimation according to the data from ECMWF, red line - geopotential; (b) time behavior of the square average integral helicity according to the data from ECMWF and to the results of numerical simulations on the WRF-ARW model. The Nordic seas. 29 – 31 March, 2013. Level 975 hPa.
4. Results and discussion

According with [35] the life cycle of the polar low may be divided into three stages: an early development stage, in which a number of small vortices appear in a shear zone; a late development stage, which is characterized by the merger of vortices and the formation of a few larger vortices; and a mature stage, in which only a single polar low is present.

As shown in figure 3 our results confirm what Shun-Ichi et al. [35] have reported in their paper. In the period of the PLs genesis the integral helicity increases. During the PLs occlusion, as from 29 March to 30 March, the helicity decreases due to the slowing down of the rotational motion when the PLs boundaries expand.

At a time of polar lows activity the local minima of the integral helicity correlate well with the local maxima of the geopotential field (a negative correlation of -0.945 for 29-31 March, 2013). Local changes in helicity are adjacent to the front of the PLs.

The first stages of PLs life cycle are shown in figure 3. It should be noted that approximately one day before the formation of PLs, a significant increase of helicity was observed.

The average helicity density of large-scale motions have the values of 0.3–0.4 ms\(^{-2}\).

The described integral helicity criterion facilitate the identification of PLs generation area. For more detailed analysis of PLs intensity the kinematic vorticity number is encouraged to apply, which is the PLs size and intensity additional indicator. At the moment of maximum intensity of PLs the kinematic vorticity number can reach values of 12–14 units.

The advantage of using the kinematic vorticity number is also a clear change in this number directly in the center of the emerging PLs, which allows to accurately indicate the boundaries of the most intense part of PLs. The results of calculating the kinematic vorticity number together with the described above integral helicity at the first stages of PLs life cycle are shown in figure 4.

Criteria associated with vorticity and helicity quite clearly manifested through the PLs genesis and development and can be used to increase the efficiency and accuracy of complex forecasting techniques.
Figure 4. (a) $W_e$ (filled colors on map), integral helicity (m s$^{-2}$, black line in graph) and geopotential (dam, blue line in graph, lines in map) in the atmospheric boundary layer. 18 UTC 28 March. $W_e$ at (b) 00 UTC 27 March; (c) 18 UTC 28 March, 2013. ECMWF. Nordic seas. Level 975 hPa.

5. Summary and conclusions
Identification criteria of PLs is commonly include several parameters and can potentially be used for forecast of PLs. For example, [17] highlighted the following identification criteria need to be verified for classifying a track as a polar low:
- T500 - SST $< -43^\circ$ (temperature at 500 hPa and the sea surface temperature), averaged over a 1° radius.
- Surface wind speed max $> 15$ m s$^{-1}$ within a circle of 2.5° radius.
- T40–T100 vorticity at 850 hPa (Polar lows are identified as relative maxima in the 3-hourly vorticity at 850 hPa with total spectral wavenumbers smaller than 40 and larger than 100 removed. This T40–T100 filtering focuses on the spatial scales characteristic of mesoscale systems (200–1000 km)) $> 6 \times 10^{-5}$ s$^{-1}$.
- Ocean fraction greater than 75%, averaged over a 1° radius.

They are just diagnostic criteria that can help to capture PLs in meteorological maps and reanalysis and simulated fields and to analyse the main features of PL’s activity. They are not intended to operational forecast. Moreover, even using these criteria only for PLs detection the results of this
procedure will depend on original fields and will represent only a part of PLs occurred. All these criteria are complicated and consume much time. They are entitled to the full range of disadvantages of traditional method (see Section 1).

The main challenge of our work is to make the operational forecast of PLs possible by selecting the diagnostic and prognostic integral characteristics of PLs, sufficient for PLs identification and analysis of their size and intensity in a convenient, usable and understandable form.

In this study the combination of the estimation of the helicity as the integral square characteristic related to aggregate vortex formations (helicity hereafter) and kinematic vorticity number has been picked up.

To indicate the main findings we have used reanalysis data and the results of numerical modeling in the WRF-ARW model for the PLs over the Nordic Seas in 2013.

It has been noted that local minima of the geopotential correlate well with local maxima of helicity during the PL (a negative correlation coefficient of -0.706 for 27-31 March and of -0.945 for 29-31 March, 2013). Local changes in helicity are adjacent to the front of the PL. One day before the formation of PL, a significant increase of helicity was observed. The average helicity density of large-scale motions have the values of 0.3–0.4 m s⁻². Kinematic vorticity number is the PL size and intensity additional indicator. It helps to single out individual polar low occurrences. At the moment of maximum intensity of PL the kinematic vorticity number can reach values of 12–14 units. These results are consistent with what we have reported above. Consequently, criteria associated with vorticity and helicity quite clearly manifested through the PL genesis and development and can be used to increase the efficiency and accuracy of complex forecasting techniques. It should be mentioned that at this time such a statement is only a hypothesis, which needs to be tested using a larger ensemble of cases. Future work will need to extend these analyses to other PLs active basins. Furthermore, it would be of interest to compare the representation of PLs by the use of any other criteria. Moreover, the spectral nudging technique would facilitate the obtaining of less intense PLs and enhance accuracy and quality of obtained statistic and integral characteristics and validate the structure and number of polar lows in high-resolution climate model either. Finally, an intention exists to use our combined criterion as a precursor of machine learning PLs identification procedure where currently analysis of satellite imagery and capturing particular cloud patterns (e.g. comma-shaped) apply in the majority of cases. It would eliminate the time consuming first stage of datasets collection.

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References

[1] Verezemskaya P Tilmina N Gulev S Renfrew I A and Lazzara M 2017 Southern Ocean mesocyclones and polar lows from manually tracked satellite mosaics Geophysical Research Letters 44(15) 7985-93.
[2] Krinitskiy M Verezemskaya P Grashchenkov K Tilmina N Gulevand S Lazzara M 2018 Deep Convolutional Neural Networks Capabilities for Binary Classification of Polar Mesocyclones in Satellite Mosaics Atmosphere 9(11) 426.
[3] Rasmussen E A 1985 A case study of a polar low development over the Barents Sea Tellus A 37(5) 407-18
[4] Rasmussen E A and Turner J 2003 Polar lows (Cambridge: Cambridge University press)
[5] Harold J M Bigg G R and Turner J 1999 Mesocyclone activity over the Northeast Atlantic. Part 2: An investigation of causal mechanisms Int. J. Climatol. 19(12) 1283-99
[6] Zahn M von Storch H and Bakan S 2008 Climate mode simulation of North Atlantic polar lows in a limited area model Tellus A: Dynamic Meteorology and Oceanography 60(4) 620-31
[7] Noer G Saetra Ø Lien T and Gusdal Y 2011 Quart. J. Roy. Meteor. Soc. 137(660) 1762-72
[8] Kolstad E W 2011 A global climatology of favourable conditions for polar lows Quarterly Journal of the Royal Meteorological Society, 137(660) 1749-61.
[9] Kolstad E W and Charlton-Perez AJ 2011 Observed and simulated precursors of stratospheric polar vortex anomalies in the Northern Hemisphere Clim Dyn 37 1443–56.
[10] Rojo M Claud C Noer Gand Carleton A M 2019 In situ measurements of surface winds, waves, and sea state in polar lows over the North Atlantic Journal of Geophysical Research: Atmospheres 124(2) 700-18.
[11] Carleton A M 1995 On the interpretation and classification of mesoscale cyclones from satellite infrared imagery Int. J. of Remote Sensing 16(13) 2457-85.
[12] Romero R and Emanuel K 2017 Climate change and Hurricane-like extratropical cyclones: Projections for North Atlantic polar lows and medicanes based on CMIP5 models J. of Climate 2017 30(1) 279-99
[13] Harley D G 1960 Frontal contour analysis of a “polar” low Meteor. Mag 89 146-7
[14] Golitsyn G S 2008 Polar lows and tropical hurricanes: Their energy and sizes and a quantitative criterion for their generation Izv., Atmos. Ocean. Phys. 44(5) 537-47
[15] Wilhelmsen K 1985 Tellus A: Dynamic Meteorology and Oceanography 37(5) 451-9
[16] Businger S 1985 The synoptic climatology of polar low outbreaks Tellus A 37(5) 419-32
[17] Zappa G Shaffrey L and Hodges K 2014 Can polar lows be objectively identified and tracked in the ECMWF operational analysis and the ERA-Interim reanalysis? Mon. Wea. Rev. 142(8) 2596-608
[18] Condron A Bigg G R and Renfrew I A 2006 Mon. Wea. Rev. 134(5) 1518-33
[19] Laffineur T Claud C Chaboureau J P and Noer G 2014 Mon. Wea. Rev. 142(6) 2271-89
[20] von Storch H Langenberg H and Feser F 2000 A spectral nudging technique for dynamical downscaling purposes. Mon. wea. Rev. 128(10) 3664-73
[21] Schielicke L Névir P Ulbrich U 2016 Kinematic vorticity number – a tool for estimating vortex sizes and circulations Tellus A: Dynamic Meteorology and Oceanography 68(1) 29464 DOI: 10.3402/tellusa.v68.29464
[22] Jeong J and Hussain F 1995 On the identification of a vortex J. Fluid Mech. 285 69-94
[23] Sinclair M R 1994 Mon. Wea. Rev.122(10) 2239-56
[24] Tory K J Dare R Davidson N McBride J and Chand S 2013 Atmos. Chem. Phys.13(4) 2115-32
[25] Golitsyn G S 2009 Tropical cyclones and polar lows: Velocity, size, and energy scales, and relation to the 26° C cyclone origin criteria Advances in Atmospheric Sciences 26(3) 585-98
[26] Boswell C A III and Schultz D M 2006 On the use of indices and parameters in forecasting severe storms. Electronic J. Severe Storms Meteor. 1(3) 1–22
[27] Onderlinde M J Nolan D S 2014 J. Atmos. Sci. 71(11) 4308-20
[28] Davies-Jones R P 1990 Test of helicity as a forecast parameter Preprints, 16th Conf. on Severe Local Storms, Kananaskis Park, AB, Canada – Amer. Meteor. Soc.
[29] Kurgansky M V 2008 Vertical helicity flux in atmospheric vortices as a measure of their intensity Izv., Atmos. Ocean. Phys. 44(4) 64-71
[30] Varentsov M I Verezemskaya P S Zabolotskikh E V Repina I A 2016 Evaluation of the quality of polar low reconstruction using reanalysis and regional climate modellingSovr. Probl. DZZ Kosm. (Current problems in remote sensing of the Earth from space)13(4) 168-191.
[31] Skamarock W C Klemp J B Dudhia J Gill D O Barker D M Wang W and Powers J G 2008 A Description of the Advanced Research WRF Version 3 (No. NCAR/TN-475+STR) University Corporation for Atmospheric Research
[32] Hide R Superhelicity 1989Geophys. Astrophys. Fluid Dyn. 48(1-3) 69-79
[33] Kurgansky M V 1989 On the relation between helicity and potential vorticity in an compressible rotating fluidIzvestia Akademii nauk SSSR. FAO25(12) 1326-9
[34] Vazaeva N V Chkhetiani O G Kouznetsov R D Kallistratova M A Kramar V F Lyulyukin V S Kouznetsov D D 2017 Izv., Atmos. Ocean. Phys. 53(2) 200-14
[35] Watanabe S I I and Niino H 2014 Mon. Wea. Rev.142(6) 2248-70