Time-dependent transport of symmetric \( \Lambda \)-type coupled triple quantum dots: competition between coherent destruction of tunneling and Fano resonance

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Abstract. With the help of the Floquet theory, we study the transport properties of symmetric \( \Lambda \)-type coupled triple quantum dots driven by an ac electric field. Under appropriate conditions, coherent destruction of tunneling (CDT) and photon-assisted Fano resonance (FR) appear because of quantum interference effects. As a consequence of competition between the super-exchange interaction and the CDT there is a drastic competition between the CDT and the FR. The phenomena represent two different ways of coherent electron trapping by which we can control the behavior of electrons and they provide a convenient way to fabricate a quantum switch.
1. Introduction

In recent years, there has been great interest in the electronic transport of quantum dot (QD) structures. In particular, time-dependent transport is one of the most important subjects and has attracted many active experimental and theoretical research works \cite{1}–\cite{5}. The external time-dependent fields applied to these nanostructures lead to more interesting phenomena and physics, such as photon-assisted tunneling \cite{6, 7}, spin pumps and spin filters \cite{8}–\cite{11}, charge pumping and photovoltaic effects \cite{12, 13}, QD turnstile \cite{14, 15}, band structure effects \cite{16}, fractional Wannier–Stark ladders \cite{17, 18}, and band suppression \cite{19}.

Coherent destruction of tunneling (CDT) \cite{20} is one of these dramatic phenomena, and it offers a convenient way to control the behavior of the electrons under the influence of a periodically driven field. Many studies have focused on closed systems \cite{21}–\cite{25}. However, the effects of environments, e.g. the leads, also play an important role in the quantum transport of the nanostructures. Different QD configurations may lead to more interesting physics. With the help of the Floquet theory \cite{26}, we studied the effects of CDT on the transport properties of a coupled QD array \cite{27} taking into account the effects of leads. In our recent work \cite{28} on serially coupled triple QDs with \( \Lambda \)-type three-level structure (in which two lower levels are coupled to a single upper level, see figure 1), we studied photon-assisted Fano resonance (FR) \cite{29}–\cite{31}, which, in contrast with a symmetric Breit–Wigner resonance, usually shows a typical asymmetric resonant profile (the so-called Fano shape) due to the interference between a set of continuous states and a discrete state \cite{32}. In such a system, the super-exchange interaction between the left and right QDs and related self-trapping are the key factors for the FR. On the other hand, CDT is another kind of trapping phenomena, which is caused by a driving ac electric field and can cause one electron to be trapped in a level. FR and CDT play opposite roles in quantum transport; therefore the competition between CDT and FR is a very interesting issue.

In this paper, we study the transport properties of symmetric \( \Lambda \)-type coupled triple dots driven by an ac electric field with the help of the Floquet theory. Through calculation of the quasi-energy spectrum, we find that under proper conditions, FR and CDT can both appear simultaneously; thus there is a drastic competition between these two phenomena, which represent two different ways of electron trapping. Thus, FR and CDT provide a convenient way to control the behavior of the electrons and can be used to fabricate a quantum switch.

The organization of this paper is as follows. In section 2, we describe the model and the method used to calculate the electronic transport. In section 3, we present the two interesting phenomena and the competition between them and then we discuss the results. The paper is summarized in section 4.
2. Model and method

We consider a symmetric $\Lambda$-type coupled triple-dot system as shown in figure 1. The continuous manifolds on the two sides of this figure correspond to electronic states with chemical potentials $\mu_L$ and $\mu_R$, and the two side dots are coupled to the leads with the dots–leads hopping rates $\Gamma_L$ and $\Gamma_R$.

In this paper, we focus only on the regime of coherent transport. In the tight binding approximation, the Hamiltonian for the dots under the action of a driving field with a frequency $\Omega$ can be written as

$$ H_{\text{dots}}(t) = \sum_{n=1}^{N} \left[ E_n + x_n e\mathcal{E}(t) \right] |n\rangle \langle n| - \Delta \sum_{n=1}^{N-1} (|n\rangle \langle n+1| + |n+1\rangle \langle n|), $$

where $N = 3$. $E_2$ is the middle energy level, $E_1 = E_3$ are the left and right energy levels and $\mathcal{E}(t) = A \cos(\Omega t)$ is the ac electric field applied on all the dots. $x_n e\mathcal{E}(t)$ is the time-dependent level shift caused by the oscillating dipole field, where $x_n = (N + 1 - 2n)/2$ denotes the scaled position of the dot with energy level $E_n$, $e$ is the electron charge and $d$ is the distance between two neighboring dots. $\Delta$ is the tunneling energy between the dots.

The transport properties of this ac driving system can be obtained using the Floquet approach [26]. Through rigorous derivation, the average current of this system can be written as

$$ I = \frac{e}{\hbar} \sum_{k=-\infty}^{+\infty} \int d\varepsilon \left[ T_{LR}^{(k)}(\varepsilon) f_R(\varepsilon) - T_{RL}^{(k)}(\varepsilon) f_L(\varepsilon) \right], $$

where

$$ T_{LR}^{(k)}(\varepsilon) = \Gamma_L(\varepsilon + k\hbar\Omega) \Gamma_R(\varepsilon) |\langle 1 | G^{(k)}(\varepsilon) | N \rangle|^2, $$

$$ T_{RL}^{(k)}(\varepsilon) = \Gamma_R(\varepsilon + k\hbar\Omega) \Gamma_L(\varepsilon) |\langle N | G^{(k)}(\varepsilon) | 1 \rangle|^2 $$

denote the transmission probabilities for electrons with initial energy $\varepsilon$ and final energy $\varepsilon + k\hbar\Omega$ from the right and left leads, respectively, $f_L(\varepsilon) = (1 + \exp[(\varepsilon - \mu_L)/k_B T])^{-1}$ denotes the Fermi function:

$$ G^{(k)}(\varepsilon) = -\frac{i}{\hbar} \int_0^T \frac{dT}{T} e^{ik\Omega T} \int_0^\infty d\tau e^{i\varepsilon \tau / \hbar} U(t, t - \tau) $$

Figure 1. Schematic diagram of a symmetric $\Lambda$-type coupled triple-dot system.
is the Fourier coefficient of the retarded Green function \( G(t, t') = -(i/\hbar)U(t, t')\Theta(t - t') \), which obeys
\[
(ii - \frac{d}{dt} - H_{\text{dots}}(t)) G(t, t') + i \int_0^\infty d\tau \Gamma(\tau) G(t - \tau, t') = \delta(t - t'),
\]
where \( \Gamma(t) = |1\rangle \Gamma_L(t) \langle 1| + |N\rangle \Gamma_R(t) \langle N| \). In the wide-band limit, \( \Gamma(\epsilon) \) can be chosen as energy-independent; thus \( \Gamma_{L,R}(t) = \Gamma_{L,R} \delta(t) \). Once we get the Fourier coefficients of the Green function (5), the average current (2) can be obtained.

With the help of the Floquet theorem, the Fourier coefficients of the Green function (5) can be written as [26]
\[
G^{(k)}(\epsilon) = \sum_{\alpha,k} \frac{|u_{\alpha,k}^+|}{\epsilon - (\epsilon_{\alpha} + k^2\hbar \Omega - i\hbar \gamma_{\alpha})}.
\]

where \( |u_{\alpha,k}\rangle \) are the Fourier coefficients of the Floquet states \( |u_{\alpha}(t)\rangle \). The Floquet states \( |u_{\alpha}(t)\rangle \) have the same time periodicity as \( H_{\text{dots}} \) and they obey the Floquet eigenvalue equation (with the eigenvalues \( \epsilon_{\alpha} - i\hbar \gamma_{\alpha} \))
\[
\left( H_{\text{dots}}(t) - i\Sigma - i\hbar \frac{d}{dt} \right) |u_{\alpha}(t)\rangle = (\epsilon_{\alpha} - i\hbar \gamma_{\alpha}) |u_{\alpha}(t)\rangle,
\]
where the self-energy \( \Sigma = |1\rangle \Gamma_L \langle 1| + |N\rangle \Gamma_R \langle N| \) results from the coupling to the leads.

### 3. Results and discussions

In this section, we present our results for the transport properties of the symmetric \( \Lambda \)-type coupled triple dots. In our calculation, we assume that \( k_B T = 0 \), and \( 2\Delta \) serves as the energy unit. The energy gap between the middle and the left (right) dots \( E_2 - E_{1(3)} = 5.15 \), the applied voltage \( \mu_L - \mu_R = 2.90 \), and \( \Gamma_L = \Gamma_R = \Gamma = 0.04 \). In addition, we define \( e\hbar A = \mu \).

First, we study the transport properties of our model under the resonant condition that the frequency \( \Omega_0 \) of the ac field equals the energy difference between the middle and the left (right) dots, i.e. \( \Omega_0 = E_2 - E_{1(3)} = 5.15 \). In figure 2, we show the average current \( I \) and the real part of the quasi-energy spectrum \( \epsilon_a \) of the system as functions of the amplitude of the oscillating field at the given resonant frequency. In the calculation of the quasi-energy, no state from the leads was explicitly included. A partial effect from the leads is taken into account by the \( \Gamma \) term in equation (7). From figure 2(b), we see that our analytical results for the real part of the quasi-energy \( \epsilon_a = E_1, E_1 \pm 2J_1^2(\mu/\Omega_0) + (\Gamma^2/4), \alpha = 1, 2, 3, \) and \( J_1 \) is the first-order Bessel function) agree well with numerical results. Moreover, we see that the minima of the average current appear at the positions \( \mu/\Omega_0 = 3.83, 7.02 \) and 10.17 (the roots of the first Bessel function), which correspond to the collapse points of the quasi-energy spectrum. In addition, due to the presence of time-dependent energy difference between the dots and the leads, the tunneling between them is coherently destroyed under the conditions \( \mu/\Omega_0 = 2.405, 5.52, \ldots \) (the roots of the 0th Bessel function), leading to an extra minimum of the average current

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Figure 2. (a) Average current $I$ as a function of the amplitude of the ac electric field $\mu / \Omega_0$. (b) The real part of the quasi-energy spectrum $\varepsilon_\alpha$ of the system. The solid line is for the numerical results and the dotted line is for the analytical results. $R_0$ and $R_1$ refer to the roots of the 0th and first Bessel functions, respectively.

Figure 3. Average current as a function of the driving frequency $\Omega$: (a) $\mu / \Omega_0 = 0.1$ and (b) $\mu / \Omega_0 = 0.5$.

(see also the discussion in [33]). From our previous work [27], we can conclude that CDT takes place perfectly at the above field magnitude for an initial localized electron, which will be delocalized at other parameters.

Next, we present another interesting phenomenon in this symmetric $\Lambda$-type coupled triple-dot system. We calculate the average current as a function of the frequency of the field at certain ac field magnitudes $\mu / \Omega_0 = 0.1$ and 0.5. The results are shown in figures 3(a) and (b). We find an apparent asymmetric Fano line shape under resonant condition in the two graphs, which results from the formation of ‘trapping dark states’ as we have studied in our recent work [28]. Comparing the two panels, we observe that the amplitude of FR increases remarkably with increasing driving field magnitude. Interestingly, we also find that the two-photon-assisted FR
Figure 4. Average current as a function of the driving frequency $\Omega$: (a) $\mu/\Omega_0 = 3.41$, (b) $\mu/\Omega_0 = 3.70$, (c) $\mu/\Omega_0 = 3.75$ and (d) $\mu/\Omega_0 = 3.83$.

occurs clearly at the left side of figure 3(b) when we apply an ac electric field of larger magnitude to the system.

In the above discussion, both of the interesting phenomena (CDT and FR) appear, but the conditions for them to occur are different: CDT appears at a high value of the field magnitude to frequency ratio, whereas FR appears strongly at a low value of the ratio. Hence, we investigate the relation between CDT and FR in the following. We choose a small region around the first position $\mu/\Omega_0 = 3.83$ at which CDT appears perfectly as shown in figure 2(a). To determine whether FR can appear and how one phenomenon affects the other, we study the average current $I$ as a function of field frequency $\Omega$ with the field magnitude $\mu/\Omega_0$ increasing to 3.83. The current–frequency curves with $\mu/\Omega_0 = 3.41$, 3.70, 3.75 and 3.83 are shown in figures 4(a), (b), (c) and (d), respectively. In figure 4(a), we see an asymmetric line shape clearly. We fit the curve with the Fano function $f(\epsilon) = A(\epsilon + q)^2/(1 + \epsilon^2)$, where $\epsilon = (\Omega - \Omega_0)/\gamma/2$, and we find that the asymmetric Fano factor $q = 3.0$, indicating that FR still occurs under resonant conditions at a large ratio of $\mu/\Omega_0$, and CDT does not have an obvious impact on FR in this case. In figure 4(b), the intensity of the average current decreases and the Fano line shape is destroyed a little, suggesting that CDT begins to modify FR. In figure 4(c), the current intensity decreases further, and the Fano line shape is destroyed too much to be identified, showing that the Fano peak is restrained by CDT greatly. In figure 4(d), we cannot see FR any more in the current curve; instead, two peaks appear, and in between the two peaks, the average current decreases to the minimum around the resonant frequency. In this case, the Fano peak is suppressed completely and CDT appears perfectly in the system. From figures 4(a)–(d), we find
that FR is weakened gradually while CDT is strengthened as the field magnitude moves to the point where CDT appears perfectly.

Then we continue to increase the ac electric field magnitude and observe the competition between FR and CDT. In figures 5(a)–(c) and (d), we show four current–frequency curves with $\mu/\Omega_0 = 3.86, 3.90, 4.00$ and 4.30, respectively. From the figures, we can see that the effect of CDT is weakened slowly and that the Fano peak is strengthened gradually. In figure 5(d), FR appears completely again under resonant conditions, getting rid of the control of CDT. We fit the line with the Fano function and find that the asymmetric Fano factor $q = -4.0$.

To gain better understanding of the physical picture, we present in figure 6 our numerical results for the time dependence of the occupation probability of an electron initially in the left dot. Figure 6(a) shows the time evolution of the occupation probability under FR. In this situation, the electron has a large probability in the left and right dots. Figure 6(b) shows the CDT with the electron trapped in the left dot. Figure 6(c) is for the situation of CDT competing with FR (see figure 5(b)). In this case, the electron has roughly the same time-average probability in the three dots.

From figures 4 and 5, the whole process of the drastic competition between CDT and FR is demonstrated. In essence, the two phenomena are both related to trapping. However, they are quite different, since the occurrence of FR is due to the formation of ‘trapping dark states’, which demands that the electrons almost entirely occupy the left and right dots in our system; but for the appearance of CDT, the electrons need to stay at one dot nearly all the time. FR and CDT may not always appear at the same time. One may tune the system parameters within small

Figure 5. Average current as a function of the driving frequency $\Omega$: (a) $\mu/\Omega_0 = 3.86$, (b) $\mu/\Omega_0 = 3.90$, (c) $\mu/\Omega_0 = 4.00$ and (d) $\mu/\Omega_0 = 4.30$. 
neighborhood of the Bessel zeros, so that photon-assisted FR and CDT compete with each other as shown in figures 4 and 5. This sensitivity of the current response to external field frequency and strength provides the basis for fabricating a quantum switch.

4. Conclusion

In this paper, we studied the transport properties of symmetric Λ-type coupled triple dots under the action of an ac electric field with the help of the Floquet theory. Under resonant conditions, we calculated the average current numerically and found that CDT appears at some particular field magnitude. We also observed photon-assisted FR at the resonant frequency. Then we focused on the parameter regimes around the CDT point and studied the average current as a function of the driving frequency. With proper parameters, we find that both FR and CDT can appear and there is competition between the two phenomena. They represent two different ways of electron trapping, i.e. trapping an electron in one dot (CDT) and two dots (self-trapping in the left and right dots, leading to photon-assisted FR). This paper presents the competition between these intriguing phenomena of the driven triple QD system and provides a convenient way to fabricate a quantum switch.

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