Synthesis of Cross-Coupled Resonator Filters Using Optimization Techniques

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Abstract. SolvOpt was a method to solve the local extremum of nonlinear objective function. A single local optimization method based on SolvOpt that synthesizes coupling matrix for cross-coupled microwave filters was presented. In order to solve the global minimum value of cost function, the setting rule of SolvOpt initial value was given, SolvOpt method simplified the process of synthesizing coupling matrix and provides fast convergence and good accuracy to find the final solution, compared hybrid optimization methods. Last, application examples illustrate the excellent performance and the validity of this method.

1 Introduction

Filters were widely used in various wireless communication systems, such as base station and satellite communication system. With the rapid development of wireless communication system, the spectrum became more and more crowded, which puts forward higher requirements for the design of microwave passive components. Both analytical and numerical methods for the synthesis of coupling matrices corresponding to cross-coupled filters have been extensively studied. Recently, a class of hybrid optimization methods combining local search methods with global methods had been reported [1-4]. For example, the paper in [2] presented a method consists of a Levenberg-Marquardt algorithm for a local optimizer and genetic algorithm for a global optimizer, respectively. In [3] a genetic algorithm was combined with a sequential quadratic programming local search method to form a hybrid method. These hybrid optimization methods could find a global minimum, however, they needed more iteration, and the process of synthesizing coupling matrix became very complex.

A single optimization method based on SolvOpt that synthesized coupling matrix for cross-coupled microwave filters was presented in this paper. We could easily guess a good initial values for SolvOpt algorithm to synthesize coupling matrix. We proposed the rules for setting initial values of SolvOpt optimization method. SolvOpt algorithm simplified the process of extracting coupling matrix and provided fast convergence and good accuracy to find the final solution, compared hybrid optimization methods.
2 Synthesis of coupling matrix based on single SolvOpt optimization method

For any two-port lossless filter network, the transmission function \( S_{21}(\Omega) \) and reflection function \( S_{11}(\Omega) \) may be expressed as:

\[
|S_{21}(\Omega)|^2 = \frac{1}{1 + \varepsilon^2 C_N^2(\Omega)} , \quad |S_{11}(\Omega)|^2 = 1 - |S_{21}(\Omega)|^2 \tag{1}
\]

The relation between S-parameters and the coupling matrix can be expressed that, for \( \text{“N”} \) coupling matrix without source/load-multiresonator coupling case [5].

\[
S_{21} = -2j\sqrt{R_1 R_N}[A^{-1}]_{N,1} , \quad S_{11} = 1 + 2jR_1[A^{-1}]_{1,1} \tag{2}
\]

Here, \( R_1 \) and \( R_N \) were the normalized load and source resistors respectively, \( A = [\Omega U - jR + M] \), \([U]\) is the identity matrix, \([R]\) was a matrix whose only nonzero entries are \( R_{i,j} = R_1 \) and \( R_{N,N} = R_N \), and \([M]\) was the \( N \times N \) normalized symmetric coupling matrix. \( R_1 \) and \( R_N \) can be accurately calculated in this paper using Cameron’s analytical method [6], and this was different with other optimization methods.

Once the filter order and TZs were determined, the frequency response of the filter, as computed directly from the filtering function in (1), was determined. The elements of coupling matrix \( M_{i,j} \) were known as the coupling coefficients and varying their values causes the response to change. The aim of the coupling matrix synthesis process was to select coupling matrix which cause (2) to produce a filter response coincide with the response obtained from (1).

The selection of an appropriate cost function was important for the success of any optimization method. The cost function was defined to evaluate the amplitude of ideal filtering function response in (1) at the critical frequencies such as the transmission and reflection zeros and the passband edges. The cost function given by Amari [8] was used for the current work.

\[
K(x) = \sum_{k=1}^{q}|S_{11}(\Omega_{pk})|^2 + \sum_{k=1}^{p}|S_{21}(\Omega_{zk})|^2 + \left|S_{11}(\Omega = 1) - \frac{\varepsilon}{\sqrt{1 + \varepsilon^2}}\right|^2 + \left|S_{11}(\Omega = -1) - \frac{\varepsilon}{\sqrt{1 + \varepsilon^2}}\right|^2 \tag{3}
\]

where, \( p \) and \( q \) were the number of finite transmission and reflection zeros, respectively, \( \Omega_{zk} \) and \( \Omega_{pk} \) were the location of the \( k \)-th transmission and reflection zero at the normalized frequency, respectively. The variable \( x \) represented the set of control variables at the current iteration, that was, the elements of coupling matrix. \( S_{21} \) and \( S_{11} \), defined in (2), were a function of the control variables. The nonzero coupling matrix element \( m_{i,j} \) would be used as independent variables in the optimization process.

The gradient of the cost function needed to be used in SolvOpt algorithm. The gradient of the cost function with respect to an independent variable \( M_{i,j} \) can be derived from (3)

\[
\frac{\partial K}{\partial M_{i,j}} = \sum_{k=1}^{p} 2|S_{11}(\Omega_{pk})|^2 \frac{\partial |S_{11}(\Omega_{pk})|}{\partial M_{i,j}} + \sum_{k=1}^{q} 2|S_{21}(\Omega_{zk})|^2 \frac{\partial |S_{21}(\Omega_{zk})|}{\partial M_{i,j}} \\
+ 2 \left|S_{11}(\Omega = 1) - \frac{\varepsilon}{\sqrt{1 + \varepsilon^2}}\right| \frac{\partial |S_{11}(\Omega = 1)|}{\partial M_{i,j}} + 2 \left|S_{11}(\Omega = -1) - \frac{\varepsilon}{\sqrt{1 + \varepsilon^2}}\right| \frac{\partial |S_{11}(\Omega = -1)|}{\partial M_{i,j}} \tag{4}
\]
The gradient of the $|S_{11}|$ and $|S_{21}|$ with respect to $M_{ij}$ was given in [6] for “$N$” coupling matrix case.

The SolvOpt (Solver for local optimization problems) was concerned with minimization or maximization of nonlinear, possibly non-smooth objective functions or solve a nonlinear optimization problem [9]. This algorithm was selected to take advantage of the fast local search and simple usage properties. Its usage had a form: $[x,f]=\text{solvopt}(x,'fun','\text{gradfun})$, where, the left-hand side was output variables: $x$ was the solution point and $f$ was the value of the cost function at the solution point, the right-hand side was input variables: $x$ was a vector of the starting point, 'fun' provides the name of the $M$-file ($M$-function) of the cost function and 'gradfun' provides the name of the $M$-file that returns the gradient vector of the cost function at a point $x$. More details were introduced in [9].

We could easily guess good initial values for SolvOpt algorithm to synthesize coupling matrix. Generally, magnitudes of the direct coupling coefficients were bounded by 0.1 and 1, and the cross couplings by 0 and 0.8. For “$N$” coupling matrix, all cross couplings set to specific value ranged from zero to 0.2 and all direct couplings to specific value ranged from 0.4 to 0.6. We could synthesize “$N$” coupling matrix easily and efficiently using SolvOpt to minimize a cost function based on the rules above of setting initial values. This method provided faster convergence and higher accuracy to find the final solution than hybrid method in [2-3].

Flowchart of SolvOpt method was shown in Fig.1. It began with an initial set of control variables $x$, which consists of elements of coupling matrix, according to the rules given in this paper.

![Flowchart of SolvOpt method](image)

**Figure 1.** Characteristic of low-pass prototype filter.

### 3 Examples

In this section, for the verification of the method presented in this paper, it was applied to four examples of filter designs. Coupling schemes of filters were shown in Fig. 2.

This was an example of directly synthesize “$N$” coupling matrix. We considered an asymmetric 7th-order filter with three transmission zeros at -3.050, -2.583 and -1.308 and a passband maximum return loss of 20 dB, these specification were given in [3]. The seven reflection zeros locates at 0.96289, 0.68773, 0.23889, -0.98665, -0.87371, -0.62596, -0.23810 and \( R_1=R_N =0.99062 \), which were calculated using Cameron’s method [6]. Coupling scheme of this filter was shown in Fig. 2. The initial guess of control variables, $x$, for this example consists of the following 16 variables, corresponds to setting all direct couplings, $M_{i,i+1}$, for $i=1,2, \ldots ,6$, to 0.5; the cross couplings, $M_{1,3}$, $M_{3,5}$ and $M_{5,7}$ to 0.2; the self-couplings, $M_{i,i}$, for $i=1,2, \ldots ,7$, to 0.2. The value of the cost function in (3) reaches
7.3761×10−14 when three iterations had been performed. “N” coupling matrix of filter 2 can be obtained in (5).

![Coupling schemes of filter.](image)

Both the frequency response of the filtering function as computed from (1) and that calculated directly from the coupling matrix in (5) are shown in Fig. 3. The excellent agreement between the two, the difference was not visible in the figure, showed the accuracy of the SolvOpt method.

![The frequency responses of filter 3 (synthesized), as obtained from the coupling matrix in (8) and the filtering function. The two cannot be distinguished.](image)

| 0.0209 | 0.8065 | −0.1997 | 0 | 0 | 0 | 0 |
| 0.8065 | 0.2621 | 0.5745 | 0 | 0 | 0 | 0 |
| −0.1997 | 0.5745 | −0.0673 | 0.5551 | −0.1063 | 0 | 0 |
| 0 | 0 | 0.5551 | 0.1490 | 0.5557 | 0 | 0 |
| 0 | 0 | −0.1063 | 0.5557 | −0.0997 | 0.4587 | −0.4668 |
| 0 | 0 | 0 | 0 | 0.4587 | 0.6327 | 0.6873 |
| 0 | 0 | 0 | 0 | −0.4668 | 0.6873 | −0.0209 |

Note that, the accuracy of coupling matrix had an effect on the value of cost function. Coupling matrices obtained above had been taken four significant digits. By re-substituting coupling matrix in (5) with an accuracy of four significant digits into (3), we got that the value of the cost function was equal to 3.6227×10−7. For comparison, the value of cost function was equal to 0.3776, calculated by substituting coupling matrix in [3, p. 956] with the accuracy of the same four significant digits into (3). 18317 iterations were needed to converge for this example using the hybrid method in [3], however, SolvOpt method only needed one iteration to converge. We could see that SolvOpt method not only simplifies the
process of extracting coupling matrix but also provided fast convergence and good accuracy to find the final solution, compared hybrid optimization methods.

4 Conclusion

This paper had presented the generalized synthesis method for microwave cross-coupled resonator filters with or without source/load-multiresonator coupling. A single SolvOpt algorithm that synthesizes coupling matrix for cross-coupled microwave filters had been presented, and its appropriate initial set had been proposed for fast convergence and good accuracy. The method had been applied to synthesis of filters with varied orders and symmetries and had yielded excellent results, which showed simplicity, efficiency and accuracy of SolvOpt method, even for filter responses with large numbers of control variables to be optimized.

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