Search for an optimal light-extracting surface
derived from the morphology of a firefly lantern

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Fireflies lighten up our warm summer evenings. There is more physics behind these little animals than anyone of us could imagine. In this paper we analyze from a physical point of view one structure found on the firefly lantern, the one which best improves light extraction. Moreover, simulations will be done to show why this specific structure may be more effective than a "human-thought” one.

Keywords: Firefly, light extraction, external efficiency, photonic crystal, improvement

I. LIGHT IN NATURE

Light is a major path of communication in nature. Since the emergence of the eye in the animal kingdom, the variety of species has found new opportunities to grow. Parker [1] suggested the "light switch argument" in order to explain this known "Big Bang of the evolution". With vision, animals – especially insects – had either to adapt their outer appearance to be seen or hide effectively. For intra-specific communication, males have to wear bright colors that suggest genetic richness to attract the females. In contrary, for inter-specific communication, insects often have to make themselves invisible to escape predators. Another way to discourage predators is to show bright signal colors, such as red or yellow, which is commonly known to signal toxicity in nature. All these ways of communication use sunlight. The importance of photonic communication between animals and their environment is best underlined by what happens when sunlight disappears, at night or in the deep sea. Then and there the communication is still based on vision and animal produce their own light. Deep-sea fishes use mainly blue light to attract their preys and on earth, when the sun sets, insects starts to lighten up. Different species of tropical click-beetle show three light-emitting spots on their back; fireflies emit light at all stages of their evolution. The eggs and the larvaees lighten up to prevent a predator attack and as adults use bioluminescence to improve mating occurrences.

Fireflies produce their own light by a catalyzed oxidation chemical reaction, which is called bioluminescence. This bioluminescence reaction is well known [2]: it is even possible to recreate it in vitro [3]. In view of its importance, the structure of the bioluminescent organ of the firefly has received the considerable attention [4–7]. The present paper is based on the results from a former article by Bay and Vigneron [8]. There will be a brief recall of the results. At first we give a brief account of the main results in this earlier work, which then is augmented by new data that might be useful to technical and engineering developments.

II. FIREFLY MORPHOLOGY AND ITS INFLUENCE ON LIGHT EXTRACTION

Morphological analysis by scanning electronic microscopy (SEM) has revealed different structures in and around the bioluminescent organ of the fireflies. Two of these structures seemed to be particularly interesting: (1) the spheres in the bioluminescent organ and (2) the tilted scales on the outer cuticle above the bioluminescent organ. A detailed model, that combined these two structures led to a improved light extraction which reaches a factor two compared to a flat surface reference. We will here analyse more precisely the impact of the tilted scales geometry on light extraction.

The tilted scales geometry has two distinct effects on the light extraction: (1) The interface is not plane anymore, but tilted which could help changing the critical angle phenomenon. (2) The protruding ends create sharp edges on which diffusion can take place. From the SEM analysis an average height of the protruding end of 3 µm and a periodicity of 10 µm could be determined. These mechanisms have received an appropriate modeling in all details in two dimensions [8]: the vertical corrugation profile (z direction) in this model varies only in the lateral (y direction), while the height stays invariant in the x direction.
A. Light extraction path

By studying the convergence of the light extraction as a function of the accounted number of diffraction orders in this specific structure, some precious information about the way light escapes can be obtained. New simulations have been performed with the three-dimensional transfer matrix algorithm \cite{9}. The incident azimuthal direction has been fixed perpendicular to the protruding edges ($\phi = 90^\circ$) and the incident polar angles ($\theta$) varies from 0$^\circ$ to 90$^\circ$. This simulation is conducted with a wavelength of 560 nm, close to the emission peak of the firefly \cite{10}. The incident light is considered depolarized, which is achieved by averaging TE and TM polarizations. With a period (10 $\mu$m) much longer than the wavelength, transmission can occur via many diffraction orders with emergences widely distributed in space. If in the calculation we artificially limit the number of diffraction orders accounted for, we can examine in detail the paths of the emerging light. Fig. 2 shows that for a limited number of diffraction orders (2 - 4 emerging plane waves), the tilted scales are hardly considered by our computation and the critical angle is similar to the one in a planar interface (classic). With more diffraction orders (8 - 16), the critical angle increases and disappears even completely (32-128). Low diffraction orders cannot take into account the influence of the sharp edge at the protruding edges.
end of the scales. With more diffraction orders the effect of the prism can be clearly seen. Remarkably, a huge part of
the light is extracted at high angles. This light is completely lost in the plane-interface case. The convergence study
shows as well, that 32 plane waves can already properly account for the effect of the corrugation.

B. Light extraction map

For a full understanding of the effect of the sharp edge effect, we calculate the light extraction for varying incident
polar and azimuthal angles. Polar angles ($\theta$) range from 0° to 90° and azimuthal angles ($\phi$) from 90° to 270°,
expandable to the full range (360°) by symmetry. The significance of these azimuthal angles are as follows: for
$\phi = 90^\circ$ the light impinges perpendicular to the sharp edge, for $\phi = 180^\circ$ the light impinges parallel to this edge
and for $\phi = 270^\circ$ the light impinges perpendicular to the slope. The map in Fig. 3 shows the extracted light
intensity as a function of the incident polar and azimuthal angle. Parallel to the edge ($\phi = 180^\circ$), the total reflexion
above the critical angle reduces the extraction to zero. While getting more and more perpendicular to the sharp edge
($\phi = 90^\circ$ or $\phi = 270^\circ$), this effect is attenuated and the transmission is highly enhanced: the total reflexion disappears
completely. One can see that the light extraction efficiency is not symmetric: light escapes more easily the structure
in the direction of the sharp edge ($90^\circ$) than in the direction of the slope ($270^\circ$).

![Map of the relative extraction intensity](image)

**FIG. 3:** Map of the relative extraction intensity in function of the incident polar angle and the incident azimuthal angle.
The light is highly extracted when the light rays are impinging on the surface perpendicular to the edge of the protruding
structure and perpendicular to the slope. The color scale is relative: black shows zero transmission, and white shows maximal
transmission.

This enhancement is achieved along two paths: (1) the slope of the tilted scale changes completely the geometry
of the surface and the effect of the critical angle is therefore perturbed. (2) A strong diffusion phenomenon takes
places on the sharp edges of the tilted scales. The size of this grating allows a lot of diffraction orders to escape the
source volume. We rather talk about diffusion instead of diffraction.

C. Search for the optimal geometry

Another interesting question concerns optimality: has the firefly done "the best she could" with this structure
to improve on the light extraction. From the biomimetic point of view, it would be interesting to know if another
geometry could give even better results. Therefore simulations with various values of the period and the prism height
were carried out with both parameters ranging from 1 $\mu$m to 15 $\mu$m. TE and TM polarizations have been computed,
but in view of the slight difference obtained on these maps, only the TE maps are shown in Fig. 4. There is a slight
difference between the optimization calculation for the TM polarization, but the difference is not significant to change
the result completely.

The map on Fig. 4 shows the extracted light intensity as a function of the period (x-axis) and the corrugation
height (y-axis). One can see clearly that the firefly is not far from the optimal case (grey rectangle on the map).
FIG. 4: Optimal geometry for the tilted scale (or the so-called firefly) geometry. The map shows the light extraction percentage as a function of the period and the height. The grey rectangle shows the geometry of the firefly.

In fact the differences between the highest light extraction intensity and the light extraction achieved by the firefly is less than 2%. This small discrepancy is remarkable and suggests that the optimization of the corrugation in the firefly is dominated by the light extraction process. This is astonishing, because (1) the cuticle not only has to be easily permeable for light, but it has also to show different properties, such as lightness, mechanical stiffness and hydrophobicity. The cuticle is then not only optimized for light extraction but more multi-optimized for a lot of different aspects. (2) We used many different specimens at different stages of their life in order to get average values of the geometrical parameters and the variability is at least of the order of these 2%.

The data contained in Fig. 4 and their adaptation to inorganic materials is of value for many light extraction designs including artificial ones. The originality of this work lies in its biomimetic approach which directly points to the size of the structures and an optimal profile. Instead of figuring out new designs from scratch, we use readily optimized solutions from evolution and inspiration from nature. As other living organisms [11–14], fireflies have benefitted from the trials and errors during the long process of evolution in order to find an adapted structure to improve light extraction. Engineered optimization only looks back on several decades. It is difficult to tell whether such a structure with their optimal parameters could have emerged without the natural model. It seems, that earlier studies show solutions usually of size smaller than the one revealed here. Considerations of photonic-crystal films and other structures which show inhomogeneities of the scale of the light wavelength are most often worked out [15–17].

The solution indicated by the firefly is actually multi-scale, as the period chosen is much larger, than the emitted wavelength and the abruptness of the edge responsible for the diffusion corresponds to a length scale much smaller than the wavelength. Consistent with this structural characteristics, other structures might be designed and checked for efficiency.

### III. DERIVED BIOINSPIRED STRUCTURES

Regarding these huge improvements of the prism-shaped form, we could now think about similar structures, like sharp triangles, pyramids or cones. Intuitively those structures could even be better for improving light extraction. Such structures present sharp edges and at the same time provide a gradual adaption of the refractive index.
A. New morphologies

Fig. 5(a) shows the structure found on the abdomen of the firefly which has been studied earlier. After this, we will consider three new specific structures:

1. Two-dimensional triangle-shaped structure. This structure is quite similar to the one found on the firefly, but shows a symmetry around the z-axis. Similar to the firefly, in the direction of the x-axis the corrugation exhibits a total translational invariance (Fig. 5(b)).

2. Three-dimensional pyramid structure. This shape is similar to the two-dimensional triangle structure, but varies as well along the x-axis (Fig. 5(c)).

3. Three-dimensional conical structure. This structure is similar to the three-dimensional pyramid structure, but due to the conical shape, no sharp corners appear and the variation of the refractive index is smoother (Fig. 5(d)).

We will proceed to the same calculations as before, i.e. check the convergence of the calculations, define the optimal geometry and show the map of the relative extraction in function of the incident polar and azimuthal angle. For an easier comparison, the former results will be recalled next to the new simulations.

B. New light extraction paths

A complete convergence study has been performed for these new structures. We can keep with 32 plane waves to properly account for the light propagation in the structure as justified in the following (Fig. 6). The simulations have been done, as before, at a wavelength of 560 nm and for depolarized light. The azimuthal angle is taken to be at 90° for all calculations. We can already see by inspection of these graphs, that the firefly has the most effective light extractor. The light can easily escape the material beyond the critical angle (Fig. 6(a)) and the light extraction is considerably enhanced. The triangle structure wipes out the effect of the critical angle as well (Fig. 6(b)), but the light extraction is not improved with the same effectiveness as in the firefly case. The pyramid and the conical structures also improves on the extraction angle, but the intensity, in this specific direction, is way less effective than the firefly structure.
C. Search for the new optimal geometry

The maps on Fig. 7 show the integrated extraction coefficient in function of the periodicity (x-axis) and the height (y-axis) of the geometry. There is little difference between the firefly case (Fig. 7(a)) and the two-dimensional triangle (Fig. 7(b)). The firefly structure is more efficient over a wider range of heights where the triangle structure needs bigger heights for efficient extraction. However the highest value on the scale shows only 29.5% of light extraction for the triangle shape, whereas the firefly-shape reaches 31.6% at its maximum. The firefly is still slightly better.

Fig. 7(c) shows that the highest extraction values for the pyramids are located at larger heights than the previous ones. Again, the optimal range is slightly smaller. Moreover the light extraction reaches only 28.8% in this case. The highest value occurs for a period of 8 µm and a height of 11 µm. The map of the conical structures (Fig. 7(d)) is quite different to the pyramid structure. The extraction efficiencies are a little bit lower (highest values at 27.1 %). The highest extraction values occur at a period of 8 µm and a height of 11 µm and at a period of 9 µm and a height of 11 µm.

D. New light extraction maps

The light extraction maps show more differences. Fig. 8(a) shows the light extraction for the firefly, clearly exhibiting the huge improvement on the asymmetric light extraction. Fig. 8(b) shows the light extraction map for the two-dimensional triangle-shaped structure. This shows a symmetric light extraction as expected from the symmetric shape. Fig. 8(c) shows the light extraction map for the pyramid structure. One can clearly see that the light extraction is not as efficient as in the firefly or the triangle cases. The light is able to escape a little bit above the critical angle, but not in such a spectacular way as in the previous cases. Cones have a similar behavior as the pyramid structure.
FIG. 7: Optimal geometry for the different structures. Maps showing the light extraction percentage as a function of the period and the height.

(a) Firefly. ($p=10\,\mu m$, $h=3\,\mu m$)
(b) Triangle. ($p=7\,\mu m$, $h=8\,\mu m$)
(c) Pyramid. ($p=8\,\mu m$, $h=11\,\mu m$)
(d) Cone. ($p=8\,\mu m$, $h=11\,\mu m$)

FIG. 8: Light extraction maps for the different analyzed structures. For the two-dimensional structures, the light extraction is widely enhanced above the critical angle. The three-dimensional structures are less effective. The color scale is relative: black shows zero transmission, and white shows maximal transmission.
E. Summarizing the highest extraction efficiencies

Let’s now compare the extraction efficiency for the most effective geometry for each new structure (except for the firefly, where we will stay with the geometry close to nature). For our appreciation, we will choose the most effective direction of the azimuthal angle. For the firefly structure, we will keep the periodicity of 10 \( \mu \text{m} \) and the height of 3 \( \mu \text{m} \). The triangle shows the best extraction for a period of 7 \( \mu \text{m} \) and a height of 8 \( \mu \text{m} \). These two structures are most effective for an incident azimuthal angle of \( \phi = 90^\circ \). The pyramid structure shows best results with a periodicity of 8 \( \mu \text{m} \) and a height of 11 \( \mu \text{m} \) for an incident azimuthal angle of \( \phi = 125^\circ \). The conical structure has been shown to provide extraction very close to that of the pyramid structure and is therefore not shown. Clearly, the firefly achieves the best extraction behind the critical angle, but is a little bit less effective for smaller incidences. The triangle structure is evenly distributed over the whole range of incident polar angle, but doesn’t reach an extraction percentage as high as the firefly. The pyramid structure shows the best extraction efficiency around the critical angle, but falls quickly to nearly 0% at larger incidences. The integrated values of these curves gives 34.2%, 26.6% and 24.5% for the firefly, the triangle and the pyramid respectively.

![Graph comparing extraction efficiencies](image)

**FIG. 9:** Comparison between the light extraction in the best geometry and the most effective direction for the different structures. The firefly extracts the light in the most efficient way.

IV. CONCLUSION

The two-dimensional triangle-shaped structure provides extraction efficiencies quite close to those exhibited by the firefly structure. But the firefly structure still remains the best extractor. The present study suggest that two-dimensional profiles such as firefly or triangle structures provide light extraction with a mechanism very different from three-dimensional such as pyramids or cones. Surprisingly the studied three-dimensional structures are not more efficient than two-dimensional structures. A possible explanation is that the three-dimensional sharp points in pyramids or cones appear with a surface density which is extremely weak.

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