Bolometric detection of coherent Josephson coupling in a highly dissipative environment

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The Josephson junction is a building block of quantum circuits. Its behavior, well understood when treated as an isolated entity, is strongly affected by coupling to an electromagnetic environment. In 1983 Schmid predicted that a Josephson junction shunted by a resistance exceeding the resistance quantum \( R_Q = h/4e^2 \approx 6.45 \text{ kΩ} \) for Cooper pairs would become insulating since the phase fluctuations would destroy the coherent Josephson coupling. Although this prediction has been confirmed in charge transport experiments, recent microwave measurements have questioned this interpretation. Here, we insert a small junction in a Johnson-Nyquist type setup, where it is driven by weak current noise arising from thermal fluctuations. Our heat probe minimally perturbs the junction’s equilibrium, shedding light on features not visible in charge transport. We find that while charge transport through the junction is dissipative as expected, thermal transport is determined by the inductive-like Josephson response, unambiguously demonstrating that a supercurrent survives even deep into the expected insulating regime. The discrepancy between these two measurements highlights the difference between the low frequency and the high frequency response of a junction and calls for further theoretical and experimental inputs on the dynamics of Josephson junctions in a highly dissipative environment.

Thermal transport by photons in electrical circuits arises from Johnson-Nyquist noise [1, 2] between two resistive elements at unequal temperatures. The resulting current noise flowing in the ideally lossless circuit linking the two resistors provides an efficient channel for thermalization and energy exchange at very low temperatures [3, 4]. This current noise can be modulated by adding a suitable tunable dissipationless element in the circuit, which can conveniently be implemented by a magnetically [4–6] or electrically [7] controlled Josephson device. So far, both theoretical [8, 9] and experimental approaches [4–7] of photonic heat transport had only considered on-chip resistors with resistance \( R \) much smaller than the superconducting resistance quantum \( R_Q \), where the environmental back-action effect on the junctions is weak. More fundamentally, energy transport through quantum coherent systems strongly coupled to a dissipative environment, characterized by a large effective fine structure constant \( R/R_Q \) [10], remains largely unexplored [11].

It is well-established that the electrical transport properties of a superconducting junction depend on the electromagnetic environment in which it is embedded. Charge transport through a tunnel junction in an Ohmic environment with resistance comparable to the resistance quantum is suppressed at low voltage bias and temperature because of Coulomb blockade [12], and the extension of this phenomenon to superconducting junctions, which are intrinsically phase-coherent, comes naturally [12–15]. Recent experiments using this effect include the production of antibunched photons at high rates [16] and the suppression of the Andreev bound state-induced zero bias anomaly, which could be beneficial in the arduous search for Majorana quasiparticles [17]. For a Josephson junction shunted by a resistor with resistance exceeding \( R_Q \), the supercurrent peak (i.e., current at zero applied voltage bias) is predicted to disappear, being shifted to finite voltage as a result of inelastic Cooper pair tunneling [12]. This is accompanied by a sublinear current-voltage characteristic at low voltages, which signals an insulating behavior of the junction. This dissipative transition, which can be associated with the one predicted by Schmid [18] and Bulgadaev [19], was first confirmed in dc charge transport experiments [13, 15, 20]. However, recent admittance measurements of small junctions, shunted by a highly Ohmic environment [21] called into question the scenario of a dissipative phase transition, leading to further debate about the very existence of this transition [22–25].

In this context, we present a heat transport experiment in which a small tunable junction (effectively a superconducting quantum interference device SQUID) is
embedded in a Johnson-Nyquist setup with hot and cold resistances $> R_Q$ to explore this regime. The SQUID geometry enables magnetic-flux control of the photonic heat current [4]. It is intended to demonstrate the destruction or resilience of the Josephson coupling through the observations of heat flow oscillations, or lack thereof if the junctions are truly insulating. We find that the magnitude of heat current flowing from one resistor to another remains close to the value given by the quantum limit, and it exhibits clear oscillations with the external magnetic flux, similarly to the systems embedded in a low impedance environment [4]. While this observation might point towards a survival of the dc Josephson current effect, a control experiment on dc charge transport shows clear suppression of the charge current at low bias caused by the environmental Coulomb blockade, as reported in earlier experiments [13, 20]. This apparent contradiction, highlighting the role of heat transport as a complementary probe when many-body correlations are present [26, 27], is discussed within the existing theoretical and experimental literature.

Our device (see Fig. 1a for an SEM image) consists of a SQUID between two nominally identical on-chip thin chromium resistors acting as thermal baths, from now on referred to as a source and drain with resistance $R_S$ and $R_D$, respectively. Each arm of the SQUID is galvanically connected to one source and drain resistor of volume $\Omega = 10 \times 0.1 \times 0.014 \text{\,\,\,$\mu$m}^3$ and whose resistance is nominally equal to that of an independently measured resistor with same dimensions on the same chip, with a value $R_S = R_D = 11 \pm 0.5 \text{\,\,\,k}\Omega$ (see Supplemental S1, Fig. S2). The distance between the SQUID and the resistors is kept short (a few microns) to avoid suppression of environment-induced effects via stray capacitance. The series configuration of the SQUID and resistors is further closed into a loop by a superconducting line. This warrants efficient electromagnetic heat transport through improved impedance matching [28]. The clean contact between chromium and superconducting aluminum leads serves as an Andreev-mirror [29], which enables essentially perfect conversion to charge transport by Cooper pairs in the superconducting strips while effectively suppressing quasiparticle heat diffusion along them at low temperatures ($\lesssim 200 \text{\,\,mK}$) [28]. Four external superconducting leads are contacted with the source resistor through a thin oxide barrier, forming NIS-tunnel junctions. A pair of these junctions is used to measure the electronic temperature (in the case of quasi-equilibrium where the electron temperature is well-defined [30]) by applying a small dc-current bias through it, whereas another pair is used to locally cool the resistor when voltage-biased [30]. The electron temperature of the drain resistor is measured simultaneously by another SI-NIS junction structure, as depicted in Fig. 1a. We have presented data on two samples, henceforth called Sample I and Sample II.

We first measure the current-voltage characteristics (IVC) of a reference sample (see Fig. 2a) on the same chip from now on called “Replica”, with nominally equal parameters as the main sample. This provides estimates of parameters for the heat transport experiments and enables comparison between charge and heat transport behavior. Figure 2b and 2c show the IVC for two Replica samples at a phonon temperature of $T_0 = 87 \text{\,\,mK}$ in the low bias region at two different magnetic flux values $\Phi = 0$ (solid circles) and $\Phi = \Phi_0/2$ (open circles) with $\Phi_0 = \pi h/e$ the superconducting magnetic flux quantum. Suppression, with respect to the unblocked case, is observed in the low-bias dc current through the SQUID, which is more robust for Replica II (panel c) due to its...
higher charging energy $E_C$. This observation is well understood in the framework of dynamical Coulomb blockade [12]: the resistive environment impedes charge relaxation after a Cooper-pair tunneling event through junctions with high charging energy, which translates to a conductance reduction at low energy [12]. The magnitude of $E_C$ (0.8 K for Replica I and 1.4 K for Replica II) is extracted from the current peak feature appearing in the IVC, which for small Josephson junctions in contact with an environment impedance with $R \gg R_Q$ occurs at a voltage bias $eV_b \sim 2E_C$ [13, 31]. The Josephson energy is estimated by using the Ambegaokar-Baratoff relation as $E_J = \Phi_0 \Delta / 4eR_1$ (0.07 K for Replica I and 0.03 K for Replica II), with $\Delta \approx 200 \mu$V the aluminum superconducting energy gap and $R_1$ the quasiparticle tunnel resistance of the SQUID. The dashed lines in Fig. 2b and 2c are the theoretical results obtained by the standard $P(E)$ theory for two magnetic flux values, $\Phi = 0$ (dashed black line) and $\Phi = \Phi_0/2$ (dashed gray line). The fit parameters for Replica I of panel b are: critical current $I_C = 7.9$ nA, Josephson energy $E_J \sim 0.19$ K, charging energy $E_C \sim 0.8$ K and, for Replica II of panel c: $I_C = 3.3$ nA, $E_J \sim 0.08$ K, $E_C = 1.4$ K. The resistance of Cr-strips is $R_e = 11$ kΩ for both samples. d.- Illustration of the Schmid phase diagram for a Josephson junction attached to a resistive environment $R$ at zero temperature. Here, $R = R_S + R_D = 2R_e$ is the total resistance of the environment. Our samples are well placed in the insulating part, represented by the two points.

The power flowing from drain to source $\dot{Q}_v$ under a thermal gradient is determined by measuring the electronic temperatures of the drain $T_D$ and the source $T_S$. The temperature difference is generated by dc biasing the source resistor with a voltage $V_H \lesssim 2E_C/e$ that enables electronic cooling of the source by removal of hot electrons [28, 30], as depicted in Fig. 3a and 3b. In steady-state, these temperatures involve the different energy relaxation channels in the system, as illustrated in the thermal model that accounts for our setup shown in Fig. 1b. By continuity (valid below 200 mK, when quasiparticle heat diffusion along the superconductor can be neglected) a direct relation between $\dot{Q}_v$ and the temperatures $(T_S, T_D, T_0)$ measured in the system is found

$$\dot{Q}_v(T_S, T_D, \Phi) = \dot{Q}_{ep,D}(T_D, T_0),$$

where $\dot{Q}_{ep,D} = \Sigma \Omega_0 [T_0^6 - T_0^6]$ is the electron-phonon heat current governed by the drain resistor. Here, $\Sigma$ is the electron-phonon coupling constant of the normal metal which was measured independently to be $\Sigma = (12 \pm 0.25) \times 10^9$ W K$^{-6}$ m$^{-3}$ (unpublished). With our experimental setup (see Fig. 1a), we have full control of all temperatures and, therefore, the powers involved in the system, leading to an accurate and fully calibrated measurement of the thermal conductance between the drain and source.

Figure 3c and 3d show the temperature drops $\Delta T_i = T_i(V_i^{\text{opt}}) - T_i(V_H = 0)$, $i = S, D$ at the optimum cooling bias of the SINIS $V_H^{\text{opt}} \lesssim 2E_C/e$ for the two samples at two magnetic flux values. At $V_H = 0$ the electronic temperature equals the phonon temperature $T_0$. These drops characterize the thermal coupling between the drain and source, i.e., the thermal conduc-
We then confront the data with theoretical calculations based on the Landauer relation for heat current from drain to source [3],

\[
\dot{Q}_\nu = \int_0^\infty \frac{d\omega}{2\pi} \hbar \omega \tau(\omega, \Phi) \left[ \frac{1}{e\hbar/\k_B T_D} - 1 \right] - \frac{1}{e\hbar/\k_B T_S} - 1, \tag{2}
\]

where \(\tau(\omega, \Phi)\) is the transmission probability of the thermal radiation from the drain to the source at angular frequency \(\omega\). In the limit of small phase fluctuations around a given average phase bias \(\varphi\), the SQUID can conveniently be approximated by a harmonic oscillator. In electrical terms, this translates to an effective phase-dependent Josephson inductance \(L_{\text{eff}}(\Phi) = h/(2e|I_c(\Phi)|\langle \cos \varphi \rangle)\) in parallel with a capacitance (see Fig. 4e), where \(\langle \cos \varphi \rangle\) is an average over phase fluctuations [24, 33]. Within this linear model, and by assuming the lumped approximation (valid since the dominant radiation wavelength \(\lambda_b = \hbar c/k_B T \sim 10 \text{ cm at 150 mK}\) is much larger than the circuit characteristic dimensions \(\sim 50 \mu\text{m}\)), the power transmission coefficient can be explicitly written [3, 9] as \(\tau(\omega, \Phi) = 4R_S R_D/Z_T(\omega, \Phi)^2\) (see methods), with \(Z_T(\omega, \Phi)\) the frequency-dependent total series impedance of the circuit. In this framework, the maximum heat transfer is expected for a perfect impedance matching when \(R_S = R_D\) and when the phase fluctuations are small, i.e., \(\langle \cos \varphi \rangle \approx 1\) under no net electrical bias. However, for strong phase fluctuations at high resistances \(R_S + R_D > R_Q\), one naively expects the average value of the cosine to almost vanish, \(\langle \cos \varphi \rangle \approx 0\). Indeed, we have shown above that the dc charge transport measurements of the Replica samples are well described by the usual \(P(E)\) theory of Coulomb blockade, which implies \(\langle \cos \varphi \rangle \approx 0\). Assuming this, the SQUID can be regarded as a parallel connection of a capacitor \(C_J\) and of high effective impedance \(Z_T(\omega, \Phi) \approx 1/E^2(\Phi)\), see Fig. 4f. Then the modulation of the Josephson coupling by flux almost does not affect the transmission probability \(\tau(\omega, \Phi)\), and the oscillations of the heat flow are expected to be very small. Below we will show that the strong modulation of the heat flow observed in our experiment is consistent with the assumption of nonvanishing \(\langle \cos \varphi \rangle\) rather than with \(\langle \cos \varphi \rangle \approx 0\).

The dashed lines displayed in figures 4a and 4b are the theoretical results of \(\dot{Q}_\nu\) obtained within the linear model for the corresponding magnetic fluxes applied. For Sample I, reasonable agreement with the experimental data at \(\Phi = 0\) is found if we use the bare Josephson junction inductance in the calculated \(Z_T\). Nevertheless, for Sample II, a significant deviation from the data is observed as the temperature is lowered. This deviation can be captured if we set the re-normalization parameter \(\langle \cos \varphi \rangle = 0.18\). These results indicate that the photonic heat exchange from the drain to the source at lower thermal frequencies is mainly transmitted through the Josephson induc-

![FIG. 3. a, b.- Electronic temperature of the source \(T_S\) (blue points) and drain \(T_D\) resistor (red points) at \(\Phi = 0\) for Sample I and Sample II, respectively, as a function of the bias voltage recorded at phonon temperature \(T_0\) of 151 mK (Sample I) and 180 mK (Sample II). c, d.- Temperature drops \(\Delta T_S\) (blue circles) and \(\Delta T_D\) (red circles) recorded at two magnetic flux values \(\Phi = 0\) (solid circles) and \(\Phi = \Phi_0/2\) (open circles) measured at different values of \(T_0\).](image-url)
tor channel. At \( \Phi = \Phi_0/2 \) (dashed purple and dark red lines), the power is reduced as expected, in fair agreement with the data. In this regime, the Josephson critical current is vanishingly small (making the inductance essentially infinite at the relevant frequencies); consequently, the power transmitted from the drain to the source takes place mainly through the junction capacitance \( C_J \), which acts as a high-pass filter in the transmission and thus only enables a small fraction of the thermal fluctuations to be transmitted as current in the circuit. Furthermore, the heat current calculated within the \( P(E) \) theory at \( \Phi = 0 \) shows a decrease of \( Q_\nu \) to the level of the power obtained in the linear model at \( \Phi = \Phi_0/2 \) (see Supplemental Figs. S3b and S3c).

Figure 4c and Fig. 4d show the heat current modulations measured at given phonon temperatures \( T_0 \) for Sample I and Sample II, respectively. Clear oscillations with period \( \Phi_0 \) are observed. This result unequivocally demonstrates that the inductive response of the junction persists in the presence of strong environmental back-action, in contrast to what is observed for charge transport measurements in the Replica samples. The data is again compared to the theoretical models proposed. For simplicity, we have assumed a symmetric SQUID in the models. On the one hand, for the two examples presented, the heat current modulations are well captured with the linear model if we use a value of \( \langle \cos \varphi \rangle = 0.23 \) and \( \langle \cos \varphi \rangle = 0.18 \) for Sample I and Sample II, respectively. Besides, to fit the data of Sample II at other temperatures, values of \( \langle \cos \varphi \rangle \) in the range 0.3-0.6 are used. On the other hand, the oscillation amplitude predicted by \( P(E) \) theory \( \langle \cos \varphi \rangle \approx 0 \) is much smaller than the amplitudes observed (see Supplemental Figs. S3d and S3e).

Let us now focus on the discrepancy between charge and heat transport measurements. One obvious difference resides in the relevant frequency range: at zero frequency, \( P(E) \) theory of Coulomb blockade describes inco-
herent Cooper pair tunneling through the junction, and the transition to an insulating state predicted by Schmid and Bulgadaev is observed as $R$ becomes greater than $R_Q$. On the other hand, heat transport deals with non-zero frequency current fluctuations flowing through the junction at zero net voltage bias. This was considered previously [34] through an extension of the static version of $P(E)$ theory to finite frequency transport. The derivation relies on the hypothesis of very weak Josephson coupling, $2E_J \ll k_B T$, which is not perfectly satisfied for Sample I, unlike for Sample II, where the contrast is the strongest (as highlighted by the sharp Coulomb gap observed in charge transport, see Fig. 2c). An alternative, motivated by the microscopic description of the Josephson junction [33], is the existence of an inductive-like shunt in the junctions' environment, fundamentally due to the BCS gap, which protects the ground state of the junction from strong phase diffusion. The presence of such a shunt would translate as a finite supercurrent peak, which indeed was reported recently [16]. Its absence in our charge measurement (and previous ones [13, 20]) contradicts this interpretation. We do not offer a definite conclusion, as it might be explained by noise and imperfections of measurement setups. Our data provide a piece of evidence on the overall puzzle. In that respect, recent high-frequency measurements of a small Josephson junction in an engineered high impedance environment have revealed the inelastic nature of the scattering process of a photon off the junction [35]. The conceptual similarity between the setup considered there and ours suggests that the description of the junction as a renormalized inductor [33], while appealing and useful for a basic understanding, is too simplistic because the non-linearities of the junctions are explored by strong phase fluctuations. Nevertheless, our bolometric technique enables to collect energy over a bandwidth $\sim k_B T/\hbar$ and, therefore, would not distinguish between several down-converted photons out of an inelastic process or a single elastically scattered photon.

In summary, we have experimentally demonstrated through heat transport measurements that a Josephson junction acts as an inductor even in the presence of a highly resistive environment. Though the interpretation of the dissipative transition can be debated [21, 24], the discrepancy between the heat transport measurements and the control charge transport measurements by us here and in the previous works [13, 20, 36], cannot be accounted for by the existing theory, and calls for further developments, both experimental and theoretical. Our findings are important not only from the fundamental physics point of view but also for future applications such as microbolometers or heat sink designs in quantum circuits. On a practical side, we note that any design aiming at increasing resistances for improved, quantum-limited tunable remote electronic cooling [4, 28] is much less sensitive to back-action effects than initially antici-

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**AUTHOR CONTRIBUTIONS**

The experiment was conceived by D.S., O.M., and J.P.P. and carried out by D.S. with contribution from O.M. and technical support by J.T.P. Sample fabrication was made by D.S. The theoretical model for heat transport based on the $P(E)$ theory was proposed by D.S.G, and the simulations were performed by D.S. The data were analyzed, and the manuscript was written by D.S. with important contributions from all the authors.

**COMPEING INTERESTS**

The authors declare no competing interests.

**METHODS**

Device fabrication and measurement

The devices were fabricated on 4-inch silicon substrates covered by 300 nm of Si/SiO$_2$ in an electron beam lithography (EBL, Vistec EBPG500 + operating at 100 kV) using a Ge-based hard mask process and the conventional shadow evaporation technique [37]. The silicon wafer was coated with 400 nm layers of poly(methylmethacrylate-methacrylate acid) P(MMA-MAA) resist spun for 1 min at 5500 rpm and baked at 160 °C for 20 min, twice. Then, on top of it 22 nm Ge layer was deposited in an electron-beam evaporator, and right after, approximately 50 nm thick of PMMA was coated with spin at 2500 rpm for 1 min and baked at 160 °C for 1 min. The devices were patterned on the PMMA layer by using electron beam lithography, and afterward, it was developed using a mixture solution with a concentration of 1:3 of methyl-isobutylketone+isopropanol (MIBK). This pattern is transferred to the Ge mask using reactive ion etching (RIE) with tetrafluoromethane CF$_4$ plasma. The undercut in the MMA resist was created by oxygen plasma in the same RIE chamber. The metallic parts were made in three evaporation steps: first, a 20 nm layer of Al is evaporated at an evaporation angle of $-22^\circ$. Then, static oxidation in-situ with pressure around 3 mbar for 3 minutes is made. This step defines the superconducting finger used as a thermometer, heater, and branch of the SQUID. In the second step, a 20 nm layer of Al is evaporated at an angle of $-7^\circ$, forming the SQUID and the clean superconducting contact. Finally, a 14 nm layer of Cr is evaporated at an angle of 24° comprising the thermal bath. The nominal loop area of the SQUID for the two samples was 25 $\mu$m$^2$. The main difference between them lies in the overlap area of the Josephson junction (JJ), which for Sample I is nominally 130×140 nm$^2$ and for Sample II is 85×85 nm$^2$. The resist was lift-off in
acetone at 52°C. Then, the sample is attached to a sample carrier to be electrically connected to it by Al wire bonds for being measured. The bonded sample is placed on a stage with a double brass enclosure that acts as a radiation shield. It is connected to the mixing chamber of a custom-made plastic dilution refrigerator with a base temperature of approximately 40 mK. Dc signals were applied through cryogenic signal lines filtered with lossy coaxial cables with 0-10 kHz bandwidth connected to the bonded sample through a room-temperature breakout box. In order to sweep the SQUID Josephson energy, a perpendicular magnetic field is supplied by applying dc current to an external superconducting magnet inserted around the vacuum can. All the input signals were applied and read out using programmable sources and multimeters. Amplifying current and voltage output signal was accomplished using a room temperature low noise current amplifier Femto DDPCA-300 and voltage amplifier Femto DLVPA-100-F-D, respectively. The cryostat temperature is controlled by applying a voltage across the SINIS configuration (current biased $I_{th} = 15$ pA) at zero heating bias voltage while varying the cryostat temperature up to 500 mK [30].

**Photon transmission coefficient**

As mentioned in the main text, the transmission probability of the thermal radiation from the source and drain $\tau(\omega, \Phi)$ used is Eq. (2) has been calculated within the two models. In the linear model approximation, $\tau(\omega, \Phi)$ can be written as

$$\tau(\omega, \Phi) = \frac{4R_S R_D}{|Z_T(\omega, \Phi)|^2},$$

with

$$Z_T(\omega, \Phi) = R_S + R_D + \frac{1}{-i\omega C_j + \frac{2\pi T}{e^2}} \langle \cos \phi \rangle.$$  

(4)

In the charge dominated regime ($E_C \gg k_B T_{S,D}$) and taking into account the effect of the environment resistors through $P(E)$ function, the transmission probability $\tau(\omega, \Phi)$ for the system studied reads [11],

$$\tau(\omega, \Phi) = \frac{4R_S R_D}{|R_S + R_D + \frac{1}{-i\omega C_j + Z^{-1}(\omega, \Phi)}|^2} + \frac{\pi^2 I_c^2}{2e^2} \left[ P_S(\omega) - P_S(-\omega) \right] \left[ P_D(\omega) - P_D(-\omega) \right],$$

(5)

where $\tilde{Z}(\omega, \Phi)$ is the effective frequency-dependent impedance and, the functions $P_S(\omega)$ and $P_D(\omega)$ represent the probability of photon absorption in the source and the drain resistors, respectively. These functions are defined as

$$P_l(\omega) = \int \frac{dt}{2\pi} e^{i\omega t} e^{-J_l(t)},$$

(6)

where $l = S, D$ and $J_l$ is the phase-phase correlation function given by [12]:

$$J_l(\omega) = \frac{4e^2 R_l}{\pi \hbar} \int_0^\infty d\omega \frac{\coth \frac{\hbar \omega}{2k_B T} (1 - \cos \omega t) - i \sin \omega t}{\omega (1 + \omega^2 (R_S + R_D)^2 C_j^2)}.$$  

(7)

The effective impedance $\tilde{Z}(\omega, \Phi)$ is defined as

$$\frac{1}{Z(\omega, \Phi)} = \frac{\pi I_c^2}{2\hbar \omega} \left[ P(\omega) - P(-\omega) - i (P(\omega) + P(-\omega) - 2P(0)) \tan \frac{\pi (R_S + R_D)}{R_Q} \right],$$

(8)

where $P(\omega)$ is the $P$-function of the effective environment defined by the convolution of the $P(E)$-function of the two resistors,

$$P(\omega) = \int d\omega' P_S(\omega - \omega') P_D(\omega').$$

(9)
S1. CURRENT-VOLTAGE CHARACTERISTIC OF THE REPLICA SAMPLE

![Graphs showing current-voltage characteristics](image)

**FIG. S1.** a, c.- Current-voltage characteristic at large voltage bias of the Replica I and Replica II. The dashed black line is the linear fit. b, d.- Bias dependence of the temperature resistors in the low voltage bias regime at two magnetic flux values $\Phi = 0$ (solid circles) and $\Phi = \Phi_0/2$ (open circles), obtained from Eq. S4.

The quasiparticle tunnel resistance of the SQUID $R_J$ and the superconducting gap were obtained from standard current-voltage measurements as depicted in Figs. S1a and S1c. The total resistance in series $R_T = R_S + R_D + R_J$ of the Replica was obtained by a linear fit, dashed black line in Figs. S1a and S1c. These resistances were: 122 kΩ for...
Replica I and 280 kΩ for Replica II. Source and drain resistance were independently measured (see Fig. S2b) on-chip in a Cr-strip having the same size as those placed in the main and Replica sample and evaporated with the same target to be $R_S = R_D = 11 \, \text{kΩ}$. The electronic setup to measure this resistance is shown in Fig. S2a. Therefore, the resistance $R_J$ was determined by subtracting from $R_T$ the source $R_S$ and drain $R_D$ resistance values. Additionally, the superconducting gap was measured to be $\Delta = 200 \, \mu\text{eV}$. Thus, the Josephson energy is calculated using the Ambegaokar-Baratoff relation $E_J = 4.14 \Delta / R_J$. The single charging energy of the junction $E_C = e^2 / 2C_J$ is extracted from the current-voltage at a low bias, as explained in the main text, from which we estimate the SQUID capacitance to be 1.2 fF for Replica I and 0.7 fF for Replica II.

FIG. S2. a.- Colored scanning electron micrograph (scale bar: 5 µm) highlighting the thin chromium normal metal (blue), with the schematics of the IV measurement. b.- Current-voltage curve measured at a phonon temperature of $T_0 = 87 \, \text{mK}$. The dashed line is the linear fit with a resistance value of $R = 11 \, \text{kΩ}$.

**S2. JOSEPHSON CURRENT OF A SMALL JUNCTION EMBEDDED IN AN ELECTROMAGNETIC ENVIRONMENT**

It is well established that the finite voltage bias current-voltage characteristic of a Josephson junction with a small critical current embedded in an electromagnetic environment at low bias is written as [S1, S2],

$$I = \frac{\pi e E_J^2(\Phi)}{\hbar} [P(2eV) - P(-2eV)], \quad (S1)$$

where $P(E)$ is the probability function that describes the energy exchange in the inelastic Cooper pair tunneling with the environment. This probability density is given by

$$P(E) = \frac{1}{2\pi\hbar} \int e^{iEt} \langle e^{i\hat{\phi}(t)} e^{-i\hat{\phi}(0)} \rangle = \frac{1}{2\pi\hbar} \int dt e^{iEt} - J(t), \quad (S2)$$

where

$$J(t) = \frac{4e^2}{\pi\hbar} \int_0^\infty d\omega \text{Re} [Z_T(\omega)] \left[ \text{coth} \left( \frac{\hbar\omega}{2k_B T(V_b)} \right) \frac{1 - \cos \omega t}{\omega} + \frac{\sin \omega t}{\omega^2} \right], \quad (S3)$$

is the phase-phase correlation function. Here, $Z_T(\omega)$ and $T(V)$ are the impedance seen by the junction and the bias voltage dependent temperature of the resistors. The latter is modeled in the usual way,

$$T(V) = \left[ T_0^6 + \frac{IV_b}{2\Sigma \Omega} \right]^{1/6}, \quad (S4)$$

and its behavior is shown in Figs. S1b and S1d. Here, $T_0$ is the phonon temperature, $\Sigma = 12 \times 10^9 \, \text{WK}^{-1}\text{m}^{-3}$ is the electron-phonon constant of the normal metal, and $\Omega = 1.4 \times 10^{-20} \, \text{m}^3$ is the volume of each resistor. Factor 2 in Eq. (S4) accounts for the two resistors surrounding the SQUID. The impedance $Z_T(\omega)$ is derived from the resistively
and, capacitively shunted junction (RCSJ) model,

$$Z_T(\omega) = \frac{1}{-i\omega C_J + (R_S + R_D)^{-1}}, \quad \text{Re}[Z_T(\omega)] = \frac{R_S + R_D}{1 + \omega^2 (R_S + R_D)^2 C_J^2}. \quad (S5)$$

Note that the asymmetry of the critical current of the SQUID in Eq. (S1) is taken into account through the parameter $d$ as [S3]

$$E_J(\Phi) = |E_J(0)| \cos(\pi \Phi / \Phi_0) \sqrt{1 + d^2 \tan^2(\pi \Phi / \Phi_0)}. \quad (S6)$$

S3. HEAT CURRENT THROUGH A SQUID CONNECTED IN SERIES WITH RESISTORS BASED ON THE $P(E)$-THEORY

Here we consider the system shown in Fig. S3a. The photonic heat flux between the two resistors can be expressed as (see Ref. [S4])

$$\dot{Q}_\nu = \int_{0}^{\infty} \frac{d\omega}{2\pi} \hbar \omega \tau(\omega) \left[ \frac{1}{e^{\hbar \omega/k_B T_D} - 1} - \frac{1}{e^{\hbar \omega/k_B T_S} - 1} \right] + \frac{\pi \hbar I_c^2}{4e^2} \int_{-\infty}^{+\infty} \frac{d\omega}{\omega} P_D(\omega) P_S(-\omega). \quad (S7)$$

This equation is similar to the one deduced by Thomas, et.al. [S4] for a SQUID connected in parallel with resistors. However, one should re-define the photon transmission probability $\tau(\omega)$ and the functions $P_S(\omega)$ and $P_D(\omega)$ for the
circuit shown in Fig. S3a. Namely, the transmission probability is expressed as
\[
\tau(\omega) = \frac{4R_SR_D}{\left| R_S + R_D + \frac{1}{i\omega C_J + \frac{1}{2\pi(\omega)}} \right|^2},
\]
(S8)
where the effective impedance of the junction \( \tilde{Z}(\omega) \) was already defined in the methods section of the main text (see Eq. (8)). To find the expressions for the functions \( P_j(\omega) \) (\( j= S, D \)) we write down the classical equation of motion for the Josephson phase in the circuit of Fig. S3a,
\[
C\left[ \frac{\hbar}{2e} \dot{\phi} + \frac{1}{R_S + R_D} \frac{\hbar}{2e} \dot{\phi} + I_C \sin \varphi = \frac{R_S \xi_S + R_D \xi_D}{R_S + R_D}. \right.
\]
(S9)
In order to obtain the equation for the phase, we introduce the new effective resistances \( R_S \) and \( R_D \) such that the equation (S9) reads,
\[
C_1\left[ \frac{\hbar}{2e} \dot{\phi} + \frac{1}{R_S} \frac{\hbar}{2e} \dot{\phi} + iC(\Phi) \sin \varphi = \eta_S + \eta_D \right.
\]
(S10)
where
\[
\mathcal{R}_j = \frac{(R_S + R_D)^2}{R_j}, \quad \eta_j = \frac{R_j \xi_J}{R_S + R_D}, \quad |\eta_j| = \frac{R_j}{(R_S + R_D)^2} \omega \coth \frac{\hbar \omega}{2k_B T_j}, \quad (S11)
\]
Thus, for the circuit of Fig. S3a we find the functions \( P_j(\omega) \) by replacing the expression (S11) in the Eqs. (S2, S3)
\[
P_j(\omega) = \int \frac{dt}{2\pi} e^{i\omega t} e^{-J_j(t)},
\]
(S12)
where
\[
J_j(t) = \frac{4e^2}{\pi \hbar} \int_0^\infty d\omega \frac{\coth \frac{\hbar \omega}{2k_B T_j} (1 - \cos \omega t) + i \sin \omega t}{\mathcal{R}_j} \left[ \frac{1}{R_S} + \frac{1}{R_D} \right]^2 \left[ \frac{\coth \frac{\hbar \omega}{2k_B T_j} (1 - \cos \omega t) + i \sin \omega t}{\mathcal{R}_j} \right]^2
\]
\[
= \frac{4e^2}{\pi \hbar} \int_0^\infty d\omega \frac{R_j \left[ \coth \frac{\hbar \omega}{2k_B T_j} (1 - \cos \omega t) + i \sin \omega t \right] \left[ \coth \frac{\hbar \omega}{2k_B T_j} (1 - \cos \omega t) + i \sin \omega t \right]}{(R_S + R_D)^2} \left[ \frac{1}{R_S + R_D} \right]^2
\]
\[
= \frac{4e^2 R_j}{\pi \hbar} \int_0^\infty d\omega \left[ \coth \frac{\hbar \omega}{2k_B T_j} (1 - \cos \omega t) + i \sin \omega t \right] \left[ \frac{1}{R_S + R_D} \right]^2.
\]
(S13)

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