Sediment Transport and Morphodynamics Induced by a Translating Monopolar Vortex

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Abstract We performed laboratory experiments to describe and quantify the transport of sediment and the changes in the bed due to a generic translating monopolar vortex. Experiments were performed inside a water-filled, square tank with a particle bed on the bottom and a vertical plate attached perpendicular to one of the sidewalls. The tank was placed on top of a rotating table to create the vortex by changing its rotation rate. This change created a current inside the tank that separated at the edge of the vertical plate, with the shear layer rolling up into a vortex. Once the vortex was formed, the table was promptly stopped. Sediment particles are brought into suspension and captured by the vortex, forming a conical region that moves with the vortex until the sediment resettle in the bed, changing the original bed morphology. Three different measurement techniques were used to obtain information about the flow velocities, the sediment in suspension, and the net changes in the bed. Changes in the bed morphology occur along the trajectory of the vortex, where a region of erosion is followed by a region of deposition. The strength of a vortex is the main parameter governing the capture and suspension of particles with similar characteristics. A power law relationship is found between the vortex strength and the net displaced particle volume. Experiments were also performed without sediment to determine if the presence of sediment could affect the vortex dynamics. However, a definitive answer requires more experiments to obtain reliable statistics.

1. Introduction

Strong vortices are capable of trapping and transporting heavy particles, such as sediment, over long distances. Notable examples are dust devils (Greeley et al., 2003; Snow, 1984), tornadoes (Lewellen et al., 2008; Magsig & Snow, 1998; Pepper, 1975), and hurricanes (Sapsis & Haller, 2009). While comparatively less referenced, vortices in aquatic media also play a role in the suspension and transport of sediment in beaches (Shibayama & Horikawa, 1982), tidal basins (Amoroso & Gagliardini, 2010), and along coasts (MacMahan et al., 2006).

Vortices are capable of trapping particles due to a radially inward flow in the boundary layer below the vortex, which moves particles if the shear stress exerted $\tau$ exceeds a critical value $\tau_{cr}$ (Fleming & Hunt, 1977; González-Vera et al., 2018). Due to the radial inflow, a vertically upward flow is driven above the boundary layer. Furthermore, the pressure deficit in the center of the vortex provides an increase in the lift force, enhancing the suspension of particles (Greeley et al., 1981; Greeley & Iversen, 1987). Similar dynamics have been studied in “tornado-like” vortices (Jischke & Parang, 1974; Kuo, 1971; Nolan & Farrell, 1999; Rotunno, 1979), which are characterized by very strong swirling motions and the radial convergence of fluid at the bottom.

Although the presence of vortices over granular beds is common, previous studies on the sediment transport capabilities of a single vortex have been mainly limited to the case of dust devils (Greeley et al., 2003). To our knowledge, there have been no studies regarding the transport capabilities of generic underwater vortices. Furthermore, no quantification of the net transported sediment, the volume occupied by the suspended particles, or the changes in the morphology of the sediment bed have been made. These topics are of interest, for example, in the conservation of coasts and navigation channels since the generation of vortices can lead to significant changes in the morphology of the region over time (see, e.g., Berthot & Pattiaratchi, 2006; Takasugi et al., 1994). Furthermore, these changes in the morphology can, in turn, affect the flow.

Commonly, studies on the transport properties of vortices have been limited to two-dimensional (2-D) flows, where particles are treated as passive tracers (Babiano et al., 1987; Boffetta et al., 1996; Crisanti et al., 1991, ...
GONZÁLEZ-VERA ET AL. 2020. The vortex field and have shown that vortices have not been accounted for.

In the current paper, we study experimentally the sediment transport by a single, underwater, monopolar vortex traveling over a flat sediment bed. Here, monopolar indicates a simple and single vortex structure. Particles at the bed are set into motion and brought into suspension when a strong enough vortex passes over them (Fleming & Hunt, 1977); the particles in suspension travel with the vortex as it moves and settle to the bed as the vortex decays. We describe qualitatively and quantitatively the characteristics of the volume occupied by the sediment in suspension, the net amount of sediment transported, and the changes in the morphology of the particle bed as a function of the characteristics of the vortex. The volume occupied by the suspended sediment was dynamically measured using a vertically moving horizontal laser sheet which illuminated cross sections of the region where particles were suspended. Changes in the particle bed were measured using a single-camera photogrammetric technique (González-Vera et al., 2020). The vortex characteristics (size, strength, trajectory, and velocity) were obtained using particle image velocimetry (PIV). Experiments without a sediment bed were also performed and compared to their counterparts with sediment to determine possible effects of the changes in the sediment bed and particle loading on the vortex.

The present paper is organized as follows. The experimental setup and experimental techniques are described in section 2. The typical characteristics of the vortices are presented in section 3.2. The description of the evolution of the sediment in suspension is presented in section 4.1 and the description of the changes in the bed in section 4.2. Results are discussed and conclusions presented in section 5.

2. Experimental Setup

The experimental setup is shown in Figure 1. It consisted of a 1 × 1 × 0.35 m³ square plexiglass tank placed on top of a computer-controlled rotating table. At the bottom of the tank, a 0.95 × 0.3 × 0.016 m³ plastic panel was placed to produce a difference in height within the tank, separating the bottom into a shallow and a deep region. Above the plastic panel, a flat vertical plate of 25 cm in length was fixed perpendicularly to one of the side walls of the tank at 3 cm from the edge of the plastic panel. The tank was filled with water to a height $H = 21.6$ cm from the tank’s bottom, having a density $\rho_f = 1.0$ g cm$^{-3}$ and kinematic viscosity $\nu \approx 1.0 \times 10^{-6}$ m² s$^{-1}$. For experiments without sediment, the additional plastic panel at the bottom was removed to have a perfectly flat bottom. Additionally, for all experiments to have the same water depth, the tank was filled with water to a height $H = 20$ cm from the tank’s bottom.

Two cameras were positioned above the tank at a height $H_c = 2.06$ m from the tank’s bottom ($z = 0$), looking vertically down. This gave a field of view of approximately 0.76 × 0.61 m², which was displaced toward the wall of the tank, as shown in Figure 1. Small particles were added to the deeper region inside of the tank until a sediment layer of thickness $\approx 1.6$ cm (i.e., equal to the thickness of the plastic panel) was formed, leaving a water layer of 20 cm. Consistent initial conditions were achieved for all experiments by flattening the sediment bed to a uniform thickness before the beginning of each experiment. Two different types of particles were used in separate experiments since it is known that particle properties (e.g., density and size) affect the net transport of sediment produced by a flow (Le Roux, 2005; Paphitis, 2001). The first type of particles (from now on referred to as PS particles) were translucent, spherical, polystyrene particles with an average diameter $d_p = 580$ μm and a density $\rho_p = 1.050$ g cm$^{-3}$. The second type of particles (referred to as PMMA particles) were smaller and heavier, with a diameter 250 μm $< d_p < 300$ μm and a density $\rho_p = 1.200$ g cm$^{-3}$. Since the density and size of the types of particles were different, the $\tau_{cr}$ value required before particles initiate movement is different as well. On the other hand, the settling velocity of the particles, approximated by

$$w_s = \frac{1}{18} \frac{(\rho_p - \rho_f)gd_p^2}{\rho_f \nu} \ (1)$$

where $g$ is the gravitational acceleration, was similar in both cases with $w_s = 9.16$ mm s$^{-1}$ for PS particles, and $w_s = 6.81$ to 9.81 mm s$^{-1}$ for PMMA particles. Although Equation 1 is based on the Stokes drag
relationship, which is only valid for $Re \ll 1$, it allows us to characterize the particles based on their size and density. Further characterization is obtained by calculating the particle response time or the so-called Stokes time (Bec et al., 2007)

$$t_0 = \frac{\rho_p d_p^2}{\rho_f 18 \nu}$$

The relaxation time of the particle types differs by almost an order of magnitude with $t_0 = 0.019$ s for the PS particles and $0.004 < t_0 < 0.006$ s for the PMMA particles.

2.1. Measurement Techniques

The effect of the vortex on the sediment bed was determined by measuring the properties of the vortex (radius, strength, trajectory, and velocity), the region occupied by the sediment in suspension, and the changes in the particle bed height. For this, three separate measurement techniques were implemented: PIV to determine the flow velocity field and the vortex characteristics at the water surface, a laser scanning technique to determine the regions of suspended sediment, and a photogrammetric technique to determine the changes in the bed.

2.1.1. PIV and Laser Scanning

To avoid interference from the sediment particles in suspension, PIV was performed at the water surface by using light, fluorescent, polyethylene microspheres (COSPHERIC UVYPMS-0.97) as tracer particles. These particles have a density $\rho_i = 0.9800$ g cm$^{-3}$ and a diameter of 600–710 $\mu$m. They were illuminated by six pulsed ultraviolet/light-emitting diodes (UV-LED) lights (as shown in Figure 1), causing them to emit a yellow-green light (with wavelength $\approx 365$ nm). Calculations of the 2-D velocity field $\vec{u}v = (u, v)$ were performed using commercial software (PIVview3C), developed by PIVTEC. We used interrogation windows of 32 × 32 pixels with an overlap of 50%, which resulted in an average resolution of 7.8 mm. The method proposed by (Garcia, 2010, 2011) was used to replace the velocity outliers and smooth the velocity data to compute the vortex size. The smoothing parameter was chosen as $\sim 0.25$, without the “robust” option to avoid oversmoothing.

For the laser scanning measurements, a laser shining a horizontal light sheet was attached to a vertically moving linear motor outside of the tank to measure the region occupied by the particles in suspension. The laser illuminated horizontal cross sections of the tank at different depths. The laser moved
sinusoidally up and down with a period \( T = 1 \) s and amplitude \( L_{\text{motor}} = 15.2 \) cm. The laser's vertical movement did not cover the entire water column to avoid illuminating the tracer particles used for PIV measurements at the water surface. The cross-sectional area illuminated by the laser remained constant throughout all experiments. By combining each illuminated plane, a three-dimensional reconstruction of the regions of high particle suspension was obtained. Due to the movement of particles between each illuminated plane and the large amount of particles that were displaced, no PIV measurements using the sediment particles was possible.

To measure the region of suspended sediment as a function of the vortex characteristics, the velocity field of the flow and the volume occupied by the suspended sediment were measured in time for each experiment. We achieved this by illuminating and capturing the images used for PIV out of phase with those used by the volumetric reconstruction. Essentially, the cross sections of the volume occupied by the suspended sediment were recorded only when the tracer particles at the surface were not illuminated by the pulsed LEDs. Since the laser sheet illuminated continuously, a B+W F-Pro orange 550 filter was placed on the camera used for PIV measurements. This filter blocked the light from the laser, reduced the reflection of the UV-LEDs on the water surface, and increased the contrast between the background and the fluorescent particles emitting yellow-green light.

The synchronization of both cameras, the linear motor, and the UV-LED lights was done by using a PIC microcontroller (PIC16F877A), an infrared switch, and two controller boxes. At the start of each recording, the microcontroller is activated by the linear motor controller, sending a signal pulse to the first controller box. Once the signal is received, Camera 1 captures an image illuminated by the laser sheet. After a 20 μs delay, a pulse is sent from Controller Box 1 to Controller Box 2, activating the UV-LED lights and capturing an image for PIV measurements with Camera 2. Both the acquisition of images used for the PIV measurements and the volumetric reconstruction were fixed at 30 fps and were equidistant in time. This meant that 15 images were taken as the laser moved downward and 15 images as it moved upward. However, due to the acquisition time of the images, there is a higher resolution near the bottom and top of the water column (±\( L_{\text{motor}} \)) than in the middle (\( L_{\text{motor}}/2 \)). To ensure that the period \( T \) of the linear motor remained constant, a pulse is sent from a photo sensor each time the laser crossed \( L_{\text{motor}}/2 \). By using the time between each signal, \( T \) is estimated and corrected if necessary.

2.1.2 Bed Morphology Measurements

Measurements of the sediment bed height were obtained via a single-camera photogrammetric technique. The photogrammetric technique is described in detail by González-Vera, Wilting, et al. (2020). Hence, only a general description and the most relevant details will be given here. These measurements were performed only before the start and after the end of each experiment. The changes in the bed morphology due to the passing vortex were obtained by subtracting the almost flat reference height distribution before the start of the experiment from the bed height after the end of the experiment.

For these measurements, a digital projector with a maximum resolution of 1,920 × 1,080 pixels, was placed at a height \( H_P \sim 109 \) cm (from the bottom of the tank). The projector was set to an angle of either \( \theta \sim 21° \) or \( 25° \), which illuminated an area of approximately 0.73 × 0.375 m² downstream of the plate. The illuminated region inside the field of view of the camera defined the effective measurement area or region of interest for the measurements of the changes in the sediment bed.

The projector is set at an angle so that images projected on the sediment bed deform with changes in the bed. This deformation depends on the projector height and angle. Projected images contained between 50 and 60 elliptical patches over a black background. A light ray is defined for each projected dot, originating from the projector and intersecting the sediment bed, where a light patch is produced. The location and shape of these light patches depend on both the angle and position of the projector and the bed height. In our case, the size of
3. Vortex Generation and Characteristics

3.1. Vortex Generation

To generate a single columnar vortex, boundary layer separation was induced at the perpendicular plate by changing the velocity of the rotating table. As discussed by van Heijst et al. (1990), when the rotation rate of a fluid-filled tank in solid-body rotation is changed, a domain filling circulation is generated. Due to the presence of the plate perpendicular to one of the sidewalls, the circulation is deflected. This causes the boundary layer to detach at the sharp edge of the plate and roll up, producing an intense vortex on the leeside of the plate. This vortex generation method is reminiscent of the technique implemented by Meunier and Leweke (2005), Riedinger et al. (2010), and David et al. (2018), where a columnar vortex is generated by a single columnar vortex.

The velocity of the rotating table was changed as follows. First, the tank was spun up to a predetermined rotation rate \( \Omega_i < 0 \) in the clockwise direction. This was done very slowly so that the sediment bed was not disturbed. This rotation rate remained constant until the water reached a state of solid-body rotation. Then, the rotation rate of the table was suddenly changed to \( \Omega_f > 0 \) in the counterclockwise direction, producing the vortex on the leeside of the plate. After two seconds, the table was promptly stopped so that the final rotation rate \( \Omega_f = 0 \). This deceleration weakens the shear instability that develops at the edge of the plate. Since \( \Omega_f > \Omega_i \), a residual current remains, advecting the vortex away from the plate. Note that there is no background rotation once the vortex forms and propagates over the sediment bed since \( \Omega_i = 0 \).

We define two control parameters: the initial velocity change \( \Delta \Omega_i = \Omega_m - \Omega_i \) and the velocity difference between the initial and final rotation rate of the table \( \Delta \Omega_f = \Omega_f - \Omega_i = -\Omega_i \). The explored region of the parameter space is shown in Figure 3.

After the first velocity change, an Ekman boundary layer would be again expected to form. However, its formation time of \( O(1/\Omega_m) \) (Benton, 1966; Weidman, 1976) is in general large compared to the time before the next velocity change (2 s). Hence, the boundary layer still resembles the boundary layer under a flow in straight channel.

After the second velocity change, a large-scale circulation cell forms. The magnitude of the flow velocity of this cell is approximately \( r \Delta \Omega_2 \) with \( r \) the distance to the center of the tank (van Heijst et al., 1990). The
experiments are limited in the value of $\Delta \Omega_2$ so that the large-scale circulation does not disturb the sediment bed anywhere at any time. With the vortex located at $r \approx 250 \text{ mm}$, it can be later seen that the velocities associated to the vortex are at least 1 order of magnitude larger than those of the large recirculation cell. Since there is no background rotation, the boundary layer that develops under this large recirculation cell is a Bödewadt-type boundary layer. This type of boundary layer has a secondary motion with a radial velocity inward close to the bottom associated to them, and this makes them different to the boundary layer in a straight channel. The magnitude of the radial velocity is about $r \Delta \Omega_2/2$, hence also much smaller than the velocities associated to the vortex.

3.2. Vortex Characteristics

In the current paper, we determine the transport of sediment as a function of the measured characteristics of the vortex (radius $a$, strength $\Gamma$, and trajectory of the vortex) as it travels over the sediment. When conceiving the experiments, it was expected that $\Delta \Omega_1$ would control the strength of the vortex, and $\Delta \Omega_2$ would control the residual current and the propagation speed of the vortex. However, this was not the case, as is shown below. For this reason, it is necessary to compute the vortex characteristics for each combination of the control parameters ($\Delta \Omega_1, \Delta \Omega_2$).

For all experiments, a well-defined vortex consistently formed at the tip of the plate. An example of the vertical vorticity field $\omega(x, y)$ is shown in Figure 4a. Due to a shear instability, a number of satellite vortices also formed around the main vortex for high values of $\Delta \Omega_1$. However, these satellite vortices were much weaker and seemed to decay much faster than the primary vortex. To distinguish the primary vortex signal from the satellite vortices, shear regions, and background noise, we used the Okubo-Weiss parameter

$$\text{OW} = \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 - \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)^2 = S_n^2 + S_s^2 - \omega^2; \quad (3)$$

where $S_n$ and $S_s$ are the normal and shear component of strain respectively (Okubo, 1970; Weiss, 1991). This parameter compares strain and vorticity, allowing the separation of the flow into vorticity-dominated regions ($\text{OW} < 0$) and strain-dominated regions ($\text{OW} > 0$). By discarding $\text{OW}$ values above a threshold value equal to 10% of the minimum value of $\text{OW}$, the primary vortex was identified since the lowest value of $\text{OW}$ is close to the vortex center.
The characteristics of each vortex were obtained by first locating the center of the vortex. Then, we computed its radial vorticity distribution, as shown for one example in Figure 4b. This radial vorticity distribution is well approximated by a Gaussian (or Lamb-Oseen) vortex profile given by

$$\omega(r) = \omega_0 \exp\left(-r^2/a^2\right),$$  \hspace{1cm} (4)

where $\omega_0$ is the maximum vorticity and $r$ is the radial distance from the center of the vortex. By doing a fit, the radius of the vortex was estimated at each instant. In Figure 4b, the scatter at $r \approx 10$ cm is due to the satellite vortices. To compute the vortex strength, the circulation $\Gamma$ of the vortex was calculated by the line integral:

$$\Gamma(t) = \oint_{C} \mathbf{v} \cdot d\mathbf{l},$$  \hspace{1cm} (5)

where $C$ is a closed curve encapsulating the vortex and $\mathbf{v}$ is the 2-D vector velocity field of the fluid. The contour $C$ is a square with side $4a$ at each instant. Both the strength and the radius of the vortex change in time. For this reason, we have chosen to characterize the vortex using the maximum strength $\Gamma^*$ and the radius of the vortex $a^*$ when this maximum strength is reached. Lastly, to determine the initial propagation velocity of the vortex, the center of the vortex is obtained for each PIV image pair. We define an average initial propagation velocity of the vortex $U$ by performing a linear fit of the distance traveled by the vortex in the $x$ direction for $t < 15$ s. At the start of the experiments, a constant propagation velocity was observed for about $t \lesssim 20$ s. Hence, the choice of 15 s or a shorter time has little to no influence on the results.

Figure 5 shows the maximum strength $\Gamma^*$ as a function of a linear combination of both $\Delta\Omega_1$ and $\Delta\Omega_2$, which leads to a coefficient of determination $R^2 = 0.93$. It can be seen here that both control parameters are important in determining the strength of the vortex and that a linear relationship is a good approximation. In the multivariate linear regression, the value of $C_\Gamma$ is different from 0 since a minimum $\Delta\Omega_1$ value is required for the boundary layer to separate at the plate and generate a vortex. Independently of the presence of sediment, all vortices generated show the same relation between $\Gamma^*$ and $(\Delta\Omega_1, \Delta\Omega_2)$. Overall, a comparison between experiments with and without sediment show differences in $\Gamma^*$ of about 5% to 15%. However, the weakest vortices or those with the lowest $\Delta\Omega_2$ values show larger differences in $\Gamma^*$. These differences exceed 50% in the weakest vortices since the signal-to-noise ratio is larger.

The vortex radius $a$ did not show a clear relationship with neither $\Delta\Omega_1$, $\Delta\Omega_2$, nor with a linear combination of both. This might be explained in part by the temporal variability being larger than the effect of changing the values of the control parameters. For all the experiments, the average value of the characteristic radius $a^*$ is 27 mm with a standard deviation of 5 mm.

For the vortex trajectories, we observed three distinct behaviors which depend on the value of $\Delta\Omega_2$. One example of each of these behaviors is shown in Figure 6 for experiments with $\Delta\Omega_1 = 0.15$ rad s$^{-1}$. For small values of $\Delta\Omega_2$ (e.g., $\Delta\Omega_2 = 0.025$ rad s$^{-1}$), the vortices initially travel to the right (negative $y$ direction). This is explained by the formation of a small stopping vortex with opposite (negative) vorticity that joins the main vortex to form an asymmetric dipolar structure. Due to the larger strength of the main vortex, this dipole then turns and propagates in the positive $x$ direction. For large $\Delta\Omega_2$ values ($\Delta\Omega_2 \geq 0.07$ rad s$^{-1}$), the vortices are advected by the current and travel initially to the left (negative $y$ direction). For intermediate $\Delta\Omega_2$ values, there is a combination of both advection mechanisms, and the vortex initially travels in the $x$ direction. For early times, these three behaviors are independent of the type and presence of sediment.
Figure 7 shows the initial velocity of the vortex in the $x$ direction $U_x$ as a function of $\Delta \Omega_2$. For $\Omega_2 = 0.025$ rad s$^{-1}$, the speed increases as a function of $\Delta \Omega_1$ because velocity of the dipolar structure is proportional to its strength. For $\Delta \Omega_2 = 0.075$ rad s$^{-1}$, the speed of the vortex is independent of $\Delta \Omega_1$. It is only

Figure 8. Photographs of the particle suspension produced by a vortex for an experiment with $\Delta \Omega_1 = 0.15$ rad s$^{-1}$ and $\Delta \Omega_2 = 0.10$ rad s$^{-1}$. To take the photographs, no measurements were taken. Fluorescent particles used in the PIV measurements are observed on the water surface, at 20 cm from the sediment bed. (a) Photograph of the initial shape of the region of particle suspension, taken at $t = 3$ s. (b) Photograph of the fully developed region of particle suspension taken at $t = 10$ s. The plate is on the right side of the picture, and the vortices are propagating from right to left.

Figure 9. Example of the reconstructed volume occupied by the suspension of particles produced by a vortex with $\Delta \Omega_1 = 0.12$ rad s$^{-1}$ and $\Delta \Omega_2 = 0.09$ rad s$^{-1}$ ($\Gamma^* = 1.6 \times 10^4$ mm$^2$ s$^{-1}$, $U = 10.47$ mm s$^{-1}$) at $t = 7.53$ s. The vorticity field of the flow is superimposed at the height of the water surface. The colors of the contours indicate the vorticity magnitude.
from this point on that our initial assumption that the propagation speed of the vortex is due to the background current and is governed exclusively by \( \Delta \Omega_2 \). Clearly for intermediate \( \Delta \Omega_2 \) values, the speed still depends on \( \Delta \Omega_1 \) but not as strongly as for smaller values.

Just as in the case of \( \Gamma^* \), the propagation velocity of the vortex is similar between experiments with and without sediment, having a 5% to 15% difference, except for weak vortices or low \( \Delta \Omega_2 \) values. We conclude that, in general, the presence of sediment does not influence the initial characteristics of the vortex.

4. Sediment Transport

4.1. Sediment in Suspension

Once the vortex reaches the sediment bed, particles under the vortex are set into motion if the vortex is strong enough. These particles follow a spiral trajectory as they approach the vortex core. During the particle convergence, small bands appear in the sediment bed due to the swirling motion but vanish as more and more particles accumulate. Once particles reach the core of the vortex, a significant amount of them move upward driven by the vertical flow velocities associated to the secondary motion. Close to the bed, we also observed turbulent motion which is known to be able to bring particles in suspension due to turbulent diffusion; see, for example, Bennett and Best (1995), Noh and Fernando (1991), and Yang and Shy (2003) for generic situations and González-Vera et al. (2018) for the comparable situation of a spin-down flow. This forms a region of high concentration of suspended particles below the vortex core, as seen in Figure 8. Helical concentration bands form as the vortex captures more particles. These bands are more visible for stronger vortices. As more particles are trapped, the region of suspended particles takes a conical shape, as shown in Figure 8b.

To estimate the response of the particles to the flow, we defined the Stokes number following IJzermans and Hagmeijer (2006),

**Figure 10.** Maximum (a) bottom area \( A_s^* \), and (b) volume \( V_s^* \) of the region occupied by the suspended particles as a function \( \Gamma^* \). The black line represents a linear fit. The coefficient of determination for each fit is \( R^2 = 0.83 \) and \( 0.86 \), respectively.

**Figure 11.** Typical settling time of the particles: the time that it takes for the volume of the cone to decrease from its maximum value to 25% of it.
which represents the ratio of the characteristic time of the particles (defined in Equation 2) to a characteristic time of the vortex. For the PS particles \( St = [0.12, 0.54] \), while for the PMMA particles \( St = [0.14, 0.21] \). For both particle types, the values of the Stokes number are similar and smaller than unity, meaning that both types of particles are expected to follow the vortex well.

Figure 9 shows an example of a reconstruction of the region of suspended particles using the scanning laser together with the vorticity field obtained with PIV. From this reconstruction, it is possible to calculate, for example, the volume occupied by the suspended particles, the height that they reach, and the area of the bed where particles are in suspension. Furthermore, Figure 9 shows that the cone of suspended particles is aligned with the vortex.

Figure 10 shows the maximum bottom area \( A_s^* \) and volume \( V_s^* \) of the region occupied by the suspended sediment as a function of the maximum vortex strength \( \Gamma^* \) for all experiments. No clear distinction is observed between the two types of particles, probably because of the comparable settling velocity \( w_s \), relaxation time \( t_0 \). For both the bottom area and the volume, a clear linear dependence with \( \Gamma^* \) is observed. The respective fits have a coefficient of determination \( R^2 = 0.83 \) and 0.86. These results suggest that the strength of the vortex determines the volume occupied by the particles in suspension, provided that the particle settling velocity \( w_s \) is similar and the Stokes number is small.

The magnitude of the bottom shear stress exerted on the bed, the upward flow lifting particles, and the turbulent intensity decrease as the vortex weakens. Consequently, the suspension of new particles ceases, and those particles already in suspension begin to settle. This leads to the flattening of the cone of suspended particles as it moves with the vortex until, finally, all particles settle. To estimate a time scale \( T_V \) for the settling of the particles, we measure the time it takes for the volume of the cone to decrease from its maximum value to 25% of it. The limit of 25% was chosen because, as will be discussed in detail in section 4.2, a mound of sediment forms at the bed as the sediment deposits, and it is difficult for some experiments to differentiate if the sediment is still in suspension or is already part of the bed. As shown in Figure 11, the settling time \( T_V \) is larger for stronger vortices. For two extreme weak cases, the shortest times are in the order of 5 s, but for most vortices, the settling time lies between 15 and 40 s. As a reference, 40 s is about double the time that it would take a PS particle to fall freely through the whole water depth of 20 cm. This implies that the flow slows down the settling of the particles, even as the vortex weakens.

### 4.2. Changes in the Sediment Bed

We distinguish changes in the sediment bed due to the vortex strength and due to the vortex propagation. The role of the vortex strength is exemplified by comparing the effect on the sediment bed of two vortices with similar trajectories and propagation speeds but different values of \( \Gamma^* \), as shown in Figure 12. For these two examples, \( \Delta\Omega_2 = 0.75 \text{ rad s}^{-1} \) so that the vortices are advected by the background current at approximately the same
speed. However, the values of $\Delta \Omega_1$ differ so that they have a different strength. For both experiments, there is a region of net erosion close to the plate where the vortex forms and reaches its maximum strength. The trajectories of the two vortices are very similar at the beginning. Both vortices travel in a diagonal in the positive $x$ and $y$ directions until they reach the position $(x,y)=(160,80)$, and then, they turn to the right and start traveling in the negative $y$ direction while moving away from the plate in the $x$ direction. It is around the point where the vortices change direction that we observe a region of deposition in the bed. As the vortex travels and decays, particles settle, creating this region of net deposition. Differences in the size and height of the regions of erosion and deposition are evident, with the largest displaced particle volume for the strongest vortex. This is to be expected already from the results shown in the previous section, where it was shown that $\Gamma^*$ governs the volume occupied by the sediment in suspension. However, this volume is not equivalent to the net volume of sediment displaced at the bed.

The erosion rates and bed load transport rates of sediment are commonly modeled as power laws of the shear stress or the flow velocity (see, e.g., Van Rijn, 1984, for a comprehensive description). The erosion rate of sediment $E$ can be modeled, for example, using the Partheniades formulation (van Rijn, 1993; van Kessel et al., 2011):

$$ E \propto (|\tau| - \tau_{cr})^n, $$

where $\tau$ is the shear stress at the bottom, $\tau_{cr}$ is the critical shear for the initiation of motion, and $n$ is an exponent that is usually taken equal to $3/2$ for noncohesive sediment (van Rijn, 1993). If we consider that the boundary layer under the vortex is a Bödewadt-type boundary layer, the shear stress at the bottom is related to the circulation such that $|\tau| \propto \Gamma^{3/2}$ (González-Vera et al., 2018). Since the evolution of the vortices is similar during the time that they erode the bed, it is to be expected that the volume of sediment eroded is

$$ V_D = M(\Gamma^* - \Gamma_{cr})^m, $$

where $\Gamma_{cr}$ is the smallest value of the circulation at which erosion occurs and $m = 9/4 = 2.25$.

To test the validity of Equation 8 and the value of the exponent $m$, Figure 13 shows $(V_D/M)^{4/9}$ as a function of $(\Gamma^* - \Gamma_{cr})$. The values of $M$ and $\Gamma_{cr}$ were obtained through a linear regression of $V_D^{4/9}$ as a function of $M^{4/9}(\Gamma^* - \Gamma_{cr})$. This resulted in $M = (3.7 \pm 1.0) \times 10^{-4} \text{ s}^{9/4} \text{ cm}^{-3/2}$ and $\Gamma_{cr} = (2.7 \pm 16.4) \text{ cm}^2 \text{ s}^{-1}$ for the lighter and larger PS particles, and $M = (1.8 \pm 0.5) \times 10^{-4} \text{ s}^{9/4} \text{ cm}^{-3/2}$ and $\Gamma_{cr} = (37 \pm 22) \text{ cm}^2 \text{ s}^{-1}$ for the heavier and smaller PMMA particles. A collapse of the results for both types of particles and a good agreement with Equation 8 is observed in Figure 13. The difference in the value of $M$ suggests that the PS particles are easier to transport than the PMMA particles. Here we have fixed the value of the exponent $m$. It is then reasonable to ask if other values of the exponent $m$ would yield similarly good results. There is only a narrow range of values for $m$ where physically meaningful results are obtained. For $m \geq 2.5$, the obtained values of $\Gamma_{cr}$ become negative for the PS particles, and for $m \leq 1.5$ the value of $\Gamma_{cr}$ becomes larger than the minimum value of $\Gamma^*$ at which erosion is observed.

It can be seen that the PMMA particles are more difficult to transport because the critical circulation for erosion is larger and the value of the erosion coefficient $M$ is smaller. The clear difference in the value of $M$ indicates that, for the same value of $\Gamma^* - \Gamma_{cr}$, the vortices are about 2 times less efficient in displacing the smaller,

![Figure 14](image-url) Changes in the particle bed for three experiments with different $\Delta \Omega_2$ values but with $\Gamma^* = 1.6 \times 10^4 \text{ mm}^2 \text{ s}^{-1}$. (a) Experiment with $\Delta \Omega_1 = 0.017 \text{ rad} \text{ s}^{-1}$, $\Delta \Omega_2 = 0.025 \text{ rad} \text{ s}^{-1}$ and $\Gamma^* = 1.59 \times 10^4 \text{ mm}^2 \text{ s}^{-1}$ ($U_x = 11.1 \text{ mm} \text{ s}^{-1}$), (b) $\Delta \Omega_1 = 0.136 \text{ rad} \text{ s}^{-1}$, $\Delta \Omega_2 = 0.068 \text{ rad} \text{ s}^{-1}$ and $\Gamma^* = 1.57 \times 10^4 \text{ mm}^2 \text{ s}^{-1}$ ($U_x = 8 \text{ mm} \text{ s}^{-1}$), and (c) $\Delta \Omega_1 = 0.12 \text{ rad} \text{ s}^{-1}$, $\Delta \Omega_2 = 0.09 \text{ rad} \text{ s}^{-1}$ and $\Gamma^* = 1.6 \times 10^4 \text{ mm}^2 \text{ s}^{-1}$ ($U_x = 10.5 \text{ mm} \text{ s}^{-1}$). Positive values indicate the area of deposition, and negative values indicate the area of erosion.
heavier PMMA particles. Since the volume with particles in suspension is similar for both types of particles, this means that the concentration of PMMA particles in suspension is lower than that of PS particles.

Next, the effect of the trajectory of the vortex on the sediment bed is exemplified by comparing experiments with the same strength (i.e., $\Gamma^*$ value) but different trajectories, as shown in Figure 14. For all cases, there is an erosion region close to the plate followed by a deposition region along the trajectory of the vortex. For the experiment with smallest $\Delta\Omega$ value, for which a dipole forms, the vortex first moves quickly in the negative $y$ direction, and then turns to propagate in the in a straight line in the positive $x$ direction. Then it stops for some time while most of sediment is deposited. For the other two experiments, the vortex is initially taken by the current in the positive $x$ and $y$ directions. After $x \approx 100$ mm, the sediment starts to be deposited in the bed until $x \approx 200$ mm. Within this area of sediment deposition, the vortex changes direction and starts to travel in a diagonal but now in the negative $y$ direction. With the exception of two extreme cases, the distance traveled by the vortex within the time scale $T_i$, $L = U_x T_i$, is much larger than the radius of the vortex, as shown in Figure 15. This explains the fact that the areas of the bed affected by the vortex tend to be long and that we can clearly distinguish erosion and deposition areas. For much slower translation velocities compared to the settling speed of sediment, sediment could deposit back on the same place where it was eroded from.

Despite the variation in the trajectories of different vortices and different particle types, a clear change in the direction of movement of the vortex consistently occurred around the largest peak in the area of deposition. While the changes in the bathymetry can affect the vortex (see, e.g., Zavala Sansón et al., 2012), it is unknown what the minimum size of the deposition mound is to have a significant effect on a vortex trajectory. Furthermore, following classical explanations of conservation of vorticity (Pedlosky, 1987), changes in the height of the sediment bed can decrease or increase the vorticity of the vortex due to vortex stretching. This can lead to the increase of the vertical velocity which in turn can enhance particle suspension. Such a mechanism can be an explanation for the few cases where the vortex resuspended a small amount of particles when the peak suspended sediment began to decrease.

It is unclear if the changes in the direction of the vortex trajectory around the location of the largest peak in the sediment bed are due to the changes in the bed or the particle loading, or if it is independent of the sediment and the maximum in the deposition area is a result of the vortex stopping or changing direction. When

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**Figure 15.** Ratio between the time required for each vortex to travel the distance equivalent to its radius ($a^*/U_x$) and the effective settling time of the volume ($T_i$) of the region occupied by the suspended particles as a function of $\Gamma^*$.

**Figure 16.** Comparison of the vortex trajectories between experiments without sediment and those with the two different sediment types used, PS and PMMA particles. Two different cases are shown: (a) $\Delta\Omega_1 = 0.088$ rad s$^{-1}$ with $\Delta\Omega_2 = 0.07$ rad s$^{-1}$ and (b) $\Delta\Omega_1 = 0.15$ rad s$^{-1}$ with $\Delta\Omega_2 = 0.025$ rad s$^{-1}$.
comparing with experiments without sediment, vortices present similar trajectories with a change in direction, as shown in Figure 16a. This similarity suggests that the peak in the sediment is rather a result of the vortex trajectory. However, as the value of $\Delta \Omega_1$ increases, the differences in trajectory between vortices over a sediment bed and those over a solid bottom increase, as shown in Figure 16b.

The volume of PMMA particles displaced is about 1 order of magnitude smaller than that of PS particles. It would then be expected that the largest differences in trajectory would be observed between experiments with PS particles and without particles. However, this is not always the case, as seen in Figure 16b. Since this difference occurs for the strongest vortices, other nonlinear mechanisms might overshadow the effect of sediment. More experiments are necessary to ascertain the effect of the sediment on the vortex. In other words, statistics on the deviations from a mean or typical vortex behavior should be obtained.

5. Discussion and Conclusion

The effect of a translating single vortex on a particle bed was studied experimentally. PIV was used to measure the flow velocities at the water surface to characterize and follow the vortex, while a vertically moving laser sheet was used to obtain horizontal cross sections of the region occupied by particles in suspension. Lastly, changes in the particle bed were measured using a single-camera photogrammetric technique. By combining these three distinct measurement techniques, insight was gained into the sediment transport processes that occur due to the vortex: their suspension, their transport and the net changes in the sediment bed.

The quantities characterizing the region occupied by suspended particles were primarily governed by the vortex strength ($\Gamma^*$). The two types of particles used in the experiments had similar settling velocities, which meant that no discernible difference was observed in the volume occupied by the particles in suspension. In particular, the maximum volume and bottom area of the region occupied by the suspended particles appears to scale linearly with $\Gamma^*$. The volume of particles displaced was also primarily governed by the vortex strength showing a power law relationship. Although the values of the exponent are in agreement with classical descriptions, such sediment transport is solely explained by the horizontal shear stress exerted at the sediment bed. However, particles are then transported due to the the radially inward secondary motion toward the vortex core, where significant upward flow velocities occur. As discussed by Alfadhli et al. (2014), vertical velocities influence the particle transport rate in sediment beds due to the changes in critical shear stress, increasing the transport if upward and decreasing it if downward. Therefore, in contrast with studies of the radial exerted shear stress at the bottom boundary layer of a rotating flow (Caps & Vandewalle, 2003; González-Vera et al., 2018; Thomas, 1994; Thomas & Zoueshtiagh, 2007), the vertical velocity induced by the pressure gradient effect (as described by Greeley et al., 1981; Greeley & Iversen, 1987) could significantly enhance the suspension of particles. The results in this paper suggest that the difference in the transport of sediment by vortices as compared to that of a plane channel flows might be modeled by a larger erosion coefficient or a smaller critical shear stress.

The measurements of the vortex and the sediment bed reveal that the changes in the morphology occur primarily along the trajectory of the vortex. It can be understood that the main displacement of particles is localized under the vortex translating path, with a smaller portion occurring due to dispersion. The distance the particles are transported is found to be related to velocity of the vortex since it will determine the distance traveled before particles settle. However, the trajectories are complex, showing drastic changes in direction and speed. In some cases, the vortex even remains in a single place for some time before resuming its translation.

Even though the effect of vortices on the sea floor has been previously observed (see, e.g., Berthot & Pattiaratchi, 2006; Takasugi et al., 1994), how the results reported in the current paper translate to large scales in the ocean and along the coast requires further research. On the one hand, such large-scale vortices tend to be much shallower with the ratio of the depth to the radius of the vortex much smaller than one. On the other hand, large-scale vortices are associated to much stronger turbulence. For shallower vortices, the vertical upward flow is expected to be weaker (see, e.g., Duran-Matute et al., 2010), but it should remain a relevant mechanism as long as the flow velocities are larger than the settling velocity of the particles. However, for these larger and shallower vortices, turbulent diffusion should play a larger role in the upward transport of sediment. Even if the relative importance of the mechanism responsible for bringing the sediment high in the water column is different for large-scale vortices, the experiments described in the
current paper show the large and localized effect that vortices can have on the transport of sediment. This effect could be significant for the morphology evolution in coastal areas where vortices tend to form.

Data Availability Statement

Data sets for this research are available from González-Vera, Duran-Matute, and van Heijst (2020) under a Creative Commons BY-NC license.

Acknowledgments

This study was financially supported by CONACYT (Mexico) through a scholarship grant for A. S. G. V. (383903) and by the NWO/VENI (the Netherlands) grant of M. D. M. (863.13.022).

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