Reliability-based optimization of maintenance scheduling of mechanical components under fatigue

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ABSTRACT

This study presents the optimization of the maintenance scheduling of mechanical components under fatigue loading. The cracks of damaged structures may be detected during non-destructive inspection and subsequently repaired. Fatigue crack initiation and growth show inherent variability, and as well the outcome of inspection activities. The problem is addressed under the framework of reliability based optimization. The initiation and propagation of fatigue cracks are efficiently modeled using cohesive zone elements. The applicability of the method is demonstrated by a numerical example, which involves a plate with two holes subject to alternating stress.

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1. Introduction

The application of alternating loading to metallic components may lead to fatigue failure. One or several fatigue cracks initiate and grow within the structure, and finally lead to loss of serviceability or eventually to structural collapse. The occurrence of initial crack, the initiation and propagation of fatigue cracks is a highly uncertain phenomenon [1] and thus, must be addressed within an appropriate concept that accounts for this uncertainty [2–4]. In particular, the effects of uncertainty can be quantified in terms of structural reliability. As cracks develop and grow during the life time of a structure, a time variant decay of the reliability is to be expected. The harmful effects of propagating cracks can be avoided by scheduling maintenance activities [5,6]. The scheduling of these activities involves selecting a crack detection technique (e.g. visual inspection, ultrasonic methods, etc.) and an inspection periodicity (e.g. monthly, annual inspection, etc.) [7–9]. Among different inspection approaches, Non Destructive Inspection (NDI) techniques play a fundamental role. However, these techniques can fail in detecting cracks. Thus, they are characterized by the probability of detection, which depends on the crack length (see e.g. [10]).

Maintenance activities are necessary to ensure sufficient reliability. However, such activities contribute significantly to the costs associated with the operation of the structure [11]. The best maintenance schedule can be interpreted as a trade off between the costs related to the inspection and repair activities and the level of reliability (see e.g. [12–14]). The high level of uncertainties inherent in the fatigue strength of the material and in the outcome of non-destructive inspection entails the use of reliability based optimization in order to identify an adequate maintenance scheduling [9,15,7,13,5,16].

Most contributions on this area apply the so-called First Order Reliability Method, see e.g. [9,7,13,17,18]. However, the First Order Reliability Method may be inaccurate in case the performance function is strongly nonlinear or in case it is a high dimensional problem [19,20]. In this contribution, the evaluation of the reliability is performed by means of advanced simulation methods, in particular, by means of Subset Simulation [21]. Advanced simulation methods have been successfully applied in structural dynamics and stochastic finite elements (see, e.g. [22]) and to fatigue analysis [23].

In this work, a numerical strategy for designing an optimal maintenance scheduling for a structure, accounting explicitly for the effects of uncertainty is suggested. This contribution, which can be regarded as an extension of the methods developed in [23], presents several novel aspects over similar approaches proposed in the literature. Firstly, the initiation and propagation of fatigue crack is modeled efficiently by means of cohesive zone elements [24–26]. The application of this class of elements allows modeling the crack initiation and propagation within a unified framework. It should be noted that cohesive zone elements have already been used for uncertainty quantification of the crack propagation phenomenon [27,28]. However its application within the
context of maintenance scheduling constitutes a novelty. The second innovative aspect of this contribution refers to the assessment of the reliability sensitivity with respect to the variables that define the maintenance scheduling. The estimation of this sensitivity, which is required in order to determine the optimal maintenance schedule within the proposed framework, can be quite demanding as the model characterizing repair of a cracked structure leads to a discontinuous performance function associated with the failure probability. A new approach for modeling this function is proposed herein. The continuous and discontinuous parts respectively of the function are considered separately to estimate accurately the gradients of the failure events.

This manuscript is organized as follows. Section 2 presents respectively the mechanical model, the definition of the performance function and of the objective function. In Section 3, the numerical methods used in this study are described. The implementation of a formulation of a cohesive zone element is proposed. Meta-models are used to reduce the computational time. Numerical methods to estimate reliability and its sensitivity are discussed. The methods developed in this study are then applied to a numerical example, which is described in Section 4.

2. Description of the problem

2.1. Crack propagation phenomenon

Mechanical components may deteriorate under cyclic loadings. One or several cracks may initiate and propagate through the structure, leading to an eventual structural failure of the component, or to a loss of serviceability. The fatigue life is characterized by three different stages: fatigue crack initiation, stable crack growth and unstable crack growth. During the crack initiation stage, damage accumulates at the microscopic level. In the case of a metallic material, one or several micro-cracks initiate at stress concentration points or at the defects of the material (inclusions, grain boundaries, etc.). These micro-cracks progressively grow and coalesce until a macroscopic crack appears. The crack initiation is strongly affected by the micro-structural parameters (size of the inclusions or grain orientation at the stress concentration, etc.) [29]. Thus the time to crack initiation depends on parameters that cannot be fully controlled at the macroscopic level and can be modeled as an uncertain process [30,31].

The propagation stage is first characterized by stable crack growth. The crack length increases progressively during the fatigue life, and the crack partially propagates through the cross-section of a structural component. The crack propagation stage is also an uncertain process since it is influenced by the microscopic structure. Once the cracks reach a critical size, the cross section of the structure is so reduced that it can no longer sustain the applied load. The structure is partially or fully destroyed by brittle failure or ductile collapse.

The most widely used model to predict fatigue crack growth is expressed by the Paris–Erdogan equation [32] or any of its further implementations (see e.g. [33,34]). They consist of a phenomenological relation between the crack growth rate and the stress intensity factor range. Numerical methods have been developed in order to determine the stress intensity factor of complex structures incorporating one or several cracks, such as the extended finite element method [35]. This method can be used in combination with the Paris–Erdogan equation to model fatigue crack growth (see for instance [36]). However, specific requirements have to be met to ensure that Paris–Erdogan equation is predictive. The crack must exhibit a certain minimum initial length and the yielding at the crack tip should not be excessive. However, these conditions do not apply to most engineering structures.

Cohesive zone elements are an alternative method to account for crack growth by means of finite element simulation. Such models have been pioneered by Dugdale [37] and Barrenblatt [38]. They consist of zero-thickness elements that are inserted between the bulk elements and account for the resistance to crack opening by means of a dedicated traction-displacement law. This cohesive force dissipates, at least partially, the energy related to crack formation.

Unfortunately, the cohesive zone elements as described above are not suitable for modeling fatigue crack growth. In such cases, the stiffness of the cohesive elements does no longer evolve after few cycles, leading to crack arrest (i.e. the crack length is no longer increasing). Nguyen et al. [25] extended the cohesive law to include fatigue crack growth, which is modeled by the means of a deterioration of the material properties at each cycle. During the unloading–reloading process, the cohesive law shows a hysteresis loop, the slight decay of the stiffness simulates fatigue crack propagation. Such cohesive elements account for both the crack initiation and the crack propagation, respectively. In view of the above discussion, the crack growth phenomenon is modeled in this contribution using cohesive zone elements. The uncertainties inherent in the fatigue crack initiation and propagation are modeled by means of random variables (grouped in a vector \( \theta \)) for the material parameters of the cohesive zone elements. Thus, the uncertainty in these material parameters propagates to the crack initiation and propagation phenomena. Details on the implementation of this model are discussed in Section 3.

2.2. Modeling of non-destructive inspection

The deterioration of mechanical components subject to fatigue leads to a decrease of reliability. In order to ensure sufficient reliability during lifetime, two different strategies may be adopted [6].

- The reliability completely relies on the design of the structure, involving appropriate sizing of the components, quality assurance of the parts during manufacturing or with the use of conservative safety factors.
- Sufficient reliability is maintained by a program of periodic inspections, which allows to assess the service conditions of the structure. The damaged components can be replaced, repaired or strengthened when necessary, which guarantees an extended service life or a less costly design.

The selection of one of these strategies depends on the service conditions and on costs considerations. The second strategy is suitable for components that can be easily accessed and replaced. Furthermore, the application of this second strategy requires the definition of a particular inspection technique and also its periodicity. Several inspection techniques are available to evaluate the degradation of aging structures. The most common ones are visual inspection, penetrant inspection, eddy current, radiographic inspection, ultrasonic inspection, etc. [10,6]. Each method shows particular benefits and also drawbacks. For instance visual inspection can be easily performed and does not require many tools, however it relies mainly on the skills of the inspector and hence human errors cannot be avoided. Radiographic inspection can efficiently detect cracks with a low risk of error, but this method requires costly equipment investments.

All the non-destructive inspection techniques show variability in their outcome. The results are affected by the conditions of inspections, e.g. the flaw size, the geometry of the structure, the particular inspection technique, the inspectors skills, etc. The uncertainties inherent in the non-destructive inspection techniques can be modeled within the framework of probabilistic
approaches, see e.g. [6]. In particular, the probability of detection of a crack can be regarded as dependent on the inspection technique and on the flaw size [39]. Several formulations of the probability of detection function have been proposed [40,41]. Typically, the probability of detection increases with the crack length and reaches a limit value, which might be less than one, for instance because of human errors during the non-destructive inspection.

In this contribution, the probability of detection is modeled with an exponential distribution [39]. The parameter of the distribution is assumed to depend on the respective quality of inspection:

$$POD(l(t, \theta), q) = 1 - \exp(-q \cdot l(t, \theta)),$$

where $POD$ denotes the probability of detection, $l(t, \theta)$ denotes the crack length (that depends on time $t$ and the vector of random variables $\theta$) and $q$ is the scalar value modeling the quality of inspection. Using Eq. (1), the probability of detection of short cracks is very low, and increases as the crack length $l(t, \theta)$ increases. The parameter $q$ describes the characteristics of the non-destructive inspection. As a matter of fact, an increase of the value of the parameter $q$ corresponds to increased chances of detecting a crack of a given length. For instance, in case $q = 1 \text{ mm}^{-1}$, the probability of detecting a crack with a length of 1 mm is approx. 63%. In case $q = 5 \text{ mm}^{-1}$, the probability of detecting a crack with the identical length is approx. 99%. Hence, in this model, selection of the value of $q$ is regarded as equivalent to choosing a particular inspection technique.

The crack length estimated during the non-destructive inspection is affected by sizing errors, i.e. the measured crack size is different from its true size. Several sources of uncertainties may cause the measurement errors (see e.g. [42]): the lack of repeatability of the inspection procedure, an inadequate calibration of the measurement device, the geometry of the flaw, the influence of the temperature and humidity, etc. The error in sizing is modeled using a Gaussian distribution [6], the measured crack length is the sum of the actual crack length and the sizing error.

### 2.3. Life-time events and effects of maintenance

During the lifetime of a structure, different events may occur, i.e. due to crack growth, inspection and eventual repair may be performed. In order to illustrate these events, assume a structure with a single crack where inspection is performed at the time $t_I$ and the critical crack length (i.e. the crack length at which the structure collapses) is denoted as $l_c$. Thus, the following sequences of events, illustrated schematically in Fig. 1, may occur.

- Fracture occurs before inspection (case 1 in Fig. 1).
- In case the structure has not failed before time $t_f$, non-destructive inspection is performed. Two situations may occur depending on the outcome of the inspection:
  1. The structure is not repaired, either because the structure is not jeopardized by the level of damage or because of detection errors (case 2 in Fig. 1). Fracture may or may not occur before the end of the service life at a time $t > t_f$. For case 2 illustrated in Fig. 1, fracture does not occur.
  2. In case the structure is repaired (case 3 in Fig. 1), imperfect removal is considered (i.e. another crack may initiate and grow at the same location). As previously, the crack may lead to failure before the end of the service life or the structure may survive.

The specific repair activity to be performed on a structure is problem dependent. For example, the cracked parts of a system can be replaced. In metallic structures, another possible strategy consists of welding the cracks [43]. Alternatively, a patch can be applied to the structure [44], which consists of a metallic or composite plate glued on the damaged area and which partially carries the load.

Fig. 2 summarizes the events which may happen during the service life of a structure. The repair event and the fracture event are not fully correlated. For instance, the structure may fail before the end of the service life, even though it has been repaired. Indeed, imperfect removal is considered (i.e. a crack may initiate and propagate after repair) and multiple site damage may happen (i.e. several cracks may propagate through the structure and some of them may not be repaired). The structure can as well be safe even though it has not been repaired during its service life.

![Fig. 1. Aspect of the evolution of the crack length, with a maintenance operation.](image1)

![Fig. 2. Event tree associated for the service life of a component, with a single inspection time.](image2)
According to the event tree illustrated in Fig. 2, two notable events may take place.

- The structure may not be repaired during the maintenance activities, leading to failure before the end of the service life. Such event is caused for instance by the uncertainties inherent in the non-destructive inspection (i.e., the probability of detecting a crack is not equal to one), or by inadequate scheduling of the maintenance activities (i.e., if the maintenance activities are planned too early in the service life, the cracks may be too short to be detected).

- The structure may be repaired although it is not required (i.e., the structure does not fail during its service life without maintenance activities). This situation may be caused by a too conservative maintenance scheme.

The two events described above constitute outcomes that are undesirable. This is because the first event implies failure even though efforts on inspection are being performed while the second event implies an unnecessary repair effort.

As an additional remark, it should be noted that due to the effects of repair, a crack can be removed. Thus, the crack length \( l \) does not depend solely on time \( t \) and the random variables \( \theta \) associated with the crack propagation process but also on the time of inspection \( t \) and the quality of inspection \( q \). Thus, \( l = l(t, x, \theta) \), where \( x = (q, t)^T \).

2.3.1. Formulation of the performance functions

In order to characterize the occurrence of the repair and failure events for structural reliability analysis, the so-called performance function is defined with respect to the random variables. The value of this function is less than or equal to zero for those realizations of \( \theta \) that cause the event of interest (either repair or failure) and larger than zero otherwise.

The performance function is frequently expressed as the difference between the capacity and the demand functions (see for instance [21]). Following [21,23], the performance functions are defined as the difference between a normalized capacity and a normalized demand, as shown in Eq. (2). The normalized capacity is equal to one and the normalized demand \( d_x(x, \theta) \) is a dimensionless function expressed in terms of the random variables:

\[
g_x(x, \theta) = 1 - d_x(x, \theta),
\]

where \( x \) is the vector of variables defining the maintenance scheme (recall that \( x = (q, t)^T \)), \( X \) denotes the life-time events associated with the structure, e.g., the failure or the repair event. \( d_x \) and \( g_x \) denote the normalized demand and the performance function associated with the event \( X \), respectively, and \( \theta \) denotes the uncertain parameters.

The performance function associated with fatigue prone components is typically expressed with respect to the actual fatigue life and the target fatigue life [45] (referred to as \( t_c \) and \( t_F \), respectively, in Fig. 3). Following an approach similar to the one developed in [23], the performance function is expressed with respect to the crack length at the end of the service life \( l_F \) and the critical crack length \( l_c \). In case fracture occurs before the end of the service life, the crack is artificially propagated beyond its critical length, as depicted in Fig. 3. Clearly, this does not possess any physical meaning. Nonetheless, the cracks are propagated beyond their physical limit as a means for formulating the performance functions associated with repair and failure, respectively. For instance, in case repair is not considered, it suffices to check the crack length at the time \( t_F \) in order to determine whether fracture occurs, instead of checking the crack length for any time instant \( t \in [0, t_F] \). In this study, the artificial crack length increases with a constant rate with time, which is equal to the crack growth rate at the last cycle before fracture occurs.

2.3.2. Performance function associated with the repair event

In the numerical model, the decision to repair a crack is taken in case the following requirements are fulfilled:

- The crack is detected during inspection at time \( t_I \). The uncertainties inherent in the crack detection procedure are modeled using an extra random variable \( \theta_d \) with a uniform distribution in the range [0,1]. The non-destructive inspection fails in detecting the crack if \( \theta_d \geq \text{POD}(l(t_I, x, \theta)) \); otherwise the crack is detected [23]. This formulation leads to the detection of a crack of the length \( l(t_I, x, \theta) \) with a probability equal to \( \text{POD}(l(t_I, x, \theta)) \).

- The structure is repaired only if the measured crack length \( l_{\text{meas}}(t_I, x, \theta) \) exceeds a given threshold length \( l_{\text{th}} \). This is equivalent to performing repair if \( l_{\text{meas}}(t_I, x, \theta)/l_{\text{th}} \geq 1 \). The estimation of the crack length is affected by measurement errors, which is modeled with an additive variable: \( l_{\text{meas}}(t_I, x, \theta) = l(t_I, x, \theta) + \epsilon \), where \( \epsilon \) denotes the error in sizing of the crack, its value may be greater than zero (the crack length is overestimated) or less than zero (the crack length is underestimated). In this contribution, \( \epsilon \) is modeled by a Gaussian distribution.

- The structure is repaired in case fracture has not occurred before the inspection time, which is equivalent to having \( l_F(t_I, x, \theta)/l_{\text{th}} \geq 1 \). Recall that according to Section 2.3.1, a crack is artificially propagated beyond its critical length. Thus, the condition of no failure before the inspection time can be checked by means of the inequality \( l_F(t_I, x, \theta)/l_{\text{th}} \geq 1 \).

Each crack of the structure is repaired if the three conditions stated above are fulfilled. Hence, the associated normalized demand associated with a crack \( d_{x,i} \) is defined as:

\[
d_{x,i}(x, \theta) = \min \left( \frac{\text{POD}(l_F(t_I, x, \theta), \theta)}{\theta_d}, \frac{l_{\text{meas}}(t_I, x, \theta)}{l_{\text{th}}}, \frac{l_F(t_I, x, \theta)}{l(t_I, x, \theta)} \right),
\]

where the subscript \( i \) refers to the \( N_c \) cracks present in the structure, \( \theta_d \) is the uncertain parameter associated with crack detection, \( l_{F,i}(t_I, x, \theta) \) is the actual crack length, \( l_{\text{th}} \) threshold crack length at which the decision of repair is taken, \( l(t_I, x, \theta) \) is the critical crack length.

In case repair actions are taken, all the cracks that fulfill the three requirements stated above are removed from the model.
The other cracks have identical lengths before and after the inspection.

Repair actions may be necessary in case at least one of the cracks fulfills the requirements stated above and the associated normalized demand is expressed as:

$$d_{kr}(\mathbf{x}, \theta) = \max_i (d_{kr}(\mathbf{x}, \theta)), \quad i = 1 \ldots N_c,$$

where $d_{kr}$ denotes the performance function associated with the repair of one of the cracks. The structure is not repaired in case fracture occurs before the time of inspection, i.e. in case the length of one of the cracks exceeds its critical value before the time of inspection $t_I$, which is expressed as:

$$d_{kr}(\mathbf{x}, \theta) = \max_i \left( \frac{l_i(t_I, \mathbf{x}, \theta)}{l_c(\mathbf{x}, \theta)} \right), \quad i = 1 \ldots N_c,$$

where $d_{kr}$ denotes the normalized demand associated with fracture before the time of inspection $t_I$.

Subsequently, the performance function associated with the repair event $d_k$ is expressed as:

$$d_k(\mathbf{x}, \theta) = \min_i \left( d_{kr}(\mathbf{x}, \theta), \frac{1}{d_{kr}(\mathbf{x}, \theta)} \right), \quad i = 1 \ldots N_c.$$

### 2.3.3. Performance function associated with fracture

Failure occurs during the service life if the length of one of the cracks $l_i(t, \mathbf{x}, \theta)$ exceeds a critical value $l_c(\mathbf{x}, \theta)$, thus leading to unstable crack growth. The normalized demand associated with failure can be expressed as:

$$d_f(\mathbf{x}, \theta) = \max_i \left( \frac{l_i(t_I, \mathbf{x}, \theta)}{l_c(\mathbf{x}, \theta)} \right), \quad i = 1 \ldots N_c,$$

where $t_I$ is the inspection time and $t_F$ is the target life time. Note that the normalized demand function introduced in Eq. (7) checks the occurrence of failure at two specific times only instead of checking failure at each time $t \in [0, t_F]$. Nonetheless, this strategy is still valid due to the fact that in this contribution, cracks are artificially propagated beyond their critical length and that the crack length is a function increasing with time. Thus, the failure condition can be still captured by Eq. (7), regardless failure occurs at some time $t$ different from $t_I$ or $t_F$.

### 2.4. Design of a maintenance scheduling by means of reliability-based optimization

As stated previously, the effects of uncertainties cannot be neglected for scheduling of maintenance activities. Uncertainties are considered in the non-destructive inspection, as well as in the crack initiation and growth processes. Hence, the costs associated with repair and fracture are not fixed, but they are influenced by the uncertain parameters. The optimum of a function including uncertainties can be found in the framework of reliability based optimization. Several definitions of reliability-based optimization have been proposed in the literature [46–48]. The outcomes from reliability analysis can be considered in the performance function, or in the constraints, or in both. Herein, a function whose expression includes a linear combination of outcomes from reliability analysis is minimized. The problem of reliability based optimization is formally stated as:

$$\min_{\mathbf{x}=(h_i)} C_r(\mathbf{x}),$$

Subject to $h_i(\mathbf{x}) \leq 0, \quad i = 1 \ldots N_c,$

where $C_r$ denotes the total life time costs of the structure, which have to be minimized, $h_i(\mathbf{x})$ denote the constraint functions, which are fulfilled as long as their value is less than (or equal to) zero and $N_c$ is the total number of constraints. The time of inspection $t_I$ and quality of inspection $q$ are introduced as the design variables of the optimization procedure (i.e. the objective of the study is finding the values of these parameters leading to minimized total costs). Only deterministic constraint functions are considered herein (i.e. they do not depend on the outcome of a reliability analysis).

In this study, total costs are expressed as the summation of the costs of inspection, repair and failure:

$$C_r(\mathbf{x}) = C_i(\mathbf{x}) + C_F(\mathbf{x}) + C_r(\mathbf{x}),$$

where $C_i$, $C_F$ and $C_r$ denote the cost functions associated with inspection, repair and failure respectively. Following the approach developed in [9,15], no additional information about the relative costs is considered and it is assumed that the there is a linear relation between the costs associated with the uncertain events (fracture, repair) and their respective probability of occurrence. Similarly, the costs associated with inspection are assumed to be proportional to the parameter $q$. The proportionality coefficients weigh the different events (inspection, repair and fracture) according to their contribution to the total costs.

The costs associated with inspection are assumed to be proportional to the quality of inspection:

$$C_i(\mathbf{x}) = C_i \cdot q,$$

where $C_i$ is a coefficient weighting the contribution of the inspection to the total costs.

The costs associated with repair and failure are expressed as:

$$C_F(\mathbf{x}) = C_F \cdot p_f(\mathbf{x}),$$

$$C_r(\mathbf{x}) = C_r \cdot p_r(\mathbf{x}),$$

where $p_f$ and $p_r$ are the probability of repair and the probability of fracture during the service life respectively, $C_r$ and $C_f$ are coefficients weighting the contribution of the repair and of the failure of the structure within the total costs respectively. In the formulation of Eq. (11), the number of repaired cracks does not affect the costs associated with the repair activities.

Within the scope of this manuscript, the outcome of an inspection is used to decide whether or not repair should be carried out. Hence, the information collected at inspection time is used solely for deciding the most appropriate time for inspection and also the best strategy for performing that inspection (which is related to the quality parameter). In other words, the problem is designing an optimal maintenance schedule for a generic mechanical component subject to fatigue damage. However, it is important to note that the outcome of an inspection can be also used for updating the reliability of a particular structure by means of, e.g. Bayesian approaches. That is, for a structure that has been built and where one has some prior knowledge on its state involving fatigue damage, the information gathered by inspection activities may allow updating the knowledge on the state of the component and taking decisions on repair for that particular structure. The latter approach is outside the scope of this contribution. The interested readership is referred to e.g. [83] on this issue.

### 3. Solution strategy

#### 3.1. Modeling of fatigue cracks using cohesive elements

This study is focused on investigating the fatigue life of a structure. Fatigue cracks are expected to initiate at the rivet holes and propagate through the structure until fracture occurs.

The use of cohesive zone elements allows to treat cracks by means of finite element simulation. They consist of zero-thickness elements that are inserted between the bulk elements (see Fig. 4(a)) and account for the resistance to crack opening using a
specific traction-displacement law. The cohesive force dissipates, at least partially, the energy related to crack formation. The use of such elements to account for fracture has been pioneered by Dugdale [37] and Barenblatt [38]. In this context, the crack growth is seen as a gradual phenomenon, with the progressive separation of the lips of an extended crack.

Nguyen et al. [25] extended the cohesive law to include fatigue crack growth. If the classical cohesive elements are used to model a cracked body undergoing alternating stress, the parameters of the finite element model do no longer evolve after few cycles, leading to crack arrest. The effects of the history are modeled using deterioration of the stiffness with time. During the unloading–reloading process, the cohesive law shows a hysteresis loop. A slight decay of the stiffness is introduced to simulate fatigue crack propagation (see Fig. 4(b)). This approach has been successfully used to model fatigue crack growth [see e.g. [49–51]].

Using the principle of virtual work, the mechanical equilibrium of a solid containing a cohesive surface can be expressed as:

$$
\int_V \sigma : \delta \varepsilon dV - \int_{S_{ext}} T_{coh} : \delta \sigma dS = \int_{S_{ext}} T_{ext} \delta u ds,
$$

where $V$, $S_{int}$ and $S_{ext}$ are the bulk volume, the cohesive and external surface respectively, $\sigma$, $T_{coh}$ and $T_{ext}$ denote the stress tensor, the cohesive traction vector and the external traction vector respectively, $\delta \varepsilon$ is the symmetric gradient of the test displacement field $u$. $A$ denotes the relative displacement between adjacent cohesive surfaces. The second term of the left-hand side of Eq. (13) represents the contribution of cohesive elements to the total mechanical energy.

The resistance of a material to crack formation can be expressed considering the energy dissipated during the formation of a new surface within the material. The total amount of energy dissipated during the formation of this surface is expressed as the sum of the energy related to destroying the chemical bonds between the atoms (or molecules) constituting the material and the energy associated with the plastic strain at the vicinity of the interface (e.g. the energy associated with the crack tip plasticity):

$$
\Gamma_t = \Gamma_d + \Gamma_{pl},
$$

where $\Gamma_t$ denotes the total amount of energy associated with the creation of the interface, $\Gamma_d$ is the energy associated with plastic strain and $\Gamma_d$ is the energy associated with debonding. Regarding cohesive zone element models, the plastic strain in the bulk elements at the crack tip accounts for $\Gamma_p$ and the traction-displacement law dedicated to the cohesive elements [see Eq. (15)] accounts for $\Gamma_d$.

During a finite element simulation, stable crack growth occurs as long as the mechanical energy associated with the boundary condition can be dissipated by the elements. When this energy can no longer be dissipated, unstable crack growth occurs. In this contribution, the critical crack length is defined as the crack length at the last instant before fracture.

The mechanical model proposed by Needleman [24] is used in the case of monotonic loading. The cohesive stress is expressed as:

$$
T_n = a \cdot \delta_h \cdot \exp \left( - \frac{\delta_h}{b} \right),
$$

where $\delta_h$ denotes the displacement of the opposite nodes of an element in the normal direction, $T_n$ is the normal stress within a cohesive element, $a$ and $b$ are material parameters. The features of the stress-displacement law are shown in Fig. 4(b). When such an element undergoes separation, the cohesive force first increases, which models the resistance of material to crack propagation. If the displacement exceeds a critical value, the cohesive force decreases, which accounts for the loss of strength of the damaged material (i.e. voids or micro-cracks appear in front of the crack tip).

Eq. (15) does not apply though when unloading is considered. Indeed, the behavior of the cohesive elements has to account for the irreversibility of crack growth. The stiffness of the cohesive elements is reduced by damage and unloading occurs linearly at constant stiffness so that stress vanishes when the separation is equal to zero.

In conventional formulations of cohesive zone elements, an unloading–reloading cycle is performed at constant stiffness values. Such formulations are applicable to fracture mechanics only. The cohesive law, as presented up to now, is non-dissipative, since there is no degradation of the material properties over a cycle, leading to crack arrest after few cycles. The material law proposed in Eq. (15) is extended to cyclic loading in the implementation of the cohesive zone element. The material law consists of a cohesive envelope describing the behavior of an element under monotonic loading and a hysteresis loop accounts for the damage accumulation at each fatigue cycle. When a cohesive element undergoes unloading and then reloading, the stiffness decreases slightly as the stress is increased. The loss of stiffness of damaging material can be assessed with a scalar damage parameter $D$ whose value is within the range $[0–1]$ [52]. Several authors used such a scalar parameter in the context of cohesive zone elements under fatigue loadings [53–55]. The rate of loss of stiffness is expressed as:

$$
\frac{dD}{dt} = \alpha \cdot (\Gamma(t))^{\beta} \cdot \max (\Gamma(t) - T_0, 0)^{\gamma},
$$

where $D$ is the total damage accumulated within an element, $\alpha$, $\beta$, $\gamma$ and $T_0$ are material parameters and $t$ is the time. The parameter $T_0$ is...
the stress at which damage does no longer accumulate within the material. In case of homogeneous repartition of the stress (at least among the crack path), the fatigue limit is equal to the value of \( T_0 \). The coefficient \( \alpha \) monitors the rate at which damage accumulates. The coefficients \( \beta \) and \( \gamma \) monitor the sensitivity of damage rate to the stress.

At any instant during cyclic loading, the stress in a cohesive element is equal to:

\[
T_n(t) = \frac{\sigma}{b} \cdot (1 - D(t)) \cdot \delta_n, \tag{17}
\]

where \( a \) and \( b \) are the parameters of the cohesive envelop law, given by Eq. (15), \( \delta_n \) is the relative displacement in the direction normal to the center-line of the element.

In case of monotonic loading, the traction-displacement law is that of the cohesive envelop (see Fig. 4(b)).

Unloading of a structure can be defined as a decrease of the applied stress. However, this definition cannot be systematically generalized to the behavior of one single cohesive element. Local unloading can be caused by global unloading of the structure, by a change in the repartition of stress as a crack propagates or by interactions between cracks. Since cohesive elements show softening, loading (resp. unloading) is defined as an increase (resp. decrease) of the separation (i.e. displacement of opposite nodes of the element).

The case \( D = 0 \) corresponds to virgin material. When the first loading is applied, the behavior of the element is determined by Eq. (15) until unloading occurs. The case \( D = 1 \) corresponds to completely damaged elements, which do not transfer any stress. Such elements correspond to the physical crack.

The value of the damage parameter is equal to one in the elements at the crack location and its value decreases progressively with the distance from the crack. However, there is a progressive transition between the cracked material and the uncracked material. As suggested in [27], the elements with a damage parameter greater than 0.99 are assumed to be fully damaged, and a cohesive transition between the cracked material and the uncracked material. In case of homogeneous repartition of the stress (at least near a sharp angle, a hole, etc.).

The finite element simulation using cohesive zone elements is extremely demanding from a computational viewpoint. Three factors contribute to the computational time associated with the numerical simulation of fatigue crack growth using cohesive elements:

- The formulation proposed in Section 3.1 is strongly non-linear. Hence, several inversions of the tangent matrix are required to model the behavior of a structure over one fatigue cycle.
- It is necessary to repeat a large number of simulations of the behavior over one single cycle in order to describe accurately the behavior. The simulations can be accelerated by the means of special algorithms (see e.g. [60]). However, it is necessary to repeat many times the finite element simulations of the behavior of the structure over an individual cycle in order to model accurately the fatigue crack growth. Most of the computational efforts are spent on these successive simulations over a cycle.
- Most of the fatigue life is spent during the crack initiation or during the growth of short cracks. In order to accurately model these processes, the finite element mesh must be refined at the crack initiation sites.

The use of meta-models (or surrogate models), such as response surface models [61], Gaussian process [62] or Kriging interpolation [63] allows to approximate the crack length or the fatigue life with limited computational efforts. The use of meta-models is well adapted to reliability analysis, which requires a large number of computations of the performance function [64].

In this study, linear regression is used to approximate the outcomes of time consuming finite element simulations. A set of \( N_{reg} \) independent basis functions \( T_{reg} = \{ T_{reg,1}, \ldots, T_{reg,N_{reg}} \} \) is selected. The meta-model is expressed as:

\[
\tilde{F}(t, \theta, l_0, B) = \sum_{j=1}^{N_{reg}} B_j \cdot T_{reg,j}(t, \theta, l_0) + e_{reg}, \tag{18}
\]

where \( \tilde{F} \) denotes the response surface, \( T_{reg,j}, j = 1 \ldots N_{reg} \) denotes the basis functions used in the regression, \( e_{reg} \) is the regression error. The regression variables consist of the time \( t \) the uncertain parameters \( \theta \) and the initial crack lengths \( l_0 \). During a simulation of the fatigue life, no crack is initially present in the model and the terms of \( l_0 \) are all equal to zero. The consideration of initial cracks allow to model fatigue crack growth after repair activities, in case cracks are removed from the model. The details of the implementation are described in appendix. \( B = \{ B_1, \ldots, B_{N_{reg}} \} \) is the vector of the

### 3.2. Meta-modeling

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| Variable | Value | Unit |
|----------|-------|------|
| Young’s modulus | 70000 | MPa |
| Poisson ratio | 0.3 |  |
| Yield stress | 330 | MPa |
| Ultimate stress | 650 | MPa |
| Coefficient \( a \) of Eq. (15) | 1500 | MPa |
| Coefficient \( b \) of Eq. (15) | 0.05 | mm |
regression parameters, which has to be determined in order to minimize the regression error.

The least square estimate $\hat{B}$ of the regression parameters can be expressed as [65]:

$$\hat{B} = (X'X)^{-1}Xy_{\text{full model}}, \quad (19)$$

where $y_{\text{full model}}$ denotes the set of outcomes of the finite element simulation corresponding to the training points, $X$ is a matrix containing the value of the basis functions for the different values of the training points, i.e. $X_j = T_{reg_j}(t^0, \theta^0, t^0_i), i = 1 \ldots N_{SP}, j = 1 \ldots N_{reg}$, where $N_{SP}$ denotes the number of support points and $(t^0, \theta^0, t^0_i)$ denotes the support points. Response surfaces are calibrated to approximate the actual length of the cracks $l_i$ at any instant of the service life, and the critical length of the cracks $L_{cr}$. The response surfaces are directly used in the formulation of the performance functions (Eqs. (3, 7, 29)) instead of the outcome of the finite element simulations.

Response surfaces approximating the crack lengths in the time range $[t_i, t_F]$ are required. At the instant $t_i$ the structure may include cracks at some of the initiation sites (these cracks have not been repaired during the maintenance activities). At the other initiation sites, there may be no crack at the instant $t_i$ since repair activities have been performed. In order to approximate accurately the crack lengths in the time range $[t_i, t_F]$, training points with initial cracks are considered in order to calibrate the response surface. However the initial crack lengths $l_i$ are not included in the model of uncertainties, since cohesive zone elements account for fatigue crack initiation.

Efficient methods allow to use meta-models in order to perform reliability analysis without systematic bias by means of Subset Simulation [82,81]. However, in the context with reliability-based optimization, the performance function is expressed with respect to the random variables and the design variables, respectively. Hence, it is necessary to calibrate as well a meta-model accounting for the random variables and the design variables, respectively. Yet, these algorithms are not considered in this manuscript.

Details on the implementation of the meta-model considered in this contribution (such as training points, basis functions, etc.) are described in depth in Appendix A.2.

### 3.3. Assessment of reliability

Reliability analysis aims at determining the probability that a component reaches a given state condition. In this study, the state conditions of interest are respectively repair and failure of the structure.

The uncertain parameters are modeled with random variables and the probability can be expressed through the following multi-dimensional integral:

$$p(X) = \int_{X \times \theta \times 0} f(\theta) d\theta, \quad (20)$$

where $\theta$ denotes the uncertain parameters, $f$ is the joint probability density function and $g$ represents the performance function. Reliability analysis can be performed e.g. by means of Monte Carlo simulation, that consists of generating samples of the random variables and counting the number of outcomes within the failure region:

$$\hat{p}(X) = \frac{1}{N} \sum_{i=1}^{N} I_i(X, \theta^0), \quad (21)$$

where $\hat{p}$ is the approximation of the failure probability, $N$ is the number of samples generated, $\theta^0$ denotes the samples and $I$ is the indicator function, which is equal to one for the samples in the failure region and zero elsewhere. However, Monte Carlo simulation generally requires to generate a very large number of samples, which is computationally prohibitive when small failure probabilities (e.g. $10^{-6}$) have to be estimated.

The advanced procedure of Subset Simulation [21] allows to estimate small failure probabilities with a limited number of evaluations of the performance function. It is based on a decomposition in intermediary failure events. A set of intermediary failure regions is defined so that $F_1 \supset F_2 \supset \cdots \supset F_m$, where $F_m$ is the failure region whose probability of occurrence has to be determined. The probability associated with the intermediary failure region can be estimated with limited computational efforts. The final failure probability can be determined by conditional probabilities:

$$p \simeq P(F_1) \prod_{i=1}^{m-1} P(F_{i+1}|F_i), \quad (22)$$

where $P(\cdot)$ denotes the probability associated with an event

#### 3.4. Reliability sensitivity estimation

Besides determining the probability of repair and failure, the sensitivities (gradients) of each of these probabilities with respect to time of inspection and quality of inspection are required for determining an optimal maintenance schedule according to the optimization strategy considered. In this study, the sensitivity of the reliability is performed following the procedure described in [23,66]. This procedure allows to estimate the gradients of the failure probabilities at reduced computational costs.

The partial derivative of the probability with respect to $x_i$ is defined as:

$$\frac{\partial p}{\partial x_i} = \lim_{\delta x \to 0} \frac{P(g(X + \delta x \cdot e_i, \theta) \leq 0) - P(g(X, \theta) \leq 0)}{\delta x}, \quad (23)$$

where $e_i$ is a vector with the same size as the set of design variables $x$, the $i^{th}$ element of $e_i$ is equal to one, the other terms are all equal to zero. Recall the design variables consist of the time and quality of inspection (i.e. $x = (q, t_i)^T$). In order to evaluate the partial derivative of Eq. (23) efficiently, two approximations are introduced. First, a local linear approximation of the performance function is performed in the vicinity of the design variables of interest:

$$g(X + \delta x \cdot e_i, \theta) \simeq g(X, \theta) + \beta_{0,i} \cdot \delta x, \quad (24)$$

where $\beta_{0,i}$ is a scalar parameter. The procedure for computation of the parameter $\beta_{0,i}$ is described in depth in [66] and can be summarized as follows: (i) a subset of samples within the vicinity of the limit state is selected; (ii) the performance function is computed for these samples considering perturbed design variables (i.e. the value of $g(X + \delta x \cdot e_i, \theta)$ is determined); (iii) the coefficient $\beta_{0,i}$ is computed from the results of the previous steps, for instance using linear regression.

The second approximation introduced to estimate the partial derivative of Eq. (23) is:

$$P(g(X, \theta) - \Xi \leq 0) \simeq e^{x_2 \Xi}, \quad (25)$$

where $x_1$ and $x_2$ are two scalars determined using linear regression, $\Xi$ denotes a perturbation term, which is set to have a locally
imation of the probability of interest (for instance, $\Xi \in [-0.1,0.1]$).
At first, the probability in the left hand side of Eq. (25) is computed for various values of $\Xi$. Then the coefficients $x_1$ and $x_2$ are determined using linear regression. This approximation has been used successfully in several publications within the area (see e.g. [67–70]). If the reliability analysis has been performed beforehand (for instance to estimate the objective function), the coefficient $x_1$ and $x_2$ are estimated without extra performance function evaluations. The results obtained from the reliability analysis performed previously are reused and only the count of the samples leading to a performance function value below the threshold level $\Xi$ is computed [23].

Considering the two approximations described above, it can be shown [66] that the sought partial derivative can be estimated by means of the following expression:

$$
\frac{\partial p}{\partial x_i} = -\beta_{0,i}x_i p(x). 
$$

(26)

### 3.5. Sensitivity of the failure probability

One of the main assumptions behind the approach described above for estimating the sensitivity of the probability is that the associated performance function is continuous. However, the performance function associated with the failure event may not fulfill this condition [23]. In order to overcome this issue, a strategy is proposed in the following.

It should be noted that the performance function associated with probability of fracture shows discontinuities with respect to the random variables monitoring fatigue crack growth and with respect to the parameters associated with crack detection. Indeed, in the numerical model, a slight variation of one of these parameters may lead to detection and repair of a crack that was initially not repaired (and reciprocally), leading to a discontinuity in the performance function. In order to clarify this issue, consider the following qualitative example. Assume a plate with an edge crack, which undergoes an inspection with perfect sizing of the crack and the probability of detecting the crack may be defined by Eq. (1). Uncertainties are considered in the crack growth rate and in the outcome of non-destructive inspection. Fig. 5(a) illustrates the shape of the performance function associated with failure.

Fig. 5(b) shows the division of the random variables space in 4 zones. A1 denotes the zone for which the random variables values always lead to a safe structure, structural failure does not occur during the entire service life and the structure is not repaired. The zone A2 denotes the region of values leading to repair of the structure, although the structure is safe without repair (i.e. the zone A2 corresponds to the second undesirable outcome described in Section 2.3). The zone A3 denotes the region of values leading to failure when no maintenance is performed and leading on the other hand to a safe structure if maintenance activities are performed. The zone A4 denotes the region of values leading to failure, either because the crack fails in being detected during the non-destructive inspection, or because the crack reaches its critical length before the time of inspection $t_c$.

The zones A2 and A3 correspond to the regions of values leading to repair of the structure, and the performance function associated with fracture is discontinuous at the border of these regions. This discontinuity imposes a major challenge when analyzing the sensitivity of the failure event with respect to the different parameters relevant to the model. On the contrary, the discontinuity between the zones A1 and A2 does not affect the analysis, since these zones are both in the safe domain.

In the simple example proposed in this section, the discontinuity is due to the fact that whenever a crack is repaired, its length changes suddenly from a given value to zero (when perfect repair is considered). In the example presented in Section 4, imperfect repair is considered, another crack initiates and grows. Its final length is not correlated with the crack length without repair. This sudden change clearly introduces a discontinuity in the associated performance function. In order to cope with the discontinuity discussed previously two artificial performance functions are introduced, which are associated with the subsets of the space of the random variables described above.

The first function is related to the continuous part of the performance function (i.e. the safe domain consist of area A1 and A2 in Fig. 5(b)). The second performance function is related to the discontinuous part of the performance (i.e. the failure domain consist of area A3 in Fig. 5(b)). It can be expressed as the probability of performing necessary repair (i.e. the structure would fail if it is not repaired and it is safe after perfect repair). Both of these performance functions are continuous and hence suitable for sensitivity estimation [66]. The probability of failure (and its gradients) is estimated as the difference of the probabilities defined by the performance functions described above:

$$
p_F(x) = p_0 - p_{NR}(x),
$$

(27)

where $p_F$ denotes the probability of failure (fracture before the target life), $p_{NR}$ defines the probability of necessary repair and $p_0$ denotes the probability of failure without repair activities, which is not expressed in terms of $x$ (since the structure is not repaired). Hence, the probability $p_0$ is determined before starting the optimization and its value does no need to be updated at each iteration.

Using Eq. (27), in case the structure is repaired, the simulation of the life time events have to be performed twice i.e. with and without the repair activities.

The normalized demand associated with the probability of failure without repair $d_0$ can be expressed as:

$$
d_0(\theta) = \max_i \left( \frac{l_i(t_i, \theta)}{l_i(\theta)} \right) \quad i = 1 \ldots N_c.
$$

(28)

In this study, the necessary repair operation is defined as fulfilling the following requirement.

- The crack is detected and repaired right after the inspection.
- Fracture occurs before the end of the service life without repair.

The normalized demand associated with the probability of necessary repair $p_{NR}$ can be expressed as:

$$
d_{NR}(x, \theta) = \min\left( d_0(\theta), d_{FR}(x, \theta) \right).
$$

(29)

### 3.6. Optimization strategy

The objective of this study is to determine the maintenance schedule by minimizing the total costs associated with the maintenance and eventual failure of the structure, which are expressed by Eq. (9). As discussed in the previous section, the sensitivity associated with the reliability analysis can be determined efficiently. Hence gradient-based optimization algorithms are well suited for solving the reliability based optimization problem. In particular, a first order scheme based on feasible directions is applied in this contribution (see e.g. [71,72]). This scheme is implemented due to its simplicity and robustness but certainly other optimization schemes based on gradients that are more efficient could be applied as well.

The method of feasible directions involves two main steps. In the first one, for a given a feasible design $x_k$ (i.e. a design fulfilling the constraints of the optimization problem) a search direction $d_k$ is determined such that it is possible to find a sufficiently small step $\xi > 0$ fulfilling the condition $C_F(x_k + \xi d_k) < C_F(x_k)$. The search direction $d_k$ can be determined by solving a linear programming prob-
lem involving the gradient of the objective function and the active constraints (for details on this issue, it is referred to e.g. [71]).

The second step of the method of feasible directions consists in exploring the one dimensional space defined by the search direction \( d_i \), i.e. a line search is performed. The objective is determining an optimal step \( \xi^{\text{opt}} \) that solves the following one-dimensional optimization problem:

\[
\min_{\xi} \quad C_f(\xi) = C_f(x_i + \xi d_i)
\]

Subject to \( \xi > 0 \), \( h_i(x_i + \xi d_i) \leq 0 \), \( i = 1 \ldots N_c \),

\[\text{(30)}\]

where \( C_f(\cdot) \) is the total costs function along the search direction. For solving this one dimensional optimization problem, the step \( \xi \) to the nearest active constraint is determined using any appropriate search scheme such as bisection [73,74]. Once \( \xi \) has been found, the optimal step \( \xi^{\text{opt}} \) is calculated by means of the following criterion. In case the derivative of \( C_f(\cdot) \) is negative at \( \xi \), then \( \xi^{\text{opt}} = \xi \) and the new feasible design is \( x_i + \xi^{\text{opt}} d_i \). In case the derivative of \( C_f(\cdot) \) is positive at \( \xi \), then the optimal step is located in the interval \([0, \xi] \). Thus, the value of the optimal step can be determined using again a bisection scheme [73,74].

For the actual implementation of the line search step described above, it should be noted that it might be necessary to evaluate \( \xi^{\text{opt}} \) several times. As its evaluation is numerically demanding (because it implies calculating probabilities), it is proposed to approximate this function by a polynomial:

\[
C_f(\xi) \approx C_f(\xi) = C_0 + C_1 \xi + C_2 \xi^2.
\]

\[\text{(31)}\]

where \( C_j, j = 0, 1, 2 \) are real coefficients. These coefficients are determined using the values of the cost function and of its sensitivity along the search direction (directional derivative) evaluated at three points \((\xi, \xi_1, \xi_2) \in [0, \xi] \), following a procedure suggested in [75]. It is clear that only the function values at three points would be required for determining the sought coefficients. However, it should be kept in mind that there is an inherent variability associated with the evaluation of the total costs function as it depends on probabilities that are evaluated by means of simulation. Thus, the extra data (directional derivative) improve the robustness of the method by coping, at least partially, with the variability inherent to simulation methods. Details on the construction of the interpolation of Eq. (31) can be found in [76,23].

4. Numerical example

4.1. Description

The objective of this example is designing a maintenance schedule for a metallic component subject to cyclic loading. The structure studied consists of a plate with two rivet holes with a diameter of 4 mm each (see Fig. 6). The plate has a height of 400 mm, a width of 64 mm and a thickness of 2.3 mm. The loading is applied in the longitudinal direction, with a maximum stress of 200 MPa and a minimum stress of 40 MPa. The symmetry of the structure among its center-line in the transverse direction (represented by a dashed line on Fig. 6) is considered and the finite element model consists of half of the plate. The mesh is refined at the rivet holes in order to describe accurately the repartition of the stress at the rivet holes. The mesh refinement also improves the accuracy of the modeling of fatigue crack initiation and of the propagation of short cracks.

Cohesive zone elements are inserted at the crack path, as indicated in Fig. 6.

As discussed in Section 2.1, the uncertainties inherent in the fatigue crack initiation and propagation are influenced by parame-

Fig. 6. Geometry of the structure.
ters showing spatial variation within the structure (such as the micro structural properties). In case the coefficient of Eq. (16) monitoring the fatigue crack initiation and growth is modeled using a single random variable, the time to crack initiation is the same for the four sites where cracks initiate (at the holes of the structure shown on Fig. 6). Thus, all the cracks have the same length at any instant of the service life. This is obviously incorrect, since one could expect to have a single crack initiating from one of the sites and then propagating through the structure. Thus the parameters \( \alpha, \beta \) and \( \gamma \) of Eq. (16) are modeled with spatial variation within the structure. Four independent random variables are used, where each of them is devoted to one of the crack initiation sites. At each extremity of the central ligament, the coefficient \( \alpha \) is equal to the realization of the random variable devoted to this crack initiation site. This coefficient shows a linear variation within the central ligament. In each of the ligaments at the extremities of the structure, the coefficient \( \alpha \) is constant (i.e. there is no spatial variation within each of the ligaments). The coefficient \( \alpha \) is equal to the realizations of the random variable devoted to this location. Recall that parameters \( \alpha, \beta \) and \( \gamma \) are modeled as fully correlated, thus \( \beta \) and \( \gamma \) are fully characterized once \( \alpha \) has been defined, as stated in Section 3.1.

The error in sizing of the crack is modeled with Gaussian distribution with zero mean and a standard deviation equal to \( 2.4 \times 10^{-3} \) mm. The sizing error may be different for each of the cracks present in the structure. Hence, four independent random variables are used in the model. Similarly, the independent random variables are used in the formulation of the probability of detecting a crack, described in Eq. (3). The uncertainties inherent in the detection, sizing, initiation and propagation of the cracks are modeled using 12 random variables in total.

The structure has a target fatigue life of 250,000 cycles. The structure is considered safe if fracture does not occur, i.e. none of the cracks has reached the critical length leading to unstable crack growth. One inspection activity is considered during the total life. It is assumed that the coefficient \( q \) can assume any (real) value. The threshold crack length \( l_{th,i} \) is equal to 1 mm. It is assumed that the cracks with a length below this value do not jeopardize the structure and are not repaired after the inspection, even though these cracks may be successfully detected. The same threshold length is used for all the cracks of the model.

The coefficient related to the costs of inspection, repair and failure are equal to \( C_i = 5 \times 10^{-3} \), \( C_r = 2.5 \) and \( C_f = 100 \), expressed in arbitrary monetary unit. The objective of the reliability based optimization is minimizing the total costs. The side constraints for the design variables are \( 140,000 \leq t_i \leq 250,000 \) and \( 1 \leq q \leq 30 \).

For launching the optimization procedure, the maintenance schedule is selected such that the inspection is performed after 148,000 cycles and the coefficient \( q \) is equal to 28.6 mm\(^{-1}\).

In case a crack initiates at the side of a rivet hole, it is likely to have another crack emanating from the opposite side of the hole. Proppe and Schüeller [77] modeled the initiation of cracks emanating from the same rivet hole with correlated random variables. However, such approach is not required herein. The parameter \( x \) monitors the initiation and the growth of the cracks, and is modeled with independent random variables (at each site of crack initiation). However, the lengths of the cracks at the different sites are actually correlated (see Fig. 7). This correlation is caused by the repartition of the stress in the structure in the presence of cracks. As an example, when a crack appears at a side of a hole, the stress at the opposite side is increased, which speeds up the initiation and growth of a crack at this location.

4.2. Results

The procedure for reliability based optimization described in Section 3 has been applied to the model described above. Fig. 8(a)–(c) show the costs associated with fracture, repair and inspection respectively as a function of the time of inspection and quality of inspection. The costs associated with inspection increase linearly with the quality of inspection respectively. The costs associated with repair are strongly affected by the time of inspection. Indeed, the latter the inspection is performed, the longer the cracks are in the structure, which require repair, causing the increase of the associated costs. The costs of repair are slightly affected by the quality of inspection, which increase the chances of detecting cracks. The costs associated with fracture are strongly affected by the time of inspection. In case the inspection is performed too early, the likelihood of detecting a crack is very low, and/or the decision to repair the structure is not taken. In case the inspection activities are performed too late, the probability of failure before the inspection is rather high, which leads to an increase of the associated costs.

The total costs are shown in Fig. 8d by means of contour lines. The function of the total costs shows one minimum, and is relatively flat at the vicinity of its minimum. The same figure illustrates

![Fig. 7. Correlation matrix between the cracks lengths after 200,000 cycles, obtained using Monte-Carlo simulation with 200 samples.](image)
the trajectory of optimization algorithm in the space of the design variables. The details on each point along this trajectory are summarized in Table 3. The procedure converges efficiently towards the optimum (see Fig. 8(d)). At the first iteration, the total costs are greatly reduced. The procedure arrives to the vicinity of the optimum and the costs are further reduced at subsequent iterations. The minimum costs could be found after three iterations. In total, three line searches were necessary, which represents 10 successive runs of Subset Simulation. The total computer time required to perform reliability sensitivity (computation of the gradients) is negligible when compared to the computational time associated with reliability analysis.

In addition to the information provided in Fig. 8 and Table 3, the first two columns of Fig. 9 provide details on the costs associated with inspection, repair and failure for the initial design and optimal design, respectively. It is seen that the optimal maintenance schedule is a compromise between the costs associated with these three events. The initial maintenance strategy is not appropriate and the costs related with failure are the dominant ones. As the optimization progresses and the optimal maintenance schedule is found, the costs associated with repair increase, but this allows a subsequent decay of the costs associated with failure, leading to a decrease of the total costs associated to the structure.

Considering the optimal maintenance scheduling, the inspection is performed late during the service life. Indeed, the optimal value of the time of inspection is equal to 191,000 cycles, which corresponds to approx. 76% of the service life of the structure. The structure is not damaged at the beginning of its service life, and the amount of damage progressively increases.

In order to gain insight about the trade off that arises between inspection, repair and maintenance costs when looking for an optimal maintenance schedule, two additional cases were analyzed. The first case involves minimizing the costs of failure alone and

![Fig. 8. Costs associated to the model. (a) Costs associated with fracture. (b) Costs associated with repair. (c) Costs associated with inspection. (d) Evolution of the design variables during the reliability based optimization procedure. The contour lines show the total costs (in arbitrary monetary units), the solid lines show the successive search directions, the dots represent the intermediary designs, the cross shows the coordinates of the optimum.](image)

![Table 3](image)

| Iteration     | \( q \) (mm\(^{-1}\)) | \( t_I \times 10^3 \) Cycles | Total costs |
|---------------|------------------------|-----------------------------|-------------|
| Initial design| 28.6                   | 148                         | 0.51        |
| Intermediary design 1 | 9.8             | 163                          | 0.21        |
| Intermediary design 2 | 7.6             | 192                          | 0.11        |
| Final design   | 6.7                    | 191                          | 0.09        |

![Fig. 9. Total costs associated with the structure.](image)
determining the associated optimal maintenance schedule. Then, the total costs associated for that optimal maintenance schedule are calculated and plotted in the third column of Fig. 9. The second additional case studied corresponds to calculating the total costs when no maintenance activities are considered. As no maintenance activities are considered, the total costs for this second case are equal to the failure costs. This last result is plotted in the fourth column of Fig. 9. It is most interesting to note that minimizing the failure costs alone leads to total costs that are considerably larger than the case where all costs are considered for optimization (second column of the Figure). In addition, it can be noted that suppressing maintenance activities (fourth column of the Figure) causes a dramatic increase of the total costs. Details on the costs associated with the second, third and fourth columns of Fig. 9 are summarized in Table 4. These results highlight the importance of considering all costs when searching for an optimal maintenance schedule, as the optimal solution is evidently a trade off between different factors.

5. Conclusions

A method for determining optimal maintenance scheduling of metallic structures considering uncertainties has been proposed herein. Cohesive zone elements provide a framework to investigate fatigue crack growth. Contrary to approaches based on linear fracture mechanics, cohesive elements do not require to introduce explicitly initial cracks. The degradation associated with cyclic load is modeled by means of an internal damage parameter which increases during the fatigue life. Cracks appear once the elements are fully damaged. This approach accounts for fatigue crack initiation and propagation using the same phenomenological model. The variability inherent in fatigue of a structure has been assessed using a stochastic model for the parameters monitoring the evolution of the damage. The uncertainties related to fatigue crack initiation and to crack propagation are accounted for using a single model for uncertainties. Moreover, the model describing the crack detection includes its inherent variability. It is assumed that the outcome of non-destructive inspection can be fully represented by its probability of detection.

The performance function associated with fracture is discontinuous, which is not suitable for the estimation of reliability sensitivity. The gradients have been estimated by introducing two auxiliary performance functions, one of them is accounting for the continuous part of the gradient, the second one is accounting for the effects of the discontinuities.

The methods presented here allowed to find the optimal schedule for the maintenance activities. The time and quality parameters of the inspection leading to the minimum costs associated with the structure were determined. The evaluation of the cost function over a grid showed that the costs associated with such structure are mainly affected by the time of inspection and in less degree by the quality of inspection.

The computational efforts are greatly reduced by introducing a meta-model (e.g. a response surface), using an advanced simulation method for the reliability analysis and an efficient algorithm for computing the gradients of the failure probabilities.

Concerning the numerical example and the results obtained, it is most interesting to observe that the determination of an optimal maintenance schedule with respect to total costs implies finding a trade off between the costs of inspection, repair and eventual failure. Thus, it is not sufficient to consider one of these three events by itself, as it may lead to a suboptimal scheduling of maintenance activities.

Future work is directed towards the extension of the study to a more general case. For instance, additional inspections may be added, a larger structure with more cracks may be investigated, imperfect repair may also be considered. However, it should be kept in mind that despite all the efforts to reduce computational time, reliability based optimization still remains a demanding procedure.

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Appendix A

A.1. Implementation of a cohesive element

The relative displacement between adjacent cohesive surfaces \( \Lambda \) can be expressed independently from the orientation of a cohesive element as:

\[
\Lambda = \begin{bmatrix}
\delta_i \\
\delta_n
\end{bmatrix} = \begin{bmatrix}
\cos(\theta_{el}) & \sin(\theta_{el}) \\
-\sin(\theta_{el}) & \cos(\theta_{el})
\end{bmatrix} \begin{bmatrix}
\upsilon_{\text{surface } 1} \\
\upsilon_{\text{surface } 2}
\end{bmatrix} = R \begin{bmatrix}
\upsilon_{\text{surface } 1} \\
\upsilon_{\text{surface } 2}
\end{bmatrix},
\]

where \( \theta_{el} \) denotes the angle of a cohesive element with respect to the horizontal (see Fig. 10), \( \delta_i \) and \( \delta_n \) denote the tangential and the normal component of the relative displacement between adjacent cohesive surfaces (in the coordinate system attached to the element of interest) respectively. \( \upsilon_{\text{surface } i} \) denotes the displacement of the cohesive surface \( i \) in the direction \( j \) (i.e. in Fig. 10 the cohesive surfaces are the segments AB and CD).

Considering an element as shown in Fig. 10, the displacement of the cohesive surfaces can be related to the nodal displacements:

\[
\begin{bmatrix}
\upsilon_{1}^{\text{surface } 1} \\
\upsilon_{2}^{\text{surface } 1} \\
\upsilon_{1}^{\text{surface } 2} \\
\upsilon_{2}^{\text{surface } 2}
\end{bmatrix} = \begin{bmatrix}
N_1 & 0 & N_2 & 0 & 0 & 0 & 0 & 0 \\
0 & N_1 & 0 & N_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & N_1 & 0 & N_2 & 0 \\
0 & 0 & 0 & 0 & 0 & N_1 & 0 & N_2
\end{bmatrix} \begin{bmatrix}
\upsilon_1 \\
\upsilon_2 \\
\upsilon_3 \\
\upsilon_4
\end{bmatrix} = \mathbf{N} \begin{bmatrix}
\upsilon_1 \\
\upsilon_2 \\
\upsilon_3 \\
\upsilon_4
\end{bmatrix}.
\]
where $N_1$ and $N_2$ denote the shape functions, $u_i^X$ is the displacement of the node $X$ in the direction $i$ ($i$ being the horizontal or vertical direction in this study, $X$ being the node A, B, C or D in Fig. 10). Numerical integration was performed according to Newton–Cotes scheme. Indeed, the integration points are located at the extremities of the center-line of a cohesive element, as shown on Fig. 10(a).

Such integration scheme provides better robustness of the implementation by avoiding spurious oscillations in the stress field of the cohesive elements [78]. Fig. 10(b) presents the aspect of the shape functions.

The nominal traction rates are expressed as:

$$
\begin{bmatrix}
\dot{T}_t \\
\dot{T}_n
\end{bmatrix} = \begin{bmatrix}
\frac{\partial T_t}{\partial X^1} & \frac{\partial T_t}{\partial X^2} \\
\frac{\partial T_n}{\partial X^1} & \frac{\partial T_n}{\partial X^2}
\end{bmatrix} \cdot \begin{bmatrix}
\dot{\delta}_1 \\
\dot{\delta}_2
\end{bmatrix} = S \cdot \begin{bmatrix}
\dot{\delta}_1 \\
\dot{\delta}_2
\end{bmatrix},
$$

(34)

where $T_t$ and $T_n$ denote the stress in the normal and tangential direction respectively, $\delta_n$ (resp. $\delta_t$) denotes the normal (resp. tangential) traction rate (see Eq. (32)). $S$ denotes the matrix of the material properties independently from the geometry of the element. Using Eqs. (32)–(34) the stiffness matrix of one cohesive element can be expressed as:

$$
K = \int_{\Omega} N^T \cdot R \cdot S \cdot R \cdot N dS.
$$

(35)

Eq. (35) was used as the basis for implementation of the user defined element subroutine.

In this study, the cracks are loaded according to mode 1 (opening mode, the stress is perpendicular to the crack direction). Hence the tangential stiffness was neglected and it was not implemented in the formulation proposed here.

Regarding the computational implementation, cohesive zone elements have been modeled in the finite element code FEAP [79] by means of a user defined element subroutine available in this software.

A.2. Training of a Meta model

The crack growth is influenced by the variables defining the maintenance scheme $x = (q,t)^T$, among others. When a crack is repaired and removed from the model, the repartition of the stress within the structure changes, which in turns affects the growth rate. This is accounted for by first training a response surface with the uncertain parameters $\theta$, the time $t$ and the initial crack lengths $l_0$ as regression variables. Subsequently, a new meta-model is calibrated, which approximates the crack length in terms of the uncertain parameters $\theta$, the time instant $t$ and the variables defining the maintenance scheme $x$.

Typically, the fatigue crack growth rate is first very low at the beginning of the fatigue life, and then increases (see for instance the experimental results available in [1], or the results from the finite element simulations using cohesive zone elements shown on Fig. 12). Hence an exponential growth is used as the basis function approximating the crack length with respect to time. The dependence of the crack growth with respect to the uncertain parameters of Eq. (16) and with respect to the length of the initial cracks is modeled by polynomials (see Eq. (36)).

Calibration samples from the uncertain parameters and from the initial crack lengths are generated using Koschal design [80], which allows to define polynomial response surfaces with an optimal number of simulations. The initial cracks are from zero to eight millimeters long. The samples from the uncertain parameters $\theta$ are in the range $[2 \times 10^{-7}, 7 \times 10^{-3}]$. In total 2355 calibration points are used. For each calibration sample, the finite element simulation is performed until fracture occurs. The time history of the crack lengths are used for the calibration of the response surface, which is expressed as:

$$
l_{RS}(t, \theta, l_0) = \exp \left( P_1(\theta, l_0) \cdot t + P_2(\theta, l_0) \cdot t^2 \right),
$$

(36)

where $\theta$ denotes the uncertain parameters devoted to the coefficient $C$ of Eq. (16) (monitoring fatigue crack initiation and growth) and $l_0$ is the initial crack length. Samples are generated with initial crack lengths $l_0$ which are greater than zero to account for the crack initiation and growth after repair, the subscript $i$ refers to the $N_i$ cracks present in the structure. The coefficients of Eq. (36) are determined using linear regression, $P_1$ and $P_2$ are polynomials of degree five.

Then a new meta-model $\tilde{l}_i(t, x, \theta)$ is calibrated, the crack length is expressed in terms of the uncertain parameters $\theta$, the time $t$ and the parameters defining the maintenance scheme $x$. Fig. 11 shows a schematic representation of the use of Eqs. (38) and (37) to approximate the crack length. In case the crack length is approximated at the instant $t$ occurring before inspection ($t < t_I$), or in case the structure is not repaired, this meta-model is equivalent to the response surface described in Eq. (36), and the length of a crack is expressed as:

$$
\tilde{l}_i(t, x, \theta) = l_{RS}(t, \theta, 0),
$$

(37)

where $\tilde{l}$ denotes the approximation of the crack length using the meta-model, $0$ is a vector of which all the terms are equal to zero (since Eq. (37) considers a structure without initial cracks), $x$ denotes the design variables, $\theta$ denotes the uncertain parameters.

The lower part of Fig. 11 represents the approximation of the crack length after repair. First, the length of the cracks is determined at the time of inspection $t_I$ and the repair actions are taken where necessary. As shown on Fig. 11, in case repair actions are performed, the length of the corresponding cracks is equal to zero just after repair (i.e. there is no crack at this location), and another
crack may initiate and propagate at the same location (imperfect repair). Otherwise, the length of the cracks is identical before and after inspection. Then, the length of the cracks is approximated using the response surface defined in Eq. (36) for \( t \in [t_I, t_F] \), the value of the regression variable \( l_0 \) consists of the crack lengths after repair and the time of reference is \( t_I \). The length of the cracks is expressed as:

\[
\hat{l}_i(t, x, \theta) = \frac{l_{RS}(t - t_I, \theta_1, l_{t_I})}{C_0(t_I, x, \theta)}.
\]

where \( l_{t_I} \) denotes the crack lengths immediately after repair. Obviously, Eq. (38) is not valid if \( t < t_I \), which is represented by a gray area on Fig. 11.

Fig. 12 shows the time evolution of the crack length obtained by the means of finite element simulation (reference solution) and the approximation by the response surface described above. The results of the finite element simulation show a discontinuity in the crack length, which is caused by the discretization. Each sudden increase of the crack length corresponds to an extra element becoming fully damaged. The length of the crack is then increased by the size of the element.

The critical crack length \( l_{c,i} \) is expressed in terms of the uncertain parameters and in terms of the variables defining the maintenance scheme. The strategy adopted to approximate the critical crack length is similar to that developed above. At first, a response surface approximating the critical crack length is calibrated for each crack growth site. The regression variables consist of the uncertain parameters \( \theta_1 \) and the initial length of the cracks \( l_I \). Polynomials of degree five are used as basis functions and determined using linear regression. The approximation of the critical crack length is performed using the same calibration samples as those used for the approximation of the crack length. Subsequently, a meta-model of the critical crack length \( \hat{l}_{c,i}(x, \theta) \) is calibrated, it is expressed in terms of the uncertain parameters \( \theta \) and the parameters defining the maintenance scheme \( x \). The approach is similar to the one developed in Eqs. (37) and (38). In case the structure is not repaired, the polynomial response surface is directly

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**Fig. 11.** Schematic representation of the use of Eqs. (37) and (38) in terms of the life time events. The upper part of the figure denotes the cases where Eq. (37) is used, the lower part of the figure denotes the cases where Eq. (38) is used. Eq. (38) is not valid if \( t < t_I \), which is represented by a gray area.

**Fig. 12.** Evolution of the crack length with respect to the time (expressed in thousand of cycles). Dots denote the crack length obtained by the finite element simulations, lines denote the response surface used to approximate the crack length.
used. Otherwise, the length of the cracks immediately after inspection \( L_0 \) is determined first, and the polynomial response surface used is used subsequently.

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