CONFINE M ENT OF COLOR: RECENT PROGRESS.

ADRIANO DIGIACOMO
Dipartimento di Fisica, Università di Pisa, INFN, Sezione di Pisa,
Via Buonarroti 2, 56127 Pisa, Italy
E-mail: adriano.digiacomo@df.unipi.it

Recent progress done in Pisa on the subject is presented. It is shown that dual superconductivity of the vacuum or absence of it is an intrinsic property of QCD vacuum, independent of the choice of the abelian projection. The order of the deconfining phase transition in $N_f=2$ QCD is studied as a key to understand the mechanism of confinement.

1. Dual superconductivity: dependence on the abelian projection.

The mechanism of confinement by dual superconductivity of the vacuum requires the identification in QCD of a U(1) bundle, which has to be color gauge invariant and color singlet if color is not broken by monopole condensation in the vacuum. The procedure for that is known as "Abelian Projection".

A disorder parameter is then introduced, $h_i$ which is the vev of a magnetically charged operator. $h \neq 0$ means Higgs breaking of the magnetic U(1) gauge symmetry, or dual superconductivity, and is expected to hold in the confined phase $T < T_c$. For $T > T_c$ (deconfined phase) instead the U(1) is not broken and $h_i = 0$.

It can be instructive to start from the SU(N) Higgs model, which is defined by the Lagrangean

$$L = \frac{1}{4} \text{Tr} F G + g \text{Tr} F D' g + V(\phi)$$  (1)

The Higgs field is an $N$ by $N$ matrix and transforms in the adjoint representation. In the Higgs phase $h_i = 0$. The index $a$ labels different minima of $V(\phi)$.

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The relevant quantity for monopole solutions is\(^5\)
\[ F = \text{Tr} \frac{g}{\partial} \text{Tr} \frac{D}{\partial} g \quad (2) \]

A theorem can be proved on \( F \)\(^6\):

A necessary and sufficient condition (i) for the cancellation of bilinear terms in \( A \) \( A \) between the two terms on the rhs of eq(2) and (ii) for the validity of the Bianchi identities \@\( F = 0 \) is that
\[ a(\chi) = U(\chi) a_{\text{diag}} U^Y(\chi) \quad (3) \]

with
\[ a_{\text{diag}} = \text{diag} \begin{pmatrix} 0 & \ldots & 0 \\ N & \ldots & N \\ a & \ldots & a \\ \vdots & \ddots & \ddots \\ N & \ldots & N \\ a & \ldots & a \end{pmatrix} \quad (4) \]

and \( U(\chi) \) an arbitrary gauge transformation. The little group of \( a_{\text{diag}} \) is \( \text{SU}(a) \) \( \text{SU}(N - a) \) \( U(1) \). It identifies a symmetric space\(^7\). If \( L_0 \) is the corresponding Lie algebra, \( L \) is the Lie algebra of \( \text{SU}(N) \), and \( L_1 = L - L_0 \), \([L_0;L_0]\), \([L_0;L_1]\), \([L_1;L_1]\), \([L_1;L_0]\). It can be shown that all possible symmetric subspaces of \( \text{SU}(N) \) have the form eq(3).

Vice versa, if the Higgs field belongs to the adjoint representation then the breaking identifies a symmetric space, i.e., \( a \) has the form eq(3)\(^8\).

For a Higgs field of the form eq(3) one has identically
\[ F^a = \partial \text{Tr} a_A g \partial \text{Tr} a_A g \frac{i}{g} \text{Tr} a_\partial \partial g \quad (5) \]

\( F^a \) is gauge invariant by construction. In the unitary gauge \( a = 0 \), the second term of eq(5) vanishes and
\[ F^a = \partial \text{Tr} ( a_A ) \partial \text{Tr} ( a_A ) \quad (6) \]

assumes an abelian form. The transformation to the unitary gauge is called abelian projection.

Expanding the diagonal part of the fields \( A_{\text{diag}} = iA^i \), in terms of the roots
\[ i = \text{diag}(0;0;\ldots;1;0;\ldots;0) \quad \text{tr} f i A_{\text{diag}} = iA \quad (7) \]
gives
\[ F^a = \partial A^a \partial A^a \quad (8) \]
A monopole solution exists in the SU(2) subspace spanned by the diagonal elements $i$ and $i+1$, of the form of the SU(2) solution\(^5\). For this solution $E_i = F_{0i} = 0$ and

$$H = \frac{2}{g} \frac{x^i}{r^3} + D \text{ Dirac string}$$

whence the name of a monopole to the soliton.

A magnetic current can be defined

$$J^a = \@ F^a$$

Bianchi identities require that $J^a = 0$ but it can be different from zero in a compact formulation like lattice, in which the Dirac string is invisible. In any case

$$\@ J^a = 0$$

N 1 U(1) magnetic symmetries are thus defined, which are topological symmetries, which do not correspond directly to invariances of the Lagrangian. In QCD they will be the magnetic symmetries which will eventually be Higgs broken in the confined phase, producing dual superconductivity.

The construction of the magnetically charged operators $^a$ which will provide the disorder parameters for dual superconductivity is the following\(^4\).

The denomination "disorder" comes from statistical mechanics and means that $^a$ is expected to be different from zero in the confined phase, which is strong coupling (disordered), and to be equal to zero in the ordered phase.

$$^a (x; t) = e^{i \frac{a^i}{2} \gamma_\tau (\gamma \gamma_t) \vec{b} \ , \ x \gamma}$$

with

$$\vec{F} \cdot \bar{b} = 0 \ ; \vec{F} \wedge \bar{b} = \frac{2}{g} \frac{x^i}{r^3} + D \text{ Dirac string}$$

$^a$ is gauge invariant if $^a$ belongs to the adjoint representation.

In the abelian projected gauge $^a = \frac{^a}{\text{diag}} \tau \gamma_\tau (\gamma \gamma_t) \frac{^a}{\text{diag}} = E^a$ and $^a (x; t) = \exp \frac{i}{\gamma_\tau} \frac{a^i}{2} \gamma_\tau (\gamma \gamma_t) \bar{b} \ , \ x \gamma)$$

where only the transverse part of $E_x^a$ survives in the convolution with $b_t$.

In whatever quantization scheme $E_x^a$ is the conjugate momentum to $\vec{F}_\gamma^a$, and hence

$$^a (x; t) \vec{F}_\gamma^a (y; t) = \frac{a^i}{2} \gamma_\tau (\gamma \gamma_t) \gamma_\gamma \gamma + \bar{b} \ , \ y \gamma)$$
a creates a monopole in the $U(1)$ generated by $a$ in the abelian projected gauge.

The construction can be repeated unchanged in the Coulomb phase, in spite of the fact that there are no monopoles as solitons, by taking any $a(x) = U(x) \tilde{\text{diag}} U(x)y$ in the adjoint representation. If $U(x)$ is defined as the gauge transformation which diagonalizes the Higgs field $\phi(x)$, then $a(x)$ is diagonal with $\phi(x)$, but any other choice provides an abelian projection: for example $a(x)$ can be diagonal in the maximal abelian gauge.

$a$ depends on the choice of $U(x)$

$$a(x;t) = e^{i \int dx^3 U(x)^T \phi(x)U(x)B(x,y)}$$  \hspace{1cm} (15)

If $U(x)$ does not depend on the gauge field configuration, when computing correlators of $a(x)$ it can be reabsorbed by a change of variables which leaves the measure invariant, and

$$a(x;t) = e^{i \int dx^3 \tilde{\text{diag}} U(x)^T \phi(x)U(x)B(x,y)}$$  \hspace{1cm} (16)

All memory of $U(x)$ has disappeared, and $h^{\alpha \beta} = 0$ or $h^{\alpha \beta} = 0$ are statements independent on the abelian projection.

If $U(x)$ depends on $\phi(x)$ in general the measure is not invariant and a non-trivial Jacobian can appear after gauge transformation by $U(x)$, so that the correlation functions of $a(x)$, and in particular its vev are projection dependent. However if the number density of monopoles is finite the operator $a(x)$ defined by eq(16) will create a monopole in all abelian projections, since the gauge transformation to any abelian projection will be continuous in a neighbourhood of $x$ and will preserve topology. Hence, if the number density of monopoles is finite the statement $h^{\alpha \beta} = 0$ and $h^{\alpha \beta} = 0$ are abelian projection independent.

Dual superconductivity (or non-superconductivity) of the vacuum is an intrinsic property, independent on the particular choice of the abelian projection.

In QCD there are no Higgs fields, but, as discussed above, any operator in the adjoint representation $O(x)$ can provide an abelian projection, in the sense that the operator $U(x)$ of eq(2) can be chosen as the one which diagonalizes $O(x)$. Again, if the number density of monopoles is finite dual superconductivity is an intrinsic property, independent of the choice of the abelian projection. The density of monopoles can be estimated by looking at the distribution of the difference of the eigenvalues of any operator in the adjoint representation on the sites of a lattice. The location of monopoles
coincides indeed with such zeros. We have studied that distribution on samples of lattice configurations, with different lattice spacings and for a number of operators. A typical distribution is shown in Fig. 1, which refers to the Polyakov line as operator, $10^3$ configurations on a $16^4$ lattice quenched SU(3) and $\beta = 6.4$. The number of sites on which there is a monopole is zero.

Figure 1. An example of probability distribution of the difference of the two highest eigenvalues of the phase of the Polyakov line $e^{i}$, at the lattice sites. SU(3) gauge group, $\beta = 6.4$, lattice $16^4$, $10^3$ configurations.

An independent test can be made by numerical comparison of the order parameter for different abelian projections, which confirms the independence of dual superconductivity on the choice of the abelian projection.\(^{13}\)

The measurement of the disorder parameters $h^{\alpha}i$ in the quenched case works as follows.\(^4\) Instead of $h^{\alpha}i$ one determines the quantity $\frac{d}{d\beta}\text{ln}h^{\alpha}i$. $\alpha$ is a susceptibility. At the deconfining transition where $h^{\alpha}i$ has a sharp drop $\alpha$ has a negative peak. A phase transition can only take place in the infinite volume limit.\(^{12}\) As the volume increases the drop of $h^{\alpha}i$ becomes sharper and sharper and the negative peak of $\alpha$ higher and higher. Since\(^4\)

$$Z \quad h^{\alpha}i = \exp(\int_0^{\beta_0} \text{d} \lambda \quad \lambda)$$

and for $T < T_c$ $\alpha$ becomes volume independent within numerical errors

\(\beta = \)
Figure 2. The phase diagram of two flavor QCD.

with increasing volume, one concludes that for $T < T_c$, $h^a i = 0$, implying dual superconductivity. For $T > T_c$, with $N_s$ the spatial extension of the lattice, implying that $h^a i = 0$ strictly in the thermodynamic limit (norm al vacuum). In the critical region the correlation length $\xi$ goes large, the ratio of the lattice spacing $a$ to $\xi$, $a/\xi$ can be put to zero, and then the disorder parameter only depends on the ratio $N_s$:

$$h^a i = \frac{N_s}{a} = 0$$

whence the scaling law follows

$$a = N_s^{1/2} = f\left(\frac{N_s}{a}\right)$$

In particular the peak height scales as $N_s^{1/2}$, whence can be determined. The result is consistent with the values obtained by use of the Polyakov line $^{14,15}$, or $= \xi/2$ for pure gauge SU (2) (3d Ising universality class), and $\xi$ for SU (3), 1st order transition.

2. Two flavor QCD.

In quenched theory one uses the Polyakov criterion to define confinement, which refers to the static potential between a quark and an antiquark.

The order parameter is $h^a i$ the Polyakov line: when $h^a i = 0$ the potential grows linearly with distance, when $h^a i = 0$ it goes to a constant.
Of course one should in principle show that con nement defined in this way implies the absence of any colored particle in asymptotic states, which is not easy to do, but the criterion is reasonable anyway. As shown above it is with identifying con nement with dual superconductivity of the vacuum.

In the presence of dynamical quarks $Z_3$ symmetry is explicitly broken and $hLi$ cannot be an order parameter. Moreover string breaking is expected to occur: the potential energy stops growing with distance, due to the instability for production of quark antiquark pairs, even if there is con nement. At $m_q = 0$ there is chiral symmetry, which is known to be spontaneously broken at zero temperature, the pseudoscalar mesons being the Goldstone particles. The symmetry is restored at some temperature $T_c$, where the order parameter $hL$ goes to zero. It is not known what is the relation between chiral symmetry breaking and con nement. In any case at $m_q \neq 0$ chiral symmetry is explicitly broken and $hL$ is not an order parameter either. The situation for $N_f = 2m_u = m_d = m$ is depicted in $g2$.

A number of susceptibilities can be measures on the lattice as functions of the temperature $T$ at given $m$ (The susceptibility of $hL$, the specific heat $^{17,18}$). All of them show a peak at the same $T(m)$, which defines the curve in the phase diagram of $g2$. By convention the region below that curve is called con ned, the region above it decon ned. A renormalization group analysis can be made $^{16}$ of the chiral transition assuming that the Goldstone particles are the relevant degrees of freedom at the transition, with the following result. For $N_f = 3$ the chiral transition is 1st order and such is the transition at $m \neq 0$. For $N_f = 2$ if the anomaly of the U(1) axial current vanishes below $T_c$, the transition is 1st order and such is the transition at $m \neq 0$; if instead the anomaly persists up to $T_c$, the transition is 2nd order in the universality class of $O(4)$ and the line at $m \neq 0$ is a crossover. Lattice data are not yet conclusive on this issue, but for some reason the second possibility is usually assumed to be true.

A possible criterion for con nement could be dual superconductivity of the vacuum, which is already valid in the quenched case. Indeed the disorder parameter $h$ can equally well be defined in the presence of dynamical quarks. Lattice simulations show $^{19}$ that $h$ is non zero below the critical line of $g2$, and is strictly zero above it in the thermodynamic limit. Of course in principle one should show that dual superconductivity implies absence of colored particles in asymptotic states, which is not trivial to do: but the situation is not different than that of the Polyakov criterion,
as discussed in the previous section.

A finite size scaling analysis around \( T_c \) can be performed to get information on the order of the phase transition. The issue is very relevant to understand con nem ent. Indeed if the determination gives a result consistent with what is obtained by studying the specific heat a legitimat ion results for \( h^a_i \) as an order parameter and for dual superconductivity as a mechanism of con nem ent. Preliminary data indicate that the chiral transition is 1st order and certainly not in the universality class of O (4). A careful analysis is being completed, which will give an unambiguous answer to the question. A careful analysis of the anomaly around \( T_c \) is also on the way to check consistency with ref.\(^{16} \). Some details on the analysis. A new scale is present in the problem with respect to the quenched case. Eq (18) now reads

\[
h^a_i = \langle N_z = m N_z \rangle
\]

The problem can be reduced to a single scale by choosing masses and sizes such that \( m N_z \) is constant, assuming for \( y_h \) alternatively the value corresponding to O (4) universality class \( (y_h = 2.49) \) or the value for a 1st order transition \( y_h = 3 \). For the same values different susceptibilities and the specific heat can be determined, and the critical indices can be measured consistently. This program is being completed.

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