Closing the Neutrinoless Double Beta Decay Window into Violations of the Equivalence Principle and/or Lorentz Invariance

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Abstract

We have examined Lorentz invariance and equivalence principle violations in the neutrino sector as manifested in neutrinoless double beta decay. We conclude that this rare decay cannot provide a useful view of these exotic processes.
I. INTRODUCTION

During the past few years, neutrino oscillations have been used to explore exotic properties of neutrinos such as possible violations of Lorentz invariance (VLI) [1–3] and/or violations of the equivalence principle (VEP) [4–9]. Since neutrinoless double beta decay has served as a window into neutrino masses for the last two decades [10–12], it is natural to enquire if this rare decay can tell us anything about VLI and VEP processes.

We begin with the observation that the properties of neutrinos enter into the neutrino exchange diagram for the neutrinoless double beta decay amplitude (without right handed currents) in the form of the factor [13]

\[ A_\nu = (1 - \gamma_5)P(1 - \gamma_5), \]  

where P is a linear combination of the propagators for each of the Majorana neutrino fields that constitute the \( \nu_e \) field,

\[ P = U^2_{ea}P_a. \]  

The \( U_{ea} \) are elements of the unitary matrix connecting mass and weak eigenstate neutrinos. In the absence of external fields, and in the absence of Lorentz invariance violation, the Majorana fields have definite masses, \( m_a \), and all neutrinos have the same limiting velocity, \( c \). In that case the \( P_a \) are, of course, given by

\[ P_a = (\gamma^0 E - \gamma^k p_k c + m_a c^2)^{-1}. \]  

II. MODIFICATIONS DUE TO VIOLATION OF LORENTZ INvariance.

In the Lorentz invariance violating scheme of Coleman and Glashow, the limiting velocity of each neutrino may be distinct, so that \( c \to c_a \). Our conclusions are unaltered by the introduction of distinct mass and velocity bases, and for the sake of clarity such a complication will be ignored. To first order in \( m_a \), the VLI modified \( A_\nu \) is then given by

\[ A_{\nu,VLI} \simeq -\frac{2U^2_{ea}(1 - \gamma_5)m_a c^2_a}{E^2 - (pc_a)^2}, \]  

where the chirality factors have been used to eliminate contributions from factors of \( E\gamma^0 \) or \( \gamma^k p_k \) in the numerator.

For neutrinoless double beta decay in nuclei we usually make the approximation of ignoring nuclear recoil, so the energy of the exchanged neutrino is set equal to zero, that is \( E = 0 \). With this standard approximation,

\[ A_{\nu,VLI} \simeq 2U^2_{ea}(1 - \gamma_5)m_a/p^2. \]  

Since this expression is independent of limiting velocities, we conclude that VLI cannot enter into neutrinoless double beta decay in any significant way.
III. MODIFICATIONS DUE TO VIOLATION OF THE EQUIVALENCE PRINCIPLE.

Following the formalism of Ref. [6], the Dirac equation governing neutrino $a$ is modified by the presence of an external gravitational field which couples to neutrinos with strength $f_a$ relative to the usual universal Newtonian coupling. For the sake of clarity, we take the mass and gravitational coupling bases to be the same. In the presence of a constant Newtonian potential, $\Phi$, the neutrino propagator becomes

$$P_a = [(1 + f_a \Phi)E\gamma^0 - (1 - f_a \Phi)\gamma^k p_k + m_a]^{-1}$$

where we have set the common limiting vacuum (i.e. $\Phi = 0$) velocity equal to 1.

To first order in $m_a$ we then have, making use of the chirality factors as before,

$$A_{\nu VEP}^\nu \simeq -\frac{2U_{e\alpha}^2 (1 - \gamma_5) m_a}{(1 + f_a \Phi)^2 E^2 - (1 - f_a \Phi)^2 p^2}.$$  

Making the zero recoil approximation as above and retaining only to first order in $\Phi$ for consistency, we then have

$$A_{\nu VEP}^\nu \simeq 2U_{e\alpha}^2 (1 - \gamma_5) m_a (1 + 2f_a \Phi)/p^2.$$  

The expression above includes only the modification to the neutrino propagator due to the presence of $\Phi$. To this we must also add modifications to the W- boson, quark and electron lines due to $\Phi$. Assuming that only neutrinos have anomalous gravitational couplings, restoration of gravitational gauge invariance in the limit that all $f_a$ are equal guarantees that the final $\Phi$-dependent contribution depends only upon $\Delta f_a = f_a - f_0$, where $f_0 = 1$ in Einsteinian gravity. The $\Phi$-dependent contribution to the total neutrinoless double beta decay rate will then be proportional to $U_{e\alpha}^2 m_a \Delta f_a$. Thus, the VEP effect is proportional to both $m_a$ and $\Delta f_a$ and is therefore extremely small.

IV. CONCLUSIONS

We have examined the modifications to the usual neutrino exchange diagram for the neutrinoless double beta decay amplitude arising from violations of Lorentz invariance and/or the equivalence principle in the neutrino sector. We find that the VLI parameters disappear from the decay amplitude in the usual zero recoil approximation and that the VEP parameters enter the amplitude only in combination with with neutrino mass factors. We therefore conclude that neutrinoless double beta decay cannot provide a significant window into VLI and VEP neutrino processes. This result appears to contradict the conclusions of a recent paper [14].

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