An algorithm for the numerical calculation of nonlinear lens-shaped membrane-pneumatic systems created by the incremental-iterative method with account for the air pressure aftereffect

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Abstract. The authors have developed a method for the computer-aided static calculation of the large span structures geometrically and physically nonlinear lens-shaped membrane-pneumatic systems using the incremental-iterative algorithm with a stepwise application of the finite elements method in the displacement method form, the universal gas equation, and the improved Euler-Cauchy numerical procedure of the third order accuracy.

Introduction
The algorithm for calculating the lens-shaped nonlinear membrane-pneumatic systems was developed by A.Yu Kim with account for the air elastic behavior [1].

S. V. Polnykov calculated the investigated lens-diaphragm-pneumatic systems influence on the power, kinematic, thermal and pneumatic loads taking into account the excess air pressure aftereffect in the lenses [2].

M. F. Amoyan conducted the investigations into the lens-shaped covering systems stability and the buckling analysis depending on the air overpressure in the lenses, provided the effect assessment of geometric and physical nonlinear systems, which are for the computer program algorithms [3].

Materials and methods
The authors are designing an improved version of the incremental-iterative method for calculating pneumatic systems, whereas the basis for the development of the given method is laid in the works of Prof. D. F. Davidenko [4], Prof. V. V. Petrov [5 et al.] and other scientists.

Figure 1. A single-span lens-shaped membrane-pneumatic covering of the rectangular building
The work by D. F. Davidenko “The Parameter Method Variation Application to the Nonlinear Functional Differential Equations Theory” and the work by V. V. Petrov “The Method of Incremental Loading in the Nonlinear Theory of Plates and Shells” started a rapid development of different versions to the incremental method. The given methods formed the basis for the universal iterative-incremental method, which allows the different parameters variations in nonlinear systems.

**Figure 2.** A computational model for a single-span lens-shaped membrane-pneumatic system

We will focus on the idea behind the method of incremental loading developed by Prof. V. V. Petrov [5]. Let us describe the system stress-strain state by the nonlinear functional

\[ A(x, y) = 0, \]

which includes continuously dependent on numerous parameters of the system accumulation functions, reflecting the changes in the system stiffness in the course of its deformation, and the required functions \( Z \), expressing the displacements, deformations and stresses in the system.

With the following assumption

\[ \frac{\partial Z}{\partial x} = A'_m(x, y) \]

we write the solution to the equation \( A(x, y) = 0 \) in line with the incremental loadings method using the Euler procedure

\[ \Delta Z_m = \Delta x_{n \mu} \cdot A'_m(x_{n-1, \mu}, y_{n-1}), \tag{1} \]

where \( n \) is the current step number; \( \Delta x_{n \mu} \) is the parameter increment \( x_{n \mu} \) at the step \( n \); \( x_{n-1, \mu} \) is the parameter value \( \mu \) at the end of the previous step \( n-1 \); \( y_{n-1} \) is the value of some accumulation function \( y \) at the end of step \( n-1 \).

The solution at the starting point \( x_{0, \mu}, y_0 \) is regarded as known. The error of the solution at the step \( n \) is \( O(h) \), where \( h = \Delta x_{n \mu} \).

A way for further incremental method development is the incremental-iterative algorithm creation with application of the iterative Euler-Cauchy procedure. This method, first applied for nonlinear calculations of pneumatic structures by Prof. A. Yu. Kim [3, 4], includes the following sequence of operations:

1. The problem solution at a first approximation, i.e. by the incremental technique of the first accuracy order using the Euler formula

\[ \Delta Z_{m}^{(1)} = \sum_{\mu=1}^{\mu_{n}} \Delta x_{n \mu} \cdot A'_m(x_{n-1, \mu}, y_{n-1}). \tag{2} \]

2. The problem solution at s-approximation according to the formula

\[ \Delta Z_{m}^{(c)} = \sum_{\mu=1}^{\mu_{n}} \Delta x_{n \mu} \cdot A'_m(x_{n-1, \mu} + \frac{\Delta y_{n \mu}}{2}, y_{n-1} + \frac{\Delta y_{n \mu}(c-1)}{2}), \tag{3} \]

where \( 2 \leq c \leq 4 \).

Meanwhile, the accumulation functions \( \Delta y_n \) increments are averaged applying the Runge-Kutta method within the step at \( C = 2 \). When \( C = 4 \), the calculation formula (2) of the incremental-iterative method provides an error in the results of the third order accuracy, i.e. \( O(h^4) \).
Note that accumulation functions include the temperature, kinematic, and pneumatic loads. Additionally, only pneumatic loads refer to the guiding type of loads, and on application at the loading stage the systems acquire the aftereffect, i.e. the subsequent relaxation and are under stress due to additional air pressure. Only after relaxation the systems acquire the equilibrium status.

As it follows from the numerical calculations of weakly nonlinear systems, a coincidence in the required node deflections iterations in the nonlinear system of the third order accuracy is observed under three or four iterations.

Averaged at the incremental step, the accumulation functions \( Y_n \) and, consequently, averaged at the incremental step the Frechet derivatives \( A'(\Delta X_n^{(c)}, Y_n^{(c)}) \) are selected from the condition, where the solution is equivalent to the linearized problem and coincides with the nonlinear problem solution with the \( \varepsilon \)-th order accuracy (Fig. 1). The principle of the nonlinear problem equivalent linearization allows the superposition principle application at each iterative step.

The authors have improved the incremental-iterative method using the numerical Euler-Cauchy procedure of the third order accuracy at the ransom step, which is described in the doctoral dissertation of Prof. A. Yu. Kim. The objective was to take account of the aftereffect in the system after the air pressure changes in the system.

In order to consider the aftereffect, that is the system relaxation caused by the changes of excess air pressure in the closed cavity, it is necessary to increase the number of iterations in the computational process, in particular, especially under an insufficient excess air pressure in the lens and at the large spans of the covering system. A positive feature of the Euler-Cauchy numerical procedure of the third order accuracy is its iterative character, which allows application of the aftereffect of the pressure increment in the systems.

When calculating the lens-shaped system relating the pneumatic load in terms of the aftereffect, it is necessary to adjust the load at each iterative procedure.

The authors apply the incremental-iterative method coupled with a step-by-step application of finite elements method, the universal gas equation, and the Euler-Cauchy procedure of the third order accuracy, taking into account that the pneumatic system stiffness depends on the excess air pressure value in the lens. The system is estimated using the well-known finite elements method in the displacement method form.

The Euler-Cauchy procedure is applied at increased number of iterations corrected in case of the changes in excess air pressure in the lens. The new methodology for calculating pneumatic systems of large span structures is effective due to the excess air pressure increase in the lenses to the required degree. The pressure increases the large-span covering to ensure its stability and bearing capacity during the hurricanes.

We consider one step coupled with numerous iterations, when the pneumatic loading rate is corrected at each iteration procedure. The number of iterations is assigned as large enough as to adequately describe the pneumatic system operation process in its development and obtain the prescribed accuracy of the outcome. In this case, the aftereffect phenomenon is estimated quite accurately, which is accompanied by the changes in excess air pressure in the lens, and creates an internal pneumatic load forcing deformations in the pneumatic system, redistribution of forces in the elements, and pressure changes.

In the loads set, the pneumatic load \( P \) holds a specific place. It is a system of pumping air into a closed cavity of the lens coating or air leakage from the lens under operation disturbances of the facility. The aftereffect of the applied pneumatic load is significant enough that it changes the required results significantly.

For the aftereffect accurate estimation at each iteration of the applied formula which corrects the external pneumatic load value applied to the system in line with the formula

\[
P^{(c)} = P^{(c)} - \Delta P^{(c)}
\]

Here \( P \) refers to the pneumatic load that occurs when the air is pumped into the pneumatic lens, or when the air leaks from the lens. Note that according to the formula (3) at the current iteration, the
pneumatic load $P$ is improved and will be used for the next iteration. The following rules are applied for the signs: at increasing the pressure in the lens, the pressure increment is considered positive, whereas at decreasing the pressure it is negative. If the pneumatic load increment is positive, then in the course of the aftereffect the excess pressure in the lens decreases. Since the value of the required pneumatic load $P$ at each iteration procedure should be constant and equal to the required value of the pneumatic load, the sign in the correction formula (3) should be “minus”. The load $R$ will increase by $\Delta P^{(c)}$, and remain constant and equal to $P^{(c+1)} = P + \Delta P^{(c)}$.

**Summary**

Thus, the numerical procedure of the considered version of the incremental-iterative method with a step-by-step application of the improved Euler-Cauchy procedure of the third order accuracy will include the following sequence of operations:

1. Solution to the problem in the first approximation, i.e. by the incremental method of the first order accuracy using the Euler formula

   $$\Delta Z_{n
u}^{(1)} = \sum_{\mu=1}^{m\nu} \Delta x_{n\mu} \cdot A'_{n\mu}(x_{n-1,\mu}, y_{n-1}),$$  

   (5)

2. Solution to this problem in $c$ approximation according to the formula

   $$\Delta Z_{n
u}^{(c)} = \sum_{\mu=1}^{m\nu} \Delta x_{n\mu} \cdot A'_{n\mu}(x_{n-1,\mu}, y_{n-1} + \frac{\Delta y_{n\mu}^{(c-1)}}{2}),$$

   where $2 \leq c \leq C_k$.

   $$P^{(c+1)} = P - \Delta P^{(c)}.$$ 

Here $C_k$ is the final iteration, where the convergence process relating the required accuracy achievement is completed.

The iterative process continues until the predetermined overpressure in the lens is achieved at the loading process end. The Euler-Cauchy iterations convergence procedure is proved by Cauchy.

It can be concluded that the authors have created a new calculation method, which allows to take into account the air as an elastic component enclosed between the membrane belts of the lens, while dependence of its properties on the temperature, volume of the lens, initial pressure, and increments of all these values in the process of the pneumatic load increment greatly affects the VAT zones of the system. According to the new method, an excess air pressure increment in the lens is calculated more accurately, since we use as many iterations for the estimation as required in order to achieve a full convergence of the process in relation to the estimated excess air pressure in the pneumatic systems. The prospects for further development of the given trend is related with complete solution of the aftereffect problem in the near future under effect of traditional and servo–pneumatic loads, that will significantly upgrade the accuracy of estimating these systems.

**References**

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