Pound–Drever–Hall locking scheme free from Trojan operating points

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Manuel Zeyen,1,a) Lukas Affolter,1 Marwan Abdou Ahmed,2 Thomas Graf,2 Oguzhan Kara,1 Klaus Kirch,1,2 Miroslaw Marszalek,3 François Nez,4 Ahmed Ouf,1 Randolf Pohl,5,6 Siddharth Rajamohanan,6 Pauline Yzombard,4 Aldo Antognini,1,3,a) and Karsten Schuhmann1

AFFILIATIONS
1 Institute for Particle Physics and Astrophysics, ETH, 8093 Zurich, Switzerland
2 Institut für Strahlwerkzeuge, Universität Stuttgart, Pfaffenwaldring 43, 70569 Stuttgart, Deutschland
3 Paul Scherrer Institute, 5232 Villigen, Switzerland
4 Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-Université PSL, Collège de France, 75252 Paris Cedex 05, France
5 QUANTUM, Institute of Physics, Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany
6 Excellence Cluster PRISMA+, Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany

a)Authors to whom correspondence should be addressed: zeyenm@phys.ethz.ch and aldo.antognini@psi.ch

ABSTRACT

The Pound–Drever–Hall (PDH) technique is a popular method for stabilizing the frequency of a laser to a stable optical resonator or, vice versa, the length of a resonator to the frequency of a stable laser. We propose a refinement of the technique yielding an “infinite” dynamic (capture) range so that a resonator is correctly locked to the seed frequency, even after large perturbations. The stable but off-resonant lock points (also called Trojan operating points), present in conventional PDH error signals, are removed by phase modulating the seed laser at a frequency corresponding to half the free spectral range of the resonator. We verify the robustness of our scheme experimentally by realizing an injection-seeded Yb:YAG thin-disk laser. We also give an analytical formulation of the PDH error signal for arbitrary modulation frequencies and discuss the parameter range for which our PDH locking scheme guarantees correct locking. Our scheme is simple as it does not require additional electronics apart from the standard PDH setup and is particularly suited to realize injection-seeded lasers and injection-seeded optical parametric oscillators.

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I. INTRODUCTION

The Pound–Drever–Hall (PDH) locking technique1 is a popular method to stabilize the frequency of a laser to a stable optical resonator. It has been used to achieve lasers with sub-Hertz linewidth2,3 and is applied in a wide range of fields, such as gravitational wave detection,4 atomic physics,5,6 and metrology,7 just to name a few. The PDH method can also be used in the opposite way to stabilize the length of an optical resonator to a stable single-frequency laser with equally numerous applications.5–11

Despite its widespread and long-standing application, this technique is continuously refined and adapted to specific applications.11–16 The dynamic range of the standard PDH technique is limited by the additional zero crossings of the error signal at off-resonant frequencies. This typically limits the dynamic range (or capture range) of the lock to a fraction of the free spectral range (FSR) of the resonator, which is given by

$$\Delta f = \frac{c}{2L},$$

(1)

where $c$ is the speed of light in the resonator and $L$ is the length of the (linear) resonator. The PDH error signal has a zero crossing right in the middle between two adjacent resonator modes at frequency detuning $\nu = \Delta f/2$ from the resonator mode. This zero crossing represents a stable lock point, where the laser and resonator are stabilized in a totally off-resonant state. Such undesired stable operating points are also called Trojan operating points.17,18 Since
The modulation frequency is typically much smaller than the FSR of the resonator, a large disturbance (causing large laser frequency or resonator length variations) may lead to an erroneous stabilization on the Trojan operating point. When this occurs, the correct lock point must be restored either manually or via an automated process, which requires dedicated electronics and can take up to several ms.\(^{25-27}\)

In this paper, we demonstrate a simple way to avoid off-resonant Trojan operating points in a PDH error signal, by modulating the seed laser at \(v_M = \Delta f/2\), i.e., at half the FSR of the resonator. In doing so, the off-resonant lock point between two resonances is made unstable, resulting in an “infinite” capture range so that re-locking always occurs on a resonance independently of the size of the perturbation. Our scheme is particularly well suited for injection-seeding lasers or optical parametric oscillators.

The paper is organized as follows: In Sec. II, we present the theory of the PDH error signal, highlighting the peculiarities related to the use of \(v_M = \Delta f/2\). We also emphasize the parameter range in which the locking scheme works best and we link our scheme to recent ideas that extend the linear range of the PDH error signal.\(^{25-28}\)

In Sec. III, we present an implementation of our scheme in an injection-seeded thin-disk laser (TDL).

### II. PDH Scheme Without Trojan Operating Points

#### A. Analytical expression for the classic PDH error signal

In general, a resonator, where the losses mainly occur at the end-mirrors, can be simplified to a two-mirror resonator, where the mirrors have power reflectivities \(R_1\) and \(R_2\). Without loss of generality, the possible intra-resonator gain can be included in \(R_2\); so that \(R_2 > 1\) is possible. For such a general resonator, we define its finesse\(^{25}\) as

\[
\mathcal{F} = \frac{2\pi}{\text{Im}(R_1 R_2)}.
\]

Where \(\delta f\) is the FWHM linewidth of the resonances. In this study, \(v\) denotes the relative detuning between the seed laser frequency and the nearest resonance frequency of the resonator TEM\(_{00}\) modes.

Depending on \(v\), part of the phase modulated seed light is reflected from the resonator. Its electric field amplitude and the nearest resonance frequency of the resonator TEM\(_{00}\) modes.

\[
F(v) = \sqrt{R_1} = \frac{1 - R_1 \sqrt{R_2} \exp\left[2\left(\pi \frac{v}{\mathcal{F}} + \phi\right)\right]}{1 - \sqrt{R_1 R_2} \exp\left[2\left(\pi \frac{v}{\mathcal{F}} + \phi\right)\right]},
\]

where \(\phi\) is an additional phase shift that the light might acquire over one resonator round trip (e.g., by propagating through a gain medium). For simplicity, in the following, we set \(\phi = 0\).

From this reflection coefficient, the well known PDH error signal is obtained as\(^{27}\)

\[
e(v, v_M) = -2\sqrt{P_e P_s} \text{Im}\left[F(v) F^* (v + v_M) - F^* (v) F(v - v_M)\right],
\]

where \(v_M\) is the modulation frequency, \(P_e\) and \(P_s\) are the power in the carrier and the sidebands of the seed laser, respectively, \(\text{Im}[\ldots]\) takes the imaginary part, and \(^*\) denotes complex conjugation. Note that this is the error signal only for the case of demodulation at \(v_M\) and phase delay \(\Delta \varphi = 0\) (see Subsection II E).

The modulation frequency \(v_M\) can be expressed in terms of the FSR as \(v_M = \xi \Delta f\), where \(0 < \xi < 1\) so that the PDH error signal can be re-written as

\[
e(v, \xi) = -2\sqrt{P_e P_s} \text{Im}\left[F(v) F^* (v + \xi \Delta f) - F^* (v) F(v - \xi \Delta f)\right],
\]

Inserting Eq. (4) into Eq. (6), we find

\[
e(v, \xi) = 4\sqrt{P_e P_s} \sin(\xi \pi) \frac{G_2(v, \xi)}{G_1(v) + G_1(v + \xi \Delta f)} \frac{G_2(v - \xi)}{G_1(v - \xi \Delta f)}
\]

with

\[
G_1(v) = 1 + y^2 - 2y \cos\left(\frac{2\pi v}{\Delta f}\right)
\]

and

\[
G_2(v, \xi) = (y'^2 R_1 - R_2) \cos(\xi \pi) + \frac{[y - y' + y(R_2 - R_1)] \cos\left(\frac{2\pi v}{\Delta f} + \xi \pi\right)}{1 + y^2 - 2y \cos\left(\frac{4\pi v}{\Delta f}\right)}
\]

where

\[
y = \sqrt{R_1 R_2}.
\]

Instead of using the parameter \(y\), these equations can also be expressed in terms of the finesse by performing the substitution,

\[
y = \exp\left(-\frac{\pi}{\mathcal{F}}\right)
\]

#### B. Modulating at half the free spectral range

If we modulate the seed laser at half the FSR of the resonator, i.e., \(v_M = \Delta f/2\) or \(\xi = 1/2\), Eq. (7) simplifies to

\[
e\left(v, \frac{1}{2}\right) = 8\sqrt{P_e P_s} \frac{[y' - y(1 + R_2 - R_1)] \sin\left(\frac{2\pi v}{\Delta f}\right)}{1 + y^2 - 2y \cos\left(\frac{4\pi v}{\Delta f}\right)}.
\]

In this case, the blue and the red sidebands from neighboring resonances overlap, and the error signal is free from Trojan operating
points, as stressed by the shaded area spanning the whole region between two resonances in Fig. 2.

Since $0 < \gamma < 1$, we find

$$
\epsilon\left(\nu, \frac{1}{2}\right) \leq 0 \quad \text{for } \nu \in [-\Delta f/2, 0]
$$

$$
> 0 \quad \text{for } \nu \in [0, \Delta f/2]
$$

as the denominator in Eq. (12) is a positive quantity. This guarantees that only stable lock points coincide with the resonator resonances. The feedback loop will thus always re-lock on the nearest resonance independently of the size of the perturbation.

### C. Tolerable mismatch in the case $\nu_M \neq \Delta f/2$

For the practical realization of this locking scheme, it is key to understand the sensitivity of the error signal to a mismatch $\nu_M \neq \Delta f/2$. The maximal mismatch allowed to still achieve correct locking over the whole range is reached when the error signal has a saddle point at $\nu = \Delta f/2$ so that the sign of the error signal still only flips at $\nu = \Delta f/2$ (see Fig. 3). To find the modulation frequency for which the saddle point appears in the error signal, we calculate the derivative of the error signal at $\nu = \Delta f/2$,

$$
\frac{\partial \epsilon(\nu, \xi)}{\partial \nu} \bigg|_{\nu = \Delta f/2} = \frac{16\sqrt{P_c}P_s\gamma^2}{(1 + \gamma)^2(1 + \gamma^2 + 2\gamma \cos(2\xi\pi))^2} \times \sin(\xi\pi),
$$

(14)

where $\gamma_0 = 1 - \gamma^4 - 2\gamma(R_1 + R_2) + (\gamma^2 - 1)(R_1 R_2)$. The non-trivial zeros of Eq. (14) are

$$
\xi_k = 2\pm \frac{1}{2\pi} \arccos \frac{\gamma_0 - 2\gamma(1 - \gamma^2)}{2\gamma^2 R_1 R_2},
$$

(15)

which correspond to the minimal and maximal values that $\xi$ can take in order to avoid additional zero crossing in the PDH error signal. The allowed mismatch for a general resonator is illustrated in Fig. 4. The figure shows the two solutions obtained in Eq. (15) vs the gain factor $\gamma$. The gray shaded area between the two curves indicates
the allowed region of modulation frequencies in terms of $\xi$, where $\xi = 0.5$ means modulation at exactly half the FSR. Clearly, higher values of $F$ (or $\gamma$) reduce the tolerance for a mismatch between $\nu_M$ and $\Delta f/2$. For this reason, our scheme is particularly suited for injection seeding of laser resonators with not too high finesse. In this case, red is indicated a typical value of $F \approx 19$ (or $\gamma \approx 0.85$, with $R_1 = 0.5$ and $R_2 = 1.44$), which we used in our experimental verification (see Sec. III). In this case, we could tolerate $\nu_M \in [0.475\Delta f, 0.525\Delta f]$, i.e., a mismatch between $\nu_M$ and $\Delta f$ of about $\pm 5\%$. This requirement is easy to satisfy either by mechanical design or by using a frequency-tunable modulator.

D. Dependence of the locking scheme on resonator finesse

Our PDH locking scheme with $\nu_M = \Delta f/2$ is best suited for locking a low-finesse resonator to a seed laser. Indeed, Fig. 5 illustrates how the error signal approaches zero between the carrier and the sideband for increasing $F$. If the electronic noise on the error signal is too large, the error signal can have random zero crossings that would lead to Trojan operating points and jeopardize the whole idea of the half FSR locking; for example, a (conservative) noise level of 10%, as indicated by the dashed horizontal line in Fig. 5, requires the finesse of the seeded resonator to be $F \leq 55$ (or $\gamma \leq 0.95$).

E. Dependence of the locking scheme on the demodulation phase shift

Strikingly, our scheme is rather insensitive to variations of the phase shift $\Delta \phi$ used in the demodulation (see Sec. III). While the amplitude of the error signal decreases for $\Delta \phi \neq 0$, the overall shape stays the same as shown in Fig. 6. This property simplifies the practical implementation and optimization of the PDH loop parameters.

F. Comparison to schemes with a large linear locking range

Recently, a PDH scheme has been proposed in which the conventional error signal is divided by the transmitted power $T(\nu, \nu_M)$ from the resonator\textsuperscript{22,23} to obtain a new error signal,

$$\tilde{\epsilon}(\nu, \nu_M) = \frac{\epsilon(\nu, \nu_M)}{T(\nu, \nu_M)}, \quad (16)$$

with an increased linear dynamic range. Our scheme can be combined with this approach, resulting in an error signal as shown in Fig. 7. The modified error signal merges the advantage of both schemes: an “infinite” (over the whole FSR) capture-range and more linear behavior. The new error signal is more noisy between the resonance and the sidebands since $T(\nu, \nu_M)$ [i.e., the denominator of Eq. (16)] is small in this region. However, for this large detuning, the noise should not excessively disturb the proper working of the feedback loop, especially for low-finesse resonators. Moreover, non-linear filtering\textsuperscript{24} may be employed to reduce this noise.

III. EXPERIMENTAL VERIFICATION

We tested the PDH lock at $\nu_M = \Delta f/2$ for an injection seeded pulsed thin-disk laser (TDL) at 1030 nm. While details on the TDL will be published elsewhere, its simplified scheme is sketched in Fig. 8. A single-frequency laser with linewidth $<30$ kHz (Toptica DL Pro) at 1030 nm wavelength was used to seed the TDL resonator. Resonant incoupling of the seed was achieved by stabilizing the length of the TDL resonator via the PDH method to the seed laser.
frequency. The TDL resonator length was adjusted by having one end-mirror mounted on a piezo-electric actuator (piezo). The resonator length was $L = 1.85\, \text{m}$ corresponding to a FSR of $\Delta f = c/2L = 81\, \text{MHz}$. Phase modulating the seed laser at $\nu_M = \Delta f/2 = 40.5\, \text{MHz}$ thus allowed us to obtain the PDH error signal free from Trojan operating points as shown in Fig. 9. The phase modulation was achieved by current modulating the seed laser with the output from one channel of a function generator (Tektronix AFG1062). The second channel was set to the same frequency and used as a reference to demodulate the signal from the fast photodiode (PD) measuring the light reflected from the TDL resonator. The relative phase $\Delta \phi$ between both channels could be adjusted on the function generator.

We first tested the robustness of our PDH locking scheme operating the TDL below the lasing threshold in CW-mode. The finesse of the TDL resonator was $\mathcal{F} \approx 19$, and its length was locked to the seed while the TDL crystal was pumped at 350 W and water impingement-cooled from the back side. Figure 10 shows the intra-resonator intensity (red), the closed loop PDH error signal (black), and the feedback voltage signal to the piezo (blue) over more than 15 h. During this time, five relocks occurred (marked by arrows in the figure) when the high-voltage amplifier of the proportional-integral-derivative (PID) control unit ran into its limit. No false re-locking was observed.

We then tested the transient behavior of our PDH lock by seeding the TDL in CW and pulsing it with a repetition rate of 100 Hz...
to introduce a large disturbance in the laser dynamics. As expected, a laser pulse strongly perturbs the PDH error signal and, for a short time (<1 μs), also saturates the fast PD. However, saturation is beneficial since it limits the sensitivity of the feedback loop to the pulse because the constant (saturated) signal from the PD is mixed with the reference and low-pass filtered down to zero during this time. Hence, the error signal is zero while the PD is saturated, which prevents the feedback loop from correcting the TDL resonator length by an excessive amount, essentially leaving the resonator freely drifting until the PD recovers.

Figure 11 shows the intra-resonator power (red), the closed loop error signal (black), and the applied feedback voltage to the piezo (blue) during and just after a pulse extraction from our TDL. No false re-locking was observed also in this pulsed operating mode. The 20 ns long laser pulse (coupled out at t = 0) disturbs the error signal for a short time. The fast saturation of the diode signal limits the excursion of the error signal so that the error signal stabilizes already after 40 μs, and the TDL resonator is again locked on resonance with the seed. Such a short stabilization time could, in principle, allow our system to deliver injection seeded pulses at a 10–20 kHz repetition rate.

IV. DISCUSSION AND CONCLUSION

We proposed a modified PDH locking scheme, where the error signal is free from Trojan operating points, by phase modulating the seed laser at half the FSR of the seeded resonator. In this way, the dynamic range of the PDH lock is broadened to “infinity” (the full FSR), ensuring correct re-locking to a resonator TEM00 mode even after large perturbations.

We applied this technique to seed a thin-disk laser at 1030 nm, from which we obtained single-frequency pulses of 20 ns length and 35 mJ pulse energy. The disk-laser was tested for >15 h, and no false locking was observed.

Our method requires good alignment and mode-matching of the seed to the resonator since higher-order transverse modes distort the PDH error signal and can lead to Trojan operating points. However, operating seeded laser resonators in TEM00 is not problematic, thanks to aperture effects (e.g., soft aperture of the gain medium) and since the typical seed lasers already run in TEM00. The reduction of the finesse due to the aperture effects is advantageous in our approach since a high finesse would lead to a PDH error signal very close to zero for detunings between the carrier and the sidebands, which, in the presence of excessive noise, might trap the feedback loop and behave like a Trojan operating point.

Our locking scheme is particularly useful in situations where single-frequency operation of a master-slave system is required under rough conditions or over an extended period of time; for example, high-power/energy injection-seeded laser systems subject to large fluctuations and drifts of the running conditions would benefit from such a robust locking mechanism. Compared to widely used ramp-fire techniques, our method avoids the disadvantage of scanning the position of the end-mirror to obtain the resonance condition. Better stability and higher repetition rates might thus be feasible. Recently, high-frequency RF-drivers, EOMs, and photodiodes with bandwidths >50 GHz have become available so that our technique can be applied to resonators shorter than 1 cm. This extends the range of applications of our technique to, for example, injection-seeded OPOs.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Manuel Zeyen: Conceptualization (equal); Data curation (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Project administration (equal); Visualization (equal); Writing – original draft (equal). Lukas Affolter: Data curation (equal); Investigation (supporting). Marwan Abdou Ahmed: Resources (supporting). Thomas Graf: Resources (supporting). Ouguzhan Kara: Investigation (equal). Klaus Kirch: Conceptualization (supporting); Funding acquisition (equal); Project administration (supporting); Resources (equal). Miroslaw Marszałek: Data curation (equal); Visualization (supporting); Writing – review & editing (lead). François Nez: Conceptualization (equal); Funding acquisition (equal). Ahmed Ouf: Validation (equal); Writing – review & editing (supporting). Randolph Pohl: Funding acquisition (equal); Resources (equal). Siddharth Rajamohanan: Validation (equal). Pauline Yzambard: Writing – review & editing (supporting). Aldo Antognini: Conceptualization (equal); Funding acquisition (equal); Methodology (equal); Project administration (equal); Supervision (equal). Karsten Schuhmann: Conceptualization (lead); Formal analysis (supporting); Investigation (supporting); Methodology (equal); Project administration (supporting); Supervision (lead); Writing – review & editing (supporting).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

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