Prediction of the CP violation in the Phenomenological Quark-Lepton Mass Matrix Approach

M. Fukugita\textsuperscript{1,2}, M. Tanimoto\textsuperscript{3} and T. Yanagida\textsuperscript{4}

\textsuperscript{2} Institute for Advanced Study, Princeton, NJ 08540, U. S. A.
\textsuperscript{1} Institute for Cosmic Ray Research, University of Tokyo, Tanashi, Tokyo 188, Japan
\textsuperscript{3} Faculty of Education, Ehime University, Matsuyama 790-8577, Japan
\textsuperscript{4} Department of Physics and RESCEU, University of Tokyo, Tokyo 113-0033, Japan

Abstract

We explore the phenomenological quark-lepton mass matrices, which are devised following the S\textsubscript{3} flavour symmetry principle, yet fully consistent with SU(5) gauge models with the Higgs particles of 5, 45 and their conjugates. The model contains 10 free parameters altogether. When 6 parameters are fixed by charge 2/3 quark and charged lepton masses, charge\((-1/3)\) quark masses and all quark mixing matrix elements are predicted to be close to experiment, leaving some narrow ranges still as freedom. Further specification of 3 more parameters (using, m\textsubscript{d}, Cabibbo angle and |V\textsubscript{cb}|) suffices to fix the quark mixing matrix nearly completely, and all elements come out to be in accurate agreement with experiment. We obtain the CP violation phase as a prediction.
Grand unification of gauge theories has not given much hint to our understanding of the quark-lepton mass spectrum. The most successful among many attempts is perhaps the “prediction” of the mass relations between the charge $-1/3$ quarks (referred simply to as down quarks) and the charged leptons by an introduction of $45$-plet Higgs in addition to the standard $5$-plet within the SU(5) grand unification \[1\]. The relation reads,

$$m_d = 3 m_e, \quad m_s = \frac{1}{3} m_\mu, \quad m_b = m_\tau,$$

which is often called Georgi-Jarlskog mass relation. No successful prediction, however, has been known for the charge $2/3$ quark spectrum and hence for quark mixing. Even more difficult is to understand the CP violation phase and its origin.

The only approach which turns out to be “successful” in giving correct mass-mixing relations for quarks is an empirical approach, where some discrete symmetry is imposed on the form of mass matrices and fix parameters using some quark masses as input \[2, 3, 4\]. One of the most unsatisfactory aspect of such approaches was that its consistency is not clear with the unified gauge model, which anyway we must impose at some level. If one would impose the compatibility with a gauge model in a straightforward way, we are usually led to unwanted relations for quark and lepton masses as a remnant of prototype unified gauge models.

We have devised in a previous paper \[5\] phenomenological quark-lepton mass matrices based on the $S_3$ permutation symmetry principle in a manner fully compatible with SU(5) grand unification. This model results in an approximate Georgi-Jarlskog relation and a mixing angle pattern for the quarks, which are in decent agreement with experiment. The model also successfully applies to the neutrino mass-mixing problem with bimaximal mixing as a natural outcome \[6\]. For sake of simplicity of the argument, we have assumed in \[5\] the matrix elements to be all real, ignoring all phases which could in principle appear therein; also, accurate agreement with experiment was not sought for the mixing angles.
In this paper we extend our analysis allowing for full degrees of freedom of the mass matrices, trying to cure the defects of the previous model. We have now more parameters, but they are still tightly constrained within the model, and this leads to a prediction of the CP violating phase. Here we should quote an earlier work of Fritzsch [7], who has also derived the CP violating phase in his matrix model approach.

We begin with the Yukawa coupling in the SU(5) model:

\[ L_{\text{Yukawa}} = Y(5_H)_{ij} 10_i 10_j 5_H + Y(45_H)_{ij} 10_i 10_j 45_H + \kappa(5_H 5_H)_{ij} 5_i^* 5_j^* \frac{5_H 5_H}{M_R}, \]  

(2)

where bold face symbols with suffix \( H \) denote Higgs scalars of a specified multiplet, and those with suffix \( i \) or \( j \) (refer to flavour) are SU(5) matter fields, \( 5_i^* = (d_e^c, d_d^c, d_d^c, e^- \nu_e)_L \) and \( 10_j = (u^c_1, ..., u^c_1, ..., d^c_i, ..., e^+)_L \). We specify down quarks and charged leptons with suffix \( D/E \) as they are unified. The last term of eq. (2) is an effective neutrino coupling where the neutrino is assumed to be of the Majorana type. We suppose that it is induced from heavy Majorana right-handed neutrinos \( N_i \) (SU(5) singlet) with mass \( M_R \), so that \( 45_H 45_H \) does not appear in (2).

We postulate the mass matrices of the form [8]:

\[ M_D = \frac{K_D}{3} \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right) + \left[ \begin{array}{cc} -\epsilon_D & 0 \\ 0 & \epsilon_D \\ \epsilon_D & 0 \end{array} \right], \]  

(3)

for the down-quarks, and

\[ M_E = \frac{K_D}{3} \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right) + \left[ \begin{array}{cc} -\epsilon_E & 0 \\ 0 & \epsilon_E \\ \epsilon_E & 0 \end{array} \right], \]  

(4)

for the charged leptons. Here, the main part of the mass matrices is induced by a 5 plet, which is \( S_3 \) permutation symmetry invariant. We write it as \( S_3^{10} \times S_3^5 \) where \( 10 \) and \( 5 \) refer to representations of fermions. We break \( S_3 \) symmetry in a hierarchical manner. We introduce \( \delta \) terms to break \( S_3^{10} \times S_3^5 \) down to \( S_2^{10} \times S_2^5 \). We assume that \( \delta_D \) and \( \delta_E \) elements
are generated from the coupling to a $45^*_H$-plet Higgs scalar. This assignment removes the unwanted down-quark charged lepton mass degeneracy of the minimal SU(5) model, but produces the Georgi-Jarlskog mass relation. Further symmetry breaking is caused by $\epsilon$ terms ($\epsilon \ll \delta$) in a way consistent with $5^*_H$ to allow further adjustment of mass hierarchy. We have then

$$\epsilon_E = \epsilon_D, \quad \delta_E = -3\delta_D. \quad (5)$$

For the up quark masses, respecting the same principle of the symmetry breaking pattern as with the down quark/charged lepton sector, we write

$$M_U = \frac{K_U}{3} \left[ \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right] + \left[ \begin{array}{ccc} -\epsilon_U & 0 & \delta_U \\ 0 & \epsilon_U & \delta_U \\ -\delta_U & -\delta_U & 0 \end{array} \right]. \quad (6)$$

where the main term and $\epsilon$ terms arise from $5_H$ and $\delta$ terms from $45_H$. Note that the texture of $\delta_U$ in eq.(6) is an unique invariant of $S_2^{10} \times S_2^5$ among the anti-symmetric $(45)$ mass matrix.

In general, parameters $\epsilon_D$, $\delta_D$, $\epsilon_U$ and $\delta_U$ are complex, and we can express them as $\epsilon_i = |\epsilon_i|e^{i\alpha_i}$ and $\delta_i = |\delta_i|e^{i\beta_i}(i = U, D)$, whereas we assumed them to be all real in [5]. We write the matrices in the hierarchical base [8] by applying an orthogonal transformation in order to investigate the structure of the phase,

$$F^T M_D F \equiv \overline{M}_D = \frac{K_D}{3} \left[ \begin{array}{ccc} 0 & -\frac{2}{\sqrt{3}}\epsilon_D & -\frac{1}{\sqrt{6}}\epsilon_D \\ -\frac{2}{\sqrt{3}}\epsilon_D & \frac{2}{3}(\delta_D - \epsilon_D) & \frac{1}{3\sqrt{2}}(2\delta_D + \epsilon_D) \\ -\frac{1}{\sqrt{6}}\epsilon_D & \frac{2}{3\sqrt{2}}(2\delta_D + \epsilon_D) & 3 + \frac{4}{3} + \frac{2\epsilon_D}{3} \end{array} \right], \quad (7)$$

and

$$F^T M_U F \equiv \overline{M}_U = \frac{K_U}{3} \left[ \begin{array}{ccc} 0 & \frac{1}{\sqrt{3}}\epsilon_U & -\sqrt{2}\epsilon_D \\ \frac{1}{\sqrt{3}}\epsilon_U & 0 & \sqrt{2}\delta_U \\ \frac{1}{\sqrt{3}}\epsilon_D & \sqrt{2}\delta_U & 3 \end{array} \right], \quad (8)$$

where

$$F = \left[ \begin{array}{ccc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{array} \right]. \quad (9)$$
Extra phases in $M_D$ and $M_U$ are removed by a phase transformation applied to the quark fields $q_L$ and $q_R$. With diagonal phase matrices $P_D$ and $P_U$, we obtain

$$
\hat{M}_D = P_D M_D P_D \simeq \frac{K_D}{3} \begin{bmatrix}
0 & -\frac{2}{\sqrt{3}}|\epsilon_D| & -\frac{1}{\sqrt{6}}|\epsilon_D|e^{i\beta_D} \\
-\frac{2}{\sqrt{3}}|\epsilon_D| & \frac{2}{3}|\delta_D|e^{-i\beta_D} & -\frac{\sqrt{2}}{3}|\delta_D| \\
-\frac{1}{\sqrt{6}}|\epsilon_D|e^{i\beta_D} & -\sqrt{\frac{2}{3}}|\delta_D| & 3
\end{bmatrix}, \quad (10)
$$

and

$$
\hat{M}_U = P_U M_U P_U = \frac{K_U}{3} \begin{bmatrix}
0 & -\frac{1}{\sqrt{3}}|\epsilon_U| & 0 \\
-\frac{1}{\sqrt{3}}|\epsilon_U| & \frac{2}{3}|\delta_U|e^{i\beta_U} & 0 \\
-\frac{2}{\sqrt{6}}|\epsilon_U|e^{i\beta_U} & 0 & 3
\end{bmatrix}, \quad (11)
$$

where

$$
P_D = \begin{bmatrix}
e^{-i(\alpha_D - \beta_D)} & 0 & 0 \\
0 & e^{-i\beta_D} & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad P_U = \begin{bmatrix}
e^{-i(\alpha_U - \beta_U)} & 0 & 0 \\
0 & e^{-i\beta_U} & 0 \\
0 & 0 & 1
\end{bmatrix}. \quad (12)
$$

In eq.(10), only leading terms are retained in the (2,2), (2,3), (3,2) and (3,3) elements for simplicity of our expressions, while we keep all terms when we carry out a numerical analysis. The expression of eq.(11) is exact. Phase matrices $P_D$ and $P_U$ contribute to the Cabibbo-Kobayashi-Maskawa (CKM) matrix as:

$$
V_{CKM} = U_D^\dagger P_U P_D U_D, \quad (13)
$$

where $U_D$ and $U_U$ are unitary matrices, which diagonalize $\hat{M}_D$ and $\hat{M}_U$. For convenience, we define the phase matrix $Q$ by

$$
Q = P_U^\dagger P_D = \begin{bmatrix}
e^{i\sigma} & 0 & 0 \\
0 & e^{i\tau} & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad (14)
$$

where $\sigma = (\alpha_U - \beta_U) - (\alpha_D - \beta_D)$ and $\tau = \beta_U - \beta_D$.

The unitary matrix $U_D$ is given by

$$
U_D = \begin{bmatrix}
1 & 0 & 0 \\
0 & c_2^D & s_2^D \\
0 & -s_2^D & c_2^D
\end{bmatrix} \begin{bmatrix}
e^{-i\beta_D} & 0 & 0 \\
0 & e^{i\beta_D} & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
c_1^D & s_1^D & 0 \\
-s_1^D & c_1^D & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
e^{-i\beta_D} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
c_3^D & 0 & s_3^D \\
0 & 1 & 0 \\
-s_3^D & 0 & c_3^D
\end{bmatrix}, \quad (15)
$$
where $s_i^D = \sin \theta_i^D$ and $c_i^D = \cos \theta_i^D$ with

$$
s_1^D \simeq -\sqrt{3} \frac{|\epsilon_D|}{|\delta_D|} \simeq -\sqrt{\frac{m_d}{m_s}}, \quad s_2^D \simeq -\frac{\sqrt{2}}{9} |\delta_D| \simeq -\frac{1}{3} \frac{m_s}{\sqrt{2} m_b}, \quad s_3^D \simeq -\frac{|\epsilon_D|}{3\sqrt{6}} \simeq -\frac{1}{2\sqrt{2}} \sqrt{\frac{m_d m_s}{m_b}}.
$$

The down-quark masses are then,

$$
m_b \simeq K_D (1 + \frac{1}{9} |\delta_D| \cos \beta_D),
m_s \simeq \frac{2}{9} K_D |\delta_D| \left(1 - \frac{1}{18} |\delta_D| \cos \beta_D\right),
m_d \simeq -\frac{2}{3} K_D \frac{|\epsilon_D|^2}{|\delta_D|^2} \left(1 - \frac{1}{4} |\delta_D| \cos \beta_D\right).
$$

The charged-lepton masses are given as

$$
m_\tau \simeq K_D (1 - \frac{1}{3} |\delta_D| \cos \beta_D),
m_\mu \simeq -\frac{2}{3} K_D |\delta_D| \left(1 - \frac{1}{6} |\delta_D| \cos \beta_D\right),
m_e \simeq \frac{2}{9} K_D \frac{|\epsilon_D|^2}{|\delta_D|^2} \left(1 - \frac{3}{4} |\delta_D| \cos \beta_D\right).
$$

The matrix for the up quark sector $U_U$ is obtained as

$$
U_U = \begin{bmatrix}
1 & 0 & 0 & \frac{c_1^U}{s_1^U} & 0 & 0 & \frac{c_2^U}{s_2^U} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{c_2^U}{s_2^U} & \frac{s_2^U}{c_2^U} & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{c_3^U}{s_3^U} & 0 & s_3^U & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\frac{c_3^U}{s_3^U} & 0 & s_3^U & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix},
$$

where

$$
s_1^U \simeq -\sqrt{\frac{3}{2}} |\epsilon_U| \simeq -\sqrt{\frac{m_u}{m_c}}, \quad s_2^U \simeq -\sqrt{3} |\delta_U| \simeq -\sqrt{\frac{m_c}{m_t}}, \quad s_3^U \simeq -\frac{2}{3\sqrt{6}} |\epsilon_U| \simeq -\sqrt{2} \frac{m_u m_c}{m_t^2}.
$$

The up-quark masses are

$$
m_t \simeq K_U \left(1 - \frac{2}{9} |\delta_U|^2\right), \quad m_c \simeq \frac{2}{3} K_U |\delta_U|^2, \quad m_u \simeq -\frac{1}{6} K_U \frac{|\epsilon_U|^2}{|\delta_U|^2}.
$$

Substituting $U_D$ and $U_U$ into eq.(13), we obtain the CKM matrix in terms of the quark masses and the phase parameters. After taking $c_1^i \simeq c_2^i \simeq c_3^i \simeq 1 (i = D, U)$ and
neglecting $s_3^i s_3^i$, $s_2^i s_3^i$ and $s_1^i s_2^i$ ($i = D, U$) terms, we obtain the CKM matrix elements approximately as

\[
V_{ud} \simeq \sqrt{1 - \frac{m_d}{m_s}} e^{i(\sigma - \beta_D)},
\]

\[
V_{us} \simeq \frac{m_d}{m_s} e^{i(\sigma - \beta_D)} - \sqrt{\frac{m_u}{m_c}} e^{i\tau},
\]

\[
V_{ub} \simeq -\frac{1}{2\sqrt{2}} \sqrt{\frac{m_d m_s}{m_b^2}} e^{i(\sigma - \beta_D)} - \frac{1}{\sqrt{2} m_b} \sqrt{\frac{m_u}{m_c}} e^{i\tau} + \sqrt{\frac{m_u}{m_t}} + \sqrt{\frac{2 m_u m_c}{m_t^2}} e^{-i\beta_U},
\]

\[
V_{cd} \simeq \sqrt{\frac{m_d}{m_s}} e^{i\tau} - \frac{m_u}{m_c} e^{i(\sigma - \beta_D)},
\]

\[
V_{cs} \simeq \frac{1}{\sqrt{2}} \sqrt{\frac{m_u}{m_c}} e^{i(\sigma - \beta_D)},
\]

\[
V_{cb} \simeq -\frac{1}{\sqrt{2}} \sqrt{\frac{m_u}{m_c}} e^{i(\sigma - \beta_D + \beta_U)},
\]

\[
V_{td} \simeq \frac{1}{2\sqrt{2}} \sqrt{\frac{m_d m_s}{m_b^2}} + \frac{1}{\sqrt{2} m_b} \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_d m_c}{m_t^2}} e^{i\tau} - \sqrt{\frac{2 m_u m_c}{m_t^2}} e^{i(\sigma - \beta_D + \beta_U)},
\]

\[
V_{ts} \simeq \frac{1}{\sqrt{2}} \sqrt{\frac{m_u}{m_c}} e^{i\tau},
\]

\[
V_{tb} \simeq 1.
\]

For the CP violation parameter, the angle $\gamma$ that enters in the unitarity triangle \([9]\) reads,

\[
\gamma \simeq -(\sigma - \beta_D) + \sin^{-1}\left(-\frac{1}{2\sqrt{2}} \frac{\sqrt{m_d m_s}}{m_b} \sin(\sigma - \beta_D) \right) - \sin^{-1}\left(\sqrt{\frac{m_u}{m_c}} \frac{\sin(\sigma - \beta_D)}{|V_{13}|} \right) - \sin^{-1}\left(\sqrt{\frac{m_u}{m_c}} \frac{\sin(\sigma - \beta_D)}{|V_{12}|} \right),
\]

(23)

in the case of a small $\tau$.

These expressions agree with the result of a numerical calculation (without any approximations) within 10% error. Although there are four phase parameters $\sigma$, $\tau$, $\beta_D$ and $\beta_U$ in our matrices, the CKM matrix is determined in practice by only two of them, $\sigma - \beta_D$ and $\tau$, because the last terms of $V_{ub}$ and $V_{td}$, which contain phases other than the two, are strongly suppressed compared with other terms. In \([9]\) we have shown that these matrix elements, where all phases are completely dropped, yield a resonable description for all
CKM matrix elements. The point of the present paper is that all CKM matrix elements, including the CP violation phase, are completely determined, once the above two phases are fixed by the adjustment of two of the CKM matrix element, say $V_{us}$ and $V_{cb}$.

While proceeding to a numerical analysis, we need some care as to the input data. Since the CKM matrix elements in eq.(22) and masses eqs.(20,21) are discussed at the $SU(5)$ GUT scale, predictions should also be compared at the GUT scale, rather than at the electroweak scale. Since a supersymmetric extension is the only way to make the $SU(5)$ GUT viable, we take quark and lepton masses at the GUT scale obtained in the minimal SUSY model (MSSM) with the aid of renormalisation group equation (RGE) incorporating two-loop\[10\]. These mass parameters are given in Table 1.

Let us first discuss charge $-\frac{1}{3}$ quark and charged lepton masses. With the three charged lepton masses as input, the parameters in the charged lepton/down quark sector are determined to be $K_D = 1.203$GeV, $|\delta_D| = 0.080$ and $|\epsilon_D| = 0.0104$, which in fact satisfies $K_D \gg |\delta_D| \gg |\epsilon_D|$. The ratio of $d$-quark mass to electron mass is given, using eqs.(17) and (18), by

$$\frac{|m_d|}{m_e} \approx 3 \frac{1 + \frac{1}{4}|\delta_D|\cos\beta_D}{1 - \frac{3}{4}|\delta_D|\cos\beta_D}. \quad (24)$$

This shows the dependence of $|m_d/m_e|$ on further two parameters. By carrying out an accurate numerical calculation with $|\delta_D| = 0.08$, we see that this ratio takes a value between 2.3 and 3.7, which corresponds to $m_d = 0.76 - 1.20$MeV, when $\beta_D$ is varied form 0 to $+\pi$. This is compared with the “experimental value”, $m_d = 1.3 \pm 0.2$MeV (i.e., the ratio is 3.4-4.6). Requiring an agreement with experiment leads to $0.7 \leq \cos\beta_D$. Here we take $\cos\beta_D = 1$ for further analysis. For this case we have $m_d = 1.20$MeV. The prediction for other down quark masses is given in Table 1. The values of $m_b$ and $m_s$ are somewhat larger, but taken as acceptable when we consider the uncertainty in the mass analysis using low-order perturbation theory.

Now we are concerend with the CKM matrix element. We obtain the three parameters
of the up quark sector to be $K_U = 129 \text{GeV}$, $|\delta_U| = 0.1026$ and $|\epsilon_U| = 0.00071$ ($K_U \gg |\delta_U| \gg |\epsilon_U|$ being satisfied) if the central values in Table 1 are adopted for up-quark masses. We note, however, a large uncertainty in the estimate of the top quark mass at the GUT scale, arising from the fact that the top quark mass is near the fixed point of RGE at the electroweak scale. The prediction of the CKM matrix depends on top quark mass used as input; we take account of this large error, $89 - 325 \text{GeV}$ (for a given pole mass $m_t(\text{pole}) = 180 \pm 12 \text{GeV}$), for further analysis of the matrices. As with the case of the mass, we must take account of the running effect of the CKM matrix elements. It is known that the elements $V_{ud}, V_{us}, V_{cd}, V_{cs}$ and $V_{tb}$ are nearly constant during running between the electroweak and the GUT scales. On the other hand, all others are affected by the large Yukawa coupling of the top quark by 10-20%. We may use $|V_{us}| = 0.217 - 0.224$ (GUT scale) to constrain the phase $\phi = \sigma - \tau - \beta_D$. We then obtain $\phi = (\pm 60.5^\circ) - (\pm 68.5^\circ)$. Another phase $\tau$ is fixed by $|V_{cb}| = 0.0347 - 0.036$; we obtain $\tau = 0 - (\pm 22)^\circ$ for $m_t = 129 \text{GeV}$ at the GUT scale. All parameters are now fixed, and a prediction is given for the full CKM matrix elements, as presented in Table 2. In this table we also give experimental values and estimates at the GUT scale after running [10]. All predictions are within the uncertainty of the empirical values. The agreement with experiment is not spoiled even if we take the upper limit value $m_t = 325 \text{ GeV}$ at the GUT scale. On the other hand, if we decrease the top quark mass the agreement is disturbed for $|V_{cb}|$: the prediction goes out of the upper limit of the range allowed by experiment, $0.030 - 0.036$. This limits our consideration to $123 \text{GeV} \leq m_t \leq 325 \text{GeV}$.

The most interesting prediction is perhaps that of the CP violation parameter. We obtain

$$\gamma = 106^\circ - 114^\circ,$$

for $m_t = 129 \text{ GeV}$. This is equivalent to the vertex position of $\rho = -0.098 \sim -0.086$.
and $\eta = 0.221 \sim 0.309$ in Wolfenstein's $\rho - \eta$ plane [11]. If the range of the input $m_t$ is relaxed as discussed above, the prediction for $\gamma$ becomes $76^\circ - 114^\circ$. This value is similar to the prediction of Fritzsch [7], $\gamma = 72^\circ \sim 76^\circ$. In Table 3, we present the prediction of the several key parameters for various top-quark masses. Here, we take $|V_{ub}/V_{cb}|$ as one of the indicators. Fig. 1 is a summary of our prediction for the CKM matrix in the $\rho - \eta$ plane, where currently available experimental constraints are also plotted: (i) $\epsilon_K$ parameter with $B_K = 0.6 - 0.9$ (ii) $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ and (iii) $\Delta M_{B_d}$ with $\sqrt{B_{B_d} F_{B_d}} = 200 \pm 40$MeV.

We have explored the full content of the phenomenological quark-lepton mass matrices, which are derived from $S_3$ flavor symmetry principle and its hierarchical breaking in the framework of SU(5) gauge models, proposed in [4]. Our mass matrices have 10 free parameters altogether, 6 real $K_i$, $|\delta_i|$, $|\epsilon_i|(i = D, U)$ and 4 phases $\beta_D$, $\sigma$, $\beta_D$, $\tau$, excluding neutrino parameters. Using $(m_u, m_c, m_t)$ and $(m_e, m_\mu, m_\tau)$ as input, we have a prediction in decent agreement with experiment for the rest of the physical parameters, irrespective of the free parameters left unspecified. Requiring more precise agreement for $m_d$ and two mixing angles, say $|V_{us}|$ and $|V_{cb}|$, we have a complete determination of the CKM matrix elements including phases. The obtained matrix shows an excellent agreement with experiment within the present experimental accuracy. The most interesting among others is the prediction of the CP violation phase, which would soon be tested to higher accuracy with B-factories.

We have also studied the mixing problem for the lepton sector including neutrinos in a way parallel to the quark sector, but have found little to add to the previous work of ref.[3], except for phases. With the allowed parameter range the net CP violating phase, as defined by Jarlskog [12], is as small as $J_{CP} < 10^{-4}$ because the CP violating phase enters

\footnote{These predictions should in principle be compared with the corresponding values at the GUT scale. Fortunately, the energy scale dependence of these quantities is very weak [10], and we can safely neglect the running effect.}
only the symmetry breaking terms. This is too small to arouse any phenomenological interests.

Acknowledgements

MF thanks the Raymond and Beverly Sackler Fellowship and the Alfred P. Sloan Foundation for the support for the work in Princeton. MT and TY are supported in part by the Grants-in-Aid of the Ministry of Education of Japan (Nos.10640274, 7107).
References

[1] H. Georgi and C. Jarlskog, Phys. Lett. 86 B (1979) 297.

[2] H. Fritzsch, Phys. Lett. 70 B (1977) 436; Nucl. Phys. B 155 (1979) 189.

[3] H. Harari, H. Haut and J. Weyers, Phys. Lett. 78 B (1978) 459;
    Y. Koide, Phys. Rev. D 28 (1983) 252; D 39 (1989) 1391;
    P. Kaus and S. Meshkov, Mod. Phys. Lett. A 3 (1988) 1251;
    M. Tanimoto, Phys. Rev. D 41 (1990) 1586;
    G.C. Branco, J.I. Silva-Marcos and M.N. Rebelo, Phys. Lett. 237 B (1990) 446;
    H. Fritzsch and J. Plankl, Phys. Lett. 237 B (1990) 451;
    P. Ramond, R.G. Roberts and G.C. Ross, Nucl. Phys. B 406 (1993) 19.

[4] L.J. Hall and A. Rašin, Phys. Lett. 315 B (1993) 164.

[5] M. Fukugita, M. Tanimoto and T. Yanagida, hep-ph/9809554, to be published in
    Phys. Rev. D.

[6] M. Fukugita, M. Tanimoto and T. Yanagida, Phys. Rev. D 57 (1998) 4429.

[7] H. Fritzsch, Nucl. Phys. B Proc. Suppl., 64 (1998) 271 (hep-ph/9709300).

[8] H. Fritzsch and J. Plankl, ref. 3.

[9] C. Jarlskog and R. Stora, Phys. Lett. 208 B (1988) 268;
    R. Aleksan, B. Kayser and D. London, Phys. Rev. Lett. 73 (1994) 18.

[10] H. Fusaoka and Y. Koide, Phys. Rev. D 57 (1998) 3986.

[11] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.

[12] C. Jarlskog Phys. Rev. Lett. 55 (1985) 1839.
Table 1: Input quark-lepton mass parameters and the prediction of our model at the GUT scale. $m_d$ is the value with $\cos \beta_D = 1$.

| Parameter | Value (MeV) | Prediction (MeV) |
|-----------|-------------|------------------|
| $m_e$     | 0.325       |                  |
| $m_{\mu}$ | 68.60       |                  |
| $m_{\tau}$| 1.171       |                  |
| $m_d$     | 1.3 ± 0.2   |                  |
| $m_s$     | 26.5$^{+3.4}_{-3.7}$ | |
| $m_b$     | 1.00 ± 0.04 | |
| $m_u$     | 1.0 ± 0.2   | |
| $m_c$     | 302$^{+25}_{-27}$ | |
| $m_t$     | 129$^{+196}_{-40}$ | |

Table 2: The CKM matrix elements. The first column shows experiment, the second is the values estimated at the GUT scale. The third column is the prediction of our model with $m_t = 129$GeV at GUT scale. The underlined values are input.

| Parameter | Value at GUT scale | Prediction at GUT scale |
|-----------|--------------------|-------------------------|
| $|V_{ud}|$       | 0.975 – 0.976       | 0.975 – 0.976           |
| $|V_{us}|$       | 0.217 – 0.224        | 0.217 – 0.224           |
| $|V_{ub}|$       | 0.0018 – 0.0045      | 0.0015 – 0.0040         |
| $|V_{cd}|$       | 0.217 – 0.224        | 0.217 – 0.224           |
| $|V_{cs}|$       | 0.974 – 0.976        | 0.974 – 0.976           |
| $|V_{cb}|$       | 0.036 – 0.042        | 0.030 – 0.036           |
| $|V_{td}|$       | 0.004 – 0.013        | 0.0035 – 0.011          |
| $|V_{ts}|$       | 0.035 – 0.042        | 0.030 – 0.036           |
| $|V_{tb}|$       | 0.999                | 0.999 – 1.000           |

Table 3: Predictions of the CKM matrix elements and the unitarity triangle. $m_t$ is the value at the GUT scale.
Figure 1: Predicted vertex of the unitarity triangle on the $\rho - \eta$ plane for $m_t(GUT)$ varying between 123–325 GeV. Experimental constraints from $\epsilon_K$, $|V_{ub}/V_{cb}|$ and $\Delta M_{B_d}$ are also plotted.