Study of charmonium-nucleon interaction in lattice QCD

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Abstract. We report preliminary results for charmonium-nucleon potential $V_{c\bar{c}N}(r)$ from quenched lattice QCD, which is calculated from the equal-time Bethe-Salpeter amplitude through the effective Schrödinger equation. Our simulations are performed at a lattice cutoff of $1/a=2.0$ GeV in a spatial volume of $(3 \text{ fm})^3$ with the nonperturbatively $O(a)$ improved Wilson action for the light quarks and a relativistic heavy quark action for the charm quark. We have found that the potential $V_{c\bar{c}N}(r)$ is weakly attractive at short distance and exponentially screened at long distance.

Keywords: Lattice QCD, Hadron-hadron interaction

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The heavy quarkonium state such as the charmonium ($c\bar{c}$) state does not share the same quark flavor with the nucleon ($N$). This suggests that the heavy quarkonium-nucleon interaction is mainly induced by the genuine QCD effect of multi-gluon exchange [1, 2]. As an analog of the van der Waals force, two-gluon exchange contribution gives a weakly attractive, but long-ranged interaction between the heavy quarkonium state and the nucleon. However, the validity of the calculation based on the perturbative theory is questionable for QCD where the nature of the strong coupling appears in the long distance region.

The $c\bar{c}$-$N$ scattering at low energies has been studied from first principles of QCD. The $s$-wave $J/\psi$-$N$ scattering length is about $0.1$ fm by using QCD sum rules [4] and $0.71\pm0.48$ fm ($0.70\pm0.66$ fm for $\eta_c$-$N$) by lattice QCD [5], while it is estimated as large as $0.25$ fm from the gluonic van der Waals interaction [1]. All studies suggest that the $c\bar{c}$-$N$ interaction is weakly attractive. This indicates that the possibility of the formation of charmonium bound to nuclei is enhanced. In 1991, Brodsky et al. had argued that the $c\bar{c}$-nucleus ($A$) bound system may be realized for the mass number $A \geq 3$ if the attraction between the charmonium and the nucleon is sufficiently strong [3]. Therefore, precise information on the $c\bar{c}$-$N$ potential $V_{c\bar{c}N}(r)$ is indispensable for exploring nuclear-bound charmonium state like $\eta_c$-$^3$He or $J/\psi$-$^3$He bound state in few body calculations [6].

We recall a recent great success of the $N$-$N$ potential from lattice QCD [7]. In this new approach, the potential between hadrons can be calculated from the equal-time Bethe-Salpeter (BS) amplitude through the effective Schrödinger equation. Thus, the direct measurement of the $c\bar{c}$-$N$ potential is now feasible by using lattice QCD. It should be very important to give a firm theoretical prediction about nuclear-bound charmonium, which is possibly investigated by experiments at J-PARC and GSI.

The method utilized here to calculate the hadron-hadron potential in lattice QCD is based on the same idea originally applied for the $N$-$N$ potential [7, 8]. We first calculate
the equal-time BS amplitude of two local operators (hadrons \( h_1 \) and \( h_2 \)) separated by given spatial distances \( r = |x - y| \) from the four-point correlator \( G^{h_1,h_2}(r,t_4,t_3,t_2,t_1) = \sum_{x,y} \langle \phi^{h_1}(x,t_4) \phi^{h_2}(y,t_3) (\phi^{h_1}(x',t_2) \phi^{h_2}(y',t_1)) \rangle \), which becomes asymptotically proportional to \( \phi_{h_1,h_2}(r) e^{-E(t_3-t_1)} \) for \( |t_3-t_1| \gg 1 \) with fixed \( t_2 \) and \( t_4 \), but keeping \( |t_4-t_2| \gg 1 \). Here \( \phi_{h_1,h_2}(r) = \langle 0 | \phi^{h_1}(x) \phi^{h_2}(y) | h_1 h_2; E \rangle \) with the total energy \( E \) for the ground state of the two-particle \( h_1-h_2 \) state corresponds to a part of the BS amplitude and are called as the BS wave function \([9, 10]\). After an appropriate projection with respect to discrete rotation of the cubic group, which is now “rotational symmetry” on the lattice, one can get the BS wave function projected in the \( s \)-wave. Once the BS wave function \( \phi_{h_1,h_2}(r) \) and the total energy \( E \) are calculated in lattice simulations, the hadron-hadron potential can be obtained by

\[
V_{h_1,h_2}(r) = E + \frac{1}{2 \mu} \nabla^2 \phi_{h_1,h_2}(r)
\]

where \( \mu \) is the reduced mass of the \( h_1-h_2 \) state and \( \nabla^2 \) is defined by the discrete Laplacian with nearest-neighbor points. More details of this method can be found in Ref. \([8]\).

In this study, we only consider the low energy \( \eta_c-N \) interaction, which doesn’t possess the spin dependent part. We have performed quenched lattice QCD simulations on two different lattice sizes, \( L^3 \times T = 32^3 \times 48 \) and \( 16^3 \times 48 \), with the single plaquette gauge action at \( \beta = 6/g^2 = 6.0 \), which corresponds to a lattice cutoff of \( a^{-1} \approx 2.1 \) GeV. Our main results are obtained from the data taken on the larger lattice (\( La \approx 3.0 \) fm). A supplementary data with a smaller lattice size (\( La \approx 1.5 \) fm) are used for a test of the finite size effect. The number of statistics is \( O(600) \) for \( L = 32 \) and \( O(200) \) for \( L = 16 \), respectively.

We use non-perturbatively \( \mathcal{O}(a) \) improved Wilson fermions for the light quarks \( (q) \) and a relativistic heavy quark (RHQ) action for the charm quark \( (Q) \) \([12]\). The RHQ action is a variant of the Fermilab approach \([11]\), which can remove large discretization errors for heavy quarks. The hopping parameter is chosen to be \( \kappa_q = 0.1342, 0.1339, 0.1333 \), which correspond to \( M_q = 0.64, 0.73, 0.87 \) GeV, and \( \kappa_Q = 0.1019 \) which is reserved for the charm-quark mass \( (M_{\eta_c} = 2.92 \) GeV) \([13]\). Each hadron mass is obtained by fitting corresponding two-point correlation functions with a single exponential form. We calculate quark propagators with wall sources, which are located at \( t_{src} = 5 \) for the light quarks and at \( t_{src} = 4 \) for the charm quark, with the Coulomb gauge fixing. The ground state dominance in the four point function is checked by the effective mass plot of the total energy of the \( \eta_c-N \) system.

The left panel of Fig.1 shows a typical result of the projected BS wave function at the smallest quark mass, which is evaluated by a weighted average of data in the time-slice range of \( 16 \leq t - t_{src} \leq 35 \). The wave function is normalized to unity at a reference point \( r = (16, 16, 16) \), which is supposed to be outside of the interaction region. As shown in Fig.1, the wave function is enhanced from unity near the origin so that the low-energy \( \eta_c-N \) interaction is certainly attractive. This attractive interaction, however, is not enough strong to form a bound state as is evident from this figure, where the wave function is not localized, but extended at long distances.

In the right panel of Fig.1, we show the effective central \( \eta_c-N \) potential, which is evaluated by the wave function through Eq. (1) with measured \( E \) and \( \mu \). As is expected,
FIGURE 1. The wave function (left) and the effective central potential (right) in the s-wave $\eta_c$-$N$ system for $m_\pi = 0.64$ GeV as a typical example. In the right panel, the solid and dotted curves represent a fit result with the Yukawa form and the phenomenological potential adopted in Ref. [3], respectively.

the $\eta_c$-$N$ potential clearly exhibits the entire attraction between the charmonium and the nucleon without any repulsion at either short or long distance. It also can be observed that the interaction is exponentially screened in the long distance region $r \gtrsim 1$ fm. This is consistent with what we expected for the color van der Waals force in QCD theory, where the strong confining nature of the color electric field must emerge [14, 15].

In detail, the long-range screening of the color van der Waals force is confirmed by the following analysis. We have tried to fit data with two types of fitting functions: i) exponential type function as $-\exp(-r^m)/r^n$, which includes the Yukawa form ($m = 1$ and $n = 1$), and ii) inverse power law function as $-1/r^n$, where $n$ and $m$ are not restricted to be integers. The former case can easily accommodate a good fit with a small $\chi^2/ndf$ value, while in the latter case we cannot get any reasonable fit. For examples, functional forms $-\exp(-r)/r$ and $-1/r^7$ give $\chi^2/ndf \simeq 2.5$ and 34.3 for fittings, respectively. It is clear that the long range force induced by a normal “van der Waals” type potential based on two-gluon exchange [15] is non-perturbatively screened.

If we adopt the Yukawa form $-\gamma e^{-\alpha r}/r$ to fit our data of $V_{c\bar{c}N}(r)$, we obtain $\gamma \sim 0.1$ and $\alpha \sim 0.6$ GeV. These values should be compared with the phenomenological $c\bar{c}$-$N$ potential adopted in Refs. [3], where parameters ($\gamma = 0.6$, $\alpha = 0.6$ GeV) are barely fixed by a Pomeron exchange model. The strength of the Yukawa potential $\gamma$ is six times smaller than the phenomenological one, while the Yukawa screening parameter $\alpha$ obtained from our data is comparable to the corresponding one. The observed $c\bar{c}$-$N$ potential from lattice QCD is rather weak.

We next show the finite size dependence and the quark-mass dependence of the $\eta_c$-$N$ potential in Fig. 2. Firstly, as shown in the left panel of Fig. 2, there is no significant difference between potentials computed from lattices with two different spatial sizes ($La \approx 3.0$ and 1.5 fm). This observation is simply because of the fact that the $\eta_c$-$N$ potential is quickly screened to zero and turns out to be somehow short ranged. In principle, the short range part of the potential, which is represented by the ultraviolet physics, should be insensitive to the spatial extent associated with an infrared cutoff. As a result, it is assured that the larger lattice size is large enough to study the $\eta_c$-$N$ system. The appreciable quark-mass dependence is also not observed in the right panel.
FIGURE 2. The volume dependence (left) and the quark-mass dependence (right) on the $\eta_c$-$N$ potential.

of Fig. 2. This is expected from the fact that the $c\bar{c}$-$N$ interaction is mainly governed by multi-gluon exchange. However, it is worth mentioning that the ordinary van der Waals interaction is sensitive to the size of the charge distribution. Indeed, it is reminded that our simulations are performed in quenched approximation and at rather heavy quark masses. This suggests that the $c\bar{c}$-$N$ potential from the dynamical simulations would become more strongly attractive in the vicinity of the physical point, where the size of the nucleon is much larger than at the simulated quark mass in this study.

We have studied the $c\bar{c}$-$N$ potential $V_{c\bar{c}N}(r)$ from quenched lattice QCD, which is calculated from the equal-time BS amplitude through the effective Schrödinger equation. It is found that potential $V_{c\bar{c}N}(r)$ is weakly attractive at short distance and exponentially screened at long distance. In order to make a reliable prediction about nuclear-bound charmonium, an important step in the future is clearly an extension to dynamical lattice QCD simulation. Such planning is now underway.

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