Phase transition of the dissipative double-well quantum mechanics

Ken-Ichi Aoki \(^1\) and Tamao Kobayashi \(^2\)

\(^1\)Institute for Theoretical Physics, Faculty of Mathematics and Physics, Kanazawa University, Kanazawa, Ishikawa, Japan

\(^2\)General Education, Yonago National College of Technology, Yonago, Tottori, Japan

abstract

We investigate the critical dissipation of the double-well quantum mechanics. We adopt two-state approximation to define effective Ising models and apply the block decimation renormalization group and the finite range scaling method recently proposed for the long range Ising model. We briefly report the numerical results of the critical dissipation for various model parameters.

1 Introduction

The phase transition here is the quantum-to-classical phase transition. Consider a quantum system of the double well potential. Due to the tunneling, the system is oscillating and symmetric, which we call ‘quantum’. With dissipative effects, the tunneling is suppressed, and the ground state becomes a localized state to break the original Parity symmetry, which we call ‘classical’. Then, there must be a ‘phase transition’ between these two states. Our subject is to evaluate the critical dissipation for this phase transition using a new renormalization group method and a new scaling ansatz according to the range of interactions.

The above picture about the phase transition has been common now by many researches with various methods \([1, 16, 17, 18]\). This type of transition is expected to be observed as decoherence effects in the recent quantum mechanical few body experiments \([10]\) or future quantum devices. Quantitative treatment of decoherence processes is essential in planning realistic quantum devices, but it must be a difficult subject since it need to bridge between the quantum and classical notions. Certainly the double-well quantum mechanics with dissipative effects is a first step model which we must fully understand quantitatively.

The actual value of the criticality for the model as a function of the parameters of the double-well potential and the dissipative interactions, are not well established. Also, the double-well system has been frequently compared to the one-dimensional Ising model especially for the deep well case. However, the detailed and quantitative relations between the double-well quantum mechanics and the Ising model with general type of interactions have not been clear yet.

In this article, we develop new methods of handling the infinite range interactions, that is, the block decimation renormalization group and the finite range scaling. These two methods are new way of evaluating the infinite range system with recourse to the intermediate finite range systems. The block decimation renormalization group is a real space renormalization group

\(^1\)aoki@hep.s.kanazawa-u.ac.jp

\(^2\)kobayasi@yonago-k.ac.jp
method to solve finite range system exactly. Then the finite range scaling ansatz is applied to the susceptibility calculated exactly for finite range systems, and we evaluate its scaling exponent. The finite range scaling exponent finally tells us the value of the critical coupling constant through the pole position of the zeta function. These new methods may shed light on other infinite range systems even in higher dimensional space, and may open new analyzing power in the renormalization group world.

First of all we should recall that it is non-trivial to include dissipative effects in the quantum mechanics, since there is no simple Hamiltonian to realize friction proportional to the velocity. We take the Caldeira-Leggett (CL) model[1], where dissipation comes out of the microscopic origin. The model consists of a target system and environmental degrees of freedom of infinitely many harmonic oscillators,

\[ S[q, \{x_\alpha\}] = \int dt \left\{ \frac{1}{2}M \dot{q}^2 - V_0(q) + \sum_\alpha \left[ \frac{1}{2}m_\alpha \dot{x}_\alpha^2 - \frac{1}{2}m_\alpha \omega_\alpha^2 x_\alpha^2 - qC_\alpha x_\alpha \right] \right\} . \] (1)

The target variable \( q \) and each environmental variable \( x_\alpha \) are coupled linearly with coupling constant \( C_\alpha \). Due to these couplings, the energy of the target system is transferred to the environmental oscillators, and it exhibits as dissipative effects.

To describe the effective target dynamics, we path integrate out the environmental degrees of freedom. Then the non-local interactions of the target variable emerge as,

\[ \Delta S_{NL} = \frac{\eta}{4\pi} \int ds \int d\tau \frac{(q(s) - q(\tau))^2}{|s - \tau|^p} , \] (2)

where \( \eta \) is an effective coupling constant to represent the strength of dissipation and parameter \( p \) denotes the damping rate of the non-local interactions. Of course these parametrization is a simplified expression of non-local interactions which are determined by micro parameters \( m_\alpha, \omega_\alpha \) and \( C_\alpha \). It is known that with quadratic damping \((p = 2)\) interactions, the classical equation of motion of \( q(t) \) suffers effectively a friction proportional to the velocity \( \frac{dq}{dt} \), and thus this case is called the Ohmic case.

In the Euclidean path integral formalism, the quantum mechanical system is equivalent to a one-dimensional statistical system. If we approximate the system using the minimum degrees of freedom, that is, two states per site (after discretization in time with a finite slice), it is transformed into the Ising spin chain with long range interactions.

Long range Ising models with the above type of simple interactions,

\[ -\beta H = \sum_k \sum_n K_n \sigma_k \sigma_{k+n} = \sum_k \sum_n \frac{\eta_n}{\eta_{p}} \sigma_k \sigma_{k+n} , \quad \sigma_k = \pm 1 , \] (3)

have its own long history pioneered by Refs.[4, 2, 3], and it has been understood that with strong enough long range interactions there occurs the spontaneous magnetization. The known facts[5, 6, 7, 8] about the phase transition are that the critical coupling constant \( \eta_k \) is finite only for \( 1 < p \leq 2 \), that is, \( \eta_k = 0 \) for \( p \leq 1 \) and \( \eta_k = \infty \) for \( p > 2 \). Particularly the \( p = 2 \) case, which is called as "Ohmic" because it corresponds to the velocity proportional friction in the classical mechanical situation, is a very special boundary where it is proved that the Kosterlitz-Thouless transition occurs [9].

It is difficult, however, to evaluate the value itself of \( \eta_k \). In the previous article we successfully evaluated the critical coupling constant of this long range Ising model, where we developed a new method of investigating the infinite range system, Finite Range Scaling (FRS) hypothesis[11].
In this article we apply this method to the double-well quantum mechanics. Here we briefly report the prototype calculation using two-state approximation of the system. Even with two-state approximation, the effective Ising spin interactions are not a straightforward mapping of the original non-local interactions. We will confirm that FRS method does work to evaluate the critical dissipation and our results are consistent with those obtained by other methods. The full calculation without state reduction will be reported elsewhere.

2 Finite Range Scaling method in the long range Ising model

For the long range Ising models defined in Eq. (3), we successfully evaluated the critical dissipative interactions to bring about the spontaneous magnetization. We developed a new method of the Block Decimation Renormalization Group (BDRG) and the Finite Range Scaling (FRS) ansatz [11]. Our method utilizes simple calculation with very small size of the computational resources. There are some large scale Monte Carlo studies [12, 13, 14] for the limited parameter space, and their results are perfectly consistent with ours, which indicates our ansatz of FRS is just on the right way of accessing properties of infinite range interactions.

We explain the BDRG method in Fig.1 in case of range $n = 3$. The DRG (Decimation Renormalization Group) is a block spin transformation method formulated by Wilson to calculate the partition function approximately [15]. In one dimensional nearest-neighbor Ising system, it works without any approximation and we can get the exact partition function very easily. In the case that there are non-nearest-neighbor interactions, the original DRG does not work. We developed an extended version, the Block DRG, which may accommodate long range interactions.

We consider a system with long range interactions with coupling constants $K_m$ which couples $m$-separated two spins. We set the maximal finite range of interactions to be $n$, and investigate the infinite range limit by increasing $n$, one by one. First, we divide spins into blocks of size $n$. Then there are interactions only between nearest neighbor blocks, that is, looking at the model ‘block-wisely’, it is nothing but the nearest-neighbor model where one block plays a site of $2^n$ states.

Next, all the spins in the middle block in Fig.1 are decimated, namely, are integrated for all possible $2^n$ states. After decimation we obtain new inter-block interactions which still keeps nearest-neighbor-block property. Now the renormalization transformation is defined and can
be repeated without approximation.

We represent inter-block interactions by the transfer matrix $T$ of $2^n \times 2^n$. Then the renormalization transformation is nothing but to make a product of $T$ matrices,

$$T^{(k+1)} = T^{(k)} \cdot T^{(k)},$$

(4)

where $T^{(k)}$ matrix is inter-block interactions between $k$-th renormalized neighboring blocks.

By BDRG we calculate the susceptibility of the system $\chi(n)$ with a finite range interactions exactly. We impose the extend field $h$ as,

$$-\beta H = \sum_k \left[ \sum_n K_n \sigma_k \sigma_{k+n} + h \sigma_k \right].$$

(5)

Then the susceptibility of the system is given by,

$$\chi(n) = \lim_{k \to \infty} \frac{1}{n \cdot 2^k} \left. \frac{\partial^2 \left( \ln \mathrm{Tr} T^{(k)} \right)}{\partial h^2} \right|_{h=0}.$$

(6)

We increase the range $n$ by 1 to see the change of the susceptibility and define the corresponding scaling exponent $\beta$ as follows,

$$\log \chi(n) - \log \chi(n - 1) \propto \left( \frac{1}{n} \right)^{\beta(p,\eta)}.$$

(7)

We expect that this exponent $\beta$ will be asymptotically a constant for enough large $n$. This is a hypothesis and we call it Finite Range Scaling (FRS).

If the FRS does work, then the infinite $n$ behavior of the susceptibility apart from the finite part can be evaluated as

$$\log \chi(n \to \infty) = \sum_n \left[ \log \chi(n) - \log \chi(n - 1) \right] \simeq \sum_n \left( \frac{1}{n} \right)^{\beta(p,\eta)} = \zeta(\beta(p,\eta)) .$$

(8)

Using the exponent we estimate the divergent point of the susceptibility and it gives the critical dissipation. That is, the infinite $n$ behavior of the susceptibility is controlled by the zeta function $\zeta(\beta)$, which has a pole singularity at $\beta = 1$. Therefore, the critical $\eta$ is determined by the condition $\beta(p,\eta_c) = 1$. This method of using finite range systems to evaluate the infinite range divergence is the Finite Range Scaling method we have developed.

3 Effective Ising model of the double-well quantum mechanics

We move on to the double well quantum mechanics,

$$V(x) = -\frac{1}{2} x^2 + \lambda x^4 .$$

(9)

Note that the number of states on a site is infinite in quantum mechanics and $T$ matrix is replaced by a bi-local function defined by a bi-local potential $W$ as,

$$T(x, y) = e^{-W(x,y)} = \langle x \mid \hat{U} \mid y \rangle .$$

(10)
The operator $\hat{U}$ is the imaginary time evolution operator and this bi-local function is nothing but the Feynman path integral kernel.

Adding the non-local interactions generated by the path integration of environmental degrees of freedom, we have to compose $n$ sized blocks to make the system 'nearest-neighbor'. Then the inter-block $T$ matrix is a multi variable function with $2n$ variables as follows,

$$T = e^{-W(x_1 \cdots x_n, y_1 \cdots y_n)} = \langle x_1 x_2 \cdots x_n | \hat{U} | y_1 y_2 \cdots y_n \rangle .$$

Here we introduce a new complete set of states $| a_n \rangle$ and express $T$ matrix with this new base system,

$$T = e^{-\tilde{W}(a_1 \cdots a_n, b_1 \cdots b_n)} = \langle a_a \cdots a_n | \hat{U} | b_1 \cdots b_n \rangle = \int dx_1 \cdots dx_n dy_1 \cdots dy_n \langle a_a \cdots a_n | x_1 \cdots x_n | \hat{U} | y_1 \cdots y_n \rangle \times \langle y_1 \cdots y_n | b_1 \cdots b_n \rangle$$

$$= \int dx_1 \cdots dx_n dy_1 \cdots dy_n \psi^*_a(x_1) \cdots \psi^*_a(x_n) e^{-W(x_1 \cdots x_n, y_1 \cdots y_n)} \times \psi_b(y_1) \cdots \psi_b(y_n),$$

where $\psi_a(x)$ is the wave function of state $| a \rangle$. Our site variables $x, y$ are converted into other complete set suffix $a, b$.

Now we approximate the states on a site with only two states, that is, the summation in $a, b$'s are just two-fold. This defines an effective Ising model of one-dimension which has general interactions between spins, and we can write the multi-local potential as $\tilde{W}(\sigma_a, \sigma_b)$ where $\sigma_a, \sigma_b = \{-1, +1\}$.

The simplest version of two-state approximation is to take the completely localized states at the left and right well bottoms, where the wave functions are the $\delta$-function. Then the converted $\tilde{W}(\sigma_a, \sigma_b)$ function is directly related to the original $W(x, y)$ function as follows,

$$\tilde{W}(\sigma_a, \sigma_b) = W(v \sigma_a, v \sigma_b) , \quad v = \frac{1}{2\sqrt{\lambda}},$$

where $\pm v$ is the position of the double well bottoms. The effective spin interactions in this case are given by,

$$\epsilon^{2-p} \frac{\eta}{4\pi \lambda} \sum_k \sum_n \frac{1}{n^p} \sigma_k \sigma_{k+n} + \frac{m}{4\lambda} \sum_k \sigma_k \sigma_{k+1} ,$$

which is exactly the same form of the normal long range Ising model defined in Eq. (13) except for the total rescaling of the coupling constant $\eta$ and the nearest-neighbor interactions due to the kinetic term of the quantum system. Then we can easily estimate the critical dissipation by referring to the Ising model result calculated in Ref. [11, 13] as follows:

$$\eta_c = 4\pi \lambda \times \eta_c [\text{Ising}] ,$$

where we assume that the criticality does not strongly depend on the size of the nearest neighbor term. This relation holds for any $p$ and in the Ohmic case ($p = 2$), for example, we have

$$\eta_c \simeq 4\pi \lambda \times 0.66 .$$

This result is quite similar in the $\lambda$ dependence to that obtained by the dilute instanton calculation [16],

$$\eta_c = 2\pi \lambda ,$$

5
and we note that it is larger than this instanton approximation.

However this selection of states makes difficulty that there appears divergences due to the vanishing time discretization slice $\epsilon$. For example the correlation length $\xi$ of the system diverges as

$$\xi \approx \frac{\epsilon}{2} \exp \left( \frac{m}{2} \frac{1}{\epsilon} \right).$$

(18)

because of the diverging nearest neighbor interactions. This is not consistent with the fact that the quantum mechanics does not need any renormalization of bare parameters.

Instead, we take linear combination of two states, the ground state $|0\rangle$ and the 1st excited state $|1\rangle$ of the double well potential without dissipation ($\eta = 0$), $\psi_{\uparrow,\downarrow} = (|0\rangle \pm |1\rangle)/\sqrt{2}$, which are regarded as left or right states corresponding to up or down of Ising spin. We call this type of two-state approximation as the ground state approximation. In this approximation the correlation length of the system is evaluated as

$$\xi = -\epsilon \left[ \log \tanh \left( -\frac{1}{2} \log \tanh \left( \frac{\epsilon \delta}{2} \right) \right) \right]^{-1},$$

(19)

where $\delta$ is the energy gap between the ground and the 1st excited states. At the vanishing time slice, it converges as

$$\lim_{\epsilon \to 0} \xi(\epsilon) = \frac{1}{\delta},$$

(20)

which is the correct value of the continuum system.

Hereafter, our aim is to evaluate the plausibility of our BDRG and FRS methods in the effective Ising model with the ground state approximation. Note that the effective Ising models in this approximation have all possible spin interactions including multi-spin products, which might be far away from the normal long range Ising interactions in Eq. (3). Therefore it is non-trivial that FRS method works with such general type of long range Ising models.

For example, in $n = 2$, initial $T$ matrix is calculated as follows,

$$T = e^{-W(a_1 a_2, b_1 b_2)} = \int dx_1 dx_2 dy_1 dy_2 \psi_{a_1}^*(x_1) \psi_{a_2}^*(x_2) \psi_{b_1}(y_1) \psi_{b_2}(y_2) \times \exp \left[ -\frac{m(x_1 - x_2)^2}{4\epsilon} - \frac{m(x_2 - y_1)^2}{2\epsilon} - \frac{m(y_1 - y_2)^2}{4\epsilon} - \frac{\epsilon}{2} (V(x_1) + V(x_2) + V(y_1) + V(y_2)) \right]$$

$$-\frac{\eta}{2\pi} \epsilon^{2-\nu} \left[ \frac{1}{2} (x_1 - x_2)^2 + (x_2 - y_1)^2 + \frac{1}{2} (y_1 - y_2)^2 \right] + \frac{1}{2\nu} \left( (x_1 - y_1)^2 + (x_2 - y_2)^2 \right),$$

(21)

where $\epsilon$ is the discretization step for the imaginary time and $a, b$ are Ising variables $\{\uparrow, \downarrow\}$. The first group of terms are the original kinetic terms, the second are the potential terms, and the last are dissipation terms given by the CL non-local interactions.

In the ground state approximation, generally, $2^n$ integrations of $2^n$-dimensions are necessary to get the initial $T$ matrix of BDRG. To evaluate these large dimensional integration, we adopt the Monte Carlo integration method. We set up random numbers obeying the probability distribution function defined by all local terms in the integral, namely, wave functions and potential terms. Wave functions are not positive semi-definite and we introduce sign functions.
to take account of the negative region of the wave functions. Kinetic terms and non-local interaction terms are evaluated by random numbers defined above.

After obtaining the effective spin interactions, we modify it so that the $T$ matrix respects the Parity symmetry and mirror symmetry. For example, in $n = 2$, these symmetries are,

$$W(\bar{a}_1\bar{a}_2, b_1b_2) = W(a_1a_2, b_1b_2), \quad W(b_2b_1, a_2a_1) = W(a_1a_2, b_1b_2),$$

(22)

where $\uparrow = \downarrow, \downarrow = \uparrow$. This symmetrization is important and necessary, since if not, the renormalization transformation enhances the symmetry breaking components and the resultant renormalized $T$ matrix will be out of reality. We checked the validity and precision of our Monte Carlo integration by comparing our results with those obtained by Simpson’s integration method which can be done for very low $n$ cases.

4 Results

We show our numerical results in order. Fig.2 shows a typical behavior of the Finite Range Scaling exponent $\beta$ versus $\eta$ for $n = 5, 6, 7$. Here we take $\lambda = 0.04$, the damping rate $p = 1.99$, the discretization step $\epsilon = 0.9$, the number of configurations for Monte Carlo integration $=1.28$ million. We calculated $\beta$ with 16 different set of random numbers and evaluated the standard deviation of the data which are plotted in the figure as $1\sigma$ bar. This initial evaluation is called rough estimate.

![Figure 2: Typical behavior of $\beta$.](image)

The global behavior of $\beta$ is very smooth and natural. It decreases monotonically with respect to $\eta$ and crosses the critical value $\beta = 1$. According to the FRS hypothesis, the crossing point is the critical coupling constant $\eta_c$. Note that we need estimates of $\beta$ with enough large $n$, and $n = 6$ or $n = 7$ are actually ‘large’ since it seems they are already stable against $n$. This exhibits a surprising feature that with relatively short range system we can evaluate the criticality which can emerge only for infinite range interactions. Such features of FRS method is common to the normal long range Ising models.
Thus we can obtain the critical coupling constant by FRS method. This holds for all cases we calculated with various system parameters $\lambda, p, \epsilon$.

After checking the critical region in the above ‘rough’ data, we increase the Monte Carlo data points to be 10.24 million and take 64 different set of random numbers, which we call ‘detailed’ data. In fig.3 the results of ‘detailed’ $\beta$ are plotted with 1$\sigma$ errors, which appears consistent with the ‘rough’ data whose 1$\sigma$ region are expressed by curved lines.

Using the 1$\sigma$ error for each $\eta$ point, we get 1$\sigma$ region of $\beta$ estimate as in fig.4. Then the crossing period of $\beta = 1$ line with this region gives our estimate of $\eta_c$. 

Next we check the $\epsilon$ dependence of $\eta_c$. The discretization skip $\epsilon$ should be small enough. However, the situation is not so simple and we cannot take the vanishing $\epsilon$ limit straightforwardly. The small $\epsilon$ means the length scale of the distance between spins becomes small, and the long range interaction nature of the system is being lost with fixed finite $n$. Actually we can calculate only rather short range interaction cases around $n = 6, 7$ because of the computer resource limit. Then the too-small $\epsilon$ finally breaks the FRS framework.

There must be an optimized finite $\epsilon$ which gives the best value for the physical results within the limited calculational resources. Investigating the $\epsilon$ dependence of $\eta_c$ and size of their errors in fig.5 ($p = 1.99, n = 6$) and fig.6 ($p = 1.99, n = 7$), we adopt $\epsilon = 0.9(n = 6)$.
and $\epsilon = 0.7(n = 7)$ for major estimates with whole range of parameter values, mainly taking account of total stability of calculations.

![Figure 7: $p$ dependence of $\eta_c$](image)

Then we plot the $p$ dependence of $\eta_c$ in fig.7 ($\epsilon = 0.9$, $n = 7$). Larger $p$ is equivalent to weaker interactions among spins. Therefore $\eta_c$ should increase according to $p$. The dependence resembles to the case of the normal long range Ising models\[11\].

However we observe still finite $\eta_c$ in case of large $p > 2$, which is quite different from the normal long range Ising models defined in Eq.(3) where $\eta_c$ is proved to be infinite for $p > 2$. This issue of finite critical coupling constants seen for $p > 2$ is an interesting problem indicating some new features of the model or the important defects of our approximated model. Actually our model here does not correspond to a usual simple long range Ising model. Actually we may define and evaluate effective Ising spin interactions given by the ground state approximation. They are quite different from the simple-minded interactions which are actually realized in case of the delta-function states. There appear strong multi spin ($\sigma_4, \sigma_6$ etc.) interactions and enhanced long range interactions.

In this sense it is non-trivial that our approximation of the quantum double-well system may show up the FRS behaviors just as in the normal long range Ising model, and even more the quantum double-well system with long range interactions has the same boundary structure at $p = 2$ as the normal long range Ising model. To conclude something more definite about this issue we need to proceed beyond the two-state approximation where FRS will be examined with the full system.

To compare our results with earlier researches, we plot $\eta_c$ versus $\lambda$ in Figs.8 and 9. Here we take $p = 1.99$, which is for the future comparison with normal long range Ising calculation with FRS since $p = 2$ there is hard to evaluate due to the boundary Kosterlitz-Thouless transition. As for other parameters, we adopt two sets: $\epsilon = 0.9, n = 6$ (FRS(6) in figures) and $\epsilon = 0.7, n = 7$ (FRS(7) in figures). We show data with error bars which show statistical errors because of the Monte Carlo integration part in our method. We see some variation of results between these two sets of parameters, which indicate size of systematic errors of our method of calculation in this stage.
Our FRS results and those of sophisticated Monte Carlo simulation (MC in figures)\cite{17} support complementary parameter regions. At the larger $\lambda (> 0.05)$ area, our results by 2-state approximation seems to deviate from the MC, and we consider that it reveals the insufficientness of our approximation of picking up only 2-state at each site in this region. The reason is that at larger $\lambda$ region, the well becomes shallower and the energy gaps between the 1st and the 2nd excited states are not so large as that between the ground and the 1st excited states.

On the other hand, the dilute gas approximated instanton results (Instanton in figures)\cite{16} look very much consistent with our results at small $\lambda (< 0.02)$ region where both 2-state approximation and the dilute gas instanton are expected to work well there. The dilute gas instanton calculation keeps only the position of instantons and anti-instantons for configurations and neglect energy between instantons, that is, non-interacting gas. Then the dissipation effects are evaluated for instanton configurations. This approach can be regarded as an another type of two-state approximation, and therefore its predictable region resembles to ours. Thus, the dilute gas instanton at larger $\lambda (> 0.04)$ is not a good approximation.

We plot the 2-state approximation results with the delta function wave function ($\delta$-wf in figures) explained in Eq.(16) though it suffers a serious problem for vanishing $\epsilon$ limit. It is just above the instanton results and seems better than that.

Accordingly our method is a good candidate to give the critical coupling constants in the intermediate region between the MC plausible region and the dilute gas instanton region. However, it seems that within 2-state approximation, our method is not good enough to smoothly connect the MC and instanton. We have to proceed to multi-state or full-state calculations, to which we will apply the Finite Range Scaling.

We also plot a non-perturbative renormalization group approach (NPRG-WH in figures)\cite{19} with the local potential approximation, which gives results at very large $\lambda (> 0.1)$ region. This approach uses the Wegner-Houghton equation where the effective Wilsonian action consists of
the fixed kinetic term and the general potential term ignoring all other derivative interactions. It is known to give very good results of the susceptibility for large $\lambda (> 0.1)$ case [20]. Ignoring the derivative terms in the Wilsonian effective potential means it is not a good approximation for configurations with a sharp change of $x(\tau)$, and it fails for small $\lambda (> 0.1)$ region. In total, it looks consistent with other approaches, though there is no other results in the large $\lambda (> 0.1)$ region ever.

In the appendix we list our results of $\eta_c$ for various parameter values of $\lambda, p, \epsilon, n$.

Acknowledgments

@We thank fruitful discussions with Daisuke Sato and Kazuhiro Miyashita, and also collaboration of Masakazu Arimoto, Yasuhiro Fujii and Hiroshi Tomita at the early stage of this work.

This research was partially supported by the Ministry of Education, Culture, Sports, Science and Technology through a Grant-in-Aid for Challenging Exploratory Research (No.12011251, 2012).

Appendix

@We list the critical dissipation for various model parameters. The $1\sigma$ errors are put in the parentheses.

Critical dissipation $\eta_c$
\[
\lambda \ (\epsilon = 0.9, n = 6)
\]

| p   | 0.02          | 0.04          | 0.06          | 0.10          |
|-----|---------------|---------------|---------------|---------------|
| 1.2 | 0.01(0.00097) | 0.041(0.0005) | 0.057(0.0008) | 0.069(0.0026) |
| 1.7 | 0.044(0.0049) | 0.18(0.002)   | 0.26(0.004)   | 0.32(0.012)   |
| 1.9 | 0.097(0.011)  | 0.37(0.007)   | 0.59(0.011)   | 0.70(0.024)   |
| 1.95| 0.13(0.016)   | 0.48(0.009)   | 0.76(0.017)   | 0.87(0.031)   |
| 1.99| 0.16(0.019)   | 0.59(0.012)   | 0.95(0.024)   | 1.05(0.040)   |

\[
\lambda \ (\epsilon = 0.9, n = 7)
\]

| p   | 0.02          | 0.04          | 0.06          | 0.10          |
|-----|---------------|---------------|---------------|---------------|
| 1.2 | 0.0039(0.0012)| 0.030(0.0006)| 0.040(0.0009)| 0.053(0.0037)|
| 1.7 | 0.017(0.0056) | 0.14(0.0035)  | 0.20(0.005)  | 0.28(0.017)  |
| 1.9 | 0.038(0.013)  | 0.33(0.0075)  | 0.48(0.014)  | 0.63(0.036)  |
| 1.95| 0.049(0.017)  | 0.39(0.012)   | 0.62(0.018)  | 0.79(0.043)  |
| 1.99| 0.060(0.022)  | 0.47(0.016)   | 0.77(0.023)  | 0.95(0.055)  |

\[
\lambda \ (\epsilon = 0.7, n = 6)
\]

| p   | 0.02          | 0.04          | 0.06          | 0.10          |
|-----|---------------|---------------|---------------|---------------|
| 1.99| 0.18(0.047)   | 0.65(0.026)   | 1.12(0.045)   | 1.23(0.060)   |

\[
\lambda \ (\epsilon = 0.7, n = 7)
\]

| p   | 0.02          | 0.04          | 0.06          | 0.10          |
|-----|---------------|---------------|---------------|---------------|
| 1.99| 0.011(0.031)  | 0.4(0.024)    | 0.80(0.030)   | 0.99(0.065)   |

References

[1] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. 46, 211(1981); Ann. of Phys. 149, 374(1983).

[2] D. Ruelle, Commun. Math. Phys. 9, 267(1968).

[3] F. J. Dyson, Commun. Math. Phys. 12, 91(1969).

[4] R. B. Griffiths, Commun. Math. Phys. 6, 121(1967).

[5] J. Froehlich and T. Spencer, Commun. Math. Phys. 84, 87(1982).

[6] M. Aizenman and R. Fernández, Let. Math. Phys. 16, 39(1988).

[7] M. Aizenman, J. T. Chayes, L. Chayes and C. M. Newman, J. Stat. Phys. 50, 1(1988).

[8] J. Z. Imbrie and C. M. Newman, Commun. Math. Phys. 118, 303(1988).

[9] P. W. Anderson and G. Yuval, J. Phys. C4 607(1971). J. M. Kosterlitz and D. J. Thouless, J. Phys. C6 1181(1973). J. M. Kosterlitz, J. Phys. C7 1046(1974). J. M. Kosterlitz, Phys. Rev. Lett. 37 1577(1976). J. L. Cardy, J. Phys. A14 1407(1981).

[10] D. L. Haycock, P. M. Alsing, I. H. Deutsch, J. Grondalski, and P. S. Jessen, Phys. Rev. Lett. 85 3365(2000).
[11] K-I. Aoki, T. Kobayashi and H. Tomita, Prog. Theor. Phys. 119, 509(2008).

[12] J. Bhattacharjee, S. Chakravarty, J. L. Richardson and D. J. Scalapino, Phys. Rev. B24 3862(1981). S. A. Cannas and A. C. N. de Magalhaes, J. Phys. A30 3345(1997). E. Bayong, H. T. Diep, and V. Dotsenko, Phys. Rev. Lett. 83 14(1999).

[13] Erik Luijten and Henk W. J. Blöte, Phys. Rev. B 56, 8945(1997)

[14] Erik Luijten and Holger. Meßingfeld, Phys. Lev. Lett. 86, 5305(2001)

[15] K. G. Wilson, Rev. Mod. Phys. 47, 773(1975).

[16] S. Chakravarty, Phys. Rev. Lett. 49, 681(1982). A. J. Bray, M. A. Moore, Phys. Rev. Lett. 49, 1545(1982).

[17] T. Matsuo, Y. Natsume and T. Kato, J. Phys. Soc. Jpn. 75, 103002(2006); Phys. Rev. B 77, 184304(2008).

[18] K. Fujikawa, S. Iso, M. Sasaki and H. Suzuki, Phys. Rev. Lett. 68 1093(1992). K. Fujikawa, S. Iso, M. Sasaki and H. Suzuki, Phys.Rev. B46 10295(1992).

[19] K-I. Aoki and A. Horikoshi, Phys. Rev. A66 042105(2002); Phys. Lett. A 314, 177(2003).

[20] A. S. Kapoyannis and N. Tetradis, Phys. Lett. A276 225(2000). D. Zappala, Phys. Lett. A290 35(2001). Ken-Ichi Aoki, Atsushi Horikoshi, Masaki Taniguchi and Haruhiko Terao, Prog. Theor. Phys. 108 571(2002).