Kinematic synthesis of a reconfigurable robot manipulator with lattice morphing mechanisms based on enhanced QPSO

Bo Yin, Zhenyu Zhao, Haibing Xiao, Mingjun Liu and Wen Hu

School of intelligent manufacturing and equipment, Shenzhen Institute of Information Technology, 2188 Longxiang Road, Shenzhen 518172, China

Corresponding author’s e-mail: zhaozy@sziit.edu.cn

Abstract. A kinematic synthesis approach of a modular reconfigurable robot manipulator with lattice morphing mechanisms is proposed to find the optimal robot configuration with the minimum weight. First, the structure features and types of lattice mechanisms are introduced. Then, a synthesis approach using an enhanced QPSO (Quantum-behaved Particle Swarm Optimization) to get the optimal robot configuration with the minimum lattice is presented. Finally, a computation example is given to show the optimization process. The obtained results demonstrate the feasibility of this approach.

1. Introduction

Reconfigurable robot manipulators have the potential to exceed specialized robots in multi-functionality, flexibility, and robustness. However, reconfigurable robots have too much redundant mass to perform more deftly than specialized robots. Hence, finding an optimal robot configuration with minimum weight by kinematic synthesis becomes essential for reconfigurable robots. Finistauri provided guidance on the topological reconfiguration of a fully reconfigurable parallel robot to achieve task-based reconfiguration[1]. Valsamos introduced an approach for the determination of the best configuration of a reconfigurable manipulator based on the location of the task[2]. Zhao proposed a universal approach for configuration synthesis of reconfigurable robots based on fault tolerant indices[3]. Baca proposed a heterogeneous reconfigurable robot design approach based on three types of modules for fast response to a diversity of tasks[4]. Tarkian presented a multidisciplinary design optimization framework for automated design of a modular industrial robot[5]. Yin proposed a configuration optimization approach including a kinematic description of joints for a lattice distortable reconfigurable robot and an optimization method based on Genetic Algorithm[6]. In this paper, inspired by the above approaches, the kinematic synthesis model is built for the reconfigurable robot manipulator with lattice morphing mechanisms[7]. Different from the above methods, an enhanced QPSO is selected to solve the optimization problem because of its fast convergence.
2. Types of lattices

2.1. Features of the module in lattices
As shown in Figure 1, each module has six connectors and can connect other modules on its six faces. Figure 1. The structure of a single module. A single module has two actuators driven by two pivot joints that their rotation axes are parallel each other. Two motors placed in the module body provide the driving torque to yaw. In the end of each pivot joint, there is a prismatic joint.

2.2. The morphing rule of the lattice morphing mechanism
Four modules can configure a planar lattice morphing mechanism. When the joint of each module of a lattice rotates a same angle, the telescopic connector on one side of the lattice is extruded with a corresponding length, and then the shape of the lattice is morphed to an isosceles trapezoid from a square, shown in Figure 2. Figure 2. The illustration of the lattice morphing rule. A morphing lattice can be simplified to a “Prismatic–Rotation-Prismatic (PRP)” mechanism, which has 3 degrees of freedom including one rotation and two translations. \( \mathbf{L} = 0.156 \text{m} \) is the length of a module; \( \theta \) is the rotation angle of the lattice morphing mechanism. \( \mathbf{L}_p \) is the sliding length of each prism and its maximum sliding length is 0.039m.

2.3. Types and kinematic models of lattices
Based on their role in the configuration of a robot manipulator, lattices can be distinguished into five types as shown in Table 1. In this paper, the D-H formulation is used to describe the kinematic model of lattice mechanisms.

| Type | Rotation axis or Connection direction | Transformation matrix | Model |
|------|--------------------------------------|-----------------------|-------|
| P-R-P joint | Y-axis | \( T^n = \begin{bmatrix} \cos \theta & 0 & \sin \theta & (L + L_p) \sin \theta \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & (L + L_p)(1 + \cos \theta) \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | ![Image] |
| P-R-P joint | X-axis | \( T^n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & (L + L_p) \sin \theta \\ 0 & -\sin \theta & \cos \theta & (L + L_p)(1 + \cos \theta) \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | ![Image] |
| Straight link | Z-direction | \( T^n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | ![Image] |
3. Kinematic synthesis for a manipulator

3.1. Optimization model

The goal of the synthesis problem is to determine the types of joints and the dimensions of links in a manipulator that can position a tool held by its end effector in a given set of task positions. The synthesis equation can be written in the form:

\[
E_i = \|P_i(\theta) - P_{\text{LT}} L\| \quad i = 1, 2, n
\]

Where \( \theta \) is the vector of rotation angle of PRP joints; \( P_i(\theta) = [x_i, y_i, z_i]^T \) is the \( i \)-th work position point of the end actuator; \( P_{\text{LT}} L \) is the identified \( i \)-th task position point; \( E_i \) is the acceptable bias between the work position of the actuator and the identified task position.

In order to count easily, a lattice with “PRP joint + straight link + elbow link + straight link” can be regarded as a lattice group as shown in table 2. \( x_1 \in \{0, 1\} \) represents the state of PRP joint, 0- no PRP joint, 1- a PRP joint; \( x_2, x_4 \in \{0, 1, 2, 3\} \) represent the length of straight link, 0- no straight link; 1- a straight link; 2- two straight links; 3- three straight links; \( x_3 \in \{0, 1, 2\} \) represents the state of elbow links, 0- elbow link; 1- a Z-Y elbow link, 2- a Z-X elbow link.

Table 2. A lattice group

| State of PRP chain | Length of straight link | State of Elbow link | Length of straight link |
|--------------------|-------------------------|--------------------|------------------------|
| \( x_1 \) | \( x_2 \) | \( x_3 \) | \( x_4 \) |

Then, the kinematic model \( T_{L^{G}} \) of a lattice group can be represented as:

\[
T_{L^{G}} = \begin{bmatrix}
1 - x_1 \cos \theta & -x_1 \sin \theta & 0 & x_i L_p \sin \theta + x_4 L \sin \theta + x_3 L \sin \theta / 2 \\
0 & 1 - x_3 & 1 & x_i L / 2 + x_4 L \\
x_1 \sin \theta & x_3 - x_1 - x_i \sin \theta & 0 & x_i (L + L_p + L_p \cos \theta) + x_4 L(1 - x_i + x_3 \cos \theta) + x_3 L(1 - x_i + x_4 \cos \theta) / 2 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

To minimize the weight of the manipulator, the number of PRP joints and links must be the least.

\[
\min \text{ weight} \rightarrow \min \text{ number}(x_1, x_2, x_3, x_4) = \sum_{j=1}^{m} (\text{sign}(x_{1j}) + x_{2j} + x_{3j} + x_{4j})
\]

Where \( m \) is the minimum number of lattice groups; \( x_{1j} \) is the state of the PRP joint in the \( j \)-th lattice group; \( x_{2j} \) and \( x_{4j} \) are the states of straight links in the \( j \)-th lattice group; \( x_{3j} \) is the state of elbow links in the \( j \)-th lattice group.

Here, the kinematic synthesis problem is converted to an optimization problem. States of PRP joints, straight links and elbow links are regarded as design variables. The best configuration with the least lattice that can perform the given task accurately is the aim. The optimization model is:
\[ \text{find } x_1, x_2, x_3, x_4 \]
\[ \min f(x_1, x_2, x_3, x_4) = \sum_{i=1}^{m} \| \mathbf{P}_i(0) - \mathbf{P}_c \| + \sum_{j=1}^{m} (\text{sign}(x_j) + x_2 + x_3 + x_4) \]
\[ \text{s.t. } x_1 = x_2 = 0, 1, 2 \]
\[ x_2 = x_4 = 0, 1, 2, 3 \]
\[ -\pi / 12 < \theta < \pi / 12 \]

### 3.2. Optimization algorithm

Quantum-behaved Particle Swarm Optimization (QPSO) can search the solution quickly and efficiently [8]. Therefore, QPSO is adopted to solve the optimization problem.

#### 3.2.1. Quantum-behaved Particle swarm optimization

The particle of QPSO algorithm moves according to the following equation:

\[ m_{\text{best}} = \frac{1}{M} \sum_{i=1}^{M} P_{i, \text{best}} \]  

(5)

\[ P_i = \phi \cdot P_{i, \text{best}} + (1 - \phi) \cdot G_{\text{best}} \]  

(6)

\[ X_i(t) = P_i \pm \alpha \cdot |m_{\text{best}} - X_i(t-1)| \cdot \ln(1/u) \]  

(7)

Where \( m_{\text{best}} \) is the mean best position among the particles; \( M \) is the population size; \( P_i \) is the local attractor of the \( i \)th particle and is a stochastic point between \( P_{i, \text{best}} \) and \( G_{\text{best}} \); \( \phi \) is a random number distributed uniformly on \([0, 1]\); \( u \) is another uniformly-distributed random number on \([0, 1]\); \( \alpha \) is a parameter of QPSO that is called Contraction-Expansion Coefficient.

\[ \alpha = (\alpha_{\text{max}} - \alpha_{\text{min}}) \cdot (\text{Miteration} - t) / \text{Miteration} + \alpha_{\text{min}} \]  

(8)

\( \text{Miteration} \) is the max iteration number; in general, \( \alpha_{\text{max}}=1, \alpha_{\text{min}}=0.5 \).

#### 3.2.2. Enhanced QPSO for kinematic synthesis of manipulators

To avoid falling into a local optimal solution, we set up a judgment rule in the optimization process: if the function values are same in three continuous iterations, then the algorithm reset the initial random particles and reiterate from the beginning. The process include two stages: one stage is to calculate the difference between the reachable workspace of every population and the task points; the other stage is to search the best configuration with the least lattices.

### 4. Computation example

The task trajectory is a spatial V-type polyline which starts from point A, crosses through point B, and ends at point C. The coordinates of these points are (0.376, 0.482, 0.313), (0.293, 0.478, 0.467), (0.211, 0.471, 0.572), unit: m. The base of the manipulator is located at the origin of the task coordinate system. According to the coordinates of the task points, the max distance in x direction is 0.165m, and then at least three joint lattices and one elbow link are needed in the configuration, because the max translation distance of one joint lattice is 0.08m. The max distance in y direction is 0.011m, and then there exists one joint lattice in the configuration. The max distance in z direction is 0.259m, and then there are two joint lattices in the configuration. Considering the above cases, the required configuration must have at least three groups and at least one elbow link. Then, the initial number of lattice groups \( N_{LG} = 3 \). The other parameters of the enhanced QPSO are listed in Table 3.
Table 3. Parameters of QPSO for kinematic synthesis

| Parameter                      | Value       |
|--------------------------------|-------------|
| maximum contraction-expansion coefficient $\alpha_{\text{max}}$ | 1           |
| minimum contraction-expansion coefficient $\alpha_{\text{min}}$ | 0.5         |
| Allowable position tolerance $E_1$ | $0.1 \times 10^{-6}$ m |
| population size $M$            | 60          |
| maximum generation $M_{\text{iteration}}$ | 20          |

The computing result is obtained as shown in Figure 3. The output is $x_1=(1\ 1\ 1)$, $x_2=(0\ 0\ 0)$, $x_3=(0\ 1\ 2)$, $x_4=(0\ 0\ 1)$. The configuration of the manipulator is shown in Figure 4.

5. Conclusion
The enhanced QPSO developed in this paper is valid to solve the kinematic synthesis of manipulators. The computing time of the algorithm can be reduced by analyzing the characteristic and the size of the given task and setting suited parameters.

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