Supersymmetry and cosmic censorship

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Abstract

We show that requiring unbroken supersymmetry everywhere in black-hole-type solutions of \( N = 2, d = 4 \) supergravity coupled to vector supermultiplets ensures in most cases absence of naked singularities. We show that the requirement of global supersymmetry implies the absence of sources for NUT charge, angular momentum, scalar hair and negative energy, for which there is no microscopic interpretation in String Theory. These conditions exclude, for instance, singular solutions such as the Kerr-Newman with \( M = |q| \), which fails to be everywhere supersymmetric. There are, nevertheless, everywhere supersymmetric solutions with global angular momentum and non-trivial scalar fields.

We also present similar preliminary results in \( N = 1, d = 5 \) supergravity coupled to vector multiplets.

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1 Introduction: the 1992 SUSY versus cosmic censorship conjecture

A possible relation between cosmic censorship and supersymmetry follows from the observation that in the simplest supergravity theories the BPS bound coincides with the condition of existence of event horizons in static black-hole-type solutions. Thus, for the Schwarzschild solution, which is a solution of pure $N=1$, $d=4$ supergravity, the BPS bound $M \geq 0$ ensures the existence of a regular horizon. When the bound is saturated $M = 0$, there is unbroken supersymmetry and a regular solution: Minkowski spacetime.

The Reissner-Nordström solution

$$ds^2 = \frac{(r - r_+)(r - r_-)}{r^2} dt^2 - \frac{r^2}{(r - r_+)(r - r_-)} dr^2 - r^2 d\Omega^2_{(2)}, \quad r_{\pm} = M \pm \sqrt{M^2 - q^2}, \quad (1)$$

is a solution of pure $N = 2$, $d = 4$ supergravity and has a regular event horizon for $M^2 \geq q^2$, which is the BPS bound. When the bound is saturated, there is unbroken supersymmetry and a regular solution: the extreme Reissner-Nordström black hole.

It was then proposed that supersymmetry works as a cosmic censor $[1]$. The conjecture is supported by the relation between these two concepts and the positivity of the energy.

It was, however, quickly realized that the conjecture fails for the simplest black-hole-type stationary supersymmetric solutions of pure $N = 2$, $d = 4$ supergravity, which have the general form $[2,3,4]$

$$ds^2 = |V|^2 (dt + \omega)^2 - |V|^{-2} d\vec{x}^2, \quad (2)$$

where $d\omega = i \star (|V|^{-2} [\bar{V}^{-1} dV^{-1} - V^{-1} d\bar{V}^{-1}]), \quad \nabla^2_{(3)} V^{-1} = 0.$

For instance, the extreme Reissner-Nordström-Taub-NUT solution $[5]$, which is characterized by the complex harmonic function

$$V^{-1} = 1 + \frac{M + iN}{r} \quad \Rightarrow \quad \omega = 2N \cos \theta d\phi, \quad (3)$$

has wire singularities at $\theta = 0, \pi$. They are always associated to the integral

$$\int_{\partial \Sigma^3} d\omega = \int_{\Sigma^3} d^2 \omega = -8\pi N, \quad (4)$$

and can be removed, but only at the expense of asymptotic flatness $[6]$.

Another example is provided by the supersymmetric ($M = |q|$) Kerr-Newman solution $[7,8] J = M\alpha$, characterized by

$$V^{-1} = 1 + \frac{M}{\sqrt{x^2 + y^2 + (z - i\alpha)^2}}, \quad (5)$$

which has a naked singularity in the ring $x^2 + y^2 = \alpha^2, \quad z = 0.$
Finally, the multipole solutions of this theory

$$V^{-1} = 1 + \sum_n \frac{M_n + iN_n}{|\vec{x} - \vec{x}_n|},$$

always seem have wire singularities associated to the points at which $d^2\omega \neq 0$, which can be seen as sources of NUT charges $N_n$, and there seems to be no choice of $M_n, N_n, \vec{x}_n$ that eliminates them. It was then conjectured by Hartle and Hawking [9] that the only regular black-hole solutions of this class were the Papapetrou-Majumdar [10, 11] solutions ($N_n = 0, \forall n$) describing static extreme Reissner-Nordström black holes in equilibrium [12].

Observe that, coincidentally, no microscopic String Theory descriptions of NUT charge or angular momentum (preserving supersymmetry) seem to exist.

We will show that, actually, only the regular solutions which can be described by String Theory are truly supersymmetric everywhere.

2 The 2006 SUSY versus cosmic censorship conjecture

The explicit knowledge of the most general supersymmetric black-hole-type solutions of $N = 2, d = 4$ SUGRA theories [13, 14] has led as to reformulate the 1992 conjecture extending the requirement of supersymmetry to the sources that give rise to the macroscopic fields [15]. The conjecture says now that supersymmetric, asymptotically-flat, black-hole-type solutions satisfying the following conditions will be regular black holes without naked singularities:

I The solutions must be everywhere supersymmetric. In particular this implies that

1. The integrability conditions of the Killing spinor equations (KSIs [16, 17]) must be satisfied everywhere. We will see that this always requires $d^2\omega = 0$ everywhere.

2. The masses of each of the sources of the solutions should be positive.

II In presence of scalars, the attractor equations [18, 19, 20, 21]

$$\mathcal{D}_i Z|_{Z^i = z^i_{\text{fix}}} = 0,$$

must be satisfied at each of the sources for admissible values of the scalars (no hair) and the value of the central charge must be finite.

These conditions should be enough to ensure the finiteness and positivity of $-g_{rr}$ everywhere. Now let us see how these conditions select regular solutions that can be described microscopically by String Theory.
2.1 Pure $N = 2, d = 4$ SUGRA

The KSIs are relations between the equations of motion of the bosonic fields $\mathcal{E}^{\mu\nu} \equiv \delta S/\delta g_{\mu\nu}$, $\mathcal{E}^\mu \equiv \delta S/\delta A_\mu$, evaluated on supersymmetric configurations. For black-hole-type configurations they are \[14\]

$$\mathcal{E}^{0m} = \mathcal{E}^m = *\mathcal{B}^m = \Im[e^{-i\alpha}(\mathcal{E}^0 - i*\mathcal{B}^0)] = 0, \quad \mathcal{E}^{00} = \Re[e^{-i\alpha}(\mathcal{E}^0 - i*\mathcal{B}^0)]. \tag{7}$$

If we deal with classical solutions, then $\mathcal{E}^{\mu\nu} = \mathcal{E}^\mu = 0$ and, where this is not so, (singularities), we can think on the presence of sources. Then the KSIs become constraints for supersymmetric sources:

- $\mathcal{E}^{0m} = 0 \Rightarrow$ no sources of angular momentum or NUT charge.
- $\mathcal{E}^m = *\mathcal{B}^m = 0 \Rightarrow$ no sources of electric or magnetic dipole momenta.
- $\Im[e^{-i\alpha}(\mathcal{E}^0 - i*\mathcal{B}^0)] \sim d^2\omega = 0 \Rightarrow$ no sources of angular momentum or NUT charge.

These are precisely the supersymmetric sources String Theory does not account for. This excludes automatically the pathological solutions that we studied in the introduction because all of them correspond to sources of angular momentum, NUT charge or dipole momenta.

It should be stressed that Solutions with global angular momentum and dipole momenta are not excluded \textit{a priori}, but their sources must be static electric and magnetic monopoles (Reissner-Nordström black holes). This is exactly the opposite to what happens with nuclear magnetic dipole momenta which always correspond to dipole sources (spin) and not to pairs of magnetic monopoles [22]. It is, however, possible to show that in pure $N = 2, d = 4$ supergravity multipole sources such as those in Eq. (6) satisfying all the supersymmetry constrains will only give rise to static fields, proving the conjecture made by Hartle and Hawking [9].

To have solutions with angular momentum we need to add matter fields.

2.2 $N = 2, d = 4$ SUGRA coupled to vector multiplets

The KSIs are basically identical to those of the pure supergravity theory but now they contain symplectic-invariant combinations of electric and magnetic charges and dipole momenta. Again, one of them is related to the condition $d^2\omega = 0$ which is the integrability condition of the differential equation defining $\omega$ [23,24].

Now there is a new KSI involving the equations of motion of the scalars $\mathcal{E}_i \equiv \delta S/\delta Z^i$:

$$\langle \mathcal{U}_i | \mathcal{E}^0 \rangle = \frac{1}{2} e^{-i\alpha} \mathcal{E}_i. \tag{8}$$

This equation means that \textit{when the attractor equations are satisfied}, which is one of the conditions that we require, the l.h.s. vanishes everywhere and then, by supersymmetry, the
r.h.s. also vanishes and there are no scalar sources. It is not known how to account for these sources in String Theory.

It is worth giving an explicit example of how the conditions we impose lead in this context to regular solutions. Let us consider the simple prepotential $F = -iX^0X^1$: choose the four real harmonic functions

$$I_0 = \frac{1}{\sqrt{2} + q r_1 + q r_2},$$

$$I_1 = \frac{1}{\sqrt{2} + \frac{8q}{r_1} + \frac{8q}{r_2}},$$

$$r_{1,2} \equiv |\vec{x} - \vec{x}_{1,2}|,$$

where $q > 0$. With this choice we get the metric component

$$-g_{rr} = 1 + \frac{9\sqrt{2}q}{r_1} + \frac{10\sqrt{2}q}{r_2} + \frac{16q^2}{r_1^2} + \frac{8q^2}{r_2^2} + \frac{40q^2}{r_1 r_2},$$

which is finite everywhere outside $r_{1,2} = 0$, the positions of the would-be sources which in the end are regular horizons. In particular the “mass” of each of the two objects is positive

$$M_1 = 9q/\sqrt{2}, \quad M_2 = 5\sqrt{2}q, \quad M = M_1 + M_2 = 19q/\sqrt{2}. \quad (11)$$

In the $r_{1,2} \rightarrow 0$ limits we find spheres of finite areas

$$\frac{A_1}{4\pi} = 16q^2 = 2|Z_{\text{fix},1}|^2, \quad \frac{A_2}{4\pi} = 8q^2 = 2|Z_{\text{fix},2}|^2. \quad (12)$$

To have zero NUT charge, we must fix

$$r_{12} \equiv |\vec{x}_2 - \vec{x}_1| = 12\sqrt{2}q, \quad (13)$$

and we get a finite global angular momentum given by

$$|J| = 12q^2. \quad (14)$$

The origin of this angular momentum is the angular momentum of the electromagnetic fields, which is due to the simultaneous presence of electric and magnetic charges. Therefore, it is naturally quantized.

### 2.3 Preliminary results in $N = 1, d = 5$ SUGRA

In this theory the KSIs are qualitatively different, which is to be expected since in $d = 5$ there are regular supersymmetric rotating black holes and black rings [25, 26]. In particular we have a KSI of the form [27]

$$\mathcal{E}^m = -\frac{\sqrt{3}}{4} h^i \mathcal{E}_i^m, \quad \rightarrow \text{source of angular and electric dipole momenta are related.} \quad (15)$$
The metrics of all supersymmetric black holes and rings are of the form [28, 29, 30]
\[ ds^2 = f^2(dt + \omega)^2 - f^{-1}h_{mn}dx^m dx^n, \]
and the information about “NUT charge” (its \(d = 5\) equivalent), angular momentum etc. is, again, contained in the 1-form \(\omega\) which is determined by a differential equation in \(d\omega\). The above KSI turns out to be the integrability of the \(\omega\) equation:
\[ E^m + \sqrt{\frac{3}{4}} h^{il} E_l^m = \frac{1}{2} f^{-5/2}[*_4 d^2 \omega]^m. \]
Again, requiring supersymmetry \(\text{everywhere}\) in these 5-dimensional theories implies that the integrability condition \(d^2 \omega = 0\) \(\text{everywhere}\), but now, as different from what happens in \(d = 4\), this does not imply total absence of sources of angular momentum, but a specific relation between these and electric dipole sources. We have checked that the regular supersymmetric BMPV black hole [25] and ring [26] solutions do satisfy the integrability condition \(d^2 \omega = 0\) \(\text{everywhere}\).

3 Final comments

Some problems have been left aside in the above discussion.

Why do we impose the attractor equations? Sometimes there is no attractor, but the KSI Eq. (8) is still satisfied. This is typically what happens for \(\text{small black holes}\), but in that case they seem to always satisfy a new quantum-corrected attractor equation and the quantum-corrected geometry is regular [31].

There are other possible pathologies of a supersymmetric solution that do not seem to be eliminated by our conditions. The most important of them is the existence of closed timelike curves (CTCs) in some 5-dimensional solutions. The everywhere regular (it is a Riemannian homogenous space) and maximally supersymmetric Gödel solution of \(n = 1, d = 5\) supergravity has CTCs and it is not clear how the should be interpreted.

Finally, the Calabi-Yau compactifications that give rise to \(d = 4, 5\) supergravities with 8 supercharges give not only vector multiplets but also hypermultiplets. While it is always consistent to truncate them, these truncations do not correspond to the generic situation and the modifications induced by turning on the hyperscalars need to be studied in depth. The first steps in this direction have been taken in [32, 33], where all the supersymmetric solutions of these ungauged supersymmetric theories with hypermultiplets have been found. In [33] an asymptotically flat 1/8 supersymmetric “deformation” of the 1/2 supersymmetric \(d = 5\) Reissner-Nordström solution induced by the presence of non-trivial hyperscalars was constructed and found to be singular.

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**References**

[1] R. Kallosh, A.D. Linde, T. Ortín, A.W. Peet and A. Van Proeyen, Phys. Rev. D **46** 5278 (1992) [arXiv:hep-th/9205027].

[2] Z. Perjés, Phys. Rev. Lett. **27** 1668 (1971).

[3] W. Israel and G.A. Wilson, J. Math. Phys. **13**, 865 (1972).

[4] K.P. Tod, Phys. Lett. B **121** 241 (1983).

[5] D. Brill, Phys. Rev. **133** B845-B848 (1964).

[6] C. Misner, J. Math. Phys. **4** 924 (1963).

[7] R. P. Kerr, Phys. Rev. Lett. **11** 237 (1963).

[8] E. T. Newman, E. Couch, K. Chinnapared, A. Exton, A. Prakash and R. Torrence, J. Math. Phys. **6** 918 (1965).

[9] J.B. Hartle and S.W. Hawking, Commun. Math. Phys. **26** 87 (1972).

[10] A. Papapetrou, Proc. Roy. Irish. Acad. **A51** 191 (1947).

[11] S.D. Majumdar, Phys. Rev. **72** 390 (1947).

[12] D.R. Brill and R.W. Lindquist, Phys. Rev. **131** 471-476 (1963).

[13] K. Behrndt, D. Lüst and W.A. Sabra, Nucl. Phys. B **510** 264 (1998) [arXiv:hep-th/9705169].

[14] P. Meessen and T. Ortín, Nucl. Phys. B **749** 291 (2006) [arXiv:hep-th/0603099].

[15] J. Bellorín, P. Meessen and T. Ortín, arXiv:hep-th/0606201.

[16] R. Kallosh and T. Ortín, arXiv:hep-th/9306085.

[17] J. Bellorín and T. Ortín, Phys. Lett. B **616** 118 (2005) [arXiv:hep-th/0501246].

[18] S. Ferrara, R. Kallosh and A. Strominger, Phys. Rev. D **52** 5412 (1995) [arXiv:hep-th/9508072].

[19] A. Strominger, Phys. Lett. B **383** 39 (1996) [arXiv:hep-th/9602111].
[20] S. Ferrara and R. Kallosh, Phys. Rev. D 54 1514 (1996) [arXiv:hep-th/9602136].

[21] S. Ferrara and R. Kallosh, Phys. Rev. D 54 1525 (1996) [arXiv:hep-th/9603090].

[22] J.D. Jackson, Yellow Report CERN-77-17 [SPIRES entry] Reprinted in V. Stefan and V.F. Weisskopf, eds., Physics and Society: Essays in Honor of Victor Frederick Weisskopf (AIP Press, New York, Springer, Berlin, 1998) 236pp.

[23] F. Denef, JHEP 0008 050 (2000) [arXiv:arXiv:hep-th/0005049].

[24] B. Bates and F. Denef, arXiv:hep-th/0304094.

[25] J.C. Breckenridge, R. C. Myers, A. W. Peet and C. Vafa, Phys. Lett. B 391 93 (1997) [arXiv:hep-th/9602065].

[26] H. Elvang, R. Emparan, D. Mateos and H.S. Reall, JHEP 0508 042 (2005) [arXiv:hep-th/0504125].

[27] J. Bellorín, P. Meessen and T. Ortín, (to appear).

[28] J. P. Gauntlett, J. B. Gutowski, C. M. Hull, S. Pakis and H. S. Reall, Class. Quant. Grav. 20 4587 (2003) [arXiv:hep-th/0209114].

[29] J.P. Gauntlett and J.B. Gutowski, Phys. Rev. D 71 025013 (2005) [arXiv:hep-th/0408010].

[30] J. P. Gauntlett and J. B. Gutowski, Phys. Rev. D 71 045002 (2005) [arXiv:hep-th/0408122].

[31] A. Dabholkar, R. Kallosh and A. Maloney, JHEP 0412 059 (2004) [arXiv:hep-th/0410076].

[32] M. Hübscher, P. Meessen and T. Ortín, arXiv:hep-th/0606281.

[33] J. Bellorín, P. Meessen and T. Ortín, arXiv:hep-th/0610196.