IMPLICATIONS OF A HIGGS DISCOVERY AT LEP

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If the Higgs boson has a mass below 130 GeV, then the standard model vacuum is unstable; if it has a mass below 90 GeV (i.e. within reach of LEP within the next two years), then the instability will occur at a scale between 800 GeV and 10 TeV. We show that precise determinations of the Higgs and top quark masses as well as more detailed effective potential calculations will enable one to pin down the location of the instability to an accuracy of about 25 percent. It is often said that “the standard model must break down” or “new physics must enter” by that scale. However, by considering a toy model for the new physics, we show that the lightest new particle (or resonance) could have a mass as much as an order of magnitude greater than the location of the instability, and still restabilize the vacuum.
1 Introduction

The large value of the top quark mass has intensified interest in Higgs mass bounds arising from the requirement of vacuum stability. Since the contribution of the top quark Yukawa coupling to the beta function of the scalar self-coupling, $\lambda$, is negative, a large top quark mass will drive $\lambda$ to a negative value (thus destabilizing the standard model vacuum) at some scale, generally denoted as $\Lambda$. The only way to avoid this instability is to require that the Higgs mass be sufficiently large (thus the initial value of $\lambda$ is large) or to assume that the standard model breaks down before the scale $\Lambda$ is reached \[1\].

If one assumes that the standard model is valid up to the unification or Planck scale (the difference between the two does not appreciably affect the bounds), then a lower bound on the Higgs mass can be obtained by requiring that the standard model vacuum be the only stable minimum up to that scale. Many papers in recent years\[2, 3, 4\] have gradually refined this lower bound; the most recent are the works of Casas, Espinosa and Quiros\[3\][CEQ] and of Altarelli and Isidori\[4\][AI], who show that the requirement of vacuum stability up to the Planck scale gives, for a top quark mass of 175 GeV, a lower bound of 130 GeV on the Higgs mass. If the Higgs mass is lighter than this bound, then the standard model must break down at a lower scale; the farther below the bound, then the lower this scale. In fact, as emphasized by AI and CEQ, if the Higgs has a mass just above its current experimental limit, then the standard model must break down at a scale of roughly a TeV. Since the standard model is defined by the assumption that there is no new physics until a scale of several TeV (at least), one concludes that the discovery of a Higgs boson at LEPII could, depending on the precise top quark mass, rule out the standard model!!

In this Letter, we will examine this question in more detail. First, we will discuss the scale, $\Lambda$, at which the Higgs potential turns negative, thus destabilizing the standard model vacuum. We will consider Higgs masses within reach of LEPII. Then, the uncertainties and difficulties associated with determining this scale precisely, given the Higgs and top quark masses, will be discussed. The standard statement is that “the standard model must break down before the scale $\Lambda$” or that “new physics must operate before the scale $\Lambda$”. We will examine the meaning of this statement by considering a toy model in which a scalar boson of mass $M$ is added to the standard model, and we will show that it is not necessary that this mass be less than $\Lambda$, that it could even have a mass as high as $5 - 10$ times larger and still prevent the vacuum instability. Thus, even if one
were to conclude that \( \Lambda \) is only a TeV, this would not necessarily imply that a particle or resonance must be at or below this scale.

\section{The Higgs potential and location of the instability}

For top quark and Higgs masses of interest, the Higgs potential has its usual electroweak minimum at 246 GeV, and then at some larger scale, \( \Lambda \), turns around (sharply) and becomes negative, destabilizing the electroweak vacuum\[^1\]. In order to calculate the potential as accurately as possible, one must sum all leading and next-to-leading logarithms. This is done\[^5\] by improving the one-loop effective potential by two-loop renormalization-group equations. The most recent and detailed calculations of the bounds are those of Altarelli, et al.\[^4\] and Casas, et al.\[^3\]. The reader is referred to those papers for details, we will simply present their results here.

The procedure is straightforward. One begins with values of the scalar self-coupling, \( \lambda \), and the top quark Yukawa coupling, evaluated at some scale (usually \( m_Z \)). One then integrates these using the two-loop renormalization group equations. The running couplings are then inserted into the one-loop Higgs potential, which is then examined to see if it goes negative, and if so, at what scale. Finally, the Yukawa coupling and \( \lambda \) must be converted into pole masses for the physical Higgs boson and top quark. Many issues involving the choice of renormalization scale and the renormalization procedure must be considered\[^3, 6\].

The most important results of the papers of AI and CEQ was the bound on the Higgs mass assuming that the standard model is valid up to the Planck scale. They obtained (using a value of the strong coupling given by \( \alpha_s(m_Z) = 0.124 \)):

\[
m_H > 130.5 + 2.1(m_t - 174) \quad (1)
\]

for AI (all masses are in GeV) and

\[
m_H > 128 + 1.92(m_t - 174) \quad (2)
\]

for CEQ. These results are in agreement to well within the stated 3 – 5 GeV errors of the two calculations.

\[^1\]We will define \( \Lambda \) to be the point at which the potential drops below the value of the electroweak minimum, however the drop is so rapid that this does not appreciably differ from the point at which it turns around.
Suppose one looks at the values of $m_H$ and $m_t$ which give an instability at $\Lambda = 1$ TeV. AI obtain

$$m_H = 71 + .9(m_t - 174)$$

(3)

We have reproduced this result and have generalized it to different $\Lambda$. We find that the factor of $.9(m_t - 174)$ is unchanged, and the factor of 71 GeV changes to 77 GeV for $\Lambda = 1.5$ TeV, 81 GeV for $\Lambda = 2$ TeV, and 89 GeV for $\Lambda = 4$ TeV. Thus, knowing the experimental values of the Higgs mass and top quark masses to an accuracy of 1 GeV each, which may be possible at an NLC, will enable one to determine $\Lambda$ to roughly 25 percent accuracy.

Before discussing the significance of knowing a particular value of $\Lambda$, it is important to discuss the uncertainty in this formula. As discussed very clearly by CEQ, the choice of the scale-dependence of the renormalization scale introduces uncertainties of roughly 3 – 5 GeV in the Higgs mass. In addition, one must be very careful in determining the condition for the instability. For example, AI took the instability to occur when $\lambda$ became negative, whereas CEQ took the instability to occur when $\tilde{\lambda}$, given by (ignoring, for illustrative purposes, the electroweak gauge couplings)

$$\tilde{\lambda} = \lambda - \frac{1}{32\pi^2} \left[ 6h_t^4 \left( \ln \frac{h_t^2}{2} - 1 \right) \right]$$

(4)

goes negative, which minimizes uncertainties due to higher orders. This can make a small difference in $\Lambda$, as CEQ show, which is irrelevant for large $\Lambda$ but can be very important for smaller $\Lambda$.

Finally, there is another, potentially serious, uncertainty. In running the scalar self-coupling from $m_Z$ to the location of the instability, one includes top quark contributions at all scales, and thus the beta function is negative at $m_Z$ and drives the scalar self-coupling towards negative values immediately. However, suppose one were to argue that the top quark loop contributions to the beta function should not enter until $2m_t$ is reached, as is usually the case for the QCD beta function. Then, $\lambda$ would not change much

2The results of CEQ differ significantly, by as much as 15 GeV. The reason appears to be related to the work of Willey and Bochkarev[7]. They noted that the contribution of the finite $\overline{MS}$ electroweak tadpoles to the relation between the top quark pole mass and the mass defined in terms of the $\overline{MS}$ Yukawa coupling is much larger than the well-known QCD correction. A similar contribution exists in relating the Higgs pole mass to the potential. Willey[8] has pointed out that this contribution cancels in the Higgs-top mass ratio. In the paper of CEQ, it was included in the Higgs mass relation, but not in the top mass relation. By including the term in the top mass relation in the CEQ work, Willey[9] has found that the discrepancy becomes much smaller (and the agreement for large $\Lambda$ persists).
between $m_Z$ and $2m_t$, increasing the location of the instability by (very roughly) a factor of $2m_t/m_Z \sim 4$. Of course, the $\overline{MS}$ renormalization scheme is mass-independent, and thus the contribution should be included at all scales, but this does indicate that another renormalization scheme which is more sensitive to threshold effects could give significantly different results. This would imply that uncalculated higher order contributions could become important if threshold effects are included.

In principle, all of these issues can be dealt with (and certainly will be if the Higgs boson is discovered at LEP). In that case, the results of Eq. 3 will be accurate to within a couple of GeV. If the Higgs boson is discovered next year at LEP, then the standard model vacuum will be known to be unstable at a scale of somewhere between 0.8 and 10 TeV. The biggest uncertainty in pinning down this number is the top quark mass. Once it is known to an accuracy of around 5 GeV, then the biggest uncertainty will be in the above calculations. When these uncertainties are removed, then the location of the instability, $\Lambda$, will be known to roughly a factor of 2. Finally, as the experimental values of the Higgs and top quark masses are narrowed down to 1 GeV each, the location of the instability will eventually be determined to roughly 25% accuracy.

We now consider the following question. Let us suppose that this happens, and one concludes that the instability occurs at, say, 1000-1400 GeV. What does this imply for new physics? Must a new particle or resonance occur with a mass below or near this scale?

### 3 Model of New Physics

In order to examine the effects of new physics, a simplified version of the standard model Higgs potential will be considered in which the renormalization scale dependence of the parameters is ignored. The resulting potential can then be written as

$$V = -\frac{1}{2}m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 - \frac{1}{4} B \phi^4 \left( \ln \frac{\phi^2}{\mu} - C \right)$$

where

$$B = \frac{1}{16 \pi^2} \left( 3h_t^4 - 3g^4/8 - 3(g^2 + g'^2)^2/16. \right)$$

This simplified potential has all of the qualitative features of the full renormalization-group improved potential, and the results obtained from its use will not be substantially changed by using the full potential. Differentiating the potential, one can replace $m^2$ and
by the Higgs mass $m_h$ and the electroweak minimum, $\sigma$:

$$V = -\frac{1}{4} m_h^2 \phi^2 - \frac{1}{2} B \sigma^2 \phi^2 + \frac{3}{8} B \phi^4 + \frac{1}{8 \sigma^2} m_h^2 \phi^4 - \frac{1}{4} B \phi^4 \ln \frac{\phi^2}{\sigma^2}$$  \hspace{1cm} (7)

Plugging in a top quark mass of 190 GeV\(^3\) and a Higgs mass of 70 GeV, one can see that the potential turns around and becomes negative at a scale, $\Lambda$, of 1250 GeV. This would seem to imply that “new physics” must enter by that scale.

In order to model the “new physics”, we will add to the model a scalar field, with bare mass $M$, which couples to the standard model Higgs field with coupling $\delta$ (so that the mass-squared of the scalar is $M^2 + \delta \sigma^2$). In addition, the multiplicity of the scalar field will be $N$. This is fairly general. We know that additional fermionic degrees of freedom will further destabilize the vacuum, so that only bosonic degrees of freedom need to be considered. These degrees of freedom must couple to the Higgs field (to have any effect on the potential), and one would expect a number of such fields (if they are vector fields, of course, the multiplicity of each would be 3).

What are reasonable values for $N$ and $\delta$? The value of $N$ will be taken to be anywhere between 1 and 100. Such large values of $N$ are not implausible. In the minimal supersymmetric model, for example, the multiplicity of scalar quarks is $N = 72$ (6 for flavor, 3 for color, 4 for particle/antiparticle and left/right); in left-right models, the multiplicity of the new gauge bosons and Higgs bosons is $N \sim 25$. $\delta$ will be taken to be between 0.1 and 10. In the next section, the unitarity bound on $N$ and $\delta$ will be found and we will only assume that the values must be lower than that bound. It is plausible that the value of $\delta$ would be close to the unitarity bound, if the effective new physics is strongly coupled.

The effects of the scalar on the Higgs potential is to add a term

$$\frac{N}{64 \pi^2} (M^2 + \delta \phi^2)^2 \left( \ln \frac{M^2 + \delta \phi^2}{\mu^2} - C \right)$$  \hspace{1cm} (8)

to the potential. Differentiating the potential, one can replace $m^2$ and $\lambda$ with $m_h$ and $\sigma$, yielding

$$V = -\frac{1}{4} m_h^2 \phi^2 - \frac{1}{2} B \sigma^2 \phi^2 + \frac{3}{8} B \phi^4 + \frac{1}{8 \sigma^2} m_h^2 \phi^4 - \frac{1}{4} B \phi^4 \ln \frac{\phi^2}{\sigma^2}$$  

$$+ \frac{N}{64 \pi^2} (M^2 + \delta \phi^2)^2 \ln \frac{M^2 + \delta \phi^2}{M^2 + \delta \sigma^2}$$  \hspace{1cm} (9)

\(^3\)The pole mass is chosen to be 190 GeV, so that the Yukawa coupling corresponds to a mass which is 5-6 % smaller.
where

\[ B_1 = B - \frac{N}{32\pi^2}(2\delta^2 - \delta \frac{M^2}{\sigma^2}) \]
\[ B_2 = B - \frac{N}{16\pi^2}\delta^2. \]  

(10)

This potential can be plotted, and examined for various values of \( N, \delta \) and \( M \) to see if the instability remains. Some features are easy to see. Consider the limit in which \( M = 0 \), so the additional scalars are very light. In this case, the coefficient of the \( \phi^4 \ln \phi^2 \) term is \( N\delta^2/64\pi^2 - B/4 \). Thus, if \( N\delta^2 \) is too small, this coefficient will be negative and the instability will remain. Thus, a lower bound on \( N\delta^2 \), in order to remove the instability, is

\[ N\delta^2 > 3h_t^4 - 3g^4/8 - 3(g^2 + g'^2)^2/16. \]  

(11)

It is also interesting to consider the limit in which \( M \to \infty \). In this case, the logarithm can be expanded and one can see that the effects of the extra term vanishes completely, as expected from the decoupling theorem. In this case, the scalar field will not restabilize the potential, regardless of the values of \( N \) and \( \delta \).

Thus, for any given values of \( N \) and \( \delta \) (above the critical value), there will be some critical value of the scalar mass; if \( M \) is below this value, the potential will be restabilized; if \( M \) is above this value, it will not be. This is illustrated in Fig. 1. We have chosen \( N = 60 \) and \( \delta = 1 \), and have plotted the potential for various values of \( M \). In this case, \( M \) can be as large as 5.3 TeV, and still restabilize the potential. Note that this value for \( M \) is four times the value of \( \Lambda \), the point by which “new physics must enter”.

The critical value of the scalar mass is shown as a function of \( N \) and \( \delta \) in Fig. 2. Note that the \( M = 0 \) line corresponds to the critical value of \( N\delta^2 \) shown above. We see that for the largest values of \( N \) and \( \delta \), the scalar mass could be as large as 100 times \( \Lambda \), and still restabilize the potential! Of course, one would question the validity of perturbation theory for such values, and we now turn to the question of the unitarity bounds on \( N \) and \( \delta \).

## 4 Unitarity Bounds

There are two types of unitarity bounds that we can consider; a bound on \( \delta \) from tree-level unitarity and a bound on \( N\delta^2 \) from one-loop unitarity.

The first is the bound on \( \delta \) arising from the requirement of tree-level unitarity. If we call the scalar \( S \) and the Higgs boson \( H \), then one will obtain a bound on \( \delta \) by
requiring that the real part of each $H S \rightarrow H S$ partial wave scattering amplitude be less than $1/2$. This calculation can be easily done, and in the limit that the quartic $S^4$ coefficient is small, we find the bound $\delta < 4\pi$. This is a fairly weak bound, which is weaker than the bound obtained from the following bound on $N\delta^2$.

A bound on $N\delta^2$ will arise by considering $H H \rightarrow H H$ at one-loop in which a loop with $S$ bosons is in the diagram—this will be proportional to $N\delta^2/8\pi^2$; so one might expect a bound on $N\delta^2$ somewhat less than $8\pi^2$. One-loop unitarity is a more difficult problem, and the bound will always depend on $\sqrt{s}$ of the scattering process. We will estimate the bound in two very different ways.

The first, and simplest, method is to note that the beta function for $\lambda$ can be read off from the potential of Eq. (9), and clearly has a term proportional to $N\delta^2$. Thus, we can integrate $\lambda$ from the electroweak scale (or the $Z$ mass—the choice doesn’t significantly affect the result) up to the scale given by $M$, and simply require that $\lambda$ not exceed its unitarity limit by that scale. Since we are using one-loop beta functions here, we only require that $\lambda$ not exceed its tree level unitarity bound, given by $\lambda \sim 4$ (this value corresponds to a Higgs mass of 700 GeV, which is the Lee-Quigg-Thacker bound[10]). When we do so, we find the bound given by the solid line of Fig. 2, which corresponds to $N\delta^2$ varying between 30 and 60.

The second method is to compute the scattering amplitude for $HH \rightarrow HH$ at one loop. Since we are interested in $m_h \leq 90$ GeV, the Higgs self-coupling $\lambda$ is small ($\leq 0.067$) and the one-loop contributions to the above process coming from $H$ and the Goldstone bosons $w, z$ can be neglected. The dominant contribution comes from $S$. In the limit that the quartic $S^4$ coefficient is small compared with $\delta$, we can ignore the one-loop contribution to $HS \rightarrow HS$.

The one-loop contribution of $S$ to $HH \rightarrow HH$ can be straightforwardly computed. The renormalization consists of two parts. One comes from the one-loop self energy of $H$ due to $S$ where a factor of $N\delta^2$ is present. (We are again ignoring the contributions due to $H$ and $w, z$ which are proportional to $\lambda^2$.) This contributes to the wave function renormalization constant for $H$ and to the renormalization of $\lambda$. The other comes from the bubble diagram for $HH \rightarrow HH$ involving $S$. The final physical scattering amplitude is, of course, finite. In the limit $\sqrt{s} >> M, m_h$, the real and imaginary parts of the $S$-wave partial wave amplitude, $a_0$, are given by

$$\text{Re } a_0 = -(3/8\pi)\lambda_s + (N\delta^2/64\pi^3)(1 + 3m_h^2 I_s(m_h^2)),$$
\[
\text{Im } a_0 = \frac{N\delta^2}{128\pi^2},
\]
where
\[
\lambda_s = \lambda + \frac{N\delta^2}{8\pi^2}(\ln \frac{\sqrt{s}}{M} - 1 + \frac{1}{2} I_s(m_h^2)),
\]
with \(I_s(p^2) = \int_0^1 dx \ln(1 + 4x(1 - x)/\beta)\) and \(\beta = 4M^2/p^2\) (\(I'_s\) is the derivative of \(I_s\) with respect to \(p^2\)). Numerically, \(I_s(m_h^2)\) and \(m_h^2 I'_s(m_h^2)\) are small.

By including the (tree level) real S-wave amplitude \((\delta/8\pi)\) for \(HS \rightarrow HS\) and diagonalizing the real part of the \(2 \times 2\) matrix, we can plot the Argand diagram for the largest eigenvalue (see Durand, et al. [11] for a detailed discussion). The upper limits on \(N\delta^2\) are found by looking at the point where the amplitude deviates significantly from the unitarity circle. We find the following results which depend on \(\sqrt{s}\): \(N\delta^2 < 30\) (\(\sqrt{s}/M \approx 100\)); \(N\delta^2 < 60\) (\(\sqrt{s}/M \approx 40\)); \(N\delta^2 < 100\) (\(\sqrt{s}/M \approx 20\)). This corresponds respectively to \(\lambda_s = 1.42, 2.07, 2.58\). This approach gives results which are basically consistent (within a factor of two in \(N\delta^2\)) with the ones obtained by “running” \(\lambda\) as discussed above.

5 Conclusions

From Fig. 2, we see that a single scalar boson with a coupling \(\delta \sim 6\) to the standard model Higgs (which is at, but not above, the unitarity bound) can have a mass as high as 10 TeV, and still succeed in eliminating the instability which would occur at 1250 GeV. If there were a strongly interacting sector, one might expect just such a coupling.

Thus, should LEP discover a Higgs boson in the near future, one will conclude that the standard model must “break down” at some calculable scale, \(\Lambda\), which could be between 1 and 10 TeV (depending on the top quark mass). In this letter, we have shown that this does not necessarily mean that a new particle(s) or resonance(s) must exist at this scale, but that the new states could be close to a factor of 10 higher in mass. There is no guarantee that an accelerator which reaches the scale \(\Lambda\) will find any direct evidence of new physics.

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Figures

1. The Higgs potential is plotted for three different values of $M$, with $N = 60$ and $\delta = 1$. Between the origin and $\phi = 500$ GeV, the three curves are essentially identical, and look like the conventional Higgs potential. For very large $M$, the scalar field decouples and the potential develops an instability at 1250 GeV. As $M$ decreases to 5.5 TeV, the instability point moves outward and then disappears for $M = 5.3$ TeV.

2. For various values of $N$ and $\delta$, the largest value of $M$ which will eliminate the instability (which occurs at 1250 GeV in the absence of the additional scalar field). The shaded region covers the values of $N$ and $\delta$ which violate the unitarity bound, as discussed in the text.
