CYCLOSTATIONARY IN THE TIME VARIABLE UNIVERSE

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ABSTRACT

Cyclostationary processes are those signals whose have vary almost periodically in statistics. It can give rise to random data whose statistical characteristics vary periodically with time although these processes not periodic functions of time. Intermittent pulsar is a special type in pulsar astronomy which have period but not a continuum. The Rotating RA dio TransientS (RRATs) represent a previously unknown population of bursting neutron stars. Cyclical period changes of variables star also can be thought as cyclostationary which are several classes of close binary systems. Quasi-Periodic Oscillations (QPOs) refer to the way the X-ray light from an astronomical object flickers about certain frequencies in high-energy (X-ray) astronomy. I think that all above phenomenon is cyclostationary process. I describe the signal processing of cyclostationary, then discussed that the relation between it and intermittent pulsar, RRATs, cyclical period changes of variables star and QPOs, and give the perspective of finding the cyclostationary source in the transient universe.

Subject headings: Cyclostationary — methods: data analysis: Intermittent pulsar, RRATs, Cyclical period changes of variables star, QPOs

1. INTRODUCTION

It was first mentioned by the design of synchronization algorithms for communications systems that cyclostationary process (Serpedin et al. 2005). Many processes encountered in nature arise from periodic phenomena. For example, in telecommunications, telemetry, radar, and sonar applications, periodicity is due to modulation, sampling, multiplexing, and coding operations. In mechanics it is due, for example, to gear rotation. In econometrics, it is due to seasonality; and in atmospheric science it is due to rotation and revolution of the earth (Gardner et al. 2006; Serpedin et al. 2005; Gardner 1994). Cyclostationary processes are named in multiple different ways such as periodically correlated, periodically nonstationary or cyclic correlated processes in literature (Hurd 1989, 1997).

In astronomy, now just in removing cyclostationary radio frequency interferences (RFI) from radio astronomical data (Leshem et al. 2000; Bretteil & Weber 2002) and the galactic white-dwarfs background that will be observed by LISA (Edlund et al. 2005), cyclostationary process be mentioned. Some radio pulsar have been discovered the phenomenon of intermittent period in recently (Kramer et al. 2006). The nulling of the pulsar be thought a characteristic of the older pulsars. In X-ray pulsar, the phenomenon of intermittent far-flung exist for the effect of companion. It also show almost period in time series. The Rotating RA dio TransientS (RRATs) demonstrate a previously unknown population of bursting neutron stars (McLaughlin et al. 2006). Cyclical period changes of variables star that including Algol, WUrsaeMajoris, and RS Canum Venaticorum systems and the cataclysmic variables, and RR Lyrae star et al. also can be thought as cyclostationary. The fastest variability components in X-ray binaries are the kilohertz quasi-periodic oscillations (kHz QPOs), which occur in a wide variety of low magnetic-field neutron star systems (van der Klis 2005). The signal process of the above phenomenon, in a statistical sense, as a periodic function of time. These kind of processes have been studied for many years, and are usually referred to as cyclostationary random processes (see Gardner et al. 2006; Serpedin et al. 2005; Gardner 1994) for a comprehensive overview of the subject and for more references.

In what follows we will briefly summarize the properties of cyclostationary processes, then discuss the cyclostationary signal process in some astronomical phenomenon, in the end give the perspective of finding cyclostationary processes in the time variable universe.

2. THE BASIC OF CYCLOSTATIONARY PROCESSES

The continuous stochastic process \( X(t) \) having finite second order moments is said to be cyclostationary with period \( T \) if the following expectation values

\[
E[X(t)] = m(t) = m(t + T), \tag{1}
\]

\[
E[X(t') X(t)] = C(t', t) = C(t' + T, t + T) \tag{2}
\]

are periodic functions of period \( T \), for every \( (t', t) \in \mathbb{R} \times \mathbb{R} \). We will assume \( m(t) = 0 \) for simplicity now (Gardner & Franks 1973; Hurd 1984, 1997). The important special case of cyclostationary signals are those which exhibit cyclostationary in second-order statistics (e.g., the autocorrelation function). It is called wide-sense cyclostationary signals, and is analogous to wide-sense stationary processes. The exact definition differs depending on whether the signal is treated as a stochastic process or as a deterministic time series (Gardner 1978).

If \( X(t) \) is cyclostationary, then the function \( B(t, \tau) \equiv C(t + \tau, t) \) for a given \( \tau \in \mathbb{R} \) is periodic with period \( T \), and it can be represented by the following Fourier series

\[
B(t, \tau) = \sum_{r=-\infty}^{\infty} B_r(\tau) e^{i2\pi \frac{r}{T}}, \tag{3}
\]
where the functions $B_r(\tau)$ are given by

$$B_r(\tau) = \frac{1}{T} \int_0^T B(t, \tau) e^{-i2\pi \tau t} \, dt .$$  \hfill (4)

The Fourier transforms $g_r(f)$ of $B_r(\tau)$ are the so-called “cyclic spectra” of the cyclostationary process $X(t)$ [Hurd 1989].

$$g_r(f) = \int_{-\infty}^{\infty} B_r(\tau) e^{-i2\pi \tau f} \, d\tau .$$  \hfill (5)

If a cyclostationary process is real, the following relationships between the cyclic spectra hold

$$B_{-r}(\tau) = B_{r}^*(\tau) ,$$  \hfill (6)

$$g_{-r}(-f) = g_{r}(f) ,$$  \hfill (7)

where the symbol $\ast$ means complex conjugation. This implies that, for a real cyclostationary process, the cyclic spectra with $r \geq 0$ contain all the information needed to characterize the process itself.

The function $\sigma^2(\tau) \equiv B(0, \tau)$ is the variance of the cyclostationary process $X(t)$, and it can be written as a Fourier decomposition as a consequence of Eq. (4)

$$\sigma^2(\tau) = \sum_{r=\infty}^{\infty} H_r e^{i2\pi r \tau} ,$$  \hfill (8)

where $H_r \equiv B_r(0)$ are harmonics of the variance of $\sigma^2$. From Eq. (4) it follows that $H_{-r} = H_r^*.$

For a discrete, finite, real time series $\{X_i, t = 1, \ldots, N\}$ the cyclic spectra can be estimated by generalizing standard methods of spectrum estimation used with stationary processes. Assuming again the mean value of the time series $X_i$ to be zero, the cyclic autocorrelation sequences are defined as

$$s_t^r = \frac{1}{N} \sum_{i=1}^{N} X_i X_{i+\lfloor r \rfloor} e^{-i2\pi \tau(t-1)} .$$  \hfill (9)

The cyclic autocorrelations are asymptotically (i.e. for $N \to \infty$) unbiased estimators of the functions $B_r(\tau)$ [Hurd 1989]. The Fourier transforms of the cyclic autocorrelation sequences $s_t^r$ are estimators of the cyclic spectra $g_r(f)$. These estimators are asymptotically unbiased, and are called “inconsistent estimators” of the cyclic spectra, i.e. their variances do not tend to zero asymptotically. In the case of Gaussian processes [Hurd 1989] consistent estimators can be obtained by first applying a lag window to the cyclic autocorrelation and then perform a Fourier transform.

The alternative procedure for identifying consistent estimators of the cyclic spectra is to first take the Fourier transform [Gardner et al. 2003, Gardner 1991], $\tilde{X}(f)$, of the time series $X(t)$

$$\tilde{X}(f) = \sum_{t=1}^{N} X_t e^{-i2\pi f(t-1)}$$  \hfill (10)

and then estimate the cyclic periodograms $g_r(f)$

$$g_r(f) = \frac{\tilde{X}(f) \tilde{X}^*(f - \frac{2\pi r}{T})}{N} .$$  \hfill (11)

By finally smoothing the cyclic periodograms, consistent estimators of the spectra $g_r(f)$ are then obtained. The estimators of the harmonics $H_r$ of the variance $\sigma^2$ of a cyclostationary random process can be obtained by first forming a sample variance of the time series $\{X_i\}$. The sample variance is obtained by dividing the time series $X_i$ into contiguous segments of length $\tau_0$ such that $\tau_0$ is much smaller than the period $T$ of the cyclostationary process, and by calculating the variance $\sigma^2$ over each segment. Estimators of the harmonics are obtained either by Fourier analyzing the series $\sigma^2$ or by making a least square fit to $\sigma^2$ with the appropriate number of harmonics. Note that the definitions of (i) zero order ($r = 0$) cyclic autocorrelation, (ii) periodogram, and (iii) zero-order harmonic of the variance, coincide with those usually adopted for stationary random processes. Thus, even though a cyclostationary time series is not stationary, the ordinary spectral analysis can be used for obtaining the zero order spectrum. Note, however, that cyclostationary random processes provide more spectral information about the time series they are associated with due to the existence of cyclic spectra with $r > 0$ [Edlund et al. 2003].

For stationary and cyclostationary time-series, there is two alternative philosophical frameworks for the two problems of estimating the time-invariant or time-variant autocorrelation function and its Fourier transform. One is based on the stochastic process model, and the other is based on the nonstochastic time-series model. Gardner, W.A. compared it, and explained that results on estimator bias and variance for these two problems couched within the stochastic process framework have analogs within the nonstochastic framework. The bias and variance results for cyclostationary time-series that are available within these two frameworks [Gardner 1991].

As an important and practical application, assuming consider a time series $y_t$ consisting of the sum of a stationary random process, $n_t$, and a cyclostationary one $X_t$ (i.e. $y_t = n_t + X_t$). Let the variance of the stationary time series $n_t$ be $\nu^2$ and its spectral density be $E(f)$. It is easy to see that the resulting process is also cyclostationary. If the two processes are uncorrelated, then the zero order harmonic $\Sigma_0$ of the variance of the combined processes is equal to

$$\Sigma_0^2 = \nu^2 + \sigma^2$$  \hfill (12)

and the zero order spectrum, $G_0(f)$, of $y_t$ is

$$G_0(f) = E(f) + g_0(f) .$$  \hfill (13)

The harmonics of the variance as well as the cyclic spectra of $y_t$ with $r > 0$ coincide instead with those of $X_t$. In other words, the harmonics of the variance and the cyclic spectra of the process $y_t$ with $r > 0$ contain information only about the cyclostationary process $X_t$, and are not “contaminated” by the stationary process $n_t$ [Edlund et al. 2003].

3. CYCLOSTATIONARY IN PULSAR, QPOS AND VARIABLES STAR

Recently, Kramer, M. et al. discovered one class of neutron stars that are seemingly ordinary radio pulsars, but which are only active for some short time and in a quasi-periodic fashion. So they call these "Intermittent
Pulsars”. These pulsar that is only periodically active. It appears as a normal pulsar for about a week and then "switches off" for about one month before emitting pulses again. The pulsar, called PSR B1931+24, is unique in this behaviour and affords astronomers an opportunity to compare its quiet and active phases. Most surprisingly, the pulsar rotation slows down faster when the pulsar is on than when it is off (Kramer et al. 2006). In the Figure 4, part a) show a typical sequence of observations covering a 20-month interval is indicated by the black lines. It shows respectively the times of observation and the times when PSR B1931+24 was on. It is clear that the pulsar is not visible for ~80% of the time, part b) give the appearance of the pulsar is quasi-periodic nature, demonstrated by the power spectrum of the intensity obtained from the Fourier Transform of the autocorrelation function of the mean pulse flux density obtained over the same 20-month interval, part c) is histograms of the durations of the on (solid) and off (hatched) phases. In off phases, integration over several weeks shows that any pulsed signal has a mean flux density of less than 2 µJy at 1400 MHz (Kramer et al. 2006).

McLaughlin, M.A. et al. discovered a class of Rotating Radio Transients (RRATs) which are identified as rotating neutron stars that send out very short flashes of radio light. These flashes are very short and very rare: one hundredth of a second long, the total time the objects are visible amounts to only about one tenth of a second per day. The isolated flashes last for between 2 and 30 milliseconds. In between, for times ranging from 4 minutes to 3 hours, the new stars are silent. They current estimates suggest that these objects are four times more common in the Galaxy than radio pulsars. In the Figure 2 from top to bottom, it is show that the original detections of J1317–5759, J1443–60 and J1826–14 in the Parkes Multibeam Survey data (McLaughlin et al. 2006). It is show that RRATs still have period in short time scale. They have identified periodicities in the range of 0.4 to 7 s in 10 of the 11 objects (Lyn 2007).

Camilo, F. et al. show that XTE J18102197 emits bright, narrow, highly linearly polarized radio pulses, observed at every rotation, thereby establishing that magnetars can be radio pulsars. There is no evidence of radio emission before the 2003 X-ray outburst (unlike ordinary pulsars, which emit radio pulses all the time), and the flux varies from day to day. The flux at all radio frequencies is approximately equal and at > 20GHz (Camilo et al. 2006).

In wide-sense, if one pulsar have nulling or giant pulse phenomenon, it will not have strict period function with time, just in statistics have periodic. Another pulsar cy-clostationary process is Shabanova, T.V. find that the nature of the observed cyclical changes in the timing residuals from PSR B1642 03 is a continuous generation of peculiar glitches whose amplitudes are modulated by a periodic large-scale sawtooth-like function. As the modulation function is periodical, the picture of cyclical timing residuals will be exactly repeated in each modulation period or every 60 years (Shabanova 2009).

QPOs were first identified in white dwarf systems (van der Klis et al. 1985) and then in neutron star systems (van der Klis 2005). Two QPO peaks (the ‘twin peaks’) occur in the power spectrum of the X-ray flux variations. They move up and down in frequency together in the 300-1200Hz range in correlation with source state and often, luminosity. The typically 300-Hz peak separation usually decreases by a few tens of Hz when both peaks move up by hundreds of Hz (van der Klis 2003). In the Figure 3 show that Keck II spectroscopy of optical mHz quasi-periodic oscillations (QPOs) in the light curve of the X-ray pulsar binary Hercules X-1 (O’Brien et al. 2001).

Year- to decade-long cyclic orbital period changes have been observed in several classes of close binary systems, including Algol, W Ursae Majoris, and RS Canum Venaticorum systems and the cataclysmic variables. The origin of these changes is unknown, but mass loss, apsidal motion, magnetic activity, and the presence of a third body have all been proposed. In the Figure 4 show that cyclical period changes in the dwarf novae V2051 Oph.

The other two statistics periodic phenomenon also relate with stellar. One is Hallinan, G. et al. detected periodic bursts of extremely bright, circularly polarized, coherent radio emission from the ultracool dwarf (Hallinan et al. 2007). Another is Double Periodic Variables (DPVs) that are blue stars characterized by a short periodicity (1-16 days) and a long periodicity (50-600 days) in their light curves. They were discovered in the Magellanic Clouds after a search for Be stars in the OGLE variable star catalog (Mennickent et al. 2003).

4. DISCUSSION AND CONCLUSIONS

Signal detection techniques designed for cyclostationary signals take account of the periodicity or almost periodicity of the signal autocorrelation function. Single-cycle and multicycle detectors exploit one or multiple cycle frequencies, respectively (Gardner et al. 2000). Some search-efficient methods of detection of cyclostationary signals (Yeung & Gardner 1996) and higher-order cyclostationary for weak-signal detection (Spooner & Gardner 1992) be developed. These will benefit of search transient source in the time variable universe, especially that have periodic in statistics.

Although no periodicities were detected in any of the sources using standard Fourier or folding methods, for ten of the sources (RRATs) McLaughlin, M.A. et al. identify a periodicity from the arrival times of the individual bursts that used search techniques similar to those described in (Cordes & McLaughlin 2003). In short, the 35-minute time series were dedispersed for a number of trials values of DM. The time series were smoothed by convolution with boxcars of various widths to increase sensitivity to broadened pulses, with a maximum boxcar width of 32 ms. Because the optimal sensitivity is achieved when the smoothing window width equals the burst width, our sensitivity is lower for burst durations greater than 32 ms. Each of these time series was then searched for any bursts above a threshold five standard deviations, computed by calculating a running mean and root-mean-square deviation of the noisy time series. All bursts detected above a 5-σ threshold are plotted as circles, with size proportional to the signal-to-noise ratio of the detected burst. The ordinate shows arrival time while the abscissa shows arrival time while the ordinate shows the DM. Because of their finite width, intense bursts are detected at multiple DMs and result in vertical broadening of the features. Bursts which are strongest at zero DM and therefore likely to be impulsive terrestrial interference are not shown. In general
these were easily identified by their detection in multiple beams of the 13-beam receiver\cite{McLaughlin2006}. More recently, Deneva, J.S. et al. use the above method and a friends-of-friends algorithm perform the ongoing Arecibo Pulsar ALF A (PALFA) survey of the Galactic planethen discover seven objects\cite{Deneva2009}.

The discovery of RRATs increases the current Galactic population estimates of radio pulsar by at least several times. It seems also that there will be many candidates for whom it will be impractical or impossible to follow up at present with current observing facilities. These will require followup with instruments like LOFAR, FAST or the SKA. Keane, E.F. et al. note that these instruments will produce extraordinarily large volumes of data so that searching for transient RRAT-like sources will necessitate the development of automated algorithms which will use the steps as outlined above\cite{Keane2009}. So I think we should use the detection methods of cyclostationary process integrate with DM search, that will be one automated and effective algorithms to seek RRATs.

If we do similar things in find the other astronomical cyclostationary sources that include Intermittent Pulssars, QPOs et al., it will lead to discover more unusual astronomical phenomenon.

DJ thanks

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Fig. 1.—Time variation of the radio emission of PSR B1931+24. During the on phases, the pulsar is easy to detect and has the stable long-term intrinsic flux density associated with most normal pulsars. Since 1998, the pulsar has been observed as frequently as twice a day. (see Kramer et al. 2006)
Fig. 2.— The observational signatures of the new radio transient sources. (see McLaughlin et al. 2006)

Fig. 3.— Left panel, lightcurve of the optical variability in Hercules X-1 for the entire data-set. Right panel, lightcurve of a subset of the data clearly showing the QPO. (O’Brien et al. 2001)
Fig. 4.— The dashed and dot-dashed lines in the lower panel show the best-fit 11 yr cycle period sinusoidal ephemeris, respectively, for the data in the first and the second halves of the time interval. (see Baptista et al. 2003)