Rationalization of cross-sections of the composite reinforced concrete span structure of bridges with a monolithic reinforced concrete roadway slab

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Abstract. The article presents the developed rationalization technique of composite steel reinforced concrete sections with steel open section beams based on the criterion of equal strength of the section elements that are extremely distant from the neutral line. Algorithms for search for geometric parameters of a composite section limited to a certain range of values are implemented to achieve the equal strength condition. The dimensions of the individual elements which are parts of the cross-section are obtained from the condition of the constant ratio of the distances from the neutral axis to the extreme concrete and steel fibers. The numerical methods were used for calculation of continuous three-span composite reinforce concrete bridge. The technique implements the steps of bridge construction, taking into account the contact yield of the composite section, the redistribution of forces between the elements, and the effect of elastic-plastic and rheological properties of materials. The generalized kinetic curve was utilized for evaluation of concrete creep together with the phenomenological equations for the development of deformations based on a colloid-chemical representation of the mechanism for long-term concrete deformation. The proposed methodology is implemented in the LIRA-SAPR software package based on the Building Information Model Technology (BIM) and the Finite Element Method (FEM).

1. Introduction
The efficiency of composite reinforced concrete span structures with the use of a monolithic reinforced concrete roadway slab is verified by calculation at the design stage, and will primarily depend on the adequacy of the simulation of the task. However, the applicable Ukrainian building codes [1] do not contain a technique of such calculations, which complicates the application of this type of structures. A rational choice of preliminary design parameters of the composite cross-section (such as beam height, slab thickness, upper and lower chord width, wall thickness, etc.) is also problematic. The article describes the practical experience of finite-element simulation of the work of a span structure using the example of a continuous composite reinforced concrete road bridge, taking into account the stages of operation of the construction, from the construction completion to the late service life. The proposed methodology implements the technique of rationalization and evolutionary transformations of the model (calculation based on the phases of construction) taking into account the influence of the elastic-plastic and rheological properties of the materials.
2. The rationalization technique for a composite cross-section

2.1. Problem statement
Figure 1 shows a composite beam cross-section taking bending moment. Based on the analysis of [2-5] with proposals on reinforcement of steel bars with concrete, the article [6] presented the rationale that the location of the slab in the compressed area of I-section is the effective reinforcement in case of perception of deformation during bending is (figure 1). This shape of the cross-section is acceptable, in particular, during the reconstruction of the structure for reinforcement purpose.

![Figure 1. Transverse cross-section.](image)

The section will be rational provided that one of the requirements for optimal design – equal strength – is taken into account. Equal strength for a section means that the stresses in the most distant from the neutral axis of the fibers reach their limiting values simultaneously. For concrete, this is \( \sigma_c = f_{cd} \) (estimated value of compressive strength of concrete) in compressed fibers, in steel \( \sigma_a = fyd \) (estimated resistance at the yield limit) in stretched fibers.

2.2. Rationalization algorithm
Let us determine the position of the neutral axis using the transformed section method described in [7]. Cumulative axial force acting in the transverse section should be zero, i.e.

\[
\int \sigma_a dA + \int \sigma_c dA = 0,
\]

where \( dA \) – cross-section area.

In bending:

\[
\sigma_a = \frac{E_c z}{\rho}; \quad \sigma_c = \frac{E_c z}{\rho},
\]

where \( E_a, E_c \) – elastic modulus of steel and concrete; \( z \) – distance from the described point to the neutral axis; \( \rho \) – curvature radius of neutral layer.

When (2) is substituted in (1), we get:

\[
\int z dA + n \int z dA = 0,
\]

where \( n = E_c / E_a \).

Determination of the maximum bending moment (figure 1). According to the axis deflection equation, maximum estimated bending moment \( M_{Ed} \) is determined by expression:
where \( \gamma_M \) – reliability factor of the steel; \( J_{tr} \) – inertia moment of the transformed section, determined by formula:

\[
J_{tr} = J_a + n J_c,
\]

where \( J_a, J_c \) – inertia moments of the steel and concrete parts of the section relative to the neutral axis of the section.

Substituting in condition (3) the integrands in accordance with figure 1, we obtain an equation for \( z_a \), the solution of which is the position of the neutral axis:

\[
2 b_1 t f_1 + t_w \left( h_{a1}^2 - t_{f1}^2 \right) + b_{12} t_{f2} \left( 2 h_{a1} + t_{f1} \right) + nb_{2} h_{f} \left( h_{a1} + t_{f2} + \frac{h_f}{2} \right)^2 = 2 \left[ b_1 t f_1 + t_w \left( h_{a1} - t_{f1} \right) + b_{12} t_{f2} \right] + nb_{2} h_{f}.
\]

The calculated cross-section corresponds to the maximum bending moment. Here, the greatest stresses occur in the most distant from the neutral axis fibers, which, according to the strength condition, should not exceed the estimated values:

\[
\sigma_{a \max} = \frac{M_{Ed} z_a}{J_{tr}} \leq \gamma_M f_{yd},
\]

\[
\sigma_{c \max} = n \sigma_{tr \max} = n \frac{M_{Ed} z_c}{J_{tr}} \leq f_{cd},
\]

where \( \sigma_{tr \max} \) – stress of transformed cross-section.

To achieve the equal strength of the cross-section and, thus, the optimal design, the strength conditions of concrete and steel at the same bending moment \( M_{Ed} \) should be met in the form of equalities. Hence:

\[
\frac{M_{Ed}}{J_{tr}} = \frac{f_{cd}}{n z_c} = \gamma_M f_{yd},
\]

and the dimensions of individual elements constituting the cross-section should be found from the condition of constant ratio of the distances to the extreme fibers of concrete and steel from the neutral axis:

\[
\frac{z_c + h_f + t_{f2} - z_a}{n \gamma_M f_{yd}} = \frac{f_{cd}}{n \gamma_M f_{yd}}.
\]

Figure 1 shows that

\[
z_c = h_{a1} + h_f + t_{f2} - z_a.
\]

From (10) and (11), we find

\[
z_a = \frac{\left( h_{a1} + h_f + t_{f2} \right) n \gamma_M f_{yd}}{f_{cd} + n \gamma_M f_{yd}}.
\]

The equation for determining the optimal dimensions of the composite section is obtained by equating the right member of the condition of equal strength (12) and expression (6) for the neutral axis:
\[
\frac{b_1t_{f1}^2 + t_w(h_{ai} - t_{f1}) + b_2t_{f2}(2h_{ai} + t_{f2}) + nb_2h_f(h_{ai} + t_{f2} + \frac{h_f}{2})}{2[b_1t_{f1} + t_w(h_{ai} - t_{f1}) + b_2t_{f2}]} + nb_2h_f.f_{cd} + n\gamma_{M0}f_{cd}. \tag{13}
\]

Fulfillment of condition (13) is achieved numerically by exhaustive search through all optimized cross-section dimensions.

Since equation (13) is non-linear, various combinations of dimensions of equal-strength sections can be obtained. Of these, the one must be chosen for which \(M_{max} = M_{Ed}\), where \(M_{max}\) is maximum bending moment occurring in a beam from a given external load.

Value \(M_{max}\) is chosen among the maximum values of moment on spans 1 and 2 \(M_{max1}\) and \(M_{max2}\), as well as the values of moment at support \(|M_{sup}|\). Since the beam section is piecewise constant within each span, the method of forces is used to determine bending moments.

To determine the maximum design moment \(M_{Ed}\) (4), moments of inertia of the steel and concrete components of the section are expressed (figure 1):

\[
J_a = 2 \left[ \frac{b_1t_{f1}^3}{12} + b_1h_f(t_a - t_{f1})^2 \right] + 2 \left[ \frac{t_w(h_{ai} - t_{f1})}{12} + t_w(h_{ai} - t_{f1}) \left( z_a - \frac{h_{ai}}{2} - \frac{t_{f1}}{2} \right)^2 \right] + \\
2 \left[ \frac{b_2t_{f2}^3}{12} + b_2h_f(t_a + t_{f2} - z_a)^2 \right]; \tag{14}
\]

\[
J_c = \frac{b_2h_f^3}{12} + b_2h_f(t_a + t_{f2} - z_a + \frac{h_f}{2} - z_a)^2. \tag{15}
\]

When inertia moments (14), (15) are known, we can find inertia moment of the transformed section \(J_{tr}\) (5) and the maximum bending moment (4).

The per-meter cost of the beam is determined by formula:

\[
C = V_a\gamma_a c_a + V_c c_c = (A_a \cdot 1)\gamma_a c_a + (A_c \cdot 1)c_c, \tag{16}
\]

where \(c_a\) – cost of the steel (UAH/ton); \(c_c\) – cost of the concrete (UAH/m³); \(V_c\) and \(A_c\) – volume of lineal meter (m³) and area of the concrete part of the cross-section (m²); \(V_a\) and \(A_a\) – volume of lineal meter (m³) and area of the steel part of the cross-section (m²); \(\gamma_a\) – volume-weight of the steel.

### 3. Concrete creep

Inelastic deformations are known to occur in concrete structures perceiving continuous action of loads; such deformations can exceed the initial, conditionally elastic deformations several times. The most critical problem of creep is in composite reinforced concrete beams [8]. The presence of a rigid steel part in the steel and reinforced concrete section, in contrast to reinforced concrete beams, determines a different mechanism of concrete creep influence, which is the redistribution of forces between concrete and steel. As a result, compressive forces in concrete are reduced and bending moment increases in a steel beam.

The Building Norms of Ukraine harmonized with Eurocode 2 [9] are based on the simplest approach establishing dependencies between deformations and time with the creep coefficient \(\phi(t, t_0)\).

According to this method, ultimate deformations of creep depend on concrete strength, age and relative moisture, and creep development in time \(t\) depends on the creep coefficient \(\phi(t, t_0)\).

\[
\varepsilon_{c,t} = [1 + \phi(t, t_0)\beta(t, t_0)]\sigma_c/E_c, \tag{17}
\]

where \(\beta(t, t_0)\) is coefficient describing the development of creep over time; \(t_0\) – age of the concrete at the time of the first load; \(\phi_0\) – theoretical creep coefficient.
This technique implements 90% of the creep deformations of concrete in 20-25 years of operation of the structure.

Due to the acceptable integrity and good implementation using the state-of-the-art software systems, we have adopted this method as the basis for further calculations of composite reinforced concrete spans.

At the same time, an alternative method for assessing the long-term deformation of concrete, based on the colloid-chemical presentation of the creep mechanism, has been acknowledged earlier in studies of CFST columns [10] and reinforced concrete spans structures [11].

4. Implementation of the technique

4.1. Design solution

The presented rationalization technique was taken as the basis for designing a continuous composite reinforced concrete span bridge with a monolithic reinforced concrete roadway slab (figure 2). The bridge diagram is 34.0+45.0+34.0 m, the total length is 117.30 m. The bridge dimension is G 0.75(pedestrian part)+1(right-of-way)+11.7(traffic way)×2+1(right-of-way)+1.5(pedestrian part) m. The custom-design intermediate poles are made of monolithic reinforced concrete on the foundation of cast in situ piles.

Design load: heavy single load – NK-100 (according to State Building Norms in Production DBN V.1.2-15:2009), motor vehicles load – six A-15 bands (the load from a convoy of two-axle 30-ton trucks), pedestrian load – 2.0 kPa.

The span structure in the cross-section is taken as six box-shaped main beams with chords of variable cross-section and dimensions established in the first approximation according to the results of the proposed rationalization technique. The distance between the beams is 4.4 m. Plate diaphragms with apertures are installed with a pitch of 1.5 m in a checkerboard pattern. Transverse beams are installed with a pitch of 3 m.

The reinforced concrete slab of the span structure with a thickness of 220 mm is combined with the main beams using shear bolts KKB-22/150 according to DIN32000-3. The compressive strength of concrete is C28/35. The transverse beams are made of welded I-beam and installed with a pitch of 3 m.

The following hypotheses were taken for designing a finite element model of the span.

4.2. Design scheme

Modern design of transport structures, as a rule, involves the use of industrial software systems as the main tool for computer-aided modeling [12-15]. The final calculation of the span can be fully conducted by means of LIRA-SAPR 2018 Pro software complex [16].

Computer-aided modeling is carried out to verify and finalized the obtained results taking into account the multistage work of the span (structural non-linearity), physical non-linearity and concrete creep, redistribution of forces due to the compliance of a flexible connection over the contact between concrete and steel.

The following hypotheses were taken for designing a finite element model of the span.
1. Calculation involves plotting a plate and rod design scheme using universal rod FE for simulation of a girder network and physically non-linear FE of the shell for simulation of a roadway slab.

2. The model is built within the entire span structure. The elements of the section are combined into joint work with the help of absolutely rigid bodies, as well as binodal finite elements of elastic coupling with a given stiffness to simulate the operation of shear bolts (figure 3).

3. The strength and deformability of concrete is characterized by parabolic and linear deformation diagrams $\sigma_c - \varepsilon_c$ depending on its class, as well as long-term deformation diagrams $\varphi(t) - t$. The creep calculation considered the relative humidity RH = 74% and the age of concrete of the mounted slab to be 28 days. Diagram $\sigma_a - \varepsilon_a$ without a yield plateau is used to account for reinforcing steel.

4. The structural non-linearity of the system was revealed with the design method “Mounting” which considered the evolutionary transformations. Besides, each building stage had the finite elements specified for it. And the load corresponding to this stage was calculated. For finite elements included for several stages all the efforts were summed up. In this case, three loading (assembly) stages are considered. The loads perceived by the beams at the first stage (1st part of constant loads) include the deadweight of the beams, diaphragms, as well as the load from the weight of the reinforced concrete roadway slab. The loads perceived by the combined cross-section in the second stage (the 2nd part of constant loads) include the deadweight of asphalt concrete, railings and barriers. The loads perceived in the third stage include one of the imposed loads.

5. The model of the span structure is made in accordance with the statically indeterminate scheme and hinged support along the axes of the supports.

4.3. Calculation results

At the stage of analysis of the calculation results, the problem was solved to finalize the accepted dimensions of the combined cross section at a constant thickness and the concrete class of the roadway slab, for which reinforcement was selected. As a result, two sizes were selected with the solution of the search problem. The first one is for the 18 m long sections of the span symmetrically above the supports, and the second one is for the 25 m and 27 m long sections located in the spans.

The calculation results in this article are presented in the form of graphs of forces plotted for cross sections in the central span of the bridge and on the support (figure 4), as well as graphs of shear forces over the contact of concrete and steel (figure 5).

The results of calculations are summarized in table 1.

The estimated cross-sections obtained by combining the rationalization algorithm and the finite-element simulation method are shown in figure 6 and table 2.
Figure 4. Diagrams of forces (a – moment and b – lateral forces) in transverse cross-sections of the bridge span structure.

Table 1. The results of calculations of the span structure.

| Force                                                                 | Value       | Units     |
|-----------------------------------------------------------------------|-------------|-----------|
| Bending moment in the main beam on the support                        | 22.60       | MNm       |
| Lateral force in the main beam on the support                         | 2.82        | MN        |
| Bending moment in the main beam in the span                           | 14.90       | MNm       |
| Lateral force in the longitudinal beam in the span                    | 0.34        | MN        |
| Strength index of the combined cross-section                          | 1.16        | –         |
| Shear force along the span structure over the contact between the main beam and the slab | 650.00      | kN        |
| Shear force across span structure and the slab                        | 269.00      | kN        |
| Bending moment in the roadway slab                                    | 27.40 (kNm)/m|          |
| The required number of shear bolts along the span structure           | 24.00       | pcs/m     |

Figure 5. Diagram of shear forces over the concrete-steel contact.

Table 2. Results of section sizes calculation.

| Rationalization parameter       | Combined section obtained by rationalization algorithm | Combined section obtained by LIRA SAPR in the bridge span | Combined section obtained by LIRA SAPR above the bridge support |
|---------------------------------|----------------------------------------------------------|----------------------------------------------------------|---------------------------------------------------------------|
| Slab thickness, cm              | 15.5                                                     | 22.5                                                     | 22.5                                                          |
| Wall height, cm                 | 206                                                      | 213                                                      | 214                                                          |
| Upper chord width, cm           | 53                                                       | 60                                                       | 60                                                           |
| Lower chord width, cm           | 63                                                       | 70                                                       | 90                                                           |
| Upper chord thickness, cm       | 2                                                        | 2                                                        | 2                                                            |
| Lower chord thickness, cm       | 3                                                        | 3                                                        | 4                                                            |
| Stiffness moment, cm³           | 107388                                                   | 131703                                                   | 158400                                                        |
5. Conclusion

The calculations show that when the loading sequence of the span model and the contact yield of concrete and steel are taken into account, it leads to a decrease in the influence of the roadway slab on the load-bearing capacity of the structure. As a result the cross-section obtained by the rationalization algorithm was corrected with finite-element modeling by stiffness moment increase for combined section in the bridge span on 18.5% and on 32% – above the bridge support. It has also been established that the plastic properties of materials and long processes that occur in the combined cross-section affect the stress-strain state, increasing bending moment and deformation in steel beams. The efficiency of the composite reinforced concrete will not decrease with time, and will depend on the rational composition of the concrete to minimize creep deformations and the optimal ratios of the design parameters. Using the proposed simulation technique, the actual load-bearing capacity of the bridge span structure was established and a rational design of the composite cross-section was assigned to meet the modern loads of NK-100 and A-15.

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