UPDATED BOUNDS ON CP ASYMMETRIES IN $B^0 \to \eta'K_S$ AND $B^0 \to \pi^0 K_S$  

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New rate measurements of $B^0$ decays into $\pi^0\pi^0$, $\pi^0\eta$, $\pi^0\eta'$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$ and $K^+K^-$ are used in conjunction with flavor SU(3) to constrain the coefficients $S$ and $C$ of $\sin \Delta m t$ and $\cos \Delta m t$ in the time-dependent CP asymmetries of $B^0 \to \eta'K_S$ and $B^0 \to \pi^0K_S$. Experimental values of $S_{\eta'K}$ are now seen to be closer to the Standard Model expectations, fully consistent with the new improved bounds.

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I INTRODUCTION

Time-dependent CP asymmetries in $B^0$ decays to CP eigenstates dominated by the $b \to s$ penguin amplitude have for several years been fertile ground for exploring signatures of new physics [1]. The decay $B^0 \to \eta'K_S$, as one example, attracted attention because of the possible deviation of the coefficient $S_{\eta'K_S}$ of the $\sin \Delta m t$ term from its predicted value of $\sin 2\phi_1 = \sin 2\beta$, where $\beta \equiv \arg(-V_{tb}V_{td}^*V_{cd}V_{cb}^*)$ is one of the phases in the standard unitary triangle constructed from the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. A value $\sin 2\beta = 0.674 \pm 0.026$ is extracted from $B \to J/\psi K_{S,L}$ decays [2].

In Refs. [3] and [4] correlated bounds on $S$ and $C$ parameters in time-dependent decays $B^0 \to \eta'K_S$ and $B^0 \to \pi^0K_S$ were obtained using branching ratio measurements of SU(3)-related $B^0$ decays. The BaBar Collaboration now has updated its

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measurements of a number of branching ratios which contributed to the bounds in Ref. [3], leading to a further strengthening of these bounds within the Standard Model. These new results include branching ratios for $B^0 \to \eta'\pi^0$, $\eta\pi^0$, $\eta'\pi^0$ based on 232 million $BB$ pairs [5], for $B^0 \to \eta\pi$, $\eta'\eta'$ based on 324 million $BB$ pairs [6], and $B^0 \to \pi^0\pi^0$ based on 347 million $BB$ pairs [7]. Belle has also updated its branching ratio for $B^0 \to \pi^0\pi^0$ based on 532 million $BB$ pairs [8]. At the same time BaBar has presented new values for $S_{\eta'K}$ and $C_{\eta'K}$ which are closer to the predictions of the Standard Model [9], while Belle has updated its values based on more data [2, 10, 11]. Finally, new measurements of $S_{\pi^0 K_S}$ and $C_{\pi^0 K_S}$ were presented by both Belle [10] and BaBar [12]. The purpose of this work is to compare the new predictions with the new measurements. The considerable improvements in bounds in comparison with our earlier treatments [3, 4] deserve to be noted despite the fact that we break no new theoretical ground here. Where otherwise unspecified we use values of branching ratios quoted by the Heavy Flavor Averaging Group [13].

Alternative approaches for studying the asymmetries $S$ and $C$ in $b \to s$ penguin-dominated $B^0$ decays have been adopted in other works by calculating hadronic amplitudes for these processes within the frameworks of QCD Factorization [14], Soft Collinear Effective Theory (SCET) [15] and a model for final state interactions [16].

In Section II we briefly sketch the formalism for the case of time-dependent asymmetries in $B^0 \to \eta' K_S$, referring to [3] for details, and present the new bounds on $S_{\eta'K_S}$ and $C_{\eta'K_S}$. In Section III the updated bounds for $B^0 \to \pi^0 K_S$ will then be given. We summarize in Section IV, stressing the fact that our bounds may be approaching their optimum limits.

II BOUNDS FOR $S_{\eta' K_S}$ AND $C_{\eta' K_S}$

For $\eta' K_S$ the asymmetry has the form [17]:

$$A(t) \equiv \frac{\Gamma(B^0(t) \to \eta' K_S) - \Gamma(B^0(t) \to \eta K_S)}{\Gamma(B^0(t) \to \eta K_S) + \Gamma(B^0(t) \to \eta' K_S)} = -C_{\eta'K} \cos(\Delta m t) + S_{\eta'K} \sin(\Delta m t),$$

with

$$S_{\eta'K} \equiv \frac{2 \text{Im}(\lambda_{\eta'K})}{1 + |\lambda_{\eta'K}|^2}, \quad C_{\eta'K} \equiv \frac{1 - |\lambda_{\eta'K}|^2}{1 + |\lambda_{\eta'K}|^2}, \quad \lambda_{\eta'K} \equiv -e^{-2i\beta} \frac{A(B^0 \to \eta' K^0)}{A(B^0 \to \eta K^0)}.$$  \hspace{1cm} (1)

We decompose the $B^0 \to \eta' K^0$ amplitude into two terms $A_P$ and $A_C$, containing respectively the CKM factors $V_{cb}^* V_{cs}$ and $V_{ub}^* V_{us}$:

$$A(B^0 \to \eta' K^0) = A_P + A_C = |A_P'| e^{i\delta} + |A_C'| e^{i\gamma},$$

where $\delta$ and $\gamma$ in the last equality are respectively the strong and the weak phase. In the diagrammatic language $A_P'$ is the dominant $b \to s$ penguin amplitude and $A_C'$ is a color-suppressed amplitude.\footnote{The normalization of $A_{P,C}'$ differs from the one in [3] by $\sqrt{6}$. This normalization cancels in the results for $S_{\eta'K}, C_{\eta'K}$.}
The asymmetries $S_{\eta'K}$ and $C_{\eta'K}$ are

\begin{align}
S_{\eta'K} &= \frac{\sin 2\beta + 2|A_C'/A_P'| \cos \delta \sin(2\beta + \gamma) - |A_C'/A_P'|^2 \sin 2\alpha}{R_{\eta'K}}, \quad (4) \\
C_{\eta'K} &= \frac{2|A_C'/A_P'| \sin \delta \sin \gamma}{R_{\eta'K}}, \quad (5) \\
R_{\eta'K} &= 1 + 2|A_C'/A_P'| \cos \delta \cos \gamma + |A_C'/A_P'|^2. \quad (6)
\end{align}

The amplitudes $A_P'$ and $A_C'$ are expected to obey $|A_C'| \ll |A_P'|$ [18]. If $A_C'$ were neglected one would have $S_{\pi K} = \sin 2\beta$, $C_{\pi K} = 0$. Keeping only linear terms in $|A_C'/A_P'|$ [17] one would have an allowed region in the $(S_{\eta'K}, C_{\eta'K})$ plane lying inside an ellipse centered at $(\sin 2\beta, 0)$. We use the exact expressions (4)–(6). Bounds on $\gamma$ from global CKM analyses [19] lead to asymmetries in the approximately elliptical regions surrounding the Standard Model point.

Using the flavor-SU(3) decomposition of Refs. [18, 20–25] one can express the ratio $A_C'/A_P'$ in terms of SU(3)-related amplitudes $A_C/A_P$ for $\Delta S = 0 B^0$ decays as pointed out in [26]. The bounds on $\Delta S_{\eta'K} \equiv S_{\eta'K} - \sin 2\beta$ and $C_{\eta'K}$ then arise because $A'_C = \lambda A_C$ is CKM-suppressed, while $A'_P = -\lambda^{-1} A_P$ is CKM-enhanced compared to the $\Delta S = 0$ amplitudes (here $\lambda = -V_{cd}/V_{cs} = 0.230$). Writing $A_{P,C}$ in terms of the $\Delta S = 0$ $B \to f$ amplitudes $A_f$

$$\Sigma_f a_f A(f) = A_P + A_C,$$

one then obtains the bounds (see [3] for details)

$$\frac{|R - \bar{\lambda}^2|}{1 + R} \leq |A_C'/A_P'| \leq \frac{R + \bar{\lambda}^2}{1 - R}. \quad (8)$$

The ratio $R$ is

$$R^2 \equiv \frac{\bar{\lambda}^2 \left[ |\Sigma_f a_f A(f)|^2 + |\Sigma_f a_f \bar{A}(f)|^2 \right]}{|A(B^0 \to \eta'K^0)|^2 + |A(B^0 \to \eta'K^0)|^2}, \quad (9)$$

and is bounded by

$$R \leq \bar{\lambda} \Sigma_f |a_f| \sqrt{\frac{\bar{B}_f}{B(\eta'K^0)}}. \quad (10)$$

For a given set of coefficients $a_f$, nonzero branching ratio measurements and upper limits on CP averaged branching ratios $\bar{B}_f$ provide an upper bound on $R$, for which the right-hand-side of (8) gives an upper bound on $|A_C'/A_P'|$.

Since there are more physical amplitudes $A(f)$ than SU(3) contributions, one may form a variety of combinations satisfying (7). We consider two of the cases noted in Ref. [3]:

1. A combination involving pairs including $\pi^0$, $\eta$ and $\eta'$ in the final state was proposed in [26] by using a complete SU(3) analysis, and in [27] by applying
Table I: Branching ratios in $10^{-6}$ and 90% C.L. upper limits on branching ratios

| Mode       | $\eta'K^0$ | $\pi^0\pi^0$ | $\pi^0\eta$ | $\pi^0\eta'$ | $\eta\eta'$ | $\eta\eta'$ |
|------------|------------|---------------|--------------|--------------|-------------|-------------|
| This work  | 64.9 ± 3.5 | 1.31 ± 0.21   | < 1.3        | 1.5$^{+0.6}_{-0.6}$ | < 1.8       | < 2.4       | < 1.7       |
| Ref. [3]   | 65.2$^{+0.0}_{-0.9}$ | 1.9 ± 0.5     | < 2.5        | < 3.7        | < 2.8       | < 10        | < 4.6       |

U-spin symmetry arguments:

$$\sum_f a_f A(f) = \frac{1}{4\sqrt{3}} A(\pi^0\pi^0) - \frac{1}{3} A(\pi^0\eta) + \frac{5}{6\sqrt{2}} A(\pi^0\eta')$$

$$+ \frac{2}{3\sqrt{3}} A(\eta\eta) - \frac{11}{12\sqrt{3}} A(\eta'\eta') - \frac{5}{3\sqrt{3}} A(\eta\eta').$$ \hspace{1cm} (11)

2. Another superposition, satisfying (7) in the limit in which small amplitudes involving the spectator quark may be neglected, involves only three strangeness-conserving amplitudes:

$$\sum_f a_f A(f) = -\frac{5}{6} A(\pi^0\eta) + \frac{1}{3\sqrt{2}} A(\pi^0\eta') - \frac{\sqrt{3}}{2} A(\eta\eta').$$ \hspace{1cm} (12)

The coefficients $a_f$ in these cases can be read off Eqs. (11) and (12).

As mentioned before, the upper bounds for a number of the relevant decays have been strengthened recently. In units of $10^{-6}$, we use the value [13] $B(\eta'/K^0) = 64.9$ (we ignore the error ±3.5 as in Ref. [3]) and the 90% c.l. upper limits $B(\pi^0\pi^0) < 1.58$, $B(\pi^0\eta) < 1.3$, $B(\pi^0\eta') < 2.4$, $B(\eta\eta) < 1.8$, $B(\eta'\eta') < 2.4$, and $B(\eta\eta') < 1.7$. These inputs are compared with those used in Ref. [3] in Table I. The bounds on $\mathcal{R}$ obtained in the above two cases are then as follows:

1. Assuming exact SU(3) and applying (11) we find, using the central value for $B(\eta'/K^0)$,

$$\mathcal{R} < 0.116 \text{ (formerly 0.18).}$$ \hspace{1cm} (13)

2. Using (12), which contains three processes, one finds

$$\mathcal{R} < 0.070 \text{ (formerly 0.10).}$$ \hspace{1cm} (14)

The approximation involved in deriving (14), where SU(3) breaking and small amplitudes were neglected, is comparable to that associated with (13) which only neglects SU(3) breaking effects.

In order to study constraints in the $(S_{\eta'K}, C_{\eta'K})$ plane, we now apply the upper bounds (13) and (14). The exact expressions (4)–(6) imply correlated bounds on these two quantities associated with fixed values of $\mathcal{R}$. We scan over $-\pi \leq \delta \leq \pi$, taking a central value $\beta = 21.2^\circ$, values of $\gamma$ satisfying $52^\circ \leq \gamma \leq 74^\circ$ [19], and
Figure 1: Regions in the \((S_{\eta'K}, C_{\eta'K})\) plane satisfying limits on the ratio \(|A'_C/A'_P|\) and bounds (13) (region enclosed by the solid curve) or (14) (region enclosed by the dashed curve). The small plotted point denotes \((S_{\eta'K}, C_{\eta'K}) = (\sin 2\beta, 0)\). The points with experimental errors denote values from BaBar [9] (plain point) and Belle [11] (small square).

The greatest range of \(\Delta S_{\eta'K_S}\) is obtained for \(C_{\eta'K_S} = 0\). For the inner ellipse in Fig. 1, based on Eq. (14), one finds

\[-0.046 < \Delta S_{\eta'K_S} < 0.094\]  

while for the outer ellipse based on Eq. (13), the limits are

\[-0.133 < \Delta S_{\eta'K_S} < 0.152\]  

Note that the conservative bound (16) uses only SU(3) symmetry. In obtaining
Table II: Time-dependent asymmetries in $B^0 \to \eta' K_S$.

| Parameter | BaBar [9] | Belle [11] |
|-----------|-----------|------------|
| $S_{\eta'K_S}$ | $0.58 \pm 0.10 \pm 0.03$ | $0.64 \pm 0.10 \pm 0.04$ |
| $C_{\eta'K_S}$ | $-0.16 \pm 0.07 \pm 0.03$ | $0.01 \pm 0.07 \pm 0.05$ |

Table III: Time-dependent asymmetries in $B^0 \to \pi^0 K_S$.

| Parameter | BaBar [12] | Belle [2] |
|-----------|-----------|------------|
| $S_{\pi^0K_S}$ | $0.33 \pm 0.26 \pm 0.04$ | $0.33 \pm 0.35 \pm 0.08$ |
| $C_{\pi^0K_S}$ | $0.20 \pm 0.16 \pm 0.03$ | $0.05 \pm 0.14 \pm 0.05$ |

the more restrictive bound (15) further dynamical assumptions were made: that the annihilation-like amplitudes $p\alpha$ and $e$ [18] can be neglected (this can be justified by taking the $m_b \to \infty$ limit [15, 28]) and furthermore that the singlet annihilation-like amplitudes $c_{\eta}$ and $s_0$ [15] that depend on the gluonic content of $\eta'$ can be neglected (the latter do not vanish in the $m_b \to \infty$ limit, while it is not clear whether or not they are small numerically [15]). The explicit calculations in QCD Factorization [14], SCET [15], and a model for final state interactions [16] give results that lie well within both of the above ranges.

### III BOUNDS FOR $S_{\pi^0K_S}$ AND $C_{\pi^0K_S}$

We next turn to $B \to \pi^0 K_S$ decay. Measured asymmetries are summarized in Table III. The analysis is similar to the one presented above, with the details given in [4]. Using the same notation as for $B \to \eta' K_S$ we have

$$\sum_f a_f A_f = A(B^0 \to \pi^0 \pi^0) + A(B^0 \to K^+ K^-)/\sqrt{2},$$

so that (9) gives now

$$R_{\pi/K}^2 \equiv \frac{\overline{\lambda}^2 |A_{\pi\pi} + A_{KK}/\sqrt{2}|^2 + |\bar{A}_{\pi\pi} + \bar{A}_{KK}/\sqrt{2}|^2}{|A_{\pi K}|^2 + |\bar{A}_{\pi K}|^2}.$$ (18)

As in [4] we now distinguish two cases:

1. Neglect the $1/m_b$ suppressed $B^0 \to K^+ K^-$ amplitude for which the experimental value is $\overline{B}(B^0 \to K^+ K^-) = (0.07^{+0.12}_{-0.11}) \cdot 10^{-6}$ [13]. Then with $\overline{B}(B^0 \to \pi^0 \pi^0) = (1.31 \pm 0.21) \times 10^{-6}$ and $\overline{B}(B^0 \to \pi^0 K_S) = (10.0 \pm 0.6) \times 10^{-6}$, we find

$$R_{\pi/K} = \overline{\lambda} \sqrt{\frac{\overline{B}(B^0 \to \pi^0 \pi^0)}{\overline{B}(B^0 \to \pi^0 K_S)}} = (8.3 \pm 0.7) \cdot 10^{-2},$$ (19)
to be compared with $\mathcal{R}_{\gamma/K} = (9.1 \pm 1.2) \cdot 10^{-2}$ in [4].

2. Keeping $A(B^0 \to K^+K^-)$ increases the error on $\mathcal{R}_{\gamma/K}$ which now lies in the range

$$\mathcal{R}_- \leq \mathcal{R}_{\gamma/K} \leq \mathcal{R}_+, \quad (20)$$

where

$$\mathcal{R}_\pm \equiv \bar{\lambda} \left( \sqrt{\frac{\mathcal{B}(B^0 \to \pi^0\pi^0)}{\mathcal{B}(B^0 \to \pi^0K^0)}} \pm \sqrt{\frac{\mathcal{B}(B^0 \to K^+K^-)}{2\mathcal{B}(B^0 \to \pi^0K^0)}} \right) \equiv \mathcal{R}_{\gamma/K}(1 \pm r). \quad (21)$$

With $\mathcal{B}(B^0 \to K^+K^-) < (0.224 \times 10^{-6}$ (90% c.l.) and the central value of $\mathcal{B}(B^0 \to \pi^0\pi^0)$ we find $r < r_{\text{max}} = 0.292$. Then the lower limit on $\mathcal{R}_{\gamma/K}$ becomes $\mathcal{R}_- = (0.076)(1 - r_{\text{max}}) = 0.054$, while the upper limit becomes $\mathcal{R}_+ = (0.090)(1 + r_{\text{max}}) = 0.117$. These are to be compared with $\mathcal{R}_- = 0.055$, $\mathcal{R}_+ = 0.126$ obtained in [4] using central values for $\mathcal{B}(B^0 \to \pi^0\pi^0), \mathcal{B}(B^0 \to \pi^0\pi^0)$ and the upper limit on $\mathcal{B}(B^0 \to K^+K^-)$.

The results of these two cases are shown in Figs. 2. A small region of parameter space near the value $(S, C) = (\sin 2\beta, 0, \gamma)$ is actually excluded, as in the case considered in Ref. [4] when the small $B^0 \to K^+K^-$ decay amplitude was ignored. Here, a small region is excluded even when $B^0 \to K^+K^-$ is taken into account. This is due in part to the improved upper bounds on this process but also to the more restricted range assumed for $\gamma$: $52^\circ \leq \gamma \leq 74^\circ$ [19] compared with $38^\circ \leq \gamma \leq 80^\circ$ taken in Ref. [4].

IV SUMMARY

SU(3) bounds on the time-dependent CP asymmetries in $B^0 \to \eta'K_S$ and $B^0 \to \pi^0K_S$ continue to improve as one incorporates improved bounds on rare $B^0 \to \pi^0\eta^{(')}$, $\eta^{(')}\eta^{(')}$, and $K^+K^-$ decay branching ratios. The bounds presented in this work will thus reach their minimal values once all the above decay branching ratios are measured. These minimal bounds can be estimated using theoretical predictions for $B^0 \to \pi^0\eta^{(')}$, $\eta^{(')}\eta^{(')}$, and $K^+K^-$ within QCD Factorization [14], SCET [15] and perturbative QCD (PQCD) [29]. While the central values in these calculations are typically smaller than the current experimental upper bounds, their theoretical uncertainties are large, permitting values close to these bounds. For example, gluonic contributions to $B \to \eta^{(')}$ form factors may enhance the relevant branching ratios. Global SU(3) fits for $B$ decays into two pseudoscalars obtain values which are within errors near the upper bounds [27,30,31]. A first indication that the actual branching ratios are not far below current bounds is the measurement $\mathcal{B}(B^0 \to \pi^0\eta') = (1.5_{-0.7}^{+0.3}) \times 10^{-6}$, lying significantly higher than central values calculated in QCD Factorization and PQCD. This may indicate that the bounds (13) and (14) will not improve significantly in the future.

The present constraint on the region around $(S, C) = (\sin 2\beta, 0)$ consistent with the Standard Model is shown in Figs. 1 and 2. With the new measurements the experimental deviations from the Standard Model for $B^0 \to \eta'K_S$ have decreased, while those for $B^0 \to \pi^0K_S$ are not yet statistically compelling.
Figure 2: Top: points in the $S_{\pi K} - |C_{\pi K}|$ plane satisfying $\pm 1\sigma$ limits (19) on the ratio $R_{\pi/K}$, with the small $B^0 \to K^+K^-$ contribution ignored. The small plotted point denotes the pure-penguin value $S_{\pi K} = \sin 2\beta$, $C_{\pi K} = 0$. Points with experimental errors denote values from BaBar [12] (plain point) and Belle [10] (small square). The dashed arc denotes the boundary of allowed values: $S^2_{\pi K} + C^2_{\pi K} \leq 1$. Bottom: small $B^0 \to K^+K^-$ contribution included.
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