Edge states in graphene quantum Hall system with bond vs potential disorder

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Abstract. While usual edge states in the quantum Hall effect (QHE) reside between adjacent Landau levels, QHE in graphene has a peculiar edge mode at $E = 0$ that resides right within the $n = 0$ Landau level as protected by the chiral symmetry. In real graphene, disorder always exists, and here we study how the edge states appear in disordered graphene QHE. We have to discriminate bond disorder and potential disorder, since the former respects the chiral symmetry in the bipartite lattice while the latter does not. We have found that, for a bond disorder, the charge accumulation along zigzag edges persists to occur, while a potential disorder destroys them. The charge accumulation along zigzag edges should be measured with an STM imaging in magnetic fields.

Quantum Hall effect (QHE) in graphene is interesting\cite{1, 2}, particularly because the chiral symmetry for the honeycomb lattice dictates the way in which the “massless Dirac” dispersion and the associated $n = 0$ Landau level in magnetic fields appear. Due to the intimate relation between the edge and bulk states in QHE\cite{3}, which is a topological effect, edge states are of crucial interest. Usual edge states in the QHE reside between adjacent Landau levels, which is related to an important general question is how the bulk and edge QHE conductions are related for finite samples. Many authors have addressed this question \cite{4, 5}, where one of the present authors has shown that the bulk QHE conductivity, a topological quantity, coincides with the edge QHE conductivity, itself another topological quantity. This exemplifies phenomena that, when a bulk system has a topological order\cite{6, 7, 8, 9} that reflects the geometrical phase of the system\cite{10}, this should be reflected and become visible in the edge states in a bounded system \cite{3, 11}. We have previously shown for a clean graphene that an edge mode exists at $E = 0$, and that, although the mode is embedded in the bulk Landau level, the charge is accumulated along a zigzag edge with an appreciable charge re-distribution involving the bulk states which we have called a “topological compensation”\cite{12}.

In real graphene, disorder should always exist. Especially, even when a single layer is atomically clean, there are ripples, i.e., long-ranged corrugation of the graphene plane\cite{13}. Since the ripples consist of random bending of the honeycomb lattice, its main effect should be a modification of the hopping integral between neighboring sites in the tight-binding model\cite{14}. The quantum Hall transition at $n = 0$ Landau level in the graphene has been shown to be anomalously sensitive to the spatial correlation of the random bonds, where a bond disorder correlated over a few lattice constatns recovers a step-function like QHE transition\cite{15}. Here
Figure 1. (a) Honeycomb lattice with zigzag edge with $\mathbf{e}_1$, $\mathbf{e}_2$ being unit translation vectors. Energy dispersion for a zigzag edge against momentum along the edge in magnetic fields $\phi = 1/5$ (b) and $\phi = 1/21$ (c) for a clean sample. For $\phi = 1/5$, here for clarity, with shaded regions represent the bulk energy spectra, while red (blue) curves are the modes localized on the zigzag (bearded) edge.

we combine these to ask a question: how the edge states appear in disordered graphene QHE. We have to discriminate bond disorder and potential disorder, since the former respects the chiral symmetry in the bipartite lattice, while the latter does not. So we have examined the dependence on the nature of disorder for the charge density along a zigzag edge. We consider the tight-binding model with nearest-neighbor hopping on the honeycomb lattice in a magnetic field with bond or potential disorder:

$$
\mathcal{H} = - \sum_{\langle i,j \rangle} \left\{ t_{ij} c_{i}^{\dagger} c_{j} + H.c. \right\} + \sum_{\alpha = \bullet, \circ} \sum_{i} \mu_{\alpha,i} c_{\alpha,i}^{\dagger} c_{\alpha,i}.
$$

The summation $\langle i,j \rangle$ is taken over the nearest neighbor sites. Since honeycomb is a non-Bravais, bipartite lattice with two sublattice sites $\bullet$ and $\circ$ per unit cell, we can define two fermion operators $c_{\bullet,j}$ and $c_{\circ,j}$ with $j = j_1 \mathbf{e}_1 + j_2 \mathbf{e}_2$ defined in Fig. 1(a) specifies the position of unit cell. The $j_1$ is taken as $j_1 = 0, \cdots, L_1$ for $\bullet$ lattice, and $j_1 = 0, \cdots, L_1 - 1$ for $\circ$ lattice, where the opposite side of the zigzag edge is bearded one. We apply the periodic boundary condition along the direction ($\mathbf{e}_2$) parallel to the zigzag edge. The magnetic field is introduced in terms of the Peierls phase $\phi_{ij}$, so that the flux in units of the magnetic flux quantum is $\phi \equiv B S_6/(2\pi)$ in each hexagon with an area $S_6 = (3\sqrt{3}/2)a^2$. When $t_{ij} = t$ and $\mu_{\alpha,i} = 0$, the system is reduced to the clean system. Hereafter we choose $t$ as the unit of energy. Figure 1 shows the energy spectrum for a zigzag edge in magnetic fields $\phi = 1/5$ (b) or $\phi = 1/21$ (c). For a magnetic field ($\phi = 1/21$) the $n = 0$ Landau level around $E = 0$, with a narrow width, almost looks like a line spectrum on
Figure 2. Charge density profile (\(\propto\) area of each circle) for the energy window \(|E| < 0.05\) in a magnetic field \(\phi = 1/21\) with bond disorder \((W_1, \sigma) = (0.5, 0.2)\) (a) and potential disorder \((W_2, \sigma) = (0.5, 0.2)\) (b). For both cases, the system size is \((L_1, L_2) = (21, 21)\).

The charge density profiles are displayed in Fig. 2. The energy window \(|E| < 0.05\) is set to just cover the \(n = 0\) bulk Landau level with bond and potential disorders. For a bond disorder, the charge is accumulated along the zigzag edge (see Fig.2(a)), whose behavior is similar to those in the clean system. By contrast, for a potential disorder the charge density is depleted in the edge region, whose width scales with the magnetic length, \(l_B\) [12].

The bond disorder is realized when \(\mu_{\alpha,1} = 0\) and \(t_{ij} = t + \delta t_{ij}\), where \(\delta t_{ij} = W_1 f_{ij}(\sigma)\) is the strength of bond randomness. The \(f_{ij}(\sigma)\) is the normal Gaussian random variable with standard deviation \(\sigma\). The potential order is realized when \(t_{ij} = t\) and \(\mu_{\alpha,1} = W_2 f_{ii}(\sigma)\) is the strength of potential randomness. We define the operator \(\gamma = \exp[i \sum_j c_{i,j}^\dagger c_{i,j}]\), which satisfies \(\gamma^2 = 1\). For a bond disorder we retain the chiral symmetry, \(\{\gamma, \hat{H}\} = 0\), where \(\{,\}\) is an anticommutation, whereas a potential disorder destroys the symmetry. Here we consider spatially uncorrelated disorders.

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In summary, we have found that, for a bond disorder, the edge mode, being protected by the chiral symmetry, still exists around \(E = 0\) and embedded in the \(n = 0\) bulk Landau level, where the charge is accumulated along the zigzag edge. This is no longer the case with a potential disorder. The dependence of the disorder in the charge density accumulated along the zigzag edge may be examined with the STM imaging in magnetic fields [19].
**Figure 3.** Local density of states at a zigzag edge site in a magnetic field $\phi = 1/21$ with bond/potential disorder. The parameters and the system size are same as in Fig.2.

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