Research Note

Signatures of highly inclined accretion disks in Galactic Black Hole Candidates and AGNs

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Abstract. Recent X-ray observations of microquasars and Seyfert galaxies reveal the broad emission lines in their spectra, which can arise in the innermost parts of accretion disks. Simulation indicates that at low inclination angle the line is registered by distant observer with characteristic two-peak profile. However, at high inclination angles (> 85°) two additional peaks arise. This phenomenon was discovered by Matt, Perola & Stella (1993) using the Schwarzschild black hole metric to analyze such effect. They assumed that the effect is applicable to a Kerr metric far beyond of a range of parameters that they exploited. We check and confirm their hypothesis about such structure of the spectral line shape for a Kerr metric case. We use no astrophysical assumptions about physical structure of the emission region except assumption that the region should be narrow enough. Positions and heights of these extra peaks drastically depends on both the radial coordinate of the emitting region (circular hot spot) and the inclination angle. We find that best conditions to observe this effect are realized at θ > 85° and r > 5r_g and may exist in microquasars or low-mass black holes in X-ray binary systems, because there is some precession (and nutation of accretion disks) with not very long time periods (see, for example, SS433 binary system). The line profiles for different inclination angles and radial coordinates are presented. To analyze an influence of disk models on the spectral line shapes we simulate the line profiles for Shakura – Sunyaev disk model for accretion disks with the high inclination.

Key words. black hole physics; line: profiles; X-ray individuals:SS433

1. Introduction

More than ten years ago it was predicted that profiles of lines emitted by AGNs and X-ray binary systems could have asymmetric double-peaked shape (e.g. Chen, Halpern & Filippenko (1989); Fabian et al. (1989); Robinson, Perez & Binette (1990); Dumont & Collin-Souffrin (1990); Matt, Perola & Stella (1993)). Generation of the broad Kα fluorescence lines as a result of irradiation of cold accretion disk was discussed by many authors (see, for example, Matt, Perola & Pird (1991); Matt et al. (1992a,b); Matt & Perola (1992); Matt, Fabian & Ross (1993).

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1 Some of them are microquasars (for details see, for example, Greiner (1999); Mirabel (2000); Mirabel & Rodriguez (2002)).
Numerical simulations of the line structure could be found in a number of papers: [Kojima (1991); Lao (1991); Bao & Stuchlik (1992); Bad (1993); Bao, Hadrava & Ostgaard (1994); Rauch & Blandford (1993, 1994); Bromley, Chen & Miller (1997); Fanton et al. (1997); Pariev & Bromley (1997, 1998); Pariev, Bromley & Miller (2001); Ruszkowski M. (2000); Ma (2002)]. They indicate that the accretion disks in Seyfert galaxies are usually observed at the inclination angle $\theta$ close to $30^\circ$ or less. It occurs because according to the Seyfert galaxy models, the opaque dusty torque, surrounding the accretion disk, so, such structure does not allow us to observe the disk at larger inclination angles.

However, at large inclination angles $\theta > 80^\circ$ the new observational manifestations of GR could arise. [Matt, Perola & Stella (1993)] discovered such phenomenon for a Schwarzschild black hole, moreover the authors predicted that their results could be applicable for a Kerr black hole over the range of parameters which were exploited. The authors mentioned that this problem was not analyzed in details for a Kerr metric case and it would be necessary to investigate this case. Below we do not use some specific model on surface emissivity of accretion (we only assume that the emitting region is narrow enough). But general statements (which will be described below) could be generalized on a wide disk case without any problem. Therefore, in this paper we check and confirm their hypothesis for Kerr metric case and for a Schwarzschild black hole using another assumptions about surface emissivity of accretion disk. In principle, such phenomenon could be observed in microquasars and X-ray binary systems where there are neutron stars and black holes with stellar masses.

In Subsection 3° we discuss disk models used for simulations. In Subsection 4° we present the results of simulations with two extra peaks in the line profile. These extra peaks exist because of the gravitational lens effect in the strong field approximation of GR. In Subsection 5° we discuss results of calculations and present some conclusions.

2. Numerical methods

We used an approach which was discussed in details in papers by [Zakharov (1991, 1994); Zakharov & Repin (1999, 2000)]; [Zakharov & Repin (2002)]; [Zakharov et al. (2002)]. The approach was used in particular to simulate spectral line shapes. For example, [Zakharov et al. (2002)] used this approach to simulate an influence of magnetic field on spectral line profiles. This approach is based on results of qualitative analysis (which was done by [Zakharov (1986, 1989)] for different types of geodesics near a Kerr black hole). The equations of photon motion in Kerr metric are reduced to the following system of ordinary differential equations in dimensionless Boyer – Lindquist coordinates (Zakharov (1991, 1994):

$$\begin{align*}
\frac{dt}{d\sigma} &= -a \left( \sin^2 \theta - \xi \right) + \frac{r^2 + a^2}{\Delta} \left( r^2 + a^2 - \xi a \right), \\
\frac{dr}{d\sigma} &= r_1, \\
\frac{d\theta}{d\sigma} &= \cos \theta \left( \frac{\epsilon^2}{\sin^2 \theta} - a^2 \sin \theta \right), \\
\frac{d\phi}{d\sigma} &= -\left( a - \frac{\xi}{\sin^2 \theta} \right) + \frac{a}{\Delta} \left( r^2 + a^2 - \xi a \right),
\end{align*}$$

where $\Delta = r^2 - 2r + a^2; \eta = Q/M^2E^2$ and $\xi = L_z/ME$ are the Chandrasekhar constants (Chandrasekhar (1983)) which are derived from the initial conditions of the emitted quantum in the disk plane. The system (1)–(6) has two first integrals

$$\begin{align*}
\epsilon_1 &\equiv r_1^2 - r^4 - \left( a^2 - \xi^2 - \eta \right) r^2 - 2 \left( a - \xi \right) + a^2 \eta = 0, \\
\epsilon_2 &\equiv \theta_1^2 - \eta - \cos^2 \theta \left( a^2 - \frac{\epsilon^2}{\sin^2 \theta} \right) = 0,
\end{align*}$$

which can be used for the accuracy control of computation. Solving Eqs. (1)–(6) for monochromatic quanta emitted by a ring we can calculate a spectral line shape $L_s(r; \theta)$ which is registered by a distant observer at inclination angle $\theta$.

The numerical integration has been performed using the combination of the Gear and Adams methods (Gear (1971)) and realized as the standard package by Hindmarsh (1983); Petzold (1983); Hiebert & Shampine (1983). We obtain the entire disk spectrum by summation of sharp ring spectra.

3. Disk model

To simulate the structure of the emitted line it is necessary first to choose a model for an emissivity of an accretion disk. We exploit two different models, namely we consider narrow and thin disk moving in the equatorial plane near a Kerr black hole as the first model and as we analyze the inner wide part of accretion disk.

The authors predicted that their results could be applicable for a Kerr black hole over the range of parameters which were exploited. The authors mentioned that this problem was not analyzed in details for a Kerr metric case and it would be necessary to investigate this case. Below we do not use some specific model on surface emissivity of accretion (we only assume that the emitting region is narrow enough). But general statements (which will be described below) could be generalized on a wide disk case without any problem. Therefore, in this paper we check and confirm their hypothesis for Kerr metric case and for a Schwarzschild black hole using another assumptions about surface emissivity of accretion disk. In principle, such phenomenon could be observed in microquasars and X-ray binary systems where there are neutron stars and black holes with stellar masses.

In Subsection 3° we discuss disk models used for simulations. In Subsection 4° we present the results of simulations with two extra peaks in the line profile. These extra peaks exist because of the gravitational lens effect in the strong field approximation of GR. In Subsection 5° we discuss results of calculations and present some conclusions.
So, at the first, we assume that the source of the emitting quanta is a narrow and thin disk rotating in the equatorial plane of a Kerr black hole. We also assume that the disk is opaque for radiation, so that a distant observer situated on one disk side cannot register the quanta emitted from its other side.

For the sake of computational simplicity we suggest that the spectral line is monochromatic in its co-moving frame. It can really be adopted because even at $T = 10^8$ K the thermal line width

$$\frac{\delta f}{f} \sim \frac{v}{c} \sim \frac{1}{c} \sqrt{\frac{kT}{m_{\text{pl}}}} \approx 10^{-3}$$

appears to be much less than the Doppler line width associated with the Kepler velocity of the disk rotation.

For second case of the Shakura – Sunyaev disk model, we assume that a local emissivity is proportional to surface element and and $T^4$, where $T$ is a local temperature. The emission intensity of the ring is proportional to its area. The area of emitting ring (width $dr$) differs in the Kerr metric from its classical expression $dS = 2\pi r dr$ and should be replaced with

$$dS = \frac{2\pi (r^2 + a^2)}{\sqrt{r^2 - r_g a + a^2}} dr.$$  \hfill (9)

Thus, the total flux density emitted by the disk and registered by distant observer is proportional to the integral

$$I_\nu(\theta) = \int_{r_{\text{in}}}^{r_{\text{out}}} I_\nu(r, \theta) T^4(r) dS,$$  \hfill (10)

where $I_\nu(r, \theta)$ is obtained from the solution of equations \(1\) - \(4\), $T(r)$ – from the appropriate dependence for a hot disk and $dS$ – from Eq. \(9\).

For simulation we assume that the emitting region lies entirely in the innermost region of $\alpha$-disk (zone $a$) from $r_{\text{out}} = 10 r_g$ to $r_{\text{in}} = 3 r_g$ and the emission is monochromatic in the co-moving frame.\(^2\) The frequency of this emission set as a unity by convention.

### 4. Simulation results

Spectral line profiles of a narrow ring observed at large inclination angles $\theta > 85^\circ$ and different radii $r$ are shown in Fig. \(1\) (one could say that is a Kerr metric generalization of Fig. 1 from Matt, Perola & Stella 1993 which was drawn using calculations for the Schwarzschild case). The ring is assumed to move in the equatorial plane of a Kerr black hole with almost extreme rotation parameter $a = 0.9981$. The inclination angle increases there from left to right and radial coordinate – from bottom to top. The figure indicates that practically there are no new specific features of profiles, thus, the line profile remains one-peaked with a maximum close to $1.6 E_{\text{lab}}$ and very long red wing without any significant details. For lowest radii there are no signatures of multiple peaks of spectral line shapes even for high inclination angles (the bottom raw in Fig. 1 which corresponds to $r = 0.8 r_g$).

With increasing the radius to $r = 1.2 r_g$ the additional blue peak arises in the vicinity of the blue maximum at the highest inclination angle $\theta = 89^\circ$. The red maximum is so small that no details can be distinguished in its structure. At lower inclination angles $\theta \leq 88^\circ$ the blue maximum also has no details and the entire line profile remains essentially one-peaked.

For $r = 3 r_g$ the additional detail in the blue peak appears for $\theta \geq 85^\circ$. Thus, for $\theta = 85^\circ$ we have a fairly distinguished bump, for $\theta = 88^\circ$ it changes (transforms) into a small complementary maximum and for $\theta = 89^\circ$ this maximum becomes well-distinguished. Its position in the last case differs significantly from the main maximum: $E_3 = 1.12 E_{\text{lab}}, E_4 = 1.34 E_{\text{lab}}$.

With further increasing the radius the red maximum bifurcates also. This effect becomes apparently visible for $r = 5 r_g$ and $\theta \geq 88^\circ$. Thus, for $\theta = 85^\circ$ we have only a faintly discernible (feebly marked) bump, but for $\theta = 88^\circ$ both complementary maxima (red and blue) arise. For $\theta = 89^\circ$ we have already four maxima in the line profile: $E_1 = 0.63 E_{\text{lab}}, E_2 = 0.66 E_{\text{lab}}, E_3 = 1.07 E_{\text{lab}}, E_4 = 1.28 E_{\text{lab}}$.

Note that the splitting for the blue and red maxima is not equal, moreover, $E_4 - E_3 \approx 7 (E_2 - E_1)$.

For $r = 10 r_g$ the profile becomes more narrow, but the complementary peaks appear very distinctive. We have four-peak structure for $\theta \geq 88^\circ$. It is interesting to note that for $\theta = 89^\circ$ the energy of the blue complementary peak is close to its laboratory value.

Note that the effect almost disappears when the radial coordinate becomes less than $r_g$, i.e. for the orbits which could exist only near a Kerr black hole.

Fig. \(2\) demonstrates the details of the spectrum presented in the top row in Fig. \(1\). Thus, the left panel includes all the quanta emitted by a hot spot at $r = 10 r_g$ with $\theta > 89^\circ$ at infinity (the mean value could be counted as $85^\circ$ if the quanta distribution would be uniform there) but with much higher resolution than in Fig. \(1\). The spectrum has four narrow distinctive maxima separated by lower emission intervals. In the right panel which includes all the quanta with $\theta > 85^\circ$ the right complementary maxima is even higher than the main one. The blue complementary maxima remains still lower than its main counterpart, but it increases rapidly its intensity with increasing the inclination angle. It follows immediately from the comparison of left and right panels. Some “oscillation behavior” of the line profile between the maxima has a pure statistical origin and does not refer to physics.

As an illustration a spectrum of an entire accretion disk at high inclination angles in the Schwarzschild metric is shown in Fig. \(3\). (In reality we’ve calculated geodesics for the quanta trajectories in the Schwarzschild metric using the same Eqs. \(1\) - \(6\) as for a Kerr metric, but assuming there $a = 0.01$.) The emitting region (from 3 to 10 $r_g$) lies as a whole in the in-

\(^2\) We use as usual the notation $r_g = 2GM/c^2$. 

Fig. 1. Line profiles with specific details, which arise for high inclination angles $\theta > 85^\circ$ due gravitational lens effect in the strong gravitational field approach. The Kerr (rotation) parameter was chosen like $a = 0.9981$. The radii decrease from top to bottom, the inclination angles increase from left to right; their values are shown in each panel along with the number of quanta included in spectrum.
Fig. 2. Details of a hot spot line profile for the most distinctive case with $r = 10r_g$ and $a = 0.9981$ (see the top row of Fig. 1). The images of all orders are counted. Left panel includes all the quanta, registered at infinity with $\theta > 89^\circ$. Right panel includes the quanta with $\theta > 89.5^\circ$.

Fig. 3. Details of a line structure for $\alpha$-disk in the Schwarzschild metric with outer and inner edges of emitting region equal to $r_{\text{out}} = 10r_g$ and $r_{\text{in}} = 3r_g$, respectively. Left panel includes all the quanta, registered at infinity with $\theta > 89^\circ$. Right panel includes the quanta with $\theta > 89.5^\circ$.

nermost region of $\alpha$-disk (the detailed description of this model was given by Shakura & Sunyaev 1973; Lipunova & Shakura 2002).

As it follows from the Fig. 3 the blue peak may consists of the two components, whereas the red one remains unresolved.

5. Discussion and conclusions

The complicated structure of the line profile at large inclination angles is explained by the multiple images of some pieces of the hot ring. One could point out that the result was obtain in the framework of GR without any extra physical and astrophysical assumptions about character of radiation etc. For a Kerr black hole we assume only that radiating ring is circular and narrow.
The problem of multiple images in the accretion disks and extra peaks has first been considered by Matt, Perola & Stella (1993); see also Bao & Stuchlik (1992); Bao (1993); Bao, Hadrava & Ostgaard (1994, 1996). Using numerical simulations Matt, Perola & Stella (1993) proved the statement for the Schwarzschild metric and suggested that the phenomenon is applicable to a Kerr metric over the range of parameters that the authors have analyzed. Matt, Perola & Stella (1993) noted also that it is necessary to realize detailed calculations to confirm their hypothesis. We verified and confirmed their conjecture without any assumptions about a specific distribution of surface emissivity or accretion disk model (see, Fig. 12).

We confirmed also a conclusion of Matt, Perola & Stella (1993) that extra peaks are generated by photons which are emitted by the far side of the disk, therefore we have a manifestation of gravitational lensing in the strong gravitational field approach for GR.

Some possibilities to observe considered features of spectral line profiles was considered by Matt, Perola & Stella (1993); Bao (1993). The authors argued that there are non-negligible chances to observe such phenomenon in some AGNs and X-ray binary systems. For example, Bao (1993) suggested that NGC 6814 could be a candidate to demonstrate such phenomenon (but Madejski et al. (1993) found that the peculiar properties of NGC6814 are caused by a cataclysmic variable like an AM Herculis System).

However, it is clear that in general a probability to observe objects where one could find such features of spectral line is small. Moreover, even if the inclination angle is very close to 90°, a thickness of disk (shield or a torus around accretion disk) may not give us a possibility to look at the inner part of accretion disk. Let us discuss astrophysical situations when the inclination angle of accretion disk is high enough.

About 1% of all AGN or microquasar systems could have an inclination angle of accretion disk > 89°. For example, Kormendy et al. (1996) found that NGC 3115 has very high inclination angle about 81° (Kormendy & Richstone (1992) discovered a massive dark object $M \sim 10^9 M_\odot$ (probably, massive black hole) in NGC3115). Perhaps, we have much higher probability to observe such phenomenon in X-ray binary systems where black holes with stellar masses could be. Taking into account an precession which actually observed for some X-ray binary systems (for example, there is a significant precession of the accretion disk for SS433 binary system Cherepashchuk (2002), moreover since the inclination of the orbital plane is high (i $\sim$ 79° for this object), so, sometimes we could have a possibility to observe almost edge-on accretion disks for such objects. Observations indicated that there is a strong evidence that optically bright accretion disk in SS433 is in supercritical regime of accretion.

First description for supercritical accretion disk was given by Shakura & Sunyaev (1973). Even now such model is discussed in to explain observational data for SS433 Cherepashchuk (2002). We used a temperature distribution from Shakura – Sunyaev model (Shakura & Sunyaev (1973); Lipunova & Shakura (2002)) for inner part of accretion disk to simulate shapes of lines which could be emitted from this region (Fig. 3). Therefore, we should conclude that the properties of spectral line shapes discovered by Matt, Perola & Stella (1993) are confirmed also for such emissivity (temperature) distributions which correspond to Shakura – Sunyaev model.

Thus, such properties of spectral line shapes are robust enough in respect to wide variations of rotational parameters of black holes and a surface emissivity of accretion disks as it was predicted by Matt, Perola & Stella (1993). In this work their conjecture it was verified and confirmed not only for a Kerr black hole case but also for another dependence of surface emissivity of accretion disk.

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