MiniBooNE and MicroBooNE Joint Fit to a 3+1 Sterile Neutrino Scenario

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This letter presents the results from the MiniBooNE experiment within a full “3+1” scenario where one sterile neutrino is introduced to the three-active-neutrino picture. In addition to electron-neutrino appearance at short-baselines, this scenario also allows for disappearance of the muon-neutrino and electron-neutrino fluxes in the Booster Neutrino Beam, which is shared by the MicroBooNE experiment. We present the 3+1 fit to the MiniBooNE electron-(anti)neutrino and muon-(anti)neutrino data alone, and in combination with the MicroBooNE electron-neutrino CCQE data. The best-fit parameters of this joint fit are $\Delta m^2 = 0.209\, \text{eV}^2$, $|U_{e4}|^2 = 0.502$, $|U_{\mu 4}|^2 = 0.0158$, and $\sin^2(2\theta_{\mu e}) = 0.0316$. Comparing the no-oscillation scenario to the 3+1 model, the data prefer the 3+1 model with a $\Delta\chi^2/\text{dof} = 24.7/3$.

INTRODUCTION

The MiniBooNE low-energy excess (LEE) is a longstanding anomaly in neutrino physics. This excess of electron-like events was observed in reconstructed neutrino energy. Initially reported in [1], the excess reached a significance of 4\,\sigma in the energy range 200\,MeV < $E_{\nu}$ < 1250\,MeV with the full MiniBooNE $\nu$ and $\bar{\nu}$ data set [2]. We note that this significance is derived from a direct comparison between MiniBooNE data and the Standard Model (SM) prediction, and is thus independent of any physics model, including the 3 + 1 model explored in this paper. A wide range of explanations for the excess have been put forward, but the initial, and still most referenced, new physics explanations invoke $\nu_{\mu} \rightarrow \nu_{e}$ oscillations.

The BNB flux is produced through 8\,GeV protons impinging on a beryllium target that is located inside a magnetic focusing horn, which can reverse polarity to run in neutrino or antineutrino mode, followed by a 50\,m meson decay pipe. The MiniBooNE detector, which is a 450\,t fiducial mass, mineral-oil-based Cherenkov detector, is located 541 m downstream of the beryllium target. The detector is sensitive to neutrinos with energies between 100\,MeV and 3\,GeV. This combination of energy and baseline makes MiniBooNE an ideal experiment to probe the appearance of electron-neutrinos from $\nu_{\mu} \rightarrow \nu_{e}$ oscillations in a mass-squared splitting region greater than $1 \times 10^{-2}\,\text{eV}^2$. The full data set taken in a series of runs between 2002 and 2019 yields a 1$\sigma$ allowed region in $\Delta m^2$ between 0.04\,eV$^2$ and 0.4\,eV$^2$, with mixing angles varying from 1.0 to 0.01 [2].

These mass-squared splittings are more than an order of magnitude larger than the splitting of atmospheric neutrino oscillations, $\Delta m^2_{\text{atmos}} \approx 2.5 \times 10^{-3}\,\text{eV}^2$ [3]—associated with the largest mass splitting in three neutrino oscillation models. Therefore, to accommodate such oscillations, it is necessary to postulate the exis-
tence of a fourth neutrino mass, and a fourth neutrino flavor that must be non-weakly-interacting (or “sterile”) to avoid constraints from $Z$ decay \cite{4}. In such a model, the sterile neutrino flavor and the three active flavors are connected to a fourth mass state through an extension of the PMNS mixing matrix. Such a model introduces a combination of three possible experimental signatures: 1) electron flavor disappearance to other flavors, leading to fewer $\nu_e$ events than expected (“$\nu_e \rightarrow \nu_e$”); 2) muon flavor disappearance to other flavors (“$\nu_\mu \rightarrow \nu_\mu$”) reducing the $\nu_\mu$ rate; and 3) $\nu_\mu \rightarrow \nu_e$ appearance, where an excess of $\nu_e$ events is observed. Past MiniBooNE $\nu_\mu \rightarrow \nu_e$ appearance analyses have assumed that the $\nu_e$ and $\nu_\mu$ disappearance effects were negligible\footnote{This assumption was justifiable in the context of the best-fit LSND two-neutrino oscillation oscillation scenario, and the unitarity constraints of the model.}. However, this approach has been considered to be overly simplified, since MiniBooNE uses the $\nu_\mu$ data to predict the $\nu_e$ backgrounds in the beam, while disappearance will affect these predictions. In response to this, in this paper we expand the analyses of the full MiniBooNE data sets and simulation samples, to present the first full 3+1 sterile-neutrino oscillation model by the collaboration.

In 2015, the MicroBooNE experiment joined the MiniBooNE experiment as a user of the BNB beamline. The MicroBooNE experiment was designed with the primary goal of investigating the LEE by using the detailed information from its liquid-argon time-projection-chamber (LArTPC) to distinguish between electron induced events and photon induced events. This allows the rejection of many mis-identified backgrounds in the MiniBooNE data set. MicroBooNE has recently released results of a search for a generic $\nu_e$ excess, assuming the median shape of the MiniBooNE excess, in a strategy that is agnostic to particular oscillation models. External analyses have applied more focused studies, placing limits on $\nu_e$ disappearance \cite{5}, expanding the MicroBooNE analysis to all systematically allowed shapes of the MiniBooNE excess \cite{6}, and considering how the MicroBooNE data constrain the parameters of a 3+1 sterile neutrino model \cite{6}. However, until now, there has been no MiniBooNE-MicroBooNE joint analysis.

Because MiniBooNE and MicroBooNE share the same beamline, we can use MiniBooNE tools to perform a joint fit to the data of the two experiments. On the other hand, because the detectors are substantially different, the two experiments have complementary capabilities. MicroBooNE is an 85 t active mass LArTPC \cite{7}, which allows for detailed reconstruction of neutrino interactions that is not possible using the MiniBooNE Cherenkov detector.

The MiniBooNE experiment has a large sample size, but relatively high backgrounds from mis-identification backgrounds that dominate MiniBooNE’s electron neutrino sample. The MicroBooNE experiment uses a relatively small detector, but can remove most misidentification backgrounds \cite{8}. Also, the imaging capability of the LArTPC has allowed the MicroBooNE experiment to select a high purity sample of charged current quasi-elastic (CCQE) interactions, which has low systematic uncertainty compared to a semi-inclusive or fully inclusive cross section \cite{8–11}. Thus, in principle, the MicroBooNE data allow for a clean test, albeit with a small sample, of the hypothesis that the MiniBooNE excess events are due to $\nu_e$ charged-current quasi-elastic interactions. Therefore, we consider only MicroBooNE’s exclusive CCQE $\nu_e$ search \cite{8} in this paper, which presents the first MiniBooNE/MicroBooNE joint fit.

\section*{FIT DETAILS}

The model of interest is a three-active plus one-sterile neutrino model called “3+1.” This model expands the $3 \times 3$ neutrino mixing matrix to $4 \times 4$:

\begin{equation}
U_{3+1} = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} & U_{e4} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\
U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\
U_{s1} & U_{s2} & U_{s3} & U_{s4}
\end{pmatrix}.
\end{equation}

In such a model, both $\nu_\mu$ and $\nu_\tau$ disappearance are expected to occur with the same $\Delta m^2$ as the $\nu_\mu \rightarrow \nu_e$ appearance signal, as long as both $U_{e4}$ and $U_{\mu4}$ are non-zero. The three processes are related through their effective mixing angles, which are expressed as:

\begin{align}
\sin^2(2\theta_{\mu\nu_e}) &= 4(1 - |U_{\mu4}|^2)|U_{\mu4}|^2, \\
\sin^2(2\theta_{ee}) &= 4(1 - |U_{e4}|^2)|U_{e4}|^2, \\
\sin^2(2\theta_{\mu\tau}) &= 4|U_{\mu4}|^2|U_{\mu4}|^2,
\end{align}

which appear within the oscillation probability formulae:

\begin{align}
P(\nu_\mu \rightarrow \nu_e) &= \sin^2 2\theta_{\mu\nu_e} \sin^2(\Delta m^2_{41} L/E) , \\
P(\nu_e \rightarrow \nu_e) &= 1 - \sin^2 2\theta_{ee} \sin^2(\Delta m^2_{41} L/E) , \\
P(\nu_\mu \rightarrow \nu_\mu) &= 1 - \sin^2 2\theta_{\mu\nu} \sin^2(\Delta m^2_{41} L/E).
\end{align}

There are three physics parameters in the 3+1 model relevant to these two experiments: the sterile mass splitting $\Delta m^2_{41} \equiv \Delta m^2$ (where we assume degeneracy for $i \in \{1, 2, 3\}$) and the two mixings of the new mass eigenstate to the electron weak eigenstate $|U_{e4}|^2$ and muon weak eigenstate $|U_{\mu4}|^2$. Different combinations of these parameters will induce different rates of $\nu_e$ appearance as well as $\nu_\mu$ and $\nu_\tau$ disappearance in the MiniBooNE and MicroBooNE detectors. In each case the oscillation probability depends upon the true neutrino energy, $E$, and baseline of each event, $L$.

The oscillation prediction in MiniBooNE is determined by a simple reweighting of the MiniBooNE $\nu_\mu \rightarrow \nu_e$ simulation using the oscillation formulae (Eqs. 3). This direct method is not possible for the MicroBooNE $\nu_\mu \rightarrow \nu_e$ simulation, as only limited simulation information for this analysis is available \cite{12}. Instead, for MicroBooNE, we
use the MiniBooNE BNB simulation to obtain a ratio between the nominal intrinsic $\nu_e$ background prediction and the $\nu_e$ appearance prediction at the MicroBooNE baseline as a function of true neutrino energy, using the BNB flux prediction at the MicroBooNE location. This ratio, combined with the intrinsic $\nu_\mu$ simulation provided by MicroBooNE allows us to obtain a $\nu_e$ appearance prediction in MicroBooNE. We use the same procedure to account for $\nu_\mu$ disappearance. However, we neglect $\nu_\mu$ disappearance in the MicroBooNE prediction, as the $\nu_\mu$ background contamination in MicroBooNE’s $1e1p$ analysis is sub-dominant and the simulation information for the $\nu_\mu$ contribution is not provided by MicroBooNE. We also note that $\nu_\mu \to \nu_\tau$ neutral-current backgrounds in MiniBooNE’s electron neutrino measurement are not included in the prediction; however, this effect is expected to be small. An example of this oscillation prediction is shown in Figure 1.

For the MiniBooNE likelihood we compare the fixed observation to the theoretical expectation with a multivariate normal distribution that includes systematic uncertainties, Poisson statistical uncertainties on the expectation, and finite Monte-Carlo statistical uncertainties. With the large MiniBooNE sample size, the multivariate normal distribution is a reasonable approximation for the likelihood. The MiniBooNE systematic errors of this analysis remain the same as in [2], with one exception. The correlated systematic errors from uncertainties in the MiniBooNE optical model are limited to the three principal components of the corresponding covariance matrix with the largest eigenvalues, and the remaining optical model errors are assumed to be uncorrelated with no covariance among energy bins. For MicroBooNE, we use a Poisson-derived likelihood that accounts for finite Monte-Carlo size [13]; additionally, the expectation in each bin is treated as a nuisance parameter that is constrained by the systematics covariance matrix [14]. The total likelihood is then composed of these two experimental likelihoods. We note that the fit presented here does not account for systematic correlations between MiniBooNE and MicroBooNE. Additionally, we allow the MicroBooNE $\nu_\mu$ measurement to constrain the MicroBooNE $\nu_\mu$ prediction and uncertainties, and do not account for oscillations in MicroBooNE’s $\nu_\mu$ prediction. Ignoring $\nu_\mu$ disappearance in MicroBooNE is a reasonable assumption for small $U_{\mu 4}$ given the limited sample statistics from MicroBooNE.

RESULTS

With the methods described in the preceding section we can examine the MiniBooNE LEE in the context of a 3+1 sterile neutrino model, both with the MiniBooNE data alone and together with the MicroBooNE electron-neutrino data. We show the no-oscillation SM prediction as a dashed purple line in Figure 1. In the SM case the MiniBooNE prediction lies substantially below the data}

FIG. 1. The comparison predictions assuming the SM and 3+1 “Combination” fit parameters of Table I for each experiment. Black crosses show the observed data and the statistically allowed band of per bin expectations. The SM prediction is represented as a dashed purple line, and the 3+1 prediction shown with either purple crosses or a solid line. The top panel shows unconstrained predictions and errors. The middle panel shows predictions and errors in purple after being constrained by the $\nu_\mu + \bar{\nu}_\mu$ data, and shows the unconstrained 3+1 prediction as the stacked histogram. The displayed errors on the predictions contain only systematic and finite Monte-Carlo errors. The bottom panel shows predictions after the allowed systematic variations have been fit to data, and thus does not have systematic error bars shown.
in the electron-neutrino channel. For MicroBooNE, the data lie scattered above and below the SM prediction, in part due to the small sample size. The disparity between the data and SM prediction in MiniBooNE shows the inability of the SM to accommodate the MiniBooNE low energy excess in the electron-neutrino data while remaining in agreement with the MiniBooNE muon-neutrino data.

In contrast to the SM, the 3+1 oscillation model provides the additional freedom necessary to potentially better accommodate the MiniBooNE muon neutrino data, and low energy excess, within systematic errors. In the 3+1 scenario we expect $\nu_\mu \rightarrow \nu_e$ oscillations to increase the prediction in the electron-neutrino channels of both experiments, while $\nu_e$ disappearance will reduce the intrinsic electron-neutrino backgrounds, and $\nu_\mu$ disappearance will reduce the muon-neutrino prediction as well as the contribution of misidentified events in the electron-neutrino observable channel. The prediction for the best-fit 3+1 scenario across both experiments is shown in Figure 1, separated by component, experiment, and observable channel. Figure 1a compares the MiniBooNE unconstrained muon neutrino and antineutrino prediction to observed data, where the crosses denote the unconstrained 3+1 prediction and the dashed line denotes the unconstrained SM prediction. Figure 1b compares the MiniBooNE electron neutrino and antineutrino prediction to data; the prediction and errors are shown after being constrained by the muon neutrino data for the 3+1 and SM scenarios in purple, whereas the unconstrained 3+1 prediction is shown by the stacked histogram.

While the best-fit 3+1 scenario is preferred to the no-oscillation scenario, it still cannot perfectly describe MiniBooNE’s low energy excess, especially at the lowest energies. This is consistent with the recent MicroBooNE results, which indicate that the low energy excess cannot be explained entirely by electron neutrinos [9]. This is also consistent with previous MiniBooNE studies indicating a forward-peaked angular distribution of the low energy excess [2].

The best-fit 3+1 parameters and the $\Delta \chi^2$ between the SM and 3+1 scenarios are given in Table I. We obtain a best-fit that includes substantial sterile-electron-mixing, with $|U_{e4}|^2$ near 0.5, and moderate sterile-muon mixing, with $|U_{\mu4}|^2$ near 0.02, for both the MiniBooNE-only and combined fits. The best-fit $\Delta m^2$ is near 0.2 eV$^2$ as well. The large sterile-electron mixing at the best-fit point is in tension with constraints on unitarity in the neutrino sector [15]. However, a broad region in parameter space is allowed within the estimated 1$\sigma$ confidence region, as is visualized in Figure 2, extending to regions of parameter space which are not in tension with unitarity constraints. The 1$\sigma$ allowed region in $\Delta m^2$ and $\sin^2(2\theta_{\mu e})$ is similar to that reported in [2], and takes the form of a diagonal band because the MiniBooNE LEE spans a broad energy range and extends down to the 200 MeV bounding energy. The excess drives the allowed values of $\sin^2(2\theta_{\mu e})$, but large deviations from the best-fit in $|U_{e4}|^2$ and $|U_{\mu4}|^2$ are allowed, provided the combination produces enough $\nu_\mu \rightarrow \nu_e$ appearance to describe the excess. This freedom is present in part because the systematic errors of the prediction allow large changes to the muon-neutrino channel with little penalty, which in turn provides only a weak constraint on $|U_{\mu4}|^2$ through $\nu_\mu$ disappearance.

MicroBooNE’s electron-neutrino data do not exhibit an excess at the lower end of their energy spectrum, as MiniBooNE’s electron-neutrino data do, and MicroBooNE overall observes a lower event rate than predicted by the nominal no-oscillation model [9]. However, the data sample from MicroBooNE does not have the statistical power needed to rule out a 3+1 $\nu_\mu \rightarrow \nu_e$ explanation of the MiniBooNE low-energy-excess. The observed event-rate from MicroBooNE’s electron-neutrino channel precludes very large $\nu_\mu \rightarrow \nu_e$ appearance at values of $\Delta m^2$ and $\sin^2(2\theta_{\mu e})$ higher than the MiniBooNE allowed region. This manifests in Figure 2 as a small shift in the allowed region to lower $\Delta m^2$ and lower $\sin^2(2\theta_{\mu e})$. In Figure 1c, the best-fit 3+1 oscillation prediction increases the expected number of events in a region that MicroBooNE observes a deficit, suggesting that the fit is primarily driven by the larger MiniBooNE data sample, in line with our expectation.

The 3+1 scenario is preferred over the no-oscillation model in both the MiniBooNE-only and joint-fit cases. In the MiniBooNE-only fit we obtain a $\Delta \chi^2 = 27.8$ between the two models, whereas in the joint-fit fit we obtain a $\Delta \chi^2 = 24.7$ for 3 additional degrees of freedom introduced in the fit. If we assume the asymptotic approximation to the test-statistic distribution provided by Wilks’ theorem [16] with a difference of three degrees of freedom between the models, then we obtain p-values of $4.69 \times 10^{-6}$ and $1.77 \times 10^{-5}$ in favor of the 3+1 scenario for the MiniBooNE-only and joint analyses, respectively. However, we expect the true difference in degrees of freedom between the models to be less than three, based on both the degeneracy inherent in the 3+1 model and the smaller difference in degrees of freedom observed in the two-neutrino MiniBooNE oscillation study [2]. A reduction in the difference in degrees of freedom between the models would increase the significance of these two statistical tests. Therefore, we conservatively estimate that the MiniBooNE-only 3+1 model test prefers the 3+1 model to the SM at approximately 4.6$\sigma$, and the addition of the MicroBooNE electron-neutrino CCQE data reduces this significance to approximately 4.3$\sigma$.

**CONCLUSION**

This letter has explored a full 3+1 sterile-neutrino oscillation model within the context of results from the MiniBooNE and MicroBooNE experiments. In the MiniBooNE electron-like analysis, we consider $\nu_\mu \rightarrow \nu_e$ appearance alongside both $\nu_e$ and $\nu_\mu$ disappearance. In the MicroBooNE CCQE analysis, we consider $\nu_e$ appearance and $\nu_e$ disappearance. In an analysis of the
FIG. 2. The results of the MiniBooNE-only and combined fits. The likelihood is obtained by profiling over all parameters except $\Delta m^2$ and $\sin^2(2\theta_{\mu e})$. The two best-fit points are shown as appropriately colored stars, and the contours are obtained by comparing the profile-likelihood-ratio test-statistic to the asymptotic distribution provided by Wilks’ theorem, and assuming a difference of two degrees of freedom.

| 3+1 Fit | $|U_{e4}|^2$ | $|U_{\mu 4}|^2$ | $\Delta m^2$ | $\Delta \chi^2 / \text{dof}$ |
|---------|--------------|--------------|-------------|-----------------|
| MiniBooNE only | 0.508 | 0.0205 | 0.191 | 27.8 / 3 |
| Combination | 0.502 | 0.0158 | 0.209 | 24.7 / 3 |

TABLE I. Summary of results. The $\Delta \chi^2 / \text{dof}$ in the last column compares the 3 + 1 model to the no-oscillation model.

MiniBooNE-only data, we find a best fit to the 3+1 model of $\Delta m^2 = 0.191 \text{eV}^2$, $|U_{e4}|^2 = 0.508$, $|U_{\mu 4}|^2 = 0.0205$, and $\sin^2(2\theta_{\mu e}) = 0.0417$. A joint-fit to both analyses finds a best fit to the 3+1 model at oscillation parameters of $\Delta m^2 = 0.209 \text{eV}^2$, $|U_{e4}|^2 = 0.502$, $|U_{\mu 4}|^2 = 0.0158$, and $\sin^2(2\theta_{\mu e}) = 0.0316$. In the MiniBooNE only analysis, the 3+1 scenario is preferred over the no-oscillation case with a $\Delta \chi^2 / \text{dof}$ of 27.8/3, whereas in the combined analysis we obtain $\Delta \chi^2 / \text{dof} = 24.7/3$. Although the 3+1 model is not a perfect description of the low-energy MiniBooNE electron neutrino data, we find that a 3+1 sterile neutrino oscillation scenario is a better description of the MiniBooNE data than the no-oscillation scenario and is not in tension with MiniBooNE’s muon neutrino data. We also find that the MicroBooNE electron-neutrino data do not rule out the allowed 3+1 interpretations for the MiniBooNE data, but do slightly reduce the significance of the result and make only a small modification to the allowed regions.

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[1] A. A. Aguilar-Arevalo et al. (MiniBooNE), A Search for Electron Neutrino Appearance at the $\Delta m^2 \sim 1 \text{eV}^2$ Scale, Phys. Rev. Lett. 98, 231801 (2007), arXiv:0704.1500 [hep-ex].
[2] A. A. Aguilar-Arevalo et al. (MiniBooNE), Updated MiniBooNE neutrino oscillation results with increased data and new background studies, Phys. Rev. D 103, 052002 (2021), arXiv:2006.16883 [hep-ex].
[3] P. A. Zyla et al. (Particle Data Group), Review of Particle Physics, PTEP 2020, 083C01 (2020).
[4] S. Schael et al. (ALEPH, DELPHI, L3, OPAL, SLD, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group), Precision electroweak measurements on the $Z$ resonance, Phys. Rept. 427, 257 (2006), arXiv:hep-ex/0509008.
[5] P. B. Denton, Sterile Neutrino Searches with MicroBooNE: Electron Neutrino Disappearance, (2021), arXiv:2111.05793 [hep-ph].
[6] C. A. Argüelles, I. Esteban, M. Hostert, K. J. Kelly, J. Kopp, P. A. N. Machado, I. Martinez-Soler, and Y. F. Perez-Gonzalez, MicroBooNE and the $\nu_e$ Interpretation of the MiniBooNE Low-Energy Excess, (2021), arXiv:2111.10359 [hep-ph].
[7] R. Acciarri et al. (MicroBooNE), Design and Construction of the MicroBooNE Detector, JINST 12 (02), P02017, arXiv:1612.05824 [physics.ins-det].
[8] P. Abratenko et al. (MicroBooNE), Search for an anomalous excess of charged-current quasi-elastic $\nu_e$ interactions with the MicroBooNE experiment using Deep-Learning-based reconstruction, (2021), arXiv:2110.14080 [hep-ex].
[9] P. Abratenko et al. (MicroBooNE), Search for an Excess of Electron Neutrino Interactions in MicroBooNE Using Multiple Final State Topologies, (2021), arXiv:2110.14054 [hep-ex].
[10] P. Abratenko et al. (MicroBooNE), Search for an anomalous excess of inclusive charged-current $\nu_e$ interactions in the MicroBooNE experiment using Wire-Cell reconstruction, (2021), arXiv:2110.13978 [hep-ex].
[11] P. Abratenko et al. (MicroBooNE), Search for an anomalous excess of charged-current $\nu_e$ interactions without pions in the final state with the MicroBooNE experiment, (2021), arXiv:2110.14065 [hep-ex].

[12] Search for an anomalous excess of charged-current quasi-elastic $\nu_e$ interactions with the MicroBooNE experiment using Deep-Learning-based reconstruction (2021).

[13] C. A. Argüelles, A. Schneider, and T. Yuan, A binned likelihood for stochastic models, JHEP 06, 030, arXiv:1901.04645 [physics.data-an].

[14] 'NuE background constrained fractional covariance matrix' of 'Search for an anomalous excess of charged-current quasi-elastic $\nu_e$ interactions with the MicroBooNE experiment using Deep-Learning-based reconstruction' (2021).

[15] S. A. R. Ellis, K. J. Kelly, and S. W. Li, Current and Future Neutrino Oscillation Constraints on Leptonic Unitarity, JHEP 12, 068, arXiv:2008.01088 [hep-ph].

[16] S. S. Wilks, The large-sample distribution of the likelihood ratio for testing composite hypotheses, Ann. Math. Statist. 9, 60 (1938).
Supplemental Materials: MiniBooNE and MicroBooNE Joint Fit to a 3+1 Sterile Neutrino Scenario

I. LIKELIHOOD

The physics parameters of the model are the mass squared splitting $\Delta m^2$, electron-sterile mixing $|U_{e4}|^2$, and muon-sterile mixing $|U_{\mu 4}|^2$. The mixing parameters $(|U_{e4}|^2, |U_{\mu 4}|^2)$ are allowed to vary between 0 and 1 while maintaining unitarity of the mixing matrix through the condition $|U_{e4}|^2 + |U_{\mu 4}|^2 \leq 1$. The additional nuisance parameters of the model are the MicroBooNE per-bin systematic scalings $\alpha_i$. Here the set of physics parameters are denoted by $\vec{\theta}$, and the set of nuisance parameters denoted by $\vec{\eta}$. The combined MiniBooNE-MicroBooNE likelihood is the product the two experimental likelihoods such that

$$L(\vec{\theta}, \vec{\eta} | \vec{x}) = L_{MB}(\vec{\theta} | \vec{x}_{MB}) \times L_{uB}(\vec{\theta}, \vec{\eta} | \vec{x}_{uB}),$$

where $L_{MB}$ is the MiniBooNE likelihood, $L_{uB}$ is the MicroBooNE likelihood, $\vec{x}_{MB}$ is collection of the MiniBooNE data counts, $\vec{x}_{uB}$ is the collection of MicroBooNE data counts, and $\vec{x} = \vec{x}_{MB} \cup \vec{x}_{uB}$ is the collection of all data counts. The MiniBooNE likelihood is approximated as a multivariate normal distribution

$$L_{MB}(\vec{\theta}, \vec{\eta} | \vec{x}_{MB}) = \mathcal{N}(\vec{x}_{MB} | \vec{\mu}_{MB}(\vec{\theta}), \Sigma_{MB}(\vec{\theta})),$$

where $\vec{\mu}_{MB}$ is the predicted number of data counts in each bin, and $\Sigma_{MB}$ is the MiniBooNE covariance matrix. In this case the MiniBooNE covariance matrix includes systematic errors, Poisson statistical errors, and Monte-Carlo statistical errors. The MicroBooNE likelihood is given by

$$L_{uB}(\vec{\theta}, \vec{\eta} | \vec{x}_{uB}) = \mathcal{N}(\vec{\alpha} | 1, \Sigma_{uB}) \times \prod_i L_{\text{Eff}}(\alpha_i, \mu_i, \sigma_i, \sigma_i^{\text{inc}} | x_i),$$

where $\mathcal{N}(\vec{\alpha} | 1, \Sigma_{uB})$ is the multivariate normal prior on the MicroBooNE systemsatic scalings, $\Sigma_{uB}$ is the MicroBooNE fractional covariance matrix, $\mu_i$ is the predicted number of data counts in each bin before systematic modifications, and $\sigma_i^{\text{inc}}$ is the Monte-Carlo statistical error on the per-bin data count prediction after the systematics scalings have been applied. The MicroBooNE fractional covariance matrix, $\Sigma_{uB}$, is the constrained fractional covariance matrix from [8, 14]. The likelihood $L_{\text{Eff}}$ is a Poisson-based likelihood that accounts for finite Monte-Carlo sample errors, and is described in [13].