Kinematics and Error Analysis of Single Motor Driving Multi-Stage Metamorphic Mechanism

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Abstract. Traditional series robots have the disadvantages of complex structure and complicated control. Almost all of them have no adaptive ability. Although soft robots have good adaptive characteristics, they also have the limitations of poor bearing capacity and low stiffness. In addition, their motion trajectory and motion precision are difficult to achieve accurate control. In order to solve the above problems, based on a single motor-driving multi-stage metamorphic mechanism, this paper carries out kinematic analysis and error analysis to determine the motion trajectory and motion accuracy of the multi-stage metamorphic mechanism. And this paper uses ADAMS to carry out simulation calculation. Comparing the theoretical calculation results and simulation calculation results, and determining the errors between the multi-stage metamorphic mechanisms, so as to ensure the accurate position of the multi-stage metamorphic mechanism during work. The multi-stage metamorphic mechanism realizes stable and efficient work.

1. Introduction
With the continuous research on the field of robots and related theories, more and more robots are applied in real life and production. Serial robots have problems such as large errors and insufficient motion accuracy. Especially for serial robots with less drive sources and less control, it is necessary to study their motion accuracy and motion error. In 1987, E. Rank and O.C. Zienkiewicz proposes an error calculation and evaluation method based on finite element method [1]. Using a bilinear finite element approximation method for second-order linear elliptic boundary value problem, the error index is estimated and the estimation amount is given. Therefore, from a mathematical point of view, the up-and-down predictor in the energy norm is transferred to the new predictor. In 1988, Zhang Ce et al. obtain the numerical
solution of the motion equation of a planar four-bar mechanism with four motion pair gaps under the continuous contact model. They also discuss the influence of the dynamic performance of the multi-gap mechanism and the change of this influence with the gap amount [2]. In 1995, Zechang Shi and Shen Liu write a book on mechanism accuracy [3]. There are errors in the machining process of mechanical structures and errors in the assembly process. Such errors affect the overall accuracy and precision in the process of robot movement and work. Among them, vector method has carried out a detailed study on static error analysis and so on. In 2001, G. De bunne and A. Barr propose an adaptive method using elastic materials [4]. Combining the large displacement strain tensor formula with finite element method, a model composed of continuous differential equations is established. The accuracy in the adaptive process is also analyzed. In 2004, Thomas Gratsch and Klaus-Jurgen Bathe propose the basic concept of posterior error estimation for finite element solutions of elliptic linear model problems [5]. They establish the global error estimation of energy norm and the target-oriented error estimation, and propose the error evaluation states and hybrid methods used in nonlinear and transient analysis. In 2014, Umut Kocak and Karl Johan Lundin Palmerius etc. propose a model to analyze the error of different types of small deformation [6]. They need to consider different simulation parameters, and use the adaptability of different parameters. In addition, they compare the force output with the reference force to estimate the error of given parameters. In 2017, Gang Cheng and others analyze the inverse dynamics of a rigid body of a 3UPS-PRU parallel robot [7]. Position, velocity, acceleration and singularity are considered in inverse kinematics analysis. Rigid body dynamics model is established by virtual work principle and Jacobian matrix. On this basis, in 2018, Shantou University's Zhao Yongjie team conduct inverse kinematics analysis on the displacement of the super redundant bionic elephant trunk robot [8].

The above-mentioned in-depth research on the robot motion error and motion accuracy has been carried out, but the research on the motion accuracy of the multi-stage metamorphic mechanism without feedback is less. Therefore, this paper mainly analyzes the kinematic characteristics and accuracy of the multi-stage metamorphic mechanism driven by a single motor.

2. Kinematic Analysis

2.1. Single-stage Kinematic Analysis

Kinematic analysis is carried out for the multi-stage metamorphic mechanism driven by a single motor that has been developed as shown in Fig. 1.

![Figure 1. Multilevel metamorphic mechanism model](image)

In the kinematic analysis, in order to facilitate the analysis, the scissor mechanism can be equivalent to a telescopic rod,
From the equivalent model shown in Fig. 2, the following relation can be obtained.

\[ c'' = \sqrt{c'^2 - c^{\sigma 2}} \]  

(1)

After the relevant parameters are brought in, it can be seen that the length of the telescopic rod has a nonlinear relationship with time.

From the vector method,

\[ l_1^2 = l_1^2 + l_2^2 + l_3^2 - 2 \cdot l_1 \cdot l_2 \cdot \sin \phi_1 - 2 \cdot l_1 \cdot l_3 \cdot \sin \phi_2 + 2 \cdot l_1 \cdot l_2 \cdot \cos \phi_1 \cdot \cos \phi_2 + 2 \cdot l_1 \cdot l_3 \cdot \sin \phi_1 \cdot \sin \phi_2 \]  

(2)

Further, the velocity equation can be obtained as follows,

\[ J \cdot v = b \]  

(3)

Among them,

\[ J = \begin{bmatrix} \cos \phi_1 & -l_1 \cdot \sin \phi_1 & -l_2 \cdot \cos \phi_2 \\ -\sin \phi_1 & -l_1 \cdot \cos \phi_1 & l_2 \cdot \sin \phi_2 \\ \end{bmatrix} \]  

(4)

\[ v = \begin{bmatrix} v_4 \\ \phi_4 \\ \phi_5 \\ \end{bmatrix} \]  

(5)
Similarly, the acceleration equation can also be solved.

\[ \mathbf{J} \cdot \mathbf{a} = \mathbf{c} \]  

Among them,

\[ \mathbf{c} = \begin{bmatrix} 2 \cdot v_4 \cdot \alpha_1 \cdot \sin \varphi_1 + l_4 \cdot \alpha_1^2 \cdot \cos \varphi_1 - l_1 \cdot \alpha_2^2 \cdot \sin \varphi_2 \\ 2 \cdot v_4 \cdot \alpha_3 \cdot \cos \varphi_3 - l_4 \cdot \alpha_3^2 \cdot \sin \varphi_3 - l_1 \cdot \alpha_2^2 \cdot \sin \varphi_2 \end{bmatrix} \]

2.2. Overall Kinematic Analysis

When the multi-stage metamorphic mechanism is not in contact with the object, the spring will always be in the original long state. During the extension process of the scissor mechanism, \( \varphi_i \) of each section in the whole will remain unchanged. The coordinate system is established as shown in Fig. 4.

According to coordinate transformation, the displacement of \( C_{i+1} \) point in \( O-xyz \) is,

\[
\mathbf{r}_i = \begin{bmatrix} BC_i + \sum_{n=2}^{i} C_{n-1}C_n \cos \left( \varphi_{i,n-1} + \sum_{m=1}^{n-1} \varphi_{2,m} - \frac{n-1}{2} \pi \right) \\ \sum_{n=2}^{i} C_{n-1}C_n \sin \left( \varphi_{i,n-1} + \sum_{m=1}^{n-1} \varphi_{2,m} - \frac{n-1}{2} \pi \right) \\ 0 \end{bmatrix}
\]

The end trajectory is shown in Fig. 5.

Similarly, the equation of velocity and acceleration can be obtained.

Figure 4. Overall model
3. Error Analysis

When a corresponding coordinate system is established on each level of metamorphic mechanism, the relationship between two adjacent coordinate systems can be established by using the rotation transformation and translation transformation.

The $i$ level metamorphic mechanism is expressed in the form of homogeneous coordinate transformation connected product with respect to the $i-1$ level metamorphic mechanism, and is denoted as matrix $A_i$,

$$A_i = \begin{bmatrix}
\sin \phi_{2i-1} & \cos \phi_{2i-1} & 0 & 0 \\
\cos \phi_{2i-1} & \sin \phi_{2i-1} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

Therefore, the homogeneous coordinate of the extreme position of the multistage metamorphic mechanism with respect to the $i$-1st coordinate system is transformed into,

$$i-1T_n = A_i \cdot A_{i-1} \cdots A_1$$

Homogeneous coordinate transformation for the base coordinate system is as follows,

$$T_n = A_i \cdot A_{i-1} \cdots A_1$$

Differentiate each quantity of formula (10) to obtain,

$$dA_i = i-1A_iA_i^T$$

According to the error theory,

$$A_i = (dT_n)T_n^{-1}$$
is the pose error of the extreme position of the multistage metamorphic mechanism relative to the base coordinate system.

\[ \Delta_a = \left( \mathbf{d} \mathbf{T}_a - \mathbf{A}_n \mathbf{T}_a \right) \mathbf{T}_n^{-1} \]  

(16)

\[ \dot{\mathbf{A}}_n = \left( \mathbf{d} \mathbf{T}_a - 2 \mathbf{A}_n \mathbf{T}_a - \mathbf{A}_n \mathbf{T}_a \right) \mathbf{T}_n^{-1} \]  

(17)

\[ \ddot{\mathbf{A}}_n \] is the acceleration error of the extreme position of the multistage metamorphic mechanism relative to the base coordinate system.

4. Software Simulation

After ADAMS software is used to analyze the kinematic characteristics of the metamorphic mechanism. As the rod length and other parameters of the metamorphic mechanisms at all levels are in an equal proportion relationship, the changes of the angle and other parameters of the metamorphic mechanisms at all levels are the same, and the changes of the parameters related to the rod length are in a proportional relationship. Therefore, the motion form of the single-stage metamorphic mechanism needs to be analyzed, and the overall motion parameters only need to simply superimpose the single-stage metamorphic mechanism. Therefore, software is needed to verify the correctness of theoretical derivation of the single-stage metamorphic mechanism, and the correctness of the whole metamorphic mechanism is also verified.

Taking the model of the 8th stage metamorphic mechanism as an example, it is modeled in ADAMS software, as shown in Fig. 6.

![Figure 6. Results comparison](image)

The theoretical calculation and simulation results are only slightly different in the process of motion, so the correctness of the theoretical results can be verified.
Acknowledgments
The authors thanks the National Key Laboratory of Robotics for supporting this research work.

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