We give a description of supermembranes in nontrivial target-space geometries. A special class are the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ spaces that have the maximal number of 32 supersymmetries.

Lecture presented at the 22-nd Johns Hopkins Workshop, Novelties in String Theory, Göteborg, August 20-22, 1998.

1 Introduction

The original study of supermembranes \cite{1} was motivated by the desire to find a consistent quantum-mechanical extension of 11-dimensional supergravity \cite{2} along the same lines that string theory defines a quantum-mechanical extension of 10-dimensional supergravity theories. In 11 spacetime dimensions the supermembrane can consistently couple to a superspace background that satisfies a number of constraints which are equivalent to the supergravity equations of motion. The supermembrane action can also exist in 4, 5 and 7 spacetime dimensions, just as the Green-Schwarz superstring \cite{3} is classically consistent in 3, 4, 6 and 10 dimensions. Guided by string theory it was natural to expect that the massless states of the supermembrane would correspond to those of 11-dimensional supergravity. However, the supermembrane mass spectrum turned out to be continuous \cite{4} and in that situation it was cumbersome to directly prove or disprove the possible existence of massless states \cite{5,6}. The difficulties in making sense of the supermembrane mass spectrum and the fact that 11-dimensional supergravity seemed to have no place in string theory, formed an obstacle for subsequent development of the theory. More recently, however, interest in supermembranes was rekindled by the realization that 11-dimensional supergravity does have its role to play as the long-distance approximation to M-theory \cite{7,8,9,10}. M-theory is the conjectured framework for unifying all five superstring theories and 11-dimensional supergravity. It turns out that supermembranes, M-theory and super matrix theory are all intricately related.

An important observation was that it is possible to regularize the supermembrane in terms of a super matrix model based on some finite group, such as
U(N). In the limit of infinite $N$ one then recovers the supermembrane \([5]\). These supersymmetric matrix models were constructed long ago \([11]\) and can be obtained from supersymmetric Yang-Mills theories in the zero-volume limit. More recently it was realized that these models describe the short-distance behaviour of $N$ Dirichlet particles \([12]\). The continuity of the spectrum is then understood directly in terms of the spectrum of $N$-particle states. A bold conjecture was that the super matrix models capture the degrees of freedom of M-theory \([13]\). In the large-$N$ limit, where one considers the states with an infinite number of particles, the supermembranes should then re-emerge. Furthermore there is new evidence meanwhile that the supermembrane has massless states, which will presumably correspond to the states of 11-dimensional supergravity, although proper asymptotic states do not exist. The evidence is based on the matrix model regularization of the supermembrane \([14]\). For fixed value of $N$ the existence of such states was foreseen on the basis of identifying the Kaluza-Klein states of M-theory compactified on $S^1$ with the Dirichlet particles and their bound states in type-IIA string theory.

From this viewpoint it is natural to consider the supermembrane in curved backgrounds associated with 11-dimensional supergravity and it is the purpose of this talk to report on progress in this direction. Such backgrounds consist of a nontrivial metric, a three-index gauge field and a gravitino field. This provides us with an action that transforms as a scalar under the combined (local) supersymmetry transformations of the background fields and the supermembrane embedding coordinates. Here it is important to realize that the supersymmetry transformations of the embedding coordinates will themselves depend on the background. When the background is supersymmetric, then the action will be supersymmetric as well. In the light-cone formulation this action leads to models invariant under area-preserving diffeomorphisms, which in certain situations can be approximated by matrix models in curved backgrounds. The area-preserving diffeomorphisms are then replaced by a finite group, such as U($N$), but target-space diffeomorphisms are no longer manifestly realized. Matrix models in curved space have already been the subject of independent studies \([15]\). Also in view of the relation between near-horizon geometries and conformal field theories \([16]\) interesting classes of backgrounds are the ones where the target space factorizes locally into the product of an anti-de Sitter space and some compact space. Examples of these are the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ backgrounds that we discuss at the end of the talk.

### 2 Supermembranes

Fundamental supermembranes can be described in terms of actions of the Green-Schwarz type, possibly in a nontrivial but restricted (super)spacetime background \([1]\). Such actions exist for supersymmetric $p$-branes, where $p = 0, 1, \ldots, d−1$ defines the spatial dimension of the brane. Thus for $p = 0$ we have a superparticle, for $p = 1$ a superstring, for $p = 2$ a supermembrane, and so on. The dimension of spacetime in which the superbrane can live is very restricted. These
restrictions arise from the fact that the action contains a Wess-Zumino-Witten term, whose supersymmetry depends sensitively on the spacetime dimension. If the coefficient of this term takes a particular value then the action possesses an additional fermionic gauge symmetry, the so-called \( \kappa \)-symmetry. This symmetry is necessary to ensure the matching of (physical) bosonic and fermionic degrees of freedom. In the following we restrict ourselves to supermembranes (i.e., \( p = 2 \)) in 11 dimensions.

The 11-dimensional supermembrane is written in terms of superspace embedding coordinates \( Z^M(\zeta) = (X^\mu(\zeta), \theta^a(\zeta)) \), which are functions of the three world-volume coordinates \( \zeta^i \) (\( i = 0, 1, 2 \)). It couples to the superspace geometry of 11-dimensional supergravity, encoded by the supervielbein \( E_M^A \) and the antisymmetric tensor gauge superfield \( B_{MNP} \), through the action

\[
S[Z(\zeta)] = \int d^3\zeta \left[ -\sqrt{-g(Z(\zeta))} - \frac{1}{6} \epsilon^{ijk} \Pi_i^A \Pi_j^B \Pi_k^C B_{CBA}(Z(\zeta)) \right],
\]

where \( \Pi_i^A = \partial Z^M / \partial \zeta^i \) \( E_M^A \) is the pull-back of the supervielbein to the membrane worldvolume. Here the induced metric equals \( g_{ij} = \Pi_r^i \Pi_s^j \eta_{rs} \), with \( \eta_{rs} \) being the constant Lorentz-invariant metric. This action is invariant under local fermionic \( \kappa \)-transformations alluded to above, given that certain constraints on the background fields hold, which are equivalent to the equations of motion of 11-dimensional supergravity \[1\].

Flat superspace is characterized by

\[
\begin{align*}
E_{\mu r} &= \delta_{\mu r}, & E_{\mu a} &= 0, & E_{r a} &= \delta_{r a}, & E_{r r} &= \delta_{r r}, \\
B_{\mu\nu\alpha} &= (\bar{\theta} \Gamma_{\mu\nu})_{\alpha}, & B_{\mu\nu\beta} &= (\bar{\theta} \Gamma_{\mu\nu})_{\beta}, & B_{\mu r} &= 0.
\end{align*}
\]

The gamma matrices are denoted by \( \Gamma^r \); gamma matrices with more than one index denote antisymmetrized products of gamma matrices with unit weight. In flat superspace the supermembrane Lagrangian, written in components, reads (in the notation and conventions of \[2\]),

\[
L = -\epsilon_{ij} \bar{\theta} \Gamma_{\mu\nu} \partial_{\theta} \left[ \frac{1}{2} \partial_\mu X^\nu (\partial_\nu X^\mu + \bar{\theta} \Gamma^\nu \partial_\nu \theta) + \frac{1}{6} \bar{\theta} \Gamma^\mu \partial_{\theta} \bar{\theta} \Gamma^\nu \partial_{\theta} \right],
\]

The target space can have compact dimensions which permit winding membrane states. In flat superspace the induced metric,

\[
g_{ij} = (\partial_\mu X^\nu + \bar{\theta} \Gamma^\nu \partial_\mu \theta) (\partial_\nu X^\mu + \bar{\theta} \Gamma^\mu \partial_\nu \theta) \eta_{\mu\nu},
\]

is supersymmetric. Therefore the first term in \( \[3\] \) is trivially invariant under spacetime supersymmetry,

\[
\delta X^\mu = -\epsilon \Gamma^\mu \theta, \quad \delta \theta = \epsilon.
\]

1Our notation and conventions are as follows. Tangent-space indices are \( A = (r, a) \), whereas curved indices are denoted by \( M = (\mu, \alpha) \). Here \( r, \mu \) refer to commuting and \( a, \alpha \) to anticommuting coordinates. Moreover we take \( \epsilon_{012} = -\epsilon^{012} = 1. \)
In 4, 5, 7, or 11 spacetime dimensions the second term in the action proportional to $\varepsilon^{ijk}$ is also supersymmetric (up to a total divergence) and the full action is invariant under $\kappa$-symmetry.

Let us now consider the supermembrane action for nontrivial backgrounds, such as those induced by a nontrivial target-space metric, a target-space tensor field and a target-space gravitino field, corresponding to the fields of (on-shell) 11-dimensional supergravity. This background can in principle be cast into superspace form by a procedure known as 'gauge completion'. For 11-dimensional supergravity, the first steps of this procedure were carried out long ago [17] and recently [18] the results were determined to second order in the fermionic coordinates $\theta$.

To elucidate the generic effects of nontrivial backgrounds for membrane theories, let us confine ourselves for the moment to the purely bosonic theory and present the light-cone formulation of the membrane in a background consisting of the metric $G_{\mu\nu}$ and the tensor gauge field $C_{\mu\nu\rho}$. In the subsequent sections we will include the fermionic coordinates. The Lagrangian density for the bosonic membrane follows directly from (1),

$$L = -\sqrt{-g} - \frac{1}{6} \varepsilon^{ijk} \partial_i X^\mu \partial_j X^\nu \partial_k X^\rho C_{\rho\mu\nu},$$

(6)

where $g_{ij} = \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}$. In the light-cone formulation, the coordinates are decomposed in the usual fashion as $(X^+, X^-, X^a)$ with $a = 1 \ldots 9$. Furthermore we use the diffeomorphisms in the target space to bring the metric in a convenient form [19],

$$G_{--} = G_{a-} = 0.$$  

(7)

Just as for a flat target space, we identify the time coordinate of the target space with the world-volume time, by imposing the condition $X^+ = \tau$. Moreover we denote the spacesheet coordinates of the membrane by $\sigma^r, r = 1, 2$. Following the same steps as for the membrane in flat space, one arrives at a Hamiltonian formulation of the theory in terms of coordinates and momenta. These phase-space variables are subject to a constraint, which takes the same form as for the membrane theory in flat space, namely,

$$\phi_r = P_a \partial_r X^a + P_- \partial_\tau X^- \approx 0.$$  

(8)

Of course, the definition of the momenta in terms of the coordinates and their derivatives does involve the background fields, but at the end all explicit dependence on the background cancels out in the phase-space constraints.

The gauge choice $X^+ = \tau$ still allows for $\tau$-dependent reparametrizations of the world-space coordinates $\sigma^r$. In addition there remains the freedom of performing tensor gauge transformations of the target-space three-form $C_{\mu\nu\rho}$. In order to write the theory as a gauge theory of area-preserving diffeomorphisms it is desirable to obtain a Hamiltonian which is polynomial in momenta and coordinates. Obviously one wants the background fields to be independent of $X^\pm$. With suitable choices for the gauge condition one thus derives [18],

$$H = \int d^2 \sigma \left\{ \frac{G_{++}}{P_- - C_-} \left[ \frac{1}{4} \left( P_a - C_a - \frac{P_- - C_-}{G_{++}} G_{a+} \right)^2 + \frac{1}{4} \left( \varepsilon^{rs} \partial_r X^a \partial_s X^b \right)^2 \right] \right\}$$

4
\[-\frac{P_- - C_-}{2G_{+-}} G_{++} - C_+ \]
\[+ \frac{1}{P_- - C_-} \left[ \varepsilon^{rs} \partial_r X^a \partial_s X^b P_a C_{+-} + C_- C_+ \right], \quad (9)\]

where \(P_- - C_-\) equals a constant times \(\sqrt{w}\) and \(C_{-ab}\) has been set to a constant by a choice of gauge. Furthermore we made use of the definitions,
\[C_a = -\varepsilon^{rs} \partial_r X^r \partial_s X^b C_{-ab} + \frac{1}{2} \varepsilon^{rs} \partial_r X^b \partial_s X^c C_{abc}, \]
\[C_{\pm} = \frac{1}{2} \varepsilon^{rs} \partial_r X^a \partial_s X^b C_{\pm ab}, \quad C_{+-} = \varepsilon^{rs} \partial_r X^- \partial_s X^a C_{+-a}. \quad (10)\]

At this point one can impose further gauge choices and set \(G_{+-} = 1\) and \(C_{+-a} = 0\). Taking also \(C_{-ab} = 0\) the corresponding Hamiltonian can be cast in Lagrangian form in terms of a gauge theory of area-preserving diffeomorphisms\[20\],
\[w^{-1/2} L = \frac{1}{2} \left( D_0 X^a \right)^2 + D_0 X^a \left( \frac{1}{2} C_{abc} \{ X^b, X^c \} + G_{a+} \right) - \frac{1}{4} \{ X^a, X^b \}^2 + \frac{1}{2} G_{++} + \frac{1}{2} C_{+ab} \{ X^a, X^b \}, \quad (11)\]

where the derivative is covariant with respect to area-preserving diffeomorphisms. For convenience we have set \((P_-)_0 = 1\). In the case of compact dimensions, it may not always be possible to set \(C_{+-a}\) and \(C_{-ab}\) to zero, although they can be restricted to constants. One can still follow the same procedure as above, but the Lagrangian then depends explicitly on \(X^-\), a feature that was already exhibited earlier for the winding membrane. However, in the case at hand, the constraint makes the resulting expression for \(X^-\) extremely nontrivial. The antisymmetric constant matrix \(C_{-ab}\) was conjectured to play a role for the matrix model compactification on a noncommutative torus\[21\]. It should be interesting to see what the role is of \((9)\) in this context.

With a reformulation of the membrane in background fields as a gauge theory of area-preserving diffeomorphisms at one’s disposal, one may consider its regularization through a matrix model by truncating the mode expansion for coordinates and momenta. This leads to a replacement of Poisson brackets by commutators, integrals by traces and products of commuting fields by symmetrized products of the corresponding matrices. At that point the original target-space covariance is affected, as the matrix reparametrizations in terms of symmetrized products of matrices do not possess a consistent multiplication structure; this is just one of the underlying difficulties in the construction of matrix models in curved space\[15\]. With the above results at hand one may study interactions between membranes by considering the behaviour of a test membrane in a background field induced by another membrane\[22\].

3 Superspace backgrounds

It is possible to express superspace backgrounds in terms of the component fields of 11-dimensional supergravity, by means of a technique called gauge completion.
We refer to ref. 18 for the details. The superspace geometry with coordinates $Z^M = (x^\mu, \theta^\alpha)$ is encoded in the supervielbein $E_M^A$ and a spin-connection field $\Omega^{AB}$. In what follows we will not pay much attention to the spin-connection, which is not an independent field. Furthermore we have an antisymmetric tensor gauge field $B_{MNP}$, subject to tensor gauge transformations,

$$\delta B_{MNP} = 3 \partial_{[M} \Xi_{NP]}.$$  \hfill (12)

Unless stated otherwise the derivatives with respect to $\theta$ are always left derivatives.

Under superspace diffeomorphisms corresponding to $Z^M \rightarrow Z^M + \Xi^M(Z)$, the super-vielbein and tensor gauge field transform as

$$\delta E_M^A = \Xi^N \partial_N E_M^A + \partial_M \Xi^N E_N^A,$$
$$\delta B_{MNP} = \Xi^Q \partial_Q B_{MNP} + 3 \partial_{[M} \Xi^Q B_{Q|NP]}.$$ \hfill (13)

Tangent-frame rotations are $Z$-dependent Lorentz transformations that act on the vielbein according to

$$\delta E_M^A = \frac{1}{2}(\Lambda^{rs} L_{rs})^A_B E_B^M,$$ \hfill (14)

where the Lorentz generators $L_{rs}$ are defined by

$$\frac{1}{2}(\Lambda^{rs} L_{rs})^t_u = \Lambda^t_u,$$  \hfill (15)
$$\frac{1}{2}(\Lambda^{rs} L_{rs})^a_b = \frac{1}{4} \Lambda^{rs}(\Gamma^{rs})^a_b.$$

The superspace that we are dealing with is not unrestricted but is subject to certain constraints and gauge conditions. Furthermore, we will not describe an off-shell situation as all superfields will be expressed entirely in terms of the three component fields of on-shell 11-dimensional supergravity, the elfbein $e_{\mu r}$, the antisymmetric tensor gauge field $C_{\mu
u\rho}$ and the gravitino field $\psi_\mu$. As a result of these restrictions the residual symmetry transformations are confined to 11-dimensional diffeomorphisms with parameters $\xi^\mu(x)$, local Lorentz transformations with parameters $\lambda^{rs}(x)$, tensor-gauge transformations with parameters $\xi_{\mu\nu}(x)$ and local supersymmetry transformations with parameters $\epsilon(x)$. To derive how the superfields are parametrized in terms of the component fields it is necessary to also determine the form of the superspace transformation parameters, $\Xi^M$, $\Lambda^{rs}$ and $\Xi_{MN}$, that generate the supersymmetry transformations. Here it is important to realize that we are dealing with a gauge-fixed situation. For that reason the superspace parameters depend on both the $x$-dependent component parameters defined above as well as on the component fields. This has the consequence that local supersymmetry transformations reside in the superspace diffeomorphisms, the Lorentz transformations and the tensor gauge transformations. Thus, when considering supersymmetry variations of the various fields, one must in principle include each of the three possible superspace transformations.

For further details we refer to ref.18 and we proceed directly to the results. For the supervielbein $E_M^A$ the following expressions were found,

$$E_\mu^r = \epsilon_\mu^r + 2 \partial^r \Gamma^r \psi_\mu,$$
from the flat-superspace results (2). For completeness we included the \( \theta \)

\[ \text{phisms here and refer to the literature for more complete results} \text{ [18].} \]

by superspace diffeomorphisms, local Lorentz transformations and tensor gauge

transformations consistent with the fields specified above, are generated

\[ B \]

the tensor field

Observe that \( E_\mu^a \) was determined only up to terms of order \( \theta^2 \). The result for the tensor field \( B_{\mu \nu \rho} \) reads as follows,

\[ B_{\mu \nu \rho} = C_{\mu \nu \rho} - 6 \bar{\vartheta} \Gamma_{\mu [\nu \rho]} + 3 \frac{1}{2} T_{\rho [\mu \nu]} \theta + \Theta \Gamma_{\lambda \kappa \tau} \bar{F}_{\lambda \kappa \tau} \theta + O(\theta^3), \]

\[ B_{\mu \nu \alpha} = (\bar{\vartheta} \Gamma_{\mu \nu})_{\alpha} - \frac{1}{2} \bar{\vartheta} \Gamma_{\rho [\mu \nu]} (\bar{\vartheta} \Gamma_{\nu \rho})_{\alpha} + 3 \bar{\vartheta} \Gamma_{\rho [\mu \nu]} \theta + O(\theta^3), \]

\[ B_{\mu \nu \alpha} = (\bar{\vartheta} \Gamma_{\mu \nu})_{(\alpha} \bar{\vartheta} \Gamma_{\nu ) \beta} + O(\theta^3), \]

\[ B_{\alpha \beta \gamma} = (\bar{\vartheta} \Gamma_{\mu \nu})_{(\alpha} (\bar{\vartheta} \Gamma_{\nu \rho})_{\beta} + O(\theta^3). \]

For completeness we included the \( \theta^3 \)-term in \( B_{\alpha \beta \gamma} \) which were already known from the flat-superspace results [13].

Then we turn to some of the transformation parameters. The supersymmetry transformations consistent with the fields specified above, are generated by superspace diffeomorphisms, local Lorentz transformations and tensor gauge transformations. We only present the parameters for the superspace diffeomorphisms here and refer to the literature for more complete results [18].

\[ \Xi^\mu (\epsilon) = \bar{\vartheta} \Gamma^\mu \epsilon - \bar{\vartheta} \Gamma^\nu \epsilon \bar{\vartheta} \Gamma^\rho \psi_\rho + O(\theta^3), \]

\[ \Xi^\alpha (\epsilon) = \epsilon^\alpha - \bar{\vartheta} \Gamma^\mu \epsilon \psi_\mu^\alpha + \frac{1}{2} \bar{\vartheta} \Gamma_{\rho [\mu \nu]} (\bar{\vartheta} \Gamma_{\nu \rho})_{\alpha} + \epsilon^3 N_{\beta}^\alpha + O(\theta^3), \]

where \( N_{\beta}^\alpha \) encodes unknown terms proportional to \( \bar{F} \theta^2 \).

Substituting the above results into the initial supermembrane action [11] we obtain the action for a supermembrane in a supergravity background to order \( \theta^2 \). While the original action was manifestly covariant under independent superspace diffeomorphisms, tangent-space Lorentz transformations and tensor gauge transformations, this is no longer the case and one has to restrict oneself to the superspace transformations corresponding to the component supersymmetry, general-coordinate, local Lorentz and tensor gauge transformations. When writing [11] in components, utilizing the expressions found above, one thus obtains an action that is covariant under the restricted superspace diffeomorphisms [18] acting on the superspace coordinates \( Z^M = (X^\mu, \theta^a) \) (including the space-time arguments of the background fields) combined with usual transformations on the component fields. Note that the result does not constitute an invariance. Rather it implies that actions corresponding to two different sets of background fields that are equivalent by a component gauge transformation, are the same modulo a reparametrization of the supermembrane embedding coordinates. In
order to be precise let us briefly turn to an example and consider the action of a particle moving in a curved spacetime background with metric \(g_{\mu\nu}\),

\[
S[X^\mu, g_{\mu\nu}(X)] = -m \int dt \sqrt{-g_{\mu\nu}(X(t)) X^\mu(t) X^\nu(t)}.
\]  

(19)

This action, which is obviously invariant under world-line diffeomorphisms, satisfies \(S[X'^\mu, g'_{\mu\nu}(X')] = S[X^\mu, g_{\mu\nu}(X)]\), where \(X'^\mu\) and \(X^\mu\) are related by a target-space general coordinate transformation which also governs the relation between \(g'_{\mu\nu}\) and \(g_{\mu\nu}\). Of course, when considering a background that is invariant under (a subset of the) general coordinate transformations (so that \(g = g'\)), the action will be invariant under the corresponding change of the coordinates.

This is the situation that we will address in the next section, where we take a specific background metric with certain isometries. In that context the relevant target space for (19) is an anti-de Sitter (AdS) space, which has isometries that constitute the group \(\text{SO}(d - 1, 2)\), where \(d\) is the spacetime dimension. Then (19) describes a one-dimensional field theory with an \(\text{SO}(d - 1, 2)\) invariance group. In the particular case of \(d = 2\) this invariance can be re-interpreted as a conformal invariance for a supersymmetric quantum mechanical system.

Using the previous results one may now write down the complete action of the supermembrane coupled to background fields up to order \(\theta^2\). Direct substitution leads to the following result for the supervielbein pull-back,

\[
\Pi_i' = \partial_i X^\mu \left( \epsilon^\mu_\nu + 2 \bar{\theta} \Gamma^\nu_\mu \psi_\mu - \frac{1}{4} \bar{\theta} \Gamma^r s t \theta \hat{\omega}_{rst} + \bar{\theta} \Gamma^r T_{\mu}^{\nu \rho \sigma \lambda} \theta \hat{F}_{\nu \rho \sigma \lambda} \right)
\]  

\[+ \bar{\theta} \Gamma^r \partial_i \theta + O(\theta^3),\]

\[
\Pi_i^a = \partial_i X^\mu \left( \psi_\mu - \frac{1}{4} \hat{\omega}_{\mu \nu} (\Gamma_{rs} \theta)^a + (T_{\mu}^{\nu \rho \sigma \lambda} \theta)^a \hat{F}_{\nu \rho \sigma \lambda} \right)
\]  

\[+ \partial_i \theta^a + O(\theta^2).\]  

(20)

Consequently the induced metric is known up to terms of order \(\theta^3\). Furthermore the pull-back of the tensor field equals

\[
-\frac{1}{6} \varepsilon^{ijk} \Pi_i^A \Pi_j^B \Pi_k^C B_{BCA} = -\frac{1}{6} \varepsilon^{ijk} \partial_i Z^M \partial_j Z^N \partial_k Z^P B_{PMN} =
\]  

\[
\frac{1}{6} dX^{\mu \nu \rho} \left[ C_{\mu \nu \rho} - 6 \bar{\theta} \Delta_{\mu \nu} \psi_\rho + \frac{1}{4} \bar{\theta} \Gamma_{rs} T_{\mu}^{\nu \rho \sigma \lambda} \theta \hat{F}_{\nu \rho \sigma \lambda}
\]  

\[+ 3 \bar{\theta} \Gamma_{\mu \nu} T_{\rho}^{\lambda \kappa \tau} \theta \hat{F}_{\lambda \kappa \tau} - 12 \bar{\theta} \Gamma_{\sigma \mu} \psi_\nu \theta \hat{F}_{\sigma \kappa \tau} \right]
\]  

\[- \varepsilon^{ijk} \bar{\theta} T_{\mu \nu} \partial_k \theta \left[ \frac{1}{2} \partial_i X^\mu (\partial_j X^\nu + \bar{\theta} \Gamma^\nu \partial_j \theta) + \frac{1}{6} \bar{\theta} \Gamma^\mu \partial_i \theta \partial \Gamma^\nu \partial_j \theta \right]
\]  

\[+ \frac{1}{6} \varepsilon^{ijk} \partial_i X^\mu \partial_j X^\nu \left[ 4 \bar{\theta} \Gamma_{\mu \nu} \partial_k \theta \partial \Gamma^\rho \psi_\nu - 2 \bar{\theta} \Gamma^\rho \partial_k \theta \partial \Gamma_{\mu \nu} \psi_\nu \right] + O(\theta^3),\]

(21)

where we have introduced the abbreviation \(dX^{\mu \nu \rho} = \varepsilon^{ijk} \partial_i X^\mu \partial_j X^\nu \partial_k X^\rho\) for the world-volume form. Observe that we included also the terms of higher-order.

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\(^2\)This situation arises generically for any \(p\)-brane moving in a target space that is locally the product of \(\text{AdS}_{p+2}\) and some compact space. The conformal interpretation was emphasized in ref. 23.
θ-terms that were determined in previous sections and listed in (17). The first formula of (20) and (21) now determine the supermembrane action (1) up to order $\theta^3$. It has been shown that the resulting expressions are consistent with supersymmetry and $\kappa$-invariance [18].

4 Near-horizon geometries

In the previous section we discussed the determination of superspace quantities, i.e. the superspace vielbein and the tensor gauge field, in terms of the fields of 11-dimensional on-shell supergravity. The corresponding expressions are obtained by iteration order-by-order in $\theta$ coordinates, but except for the leading terms it is hard to proceed with this program. Nevertheless these results enable one to write down the 11-dimensional supermembrane action coupled to a nontrivial supergravity component-field background to second order in $\theta$, so that one can start a study of the supermembrane degrees of freedom in the corresponding background geometries.

However, in specific backgrounds with a certain amount of symmetry, it is possible to obtain results to all orders in $\theta$. Interesting candidates for such backgrounds are the membrane [24] and the five-brane solution [25] of 11-dimensional supergravity, as well as solutions corresponding to the product of anti-de Sitter spacetimes with compact manifolds. Coupling to the latter solutions, which appear near the horizon of black D-branes [26], seem especially appealing in view of the recent results on a connection between large-$N$ superconformal field theories and supergravity on a product of an $AdS$ space with a compact manifold [17]. The target-space geometry induced by the $p$-branes interpolates between $AdS_{p+2} \times B$ near the horizon, where $B$ denotes some compact manifold (usually a sphere), and flat $(p+1)$-dimensional Minkowski space times a cone with base $B$.

This program has been carried out recently for the type-II superstring and the D3-brane in a IIB-supergravity background of this type [27, 28, 29]. In the context of 11-dimensional supergravity the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ backgrounds stand out as they leave all 32 supersymmetries invariant [30, 31]. These backgrounds are associated with the near-horizon geometries corresponding to two- and five-brane configurations and thus to possible conformal field theories in 3 and 6 spacetime dimensions with 16 supersymmetries, whose exact nature is not yet completely known. In this section we consider the supermembrane in these two backgrounds [32]. As the corresponding spaces are local products of homogeneous spaces, their geometric information can be extracted from appropriate coset representatives leading to standard invariant one-forms corresponding to the vielbeine and spin-connections. The approach of ref. 32 differs from that of ref. 33; in the latter one constructs the geometric information exploiting simultaneously the $\kappa$-symmetry of the supermembrane action, while in ref. 32 the geometric information is determined independently from the supermembrane action. The results for the geometry coincide with those of ref. 34.
As is well known, the compactifications of the theory to $AdS_4 \times S^7$ and $AdS_7 \times S^4$ are induced by the antisymmetric 4-rank field strength of M-theory. These two compactifications are thus governed by the Freund-Rubin field $f$, defined by (in Pauli-Källén convention, so that we can leave the precise signature of the spacetime open),

$$F_{\mu\nu\rho\sigma} = 6f \epsilon_{\mu\nu\rho\sigma},$$

with $\epsilon$ the vierbein determinant. When $f$ is purely imaginary we are dealing with an $AdS_4 \times S^7$ background while for real $f$ we have an $AdS_7 \times S^4$ background. The nonvanishing curvature components corresponding to the 4- and 7-dimensional subspaces are equal to

$$R_{\mu\nu\rho\sigma} = -4f^2(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma}),$$
$$R_{\mu'\nu'\rho'\sigma'} = f^2(g_{\mu'\nu'}g_{\rho'\sigma'} - g_{\mu'\rho'}g_{\nu'\sigma'}).$$

Here $\mu, \nu, \rho, \sigma$ and $\mu', \nu', \rho', \sigma'$ are 4- and 7-dimensional world indices, respectively. We also use $m_{4,7}$ for the inverse radii of the two subspaces, defined by $|f|^2 = m_7^2 = \frac{1}{4} m_4^2$. The Killing-spinor equations associated with the 32 supersymmetries in this background take the form

$$(D_\mu - f\gamma_\mu \gamma_5 \otimes 1)\epsilon = (D_{\mu'} + \frac{1}{2} f1 \otimes \gamma_{\mu'})\epsilon = 0,$$

where we make use of the familiar decomposition of the (hermitean) gamma matrices $\gamma_\mu$ and $\gamma_{\mu'}$, appropriate to the product space of a 4- and a 7-dimensional subspace. Here $D_\mu$ and $D_{\mu'}$ denote the covariant derivatives containing the spin-connection fields corresponding to $SO(3,1)$ or $SO(4)$ and $SO(7)$ or $SO(6,1)$, respectively.

The algebra of isometries of the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ backgrounds is given by $osp(8|4)$ and $osp(6,2|4)$. Their bosonic subalgebra consists of $so(8) \oplus sp(4) \simeq so(8) \oplus so(3,2)$ and $so(6,2) \oplus usp(4) \simeq so(6,2) \oplus so(5)$, respectively. The spinors transform in the $(8,4)$ of this algebra.

One may decompose the generators of $osp(8|4)$ or $osp(6,2|4)$ in terms of irreducible representations of the bosonic $so(7) \oplus so(3,1)$ and $so(6,1) \oplus so(4)$ subalgebras. In that way one obtains the bosonic (even) generators $P_\mu$, $M_{rs}$, which generate $so(3,2)$ or $so(5)$, and $P_{\mu'}$, $M_{r's'}$, which generate $so(8)$ or $so(6,2)$. All the bosonic generators are taken antihermitean (in the Pauli-Källén sense). The fermionic (odd) generators $Q_{a\alpha'}$ are Majorana spinors, where we denote the spinorial tangent-space indices by $a, b, \ldots$ and $a', b', \ldots$ for 4- or 7-dimensional indices. The commutation relations between even generators are

$$[P_\mu, P_\nu] = -4f^2 M_{\mu\nu}, \quad [P_\mu', P_{\nu'}] = f^2 M_{\mu'\nu'},$$
$$[P_\mu, M_{rs}] = \delta_{rs} P_\mu - \delta_{rt} P_s, \quad [P_{\mu'}, M_{r's'}] = \delta_{r's'} P_{\mu'} - \delta_{r't'} P_{s'},$$
$$[M_{rs}, M_{tu}] = \delta_{ru} M_{st} + \delta_{st} M_{ru} \quad [M_{r's'}, M_{t's'}] = \delta_{t's'} M_{r's'} + \delta_{r't'} M_{r's'},$$
$$-\delta_{rt} M_{su} - \delta_{su} M_{rt}, \quad -\delta_{r't'} M_{s't'} - \delta_{s't'} M_{r't'}. \tag{25}$$
The odd-even commutators are given by
\[ [P_r, Q_{aa'}] = -f(\gamma_r\gamma_5)_{ab} Q_{ba'}, \quad [P_{r'}, Q_{aa'}] = -\frac{1}{2}f(\gamma_{r'})_{a'b'} Q_{ab'}, \]
\[ [M_{rs}, Q_{aa'}] = -\frac{1}{2}(\gamma_{rs})_{ab} Q_{ba'}, \quad [M_{r's'}, Q_{aa'}] = -\frac{1}{2}(\gamma_{r's'})_{a'b'} Q_{ab'}. \]

Finally, we have the odd-odd anti-commutators,
\[ \{Q_{aa'}, Q_{bb'}\} = -(\gamma_5 C)_{ab} \left( 2(\gamma_{r'} C')_{a'b'} P^{r'} - f(\gamma_{r's'} C')_{a'b'} M^{r's'} \right) \]
\[ - C'_{a'b'} \left( 2(\gamma_r C)_{ab} P^r + 2f(\gamma_{rs} \gamma_5 C)_{ab} M^{rs} \right). \]

All other (anti)commutators vanish. The normalizations of the above algebra were determined by comparison with the supersymmetry algebra in the conventions of [18] in the appropriate backgrounds.

However, one can return to 11-dimensional notation and drop the distinction between 4- and 7-dimensional indices so that the equations obtain a more compact form. In that case the above (anti)commutation relations that involve the supercharges can be concisely written as,
\[ [P_r, \bar{Q}] = \frac{1}{2} Q T^{stuv}_{f} F_{stuv}, \quad [M_{rs}, \bar{Q}] = \frac{1}{2} Q \Gamma_{rs}, \]
\[ \{Q, \bar{Q}\} = -2 \Gamma_r P^r + \frac{1}{144} \left[ \Gamma^{r stuvw} F_{tuvw} + 24 \Gamma_{tu} F^{rstu} \right] M_{rs}, \]
where the tensor \( T \) equals
\[ T^{stuv}_{f} = \frac{1}{288} \left( \Gamma^{r stuvw} - 8 \delta^{[s}_{f} \Gamma^{tuvw}] \right). \]

Note, however, that the above formulae are only applicable in the background where the field strength takes the form given in (22). In what follows, we will only make use of (28).

5 Coset-space representatives of \( AdS_4 \times S^7 \) and \( AdS_7 \times S^4 \)

Both backgrounds that we consider correspond to homogenous spaces and can thus be formulated as coset spaces. In the case at hand these (reductive) coset spaces \( G/H \) are \( OSp(8|4; \mathbb{R})/SO(7) \times SO(3,1) \) and \( OSp(6,2|4)/SO(6,1) \times SO(4) \). To each element of the coset \( G/H \) one associates an element of \( G \), which we denote by \( L(Z) \). Here \( Z^A \) stands for the coset-space coordinates \( x^r, \theta^a \) (or, alternatively, \( x^r, y^{r'} \) and \( \theta^{aa'} \)). The coset representative \( L \) transforms from the left under constant \( G \)-transformations corresponding to the isometry group of the coset space and from the right under local \( H \)-transformations: \( L \rightarrow L' = g L h^{-1} \).
The vielbein and the torsion-free $H$-connection one-forms, $E$ and $\Omega$, are defined through
\[ dL + L\Omega = LE, \] (30)
where
\[ E = E^r P_r + \bar{E} Q, \quad \Omega = \frac{1}{2} \Omega^{rs} M_{rs}. \] (31)
The integrability of (30) leads to the Maurer-Cartan equations,
\[ d\Omega - \Omega \wedge \Omega - \frac{1}{2} E^r \wedge E^s [P_r, P_s] \]
\[ - \frac{1}{288} \bar{E} \left[ \Gamma^{rstuvw} F_{tuvw} + 24 \Gamma_{tu} F^{rstu} \right] E M_{rs} = 0, \]
\[ dE - \Omega^s \wedge E_r - \bar{E} \Gamma^r \wedge E = 0, \]
\[ dE + E^r \wedge T^tuvw E F_{tuvw} - \frac{1}{4} \Omega^{rs} \wedge \Gamma_{rs} E = 0, \] (32)
where we suppressed the spinor indices on the anticommuting component $E^a$. The first equation in a fermion-free background reproduces (23) upon using the commutation relations (25).

Now the question is how to determine the vielbeine and connections to all orders in $\theta$ for the spaces of interest. First, observe that the choice of the coset representative amounts to a gauge choice that fixes the parametrization of the coset space. We will not insist on an explicit parametrization of the bosonic part of the space. It turns out to be advantageous to factorize $L(Z)$ into a group element of the bosonic part of $G$ corresponding to the bosonic coset space, whose parametrization we leave unspecified, and a fermion factor. Hence one may write
\[ L(Z) = \ell(x) \hat{L}(\theta), \quad \text{with } \hat{L}(\theta) = \exp[ \theta Q]. \] (33)
There exists a convenient trick [27, 28, 35] according to which one first rescales the odd coordinates according to $\theta \rightarrow t \theta$, where $t$ is an auxiliary parameter that we will put to unity at the end. Taking the derivative with respect to $t$ of (30) then leads to a first-order differential equation for $E$ and $\Omega$ (in 11-dimensional notation),
\[ \dot{E} - \dot{\Omega} = d\bar{\theta} Q + (E - \Omega) \bar{\theta} Q - \bar{\theta} Q (E - \Omega) \] (34)
After expanding $E$ and $\Omega$ on the right-hand side in terms of the generators and using the (anti)commutation relations (28) we find a set of coupled first-order linear differential equations,
\[ \dot{E}^a = \left( d\theta + E^r T_{r}^{stuv} \theta F_{stuv} - \frac{1}{4} \Omega^{rs} \Gamma_{rs} \theta \right)^a, \]
\[ \dot{E}^r = 2 \bar{\theta} \Gamma^r E, \]
\[ \dot{\Omega}^{rs} = \frac{1}{12} \bar{\theta} \left[ \Gamma^{rstuvw} F_{tuvw} + 24 \Gamma_{tu} F^{rstu} \right] E. \] (35)
\[ ^3A \text{ one-form } V \text{ stands for } V \equiv dZ^A V_A \text{ and an exterior derivative acts according to } dV \equiv -dZ^B \wedge dZ^A \partial_A V_B. \text{ Fermionic derivatives are thus always left-derivatives.} \]
These equations can be solved straightforwardly [28] and one finds

\[ E(x, \theta) = \sum_{n=0}^{16} \frac{1}{(2n+1)!} \mathcal{M}^{2n} D\theta, \]

\[ E^r(x, \theta) = dx^\mu e^r_\mu + 2 \sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\theta} \Gamma^r \mathcal{M}^{2n} D\theta \] (36)

\[ \Omega^r(x, \theta) = dx^\mu \omega^r_\mu \]

\[ + \frac{1}{72} \sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\theta} [\Gamma^{rstuvw} F_{tuvw} + 24 \Gamma_{tu} F^{rstu}] \mathcal{M}^{2n} D\theta, \]

where the matrix \( \mathcal{M}^2 \) equals,

\[ (\mathcal{M}^2)^a_b = 2 \langle T_r^{stuv} \theta^a F_{stuv} (\bar{\theta} \Gamma^r)_b \]

\[ - \frac{1}{288} (\Gamma_{rs} \theta)^a_b (\bar{\theta} [\Gamma^{rstuvw} F_{tuvw} + 24 \Gamma_{tu} F^{rstu}])_b. \] (37)

and

\[ D\theta^a \equiv \dot{E}^a \bigg|_{t=0} = \left( \frac{d\theta + e^r T_r^{stuv} \theta F_{stuv} - \frac{1}{4} \omega^r \Gamma_{rs} \theta}{\bar{\theta}} \right)^a. \] (38)

It is straightforward to write down the lowest-order terms in these expansions,

\[ E^r = e^r + \bar{\theta} \Gamma^r d\theta + \bar{\theta} \Gamma^r (e^m T_m^{stuv} F_{stuv} - \frac{1}{4} \omega^{st} \Gamma_{st}) \theta + \mathcal{O}(\theta^4), \]

\[ E = d\theta + (e^r T_r^{stuv} F_{stuv} - \frac{1}{4} \omega^r \Gamma_{rs} \theta + \mathcal{O}(\theta^3), \]

\[ \Omega^r = \omega^r + \frac{1}{144} \bar{\theta} [\Gamma^{rstuvw} F_{tuvw} + 24 \Gamma_{tu} F^{rstu}] d\theta + \mathcal{O}(\theta^4), \] (39)

which agree completely with those given in section 8 (and, for the spin-connection field, in ref. 17).

This information can now be substituted into the first part of the supermembrane action (1). By similar techniques one can also determine the Wess-Zumino-Witten part of the action by first considering the most general ansatz for a four-form invariant under tangent-space transformations. Using the lowest-order expansions of the vielbeine (39) and comparing with ref. 18 shows that only two terms can be present. Their relative coefficient is fixed by requiring that the four-form is closed, something that can be verified by making use of the Maurer-Cartan equations (32). The result takes the form

\[ F^{(4)} = \frac{1}{4!} \left[ E^r \wedge E^s \wedge E^t \wedge E^u F_{rstu} - 12 \bar{\theta} \wedge \Gamma_{rs} E \wedge E^r \wedge E^s \right]. \] (40)

To establish this result we also needed the well-known quartic-spinor identity in 11 dimensions. The overall factor in (40) is fixed by comparing to the normalization of the results given in ref. 18.

Because \( F^{(4)} \) is closed, it can be written locally as \( F^{(4)} = dB \). The general solution for \( B \) can be found by again exploiting the one-forms with rescaled \( \theta \).
coordinates according to $\theta \to t\theta$ and deriving a differential equation. Using the lowest order result for $B$ this equation can be solved and yields

$$B = \frac{1}{6} e^\tau \wedge e^s \wedge e^r C_{rst} - \int_0^1 dt \bar{\theta} \Gamma_{rs} E \wedge E^r \wedge E^s,$$  \hfill (41)

where the vielbein components contain the rescaled $\theta$’s. This answer immediately reproduces the flat-space result upon substitution of $F_{rstu} = \omega^{rs} = 0$.

In order to obtain the supermembrane action one substitutes the above expressions in the action (1). The resulting action is then invariant under local fermionic $\kappa$-transformations as well as under the superspace isometries corresponding to $osp(8|4)$ or $osp(6,2|4)$.

We have already emphasized that the choice of the coset representative amounts to adopting a certain gauge choice in superspace. The choice that was made in ref. 32 connects directly to the generic 11-dimensional superspace results, written in a Wess-Zumino-type gauge, in which there is no distinction between spinorial world and tangent-space indices. In specific backgrounds, such as the ones discussed here, gauge choices are possible which allow further simplifications. For this we refer to refs. 28 and 36.

The results of this section provide a convincing check of the low-order $\theta$ results obtained by gauge completion for general backgrounds. A great amount of clarity was gained by expressing our results in 11-dimensional language, so that both the $AdS_4 \times S^7$ and the $AdS_7 \times S^4$ solution could be covered in one go. Note that in both these backgrounds the gravitino vanishes.

We have no reasons to expect that the 11-dimensional form of our results will coincide with the expressions for a generic 11-dimensional superspace (with the gravitino set to zero) at arbitrary orders in $\theta$.

From a more technical viewpoint it is gratifying that explicit constructions of supermembranes in certain nontrivial backgrounds are now possible. The complete M-theory two-brane action in $AdS_4 \times S^7$ and $AdS_7 \times S^4$ to all orders in $\theta$ represents a further step in the program of finding the complete anti-de-Sitter background actions for the superstring and the M2-, D3-, and M5-branes initiated for the bosonic part in ref. 23. Furthermore, by studying the interaction between a test membrane in the background of an M2- or an M5-brane, one may hope to learn more about the interactions between branes. Some of these issues have already been considered recently.

Acknowledgments

The results described in this talk were obtained in collaboration with Kasper Peeters, Jan Plefka and Alexander Sevrin.

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