The analysis of the working surface of the abrasive tool for wood grinding

V V Sergeevichev, A V Sergeevichev, V A Sokolova, V D Kebko and E A Alekseeva

Saint-Petersburg State Forest Technical University, Institutskiy per., 5, lit U, Saint-Petersburg, 194021, Russia

E-mail: sokolova_vika@inbox.ru

Abstract. The analysis of the existing models of the cutting elements of abrasive grains shows that the model in the form of a parabola is in comparison with others more preferable and with a fine precision corresponds to the experimental geometrical parameters of abrasive grains. The natural increase in radius of top due to wear is revealed.

1. Introduction
Grinding is carried out for the purpose of alignment of a surface after the previous operations and giving of a required roughness to it, and also by removals of a layer of wood or wood material for providing a given size of a product.

Studying and determination of surface energy of wood and wood materials is an actual problem of the modern woodworking connected with formation of new surfaces as a result of technological processes.

The studied processing of details abrasive tools according to the description and modeling is the most difficult. They are characterized by the developed spatio-temporal bonds which detailed studying has to be carried out on the basis of system approach.

Wood cutting by an abrasive tool – grinding abrasive paper, has the mass character of aggression connected with disperse structure of grinding skins.

Difference of grinding from other types of machining of wood cutting (planing, milling, drilling, etc.) is absence at abrasive tools of constants, certain geometrical parameters of the cutting part of grinding abrasive paper.

Cutting of wood and wood materials when grinding is carried out by abrasive grains. In a form abrasive grains can be isomeric, lamellar and xiphoidal. Isomeric grains have the rounded symmetric shape (approximately equal sizes on height, width and thickness), and grains lamellar and xiphoidal have the pronounced asymmetrical form. The main form of abrasive grains is isomeric [1].

2. Methods of research
In figure 1, the scheme of cutting is submitted by abrasive grain. Abrasive grain has an edge, as a rule, with a negative face angle – γ and a sharpening angle β within 40° … 145°, and at the majority of grains this angle exceeds 90 °. During contact with the ground material the edge removes from the processed surface a microlayer thickness of \(a_i\), mm.
Average radius of a curve of the cutting edges of abrasive grains $\rho = 3 \ldots 30$ microns also depends, respectively, on the size of grains. This circumstance, in particular, indicates that it of reduction of a surface roughness when grinding by fine-grained skins is reached not only due to reduction of thickness of the cut-off layers, but also due to higher sharpness of the cutting edges of fine grains [2].

The number of the abrasive grains which are at the same time interacting with the processed surface depends on the size of grains (granularity), extent of their blunting, the area of contact with the processed product and characteristics of the mode of grinding. The same parameters, the size of a roughness of a worked surface, productivity of process and firmness of grinding abrasive paper generally is defined.

Abrasive grains are made of abrasive material by its crushing. Abrasive materials have to have the high hardness, durability and have to be rather brittle. Abrasive materials are subdivided into natural and artificial (synthetic). Quartz, a sandpaper, flint, corundum, etc. belong to natural. The main lack of natural abrasive materials is availability of impurity, the cutting properties significantly reducing them.

The description of a basic site of a working surface of an abrasive tool provides determination of the geometrical sizes, numbers and distributions of form-building elements. The form of elements depends on their nature and a type. For processing, it is set by the tool with edge in the form of nonrandom parameters. For processes of the second group elements have no regular geometry, their form can be simulated by stochastic functions.

Radius vector of points of a contour of abrasive grain $R_g(\varphi)$ according to the author [3] is stochastic function of an angle $\varphi$, figure 2, a.

\[
R_g(\varphi, 0) = r_g + \sum_{k=2}^{p} (x_k \sin \omega_k \varphi + y_k \cos \omega_k \varphi).
\]
where \( r_s \) – the radius of an average circle of a profile, micron, defined as average value of the function \( R_0(\phi, 0) \); \( x_k, y_k \) – the accidental uncorrelated amplitudes which are corresponding to \( k \) harmonic and having equal dispersions; \( \omega_k \) – randomly the chosen frequency.

The equation (1) is result of decomposition of stochastic function in a row of Fourier and is acceptable for the description of a form practically of any single elements of a working surface of the tool.

At record of the equation (1) in the form of

$$ R_g(\phi, 0) = r_g + \sum_{k=2}^{p} A_k \sin(k\phi + \psi_k), $$

where

$$ A_k = \sqrt{x_k^2 + y_k^2}; \quad \tan \psi_k = y_k / x_k, $$

are calculated as a population mean and correlation function the radius vector of points of a contour of abrasive grain.

The equation (1) is result of decomposition of stochastic function in a row of Fourier and is acceptable for the description of a form practically of any single elements of a working surface of the tool.

The equations (1) and (2) describe a static profile of the cutting edges. For a number of processes the form of edges does not remain to a constant, and significantly changes during firmness of the tool. For processing of details abrasive tools and tools with edge, change of a form of edges is defined by their wear (figure 2, b). For single abrasive grain the current polar radius \( R_g(\phi, \tau) \) during \( \tau \):

$$ R_g(\phi, \tau) = R_g(\phi, 0) - h_g(\phi, \tau). $$

The elementary increment of size of wear in the direction of polar radius can be accepted proportional to relative wear \( h_0 \) and track length of cutting \( dL_p(\phi, \tau) \):

$$ dh_g(\phi, \tau) = h_0 dL_p(\phi, \tau) = h_0(V_k \pm V_a) P_k(\phi, \tau) d\tau, $$

where \( V_k \) and \( V_a \) – speeds of an abrasive tool and preparation, m/s; \( P_k(\phi, \tau) \) – probability of contact of a point of a profile with material.

When turning continuous surfaces \( P_k = 1, V_k = 0 \), from the equation (4) we receive dependence, known in cutting:

$$ h = h_0 V_a \tau. $$

When processing details abrasive tools \( P_k(\phi, \tau) \) always happens less or is equal 1 because there are risks from an initial roughness or from earlier passed grains on a surface. Generally, the probability of contact is defined as a ratio of the sum of the pieces cut in material at this level to the basic length \( L \rightarrow \infty \) and in size matches a population mean of relative basic length of a profile. When microcutting a surface with a regular profile in the form of triangular scratch (figure 2, a), the probability of contact of a point of a contour of grain is calculated from a ratio:

$$ P_k(\phi, \tau) = \begin{cases} 0, & \text{at } R_g(\phi, \tau) \cos \phi < u_R - H \\ \frac{R_g(\phi, \tau) \cos \phi + H - u_R}{H}, & \text{at } R_g(\phi, \tau) \cos \phi \geq u_R - H \\ 1, & \text{at } R_g(\phi, \tau) \cos \phi \geq u_R \end{cases} $$

and the differential equation (4) for the points of a profile which are in a zone of microroughnesses takes a form:
where $H$ – the size of a layer of a surface roughness, micron; $u_R$ – distance from the lower bound of a layer to the center of grain, micron.

For the solution of the differential equation (5) it is necessary to have the law of change of distance from the deepest hollow of a profile to the center of abrasive grain. At $u_R = \text{const}$:

$$h_g(\varphi, \tau) = R_g(\varphi, 0) + \frac{H-u_R}{\cos \varphi} \left[ 1 - \exp \left( - \frac{h_0(V_k+V_u)\tau}{H} \cos \varphi \right) \right] = R_g(\varphi, 0) + \frac{H-u_R}{\cos \varphi} \psi(\varphi, \tau).$$

When $u_R = u_{R0} + V_R \tau$, where $V_k$ – the motion speed of the center of grain in the direction perpendicular surfaces, m/s, similar dependence takes a form:

$$h_g(\varphi, \tau) = R_g(\varphi, 0) + \frac{H-u_{R0}}{\cos \varphi} - \frac{V_R H}{h_0(V_k+V_u)\cos \varphi} \psi(\varphi, \tau) + \frac{V_R \tau}{\cos \varphi}.$$

The population mean and correlation function of a profile of worn-out part of a form-building element in a general view are calculated:

$$M[R_g(\varphi, \tau)] = M[R_g(\varphi, 0)] - M[h_g(\varphi, \tau)],$$

$$k_R(\varphi_1, \varphi_2, \tau) = M[(R_g(\varphi_1, \tau) - m_R(\varphi_1, \tau))(R_g(\varphi_2, \tau) - m_R(\varphi_2, \tau))],$$

where $m_R(\varphi_1, \tau)$ and $m_R(\varphi_2, \tau)$ – corresponding population means of random variables $R_g(\varphi_1, \tau)$ and $R_g(\varphi_2, \tau)$.

For abrasive grain $M[R_g(\varphi, 0)] = r_{g0}$, and wear of a profile in the direction radius vector at the fixed values $\tau$ and $\varphi$ is function of a random variable $R_g(\varphi, 0)$.

The population mean of stochastic function $h_g(\varphi, \tau) = \eta(\rho)$ is determined by density of probabilities radius vector $f_\rho(\rho)$.

$$M[h_g(\varphi, \tau)] = \int_0^\infty \eta(\rho)f_\rho(\rho)d\rho,$$

where $\rho$ – polar coordinate. For processes and angles, at which almost all possible implementations of profiles lie in a surface roughness layer, dependence (8) can be replaced with the equation:

$$M[h_g(\varphi, \tau)] = M \left[ R_g(\varphi, 0) + \frac{H-U_R}{\cos \varphi} \right] \psi(\varphi, \tau) = r_{g0} \psi(\varphi, \tau) + \frac{H-U_R}{\cos \varphi} \psi(\varphi, \tau).$$

We will finally receive

$$M[R_g(\varphi, \tau)] = r_{g0} \left( 1 - \psi(\varphi, \tau) \right) - \frac{H-U_R}{\cos \varphi} \psi(\varphi, \tau),$$

$$k_R(\varphi_1, \varphi_2, \tau) = k_R(\varphi_1, \varphi_2) \exp \left[ \left( - \frac{h_0(V_k+V_u)\tau}{H} \right)(\cos \varphi_1 + \cos \varphi_2) \right].$$

Unlike initial, the profile of worn-out abrasive grain is not stationary function. Its correlation function depends not only on a difference of values of angles $\varphi_1$ and $\varphi_2$, but also on their absolute values. When $\tau = 0$ population mean and correlation function of radius vector $R_g(\varphi, \tau)$ at calculation on dependences (6) and (7) are equal to a population mean and correlation function the radius vector of points of an initial profile of grain. Thus, the offered mathematical model reflects kinetics of change of a profile of a form-building element during firmness of the tool.

3. Results and discussion

Other possible option of modeling of an element of a working surface of the tool is the description only that its part which directly defines process of a shaping. In the analysis of operations of processing of preparations by abrasive tools assume that the grain top acting over sheaf level can have the form of a
cone, a cone with the rounded-off top, a cone with the truncated top, a sphere. More perfect model of top of grain is the model in the form of a paraboloid of rotation, figure 3.

![Image of models of abrasive grain: a. grain contour; b. rotation paraboloid.](image)

Figure 3. Models of abrasive grain: a. grain contour; b. rotation paraboloid.

The paraboloid equation at combination of the beginning of coordinates with a peak point of a profile registers:

\[
z = \frac{x^2 + y^2}{a^2}, \text{ if } y = 0 \text{ and } a^2 = 2\rho_g x = \sqrt{2\rho_g z},
\]

where \(a\) – paraboloid parameter; \(\rho_g\) – rounding radius at top of grain, micron.

The analysis of the existing models of the cutting elements of abrasive grains executed in work [4] shows that the model in the form of a parabola is in comparison with others more preferable and with a fine precision corresponds to experimental geometrical parameters of abrasive grains. If to designate distance from grain top to the considered level like \(h\), then top profile width \(b_g\) at this level (figure 3) will be defined, micron:

\[
b_g = 2\sqrt{2\rho_g h}.
\]

(9)

Considering variety of forms abrasive, diamond, elbor grains, the equation (9) write down more generally:

\[
b_g = C_b h^m,
\]

(10)

in which the coefficient \(C_b\) and an exponent of \(m\) define experimentally.

The top of abrasive grain can be considered in its general profile as one of emissions of the accidental field with \(R_g(\varphi) > \rho\), [7]. For any point of a profile of abrasive grain curvature radius in polar coordinates is calculated on the equation:

\[
\rho_g(\tau) = \frac{[R_g^2(\varphi, \tau) + R_g'^2(\varphi, \tau)]^{3/2}}{R_g^2(\varphi, \tau) + 2R_g'(\varphi, \tau)R_g''(\varphi, \tau)}.
\]

(11)

Curvature radius on sites of emissions can differ from value of radius of an average circle considerably. Analytical determination of radius of curvature leads to studying main curvatures \(k_1\) and \(k_2\) of a profile in points of the local maxima exceeding some level. For a Gaussian homogeneous and isotropic accidental surface, they are found in work [5]. In practice it is often much simpler to study directly geometrical parameters of tops of form-building elements. According to many researchers, radius at top of grain depends on material of abrasive grain, a way of production, granularity of the tool. Use of the second approach when modeling elements of a working surface of the tool in some cases is more preferable as on geometry of tops more practical data, than on geometry of grains in general are saved up [6].

The current radius of rounding depends on its reference value, conditions of contact of abrasive grain with the processed material, the mode of cutting and an operating time of the tool. With increase in an operating time of the tool the current radius of rounding naturally increases, at top of grain there is a radial site of wear, blunting of abrasive grain is observed.
Experimental check of patterns of wear of tops of form-building elements [7] is made when microcutting samples of C8 steel by grains from green carbide of silicon and firm alloy T15K10. The initial surface of samples was prepared grinding with longitudinal giving of 0,31 mm sharply ground cutter with a point angle 75°. The size of a layer of a roughness of an initial surface made 0,2 mm. Experiments were made at rotating speed of a detail 20,83 Hz, longitudinal giving 0,21 mm, depth of microcutting 0,16 mm. The profile of single grains was sketched on a tool microscope at 50 multiple increase. The typical picture of change of a profile of tops of grains of green carbide of silicon after one, two … five passes is presented in figure 2, by curves 2, 3, 4, 5 and 6 respectively.

4. Conclusions
Based on the carried-out analysis of the existing models of the cutting elements of abrasive grains, it is possible to draw a conclusion that the model in the form of a parabola is in comparison with others more preferable and with a fine precision corresponds to experimental geometrical parameters of abrasive grains. Natural increase in radius of top due to wear is revealed. For one pass it increases in 1,2 … 1,5 times. Grain obtusion naturally increases up to its destruction or ejection from a sheaf.

References
[1] Sergeevichev A, Semenov A 2018 The analysis of ways to increase the resistance of abrasive belts in the processing of wood and wood materials News of the St. Petersburg forest technical academy 222 213-27
[2] Hromchak I 1996 Abrasive processing of plate materials on mineral binders [In Russian: Abrazivnaya obrabotka plitnyh materialov na mineral'nyh vyazhushchih] (L'vov) p 47
[3] Gdalevich A 1994 Finishing with petal circles (Moscow) p 112
[4] Bratan S 2000 Identification of parameters of removal at the combined grinding Progressive technologies and systems of mechanical engineering (Materials of the international collection of scientific works Donetsk: Donetsk state technological university) pp 24-32
[5] Sanev V, Kamenev B and Sergeevichev A 2018 Wood cutting (St. Petersburg: Lan’) p 456
[6] Ostrovskij V 1981 Theoretical basis of the grinding process [In Russian: Teoreticheskie osnovy processa shlifovaniya] (Leningrad) p 144
[7] Carranoa A and James B 2005 Geometric Modeling of Engineered Abrasive Processes Taylor Journal of Manufacturing Processes 7(1) 17-27