Research of the labor resource redeployment by mathematical methods of optimal management

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Abstract. The problem statement of labor resource redeployment to ensure the sustainability and balance of positive territorial transformations is due to the necessary in order to study the socio-economic factors of the labor resource development in modern economic conditions. The paper presents mathematical models of the labor resource redeployment. In the dynamic model of staff resource redeployment, the speed of redeployment, the dynamics of inter-resource composition and the limitations of the dynamic characteristics of the redeployment process are studied. The model of the territorial labor resource redeployment is a solution to the problem of placing workers with their multi-agent interaction and any finite number of employers on the network with a given set of vacancies for each employee that he wants to receive.

1. Introduction
The solution of complex problems associated with the management of labor resources is correlated with the development of new theoretical and methodological approaches to building an economic management system adequate to the properties of resources. This requires the creation of appropriate economic and mathematical models, management and optimization methods; definition of transient quality criteria; improved laws of labor management and software implementation of the developed models [1].

2. Dynamic model of labor resource redeployment
Consider the process of labor resource redeployment. The driving force behind this process is the oversaturation of the labor market. For a quantitative description the process of labor resource allocation of a certain qualification, we consider this phenomenon in the “conditional volume” of the total number of labor resources assigned to a certain point x of the workspace (labor market) and to the time period [t, t + Δt] [2].

2.1. The rate of labor resource redeployment
Let there be N labor resources of a certain qualification in this volume, each with a mass of \( m_K = m_K(t) \), \( k = 1, N \) and an inter-resource composition of the mass \( M = M(t) \). The composition of the inter-resource composition is characterized by the share of \( B = B(t) \) labor resources of different qualifications and the share of qualification \( \tau = \tau(t) \) in them (good quality of the inter-resource
composition). The values of B and τ will be understood precisely as fractions (rather than percentage), so that the mass of the inter-resource composition is $M_{CB}$ and the mass of qualified personnel in $M_S$ are expressed accordingly:

$$M_{CB} = MB, \quad M_S = M_{CB}\tau = MB\tau.$$  \hfill (1)

The need for labor resources and their qualitative growth depend, first of all, on the coefficient of supersaturation $\Pi = \frac{H}{H_0} = \frac{M_S(1-B)}{M_H0}$ \hfill (2)

where $H$ is the labor force per unit mass of the labor market in the inter-resource composition, $H_0$ is the coefficient of labor demand, depending on the value of T, i.e. $\tau$ \hfill [3].

Empirical dependence will take the form $H_0 = H_0(T, \tau) = 3.33 + 0.187T - 18.8r^2 + 167.5\tau^2$  \hfill (3)

With values of $\Pi \geq 1.2$ there is a redeployment of labor.

Next, we will consider the process of labor resource redeployment of $\Pi \in [1.05, 1.10]$, when the labor resource redeployment practically does not occur, but there is an effective growth in the qualification of labor resources. Moreover, the increase $\Delta m_k$ of the each resource mass $k$ is proportional to its surface area $F_k$, the time of labor resources redeployment $\Delta t$ and is expressed by the formula (Fick's law):

$$\Delta m_k = K_v \frac{T(\Pi-1)}{\eta} F_k\Delta t = vF_k\Delta t$$  \hfill (4)

where $K_v$ is the coefficient of the rate of labor resource redeployment, $\eta$ is the density of the labor market. Next we have

$$K_v = -550 + 1050\tau + 245000\tau(\Pi - 1)^2 + 1910^4(\Pi - 1)^2$$  \hfill (5)

$$\eta = (a_0 + a_1T + a_2T^2)A^{10(\Pi-1)}$$  \hfill (6)

$$a_0 = 130.12 - 250\tau + 122\tau^2$$

$$a_1 = (272.7 + 505.8\tau - 273\tau^2)10^{-2}$$

$$a_2 = (18.6 - 33\tau + 14.8\tau^2)10^{-3}$$

$$A = 0.525(1 - \tau)\tau^{-1} + 1.65$$

The quantity $v = K_vT(\Pi-1)/\eta$ in formula (4) is called the rate of labor resource redeployment. We use in (4) the relationship between mass and the surface area of the labor market:

$$F_K = 4.12^3\sqrt{m_k^2}$$  \hfill (7)

Dividing equality (4) by $\Delta t$ and passing to the aisle at $\Delta t$, we obtain, taking into account (7), a differential equation for changing the mass of labor resources of a certain qualification

$$\frac{dm_k}{dt} = K_v \frac{T(\Pi-1)}{\eta} 4.12^3\sqrt{m_k^2}$$  \hfill (8)

Let $M_z = M_z(t) = \sum_{k=1}^{N} m_k(t)$ is the mass of labor resources of a certain qualification in the labor market. In accordance with this, the increase $\Delta M_z$ of the mass $M_z$ during the time $\Delta t$ will be equal (if we assume that the masses of all resources are the same, then we calculate from the passage to the limit in $\Delta t \to 0$)

$$M_z = 4.12vN^2\sqrt{M_z^2N^{-2}}$$  \hfill (9)
2.2. The dynamics of the inter-resource composition of the labor resources

Let \( b_i \) and \( \tau_i \) be the qualitative characteristics of the labor resources and the qualitative characteristics, respectively, and \( u = u(t) \) is a rate of their formation in the inter-resource composition under consideration, i.e. \( \Delta Q = u \Delta t \) - qualitative characteristics for the time \( \Delta t \). During this time, \( W \Delta t \) resources were removed from the labor market. Thus, over time from the moment \( t \), the change in the composition of \( M, B, \tau \) of the inter-resource composition is expressed taking into account (1) the formulas

\[
\begin{align*}
\Delta M &= M(t + \Delta t) - M(t) = u \Delta t - \Delta M_z - w \Delta t - M(t) \\
\Delta B &= B(t + \Delta t) - B(t) = (MB + ub_i \Delta t - \Delta M_z) (M + \Delta M)^{-1} - B(t) \\
\Delta \tau &= \tau(t + \Delta t) - \tau(t) = (MB \tau + ub_i \tau_i \Delta t - \Delta M_z) (MB + \Delta B)^{-1} - \tau(t)
\end{align*}
\]

Dividing these relations by \( \Delta t \) and passing to the limit \( \Delta t \to 0 \), we obtain a system of differential equations

\[
\begin{align*}
M &= u + W - Mz & \text{(10)} \\
B &= (ub_i - M_z - BM) M^{-1} & \text{(11)} \\
\tau &= (ub_i \tau_i - M_z - \tau B) (MB)^{-1} & \text{(12)}
\end{align*}
\]

So, the solution of system (9) - (12) is unique to the dynamics of the process of redistribution of labor resources in the “conditional volume”, if you set the initial state - \( M_z(t_0), M(t_0), B(t_0), \tau(t_0) \), output stream - the function \( u(t) \) is the rate of redistribution of labor resources with the parameters \( b_i(t), \tau_i(t) \) and the rate of redeployment of labor resources \( W \), which is expressed by the formula

\[
W = K_w F_w (T_w - T) \quad \text{(13)}
\]

where \( F_w \) is the volume of the labor market (in the "conditional volume"), \( T_w \) is the degree of redeployment, \( K_w \) is the qualification coefficient:

\[
K_w = 219 \left( \frac{1-B}{1.18-B} \right)^2 \left( \frac{260-P}{736} \right)^{0.84} (T_w - T) \quad \text{(14)}
\]

\( P \) - labor market density. It is clear that the model of the process of redeployment of labor resources under consideration is essentially non-linear due to relations (2), (3), (5), (6), (13) and (14) [3].

2.3. Dynamic characteristics limitations of the process of labor resources redeployment

We will show the most significant limitations on the process of labor resources redeployment.

a) The working space of the labor market is limited by a predetermined volume \( \bar{V} \). We will consider the labor market of periodic action and the process of labor resources redeployment to be homogeneous throughout the workspace of the labor market and described by system (9) - (12) [4-9]. Then the volume \( V(t) \), occupied by labor resources and the inter-resource composition is determined by the specific weights \( p_z \) - labor resources, \( p_s \) - qualification, \( p_h \) - unearned resources, \( p_w \) - quantitative characteristics for a given degree of redeployment \( T \) and labor market density. Apparently,

\[
V(t) = \frac{M_z}{p_z} + \frac{MB \tau}{p_s} + \frac{MB(1-z)}{p_h} + \frac{M(1-B)}{p_w} \quad \text{(16)}
\]

Thus, the limitations on volume is expressed by the inequality

\[
V(t) \leq \bar{V} \quad \text{(17)}
\]

b) The changing surface of the labor market must be covered by resources:

\[
\bar{V} \leq V(t) \quad \text{(18)}
\]
c) The size (mass) of labor resources does not exceed a given:
\[ m_k(t) \leq \bar{m} \]  
(19)

d) Limitations on the degree of redeployment
\[ \tau(t) \leq \bar{\tau}(1 - P)M(M + M_2)^{-1} \geq \gamma > 0 \]  
(20)

e) The coefficient of saturation lies in the zone:
\[ 1.05 \leq \Pi \leq 1 \]  
(21)

Violation of each of the conditions (17) - (21) essentially means the end of the process of labor resources redeployment. Thus, a certain moment \( t = \bar{t} \), we will consider the end of the process of labor resources redeployment in the labor market.

In the model (9) - (12), the control parameters can be considered: the input flow rate \( u(t) \) and its composition \( b_i(t) \), \( \tau_i(t) \), the redeployment degree \( T \) of inter-resource composition, and also \( N \) - labor resources. The set of these parameters will be reduced to the vector \( u = (u(t), b_i(t), \tau_i(t), T(t), N, P) \) will be called a control. A control is called admissible if a solution to system (9) - (12) corresponding to it exists on a certain interval \([ t_0, \bar{t} ]\) if limitations (17) - (20) are satisfied. Here the numbers \( t_0, \bar{t} \) are the beginning and end of the process of labor resources redeployment. The end of the process \( t = \bar{t}(u) \) can be defined as above or in another way, in particular, we can assume that \( \bar{t} - \tau_i = \Pi \) is the given duration of the process [10-15].

The class of controls \( \mathcal{U} \) is denoted by \( \mathcal{V} \). This means that the components of the vector functions \( \mathcal{U} = \mathcal{U}(t) = (u, i, \bar{\tau}, T, N, P) \) are continuous and constrained by additional limits. For example,
\[ u^- \leq u(t) \leq u^+ \]  
(22)
\[ b_i(t) \in [b_i^-, b_i^+], \tau_i(t) \in [\tau_i^-, \tau_i^+] , T(t) \in [T_-, T_+], p \in [p_-, p_+] \]  
(23)

Here the quantities \( b_i^-, b_i^+, \tau_i^-, \tau_i^+, T_-, T_+, p_-, p_+ \) are the given constants. There may be restrictions on the nature of the change in control functions. For example, the degree of redeployment \( T(t) \) changes without differences, that is, \([+ (t)]\) is continuous and limited by a given limit.

Consider several criteria for controllability of the redeployment process of labor resources. Let \( t_0 = 0 \) and the redeployment process of labor resources ends at time \( t = \bar{t}(u) \), defined by the equality
\[ m_k(\bar{t}) = \bar{m}. \]  
(24)

3. The territorial distribution model of labor resources

3.1. Informal statement of the problem of placing employees in multi-agent interaction and any finite number of employers on the network with a given set of vacancies for each employee that he wants to receive

The following problem is considered: a network is defined on the flat. At some points in the network are employers and employees. For each edge two functions of transport costs are set, one for employees, another for employers, which indicate the cost of moving along this edge. All network points are connected, i.e. from any point on the network there is also a possibly non-unique way to any other point on the network. Each employee has an assortment consisting of several types of skills that he can offer. The cost of employee skills is equal to the sum of his cost and various expenses. For each skill type, the moving cost along the edges of the network is different. For each employer, a set of employees skills that he wants to acquire is defined. To meet their demand, the employer will need to consider several employees. The sum of the costs to meet their demand for each of the employers is equal to the sum of all the money spent by him on the purchase of employees, and the sum of all the expenses that he spends to find employees. We believe that each employer preliminarily calculates all possible options for meeting his demand and chooses the least costly option. To do this, we need to set an algorithm by which each employer chooses for himself the least costly way to meet his demand,
depending on the location of the workers. There is a problem of placing workers in this network, in accordance with some principle of optimality. In this paper, we find a compromise solution and Stackelberg equilibrium [16-19].

3.2. Task formalization of placing employers with their multi-agent interaction and any finite number of employees on the network with a specific set of employees for each employer that he wants to acquire

A network is on a flat torus, with transport costs functions defined on the edges of the network \((N, a, b)\), which consists of a finite nodes set (vertices), which will be denoted by the letters \(x_0, x_1, ...\) etc. An ordered pair of nodes \((x_i, x_j)\) is called the edge of the network. At some points in the network are \(m\) employers \(M_1, M_2, ..., M_m\). There are \(r\) types of skills \(k_1, k_2, ..., k_r\) each of which is delivered from \(K_1, K_2, ..., K_r\) of different workers to each of the employers. Each type of skill has its own value in the labor market \(q_1, q_2, ..., q_r\). For each edge of the network, the transport cost functions for moving the unit of skills \(a_1, a_2, ..., a_r\) are defined, which associates with each edge \((x_i, x_j)\) the non-negative number \(a_1(x_i, x_j), a_2(x_i, x_j), ..., a_r(x_i, x_j)\), for each type of skill. In the problem under consideration, \(n\) employers are located at points with coordinates \(x_1, x_2, ..., x_n\). For each employer, a set of skills \((k_1, k_2, ..., k_r)\) is defined, which he wants to acquire [20, 21]. For each edge of the network, a cost function for moving employers \(b\) along it is given, which associates with each edge \((x_i, x_j)\) a non-negative number \(b(x_i, x_j)\) for employers. Thus, the costs of each employer to meet their demand are equal to the costs sum of all the skills acquired by employees and the costs sum of all transfers between employees:

\[
C = \sum_{i=1}^{r} k_i + \sum_{i} b(x_i, y_i).
\]

Each employer tries to minimize the cost of meeting his demand. If at any step it turns out that the cost of buying the next skill from two or more employees is the same, the employer chooses the employee who is cheaper. Further, \(m\) workers can be located in free nodes of the network, each of which possesses each of \(k\) skills types. The cost of each employee skill is equal to the sum of his skills cost and expenses when delivering it to the employee:

\[
P_r = q_r + \sum_{h=1}^{r} a_h(x_i, x_j).
\]

Each employee is interested in increasing their profits, i.e. so that it is located at a point where the sum of skills relative to other points on the network is greater. Thus, the problem arises of placing workers at some points in the network in accordance with some principle of optimality. As a solution, a compromise solution and Stackelberg equilibrium are proposed [22-25].

3.3. Formalization of the algorithm for solving the problem

Stage 1. Finding the matrix of shortest paths between all pairs of points on the network using the Floyd’s algorithm. Separately, we select the weights matrix of the shortest paths between all employees.

Stage 2. Finding the winning functions of the players. Description of the algorithm for finding player winning functions.

Stage 3. Finding a compromise solution. Description of the algorithm for finding a compromise solution. Implementation of the algorithm in a program written in a programming language.

Given the payoff functions of the players, a compromise solution can be found. To quickly find a compromise solution, a program was also written [26-28].
4. Conclusion
In connection with the above, it becomes relevant to study the patterns of change in labor resources, assess the impact of workforce planning on the economic development of the region, develop and justify methods of labor management. Labor resources, high cultural and professional level of workers, their production potential are internal reserves for reforming the economy as a whole and its individual sector. The result of the study may be the concept of economic development based on analysis, taking into account the assessment and forecasting the state of labor resources.

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