A new bigravity model with exclusively positive branes

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\textbf{Abstract}

We propose a new “bigravity” model with two positive tension $AdS_4$ branes in $AdS_5$ bulk and no negative tension branes. The bounce of the “warp” factor mimics the effect of a negative brane and thus gives rise to an anomalously light graviton KK mode. This configuration satisfies the weak energy condition and has no ghost state. In addition, the extra polarization states of the massive graviton practically decouple and thus it does not contradict to Einsteinian gravity. However, the model has certain phenomenological difficulties associated with the presence of a negative cosmological constant on the branes.

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1 Introduction

Brane universe models in more than four dimensions have been extensively studied over the last three years because they provide novel ideas for resolutions of long standing problems to particle physics such as the Planck hierarchy one. Moreover, mechanisms to localize gravity on a brane have led to the realization that if extra dimensions exist, they need not be compact. Also “bulk” (transverse to the 3-brane space) physics turns out to be very interesting giving alternative possible explanations to other puzzles of particle physics, like the smallness of the neutrino masses, the neutrino oscillations, or the pattern of the SM fermion mass hierarchy (see for example and references therein). The phenomenological implication of these constructions are radical and the fact that these ideas can, in principle, be put to test in current and future experiments, makes them very attractive. A comprehensive account of these ideas can be found in .

In this paper, we present a model which belongs to a class of brane universe models that suggest that a part or all of gravitational interactions come from massive gravitons. In the first case (“bigravity”), gravitational interactions are the net effect of a massless graviton and a finite number of KK states that have sufficiently small mass (or/and coupling) so that there is no conflict with phenomenology. In the second case (“multigravity”), in the absence of a massless graviton, the normal Newtonian gravity at intermediate scales is reproduced by special properties of the lowest part of the KK tower (with mass close to zero). These models predict that at sufficiently large scales, which correspond to the Compton wavelength of the KK states involved, modifications to either the Newtonian coupling constant or the inverse square law will appear due to the Yukawa suppression of the KK states contribution. These modifications affect the CMB power spectrum and thus are, in principle, testable .

The prototype model of this class was the “bigravity” model where we considered a modification of the compact Randall-Sundrum model (RS1) with two positive tension flat branes (“+” branes) separated by one intermediate negative tension flat brane (“−” brane) in AdS bulk. The task of finding the KK spectrum reduces to a simple quantum mechanical problem. It is simple to see that the model (as every compact model) has a massless graviton that corresponds to the ground state of the system whose wavefunction follows the “warp” factor. It is also easy to see as in figure 1 that there should be a state with wavefunction antisymmetric with respect to the minimum of the “warp” factor, whose mass splitting from the massless graviton will be very small compared to the ones
Figure 1: The graviton (solid line), first (dashed line) and second (dotted line) KK states wavefunctions in the symmetric "+ " model. The wavefunctions are not smooth on the " " branes. The same pattern have also the " + " model wavefunctions with the position of the " " brane corresponding to the minimum of the warp factor. The wavefunctions are then smooth.

of the higher levels. Because the “warp” factor is exponential the difference of the mass behaviour of the first and the rest of the KK states is also exponential. This allowed for the construction of a “bigravity” model in which the remainder of the KK tower does not affect gravity beyond the millimeter bound. In this case gravity at large scales beyond the Compton wavelength of the first state will be modified in the sense that Newton’s constant will be reduced. In particular, it is reduced by one half in the symmetric configuration and it can be effectively switched off in the highly asymmetric case.

Independently, Gregory, Rubakov and Sibiryakov [9] (GRS) suggested a construction in which gravity is also modified at ultra-large scales. The GRS model modifies the decompactified Randall-Sundrum model [3] (RS2) by adding a "-" brane of half the tension of the "+" brane and requiring flat space to the right of the new "-" brane. This model does not have a normalizable 4D graviton but generates 4D gravity at intermediate distances due to a resonance-like behaviour [13, 14] of the wavefunctions of the KK states continuum. Gravity in this picture appears to be “quasi-localized” and this time it is modified at ultra-large scales in the sense that the inverse square law becomes inverse cube, i.e. five dimensional. Although the " + - + " and GRS models look quite different it was shown in [10] and [11] that these two models are the limiting cases of a more general " + - + + " multi-brane model that interpolates between the “bigravity” " + - + " model and the “quasi-localized” gravity
GRS model. The key element of both models that made “multigravity” possible was the bounce of the “warp” factor generated by the “−” branes.

However, one should be careful when dealing with models with ultralight massive KK states because it is known that the extra polarizations of the massive gravitons do not decouple in the limit of vanishing mass, the famous van Dam - Veltman - Zakharov discontinuity. This could make these models disagree with standard tests of General Relativity, as for example the bending of the light by the sun. Furthermore, the moduli (radions) associated with the perturbations of the “−” branes are necessarily physical ghost fields, therefore unacceptable. The latter problem is connected to the violation of the weaker energy condition on “−” branes sandwiched between “+” branes. In the GRS model this radion cancels the extra polarizations of the massive gravitons and gives the graviton propagator the correct tensorial structure at intermediate distances. However, the model has still an explicit ghost in the spectrum which reveals itself as scalar antigravity at cosmological scales. A mechanism of cancelling both the extra massive graviton polarizations and the radion field contribution was suggested in and involves some bulk dynamics which are necessary to stabilize the system, based on a scenario described in . This mechanism is however non-local in the extra dimension and because of this may not be very attractive.

In the present paper we will demonstrate that there is actually a way out of both these problems. We will use a two brane model with only “+” branes which was known to exhibit a bounce of the “warp” factor, and therefore is bound to have “bigravity” by general arguments presented in . This can be achieved if we consider two AdS4 branes in AdS5 bulk. Motivated by quasi-localization of gravity, Randall and Karch studied the non-compact case of a single “+” brane, which is a limiting case of the asymmetric “++” model. The weaker energy condition is satisfied so there is no ghost modulus in this setup. Furthermore, as was shown in , in AdS space it is possible to circumvent the van Dam - Veltman - Zakharov no go theorem about the non-decoupling of the massive graviton extra polarization states. The price we pay is that there is a remnant negative cosmological constant on the brane. This sets an horizon scale and unfortunately the Compton wavelength of the light state of the system lies exponentially far from this horizon. This does not change even if we consider a highly asymmetric version of this model. As a result, this “bigravity” model makes no predictions for observable deviations.

\footnote{One day after this paper had appeared in the hep-archives, ref. appeared, reaching the same conclusions by a different method.}
from Newtonian gravity at ultra-large distances. In addition, although theoretically there exist modifications of General Relativity at all scales due to the additional polarization states of the massive graviton, they are so highly suppressed that they are not observable. Additionally, let us note that recently models with only positive tension branes and similar “warp” factors (created in a different and dynamical way) were discussed in [31, 32].

2 The two positive brane model

The model consists of two 3-branes with tensions $V_1$ and $V_2$ respectively, in an $AdS_5$ space with five dimensional cosmological constant $\Lambda < 0$. The 5-th dimension has the geometry of an orbifold and the branes are located at its fixed points, i.e. $L_0 = 0$ and $L_1 = L$. Due to orbifolding, we can restrict ourselves to the region $0 \leq z \leq L$, imposing the suitable boundary conditions to our solutions. Firstly, we find a suitable vacuum solution. The action of this setup is:

$$S = \int d^4x \int_{-L}^L dz \sqrt{-G}\{-\Lambda + 2M^3 R\} + \sum_i \int_{z=L_i}^{z=L} d^4x V_i \sqrt{-\hat{G}^{(i)}}$$

where $\hat{G}^{(i)}_{\mu\nu}$ is the induced metric on the branes. The notation is the same as in Ref. [4]. The Einstein equations that arise from this action are:

$$R_{MN} - \frac{1}{2} G_{MN} R = -\frac{1}{4M^3} \left( \Lambda G_{MN} - \sum_i V_i \sqrt{-\hat{G}^{(i)}} \hat{G}_{\mu\nu}^{(i)} \delta^\mu_M \delta^\nu_N \delta(z - L_i) \right)$$

In order to find a specific form of the equations of motion we need to write a metric ansatz which will take into account the spacetime symmetries of the 3-brane. Since we would like not to restrict our model to flat solutions on the branes, we should make a choice which will let us interpolate between the maximally symmetric space-times in four dimensions, i.e. de-Sitter, Minkowski and Anti-de-Sitter. The metric ansatz [19] that accomplishes this is the following:

$$ds^2 = a^2(z)(-dt^2 + e^{2Ht}d\vec{x}^2) + b^2(z)dz^2$$

where $H$ is the “Hubble” parameter and is determined in terms of the brane tension $V_i$ and the bulk cosmological constant $\Lambda$ from Einstein’s equations. The $z$-dependent function $a(z)$ is the “warp” factor that is essential for the localization of gravity and also for producing the hierarchy between the two branes. In the case of flat brane solution, i.e. the effective
cosmological constant on the brane is zero, we have $H = 0$. On the other hand if we demand a de-Sitter solution on the brane, i.e. the effective cosmological constant on the branes is positive, we have $H^2 > 0$. In the case of Anti-de-Sitter solution, i.e. the effective cosmological constant on the branes is negative, we have $H^2 < 0$ and thus $H$ is imaginary. In order to get a physical interpretation of the latter case it is necessary to analytically continue the solution by a coordinate transformation of the form $t = -ix'_1$, $x_1 = it'$, $x_2 = x'_2$ and $x_3 = x'_3$. After this transformation the metric ansatz can be written in the following form:

$$ds^2 = a^2(z)(dx_1'^2 + e^{2Hz'_1}(-dt'^2 + dx_2'^2 + dx_3'^2)) + b^2(z)dz^2$$

Furthermore, in order to have a more compact notation for all cases of maximally symmetric spaces and simplify our calculations, it is useful to bring the metric ansatz in the form:

$$ds^2 = \frac{a^2(z)}{(1 - \frac{H^2 x^2}{4})^2} \eta_{\mu\nu}dx^\mu dx^\nu + b^2(z)dz^2$$

where $x^2 = \eta_{\mu\nu}x^\mu x^\nu$. It can be shown that the Ricci scalar for this metric is $R = -12H^2$. Thus this metric represents all maximally symmetric spaces: Minkowski for $H^2 = 0$, Anti-de-Sitter for $H^2 > 0$ and de-Sitter for $H^2 < 0$. From now on we shall choose the gauge $b(z) = 1$ where our coordinate system is Gaussian Normal. A straightforward calculation of the Einstein’s equations gives us the following differential equations for $a(z)$:

$$a^2(z) = H^2 - \frac{\Lambda}{24M^3}a^2(z)$$

$$a''(z) = -\sum_i \frac{V_i}{12M^3}a(z)\delta(z - L_i) - \frac{\Lambda}{24M^3}a(z)$$

By solving the above equations we find that the solution can be written in the form:

$$a(z) = \cosh(k|z|) + \frac{V_1k}{\Lambda} \sinh(k|z|)$$

with

$$|H^2| = \begin{cases} \frac{k^2}{\Lambda^2}(V_1^2k^2 - \Lambda^2) & , \frac{|\Lambda|}{k} < V_1 \text{ for } dS_4 \text{ branes} \\ 0 & , \frac{|\Lambda|}{k} = V_1 \text{ for flat branes} \\ \frac{k^2}{\Lambda^2}(\Lambda^2 - V_1^2k^2) & , \frac{|\Lambda|}{k} > V_1 \text{ for } AdS_4 \text{ branes} \end{cases}$$
where we have normalized $a(0) = 1$ and assumed $V_1 > 0$. Also we have defined $k \equiv \sqrt{\frac{\Lambda}{24 M^3}}$.

Additionally, in order to have this solution, the brane tensions $V_1, V_2$, the bulk cosmological constant $|\Lambda|$ and the position of the second brane $L$ must be related through the equation:

$$\tanh(kL) = k|\Lambda| \frac{V_1 + V_2}{|\Lambda|^2 + k^2 V_1 V_2} \quad (10)$$

Let us now restrict ourselves to the case of $AdS_4$ spacetime on the two branes which will turn out to be the most interesting. In this case the condition $\frac{|\Lambda|}{k} > V_1$ must hold. Hence, we can define $\tanh(kz_0) \equiv \frac{kV_1}{|\Lambda|}$ and write the solution in the form:

$$a(z) = \frac{\cosh(k(z_0 - |z|))}{\cosh(kz_0)} \quad (11)$$

from which it is clear that the “warp” factor has a minimum at $z = z_0$. From this point we can see the role of the $AdS_4$ on the branes, i.e. the role of the condition $\frac{|\Lambda|}{k} > V_1$. This condition allows us to have the bounce form of the “warp” factor (i.e. a minimum in the “warp” factor) allowing the second brane to have positive tension and give us, as we will see shortly, a phenomenology quite similar to the $''+--+''$ “bigravity” model \cite{8}. This can be easily seen from the eq.(10) which relates the brane tensions and the distance between the branes. From this equation we indeed see that by placing the second brane after the minimum of the “warp” factor we can make the tension of the second brane positive and thus both branes that appear in the model have positive tension avoiding the problems associated with negative tension branes. In fact it is clear that the present model mimics the characteristics of the $''+--+''$ “bigravity” model since what we effectively do is to reproduce the effect of the presence of a negative tension brane, i.e. the bounce form of the “warp” factor, with another mechanism allowing a negative four dimensional cosmological constant on the brane. Note that in the limit that $V_1 \to \frac{|\Lambda|}{k}$ (flat limit) the minimum of the “warp” factor tends to infinity and if we wish to have a brane at a finite point, it will necessarily have negative tension.

The relationship between the 4D effective fundamental scale $M_5$\footnote{the factor $2 M_5^2$ multiplies the four dimensional Ricci scalar in the Lagrangian after dimensionally reducing} and the five dimensional fundamental scale $M$ can be easily found by dimensional reduction to be:
\[ M_*^2 = \frac{M^3}{k \cosh^2(kz_0)} [kL + \sinh(kL) \cosh(k(L - 2z_0))] \] (12)

The above formula tells us that for finite \( L \) the compactification volume is finite and thus the zero mode is normalizable. In the case where we send the second brane to infinity, the compactification volume becomes infinite which indicates that the zero mode becomes non-normalizable. Note that \( M_* \) is not necessarily equal to \( M_{Pl} \) since as will see shortly, at least for a sector of the parameter space of our model, gravity is the result not only of the massless graviton but also of an ultralight KK state.

The “warp” factor renormalizes the physical scales of the theory as in [4]. Thus, all mass parameters \( m_0 \) on a brane placed at the point \( z \) are rescaled as

\[ m = a(z)m_0 \] (13)

Hence, assuming some kind of stabilization mechanism which fixes the positions of the branes, one can choose a distance between the two branes such that this rescaling leads to the creation of a desired mass hierarchy.

However, since we consider non-flat solutions on the branes, we have to make sure that the four dimensional effective cosmological constant does not contradict present experimental and observational bounds. Recent experimental data favour a positive cosmological constant, nevertheless since zero cosmological constant is not completely ruled out it can be argued that also a tiny negative cosmological constant can be acceptable within the experimental uncertainties. The effective cosmological constant on the two branes is:

\[ \Lambda_{4d} = -12H^2M_*^2 = -\frac{12}{\cosh^2(kz_0)} k^2M_*^2 \] (14)

From the above formula we can see that we can make the cosmological constant small enough \( |\Lambda_{4d}| \lesssim 10^{-120}M_{Pl}^4 \) if we choose large enough \( kz_0 \), i.e. \( kz_0 \gtrsim 135 \). This however will make observable deviations from Newtonian gravity at ultra-large scales impossible as we will see in the next section.

To determine the phenomenology of the model we need to know the KK spectrum that follows from the dimensional reduction. This is determined by considering the (linear) fluctuations of the metric around the vacuum solution that we found above. We can write the metric perturbation in the form:
\[ ds^2 = \left[ a(z)^2 g_{\mu\nu}^{AdS} + \frac{2}{M f^{3/2}} h_{\mu\nu}(x, z) \right] dx^\mu dx^\nu + dz^2 \] (15)

where \( g_{\mu\nu}^{AdS} \) is the vacuum solution. Here we have ignored the radion mode that could be used to stabilize the brane positions \( z = L_0 \) and \( z = L_1 \), assuming some stabilization mechanism. We expand the field \( h_{\mu\nu}(x, z) \) into graviton and KK plane waves:

\[ h_{\mu\nu}(x, z) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x) \Psi^{(n)}(z) \] (16)

where we demand \( (\nabla^2 + 2H^2 - m_n^2) h_{\mu\nu}^{(n)} = 0 \) and additionally \( \nabla^\alpha h_{\alpha\beta}^{(n)} = h_{\mu\nu}^{(n)} = 0 \). The function \( \Psi^{(n)}(z) \) will obey a second order differential equation which after a change of variables and a redefinition of the wavefunction reduces to an ordinary Schrödinger-type equation:

\[ \left\{ -\frac{1}{2} \partial^2_w + V(w) \right\} \hat{\Psi}^{(n)}(w) = \frac{m_n^2}{2} \hat{\Psi}^{(n)}(w) \] (17)

where the potential is given by:

\[
V(w) = - \frac{9\hat{k}^2}{8} + \frac{15\hat{k}^2}{8} \cos^2\left(\hat{k}(|w| - w_0)\right) - \frac{3\hat{k}}{2} \left[ \tanh(kz_0)\delta(w) + \frac{\sinh(k(L - z_0))}{\cosh(kz_0)}\delta(w - w_1) \right]
\] (18)

with \( \hat{k} \) defined as \( \hat{k} \equiv \frac{k}{\cosh(kz_0)} \). The new variables and the redefinition of the wavefunction are related with the old ones by:

\[ w \equiv \text{sgn}(z) \frac{2}{\hat{k}} \left[ \arctan\left( \frac{\tanh(\frac{|z| - z_0}{2})}{2} \right) + \arctan\left( \frac{\tanh(\frac{kz_0}{2})}{2} \right) \right] \] (19)

\[ \hat{\Psi}^{(n)}(w) \equiv \frac{1}{\sqrt{a(z)}} \Psi^{(n)}(z) \] (20)

Thus in terms of the new coordinates, the branes are at \( w_{L_0} = 0 \) and \( w_L \), with the minimum of the potential at \( w_0 = \frac{2}{\hat{k}} \arctan\left( \frac{kz_0}{2} \right) \). Also note that with this transformation the point \( z = \infty \) is mapped to the finite point \( w_\infty = \frac{2}{\hat{k}} \left[ \frac{\pi}{4} + \arctan\left( \frac{\tanh(\frac{kz_0}{2})}{2} \right) \right] \).
From now on we restrict ourselves to the symmetric configuration of the two branes with respect to the minimum $w_0$ (i.e. the first brane at $0$ and the second at $2w_0$), since the important characteristics of the model appear independently of the details of the configuration. Thus, the model has been reduced to a “quantum mechanical problem” with $\delta$-function potentials wells of the same weight and an extra smoothing term in-between (due to the AdS geometry). This gives the potential a double “volcano” form.

An interesting characteristic of this potential is that it always (for the compact cases i.e. $w_L < w_\infty$) gives rise to a normalizable massless zero mode, something that is expected since the volume of the extra dimension is finite. The zero mode wavefunction is given by:

$$\hat{\Psi}^{(0)}(w) = \frac{A}{\left[\cos(\tilde{k}(w_0 - |w|))\right]^{3/2}}$$

(21)

where the normalization factor $A$ is determined by the requirement $\int_{-w_L}^{w_L} dw \left[\hat{\Psi}^{(0)}(w)\right]^2 = 1$, chosen so that we get the standard form of the Fierz-Pauli Lagrangian.

The form of the zero mode resembles the one of the zero mode of the $'' + - +''$ model, i.e. it has a bounce form with the turning at $w_0$ (see figure 1). In the case of the $'' + - +''$ the cause for this was the presence of the $''-''$ brane. In the present model it turns out that by considering AdS spacetime on the branes we get the same effect.

In the case that we send the second brane to infinity (i.e. $w \to w_\infty$) the zero mode fails to be normalizable due to singularity of the wavefunction exactly at that point. This can be also seen from eq.(12) which implies that at this limit $M_*$ becomes infinite (i.e. the coupling of the zero mode becomes zero). Thus in this limit the model has no zero mode and all gravitational interactions must be produced by the ultralight first KK mode. The spectrum in this case was discussed by Randall and Karch at [27].

Considering the Schrödinger equation for $m \neq 0$ we can determine the wavefunctions of the KK tower. It turns out that the differential equation can be brought to a hypergeometric form, and hence the general solution is given in terms two hypergeometric functions:

$$\hat{\Psi}^{(n)} = \cos^{5/2}(\tilde{k}(|w| - w_0)) \left[ C_1 F(\tilde{a}_n, \tilde{b}_n, \frac{1}{2}; \sin^2(\tilde{k}(|w| - w_0))) \\
+ C_2 |\sin(\tilde{k}(|w| - w_0))| F(\tilde{a}_n + \frac{1}{2}, \tilde{b}_n + \frac{1}{2}, \frac{3}{2}; \sin^2(\tilde{k}(|w| - w_0))) \right]$$

(22)
where

\[
\tilde{a}_n = \frac{5}{4} + \frac{1}{2} \sqrt{\left(\frac{m_n}{\tilde{k}}\right)^2 + \frac{9}{4}} \\
\tilde{b}_n = \frac{5}{4} - \frac{1}{2} \sqrt{\left(\frac{m_n}{\tilde{k}}\right)^2 + \frac{9}{4}}
\]  

(23)

The boundary conditions (i.e. the jump of the wave function at the points \(w = 0, w_L\)) result in a \(2 \times 2\) homogeneous linear system which, in order to have a non-trivial solution, leads to the vanishing determinant. In the symmetric configuration which we consider, this procedure can be simplified by considering even and odd functions with respect to the minimum of the potential \(w_0\).

In more detail, the odd eigenfunctions obeying the b.c. \(\hat{\Psi}^{(n)}(w_0) = 0\) will have \(C_1 = 0\) and thus the form:

\[
\hat{\Psi}^{(n)} = C_2 \cos^{5/2}(\tilde{k}(|w| - w_0))|\sin(\tilde{k}(|w| - w_0))| F(\tilde{a}_n, \tilde{b}_n + \frac{1}{2}, \frac{3}{2}; \sin^2(\tilde{k}(|w| - w_0)))
\]  

(24)

On the other hand, the even eigenfunctions obeying the b.c. \(\hat{\Psi}^{(n)\prime}(w_0) = 0\) will have \(C_2 = 0\) and thus the form:

\[
\hat{\Psi}^{(n)} = C_1 \cos^{5/2}(\tilde{k}(|w| - w_0)) F(\tilde{a}_n, \tilde{b}_n, \frac{1}{2}; \sin^2(\tilde{k}(|w| - w_0)))
\]  

(25)

The remaining boundary condition is given by:

\[
\hat{\Psi}^{(n)\prime}(0) + \frac{3k}{2} \tanh(kz_0) \hat{\Psi}^{(n)}(0) = 0
\]  

(26)

and determines the mass spectrum of the KK states. From this quantization condition we get that the KK spectrum has a special first mode similar to the one of the \(^{"+}^{+}^{"}+\) “bigravity” model. For \(kz_0 \gtrsim 5\) the mass of the first mode is given by the approximate relation:

\[
m_1 = 4\sqrt{3} k e^{-2kz_0}
\]  

(27)

In contrast, the masses of the next levels, if we put together the results for even and odd wavefunctions, are given by the formula:

\[
m_{n+1} = 2\sqrt{n(n+3)} k e^{-kz_0}
\]  

(28)
with \( n = 1, 2, \ldots \).

We note that the first KK state has a different scaling law with respect to the position of the minimum of the “warp” factor compared to the rest of the KK tower, since it scales as \( e^{-2kz_0} \) while the rest of the tower scales as \( e^{-kz_0} \). Thus the first KK state is generally much lighter than the rest of the tower. It is clear that this mass spectrum resembles the one of the “+--” “bigravity” model. The deeper reason for this is again the common form of the “warp” factor. In both cases the “warp” factor has a minimum due to its “bounce” form. The graviton wave function follows the form of the “warp” factor, i.e. it is symmetric with respect to \( w_0 \), while the wavefunction of the first KK state is antisymmetric in respect to \( w_0 \) (see figure 1). The absolute values of the two wavefunctions are almost identical in all regions except near \( w_0 \) (the symmetric is nonzero while the antisymmetric is zero at \( w_0 \)). The graviton wavefunction is suppressed by the factor \( \frac{1}{\cosh^2(kz_0)} \) at \( w_0 \) which brings it’s value close to zero for reasonable values of \( kz_0 \). Thus, the mass difference which is determined by the wavefunction near \( w_0 \) is expected to be generally very small, a fact which formally appears as the extra suppression factor \( e^{-kz_0} \) in the formula of \( m_1 \) in comparison with the rest of the KK tower.

In the case that we consider an asymmetric brane configuration, for example \( w_L > 2w_0 \) the spectrum is effectively independent of the position of the second brane \( w_L \) (considering \( kz_0 \gtrsim 5 \)). Thus, even in the case that we place the second brane at \( w_\infty \), i.e. the point which corresponds to infinity in the \( z \)-coordinates, the spectrum is given essentially by the same formulas. In the case that the second brane is placed at \( w_0 < w_L < 2w_0 \), some dependence on the position of the second brane (i.e. dependence on the scale hierarchy between the branes) is present. Nevertheless, the main characteristics of the spectrum remain the same, i.e. the first KK state is special and always much lighter than the others. In conclusion, the key parameter which determines the spectrum is the position of the minimum of the “warp” factor.

Returning to our wavefunction solutions, we should note that for each eigenfunction the normalization constants \( C_1 \) and \( C_2 \) can be determined by the normalization condition

\[
\int_{w_0}^{w_L} dw \left[ \Psi^{(n)}(w) \right]^2 = 1 \]

which is such that we get the standard form of the Fierz-Pauli Lagrangian for the KK states. Knowing the normalization of the wavefunctions, it is straightforward to calculate the strength of the interaction of the KK states with the SM fields confined on the brane\(^6\). This can be calculated by expanding the minimal gravitational

\(^6\)In the symmetric configuration it does not make any difference which brane is our universe.
coupling of the SM Lagrangian $\int d^4x \sqrt{-\hat{G}} \mathcal{L} \left( \hat{G}, SMfields \right)$ with respect to the metric. In this way we get:

$$\mathcal{L}_{int} = -\frac{1}{M^{3/2}} \sum_{n \geq 0} \hat{\Psi}^{(n)}(w_{brane}) h^{(n)}_{\mu\nu}(x) T_{\mu\nu}(x) =
$$

$$= -\frac{A}{M^{3/2}} h^{(0)}_{\mu\nu}(x) T_{\mu\nu}(x) - \sum_{n > 0} \frac{\hat{\Psi}^{(n)}(w_{brane})}{M^{3/2}} h^{(n)}_{\mu\nu}(x) T_{\mu\nu}(x) \quad (29)$$

with $T_{\mu\nu}$ the energy momentum tensor of the SM Lagrangian. Thus, the coupling of the zero and the first KK mode to matter are respectively:

$$\frac{1}{c_0} = \frac{A}{M^{3/2}} = \frac{1}{M_*} \quad (30)$$

$$\frac{1}{c_1} = \frac{\hat{\Psi}^{(1)}(w_{brane})}{M^{3/2}} \approx \frac{1}{M_*} \quad (31)$$

where $A$ is the zero mode normalization constant which turns out to be $\frac{M^{3/2}}{M_*}$. We should also note that the couplings of the rest of the KK states are much smaller and scale as $e^{-kz_0}$.

Exploiting the different mass scaling of the first KK relative to the rest we can ask whether it is possible to realize a “bigravity” scenario similar to that in “$++$” “bigravity” model. In that model by appropriately choosing the position of the minimum of the “warp” factor, it was possible to make the first KK state have mass such that the corresponding wavelength is of the order of the cosmological scale that gravity has been tested and at the same time have the rest of the KK tower wavelengths below 1mm (so that there is no conflict with Cavendish bounds). In this scenario the gravitational interactions are due to the net effect of the massless graviton and the first ultralight KK state. From eq.(30), (31) it can be understood that in the symmetric configuration the massless graviton and the special KK state contribute by the same amount to the gravitational interactions. In other words:

$$\frac{1}{M_{Pl}^2} = \frac{1}{M_*^2} + \frac{1}{M_*^2} \quad \Rightarrow \quad M_{Pl} = \frac{M_*}{\sqrt{2}} \quad (32)$$

In the present model, the fact that the effective four dimensional cosmological constant should be set very close to zero, requires that the “warp” factor is constrained by $kz_0 \geq 135$
and thus, in this case, the spectrum of the KK states will be very dense (tending to continuum) bringing more states close to zero mass. The KK states that have masses which correspond to wavelengths larger than 1\(\text{mm}\) have sufficiently small coupling so that there is no conflict with phenomenology (the situation is exactly similar to the RS2 case where the coupling of the KK states is proportional to their mass and thus it is decreasing for lighter KK states). The fact that the spectrum tends to a continuum shadows the special role of the first KK state. Moreover, it is interesting to note that at the limit where the minimum of the “warp” factor is sent to infinity \((w_\infty)\) the special behaviour of the first KK persists and does not catch the other levels (by changing its scaling law) as was the case in [11]. This means that the limit \(w \to w_\infty\) will indeed be identical to two RS2, but on the other hand it is interesting to note that what we call graviton in the RS2 limit is actually the combination of a massless graviton and the “massless” limit of the special first KK state. This “massless” limit exists as we will see in the next section and ensures that locality is respected by the model, since physics on the brane does not get affected from the boundary condition at infinity.

### 3 Discussion and conclusions

The fact that we have a ultralight graviton in our spectrum is at first sight worrying because it is well known that in the flat space the tensor structure of the propagators of the massless and of the massive graviton are different [15] and that there is no smooth limit between them when \(m \to 0\). The bending of the light by the sun agrees with the prediction of the Einstein theory to 1% accuracy. This is sufficient to rule out any scenario which a significant component of gravity is due to a massive graviton, however light its mass could be. However, as was shown in [28], the situation in \(AdS\) space is quite different. There it was shown that if we could arrange \(\frac{m_1}{H} \lesssim 0.1\) there is no discrepancy with standard tests of Einsteinian gravity as for example the bending of the light by the sun.

In the particular model we have at hand, it is \(\frac{m_1}{H} \sim e^{-kz_0}\) so we can easily accommodate the above bound. Then, the Euclidean propagator (in configuration space) of the massive KK states for relatively large \(z_0\) will be given by:

\[
G^m_{\mu \nu; \mu' \nu'}(x, y) = \frac{1}{4\pi^2 \mu^2} (\delta_{\mu \nu} \delta_{\nu \mu'} + \delta_{\mu \nu'} \delta_{\nu \mu} - \left(1 - \frac{1}{6} e^{-2kz_0}\right) \delta_{\mu \nu} \delta_{\mu' \nu'})
\]  

where \(\mu\) is the geodesic distance between two points. In the above, we have omitted terms...
that do not contribute when integrated with a conserved $T_{\mu\nu}$. For $kz_0 \gtrsim 2.3$ there is no problem with the bending of light. However, if our aim is to see modifications of gravity at ultra-large distances, this is impossible because the Compton wavelength of our ultralight graviton will be $e^{kz_0}$ times bigger than the horizon $H^{-1}$ of the $AdS_4$ space on our brane due to equations (27), (28). The “Hubble” parameter follows $m_2$ rather than $m_1$.

What happens if one takes an asymmetric version of this model where $L > 2z_0$ is that the spectrum does not get significantly modified so we are effectively in the same situation. In the case where $z_0 < L < 2z_0$ the spectrum will behave like the one of the "$+ - +$" model. Then for $\omega \ll 1$ we will have $m_1 \sim \omega e^{-2kz} M \sim e^{-2kx} M_{Pl}$, $H \sim e^{-kz_0} M \sim e^{-kx} M_{Pl}$ where $M \approx M_{Pl}/\omega$ is the fundamental scale, $\omega \approx e^{-(2z_0 - L)}$ is the “warp” factor and $x = L - z_0$. Again, the ultralight graviton is hiding well beyond the $AdS$ horizon. However, the coupling of the remaining of the KK tower to matter will be different than the symmetric case and one may have different corrections to Newton’s law on the left and right branes.

In summary, in this paper we presented a new “bigravity” model with two $AdS_4$ branes in $AdS_5$ bulk which has a lot of similarities with the "$+ - +$ “bigravity” model. The fact that we have no "$-"$ branes removes the ghost state problem and furthermore, due to some amazing property of the $AdS$ space we are able to circumvent the van Dam - Veltman - Zakharov discontinuity of the graviton propagator. This makes the model compatible with the predictions of General Relativity in the small graviton mass limit since the extra degrees of freedom of the massive graviton practically decouple. However, the presence of the $AdS$ horizon prevents the modifications of gravity at large distances becoming observable. In the future it would be interesting to explore similar models where it would be possible to obtain observable modifications of gravity at large (cosmological) scales. In particular, it would be interesting to see if the above characteristics of this model persist when we add matter density on the branes. Then it is interesting to examine if/how the ultralight graviton is going to reveal itself in the cosmological solutions discussed by [25].

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**Addendum:** One day after this work had appeared in the hep-archives, ref. [33] appeared, being the published version of [27]. In that paper the non-compact case of a single "+" brane was studied.

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