Simulation of Two-Dimensional Unsteady Non-Newtonian Nonisothermal Fluid Flow in Micro Channels

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Abstract. This paper presents a numerical model for simulation of two-dimensional moving-interface flow of non-Newtonian fluid under non-isothermal conditions. The level-set method is used to capture the moving interface implicitly. The finite volume method is used to solve the governing equations and the level-set equations. The development of the moving interface is simulated successfully. The velocity and temperature profiles are also investigated.

1. Introduction
The transient flow is often found in the micro-fluidics devices. Compared to the fluid in macro channels, the fluid in micro channels may exhibit significant different phenomenon. For example, some micro-scale effects such as surface tension, wall roughness, etc. may become significant, which are often neglected in the macro-scale analysis [1]. It is therefore necessary to study the fluid behavior in the micro-scale.

This paper presents a numerical model for simulation of the two-dimensional transient flow in micro channels taking into the non-Newtonian behavior and non-isothermal condition.

2. Modeling
2.1. Schematic of the problem
Figure 1 shows the schematic of the problem. The entering fluid used in this study is Polyoxymethylene (POM), of which the density is 1154 kg/m³. The viscosity of air is artificially increased to 0.01 Pa.s to enhance the numerical stability.
2.2. Governing equations

The motion of fluid is governed by the governing equations. The effect of gravity and surface tension are neglected. The governing equations can be expressed using tensor as:

$$\nabla \cdot \vec{v} = 0$$ (1)

$$\rho \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \nabla \cdot (\eta \dot{\gamma})$$ (2)

$$\dot{\gamma} = \nabla \vec{v} + (\nabla \vec{v})^T$$ (3)

$$\rho C_v \left( \frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \eta \dot{\gamma}$$ (4)

$$\dot{\gamma} = \sqrt{\frac{1}{2} \dot{\gamma} : \dot{\gamma}}$$ (5)

2.3. Level-set equations

The level-set method is used to capture the interface. The level-set transport equation and the re-initialization equation are given as:

$$\frac{\partial \phi}{\partial t} + \vec{v} \cdot \nabla \phi = 0$$ (6)

$$\frac{\partial \xi}{\partial t} = \text{sign}(\phi)(1 - |\nabla \xi|)$$ (7)

$$\xi(\vec{x}, 0) = \phi(\vec{x}, t + \Delta t)$$ (8)

2.4. Density and viscosity

The viscosity of the entering non-Newtonian fluid is modeled using the Modified Arrhenius Power Law (MAPL):

$$\eta = \frac{\eta^{(T)}}{1 + \left( \frac{\eta^{(T)}}{K} \right)^{\dot{\gamma}}}$$
\[ \eta_s = Be^x, \]

The density and viscosity are approximated using:

\[ \alpha = (1 - h)\alpha_1 + h\alpha_2 \]

where, \( \alpha_1 \) and \( \alpha_2 \) can stand for densities and viscosities of each phase of the fluids. \( h \) is the Heaviside function.

2.5. Initial and boundary conditions

Initially, velocity and temperature of the entering fluid are specified at the inlet. The flow front position is also given. For the entering fluid occupied region, the no-slip boundary condition is imposed at the wall and free-slip boundary condition at axisymmetry. However, for the air occupied region, the free-slip boundary condition is imposed. At the outlet, the outflow condition is used. Moreover, the wall temperature is set as a constant.

3. Solution procedure

The governing equations and the level set equations are special cases of the following general equation:

\[ \frac{\partial}{\partial t} (\rho \phi) + \frac{\partial}{\partial x_j} (\rho u_j \phi) = \frac{\partial}{\partial x_i} (\Gamma \frac{\partial \phi}{\partial x_i}) + S_\phi \]

To solve this general equation, the finite volume method [3] is used. The SIMPLER algorithm [3] is used to deal with the coupling between the velocity and pressure. The diffusion-convection effect in the momentum equation is modeled using the Power-Law scheme [3]. The fully implicit scheme is used to discretize the temporal term. The level set equations are discretized using the upwind scheme. The resulting algebraic equations are solved using Tri-Diagonal Matrix Algorithm (TDMA).

4. Results and discussion

Figure 2 shows the simulated development of the moving interface and Figure 3 shows the fountain effect for viscous fluid.
Figures 4-6 show the temperature contours at different times, i.e., 0.75ms, 1.125ms and 1.5ms, where the temperature at the wall is 440K and the temperature of the entering fluid is 490K. It can be seen that the temperature of the entering fluid is not affected by the temperature at the wall due to the relatively high inlet velocity. This indicates that the non-isothermal condition does not contribute to the change in the velocity field although there is a change in the viscosity of the non-Newtonian fluid.

Figure 7 shows the comparison of the simulated development of the flow front between isothermal and non-isothermal conditions. It can be seen that there is little difference in the results between the isothermal condition and non-isothermal condition, except near the wall. Near the wall, the viscosity is higher in the non-isothermal condition and this results in lower calculated velocity.
When the wall temperature is set at a lower temperature of 400K while the inlet velocity is kept the same, i.e. 1.0 m/s, the simulation result also does not change significantly. Therefore, it can be concluded that if the inlet velocity is very high relatively to the part size, the temperature of the wall has no significant effect on the velocity field. This result is true only when the temperature of the wall is kept higher than the glass transition temperature of the entering fluid.
The temperature profiles at different cross sections at 1.5ms are shown in Fig. 8. It can be seen that the change in temperature along the y-direction is minimal. The development of temperature at the cross section x=0.175mm is shown in Fig. 9. The maximum temperature is slightly higher than the steps wall temperature, which indicates that the viscous dissipation is not much significant in this case.

5. Conclusion
The proposed model is valid in the study of the transient flow in micro channels. It can be applied to both Newtonian and non-Newtonian fluid flow under the non-isothermal condition. The development of the moving interface is captured successfully, and the fountain effect of viscous fluid is also shown clearly. Moreover, the inlet velocity has significant effect on the velocity field of the non-Newtonian fluid, i.e., higher inlet velocity will reduce the effect of the non-isothermal condition.

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