The quaternion spaces can be used to describe the property of electromagnetic field and gravitational field. In the quaternion space, some coordinate transformations can be deduced from the feature of quaternions, including Lorentz transformation and Galilean transformation etc., when the coordinate system is transformed into others. And some coordinate transformations with variable speed of light can be obtained in the electromagnetic field and gravitational field.

PACS numbers: 03.30.+p ; 03.50.De ; 02.20.Hj .

Keywords: quaternion; Lorentz transformation; Galilean transformation.

I. INTRODUCTION

The quaternion was invented by W. R. Hamilton [1]. He spent a lot of time on the theoretical analysis of the quaternions, and tried to apply quaternions to describe different physical phenomena. Later, J. C. Maxwell in the electromagnetic theory [2] applied the quaternion to describe the properties of the electromagnetic field [3].

In the late 20th century, the quaternions have had a revival due to their utility in describing spatial rotations primarily. The quaternion representation of rotations are more compact and faster to compute than the representations by matrices. And they find uses in both theoretical and applied physics, in particular for calculations involving three-dimensional rotations, such as in 3D computer graphics, control theory, signal processing, and orbital mechanics etc., although they have been superseded in many applications by vectors and matrices. Moreover, there are other quaternion theories, such as quaternion quantum mechanics [4], quaternion optics, quaternion relativity theory [5], etc.

With the property of quaternions, we obtain Galilean transformation and Lorentz transformation [6], and some other transformations of coordinate system, including the transformations with the variable speed of light. In the quaternion spaces, the speed of light will be varied with the electromagnetic field potential as well as gravitational field potential.

II. TRANSFORMATIONS IN THE QUATERNION SPACE

The electromagnetic theory can be described with the quaternions. In the treatise on electromagnetic theory, the algebra of quaternion was first used by J. C. Maxwell to describe the various properties of the electromagnetic field. At present, the gravitational field can be described by the algebra of quaternions as well.

A. Coordinate transformation

In the quaternion space, we have the radius vector \( \mathbf{R} = (r_0, r_1, r_2, r_3) \), and the basis vector \( \mathbb{E} = (1, i_1, i_2, i_3) \).

\[
\mathbf{R} = r_0 + i_1 r_1 + i_2 r_2 + i_3 r_3 \tag{1}
\]

where, \( r_0 = v_0 t \); \( t \) denotes the time; \( v_0 \) is the speed of gravitational intermediate boson, which is the first part of the photon.

The physical quantity \( \mathbb{D}(d_0, d_1, d_2, d_3) \) in quaternion space is defined as

\[
\mathbb{D} = d_0 + i_1 d_1 + i_2 d_2 + i_3 d_3 \tag{2}
\]

When we transform one form of the coordinate system into another one, the physical quantity \( \mathbb{D} \) is transformed into \( \mathbb{D}'(d'_0, d'_1, d'_2, d'_3) \).

\[
\mathbb{D}' = \mathbb{K}^* \circ \mathbb{D} \circ \mathbb{K} \tag{3}
\]

where, \( \mathbb{K} \) is the quaternion, and \( \mathbb{K}^* \circ \mathbb{K} = 1 \); * denotes the conjugate of quaternion.

In case of the coordinate system is transforming, the quaternions in the above satisfy the relation as follows.

\[
\mathbb{D}^* \circ \mathbb{D} = (\mathbb{D}')^* \circ \mathbb{D}' \tag{4}
\]

When the scalar part of quaternion physical quantity \( \mathbb{D} \) does not take part in the coordinate transformations, the scalar part \( d_0 \) remains the same.

\[
d_0 = d'_0 \tag{5}
\]

From Eqs. (4) and (5), we can obtain many kinds of coordinate transformations in the quaternion space.

B. Galilean transformation

In the quaternion space, the velocity \( \mathbb{V}(v_0, v_1, v_2, v_3) \) is

\[
\mathbb{V} = v_0 + i_1 v_1 + i_2 v_2 + i_3 v_3 \tag{6}
\]

When the coordinate system is transformed into another one, we have a radius vector \( \mathbb{R}'(r'_0, r'_1, r'_2, r'_3) \) and velocity \( \mathbb{V}'(v'_0, v'_1, v'_2, v'_3) \) respectively from Eq. (3).
From Eqs. (1), (3), and (6), we have
\[ r_0 = r'_0 \]  
and \( v_0 = v'_0 \) \( (7) \) \( (8) \) and then \( (j = 1, 2, 3) \)
\[ t_0 = t'_0 \]  
\[ \Sigma(r_j)^2 = \Sigma(r'_j)^2 \]  
(9) \( (10) \)
The above means that if we emphasize especially the important of the radius vector Eq. (1) and the velocity Eq. (6), we will obtain the Galilean transformation of the coordinate system from Eqs. (1), (3), and (6).

C. Lorentz transformation

The physical quantity \( \mathbb{D}(d_0, d_1, d_2, d_3) \) in quaternion space is defined as
\[ \mathbb{D} = \mathbb{R} \circ \mathbb{R} \]  
\[ = d_0 + i_1 d_1 + i_2 d_2 + i_3 d_3 \]  
(11)
In the above equation, the scalar part remains the same during the quaternion coordinate system is transforming. From Eq. (5) and the above, we have
\[ (r_0)^2 - \Sigma(r_j)^2 = (r'_0)^2 - \Sigma(r'_j)^2 \]  
(12)
The above means the spacetime interval \( d_0 \) remains unchanged, when the coordinate system rotates. From Eqs. (8) and (12), we obtain the Lorentz transformation of the coordinate system.
\[ (r_0)^2 - \Sigma(r_j)^2 = (r'_0)^2 - \Sigma(r'_j)^2 \]  
\[ v_0 = v'_0 \]  
(13)
The above means that the Galilean transformation and Lorentz transformation of the coordinates depend on the choosing from different combinations of the basic physical quantities. When \( r_0^2 \gg \Sigma(r_j)^2 \) and \( (r'_0)^2 \gg \Sigma(r'_j)^2 \), we have \( r_0^2 \approx (r'_0)^2 \). And then Eq. (12) is reduced to Eq. (7).

D. Variable speed of light

The physical quantity \( \mathbb{Q}(q_0, q_1, q_2, q_3) \) in quaternion space is defined as
\[ \mathbb{Q} = \mathbb{V} \circ \mathbb{V} \]  
\[ = q_0 + i_1 q_1 + i_2 q_2 + i_3 q_3 \]  
(13)

| TABLE I: The quaternion multiplication table. |
|------------------|------------------|------------------|------------------|
| 1                | \( i_1 \)        | \( i_2 \)        | \( i_3 \)        |
| \( i_1 \)        | \( i_1 \)        | \( i_2 \)        | \( i_3 \)        |
| \( i_2 \)        | \( i_2 \)        | \( -i_3 \)       | \( -i_2 \)       |
| \( i_3 \)        | \( i_3 \)        | \( i_2 \)        | \( -i_1 \)       |

When the coordinate system is transformed into other one, we have one physical quantity \( \mathbb{Q}'(q'_0, q'_1, q'_2, q'_3) \) from Eq. (3). In the above equation, the scalar part remains the same during the quaternion coordinate system is transforming. From Eq. (5) and the above, we have
\[ (v_0)^2 - \Sigma(v_j)^2 = (v'_0)^2 - \Sigma(v'_j)^2 \]  
(14)
The above equation represents the physical quantity \( q_0 \) remains unchanged when the coordinate system rotates. From Eqs. (7) and (14), we obtain the transformation A with variable speed of light.
\[ r_0 = r'_0 \]  
\[ (v_0)^2 - \Sigma(v_j)^2 = (v'_0)^2 - \Sigma(v'_j)^2 \]  
From Eqs. (12) and (14), we obtain the transformation B with variable speed of light.\[ (v_0)^2 - \Sigma(v_j)^2 = (v'_0)^2 - \Sigma(v'_j)^2 \]  
\[ (v_0)^2 - \Sigma(v_j)^2 = (v'_0)^2 - \Sigma(v'_j)^2 \]  
When \( v_0^2 \gg \Sigma(v_j)^2 \) and \( (v'_0)^2 \gg \Sigma(v'_j)^2 \), we obtain \( v_0^2 \approx (v'_0)^2 \). And then Eq. (14) is reduced to Eq. (8). In a similar way, the quantity \( \mathbb{D} \) and \( \mathbb{Q} \) can be defined as other kinds of functions of radius vector \( \mathbb{R} \) or velocity \( \mathbb{V} \), such as \( \mathbb{D} = \mathbb{R} \circ \mathbb{R} \circ \mathbb{R} \circ \mathbb{R} \) or \( \mathbb{Q} = \mathbb{V} \circ \mathbb{V} \circ \mathbb{V} \), etc. And then we have some kinds of complicated coordinate transformations in the quaternion spaces.

III. TRANSFORMATIONS IN THE OCTONION SPACE

The gravitational field and electromagnetic field both can be demonstrated by quaternions, but they are quite different from each other indeed. We add another four-dimensional basis vector to the ordinary four-dimensional basis vector to include the feature of the gravitational and electromagnetic fields [7].

| TABLE II: Some coordinate transformations in the quaternion space. |
|------------------|------------------|------------------|
| transformations  | radius vector & velocity |
| Galilean         | \( r_0 = r'_0 \), \( \Sigma(r_j)^2 = \Sigma(r'_j)^2 \) |
|                  | \( v_0 = v'_0 \) |
| Lorentz          | \( (r_0)^2 - \Sigma(r_j)^2 = (r'_0)^2 - \Sigma(r'_j)^2 \) |
|                  | \( v_0 = v'_0 \) |
| transformation A | \( r_0 = r'_0 \), \( \Sigma(r_j)^2 = \Sigma(r'_j)^2 \) |
|                  | \( (v_0)^2 - \Sigma(v_j)^2 = (v'_0)^2 - \Sigma(v'_j)^2 \) |
| transformation B | \( (r_0)^2 - \Sigma(r_j)^2 = (r'_0)^2 - \Sigma(r'_j)^2 \) |
|                  | \( (v_0)^2 - \Sigma(v_j)^2 = (v'_0)^2 - \Sigma(v'_j)^2 \) |
| others           | \( \mathbb{D} = \mathbb{R} \circ \mathbb{R} \circ \mathbb{R} \circ \mathbb{R} \), etc. |
|                  | \( \mathbb{Q} = \mathbb{V} \circ \mathbb{V} \circ \mathbb{V} \), etc. |
A. Coordinate transformation

The basis vector of the quaternion space for the gravitational field is $E_g = (1, i_1, i_2, i_3)$, and that for the electromagnetic field is $E_e = (I_0, I_1, I_2, I_3)$. The $E_e$ is independent of the $E_g$, with $E_g = (1, i_1, i_2, i_3) \circ I_0$. The basis vectors $E_g$ and $E_e$ can be combined together to become the basis vector $E$ of the octonion space.

$$E = E_g + E_e = (1, i_1, i_2, i_3, I_0, I_1, I_2, I_3)$$

(15)

The radius vector $R(r_0, r_1, r_2, r_3, R_0, R_1, R_2, R_3)$ in the octonion space is

$$R = r_0 + i_1 r_1 + i_2 r_2 + i_3 r_3 + I_0 R_0 + I_1 R_1 + I_2 R_2 + I_3 R_3$$

(16)

and the velocity $V(v_0, v_1, v_2, v_3, V_0, V_1, V_2, V_3)$ is

$$V = v_0 + i_1 v_1 + i_2 v_2 + i_3 v_3 + I_0 V_0 + I_1 V_1 + I_2 V_2 + I_3 V_3$$

(17)

where, $R_0 = V_0 T$; $T$ is one time-like quantity; $V_0$ is the speed of electromagnetic intermediate boson, which is the second part of the photon.

When the coordinate system is transformed into other one, the octonion physical quantity $D$ will be transformed into $D' (d'_0, d'_1, d'_2, d'_3, D'_0, D'_1, D'_2, D'_3)$.

$$D' = K^* \circ D \circ K$$

(18)

where, $K$ is the octonion, and $K^* \circ K = 1$; $*$ denotes the conjugate of octonion.

In case of the coordinate system is transforming, the octonions in the above satisfy the relation as follows.

$$D^* \circ D = (D')^* \circ D'$$

(19)

When the scalar part of octonion does not take part in the coordinate transformation, the $d_0$ remains the same.

$$d_0 = d'_0$$

(20)

From Eqs.(19) and (20), we can obtain many kinds of coordinate transformations in the octonion space.

B. Galileean transformation

When the coordinate system is rotated, we have one radius vector $R'(r'_0, r'_1, r'_2, r'_3, R'_0, R'_1, R'_2, R'_3)$ and velocity $V'(v'_0, v'_1, v'_2, v'_3, V'_0, V'_1, V'_2, V'_3)$ respectively from Eq.(18).

From Eqs.(16), (17), and (18), we have

$$r_0 = r'_0$$

(21)

$$v_0 = v'_0$$

(22)

and then ($i = 0, 1, 2, 3$)

$$t_0 = t'_0$$

(23)

$$\Sigma(r_j)^2 + \Sigma(R_i)^2 = (r'_j)^2 + \Sigma(R'_i)^2$$

(24)

The above means that if we emphasize especially the important of the radius vector Eq.(16) and the velocity Eq.(17), we obtain the Galilean transformation of the coordinate system from Eqs.(16), (17), and (18).

The above states also that the $r_0$ remains unchanged when the coordinate system rotates, but the $R_0$ keeps changed as a vectorial component. When $R_i \approx R'_i \approx 0$, Eq.(24) is reduced to Eq.(10).

C. Lorentz transformation

The physical quantity $D(d_0, d_1, d_2, d_3, D_0, D_1, D_2, D_3)$ in the octonion space is defined as

$$D = R \circ \hat{R}$$

$$= d_0 + i_1 d_1 + i_2 d_2 + i_3 d_3 + I_0 D_0 + I_1 D_1 + I_2 D_2 + I_3 D_3$$

(25)

By Eqs.(20) and (25), we have

$$(r_0)^2 - \Sigma(r_j)^2 - \Sigma(R_i)^2 = (r'_0)^2 - \Sigma(r'_j)^2 - \Sigma(R'_i)^2$$

(26)

The above represents that the spacetime interval $d_0$ keeps unchanged when the coordinate system rotates in the octonion space. When the octonion space is reduced to the quaternion space, the above equation should be reduced to Eq.(12) in the quaternion space. By Eqs.(22) and (26), we have Lorentz transformation.

$$(r_0)^2 - \Sigma(r_j)^2 - \Sigma(R_i)^2 = (r'_0)^2 - \Sigma(r'_j)^2 - \Sigma(R'_i)^2$$

$$v_0 = v'_0$$

(27)

In the octonion space, when $R_i \approx R'_i \approx 0$, Eq.(26) is reduced to Eq.(12) in the quaternion space.

D. Variable speed of light

The physical quantity $Q(q_0, q_1, q_2, q_3, Q_0, Q_1, Q_2, Q_3)$ in octonion space is defined as

$$Q = V \circ V$$

$$= q_0 + i_1 q_1 + i_2 q_2 + i_3 q_3 + I_0 Q_0 + I_1 Q_1 + I_2 Q_2 + I_3 Q_3$$

(27)
When the coordinate system is rotated, we have one physical quantity \( \Omega'(q'_0, q'_1, q'_2, q'_3, Q'_0, Q'_1, Q'_2, Q'_3) \) from Eq.(16). In the above, the scalar part remains the same during the octonion coordinate system is transforming. From Eq.(18) and the above, we have

\[
(v'_0)^2 - \Sigma(v'_j)^2 - \Sigma(V'_i)^2 = (v'_0)^2 - \Sigma(v'_j)^2 - \Sigma(V'_i)^2
\] (28)

The above equation represents the physical quantity \( q' \) of the gravitational field and electromagnetic field; \( v'_0 \) is a coefficient of gravitational field and electromagnetic field, and \( v'_0 \) is the gravitational field from Eq.(36).

From Eq.(21) and (28), we obtain the transformation \( A \) with variable speed of light.

In the same way, we have other kinds of the coordinate transformations in the octonion space. In the octonion space, when \( V_i \approx V'_j \approx 0 \), Eq.(28) is reduced to Eq.(14) in the quaternion space.

IV. TRANSFORMATIONS IN THE OCTONION COMPOUNDING SPACE

In the gravitational field and electromagnetic field demonstrated by quaternions, the vector radius \( R \) will be extended to \( R + k_{rx}X \) to cover the different definitions of energy. And the octonion space with \( R \) is extended to the octonion compounding space with \( R + k_{rx}X \), although their basis vector \( E \) remains the same.

A. Coordinate transformation

In the octonion compounding space, the basis vector

\[
E = (1, i_1, i_2, i_3, I_0, I_1, I_2, I_3)
\]

and the radius vector \( R \) will be extended.

\[
R \rightarrow R + k_{rx}X
\] (29)

where, \( X(x_0, x_1, x_2, x_3, X_0, X_1, X_2, X_3) \) is the octonion.

Therefore, the components of the radius vector \( R \) in Eq.(16) will be extended,

\[
r_i \rightarrow r_i + k_{rx}x_i , \quad R_i \rightarrow R_i + k_{rx}X_i
\] (30)

and that of the velocity \( V \) in Eq.(17) will be extended.

\[
v_i \rightarrow v_i + k_{rx}a_i , \quad V_i \rightarrow V_i + k_{rx}A_i
\] (31)

where, \( A(a_0, a_1, a_2, a_3, A_0, A_1, A_2, A_3) \) is the potential of gravitational field and electromagnetic field; \( x_0 = a_0; k_{rx} \) is one coefficient, and \( k_{rx} = 1/v_0 \).

When the coordinate system is transformed into other one, the octonion \( D \) in the compounding space will be transformed into \( D'(d'_0, d'_1, d'_2, d'_3, D'_0, D'_1, D'_2, D'_3) \).

\[
D' = K^* \circ D \circ K
\] (32)

where, \( K \) is the octonion, and \( K^* \circ K = 1; * \) denotes the conjugate of octonion.

In case of the coordinate system is transforming, the octonions in the above satisfy the relation as follows.

\[
D^* \circ D = (D')^* \circ D'
\] (33)

When the scalar part of physical quantity \( D \) does not take part in the coordinate transformation, the scalar part \( d_0 \) remains the same.

\[
d_0 = d'_0
\] (34)

We can obtain some coordinate transformations in the octonion compounding space from Eqs.(33) and (34).

B. Galilean transformation

When the coordinate system is rotated, we have one radius vector \( R'(r'_0, r'_1, r'_2, r'_3, R'_0, R'_1, R'_2, R'_3) \) and velocity \( V'(v'_0, v'_1, v'_2, v'_3, V'_0, V'_1, V'_2, V'_3) \) respectively from Eq.(32).

In the same way, we have correspondingly the new physical quantity \( X'(x'_0, x'_1, x'_2, x'_3, X'_0, X'_1, X'_2, X'_3) \) and the potential \( A'(a'_0, a'_1, a'_2, a'_3, A'_0, A'_1, A'_2, A'_3) \).

From Eqs.(30), (31), and (32), we have

\[
r_0 + k_{rx}x_0 = v'_0 + k_{rx}x'_0
\]

\[
v_0 + k_{rx}a_0 = v'_0 + k_{rx}a'_0
\] (35) (36)

and then

\[
t_0 = t'_0
\] (37)

\[
\Sigma(r_j + k_{rx}x_j)^2 + \Sigma(R_i + k_{rx}X_i)^2 = \Sigma(r'_j + k_{rx}x'_j)^2 + \Sigma(R'_i + k_{rx}X'_i)^2
\] (38)

The above means that if we emphasize especially the important of the radius vector Eq.(30) and the velocity Eq.(31), we obtain the Galilean transformation of the coordinate system from Eqs.(30), (31), and (32).

The above states also that the \( (r_0 + k_{rx}a_0) \) remains unchanged when the coordinate system rotates, but the speed of light, \( v_0 \), will be changed with the potential, \( a_0 \), of the gravitational field from Eq.(36).

C. Lorentz transformation

The physical quantity \( D(d_0, d_1, d_2, d_3, D_0, D_1, D_2, D_3) \) in the octonion space is defined as

\[
D = (R + k_{rx}X) \circ (R + k_{rx}X)
\]

\[
= d_0 + i_1d_1 + i_2d_2 + i_3d_3
+ I_0D_0 + I_1D_1 + I_2D_2 + I_3D_3
\] (39)
By Eqs.(30), (31), (32), and Eq.(39), we have
\[
(r_0 + k_{rx}a_0)^2 - \Sigma(v_j + k_{rx}a_j)^2 - \Sigma(V_i + k_{rx}A_i)^2
= (r_0' + k_{rx}a_0')^2 - \Sigma(v_j' + k_{rx}a_j')^2 - \Sigma(V_i' + k_{rx}A_i')^2
\] (40)

The above equation states the \( d_0 \) remains unchanged when the coordinate system rotates in the octonion compounding space. When the compounding octonion space is reduced to the compounding quaternion space, the Eq.(40) will be reduced to that in the latter space.

The above means also that the speed of light \( v_0 \) will be changed with the potential of either the gravitational field or the electromagnetic field. When the potential \( a_i \) and \( A_i \) are both equal approximately to zero, the Eq.(42) will be reduced to Eq.(28). By Eqs.(36) and (40), we can obtain the Lorentz transformation.

D. Variable speed of light

The physical quantity \( Q(q_0, q_1, q_2, q_3, Q_0, Q_1, Q_2, Q_3) \) in octonion space is defined as
\[
Q = (V + k_{rx}A) \circ (V + k_{rx}A)
= q_0 + i_1 q_1 + i_2 q_2 + i_3 q_3 + I_0 Q_0 + I_1 Q_1 + I_2 Q_2 + I_3 Q_3
\] (41)

When the coordinate system is rotated, we have a new physical quantity \( Q'(q_0', q_1', q_2', q_3', Q_0', Q_1', Q_2', Q_3') \) from Eq.(32). In the above equation, the scalar part remains the same during the octonion coordinate system is transforming. From Eq.(32) and the above, we have
\[
(v_0 + k_{rx}a_0)^2 - \Sigma(v_j + k_{rx}a_j)^2 - \Sigma(V_i + k_{rx}A_i)^2
= (v_0' + k_{rx}a_0')^2 - \Sigma(v_j' + k_{rx}a_j')^2 - \Sigma(V_i' + k_{rx}A_i')^2
\] (42)

The above equation represents the physical quantity \( q_0 \) remains unchanged when the coordinate system rotates. From the above and Eq.(38) or Eq.(40), we obtain the transformations with variable speed of light.

The above means also that the speed of light \( v_0 \) will be changed with either the gravitational potential or the electromagnetic potential. So we find different kinds of light speed in various optical waveguide materials. And there exist negative refractive indexes in optical waveguide materials, when the field potentials are switched from positive to negative. When the potential \( a_i \) and \( A_i \) both are equal approximately to zero, the Eq.(42) will be reduced to Eq.(28).

V. Conclusions

In the quaternion spaces, Galilean transformation and Lorentz transformation can be deduced from the feature of quaternions. This states that Lorentz transformation is only one of several coordinate transformations in the electromagnetic field and gravitational field.

In the octonion spaces, there exist some coordinate transformations with the variable speed of light. In the electromagnetic and gravitational fields, the gravitational intermediate boson and electromagnetic intermediate boson can be combined together to become the photon. So the speed of light will be changed with the electromagnetic potential and gravitational potential.

It should be noted that the study for the coordinate transformation has examined only some simple cases, including Galilean transformation and Lorentz transformation etc. Despite its preliminary character, this study can clearly indicate that there are several kinds of coordinate transformations with the variable speed of light. For the future studies, the investigation will concentrate on only some suitable predictions about the complicated coordinate transformations in the electromagnetic field and gravitational field.

Acknowledgments

This project was supported partly by the National Natural Science Foundation of China under grant number 60677039, Science & Technology Department of Fujian Province of China under grant number 2005HZ1020 and 2006H0092, and Xiamen Science & Technology Bureau of China under grant number 350220055011.

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