Study of the branching ratio of $\psi(3770) \to D\bar{D}$ in $e^+e^- \to D\bar{D}$ scattering

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Based on the data of BES and Belle, the production of $D\bar{D}$ in the $e^+e^- \to D\bar{D}$ scattering process is studied in this paper. We analyze the continuum and resonant contributions in the energy region from 3.7 to 4.4 GeV. In the $\chi^2$ fit to data, we obtain the resonance parameters of $\psi(3770)$, the branching ratio of $\psi(3770) \to D\bar{D}$ decay by confronting the data to the theoretical formula where both the contributions of the resonances, continuum and interference effects are included. We obtain the branching ratio of $\psi(3770) \to D\bar{D}$ decay is 97.2% ± 8.9%, as well as the branching ratio of $\psi(4040) , \psi(4160) \to D\bar{D}$ decays.

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Physics of $e^+e^-$ annihilation at the energy region of $3 - 5$ GeV is interesting. It attracts the focus of both experimental and theoretical studies. The physics at this energy region involves several well-established vector resonances, $J/\psi, \psi(2S), \psi(3770), \psi(4040), \psi(4160), \ldots$, which are bound states of quark and antiquark pair $c\bar{c}$. Studying the production and decays of these resonances can deepen our knowledge about dynamics of interactions between quarks. The resonance $\psi(3770)$ has a mass just slightly above the threshold of $D\bar{D}$ pair production. Its decays evade the suppression of the Okubo-Zweig-Iizuka (OZI) rule \[1\]. This is consistent with the fact that the width of $\psi(3770)$ is about 2 orders larger than those of $J/\psi$ and $\psi(2S)$ \[2\]. The $J/\psi$ and $\psi(2S)$ can only decay into non-$D\bar{D}$ final states, which is suppressed according to the OZI rule. It is believed that $\psi(3770)$ decays dominantly into $D\bar{D}$ pair. All the well-established non-$D\bar{D}$ decay modes of $\psi(3770)$ only show up with the branching ratios at the order of $10^{-3} - 10^{-4}$. The measurement of many other decay modes of $\psi(3770)$ only gives upper bounds, which shows that decay rates of these non-$D\bar{D}$ decay modes should be smaller than $10^{-3} - 10^{-4}$. The sum of all these well-measured decay rates is at most at the order of several percent.

In the $e^+e^-$ collider the properties of the resonance $\psi(3770)$ are measured through the scattering process $e^+e^- \to \psi(3770) \to f$, where $f$ can be any final states like $D\bar{D}$ or any other hadrons. The branching ratio of $\psi(3770) \to D\bar{D}$ and $\psi(3770) \to$ non-$D\bar{D}$ can be derived from the measured scattering cross section of $e^+e^- \to D\bar{D}$ and $e^+e^- \to$ hadrons. Both BES and CLEO-c Collaborations measured the cross section of $e^+e^- \to D\bar{D}$ at the center-of-mass energy $E_{c.m.} = 3773$ MeV several years ago \[4, 5\], and their results are in good agreement with each other.

The CLEO-c Collaboration also measured the cross section of $e^+e^- \to \psi(3770) \to$ hadrons at $E_{c.m.} = 3773$ MeV \[6\]. The difference between this and the cross section of $e^+e^- \to \psi(3770) \to D\bar{D}$ is found to be $(-0.01 \pm 0.08^{+0.41}_{-0.30})$ nb, which indicates that the decay rate of $\psi(3770)$ to non-$D\bar{D}$ is tiny. However, the measurement of the BES Collaboration gives that the branching ratio of $\psi(3770) \to D\bar{D}$ is $(85.5 \pm 1.7 \pm 5.8)\%$ or $(83.6 \pm 7.3 \pm 4.2)\%$, and the decay branching ratio of $\psi(3770)$ to non-$D\bar{D}$ is $(14.5 \pm 1.7 \pm 5.8)\%$ or $(16.4 \pm 7.3 \pm 4.2)\% \[7, 8\]$, which is not consistent with CLEO-c’s measurement.

On one hand, a large branching ratio of $\psi(3770)$ to non-$D\bar{D}$ contradicts the fact that the sum of the branching ratios of all the well-established exclusive non-$D\bar{D}$ decays is not large enough to give such a large decay rate for the inclusive decay mode. On the other hand, it is difficult to understand such a large branching ratio for the non-$D\bar{D}$ decays theoretically. Calculations based on the method of QCD predict that the branching ratios of both exclusive and inclusive decays of $\psi(3770)$ to non-$D\bar{D}$ final states are very tiny, the sum of them is at most about 5% \[9, 11\]. Therefore, the decays of $\psi(3770) \to D\bar{D}$ (or non-$D\bar{D}$, an equivalent expression for one problem) is still an unsolved problem for both theory and experiment.

At the $e^+e^-$ collider, the branching ratio of $\psi(3770) \to D\bar{D}$ can be derived from the measured cross section of $e^+e^- \to D\bar{D}$. In this work, we reanalyze the experimental data of the $e^+e^- \to D\bar{D}$ cross section in the center-of-mass energy region from 3.74 to 4.4 GeV measured by the BES \[12\] and the Belle \[13\] Collaborations. We will include not only the contribution of $\psi(3770)$ itself, but also the contributions of the other resonances with masses near 3770 MeV. We also include the continuum contribution and its interference effect with the resonances in the energy region 3.74 to 4.4 GeV. We finally find that the continuum contribution can be explained as an effect of the tail of $\psi(2S)$, whose mass is about 40 MeV lower than the threshold of $D\bar{D}$ pair production. By including the contributions of all the resonances near 3770 MeV and the interference effects, the branching ra-
ratio of $\psi(3770) \to D\bar{D}$ we derived is apparently different from that of the BES experiment \[13\]. The contributions of the resonances with masses below and above that of $\psi(3770)$ are not included in analyzing the data of the $e^+e^- \to D\bar{D}$ cross section by the BES Collaboration \[14\]. The effect of these contributions is important for deriving the branching ratio of $\psi(3770) \to D\bar{D}$ decay.

In Ref. \[14\], one of us analyzed the data of the $e^+e^- \to D\bar{D}$ cross section measured by the BES and CLEO-c Collaborations at $E_{c.m.} = 3773$ MeV \[4, 8\]. The formula describing the $e^+e^- \to D^0\bar{D}^0$ or $D^+D^-$ cross section is derived

$$\sigma(e^+e^- \to D^0\bar{D}^0, D^+D^-) = \frac{\pi}{3} \frac{(s - 4m_{D^0}^2)^{3/2}}{s^{5/2}} \alpha^2 \left| -F_{D\bar{D}}(s) + \sum_i \frac{g_{\psi_iDD}Q_i f_{\psi_i} m_{\psi_i}}{s - m_{\psi_i}^2 + im_{\psi_i} \Gamma_i} e^{i\phi_i} \right|^2,$$  

(1)

where $s = (p_1 + p_2)^2$, $p_1$ and $p_2$ are the momenta of $D$ and $D$ mesons, respectively, $m_D$ is the mass of $D^+$ or $D^0$, $D^0$, $m_{\psi_i}$, the mass of the $i$th resonance, $\alpha = 1/137$ is the electromagnetic fine-structure constant, and $Q_s = 2/3$ is the electric charge of the $c$ quark. The first term in the absolute-value squared $F_{D\bar{D}}(s)$ describes the continuum contribution. The second terms in the summation are the contributions of all the possible resonances, which are described by the Breit-Wigner form. The $\Gamma_i$'s are the total decay widths of resonances $\psi_i$'s, and $\phi_i$'s are the relevant phases of the resonance contributions. $g_{\psi_iDD}$ is the coupling of the resonance $\psi_i$ and $D\bar{D}$, which is defined by

$$(D(p_1)\bar{D}(p_2)|\psi(p_i)) = -ig_{\psi_iDD} \epsilon^{(3)} \cdot (p_1 - p_2)(2\pi)^4 \delta^4(p - p_1 - p_2),$$ 

(2)

where $\epsilon^{(3)}$ is the polarization vector of $\psi_i$, and $\lambda$ stands for the polarization state. $f_{\psi_i}$ is the decay constant of the $i$th resonance $\psi_i$, which is defined by

$$\langle 0|\bar{c}\gamma_\mu c|\psi_i \rangle = f_{\psi_i} m_{\psi_i} \epsilon^{(\mu)}_{\lambda}.$$ 

(3)

Using the definition of the decay constant, the leptonic decay width of the resonance $\psi_i$ is

$$\Gamma_{lei} = \frac{4\pi Q_i^2 \alpha f_{\psi_i}^2}{3 m_{\psi_i}}.$$ 

(4)

With isospin symmetry, the coupling $g_{\psi_iDD}$ and the continuum function $F_{D\bar{D}}(s)$ are the same for both the production of $D^+D^-$ and $D^0\bar{D}^0$. The difference of the cross sections of $e^+e^-$ to $D^0\bar{D}^0$ and to $D^+D^-$ is caused by the phase-space difference of $D^0\bar{D}^0$ and $D^+D^-$. With the coupling $g_{\psi_iDD}$, the branching ratio of the vector resonance $\psi_i$ can be obtained

$$BR(\psi_i \to D^0\bar{D}^0, \text{or } D^+D^-) = \frac{g_{\psi_iDD}^2 (m_{\psi_i} - 4m_{D^0}^2)^{3/2}}{48\pi\Gamma_i m_{\psi_i}^2}.$$ 

(5)

One can define the branching ratio of $\psi_i \to D\bar{D}$ as the sum of $\psi_i \to D^0\bar{D}^0$ and $D^+D^-$, and then the branching ratio of $\psi_i \to D\bar{D}$ is

$$BR(\psi_i \to D\bar{D}) = g_{\psi_iDD}^2 [(m_{\psi_i}^2 - 4m_{D^0}^2)^{3/2} + (m_{\psi_i}^2 - 4m_{D^+}^2)^{3/2}] / 48\pi\Gamma_i m_{\psi_i}^2.$$ 

(6)

The summed cross section of $e^+e^- \to D^0\bar{D}^0$ and $D^+D^-$ can be obtained from Eq.(1)

$$\sigma(e^+e^- \to D^0\bar{D}^0) = \frac{\pi}{3} \frac{(s - 4m_{D^0}^2)^{3/2} + (s - 4m_{D^+}^2)^{3/2}}{s^{5/2}} \alpha^2 \left| -F_{D\bar{D}}(s) + \sum_i \frac{g_{\psi_iDD}Q_i f_{\psi_i} m_{\psi_i}}{s - m_{\psi_i}^2 + im_{\psi_i} \Gamma_i} e^{i\phi_i} \right|^2.$$ 

(7)

With the expressions of the leptonic decay width and the branching ratios of $\psi_i \to D\bar{D}$ in Eqs. (4) and (5), one can reexpress the cross section of $e^+e^- \to D\bar{D}$ in Eq. (1) as

$$\sigma(e^+e^- \to D\bar{D}) = \frac{\pi}{3} \frac{(s - 4m_{D^0}^2)^{3/2} + (s - 4m_{D^+}^2)^{3/2}}{s^{5/2}} \alpha^2 \left| -F_{D\bar{D}}(s) + \sum_i \frac{g_{\psi_iDD}Q_i f_{\psi_i} m_{\psi_i}}{s - m_{\psi_i}^2 + im_{\psi_i} \Gamma_i} e^{i\phi_i} \right|^2,$$ 

(8)

where the function $g(x)$ is defined as

$$g(x) = \frac{1}{\sqrt{(x - 4m_{D^0}^2)^{3/2} + (x - 4m_{D^+}^2)^{3/2}}}.$$ 

(9)

and $BR_i$ is the branching ratio of $\psi_i \to D\bar{D}$.

The data of the cross sections of $e^+e^- \to D\bar{D}$ measured by BES \[12\] and Belle \[13\] are analyzed with the formula in Eq.(8). The BABAR Collaboration \[15\] has

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also made similar measurements before Belle’s experiment [13], and their data are consistent with Belle’s, but for simplicity, we only use Belle’s data in our analysis since they have higher statistics. The data require that the continuum term should be chosen as

$$-F_{DD}(s) = \frac{F_0 m^2_{\psi(3770)}}{s - a},$$  \hspace{1cm} (10)$$

where $F_0$ and $a$ are parameters to be fitted. The value of the parameter $a$ is found to be approximately the mass squared of $\psi(2S)$. Therefore the continuum term can be identified as the virtual contribution of the resonance $\psi(2S)$, whose mass is below the threshold of $DD$ production. Then the continuum term is chosen to be

$$-F_{DD}(s) = \frac{c_0}{s - m_{\psi(2S)}^2 + i m_{\psi(2S)} \Gamma_{\psi(2S)}},$$  \hspace{1cm} (11)$$

where $c_0$ is the parameter to be fitted.

The other resonances included in analyzing the $e^+e^- \to DD$ scattering cross section are $\psi(3770), G(3900), \psi(4040)$, and $\psi(4160)$. Both the resonances are parametrized as Breit-Wigner form except for $G(3900)$, for which the square root times a phase factor is used according to $BABAR’s$ finding [13]. The cross section of $e^+e^- \to DD$ scattering in the energy range from 3.7 to 4.4 GeV can be expressed as

$$
\sigma(e^+e^- \to DD) = \frac{\pi}{3} \frac{(s - 4m_{\psi(2S)}^2)^{3/2} + (s - 4m_{\psi(3770)}^2)^{3/2}}{s^{5/2}} \frac{\alpha^2}{\alpha} \frac{c_0}{s - m_{\psi(2S)}^2 + im_{\psi(2S)} \Gamma_{\psi(2S)}} \left( \frac{1}{\sqrt{2\pi \sigma_{G(3900)}}} e^{-\frac{c_G^2 m_{G(3900)}^2}{2 \sigma_G^2}} e^{i\phi_1} + \frac{g(m_{\psi(3770)}^2)}{\alpha} \frac{\Gamma_{ee1} BR_{1} m_{\psi(3770)}^{5/2} \psi(3770)}{s - m_{\psi(3770)}^2 + im_{\psi(3770)} \Gamma_{\psi(3770)}} e^{i\phi_2} + \frac{g(m_{\psi(4040)}^2)}{\alpha} \frac{\Gamma_{ee2} BR_{2} m_{\psi(4040)}^{5/2} \psi(4040)}{s - m_{\psi(4040)}^2 + im_{\psi(4040)} \Gamma_{\psi(4040)}} e^{i\phi_3} \right)^2,
$$

where $c_0, c_1$ and the phases $\phi_i$’s are free parameters which will be fitted in the $\chi^2$ fit to the experimental data. In our fitting the values of the following quantities are fixed and taken from the PDG [2]: $m_{\psi(2S)} = 3686.09 \pm 0.04$ MeV, $\Gamma_{\psi(2S)} = 317 \pm 9$ keV; the leptonic width of $\psi(3770)$ is $\Gamma_{ee1} = 0.265 \pm 0.018$ keV; $m_{\psi(3770)} = 4039 \pm 1$ MeV, $\Gamma_{\psi(3770)} = 80 \pm 10$ MeV, the leptonic width of $\psi(4040)$ is $\Gamma_{ee2} = 0.86 \pm 0.07$ keV; $m_{\psi(4040)} = 4153 \pm 3$ MeV, $\Gamma_{\psi(4160)} = 103 \pm 8$ MeV, the leptonic width of $\psi(4160)$ is $\Gamma_{ee3} = 0.83 \pm 0.07$ keV. The quantities $m_{\psi(3770)}$, $\Gamma_{\psi(3770)}$, and the branching ratios of $\psi(3770) \to DD, \psi(4040) \to DD$ and $\psi(4160) \to DD$ are set free and fitted from the data of BES [12] and Belle [13]. The values of $M_{\psi(3900)}$ and $\sigma_G(3900)$ are varied and finally fixed, which can give the best fit to the data. Figure 1 shows the nominal fit to the data of the cross section of $e^+e^- \to DD$ measured by BES [12] and Belle [13]. The data at 14 different energy points under the $\psi(3770)$ peak between 3.73 and 3.80 GeV are from BES experiments by using the $e^+e^-$ scan [12], while the rest of the data at 27 different energy points above 3.80 GeV are from the exclusive initial state radiation (ISR) production of $DD$ events from electron-positron annihilation at a center-of-mass energy of 10.58 GeV at Belle with an integrated luminosity of 673 fb$^{-1}$ [13]. The data are corrected by the ISR [4] [10] [18], and are all Born cross sections. The solid curve is the best fit to the data by using the formula in Eq. (12). The dashed curve is the contribution of the resonance $\psi(3770)$, and the dotted curve is the continuum contribution from the tail of $\psi(2S)$. The very asymmetric line shape (dashed line in Fig. 1) is caused by the phase-space factor in Eq. (12).

The fit gives the following values for the parameters $c_0, c_1$ and the phases:

$$c_0 = 8.75 \pm 0.71 \text{ GeV}^2, \quad c_1 = 1.00 \pm 0.35 \text{ GeV}^{1/2},$$

$$\phi = -2.63 \pm 0.09, \quad \phi_1 = -1.89 \pm 0.33,$$

$$\phi_2 = -2.14 \pm 0.14, \quad \phi_3 = 1.91 \pm 0.44,$$

(13)
where the unit for $\phi_i$'s is radian. The results for the mass and width of $\psi(3770)$ and the branching ratios of $\psi(3770)$, $\psi(4040)$, and $\psi(4160) \to D\bar{D}$ are

$$m_{\psi(3770)} = 3776 \pm 1 \text{ MeV},$$

$$\Gamma_{\psi(3770)} = 28.5 \pm 2 \text{ MeV},$$

$$\text{BR}(\psi(3770) \to D\bar{D}) = (97.2 \pm 8.9)\%,$$

$$\text{BR}(\psi(4040) \to D\bar{D}) = (25.3 \pm 4.5)\%,$$

$$\text{BR}(\psi(4160) \to D\bar{D}) = (2.8 \pm 1.8)\%,$$

and the parameters for $G(3900)$ are $M_{G(3900)} = 3900 \pm 20$ MeV and $\sigma_{G(3900)} = 52 \pm 23$ MeV which are fixed in the fit. In the above fit, there are totally 41 data points measured by BES and Belle and 11 free parameters are floated as shown in Eqs. (13) $\sim$ (15). The fitted quality is $\chi^2/n_\text{d.o.f} = 1.06$.

In the fit, we find that the correlation between the phase $\phi$ and $\text{BR}(\psi(3770) \to D\bar{D})$ is 0.73 which is large and mainly due to the component of the tail of $\psi(2S)$. There is also a large correlation ($\sim -0.42$) between the parameter $c_1$ for the structure $G(3900)$ and the $\text{BR}(\psi(3770) \to D\bar{D})$. The structure of $G(3900)$ had been suggested in Ref. [19] and confirmed by BABAR data [13].

To test the significance of $\psi(2S)$ in the fit, a fit has been done without the contribution of $\psi(2S)$. We find the quality $\chi^2/n_\text{d.o.f} = 1.5$ (the $\chi^2$ in this fit is worse by 14 for 2 degrees of freedom by comparing to the $\chi^2$ value in the nominal fit), while the value of $\chi^2/n_\text{d.o.f} = 1.06$ in the nominal fit. Furthermore, in the fit without the contribution of $\psi(2S)$, we obtain the $\text{BR}(\psi(3770) \to D\bar{D}) = (85.9 \pm 7.0)\%$, which is significantly smaller than the result obtained in the nominal fit.

Figure (a) shows that the virtual contribution of $\psi(2S)$ is very large (the dotted curve). It is even larger than the contribution of $\psi(3770)$ when the colliding energy is above the resonance region of $\psi(3770)$. The contribution of any resonance to $e^+e^- \to D\bar{D}$ depends on both the coupling of this resonance with the virtual photon and with the $D\bar{D}$ pair. The coupling of the resonance with virtual photon can be described by its decay constant, which can be extracted from the measured leptonic decay width of the resonance. From the data of the leptonic width of $\psi(2S)$ and $\psi(3770)$, one can obtain the decay constant of $\psi(2S)$ is $f_{\psi(2S)} = 297$ MeV, while the decay constant of $\psi(3770)$ is $f_{\psi(3770)} = 100$ MeV. This indicates that the coupling of $\psi(2S)$ with the virtual photon is approximately 3 times of that of $\psi(3770)$. One can also obtain the coupling of $\psi(3770)$ and $\psi(2S)$ with $D\bar{D}$ from the fitted result of $\text{BR}(\psi(3770) \to D\bar{D})$ and the parameter $c_0$ with identifying $c_0 = g_{\psi(2S)?D\bar{D}Q\bar{Q}} f_{\psi(2S)} m_{\psi(2S)}$, which is indicated by the numerator of the second term of Eq. (7). The obtained couplings of the resonances with $D\bar{D}$ are $g_{\psi(3770)D\bar{D}} = 12.8$ and $g_{\psi(2S)D\bar{D}} = 12.0$, i.e., the couplings of $\psi(3770)$ and $\psi(2S)$ with $D\bar{D}$ are approximately the same. Therefore the reason for the large contribution of $\psi(2S)$ to $e^+e^- \to D\bar{D}$ scattering comes from the large coupling of $\psi(2S)$ with the virtual photon.

In Fig. (b) we show one of the best solutions. With the current data, multisolutions in the fit are possible since both the four phases and branching ratios are floated in the nominal fit. We investigate these effects, and find 8 solutions with comparable fit quality. However 6 of the 8 solutions are with the $\text{BR}(\psi(3770) \to D\bar{D})$ less than 70%, which are not consistent with the fact that no non-$D\bar{D}$ decay modes with significant branching fraction have been found in experiment. The sum of the branching ratios of all the well-established non-$D\bar{D}$ decay modes

![Figure 1: The cross section of $e^+e^- \to D\bar{D}$](image)

(a) The cross section of $e^+e^- \to D\bar{D}$. The dots with error bars are the data measured by BES [12] and squares with error bars are data from Belle [13]. The data are cross sections corrected by the ISR. The solid curve is the best fit to the data, the dashed curve is the contribution of the resonance $\psi(3770)$, and the dotted curve is the continuum contribution of $\psi(2S)$. (a) is the cross sections in the whole energy range from 3.7 to 4.4 GeV, while (b) is the detail in the region of the resonance $\psi(3770)$. (b)
are at most 2%-3%. Hereafter we discard the non-physical results. By assuming a constant width $\Gamma_{\psi(3770)}$ for $\psi(3770)$ resonance in Eq. (12), we show two possible physical solutions for comparison in Table I. While, by assuming an energy-dependent width for $\psi(3770)$ in the fit, we obtain similar results which are listed in the third column in Table I. The energy-dependent width $\Gamma_{\psi(3770)}(s)$ is defined as \[ \Gamma_{\psi(3770)}(s) = \Gamma_{D^+D^-}(s) + \Gamma_{D^0\bar{D}^0}(s) + \Gamma_{non-DD}(s), \] where $\Gamma_{D^+D^-}(s)$, $\Gamma_{D^0\bar{D}^0}(s)$ and $\Gamma_{non-DD}(s)$ are the partial widths for $\psi(3770) \rightarrow D^+D^-$, $\psi(3770) \rightarrow D^0\bar{D}^0$ and $\psi(3770) \rightarrow non-DD$, respectively; and are the step functions to account for the thresholds of $\psi(3770)$ resonance. The last column is one solution for a fit to data with energy-dependent width ($s$ dependent) as defined in Eq. (19).

\begin{align}
\Gamma_{D^+D^-}(s) &= \Gamma_{\psi(3770)}(s)(E_{c.m.} - 2m_{D^0}) \\
&\times \left( \frac{p_D^+}{p_D^0} \right)^2 \frac{1 + (rp_D^0)^2}{1 + (rp_D^0)^2} B_{+-}, \\
\Gamma_{D^0\bar{D}^0}(s) &= \Gamma_{\psi(3770)}(s)(E_{c.m.} - 2m_{D^0}) \\
&\times \left( \frac{p_{D^0}}{p_{D^0}} \right)^2 \frac{1 + (rp_{D^0})^2}{1 + (rp_{D^0})^2} B_{00},
\end{align}
and

\begin{equation}
\Gamma_{non-DD}(s) = \Gamma_{\psi(3770)}(1 - B_{+-} - B_{00}),
\end{equation}
where $p_D^0$ and $p_D$ are the momentum of the $D$ mesons produced at the peak of $\psi(3770)$ and at the center-of-mass energy $\sqrt{s}$, respectively; $r$ is the interaction radius of the $cc$, which is set to be 1.0 fm here; $B_{+-}$ and $B_{00}$ are the branching ratios for $\psi(3770) \rightarrow D^+D^-$ and $D^0\bar{D}^0$, respectively; and $\theta(E_{c.m.} - 2m_{D^0})$ and $\theta(E_{c.m.} - 2m_{D^0})$ are the step functions to account for the thresholds of $DD$ production. In the fit, we fix the ratio $B_{00}/B_{+-} = 1.33$, so that no additional free parameter is introduced.

Our fitted result for the branching ratio $\psi(3770) \rightarrow DD$ is different from that of the BES Collaboration. In our fitting we find that the contribution of the continuum term from the tail of $\psi(2S)$, the contributions of the resonances $\psi(4040)$, $\psi(4140)$, the structure $G(3900)$ and their interference effects with the resonance $\psi(3770)$ are important; they can seriously affect the fitting result of the branching ratio of $\psi(3770) \rightarrow DD$. If these effects are not included, a smaller result for the decay rate of $\psi(3770)$ to $DD$ will be introduced. The smaller decay rate of $\psi(3770) \rightarrow DD$ means a larger decay rate of $\psi(3770) \rightarrow non-DD$. However, no large exclusive non-$DD$ decay mode of $\psi(3770)$ has been seen in experiment up to now. The sum of all the well-established exclusive non-$DD$ decay rates is less than 2%-3%. This makes a puzzle for $DD$ or non-$DD$ decays of $\psi(3770)$. Our new analysis can solve this problem. With the branching ratio of $\psi(3770) \rightarrow DD$ is (97.2 ± 8.9)%, the long-standing problem for $\psi(3770)$ decay disappears. Our result can also be well understood theoretically. Theoretical prediction based on QCD to the branching ratio of the non-$DD$ decay of $\psi(3770)$ is at most 5%. This is consistent with our result of a large branching ratio of $\psi(3770) \rightarrow DD$ decay [see Eq. (16)].

| Variables | Constant width |
|-----------|---------------|
| $m_{\psi(3770)}$ (MeV) | $376.1 \pm 1$ | $376.1 \pm 1$ |
| $\Gamma_{\psi(3770)}$ (MeV) | $28.5 \pm 2.1$ | $28.7 \pm 2.1$ |
| $\Gamma_{BR_{1}}$ (%) | $97.2 \pm 8.9$ | $101.1 \pm 9.0$ |
| $\Gamma_{BR_{2}}$ (%) | $25.3 \pm 4.5$ | $34.7 \pm 4.8$ |
| $\Gamma_{BR_{3}}$ (%) | $2.8 \pm 1.8$ | $40.4 \pm 3.8$ |
| $c_{0}$ | $8.75 \pm 0.71$ | $8.67 \pm 0.67$ |
| $c_{1}$ | $1.00 \pm 0.35$ | $0.82 \pm 0.29$ |
| $\phi$ (rad.) | $-2.63 \pm 0.09$ | $-2.56 \pm 0.09$ |
| $\phi_{1}$ (rad.) | $-1.89 \pm 0.33$ | $-1.55 \pm 0.36$ |
| $\phi_{2}$ (rad.) | $-2.14 \pm 0.14$ | $-1.62 \pm 0.11$ |
| $\phi_{3}$ (rad.) | $1.91 \pm 0.44$ | $-3.03 \pm 0.1$ |

| non-dependent |
|---------------|
| Solution 1    | Solution 2    |
| $m_{\psi(3770)}$ (MeV) | $376.1 \pm 1$ | $376.1 \pm 1$ |
| $\Gamma_{\psi(3770)}$ (MeV) | $28.5 \pm 2.1$ | $28.7 \pm 2.1$ |
| $BR_{1}$ (%) | $97.2 \pm 8.9$ | $101.1 \pm 9.0$ |
| $BR_{2}$ (%) | $25.3 \pm 4.5$ | $34.7 \pm 4.8$ |
| $BR_{3}$ (%) | $2.8 \pm 1.8$ | $40.4 \pm 3.8$ |
| $c_{0}$ | $8.75 \pm 0.71$ | $8.67 \pm 0.67$ |
| $c_{1}$ | $1.00 \pm 0.35$ | $0.82 \pm 0.29$ |
| $\phi$ (rad.) | $-2.63 \pm 0.09$ | $-2.56 \pm 0.09$ |
| $\phi_{1}$ (rad.) | $-1.89 \pm 0.33$ | $-1.55 \pm 0.36$ |
| $\phi_{2}$ (rad.) | $-2.14 \pm 0.14$ | $-1.62 \pm 0.11$ |
| $\phi_{3}$ (rad.) | $1.91 \pm 0.44$ | $-3.03 \pm 0.1$ |
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