ABSTRACT
We have measured the topology (genus) of the fluctuations in the cosmic microwave background seen in the recently completed (four-year) data set produced by the COBE satellite. We find that the genus is consistent with that expected from a random-phase Gaussian distribution, as might be produced naturally in inflationary models.

Key words: cosmic microwave background – cosmology: observations – large-scale structure of the universe – methods: statistical

1 INTRODUCTION
The microwave background radiation (hereafter MBR) discovered by Penzias and Wilson (1965), has a spectrum close to that of a blackbody with mean temperature $T = 2.726 \pm 0.01K$ (Mather et al. 1993), with small scale fluctuations of order $\Delta T/T \sim 10^{-5}$ at $\geq 10^\circ$ as observed by COBE (Smoot et al. 1992), in agreement with inflationary models with various values of $\Omega$ (cf., Gott 1982, 1986; Bardeen, Steinhardt, and Turner 1983; Bond 1988; Vittorio and Juszkiewicz 1987). Recently, the final (four-year) data have been reported by COBE on the World Wide Web (COBE 1996).

The geometric nature of the MBR anisotropy may be assessed by mapping isotemperature contours. Local geometrical information about structures in the map is contained in three quantities: total area above (or below) a threshold, total contour length and curvature of the contour (cf., Willmore 1982). These are local and invariant quantities of the contours in the sense that they can be calculated even from a temperature map with incomplete and patchy coverage and do not change under translation and rotation of the coordinate frame. At high threshold levels the excursion regions of a random temperature field will appear as isolated hot spots surrounded by a cold background and the total curvature will be positive. At a low-temperature threshold the contours surround cold spots and the total curvature is negative. Near the mean temperature the hot and the cold regions are nearly symmetric, and the total curvature is close to zero.

We define the genus $G$ of the excursion set for a random temperature field on a plane as

$$G = \frac{\text{number of isolated high-temperature regions}}{-\text{number of isolated low-temperature regions}}$$

Equivalently, the genus can be defined as the total curvature of the contours. Assuming a contour defines a differentiable curve $C$ on the map, its total curvature is given by the integral

$$K = \int_C \kappa ds \equiv 2\pi G$$

where $\kappa$ is the local curvature, $s$ parameterizes the curve, and $G$ is the genus of a single contour.

A hot spot will contribute +1 to the total map genus and a cold spot (“hole”) in it will decrease the genus by 1. Therefore the genus can be considered as the total number of connected hot regions minus the total number of holes in them. In practice, contours may cross the edge of the survey region, in which case the partial curves contribute non-integer rotation indices to the genus.

According to (Gott et al. 1990), a two-dimensional random-phase Gaussian temperature field will generate a genus per unit area

$$g \propto \nu e^{-\nu^2/2}$$

where $\nu$ is the threshold value, above which a fraction, $f$ of the area has a higher temperature

$$f = (2\pi)^{-1/2} \int_\nu^\infty \exp(-x^2/2)dx$$

(cf., also Adler 1981; Melott et al. 1989; Coles 1988; Gott et al. 1992; Park et al. 1992).

2 COBE OBSERVATIONS
For our analysis we have used the linear combination of the 6 COBE (four-year) DMR maps provided by the COBE team, which properly subtracts the Galaxy while minimizing the noise (COBE Archive 1996), (for method, see Bennett et al. 1992). This map is then smoothed with a Gaussian
Figure 1. Total genus (North–South) is plotted as a function of $\nu$. The size of the error-bars has been estimated from the observed rms of all (North–South) genus differences at fixed $\nu$. The best-fit theoretical curve for a Gaussian random-phase distribution ($g \propto \exp(-\nu^2/2)$) is shown as a solid line. The temperature-scale is given at the top.

window function with a FWHM of 7°. Since the COBE antennas have a FWHM beam width of 7° this amounts to an effective Gaussian smoothing window function with FWHM of 10° for any real signal. To eliminate any residual galactic contamination at low galactic latitude we will only consider the North and South galactic caps with $|b| > 30°$.

The genus of the temperature anisotropies is plotted as a function of $\nu$ in figure 1. Across the top of the figure the temperature scale is shown—that it is nearly linear shows that the temperature histogram in the smoothed map is approximately Gaussian. The best-fit theoretical genus curve expected for a random-phase Gaussian distribution: $g(\nu) \propto \nu \exp(-\nu^2/2)$ (eq. 3) is shown as a solid line. The total genus (North+South) is plotted, and a single estimate of the size of the error bars (in the total genus) is made from the rms of all the observed (North–South) genus differences at fixed $\nu$. As figure 1 shows, the observed genus curve is consistent with the random-phase theoretical curve within the errors (thus confirming similar findings from the COBE one-year data [Torres 1993; Smoot et al. 1994; Park and Gott 1993; Kogut 1993]).

3 DISCUSSION

For comparison, we have constructed and analyzed simulated pure noise maps using COBE error estimates at each position on the sky, which gives $G(\nu = 1) = 48 \pm 4$. Our observed best-fit value for $G(\nu = 1) = 31$ (figure 1) is over four standard deviations below that expected for a pure noise map. This independently confirms the Smoot et al. 1992 conclusion that we are observing real signal here and not just receiver noise.

If the real signal is Gaussian random-phase, then addi-

tion of instrumental noise (which is itself presumably locally Gaussian random-phase) should leave us with a genus curve which approximately obeys the Gaussian random-phase law, $g \propto \exp(-\nu^2/2)$ as seen in figure 1. Since the signal-to-noise ratio here is approximately 0.4, only gross deviations from Gaussian random-phase in the real signal could possibly reveal themselves in the figure. Nonetheless, the agreement between the data and the random-phase theory shown in figure 1 demonstrates conclusively that the topology of the MBR from the COBE results does not falsify the Gaussian random-phase hypothesis.

We have presented here the genus measurement of temperature anisotropies in the microwave background as measured by the COBE satellite and found it to be consistent with the inflationary prediction of Gaussian random-phase anisotropies.

4 ACKNOWLEDGEMENTS

The COBE datasets were developed by the NASA Goddard Space Flight Center under the guidance of the COBE Science Working Group and were provided by the NSSDC. WNC thanks the Fannie and John Hertz Foundation for its continued and gracious support. This research has been supported by NASA Grant NAGS-2759.

REFERENCES

Adler, R.J., 1981, The Geometry of Random Fields, Wiley, New York
Bennett, C.L. et al., 1992, ApJ, 396, L7
Bardeen, J.M., Steinhardt, P.J., & Turner, M.S. 1983, Phys Rev, 28, 679
Bond, J.R., 1988, in C.F. Frenk et al., eds., The Epoch of Galaxy Formation, Proc. NATO Advanced Research Workshop (Durham), Kluwer, Dordrecht
COBE Archive, 1996, internet URL [http://nssdc.gsfc.nasa.gov/htbin/htdir/anon_dir/cobe/asds/dmr/dat/dmr_dcmh.fits]
Coles, P., 1988, MNRAS, 234, 509
Gott, J.R., 1982, Nature 295, 304
Gott, J.R., 1986, in E.W. Kolb, M.S. Turner, D. Lindley, K. Olive, & D. Seckel, eds., Inner Space/Outer Space, The Interface Between Cosmology and Particle Physics, Univ. Chicago Press, Chicago, p. 362
Gott, J.R., Park, C., Juskiewicz, R., Bies, W.E., Bennett, D.P., Bouchet, F.R., & Stebbins, A., 1990, ApJ, 352, 1
Gott, J.R., Mao, S., Park, C. & Lahav, O., 1992, ApJ, 385, 26
Kogut, A., 1993, Bull. Amer. Astron. Soc., 183, 121.03
Mather, J., et al., 1993, Talk at January AAS Meeting, Phoenix
Melott, A.L., Cohen, A.P. Hamilton, A.J.S., Gott, J.R. & Weinberg, D. H., 1989, ApJ, 345, 618
Park, C. & Gott, J.R., 1993, (unpublished)
Park, C., Gott, J.R., Melott, A., Karachentsev, I.D., 1992, ApJ, 387, 1
Penzias, A.A., & Wilson, R.W. 1965, ApJ, 142, 419
Smoot, G.F. et al. 1992, ApJ 396, L1
Smoot, G.F. et al. 1994, ApJ 437, 1
Torres, S. 1993, ApJ, 423, L9
Vittorio, N. & Juszkiewicz, R. 1987, ApJL, 314, L29
Willmore, T.J., 1982, Total Curvature in Riemannian Geometry, Ellis Horwood, New York
