The task of coordinating social and economic indicators of the development of the region and the mathematical approach to its solution

Yu V Bondarenko¹, I V Goroshko² and I L Kashirina¹

¹Department of Applied Mathematics, Informatics and Mechanics, Voronezh State University, 1, University Square, Voronezh, 394018, Russia
²V A Trapeznikov Institute of Control Sciences, Russian Academy of Science, Moscow, Ulitsa Profsoyuznaya, 65, Russia
³The Academy of Management of Ministry Internal Affairs, Moscow, Ulitsa Zoi i Aleksandra Kosmodemjanskih, 8, Russia

E-mail: bond.julia@mail.ru

Abstract. The article is devoted to the issues of coordinating the values of social indicators of the region and economic indicators of the activities of economic entities in the context of improving the quality of life of the population. The paper presents the general mathematical problem of coordinating the social and economic indicators of the region and the general mathematical problem of regulating an economic entity. The peculiarity of the proposed approach is to take into account the different attitudes of economic entities to the solution of social problems of the region, the parameter of loyalty is adopted as a quantitative assessment. For the practical implementation of the tasks, mechanisms and algorithms have been developed that allow regional authorities to select the type of regulation for a particular business entity and calculate the values of the regulation parameters that ensure an improved quality of life. The software implementation of the algorithms makes it possible to make an informed choice of the parameters of the regulatory action.

1. Introduction

The socio-economic development of the region as a complex process of qualitative transformations has a certain direction. Despite the uniqueness of each territorial entity, the vector of priority directions of regional development is determined by the system of goals, the achievement of which will ensure the solution of the tasks relevant to the state.

Currently, one of the most important social and economic tasks of any state is the creation of conditions that ensure a decent standard and high quality of life for its citizens [1, 2]. Its solution implies, in our opinion, not so much the overcoming of a whole complex of social problems specific to the country as a whole, but rather an orientation of the policy of each of its regions towards the achievement of such values of social development indicators that would meet the needs of the majority of the population. The primary role in choosing the priority directions of such a policy belongs, of course, to regional authorities.

Our analysis of the current state of a number of Russian regions made it possible to identify a number of pressing social problems, among which is a rather high level of unemployment, crime, inequality and poverty, low fertility, etc. [3]. Note that a significant part of these problems has
an economic basis. Therefore, their decision at the level of the regional administration should be based on the active involvement of economic entities, their economic capabilities and resources in solving pressing social problems. Convincing evidence of the effectiveness of such a policy is served by global management experience in the most successful regions [3, 4].

The processes of interaction between regional authorities and business representatives are in their essence processes of coordinating interests, and require from regional authorities the availability of a developed and effective system of incentives, motivating and regulating influences. The positive result of this interaction is the achievement of the values of social indicators of the region specified by the administration (planned) levels enshrined in regional development strategies and programs. The problem of harmonization of interests in socio-economic systems has repeatedly become the subject of discussion, versed in many studies and in various aspects.

General theoretical foundations for coordinating the interests of participants in economic relations are laid in works on the theory of economic mechanisms (L. Gurwitz, E. Maskin, R. Myerson, and others [5, 6], as well as in research on incentives and economic behavior (J. Stigler, J. Tyrol and others) [7].

Among the studies on the development of specific mechanisms for the coordination of interests in domestic management practice, we note the work of K. A. Bagrinovsky, affecting the task of coordinating planning decisions [8], the works of V. N. Burkov, D. A. Novikov, V. N. Kuznetsov, which present mechanisms for coordinated management of active organizational systems [9, 10] and many more [11, 12]. The issues of coordinating interests in the socio-economic systems of the regional level were considered in the works of E. V. Orlova [13], devoted to the development of agreed resource allocation mechanisms, M. I. Geraskin, affecting the coordination of economic interests in corporate structures and the process of interregional interaction [14] and many more. Other General ideas for harmonizing the interests of the social and economic systems of the region in the context of solving pressing social problems for the region were previously discussed in the works of I. V. Goroshko, Yu. V. Bondarenko [3, 15].

The purpose of this article is a formalized description of the task of coordinating social and economic indicators of the regions development and the development of a set of algorithms and mechanisms for its solution, the basis of which are mathematical models and methods. A distinctive feature of the proposed approach is to take into account the different attitudes of economic entities to the solution of social problems of the region, the parameter of loyalty is taken as a quantitative assessment.

2. Materials and methods
2.1. General description of the approach
The socio-economic system of the region will be considered as a set of interrelated elements and relations arising in the process of production, distribution, exchange and consumption of tangible and intangible benefits while ensuring the livelihoods and development of the regions society. The structure of such a system will be represented as a combination of the following elements:

1) regional authorities (regional administration);
2) social system of the region;
3) regional economic system.

**Social system of the region** — community of individuals living in the region, united by forms of joint activity, as well as interests and needs for a high quality of life in the region.

**The economic system of the region** is considered as a set of economic entities located on the territory of the region and realizing economic interests in the process of economic activity. We will identify his leadership with each business entity. We believe that the management is
interested in the choice and implementation of such a direction of development of the enterprise, which will ensure maximum profit.

The state of the social system in the region is diagnosed by regional authorities based on an analysis of a number of social indicators (indicators). In the Russian regions, the most important are such indicators as: the number of registered crimes, the unemployment rate, real and nominal monetary incomes, the subsistence minimum, the level of poverty, the overall fertility rate, etc. The planned (“ideal”) values of these indicators are determined by the regional authorities and are displayed in the regional development strategy.

We believe that $K$ social indicators have been chosen to assess the socio-economic status and development of one specific region. The set of their values $c_{kt} \ (k = 1, \ldots, K)$ at each moment $t$ of the considered discrete time interval $t = 1, \ldots, T$ is represented in the form of a vector $c_t = (c_{1t}, \ldots, c_{Kt})$.

Considering that the time clock is one year, we assume that the administration of the region at each time moment of the period under review determines:

- $\ddot{c}_{kt}$ — ”ideal” value of each $k$-th social indicator, on the basis of which the vector of ”ideal” values is formed $\ddot{c}_t = (\ddot{c}_{1t}, \ldots, \ddot{c}_{Kt})$;
- $\Omega_{kt}$ — the area of permissible deviations of the actual value of each social indicator $c_{kt}$ from the ”ideal” $\ddot{c}_{kt}$, Cartesian product of which $\Omega_t = \prod_{k=1}^{K} \Omega_{kt}$ we call the ideal region.

Among the economic indicators of the activities of business entities, we will single out those that have the most significant impact on the values of social indicators (as confirmed by the estimates of $t$ — Student statistics). These primarily include: the value of the average monthly wage (nominal and real); the number of vacancies, as well as the number of employees in enterprises; prices for products of regional producers (in particular, for the goods included in the minimum consumer basket).

We introduce the following notation:

- $M$ — the number of selected economic indicators of activity of economic entities in the region, $m$ — index number ($m = 1, \ldots, M$);
- $s_{mt}$ — value of $t$ $m$-th economic indicator of the activity of the economic entity $i \ (i=1, \ldots, n)$ at the time $t$, where $m = 1, \ldots, M$, $i = 1, \ldots, n$.

Based on the values $M$ of economic indicators, for each economic entity $i \ (i=1, \ldots, n)$ we will form a multitude $\{s_{it} = (s_{1it}, \ldots, s_{Mit}), \ t = 1, \ldots, T\}$. Each element $s_{it}$ will be defined as a set of values of economic indicators of the activity of the $i$-th regional economic entity at a time $t$.

In accordance with the stated approach, the harmonization of the values of social and economic indicators of development at each moment in time $t$ is carried out through the targeted impact of regional authorities on the economic activities of each economic entity (through regulation), which we denote by $v_{it}, \ i = 1, \ldots, n$. An example of such regulatory impacts on the part of the regional administration can be a change in the corporate income tax rate for enterprises, land tax or property tax, the provision of special lending conditions, administrative coercion, the allocation of a resource (including financial), etc.

For convenience, we consider $v^i = (v_{1t}^i, \ldots, v_{Tt}^i)$ — the sequence of regulatory actions of regional authorities on the economic activities of the $i$-th of the economic entity over the entire selected time interval, and also $\tilde{v}_t = (v_{1t}^1, \ldots, v_{Tt}^n)$ — a set of regulatory actions of the managing center on the economic entities of the region at the moment of time $t$. In the notation adopted by us, the admissibility of regulatory action means the belonging $\tilde{v}_t$ to a given (permissible) set $\tilde{V}_t$, that is $\tilde{v}_t \in \tilde{V}_t, \ \forall \ t = 1, \ldots, T$. 

3
Depending on the regulation \( v^i = (v^i_1, \ldots, v^i_T) \), the management of the enterprise \( i \) forms the control \( u^i_t \), that is permissible under the given regulatory impact (i.e. \( u^i_t \in U^i_t(v^i_t) \)). As a result of this management, the economic indicators \( s^i_t \) of an economic entity change in accordance with a certain rule \( s^i_t = g^i_t(s_{i-1}^i, u^i_t) \). Such changes entail changes in the social indicators of the regions development. Without loss of generality of reasoning, we will assume a zero delay for these changes, that is:

\[
    c_t = \varphi \left( c_{t-1}; s^1_t, s^2_t, \ldots, s^n_t \right).
\]  

Dependence (1) shows that a correspondence can be established between the "ideal" values of social indicators \( \overline{c}_t \) (\( t = 1, \ldots, T \)) and the "ideal" values of economic indicators ensuring their achievement for each economic entity \( \overline{s}^i_t = (\overline{s}^i_{1t}, \ldots, \overline{s}^i_{Mt}) \) (where \( \overline{s}_{mt} \) — "ideal" value of the \( m \)-th of the economic indicator of an economic entity with an index \( i \) at the moment of time \( t \) (\( m = 1, \ldots, M, i = 1, \ldots, n \)).

In this case, we will assume that the management of each enterprise behaves rationally, and the choice of the best management is determined by it based on the optimality criterion, which takes into account the following conditions:

1) management is interested in getting the company maximum profit;
2) carrying out activities in the region, the management of the enterprise should be interested in the high quality of life of the population, and therefore one or another degree seeks to support the administration of the region in solving problems that ensure the achievement of the required values of social indicators. This property, in our opinion, is objectively inherent in varying degrees, economic entities of the region. In [3], we called this property loyalty, and as its quantitative assessment we suggested using the loyalty parameter \( \lambda_i \): \( \lambda_i \in [0, 1] \), \( i = 1, \ldots, n \).

Mathematically, such an optimality criterion can be represented by a pair of functions:

- \( F_1(s^i) = -\frac{1}{T \cdot M} \sum_{t=1}^{T} \sum_{m=1}^{M} \left( \frac{\overline{s}^i_{mt} - \overline{s}_{mt}}{\overline{s}_{mt}} \right) \) — normalized estimate of the total deviation of economic indicators of the \( i \)-th of economic entity from the "ideal" values of these indicators, where \( s^i = (s^i_1, \ldots, s^i_T) \) — a set of values of economic indicators of the \( i \)-th of economic entity, ordered by time;
- \( F_2(v^i, u^i, s^i) = \frac{1}{\pi} \sum_{t=1}^{T} \pi_t(v^i_t, u^i_t, s^i_t) \) — normalized total profit of the economic entity for the considered time interval, where \( \pi_t \) — the amount of profit of the economic entity at time \( t \), \( u^i = (u^i_1, \ldots, u^i_T) \) — set of management offices of the \( i \)-th of the enterprise, \( \pi \) — estimate of the maximum possible total profit for the period.

The value of the loyalty parameter of the business entity allows to evaluate the priority of each of the components of the optimality criterion described by us and proceed reasonably to the following form of its presentation:

\[
    G_t(v^i, u^i, s^i) = \lambda_i F_1(s^i) + (1 - \lambda_i) F_2(v^i, u^i, s^i) \rightarrow \max.
\]

2.2. Statement of the task of coordinating social and economic indicators of the development of a region

Let us formulate a general mathematical problem of coordinating social and economic indicators of the development of a region:

- **The task of the regional authorities (top level):** find the regulation \( \hat{v} = \hat{v}_1, \ldots, \hat{v}_T \in \prod_{t=1}^{T} \hat{V}_t \) of economic entities, providing at each time point \( t = 1, \ldots, T \) a change in the values of social indicators of the development of a region \( c_t \) within an established ideal area: \( c_t \in \Omega_t \), \( \forall t = 1, \ldots, T \).
At the same time, according to (1), the values of social indicators at each moment of time $t$ are determined by the function $c_t = \varphi(c_{t-1}; s^1_t, s^2_t, \ldots, s^n_t)$, $n$ arguments which — the economic performance of economic entities $s^1_t, s^2_t, \ldots, s^n_t$ — calculated on the basis of solving $n$ the tasks of the management of economic entities.

- **Tasks of management of business entities (lower level):** for each of $i$-th of economic entity ($i = 1, \ldots, n$) the regulation $v^i = (v^i_1, \ldots, v^i_T)$ determines the best management of the economic entity, taking into account its loyalty parameter. Such management for the management of an enterprise with an index $i$ is formed as a solution to the next optimization problem:

$$G_i(v^i, u^i, s^i) = \lambda_i \cdot F_1(s^i) + (1 - \lambda_i) \cdot F_2(v^i, u^i, s^i) \to \max,$$

with:

$$\begin{align*}
&u^i_t \in U^i_t(v^i_t); \\
&s^i_t = g_i(s^i_{t-1}, u^i_t), \quad t = 1, \ldots, T; \\
&s^i_0 = \tilde{s}^i.
\end{align*}$$

In model (2), the values of economic indicators are indicated $\tilde{s}^i$ at the moment of time $t = 0$ (initial conditions).

It should be noted that the practical implementation of the general mathematical task presented by us to harmonize the social and economic indicators of the regions development is associated with a number of difficulties caused by the need to take into account a large number of business entities operating in the region. To overcome this problem, we propose to look in the direction of identifying $n'$ ($n' \ll n$) the most significant economic entities in the region — large city-forming enterprises and medium-sized enterprises. In this case, the solution of the problem is reduced to the solution of $n'$ problems, each of which we define as a *general mathematical task of regulating an economic entity in order to the coordination of social and economic indicators of the development of a region*.

In order to implement such a transition, the regional authorities should, for each of the economic entities allocated to them, determine the set $\tilde{S}_t$ of permissible changes in the economic performance indicators at a point in time $t = 1, \ldots, T$. The belonging of economic indicators of an economic entity to such a multitude ensures in its totality the achievement by social indicators of an ideal area. It is obvious that the "ideal" values of economic indicators of economic entities belong to this area.

The presented assumption makes it possible, without loss of generality, to proceed to the consideration of some selected business entity, the index $i$ of which, for convenience, we will omit, while maintaining the accepted notation.

The general mathematical task of regulating an economic entity in order to ensure the coordination of social and economic indicators of the development of a region:

- **The task of the regional authorities (top level):** find the regulation of the economic entity of the region $v = (v_1, \ldots, v_T)$, ensuring the change in the values of indicators of its economic activity $s_t$ within the established area: $s_t \in \tilde{S}_t$, $\forall t = 1, \ldots, T$.

At the same time, the set of values of economic indicators of an economic entity $s = (s_1, \ldots, s_T)$, to which regional authorities are guided, forming a regulatory impact, is calculated on the basis of solving the lower level problem.

- **The task of managing an economic entity (lower level):** under regulation $v$ to find such management $u = (u_1, \ldots, u_T)$ by an economic entity, which is the solution to the problem:
\[ G(v, u, s) = \lambda \cdot F_1(s) + (1 - \lambda) \cdot F_2(v, u, s) \rightarrow \text{max}, \]

\[
\begin{align*}
  u_t &\in U_t(v_t); \\
  s_t & = g(s_{t-1}, u_t), \quad t = 1, \ldots, T; \\
  s_0 & = \tilde{s}.
\end{align*}
\]

The formulated tasks formed the basis of the proposed mechanisms for the coordination of social and economic indicators of the regions development. A feature of such mechanisms is that the choice of the type of regulatory impact for a particular business entity is based on the value of its loyalty parameter.

**Loyalty parameter of each business entity** \( \lambda \) we understand as an indicator characterizing the compliance of the parameters of an economic entity with the requirements (tasks, installations) formed by the regional authorities [14]. Its values range from 0 to 1 (\( \lambda \in [0, 1] \)) — from complete nonconformity (\( \lambda = 0 \)) to full compliance (\( \lambda = 1 \)).

For each business entity, the loyalty parameter \( \lambda \) is determined based on the ratio:

\[ \lambda = \min\{\lambda^{soc}, \lambda^{econ}\}, \]

where \( \lambda^{soc} \) — integral assessment of the compliance of the parameters of the municipality in which the entity operates, with the social requirements of the regional administration; \( \lambda^{econ} \) — integral assessment of the compliance of economic indicators of the business entity with the economic requirements of the regional administration; \( \lambda^{soc}, \lambda^{econ} \in [0, 1] \).

Each component of formula (4) is defined by us as an integral assessment of the compliance of object indicators with the requirements; therefore, the methods for calculating them will be identical. In the general case, we denote such an estimate by \( \lambda' \), where \( \lambda' \in \{\lambda^{soc}, \lambda^{econ}\} \).

When conducting practical calculations \( \lambda^{soc} \) and \( \lambda^{econ} \) reasonably multiplicative representation of each of them based on the values \( J \) of particular indicators:

\[ \lambda' = \prod_{j=1}^{J} \lambda_j, \]

where \( \lambda_j = 1 - \varepsilon_j(1 - \mu_j) \), \( \mu_j = \frac{z_j - z_j^{\min}}{z_j^{\max} - z_j^{\min}} \), \( \varepsilon_j = \frac{z_j^{\star} - z_j^{\min}}{z_j^{\max} - z_j^{\min}} \) — accordingly, the assessment of compliance of \( j \)-th of a particular private indicator \( z_j \) with the requirements \( z_j^{\star} \); normalized value of the indicator; normalized value requirements for the indicator.

### 2.3. Mechanisms for coordinating social and economic indicators of the regions development

The mechanisms for coordinating social and economic development indicators will be considered in the context of a set of procedures that allow regional authorities:

1) to choose the type of regulation of a specific business entity in order to ensure the harmonization of social and economic indicators of development;

2) to determine the values of the regulatory parameters of economic entities, ensuring the coordination of social indicators of the development of the region and economic indicators of their activities.

The developed complex of mechanisms for coordinating social and economic indicators of regional development is implemented on the basis of a set of algorithms and mechanisms:

- loyalty mechanism (regulation);
- regulatory effects generation algorithms.
Loyalty mechanism \((\lambda — \text{regulation})\) developed by us as a procedure that allows you to organize the process of choosing the types of regulation of economic entities in accordance with the value of the loyalty parameter of each of them and motivating in the long term to increase it.

The typology of regulatory impacts of the regional administration is carried out on the basis of the analysis of the most common approaches used in practice. We will distinguish:

- motivation, as an influence of the regional administration, corresponding to a change in the pre-respects of the management of an economic entity in such a way that it becomes advantageous to focus on achieving the goals of the managing center;
- coercion, which is an impact on the permissible number of directorates of the management of an economic entity by imposing strong fines, imposing additional restrictions, etc.;
- support — impact aimed at expanding the permissible number of managements of the management of an economic entity by proposing strategies that benefit both economic entities and the region;
- the conviction of building such a system of interaction between the administration of the region and the management of the enterprise, in which the latter independently carry out the solution of regional tasks.

Here are the possible combinations of types of regulatory actions that correspond to certain values of the loyalty parameter of an economic entity:

- \(\lambda = 0\) — motivation and coercion in the form of organizational, personnel decisions, etc.;
- \(\lambda \in (0, 1)\) — motivation; coercion; support;
- \(\lambda = 1\) — support and persuasion in the form of promotion, replication of experience, etc.

The practical implementation of each of the above types of regulation is based on the developed algorithmic and software complex, which makes it possible to find the parameters of the regulatory impact, ensuring the solution of the coordination problem. One of the constituent elements of the complex is an algorithm for forming regulation based on a change in the tax rate on profits. We are talking about a reasonable reduction of the regional component of the income tax rate in order to induce (motivate) an economic entity to achieve the required values of economic indicators of development.

The algorithm for forming regulation based on a change in the income tax rate is based on the "reward/punishment" rule. If the requirements of the regional administration are met, the business entity is guaranteed a reduction in the income tax rate. The magnitude of such a reduction should, on the one hand, motivate the leadership of an economic entity to support the solution of social problems (the achievement of economic indicators \(S_t, \forall t = 1, \ldots, T\)), on the other hand, to be as low as possible. To describe the steps of the algorithm, we make additional assumptions:

- the profit of the economic entity at each moment in time \(t = 1, \ldots, T\), as a component of the objective function of the economic entity of the task (3), is presented in the following form:
  \[
  \pi_t(\alpha_t, u_t, s_t) = (1 - \alpha_t) \cdot \tilde{\pi}(u_t, s_t),
  \]
  where \(\tilde{\pi}_t(\cdot)\) — enterprise profit at time \(t\) without profit tax; \(\alpha_t\) — corporate tax rate at time \(t\).

The objective function of the task of the lower level (3) in this case will have the following form:

\[
G(\alpha, u, s) = \lambda \cdot F_1(s) + (1 - \lambda)F_2(\alpha, u, s), \quad \text{where} \quad F_1(s) = -\frac{1}{T \cdot M} \sum_{t=1}^{T} \sum_{m=1}^{M} \left( \frac{s_{mt} - \tilde{s}_{mt}}{s_{mt}} \right)^2;
\]
\[ F_2(\alpha, u, s) = \frac{1}{T} \sum_{t=1}^{T} (1 - \alpha_t) \tilde{p}_t(u_t, s_t); \quad \alpha = (\alpha_1, \ldots, \alpha_T); \]

- the minimum tax rate on the profit of the enterprise is \( \alpha; \) the maximum size of the income tax rate is equal to the one in force at the initial moment of time, i. e. is \( \eta. \)

**Algorithm of formation of regulation based on the change in the tax rate on profits**

**Stage 0.** Setting the initial conditions: the lower limit of the tax rate \( \alpha; \) constants \( \varepsilon > 0 \) — the value of "encouragement" (perhaps arbitrarily small) for the achievement of economic indicators of the established values.

**Stage 1.** The calculation of the optimal result \( G^*, \) which can provide an economic entity with the current (fixed in size \( \eta) \) tax rate on profits, as the optimal value of the objective function of the task:

\[
G(\eta, u, s) = \lambda \cdot F_1(s) + (1 - \lambda) \cdot F_2(\eta, u, s) \rightarrow \max,
\]

\[
\left\{ \begin{array}{l}
u_t \in U_t; \quad s_t = g(s_{t-1}, u_t), \ t = 1, \ldots, T; \\
\lambda \cdot F_1(s) + (1 - \lambda) \cdot F_2(\alpha, u, s) \geq G^* + \varepsilon; \\
s_0 = \tilde{s}.
\end{array} \right.
\]

**Stage 2.** Determination of the size of the motivating rate of income tax based on the solution of the optimization problem:

\[
\sum_{t=1}^{T} \alpha_t \rightarrow \max,
\]

\[
\left\{ \begin{array}{l}
u_t \in U_t; \quad s_t = g(s_{t-1}, u_t), \ t = 1, \ldots, T; \\
\lambda \cdot F_1(s) + (1 - \lambda) \cdot F_2(\alpha, u, s) \geq G^* + \varepsilon; \\
s_0 = \tilde{s}, \ s_t \in \tilde{S}_t, \ t = 1, \ldots, T; \quad \alpha \leq \alpha_t \leq \eta, \ t = 1, \ldots, T.
\end{array} \right.
\]

If the problem has no solution, change the initial conditions by means of an acceptable reduction of constants \( \alpha \) and \( \varepsilon, \) go to Stage 1. Otherwise — \( \alpha^* = (\alpha_1^*, \ldots, \alpha_T^*) \) — optimal solution to the problem, go to Stage 3.

**Stage 3.** The administration of the region establishes the following tax rate for an economic entity:

\[
v_t = \left\{ \begin{array}{ll}
\alpha_t^*, & s_t \in \tilde{S}_t, \\
\eta, & s_t \notin \tilde{S}_t.
\end{array} \right. \quad t = 1, \ldots, T. \quad \text{Stop.}
\]

3. **Results and discussion**

The practical implementation of the described mechanisms was carried out on the basis of data from a number of enterprises in the Voronezh Region in cooperation with the municipal administrations. In this article, detailed calculations carried out on the basis of one of the enterprises of the Rossoshansky district.

Initially, we will calculate the enterprise loyalty parameter.

As the particular indicators of the integral assessment of the compliance of the parameters of the Rossoshansky district with social requirements \( \lambda^\text{soc} \) we choose: 1 — "crisis" quality of life index; 2 — labor market tension ratio; 3 — average monthly nominal wages in the district, rubles; 4 — number of registered crimes per 10 thousand people. Through \( z_1^\text{soc}, z_2^\text{soc}, z_3^\text{soc} \) and \( z_4^\text{soc} \) we denote the corresponding actual values of the given indicators. The minimum and maximum values of these indicators required to calculate the loyalty parameter are denoted as \( (z_j^\text{soc})^{\min} \) and \( (z_j^\text{soc})^{\max}, \) and the required values are as \( (z_j^\text{soc})^*, j = 1, \ldots, 4. \)
As private indicators of the integral assessment of the compliance of the economic indicators of the enterprise with the economic requirements, we select the following: 1 — index of the average number of employees, %; 2 — the size of the average monthly nominal wage, rubles; 3 — the index of the average monthly nominal wage, %. The actual values corresponding to them are denoted by $z_{econ}^1$, $z_{econ}^2$, and $z_{econ}^3$. The minimum and maximum values of these indicators we denote as ($z_{p econ}^1$)$_{min}$ and ($z_{p econ}^1$)$_{max}$, and the required value is as ($z_{p econ}^1$)$_{p}$, $p = 1, 2, 3$.

Social indicators of the four municipalities of the Voronezh region, as well as data for the calculation $\lambda_{soc}$, including private estimates $\lambda_{soc}^j$, are presented in table 1; the economic indicators of the five economic entities (enterprises) of the Rossoshansky district, as well as data for the calculation of the integrated assessment $\lambda_{econ}$, including private evaluations $\lambda_{econ}^p$ of the selected enterprise 1, are presented in table 2.

| District name              | $z_{1 soc}$ | $z_{2 soc}$ | $z_{3 soc}$ | $z_{4 soc}$ |
|----------------------------|-------------|-------------|-------------|-------------|
| Rossoshansky district      | 0.37        | 2.2         | 17 962      | 98.5        |
| Semiluksky district        | 0.29        | 2.5         | 13 578      | 104.8       |
| Anninsky district          | 0.27        | 8.6         | 12 770      | 132.9       |
| Bobrovsky district         | 0.21        | 2.7         | 13 050      | 125.3       |

($z_{j soc}^1$)$_{min}$ = 0.21, ($z_{j soc}^1$)$_{max}$ = 0.37, ($z_{j soc}$)$_{p}$ = 0.3, ($z_{soc}^j$)$_{min}$ = 0.21, ($z_{soc}^j$)$_{max}$ = 0.37, ($z_{soc}^j$)$_{p}$ = 0.3, ($z_{soc}^j$)$_{p}$ = 0.3.

$\lambda_{j soc}$ (for Rossoshansky district) = 0.77

| Enterprises | $z_{1 econ}$ | $z_{2 econ}$ | $z_{3 econ}$ |
|-------------|--------------|--------------|--------------|
| P1          | 102.6        | 21 450       | 117.3        |
| P2          | 100.1        | 26 600       | 101.9        |
| P3          | 92.2         | 16 400       | 113.1        |
| P4          | 123          | 18 800       | 134.7        |
| P5          | 105.7        | 18 250       | 115.6        |

($z_{p econ}^1$)$_{min}$ = 92.2, ($z_{p econ}^1$)$_{max}$ = 123, ($z_{p econ}$)$_{p}$ = 100, ($z_{p econ}$)$_{p}$ = 100.

$\lambda_{econ}^p$ (for enterprise 1) = 0.27

Considering all private indicators are equivalent, for Rossoshansky district, by the formula (5) it is calculated that $\lambda_{soc} = 0.77 \cdot 0.81 \cdot 0.9 \cdot 1 \approx 0.56$.

The calculation results for enterprise 1 are as follows: $\lambda_{econ} = 0.27 \cdot 0.941 \cdot 0.72 \approx 0.19$. The coefficient of loyalty of enterprise 1 is: $\lambda = \min\{\lambda_{soc}, \lambda_{econ}\} = \lambda_{econ} = 0.19$.

The basis of the calculation of the loyalty of the enterprise based on private indicators:
the district’s compliance with social requirements: a “crisis” quality of life index; coefficient of tension in the labor market; average monthly nominal wages in the district, the number of registered crimes per 10 thousand people;

- compliance of economic indicators of economic entities with economic requirements: the index of the average number of employees; average monthly nominal wage; monthly average nominal wage index.

In order to reduce the unemployment rate in the area, the administration of the municipality defined the task of creating 25 and 30 new jobs at the enterprise in the next two years, respectively — $t = 1$ and $t = 2$.

In accordance with the loyalty control mechanism, coercion or inducement is recommended for this enterprise. The administration of the municipality selected the method of incentives, respectively defined the task of creating 25 and 30 new jobs at the enterprise in the next two years, respectively.

For the practical implementation of the proposed regulation generation algorithm based on the change in the income tax rate, an objective need arose to specify the lower level task (3) for the selected business entity.

Developed the next task of the subject of management of the economic system:

$$G = \lambda \cdot \left[-\frac{1}{T} \sum_{t=1}^{T} \left( \frac{L_t - \tilde{L}_t}{L_t} \right)^2 \right] + (1 - \lambda) \cdot \left[ \frac{1}{\pi} \sum_{t=1}^{T} (1 - \eta) \pi_t \right] \rightarrow \max_{\{\delta, \nu, \alpha \}}$$  

I. \[ \begin{aligned} L_t &= L_{t-1} + \Delta L_t, \\ \Delta L_t &\leq \tilde{N}_t, \quad L_t \geq \tilde{L}_t, \quad \Delta L_t = \frac{\nu_t \Phi_t}{\omega_t}, \quad t = 1, \ldots, T; \end{aligned} \]

II. \[ \begin{aligned} y_t &= f(K_t, \omega L_t), \\ Q_t \leq y_t \leq \tilde{Q}_t, \quad t = 1, \ldots, T; \end{aligned} \]

III. \[ \begin{aligned} \Phi_t^c &= \Phi_{t-1}^c - (\delta_{t-1} + \nu_{t-1}) \Phi_{t-1}^c + (1 - \eta) \pi_{t-1}, \quad t = 2, \ldots, T, \\ \Phi_t^b &= \Phi_{t-1}^b - (\delta_{t-1} + \nu_{t-1}) \Phi_{t-1}^b + \Delta \Phi_t^b, \quad t = 2, \ldots, T; \end{aligned} \]

IV. \[ \begin{aligned} \pi_t &= y_t - \left( \sum_{r=1}^{R} b_r y_t + \frac{1}{1 - \rho} \omega L_t + \gamma K_t \right), \quad \pi_t \geq \pi_0; \\ K_t &= (1 - \gamma) K_{t-1} + \delta_t \Phi_t, \quad t = 1, \ldots, T; \end{aligned} \]

V. \[ \begin{aligned} 0 \leq \delta_t \leq \delta_t \leq \tilde{\delta} \leq 1, \quad 0 \leq \nu_t \leq \nu_t \leq \tilde{\nu} \leq 1, \quad \delta_t + \nu_t \leq 1, \end{aligned} \]

VI. \[ \begin{aligned} K_0 &= \tilde{K}, \quad L_0 = \tilde{L}, \quad \Phi_0^c = \tilde{\Phi}_0^c, \quad \Phi_0^b = \tilde{\Phi}_0^b. \end{aligned} \]

The constraints of the problem (6) are arranged in the following semantic blocks: I describes the change in the number of people employed in the enterprise due to the creation of new jobs; II is associated with the volume of release; III reflects changes in financial assets; IV is associated with the formation of profits and changes in share capital; V includes constraints on model variables $\delta_t$, $\nu_t$; VI sets the initial conditions.

In model (6), the following notation is used: $1, \ldots, T$ — considered time interval, $t$ — moment of time ($t = 1, \ldots, T$); $\lambda$ — loyalty parameter; $L_t$ — workforce at time $t$; $\tilde{L}_t$ — established by the administration of the municipality (in accordance with social objectives) the number of jobs for the company at a time $t$; $\Delta L_t$ — number of jobs created at time $t$; $\tilde{N}_t$ — the number of unemployed able-bodied population of the region, allowing employment at the enterprise; $\tilde{L}_t$ — minimum amount of manpower at time $t$; $y_t$ — gross output at time $t$; $K_t$ — share capital at
the time $t$; $\omega$ — the average value for the period of nominal wages; $f(\cdot)$ production function; $Q_t$, $\overline{Q}_t$ — the boundaries of the projected demand for cholesterol products at the moment and $t$; $\Phi_t$ — production development fund at a point in time $t$, formed from own funds $\Phi^c$ and external sources $\Phi^b_t$ (at the expense of $\Delta \Phi^b$); $\delta_t$ — share of financial resources spent on capital increase; $\delta_t$, $\overline{\delta}_t$ — respectively, the lower and upper limits of the allowable change in magnitude $\delta_t$; $v_t$ — the proportion of funds allocated for the creation of new jobs, including an increase in the wage fund; $\overline{v}_t$, $\underline{v}_t$ — respectively, the lower and upper limits of the allowable change in magnitude $v_t$; $\eta$ — current income tax rate; $\pi_t$ — profit at the moment $t$, $\underline{\pi}_t$ — lower limit of allowable profit; $\rho$ — share of social contributions from the wage fund; $b_r$ — the ratio of the cost of $r$-th of the type of material resource per unit of production, $r = 1, \ldots, R$, where $R$ — amount of material resources; $\gamma$ — rate of depreciation of fixed capital (fixed assets); $\omega'_t$ — average cost of creating one workplace at a time $t$; $\overline{K}$, $\overline{L}$, $\overline{\Phi^c}$, $\overline{\Phi^b}$ — initial values (at time $t = 0$) of the relevant indicators.

Model (6) is specified on the basis of data of statistical and accounting reports for the period 1996–2016, and practically implemented as a scalar optimization problem with a convex objective function and a nonlinear constraint system. Programmatic calculations showed that under the conditions, if at the initial moment of time the number of employees of the enterprise was $L = 305$ people, and the regulating influence of the regional administration is absent, this indicator will be, respectively $L_1 = 332$ people ($t = 1$) and $L_2 = 351$ people ($t = 2$).

The implementation of the regulation formation algorithm based on a change in the income tax rate showed that at the time $t = 1$ point a change in the income tax rate is not required, but at the time $t = 2$ is necessary. The algorithm for forming regulation based on a change in the income tax rate yielded the following results: reducing the regional component of the enterprise income tax rate at a time $t = 2$ from $\eta = 18\%$ to $\alpha^* = 14.6\%$ motivates the company to create the required number of jobs (in this case, the estimated value of the workforce $L_2^*$ was 363 people).

4. Conclusion

The approach proposed in this paper for solving the urgent problem of regional development — achieving a high quality of life — is based on an understanding of the need to carry out a targeted impact on economic entities of the region, motivating them to achieve such indicators of economic activity that will provide the required values of social indicators of regional development. For this purpose, a general mathematical problem has been formulated for coordinating social and economic indicators of development, which made it possible in theory to substantiate coordination mechanisms. A distinctive feature of our proposed approach is taking into account the interest of business entities not only in the implementation of purely economic goals (primarily profit maximization), but also in improving the quality of life of the population of the region in whose territory they operate. This property is designated as loyalty, and as its quantitative assessment a special indicator is adopted — the loyalty parameter, the methods for calculating which are presented in this paper. The formed mechanisms for the harmonization of indicators provide support for decision-making on the regulation of the economic activities of economic entities in order to achieve the required values of social indicators. The introduction of the developed mechanisms and algorithms into management practice allowed not only to justify the use of certain forms and methods of regulation, to calculate specific values of parameters of regulatory influences, but at the same time, and determined the future directions of development of the proposed approach.
References

[1] Gavrilova I A and Makarov A D 2017 Fundamental’nye issledovaniya (Fundamental research) 4-1 133–137
[2] Semova N G, Mataev A S and Vorontsova A V 2010 Vestnik moskovskogo gosudarstvennogo oblastnogo universiteta. Seriya: ekonomika (Bulletin of the Moscow State Regional University. Series: economics) 4 137–139
[3] Bondarenko Yu V and Goroshko I V 2015 Mekhanizmy soglasovaniya pokazatelej social’no-ekonomicheskogo razvitiya regiona i rol’organov vnutrennih del v ih realizacii (Mechanisms for the Coordination of Indicators of Socio-economic Development of the Region and the Role of Internal Affairs Bodies in Their Implementation) (Moscow: Academy of Management of the Ministry of Internal Affairs of Russia)
[4] Mc Dermott G A 2007 Politics & Society 35(1) 103–143
[5] Maskin E S 2008 American Economic Review 98(3) 567–576
[6] Myerson R 1986 Econometrica 54 323–358
[7] Maskin E and Tirole J 1990 Econometrica 58 379–409
[8] Bagrinovskiy K A 1977 Osnovy soglasovaniya planovyh reshenij (Basics of Coordinating Planning Decisions) (Moscow: Science)
[9] Burkov V, Shichepin A, Irikov V and Kondratiev V 2019 Studies in Systems, Decision and Control 181 29–38
[10] Novikov D A 2007 Teoriya upravleniya organizacionnymi sistemami (Theory of Management of Organizational Systems) (Moscow: Publishing house of physical and mathematical literature)
[11] Aleskerov F and Subochev A 2013 Journal of Global Optimization 56 737–756
[12] Burkov V N, Kuznetsov V N and Pavlov V A 2009 Teoriya aktivnyh sistem: Tr. Mezhdunar. nauchno-praktich. Konf. (The Theory of Active Systems: Works of International Scientific and Practical Conf) vol 1 (Moscow, Russia) pp 55-58
[13] Orlova E V 2017 Proceedings of the Mathematical Modeling Session at the International Conference Information Technology and Nanotechnology (MM-ITNT 2017) 1904 1–6
[14] Geraskin M I 2005 Soglasovanie interesov v korporativnyh strukturah (Coordination of Interests in Corporate Structures) (Moscow: IPU RAS)
[15] Goroshko I V, Bondarenko Yu V and Sidorova V G 2014 Ekonomika i menedzhment sistem upravleniya (Economics and management of management systems) 13(3-1)