Cross-correlation of the Polarizations of the 21-cm and Cosmic Microwave Backgrounds

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The polarization of the 21-cm radiation from the epoch of reionization arises from Thomson scattering of 21-cm photons from free electrons and provides information that complements that from the intensity fluctuation. Previous work showed that a direct detection of this signal will be difficult, and hinted that the signal might be enhanced via correlation with other tracers. Here, we discuss the cross-correlation between the cosmic microwave background (CMB) polarization and the 21-cm polarization. We treat reionization using an analytical model with parameters calibrated by semi-numerical simulations. We then derive the cross-correlation angular power spectrum using the total-angular-momentum formalism. We also provide a noise analysis to test against two closely related, but subtly different, null hypotheses. First, we assume no reionization as a null hypothesis, and determine how well this null hypothesis could be ruled out by an observed 21cm-CMB polarization correlation. Second, we determine how well the null hypothesis of no 21-cm polarization can be ruled out by seeking the cross-correlation, assuming reionization is established from the CMB. We find that the first question could be answered by a synergy of ambitious next-generation 21-cm and CMB missions, whereas the second question will still remain out of reach.

I. INTRODUCTION

The redshifted 21-cm line of the hydrogen hyperfine transition provides both a spatially and temporally resolved image of the baryons’ growth in inhomogeneity, collapse, heating, and reionization [1–3]. A wide range of cosmological and astrophysical information can be derived from it. Previous work focused primarily on the intensity signal, showing its potential power in constraining fundamental physics [4, 5], star and galaxy formation [6], and intergalactic medium [7].

More can be learned from the polarization signal. The dominant contribution to the 21-cm polarization arises in the same way as the polarization of the cosmic microwave background (CMB). In the reionized Universe, the Thomson scattering between of a radiation quadrupole from a free electron generates a linear polarization. This effect has first been explored in Refs. [8] and [9]. Ref. [8] estimates the strength of this signal by assuming a relatively simple model of reionization, and claims that it can be detected by Square Kilometre Array (SKA) with a one-month observation time. Ref. [9], with refined reionization modeling via 21cmFAST, concludes that this signal is smaller than the SKA sensitivity. However, Ref. [9] also points out that the signal may still be detected by cross-correlation with other probes.

In this paper, we discuss the cross-correlation between the 21-cm polarization and the CMB polarization. The CMB polarization, produced also by the Thomson scattering, is generated in roughly two different epochs—one right before recombination, the other after reionization. The latter, as we will show, gains contribution from density perturbations within a certain comoving-wavelength range that also give rise to the 21-cm polarization. This generates a cross-correlation. By correlating the well-established CMB signal with a to-be-detected signal, we enhance the sensitivity to the target signal.

To assess the prospects to detect this cross-correlation, we evaluate the ability of future measurements to answer two related but subtly different questions. In the first, we determine the possibility to rule out the null hypothesis of no reionization from this cross-correlation alone. In the second, we assume that reionization has been well established from the CMB-polarization measurement and ask whether the cross-correlation can be detected under this assumption. We conclude that the amplitude of the cross-correlation is large enough to distinguish it from the null hypothesis of no reionization, but it cannot be detected under the null hypothesis of a well established detection of the reionization in the CMB polarization.

The rest of the paper is organized as follows. In Section II, we present the theoretical calculation of the cross-correlation signal. In Section III, we analyze the detection prospect of this signal by next-generation CMB and 21-cm observations, while pointing out potential hurdles. We end in Section IV with some concluding remarks.

II. THEORY

The physical picture is illustrated schematically in Fig. 1, which shows the observable Universe in comoving coordinates. As described in the figure caption, we start by discussing the remote intensity anisotropy in Section II A; we then move to compute the induced linear polarization in Section II B. These steps are done for both the CMB polarization and the 21-cm polarization. Next, we use the results from these previous steps to cal-
account for the fractional perturbation $\delta_{21} \equiv T_{21}/\bar{T}_{21} - 1$, where $\bar{T}_{21}$ is the spatial average of $T_{21}$.

Since the Thomson scattering is achromatic, the linear polarization of a given frequency $\nu_0$ we observe is only due to the scattering of those 21-cm photons emitted at redshift $z_e = \nu_e/\nu_0 - 1$, and is independent of the redshift of the scattering. Here, $\nu_e = 1420\text{MHz}$ is the proper frequency of the hydrogen line. Thus the relevant intensity-anisotropy pattern (i.e. responsible for the observed polarization at frequency $\nu_0$) seen by an electron at $(\vec{x}, \eta)$ from direction $\hat{u}$ is

$$\Theta_{21, \nu_0}(\hat{u}; \vec{x}, \eta) \equiv \delta_{21}[\vec{x} + (\eta - \eta_e)\hat{u}, \eta_e],$$

where $\eta_e$ is the conformal time at redshift $z_e$. We define the quadrupole tensor

$$t_{\alpha\beta}^{21, \nu_0}(\vec{x}, \eta) \equiv \int d^2\hat{u} (3u_\alpha u_\beta - \delta_{\alpha\beta})\Theta_{21, \nu_0}(\hat{u}; \vec{x}, \eta).$$

(2)

It will be useful later to express $t_{\alpha\beta}^{21, \nu_0}$ using $\delta_{21}$ in the Fourier space. With detailed derivation given in Appendix A, we write the result here,

$$t_{\alpha\beta}^{21, \nu_0}(\vec{k}, \eta) = -12\pi j_2[k(\eta - \eta_e)] \left( k_\alpha k_\beta - \delta_{\alpha\beta}/3 \right) \delta_{21}(\vec{k}, \eta).$$

(3)

Here, $j_2(x)$ is the spherical Bessel function; $\delta_{\alpha\beta}$ is the Kronecker delta symbol; $k \equiv |\vec{k}|$ and $\vec{k} \equiv \vec{k}/k$ are the magnitude and direction of the wavevector $\vec{k}$.

Next, we discuss the the CMB intensity quadrupole. The polarization of CMB is generated both right before recombination and after reionization. However, only the latter contributes significantly to the cross-correlation with the 21-cm polarization. To see this, we note that, for a given free electron, the intensity quadrupole—thus the resulting polarization—is dominated by perturbations with a wavelength roughly the size of the electron’s last-scattering-surface radius. For those free electrons right before recombination, this size is roughly the duration $\Delta\chi_\star \sim 15\text{Mpc}$ of recombination (assuming recombination at $z_\star = 1090$ with a duration of $\Delta z_{\star} = 80$).

So, the recombination polarization pattern will peak at multipole $l \sim \chi_\star/\Delta\chi_\star \sim 10^3$, where $\chi_\star \sim 1.4 \times 10^4\text{Mpc}$ is the comoving distance to the last-scattering surface. At these high multipoles, the Limber approximation [10] implies that the 21-cm polarization, coming from a comoving distance different from $\chi_\star$, will have very limited correlation with the recombination CMB polarization.

The relevant CMB intensity quadrupole, seen by the free electrons in the reionized Universe, can then be simply calculated using the Sachs-Wolfe effect. Similar to the 21-cm case, the intensity-anisotropy pattern,

$$\Theta_{\text{CMB}}(\hat{u}; \vec{x}, \eta) \equiv -\frac{1}{3}\Phi[\vec{x} + (\eta - \eta_e)\hat{u}, \eta_e],$$

(4)

determines the quadrupole tensor

$$t^{\text{CMB}}_{\alpha\beta}(\vec{x}, \eta) \equiv \int d^2\hat{u} (3u_\alpha u_\beta - \delta_{\alpha\beta})\Theta_{\text{CMB}}(\hat{u}; \vec{x}, \eta).$$

(5)
Here, $\Phi$ is the conformal-Newtonian-gauge gravitational potential, and $\eta_*$ is the conformal time at recombination. This relation can be written in Fourier space, following the derivation in Appendix A, as

$$t_{ab}^{\text{CMB}}(\vec{k}, \eta) = 4\pi j_2[k(\eta - \eta_*)] \left( \hat{k}_a \hat{k}_b - \frac{\delta_{ab}}{3} \right) \Phi(\vec{k}, \eta_*) .$$  \hspace{1cm} (6)

We can re-express this equation in terms of the matter-density perturbation $\delta$ at conformal time $\eta$, by realizing that

$$\Phi(\vec{k}, \eta) = \frac{3H_0^2\Omega_m}{k^2a(\eta)} \delta(\vec{k}, \eta)$$  \hspace{1cm} (7)

is constant deep in the matter-dominated era, giving [11]

$$t_{ab}^{\text{CMB}}(\vec{k}, \eta) = \frac{6\pi H_0^2\Omega_m}{k^2a(\eta)} j_2[k(\eta - \eta_*)] \left( \hat{k}_a \hat{k}_b - \frac{\delta_{ab}}{3} \right) \delta(\vec{k}, \eta_*).$$  \hspace{1cm} (8)

Here, $H_0$ is the current Hubble parameter; $\Omega_m$ is the matter-density parameter; $a(\eta)$ is the scale factor at conformal time $\eta$. We formally switch the time slice from $\eta_*$ to $\eta$, so that we only need to refer to the equal-time correlation between $\delta$ and $\delta_{21}$ at the conformal time $\eta_*$ of the 21-cm emission.

### B. Polarization

The linear polarization tensor, as a function of position $\hat{n}$ on the sky, is

$$P_{AB}(\hat{n}) \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} Q(\hat{n}) & U(\hat{n}) \sin \theta \\ U(\hat{n}) \sin \theta & -Q(\hat{n}) \sin^2 \theta \end{pmatrix} ,$$  \hspace{1cm} (9)

where, $Q(\hat{n})$ and $U(\hat{n})$ are the Stokes parameters measured in the local coordinate frame \{\(\theta, \phi\}\}. The components of $P_{AB}$ are given in the coordinate chart \{\(\theta, \phi\)\}, indicated by the capital subscripts \(AB\). The $\sin \theta$-related factors follow from the fact that the chart \{\(\theta, \phi\)\} is orthogonal but not orthonormal. Later, we will mainly use the Cartesian components $P_{ab}$ of this polarization tensor, which can be obtained by embedding the unit sphere into the three dimensional space and performing a general coordinate transformation between \{\(\theta, \phi\)\} and \{\(x, y, z\)\}. In both representations, the polarization tensor is symmetric ($P_{AB} = P_{BA}$ and $P_{ab} = P_{ba}$) and trace free ($g^{AB}P_{AB} = 0$ and $g^{ab}P_{ab} = 0$, with $g_{AB} = \text{diag}(1, \sin^2 \theta)$ and $g_{ab} = \delta_{ab}$). The embedding also implies that $P_{ab}$ is transverse ($\hat{n}_aP_{ab} = 0$).

Both the CMB and the 21-cm polarization are produced by Thomson scattering between free electrons and unpolarized incident radiation possessing a quadropolar intensity anisotropy. This indicates that, for the probe $X = \text{CMB}$ or $21(\nu_o)$, we have

$$P^{X}_{ab}(\hat{n}) = \frac{\sqrt{2}}{16\pi} \int g(\eta) d\eta \Pi_{ab,ij}(\hat{n}) T^X_{ij}(\hat{n} - \eta, \eta),$$  \hspace{1cm} (10)

where the integral starts from $\eta_*$ for $X = \text{CMB}$ and $\eta_*$ for $X = 21(\nu_o)$, to the current conformal time $\eta_0$, and will be omitted from now on. Here, $g(\eta) \equiv e^{-\tau} d\tau/d\eta$ is the photon-visibility function, where the Thomson-scattering optical depth is defined as

$$\tau(\eta) = \int_\eta^{\infty} \frac{\sigma_T n_b x_e(\eta')}{a(\eta')^2} d\eta'.$$  \hspace{1cm} (11)

Here, $\sigma_T$ is the total cross section of Thomson scattering; $n_b$ the comoving number density of baryons; $x_e(\eta)$ the mean ionization fraction [see Eq. (34)]. Note that $\sigma_T n_b = 2.307 \times 10^{-5} (\Omega_b h^2)$ Mpc$^{-3}$ in terms of the baryon-density parameter $\Omega_b$. The tensor projector is

$$\Pi_{ab,ij}(\hat{n}) \equiv \mathcal{P}_{ai}(\hat{n})\mathcal{P}_{bj}(\hat{n}) - \frac{1}{2} \delta_{ij} \mathcal{P}_{ab}(\hat{n}),$$  \hspace{1cm} (12)

where $\mathcal{P}_{ij}(\hat{n}) \equiv \delta_{ij} - \hat{n}_i\hat{n}_j$. Eq. (10) must take this form because the integrand is the only possible way to project a three dimensional quadrupole tensor $t_{ij}$ to a polarization tensor $P_{ab}$ that is simultaneously symmetric, trace free, and transverse. The pre-factor in Eq. (10) can be determined by taking $\hat{n} = -\hat{z}$ and comparing to Eqs. (2.10) in Ref. [12].

### C. Angular power spectrum

The statistical property of $P^{X}_{ab}(\hat{n})$ is encoded in the angular power spectrum $C^{XY}_{LM} = \langle (P^{X}_{JM})^* P^{Y}_{JM} \rangle$, where the E-mode spherical-harmonic expansion coefficients are defined as

$$C^{XY}_{LM} \equiv \int d^2\hat{n} \left[ Y^{TE}_{(JM)ab}(\hat{n}) \right]^* P^{X}_{ab}(\hat{n}).$$  \hspace{1cm} (13)

Using the tensor spherical harmonics $Y^{TE}_{(JM)ab}(\hat{n})$ defined in Ref. [13]. There exists another family of B-mode expansion coefficients, obtained by projection onto $Y^{TB}_{(JM)ab}(\hat{n})$ (defined also in Ref. [13]), that will be zero under our assumption of no primordial gravitational waves.

The next step is to plug Eq. (10) into Eq. (13) and calculate the angular power spectrum. This task is immensely simplified by the use of total-angular-momentum (TAM) waves defined in Ref. [13], whose notation we shall now follow. Assuming only primordial scalar perturbation, the symmetric and trace-free tensor $t^{X}_{ab}$ can be expanded as

$$t_{ab}(\vec{x}, \eta) = \sum_{k,LM} t_{kLM}(\eta \vec{x}) \left[ \frac{4\pi i^k}{k^2} \Psi^{kL}_{(JM)ab}(\vec{x}) \right]$$  \hspace{1cm} (14)

in terms of the L-mode tensor TAM waves. Here, $\sum_k$ is a shorthand for $\int k^2dk/(2\pi)^3$. The projection in Eq. (10) is then simplified by the property

$$\Pi_{ab,ij}(\hat{x})\Psi^{kL}_{(JM)ij}(\hat{x}) = H^{L,TE}_{k}(kx)Y^{TE}_{(JM)ab}(\hat{x})$$  \hspace{1cm} (15)
of the TAM waves. Here the radial function $R^{L,TE}_J(kx)$ can be inferred from Eq. (38) of Ref. [14]. It will also be helpful to expand the matter-density perturbation $\delta$ and the 21cm-temperature perturbation $\delta_{21}$ in terms of the scalar TAM waves. To simplify the writing of parallel equations, we formally rename

$$\delta_{\text{CMB}}(\vec{x}, \eta) = \delta(\vec{x}, \eta) \quad \text{and} \quad \delta_{21, \nu_c}(\vec{x}, \eta) = \delta_{21}(\vec{x}, \eta),$$

and exclusively refer to the symbols on the left-hands sides when we abstractly express the probe using $X$ and $Y$. Now, the expansions can be compactly written as

$$\delta_X(\vec{x}, \eta) = \sum_{k, J, M} \delta^X_{k, J, M}(\eta) \left[ 4\pi i^J \Psi^k_{J, M}(\vec{x}) \right].$$

(17)

Similar to the Fourier-space relations, Eqs. (3) and (8), $t^X_{k, J, M}$ can also be related to $\delta^X_{k, J, M}$. With details given in Appendix B, we list the result here,

$$t^X_{k, J, M}(\eta) = T_X(k; \eta, \eta_c) \delta^X_{k, J, M}(\eta_c).$$

(18)

Here the quadrupole transfer functions $T_X$ are defined as

$$T_{21, \nu_c}(k; \eta, \eta_c) = 4\pi \sqrt{6} j_2[k(\eta - \eta_c)];$$

$$T_{\text{CMB}}(k; \eta, \eta_c) = -\frac{2\pi \sqrt{6} \Omega_m H_0^2}{k^2 a(\eta_c)} j_2[k(\eta - \eta_c)].$$

(19)

With this setup, we can plug Eq. (18) into Eq. (14), and then the result into Eq. (10) with the aid of Eq. (15). Finally, the orthonormality of $Y^T_{J, M(\alpha \beta)}(\vec{r})$ implies that

$$\mathcal{P}^X_{J, M} = \frac{\sqrt{2}}{4} i^J \int \frac{k^2 dk}{(2\pi)^3} T^X_J(k; \eta_c) \delta^X_{k, J, M}(\eta_c),$$

(21)

where the polarization transfer functions $T^X_J$ (not to be confused with $T_X$) are defined as

$$T^X_J(k; \eta_c) = g(\eta) d\eta \int P_{L, TE}^{L, TE}(k[\eta - \eta_c]) T_X(k; \eta, \eta_c).$$

(22)

The angular power spectrum then evaluates to

$$C^X_{J,Y} = \frac{1}{8} \int \frac{k^2 dk}{(2\pi)^3} P_{XY}(k; \eta_c) T^X_J(k; \eta_c) T^Y_J(k; \eta_c),$$

(23)

where the power spectrum is defined such that the products of the TAM coefficients have the expectation values

$$\langle \delta^X_{k, J, M}(\eta_c) \delta^Y_{k', J', M'}(\eta_c) \rangle = \delta_{kk'} \delta_{JJ'} \delta_{MM'} P_{XY}(k; \eta_c).$$

(24)

Here $\delta_{kk'}$ is a shorthand for $(2\pi)^3 \delta(k - k')/k^2$. Note that this definition coincides with the canonical definition using Fourier coefficients, as detailed in Ref. [13],

$$\langle \delta_X(\vec{k}, \eta_c) \delta_Y(\vec{k'}, \eta_c) \rangle = (2\pi)^3 \delta(\vec{k} - \vec{k'}) P_{XY}(k; \eta_c).$$

(25)

D. Reionization model

The question now reduces to obtaining $P_{XY}$, the cross- and auto-spectrum of the matter-density perturbation $\delta$ and the 21cm-temperature perturbation $\delta_{21}$. We employ a simple but effective “bubble model” provided in Ref. [16], with its parameters calibrated to semi-numerical simulations via 21cmFAST. We emphasize that here we are, in a sense, only using the bubble model as a fitting template of the simulation results, so we avoid interpreting the fitted values of the model parameters in a physical way. This approach is similar to that used in Ref. [9] (i.e. assuming a specific bias scaling outside the $k$-range of the simulation).

Ref. [16] models the ionization-fraction field via a collection of (possibly overlapping) fully ionized bubbles. The bubbles are uniformly distributed across space, and have a log-normal radius $R$ distribution $P(R)$,

$$P(R) = \frac{1}{R \sigma_{\ln R} \sqrt{2\pi}} \exp \left[ -\frac{(\ln R - \mu_{\ln R})^2}{2\sigma_{\ln R}^2} \right],$$

(26)

where $\mu_{\ln R}$ and $\sigma_{\ln R}$ are the mean and the standard deviation of $\ln R$, respectively. Note that the mean bubble radius is then $\langle R \rangle = \exp(\mu_{\ln R} + \sigma_{\ln R}^2/2)$.

The 21cm-temperature perturbation, in the limit detailed in Section II A, becomes $\delta_{21} = n_{HI}/\bar{n}_{HI} - 1$. The bubble model then predicts its auto-spectrum

$$P_{\delta_{21}, \delta_{21}} = \left[ b(W_R(k)) \ln(1 - x_e) + 1 \right]^2 \mathcal{P}_{\delta \delta}$$

$$+ \frac{x_e}{1 - x_e} [\langle V_b \rangle \langle W_R^2(k) \rangle + \tilde{P}_{\delta \delta}],$$

(27)

and its cross-spectrum with matter-density perturbation

$$P_{\delta_{21}, \delta} = \left[ b(W_R(k)) \ln(1 - x_e) + 1 \right] \mathcal{P}_{\delta \delta}.$$  

(28)

In these equations, we define the mean bubble volume

$$\langle V_b \rangle = \int P(R) dR V_b(R),$$

(29)

where $V_b(R) \equiv 4\pi R^3/3$ is the volume of single bubble with radius $R$. We also define the volume-weighted-window function as

$$\langle W^n_R(k) \rangle = \frac{1}{\langle V_b \rangle} \int P(R) dR V^n_b(R) W^n_R(k) \quad (n = 1, 2),$$

(30)

where $W^1_R(k) = 3[\sin(kR) - kR \cos(kR)]/(kR)^3$ is the spherical top hat window function in the Fourier space. In addition, we define

$$\tilde{P}_{\delta \delta} \equiv \frac{\mathcal{P}_{\delta \delta}(\langle V_b \rangle \langle \sigma_{\delta}^2 \rangle)}{\sqrt{\mathcal{P}_{\delta \delta}^2 + (\langle V_b \rangle^2 \langle \sigma_{\delta}^2 \rangle)^2}},$$

(31)

1 Note that Ref. [16] predicts spectra involving $\delta_{HI} \equiv (n_{HI} - \bar{n}_{HI})/\bar{n}_{HI}$, which is normalized differently from $\delta_{21} \equiv n_{HI}/\bar{n}_{HI} - 1$ interested in here. The conversion is $\delta_{21} = \delta_{HI}/(1 - x_e)$. Here, Eqs. (27) and (28) have already been adjusted with appropriate powers of $(1 - x_e)$. 


where \( P_{\delta \delta} \) is the matter power spectrum, and

\[
\langle \sigma_R^2 \rangle \equiv \frac{1}{(V_b)^2} \int P(R) dR V_b^2(R) \sigma_R^2
\]

is the volume-weighted average of

\[
\sigma_R^2 \equiv \int \frac{dk}{k} W_R^2(k) \frac{k^3}{2\pi^2} P_{\delta \delta}(k),
\]

the variance of the mean fluctuation in a spherical region of radius \( R \). Eqs. (27) and (28), plus the matter power spectrum \( P_{\delta \delta} \), can then be used to compute the angular power spectrum via Eq. (23). We model the redshift evolution of the mean ionization fraction \( x_e \) with a hyperbolic tangent as

\[
x_e(z) = \frac{1}{2} \left\{ 1 - \tanh \left[ \frac{y(z) - y_e}{\Delta_y} \right] \right\},
\]

where \( \Delta_y \) and \( y_e \) are model parameters and \( y(z) = (1 + z)^{3/2} \). We illustrate the results of our bubble-model calculation—whose parameters are chosen to match the semi-numerical 21cmFAST simulations—in Fig. 2, together with the results from simulations, for a set of redshifts where the signal-to-noise of the effect is highest. In order to match the redshift evolution of the mean ionization fraction, we have chosen \( y_e = 25.6 \) and \( \Delta_y = 4.9 \), which match well to the simulations at \( z < 7.2 \).

### III. FORECAST

Here, we assess the detectability of the cross-correlation signal between the 21-cm polarization and the CMB. Upcoming 21-cm power-spectrum experiments, such as HERA [17] and SKA1-low [18, 19], will generate a wealth of high-precision data in the following years. These experiments are primarily designed as interferometers and the bulk of the scientific effort in modelling the involved systematics focuses on increasing the measurement quality on scales relevant to the baryon-acoustic oscillations, 0.1 h/Mpc < \( k < 1.0 \) h/Mpc. These radio interferometers are limited on large scales, however, by their minimum dish (or antenna) separation. This sets a limit on the minimum \( \ell \) multipole that can be measured. The anticipated designs of HERA and SKA experimental configurations suggest that multipoles lower that \( \sim 10 \) will not be accessible when operated as interferometers. Nevertheless, experiments like HERA can also be operated in a single-dish mode, allowing a solution to this problem [see e.g. 20].

Another cause for concern is the observational systematics, which includes the beam-profile, the 1/\( f \) noise and the foreground wedge [21–23] of the 21-cm experiment at hand. These have been primarily studied in the context of interferometer experiments. Encouragingly, however, phase-1 experiments like MeerKAT (an SKA precursor on the planned site of SKA1-MID [24]) are already taking data, albeit on low redshifts \( z < 3 \). Nevertheless, overcoming these systematic challenges should be equally plausible for intensity mapping at higher redshifts. It is not difficult to imagine that path-finder experiments like MeerKAT will provide the road-map for using HERA (or similar precision) experiments in single-dish mode when measuring the epoch of reionization.

Finally, it is possible for the 21-cm polarization measurements to suffer unique (and major) challenges in addition to intensity. These include calibration [25–27], foregrounds from polarized synchrotron emission [28–30], instrumental leakage of intensity into polarization [31], Faraday rotation from various sources of magnetic fields [32–34] and depolarization effects [35, 36]. While there is ongoing research concerning these systematics, their effective mitigation may be challenging for the near-future surveys. Nevertheless, since these effects are not correlated with the CMB data, the cross-correlation signal that we study in this work should be less susceptible to systematics and may prove a promising direction forward.

In what follows we model the anticipated noise (per multipole) from a single-dish 21-cm power-spectrum measurement as [37–39]:

\[
N^2_{\ell 21} = \frac{\Omega_{\text{pix}} T^2_{\text{sys}}}{B t_o T_B} \exp \left\{ \ell (\ell + 1)(\theta_B \sqrt{\ln 2})^2 \right\},
\]

where \( \theta_B = \lambda / D_{\text{dish}} \) is the beam full-width at half-maximum of a single dish with diameter \( D_{\text{dish}} \) at wavelength \( \lambda = \lambda_2 (1 + z) \), \( \Omega_{\text{pix}} = 1.13 \theta_B^2 \) for a Gaussian beam, \( T_{\text{sys}} \) is the system temperature, \( B \) is the frequency bandwidth of observation, \( t_o \) is the total observation time and \( T_B(z) \) is the measured mean baryonic temperature at redshift \( z \). We ignore the so-called foreground wedge [see e.g. 40] since, in principle, foregrounds do not lead to a loss of information (a similar approach is taken in e.g. Ref. [41]), and the wedge can potentially be removed with better understanding of the instrument [43, 44]. We demonstrate the anticipated noise on the 21-cm polarization in Fig. 3 along with the anticipated polarization signals from the CMB and 21-cm and the CMB intensity noise for Planck.

We define the detection signal-to-noise (SNR) of the cross-correlation as:

\[
\text{SNR}^2 = \sum_{l \ell'} C^E_{\ell} E_{21} \text{cov}^{-1}(C^E_{\ell} E_{21}, C^E_{\ell'} E_{21}) C^E_{\ell'} E_{21},
\]

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where

\[ \text{cov}(\hat{C}_\ell^{E_{\text{CMB}}E_{\text{CMB}}}, \hat{C}_\ell^{E_{21}E_{21}}) = \frac{\delta \ell \delta \ell}{2\ell + 1} \left( C_{\ell}^{E_{\text{CMB}}E_{\text{CMB}}} \hat{C}_\ell^{E_{21}E_{21}} + \hat{C}_\ell^{E_{\text{CMB}}E_{21}} \hat{C}_\ell^{E_{21}E_{21}} \right). \]  

Here, \( f_{\text{sky}} \) is the sky fraction, \( \hat{C}_\ell^{E_{\text{CMB}}E_{21}} \) is the cross-correlation between the CMB polarization and the 21-cm polarization, \( C_{\ell}^{E_{\text{CMB}}E_{\text{CMB}}} \) and \( C_{\ell}^{E_{21}E_{21}} \) are the auto-correlations of the CMB and 21-cm polarization, respectively, and spectra denoted with 'tilde's are observed values, i.e., \( \tilde{C}_\ell = C_{\ell} + N_\ell \).

In what follows, we test two hypotheses. First, to access the detectability of the polarization cross-correlation signal between 21-cm and CMB, we take a null hypothesis scenario in which there is no reionization bump signal in the CMB auto-correlation in the denominator of Eq. (36) and exclude the cosmic variance from the 21-cm polarization auto-correlation. We also take the noise and signal to be diagonal in redshift bins and no signal or noise in the cross-correlation signal in the denominator of Eq. (36). Second, we ask: ‘knowing that there is indeed a period of reionization, would we see the 21-cm-CMB E mode correlation?’ For the latter, we include the reionization bump signal in the CMB auto-correlation cosmic variance.

We consider a fiducial experiment with \( T_{\text{sys}} = 40 \text{ K} \) and satisfying \( \theta_B(z) = 20 \text{ arcmin} \) at \( z = 6.1 \). We take 5 redshift bins in the range \( z \in [4, 11] \) with redshift depth \( \Delta z = 1 \), and model the CMB noise as \( N_{\ell E}^2 = 2 \Delta \ell^2 \exp[\ell(\ell + 1)/\Delta_T^2] / 8 \ln 2 \) where \( \theta_{\text{FWHM}} \) is the beam size (FWHM) of the CMB experiment, satisfying \( \theta_{\text{FWHM}} \approx 5 \text{ arcmin} \) for *Planck*.

*Planck*. While upcoming CMB experiments such as SO and CMB-S4 will lower the thermal noise on small scales significantly, improving the fidelity of large-scale measurements (\( \ell < 30 \)) requires satellite experiments that do not suffer the atmospheric noise. A promising next-generation experiment was proposed in Ref. [45], which may play a crucial role in measuring the cross-correlation signal we study in this paper. For the null hypothesis the SNR depends on survey specifications as

\[ \text{SNR} \approx 2.1 f_{\text{sky}} \left( \frac{20}{\theta_B^*} \right)^2 \left( \frac{1\mu K}{\Delta_T} \right)^2 \left( \frac{40\text{K}}{T_{\text{sys}}} \right)^2 \left( \frac{t_0}{2\text{Yrs}} \right), \]

where \( \theta_B^* \) is the beam at \( z = 6.1 \). Here, we have taken the minimum multipole as \( \ell_{\text{min}} = 2 \). For our second hypothesis, we find the coefficient of Eq. (38) degrade by over an order of magnitude to SNR \( \approx 0.017 \) due to the large cosmic-variance of the reionization bump. For the first hypothesis, we find that the SNR depends on the choice of \( \ell_{\text{min}} \) as \( \text{SNR} \propto \exp[-4\ell_{\text{min}}/5] \), suggesting that the measurement of the largest scales will be crucial for detection. We find the decrease with increasing multipoles is less rapid for the second hypothesis, where the SNR depends on choice of \( \ell_{\text{min}} \) as \( \text{SNR} \propto \exp[-0.3\ell_{\text{min}}] \). Since reaching any of these specifications remains challenging for the near-future experiments, we conclude detecting the cross-correlation signal will be difficult, but not impossible with dedicated next-generation surveys.

**IV. CONCLUSION**

In this paper, we propose to enhance the detectability of the 21-cm polarization by cross-correlation with the CMB polarization. We present the theoretical procedure to compute this cross-correlation from reionization physics, which is then modeled by the bubble model.
the Faraday rotation induced by the magnetic fields. Vide a more realistic account for the 21-cm observational radiation field before the saturation of heating; probability.

as that is where the correlation gains most of its contribution. In the first scenario, the observation at large angular scales is crucial, as that is where the correlation gains most of its contribution.

Future work could improve on the reionization modeling with larger numerical simulations; include the effect of redshift-space distortion; discuss the 21-cm radiation field before the saturation of heating; provide a more realistic account for the 21-cm observational systematics; devise strategies to mitigate foregrounds unique to the 21-cm polarization measurement, especially the Faraday rotation induced by the magnetic fields.

Acknowledgments

This work was supported by NSF Grant No. 1818899 and the Simons Foundation. This work was performed in part at Aspen Center for Physics, which is supported by National Science Foundation grant PHY-1607611, and was partially supported by a grant from the Simons Foundation. SCH is supported by the Horizon Fellowship from Johns Hopkins University

Appendix A: Derivation of the quadrupole tensor and its TAM coefficients

When the observed anisotropy at \((\vec{x}, \eta)\) is determined by the value of some field \(\phi\) on the emission shell \(\vec{x}_e \equiv \vec{x} + (\eta - \eta_e)\hat{u}\) at conformal time \(\eta_e\),

\[
\Theta(\hat{u}, \vec{x}, \eta) = \phi[\vec{x} + (\eta - \eta_e)\hat{u}, \eta_e],
\]

the associated quadrupole tensor is

\[
t_{ab}(\vec{x}, \eta) \equiv \int d^2u \left(3u_a u_b - \delta_{ab}\right)\phi[\vec{x} + (\eta - \eta_e)\hat{u}, \eta_e].
\]

In Fourier space, this relation takes the form

\[
\hat{t}_{ab}(\vec{k}, \eta) = \int d^2u \left(3u_a u_b - \delta_{ab}\right) \int d^3x \exp(-i\vec{k} \cdot \vec{x}) \times \phi[\vec{x} + (\eta - \eta_e)\hat{u}, \eta_e]
\]

\[
= \tilde{\phi}(\vec{k}, \eta_e) \int d^2u \left(3u_a u_b - \delta_{ab}\right) \exp[i\hat{k} \cdot \hat{u}(\eta - \eta_e)]
\]

\[
= -12\pi j_2[k(\eta - \eta_e)] \left(\frac{k_a k_b - \delta_{ab}}{3}\right) \tilde{\phi}(\vec{k}, \eta_e).
\]

Here, in the second equal sign, we have used the shift formula of the Fourier transform. In the third equal sign, the integral can be evaluated by taking \(\vec{k} = k\hat{z}\), and then be restored to its general form via symmetry.

Appendix B: Transfer functions

Eqs. (8) and (104) of Ref. [13] can convert the Fourier amplitudes to the TAM coefficients, giving

\[
t_{k,\text{JM}}(\eta_e) = 4\pi \sqrt{6} \tilde{j}_2[k(\eta - \eta_e)] \phi_{k,\text{JM}}(\eta_e).
\]

For the 21-cm signal and the CMB, let \(\phi\) be \(\phi_{21}\) and \(-\Phi/3\), respectively. We have

\[
t_{k,\text{JM}}^{21,\nu}(\eta_e) = T_{21,\nu}(k; \eta, \eta_e) \phi_{k,\text{JM}}(\eta_e);
\]

\[
t_{k,\text{JM}}^{\text{CMB}}(\eta_e) = T_{\text{CMB}}(k; \eta, \eta_e) \delta_{k,\text{JM}}(\eta_e);
\]

where

\[
T_{21,\nu}(k; \eta, \eta_e) = 4\pi \sqrt{6} \tilde{j}_2[k(\eta - \eta_e)];
\]

\[
T_{\text{CMB}}(k; \eta, \eta_e) = \frac{2}{k^2a(\eta_e)} j_2[k(\eta - \eta_e)].
\]
In the second formula we used the relation $\tilde{\Phi}(\vec{k}, \eta) = 3\Omega_m H_0^2 \hat{\delta}(\vec{k}, \eta)/[2k^2 a(\eta)]$ and the fact that $\tilde{\Phi}(\vec{k}, \eta)$ is approximately constant deep in the matter-dominated era.

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