A mathematical framework
for a standard theory
using extended representations
of paths and world lines

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Abstract

An analysis using a composition of currently-accepted theories is given. Starting with a synthesis of what may be generically termed “paths”, analysis of representations for these “paths” is developed. Foreground and background interactions are explicitly treated by using a local representation that treats the two representations equally and symmetrically. A restriction coupling from the global space-time representation to local interaction source terms is treated in terms of mass and charge couplings. Rewriting the connection in terms of the global manifold and the coupled terms yields compatibility with Dirac and Klein-Gordon equations for electro-weak coupled particles and fields. Compatibility with currently-accepted theories that includes standard charge assignments, SU(3) confinement, and a definition for particle flavor generations is used to constrain and validate the composition and the analysis.
1 Introduction

This paper will try to address the following question: to what extent is it possible to take the elements of currently-accepted theories that are experimentally verified and combine them to produce real, consistent physics? This assumes proceeding with the minimum possible usage of speculative higher symmetries and groups, dimensions, new particles, and fields. The idea is to combine existing elements in ways compatible with quantum field theory and general relativity to the extent that this is possible.

Some key practices include: working with known symmetries and groups such as U(1), SU(2), SU(3), and SO(3), working with four or 3+1 dimensions so that a cross product is possible, working with a holomorphic, analytic-continuation representation, working with classical concepts such as phase and configuration space trajectories (paths) and world lines, and not assuming any new particle or field content. It must be acknowledged that, currently, any composition of the key elements of accepted theories is, in a real sense, speculative in its own right.

1.1 Synthesis

It is already known that phase trajectories (also known as phase paths) and world lines are concepts that are compatible with each other [1]. The paths (or orbits or trajectories) of particles in space-time, their phase or configuration space paths, and world lines are different representations or aspects of the same thing. These can also be understood using the concepts of symmetry group (gauge) transformations and orbits. Another type of fundamental representation that is essential is field representations such as probability wave function representations. These and their associated machinery such as Lagrangians and Hamiltonians are all complementary methods that express and manage different aspects or facets of what is being studied.

Some other aspects that are important to consider are whether the representations are local or global, and whether the underlying representation (metric) is considered to be flat or curved. Analytic-continuation representations and Wick rotations also provide means of representing and working with different aspects. The most realistic approach is to combine the usage of different methods and representations in ways that allow the different aspects to be handled best.

To have the best chance of fitting different elements of current theories together, a generalized analysis will be given. The local representations will
be setup to maintain consistency with current theories. The relationships required by current theories will then be imposed. This allows for the best consistency with current theories while allowing for any implicit relationships between current theories to become manifest.

Compatibility with the currently-accepted theories will provide constraint and validation for the development of the synthesis and analysis. The compatibility of phase paths with quantum field theory, when used carefully, will be used as the basis for maintaining compatibility with both quantum field theory and general relativity. Analytic-continuation representations and Ashtekar’s new variables [2] will also be assumed to help form a common basis between quantum field theory and general relativity. In the interest of brevity, the statement of the ties to currently-accepted theories will be restricted to the minimum necessary to constrain the analysis and validate compatibility.

1.2 Path, representation, and combination approach

The approach this paper will use will be to treat paths, orbits, trajectories, and world-lines of the various sorts such as phase, configuration, space-time, field, gauge group, or relativistic as similar entities whose representations capture the differences in the facets or aspects of the different entities. To this end, a synthesis that abstracts the treatment of “paths” and their “representations” will be done. The probability relations (functions and functionals) for these entities are also an important aspect or facet.

This approach starts by considering numbers to be capable of being decomposed into two “dual” parts—each part being tangent and cotangent four-dimensional representations. The next step is to consider merging these numbers together to create extended objects—but preferring that the numbers be put together only if their representations merge together smoothly and continuously. These (complex) numbers are identifiable as system points and their extended objects as paths. The local representations are constructed as symmetric, holomorphic, analytic-continuation representations so that either representation can be a foreground or background representation. This allows full foreground and background (self) interaction to be present. The smooth merging of the paths and their local chart representations allows these local charts to be “glued” together to form a global manifold solution[3]. The global manifold solution is a background representation that does not participate in the local foreground and background symmetry. The global manifold solution is given as a restriction of the holomorphic, analytic-continuation (symmetric) representation.
Permutations of the possible paths, the possible representations for the paths, and combinations and transitions of the representations constitute an initial local superset realm\([4]\). Not all of this realm can be effectively realized as (directly) observable solutions. We will follow the convention of calling the observable entities “effective” entities. Other entities we will call either “initial” or “local” entities. We assume initial entities have only local structure while effective entities have both local and global structure.

For full foreground and background symmetry, holomorphic, analytic-continuation representations for the paths are introduced. Explicit mappings between the conjugate representations is given. Local representation quantities are expressed conjugate to each other at each point but the physical Hilbert structure is only imposed on the effective representation\([4]\) and is not considered in this paper. Local interactions within and between these representations are explored. These representations are then mapped (restricted) to a target space-time manifold solution.

When the combinatorial sum composition is taken, some global solutions have the possibility of separating so that they have no direct coupling to the local interactions. Most other global solutions do not have the local interaction coupling eliminated. The coupled remainder for the local interactions leads to what may be identified as mass terms. The particles have local interaction terms that couple to the mass unless they are capable of the cancellations necessary to decouple and to become independent of any mass terms (or other local coupled terms).

### 1.3 Conventions

We will not follow the convention of setting \(\hbar = 1, \ c = 1, \ m_e = 1, \) etc. It is also part of the purpose of this paper to track the origin and combinations of the fundamental constants. That is easier to do when they are explicit in the equations. The range (dimensionality) of indexes will be specified in the paper when it is appropriate to assume a restricted range, except that the indices \(i\) and \(j\) are assumed to run from one to three.

### 2 Paths (and trajectories and world lines)

An essential ingredient in the mathematics for physics is the combination of entities that produce a (possibly complex) number or numeric (scalar) field as a result. The general form in mathematics is to map two entities to a number. The map of vectors \(V\) and differential forms \(\omega\) to (possibly
complex) numbers $Z, V : \omega \rightarrow Z$, will be the basic form considered here. The remainder of this section will state some general machinery for considering “paths”.

To analyze paths, start with an unordered set of complex numbers that does not contain zero: $Z \subset \mathbb{C}, 0 \notin Z$. Please do not yet think of these numbers $Z$ as points as there is not yet an associated space or manifold. Consider different permutated subsets of the values in the set $Z$ of $\lambda \subset Z$, in which $\lambda \neq \emptyset$. Define these (range) subsets as the images of different functions $\lambda(\omega)$. Define the functional that is constructed by using a domain set consisting of all functions whose (range) images $\lambda$ are all the possible permutations of (domain) subsets of $Z$

$$\Lambda[\Omega] = \{ \lambda(\omega) \mid \forall z, \forall \lambda, z = \lambda(\omega), z \in \lambda \}. \quad (1)$$

The $\lambda(\omega)$ are then defined as path functions whose values are the complex numbers $z$ in some permutation. There is not a priori a requirement for the values along a path $\lambda(\omega)$ to be smooth or continuous—in the general case, they could jump around in any order and be discontinuous.

The path element $\triangle \lambda$ is a discrete numerical representation. The differential form $d\lambda$ represents an infinitesimal (continuous) piece of numeric path. For ease of notation, represent both $\triangle \lambda$ and $d\lambda$ by $d\lambda$ and create a path

$$\lambda_n(\omega) = \sum_m d\lambda_{n(m)}(d\omega_{n(m)}) \quad \text{(2)}$$

in which $n(m)$ is some different arrangement of the $\lambda$. As defined so far, these paths are possibly very chaotic—allowing continuous and discrete changes along the path and different and incompatible representations of elements along the path. In the general case, the different representation spaces for each path element could include changes in scale, orientation, dimension, and structure such as algebra and groups.

### 3 Representations

Giving quantum field theory preeminent place, the probability representation for a path will be considered to be a primary representation. The problem of foreground and background representation interaction will next be explicitly addressed. Differential form representations are considered next in importance with their particular representations constrained or validated by compatibility with current theory.
3.1 Probability amplitude functions

For probability representations it is important to define probability amplitude functions and functionals. Define $\Phi[\Lambda]$ as a functional whose domain is the set of functions that assigns complex probability amplitude images (the function’s range) to each path $\lambda$ (the function’s domain):

$$\Phi[\Lambda] = \{ \phi(\lambda) \mid \forall c, c = \phi(\lambda), c \in \mathbb{C} \}. \tag{3}$$

Allow the value of the probability amplitude to be everywhere zero for particular paths $\lambda$. In other words, some particular paths will be so pathological that they do not effectively exist.

3.2 Foreground and background representations

A key problem to address explicitly up-front is the problem of foreground and background definitions and their interactions. The goal is to try to treat the foreground and background representations equally and symmetrically. We will start as usual with local definitions called coordinate patches or charts. Define $V$ as the foreground component. Define $d\omega$ in differential form as the background representation that has the conjugate coordinate basis $\frac{\partial}{\partial \omega}$. These definitions are such that $V : d\omega \to \lambda$ and

$$\Phi(\lambda) \frac{\partial \Phi(\lambda)}{\partial \omega} = V, \tag{4}$$

where $\Phi(\lambda)$ is the probability amplitude wave function of the entity that is to be represented.

Now consider allowing a portion $B$ of the foreground $V = g_B BV_1$ to act on the background representation

$$\Phi(\lambda) B \frac{\partial \Phi(\lambda)}{\partial \omega} = V_2. \tag{5}$$

Write $B$ in differential form $B = b(d\omega_B)d\omega_B$ and write a combined holomorphic, analytic-continuation representation using the conjugate of $d\omega_B$

$$\Phi(\lambda) \left( \frac{\partial}{\partial \omega} + i g_B \tau_B \frac{\partial}{\partial \omega_B} \right) \Phi(\lambda) = V_2. \tag{6}$$

The use of a dimensional constant $g_B$, placing the $\frac{\partial}{\partial \omega_B}$ representation on the imaginary axis, and/or usage of a group vector $\tau_B$ can be important to make sure that there is not inappropriate mixing of terms. If it is appropriate, then $g_B$ and $\tau_B$ can be set to one.
Another way of viewing this is that the “foreground” definition is a representation in its own right and to treat it symmetrically [5]. To this end, two local representations, \( d\omega_1 \) and \( d\omega_2 \), will be introduced in which one can be thought of as a local foreground definition and the other as a local background definition—and with nothing (yet) selecting which one is which. This type of formulation allows us to try to simultaneously fit both the \( d\omega_1 \) and \( d\omega_2 \) representations to smooth transitions. In the case that \( \omega_1 \) is a spatial coordinate, then included in \( \omega_2 \) are conjugate momentum terms that correspond to translational transformations of \( \omega_1 \) and conjugate angular momentum terms that correspond to rotational transformations of \( \omega_1 \). This program of extending the representations corresponds to a program of embedding the possible transformations of a representation into a larger, extended representation. The internal and external transformations will also be embedded into extended representations with (gauge) fields providing a mapping restriction back to a restricted representation.

Write a holomorphic, analytic-continuation representation in terms of coordinate patch bases

\[
\frac{\partial}{\partial \omega} = \frac{\partial}{\partial \omega_1} + ig_\omega \tau_\omega \frac{\partial}{\partial \omega_2}.
\]

or

\[
\partial = \partial_1 + ig_\omega \tau_\omega \partial_2.
\]

Quantum conditions can be assumed by excluding a region of the complex plane (which could be located at the origin). If an area of a particular size \( d\omega_1 d\omega_2 \) is excluded from the complex plane, then the quantum condition

\[
d\omega_1 a \omega_2 \geq \hbar/2
\]

has been applied (no summation on \( a \)—these are separate conditions). For classical conditions, either take the limit as \( \hbar \) goes to zero or scale up to a scale very much larger than \( \hbar \) so that \( \hbar \) is effectively negligible (zero).

To highlight the key differences from the usual definitions, start by considering a standard definition of a bundle \( \mathcal{B} \) with manifold \( \mathcal{M} \), fiber \( \mathcal{V} \), and group \( \mathcal{G} \). Next we see that for local, full symmetry we want a “bundle” with fibers \( \mathcal{V}_1 \) and \( \mathcal{V}_2 \)—not the usual definition of a bundle. In the case that foreground and background symmetry is ignored, then \( \mathcal{V}_1 \) may be mapped to a (global) manifold \( \mathcal{M} \) and a standard bundle \( \mathcal{B} \) results.
3.3 Transformations on holomorphic representations

This subsection states some properties of transformations and gauge field representations of holomorphic representations[6]. Please consider external (group) transformations $\Theta_E \in G_E$ operating on the representation

\[ D_\omega = \Theta_E \partial = \Theta_E (\partial_1 + ig_\omega \tau_\omega \partial_2). \]

These correspond to the internal (conjugate) transformations that transform between $\omega_1$ and $\omega_2$. These transformations are such that $\Theta_E : \omega_1 \rightarrow \omega_2$ or $\Theta_E : \omega_2 \rightarrow \omega_1$ which, when summed (if possible) are

\[ f'(\omega_2) = \int \Theta_E f(\omega_1) d\omega_1 \]  

and

\[ f(\omega_1) = \int \Theta_E f'(\omega_2) d\omega_2. \]

If $g_\omega = 1$, $\tau_\omega = 1$, and the $\Theta_E$ transformations are U(1) external transformations $e^{\pm i\theta}$ that correspond to the internal (conjugate) transformations $\frac{1}{\sqrt{2\pi}} e^{\pm i\omega_1}$ then (10a) and (10b) are Fourier and inverse Fourier transformations as (internal) rotations in the complex plane. If the group structure of $\tau_\omega$ corresponds to some group SU(N), of which SU(2) or SU(3) are used in this paper, then corresponding (internal) transformation equations for (10a) and (10b) may be obtained.

Defining the “momentum” (gauge field) representation that is conjugate to the external transformations $\Theta_E$ representation as $\vec{B}$ gives another form of representation

\[ D_B = \partial + ig\vec{\tau}\vec{B}. \]

3.4 Local and global representations couplings

The preceding representations are local definitions. These are partial derivatives with no transition functions (yet) defined to other (local) derivative representations (coordinate patches or charts). For a global definition, a preferred background representation will have to be identified and (summed combination) transition functions and operations will have to be identified and used.

The global representation is a (restriction) subset of the degrees of freedom and interactions that can be present locally. When going from local to
global representations, some local (internal) symmetries or group transformations may show closure—the group orbits within their phase or configuration volumes may be shrunk to “points” from a global perspective. The local interactions become hidden and some part of the system acts as a composite single entity. This means that not all quantities that are present locally are quantities that must be representable directly on the global representation—especially with full foreground and background interaction symmetry. These local representations can be written as a composite representation term $\epsilon$ that couples to the global space-time representation. This coupling of a global manifold to locally coupled terms can be written as

$$\Phi^* D_a^c \Phi \leftrightarrow \Psi^* D_{X^a} \Psi + \epsilon(\lambda)$$

where $\epsilon(\lambda)$ are the local coupled terms and $D_{X^a} = \frac{\partial}{\partial X^a}$ are the coordinates of a global manifold.

4 Local representations

For two sets of entities $\omega_1$ and $\omega_2$ that have maps to numbers $\Upsilon : \Omega \to Z$, define two complementary conjugate representations. Define a mapping on $V_1$ to $d\lambda \in d\lambda_{n(m)},$

$$(13) \quad d\omega_1 : V_1 \to d\lambda,$$

so that $d\omega_1 V_1 = d\lambda$, in which $V_1 \in \Upsilon$ and $V_1 \notin \Omega$. Also define a mapping from $V_2$ to a $d\lambda$

$$(14) \quad V_2 : d\omega_2 \to d\lambda$$

so that $V_2 d\omega_2 = d\lambda$, in which $V_2 \in \Omega$ and $V_2 \notin \Upsilon$. The terms $V_1$ and $V_2$ are assumed to be complex vectors and the terms $d\omega_1$ and $d\omega_2$ are assumed to be complex differential forms. They are assumed to have properties local to $d\lambda_{n(m)}$.

4.1 Differential form representation

For each $d\omega$ there are many possible choices for their representation as coordinate one-form representations

$$d\omega = u(\xi^a) d\xi^a$$

with the numeric tuples $u(\xi^a)$. In the general case, these different choices of coordinates could include differences in scale, orientation, dimension, and structure such as algebra and groups depending on different choices for $\xi$. 
Some relations that follow are:

\[(16a) \quad d\lambda = V d\omega = V_a u(\xi^a) d\xi^a,\]

which can be integrated to give

\[(16b) \quad \lambda(\xi) = \int V_a u(\xi^a) d\xi^a,\]

and whose derivative with respect to \(\xi^a\) can be taken to give

\[(16c) \quad \frac{\partial \lambda(\xi)}{\partial \xi^a} = V_a u(\xi^a).\]

Using (3) gives

\[(16d) \quad \phi^\dagger(\lambda) \frac{\partial \phi(\lambda)}{\partial \xi^a} = \phi^\dagger(\lambda) \frac{d\phi(\lambda)}{d\lambda} V_a u(\xi^a),\]

in which \(\frac{d\phi(\lambda)}{d\lambda}\) is a total derivative since \(\phi\) is only a function of \(\lambda\). In general, the \(\phi(\lambda)\) might be chosen to be in a form in which \(\frac{d\phi(\lambda)}{d\lambda}\) contributes factors such as \(\hbar\) and \(\pm i\). Unless stated otherwise, in this paper it will be assumed that \(\phi(\lambda)\) are in a form in which

\[(17) \quad \phi^\dagger(\lambda) \frac{d\phi(\lambda)}{d\lambda} = 1\]

and

\[(18) \quad \phi^\dagger(\lambda) D_{\xi^a} \phi(\lambda) = \phi^\dagger(\lambda) \frac{\partial \phi(\lambda)}{\partial \xi^a} = V_a u(\xi^a).\]

### 4.2 Compatible representation assumptions

For ease of notation, the representation of \(\omega_1\) will be given as \(\chi^a\) and the representation of \(\omega_2\) will be given as \(\pi^a\) and if the discussion applies to both equally, their representations will be referred to as \(\xi^a\). The conjugate for \(\chi^a\) will be given as \(\kappa_a\) and the conjugate for \(\pi^a\) will be given as \(\Xi^a\).

For compatibility, assume four dimensions for \(\chi^a\), \(\pi^a\), and \(\xi^a\) and all related dimensional quantities for the remainder of the paper. It is fairly standard to treat the four dimensions as 3+1 dimensions. Use the zero index for the separate dimension and give \(a\) and \(b\) the ranges

\[(19) \quad a, b = 0, 1, 2, 3.\]

For \(\xi^a\) (for each of \(\chi^a\) and \(\pi^a\)), arbitrarily pick one axis \(\xi^0\) and assume a (scaled) identity alignment mapping within the different \(\xi^0\)'s. Assume the
functional derivatives between possible coordinate basis sets is

\[ g_\mu \sigma^\mu = \frac{\delta \xi^{a'}}{\delta \xi^a} \]  

(no summation), in which \( \sigma^0 = 1 \), the \( \sigma^i \) are Pauli matrices, and the \( \sigma^\mu \) are a representation of SU(2). Assume Gell-Mann matrices \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) that are isomorphic to the matrices \( \sigma^i \) and that belong to the group \( G_s(3) \) of SU(3) rotations on the \( \xi^i \) axes.

Since the zero axis was arbitrarily picked, there is a gauge transformation in SU(2) for the \( \xi \) basis that corresponds to multiplying through by an element of SU(2) to pick a different \( \xi^0 \) axis associated with the identity element. Designate by \( \tau_\mu^\nu \) the set of SU(2) elements for this gauge transformation.

The \( \xi^a \) (\( \chi^a \) and \( \pi^a \)) bases are local to each point \( z \) with partial derivatives that commute and, therefore, locally flat metrics \( \eta_{ab} \). The possible metric signatures for \( \xi^a \) are assumed to be \([- + , + , +] \) or \([ +, -, -, -] \) (\( \xi^a \) is indexed from zero). These correspond (respectively) to the \( \xi^i \) (\( \chi^i \) and \( \pi^i \)) being positive definite and having a right-handed chirality or negative definite and having a left-handed chirality.

### 4.3 Group closure restriction to idempotent basis

Define the mapping \( G_s : \xi^i \rightarrow \xi^0 \) so that

\[ \sum_i \frac{\partial \xi^i}{\partial \xi^0} = \pm G_s \vec{\nu}_\xi. \]

The positive or negative sign comes from the metric signature and \( G_s \) is a new dimensional constant that ensures \( \vec{\nu}_\xi \) is a vector value.

Using the rotation \( \lambda_n \), identity combinations of the trivial representation can be constructed. The group trivial representation \( \lambda_0 = 1 \) is

\[ I_D \xi^0 = \frac{\partial}{\partial \xi^0}. \]

The group inverse representation \( \lambda_n^{-1} \lambda_n = 1 \) gives

\[ GID \xi^0 = G_s^{-1} \vec{\nu}_\xi^{-1} G_s \vec{\nu}_\xi \frac{\partial}{\partial \xi^0}. \]

The group closure representation \( \lambda_i \lambda_j \lambda_k = 1 \) gives

\[ GC \xi^0 = -G_s \vec{\nu}_\xi^i G_s \vec{\nu}_\xi^j G_s \vec{\nu}_\xi^k \frac{\partial}{\partial \xi^0}. \]
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in which \( i \neq j, j \neq k, i \neq k \). The representation on \( \xi^a \) that does not export a dependency on \( \lambda_n \) (an interaction with \( \lambda_n \)) is the one whose group element is \( \lambda_0 \). A subset representation of \( \xi^a \) that does not have an exported dependency on \( \lambda_n \neq 0 \) is the combination of the group identity as given by (22), the group inverse as given by (23), and the group closure as given (24). This is

\[
D_{\xi^0} = I D_{\xi^0} + G^T D_{\xi^0} + C \).
\]

4.4 Transformation based extended representations

As given in section 3.3, transformations \( \Theta_\mu \) between the two representations \( d\chi \) and \( d\pi \) can be written as additional representations

\[
\Theta_\mu \partial \rightarrow D_B = \partial + ig_B \tau_\mu B_\mu.
\]

Including the U(1) identity orientation \( \tau_\mu^0 = 1 \), there are four possible orientations \( \tau_\mu \) between \( d\chi \) and \( d\pi \). Corresponding to the four \( \tau_\mu \) there are different maps \( \Theta_\mu : d\chi \leftrightarrow d\pi, \Theta_\mu = \{ \partial_\mu, \zeta_\mu \} \) between \( \xi^0_\chi \) and \( \xi^0_\pi \):

\[
\begin{align*}
(27a) & \quad \tau_\mu \frac{\partial \xi^0_\pi}{\partial \xi^0_\chi} = g \partial_\mu \\
(27b) & \quad \xi_\mu \frac{\partial \xi^0_\pi}{\partial \xi^0_\chi} = g \zeta_\mu,
\end{align*}
\]

with corresponding vector potentials \( B_\mu \) and \( \tilde{B}_\mu \).

The translational transformations and rotational transformations for the target manifold coordinates will be assumed to already be included in the momentum conjugate to the target manifold coordinates. The SU(2) basis for \( \omega_{1i} \), has SU(3) transformations so that

\[
G_f \partial_{\omega_{1i}} \rightarrow D_F = \partial_{\omega_{1i}} + ig_F \tau_F \tilde{B}_F.
\]

The SU(2) basis for \( \omega_{2i} \), also has SU(3) transformations so that

\[
G_S \partial_{\omega_{2i}} \rightarrow D_S = \partial_{\omega_{2i}} + ig_S \tau_S \tilde{B}_S.
\]

To contain the scope of the paper, unless stated otherwise in the remainder of the paper, the restricted forms of (28) and (29) in the form given by (25) will be used.
4.5 Mapped representations

A mapped representation can be written in terms of transformation fields $\Theta_\mu$ as

$$(30a) \quad D^A_a = \Theta_\mu [(g_\zeta \zeta_\mu)^m \vec{\zeta}^\mu D^\chi_a + (g_\vartheta \vartheta_\mu)^n \tau^\mu D^\pi_a],$$

in which $m \in \{0, 1\}$, $n \in \{0, 1\}$, $n \neq m$ and

$$(30b) \quad \Theta_\mu = \prod \varpi_\mu, \varpi_\mu \in \{\vartheta_\mu, \zeta_\mu\}.$$  

This expresses one of the representations as a native representation and the other as a mapped representation. To preserve foreground and background symmetry by not writing a preferred form, write an analytic extension

$$(31) \quad D^L_a = D^A_a - iD^B_a$$

by the use of (30a) and

$$(32) \quad D^B_a = \Theta'_\mu [(g_\zeta \zeta_\mu)^{m'} \vec{\zeta}^{m'} D^\chi_a + (g_\vartheta \vartheta_\mu)^{n'} \tau^{n'} D^\pi_a],$$

in which it is assumed that the $m \neq m'$ and $n \neq n'$.

Equations (30b) and (31) allow transformations (rotations) between representations $D^A_a$ and $D^B_a$ of $D^L_a$ by taking the proper transformations $\Theta_\mu$ and $\Theta'_\mu$ (which include the U(1) transformations of the form $e^{i\theta}$) to give a transformed representation

$$(33) \quad D^{L'}_a = \delta \Theta D^L_a,$$

in which $\delta \Theta$ is a functional transformation of $\Theta_\mu$ and $\Theta'_\mu$.

5 Local coupled terms

Up to this point, local, full symmetry has been considered with a “bundle” with fibers $V_1$ and $V_2$. At this point, consider mapping $V_1$ to a (global) manifold $M$ and a standard bundle $B$ with manifold $M$, fiber $V$, and group $G$.

If the phase and configuration spaces $\alpha_R$ for this restriction to the bundle $B$ are separable from the extended possibilities of the local, full symmetry $\alpha_R$, then the properties of the bundle are intrinsic. If the phase and configuration spaces $\alpha_R$ for this restriction to the bundle $B$ are not cleanly separable from the extended possibilities (phase and configuration space $\alpha_F$) of the local, full symmetry, then the properties of the bundle will be considered to have remaining extrinsic terms $\epsilon$. This section will define terms $\epsilon$ with the intrinsic and extrinsic possibilities built into the definition.
Define the unit configuration space volume element as the volume element in which each dimension has a unit value of $\hbar c$ for a volume element of $\alpha_{\text{unit}} = (\hbar c)^N$.

5.1 Local internal interaction coupling

Designate “local internal interaction” (real) terms as

$$
\epsilon_m(\lambda) = E_m M(\lambda) \begin{pmatrix} L_M(\alpha_I) \\ L_M(\alpha_E) \end{pmatrix}
= E_m M(\lambda) \iota_M,
$$

in which $\alpha_I$ and $\alpha_E$ are intrinsic and extrinsic configuration volume values and $E_m$ contains a dimensional coupling constant $m$ to these terms. Assume $m$ is scaled equal to the electron mass $m = m_e$. Assuming a velocity of $c$ and scaling over a unit value of $\hbar c$, lets us assume the term is $E_m = \frac{mc^2}{\hbar c}$. Assume the boundary of the configuration volume for a particular $D_xa$ for intrinsic $L_M(\alpha_I)$ is such that the local coupled configuration volume is zero. In this case, the $D_xa$ decouples from the local configuration volume so that $\alpha_I = 0$ and $L_M(\alpha_I) = 0$. Otherwise, it is the case that $\alpha_E \neq 0$ and $L_M(\alpha_E) = 1$. The “local internal interaction” coupling term is then written as

$$
\epsilon_m(\lambda) = \frac{mc^2}{\hbar c} M(\lambda) \begin{pmatrix} 0 \\ 1 \end{pmatrix}
= \frac{mc^2}{\hbar c} M(\lambda) \iota_M.
$$

5.2 Local external interaction coupling

Any global target manifold need not be aligned with a particular $D^\mathcal{E}_a$ corresponding to $\Theta_\mu$. Designate the local difference in alignment as a “local external interaction” alignment variance

$$
\epsilon_e(\lambda) = \Theta_e Q(\lambda) \begin{pmatrix} L_Q(\Theta_\mu) \\ L_Q(\Theta_\mu) \end{pmatrix}
= \Theta_e Q(\lambda) \iota_Q,
$$

in which $\Theta_e$ contains a dimensional coupling constant that is normalized over $\hbar c$ of

$$
\Theta_e = \pm \frac{e}{\hbar c}.
$$
Assume $L_Q(\Theta_\mu) = 1$. The representation alignment coupling term is then

$$
(38) \quad \epsilon_e(\lambda) = \pm \frac{e}{\hbar c} Q(\lambda) \begin{pmatrix} 1 \\ 1 \end{pmatrix}
$$

$$
= \pm \frac{e}{\hbar c} Q(\lambda) i_Q.
$$

The $\Theta_\mu$ transformation is properly both positive and negative because it rotates between chirality—it rotates between metric signatures of $[-, +, +, +]$ for $D_\chi^a$ and $[+, -, -,-]$ for $D_\pi^a$.

5.3 Total local interaction coupling

The total local coupled “local interaction” terms for $\Phi^* D_{\lambda a} \Phi$ is

$$
(39) \quad \epsilon(\lambda) = \epsilon_m(\lambda) \Theta^{\lambda M}_\mu - i \epsilon_e(\lambda) \Theta^{\epsilon}_\mu.
$$

The $\epsilon(\lambda)$ are coupled to $\Phi$ by way of $D_{\lambda a}$ but local fields do not have to go through this coupling to interact with other local fields.

6 Global manifold

Introduce entities $x^A$, $p_A$, $X^a$, $P_a^A$, and $W_\mu^a$ where $X^a$ and $P_a^A$ are assumed to have diffeomorphic, smooth, continuous transition functions on the intersections of the $x^A$ and $p_A$. The $x^A$ and $p_A$ are, respectively, tangent space and cotangent space effective representations of the smooth, continuous, effective phase path solutions $\lambda(X, P) \in Z$. The $X^a$ are the coordinates of the global manifold solution and are a transform solution from the $x^A$, which are variants of the $\chi^a$. The $W_\mu^a$ are the equivalent transformation from the $B_\mu^a$ that map to $x^A$ instead of to $\chi^a$ and $\pi^a$. Equations equivalent to (13) and (15) for $X^a$ are a mapping from $X$ to $Z$ of

$$
(40a) \quad X : P \to Z
$$

so that $XP = Z$ with a representation of

$$
(40b) \quad X = \{ X^a \mid X^a = g(X^a) dX^a, g(X^a) \in C\}
$$

and the numeric tuple $g(X^a)$.

Assume there is an isomorphism $\sigma_\chi \to \sigma_X$ and that the $i\sigma_X$ are the generators of the group $SO(3)$ of rotations on the $X^i$ axes. We will use the standard convention for writing the $x$ coordinate representation

$$
(41) \quad D_{xa} = \gamma^a \partial_0 = \gamma^a \frac{\partial}{\partial x^a}.
$$
with the manifold coordinate representation written as
\[ D_{X^a} = \gamma^a \partial_{X^a} = \gamma^a \frac{\partial}{\partial X^a}, \]
in which \( \gamma = \varsigma \otimes \sigma_X \) are the Dirac matrices and \( \varsigma \) is a factor from (33).

Equations equivalent to (16a), (16b), and (18) for \( X^a \) and \( P \) are (where \( e^A \) are \( x^A \) orthonormal frame sets)

\[ e^A d\lambda = P^A_a dX^a, \]
\[ e^A \lambda(X, P) = \int P^A_a g(X^a) dX^a, \]
and
\[ e^A \Phi(X, P) \frac{\partial \Phi(X, P)}{\partial X^a} = g(X^a) P^A_a. \]

Writing the distance along an effective phase path as a world line as
\[ ds^2 = d\lambda^* d\lambda = g^*(X^a)g(X^b) P^A_a P^B_b \eta_{AB} dX^* a dX^b \]
gives
\[ ds^2 = g_{ab} dX^a dX^b, \]
in which the metric has the form[2]
\[ g_{ab} = g^*(X^a)g(X^b) P^A_a P^B_b \eta_{AB} \]
and \( P^A_a \) is in the form of an Ashtekar tetrad “soldering form” that links to the neighborhood of tangent spaces \( x_A \) that define the manifold value of \( X_a \). Because a neighborhood \( X^a \) of the manifold may be composed of multiple coordinate patch sets of \( (x^a, p_a) \), \( P^A_a \) can represent a value that is not on the mass-shell.

### 6.1 Composite coupling

The restriction to a background manifold derivative fixes the direction of \( W^a_\alpha \) and transforms \( \Xi_\alpha \) and \( \kappa_\alpha \) to the coupled “local interaction”
\[ \Phi^* D^\xi \alpha \Phi \rightarrow \Psi^* D_{\alpha} \phi \Phi + \Phi^*(D^A_a - iD^B_a)\Phi = \Psi^* D_{\alpha} \phi \Phi + \epsilon(\lambda), \]
so that (using (39))
\[ \Phi^*(D^A_a - iD^B_a)\Phi = \epsilon_m(\lambda) \Theta^M_\nu - i\epsilon_\nu(\lambda) \Theta^\xi_\nu. \]
6.2 Internal interaction coupling

Using (30a), the real part transforms (restricts) as a variation of $\chi^a$ to give
the manifold and a term $\epsilon_m(\lambda) = \Psi^* D_A^a \Psi$ in which

$$\epsilon_m(\lambda) \Theta^M_{\nu} = \Phi^* (\zeta_{\nu_1} D_{\chi^a} \zeta_0 + g_\theta \vartheta_{\nu_1} \vec{\tau}^1 D_{\pi^a}) \Phi$$

so that $m = 0, n = 1$. An alternate transform is

$$\epsilon_m(\lambda) \Theta^M_{\nu} = \Phi^* [g_\theta \vartheta_{\nu_1} (g_\zeta \zeta_{\nu_2} \vec{\varsigma}_0 D_{\chi^a} + \vec{\tau}^1 D_{\pi^a})] \Phi$$

so that $m = 1, n = 0$. Fix the gauge of the corresponding gauge field $Z_a$
by assuming a gauge fixing transformation $\vartheta_{\nu_1} \to 0$, $\Theta^M_{\nu} = 1$. Using (35)
gives

$$\epsilon_m(\lambda) = \Phi^* \zeta_{\nu_0} D_{\chi^a} \Phi = \frac{mc^2}{\hbar c} M(\lambda) \iota_M.$$ 

6.3 External interaction coupling

Using (32), the imaginary part transforms (restricts) to give

$$\epsilon_e(\lambda) \Theta^E_{\nu} = \Phi^* [g_\theta \vartheta_{\nu_1} (g_\zeta \zeta_{\nu_2} \vec{\varsigma}_0 D_{\chi^a} + \vec{\tau}^3 D_{\pi^a})] \Phi$$

so that $m' = 1, n' = 0$. Rewriting using (38) gives

$$\frac{e}{\hbar c} Q(\lambda) \iota_Q = \Phi^* [g_\theta (g_\zeta \zeta_{\nu_2} \vec{\varsigma}_0 D_{\chi^a} + \vec{\tau}^3 D_{\pi^a})] \Phi,$$

in which the gauge field $A_a$ is introduced as

$$\Theta^E_{\nu} = \vartheta_{\nu_3} = \gamma^a A_a.$$ 

7 Standard Model compatibility

This section will establish compatibility with the Standard Model in terms
of electroweak fields, charges, and Dirac and Klein-Gordon equations.

7.1 Interaction field

Arbitrarily assigning $W_{a}^-$ the index 1 location and $W_{a}^+$ the index 2 location,
the new $W_a^\mu$ may be given as

$$W_a^\mu = (A_a, W_a^-, W_a^+, Z_a),$$
in which the Standard Model form[7, 8, 9] is assumed:

$$A_a = \sin \theta_W B_a^3 + \cos \theta_W B_a^0,$$
(53c) \[ W_a^\pm = \frac{B_a^1 \pm iB_a^2}{\sqrt{2}} , \]
and
(53d) \[ Z_a = \cos \theta_W B_a^3 - \sin \theta_W B_a^0 . \]
It is reasonable to assume \( A_a \) has the form
(54) \[ A_a \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \]
so that, using (35), it is massless
(55) \[ A_a \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) \epsilon_m(\lambda) = A_a \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \frac{mc^2}{\hbar c} M(\lambda) \left( \begin{array}{c} 0 \\ 1 \end{array} \right) = 0 . \]

Charge is a perpendicular transformation (rotation) in the direction of the local axis \( D^B_a \), with \( \epsilon_a(\lambda) \) and \( A_a \) having perpendicular rotation in the direction of that axis, and with \( A_a \) centered (gauge fixed) at zero. The \( A_a \) has zero net charge and has a rotation alignment of zero with respect to the \( x \) manifold. The \( A_a \) is four-dimensional and has no mass coupling from (55). Mass is along the local real axis \( D^A_a \), with \( \epsilon_m(\lambda) \) and \( Z_a \) along that axis, and with \( Z_a \) displaced from zero. The \( Z_a \) has no charge coupling and has a mass. The \( W_a^\pm \) are along, and off-axis, for both axes. The \( W_a^\pm \) have a charge coupling and have masses. The \( W_a^+ \), its antiparticle \( W_a^- \), and \( Z_a \) can be easily shown to couple to mass using (35).

It is possible that the preceding description does not preserve complete Lorentz invariance, but SIM(2) plus strong CP preservation (local Lorentz invariance) as given in Very Special Relativity\[10\] should be preserved.

The assignments that implement what has just been discussed are to assign \( g_A = g_\theta \) and set \( \zeta^0 = \zeta^0, \zeta^5 = \zeta^5, \zeta^1 = \zeta^3, \zeta^2 = \zeta^2, \) and \( \tau^3 = \tau^3, \) and \( \zeta^0 = \zeta^0, \) and \( \zeta^3 = \zeta^3, \) and \( \tau^5 = \tau^5. \) Rewriting (49) with these assignments gives
(56) \[ \epsilon_m(\lambda) = \Phi^* \zeta^0 D_{x^0} \Phi = \frac{mc^2}{\hbar c} M(\lambda)_{\mu M} . \]
Rewriting (51) with these assignments gives
(57) \[ \frac{e}{\hbar c} Q(\lambda)_{\epsilon Q} = \Phi^* (g_A g_\zeta \zeta^3 \zeta^3 D_{x^0} + g_A \tau^0 D_{x^0}) \Phi . \]

The new “direction” of \( W_a^\mu \) is set relative to \( x^0 \). Since \( D_{x^0} \) is a variation (restriction transformation) of \( D_{x^0} \), assume
(58) \[ \Phi^* g_A \tau^0 \Phi = \frac{1}{2} \left( \begin{array}{c} 1 \\ 1 \end{array} \right) . \]
7.2 Weak mixing angle

The (restriction) transform from $B^\mu_a$ to $W^\mu_a$ goes from groups SU(2) to SU(2)/U(1) with a preferred duality rotation direction $e$. The charge $e$ is a scale factor with respect to $\theta_W$—the $g'/g$ set the direction of $\theta_W$ and $e$ is the (inverse unit) magnitude along the direction. The charge $e$ then effects a scaling to make it a partial (restriction) transformation instead of a complete (restriction) transformation. The charge is then as given in the Standard Model as $e^2 = g^2 \sin^2 \theta_W$ with directions $\sin^2 \theta_W = e^2 / g^2$, $\cos^2 \theta_W = e^2 / g'^2$, and $\tan^2 \theta_W = g'^2 / g^2$ in which $\sin^2 \theta_W$ is the Standard Model [7, 8, 9] weak mixing angle.

7.3 Charge assignments

Using (57) and (58) lets us write

$$\Phi^* g_A \bar{\eta} D_{\pi^0} \Phi = -\frac{1}{2} \frac{e}{\hbar c} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{e}{\hbar c} \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix},$$

in which there is a change of sign for $\Phi^* D_{\pi^0} \Phi$ due to the sign of the metric of $\pi$. Writing the term for $D_{\pi^i}$ with a factor of $\frac{1}{3}$ to account for the three terms of $\pi^i$ gives

$$\Phi^* g_A \bar{\eta} D_{\pi^i} \Phi = \frac{1}{3} \frac{e}{\hbar c} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{e}{\hbar c} \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \end{pmatrix}.$$

Using (57) allows us to write

$$\Phi^* g_A g_\zeta \bar{\zeta} D_{\chi^3} \Phi = \frac{1}{2} \frac{e}{\hbar c} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{e}{\hbar c} \begin{pmatrix} \frac{1}{3} \\ -\frac{1}{3} \end{pmatrix}.$$

Using (57) and (61) and using (59) for leptons and using (60) for quarks then gives Standard Model compatible charge assignments:

- A charge for leptons

$$Q(\lambda_{\Xi^0}) \iota_Q = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$

- A charge for quarks

$$Q(\lambda_{\Xi^i}) \iota_Q = \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{6} \end{pmatrix} + \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix}.$$
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7.4 Dirac and Klein-Gordon equations

The restriction transformation (45) may be rewritten as
\[ \Phi^* D^\alpha_a \Phi \rightarrow \Psi^* D_a \Psi, \]
in which
\[ D_a = D_a^\alpha + \frac{mc^2}{\hbar c} M(\lambda)_{\mu\mu} - i\gamma^\alpha A^\mu_a \frac{e}{\hbar c} Q(\lambda)_{\mu Q}. \]
Substituting (62) into (65) gives, for leptons,
\[ \Xi_a D_a = \gamma^0 \partial_a + \frac{mc^2}{\hbar c} M(\lambda) \begin{pmatrix} 0 \\ 1 \end{pmatrix} - i\gamma^\alpha A^\mu_a \frac{e}{\hbar c} \begin{pmatrix} 0 \\ -1 \end{pmatrix}. \]
Splitting (66) into its two parts gives a neutrino equation,
\[ \Psi^* D^\nu_a \Psi = \Psi^* \gamma^0 \partial_a \Psi, \]
and an electron-like equation (in which \( D^E_a \) stands for all three “electron-like” particles),
\[ \Psi^* D^E_a \Psi = \Psi^* (\gamma^\alpha \partial_a + \frac{mc^2}{\hbar c} M(\lambda) + i\gamma^\alpha A^\mu_a \frac{e}{\hbar c}) \Psi. \]
This shows that the three neutrinos can decouple and the electron, muon, and tauon do not. The coupled local interactions \( \epsilon(\lambda) \) are seen to contain all of the source terms.

The equation for the global derivative is (a type of Dirac equation[11]), from (65),
\[ \Psi^* D_a \Psi = \Psi^* (\gamma^\alpha \partial_a - \frac{ie}{\hbar c} q(\lambda) A_a) + \frac{mc^2}{\hbar c} m(\lambda) \Psi, \]
in which \( q(\lambda) \) are allowed combinations of \( Q(\lambda_{\Xi\Xi}) \) and \( Q(\lambda_{\Xi i}) \) and \( m(\lambda) \) are allowed combinations of \( M(\lambda) \). When dealing with the group inverse terms of \( \pi \) (scalar equations), a Klein-Gordon form is more appropriate:
\[ \Psi^* D^\alpha_a D_a \Psi = \Psi^* (\partial_a^2 + \frac{e^2}{\hbar^2 c^2} q(\lambda) A_a^2 + \frac{m^2 c^4}{\hbar^2 c^2} m(\lambda) \omega^2) \Psi. \]

8 Totals and time

So far in this paper, care has been taken to not identify any particular dimension or variable as time. The 3+1 dimensions and group structures that have been used so far are compatible with the selection of a preferred time dimension but full symmetry has been maintained. Some sort of parameterization of the paths in terms of an explicit separate parameter or in terms of time could be assumed to allow usage of the functional path integral[12, 13].
This is done to make sure that the integrals are well defined and do not diverge.

For the path representation overlap to be smooth, the paths must be properly ordered and joined together in sets of compatible “histories”. Also, for the overlap to be smooth, the transition functions for path representations must be smooth or differentiable. Another way to put this is that the path representations must be diffeomorphic at the overlap. If the path representations are (indirectly) diffeomorphic to a common representation then that establishes a diffeomorphic relationship between them. The diffeomorphism of the common representation to a total derivative such as time helps to parameterize and order the permutations of the paths. The presence of a total derivative such as time implies smoothness and continuity of representations and paths that select for permutations of paths that are effective. Therefore, the paths that cannot be parameterized in terms of a monotonically increasing parameter must be excluded from the set of considered paths.

8.1 Composite entities

The configuration volume can contain products of terms such as \( \lambda_{\chi a} \kappa^a \) and \( \lambda_{\pi a} \Xi^a \). Constructing enclosed configuration subspaces for these give the terms defined in (23) and (24). It is possible to assume \( \lambda_{\chi a} \) and \( \lambda_{\pi a} \) are confined symmetries by assuming that the external transformations \( \Theta_{\mu} \) do not project \( \lambda_{\chi a} \kappa^a \) and \( \lambda_{\pi a} \Xi^a \) onto the underlying basis—that these terms disappear in the restriction projection

\[
\Theta_{\mu} \kappa^a = 0, \quad a \neq 0
\]

and

\[
\Theta_{\mu} \Xi^a = 0, \quad a \neq 0.
\]

The content of this assumption is that the external transformations \( \Theta_{\mu} \) interact only with the zero-indexed terms \( \kappa^0 \) and \( \Xi^0 \).

The following dimensional basis-state constructs are then possible:

- A four-dimensional configuration consisting of only \((x^a, p^a)\). The local configuration volume coupling \( \epsilon(\lambda) \) is zero.
- A five-dimensional configuration consisting of the \((x^a, p^a)\) and also \( \epsilon(\lambda) \) coupled to the \((\Xi^0, \kappa^0)\) terms.
- A seven-dimensional configuration consisting of the \((x^a, p^a)\) and also \( \epsilon(\lambda) \) coupled to the \((\Xi^i, \kappa^i)\) terms. Combinations of the \((\Xi^i, \kappa^i)\) terms are first mapped to the \((\Xi^0, \kappa^0)\) terms by (combinations of) group inverse and group closure restrictions.
• Combinations of the terms \((\Xi^a, \kappa^a)\) that do not map or couple to the \((x^a, p^a)\) and are therefore hidden, local transition terms.

If the \(\epsilon(\lambda)\) are zero, then time and energy are intrinsic and the representation is four-dimensional. If \(\epsilon(\lambda)\) is not zero then, then effectively the representation has more than four dimensions. The \(\Xi^0\) and \(\kappa^0\) constitute part of a five-dimensional representation. The \(\Xi^i\) and \(\kappa^i\) and their group inverses and group closure restrictions constitute a seven-dimensional representation mapped to five dimensions and then coupled to four dimensions.

The terms \(I_D\chi, G_D\chi,\) and \(G_D\chi^0\) are compatible with characterizing different flavor or generation terms. The \(I_D\chi^0\) term is assumed to correspond to the first generation particles (electron, electron neutrino, and down and up quarks). The \(G_D\chi^0\) term is assumed to correspond to the second generation particles (muon, muon neutrino, and strange and charm quarks). The \(G_D\chi^0\) term is assumed to correspond to the third generation particles (tauon, tauon neutrino, and top and bottom quarks).

The terms \(I_D\pi, G_D\pi,\) and \(G_D\pi^0\) are compatible with leptons, mesons, and baryons respectively.

8.2 Energy

Assume in the standard way that there is a representation (restriction) where the coordinate \(X^0\) is diffeomorphic (with a Wick rotation) to the total derivative—that the coordinate \(X^0\) is diffeomorphic to time \(\frac{\partial X^0}{\partial t} = ic\). Assume its dual \(P^0\) is equal to the total energy

\[(73) \quad P^0 = iE/c.\]

Using (70), (43c), (44c), and (73), assuming the gauge condition \(A_a = 0\), and setting (70) equal to a constant

\[(74) \quad \frac{1}{\hbar^2 c^2} \Lambda_C = \Psi^* D^a_D a \Psi\]

gives

\[(75) \quad \frac{1}{\hbar^2 c^2} \Lambda_C = g(X_0)^2 P^a P_a + \frac{m^2 c^4}{\hbar^2 c^2} m(\lambda)^2
= -g(X_0)^2 E^2 + g(X)^2 P^a P_a + \frac{m^2 c^4}{\hbar^2 c^2} m(\lambda)^2.\]
If $g(X_a) = \frac{1}{R}$, then

\begin{equation}
\frac{1}{\hbar^2 c^2} \Lambda_C = -E^2 \frac{1}{\hbar^2 c^2} + P^i P_i c^2 + \frac{m^2 c^4}{\hbar^2 c^2} m(\lambda)^2
\end{equation}

or

\begin{equation}
E^2 + \Lambda_C = P^i P_i c^2 + m^2 c^4 m(\lambda)^2.
\end{equation}

For the neutrino, this becomes $E^2 = P^i P_i c^2 - \Lambda_C$. For the electron, this becomes $E^2 = P^i P_i c^2 + m^2 c^4 - \Lambda_C$. Using a Wick rotation $\Lambda_C = i\epsilon_F$, gives the standard form needed for the convergence of the path integral[14].

9 Discussion

This paper has demonstrated a holomorphic representation that is compatible with general relativity and Dirac and Klein-Gordon equations. An interpretation that is compatible with Standard Model group structure, fields, charge assignments, and particle content has also been shown.

Time is introduced on the physical restriction (effective) representation but full four-dimensional covariance and group structures are maintained on all of the “underlying” (local and analytic continuation) representations. The local foreground and background conjugate symmetry, when restricted to the effective (physical) representation is seen to be able to impose the proper properties and structure on the spacetime manifold and its conjugate particle content.

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