Nonperturbative parton distributions and the proton spin problem

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December 5, 2014

Abstract

The Lorentz contracted form of the static wave functions is used to calculate the scaled parton distributions for mesons and baryons, boosting the rest frame solutions of the path integral Hamiltonian. Both unpolarized and polarized parton distributions are derived, and the proton spin problem is solved in the new setting. It is argued, that the DIS parton densities are not related to the ground state nucleon.

1 Introduction

The partonic model, which allows to express the physical amplitudes in terms of quark and gluon densities, is now widely used in many processes [1, 2, 3].

One of basic principles in this approach is the assumption, that at high momentum $P$ the wave function of a hadron can be represented as an assembly of quasi free partons – quarks and gluons – which interact perturbatively and are subject to a DGLAP evolution [3, 4, 5]. E.g. at the first step the incident object (photon) interacts with one of free quarks of the hadron, which subsequently emits gluons etc.

This idea implies, that the nonperturbative interaction in the wave function of a very fast hadron can be neglected as compared to the high kinetic energy of every parton (quark or gluon). The results of this approach seem to
be quite successful in many cases and the whole industry of the parton density calculations is now operating to exploit and predict experimental data [6, 7].

From the theoretical point of view this method being intuitively persuasive, still lacks rigorous foundations. It is clear, that in the rest frame the hadron wave function is governed by nonperturbative interactions, such as confinement, and it is not understood how it changes with increasing velocity of the hadron.

One would like to calculate the hadron wave function in any moving frame and demonstrate the resulting transformation and transition to the pure parton picture.

Recently the possibility of this procedure was discovered in [8], where it was shown, that the Lorentz contraction condition on the wave function moving with velocity \( v \), automatically brings it into a scaled partonic-like form \( \psi(p_\perp^{(1)}, \ldots p_\perp^{(n)}; x_1, \ldots, x_n) \), depending on transverse momenta \( p_\perp^{(i)} \) and longitudinal momenta \( p_\parallel^{(i)} = x_i P \).

The new element of this “partonic wave function” is the full scale nonperturbative interaction, governing dependence of \( \psi \) on its arguments. In particular, for the simplest s-wave meson wave function one obtains the form of valence component \( \varphi \left( P_\perp, M_0 (x - \frac{1}{2}) \right) = \varphi(\sqrt{P_\perp^2 + M_0^2 (x - \frac{1}{2})^2}) \) where \( M_0 \) is the meson rest mass and \( \varphi(k) \) is the Fourier transform of the rest frame meson wave function.

It seems interesting, that the shape of the resulting quark density for this purely valence component (without Regge ladder contribution) partly resembles the known examples (at low \( Q^2 \) the maximum around \( x = \frac{1}{2} \) for mesons and around \( 1/3 \) for baryons) however other features, such as admixture of antiquarks, gluons and behavior around \( x = 0 \) and \( 1 \), are different. Thus the use of the Lorentz contracted form of nonperturbative hadron might be an interesting step in establishing of the new “nonperturbative parton model” and nonperturbative quark densities. In addition, it may resolve the existing difficulties in the present theory, such as the proton spin problem [9], as we discuss in what follows.

At this point one should stress the difference between our relativistic Hamiltonian approach and standard parton theory. In our case we are dealing with the Fock column solution of the Hamiltonian, and to each isolated hadron there corresponds a unique Fock column of a fixed energy. We show, that in all inertial frames the structure of the Fock column is the same (i.e
the coefficients in front of each Fock component do not depend on the frame, if the normalization condition is boost-invariant, while the wave functions of the components transform according to the Lorentz contraction rule for the instantaneous frames). This set of Fock columns produces the set of partonic wave functions, when the boost momentum $P$ tends to infinity (see appendix of [8] for an explicit correspondence). Since we consider hadrons of finite mass, it is conceivable that all Fock columns consists of one dominant Fock component $\Psi_n^{(0)}; \{\Psi_n\} = \{\Psi_n^{(0)}, \Psi_n^{(1)}, ...\}$ while higher components are strongly suppressed.

This type of Fock columns can be called 2-point Fock columns, and the corresponding parton sets can be called 2-point, or one-hadron set and it is (in the sense outlined above) boost invariant.

However in the standard parton model, e.g. used for DIS or proton spin problem, more general sets of Fock columns are used, which we shall call below 4-point Fock columns and 4-point parton sets.

Take first an example of the Regge ladder set, which contributes for high energy in 4-point amplitudes, such as $hh, eh, \nu h$ etc., and is absent for the one-hadron (2-point) set, since it appears in the 4-point amplitude $A(s,t)$.

Note, that this is a sum of Fock components for the crossed channel $s \to t$, where $\sqrt{t}$ plays the role of the energy.

Another example is the set of DGLAP (BFKL) diagrams (e.g. the gluon ladder in the axial gauge), which can be considered as the set of multigluon hybrid states in the $s$ channel and one is interested in the sum of these for very high incident energy.

The corresponding Fock column also exists for a 2 point pure hadron state, however for the ground or low excited states the coefficients strongly decrease due to a small hadron mass and high multigluon hybrid masses, in addition to small $\alpha_s$, while for DIS and high energy $hh$ amplitudes these coefficients contain only $\alpha_s$ as a small parameter. Thus we conclude, that one cannot use the 4 point Fock and parton sets for the 2 point (ground) states, since even for high $P$ the structure of two set is completely different.

We repeat, that the structure of a single hadron set does not change with $P$, the individual components being Lorentz contracted.

The plan of the paper is as follows. In the next section we write the general equations for the scaled parton distributions from the nonperturbative (NP) rest frame wave functions for two and three partons. In section 3 the QCD Hamiltonian and Fock components are discussed both in the rest frame and at high $P$. 

3
In section 4 the proton spin problem is discussed for the relativistic proton wave function which ensures the correct $g_A/g_V$ ratio.

The last section is devoted to the Summary of the results and perspectives.

## 2 Parton densities from the Lorentz contracted wave functions

The multiparton wave function normalized in a standard way \[2\]

$$E(P) \int \prod_{i=1}^{N} \frac{d^3p_i}{\varepsilon_i} \delta^{(3)} \left( P - \sum_{k=1}^{n} p_k \right) |\psi(p_1, ... p_N)|^2 = 1, \varepsilon_i = \sqrt{p_i^2 + m_i^2} \quad (1)$$

in the limit of large $P$ is written as

$$\int \prod d^2p_{i \perp} \frac{d\varepsilon_i}{\varepsilon_i} \delta^{(2)} \left( \sum_{i=1}^{N} p_{i \perp} \right) \delta \left( 1 - \sum_{i=1}^{N} \varepsilon_i \right) |\psi(p_{i \perp}, x_i)|^2 = 1. \quad (2)$$

It can be connected to the Lorentz contracted rest frame wave function \[\tilde{\varphi}_0(k_{\perp}^{1}, k_{\perp}^{2}, ...), k_{\parallel}^{(1)} \sqrt{1 - \nu^2}, ... \] which is normalized in the rest frame as

$$M_0 \int |\tilde{\varphi}(k^{(1)}k^{(2)}...)|^2 \frac{d^3k^{(1)}}{(2\pi)^3} ... \frac{d^3k^{(N)}}{(2\pi)^3} \delta^{(2)}(\sum_{i=1}^{N} k_{\perp}^{(i)}) \delta \left( 1 - \sum_{i=1}^{N} x_i \right) = 1, \quad (3)$$

where $k_{\parallel}^{(i)} = M_0(x_i - \nu_i)$.

The parton distribution in the hadron $h$ is $D^{q}_{h}(x, k_{\perp}^{j})$, which is defined as

$$D^{q}_{h}(x, k_{\perp}^{j}) = \sum_{n} \prod_{r} \frac{d^3k_{r}}{\varepsilon_r} E^{2}_h \delta^{(3)}(P - \sum_{r} k_{r}) |\psi_h^{(n)}(k_{r}, \lambda_{r})|^2 \sum_{r\langle j \rangle} \delta^{(3)}(k - k_{j}) \quad (4)$$

$D^{q}_{h}(x, k_{\perp}^{j})$ satisfies the following conditions \[2\]

$$\int d^2k_{\perp} dx D^{q}_{h}(x, k_{\perp}) = N^{j}_{h}, \quad (5)$$

where $N^{j}_{h}$ is the number of partons of the type $j$ in the hadron $h$, and normalization condition
\[ \sum_j \int d^2 k_\perp dx x D^q_h(x, k_\perp) = 1. \]  \hspace{1cm} (6)

One can also define the parton density

\[ D^q_h(x) = \int d^2 k_\perp D^q_h(x, k_\perp) \]  \hspace{1cm} (7)

In terms of the meson rest frame wave function one can write, taking into account, that it depends on the relative momentum \( \mathbf{k} \) in the rest frame

\[ D^q_M(x) = \frac{M_0^2}{(2\pi)^3} \left| \tilde{\varphi}^{(2)}_0 \left( k_\perp, M_0 \left( x - \frac{1}{2} \right) \right) \right|^2, \]  \hspace{1cm} (8)

where the meson wave function is normalized as

\[ \frac{M_0^2}{(2\pi)^3} \int \left| \tilde{\varphi}^{(2)}_0 \left( k_\perp, M_0 \left( x - \frac{1}{2} \right) \right) \right|^2 d^2 k_\perp dx = \frac{M_0}{(2\pi)^3} \int \left| \tilde{\varphi}^{(2)}_0 \left( k_\perp, k_\parallel \right) \right|^2 d^3 k = 1 \]  \hspace{1cm} (9)

From the condition \( \int \left( x - \frac{1}{2} \right) |\tilde{\varphi}^{(2)}_0| d^2 k_\perp dx = 0 \) and normalization condition (9) one obtains both relations (5) and (6).

For the 3q valence wave function of the baryon one can write, e.g. for the \( u \) quark distribution in the proton (ignoring spin degrees of freedom, see section 4)

\[ u(x, k_\perp) = \int \delta^{(2)} \left( \sum_{i=1}^3 k_{\perp i} \right) \prod_{i=1}^3 d^2 k_{\perp i} dx_1 dx_2 dx_3 \delta \left( 1 - \sum x_i \right) \times \]
\[ \times \frac{M_0^3}{(2\pi)^6} |\tilde{\varphi}^{(3)}_0|^2 \left[ (\delta^{(2)}(k_\perp - k_{\perp 1})\delta(x - x_1) + (1 \leftrightarrow 2)) \right] \]  \hspace{1cm} (10)

where \( \tilde{\varphi}^{(3)}_0 \) is \( \tilde{\varphi}^{(3)}_0 \left( k_{\perp 1}, ..., k_{\parallel 1}^{(0)}, ... \right) \).

The normalization of the \( \tilde{\varphi}^{(3)}_0 \) is

\[ \frac{M_0^3}{(2\pi)^6} \int \prod_{i=1}^3 d^2 p^{(i)}_\perp dx_i \delta^{(2)} \left( \sum_{i=1}^3 p^{(i)}_{\perp} \right) \delta \left( 1 - \sum_{i=1}^3 x_i \right) |\tilde{\varphi}^{(3)}_0 (p_{\perp 1}, ..., x_i) |^2 = 1. \]  \hspace{1cm} (11)
In a similar way one defines the $d$ quark distribution, in which case one will have one product of $\delta$ functions instead of the sum of two products in the square brackets in (10).

For the 3-particle wave function and Hamiltonian it is convenient to introduce the total momentum $P$ and two relative momenta $\pi, q$ defined as follows \[11, 12\] (we assume for simplicity particles 1 and 2 to be identical)

$$
\eta = \frac{z^{(1)} - z^{(2)}}{\sqrt{2}}, \quad \xi = \sqrt{\frac{\omega_3}{2\omega}}(z^{(1)} + z^{(2)} - 2z^{(3)})
$$

$$
R = \frac{1}{\Omega} \sum_{i=1}^{3} \omega_i z^{(i)}, \quad \Omega = \sum_{i=1}^{3} \omega_i, \quad \omega_1 = \omega_2 = \omega
$$

$$
P = \frac{\partial}{i\partial R}, \quad q = \frac{\partial}{i\partial \xi}, \quad \pi = \frac{\partial}{i\partial \eta}
$$

$$
H = \frac{P^2}{2\Omega} + \frac{q^2 + \pi^2}{2\omega} + \sum_{i=1}^{3} \frac{m_i^2 + \omega^2}{2\omega_i} + V(\eta, \xi).
$$

(13)

In terms of individual momenta $p^{(i)} = \frac{1}{\omega z^{(i)}}$ one has the following connection

$$
p^{(1)} = \frac{\omega}{\Omega} P + \sqrt{\frac{\omega_3}{2\Omega}} q - \frac{\pi}{\sqrt{2}}
$$

$$
p^{(2)} = \frac{\omega}{\Omega} P + \sqrt{\frac{\omega_3}{2\Omega}} q + \frac{\pi}{\sqrt{2}}
$$

$$
p^{(3)} = \frac{\omega_3}{\Omega} P - \sqrt{\frac{2\omega_3}{\Omega}} q
$$

(14)

Note, that $\omega_i$ are found from the stationary point analysis of $M_n(\omega_1, \omega_2, \omega_3)$ the eigenvalue of $H$, namely from the relations

$$
\left. \frac{\partial M_n(\{\omega_i\})}{\partial \omega_k} \right|_{\omega_k = \omega_k^{(0)}} = 0.
$$

(15)

Since $\tilde{\phi}_0^{(3)}$ is the Fourier transform of the rest frame wave function, we shall use finally the values of $\omega_i^{(0)}$ obtained in the rest frame.

Using (14) one can write in (11)

$$
\delta^{(2)}(\sum_{i=1}^{3} p^{(i)}_{\perp}) \prod d^2 p^{(i)}_{\perp} = \frac{\omega_3}{\Omega} d^2 P_{\perp} d^2 q_{\perp} d^2 \pi_{\perp} d^2 \pi_{\perp} \delta(P_{\perp}) = \frac{\omega_3}{\Omega} d^2 q_{\perp} d^2 \pi_{\perp}. \quad (16)
$$
To find the arguments \( p_{||i}^{(0)}(x_i) \) in (10), which are the longitudinal components of parton momenta in the rest frame, we use the Lorentz connection of these to the momenta in the moving frame

\[
p_{||i} = \frac{p_{||i}^{(0)} + v \tilde{\omega}_i^{(0)}}{\sqrt{1 - v^2}} = P x_i,
\]

which yields

\[
p_{||i}^{(0)} = \left( x_i - \frac{\tilde{\omega}_i^{(0)}}{M_0} \right) M_0.
\]

In what follows we shall accept for simplicity the relation for massless quark \( \tilde{\omega}_i^{(0)} \equiv \tilde{\omega}_i \), \( i = 1, 2, 3 \) and the relation for the total mass in the case of free quarks, which holds approximately true in the interacting case

\[
M_0 = \sum_{i=1}^{3} \tilde{\omega}_i = 3 \tilde{\omega},
\]

hence \( p_{||i}^{(0)} = \left( x_i - \frac{1}{3} \right) M_0 \). For the relative momenta one obtains

\[
q_{||i}^{(0)} = \frac{M_0}{\sqrt{6}}(x_1 + x_2 - 2x_3), \quad \pi_{||i}^{(0)} = \frac{M_0(x_2 - x_1)}{\sqrt{2}}.
\]

One can rewrite the normalization condition (11) as

\[
1 = \frac{M_0^3}{(2\pi)^6} \cdot \frac{1}{3} \int d^2q_{\perp} d^2\pi_{\perp} dx_1 dx_2 dx_3 \delta \left( 1 - \sum_{i=1}^{3} x_i \right) |\tilde{\varphi}_0^{(3)}(q_{\perp}, \pi_{\perp}, q_{||}^{(0)}, \pi_{||}^{(0)})|^2
\]

and Eq. (10) acquires the form

\[
u(x, k_{\perp}) = \frac{4M_0^3}{(2\pi)^3} \int d^2\pi_{\perp} \int_0^1 \right.
\]

\[
\left. dx_2 \left. |\tilde{\varphi}_0^{(3)}\left( \sqrt{3} \pi_{\perp} + \sqrt{6} k_{\perp}; \pi_{\perp}; \sqrt{\frac{3}{2}} M_0 \left( x + x_2 - \frac{2}{3} \right); \left( \frac{M_0(x_2 - x)}{\sqrt{2}} \right) \right) \right|^2.
\]

It is known, however, that the nucleon wave function can be expanded in the series of hyperspherical harmonics [12], and the leading term accounts for \( \sim 90\% \) of the normalization condition. This means, that \( \tilde{\varphi}_0^{(3)} \) can be considered as the function of

\[
Q^2 = \pi^2 + q^2 = \pi_{\perp}^2 + q_{\perp}^2 + (\pi_{||}^{(0)})^2 + (q_{||}^{(0)})^2 = \pi_{\perp}^2 + q_{\perp}^2 + \frac{M_0^2}{3} \sum_{i>j} (x_i - x_j)^2
\]
As a result the argument of $\tilde{\varphi}^{(3)}_0$ in (21) can be written as

$$
\tilde{\varphi}^{(3)}_0(Q^2) \rightarrow \tilde{\varphi}^{(3)}_0(4\pi_-^2 + 6k_-^2 + 6\sqrt{2}\pi_- k_- + \frac{2M_0^2}{3} f(x, x_2)) \quad (23)
$$

where

$$
f(x, x_2) = 1 + 3x^2 + 3x_2^2 + 3xx_2 - 3x - 3x_2 = 3\left((x_2 - \frac{1 - x}{2})^2 + c(x)\right), \quad (24)
$$

$$
c(x) = \frac{9x^2 - 6x + 1}{12}.
$$

It is conceivable, that after the integration over $d^2\pi_-$ and $dx_2$ the result will depend mostly on $c(x)$, which has the minimum at $x = \frac{2}{3}$ and hence $u(x, k_-)$ as a function of $x$ would have a maximum around that point.

This is close to that obtained for the valence quark density in [13] and has a form similar to the DIS data [7], [14] at low $Q^2$. In Fig. 1 we compare our results with those of [13]. We shall argue however, that the DIS data refer to the high excited baryon state, and hence could differ from the pure proton case.

### 3 The QCD Hamiltonian and Fock components

We assume in this section that one can construct a QCD Hamiltonian, which provides eigenvalues and eigenfunction for all Fock components, e.g. in case of a baryon state, the pure valence state $(qqq)$, also with any number of additional gluons $(qqqg)$, $(qqqgg)$, ..., which are actually hybrid states, and with additional $q\bar{q}$ pairs:$(qqq(q\bar{q}))$ etc.

One example of such Hamiltonian is provided by the path integral Hamiltonian derived in the framework of the Fock-Feynman-Schwinger Representation (FFSR) [15], and developed further in [16], [17]. It was used for mesons [18], baryons [12], [19], hybrids [20] and glueballs [21], yielding in all cases spectra in good agreement with experimental and lattice data.

The full Fock matrix Hamiltonian in this case consists of the diagonal elements $H_n^{(0)}$ for each $n$-th Fock component and of the nondiagonal elements $H_{nk}$, $n \neq k$, which are actually elements of the interaction vertices $V_{nk}$. In what follows we are using the line of reasoning from [22], [23] and start with
Figure 1: The valence parton distribution in proton (red solid line) in comparison with the results of [13] for fixed values of quark condensate

the center of mass frame, $P = 0$. Note, that we have the instantaneous dynamics, so that all operators $\hat{H} = \hat{H}^{(0)} + \hat{V}$ do not depend on time, and $\hat{V}$ acts at the instantaneous moment of creation or destruction of an additional particle.

In Fig. 2 it is shown, how subsequent Fock components appear in the meson Green’s function, where additional gluons and a $q\bar{q}$ pair are created by the interaction $V_{nk}$.

One can simplify, as in [22], the structure of $\hat{H}$, and taking the limit of large $N_c$, so that an additional $q\bar{q}$ pair “costs” a factor of $1/N_c$, while an additional glueball gives $1/N_c^2$, so that all Fock components reduce to the pure valence states and its hybrid excitations.

The basic matrix equation is simply

$$\hat{H}\Psi_N = (\hat{H}^{(0)} + \hat{V})\Psi_N = E_N\Psi_N,$$  \hspace{1cm} (25)

and the Fock column $\Psi_N\{P, \xi, n\}$ has quantum numbers $N = 0, 1, 2...$ of the ground and excited states, $n$ refers to the Fock column number, and $\xi$ denotes additional internal numbers for a given type of excitation. In the diagonal
Figure 2: The meson Green’s function, containing the Fock components: $q\bar{q}$ (sector I), $q\bar{q}g$ (sector II and sector III), $q\bar{q}gg$ (sector IV) and $q\bar{q}qq$ (sector V). Note, that all surface between external solid lines is filled in by the string world sheet, except for the narrow gap between $q\bar{q}$ lines in sector V.

approximation $\Psi_N \to \Psi_{N\{P,\xi,n\}}$ with $n = n^{(0)}$. Note, that for $N_c \to \infty$ and baryon number $B = 1$ one has only discrete spectrum of a valence and hybrid states. In this case the eigenvalues for $P \neq 0$ are simply

$$E^{(0)}_N = E^{(0)}_n(P) = \sqrt{P^2 + M^2_{n\{k\}}}.$$  \hfill (26)

and the eigenfunctions are $\psi_n(P, \xi, k) \equiv \psi_{n,k}$, where $\xi, k$ comprise radial and angular $(k)$, as well as additional $(\xi)$ quantum numbers. The set $\psi_{n\{k\}}$ can be used to expand the total wave function $\Psi_N$, (discrete spectrum at $N_c \to \infty$)

$$\Psi_N = \sum_{m\{k\}} c_{m\{k\}}^N \psi_{m\{k\}}, \quad \int \Psi_{N}^{\dagger} \Psi_M d\Gamma = \sum_{m\{k\}} c_{m\{k\}}^N c_{m\{k\}}^M = \delta_{NM}. \hfill (27)$$

As in [22] [23], we use the orthonormality condition

$$\int \psi_{m\{k\}}^{\dagger} \psi_{n\{p\}} d\Gamma = \delta_{mn} \delta_{\{k\}\{p\}}.$$  \hfill (28)

to find the equation for $c$ and $\Psi$

$$c_{n\{p\}}^N (E_N - E^{(0)}_{n\{p\}}) = \sum_{m\{k\}} c_{m\{k\}}^N V_{n\{p\},m\{k\}}.$$  \hfill (29)

with

$$V_{n\{p\},m\{k\}} = \int \psi_{n\{p\}}^{\dagger} \hat{V} \psi_{m\{k\}} d\Gamma.$$  \hfill (30)
to the first order in $V$ one has

$$e^{N(1)}_{n\{p\}} = \frac{V_{n\{p\} \nu\{\kappa\}}}{E^{(0)}_{\nu\{\kappa\}} - E^{(0)}_{n\{p\}}}$$  \hspace{1cm} (31)

and for the high Fock component with $l$ gluons in addition to the valence quarks one has in the lowest approximation

$$C^{N(\nu\{\kappa\})}_{\nu+l\{k\}} = \sum_{\{k_1\}...\{k_l\}} \frac{V_{\nu+l\{k\},\nu+l-1\{k_1\}} V_{\nu+l-1\{k_1\},\nu+l-2\{k_2\}}}{E^{(0)}_{\nu\{\kappa\}} - E^{(0)}_{\nu+l\{k\}}} ... \frac{V_{\nu+1\{k_1\},\nu\{\kappa\}}}{E^{(0)}_{\nu\{\kappa\}} - E^{(0)}_{\nu+1\{k_1\}}} + O(V^{l+2}). \hspace{1cm} (32)$$

Using (27), (28), one can obtain the equality

$$\sum_{n\{k\}} \left| C^{N_{n\{k\}}}_{n\{k\}} \right|^2 = 1, \hspace{1cm} (33)$$

and each state $N$ can be characterized by the sequence

$$\{|C^0_{0}|^2, |C^N_{i\{k\}}|^2, ...\} \equiv \{C^N\}.$$

Note, that $\hat{V}$ is $O(g)$, and hence in the limit $g \to 0$ one has the unmixed states $\{1,0,0,...\}, \{0,1,0,...\}$ etc., while the inclusion of $\hat{V}$ starts the “evolution” of the basic state along the chain of neighboring states. This can be done in principle in accordance with the DGLAP evolution equation, to be written in terms of parton distribution functions (pdf), i.e. in terms of $|\psi_{n\{k\}}|^2$ integrated over all pairs $p^{(i)}_\perp$ and $x_i$, except one, $p^{(i)}_\perp \equiv k_\perp, x_1 \equiv x$.

Thus the nonperturbative $N_c \to \infty$ equivalent of the DGLAP or BFKL evolution is given by (29), (31). At this point it is important to stress, that sequences $\{C^N\}$ can be completely different for the baryon ground state with $E_0 = m_p$, and the highly excited baryon state with a large c.m. energy $E_N$. In our case ($N_c = \infty$) this refers to the discrete spectrum, while allowing for the $q\bar{q}$ pairs one has a continuous spectrum. Indeed, as was estimated in [23, 24] for the meson-hybrid mixing coefficient $V_{Mh} \sim g \cdot 0.08$ GeV and

$$C_{Mh} = \frac{V_{Mh}}{E^{(0)}_M - E^{(0)}_h} = \frac{V_{Mh}}{\Delta M_{Mh}}, \quad \Delta M_{Mh} \sim O(1 \text{ GeV}) \hspace{1cm} (34)$$

11
which gives the hybrid admixture \(|C_{Mh}|^2 \approx O(1\%)\). As shown in \([22, 23]\) the addition of one gluon to the hybrid state "costs" around 1 GeV, hence multigluon states contribute very little to the ground state wave function. For the high excited states the denominator in \((34)\) can be much smaller and the total number of mixing states grows, so that the gluon admixture should grow substantially.

This is exactly what happens in DIS. Indeed, the c.m. energy of the baryon state, with initial momentum \(p\), exited by the incident virtual \(\gamma\) or \(W, Z\) with momentum \(q\), is very high in the Bjorken limit.

\[
s = m_B^2 + 2\nu(1 - x), \quad x = Q^2/2\nu, \quad s - m_B^2 = 2\nu(1 - x) \gg m_B^2.
\]

In Fig. 3 we show schematically how the excited baryon state emerges in DIS.

Figure 3: The excited baryon state created in DIS has the excitation energy in the rest frame \(E_{CM} = \sqrt{M_B^2 + \frac{Q^2(1-x)}{x}}\), which is much larger than \(M_B\) and the boost momentum \(Q\) for small \(x\).

It is actually the baryon state with the c.m. energy \(E_{cm} = \sqrt{s}\), which is tested in DIS and the resulting pdf refer to this \(E_{cm}\), which is close to \(m_B^2\) only at the end point \(x = 1\). Therefore one should expect, that at growing \(s = m_B^2 + \frac{Q^2}{x}(1 - x)\) the admixture of gluons should grow fast, since the coefficients \((32)\) increase with energy, or equivalently, DGLAP evolution at high \(Q^2\) produce a large gluon component, see e.g. \(6, 14\). We stress again,
that this fact refers not to the ground state described above. For the ground state baryon (or meson) the sequence \( \{C^N\} \) is fast decreasing and is given by the rest frame wave functions, described in the previous section.

One of the consequences of this discussion is the possible resolution of the proton spin problem in the next section (see [9] for discussion and references). Indeed, insertion in the proton wave function the polarized parton distributions, obtained from the highly excited baryon states, results in the high admixture of the antiquark and gluon components (which are suppressed in the genuine proton wave function). As a result the contribution of the valence quarks is very small, and one faces the strange picture of the almost quarkless proton. If instead one uses the pdf of the proper proton, described in the previous section, this discrepancy disappears, as we show in the next section.

One should stress in addition, that the virtual photon in DIS is able to transfer any amount of the angular momentum (similarly to the process of the electroexcitation of nuclei), so that the excited baryon can have any half integer spin, suppressing in this way the contribution of the \( s = 1/2 \).

We now turn to the boosted form of the hadron wave function. We assume, as in [8], that the boost acts on the spacial wave function \( \Psi_N \) as the Lorentz contraction, while the interaction term \( \hat{V} \) behaves as \( \hat{L}\hat{V} = CV, \quad C_0 = \sqrt{1 - v^2} \).

This becomes clear from (25), if one writes the Hamiltonian in (25) in the off-shell form as in (13)

\[
\hat{H}^{(0)} = \frac{P^2 + \Omega^2}{2\Omega} + \frac{\sum_{i=1}^{N} m_i^2 + \omega_i^2 + P_i^2}{2\omega_i} + \frac{C(V_0 + \Delta V)\hat{M}}{2\Omega},
\]

where we have splitted the interaction \( \hat{V} = V_0 + \Delta V \) into a diagonal and nondiagonal parts in the number \( n \) of constituents.

As a result

\[
E_n^{(0)} = \sqrt{P^2 + (M_n^{(0)})^2} \simeq P + \frac{(M_n^{(0)})^2}{2P} + ...
\]

and

\[
C_n\{P\}^{(1)} \simeq \frac{CV_n^{(0)}_{n,n+1}}{E_n^{(0)} - E_{n+1}^{(0)}} \simeq \frac{C_0 PV_n^{(0)}_{n,n+1}}{(M_n^{(0)})^2 - M_n^{(0)} + (M_n^{(0)})^2} \simeq \frac{M_n^{(0)} V_n^{(0)}_{n,n+1}}{(M_n^{(0)})^2 - (M_n^{(0)})^2}.
\]
Hence the set \( \{C^N\} \) is boost invariant and the nucleon mixing contents does not change, when going from the rest frame to the infinite momentum frame. The same can be said about all excited baryon states, which are measured using 4 point amplitudes, as it is done in DIS, and the high excited energy states (about tens of GeV on average) are very different from the ground state nucleon both in the c.m. and infinite momentum frames. In the DIS partonic set \( \{C^N\} \) one has much higher admixture of antiquark and gluon (hybrid) components, as compared to the ground state nucleon partonic set. This explains the long standing proton spin puzzle \( [9] \), where the use of the polarized DIS data yields high antiquark admixture, cancelling the valence quark contribution, and high gluon contribution.

At this point it is essential to note, that the very idea, that the high excited hadron can be represented by an assembly of almost free partons seems to be reasonable, but the idea, that the fast moving ground state hadron can be represented by the same set of free partons from our point of view has no foundations.

It is important, that in DIS we talk about 4 point Green’s function and 4 point parton distributions, which is especially evident, when one considers the Regge exchange contribution to the parton densities.

In some cases, as in DIS, quantum numbers of this 4 point object can be the same, as for a nucleon, but we stress, that the ground state nucleon has nothing to do with this 4 point quark densities, while it is likely, that the high excited hadron can simulate the 4 point quark distribution. It is even likely, that at very high excitation the nonperturbative contributions can be treated as the initial state to the evolution of partons subject to the DGLAP or BFKL procedure, see e.g. \( [25] \).

4 Polarized parton distributions and \( g_A/g_V \) in the proton

In this section we shall derive the polarized parton distributions and nucleon axial charges, starting with the rest frame nucleon wave function. In section 2 we have defined the unpolarized parton distributions using the nucleon wave function, which has a simple structure without lower Dirac components. However for a reliable description of spin degrees of freedom and axial charge this is not enough and one must use the full 4 component Dirac structure
of every quark. To this end we use the decomposition of the 3q wave function in the products of Dirac quark bispinors – the so-called Dirac orbital model [26, 27] and keep for simplicity only the first dominant term in the sum over spin, isospin and angular momenta.

\[
\Psi(r_1, r_2, r_3) = \sum_{\{n_i\}} \prod_{i=1}^{3} \psi_{n_i}(r_i) C_{n_1n_2n_3}.
\]  

In the momentum space one can write for the nucleon

\[
\tilde{\Psi}_N(p_1, p_2, p_3) = \sum_{\alpha_i} \prod_{i=1}^{3} \phi_{\alpha_i}(p_i) C_{\alpha_1\alpha_2\alpha_3},
\]

where \(\sum p_i = 0\) and \(\alpha_i\) stand for spin-isospin and the angular momentum variables. As it is known from the actual calculations [12, 19, 28] the dominant contribution in the 3 body nucleon wave function is given by the symmetric in quarks component, which can be written for the proton with the spin up as

\[
\Psi_p \equiv \langle p \uparrow \rangle = \frac{1}{\sqrt{18}} \left[ -2(|u \uparrow u \uparrow d \downarrow \rangle + \text{perm}) + (|u \uparrow u \downarrow d \uparrow \rangle + \text{perm}) \right]
\]

and perm implies the permutation of the quark positions in the triade, while each of quark functions is a Dirac bispinor, viz.

\[
\chi_{\uparrow}(r, \theta, \phi) = \frac{1}{r} \begin{pmatrix}
G(r)\Omega^{00}_{\frac{1}{2}\frac{1}{2}} \\
F(r)\Omega^{11}_{\frac{1}{2}\frac{1}{2}}
\end{pmatrix},
\]

normalized as

\[
\int_0^{\infty} (G^2(r) + F^2(r)) dr = 1.
\]

Now one can define the proton axial charge [27]

\[
g_A = \langle p \uparrow | \hat{u}^+ \Sigma_3 \hat{u} - \hat{d}^+ \Sigma_3 \hat{d} | p \uparrow \rangle,
\]

where \(\Sigma_3 = \begin{pmatrix}
\sigma_3 & 0 \\
0 & \sigma_3
\end{pmatrix}\), and one obtains

\[
g_A = \frac{4}{3} \langle \chi_\uparrow | \Sigma_3 | \chi_\uparrow \rangle - \frac{1}{3} \langle \chi_\downarrow | \Sigma_3 | \chi_\downarrow \rangle = \frac{5}{3} \int_0^{\infty} \left( G^2(r) - \frac{1}{3} F^2(r) \right) dr = \frac{5}{3} \left( 1 - \frac{4}{3} \eta \right).
\]
with $\eta = \int_0^\infty F^2(r)dr$. As was shown in [27], the calculation of $\eta$ for $\alpha_s = 0.39$ yields $g_A = 1.27$ in good agreement with experiment [29, 30].

In a similar way one can define the proton spin projection, which has a general form [9], which is made gauge and boost invariant, using Fock space Hamiltonian solutions,

$$J_3 = \frac{1}{2} \Sigma_3 + \Delta L_q + \Delta G + \Delta L_g = \frac{1}{2},$$  \hspace{1cm} (45)

where for all operators one should take matrix element between the states $|p\uparrow\rangle$, and $\Delta L_{q,g}$ refer to the quark and gluon angular momentum, while $\Delta G$ refers to the gluon (hybrid) spin operator. As was shown in the previous section, the gluon (hybrid) contribution to the ground state proton is small ($O(1\%)$) and we shall neglect the last two terms in (45). For the first two terms using quark wave functions (44) one obtains

$$\langle \chi\uparrow|\Sigma_3|\chi\uparrow\rangle = \int_0^\infty \left( G^2(r) - \frac{1}{3} F^2(r) \right) dr = 1 - \frac{4}{3}\eta,$$  \hspace{1cm} (46)

$$\langle \chi\uparrow|\Delta L_q|\chi\uparrow\rangle = \frac{2}{3} \int_0^\infty F^2 dr,$$  \hspace{1cm} (47)

and hence

$$\langle \chi\uparrow|J_3|\chi\uparrow\rangle = \left\langle \chi\uparrow|\frac{1}{2} \Sigma_3 + \Delta L_q|\chi\uparrow\right\rangle = \int \frac{1}{2} \left( G^2 - \frac{1}{3} F^2 \right) + \frac{2}{3} F^2 dr = \frac{1}{2}. \hspace{1cm} (48)$$

Hence one does not have proton spin problems in the rest frame, however the quark orbital momentum is essential, indeed defining $\eta$ from (44), where $g_A = 1, 27$ one obtains that the first two terms contribute as follows

$$\frac{1}{2} \langle \Sigma_3 \rangle \simeq 0.38; \langle \Delta L_q \rangle \simeq 0.12. \hspace{1cm} (49)$$

We now turn to the polarized quark distributions (PQD) in the nucleon with the spin along $z$ direction,

$$\Delta u(x) = (u_\uparrow(x) - u_\downarrow(x)), \hspace{0.5cm} \Delta d(x) = (d_\uparrow(x) - d_\downarrow(x)) etc. \hspace{1cm} (50)$$

and the PQD of the proton is

$$g_1(x) = \sum_i \frac{e_i^2}{2} \Delta q_i(x) = \frac{2}{9} (\Delta u(x) + \Delta \bar{u}(x)) + \frac{1}{18} (\Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}). \hspace{1cm} (51)$$
One can connect PQD with \( g_A \), namely \[ 9 \]

\[ g_A = \int_0^1 [\Delta u + \Delta \bar{u} - (\Delta d + \Delta \bar{d})]dx. \] \tag{52} \]

To calculate \( \Delta u(x, k_\perp) \) one can use Eq. (10), where the square brackets should be rewritten for the chosen spin projection \( \mu \) as

\[ [ ] \rightarrow [ ]_{\mu,q} \equiv \sum_q \delta(x - x(q))\delta(s_z(q) - \mu)\delta^{(2)}(k_\perp(q) - k_\perp), \] \tag{53} \]

So that for \( \Delta u \) one can write

\[ \Delta u(x, k_\perp) = \int \delta^{(2)}(\sum_{i=1}^3 k_{\perp i}) \prod_{i=1}^3 d^2 k_{\perp i} dx_1 dx_2 dx_3 \delta(1 - \sum_{i=1}^3 x_i) \times \]

\[ \times \frac{M_0^3}{(2\pi)^3} \left| \Psi_N \left( k_{\perp 1}, k_{\perp 2}, k_{\perp 3}, M_0 \left( x_1 - \frac{1}{3} \right), M_0 \left( x_2 - \frac{1}{3} \right), M_0 \left( x_3 - \frac{1}{3} \right) \right) \right|^2 \frac{1}{2} \left( [ \frac{1}{2}, u] - [ \frac{1}{2}, u] \right), \] \tag{54} \]

and \( \Psi_N \) for the proton is given by the sum (40) of the products of single-quark wave functions (41).

Both contributions (\( \Sigma_3 \) and \( \Delta L_q \)) are boost and gauge invariant \[ 9 \] and using the PQD of (54) and our result of the previous section, that the Fock sequence \( \{ C_N \} \) is boost invariant, one can conclude, that the proton spin condition (45) is satisfied also in the boosted system. One of the main conclusion of the previous section retains, that the Fock sequences of the ground state nucleon and the DIS Fock sequence refer to different objects and hence the “proton spin problem” with DIS data is actually the 4-point or “excited baryon spin problem”. Our conclusion does not disprove or invalidate the enormous experimental and theoretical efforts, which have provided important information on DIS structure functions. The latter can be used in many proper places, where the intermediate high excited baryon states appear.

5 Conclusions and prospectives

We have presented in the paper the new formalism for the calculation of the boosted valence wave functions of mesons and baryons. It was shown above, how one derives the valence parton distributions from these wave functions, which can be called the proper valence parton distributions. We
have stressed, that they are different from the 4-point distributions, such as DIS, the latter contain the parton distributions of high excited hadrons. In this way our results contradict the implicit assumption of the parton model, that the pdf of the highly boosted hadron resembles that of the DIS affected hadron. Contrary to that, we show, that the Fock sequence of a hadron is boost invariant, and consequently contains the same dominant components, as in the rest frame.

The Fock sequence of the DIS affected hadron is much more rich in higher components, the latter are given e.g. by the DGLAP formalism, or by the Fock space Hamiltonian.

Another result of our approach is the nonperturbative character of the valence component, which can be used as an initial step in the DGLAP or BFKL evolution.

The possible importance of the nonperturbative input can be seen in many examples of inconsistencies of the purely perturbative parton model, such as the high $p_t$ hadron reactions, Drell-Yan processes, breakdown of factorization theorems etc., see [10, 31] for discussions and references.

As a good check of our formalism we have chosen the proton spin problem, which was not resolved in the standard parton approach, using the DIS parton distributions. We have shown that this problem is solved in the proton c.m. system, where the admixture of antiquarks and gluons is negligible, and then have used the boost invariance of our parton distributions to formulate the same solution in an arbitrary system. In fact, the present paper together with the preceding one [8], is the first step in an attempt to construct the new formalism of nonperturbative QCD at high momenta and energies, especially in the highly boosted systems. As it is, we suggest the way, where the treatment of the boost is extremely simple, so that one can directly reformulate the results obtained in the rest frame. This work for the PDF’s of different baryons is now in the process [32]. The next step: the interaction of two complexes boosted with respect to each other is a much more complicated problem, the first conclusions on the behavior of the decay amplitudes and formfactors were given in [8].

This work was supported by the RFBR grant 1402-00395. The author is grateful to M.A.Trusov for the help in preparing of Fig. 1.
References

[1] R.P.Feynman, Photon-Hadron Interactions, W.A.Benjamin Inc. Reading MA, 1972.

[2] B.L.Ioffe, V.A.Khose, and L.N.Lipatov, Deep Inelastic Processes, North-Holland, 1984.

[3] F.J.Yndurain, The Theory of Quark and Gluon Interactions, 4th edition, Springer, 2006.

[4] V.N.Gribov and L.N.Lipatov, Sov. J. Nucl. Phys. 15, 438 (1972); Yu.M.Dokshitzer, Sov. Phys. JETP 46, 641 (1977).

[5] G.Altarelli and G. Parisi, Nucl. Phys. B 126, 298 (1977).

[6] J.Blümlein, Progr. Part. Nucl. Phys. 69, 28 (2013); arXiv:1208.6087
S.Alekhin, J.Blümlein, and S.Moch, arXiv: 1303.1073.

[7] B.Foster, A.D.Martin and M.G.Vincter in J.Beringer et al., (Particle Data Group), Phys. Rev. D 86, 010001 (2012). M.Glück, E.Reya and A.Vogt, Eur. Phys. J. C 5, 461 (1998); P.Jimenez-Delgado, A.Accardi and W.Melnitchouk, phys. Rev. D 89, 034025 (2014).

[8] Yu.A.Simonov, arXiv:1409.4964 [hep-ph].

[9] R.L.Jaffe and A.Manohar, Nucl. Phys. B 337, 509 (1990); X.Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249
X.Ji, J.-H.Zhang and Y.Zhao, arXiv: 1409.6329.

[10] S.J.Brodsky, G.deTéramond and M.Karliner, Ann. Rev. Nucl. Part. Sci. 2012, 62 (2012).

[11] Yu. A. Simonov, Phys. Lett. B 719, 464 (2013).

[12] Yu. A. Simonov, Phys. At.Nucl. 66, 338 (2003); Phys. Rev. D 65, 116004 (2002).

[13] B.L.Ioffe and A.G.Oganesian, Nucl. Phys. A 714, 145 (2003).

[14] J.Botts et al., Phys. Lett. B 304, 159 (1993).
[15] Yu. A. Simonov, Nucl. Phys. B 307, 512 (1988); Yu. A. Simonov and J.A.Tjon, Ann. Phys. (N.Y) 300, 54 (2002).

[16] Yu. A. Simonov, Phys.Rev. D 88, 025028 (2013).

[17] Yu. A. Simonov, Phys.Rev. D 88, 053004 (2013); arXiv:1304.0365.

[18] Yu.A. Simonov, in: "QCD: Perturbative or Nonperturbative?" eds. L. Ferreira., P. Nogueira, J.I. Silva-Marcos, World Scientific, Singapore, 2001, hep-ph/9911237

A.M.Badalian, Yu.A. Simonov, and V.I. Shevchenko, Yad.Fiz. 69, 1818 (2006).

[19] I.M.Narodetskii and M.A.Trusov, Phys. At. Nucl. 67, 762 (2004).

[20] Yu.A.Simonov, Nucl. Phys. B (Proc. Suppl.) 23, 283 (1991); Yu.S. Kalashnikova and Yu.B. Yufryakov, Phys. Lett. B 359, 175 (1995); Phys. At. Nucl. 60, 307 (1997); Yu.S. Kalashnikova and D.S. Kuzmenko, Phys. At. Nucl. 67, 538 (2004); ibid 66, 955 (2003).

[21] A.B. Kaidalov and Yu.A. Simonov, Phys. Lett. B 477, 163 (2000); ibid B 636, 101 (2006); Phys. At. Nucl. 63, 1428 (2000).

[22] Yu.A. Simonov, Phys. At. Nucl. 67, 553 (2004); hep-ph/0306310

[23] Yu.A. Simonov, Phys. At. Nucl. 64, 1876 (2001); hep-ph/0110033.

[24] A.Le Yaouanc, L. Oliver, O. Péne, J.-C. Raynal, and S. Ono, Z. Phys. C 28, 309 (1985);
F. Iddir, S. Safir, and O. Péne, Phys. Lett. B 433 125 (1998).

[25] F.Hautmann and H.Jung, arXiv:1312.7875; P.Jimenez-Delgado, arXiv:1410.2431

[26] Yu.A. Simonov, J.A. Tjon and J.Weda, Phys. Rev. D 65, 094013 (2002); J.A. Tjon and J.Weda, Phys. At.Nucl. 68, 591 (2005).

[27] Yu.A.Simonov and M.A.Trusov, Phys. At. Nucl. 72, 1058 (2009); arXiv:hep-ph/0607075

[28] M.Fabre and Yu.A.Simonov, Ann. Phys.(NY) 212, 235 (1991).
[29] V.Barone, A.Drago and P.C.Ratcliffe, Phys. Rept. 359, 1 (2002); hep-ph/0104283.

[30] J.Beringer et al., [Particle Data Group Collaboration], Phys. Rev. D 86, 010001 (2012).

[31] D. Amati, R. Petronzio, G. Veneziano. Nucl. Phys. B 140 (1978) 54; A.V. Efremov, A.V. Radyushkin. Teor. Mat. Fiz. 42 (1980) 147; Theor. Math. Phys. 44 (1980) 573; Teor. Mat. Fiz. 44 (1980) 17; Phys. Lett. B 63 (1976) 449; Lett. Nuovo Cim. 19 (1977) 83; S. Libby, G. Sterman. Phys. Rev. D 18 (1978) 3252. S.J. Brodsky and G.P. Lepage. Phys. Lett. B 87 (1979) 359; Phys. Rev. D 22 (1980) 2157; J.C. Collins and D.E. Soper. Nucl. Phys. B 193 (1981) 381; J.C. Collins and D.E. Soper. Nucl. Phys. B 194 (1982) 445; J.C. Collins, D.E. Soper and G. Sterman. Nucl. Phys. B 250 (1985) 199; A.V. Efremov and I.F. Ginzburg. Fortsch. Phys. 22 (1974) 575; A.V. Efremov and A.V. Radyushkin. Report JINR E2-80-521; Mod. Phys. Lett. A 24 (2009) 2803; S. Catani, M. Ciafaloni, F. Hautmann. Phys. Lett. B 242 (1990) 97; Nucl. Phys. B 366 (1991) 135; B.I. Ermolaev, M. Greco, S.I. Troyan, Phys. Part. Nucl./ 44, 260 (2013); arXiv:1005.2829; V.A.Khoze, A.D.Martin and M.G.Ryskin, arXiv: 1409.8451.

[32] Yu.A.Simonov and M.A.Trusov (in preparation).