EVOLUTION OF PROGENITORS FOR ELECTRON CAPTURE SUPERNOVAE

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ABSTRACT

We provide progenitor models for electron capture supernovae (ECSNe) with detailed evolutionary calculation. We include minor electron capture nuclei using a large nuclear reaction network with updated reaction rates. For electron capture, the Coulomb correction of rates is treated and the contribution from neutron-rich isotopes is taken into account in each nuclear statistical equilibrium (NSE) composition. We calculate the evolution of the most massive super asymptotic giant branch stars and show that these stars undergo off-center carbon burning and form ONe cores at the center. These cores become heavier up to the critical mass of 1.367 $M_\odot$ and keep contracting even after the initiation of O+Ne deflagration. Inclusion of minor electron capture nucleus causes convective URCA cooling during the contraction phase, but the effect on the progenitor evolution is small. On the other hand, electron capture by neutron-rich isotopes in the NSE region has a more significant effect. We discuss the uniqueness of the critical core mass for ECSNe and the effect of wind mass loss on the plausibility of our models for ECSN progenitors.

Key words: nuclear reactions, nucleosynthesis, abundances – stars: evolution – stars: interiors – supernovae: general

Online-only material: color figures

1. INTRODUCTION

Electron capture supernova (ECSN) is a distinct class in core collapse supernova (CCSN). An ECSN progenitor is a super asymptotic giant branch (SAGB) star with a mainly oxygen and neon core, surrounded by a thin helium shell and diffuse hydrogen envelope (Nomoto 1987). In an ONe Chandrasekhar mass core, electron capture reactions by $^{24}$Mg and $^{20}$Ne heat the surroundings. As a result, O+Ne burning ignites at the center and generates energy, and O+Ne deflagration propagates outward. However, the released energy is too small to explode the highly bound core (Miyaji et al. 1980). Further electron capture reactions in the central nuclear statistical equilibrium region (NSE) accelerate core contraction. Finally, a proto-neutron star forms and becomes a weak Type II SN (Kitaura et al. 2006).

The most distinct point of the progenitor may be its contrasting structure of a highly concentrated core and a diffuse envelope. While the prompt explosion reported in an earlier work of Hillebrandt et al. (1984) was not confirmed in other groups’ simulations (Burrows & Lattimer 1985; Baron et al. 1987), a hydrodynamical simulation of collapsing ONe core showed that the delayed explosion powered by neutrino heating takes place even in one-dimensional calculations (Mayle & Wilson 1988; Kitaura et al. 2006). The successful explosion is found by recent multi-dimensional calculations as well (Janka et al. 2012), and properties of ECSNe such as nucleosynthesis (Wanajo et al. 2011, 2013) have been studied.

For observations as well as for theory, a model of ECSN has important implications. Some low luminosity SNe, e.g., SN1997D (Turatto et al. 1998), SN2005cs (Pastorello et al. 2006, 2009), can be explained by the explosion model of an ECSN which has a low explosion energy and synthesizes a small amount of $^{56}$Ni. Also observed peculiar compositions in the well-known Crab nebula, such as abundant He and less abundant O, indicate that the Crab supernova SN1054 arose from a collapse of an SAGB star (Nomoto et al. 1982). Type IIn SN, which is an SN explosion enshrouded by a dense circumstellar medium, can be explained by an ECSN as well as a CCSN from very massive star that has experienced an intense mass-loss phase. Recently, a progenitor of a dust-shrouded transient SN2008S is found in a pre-explosion image (Botticella et al. 2009) and it would have a mass of $\sim 10 M_\odot$, which is a plausible mass for an ECSN progenitor.

However, there has not been a consistent progenitor calculation from zero-age main sequence (ZAMS) to collapse because of the numerical difficulties in calculating the full evolution of SAGB stars. The main difficulties are off-center C burning, thermal pulses, contraction of a highly degenerate core, calculation of electron capture, and propagation of deflagration. These phases have been separately studied by several authors.

The theoretical work on a collapsing ONe core was initiated by Nomoto and collaborators in the 1980s (Miyaji et al. 1980; Miyaji & Nomoto 1987; Nomoto 1984, 1987). Miyaji et al. (1980) investigated effects of electron capture by $^{24}$Mg and $^{20}$Ne and showed that these effects can be summarized as follows: first, reduction of the electron mole fraction induces core contraction. Second, reduction of the electron mole fraction reduces the Chandrasekhar mass. Finally, electron capture affects the energy equation endothermically and exothermically. Moreover, Nomoto (1987) followed the core evolution after the initiation of O+Ne deflagration using a He star model and provided the progenitor model for an ECSN. Until now, this model was the only one which could be used for an explosion simulation. In the 1990s, non-explosive evolutionary calculation of solar-metal SAGB stars was investigated by García-Berro and collaborators (García-Berro & Iben 1994; Ritossa et al. 1996; García-Berro et al. 1997; Iben et al. 1997; Ritossa et al. 1999). Non-solar metallicity (Gil-Pons et al. 2005; Siess 2007), as well as detailed physics such as overshooting (Gil-Pons et al. 2007) and thermohaline convection (Siess 2009) were considered in recent studies. Off-center carbon burning and thermal pulses, which require expensive calculations, were extensively investigated by Siess (2006, 2010). The contraction
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phase of an ONe core was investigated by recent works as well, concerning the different nuclear reaction rates (Hashimoto et al. 1993), different convective assumptions (Gutiérrez et al. 1996), and different compositions (Gutiérrez et al. 2005). These simulations stopped at the ignition of O burning, and did not model the continuous deflagration phase.

The main purpose of this work is to calculate a progenitor model for an ECSN from a detailed stellar evolutionary simulation. This calculation treats the main-sequence phase, which was omitted in Nomoto (1987). It also models the off-center C burning phase, one of the important improvements in the evolutionary theory for SAGB stars. Using a large nuclear reaction network, we include minor isotopes that are synthesized during the C burning phase and additional electron capture reactions by these isotopes are treated with the Coulomb correction. Updated nuclear reaction rates, especially the new electron capture rate for each NSE composition by Juodagalvis et al. (2010), are taken into account. The increasing ONe core mass is assumed to result from stationary He burning. This enables us to avoid numerical difficulties during the shell He burning phase and to investigate the full evolution of an ONe core.

The critical core mass for ECSNe, past which the ONe core is unstable due to the initiation of electron capture reactions, is considered to be a uniquely determined quantity. The value, calculated by Nomoto (1987), is used for estimates of the initial mass range for ECSNe (Siess 2007; Poelarends et al. 2008; Pumo et al. 2009). However, the uniqueness on various parameter settings has not yet been confirmed, owing to a lack of calculations. Moreover, an update of both numerical prescriptions and physical effects such as diffusive convective mixing and the Coulomb correction on electron capture rates possibly affects the value. Investigation on the parameter dependence of the critical core mass for ECSNe is intended in this work as well.

We report the stellar evolution of the most massive SAGB stars with solar composition from its main sequence through ONe core contraction and further deflagration phase, in which the central NSE region extends outward. In the next section, the method of calculation and input physics are explained. The predicted evolutionary path from the ZAMS phase to the formation of the ONe core is presented in Section 3.1. The core contraction including propagation of the deflagration front is explained in Section 3.2. Discussions and conclusions are given in Section 4.

2. METHODS

We modify the stellar evolution code in Yoshida & Umeda (2011, hereafter Y&U11) and Umeda et al. (2012) in order to calculate the late phase of ONe core evolution. The mixing length is set to be 1.5 times pressure scale height. In the following, the main modified points are described.

2.1. Spacial Resolution

Mesh points are automatically replaced to achieve required resolutions for calculations of stellar structures. Different conditions are set for different environments as well as different evolutionary phases. Especially, for the case of propagation of shell C burning, which requires careful treatment of mesh refining,

1. $|\Delta \log P| < 0.05$,
2. $|\Delta \log T| < 0.1$,
3. $|\Delta \log r| < 0.1$,
4. $|\Delta L_r/L_r| < 0.1$, and
5. $\Delta M_r/M < 10^{-4}$

are taken as the constraints, where $\Delta f$ means the difference of $f$ between two mesh points and the symbols have their usual meanings.

2.2. Nuclear Reaction Network

We include 300 isotopes in a reaction network from $n$, $p$ to Br for calculations of chemical composition evolution and nuclear energy generation. Newly added isotopes from Y&U11 (see also Yoshida et al. 2013) are shown in Figure 1.

When the temperature exceeds the value of $5.0 \times 10^9$ K, NSE is assumed to be achieved in the region and the NSE composition is used for calculations of the thermodynamical quantities and nuclear generation rate. In an NSE region in a highly dense ONe core, both density and electron mole fraction vary over wide ranges of $(9.0 \lesssim \log \rho \lesssim 11.0)$ and $(0.25 \lesssim Y_e \lesssim 0.5)$. In order to calculate an NSE composition consistently in such a wide parameter space, 3091 isotopes are treated (Figure 2). For electron capture reactions in the NSE region, we applied the rate by Juodagalvis et al. (2010) which takes into account roughly 2700 isotopes including very neutron-rich and heavy ones with screening corrections. Though the contributions from positron capture and decay as well as $\beta^-$-decay in the NSE region may affect the stellar evolution, we omit these effects because of the lack of data tables.

2.3. Convective Criterion and Diffusive Mixing Approximation

In our code, convective mixing is treated as a diffusive process. Mixing beyond the convective boundaries, as in overshooting, is not treated. Convective boundaries are determined by the Schwarzschild criterion, and semi-convective diffusion coefficient given by Spruit (1992) is also applied, thus

$$D_{mix} = \begin{cases} \frac{1}{3} \nu_{cv} l_{cv} & \text{for } \nabla_{rad} - \nabla_{ad} > \max \left(0, \frac{\delta}{\mu} \nabla_{\mu} \right) \\ f_{sc} D_{therm} \frac{\nabla_{rad} - \nabla_{ad}}{\nu_{sc}} & \text{for } 0 < \nabla_{rad} - \nabla_{ad} \leq \frac{\delta}{\mu} \nabla_{\mu} \end{cases},$$

where $\nu_{cv}$ and $l_{cv}$ are convective velocity and scale length determined by the mixing length theory, respectively. $D_{therm} \equiv (1/C_P \rho)(4\alpha c T^3/3c \rho)$ is the thermal diffusivity, and $f_{sc}$ is a free parameter taken as 0.3 according to Umeda & Nomoto (2008). In order to take into account the effect of degeneracy of electrons both in the criterion and the coefficient of convective mixing, we define

$$\phi_{\delta} \nabla_{\mu} \equiv \frac{1}{\delta} (\phi_{i} \nabla_{i} + \phi_{e} \nabla_{e}) \quad (2)$$

by extending the work in Kato (1966), where

$$\delta = -\frac{\partial \ln \rho}{\partial \ln T}, \quad \phi_{i} = \frac{\partial \ln \rho}{\partial \ln \mu_{i}}, \quad \phi_{e} = \frac{\partial \ln \rho}{\partial \ln \mu_{e}}, \quad \nabla_{i} = \frac{d \ln \mu_{i}}{d \ln P}, \quad \nabla_{e} = \frac{d \ln \mu_{e}}{d \ln P}. \quad (3)$$

Note that thermohaline convection which would take place in a region of $(\phi/\delta) \nabla_{\mu} < 0$ is not treated for the sake of simplicity.

2.4. Electron Capture and $\beta^-$-decay

2.4.1. The Energy Equation

In a dense ONe core with a density of $>10^9$ g cm$^{-3}$, the energy release by electron capture and $\beta^-$-decay by C burning products
becomes important in the energy equation. The divergence of the energy flux $L_r$ in the stellar equation is written as

$$\frac{dL_r}{dM_r} = \epsilon_n - \epsilon_v + \epsilon_{\text{weak}} - \epsilon_{\text{mix}}, \tag{4}$$

where $s_k$ is the specific entropy for the $k$th particle: nuclei, electrons, and photons (Miyaji et al. 1980; see also Ritossa et al. 1999). $\epsilon_n$ and $\epsilon_v$ are the nuclear energy generation rate and the neutrino energy loss rate due to processes other than electron capture and $\beta^-$-decay, respectively. $\epsilon_{\text{weak}}$ is the energy generation rate of electron capture and $\beta^-$-decay, and $\epsilon_{\text{mix}}$ represents a cooling term owing to the work by convection (Couch & Arnett 1975; Iben 1978). For radiation, the chemical potential is assumed to vanish, while the one of nuclei is ignored up to the achievement of NSE. For electrons, the chemical potential affects the energy equation through both electron capture and $\beta^-$-decay because of their small effects.

The energy generation rate of both electron capture and $\beta^-$-decay consists of three terms: mass difference, neutrino emission, and chemical potential excluding subatomic energy of relevant particles (Miyaji et al. 1980). Since we ignore the chemical potential of nuclei in the energy term, the total rate

$$\epsilon_k \equiv -T \left( \sum_{j} d\lambda_{j} n_{j} \right), \tag{5}$$

where $n_j$ denotes the specific number density of the $j$th nucleus, and the sign of the reaction rate of electron capture or $\beta^-$-decay, $\lambda_j$, is taken to be negative for electron capture and positive for $\beta^-$-decay, so that the emission of a neutrino should always take some energy away.

According to Iben (1978), we define

$$\epsilon_{\text{mix}} = \frac{\partial \mu_e}{\partial r} F_e(M_r), \tag{6}$$

where $F_e(M_r)$ is the flux of electrons and is defined as

$$F_e(M_r) = -(4\pi r^2 \rho) D_{\text{mix}} \frac{\partial n_e}{\partial M_r}. \tag{7}$$

Practically, the electron flux is calculated explicitly in our code, integrating the result of chemical mixing, thus,

$$(4\pi \rho r^2) F_e = -\int_{0}^{M_r} \frac{d\epsilon_{\text{mix}}}{dt} dM_r, \tag{8}$$

where $(d\epsilon_{\text{mix}}/dt)$ is a time derivative of a specific electron number density owing to chemical mixing. In this expression, the energy loss vanishes at the boundaries of the convective region where the net flow of electrons should be zero.

2.4.2. Correction for the Reaction Rate of Electron Capture and $\beta^-$-decay

The Coulomb screening for the electron capture rate (Couch & Loumos 1974; Gutiérrez et al. 1996; Juodagalvis et al. 2010)
charge of ions, and the four constants are $c_1 = 0.2843$, $c_2 = -0.054$, $d_0 = -9/16$, and $d_1 = 0.460$, respectively (DeWitt et al. 1973). Therefore, the effective threshold energy becomes

$$\epsilon_{\text{ec,eff}}^0 = \epsilon_{\text{ec}}^0 + \Delta \epsilon_{\text{ec}}^0.$$  (14)

We assume that the energy distribution function for degenerate electrons does not change its shape for a small difference in electron density. Thus, to take into account the effect of the Coulomb correction of the rates, we first evaluate the effective electron density that reproduces the effective chemical potential as the corrected Fermi energy,

$$\epsilon_F^0 = \epsilon_F - \Delta \epsilon_{\text{ec}}^0$$  (15)

$$\equiv \mu_{\text{ec,eff}}((\rho Y_e)\text{eff}),$$  (16)

Then the reaction rate with effective electron density and fixed temperature is applied as the corrected electron capture rate,

$$\lambda_{\text{ec,eff}} \equiv \lambda_{\text{ec}}((\rho Y_e)\text{eff}, T).$$  (17)

Since $\Delta \epsilon_{\text{ec}}^0$ is positive, this correction increases the effective threshold energy, reduces the effective Fermi energy, and thus reduces the rate of electron capture.

The same correction is also applied to $\beta^{-}$-decay. In the case of $(A, Z) \rightarrow (A, Z + 1) + e^- + \bar{v}_e$, change in the threshold $\Delta \epsilon_{\text{bd}}^0$ is given as

$$\Delta \epsilon_{\text{bd}}^0 = \mu(Z + 1) - \mu(Z)$$  (18)

which takes a negative value and reduces the threshold energy. For $\beta^{-}$-decay, the decay rate is correlated to the number of unfilled electron states in which the kinetic energy is between the threshold energy and the chemical potential, i.e., $\mu_e < \epsilon_e < \epsilon_{\text{bd,eff}}^0$. Therefore, the decrease of the effective threshold energy reduces the rate of $\beta^{-}$-decay as well electron capture as mentioned above.

2.5. An Approximate Treatment of Core Growth

After the completion of C burning in a forming ONe core, we approximate core mass growth by shell He burning in a constant rate in order to avoid some numerical difficulties given below. We assume that the envelope will remain and shell He burning will continue until core collapse. We also treat the core as if it were a single star, and the entropy structure at the edge is assumed to retain geometric similarity through the later evolutionary phases. Since the index of entropy structure is typically expressed by using the homology invariants $V \equiv -(\partial \ln P/\partial \ln r)$ and $U \equiv (\partial \ln M_r/\partial \ln r)$ (Sugimoto et al. 1981), the boundary conditions can be taken as

$$\sigma = \sigma_1(\text{const.}),$$  (19)

$$\frac{V}{U} \equiv -\left(\frac{\partial \ln P}{\partial \ln M_r}\right)$$  (20)

$$\frac{\partial \sigma}{\partial \ln M_r} = (\text{const.}),$$  (21)

where $\sigma \equiv (m_n/k) \Sigma s_i$ is the specific entropy per baryon in units of the Boltzmann constant, and $\sigma_1$ is the specific entropy
at the edge of the core, defined to be a constant. Since the edge structure is extremely steep, this approximation would not affect the later core evolution, especially at the central region.

At the H/He boundary, merging of convective regions, or the dredge-out episode named by Iben et al. (1997), takes place in our calculations (Section 3.1.3.). Due to an extended convective region in the hydrogen and helium layers merge together. Some envelope hydrogen is mixed into the base of the helium burning shell (HeBS), resulting in H burning with significant energy production. As described in Poelarends et al. (2008), this energy production makes numerical convergence difficult, and requires a scheme which can simultaneously solve for mixing and reactions. Our code is not equipped with such a scheme at present. Next, the star enters a thermal pulse phase, if stationary H burning is too short to influence error in the core mass. Also, the relaxation time from the rapid H mixing to the stationary burning is too short to influence error in the core mass growth rate. The middle one is the most likely rate calculated. This requires an expensive calculation and a full simulation of the phase is difficult.

The approximation of a constant core mass growth is valid. Because of the large number of pulses, it will be plausible to consider the discrete growths as a time-averaged continuous effect. Also, the relaxation time from the rapid H mixing to the stationary burning is too short to influence error in the core mass growth.

Under these assumptions, three rates, $1.0 \times 10^{-5} M_\odot$ yr$^{-1}$, $1.0 \times 10^{-6} M_\odot$ yr$^{-1}$, and $1.0 \times 10^{-7} M_\odot$ yr$^{-1}$ are taken as the core growth rate. The middle one is the most likely rate for core growth from shell He burning. This rate is consistent with the work by Nomoto (1987) in which steady He burning is assumed, and with recent studies by Siess (2010) and Poelarends et al. (2008) in which the thermal pulse phase is calculated. The results shown in Section 3 are cases using this likely rate.

### 2.6. Late Phase of Core Evolution

When the timescale of core evolution becomes shorter than that of convection, the well-known mixing length theory (Böhm-Vitense 1958), which assumes stationary convection, becomes invalid. In order to determine the temperature gradient in such cases, the time-dependent mixing length theory formulated by Unno (1967) is adopted in our calculation. In this scheme, two time differential equations for convective velocity $v_{cv}$ and temperature fluctuation $\Delta T$ are given as

$$
\left(\frac{d}{dt} + \frac{v_{cv}}{l_{cv}/2}\right) v_{cv} = \frac{\Delta T}{2\rho T} \frac{\partial \log \rho}{\partial \log T} \frac{dP}{d\rho}, \quad (22)
$$

$$
\left(\frac{d}{dt} + \frac{v_{cv}}{l_{cv}/2}\right) \Delta T = \frac{v_{cv}}{l_{cv}/2} \frac{l_{cv}}{2H_p} T(\nabla - \nabla_{ad}), \quad (23)
$$

where $\nabla - \nabla_{ad}$ represents the excess of temperature gradient compared with the adiabatic gradient. Following Nomoto (1984), we take the length scale of time-dependent mixing $l_{cv}$ to be shorter than radial distance. The convective energy flux $F_{cv}$ is written as

$$
F_{cv} = c_p \rho \Delta T v_{cv}, \quad (24)
$$

where $c_p$ is the specific heat at constant pressure. Then, identical to the mixing length theory, equations of total luminosity

$$
L_r = L_{rad} + 4\pi r^2 F_{cv}, \quad (25)
$$

$$
L_{rad} = \frac{16\pi acGM_rT^4}{3\kappa P} \nabla \quad (26)
$$

are solved to obtain the temperature gradient.

As the timescale of evolution decreases and becomes comparable with the free-fall timescale, the assumption of hydrostatic structure becomes invalid. In this work, an inertia term is included in the equation of motion and also in the radiative temperature gradient (Heger et al. 2000) as

$$
\frac{dP}{dM_r} = -\frac{GM_r}{4\pi r^4} \left[1 + \frac{r^2}{GM_r} \frac{\partial^2 r}{\partial \tau^2}\right]. \quad (27)
$$

$$
\nabla_{rad} = \frac{3\kappa P L_r}{16\pi acGM_rT^4} \left[1 + \frac{r^2}{GM_r} \frac{\partial^2 r}{\partial \tau^2}\right]^{-1}. \quad (28)
$$

### 3. RESULTS

We calculated the evolution of 10.4–11.2 $M_\odot$ stars with a metallicity of $Z = 0.02$, from ZAMS to O+Ne deflagration for 10.4–10.8 $M_\odot$ models and from ZAMS to off-center Ne ignition for 11.0 and 11.2 $M_\odot$ models. Figures 3 and 4 show
Figure 5. Time evolution of convective regions of a 10.8 $M_\odot$ model until completion of the dredge-out. The convective regions are shown in green hatched region in the figure, and log scaled net nuclear energy generation is shown in color. (A color version of this figure is available in the online journal.)

the time evolution in the H-R diagram, and in the central density–temperature plane, respectively. In Figure 4, spikes are shown at $\rho_c \sim 10^8$ g cm$^{-3}$ for both 11.0 and 11.2 $M_\odot$ models, representing off-center neon ignitions. In our calculation, the minimum initial mass for Ne ignition is 11.0 $M_\odot$ and the CO core mass is 1.35 $M_\odot$. A star with a larger initial mass than this critical mass will form an Fe core and will end up as a normal CCSN (Nomoto & Hashimoto 1988). Since evolutionary properties of both 10.4 and 10.6 $M_\odot$ models are similar to 10.8 $M_\odot$ model, here we mostly show the results of a 10.8 $M_\odot$ model, which has the largest core in these less massive stars and provides the most plausible progenitor model for an ECSN.

The minimum mass for Ne ignition of $\sim 11$ $M_\odot$ is large compared with other recent evolutionary calculations (Poelarends et al. 2008; Pumo et al. 2009). This is because our calculations do not take into account the effect of additional mixing such as overshooting. If an exponentially decreasing diffusion (Herwig 2000) is considered, our calculation shows that this minimum mass is reduced by $\sim 2$ $M_\odot$ with an overshooting parameter of $f_{\text{ oversh}} = 0.02$. This behavior is quite consistent with the calculation by Siess (2007). In fact, the inclusion of overshooting significantly alters the relation between initial mass and He core mass. While this severely affects an estimation of an initial mass range for ECSNe, effects on evolutionary results of progenitor calculation are much more mild. Especially, due to a lack of observational constraints, most evolutionary calculations for massive stars only take into account the overshooting before core C burning stages (Hirschi et al. 2004; Limongi & Chieffi 2006). In this case, our results on the progenitor evolution become fully consistent with these calculations.

3.1 Pre-SAGB Evolution

In this section, we summarize the evolutionary results of a 10.8 $M_\odot$ star from its ZAMS phase to the completion of the dredge-out (Iben et al. 1997), or merging of convective regions which surround the ONe core. Figure 5 shows the time evolution of convective regions.

3.1.1 The Hydrogen Burning Stage

The duration of core H burning is $1.74 \times 10^7$ yr. In this stage, convection develops in the central region owing to the luminosity generated by the CNO-cycle. Core H burning continuously shifts to shell H burning as the central hydrogen burns out, while the H-depleted core contracts and is heated by the release of gravo-thermal energy. The increasing luminosity in the outer shell burning region expands and cools the envelope. As the opacity increases, a convectively unstable region emerges at the surface. The base of the convective region extends inward, and the star becomes a red giant. When the first dredge-up episode occurs, $^{14}$N, the second main product of the CNO-cycle, is dredged up to the surface. Surface composition of CNO isotopes at the end of this episode are summarized in Table 1.

3.1.2 The Helium Burning Stage

When the central density and temperature reach $\log \rho_c = 3.50$ and $\log T_c = 8.16$, core He burning takes place. The luminosity of shell H burning decreases, and the base of the convective envelope retreats outward in mass. The entire hydrogen envelope becomes convectively stable, and the star enters a blue-loop on the H-R diagram. The core He burning phase continues for $2.76 \times 10^6$ yr, followed by shell He burning after core helium depletion. Since the large luminosity by shell He burning expands and cools the H burning layer, shell H burning dies out. This luminosity re-heats the envelope and induces convection. The star becomes an AGB star, in which a partially degenerate CO core has formed, surrounded by the HeBS. At the center of the core, the mass fraction ratio of carbon to oxygen becomes $X(C)/X(O) = 0.5728$.

3.1.3 The Carbon Burning Stage

In the partially degenerate CO core, off-center C flashes take place. In our calculation, the first two flashes ($C_1$ and $C_2$ in Figure 6) arise near the center and the last seven flashes burn outward (from $C_3$ to $C_9$ in Figure 6). These C flashes transform core carbon into neon and other intermediate-mass isotopes.
After the end of the sixth C burning, the second dredge-up reduces the mass of the helium layer. These results are consistent with other calculations (Siess 2006, 2007). Figure 7 shows the evolution of the core structure in terms of electron pressure fraction (top), temperature (middle), and density (bottom) as functions of mass coordinate at five different stages: disappearance of convective core He burning (1a), commencement of the first shell C burning (1b), ignition at the center of the core (1c), ignition of the eighth C burning (1d), and the dredge-out (1e).

In advance of the off-center C ignition, the gravo-thermal heating increases the temperature of the contracting CO core (Figure 7, 1a). When the central density reaches \( \rho_c = 6.28 \) and the maximum temperature in the core reaches \( T_{\text{max}} = 8.81 \) at \( M_r = 0.05 M_\odot \), off-center carbon ignition takes place (Figure 7, 1b). This is because relatively high temperature activates the cooling process via neutrino emission in the CO core. Neutrino cooling efficiently removes local heat and suppresses temperature increase. The efficiency becomes greater with higher density, so for the inner region of the core, the cooling becomes more effective. The inner region is more degenerate and thus is harder to contract. On the other hand, the still mildly degenerated outer region liberates gravo-thermal energy and increases its temperature, owing to contraction by neutrino cooling. As a result, an inverse temperature gradient appears in a degenerate core.

Since the first C burning expands and cools the central region, the burning front does not propagate inward and the chemical composition remains unchanged.
Figure 8. Profiles of the energy generation rates and luminosity with mass coordinate at different stages: occurrence of convection on HeBS (2a), ignition of the seventh C burning (2b), shell He burning at its maximum (2d), 6.84 × 10^7 yr after (2d) (2e), and occurrence of the dredge-out (2f). The different line types are the nuclear energy generation (red, solid line), the neutrino energy loss (green, dashed line), the gravo-thermal energy release (blue, dotted line), and the luminosity (cyan, dash-dotted line) respectively. The gray regions represent convective regions.

Table 2

| Isotope          | Mass Fraction (in terms of mass fractions) |
|------------------|--------------------------------------------|
| ^{16}O           | 4.783 × 10^{-1}                            |
| ^{20}Ne          | 4.074 × 10^{-1}                            |
| ^{24}Mg          | 4.255 × 10^{-2}                            |
| ^{23}Na          | 3.217 × 10^{-2}                            |
| ^{25}Mg          | 1.451 × 10^{-2}                            |
| ^{26}Mg          | 7.952 × 10^{-3}                            |
| ^{27}Al          | 7.245 × 10^{-3}                            |
| ^{22}Ne          | 2.836 × 10^{-3}                            |
| ^{28}Si          | 2.330 × 10^{-3}                            |
| ^{12}C           | 1.277 × 10^{-3}                            |
| ^{21}Ne          | 8.384 × 10^{-4}                            |
| ^{29}Si          | 3.621 × 10^{-4}                            |
| ^{32}S           | 3.362 × 10^{-4}                            |
|                  | 1.463 × 10^{-4}                            |

Composition at the center does not change. On the other hand, the second shell C burning, which takes place as the first burning dies out, propagates inward and the center of the core ignites at last (Figure 7, 1c). The surrounding region of the second C burning shell has lower mass fraction of ^{12}C owing to the first C burning. This weakens the second shell burning, resulting in a smaller expansion of the central region. This enables the second burning flame to propagate inward (Siess 2006). The propagation is caused by heat conduction from the base of the nearly stationary flame, the mean propagation speed becomes 9.7 × 10^{-3} cm s^{-1}. Final composition at the center of the core is shown in Table 2 in terms of mass fractions.

During the following four C flashes (C_{3-6}), the core continues contracting. At the ignition of the seventh C burning, the center of the core is supported only by perfectly degenerate electrons, and the central region clearly shows a temperature inversion (Figure 7, 1d). On the other hand, radiative pressure has a major fraction of the total pressure both at the flame front (M_r = 1.24 M_⊙) and at the edge of the core (M_r = 1.35 M_⊙). This is due to the high temperature and low density at these regions. Figure 8 shows the evolution of the energy structure and convective region around the edge of the core. Core contraction heats the base of the surrounding HeBS and increases its luminosity. This He shell burning induces convection at the base of the HeBS (Figure 8, 2a), and shell He burning keeps supporting the convection during the seventh and eighth C burning phases (Figure 8, 2b and 2c).

After the end of the eighth C burning, luminosity from the shell He burning takes its maximum value owing to the temperature rise at the base of the HeBS (Figure 8, 2d). However,
soon the luminosity decreases, and convection in the helium layer is supported by escaping energy from core-edge C burning (Figure 8, 2e). The growing convective region in the helium layer merges with the outer convection in the hydrogen envelope. At the moment of the dredge-out episode (Iben et al. 1997), the temperature and density steeply drop at the edge of the core from \( T = 9.0 \rightarrow 6.8 \) and \( \rho = 4.8 \rightarrow -2.4 \) in a narrow mass range of \( 5 \times 10^{-3} M_\odot \) (Figure 7, 1e). Some envelope hydrogen is mixed into the base of the HeBS. The resulting H burning releases energy at a very high rate of \( \approx 10^{41} \) erg s\(^{-1}\) g\(^{-1}\) (Figure 8, 2f). The ONe core mass, defined by the mass coordinate of maximum energy generation by He burning, is \( 1.347 M_\odot \) at the end of the C burning stage. Since the dredge-out episode mixes CNO products with the hydrogen envelope, the surface composition changes after the convective merging (see Table 1).

### 3.2. Evolution of a Contracting ONe Core

In this section, we show that the evolution of a contracting ONe core can be divided into four sub-phases in terms of driving mechanisms; neutrino cooling, core mass growth, electron capture by \(^{24}\)Mg and \(^{20}\)Ne, and O+Ne deflagration. The efficiency of each mechanism is related to evolutionary timescales shown in Figure 9. The definitions of timescales are given by

\[
\tau_{\text{con}} \equiv \frac{dt}{d\ln \rho_c}, \quad \tau_{\text{KH}} \equiv \frac{GM_{\text{core}}^2}{R_{\text{core}}(L + L_\nu)}, \quad \tau_{\text{growth}} \equiv \frac{dt}{d\ln M_{\text{core}}}, \\
\tau_{\text{elec}} \equiv \frac{dt}{d\ln T_e}, \quad \tau_{\text{dyn}} \equiv \sqrt{\frac{R_{\text{core}}^3}{GM_{\text{core}}}},
\]

and they represent the timescale of core contraction, the Kelvin–Helmholtz timescale, the timescale of core mass growth, the timescale of electron capture, and the dynamical timescale, respectively.

#### 3.2.1. Contraction Due to Neutrino Cooling

From the beginning of contraction until the central density reaches \( \log \rho_c = 9.39 \), contraction is caused by neutrino cooling. Figure 9 shows that the contraction timescale is close to the Kelvin–Helmholtz timescale, corresponding to the neutrino-cooling timescale. In this phase, the central temperature decreases as the entropy is radiated by thermally activated neutrino emission. As the central temperature decreases, the cooling rate of neutrino emission decreases as well. The timescale of density evolution simultaneously becomes longer.

Although electron captures by \(^{27}\)Al, \(^{25}\)Mg, and \(^{23}\)Na proceed in this stage (Figure 10), these reactions have only a minor effect on density evolution. First, this is because the mass fractions of these isotopes are so small that reduction of electron mole fraction does not induce contraction. Second, thermal contributions from these reactions are smaller than neutrino cooling. Since the energy production by these electron captures is small, the convective URCA cooling, described in Ritossa et al. (1999), does not affect our calculation.
3.2.2. Contraction Due to Core Mass Growth

As the core mass increases, both the required pressure to support the core and the actual pressure increase, and thus the central density increases. While the efficiency of neutrino cooling is suppressed by entropy reduction, the efficiency of core mass growth is independent from core structure and becomes constant. After neutrino cooling becomes less effective than core mass growth, the timescale of core growth $\tau_{\text{growth}}$ starts to limit the contraction timescale.

The stationary core growth forces the central density to increase in a constant rate, and the production rate of gravothermal energy becomes constant. Owing to this constant heating, and to less effective neutrino cooling, the central temperature increases proportionally to the central density.

3.2.3. Contraction Due to Electron Capture

After the central density exceeds $\log \rho_c = 9.88$, core contraction is driven by electron capture and thus $\tau_{\text{elec}}$ starts to dominate the evolution timescale. The contraction timescale decreases with increasing density owing to the increasing electron capture rate. Soon core growth becomes negligible since $\tau_{\text{elec}}$ becomes much smaller than $\tau_{\text{growth}}$. After that, the core mass is frozen and is considered to be at the critical core mass for an ECSN, $M_{\text{EC}}$.

Figure 4 shows that the central temperature increases with the central density more steeply in this core growth stage. The increase of temperature is due to heating via electron capture and the main reaction sequence is $^{24}\text{Mg} \rightarrow ^{24}\text{Na} \rightarrow ^{24}\text{Ne}$ at that time. Electron capture by $^{24}\text{Mg}$ forms an excited $^{24}\text{Na}^*$ as a daughter nucleus. When this $^{24}\text{Na}^*$ decays to the ground state, a $\gamma$-ray results and heats the surroundings. Owing to this additional heating, the electron capture by $^{24}\text{Mg}$ becomes exothermic. Moreover, both $^{24}\text{Na}^*$ and $^{24}\text{Na}$ at the ground state can capture another electron with lower threshold density than with $^{24}\text{Mg}$. Therefore, electron capture by $^{24}\text{Mg}$ results in double-electron capture and releases a large amount of heat. It is noteworthy that the second daughter nucleus $^{22}\text{Ne}$ becomes the dominant product of electron capture by $^{24}\text{Mg}$ (Figure 10). As a result, the amount of $\nabla_T$ becomes sufficiently large compared to $(\phi/\beta)\nabla\mu$, and the central region is fully mixed by convection. The growing convection supplies fresh $^{24}\text{Mg}$ to the center where electrons are quickly captured. As a result, the electron mole fraction decreases in the convective region. In accordance with the discussion by Miyaji et al. (1980), convective URCA cooling by $^{24}\text{Mg}$-$^{24}\text{Ne}$ does not take place in this phase, though the reaction drives convection. This is because the threshold density of $\beta^-$-decay for the daughter nucleus $^{24}\text{Ne}$, $\sim 10^8 \text{ g cm}^{-3}$, is too small to occur in this phase. While minor electron capture nuclei, such as $^{21}\text{Na}$ and $^{29}\text{Si}$, can induce convective URCA processes, the average energies lost by these reactions are reasonably smaller than the energy released by electron capture by $^{24}\text{Mg}$, and thus these processes should be neglected.

After the central density reaches $\log \rho_c = 10.3$, $^{20}\text{Ne}$ starts to capture electrons (Figure 10). This electron capture continues to drive convection in the same way as electron capture by $^{24}\text{Mg}$. The timescale of contraction becomes shorter than the convection timescale, and partial mixing allows a gradient of $X(\text{Ne})$ to exist in the central region. However, since $^{20}\text{Ne}$ has a large mass fraction of $\sim 0.4$, until the commencement of Ne-O deflagration, the fuel is not consumed and the reaction continues to heat the core.

![Figure 11](image-url). Profiles of temperature, density, and electron mole fraction during the O+Ne deflagration phase are shown as a function of both mass and radius coordinates. These profiles are taken at $1.13 \times 10^{-2}$, $5.40 \times 10^{-2}$, $9.92 \times 10^{-2}$, $1.49 \times 10^{-1}$, $2.00 \times 10^{-1}$, and $2.34 \times 10^{-1}$ s after the ignition at the center of the core.

(A color version of this figure is available in the online journal.)

3.2.4. O+Ne Deflagration

When the central temperature reaches $T_c = 9.2$, O+Ne burning takes place at the center and the central temperature rises with very short timescale. Since reaction rates of the nuclear burning highly depend on temperature, thermal runaway takes place in the extremely degenerate region. While the temperature steeply increases at the moment of ignition, the variation of pressure and of density becomes small. When the central temperature exceeds $5 \times 10^9 \text{ K}$, NSE is assumed to be achieved.

At the boundary of the NSE region and the surrounding ONe region, there must be a negative steep gradient of entropy as a result of the O+Ne burning, and convection must exist. In our calculation, heat transportation by convection is solved (see Section 2.6), and the resulting heating at the base of the ONe region becomes much more significant than heating by electron capture by Ne and Mg. Healing increases the temperature, and soon O+Ne burning ignites and NSE is achieved. The top panels of Figure 11 show the evolution of temperature distribution. The location of the steep temperature gradient is identical to the burning front and propagates outward. The propagation velocity, $1.6 \times 10^3 \text{ km s}^{-1}$, becomes smaller than the sound velocity. In other words, the NSE region extends outward in mass by the O+Ne deflagration.

In the NSE region, electron capture reactions by both free protons and heavy isotopes take place (Juodagalvis et al. 2010). The timescale of electron capture in the NSE region ($\sim 0.1$ s) becomes much shorter than in the outer ONe region ($\sim 10^2$ s; see bottom panels of Figure 11). The reduction of the electron mole fraction, coupled with the extension of the NSE region, affects the dynamical evolution of the core. Contrary to the electron capture in the ONe region, the electron capture in the NSE region becomes an endothermic reaction. This is due to the global compositional change in the NSE; unstable neutron-rich isotopes are preferred in the lower $Y_e$ environment, and thus reduction of $Y_e$ causes the free energy to be restored in terms of nuclear binding energy. Because of the resulting cooling,
a positive entropy gradient appears in the NSE region. The center of the core becomes convectively stable.

The ONe core has been strongly bound by gravity and has a binding energy of \(-6.534 \times 10^{51}\) erg and a total energy of \(-5.791 \times 10^{50}\) erg at the commencement of O+Ne ignition. While energy injection by the O+Ne burning is too small to disrupt the whole star, fast reduction of electrons in the NSE region accelerates core contraction. As a result of both energy injection and electron reduction, the contraction timescale slowly decreases during the deflagration phase. After \(2.36 \times 10^{-1}\) s from the ignition at the center of the core, the central density reaches \(\log \rho_c = 11.0\) and the deflagration front reaches \(M_{\text{NS}} = 0.12 M_\odot\) and \(r_{\text{NS}} = 1.51 \times 10^{-3} R_\odot\). The binding energy and the total energy of the core become \(-7.739 \times 10^{51}\) erg and \(-9.158 \times 10^{50}\) erg, respectively.

### 3.3. Parameter Dependences on \(M_{\text{EC}}\)

In order to investigate the uniqueness of the critical core mass for ECSNe, \(M_{\text{EC}}\), in various situations, additional test calculations on ONe core contraction are done with different settings. As an initial condition, ONe cores of 1.346, 1.332, and 1.288 \(M_\odot\) are taken to be formed in SAGB stellar models of 10.8, 10.6, and 10.4 \(M_\odot\). For each ONe core, the later contraction phase is calculated with three different rates of core mass growth \(\dot{M}_{\text{EC}}\). In various situations, additional test calculations on ONe core contraction are done with different settings. For each ONe core, the later contraction phase is calculated with three different rates of core mass growth \(\dot{M}_{\text{EC}}\).

The mean value of our results with the Coulomb correction is 1.367 \(M_\odot\). We estimate the uncertainty of \(M_{\text{EC}}\) at about \(\pm 0.005 M_\odot\), taking into account the error from the determination of \(M_{\text{EC}}\) and the variances of the results. When uncertainties from the C burning phase are considered, the uncertainty should increase in total. We assume that the amount of this uncertainty is as large as that from later stages of evolution. Thus, the uncertainty of our result is about \(\pm 0.01 M_\odot\) in total.

### 4. DISCUSSIONS AND CONCLUSIONS

#### 4.1. The Critical Core Mass for ECSNe

In our calculation, \(M_{\text{EC}}\) is 1.367 \(M_\odot\) with a relatively small error of \(\pm 0.01 M_\odot\), and is consistent with the result by Nomoto (1987), \(M_{\text{EC}} = 1.375 M_\odot\). In a critical Chandrasekhar mass object with given temperature and composition distributions, the central density can be represented only by the core mass, \(\rho_c = \rho_c(M_{\text{core}})\). From different \(M_{\text{ZAMS}}\), different temperature and composition distributions are resulted. The change in \(M_{\text{core}}\) results in different durations for neutrino cooling, and leads to different temperature distributions. However, because of the small temperature dependence on degenerate electron pressure, and because of the similar composition of ONe cores for ECSN progenitors from a narrow initial mass range, variations in \(M_{\text{ZAMS}}\) and \(M_{\text{core}}\) have little effect on the relation of \(\rho_c = \rho_c(M_{\text{core}})\). Therefore, the critical mass for ECSNe is almost uniquely determined by solving the relation of \(\rho^{\nu\text{Mg}} = \rho_c(M_{\text{EC}})\), where \(\rho^{\nu\text{Mg}}\) is the threshold density for electron capture by \(^{24}\text{Mg}\).

Our result shows that inclusion of the Coulomb correction increases \(\rho^{\nu\text{Mg}}\) by \(\sim 10\%\) and increases \(M_{\text{EC}}\) by \(\sim 0.003 M_\odot\) for mean values, but the difference is smaller than the error. Since small variations in \(M_{\text{EC}}\) do not show tendencies on changes in \(M_{\text{ZAMS}}\) and \(M_{\text{core}}\), changes in \(M_{\text{EC}}\) due to changes in these parameters would be much smaller than the error. Thus, the major component of the \(\pm 0.01 M_\odot\) error should have come from the accumulation of uncertainties in numerical calculations and difficulties in the analysis described in Section 3.3.

#### 4.2. Convective URCA Cooling by Minor Electron Capture Nuclei

When electron capture by \(^{24}\text{Mg}\) drives convection, convective URCA cooling is induced by minor electron capture nuclei such as \(^{23}\text{Na}\) and \(^{29}\text{Si}\). In a central convective region, these daughter nuclei, which are formed at the center of the core, are mixed with the outer less dense region. Sometimes the density at the outer edge of the convective region is much less than the threshold densities for \(\beta^-\) decay of these nuclei. In such cases, reaction rates of \(\beta^-\)-decay become significantly large, resulting in large energy loss only at the edge of the convection. Some of our models are halted by numerical difficulties caused by this energy loss.

However, this behavior seems unphysical, and slower reactions should occur in reality. Such a large energy loss should immediately stop the convective mixing around the region, and mass fractions of these nuclei are so small that reaction durations should be limited in such a non-convective environment. Therefore, if small fractions of unstable \(\beta^-\)-decay nuclei are mixed into less dense regions, the resulting endothermic reaction will stop immediately and have only a minor effect on overall evolution. We expect that the convective URCA process during the electron capture phase will not affect later evolution of an ONe core as shown in the 10.8 \(M_\odot\) model. To prove this statement, simultaneous solving of the coupling of convective mixing, nuclear reactions, and stellar structure is necessary.
4.3. Deflagration and Core Collapse

Figure 9 shows that the contraction timescale $\tau_{\text{con}}$ is longer than the dynamical timescale $\tau_{\text{dyn}}$ even at the end of calculation. Such quasi-static contraction will be altered into dynamical collapse during the deflagration phase, and thus, the core collapse in an ECSN is caused by electron capture in the extending NSE region after the initiation of O+Ne deflagration, rather than by electron capture by $^{24}\text{Mg}$ and $^{20}\text{Ne}$. In order to provide a plausible progenitor model for an ECSN, the deflagration phase should be considered carefully because propagation of the deflagration front and both energy generation and electron capture in the NSE region can affect the core evolution importantly.

The obtained deflagration velocity $1.6 \times 10^3$ km s$^{-1}$ at the end of the calculation ($\rho_{c} = 10^{11}$ g cm$^{-3}$) is consistent with the velocity $\sim 10^3$ km s$^{-1}$ described in Miyaji et al. (1980). On the other hand, the extension of the NSE region, 0.12 $M_{\odot}$, is smaller than 0.354 $M_{\odot}$, obtained by Miyaji et al. (1980) and $\sim 0.3$ $M_{\odot}$ by Nomoto (1987). This is due to a different treatment of electron capture in the NSE region. Nuclei included in the calculation by Juodagalvis et al. (2010) are much more extended than those used in Miyaji et al. (1980) or Nomoto (1987). At high $Y_{e}$ environment with high proton fraction, electron capture rates are dominated by a contribution from free protons. However, at low $Y_{e}$ environment with extremely low proton fractions, contribution from neutron-rich isotopes becomes dominant and the importance is significant. In order to confirm the effect of electron capture by neutron-rich isotopes, we calculate the deflagration phase while taking into account only electron capture by free protons in NSE. The resulting propagation of the deflagration front is shown in Figure 13. Comparing with the case of full electron capture, contraction becomes slower owing to a smaller reduction of electrons. Then, the duration of contraction becomes longer and the flame front propagates far from the center by the end of the calculation, even though the flame front has a smaller mean velocity of $8.4 \times 10^2$ km s$^{-1}$. The importance of the mechanism of neutrino heating was suggested in Kitaura et al. (2006). Since the efficiency is affected by the density profile, a central concentrated density profile in our calculation would affect its explosion and the inclusion of electron capture by neutron-rich isotopes will be important.

4.4. Constraints by Mass Loss

It is considered that not all SAGB stars end up as ECSNe because of intense wind mass loss during the SAGB phase. As the core mass increases, a more significant amount of envelope mass will be lost by wind. This limits the duration of the SAGB phase, and if the core mass is less than $M_{\text{EC}}$ at the end of the SAGB phase, the star ends up as an ONe WD. Therefore, the ratio $\xi \equiv M_{\text{core}}/M_{\text{env}}$ is important to determine stellar fates. The minimum $\xi$, with which a star becomes an ECSN just before completely losing its envelope, can be defined as

$$\xi_{\text{crit}} = \frac{M_{\text{EC}} - M_{\text{core}}^{\text{ini}}}{M_{\text{tot}}^{\text{ini}} - M_{\text{EC}}}$$

where $M_{\text{core}}^{\text{ini}}$ and $M_{\text{tot}}^{\text{ini}}$ represent an ONe core mass and a total mass at the beginning of the SAGB phase. When an actual $\xi$ for an SAGB star is larger than $\xi_{\text{crit}}$, the star ends up as an ECSN, and vice versa. If core growth rates are specified by certain simulations, this prescription becomes identical to the definition of the critical mass-loss rate $|M_{\text{env, crit}}|$, thus

$$|M_{\text{env, crit}}| = \frac{1}{\xi_{\text{crit}}} M_{\text{core}}^{\text{ini}}$$

and the star with $|M_{\text{env}}| \leq |M_{\text{env, crit}}|$ can become an ECSN. $\xi_{\text{crit}}$ becomes $2.41 \times 10^{-3}$, 4.01 $\times 10^{-3}$, and 9.25 $\times 10^{-3}$ for 10.8, 10.6, and 10.4 $M_{\odot}$ models, respectively. Because of large uncertainties for both $M_{\text{core}}$ and $M_{\text{tot}}$, a current estimate of $\xi$ is highly uncertain. Here, according to the results by Poelarends et al. (2008), we limit the range of $\xi$ as $2.54 \times 10^{-3} - 1.80 \times 10^{-2}$. This estimate includes wide ranges of mass-loss rates, initial masses, and thus mass growth rates, and the minimum value will be too small to be applied to the most massive SAGB stars in our calculation. Even from this crude estimate, our model of a 10.8 $M_{\odot}$ star is plausible for the progenitor of an ECSN. This conclusion will be robust if the inclusion of overshooting reduces the total mass by $\sim 2 M_{\odot}$ and rises $\xi_{\text{crit}}$ to $3.13 \times 10^{-3}$.

4.5. Conclusions

Stellar evolution for the most massive SAGB stars having solar abundances is calculated in this work, and we provide progenitor models for ECSNe with detailed evolutionary calculation for the first time. In order to avoid the numerical difficulties, and an expensive calculation for thermal pulses, we assume constant core mass growth rate as a result of He shell burning after the completion of core C burning. After this, the core is assumed to retain a geometrically similar edge structure. As to the high temperature region with $> 5.0 \times 10^8$ K which appears after the initiation of the O+Ne deflagration, NSE is assumed to be achieved.

In such a star, nuclear burning of hydrogen, helium, and carbon takes place step by step, then a core mainly made of oxygen and neon forms. The ONe core, which is supported by degenerate electron gas, contracts due to neutrino cooling, core mass growth by the surrounding shell He burning, and reduction of electron mole fraction by electron capture reactions. When the central temperature increases enough to ignite oxygen, O+Ne deflagration takes place and NSE is achieved. Although the O+Ne burning heats the region, the released energy is too small to explode the highly gravitationally bound core. As electron
capture reactions by neutron-rich isotopes as well as by free protons accelerate the core contraction, the deflagration front propagates outward. The core will continue to contract up to the formation of a proto-neutron star. The fate will be a weak Type II SN.

In our results, the critical mass for ECSNe, \( M_{\text{EC}} \), is 1.367 \( M_\odot \) with a relatively small uncertainty of about ±0.01 \( M_\odot \). We show the uniqueness of the value under various initial core masses and various core growth rates. The uncertainty comes from the numerical errors and the analytical error as discussed in Section 4.1. Inclusion of the Coulomb correction increases \( M_{\text{EC}} \), however, the effect is smaller than the numerical uncertainties.

The inclusion of minor intermediate-mass isotopes such as \(^{29}\text{Si}, ^{23}\text{Na}, ^{25}\text{Mg}, \) and \(^{27}\text{Al} \) does not affect the core evolution and thus \( M_{\text{EC}} \). Though the convective URCA process by these isotopes sometimes causes numerical difficulties in an electron capture phase, this process will have only a minor effect on the whole evolution.

Since the ONe core keeps contracting quasi-statically even during the deflagration phase, the core evolution is importantly affected by propagation of the deflagration front and electron capture in the NSE region. We showed that the assumption of electron capture only by free protons leads to slower contraction and electron capture by neutron-rich isotopes should be incorporated in the model.

Owing to intense wind mass loss during SAGB phase, not all our models may end up as ECSNe. Accurate estimates for the core growth rate and the mass-loss rate are difficult and further investigations are still needed on this topic. However, even under the most strict condition taken from Poelarends et al. (2008), our 10.8 \( M_\odot \) model is plausible for a progenitor model of an ECSN.

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