Probing quantum vacuum geometrical effects with cold atoms

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The lateral Casimir-Polder force between an atom and a corrugated surface should allow one to study experimentally non trivial geometrical effects in quantum vacuum. Here, we derive the theoretical expression of this force in a scattering approach that accounts for the optical properties of the corrugated surface. We show that large corrections to the “proximity force approximation” could be measured using present-day technology with a Bose-Einstein condensate used as a vacuum field sensor.

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The Casimir force between bulk surfaces as well as the Casimir-Polder force between an atom and a surface are due to the reshaping of vacuum field fluctuations imposed by the boundary conditions. In recent years there has been considerable progress in measurements of these forces, opening the way for various applications in nanotechnology and quantum science.

Most studies of the effect of geometry on the Casimir force are based on the proximity force approximation (PFA), which essentially amounts to an averaging over plane geometries. The related pairwise summation approach was used to calculate roughness corrections to the force between an atom and a plate. It is clear however that Casimir forces are not additive, except in the special case of very dilute media. Predictions beyond the PFA have recently been obtained, for the normal component of the Casimir force between non-planar surfaces, the effect of roughness, and the lateral Casimir force between corrugated surfaces.

In this Letter we calculate non-trivial effects of geometry, i.e. effects beyond the PFA, on the Casimir-Polder force between an atom in its ground state and a corrugated surface. In contrast to the case of the lateral Casimir force between corrugated surfaces, where the effect of reshaping vacuum field fluctuations is averaged over the surfaces, an atom is a local probe of the lateral Casimir-Polder force. We show that deviations from the PFA can thus be much larger than for the force between two surfaces. We first consider the simple case of sinusoidal corrugations and come then to a periodically grooved surface where the deviation can be even more impressive. We present estimations of the order of magnitude of the lateral Casimir-Polder force for a Bose-Einstein condensate trapped close to a corrugated surface. These estimations indicate that an experimental demonstration of non trivial effects of geometry on quantum vacuum should be within reach.

Our calculations will be based upon the scattering approach that has been developed to study the Casimir force between two surfaces in presence of stochastic roughness or corrugation. This approach is used in a perturbative expansion with respect to the corrugation amplitude, which should be the smallest length-scale in the problem. It allows one to deal with arbitrary relative values of the atom/surface distance, the corrugation wavelength and the typical wavelengths characterizing the optical responses of atom and surface respectively. It will thus be possible to recover the trivial PFA limit as well as to explore non-trivial (beyond-PFA) effects, both for the van der Waals and Casimir-Polder (both regimes).

Sinusoidal corrugation — We first consider the case of an atom located above a sinusoidally corrugated surface. The position components of the atom are denoted and the uni-axial corrugation described by a profile function , measured from the plane (amplitude and wavelength ). In an expansion in powers of , the scattering upon the corrugated surface is given by zeroth-order specular reflection amplitudes and first-order non specular amplitudes. Higher-order terms will not be considered in this letter.

The Casimir-Polder energy of the atom above the surface is then written as

\[ U_{\text{CP}} = U_{\text{CP}}^{(0)}(z_A) + U_{\text{CP}}^{(1)}(z_A, x_A). \]  (1)

The zeroth-order term is the standard Casimir-Polder potential between the atom and a plane plate, it depends only on specular reflection amplitudes. The first-order correction depends on and thus gives rise to the lateral Casimir-Polder force. It can be written in terms of the non specular reflection amplitudes by using the techniques developed in.

For the simple sinusoidal case, we find

\[ U_{\text{CP}}^{(1)} = h_0 \cos(k_c x_A) g(k_c, z_A). \]  (2)
The response function $g$ can be expressed in terms of the dynamic atomic polarizability $\alpha$ of the atom (whose ground state is assumed to be spherically symmetric) and of the first-order nonspecular amplitudes (same notation $R_{pp'}$ as in [2])

$$g(k, z_A) = \frac{\hbar}{c^2\varepsilon_0} \int_0^{\infty} \frac{d\xi}{2\pi} \alpha(i\xi) \int \frac{d^2k'}{(2\pi)^2} a_{k',k,-k} \cdot \frac{\hat{e}_{p'} \cdot \hat{e}_{p} e^{-i(k' + k') z_A}}{2k''} R_{p'p}(k', k''),$$

and the retarded (Casimir-Polder) regimes respectively

$$g(k_c, z_A) = \frac{-\hbar G(k_c z_A)}{64\pi^2 \varepsilon_0^2} \int_0^{\infty} d\xi \alpha(i\xi), \quad z_A \ll \lambda_A$$

$$G(Z) = Z^2[2K_2(Z) + Z K_3(Z)];$$

$$g(k_c, z_A) = \frac{-3\hbar c\alpha(0)}{8\pi^2 z_A^5} F(k_c z_A), \quad z_A \gg \lambda_A$$

$$F(Z) = e^{-Z}(1 + Z + 16Z^2/45 + Z^3/45).$$

$K_2$ and $K_3$ are the modified Bessel functions of second and third order. As is usual, the Casimir-Polder expression depends only on the static polarizability $\alpha(0)$ of the atom. Note that the description of the surface as perfectly reflecting is more adapted to the retarded regime than to the non retarded one. The PFA result corresponds to the limit $Z \to 0$ in expressions (5). The deviations from the PFA prediction should be noticeable as soon as $g(k_c, z_A)$ differs from $g(0, z_A)$. The deviations can be quantified by the ratio $\rho = g(k_c, z_A)/g(0, z_A)$.

We will see below that $\rho$ is smaller than unity for a sinusoidal corrugation, which implies that the PFA overestimates the magnitude of the lateral effect in this case. Then, we will consider different kinds of corrugations where this conclusion may be spectacularly modified.

**Perfect and real materials** — Assuming first that the corrugated surface can be described as perfectly reflecting, we deduce the nonspecular amplitudes from [2] and derive an explicit expression for the response function $g$. We write the results for the non-retarded (van der Waals) and the retarded (Casimir-Polder) regimes respectively.

In Fig. 1 we plot the lateral potential $U^{(1)}$ for a rubidium atom in front of a perfectly reflecting surface with a sinusoidal corrugation of wavelength $\lambda_c = 10\mu m$. The inset shows the ratio $\rho$ that measures the deviation from the PFA.
As for the perfectly conducting surface, it turns out that PFA overestimates the lateral Casimir-Polder force in the case of a sinusoidal corrugation, and that the function $\rho$ decreases exponentially to zero as $k_c z_A$ grows. This is due to the fact that the reflected field modes thus correspond to an exponentially small propagation factor $\approx \exp(-k_c z_A)$ in Eq. (3). Specific results obtained for some materials of interest will be shown below.

**BEC above a grooved surface** — We come now to the discussion of a configuration which should allow one to show experimental non trivial effects of geometry on quantum vacuum, using the novel possibilities offered by cold atoms techniques. A Bose-Einstein condensate (BEC) trapped in close proximity to a surface has already been successfully used to observe the normal component of the Casimir-Polder force by measuring the frequency shift of the dipolar oscillation of the center-of-mass of the BEC [12]. For the lateral force we have in mind a setup where the long axis of an elongated BEC would be parallel to the corrugations, while the lateral (along the x direction) dipolar oscillation of the center-of-mass would be monitored as a function of time (see Fig. 2). An interesting corrugation profile corresponds to periodical grooves. If the atom is located above one plateau, its dipole oscillation frequency along the x direction is shifted due to the lateral force effect.

For the periodical grooves of Fig. 2, the potential [2] has to be replaced by a sum over Fourier components $a_n$ of the corrugation profile (assumed to be even for simplicity) $h(x) = \sum_n a_n \cos(n k_c x)$

$$U^{(1)}_{\text{CP}} = \sum_{n=0}^{\infty} a_n \cos(n k_c x_A) g(n k_c, z_A).$$

The PFA is now recovered when the response function $g(n k_c, z_A)$ may be replaced by the limiting expression $g(0, z_A)$ for all values of $n$ significantly contributing to the profile. When this is the case, $U^{(1)}_{\text{CP}}$ has the same profile as the corrugation itself. Otherwise, as $z_A$ increases multiplication by $g(n k_c, z_A)$ renders the contribution of higher orders comparatively smaller. When $k_c z_A$ is much larger than unity in particular, the exponential decrease already discussed for $g$ implies that the first order $n = 1$ dominates the sum (apart from the irrelevant $n = 0$ term) and that the potential is approximately sinusoidal.

**Frequency shifts for the dipolar oscillation** — In order to discuss the feasibility of the experiment, we first evaluate the relative frequency shift $\gamma_0 = (\omega_{\text{CP},x} - \omega_x)/\omega_x$ for a single rubidium atom of mass $m$ located at a distance $z_A$ from the plateau shown in Fig. 2. Here $\omega_x$ is the trap frequency along the x direction and $\omega_{\text{CP},x}$ is the oscillation frequency in the presence of the Casimir-Polder force. In Fig. 3 we plot the relative frequency shifts as functions of $k_c z_A$ for $z_A = 2 \mu m$, groove width $s = \lambda_c/2$, and depth $a = 250 \mu m$. We consider different materials for the bulk surface, namely a good conductor (gold), a semiconductor (silicon), and a dielectric (fused silica).

Assuming PFA, the shift would vanish since the potential is locally flat on top of the plateau in this case. Fig. 3 shows that $\gamma_0$ is indeed very small for $k_c z_A < 1$. But as $k_c z_A$ increases, $\gamma_0$ develops a peak and then decays exponentially for $k_c z_A \gg 1$. The maximal frequency shift decreases as the atom-surface separation grows, reaching values lower than $10^{-5}$ for distances greater than $z_A = 3 \mu m$. Given the reported sensitivity of $10^{-5} - 10^{-4}$ for relative frequency shift measurements [12], we expect that the effect could be measured for distances below $3 \mu m$ both for gold and silicon surfaces, while being on the border of detectability for fused silica. Different materials induce a variety of error sources, including stray electric fields (especially from spurious charges in insulating surfaces) and magnetic fields (from adsorbed atoms in conducting and semiconducting surfaces). Ultimately the optimal choice of material would come from a trade-off between maximizing the signal and minimizing the error sources [12].

The single-atom case described up to now can be applied to a BEC of Thomas-Fermi radius $R$ only in the “point-like” limit $R \ll z_{\text{CM}}, \lambda_c$, with $z_A$ merely replaced by the center-of-mass coordinate $z_{\text{CM}}$. Otherwise, we have to calculate finite-size corrections. To this aim, we consider an elongated rubidium BEC with trap frequencies $\omega_y \ll \omega_x = \omega_z$, and a two-dimensional density profile in the Thomas-Fermi limit given by
different materials. The unperturbed frequency is $\omega_0 = 2\pi$ Hz, the groove width is $s = \lambda_c/2$, and the depth $d = 250$ nm. The inset shows the corrections to the relative frequency shift for a finite size BEC of Thomas-Fermi radius $R$ oscillating at a distance $z_{CM} = 2\mu$m from a silicon grooved surface of period $\lambda_c = 4\mu$m. For gold and fused silica we obtain very similar results, which are indistinguishable on the scale of the inset.

$$(15/6\pi) \left( R^2 - (x^2 + z^2) \right)^{3/2} / R^5.$$ When averaging the single-atom frequency shift $\gamma_0$ over the two-dimensional density, the resulting shift $\gamma$ is found to increase as a function of $R$ as shown in the inset of Fig. 3. The frequency shift due to the lateral Casimir-Polder force should thus be detectable for distances below $3\mu$m and a radius of, say, $R \approx 1\mu$m. Note that non-linear corrections due to the finite amplitude $\delta_x$ of oscillations along the $x$ direction can be estimated along the lines of [17]. For the lateral Casimir-Polder force they are proportional to $k_x^2 \delta_x^2 / 8$, and for typical values $\delta_x = 0.5\mu$m and $\lambda_c = 4\mu$m they induce only a small decrease of the relative frequency shift of about 8%.

Conclusion — Novel cold atoms techniques open a promising way of investigating nontrivial geometrical effects on quantum vacuum. An interesting feature of atoms, with respect to macroscopic objects, is that they can be used as local probes of quantum vacuum fluctuations as they are reshaped when scattered by nontrivial boundaries. For example an atom or a BEC above the plate of a grooved plate would see no lateral effect, should the widely used proximity force approximation be exact. The results of the present letter, based upon a scattering approach and accounting for a realistic optical response for both atoms and bulk material, show that the non-trivial beyond-PFA effect should be measurable with a BEC using available technology.

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