Spectral evolution and stimulated Brillouin scattering suppression in phase-modulated fiber amplifier

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Abstract
The model combining power balance equations, coupling amplitude equations and stimulated Brillouin scattering (SBS) equation is established to simulate the spectrum evolution and SBS threshold in the amplifier stage. Four typical phase modulation seeds are employed to calculate the SBS spectrum evolution in fiber amplifier. The spectrum evolution results show that the top-hat shaped spectrum seed is optimal to suppress the SBS effect and scale the SBS threshold. The SBS threshold is studied in co-pumped and counter-pumped amplifiers. Simulation results show that counter-pumped amplifier is capable of generating about 2.5 times more output than co-pumped amplifier. The results provide theoretical basis on the design and experiment of continuous wave high power narrow linewidth fiber laser.

1. Introduction
Continuous wave (CW) fiber laser has made great progress due to its advantages such as high efficiency, high beam quality, compactness, and maintenance. In recent decades, high power narrow linewidth CW fiber lasers have been widely applied in many fields such as beam combining [1, 2], nonlinear frequency conversion [3, 4], and so on. The power scaling of high-power narrow linewidth fiber amplifier is mainly limited by stimulated Brillouin scattering (SBS), which is the lowest threshold nonlinear effect in narrow linewidth CW fiber laser. Early works have reported many techniques to suppress SBS such as phase modulation [5–7], novel fiber designs [8], longitudinal thermal gradients [9], and laser gain competition [9]. To date, with the development of phase modulators, phase modulation has been the predominant method to suppress SBS effect in high power narrow linewidth fiber laser.

Phase modulating to signal on time scales shorter than the phono lifetime can decrease the Brillouin gain and increase the SBS threshold. The prevalent phase modulation seeds consist of sinusoidal [7, 10], white noise signal (WNS) [11–13], pseudo-random binary sequence (PRBS) [14] and nearly top-hat shaped spectrum seed [15, 16]. The theoretical models predicting and comparing the SBS enhancement capacity of sinusoidal, WNS and PRBS phase modulation techniques have been reported [17–19]. The previous researches mainly concentrate on scaling the SBS threshold, but the study on SBS spectrum evolution is seldom seen, which is benefit for the detail analysis on SBS suppression. In this paper, we use the SBS model in the frequency domain to simulate the SBS spectrum evolution along the fiber. These typical phase modulation seeds are employed to compute the SBS spectrum evolution in fiber amplifier. The detailed laser and Stokes spectrum evolution process is analyzed. According to the results, the top-hat shaped spectrum seed is optimal to suppress the SBS effect and scale the SBS threshold.

Comparing to co-pumped amplifier, stimulated Raman scattering and four-wave mixing can be suppressed in the counter-pumped scheme [20]. We guess that the SBS can also be suppressed with counter-pumped scheme. In this paper, we computed SBS thresholds in co-pumped and counter-pumped schemes with four phase modulation seeds proposed above. Through the simulation, it is found that the SBS can be suppressed effectively by counter-pumped scheme, and the SBS threshold be increased significantly. According to the
simulation results, 3 ~ 5 kW, GHz linewidth fiber amplifier with phase modulation and counter-pumped scheme can be obtained, providing the theoretical basis for the development of higher power narrow linewidth fiber amplifier.

2. Theory

2.1. Phase modulation

For a single frequency (SF) sinusoidal phase modulation function with modulation amplitude \( m_d \) and modulation frequency \( f_m \). The complex amplitude of the light wave is:

\[
E(t) = E_0 \exp(j\omega t) \exp(-jm_\lambda (2\pi f_m t))
\]

(1)

where \( E_0 \) is amplitude, \( \omega \) is angular frequency. The output optical signal after modulation can be written as:

\[
\frac{E(t)}{E_0} = \exp(j\omega t) \sum_{p=-\infty}^{\infty} (j)^p f_p(m_d) \exp(p\Omega t)
\]

(2)

where \( f_p(m) \) is the first kind of Bessel function, \( \Omega = 2\pi f_m \). Therefore, the spectral power exhibits a series of sidebands with integer multiples of \( p \). The broadened linewidth of the sinusoidal modulated signal depends on the modulation depth \( m_d \) and modulation frequency \( f_m \) [10].

As for the WNS phase modulation, a white noise source is utilized to produce a Gaussian broadened signal, and the spectrum broadened with Lorentz profile is generated [13]. The linewidth and spectrum of the optical signal depend on the frequency distribution and power applied to the electro-optics modulator.

For PRBS phase modulation, a pseudo-random binary voltage is applied to a phase modulator. The pseudo-random pattern \( n \) produces a length of \( 2^n \) - 1 bits with a value of 0 or 1 chosen randomly with 50% probability. The series repeats itself after \( 2^n \) - 1 bits. In this study, the 0 and 1 are taken to represent modulation depths of 0 and \( \pi \), respectively. For the optical field phase modulated with a PRBS pattern \( n \) at \( f_m \), the spectrum combines of \( 2^n \) - 1 modes with a spacing of \( \Delta f = f_m/(2^n - 1) \), where \( f_m \) is the PRBS modulation frequency [14]. For PRBS phase modulation, the modulation frequency \( f_m \) and modulation pattern \( n \) influence the linewidth and power distribution of signal. The resulting optical spectrum consists of a sinc\(^2\) envelope with nulls located at integer multiples of \( f_m \).

The ideal top-hat shaped spectrum cannot be obtained with a pure phase modulation, as shown in paper [15], but it is possible to come close by optimizing the modulation signal. The principle of the approaches to achieve nearly top-hat shaped spectrum is judiciously select the modulation signal with optimal frequencies and amplitudes. The modulation signal could be multiple sinusoidal wave and arbitrary wave [15, 16].

2.2. Numerical model

The SBS threshold for phase modulated fiber laser generally is simulated by the coupling equations with pump, signal and Stokes signal in the time domain [17, 19]. For CW fiber laser, we pay more attention to the spectrum evolution process than the time evolution, so this paper, the SBS model in the frequency domain is used to simulate the spectrum evolution and SBS threshold. In this paper, we define the SBS threshold as the signal power when the Stokes power exceeds to 1 W. Because the Stokes power is low enough, the effect on laser signal by Stokes signal is neglected. The SBS is simulated in the frequency domain and the Stokes power is calculated in a series of frequencies, so the Stokes spectrum evolution can be obtained. The concrete model is described as follows.

The seed signal is amplified in amplifier stage. The amplification process can be simulated by power balanced equations (PBEs).

\[
N_2(z) = \frac{N_0(\Gamma_1\sigma_T^e P_L \lambda_\ell + \Gamma_\ell \sigma_T^a (P_p^+ + P_p^-) \lambda_p)}{\Gamma_1(\sigma_T^e + \sigma_T^a) P_L \lambda_\ell + \Gamma_\ell (\sigma_T^a + \sigma_T^e) (P_p^+ + P_p^-) \lambda_p + hcA_{eff}/\tau}
\]

\[
\frac{dP_p^+}{dz} = \pm(\Gamma_p(N_2\sigma_T^r - N_1\sigma_T^r) P_p^+ - \alpha_p P_p^+ + \alpha_p P_p^-)
\]

\[
\frac{dP_p^-}{dz} = (\Gamma_p(N_2\sigma_T^r - N_1\sigma_T^r) P_p^- - \alpha_p P_p^+ - \alpha_p P_p^-)
\]

(3)

where index \( p \) and \( L \) stand for pump wave and signal wave respectively; \( + \) and \( - \) correspond to forward and backward propagation wave respectively; \( \sigma_T^a \) and \( \sigma_T^e \) are the corresponding absorption and emission cross section; \( N_1 \) and \( N_2 \) represent the numbers of Yb-ions in ground state and excited state; \( \Gamma \) is the overlap factor; \( \lambda \) is the wavelength; \( \alpha \) is the loss coefficient; \( \tau \) is the life of the excited state population.
PBEs can be solved by numerical simulation simply, and the gain coefficient of the signal wave is obtained:

$$g_i(z) = \Gamma_i (N_i \sigma_a^i + N_i \sigma_i^i)$$

(4)

The seed signal will suffer nonlinear propagation in amplifier stage. The nonlinear propagation of the optical field can be simulated by the nonlinear coupling amplitude equation [21, 22]. For the seed with a series of longitudinal modes generated by phase modulation, coupling amplitude equation needs to consider a series of light waves. The gain coefficient in equation (4) is used to connect the PBEs and the coupling amplitude equations:

$$\frac{\partial A_n}{\partial z} = \left(\frac{g_i(z) - \phi_n}{2}\right) A_n + i\gamma \sum_{i,j} A_i A_j^* A_m^*$$

(5)

Where $A_n$ is the complex amplitude of the envelope of the $n$-th longitudinal mode, $A_0 = \sqrt{P_n} \exp(i\phi_n)$, where $P_n$ and $\phi_n$ are the power and phase of the $n$-th longitudinal mode, respectively. The $\Sigma_{i,j} A_i A_j A_m^*$ means sum up all the combination of possible wave $i,j,m$ which satisfy $\omega_i + \omega_j = \omega_m + \omega_m \gamma$ is the nonlinear Kerr coefficient: $\gamma = n_2 \omega_0 / (c A_{eff})$. The coupling amplitude equations (5) expresses the interaction between different longitudinal modes. Each mode power will change in the propagation process along the optical fiber, but the propagation of total power always stratifies the third of equations (3).

With the amplification of signal power, the SBS generated. The Stokes spectrum has a linewidth consisting of many frequencies, so we calculate the power with a series of Stokes waves. The SBS threshold for phase modulated fiber laser generally is simulated by the coupling equations with pump, signal and Stokes signal in the time domain [17, 19]. In this paper, because the Stokes power is low enough, the effect on laser signal by Stokes signal is neglected. Therefore, the system equation is expressed as:

$$\frac{\partial P^j_i}{\partial z} = -\sum_i g_b(\nu_s, \nu^j_s) P_i P^j_i + \alpha^j_s P^j_i - (N_i \sigma_s^j - N_i \sigma_s^j) \Gamma_0 P^j_i - g_a \hbar \nu_s \Delta_\nu_s$$

(6)

Where $P_i$ is the signal power at longitudinal mode $i$, and $P_i = A_i A_i^*$; $P^j_i$ is the $j$-th Brillouin wave power at frequency $\nu^j_s, \Delta_\nu_s$ the Brillouin gain bandwidth; $g_a$ is the gain coefficient caused by spontaneous emission in fiber. The total Brillouin wave power is:

$$P_s = \sum_j P^j_s$$

(7)

When the inhomogeneous spectral broadening of Brillouin wave and the temperature are taken into account, the Brillouin gain coefficient is given by [23]:

$$g_b(\nu_s, \nu^j_s) = g_b \frac{\Gamma_0 / 2}{F_0 - F_s} \left[ \tan^{-1} \left( \frac{F_0 - (\nu_s - \nu^j_s - C\Delta T)}{\Gamma_0 / 2} \right) - \tan^{-1} \left( \frac{F_s - (\nu_s - \nu^j_s - C\Delta T)}{\Gamma_0 / 2} \right) \right]$$

(8)

where $g_b$ is the SBS gain coefficient for bullk silica and $\Gamma_0$ is the homogeneous full width at half maximum of Brillouin spectral line. $F_0$ and $F_s$ are the stokes frequency shifts related to numerical aperture (NA) and can be expressed as: $F_0 = 2n\nu / \lambda_s$ and $F_s = 2n\nu [1 - (NA/n)^2]^{1/2} / \lambda_s$ separately, here $\nu$ is the velocity of sound; $n$ is the refractive index of the core; $\lambda_s$ is the signal wavelength; $C$ is the temperature slope coefficient: $C = 1.6$ MHz/K; and $\Delta T$ is the fiber core temperature difference which is directly proportional to the pump distribution.

### 3. Simulation results and discussion

For a given root-mean-square (RMS) linewidth 5 GHz, the SF seed and the typical four phase modulated seeds: sinusoidal signal, WNS, PRBS modulated seeds and nearly top-hat shaped spectrum seed are applied to amplifiers with different pumped schemes to study SBS threshold and SBS spectrum evolution. In the following work, the parameters in table 1 are used to simulate the spectral evolution and the SBS threshold.

#### 3.1. SF seed

For a SF signal, the output power and SBS power calculated with the model are shown in figure 1. The figures (a) and (b) depict power transmission with co-pumped and counter-pumped amplifier, respectively. The simulated results show that the SBS threshold for SF seed develops from 45 W to 105 W by using counter-pumped scheme.
Comparing the SBS threshold for phase modulated seeds to SF seed, the SBS enhance capacity for different modulation formats can be obtained.

### 3.2. Sinusoidal modulated seed

For sinusoidal modulated seed, the modulation depth $m_d$ and modulation frequency $f_m$ are used with different combines to simulate the SBS threshold. To control the RMS linewidth as 5 GHz, the combines: $f_m = 1$ GHz, $m_d = 3.6$, $f_m = 0.5$ GHz, $m_d = 7.2$, and $f_m = 0.1$ GHz, $m_d = 36$ are used. The simulation results are shown in figure 2. Obviously, the SBS thresholds are enhanced with co-pumped and counter-pumped schemes simultaneously, shown in table 2. Comparing the three seeds, it can be found that for a given RMS linewidth, the SBS threshold can be enhanced by using higher modulation amplitude $m_d$ and less modulation frequency $f_m$.

### Table 1. The parameters used in the simulation.

| Symbol | Parameter | Value and units |
|--------|-----------|-----------------|
| $L$    | Length of active fiber | 8 m |
| $D$    | Diameter of active fiber | 25/400 um |
| $P_s$  | Power of the seed | 10 W |
| $\lambda_p$ | Pump wavelength | 976 nm |
| $\lambda_L$ | Signal wavelength | 1064 nm |
| $N_0$  | Yb ion density | $6.52 \times 10^{25}$ m$^{-3}$ |
| $\tau$ | Fluorescence lifetime | 0.84 ms |
| $\sigma_a^p$ | Absorption cross section of pump | $2 \times 10^{-24}$ m$^2$ |
| $\sigma_e^p$ | Emission cross section of pump | $2 \times 10^{-24}$ m$^2$ |
| $\sigma_a^L$ | Absorption cross section of signal | $6.4 \times 10^{-27}$ m$^2$ |
| $\sigma_e^L$ | Emission cross section of signal | $3.98 \times 10^{-25}$ m$^2$ |

**Figure 1.** The signal and SBS power versus pump power for (a) co-pumped and (b) counter-pumped amplifier with SF seed.

**Figure 2.** The SBS power versus laser power with co-pumped and counter-pumped amplifier for sinusoidal modulated seeds.

Comparing the SBS threshold for phase modulated seeds to SF seed, the SBS enhance capacity for different modulation formats can be obtained.
Considering the pumped schemes, the SBS threshold with counter-pumped scheme is about 2.4 times larger than that with co-pumped amplifier for the same seed.

The seed modulated with $f_m = 0.5$ GHz, $m_d = 7.2$ is selected to present the laser spectrum and Stokes spectrum evolution process (Figure 3). It is found that as the laser power is amplified, and the spectrum shape and linewidth remain unchanged. The SBS signal is amplified in the laser direction. The SBS spectrum presents multi-peaks structure at the first z = L, and the peaks corresponding to higher laser power increase sharply in the SBS spectrum evolution process. Ultimately, the SBS power concentrates on few main peaks corresponding to laser peak power.

### 3.3. WNS modulated seed

The amplification process with the seeds modulated by WNS with different RMS linewidth is simulated, the SBS threshold versus the RMS linewidth of seed are expressed in Figure 4. The co-pump and counter-pump thresholds are enhanced with the increasing of the RMS linewidth (Table 2). The counter-pumped scheme can provide an about 2.7 times SBS threshold increase compared to the co-pumped scheme. When the RMS linewidth is 5 GHz, the SBS thresholds for co-pumped and counter-pumped amplifiers are 1274 W and 3456 W, which is higher than the seed modulated by sinusoidal signal with $f_m = 0.1$ GHz, $m_d = 36$.

For the seed modulated by WNS with 5 GHz RMS linewidth, the evolutions of laser spectrum and Stokes wave normalized spectrum are shown in Figures 5(a) and (b), respectively. The laser spectrum and SBS spectrum evolutions present the same rule as the seed modulated by sinusoidal signal.

### Table 2. The comparison of SBS threshold for sinusoidal modulated seeds to SF seed.

| Power/W | SF | Sinusoidal modulation |
|---------|----|-----------------------|
|         |    | $f_m = 1$ GHz, $m_d = 3.6$ | $f_m = 0.5$ GHz, $m_d = 7.2$ | $f_m = 0.1$ GHz, $m_d = 36$ |
| Co-pump | 45 | 239 | 396 | 992 |
| Counter-pump | 105 | 587 | 923 | 2347 |

**Figure 3.** (a) The spectrum evolution of laser signal and (b) the normalized spectrum evolution of Brillouin wave for the sinusoidal phase modulated seed with counter-pumped scheme.

**Figure 4.** The SBS threshold with co-pumped and counter-pumped amplifier for WNS modulated seeds.
3.4. PRBS modulated seed

For PRBS modulated seed, the PRBS pattern $n$ and modulation frequency $f_m$ determine the SBS threshold enhancement capacity. The enhancement factor with different $n$ and $f_m$ has been studied in paper [15]. In this paper, the three seeds with $n = 3, f_m = 3$ GHz, $n = 5, f_m = 3$ GHz, and $n = 7, f_m = 3$ GHz are amplified with co-pumped and counter-pumped schemes, and the RMS linewidth is almost 5 GHz. The simulation results are shown in figure 6. The SBS threshold enhancement factors (divide the SBS threshold for modulation seed by that for SF seed) for $n = 5$ is 24.3 which satisfies the results 25 in paper [14]. It is found that the SBS threshold can be enhanced about $2.45 \times$ by using counter-pumped amplifier (table 4).

The seed modulated by PRBS with pattern $n = 5$, modulation frequency $f_m = 3$ GHz are selected to present the laser spectrum and Stokes signal spectrum evolution process, shown in figure 7. Results show that the amplification process can change the laser spectrum shape and linewidth. The SBS power is amplified inverse the fiber direction. The SBS spectrum presents multi-peaks structure at $z = L$, and the SBS power concentrate on few frequencies in the evolution process, the same as the seeds modulated by sinusoidal signal and WNS.

3.5. Top-hat shaped spectrum seed

As the ideal top-hat shaped spectrum cant be generated by phase modulation [15], so the nearly top-hat shaped spectrum with 10% power fluctuation are used to simulate the SBS spectrum evolution and the SBS threshold. The nearly top-hat shaped spectrum seed with 5 GHz RMS linewidth and 60 longitudinal modes are amplified in

| Power / W | SF | WNS modulation (RMS / GHz) |
|-----------|----|---------------------------|
|           |    | 1.27 | 2.36 | 4.95 | 6.54 | 7.74 |
| Co-pump   | 45 | 636  | 933  | 1274 | 1698 | 1954 |
| Counter-pump | 105 | 1729 | 2593 | 3456 | 4663 | 5183 |

Figure 5. (a) The spectrum evolution of laser signal and (b) The normalized spectrum evolution of Brillouin wave for the WNS modulated seed with counter-pumped scheme.

Figure 6. The SBS power versus laser power with co-pumped and counter-pumped amplifier for PRBS modulated seeds.
both the co-pumped and the counter-pumped scheme, and the SBS thresholds are 2188 W and 5145 W respectively, as shown in figure 8. The SBS threshold with counter-pumped amplifier can be scaled $2.35 \times$ compared to co-pumped amplifier. The SBS threshold with nearly top-hat shaped spectrum seed is higher than sinusoidal, WNS and PRBS modulated seeds, and it can be further enhanced by increasing the number of longitudinal modes and linewidth.

The laser spectrum and Stokes signal normalized spectrum evolution process for the nearly top-hat shaped spectrum seed is shown in figure 9. Obviously, the SBS spectrum evolution is different from the seeds modulated with sinusoidal signal, WNS and PRBS. The SBS power distributes almost equally in the range of spectrum, and the SBS spectrum power increases steadily, without ultra-high peak produced. Paper [15] illustrates qualitatively that the top-hat shaped spectrum is optimal to suppress SBS. In this paper, it is more clear that the top-hat shaped spectrum seed is optimal by analyzing the SBS spectrum evolution process.

### 3.6. Discussion
From figures 3(a), 5(a) and 7(a), it can be concluded that for the phase modulated seeds, the amplifier stage only can amplify the spectrum intensity, and the spectrum shape and linewidth remain unchanged, which is different from the seed based on narrow linewidth fiber Bragg grating (FBG). So using the phase modulation signal as
seed, the spectrum and linewidth can be precisely controlled, which is a predominant advantage for high power narrow linewidth fiber amplifier.

The evolutions of Stoke spectrum are shown in the figures 3(b), 5(b), 7(b) and 9(b). It is found that for sinusoidal, WNS, PRBS modulated seeds, the Stokes spectrum presents multi-peaks structure, for the influence of the laser signal. And because of the coupling term of $P_L \times P_S$ in SBS equation, the SBS power nonlinear increases in the evolution process, and the SBS power concentrates on few main peaks ultimately. For the nearly top-hat shaped spectrum, the SBS power distributes equally in the range of spectrum, so the SBS spectrum has no ultra-high peak, and the SBS spectrum power increases steadily. So the SBS power is lower for top-hat shaped spectrum, proving that the top-hat shaped spectrum is the optimal modulation format to suppress SBS.

Together figures 2, 4, 6, and 8, one can conclude that the SBS threshold can be enhanced dramatically with counter-pumped scheme comparing to co-pumped amplifier, and the enhancement factors for sinusoidal, WNS, PRBS and top–hat shaped spectrum are all about 2.5. The reason for SBS suppression is the difference of laser power distribution in counter-pumped and co-pumped amplifier. The pump and signal power distribution along the fiber are illustrated in figures 10(a) and (b). Compared with the co-pumped amplifier in which the signal needs more than 5 m to double its signal power, in counter-pumped amplifier, the signal doubles its power and reaches the maximum in the last 1 m. This indicates that the short section of fiber in the counter-pumped amplifier is occupied by the high intensity signal which acts as a pump source for Brillouin Stokes waves, instead of the majority of fiber in co-pumped amplifier. As a result, the co-pumped amplifier generates more SBS than that of the counter-pumped amplifier, and the SBS threshold with counter-pumped amplifier is much larger than that of the co-pumped amplifier. According to the current experiment results (the maximum output power of GHz fiber laser is about $2 \sim 3$ kW), it can be concluded that the $3 \sim 5$ kW, GHz linewidth fiber amplifier with phase modulation seed and counter-pumped scheme can be obtained.

4. Conclusion

In summary, the model in frequency domain is used to simulate the SBS spectrum evolution along the fiber in fiber amplifier. Four typical phase modulation seeds: sinusoidal, WNS, PRBS phase modulation seeds and nearly top-hat shaped spectrum seed are employed to study the spectrum evolution and SBS threshold. Both co-
pumped and counter-pumped amplifier are used to study the SBS threshold. Simulation results show: (1) different phase modulation formats have different SBS threshold enhancement capacity. The Stokes spectrum evolution with nearly top-hat shaped spectrum seed presents different structure comparing to the sinusoidal, WNS, PRBS modulated seeds, proving that the top-hat shaped spectrum is the optimal modulation format to suppress SBS; (2) as the amplification of the modulated signal power in the amplifier stage, the spectrum shape and linewidth of seed cant be changed. Thus, the narrow linewidth fiber amplifier with phase modulated seed can precisely control the output spectrum; (3) comparing to co-pumped amplifier, the SBS can be suppressed with counter-pumped scheme tremendously.

Combining the simulation results, it can be found that by using counter-pumped amplifier, 25/400 fiber and phase modulation seed, the output power of narrow linewidth fiber laser can break through the current 3 kW threshold and reach to 5 kW. The theoretical results obtained in this work have a certain clinical significance to the design and experiment of CW high power narrow linewidth fiber laser.

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