New Predictive Framework for Fermion Masses in SUSY SO(10)

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Abstract

We present a new predictive approach based on SUSY SO(10) theory. The inter-family hierarchy is first generated in the sector of hypothetical super-heavy fermions and then transferred inversely to ordinary quarks and leptons by means of the universal seesaw mechanism. The obtained mass matrices are simply parametrized by two small complex coefficients $\varepsilon_u$ and $\varepsilon_d$, which can be given by the ratio of the GUT scale $M_G \approx 10^{16}$ GeV and some higher scale $M \approx 10^{17} - 10^{18}$ GeV (presumably superstring scale). The model provides a possibility for doublet-triplet splitting without fine tuning and the Higgsino mediated $d = 5$ operators for the proton decay are naturally suppressed. Our ansatz provides the correct qualitative picture of fermion mass hierarchy and mixing pattern, provided that $\varepsilon_d/\varepsilon_u \sim 10$. The running masses of the first family fermions: electron, u-quark and d-quark obey an approximate SO(10) symmetry limit. At GUT scale we have: $u \sim d \sim 3e$, $(\frac{\varepsilon_u}{\varepsilon_d}) c_s \sim s \approx \frac{1}{3} \mu$ and $(\frac{\varepsilon_u}{\varepsilon_d})^2 t \sim b \approx \tau$. The Cabibbo angle is large: $s_{12} \approx \sqrt{m_d/m_s}$ while other mixing angles have their natural size: $s_{23} \approx m_s/m_b$ and $s_{13} \approx m_d/m_b$. We have many strong quantitative predictions though no special ‘zero’ texture is utilized (in contrast to the known predictive frameworks). Namely, taking as input the lepton, c-quark and b-quark masses, $m_s/m_d$ mass ratio and Cabibbo angle, we can obtain the light (u,d,s) quark masses, top mass and $\tan \beta$. The top quark is naturally in the 100 GeV range, but not too heavy: $m_t < 165$ GeV. The lower bound $M_t > 150$ GeV (160 GeV) implies $m_s/m_d > 19$ ($> 22$). $\tan \beta$ can vary from 1.2 to 1.7. For light quark masses we have $m_s \approx 150$ MeV, $m_d \approx 7$ MeV, and $m_u/m_d \approx 0.5$.

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1. Family Puzzles and SUSY GUT

Understanding the origin of the observed pattern of fermion masses and mixing is one of the main issues in modern particle physics. The Standard Model accommodates all the observed quarks (including Top) and leptons in a consistent way – three families bear the same quantum numbers under the $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry. The fermion mass scale is set by the same Higgs field that gives masses to $W^\pm$ and $Z$ bosons. However, the pattern of fermion masses and mixing remains undetermined due to arbitrariness of the Yukawa couplings: there is no explanation, what is the origin of hierarchy of their eigenvalues, why they are approximately alligned for the up and down quarks thus providing the small mixing, what is the origin of their complex structure, why the $\Theta$-term is vanishing in spite of this complex structure etc. Understanding of all these issues calls for a more fundamental flavour theory beyond the Standard Model.

There is an almost holy trust that all the fundamental problems, and among them the problem of fermion masses, will find a final solution within the Superstring Theory – 'Theory of Everything'. In principle, it should allow us to calculate all Yukawa couplings. Unfortunately, we do not know how the superstring can be linked to lower energy physics in unambiguous way. Many theorists try to attack the problem in whole, or its certain aspects, in the context of particular (among many billions) superstring inspired models. However, the problem remains far away from being solved and all what we know at present from superstring can be updated in few important but rather general recommendations.

Nevertheless, one may think that many aspects of the flavour problem can be understood by means of more familiar symmetry properties. A relevant theory could take place at some intermediate energies between the electroweak and Planck scales. Nowadays the concepts of grand unification and supersymmetry are the most promising ideas towards the physics beyond the SM, providing a sound basis for understanding the issues of coupling unification and stability of gauge hierarchy. In particular, the famous coupling crossing phenomenon in the Minimal Supersymmetric Standard Model (MSSM) points to the GUT scale $M_G \simeq 10^{16}$ GeV.\[1\]

The $SO(10)$ model is an outstanding candidate for GUT. It unifies all quark and lepton states in one family into one irreducible representation $16$, providing thereby possibility to link their masses with specific group-theoretical relations. This can be clearly seen by considering the $G_{PS} = SU(4) \otimes SU(2)_L \otimes SU(2)_R \otimes P$ subgroup of $SO(10)$: $SU(4) \supset SU(3)_C \otimes U(1)_{B-L}$ unifies the lepton with quarks as a fourth colour, whereas the $SU(2)_L \otimes SU(2)_R$ provides an "isotopic" symmetry interchanging the up and down fermions, so that their mass matrices are somehow alligned even if this symmetry is spontaneously broken. In this way the smallness of quark mixing angles can be naturally linked to the quark mass 'horizontal' hierarchy, though the origin of this hierarchy in itself is beyond the scope of the $SO(10)$ model. The automatic discrete $P(L \leftrightarrow R)$ symmetry, essentially P-parity, can be also used for constraining the fermion mass matrices. Therefore, the $SO(10)$
naturally contains all types of the simplest fermionic symmetries: the isotopic and quark-lepton symmetries as well as P-parity, which appear to be necessary but not sufficient tools for the fermion mass model building: in order to constrain the fermion mass matrices at the needed degree, also some inter-family (horizontal) symmetry $\mathcal{H}$ should be invoked. It is natural to expect that $\mathcal{H}$ is also broken at the GUT scale $M_G$. Such a $\text{SUSY} \otimes \text{SO}(10) \otimes \mathcal{H}$ theory can be regarded as a Grand Unification of fermion masses.

On the other hand, we believe that the Standard Model is literally correct at lower energies. This sets a 'boundary' condition for any hypothetical theory of the flavour: in the low energy limit it should reduce to the minimal Standard Model (or, better to say to MSSM [2]) in all sectors (gauge, fermion and Higgs) – all the possible extra degrees of freedom must decouple. Since the decoupling is expected to occur at superhigh ($\sim M_G$) energies, there is practically no hope to observe experimentally any direct dynamical effect of such a flavour theory. However, it could manifest itself in the sense of flavour statics, through certain constraints on the Yukawa sector of the resulting MSSM, or, in other words, through the testable predictions for the fermion masses and mixing angles.

2. Deducing the Mass Matrix: Inverse Hierarchy and Seesaw

The mass spectrum of the quarks and charged leptons is spread over five orders of magnitude, from MeVs to 100 GeVs. In order to understand its shape it is necessary to compare the fermion running masses at the scale $\mu \sim M_G$, where the relevant new physics could take the place. In what follows, with an obvious notation we indicate by $u, d, ...$ the fermion running masses at the GUT scale, and by $m_u, m_d, ...$ their physical masses. For the light quarks ($u,d,s$) the latter traditionally are taken as running masses at $\mu = 1 \text{ GeV}$ [3]. In doing so, we see that the horizontal hierarchy of quark masses exhibits the approximate scaling low (see Fig. 1)

$$t : c : u \sim 1 : \varepsilon_u : \varepsilon_u^2 , \quad b : s : d \sim 1 : \varepsilon_d : \varepsilon_d^2$$

where $\varepsilon_u^{-1} = 200 - 300$ and $\varepsilon_d^{-1} = 20 - 30$. As for the charged leptons, they have a mixed behaviour:

$$\tau : \mu : e \sim 1 : \varepsilon_d : \varepsilon_u \varepsilon_d$$

One can also observe that the vertical mass splitting is small within the first family of quarks and is quickly growing with the family number:

$$\frac{u}{d} \sim 1, \quad \frac{c}{s} \sim 10, \quad \frac{t}{b} \sim 10^2,$$

whereas the splitting between the charged leptons and down quarks (at large $\mu$) remains considerably smaller – the third family is almost unsplit:

$$b \approx \tau,$$
whereas the first two families are split by a factor about 3 but

\[ ds \approx e \mu. \]  

(5)

One can also exploit experimental information on the quark mixing. The weak transitions dominantly occur inside the families, and are suppressed between different families by the small Cabibbo-Kobayashi-Maskawa (CKM) angles [4]:

\[ s_{12} \sim \sqrt{\varepsilon_d}, \quad s_{23} \sim \varepsilon_d, \quad s_{13} \sim \varepsilon_d^2, \]  

(6)

This shows that the quark mass spectrum and weak mixing pattern are strongly correlated. Moreover, there are intriguing relations between masses and mixing angles, such as the well-known formula \( s_{12} = \sqrt{d/s} \) for the Cabibbo angle. All of the observed CP-violating phenomena can be successfully described in the frames of the Standard Model with the mixing angles in the range of Eq. (5) and the sufficiently large (\( \delta \sim 1 \)) CP-phase in the CKM matrix.

It is tempting to think that the certain structure of the mass matrices is resonsible for several mass relations and the CKM angles can be expressed (explicitly or implicitly) as functions of the fermion masses. It is natural to think that these functions have the following ‘analytic’ properties [5]:

**Decoupling:** The mixing angles of the first quark family with others, \( s_{12}, s_{13} \), vanish in the limit \( u, d \to 0 \). At the next step, when \( c, s \to 0, s_{23} \) also vanishes.

**Scaling:** In the limit when the masses of the up and down quarks are proportional to each other: \( u : c : t = d : s : b \), all mixing angles: \( s_{12}, s_{13} \) and \( s_{23} \) are vanishing.

Motivated by observations made above, let us consider for the mass matrices at the GUT scale the following ansatz:

\[ \hat{m}_f = \mu_f (\hat{Q}_3 + \varepsilon_f \hat{Q}_2 + \varepsilon_f^2 \hat{Q}_1), \quad f = u, d, e \]  

(7)

where \( \mu_{u,d,e} \) are some ‘starting’ mass parameters, essentially the masses of the third family fermions: top, bottom and tau. The small expansion parameters \( \varepsilon_f = \varepsilon_u, \varepsilon_d, \varepsilon_e \) are also different for the upper quark, down quark and lepton mass matrices. \( \hat{Q}_{1,2,3} \) are some rank-1 matrices with \( O(1) \) elements, not orthogonal in general. These are axis fixing the same ‘skeleton’ for all mass matrices \( \hat{m}_f \). Without loss of generality one can take

\[ \hat{Q}_1 = (0, 0, 1)^T \bullet (0, 0, 1), \quad \hat{Q}_2 = (0, a, b)^T \bullet (0, a', b'), \quad \hat{Q}_3 = (x, y, z)^T \bullet (x', y', z') \]  

(8)

so that the mass matrices of Eq. (7) have the form

\[ \hat{m}_f \propto \mu_f \begin{pmatrix}
O(\varepsilon_f^2) & O(\varepsilon_f^2) & O(\varepsilon_f^2) \\
O(\varepsilon_f^2) & O(\varepsilon_f) & O(\varepsilon_f) \\
O(\varepsilon_f^2) & O(\varepsilon_f) & O(1)
\end{pmatrix}, \quad f = u, d, e \]  

(9)

In lowest order their eigenvalues are given by diagonal entries and their ratios are \( O(\varepsilon^2) : O(\varepsilon) : O(1) \), which reproduces the quark mass pattern of Eq. (1). On
the other hand, since $\varepsilon_u \ll \varepsilon_d$, the quark mixing arises dominantly due to diagonalization of the down quark mass matrix $\hat{m}_d$ — the up quark mass matrix $\hat{m}_u$ is much more "stretched" and essentially close to its diagonal form, so that it brings only $O(\varepsilon_u/\varepsilon_d)$ corrections to the CKM mixing angles. Thus, we expect that $s_{12}, s_{23} \sim \varepsilon_d$ and $s_{13} \sim \varepsilon_d^2$. Clearly, both the decoupling and scaling hypothesis are naturally fulfilled. In the limit when $\varepsilon_u = \varepsilon_d$ we have the scaling: $u : c : t = d : s : b$ and all CKM angles are vanishing. The decoupling can be seen in the following way: by putting $\varepsilon_u^2$ to zero (as parametrically smaller values compared to $\varepsilon_{u,d}$), we see that $u, d \rightarrow 0$ and at the same time $s_{12}, s_{13} \rightarrow 0$. At the next step, by putting $\varepsilon_{u,d}$ to zero, we see that $c, s \rightarrow 0$ and also $s_{23} \rightarrow 0$.

The mass matrices presented in a manner of Eq. (3) imply that the mass generation starts from the heaviest third family and propagates to the lighter ones, as it is commonly accepted in a mass matrices model building. However, this situation, especially in the context of the $SO(10)$ model, looks somewhat paradoxical. If the third family masses appear from the tree level Yukawa couplings, then only Higgs 10-plet is good (among the possible 10, 120 and 126) for understanding the $b - \tau$ unification: $\mu_\tau \approx \mu_e$. On the other hand, the top and bottom masses are extremely strongly split: $\mu_u \gg \mu_d$. These can be reconciled at a price of extremely large $\tan \beta$, of about two orders of magnitude. However, then it is difficult to understand why are the other parameters in mass matrices adjusted so that $c/s$ splitting becomes order of magnitude less compared to $t/b$, and $u$ and $d$ are almost unsplit versus giant $\tan \beta$. The pattern of vertical splitting (3) suggests that the ‘large’ mass parameters $\mu_f$ are linked with the expansion parameters $\varepsilon_f$ by relations $\mu_f \approx m/\varepsilon_f^2$, where the ‘small’ mass parameter $m$ is the same for all $f = u, d, e$. The structure which appears in this case can be easier seen by inverting the mass matrices of Eq. (4):

$$ \hat{m}_f^{-1} = \frac{1}{m} (\hat{P}_1 + \varepsilon_f \hat{P}_2 + \varepsilon_f^2 \hat{P}_3), \quad f = u, d, e $$

(10)

where $m \approx u, d, e$ is a mass scale of the first family fermions. The rank-1 matrices $\hat{P}_{1,2,3}$ can be taken so that the inverted mass matrices have the form

$$ \hat{m}_f^{-1} \propto \frac{1}{m} \begin{pmatrix} O(1) & O(\varepsilon_f) & O(\varepsilon_f^2) \\ O(\varepsilon_f) & O(\varepsilon_f) & O(\varepsilon_f^2) \\ O(\varepsilon_f^2) & O(\varepsilon_f^2) & O(\varepsilon_f) \end{pmatrix}, \quad \varepsilon_f = \varepsilon_u, \varepsilon_d, \varepsilon_e $$

(11)

In this way, quark mass pattern of Eqs. (11) is understood by means of one parameter $\varepsilon_d/\varepsilon_u > 1$: we have $u \sim d \sim m$, $c/s \sim (\varepsilon_d/\varepsilon_u) > 1$ and $t/b \sim (\varepsilon_d/\varepsilon_u)^2 \gg 1$. We call the mass matrix pattern given by Eq. (11) the inverse hierarchy pattern.

The above consideration suggests that the mass generation proceeds from the lightest family to heavier ones. How one could realize such a situation?

Now it is widely accepted the idea [5] that the fermion mass generation is related to higher order operators induced by the exchange of some hypothetical heavy fermions in vectorlike representations of $G_{SM}$. One can imagine, that there are some charged fermion states which are allowed to be superheavy by the GUT symmetry, or
become superheavy after GUT breaking (like the right handed neutrinos in the context of the $SO(10)$ model). Let us remark that such fermions automatically appear in the context of extended GUTs ($E_6$, Georgi’s $SU(11)$-type models etc.) as well as in the context of superstring derived models. that these heavy fermions participate in the mass generation phenomenon. Namely, one can consider the the simplest possibility that the ordinary quark and lepton masses appear through their mixing with the heavy states, just in analogy with the famous seesaw scenario for neutrinos \(^1\). Such a mechanism, named subsequently universal seesaw mechanism, was suggested in \([8]\), and it is indeed capable to provide naturally the inverse hierarchy pattern.\(^2\)

Let us consider the simple case \([8, 9]\) when along with three families of the standard fermions $f_i$ ($SU(2)$ doublets) and $f^c_i$ (singlets), $i = 1, 2, 3$, we introduce also three additional families of vectorlike 'quarks' and 'leptons': $F_i + F^c_i$ (these are weak singlets, like $f^c_i$'s). In the context of the Standard Model they are allowed to be superheavy. However, in is natural to think that their mass generation is related to some flavour physics beyond the standard model and therefore their mass matrices $\hat{M}_F$ carry the structure dictated by the spontaneous breaking pattern of the relevant symmetry (underlying GUT or horizontal symmetry). One can imagine also that the same symmetry reasons forbid direct mass terms of usual fermions $f - i.e. f f^c H$ couplings with the Standard Model Higgs $H$, allowing instead couplings of the type $\Gamma f F^c H$ and $G f^c F S$, where $S$ is electroweak singlet fragment of some GUT scalar, and $\Gamma$ and $G$ are the corresponding Yukawa matrices. Then the total mass matrix of $f$ and $F$ type fermions takes the form

$$
\hat{M}_{tot} = \begin{pmatrix} f^c & F^c \\ 0 & \Gamma \langle H \rangle \\ G^T \langle S \rangle & \hat{M}_F \end{pmatrix}
$$

Assuming that $G \langle S \rangle < \hat{M}_F$, after integrating out the heavy fermion states we receive the light fermion mass matrix of the following form:

$$
\hat{m}_f = \Gamma \hat{M}_F^{-1} G^T \langle S \rangle \langle H \rangle
$$

It is quite natural to assume that all the Yukawa couplings are $O(1)$, and all the flavour structure is contained in the 'heavy' mass matrices $\hat{M}_F$. In other words, the fermion mass hierarchy is initiated in the heavy fermion sector, while the usual light fermions are just the spectators of this phenomenon. The inverse power of $\hat{M}_F$ in Eq. (13) is crucial: by means of the seesaw mixing the heavy fermion mass hierarchy is transferred to ordinary fermions in an inverted way. This can provide a firm basis to the inverse hierarchy pattern of Eq. (11). It is clear, that the heaviest ones among the heavy fermions are just the partners of the first standard family, and its small mass splitting can be just a reflection of the symmetry limit: namely, the heaviest

\(^1\)The idea of universal seesaw mechanism was also explored in a number of papers \([10]\). The inverse hierarchy, however, corresponds to the spirit of the original paper \([8]\).
of heavies can be heavier than the relevant GUT scale, so that their mass terms
obey the isotopic and quark-lepton symmetries, which are natural subsymmetries of
the \(SO(10)\).

The inverse hierarchy pattern of the type \([11]\) was explored in the context of
radiative mass generation scenario, and several intriguing predictions were obtained
for the fermion masses and mixing \([11]\). It is clear, however, that the use of a
radiative mechanism to ensure the mass hierarchy in a heavy fermion sector is in
obvious contradiction with the idea of low-energy supersymmetry. Within SUSY
scheme one should think of some tree level mechanism that could generate the masses
of heavy fermions by means of the effective operators of successively increasing
dimension, providing thus a hierarchical structure to their mass matrix.

3. Inverse Hierarchy in SUSY SO(10) Model

We intend to build a predictive SUSY \(SO(10)\) model for the fermion masses,
pursuing the universal seesaw mechanism in order to obtain naturally the inverse
hierarchy pattern. For this purpose one has to appeal to some symmetry properties.
We assume that there is some extra ‘family’ symmetry \(\mathcal{H}\), that distinguishes the
superfields involved into the game. In the following we will not specify the exact
form of \(\mathcal{H}\), describing only how it should work. We also demand that our model
fulfills the following fundamental conditions:

\(A.\) Unification of the strong, weak and weak hypercharge gauge couplings – cor-
rect prediction for \(\sin^2\theta_W\) or \(\alpha_s\) at lower energies.

\(B.\) Natural (not fine-tuned) gauge hierarchy and doublet-triplet splitting – a cou-
ple of Higgs doublets should remain light whereas their colour triplet partners in
GUT supermultiplet should be superheavy.

\(C.\) Sufficiently long-lived proton – proton lifetime should be above the current
experimental lower bound \(\tau_p > 10^{32}\) yr. In particular, we have to make safe the \(X, Y\)
gauge boson mediated \(d = 6\) operators, as well as the colour Higgsino mediated \(d = 5\)
operators and the squark mediated \(d = 4\) operators.

\(D.\) Natural suppression of the Flavour changing Neutral currents (FCNC).

Let us construct such a SUSY \(SO(10) \otimes \mathcal{H}\) model\([7]\). We know that three families
of quarks and leptons should be arranged within 16-plets \(16_i^f\), \(i = 1, 2, 3\). Besides
them, I exploit three families of superheavy fermions \(16_k^F + \overline{10}_k^F\). All these have cer-
tain transformation properties under \(\mathcal{H}\) symmetry. For the following it is convenient
to describe them in the terms of \(SU(4) \otimes SU(2)_L \otimes SU(2)_R\) decomposition:

\[
16_i^f = f_i(4, 2, 1) + f_i^c(\bar{4}, 1, 2) \quad (14)
\]

\[
16_i^F = F_i(4, 2, 1) + F_i^c(4, 1, 2), \quad \overline{16}_i^F = \overline{F}_i(4, 2, 1) + F_i(4, 1, 2) \quad (15)
\]

\(^2\) For more details see ref. \([12]\).
Motivated by the coupling crossing phenomenon in MSSM \cite{4}, we suggest that \(SO(10)\) is broken down to \(G_{SM}\) at once, by VEVs of Eq. (15) \(\sim M_G\). Below this scale the theory is just MSSM, with three fermion families \((f_i)\) and one light couple of the Higgs doublets \((\phi = H_1 + H_2)\). In other words, from the SUSY scale \(M_S\) up to GUT scale \(M_G\) we have \textit{Grand Desert}. Many conditions of the series \(A - D\) are immediately fulfilled in this way: (i) \(\sin^2 \theta_W(\mu)\) and \(\alpha_s(\mu)\) are correctly related at \(\mu = M_Z\) (ii) the FCNC are suppressed provided that the SUSY breaking sector has \textit{universal} structure (let us remind that in Standard model FCNC are naturally suppressed in both Z-boson and Higgs exchanges \cite{4} (iii) large unification scale \((M_G \approx 10^{16} \text{ GeV})\) saves the proton from unacceptably fast decay mediated

\[\langle 54 \rangle = I \otimes \text{diag}(1,1,1, -3/2, -3/2) \cdot V_G\]
\[\langle 45_{BL} \rangle = \sigma \otimes \text{diag}(1,1,1,0,0) \cdot V_{BL}\]
\[\langle 45_R \rangle = \sigma \otimes \text{diag}(0,0,0,1,1) \cdot V_R\]
\[\langle 45_X \rangle = \sigma \otimes \text{diag}(1,1,1, x,x) \cdot V_X\]
\[\langle 126_{(10,1,3)} \rangle = \langle 126 \rangle_{(10,1,3)} = v_R\]

\[I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\]

For some details of the Higgs superpotential analysis see e.g. refs. \cite{13, 14}. Let us remark that 'general' form of the VEV \(\langle 45_X \rangle\) with \(x \neq 1\) can be obtained if the superpotential includes the terms \(45_X^254\) and \(45_X 126\). As for \(45_{BL}\) and \(45_R\), in order to ensure the strict 'zeroes' in their VEVs, they should couple only to 54-plet, but not to 126-plets.
by $X,Y$ gauge bosons ($d = 6$ operators). Let us note also, that the SUSY $SO(10)$ theory has automatic matter parity, under which the spinor representations change the sign while the vector ones stay invariant. (In fact, this gives a natural ground to refer the spinor representations as fermionic superfields, and the vector ones as Higgs superfields.) Provided that non of the 16-plets has the VEV, this implies an automatic $R$-parity conservation for the resulting MSSM. It is well-known, that the proton decaying $d = 4$ operators mediated by squarks are absent in this case.

Before addressing to the fermion mass issues let us first outline the ways to solve the Doublet-Triplet splitting puzzle and the problem of dangerous $d = 5$ operators.

**Doublet-Triplet splitting.** We require that the $\phi(1,2,2)$ component of 10-plet, which consists of the MSSM Higgs doublets $H_1$ and $H_2$, remains massless in the SUSY limit. On the other hand, the $T(6,1,1)$ fragment, containing colour triplets, should acquire the mass of the order of $M_G$: otherwise it would cause unacceptably fast proton decay and also would affect the unification of the gauge couplings. In order to resolve this famous problem without fine tuning of the superpotential parameters, one can address to the Dimopoulos-Wilczek ‘missing VEV’ mechanism utilizing the Higgs 45$_{BL}$-plet [16]. The VEV of $\phi$ arises after the SUSY breaking and breaks the $SU(2)_L \otimes U(1)_Y$ symmetry:

$$\langle \phi \rangle = \begin{pmatrix} v_2 & 0 \\ 0 & v_1 \end{pmatrix}, \quad (v_1^2 + v_2^2)^{1/2} = v = 174 \text{ GeV} \quad (19)$$

where $\tan \beta = v_2/v_1$ is the famous up-down VEV ratio in MSSM [2].

**d=5 operators [17].** The universal seesaw can be successfully implemented for the needed suppression of these operators in a manner pointed out in [15]. (For the analogous mechanism using heavy 144 + 144−-plets see also [19].) One can assume that $H$ symmetry does not allow direct mass terms for light fermions $f$ (i.e. the Yukawa couplings $16^f 16^f 10$), and they are generated through the seesaw mixing with heavy ones due to the following terms in the superpotential:

$$\Gamma_{ik} 10^f 16_k^f + G_{ik} 45_R 16_i^f \quad (20)$$

After substituting the large VEVs the whole $9 \times 9$ Yukawa matrix for the fermions of different charges acquires the form

$$W_{Yuk}^f = \begin{pmatrix} f^c & F^c & \hat{F}^c \\ 0 & \hat{\Gamma}_\phi & 0 \\ \hat{\Gamma}^+ \phi & \hat{M}_F & 0 \end{pmatrix} \quad (21)$$

(each entry of this matrix is $3 \times 3$ matrix in itself, $f = u,d,e,\nu$). The choice of the VEV $\langle 45_R \rangle$ towards the $(1,1,3)$ direction implies that the $(1,3)$-block of this matrix is vanishing. Therefore, only the $SU(2)_L$-singlet $F$-type fragments of eq. (13) are
important for the seesaw mass generation, whereas the $F$-type ones are irrelevant. As it was shown in \cite{18}, this feature is decisive for the natural suppression of the $d = 5$ operators dangerous for the proton. Indeed, the $f$ and $F$ states are unmixed, so the colour triplets in $T(6, 1, 1)$ can cause transitions of $f$’s only into the superheavy $F$’s. Therefore, the baryon number violating $d = 5$ operators of the type $[ffe]_F$ ($f = q, l$) – so called $LLLL$ operators which bring the dominant contribution to the proton decay after being dressed by the Winos, are automatically vanishing. As for the $RRRR$ type operators $[f^c f^c f^c f^c]_F$ ($f^c = u^c, d^c, e^c$), they clearly appear due to the $f^c - F^c$ mixing and have their usual size. However, they are known to be much more safe for the proton (see e.g. \cite{21} and refs. therein).

**Flavour Structure.** The structure obtained in Eq. (21) is a good starting point in order to proceed towards the fermion mass generation. It tells that the $(2, 1)$-blocks of the matrices (21) are essentially the same for the fermions $f$ of all charges – they differ only by the sign for the up-type and down-type fermions: it is $+G^T V_R$ for $f = u, \nu$ and $-G^T V_R$ for $f = d, e$. The $(1, 2)$ blocks are also the same: the matrix $\Gamma$ stands for the coupling of up-type and down-type fermions with the MSSM Higgses $H_2$ and $H_1$, respectively. On the other hand, as we already noted, the $(1, 3)$-block is vanishing, so that the $F$-type fermions are irrelevant for the seesaw mass generation. In what follows, we assume for the simplicity that $G = \Gamma$ (one can check that this assumption will not change essentially our results). We do not specify the form of $\Gamma$, suggesting that it is some general complex and non-degenerated matrix with elements $O(1)$. Without loss of generality, by suitable redefinition of the $f$ fermion basis, we always can bring it to a skew-diagonal form:

$$\Gamma_{ik} = 0, \quad \text{if } i < k. \quad (22)$$

Therefore, Assuming $\hat{M}_F \gg \Gamma V_R$, the seesaw block-diagonalization of Eq. (21) reduces our theory to the MSSM with the following Yukawa coupling matrices for the ordinary quarks and leptons:

$$\hat{\lambda}_f = \Gamma (V_R \hat{M}_F^{-1}) \Gamma^T \quad (23)$$

Therefore, all the flavour structure is contained in the heavy $F$ fermion mass matrices and the light fermion ones are $\hat{m}_f \propto \hat{M}_F^{-1}$. What remains is to obtain the needed hierarchical pattern for the heavy mass matrices $\hat{M}_F$.

Let us assume that the $\mathcal{H}$ symmetry allows the bare mass term ($M \gg M_G$) for the first heavy family $F_1$ and the mass of the 2$^{nd}$ one ($F_2$) is generated via 45$X$:

$$M_{16}^F 16^F_1 \bar{16}^F_1 + g_{45_1} 16^F_2 \bar{16}^F_2, \quad (24)$$

while the 3$^{rd}$ family becomes massive through the effective operator $(45^2_3/M) 16^F_3 \bar{16}^F_3$. In this case the fermion mass hierarchy will be explained due to small parameter
However, it is not enough restrictive to use this operator without specifying to which of the possible \( SO(10) \) channels it acts: \( 45 \times 45 \to 1 + 45 + 210 \). In order to be less vague, let us introduce the additional couple \( 16_F^c + \overline{10}_0 \) with mass \( M' \sim M \) and Yukawa couplings \( g'16_F^c \overline{10}_0 + g''16_F^c \overline{10}_3 \). Then the mass terms of the \( F_3 \) states appear at the decoupling of the heavier \( F_0 \) states, i.e. after the diagonalization of the mass matrix

\[
\begin{pmatrix}
F_3^c & F_0^c \\
0 & g'\langle 45_X \rangle \\
\end{pmatrix}
\]

As a consequence, we obtain the mass matrices \( \hat{M}_F \) of the desired form:

\[
\hat{M}_F = M(\hat{P}_1^0 + \varepsilon_f \hat{P}_2^0 + \sigma \varepsilon_f^2 \hat{P}_3^0),
\]

where \( \sigma \sim M/M' \sim 1 \), since all Yukawa couplings are assumed to be \( O(1) \). What is new, is that the complex expansion parameters \( \varepsilon_f \) (\( f = u, d, e, \nu \)) are related due to the VEV pattern (18) of the \( 45_X \):

\[
\begin{align*}
\varepsilon_d &= \varepsilon_1 + \varepsilon_2, \\
\varepsilon_e &= -3\varepsilon_1 + \varepsilon_2, \\
\varepsilon_u &= \varepsilon_1 - \varepsilon_2, \\
\varepsilon_\nu &= -3\varepsilon_1 - \varepsilon_2
\end{align*}
\]

Hence, only two of these four parameters are independent:

\[
\begin{align*}
\varepsilon_e &= -\varepsilon_d - 2\varepsilon_u, \\
\varepsilon_\nu &= 2\varepsilon_e + 3\varepsilon_u
\end{align*}
\]

As we have already noted, the heavy states \( F \) decouple at the scale \( V_R \): below this scale the effective theory is the MSSM with the Yukawa couplings given by Eq. (23). Then the ratio \( V_R/M \) is given by \( m_e/v \sim 10^{-5} \). Taking into account that \( M/M_G \sim \varepsilon_d^{-1} \sim 30 \), this implies \( V_R \sim 10^{13} \) GeV, i.e. some three orders of magnitude below the GUT scale \( M_G \). The seesaw limit \( M_F \gg V_R \) is certainly very good for large \( \varepsilon \sim V_G/M \)\footnote{For this \( M \) should be about few times \( 10^{17} \) GeV, which corresponds to superstring scale. In fact, there is no contradiction in treating our SUSY \( SO(10) \otimes H \) theory as a superstring derived one. Obviously, since we utilize the higher dimensional representations of \( SO(10) \), such a theory should be realized at some higher Kac-Moody level. In particular, \( \kappa > 5 \), if we use the 126-dimensional representation for the symmetry breaking and neutrino mass generation purposes \cite{20}.}

\footnote{This is not much of a problem neither for gauge coupling unification nor for other issues, provided that the VEV of the 126-plet \( v_R \) is of the order of \( M_G \). Nevertheless, one may find such a small \( V_R \) as unappealing. In this case we can suggest that the \((2,1)\)-block of the "big" matrix (21) appears due to the effective operators \((1/M)(16_f 45_R)(45_R 16_F)\) rather than the direct Yukawa couplings of the eq. (20). These operators can be obtained by exchange of the \( 16 + 16 \) states with \( \sim M \) masses. Then \((2,1)\)-block is naturally \( \sim 10^{13} \) GeV when \( V_R \sim M_G \), and it is the same for all types of fermions.}
all light states apart from t-quark, since their Yukawa couplings are much smaller than 1. However, since $\lambda_t = O(1)$, we expect the mass of its F-partner $M_T$ to be of the order of $V_R$ (remember that the Yukawa couplings are considered to be $O(1)$). Thus, to evaluate $\lambda_t$, we have to diagonalize the matrix (32) without the restriction $M_F \gg V_R$. The result is given by

$$\hat{\lambda}_f \hat{\lambda}_f^\dagger = \Gamma \left[ 1 + \hat{M}_F^\dagger (\Gamma^T \Gamma^* V_R^2)^{-1} \hat{M}_F \right]^{-1} \Gamma^\dagger.$$  \hspace{1cm} (29)

Notice that, when $V_R \gg \hat{M}_F$, this equation gives the obvious result $\hat{\lambda}_f = \Gamma$. On the other hand, when $V_R \ll \hat{M}_F$, it reduces to the Eq. (23).

4. Mass and Mixing Pattern

First it is useful to study $\hat{\lambda}_f$ in the seesaw limit (23). The exact formula (23) will only be relevant to evaluate the top quark Yukawa constant $\lambda_t$. Once again, the inverse matrices are easier to analyse. From the eqs. (26) and (23) we have

$$\hat{\lambda}_f^{-1} = \frac{M}{V_R} (\Gamma^T)^{-1} (\hat{P}_1^0 + \epsilon_f \hat{P}_2^0 + \epsilon_f^2 \hat{P}_3^0) \Gamma^{-1} = \frac{1}{\lambda} (\hat{P}_1 + \epsilon_f \hat{P}_2 + \epsilon_f^2 \hat{P}_3),$$  \hspace{1cm} (30)

where $\hat{P}_n \propto (\Gamma^T)^{-1} \hat{P}_n^0 \Gamma^{-1}$ are still rank-1 matrices, but not orthogonal anymore. We can also choose a basis of eq. (22) and use a relation $(\Gamma^T)^{-1} \hat{P}_1^0 \Gamma^{-1} = (\Gamma_{11})^{-2} \hat{P}_1^0$, to define $\hat{P}_1 = \hat{P}_1^0$ and $\lambda = \Gamma_{11}^2 V_R / M$. In other words, without loss of generality we can take

$$\hat{P}_1 = (1, 0, 0)^T \bullet (1, 0, 0), \; \hat{P}_2 = (a, b, 0)^T \bullet (a, b, 0), \; \hat{P}_3 = (x, y, z)^T \bullet (x, y, z)$$  \hspace{1cm} (31)

so that the inverse Yukawa matrices can be rewritten as the following:

$$\hat{\lambda}_f^{-1} = \frac{1}{\lambda} \begin{pmatrix} 1 + a^2 \epsilon_f + x^2 \epsilon_f^2 & ab \epsilon_f + xy \epsilon_f^2 & xz \epsilon_f^2 \\ ab \epsilon_f + xy \epsilon_f^2 & b^2 \epsilon_f + y^2 \epsilon_f^2 & yz \epsilon_f^2 \\ xz \epsilon_f^2 & yz \epsilon_f^2 & \epsilon_f^2 \end{pmatrix}.$$  \hspace{1cm} (32)

One can easily see that the $O(\epsilon)$ corrections can be neglected in all entries of the matrix (32) except the 1,1 element. Indeed, the eigenvalues of the matrices demonstrate the approximate behaviour of Eqs. (1), so that $\epsilon_u \ll \epsilon_d$. Then the CKM mixing angles are essentially determined by the down quarks Yukawa matrix $\hat{\lambda}_d$, and if $O(\epsilon)$ corrections would be negligible for its all entries, we should have $s_{12}, s_{23} \sim \epsilon_d$ and $s_{13} \sim \epsilon_d^2$. For $s_{23}$ and $s_{13}$ this is really the case. For example, from Eq. (32) we have $s_{23} \sim |yz/b^2| \epsilon_d$ and $\lambda_u/\lambda_b \approx |z/b^2| \epsilon_d$. Then the experimental observation $s_{23} \sim \lambda_u/\lambda_b \sim \epsilon_d$ implies that $z \sim y \sim b$. Thus, the $\epsilon_d^2 y^2$ term leads only to a few percent correction to the leading term $\epsilon_d b^2$ in 2,2 element of the matrix (32) and can be safely neglected. The similar consideration for $s_{13}$ allows one to neglect also $O(\epsilon^2)$ corrections in 1,1 and 1,2 elements. As for the Cabibbo angle, we have

$$s_{12} \approx \frac{|\epsilon_d ab|}{1 + \epsilon_d a^2} \approx \sqrt{\frac{\lambda_d}{\lambda_u}} |\epsilon_d a^2|.$$  \hspace{1cm} (33)
Hence, due to experimentally observed relation \( s_{12} \approx \sqrt{d/s} \) we have to assume that \(|\varepsilon_d a^2| \approx 1\). In turn, this implies that these \(\varepsilon_d a^2\) terms in the 1,1 element can split the d-quark and electron masses by the needed amount.

One may wonder how to achieve \(\varepsilon_d a^2 \approx 1\), if the Yukawa couplings are assumed to be \(O(1)\) and \(\varepsilon\) is a small parameter: \(\varepsilon_d \sim 1/20 - 1/30\) (see eq. (1)). However, here we still see the advantage of seesaw mechanism: \(a\) is not the coupling constant but rather their ratio, due to the "sandwiching" between \(\Gamma\)’s in Eq. (23). Thus, it should not come as a surprise if \(a^2 \sim 20 - 30\) due to some spread in the Yukawa coupling constants, \(a = \Gamma_{21}/\Gamma_{11} \sim 4-5\), while the Yukawa constants themselves are small enough to fulfill the triviality bound \(\Gamma_{21}/4\pi < 1\). On the other hand, the pattern of the fermion masses and mixing suggests that such a random enhancement does not happen for other entries in the matrix (23), so that the corresponding \(O(\varepsilon)\) corrections are negligible.

Thus, in order to split fermion masses within the first family and to accommodate large (~\(\varepsilon_d^{1/2}\)) Cabibbo angle, the matrix (32) must be diagonalized assuming that \(a^2\varepsilon_d,e \sim 1\). Then for the Yukawa eigenvalues at the GUT scale we have

\[
\begin{align*}
\lambda_u &= \frac{1}{\lambda} \left[ 1 + \varepsilon_u a^2 \right], & \lambda_e &= \frac{1 + \varepsilon_u a^2}{\lambda |\varepsilon_e b^2|}, & \lambda_t &= \frac{1}{\lambda |\varepsilon_e z^2|} \\
\lambda_d &= \frac{1}{\lambda} \left[ 1 + \varepsilon_d a^2 \right], & \lambda_s &= \frac{1 + \varepsilon_d a^2}{\lambda |\varepsilon_e b^2|}, & \lambda_b &= \frac{1}{\lambda |\varepsilon_e z^2|} \\
\lambda_e &= \frac{1}{\lambda} \left[ 1 + \varepsilon_e a^2 \right], & \lambda_u &= \frac{1 + \varepsilon_e a^2}{\lambda |\varepsilon_e b^2|}, & \lambda_\tau &= \frac{1}{\lambda |\varepsilon_e z^2|}
\end{align*}
\]

where \(\tilde{\lambda}_t\) is ‘would-be’ Yukawa coupling given from the ‘seesaw’ formula (23). According to Eq. (29) it is related to the top quark genuine Yukawa constant through the following relation:

\[
\lambda_t = \frac{\tilde{\lambda}_t}{\sqrt{1 + (\lambda_t/\Gamma_{33})^2}} < \tilde{\lambda}_t
\]

From Eqs. (34) we can immediately obtain the ‘mass’ relations

\[
\begin{align*}
|\varepsilon_u| &= \frac{\lambda_u \lambda_\mu}{\lambda_u \lambda_e}, & |\varepsilon_u|^2 &= \frac{\lambda_\tau}{\lambda_t} \Rightarrow \tilde{\lambda}_t = \left( \frac{\lambda_u \lambda_e}{\lambda_u \lambda_\mu} \right)^2 \\
|\varepsilon_d| &= \frac{\lambda_u \lambda_\mu}{\lambda_d \lambda_e}, & |\varepsilon_d|^2 &= \frac{\lambda_\tau}{\lambda_b} \Rightarrow \lambda_b = \left( \frac{\lambda_d \lambda_s}{\lambda_d \lambda_\mu} \right)^2
\end{align*}
\]

These equations are valid at the GUT scale. In order to discuss their implications for the fermion masses the renormalization group (RG) running of the coupling constants has to be considered. We have (see e.g. (23)):

\[
\begin{align*}
m_u &= \lambda_u \eta_u A_u \beta^2 v \sin \beta, & m_d &= \lambda_d \eta_d A_d \beta \cos \beta, & m_e &= \lambda_e \eta_e A_e \beta \\
m_c &= \lambda_c \eta_c A_c \beta^2 v \sin \beta, & m_s &= \lambda_s \eta_s A_s \beta \cos \beta, & m_\mu &= \lambda_\mu \eta_\mu A_\mu \beta \\
m_t &= \lambda_t A_t \beta^2 v \sin \beta, & m_b &= \lambda_b \eta_b A_b \beta \cos \beta, & m_\tau &= \lambda_\tau \eta_\tau A_\tau \beta
\end{align*}
\]
where the factors \( A_f \) account for the running from the scale \( M_G \) to the SUSY breaking scale \( M_S \), induced by the gauge couplings, and \( B_t \) includes the running induced by the top quark Yukawa coupling:

\[
B_t = \exp \left[ -\frac{1}{16\pi^2} \int_{\ln m_t}^{\ln M_G} \lambda_t^2(\mu) d(\ln \mu) \right] \tag{39}
\]

(for definiteness we take \( M_S \sim m_t \)). The factors \( \eta_f \) encapsulate the QCD+QED running from \( m_t \) down to \( m_f \) (or to \( \mu = 1 \) GeV for the light quarks u,d,s).

The Eq. (34) shows that the \( \varepsilon_u/\varepsilon_d \) ratio is small: \( |\varepsilon_u/\varepsilon_d| < 0.15 \). Then, by invoking the SO(10) relation (28), the Eq. (37) approximately gives the ‘mass’ relationships between the down quarks and leptons:

\[
\sqrt{\frac{\lambda_b}{\lambda_\tau}} = \frac{\lambda_d}{\lambda_e} \frac{\lambda_s}{\lambda_\mu} = \frac{|\varepsilon_e|}{|\varepsilon_d|} = \left| 1 + 2 \frac{\varepsilon_u}{\varepsilon_d} \right| \approx 1 \tag{40}
\]

Thus, we received the approximate GUT relationships (4) and (3) between the down quark and lepton masses, where \( O(\varepsilon_u/\varepsilon_d) \) corrections induce about 30 % uncertainty. Although the uncertainty is large, this is a remarkable result!!! It shows that the main ‘grand’ prise, \( b - \tau \) unification, is not completely lost in our inverse hierarchy approach, as was expected naively (Indeed, \( \lambda_b \) and \( \lambda_\tau \) appear at \( O(\varepsilon_d,e) \) level and e.g. the factor 2 difference among the coefficients \( \varepsilon_d \) and \( \varepsilon_e \) would cause already factor 4 splitting between \( b \) and \( \tau \), analogously to the case of up and down quark splitting). However, the SO(10) group-theoretical relation (28) ensures that \( |\varepsilon_e| \approx |\varepsilon_d| \). Running the relation (40) from the GUT scale down we obtain that \( m_b = 5 \pm 2 \) GeV and \( m_d m_s = 700 - 1300 \) MeV².

Moreover, from the equation \( \lambda_d \lambda_s \approx \lambda_e \lambda_\mu \) we derive \( \lambda_d/\lambda_e \approx 3 \), where we have used the current algebra relation \( \lambda_s/\lambda_d = m_s/m_d \approx 22 \). Since \( |\varepsilon_e| \approx |\varepsilon_d| \), the Eqs. (34) show that such a splitting is possible only if \( |\varepsilon_d a^2| = 0.5 - 1 \). Then Eq. (33) leads to \( s_{12} \sim \sqrt{d/s} \sim \varepsilon_d^{1/2} \) as compared to what was naively expected from the ansatz (11): \( s_{12} \sim \varepsilon_d \). Thus, the mass splitting within the first family implies that the Cabibbo angle is parametrically larger as compared to other mixing angles (which have their natural size \( s_{23} \sim \varepsilon_d \) and \( s_{13} \sim \varepsilon_d^2 \)) and vice versa, the large Cabibbo angle points that \( |\varepsilon_d a^2| \sim 1 \) and thereby to the substantial mass splitting between electron and d-quark. The relation \( \varepsilon_d \approx -\varepsilon_e \) is crucial, since it splits \( \lambda_d \) and \( \lambda_e \) to different sides from \( \lambda \approx \lambda_u \) by about a factor 2. Then, according to Eq. (34), the order of magnitude difference between \( \mu/e \) and \( s/d \) follows automatically. In this way, owing to the numerical coincidence \( (\lambda_u/\lambda_d)^2 \sim \varepsilon_u/\varepsilon_d \sim 0.1 \), we reproduce the mixed behaviour of leptons (see Eq. (4)). The Eqs. (34) also imply

\[
\frac{\lambda_u}{\lambda_d} = |1 + \varepsilon_e a^2| \left( \frac{m_e m_s}{m_\mu m_d} \right)^{1/2} = 0.6 - 0.7 \tag{41}
\]

Thereby, the mass splitting between u- and d-quarks, \( m_u/m_d \approx (\lambda_u/\lambda_d) B_t^3 \tan \beta \) is \( O(1/2) \), if \( \tan \beta \) is small and \( \lambda_t \geq 1 \). 

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Until now, we have given only the qualitative description of the fermion mass and mixing pattern following from our ansatz. The detailed analysis leads to more concrete quantitative results. We take as an input the masses of leptons, c-quark and b-quark, the ratio \( \zeta = m_s/m_d = \lambda_s/\lambda_d \) and the Cabibbo angle \( s_{12} \). As we know, only two of complex expansion parameters are independent: \( \varepsilon_e = -\varepsilon_d - 2\varepsilon_u \). The value of Cabibbo angle fixes the modulus \(|\varepsilon_d a^2|\) in terms of \( \zeta \). Using the RG relations (38) we find \( \lambda_e, \lambda_\mu, \lambda_\tau, \lambda_c \) and \( \lambda_b \) in terms of \( m_e, m_\mu, m_\tau, m_c, m_b \), \( \tan \beta \) and \( \lambda_t \). Then we can proceed as follows. From the second equations in (38) and (40) we express \(|\varepsilon_u/\varepsilon_d|\) and \( \arg(\varepsilon_u/\varepsilon_d) \) as functions of \( \lambda_t \) and \( \tan \beta \) (for some fixed \( \Gamma_{33} \)). Using Eqs. (34) and (37) we obtain

\[
\frac{|1 - (\varepsilon_d + 2\varepsilon_u)a^2|}{|1 + \varepsilon_d a^2|} = \frac{\lambda_d}{\lambda_e} = \left( \frac{m_\mu/m_e}{\varepsilon_d} \right)^{1/2} \left( \frac{m_b A_\varepsilon A_\eta_r}{m_\tau A_\lambda A_\eta_B} \right)^{1/4}
\]

out of which we can find \( \arg(\varepsilon_d a^2) \). Therefore, we can express the complex parameters \( \varepsilon_u a^2 \) and \( \varepsilon_d a^2 \), and thereby \( \lambda_u/\lambda_e \), in terms of as yet unknown \( \tan \beta \) and \( \lambda_t \). Finally, using the last equation in (34) we can find \( \tan \beta \) in terms of \( \lambda_t \). This can compared with the relation between \( \tan \beta \) and \( \lambda_t \) given by fixing the top quark physical mass \( M_t = m_t[1 + 4\alpha_3(m_t)/3\pi] \).

The results of numerical calculations are shown on Fig. 2. We have taken \( \alpha_3(M_Z) = 0.11, m_b = 4.4 \text{ GeV} \) and \( m_c = 1.32 \text{ GeV} \). For the RG running factors we have used the results of ref. [22]. The constant \( \Gamma_{33} \) is taken to be 3.3, just at the perturbativity border. The solid curves show the \( \tan \beta \) and \( \lambda_t \) correlation resulting from our ‘mass’ relations for various values of \( \zeta = m_s/m_d \). The dashed curves are isolines for the fixed values of \( M_t \). We see, that by setting the lower bound on the top mass as \( M_t > 150 \text{ GeV} (160 \text{ GeV}) \) we obtain the lower bound \( \zeta > 19 (\zeta > 22) \) on the \( m_s/m_d \) ratio. The maximal top mass that we can achieve in our model by taking the extreme value \( \zeta = 25 \), is about 165 GeV.

We would like to remark that the above mentioned values of \( \alpha_3, m_b, m_c \) and \( \Gamma_{33} \) are taken on their extremes, so that for given \( \zeta \) the solid curves on Fig. 2 actually mark the upper borders of allowed regions. The Fig. 3 shows the \( \Gamma_{33} \) dependence of our results for \( \zeta = 22 \). We see that the constant \( \Gamma_{33} \), which sets the seesaw ‘cutoff’ (see Eq. (35)) should be quite close to the perturbativity bound in order to ensure the sufficiently large \( M_t \). For comparison, we have also shown the curve for \( \Gamma_{33} = 6.6 \) violating the perturbativity bound. From Figs. 2 and 3 we also see that the preferable values \( \lambda_t \), at which \( M_t \) reaches the maximum, are in the interval 1 – 2, which corresponds to the infrared fix-point [24]. As for \( \tan \beta \), it varies from 1.7 \( (M_t = 165 \text{ GeV}) \) to 1.2 \( (M_t = 150 \text{ GeV}) \). For such a small values of \( \tan \beta \) the Higgs sector of MSSM can be tested on LEP200 [25].

\(^6\) Let us remind that the condition \( G = \Gamma \) on the couplings of eq. (20), which ensures the symmetric form of the Yukawa matrices in eq. (32), was imposed by hands. In fact, this condition also leads to the upper bound on \( M_t \) provided that the ‘right’ Cabibbo angle does not exceed the physical ‘left’ one which in itself is much above the naively expected size \( O(\varepsilon_d) \) [12].
Running Eq. (42) from GUT scale down to µ = 1 GeV, we obtain
\[ m_d m_s = 1050 \text{ MeV}^2 \]  
(43)

where the uncertainty in λt gives only 1-2 % correction, provided that λt < 3. Then for fixed value of ζ we get
\[ m_d = 7.0 \cdot (22/ζ)^{1/2} \text{ MeV}, \quad m_s = 150 \cdot (ζ/22)^{1/2} \text{ MeV} \]  
(44)

Finally, we can get some information on the u- and d-quark mass splitting. The Fig. 3 shows various isocurves (dotted) of the mass ratio \( \rho = m_u/m_d \) for the fixed ratio ζ = 22. We see that the condition \( M_t > 150 \text{ GeV} \) allows to \( \rho \) to vary between 0.4 and 0.8. For \( M_t \approx 160 \text{ GeV} \) the allowed interval becomes smaller, \( \rho = 0.5 - 0.6 \).

We also show the \( \rho \)-isocurves for \( ζ = 19 \) (Fig. 4a) and \( ζ = 25 \) (Fig. 4b). From Fig. 4a we see that the maximal possible top mass \( M_t = 150 \text{ GeV} \) implies \( ρ \approx 0.4 \). Fig. 4b demonstrates that for the allowed region the values of \( ρ \) is \( ρ = 0.4 - 1 \) (for \( M_t = 150 \text{ GeV} \)) and \( ρ = 0.5 - 0.9 \) (for \( M_t = 160 \text{ GeV} \)). For \( ζ = 25 \) these values are somewhat higher than is allowed by the current knowledge on the light quark mass ratios [26].

Taking all these into account, we can conclude that the preferable choice for our ansatz corresponds to the parameter region \( m_s/m_d = 19 - 22 \), when \( M_t = 150 - 160 \text{ GeV} \) (\( \tan β = 1.2 - 1.4 \)) and \( m_u/m_d = 0.4 - 0.6 \). These values of top mass are somewhat lower as compared to the CDF result \( M_t = 174 \pm 10 \pm 13 \text{ GeV} \) [23]. One can remark, however, that this preliminary estimate by CDF is somewhat controversial. Namely, their \( t\bar{t} \) production cross section corresponds to \( M_t \approx 150 \text{ GeV} \) rather than 170 GeV. This may hint that the top mass central value resulting from the combined fit can be shifted towards 160 – 165 GeV.

5. Discussion and Outlook

One could imagine that our SUSY \( SO(10) \otimes \mathcal{H} \) theory is what remains from the superstring after compactification. Obviously, such a theory should be realized at some higher Kac-Moody level, since we utilize the higher dimensional representations of \( SO(10) \): 54, 45 etc. The fermionic sector includes 5 zero modes of 16-plets: \( 16^f_{1,2,3} \) and 16\( _F^f_{2,3} \), and 2 zero modes of \( \overline{16}^F \)-plets: \( \overline{16}^F_{2,3} \). We have also included in game non-zero modes like \( 16^F_1 + \overline{16}^F_1 \), with masses of the order of the compactification scale \( M \sim \text{few times } 10^{17} \text{ GeV} \). Taking seriously the coupling crossing phenomenon in MSSM, we suggest that the breaking of \( SO(10) \otimes \mathcal{H} \) symmetry down to \( SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \) occurs at one step, at the scale \( M_G \sim 10^{16} \text{ GeV} \). All what remains below is just MSSM with three quark-leptonic families.

We assumed that the generation of fermion masses occurs due to universal seesaw mechanism. Once again we would like to stress that in seesaw picture the ordinary light fermions \( f \) are just the spectators of the phenomena that determine the flavour structure. This structure arises in a sector of the superheavy \( F \) fermions and is
transferred to the light ones at their decoupling. Namely, the heaviest $F$ family $F_1$ is unsplit since it has $SO(10) \otimes \mathcal{H}$ invariant mass of the order of $M$. The lighter ones $F_2$ and $F_3$ get the masses of the order of $M_G$ and $M_G^2/M$ respectively, due to effective operators involving the Higgs $45_X$ with successively increasing power, and are thereby split. As a result, the $f$’s mass matrices, given by a seesaw mixing with the $F$’s, have the inverse hierarchy form given by eq. (10).

These mass matrices reproduce the fermion spectrum and mixing pattern in a very economical way. They differ only due to different, in general complex expansion parameters $\varepsilon_f \sim M_G/M \sim 10^{-1} - 10^{-2}$, where $f = u, d, e, \nu$. These parameters are related through the $SO(10)$ symmetry properties (see eq. (28)), so only two of them, say $\varepsilon_d$ and $\varepsilon_u$ are independent. Due to common mass factor $m$, the first family plays a role of a mass unification point, and the $e - u - d$ mass splitting is understood by the same mechanism that enhances the Cabibbo angle up to the $O(\sqrt{\varepsilon_d})$ value. Other mixing angles naturally are in the proper range (see eq. (6)).

We have obtained a number of interesting mass formulas, from which it follows that $m_t \sim 150$ GeV, $m_b \sim 5$ GeV, $m_s \sim 150$ MeV, $m_d \sim 7$ MeV and $m_u \sim 4$ MeV. The upper limit on the top mass in our scheme is about 165 GeV. On the other hand, the CDF lower bound $M_t > 150$ GeV (or $M_t > 160$ GeV) implies the lower bound on the $s/d$ mass ratio: $\zeta > 9$ (or $\zeta > 22$). Using eq. (28) for $\varepsilon_\nu$, we can calculate also the neutrino Dirac masses. However, in order to provide any predictions for the neutrino mass and mixing pattern we have to fix also the Yukawa couplings of the 126-plet. We have also the interesting prediction $\tan \beta = 1.2 - 1.7$. This implies interesting phenomenology for the MSSM Higgs sector which can be tested on the accelerators under construction.

On the other hand, our seesaw pattern (24) leads to the automatic suppression of the $LLLL$-type $d = 5$ operators dangerous for proton decay, and only much weaker $RRRR$-type ones remain to be effective. This leaves us with the chance to observe the proton decay at the level of present experimental bound. It is worth to remark also, that in our scheme we can evaluate the branching ratios of the different decay modes, since we are able to calculate all mixing angles, including the ones for the charged leptons. This can be rather important for testing the inverse hierarchy scheme, if the proton decay will be observed in the future.

We have not suggested any concrete example of our mysterious symmetry $\mathcal{H}$ that could support the inverse hierarchy pattern of the fermion mass matrix, the Dimopoulos-Wilczek ansatz for natural doublet-triplet splitting and, finally, the needed VEV pattern. In fact, $\mathcal{H}$ could contain some set of discrete or abelian (Peccei-Quinn type) symmetries. Non-abelian horizontal symmetries like $SU(3)$ or $SO(3)$ can be also implemented. Alternatively, one can try to utilize global or discrete $R$-symmetries. I believe that to find a viable example of $\mathcal{H}$ symmetry is a rather cumbersome but feasible task for a smart model-builder.

\footnote{In fact, the existing calculations of the proton decay modes (see e.g. [21]) cannot be satisfactory, since they are performed within the framework of the minimal SUSY SU(5) model with obviously wrong mass relations $d = e$ and $s = \mu$.}
We find it amusing that the idea of inverse hierarchy, implemented in SUSY $SO(10)$ theory in a natural way, can explain the key features of the fermion mass spectrum and weak mixing. Notice, that in contrast to all the known predictive frameworks for fermion masses (see e.g. [27, 28]), we did not exploit any particular ‘zero’ texture for the Yukawa matrices. Moreover, it is clear that in our matrices there can be no ‘zeros’ - this would immediately lead us to obviously wrong predictions. However, a clever horizontal structure in $\Gamma$ can reduce the number of input parameters and thus enhance a predictive power of our approach. In particular, the $H$ symmetry could induce the matrix $\Gamma$ with certain ‘zero’ texture. These ‘zeros’ will not be seen directly in the quark and lepton mass matrices of Eq. (31), but will manifest themselves through imposing certain relations between the parameters of Eq. (31). We can also restrict the matrix $\Gamma$ to be real by imposing the CP-ivariance (spontaneously broken by the complex VEVs in (18)). This will allow us to calculate also the CP-violating phase in CKM matrix.

Our approach is an alternative to the other popular predictive SUSY $SO(10)$ frameworks (see e.g. [28]), in which the third family masses appear via direct Yukawa coupling to the Higgs 10-plet, and the lighter fermion masses are generated by certain higher order operators with specific $SO(10)$ Clebsch structures. These models have interesting predictions for fermion masses and mixing angles. However, they seem to be too much ‘fine tuned’ in order to describe the observed mass pattern. Indeed, one ‘fine tuning’ should be payed for the extremely large $\tan \beta$ (of about 60) in order to split top from bottom in spite of equal Yukawa couplings (see e.g. ”The good, the bad and the ugly” paper by Rattazzi et al. [29]). Furthermore, the judicious selection of the $SO(10)$ Clebsch structures is needed to achieve, despite giant $\tan \beta$, the order of magnitude less splitting between the charm and strange quark masses, and especially, to obtain almost unsplit up and down quarks. These puzzles are completely absent in our ‘bottom-up’ inverted way of looking on the fermion mass spectrum, which in fact advocates the small $\tan \beta$ scenario in SUSY $SO(10)$. We assume that the Yukawa unification takes place for the first family rather than for the third one. The Yukawa couplings for the second and third families involve the ‘up-down’ symmetry breaking VEVs in successively increasing powers, which naturally explains their increasing splitting. Last but not least, we are not loosing the understanding of $b-\tau$ unification in spite of naive expectation. It is more precise the more $t-b$ are split, and this is granted by the $SO(10)$ symmetry relation (28).

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Figure Captions

Fig. 1. Logarithmic plot of fermion running masses at the GUT scale versus family number. Points corresponding to the fermions with the same electric charge are joined.

Fig. 2. Predictions of the model corresponding to different values of $m_s/m_d$: $\zeta = 19, 22$ and 25 for $\Gamma_{33} = 3.3$ (solid). Isocurves corresponding to fixed top mass: $M_t = 150, 160, 170$ and 180 (in GeVs) are also shown (dashed).

Fig. 3. Predictions of the model with different 'seesaw cutoff': $\Gamma_{33} = 1.5, 2.2, 3.3$ and 6.6, for $\zeta = 22$ (solid). The isocurves corresponding to different values of $m_u/m_d$ are also shown: $\rho = 0.4, 0.5, 0.6, 0.7$ and 0.8 (dotted).

Fig. 4. Predictions for $\rho = m_u/m_d$ (dotted curves) versus $\zeta = m_s/m_d$: $\zeta = 19$ (Fig. 4a) and $\zeta = 25$ (Fig. 4b).
Figure 1

\[ \log(\frac{m}{me}) \]

family number

\[ u \]
\[ e \]
\[ d \]
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