A Tableau Construction for Finite Linear-Time Temporal Logic

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Abstract. This paper describes a method for converting formulas in finite propositional linear-time temporal logic (Finite LTL) into finite-state automata whose languages are the models of the given formula. Finite LTL differs from traditional LTL in that formulas are interpreted with respect to finite, rather than infinite, sequences of states; this fact means that traditional finite-state automata, rather than ω-automata such as those developed by Büchi and others, suffice for recognizing models of formulas. The approach considered is based on well-known tableau-construction techniques developed for LTL, which we adapt here for the setting of Finite LTL. The resulting automata may be used as a basis for model checking, satisfiability testing, and property monitoring of systems.

1 Introduction

Since its introduction into Computer Science by Amir Pnueli in a landmark paper [10], propositional linear temporal logic, or LTL, has played a prominent role as a specification formalism for discrete systems. LTL includes constructs for describing how a system’s state may change over time; this fact, coupled with its simplicity and decidability properties for both model and satisfiability checking, have made it an appealing framework for research into system verification and analysis. The notation has also served as a springboard for the study of other temporal logics in computing.

Traditional LTL formulas are interpreted with respect to infinite sequences of states, where each state assigns a truth value to the atomic propositions appearing in the formula. Such infinite sequences are intended to be viewed as runs of a system, with each state representing a snapshot of the system as it executes. However, applications of so-called finite variants of LTL have emerged as well; in finite LTL finite, rather than infinite, state sequences are the models. Domains as varied as robotic path planning and automated run-time monitoring have used finite versions of LTL to precisely specify the desired behavior of systems.

The purpose of this paper is to define a construction for a specific finite variant of LTL, which we call Finite LTL, that renders formulas into finite-state automata that accepts the models, or “satisfying state sequences,” of the corresponding formula. Such constructions have been given for traditional LTL (e.g.
and have played a pivotal role in practical techniques for both model checking (i.e. determining if every execution of a system satisfies an LTL formula) and satisfiability checking (i.e. determining if a given formula has any models). To the best of our knowledge, no such construction has been given in the literature for Finite LTL; our goal in providing such a construction in this paper is to provide researchers with a basis for automata-theoretic techniques for studying Finite LTL as well. The construction exploits specific features of Finite LTL to simplify the traditional tableau constructions found for classical LTL; in particular, the state construction relies on semantics-preserving syntactic formula transformations, and the acceptance condition for the resulting automaton can be determined syntactically based purely on the formulas associated with a state.

The remainder of the paper is organized as follows. Section 2 provides some background literature related to finite semantics of LTL, as well as some contrasting points and motivation for our own work. Section 3 introduces the syntax and semantics of our version of Finite LTL and discusses some nuances of the language. Section 4 discusses several syntactic normal forms that formulas in Finite LTL may be translated into; these are used in describing the actual automaton construction presented in Section 5. Section 6 describes our implementation of the tableau construction and the results of empirical case study conducted using a benchmark from the LTL literature. Finally, Section 7 concludes the paper.

2 Background

This section reviews existing work on the use of finite versions of LTL. There have been a number of different LTL constructions intended to bridge the gap between finite-length real world data sequences and the automated reasoning of LTL vis à vis infinite sequences in the formal-verification community. Some have arisen originating with an intended application in mind (i.e. planning/robotics), while others approach from a more foundational direction. As such, different works have yielded several variants of so-called “finite LTL.” With varying semantics syntactically similar logics, there are a number of nuances that are elicited when comparing these different works.

De Giacomo and Vardi [4] provide complexity analysis of their finite semantic logic $\text{LTL}_f$. They also devise a PDL-inspired logic into which $\text{LTL}_f$ can be translated in linear time. It was shown for both logics that determining satisfiability of a formula is PSPACE-complete. De Giacomo et al. [3] focus on the interplay between finite and infinite LTL. They address some risks of directly transferring approaches from the infinite to the finite case, and specifically formalize when a finite trace is “insensitive to infiniteness,” where an infinite suffix is constructed to pad a finite sequence. This property is shown to be provable using a standard LTL reasoner.

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historically they are in standard LTL). Roşu [11] poses a sound and complete
proof system for his version of finite-trace temporal logic.

Fionda and Greco [7] investigate the complexity of satisfiability for restricted
fragments of LTL over finite semantics. They also provide an implementation of
a finite LTL reasoner which utilizes their complexity analysis to identify when
satisfiability for a specified formula is computable in polynomial time, and per-
forms the computation in the case; otherwise they convert the input formula to
a SAT representation and invoke a SAT solver. Notably, their maximal fragment
does not include the dual operator of Next or any binary operator with tempo-
ral modality such as Until, and only permits negation to be applied to atomic
propositions.

A recent paper by Li et al [8] addresses Mission-Time LTL (MLTL), an LTL
logic with time intervals supported for the Until and Release operators. They
developed a satisfiability checking tool for MLTL by first using a novel transfor-
mation from MLTL to LTL/LTL$f$ and then turning this resulting formula into
SAT. The authors also observe that there is a need for more solvers of these re-
lated languages. This fact, along with the other aforementioned work, motivates
our work. Across the surveyed literature, if any construction for an automaton
representing a finite semantic LTL formula was provided, the acceptance cri-
teron was recursively defined based on reachability from the initial state. In
our case, we have also provided a purely syntactic method of determining state
acceptance.

In this paper, we have adopted a version of Finite LTL that is similar to that
as used in [3,4]. Our choice is based firstly on a desire to appeal to the mature and
well-studied similarities this representation has to the case of standard (infinite)
LTL, and secondly to facilitate supplementing the logic to support the task of
query checking [1] for finite sets of finite data streams.

3 Finite LTL

This section introduces the specific syntax and semantics of Finite LTL. In what
follows, fix a (nonempty) finite set $\mathcal{A} P$ of atomic propositions.

3.1 Syntax of Finite LTL

Definition 1 (Finite LTL Syntax). The set of Finite LTL formulas is defined
by the following grammar, where $a \in \mathcal{A} P$.

$$\phi ::= a | \neg \phi | \phi_1 \land \phi_2 | X \phi | \phi_1 U \phi_2$$

We call the operators $\neg$ and $\land$ propositional and $X$ and $U$ modal. We use $\Phi^{\mathcal{A} P}$
to refer to the set of all Finite LTL formulas and $\Gamma^{\mathcal{A} P} \subseteq \Phi^{\mathcal{A} P}$ for the set of all
propositional formulas, i.e. those containing no modal operators. We often write
$\Phi$ and $\Gamma$ instead of $\Phi^{\mathcal{A} P}$ and $\Gamma^{\mathcal{A} P}$ when $\mathcal{A} P$ is clear from context.
Finite LTL formulas may be constructed from atomic propositions using the
traditional propositional operators $\neg$ and $\land$, as well as the modalities of “next”
($X$) and “until” ($U$). We also use the following derived notations:

$$\text{false} = a \land \neg a$$
$$\text{true} = \neg \text{false}$$
$$\phi_1 \lor \phi_2 = \neg((\neg \phi_1) \lor (\neg \phi_2))$$

The constants false and true, and the operators $\land$ and $\lor$, and $U$ and $R$, are
duals in the usual logical sense, with $R$ sometimes referred to as the “release”
operator. We introduce $X$ (“weak next”) as the dual for $X$. That this operator
is needed is due to the semantic interpretation of Finite LTL with respect to
finite sequences, which means that, in contrast to regular LTL, $X$ is not its own
dual. This point is elaborated on later. Finally, the duals $F$ and $G$ capture the
usual notions of “eventually” and “always”, respectively.

### 3.2 Semantics of Finite LTL

The semantics of Finite LTL is formalized as relation $\pi \models \phi$, where $\pi \in (2^{\AP})^*$
is a finite sequence whose elements are subsets of $\AP$. Such a subset $A \subseteq \AP$
represents a state $\sigma_A \in \AP \rightarrow \{0, 1\}$, or assignment of truth values to atomic
propositions, in the usual fashion: $\sigma_A(a) = 1$ if $a \in A$, and $\sigma_A(a) = 0$ if $a \notin A$.
We first introduce some notation on finite sequences.

**Definition 2 (Finite-Sequence Terminology).** Let $X$ be a set, with $X^*$ the
set of finite sequences of elements of $X$. Also assume that $\pi \in X^*$ has form
$x_0 \ldots x_{i-1}$ for some $i \in \mathbb{N} = \{0, 1, \ldots\}$. We define the following notations.

1. $\varepsilon \in X^*$ is the empty sequence.
2. $|\pi| = i$ is the number of elements in $\pi$. Note that $|\varepsilon| = 0$.
3. For $j \in \mathbb{N}$, $\pi_j = x_j \in X$, provided $j < |\pi|$, and is undefined otherwise.
4. For $j \in \mathbb{N}$, the suffix, $\pi(j)$, of $\pi$ beginning at $j$ is taken to be $\pi(j) =
   x_j \ldots x_{i-1} \in X^*$, provided $j \leq |\pi|$, and is undefined otherwise. Note that
   $\pi(0) = \pi$ and that $\pi(|\pi|) = \varepsilon$.
5. If $x \in X$ and $\pi \in X^*$ then $x\pi \in X^*$ is the sequence such that $(x\pi)_0 = x$ and
   $(x\pi)(1) = \pi$.

**Definition 3 (Finite LTL Semantics).** Let $\phi$ be a Finite LTL formula, and
let $\pi \in (2^{\AP})^*$. Then the satisfaction relations, $\pi \models \phi$, for Finite LTL is defined
inductively on the structure of $\phi$ as follows.

$\pi \models a$ iff $|\pi| \geq 1$ and $a \in \pi_0$
$\pi \models \neg \phi$ iff $\pi \not\models \phi$
$\pi \models \phi_1 \land \phi_2$ iff $\pi \models \phi_1$ and $\pi \models \phi_2$
$\pi \models X \phi$ iff $|\pi| \geq 1$ and $\pi(1) \models \phi$
$\pi \models \phi_1 \lor \phi_2$ iff $\exists j: 0 \leq j \leq |\pi|: \pi(j) \models \phi_2$ and $\forall k: 0 \leq k < j: \pi(k) \models \phi_1$
We write \([\phi]\) for the set \(\{\pi \mid \pi \models \phi\}\). We also say that \(\phi_1\) and \(\phi_2\) are logically equivalent, notation \(\phi_1 \equiv \phi_2\), if \(\llbracket \phi_1 \rrbracket = \llbracket \phi_2 \rrbracket\).

Intuitively, \(\pi\) can be seen as an execution sequence of a system, with \(\pi_0\), if it exists, taken to be the current state and \(\pi_i\) for \(i > 0\), if they exist, referring to states \(i\) time steps in the future. In this interpretation \(\varepsilon\) can be seen as representing an execution with no states in it. Then \(\llbracket \pi \rrbracket = \phi\) holds if the sequence \(\pi\) satisfies \(\phi\). Formula \(a \in \mathcal{AP}\) can only be satisfied by non-empty \(\pi\), as the presence or absence of \(a\) in the first state \(\pi_0\) contained in \(\pi\) is used to determine whether \(a\) is true \((a \in \pi_0)\) or not \((a \notin \pi_0)\). Negation and conjunction are defined as usual. The \(X\) operator is the next operator; a sequence \(\pi\) satisfies \(X\) if it is non-empty (and thus has a notion of “next”) and the suffix of \(\pi\) beginning after \(\pi_0\) satisfies \(\phi\). Finally, \(U\) captures a notion of until: \(\pi\) satisfies \(\phi_1 U \phi_2\) when it has a suffix satisfying \(\phi_2\) and every suffix of \(\pi\) that strictly includes this suffix satisfies \(\phi_1\).

### 3.3 Properties of Finite LTL

Despite the close similarity of Finite LTL and LTL, the former nevertheless possesses certain semantic subtleties that we will address in this section. Many of these aspects of the logic have to do with properties of \(\varepsilon\), the empty sequence, as a potential model of formulas. (Indeed, some other finite versions of LTL explicitly exclude non-empty sequences as possible models.) The inclusion of \(\varepsilon\) as a possible model for formulas simplifies the tableau construction given later in this paper, however; indeed, the definition of the acceptance condition that we give demands it. Accordingly, this section also shows how the possibly counter-intuitive features of Finite LTL can be addressed with proper encodings.

\(X\) is not self-dual. In traditional LTL the \(X\) operator is self-dual. That is, for any \(\phi\), \(X\phi\) and \(\neg X(\neg \phi)\) are logically equivalent. This fact simplifies the treatment of notions such as positive normal form, since no new operator needs to be introduced for the dual of \(X\).

In Finite LTL \(X\) does not have this property. To see why, consider the formula \(X\text{true}\). If \(X\) were self-dual then we should have that \(\llbracket X\text{true} \rrbracket = \llbracket \neg X(\neg \text{true}) \rrbracket\), i.e. that \(\llbracket X\text{true} \rrbracket = \llbracket \neg X(\neg \text{true}) \rrbracket\). However this fact does not hold. Consider \(\llbracket X\text{true} \rrbracket\). Based on the semantics of Finite LTL, \(\pi \models X\text{true} \) iff \(|\pi| \geq 1\) and \(\pi(0) \models true\). Since any sequence satisfies \(true\), it therefore follows that

\[
\llbracket X\text{true} \rrbracket = \{\pi \in (2^\mathcal{AP})^* \mid |\pi| \geq 1\};
\]

note that \(\varepsilon \notin \llbracket X\text{true} \rrbracket\). Now consider \(\llbracket \neg X\neg\text{true} \rrbracket\). From the semantics of Finite LTL one can see that \(\llbracket \neg\text{true} \rrbracket = \emptyset = \llbracket X\neg\text{true} \rrbracket\). It then follows that

\[
\llbracket \neg X\neg\text{true} \rrbracket = (2^\mathcal{AP})^*;
\]

and thus \(\varepsilon \in \llbracket \neg X\neg\text{true} \rrbracket\).
The existence of duals for logical operators is used extensively in the tableau construction, so for this reason we have introduced $\overline{X}$ as the dual for $X$. Using the semantics of $X$ and $\neg$ it can be seen that $\pi \models X\phi$ iff either $|\pi| = 0$ or $\pi(1) \models \phi$. Indeed, $[[\overline{X} \phi]] = [[X \phi]] \cup \{\varepsilon\}$; we sometimes refer to $\overline{X}$ as the weak next operator for this reason.

Including / excluding $\varepsilon$. The discussion about $X$ and $\overline{X}$ above leads to the following lemma.

**Lemma 1 (Empty-sequence Formula Satisfaction).** Let $\pi \in (2^{AP})^*$.

1. $\pi \neq \varepsilon$ iff $\pi \models X \text{true}$.
2. $\pi = \varepsilon$ iff $\pi \models \overline{X} \text{false}$.

*Proof.* Immediate from the semantics of $X$, $\overline{X}$.

This lemma suggests a way for including / excluding $\varepsilon$ as a model of formula.

**Corollary 1.** Let $\pi \in (2^{AP})^*$ and $\phi \in \Phi^{AP}$. Then the following hold.

1. $\pi \models \phi \land X \text{true}$ iff $\pi \models \phi$ and $\pi \neq \varepsilon$; and
2. $\pi \models \phi \lor \overline{X} \text{false}$ iff $\pi \models \phi$ or $\pi = \varepsilon$.

**Literals and $\varepsilon$.** A literal is a formula that has form either $a$ or $\neg a$ for $a \in AP$. A positive-normal-form result for a logic asserts that any formula can be converted into an equivalent one in which all negations appear only as literals.

The semantics of Finite LTL dictates that for $\pi \models a$ to hold, where $a \in AP$, $\pi$ must be non-empty. Specifically, the semantics requires that $|\pi| \geq 1$ and $a \in \pi_0$. Based on the semantics of $\neg$, it therefore follows that $\pi \models \neg a$ iff either $|\pi| = 0$ or $a \notin \pi_0$. It follows that $\varepsilon \models \neg a$ for any $a \in AP$.

This may seem objectionable at first glance, since $\neg a$ can be seen as asserting that $a$ is false “now” (i.e. in the current state), and $\varepsilon$ has no current state. Given the semantics of Finite LTL, however, the conclusion is unavoidable. However, using Corollary 1 we can give a formula that captures what might be the desired meaning of $\neg a$, namely, that a satisfying sequence must be non-empty. Consider $\neg a \land X \text{true}$. From the corollary, it follows that $\pi \models \neg a \land X \text{true}$ iff $\pi$ is non-empty and $a \notin \pi_0$.

The relationship between $\varepsilon$ and literals also influences the semantics of formulas involving temporal operators. For example, consider $F \neg a$ for literal $\neg a$, which intuitively asserts that $a$ is eventually false. More formally, based on the definition of $F$ in terms of $U$ and the semantics of $U$, it can be seen that $\pi \models F \neg a$ iff there exists $i$ such that $0 \leq i \leq |\pi|$ and $\pi(i) \models \neg a$. Since for any $\pi$, $\pi(|\pi|) = \varepsilon$, it therefore follows that $\pi(|\pi|) \models \neg a$ for any $\pi$, and thus that every $\pi$ satisfies $\pi \models F \neg a$. This can be seen as offending intuition. However, Corollary 1 again offers a helpful encoding. Consider the formula $F((\neg a) \land X \text{true})$. It can be seen that $\pi \models F((\neg a) \land X \text{true})$ iff there is an $i$ such that $0 \leq i < |\pi|$ and $\pi(i) \models \neg a$, meaning that there must exist an $i$ such that $a \notin \pi_i$. 

A similar observation highlights a subtlety in the formula $G\, a$ when $a \in \mathcal{AP}$. It can be seen that $\pi \models G\, a$ iff for all $i$ such that $0 \leq i \leq |\pi|$, $\pi(i) \models a$. Since $\pi(|\pi|) = \varepsilon$ and $\varepsilon \not\models a$, it therefore follows that $G\, a$ is unsatisfiable. This also seems objectionable, although Corollary 1 again offers a workaround. Consider $G(a \lor X\, false)$. In this case $\pi(|\pi|) \models a \lor X\, false$, and for all $i$ such that $0 \leq i < |\pi|$, $\pi(i) \models a$ iff $a \in \pi_i$. This formula captures the intuition that for $\pi$ to satisfy $a$, $a$ must be satisfied in every subset of $\mathcal{AP}$ in $\pi$.

Propositional Formulas. We close this section with a discussion about the semantics of propositional formulas (i.e. those not involving any propositional operators) in Finite LTL. Later in this paper we rely extensively on traditional propositional identities, including De Morgan’s Laws and distributivity, and the associated normal forms — positive normal form and disjunctive normal form in particular — that they enable. In what follows we show that for the set $\Gamma^{\mathcal{AP}}$ of propositional formulas in Finite LTL, logical equivalence coincides with traditional propositional equivalence. The only subtlety in establishing this fact has to do with the fact that in Finite LTL, $\varepsilon$ is allowed as a potential model.

We begin by recalling the traditional semantics of propositional formulas.

**Definition 4 (Semantics for Finite LTL Propositional Subset).** Given a (finite, non-empty) set $\mathcal{AP}$ of atomic propositions, the propositional semantics of formulas in $\Gamma^{\mathcal{AP}}$ is given as a relation $\models_p \subseteq 2^{\mathcal{AP}} \times \Gamma^{\mathcal{AP}}$ defined as follows.

1. $A \models_p a$, where $a \in \mathcal{AP}$, iff $a \in A$.
2. $A \models_p \neg \gamma$ iff $A \not\models_p \gamma$.
3. $A \models_p \gamma_1 \land \gamma_2$ iff $A \models_p \gamma_1$ and $A \models_p \gamma_2$.

We write $[\gamma]_p$ for $\{A \subseteq \mathcal{AP} \mid A \models_p \gamma\}$ and $\gamma_1 \equiv_p \gamma_2$ when $[\gamma_1]_p = [\gamma_2]_p$.

In this section we show that for any $\gamma_1, \gamma_2 \in \Gamma^{\mathcal{AP}}$, $\gamma_1 \equiv_p \gamma_2$ iff $\gamma_1 \equiv \gamma_2$; in other words, logical equivalence of propositional formulas in Finite LTL coincides exactly with traditional propositional logical equivalence. In traditional LTL, this fact follows immediately from the fact that for infinite sequence $\pi$, $\pi \models \gamma$ iff $\pi_0 \models_p \gamma$. In the setting of Finite LTL we have a similar result for non-empty $\pi$, but care must be taken with $\varepsilon$.

**Lemma 2 (Non-empty Sequence Propositional Satisfaction).** Let $\pi \in (2^{\mathcal{AP}})^*$ be such that $|\pi| > 0$, and let $\gamma \in \Gamma^{\mathcal{AP}}$. Then $\pi \models \gamma$ iff $\pi_0 \models_p \gamma$.

**Proof.** Follows by induction on the structure of $\gamma$.

The next lemma establishes a correspondence between $\varepsilon$ satisfying propositional Finite LTL formulas and the propositional semantics of such formulas.

**Lemma 3 (Empty Sequence Propositional Satisfaction).** Let $\gamma \in \Gamma^{\mathcal{AP}}$ be a propositional formula. Then $\varepsilon \models \gamma$ iff $\emptyset \models_p \gamma$.

**Proof.** The result follows by structural induction on $\gamma$. There are three cases to consider
1. \( \gamma = a \) for some \( a \in \mathcal{AP} \). In this case \( \varepsilon \not| a \) and \( \emptyset \not|_p a \), so the desired bi-implication follows.

2. \( \gamma = \neg \gamma' \) for some \( \gamma' \in \Gamma^{\mathcal{AP}} \). In this case the induction hypothesis says that
\( \varepsilon \models \gamma' \) iff \( \emptyset \models_\gamma \neg \gamma' \). The reasoning proceeds as follows.

\[
\begin{align*}
\varepsilon \models \gamma & \iff \varepsilon \models \neg \gamma' \quad \text{Semantics of Finite LTL} \\
\text{iff } \varepsilon \not| \gamma' & \quad \text{Induction hypothesis} \\
\text{iff } \emptyset \models_\gamma \neg \gamma' & \quad \text{Propositional semantics}
\end{align*}
\]

3. \( \gamma = \gamma_1 \land \gamma_2 \) for some \( \gamma_1, \gamma_2 \in \Gamma^{\mathcal{AP}} \). In this case the induction hypothesis guarantees the result for \( \gamma_1 \) and \( \gamma_2 \). We reason as follows.

\[
\begin{align*}
\varepsilon \models \gamma & \iff \varepsilon \models \gamma_1 \land \gamma_2 \quad \gamma = \gamma_1 \land \gamma_2 \\
\text{iff } \varepsilon \models \gamma_1 \text{ and } \varepsilon \models \gamma_2 & \quad \text{Semantics of Finite LTL} \\
\text{iff } \emptyset \models_\gamma \gamma_1 \text{ and } \emptyset \models_\gamma \gamma_2 & \quad \text{Induction hypothesis (twice)} \\
\text{iff } \emptyset \models_\gamma \gamma_1 \land \gamma_2 & \quad \text{Propositional semantics}
\end{align*}
\]

We can now state the main result of this section.

**Theorem 1 (Propositional / Finite LTL Semantic Correspondence).**

Let \( \gamma_1, \gamma_2 \in \Gamma^{\mathcal{AP}} \). Then \( \gamma_1 \equiv \gamma_2 \) iff \( \gamma_1 \equiv_\gamma \gamma_2 \).

**Proof.** We break the proof into two pieces.

1. Assume that \( \gamma_1 \equiv \gamma_2 \); we must show that \( \gamma_1 \equiv_\gamma \gamma_2 \), i.e. that for any \( A \subseteq \mathcal{AP} \), \( A \models_\gamma \gamma_1 \iff A \models_\gamma \gamma_2 \). We reason as follows.

\[
\begin{align*}
A \models_\gamma \gamma_1 \text{ iff } & \text{ all } \pi \text{ such that } |\pi| > 0 \text{ and } \pi_0 = A \\
A \models_\gamma \gamma_2 & \text{ Lemma 2}
\end{align*}
\]

2. Assume that \( \gamma_1 \equiv_\gamma \gamma_2 \); we must show that \( \gamma_1 \equiv \gamma_2 \), i.e. that for any \( \pi \in (2^{\mathcal{AP}})^* \), \( \pi \models_\gamma \gamma_1 \iff \pi \models_\gamma \gamma_2 \). So fix \( \pi \in (2^{\mathcal{AP}})^* \); we first consider the case when \( |\pi| > 0 \).

\[
\begin{align*}
\pi \models \gamma_1 \text{ iff } & \pi_0 \models_\gamma \gamma_1 \quad \text{Lemma 2} \\
\text{iff } \pi_0 \models_\gamma \gamma_2 & \quad \gamma_1 \equiv \gamma_2 \\
\text{iff } \pi \models_\gamma \gamma_2 & \quad \text{Lemma 2}
\end{align*}
\]

We now consider the case when \( |\pi| = 0 \), meaning \( \pi = \varepsilon \).

\[
\begin{align*}
\varepsilon \models \gamma_1 \text{ iff } & \emptyset \models_\gamma \gamma_1 \quad \text{Lemma 3} \\
\text{iff } \emptyset \models_\gamma \gamma_2 & \quad \gamma_1 \equiv \gamma_2 \\
\text{iff } \varepsilon \models_\gamma \gamma_2 & \quad \text{Lemma 3}
\end{align*}
\]

Because of this lemma, propositional formulas in Finite LTL enjoy the usual properties of propositional logic. In particular, in a logic extended with \( \lor \) formulas can be converted into positive normal form, and disjunctive normal form, while preserving their semantics, including their behavior with respect to \( \varepsilon \).
4 Normal Forms for Finite LTL

The purpose of this paper is to define a construction for converting formulas in Finite LTL into non-deterministic finite automata (NFAs) with the property that the language of the NFA for a formula consists exactly of the finite sequences that satisfy the formula. Such automata have many uses: they provide a basis for model checking against Finite LTL specifications and for checking satisfiability of Finite LTL formulas. The approach is adapted from the well-known tableau construction [12] for LTL. Our presentation relies on showing how Finite LTL formulas may be converted into logically equivalent formulas in a specific normal form; this normal form will then be used in the construction given in the next section.

4.1 Extended Finite LTL and Positive Normal Form

Our construction works with Finite LTL formulas in positive normal norm (PNF), in which negation is constrained to be applied to atomic propositions. The PNF formulas in Finite LTL as given in Definition 1 are not as expressive as full Finite LTL; there are formulas \( \phi \) in Finite LTL such that \( \phi \not\equiv \phi' \) for any PNF \( \phi' \) in Finite LTL. However, if we extend Finite LTL by including duals of all operators in Finite LTL, we can obtain a logic whose formulas are as expressive as those in Finite LTL.

Definition 5 (Extended Finite LTL Syntax). The set of Extended Finite LTL formulas is given by the following grammar, where \( a \in \mathcal{AP} \).

\[
\phi ::= a \mid \neg \phi \mid \phi_1 \land \phi_2 \mid X \phi \mid \phi_1 U \phi_2 \mid \phi_1 \lor \phi_2 \mid \overline{X} \phi \mid \phi_1 R \phi_2
\]

We use \( \Phi_{c}^{\mathcal{AP}} \) to refer to the set of all Extended Finite LTL formulas, and \( \Gamma_{c}^{\mathcal{AP}} \) for the set of propositional Extended Finite LTL formulas (i.e. formulas that do not include any use of \( X, U, \overline{X} \) or \( R \)).

Extended Finite LTL extends Finite LTL by including the duals of \( \land, X \) and \( U \), namely, \( \lor, \overline{X} \) and \( R \), respectively. Note that \( \Phi_{c}^{\mathcal{AP}} \subseteq \Phi_{c}^{\mathcal{AP}} \): every Finite LTL formula is syntactically an Extended Finite LTL formula, but not vice versa.

The semantics of Extended Finite LTL is given as follows.

Definition 6 (Extended Finite LTL Semantics). Let \( \phi \) be an Extended Finite LTL formula, and let \( \pi \in (2^{\mathcal{AP}})^{\ast} \). Then the semantics of Extended Finite LTL is given as a relation \( \pi \models_{c} \phi \) defined as follows.

\[
\begin{align*}
\pi \models_{c} a & \text{ iff } |\pi| \geq 1 \text{ and } a \in \pi_{0}. \\
\pi \not\models_{c} \neg \phi & \text{ iff } \pi \not\models_{c} \phi. \\
\pi \models_{c} \phi_1 \land \phi_2 & \text{ iff } \pi \models_{c} \phi_1 \text{ and } \pi \models_{c} \phi_2. \\
\pi \models_{c} X \phi & \text{ iff } |\pi| \geq 1 \text{ and } \pi(1) \models_{c} \phi. \\
\pi \models_{c} \phi_1 U \phi_2 & \text{ iff } \exists j: 0 \leq j \leq |\pi|: \pi(j) \models_{c} \phi_2 \text{ and } \forall k: 0 \leq k < j: \pi(k) \models_{c} \phi_1.
\end{align*}
\]
We define \( J^e \phi K_e = \{ \pi \in (2^{AP})^* | \pi \models e\phi \} \) and \( \phi_1 \equiv_e \phi_2 \) iff \( J^e \phi_1 K_e = J^e \phi_2 K_e \).

The next lemmas establish relationships between Finite LTL and Extended Finite LTL. The first shows that the semantics of Extended Finite LTL, when restricted to Finite LTL formulas, matches the semantics of Finite LTL.

**Lemma 4 ((Extended) Finite LTL Semantic Correspondence).** Let \( \phi \) be a formula in Finite LTL and \( \pi \in (2^{AP})^* \). Then \( \pi \models e\phi \) iff \( \pi \models \phi \).

**Proof.** Immediate.

The next result establishes duality properties between the new operators in Extended Finite LTL and the existing ones in Finite LTL.

**Lemma 5 (Dualities in Extended Finite LTL).** Let \( \phi, \phi_1, \phi_2 \) be formulas in Extended Finite LTL, and let \( \pi \in (2^{AP})^* \). Then the following hold.

1. \( \pi \models e\phi_1 \lor \phi_2 \) iff \( \pi \models e(\neg \phi_1 \land \neg \phi_2) \).
2. \( \pi \models eX\phi \) iff \( \pi \models e\neg X\neg \phi \).
3. \( \pi \models e\phi_1 R \phi_2 \) iff \( \pi \models e(\neg \phi_1 U \neg \phi_2) \).

**Proof.** Follows from the definition of \( \models e \).

The next lemma establishes that although Extended Finite LTL includes more operators than Finite LTL, any Extended Finite LTL formula can be translated into a logically equivalent Finite LTL formula. Thus, the two logics have the same expressive power.

**Lemma 6 (Co-expressiveness for (Extended) Finite LTL).** Let \( \phi \) be an Extended Finite LTL formula. Then there is a Finite LTL formula \( \phi' \) such that \( [\phi]_e = [\phi'] \).

**Proof.** Follows from Lemmas 4 and 5. The latter lemma in particular establishes that each non-Finite LTL operator in \( \phi \) (\( \lor, X, R \)) can be replaced by appropriately negated versions of its dual. Specifically, \( \phi_1 \lor \phi_2 \) can be replaced by \( \neg((\neg \phi_1) \land (\neg \phi_2)) \), \( X\phi \) by \( \neg X\neg \phi \), and \( \phi_1 R \phi_2 \) by \( \neg(\neg \phi_1 U \neg \phi_2) \).

Although Extended Finite LTL does not enhance the expressive power of Finite LTL, it does enjoy a property that Finite LTL does not: its formulas may be converted in positive normal form. This fact will be useful in defining the tableau construction; the relevant mathematical results are presented here.

**Definition 7 (Positive Normal Form (PNF)).** The set of positive normal form (PNF) formulas of Extended Finite LTL is defined inductively as follows.

\[- \text{If } a \in AP \text{ then } a \text{ and } \neg a \text{ are in positive normal form.} \]
– If \( \phi \) is in positive form then \( X \phi \) and \( X \phi \) are in positive normal form.
– If \( \phi_1 \) and \( \phi_2 \) are in positive normal form then \( \phi_1 \land \phi_2 \), \( \phi_1 \lor \phi_2 \), \( \phi_1 \, U \, \phi_2 \) and \( \phi_1 \, R \, \phi_2 \) are in positive normal form.

We now have the following.

**Lemma 7 (PNF and Extended Finite LTL).** Let \( \phi \in \Phi^{A_P} \) be an Extended Finite LTL formula. Then there is a \( \phi' \in \Phi^{A_P} \) in PNF such that \( \phi \equiv_e \phi' \).

**Proof.** Follows from the fact that \( \neg \neg \phi \equiv_e \phi \) and the existence of dual operators in Extended Finite LTL, which enable identities such as \( \neg (\phi_1 \, U \, \phi_2) \equiv_e (\neg \phi_1) \, R \, (\neg \phi_2) \) to be used to “drive negations” down to atomic propositions.

### 4.2 Automaton Normal Form

Propositional logic exhibits a number of logical equivalences that support the conversion of arbitrary formulas into various normal forms that are then the basis for algorithmic analysis, including satisfiability checking. Disjunctive Normal Form (DNF) is one such well-known normal form. In this section we show how Extended LTL formulas in PNF can be converted into a normal form related to DNF, which we call **Automaton Normal Form (ANF)**; ANF will be a key vehicle for the automaton construction in the next section. We begin by reviewing the basics of DNF in the setting of the propositional fragment of Extended LTL.

We first lift the definitions of \( \lor \) and \( \land \) to finite sets of formulas as follows.

**Definition 8 (Conjunction / Disjunction for Sets of Formulas).** Let \( P = \{ \phi_1, \ldots, \phi_n \} \), \( n \geq 0 \) be a finite set of Extended LTL formulas. Then \( \land P \) and \( \lor P \) are defined as follows.

\[
\land P = \begin{cases} 
\text{true} & \text{if } n = 0 \ (i.e. \ P = \emptyset) \\
\phi_1 & \text{if } n = 1 \ (i.e. \ P = \{ \phi_1 \}) \\
\phi_1 \land (\land \{ \phi_2, \ldots, \phi_n \}) & \text{if } n \geq 2
\end{cases}
\]

\[
\lor P = \begin{cases} 
\text{false} & \text{if } n = 0 \ (i.e. \ P = \emptyset) \\
\phi_1 & \text{if } n = 1 \ (i.e. \ P = \{ \phi_1 \}) \\
\phi_1 \lor (\lor \{ \phi_2, \ldots, \phi_n \}) & \text{if } n \geq 2
\end{cases}
\]

We now define disjunctive normal form as follows.

**Definition 9 (Disjunctive Normal Form (DNF)).**

1. A literal is a formula of form \( a \) or \( \neg a \) for some \( a \in A_P \).
2. A DNF clause is a formula \( C \) of form \( \land \{ \ell_1, \ldots, \ell_n \} \), \( n \geq 0 \), where each \( \ell_i \) is a literal.
3. A formula in \( \Gamma^{A_P}_e \) is in disjunctive normal form (DNF) if it has form \( \lor \{ C_1, \ldots, C_k \} \), \( k \geq 0 \), where each \( C_i \) is a DNF clause.

The following is a well-known result in propositional logic that, due to Theorem 1, is also applicable to the propositional fragment of Extended Finite LTL.
Theorem 2 (DNF Conversion for Extended Finite LTL). Let \( \gamma \in \Gamma_e^{AP} \). Then there is a DNF formula \( \gamma' \in \Gamma_e^{AP} \) such that \( \gamma \equiv_e \gamma' \).

Automaton normal form (ANF) can be seen as an extension of DNF in which each clause is allowed to have a single subformula of form \( X \phi \) or \( X \bar{\phi} \), where \( \phi \) is an formula in full Extended Finite LTL. A clause in an ANF formula can be seen as defining whether or not a sequence \( \pi \) satisfies the formula in terms of conditions that must hold on the first element of the sequence, if there is one, (the literals in the clause), and the rest of the sequence (the “next-state” formula in the clause). This feature will be exploited in the automaton construction in the next section. The formal definition of ANF is as follows.

Definition 10 (Automaton Normal Form (ANF)).

1. An ANF clause \( C \) has form \((\bigwedge \{\ell_1, \ldots, \ell_k\}) \land N(\bigwedge \{\phi_1, \ldots, \phi_n\})\), where each \( \ell_i \) is a literal, \( N \in \{X, X\} \) and each \( \phi_j \in \Phi_e^{AP} \) is an arbitrary Extended Finite LTL formula.

2. A formula in Extended Finite LTL is in automaton normal form (ANF) iff it has form \( \bigvee \{C_1, \ldots, C_k\} \), \( k \geq 0 \), where each \( C_i \) is an ANF clause.

We often represent clauses as \((\bigwedge L) \land N(\bigwedge F)\), where \( L \) is a finite set of literals and \( F \) a finite set of Extended LTL formulas. If \( C = (\bigwedge L) \land N(\bigwedge F) \) we write \( \text{lits}(C) = L \) \( \text{nf}(C) = F \) for the set of literals and the set of “next formulas” following the next operator \((X \text{ or } X)\) in \( C \).

The next lemma establishes a key feature of formulas in ANF vis à vis the sequences in \((2^{AP})^*\) that model it.

Lemma 8 (Sequence Satisfaction and ANF).

1. Let \( C \) be an ANF clause. Then for any \( \pi \in (2^{AP})^* \) such that \( |\pi| > 0 \), \( \pi \models_e C \) iff \( \pi_0 \models_p L \land \text{lits}(C) \) and \( \pi(1) \models_e \land \text{nf}(C) \).

2. Let \( \phi = \bigvee_i C_i \) be in ANF. Then for every \( \pi \in (2^{AP})^* \), \( \pi \models_e \phi \iff \pi \models_e C_i \).

Proof. For Part 1, let \( \pi \in (2^{AP})^* \) be such that \( |\pi| > 0 \). Also let \( L = \text{lits}(C) \) and \( F = \text{nf}(C) \). We reason as follows.

\[
\begin{align*}
\pi \models_e C & \iff \pi \models_e (\bigwedge L) \land N(\bigwedge F) & \text{Definition 10} \\
& \text{iff } \pi \models_e \bigwedge L \text{ and } \pi \models_e N(\bigwedge F) & \text{Semantics of } \land \\
& \text{iff } \pi_0 \models_p \bigwedge L \text{ and } \pi \models_e N(\bigwedge F) & \text{Lemma 2}, \bigwedge L \in \Gamma_e^{AP} \\
& \text{iff } \pi_0 \models_p \bigwedge L \text{ and } \pi(1) \models_e \bigwedge F & \text{Semantics of } N \in \{X, X\}
\end{align*}
\]

Part 2 follows immediately from the semantics of \( \bigvee \).
The import of this lemma derives especially from its first statement. This asserts that determining if an ANF clause is satisfied by a non-empty sequence can be broken down into a propositional determination about its initial state \( \pi_0 \) and the literals in the clause, and a determination about the rest of the sequence \( \pi(1) \) and the “next formulas” of the clause. This observation is central to the construction of automata from formulas that we give later.

In the rest of this section we will show that for any Extended Finite LTL formula \( \phi \) there is a logically equivalent one in ANF. We start by stating some logical identities that will be used later.

**Lemma 9 (Distributivity of \( X \), \( \overline{X} \)).** Let \( \phi_1, \phi_2 \in \Phi^e_{AP} \).

1. \( (X \phi_1) \wedge (X \phi_2) \equiv e X(\phi_1 \wedge \phi_2) \).
2. \( (\overline{X} \phi_1) \wedge (\overline{X} \phi_2) \equiv e \overline{X}(\phi_1 \wedge \phi_2) \).
3. \( (X \phi_1) \vee (X \phi_2) \equiv e X(\phi_1 \vee \phi_2) \).
4. \( (\overline{X} \phi_1) \vee (\overline{X} \phi_2) \equiv e \overline{X}(\phi_1 \vee \phi_2) \).

**Proof.** Immediate from the semantics of Extended Finite LTL.

The next lemma establishes that in a certain sense, \( X \) “dominates” \( \overline{X} \) in the context of conjunction.

**Lemma 10 (X Dominates \( \overline{X} \)).** The following holds for any Extended Finite LTL formulas \( \phi_1, \phi_2 \).

\[
(X \phi_1) \wedge (\overline{X} \phi_2) \equiv e X(\phi_1 \wedge \phi_2)
\]

**Proof.** Follows from the fact that if \( \pi \models (X \phi_1) \wedge (\overline{X} \phi_2) \) then \(|\pi| > 0\).

The final lemma is key to our ANF transformation result. It states that operators \( U \) and \( R \) may be rewritten using operators \( \wedge \), \( \vee \), \( X \) and \( \overline{X} \).

**Lemma 11 (Unrolling U and R).** The following holds for any Extended Finite LTL formulas \( \phi_1, \phi_2 \).

1. \( \phi_1 \ U \phi_2 \equiv e \ (\phi_1 \wedge X(\phi_1 \ U \phi_2)) \).
2. \( \phi_1 \ R \phi_2 \equiv e \ (\phi_1 \vee \overline{X}(\phi_1 \ R \phi_2)) \).

**Proof.** We prove Part 1 by showing that \( [\phi_1 \ U \phi_2]_e = [\phi_2 \wedge (\phi_1 \vee \overline{X}(\phi_1 \ R \phi_2))]_e \).
To prove Part 2, we can rely on the duality of $R$ and $U$ and Part 1. Again, it suffices to show that $\lbrack \phi_1 \text{ R } \phi_2 \rbrack_e = \lbrack \phi_2 \wedge (\phi_1 \text{ U } \phi_2) \rbrack_e$. We reason as follows.

$$
\lbrack \phi_1 \text{ R } \phi_2 \rbrack_e = \{ \pi \mid \pi \models_e \phi_1 \text{ R } \phi_2 \}
$$

1. $\phi \equiv_e \text{ anf}(\phi)$.
2. Suppose $\text{ anf}(\phi) = \bigvee C_i$. Then for each $C_i$ and each $\phi' \in \text{ nf}(C_i)$, $\phi'$ is a subformula of $\phi$.

Theorem 3 (Conversion to ANF). Let $\phi$ be an Extended Finite LTL formula in PNF. Then there exists a transformation $\text{ anf }$ such that $\text{ anf}(\phi)$ is in ANF and the following hold.

1. $\phi \equiv_e \text{ anf}(\phi)$.
2. Suppose $\text{ anf}(\phi) = \bigvee C_i$. Then for each $C_i$ and each $\phi' \in \text{ nf}(C_i)$, $\phi'$ is a subformula of $\phi$.

This theorem states that any PNF Extended LTL formula $\phi$ can be converted into ANF formula $\text{ anf}(\phi)$, and in such a way that each clause’s “next-state subformula” consists of a conjunction of subformulas of $\phi$. As any Extended LTL
To prove this theorem, we define several formula transformations that, when applied in sequence, yield a formula in ANF with the desired properties. The first transformation ensures that all occurrences of $U$ and $R$ are guarded in the resulting formula, in the following sense.

**Definition 11 (Guardedness).** Let $\phi$ be an Extended Finite LTL formula.

1. Let $\phi'$ be a subformula of $\phi$. Then $\phi'$ is guarded in $\phi$ iff for every occurrence of $\phi'$ in $\phi$ is within an occurrence of a subformula of $\phi$ of form $N\phi''$, where $N \in \{X, \overline{X}\}$.
2. Formula $\phi$ is guarded iff every subformula of $\phi$ of form $\phi_1 U \phi_2$ or $\phi_1 R \phi_2$ appears guarded in $\phi$.

As an example of the above definition, consider formula $\phi = (a U b) \land X(a U b)$. This formula is not guarded, because the left-most occurrence of $(a U b)$ does not appear within an occurrence of a subformula of form $X\phi''$. However, $\phi' = (b \lor (a \land X(a U b))) \land X(a U b)$ is guarded, and indeed $\phi' \equiv_e \phi$ due to Lemma 11(1).

We now define a transformation $gt$ on formulas; the intent of this transformation is that $gt(\phi)$ is guarded, and $gt(\phi) \equiv_e \phi$.

**Definition 12 (Guardedness Transformation).** Extended Finite LTL formula transformation $gt$ is defined inductively as follows.

$$
gt(\phi) = \begin{cases}
a & \text{if } \phi = a \\
\neg(gt(\phi')) & \text{if } \phi = \neg\phi' \\
gt(\phi_1) \land gt(\phi_2) & \text{if } \phi = \phi_1 \land \phi_2 \\
gt(\phi_1) \lor gt(\phi_2) & \text{if } \phi = \phi_1 \lor \phi_2 \\
\phi & \text{if } \phi = X\phi' \text{ or } \phi = \overline{X}\phi' \\
gt(\phi_2) \lor (gt(\phi_1) \land X\phi) & \text{if } \phi = \phi_1 U \phi_2 \\
gt(\phi_2) \land (gt(\phi_1) \lor \overline{X}\phi) & \text{if } \phi = \phi_1 R \phi_2 
\end{cases}
$$

We have the following.

**Lemma 12 (Properties of $gt$).** Let $\phi$ be an Extended Finite LTL formula. Then:

1. $gt(\phi)$ is guarded.
2. $gt(\phi) \equiv_e \phi$.
3. If $\phi$ is in PNF, then so is $gt(\phi)$.
4. Let $N\phi'$ be a subformula of $gt(\phi)$, where $N \in \{X, \overline{X}\}$. Then $\phi'$ is a subformula of $\phi$.

**Proof.** Immediate from the definition of $gt$ and Lemma 11.

The next transformation we describe converts guarded Extended Finite LTL formulas into pseudo-ANF.
**Definition 13 (Pseudo ANF).**

1. An ANF pseudo-literal has form \( a, \neg a \) or \( N_\phi \), where \( N \in \{X, X_\neg\} \) and \( \phi \in \Phi_{eAP} \).
2. An ANF pseudo-clause \( C \) has form \( \bigwedge \{\alpha_1, \ldots, \alpha_n\} \), \( n \geq 0 \), where each \( \alpha_i \) is an ANF pseudo-literal.
3. A formula is in Pseudo-ANF if it has form \( \bigvee \{C_1, \ldots, C_n\} \), \( n \geq 0 \), where each \( C_i \) is an ANF pseudo-clause.

Note that every literal is also an ANF pseudo-literal. Moreover, an ANF pseudo-clause differs from an ANF clause in that the former may have multiple (or no) instances of pseudo-literals of form \( N_\phi \), while the latter is required to have exactly one, of form \( N \bigwedge F \). We have the following.

**Lemma 13 (Conversion to Pseudo ANF).** Let \( \phi \) be a guarded Extended Finite LTL formula in PNF. Then there exists a formula \( \text{pa}(\phi) \) such that:

1. \( \text{pa}(\phi) \) is in Pseudo ANF.
2. \( \text{pa}(\phi) \equiv_e \phi \).

**Proof.** Transformation \( \text{pa} \) is a version of the classical DNF transformation for propositional formulas in which ANF pseudo-literals are treated as literals.

The final transformation, \( \text{an} \), converts formulas in pseudo-ANF into semantically equivalent formulas in ANF.

**Definition 14 (Pseudo-ANF to ANF Conversion).**

1. Let \( C = \bigwedge P \), where \( P = \{\alpha_1, \ldots, \alpha_n\} \) is a set of ANF pseudo-literals and \( n \geq 0 \), be an ANF pseudo-clause. Also let \( L(P) \) be the literals in \( P \) and \( N(P) = P - L(P) = \{N_1\phi_1, \ldots, N_i\phi_i\} \) for some \( 0 \leq i \leq n \), each \( N_i \in \{X, X_\neg\} \), be the non-literals in \( P \). Then \( \text{ct}(C) \) is defined as follows.

\[
\text{ct}(C) = \begin{cases} 
(\bigwedge L(P)) \land X(\bigwedge \{\phi_1, \ldots, \phi_i\}) & \text{if } N_j = X \text{ for some } 1 \leq j \leq i \\
(\bigwedge L(P)) \land X_\neg(\bigwedge \{\phi_1, \ldots, \phi_i\}) & \text{otherwise}
\end{cases}
\]

2. Let \( \phi = \bigvee\{C_1, \ldots, C_n\} \), \( n \geq 0 \), be an Extended Finite LTL formula in Pseudo ANF. Then transformation \( \text{an}(\phi) = \bigvee\{\text{ct}(C_1), \ldots, \text{ct}(C_n)\} \).

The next lemma and its corollary establish that \( \text{ct} \) and \( \text{an} \) convert pseudo-ANF clauses and formulas, respectively, into ANF clauses and formulas.

**Lemma 14 (Conversion from Pseudo ANF to ANF Clauses).** Let \( C \) be a pseudo-ANF clause. Then \( \text{ct}(C) \) is an ANF clause, and \( C \equiv_e \text{ct}(C) \).

**Proof.** Follows from Lemmas 9 and 10.

**Corollary 2 (Conversion from Pseudo ANF to ANF Formulas).** Let \( \phi \) be a pseudo-ANF formula. Then \( \text{an}(\phi) \) is in ANF, and \( \phi \equiv_e \text{an}(\phi) \).
Proof. Follows from Lemma 14.
We now have the machinery necessary to prove Theorem 3.
Proof (Theorem 3). Let $\phi$ be an Extended Finite LTL formula in PNF. We must show how to convert it into an ANF formula $\text{anf}(\phi) = \bigvee C_i$ such that $\phi \equiv_{e} \text{anf}(\phi)$, and such that for each $C_i$ and each $\phi' \in \text{nf}(C_i)$, $\phi'$ is a subformula of $\phi$.

We define $\text{anf}(\phi) = \text{an}(\text{pa}(\text{gt}(\phi)))$; obviously $\text{anf}(\phi)$ is in ANF. We now reason as follows.

\[
\begin{align*}
\phi & \equiv_{e} \text{gt}(\phi) & \text{Lemma 12; note gt(\phi) is PNF} \\
& \equiv_{e} \text{pa}(\text{gt}(\phi)) & \text{Lemma 13} \\
& \equiv_{e} \text{an}(\text{pa}(\text{gt}(\phi))) & \text{Corollary 2} \\
& \equiv_{e} \text{anf}(\phi) & \text{Definition of anf}
\end{align*}
\]

Thus $\text{anf}(\phi)$ is in ANF, and $\text{anf}(\phi) \equiv_{e} \phi$.

For the second part, we note that in the construction of $\text{anf}(\phi)$ we first compute $\text{gt}(\phi)$, which has the property that every subformula of form $\text{N}\phi''$ is such that $\phi''$ is a subformula of $\phi$. The definition of $\text{pa}$ guarantees that this property is preserved in $\text{pa}(\text{gt}(\phi))$. Finally, the definition of $\text{an}$ ensures the desired result.

Example 1 (Conversion to ANF). We close this section with an example showing how our conversion to ANF works. Consider $\phi = a U (b R c)$; we show how to compute $\text{an}(\text{pa}(\text{gt}(\phi)))$. Here is the result of $\text{gt}(\phi)$.

\[
\text{gt}(\phi) = \text{gt}(a \ U (b R c))
= \text{gt}(b R c) \lor (\text{gt}(a) \land X \phi)
= (\text{gt}(c) \land (\text{gt}(b) \lor X (b R c))) \lor (a \land X \phi)
= (c \land (b \lor X (b R c))) \lor (a \land X \phi)
\]

Note that this formula is guarded. We now consider $\text{pa}(\text{gt}(\phi))$.

\[
\text{pa}(\text{gt}(\phi)) = \text{pa}((c \land (b \lor X (b R c))) \lor (a \land X \phi))
= \text{pa}((c \land b) \lor (c \land X (b R c))) \lor (a \land X \phi))
= \bigvee \{c \land b, c \land X (b R c), a \land X \phi\}
\]

Note that two of the three clauses in $\text{pa}(\text{gt}(\phi))$ are already ANF clauses; the only that is not is $c \land b$. This leads to the following.

\[
\text{anf}(\phi) = \text{an}(\text{pa}(\text{gt}(\phi)))
= \text{an}(\bigvee \{c \land b, c \land X (b R c), a \land X \phi\})
= \bigvee \{\text{ct}(c \land b), \text{ct}(c \land X (b R c)), \text{ct}(a \land X \phi)\}
= \bigvee \{c \land b \land \text{true}, c \land X (b R c), a \land X \phi\}
\]

Note that this formula is in ANF. Also note that $\text{ct}(c \land b) = c \land b \land \text{true}$ due to the fact that in pseudo-ANF clause $c \land b$ has no next-state pseudo-literals. The definition of $\text{ct}$ ensures that $X \land \emptyset = \text{true}$ is added to ensure that the result satisfies the syntactic requirements of being an ANF clause.
5 A Tableau Construction for Finite LTL

In this section we show how Finite LTL formulas may be converted into finite-state automata whose languages consist of exactly the sequences making the associated formula true. Based on Lemma 7 we know that any Finite LTL formula can be converted into an Extended Finite LTL formula in PNF, so in the sequel we show how to build finite automata from Extended Finite LTL formulas in PNF. We begin by recalling the definitions of non-deterministic finite-state automata.

Definition 15 (Non-deterministic Finite Automata (NFA)).

1. A non-deterministic finite automaton (NFA) is a tuple \((Q, \Sigma, q_I, \delta, F)\), where:
   - \(Q\) is a finite set of states;
   - \(\Sigma\) is a finite non-empty set of alphabet symbols;
   - \(q_I \in Q\) is the start state;
   - \(\delta \subseteq Q \times \Sigma \times Q\) is the transition relation; and
   - \(F \subseteq Q\) is the set of accepting states.

2. Let \(M = (Q, \Sigma, q_I, \delta, F)\) be a NFA, let \(q \in Q\), and let \(w \in \Sigma^*\). Then \(q\) accepts \(w\) in \(M\) iff one of the following hold.
   - \(w = \varepsilon\) and \(q \in F\)
   - \(w = \sigma w'\) for some \(\sigma \in \Sigma, w' \in \Sigma^*\) and there exists \((q, \sigma, q') \in \delta\) such that \(q'\) accepts \(w'\) in \(M\).

3. Let \(M = (Q, \Sigma, q_I, \delta, F)\) be a NFA. Then \(L(M)\), the language of \(M\), is
   \[L(M) = \{ w \in \Sigma^* \mid q_I \text{ accepts } w \text{ in } M \}.\]

We now state the theorem we will prove in the rest of this section.

Theorem 4 (NFAs from Extended LTL Formulas). Let \(\phi \in \Phi^{AP}_e\) be in PNF. Then there is a NFA \(M_\phi\) such that \(L(M_\phi) = [\phi]_e\).

5.1 The Construction

In this section we describe our construction for building NFA \(M_\phi\) from PNF Extended Finite LTL formula \(\phi\). We have been referring to this construction as a tableau construction, and indeed it makes essential use of identities, such as those in Lemmas 5–11, that also underpin classical tableau constructions. However, because of our use of ANF we are able to avoid other aspects of tableau constructions, such as the need for maximally consistent subsets as automaton states.

In what follows we use \(S(\phi)\) to refer to the set of (not necessarily proper) subformulas of \(\phi\). States in \(M_\phi\) will be associated with subsets of \(S(\phi)\), and defining accepting states will require checking if \(\varepsilon \models_\varepsilon \phi'\) for arbitrary \(\phi' \in S(\phi)\). The next lemma establishes that this latter check can be computed on the basis of the syntactic structure of \(\phi'\).
Lemma 15 (Empty-sequence Check). Let $\phi \in \Phi_{\text{AP}}^e$ be in PNF. Then $\varepsilon \models_e \phi$ iff one of the following hold.

1. $\phi = \neg a$ for some $a \in \text{AP}$
2. $\phi = \phi_1 \land \phi_2$, $\varepsilon \models_e \phi_1$, and $\varepsilon \models_e \phi_2$
3. $\phi = \phi_1 \lor \phi_2$ and either $\varepsilon \models_e \phi_1$ or $\varepsilon \models_e \phi_2$
4. $\phi = X \phi'$
5. $\phi = \phi_1 R \phi_2$ and $\varepsilon \models_e \phi_2$

Proof. Immediate from the definition of $\models_e$.

We now define our construction for $M_\phi$.

Definition 16 (NFA $M_\phi$). Let $\phi \in \Phi_{\text{AP}}^e$ be in PNF. Then we define NFA $M_\phi = (Q_\phi, \Sigma_{\text{AP}}, q_1, \delta_\phi, F_\phi)$ as follows.

- $Q_\phi = 2^{S(\phi)}$
- $\Sigma_{\text{AP}} = 2^{\text{AP}}$
- $q_1 = \{\phi\}$
- Let $q, q' \in Q_\phi$ (so $q, q' \subseteq S(\phi)$) and $A \in \Sigma_{\text{AP}}$ (so $A \subseteq \text{AP}$). Also let $\text{anf}(\bigwedge q) = \bigvee\{C_1, \ldots, C_n\}$ be the ANF conversion of $\bigwedge q$. Then $(q, A, q') \in \delta$ iff there exists $C_i$ such that:
  - $A \models_p \bigwedge \text{lits}(C_i)$; and
  - $q' = \text{nf}(C_i)$.
- $F_\phi = \{q \in Q_\phi \mid \varepsilon \models_e \bigwedge q\}$

Theorem 4 states that the above construction is correct. In the rest of this section, we will prove this claim. We first establish the following useful lemma.

Lemma 16 (Well-Formedness of $M_\phi$). Let $\phi \in \Phi_{\text{AP}}^e$ be in PNF, and let $M_\phi = (Q_\phi, \Sigma_{\text{AP}}, q_1, \delta_\phi, F_\phi)$. Fix arbitrary $q \in Q_\phi$, and let

$$\text{anf}(\bigwedge q) = \bigvee C_i,$$

Then for each $C_i$, $\text{nf}(C_i) \in Q_\phi$.

Proof. Follows from Lemma 12(4) and the fact that every subformula of every $\phi' \in \text{nf}(C_i)$ of form $N \phi'', \phi_1 U \phi_2'$ or $\phi_1 R \phi_2'$ is also a subformula of $\phi$.

This lemma in effect says that every clause occurring in $\text{anf}(\bigwedge q)$ (recall $q$ is a set of subformulas of $\phi$) gives rise to transitions in $M_\phi$, because the “next-state” formulas in such a clause involve subformulas of $\phi$.

Proof (Theorem 4). We now prove Theorem 4 as follows. Let $\phi \in \Phi_{\text{AP}}^e$ and $M_\phi = (Q_\phi, \Sigma_{\text{AP}}, q_1, \delta_\phi, F_\phi)$. We recall that $\Sigma_{\text{AP}} = 2^{\text{AP}}$, and thus $(\Sigma_{\text{AP}})^* = (2^{\text{AP}})^*$. Thus, the words accepted by $M_\phi$ come form the same set as the sequences to interpret Extended Finite LTL formulas. To emphasize this connection, we use $A \in \Sigma_{\text{AP}}$ and $\pi \in (\Sigma_{\text{AP}})^*$ in the following. We will in fact prove a stronger
result: for every $\pi \in (\Sigma_{\text{AP}})^*$ and $q \in Q$, $q$ accepts $\pi$ in $M_\phi$ iff $\pi \models_e \bigwedge q$. The desired result then follows from the fact that this statement holds in particular for the start state, $q_{I,\phi}$, that $\bigwedge q_{I,\phi} = \phi$, and that as a result, $L(M_\phi) = [\phi]_e$.

The proof proceeds by induction on $\pi$. For the base case, assume that $\pi = \varepsilon$ and fix $q \in Q$. We reason as follows.

$$q \text{ accepts } \varepsilon \text{ in } M \text{ iff } q \in F_\phi \quad \text{(Definition of acceptance)}$$
$$\quad \text{iff } \varepsilon \models_e \bigwedge q \quad \text{(Definition of } F_\phi)$$

In the induction case, assume $\pi = A\pi'$ for some $A \in \Sigma_{\text{AP}}$ (so $A \subseteq \text{AP}$) and $\pi' \in (\Sigma_{\text{AP}})^*$. The induction hypothesis states for any $q' \in Q$, $q'$ accepts $\pi'$ in $M_\phi$ iff $\pi' \models_e \bigwedge q'$ (recall $q' \subseteq S(\phi)$). Now fix $q \in Q$; we must prove that $q$ accepts $\pi$ in $M_\phi$ iff $\pi \models_e \bigwedge q$. We reason as follows.

$$\pi \models_e \bigwedge q$$
$$\text{iff } A\pi' \models_e \bigwedge q \quad \text{iff } \pi = A\pi'$$
$$\text{iff } A\pi' \models_e \varepsilon \bigwedge q \quad \text{(Theorem 3)}$$
$$\text{iff } A\pi' \models_e \varepsilon \bigvee C_i \quad \text{ iff } \pi' \models_e \bigvee C_i \text{ in ANF}$$
$$\text{iff } A\pi' \models_e \varepsilon \bigvee C_i \text{ some } i \quad \text{ (Lemma 8(2))}$$
$$\text{iff } A \models_p \bigwedge L \text{ and } \pi' \models_e \bigwedge \mathcal{F} \quad \text{ (Lemma 8(1), } L = \text{lits}(C_i), \mathcal{F} = \text{nf}(C))$$
$$\text{iff } A \models_p \bigwedge L \text{ and } \pi' \models_e \bigwedge q' \text{ some } q' \in Q_\phi \quad \text{ (Lemma 16)}$$
$$\text{iff } (q, A, q') \in \delta_\phi \text{ and } \pi' \models_e \bigwedge q' \text{ some } q' \in Q_\phi \quad \text{ (Definition of } \delta_\phi)$$
$$\text{iff } (q, A, q') \in \delta_\phi \text{ and } q' \text{ accepts } w' \text{ in } M \quad \text{ (Induction hypothesis)}$$
$$\text{iff } q \text{ accepts } w \text{ in } M \quad \text{ (Definition 15(2))}$$

5.2 Discussion of Construction $M_\phi$

We now comment of some aspects of $M_\phi$, both from the standpoint of its complexity but also in terms of heuristics for improving the construction in practice.

Size of $|M_\phi|$. The key drivers for the size of $|M_\phi|$ are the size of its state space, $Q_\phi$, and its transition relation, $\delta_\phi$. The next theorem characterizes these.

**Theorem 5 (Bounds on Size of $M_\phi$).** Let $\phi \in \Phi^\text{AP}_e$ be in PNF, and let $M_\phi = (Q_\phi, \Sigma_{\text{AP}}, q_{I,\phi}, \delta_\phi, F_\phi)$. Then we have the following.

1. $|Q_\phi| \leq 2^{|\phi|}$.
2. $|\delta_\phi| \leq 4^{|\phi|} \cdot 2^{|\text{AP}|}$.

**Proof.** For the first statement, we note that there is a state in $M_\phi$ for each subset of $S(\phi)$, and that there are at most $2^{|\phi|}$ such subsets. The second follows from the fact that each pair of states can have at most $2^{|\text{AP}|}$ transitions between them.

It is worth noting that in the above result, the bound on the number of states is tight: it is $2^{|\phi|}$, not e.g. $2^{O(|\phi|)}$, which some tableau constructions for LTL yield. Also note that if $\phi$ contains multiple instances of the same subformula, then $|S(\phi)| < |\phi|$; this explains the inequality in Statement (1).
Optimizing $M_\phi$. The size results in Theorem 5 are consistent with other tableau constructions; they are in the worst case exponential in the size of the formulas for which automata are constructed. This worst-case behavior cannot be avoided over-all, but it can often be mitigated heuristically. In the remainder of this section we consider different methods for doing so.

**On-the-fly Construction of $Q_\phi$.** The construction in Definition 16 may be seen as pre-computing all possible states of $M_\phi$. In practice many of these states are unreachable, and adding them to $Q_\phi$ and computing their transitions is unnecessary work. One method for avoiding this work is to construct $Q_\phi$ in a demand-driven, or on-the-fly manner. Specifically, one starts with the state $q_{I,\phi}$ and adds this to $Q_\phi$. Then one repeatedly does the following: select a state $q$ in the current $Q_\phi$ whose transitions have not been computed, compute $q$’s transitions, adding states into $Q_\phi$ so that each transition has a target in $Q_\phi$. Stop when there are no states in $Q_\phi$ whose transitions have not been computed. The result of this strategy is that only states reachable from $q_{I,\phi}$ will be added into $Q_\phi$.

**Symbolic Representation of Transitions.** In Definition 16 transition labels are represented concretely, as sets of atomic propositions. One can instead allow transition labels that are symbolic: these labels have form $\gamma \in \Gamma_\text{expr}$ for some propositional Extended LTL formula $\gamma$. A transition labeled by such a gamma can be seen as summarizing all transitions in $M_\phi$ labeled by $A \subseteq \mathcal{AP}$ such that $A \models_p \gamma$. The construction given in Definition 16 suggests an immediate method for doing this: rather than labeling transitions by $A \subseteq \mathcal{AP}$ such that $A \models_p \bigwedge \text{lits}(C_i)$, instead label a single transition by $\text{lits}(C_i)$.

**Relaxation of ANF.** Our definition of ANF says that a formula is in ANF iff it has form $\bigvee C_i$, where each clause $C_i$ has form $(\bigwedge L) \land N(\bigwedge F)$. The method we give for converting formulas into ANF involves the use of a routine for converting formulas into DNF, which can itself be exponential. We adopted this mechanism for ease of exposition, and also because in the worst case this exponential overhead is unavoidable. However, requiring the propositional parts of clauses to be of form $\bigwedge L$, where $L$ consists only of literals, is unnecessarily restrictive: all that is needed for the construction of $M_\phi$ is to require clauses to be of form $\gamma \land N(\bigwedge F)$, where $\gamma \in \Gamma_\text{expr}$ is a proposition formula in Extended Finite LTL. Relaxing ANF in this manner eliminates the need for full DNF calculations in general, and can lead to time and space savings when transitions are being represented symbolically.

### 6 Implementation and Empirical Results

We have implemented our tableau construction as a C++ package. The user specifies a formula $\phi \in \Phi_\text{expr}$ and the corresponding NFA $M_\phi$ is constructed as defined in Definition 16. For convenience, we also support the boolean implication operator $\rightarrow$ by immediately performing the syntactic rewrite $\phi \rightarrow \psi \equiv \neg \phi \lor \psi$ upon parsing the formula. Our current implementation utilizes an on-the-fly construction of $Q_\phi$ such that only states reachable from the initial state
are constructed and recursed upon. We also support the option to represent transitions symbolically by bundling edges together as a single transition whose label is the disjunction of all merged edge labels.

We used the Spot [6] platform (v.2.8.1) to handle the parsing of input formulas as well as the syntactic representation of the constructed output graph. To support proper parsing and construction of a Finite LTL formula, we added support for the $\mathbf{X}$ (Weak Next) operator. Additionally, several Spot-provided automatic formula rewrites are based on standard LTL identities and do not hold under finite semantics; these were disabled (such as $\mathbf{X} \mathbf{true} \equiv \mathbf{true}$).

We performed our tableau construction on the benchmark set of 184 standard LTL formulas (92 formulas and their negations) used by Duret-Lutz [5]. As our semantics for Finite LTL differs from traditional LTL, a direct comparison of times needed for automaton construction are not appropriate. We imposed a 60 minute time-out for each formula, marked by "t/o" when applicable (this only occurred for formulas 24 and 109). We also report formula complexity ($|\phi|$), which is the number of subformulas in $\phi$. Experiments were carried out on a single machine with an Intel Core i5-6600K (4 cores), with 32 GB RAM and a 64-bit version of GNU/Linux.

Figures 2 and 3 contain the results of our experiments. For each formula in the testbed, four pieces of data are reported: the size of the formula, the number of states in the resulting NFA, the number of transitions in the NFA, and the time needed to build the automaton. In most cases the construction time is negligible, although in two cases — Formulas 24 and 109 — our tool did not terminate before the 60-minute time-out we imposed. The reasons for this behavior are under further investigation. We have presented a summarized set of the results in Figure 1. Formulas are ordered by complexity.

7 Conclusion

This paper has defined the logic Finite LTL, which uses the syntax of LTL but employs a semantics based on finite, rather than infinite, state sequences. It also has provided a tableau-inspired construction for converting formulas into non-deterministic finite automata whose languages consist of exactly the sequences making the corresponding formulas true. In the construction states are equivalent to subsets of subformulas of the input formula, and accepting states are defined as those where all contained subformulas satisfy the empty sequence. We have also observed that the check for satisfaction by the empty sequence can be done purely syntactically on the basis of the structure of the formula. We also have described a prototype implementation of the approach and given empirical results on an existing benchmark for standard LTL. Our methodology, while heavily inspired by the standard (infinite semantics) LTL community, transforms a Finite LTL formula directly into an NFA, and does not rely upon any intermediate representation from the standard LTL realm. The continued development of such “native” approaches will allow us to appropriately and accurately reason over domains with finite sequences.
Fig. 1: Summarized results of experiments. Formulas are ordered by complexity. (a) Formula complexity vs nodes in automaton $M_\phi$. (b) Formula complexity vs edges in automaton $M_\phi$. (c) Formula complexity vs nodes + edges in automaton $M_\phi$. (d) Formula complexity vs computation time.
Fig. 2: Experimental results: Formulas 0–91.

| ID | φ | states | edges | time(s) |
|----|---|--------|-------|---------|
| 0  | 2 | 1      | 0.00311377 |
| 1  | 3 | 2      | 0.00326687 |
| 2  | 5 | 4      | 0.00331742 |
| 3  | 6 | 5      | 0.00342465 |
| 4  | 5 | 2      | 0.00342077 |
| 5  | 6 | 3      | 0.00344014 |
| 6  | 7 | 4      | 0.00357705 |
| 7  | 8 | 5      | 0.00369425 |
| 8  | 8 | 4      | 0.00449891 |
| 9  | 9 | 5      | 0.00369425 |
| 10 | 1 | 2      | 0.00306629 |
| 11 | 7 | 3      | 0.00317363 |
| 12 | 8 | 4      | 0.00357145 |
| 13 | 6 | 5      | 0.00347468 |
| 14 | 7 | 5      | 0.00367739 |
| 15 | 10| 4      | 0.00477515 |
| 16 | 11| 6      | 0.0152335  |
| 17 | 12| 8      | 0.00672642 |
| 18 | 13| 8      | 0.00995686 |
| 19 | 14| 9      | 0.0053067  |
| 20 | 15| 7      | 0.00367739 |
| 21 | 24| 8      | 0.00672642 |
| 22 | 12| 7      | 0.00311377 |
| 23 | 8 | 4      | 0.00305685 |
| 24 | 4 | 2      | 0.00308841 |
| 25 | 2 | 3      | 0.00311377 |
| 26 | 5 | 2      | 0.00340377 |
| 27 | 10| 6      | 0.00374102 |
| 28 | 11| 7      | 0.00477528 |
| 29 | 12| 8      | 0.00477528 |
| 30 | 13| 7      | 0.00477528 |
| 31 | 14| 9      | 0.00477528 |
| 32 | 15| 10     | 0.00477528 |
| 33 | 16| 3      | 0.00311377 |
| 34 | 17| 12     | 0.00311377 |
| 35 | 18| 4      | 0.00311377 |
| 36 | 19| 6      | 0.00311377 |
| 37 | 20| 8      | 0.00311377 |
| 38 | 21| 4      | 0.00311377 |
| 39 | 22| 5      | 0.00311377 |
| 40 | 23| 6      | 0.00311377 |
| 41 | 24| 7      | 0.00311377 |
| 42 | 25| 8      | 0.00311377 |
| 43 | 26| 9      | 0.00311377 |
| 44 | 27| 10     | 0.00311377 |
| 45 | 28| 11     | 0.00311377 |
| ID | $\varphi$ | states | edges | time(s) | ID | $\varphi$ | states | edges | time(s) |
|----|---------|-------|-------|--------|----|---------|-------|-------|--------|
| 92 | 15     | 16    | 93    | 0.0744316 | 138 | 4      | 2     | 4     | 0.00357135 |
| 93 | 16     | 11    | 43    | 0.0076845  | 139 | 5      | 2     | 3     | 0.00330855 |
| 94 | 20     | 24    | 267   | 2.07052   | 140 | 6      | 3     | 5     | 0.00369162 |
| 95 | 21     | 21    | 117   | 0.0196223  | 141 | 7      | 2     | 4     | 0.00365836 |
| 96 | 9      | 4     | 13    | 0.0042581  | 142 | 7      | 4     | 12    | 0.00345923 |
| 97 | 10     | 3     | 7     | 0.0033977  | 143 | 8      | 5     | 11    | 0.00406279 |
| 98 | 16     | 10    | 29    | 0.00519818 | 144 | 4      | 4     | 16    | 0.00372749 |
| 99 | 17     | 11    | 42    | 0.0119839  | 145 | 5      | 3     | 5     | 0.00332615 |
| 100| 11     | 3     | 7     | 0.00400027 | 146 | 4      | 4     | 8     | 0.00323916 |
| 101| 12     | 4     | 10    | 0.00386793 | 147 | 5      | 3     | 4     | 0.00342545 |
| 102| 18     | 16    | 93    | 0.0812396  | 148 | 8      | 33    | 312   | 0.0902802 |
| 103| 19     | 11    | 43    | 0.0129342  | 149 | 9      | 12    | 52    | 0.00509586 |
| 104| 25     | 24    | 267   | 2.4522    | 150 | 13     | 12    | 52    | 0.0218034 |
| 105| 26     | 21    | 162   | 0.134175  | 151 | 14     | 25    | 132   | 0.00808484 |
| 106| 19     | 6     | 20    | 0.00432272 | 152 | 13     | 19    | 63    | 0.0280241 |
| 107| 20     | 64    | 740   | 0.289294  | 153 | 14     | 25    | 150   | 0.00879734 |
| 108| 25     | 9     | 36    | 0.0043379  | 154 | 10     | 4     | 8     | 0.00443243 |
| 109| 26     | -     | -     | t/o       | 155 | 11     | 6     | 13    | 0.00404387 |
| 110| 1      | 2     | 3     | 0.00336265 | 156 | 6      | 2     | 2     | 0.00348334 |
| 111| 2      | 2     | 3     | 0.0030478  | 157 | 7      | 4     | 6     | 0.00364548 |
| 112| 2      | 3     | 6     | 0.00385779 | 158 | 2      | 3     | 5     | 0.00326534 |
| 113| 3      | 4     | 11    | 0.00351073 | 159 | 3      | 3     | 5     | 0.00310239 |
| 114| 5      | 4     | 11    | 0.00418075 | 160 | 3      | 2     | 3     | 0.00371879 |
| 115| 6      | 3     | 6     | 0.00337304 | 161 | 4      | 4     | 11    | 0.00377196 |
| 116| 6      | 5     | 11    | 0.00346558 | 162 | 4      | 3     | 5     | 0.00368115 |
| 117| 7      | 5     | 16    | 0.00388114 | 163 | 5      | 4     | 11    | 0.00367594 |
| 118| 3      | 4     | 10    | 0.00348359 | 164 | 14     | 7     | 13    | 0.00375197 |
| 119| 4      | 4     | 11    | 0.00378654 | 165 | 15     | 20    | 90    | 0.065343  |
| 120| 2      | 4     | 6     | 0.00404544 | 166 | 4      | 2     | 3     | 0.00326375 |
| 121| 3      | 3     | 6     | 0.00327689 | 167 | 5      | 2     | 4     | 0.0031863 |
| 122| 9      | 4     | 6     | 0.00387858 | 168 | 6      | 4     | 6     | 0.00379064 |
| 123| 10     | 6     | 13    | 0.00336561 | 169 | 7      | 4     | 10    | 0.00358385 |
| 124| 6      | 4     | 12    | 0.00392436 | 170 | 4      | 3     | 5     | 0.00364375 |
| 125| 7      | 5     | 11    | 0.00403966 | 171 | 5      | 2     | 4     | 0.00378301 |
| 126| 13     | 9     | 32    | 0.00489335 | 172 | 5      | 5     | 10    | 0.00338291 |
| 127| 14     | 17    | 55    | 0.00644804 | 173 | 6      | 5     | 20    | 0.0040754 |
| 128| 2      | 2     | 3     | 0.00396178 | 174 | 4      | 2     | 4     | 0.00345263 |
| 129| 3      | 2     | 3     | 0.00344865 | 175 | 5      | 3     | 6     | 0.00362208 |
| 130| 8      | 5     | 9     | 0.00355222 | 176 | 10     | 5     | 9     | 0.00343033 |
| 131| 9      | 7     | 10    | 0.00391759 | 177 | 11     | 4     | 7     | 0.0040377 |
| 132| 9      | 4     | 8     | 0.00336949 | 178 | 11     | 32    | 1024  | 0.109338  |
| 133| 10     | 7     | 10    | 0.0040027  | 179 | 12     | 11    | 25    | 0.00371685 |
| 134| 14     | 7     | 26    | 0.00616796 | 180 | 7      | 8     | 23    | 0.00456058 |
| 135| 15     | 9     | 15    | 0.00367025 | 181 | 8      | 65    | 1305  | 0.150071  |
| 136| 13     | 9     | 28    | 0.00601231 | 182 | 7      | 8     | 28    | 0.006194  |
| 137| 14     | 9     | 15    | 0.00418518 | 183 | 8      | 9     | 50    | 0.00451803 |

Fig. 3: Experimental results: Formulas 92–183.
References

1. William Chan. Temporal-logic queries. In E. Allen Emerson and A. Prasad Sistla, editors, *Computer Aided Verification, 12th International Conference, CAV 2000, Chicago, IL, USA, July 15-19, 2000, Proceedings*, volume 1855 of *Lecture Notes in Computer Science*, pages 450–463. Springer, 2000.

2. Jean-Michel Coulveur. On-the-fly verification of linear temporal logic. In Jeannette M. Wing, Jim Woodcock, and Jim Davies, editors, *FM’99 — Formal Methods*, pages 253–271, Berlin, Heidelberg, 1999. Springer Berlin Heidelberg.

3. Giuseppe De Giacomo, Riccardo De Masellis, and Marco Montali. Reasoning on ltl on finite traces: Insensitivity to infiniteness. In *Proceedings of the Twenty-Eighth AAAI Conference on Artificial Intelligence*, AAAI’14, pages 1027–1033. AAAI Press, 2014.

4. Giuseppe De Giacomo and Moshe Y. Vardi. Linear temporal logic and linear dynamic logic on finite traces. In *Proceedings of the Twenty-Third International Joint Conference on Artificial Intelligence*, IJCAI’13, pages 854–860. AAAI Press, 2013.

5. Alexandre Duret-Lutz. LTL translation improvements in Spot 1.0. *International Journal on Critical Computer-Based Systems*, 5(1/2):31–54, March 2014.

6. Alexandre Duret-Lutz, Alexandre Lewkowicz, Amaury Fauchille, Thibaud Michaud, Etienne Renault, and Laurent Xu. Spot 2.0 — a framework for LTL and \(\omega\)-automata manipulation. In *Proceedings of the 14th International Symposium on Automated Technology for Verification and Analysis (ATVA’16)*, volume 9938 of *Lecture Notes in Computer Science*, pages 122–129. Springer, October 2016.

7. Valeria Fionda and Gianluigi Greco. The complexity of ltl on finite traces: Hard and easy fragments. In *Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence*, AAAI’16, pages 971–977. AAAI Press, 2016.

8. Jianwen Li, Moshe Y. Vardi, and Kristin Y. Rozier. Satisfiability checking for mission-time ltl. In Isil Dillig and Serdar Tasiran, editors, *Computer Aided Verification*, pages 3–22, Cham, 2019. Springer International Publishing.

9. Jianwen Li, Lijun Zhang, Geguang Pu, Moshe Y. Vardi, and Jifeng He. Ltlf satisfiability checking. *CoRR*, abs/1403.1666, 2014.

10. A. Pnueli. The temporal logic of programs. In 18th Annual Symposium on Foundations of Computer Science (sfcs 1977), pages 46–57, Oct 1977.

11. Grigore Roșu. Finite-trace linear temporal logic: Coinductive completeness. In Yliès Falcone and César Sánchez, editors, *Runtime Verification*, pages 333–350, Cham, 2016. Springer International Publishing.

12. Moshe Y Vardi and Pierre Wolper. An automata-theoretic approach to automatic program verification. In *Proceedings of the First Symposium on Logic in Computer Science*, pages 322–331. IEEE Computer Society, 1986.