We discuss and compare different approaches to include gluon transverse momenta for heavy quark-antiquark pair and meson production. The results are illustrated with the help of different unintegrated gluon distributions (UGDF) from the literature. We compare results obtained with on-shell and off-shell matrix elements and kinematics. The results are compared with recent experimental results of the CDF collaboration.

1. Introduction

The heavy quark-antiquark production in hadroproduction is known as one of the crucial tests of conventional gluon distributions within a standard factorization approach. At high energies one tests gluon distributions at low values of longitudinal momentum fraction. Standard collinear approach does not include transverse momenta of initial gluons, the method to include transverse momenta is $k_t$ - factorization approach\cite{1,2,3}. In the first step of our approach the single particle spectra of charmed quarks and antiquarks are obtained assuming gluon-gluon fusion\cite{4}. Different unintegrated gluon distributions from the literature are used\cite{5,6,7,8,9}. To obtain the single particle spectra of meson from those of quarks/antiquarks a standard hadronization procedure with Peterson fragmentation function is applied. Conclusions about unintegrated gluon distributions are drawn.
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2. Heavy quark production

Let us consider the reaction $h_1 + h_2 \rightarrow Q + \bar{Q} + X$, where $Q$ and $\bar{Q}$ are heavy quark and heavy antiquark, respectively.

In the leading-order approximation within collinear approach the triple-differential cross section in rapidity of $Q (y_1)$, in rapidity of $\bar{Q} (y_2)$ and transverse momentum of one of them ($p_t$) can be written as

$$\frac{d\sigma}{dy_1 dy_2 dp_t^2} = \frac{1}{16\pi^2 s^2} \sum_{i,j} x_1 p_i(x_1, \mu^2) x_2 p_j(x_2, \mu^2) |M_{ij}|^2 .$$  \hspace{1cm} (1)

Above $p_i(x_1, \mu^2)$ and $p_j(x_2, \mu^2)$ are familiar (integrated) parton distributions in hadron $h_1$ and $h_2$, respectively.

The parton distributions are evaluated at: $x_1 = \frac{m_{t_1}}{\sqrt{s}} (\exp(y_1) + \exp(y_2))$, $x_2 = \frac{m_{t_2}}{\sqrt{s}} (\exp(-y_1) + \exp(-y_2))$. The formulae for matrix element squared averaged over initial and summed over final spin polarizations can be found e.g. in Ref.\textsuperscript{10}.

If one allows for transverse momenta of the initial partons, the transverse momenta of the final $Q$ and $\bar{Q}$ no longer cancel. Formula (1) can be easily generalized if one allows for the initial parton transverse momenta. Then

$$\frac{d\sigma}{dy_1 dy_2 dp_{1,t}^2 dp_{2,t}^2} = \sum_{i,j} \int \frac{d^2 \kappa_{1,t}}{\pi^2} \frac{d^2 \kappa_{2,t}}{\pi^2} \frac{1}{16\pi^2 (x_1 x_2 s)^2} |M_{ij}|^2 \delta^2 (\vec{\kappa}_{1,t} + \vec{\kappa}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t}) f_i(x_1, \kappa_{1,t}^2) f_j(x_2, \kappa_{2,t}^2) ,$$ \hspace{1cm} (2)

where now $f_i(x_1, \kappa_{1,t}^2)$ and $f_j(x_2, \kappa_{2,t}^2)$ are so-called unintegrated parton distributions. The extra integration is over transverse momenta of the initial partons. The two extra factors $1/\pi$ attached to the integration over $d^2 \kappa_{1,t}$ and $d^2 \kappa_{2,t}$ instead over $dk_{1,t}^2$ and $dk_{2,t}^2$ as in the conventional relation between unintegrated and integrated parton distributions. The two-dimensional delta function assures momentum conservation. Now the unintegrated parton distributions must be evaluated at: $x_1 = \frac{m_{k_{1,t}}}{\sqrt{s}} \exp(y_1) + \frac{m_{k_{2,t}}}{\sqrt{s}} \exp(y_2)$, $x_2 = \frac{m_{k_{1,t}}}{\sqrt{s}} \exp(-y_1) + \frac{m_{k_{2,t}}}{\sqrt{s}} \exp(-y_2)$. In general,

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the matrix element must be calculated for initial off-shell partons. The corresponding formulae for initial gluons were calculated in \cite{12,3}. In the present paper for illustration we shall compare results obtained for both on-shell and off-shell matrix elements.

In Fig. 2 we collected results for $d\sigma/dp_t$ obtained with different unintegrated gluon distributions from the literature. In this case consequently the off-shell matrix element and off-shell kinematics were used. The GBW gluon distribution leads to a much smaller cross section. The KL gluon distribution produces the hardest $p_t$ spectrum. Rather different slopes in transverse momentum of $c$ (or $\bar{c}$) are obtained for different UGDFs. This differences survive after convoluting the inclusive quark/antiquark spectra with fragmentation functions. Thus, in principle, precise distribution in transverse momentum of charmed mesons should be useful to select a "correct" model of UGDF. A detailed comparison with the experimental data requires, however, a detailed knowledge of fragmentation functions.

The inclusive spectra are not the best observables to test UGDF \cite{11}. Let us come now to correlations between charm quark and charm antiquark.

In Fig.3 we compare results for different unintegrated gluon distribution from the literature. Quite different results are obtained for different UGDFs. The non-perturbative GBW glue leads to strong azimuthal correlations between $c$ and $\bar{c}$. In contrast, BFKL dynamics leads to strong decorrelations of azimuthal angles of...
charm and anticharm quarks. The saturation idea inspired KL distribution leads to an local enhancement for $\phi_{c\bar{c}}$ which is probably due to simplifications made in parametrizing the UGDF. In the last case there is a sizeable difference between the result obtained with on-shell (left panel) and off-shell (right panel) matrix elements. All this is due to an interplay of the matrix element and the unintegrated gluon distributions.

3. From unintegrated parton distributions to meson production

The inclusive distributions of hadrons are obtained through a convolution of inclusive distributions of heavy quarks/antiquarks and $Q \rightarrow h$ fragmentation functions

$$
\frac{d\sigma(y_h, p_{t,h})}{dy_h d^2p_{t,h}} \approx \int_1^1 \frac{dz}{z^2} D_{Q\rightarrow h}(z) \left. \frac{d\sigma_{gg\rightarrow Q}(y_Q, p_{t,Q})}{dy_Q d^2p_{t,Q}} \right|_{y_Q = y_h, p_{t,h}/z}.
\tag{3}
$$

In the present paper we have used so-called Peterson fragmentation functions with parameters from last issue of PDG (Particle Data Group). We have neglected a possibility that charmed mesons are produced from light quarks and/or gluons. This approximation should be better for heavier quarks/mesons.

In the present analysis we show dependence of the total cross section on transverse momenta for $D^{*+}$ production

$$
\frac{d\sigma(p_t)}{dp_t} = \int_{-1}^{1} dy \frac{d\sigma(y, p_t)}{dy dp_t} \approx 2 \frac{d\sigma(y = 0, p_t)}{dy dp_t}.
\tag{4}
$$
Fig. 4. Inclusive $d\sigma/dp_t$ for $D^{*+}$ production at $W = 1.96$ TeV for different UGDF. The results with $\alpha_s = \alpha_s(4m_c^2)$ is in the left panel and the results with $\alpha_s = \alpha_s(\kappa_1^2 t)$ or $\alpha_s(\kappa_2^2 t)$ in the right panel.

In Fig. 4 the theoretical results are compared with experimental data of the CDF collaboration\textsuperscript{13}. We collected results obtained with different unintegrated gluon distributions from the literature.

We have made calculations for two different choices of renormalization scale. The results obtained with $\alpha_s(4m_c^2)$ (left panel) are below the experimental data, while results with $\alpha_s = \alpha_s(\kappa_1^2 t)$ or $\alpha_s(\kappa_2^2 t)$ (see\textsuperscript{12}) better describe the data. There is also a dependence on UGDF used in the calculation. Even if the second choice is made, there seem to be a deficit of the cross section.

It is not clear to us if the deficit is due to the omission of $q \rightarrow D^*$ or $g \rightarrow D^*$ fragmentation, or due to higher order terms in heavy quark production.

Similar situation is for $B^+$ production, as shown in Fig. 5.

4. Summary

Inclusive cross section for heavy quark - antiquark and heavy mesons in proton -(anti)proton collisions have been calculated in the $k_t$-factorization approach. We have compared quantitatively different methods to include gluon transverse momenta and their effect on inclusive spectra as well as on $c\bar{c}$ correlations. Different UGDF from the literature were used.

The inclusive spectra of heavy mesons were obtained via convolution of heavy quark spectra with so-called Peterson fragmentation functions. The results depend on the choice of the factorization scale. The “best” results are obtained with the
Fig. 5. Inclusive $d\sigma/dp_t$ for $B^+$ production at $W = 1800$ GeV for different UGDF. The experimental data are from Ref. 13. The results with $\alpha_s = \alpha_s(4m_c^2)$ is in the left panel and the results with $\alpha_s = \alpha_s(\kappa_1^2 t)$ or $\alpha_s(\kappa_2^2 t)$ in the right panel.

BFKL UGDF. We observe a deficit of the cross section for almost all UGDFs used. It is not clear to us if the deficit is due to neglecting some hadronization components or due to next-to-leading order effects.

References
1. S. Catani, M. Ciafaloni and F. Hautmann, Nucl. Phys. 366 (1991) 135.
2. J.C. Collins and R.K. Ellis, Nucl. Phys. B360 (1991) 3.
3. R.D. Ball and R.K. Ellis, J.H.E.P. 0105 (2001) 053.
4. M. Luszczak and A. Szczurek, Phys. Rev. D73 (2006) 054028
5. D. Kharzeev and E. Levin, Phys. Lett. B523 (2001) 79.
6. A.J. Askew, J. Kwiecinski, A.D. Martin and P.J. Sutton, Phys. Rev. D49 (1994) 4402.
7. K. Golec-Biernat and M. Wüsthoff, Phys. Rev. D60 (1999) 114023-1.
8. M.A. Kimber, A.D. Martin and M.G. Ryskin, Eur. Phys. J. C12 (2000) 655;
   M.A. Kimber, A.D. Martin and M.G. Ryskin, Phys. Rev. D63 (2001) 114027-1.
9. E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP 45 (1977) 199;
   Ya.Ya. Balitskij and L.N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822.
10. V.D. Barger and R.J.N. Phillips, “Collider physics”, Addison-Wesley Publishing Company, 1987
11. M. Luszczak and A. Szczurek, Phys. Lett. B594 (2004) 291.
12. A.V. Lipatov, V.A. Saleev and N.P. Zotov, hep-ph/0112114
13. D. Acosta et al. (CDF Collaboration), Phys. Rev. Lett. 91 (2003) 241804.
14. D. Acosta et al. (CDF Collaboration), Phys. Rev. D65 (2005) 052005.