Scalar mesons and polarizability of the nucleon

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Abstract. It is shown that the scalar mesons \( \sigma, f_0(980) \) and \( a_0(980) \) as \( t \)-channel exchanges quantitatively solve the problem of diamagnetism and give an explanation of the large missing part of the electric polarizability \( \alpha \) showing up when only the pion cloud is taken into account. The electric polarizability of the proton \( \alpha_p \) confirms a two-photon width of the \( \sigma \) meson of \( \Gamma_{\sigma\gamma\gamma} = (2.58 \pm 0.26) \) keV.

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INTRODUCTION

Due to recent experiments very precise data are available for the polarizabilities of the nucleon [1]. The interpretation of these data in terms of the structure of the nucleon was a problem up to very recently. Calculations using nucleon models or chiral perturbation theory were not sufficient. Also calculations in terms of dispersion theory failed, unless contributions from a scalar \( t \)-channel were taken into account which, however, were only vaguely known. Recently essential progress has been made by showing [2, 3, 4] that the largest part of the \( t \)-channel contributions can be predicted from the \( \sigma \) meson having properties as proposed by Delbourgo and Scadron [5]. Minor contributions are from the \( f_0(980) \) and \( a_0(980) \) mesons.

COMPTON SCATTERING AND POLARIZABILITIES

The polarizabilities of the nucleon are determined via Compton scattering. For the forward and backward direction the amplitudes are given by (1) and (2), respectively,

\[
\frac{1}{8\pi m} [T_{fi}]_{\theta=0} = f_0(\omega) \varepsilon^* \cdot \varepsilon + g_0(\omega) i \sigma \cdot (\varepsilon'^* \times \varepsilon),
\]

\[
\frac{1}{8\pi m} [T_{fi}]_{\theta=\pi} = f_\pi(\omega) \varepsilon^* \cdot \varepsilon + g_\pi(\omega) i \sigma \cdot (\varepsilon'^* \times \varepsilon),
\]

where \( m \) is the nucleon mass, \( \omega \) the energy of the incoming photon in the LAB system, \( \sigma \) the spin operator of the nucleon and \( \varepsilon \) and \( \varepsilon' \) the polarization vectors of the incoming and outgoing photon. A definition of the electric (\( \alpha \)) and magnetic (\( \beta \)) polarizabilities and the spin polarizabilities \( \gamma_0 \) and \( \gamma_\pi \) for the forward and backward directions is obtained.
by expanding the amplitudes in terms of the photon energy $\omega$

$$f_0(\omega) = -(e^2/4\pi m)q^2 + \omega^2(\alpha + \beta) + \mathcal{O}(\omega^4), \quad (3)$$

$$g_0(\omega) = \omega \left[ -(e^2/8\pi m^2) \kappa^2 + \omega^2 \gamma_0 + \mathcal{O}(\omega^4) \right], \quad (4)$$

$$f_\pi(\omega) = (1 + (\omega'\omega/m^2))^{1/2} \left[ -(e^2/4\pi m)q^2 + \omega\omega'(\alpha - \beta) + \mathcal{O}(\omega^2\omega'^2) \right], \quad (5)$$

$$g_\pi(\omega) = \sqrt{\omega\omega'}[(e^2/8\pi m^2)(\kappa^2 + 4q^2\kappa + 2q^2) + \omega\omega'\gamma_\pi + \mathcal{O}(\omega^2\omega'^2)], \quad (6)$$

$$\omega' = \omega/(1 + (2\omega)/m),$$

where $qe$ is the electric charge ($e^2/4\pi = 1/137.04$) and $\kappa$ the anomalous magnetic moment of the nucleon. The relevant graphs are shown in Figure 1. Graph a) represents the Born terms, b) the single pion meson-cloud contribution, c) the contributions of nucleon resonances, d) higher-order $s$-channel contributions, e) the pseudoscalar $t$-channel and f) the scalar $t$-channel.

The $s$-channel parts of the electromagnetic polarizabilities can be calculated from the multipole components of the photoabsorption cross section of the nucleon using the BL sum rule (see [1])

$$(\alpha + \beta)^s = \frac{1}{2\pi^2} \int_{\omega_{thr}}^{\infty} \frac{\sigma_{tot}(\omega)}{\omega^2} d\omega \quad (7)$$

for the forward direction and the BEFT sum rule (see [1])

$$(\alpha - \beta)^s = \frac{1}{2\pi^2} \int_{\omega_{thr}}^{\infty} \sqrt{1 + \frac{2\omega}{m}(\sigma_{E1,M2,\ldots}(\omega) - \sigma_{M1,E2,\ldots}(\omega))} \frac{d\omega}{\omega^2} \quad (8)$$

for the backward direction. The results are given in Table I.
In our present approach the $\sigma$ meson simultaneously leads to mass generation of the constituent quarks and to the main part of the $t$-channel contribution to $\alpha - \beta$. Therefore, the polarizabilities provide us with access to the off-shell $\sigma$-meson as entering into dynamical symmetry breaking. Dynamical symmetry breaking starts from equations for the pion decay constant $f_\pi = 89.8$ MeV in the chiral limit (cl) and the mass $M$ of the constituent quark in the chiral limit \cite{2,5}.

\begin{equation}
 f_\pi^\text{cl} = -4iN_c gM \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - M^2}, \quad M = -\frac{8iN_c g^2}{m_\sigma^2} \int \frac{d^4 p}{(2\pi)^4} \frac{M}{p^2 - M^2}, \tag{9}
\end{equation}

where $m_\sigma^2 = 2M$ is the $\sigma$ meson mass in the chiral limit and $N_c = 3$ the number of colors. Using dimensional regularization and making use of the Goldberger-Treiman relation this leads to

\begin{equation}
 g = M/f_\pi^\text{cl} = \frac{2\pi}{\sqrt{3}} = 3.63 \quad \text{and} \quad M = 325.8. \tag{10}
\end{equation}

Furthermore, using the current quark masses $m_u^\text{curr} = 3.0$ MeV and $m_d^\text{curr} = 6.5$ MeV we arrive at the constituent quark masses $m_u = 328.8$ MeV, $m_d = 332.3$ MeV. The mass of the $\sigma$ meson is given by $m_\sigma = (4M^2 + \hat{m}_\pi^2)^{1/2} = 666.0$ MeV, where $\hat{m}_\pi$ is the average pion mass. This value $m_\sigma = 666.0$ MeV for the sigma meson mass will be used throughout in the following calculations.

For pseudoscalar and scalar mesons having the constituent quark structure

\begin{equation}
 |q\bar{q}\rangle = a|u\bar{u}\rangle + b|d\bar{d}\rangle + c|s\bar{s}\rangle, \quad a^2 + b^2 + c^2 = 1, \tag{11}
\end{equation}

the two-photon decay amplitude may be given in the form

\begin{equation}
 M(M \to \gamma\gamma) = \frac{\alpha_e}{\pi f_\pi} N_c \sqrt{2} \langle e_q^2 \rangle, \quad \text{with} \quad \langle e_q^2 \rangle = a e_u^2 + b e_d^2 + c (\hat{m}/m_s) e_s^2, \tag{12}
\end{equation}

where $\alpha_e = 1/137.04$, $f_\pi = (92.42 \pm 0.26)$ MeV the pion decay constant, $\hat{m}$ the average constituent mass of the light quarks and $m_s$ the constituent mass of the strange quark. Numerically we have $m_s/\hat{m} \simeq 1.44$ \cite{6}. Using (12) and adjusting the two-photon widths

\begin{equation}
 \Gamma_{M\gamma\gamma} = \frac{m_M^3}{64\pi} |M(M \to \gamma\gamma)|^2 \tag{13}
\end{equation}
to the experimental data [7] we arrive at the following \(|q\bar{q}\rangle\) structures of pseudoscalar and scalar mesons

\[
|\pi^0\rangle = \frac{1}{\sqrt{2}}(-u\bar{u} + d\bar{d}) \quad ^1S_0
\]

\[
|\eta\rangle = \frac{1}{\sqrt{2}}(1.04 |IS\rangle - 0.96 |s\bar{s}\rangle) \quad ^1S_0
\]

\[
|\eta'\rangle = \frac{1}{\sqrt{2}}(0.83 |IS\rangle + 1.15 |s\bar{s}\rangle) \quad ^1S_0
\]

|IS\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}| + |d\bar{d}|)

|IV\rangle = \frac{1}{\sqrt{2}}(-|u\bar{u}| + |d\bar{d}|)

|\sigma\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle \quad ^3P_0

|f_0(980)\rangle = \frac{1}{\sqrt{2}}(0.52 |IS\rangle - 1.32 |s\bar{s}\rangle) \quad ^3P_0

|a_0(980)\rangle = \frac{1}{\sqrt{2}}(0.83 |IV\rangle + 1.15 |s\bar{s}\rangle) \quad ^3P_0

|IS\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}| + |d\bar{d}|) = -\frac{1}{\sqrt{2}}(|K^+K^-| + |K^0\bar{K}^0|)

|IV\rangle = \frac{1}{\sqrt{2}}(-|u\bar{u}| + |d\bar{d}|) = \frac{1}{\sqrt{2}}(|K^+K^-| - |K^0\bar{K}^0|)

In these adjustments we have kept the structures of the \(\pi^0\) and \(\sigma\) meson unmodified as given above. The structure proposed for the \(a_0(980)\) meson contains the problem of isospin violation. This problem can be solved by noting that the mesons \(f_0(980)\) and \(a_0(980)\) and the \(K\bar{K}\) meson pairs have almost equal masses: \(m(f_0) = 980 \pm 10\) MeV, \(m(a_0) = 984.8 \pm 1.4\) MeV, \(m(K^+K^-) = 987.4\) MeV, \(m(K^0\bar{K}^0) = 995.3\) MeV. This means that in addition to a \(|q\bar{q}\rangle\) expansion the states \(|IS\rangle\) and \(|IV\rangle\) may also be expanded in terms of \(|K\bar{K}\rangle\) states where the latter are located in the continuum. Taking into account the mass differences between \(|K^+K^-\rangle\) and \(|K^0\bar{K}^0\rangle\) we note that the isospin is an oscillating and therefore not definite quantity:

\[
|IS, IV\rangle = \frac{1}{\sqrt{2}}(-e^{i\Delta m t}|K^+K^-\rangle - |K^0\bar{K}^0\rangle)
\]

where \(\Delta m\) is the mass difference between the two \(|K\bar{K}\rangle\) components. It is important to note that the pseudoscalar mesons are in \(^1S_0\) angular momentum states whereas the scalar mesons are in \(^3P_0\) angular momentum states. For the \(\pi\) and the \(\sigma\) mesons this means that the \(\pi\) mesons are located in the ground state of a confining potential whereas the \(\sigma\) meson is located in the first excited state. This implies that at least qualitatively confinement provides a complementary explanation of chiral symmetry breaking.

Table 2 summarizes the properties of the pseudoscalar and scalar mesons as obtained from the \(|q\bar{q}\rangle\) structures given above. The decay amplitude is obtained using (12) and (13). The calculation of the meson-nucleon coupling constant is described in [4]. The quantity \(\Gamma_{MN\gamma\gamma}\) is the experimental two-photon width, except for the \(\sigma\) meson where the prediction of dynamical symmetry breaking is given. Within the framework of this theory the only error entering into the result is due to the pion decay constant \(f_\pi\) which is known with a precision of \(\sim 0.3\%\). This leads to an error of \(\sim 1\%\) for the predicted two-photon width \(\Gamma_{\sigma\gamma\gamma}\). An experimental error is obtained from the electric polarizability of
the proton which is known with a precision of $\sim 5\%$. Using $\Delta \Gamma_{\sigma\gamma\gamma}/\Gamma_{\sigma\gamma\gamma} \approx 2\Delta \alpha_p/\alpha_p \approx 10\%$ the error adopted in Table 2 is obtained. The $t$-channel pole contributions due to scalar mesons are given by

\[ \frac{\sigma\gamma\gamma}{\pi^0} = \frac{g_{\sigma NN} M(\sigma \rightarrow \gamma\gamma)}{2\pi m_{\sigma}^2} + \frac{g_{f_0 NN} M(f_0 \rightarrow \gamma\gamma)}{2\pi m_{f_0}^2} + \frac{g_{a_0 NN} M(a_0 \rightarrow \gamma\gamma)}{2\pi m_{a_0}^2} \tau_3 \] (15)

and

\[ (\alpha + \beta)^t = 0. \] (16)

Using these pole contributions the final results given in Table 3 are obtained. The pseudoscalar $t$-channel contributions are given by

\[ \gamma_{\pi}^t = \frac{1}{2\pi m} \left[ \frac{g_{\pi NN} M(\pi^0 \rightarrow \gamma\gamma)}{m_{\pi^0}^2} \tau_3 + \frac{g_{\eta NN} M(\eta \rightarrow \gamma\gamma)}{m_{\eta}^2} + \frac{g_{\eta' NN} M(\eta' \rightarrow \gamma\gamma)}{m_{\eta'}^2} \right]. \] (17)

Inserting the numerical values given in Table 2 we arrive at the spin polarizabilities given in Table 4.

**SUMMARY AND DISCUSSION**

In the foregoing we have presented information on the structure of pseudoscalar and scalar mesons as entering into the $t$-channel contributions of the polarizabilities of the
TABLE 4. Spin polarizabilities for the backward direction. Line 2 contains resonant and nonresonant contributions due to the nucleon structure (s-channel). The t-channel pole contributions may be viewed as properties of the constituent quarks.

| spin polarizabilities | \( \gamma^{(p)} \) | \( \gamma^{(n)} \) |
|-----------------------|-----------------|-----------------|
| resonant + nonresonant| \(+ (7.1 \pm 1.8)\) | \(+ (9.1 \pm 1.8)\) |
| \( \pi^0 \) pole      | \(-46.7\)       | \(+46.7\)       |
| \( \eta \) pole       | \(+1.2\)        | \(+1.2\)        |
| \( \eta' \) pole      | \(+0.4\)        | \(+0.4\)        |
| sum                   | \(-38.0\)       | \(+57.4\)       |
| average exp. result   | \(-38.7 \pm 1.8\) | \(+57.6 \pm 1.8\) |

nucleon. As t-channel exchanges these mesons are probed far off-resonance and, therefore, may be described in terms of a \( q\bar{q} \) expansions of their possibly rather complicated wave-functions. These \( q\bar{q} \) expansions make it possible to make predictions of their decay matrix elements \( M(M \to \gamma\gamma) \) including their signs and of the meson-nucleon coupling constants \( g_{MNN} \) as entering into the expressions for the t-channel exchanges. For the \( \sigma \) meson dynamical symmetry breaking leads to a precise value \( m_\sigma = 666.0 \) MeV for the mass which is confirmed through the agreement between the predicted t-channel contribution to \( (\alpha - \beta) \) and the experimental data. For all polarizabilities there is a general excellent agreement between the experimental and predicted data, showing that the polarizabilities are known and understood on a high level of precision.

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