A Generalized Labelled Multi-Bernoulli Filter for Extended Targets With Unknown Clutter Rate and Detection Profile

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ABSTRACT A prior knowledge of the background parameters such as clutter rate and detection profile is of critical importance in the tracking algorithms under the theory of random finite sets for extended objects which would lead to restrictions in the application. To accommodate this problem, a multiple extended target tracking algorithm based on the generalized labelled multi-Bernoulli (GLMB) filter under the circumstance of unknown clutter rate and detection profile is proposed in this article. After introducing a clutter generator, this new algorithm establishes augmented state space model for targets and clutter and propagates them in parallel by applying multi-class GLMB theory. We then employ Beta to describe detection probability. Target extension is modelled as an ellipse by using gamma Gaussian inverse Wishart distribution. Simulation results indicate that the proposed algorithm has better performance in estimating trajectories and extended shapes compared with the conventional filter having prior knowledge.

INDEX TERMS Multitarget tracking, extended targets, generalized labelled multi-Bernoulli (GLMB) filter, unknown clutter rate and detection.

I. INTRODUCTION
Multiple target tracking (MTT) aims to estimate the exact number of targets and the states of each target from a stack of measurements with unknown sources. The main challenges in MTT include detection uncertainty, clutter and data association uncertainty. Combined with the inaccuracy of the sensor detection, the set of measurements is corrupted by clutter, background noise and measuring error of sensor, resulting in a bias with the real state of the target. Random Finite Sets (RFS) theory [1], proposed by Mahler, provides a simple and effective form of Bayesian recursion for MTT treating the multi-target state as a finite set. Recursive Bayesian approach shows good features in enhancing the state estimation accuracy even in the case of insufficient measurements [2]. The Bayesian framework has become a significant estimation approach with applications including radar or sonar [3], computer vision [4], cell biology [5], autonomous vehicle [7] and sensor network [8]. Taking the numerical complexity of Bayes multi-target filter into consideration, the Probability Hypothesis Density (PHD) [9] filter, Cardinalized PHD (CPHD) [10] filter and multi-target multi-Bernoulli (MeMBer) filter [11] have been developed as approximations. However, one common issue of these filters is that the estimated targets are indistinguishable. A notion of labelled RFS is introduced and the analytic solution to address target trajectories known as the generalized labelled multi-Bernoulli (GLMB) filter is proposed by Vo in [12] and [13]. It outperforms the RFS-based filters mentioned earlier by introducing hypothesis method into the iteration. Implementations such as cell migration [5], multi-sensor framework [8] and joint detection, tracking and classification [14] have been investigated using this state-of-the-art filter. A cheaper method is employed to reduce the complexity in [15] and it is proposed in [16] to adopt trajectories in an interval to describe the multi-target state.

In real scenarios, the clutter rate and detection profile are usually unknown. But knowledge of parameters in the background such as clutter rate and detection profile is crucial in Bayesian multi-target filtering. In general, clutter is usually modelled as a Poisson point process and detection profile is characterized by the detection probability. These parameters are actually scenario-dependent and cannot be assumed in advance. Therefore, a multi-object filter which can learn the
clutter rate and the detection probability adaptively while tracking is discussed in [17]. It assumes that clutter is generated by clutter generator, which can be tracked during the propagation together with the real target. We assume the unknown detection probability as a variate following a Beta distribution and then extend the target state space so that a Beta-Gaussian mixture implementation is given to track multiple objects under the CPHD recursion. Beta distributions are introduced naturally as an implementation strategy [18]. A multi-Bernoulli filter with the same assumption is presented in [18]. Simulation results illustrate that the one in [18] gives a better performance in Sequential Monte Carlo approximation compared with the algorithm presented in [17]. Nonetheless, none of these filters differentiate trajectories. After introducing labelled RFS method into this problem, an efficient implementation of GLMB for point targets with unknown background parameters is described in [19] and it shows better robustness in high clutter rate or low detection probability situations compared with CPHD filter in [17].

However, the theory to learn the unknown detection profile relies on the premise that the current detection probability is related to the previous estimation. It would require more time and lead to an unsatisfactory performance if a poor initialization is given. Besides, the detection probability is usually affected by multiple factors, such as the sensor property, the movement of the target, the environment of the observation area and the amplitude feature received by the sensor along with the detections. Usually the environment factor implies the signal-to-noise ratio (SNR) and the amplitude feature, to be specific, is often known as the signal strength or amplitude of a signal, which can be used as a discriminant index since the amplitude feature coming from the target is normally stronger than the one from clutter. As we all know, high detection probability depends on high SNR and strong features. As a result, these parameters can be applied to estimate the unknown detection profile. A new approach to incorporate the feature information into the Bayesian framework is demonstrated in [20].

Conventional multi-object filters rely on the standard point target model, which is based on the assumption that each target produces at most one measurement at a given time. The origin of each measurement is either one particular target or a clutter. With the development of high-resolution sensors, multiple detections can be received from a target and an extended target model [22] is needed to demonstrate more details about the size, shape and orientation of the observed targets, which are essential in recognition and classification application. An overview of the field of extended target tracking is demonstrated in [23].

There are two critical components required when dealing with extended target. One is a model describing the number of measurements generated by each target and the other is a model describing target spatial distribution. As for the former one, [24] provides an approach assuming the number of measurements is modelled as an inhomogeneous Poisson distribution characterized by a rate parameter. With regards to modelling the extent of the target, an approach assuming an elliptical extent is proposed in [25] by utilizing random matrix theory. This is termed as a Gaussian inverse Wishart (GIW) distribution and the GIW-PHD filter presented in [26] providing a concise and effective recursion under RFS theory. Further, by treating the number of measurements as a new random variable, which is modelled using the gamma probability density function (pdf), a modification of the GIW approach known as the gamma Gaussian inverse Wishart (GGIW) distribution is described in [27]. The tracker implemented in [27] is a CPHD filter. The formulation of CPHD does not explicitly accommodate the estimation of target trajectories although it avoids data association. Therefore, an extended target GLMB algorithm based on a GGIW method is developed in [28], allowing for the realization of continuous tracking of target trajectories identified by distinct labels. Yet it seems that a better trade-off method is essential to reduce the complexity of the proposed GLMB filter and to maintain the accuracy of the estimation of target state together with shape. Up till now, the two approaches mentioned above require prior knowledge of clutter rate and detection probability.

In this article, we propose an extended target GLMB filter that can adaptively learn the unknown clutter rate and detection profile while tracking. The proposed filter establishes an augmented state space model with Beta distribution modelling the unknown detection probability and treats clutter as detections originated from the clutter generator. And the multi-class GLMB filter which is able to jointly track and classify multiple objects is applied to estimate the hybrid multi-object posterior probability density. We then use the GGIW approach to estimate the extend as the extended target is assumed to be an ellipse. Measurements received at each time are clustered using a shape selection method [29]. An efficient implementation is given by employing the jointly predict and update method detailed in [15]. Simulation results confirm the proposed algorithm has apparent improved performance in unknown background.

The remainder of this article is organized as follows. In section 2, we provide some background information related to the extended object model and multi-class GLMB filter. The proposed GLMB filter for extended targets with unknown background parameters is detailed in section 3. Section 4 contains the simulation results highlighting the performance of our proposed filter compared with the original one with prior knowledge of clutter rate and detection probability and the one with mismatching parameters. Concluding remarks are presented in section 5.

II. BACKGROUND
A. LABELLED RANDOM FINITE SETS
At first, we give some notions and definitions related to the labelled random finite sets.

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**Notion 1:** For a real valued function \( h \), its multi-object exponential is \( h^X \triangleq \prod_{x \in X} h(x) \) with \( h^\emptyset = 1 \), where \( X \) is a set whose elements could be any type such as vectors, scalars or sets as long as the function \( h(\cdot) \) takes an argument of that type.

**Notion 2:** The generalized Kronecker delta function and the set inclusion function is defined as
\[
\delta_Y(X) = \begin{cases} 1, & \text{if } X = Y \\ 0, & \text{otherwise} \end{cases} \quad (1)
\]
\[
\mathbf{1}_Y(X) = \begin{cases} 1, & \text{if } X \subseteq Y \\ 0, & \text{otherwise} \end{cases} \quad (2)
\]
where \( X \) and \( Y \) can be any type mentioned in Notion 1.

**Notion 3:** \( \mathcal{F}(X) \) is the class of finite subsets of \( X \) whose elements can be any type such as vectors mentioned earlier.

**Definition 1:** An RFS can be defined as a simple finite-set-valued random variable whose number of components is random and the components themselves are random.

**Definition 2:** A labelled RFS \( X \) with state space \( \mathcal{X} \) and discrete label space \( \mathcal{L} \) is an RFS on \( \mathcal{X} \times \mathcal{L} \) and the labels are always distinct. That is, if \( \mathcal{L}(X) \) is a set of unique labels in \( X \) and we define the distinct label indicator function as \( \Delta(X) = \begin{cases} 1, & \text{if } \left| \mathcal{L}(X) \right| = \left| X \right| \\ 0, & \text{if } \left| \mathcal{L}(X) \right| \neq \left| X \right| \end{cases} \) with \(|\cdot|\) denoting the number of a set, and a labelled RFS always satisfies \( \Delta(X) = 1 \).

**Definition 3:** A generalized labelled multi-Bernoulli RFS is a labelled RFS with state space \( \mathcal{X} \) and discrete label space \( \mathbb{L} \), which is distributed according to [12] and [13]

\[
\pi(X) = \Delta(X) \sum_{c \in \mathcal{C}} \mathbf{w}^c(L) (\mathcal{L}(X)) \mathbf{p}^c(\cdot)
\]

where \( \mathbb{C} \) is a discrete index set, \( \mathbf{w}^c(L) \) and \( \mathbf{p}^c(x, l) \) satisfy

\[
\sum_{L \in \mathcal{L}, c \in \mathbb{C}} \mathbf{w}^c(L) = 1 \quad \text{and} \quad \int_{x \in \mathcal{X}} \mathbf{p}^c(x, l) \, dx = 1, \quad \text{with } x \in \mathcal{X} \quad \text{and} \quad l \in \mathcal{L} \text{ denoting the target state and the corresponding label, respectively.}

**B. EXTENDED TARGET TRACKING MODEL BASED ON LABELLED RFS**

At time \( k \), the labelled RFS of extended targets is denoted as \( X = \{x_1, l_1, \ldots, x_{\mathcal{I}}, l_{\mathcal{I}}\} \), where \( (x_i, l_i) \) represents the \( i \)-th target with state \( x_i \) and label \( l_i \) and \( i \in \{1, \ldots, |\mathcal{X}|\} \). Each state \( x_i \) consists of two parts describing the centroid of the target and the extended shape. In [28], the GGIW method is employed to model a single extended target. To start, some notations should be introduced:

We thus model the extended target state along with its shape as \( x_i = (x_{C, i}, x_{C, i}, x_{C, i}) \in \mathbb{R}^2 \times \mathbb{R}^d \times \mathbb{S}^n_+ \), where \( x_{C, i} \sim \mathcal{G}(x_{C, i}; \alpha, \beta) \) is the poisson rate parameter of a Poisson distribution that models the number of measurements generated by the target, \( x_{C, i} \sim \mathcal{N}(x_{C, i}; m, P) \) is a vector that describes the state of the target centroid and \( x_{C, i} \sim \mathcal{IW}(x_{C, i} \sim f(x), v, V) \) is a covariance matrix that describes the target extent. Furthermore, the probability density function of the extended target state is a GGIW distribution on the space \( \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{S}^d_+ \), given by

\[
p'(x) = \mathcal{G}(x; \alpha, \beta) \mathcal{N}(x; m, P \otimes x) \mathcal{IW}(x; v, V) \equiv \mathcal{GGIW}(x; \zeta)
\]

where \( \zeta = (\alpha, \beta, m, P, v, V) \) represents an array that encapsulates all the GGIW density parameters.

Each particular extended target \( (x_i, l_i) \) may be detected with probability \( p_D(x_i, l_i) \) and generate a set of measurements \( \mathcal{W}_i \) which is assumed to be independent with all other targets with likelihood function \( g(W_i | x_i, l_i) \) or misdetected with probability \( q_D(x_i, l_i) = 1 - p_D(x_i, l_i) \). In addition, over the whole observation area, the set of false observations received by the sensor is represented as a Poisson RFS \( X_{FA} \) with intensity function \( \kappa(\cdot) \), independent of the target measurements (i.e., \( X_{FA} \) is distributed according to \( \mathcal{G}(X_{FA}) = e^{-\kappa(x)} \mathcal{X}_{FA} \), where \( f, g, \kappa \) represents the standard inner product). Thus, the total detections at time \( k \) is denoted as \( Z = \bigcup_{(x_i, l_i)} W_i \cup X_{FA} \).

**C. MULTI-CLASS GLMB**

A Jump Markov system (JMS) is composed by a set of parameterized state space models, whose parameters evolve with time according to a finite state Markov chain. It can be employed in Bayesian filtering to jointly track multiple targets with different kinematic models. For instance, there are two objects in the observation area. One is moving with a constant speed and the other follows a uniformly accelerated motion. JMS theory allows us to track and classify these two targets in parallel.

In [19], a new filter to process objects with multiple kinematic model called multi-class GLMB is presented based on
JMS theory. By using the kinematic model as a class label and assuming that targets with different models do not interact with each other, the problem of jointly tracking and classification can be addressed by applying Multi-Class GLMB recursion. The following two conditions explain how the class label work:

1) All possible states of a new target with the same object label share a common class label;

2) A target cannot switch between different models from one step to the next.

Let $m \in \mathbb{M}$ be the discrete index set of modes in the system representing the class label set and a GLMB RFS is an RFS on $\mathbb{X} \times \mathbb{L} \times \mathbb{M}$. Let $\mathbb{L}^{(m)}$ represents the set of labels of all the elements with mode $m \in \mathbb{M}$ and different label sets $\mathbb{L}^{(m)}$ and $\mathbb{L}^{(m')}$. Let $\mathbb{L}^{(m)}$ and $\mathbb{L}^{(m')}$. Let $\mathbb{X}^{(m)}$ donates all the possible target states with mode $m$, and follows $\mathbb{X} = \cup_{m \in \mathbb{M}} \mathbb{X}^{(m)}$.

In a GLMB filtering algorithm, the new born target is modelled as a labelled multi-Bernoulli (LMB) RFS. A new object is classified as class $m$ if and only if its label falls into $\mathbb{L}^{(m)}$. Thus, an LMB birth model can be parameterized as

$$r_{B,+}(l_+) = \sum_{m \in \mathbb{M}} r_{B,+}^{(m)} (l_+)$$

(5)

and

$$p_{B,+}^{(m)} (\xi_+, l_+) = p_{B,+}^{(m)} (x_+, l_+) l_{B,+}^{(m)} (l_+)$$

(6)

where $r_{B,+}^{(m)}$ and $p_{B,+}^{(m)} (x_+, l_+)$ are the existence probability and probability density of the kinematics $x_+$ of a new born target given mode $m$, respectively. Note that variate with subscript $+$ denotes the value of itself at the next time.

The multi-target density for $X^{(m)}$ is the GLMB

$$\pi_{\\mathbb{L}^{(m)}} \left( X^{(m)} \right) = \Delta \left( X^{(m)} \right) \delta_{I^{(m)}} \left[ \mathcal{L} \left( X^{(m)} \right) \right] \times \left[ p^{(m)} \right] X^{(m)}$$

(7)

where $\mathbb{L}^{(m)}$ represents all the history association between $I^{(m)}$ and the measurements with $\mathbb{Z}$ denoting the history association space, $I^{(m)} = I \cap \mathbb{L}^{(m)}$ denotes all the labels of objects with mode $m$ that still exist at time $k$ and $I \subseteq \mathbb{L} = \cup_{m \in \mathbb{M}} \mathbb{L}^{(m)}$ is the set of labels of objects that still exist at this current time. Additionally, the hybrid multi-target density for $X$ can be written as

$$\pi (X) = \sum_{\xi, I} 1_{\Theta (I)} (\xi, \perp \Theta) \prod_{m \in \mathbb{M}} \pi_{\\mathbb{L}^{(m)}} \left( X^{(m)} \right)$$

(8)

where $\xi, \perp \Theta$ represents the projection of $\xi$ into the space $\Theta$, $\xi^{(m)} = \xi |_{\mathbb{L}^{(m)} \times \cdots \times \mathbb{L}^{(m)}}$ with the subscript $\mathbb{L}^{(m)}$ representing the set of labels with mode $m$ at time $k$, and $\Theta$ is the positive 1-1 map space, indicating possible assumptions of the association between labels and measurements, i.e. $\theta \in \Theta : \mathbb{L} \rightarrow \{0 : |Z|\}$.

For any mode $m$, the multi-target density at time $k + 1$ is a GLMB of the form

$$\pi_{Z+} \left( m, I^{(m)}, \xi^{(m)}, l^{(m)} \right) \left( X^{(m)} \right) = \Delta \left( X^{(m)} \right) w_{Z+} \left( m, I^{(m)}, \xi^{(m)}, l^{(m)} \right) \left[ \mathcal{L} \left( X^{(m)} \right) \right] \times \delta_{l_k} \left[ \mathcal{L} \left( X^{(m)} \right) \right] \left[ p_{Z+}^{(m), \theta^{(m)}} \right] X^{(m)}$$

(9)

where

$$w_{Z+} \left( m, I^{(m)}, \xi^{(m)}, l^{(m)} \right) = \left[ \Psi_{Z+}^{(m)} (m, \cdot) \right] \left[ 1 - r_{B,+} \right] \left[ \mathbb{I} - I \right] \left[ \mathbb{I} \right]$$

and

$$\Psi_{Z+}^{(m)} \left( m, l \right) = \left[ p_{Z+}^{(m)} (\xi, m, l), p_{Z+}^{(m)} (\xi, m, l) \right] \left[ \mathcal{L} \left( X^{(m)} \right) \right] \left[ p_{Z+}^{(m), \theta^{(m)}} \right] X^{(m)}$$

(10)

$$\Psi_{Z+}^{(m)} \left( m, l \right) = \left[ p_{Z+}^{(m)} (\xi, m, l), p_{Z+}^{(m)} (\xi, m, l) \right] \left[ \mathcal{L} \left( X^{(m)} \right) \right] \left[ p_{Z+}^{(m), \theta^{(m)}} \right] X^{(m)}$$

(11)

III. GLMB FILTERING WITH UNKNOWN BACKGROUND PARAMETERS FOR EXTENDED TARGET

To jointly accommodate the unknown clutter rate and detection profile, the basic idea is to hybridize and augment the single extended target state. Similar to [19], clutter is assumed to be generated by another kind of target with no dynamics, i.e. clutter generator, and multi-class GLMB is applied to
provide traceable solution of jointly tracking multiple targets with different kinematic models. In subsection 3.1, we introduce beta distribution for the detection probability. In 3.2, we augment the state space model for the disjoint union of the true target and the clutter generator. And the detailed derivation of the proposed algorithm under unknown clutter rate and detection probability scenario is detailed in 3.3, along with the algorithm flow provided in 3.4.

A. BETA DISTRIBUTION FOR THE DETECTION PROBABILITY

In order to estimate the unknown detection probability, we define a state independent variate \( a \in X^\Lambda = [0, 1] \), where \( X^\Lambda \) denotes the space of the unknown detection probability. And the unknown detection probability is assumed to follow a beta distribution, ie. \( a \sim \beta(\cdot; s, t) \) with parameters \( s > 1 \) and \( t > 1 \).

According to [19], it is clear that the prediction of \( a \) is completely governed by the Beta distribution. For instance, at time \( k - 1 \), Beta distribution function can be parameterized as \( \beta(a; s_k - 1, t_k - 1) \) with mean and covariance denoted as \( \mu_k \) and \( \sigma_k^2 \). Then, for the purpose of increasing the uncertainty, the predicted density can be obtained by preserving its mean and dilating the covariance,

\[
\mu_{k|k-1} = \mu_k, \quad \sigma^2_{k|k-1} = C_B \sigma^2_k, \quad C_B > 1
\]

The parameters satisfying this change can be verified as

\[
S_{k|k+1} = \left( \frac{1}{\sigma_k^2} \frac{1}{\mu_k^2} \right) - 1 \mu_{k|k-1}
\]

\[
t_{k|k+1} = \left( \frac{1}{\sigma_k^2} \frac{1}{1 - \mu_k^2} \right) \left( 1 - \mu_{k|k-1} \right)
\]

Besides, at the update step, the transition density of this variate \( a \) is given by \( f^0_+ (a|x, a) \), and consequently, the updated density is represented as

\[
\beta (a; s_+, t_+) = \int \beta (a; s_k-1, t_k-1) f^0_+ (a|x, a) da
\]

Analytic computation of updating Beta distribution includes particular forms of calculation based on different conditions such as whether the corresponding target is detected or not,

\[
a \beta (a; s, t) = \frac{B(s + 1, t)}{B(s, t)} \beta (a; s + 1, t + 1)
\]

\[
(1 - a) \beta (a; s, t) = \frac{B(s, t + 1)}{B(s, t)} \beta (a; s, t + 1)
\]

\[
B(s, t) = \int_0^1 a^{s-1} (1 - a)^{t-1} da
\]

resulting in a weighted Beta function in variate \( a \).

B. JOINT OBJECT-CLUTTER STATE MODEL FOR EXTENDED TARGET

Under the assumption in multi-class GLMB, the sets for the state of extended target and clutter generator are two disjointed classes of targets and can be distinguished by their kinematic models. Let \( m \in \mathbb{M} = \{0, 1\} \) denote the mode space with \( m = 0 \) and \( m = 1 \) representing clutter generator and extended target, respectively. As a result, the joint object-clutter state model can be defined as \( X^+ = X^{(1)} \cup X^0 \).

1) THE CLUTTER GENERATOR

In view of the clutter part, the transition function of clutter which has no dynamics is independent from the previous state and the measurement likelihood function follows a uniform distribution in the whole observation area \( V \), given as

\[
f^0_+ (x_+, x, l) = s (x, l), \quad g^0 (z|x, l) = u (z) V^{-1}
\]

where \( s (\cdot) \) is a constant function and \( u (z) \) represents a uniform distribution. Additionally, for Gaussian implementation, it is usually assumed that the survival probability and detection probability for clutter are independent from the state, so that they can be simplified as \( P^0_S (x_+, l) = P^0_S \) and \( P^0_D (x_+, l) = P^0_D \). When adopting multi-class GLMB filter to update clutter, it can be seen that the posterior probability density is irrelevant with the clutter state, which can also be denoted as a constant function

\[
p^0_R (x, l) = p^0_R (x, 0, l) = p^0_Z (x, 0, l) = s (x)
\]

Hence, the propagation of clutter can be reduced to its weights \( w_{Z^0} \).

2) THE TARGET MODEL

From above, the joint state model can be rewritten as \( X^+ = (X^1 \times X^\Lambda) \cup X^0 \). Combining with section 2.2, the extended target state can be described as \( x = (x_{r}, x_{c}, x_{\xi}, x_{\zeta}, a) \in \mathbb{R}^2 \times \mathbb{R}^d \times S^2_{\mathbb{R}^2} \times \Lambda \). Since the detection probability follows a beta distribution and the extended target state follows a GGIW (gamma Gaussian inverse Wishart) distribution, which are independent, the density of the augmented target state can be modelled as a hybrid distribution of the form as beta gamma Gaussian inverse Wishart (\( \beta - GGIW \)) distribution,

\[
p (x) \triangleq \beta (a; s, t) \mathcal{G}\mathcal{G}\mathcal{I}W (x; \zeta)
\]

in which the two parts can be propagated separately.

C. IMPLEMENTATION IN EXTENDED TARGET FILTERING

As mentioned above, it can be seen that the key challenge in multi-class GLMB recursion is to propagate the GLMB components involved. For each parent component \((I, \xi)\), the main idea is to search the space \( F (\mathbb{L}_+ \times \Theta_+) \) and to find a set of \((I_+, \theta_+)\) so that the children components \((I, \xi, I_+, \theta_+)\) would have significant weight \( w_{Z^0} \).

Let \( X^+_k = \left( X^1_k \times X^\Lambda_k \right) \cup X^0_k \) and \( Z_k \) denote the set of joint object-clutter state space and the set of detections at time \( k \) with \( X^\Lambda_k \) representing the space for additional
information adopted to estimate the detection probability. For all the objects in $X^k_+$, the set of $(I_+, \theta_+)$ is detailed as
\begin{equation}
(I_+, \theta_+) \triangleq \left( I^0_+ \cup I^1_+, \xi_+, \tilde{\eta}_+(\cdot, \cdot), \eta_+(\cdot, \cdot) \right) \in \mathcal{F}(L_+) \times \Theta_+
\end{equation}
where $L_+ = L^0_+ \cup L^1_+, \Theta_+ = \Theta^0_+ \cup \Theta^1_+$. And the weight of the children components $(I, \xi, I_+, \theta_+)$ is in the form of
\begin{equation}
w_{Z_k}(I^0_+, \xi^0_+, I^1_+, \theta^0_+, \theta^1_+) = 1_{\Theta(I_+)} \left( \theta_+ \right) w_{Z_k} \left( 1, I^0_+, \xi^0_+, I^1_+, \theta^0_+, \theta^1_+ \right)
\end{equation}

To focus on the calculation of the extended target, an efficient implementation is detailed as follows:

It is obvious in (20) and (21) that the searching in space $\mathcal{F}(L_+) \times \Theta_+$ can be divided into two non-interacting parts: $(I^0_+, \theta^0_+)$ and $(I^1_+, \theta^1_+)$. As a result, instead of computing the hybrid multi-target posterior density, we process extended target and clutter in separate procedure.

For the extended target, we seek $(I^0_+, \theta^0_+)$ with significant weight $w_{Z_k}(I^0_+, \xi^0_+, I^1_+, \theta^0_+)$, Murty’s algorithm [15] gives us an alternative way to generate a set of $(I^0_+, \theta^0_+)$ without enumerating all the hypotheses and their weights, but this requires the computation of the hybrid multi-target filtering density. We treat clutter as Poisson distribution with matching intensity so that Murty’s algorithm can be applied to obtain the required set of $(I^0_+, \theta^0_+)$. To be specific, assuming that there are $|I^0_+|$ clutter generated from previous time with survival probability $P^0_+$, and $|I^1_+|$ clutter born with existence probability $r^0_{I_+}$, the matching intensity for clutter is computed as $\tilde{\xi}_+ = \left( P^0_+ |I^0_+| + r_{B_+} |I_+^1| \right) P^0_D V^{-1}$.

Note that the received measurement set is $Z_k = [Z_1, N]$, $N = |Z|$, the label set of extended targets survived from previous time is $I^1_+ = \{l_1, R\}$ with $R = |I^1_+|$, and the label set for new born extended targets is denoted as $\Xi^0_+ = \{R+1, P\}$, where $P = |I^1_+| + |\Xi^1_+|$. Following [15], we apply jointly predicting and updating approach to accommodate the increasing computation and $(I^0_+, \theta^0_+)$ with significant weights which are determined by solving the ranked assignment problem with cost matrix $\eta_{I^0_+}(\cdot, j)$ with $i = 1 : P$ and $j = -1 : N$, which can be computed as
\begin{equation}
\eta_{I^0_+}(\cdot, j) = \begin{cases}
1 - \eta^0_{I^0_+}(\cdot, j), & l_i \in I^1_+, j < 0 \\
\eta^0_{I^0_+}(\cdot, j), & l_i \in I^1_+, j \geq 0 \\
1 - r^0_{B_+}(l_i), & l_i \in \Xi^0_+, j < 0 \\
r^0_{B_+}(l_i), & l_i \in \Xi^0_+, j \geq 0 
\end{cases}
\end{equation}

where $\eta^0_{I^0_+}(\cdot, j) = \left( \begin{array}{c}
1 - \eta^0_{I^0_+}(\cdot, j) \\
\eta^0_{I^0_+}(\cdot, j) \\
1 - r^0_{B_+}(l_i) \\
r^0_{B_+}(l_i)
\end{array} \right)$.

Consequently, seeking the best assumption $(I^0_+, \theta^0_+)$ reduces to seek the best $(N^0_-, N^0_+)$ subject to the constraints $0 \leq N^0_- \leq |I^0_+|$, $0 \leq N^0_+ \leq |\Xi^0_+|$ and $N^0_- + N^0_+ \geq |Z_k^0|$.
After carrying out all the possibilities, the expected GLMB components can be constructed via (20). The implementation using Beta distribution is simply a combination of the propagation of Beta distribution and multi-class GLMB iteration. The prediction and update of the parameters of Beta distribution is merged with the process of GGIW density parameters and then the detection probability can be acquired by these updated parameters. Consequently, the estimation of the unknown detection probability is calculated by

\[
P_{D,+}^{(k)}(x^{(1)}, I_t) = \frac{1}{N_x} \sum_{(x^{(a)}, a) \in X_k^I \times X_k} \mu_+ (28)
\]

where \(N_x\) denotes the number of the extended targets, and \(\mu_+ = \frac{x_k}{s_k^2}\) is the mean of the Beta distribution.

D. ALGORITHM FLOW

Algorithm 1 The Main Steps of the Algorithm

1: Initial parameters: set initial target state set \(X_k^{(1)}\), clutter set \(X_k^{(0)}\), s, t for Beta distribution, measurement set \(Z_k\)
2: Establish the joint object-clutter state model with \(X_k^{(1)} = X_k^{(1)} \times X_k a/\Omega\) and \(X_k^{(0)} = X_k^{(0)}\)
3: Compute cost matrix according to (22), then generate a set of \((t_{k,+}^{(1)}, \theta_{k,+}^{(1)})\) by Murty’s algorithm
4: Update the GGIW parameter and Beta distribution parameter according to (28) for each given hypothesis \((t_{k,+}^{(1)}, \theta_{k,+}^{(1)})\)
5: Find clutter measurement \(Z_k^{(0)}\) to compute \((t_{k,+}^{(0)}, \theta_{k,+}^{(0)})\) according to (27)
6: Construct \((I_{k,+}, \theta_{k,+})\), compute the corresponding weight via (20) and (21)
7: Extract extended target state and shape parameters.

IV. SIMULATION EXAMPLE

In this section, \(\beta\)-GGIW-GLMB is given to represent the proposed GLMB filter in unknown background, while the Beta distribution is used for estimating the unknown detection probability. Performance of the \(\beta\)-GGIW-GLMB, the conventional GLMB filter with prior knowledge of the clutter rate and detection profile (P-GGIW-GLMB) in [28] and the standard filter but given the mismatching clutter rate (M-GGIW-GLMB) are illustrated and compared under the complex background by applying a Gaussian mixture implementation. The optimal subpattern assignment (OSPA) matrix [30] is used for performance assessments. The CPU and RAM of the computer used in our simulation experiment are Intel(R) Core(TM) i5-4430 and 8GB, respectively.

A. SCENARIO SET-UP

We consider a four extended targets scenario on a \([-500 \times 500] m \times [-500 \times 500]\) m 2D surveillance area. The whole observation process lasts 40 time steps and the time when the targets appear are given as: \(t_b^{(1)} = 1, t_b^{(2)} = 1, t_b^{(3)} = 4, t_b^{(4)} = 25\). The ground truth of the kinematic state of the extended targets is shown in Fig. 1 with target 1 and 3 coming cross at time step 8 and 9 and target 3 and 4 coming cross at time step 36 and 37. All the targets are modelled as an ellipse with the major and minor axes setting at \(a = 20m\) and \(b = 10m\).

All extended targets follow the same dynamic model [28]

\[
x_{k+1} = (F_{k+1|k} \otimes I_n) x_k + w_k \quad (29)
\]

where \(w_k \sim \mathcal{N}(0, Q_{k+1|k})\) denotes the Gaussian process noise and \(I_n\) is the identity matrix of dimension \(n\)

\[
F_{k+1|k} = \begin{bmatrix} 1 & T_s & \frac{1}{2} T_s^2 \\ 0 & 1 & T_s \\ 0 & 0 & e^{-\frac{T_s}{\tau}} \end{bmatrix} \quad (30)
\]

\[
Q_{k+1|k} = \begin{bmatrix} \Sigma_2 \left(1 - e^{-\frac{T_s}{\tau}}\right) \text{diag}\left(\left[0 \quad 0 \quad 1\right]\right) \end{bmatrix} \quad (31)
\]

with parameters: \(T_s = 1s, \theta = 1s\) and \(\Sigma = 0.1m/s^2\). The measurement model is given by

\[
z_k = (H_k \otimes I_n) x_k + v_k \quad (32)
\]

where \(H_k = \begin{bmatrix} 1 \quad 0 \quad 0 \end{bmatrix}\) and \(v_k\) is the Gaussian white noise with zero mean and covariance \(R_t = \text{diag}(0.09, 0.09)\).

Three experiments corresponding to three different pairings of clutter rate and detection probability (see Table 1) are studied in the scenario.

| Experiment | Clutter rate | Detection probability |
|------------|--------------|-----------------------|
| 1          | 30           | 0.95                  |
| 2          | 30           | 0.85                  |
| 3          | 60           | 0.95                  |

The parameters of OSPA are set as \(c = 60\) and \(p = 2\). All results are obtained from 100 Monte Carlo runs with random measurements generated at every simulation run. The extended targets follow a labelled multi-Bernoulli distribution with four components with birth probability of
For the birth parameters of each component, the initial gamma pdf parameters are set as $\alpha = 10$ and $\beta = 1$, the inverse Wishart component parameters are $\nu = 10$ and $V = 100 \times I_2$, and the mean and covariance of kinematic components are $m = \begin{bmatrix} (x^T, y^T) \end{bmatrix}^T$ and $P = \text{diag}(\begin{bmatrix} 100 & 25 & 25 \end{bmatrix})^T$ with the initial assumption being close to the true target starting positions. And the detection model parameters for all new born extended targets are $s = 9$ and $t = 1$. The survival probability for extended targets are set to be 0.95.

In addition, the clutter is also modelled as a Bernoulli RFS with the birth probability of 0.5 and follows a uniform distribution in the observation area. Similar to the extended target birth parameters, at the initial time step, the RFS is assumed to have 120 components and 30 for the following time steps. The true probability of survival and detection of clutter are both set as 0.9.

The parameters of OSPA is set as $c = 60$ and $p = 2$. All the results below are obtained from 100 Monte Carlo runs with random measurements generated in every simulation.

### B. SIMULATION RESULTS

Tracking results and comparison of the performance of all three filters are shown in this subsection.

Fig. 2 depicts the results of experiment 1, a relatively friendly environment, with Fig. 2(a), Fig. 2(b) describing the average result for the cardinality of extended targets, the average OSPA error, respectively. Evaluation of the tracking performance of the shape parameters, such as the semi-major axis’s length and semi-minor axis’s length, are also given in Fig. 2(b). It can be seen that the proposed $\beta$-GGIW-GLMB filter has similar performance comparing with the traditional P-GGIW-GLMB filter except at time step 5-10. Additionally, given a mismatching clutter rate, the capacity of M-GGIW-GLMB filter to accurately track the extended targets drops especially when targets are close to each other or come cross. More time is required to overcome the change of target number in this whole process. Meanwhile, the chance of missing a target or tracking a false alarm increases.

Furthermore, after statistics, Fig. 3 exhibits the assessments of the unknown parameters. The estimation results for clutter rate are nearly equal to each other with a slightly fluctuation around the actual value. As the initial mean value of Beta distribution of the new born target is not equal to the current estimation of detection probability, a drop of the estimation would occur if we apply (28) to get the evaluation of the detection probability.

We further investigate the performance of the proposed filter by reducing the detection probability in experiment 2. The average results for the cardinality estimation, OSPA error and the estimated clutter rate and detection probability are demonstrated in Fig. 4(a) and Fig. 4(b), respectively. Due to the low detection probability, M-GGIW-GLMB filter shows a poor tracking performance and completely breaks down and we omit this part. On the contrary, the proposed $\beta$-GGIW-GLMB filter is capable of tracking the extended targets precisely as well as learning the unknown clutter rate and detection probability, even though the OSPA error rises slightly when targets come cross during this period. Fig. 5 provides the estimated results of the unknown clutter rate.
Compared to experiment 1, we increase the clutter rate in experiment 3 to examine the robustness of the proposed algorithm. The results is shown in Fig. 6 with Fig. 6(a). Fig. 6(b) describing the average result for the cardinality, the average OSPA error, respectively. Evaluation of the tracking performance of the shape parameters, such as the semi-major axis’s length and semi-minor axis’s length, are also given in Fig. 6(b). The estimated clutter rate and detection probability are shown in Fig. 7.

In Table 2, we list the simulation time of the three algorithms in a single time step. It is noted that the running time of the algorithm is mainly spent on the measurement partition. The three algorithms used same measurement partition method, DBSCAN (Density-Based Spatial Clustering of Applications with Noise) which usually has a time complexity of O(N). For the filtering parts, the order of time complexity of the compared algorithms is the same, resulting to a comparable time cost. Furthermore, in $\beta$-GGIW-GLMB, the key parameters of clutter rate and detection probability need to be predicted at each time step. It is not surprising that the calculation time of the proposed $\beta$-GGIW-GLMB is slightly longer than the standard algorithm.
one. Meanwhile, In M-GGIW-GLMB, after given the wrong parameter, the filtering would diverge as the number of detected targets and hypotheses decrease accordingly. It will lead to an erroneous estimation and a reduced time consumption. In summary, \( \beta \)-GGIW-GLMB sacrifices the time complexity to improve the accuracy of target tracking and realizes adaptive estimation of unknown detection probability. This property of \( \beta \)-GGIW-GLMB is acceptable for the real tracking scenario.

V. CONCLUSION

In this article, we propose an algorithm for tracking multiple extended targets based on the GLMB filter with unknown background parameters such as clutter rate and detection profile. The proposed method treats clutter as another kind of target with no dynamics and establishes joint object-clutter state space model to jointly estimate the hybrid posterior probability density of the unknown detection probability, two approaches are adopted in the filtering process. One is to make use of the Beta distribution to directly model the augmented detection probability space in the extended target of this joint model, with the other utilizing the amplitude feature received by sensor along with the observed measurements. An inverse gamma distribution is applied to model the nonnegative feature in the second method. Specifically, jointly prediction and update method and Murty’s algorithm are employed to reduce the complexity of this tracker. Simulation results demonstrate that the proposed filter has a good performance in the presence of unknown background parameters compared with the conventional filter that requires priori knowledge of those parameters. Good robustness is shown in addressing high clutter or low detection probability conditions.

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![FIGURE 7. Estimated parameters for experiment 3.](image)

**TABLE 2.** Time on Simulation Experiments in Single Time Step.

| Approaches       | Time(s) |
|------------------|---------|
| \( \beta \)-GGIW-GLMB | 2.4307  |
| P-GGIW-GLMB      | 2.0186  |
| M-GGIW-GLMB      | 2.1900  |
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