What is Nonreciprocity? – Part II

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Abstract—This paper is the second part of a two-part paper on Electromagnetic (EM) Nonreciprocity (NR). Part I has defined NR, pointed out that linear NR is a stronger form of NR than nonlinear (NL) NR, explained EM Time-Reversal (TR) Symmetry Breaking (TRS-B), described linear Time-Invariant (TI) NR media, generalized the Lorentz reciprocity theorem for NR, and provided a physical interpretation of the resulting Onsager-Casimir relations [1]. This part first explains the TR specificity of lossy and open systems. Next, it proposes an extended version of the S-parameters for all NR systems. Then, it presents the fundamentals of linear-TI (LTI) NR, linear Time-Variant (LTV) Space-Time (ST) modulated NR and NL NR systems. Finally, it addresses confusions between with systems.

Index Terms—Nonreciprocity (NR), Time Reversal Symmetry Breaking (TRS-B), loss irreversibility, Onsager-Casimir relations, extended scattering parameters, Linear Time-Invariant (LTI) and Linear Time-Variant (LTV) systems, Space-Time (ST) varying systems, nonlinear (NL) systems, metamaterials, asymmetry.

I. INTRODUCTION

Part I [1] has introduced the overall paper and covered the topics enumerated in the abstract. This second part, after subdividing the linear category of NR systems (Tab. [1].I) into Linear Time-Invariant (LTI) and Linear Time-Variant (LTV)1 systems, expands the concepts of Part I to lossy and open systems, extends the scattering parameter concept from LTI systems to LTV Space-Time (ST) and NL systems, presents the fundamental NR characteristics and devices for these three types of systems, and finally warns against fallacious similarities between NR and other forms of asymptotic transmission.

| TIME REVERSAL Secs. [1].III-VI | LINEAR | TV (SPACE-TIME) | NONLINEAR |
|-------------------------------|--------|----------------|----------|
| ONSAGER-CASIMIR Eqs. [1].(11) | ✓      | ✓              | ✓        |
| LORENTZ NR Secs. [1].VII-IX  | ✓      | ✓              | ✓        |
| EXTENDED S-PAR. Sec. IV       | ✓      | ✓ Δ            | ✓ Δ Δ    |

TABLE I

This paradox originates in the looseness2 of the assumption that “TRS/A is equivalent to reciprocity/NR” in Sec. [1].III. This assumption is, as pointed out in Fn. [1].8, stricto senso incorrect, as the equivalence only holds in terms of absolute field levels but not field ratios! In the case of loss, as just seen, the field ratios are equal \(|\frac{P_0}{P_1}/\frac{P_1}{P_2} = 0.5 = \frac{P_0}{P_2}/\frac{P_0}{P_1}\)|, consistently with the general definition of reciprocity in Sec. [1].II, but the field levels are not \((P_0/4 \neq P_0)\), in contradiction with the definition of TR in Sec. [1].III. In this sense, a simple lossy system is perfectly reciprocal despite breaking TRS. This TRA may be seen as an expression of thermodynamical macroscopic irreversibility3.

II. RECIPROCITY DESPITE TRS-B IN LOSSY SYSTEMS

Figure 1 monitors the process of EM wave propagation in a lossy bias-less waveguide system. Let us see how such a system responds to TR (Secs. [1].III to [1].IV) by applying the TRS-B test in Sec. [1].V, as for the Faraday system in Figs. [1].2(b),(c).

![Fig. 1. TRS-B in a lossy reciprocal waveguide of length l (Sec. [1].V). Assuming that the process under consideration is the propagation of a modulated pulse, we have \(\psi^{-1}(t) = [\psi_1^{-1}(-t), \psi_2^{-1}(-t)]^T \neq [\psi_1(t), \psi_2(t)]^T = \psi(t)\), and in particular \(\psi(0) = [P_0/4, 0]^T \neq [P_0, 0]^T = \psi(0)\).

In the direct part of the process, the wave is attenuated by dissipation as it propagates from port P1 to port P2 (red curve), say from \(P_0\) to \(P_0/2\) (3 dB loss). Upon TR, the propagation direction is reversed, and loss is transformed into gain (Tab. [1].II). As a result, the wave propagates back from P2 to P1 and its power level is restored (green curve), from \(P_0/2\) to \(P_0\). However, the system has been altered. Maintaining lossy leads to further attenuation on the return trip, from \(P_0/2\) to \(P_0/4\) (6 dB loss), and hence breaks TRS. According to Sec. [1].V, this would imply NR, which is at odds with the generalized reciprocity theorem (Sec. [1].IX)!

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1Consider the isotropic dielectric medium system: \(D = \varepsilon E\). The system is clearly NL if \(\varepsilon = \varepsilon(E)\), since it excludes superposition because \(\varepsilon = \varepsilon_1(E_1 + E_2)\) [2]. However, we call here linear-TV a system with \(\varepsilon = \varepsilon(t) \neq \varepsilon(E)\) insofar such a system supports superposition, although it is often called NL due to the generation of new frequencies.

2This looseness has been tolerated because it is the ratio definition of Sec. [1].II, and not its restricted level form, that is commonly used in practice.

3Consider for instance an empty metallic waveguide. The transfer of charges from the waveguide lossy walls results in EM energy being transformed into heat (Joule first law) [3]. In theory, a Maxwell demon [4] could reverse the velocities of all the molecules of the system, which would surely reconvert that heat into EM energy. In this sense, all systems are microscopically irreversible, which is the fundamental assumption underpinning Onsager reciprocity relations [5]–[8] (Fn. [1].6). However, such reconversion is prohibited by the second law of thermodynamics, which stipulates that the total entropy in an isolated system cannot decrease over time. It would at least require injecting energy from the outside of the system! So, such a lossy system is microscopically – and hence practically! – irreversible. Loss cannot be undone; it ever accumulates over time, as in Fig. 1.
III. OPEN SYSTEMS AND THEIR TR-“LOSSY” BEHAVIOR

Consider the two-antenna open system in Fig. 2, showing the original [Fig. 2(a)], TR [Fig. 2(b)] and reciprocal [Fig. 2(c)] problems. The nature of the system is clearly altered upon TR, where the intrinsic impedance of the surrounding medium becomes negative (Tab. [1].II). This results from the fact that the radiated and scattered energy escaping the antennas in the original problem is equivalent to loss relatively to the two-port system. Such loss transforms into gain upon TR, as in Sec. II, leading to fields emerging from infinity. Upon replacing TR by reciprocal TR so as to avoid denaturing the system, one would find, as in the lossy case, reduced field levels but conserved field ratios. An open system is thus TR-wise similar to a lossy system (Sec. II).

Fig. 2. TRA and apparent (restricted) nonreciprocity of an open system composed of a dipole antenna and a patch antenna in free space.

IV. EXTENDED SCATTERING PARAMETER MODELING

The lossy/open system paradox (Secs. II/III) and the common definition in Sec. [1].II, suggest describing LTI (non)reciprocal systems in terms of field ratios. This leads to the scattering parameters or S-parameters, introduced in quantum physics in 1937 [9], used for over 70 years in microwave engineering [10]–[12], extended to power parameters for arbitrary loads in the 1960es [12], [13], and to the cross-coupled matrix theory for topologically-coupled resonators in the 2000s [14], [15]. We shall attempt here an extension of these parameters to LTV and NL systems.

Figure 3 defines an extended arbitrary P-port network as an electromagnetic structure delimited by a surface S with N waveguide terminals, Tn, supporting each a number of mode-frequency ports, Pp = Pn(ω, ω) with p = 1, 2, . . . , P (e.g. if T1 is a waveguide with the M1 = 2 modes TE10 and TM11 and the Ω1 = 2 frequencies ω and 2ω, it includes the M1Ω1 = 4 ports

\[ P_1 = P_{TR,1,2ω} \]

\[ P_2 = P_{TR,0,2ω} \]

\[ P_3 = P_{TM,1,ω} \]

\[ P_4 = P_{TM,1,2ω} \]

The transverse fields in the waveguides have the frequency-domain form [11], extended here to LTV and NL systems,

\[ \begin{bmatrix} E_{\text{L},p}(x, y, z) \\ H_{\text{L},p}(x, y, z) \end{bmatrix} = \left( a_p e^{-jβ_0 z} \pm b_p e^{+jβ_0 z} \right) \begin{bmatrix} \hat{E}_{\text{L},p}(x, y) \\ \hat{H}_{\text{L},p}(x, y) \end{bmatrix} \]

where \( \hat{E}_{\text{L},p}(x, y) \times \hat{H}_{\text{L},p}(x, y) \) is the port input/output complex wave amplitudes, related by the extended S-matrix, S, as

\[ b = S a, \quad \text{with} \quad \begin{bmatrix} a_{1} \\ a_{2} \\ \vdots \\ a_{P} \end{bmatrix} = \begin{bmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{P} \end{bmatrix} \]

\[ S = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1P} \\ S_{21} & S_{22} & \cdots & S_{2P} \\ \vdots & \vdots & \ddots & \vdots \\ S_{P1} & S_{P2} & \cdots & S_{PP} \end{bmatrix} \]

If the system is linear, each entry of the matrix can be expressed by the simple transfer function \( S_{ij} = b_i/a_j \) for all \( i \neq j \), which corresponds to the conventional definition of the S-parameters, except for the frequency port definition extension in the ST (LTV) case. If the system is NL, then \( S_{ij} = S_{ij}(a_1, a_2, \ldots, a_P) \), and therefore all the (significant) input signals must be simultaneously present in the measurement of the transfer function \( S_{ij} \), as done in Broadband Poly-Harmonic Distortion (PHD), used in the Keysight microwave Nonlinear Vector Network Analyzer (NVNA) [16]–[18].

In a LTI medium, the bianisotropic reciprocity relations [19], [20] excited by the two states \( a' \) and \( a'' \) with responses \( b' \) and \( b'' \) read now (sources are outside of the system) [21]

\[ \nabla \cdot \left( \hat{E}' \times \hat{H}' - \hat{E}'' \times \hat{H}'' \right) = j \omega \left( \hat{E}' \cdot \left( \hat{E}'' \times \hat{H}' \times \hat{H}'' \right) - \hat{H}' \cdot \left( \hat{E}'' \times \hat{E}' \times \hat{H}'' \right) \right) = 0. \]

Inserting the sum (\( \sum_{p=1}^{P} \)) of fields (1) into this equation, taking the volume integral of the resulting relation, applying the Gauss theorem and the orthogonality relation, and using the Onsager-Casimir relations [Eqs. (1),(11)], yields [21]

\[ \sum_{p=1}^{P} (b_p' a_p'' - a_p' b_p'') = b a'' - a b'' = a' a'' (S^T - S), \]

where (2) has been used to eliminate \( b''(a'', a'') \) in the last equality. This leads to the reciprocity condition \( S = S^T \), and hence to the convenient scattering-parameter NR condition

\[ S \neq S^T \Rightarrow \exists(i, j) \text{ s.t. } S_{ij} \neq S_{ji}, i = 1, 2, \ldots, N, \]

(e.g. \( S_{21} \neq S_{12} \)) which also applies to LTV and NL systems, although the current demonstration is restricted to LTI media.

In the microwave regime, these S-parameters can be directly measured (magnitude and phase) with a VNA [12] or with an NVNA. In contrast, in the optical regime no specific instrumentation is available for that and a special setup, with nontrivial phase handling, is therefore required [22].

\[ \text{Example: } S_{21} = b_2/a_1 |a_2-a_3=0 \neq S_{21} (a_3); \quad S_{21} = S_{21} (a_3, LO). \]
V. ENERGY CONSERVATION - THERMODYNAMIC PARADOX

In a lossless system, energy conservation requires that the total output power equals the total input power, or $\sum_{p=1}^{P} |b_p|^2 = \sum_{p=1}^{P} |a_p|^2$ in Fig. 3, since no power is dissipated in the system. In terms of S-matrix, this requirement translates into the unitary relation $SS^\dagger = I$ ($I$: unit matrix). Many fundamental useful facts on multi-port systems straightforwardly follow from energy conservation (e.g. [12]).

Some immediate consequences for NR are: 1) A lossless 1-port system can be only totally reflective, from $|S_{11}|^2 = 1$, even if it includes NR materials, contrary to claims in [23]. 2) A 2-port system can be magnitude-wise NR only if it is lossy; specifically, a purely reflective 2-port isolator $S = [0, 0; 1, 1]$ is impossible, since energy conservation requires $b_2^2 = a_1^2 + a_2^2$, whereas the device would exhibit $b_2^2 = a_1^2 + a_2^2 + 2a_1a_2$, with the additional term $2a_1a_2$ that may bring the total energy to be larger than the input power, depending on the relative phases of $a_1$ and $a_2$. 3) A lossless 2-port system can still be NR in phase, since $SS^\dagger = I$ does not demand $\angle S_{21} = \angle S_{12}$ [24]. 4) A lossless 3-port system can be matched simultaneously at all port only if it is NR, according to the system $SS^\dagger = I$ [12].

The case 2) has raised much perplexity in the past\(^8\), and led to the so-called “thermodynamic paradox”, according to which such a system would ever increase the temperature of the load at the passing end at the detriment of the load at the isolated end, hence violating the second law of thermodynamics, which prescribes heat transfer from hot to cold bodies. The paradox resurfaced in 1955, as Lax and Bunt pointed out the existence of lossless unidirectional eigenmodes in some ferrite-loaded waveguide structures [28], but it was resolved by Ishimaru, who showed that such a waveguide would necessarily support loss in its terminations, even in the limit of negligible material loss [29], [30].

VI. LINEAR-TI (LTI) NR SYSTEMS

LTI NR systems have the following characteristics: 1) TRS-B by TR-odd external bias $F_0$, which is often a magnetic field, $B_0$ [Fig. 1.2(c)]; 2) applicability, by linearity, to arbitrary excitations and intensities – strong NR (Tab. [1].I); 3) frequency conservation, and hence unrestricted frequency-domain description (Secs. [1], VII and [1], VIII), applicability of the Lorentz reciprocity theorem (Sec. [1].IX) and of S-parameters (Sec. IV); 4) generally based on LTI materials [31]–[39], including 2DEGs and graphene [40]–[46], or metamaterials [47]–[57] (Sec. [1], VII).

The main LTI NR systems are isolators, NR phase shifters and circulators [12], [31], [34]. **Isolators** ($S = [0, 0; 1, 0]^{10}$) are typically used to shield equipment (e.g. VNA or laser) from detuning, interfering and even destructive reflections. **NR phase shifters** ($S = [0, e^{j\Delta \varphi}; 1, 0]^{10}$), especially for gyrotors ($\Delta \varphi = \pi$) [58], combine with couplers to form isolators or circulators, provide compact simulated inductors and filter inverters, and enable NR pattern and scanning arrays. **Circulators**

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\(^8\)This started in 1885 with the comment by Rayleigh that the system composed of two Nicols sandwiching a magnetized dielectric would be “inconsistent with the second law of thermodynamics” [25], to be overruled by Rayleigh himself 16 years later [26], in reaction to studies of Wiener [27].

\(^9\)They are of resonance, field-displacement or matched-port-circulator type, and may involve resistive sheets, quarter-wave plates or polarizing grids.

\(^10\)They may be of latching (hard magnetic hysterisis) or Faraday rotation (Sec. [1].VI) type, and may involve quarter-wave plates.

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VII. LINEAR-TV (LTV) SPACE-TIME (ST) NR SYSTEMS

LTV ST\(^12\) NR systems have the following characteristics: 1) TRS-B by TR-odd external bias ($F_0$) velocity, $\nu_0$; 2) strong NR, as in Sec. VI; 3) generation of new, possibly anharmonic, frequencies, and hence restricted applicability of S-parameters (Fn. 5); 4) moving medium (moving matter, e.g. opto-mechanical) [19], [59]–[62] or modulated medium (moving perturbation, e.g. electro/acousto/NL-optic)\(^13\) [66], [67], 5) pulse/periodic and abrupt/smooth mat./pert. motion.

As an illustration, Fig. 4 graphically depicts, using an extended Minkowski diagram [65], a step ST-modulated (STM) system with interface between media of refractive indices $n_1$ and $n_2$ ($n_2 > n_1$) moving in the $-z$-direction with the constant velocity $\nu_0 = v\hat{z}$ ($v < 0$) and excited by an incident (i) wave propagating in the $+z$-direction. Upon full-TR, $\nu_0$ is reversed, which leads to identical Doppler shifts [63] in the reflected ($r$) and transmitted ($t$) waves\(^14\), as in Fig. 4(a), but alters the system. The unaltered system is TRA, and hence breaks TRS, which leads to the NR scattering in Fig. 4(b).\(^15\)

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\(^11\)They may be of 4-port Faraday rotation or 3-port junction rotation type.

\(^12\)As will be seen, NR requires also spatial inversion symmetry breaking.

\(^13\)Moving and modulated media both produce Doppler shifts [63] and NR. In contrast, only the former supports Fizeau drag [61], [64] and bianisotropy (moving perturbation, e.g. electro/acousto/NL-optic)\(^13\) [66], [67].

\(^14\)Transitions follow frequency conservation lines in the moving frame [65].

\(^15\)Purely temporal modulation (horizontal interface) [68]–[70] would clearly be insufficient for NR: spatial inversion symmetry breaking, provided here by the moving modulation, is also required to break TRS: $\tilde{\chi}^{(\nu_0)} \neq \tilde{\chi}^{(-\nu_0)}$. 

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Fig. 4. Step STM system, $t > 0$. (a) $t < 0$: TRS. (b) $t < 0$: TRA-NR. A great diversity of useful STM NR systems have been reported in recent years [71]–[89]. They are all based on...
the production of different traveling phase gradient in different directions and are, in that sense, more or less lumped/distributed [12] variations of parametric systems developed by microwave engineers in the 1950s [90]–[96].

Figure 4 shows a NR metasurface reflector application [85], [97] based on the ST modulation $n(x) = n_0 + n_m \cos(\beta_m x + \omega_m t)$, where $\omega_m/\beta_m \approx \omega_0$. The ST metasurface breaks reciprocity and hence provides a quite unique NR device by adding the spatial and temporal momenta $K_{MS}$ and $\omega_{MS}$ to those of the incident wave.

VIII. NONLINEAR (NL) NR SYSTEMS

NL NR systems have the following characteristics: 1) TRS-B by spatial asymmetry and NL self-biasing (NL triggering by wave) [98], [99]; 2) limitation to restricted excitations, intensities and isolation – weak NR (Tab. [1], I); 3) generation of new, only harmonic, frequencies, and inapplicability of superposition, and hence very restricted applicability of S-parameters (Fns. 5 and 7); 4) diversity of TRS-B approaches. 

Fig. 6 shows a simple way to achieve NL NR by pairing a linear medium and a NL lossy medium. The two media are strongly mismatched, with reflection $\Gamma$. A wave injected at port $P_1$ experiences a transmittance of $|T|^2 = 1 - |\Gamma|^2 \ll 1$, yielding a much smaller field level in the NL medium. If this level is insufficient to trigger NL loss, all the power transmitted through the interface $(|T|^2)$ reaches port $P_2$, so that $|S_{21}| = |T|$ and $|S_{11}| = |\Gamma|$. The same wave injected at $P_2$, assuming sufficient intensity to trigger NL loss, undergoes exponential attenuation $e^{-\alpha_{NL}}$ (NL length), so that $|S_{12}| = |T|e^{-\alpha_{NL}} \approx 0$. The system is thus NR, but it is a pseudo-isolator [18]: 1) it is restricted to a small range of intensities; 2) it works only for one excitation direction ($P_1 \rightarrow P_2$ or $P_2 \rightarrow P_1$) at a time, since the $P_2 \rightarrow P_1$ wave would trigger NL loss and hence also the $P_1 \rightarrow P_2$ wave; 3) it often suffers from poor isolation ($|S_{21}|/|S_{12}| = e^{\alpha_{NL}}$) and poor isolation to insertion loss ratio ($|S_{21}|/|S_{12}|$).

Ingenious variations of the NL NR device in Fig. 6 have been reported [102]–[113]. Some of them mitigate some of the aforementioned issues, but these improvements are severely restricted by fundamental limitations of NL NR [22], [101].

IX. DISTINCTION WITH ASYMMETRIC PROPAGATION

A NR system is a system that exhibits TRA field ratios between well-defined ports (Eq. (5)), which is possible only under external biasing (linear NR) or self-biasing plus spatial asymmetry (NL NR). Any system not satisfying this condition is necessarily reciprocal, despite possible fallacious asymmetries in transmission [22], [101], [114]–[120]. For instance, the system in Fig. 7 exhibits asymmetric ray propagation, but it is fully reciprocal since only the horizontal ray gets transmitted between the array ports, the $P_1 \rightarrow P_2$ oblique rays symmetrically canceling out on the right array due to opposite phase gradients. Other deceptively NR cases include: asymmetric field rotation-filtering (e.g. $\pi/2$ rotator + polarizer), where $S_{21} \neq S_{21}^{\pi/2}$, but $S_{21} = S_{21}^{\pi/2}$; asymmetric waveguide junction (e.g. step width variation), with full transmission to larger side distributed over multi-modes and small transmission from the same mode to the smaller (single-mode) side, but reciprocal mode-to-mode transmission (e.g. $S_{11} = S_{11}$, 1: small side single-mode port, 2: large side mode/port 5) [22]; asymmetric mode conversion (e.g. waveguide with nonuniform load), where even mode transmits in opposite directions with and without excitation of odd mode, without breaking reciprocity, since $S_{21} = S_{21}$ and $S_{21} = 0 = S_{21}$ [117]. In all cases, reciprocity is verified upon exchanging the source and detector.

X. CONCLUSION

This paper has presented, in the context of novel magnetless NR systems aiming at repelling the frontiers of NR technology, a global perspective of NR, with the following conclusions: 1) NR systems may be classified into linear (TI and TV-ST) and NL systems, based on TRS-B by external biasing and self-biasing plus spatial asymmetry, respectively; 2) NL NR is a weaker form of NR than linear NR, as it suffers from restricted intensities, one-way-at-a-time excitations, and poor isolation and high insertion loss; 3) lossy and open systems, although reciprocal in terms of field ratios, are TRS-A, due macroscopic irreversibility; 4) S-parameters are advantageously generalized to all types of NR systems, with some restrictions; 5) Care must be exercised to avoid confusing asymmetric transmission with NR in some fallacious systems. Nonreciprocity is a rich and fascinating concept, that has already and will continue to open new scientific and technological horizons.

16 The main difference is that the STM or parametric systems of that time were developed mostly for amplifiers or mixers, rather than NR devices.

17 This system actually includes infinitely many ports, $S_{n+1,n} = 0$ and $S_{n+1,n} = 0 \omega_{n+1} > \omega_n (n = 1, ..., x)$, but its functional reduction in the figure is meaningful the power transfer beyond $P_3$ is of no interest.

18 The device is also not a diode [100], whose NR consists in different forward/backward spectra due to positive/negative wave cycle clipping.

19 This precludes most of the applications of real isolators (Sec. VI) [101].

20 These two parameters are typically smaller than 20 dB/25 dB in NR NL structures, whereas they commonly exceed 45 dB/50 dB in NR LTI isolators.
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