Heterogeneity: method and applications for complex systems analysis

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Abstract. Socio-ecological systems like another physical systems are complex systems in which are required methods for analyzes their non-linearities, thresholds, feedbacks, time lags, and resilience. This involves understanding the heterogeneity of the interactions in time and space. In this article, we carry out the proposition and demonstration of two methods that allow the calculation of heterogeneity in different contexts. The practical effectiveness of the methods is presented through applications in sustainability analysis, land transport, and governance. It is concluded that the proposed methods can be used in various research and development areas due to their ease of being considered in broad modeling frameworks as agent-based modeling, system dynamics, or machine learning, although it could also be used to obtain point measurements only by replacing values.

1. Introduction

Understanding the complexity of the socio-ecological systems in which human beings and natural systems interact requires the identification of methods that allow the identification of non-linearities, thresholds, feedbacks, time lags, and the resilience of these systems, through integrated studies [1] that involve modeling processes in which the heterogeneity of the interactions is represented and how they affect behavior in time and space [2]. Therefore, methodologies are required to approximate the heterogeneity inherent in socio-ecological systems [3] through interdisciplinarity [4].

In landscape studies, heterogeneity has many applications on community structure ecosystem processes, and population dynamics [5]. In urban landscapes, urbanization patterns turn heterogeneity into a unique and distinctive imprint of their sociocultural characteristics [6] that, on the one hand, makes sustainable soil management difficult [7], while it provokes a game of interactions that can lead to conflict or the emergence of novelty, creativity, and innovation [8].

In socio-ecological systems, habitat heterogeneity is one of the most important variables in the study of ecosystem services [9] because it generates biodiversity conservation [10], due to the restoration and sustainability capacity it confers even in the presence of intensive agricultural systems [11], reducing the probability of invasion by non-native organisms, especially of ectotherms such as insects [12] and decreasing vulnerability to global scale disturbances such as...
as climate change [3]. Other applications also include spatial-temporal heterogeneity of diseases or their vectors [13], resilience related to phenotypic heterogeneity [14], social heterogeneity as a requirement of governance [15] and means to achieve the goals of sustainable development [8,16] the heterogeneity of fishers [17] and the heterogeneity among tourism stakeholders [18].

Among the techniques for the study of heterogeneity, the shannon diversity index (SDI) stands out, through which a synthetic measure of landscape heterogeneity is offered [19] and from others formulas have been defined [20]. Other techniques for analyzing heterogeneity include spatial techniques [13, 21], bayesian networks [3, 22] or own formulas [10] for the study of landscapes; Agent-based modeling for the study of social interactions [2, 18, 23, 24]; or the use of software such as the i-Tree Eco tool to calculate the heterogeneity of ecosystem services [6].

The purpose of this document is to present two methods that allow the calculation of the heterogeneity of socio-ecological systems and that can be easily articulated to modeling techniques such as agent-based simulation, system dynamics, machine learning, or layer algebra, among others, presenting examples that show possible results in different areas. This document is organized as follows; in section 2 two propositions are related for the evaluation of heterogeneity, in section 3 shows applications for sustainability, road transport and governance. Finally, in section 4 the conclusions of this chapter are presented.

2. Methodology

A first process has been carried out to formulate the heterogeneity in which it was considered to have comparable weighted magnitudes under some criterion that would be defined in the scope of application, giving rise to Proposition 1. As will be shown in the applications, this case can be used to establish the heterogeneity in coverage that has the weighting of the area in section 3.1, or in the analysis of land transport in section 3.2.

Proposition 1. Weighted heterogeneity; let a magnitudes set \( X = \{x_1, \ldots, x_n\} \), with weights \( w_1, \ldots, w_m, \ldots, w_n \), where \( w_m \) is the weight of \( x_m = \max(X) \), such that \( k = \sum_{i=1}^{n} w_i \), \( a = n/k \), \( a \cdot w_m > 1 \), and \( x_T = \sum_{i=1}^{n} x_i \). The heterogeneity \( H \) of the set \( X \) is given by the Equation (1).

\[
H = \frac{n \cdot (H_v - 1) + a \cdot w_m}{a \cdot w_m - 1} \cdot 100\% ,
\]

where \( H_v \) is defined by Equation (2).

\[
H_v = 1 - \frac{1}{k \cdot x_T} \sum_{i=1}^{n} x_i \cdot w_i .
\]

Proof. For the demonstration, we start from Equation (2) of weighted averages considering \( \bar{x}_i = x_i/x_T \); then, we obtain the Equation (3).

\[
\bar{x} = \frac{\sum_{i=1}^{n} \bar{x}_i w_i}{\sum_{i=1}^{n} w_i} = \frac{1}{k \cdot x_T} \sum_{i=1}^{n} x_i \cdot w_i .
\]

In Equation (3), note that \( \lim_{n \to \infty} \bar{x} = 0 \), then let us propose that there is a heterogeneity that we call virtual \( H_v = 1 - \bar{x} \), like Equation (2). First case (minimum heterogeneity): if \( x_i \to x_T \), \( w_m \) is the weight of \( x_m = \max(X) \), \( x_T = \sum_{i=1}^{n} x_i \), and \( k = \sum_{i=1}^{n} w_i \), then \( H_v = 1 - w_m/k \). But we can consider \( a > 0 \) such that \( k = n/a \), so, virtual heterogeneity is: \( H_{v1} = 1 - a w_m/n \).
Second case (maximum heterogeneity): if \( x_i = x_j \), and taking into account that \( n = x_T/x_i \), then virtual heterogeneity is: \( H_{v2} = 1 - 1/n \). Si \( a w_m > 1 \), the curves \( H_{v1} \) and \( H_{v2} \) delimit a region in which a \( H_v \) value between the two allows the heterogeneity to be defined in percentage terms, as Equation (4).

\[
H = \frac{H_v - H_{v1}}{H_{v2} - H_{v1}} \cdot 100\%.
\] (4)

Replacing the values of \( H_{v1} \) and \( H_{v2} \) we obtain the Equation (1). In a second process, the heterogeneity of a magnitudes set in which there was no weighting was taken into account, as in the case of governance symmetries that will be presented in section 3.3.

Proposition 2. Non-weighted heterogeneity; let a magnitudes set \( X = \{x_1, \ldots, x_n\} \), where \( x_T = \sum_{i=1}^{n} x_i \). The heterogeneity \( H \) of the set \( X \) is given by the Equation (5).

\[
H = \frac{n^2 \cdot (H_v - 1) + n}{n - 1},
\] (5)

where \( H_v \) is defined by Equation (6).

\[
H_v = 1 - \frac{1}{n} \sum_{i=1}^{n} \left( \frac{x_i}{x_T} \right)^2.
\] (6)

Proof. For the demonstration, we start from Equation (7).

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{x_i}{x_T} \right)^2.
\] (7)

Note that \( \lim_{n \to \infty} x_i = 0 \), then let us propose that there is a heterogeneity that we call virtual \( H_v = 1 - \bar{x} \), as in Proposition 1, like Equation (6). First case (minimum heterogeneity): if \( x_i \to x_T \), and \( x_T = \sum_{i=1}^{n} x_i \), then \( H_v = 1 - 1/n \). Second case (maximum heterogeneity): if \( x_i = x_j \), and taking into account that \( n = x_T/x_i \), then virtual heterogeneity is: \( H_{v2} = 1 - 1/n^2 \). Si \( n > 1 \), the curves \( H_{v1} \) and \( H_{v2} \) delimit a region in which a \( H_v \) value between the two allows the heterogeneity to be defined in percentage terms, as Equation (8).

\[
H = \frac{H_v - H_{v1}}{H_{v2} - H_{v1}} \cdot 100\%.
\] (8)

Replacing the values of \( H_{v1} \) and \( H_{v2} \) we obtain the formula in Equation (5). Next, applications of these heterogeneity methods proposed in Proposition 1 and Proposition 2 will be presented for situations of sustainability, land transport, and governance.

3. Results and discussion
Socio-ecological systems are complex systems in which various metrics such as heterogeneity are measured; in this section, we will present results in which the method of Proposition 1 and Proposition 2 are applied for the heterogeneity calculation and the result obtained will be discussed.
3.1. Sustainability

Sustainability is a concept with a broad scope in socio-ecological systems; among the aspects studied in a sustainability analysis for agricultural landscapes [25] is found the heterogeneity of land uses, which considered the existence of the use and its extension. In this case, we will show the results of the heterogeneity calculation in a hypothetical landscape in which must be ten agricultural uses, using the method proposed in Proposition 1. Each agricultural use has an area and a coverages number that assign the weighting, as can be seen in Table 1.

Table 1 presents the results of the extreme cases of a heterogeneity evaluation in the agricultural uses of a rural landscape. The left side of this table are presented the data of 10 agricultural uses in which only one of them dominates the study area and the others are so small, showing the study area is homogeneous, while the right side shows all the uses have the same extension, showing the area is heterogeneous. Note on the left side that, if there is an agricultural use that almost totalizes the entire landscape, the heterogeneity is 0%, while, on the right side, if all the agricultural uses have the same extension, then the heterogeneity will be 100%, as expected.

Results in Table 1 shows that in extreme cases, the method of Proposition 1 responds adequately to what is expected and can be recommended for calculating the heterogeneity of agricultural uses in rural landscapes in the sustainability analysis framework.

Table 1. Heterogeneous and homogeneous comparison in agricultural landscape.

| Homogeneous agricultural landscape | Heterogeneous agricultural landscape |
|-----------------------------------|--------------------------------------|
| Agriculture use | Area | Covers weight | Agriculture use | Area | Covers weight |
| 1 | 0 | 3 | 1 | 100 | 5 |
| 2 | 0 | 5 | 2 | 100 | 4 |
| 3 | 0 | 2 | 3 | 100 | 1 |
| 4 | 0 | 1 | 4 | 100 | 5 |
| 5 | 0 | 1 | 5 | 100 | 5 |
| 6 | 0 | 3 | 6 | 100 | 3 |
| 7 | 0 | 5 | 7 | 100 | 2 |
| 8 | 0 | 1 | 8 | 100 | 4 |
| 9 | 0 | 4 | 9 | 100 | 3 |
| 10 | 100 | 2 | 10 | 100 | 3 |
| Total | 100 | 27 | Total | 1000 | 35 |
| Heterogeneity | 0% | | Heterogeneity | 100% | |

3.2. Land transport

Population growth, dynamism in cities, and electronic commerce have increased the need for transportation [26]. Transport and social development are directly related to the territorial expansion that occurred thanks to transport, this given that the cities were compact since all movements were carried out on foot until the railroad and later the automobile opened the road to expansion we see today [27]. This sector is susceptible to both technological and energy transitions and can be used to show the heterogeneity proposed in the methodology.

3.2.1. Vehicle fleet. Now, we will show the measure of heterogeneity of the vehicle fleet using official data of the Colombian national fleet for 2019, obtained from the “Ministerio de Transporte” [28]; here is considered six of the fourteen vehicles categories of the national fleet: (1) automobiles, (2) pickup truck, (3) microbus, (4) bus (BRT), (5) medium bus, and (6) motorcycles. According to the number of people it is capable of transporting has been assigned a weight as shown in Table 2; the parameters were: \( n = 6 \), \( a = 0.0447 \), \( w_m = 2 \), \( H_v = 0.9716 \).
According to the weighting given to each of the vehicle categories, was obtained a heterogeneity of 8.82% from the Proposition 1 which shows the little variety in the participation of some categories in the total vehicle fleet, particularly the categories that transport fewer people are those with the highest participation in the number of vehicles. It is a relevant result for transport public policies aimed at sustainable transport because it is a strategy for reducing emissions.

Table 2. Heterogeneity evaluation of the vehicle fleet, case study Colombia [28].

| Categories of vehicles | Number of vehicles 2019 | Weighing |
|------------------------|-------------------------|----------|
| Automobiles            | 3,601,446               | 5        |
| Medium bus             | 35,636                  | 30       |
| Pickup truck           | 1,368,567               | 7        |
| Microbus               | 82,946                  | 30       |
| Bus (BRT)              | 80,475                  | 60       |
| Motorcycles            | 9,117,525               | 2        |
| **Total**              | 14,286,595              | 134      |
| **Heterogeneity**      |                         | 8.82%    |

3.2.2. Energetic. In other application about land transportation, have been considered the heterogeneity measure of the energy share in the vehicle fleet. The data and weights are found in Table 3; on the right side of the table, the weights given to each of the energy sources are shown. This case is very particular, given the interest in achieving an energy transition in transport, evaluating the participation of each of the energetics in the fleet allowed us to know their heterogeneity. Fossil fuels are responsible for a large number of emissions, the transport sector is responsible for more than 20% of global greenhouse gas emissions, mostly emitted in cities [29, 30]. Therefore, the diversification of energy in transport becomes a relevant issue for the management of climate change; the parameters were: \( n = 5 \), \( a = 0,5555 \), \( w_M = 1 \), \( H_v = 0,8876 \).

In Table 3, the weighting corresponds to a value from 1 to 3, in which 3 corresponds to zero-emission fuels (electricity), 2 to low-emission fuels (hybrids and natural gas), and 1 to fossil fuels (gasoline and diesel). It is relevant to highlight that for this exercise, we consider energy sources with zero and low emissions. In the heterogeneity evaluation, from the Proposition 1, the result obtained was 1.35%, which shows a low percentage of heterogeneity for zero and low-emission energetics; this reflects how non-participatory electric vehicles are compared to vehicles that run on fossil fuels. We could conclude that the energy transition towards zero and low emission fuels in Colombia is hit back and it may take time to achieve it.

Table 3. Heterogeneity evaluation of the energetic in the vehicle fleet, case study Colombia [28].

| Energetic     | Number of vehicles 2019 | Weighing |
|---------------|-------------------------|----------|
| Gasoline      | 14,300,292              | 1        |
| Diesel        | 1,058,988               | 1        |
| Hybrids       | 131,825                 | 2        |
| Natural gas   | 17,179                  | 2        |
| Electric      | 9,431                   | 3        |
| **Total**     | 15,517,715              | 9        |
| **Heterogeneity** |                     | 1.35%    |
3.3. Governance

Governance emerges as an alternative to state control and privatization in the management of common use resources, allowing stakeholders to establish agreements that allow them to obtain symmetrical benefits [31]. In this framework, two expected symmetries are gender heterogeneity and actors heterogeneity in decision-making.

Using the method of Proposition 2, the results for the extreme cases evaluation of gender participation are presented in Table 4 and the results of a usual situation of governance in which only the government and investors participate, leaving aside the other actors in the landscape (left side of the Table 5), and the case in which all actors participate in equitable conditions (right side of the Table 5). In all cases, the expected heterogeneity values are obtained.

The Table 4 presents the results of the extreme cases of a heterogeneity evaluation in the governance participation; the left side shows the heterogeneity with a dominant participation of the male gender, while the right side shows the heterogeneity if both genders have the same participation.

Table 5 presents the results of a usual situation of governance. The left side shows the heterogeneity with dominant participation of government and investors, while the right side shows the heterogeneity if all actors participate in equitable conditions.

This results show that the method of Proposition 2 also responds adequately to what is expected and can be recommended for calculating the heterogeneity in governance analysis.

Table 4. Extreme cases of a heterogeneity evaluation in the governance participation.

| Gender | Participation | Gender | Participation |
|--------|---------------|--------|---------------|
| Male   | 50            | Male   | 26            |
| Female | 0             | Female | 26            |
| Total  | 50            | Total  | 52            |
| Heterogeneity | 0.00% | Heterogeneity | 100.00% |

Table 5. Possible situations in a governance analysis.

| Actor type | Participation | Actor type | Participation |
|------------|---------------|------------|---------------|
| Indigenous | 0             | Indigenous | 1             |
| Peasant    | 0             | Peasant    | 1             |
| Afro       | 0             | Afro       | 1             |
| NGO’s*     | 0             | NGO’s*     | 1             |
| Investor   | 1             | Investor   | 1             |
| Manufacturer | 0       | Manufacturer | 1         |
| Government | 6             | Government | 1             |
| Total      | 7             | Total      | 7             |
| Heterogeneity | 28.57% | Heterogeneity | 100.00% |

* Non-governmental organization (NGO).
4. Conclusions
Two different methods for calculating heterogeneity in systems were proposed and demonstrated. In the first case, the elements of the system had weights that had to be considered, as in the examples shown in the sustainability and transport analyzes. In the second case, weights on the elements of the system are not considered, as in the examples shown for the governance analysis. Using extreme value tests in the applications and showing results with real data, it was shown that the performance of the methods was adequate.

In this way, it is concluded that the methods presented in Proposition 1 and Proposition 2 can be used in various research and development areas. In addition, the methods proposed in this article allow their consideration in broad modeling frameworks such as agent-based modeling, system dynamics, or machine learning, in this sense, the inclusion of heterogeneity methods in these frameworks is proposed as future work.

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