Small Superimposed Radial Oscillations of an Arterial Tissue: Before and After Angioplasty

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Abstract. Atherosclerosis is a major topic in cardiovascular disease, wherein the arterial wall thickens and subsequent hardening is observed. Lumen size is reduced during this and thus the flow of blood is adversely affected. Angioplasty is a mechanical procedure to widen blocked arterial tissue. Expansion of the arterial wall during angioplasty causes some micro-damage and is the reason for a successful angioplasty. The proposed work shed some light in the context of angioplasty, by mimicking the arterial tissue as an incompressible, isotropic, and homogeneous damaged Demiray cylindrical material tube. It is noted that the in vivo oscillations of the arterial tissue is under the physiological loading domain. The effect of damage on the small superimposed radial oscillations of the Demiray tube is explored. The obtained results for the Demiray tube are interpreted with the clinical data of the existing literature. It is quantified that both the pressure and frequency ratio for the virgin and damaged arterial tissue, respectively, are more before angioplasty and vice-versa.

List of Symbols

B: Left Cauchy-Green deformation tensor
P: Normalized pressure
b: Damage parameter
u: Square of the radial stretch
x(t): Motion of the inner surface of the cylindrical tube
I₁: First invariant of the left Cauchy-Green deformation tensor
I₁M: Maximum value of the first invariant of the left Cauchy-Green deformation tensor in its deformation history
R₁ and R₂: Undeformed inner and outer radius in reference configuration
p₁: Internal pressure applied to the tube
p₂: External pressure applied to the tube
p₀ = p₁ − p₂: Inflation pressure
r₁ and r₂: Undeformed inner and outer radius in current configuration
α: Damage function
γ: Demiray material constant
λ: Stretch
µ: Thickness of the cylindrical tube
ω²: Squared normalized frequency
1. Introduction

Atherosclerosis is an inflammatory disease of the arterial wall and is a main topic in cardiovascular disease. Lumen size is reduced during this and thus flow of blood is adversely affected. Angioplasty is an effective technique to treat the same at early stages. Expansion of the arterial wall during angioplasty causes some micro-damage and is the reason for a successful angioplasty [1]. Proposed study mimicks the arterial tissue as an incompressible, isotropic, and homogeneous Demiray cylindrical material tube and investigates the effect of damage on the small amplitude radial oscillations of the tube.

Knowles’ in [2] and [3] first solved the problem of finite amplitude pure radial oscillations of an axially constrained, incompressible, isotropic hyperelastic long thick-walled rubber-like tube. Later in [4], Beatty examines the forced radial oscillations of an incompressible tube for the undamaged virgin case. Driven by this and the surgical procedure of angioplasty, the present work investigates the radial oscillations of a damaged Demiray tube under pressure. The concerned problem is formulated next, followed by the results and discussions in Section 3. Several inferences are then remarked in Section 4.

2. Problem Formulation

An incompressible, isotropic and homogeneous damaged Demiray cylindrical tube is considered under constant inflation pressure.

![Figure 1. Schematic diagram of a homogeneous damaged Demiray cylindrical tube under constant inflation pressure.](image)

For the tube problem, we define the following ratios:

\[ x(t) \equiv \frac{r_1(t)}{R_1} = \lambda_1(t), \tag{1} \]

\[ \mu \equiv \frac{R_2^2}{R_1^2} - 1, \tag{2} \]

and

\[ u = \frac{r_2^2}{R_2^2} = \lambda^2, \tag{3} \]

wherein \( x(t) \) defines the motion of the inner surface of the cylindrical tube, \( \mu \) is the tube wall thickness parameter and \( u \) is square of the stretch. Following [5] and [6] the equation of motion for the damaged incompressible tube is obtained as

\[ x \log \left( 1 + \frac{\mu}{x^2} \right) \ddot{x} + \left( \log \left( 1 + \frac{\mu}{x^2} \right) - \frac{\mu}{\mu + x^2} \right) \dot{x}^2 + g(\alpha, x, \mu) = 0, \tag{4} \]
wherein \( g(\alpha, x, \mu) \)
\[
g(\alpha, x, \mu) \equiv \frac{2}{\rho R^2} (z(\alpha, x, \mu) - p_0), \tag{5}
\]
\( z(\alpha, x, \mu) \) is defined as
\[
z(\alpha, x, \mu) \equiv \int_{(\mu+x_e^2)/(\mu+1)}^{x_e^2} \left( \frac{1}{u^2} \right) e^\gamma(I_1-3) \alpha(u) \ du, \tag{6}
\]
and \( p_0 = p_1 - p_2 \) defines the inflation pressure. Herein, \( I_1 = 1 + u + 1/u \) is the first invariant of the left Cauchy-Green deformation tensor \( B \) and
\[
\alpha = e^{-b\sqrt{I_{1M}-I_1}}, \tag{7}
\]
is the front factor damage function ([7]) and exhibits the selective memory property of the arterial tube which has undergone angioplasty. Further, \( \gamma, b \) signifies the dimensionless positive material constants and \( I_{1M} \) is the maximum previous ever deformation.

Proceeding as in [6], the respecting expressions for the normalized pressure and the squared normalized frequency for a damaged Demiray tube are obtained explicitly as
\[
P(x_e) = \int_{(\mu+x_e^2)/(\mu+1)}^{x_e^2} \left( \frac{1}{u^2} \right) e^\gamma(I_1-3) \alpha(u) \ du, \tag{8}
\]
and
\[
\tilde{\omega}^2 = \frac{1}{2 \log \left( 1 + \frac{\mu}{x_e^2} \right)} \left[ \frac{1}{x_e^2} e^\gamma(I_1(x_e^2)-3) \alpha(x_e^2) \right. \\
-\left. \frac{(2\mu + 1 + x_e^2)}{(\mu + x_e^2)^2} e^\gamma(I_1(\mu+x_e^2)/\mu+1)-3) \alpha \left( \frac{\mu + x_e^2}{\mu + 1} \right) \right]. \tag{9}
\]

3. Results and Discussions
The normalized pressure and frequency responses for the Demiray biomaterial tube are presented in Figures 2 and 3 below. The oscillations of an arterial tissue under physiological loading domain are clinically verified. The shaded vertical strip here signifies the physiological loading domain. Several values of the damage parameter \( b \) are considered as \( b = [1.6; 1.986; 3.402] \). It is easily observed that with the increasing value of the damage parameter \( b \) the difference between the virgin and damaged curves increases. Hence, if we obtain the ratio between virgin and damaged pressure at a given stretch \( x_e \) then the same will increase with the increasing extent of the damage. The similar increase in frequency ratio is also observed. Additionally, we may observe that both the pressure and frequency values for the virgin case are evaluated at low stretch values because of the reduced compliance of the arterial wall. We take, say, \( x_e = 1.1 \) for the virgin analysis. The corresponding pressure and frequency values for the damaged case are evaluated at \( x_e = 1.48 \) which is quite near to the mean stretch value in the physiological loading domain.

Further, clinically the normalized pressure \( P \) is relevant as the difference \( (P_a - P_d) \); wherein \( P_a \) is the mean aortic pressure and \( P_d \) is the pressure distal to the narrowed part of artery. While the frequency ratio is admissible as \( \tilde{\omega}_v^2/\tilde{\omega}_d^2 = (P/V)_v/(P/V)_d \); wherein \( P \) is the mean pressure gradient per beat given as \( P \approx \frac{dP}{dt} \approx \frac{(P_a - P_d)}{\text{beat} - \text{interval}} \) and \( V \) is the average blood flow velocity.
For $b = 1.986$ the obtained pressure and frequency ratios from Figures 2 and 3 are 1.5481 and 2.7524, respectively. These data matches well with the clinical data in [8], having the corresponding pressure ratio as 1.7406 and the frequency ratio 2.5538. All our obtained data are tabulated in Table 1.

Table 1. Table showing the data for normalized pressure and squared normalized frequency ratios $P_v/P_d$ and $\tilde{\omega}_v^2/\tilde{\omega}_d^2$ with $\mu = 1.083$, $\gamma = 0.908$, $I_{1M} = 4.3$ and $b = [1.6, 1.986, 3.402]$.

| $x_{ev}$ | $x_{ed}$ | $b$ | $P_v/P_d$ | $\tilde{\omega}_v^2/\tilde{\omega}_d^2$ |
|----------|----------|-----|-----------|---------------------------------|
| 1.6      | 1.48     |     | 1.0815    | 2.1078                          |
| **1.986**| **1.5481**| **2.7524**|            |                                 |
| 3.402    | 5.7362   | 7.6410                                 |

Figure 2. Graphical representation between the normalized pressure and static stretch of a thick Demiray tube for fixed $\mu = 1.083$, $\gamma = 0.908$, $I_{1M} = 4.3$ and $b = [1.6, 1.986, 3.402]$. 
Figure 3. Graphical representation between the squared normalized frequency and static stretch of a thick Demiray tube for fixed $\mu = 1.083$, $\gamma = 0.908$, $I_{1M} = 4.3$ and $b = [1.6, 1.986, 3.402]$.

4. Concluding Remarks

In the physiological loading domain [9] the amplitude of oscillation of an arterial tissue is small. The obtained results are matched with the clinical data in the existing literature and it is concluded that in the physiological loading domain, the small amplitude frequency of a virgin artery is always greater than that of the corresponding damaged artery after angioplasty. Moreover, the developed pressure gradient across narrowed part of the artery is always greater before angioplasty and vice versa. The obtained results in Table 1 above matched well with the clinical data observed in [8]. Additionally, the data in other clinical literature may be matched by suitable choice of the damage parameter $b$ values.

The local radial bulging characteristic of arterial tissue at high pressure is a very important phenomenon to look for. But the damaged Demiray tube considered here does not exhibited the required bulging. The same is observed for the virgin case, too. Consequently, any arbitrary Demiray material tube is infinitesimally stable at any given equilibrium pressure and applies to both the damaged and virgin cases.

References

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