Optimize TSK Fuzzy Systems for Big Data Classification Problems: Bag of Tricks

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Abstract—Takagi-Sugeno-Kang (TSK) fuzzy systems are flexible and interpretable machine learning models; however, they may not be easily applicable to big data problems, especially when the size and the dimensionality of the data are both large. This paper proposes a mini-batch gradient descent (MBGD) based algorithm to efficiently and effectively train TSK fuzzy systems for big data classification problems. It integrates three novel techniques: 1) uniform regularization (UR), which is a regularization term added to the loss function to make sure the rules have similar average firing levels, and hence better generalization performance; 2) random percentile initialization (RPI), which initializes the membership function parameters efficiently and reliably; and, 3) batch normalization (BN), which extends BN from deep neural networks to TSK fuzzy systems to speedup the convergence and improve generalization. Experiments on nine datasets from various application domains, with varying size and feature dimensionality, demonstrated that each of UR, RPI and BN has its own unique advantages, and integrating all three together can achieve the best classification performance.

Index Terms—Batch normalization, big data, mini-batch gradient descent, random percentile initialization, TSK fuzzy classifier, uniform regularization

I. INTRODUCTION

Takagi-Sugeno-Kang (TSK) fuzzy systems have achieved great success in numerous applications, including both classification and regression problems. However, they may not be easily applicable to big data problems, due to the lack of efficient training algorithms. Our very recent research proposed an efficient and effective training algorithm for TSK fuzzy systems for big data regression problems. This paper focuses on TSK fuzzy systems for big data classification problems.

Let the training set be $\mathcal{D} = \{x_n, y_n\}_{n=1}^N$, in which $x_n = [x_{n,1}, ..., x_{n,D}]^T \in \mathbb{R}^{D \times 1}$ is a $D$-dimensional feature vector, and $y_n$ the corresponding class label. For big data applications, both the size ($N$) and the feature dimensionality ($D$) of the data may be very large. The goal is to train a TSK fuzzy classifier efficiently and effectively on $\mathcal{D}$ so that it can have good generalization performance on unseen test data.

The first step in training a TSK fuzzy system is to define the objective function to be optimized. For regression problems, the objective is usually to minimize the root mean squared error. For classification problems, the objective can be maximizing the classification accuracy, or minimizing the cross-entropy between the estimated class probabilities and the true class probabilities. Furthermore, regularization terms can be added to the classification accuracy or cross-entropy objective function so that the fuzzy classifier parameters can possess some desirable properties, such as sparsity ($\ell_1$ regularization) or small magnitude ($\ell_2$ regularization). This paper proposes a novel uniform regularization (UR) term, which forces each rule to contribute somewhat equally to the output, and hence helps generalization.

The second step in training a TSK fuzzy system is to determine the number and format of the rules. Assume for a $C$-class classification problem on $\mathcal{D}$, the TSK fuzzy system has $R$ rules, each in the form of:

$$\text{Rule}_r : \text{IF } x_1 \text{ is } X_{r,1} \text{ and } ... \text{ and } x_D \text{ is } X_{r,D},$$

$$\text{THEN } y_r^c(x) = b_{r,0}^c + \sum_{d=1}^{D} b_{r,d}^c \cdot x_d \text{ and } ...$$

(1)

where $X_{r,d}$ ($r = 1, ..., R; d = 1, ..., D$) is the membership function (MF) for the $d$-th attribute in the $r$-th rule, and $b_{r,0}^c$ and $b_{r,d}^c$ ($c = 1, ..., C$) are the consequent parameters for the $c$-th class. The easiest way to determine $R$ is to specify it according to the designer’s experience or knowledge about the problem. Another approach is to first specify the number of MFs in each input domain, and then $R$ is the number of input space partitions. For example, if each input has $k$ MFs, then $R = k^D$. In this paper, we assume $R$ is already given, and the TSK rules are in the format of (1).

Next, we need to optimize the antecedent and consequent parameters of the rules. There are two steps: initialization and fine-tuning.

The rule parameters can be initialized randomly, subject to some constraints, e.g., the mean and standard deviation of a Gaussian MF should be within a certain range, and the three parameters for determining a triangular MF should be in ascending order, etc. The antecedent parameters can also be initialized through clustering, or by partitioning each input domain according to the user’s preference, e.g., uniformly. However, all these approaches have limitations. As will be shown later in this paper, random initialization may lead to a TSK fuzzy system with many zero rule firing levels, and hence many parameters cannot be updated in gradient descent (GD) based optimization algorithms. Clustering based initialization, e.g., $k$-means clustering and fuzzy $C$-means clustering, are very inefficient when the size of the data is large, and they also suffer from the curse of dimensionality, be-
cause the distinguishing power of Euclidean distance vanishes in a high dimensional space (the distance differences between different pairs of samples become very small). Finally, uniform initialization, also called grid partitioning in fuzzy systems literature, can easily result in rulebase explosion (recall that $R = k^D$, where $k$ is the number of MFs in each input domain), and cannot be used when a specific $R$ is given. This paper proposes a random percentile initialization (RPI) approach, which is both efficient and effective for big data applications.

As summarized in [2], there are generally three different strategies for fine-tuning the parameters of a TSK fuzzy system, after initialization: evolutionary algorithms [3], GD [2], and GD plus least squares estimation (LSE) [8]. They all have their limitations. Evolutionary algorithms need to maintain a population of candidate solutions, and usually the entire training set $D$ is used to evaluate the fitness of each candidate solution, and hence may be too slow and too memory-hungry for big data applications. Batch GD (using the entire $D$ to compute the gradients) cannot be used either when $N$ is large, again because there may not be enough memory to load the entire training set for computing the gradients. Stochastic GD (using one training example each time to compute the gradients) may be used to alleviate this problem, but the stochastic gradients usually have very large variance, and hence the training is slow and unstable. GD+LSE is made popular by the adaptive-network-based fuzzy inference system (ANFIS) [8], where the antecedent parameters are optimized by GD, and the consequent parameters by LSE. It converges very fast; however, the GD part suffers from the same problems as batch GD described just now. Additionally, as shown in [2], it can result in significant overfitting in regression problems.

Our previous research [2] introduced mini-batch GD (MBGD), which is pervasively used in the training of deep neural networks [9], [10], to the training of TSK fuzzy systems for big data regression problems. MBGD randomly selects a small number (typically 32 or 64) of training examples in each mini-batch to compute the gradients and update the model parameters, and hence can handle big data better than batch GD, and is also more stable and more efficient than stochastic GD. This paper again uses MBGD to train TSK fuzzy systems for big data classification problems. Moreover, inspired by the great success of batch normalization (BN) [11] in deep learning, we also propose a BN approach for training TSK fuzzy classifiers, and validates its superior performance in big data applications.

In summary, the main contributions of this paper are:

1) We introduce a novel UR term to the cross-entropy loss function in training TSK fuzzy classifiers, and show that it improves the generalization performance of TSK fuzzy classifiers.

2) We propose an RPI approach to initialize the parameters of a TSK fuzzy classifier, and demonstrate its efficiency and robustness.

3) We extend BN from the training of deep neural networks to the training of TSK fuzzy classifiers, and show that it can speed-up the convergence and improve the generalization performance.

The remainder of this paper is organized as follows: Section II introduces the proposed UR, RPI and BN approaches. Section III presents the experimental results to validate the performances of UR, RPI and BN. Section IV draws conclusion.

II. THE PROPOSED ALGORITHMS

This section introduces the details of the TSK fuzzy classifier under consideration, our proposed UR for regularizing the loss function, RPI for rule antecedent parameter initialization, and BN for more efficient and effective training of the TSK fuzzy classifier.

A. The TSK Fuzzy System

The TSK fuzzy system has $R$ rules, each in the form of (1). Gaussian MFs are considered in this paper. The membership grade of $x_d$ on $X_{r,d}$ is:

$$\mu_{X_{r,d}}(x_d) = \exp \left(-\frac{(x_d - m_{r,d})^2}{2\sigma_{r,d}^2}\right),$$  \hspace{1cm} (2)

where $m_{r,d}$ and $\sigma_{r,d}$ are the center and the standard deviation of the Gaussian MF, respectively.

The output of the TSK fuzzy system for the $c$-th class is:

$$y^c(x) = \frac{\sum_{r=1}^{R} f_r(x) y^c_r(x)}{\sum_{r=1}^{R} f_r(x)}, \hspace{1cm} (3)$$

where

$$f_r(x) = \prod_{d=1}^{D} \mu_{X_{r,d}}(x_d) = \exp \left(-\sum_{d=1}^{D} \frac{(x_d - m_{r,d})^2}{2\sigma_{r,d}^2}\right)$$ \hspace{1cm} (4)

is the firing level of Rule $r$. We can also re-write (3) as:

$$y^c(x) = \sum_{r=1}^{R} T_r(x) y^c_r(x),$$ \hspace{1cm} (5)

where

$$T_r(x) = \frac{f_r(x)}{\sum_{i=1}^{R} f_i(x)}$$ \hspace{1cm} (6)$$

is the normalized firing level of Rule $r$.

Once the output vector $y(x) = [y^1(x), ..., y^C(x)]^T$ is obtained, the input $x$ is assigned to the class with the largest $y^c(x)$.

To optimize the TSK fuzzy system, we need to fine-tune the antecedent MF parameters $m_{r,d}$ and $\sigma_{r,d}$, and the consequent parameters $b_{c,r}$ and $b_{c,r,d}$, where $r = 1, ..., R$, $d = 1, ..., D$, $c = 1, ..., C$.

B. Uniform Regularization (UR)

A mixture of experts (MoE) [12] model is shown in Fig. I

It trains multiple local experts, each taking care of only a small local region of the input space. For a new input, the gating network determines which experts should be used for it, and then aggregates the outputs of these experts by a weighted average.

1However, our algorithm can also be extended to other types of MFs, as long as they are differentiable.
we first define \( K \) set to \( 1 \). softmax probabilities [obtained by applying a training samples, \( M \) training the TSK fuzzy classifier, i.e., for each mini-batch with where \( \ell \) is the cross-entropy loss between the estimated class true class probabilities, \( \ell \) and the trade-off \( \gamma \) of similarity [17], we can extend BN to the optimization corresponding to the linguistic terms low and high, respectively. For \( K = 3 \), we can set the split points as \( s = (5, 50, 95) \), corresponding to the linguistic terms low, medium, and high, respectively.

\[ \ell_{UR} = \frac{1}{N} \sum_{n=1}^{N} \left( \frac{1}{R} \sum_{r=1}^{R} f_r(x_n) - \tau \right)^2, \tag{7} \]

\[ \ell = \ell + \alpha \ell_2 + \lambda \sum_{r=1}^{R} \left( \frac{1}{M} \sum_{n=1}^{M} f_r(x_n) - \frac{1}{R} \right)^2, \tag{8} \]

where \( \ell \) is the cross-entropy loss between the estimated class probabilities [obtained by applying a softmax operation to \( y(x) \)] and the true class probabilities, \( \ell_2 \) the L2 regularization of the rule consequent parameters, and \( \alpha \) and \( \lambda \) the trade-off parameters.

\( C. \) Random Percentile Initialization (RPI)

Clustering algorithms, such as \( k \)-means [5], [18] and fuzzy \( c \)-means [3], are frequently used to initialize the MFs in the rules. However, both approaches are time-consuming on big data, and suffer significantly from the curse of dimensionality [19], [20].

Next, we propose a more efficient and reliable RPI approach to initialize Gaussian MFs. For input training data \( \{x_n\}_{n=1}^{N} \), we first define \( K \) split points \( \{s_k\}_{k=1}^{K} \), where \( 0 < s_k < 100 \).

Then, we find the corresponding \( s_k \) percentile value on the \( d \)-th attribute, which is denoted as \( p_{k,d} \). The center of the Gaussian MF \( X_{r,d} \) is then randomly selected from \( p_{k,d} \), \( k = 1, \ldots, K \).

For \( K = 2 \), we can set the split points as \( s = (25, 75) \), corresponding to the linguistic terms low and high, respectively. For \( K = 3 \), we can set the split points as \( s = (5, 50, 95) \), corresponding to the linguistic terms low, medium, and high, respectively.

\[ x'_n = \frac{x_n - m_B}{\sqrt{\sigma_B^2 + \epsilon}} + \beta, \tag{9} \]

where \( m_B \) and \( \sigma_B \) are the mean and standard deviation of the mini-batch, respectively, \( \gamma \) and \( \beta \) are parameters to be learned during training, and \( \epsilon \) is usually set to 1e-8 to avoid being divided by zero. During training, an exponential weighted average of \( m_B \) and \( \sigma_B \) are recorded so that they can be used at the test phase.

There has not been an agreement on exactly why BN works in deep learning [11], [24], [25]. Some authors argued that it smoothes the loss landscape, and also reduces the dependency on the training hyper-parameters, such as the learning rate and the regularization weights [25]. However, that is beyond the scope of this paper.

Since a TSK fuzzy system and a neural network share lots of similarity [17], we can extend BN to the optimization of TSK fuzzy classifiers and expect good performance. A straightforward extension is shown in Fig. 2(a) where we perform BN on the input \( x_n \) to obtain \( x'_n \), and use \( x'_n \) in both the antecedents and consequents of the rules. However, our experiments in Section III-E show that this approach may not result in improved performance, because the MFs, which are initialized from \( x_n \), are not consistent with \( x'_n \). A better

\[ \text{Fig. 1. Mixture of experts (MoE).} \]
BN approach is illustrated in Fig. 2(b), where the normalized \( x_n' \) is only used in the consequents.

**Fig. 2. Two possible approaches for performing BN in a TSK fuzzy system.**
(a) BN for both the antecedents and the consequents; (b) BN for only the consequents. The latter is used in this paper.

At the testing phase, the BN operation can be merged into the consequent layer. Assume that after the training, we obtain a BN layer with learned \( m = (m_1, ..., m_D)^T \), \( \sigma = (\sigma_1, ..., \sigma_D)^T \), \( \gamma \) and \( \beta \). Then, the output \( y_r \) of the \( r \)-th rule with BN is:

\[
y_r(x_n) = b_{r,0} + \gamma \sum_{d=1}^{D} b_{r,d} \frac{x_{n,d} - m_d}{\sigma_d} + \beta D \tag{10}
\]

which can be re-written as:

\[
y_r(x_n) = b_{r,0}' + \sum_{d=1}^{D} b_{r,d}' x_{n,d}, \tag{11}
\]

where

\[
b_{r,0}' = b_{r,0} + \beta D - \gamma \sum_{d=1}^{D} \frac{m_d b_{r,d}}{\sigma_d}, \tag{12}
\]

\[
b_{r,d}' = \frac{b_{r,d}}{\sqrt{\sigma_d} + \epsilon}. \tag{13}
\]

By doing this, the original architecture of the TSK fuzzy system is kept unchanged.

### III. Experiments and Results

This section validates the performances of our proposed UR, RPI and BN on multiple datasets from various applications domains, with varying size and feature dimensionality.

#### A. Datasets

We evaluated our proposed algorithms on nine classification datasets. One of them was MNIST, and the other eight were selected from the UCI Machine Learning Repository. Their characteristics are summarized in Table I.

| Dataset       | No. of samples | No. of features | No. of classes |
|---------------|----------------|-----------------|---------------|
| Yeast\(^1\)   | 1,484          | 8               | 10            |
| Steel\(^2\)   | 1,941          | 27              | 7             |
| Abalone\(^3\) | 4,177          | 8               | 3             |
| Waveform-noise\(^4\) | 5,000      | 40              | 3             |
| Page-blocks\(^5\) | 5,473        | 10              | 5             |
| Satellite\(^6\) | 6,435          | 36              | 7             |
| Clave\(^7\)   | 10,800         | 16              | 4             |
| MAGIC\(^8\)   | 19,020         | 10              | 2             |
| MNIST\(^9\)   | 60,000         | 784             | 10            |

\(^1\) https://archive.ics.uci.edu/ml/datasets/Yeast
\(^2\) https://archive.ics.uci.edu/ml/datasets/Steel+Plates+Faults
\(^3\) https://archive.ics.uci.edu/ml/datasets/Abalone
\(^4\) https://archive.ics.uci.edu/ml/datasets/Waveform+Database+Generator+(Version+1)
\(^5\) https://archive.ics.uci.edu/ml/datasets/Page+Blocks+Classification
\(^6\) https://archive.ics.uci.edu/ml/datasets/Statlog+(Landsat+Satellite)
\(^7\) https://archive.ics.uci.edu/ml/datasets/Firm+Teacher+Clave-Direction+Classification
\(^8\) https://archive.ics.uci.edu/ml/datasets/MAGIC+Gamma+Telescope
\(^9\) http://yann.lecun.com/exdb/mnist/

Some datasets contain both numerical features and categorical features. The categorical features were converted into numerical ones by one-hot coding. For the eight datasets from the UCI Machine Learning Repository, we randomly sampled 70% samples as the training set, and the remaining 30% samples as the test set. We \( z \)-normalized each feature using the mean and standard deviation computed from the training set. For MNIST, we kept the original train-test split: 50,000 samples for training and 10,000 for testing. Note that we did not perform \( z \)-normalization on MNIST, because some features (e.g., those pixels at the corners of the images) have the same value in all samples, and hence the standard deviations are zero. Instead, we followed a commonly used preprocessing approach, which normalizes the range of each feature to \([0, 1]\), by dividing the inputs by 255.

#### B. Algorithms

We compared the performances of the following nine algorithms:

1) kM, whose loss function involved only the first two terms in (8). k-means clustering was used to initialize the Gaussian MF centers, and the ordinary MBGD (without BN) was used to refine the rule parameters.

\(^2\) http://archive.ics.uci.edu/ml/index.php
2) UR-kM, which was almost identical to kM, except that the loss function [8] was used, i.e., it also considered UR.
3) UR-kM-BN, which was almost identical to UR-kM, except that BN was also used in tuning the rule parameters.
4) FCM, which was almost identical to kM, except that k-means clustering was replaced by fuzzy C-means clustering in initializing the Gaussian MF centers.
5) UR-FCM, which was almost identical to FCM, except that the loss function [8] was used, i.e., it also considered UR.
6) UR-FCM-BN, which was almost identical to UR-FCM, except that BN was also used in tuning the rule parameters.
7) RPI, which was almost identical to kM, except that k-means clustering was replaced by RPI in initializing the Gaussian MF centers.
8) UR-RPI, which was almost identical to RPI, except that the loss function [8] was used, i.e., it also considered UR.
9) UR-RPI-BN, which was almost identical to UR-RPI, except that BN was also used in tuning the rule parameters.

Following the settings in [18], \( R \), the number of rules for each dataset, was set to the number of classes. The initial standard deviations of the Gaussian MFs, \( \sigma_{r,d} \), \( r = 1, \ldots, R \), \( d = 1, \ldots, D \), were all set to 1. We used \( \alpha = 0.05 \) in [8] for the UCI datasets, and \( \alpha = 0.005 \) for MNIST. The optimal \( \lambda \) in [8] was identified from a candidate set of \( \{0.1, 1, 10, 20, 50\} \). For the eight UCI datasets, we used 10% samples from the training set to choose \( \lambda \). For MNIST, we used a pre-partitioned 10% validation set to choose \( \lambda \). The values of \( \alpha \) and the optimal \( \lambda \) are summarized in Table II.

| Dataset          | \( \lambda \) | \( \alpha \) |
|------------------|--------------|-------------|
| Yeast            | 20           | 0.05        |
| Steel            | 1            | 0.05        |
| Abalone          | 20           | 0.05        |
| Waveform-noise   | 20           | 0.05        |
| Page-blocks      | 1            | 0.05        |
| Satellite        | 10           | 0.05        |
| Clave            | 20           | 0.05        |
| MAGIC            | 0.1          | 0.05        |
| MNIST            | 10           | 0.005       |

As in [2], AdaBound [26] optimizer with learning rate 0.01, \( \beta_1 = 0.9 \), and \( \beta_2 = 0.999 \) was used in MBGD. The batch size was 64 for the UCI datasets, and 1,024 for MNIST. Early stopping was used to obtain the final optimization performance. The maximum number of epochs was 500 for UCI datasets and 300 for MNIST. We repeated each algorithm five times and report the average performance.

When the feature dimensionality is very high, computing the rule firing levels may result in numerical underflow. For instance, the MNIST dataset has 784 attributes, and hence each rule firing level is the product of 784 membership grades. Each membership grade is a value in \([0, 1] \), and the product of 784 such small values may be smaller than the smallest number a computer can represent. In order to avoid numerical underflow on MNIST, we added a bias to the exponent of [4]:

\[
f_r(x) = \exp \left( -\sum_{d=1}^{D} \frac{(x_d - m_{r,d})^2}{2\sigma_{r,d}^2} + \delta \right),
\]

where \( \delta \) was empirically set to 75.

C. Overall Performance

The test classification accuracies on the nine datasets, after early-stopping, are shown in Table III. The highest accuracy on each dataset is marked in bold. To facilitate the comparison, we also show the ranks of the classification accuracies in Table IV.

The following observations can be made from the above two tables:

1) Generally, UR improved the classification accuracy. Comparing each even column of Table III (or Table IV) with the odd column immediately before it, we can observe that generally UR improved the classification performance, regardless of which initialization method was used, and also regardless of whether or not BN was used. The average ranks in Table IV demonstrate this more clearly: the average rank in each even column (with UR) is always smaller than the average rank in the odd column immediately before it (without UR), suggesting that on average an algorithm with UR outperformed a corresponding algorithm without UR.

2) Generally, RPI outperformed both k-means clustering initialization and fuzzy C-means clustering initialization. Comparing the \((4 + i)\)-th \((i = 1, 2, 3, 4)\) column of Table III (or Table IV) with the \(i\)-th and the \((4 + i)\)-th columns in the same table, we can observe that generally RPI achieved better performance than k-means clustering initialization and fuzzy C-means clustering initialization, regardless of whether UR and/or BN were used or not. The average ranks in Table IV demonstrate this more clearly: the average rank in the \((4 + i)\)-th column (with RPI) is always smaller than or equal to the average ranks in the \(i\)-th column (with k-means clustering initialization) and the \((4 + i)\)-th column (with fuzzy C-means clustering initialization), suggesting that on average an algorithm with RPI outperformed a corresponding algorithm with k-means clustering initialization, or fuzzy C-means clustering initialization.

3) Generally, BN improved the classification accuracy. Comparing the \((4i + j + 3)\)-th \((i = 0, 1; j = 0, 1)\) column of Table III (or Table IV) with the \((4i + j + 1)\)-th column in the same table, we can observe that generally BN improved the classification performance, regardless of which initialization method was used, and also regardless of whether or not UR was used. The average ranks in Table IV demonstrate this more clearly: the average rank in the \((4i + j + 3)\)-th column (with BN) is always smaller than the average rank in the \((4i + j + 1)\)-th column (without BN), suggesting that on average an algorithm with BN outperformed a corresponding algorithm without BN.
4) Generally, the performance improvements introduced by UR and BN were complementary, and hence integrating them can result in even better performance. This is demonstrated by the fact that the result in the 4i-th (i = 1, 2, 3) column (with both UR and BN) of Table III (or Table IV) is better than the results in the two columns immediately before it (with UR or BN alone) in the same table, regardless of which initialization method was used. Again, this can be seen more clearly from the average ranks in Table IV.

5) On average, UR-RPI-BN, which integrates all three proposed novel techniques, achieved the best performance among the 12 algorithms. As shown in Table IV, UR-RPI-BN achieved the best performance on four out of the nine datasets, and the second best on another three. Its average rank was the smallest among the 12 algorithms.

Next, we study the individual effects of UR, RPI and BN in more details.

D. Effectiveness of UR

Fig. 5 shows the test classification accuracies with and without UR, when RPI and RPI-BN were used. At convergence, UR-RPI outperformed RPI in six out of the nine datasets, and they achieved comparable performances on another dataset. UR-RPI-BN outperformed RPI-BN in six datasets, and they achieved comparable performances on the remaining three datasets. These results demonstrate that UR was generally effective. Especially, when it was integrated with our proposed RPI and BN, the resulting UR-RPI-BN can always achieve a classification accuracy higher than or comparable with that without using it (RPI-BN), i.e., though UR may not always be helpful, it does not hurt anyway. So, it is recommended to always integrate UR with RPI and BN.

Next, we study if UR can indeed result in more uniform rule firing levels. We recorded the average firing levels of the rules in the initialization of the MF parameters, and the results can be helpful, it does not hurt anyway. So, it is recommended to always integrate UR with RPI and BN.

In summary, we have shown that our proposed UR indeed made the firing levels of the rules more uniform, and hence improved the generalization performance of the resulted TSK fuzzy classifier.

E. Effectiveness of RPI

Fig. 5 shows the test classification accuracies when different initialization methods were used. KM, FCM and RPI seemed to have overall comparable performances, regardless of whether they were integrated with UR and BN or not, though Table IV shows that on average RPI slightly outperformed KM and FCM. This is reasonable, as KM, FCM and RPI were only fuzzy classifier.

Nevertheless, it’s worthwhile to emphasize that a significant advantage of RPI over KM and FCM is its efficiency. The computing time of KM, FCM and RPI on the nine datasets is significantly lower than that of RPI-BN.

3Note that a mini-batch iteration is different from an epoch in training: an epoch consists of multiple mini-batch iterations.

4All three approaches were implemented in Python. The code for KM was downloaded from https://scikit-learn.org/stable/modules/generated/sklearn.cluster.KMeans.html, and the code for FCM was downloaded from https://pythonhosted.org/scikit-fuzzy/api/skfuzzy.cluster.html. We wrote our own code for RPI.
is shown in Table V. Clearly, RPI ran much faster than both kM and FCM.

Finally, we need to point out that the antecedent MF parameters may also be initialized completely randomly when the feature dimensionality is low. However, we observed that when the feature dimensionality is very high, random initialization may result in zero firing levels for certain rules. Thus, some parameters never got updated in MBGD, and the training did not converge. We did not observe such phenomenon when kM, FCM or RPI was used in the initialization.

In summary, we have shown that our proposed RPI had comparable, if not better, performance with $k$-means clustering initialization and fuzzy $C$-means clustering initialization, but it ran much faster than the latter two. It is also more robust than random initialization. So, it is overall the best MF initialization approach for training TSK fuzzy classifiers.

| Dataset        | kM     | FCM    | RPI  |
|----------------|--------|--------|------|
| Yeast          | 0.0565 | 0.0427 | **0.0035** |
| Steel          | 0.0174 | 0.1054 | **0.0048** |
| Abalone        | 0.0103 | 0.2927 | **0.0006** |
| Waveform-noise | 0.0086 | 0.1389 | **0.0011** |
| Page-blocks    | 0.0117 | 0.0472 | **0.0049** |
| Satellite      | 0.0235 | 0.2960 | **0.0058** |
| Clave          | 0.0106 | 0.4452 | **0.0015** |
| MAGIC          | 0.0129 | 0.0943 | **0.0017** |
| MNIST          | 10.2954| 5.3512 | **2.4258** |

5Theoretically, when Gaussian MFs are used, the rule firing levels can never be zero; however, when the feature dimensionality is very high, some rule firing levels become very small, and cause numerical underflow. As a result, they are treated as zero by the computer.
Table IV already shows that on average an algorithm with BN always outperformed the corresponding algorithm without BN. More detailed analysis on the effectiveness of BN is given in this subsection.

Fig. 6 shows the test classification accuracies when different initialization approaches were used, with and without BN. At convergence, UR-kM-BN outperformed UR-kM on eight out of the nine datasets (except Abalone); UR-FCM-BN outperformed UR-FCM on six datasets, and they had comparable performances on another two datasets (except Abalone); and, UR-RPI-BN outperformed UR-RPI on seven datasets (except Yeast and Abalone). These results again confirmed the effectiveness of BN.

As mentioned in Section II-D, performing BN for both the antecedents and consequents [shown in Fig. 2(a); denoted as BN2 in the sequel] may not lead to improved performance. We conducted an experiment on MNIST to verify this, in which UR-RPI, UR-RPI-BN and UR-RPI-BN2 were compared. The results are shown in Fig. 7. UR-RPI-BN2 achieved even worse performance than UR-RPI, not to mention UR-RPI-BN.

We also tried to understand why BN can improve the classification performance, whereas BN2 cannot. The 1-norm of the gradients of the consequent parameters $b_{r,d}$ ($r = 1, \ldots, R; \ d = 0, \ldots, R$) are shown in Fig. 8(a) and the 1-norm of the gradients of $m_{r,d}$ (the centers of the antecedent MFs) are shown in Fig. 8(b), for the first 200 mini-batch iterations on MNIST. Generally, UR-RPI-BN gave the largest gradients, which helped the convergence, and maybe also the generalization. On the contrary, UR-RPI-BN2 gave the smallest gradients, which may cause the solution to be easily trapped at a local minimum. These observations may help explain why BN, instead of BN2, should be used in optimizing a TSK fuzzy classifier.

In summary, we have shown that our proposed BN, which applies only to the rule consequents, gave larger gradients in optimization, and also improved generalization performance of the resulted TSK fuzzy classifier.

IV. CONCLUSION

TSK fuzzy systems are powerful and frequently used machine learning models, for both regression and classification. However, they may not be easily applicable to big data problems, especially when the size and the dimensionality of the data are both large. Our very recent research [2] proposed an MBGD-based efficient and effective training algorithm for
TSK fuzzy systems for big data regression problems. This paper has proposed an MBGD-based algorithm to train TSK fuzzy systems for big data classification problems. It integrates three novel techniques, which are also first proposed in this paper: 1) UR, which is a regularization term in the loss function to make sure the rules have similar average firing levels, and hence better generalization performance; 2) RPI, which initializes the MFs efficiently and reliably; and, 3) BN, which normalizes the rule consequents to speedup the convergence and improve generalization. Experiments on nine datasets from various domains, with varying size and feature dimensionality, demonstrated that each of UR, RPI and BN has

![Graphs showing test classification accuracies for different datasets](image)

**Fig. 6.** Test classification accuracies when different initialization approaches were used, with and without BN.

![Graphs showing test classification accuracies for MNIST](image)

**Fig. 7.** Test classification accuracies of UR-RPI, UR-RPI-BN, and UR-RPI-BN2 on MNIST.

![Graphs showing 1-norm of gradients](image)

**Fig. 8.** (a) 1-norm of the gradients of the consequent parameters; (b) 1-norm of the centers of the antecedent MFs. Showing the first 200 mini-batch iterations on MNIST.
its own unique advantages, and integrating all three together can achieve the best classification performance for big data applications.

This algorithm, together with the one in \[2\], shall greatly promote the applications of TSK fuzzy systems in big data classification and regression problems.

**References**

[1] A.-T. Nguyen, T. Taniguchi, L. Eciolaza, V. Campos, R. Palhares, and M. Sugeno, “Fuzzy control systems: Past, present and future,” *IEEE Computational Intelligence Magazine*, vol. 14, no. 1, pp. 56–68, 2019.

[2] D. Wu, Y. Yuan, and Y. Tan, “Optimize TSK fuzzy systems for big data regression problems: Mini-batch gradient descent with regularization, DropRule and AdaBound (MBGD-RDA),” *IEEE Trans. on Fuzzy Systems*, 2019, submitted. [Online]. Available: https://arxiv.org/abs/1903.10951

[3] J. C. Bezdek, *Pattern Recognition with Fuzzy Objective Function Algorithms*. New York, NY: Plenum Press, 1981.

[4] S. L. Chiu, “Fuzzy model identification based on cluster estimation,” *Journal of Intelligent & Fuzzy Systems*, vol. 2, no. 3, pp. 267–278, 1994.

[5] N. Kasabov and Q. Song, “DENFIS: Dynamic evolving neural-fuzzy inference system and its application for time-series prediction,” *IEEE Trans. on Fuzzy Systems*, vol. 10, no. 2, pp. 144–154, 2002.

[6] D. Wu and W. W. Tan, “Genetic learning and performance evaluation of interval type-2 fuzzy logic controllers,” *Engineering Applications of Artificial Intelligence*, vol. 19, no. 8, pp. 829–841, 2006.

[7] L.-Y. Wang and J. M. Mendel, “Back-propagation of fuzzy systems as nonlinear dynamic system identifiers,” in *Proc. IEEE Int’l Conf. on Fuzzy Systems*, San Diego, CA, 1992, pp. 1409–1418.

[8] J. S. R. Jang, “ANFIS: Adaptive-network-based fuzzy inference system,” *IEEE Trans. on Systems, Man, and Cybernetics*, vol. 23, no. 3, pp. 665–685, 1993.

[9] I. Goodfellow, Y. Bengio, and A. Courville, *Deep Learning*. Boston, MA: MIT press, 2016.

[10] S. Ruder, “An overview of gradient descent optimization algorithms,” *arXiv preprint arXiv:1609.04747*, 2016.

[11] S. Ioffe and C. Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift,” in *Proc. Int’l Conf. on Machine Learning*, Lille, France, Jul. 2015.

[12] R. A. Jacobs, M. I. Jordan, S. J. Nowlan, and G. E. Hinton, “Adaptive mixtures of local experts,” *Neural Computation*, vol. 3, no. 1, pp. 79–87, 1991.

[13] T. Shen, M. Ott, M. Auli, and M. Ranzato, “Mixture models for diverse machine translation: Tricks of the trade,” *arXiv preprint arXiv:1902.07816*, 2019.

[14] N. Shazeer, A. Mirhoseini, A. Davis, Q. Le, G. Hinton, and J. Dean, “Outrageously large neural networks: The sparsely-gated mixture-of-experts layer,” *arXiv preprint arXiv:1701.06538*, 2017.

[15] H. Bersini and G. Bontempi, “Now comes the time to defuzzify neuro-fuzzy models,” *Fuzzy Sets and Systems*, vol. 90, no. 2, pp. 161–169, 1997.

[16] H. Andersen, A. Lotfi, and L. Westphal, “Comments on ‘functional equivalence between radial basis function networks and fuzzy inference systems’ [and author’s reply],” *IEEE Trans. on Neural Networks*, vol. 9, no. 6, pp. 1529–1532, 1998.

[17] D. Wu, C.-T. Lin, J. Huang, and Z. Zeng, “On the functional equivalence of TSK fuzzy systems to neural networks, mixture of experts, CART, and stacking ensemble regression,” *IEEE Trans. on Fuzzy Systems*, 2019, submitted.

[18] Y. Peng, Z. Ren, Y. Kong, F. Bao, and Q. Dai, “A hierarchical fused fuzzy deep neural network for data classification,” *IEEE Trans. on Fuzzy Systems*, vol. 25, no. 4, pp. 1006–1012, 2016.

[19] R. Winkler, F. Klawonn, and R. Kruse, “Fuzzy c-means in high dimensional spaces,” *Int’l Journal of Fuzzy System Applications*, vol. 1, no. 1, pp. 1–16, 2011.

[20] K. Beyer, J. Goldstein, R. Ramakrishnan, and U. Shaft, “When is ‘nearest neighbor’ meaningful?” in *Proc. Int’l Conf. on Database Theory*. Jerusalem, Israel: Springer, Jan. 1999, pp. 217–235.

[21] K. He, X. Zhang, S. Ren, and J. Sun, “Deep residual learning for image recognition,” in *Proc. IEEE Conf. on Computer Vision and Pattern Recognition*, Las Vegas, NV, Jun. 2016, pp. 770–778.

[22] S. Zagoruyko and N. Komodakis, “Wide residual networks,” *arXiv preprint arXiv:1605.07146*, 2016.