The impacts of varying magnetic field and free convection heat transfer on an Eyring–Powell fluid flow with peristalsis: VIM solution

Y. A. S. El-Masry a,b, Y. Abd Elmaboud c,d and M. A. Abdel-Sattar e,f

aDepartment of Mathematics, Faculty of Science, King Khalid University, Abha, Saudi Arabia; bDepartment of Mathematics, Faculty of Science, Mansoura University, Mansoura, Egypt; cDepartment of Mathematics, College of Science and Arts at Khulais, University of Jeddah, Jeddah, Saudi Arabia; dDepartment of Mathematics, Faculty of Science, Al-Azhar University (Assiut Branch), Assiut, Egypt; eDepartment of Mathematics, Faculty of Science, Beni-Suef University, Beni-Suef, Egypt

ABSTRACT

A mathematical model of a non-Newtonian fluid (Eyring–Powell fluid) flow with peristalsis under the effect of varying magnetic field and free convection heat transfer has been discussed. Most of the previous attempts of peristaltic flow problems for a non-Newtonian fluid have been solved by using the perturbation technique for a small non-Newtonian parameter, which gives some limits for the results. To avoid such restriction, the variational iteration method (VIM) is applied to solve the current model. A code is established by using the symbolic software MATHEMATICA to get the successive solutions. Semi-analytical solutions are found by VIM for the velocity, heat transfer, pressure gradient and stream function, which include Eyring–Powell fluid parameters. Moreover, a comparison between VIM and numerical solutions is discussed. The obtained results show that the variation of the heat transfer for the Newtonian fluid is large compared with the Eyring–Powell fluid. In addition, it is noteworthy that the peristaltic transport overcomes on Lorentz force nearby the peristaltic walls.

1. Introduction

In modern medicine, there are some procedures that help the physician to make the right decision for the patient. One of the most renowned procedures is the magnetic resonance imaging (MRI) technique [1]. This technique is applied to implement 3D, noninvasive scans of the human body. The MRI procedure is vastly used especially in treatment and diagnosis of most diseases of the human body [2]. The patient is put within a transverse radiofrequency (RF) signal and magnetic field to generate images of the organs in the body.

The study of magnetohydrodynamics on different kinds of fluids has attracted the researchers’ attention. Ellahi et al. [3] studied the problem of electroosmotic Couette–Poiseuille flow of MHD power law nanofluid with entropy generation. The impact of the magnetic field on titanium dioxide-water nanofluid with entropy generation was discussed by Zeeshan et al. [4]. The authors in [5] introduced a mathematical model to investigate Kerosene-Al2O3 nanofluid on MHD Poiseuille flow with variable thermal conductivity. Shehzad et al. [6] studied a theoretical model of electroosmotic Couette–Poiseuille flow of power law Al2O3-PVC nanofluid through a channel when the upper wall is moving with a constant velocity.

Peristaltic motion of the circular and longitudinal flexible muscles appears not only in the digestive system but also in the other hollow tubes of the human body, which transport fluids by its propulsive

NOMENCLATURE

(\(x, y\)) Fixed frame axes [m]  \(\dot{p}\) Pressure in the fixed frame [kg/(m²s)]

(\(\bar{x}, \bar{y}\)) Wave frame axes [m]  \(\bar{p}\) Pressure in the wave frame

2a Width of the channel [m]  \(\mathcal{F}\) Flow rate [m³/s]

b Amplitude of the sinusoidal wave [m]  \(\bar{t}\) Time [s]

c Wave speed [m/s]  \(\lambda\) Wavelength [m]

(\(\bar{u}, \bar{v}\)) Velocity components [m/s]  \(\delta\) Dimensionless wave number

g Gravitational acceleration [m/s²]  \(\gamma_{1, 2}\) Eyring–Powell fluid parameters

\(\bar{T}\) Temperature [K]  \(\rho\) Density [kg/m³]

\(C_p\) Specific heat at constant pressure [J/(kgK)]  \(\beta\) Coefficient of thermal expansion [K⁻¹]

K Thermal conductivity [W/(mK)]  \(\phi\) Amplitude ratio

Br Brinkman number  \(\alpha\) Pattern distribution of the magnetic field

Re Reynolds number  \(\nu\) Kinematic viscosity [m²/s]

Pr Prandtl number  \(\bar{u}\) Quantity before the dimensionless

Gr Grashof number

KEYWORDS

MHD; biofluids; variational iteration technique; heat transfer

CLASSIFICATION

76Z05; 76A05

ARTICLE HISTORY

Received 26 August 2019
Revised 9 November 2019
Accepted 20 November 2019

CONTACT

Y. Abd Elmaboud yass_math@yahoo.com

© 2019 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group
This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.
movements. Transportation of urine from kidney to bladder and chyme in the gastrointestinal tract occurs under the responsibility of peristaltic transport. Latham [7] introduced an experimental investigation to study the peristaltic transport. Mathematical model on peristaltic pumping using a perturbation technique is presented by Fung and Yih [8]. The trapping and reflux phenomenon through the channel (tube) having sinusoidal wall propagation are evaluated theoretically by Shapiro et al. [9]. The above studies have taken the fluid model as a Newtonian fluid. Recently, the Newtonian model has been replaced by the non-Newtonian model because many fluids do not follow Newton’s law of viscosity. The study of the motion of non-Newtonian fluid is stepping into the spotlight of the researchers as a model of biofluids. Peristalsis of a non-Newtonian fluid in a tube, a channel and an annulus is discussed by many researchers [10–15]. Among the types of the non-Newtonian fluids, the Eyring–Powell fluid has attracted researchers’ attention. Eyring–Powell fluid is extremely very sensitive to the tiny variations in the zero shear rate viscosity and moderate sensitive to the variations in the infinite shear rate viscosity [16]. There are many contributions which discussed the peristaltic flow of Eyring–Powell fluid with or without heat transfer in a channel or a tube. Hina [17] discussed the effects of wall properties and wall slip on peristaltic transport with heat and mass transfer for Eyring–Powell fluid model. In a symmetric channel, Hayat et al. [18] studied the slip effects on the peristaltic movements of Eyring–Powell fluid. In the annular region between two coaxial tubes, Akbar and Nadeem [19] discussed peristaltic flow of Powell–Eyring fluid with heat and mass transfer for Eyring–Powell fluid model. Peristaltic transport of Eyring–Powell fluid in a curved channel was discussed by Abbasi et al. [20]. The effects of chemical reaction and convective conditions on peristaltic flow of Eyring–Powell fluid were discussed by Hayat et al. [21]. Prakash et al. [22] introduced a mathematical model for blood flow as nanofluid in tapered channel with peristalsis.

The human body is usually in a thermal steady state regarding its surroundings. Heat made by metabolic operations is lost in the surroundings through several mechanisms: convection, conduction and evaporation. In free convection, the buoyancy of the warmer fluid near the cooler surrounding fluid leads to fluid motion. Many researchers have studied the effect of the free convection process on the fluid flow. Oscillatory flow of a couple stress fluid under the free convection process is discussed in [23]. Convection of hybrid nanofluid with magnetic effects was investigated mathematically by Tlili et al. [24]. In review of its importance, the free convection of biofluid flow has been investigated by numerous authors. Mekheimer and Abdelmaboud [25] studied the influence of the buoyancy force on peristaltic transport for a Newtonian fluid in an annulus as application of an endoscope. Mekheimer et al. [26] modelled the peristaltic flow in a vertical asymmetric channel and they discussed heat transfer and space porosity effects. Tripathi and Beg [27] have studied peristaltic transport of nanofluid as an application of drug delivery. Bioconvection flow of nanofluid containing a motile gyrotactic microorganism under the effect of heat transfer and magnetic field is simulated by Waqas et al. [28]. The third-grade MHD fluid flow with chemical reaction taking into account the heat and mass transfer was investigated by Khan et al. [29].

Variational iteration method (VIM) gives rapidly convergent and successive approximations without any restrictive assumptions. For nonlinear systems that arise frequently in fluid mechanics, VIM facilitates the computational work and gives accurate solutions. Few researchers in the field of fluid mechanics used VIM to get the solutions of their mathematical models. Shahmohamadi and Rashidi [30] used VIM to get the solutions of the squeezing MHD nanofluid flow in a rotating channel. Moosavi et al. [31] used VIM to solve the problem of non-Newtonian fluid, namely, a Sisko fluid on a moving belt. Abd elmaboud [32] discussed the problem of hemodynamics flow with varying magnetic field in a semi porous vertical channel and used VIM to obtain approximate solutions.

From the previous survey, we conclude that no attempt has been made yet to discuss the impacts of varying magnetic field and convection heat transfer (as a coupled system of the governing equations) on peristaltic transport of an Eyring–Powell fluid. Moreover, to the best of our knowledge, no attempt to apply VIM for peristaltic flow has been published in the literature. So, the aim of this study is to discuss this problem and applying VIM to avoid the restriction of the perturbation technique.

2. Formulation of the problem

We study the movement of an incompressible viscous Eyring–Powell fluid in 2D vertical wavy channel (see Figure 1). The fluid is subjected to the impact of varying transverse magnetic field of strength $B_0(\tilde{Y})$, which is assumed to be applied in the positive $\tilde{Y}$ direction. The wave trains sinusoidally propagate with speed $c$ which is constant along the channel walls. The mathematical model of the wall surface is formulated as

$$\tilde{h}(\tilde{X}, \tilde{Y}) = a + b \sin \frac{2\pi}{\lambda} (\tilde{X} - c \tilde{t}).$$

(1)

Taking the wave frame moves with velocity $c$ away from the fixed frame by the transformation:

$$\tilde{y} = \tilde{Y}, \quad \tilde{x} = \tilde{X} - c \tilde{t}, \quad \tilde{v} = \tilde{V}, \quad \tilde{u} = \tilde{U} - c,$n

\quad \tilde{p}(\tilde{x}) = \tilde{p}(\tilde{X}, \tilde{Y}).$$

(2)

Under the consideration that the wavelength ($\lambda$) is large and the curvature impacts are measly, the pressure $\tilde{p}$
where $\beta$, $c_1$ are the Eyring–Powell fluid parameters. Up to the second order $\sinh^{-1}((1/c_1)\vec{V} \cdot \vec{V}) \cong ((1/c_1)\vec{V} \cdot \vec{V}) - \frac{1}{6}((1/c_1)\vec{V} \cdot \vec{V})^3$. Consider the following non-dimensional variables:

$$x = \frac{\tilde{x}}{\lambda}, \quad y = \frac{\tilde{y}}{a}, \quad u = \frac{\tilde{u}}{c}, \quad v = \frac{\tilde{v}}{\delta c}, \quad \rho = \frac{\alpha^2}{\lambda \mu \tilde{p}},$$

$$\vec{S} = \frac{\alpha}{\mu c} \tilde{\vec{S}}, \quad t = \frac{\tilde{t}}{\lambda}, \quad h = \frac{\tilde{h}}{a}, \quad \theta = \frac{\tilde{\theta} - T_0}{(T_1 - T_0)}.$$  
(8)

By using Equation (8), Equations (1), (3)–(6) become

$$y = h + 1 + \phi \sin \{2\pi x\},$$
(9)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$
(10)

$$\text{Re}_h \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = - \frac{\partial p}{\partial x} + \delta \frac{\partial^2 S_{11}}{\partial x^2} + \frac{\partial S_{12}}{\partial y} - \frac{\bar{M}(y)(u + 1) + Gr\theta}{\bar{M}^2},$$
(11)

$$\text{Re}_3 \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = - \frac{\partial p}{\partial y} + \delta \frac{\partial^2 S_{11}}{\partial x^2} + \frac{\partial S_{22}}{\partial y},$$
(12)

$$\text{RePr} \left[ u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right] = \left( \frac{\partial^2 \theta}{\partial y^2} + \delta \frac{\partial^2 \theta}{\partial x^2} \right) + Br \left[ \delta \left( S_{11} \frac{\partial u}{\partial x} + S_{22} \frac{\partial v}{\partial x} \right) + S_{12} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right],$$
(13)

with dimensionless parameters:

$$\phi = \frac{b}{a^2}, \quad \delta = \frac{\alpha}{\lambda}, \quad M^2(y) = \frac{B_0^2(y)a^2\gamma}{\mu}, \quad \text{Re} = \frac{\rho\alpha a^2c_1}{\mu},$$
$$Gr = \frac{g\beta a^2(T_1 - T_0)\rho^2}{\nu^2},$$
$$\gamma_1 = \frac{1}{\mu^2c_1}, \quad \gamma_2 = \frac{\gamma_1 c_2}{\beta_2a^2c_1}, \quad \text{Pr} = \frac{\mu c_1}{K},$$
$$Br = \frac{\mu c_1^2}{K(T_1 - T_0)}.$$  
(14)

The variation of the magnetic field through the fluid layers varies with $y$-axis, taking the following form:

$$\bar{M}(y) = M^2(1 - a^2y^2),$$
(15)

where $M$ is the magnetic parameter. For simplification, we use the long wavelength approximation ($\delta << 1$) with lubrication approach, so that Equations (10)–(13) become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$
(16)

$$0 = - \frac{\partial p}{\partial x} + \frac{\partial S_{12}}{\partial y} - \bar{M}(y)(u + 1) + Gr\theta,$$  
(17)

$$0 = - \frac{\partial p}{\partial y},$$
(18)

$$\frac{\partial^2 \theta}{\partial y^2} + Br \left[ S_{12} \left( \frac{\partial u}{\partial y} \right) \right] = 0,$$  
(19)

where $S_{12} = (1 + \gamma_1)(\delta u/\delta y) - \gamma_2(\delta u/\delta y)^3$. Eliminate the pressure between Equations (17) and (18) and use the dimensionless stream function $\psi(x,y)$ where $u = \frac{\partial \psi}{\partial y}$, $v = -\delta \partial \psi/\partial x$. The combined equation takes the following form:

$$(1 + \gamma_1) \frac{\partial^4 \psi}{\partial y^4} - \gamma_2 \frac{\partial^2 \psi}{\partial y^2} \left( \frac{\partial^2 \psi}{\partial y^2} \right)^3 - \bar{M}(y) \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial M}{\partial y},$$

$$\times \left( \frac{\partial \psi}{\partial y} + 1 \right) + Gr \frac{\partial \theta}{\partial y} = 0,$$  
(20)
and (19) becomes
\[
\frac{\partial^2 \theta}{\partial y^2} + Br \left[ (1 + \gamma_1) \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + \gamma_2 \left( \frac{\partial^2 \psi}{\partial y^2} \right)^4 \right] = 0.
\]  
(21)

The dimensionless time-mean flows \( \psi \) and \( F \) respectively in the fixed and wave frame related as
\[
\psi = F + 1,
\]  
where
\[
F = \int_0^h \frac{\partial \psi}{\partial y} \, dy = \psi(h) - \psi(0).
\]  
(23)

The non-dimensional boundary conditions for the stream function in the wave frame are:
\[
\begin{align*}
\psi &= 0, \quad \frac{\partial \psi}{\partial y} = 0, \quad \theta = 0, \quad \text{on } y = 0; \\
\psi &= F, \quad \frac{\partial \psi}{\partial y} = -1, \quad \theta = 1, \quad \text{on } y = h(x).
\end{align*}
\]  
(24)

The heat transfer coefficient (Nusselt number) is defined as follows:
\[
Nu = -\frac{ah_2}{k(T_1 - T_0)} = \frac{\partial \theta}{\partial y}.
\]  
(25)

where \( h_2 \) represents the heat transfer coefficient.

### 3. Solution technique

Most of the previous attempts of peristaltic flow problems for a non-Newtonian fluid have been solved by using the perturbation technique for a small non-Newtonian parameter, which gives some limits for the results. To avoid such restriction, the VIM is applied to solve the current model. The VIM is used to obtain approximate solutions for Equations (20) and (21) with the boundary conditions (Equation 22). The walls should depend on the position which must be taken into account. Because of this, the dependence of the surface on the position is handled here by changing the coordinate system such that it follows the motion of the wall. Consequently, the new coordinates are
\[
\eta = \frac{y}{h}, \quad \chi = \chi.
\]  
(26)

Using (26) in (20) and (21), one can get
\[
\begin{align*}
\frac{\partial^4 \psi}{\partial \eta^4} &= \frac{\gamma_2}{h^4(1 + \gamma_1)} \left[ 3 \left( \frac{\partial^2 \psi}{\partial \eta^2} \right)^2 \left( \frac{\partial^4 \psi}{\partial \eta^4} \right) \right. \\
&\quad + 6 \left( \frac{\partial^2 \psi}{\partial \eta^2} \right)^2 \left( \frac{\partial^3 \psi}{\partial \eta^3} \right)^2 \\
&\quad - M^2 h^2 (1 - ah^2 \eta^2) \frac{\partial^2 \psi}{\partial \eta^2} \\
&\quad + \frac{2M^2 h^2 \alpha \eta}{(1 + \gamma_1)} \frac{\partial \psi}{\partial \eta} + \frac{h^3 Gr}{(1 + \gamma_1)} \frac{\partial \theta}{\partial \eta} \bigg] = 0.
\end{align*}
\]  
(27)

\[
\begin{align*}
\frac{\partial^2 \theta}{\partial \eta^2} + Br \left[ (1 + \gamma_1) \left( \frac{\partial^2 \psi}{\partial \eta^2} \right)^2 - \gamma_2 \left( \frac{\partial^2 \psi}{\partial \eta^2} \right)^4 \right] &= 0.
\end{align*}
\]  
(28)

The boundary conditions are:
\[
\psi = 0, \quad \frac{\partial^2 \psi}{\partial \eta^2} = 0, \quad \theta = 0, \quad \text{on } y = 0; \\
\psi = F, \quad 1 \frac{\partial \psi}{h \partial \eta} = -1, \quad \theta = 1, \quad \text{on } y = 1.
\]  
(29)

We transform Equations (27) and (28) to a first-order system of differential equations as follows:
\[
\begin{align*}
\psi' &= \psi_1, \\
\psi_1' &= \psi_2, \\
\psi_2' &= \psi_3, \\
\psi_3' &= \frac{\gamma_2}{h^4(1 + \gamma_1)} \left[ 3(\psi_2)^2(\psi_3') + 6(\psi_2)(\psi_3')^2 \right] \\
&\quad + \frac{M^2 h^2 (1 - ah^2 \eta^2)}{(1 + \gamma_1)} \psi_2 - \frac{2M^2 h^4 \alpha \eta}{(1 + \gamma_1)} (\psi_1 + h) \\
&\quad - \frac{h^3 Gr}{(1 + \gamma_1)} \theta_1, \\
\theta' &= \theta_1.
\end{align*}
\]  
(30)

The above system of differential equations can be written as a system of integral equations (variational iteration formula) as follows:
\[
\begin{align*}
\psi(n) &= \psi(n-1) - \int_0^C \psi_3(n-1) \, d\eta, \\
\psi_3(n) &= \psi_3(n-1) - \int_0^C \psi_3(n-1) \, d\eta, \\
\psi_2(n) &= \psi_2(n-1) - \int_0^C \psi_2(n-1) \, d\eta, \\
\psi_1(n) &= \psi_1(n-1) - \int_0^C \psi_1(n-1) \, d\eta,
\end{align*}
\]  
(31)

According to the boundary conditions (Equation 29), it is straightforward to choose initial guesses as
\[
\psi_0 = 0, \quad \psi_1(0) = A, \quad \psi_2(0) = 0, \quad \psi_3(0) = B,
\]
Table 1. A comparison between the variational iteration method (VIM) and numerical solutions at $\gamma_2 = 0.01$, $\gamma_1 = 0.02$, $Gr = 0.1$, $M = 0.1$, $\alpha = 0.1$, $x = 0.5$, $\varphi = 1.3$, $\phi = 0.4$ and $Br = 0.1$.

| $y$  | $u$ (VIM solution) | $u$ (Numerical solution) | $\theta$ (VIM solution) | $\theta$ (Numerical solution) |
|------|--------------------|--------------------------|--------------------------|--------------------------|
| 0    | 0.94692            | 0.946916                 | 0                        | 0                        |
| 0.2  | 0.869787           | 0.869785                 | 0.225474                 | 0.225474                 |
| 0.4  | 0.637717           | 0.637717                 | 0.448105                 | 0.448105                 |
| 0.6  | 0.249583           | 0.249585                 | 0.660527                 | 0.660526                 |
| 0.8  | -0.295874          | -0.295871                | 0.850338                 | 0.850338                 |
| 1    | -1                 | -0.9998                  | 1                        | 0.9997                   |

$\theta_{(0)} = 0$, $\theta_{1(0)} = C$. 

4. Discussion of results

The following sections investigate the impacts of the included parameters on axial velocity $u$, shear stress $S_{12}$, pressure gradient $dP/dx$, temperature $\theta$, Nusselt number $Nu$ and stream function $\psi(x, y)$.

4.1. Velocity and shear stress

The main reason for this part is to investigate the impacts of included parameters on the axial velocity $u$ (Figures 2–6) and shear stress $S_{12}$ (Figure 7).

The influences of the Eyring–Powell fluid parameters $\gamma_2$, $\gamma_1$ are discussed through Figures 2 and 3, respectively. It is observed that the peristalsis phenomenon

Figure 2. The longitudinal velocity distribution $u$, across the channel with different values of $\gamma_2$ at $\gamma_1 = 0.2$, $Gr = 0.1$, $M = 0.1$, $\alpha = 0.1$, $x = 0.5$, $\varphi = 1.3$, $\phi = 0.4$ and $Br = 0.01$.

Figure 3. The longitudinal velocity distribution $u$, across the channel with different values of $\gamma_1$ at $\gamma_2 = 0.2$, $Gr = 0.1$, $M = 0.1$, $\alpha = 0.1$, $x = 0.5$, $\varphi = 1.3$, $\phi = 0.4$ and $Br = 0.01$.

Figure 4. The longitudinal velocity distribution $u$, across the channel with different values of $Gr$ at $M = 0.1$, $\alpha = 0.1$, $x = 0.5$, $\varphi = 1.3$, $\phi = 0.4$ and $Br = 0.01$.

Figure 5. The longitudinal velocity distribution $u$, across the channel with different values of $M$ at $\gamma_1 = 0.1$, $Gr = 0.5$, $\gamma_2 = 0.2$, $\alpha = 0.1$, $x = 0.5$, $\varphi = 1.3$, $\phi = 0.4$ and $Br = 0.01$. 
appears near the walls of the channel and more efficient on the non-Newtonian fluid. Even so, in the centre of the channel, the effect of peristalsis disappears and the fluid moves under the pressure gradient. Moreover, Figure 2 shows that the Newtonian fluid is accelerating more than the non-Newtonian fluid away from the peristaltic walls due to the fluid movement is highly reliant on the viscosity of fluids.

Figure 4 shows the impact of the Grashof number \( \text{Gr} \) (i.e. the free convection process) on the axial velocity. It is known that the viscous force of the Newtonian fluid is less than that of the non-Newtonian fluid, and this leads to the process of free convection is more efficient on the Newtonian fluid as seen in the centre of the channel away from the peristaltic walls. The impacts of the magnetic parameter \( M \) and the parameter that determines the pattern of distribution of the magnetic field through the fluid \( \alpha \) are displayed through Figures 5 and 6 respectively. Clearly, as the magnetic parameter increases the axial velocity reduces due to the impact of Lorentz force, which opposes the flow. It is noteworthy that the peristaltic transport overcomes on Lorentz force near the walls of the channel. From Figure 6, it is noticed that the quantitative increase in the parameter \( \alpha \) (decrease the magnitude of the magnetic field) leads to accelerate the axial velocity in the core stream.

Figure 7 displays the impacts of the magnetic parameter \( M \), the Eyring–Powell fluid parameters \( \gamma_2 \), \( \gamma_1 \) and the Grashof number \( \text{Gr} \) on the shear stress \( S_{12} \). It is noticed that the shear stress takes the same wave shape in the narrow and wide sections of the channel and decreases by increasing the value of the magnetic parameter, the Grashof number and Eyring–Powell fluid parameter \( \gamma_1 \) (inverse result is seen in the narrow part for the parameter \( \gamma_1 \)). Moreover, the shear stress is large in the non-Newtonian fluid compared with Newtonian fluid as seen from Figure 7(b).
4.2. Pressure gradient

Figures 8–13 display the effects of the different parameters on the pressure gradient \( \frac{dp}{dx} \). Figure 8 shows a comparison between the Eyring–Powell fluid and a viscous fluid. It is clear that the Powell–Eyring fluid needs a great pressure gradient to flow in the narrow part of the channel compared with a viscous fluid and vice versa in the wider part. Moreover, in the wider part of the channel, the pressure gradient is relatively small compared with the narrow part. This phenomenon happens due to the flow passes easily through the wider part
of the channel but in the narrow part, a large pressure gradient is required to maintain the same flux. The influence of the Eyring–Powell fluid parameter $\gamma_1$ is seen in Figure 9. It is observed that the pressure gradient increases in the narrow part by increasing the material parameter $\gamma_1$ but in the wider part the inverse result is seen. Figure 10 illustrates that, with increasing the material parameter $\gamma_2$, the pressure gradient decreases in the narrow section of the channel and in the wider section, no variation is seen. In narrow and wider parts in the channel an increase in the Grashof number $Gr$
Figure 20. Nusselt number at the walls for fixed values of $M = 1, \varphi = 1.3, \phi = 0.4$ and $\alpha = 0.1$ and different values of (a) $Gr$ at $\gamma_1 = 0.1, \gamma_2 = 0.02, Br = 0.1$, (b) $\gamma_2$ at $\gamma_1 = 0.1, Gr = 2, Br = 0.1$, (c) $\gamma_1$ at $\gamma_2 = 0.03, Gr = 2, Br = 0.1$, (d) $Br$ at $\gamma_1 = 0.1, Br_2 = 0.01, Gr = 2$.

(i.e. the free convection process) leads to increase in the pressure gradient as shown in Figure 11. Figure 12 indicates that by increasing the magnetic parameter the pressure gradient increases in the narrow part and vice versa in the wider part. Figure 13 shows the effect of the parameter $\alpha$ on the pressure gradient. It is noticed that an increase in the parameter $\alpha$ leads to increase in the pressure gradient in the wider part of the channel, while, in the narrow part, an inverse result is seen.

4.3. Heat characteristics

Heat transfer across the channel under the included parameters is discussed through Figures 14–19. Moreover, the Nusselt number is discussed in Figure 20. Figure 14 shows that the variation of temperature for the Newtonian fluid is large compared with Eyring–Powell fluid, which is expected. The variation of temperature decreases by elevating the value of the Eyring–Powell fluid parameters $\gamma_2$ and $\gamma_1$ as shown in Figures 14 and 15. The effects of the magnetic parameter $M$ and the parameter that determines the pattern of distribution of the magnetic field through the fluid $\alpha$ are displayed through Figures 16 and 17 respectively. It is noticed that the two parameters have an effect on the temperature variation. The influence of Brinkman number $Br$ on the temperature variation is shown in Figure 18. It is noticed that there exists a linear relation between the temperature $\theta$ and $y$ when the Brinkman number $Br = 0$. Moreover, the higher values

Figure 21. Streamlines for different values of $\gamma_2$ ($\gamma_2 = 0.02, \gamma_2 = 0.9$, panels a,b respectively) with fixed values of $\gamma_1 = 0.2$, $Gr = 0.1, Br = 0.5, \varphi = 1.3, \phi = 0.4, M = 0.5, x = 0.5$ and $\alpha = 0.1$, where $y \in [0, h(x)]$. 
of the Brinkman number lead to a reduction in the heat produced by viscous dissipation and consequently the variation of temperature increases. An increase in the Grashof number $Gr$ leads to a reduction in the variation of temperature as shown in Figure 19.

Figure 20 depicts the variation of the Nusselt number $\lambda$ along the channel. It is clear that Nusselt number decreases in the wider part and increases in the narrow part of the channel. This happens due to, that in the narrow part, the velocity increases and facilitates the heat transport between the fluid and the walls. Moreover, the figure shows that an increase in the parameters $Gr$, $\gamma_1$ and $Br$ decreases the Nusselt number while an increase in the parameters $\gamma_2$ increases the Nusselt number in the narrow part.

4.4. Streamlines pattern

The influences of the Eyring–Powell fluid parameters $\gamma_2$ and $\gamma_1$ and magnetic parameter $M$ on the streamlines are shown in Figures 21–23. It is noticed that the trapping bolus (closed streamlines surround a volume of fluid) appears and it takes the wave shape in the upper stream. Moreover, the number of trapped bolus decreases while the size increases by increasing the Eyring–Powell fluid parameters $\gamma_2$ and $\gamma_1$ due to the variation in the shear rate viscosity. The effect of the magnetic parameter $M$ is seen through Figure 23. It is clear that by elevating the value of the magnetic parameter, the number of the trapped bolus decreases due to the impact of Lorentz force.

5. Concluding remarks

A mathematical model for peristaltic flow of an Eyring–Powell fluid with varying magnetic field and heat transfer has been discussed. Variational iteration method (VIM) is used to solve the mathematical model to avoid the restrictions in the perturbation technique. The semi-analytical solutions are found by VIM for the velocity, heat transfer, pressure gradient and stream function which include Eyring–Powell fluid parameters. Moreover, a comparison between VIM and numerical solutions was discussed. The obtained results can be summarized as follows:

- A comparison between variational iteration method (VIM) and numerical solution shows that they are in a good agreement as shown in Table 1.
- The Newtonian fluid is accelerating more than the non-Newtonian fluid far from the peristaltic walls.
The convection process is more efficient on the Newtonian fluid than that in the non-Newtonian fluid far from the peristaltic walls.

As the magnetic parameter increases the axial velocity reduces due to the impact of Lorentz force, which opposes the flow.

The Powell–Eyring fluid needs a great pressure gradient to flow in the narrow part of the channel compared with a viscous fluid and vice versa in the wider part.

The variation of the heat transfer for the Newtonian fluid is large compared with Eyring–Powell fluid.

Due to the variation in the shear rate viscosity of the Eyring–Powell fluid, the number of trapped bolus decreases while the size increases.

Disclosure statement
No potential conflict of interest was reported by the authors.

Funding
The first author extends his appreciation to the Deanship of Scientific Research at King Khalid University for funding this work through General Research Project under grant number Scientific Research at King Khalid University for funding this manuscript.

ORCID
Y. A. S. El-Masyr  http://orcid.org/0000-0002-1678-6584
Y. Abd Elmaboud  http://orcid.org/0000-0002-3703-1500
M. A. Abdel-Sattar  http://orcid.org/0000-0002-3183-1662

References
[1] Murthy K. Nanoparticles in modern medicine: state of the art and future challenges. Int J Nanomed. 2007;2(2):129–141.
[2] Kahn T, Harth T, Kiwit JCW, et al. In vivo MRI thermometry using a phase-sensitive sequence: preliminary experience during MRI-guided laser induced interstitial thermotherapy of brain tumors. J Magn Reson Imaging. 1998;8:160–164.
[3] Ellahi R, Sait SM, Shehzad N, et al. Numerical simulation and mathematical modeling of electro-osmotic Couette–Poiseuille flow of MHD power-law nanofluid with entropy generation. Symmetry. 2019;11(8):1038.
[4] Zeeshan A, Shehzad N, Abbas T, et al. Effects of radiative electro-magnetohydrodynamics diminishing internal energy of pressure-driven flow of titanium dioxide-water nanofluid due to entropy generation. Entropy. 2019;21(3):236.
[5] Ellahi R, Zeeshan A, Shehzad N, et al. Structural impact of kerosene-AI 3 O 3 nanoliquid on MHD Poiseuille flow with variable thermal conductivity: application of cooling process. J Mol Liq. 2018;264:607–615.
[6] Shehzad N, Zeeshan A, Ellahi R. Electrosomotic flow of MHD power law AI 3 O 3 -PVC nanofluid in a horizontal channel: Couette–Poiseuille flow model. Commun Theor Phys. 2018;69(6):655–666.
[7] Latham T. Fluid motion in a peristaltic pump [M.Sc. Thesis], Cambridge, MA: MIT; 1966.
[8] Fung T, Yih C. Peristaltic transport. Trans ASME J Appl Mech. 1968;35:669–675.
[9] Shapiro AH, Jaffrin MY, Weinberg SL. Peristaltic pumping with long wavelengths at low Reynolds number. J Fluid Mech. 1969;37:799–825.
[10] Abd elmaboud Y, Mekheimer Kh. S. Non-linear peristaltic transport of a second-order fluid through a porous medium. Appl Math Model. 2011;35:2695–2710.
[11] Noreen S, Hayat T, Alsaedi A. Study of slip and induced magnetic field on the peristaltic flow of pseudoplastic fluid. Int J Phys Sci. 2011;6(36):8010–8026.
[12] Noreen S, Alsaedi A, Hayat T. Peristaltic flow of pseudoplastic fluid in an asymmetric channel. J Appl Mech. 2012;79(5):054501.
[13] Nadeem S, Riaz A, Ellahi R, et al. Mathematical model for the peristaltic flow of Jeffrey fluid with nanoparticles phenomenon through a rectangular duct. Appl Nanosci. 2014;4:613–624.
[14] Hayat T, Khan M, Siddiqui AM, et al. Non-linear peristaltic flow of a non-Newtonian fluid under effect of a magnetic field in a planar channel. Commun Nonlinear Sci Numer Simulat. 2007;12:910–919.
[15] Ellahi R, Hussain F, Ishfaq F, et al. Peristaltic transport of Jeffrey fluid in a rectangular duct through a porous medium under the effect of partial slip: an approach to upgrade industrial sieves/filters. Pramana J Phys. 2019;93(3):34–43.
[16] Yoon HK, OhaJar AJ. A note on the Powell–Eyring fluid model. Int Comm Heat Mass Transf. 1987;14:381–390.
[17] Hina S. MHD peristaltic transport of Eyring–Powell fluid with heat/mass transfer, wall properties and slip conditions. J Magn Magn Mater. 2016;404:148–158.
[18] Hayat T, Shah SI, Ahmad B, et al. Effect of slip on peristaltic flow of Powell–Eyring fluid in a symmetric channel. Appl Bionics Biomech. 2014;11:69–79.
[19] Akbar NS, Nadeem S. Characteristics of heating scheme and mass transfer on the peristaltic flow for an Eyring–Powell fluid in an endoscope. Int J Heat Mass Transf. 2012;55:375–383.
[20] Abbasi F, Alsaedi A, Hayat T. Peristaltic transport of Eyring–Powell fluid in a curved channel. J Aerosp Eng. 2014;27:04014037.
[21] Hayat T, Tanveer A, Yasmin H, et al. Effects of convective conditions and chemical reaction on peristaltic flow of Eyring–Powell fluid. Appl Bionics Biomech. 2014;11:221–233.
[22] Prakash J, Tripathi D, Tiwari AK, et al. Peristaltic pumping of nanofluids through tapered channel in porous environment: applications in blood flow. Symmetry. 2019;11(7):868.
[23] Hiremath PS, Patil PM. Free convection effects on the oscillatory flow of a couple stress fluid through a porous medium. Acta Mech. 1993;98:143–158.
[24] Tlili I, Bhatti MM, Hamad SM, et al. Macroscopic modeling for convection of hybrid nanofluid with magnetic effects. Phys A Stat Mech Appl. 2019;534(15):122136.
[25] Meckheimer KhS, Abdulmaboud Y. The influence of heat transfer and magnetic field on peristaltic transport of a Newtonian fluid in a vertical annulus: application of an endoscope. Phys Lett A. 2008;372:1657–1665.
[26] Meckheimer KhS, Husseny SZA, Abd Elmaboud Y. Effects of heat transfer and space porosity on peristaltic flow in a vertical asymmetric channel. J Numer Met Part Differ Equ. 2009. doi:10.1002/nump.20451.
[27] Tripathi D, Beg OA. A study on peristaltic flow of nanofluids: application in drug delivery systems. Int J Heat Mass Transf. 2014;70:61–70.
[28] Waqas H, Khan SU, Imran M, et al. Thermally developed Falkner–Skan bioconvection flow of a magnetized nanofluid in the presence of a motile gyrotactic microorganism: Buongiorno’s nanofluid model. Phys Scripta. 2019;94(11):115304.

[29] Khan AA, Bukhari SR, Marin M, et al. Effects of chemical reaction on third-grade MHD fluid flow under the influence of heat and mass transfer with variable reactive index. Heat Transf Res. 2019;50(11):1061–1080.

[30] Shahmohamadi H, Rashidi MM. VIM solution of squeezing MHD nanofluid flow in a rotating channel with lower stretching porous surface. Adv Powder Technol. 2016;27:171–178.

[31] Moosavi M, Momeni M, Tavangar T, et al. Variational iteration method for flow of non-Newtonian fluid on a moving belt and in a collector. Alexandria Eng J. 2016;55:1775–1783.

[32] Abd elmaboud Y. Varying magneto-hemodynamics flow in a semi-porous vertical channel with heat transfer: numerical and analytical solutions. Int J Fluid Mech Res. 2016;43(2):105–118.

[33] Riaz A, Ellahi R, Bhatti MM, et al. Study of heat and mass transfer in the Eyring–Powell model of fluid propagating peristaltically through a rectangular compliant channel. Heat Transf Res. 2019;50(16):1539–1560.