Buckling of a ballasted curved track under unloaded conditions

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Abstract
There is no industry accepted analytical model to compute the critical temperature differential for the buckling of an unloaded curved track in North American literature. In this paper, the critical temperature differential for the buckling of an unloaded curved track is formulated by incorporating a value of unity for the factor of safety in the previously developed formula, which was developed considering thermal loading only. The factor of safety was the ratio between the resistance of a tie in an unloaded track against lateral displacement in the ballast and the lateral thermal load on a tie. The derived formula of the critical temperature differential for the buckling of an unloaded curved track is simple opposed to a complicated formula endorsed in the current European literature from 1969. The new formula is also validated in this paper. The critical temperature differentials for buckling of sharp and super-sharp curves have significant implications for track design and maintenance.

Keywords
Buckling of track, thermal load, critical temperature differential, rail neutral temperature, critical misalignment, ballast resistance, alignment defect

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Introduction
A couple of field tests were conducted on tangent track from 1932 to 1966 to determine buckling load.1 Esveld2 and Hasan3 suggested analytical formulae to compute buckling load of a tangent track. There is little work on buckling load of a ballasted curved track in recent literature. An analytical formula of the critical temperature differential, $\Delta T_C$, for buckling of a curved ballasted track is reviewed; this formula from 1969 is endorsed by Lichtberger.4

Hasan5 presented formulas for track stability analysis against displacement in terms of factor of safety under four loading scenarios. The factor of safety was the ratio between the resistance of a tie against lateral displacement in the ballast and lateral load on a tie. The loading scenarios were

Case I: Under vehicle and compressive thermal load,
Case II: Under vehicle and tensile thermal load,
Case III: Under vehicle load only (no thermal load), and
Case IV: Under thermal load only (no vehicle load).

The loading scenario in Case IV is the most critical for track shifting and buckling, as the resistance of an unloaded track is much lower than that of a loaded track. Thus, the loading under Case IV is the most relevant for studying the critical temperature differential or critical buckling load of an unloaded track. The factor of safety was the ratio between the resistance of a tie in...
an unloaded track against lateral displacement in the ballast and the lateral thermal load on a tie.

\( \Delta T_c \) for the buckling of a curved unloaded track is derived by incorporating a factor of safety equaling unity in the formula presented in the previous work for the loading scenario under Case IV so that the corresponding criteria for track design, maintenance, and evaluation can be formulated. The stability of the track needs to be enhanced if \( \Delta T_c \) is lower than the available temperature differential, \( \Delta T \). Rail buckling temperature, sum of \( \Delta T_c \) and stress neutral temperature, would help to determine hot weather patrolling temperature to ensure safety of rail traffic. A track may be qualified as weak or strong depending on the value of \( \Delta T_c \). The previous formula for \( \Delta T_c \) in the literature and its computed value are compared with the derived formula for different curve radii. The results appear in Table 1. A brief comparison is made between the new and exist- ing formulas for the critical temperature differential of a ballasted curved track. Then, the proposed formula is more suitable than its previous counterpart. The following assumptions and data are used to calculate \( \Delta T_c \):\(^4\)

\[
\Delta T_c = - \frac{8 \times I^*}{\alpha \times R \times f^*} + \sqrt{\frac{8 \times I^*}{\alpha \times A \times R \times f^*}} ^2 + \frac{16 \times I^* \times F_{QVW}}{\alpha^2 \times A^2 \times E \times I^*} \tag{1}
\]

The critical buckling load is calculated by\(^2\)

\[
F_0 = A \times E \times \alpha \times \Delta T_c \tag{2}
\]

The following assumptions and data are used to calculate \( \Delta T_c \):\(^4\)

- \( f^* = 2 \) cm (assumption)
- Rail profile: UIC 60 (weight 60.34 kg/m, 121.64 lb/yd)
- Concrete tie: B 70 W (pre-stressed concrete tie at Germany)
- \( f' = 2200 \) cm\(^4\)
- \( A = 2 \times 76.9 = 153.8 \) cm\(^2\)
- \( F_{QVW} = 10 \) kN/m = 100 N/cm
- \( R = 500 \) m = 50,000 cm
- \( E = 21 \times 10^6 \) N/cm\(^2\)
- \( \alpha = 0.000012/\)°C

Table 1. Application of equation (1).

| R (m) | DOC | \( F_{QVW} \) (N/cm) | \( l \) (cm\(^4\)) | \( f^* \) (cm) | \( \Delta T_c \) (°C) |
|-------|-----|-------------------|-----------------|--------------|----------------|
| 50    | 34.9°| 100               | 2200            | 2            | 12.8           |
| 75    | 23.3°| 100               | 2200            | 2            | 19.1           |
| 100   | 17.5°| 100               | 2200            | 2            | 25.1           |
| 125   | 14.0°| 100               | 2200            | 2            | 31.0           |
| 150   | 11.6°| 100               | 2200            | 2            | 36.6           |
| 200   | 8.7° | 100               | 2200            | 2            | 47.0           |
| 250   | 7.0° | 100               | 2200            | 2            | 56.2           |
| 300   | 5.8° | 100               | 2200            | 2            | 64.4           |
| 500   | 3.5° | 100               | 2200            | 2            | 88.2           |
| 700   | 2.5° | 100               | 2200            | 2            | 102.9          |

An axial compression force of 2040 kN (1020 kN in each rail) may be enough to buckle a track.\(^1\) Seven field tests on a tangent track yielded \( \Delta T_c \) of 42.5°C–50°C corresponding to a track buckling load of 1730–2000 kN; the lower buckling load was observed on geometrically imperfect test tracks.\(^4\) Esvel\(^2\) computed a critical buckling load of 2018 kN for a misaligned tangent track with UIC 60 rail exhibiting a constant lateral shear resistance of 10 kN/m; the ballast resistance and rail section were same as those in the aforementioned calculation example provided by Lichtberger.\(^4\) Temperature load of 50°C corresponds to 1800, 1906, and 2130 kN for 115 RE, UIC 60, and 136 RE rail respectively; buckling load is expected to be more but not significantly due to track grid effect. In context with aforementioned discussion, a \( \Delta T_c \) of 50°C may be taken as a conservative value for buckling of a real-life tangent track. Interestingly the actual rise in temperature, \( \Delta T \), amounts to 45°C–50°C considering the deviations that occur in practice when rails are tensioned.\(^2\) Buckling of a tangent track is very unlikely if the available temperature differential, \( \Delta T \) is less than \( \Delta T_c \). Thus, risk of thermal buckling may be reduced or removed by setting an appropriate rail neutral temperature considering the anticipated maximum rail temperature and ballast resistance.

According to Table 1, a curved track of radius \( \approx 200 \) m exhibits a critical temperature differential of \(<50°C\), which seems logical and is therefore acceptable. It also appears that a curved track of radius \( >200 \) m is thermally more stable than the tangent track; however, this is confusing; \( \Delta T_c \) is expected to be less than or equal to that of a tangent track that is, \( \approx 50°C\). With the increase of radius, thermal load on a tie reduces but ballast resistance remains same; thus the \( \Delta T_c \) for a mild or shallow curve might exceed 50°C. However, stability of a mild or shallow curve is not a concern.

A value of the equivalent moment of inertia of the UIC 60 rail track grid, \( I^* \) is given without computation.\(^4\) Probably \( I^* \) is estimated by using beam
deflection theory with assumed value of span and deflection of a buckled track beam under assumed end support condition. The equivalent moment of inertia of the UIC 60 rail track grid, \( I \), is estimated to be 2200 cm\(^4\), whereas the moment of inertia of two UIC 60 rails (2\( I \)) is 1026 cm\(^4\). This implies that the lateral stiffness of the track frame increases by 114\%. The huge increase of 114\% in lateral stiffness due to track grid effect is not in harmony with the following facts.

The longitudinal resistance of a track equals its lateral resistance\(^4\); implying that lateral bending stiffness makes no significant contribution toward the lateral resistance of the track, and almost all of the resistance is provided by the ballast. In fact, according to the American Railway Engineering and Maintenance-of-Way Association (AREMA), the longitudinal load developed by the combination of thermal stress in a continuously welded rail and due to traffic is restrained by the mass internal friction of the ballast.\(^7\) Dognet\(^8\) showed that 23 factors influence the lateral resistance of a track, and he mentioned that tie spacing has little or no influence. Zarembski\(^9\) also compiled a list of 23 parameters that influence lateral displacement resistance, and the list excluded tie spacing. Logically, the track grid cannot influence ballast resistance significantly when tie spacing does not. The torsional rigidity of the track grid exerts a minor influence on lateral resistance.\(^9\) The stiffness of the rail–tie structure does not play a very important role in the lateral rigidity of the entire track. The track frame contributes 5\%–10\% of the lateral resistance of the track.\(^2\) It has been estimated that an increase in the ballast resistance by 10\% is sufficient to compensate for the differences in stiffness of tested track panels.\(^10\) The calculated moment of inertia of a track grid, \( I \), of 2200 cm\(^4\) is not in agreement with the recent findings that the lateral resistance is equal to the longitudinal resistance of a track and that the track frame contributes 5\%–10\% of the lateral resistance of a track. Thus, the author intends to use the moment of inertia of two rails, 2\( I \), as the equivalent moment of inertia of the track grid, \( I^* \), to study the effect on the critical temperature differential. Thus, the moment of inertia of two UIC 60 rails (1026 cm\(^4\)) is applied to equation (1) (see Table 2), assuming that the track grid makes no contribution toward the lateral stiffness vis-à-vis the lateral resistance.

A comparison of the values of \( \Delta T_C \) (°C) in Tables 1 and 2 reveals that the track grid has little effect on the buckling load for a radius of up to 300 m. This reasoning is supported by the aforementioned literature. As the curvature decreases the \( \Delta T_C \) is expected to converge toward the \( \Delta T_C \) of a tangent track. Thus, the computed critical temperature differentials over 50°C in Tables 1 and 2 are questionable. It appears that the curved tracks of radius \( \geq 250 \) m are thermally more stable than the tangent track; however, this result remains confusing. However, engineers are concerned with the stability of sharp curves, not mild or shallow curves. A critical temperature differential over 50°C might be an incorrect value for a curved track, but it indicates a stable track. The issue is further investigated with a new formula, as derived below.

### Development of formula for \( \Delta T_C \) of an unloaded track

The following formula was derived to check the status of an unloaded curved track under thermal load in the context of stability in terms of factor of safety\(^5\):

\[
FS = \frac{500 \times R \times R_S}{S \times A \times E \times \alpha \times \Delta T}
\]

(3)

A minimum factor of safety of 2.5 is recommended for an unloaded curved track.\(^5\) The longitudinal thermal force in the rails exerts a lateral force on the ties along a curve. The equilibrium under a factor of safety (FS) of one (i.e. assuming that the thermal load on a tie is equal to its lateral resistance) is unstable, which may cause buckling or radial displacement in case of sharp curve. The temperature differential or the corresponding load on the track is designated as the critical temperature differential or critical load, respectively. A small increase in load over the critical value can cause a significant misalignment.

Thus, incorporating \( FS = 1 \) in equation (3) results in the following formula expressing \( \Delta T_C \) for buckling of a curved track:

\[
\Delta T_C = \frac{500 \times R \times R_S}{A \times E \times S \times \alpha}
\]

(4)

Discreet values of sleeper resistance, \( R_S \) (kN) from panel test often do not come with sleeper spacing, \( S \) (m), and/or maintenance condition of track. The recommended values of unit resistance (kN/m) are

| \( R (m) \) | \( DOC \) | \( F_{QVW} \) (N/cm) | \( l (cm^4) \) | \( f^* (cm) \) | \( \Delta T_C (\degree C) \) |
|---|---|---|---|---|---|
| 50 | 34.9\* | 100 | 1026 | 2 | 12.7 |
| 75 | 23.3\* | 100 | 1026 | 2 | 18.8 |
| 100 | 17.5\* | 100 | 1026 | 2 | 24.5 |
| 125 | 14.0\* | 100 | 1026 | 2 | 29.8 |
| 150 | 11.6\* | 100 | 1026 | 2 | 34.7 |
| 200 | 8.7\* | 100 | 1026 | 2 | 43.2 |
| 250 | 7.0\* | 100 | 1026 | 2 | 50.3 |
| 300 | 5.8\* | 100 | 1026 | 2 | 56.1 |
| 500 | 3.5\* | 100 | 1026 | 2 | 71.5 |
| 700 | 2.5\* | 100 | 1026 | 2 | 80.0 |

Rail UIC 60, \( A = 2 \times 76.9 = 153.8 \) cm\(^2\) in the equation (1).
Table 3. Application of equation (5) for comparison with equation (1).

| R (m) | DOC | $F_{QVW}$ (N/cm) | $\Delta T_C$ (°C) by the equation (5) | $\Delta T_C$ (°C) by equation (1), ref: Table 1 | Diff. (°C) |
|-------|-----|------------------|--------------------------------------|-----------------------------------------------|-----------|
| 50    | 34.9° | 100              | 12.9                                 | 12.8                                          | 0.1       |
| 75    | 23.3° | 100              | 19.4                                 | 19.1                                          | 0.3       |
| 100   | 17.5° | 100              | 25.8                                 | 25.1                                          | 0.7       |
| 125   | 14.0° | 100              | 32.3                                 | 31.0                                          | 1.3       |
| 150   | 11.6° | 100              | 38.7                                 | 36.6                                          | 2.1       |
| 200   | 8.7°  | 100              | 51.6                                 | 47.0                                          | 4.6       |
| 250   | 7.0°  | 100              | 64.5                                 | 56.2                                          | 8.3       |
| 300   | 5.8°  | 100              | 103.2                                | 64.4                                          | 38.8      |
| 500   | 3.5°  | 100              | 129.0                                | 88.2                                          | 40.8      |
| 700   | 2.5°  | 100              | 180.6                                | 102.9                                         | 77.7      |

Rail UIC 60, $A = 2 \times 76.9 = 153.8 \text{cm}^2$ in the equation (1).

available in codes and literature. Thus, discreet value of sleeper resistance is converted to unit resistance. Replacing $R_s$ (kN)/S (m) by $F_{QVW}$ (N/cm) and adjusting the units, equation (4) may be rewritten as

$$\Delta T_C = \frac{50 \times F_{QVW}}{A \times E \times \alpha}$$

Note that $A$ is the area of a single rail in equation (5), whereas in equation (1), $A$ denotes the area of two rails. The assumption underlying equation (5) is that the buckling mode is in the horizontal plane. The computation results for $\Delta T_C$ using equation (5) are presented in Table 3 and compared with the values in Table 1, which were calculated using equation (1).

Both equations (1) and (5) provide practically equal values of $\Delta T_C$ for radii up to 200 m. Both formulae indicate that curves with radii $>200$ m are thermally more stable than the tangent track; again, this reasoning might be questionable. Thus, although the critical temperature differential over 50°C seems to be a confusing value, it is inconsequential because the actual rise in temperature, $\Delta T$, amounts to 45°C–50°C considering the deviations that occur in practice when rails are tensioned, and thus, this value may be regarded as safe. Hence, equations (1) and (5) can be said to be functionally similar.

Track standards across Europe dictate that continuous welded rail (CWR) should not be installed on tracks with radii tighter or less than 500 m. This standard is mandatory for new constructions; however, it also recognizes that tracks with radii of up to 100 m exist in all networks. The UK standards mention that CWRs should not be installed on tracks with radii $<250$ m due to the increased chance of track buckling. North America has no specific mandated rules on curve radius for CWR tracks. Different manuals and agencies suggest various maximum degree of curvature, the examples of which are cited below.

For new light rail transit (LRT) construction, AREMA recommends a curvature of no more than 23° ($R = 76$ m). AREMA also notes that the curvature should not exceed 19° ($R = 92$ m) for new heavy transit systems.7 The Transit Co-operative Research Program recommends a maximum curvature of 12° ($R = 145$ m) for lead tracks and industrial side tracks. A mainline curvature of 10° ($R = 175$ m) is regarded as the nominal maximum for designing commuter railroads. Burlington Northern and Santa Fe (BNSF) Railway recommends that the curvature be limited to 9°30’ ($R = 184$ m) for industrial tracks. TCRP recommends that the curvature should not exceed 9°30’ ($R = 184$ m) for LRT track design.12 The Canadian National (CN) Railway recommends a maximum curvature of 9° ($R = 195$ m) for industrial tracks.14 The aforementioned minimum radii are probably based on curve resistance and/or the ability of rolling stock to negotiate a curve. Among aforementioned values of radius, the minimum value of recommended radius is 76 m by AREMA. Thus, a super-sharp curve is defined as a curve with radius, $R \approx 75$ m in this paper. Modern LRT vehicles can even negotiate 35-m curves. Super-sharp curves are not used on the main line. The minimum radius of a ballasted curve should be based on the thermal stability of the unloaded track.

A 34-m curved track with regular concrete ties and without tie anchors (tie spacing = 685 mm and 100 ARA-A rail) and a shoulder ballast width of 600 mm has been constructed in the Oliver Bowen Maintenance Facility (OBMF) in Calgary, Canada. A curved track with an 80-m radius, regular concrete ties with a tie spacing of 750 mm, and 54 E1 rails (without tie anchors) has been constructed in the operation and maintenance (OMC) yard of the Canada Line Project anchors) has been constructed in the Oliver Bowen Maintenance Facility (OBMF) in Richmond, BC, Canada. A curved track with an 80-m radius, regular concrete ties with a tie spacing of 750 mm, and 54 E1 rails (without tie anchors) has been constructed in the operation and maintenance (OMC) yard of the Canada Line Project anchors) has been constructed in the Oliver Bowen Maintenance Facility (OBMF) in Richmond, BC, Canada. Kerr uses a resistance of 80 N/cm (800 kgf/m) for wood tie tracks. The Paris Metro Agency uses a resistance of 90 N/cm (0.9 tonne/m) for wood tie track. It is estimated that for a track to remain perfectly stable after the passage of several hundreds of thousands of tons, the ratio of the resistance to axial displacement should be approximately 1200 daN/m for the track that is, 120 N/cm. Esselst considers a longitudinal resistance of 100–200 N/cm (10–20 kN/m) to compute the breaching length of a continuously welded rail track; it seems that the range of resistance covers the wood-to-concrete tie track. Some railways (e.g. the German Railway (DBAG) and the Norfolk Southern (NS)) consider a same ballast resistance value ($k$) of 200 N/cm for unloaded tracks (assuming that the ties in the ballast are well maintained).15 Using the aforementioned minimum radii recommended by different agencies, two radii from the field,
and a ballast resistance of 100–200 N/cm for concrete ties, the critical temperature differentials are computed and presented in Table 4.

If the available temperature differential, \( \Delta T \), is less than the critical temperature differential, \( \Delta T_C \), computed in Table 4 using equation (5), then the curve would be thermally stable; thus, a correct assessment or choice of the ballast resistance value based on ballast consolidation is very important. Table 4 shows that the recommended minimum radii by CN Railway and BNSF Railway are safe for a CWR track even if the ballast resistance is as low as 100 N/cm. It is preferable to avoid a ballasted track if \( \Delta T \) exceeds the value of \( \Delta T_C \) computed using equation (5); otherwise, tie anchors should be used to augment lateral resistance.

The 34-m radius curve without tie anchors at OBMF, Calgary, Canada, has been found to swing by approximately ±20 mm. The maximum, minimum, and stress neutral rail temperatures are 58.3°C, −40°C, and 20°C, respectively. This implies a maximum temperature differential of 38.3°C in summer and 60°C in winter in the field. The curve swings as the temperature differential in the field is higher than the critical temperature differential of 21.2°C (see the first row in Table 4). It is evident that an excessive wide shoulder (e.g. 600 mm) does not help to increase the lateral resistance.

A photo is provided in Figure 1.

The 80-m radius curve at the OMC in Richmond, BC, Canada has exhibited no visible lateral swing since its construction. The maximum, minimum, and stress neutral rail temperatures are 50°C, −20°C, and 20°C (±3°C, −6°C), respectively. This implies a maximum field temperature differential of 42°C in summer and 43°C in winter. The curve might or might not swing, as the calculated critical temperature differential of 21.2°C (see the third row in Table 3) may be more or less than the temperature differential in the field depending on the ballast resistance it encounters; the curve has never buckled since construction.

Super-sharp curves should be avoided where ties cannot be installed too closely or where the ties cannot be equipped with tie anchors to ensure an ample \( FS \) against lateral displacement. Mono-block standard concrete ties may be replaced by scalloped ties or frictional ties to enhance lateral resistance. An experiment was conducted on a track with and without frictional ties and its result showed that the lateral resistance of the railroad increased by 64% by using frictional ties. Compared to mono-block ties, bi-block ties have greater resistance to lateral actions and lighter weight. The specific lateral resistance values, in case of an unladen track, are equal to 11 kN in monoblock tie and 14 kN/m in bi-block ties. Some railways (e.g. the German Railway (DBAG) and the Dutch Railway (NS)) consider a ballast resistance value of 20 kN/m for unloaded tracks with standard mono-block ties assuming that the ties in the ballast are well maintained. The use of tie anchors increases the lateral displacement resistance and helps achieve sufficient resistance to track buckling in long welded rails, even at critical locations. Following experimental research by Prof. Eisenmann, the German railways have recently normalized the use of Vossloh SN anchoring device which is

### Table 4. Application of equation (5).

| \( R \) (m) | DOC | Rail profile | \( A \) (mm²) | \( F_{qvw} \) (N/cm) | \( \Delta T_C \) | Site/agency |
|---|---|---|---|---|---|---|
| 34 | 51.4° | ARA-A | 6357 | 100–200 | 10.6–21.2 | OBMF example |
| 76 | 23.0° | 115 RE | 7232 | 100–200 | 20.8–41.7 | AREMA |
| 80 | 21.8° | 54 EI | 6977 | 100–200 | 22.7–45.5 | OMC example |
| 92 | 19.0° | 115 RE | 7232 | 100–200 | 25.2–50.5 | AREMA |
| 145 | 12.0° | 115 RE | 7232 | 100–200 | 39.8–79.6 | TCRP |
| 175 | 10.0° | 115 RE | 7232 | 100–200 | 48.0–96.0 | – |
| 184 | 9.5° | 115 RE | 7232 | 100–200 | 50.5–101.0 | BNSF |
| 195 | 9.0° | 115 RE | 7232 | 100–200 | 53.4–107.0 | CN |

Figure 1. Thirty-four meter radius curve without tie anchor at OBMF, Calgary, Canada.
able to increase the track stability especially in the bridge transition zone and in curves with a small radius. The experimental curves have pointed out a growing increase of the lateral resistance values linked to the sliding rise; especially the shifting values equal to 2 mm show a resistance increase equal to 47%.

The author has designed several super-sharp curves of radii 65 m for lengths of up to 25 m with scalloped concrete ties and tie anchors at the Eglinton Maintenance and Storage Facility (EMSF) yard of the Eglinton Crosstown Light Rail Transit (ECLRT) Project in Toronto, Ontario, Canada; the maximum, minimum, and stress neutral rail temperatures are 55°C, −35°C, and 23 ± 5°C, respectively. No specific formula was used to compute stress neutral temperature which was fixed on the basis of local experience.

Photos of a 30-m radius curve with three tie anchors on each tie appear in Figures 2 and 3.

**Discussion on equations (1) and (5)**

Equation (5) shows that $\Delta T_C$ is directly proportional to the radius, but equation (1) depicts a nonlinear relation between $\Delta T_C$ and the radius, which is theoretically plausible. If radius $R$ in equation (1) tends to infinity, $\Delta T_C$ is calculated to be 157°C. However, buckling of a curved track due to a temperature differential above 50°C does not sound acceptable, because a tangent track may be regarded as a curved track with an infinite (practically large) radius. Moreover, equation (1) was derived while assuming a modal track shape and a critical load, as seen in Figure 4.

The modal shape of a buckled track is a half sine wave, but on a shallow curve, the modal shape might take the form of a full sine wave, as observed for tangent tracks. Thus, equation (1) does not seem to be applicable for all radii. The same is true for equation (5).

It is apparent that both equations offer sensible values of $\Delta T_C$ (<50°C) for sharp and super-sharp curves. The computed value of the $\Delta T_C$ is acceptable so long as $\Delta T_C < 50°C$. $\Delta T_C > 50°C$ is theoretically unacceptable but inconsequential, because the actual rise in temperature, $\Delta T$, amounts to 45°C–50°C, considering that deviations occur in practice when rails are tensioned. However, the limiting value of the radius for the applicability of equations (1) and (5) can be determined by setting $\Delta T_C = 50°C$ in these equations for a given ballast resistance.

The functional similarity of equations (1) and (5) has already been noted. The latter is better than equation (1) for at least three reasons: (i) it is simpler than equation (1), (ii) it does not contain the term for the equivalent moment of inertia of the track grid, $I^*$, whose value is not sensitive to the critical temperature differential and depends on many assumed and subjective parameters (e.g. rigidity of the connection between the tie and the rail, end support conditions of the track grid, length of the assumed track grid, deflection, etc.), and (iii) it is not affected by the assumed critical
misalignment value, $f^*$, which cannot be calculated by any method. The force value arising from tie displacement by 2 mm is typically considered as the lateral displacement resistance. In situations with higher displacements, the tie begins to slide as the static friction changes to sliding friction. This fact is illustrated in Figure 5.

The assumed value of critical misalignment, $f^*$, namely 2 cm (20 mm), seems to be high compared with 2 mm (Figure 2). At present, there is no suitable theoretical model to compute the critical misalignment. The critical misalignment may be any value above 2 mm in the plastic zone.

The proposed formula is based on a well-maintained curved track. The critical temperature differential of a misaligned curved track would be less than that of a properly aligned curved track. Codes usually specify acceptable limit of deviation from uniform profile. As for example, Federal Railroad Administration specifies different values of deviation from uniform profile at mid-ordinate of 31, 62 feet chords for different class of tracks. Periodic maintenance of track would take care of lining alone with surfacing and gauging to ensure ride comfort and safety which is quite adequate to guard against thermal instability of a curved track.

**Conclusion**

The critical temperature differential of an unloaded curved track is formulated in a simpler way; one need not resort to assumptions about the critical misalignment value, which cannot be calculated by any method so far, and the moment of inertia of the track grid, which is not sensitive to the critical temperature differential. The stability of sharp and super-sharp curves is a real concern, and the proposed formula offers sensible values of critical temperature differential for buckling.

If the critical temperature differential is less than the available temperature differential then the lateral resistance of a curve shall be augmented to ensure stability. If the computed value of the critical temperature differential exceeds 50°C, then the curve may be assumed to be stable.

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**References**

1. Kerr AD. Lateral buckling of railroad tracks due to constrained thermal expansions – a critical survey. In: Kerr AD (ed.) *Railroad track mechanics & technology*. London: Pergamon Press, 1978.
2. Esveld C. *Modern railway technology*. The Netherlands: MRT Productions, 2001.
3. Hasan N. Thermal buckling of ballasted tangent track. *Adv Mech Eng* 2020; 10.
4. Lichtberger B. Track compendium – formation, permanent way, maintenance, economics. Germany: Eurail Press, 2005.
5. Hasan N. Lateral stability of a ballasted curved track. *J Transport Eng Part A Syst* 2020; 146: 1–6.
6. Nemesdy E. Berechnung waagerechter Gleisverwerfugen nach neuen ungarischen Versuchen (Calculation of horizontal lateral buckling of rails according to new Hungarian experiments). *ETR Eisenbahntechnische Rundschau* 1969; 12: 514–534.
7. American Railway Engineering and Maintenance-of-Way Association (AREMA). *Manual for railway engineering*, vol. 1B. Lanham, MD: Track, 2020.
8. Dogneton P. The experimental determination of the axial and lateral track-ballast resistance. In: Kerr AD (ed.) *Railroad track mechanics & technology*. London: Pergamon Press, 1978.
9. Zarembski AM. Factors that influence resistance to lateral track shift. *Railv Track Struct* 1995; 2: 11–12.
10. Koike Y, Nakamura T, Hayano K, et al. Numerical method for evaluating the lateral resistance of sleepers in ballasted tracks. *Soils Found* 2014; 54: 502–514.
11. Union Internationale des Chemins de fer (UIC). *UIC Code 774-3, Track/bridge interaction, recommendations for calculations*. France: UIC, 2001.
12. Transit Cooperative Research Program (TCRP). *Track design handbook for light rail transit*. Washington, DC: National Academy Press, 2012.

**Figure 5.** Lateral displacement resistance diagram.
Appendix

Notations

The following symbols are used in this paper:

- \( A \) sectional area of a rail (mm\(^2\))
- \( E \) Young’s modulus (210,000 N/mm\(^2\))
- \( F_0 \) critical rail pressure force (N)
- \( f^* \) critical track defect (2–2.5 cm)
- \( F_{QW} \) lateral displacement resistance (N/cm)
- \( I \) moment of inertia of a single rail (mm\(^4\))
- \( I^* \) moment of inertia of the track grid (cm\(^4\))
- \( R \) curve radius (m)
- \( R_S \) tie resistance from a panel displacement test (kN)
- \( S \) spacing of ties (m)
- \( \dot{\sigma} \) coefficient of expansion (0.0000118/°C)
- \( \Delta T \) temperature differential (°C)
- \( \Delta T_C \) critical temperature differential (°C)