Abstract

It is shown that the massless $j = 1$ Weinberg-Tucker-Hammer equations reduce to the Maxwell’s equations for electromagnetic field under the definite choice of field functions and initial and boundary conditions. Thus, the former appear to be of use in a description of some physical processes for which that could be necessitated or be convenient.

The possible consequences are discussed.
Can the $2(2S + 1)$ Component Weinberg-Tucker-Hammer Equations Describe the Electromagnetic Field?∗

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The attractive Weinberg’s 2(2j + 1) component formalism for a description of higher spin particles \[1\] recently got developed considerably in connection with the recent works of Dr. D.V. Ahluwalia et al., ref. \[2\]-\[5\]. Some attempts have also been done in attaching the interpretation of their ideas in my papers \[6\]-\[9\].

The main equation in this formalism, which had been proposed in ref. \[1\], is

\[
\left[ \gamma_{\mu_1\mu_2...\mu_{2j}} p_{\mu_1} p_{\mu_2} \cdots p_{\mu_{2j}} + m^{2j} \right] \psi = 0, \quad (1)
\]

One can see that it is of the “2\(j\)” order in the momentum, \(p_{\mu_i} = -i\partial/\partial x_{\mu_i}\), \(m\) is a particle mass. The analogues of the Dirac \(\gamma\)-matrices are \(2(2j+1) \otimes 2(2j+1)\) matrices which have also “2\(j\)” vectorial indices, ref. \[10\].

For the moment I take a liberty to repeat the previous results. The equations (4.19,4.20), or equivalent to them Eqs. (4.21,4.22), presented in ref. \[1b,p.B888\] and in many other publications:

\[
\nabla \times [E - iB] + i(\partial/\partial t) [E - iB] = 0, \quad (4.21)
\]

\[
\nabla \times [E + iB] - i(\partial/\partial t) [E + iB] = 0, \quad (4.22)
\]

are found in ref. \[2\] to have acausal solutions. Apart from the correct dispersion relations \(E = \pm p\) one has a wrong dispersion relation \(E = 0\). The origin of this fact seems to be the same to the problem of the “relativistic cockroach nest” of Moshinsky and Del Sol, ref. \[11\]. On the other hand, “the \(m \to 0\) limit of Joos-Weinberg finite-mass wave equations, Eq. (3), satisfied by \((j,0) \oplus (0,j)\) covariant spinors, ref. \[3\], are free from all kinematic acausality.”

The same authors (D. V. Ahluwalia and his collaborators) proposed the Foldy-Nigam-Bargmann-Wightman-Wigner-type (FNBWW) quantum field theory, “in which bosons and antibosons have opposite relative intrinsic parities”, ref. \[4\]. This Dirac-like modification of the Weinberg theory is an excellent example of combining the Lorentz and the dual transformations. Its recent development, ref. \[5\], could be relevant for a description of neutrino oscillations and in realizing the role of space-time symmetries for all types of interactions.

In ref. \[6\] I concern with a connection between antisymmetric tensor fields \[12, 13\] and the equations considered by Weinberg (and by Hammer and Tucker \[14\] in a slightly different form). In the case of the choice

\[
\psi = \begin{pmatrix} E + iB \\ E - iB \end{pmatrix}
\]

(2)

the equivalence of the Weinberg-Tucker-Hammer approach and the Proca approach has been found. The possibility of a consideration of another equation (the Weinberg

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1For discussions on the needed modifications of the theory see refs. \[4\]-\[8\] and what follows.

2My earlier attempts to give a potential interpretation of the \(\psi\) were unsuccessful in a certain manner, ref. \[15\].

3I mean the equations for \(F_{\mu\nu}\), the antisymmetric field tensor, e. g., Eq. (3) in \[3\] or Eq. (A4) in \[16b\].
“double”) was pointed out. In fact, it is the equation for the antisymmetric tensor dual to $F_{\mu\nu}$, which had also been considered earlier, e.g. ref. [18]. In the paper [7] the Weinberg fields were shown to describe the particle with transversal components (i.e., spin $j = 1$) as opposed to the conclusions of refs. [12, 13] and of the previous ones [19]. The origins of contradictions with the Weinberg theorem ($B - A = \lambda$), which have been met in the old works (of both mine and others), have been partly clarified.

The causal propagator of the Weinberg theory has been proposed in ref. [8]. Its remarkable feature is a presence of four terms. This fact is explained in my forthcoming paper [9].

The aim of the present Letter is to consider the question, under what conditions the Weinberg-Tucker-Hammer $j = 1$ equations can be transformed to Eqs. (4.21) and (4.22) of ref. [1b]? By using the interpretation of $\psi$ in the chiral representation, Eq. (2), and the explicit form of the Barut-Muzinich-Williams matrices, ref. [10], I am able to recast the $j = 1$ Tucker-Hammer equation

$$[\gamma_{\mu\nu}p_{\mu}p_{\nu} + p_{\mu}p_{\mu} + 2m^2] \psi = 0,$$

which is free of tachyonic solutions, or the Proca equation, Eq. (3) in ref. [3], to the form

$$m^2E_i = -\frac{1}{c^2} \frac{\partial^2 E_i}{\partial t^2} + \epsilon_{ijk} \frac{\partial}{\partial x_j} \frac{\partial B_k}{\partial t} + \frac{\partial}{\partial x_i} \frac{\partial E_j}{\partial x_j},$$

$$m^2B_i = \frac{1}{c^2} \epsilon_{ijk} \frac{\partial}{\partial x_j} \frac{\partial E_k}{\partial t} + \frac{\partial^2 B_i}{\partial x_i^2} - \frac{\partial}{\partial x_i} \frac{\partial B_j}{\partial x_j}.$$

The D’Alembert equation (the Klein-Gordon equation in the momentum representation indeed)

$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_i^2} \right) F_{\mu\nu} = -m^2 F_{\mu\nu}. $$

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4It is useful to compare the method applied in the papers [3] with the Dirac’s way of deriving the famous equation for $j = 1/2$ particles, ref. [17]. Namely, his aim was to obtain the linear differential equation; the coefficients at derivatives and a mass term were not known ab initio and they turned out to be matrices. The second requirement which he imposed is: the equation should be compatible with the Klein-Gordon equation, i.e. with relativistic dispersion relations.

5Answering to the referee of one of my previous paper I can ascertain my statements. The Weinberg theorem is a consequence of the general kinematical structure of the theory based on the definite representation of the Lorentz group. If one of the various “gauge” constraints one places on the dynamics leads to the results which contradict with the underlying kinematical structures this can signify the only thing: the dynamical constraint is wrong! This is a known result and reproducing it once again is caused by the importance of this particular example. Namely, the Weinberg theorem permits two values of the helicity $\lambda = \pm 1$ for a massless $j = 1$ Weinberg-Tucker-Hammer field. Setting the generalized Lorentz condition (see for a discussion the footnote # 10 in ref. [7]) yields the physical excitation of the very strange nature, $\lambda = 0$, which also contradicts with a classical limit, see ref. [3]. Therefore, imposing the generalized Lorentz condition, formulas (18) of ref. [19], on the quantal physical states is impossible in the case of a quantum field consideration and it is incompatible with the specific properties of an antisymmetric tensor field.

6I restored $c$, the light velocity, at the terms.

7The reader can reveal dual equations from Eqs. (10) or (12) and parity-conjugated equations from Eqs. (18,19) of ref. [3] without any problems.
is implied.

Restricting ourselves by the consideration of the $j = 1$ massless case one can re-write them to the following form:

$$
\frac{\partial}{\partial t} \text{curl } B + \text{grad } \text{div } E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0, \quad (7)
$$

$$
\nabla^2 B - \text{grad } \text{div } B + \frac{1}{c^2} \frac{\partial}{\partial t} \text{curl } E = 0. \quad (8)
$$

Let consider the first equation (7). We can satisfy it provided that

$$
\tilde{\rho}_e = \text{div } E = \text{const}_x, \quad J_e = \text{curl } B - \frac{1}{c^2} \frac{\partial}{\partial t} E = \text{const}_t. \quad (9)
$$

However, this is a particular case only. Let me mention that the equation

$$
\frac{\partial J_e}{\partial t} = -\text{grad } \tilde{\rho}_e \quad (10)
$$

follows from (7) provided that $J_e$ and $\tilde{\rho}_e$ are defined as in Eq. (9).

Now we need to take relations of vector algebra in mind:

$$
\text{curl } \text{curl } X = \text{grad } \text{div } X - \nabla^2 X, \quad (11)
$$

where $X$ is an arbitrary vector. Recasting Eq. (7) and taking the D’Alembert equation (6) in mind one can come in the general case to

$$
J_m = -\frac{\partial B}{\partial t} - \text{curl } E = \text{grad } \chi_m, \quad (12)
$$

in order to satisfy the recasted equation (7)

$$
\text{curl } J_m = 0. \quad (13)
$$

The second equation (8) yields

$$
J_e = \text{curl } B - \frac{1}{c^2} \frac{\partial}{\partial t} E = \text{grad } \chi_e \quad (14)
$$

(in order to satisfy $\text{curl } J_e = 0$). After adding and subtracting $\frac{1}{c^2} \frac{\partial^2 B}{\partial t^2}$ one obtains

$$
\tilde{\rho}_m = \text{div } B = \text{const}_x, \quad \frac{\partial B}{\partial t} + \text{curl } E = \text{const}_t, \quad (15)
$$

provided that

$$
\nabla^2 B - \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2} = 0 \quad (16)
$$

(i. e. again the D’Alembert equation taken into account). The set of equations (15), with the constants are chosen to be zero, is “an identity satisfied by certain space-time derivatives of $F_{\mu \nu}$...; namely,

$$
\frac{\partial F_{\mu \nu}}{\partial x^\sigma} + \frac{\partial F_{\nu \sigma}}{\partial x^\mu} + \frac{\partial F_{\sigma \mu}}{\partial x^\nu} = 0. \quad (17)
$$

Ref. [20, 21].
However, it is also a particular case. Again, the general solution is
\[
\frac{1}{c^2} \frac{\partial J_m}{\partial t} = -\text{grad} \tilde{\rho}_m. \tag{18}
\]

We must pay attention at the universal case. What are the chi-functions? How should we name them? From Eqs. (11) and (14) we conclude
\[
\tilde{\rho}_e = -\frac{1}{c^2} \frac{\partial \chi_e}{\partial t} + \text{const}, \tag{19}
\]
and from (12) and (18),
\[
\tilde{\rho}_m = -\frac{1}{c^2} \frac{\partial \chi_m}{\partial t} + \text{const}, \tag{20}
\]
what tells us that \(\tilde{\rho}_e\) and \(\tilde{\rho}_m\) are constants provided that the primary functions \(\chi\) are linear functions in time (decreasing or increasing?). It is useful to compare the definitions \(\tilde{\rho}_e\) and \(J_e\) and the fact of an appearance of the functions \(\chi\) with the 5-potential formulation of the electromagnetic theory [24], see also ref. [16,22-24].

At last, I would like to note the following. We can obtain
\[
\text{div} \mathbf{E} = 0, \quad \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} - \text{curl} \mathbf{B} = 0, \tag{21}
\]
\[
\text{div} \mathbf{B} = 0, \quad \frac{\partial \mathbf{B}}{\partial t} + \text{curl} \mathbf{E} = 0, \tag{22}
\]
which are just the Maxwell’s free-space equations, in the definite choice of the \(\chi_e\) and \(\chi_m\), namely, in the case they are constants. In ref. [16] it was mentioned: The solutions of Eqs. (4.21,4.22) of ref. [1b] satisfy the equations of the type (4,5), “but not always vice versa”. A interpretation of this statement and investigations of Eq. (1) with other initial and boundary conditions (or of the functions \(\chi\)) deserve further elaboration (both theoretical and experimental).

Next, if I use Eq. (2) as field function, of course, the question arises on its transformation from one to another frame. I would like to draw your attention at the remarkable fact which follows from a consideration of the problem in the momentum representation. For the first sight, one could conclude that under a transfer from one to another frame one has to describe the field by the Lorentz transformed function \(\psi'(\mathbf{p}) = \Lambda(\mathbf{p})\psi(\mathbf{p})\). However, let us take into account the possibility of combining the Lorentz, dual (chiral) and parity transformations in the case of higher spin equations. This possibility has been discovered and investigated in [4,7]. The four spinors \(u_1^{(1)}(\mathbf{p}), u_2^{(1)}(\mathbf{p}), u_1^{(2)}(\mathbf{p})\) and \(u_2^{(2)}(\mathbf{p})\), see Eqs. (10), (11), (12) and (13) of ref. [8], form the complete set (as well as \(\Lambda(\mathbf{p})u_i^{(k)}(\mathbf{p})\)). Namely\(\text{10}\)
\[
a_1 u_1^{(1)}(\mathbf{p}) \bar{u}_1^{(1)}(\mathbf{p}) + a_2 u_2^{(1)}(\mathbf{p}) \bar{u}_2^{(1)}(\mathbf{p}) +
\]

\(^9\)The equations for the four functions \(\psi_i^{(k)}\), Eqs. (8), (10), (18) and (19) of ref. [8], reduce to the equations for \(\mathbf{E}\) and \(\mathbf{B}\), which appear to be the same for each case in a massless limit.

\(^{10}\)After completing the preliminary version of this article I learnt that equations similar to Eq. (23) for the second-type \(j = 1/2\) and \(j = 1\) bispinors have been obtained in ref. [5b,Eqs.(24,25)]. The equations (22a-23c) of the above-mentioned reference could also be relevant in the following discussions.
\[ + a_3 u_1^\sigma (p) \bar{u}_1^\sigma (p) + a_4 u_2^\sigma (p) \bar{u}_2^\sigma (p) = 1. \] 

(23)

The constants \( a_i \) are defined by the choice of the normalization of bispinors. In any other frame we are able to obtain the primary wave function by choosing the appropriate coefficients \( c_i^k \) of the expansion of the wave function (in fact, by using appropriate dual rotations and inversions):

\[ \Psi = \sum_{i,k=1,2} c_i^k \psi_i^{(k)}. \]

(24)

Of course, the same statement is valid for negative-energy solutions, since they coincide with the positive-energy ones in the case of the Hammer-Tucker formulation for a \( j = 1 \) boson, ref. [7, 14]. By using the plane-wave expansion it is easy to prove the validity of the conclusion in the coordinate representation. Thus, the question of fixing the relative phase factor by appropriate physical conditions (if exist) in each point of the space-time appears to have a physical significance for both massive (charged) and massless particles in the framework of relativistic quantum electrodynamics [12].

Finally, let me mention that in the nonrelativistic limit \( c \to \infty \) one obtains the dual Levi-Leblond’s “Galilean Electrodynamics”, ref. [25, 26].

**Conclusion**: The Weinberg-Tucker-Hammer massless equations (or the Proca equations for \( F_{\mu\nu} \)), see also [8] and [3], are equivalent to the Maxwell’s equations in the definite choice of the initial and boundary conditions, what proves their consistency. They (Eq. (1) for spin \( j \)) were shown in ref. [2] to be free from all kinematical acausality as opposed to Eqs. (4.21) and (4.22). Therefore, we have to agree with Dr. S. Weinberg who spoke out about the equations (4.21) and (4.22): “The fact that these (!) field equations are of first order for any spin seems to me to be of no great significance...” [1b, p. B888].

Meantime, I would not like to shadow theories based on the use of the vector potentials, i.e. of the representation \( D(1/2, 1/2) \) of the Lorentz group. While the description of the \( j = 1 \) massless field by using this representation contradicts with the Weinberg theorem \( B - A = \lambda \) one cannot forget about the significant achievements of these theories. The description proposed here and in my previous papers [3]-[6] could be helpful only if we shall necessitate to go beyond the framework of the Standard Model, i.e. if we shall come across the reliable experiment results which could not have a satisfactory explanation on the ground of the concept of a minimal coupling (see, e.g., ref. [3] for a discussion of the model which forbids such a form of the interaction).

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11See Eqs. (17,20) in ref. [6].

12The paper which is devoted to the important experimental consequences of this fact (e.g., the Aharonov-Bohm effect and some others) is in progress.

13This conclusion also follows from the results of the paper [2, 3, 15] and ref. [1b] provided that the fact that \( (jp) \) has no inverse one has been taken into account.
I am grateful to Zacatecas University for a professorship. In fact, this paper is the Addendum to the previous ones [1]-[8]. It has been thought on September 3-4, 1994 as a result of discussions at the IFUNAM seminar (México, 2/IX/94).

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verhöhnen was sie nicht verstehen.
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