Incipient Separation in Shock Wave / Boundary Layer Interactions as Induced by Sharp Fin

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Abstract

The incipient separation induced by the shock wave/turbulent boundary layer interaction at the sharp fin is the subject of present study. Existing theories for the prediction of incipient separation, such as those put forward by McCabe (1966) and Dou and Deng (1992), can have thus far only predicting the direction of surface streamline and tend to over-predict the incipient separation condition based on the Stanbrook’s criterion. In this paper, the incipient separation is firstly predicted with Dou and Deng (1992)’s theory and then compared with Lu and Settles (1990)' experimental data. The physical mechanism of the incipient separation as induced by the shock wave/turbulent boundary layer interactions at sharp fin is explained via the surface flow pattern analysis. Furthermore, the reason for the observed discrepancy between the predicted and experimental incipient separation conditions is clarified. It is found that when the wall limiting streamlines behind the shock wave becomes aligning with one ray from the virtual origin as the strength of shock wave increases, the incipient separation line is formed at which the wall limiting streamline becomes perpen-
dicular to the local pressure gradient. The formation of this incipient separation line is the beginning of the separation process. The effects of Reynolds number and the Mach number on incipient separation are also discussed. Finally, a correlation for the correction of the incipient separation angle as predicted by the theory is also given.

Key Words: shock wave/turbulent boundary layers interactions; incipient separation; mechanism; prediction; sharp fin

1 NOMENCLATURE

\( A \) = parameter in the “triangular model”

\( C_{fx} \) = component of local skin friction coefficient in the main-flow (streamwise) direction (dimensionless)

\( k \) = specific heat ratio (dimensionless)

\( M \) = Mach number (dimensionless)

\( M_\infty \) (or \( M_1 \)) = incoming Mach number (dimensionless)

\( p \) = static pressure (\( N/m^2 \))

\( R \) = temperature recovery factor at wall (dimensionless)

\( Re_\theta \) = Reynolds number based on momentum thickness (dimensionless).

\( T \) = absolute temperature (\( K \))

\( u \) = velocity component in the main flow direction in the boundary layer (\( m/s \))

\( w \) = velocity component in the cross-flow direction in the boundary layer (\( m/s \))

\( x \) = coordinate in the streamwise direction (\( m \))

\( y \) = coordinate in the direction normal to the flat plate (\( m \))

\( z \) = coordinate in the direction normal to the streamwise direction (\( m \))

\( \alpha \) = turning angle of the main flow, measured relative to the direction of velocity vector at the beginning of interaction (radian or degree)
\( \beta \) = angle measured from the incoming freestream direction, centered at the virtual origin (near the fin apex) (degree)

\( \beta_0 \) = angle of shock wave (degree)

\( \gamma_w \) = wall shear angle, i.e., angle between the direction of the wall limiting streamline and the external streamline direction of boundary layer (degree)

\( \theta \) = momentum thickness of boundary layer (m)

\( \nu \) = kinematic viscosity (\( m^2/s \))

\( \nu(M) \) = Prandtl-Meyer function (radian), see Eq.(7)

\( \rho \) = density (\( kg/m^3 \))

\( \sigma = \alpha + \gamma_w \), turning angle of surface streamline, i.e., the limiting streamline direction on the wall measured from the incoming flow direction (degree)

\( \tau \) = shear stress (\( N/m^2 \))

**Subscripts**

\( e \) = external of the boundary layer

\( i \) = incipient separation

\( p \) = apex of “triangular model,” Fig.5

\( w \) = at walls

\( x \) = component in the direction of the main flow

\( z \) = component in the direction of the crossflow

2 INTRODUCTION

The shock wave/turbulent boundary layers interaction(SW/TBLI) problem is a very complex flow phenomenon encountered at high speed. The dynamics of the interaction and its mechanism constitute one of the fundamental problems of modern aerodynamics, which is very important in the design of new generation of flying vehicles (like unconventional UAV(unmanned air vehicle)) and fluid machinery operating at high speeds and others. Because of its importance and implications in aeronautical engi-
neering, shock-wave/boundary layer interactions have been studied variously for about the past 50 years or so. Although remarkable progress has been achieved, there still remain observations that cannot be satisfactorily explained and physical processes that are not quite well understood (Dolling, 2001). Computational techniques have evolved and played an increasing role in the understanding of the ensuring flow physics of the interactions (Knight et al., 1992; Knight et al., 2003). However, the cost of a full Navier-Stokes calculation is still fairly exorbitant, especially for 3D flows. In addition, issues relating to the accuracy from a physical point of view (besides the numerical accuracy) for purpose of quantitative comparison to experiments and the elucidation of the complex physical flow are not always satisfactory. Therefore, some analytical work like the simple predictive methods is still imperative for an initial estimate of the main flow physics based on preliminary design and then for progress to the subsequent stages leading eventually to the final prototype. To try to do a thorough simulation at the preliminary design stage is just too costly and most probably ineffective.

In the past over 30 years or so, there are limited works carried out on the separation behaviour in SW/TBLI induced by a sharp fin on a flat plate as typified by Fig. 1 (Bogdonoff, 1987; Delery and Marvin, 1986; Green, 1970; Panaras, 1996; Settles and Dolling, 1992; Neumann and Hayes, 2002). As the onset or occurrence of flow separation invariably changes the topology of the flow field, one important and critical problem to resolve is how to judge or predict the incipient separation and its underlying flow physics. A typical topology of the surface flow pattern is shown schematically in Fig.2 (Lu, 1993; Lu et al., 1990; Settles and Dolling, 1992) for cases of reference.

Stanbrook (1960) is probably the first to define and state that incipient separation takes place when the wall limiting streamlines becomes aligning with the inviscid shock wave. Since the angle of the inviscid shock wave can be calculated by oblique shock wave theory, the prediction of incipient separation then becomes a problem of predicting the direction of the wall limiting streamline. Based on Stanbrook’s criterion,
McCabe (1966) proposed a simple inviscid theory to predict the incipient separation by calculating the deflection of the vortex tubes caused by the lateral pressure gradient when the boundary-layer passes through the shock. For engineering purposes, Korkegi (1973) made approximations to the McCabe’s theory via utilizing the corrections with measured test data and hence obtained a semi-empirical formula for incipient separation: 

\[ M_\infty \alpha_i = 0.3, \text{ for } k = 1.4 \text{ and } M_\infty \geq 1.60. \]

Subsequently, Lu (1989) took into account the stretching of the vortex tube when the boundary-layer passes through the shock and improved on McCabe’s theory. The calculated results of Lu agree reasonably with Korkegi’s formula. In a different approach, Zheltovotov et al. (1987) proposed a method to predict the incipient separation in three-dimensional (3D) interactions by utilizing a semi-empirical equation for incipient separation of two-dimensional interaction; the latter is used as the Mach number perpendicular to the shock in three-dimensional interaction. The results indicate that the incipient separation angle \( \alpha_i \) decreases with increasing \( Re_\theta \). Based on the 3D compressible boundary layer theory, Dou and Deng (1992c) proposed a method for analyzing the secondary flow within the boundary layer and predicted the incipient separation with Stanbrook’s criterion. In their analysis, Johnston’s triangle model (1960) is employed in the boundary layer and the Prandtl-Meyer function is used in the external flow outside the boundary layer. This analysis appears to have more fundamental physical basis than those by McCabe (1966), Korkegi (1973), or Lu (1989). Their results also show the trend of \( \alpha_i \) decreasing with increasing \( Re_\theta \) which had been previously discussed in Lu (1993) and Leung and Squire (1995). It may be noted that Leung and Squire’s (1995) experimental data confirmed the same trend for \( \alpha_i \) versus \( Re_\theta \) as in Dou and Deng’s theory. Furthermore, McCabe (1966) and Dou and Deng (1992c) found that the incipient separation angle \( \alpha_i \) via skin-friction line (also called “limiting streamlines” or “surface streamlines” (Lighthill, 1963)) calculation and both works correctly predicted the wall limiting streamline direction before separation (see also Deng
et al., 1994). However, there is an overprediction of the incipient separation condition, and Korkegi’s criterion was therefore re-interpreted by Lu and Settles (1990) to mean that significant separation occurs for $\alpha > \alpha_i$. This overpredicting issue has remained unsolved or not addressed satisfactory (Settles and Dolling, 1992). This provides the motivation of the present work to systematically clarify the said overprediction and the validity of reinterpretation.

It should be mentioned that the mechanism for incipient separation is still not well understood. In the past twenty years or so, although computational fluid dynamics (CFD) technique has made a significant progress in the understanding of the flow physics and the structure of the flow field (Knight et al., 1992; Thivet et al., 2001), simple analytical work is still relevant to this subject, especially in the light of behaviour trend with varying flow parameters. Therein lies further motivation for the present work. In this paper, we firstly review Dou and Deng’s (1992c) theory, and compare the theoretical prediction results with the experimental data of Lu and Settles (1990). The effect of Reynolds number is then discussed. Then, the variation of the surface flow pattern with increasing deflection angle is analyzed, and the mechanism of the incipient separation induced by SW/TBLI at sharp fin is explained. It is shown that the genesis of incipient separation is traced to by the secondary flow due to lateral pressure gradient. In this way, it is hoped that the above mentioned discrepancies between theories and experiments are clarified. In addition, a correlation for the correction of $\alpha_i$ by the theoretical prediction is also given.

3 THEORY

Dou and Deng (1992c)’s theory is based on the “skewed” boundary layer concept. We shall briefly review this theory. In a swept shock wave/boundary layer interaction, a pressure gradient generated by the shock wave is exerted on the boundary-layer. The pressure gradient in three-dimensional interactions can be decomposed into two
components, i.e., the streamwise direction component and the lateral direction component. The former generates a viscous interaction with the incoming boundary layer, similar to two-dimensional interactions, causing flow retardation; the latter makes the velocity profiles of the boundary-layer skewed, and generates a secondary flow in the boundary layer, perpendicular to the streamwise direction (see Fig. 3). The relative magnitude of these two components indicates which of the above two phenomena is more important in dominating the separation in 3D interactions. For the interactions induced by a sharp fin in supersonic flow (with $M_\infty \geq 2.0$), the role of the lateral component of the pressure gradient overwhelms that of the streamwise component of the pressure gradient as explained by Dou and Deng (1992b) (see also Dou, 1991). For high Mach number, the incipient separation is dominated by the secondary flow because of the small shock wave angle and the large lateral pressure gradient.

The boundary layer on the wall is skewed owing to the lateral pressure gradient. The fluid particles near the wall travel along the path with larger curvature than that of the inviscid flow, as shown in Fig. 3. Starting from the outer edge of the boundary layer, the velocity vector in the boundary layer gradually deviates from the mainflow direction, and the deviation angle reaches a maximum at the wall. The direction of wall limiting streamline deviates from the primary streamline by an angle $\gamma_w$ (see Fig. 3). The angle $\gamma_w$ increases with the flow from the the front towards the downstream direction due to the lateral pressure gradient. When this angle reaches a certain value, the wall limiting streamlines converge to a single line from the upstream, which can be detected by oil film technique in experiments and is commonly called “incipient separation line” (Fig. 2). At this incipient separation line, the wall limiting streamline becomes perpendicular to the direction of the local pressure gradient. This type of separation can be broadly described by the model of Maskell (1955) and Lighthill (1963). Generally, it is difficult to define when this “incipient separation line” appears in the calculation, and the Stanbrook’s criterion is normally just invoked for the pre-
diction. Since the direction of shock wave angle can be easily calculated by inviscid shock wave equations, the prediction of incipient separation line can be strictly done by calculating the direction of wall limiting streamline (Fig. 4).

For this type of pressure-driven three-dimensional turbulent boundary-layer, there is a fair amount of works which have been carried out (see White, 1974; Olcmen and Simpson, 1992). If the wall shear angle $\gamma_w$ is not very large and the lateral flow is not bi-directional, Johnston’s triangular model seems to give the best approximation (White, 1974). This said model has been widely used in many engineering problems (Smith, 1972; Swafford and Whitfield, 1985). The law of wall in the three-dimensional turbulent boundary-layer based on this model by Johnston is still one of the best description up to recent years (Olcmen and Simpson, 1992).

Johnston (1960) studied the nature of the boundary layer on a flat wall under the influence of a turning main flow. It is stated by Johnston that this type of boundary-layer problems are commonly described as secondary-flow problems because the three-dimensional perturbations in the layer are caused by the associated pressure gradient due to the curvature of the main flow. The effect as generally observed is a skewing of the boundary-layer velocity vectors toward the center of curvature of the main flow. The type of the boundary layer in SW/TBLI is of the same nature as that described by Johnston. Thus, the theory developed by Johnston can be used to analyze the boundary-layer in SW/TBLI. Indeed, Lowrie (see Green, 1970) and Myring (1977) have used this model in their studies of SW/TBLI.

Johnston divided the turbulent boundary layer into two regions along the direction of boundary layer thickness. He assumed that a collateral region near the wall exists and the direction of the velocity vector in this region is coincident with the shear stress vector. This region is called the inner region of the boundary layer. In the outer region, the behavior of flow is primarily dominated by the outer inviscid flow. According to this model, the crossflow velocity profile of the boundary layer can be
expressed as follows (see Fig.5):

\[
\frac{w}{u_e} = \frac{u}{u_e} \tan \gamma_w \quad \text{for} \quad \frac{u}{u_e} \leq \left( \frac{u}{u_e} \right)_p, \tag{1}
\]

\[
\frac{w}{u_e} = A \left( 1 - \frac{u}{u_e} \right) \quad \text{for} \quad \frac{u}{u_e} \geq \left( \frac{u}{u_e} \right)_p, \tag{2}
\]

where \( \gamma_w \) is the angle between the wall limiting streamline and the external streamline, \( \left( \frac{u}{u_e} \right)_p \) is the streamwise velocity ratio at the apex of the triangle. If the variation of the direction of external flow is known, the direction of the wall limiting streamline can be calculated by evaluating the angle \( \gamma_w \).

From Eq.(1) and Fig.5, the following expression is obtained,

\[
\tan \gamma_w = A \left[ \left( \frac{u}{u_e} \right)_p^{-1} - 1 \right]. \tag{3}
\]

It can be seen that the parameters \( A \) and \( \left( \frac{u}{u_e} \right)_p \) must be determined for the calculation of \( \gamma_w \).

### 3.1 The Parameter \( A \)

Johnston (1960) expressed the parameter \( A \) as a function of the parameters of the main flow. For the cases in which a pressure gradient exists along the main streamline direction (\( \partial p/\partial x \neq 0 \)) and the turning angle varies in the normal direction of main flow (\( \partial \alpha/\partial z \neq 0 \)), he derived the following equation,

\[
A = 2u_e^2 \int_0^\alpha \frac{d\alpha}{u_e^2}, \tag{4}
\]

where \( u_e \) is the velocity at the outer edge of the boundary-layer, \( \alpha \) is the turning angle of main flow and is measured relative to the main flow direction at the beginning where the streamline of main flow first turns. The minus sign in the above equation which appears in the original equation of Johnston (1960) has been neglected. This is because we only consider the flow deflection in a single direction and are only concerned with the magnitude of \( A \).
According to the conservation of energy, the velocity ratio across the shock wave is expressed as follows,

\[
\frac{u_e_2}{u_e_1} = \frac{M_2}{M_1} \sqrt{1 + \frac{[(k - 1)/2] M_1^2}{1 + [(k - 1)/2] M_2^2}},
\]

(5)

where the subscripts 1 and 2 represent the parameter before and after the shock wave, respectively.

At the incipient separation condition, the strength of the shock wave is generally weak. Thus, the relation of the Mach numbers across the shock can be approximated by the Prandtl-Meyer relation. In other words, the shock wave at the edge of the boundary layer may be represented by a series of isentropic processes. Using this relation, the following closed form solution for the parameter \( A \) can be easily obtained. Our calculations showed that the error due to this approximation is very small compared to the exact relation of shock wave. Introducing Eq.(5) into Eq.(4) and using the Prandtl-Meyer relation, we have,

\[
A = \frac{2M_2^2}{1 + (k - 1) M_2^2/2} \int_0^\alpha \frac{1 + (k - 1) M_2^2/2}{M_2^2} d\alpha
\]

\[
= \frac{-2M_2^2}{1 + (k - 1) M_2^2/2} \int_{M_1}^{M_2} \frac{\sqrt{M_2^2 - 1}}{M_2^4} dM_2
\]

\[
= \frac{M_2^2}{1 + (k - 1) M_2^2/2} \left( \text{arc cos} \frac{1}{M_1} - \text{arc cos} \frac{1}{M_2} - \frac{\sqrt{M_1^2 - 1}}{M_1^2} + \frac{\sqrt{M_2^2 - 1}}{M_2^2} \right),
\]

(6)

and Prandtl-Meyer relation is expressed as

\[
\nu(M) = \sqrt{\frac{k+1}{k-1}} \text{arc tan} \sqrt{\frac{k-1}{k+1}(M^2 - 1)} - \text{arc tan} \sqrt{M^2 - 1}.
\]

(7)

The deflection angle \( \alpha \) is related to Prandtl-Meyer equation by

\[
\alpha = \nu(M_1) - \nu(M_2).
\]

(8)

3.2 The Parameter \((u/u_e)_p\)

For compressible flows, Smith (1972) made an extension of the “velocity triangle” of the three-dimensional boundary layers for incompressible flows, which was first given
by Johnston (1960). The parameter \((u/u_e)_p\), in Eqs.(1) and (2), was expressed as in reference (Smith, 1972),

\[
\left( \frac{u}{u_e} \right)_p = \bar{y}_p \sqrt{\frac{\rho_e}{\rho_w}} C_{f_x} \cos \gamma_w \frac{\nu_w}{\rho_w},
\]

where \(\bar{y}_p = \frac{u_e}{u_e} \sqrt{\frac{\rho_e}{\rho_w}}\) and \(C_{f_x}\) is the component of the skin friction coefficient in the direction of main flow. The density ratio \(\rho_e/\rho_w\) can be calculated from the energy equation for compressible flow. The components of the wall shear stresses are depicted in Fig.3. Besides Smith (1972), this equation has been used by Myring (1977) and Swafford and Whitfield (1985).

From the physical relationship for the three-dimensional boundary-layer, the value of \(\bar{y}_p\) is related to the conditions of boundary layer. In Johnston (1960) and Smith (1972), \(\bar{y}_p = 14.0\) is employed based on the test data obtained at low-speed. For the case of high-speed boundary layers (supersonic flows), the velocity distribution across the layer is very different from that of low speed flows. The role of viscosity is confined to within the thinner layer next to the wall. The test results at \(M_1 = 3.0\) by Settles was introduced in reference (Delery, 1985). The experiments showed that both the relative height of the viscous-layer and the relative height of sonic line decrease with the increasing Reynolds number, and the latter drops faster than the former. Therefore, at high Reynolds number (high Mach number), the velocity vector at the edge of the viscous-layer has larger deflection. For the turbulent boundary-layer at supersonic flow range (\(Re\) is very high), the value of \(\bar{y}_p\) is expected to be less than 14 (\(\bar{y}_p < 14\)). According to the structure and the velocity distribution of turbulent boundary-layer, it is assumed in this study that \(\bar{y}_p\) is respondent to the intersection point of the viscous-sublayer and the layer of logarithmic law defined in the two-dimensional turbulent boundary-layer, i.e. \(\bar{y}_p = 11\) (Kuethe and Chow, 1986).

For the selection of \(\bar{y}_p = 11\), a detailed explanation is provided as follows. As
discussed above, within the boundary layer, it can be divided into two regions in the
direction normal to the wall: region I which is near the wall and region II which
adjacent to the external flow. In region I, the layer is very thin and the flow deflection
is constant along the direction normal to the wall. Thus, the velocity vector is along
the same direction in this region. This region is generally called “collateral flow.” In
region II, the layer is skewed and the flow direction varies gradually. The velocity
vector changes from the direction of main (streamwise) flow at the edge of boundary
layer to the direction of wall streamline in region I.

In region II, the role of viscosity is very small and the behaviour is almost controlled
by the external flow. The degree of skewness of the flow in this layer can be described
by the equation, $\partial p/\partial r = \rho u^2/r$. Since the pressure is constant in the direction normal
to the wall within the layer, the value of $\partial p/\partial r$ is constant too. Thus, the variation
of velocity will lead to the variation of radius of curvature of the streamline. As such,
the fluid with lower velocity will have smaller radius of curvature of the associated
streamline. Thus, the flow becomes more deflected as it approaches the wall, and the
layer is skewed. In region I, the flow is mainly controlled by viscosity and is almost not
influenced or affected by the main flow. The viscous stress is larger in this layer and this
gives rise to the variation of pressure distribution. In this layer, the governing equation
normal to the streamline can be expressed as $\partial p/\partial r = \rho u^2/r + \partial \tau_{r\theta}/(r\partial \theta)$. The viscous
stress balances a part of the centrifugal force and thus the pressure gradient is reduced
approaching towards the wall. Therefore, this much wider variation of pressure could
lead to a reduction of deflection within this layer. As a result, the flow direction in
this layer experiences almost no change and a “collateral flow” is formed within this
viscous layer. Since the flow in the viscous sub-layer (also called linear layer when
expressed in terms of the inner layer variable) is viscous dominated, it is not unusual
to expect that the “collateral flow” extends to the whole height of the sub-layer, i.e.,
$\bar{y}_p = 11$. Therefore, it is deemed reasonable to assume that $\bar{y}_p = 11$, at the junction
point of two layers for high speed flows.

Substituting Eq.(9) into Eq.(3), the following equation can be derived,

$$\tan \gamma_w = A \left( \frac{0.13}{\sqrt{\frac{\rho_e}{\rho_w} C_{fx} \cos \gamma_w}} - 1 \right). \quad (10)$$

The density ratio is obtained from the energy equation,

$$\frac{\rho_e}{\rho_w} = \frac{T_w}{T_e} = 1 + \frac{k-1}{2} R M_2^2, \quad (11)$$

where $R$ is the temperature recovery factor at the wall. For the adiabatic turbulent boundary layer, it takes on 0.88 generally. For $2 \leq M_1 \leq 6$, the following correlation gives a better approximation to the experimental data (Dou and Deng, 1992c),

$$R = 0.80 + 0.01 \left(8.0 - \frac{M}{2}\right). \quad (12)$$

### 3.3 The Coefficient of Skin Friction

The local skin friction coefficient towards the streamline direction was taken from that for the 2D flow and given as (Dou, 1991),

$$\frac{C_{fx}}{C_{fxi}} = \left(1 + 0.13M^2\right)^{-0.73}, \quad (13)$$

where $C_{fxi}$ is the coefficient of skin friction for incompressible turbulent boundary layer on flat plate. In this study, the Karman-Schoenherr’s equation recommended by Hopkins and Inouye (1971) is utilized. This equation is applicable to the whole range of the Reynolds number of turbulent boundary layer and given as,

$$\frac{1}{C_{fxi}} = 17.08 \left(\log_{10} Re_\theta\right)^2 + 25.11 \log_{10} Re_\theta + 6.012. \quad (14)$$
3.4 Calculation of Shock Wave Angle

The shock wave angle can be calculated by the implicit oblique shock wave theory (e.g., Kuethe and Chow, 1986), or by the following approximate equation given by Dou and Deng (1992a). This latter formula is of satisfactory accuracy over a wide ranges of the upstream Mach number and flow deflection angle. For \( \alpha \leq 15^\circ \) and \( 2 \leq M_1 \leq 5 \), the relative error to the exact value is less than 1% and given as,

\[
\tan \beta_0 = \frac{1}{\sqrt{M_1^2 - 1}} + \frac{k + 1}{4} \left( \frac{M_1^4}{(M_1^2 - 1)^2} \right) \alpha + \frac{1}{2} \left( \frac{k + 1}{4} \right)^2 \frac{M_1^6 (M_1^2 + 4)}{(M_1^2 - 1)^{3/2}} \alpha^2. \tag{15}
\]

Using Eqs.(6) to (15), the variation of \( \gamma_w \) along the streamwise direction can be calculated for a given Mach number and \( Re_\theta \) for the incoming flow with increasing \( \alpha \). Next, the turning angle \( \sigma \) of the surface streamline on the wall due to the action of shock disturbance can be evaluated. When the turning angle \( \sigma \) at the wall equals to the shock angle \( \beta_0 \), the separation of the three-dimensional boundary layer is considered to have occurred as was shown by Stanbrook (1960). Similar calculations can be carried out for various incoming flow conditions.

In shock wave/boundary layer interactions, since the streamlines converge from the upstream, the boundary layer becomes thickening along the streamwise direction within the interaction region. Thus, the streamlines are also considerably deflected away from the wall. The actual streamline deflection is not just lying in a plane parallel with the surface as assumed in the analysis. However, because the normal velocity to the wall in the boundary layer is generally very much smaller compared to the streamwise velocity, the wall-normal deflection has been neglected in this analysis owing to its small effect. The reason for this approximation can be explained as follows in detail. In the boundary layer theory, this normal velocity is usually at least one order of magnitude smaller than the streamwise component. In present study, the shock wave generated by the high speed flow (fluid flow above the sonic line) acts
on the boundary layer. The shock wave in the inviscid flow is a plane with a jump in flow parameters. When this plane interacts on the boundary layer, the section of the interaction zone starting from the leading edge of the interaction to the incipient separation line is very short. This is because the boundary layer in high speed flows is usually very thin. Within this short distance, the variation of normal velocity is also not large; note that the flow is not completely separated from the surface. Therefore, the influence of the normal velocity can be neglected in the analysis.

4 RESULTS AND DISCUSSION

4.1 Comparison of the Theories with Experiments

The experimental data on incipient separation were generally obtained by oil film visualization technique in tunnel experiments (Deng et al., 1994; Dou and Deng, 1992b; Dou and Deng, 1992c; Lu and Settles, 1990; Settles and Dolling, 1992). Detailed description of the experiments can be found in these works. Generally, the test model of a sharp fin is amounted on one flat plate and placed in the supersonic wind tunnel (Fig.1). Before the experiment, a layer of oil film is spread thinly on the flat plate. When the air flow passes the plate, the oil film moves from the upstream to the downstream, and oil streaks are formed along the streamlines on the flat plate. After the wind tunnel is shut down, one can obtain this oil streak pattern by taking photograph or by using transparency glue papers. These pictures have the features as presented schematically in Fig.2. The occurrence of incipient separation was mostly decided in terms of the formation of the convergent line of the wall limiting streamlines from the upstream as according to Lighthill’s criterion (Lighthill, 1963). Figure 6 shows the comparison of the experimental data reported by Lu and Settles (1990) and the predictions using Dou and Deng’s method for four Mach numbers. The intersection point of the turning angle $\sigma$ of surface streamlines with the shock wave angle $\beta_0$ corresponds to the condition set by Stanbrook’s criterion (A-A line), i.e., the wall
limiting streamlines becomes parallel to the inviscid shock wave. The agreement of $\sigma(\equiv \alpha + \gamma_w)$ value between the theory (Eq.(6) to (10)) and the experiments is very good before the incipient separation occurrence based on Stanbrook's criterion. The arrows at the abscissa indicate the incipient separation as judged or ascertained using Korkegi's equation (B-B line); this was reported by Lu and Settles (1990). The latter (B-B line) is a little lower than those obtained using Stanbrook's criterion (A-A line). On the other hand, the incipient separation angles reported in experiments are even lower than Korkegi’s value (B-B line) (see Korkegi, 1973). Besides this, it should be mentioned that Deng et al. (1994) found that McCabe’s theory concurs fairly well with the experimental data of Lu and Settles (1990) for the turning angle of wall limiting streamlines, but for the incipient separation. The above mentioned inconsistencies between theory and experiments have left much to be desired. Furthermore, the physical mechanism for the occurrence/initiation of the incipient separation is still yet to be fully understood. It is the intent of this work to provide a reasonable explanation for such observation and also to present a necessary and yet robust correction to achieve overall consistency.

Figure 7 shows the comparison of the results predicted by Dou and Deng (1992c)'s theory as well as others (see also Dou and Deng, 1992c). In this figure, the effect of $Re_\theta$ on $\alpha_i$ is also displayed. When $Re_\theta$ is increased, $\alpha_i$ becomes smaller. This implies that the boundary layer is more easily susceptible to separation at higher $Re_\theta$. This is somewhat similar to that found for two-dimensional interactions (Delery, 1985). In two-dimensional shock wave/boundary layer interactions (Delery, 1985; Delery and Marvin, 1986), most experiments showed that the resistance to separation (increasing strength of shock wave) decreases with the $Re_\theta$ number at low to moderate values of $Re_\theta$. Then, there is a small reversal of the incipient separation condition at high $Re_\theta$ number. For three-dimensional shock wave/boundary layer interactions, the correlation results for two-dimensional interactions indicated the following effect
of Reynolds number $Re_\theta$: the incipient separation angle $\alpha_i$ decreases with increasing $Re_\theta$ (Zheltovodov et al., 1987). Leung and Squire (1995) have discussed the $Re_\theta$ influence on incipient separation, and their experimental data confirmed the same tendency of $\alpha_i$ as that predicted with Dou and Deng's theory. However, McCabe's theory does not depict this influence of Reynolds number (It should be pointed out that the Reynolds number may vary when the Mach number changes in real flows). Almost all of the data in Fig.7 were obtained for incoming flow Reynolds number of about $10^4$ ($Re_\theta = 5 \times 10^3\sim5 \times 10^4$). It can be seen that the incipient separation angles from the experiments are lower than those predicted by McCabe's and Dou and Deng's theory.

4.2 Analysis of Surface Flow Patterns

The process of the formation of the incipient separation could be described by analyzing the evolution of the surface flow patterns. A typical schematic of surface flow from flow visualization experiments is shown in Fig. 2. The surface flow pattern formed by a sharp fin is conical (Lu, 1993; Lu et al., 1990; Settles and Dolling, 1992; Oudheusden et al., 1996), and not cylindrical (Johnston, 1960; Myring, 1977; Knight et al., 1992; Van Oudheusden et al., 1996). The topological pattern of the surface streamlines in conical interactions has been interpreted recently by Van Oudheusden et al. (1996). In Fig.2, the schematic surface pattern depicts a geometric conicity, in that rays are shown emanating from a certain origin close to the fin apex. The direction of the oil streaklines are to a large extent fairly independent of the distance along the ray. In this kind of conical flow field, all the flow quantities take on constant values on the ray through the conical center of the flow field. The variation of surface flow pattern with the increase of the strength of shock wave for a given incoming Mach number could be divided into the following six stages as shown in the schematic diagram of Fig. 8. This figure is summarized from a large quantity of experimental data on surface flow patterns in the literature (McCabe, 1966; Korkegi, 1973; Kubota and Stollery, 1982;
In Fig. 8, for simplicity, it is assumed that the virtual origin of the conical region coincides with the apex of the fin on the plate. The evolution of the surface streamlines with the increasing strength of shock wave is shown up to the formation of the primary separation line. The following observations are made:

(a) The deflection angle is small and the shock wave is weak, and the effect of secondary flow is negligible.

(b) On increasing the deflection angle, the wall limiting streamlines behind the shock turn to the shock wave trace gradually, but the shock wave strength is not large enough to deflect the surface streamline to make it parallel to the shock. The main feature of this stage is the gathering of surface streamlines from the upstream behind the shock wave.

(c) Further increasing the deflection angle, the wall limiting streamlines converge and coalesce onto a single line from the upstream. This is true when the strength of the shock wave is still not large enough to deflect the surface streamlines behind the shock wave to become parallel to the shock wave. This single line formed from the upstream is just the “incipient separation line” exhibited by oil streak pattern technique in experiments, which symbolizes the beginning of the separation process.

(d) Upon further increasing the deflection angle, the “incipient separation line” formed from the upstream rotates (shifts) continuously with the increasing shock wave angle. Meanwhile, the surface streamlines behind the shock wave becomes parallel to the shock wave. This is the condition defined by the Stanbrook’s criterion. Of course, this condition arrives later than the appearance of “incipient separation line” indicated by experiments (Fig. 8c). This is the reason why the theories overpredict the occurrence of the incipient separation compared with the experiments shown in Fig. 6 and Fig. 7.
(e) On further increase of the deflection angle, the “incipient separation line” formed from the upstream rotates (shifts) continuously with the increasing shock wave angle, and the wall limiting streamlines from the downstream of the shock wave also converge to this line from another side.

(f) When the deflection angle is increased to a certain value, the “incipient separation line” also becomes one convergent line of the wall limiting streamlines from the downstream; this is called the “primary separation line.” Some authors prefer to use the appearance of primary separation line to judge the separation (Kubota and Stollery, 1982).

Now, the whole process of the formation of convergent line in the oil film pattern can be described as follows. In the experiments in a supersonic tunnel (Lu and Settles, 1990), the color oil substance is normally painted on the test plate, on which the sharp fin model is amounted. When the air flow passes the plate, the film with the oil substance is moved from the upstream to the downstream location by the air flow. However, the oil film moving from upstream will be blocked by this ray of incipient separation line. This is because the surface streamlines forms a half-closed pattern ahead of this ray. Thus, the oil film behind this ray will be eventually dispersed. The oil film ahead of this ray will still be kept on the plate. This is the reason why the incipient separation line can be formed in their experiments. Any two streamlines from the upstream and the convergent line form a “U”-shaped like pattern. As such, this half-closed loop existing towards the downstream could prevent the oil film flow from passing through to the convergent line.

4.3 Physical Mechanism for Incipient Separation

The three-dimensional separations induced by SW/TBLI at the sharp fin can be described by the model of Maskell (1955) or Lighthill (1963). According to Lighthill’s criterion, it is considered that the boundary layer is separated when the wall limiting
streamlines converge to a single line. In terms of mass conservation, the converging from the upstream to a single line is enough to be considered as flow separation as also discussed by Kubota and Stollery (1982). From the point of view of the equilibrium of forces, the vector of skin-friction force is perpendicular to the direction of local pressure gradient at the incipient separation line. Therefore, when the skin-friction lines of incoming flow becomes perpendicular to the direction of the local pressure gradient, the formation of the “incipient separation line” becomes possible. In fact, Stanbrook’s criterion satisfies this condition. However, with the gradual increase of the shock wave angle, the flow state as expressed by Stanbrook’s criterion is not the first manifestation of this condition. This condition is satisfied indeed somewhat earlier, as is observed in for case(c) of Fig.8. This is strictly the main mechanism for the formation of incipient separation line.

Although the surface features of the cylindrical and conical interactions are different (Setteles and Dolling, 1992), they do share some common properties/features. For both the cylindrical and conical interactions to be possible, it is required the direction of the skin-friction line at incipient separation line be perpendicular to the local pressure gradient. However, for the conical interaction, it is not necessary for the “incipient separation line” to align with the shock wave, while it is so for the cylindrical interaction. The interaction region on the flat plate generated by SW/TBLI at a sharp fin is a conical zone, which stretches across the inviscid shock wave trace on the plate when the shock wave is weak (Fig.8a to Fig.8c). Thus, the maximum of the turning of the surface streamlines as well as the primary convergence line is behind the shock wave, and this convergence line makes an angle to the inviscid shock line (Fig.8c). As a result, this convergence line is formed before it becomes parallel to the shock as the shock wave angle increases. Van Oudheusden et al. (1996) argued that the far field of the conical interaction does not possess a quasi-two dimensional structure in the cross-flow plane of the radial direction. They showed that the conicity of the inviscid
flow regions in supersonic flow produces a geometrically conical surface flow pattern. There is essential difference between the cylindrical and conical interactions.

From the above discussions we can say that when the wall limiting streamlines (not only one streamline) behind the shock wave becomes aligning with one ray from the virtual origin (near the fin apex) as the strength of shock wave increases, the incipient separation line is generated. At this ray, the direction of the skin-friction vector is perpendicular to the local pressure gradient. The wall limiting streamlines of incoming flow then converge and coalesce to this ray. Thus, this ray could prevent the oil film from spreading across it and therefore can be easily detected in experiments with oil streak pattern technique. In most experiments, this ray is considered as the incipient separation line as associated with Lighthill’s criterion.

If the flow is cylindrical as analyzed in Inger (1986) and Myring (1977), there exists none of the stage between case(c) and case (d) in Fig.8 for conical interactions; in other words, case (d) coincides with case (c). The incipient separation line formed from the upstream uniquely corresponds to that defined by Stanbrook’s criterion (Inger, 1986; Myring, 1977). However, for conical interactions, there is an angle difference between case(c) and case (d) in Fig.8. Therefore, the process for the formation of the primary separation line for conical interactions is very different from that for cylindrical interactions. As a results, the Stanbrook’s criterion is applicable to cylindrical interactions, but is not directly applicable to conical interactions. This is why there is still a discrepancy between the prediction using this criterion and the experimental data.

4.4 Correction for Incipient Separation Angle

The difference of the deflection angle between the cases(c) and (d), $\Delta \alpha$, is shown in Fig.9 for two set of data. It can be found that $\Delta \alpha$ decreases with the increasing Mach number. This is in accord with the physical mechanism of the interaction because
the pressure ratio across the shock wave increases with the Mach number, and the pressure ratio at separation line is almost constant for three-dimensional separation of supersonic flows (Dou, 1991; Dou and Deng, 1992b).

Since it is difficult to find a criterion to define the condition of Fig.8(c), we still take Stanbrook’s criterion as the incipient separation criterion, and add a correction to the predicted incipient separation angle $\alpha_i$.

Assume that the correction $\Delta \alpha$ is only related to the Mach number. Thus, we can work out a correlation using the experimental data for the correction to the theoretical prediction. The result is shown below and in Fig.9,

$$\Delta \alpha = 0.20M_1^2 - 1.8M_1 + 5.70 \quad \text{for} \quad 1.6 < M_1 < 5. \quad (16)$$

The corrected incipient separation angle $\alpha_{ic}$ is,

$$\alpha_{ic} = \alpha_i - \Delta \alpha \quad \text{for} \quad 1.6 < M_1 < 5. \quad (17)$$

5 CONCLUSIONS

The conclusion can be summarized as follows:

1. Dou and Deng (1992c) developed a theoretical method to analyze the three-dimensional turbulent boundary-layer in SW/TBLI. Based on this theory, we carried out the predictions for the experiments of Lu and Settles (1990). Dou and Deng’s theory is physically founded to be better than those previous works of McCabe (1966), Korkegi (1973) or Lu (1989) in allowing a fuller understanding of the secondary flow influence and the effect of $Re_\theta$. The prediction of the wall limiting streamline direction by this theory yielded good agreement with Lu and Settles’ (1990) experimental data.

2. The “incipient separation line” formed from the upstream is generated by the secondary flow induced by the lateral pressure gradient. The incipient separation line provided by the experiments is the condition of the first appearance of the wall limiting streamline perpendicular to the local pressure gradient. When the wall limiting
streamlines from the upstream becomes aligning along with one ray from the virtual origin (near the fin apex) as the strength of shock wave increases, the incipient separation line is formed, at which the wall limiting streamline is perpendicular to the local pressure gradient.

3. The process for the formation of the primary separation line for conical interactions is very different from those for cylindrical interactions. The Stanbrook’s criterion is only applicable to cylindrical interactions, and not applicable directly to conical interactions. The disagreement between the prediction by this criterion and the experiments is attributed to the intrinsic behavior of conical interaction.

4. The difference of the deflection angle for incipient separation between the prediction and the experiments, $\Delta \alpha$, decreases with the increasing Mach number. A correlation equation for the correction to the theoretical predicted incipient separation angle $\alpha_i$ is given.
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