We generate $SU(2)$ lattice gauge fields at finite temperature and cool them in order to characterize the two phases by the occurrence of specific classical solutions. We apply two kinds of spatial boundary conditions: fixed holonomy and standard periodic b.c. For $T < T_c$ our findings concerning classical configurations semi-quantitatively agree for both types of boundary conditions. We find in the confinement phase a mixture of undissociated calorons with lumps of positive or negative half-integer topological charges.

1. INTRODUCTION

There is one scenario of confinement which envisions the ground state of QCD as a dual superconductor formed by the condensation of abelian magnetic monopoles. Nowadays links are established to a vortex condensation picture of confinement. Still, one might be puzzled by the question which role the carrier of topological charge (easily associated with chiral symmetry breaking) could play for confinement. Dealing with dilute, uncorrelated gases of instantons \cite{1} or - at finite temperature - of calorons \cite{2} cannot explain confinement. Abandoning the tacit assumption of globally non-trivial objects with lowest possible action to any known analytical solution. In this paper we report on results showing that approximately half-integer topological charges ($D\bar{D}$ pairs) \cite{6}. They do not correspond to any known analytical solution. In this paper we report on results showing that approximately $O(4)$ symmetric calorons (denoted $CAL$) as well as static $DD$ and $\bar{D}\bar{D}$ pairs are statistically significant in the confinement phase ($0 < T \leq T_c$). In this case, fixed holonomy would require to put $L = 0$ on the boundary. However, our observations show no qualitative difference to periodic boundary conditions without this constraint.

2. CLASSICAL CONFIGURATIONS

We consider again pure $SU(2)$ gluodynamics with Wilson action at finite temperature, but this time comparing two types of spatial boundary conditions, (i) periodic b.c. with time-like link variables being fixed on the spatial surface and (ii) standard periodic b.c. without constraints. We mostly used a lattice of size $16^3 \times 4$, in a few cases also $32^3 \times 4$. We took coupling values $\beta$ below and above the deconfinement transition ($\beta_c \approx 2.29$) to create equilibrium configurations to start with. The same b.c. are applied for the Monte Carlo process and for cooling.

In a first stage we have searched for topologically non-trivial objects with lowest possible ac-
tion late in the cooling history in order to find systematic dependences on the phase and the boundary conditions. Cooling was stopped at the \( n \)-th cooling step when the following criteria were fulfilled: action \( S_n < 2 S_{\text{inst}} \), change of action \( |S_n - S_{n-1}| < 0 \) and concavity \( S_n - 2 S_{n-1} + S_{n-2} < 0 \), i.e. cooling just passed a point of inflection. For each \( \beta \) we have scanned the topological content of \( O(200) \) configurations. In this late stage we find approximate classical solutions, more or less static. We define the "non-staticity" in (Euclidean) time by

\[
T_s = \sum_i |S_i - S_{i-1}| / \sum_i S_i,
\]

where \( S_i \) denotes the action in the \( i \)-th timeslice. According to Refs. [7,8], the almost-classical configurations can be classified as \( DD \), \( CAL \), \( D\bar{D} \), and purely magnetic (Dirac) single and double sheets - \( M \) and \( 2M \). Some configurations undergoing cooling do not match the stopping criteria. They turn into trivial vacuum states.

Calorons (\( CAL \)) are a limiting case of \( DD \) configurations [8,9]. The latter have opposite sign peaks of \( L(\vec{x}) \) near the centers of the lumps of action and topological charge. If the peaks of topological charge are too close to be separated in 3D space, these objects happen to be non-static as well. For the confinement phase we have monitored how frequently objects with given \( T_s \) are found. The histograms are rather similar for both types of b.c. They have a peak at \( T_s = 0.02 \pm 0.04 \) and have a long tail of widely varying \( T_s \). To enable an easy distinction between \( DD \) and (non-ideal) \( CAL \) events we have searched and verified a cut in \( T_s \). For \( T_s < 0.17 \) we may classify the objects as \( DD \) (static with two well-separated maxima of the densities of topological charge \( q(\vec{x}) \) and action \( s(\vec{x}) \)). For \( T_s > 0.17 \) the objects can be classified as \( CAL \) (non-static, approximately \( O(4) \) symmetric solutions, with a single maximum of the 3D projected \( q(\vec{x}) \) and \( s(\vec{x}) \)). \( D\bar{D} \) have been found always to consist of two well-separated static objects with nearly vanishing \( T_s = 0.004 \pm 0.002 \), much smaller than for \( DD \)'s! Purely magnetic sheets - \( M \) have a peak at \( T_s = 0.002 \pm 0.002 \), with an action quantized in units of \( S_{\text{inst}}/4 \). They are also perfectly static with \( T_s = 0.003 \pm 0.002 \).

As can be seen from Table 1, the relative frequency to obtain different types of nearly classical configurations (\( DD \), \( CAL \), \( D\bar{D} \), \( M \) and \( 2M \)) is quite different, depending on whether cooling starts from Monte Carlo configurations in the confinement phase or the deconfinement phase.

Table 1
Relative frequencies of cooled quasi-stable configurations for \( \beta = 2.2, 2.25 \) (confinement phase) and 2.35 (deconfinement phase). First and second rows refer to fixed holonomy and standard periodic b.c., respectively. The lattice size is \( 16^3 \times 4 \).

| Type | \( \beta = 2.20 \) | \( \beta = 2.25 \) | \( \beta = 2.35 \) |
|------|----------------|----------------|----------------|
| \( DD \) | .46 \pm .05 | .52 \pm .05 | .20 \pm .03 |
| \( CAL \) | .43 \pm .05 | .44 \pm .05 | .01 \pm .01 |
| \( D\bar{D} \) | .19 \pm .03 | .17 \pm .03 | .04 \pm .01 |
| \( M, 2M \) | .24 \pm .03 | .26 \pm .03 | .06 \pm .02 |
| trivial vac. | .28 \pm .04 | .26 \pm .04 | .58 \pm .05 |
| trivial vac. | .18 \pm .03 | .16 \pm .03 | .00 |
| trivial vac. | .01 \pm .01 | .01 \pm .01 | .10 \pm .02 |
| trivial vac. | .04 \pm .02 | .03 \pm .01 | .22 \pm .03 |
| trivial vac. | .06 \pm .02 | .04 \pm .02 | .08 \pm .02 |
| trivial vac. | .11 \pm .02 | .11 \pm .02 | .71 \pm .06 |

For the confinement phase we find that the relative probabilities for all objects are approximately independent of the type of b.c. For the deconfinement phase we see that the strong enhancement of \( D\bar{D} \) configurations earlier found for fixed holonomy b.c. which would be compatible with the suppression of the topological susceptibility) is not reproduced for standard periodic b.c. In the standard case, the probability to obtain any topologically non-trivial object drops sharply with \( \beta \). As for the deconfinement phase as such, the latter observation must be considered with caution since the physical 3-volume is very small. The independence of the boundary conditions, however, in the confinement phase must be taken seriously: the enforcement of \( L = 0 \) boundary conditions seems to be equivalent with the conditions under normal (thermal) boundary conditions.

\[ S_{\text{inst}} = 2\pi^2\beta \] is the action of a single instanton.
3. DILUTE GASES AT HIGHER ACTION

Applying the $L = 0$ fixed holonomy b.c. we have studied in more detail the configurations closer to equilibrium in the confinement phase which represent snapshots of the early cooling history. The stopping criterion above (however, without limitation of action) has been applied and the configurations have been automatically stored. This gives a series of subsequent "plateaux" of the action, while the cooling history in between is always reproducible. In terms of the objects classified above, we have scanned the topological content of subsequent plateau configurations. In the first plateau we find an uncorrelated gas of dyons $D$ and antidyon $\bar{D}$ carrying roughly half-integer topological charges. We identified the dyons simultaneously by various measurable quantities: extrema of the Polyakov line $L(\vec{x})$, of the topological charge and action density and of the non-Abelianicity (after the maximal Abelian gauge has been fixed) and finally by the static Abelian monopoles (defined in Abelian projection). Their world lines coincide with the world lines of dyons. If the signs of the monopole charge of an Abelian monopole and of $L(\vec{x})$ (measured in the dyon center $\vec{x}$) are the same (opposite), the topological charge of the dyon is approximately equal to $+\frac{1}{2}$ (or $-\frac{1}{2}$).

The cooling trajectory between this stage and the final stage discussed in the previous section is a series of annihilations. Between subsequent plateaux only such annihilation events of topological lumps have been observed which are monopole-antimonopole annihilations (conservation of magnetic charge). Depending on the relative sign of $L(\vec{x})$ in the center, these are $DD$ or $D\bar{D}$ annihilations. The two cases are shown in the Figure. Neither electric nor topological charge is conserved under (Wilson) cooling. In $D\bar{D}$ annihilations a sharp drop of the topological charge density can be seen while the action density and Polyakov line slowly relax to zero. $DD$ annihilations resemble how $T = 0$ lattice instantons disappear under cooling with Wilson action: topological charge density and action density increase, turning everything into a singular "dislocation" which finally collapses. At the end of the cooling process, when $S = O(1) S_{\text{inst}}$, we find either $DD$ (incl. CAL) or $D\bar{D}$ configurations. The first ones (with $Q = \pm 1$) can decay as described, while $D\bar{D}$ annihilate into $M$, $2M$ configurations or to the trivial vacuum.

4. SUMMARY

Calorons with non-trivial holonomy have motivated us to reconsider the topological content of the vacuum at finite temperature by careful cooling. We cannot claim, that finite temperature gauge fields can be understood semiclassically with the new calorons as the only background. First, with cooling also nearly classical configurations of dyon-antidyon type are obtained. Secondly, closer to the equilibrium, cooling with fixed holonomy boundary conditions produces a dilute gas of dyons and antidyons. For the confinement phase we have found that fixed holonomy and usual periodic b.c. have less influence on cooling and the detection of a semiclassical topological background.

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Figure 1. Topological charge density (a, b, c) and corresponding spatial Polyakov line distribution (a’, b’, c’) at different cooling stages for a typical gauge field configuration. The transition (a, a’) → (b, b’) shows the annihilation of a $D\bar{D}$ pair and (b, b’) → (c, c’) the annihilation of a $DD$ pair, respectively.