Asymmetrically twisted strings

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In this letter a new class of twisted strings is presented, with an asymmetry between the holomorphic and anti-holomorphic sectors parametrized by an integer $N$. Their physical content is given by the massless resonances of the closed string plus the mass-level $N$ spectrum of the open string. The appeal of this model is the singling out of the (higher spin) massive levels of string theory together with their self/gauge/gravity interactions. Motivated by the original tree level Kawai-Lewellen-Tye relation for closed strings, its asymmetrically twisted version at four points is conjectured and shown to naturally interpolate with conventional and twisted strings. The resulting four-point amplitudes have a generalized Virasoro-Shapiro dressing factor.

I. OVERVIEW

Recent years have witnessed an unprecedented interplay between string theory and quantum field theory. Perhaps the most evident example of this mutual exchange is the Cachazo-He-Yuan (CHY) formalism [1,2], which describes field theory amplitudes through moduli space integrations over a Riemann surface. Indeed, the CHY formulae were promptly shown to emerge from a chiral worldsheet model: the ambitwistor string [3].

The connection between scattering amplitudes and the worldsheet should not come as a surprise. After all, string theory has been exploring it for about half a century. However, string amplitudes are in a sense tainted by an infinite spectrum, with arbitrarily high mass levels parametrized by the string length squared $\alpha'$. This is in contrast to one striking feature of the twisted string: it is fair to say that the twisted string lies between ambitwistor and conventional strings.

Simultaneously to the debut of the scattering equations, Hohm, Siegel, and Zwiebach introduced a string-like model with manifest $T$-duality [4]. This model was later shown to have the ambitwistor string as its tensionless limit [4]. The tensionless version is characterized by a certain twist between the left- and right-moving sectors of the target space coordinates, hence the epithet twisted string (not to be confused with the twisted string states in the context of orbifolds, e.g. [7]). This twist has a dramatic effect on the string spectrum. Besides the usual massless states of the closed string, the infinite tower of massive states gives way to a single level, with the degrees of freedom of the first massive level of the open string. Spectrum wise, it is fair to say that the twisted string lies between ambitwistor and conventional strings.

While most of the current research in scattering amplitudes revolves around massless fields, the twisted string incidentally let us hope of a more general and controlled study of massive resonances. The reason is precisely its finite spectrum, which can then be described through a Lagrangian. This fact has been recently explored in [8], introducing a surprisingly compact method for computing tree level string amplitudes involving one massive multiplet at the first mass level. This method is based on field theory techniques that shortcut the traditional conformal field theory (CFT) computations intrinsic to the string scattering.

The obvious question at this point is: what is special about the first massive level of the string spectrum? From the string cohomology perspective, not much. The twisted string is just a particular case of a deformation of the target space CFT. This deformation is parametrized by a positive integer $N$ which corresponds to the mass level it singles out. In string theory, however, higher mass levels are tied to higher spins (Regge trajectories). One is then bound to find non-trivial aspects in the tree level amplitudes of such models, for they involve interactions of higher spin states. Here I will discuss this construction and some related implications.

II. THE ASYMMETRIC TWIST

The asymmetric twist is defined as a deformation of the matter CFT, encoded in the OPE of the target space coordinates $X^m$,

$$X^m(z, \bar{z}) X^n(y, \bar{y}) \sim -\frac{\alpha'}{4} \eta^{mn} \ln(z - y) + \frac{\alpha'}{4N} \eta^{mn} \ln(\bar{z} - \bar{y}). \quad (1)$$

Here, $\eta^{mn}$ is the flat space metric. The twisted string is reproduced when $N = 1$, while $N = -1$ takes us back to the conventional string.

The left- and right-moving components of the $X^m$ energy-momentum tensor are respectively given by

$$T = -\frac{i}{2} \partial X^m \partial X_m, \quad \bar{T} = \frac{N}{2} \partial X^m \partial X_m, \quad (2)$$

such that the central charge contribution from the target space coordinates does not depend on $N$.

Nevertheless, eigen-momentum operators now have different conformal weight in the two sectors: $e^{ik \cdot X}$ (normal ordering implicit) has conformal weight $\alpha' k^2 / 4$ in the holomorphic sector and $-\alpha' k^2 / (4N)$ in the anti-holomorphic sector. Relatedly, the Koba-Nielsen factor at tree level is expressed as

$$K_N \equiv \prod_{j=1}^{n} e^{ik_j \cdot X(z_j, \bar{z}_j)} = \prod_{1 \leq i < j} \left( \frac{z_{ij}}{z_{ij}} \right)^{\alpha'(k_i, k_j) / 2}, \quad (3)$$

where $z_{ij} = z_i - z_j$. The structure of $K_N$ partially reflects the physical spectrum of the theory. For
For positive integers $N$, there are only two simultaneous solutions for $V_L$ and $V_R$. The massless solution $k^2 = 0$ corresponds to $V_R = \bar{a}_m \partial X^m$, with $k^m \bar{a}_m = 0$ and $\partial m \cong \bar{a}_m + k_m \lambda$. In this case, the solution of $\psi\bar{\sigma}$ is given by $V_L = a_m \partial X^m$, with $k^m a_m = 0$ and $a_m \cong a_m + k_m \lambda$. This vertex operator describes the massless excitations of the closed bosonic string.

There is also a massive solution of $\psi\bar{\sigma}$ with $k^2 = -4N/\alpha'$ and $V_R = 1$. Its left-moving counterpart in $[8]$ matches the cohomology of the open bosonic string at mass level $N$. In other words, the massive spectrum of the bosonic ATS has the mass level $N$ degrees of freedom of the open string.

For $D < 26$, it is still possible to build a critical bosonic string via an affine algebra extension with currents $J^A$ and $\bar{J}^a$ for the holomorphic and antiholomorphic sector, respectively. The massless spectrum is then enhanced by two gauge vectors, $A^A_{\mu}$ and $A^a_{\mu}$, and a biadjoint scalar, $\phi^{Aa}$. In principle, this setup would enable the extension of the techniques developed in $[8]$ for the computation of tree-level amplitudes in conventional bosonic string theory. As will be clear soon, however, a possible field theory description of the ATS spectrum with $N \geq 2$ leads to a Lagrangian with an arbitrarily high number of derivatives.

### Heterotic ATS

For the heterotic theory, the analysis is very similar. $V_R$ is built using $\partial X^m$ and the current $J^a$. For $V_L$, the available ingredients are $\partial X^m$ and its worldsheet superpartner $\psi^m$, and a $\beta\gamma$ system. For more details, see e.g. $[13]$.

The simultaneous solutions for $V_L$ and $V_R$ can again be divided into two groups. The massless spectrum is given by the known $N' = 1$ supergravity and super Yang-Mills states of the conventional heterotic string. The massive solutions $k^2 = -4N/\alpha'$, with $V_R = 1$, match the open superstring spectrum with mass level $N$ as can be readily seen by the output of $[8]$. Just like in the bosonic case, the dynamics of the heterotic ATS states with $N \geq 2$ cannot be completely captured by a finite Lagrangian.

### IV. THE ATS KLT MATRIX

Tree level amplitudes in the asymmetrically twisted string are computed using the standard prescription of the conventional string, with the major difference given by the Koba-Nielsen factor in $[13]$.

In the case of four external states, with (dimensionless) Mandelstam variables $s_{ij} = \alpha'(k_i + k_j)^2/4$, the ATS tree level amplitudes can be expressed in terms of the following moduli space integrals,

$$I_N^{(m,n)}(s,t) = \int d^2z z^{m+s}(1-z)^{n+t}z^{m-s-N}(1-z)^{n-t/N},$$

where $\{m,n,\bar{m},\bar{n}\}$ are integer numbers, $s = s_{12}$, $t = s_{13}$, and $u = s_{14}$.
For a generic $N$, $I_N$ is not well defined in the complex plane because of the branch cuts for non-integer $s$, $t$ and $u$. For $N=1$ this problem is overcome using intersection theory \cite{11,12}. For $N>1$, the only way to make sense of the ATS scattering amplitudes is to attach to the prescription an extra input that effectively picks the physical branches when evaluating the integrals \cite{8}.

One can anticipate some general features in the related tree level amplitudes. Among them, gauge invariance (decoupling of BRST exact states), crossing symmetry, and kinematic poles matching the mass spectrum of the theory. Instead of looking for the precise definition of the extra input to solve $I_N$, I will partially replicate the original KLT analysis \cite{8} and propose a new KLT matrix. This will lead to closed-form expression for $I_N$ that satisfies the expected physical requirements and naturally interpolates with the known cases $N=\pm 1$.

Given the character of the branch cuts in \cite{8}, it is reasonable to expect the following behavior,

$$I_N \sim Q(s,t) \times \Omega_N,$$

(9)

$$\Omega_N(s,t,u) \equiv \prod_{j=1}^{1/N} \frac{\Gamma(-s/N)\Gamma(-t/N)\Gamma(-u/N)}{\Gamma(-j)} ,$$

(10)

where $Q(s,t)$ is some rational function of the Mandelstam variables. The argument to support the scalar dressing factor $\Omega_N$ is threefold. First, it reproduces a Virasoro-Shapiro-like structure for $N=-1$. Second, when $N=1$, the dressing factor collapses to the unity, in agreement with the twisted string. There are no poles for $N>1$, which matches the finite character of the physical spectrum of the ATS. In order to see this, recall that gamma functions have no zeros, just poles. The possible poles in the dressing factor come from positive $s$, $t$ or $u$ that are multiples of $N$. In this case, both numerator and denominator are divergent, but their ratio is well-defined and finite.

The KLT construction suggests the following output for the integrals in \cite{8},

$$I_{N,(m,n)}(s,t) = \int_0^\infty d\xi \xi^{m-s}(1-\xi)^{n+t} \times \prod_{j=1}^{1/N} \int_{-\infty}^\infty d\eta \eta^{-m/N}(1-\eta)^{-n}/N ,$$

(11)

where KLT$_N$ carries the information about the physical branches of $I_N$. There is then a simple solution for the modified KLT matrix compatible with the above considerations:

$$KLT_N = \sin(\pi s)\sin(\pi t)/\sin(\pi s/\pi N).$$

Using this result and Euler’s reflection formula, $I_N$ can be finally expressed as

$$I_{N,(m,n)}(s,t) = \frac{\sin(\pi s)\sin(\pi t)}{\sin(\pi s/\pi N)\sin(\pi t/\pi N)} \times \Gamma(s+m+1)\Gamma(t+n+1)\Gamma(\frac{n+t}{N} - \bar{m} - \bar{n} - 1) \Gamma(s+t+m+n+2)\Gamma(\frac{n+t}{N} - \bar{m})\Gamma(\frac{n+t}{N} - \bar{n}).$$

(13)

The term inside the square brackets essentially kills any simple pole that is not a multiple of the level $N$. In addition, equation (13) neatly recovers the conventional and twisted string results. One might worry that poles at $2N, 3N, \ldots$ might appear in the gamma functions, which would explicitly spoil unitarity. However, the values of $\{m, n, \bar{m}, \bar{n}\}$ at a given mass level are constrained by the conformal weight of the chiral blocks $V_L, V_R$. It is then straightforward to check that the ATS amplitudes obtained through (13) cannot develop poles in $s, t, u$ greater than $N$.

V. SOME EXAMPLES

As a practical example of the conjectured moduli space integrals \cite{13}, we can look at the four-point amplitude with external gluons. It has an universal character, offering several consistency checks. A straightforward computation in the heterotic ATS yields

$$A_4 = \Omega_N(A_4^{\text{SYM}} + A_4^{\text{double}}),$$

(14)

where $A_4^{\text{SYM}}$ is the field-theory four gluon amplitude in super Yang-Mills and $A_4^{\text{double}}$ encodes the double-trace contributions, given by

$$A_4^{\text{SYM}} = e_\alpha e_\beta e_\gamma e_\delta d_{\mu\nu\rho\sigma}^D T_{\mu\nu\rho\sigma}^{mnpq} \left[ \frac{f_{ace}f_{bde} - f_{ace}f_{bde}}{st} \right] + \left[ \frac{f_{ace}f_{bde} - f_{ace}f_{bde}}{tu} \right],$$

(15)

and

$$A_4^{\text{double}} = 3\kappa\epsilon_\alpha^2 e_\beta e_\gamma e_\delta d_{\mu\nu\rho\sigma}^D T_{\mu\nu\rho\sigma}^{mnpq} \left[ \frac{\delta_{abc}\delta_{bd}}{s(t+u)} + \frac{\delta_{abc}\delta_{bd}}{t(u+n)} \right],$$

(16)

where

$$T_{\mu\nu\rho\sigma}^{mnpq} = \eta_{\mu\nu}(uk^p_kk^q_k + tk^p_kk^q_k) + \eta_{\rho\sigma}(uk^m_kk^n_k + tk^m_kk^n_k) + \eta_{\rho\sigma}(sk^m_kk^n_k + uk^m_kk^n_k) + \eta_{\rho\sigma}(sk^m_kk^n_k + tk^m_kk^n_k) + \eta_{\rho\sigma}(sk^m_kk^n_k + tk^m_kk^n_k) + \eta_{\rho\sigma}(sk^m_kk^n_k + tk^m_kk^n_k),$$

(17)

The gluon polarizations are denoted by $e_\alpha$, while $f_{abc}$ are the structure constants of the gauge group with level $\kappa$. The amplitude (13) has the ATS dressing factor of (10), and the correct factorization channels. In particular, the double-trace channels match the remaining spectrum of the heterotic ATS, with contributions from the massless sector ($N=1$ supergravity) and the massless $N$ states of the open superstring.

When $N=1$, the dressing factor in (13) goes away, leaving behind the field theory amplitude characteristic of the twisted string. For $N=-1$, the apparent tachyonic poles in (13) are canceled out by the corresponding zero in $\Omega_N$. In this case, (13) reproduces the four gluon amplitude of the conventional heterotic string. Observe that the residue of (13) in the massive pole (e.g. $s+N=0$) has a remarkably simple form,
Here the ellipsis denote other linearly independent contributions. For \( N = \pm 1 \) the prefactor disappears, the residue is a polynomial in \( t \) with degree \( N + 1 \), and unitarity is granted. For generic \( N \), however, this is not the case. The prefactor in (13) implies an infinite expansion in any polynomial basis. This is a generic behaviour of the ATS amplitudes, an immediate consequence of the dressing factor \( \Omega_N \).

For the bosonic ATS, I will quickly discuss a four-point amplitude with \( N = 2 \), involving a massive state and three gluons. The massive resonance has spin \( S = 3 \), described by the vertex operator \( V_{\text{t}} = \phi_{\text{mnp}} \partial^m \partial^n \partial^p \). The totally symmetric polarization \( \phi_{\text{mnp}} \) is traceless \( (\eta^m_{\text{mnp}} = 0) \), and transversal \((k^m_{\text{mnp}} = 0)\). Its tree level scattering with three gluons can be cast as

\[
\mathcal{A}_4(\phi_1, \epsilon_2, \epsilon_3, \epsilon_4) \propto \int_{ab} \phi_{\text{mnp}}(a) \epsilon_2^a \epsilon_3^b \epsilon_4^c \times \Omega_N \left\{ 4 \eta^{rs} \eta^{tu} \eta^{ps} - 2 \eta^{r} \eta^{tu} (k^m_2 k^m_3 + k^m_4 k^m_1) \right. \\
- 2 \eta^{r} \eta^{ps} (k^m_3 k^m_4 + k^m_2 k^m_1) - 2 \eta^{rs} \eta^{tu} (k^m_2 k^m_3 + k^m_4 k^m_1) \\
+ \frac{2}{3} \eta^{r} \eta^{tu} [k^m_2 k^m_3 (t + 1) + \eta^{ps} k^m_4 (u + 1)] \\
- \frac{2}{3} \eta^{rs} \eta^{tu} [k^m_3 k^m_4 (s + 1) + \eta^{ps} k^m_2 (u + 1)] \\
+ \frac{2}{3} \eta^{r} \eta^{ps} [k^m_2 k^m_3 (t + 1) + \eta^{tu} k^m_4 (s + 1)] \\
+ \frac{2}{3} \eta^{rs} \eta^{ps} [k^m_3 k^m_4 (s + 1) + \eta^{tu} k^m_2 (t + 1)] \\
- \frac{2}{3} \eta^{r} \eta^{tu} [k^m_2 k^m_3 (t + 1) + \eta^{ps} k^m_4 (u + 1)] \\
- \frac{2}{3} \eta^{rs} \eta^{tu} [k^m_3 k^m_4 (s + 1) + \eta^{ps} k^m_2 (u + 1)] \\
+ \left( \frac{(t + 1)(u + 1)}{2} \right) \eta^{ps} \eta^{rs} k^m_2 k^m_3 + \left( \frac{t + 1}{2} \right) \eta^{ps} \eta^{rs} k^m_3 k^m_4 + \ldots \right\}.
\]

The ellipsis inside the curly brackets consists of terms of the form \((\alpha^k k^2)\) and \((\alpha^k k^3)^2\). Because of the color structure, this amplitude has only massless poles, corresponding to the exchange of gluons. The apparent poles at \( s, t, u = 1 \) are also canceled out here by the respective roots of the dressing factor.

In the heterotic ATS with arbitrary \( N \), four-point amplitudes with one massive leg at the leading Regge trajectory and three massless ones can be determined using the very general construction of Schlotterer in [14]. This is possible because the CFT correlators in the holomorphic sector of the ATS are essentially the same as in the open superstring.

VI. DISCUSSION

The introduction of a parameter \( N > 1 \) singling out a unique mass level of the open (super) string spectrum from a worldsheet theory is certainly enthralling. With a finite physical spectrum, this worldsheet model may open the doors for a more systematic study of the massive resonances of conventional string theory using field theory methods. Especially appealing is the fact that the physical content of the mass level \( N \) contains states with spin \( S = N + 1 \).

Higher spin field theories are well-known for their intricate dynamics, severely constrained by a series of consistency requirements and no-go theorems (see, for example, the review [15] and references therein). This behavior is supported by the four-point amplitudes discussed here. They encode interactions involving an arbitrarily high number of derivatives between massive higher spin fields, gauge fields and gravity, manifested through the dressing factor \( \Omega_N \). Unlike the \( N = 1 \) case, that can be exactly described by a finite Lagrangian [8], the dynamics of the ATS states can only be captured by an effective field theory (EFT) through an \( \alpha' \)-expansion. It would be really interesting to analyze such EFTs more deeply, in particular in respect to the consistency of the higher-spin interactions (for example, along the lines of [16]). Unitarity is broken, cf. equation (18), unless there is some nontrivial mechanism involving the infinite number of derivatives of the EFT description. This analysis is left for future work. The role of the KLT matrix is particularly intriguing, for it seems to connect an open string amplitude and what resembles a chiral half of a string theory on an orbifold [7].

The analysis of the high energy behaviour of the integrals \( I_N \) is straightforward, in particular the Regge limit and the hard scattering limit. When we consider the scattering \( 1 + 2 \to 3 + 4 \) of four states with mass \( m^2 = 4N\alpha' \), and of the center of mass reference frame of the states \( 1 \) and \( 2 \), the Mandelstam variables read

\[
s = -\frac{\alpha'}{E^2}, \quad t = \left( \frac{\alpha'}{E^2} - 4N \right) \sin^2 \theta, \quad u = \left( \frac{\alpha'}{E^2} - 4N \right) \cos^2 \theta,
\]

where \( E \) is the center of mass energy and \( \theta \) is the scattering angle between states \( 1 \) and \( 3 \). The high energy behaviour \((\alpha' E^2 \gg 1)\) of the amplitudes is mainly driven by their dressing factors [10], which in the above configuration can be expressed as

- **Regge limit** \((s \to -\infty, \small \text{small} \theta)\):
  \[
  \Omega_N \approx \frac{\Gamma(-t/N)}{\Gamma(-t)} s^{-(t+4N)(1-1/N)}. \tag{21}
  \]
- **Hard scattering limit** \((s \to -\infty \text{ and } s/t \text{ fixed})\):
  \[
  \Omega_N \approx e^{(1-1/N)(s \ln s + t \ln t + u \ln u)}. \tag{22}
  \]

Notice first that the usual Regge limit resonances for integer \( t \) are absent, since the spectrum is finite. Second, the hard scattering limit presents a soft string-like behaviour. This can again be explained by the arbitrary number of higher derivatives of the effective field theory for the higher spin fields.

At the level of the spectrum, the asymmetric twist has no impact on type II string theories, either with \( N = (1,1) \) worldsheet supersymmetry or explicit \( \mathcal{N} = 2 \) spacetime supersymmetry: there are only massless resonances. Intuitively, this can be explained...
by the fact that the twist in (11) emulates a tachyon vertex operator in the right-moving sector, which is not compatible with the inbuilt supersymmetry.

Alternatively, when $N$ is taken to be a negative integer, the physical spectrum again becomes infinite, albeit with an interesting change. The level matching condition leads to an asymmetrical contribution from the left- and right-moving parts of the vertex operators. Putting it differently, the BRST cohomology is no longer comprised of worldsheet scalars, unlike conventional strings. The chiral pieces $\bar{c}V_L$ and $\bar{c}V_R$ are similar to open (super) string vertex operators but at different mass levels. The scope of this construction can be extended by letting $N$ take rational values as well.

Differently from the $N = 1$ case, the asymmetric twist is not obviously connected to a singular gauge fixing of the Polyakov action [6]. Understanding the underlying gauge fixing leading to (11) might help to clarify the role of the parameter $N$ and to reveal further applications of this model.

Note also that the twisted string ($N = 1$) has a natural description in terms of a chiral worldsheet [6, 17]. The analysis of the $\alpha' \to \infty$ (tensionless) limit yielding the ambitwistor string is very simple [18–20]. The massive states in the bosonic and heterotic cases play the role of auxiliary fields helping to implement higher derivative equations of motion for the massless fields [21, 22]. For $N > 1$, there does not seem to exist a straightforward extension of the chiral map of [17], and the tensionless limit, if sensible, likely becomes more involved. In this case, the higher spin states at mass level $N$ become massless and this investigation might shed some light on the construction of ambitwistor strings for higher spin fields.

For $N \to \infty$, the OPE (11) naively resembles that of an open string. Indeed, the right-moving sector of the ATS becomes inert. This can be seen through the BRST charge $\bar{Q}$. After a field redefinition $\bar{c} \to \bar{c}/N$ and $\bar{b} \to N\bar{b}$, it can be recast as

$$\bar{Q} = \frac{1}{\alpha'} \oint \partial \bar{X}^m \partial X_m + O(1/N).$$

In the $N \to \infty$ limit, only the first term survives. Since the OPE $\partial \bar{X}^m(\bar{z})X^n(y, \bar{y})$ is now regular, the solutions of (6) become degenerate. Curiously, the BRST cohomology at ghost number one, given in (5), is enhanced and matches the physical degrees of freedom of the open (super) string. It is enticing to combine this analysis with the tensionless limit, as it hints at a possible route to define an open ambitwistor string.

Finally, a comment on string loops, i.e. higher genus surfaces. The twisted string is not modular invariant [23, 24]. On the torus, with modular parameter $\tau = \tau_1 + i\tau_2$, a simple way to see this is to notice that the zero mode contribution to the scalar partition function of $X^m$ is a function of $\tau_1$ instead of $\tau_2$. This is related to the sign flip in (11) for $N = 1$. The asymmetric twist leads to a similar structure, with a nontrivial mixing of the real and imaginary parts of the modular parameter, which naively does not lead to a modular invariant partition function. However, the loop level construction of the ATS can only be properly done after the gauge structure of the worldsheet model is understood.

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