Graphene-Based Multiband Chiral Metamaterial Absorbers Comprised of Square Split-Ring Resonator Arrays With Different Numbers of Gaps, and Their Equivalent Circuit Model

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ABSTRACT The equivalent circuit model (ECM) is developed by using a MATLAB code to analyze graphene-based multi-band chiral metamaterial absorbers composing graphene-based square split-ring resonator arrays in the terahertz (THz) range. The absorbers are simulated numerically by the finite element method (FEM) in CST Software to verify the ECM results. Our introduced multi-band absorbers can be used as suitable platforms in polarization-sensitive devices and systems in the THz range. We have designed four tunable graphene-based chiral metamaterial absorbers containing one, two, three, and four gaps in their arms, respectively. The absorber with one gap has four absorption bands (two for TE and three for TM, one band of both modes approximately overlaps) with absorption > 50%. The absorber with two gaps has three absorption bands (two for TE and two for TM, one band of both modes approximately overlaps). The absorber with three gaps has four absorption bands (three for TE and two for TM, one band of both modes approximately overlaps). The absorber with four gaps has three absorption bands (three for TE and two for TM, two bands of both modes approximately overlap). They work in the 1-5.5 THz with maximum linear dichroism (LD) responses of 98, 99, 89, and 77%, respectively. The designed absorbers are dynamically tunable. Additionally, by a 90° rotation of the incident electromagnetic fields, it is possible to switch between the number and/or location of absorption bands making these absorbers a promising candidate for future THz systems. ECM results are following the FEM ones. The proposed ECM procedure is a simple and fast way to recognize the characteristics of the designed absorbers. Our proposed absorbers could be promising enablers in future THz systems.

INDEX TERMS Simulation, circuit modeling, chirality, terahertz, metamaterials.

I. INTRODUCTION

Chiral structures or chiral metamaterials do not superimpose to their mirror images and this feature causes them to produce non-equal or polarization-sensitive responses. This non-equality is measured as chirality responses such as circular dichroism (CD: the difference in absorbance for right-handed circular-polarized and left-handed circular-polarized waves) and/or linear dichroism (LD: the difference in absorbance for TE and TM-polarized waves) [1].

Graphene, a 2D layer of the carbon atom, has excellent properties making graphene a propitious candidate in optoelectronic devices. Graphene-based unit cell resonators of chiral metamaterials have been designed and developed recently to produce tunable chirality responses [2]–[8]. These metamaterials have a chirality response of up to 96%, but the development of tunable chiral metamaterial absorbers, which are capable to switch between the number and/or location of absorption bands by only a 90° rotation of the incident electromagnetic fields is mostly unexplored field of research.

Some chiral metamaterial absorbers have been proposed recently [9]–[14] with one or two maximum absorption
bands. There is a need for multi-band chiral absorbers in THz communication systems [15] when necessary to absorb more information through multi-bands. Similar needs of multi-band absorbers are in sensing [16], [17], spectroscopy, and imaging [18] applications. However, most of the published chiral metamaterial absorbers are not graphene-based resonators, so their absorption spectra and chirality responses are not dynamically tunable. In these papers, the maximum chirality response reached 88%, and they do not provide any theoretical model for a better understanding of the chiral metamaterial absorbers.

Recently some chiral metamaterials containing split-ring resonator arrays were designed for telecommunication and spectroscopy applications [19]–[25]. The resonator designs are based on metals and those metamaterials work mostly in the gigahertz (GHz) region. Only the metamaterial in [23] works in the THz region. Those metamaterials have good absorption properties, but they are lacking dynamical tunability (not containing a graphene or transition metal dichalcogenides resonator layer). They also introduced the ECM approach for the metamaterial except [24], but their ECM approaches differ from ours as the metal resonator array and graphene resonator array have different formulas for the circuit impedances.

Our earlier paper [6] reported a multi-band graphene-based chiral metamaterial absorber containing a single-layer U-shaped resonator array with the LD chirality response reaching 94%. While the other work [7] introduced a dual-functional graphene-based chiral metamirror containing two-layered complementary 90° rotated U-shaped resonator arrays with an LD response reaching 96%. In this work, we propose multi-band graphene-based chiral metamaterial absorbers containing single-layer square split-ring resonator arrays with different numbers of gaps. Compared to the earlier published paper [6], were carried out simulations for a similar one gap structure but in a different frequency range (1–5.5 THz). Compared to the paper [7], simulation and circuit modeling procedures of this work are assumed to be less resource demanding as the proposed structure contains only a simple single resonator layer and dielectric.

In the work [6], the ECM approach of the metastructure is based on the relation between S parameters and the elements of the ABCD matrix. The impedances of the graphene pattern are obtained as a function of the pattern reflection, substrate impedance, and electrical length. The gap of the structure is modeled by a series capacitor(s) and the impedances of the patterns are obtained as a function of reflection, gap length(s), and cosine/secant of input/output incident wave angles. Then, the ECM is developed using the transmission line formula.

Proposed tunable chiral absorbers could utilized in applications such as enhancement of chirality responses for chiral biomolecules like DNA and amino acids since the natural biomolecules have weak chirality responses [7], polarization transformers, stealth technology, thermal bolometers [9], wavelength-selective absorption filters, hot-electron collection devices [10], chiral imaging to achieve distinct images by switching the polarization of the incident wave [14] in the THz range.

### II. METAMATERIAL AND EQUIVALENT CIRCUIT MODEL

The periodic and unit cell views of the designed tunable graphene-based chiral metamaterial absorbers composed of square split-ring resonator arrays with different numbers of gaps are given in Figs. 1(a-e). The metamaterials are simulated at room temperature. For the biasing procedure of the graphene resonator patterns, we have used an ion gel layer with a thickness of $d_{ig}$ and the refractive index of 1.42 [26]. The dielectric spacer is made of Teflon with a refractive index of 1.45. The dielectric spacer is backed with a gold layer with a conductivity of $4.56 \times 10^7$ S/m [27] to prevent the transmission of TE and TM modes. Simulations are done in the CST Microwave Studio 2018 [6], [7]. The dynamically tunable devices work as multi-band absorbers with the possibility to switch between the number and/or location of absorption bands by only a 90° rotation of the incident electromagnetic fields. The considered parameters and their optimized values for the designed metamaterial absorbers are reported in Table 1. We have used the parametric sweep to optimize the results to reach the maximum linear dichroism (LD) response for the proposed chiral metamaterial absorbers in the simulated THz region. In the sweep optimization procedure in CST, we considered that the unit cell dimensions, $P_x = P_y = 20 \, \mu m$, have to be smaller than $\lambda_{min} = 54.55 \, \mu m$ if $f_{max} = 5.5 \, THz$ (the maximum frequency in the simulated region) to avoid excitation of high order Floquet modes.

The relative permittivity of graphene, by the consideration of the incident electromagnetic wave $e^{i\omega t}$, is [6], [7]:

$$\varepsilon = 1 - \frac{j \sigma}{\omega \varepsilon_0 \Delta}$$  \hspace{1cm} (1)
in which $\sigma$, $\omega$, $\varepsilon_0$, and $\Delta$ are respectively the surface conductivity of graphene, angular frequency, vacuum permittivity, and graphene thickness. $\Delta$ has assumed 0.335 nm [28]. $\sigma$ is the summation of the inter- and intra-band electron transition contributions based on the Kubo formula [6]:

$$\sigma = \sigma_{\text{inter}}(\omega) + \sigma_{\text{intra}}(\omega)$$

(2a)

$$\sigma_{\text{inter}}(\omega) = \frac{e^2}{4\hbar} \left[ H\left(\frac{\omega}{2}\right) - \frac{4\omega}{\pi} \int_0^{\infty} \frac{H(\xi) - H\left(\frac{\omega}{2}\right)}{\omega^2 - 4\xi^2} d\xi \right]$$

(2b)

$$\sigma_{\text{intra}}(\omega) = \frac{2kB_e^2T}{\pi h^2} \ln \left[ \frac{2 \cosh \left( \frac{E_f}{2kB_T} \right)}{\cosh \left( \frac{h\xi}{kB_T} \right) + \cosh \left( \frac{h\xi}{2kB_T} \right)} \right]$$

(2c)

$$H(\xi) = \frac{\sinh \left( \frac{h\xi}{kB_T} \right)}{\cosh \left( \frac{h\xi}{kB_T} \right) + \cosh \left( \frac{h\xi}{2kB_T} \right)}$$

(2d)

in which $h$ is the reduced Plank’s constant, $k_B = 1.38 \times 10^{-23}$ J/K is the Boltzmann’s constant, $e = 1.6 \times 10^{-19}$ C is the electron charge, $T$ is the temperature equals to 300 K, and $\xi$ is the integral variable. $\tau$ is the relaxation time as [6], [29]:

$$\tau = \frac{\mu E_f}{e v_f}$$

(3)

in which $v_f = 10^6$ m/s is the Fermi velocity and $\mu = 2.22$ m$^2$/V.s is the carrier mobility of graphene. The propagation constant of the electromagnetic wave in a graphene-vacuum configuration is [6], [7]:

$$\beta = k_0 \sqrt{1 - \left( \frac{2}{Z_0\sigma} \right)^2}$$

(4)

where $k_0$ and $Z_0$ are the wave vector of the incident wave and the vacuum impedance.

The equivalent circuit model (ECM) procedure could be summarized as four steps: 1) The impedances (capacitances) of the gaps are calculated. 2) The conductivities of the split ring resonator arrays (the graphene layer) are calculated. 3) The impedances of the graphene sections (not considering the gaps) of the split ring resonators in each metamaterial are calculated and plotted. 4) The TE/TM absorption spectra of the whole metamaterial absorbers are calculated by use of the transmission line formula and compared with the numerically simulated ones.

The ECM procedure for the graphene-based chiral metamaterials containing different numbers of gaps is based on the modeling of the graphene resonator array as the equivalent conductivity $\sigma_{\text{SSRRA}}'$ (in which $i$ could be OG, TG, THG, or FG which respectively means the metamaterial containing...
one gap, metamaterial containing two gaps, metamaterial containing three gaps, and metamaterial containing four gaps. The reflection coefficients $r_{SSRRA-i}$ were obtained by use of CST frequency-domain simulations considering the graphene resonator array on one half-space slab made of Teflon with a thickness of 500 $\mu$m. $r_{SSRRA-i}$ is the reflection coefficient of the graphene-based square split-ring resonator arrays of Fig. 1 for (a) TM mode of resonator array containing one gap, (b) TM mode of resonator array containing two, three, and four gaps, (c) TE mode of resonator array containing one and two gaps, (d) TE mode of resonator array containing three gaps, and (e) TE mode of resonator array containing four gaps. For the states in which the $E$ field is normal to the gap, the gap is modeled by a capacitor. For the states in which the $E$ field is parallel to the gap, the gap is not modeled by any circuit element. The graphene sections of the metamaterials are modeled by series RLC in all states shown in the red dashed-line plotted rectangles.

So, for the TM mode, only the gaps in the horizontal arms could be modeled by capacitors. For the TE incident wave, the SSRRA of Figs. 1(b, c), is modeled with an RLC circuit (the graphene section) and two serials gap capacitances (shown in Fig. 2(b)). So, the TM mode only the gaps in the horizontal arms could be modeled by capacitors. For the TE incident wave, the SSRRA of Figs. 1(b, c), is modeled with an RLC circuit (the graphene section) and two serials gap capacitances (shown in Fig. 2(b)).

The TE/TM impedances of the square split-ring resonator array (SSRRA) $Z_{SSRRA-i}^{TE/TM}$ are obtained by:

$$Z_{SSRRA-i}^{TE/TM} = \frac{1}{\sigma_{SSRRA-i}^{TE/TM}}$$

(10)

where $\sin(\theta_{out}) = \frac{1}{\sqrt{\epsilon_d}} \sin(\theta_{in})$

(9)

The TE/TM impedances of the square split-ring resonator array (SSRRA) $Z_{SSRRA-i}^{TE/TM}$ are obtained by using CST frequency-domain simulations considering the graphene resonator array on one half-space slab made of Teflon with a thickness of 500 $\mu$m. $r_{SSRRA-i}$ is the reflection coefficient of the graphene-based square split-ring resonator arrays of Fig. 1 for (a) TM mode of resonator array containing one gap, (b) TM mode of resonator array containing two, three, and four gaps, (c) TE mode of resonator array containing one and two gaps, (d) TE mode of resonator array containing three gaps, and (e) TE mode of resonator array containing four gaps. For the states in which the $E$ field is normal to the gap, the gap is modeled by a capacitor. For the states in which the $E$ field is parallel to the gap, the gap is not modeled by any circuit element. The graphene sections of the metamaterials are modeled by series RLC in all states shown in the red dashed-line plotted rectangles.
in Fig. 2(d)). For the TE incident wave of the absorber in Fig. 1(e) which is contained four gaps, only $g_3$ and $g_4$ are modeled by capacitances (shown in Fig. 2(e)). In these cases, the incident electric field is normal to the gap(s). So, for the TE mode, only the gaps in the vertical arms could be modeled by capacitors. Each gap is modeled by a capacitance calculated by:

$$C_{gap_i} = \varepsilon_{eff} \frac{w}{g_i}; \quad i = 1, 2, 3, 4$$  (11)

in which $\varepsilon_{eff}$ is the effective relative dielectric permittivity, $w = \frac{(L-1)}{2}$, the gap width, is equal to 4 $\mu$m, and $g_i$ is the gap length. $\varepsilon_{eff}$ is calculated by:

$$\varepsilon_{eff} = \frac{\varepsilon_0 (\varepsilon_{ig} + \varepsilon_d)}{2}$$  (12)

in which $\varepsilon_{ig}$ is the relative ion gel permittivity. So, $C_{gap_1} = 145.78$, $C_{gap_2} = 97.19$, $C_{gap_3} = 72.89$, and $C_{gap_4} = 58.31$ PF/m.

The impedances of the gaps are calculated by:

$$Z_{gap_i} = \frac{1}{j\omega C_{gap_i}}; \quad i = 1, 2, 3, 4$$  (13)

So, the impedances of the gaps when the incident electric field is normal to the gaps are calculated by:

$$Z_{gap_i} = \frac{4g_i}{j\omega \varepsilon_0 (\varepsilon_{ig} + \varepsilon_d) (L - l)}$$  (14)

For calculation of $Z_{gOG,TM}$ (impedance of the graphene section; Fig. 2(a)), we have:

$$Z_{gOG,TM} = Z_{gOG,TM} + Z_{gap_1}$$  (15)

$$Z_{gOG,TM} = Z_{gSSRRA-OG} - Z_{gap_1}$$  (16)

So,

$$\sigma_{gOG,TM}$$

$$j \omega e_0 (\varepsilon_{ig} + \varepsilon_d) \begin{bmatrix} \sec(\theta_{in}) - \sqrt{\varepsilon_d} \sec(\theta_{out}) - \\ \frac{r_{SSRRA-OG}}{r_{TM}} \\ \frac{r_{SSRRA-OG}}{r_{TM}} + \frac{\varepsilon_d}{\varepsilon_0} \sec(\theta_{out}) \end{bmatrix}$$

$$= Z_0 \left( 1 + r_{SSRRA-TM} \right) j \omega e_0 (\varepsilon_{ig} + \varepsilon_d) - 2 (g_1 + g_2 + \ldots + g_n) \begin{bmatrix} \sec(\theta_{in}) - \sqrt{\varepsilon_d} \sec(\theta_{out}) - \\ \frac{r_{SSRRA-OG}}{r_{TM}} \frac{\varepsilon_d}{\varepsilon_0} \sec(\theta_{out}) + \frac{r_{SSRRA-OG}}{r_{TM}} \sec(\theta_{in}) + \\ 2 (g_1 + g_2 + \ldots + g_n) \end{bmatrix}$$

(17)

For calculation of $Z_{gFG,TM}$ (impedance of the graphene section; Fig. 2(b)), we have:

$$Z_{gFG,TM} = Z_{gFG,TM} + Z_{gap_1} + Z_{gap_2}$$  (18)

$$Z_{gFG,TM} = Z_{gFG,TM} - Z_{gap_1} - Z_{gap_2}$$  (19)
mirror symmetry) of the proposed chiral metamaterials.

and TE modes. This is because of the asymmetric nature (non-

In each metamaterial differ. For example, the structure

So,

$\sigma_{g,TE}$

$$
\sigma_{g,TE} = \left[ j \omega e_0 (\varepsilon_i + \varepsilon_d) \right]
\begin{bmatrix}
\cos (\theta_{in}) - \sqrt{\varepsilon_d} \cos (\theta_{out}) \\
\frac{Z_{SSRRA-THG}}{r_{TE}} (\varepsilon_i + \varepsilon_d)
\end{bmatrix}
$$

For calculation of $Z_{g,TE}$ (impedance of the graphene section; Fig. 2(e)), we have:

$Z_{g,TE} = Z_{gFG,TE} - Z_{gap_1} - Z_{gap_4}$

$$
Z_{g,TE} = \frac{Z_{TE}}{Z_{SSRRA-FG}}
$$

consequently, we could generalize that if we have $n$ number of gaps ($g_1, g_2, \ldots, g_n$) in the vertical arms of the metamaterial absorber, $\sigma_{g,TE}$ would be as:

$Z_{g,TE} = Z_{g,TE} + Z_{gap_1} + Z_{gap_2} + \ldots + Z_{gap_n}$

$$
Z_{g,TE} = Z_{SSRRA-i} - Z_{gap_1} - Z_{gap_2} - \ldots - Z_{gap_n}
$$

So,

$\sigma_{g,TE}$

$$
\sigma_{g,TE} = \left[ j \omega e_0 (\varepsilon_i + \varepsilon_d) \right]
\begin{bmatrix}
\cos (\theta_{in}) - \sqrt{\varepsilon_d} \cos (\theta_{out}) \\
\frac{Z_{SSRRA-i}}{r_{TE}} (\varepsilon_i + \varepsilon_d)
\end{bmatrix}
$$

Therefore:

$Z_{g,TE} = Z_{g,TG,TE}$

$$
Z_{g,TG,TE} = Z_{g,THG,TM} = Z_{g,FG,TE}
$$

TE/TM equivalent conductivities of the graphene sections in each metamaterial differ. For example, the structure containing one gap, equations (17) and (25) are respectively the equivalent conductivities of the graphene section in TM and TE modes. This is because of the asymmetric nature (non-mirror symmetry) of the proposed chiral metamaterials.

In Fig. 3, the designed ECM of the proposed graphene-based chiral metamaterial absorbers is presented. The equivalent transmission line model and the input impedance of each section of the proposed chiral metamaterial absorbers are given in Fig. 3. The thickness of the graphene layer is ultra-thin compared to the spectrum wavelength and it was assumed as a point load [31], [32].

The equivalent impedances of the different parts of the absorbers are as follows [31]:

$Z_{TE} = Z_{d} + jZ_{TM} \tan (\beta_d d)$

in which $Z_{TM}$, $Z_{d}$, and $\beta_d$ are respectively the TE/TM impedances of the dielectric layer, the gold layer impedance, and the propagation constant of the THz electromagnetic wave in the dielectric spacer.

$$
Z_{d} = \begin{cases} 
Z_{TE} & \text{if } \beta_d = 0 \\
Z_{TM} & \text{if } \beta_d \neq 0 
\end{cases}
$$

So,

$Z_{TE,TM}^{TM} = Z_{TM}^{TM} + jZ_{TM}^{TM} \tan (\beta_d d)$

$$
Z_{TE,TM}^{TM} = \begin{cases} 
Z_{TM}^{TM} & \text{if } \beta_d = 0 \\
Z_{TM}^{TM} & \text{if } \beta_d \neq 0 
\end{cases}
$$

in which $Z_{TM}^{TM}$ and $\beta_d$ are respectively the TE/TM impedances of the ion gel layer and the propagation constant of the THz electromagnetic wave in the ion gel layer.

$$
Z_{TM}^{TM} = \frac{Z_0}{\sqrt{\varepsilon_{ig}}} \cos (\theta_{in})
$$

$$
Z_{TM}^{TM} = \frac{Z_0}{\sqrt{\varepsilon_{ig}}} \sec (\theta_{in})
$$
FIGURE 4. (a) Real and (b) imaginary parts of $\sigma_{g,TE}$ (Equations (25), (28), and (31); conductivity of the graphene layer in TE mode) for $E_f = 0.9$ eV.

FIGURE 5. (a) Real and (b) imaginary parts of $\sigma_{g,TM}$ (Equations (17) and (20); conductivity of the graphene layer in TM mode) for $E_f = 0.9$ eV.

$$Z_{TE} = \frac{Z_0}{\sqrt{\varepsilon_d}} \cos (\theta_d)$$

in which $\theta_d$ is the electrical length of the dielectric layer:

$$\theta_d = \frac{d \omega \sqrt{\varepsilon_d}}{c}$$

$$Z_{TM} = \frac{Z_0}{\sqrt{\varepsilon_d}} \sec (\theta_d)$$

$$\beta_{ig} = \frac{\omega \sqrt{\varepsilon_{ig}}}{c}$$

$$\beta_d = \frac{\omega \sqrt{\varepsilon_{id}}}{c}$$

The scattering parameters $S_{TE/TM, 11SSRRA-1}^{TE/TM}$ are calculated by:

$$S_{11SSRRA-1}^{TE} = \frac{Z_{inSSRRA-1}^{TE} - Z_0 \cos (\theta_{in})}{Z_{inSSRRA-1}^{TE} + Z_0 \cos (\theta_{in})}$$

$$S_{11SSRRA-1}^{TM} = \frac{Z_{inSSRRA-1}^{TM} - Z_0 \sec (\theta_{in})}{Z_{inSSRRA-1}^{TM} + Z_0 \sec (\theta_{in})}$$

The reflection coefficients $R_{SSRRA-1}^{TE/TM}$ of the chiral metamaterial absorbers are calculated by:

$$R_{SSRRA-1}^{TE/TM} = \left| S_{11SSRRA-1}^{TE/TM} \right|^2$$

The absorption coefficients $A_{SSRRA-1}^{TE/TM}$ of the chiral metamaterial absorbers are calculated by:

$$A_{SSRRA-1}^{TE/TM} = 1 - R_{SSRRA-1}^{TE/TM}$$

TABLE 2. Resonance characteristics of metamaterials of fig. 1

| TE/1.9 | TE/4.56 | TM/2.44 | TM/3.6 | TM/4.56 |
|--------|--------|--------|--------|--------|
| magnetic | electric | electric | magnetic | electric |

METAMATERIAL CONTAINING TWO GAPS

| TE/1.91 | TE/4.56 | TM/3.07 | TM/4.62 |
|--------|--------|--------|--------|
| magnetic | electric | magnetic | electric |

METAMATERIAL CONTAINING THREE GAPS

| TE/1.73 | TE/2.74 | TE/4.61 | TM/3.1 | TM/4.63 |
|--------|--------|--------|--------|--------|
| magnetic | electric | magnetic | electric |

METAMATERIAL CONTAINING FOUR GAPS

| TE/2.67 | TE/3.12 | TE/4.67 | TM/3.1 | TM/4.65 |
|--------|--------|--------|--------|--------|
| magnetic | magnetic | electric | electric |

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III. RESULTS AND DISCUSSION

We performed numerical simulations by the finite element method (FEM) in the frequency domain solver of CST 2018 [5]–[7], [33], [34]. The considered simulation setups like boundary conditions and the mesh sizes are the same as in [5]–[7]. To excite the metamaterial absorbers, TE and TM modes were excited in the simulations, and the absorptions were measured as a function of frequency. The results are shown in Figure 6, which depicts the absorption spectra and E-field distributions for metamaterials containing one gap (Fig. 1(b)), two gaps (Fig. 1(c)), three gaps (Fig. 1(d)), and four gaps (Fig. 1(e)). The absorption coefficients were calculated using the formula $E_f = 0.9 \text{ eV}$.
FIGURE 7. Surface current distributions of the metamaterial containing one gap in (a) TM 3.6 THz magnetic resonance, (b) TE 4.56 THz electric resonance, the metamaterial containing two gaps in (c) TE 1.91 THz magnetic resonance, the metamaterial containing three gaps in (d) TE 1.73 THz magnetic resonance, and the metamaterial containing four gaps in (e) TE 2.67 THz magnetic resonance.

TM electromagnetic waves were launched to them in the z-direction [5]–[7], [35]–[37].

The TE/TM equivalent conductivities of the SSRRAs, \( \sigma_{R,TE/TM} \), were calculated by simulating the SSRRAs on the half-space slab made of Teflon with a thickness of 500 \( \mu \)m and by using Equations (17), (20), (25), (28), and (31). The real and the imaginary parts of the equivalent conductivities of the SSRRAs in TE mode are respectively given in Figs. 4(a) and 4(b). As it is shown, the real and the imaginary parts of the metastructures containing one and two gaps are equal. The real and the imaginary parts of the equivalent conductivities of the SSRRAs in TM mode are respectively given in Figs. 5(a) and 5(b). As it is shown, the real and the imaginary parts of the metastructures containing two, three, and four gaps are equal. As given in Figs. 4(a) and 5(a), the real parts of the equivalent conductivities are positive in the whole frequency range representing the loss and the resistive nature of the patterned graphene layers. The imaginary parts of the equivalent conductivities of the SSRRAs are given in Figs. 4(b) and 5(b). They contain both positive and negative parts which means respectively the inductive and capacitive natures of the SSRRAs are dominant. So, we model each SSRRA as a series RLC circuit. Moreover, the TE and TM equivalent conductivities are not equal in each of the metamaterials. This is because of the non-asymmetric (chirality) or polarization-sensitive nature of the proposed metamaterials.

The TE/TM reflection spectra and the E-field distributions of the proposed chiral metamaterial absorbers of Fig. 1 were given in Fig. 6. As an interesting feature, by a 90° rotation of the incident electromagnetic fields, it is possible to switch between the number and/or location of absorption bands in 1-5.5 THz which makes these designed chiral absorbers promising candidates for future THz systems. The average of maximum absorptions of the absorption bands in this work for the metastructure containing one gap (Fig. 1(b)) reaches 95% while the average of maximum absorption bands of [8] which has a U-shaped resonator array reaches 87.5%. In each metamaterial with a different number of gaps, the electric field distributions are calculated and given in one of the resonance frequencies for both TE and TM modes representing the chirality nature of the metamaterials. For example, the E-field distributions of the metamaterial containing one gap (Fig. 1(b)) for both TE and TM modes in
FIGURE 8. Comparison of CST and ECM results of the metastructure of Fig. 1(b) for (a) TE and (b) TM modes; Fig. 1(c) for (c) TE and (d) TM modes; Fig. 1(d) for (e) TE and (f) TM modes; and Fig. 1(e) for (g) TE and (h) TM modes. Results are obtained as we assumed $E_f = 0.9$ eV.
FIGURE 9. Linear dichroism (LD) spectra of the metastructure of (a) Fig. 1(b), (b) Fig. 1(c), (c) Fig. 1(d), and (d) Fig. 1(e) for three different values of $E_f$.

3.6 THz (resonance absorption for TM mode) are respectively given in Figs. 6(b) and 6(c). The distributions are not equal for TE and TM modes in 3.6 THz which shows the chirality nature (asymmetry) of the metamaterial. As shown in Fig. 6(b), the metamaterial does not absorb the TE wave noticeably. As shown in Fig. 6(c), TM mode is absorbed greatly in the metamaterial.

The maximum linear dichroism (LD) reaches 98, 99, 89, and 77% for the metamaterial absorbers containing one, two, three, and four gaps respectively (Figs. 1(b-e)). The absorption spectra and the LD responses are dynamically tunable in 1-5.5 THz by only changing the applied bias voltage without the need to refabricate the metamaterial absorbers which serve the material, cost, and time.

To determine the type of resonances (electric or magnetic) [38], the surface current distributions of the proposed metamaterials with different resonance frequencies were determined and the obtained results summarized in Table 2. Some representative examples are shown in Fig. 7. In Fig. 7(a), the surface currents on the graphene metasurface (left to right) have the opposite direction compared to those of the gold ground plane (right to left). Therefore, the resonance type is magnetic. In Fig. 7(b), the surface currents on the graphene metasurface and the ground plane are not in opposite directions. Also, the surface currents on the graphene patterns are not making a closed loop. So, this resonance is electric type. In Fig. 7(c), the surface currents on the graphene metasurface (up to down) have the opposite direction compared to those of the gold ground plane (down to up). Therefore, the resonance type is magnetic. The resonances in Figs. 7(d) and 7(e) are also magnetic types because the surface currents on the graphene metasurface make a closed loop.

The TE/TM absorption spectra of the metamaterial, obtained by CST (numerical approach) and Equation (53) (the ECM approach in MATLAB), are shown and compared in Fig. 8. The results, obtained by those two methods, are in good agreement thus proving that the proposed EMC model is a valid approach to predict their resonance performance.

The LD (the difference between TE and TM absorption/reflection spectra [39]: $LD = A^E - A^M$) vs $E_f$ spectra for the metasstructures of Figs. 1(b-e) are given in Fig. 9. As shown, by increasing $E_f$, the resonance frequency values exhibit a blueshift. This is because the real part of the $\beta$ in Equation (4) decreases as the $E_f$ increases [5]. So, the resonance values of the LD spectra increase by the increase of $E_f$.

Comparison with previously published chiral metamaterial absorbers/metamirros is summarized in Table 3.
FIGURE 10. Comparison of CST and ECM results for the incident angles of $\theta_{in} = 0^\circ$ and $30^\circ$ of the metastructure of Fig. 1(b) for (a) TE and (b) TM modes; Fig. 1(c) for (c) TE and (d) TM modes; Fig. 1(d) for (e) TE and (f) TM modes; and Fig. 1(e) for (g) TE and (h) TM modes. Results are obtained as we assumed $\varepsilon_f = 0.9 \text{ eV.}$
TABLE 3. Comparison of the chiral metamaterial absorbers/metamirrors.

| Tunable | Frequency range | CM | Max. absorption/reflection (%) | Max. chirality response (%) |
|---------|----------------|----|-------------------------------|----------------------------|
| [6] yes | 0.5–4.5 THz     | yes| 99 (absorption)               | LD/94                      |
| [7] yes | 0.3–4.5 THz     | yes| 99 (reflection)               | LD/96                      |
| [9] no  | 7–10 GHz        | no | 93.2 (absorption)             | CD/86                      |
| [10] no | 375–500 GHz     | no | 80 (absorption)               | CD/50                      |
| [11] no | 187.5–375 GHz   | no | 50 (absorption)               | CD/12                      |
| [12] no | 1.6–3.2 GHz     | no | 97 (absorption)               | CD/70                      |
| [13] no | 166.7–375 GHz   | no | not reported                  | CD/88                      |
| [14] no | 150–250 GHz     | no | 85 (absorption)               | CD/50                      |
| [40] no | 9.1–11 GHz      | no | 98 (reflection)               | CD/93                      |
| [41] no | 300–375 GHz     | no | 90 (reflection)               | CCD/43                     |
| [42] no | 30–50 GHz       | no | 99 (reflection)               | CD/94                      |
| [43] no | 285–425 GHz     | no | 90 (reflection)               | CD/50                      |
| [44] no | 211–227 GHz     | no | (reflection)                  | CD/63                      |
| [45] no | 8–12 GHz        | no | 95 (reflection)               | CD/88                      |
| This work | 1–5.5 THz      | yes| 100 (absorption)             | LD/99                      |

The fabrication of the proposed metamaterial absorbers is not in the scope of this work, but its procedure could be the same as explained in our previous work [6].

IV. CONCLUSION

In this work, an equivalent circuit modeling (ECM) approach based on equivalent conductivities of graphene in TE/TM modes and transmission lines is being proposed and developed for the tunable graphene-based chiral metamaterial absorbers consisting of square split-ring resonator arrays with different numbers of gaps by using simple and fast MATLAB code in the terahertz (THz) region. The simulations are performed using the finite element method (FEM) in CST Microwave Studio Software. Simulation results are in good agreement with the ECM ones. Our designed chiral metamaterials are dynamically tunable, and they have maximum linear dichroism (LD) responses up to 98, 99, 89, and 77% respectively for the designed metamaterials containing one, two, three, and four gaps. Our designed multi-band chiral absorbers in the 1–5.5 THz range are promising candidates for future THz systems. The possibility to switch between the number and/or location of absorption bands makes our proposed metamaterial absorbers promising enablers in tunable polarization-sensitive THz structures and systems in the future.

APPENDIX

The incident angle dependence of TE/TM absorption spectra of the metamaterial was modelled by CST and Equation (53) (ECM). Obtained results are shown for two different incident angles of \( \theta_{in} = 0^\circ \) and \( 30^\circ \) in Fig. 10. The results are in good agreement. Comparing the response for \( \theta_{in} = 0^\circ \) and \( 30^\circ \), we can see that the change of the incident angle do not influence on the resonance frequencies and their magnitude of absorption varies only slightly. Therefore, the absorber structures for both TE and TM modes are not incident angle dependent.

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