Universal Scaling Function of Velocity Rotation in Spiral galaxies

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Abstract

The data of velocity rotation curve in spiral galaxies, almost for galaxies which have close central surface brightness, collapse onto a universal scaling function. Since scaling functions are the signature of emergence in complex systems we are led to the idea that explanation of constant velocity in spiral galaxies needs considering cooperative behavior instead of interpretation based on reductionism approach.

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Velocity rotation curve of spiral galaxies, hereafter RC, is one of the wonderful and important subjects in physics. The curve represents the rotation velocity of stars in a galaxy versus their distance, \( r \), from the galaxy center. The rotation velocity of spiral galaxies increases with \( r \) and tends to a constant value. RCs do not show any Keplerian fall-off for large radii. This behavior forced physicists to present new theories and interpretations. The first interpretation is that the luminous disk is embedded within a halo of dark matter. The distribution of dark matter in galaxies should be proportional to \( r \) in large radii regime to have a constant velocity. In spite of intensive search for the components of non-baryonic dark matter, no candidate particles have been observed. Scientists have tried to search for modified gravity theories to explain galaxy dynamics without the need for exotic dark matter as the second interpretation. These theories are based on the modification of Newtonian dynamics or generalization of Einstein’s general relatively. Study of galaxies and their RCs helps us with better understanding of the universe and its component matter or of the unknown gravitational force. In the other words, galaxies are the big labs of the universe to study gravity and matter.

Most of the empirical relations of galaxies are scaling laws. One of the most firmly established empirical scaling relations of disk galaxies is the Tully-Fisher relation. This relation indicates a tight correlation between the total luminosity and the maximum rotation velocity of the galaxy, however a complete physical interpretation of this relation is still missing. Tully-Fisher relation shows a notable dependence on morphological types and distance estimation is one of its applications. In this letter other scaling relations are established beside conventional scaling laws and it is shown that the data of RCs, almost for galaxies which have close central surface brightness, collapse onto a universal scaling function. This behavior can help us to obtain a better understanding of galaxy formation and dynamics. Also, the universal function enables us to estimate an unknown astrophysical parameter by knowing other parameters.

Tully-Fisher relation has less dispersion in I-band luminosity and also we are mainly interested in mass rather than luminosity, so we concern ourselves with the Tully-Fisher and other scaling relations measured in this band. In this letter the distance and RCs data have been derived from Persic et al. paper. Furthermore, the luminosity and central surface brightness in I-band are read from a database which is related to the Courteau et al. paper. Note that the Hubble constant is assumed \( H_0 = 70 \, km \, s^{-1} \) entire this letter.
**FIG. 1:** RCs for a bunch of galaxies which have close central surface brightness.

In order to find scaling relations logarithmic scale is used for diagrams. RCs in logarithmic scale are linear for small radii (linear regime) while the velocity reaches to a constant value for large radii (constant regime). We draw RCs for different galaxies and then pick a bunch of them that have almost the same linear regimes (Fig. 1). It is seen that the galaxies of each selected bunch have close I-band central surface brightness, $\mu_{0I}$ (e.g. see table. 1). This indicator (close central surface brightness) works well specially for later types of spiral galaxies. To have a view about other bunches we can have a comparison with the selected bunch in Fig. 1. RCs related to bunches with smaller central surface brightness are placed on the right hand of the RCs in Fig. 1 while those with larger central surface brightness will be on its left hand. It will be shown that the rotation curve data of different bunches of galaxies show a universal scaling behavior with different scaling exponents.

Linear regime indicates a power low behavior for rotation velocity in small radii as

$$V \sim r^\beta \quad r \ll r_x,$$

(1)

where the slope is $\beta = 0.7175$ for the selected galaxies of table. 1. Bunches of galaxies with larger central surface brightness have a little larger $\beta$ and vise versa. Also, this exponent shows morphological dependence. Bunches of earlier types of spiral galaxies have a minute larger slope in linear regime. $r_x$ is the radius in which the velocity reaches to its constant value. We call this radius, the crossover radius and is obtained by intersecting the fitted lines for two regimes, linear and constant. The method for determination of $r_x$ is similar to the one depicted by Family and Vicsek to define the crossover time in surface growth [19].

As it is seen in Fig. 1, all galaxies have the same behavior for small radii and then their
TABLE I: Characteristics of selected bunch of galaxies

| Galaxy | Morph | \( \mu_0 \) [mag arcsec\(^{-2} \)] | log \( L_I \) [\( L_\odot \)] |
|--------|-------|----------------------------------|-----------------------------|
| 546 – G29 | Sc   | 18.82                           | 10.231                      |
| 563 – G21 | Sbc  | 18.73                           | 11.241                      |
| 569 – G22 | Sc   | 19.06                           | 10.788                      |
| 249 – G16 | Sc   | 18.76                           | 10.457                      |
| 90 – G9 | SBC  | 18.7                            | 10.531                      |
| 580 – G29 | Sc   | 18.64                           | 10.388                      |
| 34 – G12 | Sc   | 18.92                           | 10.851                      |

Velocities become constant and separate from the linear regime in different values which we call it the saturation velocity, \( V_{sat} \). Tully-Fisher relation indicates that \( V_{sat} \) and the galaxies I-band luminosity, \( L_I \), are related by

\[
V \sim L_I^\alpha, \quad r \gg r_x,
\]

where \( \alpha = 0.2994 \) for our data. Note that, the small difference between this relation and the Tully-Fisher relation is that the average velocity in constant regime is chosen instead of the maximum velocity.

There is another scaling law beside relations (1) and (2). This relation is a power law between the crossover radius and the I-band luminosity. Since the crossover radius satisfies Eq. (1) and the saturation velocity satisfies both Eqs. (1) and (2), so

\[
r_x \sim L_I^{z},
\]

where the exponent \( z = \alpha/\beta \). Galaxies with larger luminosity have larger crossover radius. The logarithmic diagram of crossover radius as a function of galaxies I-band luminosity is plotted in Fig. 2 and the exponent \( z \) is obtained as \( z = 0.4173 \) for our data which is equal to the ratio \( \alpha/\beta \).

A universal scaling function can be constructed from our three scaling relation as follow

\[
V(r, L_I) = L_I^\alpha f\left(\frac{r}{L_I^z}\right),
\]
where the function $f(x)$ has the asymptotic behavior

$$f(x) \sim \begin{cases} 
  x^\beta & x \to 0 \\
  \text{const.} & x \to \infty.
\end{cases}$$

Actually, Eq. (4) governs the motion of stars in spiral galaxies. This equation means that the data collapse onto a function $f(x)$ when plotted as $V/L^\alpha$ versus $r/L^\beta$ or correspondingly, $V/V_{Sat}$ versus $r/r_x$ according to (2) and (3) (Fig. 3).

Since, there is uncertainty in estimation of astrophysical parameters and also existing empirical relations such as Tully-Fisher relation have a considerable dispersion, it is difficult to construct a tune universal function. Note that to study these scaling relations together to find the universal function, we need to pick galaxies whose RCs in logarithmic scale include both linear and constant regimes.
This kind of diagrams and scaling functions exist in many areas of physics such as surface growth [19], turbulent flow in rough pipes [20], non-equilibrium phenomena and near critical point problems [21–23]. Generally, scaling functions are the signature of complex systems.

In addition to existing scaling function in galaxies there are other evidences which lead us to consider galaxies as complex systems. For instance there are mutual forces and interactions between stars in a galaxy. Also, random velocities [15] and fluctuations exist, especially in the core, and the number of stars in a galaxy [15] is too much (ranged from $10^9$ to $10^{12}$).

Due to the collective behavior and interactions between the components of a complex system, emergent and coherent structures occur. Unlike the traditional scientific approach, the emergent and coherent structures cannot be described within the vocabulary applicable to the parts [24]. Dynamics of a system in coherent state depends only on the nature of components interaction and the equation of the motion of its components, lower level of description, in a sense is violated. In fact, emergent levels of description absorb the properties of lower level of description into the phenomenological parameters and has own new law of physics. In a paper entitled ‘More is different’ P.W. Anderson has described that reductionism approach is not valid in emergent phenomena of complex systems [25]. In this case the whole is more than the sum of its parts. For further clarity, consider emergence of solid state from the fluid state [26]. The pairwise interaction between atoms does not change, but the correlation between them changes when the temperature is lowered. Level of description of solid state is described by continuum mechanic and has own new law of physics. Origin of this new law is the collective behavior of the atoms. Collective state exhibits novel response characteristics, and loses memory of underlying level of description. It is why it took so long for the existence of atoms to be deduced. The elastic forces in solid are characterized by phenomenological parameters such as shear modulus that can be computed from the atoms level of description.

If we use star level of description for a spiral galaxy then its arms would wind up soon as the galaxy rotates. Also, it is not possible to describe the motion of stars in a spiral galaxy via a few number of particles. Suppose we have a two body motion, one is the star and the other is the total interior mass located at the galaxy center. In this case we cannot explain circular orbits of stars in the disk sector according to this two body motion in which the orbit should be a high eccentric elliptic. In fact the approach to study complex systems is based
on considering interaction and collective effects. For instance, superconductivity cannot be explained by just thinking about atoms. It is understood by considering the cooperative behavior.

In galaxies moving stars with larger velocity try to accelerate stars with smaller ones and vice versa. Thus interactions cause irreversible transfer of momentum from points where the velocity is large to the points where it is small and try to diminish the velocity gradient. This correlation is the physical origin of constant velocity in spiral galaxies.

In addition, stars in elliptical galaxies are on orbits that are randomly oriented within the galaxy. On the other hand, because of the existence of central mutual gravitational forces between stars in a galaxy, they have a tendency to orbit in a unique plane. This kind of ordering and symmetry breaking motivates us to consider that the disk galaxies emerge from elliptical galaxies. In the other words, it seems that elliptical and disk galaxies are two states of self gravitational systems. The difference between them should be related to the correlation length between their stars. Also, self gravity between stars in the disk puts them in spiral arms when they rotate around the galaxy center and spiral structure forms.

Thus, explanation of constant velocity in spiral galaxies needs focusing on interaction between their components, stars, instead of interpretation based on reductionism approach in lower level of description which leads to exotic dark matter or modification of gravitational force.

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