VeriFx: Correct Replicated Data Types for the Masses

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Distributed systems adopt weak consistency to ensure high availability and low latency, but state convergence is hard to guarantee due to conflicts. Experts carefully design replicated data types (RDTs) that resemble sequential data types and embed conflict resolution mechanisms that ensure convergence. Designing RDTs is challenging as their correctness depends on subtleties such as the ordering of concurrent operations. Currently, researchers manually verify RDTs, either by paper proofs or using proof assistants. Unfortunately, paper proofs are subject to reasoning flaws and mechanized proofs verify a formalisation instead of a real-world implementation. Furthermore, writing mechanized proofs is reserved to verification experts and is extremely time consuming. To simplify the design, implementation, and verification of RDTs, we propose VeriFx, a high-level programming language with automated proof capabilities. VeriFx lets programmers implement RDTs atop functional collections and express correctness properties that are verified automatically. Verified RDTs can be transpiled to mainstream languages (currently Scala or JavaScript). VeriFx also provides libraries for implementing and verifying Conflict-free Replicated Data Types (CRDTs) and Operational Transformation (OT) functions. These libraries implement the general execution model of those approaches and define their correctness properties. We use the libraries to implement and verify an extensive portfolio of 35 CRDTs and reproduce a study on the correctness of OT functions.

Additional Key Words and Phrases: distributed systems, eventual consistency, replicated data types, verification

1 INTRODUCTION

Replication is essential to modern distributed systems as it enables fast access times and improves the system’s overall scalability, availability, and fault tolerance. When data is replicated across machines, replicas must be kept consistent to some extent. When facing network partitions, replicas cannot remain consistent while also accepting reads and writes, a consequence of the CAP theorem [Brewer 2012, 2000; Kleppmann 2015]. Programmers thus face a trade-off between consistency and availability. Keeping replicas strongly consistent induces high latencies, poor scalability, and reduced availability since updates must be coordinated, e.g. using a distributed consensus algorithm. By relaxing the consistency guarantees, latencies can be reduced and the overall availability improved, but users may observe temporary inconsistencies between replicas.

Distributed systems increasingly adopt weak consistency models. However, concurrent operations may lead to conflicts which must be solved in order to guarantee state convergence. Consider the case of collaborative text editors. When a user edits a document, the operation is immediately applied locally on the replica and propagated asynchronously to the other replicas. Since concurrent edits are applied in different orders at different replicas, states can diverge.

To ensure convergence, Ellis and Gibbs [1989] proposed a technique called Operational Transformation (OT) which modifies incoming operations against previously executed concurrent operations such that the modified operation preserves the intended effect. Much work focused on designing transformation functions for collaborative text editing [Ellis and Gibbs 1989; Imine et al. 2003; Ressel et al. 1996; Suleiman et al. 1997; Sun et al. 1998], but it has been shown that all of them (even some with mechanized proofs) are wrong [Imine et al. 2003; Li and Li 2004; Oster et al. 2006].

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Since conflict resolution is hard [Almeida et al. 2015; Kleppmann and Beresford 2017; Shapiro et al. 2011b], researchers now focus on designing replicated data types (RDTs) that serve as basic building blocks for the development of highly available distributed systems. Such RDTs resemble sequential data types (e.g. counters, sets, etc.) but include conflict resolution strategies that guarantee convergence in the presence of conflicts. Conflict-free Replicated Data Types (CRDTs) [Shapiro et al. 2011b] are a widely adopted family of RDTs that leverage mathematical properties (such as commutative operations) to avoid conflicts by design. Many papers [Almeida et al. 2015; Baquero et al. 2017; Bieniusa et al. 2012; Burckhardt et al. 2012; Kaki et al. 2019; Kleppmann and Beresford 2017; Shapiro 2017; Shapiro et al. 2011a,b] propose new or improved RDT designs and include a formal specification and/or pseudo code of the RDT together with a manual proof of convergence, mostly paper proofs. Unfortunately, paper proofs are subject to reasoning flaws.

To avoid the pitfalls of paper proofs, Zeller et al. [2014] and Gomes et al. [2017] propose formal frameworks to verify the correctness of CRDTs using proof assistants. However, these frameworks use abstract specifications that are disconnected from actual implementations (e.g. Akka’s CRDT implementations in Scala). Hence, a particular implementation may be flawed, even though the specification was proven to be correct. While interactive proofs are more convincing (because the proof logic is machine-checked), they require significant programmer intervention which is time consuming and reserved to verification experts [Leino and Moskal 2010; O’Hearn 2018]. Recent research efforts try to automate (part of) the verification process of CRDTs. Nagar and Jagannathan [2019] automatically verify CRDTs under different consistency models but require a first-order logic specification of the CRDTs’ operations. Liu et al. [2020] leverage an SMT solver to automate part of the verification process but significant parts still need to be proven manually. We conclude that the development of RDTs is currently reserved to experts in distributed systems and verification.

To simplify the design and implementation of correct RDTs, we propose VeriFx, a functional object-oriented programming language with extensive functional collections including tuples, sets, maps, vectors, and lists. The collections are immutable which is said to be desirable for the implementation of RDTs and their integration in distributed systems [Helland 2015]. VeriFx features a novel proof construct which enables programmers to express correctness properties that are verified automatically. For each proof, VeriFx derives proof obligations and discharges them using SMT solvers. Verified RDTs can be transpiled to one of the supported target languages (currently Scala or JavaScript). We used VeriFx to develop libraries for the implementation and verification of CRDT and OT data types. Internally, these libraries use the proof construct to define the necessary correctness properties. Programmers can also build their own libraries in VeriFx.

We designed VeriFx to be reminiscent of existing languages (like Scala) and demonstrate that it is possible to derive automated proofs from real-world RDT implementations. We argue that the ability to implement RDTs and automatically verify them within the same language allows programmers to catch mistakes early during the development process.

To demonstrate the applicability of VeriFx, we implemented and verified 35 CRDTs, including well-known CRDTs [Baquero et al. 2017; Bieniusa et al. 2012; Kleppmann 2022; Shapiro 2017; Shapiro et al. 2011a] and new variants. From these 35 CRDTs, 34 were verified in a matter of seconds and 1 could not be verified due to its recursive nature. We also applied VeriFx to OT and verified all transformation functions described by Imine et al. [2003], and some unpublished designs [Imine [n. d.]].

In summary, we make the following contributions:

- VeriFx, the first high-level programming language that enables programmers to implement RDTs by composing functional collections, express correctness properties about those RDTs within the same language, and automatically verify those properties.
• We devise VeriFx libraries that simplify the implementation of CRDT and OT data types and automatically verify the necessary correctness properties.
• We give the first fully automated and mechanized proofs for all but one CRDT proposed by Shapiro et al. [2011a], all pure op-based CRDTs [Baquero et al. 2017], and many others.
• We reproduce the study of Imine et al. [2003] regarding the verification of OT functions.

2 MOTIVATION

To motivate the need for VeriFx, consider a distributed system in Scala with replicated data on top of Akka’s highly-available distributed key-value store\(^1\). The store provides built-in CRDTs, e.g. sets, counters, etc. However, our system requires a Two-Phase Set (2PSet) CRDT [Shapiro et al. 2011a] that is not provided by Akka. We thus need to implement it and verify our implementation.

Traditionally, software verification requires a complete formalisation of the implementation and its correctness conditions which then need to be proven manually using proof assistants. The resulting interactive proofs are complex and require much expertise. For example, Gomes et al. [2017]’s formalisation and verification of a set CRDT in Isabelle/HOL required the introduction of approximately 20 auxiliary lemmas good for more than 250 LoC in total. Thus, we cannot reasonably assume that programmers have the time nor the skills to manually verify their implementation using proof assistants [Leino and Moskal 2010; O’Hearn 2018]. Alternatively, programmers could resort to Liu et al. [2020]’s extension of Liquid Haskell [Vazou et al. 2014] which automates part of the verification process. However, non-trivial RDTs still require significant manual proof efforts: 200+ LoC for a replicated set and 1000+ LoC for a replicated map [Liu et al. 2020].

In this work, we argue that verification needs to be fully automatic in order to be accessible to non-experts. Figure 1 depicts the envisioned workflow for developing RDTs. Programmers start from a new or existing RDT design and implement it in VeriFx which will then verify it automatically without the need for a separate formalisation. If the implementation is not correct, VeriFx returns a concrete counterexample in which the replicas diverge. After interpreting the counterexample, the programmer needs to correct the RDT implementation and verify it again. This iterative process repeats until the implementation is shown correct. The verified RDT implementation can then be transpiled to a mainstream language (e.g. Scala or JavaScript) where it is deployed in the system.

In the remainder of this section we cover each step of the workflow by implementing and verifying an existing 2PSet design in VeriFx, transpiling it to Scala, and deploying it on top of Akka.

2.1 Design and Implementation

Specification 1 shows the design of the 2PSet CRDT taken from Shapiro et al. [2011a]. The 2PSet is a state-based CRDT whose state (the A and R sets) thus forms a join semilattice, i.e. a partial order \(\leq_0\) with a least upper bound (LUB) \(\sqcup_0\) for all states. Elements are added to the 2PSet by adding them to the A set and removed by adding them to the R set. An element is in the 2PSet if it is in A and not in R. Hence, removed elements can never be added again. Replicas are merged by computing the LUB of their states, which in this case is the union of their respective A and R sets.

The \(\text{compare}(S, T)\) operation checks if \(S \leq_0 T\) and is used to define state equivalence: \(S \equiv T \iff S \leq_0 T \land T \leq_0 S\). Note that state equivalence is defined in terms of \(\leq_0\) on the lattice so

\(^1\)https://doc.akka.io/docs/akka/current/distributed-data.html
that replicas may be considered equivalent even though they are not identical. This is relevant for
CRDTs that keep additional information. For example, CRDTs often use a lamport clock to generate
globally unique IDs. This lamport clock is different at every replica and is not part of the lattice
even though it is part of the state.

Listing 1 shows the implementation of the 2PSet CRDT in VeriFx, which is a straightforward
translation of the specification. The TwoPSet class is polymorphic in the type of values it stores. It
defines the added and removed fields which correspond to the \( A \) and \( R \) sets respectively. The add
and remove methods return an updated copy of the state. The class extends the CvRDT trait that is
provided by VeriFx’s CRDT library for building state-based CRDTs (explained later in Section 5.1).
This trait requires the class to implement the compare and merge methods.

2.2 Verification

We now verify our 2PSet implementation in VeriFx. State-based CRDTs guarantee convergence iff the
merge function is idempotent, commutative, and associative [Shapiro et al. 2011b]. VeriFx’s CRDT
library includes several CvRDTProof traits which encode these correctness conditions (explained
later in Section 5.1). To verify our TwoPSet, we define a TwoPSetProof object that extends the
CvRDTProof1 trait and passes the type constructor of the CRDT we want to verify (i.e. TwoPSet) as
a type argument to the trait:

\[
\text{object TwoPSetProof extends CvRDTProof1[TwoPSet]} \\
\]

The TwoPSetProof object inherits an automated correctness proof for the polymorphic TwoPSet
CRDT. When executing this object, VeriFx will automatically try to verify this proof. In this case,
VeriFx proves that the TwoPSet guarantees convergence (independent of the type of elements it
holds), according to the notion of state equivalence that is derived from compare. However, VeriFx
raises a warning that this notion of equivalence does not correspond to structural equality. As
explained before, this may be normal in some CRDT designs but it requires further investigation.

VeriFx provides a counterexample consisting of two states \( S = \text{TwoPSet}((x),\{\}) \) and \( T = \text{TwoPSet}((x),\{x\}) \), which are considered equivalent \( S \equiv T \) but are not identical \( S \neq T \). These two
states should indeed not be considered equivalent since \( x \in S \) but \( x \notin T \) according to lookup.

Looking back at Spec. 1, we notice that compare defines replica \( S \) to be smaller or equal to replica
\( T \) iff \( S.A \subseteq T.A \) or \( S.R \subseteq T.R \). Since \( S.A = T.A \) it follows that \( S \leq_v T \wedge T \leq_v S \) and thus they are

\[
\text{VeriFx traits can declare abstract methods and fields, and provide default implementations for methods.}
\]
considered equal ($S \equiv T$) without even considering the removed elements (i.e. the $R$ sets). Based on this counterexample, we correct compare such that it considers both the $A$ sets and the $R$ sets:

```scala
def compare(that: TwoPSet[V]) =  
  this.added.subsetOf(that.added) && this.removed.subsetOf(that.removed)
```

We verify the implementation again to check that it still guarantees convergence according to this modified definition of equivalence. VeriFx automatically proves that the modified implementation is correct and the warning about equivalence is now gone (meaning that the definition of equality that is derived from compare corresponds to structural equality, i.e. $s_1 \equiv s_2 \iff s_1 = s_2$).

We completed the verification of the 2PSet CRDT in VeriFx without providing any verification-specific code. This example showcases the importance of automated verification as it detected an error in the specification that would have percolated to the implementation.

### 2.3 Deployment

The final step in our workflow consists of automatically transpiling the implementation from VeriFx to Scala and integrating it in our distributed system which uses Akka’s distributed key-value store.

Listing 2 shows the transpiled implementation of the 2PSet in Scala. To store the RDT in Akka’s distributed key-value store, this implementation requires two modifications which are shown in Listing 3. First, the RDT must extend Akka’s ReplicatedData trait (Line 3) which requires at least the definition of a type member $T$ corresponding to the actual type of the CRDT (Line 4) and a `merge` method for CRDTs of that type (which we already have). Second, the RDT must be serializable. For simplicity, we use Java’s built-in serializer\(^3\). Hence, it suffices to extend the Serializable trait (Line 3) and to annotate the class with a serial version (Line 1). After applying these modifications, our verified TwoPSet can be stored in Akka’s distributed key-value store and will automatically be replicated across the cluster and be kept eventually consistent.

### 3 THE VERIFX LANGUAGE

The goal of this work is to build a familiar high-level programming language that is suited to implement RDTs and automatically verify them. The main challenge consists of efficiently encoding every feature of the language without breaking automatic verification. The result of this exercise is VeriFx, a functional object-oriented programming language with Scala-like syntax and a type system that resembles Featherweight Generic Java [Igarashi et al. 2001]. VeriFx features a novel proof construct to express correctness properties about programs. For every proof construct a proof obligation is derived that is discharged automatically by an SMT solver (cf. Section 4).

\(^3\)In production it would be safer and more efficient to implement a custom serializer, e.g. using Protobuf.
VeriFx advocates for the object-oriented programming paradigm as it is widespread across programmers and fits the conceptual representation of replicated data as “shared” objects. The functional aspect of the language, in particular its immutable collections, make the language suitable for implementing and integrating RDTs in distributed systems, as argued by Helland [2015].

The remainder of this section is organised in three parts. First, we give an overview of VeriFx’s architecture. Second, we define its syntax. Third, we describe its functional collections. VeriFx’s type system is described in Appendix A as part of the additional material.

3.1 Overall Architecture

Figure 2 provides an overview of VeriFx’s architecture. Source code is parsed into an Abstract Syntax Tree (AST) representing the program. Interestingly, every piece of VeriFx code is valid Scala syntax (but not necessarily semantically correct). This enables VeriFx to use Scala Meta\(^4\) to parse the source code into an AST representing the Scala program, which is then transformed to represent the VeriFx program.

The AST representing a VeriFx program can be verified or transpiled to other languages. Transpilation is done by the compiler which features plugins. Support for new languages can be added by implementing a compiler plugin for them. These plugins dictate the compilation of the AST to the target language. Currently, VeriFx comes with compiler plugins for Scala, JavaScript, and Z3 [de Moura and Bjørner 2008] (a state-of-the-art SMT solver).

To verify VeriFx programs, the verifier derives the necessary proof obligations from the AST. It then compiles the program to Z3 and automatically discharges the proof obligations. For every proof, the outcome (accepted, rejected, or unknown) is signaled to the user. Accepted means that the property holds, rejected means that a counterexample was found for which the property does not hold, and unknown means that the property could not be verified within a certain time frame (which is configurable). Support for other SMT solvers can also be added by implementing a compiler plugin for them.

3.2 Syntax

Figure 3 defines the syntax of VeriFx. The metavariable \(C\) ranges over class names; \(O\) ranges over object names; \(I\) ranges over trait names; \(E\) ranges over enumeration names; \(K\) ranges over constructor names of enumerations; \(T\), \(P\) and \(Q\) range over types; \(X\) and \(Y\) range over type variables.

\(^4\)https://scalameta.org/
VeriFx features built-in collections for tuples, sets, maps, vectors, and lists. Remarkably, these collections are completely verifiable and can be arbitrarily composed to build custom RDTs. All collections are immutable, "mutators" thus return an updated copy of the object. Figure 4 provides

3.3 Functional Collections
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\[ L ::= \text{class } C \langle \overline{X} : \overline{T} \rangle \{ \overline{M} \} \]
\[ J ::= \text{object } O \{ \overline{A} \} \]
\[ F ::= \text{trait } I \langle \overline{X} : \overline{T} \rangle \{ \overline{B} \} \]
\[ N ::= \text{enum } E \langle \overline{X} : \overline{T} \rangle \{ \overline{K} \} \]
\[ A ::= M | R \]
\[ B ::= \text{valDecl} | \text{methodDecl} | M | R \]
\[ M ::= \text{def } m \langle \overline{X} \rangle \langle \overline{T} \rangle : T = e \]
\[ T ::= \text{int} | \text{string} | \text{bool} | C \langle \overline{T} \rangle \]
\[ E \langle \overline{T} \rangle | \overline{T} \rightarrow T \]
\[ e ::= \text{num} | \text{str} | \text{true} | \text{false} \]

Fig. 3. VeriFx syntax.

\[ v \text{ ranges over field names; } x \text{ and } y \text{ range over parameter and variable names; } m \text{ ranges over method names; } p \text{ ranges over proof names; and } e \text{ ranges over expressions.} \]

VeriFx programs consist of one or more statements which can be the definition of an object \( O \), a class \( C \langle \overline{X} \rangle \), a trait \( I \langle \overline{X} \rangle \), or an enumeration \( E \langle \overline{X} \rangle \). Objects, classes, enumerations, and traits can be polymorphic and inherit from a single trait (except enumerations). Objects define zero or more methods and proofs. Classes contain zero or more\(^5\) fields and (polymorphic) methods. The body of a method must contain a well-typed expression \( e \). Traits can declare values and methods that need to be provided by concrete classes extending the trait, and define (polymorphic) methods and proofs. Traits can express upper type bounds on their type parameters to restrict the possible extensions. Enumerations (enums for short) define one or more constructors, each of which contains zero or more fields. Programmers can deconstruct enums by pattern matching on them.

Unique to VeriFx is its proof construct which has a name and whose body must be a well-typed boolean expression. The body expresses a property that must be verified. A proof is accepted if its body always evaluates to true, otherwise it is rejected; when rejected, VeriFx provides a concrete counterexample for which the property does not hold. Proofs can be polymorphic, that means they prove a property for all possible type instantiations of their type parameters. Polymorphic proofs are useful to prove that a polymorphic RDT converges independent of the type of values it contains.

VeriFx supports a variety of expressions, including literal values, arithmetic \( \oplus \) and boolean operations \( \otimes \), boolean negation \( !e \), field accesses \( e.v \) and method calls \( e.m \langle \overline{T} \rangle \langle \overline{X} \rangle \), variable definitions, if tests, anonymous functions and function calls, class and enum instantiations, pattern matching, quantified formulas, and logical implication. Functions are first-class and take at least one argument because nullary functions are constants.

VeriFx supports single inheritance from traits to foster code re-use but imposes some limitations. For example, the arguments of a class method need to be concrete (i.e. can not be of a trait type) because proofs about these methods require reasoning about all subtypes but these may not necessarily be known at compile time. In contrast, enumerations are supported because their constructors are fixed and known at compile time.

\(^5\)An overline, e.g. \( \overline{X} \), denotes zero or more. A dashed overline, e.g. \( \overline{X} \), denotes one or more.
| Tuple: A, B |
|------------------|
| + fst : A        |
| + snd : B        |

| Set: V |
|------------------|
| + add(e: V) : Set<V> |
| + remove(e: V) : Set<V> |
| + contains(e: V) : bool |
| + isEmpty() : bool |
| + nonEmpty() : bool |
| + union(s: Set<V>) : Set<V> |
| + diff(s: Set<V>) : Set<V> |
| + intersect(s: Set<V>) : Set<V> |
| + subsetOf(that: Set[V]) : bool |
| + map<W>(f: V => W) : Set<W> |
| + filter(p: V => bool) : Set<V> |
| + forall(p: V => bool) : bool |
| + exists(p: V => bool) : bool |

| Map: K, V |
|------------------|
| + add(k: K, v: V) : Map<K, V> |
| + remove(k: K) : Map<K, V> |
| + contains(k: K) : bool |
| + get(k: K) : V |
| + getOrElse(k: K, default: V) : V |
| + keys() : Set<K> |
| + values() : Set<V> |
| + bijective() : bool |
| + map<W>(f: (K, V) => W) : Map<K, W> |
| + mapValues<W>(f: V => W) : Map<K, W> |
| + filter(p: (K, V) => bool) : Map<K, V> |
| + zip<W>(m: Map<K, W>) : Map<K, Tuple<V, W>> |
| + combine(m: Map<K, V>, f: (V, V) => V) : Map<K, V> |
| + forall(p: (K, V) => bool) : bool |
| + exists(p: (K, V) => bool) : bool |
| + toSet() : Set<Tuple<K, V>> |

| Vector: V |
|------------------|
| + size : Int |
| + get(idx: Int) : V |
| + write(idx: Int, value: V) : Vector<V> |
| + append(value: V) : Vector<V> |
| + zip<W>(f: V => W) : Vector<W> |
| + forAll(p: V => bool) : bool |
| + exists(p: V => bool) : bool |

| List: V |
|------------------|
| + size : Int |
| + get(idx: Int) : V |
| + insert(idx: Int, value: V) : List<V> |
| + delete(idx: Int) : List<V> |
| + map<W>(f: V => W) : List<W> |
| + zip<W>(l: List<W>) : List<Tuple<V, W>> |
| + forAll(p: V => bool) : bool |
| + exists(p: V => bool) : bool |

Fig. 4. An overview of VeriFx’s built-in functional collections.

an overview of the interface exposed by these collections, which is heavily inspired by functional programming. A tuple groups two elements which can be accessed using the `fst` and `snd` fields.

Sets. Support the typical set operations and can be mapped over or filtered using user-provided functions. The `forall` and `exists` methods check if a given predicate holds for all (respectively for at least one) element of the set.

Maps. Associate keys to values. Programmers can add key-value pairs, remove keys, and fetch the value that is associated to a certain key. The `contains` method returns a set containing all keys (resp. values) contained by the map. The `bijective` method checks if there is a one-to-one correspondence between the keys and the values. Maps support many well-known functional operations; `zip` returns a map of tuples containing only the keys that are present in both maps and stores their values in a tuple; `combine` returns a map containing all entries from both maps, using a user-provided function `f` to combine values that are present in both maps.

Vectors. Represent a sequence of elements which are indexed from 0 to `size-1`. Elements can be written to a certain index which will overwrite the existing value at that index. One can append a value to the vector which will write that value at index `size`, thereby, making the vector grow.
Like sets and maps, programmers can map functions over vectors, zip vectors, and check predicates for all or for one element of a vector.

Lists. Represent a sequence of elements in a linked list. Unlike vectors, \texttt{insert} does not overwrite the existing value at that index. Instead, the existing value at that index and all subsequent values are moved one position to the right. Elements can also be deleted from a list, making the list shrink.

4 AUTOMATED VERIFICATION

VeriFx leverages SMT solvers to enable automated verification. Such solvers try to (automatically) determine whether or not a given formula is satisfiable. Modern SMT solvers support various specialized theories (for bitvectors, arrays, etc.) and are very powerful if care is taken to encode programs efficiently using these theories. However, SMT-LIB, a standardized language for SMT solvers\footnote{http://smtlib.cs.uiowa.edu/}, is low-level and is not meant to be used directly by programmers to verify high-level programs. Instead, semi-automatic program verification usually involves implementing the program in an Intermediate Verification Language (IVL) which internally compiles to SMT-LIB to discharge the proof obligations using an appropriate SMT solver. IVLs like Dafny [Leino 2010], Spec# [Barnett et al. 2005], and Why3 [Filiâtre and Paskevich 2013] are designed to be general-purpose but this breaks automated verification since programmers need to specify preconditions and postconditions on methods, loop invariants, etc.

VeriFx can be seen as a specialized high-level IVL that was carefully designed such that every feature has an efficient SMT encoding; leaving out features that break automated verification. For example, VeriFx does not support traditional loop statements but instead provides higher order operations (map, filter, etc.) on top of its functional collections. The resulting language is surprisingly expressive given its automated verification capabilities.

In the remainder of this section we show how VeriFx compiles programs to SMT and derives proof obligations that can be discharged automatically by SMT solvers. Afterwards, we explain how VeriFx leverages a specialised theory of arrays to efficiently encode its functional collections. Due to space constraints, Appendix C.4 exemplifies these compilation rules using a concrete example.

4.1 Core SMT

The semantics of VeriFx are defined using translation functions from VeriFx to Core SMT, a reduced version of SMT that suffices to verify VeriFx programs. Figure 5 defines the syntax of Core SMT. The metavariable $S$ ranges over user-declared sorts\footnote{The literature on SMT solvers uses the term "sort" to refer to types and type constructors.}; $A$ ranges over names of algebraic data types (ADTs); $K$ ranges over ADT constructor names; $X$ ranges over type variables; $v$ ranges over field names; $f$ ranges over function names; $T$ ranges over types; $x$ ranges over variable names; $e$ ranges over expressions; and $i$ ranges over integers. Valid types include integers, strings, booleans, arrays, ADTs $A(T)$, and user-declared sorts $S(T)$. Arrays are total and map values of the key types to a value of the element type. Arrays can be multidimensional and map several keys to a value.

Core SMT programs consist of one or more statements which can be the declaration of a constant or sort, assertions, the definition of a function or ADT, or a call to check. Constant declarations take a name and a type. Sort declarations take a name and a non-negative number $i$ representing their arity, i.e. how many type parameters the sort takes. Declared constants and sorts are uninterpreted and the SMT solver is free to assign any valid interpretation. Assertions are boolean formulas that constrain the possible interpretations of the program, e.g. \texttt{assert age >= 18}.

Function definitions consist of a name $f$, optional type parameters $X$, formal parameters $\overline{x} : \overline{T}$, a return type $T$, and a body containing an expression $e$. Valid expressions include array accesses...
e[τ], array updates e[τ] := e, anonymous functions, quantified formulas, etc. Updating an array returns a modified copy of the array. It is important to note that arrays are total and that anonymous functions define an array from the argument types to the return type. For example, \( \lambda(x : \text{int}, y : \text{int}).x + y \) defines an Array(int, int, int) that maps two integers to their sum. Since arrays are first-class values in SMT, it follows that lambdas are also first-class.

ADT definitions consist of a name \( A \), optional type parameters \( \overline{X} \), and one or more constructors. Every constructor has a name \( T \) and optionally defines fields with a name \( v \) and a type \( T \). Constructors are invoked like regular functions and return an instance of the data type.

The decision procedure (check) checks the satisfiability of the SMT program. If the program’s assertions are satisfiable, check returns a concrete model, i.e. an interpretation of the constants and sorts that satisfies the assertions. A property \( \varphi \) can be proven by showing that the negation \( \neg \varphi \) is unsatisfiable, i.e. that no counterexample exists.

Note that our Core SMT language includes lambdas and polymorphic functions which are not part of SMT-LIB v2.6. Nevertheless, they are described in the preliminary proposal for SMT-LIB v3.0 and Z3 already supports lambdas. For the time being, VeriFx monomorphizes polymorphic functions when they are compiled to Core SMT. For example, given a polymorphic identity function \( \text{id} : \overline{X} \rightarrow \overline{X} \), VeriFx creates a monomorphic version \( \text{id}_\text{int} : \text{int} \rightarrow \text{int} \) when encountering a call to \( \text{id} \) with an integer argument.

### 4.2 Compiling VeriFx to SMT

Similarly to Dafny [Leino 2010], we describe the semantics of VeriFx by means of translation functions that compile VeriFx programs to Core SMT. Types are translated by the \([\_]_T\) function:

\[
\begin{align*}
[\text{bool}]_T &= \text{bool} & [\text{int}]_T &= \text{int} & [\text{string}]_T &= \text{string} \\
[C(T)]_T &= C([T]_T) & [E(T)]_T &= E([T]_T) & [T \rightarrow P]_T &= \text{Array}([T]_T, [P]_T)
\end{align*}
\]

Primitives are translated to the corresponding primitive type in Core SMT. Class types and enumeration types keep the same type name and their type arguments are translated recursively \([T]_T\). Functions are encoded as arrays from the argument types to the return type. Trait types do not exist in the compiled SMT program because traits are compiled away by VeriFx, i.e. only the types of the classes that implement the trait exist in the SMT program.

We now take a look at the translation function \( \text{def}[\_] \) which compiles VeriFx’s main constructs: enumerations, classes, and objects. Enumerations are encoded as ADTs:

\[
\text{def}[\text{enum } E(\overline{X})\{K(\overline{v} : \overline{T})\}] = \text{adt} E(\overline{X})\{K(\overline{v} : [\overline{T}]_T)\}
\]

For every enumeration an ADT is constructed with the same name, type parameters, and constructors. The types of the fields are translated recursively.

---

8The complete list of expressions is described in Appendix B as part of the additional material.

9http://smtlib.cs.uiowa.edu/version3.shtml
Classes are encoded as ADTs with one constructor and class methods become functions:

\[
\text{def} \left[ \text{class } C \langle \bar{X} \rangle \ (\bar{V} : \bar{T}) \ \{ \bar{M} \} \text{ extends } I \langle \bar{P} \rangle \right] \\
\text{adt} C \langle \bar{X} \rangle \ {\{ K : \bar{T} \} : \ \text{method}}[C, \bar{X}, \bar{M} : \ \text{method}}[C, \bar{X}, \bar{M}'[\bar{P}/\bar{Y}]]
\]

where \( K = \text{str}_\text{concat}(C,"\_ctor") \) and \( I \) is defined as \( \text{trait } I \langle \bar{Y} \rangle \ \{ I' ; \ldots \} \)

\[
\text{method}[C, \bar{X}, \text{def } m \langle \bar{Y} \rangle \ (\bar{x} : \bar{T}) : T_r = e] = \text{fun } f(\bar{X}, \bar{Y})(\text{this } : C \langle \bar{X} \rangle, \bar{x} : \bar{T})_l : T_r)_l = [e]
\]

where \( f = \text{str}_\text{concat}(C,"\_",m) \)

The ADT keeps the name of the class and its type parameters, and defines one constructor containing the class’ fields. Since the name of the constructor must differ from the ADT’s name, the compiler defines a unique name \( K \) which is the name of the class followed by “\_ctor”. The class methods \( \bar{M} \) are compiled to regular functions by the \text{method}[] function. Furthermore, the class inherits all concrete methods \( \bar{M}' \) that are defined by its super trait and are not overridden by itself. This requires substituting the trait’s type parameters \( \bar{Y} \) by the concrete type arguments \( \bar{P} \) provided by the class. As such, traits are compiled away and do not exist in the transpiled SMT program.

For every method, a function is created with a unique name \( f \) that is the name of the class followed by an underscore and the name of the method. In the argument list, the body, and the return type of a method, programmers can refer to type parameters of the class and type parameters of the method. Therefore, the compiled SMT function takes both the class’ type parameters \( \bar{X} \) and the method’s type parameters \( \bar{Y} \). Without loss of generality we assume that a method’s type parameters do not override the class’ type parameters which can be achieved through \( \alpha \)-conversion. The method’s parameters become parameters of the function. In addition, the function takes an additional parameter \text{this} referring to the receiver of the method call which should be of the class’ type. The types of the parameters and the return type are translated using \text{fun}[]. The body of the method must be a well-typed expression. Expressions are translated by the \text{fun}[] function:

\[
\begin{align*}
[x] & = x \\
\text{val } x : T = e_1 \text{ in } e_2 & = \text{let } x = [e_1] \text{ in } [e_2] \\
(\langle \bar{x} : \bar{T} \rangle \Rightarrow e) & = \lambda(\bar{x} : \bar{T})_l.[e] \quad \text{where typeof}(e_1) = C(\bar{P}) \\
[e_1(\bar{x}_2)] & = [e_1][\bar{x}_2] \\
\text{new } C(\bar{T})(\bar{e}) & = C'(\bar{T})_l([\bar{e}]) \\
\text{where } C' = \text{str}_\text{concat}(C,"\_\_\_ctor") \\
\end{align*}
\]

Primitive values, variable references, and parameter references remain unchanged in Core SMT. The definition of an immutable variable is translated to a let expression. Anonymous functions remain anonymous functions in Core SMT, the type of the parameters and the body are compiled recursively. Remember that anonymous functions in SMT define (multidimensional) arrays from one or more arguments to the function’s return value. Hence, function calls are translated to array accesses. To instantiate a class or ADT, the compiler calls the data type’s constructor function. For classes, the constructor’s name is the name of the class followed by “\_ctor”. To access a field, the compiler translates the expression and accesses the field on the translated expression. To invoke a method \text{m} on an object \text{e}_1 the compiler calls the corresponding function \text{m'} which by convention is the name of the class followed by an underscore and the name of the method. Recall that the function takes both the class’ type arguments \( \bar{T} \) and the method’s type arguments \( \bar{P} \) as well as an additional argument \text{e}_1 which is the receiver of the call. The complete set of compilation rules for expressions is provided in Appendix C.1 as part of the additional material.

Finally, objects are singletons that can define methods and proofs, and are compiled as follows:

\[
\begin{align*}
\text{def}[\text{object } O \text{ extends } I(\bar{T}) \ \{ \bar{M} ; \bar{R} \}] = \\
\text{def}[\text{class } O'() \ \{ \bar{M} \} \text{ extends } I(\bar{T})] : \text{const } O'O' ; \text{assert } O == O'() ; \text{def } [\bar{R}]
\end{align*}
\]
The object is compiled to a regular class with a fresh name $O'$. Then, a single instance of that class is created and assigned to a constant named after the object $O$. The proofs defined by the object are compiled to functions. How to translate proofs into functions is the subject of the next section.

### 4.3 Deriving Proof Obligations

We previously verified a 2PSet CRDT using VeriFx’s CRDT library which internally uses our novel proof construct to define the necessary correctness properties (discussed later in Section 5). However, programmers can also define custom proofs, for instance to verify data invariants.

We now explain how proof obligations are derived from user-defined proofs in VeriFx programs. Proofs are compiled to regular functions without arguments. The name and type parameters remain unchanged and the body of the proof is compiled and becomes the function’s body. Proofs always return a boolean since the body is a logical formula whose satisfiability must be checked.

\[
\text{def}\quad \text{proof } p (\langle X \rangle \{ e \}) = \text{fun } (\langle X \rangle () : \text{bool } = [e]
\]

To check if the property described by a proof holds, the negation of the proof must be unsatisfiable. In other words, if no counterexample exists it constitutes a proof that the property is correct. A (polymorphic) proof called $p$ with zero or more type parameters $i$ is checked as follows:

\[
\text{prove}(p, i) = \text{sort } S_1 \ 0 ; \ldots ; \text{sort } S_i \ 0 ; \text{assert } \neg p(S_1, \ldots, S_i)(); \text{check}() \Rightarrow \text{UNSAT}
\]

For every type parameter an uninterpreted sort is declared. Then, the proof function is called with those sorts as type arguments and we check that the negation is unsatisfiable. If the negation is unsatisfiable, the (polymorphic) proof holds for all possible instantiations of its type parameters. The underlying SMT solver can generate an actual proof which could be reconstructed by proof assistants as shown by Böhme et al. [2011]; Böhme and Weber [2010].

### 4.4 Encoding Functional Collections Efficiently in SMT

Some IVLs feature collections with rich APIs (e.g. Why3 [Filliâtre and Paskevich 2013]) but encode operations on these collections recursively. Traditional SMT solvers fail to verify recursive definitions automatically because they require inductive proofs, which is beyond the capabilities of most solvers. However, many SMT solvers support specialised array theories. A key insight of this paper consists of efficiently encoding the collections and their operations using the Combinatory Array Logic (CAL) [de Moura and Bjørner 2009] which is decidable. As a result, VeriFx can automatically verify RDTs that are built by arbitrary compositions of functional collections. In the remainder of this section we describe the encoding of the different functional collections using this array logic.

#### 4.4.1 Set Encoding

Sets are encoded as arrays from the element type to a boolean type that indicates whether the element is in the set:

\[
\text{Set}(T) = \text{Array}(\text{Set}(T), \text{bool})
\]

An empty set corresponds to an array containing false for every element. We can create such an array by defining a lambda that ignores its argument and always returns false:

\[
[\text{new Set}(T)()] = \lambda(x : \text{Set}(T)).\text{false}
\]

Operations on sets are compiled as follows:

\[
\begin{align*}
[e_1, \text{add}(e_2)] = [e_1][e_2] := \text{true} & \quad [e_1, \text{remove}(e_2)] = [e_1][e_2] := \text{false} & \quad [e_1, \text{contains}(e_2)] = [e_1][e_2] \\
[e_1, \text{filter}(e_2)] = \lambda(x : \text{Set}(T)).[e_1][x] \land [e_2][x] & \quad \text{where } \text{typeof}(e_1) = \text{Set}(T) \land \text{typeof}(e_2) = T \rightarrow \text{bool} \\
[e_1, \text{map}(e_2)] = \lambda(y : \text{Set}(P)).\exists(x : \text{Set}(T)).[e_1][x] \land [e_2][x] = y & \quad \text{where } \text{typeof}(e_1) = \text{Set}(T) \land \text{typeof}(e_2) = T \rightarrow P
\end{align*}
\]
An element \( e_2 \) is added to a set \( e_1 \) by setting the entry for \( e_2 \) in the array that results from transforming \( e_1 \) to true. Similarly, an element is removed by changing its entry in the array to false. An element is in the set if its entry is true. A set \( e_1 \) containing elements of type \( T \) can be filtered such that only the elements that fulfill a given predicate \( e_2 : T \rightarrow \text{bool} \) are retained. Calls to \( \text{filter} \) are compiled to a lambda that defines a set containing elements \( e \) whose keys are compiled to a lambda that defines a set containing elements \( x \) that are in the original set \( e_1 \) (i.e. \( [e_1][x] \)) and fulfill predicate \( e_2 \) (i.e. \( [e_2][x] \)). Similarly, a function \( e_2 : T \rightarrow P \) can be mapped over a set \( e_1 \) of \( Ts \), yielding a set of \( Ps \). Calls to \( \text{map} \) are compiled to a lambda that defines a set containing elements \( y \) of type \( [P]_T \), such that an element \( x \) exists that is in the original set \( e_1 \) (i.e. \( [e_1][x] \)) and maps to \( y \) (i.e. \( [e_2][x] = y \)). The remaining methods are similar and are described in Appendix C.2 as part of the additional material.

### 4.4.2 Map Encoding

Maps are encoded as arrays from the key type to an optional value:

\[
[\text{Map} \langle T, P \rangle]_T = \text{Array}([T], \text{Option}([P]_T))
\]

Optional values indicate the presence or absence of a value for a certain key. The option type is defined as an ADT with two constructors: \( \text{Some}(\text{value}) \) which holds a value and \( \text{None}() \) indicating the absence of a value. An empty map corresponds to an array containing \( \text{None}() \) for every key and is created by a lambda that returns \( \text{None}() \) for every key:

\[
[\text{new Map} \langle T, P \rangle()] = \lambda (x : [T]_T). \text{None}([P]_T)(x)
\]

Operations on maps are compiled as follows:

\[
\begin{align*}
\text{map}[e_m.add(e_k, e_o)] &= [e_m][e_k] := \text{Some}([e_o]) \\
\text{map}[e_m.remove(e_k)] &= [e_m][e_k] := \text{None}([V]_T) \\
\text{map}[e_m.contains(e_k)] &= [e_m][e_k] \neq \text{None}([V]_T)(e_k) \\
\text{map}[e_m.get(e_k)] &= [e_m][e_k] = \text{Some}([V]_T)(e_k). \text{value} \\
\text{map}[e_m.getOrElse(e_k, e_o)] &= \text{if}(e_m)[e_k] = \text{None}([V]_T)(e_k), [e_o], [e_m][e_k].\text{value})
\end{align*}
\]

A key-value pair \( e_k \mapsto e_o \) is added to a map \( e_m \) by updating the entry for the compiled key \( [e_k] \) in the compiled array \( [e_m] \) with the compiled value, \( \text{Some}([e_o]) \). A key \( e_k \) is removed from a map \( e_m \) by updating the corresponding entry to \( \text{None}([V]_T)(e_k) \), thereby indicating the absence of a value. Note that \( \text{None} \) is polymorphic but the type parameter cannot be inferred from the arguments; it is thus passed explicitly. A key \( e_k \) is present in a map \( e_m \) if the value that is associated to the key is not \( \text{None}([V]_T)(e_k) \). The get method fetches the value that is associated to a key \( e_k \) in a map \( e_m \). To this end, the compiled key \( [e_k] \) is accessed in the compiled map \( [e_m] \) and the value it holds is then fetched by accessing the \( \text{value} \) field of the Some constructor. Even though the entry that is read from the array is an option type (i.e. a \( \text{None} \) or a \( \text{Some} \)) we can access the \( \text{value} \) field because the interpretation of \( \text{value} \) is underspecified in SMT. If the entry is a \( \text{None} \), the SMT solver can assign any interpretation to the \( \text{value} \) field. Hence, the get method on maps should only be called if the key is known to be present in the map, e.g. after calling \( \text{contains} \). VeriFx also features a safe variant, called \( \text{getOrDefault} \), which returns a default value if the key is not present.

We now show how a selection of advanced map operations are compiled:

\[
\begin{align*}
\text{map}[e_m.keys()] &= \lambda (x : [K]_T), [e_m][x] \neq \text{None}([V]_T)(x) \quad \text{where typeof}(e_m) = \text{Map}(K, V) \\
\text{map}[e_m.map(e_f)] &= \lambda (x : [K]_T), \text{if}(e_m)[x] \neq \text{None}([V]_T)(x), \\
& \quad \text{Some}([e_f][x, [e_m][x].\text{value}]), \text{None}([W]_T)() \\
\text{where typeof}(e_m) = \text{Map}(K, V) \text{ and typeof}(e_f) = (K, V) \rightarrow W
\end{align*}
\]

The keys method returns a set containing only the keys that are present in the map. Calls to \( \text{keys} \) on a map \( e_m \) of type \( \text{Map}(K, V) \) are compiled to a lambda which defines a set of keys \( x \) of the compiled key type \( [K]_T \) such that a key is present in the set if it is present in the compiled map, i.e. \( [e_m][x] \neq \text{None}([V]_T)(x) \). Mapping a function \( e_f \) over the key-value pairs of a map \( e_m \) is encoded as a lambda that defines an array containing only the keys that are present in the compiled
map \([e_m]\) and whose values are the result of applying \(e_f\) on the key and its associated value, i.e. \(\text{Some}([e_f][x, [e_m][x].value])\). The remaining operations (cf. Fig. 4) are encoded similarly and are described in Appendix C.3 as part of the additional material.

4.4.3 Vectors and Lists. The encoding of sets and maps is very useful to build new data structures in VeriFx without having to encode them manually in SMT. For example, vectors and lists are implemented on top of maps. Internally, they map indices between 0 and \(\text{size} - 1\) to their value, and provide a traditional interface on top (cf. Fig. 4). Note that this encoding of vectors and lists on top of maps is only used when verifying proofs in SMT; when compiling to a target language (e.g. Scala or JavaScript), VeriFx leverages the language’s built-in vector and list data structures.

5 LIBRARIES FOR IMPLEMENTING AND VERIFYING REPLICATED DATA TYPES
To simplify the development of distributed systems that use replicated data, we build two libraries for implementing and automatically verifying RDTs that use the CRDT or OT approach. We first discuss the implementation of a general execution model for CRDTs and its verification library in VeriFx. Afterwards, we present a library for implementing RDTs using OT and verifying the transformation functions. VeriFx is not limited to these two families of RDTs; programmers can build custom libraries for implementing and verifying other abstractions or families of RDTs. This section describes the core of the libraries. Their implementation will be in the artifact.

5.1 CRDT Library
CRDTs guarantee strong eventual consistency (SEC), a consistency model that strengthens eventual consistency with the strong convergence property which requires replicas that received the same updates, possibly in a different order, to be in the same state. VeriFx’s CRDT library supports several families of CRDTs, including state-based [Shapiro et al. 2011b], op-based [Shapiro et al. 2011b], and pure op-based CRDTs [Baquero et al. 2017].

5.1.1 State-based CRDTs. State-based CRDTs (CvRDTs for short) periodically broadcast their state to all replicas and merge incoming states by computing the least upper bound (LUB) of the incoming state and their own state. Shapiro et al. [2011b] showed that CvRDTs converge if the merge function \(\sqcup\) is idempotent, commutative, and associative. We define these properties as follows:

- **Idempotent**: \(\forall x \in \Sigma : \text{reachable}(x) \implies x \equiv x \sqcup_0 x\)
- **Commutative**: \(\forall x, y \in \Sigma : \text{reachable}(x) \land \text{reachable}(y) \land \text{compatible}(x, y)\)
  \(\implies (x \sqcup_0 y \equiv y \sqcup_0 x) \land \text{reachable}(x \sqcup_0 y)\)
- **Associative**: \(\forall x, y, z \in \Sigma : \text{reachable}(x) \land \text{reachable}(y) \land \text{reachable}(z) \land \text{compatible}(x, y) \land \text{compatible}(x, z) \land \text{compatible}(y, z)\)
  \(\implies ((x \sqcup_0 y) \sqcup_0 z \equiv x \sqcup_0 (y \sqcup_0 z)) \land \text{reachable}((x \sqcup_0 y) \sqcup_0 z)\)

\(\Sigma\) denotes the set of all states. A state is reachable if it can be reached starting from the initial state and applying only supported operations. Two states are compatible if they represent different replicas of the same CRDT object\(^{10}\). As explained in Section 2.1, state equivalence is defined in terms of \(\leq_0\) on the lattice: \(S \equiv T \iff S \leq_0 T \land T \leq_0 S\).

VeriFx’s CRDT library provides traits for the implementation and verification of CvRDTs. Listing 4 shows the CvRDT trait that was used in Listing 1 to implement the TwoPSet CRDT. Every state-based CRDT that extends the CvRDT trait must provide a type argument which is the actual type of the CRDT and provide an implementation for the merge and compare methods. By default, all states are considered reachable and compatible, and state equivalence is defined in terms of compare. These methods can be overridden by the concrete CRDT that implements the trait.

\(^{10}\)This definition of compatibility allows replicas to keep unique information, e.g. to generate unique tags.
We use the notation $o \cdot s$ to denote the application of an operation $o$ on state $s$ if its downstream precondition holds, otherwise it returns the state unchanged.

---

11While some CmRDT designs do not require causal delivery, the overall model assumes reliable causal broadcast.
 trait CmRDT[Op, Msg, T <: CmRDT[Op, Msg, T]] {
  def prepare(op: Op): Msg
  def effect(msg: Msg): T
  def tryEffect(msg: Msg): T = if (this.enabledDown(msg)) this.effect(msg) else this.asInstanceOf[T]
  def reachable(): Boolean = true // by default all states are considered reachable
  def canConcur(x: Msg, y: Msg): Boolean = true // all ops can occur concurrently
  def compatible(that: T): Boolean = true // all states are compatible
  def enabledSrc(op: Op): Boolean = true // no source preconditions by default
  def enabledDown(msg: Msg): Boolean = true // no downstream preconditions by default
  def equals(that: T): Boolean = this == that
}

Listing 6. Polymorphic CmRDT trait to implement op-based CRDTs in VeriFx.

Listing 6 shows the CmRDT trait that must be extended by op-based CRDTs with concrete type arguments for the supported operations, exchanged messages, and the CRDT type itself. Every CRDT that extends the CmRDT trait must implement the prepare and effect methods. The tryEffect method has a default implementation that applies the operation if its downstream precondition holds, otherwise, it returns the state unchanged. By default, we assume that all states are reachable, that all operations are enabled at the source and downstream, that all operations can occur concurrently, and that all states are compatible. For most CmRDTs these settings do not need to be altered but some CmRDTs make other assumptions which can be encoded by overriding the appropriate method. For example, in an Observed-Removed Set [Shapiro et al. 2011a]) it is not possible to delete tags that are added concurrently; this can be encoded by overriding canConcur.

Similarly to state-based CRDTs, our CRDT library provides a CmRDTProof trait and several numbered versions to verify op-based CRDTs. These traits define a general proof of correctness that checks that all operations commute based on the previously described formula.

5.1.3 Pure op-based CRDTs. Pure op-based CRDTs are a family of op-based CRDTs that exchange only the operations instead of data-type specific messages. The effect phase stores incoming operations in a partially ordered log of (concurrent) operations. Queries are computed against the log and operations do not need to commute. Data-type specific redundancy relations dictate which operations to store in the log and when to remove operations from the log. VeriFx’s CRDT library provides a PureOpBasedCRDT trait for implementing pure op-based CRDTs. The implementing CRDT inherits the prepare and effect phase (which is the same for all pure op-based CRDTs) and only needs to provide an implementation of the redundancy relations. In addition, the library provides a PureCRDTProof trait (and numbered versions for polymorphic CRDTs) which checks that for any state \( s \) and any two concurrent operations \( x \) and \( y \), their effect is the same independent of the order in which they are received. This correctness condition is a simplification of the one for op-based CRDTs as pure op-based CRDTs do not define source or downstream preconditions.

5.2 OT Library
The Operational Transformation (OT) [Ellis and Gibbs 1989] approach applies operations locally and propagates them asynchronously to the other replicas. Incoming operations are transformed against previously executed concurrent operations such that the modified operation preserves the intended effect. Operations are functions from state to state: \( Op : \Sigma \rightarrow \Sigma \) and are transformed using a transformation function \( T : Op \times Op \rightarrow Op \). Thus, \( T(o_1, o_2) \) denotes the operation that results from transforming \( o_1 \) against a previously executed concurrent operation \( o_2 \). Suleiman et al. [1998] and Sun et al. [1998] proved that replicas eventually converge if the transformation function satisfies two properties: \( TP_1 \) and \( TP_2 \). Property \( TP_1 \) states that any two enabled concurrent operations \( o_i \) and \( o_j \) must commute after transforming them:

\[
\forall o_i, o_j \in Op, \forall s \in \Sigma : enabled(o_i, s) \land enabled(o_j, s) \land canConcur(o_i, o_j) \implies T(o_j)(o_i(s)) = T(o_i)(o_j(s))
\]
VeriFx: Correct Replicated Data Types for the Masses

Property \( TP_2 \) states that given three enabled concurrent operations \( o_i, o_j, \) and \( o_k \), the transformation of \( o_k \) does not depend on the order in which operations \( o_i \) and \( o_j \) are transformed:

\[
\forall o_i, o_j, o_k \in Op, \forall s \in \Sigma : enabled(o_i, s) \land enabled(o_j, s) \land enabled(o_k, s) \land canConcur(o_i, o_j) \land canConcur(o_j, o_k) \land canConcur(o_i, o_k) \implies T(T(o_k, o_i), T(o_j, o_i)) = T(T(o_k, o_j), T(o_i, o_j))
\]

Note that properties \( TP_1 \) and \( TP_2 \) only need to hold for states in which the operations can be generated, represented by the relation \( enabled : Op \times \Sigma \rightarrow \mathbb{B} \), and only if the two operations can occur concurrently, represented by the relation \( canConcur : Op \times Op \rightarrow \mathbb{B} \).

VeriFx provides a library for implementing and verifying RDTs that use operational transformations. Programmers can build custom RDTs by extending the \( OT \) trait shown in Listing 7. Every RDT that extends the \( OT \) trait must provide concrete type arguments for the state and operations, and implement the \texttt{transform} and \texttt{apply} methods. The \texttt{transform} method transforms an incoming operation against a previously executed concurrent operation. The \texttt{apply} method applies an operation on the state. By extending this trait, the RDT inherits proofs for \( TP_1 \) and \( TP_2 \). By default, these proofs assume that operations are always enabled and that all operations can occur concurrently. If this is not the case, the RDT can override the \texttt{enabled} and \texttt{canConcur} methods respectively.

Although VeriFx supports the general execution model of OT, most transformation functions described by the literature were specifically designed for collaborative text editing. They model text documents as a sequence of characters and operations insert or delete characters at a given position in the document. Every paper thus describes four transformations functions, one for every pair of operations: insert-insert, insert-delete, delete-insert, delete-delete.

Likewise, VeriFx’s OT library provides a \texttt{ListOT} trait that models the state as a list of values and supports insertions and deletions. RDTs extending the \texttt{ListOT} trait need to implement four methods (\texttt{Tii}, \texttt{Tid}, \texttt{Tdi}, \texttt{Tdd}) corresponding to the transformation functions for transforming insertions against insertions (\texttt{Tii}), insertions against deletions (\texttt{Tid}), deletions against insertions (\texttt{Tdi}), and deletions against deletions (\texttt{Tdd}). The trait provides a default implementation of \texttt{transform} that dispatches to the corresponding transformation function based on the type of operations, and a default implementation of \texttt{apply} that inserts or deletes a value from the underlying list.

6 EVALUATION

We now evaluate the applicability of VeriFx to implement and verify RDTs. Our evaluation is twofold. First, we implement and verify numerous CRDTs taken from literature as well as some new variants. Afterwards, we verify well-known operational transformation functions and some unpublished designs. We will submit an artifact including all implementations and proofs.
Table 1. Verification results for CRDTs implemented and verified in VeriFx. S = state-based, O = op-based, P = pure op-based CRDT. ☐ = timeout, □ = adaptation of an existing CRDT, ☐ = incomplete definition.

| CRDT                          | Type | LoC | Correct | Time   | Source                      |
|-------------------------------|------|-----|---------|--------|----------------------------|
| Counter                       | O    | 17  | ✓       | 3.2 s  | [Shapiro et al. 2011a]     |
| Grow-Only Counter             | S    | 33  | ✓       | 4.3 s  | [Shapiro et al. 2011a]     |
| Positive-Negative Counter     | S    | 15  | ✓       | 5.9 s  | [Shapiro et al. 2011a]     |
| Dynamic Positive-Negative     | S    | 41  | ✓       | 7.1 s  | [Shapiro et al. 2011a]     |
| Counter Flag                  | P    | 18  | ✓       | 4.0 s  | [Baquero et al. 2017]      |
| Disable-Wins Flag             | P    | 20  | ✓       | 3.9 s  | [Baquero et al. 2017]      |
| Multi-Value Register          | S    | 63  | ✓       | 8.8 s  | [Shapiro et al. 2011a]     |
| Multi-Value Register          | P    | 18  | ✓       | 4.1 s  | [Baquero et al. 2017]      |
| Last-Writer-Wins Register     | S    | 16  | ✓       | 5.3 s  | [Shapiro et al. 2011a]     |
| Last-Writer-Wins Register     | O    | 38  | ✓       | 4.4 s  | [Shapiro et al. 2011a]     |
| Grow-Only Set                 | S    | 10  | ✓       | 5.3 s  | [Shapiro et al. 2011a]     |
| Two-Phase Set                 | O    | 27  | ✓       | 4.4 s  | [Shapiro et al. 2011a]     |
| Two-Phase Set                 | S    | 26  | ✓       | 6.3 s  | [Shapiro et al. 2011a]     |
| Unique Set                    | O    | 39  | ✓       | 4.4 s  | [Shapiro et al. 2011a]     |
| Add-Wins Set                  | P    | 28  | ✓       | 4.3 s  | [Baquero et al. 2017]      |
| Remove-Wins Set               | P    | 42  | ✓       | 4.5 s  | [Baquero et al. 2017]      |
| Last-Writer-Wins Set          | S    | 36  | ✓       | 6.6 s  | [Shapiro et al. 2011a]     |
| Optimized Last-Writer-Wins Set| S    | 37  | ✓       | 6.5 s  | new data type              |
| Positive-Negative Set         | S    | 36  | ✓       | 6.9 s  | [Shapiro et al. 2011a]     |
| Observed-Removed Set          | O    | 75  | ✓       | 6.2 s  | [Shapiro et al. 2011a]     |
| Observed-Removed Set          | S    | 34  | ✓       | 7.6 s  | [Shapiro et al. 2011a]     |
| Optimized Observed-Removed Set| S    | 78  | ✓       | 30.2 s | [Bieniusa et al. 2012]     |
| Molli, Weiss, Skaf Set        | O    | 45  | ✓       | 4.7 s  | [Shapiro et al. 2011a]     |
| Grow-Only Map                 | S    | 32  | ✓       | 9.1 s  | new data type              |
| Buggy Map                     | O    | 87  | ☐       | 65.2 s | [Kleppmann 2022]           |
| Corrected Map                 | O    | 101 | ✓       | 49.4 s | [Kleppmann 2022]           |
| 2P2P Graph                    | O    | 58  | ✓       | 7.8 s  | [Shapiro et al. 2011a]     |
| 2P2P Graph                    | S    | 41  | ✓       | 7.0 s  | [Shapiro et al. 2011a]     |
| Add-Only Directed Acyclic     | O    | 42  | ✓       | 4.7 s  | [Shapiro et al. 2011a]     |
| Add-Only Directed Acyclic     | S    | 30  | ✓       | 8.7 s  | [Shapiro et al. 2011a]     |
| Add-Remove Partial Order      | O    | 61  | ✓       | 10.4 s | [Shapiro et al. 2011a]     |
| Add-Remove Partial Order      | S    | 49  | ✓       | 13.2 s | [Shapiro et al. 2011a]     |
| Replicated Growable Array      | O    | 156 | ☐       |        | /                          |
| Continuous Sequence           | O    | 108 | ✓       | 9.2 s  | [Shapiro et al. 2011a]     |
| Continuous Sequence           | S    | 53  | ✓       | 11.4 s | [Shapiro et al. 2011a]     |

All experiments reported in this section were conducted on AWS using an m5.xlarge VM with 2 virtual CPUs and 8 GiB of RAM. All benchmarks are implemented using JMH [OpenJDK [n. d.]], a benchmarking library for the JVM. We configured JMH to execute 20 warmup iterations followed by 20 measurement iterations for every benchmark. To avoid run-to-run variance JMH repeats every benchmark in 3 fresh JVM forks, yielding a total of 60 samples per benchmark.

6.1 Verifying Conflict-free Replicated Data Types

We implemented and verified an extensive portfolio comprising 35 CRDTs, taken from literature [Baquero et al. 2017; Bieniusa et al. 2012; Kleppmann 2022; Shapiro 2017; Shapiro et al. 2011a]. To the best of our knowledge, we are the first to mechanically verify all CRDTs from Shapiro et al. [2011a], the pure op-based CRDTs from Baquero et al. [2017], and the map CRDT from Kleppmann [2022].

Table 1 summarizes the verification results, including the average verification time and code size of the different CRDTs. The Dynamic Positive-Negative Counter CRDT is a variation on the traditional Positive-Negative Counter that supports a dynamic number of replicas and is based on
the implementation found in Akka’s distributed key-value store [Akka [n. d.]]. VeriFx was able to verify all CRDTs except the Replicated Growable Array (RGA) [Shapiro et al. 2011a] due to the recursive nature of the insertion algorithm. We found that the Two-Phase Set CRDT (described in Section 2) converges but is not functionally correct, that the original Map CRDT proposed by Kleppmann [2022] diverges as VeriFx found the counterexample described in their technical report, and that the Molli, Weiss, Skaf (MWS) Set is incomplete. We now focus on the latter.

Specification 2 describes the MWS Set, which associates a count to every element. An element is considered in the set if its count is strictly positive. remove decreases the element’s count, while add increments the count by the amount that is needed to make it positive (or by 1 if it is already positive). Listing 8 shows the implementation of the MWS Set in VeriFx as a polymorphic class that extends the CmRDT trait (cf. Section 5.1.2). The type arguments passed to CmRDT correspond to the supported operations (SetOps), the messages that are exchanged (SetMsks), and the CRDT type itself (MWSSet). The SetOp enumeration defines two types of operations: Add(e) and Remove(e).

The MWSSet class has a field, called elements, that maps elements to their count (Line 3). Like all op-based CRDTs, the MWSSet implements two phases: prepare and effect. prepare pattern

```
enum SetOp[V] { Add(e: V) | Remove(e: V) }
enum SetMsg[V] { AddMsg(e: V, dt: Int), RmvMsg(e: V) }

class MWSSet[V](elements: Map[V, Int]) extends CmRDT[SetOp[V], SetMsg[V], MWSSet[V]] { override def enabledSrc(op: SetOp[V]) = op match { case Add(_) => true case Remove(e) => this.preRemove(e) } def prepare(op: SetOp[V]) = op match { case Add(e) => this.add(e) case Remove(e) => this.remove(e) } def effect(msg: SetMsg[V]) = msg match { case AddMsg(e, dt) => this.addDownstream(e, dt) case RmvMsg(e) => this.removeDownstream(e) } def lookup(e: V) = this.elements.getOrElse(e, 0) > 0 def add(e: V, dt: Int): MWSSet[V] = { val count = this.elements.getOrElse(e, 0) + dt this.addDownstream(e, dt) } def remove(e: V): MWSSet[V] = { val count = this.elements.getOrElse(e, 0) - 1 this.removeDownstream(e) } override def enabledDest(op: SetOp[V]) = op match { case Add(e) => true case Remove(e) => this.preRemove(e) } def effect(msg: SetMsg[V]) = msg match { case AddMsg(e, dt) => this.addDownstream(e, dt) case RmvMsg(e) => this.removeDownstream(e) } def prepare(op: SetOp[V]) = op match { case Add(e) => this.add(e) case Remove(e) => this.remove(e) } def effect(msg: SetMsg[V]) = msg match { case AddMsg(e, dt) => this.addDownstream(e, dt) case RmvMsg(e) => this.removeDownstream(e) } def lookup(e: V) = this.elements.getOrElse(e, 0) > 0
```
matches on the operation and delegates it to the corresponding source method which prepares a SetMsg to be broadcast over the network. The class overrides the enabledSrc method to implement the source precondition on remove, as defined by Spec. 2. When replicas receive incoming messages, they are processed by the effect method which delegates them to the corresponding downstream method which performs the actual update. For example, the removeDownstream method processes incoming RmvMsgs by decreasing some count \( k' \) by 1. Unfortunately, \( k' \) is undefined in Spec. 2.

We believe that \( k' \) is either defined by the source replica and included in the propagated message (Spec. 3), or, \( k' \) is defined as the element’s count at the downstream replica (Spec. 4). We implemented both possibilities in VeriFx (Listings 9 and 10) but it is unclear which one, if any, is correct. To find out, the companion object of the MWSSet class (cf. Line 26 in Listing 8) extends the CmRDTProof trait (cf. Section 5.1.2), passing along three type arguments: the type of operations SetOp, the type of messages being exchanged SetMsg, and the CRDT type constructor MWSSet. The object extends CmRDTProof as the MWSSet class is polymorphic and expects one type argument. When executing the proof inherited by the companion object, VeriFx automatically proves that the possibility implemented by Listing 9 is wrong and that the one of Listing 10 is correct. We thus successfully completed the MWS Set implementation thanks to VeriFx’s integrated verification capabilities.

Based on the figures reported in Table 1 we conclude that VeriFx is suited to verify CRDTs since all implementations were verified in a matter of seconds.

### 6.2 Verifying Operational Transformation

We now show that VeriFx is general enough to verify other distributed abstractions such as Operational Transformation (OT). We implemented all transformation functions for collaborative text editing described by Imine et al. [2003] and verified \( TP_1 \) and \( TP_2 \) in VeriFx.

Table 2 summarizes the verification results for each transformation function and includes the average verification time and code size. The functions proposed by Ellis and Gibbs [1989] and Sun et al. [1998] do not satisfy \( TP_1 \) nor \( TP_2 \). Ressel et al. [1996]’s functions satisfy \( TP_1 \) but not \( TP_2 \). Suleiman et al. [1997]’s functions satisfy \( TP_1 \) but the proof for \( TP_2 \) times out due to the complexity.
of the transformations. These results confirm prior findings by Imine et al. [2003]. However, VeriFx found that the transformation functions proposed by Imine et al. [2003] also do not satisfy TP₂, which confirms the findings of Li and Li [2004] and Oster et al. [2006]. In addition, in a private communication Imine [n. d.] asked us to verify (unpublished) OT designs for replicated registers and stacks. Out of the three register designs verified in VeriFx, only one was correct for both TP₁ and TP₂. Regarding the stack design, it guarantees TP₂ but not TP₁. VeriFx provided meaningful counterexamples for each incorrect design.

To exemplify our approach to verifying OT, we now describe the implementation and verification of Imine et al. [2003]’s transformation functions in VeriFx, which is shown in Listing 11. The enumeration Op on Line 1 defines the three supported operations:

- Ins(p, ip, c) represents the insertion of character c at position p. Initially, the character was inserted at position ip. Transformations may change p but leave ip untouched.
- Del(p) represents the deletion of the character at position p.
- Id() acts as a no-op. This operation is never issued by users directly but operations may be transformed to a no-op.

The object Imine extends the ListOT trait and implements the four transformation functions (Tii, Tid, Tdi, Tdd) that are required for collaborative text editing (cf. Section 5.2). The implementation of these transformation functions is a straightforward translation from their description by Imine et al. [2003]. The resulting object inherits automated proofs for TP₁ and TP₂. When running these proofs, VeriFx reports that the transformation functions guarantee TP₁ but not TP₂.

Based on the results shown in Table 2, we conclude that VeriFx is suited to verify other RDT families such as OT. Due to the number of cases that have to be considered, the verification times are longer than for CRDTs but are still acceptable for static verification [Calcagno et al. 2015].

7 DISCUSSION

Our work explores a high-level programming language that is powerful enough to implement distributed abstractions (e.g. CRDTs and OT) and verify them automatically without requiring annotations or programmer intervention of any kind. VeriFx shows that automated verification of RDTs based on SMT solving removes the need for abstract specifications and verifies the actual implementation instead. Our approach enables programmers to implement RDTs, express correctness properties, and verify those properties automatically, all within the same language. This avoids mismatches between the implementation and the verification without requiring expertise in verification. We now discuss the limitations of two key features of our approach.

Traits. For simplicity, VeriFx currently only supports single inheritance from traits. This could, however, be extended to support multiple inheritance. Traits are not meant for subtyping because subtyping complicates verification as every subtype needs to be verified but these are not necessarily known at compile time. Hence, class fields, method parameters, local variables, etc. cannot be of a trait type. Programmers can, however, define enumerations as these have a fixed number of constructors, all of which are known at compile time. Note that traits can define type parameters with upper type bounds. These type bounds are only used by the type checker to ensure that every extending class or trait is well-typed. The compiled SMT program does not contain traits as they are effectively compiled away (cf. Section 4.2). Proofs, classes, and class methods cannot have type bounds on type parameters because the compiler does not know all subtypes.

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12Suleiman’s transformation functions [Suleiman et al. 1997] do not satisfy TP₂ according to Oster et al. [2006].
13We represent characters using integers that correspond to their ASCII code.
**Functional collections.** VeriFx encodes higher order functions on collections (e.g. `map`, `filter`, etc.) using arrays, which are treated as function spaces in the combinatory array logic (CAL) [de Moura and Bjørner 2009]. Hence, anonymous functions (lambdas) merely define an array and arrays are first-class. SMT solvers can efficiently reason about VeriFx’s functional collections and their higher order operations because CAL is decidable. However, some operations are encoded using universal or existential quantifiers which may hamper decidability. In practice, we were able to verify RDTs involving complex functional operations. Unfortunately, VeriFx’s collections do not yet provide aggregation methods (e.g. `fold` and `reduce`) because this is beyond the capabilities of CAL. Instead, programmers need to manually aggregate the collection by writing recursive methods that loop over the values of the collection. While looping over finite collections works, most SMT solvers will not provide inductive proofs for general properties about recursive functions.

8 RELATED WORK
Automated program verification is a vast area of research. We focus our comparison of related work on verification languages, and approaches for verifying RDTs, invariants in distributed systems, and operational transformation.

**Verification languages.** Verification languages can be classified in three categories: interactive, auto-active, and automated verification languages [Leino and Moskal 2010]. Interactive verification languages include proof assistants like Coq and Isabelle/HOL in which programmers define theorems and prove them manually using proof tactics. Although some automation tactics exist, proving complex theorems requires considerable manual proof efforts. Similarly, programmers in Liquid Haskell [Vazou et al. 2014] provide proofs using plain Haskell functions. Some proofs can be assisted or discharged by the underlying SMT solver. In contrast, proofs in VeriFx are fully automated. Auto-active verification languages like Dafny [Leino 2010] and Spec# [Barnett et al. 2005] verify programs for runtime errors and user-defined invariants based on annotations provided by the programmer (e.g. preconditions, postconditions, loop invariants, etc.). Intermediate verification languages (IVLs) like Boogie [Barnett et al. 2006] and Why3 [Filliâtre and Paskevich 2013] automate the proof task by generating verification conditions (VCs) from source code and discharging them using one or more SMT solvers. IVLs are not meant to be used by programmers directly. Instead, programs written in some verification language (e.g. Dafny, Spec#, etc.) are translated to an IVL to verify the VCs. While the aforementioned approaches aim to be general such that they can be used to prove any property of a program, VeriFx was designed to be a high-level programming language capable of verifying RDTs fully automatically.

**Verifying SEC for RDTs.** Much work has focused on verification techniques for RDTs. Burckhardt et al. [2014] propose a formal framework that enables the specification and verification of RDTs. Attiya et al. [2016] use a variation on this framework to provide precise specifications of replicated lists - which form the basis of collaborative text editing - and prove the correctness of an existing text editing protocol. Gomes et al. [2017] and Zeller et al. [2014] propose formal frameworks in the Isabelle/HOL theorem prover to mechanically verify SEC for CRDT implementations. Unfortunately, the aforementioned verification techniques require significant efforts since they are not automated.

Liu et al. [2020] extend Liquid Haskell [Vazou et al. 2014] with typeclass refinements and use them to prove SEC for several CRDT implementations. While simple proofs can be discharged automatically by the underlying SMT solver, advanced CRDTs also require significant proof efforts (as discussed in Section 2).

Liang and Feng [2021] propose a new correctness criterion for CRDTs that extends SEC with functional correctness and enables manual verification of CRDT implementations and client programs...
using them. They mainly focus on functional correctness and provide paper proofs. In contrast, VeriFx enables *automated* verification of CRDT implementations.

Wang et al. [2019] propose replication-aware linearizability, a criterion that enables sequential reasoning to prove the correctness of CRDT implementations. The authors manually encoded the CRDTs in the Boogie verification tool to prove correctness. Those encodings are non-trivial and differ from real-world CRDT implementations.

Nagar and Jagannathan [2019] developed a proof rule that is parametrized by the consistency model and automatically checks convergence for CRDTs. Unfortunately, their framework introduces imprecisions and may reject correct CRDTs. Moreover, their framework requires a first-order logic specification of the CRDT. The resulting proofs thus verify the specification instead of a concrete implementation. In contrast, VeriFx can verify high-level CRDT implementations directly.

Finally, Jagadeesan and Riely [2018] introduce a notion of validity for RDTs and manually prove it for some CRDTs. We do not consider validity in this work.

*Verifying invariants in distributed systems.* Reasoning about program invariants and maintaining them under weak consistency is challenging. Invariant confluence [Bailis et al. 2014] is a correctness criterion for coordination avoidance; invariant confluent operations maintain application invariants, even without coordination. Whittaker and Hellerstein [2018] devise a decision procedure for invariant confluence that can be checked automatically by their interactive system.

Some work has focused on verifying program invariants for existing RDTs [Balegas et al. 2018; Gotsman et al. 2016; Nair et al. 2020; Zeller et al. 2020]. Soteria [Nair et al. 2020] verifies program invariants for state-based replicated objects. Repliss [Zeller et al. 2020] verifies program invariants for applications that are built on top of their CRDT library. CISE [Gotsman et al. 2016] proposes a proof rule to check that a particular choice of consistency for the operations preserves the application invariants. IPA [Balegas et al. 2018] detects invariant-breaking operations and proposes modifications to the operations in order to preserve the invariants. Unfortunately, these approaches assume that the underlying RDT is correct. VeriFx enables programmers to verify that this is the case. In this paper, we did not consider application invariants and leave them as future work.

Other approaches [De Porre et al. 2021; Houshmand and Lesani 2019; Li et al. 2014, 2012, 2018, 2020; Milano and Myers 2018; Zhao and Haller 2018, 2020] feature a hybrid consistency model that decides on an appropriate consistency level for operations based on a static analysis. In this work, we did not consider mixed-consistency RDTs.

*Verifying operational transformation functions.* Ellis and Gibbs [1989] first proposed an algorithm for operational transformation together with a set of transformation functions. Several works [Suleiman et al. 1998; Sun et al. 1998] showed that integration algorithms like adOPTed [Ressel et al. 1996], SOCT2 [Suleiman et al. 1998], and GOTO [Sun and Ellis 1998] guarantee convergence iff the transformation functions satisfy the $TP_1$ and $TP_2$ properties. Unfortunately, Ellis and Gibbs [1989]’s transformation functions do not satisfy these properties [Ressel et al. 1996; Suleiman et al. 1998; Sun et al. 1998]. Over the years, several transformation functions have been proposed [Ressel et al. 1996; Suleiman et al. 1997; Sun et al. 1998]. Imine et al. [2003] used SPIKE, an automated theorem prover, to verify the correctness of these transformation functions and found counterexamples for all of them, except for Suleiman et al. [1997]’s transformation functions. As shown in Section 6.2, we were able to reproduce their findings using VeriFx and generate similar counterexamples. Imine et al. [2003] also proposed a simpler set of transformation functions which later was found to also violate $TP_2$ [Li and Li 2004; Oster et al. 2006]. VeriFx also found this counterexample.
9 CONCLUSION

Replicated data types (RDTs) are widespread among highly available distributed systems but verifying them remains complex, even for experts. Automated verification efforts [Liu et al. 2020; Nagar and Jagannathan 2019] have been proposed but these cannot yet produce complete correctness proofs from high-level implementations.

To address this issue, we propose VeriFx, a functional object-oriented programming language that features a novel proof construct to express correctness properties that are verified automatically. We leverage the proof construct to build libraries for implementing and verifying two well-known families of RDTs: CRDTs and OT. Programmers can also implement custom libraries to verify other approaches. Verified RDT implementations can be transpiled to mainstream languages, e.g. Scala or JavaScript. VeriFx’s modular architecture allows programmers to add support for other languages.

This work accounts for the first extensive portfolio of verified RDTs including 35 CRDTs and 9 OT designs. All were verified automatically in a matter of seconds or minutes and with minimal effort. For example, our implementation of an Observed-Removed Set CRDT required only 75 LoC and does not involve verification-specific code. In contrast, related work requires significant programmer intervention to verify similar designs, e.g. Gomes et al. [2017] needed 20 auxiliary lemmas to verify the Observed-Removed Set. VeriFx allows programmers to implement and automatically verify RDTs within the same language, thereby, enabling the adoption of RDTs by the masses.

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A VERIFX’S TYPE SYSTEM

We now present VeriFx’s type system. An environment $\Gamma$ is a partial and finite mapping from variables to types. A type environment $\Delta$ is a finite set of type variables. VeriFx’s type system consists of a judgment for type well-formedness $\Delta \vdash T_{\text{ok}}$ which says that type $T$ is well-formed in context $\Delta$, and a judgment for typing $\Delta;\Gamma \vdash e : T$ which says that in context $\Delta$ and environment $\Gamma$, the expression $e$ is of type $T$. We abbreviate $\Delta \vdash T_{\text{ok}}$, $\ldots$, $\Delta \vdash T_n_{\text{ok}}$ to $\Delta \vdash T_{\text{ok}}$, and $\Delta;\Gamma \vdash e_1 : T_1$, $\ldots$, $\Delta;\Gamma \vdash e_n : T_n$ to $\Delta;\Gamma \vdash e : T$.

Below we define well-formed types:

$$\Delta \vdash \text{string ok} \quad (WF-\text{STRING}) \quad \Delta \vdash \text{bool ok} \quad (WF-\text{BOOL}) \quad \Delta \vdash \text{int ok} \quad (WF-\text{INT})$$

$$\begin{array}{c}
\Delta \vdash \bar{T}_{\text{ok}} \\
\quad \text{class } C(\bar{X})\{\ldots\} \\
\quad \text{or } \text{class } C(\bar{X})\{\ldots\} \text{ extends } I(\ldots)\{\ldots\} \\
\quad \Delta \vdash C(\bar{T})_{\text{ok}} \\
\quad \Delta \vdash \bar{T}_{\text{ok}} \\
\quad \bar{T} <: \bar{P} \\
\quad \text{trait } I(\bar{X} <: \bar{P})\{\ldots\} \\
\quad \text{or } \text{trait } I(\bar{X} <: \bar{P})\{\ldots\} \text{ extends } I(\ldots)\{\ldots\} \\
\quad \Delta \vdash I(\bar{T})_{\text{ok}} \\
\end{array} \quad (WF-\text{CLASS})$$

$$\begin{array}{c}
\Delta \vdash \bar{T}_{\text{ok}} \\
\quad \text{enum } E(\bar{X})\{\ldots\} \\
\quad \Delta \vdash E(\bar{T})_{\text{ok}} \\
\quad X \in \Delta \\
\quad \Delta \vdash X_{\text{ok}} \\
\end{array} \quad (WF-\text{ENUM}) \quad (WF-\text{TVAR})$$

Primitive types are always well-formed. A type variable $X$ is valid if it is in scope: $X \in \Delta$, i.e. the surrounding method or class defined the type parameter. Class types and enumeration types are valid if a corresponding class or enumeration definition exists and all type arguments are well-formed.

We now define a few auxiliary definitions which are needed for the typing rules. The $\text{fields}$ function takes a class type and returns its fields and their types:

$$\text{class } C(\bar{X})\{\pi : \bar{T}\}\{M\} \quad \text{or } \text{class } C(\bar{X})\{\pi : \bar{T}\} \text{ extends } I(\bar{P})\{\bar{M}\}$$

$$\quad \text{fields}(\bar{C}(\bar{P})) = [\bar{P}/\bar{X}] \bar{\pi} : \bar{T} \quad (F-\text{CLASS})$$

The $\text{ftypes}$ function takes an enumeration type and the name of one of its constructors and returns the type of the fields of that constructor:

$$\begin{array}{c}
\text{enum } E(\bar{X})\{K(\bar{\pi} : \bar{T}), \ldots\} \\
\text{ftypes}(E(\bar{P}), K) = [\bar{P}/\bar{X}] \bar{T} \quad (FT-\text{ENUM})
\end{array}$$

The $\text{mtype}$ function takes the name of a method and the type of a class, and returns the actual type signature of the method. If the method is not found in the class (MT-class-rec rule) it is looked up in the hierarchy of super traits by the MT-trait rules. For polymorphic methods, the returned type signature is polymorphic:

$$\begin{array}{c}
\text{class } C(\bar{X})\{\ldots\}\{\bar{M}\} \quad \text{or } \text{class } C(\bar{X})\{\ldots\} \text{ extends } I(\bar{P})\{\bar{M}\} \\
\quad \text{def } m(\bar{Y}) (\bar{X} : \bar{T}) : T = e \in \bar{M} \\
\quad \text{mtype}(m, C(\bar{P})) = [\bar{P}/\bar{X}] ([\bar{Y}]T \to T) \quad (MT-\text{CLASS})
\end{array}$$
Similarly, we assume that there are functions $\text{valNames}(I \langle P \rangle)$ and $\text{declaredMethods}(I \langle P \rangle)$ that return all fields, respectively all methods, declared by a trait (and its super traits). The $\text{ctors}$ function takes an enumeration type and returns the names of its constructors.

Figure 6 shows the typing rules for expressions. Most rules are a simplification of Featherweight Generic Java [Igarashi et al. 2001] without subtyping. Quantified formulas are boolean expressions if their body also types to a boolean expression in the environment that is extended with the quantified variables ($T\text{-uni}$ and $T\text{-exi}$ rules). Logical implication is a well-typed boolean expression if both the antecedent and the consequent are boolean expressions ($T\text{-impl}$ rule).

Classes are well-formed if the types of the fields are well-formed and all its methods are well-formed ($T\text{-class1}$ rule). If the class extends a trait, it must also implement all fields and methods declared by the hierarchy of super traits ($T\text{-class2}$ rule). The typing rules for trait definitions and object definitions can be defined similarly.

When instantiating an enumeration through one of its constructors $\text{new } K \langle P \rangle(\overline{e})$, the provided arguments $\overline{e}$ need to match the types of the constructors’ fields, and the result effectively is an object of the enumeration type $E \langle P \rangle$.

Programmers can pattern match on enumerations but the cases must be exhaustive, i.e. every constructor must be matched by at least one case. If all cases are of type $T$, then the resulting pattern match expression is also of type $T$.

Finally, the body of a proof must be a well-typed boolean expression.
Fig. 6. Typing VeriFx expressions.
B CORE SMT EXPRESSIONS
We will now discuss the expressions that are supported by Core SMT. Those expressions are common to most SMT solvers, except lambdas which, as mentioned before, are described by the preliminary proposal for SMT-LIB v3.0 and are only implemented by some SMT solvers such as Z3 [de Moura and Björner 2008].

Figure 7 provides an overview of all Core SMT expressions. The simplest expressions are literal values representing integers, strings, and booleans. Core SMT supports the typical arithmetic operators (+, −, *, /) and boolean operators (∧, ∨, and negation ¬) as well as universal and existential quantification, and logical implication. Immutable variables are defined by let bindings. Pattern matching is supported but the cases must be exhaustive. For example, when pattern matching against an algebraic data type every constructor must be handled. Core SMT supports two types of patterns: constructor patterns n(e) that match a specific ADT constructor n and binds names to its fields e, and wildcard patterns that match anything and give it a name n. References v refer to variables that are in scope, e.g. function parameters or variables introduced by let binding or pattern matching. If statements are supported but an else branch is mandatory and both branches must evaluate to the same sort. Functions can be called and type arguments can be provided explicitly to disambiguate polymorphic functions. For example, we defined an ADT Option⟨T⟩ with two constructors Some and None. When calling the None constructor we need to explicitly provide a type argument since it cannot be inferred from the call, e.g. None⟨int⟩(). Finally, fields of an ADT can be accessed by their name. Arrays and lambdas were already discussed in Section 4.1.

C COMPILER SEMANTICS
We now discuss the compiler semantics that were not discussed in the main body of the paper. First, we provide all compilation rules for expressions in Appendix C.1. Then, we provide all compilation rules for sets and maps in Appendices C.2 and C.3 respectively.

C.1 Compiling Expressions
Figure 8 shows the compilation rules for expressions. The operands of binary operators ⊕ are compiled recursively. A negated expression is compiled to the negation of the compiled expression. If statements, the condition and both branches are compiled recursively. In VeriFx, this can be used inside the body of a method to refer to the current object. The reference is compiled to

\[
\begin{align*}
  e &::= \text{num} \mid \text{str} \mid \text{true} \mid \text{false} & \quad \text{(primitive values)} \\
  &\mid e[\, \bar{e} \,] \mid e[\, \bar{e} \,] := e \mid \lambda(x: T).e \\
  &\mid x \mid e \oplus e \mid e \odot e \mid \neg e \\
  &\mid \text{match}(e, \text{case}(\bar{ptn}, e)) & \quad \text{(pattern matching)} \\
  &\mid \text{let } x = e \text{ in } e & \quad \text{(let expression)} \\
  &\mid \text{if}(e, e, e) & \quad \text{(conditional expression)} \\
  &\mid e(e) & \quad \text{(function call)} \\
  &\mid f(T)(e) & \quad \text{(function call with explicit type arguments)} \\
  &\mid e.v & \quad \text{(field access)} \\
  &\mid \forall(x: T).e \mid \exists(x: T).e & \quad \text{(quantified formulas)} \\
  &\mid e \equiv e & \quad \text{(logical implication)} \\
  &\mid ptn ::= K(\bar{x}) \mid x & \quad \text{(patterns)}
\end{align*}
\]

Fig. 7. All Core SMT expressions.
a similar this reference in Core SMT which refers to the this parameter which is always the first parameter of any method (cf. compilation of class methods in Section 4.2). We explained how to compile the remaining expressions in Section 4.2.

Figure 9 shows the compilation rules for logic expressions which in VeriFx can only occur within the body of proofs. For quantified formulas the types of the variables $\mathcal{T}$ and the formula $L$ are compiled. For logical implications, the antecedent and the consequent are compiled recursively.

Finally, pattern match expressions are compiled to similar pattern match expressions in Core SMT. To this end, every pattern is compiled recursively. Core SMT supports two types of patterns: constructor patterns $n_1(\overline{n})$ that match an algebraic data type constructor $n_1$ and binds its fields to the provided names $\overline{n}$, and wildcard patterns $\_n$ that match any value and give it a name $n$. Every VeriFx pattern is compiled to the corresponding Core SMT pattern. The first pattern, $n_1(\overline{n})$, matches an ADT constructor $n_1$ and binds its fields to $\overline{n}$. It is compiled to an equivalent constructor pattern

\[
[x] = x
\]
\[
[e_1 \oplus e_2] = [e_1] \oplus [e_2]
\]
\[
[\langle e \rangle] = \neg [e]
\]
\[
[\text{val } x : T = e_1 \text{ in } e_2] = \text{let } x = [e_1] \text{ in } [e_2]
\]
\[
[\text{if } e_1 \text{ then } e_2 \text{ else } e_3] = \text{if}([e_1], [e_2], [e_3])
\]
\[
[(x : T) \Rightarrow e] = \lambda(x : \overline{T}[\overline{e}]).[e]
\]
\[
[e_1(\overline{e}_2)] = [e_1][\overline{e}_2]
\]
\[
[\text{new Set } \langle T \rangle()] = \lambda(x : \overline{T}[\overline{e}]) . \text{false}
\]
\[
[\text{new Map } \langle T, P \rangle()] = \lambda(x : \overline{T}[\overline{e}]) . \text{None}([P][\overline{e}])
\]
\[
[\text{new } C(\overline{e})] = C'(\overline{\overline{e}})
\]
\[
\text{where } C' = \text{str_concat}(C, \_\_ctor)
\]
\[
\text{new } K(\overline{e}) = K(\overline{\overline{e}})
\]
\[
[e.v] = [e].v
\]
\[
[e_1.m(\overline{e})] = m'(\overline{P}[\overline{T}], \overline{\overline{e}})([e_1], [\overline{e}])
\]
\[
\text{where } \text{typeof}(e_1) = C(\overline{P})
\]
\[
\text{and } m' = \text{str_concat}(C, \_\_m) \text{ and } P \cap T = \emptyset
\]

Fig. 8. Compiling expressions.

\[
[\forall (x : \overline{T}). e] = \forall(x : \overline{\overline{T}}).[e]
\]
\[
[\exists (x : \overline{T}). e] = \exists(x : \overline{\overline{T}}).[e]
\]
\[
[e_1 \Rightarrow e_2] = [e_1] \Rightarrow [e_2]
\]

Fig. 9. Compiling logical expressions.

\[
[e \text{ match } \{ \text{case } \_ \Rightarrow e_c \}] = \text{match}([e], \overline{\text{case } r \Rightarrow e_c})
\]
\[
\text{pat}[\text{case } K(\overline{e}) \Rightarrow e] = \text{case}(K(\overline{e}), [e])
\]
\[
\text{pat}[\text{case } x \Rightarrow e] = \text{case}(x, [e])
\]
\[
\text{pat}[\text{case } \_ \Rightarrow e] = \text{case}(\_, [e])
\]

Fig. 10. Compiling pattern match expressions.
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Section 4.4.2 explained how to encode maps in SMT using arrays and how to efficiently encode a set mapping to a lambda which defines an array of elements that are in at least one of the two sets, i.e. \([e_1][v] \lor [e_2][v]\). Similarly, the intersection of two sets \(e_1\) and \(e_2\) is compiled to a lambda which defines an array containing only elements that are in both sets, i.e. \([e_1][v] \land [e_2][v]\). For set difference, the lambda defines an array containing only elements that are in \(e_1\) and not in \(e_2\). A set \(e_1\) is a subset of \(e_2\) iff all elements from \(e_1\) are also in \(e_2\). A set \(e\) is empty if all elements \(v\) are not in the set. A predicate \(e_2 : T \to \text{bool}\) holds for all elements of a set \(e_1\) if for every element \(v\) that is in the set the predicate is true, i.e. \([e_1][v] \implies [e_p][v]\). A predicate \(e_2 : T \to \text{bool}\) holds for at least one element of a set \(e_1\) if there exists an element \(v\) that is in the set and for which the predicate holds, i.e. \([e_1][v] \land [e_p][v]\).

C.2 Compiling Sets

In Section 4.2 we explained how basic set operations (add, remove, contains) and some advanced operations (filter, map) are compiled to Core SMT. Now, we explain how the remaining operations on sets are compiled. Figure 11 shows the compilation rules for operations over sets. The union of two sets \(e_1\) and \(e_2\) is compiled to a lambda which defines an array of elements \(v\) of type \([[T]]\), containing only elements that are in at least one of the two sets, i.e. \([e_1][v] \lor [e_2][v]\). Similarly, the intersection of two sets \(e_1\) and \(e_2\) is compiled to a lambda which defines an array containing only elements that are in both sets, i.e. \([e_1][v] \land [e_2][v]\). For set difference, the lambda defines an array containing only elements that are in \(e_1\) and not in \(e_2\). A set \(e_1\) is a subset of \(e_2\) iff all elements from \(e_1\) are also in \(e_2\). A set \(e\) is empty if all elements \(v\) are not in the set. A predicate \(e_2 : T \to \text{bool}\) holds for all elements of a set \(e_1\) if for every element \(v\) that is in the set the predicate is true, i.e. \([e_1][v] \implies [e_p][v]\). A predicate \(e_2 : T \to \text{bool}\) holds for at least one element of a set \(e_1\) if there exists an element \(v\) that is in the set and for which the predicate holds, i.e. \([e_1][v] \land [e_p][v]\).

C.3 Compiling Maps

Section 4.4.2 explained how to encode maps in SMT using arrays and how to efficiently encode the basic map operations. We now explain how to encode the advanced map operations. Figure 12 defines the SMT encoding for all advanced map operations. The keys method on maps returns a set containing only the keys that are present in the map. Calls to keys on a map \(e_m\) of type \(\text{Map}\langle K, V \rangle\) are compiled to a lambda which defines a set of keys \(k\) of the compiled key type \([[K]]\), such that a key is present in the set if it is present in the compiled map: \([e_m][k] \neq \text{None}\)\(\langle\lfloor V \rfloor]\)(). A predicate \(e_p\) of type \((K, V) \to \text{bool}\) holds for all elements of a map \(e_m\) of type \(\text{Map}\langle K, V \rangle\) iff it holds for every key \(k\) that is present in the map and its associated value:

\[
[e_m][k] \neq \text{None}(\lfloor V \rfloor)() \implies [e_p][k, [e_m][k].\text{value}] \\

\]

Fig. 11. Compiling set operations.
Similarly, the values method returns a set containing all values of the map. To this end, it defines an array containing all values for which at least one key exists that maps to that value.

A predicate \( e_p \) of type \((K, V) \rightarrow \text{bool}\) holds for at least one element of a map \( e_m \) of type \( \text{Map}(K, V) \) iff there exists a key \( k \) with associated value \( v \) that is present in the map and for which the predicate holds. Mapping a function \( e_f \) over the key-value pairs of a map \( e_m \) of type \( \text{Map}(K, V) \) is encoded as a lambda that defines an array containing only the keys that are present in the compiled map \( \llbracket e_m \rrbracket \) and whose values are the result of applying \( e_f \) on the original value, i.e. \( \text{Some}(\llbracket e_f \rrbracket[k], \llbracket e_m \rrbracket[k].\text{value}) \). The mapValues method is similar except that it applies the provided function only on the value. A map \( e_m \) can be filtered using a predicate \( e_p \) such that the resulting map only contains key-value pairs that fulfill the predicate. Calls to filter are encoded as a lambda that defines an array containing only the key-value pairs that are in the compiled map:

\[
\text{if}(\llbracket e_m \rrbracket[k] \neq \text{None}(\llbracket V \rrbracket_t)() \land \llbracket e_p \rrbracket[k], \llbracket e_m \rrbracket[k].\text{value}, \text{None}(\llbracket V \rrbracket_t)() ) \quad \text{then keep the value}
\]

To zip two maps \( e_{m_1} \) and \( e_{m_2} \) the compiler creates a lambda that defines an array containing only the keys that are present in both maps and the value is a tuple holding the corresponding values from both maps:

\[
\text{Some}(\text{Tuple}_\text{ctor}(\llbracket e_{m_1} \rrbracket[k].\text{value}, \llbracket e_{m_2} \rrbracket[k].\text{value}))
\]

To combine two maps \( e_{m_1} \) and \( e_{m_2} \) with a function \( e_f \) the compiler creates a lambda that defines an array containing all the keys from \( e_{m_1} \) and \( e_{m_2} \). If a key is present in both maps their values are combined using the provided function \( e_f \):

\[
\text{Some}(\llbracket e_f \rrbracket[k], \llbracket e_{m_1} \rrbracket[k].\text{value}, \llbracket e_{m_2} \rrbracket[k].\text{value}))
\]

If a key-value pair is present in only one of the maps it is also present in the new map. If a key is not present in \( e_{m_1} \) neither in \( e_{m_2} \) then it is also not present in the resulting map.

C.4 Compilation Example

Figure 13 shows a concrete example of a polymorphic set implemented in VeriFx and its compiled code in Core SMT. The \( MSet \) class defines a type parameter \( V \) corresponding to the type of elements it holds. It also contains one field \( \text{set} \) of type \( \text{Set}(V) \) and defines a polymorphic method \( \text{map} \) that takes a function \( f : V \rightarrow W \) and returns a new \( MSet \) that results from applying \( f \) on every element. The compiled Core SMT code defines an ADT \( MSet \) with one type parameter \( V \) and one constructor \( MSet_\text{ctor} \). The constructor defines one field \( \text{set} \) of sort \( \text{Array}(V, \text{bool}) \) which is the compiled sort for sets. In addition, a polymorphic \( MSet_\text{map} \) function is defined which takes two type parameters \( V \) and \( W \) which correspond to \( MSet \)'s type parameter and \( \text{map} \)'s type parameter respectively. The function takes two arguments, the object that receives the call and the function \( f \). The function’s body calls the \( MSet \) constructor with the result of mapping \( f \) over the set.
map\left[ e_m.\text{keys}\right] = \lambda (x : [K]_I). [e_m][ x ] \neq \text{None}(\langle V\rangle_I) \quad \text{where } \text{typeof}(e_m) = \text{Map}(K, V)
mapi\left[ e_m.\text{values}\right] = \lambda (x : [V]_I). \exists (k : [K]_I). [e_m][ k ] = \text{Some}(x) \quad \text{where } \text{typeof}(e_m) = \text{Map}(K, V)
mapi\left[ e_m.\text{bijective}\right] = \forall (k_1 : [K]_I, k_2 : [K]_I). \\
\quad (k_1 \neq k_2 \land [e_m][ k_1 ] \neq \text{None}(\langle V\rangle_I) \land [e_m][ k_2 ] \neq \text{None}(\langle V\rangle_I)) \quad \Rightarrow \quad \text{typeof}(e_m) = \text{Map}(K, V)

map\left[ e_m.\text{forall}\right] = \forall (x : [K]_I). [e_m][ x ] \neq \text{None}(\langle V\rangle_I) \quad \Rightarrow \quad [e_m][ x ][ x ].\text{value} \\
\quad \text{where } \text{typeof}(e_m) = \text{Map}(K, V) \quad \text{and } \text{typeof}(e_p) = (K, V) \rightarrow \text{bool}

map\left[ e_m.\text{exists}\right] = \exists (x : [K]_I). [e_m][ x ] \neq \text{None}(\langle V\rangle_I) \land [e_p][ x ][ x ].\text{value} \\
\quad \text{where } \text{typeof}(e_m) = \text{Map}(K, V) \quad \text{and } \text{typeof}(e_p) = (K, V) \rightarrow \text{bool}

map\left[ e_m.\text{map}\right] = \lambda (x : [K]_I). \text{if} (\text{typeof}(e_m)[ x ] \neq \text{None}(\langle V\rangle_I)) \\
\quad \text{then } \text{Some}(e_f)[ x ][ x ].\text{value} \\
\quad \text{else } \text{None}(\langle W\rangle_I)) \\
\quad \text{where } \text{typeof}(e_m) = \text{Map}(K, V) \quad \text{and } \text{typeof}(e_f) = (V, V) \rightarrow W

map\left[ e_m.\text{filter}\right] = \lambda (x : [K]_I). \text{if} (\text{typeof}(e_m)[ x ] \neq \text{None}(\langle V\rangle_I)) \land [e_p][ x ][ x ].\text{value} \\
\quad \text{then } \text{Some}(e_m)[ x ][ x ].\text{value} \\
\quad \text{else } \text{None}(\langle W\rangle_I)) \\
\quad \text{where } \text{typeof}(e_m) = \text{Map}(K, V) \quad \text{and } \text{typeof}(e_p) = (K, V) \rightarrow \text{bool}

map\left[ e_m.\text{zip}\right] = \lambda (x : [K]_I). \text{if} (\text{typeof}(e_m)[ x ] \neq \text{None}(\langle V\rangle_I)) \land [e_m][ x ] \neq \text{None}(\langle W\rangle_I)) \\
\quad \text{then } \text{Tuple ctor}(\langle e_m \rangle)[ x ][ x ].\text{value}, [e_m][ x ][ x ].\text{value} \\
\quad \text{else } \text{None}(\langle \text{Tuple}(V, W)\rangle_I)) \\
\quad \text{where } \text{typeof}(e_m) = \text{Map}(K, V) \quad \text{and } \text{typeof}(e_p) = \text{Map}(K, W)

map\left[ e_m.\text{combine}\right] = \lambda (x : [K]_I). \text{if} (\text{typeof}(e_m1)[ x ] \neq \text{None}(\langle V\rangle_I)) \land [e_m2][ x ] \neq \text{None}(\langle V\rangle_I)) \\
\quad \text{then } \text{Some}(e_f)[ x ][ x ].\text{value}, [e_m1][ x ][ x ].\text{value} \\
\quad \text{else } \text{None}(\langle W\rangle_I)) \\
\quad \text{where } \text{typeof}(e_m1) = \text{Map}(K, V) \quad \text{and } \text{typeof}(e_m2) = \text{Map}(K, W) \quad \text{and } \text{typeof}(e_f) = (V, V) \rightarrow W

map\left[ e_m.\text{toSet}\right] = \lambda (x : \text{Tuple}([K]_I), [V]_I). [e_m][ x ].\text{fst} = \text{Some}(x.\text{snd}) \quad \text{where } \text{typeof}(e_m) = \text{Map}(K, V)

\textbf{Fig. 12.} Compiling advanced map operations.

\begin{verbatim}
class MSet[V](set : Set[V]) { 
  adt MSet[V]{ MSetctor(set : Array[V, bool]) } 
  fun MSet_map[V, W](this : MSet[V], f : Array[V, W]) : MSet[W] = 
    MSetctor( 
      \lambda(y : W).\exists(x : V).this.set[ x ] \land f[ x ] = y)
}

(a) A polymorphic class in VeriFx. 
(b) Compiled Core SMT code.

\textbf{Fig. 13.} Example of a polymorphic class in VeriFx and the compiled code in Core SMT.