Decoding With Erased Elements Provocation

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Abstract. Modern telecommunication technologies play a decisive role in the methods of organization and structure of the construction of existing and projected mobile IAUs or specialized control systems designed to collect a given set of information about managed objects and, in accordance with the objective function, to manage these objects.

Keywords: objective function , erased elements , IAUs

1. Introduction:

The complexity of implementing the erasure correction algorithm in non-binary codes is commensurable with the complexity of implementing the error correction algorithm, but the erasure is corrected twice as much. Therefore, when correcting erasures, it is important to achieve an accurate absence of errors among the non-erased symbols accepted for processing after the ordering of erased positions [1].

Non-binary high priority characters are not processed and for them, positions are fixed on the length of the code vector, which we denote by \( \alpha_i \), where \( i = 0; n-s \). The symbols with low priority are erased and their positions on the length of the internal code are fixed as \( s_j \), where \( j = 0; s \). Non-binary characters of precarious priority are processed using iterative transformations. After that, they can be translated into \( \{ \alpha_i \} \), or into \( \{ s_j \} \). If a certain character is reliably translated into a set \( \{ \alpha_i \} \), then its value is fixed by the decoder as \( \alpha_s \) and this character is erased. In fact, the symbol \( \alpha_s \) is a provoked erasure. Provocation - an action aimed at calling a predictable reaction. If \( \alpha_s \) during decoding gets its declared value in the course of iterative transformations, then the restoration of the generated \( d_{min} - 2 \) the external code is considered to be true. Asymptotic gain for non-binary codes is estimated as

\[
D_{nk} = 10 \log_2 \left( k \left( 1 - k/n1 + 1/n1 \right) \right) \text{dB}
\]
Thus, the energy gain in the implementation of the algorithm under consideration will be
\[ D_{nk} = 10 \log(k1(1-k1/n1)) \text{ dB}, \]
which is less in comparison with the asymptotic estimate by an amount \( 1/n1 \).

Consider the application of the method with the code RS (7,3,5) decoding example over the field \( GF(2^3) \). The transformation results of some allowed code vector \( \mathbf{H} = \alpha^2 \alpha^4 \alpha^5 \alpha^5 \alpha^2 \alpha^6 \alpha^4 \).

A doubtful priority is assigned when parity checking is performed, but at high variance values. Based on the priority values, the RS code symbols are erased for subsequent recovery. In this case, there is no need to perform a search procedure for error locators, which significantly reduces the complexity of the decoder implementation.

Having determined number \( S \) unreliable code symbols RS, decoder erases them under the condition that \( S = d_{\text{min}} - 1 \) and this number includes the selected item \( \alpha_i \). When \( d_{\text{min}} = 5 \) the RS code decoder can recover four erasures. A code vector received from the communication channel based on soft code symbol processing \( n2 \) becomes \( \alpha^2 S_i S_j S_i \alpha^2 S_i S_j \).

Based on the \( \alpha_i \) and \( S_j \) calculate syndromes for positions 1; 2; 3 and 6. It should be noted that the erased position number, the syndromes values of the erased positions (if there are four erasures) are always calculated for \( j_1 = 0; j_2 = 1; j_3 = 2; j_4 = 3 \). The calculation algorithm is presented below.

\[
\begin{align*}
\text{Factor } j_1 + 1 \rightarrow \quad S_{j=0} &= \alpha^2 \alpha^0 + \alpha^2 \alpha^4 + \alpha^6 \alpha^5 = \alpha^2 + \alpha^6 + \alpha^4 = \alpha^5; \\
\text{Factor } j_2 + 1 \rightarrow \quad S_{j=1} &= \alpha^2 \alpha^0 + \alpha^2 \alpha^8 + \alpha^6 \alpha^{10} = \alpha^2 + \alpha^3 + \alpha^2 = \alpha^3; \\
\text{Factor } j_3 + 1 \rightarrow \quad S_{j=2} &= \alpha^2 \alpha^0 + \alpha^2 \alpha^{12} + \alpha^6 \alpha^{15} = \alpha^2 + \alpha^0 + \alpha^0 = \alpha^2; \\
\text{Factor } j_4 + 1 \rightarrow \quad S_{j=3} &= \alpha^2 \alpha^0 + \alpha^2 \alpha^{16} + \alpha^6 \alpha^{20} = \alpha^2 + \alpha^4 + \alpha^5 = \alpha^6. 
\end{align*}
\]

Based on the data obtained, a syndromes polynomial of the form \( S(x) = \alpha^5 + x\alpha^1 + x^2\alpha^2 + x^4\alpha^5 \), and according to known erased positions, a polynomial of erasure locators:

\[ L(x) = (1 + x\alpha)(1 + x\alpha^2)(1 + x\alpha^3)(1 + x\alpha^4) = 1 + x^2\alpha^6 + x^3\alpha^2 + x^4\alpha^5. \]
Having obtained the values $S(x)$ and $L(x)$, find their product, in which all values $x$ with degrees equal to or older than $n - k$ in calculation are not accepted. Thus:

$$T(x) = S(x) \times L(x) = (\alpha + x\alpha^3 + x^2\alpha^2 + x^3\alpha^6) \times (1 + x^2\alpha^6 + x^3\alpha^2 + x^4\alpha^5) = \alpha^5 + x\alpha^3 + x^2\alpha^1. $$

Forney’s algorithm is applied to the product. To do this, we need to find the derivative of the polynomial $L(x)$: $L'(x) = 0 + 2x\alpha^6 + 3x^2\alpha^2 + 4x^3\alpha^5 = x^2\alpha^3$. Here it is taken into account that the even coefficients in front of the polynomial terms indicate a mutual contraction of such elements in the derivative when they are added modulo two. In this algorithm, the values of the erased positions with the number $\gamma$ are determined in accordance with expression $Y_{\gamma} = T(x^{-\gamma})/L'(x^{-\gamma})$.

$$Y_1 = \frac{\alpha^5 + \alpha^3 + \alpha^i}{\alpha^2 + \alpha^i + \alpha^5} = \frac{\alpha^4}{1} = \alpha^4; \quad Y_2 = \frac{\alpha^5 + \alpha^3 + \alpha^i + \alpha^4}{\alpha^2 + \alpha^3 + \alpha^4} = \frac{\alpha^5}{\alpha^5} = \alpha^5;$$

$$Y_3 = \frac{\alpha^5 + \alpha^3 + \alpha^i}{\alpha^2 + \alpha^3 + \alpha^i} = \frac{\alpha^2}{\alpha^2} = \alpha^2; \quad Y_4 = \frac{\alpha^5 + \alpha^3 + \alpha^i + \alpha^4}{\alpha^2 + \alpha^3 + \alpha^4} = \frac{\alpha^5}{\alpha^2} = \alpha^4.$$

As a result, if $Y_{\gamma}$ is equal to the claimed element $\alpha^i$, it is assumed that the reconstruction of the vector is carried out correctly. The presented algorithm allows in a convenient form to realize not only the processor of the receiver, but also the transmitter. Indeed, in order to construct an encoder, it is necessary to allocate digits $i = 0; n - s$, which, according to the described algorithm, are restored with the formation of discharges $j = 0; s$. This allows us to realize not only an adaptive communication system without complex switching devices, but also solve problems of information protection by changing the numbers $i = 0; n - s$, agreed with the message receiver.

The presented algorithm for processing code vectors of code products has a number of differences from the known methods for processing cascade structures. Soft processing of internal code combinations in accordance with the objective function $Q(\bullet)$ allows to realize the decoding of an external non-binary code on the boundary of asymptotic possibilities, which contributes to obtaining a greater energy gain in the communication system. At the same time, with the increase in the redundancy introduced into the code, the efficiency growth from the implementation of the proposed algorithm for processing the RS code is noticeable.

The considered algorithm, in contrast to the classical methods, does not require the use of the trial and error method when decoding the code vector at the step of calculating the syndromes polynomial,
which allows using it in adaptive communication systems for synchronous switching of codes from one generating polynomial to another.

The method helps to increase the information protection being processed, since the pseudo-random change of the generating polynomials for the RS code together with the cryptographic methods increases the overall stability of the communication direction. The complexity of the proposed algorithm is determined by the computation complexity of polynomials $S(x)$ and $T(x)$.

2. Conclusion:

The method of list decoding based on allocation of the cluster number allows you to correct erasures of increased multiplicity. The use of decoders with ordered statistics makes it possible to correct erasures of increased multiplicity due to iterative transformations of the generating matrix of the code to bring it to a systematic form and the subsequent transition to the equivalent code.

3. References:

[1] Arnold, Dynamics, statistics, and projective geometry of Galois fields / VI Arnol'd-M: MKNMO, 2005. P-72.
[2] Gallagher, Codes with a low density of checks for parity / RJ Gallagher. - Moscow: Mir, 1966. P-144.
[3] Gasanov, EE Theory of storage and retrieval of information / EE Gasanov, VB Kudryavtsev. - Moscow: Fizmalit, 2002. P-288.
[4] Gladkich A.A. Fundamentals of the theory of soft decoding of redundant codes in an erasure channel. Ulyanovsk: UlSTU, 2010. P-379.