Mass Matrix of Majorana Neutrinos

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Abstract

We present a massive Majorana neutrino model and see how it is constrained from the solar and atmospheric neutrino deficit experiments. This model incorporates the seesaw mechanism and Peccei-Quinn symmetry. Its consequence to the neutrinoless double beta decay is also discussed.

Key words: Majorana particle, Seesaw mechanism, Peccei-Quinn symmetry

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From the recent solar neutrino and atmospheric neutrino experiments [1] [2] [3] [4], it becomes very probable that the neutrinos have masses. In this letter, we propose the model of massive Majorana neutrino and its physical consequences, especially to neutrinoless double beta decays.

Our physical standpoints are as follows. The solar (atmospheric) neutrino deficit is due to \( \nu_e - \nu_\mu (\nu_\mu - \nu_\tau) \) oscillation and all neutrinos are of Majorana type. Their masses are generated by the seesaw mechanism [9] [10] and have the hierarchy of

\[
m_{\nu_e} = \frac{m_e^2}{M_R}, \quad m_{\nu_\mu} = \frac{m_\mu^2}{M_R}, \quad m_{\nu_\tau} = \frac{m_\tau^2}{M_R} \tag{1}
\]

with \( M_R \) is the order of the Peccei-Quinn symmetry breaking. Our strategy is, therefore, to construct a model which realizes the above mentioned standpoints.

Mass Lagrangian for the leptonic part includes two SU(2) doublets \( \phi_1, \phi_2 \) and one singlet \( \phi_3 \) Higgs fields. Its explicit form is given by

\[
-L_{\text{mass}} = \frac{3}{2} \sum_{i,j} f^{(l)}_{ij} \bar{l}_L^{(i)} \phi_1 e_R^{(j)} + \frac{3}{2} \sum_{i,j} f^{(\nu)}_{ij} \bar{l}_L^{(i)} \phi_2 \nu_R^{(j)} + \frac{3}{2} \sum_{i,j} f^{(M)}_{ij} \bar{\nu}_c^{(i)} \nu_R^{(j)} \phi_3 + h.c. \tag{2}
\]

Here \((i,j)\) is the generation. \( l_L \) is the left-handed doublet and \( e_R^{(i)}(\nu_R^{(i)}) \) is right-handed charged lepton (neutrino) singlet of \( i \)’th generation. The third term represents massive Majorana neutrinos which induce the seesaw mechanism. This Lagrangian has the local SU(2) \( \times \) U(1) symmetry and global Peccei-Quinn and lepton number symmetries.

At this stage we have no relation among the coupling constants though Eq.(1) suggests some relations. Later we will see how the present experiments constrain these parameters. Here we study first the structure of mass Lagrangian after the spontaneous symmetry breaking [11] [12] [13]. We expand Higgs fields as

\[
\phi_1 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \xi_1^+ + i \xi_1^- \\ \rho_1 + \chi_1 + i \xi_1^0 \end{array} \right), \\
\phi_2 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \xi_2^+ + i \xi_2^- \\ \rho_2 + \chi_2 + i \xi_2^0 \end{array} \right), \\
\phi_3 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \xi_3^+ + i \xi_3^- \\ \rho_3 + \chi_3 + i \xi_3^0 \end{array} \right), \tag{3}
\]

where \( \rho_i \) are the vacuum expectation values. Then the following combinations of \( \xi_i^j \) are gauged away by the weak boson transformations,

\[
Z'_\mu = Z_\mu - \frac{1}{\sqrt{g^2 + g'^2}} \partial_\mu \phi^Z, \\
W'_\mu = W_\mu - \frac{1}{\rho g} \partial_\mu (\phi^{W_2}_2 + i \phi^{W_1}_1), \tag{4}
\]
\[ W'_\mu = W_\mu - \frac{1}{\rho g} \partial_\mu (\frac{\phi^{W_2} - i\phi^{W_1}}{\sqrt{2}}). \]

Here
\[ \phi^Z \equiv \frac{\rho_2 \xi^Z - \rho_1 \xi^Z}{\rho}, \]
\[ \phi^{W_1} \equiv \frac{\rho_2 \xi^{W_1} - \rho_1 \xi^{W_1}}{\rho}, \]
\[ \phi^{W_2} \equiv \frac{\rho_2 \xi^{W_2} + \rho_1 \xi^{W_2}}{\rho}. \]

with \( \rho \equiv \sqrt{\rho^2_1 + \rho^2_2}. \) Here we have written the infinitesimal transformations, sufficient to see the Higgs mechanism.

Thus \( \phi^Z, \phi^{W_1} \) and \( \phi^{W_2} \) defined below remain as the dynamical variables together with \( \chi_i \),
\[ \phi^Z \equiv \frac{\rho_2 \xi^Z - \rho_1 \xi^Z}{\rho}, \]
\[ \phi^{W_1} \equiv \frac{\rho_2 \xi^{W_1} + \rho_1 \xi^{W_1}}{\rho}, \]
\[ \phi^{W_2} \equiv \frac{\rho_2 \xi^{W_2} - \rho_1 \xi^{W_2}}{\rho}. \]

So far we have not been able to restrict the parameters in Eq.(2). Fritzsch assumed some additional symmetries and predicted the flavour mixing angles in quark sector [5]. Our procedures reverse this process in lepton sector. That is, we consider first how the experiments constrain, especially, the neutrino mass matrix \( M_{light} \) after the seesaw mechanism,
\[ M_{light} = -M_D M_R^{-1} M_D^T, \]
where \((M_D)_{ij} \equiv f^{(\nu)}_{ij} \rho_2 \) and \((M_R)_{ij} \equiv f^{(M)}_{ij} \rho_3 \).

If we adopt that the atmospheric neutrino deficit is due to \( \nu_\mu - \nu_\tau \) oscillation, \( \theta_2 (\theta_2 \equiv \theta_{23} \) and analogously \( \theta_3 \equiv \theta_{31}, \theta_1 \equiv \theta_{12} \) may be constrained to be \( \theta_2 \sim \frac{\pi}{4} \) from the experiment [11]. Also we assume that the solar neutrino deficit is due to \( \nu_e - \nu_\mu \) oscillation and set \( \theta_3 \sim 0. \) That is, the orthogonal (we have not considered CP violation phases) lepton mixing matrix defined by
\[ U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_3 & 0 & s_3 \\ 0 & 1 & 0 \\ -s_3 & 0 & c_3 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]
becomes
\[ U = \begin{pmatrix} \frac{c_1}{\sqrt{2}} s_1 & \frac{s_1}{\sqrt{2}} & 0 \\ \frac{s_1}{\sqrt{2}} c_1 & \frac{c_1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} c_1 & -\frac{1}{\sqrt{2}} s_1 & \frac{1}{\sqrt{2}} \end{pmatrix}, \]
where \( s_i \) (or \( c_i \)) is \( \sin \theta_i \) (or \( \cos \theta_i \)). \( M_{\text{light}} \) is constrained to be

\[
M_{\text{light}} = U \begin{pmatrix} -m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^T
\]

\[
= \begin{pmatrix} -c_{1}^{2}m_1 + s_{1}^{2}m_2 & \frac{1}{\sqrt{2}}c_{1}s_{1}(m_1 + m_2) & -\frac{1}{\sqrt{2}}c_{1}s_{1}(m_1 + m_2) \\ \frac{1}{\sqrt{2}}c_{1}s_{1}(m_1 + m_2) & \frac{1}{2}(-s_{1}^{2}m_1 + c_{1}^{2}m_2 + m_3) & \frac{1}{2}(s_{1}^{2}m_1 - c_{1}^{2}m_2 + m_3) \\ -\frac{1}{\sqrt{2}}c_{1}s_{1}(m_1 + m_2) & \frac{1}{2}(s_{1}^{2}m_1 - c_{1}^{2}m_2 + m_3) & \frac{1}{2}(-s_{1}^{2}m_1 + c_{1}^{2}m_2 + m_3) \end{pmatrix}. \tag{10}
\]

Here we have set the sign of \( m_1 \) negative. Indeed we can always change the sign of the mass by making the transformation \( \psi_R \rightarrow -\psi_R \) and \( \psi_L \rightarrow \psi_L \). There exists a mass hierarchy of \( m_1 \ll m_2 \ll m_3 \) and we have no lower bound with respect to \( m_1 \) from the neutrino anomalies. So we assume

\[
-c_{1}^{2}m_1 + s_{1}^{2}m_2 = 0 \tag{11}
\]

That is, the lightest neutrino mass is considered to be generated only by the flavour mixing. Eq.(11) shows that we adopt the small angle solution for the solar neutrino oscillation. Then \( M_{\text{light}} \) is reduced to

\[
M_{\text{light}} = \begin{pmatrix} 0 & A & -A \\ A & B & C \\ -A & C & B \end{pmatrix}, \tag{12}
\]

where

\[
A \equiv \frac{1}{\sqrt{2}}\sqrt{m_1m_2}, \quad B \equiv \frac{1}{2}(-m_1 + m_2 + m_3)
\]

and

\[
C \equiv \frac{1}{2}(m_1 - m_2 + m_3).
\]

It may be interesting to compare this with the quark mass matrix proposed by Fritzsch [5]. Then we will see how \( M_{\text{light}} \) in Eq.(12) is affected on seesaw mechanics (7).

Firstly, let us assume that \( M_D \) in Eq.(7) is diagonalized as \( M_D \propto \text{diag}(m_e, m_\mu, m_\tau) \) for simplicity. That is,

\[
f_{ij}^{(p)}\phi_2 = \alpha \text{ diag}(m_e, m_\mu, m_\tau)
\]

in Eq.(2), where \( \alpha \) is some constant. You should be careful not to confuse this with \( f_{ij}^{(l)}\phi_1 \). The reason for this choice is to realize Eq.(1) naively. In this case \( f_{ij}^{(l)}\phi_1 = \text{diag}(m_e, m_\mu, m_\tau) \) also must be satisfied. Then from Eqs.(7) and (9) we obtain the matrix \( M_R \) as

\[
M_R = -\alpha^2 \begin{pmatrix} m_e^2m_{1m_2} & m_em_\mu\sqrt{\frac{1}{2m_1m_2}} & -m_edm_\tau\sqrt{\frac{1}{2m_1m_2}} \\ m_em_\mu\sqrt{\frac{1}{2m_1m_2}} & m_\mu^2 & -m_\mu m_\tau \frac{1}{2m_3} \\ -m_em_\tau\sqrt{\frac{1}{2m_1m_2}} & m_\mu m_\tau \frac{1}{2m_3} & m_\tau^2 \end{pmatrix}. \tag{13}
\]
This has a rather complicated structure and is unlikely to possess some symmetry. So we adopt the other option that $M_D$ and $M_R$ have the same structure as $M_{\text{light}}$ in Eq.(12). Same structure means the same relationships between the components of the respective matrix. It is remarkable that this assumption is consistent with seesaw mechanism (7). That is, if we accept

$$M_D = \begin{pmatrix} 0 & A_D & -A_D \\ A_D & B_D & C_D \\ -A_D & C_D & B_D \end{pmatrix}, \quad M_R = \begin{pmatrix} 0 & A_R & -A_R \\ A_R & B_R & C_R \\ -A_R & C_R & B_R \end{pmatrix},$$

then $M_{\text{light}}$ in Eq.(12) is given by

$$A = \frac{A_D^2}{A_R},$$

$$B = -\frac{B_R - C_R}{2A_R^2}A_D^2 + \frac{B_R - C_R}{A_R}A_D + \frac{(B_D + C_D)^2}{2(B_R + C_R)},$$

$$C = \frac{B_R - C_R}{2A_R^2}A_D^2 - \frac{B_R - C_R}{A_R}A_D + \frac{(B_D + C_D)^2}{2(B_R + C_R)}.$$  \hspace{1cm} (15)

This matrix structure is different from that of the quark mass matrix by Fritzsch [5], though there is no need for these to coincide. Then there arises a question to what extent this matrix structure Eq.(14) is unique under the following assumption:

(a) $M_{\text{light}}$, $M_D$ and $M_R$ have the same structure and that
(b) Their $(1,1)$ components are zeros.

In (a) their structure is not necessarily identical to Eq.(14).

Running the remaining components of $M_D$ and $M_R$ as free parameters, the seesaw mechanism (7) under the conditions (a) and (b) constrains the allowed mass matrix in the following four types.

(1) $\begin{pmatrix} 0 & A & A \\ A & B & C \\ A & C & C \end{pmatrix}$  (2) $\begin{pmatrix} 0 & A & A \\ A & B & B \\ A & B & C \end{pmatrix}$  (3) $\begin{pmatrix} 0 & A & A \\ A & B & C \\ A & C & B \end{pmatrix}$  (4) $\begin{pmatrix} 0 & A & -A \\ A & B & C \\ -A & C & B \end{pmatrix}$

(4) is the structure mentioned above. (3) is transformed to (4) by the interchange of $C$ to $-C$ and these are physically equivalent as follows. So far we have set $\theta_2$ to be $\frac{\pi}{4}$. If we leave $\theta_2$ as a free parameter and keep the assumption (11), then $M_{\text{light}}$ is reduced to

$$\begin{pmatrix} 0 & c_2\sqrt{m_1m_2} & -s_2\sqrt{m_1m_2} \\ c_2\sqrt{m_1m_2} & -(m_1 + m_2)c_2^2 + m_3s_2^2 & (m_1 - m_2 + m_3)c_2s_2 \\ -s_2\sqrt{m_1m_2} & (m_1 - m_2 + m_3)c_2s_2 & -(m_1 + m_2)s_2^2 + m_3c_2^2 \end{pmatrix}$$

\hspace{1cm} (16)

Therefore (3) and (4) are corresponding to $s_2 = -\frac{\pi}{4}$ and $s_2 = \frac{\pi}{4}$, respectively. $\theta_2$ has been determined from the mixing factor $\sin^22\theta_2 \sim 1$ and they are equivalent.
(1) and (2) are also substantially same and are enforced to \( m_3 = 0 \). \( m_3 \) is the heaviest neutrino mass and (1) and (2) are rejected. Thus we obtain the unique structure (14) provided that we adopt the assumptions (a) and (b).

Unfortunately our assumptions are not sufficient to realize Eq.(1) straightforwardly since Eqs.(1), (12) and (15) can not fix the parameters \( A, B, C \) with subscript \( D \) and \( R \).

Finally we consider the physical consequences of \( M_{\text{light}} \) in Eq.(11), especially to the neutrinoless double beta decay.

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**Fig. 1**

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The amplitude of this process is proportional to \( < m_\nu > \) defined by [12]

\[
<m_\nu> \equiv |\sum_{j=1}^{3} U_{1j}^2 m_j|
\]  \hspace{1cm} (17)

Here \( U_{ij} \) is in our theory given by Eq.(8) and

\[
<m_\nu> = |c_1^2 m_1 + s_1^2 m_2|
\]  \hspace{1cm} (18)

From the solar neutrino experiment \( m_2 \) is estimated to be \( m_2 \sim O(10^{-3} \text{eV}) \), whereas the experimental upper bound of \( < m_\nu > \) is of the several \( eV \) order. Therefore our estimation is lower than the present upper bound by at least \( 10^{-3} \) times.

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Figure Caption
Fig.1 Feynman diagram of the neutrinoless double beta decay. For the helicity matching of the Majorana neutrino $N_j$ emerges the small factor $\frac{m_j}{E}$. 
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