Heavy Quark Production at the TEVATRON
in the Semihard QCD Approach
and the Unintegrated Gluon Distribution

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Abstract

Processes of heavy quark production at TEVATRON energies are considered using the semihard ($k_T$ factorization) QCD approach with emphasis of the BFKL dynamics of gluon distributions. We investigate the dependence of the $p_T$ distribution of heavy quark production (presented in the form of integrated cross-sections) on different forms of the unintegrated gluon distribution. The theoretical results are compared with recent D0 and CDF experimental data on beauty production.

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1 Introduction

It is very well known that from heavy ($c$- and $b$-) quark production processes one can obtain unique information on gluon structure function of the proton and the three gluon QCD vertex because of the dominance of the gluon-gluon fusion subprocess in the framework of QCD. These issues are important for physics at future colliders (such as LHC): many processes at these colliders will be determined by small $x$ gluon distributions.

Recently D0 and CDF Collaborations have reported [1, 2] new experimental data on the cross section of inelastic open beauty production at the TEVATRON energies. A comparison of these results with NLO pQCD calculations shows that the calculations underestimate the cross section by a factor of $\sim 2$. Therefore, it would be certainly reasonable to try a different way.

In the energy range of modern colliders (HERA, TEVATRON and LHC) the interaction dynamics is governed by the properties of parton distributions in the small $x$ region. This domain is characterized by the double inequality $s \gg \mu^2 \simeq \hat{s} \gg \Lambda^2$, which shows that the typical parton interaction scale $\mu$ (mass $m_c$ or $p_T$ of heavy quark) is much higher than the QCD parameter $\Lambda$, but is much lower than the total center of mass energy $\sqrt{s}$. The situation is therefore classified as “semihard”: the processes occur in small $x$ region ($x \simeq M^2/s \ll 1$) and the cross sections of heavy quark production processes are determined by the behavior of gluon distributions in this region.

It is known also that in the small $x$ region the factorization of QCD subprocess cross section and parton structure functions in a hadron assumed by the standard parton model is broken [3]-[7]. The resummation [3, 4, 6, 7] of the terms $[\ln(\mu^2/\Lambda^2) \alpha_s]^n$, $[\ln(1/x) \alpha_s]^n$ and $[\ln(1/x) \alpha_s]^n$ in the semihard ($k_T$-factorization) approach (SHA) of QCD results in the unintegrated gluon distributions $\Phi(x, q_T^2, \mu)$, which determine the probability to find a gluon carrying the longitudinal momentum fraction $x$ and transverse momentum $q_T$ at the probing scale $\mu^2$. They obey the BFKL equation [4] and reduce to the conventional gluon density once the $q_T$ dependence is integrated out ($\mu^2 \equiv Q^2$):

$$\int_0^{Q^2} \Phi(x, q_T^2, Q_0^2) \, dq_T^2 = x G(x, Q^2).$$ (1)

To calculate the cross section of a physical process, the unintegrated functions $\Phi_i$ have to be convoluted with off-mass shell matrix elements corresponding to the relevant partonic subprocesses. In the off-mass shell matrix element the virtual gluon polarization tensor is taken in the form of the SHA prescription [3, 4]:

$$L_{\mu\nu}^{(g)} = e^\mu_1 e^\nu_2 = p^\mu p^\nu x^2 / |q_T|^2 = q_T^\mu q_T^\nu / |q_T|^2$$ (2)

Nowadays, the significance of the $k_T$ factorization (semihard) approach becomes more and more commonly recognized. Its applications to a variety of photo-, lepto- and hadroproduction processes are widely discussed in the literature [5]-[11]. Remarkable agreement is found between the data and the theoretical calculations regarding the photo- [12] and electroproduction [13, 14] of $D^*$ mesons, forward jets [15, 16], as well as for specific kinematic correlations observed in the associated $D^*+\text{jets}$ photoproduction [17] at HERA and also the hadroproduction of beauty [18, 19], $\chi_c$ and $J/\Psi$ [20, 21] at the Tevatron. The theoretical
predictions made in ref. \[22\] has triggered a dedicated experimental analysis \[23\] of the $J/\Psi$ polarization (i.e., spin alignment) at HERA conditions.

Here study of heavy quark production processes in $p\bar{p}$–collisions at TEVATRON with the SHA is presented. Similar investigations have been done by many authors \[5, 18, 19\] earlier. But in the recent papers \[18, 19\] obtained results contradict each other.

We investigate the sensitivity of the inelastic cross section of heavy quark production processes at TEVATRON to different unintegrated gluon distributions. Special attention is given to the unintegrated gluon distributions obtained from BFKL evolution equation. The outline of our paper is the following. In sect. 2, we give the formulas for the cross sections of heavy quark hadroproduction in the SHA of QCD. Then, in sect. 3, we describe the unintegrated distributions which we use for our calculations. In sect. 4 we give the matrix elements for gluon-gluon fusion QCD subprocesses. In sect. 5, we present the results of our calculations. Finally, in sect. 6, we give some conclusions\[4\].

2 SHA QCD Cross-Section for Heavy Quark Production in $p\bar{p}$ Collisions

We calculate the total and differential cross sections of heavy quark production $p\bar{p} \rightarrow Q\bar{Q}X$ via the gluon-gluon fusion QCD subprocess (Fig.1) in the framework of the SHA. First of all we take into account the transverse momentum of initial gluons ($q_{1,2T}$), their virtualities ($q_{1,2}^2 = -q_{1,2T}^2$) and the alignment of gluon polarization vectors along their transverse momenta given by (2) \[3, 4, 6, 7\]. Let us define Sudakov variables of the process $p\bar{p} \rightarrow Q\bar{Q}X$.

\[4\]The obtained results have been reported at Workshop DIS 2001, Bologna, 27 April - 1 May 2001.
\begin{equation}
p_1 = \alpha_1 p_1 + \beta_1 p_2 + p_{1T}, \quad p_2 = \alpha_2 p_1 + \beta_2 p_2 + p_{2T}
\end{equation}
\begin{equation}
q_1 = x_1 p_1 + q_{1T}, \quad q_2 = x_2 p_2 + q_{2T},
\end{equation}
where
\begin{equation}
p_1^2 = p_2^2 = M^2, \quad q_1^2 = q_{1T}^2, \quad q_2^2 = q_{2T}^2,
\end{equation}
\begin{equation}
p_1, p_2 \text{ and } q_1, q_2 \text{ are 4-momenta of the heavy quarks and the gluons respectively, } p_{1T}, \quad p_{2T} \text{ and } q_{1T}, \quad q_{2T} \text{ are transverse 4-momenta of quarks and gluons. In the center of mass frame of colliding particles we can write } P_1 = (E, 0, 0, E), \quad P_2 = (E, 0, 0, -E), \text{ where } E = \sqrt{s}/2, \quad P_1^2 = P_2^2 = 0 \text{ and } (P_1 P_2) = s/2 . \text{ Sudakov variables are expressed as follows:}
\end{equation}
\begin{equation}
\alpha_1 = \frac{M_{1T}}{\sqrt{s}} \exp(y_1^*), \quad \alpha_2 = \frac{M_{2T}}{\sqrt{s}} \exp(y_2^*),
\end{equation}
\begin{equation}
\beta_1 = \frac{M_{1T}}{\sqrt{s}} \exp(-y_1^*), \quad \beta_2 = \frac{M_{2T}}{\sqrt{s}} \exp(-y_2^*),
\end{equation}
where $M_{1,2T} = M^2 + p_{1,2T}^2$, $y_{1,2}^*$ are rapidities of heavy quarks, $M$ is heavy quark mass. From conservation laws we can easily obtain the following relations:
\begin{equation}
q_{1T} + q_{2T} = p_{1T} + p_{2T}, \quad x_1 = \alpha_1 + \alpha_2, \quad x_2 = \beta_1 + \beta_2
\end{equation}
The differential cross section of heavy quark production has the following form:
\begin{equation}
d\sigma(p\bar{p} \rightarrow Q\bar{Q} X) = \frac{d\sigma}{d\alpha_1} \Phi(x_1, q_{1T}^2, Q^2) \frac{d\alpha_2}{2\pi} d\alpha_1^2 \times
\end{equation}
\begin{equation}
\times \frac{d\alpha_2}{2\pi} \Phi(x_2, q_{2T}^2, Q^2) \frac{d\alpha_2}{2\pi} d\alpha_2, \quad d\hat{\sigma}(g^*g^* \rightarrow Q\bar{Q}),
\end{equation}
where $\Phi(x, q^2, Q^2)$ is an unintegrated gluon distribution in the proton, $d\hat{\sigma}(g^*g^* \rightarrow Q\bar{Q})$ is the differential cross section of gluon-gluon fusion subprocess. We used the following form:
\begin{equation}
d\hat{\sigma}(g^*g^* \rightarrow Q\bar{Q}) = \frac{(2\pi)^4}{2s} \sum |M|^2_{SHA}(g^*g^* \rightarrow Q\bar{Q}) \frac{d^3 p_1}{(2\pi)^3 2p_1^0} \frac{d^3 p_2}{(2\pi)^3 2p_2^0} \delta^{(4)}(q_1 + q_2 - p_1 - p_2),
\end{equation}
where $\sum |M|^2_{SHA}(g^*g^* \rightarrow Q\bar{Q})$ is the off mass shell matrix element and
\begin{equation}
\delta^{(4)}(q_1 + q_2 - p_1 - p_2) = \delta(q_1^0 + q_2^0 - p_1^0 - p_2^0) \delta(q_{1T} + q_{2T} - p_{1T} - p_{2T}) \delta(q_3^3 + q_3^3 - p_3^3 - p_3^3).
\end{equation}
In (7) $\sum$ indicates an averaging over polarizations of initial gluons and a sum over polarizations of final quarks. We have also
\begin{equation}
\frac{d^3 p_1}{(2\pi)^3 2p_1^0} \frac{d^3 p_2}{(2\pi)^3 2p_2^0} = \frac{d^3 p_{1T}}{2(2\pi)^3} dy_1^* dy_1^* dy_2^* dy_2^*.
\end{equation}
Integrating out $p_{2T}^2, x_1$ and $x_2$ dependences in (6) and (9) with accounting of (7) and (8) we obtain the following formula for the differential cross section of the process $p\bar{p} \rightarrow Q\bar{Q} X$ in the framework of the SHA:
\begin{equation}
d\sigma(p\bar{p} \rightarrow Q\bar{Q} X) = \frac{1}{16\pi (x_1 x_2 s)^2} \Phi(x_1, q_{1T}^2, Q^2) \Phi(x_2, q_{2T}^2, Q^2) \times
\end{equation}
\begin{equation}
\times \sum |M|^2_{SHA}(g^*g^* \rightarrow Q\bar{Q}) dy_1^* dy_2^* dp_{1T}^2 dp_{2T}^2 dq_{1T}^2 dq_{2T}^2 \frac{d\alpha_1}{2\pi} \frac{d\alpha_2}{2\pi} \frac{d\alpha_3}{2\pi}.
\end{equation}
If we take the limit \( q_1T \to 0, q_2T \to 0 \) and if we average (10) over the transverse directions of the vectors \( \vec{q}_1T \) and \( \vec{q}_2T \), we obtain the formula for the the differential cross section of the process \( p\bar{p} \to Q\bar{Q}X \) in the standard parton model:

\[
\frac{d\sigma(p\bar{p} \to q\bar{q} X)}{d\sigma} = \frac{x_1G(x_1, Q^2)x_2G(x_2, Q^2)}{16\pi(x_1x_2s)^2} \sum |M|_{PM}^2(gg \to Q\bar{Q})dy_1^* dy_2^* d\vec{q}_1T^2, \tag{11}
\]

where \( \sum |M|_{PM}^2(gg \to Q\bar{Q}) \) is matrix elements of gluon-gluon fusion QCD subprocess in the standard parton model (SPM). Here \( \sum \) indicates averaging over polarizations of on-shell initial gluons and a sum over polarizations of final quarks. We average over the transverse directions of \( \vec{q}_1T \) and \( \vec{q}_2T \) using the following expression:

\[
\int d\vec{q}_1^2 T \int \frac{d\phi_1}{2\pi} \Phi(x_1, \vec{q}_1T^2) \int d\vec{q}_2^2 T \int \frac{d\phi_2}{2\pi} \Phi(x_2, \vec{q}_2T^2) \sum |M|_{SHA}^2(g^*g^* \to Q\bar{Q}) = \frac{1}{2} g^{\mu\nu} \tag{13}
\]

### 3 Unintegrated Gluon Distribution Functions

Various parametrizations of the unintegrated gluon distribution used in calculations are discussed below. First, as in the publication \[3, 5\], we used the following phenomenological parametrization (LRSS-parametrization) \[3, 5\]:

\[
\Phi(x, \vec{q}_T^2) = \Phi_0 \frac{0.05}{0.05 + x}(1 - x)^3 f_1(x, \vec{q}_T^2), \tag{14}
\]

where

\[
f_1(x, \vec{q}_T^2) = \begin{cases} 
1, & \text{if } \vec{q}_T^2 \leq q_0^2(x), \\
(q_0^2(x)/\vec{q}_T^2)^2, & \text{if } \vec{q}_T^2 > q_0^2(x)
\end{cases} \tag{15}
\]

with \( q_0^2(x) = q_0^2 + \Lambda^2 \exp(3.56 \sqrt{\ln(x_0/x)}) \), \( q_0^2 = 2 \text{ GeV}^2 \), \( \Lambda = 56 \text{ MeV} \), \( x_0 = 1/3 \). The value of the parameter \( q_0^2(x) \) can be considered as a typical transverse momentum of the partons in the parton cascade which leads to natural infrared cut-off in semihard approach. The normalization factor of the structure function \( \Phi(x, \vec{q}_T^2)(\Phi_0 = 0.97 \text{ mb}) \) was obtained in \[5\] from the beauty production in \( p\bar{p} \) at \( Sp\bar{p}S \) energy\[4\]. The effective gluon distribution \( xG(x, Q^2) \), which was obtained from eq. (1) with (14) and (15), increases at not very low \( x(0.01 < x < 0.15) \) as \( xG(x, Q^2) \sim x^{-\Delta} \), where \( \Delta \approx 0.5 \) corresponds to the QCD pomeron singularity given by summation of leading logarithmic contributions \( (\alpha_s \ln \frac{1}{x})^n \). This increase continues up to \( x = x_0 \), where \( x_0 \) is a solution of the equation \( q_0^2(x_0) = Q^2 \). In the region \( x < x_0 \), there is saturation of the gluon distribution function: \( xG(x, Q^2) \approx \Phi_0 Q^2 \) (curves 3 in Fig.2).

\[5\] The experimental data \[24\] relate to relatively low energy for applicability of the SHA.
Another parametrization is based on the numerical solution of the BFKL evolution equations (RS–parametrization), which has the following form \[25\]:

\[ \Phi(x, q^2) = \frac{1}{4\sqrt{2}\pi^3} \frac{a_1}{a_2 + a_3 + a_4} \left[a_2 + a_3 \left(\frac{Q_0^2}{q^2}\right) + \left(\frac{Q_0^2}{q^2}\right)^2 + a_5 \epsilon + \ln(1/x)\right] \right]^{1/2} [1 - a_6x^{a_7} \ln(q^2/a_8)(1 + a_{11}x)(1 - x)^{a_9 + a_{10} \ln(q^2/a_8)}], \tag{16} \]

where

\[ C_q = \begin{cases} 1, & \text{if } q^2 < q_0(x), \\ q_0(x)/q^2, & \text{if } q^2 > q_0(x). \end{cases} \tag{17} \]

All parameters (see \[25\]) \((a_1 - a_{11}, \alpha, \beta, \epsilon\) and \(\epsilon\)) were found by minimization of the differences between left hand and right-hand of the BFKL-type equation for unintegrated gluon distribution \(\Phi(x, q^2)\) at \(Q_0^2 = 4\) GeV\(^2\).

Then we use the results of a BFKL-like parameterization of the unintegrated gluon distribution \(\Phi(x, q^2, \mu^2)\), according to the prescription given in \[26\]. The proposed method lies upon a straightforward perturbative solution of the BFKL equation where the collinear gluon density \(xG(x, \mu^2)\) from the standard GRV set \[27\] is used as the boundary condition in the integral (1). Technically, the unintegrated gluon density is calculated as a convolution of collinear gluon density \(G(x, \mu^2)\) with universal weight factors \[26\]:

\[ \Phi(x, q_T^2, \mu^2) = \int_x^1 G(\eta, q_T^2, \mu^2) \frac{x}{\eta} G\left(\frac{x}{\eta}, \mu^2\right) d\eta, \tag{18} \]

where

\[ G(\eta, q_T^2, \mu^2) = \frac{\bar{\alpha}_s}{\eta q_T^2} J_0(2\sqrt{\bar{\alpha}_s \ln(1/\eta) \ln(\mu^2/q_T^2)}), \quad q_T^2 < \mu^2, \tag{19} \]

\[ G(\eta, q_T^2, \mu^2) = \frac{\bar{\alpha}_s}{\eta q_T^2} I_0(2\sqrt{\bar{\alpha}_s \ln(1/\eta) \ln(q_T^2/\mu^2)}), \quad q_T^2 > \mu^2, \tag{20} \]

where \(J_0\) and \(I_0\) stand for Bessel functions (of real and imaginary arguments, respectively), and \(\bar{\alpha}_s = 3\alpha_s/\pi\). The parameter \(\bar{\alpha}_s\) is connected with the Pomeron trajectory intercept: \(\Delta = \bar{\alpha}_s 4 \ln 2\) in the LO and \(\Delta = \bar{\alpha}_s 4 \ln 2 - N\bar{\alpha}_s^2\) in the NLO approximations, respectively, where \(N \sim 18\) \[28\]. The latter value of \(\Delta\) have dramatic consequences for high energy phenomenology \[29\]. However some resummation procedures proposed in the last years lead to positive value of \(\Delta(\sim 0.2 - 0.3)\) \[30, 31\]. Therefore in our calculations with (18) we used only the solution of LO BFKL equation and considered \(\Delta\) as a free parameter varying it from 0.166 to 0.53. Pomeron intercept parameter \(\Delta = 0.35\) was obtained from the description of \(p_T\) spectrum of \(D^*\) meson electroproduction at HERA \[12\]. We used this value of the the parameter \(\Delta\) in present paper.

Finally we tried the so called differential unintegrated gluon structure function of a proton, which could be obtained by differentiation of the collinear gluon density \(xG(x, Q^2)\) \[3\] \[4\] \[32\], for example one from the standard GRV set \[27\], according to (1)

Of course LRSS and RS parametrizations are BFKL - type too.
Figure 2: Effective gluon densities $xG(x, Q^2)$ obtained from different unintegrated gluon distribution. Curve 1 correspond to the GRV collinear gluon density [27], curves 2, 3, 4, 5 correspond to RS [25] (at $Q^2_0 = 4 \text{ GeV}^2$), LRSS [5] (at $Q^2_0 = 2 \text{ GeV}^2$), BFKL [26] (at $Q^2_0 = 1 \text{ GeV}^2$) parametrizations and differential (at $Q^2_0 = 1 \text{ GeV}^2$) gluon distribution.
Figure 3: QCD diagramms of gluon-gluon fusion subprocesses for heavy quark production.

\[ \Phi(x, q_T^2, Q^2) = \frac{d x G(x, Q^2)}{d \ln Q^2} \bigg|_{Q^2=q_T^2} \]  
\[ \text{(21)} \]

In expression (1) the accounting of the contribution of the unintegrated gluon distribution to integral at low \( q_T^2 \) region \( (0 < q_T^2 < Q_0^2) \) was done by the collinear gluon density \( x G(x, Q_0^2) \), where \( Q_0^2 = (1 \div 4) \text{ GeV}^2 \) in dependence on different parametrizations of unintegrated gluon distribution.

Fig. 2 shows the \( x \) dependence of the effective gluon density \( x G(x, Q^2) \) at fixed values of \( Q^2 \) obtained with help of the definition (1) for different parametrizations of unintegrated gluon distribution discussed above. The curve (1) corresponds to the standard GRV parametrization of collinear gluon distribution function. Curve 2, 3, 4, 5 correspond to effective gluon densities obtained from unintegrated gluon distribution functions \( \Phi(x, q_T^2) \) with help of the definition (1) for RS, LRSS, BFKL parametrizations and the definition (21) at different values of \( Q_0^2 \) parameter.

The effective gluon density \( x G(x, Q^2) \) obtained for the LRSS parametrization of unintegrated gluon distribution \( \Phi(x, q_T^2) \) differs strongly from other gluon densities normalized to GRV collinear gluon density. It is result of the normalization of the LRSS parametrization of \( \Phi(x, q_T^2) \) obtained from the \( Sp\bar{p}S \) experimental data for \( b\bar{b} \) production in \( p\bar{p} \) collisions [24].

### 4 Matrix Elements for Gluon-Gluon Fusion QCD Subprocesses

The subprocess \( g^* g^* \to Q\bar{Q} \) is described in the LO pQCD by three diagramms shown in Fig. 3. In according with the standard Feynman rules for QCD diagramms:
\[ M_A = \bar{u}(p_1) (-ig\gamma^\mu) \epsilon_\mu(q_1) i\frac{\hat{p}_1 - \hat{q}_1 + m}{(p_1 - q_1)^2 - m^2} (-ig\gamma^\nu) \epsilon_\nu(q_2) v(p_2), \]

\[ M_B = \bar{u}(p_1) (-ig\gamma^\nu) \epsilon_\nu(q_2) i\frac{\hat{p}_1 - \hat{q}_2 + m}{(p_1 - q_2)^2 - m^2} (-ig\gamma^\mu) \epsilon_\mu(q_1) v(p_2), \]

\[ M_C = \bar{u}(p_1) C^{\mu\nu\lambda}(-q_1, -q_2, q_1 + q_2) g^2 \frac{\epsilon_\mu(q_1) \epsilon_\nu(q_2)}{(q_1 + q_2)^2} \gamma_\lambda v(p_2), \]  \hfill (22)

where \( \epsilon_\mu(q_1) \) and \( \epsilon_\nu(q_2) \) are the polarization vectors of the initial gluons, \( C^{\mu\nu\lambda}(q_1, q_2, q_3) \) is the standard QCD three gluon vertex:

\[ C^{\mu\nu\lambda}(q_1, q_2, q_3) = i((q_2 - q_1)\lambda^\lambda) g^{\mu\nu} + (q_3 - q_2)^\mu g^\nu\lambda + (q_1 - q_3)^\nu g^\lambda\mu. \]  \hfill (23)

The Mandelstam variables for gluon-gluon fusion subprocess \( g^*g^* \rightarrow Q\bar{Q} \) are

\[ \hat{s} = (q_1 + q_2)^2 = (p_1 + p_2)^2, \]

\[ \hat{t} = (q_2 - p_2)^2 = (p_1 - q_1)^2, \]

\[ \hat{u} = (q_1 - p_2)^2 = (p_1 - q_2)^2, \]  \hfill (24)

and

\[ \hat{s} + \hat{t} + \hat{u} = 2M^2 + q_{1T}^2 + q_{2T}^2. \]  \hfill (25)

Let present the matrix elements in the following forms:

\[ M_{A,B,C} = \epsilon_\mu(q_1) \epsilon_\nu(q_2) M_{A,B,C}^{\mu\nu}, \]  \hfill (26)

then averaging over initial gluon polarizations and summing over final quark polarizations we have

\[ \sum |M|^2 = C_A \sum |M|^2_A + C_B \sum |M|^2_B + C_C \sum |M|^2_C + 2C_{AB} \sum |M|^2_{AB} + 2C_{AC} \sum |M|^2_{AC} + 2C_{BC} \sum |M|^2_{BC}, \]  \hfill (27)

where

\[ \sum |M|^2_i = \frac{1}{4} L_{\mu\alpha}^{(g)}(q_1) L_{\nu\beta}^{(g)}(q_2) M_i^{\mu\nu} M_i^{*\alpha\beta} \]  \hfill (28)

Here the index \( i = A, B, C, AB, AC, BC \) and the color factors are \( C_A = C_B = 1/12, \)

\( C_A = C_B = 1/12, C_C = 3/16, C_{AB} = -1/96, C_{AC} = C_{BC} = 3/32 \) accordingly. The effective

gluon polarization tensor \( L_{\mu\nu}^{(g)}(q) \) is taken from (2).

The calculation of \( \sum |M|^2_{SPM}(g^*g^* \rightarrow Q\bar{Q}) \) was done analytically by REDUCE system.

The obtained results coincide with the ones from ref. [1].

For the calculations of \( \sum |M|^2_{\tilde{P}M}(gg \rightarrow Q\bar{Q}) \) in the framework of the SPM we used the following form of the gluon polarization tensor in axial gauge:

\[ L_{\mu\nu}^{(g)}(q_1, q_2) = -g^{\mu\nu} + \frac{q_1^\mu q_2^\nu + q_1^\nu q_2^\mu}{(q_1, q_2)} - q_2 \cdot \frac{q_1 q_2}{(q_1, q_2)^2}. \]  \hfill (29)

Obtained expression for \( \sum |M|^2_{\tilde{P}M}(gg \rightarrow Q\bar{Q}) \) coincides with the results of ref. [1].
5 Results of Calculations

The calculations of the heavy quark hadroproduction cross section in the SHA have been made according to eqn. (10) for $\vec{q}_{T}^{2} > q_{0}^{2}$ GeV$^{2}$. For $\vec{q}_{T}^{2} \leq q_{0}^{2}$ GeV$^{2}$ we set $|\vec{q}_{T}| = 0$ in the matrix elements of subprocesses, take $\sum |M|_{PM}^{2}(gg \rightarrow QQ)$ instead of $|M|_{SHA}^{2}(g^{*}g^{*} \rightarrow QQ)$ and use the eqn. (11) of the standard parton model (SPM). The choice of the critical value of parameter $\vec{q}_{T}^{2} = q_{0}^{2} = (1 \div 4)$ GeV$^{2}$ is determined by the requirement of the small value of $\alpha_{s}(\vec{q}_{T}^{2})$ in the region $\vec{q}_{T}^{2} > (1 \div 4)$ GeV$^{2}$, where in fact $\alpha_{s}(\vec{q}_{T}^{2}) \leq 0.26$.

The results of our calculations for the cross sections of $b\bar{b}$ production at TEAVTRON are shown in Figs. 4 $\div$ 8. These figures show the $p_{T}^{min}$ dependence of the $b\bar{b}$ production cross section at TEVATRON energies calculated with $m_{b} = 4.75$ GeV, $\Lambda_{QCD} = 150$ MeV and $|y_{1}| < 1$ (Fig. 4) [7], and $|y_{1}| < 1$, $|y_{2}| < 1$, $p_{2T} > 6.5$ GeV (Fig. 5) [2] [4]. In Fig. 4, 5 the curves 1 correspond to the calculations in the framework of the SPM using the gluon density $xG(x, Q^{2})$ from the standard GRV set [27]. The curves 2, 3, 4, 5 are the results of calculations in the framework of the SHA using the unintegrated gluon distribution in RS ($Q_{0}^{2} = 4$ GeV$^{2}$), LRSS ($Q_{0}^{2} = 2$ GeV$^{2}$), BFKL ($Q_{0}^{2} = 1$ GeV$^{2}$) parametrizations and the one (22) ($Q_{0}^{2} = 1$ GeV$^{2}$) accordingly.

One can see that the SHA curves 2, 4 and 5 describe the D0 and CDF experimental data very well in comparison with the SPM results (curves 1). The LRRS parameterization of the unintegrated gluon distribution overestimates the experimental $b\bar{b}$ production cross section [4] [4] [7].

We tried to describe these experimental data with help the LRSS parametrization fitting the $Q_{0}^{2}$, $m_{b}$ and $\Lambda_{QCD}$ parameters in region: $1$ GeV$^{2} \leq Q_{0}^{2} \leq 4$ GeV$^{2}$, $4.5$ GeV $\leq m_{b} \leq 5.0$ GeV and $100$ MeV $\leq \Lambda_{QCD} \leq 250$ MeV. We obtained well agreement of the theoretical cross section does not depend on $p_{T}^{min}$ in the region $1$ GeV $\leq p_{T}^{min} \leq 5$ GeV approximately.
Figure 5: The $b\bar{b}$ production cross section at $\sqrt{s} = 1.8$ TeV. Curves 1 - 5 are the same as in Fig. 2.

Figure 6: The $b\bar{b}$ production cross section at $\sqrt{s} = 1.8$ TeV. Curves 3 and $3^*$ correspond to the SHA calculations with the LRSS unintegrated gluon distribution at $m_b = 4.75$ GeV, $\Lambda_{QCD} = 150$ MeV, $Q_0^2 = 2$ GeV$^2$ and $m_b = 5.0$ GeV, $\Lambda_{QCD} = 100$ MeV, $Q_0^2 = 1$ GeV$^2$ accordingly.
Figure 7: The $b\bar{b}$ production cross section at $\sqrt{s} = 1.8$ TeV. Curves 3 and $3^*$ are the same as in Fig. 6.

Figure 8: The $b\bar{b}$ production cross section at $\sqrt{s} = 1.8$ TeV. Curves 2 and $2^*$ correspond to the SHA calculations with the RS parametrization of unintegrated gluon distribution using the matrix elements (22) and the ones from ref. [3].
results (curves 3′ in Figs. 6 and 7) with the TEVATRON experimental data at the following values of the parameters: $Q_0^2 = 4 \text{ GeV}^2$, $m_b = 5.0 \text{ GeV}$ and $\Lambda_{QCD} = 100 \text{ MeV}$.

In Fig. 8 we show the results obtained using the off mass shell matrix elements of gluon-gluon fusion subprocesses in the form (22) and the ones from ref. with using the RS parametrization of unintegrated gluon distribution as example. We see that these results coincide very well.

6 Conclusions

We considered the process of inelastic heavy quark production at TEVATRON in the framework of the semihard QCD approach with emphasis on the BFKL dynamics of gluon distributions. We investigated the cross section $\sigma(p_T > p_{T}^{min})$ of inelastic $b$–quark hadroproduction as a function of different unintegrated gluon distributions. It is shown that the description of the $b$–quark inelastic cross section at TEVATRON energies is achieved in the cases of the RS, BFKL parameterizations and also the parametrization (22) at realistic values of QCD parameters ($m_b = 4.75 \text{ GeV}$ and $\Lambda_{QCD} = 150 \text{ MeV}$). The LRSS parametrization describes the D0 and CDF experimental data with another values of parameters ($m_b = 5.0 \text{ GeV}$ and $\Lambda_{QCD} = 100 \text{ MeV}$) in comparison with the description of the $b\bar{b}$ quark photoproduction data at HERA in the framework of the SHA.

One of the main goals of our investigations consist of a search in the framework of the SHA of the ”universal” unintegrated gluon distribution. In our opinion the studies of many high energy heavy quark production processes in the framework of the SHA have showed that the so called BFKL unintegrated gluon distribution function is one of the candidates for a role of universal gluon distribution.

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