CALIBRATION OF A 3D LASER RANGEFINDER AND A CAMERA BASED ON OPTIMIZATION SOLUTION

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ABSTRACT. The calibration of a 3D laser rangefinder (LRF) and a camera is a key technique in the field of computer vision and intelligent robots. This paper proposes a new method for the calibration of a 3D LRF and a camera based on optimization solution. The calibration is achieved by freely moving a checkerboard pattern in front of the camera and the 3D LRF. The images and the 3D point clouds of the checkerboard pattern in various poses are collected by the camera and the 3D LRF respectively. By using the images, the intrinsic parameters and the poses of the checkerboard pattern are obtained. Then, two kinds of geometric constraints, line-to-plane constraints and plane-to-plane constraints, are constructed to solve the extrinsic parameters by linear optimization. Finally, the intrinsic and extrinsic parameters are further refined by global optimization, and are used to compute the geometric mapping relationship between the 3D LRF and the camera. The proposed calibration method is evaluated with both synthetic data and real data. The experimental results show that the proposed calibration method is accurate and robust to noise.

1. Introduction. Today, 3D color laser ranging technology has gradually been used in digitizing the real world for many application domains, such as autonomous navigation, object identification, and industrial inspection. This technology collects 3D point clouds and 2D images by using a laser rangefinder (LRF) and a color camera respectively, and then fuses 3D point clouds and 2D images into 3D color point clouds, which can record both geometry and color information of objects and describe the world more realistically.

The 2D image and 3D point cloud of an object belong to different modes. They are collected by different devices and express different aspects of meanings of the object. Thus, the fusion of 3D point clouds and images is multimodal data fusion. In the 3D color laser ranging technology, multimodal data fusion is a crucial part,
which fuses 3D point clouds and 2D images into 3D color point clouds. It builds the geometric mapping relationship (perspective projection) between a LRF and a camera, and dyes each 3D spatial point with the color of its corresponding pixel in the 2D image. This geometric mapping relationship includes two parts: intrinsic parameters and extrinsic parameters. The intrinsic parameters include the focal length, scale factor, and principal point. The extrinsic parameters include the rotation matrix and translation vector between the LRF and the camera. In general, the determination of the geometric mapping relationship between a LRF and a camera is called the calibration of a LRF and a camera.

In most situations, the calibration of a LRF and a camera takes three steps. Firstly, the intrinsic parameters are computed by the camera calibration. Then, the extrinsic parameters are obtained by the extrinsic calibration of the LRF and the camera. Finally, the geometric mapping relationship between the LRF and the camera is calculated by the intrinsic and extrinsic parameters. In this process, no matter which kind of calibration method is used, the most important factor to determine the calibration accuracy is the calibration data. The calibration data refer to the geometric elements (points, lines, and planes) in different coordinate systems, which are computed from the raw sensor data and used for constructing the geometric constraints to solve the intrinsic and extrinsic parameters, for example, the plane-line correspondences [24]. The more precise and more abundant the calibration data are, the more accurate and more reliable calibration result we can get, which will lead to a much better fusion and a more real 3D color point cloud. Therefore, how to accurately compute the calibration data from the raw sensor data and smooth out the noise is the key to the calibration of a LRF and a camera [20].

This paper presents a new method for the calibration of a 3D LRF and a camera based on optimization solution. First, we move a checkerboard pattern in front of the camera and the 3D LRF, and collect the images and the 3D point clouds of the checkerboard pattern in various poses by using the camera and the 3D LRF respectively at the same time. Based on the images, the intrinsic parameters and the poses of the checkerboard pattern are computed by the camera calibration. Then, the line-to-plane constraints and plane-to-plane constraints are constructed to determine the extrinsic parameters by linear optimization. The intrinsic and extrinsic parameters are further refined by global optimization. Finally, the geometric mapping relationship between the LRF and the camera is calculated by using the intrinsic and extrinsic parameters. The experiments with both synthetic data and real data show that the proposed method is accurate and robust to noise.

In this paper, scalars are represented by italic symbols, e.g. \(x\), \(y\), and \(z\); vectors are denoted by bold italic symbols, e.g. \(\mathbf{p}\), \(\mathbf{q}\), and \(\mathbf{n}\); sets and matrices are indicated by italic capital symbols, e.g. \(P\), \(I\), and \(H\).

The remainder of this paper is organized as follows. In Section 2, we reviews the previous work related to this research. Section 3 gives our calibration method in detailed. Experimental results are presented in Section 4 to show the performance of our method. Finally, a conclusion and future work are given in Section 5.

2. Related work. In order to fuse a 3D point cloud from a LRF and a 2D image from a camera into a 3D color point cloud, we must accomplish the calibration of the LRF and the camera and obtain the geometric mapping relationship between the LRF and the camera at first. This geometric mapping relationship consists of the intrinsic and extrinsic parameters, which are determined by the camera calibration and the extrinsic calibration of a LRF and a camera respectively.
2.1. **Camera calibration.** Camera calibration is the process of determining the internal geometric and optical characteristics (intrinsic parameters) of the camera and/or the 3D position and orientation (extrinsic parameters) of the camera relative to a certain world coordinate system (especially the coordinate system of the calibration object) by only using the 2D images from the camera [17]. Now, camera calibration has been applied to many application fields, such as machine vision, objection identification, and industry inspection, and it is a necessary step to extract the metric information from 2D images [19]. In most cases, the overall performance of an application system strongly depends on the accuracy of camera calibration.

Several methods for camera calibration have been proposed in the literature. Forty years ago, Abdel-Aziz and Karara [1] developed a classic direct linear transformation (DLT) method to perform camera calibration. In the calibration procedure, the linear transformation from the object coordinates to the image coordinates is solved based on the pinhole camera model by using a 3D calibration object. The problem of the DLT method proposed by Abdel-Aziz and Karara is that a singularity may be introduced in the solving process of the linear least squares with constraints. In order to improve the numerical stability, Faugeras and Toscani [5] suggested another new constraint which is singularity free. Based on the DLT method, Melen [13] proposed an approach to extract the intrinsic and extrinsic parameters from the DLT matrix by using the RQ decomposition. Several years later, Heikkila and Silven [9] extended the DLT method to a four-step camera calibration procedure which includes two additional steps to compensate for the distortion of perspective projection and correct the distorted image coordinates.

The most broadly used method for camera calibration was proposed by Zhang [22]. It only requires the camera to observe a planar pattern shown at a few different orientations. Either the camera or the planar pattern can be freely moved. The proposed method gives a closed-form solution, followed by a nonlinear refinement based on the maximum likelihood criterion. This technique is easy to use and flexible. Besides 2D plane-based camera calibration, Zhang [23] also proposed another camera calibration technique based on 1D calibration objects (points aligned on a line). It gives a closed-form solution if six or more observations of a 1D calibration object are made. Similarly, Wang et al. [19] presented a multi-camera calibration algorithm by using a 1D calibration object under general motion. They also show that the minimum conditions for multi-camera cases are similar to those for calibrating a single camera with a 1D calibration object.

2.2. **Extrinsic calibration of a LRF and a camera.** The extrinsic calibration of a LRF and a camera is the process of determining the relative position and orientation (extrinsic parameters) between a LRF and a camera by using the measurements of both sensors (3D points and 2D images). The position and orientation are denoted by a translation vector and a rotation matrix numerically. The extrinsic calibration of a LRF and a camera is the key for the fusion of the two sensor modalities in many tasks, including objection classification and autonomous navigation. Many researchers devoted themselves to developing the methods for the extrinsic calibration of a LRF and a camera. These methods can be divided into two kinds according to which type of LRF is used, 2D LRF or 3D LRF.

2.2.1. **Extrinsic calibration of a 2D LRF and a camera.** The 2D LRF uses the line scan technique to acquire a series of discrete points on the intersecting line of the scan plane and object surfaces, which is called the line point cloud. In comparison
with the 3D LRF, the 2D LRF outputs less point cloud information, which increases the calibration difficulty. However, the 2D laser ranging technique is inexpensive and widely used in many fields. For the extrinsic calibration of a 2D LRF and a camera, several methods have been developed by using a checkboard pattern or an orthogonal trihedron. The core idea of these methods is to compute the geometric elements from the raw sensor data and construct the geometric constraints for the extrinsic parameters between a 2D LRF and a camera, such as point-to-point constraints [10], point-to-plane constraints [21], [12], and line-to-plane constraints [24], [20].

The checkerboard pattern is the mostly used calibration object in the extrinsic calibration of a 2D LRF and a camera. Zhang and Pless [21] developed a practical method to calibrate a 2D LRF and a camera extrinsically based on point-to-plane constraints by observing a checkerboard pattern. This method is simple to execute and is the foundation of many other extrinsic calibration methods. Based on Zhang’s method, Liu et al. [12] proposed a similar point-to-plane constraint. The difference is the representation of laser point coordinates in the camera coordinate system. By freely moving a checkerboard pattern, Vasconcelos et al. [18] presented an algorithm for the extrinsic calibration of a 2D LRF and a camera, which is formulated as one of registering a set of lines and planes in the dual 3D space. The algorithm gives a minimal closed-form solution for the extrinsic parameters by using three line-to-plane constraints. However, it suffers from the multiple-solution problem and has two degeneration cases, which leads to the numerical instability. Different from solving the extrinsic parameters in the dual 3D space, Ying et al. [20] proposed a more direct approach for the extrinsic calibration of a 2D LRF and a camera by using a checkerboard pattern in the 3D space. This approach also develops a minimal closed-form solution for the relative orientation between the two sensors from three line-to-plane constraints. In order to improve the numerical stability, Zhou [24] presented a new minimal solution for the extrinsic calibration of a 2D LRF and a camera based on both line-to-plane and point-to-plane constraints by using a checkerboard pattern. This method directly exploits the algebraic structure of the polynomial system, which yields a more numerically stable and accurate result.

In order to simply the calibration procedure, the orthogonal trihedron was proposed as the calibration object by several researchers. Gomez-Ojeda et al. [7] presented the first approach for the extrinsic calibration of a 2D LRF and a camera without the need of a specific calibration pattern based on both line-to-plane and point-to-plane constraints. It only requires the observation of an orthogonal trihedron which is commonly found as a scene corner in any human-made environment. In the same year, Briales and Gonzalez-Jimenez [3] proposed a minimal solution for the extrinsic calibration of a 2D LRF and a camera based on point-to-point constraints by observing an orthogonal trihedron. This method recovers the trihedron poses with respect to the 2D LRF and the camera from the laser data and the image data respectively. Similarly, Hu et al. [10] developed a flexible method to calibrate a 2D LRF and a camera extrinsically based on point-to-point constraints by using an orthogonal trihedron, which also computes the 2D LRF and camera poses with respect to the trihedron from the laser data and the image data respectively.

2.2.2. Extrinsic calibration of a 3D LRF and a camera. The 3D LRF uses the area scan technique to acquire a set of discrete points on object surfaces, which is called the area point cloud. With the development of 3D laser ranging techniques, some methods have been proposed for the extrinsic calibration of a 3D LRF and a camera
by using special calibration objects, such as a checkerboard pattern with round holes \cite{25}, a cube with checkerboard patterns \cite{15}, and a scenario with multiple checkerboard patterns \cite{6}. These methods mainly compute the geometric elements from the raw sensor data to construct the geometric constraints and solve the extrinsic parameters between a 3D LRF and a camera.

Rushmeier et al. \cite{15} proposed a method for the extrinsic calibration of a 3D LRF and a camera based on plane-to-plane constraints by using a high-precision cube with checkerboard patterns. Then, a 3D scanning system is designed to capture both geometry and photometry of the museum artifacts. Several years later, by scanning a circle-based calibration object, Sergio et al. \cite{16} presented an approach for the extrinsic calibration of a 3D LRF and a camera based on point-to-point constraints. It estimates the center points of the circle-based calibration object in various poses in the camera and LRF coordinate systems respectively, and formulates the extrinsic calibration as an absolute orientation problem. However, how to accurately estimate the center points from the sparse laser data is still a challenge. In order to make the calibration procedure more automatic and easier to use, Geiger et al. \cite{6} developed an extrinsic calibration method of a 3D LRF and a camera based on plane-to-plane and point-to-plane constraints by using a scenario with multiple checkerboard patterns. A single image and range scan of the calibration scenario is sufficient for solving the extrinsic calibration problem. Gong et al. \cite{8} proposed a different way to address the extrinsic calibration of a 3D LRF and a camera based on the geometric constraints associated with an arbitrary trihedron. It estimates the trihedron poses with respect to the LRF and the camera from the laser data and the image data respectively to calculate the extrinsic parameters. Recently, Zhuang et al. \cite{25} proposed an automatic method for the extrinsic calibration of a 3D LRF and a camera based on point-to-point constraints by using a checkerboard pattern with round holes. The main contribution of this method is the center detection of the round holes in the calibration board. Lee et al. \cite{11} developed a novel method to calibrate a 3D LRF and multi-view cameras extrinsically by using a ball. In this method, the center point of the ball is obtained in the laser and camera coordinate systems respectively, and then the transformation relationships between the 3D LRF and the cameras are calculated.

3. Proposed methodology.

3.1. Calibration problem. Figure 1 shows a calibration board and a 3D color laser ranging system. The calibration board is a checkerboard pattern. The 3D color laser ranging system is composed of a 3D LRF and a camera. In our system, the 3D LRF is a 3D laser sensor Velodyne LiDAR Puck VLP-16, and the camera is a color camera DAHENG MER-125-30GC. The features of the 3D LRF are the horizontal field of view 360°, horizontal angular resolution 0.1° ~ 0.4°, vertical field of view 30°, vertical angular resolution 2°, scanning frequency 5 ~ 20Hz, and scanning range 0.5 ~ 100m. The features of the camera are the image resolution 1292 × 964, frame rate 30fps, and exposure time 20μs ~ 1s. In operation, the 3D LRF and the camera collect a point cloud and an image synchronously at each sampling instant. The point cloud and the image are then fused into a color point cloud by the geometric mapping relationship between the two sensors, which is determined by their calibration.

In general, a camera can be described by the pinhole camera model, as shown in Figure 2. Let $p = [x, y, z]^T$ denote the laser point in the laser coordinate system.
\[\mathbf{q} = [u, v]^T\] is its image projection in the image coordinate system \([O_u; u, v]\). Their homogeneous coordinate are \(\mathbf{p} = [x, y, z, 1]^T\) and \(\mathbf{q} = [u, v, 1]^T\). By using the pinhole camera model, the relationship between the laser point \(\mathbf{p}\) and its image \(\mathbf{q}\) is given by

\[s\mathbf{q} = \mathbf{A}[R \quad \mathbf{t}]\mathbf{p}\]  

where \(s\) is an arbitrary scale factor, \(\mathbf{A}\) is the intrinsic parameter matrix, and \([R \quad \mathbf{t}]\) is the extrinsic parameter matrix between the laser coordinate system and the camera coordinate system. The objective of the calibration of a 3D LRF and a camera is to obtain the intrinsic parameters of the camera and the extrinsic parameters between the 3D LRF and the camera. Once the intrinsic and extrinsic parameters are determined, the geometric mapping relationship \(\mathbf{H} = \mathbf{A}[R \quad \mathbf{t}]\) between the 3D LRF and the camera is calculated.

In the calibration, we put the calibration board in front of the 3D LRF and the camera in various poses, and use the camera and the 3D LRF to shoot and scan the calibration board at the same time. Then, the images \(I = \{I_j|1 \leq j \leq m\}\) and point
clouds \( P = \{P_j|1 \leq j \leq m\} \) of the calibration board in various poses are obtained. \( I_j \) is the image of the calibration board in the \( j \)th pose. \( P_j \) is the point cloud of the calibration board in the \( j \)th pose. According to the scanning row number, we extract line point clouds \( P_{jk} \) from the point cloud \( P_j \) of the calibration board, that is, \( P_j = \{P_{jk}|1 \leq k \leq m_j\} \), as shown in Figure 3. By using these images and point clouds, we can achieve the calibration of the 3D LRF and the camera.

For taking a whole scan and image of the calibration board, the minimum distance between the calibration board and our system is 1.7m according to the features of the 3D LRF and the camera. As the distance increases, the number of the laser points in the point cloud of the calibration board becomes less, the measurement noise of the 3D LRF becomes larger, and the image of the calibration board becomes smaller. All things considered, the appropriate distance between the calibration board and the 3D color laser ranging system is 2 ∼ 4m.

Figure 3. Point cloud of the calibration board and its line point clouds

3.2. Camera calibration. By using the images \( I = \{I_j|1 \leq j \leq m\} \) of the calibration board in various poses, we calibrate the camera with Matlab Camera Calibration Toolbox [2], as shown in Figure 4. Let \([O_c; \hat{x}, \hat{y}, \hat{z}]\) denote the board coordinate system, where the \( \hat{x}\hat{y} \)-plane is on the calibration board and the origin \( O_b \) is located in the top left corner of the calibration board, as shown in Figure 1. Through the camera calibration, we can get the intrinsic parameter matrix \( A \) and extrinsic parameter matrix \([R_j \ t_j]\), where \( R_j \) and \( t_j \) are the rotation matrix and translation vector between the board coordinate system and the camera coordinate system in the \( j \)th pose.

The unit normal vector of the calibration board in the \( j \)th pose is represented as \([0, 0, 1]^T\) in the board coordinate system, since the \( \hat{x}\hat{y} \)-plane is on the calibration board. Therefore, the unit normal vector \( \vec{n}'_j \) of the calibration board in the \( j \)th pose in the camera coordinate system is calculated as

\[
\vec{n}'_j = R_j \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} r^1_j \\ r^2_j \\ r^3_j \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = r^3_j,
\]

where \( r^1_j \), \( r^2_j \), and \( r^3_j \) are the column vectors of \( R_j \). The translation vector \( t_j \) between the board coordinate system and the camera coordinate system is the vector from the origin \( O_c \) of the camera coordinate system to the origin \( O_b \) of the board coordinate system. Therefore, we can compute the perpendicular distance...
Figure 4. Calibration images

$h_j$ from the origin $O_c$ of the camera coordinate system to the calibration board by projecting $-t_j$ onto $n_j'$ as follows

$$h_j = -t_j \cdot n_j' = -t_j \cdot r_j^3.$$  \hspace{1cm} (3)

Then, we have a normal vector

$$\tilde{n}_j = h_j n_j' = - (t_j \cdot r_j^3) r_j^3,$$ \hspace{1cm} (4)

which denotes the calibration board in the $j$th pose in the camera coordinate system, as shown in Figure 5.

Figure 5. Normal vector $\tilde{n}_j$ of the calibration board in $j$th pose
3.3. **Extrinsic calibration.** In order to determine the extrinsic parameters between the 3D LRF and the camera, two kinds of geometric constraints are constructed on the transformation between the laser coordinate system and the camera coordinate system.

3.3.1. **Line-to-plane constraint.** In Subsection 3.1, we have extracted the line point clouds \( \{P_{jk} | 1 \leq k \leq m_j \} \) of the calibration board in the \( j \)th pose. Each line point cloud \( P_{jk} \) is a series of sequential discrete points, that is, \( P_{jk} = \{p_{ijk} | 1 \leq i \leq m_{ijk} \} \). We fit a straight line \( L_{jk} \) to each line point cloud \( P_{jk} \) by using the least squares method in the laser coordinate system. The straight line \( L_{jk} \) is denoted by its direction vector \( \bar{l}_{jk} = [x_{jk}^l, y_{jk}^l, z_{jk}^l]^T \) and a point \( o_{jk} = [x_{jk}^o, y_{jk}^o, z_{jk}^o]^T \) on it. Then, we transform the direction vector \( \bar{l}_{jk} \) and the point \( o_{jk} \) from the laser coordinate system to the camera coordinate system as follows

\[
\bar{l}_{jk} = R \bar{l}_{jk}, \tag{5}
\]

\[
o_{jk} = Ro_{jk} + t, \tag{6}
\]

where \( R \) and \( t \) are the rotation matrix and translation vector between the laser coordinate system and the camera coordinate system. In the camera coordinate system, since the scan line is on the calibration board, the line-to-plane constraint is constructed as

\[
\bar{n}_j \cdot \bar{l}_{jk} = 0, \tag{7}
\]

\[
\bar{n}_j \cdot (o_{jk} - \bar{n}_j) = 0. \tag{8}
\]

Combining Equation 5 and Equation 6 with Equation 7 and Equation 8, we have the line-to-plane constraint

\[
\bar{n}_j \cdot R \bar{l}_{jk} = 0, \tag{9}
\]

\[
\bar{n}_j \cdot (Ro_{jk} + t) = ||\bar{n}_j||^2. \tag{10}
\]

Equation 9 and Equation 10 can be rewritten as

\[
r_{11}x_{jk}^l\tilde{x}_j^n + r_{12}y_{jk}^l\tilde{y}_j^n + r_{13}z_{jk}^l\tilde{z}_j^n + r_{21}x_{jk}^l\bar{y}_j^n + r_{22}y_{jk}^l\bar{y}_j^n + r_{23}z_{jk}^l\bar{z}_j^n + r_{31}x_{jk}^l\bar{z}_j^n + r_{32}y_{jk}^l\bar{y}_j^n + r_{33}z_{jk}^l\bar{z}_j^n = 0, \tag{11}
\]

\[
r_{11}x_{jk}^o\tilde{x}_j^n + r_{12}y_{jk}^o\tilde{y}_j^n + r_{13}z_{jk}^o\tilde{z}_j^n + r_{21}x_{jk}^o\bar{y}_j^n + r_{22}y_{jk}^o\bar{y}_j^n + r_{23}z_{jk}^o\bar{z}_j^n + r_{31}x_{jk}^o\bar{z}_j^n + r_{32}y_{jk}^o\bar{y}_j^n + r_{33}z_{jk}^o\bar{z}_j^n + t_1\tilde{x}_j^n + t_2\bar{y}_j^n + t_3\bar{z}_j^n = \tilde{x}_j^n^2 + \bar{y}_j^n^2 + \bar{z}_j^n^2. \tag{12}
\]

where \( \bar{n}_j = [\tilde{x}_j^n, \bar{y}_j^n, \bar{z}_j^n]^T \), \( R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \), and \( t = [t_1, t_2, t_3]^T \).

3.3.2. **Plane-to-plane constraint.** In Subsection 3.1, we have obtained the point clouds \( P_j = \{p_{ij}^m | 1 \leq i \leq m_j \} \) of the calibration board in various poses. Each point cloud \( P_j \) is a set of discrete points, that is, \( P_j = \{p_{ijk}^m | 1 \leq i \leq m_{ijk} \} \). We fit a plane \( D_j \) to each point cloud \( P_j \) by using the least squares method in the laser coordinate system. The plane \( D_j \) is denoted by its normal vector \( \bar{d}_j = [x_j^d, y_j^d, z_j^d]^T \) and a point \( \bar{e}_j = [x_j^e, y_j^e, z_j^e]^T \) on it. Then, we transform the normal vector \( \bar{d}_j \) and the point \( \bar{e}_j \) from the laser coordinate system to the camera coordinate system as follows

\[
\bar{d}_j = Rd_j, \tag{13}
\]

\[
\bar{e}_j = Re_j + t, \tag{14}
\]
where $R$ and $t$ are the rotation matrix and translation vector between the laser coordinate system and the camera coordinate system. In the camera coordinate system, since the scan plane is on the calibration board, the plane-to-plane constraint is constructed as

$$\bar{n}_j \parallel \bar{d}_j ,$$  \hspace{1cm} (15)

$$\bar{n}_j \cdot (\bar{e}_j - \bar{n}_j) = 0 ,$$  \hspace{1cm} (16)

where $\parallel$ denotes parallel. Combining Equation 13 and Equation 14 with Equation 15 and Equation 16, we have the plane-to-plane constraint

$$\bar{n}_j \parallel (R\bar{d}_j) ,$$  \hspace{1cm} (17)

$$\bar{n}_j \cdot (Re_j + t) = \| \bar{n}_j \|^2 .$$  \hspace{1cm} (18)

Equation 17 and Equation 18 can be rewritten as

$$r_{11}\bar{x}_j^d \bar{y}_j^n + r_{12}\bar{y}_j^d \bar{y}_j^n + r_{13}\bar{z}_j^d \bar{y}_j^n - r_{21}\bar{x}_j^d \bar{x}_j^n - r_{22}\bar{y}_j^d \bar{x}_j^n - r_{23}\bar{z}_j^d \bar{x}_j^n = 0 ,$$  \hspace{1cm} (19)

$$r_{11}\bar{x}_j^d \bar{z}_j^n + r_{12}\bar{y}_j^d \bar{z}_j^n + r_{13}\bar{z}_j^d \bar{z}_j^n - r_{31}\bar{x}_j^d \bar{x}_j^n - r_{32}\bar{y}_j^d \bar{x}_j^n - r_{33}\bar{z}_j^d \bar{x}_j^n = 0 ,$$  \hspace{1cm} (20)

$$r_{11}\bar{x}_j^c \bar{x}_j^n + r_{12}\bar{y}_j^c \bar{x}_j^n + r_{13}\bar{z}_j^c \bar{x}_j^n + r_{21}\bar{x}_j^c \bar{y}_j^n + r_{22}\bar{y}_j^c \bar{y}_j^n + r_{23}\bar{z}_j^c \bar{y}_j^n + r_{31}\bar{x}_j^c \bar{z}_j^n + r_{32}\bar{y}_j^c \bar{z}_j^n + r_{33}\bar{z}_j^c \bar{z}_j^n + t_1 \bar{x}_j^n + t_2 \bar{y}_j^n + t_3 \bar{z}_j^n = \bar{x}_j^n + \bar{y}_j^n + \bar{z}_j^n .$$  \hspace{1cm} (21)

3.4. Optimization solution.

3.4.1. Linear optimization. In Subsection 3.1, we have obtained $m$ point clouds $\{P_j\}_{1 \leq j \leq m}$ of the calibration board in $m$ different poses and extracted $m_j$ line point clouds $\{P_{jk}\}_{1 \leq k \leq m_j}$ from each point cloud $P_j$ of the calibration board. Therefore, we can construct $m_1 + m_2 + \cdots + m_m$ line-to-plane constraints and $m$ plane-to-plane constraints by using Equation 11 $\sim$ Equation 12 and Equation 19 $\sim$
Equation 21. Then, we can use these geometric constraints to form a linear system of equations as follows

\[
\begin{bmatrix}
  x_{11}^m x_{1}^n & y_{11}^m x_{1}^n & z_{11}^m x_{1}^n & \cdots & 0 & 0 & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  x_{1m}^m x_{m}^n & y_{1m}^m x_{m}^n & z_{1m}^m x_{m}^n & \cdots & 0 & 0 & 0 \\
  x_{1}^n y_1^m & y_1^m y_1^m & z_1^m y_1^m & \cdots & 0 & 0 & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  x_{m}^m y_m^m & y_m^m y_m^m & z_m^m y_m^m & \cdots & 0 & 0 & 0 \\
  x_{1}^n z_1^m & y_1^m z_1^m & z_1^m z_1^m & \cdots & 0 & 0 & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  x_{m}^m z_m^m & y_m^m z_m^m & z_m^m z_m^m & \cdots & 0 & 0 & 0 \\
  x_{1}^n y_1^m & y_1^m z_1^m & z_1^m z_1^m & \cdots & 0 & 0 & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  x_{m}^m y_m^m & y_m^m z_m^m & z_m^m z_m^m & \cdots & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
  r_{11} \\
  r_{12} \\
  r_{13} \\
  r_{21} \\
  r_{22} \\
  r_{23} \\
  r_{31} \\
  r_{32} \\
  r_{33} \\
  t_1 \\
  t_2 \\
  t_3 
\end{bmatrix}
= 
\begin{bmatrix}
  0 \\
  \vdots \\
  x_1^2 + y_1^2 + z_1^2 \\
  \vdots \\
  x_m^2 + y_m^2 + z_m^2 \\
  \vdots \\
  x_1^2 + y_1^2 + z_1^2 \\
  \vdots \\
  x_m^2 + y_m^2 + z_m^2 
\end{bmatrix}
\] (22)

This linear system of equations is overdetermined, which can be solved by using the least squares method. Finally, we obtain the rotation matrix \( R \) and translation vector \( t \) between the laser coordinate system and the camera coordinate system.

3.4.2. Global optimization. As mentioned in Subsection 3.2 and 3.3, the normal vector of the calibration board in each pose in the camera coordinate system need to be determined by the camera calibration before the extrinsic calibration. This normal vector is a prerequisite for constructing geometric constraints to solve the extrinsic parameters between the 3D LRF and the camera. Therefore, the extrinsic calibration depends on the camera calibration. The camera calibration error could affect the performance of the extrinsic calibration of the 3D LRF and the camera.
So intuitively, the intrinsic parameters and the extrinsic parameters can be refined together by performing a global optimization.

For the camera calibration, the image projection error is computed as
\[ \sum_{j=1}^{m} \sum_{i=1}^{m_i} \| q_{ij} - q \left( A, R_{ij}, t_{ij}, \hat{p}_{ij} \right) \| , \quad (23) \]
where \( \hat{p}_{ij} \) is the \( i \)th corner of the checkerboard pattern in the \( j \)th pose in the board coordinate system, \( q \left( A, R_{ij}, t_{ij}, \hat{p}_{ij} \right) \) is the image projection of \( \hat{p}_{ij} \) in the image coordinate system, and \( q_{ij} \) is the corresponding true pixel.

For the extrinsic calibration, the line-to-plane constraint also represents that two points on the scan line are on the calibration board, since two points determine a line. Similarly, the plane-to-plane constraint also represents that three points on the scan plane are on the calibration board, since three points determine a plane. Therefore, the distance error is computed as
\[ \sum_{j=1}^{m} \sum_{k=1}^{m_j} 2 \sum_{i=1}^{2} d \left( \tilde{p} \left( R, t, p_{jk}^i \right), \tilde{n} \left( R_{ij}, t_{ij} \right) \right) + \sum_{j=1}^{m} 3 \sum_{i=1}^{3} d \left( \tilde{p} \left( R, t, p_{ij}^i \right), \tilde{n} \left( R_{ij}, t_{ij} \right) \right) , \quad (24) \]
where \( \tilde{n} \left( R_{ij}, t_{ij} \right) = \tilde{n}_j, p_{jk}^i \) is the point on the scan line in the laser coordinate system, \( p_{ij}^i \) is the point on the scan plane in the laser coordinate system, \( \tilde{p} \left( R, t, p_{jk}^i \right) \) is the point on the scan line in the camera coordinate system, \( \tilde{p} \left( R, t, p_{ij}^i \right) \) is the point on the scan plane in the camera coordinate system, and \( d \left( \tilde{p}, \tilde{n} \right) \) is the perpendicular distance from the point \( \tilde{p} \) to the calibration board \( \tilde{n} \).

By using Equation 23 and Equation 24, we can do a global optimization by minimizing the combination of the image projection error and the distance error as follows
\[ \sum_{j=1}^{m} \sum_{i=1}^{m_i} \| q_{ij} - q \left( A, R_{ij}, t_{ij}, \hat{p}_{ij} \right) \|^2 + \sum_{j=1}^{m} \sum_{k=1}^{m_j} 2 \sum_{i=1}^{2} d^2 \left( \tilde{p} \left( R, t, p_{jk}^i \right), \tilde{n} \left( R_{ij}, t_{ij} \right) \right) + \sum_{j=1}^{m} 3 \sum_{i=1}^{3} d^2 \left( \tilde{p} \left( R, t, p_{ij}^i \right), \tilde{n} \left( R_{ij}, t_{ij} \right) \right) . \quad (25) \]
This nonlinear minimization problem can be solved with the Levenberg-Marquardt method [14], [4]. It requires the initial guesses of \( A, R_{ij}, t_{ij}, R, \) and \( t \), which is set as the values obtained in the previous subsections. Finally, we have the optimal geometric mapping relationship \( H = A[R \ t] \) between the 3D LRF and the camera.

3.5. Data fusion. For a real scene, a point cloud and an image are collected synchronously by using the system presented in Subsection 3.1. Let \( \mathbf{p} = [x, y, z]^T \) be a point in the point cloud. Its homogeneous coordinate is \( \tilde{\mathbf{p}} = [x, y, z, 1]^T \). The image projection \( \mathbf{q} = [u, v]^T \) of the point \( \mathbf{p} \) is obtained by
\[ u = h_1^T \tilde{\mathbf{p}} / h_3^T \tilde{\mathbf{p}} \quad \text{and} \quad v = h_2^T \tilde{\mathbf{p}} / h_3^T \tilde{\mathbf{p}} , \quad (26) \]
where \( H = A[R \ t] = [h_{11}, h_{12}, h_{13}, h_{14}; h_{21}, h_{22}, h_{23}, h_{24}; h_{31}, h_{32}, h_{33}, h_{34}] \), \( \tilde{h}_1 = [h_{11}, h_{12}, h_{13}, h_{14}]^T \), \( \tilde{h}_2 = [h_{21}, h_{22}, h_{23}, h_{24}]^T \), and \( h_3 = [h_{31}, h_{32}, h_{33}, h_{34}]^T \). Then, we can obtain the color point
\[ \mathbf{p}_c = [x, y, z, R(u, v), G(u, v), B(u, v)]^T , \quad (27) \]
where \( R(u,v), G(u,v), \) and \( B(u,v) \) denote the three-primary colors of the image projection \( q \). Due to the calibration error and the measurement errors of the 3D LRF and the camera, the fusion error becomes larger as the distance of the point \( p \) increases.

4. Experimental results. In order to show the performance of our calibration method, we experiment with both synthetic data and real data. The synthetic data are generated by using a simulated 3D LRF and a simulated camera. The real data are acquired by using our 3D color laser ranging system. For the synthetic data where the ground truth is available, we can evaluate our method and compare it with other methods quantitatively. Furthermore, the synthetic data will be corrupted by adding noise to examine the smoothing effect of our method. For the real data, we evaluate the experimental results qualitatively from the illustrations by examining data fusion at key points.

4.1. Experiments with synthetic data. Our calibration method is first evaluated by using synthetic data under the different conditions. The synthetic data are generated by using a simulated 3D LRF and a simulated camera. The simulated 3D LRF has a horizontal field of view 360°, a horizontal angular resolution 0.18°, a vertical field of view 30°, and a vertical angular resolution 2°, which are similar to the features of the 3D LRF VLP-16. The simulated camera has a focal length 8mm, an image resolution 1292 × 964, and a pixel size 3.75µm × 3.75µm, which are similar to the features of the camera DAHENG MER-125-30GC. In addition, a standard checkerboard pattern is also simulated for the simulated experiments.

As shown in Figure 8, in the 3D space, the simulated 3D LRF and the simulated camera are fixed, and the simulated calibration board is movable, which is placed in different positions and orientations. In each pose, the simulated 3D LRF scans the simulated calibration board and generates the synthetic point cloud by using the laser scanning model. And meanwhile, the simulated camera takes a picture of the simulated calibration board and generates the synthetic image by using the camera imaging model (pinhole model).

Furthermore, the synthetic point cloud is corrupted by adding the zero-mean uniform noise \( U(a,a) \) to all position coordinates, which is approximately the same as the observed noise distribution in our sensor. The amplitude \( a \) ranges from 0mm to 14mm. Similarly, the synthetic image is also corrupted by adding the zero-mean Gaussian noise \( N(0,\sigma^2) \) to all pixel coordinates, where the standard derivation \( \sigma \) is set to 0.5 pixel. By using the synthetic point clouds and the synthetic images, we compare our method with the existing methods in terms of estimating the geometric mapping relationship (the intrinsic and extrinsic parameters) between a 3D LRF and a camera, analyzing the convergence as the number of poses of the calibration board increases, and studying the smoothing effect on different noise levels. For each number of poses and each noise level, 100 trials are conducted. In each trial, the noise is added to the synthetic point clouds and images of the simulated calibration board in various poses according to the noise level. Then, a geometric mapping relationship \( H_j \) is computed by using the calibration method of a 3D LRF and a camera, and a root-mean-square (RMS) error is calculated by

\[
e_j = \left( \frac{1}{n} \sum_{i=1}^{n} \| m_i - \bar{m}_i \|^2 / n \right)^{0.5},
\]  

(28)
where $\mathbf{m}_i = [\bar{u}_i, \bar{v}_i]^T$ is the truth value of the image projection of a space point and $\mathbf{m}_i = [u_i, v_i]^T$ is the estimated value of the image projection of the same point by using the geometric mapping relationship $H_j$. For the 100 trials, the average RMS error is computed by $\frac{\sum_{j=1}^{100} \epsilon_j}{100}$. In order to conduct a comprehensive analysis, we select two classical methods based on different geometric constraints, which are proposed by Zhang and Pless [18] and Ying et al. [17]. These two methods are extended from 2D to 3D for experimental comparison.

4.1.1. Performance with respect to the number of poses of the calibration board. This experiment researches how the number of poses of the calibration board affects the performance of the calibration method of a 3D LRF and a camera. In the experiment, we vary the number of poses of the calibration board from 16 to 30 and set the noise level of the point cloud to 10 mm. Each pose has an independent position and an independent orientation, and the noise added to the synthetic data of each pose is also independent. For each number of poses, 100 trials are conducted and the average RMS errors are calculated.

Figure 9 shows the calibration results with the increasing number of poses and the fixed noise level 10 mm. As can be observed, the average RMS errors are reduced as the number of poses increases. The more the number of poses, the better calibration results we will obtain. This demonstrates that our method is convergent as the number of poses of the calibration board increases. In Figure 9, we can also find that the performance of our method on each number of poses is always the best among the experimental methods. For example, when the number of poses is 22 and the noise level is 10 mm, the average RMS errors of our method, Zhang’s method, and Ying’s method are 4.3717, 7.6996, and 4.4496 respectively. The experimental result shows that our method is accurate and convergent.

4.1.2. Performance with respect to the noise level of the point cloud. This experiment is performed to evaluate the performance of the calibration method of a 3D LRF and a camera under different noise levels of the point cloud. In the experiment,
we vary the noise level of the point cloud from $0\text{mm}$ to $14\text{mm}$ and set the number of poses of the calibration board to 30. Each pose has an independent position and an independent orientation, and the noise added to the synthetic data of each pose is also independent. For each noise level, 100 trials are conducted and the average RMS errors are calculated.

Figure 10 shows the calibration results with the increasing noise level and the fixed number of poses 30. As can be observed, the average RMS errors become larger as the noise level increases. The stronger the noise, the worse calibration results we obtain. This demonstrates that the noise has a great influence on the calibration of a 3D LRF and a camera. In Figure 10, we can also find that the performance of our method under most noise levels is better than that of the other two methods. For example, when the noise level is $7\text{mm}$ and the number of poses is 30, the average RMS errors of our method, Zhang’s method, and Ying’s method are 2.5530, 3.0815, and 2.6273 respectively. The experimental result demonstrates that our method is accurate and robust to noise.

4.1.3. Statistical analysis and discussion. In this experiment, the number of poses of the calibration board is set to 30 and the noise level of the point cloud is set to $10\text{mm}$. Then, we run 100 trials to evaluate the performance of the experimental methods. The error distribution is shown in Figure 11. For our method, the minimum RMS error is 1.3571, the maximum RMS error is 7.2579, and the average RMS error is 3.6962; for Zhang’s method, the minimum RMS error is 1.8495, the maximum RMS error is 8.6392, and the average RMS error is 4.7702; for Ying’s method, the minimum RMS error is 1.5946, the maximum RMS error is 7.4648, and the average RMS error is 3.7866, as shown in Table 1. The experimental result demonstrates that our method is more accurate and reliable.

From the three experiments, we can find that the calibration errors of Zhang’s method are the largest among the three experimental methods. This is because Zhang’s method directly uses the raw laser points to solve the extrinsic parameters without any smoothing step to reduce noise. The performance of Ying’s method
4.2. Experiments with real data. The proposed calibration method is further tested by using real data in actual operation. As mentioned in Subsection 3.1, our 3D color laser ranging system consists of a 3D LRF (VLP-16) and a camera (MER-125-30GC). The features of the 3D LRF are the horizontal field of view 360°, horizontal angular resolution 0.1 ~ 0.4°, vertical field of view 30°, vertical angular
Table 1. RMS error analysis results (unit: pixel)

| Method       | Minimum | Average | Maximum |
|--------------|---------|---------|---------|
| Our Method   | 1.3571  | 3.6962  | 7.2579  |
| Zhang’s Method | 1.8495  | 4.7702  | 8.6392  |
| Ying’s Method | 1.5946  | 3.7866  | 7.4648  |

resolution $2^\circ$, scanning frequency $5 \sim 20Hz$, and scanning range $0.5 \sim 100m$. The features of the camera are the image resolution $1292 \times 964$, frame rate $30fps$, and exposure time $20\mu s \sim 1s$. The 3D LRF and the camera are calibrated by using our method. And, the geometric mapping relationship $H$ is written into the application software that is developed to control the system, collect the data, and fuse the data. The software interface includes three parts: image window (upper left), color point cloud window (lower left), and point cloud window (right). In operation, the application software controls the 3D LRF and the camera to collect a point cloud and an image synchronously at each sampling instant. Then, the point cloud and the image are fused into a color point cloud by the geometric mapping relationship between the 3D LRF and the camera.

In order to show the performance of our method, we choose two different scenes. The first one is an indoor scene (room), where the scene scale is small and the objects are regular. The second one is an outdoor scene (housing estate), where the scene scale is large and the objects are irregular. Figure 12 shows the 3D color point cloud of the room at an angle of view. As can be observed, our method obtains the accurate fusion result matched the real scene of the room well, especially in the geometric structure of normal discontinuity, e.g., the corner between the wall and the closet (red circle). Figure 13 shows the 3D color point cloud of the housing estate at an angle of view. As can be observed, our method obtains the accurate fusion result matched the real scene of the housing estate well, especially in the geometric structure of position discontinuity, e.g., the edges of the front and back buildings (red circle). These experimental results demonstrate that our calibration method is accurate and suitable for different kinds of geometric structures.

![Figure 12. 3D color point cloud of the room](image)

5. **Conclusion.** In this paper, we present a calibration method of a 3D LRF and a camera to fuse 3D point clouds and images into color point clouds. The method uses the camera and the 3D LRF to shoot and scan the calibration board in various poses
at the same time to obtain its point clouds and images. By using the images, the camera is calibrated. The intrinsic parameters and the poses of the checkerboard pattern are obtained. The line-to-plane constraints and plane-to-plane constraints are constructed to solve the extrinsic parameters by linear optimization. And, the intrinsic and extrinsic parameters are further refined by global optimization. Finally, the geometric mapping relationship between the LRF and the camera is calculated by using the intrinsic and extrinsic parameters. The main contribution includes the smoothing processing of the raw laser data by line fitting and plane fitting, the combination of different geometric constraints, and optimization resolution of the geometric mapping relationship between the 3D LRF and the camera.

To show the performance of the proposed calibration method, we conduct experiments with both synthetic data and real data qualitatively and quantitatively. The experimental results demonstrate that the proposed calibration method is accurate, robust to noise, and suitable for different kinds of geometric structures. Our future work will use this method in scene modeling and autonomous navigation of an intelligent robot.

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