Constraining the neutron-matter equation of state with gravitational waves

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We show how observations of gravitational waves from binary neutron star (BNS) mergers over the next few years can be combined with insights from nuclear physics to obtain useful constraints on the equation of state (EoS) of dense matter, in particular, constraining the neutron-matter EoS to within 20% between one and two times the nuclear saturation density \( n_0 \approx 0.16 \text{fm}^{-3} \). Using Fisher information methods, we combine observational constraints from simulated BNS merger events drawn from various population models with independent measurements of the neutron star radii expected from x-ray astronomy (the Neutron Star Interior Composition Explorer (NICER) observations in particular) to directly constrain nuclear physics parameters. To parameterize the nuclear EoS, we use a different approach, expanding from pure nuclear matter rather than from symmetric nuclear matter to make use of recent quantum Monte Carlo (QMC) calculations. This method eschews the need to invoke the so-called parabolic approximation to extrapolate from symmetric nuclear matter, allowing us to directly constrain the neutron-matter EoS. Using a principal component analysis, we identify the combination of parameters most tightly constrained by observational data. We discuss sensitivity to various effects such as different component masses through population-model sensitivity, phase transitions in the core EoS, and large deviations from the central parameter values.

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I. INTRODUCTION ..............................................

The detection of gravitational waves from the binary neutron star (BNS) merger GW170817 by the advanced laser interferometer gravitational wave observatory (aLIGO) detectors [1] in Hanford, WA (LHO) and Livingston, LA (LLO) and the VIRGO detector [2] ushered in the era of multi-messenger astronomy with gravitational waves [3,4]. This has been instrumental in launching novel ways of constraining cosmological parameters [5–8], on the one hand, and neutron star equation of state (EoS) parameters, on the other hand [2,9]. In a BNS system the neutron star masses and their EoS determine how much quadrupolar deformation \( Q_{ij} \) their tidal fields \( \tilde{e}_{ij} \) are able to induce in each other. The two are related by the tidal deformability parameter \( \lambda \) as \( Q_{ij} = -\lambda \tilde{e}_{ij} \). It is now well understood that the tidal deformability parameters of both neutron stars in a double neutron star system affect the phase of the gravitational wave signal during the late stages of the inspiral [10].
Recent articles that followed discovery of GW170817 have shown that upper bounds on the dimensionless tidal deformability $\Lambda = \lambda c^{10}/(GM)^5$ of the neutron stars obtained from gravitational wave data analysis provide constraints on the EOS of dense matter encountered inside neutron stars [11-13]. This is a great opportunity and challenge for several reasons: neutron rich matter, although relevant for many applications, is not easily accessible in experiments, while theoretical approaches require solving the difficult quantum many-body problem and lack a precise characterization of the underlying interactions. Observational constraints provide an anchor for nuclear theory in this uncertain regime, allowing one to extrapolate low-density and symmetric properties of nuclear matter to significantly improve constraints on neutron-rich matter at higher densities. In this article we discuss how we can extract more detailed information about the properties of dense neutron-rich matter and neutron stars during the next few years with more gravitational wave detections and measurements of neutron star radii expected from x-ray astronomy, and highlight the importance of an informed parameterization of the dense matter EOS. We make the reasonable assumption that all neutron stars are described by the same EOS. Further, modern nuclear Hamiltonians based on chiral effective field theory provide a systematic momentum expansion of two- and many-body nuclear forces. This, combined with advanced computational methods to solve the non-relativistic quantum many-body problem, now allows us to calculate the EOS of pure neutron matter up to nucleon number density $n_c \approx 2n_0$, where $n_0 = 0.16$ nucleons per fm$^3$ is the average nucleon density inside large nuclei (corresponding to a mass density $\rho_0 \approx 2.7 \times 10^{14}$ g/cm$^3$) [14]. Interestingly, there is a convergence of different ab initio methods based on realistic microscopic Hamiltonians that account for two and three neutron forces [15]. These calculations suggest that the functional form of the EOS of pure neutron in the density interval $0.5n_0$ to $2n_0$ is well determined. We use this information to parameterize the EOS and show how it helps with the analysis of multiple BNS detections and provide tighter and more useful constraints for dense matter physics. In turn, these constraints for the EOS of pure neutron in the density interval where calculations are feasible will provide new insights for nuclear physics.

Our study differs from earlier work in the following aspects:

- We incorporate insights about neutron-rich matter obtained from nuclear physics by implementing a new parameterization of nuclear equation of state and identify parameters that can be best constrained by gravitational wave observations.

- We quantify how constraints on these parameters and on the pressure of neutron matter in the density interval $n_0$ to $2n_0$ will improve with the number of detections.

- Our analysis uses a numerical relativity based tidal waveform model.

- We study the effect of different population synthesis models on the accuracy with which EOS parameters can be measured with gravitational waves and use several thousand binary neutron star source simulations to assess errors in EOS parameter measurements.

- While a nearby event like GW170817 at aLIGO design sensitivity would significantly constrain the properties of neutron matter, we show that similar constraints can be obtained from about 15 events beyond 100 Mpc.

We begin with a summary of our results in section II, then describe how we have parameterized the dense matter EOS in section III. In section IV we discuss how we obtain constraints from the gravitational waveform of simulated merger events. Finally, we discuss details of the method we use to obtain these constraints in section VI.

II. RESULTS

Our main result is that even a handful of gravitational wave observations of BNS mergers will provide the most stringent constraints on the low-temperature equation of state of dense neutron matter in the density interval between $n_0$ and $2n_0$. This is summarized in Fig. 1 which shows how the constraints on the pressure of pure neutron matter $P_n(n_n)$ improve as a function of additional NICER or LIGO observations. We start from the errors listed in Table I which, for the purposes of this analysis, we interpret as uncorrelated $1\sigma$ normal errors for the parameters. This gives the upper dotted line labeled “Nuclear”.

To this, we add the following constraints:

- Constraints from a simulated binary with similar masses and distance $D \sim 40$ Mpc to GW170817 but at aLIGO design sensitivity.

- Gravitational wave observations at aLIGO design sensitivity of $N_{\text{obs}} \in \{1, 15, 100\}$ distant $D \in \{100, 400\}$ Mpc simulated merger events from population model SubSolarA as described in section IV. To estimate the variance possible within the population model, we sample 500 different populations, each containing $N_{\text{obs}}$ and plot the $1\sigma$ (68th percentile) error bands as shaded regions.

- An uncorrelated mass and radius measurement of J0437 projected to be measured at a 5% level
from NASA’s NICER mission – i.e. $1.44(7)\,M_\odot$ with a 10% measurement of $R$ [16, 18].

This analysis demonstrates several key points: A nearby event such as GW170817 is comparable to a dozen or so events from $D \geq 100\,\text{Mpc}$. The NICER constraints are comparable to a single LIGO observation from a distant population sample having low signal-to-noise ratio (SNR), however, nearby or multiple accumulated LIGO events yield significant improvement. After about $N_{\text{obs}} = 15$ observation, we observe rather limited improvement from additional $N_{\text{obs}} = 100$. This can also be seen in Fig. 2 which shows how the constraints improve as a function of the number of observations.

One caveat: these constraints assume Gaussian errors and linear error propagation. A proper analysis requires a much more expensive Bayesian approach (see, e.g., Ref. [19]). To assess the non-linear effects, we provide similar plots for comparison in [20] for the different central values listed in Table II.

To put these results in perspective, consider the nuclear symmetry energy $S_{\text{sym}}$ and the slope of its density dependence $L$,

$$S_{\text{sym}} = E_{\text{np}}(n_0, 0) - E_{\text{np}}(\frac{n_0}{2}, \frac{n_0}{2}),$$

$$L = 3n_0 \frac{\partial E_{\text{np}}(n_0, 0)}{\partial n_0} \bigg|_{n_0 = n_0} = 3 \frac{P_{\text{n}}(n_0)}{n_0},$$

where $E_{\text{np}}(n_0, n_p)$ is the energy-per-particle of uniform nuclear matter. If the so-called parabolic approximation holds at saturation ($L_2 \approx L$ – see Eq. (5) and the surrounding discussion), then upcoming neutron skin experiments [21] expect to constrain $\Delta L = 41\,\text{MeV}$ with a possible reduction to $\Delta L = 15\,\text{MeV}$ with a followup experiment. This is comparable to combined constraints from ab initio calculations [22, 25] and astrophysical observations [15, 26, 28]. From our analysis we thus see that gravitational wave observations alone could have an impact at the ~ 15% level corresponding to $\Delta L \approx 10\,\text{MeV}$.
III. PARAMETERIZATION OF THE NUCLEAR EQUATION OF STATE

To relate the nuclear equation of state to the structure of neutron stars, we must first characterized the equation of state (EoS) of nuclear matter. This is conveniently parameterized by the energy density $\varepsilon(n_B)$ as a function of the baryon number density $n_B = n_n + n_p$, which is the sum of the neutron and proton number densities. Simple approximation for this function in terms of polytropes are often a starting point for astrophysical analysis. Indeed, many families of nuclear EoS can be characterized quite well by a simple set of piecewise polytropes [34].

Our approach here, however, is to directly express $\varepsilon(n_B)$ in terms of nuclear physics parameters. This approach allows one to directly assess how observations translate into constraints on nuclear physics. We shall demonstrate this by providing constraints on the pressure of pure neutron matter $P(n_n)$, which is inaccessible from a general polytropic analysis [Fig. 1].

It is useful to divide the neutron star interior into four regions: the outer crust, the inner crust, the outer core, and the inner core. The radial extent of the outer crust, which is composed neutron-rich nuclei embedded in a electron gas, is only a few hundred meters and its contribution to the neutron star mass is negligible. The EoS of the outer crust is well understood and depends weakly on the composition of nuclei present. The inner crust extends from $n = n_{\text{drip}} \approx 2 \times 10^{-3}n_0$ to $n = n_{\text{core}} \approx 2n_0$, has radial thickness $\sim 512\text{km}$, and contains a modest fraction of the mass. Here, exotic neutron-rich nuclei are embedded in a dense liquid of neutrons and electrons, as described by the compressible liquid-drop model (CLDM) in section III A. The outer core is a liquid composed primarily of neutrons and a small (few percent) admixture of protons, electrons, and muons. It extends from $n \sim 0.5n_0$ to $n = n_{\text{core}} \sim 2n_0$ where the description of matter in terms of nucleons interacting with static potentials is expected to break down. The inner core extends to higher densities, and we switch here to the speed-of-sound parameterization discussed in section III C.

On dimensional grounds one expects the dimensionless tidal deformability $\Lambda$ to be related to $\int_0^R \varepsilon r^3 \, \text{d}r = \langle \varepsilon r^3 \rangle$ with $n \sim 7$ for $M = 1.4M_\odot$ [33]. Although the EoS around intermediate densities dominates the 7th moment of energy distribution for massive neutron stars, the inner core also makes a large contribution to $\Lambda$ for low-mass stars (which are believed to be more common in binary neutron star systems). This contribution is shown by the dashed (purple) lines at the bottom of Fig. 3. Thus, it is important to provide a unified description of the EoS of the inner crust and the outer core in any analysis that aims to constrain the EoS using gravitational wave observations of binary neutron stars.

A. Compressible Liquid Drop Model

The compressible liquid-drop model (CLDM) (see [32]) provides a unified EoS connecting a fixed outer crust for $\rho < \rho_{\text{drip}}$ (for which we use the data in Table 4 of [34]) to the inner core EoS. In the inner crust, the CLDM constructs spherical nuclei in a spherical Wigner-
dependence of the surface tension \( \sigma(n_n, n_p) = \sigma_0 (1 - C_{\text{sym}}(\beta_p)^2 + \mathcal{O}(\beta_p^4)) \), (20) (see 20 for the exact form used), where \( \beta_p = \frac{n_n - n_p}{n_n + n_p} \) is the isospin asymmetry. We fix the parameter \( \sigma_0 \) to smoothly match the tabulated outer crust equation of state, leaving free the single parameter \( \sigma_0 \). Additionally, we include as a parameter a suppression factor \( \mathcal{C} \) for the Coulomb interaction to allow for the diffusivity of the proton charge distribution (see the discussion in 19). As will be shown in section II, these parameters have negligible effects on the constructed equation of state (EoS).

This approach allows for a small first-order phase transition from the region modeled by the CLDM to homogeneous nuclear matter. With our parameters, this phase transition is weak: \( \delta n < 0.002 \text{fm}^{-3} \).

To establish \( \beta \)-equilibrium we include leptons modeled as a Fermi gas of electrons (and muons at sufficiently high densities) in the Thomas-Fermi (TF) approximation.

### B. Homogeneous Nuclear Matter

One of the main new features of our analysis is to parameterize the nuclear-matter EoS as an expansion in the proton fraction \( \chi_p = \frac{n_p}{(n_n + n_p)} \) from pure neutron matter to symmetric neutron matter. This is in contrast to the common approach of expanding about symmetric nuclear matter in powers of the isospin asymmetry \( \beta_p = \frac{n_n - n_p}{n_n + n_p} \). The common approach allows one to directly connect experimentally relevant properties of symmetric nuclear matter to properties of neutron matter. This connection, however, is generally predicated on the so-called parabolic approximation, which is valid only if quartic terms \( \beta_p^4 \) dominate over quartic \( \beta_p^4 \) and higher-order terms. While there is some support for this below saturation density from relativistic Dirac-Bruckner-Hartree-Fock (DBHF) calculations 15, Gogny forces 16, and other perturbative techniques (see 17 for a review), it is not well established at higher densities. Indeed virtually any form of neutron-matter EoS can be accommodated with quartic \( \beta_p^4 \) terms without spoiling global mass fits 18. For this reason, we start with a parameterization of pure neutron matter, then use the properties of symmetric nuclear matter to constrain the extrapolation in the proton fraction \( \chi_p \).

To describe pure neutron matter we use a double polytrope for the energy per particle:

\[
E_n(n_n) = \frac{E_n(n_n)}{n_n} = m_N c^2 + a \left( \frac{n_n}{\bar{n}_0} \right)^\alpha + b \left( \frac{n_n}{\bar{n}_0} \right)^\beta, \\
P_n = n_n \left[ a \alpha \left( \frac{n_n}{\bar{n}_0} \right)^\alpha + b \beta \left( \frac{n_n}{\bar{n}_0} \right)^\beta \right],
\]

where \( m_N \) is the nucleon mass, \( \bar{n}_0 = 0.16 \text{fm}^{-3} \) is a constant (approximately the nuclear saturation density),

\[\text{Figure 4. (color online) Mass-radius curves for the EoSs considered in Table II. The thick solid curve is our Central EoS. Dashed curves correspond to different core parameterizations. Thin curves correspond to EoSs for which astrophysical observations would provide poor constraints for nuclear physics. These include a sharp first-order transition in the core (Core_trans), and soft EoSs (Soft and Stiff_Soft) which form very compact objects with low deformability. The Low_Ec EoS also poorly constrains nuclear physics since the core appears close to the saturation density. As shown later in Fig. 9 for these types of EoSs, observations constrain the core parameters rather than the properties of neutron matter.}\]
and $a$, $b$, $\alpha$, and $\beta$ are four EoS parameters. This form was found to accurately fit QMC calculations of the EoS using nuclear Hamiltonians with realistic two- and three-body forces \cite{15}, and is consistent with recent QMC results based on chiral EFT interactions \cite{23 25 42}. For small proton fractions $x_p = n_p/(n_n + n_p)$, we perform an expansion:

$$E_{np}(n_n, n_p) = (1 - x_p)E_n(n_n) + x_p \left( m_p c^2 + \Sigma_p(n_B) \right) + h^2 (3\pi^2)^{2/3} \frac{2}{m^*} x_p^{5/3} n_B^{2/3} + x_p f_2(n_B) + x_p^3 f_3(n_B) + \cdots$$

(3)

where $m^*$ is the proton effective mass, and $\Sigma_p(n_B)$ describes the self-energy of the proton polaron. This function is presently poorly constrained by QMC and experimental data and all known results are consistent with a simple two-parameter quadratic expansion:

$$\Sigma_p(n_B) = \mu_p \frac{n_B}{n_0} u_p - \frac{n_B}{n_0} u_p - 1$$

(4)

where $\mu_p = \Sigma_p(n_0)$ and $u_p = n_B/n_0$ where $\Sigma_p(n_B) = 0$ returns to zero. (We expect $\Sigma_p(n_B)$ to curve up for higher densities due to the repulsive nature of nuclear three-body interactions).

The additional powers $f_n(n_B)$ are chosen to match the properties of nuclear matter to quadratic order in the isospin asymmetry $\beta_p$ and expansion away from saturation $\delta_n$:

$$E_{np}^{sym}(n_n, n_p) = \varepsilon_0 + \frac{K_0}{2} \delta_n^2 + \left( S_2 + L_2 \delta_n + \frac{K_2}{2} \delta_n^2 \right) \beta_p^2,$$

$$\beta_p = \frac{n_n - n_p}{n_B}, \quad \delta_n = \frac{n_B - n_0}{3n_0}.$$  

(5)

Fitting two even powers, $\beta_p^0$ and $\beta_p^2$, and the lack of odd powers uniquely defines the functions $f_2(n_B)$ through $f_5(n_B)$, completing our characterization of the nuclear equation of state in terms of the nuclear saturation density $n_0$, energy $\varepsilon_0$, and incompressibility $K_0$; the symmetry energy $S_2$, slope $L_2$ and incompressibility $K_2$. Note that a term proportional to $\beta_p^4$ is allowed in Eq. (5) but our EoS is unconstrained by this term, i.e., does not rely on the parabolic approximation Eq. (5).

C. Speed of Sound Parameterization of the Inner Core

Above densities $n_c \sim 2n_0$ the EoS is virtually unconstrained. The typical approximation at high density is in terms of a polytrope, but we choose a more physically motivated high-density EoS parameterized in terms of the square of the speed of sound: $C(\varepsilon) = c_s^2(\varepsilon)/c^2 = P'(\varepsilon)/e$ which approaches the perturbative quantum chromodynamics (PQCD) result $C(\varepsilon) \to 1/3$ at asymptotic densities. Although the form of the function $C(\varepsilon)$ is unknown at finite density, its qualitative form at finite temperature suggests that it may first peak before returning to the asymptotic value \cite{14 49}. We thus include a simple parameterization $C(\varepsilon)$ as a quadratic polynomial smoothly connecting to the homogeneous equation of state at a fixed transition energy density $\varepsilon_c$ reaching a maximum $C_{\text{max}} = 1$ at an energy density $\varepsilon_{\text{max}}$, then returning to $C = 1/3$ at which it remains for higher densities. This core EoS thus introduces three parameters $\varepsilon_c$, $C_{\text{max}}$, and $\varepsilon_{\text{max}}$. To better understand the sensitivity of our results to the properties of the core, we include one slightly different form Core$_{\text{trans}}$ which has a first-order phase transition with discontinuity $\varepsilon_{\text{trans}}$ at $\varepsilon_c$. 

Figure 5. (color online) Sensitivity of the constraint on the pressure of neutron matter $P_n(n_n)$ from $N_{\text{obs}} = 15$ simulated merger events drawn from the SubSolarA population model to variations of the core equation of state. The vertical yellow lines denote the density $n_c$ at which the EoS reverts to the core form. Inset: form of the various core speed-of-sounds functions $C(\varepsilon) = c_s^2/c^2$. 




### CLDM parameters:

\[
\sigma_0 = 1.38(138) \text{ MeV/fm}^2, \quad \xi = 0.9(1),
\]

Symmetric nuclear matter and symmetry parameters:

- \( n_0 = 0.16(1) \text{ fm}^{-3}, \quad S_2 = 31(4) \text{ MeV}, \)
- \( e_0 = -16.0(3) \text{ MeV}, \quad L_2 = 60(40) \text{ MeV}, \)
- \( K_0 = 240(40) \text{ MeV}, \quad K_2 = 30(30) \text{ MeV}, \)

Neutron matter parameters:

- \( a = 13.0(3) \text{ MeV}, \quad \alpha = 0.50(2), \)
- \( b = 3.5(15) \text{ MeV}, \quad \beta = 2.3(5), \)

Proton polaron parameters:

- \( \mu_p(n_0) = -105(10) \text{ MeV}, \quad u_p = 3.1(6), \quad \frac{m_\text{eff}}{m_p} = 0.8(1), \)

Inner-core parameters:

- \( \varepsilon_c = 350(35) \text{ MeV}, \quad \varepsilon_\text{max} = 0.8(4) \text{ GeV}, \quad C_\text{max} = 0.8(2). \)

**Table I.** Parameters defining the central EOS along with their uncorrelated 1σ covariance (expressed using the SI convention 3.5(15) \( \equiv 3.5 \pm 1.5 \)) used to define the "Nuclear" error estimates prior to including information from astrophysical observations. We take the values of the CLDM parameters from the fits to the APR EOS tabulated in [30] but assign large errors to encompass missing physics such as the possibility of pasta phases. Symmetric nuclear matter and symmetry parameters have errors taken from the extensive analysis [31]. Neutron matter parameters have errors estimated from QMC calculations with various three-body interactions [32], and are consistent with recent QMC results based on chiral EFT interactions [33] [34] [35]. Proton polaron parameters have errors estimated from the QMC calculations [31] and are consistent with estimates from chiral interactions [34]. The core parameters are chosen to allow for a 2M\(\odot\) star at the extremes of all of our models except for the Soft EOS which requires a lower core transition and are given large errors to be conservative with the exception of the parameter \( \varepsilon_c \). This is given a small error for the purposes of our statistical analysis as the dependence is highly non-linear. Variations of this parameter are considered specially in **Fig. 5**.

**D. Parameters**

Our equation of state is thus characterized by 18 parameters: \( \sigma_0 \) and \( \xi \) (CLDM), \( n_0, e_0, K_0 \) (symmetric nuclear matter), \( S_2, L_2, K_2 \) (symmetry energy), \( a, \alpha, b, \beta \) (neutron matter) \( \mu_p, u_p, m^* \) (proton polaron), and \( \varepsilon_c, \varepsilon_\text{max}, C_\text{max} \) (core). We explore various ranges of these parameters centered about the values listed in **Table I**, which defines our base central EOS model. In addition to these central values, we repeat our analysis at a handful of different parameter values, defining the models listed in **Table II**. Some of these are referred to in the text, but a complete comparison is present in the supplement [36]. We now discuss how these constraints are derived from gravitational wave observations.

**IV. GRAVITATIONAL WAVEFORM**

Gravitational waves from merging binary neutron star systems carry information about the nuclear equations of state. During late stages of inspiral tidal interactions between neutron stars can leave imprints on the gravitational wave signal that is otherwise dominated by point-mass contributions. As mentioned earlier, tidal responses of neutron stars can be quantified by the dimensionless tidal deformability parameter \( \Lambda = \frac{5}{3}k_2c^4r^5/(GM)^5 \), where the second Love number \( k_2 \) is weakly sensitive to the matter distribution inside the star [16]. The strong dependence of \( \Lambda \) on the radius \( R \) of neutron star allows us to extract information regarding nuclear EOSs. Indeed, post-Newtonian (pN) theory is able to quantitatively describe the effect of the neutron star EOS on the signal by parameterizing the waveform in terms of \( M \) and \( \Lambda \) of component stars [16] [50].

Gravitational wave observations of inspiraling compact binaries involving neutron stars can therefore constrain \( \Lambda \) [37] [38]. However, since the constraint on \( \Lambda \) from a single BNS is weak for small to medium SNR events, multiple observations of such systems will be required for remote sources to reduce the statistical error in Ms and \( \Lambda \) in order to discern the effects of similar EOSs [19] [51] [52]. Fortunately, tens-to-hundreds of binaries of this type [53]...
are expected to be observed over the next several years by the advanced (or “second generation”) LIGO.

We consider only non-spinning neutron stars here because astrophysically their spins J are expected to be small when in a BNS system; in particular it is believed that the dimensionless spin parameter \(Jc/(GM^2) \lesssim 0.04\) \(\text{(2)}\). We plan to study the effect of spin in a future follow up study.

The gravitational wave signal from a BNS system in a detector can be expressed as the strain

\[
h(t) = \Lambda(t)e^{i\Psi(t)},
\]

where \(\Lambda(t)\) and \(\Psi(t)\) denote its amplitude and phase in the time domain. For Fisher information matrix (FIM)-based parameter estimation, we work with the Fourier transform \(\tilde{h}\) of the strain above. This is constructed by adding to the point-particle part of the TaylorF2 model at 3.5\(\sigma\) \(\text{(55)}\), a phase correction that is taken here to be the Fourier domain tidal waveform, with Padé fits, as prescribed in Dietrich et al. \(\text{(56)}\).

V. POPULATION MODELS

We employ different sets of stellar evolution model parameters of zero-age main sequence (ZAMS) binary stars each of which would lead to a binary neutron star system that merges within Hubble time. The differences among stellar evolution models can be large, resulting in appreciable variation in the component mass distribution. Since the tidal deformability parameter is sensitive to the masses, we explore four cases of mass distributions produced by population synthesis studies \(\text{(17)}\). These are more realistic than the uniform or Gaussian distributions owing to the application of stellar evolution mechanism of binary stars including two important factors, namely, metallicity and the nature of the common envelop interaction in the binary.

Metallicity plays the most dominant role in determining the strength of stellar winds in main sequence stars. The larger the metallicity the larger the stellar winds, due to increased scattering cross-section of the electrons. This results in increased mass loss; therefore, the remnant mass left behind at the end of main sequence phase is reduced. This decreases the total baryonic mass content of the supernova engine at the onset of the explosion. In our study, we consider two different variants of metallicities produced by \(\text{(17)}\). In the first case, the stellar evolution model was used with metallicity abundances being the same as solar metallicity, while in the second case 1/10th of solar metallicity was used. The latter is termed to be of sub-solar metallicity. Component masses are narrowly peaked for solar metallicity systems while sub-solar metallicity system produce a wider mass distribution.

The second most important effect that can change the component masses of BNS systems is the way mass transfer takes place during the common envelop phase of stellar evolution of the binary stars. The mass transfer in the common envelop stage depends on the evolutionary phase of the two stars. In one extreme case, for example, if the common envelop phase is initiated by the star in the Hertzsprung gap stage, it is likely to transfer a significant amount of orbital angular momentum to the entire binary system. This case is denoted by “submodel A” in \(\text{(17)}\). On the other hand, depending on the nature of interaction between the core and the envelop, one possible outcome is that during each common envelop stage for Hertzsprung gap donor stars the outer envelope acquires the significant part of the orbital angular momentum and gets ejected from the system, leaving behind the cores of the two stars to inspiral. This case is denoted by “submodel B” in \(\text{(17)}\). Furthermore, a higher metallicity in the parent star can result in greater mass loss and consequently a less massive remnant. Therefore, we employ neutron star populations resulting from solar metallicity stars as well as those with 10% of solar metallicity. These different characteristics lead to the following four categories of population models studied here:

**Solar metallicity submodel A (SolarA):** These are binary neutron star populations produced by solar metallicity stars of the submodel A type.

**Subsolar metallicity submodel A (SubSolarA):** These are binary neutron star populations produced by sub-solar metallicity stars of the submodel A type.

**Solar metallicity submodel B (SolarB):** These are binary neutron star populations produced by solar metallicity stars of the submodel B type.

**Subsolar metallicity submodel B (SubSolarB):** These
are binary neutron star populations produced by sub-solar metallicity stars of the submodel B type.

**Uniform** (Uniform): Uniform sampling of neutron stars with masses between $1.2M_\odot$ and $1.8M_\odot$.

### VI. STATISTICS AND METHODS

Given a particular parameterization of the EOS, we compute the mass $M$, radius $R$, and tidal deformability parameter $\Lambda$ of a neutron star with a given central density by solving the Tolman–Oppenheimer–Volkoff (TOV) equations (see e.g. [57, 58]). The signals (gravitational waveforms) from merging neutron stars is computed with the numerical relativity based frequency-domain model [59] mentioned above. From those waveforms, we compute the corresponding FIM characterizing the correlated uncertainties of the masses, $M_1$ and $M_2$, and the tidal deformabilities $\Lambda_1$ and $\Lambda_2$ (maximizing the

- Figure 7. (color online) Principal component analysis of the simulated observational data in terms of the EOS parameters. Each column is a plot of the components of most significantly constrained eigenvector for the particular combination of observations listed at the bottom. These should be interpreted as follows: A linear combination of the log of the corresponding parameters is constrained to the tolerance shown at the top. The rightmost column shows the principal component analysis for $N_{\text{obs}}=15$ simulated merger events drawn from the SubSolarA population model, and is the same as the leftmost column of Fig. 9. The $1\sigma$ errors in the tolerances, shown as small black strips in middle of the component bars, are obtained by performing 200 independent samples and demonstrate variation within the population model. (These errors are small here, but quite visible in the second principal components of Fig. 8)

- Figure 8. (color online) First two principal components for $N_{\text{obs}}=15$ gravitational wave observations drawn from the SubSolarA (left) and Uniform (right) population models. This demonstrates the wider distribution of masses in the Uniform model as compared to SubSolarA. The narrow distribution in SubSolarA leads to tighter statistical constraints on the 1st principal component, but leaves other directions in parameter space poorly explored. In contrast, the Uniform model distributes the 15 events over a larger range of masses, reducing the constraints on the 1st principle component, but providing more information about a second direction. Even for $N_{\text{obs}}=15$ observations, the next principal is poorly constrained at a level worse than 200%: more observations would be required to constrain this component at a useful level. Thus, neutron-star observables seem to provide tight constraints in a single direction of parameter space.
parameters: neutron star observations constrain and Advanced Virgo detectors from the for a \[\text{(see, e.g., } [\text{for hundreds }\]'s \text{)} \]. This analysis makes clear the non-linear dependence of the problem on the parameters listed in Table II. The leftmost column thus corresponds to the rightmost column of Fig. 7. The Fisher method for estimating errors has limitations, one of the main being the need for a high SNR. Bayesian methods are more reliable, but computationally much more expensive. For this latter reason use Fisher methods, whose computationally efficiency allows us to reduce source selection effects on the error estimates. We are able to quickly compute the FIM for hundreds of binaries, characterizing the variance within the population models. In spite of the drawbacks, the Fisher errors quoted here make the case to invest in Bayesian methods.

b. Methodology For a given population synthesis model, we simulate ten thousand BNS systems and distribute them uniformly in comoving volume between a luminosity distance of 100 Mpc and 400 Mpc. The latter limit is not too far from the horizon distance (~450 Mpc) of the network of aLIGO and Advanced Virgo detectors...
The GW170817 event demonstrated that useful constraints on the neutron star structure can be obtained from gravitational waves. In this article we have addressed how future observations can provide more detailed constraints on the properties of dense matter. By separating the neutron star into four distinct regions, and providing a unique nuclear physics based parameterization of the equation of state (EOS) of the crust and outer-core, we have analyzed how measurements of the tidal deformability can constrain nuclear properties of dense matter. Our parameterization, which uses the same underlying EOS of neutron matter in both inner crust and outer core, allowed us to estimate for the first time constraints on the EOS of pure neutron matter in the density interval where controlled calculations are becoming feasible. These constraints, as they become available, will provide valuable guidance for nuclear physics. In the inner core, where the EOS is poorly constrained, the speed of sound is allowed to vary over a large range constrained only by causality and the requirement that the EOS produce a 2 solar mass neutron star. We have taken first steps to study how the large uncertainties associated with the EOS of the inner core limits our ability to constrain the EOS of neutron matter in the outer core. The results we obtain suggest that, in the absence of strong first-order transitions in the core, even a handful of detections can constrain the pressure of neutron matter in the density interval between \( n_0 \) and \( 2n_0 \) to better than 20%.

The principal component analysis presented in Fig. 7 suggests that future LIGO observations will provide constraints on the density dependence of the pure neutron matter EOS in the outer core. In particular, we find that the exponent \( \beta \) in the neutron matter EOS defined in Eq. (2) will be well constrained. As expected, the nuclear physics parameters are better constrained when the outer core makes the dominant contribution to the tidal deformability. This is the case when the neutron matter EOS is stiff in the dense regions of the outer core and for low-mass neutron stars. If instead, the EOS in the outer core is soft or if a strong first-order phase transition were to occur at relatively low-density, constraints on the neutron matter EOS are weaker. In these cases, the inner core has a larger impact on the tidal deformability and gravitational wave detections will provide constraints for matter encountered in the inner core.

Although our focus here was to study the impact of the most common events that occur at large distances, we find that a single close by event similar to GW170817 at 40 Mpc at design sensitivity will provide valuable constraints. However, in the absence of such a nearby event, similar constraints may be realized by a dozen or so more distant events.

One limitation of our study is the simple parameterization of the EOS of the inner core. While this is adequate as a first step, to constrain the EOS of the inner core, a parameterization that allows for larger variability at high density will be needed. In addition, to gain more confidence in the constraints we have presented for neutron matter, it will be necessary to systematically marginalize over population models for neutron star masses and spins, and the uncertainty in the EOS of the inner core. A Bayesian approach would be better suited for this purpose, and we are in the processes of developing computer programs needed for such a study.
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Appendix A: Supplementary Material

1. Surface Term in the CLDM

In our implementation of the CLDM, we use the following surface term with an effective surface tension $\sigma(n^t_i, n^b_i) = \sigma_0/(1 - C_{\text{sym}} f(x_p)) \approx \sigma_0(1 - C_{\text{sym}} \beta^2 + O(\beta^4))$ following [39] (see also [40]):

$$ E = 4\pi^2 p \sigma_0 \frac{1}{1 - C_{\text{sym}} \frac{1}{\sigma_0} (16 - x_p^3 - (1 - x_p)^3)} $$  \hspace{1cm} (A1a)

$$ = 4\pi^2 p \sigma_0 \left(1 - C_{\text{sym}} \beta^2 + O(\beta^4)\right), \hspace{1cm} (A1b) $$

where $x_p = n^i_p/(n^i_p + n^b_i)$ is the proton fraction in the nucleus, and $\beta = (n^i_n - n^i_p)/(n^i_p + n^i_n) = 1 - 2x_p$. We parameterize this as $C_{\text{sym}} = \sigma_0/\sigma_0$ where $\sigma_0 \approx 1.38 \text{MeV/fm}^2$ is held fixed as a parameter of the theory, and $\sigma_0$ is varied to smoothly match the tabulated outer-crust data.

2. Polytropes

In Fig. 10 we compare the constraints obtained on the total pressure $P(n_B)$ of nuclear matter in $\beta$-equilibrium using our Central unified parameterization with those obtained using the piecewise polytropic EoS in [44]. To better compare these, we do the following:

1. Fit the parameters of the polytrope to best match our Central EoS: log($p_1$) = 34.3, $\Gamma_1 = 2.60$, $\Gamma_2 = 3.81$, and $\Gamma_3 = 2.91$.

2. We use the same speed-of-sound core parameterization with $E_c = 350.0 \text{MeV/fm}^3$, $E_{\text{max}} = 800.0 \text{MeV/fm}^3$, $C_{\text{max}} = 0.8$ as our Central EoS.

3. We start with a bare “Nuclear” constraint by computing the 1.2$r$ covariance matrix of the parameters from Table III of [44] over the following EoS models that have a small pressure $P(n_0) \leq 3 \text{MeV/fm}^3$ at saturation density: PAL6, SLy, APR1, APR2, APR3, APR4, FPS, WFF1, WFF2, WFF3, BB2, BPAL12, ENG, MPA1, BGN1H1, PCL2, ALF1, ALF2, ALF3, and ALF4. (This excludes some models with hyperon (GNH3, H1-7), pion (PS), and kaon (GS1-2) condensates, as well as the strange-quark matter models MS1-2, which all have significantly higher saturation pressures $P(n_0) > 3 \text{MeV/fm}^3$). This gives similar bare “Nuclear” errors as our Central model at and above saturation density.

We note that the constraints on $P$ are very similar to those from our “Nuclear” parameter set. To obtain this, however, it was critical to use correlated errors in the polytrope parameters. To this end, taking a polytropic EoS with uncorrelated priors is inadvisable. Only once correlated priors are used does the polytropic equation of state provide constraints comparable to those that can be obtained from our nuclear parameterization.

3. Tabulated EoS Data

Here we present somewhat tighter constraints on tabulated EoS data, required to ensure convexity, than we have seen presented in the literature. Suppose we have

![Figure 10](image-url)
an interval with tabulated density, pressure, and energy \( n_{0,1}, P_{0,1}, \) and \( \mathcal{E}_{0,1}. \) If these data come from an equation of state that satisfies thermodynamic convexity \( \mathcal{E}''(n) = P'(n)/n \geq 0 \) and causality \( \mathcal{E}'(P) \geq 1, \) then each interval must satisfy the following conditions:

\[
P_1 - P_0 \leq \mathcal{E}_1 - \mathcal{E}_0, \quad \frac{\mathcal{E}_1 + P_1}{\sqrt{\mathcal{E}_0 + P_0}} \leq \frac{n_1}{n_0} \leq \frac{\sqrt{\mathcal{E}_1 + P_1}(\mathcal{E}_1 + 2P_0 - P_1)}{\mathcal{E}_0 + P_0}. \tag{A2}
\]

The tabulated data in [31] used for the outer crust required some minor corrections to ensure these constraints are met.

4. Thermodynamic Relationships

Here we briefly review some \( T = 0 \) thermodynamic relationships for an \( \text{EoS} \) with a single conserved component with density \( n \) and chemical potential \( \mu, \) energy density \( \mathcal{E}, \) energy per particle \( \mathcal{E}', \) and pressure \( P. \) These are used at various places throughout the text, such as relating the slope of the symmetry energy \( L = 3P_n(n_0)/n_0 \) to the pressure of neutron matter in Eq. (1b)

\[
\mathcal{E}(n) = \frac{\mathcal{E}(n)}{n}, \quad \mu = \mathcal{E}'(n), \quad P = \mu n - \mathcal{E} = n^2 \mathcal{E}'(n), \quad C = \frac{c_s^2}{c^2} = \frac{\text{d}P}{\text{d}\mathcal{E}} = \frac{P'(n)}{\mathcal{E}'(n)} = \frac{n \mu'(n)}{\mu}. \tag{A3}
\]

5. Comparison Plots

On the following pages, we provide comparison plots for all of the \( \text{EoS} \) models discussed in the text.
Stiff Soft

![Graphs showing various physical parameters and error analyses for different models and settings.]
Stiff
Low $C_{\text{max}}$

Relative error in $P_n$

$P_n$ [MeV/fm$^3$]

$\frac{n_n}{\bar{n}_0}$

$R$ [km]

$P$ [MeV/fm$^3$]

$\frac{n_B}{0.16/fm^3}$

$M$ [M$_\odot$]

$N_{\text{obs}}$

Relative error

$P_n(n_0)$

$P_n(1.5n_0)$

$1^{st}$ pc

Nuclear

+ NICER

+ 1 LIGO (40 Mpc)

+ 1 LIGO

+ 15 LIGO

+ 100 LIGO

Low $C_{\text{max}}$

$C_{\text{max}}$

$R$ [km]

$1.0M_{\odot}$

$2.0M_{\odot}$

$1.4M_{\odot}$

$1.0M_{\odot}$

Homogeneous Inner Core

(Outer Core)

$\propto \varepsilon T^7$

$\frac{n_n}{\bar{n}_0}$

Relative error

$P_n$

Nuclear

+ nicer

+ 1 ligo

+ 15 ligo

+ 100 ligo

$P_n$ [MeV/fm$^3$]

Low $C_{\text{max}}$

$R$ [km]

$1.0M_{\odot}$

$2.0M_{\odot}$

$1.4M_{\odot}$

$1.0M_{\odot}$

Homogeneous Inner Core

(Outer Core)

$\propto \varepsilon T^7$

$\frac{n_n}{\bar{n}_0}$

Relative error

$P_n$

Nuclear

+ nicer

+ 1 ligo

+ 15 ligo

+ 100 ligo

$P_n$ [MeV/fm$^3$]

Low $C_{\text{max}}$

$R$ [km]

$1.0M_{\odot}$

$2.0M_{\odot}$

$1.4M_{\odot}$

$1.0M_{\odot}$

Homogeneous Inner Core

(Outer Core)

$\propto \varepsilon T^7$

$\frac{n_n}{\bar{n}_0}$

Relative error

$P_n$

Nuclear

+ nicer

+ 1 ligo

+ 15 ligo

+ 100 ligo

$P_n$ [MeV/fm$^3$]

Low $C_{\text{max}}$

$R$ [km]

$1.0M_{\odot}$

$2.0M_{\odot}$

$1.4M_{\odot}$

$1.0M_{\odot}$

Homogeneous Inner Core

(Outer Core)

$\propto \varepsilon T^7$

$\frac{n_n}{\bar{n}_0}$

Relative error

$P_n$

Nuclear

+ nicer

+ 1 ligo

+ 15 ligo

+ 100 ligo

$P_n$ [MeV/fm$^3$]

Low $C_{\text{max}}$

$R$ [km]

$1.0M_{\odot}$

$2.0M_{\odot}$

$1.4M_{\odot}$

$1.0M_{\odot}$

Homogeneous Inner Core

(Outer Core)

$\propto \varepsilon T^7$

$\frac{n_n}{\bar{n}_0}$

Relative error

$P_n$

Nuclear

+ nicer

+ 1 ligo

+ 15 ligo

+ 100 ligo

$P_n$ [MeV/fm$^3$]

Low $C_{\text{max}}$

$R$ [km]

$1.0M_{\odot}$

$2.0M_{\odot}$

$1.4M_{\odot}$

$1.0M_{\odot}$

Homogeneous Inner Core

(Outer Core)

$\propto \varepsilon T^7$

$\frac{n_n}{\bar{n}_0}$

Relative error

$P_n$

Nuclear

+ nicer

+ 1 ligo

+ 15 ligo

+ 100 ligo

$P_n$ [MeV/fm$^3$]

Low $C_{\text{max}}$

$R$ [km]

$1.0M_{\odot}$

$2.0M_{\odot}$

$1.4M_{\odot}$

$1.0M_{\odot}$

Homogeneous Inner Core

(Outer Core)

$\propto \varepsilon T^7$

$\frac{n_n}{\bar{n}_0}$

Relative error

$P_n$

Nuclear

+ nicer

+ 1 ligo

+ 15 ligo

+ 100 ligo

$P_n$ [MeV/fm$^3$]
High $\mathcal{E}_c$
