Autonomous attitude and orbit control of a space robot inspecting a geostationary satellite

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Abstract. Methods for the motion control of a space robot flying by an information geostationary satellite during visual inspection of its technical state are considered. The robot attitude and orbit control system employs electric propulsion unit with 8 catalytic propulsion engines and pulse-width modulation of their thrust, and also a gyro moment cluster based on 4 gyrodines with digital control. The results of computer simulation demonstrating the effectiveness of the developed discrete control algorithms are presented.

1. Introduction
Information satellites \cite{1} in geostationary orbit (GEO) have a required lifetime of up to 25 years with the availability of maintenance using space robot-manipulators (SRMs). The authors’ paper \cite{2} briefly presents the attitude and orbit control system (AOCS) of the SRM, the synthesized control algorithms for its approach to a geostationary satellite (target) from the distance of 5 km to the distance of 50 m and the results of analyzing the dynamics of the AOCS during such approach. This paper deals with the problems for synthesis of the SRM guidance and control laws when flying by a geostationary satellite in the course of visual inspection of its technical state and nonlinear dynamic analysis of the AOCS.

2. Mathematical models and the problem statement
To perform the considered mode, the AOCS uses electric propulsion unit (EPU) based on eight catalytic electrojet engines (EJEs) with pulse-width modulation (PWM) the thrust \( P_m = 0.5 \) N of each EJE, gyro moment cluster (GMC) based on four gyrodines (GDs) with their own angular momentum (AM) \( h_g = 30 \) Nms; and the kinematic parameters of the SRM spatial motion relative to the target are determined using optoelectronic cameras and lidars.

We apply the inertial reference frame (IRF) \( O_{i}x^i y^i z^i \), notations \{\cdot\} = \text{col}(\cdot), (\cdot)^i, [\cdot \times] \) and \( \circ, \circ \) for vectors, matrices and quaternions, as well as \( [a]_i \) for the matrix of elementary rotation about \( i \)-th axis to an angle \( \alpha \), \( i = 1, 2, 3 \equiv 1 \div 3 \). The orbital reference frames (ORFs) of the SRM \( O_r x^o y^o z^o \) with a pole \( O_r \) and the target \( O_t x^o y^o z^o \) with a pole \( O_t \) are used, as well as the SRM body reference frame (BRF) \( O_b x y z \). We assume that the SRM is equipped with a telescope with an axis of sight parallel to the BRF \( O_b y \) axis (Fig. 1a). The location of the target in the robot’s BRF is determined by the vector \( \Delta r(t) \), see Fig. 1b.
Figure 1. The SRM flyby scheme during visual inspection of a geostationary satellite state.

Figure 2. The schemes of EPU with 8 catalytic EJEs (a) and GMC based on 4 gyrodines (b).

In the EPU scheme with 8 catalytic EJEs (Fig. 2a) we show the unit vectors \( \mathbf{e}_p \), \( p = 1 \div 8 \) of the EJE nozzle axes. Assume that the vector \( \mathbf{p}_p \) defines the point \( O_p \) at which the thrust vector of the \( p \)-th EJE is applied. Each catalytic ERE has PWM of its thrust \( p_p(t) \) which is described by a nonlinear continuous-discrete relation

\[
p_p(t) = P^m \text{PWM}(t - T^{uz}_u, t_r, \tau_{pr}) \text{ for } t \in [t_r, t_r+1), \quad t_{r+1} = t_r + T^{uz}_u
\]

with period \( T^{uz}_u \) and a time delay \( T^{uz}_u \). Here, \( P^m \) is the nominal value of thrust, similar for all catalytic EJEs, \( t_r = rT^{uz}_u, \quad r \in \mathbb{N}_0 \equiv \{0, 1, 2, \ldots\ \}\) and the function

\[
\text{PWM}(t, t_r, \tau_{pr}) = \begin{cases} 
\text{sign} \ v_{pr} & t \in [t_r, t_r + \tau_{pr}), \quad \tau_{pr} \equiv \begin{cases} 0 & |v_{pr}| \leq \tau_m, \\
\text{sat}(T^{uz}_u, |v_{pr}|) & |v_{pr}| > \tau_m.
\end{cases}
\end{cases}
\]

In the BRF, the thrust vector of the \( p \)-th EJE is calculated by relation \( \mathbf{p}_p(t) = \{-p_p(t)\mathbf{e}_p\} \), and vectors of the EPU force \( \mathbf{P}^e = \{\mathbf{P}_i\} \) and torque \( \mathbf{M}^e \) are computed by the formulas

\[
\mathbf{P}^e = \Sigma \mathbf{p}_p(t) \text{ and } \mathbf{M}^e = \Sigma \mathbf{p}_p(t) \times \mathbf{M}_g^i(t).
\]

Column \( \mathbf{H} (\beta) = h_g \Sigma \mathbf{h} (\beta_p) \) represents the AM vector for the GMC by the scheme 2-SPE based on four gyrodines (Fig. 2b) where \( |\mathbf{h}_g| = 1, p = 1 \div 4 \), and \( h_g \) is a constant own AM of each gyrodine. The vector \( \mathbf{M}^g = \{M^g_1\} \) of the GMC control torque is presented by the nonlinear relations

\[
\mathbf{M}^g = -\mathbf{H}^* = -\mathbf{A}_h(\beta) \mathbf{u}_k^g(t) \quad \beta = \mathbf{u}_k^g(t) = \{u_k^g(t)\}, \quad u_k^g(t) = \text{sat}(q_{nk}(u_{pk}^g, u_{nk}^g), u_{k}^g), T_u \text{ for } k \in \mathbb{N}_0 \text{ and a period } T_u \text{ where vector column } \beta = \{\beta_p\}, \text{ the matrix } \mathbf{A}_h(\beta) = \partial \mathbf{H}(\beta)/\partial \beta, \text{ and } (\cdot)^* \text{ is the symbol of local time derivative. In the IRF,}
\]
the robot’s orientation is determined by quaternion $\mathbf{\Lambda} = (\mathbf{\lambda}_0, \mathbf{\lambda})$, $\mathbf{\lambda} = \{\lambda_i\}, i = 1 \div 3$. We use a vector of modified Rodrigues parameters (MRP) $\mathbf{\sigma} = \mathbf{e} \tan(\mathbf{\Phi}/4)$ with Euler unit vector $\mathbf{e}$ and the angle $\mathbf{\Phi}$ of proper turn which is uniquely connected with quaternion $\mathbf{\Lambda}$ by explicit relations. If we consider the SRM to be a solid with a mass $m$ and an inertia tensor $\mathbf{J}$, then the model of its spatial motion relative to the IRF in the projection on the BRF axes has the form

$$\mathbf{r}_r^* + \mathbf{\omega} \times \mathbf{r}_r = \mathbf{v}_r, \quad m(\mathbf{v}_r^* + \mathbf{\omega} \times \mathbf{v}_r) = \mathbf{P}^e + \mathbf{F}^d; \quad \mathbf{\dot{A}} = \mathbf{\Lambda} \mathbf{\omega} / 2, \quad \mathbf{\dot{K}} + \mathbf{\omega} \times \mathbf{G} = \mathbf{M}^e + \mathbf{M}^d. \quad (1)$$

Here, $\mathbf{r}_r$ and $\mathbf{v}_r$ (index r for robot) are the vectors of the location and velocity of the SRM forward motion; vector $\mathbf{G} = \mathbf{K} + \mathbf{H}(\beta)$, where $\mathbf{K} = \mathbf{J} \mathbf{\omega}$, is the vector of the SRM’s AM, and the $\mathbf{F}^d$ and $\mathbf{M}^d$ are the vectors of external disturbing forces and torques. The vectors $\mathbf{r}_t$ and $\mathbf{v}_t$ (index t for target) represent the geostationary satellite’s location and velocity of its forward motion.

For the SC attitude guidance law $\mathbf{A}^p, \mathbf{\omega}^p$ and $\mathbf{e}^p = \mathbf{\dot{\omega}}^p$ in the IRF the attitude error quaternion $\mathbf{E} = (\mathbf{e}_0, \mathbf{e}) = \mathbf{\hat{A}}^p \mathbf{\Lambda}$ corresponds to Euler parameters’ vector $\mathbf{E} = \{\mathbf{e}_0, \mathbf{e}\}$ with the vector $\mathbf{e} = \{e_i\}$, the angular error matrix $\mathbf{C}^e \equiv \mathbf{C}(\mathbf{E}) = \mathbf{I}_3 - 2[\mathbf{e} \times] \mathbf{Q}^e$, with the matrix $\mathbf{Q}_e \equiv \mathbf{Q}(\mathbf{E}) = \mathbf{I}_3 e_0 + [\mathbf{e} \times]$, the MRP vector $\mathbf{\sigma}^e = \mathbf{e} \tan(\mathbf{\Phi}^e / 4)$ and the angular error vector $\mathbf{\delta} \mathbf{\phi} = \{\delta \phi_i\} = 2\{e_o e_i\}$. The angular rate error vector $\mathbf{\delta} \mathbf{\omega}$ is calculated by the ratio $\mathbf{\delta} \mathbf{\omega} = \{\delta \omega_i\} \equiv \mathbf{\omega} - \mathbf{C}^e \mathbf{\omega}^p$.

The vectors of range to the target $\Delta \mathbf{r} = \{\Delta r_i\}$ (Fig. 1b) and misalignment $\Delta \mathbf{v} = \{\Delta v_i\}$ between the velocities of the SRM and the target are calculated by ratios $\Delta \mathbf{r} = \mathbf{r}_t - \mathbf{r}_r$ and $\Delta \mathbf{v} = \mathbf{v}_t - \mathbf{v}_r$. The SRM guidance law determines the vectors $\mathbf{r}_r^i(t)$, $\mathbf{v}_r^i(t)$ which allow calculating the differences $\Delta \mathbf{r}_r^i(t) = \mathbf{r}_r^i(t) - \mathbf{r}_r^i(0)$, $\Delta \mathbf{v}_r^i(t) = \mathbf{v}_r^i(t) - \mathbf{v}_r^i(0)$, as well as the vector of discrete mismatch $\mathbf{\delta} \Delta \mathbf{r}_r = \Delta \mathbf{r}_r^p - \Delta \mathbf{r}_r$, between the program difference $\Delta \mathbf{r}_r^p \equiv \Delta \mathbf{r}_r^p(t_r)$ and the measured difference $\Delta \mathbf{r}_r \equiv \Delta \mathbf{r}(t_r)$ with the period $T^e_r$ at the time moments $t_r$, $r \in \mathbb{N}_0$.

The applied discrete algorithm for the pulse-width control of the SRM forward motion are presented in [3]. In this simplified algorithm, at first the command vector $\mathbf{I}_r^e$ for thrust pulse of the catalytic EPU is calculated over semi-interval $t \in [t_r, t_{r+1})$, using the formulas

$$\mathbf{g}_{r+1}^e = k^e_r \mathbf{g}_r^e - k^e_r \mathbf{\delta} \Delta \mathbf{r}_r; \quad \mathbf{p}_r = k^e_{\mathbf{p}}(\mathbf{g}_r^e - k^e_{\mathbf{p}} \mathbf{\delta} \Delta \mathbf{r}_r); \quad \mathbf{I}_r^e = T^e_r m (\mathbf{C}^e_w^p + \mathbf{p}_r), \quad (2)$$

and then, for its implementation, the durations of the PWM thrust activation for all eight EJE$s$ are computed by explicit relations [3].

Figure 3. Trajectory of the SRM movement in the target’s ORF.
The problem of the SRM spatial guidance consists in a program location flyby of a geostationary satellite with sequential observation of its state from six points from SRM pole $O_s$. As a result, the GMC digital control vector is formed as $u(t) = J_k \Delta \omega k + \delta k \times \omega k + \dot{\mathbf{m}}_k$ (3) with $\mathbf{G}_k = J_k \omega_k + \mathbf{H}_k$, and then vector $M_k^g$ is distributed between the GDs by explicit relations [4]. As a result, the GMC digital control vector is formed as $u(t) = u_k \forall t \in [t_k, t_{k+1})$.

The paper solves two problems: (i) synthesis of the SRM guidance laws for its inspection flyby of a geostationary satellite with sequential observation of its state from six points from specified distances (Fig. 1a); (ii) dynamical analysis of the robot’s AOCS during such a flyby.

3. The robot guidance laws during flyby and visual inspection of the target

In the ORF of the target, it is convenient to set the SRM movements between the points of the inspection observation of the satellite with a plane $P$, the position of which is assigned by fixed angles $\alpha$ and $\beta$ in Fig. 1b, and with the position of the SRM pole $O_t$ in this plane, which is determined by the angle $\gamma(t)$ and the modulus $s(t) = \Delta r(t)$ of the vector $s(t) = -\Delta r(t)$ (Fig. 1b). The problem of the SRM spatial guidance consists in a program location $s(t)$ of the SRM pole $O_t$ in the plane $P$ and in a program orientation of the axis $O_x,y$ along the vector $\Delta r_p$ when the axis $O_z$ of the robot’s BRF is directed along the normal to the plane of its movement.

Assume that the number $j = 1 \div 5$ of the flight (transition) between the observation points corresponds to the point of the previous observation (Fig. 1a). During the $j$-th flight of the SRM with a given duration $T_j$, the vector $s_j(t)$ of its location is determined by the angle $\gamma_j(t)$ in the plane specified by the angles $\alpha_j$ and $\beta_j$, and the distance $s_j(t)$ from the target. The angle $\gamma_j(t)$ changes from the initial $\gamma_j(0) = \gamma_j(0)$ to the final $\gamma_j(0) = \gamma_j(0) + T_j$ value, and the distance $s_j(t) - s_j(0)$ from the start $s_j(0)$ to the end $s_j(T_j)$ value. In this flight, the column of the coordinates of the SRM’s mass center is determined in the target’s ORF $O_t x_o y_o z_o$ using the orthogonal matrices $Q_j = [\alpha_j \beta_j \gamma_j]^T$. Under the boundary conditions $\gamma_j(0) = \gamma_j(0)$, $\gamma_j(0) = \gamma_j(0)$, $\gamma_j(0) = \gamma_j(0)$, and $s_j(0) = s_j(0)$, $s_j(0)$, $s_j(0)$, $s_j(0)$, $s_j(0)$, $s_j(0)$, the splines $\gamma_j(t)$ and $s_j(t)$ are assigned with specified restrictions on the modules of the first and the second time derivatives, which allows calculation of the vectors of the robot’s forward displacements, velocities, and accelerations of the SRM relative to the target’s ORF using explicit relations. These vectors are first presented in the IRF, the corresponding kinematical parameters of the target movement are added to them, and the vectors of the SRM’s location, velocity, and acceleration in the IRF are also calculated using explicit relations.

Using standard kinematic transformation, the desired laws of robot guidance in its forward motion are represented in the BRF $O_t x_o y_o z_o$ as the program values of the vectors of location $\Delta r_p(t) = -s_p(t)$, velocity $\Delta v_p(t) = \{\Delta v_p(t)\}$ and acceleration $\mathbf{w}_p(t) = \{\mathbf{w}_p(t)\}$. A similar technique with vector splines is used in the synthesis of program changes in the quaternion $\mathbf{A}(t)$, vectors of angular rate $\omega_p(t) = \{\omega_p(t)\}$ and angular acceleration $\mathbf{\alpha}_p(t) = \{\mathbf{\alpha}_p(t)\}$.

Let us assume that visual inspection of the technical state of a geostationary satellite should be performed in the time interval $t \in [14468, 24268]$ s using the observations from six points.

![Figure 4. Time diagram of required SRM transitions and satellite observations. The upper part shows the current current time in seconds, and the lower part shows the durations of observations (green) and transitions (blue).](image-url)
specified in their vicinity. Figure 3 presents the program spatial trajectory of the SRM required flights into the ORF of the target, obtained in the MatLab environment. The time diagram of flights and observations is shown in Fig. 4, where the SRM transitions between six observation points are marked in blue, and the observation areas when the SRM is stabilized in the target’s ORF are highlighted in green. Here, the duration of the \( j \)-th flight is assumed to be the same \( T_j = 1600 \) \( \forall j = 1 \div 5 \), and the duration for observation of a geostationary satellite from each point is of 300 s (5 minutes), see Fig. 1a and Fig. 4. The designed kinematic parameters of the SRM flights when checking the satellite’s technical state are shown in Table 1.

### Table 1. Parameters of the SRM flights during the target’s state inspection.

| \( j \), flight # | \( \alpha_j \) | \( \beta_j \) | \( \gamma_{1j} \) | \( \gamma_{2j} \) | \( s_{1j} \) | \( s_{2j} \) |
|-------------------|----------------|----------------|-----------------|----------------|----------------|----------------|
| 1                 | 0              | 0              | -90             | 0              | 50             | 50             |
| 2                 | 0              | -90            | 0               | 90             | 50             | 50             |
| 3                 | 0              | -90            | 90              | 180            | 50             | 50             |
| 4                 | 0              | 70             | 180             | 90             | 50             | 40             |
| 5                 | 90             | 90             | 70              | 0              | 40             | 60             |

![Figure 5. The program coordinates of the SRM in the target’s ORF.](image)

**Figure 5.** The program coordinates of the SRM in the target’s ORF.

![Figure 6. The SRM velocities relative to the target’s ORF.](image)

**Figure 6.** The SRM velocities relative to the target’s ORF.

![Figure 7. Program angular velocities of the SRM in the IRF.](image)

**Figure 7.** Program angular velocities of the SRM in the IRF.
Figure 8. Mismatches in the location of the SRM during the inspection of the target’s state.

Figure 9. Errors in the implementation of the SRM guidance by the forward motion velocities.

Figure 10. Errors in the implementation of the SRM angular velocities.

Figures 5–7 present some results of autonomous calculation of the laws for the SRM spatial guidance, obtained with the specified initial data. Here and further in Figs. 8–11, changes in all variables are highlighted in the following colors: blue for yaw axis $x$; green for roll axis $y$ and red for pitch axis $z$.

4. Dynamics of the AOCS while inspecting the geostationary satellite state

In computer simulation of nonlinear processes for the motion control of a robot with a mass $m = 3000 \text{ kg}$ and an inertia tensor $\mathbf{J} = \text{diag}(3248, 2348, 3640) \text{ kgm}^2$, taking into account the measurement noises and external disturbances, the SRM model (1) with the control laws (2) and (3) used a period $T_u = 4 \text{ s}$ for the thrust PWM of eight catalytic EJE with a time delay of $T_{zu} = 0.25 \text{ s}$ and a period $T_u = 0.5 \text{ s}$ for digital control of four gyrodines.

Figures 8–11 present some results that demonstrate the AOCS accuracy during visual
inspection. Here it is easy to see that when performing the actual inspection of the target’s state, the errors in the location and orientation of the SRM do not exceed 10 cm and 30 arc sec, respectively.

5. Conclusions
Methods for guidance and control of spatial motion of a space robot during the flyby of a geostationary satellite for its state inspection are briefly presented, as well as numerical results showing the effectiveness of the created algorithms. The main achievements of this paper are as follows: (i) an original technique for parametrization of the robot flights between visual inspection points, which significantly simplifies the synthesis of guidance laws; (ii) confirmation of the effectiveness of algorithms for pulse-width and digital control of the robot’s motion, which provide the required position and angular accuracy of the robot’s guidance when performing visual inspection of the target’s state.

Acknowledgements
This work was supported by the Russian Foundation for Basic Research, Grant no. 20-08-00779.

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