The Types and Factors of Error of Elementary School Students in Solving Mathematical Word Problems: An Analysis Using the Fong’s Method

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Abstract. This research aims to identify types of error and factors that cause 6th-grade elementary school students in rural and urban areas to make errors in solving mathematical word problems based on Fong's Schematic Model for Error Analysis. This research is a survey with a quantitative approach. The research population is all of the 6th-grade elementary school students in Kebumen, Jawa Tengah, Indonesia, totaling 1,928 students. The sample was 522 students consisting of 316 students from urban areas and 206 students from rural areas. The instrument of data collection is an essay test in mathematical word problems form. The instrument is declared valid by the content and construct validity. The instrument reliability coefficient is 0.83. The results of the study show that 6th-grade elementary school students in the urban area make errors incomplete schema with errors (E5) category while rural area students make errors in the category of irrelevant procedures (E2) and complete schemes with errors (E5). Furthermore, the cause of 6th-grade elementary school students in urban areas to make mistakes is a language factor, while in rural areas students make language and operational factors simultaneously.

1. Introduction
There are five standard mathematical abilities to be possessed by students, namely problem-solving, communication, connection, reasoning, and representation [1]. The problem-solving ability of Indonesian students is still low. It can be seen from the results of tests by two international studies, namely the Program for International Student Assessment (PISA) and Trends in the International Mathematics and Science Study (TIMSS). According to the PISA report in 2015, the math score of Indonesian students was 386 points and it was in position 63 of 70 participating countries [2]. In the 2015 TIMSS report, it is stated that Indonesian students’ score was 397 and it was in position 44 out of 49 participating countries [3].

Problems in mathematics can be presented in verbal, images, symbols or combinations of the three [4]. Mathematical problems presented in verbal cannot be separated from the application of mathematics in everyday life. Mathematical problems in the verbal form are often known as word problems or story problems.

The most common problem solving is presented in word problems [5]. Word problems here is a problem where mathematical concepts and principles are expressed in daily language [6,7]. The solution to the word problems is not only concerned about the final answer, but the process of the solution must also be considered. In solving word problems students must focus on what mathematical
operations are needed to solve the word problem. Students must also be able to change the story questions in the form of mathematical sentences, then do calculations [8]. Students are expected to solve word problems step by step through the process so that the flow of thinking shown. Steps in solving word problems consist of: determining what is known and what is asked; making mathematical models; solving mathematical models according to mathematical rules so that they get answers to what is asked and returning the answer in the context of the question being asked.

Mathematical word problem is a difficult problem for most students [6, 7, 9, 10]. Error in solving word problems is still mostly done by most elementary school students. Errors in solving word problems need to be analyzed so that where the error happens, the type of the error and what cause the errors in completing the story problems can be clearly known. The types and causes of errors made by students in solving word problems can be different. This can be caused by the location of the school or the student environment.

Fong’s schematic model for error analysis is a method to analyze the error in solving word problems. Fong’s error analysis is based on a schematic approach to know student’s strategy in solving word problems [11]. Fong classifies the error into two levels. The first level is categorized, based on the schematic approach, into five categories (Table 1).

| Type of error           | Description                                                                 |
|-------------------------|-----------------------------------------------------------------------------|
| (E1) No Solution        | Students do not answer the questions or just write what is known and what is asked without a solution. |
| (E2) Irrelevant Procedure | Students cannot process the information provided so that the problem-solving steps do not lead to the correct answer. |
| (E3) Incomplete Procedure with No Errors | Students do not write the complete steps of solution and do not make any mistakes in the solving process. |
| (E4) Incomplete Procedure with Errors | Students do not write the complete steps of solution and make any mistakes in the solving process. |
| (E5) Complete Procedure with Errors | Students write the complete solution but make any mistakes in the solving process. |

Category E2, E4, and E5 of errors are analyzed at the second level. A second level analysis was conducted to determine the cause of the errors/mistakes made by students. Fong classifies three factors causes of errors namely language, operational and conceptual. Every cause of the error has certain indicators (Table 2).

| The Indicators of the Cause of Error                  |
|-------------------------------------------------------|
| Language                                              |
| 1. students cannot read the words, units, or symbols correctly |
| 2. students do not write what is known                |
| 3. students write down what is known but not right    |
| 4. students do not write what is asked                 |
| 5. students write down what is asked but not right     |
| 6. students do not write the conclusion                |
| 7. students write down the conclusion but not right    |
| Operational                                           |
| 1. students are wrong in choosing the operation used in solving the problem |
| 2. students incorrectly use the correct mathematical rules |
| 3. students do not further process solutions to problem-solving |
| 4. students make errors in calculations                |
| Conceptual                                            |
| students do not operate according to the level in the sequence of work (e.g. do the addition (+) or subtraction (-) before multiplication (×) or division (÷) |

The purpose of this paper is to identify the types and causes of errors made by rural and urban grade VI elementary school students in solving word problems based on the Fong method.
2. Method
The research population was all of the 6th-grade elementary school students in Kebumen, Jawa Tengah, Indonesia, totaling 1.928 students. The sample was 522 students consisting of 316 urban students and 206 rural students.

The instrument for data collection was an essay test in mathematical word problems form. The word problems consist of word problems with two unequal operations and word problems with three or more operations (Table 3).

| Basic Competency | Word Problems                                                                                                               | No item |
|------------------|-----------------------------------------------------------------------------------------------------------------------------|--------|
| Word problems    | The temperature in the refrigerator is -8ºC. After being turned off, the temperature inside rises 2ºC every minute until it reaches room temperature. After reaching room temperature, the temperature in the refrigerator is fixed. What is the temperature in the refrigerator after being turned off for 9 minutes? | 1      |
| with two unequal operations | Diki brings money to the stationery shop. With that money, Diki bought 4 notebooks, 2 pencils and got a change of IDR 2000. If the price of a book is IDR 2500 and the price of a pencil is IDR 1500 then how much money does Diki bring? | 3      |
|                   | Father filled up as much as 30 liters of gasoline. Every day the car is used to go to the office. After three weeks, the remaining gasoline is 3 liters. If the father works six days a week, how many liters of gasoline are used to go to the office every day? | 4      |
| Word problems    | Exam questions consist of 25 items. The assessment of each item is as follows: correct answer gets a score of 2, the wrong answer gets a score of -1, does not answer the question gets a score of 0. Student A answer 12 questions correctly, 11 wrong, and the rest are not answered. Student B answers 9 questions correctly, 4 wrong, and the rest are not answered. Who is the student who has a higher score? | 2      |
| with three or more operations | Lily and Rose are playing games. In the game, each player has the first 0 points and the player with the most points will be the winner. Lily gets 5 points twice, loses 11 points, and gets 3 points. Rose lost 3 points twice, lost 1 point, get 4 points twice. Who is the winner in the game? There are 27 students in 6th grade of Taruna elementary school and Mrs. Nisa is the teacher. She has 35 pieces of books. She bought another 10 packs of books. Each pack contains 10 books. All books will be distributed equally to students. How many books does each student get? | 5      |

The steps taken in the data analysis process in this study were as follows: (1) identifying errors made by students in solving word problems based on student answers; (2) classifying types of error based on Fong's Schematic Model for Error Analysis. Errors made by students were classified by referring to the error indicator in each type of error; (3) calculating the types of error percentage and then sort from the largest percentage to the smallest; (4) identifying causes of mistakes made by students. To identify the causes of errors in solving word problems, the types E2, E4, and E5 of errors made by students need to be analyzed; and (5) determining the estimated proportion for each type of error. The estimation of this proportion was carried out to determine the percentage of each type of error made by the sixth-grade elementary school students in Kebumen Sub-District. If \( \hat{p} \) is the proportion type of error in \( n \) random sample and \( \hat{q} = 1 - \hat{p} \), then the confidence interval is approximately \((1 - \alpha)100\%\) for binom \( p \) parameters given by
\[
\bar{p} - \frac{z_{\alpha/2}}{\sqrt{n}} \left( \frac{\bar{p} \bar{q}}{n} \right) < p < \bar{p} + \frac{z_{\alpha/2}}{\sqrt{n}} \left( \frac{\bar{p} \bar{q}}{n} \right)
\]

\( z_{\alpha/2} \) is the \( z \) value in a standard normal table with an area of the right side equal to \( \alpha/2 \) [12].

3. Result and Discussion

Analysis of solving word problems in grade 6th elementary school students in urban and rural areas is intended to see the percentage of each type of error made by elementary students in urban and rural areas. In general, the results of the analysis of the solving of the word problems in elementary school students in urban areas can be seen in Table 4 and rural areas in Table 5.

| No | Correct Answer | Types of Error |
|----|----------------|----------------|
|    | E1  | E2  | E3  | E4  | E5  |
| 1  | 130 | 41.14 | 2 | 0.63 | 72 | 22.78 | 4 | 1.27 | 17 | 5.38 | 91 | 28.80 |
| 2  | 93  | 29.43 | 18 | 5.70 | 45 | 14.24 | 2 | 0.63 | 13 | 4.11 | 145 | 45.89 |
| 3  | 144 | 45.57 | 1 | 0.32 | 13 | 4.11 | 11 | 3.48 | 32 | 10.13 | 115 | 36.39 |
| 4  | 69  | 21.84 | 6 | 1.90 | 101 | 31.96 | 38 | 12.03 | 31 | 9.81 | 71 | 22.47 |
| 5  | 92  | 29.11 | 15 | 4.75 | 20 | 6.33 | 1 | 0.32 | 17 | 5.48 | 177 | 56.01 |
| 6  | 117 | 37.03 | 16 | 5.06 | 51 | 16.14 | 5 | 1.58 | 12 | 3.80 | 115 | 36.39 |

Table 4. Result of analysis solving word problems 6th-grade student in the urban area (n = 316)

| No | Correct Answer | Types of Error |
|----|----------------|----------------|
|    | E1  | E2  | E3  | E4  | E5  |
| 1  | 50  | 24.27 | 2 | 0.97 | 67 | 32.52 | 2 | 0.97 | 6 | 2.91 | 79 | 38.35 |
| 2  | 30  | 14.56 | 10 | 4.85 | 32 | 15.53 | 3 | 1.46 | 12 | 5.83 | 144 | 69.90 |
| 3  | 69  | 33.50 | 3 | 1.46 | 17 | 8.25 | 14 | 6.80 | 30 | 14.56 | 73 | 35.44 |
| 4  | 21  | 10.19 | 5 | 2.43 | 126 | 61.17 | 11 | 5.34 | 18 | 8.74 | 25 | 12.14 |
| 5  | 36  | 17.48 | 16 | 7.77 | 30 | 14.56 | 7 | 3.40 | 4 | 1.94 | 113 | 54.85 |
| 6  | 51  | 24.76 | 9 | 4.37 | 38 | 18.45 | 13 | 6.31 | 7 | 3.40 | 88 | 42.72 |

Based on Table 4 and Table 5, the results of the analysis generally show that the percentage of correct answers made by urban students is greater than those made by students in rural areas. It means that rural area students make more error than urban students. This is comparable with the learning achievement of rural area students that is lower than urban area students in Kebumen Sub-District, Jawa Tengah, Indonesia. School location has a significant effect on student achievement, where rural student's achievement is not as good urban student achievement [13]. The following is a discussion of each type of error made by students in urban and rural areas.

No Solution (E1). Based on Analysis result, not many students make the E1 error category. But if we compare, rural area students made the E1 error category more than urban area students. On average, there are 3.06% of urban students and 3.64% of rural students make E1 error category. The most percentage of errors with category E1 is in item number 5 for urban area students and item number 2 for rural area students. The cause of students making this error cannot be determined specifically because there is no written answer.

Irrelevant procedure (E2). Irrelevant procedures become a category of errors that are mostly carried out by both urban and rural area students. But if we compare, rural area students made the E2 error category more than urban area students. On average, there are 15.93% of urban students and 25.08%
of rural students make E2 error category. The most percentage of errors with category E2 is in item number 4 for both urban and rural area students. The cause of the error is analyzed with the second level analysis according to the students' answers.

Incomplete Schema with No Errors (E3). Not many students make the E3 error category. But, rural area students made the E3 error category more than urban area students. On average, there are 3.21% of urban students and 4.04% of rural students make an error in this category. The most percentage of errors with category E3 is in item number 4 for urban area students and item number 3 for rural area students. The cause of students making this error cannot be determined because there is no error in the second level.

Incomplete Schema with Errors (E4). This error is done a lot enough by urban and rural students. On average, there are 6.11% of urban students and 6.23% of rural students make an error in this category. The most percentage of errors with category E4 is in item number 3 for both urban and rural area students. The cause of the error is analyzed with the second level analysis according to the students' answers.

Complete schema with errors (E5). The E5 category is the most error category carried out by urban and rural students. In almost every item, the error category E5 is the most percentage of the error category. On average, there are 37.66% of urban students and 42.23% of rural students make an error in this category. The most percentage of errors with category E5 is in item number 5 for urban area students and item number 2 for rural area students. The cause of the error is analyzed with the second level analysis according to the students' answers.

Based on Table 3 we can also see that the dominant type of error that makes by urban students in solving word problems with two unequal operations (Item 1, 3 and 4) and word problems with three or more operations (item 2, 5, and 6) is complete schema with no errors (E5). In addition, based on Table 4, we can find out what kind of mistakes rural students make when solving word problems with two unequal operations (Items 1, 3 and 4) which are irrelevant procedures (E2) and word problems with three or more operations (items 2, 5 and 6), namely a complete scheme without error (E5).

The second level analysis was used to find out the cause of students making errors in solving word problems. The analysis on the second level is only in categories E2, E4, and E5. There are 3 factor that causes the errors, namely language, operational and conceptual. Based on the analysis, both urban and rural students made an error caused by language and operational. The 6th-grade elementary school students in Kebumen Sub-district, Central Java, Indonesia have understood the concept of mixed counting operations. Based on the second level analysis of urban and rural students, there are seven dominant indicators that cause errors, they are (1) students cannot read the words, units, or symbols correctly, (2) students write down what is known but not right, (3) students do not write the conclusions, (4) students are wrong in choosing the operation used in solving the problem, (5) students incorrectly use the correct mathematical rules, (6) students do not further process solutions to problem-solving, and (7) students make errors in calculations.

The main cause of students making E2 category errors is that students choose the wrong operation to solve the problem. Every student who makes an E2 category error, made an error in choosing the operation used in solving the problem. Figure 1 is an example of student error E2 category. The student is wrong in choosing the operation so that the solving of the model does not lead to correct answers. Students only use the numbers in the question then arrange them into mathematical models that are not relevant to the question.

The main cause of students making E4 category errors is that students do not write conclusions or students cannot continue the problem-solving process. This is because writing conclusion is part of the scheme. Therefore, if the student does not write the conclusion then his scheme incomplete. Similarly, if students do not finish in processing the solution, the student scheme is also incomplete. Besides that students also make other mistakes, such as writing down what is known but not right, using incorrect rules or mathematical rules and choosing the wrong operation, or miscalculating. Figure 2 is an example of student error E4 category.
Figure 1. An Example of a student’s answer in E2 Category

Figure 2. An Example of a student’s answer in E4 Category

Schema to solve item no 3 is a-b-c-d-e where: a) Write down what is known and asked, b) Determine the price of 4 books and 2 pencils, c) Determine the amount of money to be paid, d) Determine how much money is used to pay, e) Returns the answer to the original question. Based on Figure 2, student’s scheme is (a)-(c)-(d)-e. The student did not write one of the steps of solution namely, step b and make a second level error in steps a and d. In step a, students do not complete the writing of what is known and in step d, students are wrong in choosing the operation used in solving the problem.

There are several dominant factors cause students to make E5 category error, i.e. students write down what is known but not right, students cannot read the words, units, or symbols correctly, students incorrectly use the correct mathematical rules and students make errors in calculations. Figure 3 is an example of student error E5 category.

Figure 3. An Example of a student’s answer in E5 Category

Schema to solve item no 2 is a-b-c, where: a) Write down what is known and asked, b) determine each score of student A and B, c) Return the answer to the original question. Based on Figure 3, student’s scheme is a-(b)-c. Student’s scheme is complete, but he makes an error in step b, that makes an error in calculation.

The most dominant factor causing errors in rural areas are language and operations that are carried out simultaneously. Meanwhile, the cause of errors in urban students is language or operations.
Overall, urban area students are better able to understand questions than rural area students. Understanding the problem is the initial key for students to solve word problems. It is impossible for a problem solver to solve a problem without beginning with a representation of the problem. In this case, students cannot solve the word problem if the student is unable to understand the problem. In several studies, problem-solving skills can be improved only by providing training for understanding strategies [15,16].

Operational factor that cause the error that rural students do are error in choosing an operation to solve word problems and error in calculation. In the problem-solving phase revealed by Polya, error in choosing operation occurred at the stage of making a mathematical model. Scaffolding can help reduce student errors when making mathematical models [16]. Therefore, to reduce rural students’ errors in solving word problems in Kebumen Sub-District, the teacher must provide many exercises in word problem so that students are accustomed to understanding various types of problems in various stories. Second, during the process of the exercises, the teacher must guide students until they can really solve the word problems without the help of the teacher.

Urban student’s error in solving word problem caused by incorrect use of the correct mathematical rules. To minimize these errors the teacher needs to teach mathematical rules that students must and cannot do when solving mathematical problems. It is very important to give logical and easy accepted explanations so that students really understand and they do not forget easily.

Determining the percentage of errors made by 6th-grade elementary school students in Kebumen Sub-District is done through an estimate for the proportion of the population. Estimates made on proportions use estimates of confidence intervals. Table 6 shows the number of students who make mistakes for each category for the whole sample (522 students) in each item.

| No item | E1 | E2 | E3 | E4 | E5 |
|---------|----|----|----|----|----|
| 1       | 4  | 139| 6  | 23 | 170|
| 2       | 28 | 77 | 5  | 25 | 259|
| 3       | 4  | 30 | 25 | 62 | 188|
| 4       | 11 | 227| 49 | 49 | 96 |
| 5       | 31 | 50 | 8  | 15 | 290|
| 6       | 25 | 89 | 18 | 19 | 203|
| average | 17 | 102| 19 | 32 | 201|

Table 6. Number of students who make mistakes on each item and category

Based on Table 6 we can find out the average number of students who make mistakes in each category of errors. From these data, we specify a 95% confidence interval for the proportion of types of errors students makes in solving story problems in each category. Table 7 shows the 95% convention interval for each category of errors.

| JK | Amount | n  | \(\hat{p}\) | \(\hat{q}\) | \(z_{0.025} \left( \frac{\hat{p}\hat{q}}{n} \right)^{0.5} \) | Confidence interval for the proportion |
|----|--------|----|------------|------------|-------------------------------------------------|-------------------------------------|
| E1 | 17     | 522| 0.0326     | 0.9674     | 0.0152                                          | 0.0173 < \(p_1\) < 0.0478          |
| E2 | 102    | 522| 0.1954     | 0.8046     | 0.0340                                          | 0.1614 < \(p_2\) < 0.2294          |
| E3 | 19     | 522| 0.0364     | 0.9636     | 0.0160                                          | 0.0203 < \(p_3\) < 0.0525          |
| E4 | 32     | 522| 0.0613     | 0.9387     | 0.0205                                          | 0.0407 < \(p_4\) < 0.0819          |
| E5 | 201    | 522| 0.3851     | 0.6149     | 0.0417                                          | 0.3433 < \(p_5\) < 0.4268          |

Table 7. 95% Confidence interval for the proportion of error types \((p)\) in each error category
4. Conclusion

Based on the results and discussion above, several conclusions were made. First, an error made by grade 6 elementary school students in Kebumen Sub-district in urban areas in solving word problems with two unequal operations and the word problem with three or more operations is a complete scheme with errors (E5). Second, errors made by 6th-grade elementary school students in Kebumen Sub-district in rural areas in solving word problems with two unequal operations is irrelevant procedures (E2). Then, error in solving word problems with three or more operations are complete schemes with errors (E5). Third, the factor that caused errors made by 6th-grade elementary school students in Kebumen Sub-district in urban areas is a language, namely writing what is known but incomplete or operational, namely students incorrectly use mathematical rules and students make an error in calculations. Fourth, the factors that caused errors made by 6th-grade elementary school students in Kebumen Sub-district in rural areas are language and operational that are simultaneously do. Language factors that rural students do are students cannot read the words, units, or symbols correctly, students write down what is known but not right, and students do not write the conclusions. Operational factors that rural students do are students are wrong in choosing the operation used in solving the problem, students incorrectly use the correct mathematical rules, students do not further process solutions to problem-solving, and students make errors in calculations. Fifth, the percentage of each type of error E1, E2, E3, E4 and E5 conducted by 6th-grade elementary school students in Kebumen Sub-district respectively is $1.73\% < p_1 < 4.78\%$, $16.14\% < p_2 < 22.94\%$, $2.03\% < p_3 < 5.25\%$, $4.07\% < p_4 < 8.19\%$, and $34.33\% < p_5 < 42.86\%$

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