Analysis of mesh stiffness of herringbone gear considering modification

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Abstract. Herringbone gear is a key component in the reducer. Its mesh stiffness is an important parameter in gear dynamics research. In this paper, an improved Velex method is proposed to calculate the mesh stiffness of herringbone gears, and compared with Weber method and Finite Element Method (FEM), the accuracy of the method is proved. Meanwhile, the influence of helix angle and modification coefficient on the stiffness of the gear is analyzed by Weber method. The results show that the two parameters have different degrees of influence on the mesh stiffness of the herringbone gear, which provide a theoretical basis for the design of the herringbone gear transmission. Finally, by the calculation of the stiffness of the herringbone gear, the difference between the herringbone gear and the helical gear is compared and analyzed, and the conclusion that the herringbone gear is not a simple superposition of the helical gear is obtained. Because of the difference between the helical gear and the herringbone gear, it is of great value to the stiffness analysis of the herringbone gear.

1. Introduction
With the wide application of herringbone gear in aerospace, marine and other industrial fields, the dynamic problem has become increasingly prominent. Therefore, the dynamic characteristics of gear transmission has become a hotspot issue for scholars. Among them, the mesh stiffness of the gear is a key issue, and many scholars have carried out a lot of research on it.

Through the literature research, it is found that the calculation methods of gear mesh stiffness can be summarized into three categories: empirical formula, finite element method and analytic method. For the research of spur gear, Wang [1] combines the finite element method with the analytical method to study the mesh stiffness of the spur gear. Yang and Lin [2] proposed that the potential energy integration method is widely applied to the stiffness calculation of spur gear. Liang et al. [3] analyzed the mesh stiffness of the spur gear by means of potential energy method, studied the influence of the crack on the mesh stiffness, and established the mesh stiffness equation of the crack propagation. Ma et al. [4] studied the calculation of mesh stiffness of spur gear with spalling defects. By comparing the gear mesh stiffness under different spalling width, length and position with finite element method, the correctness of the analysis model is verified. Saxena et al. [5] studied the effect of time-varying friction coefficient on the total effective mesh stiffness of spur gears. Based on the static two-dimensional finite element analysis method, Maclennan [6] studied the influence of tooth profile error on the load sharing capacity and mesh stiffness of spur gears.

For the helical gears, domestic and foreign scholars have also done a lot of research. Lin et al. [7] used
the combination of potential energy method and slicing method to study the influence of friction on the mesh stiffness of helical gears. Wang et al. [8] proposed an improved time-varying mesh stiffness model of helical gear pair to study the effect of axial meshing force component on the time-varying mesh stiffness of helical gear pair. Yu et al. [9] defined two kinds of helical gear pairs based on the relationship between the lateral contact ratio of the helical gear and the overlapping contact ratio, and used the slicing principle to calculate an improved analytical model of the single mesh stiffness of the helical gear. Chang et al. [10] used the finite element method and the local contact analysis of elastic body method to study the analytical method for calculating the mesh stiffness and load distribution of helical gears. Wang et al. [11] studied a model of the influence of tooth deviation and assembly error on the mesh stiffness of helical gears, and approximated the helical gear to a series of relatively small independent straight teeth. The stiffness of the tooth is calculated by potential energy method, and the validity of the model is verified.

The above literature shows that the analysis of gear mesh stiffness is mainly focused on spur gear and helical gear. Through the great efforts of scholars, the theory and method are relatively mature. However, the research on mesh stiffness of herringbone gear is relatively few. Zhu et al. [12] studied the effect of torsional stiffness on load sharing characteristics of herringbone gear transmission system. Wang et al. [13] used the herringbone gear as the helical gear to calculate the stiffness of the herringbone gear transmission system in the vibration characteristic analysis and experimental research. At present, the study of the properties of herringbone gears is to copy the characteristics of the helical gears, and simplifies the herringbone gears as a helical gears, which will cause certain errors to the final result. In summary, there is no systematic study on the mesh stiffness of herringbone gears. Therefore, this paper analyzes the mesh stiffness of the herringbone gear under the condition of modification.

2. Theory

2.1 Improved Velex Method

The calculation method of the meshing stiffness proposed by the French scholar Velex is improved to calculate the meshing stiffness of the herringbone gear [14-15]. The method considers that the mesh stiffness of a pair of gears on the unit contact line of the meshing transmission is constant value, and the length of the total contact line is time-varying, so the product of the stiffness of the unit contact line and the length of the contact line is the mesh stiffness.

The total length of the contact line at t time is \( L(t) \). As shown in Fig.1, M is an arbitrary point on the contact line. The mesh stiffness of the unit contact line length at point M is \( k(M) \) and the contact line length is \( dL \), then the mesh stiffness of the t time is as follows:

\[
k(t) = \int_{L(t)} k(M) \, dL
\]  

(1)

![Fig.1. Contact area of herringbone gear](image)

According to the parallel connection of stiffness, the time-varying mesh stiffness of the herringbone gear is calculated as:

\[
k(t) = 2k_o L(\tau)
\]  

(2)

The dimensionless time \( \tau = t/T_m \) was introduced, \( T_m = P_b/R_b(0) \) is the time when the gear rotates a base circle pitch, then the time-varying contact line length \( L(\tau) \) is:
\[ L(\tau) = \left(1 + 2 \sum_{n=1}^{\infty} \text{Sinc}(k_x)\text{Sinc}(k_y) \cos\left(\tau k_x + \epsilon_y - 2\tau\right)\right)_{n=0} \quad (3) \]

\[ L_m = B\epsilon_\alpha / 2\cos\beta_b \] is the average contact line length, \( \text{Sinc}(x) = \sin(\pi x) / \pi x \) is the classic sine cardinal function. Where \( B \) is the tooth width of the gear, \( \epsilon_\alpha \) is the overlap contact ratio, and \( \beta_b \) is the helix angle of base circle.

The calculation method of mesh stiffness \( k_0 \) for the length of unit contact line is as follows:

\[ k_0 = \cos\beta_b 0.8 \frac{q}{q} \quad (4) \]

Where \( q \) is the flexibility coefficient and the formula is as follows:

\[ q = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 + C_7 + C_8 + C_9 \quad (5) \]

In the formula, the coefficients \( C_1, C_2, \ldots, C_9 \) are as shown in Table 1. \( Z_{n1} \) and \( Z_{n2} \) are the virtual number of teeth of gears respectively. \( x_1 \) and \( x_2 \) are the modification coefficients of gears respectively.

Table 1. Coefficient values of the flexibility coefficient.

|   |   |   |   |   |
|---|---|---|---|---|
|   | 0.04723 | 0.15551 | 0.25791 | -0.00635 |
| C_1 | C_2 | C_3 | C_4 | C_5 | C_6 | C_7 | C_8 | C_9 |
| -0.11654 | -0.00193 | -0.24189 | 0.00529 | 0.00182 |

2.2 Weber Method

According to the elastic mechanics, the gear teeth can be equivalent to a non-uniform cantilever beam. The center of the herringbone gear is considered as original point and the xoy coordinate system is established in Fig.2. It is assumed that the thickness of segment \( i \) is \( H_i \), the area is \( A_i \), the inertia distance of the section is \( I_i \), and the distance from the segment \( i \) to the load action point along the x-axis direction is \( L_{ij} \). The Poisson's ratio of the tooth material is \( \nu \), the equivalent elastic modulus is \( E_e \), the standard elastic modulus of the material is \( E \), and the angle between the load direction and the vertical direction is \( \beta_i \).

![Fig.2. Structure sketch of gear teeth cantilever beam](image)

(1) Tooth bending deformation and shear deformation

The deformation resulting from load \( F_n \) at \( i \) segment consists of two parts: bending deformation and shear deformation. The deformation \( \delta_{bij} \) can be expressed as:

\[ \delta_{bij} = \frac{F_n}{E_e} \left\{ \frac{(H^2 + 3H_i^2L_j^2)}{3l} + \frac{12H(1+\nu)}{5A} \right\} \]

\[ \sin(2\beta_j) \frac{H}{A} \sin^2 \beta_j \] (6)

Among them, \( E_e \) is the equivalent elastic modulus, which depends on the width of the teeth. The relationship between \( E_e \) and the material's standard modulus of elasticity \( E \) is expressed as:
\[ F_x = \begin{cases} \frac{E}{1 - \nu^2} & (B_{hp} > 5) \\ \frac{(B_{hp} - 5)}{E} & (B_{hp} < 5) \\ \end{cases} \]  

(7)

Where \( B_{hp} \) is the pitch circle width.

The \( n \) small segments along the root to the meshing point \( j \) are respectively solved and superposed. The deformation amount of the \( j \) point caused by the bending deformation and shear deformation of the gear teeth is obtained as follows:

\[ \delta_{nj} = \sum_{i=1}^{n} \delta_{nj} \]  

(8)

(2) Additional deformation at the meshing point caused by elastic deformation of the gear base

As the base of the gear root is also elastic, additional deformation at the meshing point caused by the deformation of the gear base should be considered. According to the analysis of gear by R.W.Cornell [16], the additional deformation of the meshing point caused by the elastic deformation of the gear base is expressed as:

\[ \delta_{nj} = \frac{F_{n} \cos^2 \beta_j}{BE_x} \left\{ \frac{16.67}{\pi} \left[ \frac{L_j}{H_j} \right]^2 + \frac{2L_j}{H_j} \frac{1 - 2\nu}{1 - \nu} \right\} + 1.534 \left[ 1 + \frac{\tan^2 \beta_j}{2.4(1 + \nu)} \right] \]  

(9)

Where,

\[ L_j = x_j - x_w - y_j \tan \beta_j \]  

(10)

\[ H_j = 2y_w \]  

(11)

(3) Contact deformation of gear tooth meshing

The contact deformation \( \delta_{cj} \) of the meshing point \( j \) is determined by the Hertz contact theory. According to the derivation of H.H.Lin, it can be expressed as:

\[ \delta_{cj} = \frac{1.275}{E_x} \frac{0.13}{b} \frac{1}{F_{c1}^{0.1}} \]  

(12)

Where, \( b \) is the half width of the Hertz contact zone.

\[ b = \sqrt{\frac{4F_n}{\pi L \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right)}} \]  

(13)

By summing up the above three kinds of deformation, the total normal deformation of gear meshing point \( j \) is obtained as follows:

\[ \delta_j = \delta_{nj} + \delta_{nj} + \delta_{cj} \]  

(14)

The single tooth normal mesh stiffness for the meshing point \( j \) is:

\[ K = \frac{F_n}{\delta_j} \]  

(15)

The mesh stiffness of the single tooth from the entry to the exit meshing is obtained, and then the overall mesh stiffness of the gear is obtained by superimposing according to the contact ratio.

\[ 2.3 \text{ Finite Element Method} \]

The mesh stiffness of gears is solved by finite element analysis. The purpose is to solve the deformation of gear teeth. The finite element analysis of the herringbone gear was performed according to herringbone gear parameters of Table 2.

| Table 2. The basic parameters of herringbone gear pair. |
|----------------------------------------------------------|
| Parameters | Driving gear | Driven gear |
| Number of teeth | 24 | 24 |
### Table 1

| Property                      | Value     |
|-------------------------------|-----------|
| Normal module (mm)            | 3.94      |
| Pressure angle (°)            | 20        |
| Helix angle (°)               | 30        |
| Tooth width (mm)              | 60        |
| Material                      | 42CrMo    |
| Young’s modulus ($Pa$)        | $2.12\times10^{11}$ |
| Poisson’s ratio               | 0.28      |
| Mass density /($kg/m^3$)      | $7.85\times10^{3}$ |

(1) Hexahedral mesh division

The quality of the mesh and the grid density of the contact area are related to the accuracy of the finite element contact analysis. An accurate herringbone gear model is established in Pro/E, and the model is imported into HyperMesh for hexahedral meshing. For a single tooth model, the quadrilateral 2D mesh is first divided. Since the tooth is the involute tooth profile, it is divided into different regions when dividing the grid. As shown in the Fig.3a, the single tooth is divided into several different regions. Based on the 2D grid, the hexahedral mesh is stretched by the Solidmap command to obtain a higher quality mesh, as shown in the Fig.3b.

![Fig.3. Single tooth mesh model](image)

(a) ![Fig.3. Single tooth mesh model](image) (b)

(2) Finite element solution

As an excellent representative of CAE software, ABAQUS plays an important role in many fields [18]. Especially for solving the problem of contact nonlinearity, ABAQUS has its own unique advantages. In
this paper, ABAQUS is used as a finite element simulation tool for analyzing gear mesh stiffness. The change values of the rotation angles $\theta_1$ and $\theta_2$ of the reference points of the driving and driven gear rotation centers are respectively measured, and the quasi-static transmission error is obtained:

$$\Delta \theta = \theta_2 - \frac{Z_2}{Z_1} \theta_1$$  \hspace{1cm} (16)

Where $Z_1$ is the number of teeth of the driving gear, $Z_2$ is the number of teeth of the driven gear. Because the influence of installation error, manufacturing precision and other factors on the system is not considered in the simulation process, the transmission error $\Delta \theta$ can be considered to be caused by the gear load deformation, and the transmission error is converted into the normal deformation amount $\delta_n$:

$$\delta_n = \frac{\Delta \theta \cdot m_z \cos \alpha_t}{2 \cos^2 \beta}$$  \hspace{1cm} (17)

The tooth surface meshing normal load $F_n$ is:

$$F_n = \frac{2000T}{m_z \cos \alpha_t \cos \beta}$$  \hspace{1cm} (18)

Finally, the normal mesh stiffness $K_n$ of the gear can be obtained as:

$$K_n = \frac{F_n}{\delta_n}$$  \hspace{1cm} (19)

According to the above three methods, the mesh stiffness curve of the herringbone gear is obtained, as shown in Fig.6. Due to the large contact ratio of the herringbone gears, the gear teeth come into meshing engagement and secede from the meshing gradually, as shown in Fig.5c. Therefore, the mesh stiffness of the herringbone gear changes gently, unlike the mesh stiffness of the spur gear, there is a significant step abrupt change.
Fig. 6. Three kinds of mesh stiffness curves

The three methods are compared and the results are shown in Table 3.

| Method              | Mean mesh stiffness (N/m) | Error |
|---------------------|---------------------------|-------|
| Weber Method        | $12.10 \times 10^8$       | —     |
| FEM                 | $11.96 \times 10^8$       | 1.2%  |
| Improved Velex Method | $12.93 \times 10^8$   | 6.8%  |

In the finite element calculation, the quasi-static simulation method is applied. The dynamic impact of the gear teeth will cause the actual deformation to be larger than the theoretical value. When calculating the stiffness, the normal load is a static load, resulting in a smaller final result. For the improved Velex method, the change trend of the mesh stiffness is similar to the other two methods, and the result is 6.8% different from the Weber method, which shows the accuracy of the improved Velex method.

3. Results and discussion

Gear mesh stiffness is a parameter that reflects the meshing state of gears from meshing to disengaging. The research shows that different gear parameters will cause the change of mesh stiffness. According to the three parameters of the helix angle and modification coefficient of the herringbone gear, the Weber method is used to analyze the influence of each parameter on the mesh stiffness. And from the perspective of stiffness, the difference between the herringbone gear and the helical gear is compared.

3.1 Effect of the helix angle
Fig. 7. Relationship between helix angle and stiffness

Through the calculation of Weber method, the curve of gear stiffness with helix angle is obtained, as shown in Fig. 7. As a result, the single tooth stiffness and the mesh stiffness of the herringbone gear change with the change of the helix angle. For the single tooth stiffness, as the helix angle of the gear increases, the stiffness of the single tooth increases gradually, and the rate of change of the stiffness curve also increases. This is due to the influence of the overlap contact ratio $\varepsilon_\beta$ of the herringbone gear.

$$\varepsilon_\beta = \frac{B \sin \beta}{\pi m_h}$$  \hspace{1cm} (20)

From Eq. (20), it can be obtained that as the helix angle increases, the overlap contact ratio gradually increases in the range of 0 to 45 degrees. The mesh stiffness of herringbone gears varies periodically with time. In order to describe the fluctuation of the mesh stiffness, the fluctuation $\delta_C$ of the mesh stiffness of the herringbone gear is defined as [19]:

$$\delta_C = \frac{\Delta C_r}{C_m} \times 100\%$$  \hspace{1cm} (21)

Where $\Delta C_r$ is the difference between the maximum and minimum values of the mesh stiffness, $C_m$ is the mean of the mesh stiffness.

According to Eq. (21), the calculation result is shown in the Table 4.

| Helix angle/° | Mesh stiffness $[10^8$(N/m)$]$ | Fluctuation value |
|-------------|-----------------|------------------|
|             | Maximum | Minimum | Mean value |                  |
| 15          | 14.83    | 11.25    | 12.91      | 27.7%             |
| 20          | 13.21    | 12.04    | 12.65      | 9.2%              |
| 25          | 12.61    | 12.33    | 12.45      | 2.2%              |
| 30          | 12.93    | 11.67    | 12.10      | 10.4%             |

A series of helix angles is calculated to obtain a scatter plot of helix angle and stiffness fluctuation, as shown in Fig. 8.

![Fig. 8. Curve of relationship between helix angle and stiffness fluctuation](image)

The fitting equation is obtained by Gauss curve fitting:
In the transmission design process of the gear reducer, the fluctuation of the gear mesh stiffness should be controlled. From the fitting equation, the fluctuation of the gear stiffness can be judged, which provides a theoretical basis for the selection of the helix angle of the herringbone gear.

3.2 Effect of the modification coefficient
According to the modification coefficient of gears, there are three forms of gear: positive modification, zero modification and negative modification. According to the classification of transmission, gear can be divided into equal modification transmission, standard transmission, positive transmission and negative transmission.

\[ y = 0.76 - 0.73e^{-\frac{(x-24.5)^2}{20.4^2}} \]  \hspace{1cm} (22)

Compared with the standard gear transmission, various modifications are analyzed. As shown in Fig.9b, when the gear has equal modification transmission, due to the change of the tooth shape, the stiffness of the gear is decreased, but the change is little, and it can be said that the equal modification transmission has no influence on the stiffness of the gear.

When the gear has positive transmission, as shown in Fig.9c. The positive modification coefficient increases the single tooth stiffness of the gear, and the mesh stiffness also increases, but the fluctuation of the mesh stiffness increases. Because the large mesh stiffness fluctuation is not conducive to achieving the load sharing properties of the reducer [20], the positive modification transmission is not conducive to improving the gear transmission properties.
Positive modification

Contact line
Elliptical contact area
Contact point

Negative modification

Fig. 10. Change of meshing point after modification

When the gear has negative transmission, as shown in Fig. 9d. The negative modification coefficient decreases the single tooth stiffness of the gear and the mesh stiffness. But the negative modification not only reduces the center distance, also reduces the fluctuating value of the mesh stiffness. This modification mode is conducive to improving gear transmission properties.

As the gear is modified, its single gear stiffness curve also changes. When a modification occurs, the mesh point of the gear will produce slight axial displacement relative to the standard gear, as shown in Fig. 10 [21]. The change of the mesh point before and after the modification, ∆L₁ > ∆L > ∆L₂, leads to the difference of the stiffness change rate of the teeth.

3.3 Comparison of herringbone gear and helical gear

For the study of herringbone gears, many scholars have equivalent the herringbone gears to helical gears. They think that the herringbone gears are the superposition of two helical gears. According to the research on herringbone gears, this statement is wrong.

Owing to the helical gear is involved, the helix angle of the gear is changed to 20° and the other parameters remain unchanged. Select the B=120mm herringbone gear and helical gear, B=120mm herringbone gear and B=60mm helical gear as the research model.

As shown in Fig. 11, the stiffness of the herringbone gear is significantly larger than that of the helical gear, and the mesh stiffness is naturally larger than that of the helical gear. On the one hand, due to the unique symmetrical structure of the herringbone gear, there is no influence of the axial force, which causes the deformation of the gear teeth to decrease. Under the same load conditions, the stiffness of the herringbone gear is greater than the stiffness of the helical gear. On the other hand, the meshing of the herringbone gear is a process in which both sides enter and gradually withdraw at the same time, while the helical gear is one side meshing. The two meshing modes cause different contact areas. Under the same
working conditions, the deformation of the teeth of herringbone gear must be decreased.

![Comparison of herringbone and helical gear](image)

(a) Herringbone gear with B=120mm  (b) Helical gear with B=60mm

Fig.12. Comparison of herringbone and helical gear

Under the same conditions, compared with the pair of B=120mm herringbone gears and the pair of helical gears with B=60mm, as shown in Fig.12, the fluctuation trend of the mesh stiffness of them is the same, but there are huge differences in numerical values. Based on the mean mesh stiffness of the two models, the mean mesh stiffness of the herringbone gear is 24.756×10^8 N/m, but the mean mesh stiffness of the helical gear is only 10.59×10^8 N/m. The authors have used a large number of examples to illustrate the quantitative relationship between the mean mesh stiffness of the two models, as shown in Table 5. It can be concluded that the mean mesh stiffness of the herringbone gear is 2.3 times that of the helical gear. Therefore, the herringbone gear is not a simple superposition of the helical gear.

Table 5. Comparison of mean mesh stiffness of gears under different factors. (Unit: 10^8 N/m)

|          | Herringbone gear B=120mm | Helical gear B=60mm | Quantitative relationship |
|----------|--------------------------|---------------------|---------------------------|
| Helix angle | 16°                      | 25.26               | 10.88                     | 23.52 | 10.09 | 2.3 |
| Helix angle | 18°                      | 25.19               | 10.85                     | 23.49 | 10.12 | 2.3 |
| Modification Coefficient | 0.235/-0.078 | 23.52               | 10.09                     | 2.3 |
| Modification Coefficient | -0.105/0.032 | 23.49               | 10.12                     | 2.3 |

4. Conclusions

In this paper, three methods are used to calculate the meshing stiffness of the herringbone gear, and the accuracy of the improved method is proved. The influence of the helix angle and the modification coefficient of the herringbone gear on the mesh stiffness is analyzed. Finally, the difference between the herringbone gear and the helical gear is compared.

(1) The mesh stiffness of the herringbone gear is calculated by three methods. Compared with the Weber method, the difference between the two methods is 6.8%, which proves the correctness of the improved Velex method.

(2) The influence of helix angle and modification coefficient of herringbone gear on mesh stiffness is analyzed. The results show that mesh stiffness fluctuation of herringbone gear has a nonlinear relationship with helix angle, and the change trend is similar to Gauss curve. Negative modification is beneficial to improving the load sharing properties of gears. Modification coefficient affects the change of mesh points and causes fluctuation of mesh stiffness.

(3) Comparing the stiffness of the herringbone gear with the helical gear, the difference between herringbone gear and helical gear is proved. The herringbone gear is not the superposition of the helical gear, which provides theoretical guidance for the future research of the herringbone gear.

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