ON UNIFICATION OF GRAVITATION AND ELECTROMAGNETISM
IN THE FRAMEWORK OF A GENERAL-RELATIVISTIC APPROACH

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We consider the unification problem for the gravitational and electromagnetic interactions and its possible solution on the basis of the existence of an effective Riemannian space in nonlinear electrodynamics

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1 Introduction

The problem of unification of all interactions in a unified theory is remaining to be one of the most important problems of theoretical physics. Such a unification in a theory of some unified field seems to deserve a serious attention.

Among the interactions on which we now can speak, it is natural to single out two long-range interactions, gravitation and electromagnetism. Obviously, in a unified field theory these two interactions should appear in a natural way.

The circumstance that the gravitational interaction is related to changes in the symmetric metric tensor of space-time is nowadays a generally accepted fact. It must be noted, however, that for an agreement with the experimental data it is sufficient to assume that, for the motion of material bodies, there is an effective Riemannian space which can be, in turn, induced by a certain field different from the metric tensor field.

Such an induced Riemannian space appears in nonlinear electrodynamics when one considers the long-range interaction of particles which are solitons of the model [1, 2].

According to different viewpoints, the electrodynamics of vacuum should be regarded nonlinear. This means that the above-mentioned effective Riemannian space must be at least taken into account in calculations related to the gravitational interaction. But a radical viewpoint will be that gravitation as a whole is a manifestation of the electromagnetic field nonlinearity.

2 A generally invariant nonlinear field model

Consider the field of a certain second-rank tensor $G_{\mu\nu}$ in a four-dimensional space-time and the following variational principle:

$$\delta \int \sqrt{|\det(G_{\mu\nu})|} \, (dx)^4 = 0,$$

where $(dx)^4 \equiv dx^0 dx^1 dx^2 dx^3$, and Greek indices take the values 0, 1, 2, 3.

Due to the rule of variable changing in an integral and the determinant transformation rule, this variational principle is invariant with respect to general coordinate transformations.

The tensor $G_{\mu\nu}$ may be represented as a sum of symmetric and antisymmetric tensors, of which the first one could be identified with the metric tensor and the second one with the electromagnetic field tensor:

$$G_{\mu\nu} = g_{\mu\nu} + \chi^2 F_{\mu\nu}, \quad F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu},$$

$A_\mu$ being the components of the electromagnetic potential.

The variational principle of the form (1) was considered by A.S. Eddington [3] and A. Einstein [4] just because of its general invariance.

M. Born and L. Infeld [5] considered the variational principle with the tensor $G_{\mu\nu}$ of the form (2), where $g_{\mu\nu}$ is the metric tensor of flat space.
3 Equations of nonlinear electrodynamics and the effective Riemannian space

The Born-Infeld set of equations has the form

$$
\frac{\partial}{\partial x^\mu} \sqrt{|g|} f^{\mu\nu} = 0,
$$

(3)

where

$$
f^{\mu\nu} \equiv \frac{\chi^{-2} \partial \mathcal{L}}{\partial (\partial_{\mu} A_{\nu})} = \frac{1}{\mathcal{L}} \left( F^{\mu\nu} - \frac{\chi^2}{2} \mathcal{J} \varepsilon^{\mu\nu\sigma\rho} F_{\sigma\rho} \right),
$$

$$
\mathcal{L} \equiv \sqrt{1 - \chi^2 \mathcal{I} - \chi^4 \mathcal{J}^2},
$$

$$
\mathcal{I} \equiv F_{\mu\nu} F^{\nu\mu}/2, \quad \mathcal{J} \equiv \varepsilon_{\mu\nu\sigma\rho} F^{\mu\nu} F^{\sigma\rho}/8,
$$

$$
\varepsilon_{0123} \equiv \sqrt{|g|}, \quad \varepsilon^{0123} = -1/\sqrt{|g|}.
$$

The Born-Infeld model has the following energy-momentum tensor:

$$
T^\mu_\nu \equiv \left( f^{\mu\rho} F_{\nu\rho} - \chi^{-2} (\mathcal{L} - 1) \delta^\mu_\nu \right) / 4\pi.
$$

(4)

Remarkable is the characteristic equation in Born-Infeld electrodynamics [6]:

$$
\tilde{g}^{\mu\nu} \frac{\partial \Phi}{\partial x^\mu} \frac{\partial \Phi}{\partial x^\nu} = 0,
$$

(5)

where $\Phi(x^\mu) = 0$ is the equation of the characteristic surface and

$$
\tilde{g}^{\mu\nu} \equiv \tilde{g}^{\mu\nu} - 4\pi \chi^2 T^{\mu\nu}.
$$

(6)

Here, $T^{\mu\nu}$ is the energy-momentum tensor of the form (5).

4 On unification of gravitation and electromagnetism

In connection with the form of Eq. (5), the symmetric tensor $\tilde{g}^{\mu\nu}$ can be named the metric of the effective Riemannian space. Indeed, in studying a multiparticle solution of the model with the perturbation method, it turns out that the rapidly oscillating soliton particle propagates along geodesics of the effective Riemannian space with the metric $\tilde{g}^{\mu\nu}$ induced by remote particles [2]. Rays of high-frequency electromagnetic waves are bended in the same way as in a gravitational field with the metric $\tilde{g}^{\mu\nu}$.

These results mean that in nonlinear electrodynamics there is an interaction which is indistinguishable from the gravitational one.

Since the components of the energy-momentum tensor depend on even powers of the electromagnetic field tensor components, this interaction appears in the second order with respect to the weak field of remote solitons.

As to the electromagnetic interaction between soliton particles in this model, it naturally appears in the first order with respect to the weak field of remote solitons [2].

Thus there are two long-range interactions in this nonlinear electrodynamic model, and they appear in the first and second orders in the weak field of remote solitons. These interactions may be identified as the electromagnetic and gravitational ones. And that is how these two well-known long-range interactions are unified.

As already indicated, this viewpoint is radical in this approach.

A more moderate viewpoint is that the effective metric gives only a part of the gravitational interaction.

Adhering to this viewpoint, it is necessary to advert to the generally invariant principle (1) in the context of A. Einstein’s article [4]. This article uses the second-rank tensor $R_{\mu\nu}$ built from the connections $\Gamma^\alpha_{\beta\gamma}$ which are the field functions of the theory. The tensor $R_{\mu\nu}$ is not symmetric. Further on, the tensor $R_{\mu\nu}$ is split into symmetric and antisymmetric parts. The symmetric part is identified with the metric and the antisymmetric one with the electromagnetic field. Variation with respect to $\Gamma^\alpha_{\beta\gamma}$ leads to the gravitational field equations and the Maxwell equations for the case of weak fields.

It is evident that the electrodynamic equation of this theory are nonlinear for fields which are not weak. It is also evident that the nonlinear electromagnetic field will create an effective metric which will contribute to the gravitational interaction.

5 On correspondence with experimental data

The effective metric $\tilde{g}^{\mu\nu}$ can give a gravitational potential which behaves as $(1/r)$ at infinity [7]. This is possible by taking into account the rapidly oscillating electromagnetic background and by averaging over the rapid oscillations [8]. In this case, the gravitational constant depends on the magnitude of the radiation background.
It is also necessary here to touch upon gravitational waves which are intensively sought for. An indirect proof of their existence is, by common views, the observed energy loss by a gravitationally bound binary system.

However, in the framework of the unified theory under consideration, the energy loss by such a system does not necessarily mean that it is transferred to gravitational waves. With any of the two viewpoints (radical or moderate in the above-mentioned sense), the lost energy of a binary system may pass (partly or completely) into the electromagnetic background.

It is well known that gravitational waves have not yet been observed directly. The absence of their direct detection may be an argument in favor of the viewpoint that gravity is entirely created by the electromagnetic field.

6 Conclusion

The above-described approach to the important problem of unification of interactions looks reasonable and requires further development. A comparison of new theoretical results with experimental data will enable us to judge upon the applicability of the theory.

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