Method of defining region of allowed configurations in multidimensional space of generalized coordinates using hypercube graph

F N Pritykin and D I Nefedov
Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia

Abstract The work is devoted to the development of a knowledge base on possible positions of an android arm mechanism with allowance for the position of the previously known forbidden zone. The knowledge base is used for intelligent control of the arm movement of an android robot that functions autonomously in an organized space without human-operator assistance. The specified knowledge base is determined by the region of allowed configurations in the multidimensional space of generalized coordinates. Based on the methods of geometrical modeling, a representation method of the allowable configuration region with the use of hypercube graph is proposed.

1. Introduction
A large number of studies are devoted to the problem of creating intelligent robots [1, 2]. In solving this problem, computer virtual simulation of mechatronic object movements in organized environments is an important tool for analyzing and verifying the efficiency of the proposed algorithms for automatic control of manipulator mechanisms. One of the problems in developing intelligent control systems is to meet the requirements when collisions of the manipulator mechanism with the environment are absent [3-6]. These requirements are determined by the specified safe distance to the forbidden zones when a manipulator mechanism moves. This paper presents the results of studies on the determination of the set of allowed android arm configurations that meet the specified requirements. For this purpose, the varieties of points designated by the region Λ of the multidimensional space of generalized coordinates are determined, these points assigning configurations that satisfy the limit values of the generalised coordinates and the conditions of non-intersection with the forbidden zones. The robot control system should contain knowledge base about regions Λ for various forbidden zones that are known in advance. First it is necessary to create this base of knowledge that characterizes the property of maneuverability of the arm mechanism with allowance for forbidden zones, and then it may be used in synthesis of movements in the velocity vector in organized environment [7-10]. Defining the regions Λ is necessary for searching configurations that do not intersect forbidden zones. The creation of this knowledge base is quite a time-consuming task, as it is obtained as a result of processing a large array of points in a multidimensional space of generalised coordinates, taking into account the kinematic and geometric parameters of the investigated robot mechanism and the parameters of the shape and position of forbidden zones.
2. Problem Statement

In this paper a method for specifying the region $\Lambda$ and on its basis the knowledge base of the robot using a graph of hypercubes is proposed. Figure 1 shows the kinematic diagram of the arm mechanism of the android robot AR-600E and the position of the forbidden zone $P$. The height of the base point $A$ of the forbidden zone for the considered example in the coordinate system $O_o$ is equal to $z_{op} = 400$ mm, the minimum safe offset of the manipulator base plate from the obstacle is equal to $x_{op} = 450$ mm (Fig. 1).

The parameter $z_p = 100$ mm determines the height of the forbidden zone (tool rack). Geometric parameters of the arm mechanism are specified in [7]. In the figure, the generalised coordinates are indicated by $q_i$. The points $O_1$– $O_{12}$ assign the coordinate origins used for specifying the kinematic chain model of the android arm mechanism. In order to determine the region of allowed configurations $\Lambda$, taking into account the position of the forbidden zone $P$, in the work [7] a set of two-dimensional sections of the specified region in the four-dimensional generalised coordinate space is constructed. Particular positions of the arm mechanism links are investigated when $q_1 = 0$. The forbidden zone $P$, assigned by two horizontal and one profile planes of level $\Delta$, $\Delta'$ and $\Sigma$ (Fig. 2), defines in the space of generalised coordinates $L$ a certain region of the allowed configurations (RAC) $\Lambda$. A method for calculating the cross-section points of RAC $\Lambda$ to create a knowledge base about the past experience of motion synthesis in relation to the manipulator mechanism of a mobile robot was previously proposed in [8 - 10]. In the work [7], a cross section representation method of the region $\Lambda$ using a set of triangles is developed. However, the accuracy of the $\Lambda$ region representation by this method requires further research in this field.

The operation of the manipulator mechanism takes place within a limited region $\Omega$ (four-dimensional space of generalised coordinates $L_q$ at $q_1 = 0$). Region $\Omega$ in a general case specified by hyperparallelepiped is that for any value of the vector $q$ ($q_2$, ..., $q_5$), specifying a point of the hypercube, the inequalities are satisfied:

$$
q_i^{\text{min}} \leq q_i \leq q_i^{\text{max}},
$$

where $q_i^{\text{min}}$ and $q_i^{\text{max}}$ are the lower and upper limits of generalised coordinates values. For this example, if $q_1 = 0$, the values of the parameter $i$ satisfy the conditions $2 \geq i \geq 5$. The values $q_i^{\text{min}}$ and $q_i^{\text{max}}$ for the mechanism of the android robot are respectively assumed to be equal to $q_i^{\text{min}}$ ($0^\circ$, $-25^\circ$, $-120^\circ$, $-120^\circ$) and $q_i^{\text{max}}$ ($120^\circ$, $120^\circ$, $120^\circ$, $120^\circ$).

Figure 2 shows the set of allowed configurations, built in increments $\Delta q_i = 20^\circ$ in the presence of the forbidden zones $P$. The figure shows the forbidden zone on the front and profile projections in the
form of quadrangles $P_2$ and $P_3$. Four-dimensional RAC $\Lambda$ may graphically be mapped by a set of two-dimensional sections. Due to the complex and diverse shape of the two-dimensional sections of the $\Lambda$ android region, the technique of analytical sections representation proposed in [8-10] cannot be applied. At the mesh spacing $\Delta q_{\min} = 20^\circ$ in the conducted research the form of various sections was studied. The table shows some of the sections of the RAC $\Lambda$.

Based on the study of the shape and position of the RAC $\Lambda$ sections, let us solve the problem of point representation of this region in a multidimensional space of generalized coordinates. The developed representation method of RAC is used to create a knowledge base about the position of the arm mechanism with allowance for the represented forbidden zone position.

**Figure 2.** Set of configurations constructed in the presence of the forbidden zone $P$ specified in the form of a rectangular parallelepiped $z_{op} = 400$ mm, $x_{op} = 450$ mm, $z_{op} = 400$ mm;

**Table 1.** Some sections of the $\Lambda$ region at different values of the parameters $q_2$ and $q_3$

| $q_3$ | $q_2$ = $-20^\circ$ | $q_2$ = $20^\circ$ | $q_2$ = $60^\circ$ |
|-------|---------------------|-------------------|-------------------|
| $q_1$ = $-100^\circ$ | ![Diagram](image1) | ![Diagram](image2) | ![Diagram](image3) |
| $q_1$ = $-60^\circ$ | ![Diagram](image4) | ![Diagram](image5) | ![Diagram](image6) |
| $q_1$ = $-20^\circ$ | ![Diagram](image7) | ![Diagram](image8) | ![Diagram](image9) |
| $q_1$ = $20^\circ$ | ![Diagram](image10) | ![Diagram](image11) | ![Diagram](image12) |
| $q_1$ = $80^\circ$ | ![Diagram](image13) | ![Diagram](image14) | ![Diagram](image15) |
| $q_1$ = $100^\circ$ | ![Diagram](image16) | ![Diagram](image17) | ![Diagram](image18) |
3. Theoretical Basis

The graph shown in figure 3 is used to assign the region Λ. Each node of this graph, which assigns hypercubes $P^1_p$ at certain levels, allows specifying only such configurations of the android robot arm that satisfy the specified conditions of non-intersection with the forbidden zones. The advantages of the presented method of assigning the sets Λ compared with the known ones, are as follows:

− as a result of representation of the generalized coordinates space by elementary cells, it is possible to work not only with convex, but also with non-convex and composite geometric objects that define the region Λ;

− the method of studying the multidimensional space of generalized coordinates for the membership of arbitrary points in the regions Λ can significantly speed up the procedure for determining the conditions of non-intersection of the mechanism and the forbidden zones.

Let us note the main stages in the operation of this algorithm, in which the representation of the regions Λ of the multidimensional space $L_q$ is performed using a hypercube graph.

At the first step, the volume of the investigated multidimensional space $L_q$ is specified by a $p$-dimensional hypercube. The side dimension of the hypercube (Fig. 4) is determined based on the ratio $P_p = 2q^\text{max}_i$, where the parameter $q^\text{max}_i$ is assigned in accordance with the specified limit values of the generalized coordinates $q^\text{max}_i$ and $q^\text{min}_i$. The points $Q_p(φ_1^k, φ_2^k, ..., φ_n^k)$ calculated according to a certain mesh step and belonging to a specified hypercube $P^1_p$ are checked for the condition of belonging to the set Λ.

![Figure 3. Hypercube graph assigning the regions under examination in the multidimensional space $L_q$, defining the region Λ](image)

The values of the parameters $q_i$, which specify points $Q_Λ$, are found on the ratios $q^t_i = q^t_i-1+Δq^i_{\text{max}}$, where $q^t_i$ are the values of the parameters $q_i$ at the current step $t$ of the calculation, $q^t_i-1$ is the value of the parameter $q_i$ at the previous step of the calculation, $Δq^i_{\text{max}}$ is the maximum, originally adopted, increment of the parameters $q_i$ (Fig. 4). For the considered example $Δq^i_{\text{max}} = 240^\circ$.

If one of the points $Q_Λ$, represented in figure 4 by circles, turns out to belong to the set Λ (i.e. this point satisfies the conditions of non-intersection with forbidden zones), the space of the hypercube $P_p$ is divided into $2^p$ $p$-dimensional hypercubes. The dimensions of new hypercubes $P_p/2$ in this case are decreased, respectively, by a factor of two (Fig. 4). Further only hypercubes $P_p/2$ are investigated, in
which points $Q\Lambda \in \varepsilon$ were discovered. The increment $\Delta q_{\text{max}}$ in this case decreases to the value $\Delta q_{\text{max}} = \Delta q_{\text{max}} / 2$. Computational work is significantly reduced then, and accuracy increases.

\[ \Delta q_{\text{max}} = \Delta q_{\text{max}} / 2 \]

**Figure 4.** Geometric parameters defining the hypercubes under study belonging to the multidimensional space $L_q$

The partition into hypercubes $P_p/2$ and examination of their points for membership to the set $\Lambda$ occur as long as $\Delta q_{\text{max}}/2 > \Delta q_{\text{min}}$. Where $\Delta q_{\text{min}}$ is specified initially. The parameter $\Delta q_{\text{min}}$ specifies the minimum mesh step in the study of points $Q\Lambda$, which is assumed to be $\Delta q_{\text{min}} = 20^\circ$ in this work.

Next, the hypercubes connection graph containing points $Q_p \in \Lambda$ is constructed (in general, the point $O_q$, which defines coordinate origins of the space $L_q$, may not belong to the region $\Lambda$). Note that if the points located on the boundaries of hypercubes belong to the set $\Lambda$, then the points of the boundary hypercubes are also investigated. The minimization of the computational time is achieved by excluding from the next calculation step those hypercubes describing the regions of the $L_q$ space in which the $Q_p \in \Lambda$ points were not found at the current calculation step. In this context, this method of calculation allows us to study the region $\Lambda$ at relatively small values of the parameter $\Delta q_{\text{min}}$ in less time.

The set of points $Q_p \in \Lambda$, which satisfies the conditions of non-intersection of configurations and forbidden zones, will determine the region $\Lambda$ (Fig. 2). Defining the regions $\Lambda$ is necessary for searching android arm configurations that do not intersect the forbidden zones. The found region $\Lambda$ contains all possible values of the vector of generalized coordinates that meet the requirements of non-intersection of the arm mechanism with the forbidden zone.

4. **Experimental Results**

The set of sections of the allowed configurations region and their analytical assignment in the form of a hypercube graph is determined in advance based on the known forbidden zone position (for example, tool racks) and geometric and kinematic parameters of the arm mechanism. The region $\Lambda$ defines the knowledge base and the model of the working environment. Based on the use of the $\Lambda$ region, it is possible to create an algorithm to predict the development of events when the robot arm performs motor tasks. The algorithm for calculating the trajectory of the mechanism in the space of generalized coordinates using the construction of movements in the velocity vector and the region $\Lambda$ is presented in [9]. The method of calculating the increments vector of generalized coordinates (the method of movement synthesis in the velocity vector) based on the use of matrices of partial gear ratios is described in [10]. The prediction of various situations when the manipulator moves is solved in paper [9] through the use of the mapping scale of the increment vector of the generalized coordinates, and the analysis of the trajectory position relative to the region $\Lambda$. Figure 4a shows the results of simulation of the android arm movement in a deadlock. The center of the output link in the test case moves along the line segment. This line passes through the start and end points of the trajectory of the output link. The specified points in Fig. 5 ab are marked as $B_a$ and $B_q$. 

[Diagram](image-url)
Figure 5. Android arm motion modeling: a – modeling in a deadlock situation, b – modeling using the vector of increments of generalized coordinates and the region Λ

Fig. 5b represents the simulated motion along the entire segment of the assigned trajectory based on the use of the Λ region and the value of the vector for generalized coordinate increments.

Specifying the region of allowed configurations of the android robot arm using the hypercube graph makes it possible to create a module for automatic detection of collisions with the environment at the virtual level. For the acquisition and proper representation of knowledge about the possible position of the arm with the pre-determined forbidden zone, RAC is used. This region allows us to present knowledge about many positions of the arm in a form that is understandable to the intellectual system. In such a case the formalization of knowledge based on the use of RAC Λ allows the creation of a knowledge base. The research results can be used in the development of intelligent control systems of autonomously functioning robots, in particular in the creation of knowledge bases about the past experience of motion synthesis.

5. Results Discussion

A more efficient way of finding the values of the generalized coordinate vector in the multidimensional space of generalized coordinates by traversing the nodes of a hypercube graph is proposed. Selection of nodes of this graph that define hypercubes $P_1$ (Fig. 3) allows one to specify only such configurations of the android robot arm that do not intersect the forbidden zones.

The advantages of the presented method of assigning the regions Λ compared with the known ones, are as follows:

- as a result of description of the generalized coordinates space by elementary cells, it is possible to work not only with convex, but also with non-convex and separate regions Λ;
- the method of investigation of the multidimensional space of generalized velocities allows one to significantly speed up the procedure for determining the membership of an arbitrary point of the multidimensional space of generalized coordinates to the region Λ.

The region Λ for the android arm performing the movement in the specified velocity vector of OL is defined. It is proved that Λ regions can consist of separate subregions containing points $Q_\Lambda$. 
6. Summary and Conclusion

The results of the research can improve the performance of computational processes in virtual computer simulation of movements of mobile and stationary robots with allowance for forbidden zone position. The method of specifying RAC and the knowledge base can be studied in the development of intelligent adaptive motion control systems of autonomous robots operating in organized environments without human-operator.

References

[1] Lokhin V M, Manko S V and Romanov M P 2015 Improving the adaptive properties of autonomous robots based on intelligent technologies *Extreme robotics* Vol 1(1) p 59-67
[2] Yusupov R M and Timofeev A V 2014 Intellectualization of control and navigation processes of robotic systems *Extreme robotics* Vol 1(1) p 16-21
[3] Hasegawa T, Suehiro T and Takase K 1992 A model-based manipulation system with skill-based execution *IEEE Trans. Rob. and Autom.* Vol 8 p 535-544
[4] Tsukamoto K, Takubo T, Ohara K et al. 2010 Virtual impedance model for obstacle avoidance in a limb mechanism robot *IEEE International Conference on Information and Automation (China, Harbin)* 729-734
[5] Bo You, Yu Zou and Wanzhe Xiao 2010 Telerobot control system based on dual-virtual model and virtual force *Strategic Technology (IFOST)* p 246–250
[6] Lopatin P 2016 Investigation of a Target Reachability by a Manipulator in an Unknown Environment *IEEE International Conference on Mechatronics and Automation*
[7] Pritykin F N and Nefedov D I 2018 Creation of a knowledge base on the past experience of synthesis of android robot arm movements based on the use of permitted configurations *Software systems and computational methods* 4 p 60-67
[8] Pritykin F N, Rogoza Y A, Zinchenko Y V and Nefedov D I 2018 Parametric Method to Define Area of Allowable Configurations while Changing Position of Restricted Zones *MEACS 2017 / IOP Conf. Series: Materials Science and Engineering* Vol 327 p 042088–1–042088–8 DOI:10.1088/1757-899X/327/4/042088
[9] Pritykin F N and Nefedov D I 2016 Computation of manipulator mechanism path in joint coordinate space with working range forbidden regions *Dynamics of Systems, Mechanisms and Machines (Dynamics)* DOI: 10.1109/Dynamics.2016.7819065
[10] Pritykin F N and Nefedov D I 2016 Study of surfaces defining the boundaries of the allowed configurations of the mobile manipulator mechanism in the presence of restricted zones *Mechatronics, automation, control* 6 p 404-413
[11] Whitney D E 2010 The Mathematics of Coordinated Control of Prosthetic Arms and Manipulators *J. Dyn. Sys. Meas. Control.* 94(4) p 303–309