Pushing Particles with Waves: Current Drive and $\alpha$-Channeling

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It can be advantageous to push particles with waves in tokamaks or other magnetic confinement devices, relying on wave-particle resonances to accomplish specific goals. Waves that damp on electrons or ions in toroidal fusion devises can drive currents if the waves are launched with toroidal asymmetry. Theses currents are important for tokamaks, since they operate in the absence of an electric field with curl, enabling steady state operation. The lower hybrid wave and the electron cyclotron wave have been demonstrated to drive significant currents. Non-inductive current also stabilizes deleterious tearing modes. Waves can also be used to broker the energy transfer between energetic alpha particles and the background plasma. Alpha particles born through fusion reactions in a tokamak reactor tend to slow down on electrons, but that could take up to hundreds of milliseconds. Before that happens, the energy in these alpha particles can destabilize on collisionless timescales toroidal Alfvén modes and other waves, in a way deleterious to energy confinement. However, it has been speculated that this energy might be instead be channeled instead into useful energy, that heats fuel ions or drives current. An important question is the extent to which these effects can be accomplished together.

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1. Introduction

This paper reviews the physics of current drive effect and the $\alpha$-channeling effect, within the theme of the ITC-25 conference, namely, “Creating the Future Innovative Science of Plasma and Fusion.” Both effects rely on how waves push particles: Waves excited in tokamaks can catalyze the $\alpha$-channeling effect as well as to drive current. There are a number of ways too in which these effects might work in synergy.

The wave-driven current drive effect relies upon imparting wave momentum or energy to electrons or ions in such a way that breaks the toroidal symmetry, and thereby causes toroidal current to flow. That current helps to create the tokamak configuration.

The $\alpha$-channeling effect represents a second reason to push particles with waves in the tokamak, namely to divert energy from $\alpha$-particles to fuel ions. This diversion, or channeling, is possible in a tokamak reactor, because, although $\alpha$-particles born through fusion reactions tend to slow down on electrons, that could take up to hundreds of milliseconds. Before that happens, the energy in these $\alpha$-particles can destabilize on collisionless timescales toroidal Alfvén modes and other waves, in a way deleterious to energy confinement.

However, it has been speculated that this energy might instead be channeled into energy in a form that heats fuel ions or drives current. This channeling can be catalyzed by waves; the waves diffuse $\alpha$-particles in energy, but fundamentally coupled to diffusion in space. If these diffusion paths in energy-position space point from high energy in the center to low energy on the periphery, then $\alpha$-particles will be cooled while forced to the periphery, with their energy captured by the waves. The amplified waves can then heat ions or drive current. This process or paradigm for extracting $\alpha$-particle energy collisionlessly has been called $\alpha$-channeling. While the effect is speculative, the upside potential for economical fusion is immense.

Insofar as creating the future goes, among the innovations that one can anticipate in utilizing waves might be: one, identifying combinations of waves that accomplish synergistically both channeling and current drive; two, oscillating resistivity to accomplish current drive with far less average power dissipated; three, applying channeling concepts to new magnetic configurations such as stellarators or mirror machines; four, applying current drive concepts to new magnetic configurations such as spherical tokamaks; and, five, applying $\alpha$-channeling concepts to very different contexts, such as plasma centrifuges for high-throughput mass separation for remediating nuclear waste.

However, before addressing the future, we must first review the past, including the basic physics behind the current drive effect and the $\alpha$-channeling effect.

The paper is organized as follows: For background, we first review the past in two sections. In Sec. 2, we review the current drive effect. In Sec. 3, we review the $\alpha$-channeling effect. Then, we imagine the future in the next 3 sections. In Sec. 4, we review the synergy that is possible...
in accomplishing simultaneously both the $\alpha$-channeling effect and the current drive effect, with particular emphasis on the LHCD effect. In Sec. 5, we review the possibilities of current drive in quasi-steady state scenarios. In Sec. 6, we review other geometries. In Sec. 7, we conclude with some final thoughts on imagining the future in the context of the $\alpha$-channeling effect and the current drive effect.

2. Past: The Current Drive Effect

There have been a number of reviews of the current drive effect. The theory of current drive and early experiments has been extensively reviewed [1]. Early reviews include a review with experimental focus [2], and a review for the lay audience [3]. More recent reviews have focused on the electron cyclotron current drive effect [4] or the lower hybrid current drive effect [5]. There have also been a number of tutorial reviews or didactic reviews [6–9], including reviews of unsolved issues [10]. The underlying physical processes rely importantly on the wave-particle resonance, which was the subject of a recent tutorial paper as well [11]. Further exploration of the wave-particle interaction can be found, for example, in the classic book of Stix [12], the more recent book by Brambilla [13], or the most recent very excellent book by Rax [14].

In view of these recent reviews, the attempt here will be to review again this topic, but from the perspective of what are the future possibilities in pushing particles with waves, not only to achieve efficient current drive, but from the perspective of utilizing the effect in synergy with $\alpha$-channeling and from the perspective of other recent developments in the field. But first let us review the past.

The most successful current drive technique to date employs lower hybrid waves [15]. In lower hybrid current drive (LHCD), the current is carried by a tail of superthermal electrons. Through the Landau damping of an electrostatic wave, the superthermal electrons are pushed in the toroidal direction. Consider then an electrostatic wave (like the the lower hybrid wave which is nearly electrostatic) with frequency $\omega$ and wavenumber $k$ traveling to the right as in Fig. 1. Electrons resonant with the wave, that is those moving near the wave phase velocity, such that $\omega - k \cdot v = 0$, are pushed, while the non-resonant electrons are not pushed. The push can be either to increase the the particle velocity or to decrease it, depending on the phase of the particle in the trough of the wave. But, on average, for distribution functions near collisional equilibrium, namely nearly Maxwellian in energy, there will be more electrons that get pushed to higher energy than those that get pushed to lower energy. So let us consider that the electron gets pushed (to higher energy) in the direction that it is going, namely $v$.

For the purposes of current drive in a tokamak, we are interested in the current carried by this electron in the direction of the main toroidal magnetic field. So consider an electron moving with velocity $v$ that gets pushed to $v + \Delta v$. The extra instantaneous current density carried by a density $n$ (each with charge $q$) of these electrons resonant with the wave is then $J = qn \Delta v$. To accomplish this extra current for small $\Delta v$ will require energy $E = nmv\Delta v$. This current $J$ will last about a collision time, $1/\nu(v)$, so to keep the current in the steady state requires repeated injections of energy $E$, which translates into an average power dissipated $P_D = \nu(v)E$. Note that it is of importance that the velocity dependency of the collision frequency of the resonant particles be indicated as $\nu(v)$. For superthermal electrons, this frequency is importantly sensitive to velocity.

Now the current drive efficiency for pushing superthermal electrons may be put as

$$J/P_D = q/m v^2 \nu(v) \sim v^2,$$

where we made use of the fact that, for superthermal electrons, the collision frequency goes as $v^{-3}$. Since superthermal electrons may have velocities about 4 or 5 times the thermal velocity, high efficiency of current drive becomes possible. This is the basis of the LHCD effect, where the current drive efficiency using lower hybrid waves in fact goes as $v^2$. From Eq. (1), it can also be seen that there is another regime where, in principle, relatively high efficiency might be reached, namely to push electrons with low (sub-thermal) parallel velocities, but at least thermal perpendicular velocities [16, 17]. However, it has been the LHCD effect, at high phase velocity, that has been routinely demonstrated in detail on many tokamaks, with up to mega-amps of current being produced with mega-watts of lower hybrid wave power.

So, in the context then of reviewing the past, note that, since this current drive effect was predicted, it has routinely been demonstrated on many tokamaks, with up to mega-amps of current being produced, as shown in Fig. 2. Particularly key, early tokamak experiments came on the WT-2 tokamak in Kyoto [18, 19] and the JFT-2 (JAERI Fusion Torus)) tokamak in Tokai [20], the JIPP T-II torus in Nagoya [21], and Versator [22] and Alcator [23] experiments at MIT. A very key series of experiments was conducted on the PLT device at Princeton [24].

It is interesting to recall, too, that it looked at first as if it were necessary to inject momentum from the wave to produce this current drive effect. However, the main effect is not the momentum input, but the change in energy. This

![Fig. 1 Resonant interaction of electrons with an electrostatic wave. The solid wavy line indicates the wave potential as a function of distance along the direction of the phase velocity at a given instant of time.](image-url)
In going to the right, while those going to the left will slow down more quickly on ions. Hence, a net flow of electrons persists in going to the right. The ions collide more with the electrons going to the left. Hence, the ions tend to go to the left, so that the electron flow going to the right is balanced by the ion flow going to the left. In such a manner, even with no parallel momentum input by the waves, current is nonetheless generated, while particle momentum is conserved. This is the principal behind electron cyclotron current drive (ECCD) [25]. It turns out that pushing electrons in the perpendicular velocity direction results in current drive with exactly $3/4$ the efficiency compared to pushing in the parallel direction. This result changes somewhat in the limit of relativistic electrons [26–28].

How much current one gets from the steady state pushing of particles of electrons from velocity space location 1 to velocity space location 2 is a function of velocity space location as well as the direction of the push. This is a powerful concept, because we often know the exact velocity space location of the electrons through resonance conditions, and we also know the direction of the push through wave properties. Hence, it is often not necessary to solve the Fokker-Planck equation for the full electron distribution function in order to determine the current drive efficiency. This was shown in comparing complete numerical simulations [29] to analytic results.

The success in arriving at the current drive efficiency so economically suggests that there may be other properties that might be similarly usefully attributed to velocity space location, or more generally, phase space location. For example, one can assign a runaway probability to each point in velocity space [30, 31], rather than simply a runaway rate to the full distribution. As another example, in the presence of both a dc electric field and rf waves, there is an rf-induced conductivity, bilinear in the rf power and the dc field [32]. This conductivity was used to explain successfully how wave energy could be converted into poloidal magnetic field energy with very high efficiency [33]. In each of these cases, the current drive efficiency, runaway probability, or conductivity is a function only of where in velocity space the key rf interaction occurs.

In fact, in establishing the theory of LHCD, a particularly detailed and informative comparison of theory with experimental data came from the PLT series of current-drive experiments in the presence of an electric field, which included the so-called current ramp-up experiments in which current increased in time [34]. In comparing these experiments to the theory, the resonance conditions were used to isolate the key dimensionless parameters that depended on velocity space location. These experiments spanned several parameter regimes, including the steady state regimes. That wave energy could be converted efficiently into poloidal magnetic field energy, consistent with the theory, was strong evidence for the theory itself. This detailed comparison was confirmed in ramp up experi-

![Fig. 2](image1.png)  
**Fig. 2** Reported steady-state LHCD vs. year. Initials correspond to various tokamak facilities. (From Ref. [2].)

![Fig. 3](image2.png)  
**Fig. 3** An electron is pushed by a wave from velocity space location 1 (green) to velocity space location 2 (red), with no input of parallel momentum. Note that the symmetry is broken, since the symmetrically counter-propagating electron at velocity location $1'$ is not pushed.

can be seen from Fig. 3, which depicts what happens when an electron is pushed in the perpendicular velocity direction $v_\perp$, but not in the parallel velocity direction $v_\parallel$, where parallel and perpendicular are with respect to the direction of the magnetic field. The electron absorbs no parallel momentum, but in going from velocity space location 1 to velocity space location 2, it becomes more energetic.

This creates an important asymmetry, since more energetic electrons collide less both with ions and with the slower electrons. If half of the electrons are going to the left and half are going to the right (in the parallel direction), but only the ones that are going to the right gain perpendicular energy, then those going to the right will persist
ments on other facilities as well [35–38], including most recently on Tokamak EAST [39–41]. These experiments serve to confirm not only the phenomenon of ramp-up, or tokamak recharging, but also serve to confirm the underlying fundamental theory itself, which includes importantly the fact that fast electrons are subject to classical Coulomb collisions.

3. Past: The $\alpha$-Channeling Effect

Again, to consider first the past before imagining the future, let us review briefly the $\alpha$-channeling effect. This review follows earlier reviews of the $\alpha$-channeling effect, including the physical processes underlying the effect [42], a tutorial [43], a review including the channeling effect in mirror machines [44], and the more recent review [45]. Other recent, relevant reviews include energetic particles in tokamaks [46, 47]. As with the current drive review in the previous section, we will only briefly review this effect, but then we will emphasize recent ideas on synergies with current drive effects as well as new applications.

The alpha channeling effect occurs when energy is channeled from an $\alpha$-particle to a wave by virtue of an inversion in the particle energy distribution function along the wave diffusion path in energy-position space [48]. The wave-particle interaction is stochastic. The idea is to extract the energy from 3.5 MeV $\alpha$-particles, before the $\alpha$-particles slow down on electrons. The inversion occurs because these paths, in energy-position space, connect high energy in the center, where there are many $\alpha$-particles, to low energy at the periphery, where there are few. The fusion ash is advantageously removed at low energy at the periphery. But the key advantage occurs should the waves deposit this energy directly to fuel ions, achieving the hot ion mode, where ions are hotter than electrons [49].

The motivation for identifying these diffusion paths came in response to a prediction that in a reactor the lower hybrid waves would actually be damped by the $\alpha$-particles [50], thereby preventing LHCD. Quasilinear calculations that took into account the diffusion in velocity space of the $\alpha$-particles by the waves confirmed this prediction [51, 52]. It is only by arranging for a diffusion path in the joint energy-radius space, such that a population inversion can be exploited, that the damping can be reversed. This alpha channeling effect is speculative, but in principle could make a significant difference in the economical feasibility of controlled fusion energy. The benefit of channeling say 75% of the $\alpha$-particle energy to fuel ions in a DT reactor could result in ions being about twice as hot as electrons, which could double the fusion reactivity [53].

As noted, in losing energy to the wave, the $\alpha$-particles diffuse in the wave fields towards the periphery, so $\alpha$-particles are advantageously removed from the plasma. Once removed, the $\alpha$-particles no longer take up valuable plasma pressure. Also $\alpha$-channeling can help in plasma fueling and plasma heating, since the same diffusion (or very similar) path connects high-energy fuel ions in the tokamak center to low-energy fuel ions at the periphery.

We also point out that the free energy removed from the $\alpha$-particles amplifies a wave, which, in addition to producing a hot-ion mode by damping on fuel ions, might also drive toroidal current by damping on ions or electrons, through a variety of current drive mechanisms [1]. The toroidal currents, of course, can enable steady state tokamak operation, or accomplish other useful things, like to stabilize tearing modes [54]. Further advantages are listed, for example, in the recent review of $\alpha$-channeling [45]. It remains, however, to examine whether these advantages can be achieved simultaneously.

Before ending this section, we re-derive the basic $\alpha$-channeling calculation in a slab, with waves propagating in the $\hat{z}$-direction, at frequency $\omega$ and wavenumber in the $y$-direction $k_y$, and interacting with magnetized $\alpha$-particles in a magnetic field of magnitude $B$ in the $\hat{z}$-direction. As a result of a random, resonant interaction with the wave, the velocity in the $\hat{y}$-direction changes like $v_y \rightarrow v_y + \Delta v_y$. This change is presumed to occur instantaneously, and precisely at the point of resonance $\omega - k_y v_y = 0$. As a result of this acceleration, the perpendicular energy also changes instantaneously, so that, for small kicks $\Delta v_y$, the perpendicular energy changes as $E_\perp \rightarrow E_\perp + \Delta E = E_\perp + m v_y \Delta v_y$, where $m$ is the $\alpha$-particle mass, and where the energy change $\Delta E$ can be written as $m v_y \Delta v_y$. Similarly, as a result of the velocity change in the $\hat{y}$-direction, the guiding center changes in the $\hat{x}$-direction like $x_{gc} \rightarrow x_{gc} + \Delta x_{gc} = x_{gc} - \Delta v_y / \Omega$. Now note that the change in the gyrocenter in the $x$-direction, $\Delta x_{gc}$, is proportional to the energy absorbed $\Delta \epsilon$, so we have

$$\frac{\Delta x_{gc}}{\Delta \epsilon} = - \frac{1}{m \Omega v_y} = - k_y / m \Omega$$

(2)

where $\Omega \equiv q B / m$ is the $\alpha$-particle gyrofrequency, and where the last equality could be written since the interaction occurs instantaneously just when $v_y = \omega / k_y$.

Note that the ratio of change in gyrocenter to change in energy is determined by wave and particle parameters only. In the slab case, upon repeated interactions with the wave, a particle will trace a line in $\epsilon - x_{gc}$ space. Such a wave couples diffusion in energy to diffusion in position. Suppose that the plasma boundary is at $x = a$, i.e., $\alpha$-particles can only leave at $x = a$. The plasma center is at $x = 0$, by which it is meant that no $\alpha$-particles can leave at $x = 0$. For efficient channeling, one would then require $\Delta x_{gc} / \Delta \epsilon \sim a / \epsilon_0$, where $a$ is the extent of the plasma and $\epsilon_0$ is the $\alpha$-particle birth energy. For waves with the right phase velocity, if collisions are negligible, then remarkably all the energetic ions along the diffusion path must exit cold, leaving their birth energy to the wave. This is a “hard” constraint. This constraint can also be extended to more general, possibly nonstationary, discrete systems [55]. The same picture holds, with modification, in toroidal geometry [56].

The hard constraint occurs when one wave is em-
plied with the precisely correct phase velocity, established with the correct phase velocity over the entire plasma cross-section. While theoretically advantageous, this hard constraint is difficult to achieve in practice. However, by using several waves, a “soft” constraint can be arranged, where the α-particle will exit cold with very high probability [57]. For example, two waves may be used in concert, one to move α-particles large distances without extracting too much energy, and one to extract most of the energy, such as the mode-converted ion-Bernstein wave [58]. The mode-converted ion-Bernstein wave has the advantage that, after growing convectively at the expense of the α-particles, this wave can damp on the tritium fuel ions [59].

On TFTR, mode-converted ion Bernstein waves diffused 80 keV beams of deuterium ions so that they could be detected at 2.2 MeV at the periphery [43, 60, 61]. Without copious numbers of α-particles, it was only possible to show the unwanted effect, that, with the wrong phasing, ions would be heated instead of cooled as they were expelled. But these experiments did show that the diffusion paths could operate as expected. There was one great surprise, namely that the experimentally measured diffusion coefficient was a factor of fifty higher than expected, possibly because of the mode-converted ion-Bernstein wave exciting an internal mode [62]. There was no attempt to verify this mode on TFTR; however, interestingly, related internal modes were later observed on NSTX [63].

4. Future: LHCD with α-Channeling

One of the futuristic possibilities in current drive is to accomplish it together with α-channeling. There have been a number of studies to optimize LHCD in and of itself, while avoiding the α-particles [64–76]. However, little effort was made to utilize the α-particle energy, even though, historically, the α-channeling paradigm was born out of the worry that lower hybrid waves would be damped by the α-particles. Rather, research on the α-channeling effect focused on the ion Bernstein wave, which seemed to hold more promise.

However, a recent proposal where the LHCD is accomplished by waves launched from the high-field side of the tokamak [77, 78] has now motivated new interest in LHCD in an environment of α-particles. Originally, it was thought to be difficult to position the waveguides for high-field launch (sometimes called inside launch). However, there may now be ways of accomplishing that, with the waveguides advantageously better protected from the plasma and the waves capable of penetrating further. If the LH waves penetrate closer to the plasma core, more α-particles will be encountered, so it becomes important to reconsider how the α-particle environment affects the LHCD.

Interestingly, it turns out that the positive effects of LHCD in an environment of α-particles can be captured specifically with inside launch [79, 80]. Note that, while in principle both α-channeling and LHCD can be accomplished with the LH wave, it is not obvious that both can be accomplished simultaneously without overly constraining the wave propagation, since both α-channeling and LHCD put conditions on the LH wavenumber. It turns out, however, that the joint accomplishment of both effects is in fact enabled by inside launch.

To see this, following Ref. [79], consider a tokamak, with minor radius direction radial direction r, where the toroidal magnetic field is in the ẑ direction, while the poloidal magnetic field is in the ẑ direction, such that ẑ × r lies in the positive ħ direction. For illustrative purposes here, we imagine the tokamak as a straight torus, with circular flux surfaces. Denote the low-field side by \( \theta = 0^\circ \), so that the innermost part of the flux surfaces lies at \( \theta = 180^\circ \).

Note first that, for α-channeling to be effective, it must also be the case that \( |k_\theta| > |k_\phi| \), in other words that the perpendicular wavenumber point substantially in the poloidal direction. This is because only the poloidal wavenumber contributes to the diffusion in the radial direction, and a large \( k_\theta \) is necessary to recognize an inversion in the α-particle distribution in the direction of the diffusion path. Therefore, in propagating the lower hybrid wave from the periphery to the center, it is important for α-channeling that \( k_\theta \) increase substantially in magnitude. This is because, for the LH wave, the perpendicular wavenumber exceeds the parallel wavenumber by a large factor, on the order of \( \sqrt{m_\alpha/m_e} \), or on the order of 50. However, at the waveguide, the magnitude of \( k_\theta \) cannot be very much different from the magnitude of \( k_\phi \), since both are determined by waveguide dimensions. Hence, near the periphery, \( k \) must point substantially in the radial direction; it follows then, that to produce α-channeling as the wave trajectory nears the plasma center where the energetic α-particles are, the LH poloidal wavenumber \( k_\theta \) must grow substantially in absolute value.

The channeling direction further constrains the LH poloidal wavenumber. To achieve the α-channeling effect, α-particles that gain energy move to the plasma center. With reference to Eq. (2), what this means is that \( k_\theta B_\phi > 0 \), so that α-particles losing energy to the wave move to the periphery. Note that this statement is independent of the choice of toroidal magnetic field direction, since upon reversing \( B_\phi \), we must also reverse \( k_\theta \) to achieve the channeling effect.

To achieve the current drive effect places a further constraint on the toroidal wavenumber, but first we must ask what is the direction in which we might want to accomplish the current drive. For total toroidal current flowing in the +ϕ direction, the poloidal magnetic field points in the −û direction. To support this current with LHCD, the electrons need to be pushed in the −ϕ (negative ϕ) direction, which requires phase velocities in the −û (negative û) direction. For positive frequencies, this implies negative \( k_\phi \). Thus, we can say that for the LHCD to be supportive of the toroidal current, we must have \( k_\phi B_\theta > 0 \). Note that
this statement is independent of the choice of toroidal current flowing in the $+\phi$ direction; for LHCD supportive of toroidal current flowing in the opposite direction, both $k_\phi$ and $B_\phi$ change sign, so their product remains positive.

Multiplying the constraints on both the channeling direction and the current drive direction, we have $(k_\phi B_\phi) > 0$. Put differently, we have that the two terms in expanding $k \cdot B = k_\phi B_\phi + k_\phi B_\phi$ are of the same sign. Note also that $k \cdot B \equiv k_\parallel B$ gives the parallel wavenumber. Now, since we know that both to support current drive and accomplish $\alpha$-channeling, $k_\parallel$ must grow in magnitude, and since both terms are of the same sign, it follows immediately that the parallel wavenumber $k_\parallel$ must grow as the wave propagates towards the center, or that the so-called $k_\parallel$-upshift must happen.

Moreover, note that, since $k_\parallel > 0$, and $k_\parallel$ must grow, it follows that we must have $dk_\parallel/\partial s > 0$, where $s$ measures the distance along the wave trajectory as the wave propagates from the waveguide to the plasma center. For LH wave propagation, over a large range of parameters, we have $dk_\parallel/\partial s \sim -\sin \theta$, while $d\theta/\partial s \sim B_\phi/B_kk_\parallel > 0$ [81]. Since $dk_\parallel/\partial s > 0$, it follows that the trajectory must be substantially below the poloidal equator ($180^\circ < \theta < 360^\circ$). And since $d\theta/\partial s > 0$ [81], it follows that inside launch ($\theta \approx 180^\circ$) tends to maximize the trajectory distance below the poloidal equator. Quantitative calculations also support these general conclusions [80].

What this means is that LHCD with inside launch is compatible with both supportive current drive and $\alpha$-channeling; in fact, it optimizes for this synergy. However, one necessarily must anticipate concomitant $k_\parallel$-upshift. Moreover, what has been pointed out here is only the direction of the effect; since the LH wave does not move $\alpha$-particles very large distances in absorbing energy, this wave must be supplemented by other waves if a large fraction of the $\alpha$-particle energy were to be channeled.

5. Future: Current Recharge

In the past, non-inductive current drive has been imagined as the means to accomplish steady state tokamak operation. However, the power dissipation is also large for the tokamak reactor regimes presently contemplated, in particular, for those regimes where compactness is valued. But this power dissipation can be rendered almost insignificant if quasi-steady methods are used, where the current is kept nearly constant, but not quite constant, and other parameters are allowed to change significantly [1]. Such operation may be speculative, but it may be highly advantageous.

This quasi-steady-state scenario contemplates two stages: a current generation stage, when the current is generated by, for example, lower hybrid current drive (LHCD), and a current relaxation stage in which there is no current drive so that the current decays in an $L/R$ time, where $L$ is the tokamak plasma inductance and $R$ is the resistivity. The generation and relaxation cycles then repeat. Although the current does not deviate much from its average value, the plasma parameters in the two stages can be different. In principle, these other parameters can be changed on time scales short compared to the $L/R$ time, because the particle and heat confinement times are on the order of a second in a tokamak reactor, whereas the $L/R$ time is about three orders of magnitude longer.

During the current-generation stage, the current is increasing, so there is an induced electric field that opposes the increase in the current. The density in this stage should be relatively small to increase the current drive efficiency. The resistivity, which is independent of the density, should be relatively large to reduce the induced counter current. This might be arranged through small electron temperature or large effective ion charge state. On the other hand, during the current decay stage, resistivity can be relatively small and the density large. Since the effects on density and resistivity are multiplicative, the current drive efficiency by this means can be orders of magnitude more efficient. At the same time, because the induced electric field supports the rf-generated current at high density, there will be high fuel reactivity over the larger part of the cycle. It also enables better the LHCD effect, because the waves can better penetrate to the tokamak center during the low density stage. To accomplish this, however, the quasi-steady operation will need to be well-controlled.

This method, in and of itself, may be very useful, and it may be deployed with a variety of current drive mechanisms. Superthermal electron-based mechanisms exploit better the variation in temperature; however, there is a tendency for these methods to produce energetic electrons during the current generation stage, thereby increasing the conductivity. Perhaps the electron conductivity might be reduced through a transport mechanism that operates solely on the energetic backward-going electrons, like the stochastic instability suggested as responsible for restraining energy in runaway electrons [82]. The idea is to remove electrons contributing to the high conductivity. One might also speculate that other means of controlling the electron conductivity might be nonlinear in origin, perhaps through controlling parametric decay instabilities. Such instabilities are thought to accompany LHCD at lower densities [83]. Alternatively, to avoid electron tail heating altogether, the current generation stage might best use electron-based methods that operate on the bulk electrons [16, 17], or an ion-based rf method, such as minority species current drive [84], or neutral beam current drive [85].

There is an additional synergy in using $\alpha$-channeling together with transformer recharging [86]. The synergy with $\alpha$-channeling occurs because, in the current generation stage, the density is low, so the electron and ion temperatures equilibrate more slowly, facilitating the hot-ion mode. In the hot-ion mode, the fusion reactivity is greater, so the fusion power production can be made more uniform in the generation and relaxation stages, reducing the ther-
channeling 

Since the most dangerous plasma centrifuge has been advanced for high-throughput plasma devices such as the plasma centrifuge [102]. The energy can support the radial potential in non-fusion rotating plate electrodes. A fixed azimuthal perturbation can also be used to cap the plasma, depending on the wave characteristics, with α-particles most likely to leave by crossing the trapped-passing boundary. The useful waves also differ; in mirror machines, contained modes can be utilized [93–95]. Other waves might be useful too if minority ions are used to catalyze the channeling effect [96].

In a variation on the mirror machine, the centrifugal mirror, an imposed axial potential creates supersonic $E \times B$ rotation of the plasma, which provides additional axial confinement [97]. The radial potential creates further possibilities. In a generalization of the channeling effect in rotating plasma, depending on the wave characteristics, some of the α-particle energy can be channeled to electric potential energy and some channeled to the wave energy [98]. Waves can be arranged that eject α-particles while capturing their energy in the wave fields [99, 100]. A fixed azimuthal perturbation can also be used to capture α-particle energy to support the radial potential [101], replacing the need for setting up the potential through endplate electrodes.

In a further application of the generalization of the α-channeling effect, instead of α-particle energy, wave energy can support the radial potential in non-fusion rotating plasma devices such as the plasma centrifuge [102]. The plasma centrifuge has been advanced for high-throughput nuclear waste remediation [103]. Since the most dangerous wastes have high mass numbers, mass separation can advantageously minimize the volume of radioactive waste needing further treatment or burial. A variation of the centrifuge, exploiting the mass dependence of the axial confining forces in rotating plasma, confers the additional advantage of axial separation [104–107]. The use of high-throughput mass separation, as a supplement to chemical separation techniques, may be critical to process economically large amounts of nuclear waste [108].

7. Concluding Remarks

In imagining the future, we should be prepared to apply the paradigms developed here to many different configurations, both for applications other than nuclear fusion, and of course for nuclear fusion, possibly using advanced fuels [109], or under exotic plasma conditions, such as high-density degenerate plasma [110]. The paradigm of producing fusion may itself change, such as through fusion-fission parks, using fusion neutrons to breed nuclear fuel for use in conventional nuclear reactors [111]. The new economics of such a paradigm change will require re-evaluation in applying the techniques here. Other future directions include highly theoretical ones, such as the question of the precise free energy available through diffusive processes [112], where recent advances have been made [113]. This theoretical question, while not directly of particular importance in applications that we can imagine today, nonetheless possesses the interesting quality of connecting problems addressed here to fundamental problems of interest in related fields in physics or in pure mathematics.

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[1] N.J. Fisch, Rev. Mod. Phys. 59, 175 (1987).
[2] W.H. Hooke, Plasma Phys. Control. Fusion 26, 133 (1984).
[3] N.J. Fisch, American Scientist 71, 27 (1983).
[4] R. Prater, Phys. Plasmas 11, 2349 (2004).
[5] R.I. Pinsker, Phys. Plasmas 22, 090901 (2015).
[6] R.I. Pinsker, Phys. Plasmas 8, 1219 (2001).
[7] D.W. Faulconer, Fusion Sci. Technol. 53, 210 (2008).
[8] E. Westerhof, Fusion Sci. Technol. 61, 312 (2012).
[9] N.J. Fisch, Fusion Sci. Technol. 65, 1 (2014).
[10] N.J. Fisch, Fusion Sci. Technol. 65, 79 (2014).
[11] J.M. Rax, Fusion Sci. Technol. 65, 10 (2014).
[12] T.H. Stix, Waves in Plasmas (Springer-Verlag, NY, 1992).
[13] M. Brambilla, Kinetic Theory of Plasma Waves in Homogeneous plasma (Oxford University Press, NY, 1998).
[14] J.M. Rax, Physique Des Plasmas (Dunod, Paris, 2005).
[15] N.J. Fisch, Phys. Rev. Lett. 41, 873 (1978).
[16] D.J.H. Wort, Plasma Phys. 13, 258 (1971).
[17] N.J. Fisch and C.F.F. Karney, Phys. Fluids 24, 27 (1981).
[18] M. Nakamura et al., J. Phys. Soc. Japan 51, 3696 (1982).
[19] S. Kubo et al., Phys. Rev. Lett. 50, 1994 (1983).
[105] R. Gueroult and N.J. Fisch, Phys. Plasmas 19, 122503 (2012).

[106] R. Gueroult and N.J. Fisch, Plasma Sources Sci. Technol. 23, 035002 (2014).

[107] R. Gueroult, J.M. Rax and N.J. Fisch, Phys. Plasmas 21, 020701 (2014).

[108] R. Gueroult, D. Hobbs and N.J. Fisch, J. Hazardous Materials 297, 153 (2015)

[109] M.J. Hay and N.J. Fisch, Phys. Plasma 22, 112116 (2015).

[110] S. Son and N.J. Fisch, Phys. Rev. Lett. 95, 225002 (2005).

[111] W. Manheimer, J. Fusion Energy 33, 199 (2014).

[112] N.J. Fisch and J.M. Rax, Phys. Fluids B 5, 1754 (1993).

[113] M.J. Hay, J. Schiff and N.J. Fisch, Phys. Plasmas 22, 102108 (2015)