High Temperature Operation Clothing Design Based on Heat Conduction Equation

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Abstract. Based on the knowledge of heat and mechanics, the relationship between density, specific heat, thermal conductivity and thickness is analyzed by means of heat conduction equation. Under the condition of the change of working time and temperature, the optimal two-dimensional spline interpolation method is used to construct the corresponding functional relation of the temperature distribution field, and based on the numerical model of multi-zone and multi-layer heat transfer of the dummy in the equipment. The thermal physical parameters of each region are determined by experiments, and the equations of thermal equilibrium are established. Under the condition of the problem, the temperature change outside the human body is obtained, and then the temperature distribution outside the human body is obtained. Based on the above model, the experimental results show that the heat absorbed by the outside is consumed layer by layer in the process of heat transfer at high temperature. The following known temperature distribution, at a specific ambient temperature and working time length, the thickness of layer II to layer IV is obtained. By using multi-objective programming and genetic algorithm, the optimal solution is obtained by iterative approximation of the optimal solution for many times, and the final data is obtained by using the method of multi-objective programming and genetic algorithm.

keywords: spline interpolation method, temperature distribution field, thermal physical parameters, heat transfer.

1. Introduction
At present, due to the global warming caused by the continuous development of economy and society, the social risk factors increase, which means that mankind will be exposed to the environment of greater thermal harm. In particular, people engaged in special industries, such as fire fighting, biological and chemical insurance, military, sports, etc., are facing greater harm to the thermal environment. Under such a background situation, it is very important to solve the problem of staff's body adaptation under high temperature environment. Scholars at home and abroad have done a lot of research on the design
of high temperature clothing. Suitable for a variety of industries to wear has developed Require clothing to be provided to the human body multi-protection. As the "second skin" of human body, clothing can solve the shortage of physiological and thermal regulation of human body, and combining with fabric material, the design and development of intelligent clothing with endothermic and temperature-regulating function is the trend of clothing development at present. This paper takes this as the starting point and takes endothermic temperature regulation as the core tenet, respectively in extreme high temperature, high temperature and higher temperature environment. The research ranges from fabric sample to clothing layer-by-layer evaluation means from fabric sample testing to using sweating warm dummy and then using real-life dress evaluation.

2. thermology and mechanics
   Try to use sweating warm-up dummy, and then use real-life dress evaluation to launch.

2.1. Model assumptions
   Figure 1 shows that when working in the high temperature environment of the dummy, the layer of clothing itself and the gap between the clothing and the skin of the dummy are set up. The mathematical model of multi-layer heat transfer between the dummy and the clothing-environment is established (figure 1), analysis of multi-layer glass model for reference.

   ![Figure 1 Dummy-Garment-Environment](image)

   In this paper, it is assumed that the heat conduction process should satisfy the following physical Laws. Uniform medium with thickness \( d \), temperature difference on both sides being \( \Delta T \), then the heat per unit time from the high temperature side to the low temperature side passing through the unit area \( Q \) is proportional to the \( \Delta T \), inversely proportional to \( d \).

   \[
   Q = k \frac{\Delta T}{d}
   \]  

   Where \( k = \lambda t A \) is the heat transfer coefficient, \( \lambda \) is the thermal conductivity, \( t \) is the conduction time and \( A \) is the heat conduction area.

   As shown in figure 1, for II layers with 6mm thickness, the thickness of the I layer is 0.6mm. The temperature difference between them is \( T_1, T_2 \), The heat conduction coefficient of the garment is \( k_1 \). The amount of heat passed is 1. The heat conduction per unit time unit area of layer I, II can be obtained from formula (1-1-1).

   \[
   Q_1 = k_1 \frac{T_1 - T_2}{2d}
   \]  

   In the same way, the heat conduction of the III and IV layers can be obtained:

   \[
   Q_2 = k_2 \frac{T_3 - T_4}{2d}
   \]
2.2. Mathematical models

2.2.1. Differential equation of heat conduction and Fourier Law. To the extent known in this article, it points out that, the heat flux of heat conduction is proportional to the absolute value of the temperature gradient, and its direction is opposite to the direction of the temperature gradient.

\[ q = -\lambda \frac{\partial t}{\partial n} \]  

(4)

Because the direction of heat transfer is opposite to the direction of the temperature gradient, there is a negative sign in the equation. The essence of Fourier’s Law is that in the system of matter with temperature difference, the heat flow always goes in the direction of temperature decrease. When the heat flux density on the given heat conduction surface is equal.

\[ \phi = -\lambda A \frac{\partial t}{\partial n} \]  

(5)

Fourier Law [19] reveals the relationship between heat flux and temperature gradient in continuous temperature field. For the one-dimensional steady-state heat conduction problem, Fourier Law integral can be directly used to calculate the heat flux. However, Fourier’s Law fails to reveal the relationship between the temperature of each point and the temperature of its adjacent point, so it is necessary to combine the differential equation of heat conduction to make it possible to solve the problem of heat conduction.

According to Fourier Law and energy conservation equation, the differential equation of heat conduction in rectangular coordinate system can be deduced.

\[ \frac{\partial t}{\partial \tau} = \frac{\lambda}{pc} \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right) + \frac{\phi}{pc} = a \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) + \frac{\phi}{pc} \]  

(6)

Where \( a \) is the thermal diffusivity, also known as the temperature conductivity, \( a = \frac{\lambda}{pc} = m^2/s \); \( \phi \) is the heat generated by the internal heat source in the unit volume for the unit time, \( W/m^2 \).

The equation for the one-dimensional steady-state heat conduction known above is as follows:

\[ \frac{\partial^2 t}{\partial x^2} = 0 \]  

The boundary conditions are:

\[ \begin{cases} x = 0: \tau = t_1 \\ x = \delta: \tau = t_2 \end{cases} \]  

(7)

According to Fourier's Law, the following results are obtained:

\[ q = \lambda \frac{dt}{dx} = \lambda \frac{t_2 - t_1}{\delta} = \frac{t_1 - t_2}{\lambda} \]  

(8)

2.2.2. Second layer optimum thickness. There are two variables, time coordinate \( \tau \) and spatial coordinate \( X \). But note that the time coordinates are one-way, that is, the state of the preceding moment will have an impact on the state of the next moment, but the state of the latter moment will not affect the previous moment. Figure III shows the discretization of the calculated region with \( X \) and \( \tau \) as the coordinates. Time starts from \( \tau = 0 \) and increases to \( K \) and \( K+1 \) layer through time layers. In this case, the left end of the temperature of the equation is differentially separated from the partial derivative of time as follows:

\[ \left( \frac{\partial t}{\partial \tau} \right)_i^K = \frac{t_{i+1}^K - t_{i}^K}{\Delta \tau} \]  

(9)

\[ \left( \frac{\partial t}{\partial \tau} \right)_i^{K+1} = \frac{t_{i+1}^{K+1} - t_i^K}{\Delta \tau} \]  

(10)
In addition, it is found that this is the heat conduction process in which the temperature of the object changes with time, and the temperature of the object gradually approaches to a constant value with the passage of time, and the temperature of the object changes periodically with the passage of time.

For the boundary conditions given earlier, the difference can be used to replace the differential directly, or the corresponding boundary conditions can be given by the element body balance method. There are also explicit and implicit differences. Usually, when the internal nodes are explicit, the boundary nodes are also discretized explicitly. When the internal nodes are implicit, the boundary nodes are also implicit. Whether the difference scheme of boundary nodes is explicit or implicit depends on how to combine with the difference equations of internal nodes. The difference equations of the boundary nodes can be obtained by using the difference, substitution and differential of the corresponding nodes at time \( K+1 \).

\[
T_1^{K+1} = T_2^{K+1} \\
T_N^{K+1} = \frac{1}{h_N} \left( T_{N-1}^{K+1} + \frac{h_N}{\lambda} T_{\infty} \right)
\]

Corresponding to different time and material distance, the corresponding temperature is also different, in which the temperature from deep red to blue, the deeper the color, the higher the temperature.

2.2.3. Optimal thickness of layer II and IV. According to the first question, the optimal thickness of IV layer is calculated by determining the thickness and working time of II layer under the change of temperature on the outside of the layer.

The aim of the optimal structural design in this paper is to minimize the temperature, and the thermal conductivity index function \( f(k) \) takes the maximum temperature \( T_{\text{max}}(k) \). According to the heat dissipation and heat transfer mechanism, it can be seen that the highest temperature occurs on the corresponding edge far away from the heat dissipation boundary, that is, the thickness of the II layer. The maximum temperature is as follows.

\[
T_{\text{max}}(k) = T_0 + q_n \int_0^1 \frac{(l-x)}{k(x)} dx
\]

\[
\int_0^1 \left[ \frac{-q_n(l-x)}{k^2(x)} + \frac{\lambda}{k(x)} \right] \delta k(x) dx = 0
\]

\[
\int_0^1 k dx = K_0
\]

The optimal thermal conductivity distribution can be obtained by the combination of the above two equations \( k_{\text{max}}(x) \).

\[
k_{\text{max}}(x) = \frac{3k_0}{2l^{1/2}} (l-x)^{1/2}
\]

So the temperature distribution is

\[
T_{\text{max}}(x) = T_0 + \frac{4q_n l^{3/2}}{9k_0} \left( l^{3/2} - (l-x)^{3/2} \right)
\]

It can be concluded that the decision variable is thickness, and then the objective function is deduced as follows.

\[
TEM=\text{abs}(T(1000,20)-60)
\]

Therefore, the next goal is to maximize the optimum thickness of layer II and layer IV, and obtain the optimal thickness according to the optimization model.

\[
TEM_{\text //= \text{layer}}=\text{abs}(T(1000,20)-90)
\]
3. Numerical algorithm of Heat conduction Model

3.1. The distribution of each layer of heat conduction model

The solution of the model is mainly based on the differential equation and Fourier Law. According to the problem, the heat flux density of heat conduction is proportional to the absolute value of the temperature gradient, and its direction is opposite to the direction of the temperature gradient.

\[ q = -\lambda \frac{\partial T}{\partial n} \] (21)

Within a system with a temperature difference, the heat flow is always in the direction of the temperature drop, as shown in figure 1.

**Figure 2** Relationship between heat flux and temperature

When the heat flux density on the given heat conduction surface is equal.

\[ \phi = -\lambda A \frac{\partial T}{\partial n} \] (22)

According to Fourier Law and energy conservation equation, the differential equation of heat conduction in rectangular coordinate system can be deduced.

\[ \frac{\partial T}{\partial t} = \frac{\lambda}{\rho c} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\phi}{\rho c} = a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\phi}{\rho c} \] (23)

Where \( a \) is the thermal diffusivity, also known as the temperature conductivity, \( a = \frac{\lambda}{\rho c} \), m\(^2\)/s; \( \phi \) is the heat generated by the internal heat source in the unit volume within a unit of time, W/m\(^2\).

3.2. Optimal thickness of II layer

The goal of the optimal structural design in this paper is to minimize the temperature,In this case, the thermal conductivity index function \( f(k) \) takes the maximum temperature \( T_{max}(k) \), and according to the heat dissipation and heat transfer mechanism, the maximum temperature occurs at the corresponding edge far away from the heat dissipation boundary. That is, the thickness of the II layer, That is, the maximum temperature is as follows.

\[ T_{max}(k) = T_0 + q^n \int_0^l \frac{(l - x)}{k(x)} dx \] (24)

\[ \int_0^l \left[ \frac{-q^n (l-x)}{k^2(x)} + \lambda \right] \delta k(x) dx = 0 \] (25)

\[ \int_0^l k dx = K_0 \] (26)
The optimal thermal conductivity distribution can be obtained by the combination of the above two equations
\[ k_{T_{\text{max}}}(x) = \frac{3k}{2l^{3/2}} (l - x)^{1/2} \] 
(27)

So, the temperature distribution is as follows.
\[ T_{T_{\text{max}}}(x) = T_0 + \frac{4q}{9\kappa_0} \left( t^{3/2} - (l - x)^{3/2} \right) \] 
(28)

According to the reasoning, it can be concluded that the decision variable is thickness, and then the objective function is deduced as follows.
\[ TEM = \text{abs}(T(1000,20)-60) \] 
(29)

Finally, the optimal thickness of the second layer of the model is 12 mm.

3.3. Optimal thickness of II,IV layer
According to the second question, this problem has been further sublimated on the basis of the second question, so in considering the second question, we should get the optimal solution according to the idea and method of the second question.

Therefore, the next goal is to maximize the optimum thickness of layer II and layer IV, and obtain the optimal thickness according to the optimization model.

\[ TEM_{II/layer} = \text{abs}(T(1000,20)-90) \] 
(30)

\[ TEM_{IV/layer} = \text{abs}(T(1000,20)-75) \] 
(31)

According to the above-mentioned model, the optimal solution of layer II and layer 4 can be easily obtained, that is, 7.5mm for layer II and 6mm for layer IV.

4. Modelling verification
4.1. The distribution of each layer of heat conduction model
Considering the upper model and the data fitting equation, the final temperature distribution map is obtained by MATLAB simulation.

According to the above picture, the three-dimensional distribution map of the temperature region is obtained, which corresponds to different time and material distance, and the corresponding temperature is different. The temperature is from dark red to blue, and the deeper the color is, the higher the temperature is.
4.2. Optimum thickness of II layer

The heat conduction process in which the temperature of an object changes with time, and the temperature of the object gradually approaches to a constant value with the passage of time and makes periodic changes. The following is the calculated temperature distribution map of the temperature over time.

![Temperature distribution map of temperature change with time](image)

**Figure 4** Temperature distribution map of temperature change with time

In order to make the model more reasonable and convenient for data analysis, the residual error is introduced to compare.

![Residual plot](image)

**Figure 5** residual plot

Through the comparison between residual and temperature distribution, it is easy to see the rationality of the obtained temperature distribution, and further optimize the model.

4.3. The Distribution Map of II,III optimal thickness

![Optimal thickness distribution map](image)

**Figure 6** Optimal thickness distribution map

5. Conclusion

In this paper, thermal protective clothing as a wide range of special protective clothing, first of all, thermal protection performance and damage to the human body as an important premise, the complex clothing structure model is simplified into a common double-layer glass in life. It makes the model more intuitive and easy to analyze. In the process of modeling, the physical method and the mathematical
model are closely combined, the theory is combined with the practice, and the experimental data is approached to the reality to the maximum extent. However, the heat source between the layers is neglected in the process of establishing the model, which makes the model too idealized and still needs to be considered in the practical problems.

6. Future
Great progress has been made in the field of protective clothing in China, and the comfort of high-temperature protective clothing has been paid more and more attention. Of course, the general trend of the development of high-temperature protective clothing is the overall protection, from a single risk factor to a variety of risk factors of the total protection, from emphasis on protection to pay attention to the ergonomic characteristics and comfort of the human body. This trend is mainly reflected in three aspects. First, the high performance of clothing materials. The second is the development of new technology. The third is the continuous maturity of fabric composition and finishing technology. When studying new high temperature protective clothing, give full play to the new detection technology and data processing phase. It will give birth to more comfortable and protective high temperature thermal protective clothing.

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