Global analysis of electroweak data in the Standard Model

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We perform a global fit of electroweak data within the Standard Model, using state-of-the-art experimental and theoretical results, including a determination of the electromagnetic coupling at the electroweak scale based on recent lattice calculations. In addition to the posteriors for all parameters and observables obtained from the global fit, we present indirect determinations for all parameters and predictions for all observables. Furthermore, we present full predictions, obtained using only the experimental information on Standard Model parameters, and a fully indirect determination of Standard Model parameters using only experimental information on electroweak data. Finally, we discuss in detail the compatibility of experimental data with the Standard Model and find a global p-value of 0.5.

In the Standard Model (SM) of ElectroWeak (EW) and strong interactions, the SU(2) L ⊗ U(1) Y gauge symmetry is hidden at low energies through the Higgs mechanism, leaving only electromagnetism as a manifest symmetry. This hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge hidden symmetry endows the SM with calculable relations among masses and couplings of EW bosons, and a huge

The recent evaluation of the leading fermionic three-loop corrections to EWPO [54, 55] results in even smaller effects, which have been neglected in our fits.

1 The HEPfit package is available under the GNU General Public License (GPL) [50].
2 For recent comprehensive reviews of both theoretical and experimental inputs to EW precision fits see Ref. [50] and Ref. [57].
Global SM EW fit (standard scenario)

| Measurement | Posterior | Prediction | 1D Pull | nD Pull |
|-------------|-----------|------------|---------|---------|
| $\alpha_s(M_Z^2)$ | 0.1177 ± 0.0010 | 0.11792 ± 0.00094 | 0.1198 ± 0.0028 | -0.7 |
| $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ | 0.02766 ± 0.00010 | 0.027627 ± 0.000096 | 0.02717 ± 0.00037 | 1.3 |
| $M_Z$ [GeV] | 91.1875 ± 0.0021 | 91.1883 ± 0.0021 | 91.2047 ± 0.0088 | -1.9 |
| $m_t$ [GeV] | 172.58 ± 0.45 | 172.75 ± 0.44 | 176.2 ± 2.0 | -1.8 |
| $m_H$ [GeV] | 125.21 ± 0.12 | 125.21 ± 0.12 | 108.3 ± 11.7 | 1.3 |
| $M_W$ [GeV] | 80.379 ± 0.012 | 80.3591 ± 0.0052 | 80.3545 ± 0.0057 | 1.8 |
| $\Gamma_W$ [GeV] | 2.085 ± 0.042 | 2.08827 ± 0.00055 | 2.08829 ± 0.00056 | -0.1 |
| $\sin^2 \theta_{\text{eff}}^{Q^2}(Q^2_{\text{FB}})$ | 0.2324 ± 0.0012 | 0.231509 ± 0.000056 | 0.231511 ± 0.000058 | 0.7 |
| $P_{\text{eff}}^{(5)} = \mathcal{A}_t$ | 0.1465 ± 0.0033 | 0.14712 ± 0.00044 | 0.14713 ± 0.00045 | -0.2 |
| $\mathcal{A}_t$ (SLD) | 0.1513 ± 0.0021 | 0.14712 ± 0.00044 | 0.14713 ± 0.00046 | 1.9 |
| $R^{(5)}_b$ | 0.21629 ± 0.00066 | 0.21587 ± 0.00010 | 0.21587 ± 0.00010 | 0.6 |
| $R^{(5)}_t$ | 0.1721 ± 0.0030 | 0.172205 ± 0.00054 | 0.172206 ± 0.000053 | 0.0 |
| $A^{(5)}_{FB}$ | 0.0996 ± 0.0016 | 0.10314 ± 0.00031 | 0.10315 ± 0.00033 | -2.2 |
| $A^{(5)}_{FB}$ | 0.0707 ± 0.0035 | 0.07369 ± 0.00023 | 0.07370 ± 0.00024 | -0.9 |
| $A_{b}$ | 0.923 ± 0.020 | 0.934738 ± 0.000040 | 0.934739 ± 0.000040 | -0.6 |
| $A_c$ | 0.670 ± 0.027 | 0.66782 ± 0.00022 | 0.66783 ± 0.000022 | 0.0 |
| $A_{s}$ | 0.895 ± 0.091 | 0.935651 ± 0.000040 | 0.935651 ± 0.000040 | -0.4 |
| $\text{BR}_{W \rightarrow \ell \nu}$ | 0.10860 ± 0.00090 | 0.108381 ± 0.000022 | 0.108380 ± 0.000022 | 0.2 |
| $\sin^2 \theta_{\ell \nu} (HC)$ | 0.23143 ± 0.00025 | 0.231509 ± 0.000056 | 0.231511 ± 0.000058 | -0.3 |
| $R_{ac}$ | 0.1660 ± 0.0090 | 0.172227 ± 0.000032 | 0.172228 ± 0.000032 | -0.7 |

TABLE I. Experimental measurement, result of the global fit, prediction, and pull for the five input parameters ($\alpha_s(M_Z^2)$, $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$, $M_Z$, $m_t$, $m_H$), and for the set of EWPO considered in the fit, in the standard scenario for $m_t$ and $m_H$. For the results of the global fit and for the predictions, the 95% probability range is reported in square brackets. The values in the column Prediction are determined without using the corresponding experimental information. Pulls are calculated both as individual pulls (1D Pull) and as global pulls (nD Pull) for sets of correlated observables, and are given in units of standard deviations. Groups of correlated observables are identified by shades of grey.
### Global SM EW fit (conservative scenario)

| Measurement | Posterior | Prediction | 1D Pull nD Pull |
|-------------|-----------|------------|-----------------|
| $\alpha_s(M_Z^2)$ | 0.1177 $\pm$ 0.0010 | 0.11793 $\pm$ 0.00094 | 0.1199 $\pm$ 0.0028 | -0.7 |
| $\Delta \alpha_{\text{had}}^{(5)}(M_Z^2)$ | 0.02766 $\pm$ 0.00010 | 0.027631 $\pm$ 0.000097 | 0.02721 $\pm$ 0.00039 | 1.1 |
| $M_Z$ [GeV] | 91.1875 $\pm$ 0.0021 | 91.1881 $\pm$ 0.0021 | 91.2045 $\pm$ 0.0094 | -1.8 |
| $m_t$ [GeV] | 172.6 $\pm$ 1.0 | 173.31 $\pm$ 0.90 | 176.1 $\pm$ 2.0 | -1.6 |
| $m_H$ [GeV] | 125.21 $\pm$ 0.21 | 125.21 $\pm$ 0.21 | 109.7 $\pm$ 12.6 | 1.2 |
| $M_W$ [GeV] | 80.379 $\pm$ 0.012 | 80.3619 $\pm$ 0.0064 | 80.3549 $\pm$ 0.0077 | 1.7 |
| $\Gamma_W$ [GeV] | 2.085 $\pm$ 0.042 | 2.08850 $\pm$ 0.00063 | 2.08849 $\pm$ 0.00063 | -0.1 |
| $\sin^2 \theta_W^{\text{eff}} (Q_{FB}^{\text{had}})$ | 0.2324 $\pm$ 0.0012 | 0.231497 $\pm$ 0.000058 | 0.231498 $\pm$ 0.000060 | 0.8 |
| $P_{Z\ell\ell}$ | 0.1465 $\pm$ 0.0033 | 0.14722 $\pm$ 0.00046 | 0.14724 $\pm$ 0.00046 | -0.2 |
| $\Gamma_Z$ [GeV] | 2.4955 $\pm$ 0.0023 | 2.49454 $\pm$ 0.00066 | 2.49434 $\pm$ 0.00070 | 0.5 |
| $\Gamma_h^0$ [nb] | 41.480 $\pm$ 0.033 | 41.4912 $\pm$ 0.0077 | 41.4931 $\pm$ 0.0080 | -0.4 |
| $R_B^0$ | 20.767 $\pm$ 0.025 | 20.7492 $\pm$ 0.0080 | 20.7458 $\pm$ 0.0087 | 0.8 |
| $A_{FB}^{\ell}$ | 0.0171 $\pm$ 0.0010 | 0.01626 $\pm$ 0.00010 | 0.01625 $\pm$ 0.00010 | 0.9 |
| $A_{\ell}$ (SLD) | 0.1513 $\pm$ 0.0021 | 0.14722 $\pm$ 0.00046 | 0.14724 $\pm$ 0.00048 | 1.9 |
| $B_{\ell}$ | 0.21629 $\pm$ 0.00066 | 0.21586 $\pm$ 0.00010 | 0.21585 $\pm$ 0.00010 | 0.7 |
| $R_{\ell}^b$ | 0.1721 $\pm$ 0.0030 | 0.172212 $\pm$ 0.00055 | 0.172212 $\pm$ 0.00054 | 0.0 |
| $A_{FB}^{b}$ | 0.0996 $\pm$ 0.0016 | 0.10321 $\pm$ 0.00033 | 0.10323 $\pm$ 0.00034 | -2.2 |
| $A_{FB}^{c}$ | 0.0707 $\pm$ 0.0035 | 0.07374 $\pm$ 0.00024 | 0.07375 $\pm$ 0.00025 | -0.9 |
| $A_{b}$ | 0.923 $\pm$ 0.020 | 0.934741 $\pm$ 0.000040 | 0.934741 $\pm$ 0.000040 | -0.6 |
| $A_{c}$ | 0.670 $\pm$ 0.027 | 0.66787 $\pm$ 0.00023 | 0.66788 $\pm$ 0.00023 | 0.1 |
| $A_{u}$ | 0.895 $\pm$ 0.091 | 0.935660 $\pm$ 0.000042 | 0.935659 $\pm$ 0.000042 | -0.4 |
| $\text{BR}_{W\rightarrow\tau\ell}$ | 0.10860 $\pm$ 0.00090 | 0.108380 $\pm$ 0.000022 | 0.108380 $\pm$ 0.000022 | 0.2 |
| $\sin^2 \theta_W^{\text{eff}}$ (HC) | 0.23143 $\pm$ 0.00025 | 0.231497 $\pm$ 0.000058 | 0.231498 $\pm$ 0.000060 | -0.3 |
| $R_{ac}$ | 0.1660 $\pm$ 0.0090 | 0.172234 $\pm$ 0.000033 | 0.172234 $\pm$ 0.000033 | -0.7 |

| TABLE II. | Same as Table I in the conservative scenario for the errors on $m_t$ and $m_H$. |
2. The value for the five-flavour hadronic contribution to the QED coupling constant at the Z-boson mass has also been recently updated by several groups, with mostly compatible results (see Ref. [57] for a more comprehensive discussion). In our study we make use of the lattice determinations of the euclidean correlation function $\hat{\Pi}_{ij}(\pm 4\text{GeV}^2)$ = 0.0712 ± 0.0002 from Ref. [59] (Table S3) and of the bottom quark contribution $\hat{\Pi}_b(\pm 4\text{GeV}^2)$ = 0.00013 from Ref. [60], which combined give

$$\Delta \alpha^{(5)}_\text{had}(\pm 4\text{GeV}^2) = 4\pi \alpha \left( \hat{\Pi}_{ij}(\pm 4\text{GeV}^2) + \hat{\Pi}_b(\pm 4\text{GeV}^2) \right) = 0.00654 \pm 0.00002.$$ 

Running perturbatively to the scale $-M_Z^2$ and continuing analytically to Minkowski spacetime according to Ref. [61] leads to $\Delta \alpha^{(5)}_\text{had}(M_Z^2) = 0.02766 \pm 0.00010$, compatible with, but more precise than, the value we previously used [53], $\Delta \alpha^{(5)}_\text{had}(M_Z^2) = 0.02750 \pm 0.00033$.

3. For the top-quark mass, Ref. [53] used the 2014 world average from ATLAS and the Tevatron experiments. Since then several updated measurements have become available, with individual uncertainties exceeding that of the 2014 average. We have therefore reconsidered and updated the value of and uncertainty on $m_t$ that is used on the current EW precision fit. In this study we consider: i) the 2016 Tevatron [62] combination; ii) the 2015 CMS Run 1 combination [63]; iii) the combination of ATLAS Run 1 results in Ref. [64]; iii) the CMS Run 2 measurements in the dilepton, lepton+jets and all-jet channels [65, 66]; and iv) the ATLAS Run 2 result from the lepton+jet channel [67]. Unfortunately, combining these different measurements is non-trivial due to the correlations between theoretical errors and several of the systematic uncertainties of the different measurements. For the purpose of this paper, we consider a correlated combination between the different measurements, assuming the linear correlation coefficient between two systematic uncertainties to be written as $\rho_{ij} = \min \{ \sigma_{i}^{\text{sys}}, \sigma_{j}^{\text{sys}} \} / \max \{ \sigma_{i}^{\text{sys}}, \sigma_{j}^{\text{sys}} \}$. This results in $m_t = 172.58 \pm 0.45$ GeV. In performing this combination, we note that, while the LHC measurements of $m_t$ are reasonably consistent with each other, there is some tension between the ATLAS and CMS lepton+jet values. This is also the case between the LHC and Tevatron $m_t$ combinations, and while these tensions could be just due to statistical fluctuations, in the worst case scenario they could indicate that some of the systematics included in these measurements have been underestimated. A common way of dealing with this issue is to use a rescaled error following the PDG average method [68]. In our case the resulting uncertainty would turn out to be unreasonably large, $\sim 1.7$ GeV. For the purpose of this paper we illustrate the impact of the top-mass uncertainty on the SM precision fits by considering two scenarios: one where we use the standard error of $\delta m_t = 0.45$ GeV and one where we consider a more conservative error of $\delta m_t = 1$ GeV. As we will see, the two different scenarios lead at the moment to very similar results since the parametric uncertainties are subleading with respect to the experimental ones.

4. The Higgs-boson mass, whose value after Run 1 was $m_H = 125.09 \pm 0.24$ GeV [18] (as used in Ref. [53]), has now also been measured in Run 2 both by ATLAS [69, 70] and CMS [71, 72] in the $4\ell$ and $\gamma \gamma$ channels. We follow the same combination procedure as for $m_t$ and combine all the different measurements to obtain $m_H = 125.21 \pm 0.12$ GeV as the standard input value for our fits. Some tension between the ATLAS and CMS Run 2 combinations is also present in this case. This tension does not have a visible impact in the fits, since the parametric uncertainty associated to $m_H$ is negligible for $\delta m_H$ up to O(10 GeV). Nevertheless, we consider an uncertainty $\delta m_H = 0.21$ GeV for the conservative scenario, obtained using the PDG scaling method [68].

5. The ATLAS collaboration presented their first direct determination of the W-boson mass, $M_W = 80.370 \pm 0.019$ GeV [43], with an uncertainty comparable to the current LEP2+Tevatron average. Assuming the absence of significant correlations between the LHC and Tevatron determinations, one can compute an approximate “world average” of $M_W = 80.379 \pm 0.012$ GeV. Although different from the LEP2+Tevatron world average used in Ref. [53], $M_W = 80.385 \pm 0.015$ GeV, the update of $M_W$ has a very minor effect on the SM fits [43].

6. The determination of the effective leptonic weak mixing angle, $\sin^2 \theta_{\text{eff}}$, at hadron colliders has also been updated. Using the same procedure as for the Higgs-boson and top-quark masses, we combine the ATLAS [44, 45] and CMS [46] measurements, the Tevatron determinations in Ref. [69], and the LHCb measurement in Ref. [73]. This combination is done separately for the measurements in the electron and muon channels, yielding $\sin^2 \theta_{\text{eff}}^{\text{e}^+\text{e}^-} = 0.23175 \pm 0.00029$ and $\sin^2 \theta_{\text{eff}}^{\mu^+\mu^-} = 0.23093 \pm 0.00039$, respectively. We also obtain a combination assuming lepton universality, $\sin^2 \theta_{\text{eff}}^{\text{e}^+\text{e}^-} = 0.23143 \pm 0.00025$. In this last case, we note that there is some tension between the CDF and D0 values, but this would only result in a small rescaling of the error and it is ignored here.

4. We do not consider the CMS measurement using the single-top channel [63], but we checked it has a negligible impact on the average.

5. The very recent result on $M_W$ from the LHCb Collaboration [69] will be included in future updates of our fit.
7. The updates in the $Z$-lineshape observables reported in Ref. [70] have been included. Compared to Ref. [32], these updates are due to the use of more accurate calculations of the Bhabha cross section, which lead to a better understanding of any systematic bias on the integrated luminosity. Only the $Z$ width, $\Gamma_Z$, the hadronic cross section at the $Z$ peak, $\sigma^0_h$, and its correlations with other $Z$-lineshape observables are noticeably affected by these updates.

8. We have included the update in the determination of the forward-backward asymmetry of the bottom quark, $A_{FB}^{0,b}$, after taking into account the massive $O(\alpha^2)$ corrections in $e^+e^- \rightarrow b\bar{b}$ at the $Z$ pole [71]. As we will see, these corrections slightly reduce the longstanding tension between the experimental measurement of this observable and its SM prediction.

Apart from these updates, we have also extended the EW fit by including the following extra observables:

9. The determination of the $s$-quark asymmetry parameter $A_s$ at SLD [72].

10. The PDG average of the different LEP experiment determinations of the ratio $R_{uc} \equiv \frac{1}{2} \frac{\Gamma_Z \rightarrow u\bar{u} + c\bar{c}}{\Gamma_Z \rightarrow \text{had}}$ [58].

11. The leptonic branching ratio of the $W$ boson, $\text{BR}_{W \rightarrow \ell\nu} \equiv \frac{\Gamma_{W \rightarrow \ell\nu}}{\Gamma_W}$ [58].

We use flat priors for all the SM input parameters, and include the information of their experimental measurements in the likelihood. We assume that all experimental distributions are Gaussian. The known \textit{intrinsic} theoretical uncertainties due to missing higher-order corrections to EWPO are also included in the fits, using the results of Ref. [31] to which we refer for more details. The main theory uncertainties we consider are:

\begin{equation}
\delta_{th} M_W = 4 \text{ MeV}, \quad \delta_{th} \sin^2 \theta_W = 5 \cdot 10^{-5}, \quad \delta_{th} \Gamma_Z = 0.4 \text{ MeV}, \quad \delta_{th} \sigma^0_{\text{had}} = 6 \text{ pb}, \\
\delta_{th} R^0_t = 0.006, \quad \delta_{th} R^0_c = 0.00005, \quad \delta_{th} R^0_b = 0.0001. \tag{1}
\end{equation}

These uncertainties are implemented in the fit as nuisance parameters with Gaussian prior distributions. Theoretical uncertainties are still small compared to the experimental ones and, therefore, they have a very small impact on the fit. The same applies to the \textit{parametric} theory uncertainties, obtained by propagating the experimental errors of the SM inputs into the predictions for the EWPO. The breakdown of these parametric errors is detailed in Table [III] except for the contributions coming from the uncertainty in $m_H$, which, even in the conservative scenario, are numerically irrelevant in the total parametric uncertainty.

For each observable, we give in Tables [I] and [II] the experimental information used as input (\textit{Measurement}), together with the output of the combined fit (\textit{Posterior}), and the \textit{Prediction} of the same quantity. The latter is obtained from the posterior predictive distribution derived from a combined analysis of all the other quantities. The compatibility of the constraints is then evaluated by sampling the posterior predictive distribution and the experimental one, by

| Prediction | $\alpha_s(M^2_Z)$ | $\Delta\sigma^0_{\text{had}}(M^2_Z)$ | $M_Z$ |
|------------|------------------|----------------------------------|-------|
| $M_W$ [GeV] | $80.3545 \pm 0.0006 \pm 0.0018 \pm 0.0027$ | $0.0027 \pm 0.0042$ | $0.0060 \pm 0.0069$ |
| $\Gamma_W$ [GeV] | $2.34914 \pm 0.00049 \pm 0.00010 \pm 0.00021$ | $0.0010 \pm 0.00056$ | $0.0023 \pm 0.00060$ |
| $\Delta \theta_{\text{eff}}^0$ [nb] | $41.4929 \pm 0.0049 \pm 0.0001 \pm 0.0020$ | $0.0003 \pm 0.00053$ | $0.0007 \pm 0.00053$ |
| $\Gamma^0_{\text{FB}}$ | $0.231534 \pm 0.000003 \pm 0.000035 \pm 0.000015$ | $0.000013 \pm 0.000041$ | $0.000030 \pm 0.000048$ |
| $\Delta \theta_{\text{FB}}^{0,b}$ | $0.14692 \pm 0.00003 \pm 0.00028 \pm 0.00012$ | $0.00010 \pm 0.00032$ | $0.00023 \pm 0.00038$ |
| $\Delta A_e$ | $0.66775 \pm 0.00001 \pm 0.00012 \pm 0.00005$ | $0.00005 \pm 0.00014$ | $0.00011 \pm 0.00017$ |
| $\Delta A_b$ | $0.934727 \pm 0.000003 \pm 0.000023 \pm 0.000010$ | $0.000003 \pm 0.000025$ | $0.000007 \pm 0.000026$ |
| $\Delta A_{FB}^{0,b}$ | $0.016191 \pm 0.000006 \pm 0.000060 \pm 0.000026$ | $0.000023 \pm 0.000070$ | $0.000052 \pm 0.000084$ |
| $\Delta A_{FB}^{0,c}$ | $0.07358 \pm 0.00001 \pm 0.00015 \pm 0.00006$ | $0.00006 \pm 0.00018$ | $0.00013 \pm 0.00021$ |
| $\Delta A_{FB}^{0,t}$ | $0.10300 \pm 0.00002 \pm 0.00020 \pm 0.00008$ | $0.00007 \pm 0.00023$ | $0.00016 \pm 0.00027$ |
| $R^0_t$ | $20.7464 \pm 0.0062 \pm 0.0006 \pm 0.0003$ | $0.0002 \pm 0.00063$ | $0.0004 \pm 0.00063$ |
| $R^0_c$ | $0.172198 \pm 0.000020 \pm 0.000002 \pm 0.000001$ | $0.000005 \pm 0.000020$ | $0.000011 \pm 0.000023$ |
| $R^0_b$ | $0.215880 \pm 0.000011 \pm 0.000001 \pm 0.000000$ | $0.000015 \pm 0.000019$ | $0.000034 \pm 0.000035$ |
| $\text{BR}_{W \rightarrow \ell\nu}$ | $0.104386 \pm 0.000024 \pm 0.000000 \pm 0.000000$ | $0.000000 \pm 0.000024$ | $0.000000 \pm 0.000024$ |
| $A_s$ | $0.935637 \pm 0.000002 \pm 0.000022 \pm 0.000010$ | $0.000009 \pm 0.000026$ | $0.000020 \pm 0.000031$ |
| $R_{uc}$ | $0.172220 \pm 0.000019 \pm 0.000002 \pm 0.000001$ | $0.000005 \pm 0.000020$ | $0.000011 \pm 0.000023$ |
FIG. 1. 1D pulls between the observed experimental values and the SM predictions (indirect determinations) for the different EWPO (SM input parameters) considered in the fit, for the standard scenario. (The different colors in the figure are simply used to distinguish the SM inputs [orange], charged-current observables [green] and neutral-current observables [blue].) Each prediction is obtained removing the corresponding observable from the fit. The transparent bars represent the corresponding nD pulls for groups of correlated observables. See text for details.

constructing the probability density function (p.d.f.) of the residuals \( p(x) \), and by computing the integral of the p.d.f. in the region \( p(x) < p(0) \). This two-sided \( p - value \) is then converted to the equivalent number of standard deviations for a Gaussian distribution. In the case of a Gaussian posterior predictive distribution, this quantity coincides with the usual pull defined as the difference between the central values of the two distributions divided by the sum in quadrature of the residual mean square of the distributions themselves. The advantage of this approach is that no approximation is made on the shape of p.d.f.’s. These 1D pulls are also shown in Figure 1. We can see a clear consistency between the measurement of all EWPO and their SM predictions. Only \( A_{0,b}^{0,b} \) shows some tension (at the 2\( \sigma \) level), which should be considered in investigating new physics but also treated with care given the large number of observables considered in the EW fit (see the discussion below for a quantitative assessment of the global significance taking the look-elsewhere effect into account).

Moreover, when interpreting this 1D pull one needs to take into account that \( A_{0,b}^{0,b} \) is actually part of a set of experimentally correlated observables. In order to check the consistency between SM and experiments in this case, one can define an nD pull by removing from the fit one set of correlated observables at a time and computing the prediction for the set of observables together with their covariance matrix. Then the same procedure described for 1D pulls can be carried out, this time sampling the posterior predictive and experimental n-dimensional p.d.f.’s. This nD pull is shown in the last column in Tables I and II, as well as in Figure 1, in which case we see that the global pull for the set of correlated observables involving \( A_{0,b}^{0,b} \) is reduced to 1.3\( \sigma \). To get an idea of the agreement between the SM
TABLE IV. Results of the full indirect determination of SM parameters using only EWPD (third column) and of the full prediction for EWPO using only information on SM parameters (fourth column). For comparison, the input values are reported in the second column. See the text for details.

| Measurement | Full Indirect Pull | Full Prediction Pull |
|-------------|--------------------|---------------------|
| $\alpha_s(M_Z^2)$ | $0.1177 \pm 0.0010$ | $0.1217 \pm 0.0046$ | $-0.8$ |
| $\Delta \Gamma_{\text{had}}(M_Z^2)$ | $0.02766 \pm 0.00010$ | $0.02752 \pm 0.00066$ | $0.2$ |
| $M_Z$ [GeV] | $91.1875 \pm 0.0021$ | $91.200 \pm 0.039$ | $-0.3$ |
| $m_t$ [GeV] | $172.58 \pm 0.45$ | $180.1 \pm 9.6$ | $-0.8$ |
| $m_H$ [GeV] | $125.21 \pm 0.12$ | $196.2 \pm 89.9$ | $-0.4$ |
| $m_W$ [GeV] | $80.379 \pm 0.012$ | $80.379 \pm 0.012$ | $0.0$ |
| $\Gamma_W$ [GeV] | $2.085 \pm 0.042$ | $2.0916 \pm 0.0023$ | $-0.1$ |
| $\sin^2\theta_{\text{eff}}(G_{\mu})$ | $0.2324 \pm 0.0012$ | $0.23147 \pm 0.00014$ | $0.8$ |
| $P_{\text{full}} = A_t$ | $0.1465 \pm 0.0033$ | $0.1474 \pm 0.0011$ | $0.5$ |
| $\Gamma_{\text{eff}}$ [GeV] | $2.4955 \pm 0.0023$ | $2.4947 \pm 0.0020$ | $0.3$ |
| $\sigma^0_{\mu}[\text{fb}]$ | $41.480 \pm 0.033$ | $41.466 \pm 0.031$ | $0.3$ |
| $R^0_{\mu}$ | $20.767 \pm 0.025$ | $20.765 \pm 0.022$ | $0.1$ |
| $A_{\mu}^{0,\ell}$ | $0.0171 \pm 0.0010$ | $0.01630 \pm 0.00024$ | $0.8$ |
| $A_t$ (SLD) | $0.1513 \pm 0.0021$ | $0.1474 \pm 0.0011$ | $1.6$ |
| $R^0_{\tau}$ | $0.21629 \pm 0.00066$ | $0.21562 \pm 0.00035$ | $0.9$ |
| $R^0_{\mu}$ | $0.1721 \pm 0.0030$ | $0.17233 \pm 0.00017$ | $-0.1$ |
| $A_{\mu}^{0,\ell}$ | $0.0996 \pm 0.0016$ | $0.10334 \pm 0.00077$ | $-2.1$ |
| $A_{\mu}^{0,\ell}$ | $0.0707 \pm 0.0035$ | $0.07386 \pm 0.00059$ | $-0.9$ |
| $A_{\mu}$ | $0.923 \pm 0.020$ | $0.93468 \pm 0.00016$ | $-0.6$ |
| $A_{\ell}$ | $0.670 \pm 0.027$ | $0.66805 \pm 0.00048$ | $0.1$ |
| $A_{\mu}$ | $0.895 \pm 0.091$ | $0.935693 \pm 0.000088$ | $-0.4$ |
| $\text{BR}_{W \rightarrow \ell\ell}$ | $0.10860 \pm 0.00090$ | $0.10829 \pm 0.00011$ | $0.3$ |
| $\sin^2\theta_{\text{eff}}(\text{HC})$ | $0.23143 \pm 0.00025$ | $0.23147 \pm 0.00014$ | $-0.1$ |
| $R_{\text{ac}}$ | $0.1660 \pm 0.0090$ | $0.17236 \pm 0.00017$ | $-0.7$ |

and EWPD, it is useful to consider the distribution of the $p$-values corresponding to the 1D pulls for the individual measurements. For purely statistical fluctuations, one expects the $p$-values to be uniformly distributed between 0 and 1. From the results in Tables I and II, we obtain in both scenarios an average $p$-value of 0.5 with $\sigma = 0.3$, fully

FIG. 2. Impact of various constraints in the $m_t$ vs. $M_W$ (left) and $\sin^2\theta_{\text{eff}}$ vs. $M_W$ (right) planes. Dark (light) regions correspond to 68% (95%) probability ranges.
parameters. Conversely, one can obtain a Full Prediction by dropping all experimental information on EWPO and just using the SM and the information on SM predictions. The transition between these approximated expressions.

The small “bump” in the posterior predictive of the $m_H$ figure arises as a result of the tension on $M_W$ when repeating the fit, as can be seen from the left panel of Fig. 2, where the impact of different constraints in shaping the two-dimensional p.d.f.’s of $\Delta\alpha_H (M_W^2)$ and $m_H$ are only mildly decreased. One would need to go beyond the SM to improve the agreement further.

The p.d.f.’s for the SM input parameters are reported in Fig. 3, together with the posterior from the fit, the indirect determination and the full indirect one. While direct measurements are more precise than indirect determinations (by orders of magnitude in the case of the Higgs mass), all indirect determinations are compatible with the measurements within 2$\sigma$. Our indirect determination of $\Delta\alpha_H (M_Z^2)$ fully agrees with the independent one recently obtained in Ref. [73] using the Gfitter library [74]. The pull between the determination of $\Delta\alpha_H (M_Z^2)$ based on the BMW lattice calculation [59] and the indirect determination is of 1.3$\sigma$ (1.1$\sigma$) in the standard (conservative) scenario, showing no evidence for new physics.

FIG. 3. Comparison among the direct measurement, the posterior, the posterior predictive (or indirect) probability distribution (denoted by ”Prediction”), and the full indirect determination of the input parameters in the SM fit. The posterior predictive and the full indirect determination distributions are obtained from the fit by assuming a flat prior for the parameter under consideration or for all SM parameters respectively. To allow for a comparison with the posterior predictive distribution, the full indirect p.d.f for the Higgs mass is truncated in the figure. Dark (light) regions correspond to 68% (95%) probability ranges. HEPfit uses different semi-analytical approximations to the full available calculations of the EWPD, depending the range of variation of the SM inputs, see e.g. [31]. The small “bump” in the posterior predictive of the $m_H$ figure arises as a result of the transition between these approximated expressions.
inconsistency between the current lattice evaluation and the EW fit. It will be very interesting to see if the good agreement between the lattice determination and the EW fit persists when the updated lattice value corresponding to the value of the hadronic vacuum polarization recently published in Ref. [19] is released. The indirect determination of the top mass is also compatible with the measurement at less than 2σ, but on the larger side, bringing the SM further away from the Planck stability bound (see e.g. Ref. [76]).

In conclusion, EWPD appear to be fully compatible with the SM, with no more tensions than expected from statistical fluctuations. In the standard scenario SM fit, the largest pull neglecting correlations is 2.2σ on 24 observables, while taking correlations into account it is 1.8σ on 14 observables. In both the full indirect and full prediction determinations, the largest pull neglecting correlations is 2.1σ on 24 observables. To quantify further the agreement of the SM, we generated 600 toy experiments centered on the full prediction with the current experimental uncertainty and computed the fraction of toys in which the largest pull was larger than the largest one observed in real data. This fraction is an estimate of the global p-value. Neglecting correlations, we obtain p = 0.53, corresponding to 0.6σ for a Gaussian distribution, while taking into account the correlations (fixed to the values observed in current data) we get p = 0.45, corresponding to 0.8σ.

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