Hawking Radiation of a Non-stationary Kerr-Newman Black Hole: Spin-Rotation Coupling Effect

S. Q. Wu* and X. Cai†

Institute of Particle Physics, Hua-Zhong Normal University, Wuhan 430079, P.R. China

Hawking evaporation of Klein-Gordon and Dirac particles in a non-stationary Kerr-Newman space-time is investigated by using a method of generalized tortoise coordinate transformation. The location and the temperature of the event horizon of a non-stationary Kerr-Newman black hole are derived. It is shown that the temperature and the shape of the event horizon depend not only on the time but also on the angle. However, the Fermionic spectrum of Dirac particles displays a new spin-rotation coupling effect which is absent from that of Bosonic distribution of scalar particles. The character of this effect is its obvious dependence on different helicity states of particles spin-1/2.

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I. INTRODUCTION

The fourth quarter of last century has witnessed various remarkable progress on several researches on black hole physics since Hawking’s remarkable discovery [1]. One of these aspects is to reveal the thermal properties of many kinds of black holes by miscellaneous methods ([2,3] and references therein). Much efforts have been devoted to the thermal radiation of scalar and Dirac fields in some static or stationary black holes [2–6]. In the case of a non-stationary axisymmetric black hole, many works have been done on the radiation of scalar particles [3,7]. Though Hawking effect of Dirac particles has been investigated in the non-static case [8], it is difficult to deal with the Hawking evaporation of Dirac particles in a non-stationary axisymmetric black hole. The difficulty lies in the non-separability of the radial and angular variables for Chandrasekhar-Dirac equations [9] in the non-stationary axisymmetry space-time. Recently this dilemma has been attacked by us [10] through considering simultaneously the asymptotic behaviors of the first-order and second-order forms of Dirac equation near the event horizon. A new term representing the interaction between the spin of Dirac particles and the angular momentum of evaporating Kerr black holes was

*E-mail: sqwu@iopp.ccnu.edu.cn
†E-mail: xcai@ccnu.edu.cn
observed in the thermal radiation spectrum of Dirac particles. The character of this spin-
rotation coupling effect is its obvious dependence on different helicity states of particles
with spin-1/2. This effect disappears [11] when the space-time degenerates to a spherically
symmetric black hole of Vaidya-type. It should be noted that this term displayed in the
Fermi-Dirac spectrum is absent in the Bose-Einstein distribution of Klein-Gordon particles.

In this paper, we extend the method developed in Ref. [10] to tackle with the thermal
radiation of Klein-Gordon and Dirac particles in a non-stationary Kerr-Newman space-time.
This analysis is of theoretical interest under current consideration. It is shown that the
location and the temperature of the event horizon depend on the time and the angle. The
Fermionic spectrum of Dirac particles displays a spin-rotation coupling effect due to the
interaction between the particles with spin-1/2 and the black holes with rotation.

The paper is outlined as follows: In Sec. 2, the location of the event horizon of a non-
stationary Kerr-Newman black hole is derived by using the method of generalized tortoise
coordinate transformation. Then Klein-Gordon equation of scalar particles and Dirac equa-
tion of spinor fields are manipulated in Sec. 3 and Sec. 4, respectively. In Sec. 5, both
equations for massive particles are recast into a standard wave equation near the event hori-
zon, and the “surface gravity” of the event horizon is obtained. Sec. 6 is devoted to derive
the thermal radiation spectra of scalar and spinor particles. In Sec. 7, we present some
discussions about the spin-rotation coupling effect.

II. GENERALIZED TORTOISE COORDINATE TRANSFORMATION METHOD

The metric of a non-stationary Kerr-Newman black hole [12,13] can be written in the
advanced Eddington-Finkelstein system as

\[ ds^2 = \Delta - a^2 \sin^2 \theta dv^2 + 2\frac{r^2 + a^2 - \Delta}{\Sigma} a \sin^2 \theta dv d\varphi - 2dvdr \\
+ 2a \sin^2 \theta dr d\varphi - \Sigma d\theta^2 - \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\varphi^2, \]  

(1)

where \( \Delta = r^2 - 2M(v)r + Q^2(v) + a^2 \), \( \Sigma = r^2 + a^2 \cos^2 \theta = \rho^* \rho, \rho^* = r + ia \cos \theta \), \( \rho = r - ia \cos \theta \),
and \( v \) is the standard advanced time. Both the mass \( M \) and the charge \( Q \) of the hole depend
on the time \( v \), but the specific angular momentum \( a \) is a constant. The metric (1) and the
electro-magnetic potential

\[ A = \frac{Q(v)r}{\Sigma} (dv - a \sin^2 \theta d\varphi), \]  

(2)

is shown by Xu [14] to be an exact solution of the Einstein-Maxwell equations.

The line element (1) of an evaporating Kerr-Newman black hole is a natural non-
stationary generalization of the stationary Kerr-Newman solution, but it is of Petrov
type-II, whereas the latter is of Petrov type-D. The geometry of this space-time is char-
acterized by three kinds of surfaces of particular interest: the apparent horizons \( r^\pm_{AH} = \)
\[ M \pm (M^2 - Q^2 - a^2)^{1/2}, \text{ the timelike limit surfaces } r_{\pm LS}^+ = M \pm (M^2 - Q^2 - a^2 \cos^2 \theta)^{1/2}, \]
and the event horizons \[ r_{\pm EH} = r_H. \]

The event horizon is necessarily a null-surface \( r = r(v, \theta) \) that satisfies the null-surface conditions \( g^{ij} \partial_i F \partial_j F = 0 \) and \( F(v, r, \theta) = 0. \) An effective method to determine the location and the temperature of the event horizon of a dynamic black hole is called the generalized tortoise coordinate transformation (GTCT) which can give simultaneously the exact values both of the location and of the temperature of the event horizon of a non-stationary black hole. Basically, this method is to reduce Klein-Gordon or Dirac equation in a known black hole background to a standard wave equation near the event horizon by generalizing the common tortoise-type coordinate \( r_* = r + (2\kappa)^{-1} \ln (r - r_H) \) in a static or stationary space-time \[ \text{[13]} \] (where \( \kappa \) is the surface gravity of the studied event horizon) to a similar form in a non-static or non-stationary space-time and by allowing the location of the event horizon \( r_H \) to be a function of the advanced time \( v = t + r_* \) and/or the angles \( \theta, \varphi. \)

As the space-time under consideration is symmetric about \( \varphi \)-axis, one can introduce the following generalized tortoise coordinate transformation (GTCT) \[ \text{[10]} \]

\[ r_* = r + \frac{1}{2\kappa(v_0, \theta_0)} \ln[r - r_H(v, \theta)], \]
\[ v_* = v - v_0 , \quad \theta_* = \theta - \theta_0 , \quad (3) \]

where \( r_H = r(v, \theta) \) is the location of event horizon, and \( \kappa \) is an adjustable parameter. All parameters \( \kappa, v_0 \) and \( \theta_0 \) characterize the initial state of the hole and are constant under the tortoise transformation.

Applying the GTCT of Eq. \[ (3) \] to the null surface equation \( g^{ij} \partial_i F \partial_j F = 0 \) and then taking the \( r \to r_H(v_0, \theta_0), v \to v_0 \) and \( \theta \to \theta_0 \) limits, we arrive at

\[ \left[ \Delta_H - 2(r_H^2 + a^2)\dot{r}_H + a^2 \sin^2 \theta_0 \ddot{r}_H^2 + r_H^2 \right] \left( \frac{\partial F}{\partial r_*} \right)^2 = 0 , \quad (4) \]

in which the vanishing of the coefficient in the square bracket can give the following equation to determine the location of the event horizon of an evaporating Kerr-Newman black hole

\[ \Delta_H - 2(r_H^2 + a^2)\dot{r}_H + a^2 \sin^2 \theta_0 \ddot{r}_H^2 + r_H^2 = 0 , \quad (5) \]

where we denote \( \Delta_H = r_H^2 - 2Mr_H - Q^2 + a^2. \) The quantities \( \dot{r}_H = \partial r_H / \partial v \) and \( \ddot{r}_H = \partial^2 r_H / \partial \theta \) depict the change of the event horizon in the advanced time and with the angle, which reflect the presence of quantum ergosphere near the event horizon. Eq. \[ (3) \] means that the location of the event horizon is shown as

\[ r_H = \frac{M}{1 - 2\dot{r}_H} \pm \left[ \frac{M^2}{(1 - 2\dot{r}_H)^2} - \frac{Q^2 + a^2 \sin^2 \theta_0 \ddot{r}_H^2 + r_H^2}{1 - 2\dot{r}_H} - a^2 \right]^{1/2} . \quad (6) \]

The plus (minus) sign corresponds to an outer (inner) event horizon.
III. KLEIN-GORDON EQUATION

In this section, we will consider the asymptotic behavior of minimally electro-magnetic coupling Klein-Gordon equation near the event horizon. The explicit form of wave equation describing the dynamic behavior of scalar particles with mass $\mu_0$ and charge $q$

$$\frac{1}{\sqrt{-g}}(\partial_k + iqA_k)[\sqrt{-g}g^{kj}(\partial_j + iqA_j)\Phi] + \mu_0^2\Phi = 0,$$

in the above space-time $[\mathbb{I}]$ is

$$[\Delta \frac{\partial^2}{\partial r^2} + 2(r^2 + a^2)\frac{\partial^2}{\partial v\partial r} + 2a\frac{\partial^2}{\partial r\partial\varphi} + 2a\frac{\partial^2}{\partial v\partial\varphi} + \frac{\partial^2}{\partial \theta^2}$$

$$+ \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial \varphi^2} + a^2 \sin^2\theta \frac{\partial^2}{\partial v^2} + \cot\theta \frac{\partial}{\partial \theta} + 2r \frac{\partial}{\partial v}$$

$$+ 2(r - M + iqQr)\frac{\partial}{\partial r} + iqQ - \mu_0^2\Sigma] \Phi = 0.$$ (8)

Under the GTCT $[\mathbb{II}]$, it can be transformed into

$$\left[\frac{r_H}{\kappa}(1 - 2\dot{r}_H) - M + 2\Delta_H - 2r_H(r_H^2 + a^2)\right] \frac{\partial^2}{\partial r_*^2} \Phi - 2r'_H \frac{\partial}{\partial r_*} \frac{\partial}{\partial \theta_*} \Phi$$

$$+ 2a(1 - \dot{r}_H) \frac{\partial}{\partial r_*} \frac{\partial}{\partial \varphi} \Phi + 2(r_H^2 + a^2 - \dot{r}_Ha^2 \sin^2\theta_0) \frac{\partial^2}{\partial r_* \partial v_*} \Phi$$

$$- (-2r_H\dot{r}_H - 2iqQr_H + r'_H \cot\theta_0 + r''_H + \dot{r}_Ha^2 \sin^2\theta_0) \frac{\partial}{\partial r_*} \Phi = 0.$$ (9)

In deriving Eq. $[\mathbb{III}]$, we have made use of the event horizon equation $[\mathbb{V}]$ to deal with an infinite form of 0/0-type to obtain a finite value

$$\lim_{r\to r_H} \frac{\Delta - 2(r^2 + a^2)r_H + a^2 \sin^2\theta r_H^2 + r_H^2}{r - r_H} = 2(r_H - M) - 4r_H\dot{r}_H.$$ (10)

By adjusting the parameter $\kappa$, Eq. $[\mathbb{III}]$ can be reduced to a standard wave equation near the event horizon. However, we leave it to section 5.

IV. DIRAC EQUATION

To write out the explicit form of Dirac equation in the Newman-Penrose (NP) $[\mathbb{II}]$ formalism, we establish the following complex null-tetrad system that satisfies the orthogonal conditions $l \cdot n = -m \cdot \overline{m} = 1$

$$l = dv - a \sin^2\theta d\varphi,$$  
$$n = \frac{\Delta}{2\Sigma}(dv - a \sin^2\theta d\varphi) - dr,$$

$$m = \frac{1}{\sqrt{2\rho}}\left\{i \sin\theta [adv - (r^2 + a^2)d\varphi] - \Sigma d\theta\right\},$$  
$$\overline{m} = \frac{1}{\sqrt{2\rho}}\left\{-i \sin\theta [adv - (r^2 + a^2)d\varphi] - \Sigma d\theta\right\}. (11)$$
and obtain the corresponding directional derivatives

\[ D = -\frac{\partial}{\partial r}, \quad \Delta = \frac{r^2 + a^2}{\Sigma} \frac{\partial}{\partial v} + \frac{\Delta}{2 \Sigma} \frac{\partial}{\partial r} + \frac{a}{\Sigma} \frac{\partial}{\partial \phi}, \]

\[ \delta = \frac{1}{\sqrt{2\rho}} (ia \sin \theta \frac{\partial}{\partial v} + \frac{\partial}{\partial \theta} + \frac{i}{\sin \theta} \frac{\partial}{\partial \phi}), \]

\[ \bar{\delta} = \frac{1}{\sqrt{2\rho}} (-ia \sin \theta \frac{\partial}{\partial v} + \frac{\partial}{\partial \theta} - \frac{i}{\sin \theta} \frac{\partial}{\partial \phi}). \] (12)

It is of no difficulty to calculate the non-vanishing NP spin coefficients of the non-stationary Kerr-Newman space-time in the above null-tetrad as follows

\[ \tilde{\rho} = \frac{1}{\rho^*}, \quad \epsilon = -\frac{ia \cos \theta}{\Sigma}, \quad \gamma = \frac{r\Delta}{2\Sigma^2} - \frac{r - M}{2\Sigma}, \]

\[ \tilde{\mu} = \frac{\Delta}{2\Sigma \rho^*}, \quad \tau = \frac{ia \sin \theta}{\sqrt{2\rho^*}^2}, \quad \tilde{\nu} = \frac{(Mr - \dot{Q} \dot{Q})ia \sin \theta}{\sqrt{2\Sigma \rho}}, \]

\[ \alpha = \tilde{\pi} - \beta^*, \quad \tilde{\pi} = -\frac{ia \sin \theta}{\sqrt{2\Sigma}}, \quad \beta = \frac{\cot \theta}{2\sqrt{2\rho^*}} + \frac{ira \sin \theta}{\sqrt{2\Sigma \rho^*}}. \] (13)

Inserting for the null-tetrad components of electro-magnetic potential

\[ A \cdot n = \frac{Qr}{\Sigma}, \quad A \cdot l = A \cdot m = -A \cdot \mathbf{m} = 0, \] (14)

and the needed spin coefficients into the four coupled Chandrasekhar-Dirac equations [9] in the Newman-Penrose formalism

\[ (D + \epsilon - \tilde{\rho} + iqA \cdot l)F_1 + (\bar{\delta} + \tilde{\pi} - \alpha + iqA \cdot \mathbf{m})F_2 = \frac{i\mu_0}{\sqrt{2}} G_1, \]

\[ (\Delta + \mu - \gamma + iqA \cdot n)F_2 + (\delta + \beta - \tau + iqA \cdot \mathbf{m})F_1 = \frac{i\mu_0}{\sqrt{2}} G_2, \]

\[ (D + \epsilon^* - \tilde{\rho}^* + iqA \cdot l)G_2 - (\bar{\delta} + \tilde{\pi}^* - \alpha^* + iqA \cdot \mathbf{m})G_1 = \frac{i\mu_0}{\sqrt{2}} F_2, \]

\[ (\Delta + \mu^* - \gamma^* + iqA \cdot n)G_1 - (\bar{\delta} + \beta^* - \tau^* + iqA \cdot \mathbf{m})G_2 = \frac{i\mu_0}{\sqrt{2}} F_1, \] (15)

where \( \mu_0, q \) are the mass and charge of the Dirac particles, respectively, we can get

\[ -\left( \frac{\partial}{\partial r} + \frac{r}{\Sigma} \right) F_1 + \frac{1}{\sqrt{2\rho}} \left( \mathcal{L} - \frac{ira \sin \theta}{\Sigma} \right) F_2 = \frac{i\mu_0}{\sqrt{2}} G_1, \]

\[ \frac{\Delta}{2\Sigma} \left( D - \frac{ia \cos \theta}{\Sigma} \right) F_2 + \frac{1}{\sqrt{2\rho^*}} \left( \mathcal{L}^\dagger - \frac{a^2 \sin \theta \cos \theta}{\Sigma} \right) F_1 = \frac{i\mu_0}{\sqrt{2}} G_2, \]

\[ -\left( \frac{\partial}{\partial r} + \frac{r}{\Sigma} \right) G_2 - \frac{1}{\sqrt{2\rho^*}} \left( \mathcal{L}^\dagger + \frac{ira \sin \theta}{\Sigma} \right) G_1 = \frac{i\mu_0}{\sqrt{2}} F_2, \]

\[ \frac{\Delta}{2\Sigma} \left( D + \frac{ia \cos \theta}{\Sigma} \right) G_1 - \frac{1}{\sqrt{2\rho}} \left( \mathcal{L} - \frac{a^2 \sin \theta \cos \theta}{\Sigma} \right) G_2 = \frac{i\mu_0}{\sqrt{2}} F_1. \] (16)
here we have defined operators

\[ \mathcal{D} = \frac{\partial}{\partial r} + \Delta^{-1} \left[ r - M + 2i q Q r + 2a \frac{\partial}{\partial \phi} + 2(r^2 + a^2) \frac{\partial}{\partial v} \right], \]

\[ \mathcal{L} = \frac{\partial}{\partial \theta} + \frac{1}{2} \cot \theta \frac{\partial}{\partial \phi} - \frac{i}{\sin \theta} \frac{\partial}{\partial \theta} - i a \sin \theta \frac{\partial}{\partial v}, \]

\[ \mathcal{L}^\dagger = \frac{\partial}{\partial \theta} + \frac{1}{2} \cot \theta \frac{\partial}{\partial \phi} + i a \sin \theta \frac{\partial}{\partial v}. \]

By substituting \( F_1 = \frac{1}{\sqrt{2\Sigma}} P_1, \) \( F_2 = \frac{\rho}{\sqrt{2\Sigma}} P_2, \) \( G_1 = \frac{\rho^*}{\sqrt{2\Sigma}} Q_1 \) and \( G_2 = \frac{1}{\sqrt{2\Sigma}} Q_2 \) into Eq. (16), we have

\[ -\frac{\partial}{\partial r} P_1 + \mathcal{L} P_2 = i \mu_0 \rho^* Q_1, \quad \Delta \mathcal{D} P_2 + \mathcal{L}^\dagger P_1 = i \mu_0 \rho^* Q_2, \]

\[ -\frac{\partial}{\partial r} Q_2 - \mathcal{L}^\dagger Q_1 = i \mu_0 \rho P_2, \quad \Delta \mathcal{D} Q_1 - \mathcal{L} Q_2 = i \mu_0 \rho P_1. \]  

(17)

An apparent fact is that the Chandrasekhar-Dirac equations (17) could be satisfied by setting

\[ Q_1 \rightarrow P_2^*, \quad Q_2 \rightarrow -P_1^*, \quad q Q \rightarrow -q^* Q. \]

(18)

So one may deal with a pair of components \( P_1, P_2 \) only. Eq. (17) can not be decoupled except in the stationary Kerr-Newman black hole case \((M = \text{const})\) or in the spherical symmetry Vaidya-Bonner case \((a = 0)\). However, to deal with the problem of Hawking radiation, one should be concerned about the asymptotic behavior of Eq. (17) near the horizon only.

First let us consider the limiting form of Eq. (17) near the event horizon. Under the transformations (3), Eq. (17) can be reduced to the following forms

\[ -\left( r'_H + i a \sin \theta_0 \hat{r}_H \right) \frac{\partial}{\partial r_*} P_1 + \left[ \Delta_H - 2(r^2_H + a^2) \hat{r}_H \right] \frac{\partial}{\partial r_*} P_2 = 0, \]

\[ \frac{\partial}{\partial r_*} P_1 + \left( r'_H - i a \sin \theta_0 \hat{r}_H \right) \frac{\partial}{\partial r_*} P_2 = 0, \]  

(19)

after being taken limits \( r \rightarrow r_H(v_0, \theta_0), \) \( v \rightarrow v_0 \) and \( \theta \rightarrow \theta_0. \) It is interesting to note that a similar form holds for \( Q_1, Q_2 \) also.

If the derivatives \( \frac{\partial}{\partial r_*} P_1 \) and \( \frac{\partial}{\partial r_*} P_2 \) in Eq. (19) are not equal to zero, the existence condition of non-trial solutions for \( P_1 \) and \( P_2 \) is that the determinant of Eq. (19) vanishes, which gives exactly the event horizon equation (3). The relations (19) play an important role to eliminate the crossing-term of the first-order derivatives in the second-order equation. It is consistent to consider the asymptotic behavior of the first-order and second-order Dirac equations in the meanwhile because the four-components Dirac spinors should satisfy both of them.

Next we turn to the second-order form of Dirac equations. A direct calculation gives
\[
\left(\Delta D \frac{\partial}{\partial r} + \mathcal{L}^\dagger - \mu_0^2 \Sigma \right) P_1 = \mu_0 (a \sin \theta Q_2 - i \Delta Q_1)
\]
\[-ia \sin \theta \left[ (2Mr - Q\dot{Q}) \frac{\partial}{\partial r} + \dot{M} - 2iQ\dot{r} \right] P_2 , \]
\[
\left( \frac{\partial}{\partial r} \Delta D + \mathcal{L}^\dagger \mathcal{L} - \mu_0^2 \Sigma \right) P_2 = \mu_0 (a \sin \theta Q_1 + iQ_2) .
\]  

(20)

Given the GTCT in Eq. (3) and after some lengthy calculations, the limiting form of Eq. (20), when \( r \) approaches \( r_H(v_0, \theta_0) \), \( v \) goes to \( v_0 \) and \( \theta \) goes to \( \theta_0 \), leads

\[
\left\{ \frac{r_H(1 - 2\dot{r}_H) - M}{\kappa} + 2\Delta - 2\dot{r}_H(r_H^2 + a^2) \right\} \frac{\partial^2}{\partial r_*^2} - 2r_H' \frac{\partial^2}{\partial r_* \partial \theta_*} + 2a(1 - \dot{r}_H) \frac{\partial^2}{\partial r_* \partial \varphi} + 2(r_H^2 + a^2 - \dot{r}_H a^2 \sin^2 \theta_0) \frac{\partial^2}{\partial r_* \partial v_*} - (M - 2iQ\dot{r}_H)
\]
\[-r_H - ia \cos \theta_0 \dot{r}_H + r_H' \cot \theta_0 + r''_H + \dot{r}_H a^2 \sin^2 \theta_0 \frac{\partial}{\partial r_*} \right\} P_1 = -2i(\dot{M} r_H - Q\dot{Q}) a \sin \theta_0 \frac{\partial}{\partial r_*} P_2
\]
\[= -2i(\dot{M} r_H - Q\dot{Q}) a \sin \theta_0 \frac{r_H' + ia \sin \theta_0 \dot{r}_H}{\Delta - 2(r_H^2 + a^2) \dot{r}_H \frac{\partial}{\partial r_*}} P_1 ,
\]  

(21)

and

\[
\left\{ \frac{r_H(1 - 2\dot{r}_H) - M}{\kappa} + 2\Delta - 2\dot{r}_H(r_H^2 + a^2) \right\} \frac{\partial^2}{\partial r_*^2} - 2r_H' \frac{\partial^2}{\partial r_* \partial \theta_*} + 2a(1 - \dot{r}_H) \frac{\partial^2}{\partial r_* \partial \varphi} + 2(r_H^2 + a^2 - \dot{r}_H a^2 \sin^2 \theta_0) \frac{\partial^2}{\partial r_* \partial v_*} - (M - 2iQ\dot{r}_H)
\]
\[-r_H - ia \cos \theta_0 \dot{r}_H + r_H' \cot \theta_0 + r''_H + \dot{r}_H a^2 \sin^2 \theta_0 \frac{\partial}{\partial r_*} \right\} P_2 = 0 ,
\]  

(22)

where we have replaced the first-order derivative term \( \frac{\partial}{\partial r_*} P_2 \) in Eq. (21) by using the first expression of relations (19).

**V. HAWKING TEMPERATURE**

In order to reduce Eqs. (4), (21) and (22) to a standard form of wave equation near the event horizon, we adjust the parameter \( \kappa \) such that it satisfies

\[
\frac{r_H(1 - 2\dot{r}_H) - M}{\kappa} + 2\Delta - 2\dot{r}_H(r_H^2 + a^2) = r_H^2 + a^2 - \dot{r}_H a^2 \sin^2 \theta_0 ,
\]  

(23)

which means the “surface gravity” of the horizon is

\[
\kappa = \frac{r_H(1 - 2\dot{r}_H) - M}{(r_H^2 + a^2 - \dot{r}_H a^2 \sin^2 \theta_0)(1 - 2\dot{r}_H) + 2r_H^2} ,
\]  

(24)

7
where we have used Eq. (6).

With such a parameter adjustment, these wave equations can be recast into a combined form near the event horizon as follows

\[
\left[ \frac{\partial^2}{\partial r^2_{*}} + 2\frac{\partial^2}{\partial r_{*} \partial \nu^*} + 2\Omega_H \frac{\partial^2}{\partial r_{*} \partial \varphi} + 2C_3 \frac{\partial^2}{\partial r_{*} \partial \theta^*} \right] + 2(C_2 + iC_1 + iq\Phi_H) \frac{\partial}{\partial r_{*}} \Psi = 0 ,
\]

(25)

where

\[
\Omega_H = \frac{a(1 - \dot{r}_H)}{r_H^2 + a^2 - \dot{r}_Ha^2 \sin^2 \theta_0}, \quad \Phi_H = \frac{Qr_H}{r_H^2 + a^2 - \dot{r}_Ha^2 \sin^2 \theta_0},
\]

\[
C_3 = \frac{-r''_H}{r_H^2 + a^2 - \dot{r}_Ha^2 \sin^2 \theta_0},
\]

while both \(C_1\) and \(C_2\) are real,

\[
C_2 = \frac{-1}{2(r_H^2 + a^2 - \dot{r}_Ha^2 \sin^2 \theta_0)} \left[ r_H(1 - 4\dot{r}_H) - M + r''_H 
+ \dot{r}_H a^2 \sin^2 \theta_0 + r'_H \cot \theta_0 + \frac{2(\dot{M}r_H - \dot{Q}\dot{Q})\dot{r}_H a^2 \sin^2 \theta_0}{\Delta_H - 2\dot{r}_H (r_H^2 + a^2)} \right],
\]

\[
C_1 = \frac{-1}{2(r_H^2 + a^2 - \dot{r}_Ha^2 \sin^2 \theta_0)} \left[ \dot{r}_H a \cos \theta_0 - \frac{2(\dot{M}r_H - \dot{Q}\dot{Q})r'_H a \sin \theta_0}{\Delta_H - 2\dot{r}_H (r_H^2 + a^2)} \right],
\]

for \(\Psi = P_1\),

\[
C_2 = \frac{-M + \dot{r}_H + r''_H + \dot{r}_H a^2 \sin^2 \theta_0 + r'_H \cot \theta_0}{2(r_H^2 + a^2 - \dot{r}_Ha^2 \sin^2 \theta_0)},
\]

\[
C_1 = \frac{\dot{r}_H a \cos \theta_0}{2(r_H^2 + a^2 - \dot{r}_Ha^2 \sin^2 \theta_0)},
\]

for \(\Psi = P_2\), and

\[
C_2 = \frac{-2r_H \dot{r}_H + r'_H \cot \theta_0 + r''_H + \dot{r}_H a^2 \sin^2 \theta_0}{2(r_H^2 + a^2 - \dot{r}_Ha^2 \sin^2 \theta_0)},
\]

\[
C_1 = 0,
\]

for \(\Psi = \Phi\). We point out that the above parameter adjustment is a crucial step to achieve a standard form of wave equation near the event horizon, which can be viewed as an ordinary differential equation because all coefficients in Eq. (25) are regarded as finite real constants.
VI. THERMAL RADIATION SPECTRUM

Eq. (25) can be treated by separating variables as
\[ \Psi = R(r)\Theta(\theta)e^{i(m\varphi - \omega v)} \]
to the radial and angular parts
\[ R'' + 2(C_0 + iC_1 + im\Omega_H + iq\Phi_H - i\omega)R' = 0, \quad \Theta' = \lambda\Theta, \]
where \( \lambda \) is a real constant introduced in the separation of variables, \( C_0 = \lambda C_3 + C_2 \). The solutions are
\[ R = R_1e^{2i(\omega - m\Omega_H - q\Phi_H - C_1)r} + R_0, \quad \Theta = e^{\lambda\theta}. \]

The ingoing wave solution and the outgoing wave solution to Eq. (25), are respectively,
\[ \Psi_{\text{in}} = e^{-i\omega v + im\varphi + \lambda\theta}, \]
\[ \Psi_{\text{out}} = e^{-i\omega v + im\varphi + \lambda\theta}e^{2i(\omega - m\Omega_H - q\Phi_H - C_1)r} - 2C_0r (r > r_H). \]

The outgoing wave \( \Psi_{\text{out}} \) is not analytic at the event horizon \( r = r_H \), but can be analytically continued from the outside of the hole into the inside of the hole by the lower complex \( r \)-plane
\[ (r - r_H) \to (r_H - r)e^{-i\pi} \]
to
\[ \tilde{\Psi}_{\text{out}} = \Psi_{\text{out}}e^{i\pi C_0/\kappa}e^{\pi(\omega - m\Omega_H - q\Phi_H - C_1)/\kappa} (r < r_H). \]

The relative scattering probability at the event horizon is
\[ \left| \frac{\Psi_{\text{out}}}{\tilde{\Psi}_{\text{out}}} \right|^2 = e^{-2\pi(\omega - m\Omega_H - q\Phi_H - C_1)/\kappa}. \]

Following the method of Damour-Ruffini-Sannan’s [13], the Hawking radiation spectra of Klein-Gordon and Dirac particles from the black hole is easily obtained
\[ \langle \mathcal{N}_{\omega} \rangle \sim \frac{1}{e^{(\omega - m\Omega_H - q\Phi_H - C_1)/T} \pm 1}, \quad T = \frac{\kappa}{2\pi}. \]

where \( m \) is the azimuthal quantum number, \( \Omega_H \) and \( \Phi_H \) can be interpreted as the angular velocity and electro-magnetic potential of the event horizon of the evaporating Kerr-Newman black hole, respectively. In Eq. (31), the upper plus symbol corresponds to the Fermi-Dirac distribution, while the lower minus symbol stands for the Bose-Einstein statistics.
VII. SPIN-ROTATION COUPLING EFFECT

The thermal radiation spectra \((31)\) demonstrate that the total interaction energy of particles with spin-\(s\) in an evaporating Kerr-Newman space-time is

\[
\omega_p = \frac{1}{r_H^2 + a^2 - r_H a \sin^2 \theta_0} \left[ ma(1 - \dot{r}_H) - pa \cos \theta_0 \dot{r}_H 
+ qQ r_H + (s + p) \dot{M} r_H \frac{a \sin \theta_0 \dot{r}'_H}{\Delta_H - 2 \dot{r}_H (r_H^2 + a^2)} \right].
\]

(32)

When \(p = s = 0\), it corresponds to the case of scalar fields \(\Psi = \Phi\); in the case of spinor fields \((s = 1/2)\), \(\Psi\) stands for \(P_1, P_2\) when \(p = 1/2, -1/2\), respectively.

The energy spectrum is composed of three parts: \(\omega_p = m \Omega_H + q \Phi_H + C_1\), the first one is the rotational energy \(m \Omega_H\) arising from the coupling of the orbital angular momentum of particles with the rotation of the black hole; the second one is the electro-magnetic interaction energy \(q \Phi_H\); another one is \(C_1\) due to the coupling of the intrinsic spin of particles and the angular momentum of the black hole, it has no classical correspondence. From the explicit expression of the “spin-dependent” term \(C_1\)

\[
C_1 = \frac{\Omega}{1 - \dot{r}_H} \left[ - p \cos \theta_0 \dot{r}_H + (s + p) \dot{M} r_H \frac{\sin \theta_0 \dot{r}'_H}{\Delta_H - 2 \dot{r}_H (r_H^2 + a^2)} \right],
\]

(33)

one can easily find that it vanishes in the case of a stationary Kerr-Newman black hole \((M = \text{const}, \dot{r}_H = r'_H = 0)\) or a Vaidya-type black hole \((a = 0, r'_H = 0, \dot{r}_H \neq 0)\).

The term \(C_1\) is obviously related to the helicity of particles in different spin states, it characterizes a new effect arising from the interaction between the spin of particles and the rotation of an evaporating black hole. Because \(\dot{r}_H\) and \(r'_H\) describe the evolution of the black hole in the time and the change in the direction, we suggest that the radiative mechanism of an evaporating Kerr-Newman black hole can be changed by the quantum rotating ergosphere which can be viewed as a mixture of the classical rotating ergosphere and quantum ergosphere.

VIII. CONCLUSIONS

Equations \((3)\) and \((24)\) give the location and the temperature of event horizon of a non-stationary Kerr-Newman black hole, which depend not only on the advanced time \(v\) but also on the angle \(\theta\). Eq. \((31)\) shows the thermal radiant spectra of Klein-Gordon and Dirac particles in the non-stationary Kerr-Newman space-time. A difference between Bosonic spectrum and Fermionic spectrum appears, that is, a new term \(C_1\) in the latter one is absent from the former one. The new effect probably arise from the interaction between the spin of Dirac particles and the rotation of the evaporating black holes. The feature of this spin-rotation coupling effect is its dependence on different helicity states of particles with spin-1/2 and its irrelevance to the mass of particles.
To summarize, we have dealt with Hawking radiation of Klein-Gordon and Dirac particles in a non-stationary Kerr-Newman black hole. The spectrum of Dirac particles displays another new effect between the spin of the particles and the angular momentum of the hole, which is absent from the spectrum of the Klein-Gordon particles. This effect is due to the coupling of the intrinsic spin of particles with the rotation of the black holes, it vanishes when the space-time becomes a stationary Kerr black hole or a Vaidya-type spherically symmetric black hole. This study encompasses previous ones \cite{10} (when $Q = 0$) and \cite{11} (when $a = 0$) as special cases.

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\[\text{[1] } Hawking, \text{ S. W. (1974). Nature, } 248, 30; \text{ (1975). Commun. Math. Phys. 43, 199.}\]

\[\text{[2] } Frolov, \text{ V. P., and Novikov, I. D. (1998). Black Hole Physics: Basic Concepts and New Developments, (Kluwer Academic Publishers, Dordrecht).}\]

\[\text{[3] } Zhao, \text{ Z. (1999). Thermal Properties of Black Holes and Singularities of Space-times: Quantum Effect near the Null Surface, (Beijing Normal University Press, Beijing, in Chinese).}\]

\[\text{[4] } Hartle, \text{ B., and Hawking, S. W. (1976). Phys. Rev. D 13, 2188; Wald, R. M. (1975). Commun. Math. Phys. 45, 9; Unruh, W. G. (1976). Phys. Rev. D 14, 870; Isreal, W. (1976). Phys. Lett. A 57, 107; Punsly, B. (1992). Phys. Rev. D 46, 1288, 1312; Brout, R., Massar, S., Parentani, R., and Spindel, Ph. (1995). Phys. Rep. 260, 329.}\]

\[\text{[5] } Khanal, \text{ U. (1983). Phys. Rev. D 28, 1291; Khanal, U., and Panchapakesan, N. (1981). Phys. Rev. D 24, 829, 835; Ahmed, M. (1991). Phys. Lett. B 258, 318; Ahmed, M., and Mondal, A. K. (1995). Int. J. Theor. Phys. 34, 1871.}\]

\[\text{[6] } Wu, \text{ S. Q., and Cai, X. (2000). IL Nuovo Cimento B 115, 143; (2000). Int. J. Theor. Phys. 39, 2215.}\]
[7] Zhao, Z., Dai, X. X., and Huang, W. H. (1993). *Acta Astrophysica Sinica*, **13**, 299 (in Chinese); Luo, M. W. (2000). *Acta Physica Sinica*, **49**, 1035 (in Chinese); Jing, J. L., and Wang, Y. J. (1997). *Int. J. Theor. Phys.* **36**, 1745.

[8] Zhao, Z., Yang, C. Q., and Ren, Q. A. (1992). *Gen. Rel. Grav.* **26**, 1055; Li, Z. H., and Zhao, Z. (1993). *Chin. Phys. Lett.* **10**, 126; Zhu, J. Y., Zhang, J. H., and Zhao, Z. (1994). *Int. J. Theor. Phys.* **33**, 2137; Ma, Y., and Yang, S. Z. (1993). *ibid.* **32** (1993) 1237.

[9] Chandrasekhar, S. (1983). *The Mathematical Theory of Black Holes*, (Oxford University Press, New York); Page, D. (1976). *Phys. Rev. D* **14**, 1509.

[10] Wu, S. Q., and Cai, X. (2001). *Chin. Phys. Lett.* **18**, 485; (2001). *Gen. Rel. Grav.* **33**, 1181.

[11] Wu, S. Q. and Cai, X. (2001). *Int. J. Theor. Phys.* **40**, 1349; (2001). *Mod. Phys. Lett. A* **16**, 1549.

[12] Gonzalez, C., Herrera, L., and Jimenez, J., (1979). *J. Math. Phys.* **20**, 837; Jing, J. L., and Wang, Y. J. (1996). *Int. J. Theor. Phys.* **35**, 1481.

[13] Carmeli, M., and Kaye, M. (1977). *Ann. Phys. (NY)* **103**, 97; Carmeli, M. (1982). *Classical Fields: General Relativity and Gauge Theory*, (John Wiley & Sons, New York).

[14] Xu, D. Y., (1998). *Class. Quant. Grav.* **15**, 153; (1998). *Chin. Phys. Lett.* **15**, 706.

[15] Damour, T., and Ruffini, R. (1976). *Phys. Rev. D* **14**, 332; Sannan, S. (1988). *Gen. Rel. Grav.* **20**, 239.

[16] Newman, E., and Penrose, R. (1962). *J. Math. Phys.* **3**, 566.

[17] Bonnor, W., and Vaidya, P. (1970). *Gen. Rel. Grav.* **1**, 127.