Mode of "small" signal of the mathematical model of an electric multipole with memresistive branches under conditions of interval uncertainty

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Abstract. In this article, a mathematical model of an electric multipole with memresistive branches is developed when operating under conditions of interval uncertainty in the mode of a small signal on alternating current.

1. Introduction
The emergence of a wide range of nanoelectronic components in recent years, on the one hand, certainly expands the capabilities of information and computing systems. However, the use of such elements in conditions other than laboratory ones has interval uncertainties in determining the electrical parameters of these devices, which is superimposed on the uncertainty of the entire system as a whole, which complicates the circuit analysis and the entire design process as a whole.

Despite the widespread use of information and computing technologies in all spheres of human activity, the tasks of increasing the performance of modern computing devices without increasing the cost and expanding their operational characteristics for use not only in laboratory and office conditions, but also for stable operation in an extreme environment remain urgent.

A new element of nanoelectronics, the appearance of which was predicted in 1971 [1-7], the memristor was able to change its resistance depending on the charge flowing through it. A change in the properties of a memristor is provided by chemical reactions in a thin two-layer TiO2 film, and in the classification, it belongs to the class of nanoionic devices. The additionally appearing hysteresis phenomenon makes it possible to use it as a memory cell. [5-21].

In connection with the above, it is necessary to take a fresh look at the electric multipole and its mathematical model using the memristor as one of the elements of the basic set. The introduction of such an addition to the basic set of elements also determines the appearance of new memresistive branches of the electric multipole.

In accordance with the adopted classification [21], it is customary to distinguish the following typical operating modes of the investigated electronic devices, which is quite true for the elements of nanoelectronics:

- Quasi-linear mode of a small signal when calculating with an alternating current;
- Quasi-linear small signal mode when calculating in the time domain;
- Large signal mode when calculating for direct current (static mode);
- Large signal mode when calculating in the time domain (dynamic mode).

Depending on the selected mode, the initial model of the investigated device will be an electrical multipole of one of the following types: linear resistive, linear reactive, nonlinear resistive and nonlinear reactive. At the same time, the general methodological basis for studying all the listed types of models is their description in the basis of finite deviations of currents and voltages of the branches of the corresponding multipole, since the main goal of our study is to assess the ability of a device to maintain its characteristics within specified limits in the presence of disturbing influences with undefined properties. This article analyzes the first small signal modes in the above classification for an electric multipole with memresistive branches in finite increments.

2. Materials and methods

Mathematical models of an electric multipole with memresistive branches in the nominal form and using finite increments were built earlier [1-3] using the method of decomposition of branches of the directed graph of the circuit described in [19-22].

A full-size mathematical model in a full hybrid basis in finite increments of currents and voltages will look like this [19]:

\[
\begin{bmatrix}
0 & \hat{L} & \frac{d}{dt} \begin{bmatrix} \Delta U_P^P_C \\ \Delta I_M^P_C \end{bmatrix} \\
\hat{L} & 0 & \frac{d}{dt} \begin{bmatrix} \Delta U_P^P_H \\ \Delta I_M^P_H \end{bmatrix} \\
0 & \frac{d}{dt} \begin{bmatrix} \Delta U_P^P_M \\ \Delta I_M^P_M \end{bmatrix} \\
\end{bmatrix} =
\begin{bmatrix}
Q_1 & \Delta U_P^P_E & \Delta U_P^P_E \\
\Delta U_P^P_E & A_1 & A_2 & A_3 & \Delta U_P^P_E \\
\Delta U_P^P_E & A_2 & A_3 & \Delta U_P^P_E & \Delta U_P^P_E \\
\Delta U_P^P_E & A_3 & \Delta U_P^P_E & \Delta U_P^P_E & \Delta U_P^P_E \\
\end{bmatrix} -
\begin{bmatrix}
H & F \\
F & 0 \\
0 & W \\
W & 0 \\
\end{bmatrix}
\]

where \( \hat{L} = \) the matrix of inductances (\( k = 1, n_{HL}^X \)), \( l = \) the number of branches and \( n_{HL}^X \) is the number of branches in parallel; \( \hat{C} = \) the capacity matrix (\( k = 1, n_{HL}^X \)), \( l = 1, n_{HL}^X, n_{HL}^X + n_{HL}^X = \) the number of branches in parallel; \( \Delta U_P^P_C, \Delta I_M^P_C \) - increments of voltages and currents on inductive branches; \( \Delta U_P^P_H, \Delta I_M^P_H \) - voltage and current increments on the capacitive edges; \( \Delta U_P^P_M, \Delta I_M^P_M \) - voltage and current increments on memresistive edges and chords; \( \Delta U_P^P_M, \Delta I_M^P_M \) - voltage and current increments on nonlinear inductances and capacities; \( D \) - the matrix of the main contours of the directed graph of the circuit; \( n \) is the number of branches and \( k \) is the number of nodes; \( E^E_k = \) equivalent vectors of current and voltage sources; where
\[
A_1 = \begin{bmatrix}
\Delta M^{-1} & Z_X(\Delta I_X^H) \\
G_p(\Delta U_{H}^P) & \Delta M
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
\Delta M^{-1} & Z_X(\Delta I_X^H) \\
G_p(\Delta U_{H}^P) & \Delta M
\end{bmatrix}, \quad A_3 = \begin{bmatrix}
-[B_{V,H}] & 0 \\
0 & -D_{V,H}
\end{bmatrix}
\]

and \( \Delta M \) are diagonal matrices of equivalent inverse and forward memresistivities; \( G \) is the identity matrix; \( E_H^P, E_{V}^H, J_H^P, J_{V}^P, E_H^N, J_H^N, E_{V}^N, J_{V}^N \) - equivalent sources of EMF and current on nonlinear chords and edges; \( G_1 = \begin{bmatrix} B_{X_1} & \hat{R} & 0 \\ \hat{\gamma} & D_{X_1} & 0 \\ 0 & 0 & -D_{X_{II}} \end{bmatrix}^{-1} \), \( G_2 = \begin{bmatrix} B_{X_1} & \hat{R} & 0 \\ \hat{\gamma} & D_{X_1} & 0 \\ 0 & 0 & -D_{X_{II}} \end{bmatrix}^{-1} \), \( G_3 = \begin{bmatrix} B_{X_{II}} & 0 & -D_{X_{II}} \\ 0 & -D_{X_{II}} & 0 \end{bmatrix}^{-1} \), \( W = \begin{bmatrix} -B_{X_{II}} & \hat{\gamma} & D_{X_1} \\ 0 & -D_{X_{II}} & 0 \end{bmatrix}^{-1} \), \( S_1 = \begin{bmatrix} -B_{X_{VI}} & M^{-1} \hat{M} \\ M^{-1} \hat{M} & D_{X_{VI}} \end{bmatrix}^{-1} \), \( S_2 = \begin{bmatrix} -B_{X_{VI}} & M^{-1} \hat{M} \\ M^{-1} \hat{M} & D_{X_{VI}} \end{bmatrix}^{-1} \), \( S_3 = \begin{bmatrix} -B_{X_{VI}} & M^{-1} \hat{M} \\ M^{-1} \hat{M} & D_{X_{VI}} \end{bmatrix}^{-1} \), \( K = \begin{bmatrix} -B_{X_{VI}} & M^{-1} \hat{M} \\ M^{-1} \hat{M} & D_{X_{VI}} \end{bmatrix}^{-1} \), \( \hat{M} = M + \Delta M \) is the matrix of the branch memresistivities, \( \hat{M}^{-1} = M^{-1} + \Delta M^{-1} \) is the matrix of inverse memresistivities of the branch; \( E_N^M, J_N^M \) - equivalent vectors of independent voltage and current sources on memresistive branches.

Mathematical model of a multipole with memresistive branches in the "small" signal mode. The calculation of a multipole in this mode for alternating current is carried out in a certain vicinity of the nominal operating mode of the corresponding electronic devices. The same approach may well be applied to nanoelectronic components of quantum information and computing systems. In this regard, the calculation uses local linear high-frequency models of nanoelectronic devices included in their composition. Replacing the reactive elements of the equivalent circuit with complex resistances or conductivities allows us to consider the obtained equivalent circuits as linear resistive multipoles with dependent and independent sources of current and voltage.

Based on the above, the mathematical model of a multipole with memresistive branches (1) in the "small" signal mode will take the form:

\[
\begin{align*}
\begin{bmatrix}
\Delta U_{H}^P \\
\Delta I_{H}^P
\end{bmatrix} &= Q^T \begin{bmatrix} G_3 & E_N^M & E_N^R \\ J_N^M & J_N^R \end{bmatrix} \begin{bmatrix} E_N^M & E_N^R \\ J_N^M & J_N^R \end{bmatrix} \begin{bmatrix} J_N^M \\
\Delta I_{M}^X
\end{bmatrix} + W \begin{bmatrix} E_N^R \\
J_N^R \end{bmatrix} \\
\end{align*}
\]

(2)

where \( Q = G_3S_3 - O; N = G_3S_3 - H; O \) and \( H \) are unit matrices.

Expression (2) belongs to the class of interval systems of linear algebraic equations, for the solution of which it is permissible to use Kahan's interval arithmetic and the Gauss-Seidel interval method.

The essence of Kahan's interval arithmetic is that operations with intervals containing zero have the same result as in the case of other intervals and allows you to keep the interval expansion of functions unchanged, and also guarantees, under certain conditions, not only the distributiveness of operations, but also inclusion monotony [20; 22].
The basis of the Gauss-Seidel method is an algorithm of sets of iterative procedures to refine the set of solutions. Usually this method is applied after preliminary pre-conditioning of an interval system of algebraic equations [6; 23].

3. Results
Computational algorithm for external estimation of the solution sets of the mathematical model of a multipole with memresistive branches in the "small" signal mode. The method involves solving a system of equations of the form $Ax = b$. Reducing the first equation of system (2) to this form, we obtain: $A = Q; b = G_3K \begin{bmatrix} E_N^M \\ J_N^M \end{bmatrix} + W \begin{bmatrix} E_N^R \\ J_N^R \end{bmatrix}$ and $x = \begin{bmatrix} \Delta U_R^p \\ \Delta l_R^x \end{bmatrix}$. Similarly, for the second equation: $A = N; b = S_3 \begin{bmatrix} E_N^R \\ J_N^R \end{bmatrix} + K \begin{bmatrix} E_N^M \\ J_N^M \end{bmatrix}$ and $y = \begin{bmatrix} \Delta U_M^p \\ \Delta l_M^x \end{bmatrix}$. Choose the bars $x = \begin{bmatrix} \Delta U_{R1}^p, ..., \Delta U_{Rn}^p \\ \Delta l_{R1}^x, ..., \Delta l_{Rn}^x \end{bmatrix}^T$ and $y = \begin{bmatrix} \Delta U_{M1}^p, ..., \Delta U_{Mn}^p \\ \Delta l_{M1}^x, ..., \Delta l_{Mn}^x \end{bmatrix}^T$ for the first and second equations of system (2) respectively.

Using the above new assigned variables, the computational algorithm can be represented as follows:

- Reduction of system (2) to the form $Ax = b$, as shown above;
- Assign the beams $x = (x_1, ..., x_n)^T = \begin{bmatrix} \Delta U_{R1}^p, ..., \Delta U_{Rn}^p \\ \Delta l_{R1}^x, ..., \Delta l_{Rn}^x \end{bmatrix}^T$ and $y = \begin{bmatrix} \Delta U_{M1}^p, ..., \Delta U_{Mn}^p \\ \Delta l_{M1}^x, ..., \Delta l_{Mn}^x \end{bmatrix}^T$ for two equations of system (2), moreover, $x, y \in \mathbb{R}^n$, limiting the desired parts of the combined set of solutions $\Xi_x(A, b)$ and $\Xi_y(A, b)$;
- Let us introduce some constant $\varepsilon > 0$, which determines the practically significant number of possible iterations.
- $d \leftarrow +\infty$;
  - do while $(d \geq \varepsilon)$:
    - do for $i = 1$ to $n$;
      - $\bar{x}_i \leftarrow x_i \cap \left( \frac{b - \sum_{j=i+1}^n a_{ij}x_j - \sum_{j=i}^{n-1} a_{ij}x_i}{a_{ii}} \right)$, where $a_{ij}$ - element of matrix $A$;
      - if $\bar{x}_i = \emptyset$ then stop, signaling "the set of solutions $\Xi_x(A, b)$ does not intersect the bar $x$";
      - end if;
    - end do;
  - $d \leftarrow$ distance between vectors $x$ and $\bar{x} = (x_1, ..., x_n)^T$;
  - $x \leftarrow \bar{x}$;
- end do.

Similarly, the algorithm for solving the second equation of system (2) is implemented for the set of solutions given by the interval bar vector $y = \begin{bmatrix} \Delta U_{M1}^p, ..., \Delta U_{Mn}^p \\ \Delta l_{M1}^x, ..., \Delta l_{Mn}^x \end{bmatrix}^T$.

4. Discussion
The "small signal" mode considered in this article for an electric multipole with memresistive branches confirms the applicability of the chosen approach to the analysis of nano-electronic circuits of both memory elements and the architecture of information-measuring systems as a whole.

5. Conclusion
The form of presentation of the mathematical model for each mode of operation is quite consistent with the forms required for using the methods of interval analysis. Of course, it is necessary to check the adequacy of the presented model of an electric multipole with memresistive branches both in general
form and in dynamic and static modes for real nanoelectronic circuits, which is the subject of further research.

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