Distributed Weighted Least Squares Estimator Based on ADMM

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Abstract—Wireless sensor network has recently received much attention due to its broad applicability and ease-of-installation. This paper is concerned with a distributed state estimation problem, where all sensor nodes are required to achieve a consensus estimation. The weighted least squares (WLS) estimator is an appealing way to handle this problem since it does not need any prior distribution information. To this end, we first exploit the equivalent relation between the information filter and WLS estimator. Then, we establish an optimization problem under the relation coupled with a consensus constraint. Numerical simulation together with theoretical analysis testify the convergence and consensus estimations between nodes.

Index Terms—consensus estimation, distributed, alternating direction method of multipliers, weighted least squares

I. INTRODUCTION

Recently, distributed estimation schemes have gained immense popularity due to their flexibility for large networks and high fault tolerance \cite{1, 2}. Compared with the centralized fashion, it does not need a fusion center. So it is capable of overcoming congestion of massive data. In view of this, distributed estimation has been used in a wide range of domains, e.g., environmental monitoring, surveillance, cooperative target tracking \cite{3}.

The objective of a distributed estimation fashion is to achieve a consensus estimation between nodes using only local noisy measurements and information from one-hop neighbors. To this end, the existing algorithms rely on different consensus strategies, e.g., average consensus (AC) scheme \cite{4}, diffusion strategy \cite{5}, minimizing weighted Kullback-Leibler divergence \cite{6}, and covariance intersection fusion rule \cite{7}. Moreover, Wang et al. constructed an equivalent consensus problem from maximum-a-posterior (MAP) perspective \cite{8}. And then the problem with a consensus constraint is handled via the alternating direction method of multiplier (ADMM) technique. A main restriction in \cite{8} is it require the posterior distribution function w.r.t estimated quantity. However, this requirement cannot always be satisfied as it needs exact prior knowledge to hold a conjugate prior structure.

Different from the aforementioned consensus schemes, this work considers the consensus problem via weighted least squares (WLS) viewpoint \cite{9}. Compared with MAP estimator, a unique requirement for WLS criterion is the first and second moment information about noise (error). Obviously, this merit is appealing because the practical prior knowledge is limited instead of sufficient. Once the prior is not available, the related filters based on MAP will be invalid. To derive a distributed filter within WLS criterion, we first establish an equivalent relation between the information filter and WLS estimator. Next, to guarantee a consensus-based solution between nodes, we construct a novel optimization function and add consensus constraints to force these variables on neighboring nodes to reach an agreement. Finally, the optimization problem is addressed by ADMM technique. Numerical simulations together with theoretical analyses testify the effectiveness of the proposed filter on node consensus and convergence.

II. PROBLEM FORMULATION

We consider a discrete-time linear dynamic system with the state vector $x_k \in \mathbb{R}^m$. At each scan time $k$, $x_k$ is observed over the sensor network $\mathcal{G} = \{J, A\}$, where $J := \{1, \ldots, J\}$ is the set of sensor nodes, $A \subseteq J \times J$ is the set of edges such that $(s, j) \in A$ if node $s$ can communicate with $j$. For each node $s \in J$, $J^s$ denotes the set of its neighboring nodes (excluding $s$ itself), i.e., $J^s := \{j : (s, j) \in A, s \neq j\}$, and let $J_s$ be the number of the neighboring nodes of node $s$. The accumulated data from all sensor nodes at time $k$ is denoted as $Y_k = \{y_{k,s}\}_{s \in J}$. The dynamic process and the local measurement model on each node $s \in J$ is given as follows:

\begin{equation}
    x_{k+1} = F_k x_k + w_k,
\end{equation}
\begin{equation}
    y_{k,s} = H_{k,s} x_k + v_{k,s},
\end{equation}

where $F_k \in \mathbb{R}^{m \times m}$ and $H_{k,s} \in \mathbb{R}^{n \times m}$ are the state transition and measurement matrices, respectively, $w_k \in \mathbb{R}^m$ and $v_{k,s} \in \mathbb{R}^n$ are the process and measurement noise sequences, respectively. Here, we assume that zero-mean noises $w_k$ and $v_{k,s}$ with covariance matrices $Q_k$ and $R_{k,s}$ are mutually uncorrelated, i.e.,

\begin{equation}
    E \left\{ \begin{bmatrix} w_k \\ v_{k,s} \end{bmatrix} \begin{bmatrix} w_p \\ v_{p,j} \end{bmatrix}^T \right\} = \begin{bmatrix} Q_k & 0 \\ 0 & R_{k,s} \end{bmatrix} \delta_{s,j} \delta_{k,p},
\end{equation}

where $\delta_{s,j}$ is the Kronecker delta function, i.e., $\delta_{s,j} = 1$ if $s = j$. 
We further define the collective measurement, measurement noise, measurement matrix, and measurement noise covariance as $y_k := [y_{k,1}^T, \ldots, y_{k,J}^T]^T$, $v_k := [v_{k,1}^T, \ldots, v_{k,J}^T]^T$, $H_k := [H_{k,1}^T, \ldots, H_{k,J}^T]^T$, and $R_k := \text{blkdiag}(R_{k,1}, \ldots, R_{k,J})$, respectively. Then, (2) is rewritten as an integrated form

$$y_k = H_k x_k + v_k,$$

where $\mathbb{E}(v_k) = [0^T, \ldots, 0^T]^T$, $\text{Cov}(v_k) = R_k$.

Since the probability distribution knowledge about noise is unknown, we cannot anchor our hope on MAP criterion. Thus, the objective of the work is to make the estimation $\hat{x}_{k,s}$ on each node $s \in J$ achieve a consensus by resorting WLS criterion. Meanwhile, the estimation $\hat{x}_{k,s}$ should approach to the centralized estimation $\hat{x}_k$ as much as possible.

### III. DISTRIBUTED WLS ESTIMATOR

This section first provides a relation between the information filter and WLS estimator. Then, under the relation, an optimization problem with consensus constraints is constructed.

#### A. Information Filter and Its Relation to WLS Estimator

Under the collective system models as shown in (1) and (3), centralized information filter (CIF) gives an optimal estimation within the linear minimum mean-square-error (LMMSE) criterion (10). The CIF consists of two steps, namely, measurement update and time update procedure. Define the estimation errors $\tilde{x}_{k|k-1} := \hat{x}_{k|k-1} - x_k$ and $\tilde{x}_{k|k} := \hat{x}_{k|k} - x_k$ and the corresponding error covariances as $P_{k|k-1} := \mathbb{E}(\tilde{x}_{k|k-1}(\tilde{x}_{k|k-1})^T)$ and $P_{k|k} := \mathbb{E}(\tilde{x}_{k|k}(\tilde{x}_{k|k})^T)$, respectively. Let information matrix of the error covariance $P_{k|k}$ as $\Omega_{k|k} := (P_{k|k})^{-1}$. When the initial estimation $\hat{x}_{0|0}$ and information matrix $\Omega_{0|0}$ are given, CIF can obtain a recursive solution over time evolution as follows:

- **Measurement Update**

  $$\Omega_{k|k} = \Omega_{k|k-1} + H_k^T R_k^{-1} H_k,$$

  $$\tilde{x}_{k|k} = (\Omega_{k|k-1} + H_k^T R_k^{-1} H_k)^{-1} \times (\Omega_{k|k-1} \tilde{x}_{k|k-1} + H_k^T R_k^{-1} y_k),$$

- **Time Update**

  $$\tilde{x}_{k|k-1} = F_k \tilde{x}_{k-1|k-1},$$

  $$\Omega_{k|k-1} = (F_k (\Omega_{k-1|k-1})^{-1} (F_k)^T + Q_k)^{-1}.$$  

Combined (4) with error $\tilde{x}_{k|k-1}$, the measurement update procedure in CIF is expressed as an equivalent form:

$$\begin{bmatrix} \tilde{x}_{k|k-1} \\ y_k \end{bmatrix} = \begin{bmatrix} 1 \\ H_k \end{bmatrix} x_k + \begin{bmatrix} \tilde{x}_{k|k-1} \\ v_k \end{bmatrix}. $$

Letting

$$\mathcal{Y}_k = \begin{bmatrix} \tilde{x}_{k|k-1} \\ y_k \end{bmatrix}, \quad \mathcal{H}_k = \begin{bmatrix} 1 \\ H_k \end{bmatrix}, \quad \beta_k = \begin{bmatrix} \tilde{x}_{k|k-1} \\ v_k \end{bmatrix},$$

we have

$$\mathcal{Y}_k = \mathcal{H}_k x_k + \beta_k,$$

where $\mathbb{E}(\beta_k) = [0^T, 0^T]^T$, $\text{Cov}(\beta_k) = \mathcal{C}_k$.

Equation (10) is a standard WLS form, where the optimal weight matrix is $\mathcal{C}_k^{-1} = (\Omega_{k|k-1} R_k^{-1})^{-1}$. According to the WLS criterion, a WLS solution of (10) is

$$\hat{x}_{WLS} = \arg \min_{\mathcal{X}_k} \left\| \mathcal{Y}_k - \mathcal{H}_k \mathcal{X}_k \right\|_{\mathcal{C}_k}^2,$$

$$= \arg \min_{\mathcal{X}_k} \left( \|y_k - H_k x_k\|_{R_k}^2 + \|x_k - \hat{x}_{k|k-1}\|_{\Omega_{k|k-1}}^2 \right),$$

(11)

where $\|a\|^2 = a^T G a$ denotes Mahalanobis norm.

Since the cost function (11) is convex on $x_k$, taking the derivative of $x_k$ on the right side of (11) yields a WLS estimation and its corresponding information covariance

$$\hat{x}_{WLS} = (\Omega_{k|k-1} + H_k^T R_k^{-1} H_k)^{-1} \times (\Omega_{k|k-1} \tilde{x}_{k|k-1} + H_k^T R_k^{-1} y_k),$$

$$\Omega_{WLS} = \Omega_{k|k-1} + H_k^T R_k^{-1} H_k.$$  

(12a, 12b)

The results in (12) equal to that shown in (5) and (9). In other words, there exists an equivalent relation between the WLS estimator and centralized information filter.

#### B. Distributed WLS Estimator

In this section, we aim to achieve (12) in a distributed manner where every node only exchanges information with its one-hop neighboring nodes. The main difficulty in doing this is that how to accomplish a consensus between nodes. To solve the difficulty, we decompose (11) into $J$ local parallel optimization problems and add constraint constraints $\hat{x}_{k,s} = \hat{x}_{k,j}, \forall s \in J, j \in J_s$ (3). That is, the problem is transformed into the following form

$$\arg \min_{\{x_{k,s}\}} \sum_{s=1}^J f_s(x_{k,s}),$$

s.t. $x_{k,s} = x_{k,j}, \forall s \in J, j \in J_s,$

with

$$f_s(x_{k,s}) = \|y_{k,s} - H_{k,s} x_{k,s}\|_{R_{k,s}}^2,$$

$$+ \frac{1}{J} \|x_{k,s} - \hat{x}_{k|k-1,s}\|^2_{\Omega_{k|k-1,s}}.$$  

(14)

We employ the ADMM technique (11), (12) to address the constrained optimization problem shown in (13). To update the variables $x_{k,s}$ in parallel among all nodes, a auxiliary variable $z_{s,j}$ is introduced, and then the problem in (13) is rewritten as

$$\arg \min_{\{x_{k,s}\}, \{z_{s,j}\}} \sum_{s=1}^J f_s(x_{k,s}),$$

s.t. $x_{k,s} = z_{s,j}, z_{s,j} = x_{k,j}, \forall s \in J, j \in J^s.$  

(15)

In (15), the auxiliary variable $z_{s,j}$ decouples local variable $x_{k,s}$ and those variables $\{x_{k,j}\}_{j \in J_s}$ from its neighboring nodes $j \in J^s$. Let $\lambda_{s,j}(x_{s,j})$ denotes the Lagrange multiplier corresponding to the constraint $x_{k,s} = z_{s,j}$.
Then, the augmented Lagrangian function for the problem \([13]\) is given as follows

\[
L_\rho (\{x_{k,s}\},\{z_{s,j}\},\{\lambda_{s,j1},\lambda_{s,j2}\}) =
\frac{1}{2} \sum_{s=1}^{J} \left( f_s(x_{k,s}) + \rho \sum_{j \in J_s} \|x_{k,s} - z_{s,j} + \frac{\lambda_{s,j1}}{\rho}\|^2_F \right)
+ \rho \sum_{j \in J_s} \left( \|z_{s,j} - x_{k,j} + \frac{\lambda_{s,j2}}{\rho}\|^2_F \right),
\]

where \(\rho > 0\) is a penalty parameter \([13]\).

We employ the ADMM technique to tackle the problem \([16]\) in a cyclic way by minimizing \(L_\rho\) w.r.t local variables \(\{x_{k,s}\}\) and auxiliary variables \(\{z_{s,j}\}\). As for the dual variables \(\{\lambda_{s,j1},\lambda_{s,j2}\}\), one can use the gradient ascent step to update their values. Taking the derivative of the Lagrangian function \([16]\) w.r.t each variable, and setting the corresponding derivative to zero, yields

\[
z_{s,j}^l = \frac{1}{2\rho} (\lambda_{s,j1}^{l-1} - \lambda_{s,j1}^l) + \frac{1}{2} (x_{k,s}^l + x_{k,j}^l),
\]

\[
\lambda_{s,j1}^l = \lambda_{s,j1}^{l-1} + \rho (x_{k,s}^l - z_{s,j}^l),
\]

\[
\lambda_{s,j2}^l = \lambda_{s,j2}^{l-1} + \rho (z_{s,j}^l - x_{k,j}^l).
\]

Substituting \((18)\) into \((19)\) and \((20)\), we get

\[
\lambda_{s,j1}^l = \frac{1}{2} (\lambda_{s,j2}^{l-1} + \lambda_{s,j1}^{l-1}) + \frac{\rho}{2} (x_{k,s}^l - x_{k,j}^l),
\]

\[
\lambda_{s,j2}^l = \frac{1}{2} (\lambda_{s,j2}^{l-1} + \lambda_{s,j1}^{l-1}) + \frac{\rho}{2} (x_{k,j}^l - x_{k,s}^l).
\]

By some mathematical calculation, we have \(\lambda_{s,j1}^l = \lambda_{s,j2}^l\) and \(\lambda_{s,j1}^l = -\lambda_{s,j1}^l,\forall s \in J, j \in J_s\). Further, the auxiliary variable can be computed as

\[
z_{s,j}^l = \frac{1}{2} (x_{k,s}^l + x_{k,j}^l).
\]

Define \(\lambda_{s,j1}^{-1} := \sum_{j \in J_s} \lambda_{s,j1}^{-1}\) as the local aggregate Lagrange multiplier. Then, substituting \((22)\) into \((17)\), we obtain \((23)\).

Next, substituting \((22)\) into \((21a)\), the local aggregate Lagrange multiplier can be updated as follows:

\[
\lambda_{s,j1}^l = \lambda_{s,j1}^{l-1} + \frac{\rho}{2} \sum_{j \in J_s} (x_{k,s}^l - x_{k,j}^{l-1}).
\]

As pointed out in \([14]\), letting all the initial Lagrange multipliers \(\lambda_{s,j1}^0\) to be 0 for all nodes, \((23)\) and \((24)\) yield a convergent result after \(L\) (\(L\) is designed a priori) inner iterations. When \(l \to \infty\), the state estimation \(\hat{x}_{k|k,s} \leftarrow x_{k|k,s}^{\infty}\) on each node \(s \in J\) will converge to the centralized WLS estimation \(\hat{x}_{k|k}^{WLS}\).

Next, we aim at computing the information matrix \(\Omega_{k|k,s}\) w.r.t \(\hat{x}_{k|k,s}\) in the measurement update step. To make \(\Omega_{k|k,s}\) also converge to the centralized information covariance \(\Omega_{k|k}^{WLS}\), it have the following form

\[
\Omega_{k|k,s} = \sum_{s=1}^{J} \left( (H_{k,s})^T R_{k,s}^{-1} H_{k,s} \right) + \Omega_{k|k-1,s}.
\]

The summation term existing in \((25)\) may cause a computational burden especially when the number of sensor nodes is large. In fact, this summation term can be computed by using the AC \([4]\) fusion on \((H_{k,s})^T R_{k,s}^{-1} H_{k,s}\) during the inner iterations. Define the initial value at each node \(s \in J\) be \(S_{k,s}^0 := (H_{k,s})^T R_{k,s}^{-1} H_{k,s}\). At the \(t\)-th iteration, each node \(s\) updates its value using the following manner

\[
S_{k,s}^t = S_{k,s}^{t-1} + \epsilon \sum_{j \in J_s} (S_{k,j}^{t-1} - S_{k,s}^{t-1}).
\]

By alternatively doing this as shown in \((26)\), the values at each node \(s\) will converge to the average value \(\frac{1}{\epsilon} \sum_{s=1}^{J} \left( (H_{k,s})^T R_{k,s}^{-1} H_{k,s} \right)\).

Here, the rate parameter \(\epsilon\) in \((26)\) is chosen between 0 and \(\frac{1}{D_{max}}\), where \(D_{max}\) is the maximum degree of the network \(G\).

When the number of iterations \(t \to \infty\), \((25)\) is rewritten as

\[
\Omega_{k|k,s} = JS_{k,s}^{\infty} + \Omega_{k|k-1,s}.
\]

Since the dynamic matrices \(F_k\) and \(Q_k\) are the same for all nodes, the prediction state and information covariance is the same as its centralized counterpart in \((7)\) and \((8)\), namely

\[
\hat{x}_{k|k-1,s} = F_k \hat{x}_{k-1|k-1,s},
\]

\[
\Omega_{k|k-1,s} = (F_k (\Omega_{k-1|k-1,s})^{-1} (F_k)^T + Q_k)^{-1}.
\]

With analysis above, the updated state \(\hat{x}_{k|k,s}\) and information covariance \(\Omega_{k|k,s}\) on each node \(s\) are identical to the centralized fashion, when the number of iterations \(l\) approaches to \(\infty\). Then, at next time instant, the predicted state and information covariance are also identical to the centralized form as shown in \((28)\) and \((29)\). Combining these two steps, the final estimations on each node \(s\) are same as the corresponding centralized one.

However, \(l \to \infty\) is not reasonable in a practical operation. Hence, the maximum iteration \(L\) is set to be a tradeoff between the performance and computational burden. The distributed WLS estimator (DWLSE) is summarized in Table \(I\).

\[
x_{k,s}^l = \left[ 2\rho J_s I + H_{k,s}^T R_{k,s}^{-1} H_{k,s} + \frac{1}{J} \Omega_{k|k-1,s} \right]^{-1} \times \left[ H_{k,s}^T R_{k,s}^{-1} y_{k,s} + \frac{1}{J} \Omega_{k|k-1,s} \hat{x}_{k|k-1,s} + \sum_{j \in J_s} (\lambda_{s,j1}^{l-1} - \lambda_{s,j1}^l + \rho (z_{s,j}^l - z_{s,j}^{l-1})) \right],
\]

\((17)\)
\[ x_{k,s}^l = \left[ 2\rho J_s I + H_{k,s}^T R_{k,s}^{-1} H_{k,s} + \frac{1}{J} \Omega_{k-1,s} \right]^{-1} \left( H_{k,s}^T R_{k,s}^{-1} y_{k,s} + \frac{1}{J} \Omega_{k-1,s} \hat{x}_{k-1,s} - 2\lambda^l + \sum_{j \in J^c, j \neq s} \rho (\hat{x}_{k-1,j}^l - x_{k,j}^l) \right). \]

(23)

TABLE I
DISTRIBUTED WLS ESTIMATOR

| Parameters                  | Specification          |
|-----------------------------|------------------------|
| Scan Time                   | T = 1 s                |
| Measurement Noise Cov.      | \( R_{k,s} = \text{diag}(200, 8) \) |
| Process Noise Cov.          | \( Q_k = \text{diag}(50, 50, 10, 10) \) |
| Initial State               | \( \hat{x}_{0,s} = [20, 20, 20, 80]^T \) |
| Initial Inf. Cov.           | \( \Omega_{10,s} = \text{diag}(900, 900, 16, 16)^{-1} \) |
| State Transition Matrix     | \( F_k = \begin{bmatrix} 1 & 0 & 0 & T \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) |
| Consensus Para.             | \( \epsilon = 0.65/D_{\text{max}} \) |
| Penalty Para. in DWLSE      | \( \rho = 0.002 \) |
| Penalty Para. in DCKF       | \( \mu = 100 \) |
| No. of Iterations on AC Fusion | 10                        |

IV. PERFORMANCE EVALUATION

This section compares the proposed DWLSE algorithm with DCKF \[4\] in a dynamic scenario. The CIF algorithm is regarded as a benchmark. The simulation is implemented in MATLAB-2019b running on a PC with processor Intel(R) Core(TM) i7-10510U CPU 1.8GHz and with 20GB RAM. We assess the state estimation errors by mean square error (MSE) and estimation bias between different nodes by averaged consensus estimate error (ACEE), respectively, over \( M = 100 \) Monte Carlo runs.

\[ \text{MSE} := \frac{1}{M} \sum_{m=1}^M \| \hat{x}_{k,s}^m - x_k \|; \]

\[ \text{ACEE} := \frac{1}{J(J-1)} \sum_{s \in J} \sum_{j \in J^c, j \neq s} \| \hat{x}_{k,s} - \hat{x}_{k,j} \|, \]

where the superscript \( m \) is the \( m \)-th simulation run, and \( \hat{x}_{k,s} \) is the estimation at node \( s \).

The topology network including 20 sensor nodes is shown in Fig. 1. The maneuvering target moves with constant velocity \( \frac{5000}{36} \text{m/s} \) following the trajectory as shown in Fig. 2. The target goes through one \( 45^\circ \) and two \( 90^\circ \) turns. True initial state is \( x_0 = [0, 0, 3500\sqrt{2}, 3500\sqrt{2}, T] \). Table I shows the detailed tracker parameter settings.

Fig. 2 shows the whole tracking results for three examined filters. As expected, DWLSE has a lower estimation error than DCKF in this scenario. To obtain a more intuitive comparison results, Fig. 3 gives the MSE on node 1 with iteration \( L = 20 \). It is seen that DWLSE has better performance even in the three turning phases. This is because that the DWLSE constructs a novel optimization function to update the variables in parallel.

Fig. 4 shows that the differences between different nodes for DWLSE are within a reasonable range. The result demon-
Fig. 3. MSE results on node 1 with iteration $L = 20$.

Fig. 4. ACEE results between different algorithms with iteration $L = 20$.

Fig. 5. Average MSE with different consensus iterations.

Fig. 6. Average ACEE with different consensus iterations.

strates the validity of the proposed ADMM-based consensus scheme. Although the differences between nodes in DCKF are slightly small, it cannot declare that its accuracy is higher compared with DWLSE. Because when MSE metric is large, even if the ACEE value is small, it doesn’t make any sense.

The inner iteration in two examined distributed algorithms have a great impact on their performance. Thus, we check how many iterations $L$ for each distributed manner can achieve a stable behavior. Fig. 5 and Fig. 6 give the average MSE and ACEE results with different consensus iterations, respectively. Both DCKF and DWLSE converge to the centralized fashion when the number of iterations $l$ increases. Obviously, DWLSE attains a convergent MSE during the first 5 iterations, while DCKF requires 10 iterations to output an optimistic outcome. This merit makes DWLSE more appealing on a computer with limited computing capability. Moreover, it is worth noting that DWLSE has a faster convergence rate.

V. CONCLUSION

This paper proposes a distributed estimator from weighted least squares perspective. To achieve consensus between nodes, we construct an optimization function with a consensus constraint. Numerical results verify that the proposed distributed estimator gives a consensus estimations between nodes. Meanwhile, the estimations converge to the corresponding centralized form.

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