Evolutionary game of cooperative decision-making in reverse supply chain based on cooperative effort

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Abstract. Aiming at a large number of end-of-life (EOF) electrical products flowing into individual recycling outlets, it will squeeze the living space of formal dismantling companies and cause secondary pollution problems. The purpose of this paper is to establish a stable cooperative relationship between them to guide EOF products flowing from individual recycling outlets into the formal dismantling process. A game model involving the participation of formal dismantling enterprises and individual recycling outlets is constructed, and the role of government punishment mechanism intervention in the formation of active cooperative relations between participants in different parameter ranges is discussed. Then, the stability of the evolutionary equilibrium point is analysed. Finally, the Matlab software is used to simulate the influence of parameter on the formation of a stable cooperative relationship, and suggestions for promoting cooperation are provided.

1. Introduction
Due to long-term extensive management, a complete industrial chain based on individual workshop dismantling has been formed in China [1]. According to statistics, only 28% of EOF electronic products in China can be recycled and dismantled through formal channels [2]. Most EOF products flows into individual recycling channels and were illegally dismantled, causing secondary pollution to the environment. At the same time, many formal dismantling companies are facing the dilemma of insufficient raw materials. Therefore, it is necessary to guide the EOF products from individual recycling channels into formal dismantling companies, which can reduce the occurrence of illegal dismantling, and also use individual recycling channels for formal dismantling.

How to establish diversified recycling channels for formal dismantling enterprises is a hot topic of recent scholars' research. The government has also realized the shortcomings of existing management. In the "Implementation Plan on Improving the Recycling and Treatment System of waste Home Appliances to Promote the Renewal of Home Appliances", large recycling companies are encouraged to absorb individual recyclers and establish long-term and stable cooperative relationships. The relationship between formal dismantling companies and individual recyclers is no longer in life-and-death situation.

2. Literature review
In reality, the subjects are all bounded rational groups, and it can be considered that the existing formal dismantling enterprises and individual recyclers are limited but large groups. Therefore, we consider the evolutionary game to study subject behavior strategies and equilibrium issues. Scholars such as Ping Ji[3] focused on the evolutionary game analysis of whether manufacturers and suppliers cooperate on circular
manufacturing in the context of green supply chain. In recent years, the government has successively introduced relevant laws and regulations to improve management. Scholars such as Wanting Chen and Zhi-Hua Hu [4] studied the impact of tax police and subsidy of the government on producers' low-carbon production. Fan Dingxiang and Li Chonglian [5] considered a two-level recycling supply chain composed of recycling companies and consumers, and analyzed the impact of the government's penalty subsidy policy. Xie Tianshuai [6] used Stackelberg game theory to study the optimal decision-making of the government, formal and informal recyclers under the government investigation policy.

The existing literature mostly focuses on the games between the government and citizens/formal recycling channels, and rarely involves cooperation strategies between formal and informal recycling channels.

3. Model building

3.1 Model assumptions

The following are the assumptions and symbol meanings required by the model.

Assumption 1: There are two types of personal recycling outlets, that one is willing to cooperate with formal dismantling companies \((x)\), and the other only conduct illegal dismantling \((1-x)\).

Assumption 2: Under the negative state of formal dismantling enterprises, the proportion of EOF products sold at personal recycling outlets is \(k \in [0,1]\), and the remaining personal recycling outlets still conduct illegal dismantling.

Assumption 3: There are also active and passive cooperation types in formal dismantling companies. The numbers of the two types are \(y\) and \(1-y\), respectively. Under active cooperation, formal dismantling companies invest in the purchase of fixed equipment \(C_x\).

Assumption 4: When the two parties reached a positive cooperative relationship, the effort of the formal dismantling company in strengthening information exchanges and improving related management is \(H(x)(H(0.1-k))\), and it is proportional to the number of individual recycling outlets willing to cooperate \(H(x)=hx, h \in (0.1-k)\).

Assumption 5: The profit of dismantling enterprises under active cooperation is proportional to the number of cooperative personal recycling outlets \((\partial z'_x/\partial x > 0)\).

Assumption 6: The idle cost caused by the raw material gap generated when all personal recycling products are illegally dismantled is \(C_x\), and the idle cost under passive cooperation is \(H(x)C_x\).

Assumption 7: The holders’ awareness of the environmental influence for formal or illegal dismantling is \(e \in [0,1]\) (\(e_1 > e_2 > 0\)). \(\theta\) represents the preference coefficient for environmental protection.

Assumption 8: At present, individual recycling outlets can be as close to the market as possible by walking through the streets and lanes, so assuming the convenience of recycling \(f_1 > f_1 > 0\). \(\eta\) represents the preference coefficient for the convenience of holders to selling EOF products.

Table 1 shows the meaning of model symbols.

| symbols | meaning |
|---------|---------|
| \(\rho\) | Recycling price provided by formal dismantling companies |
| \(\rho\) | Recycling price provided by personal recycling outlets |
| \(F\) | The fines on illegally dismantled personal recycling outlets |
| \(s\) | Profit per unit of illegal dismantling |
| \(r\) | Profit per unit of legal dismantling |
| \(a\) | The unit price formal dismantling companies provided to personal recycling outlets, \(a < r\) |
| \(l\) | Cost conversion coefficient of the cooperation effort |
| \(\pi^j\) | Profits, where \(j \in \{c,h\}, i \in \{1,2\}\), \(c\) represents a formal dismantling enterprise, \(h\) represents a personal recycling outlet |
3.2 Proportion of the recycling number

It is assumed that each holder has one and only one EOF product to be recycled, and the initial attitude is evenly distributed on the straight line \([0,1]\). Suppose that the formal dismantling channel is located at the position 0 on the straight line, and the personal recycling outlets are located at the position 1. If the valuation of a EOL product holder is in the range \(X \in [0,1]\), he will feel the lost cost \(TX\) when he sells the EOF product through the formal dismantling enterprise recycling channel; the lost cost \(T(1-X)\) through a personal recycling network. Let \(v_1\) and \(v_2\) represent the basic utility value of EOF product holders selling EOF products through formal dismantling enterprise recycling channels or personal recycling outlets. EOL products holders' environmental protection utility value \(\theta_1\) that can be obtained from the formal dismantling channel recycling, the convenience utility value \(\eta_1\); \(\theta_2\) obtained from the personal recycling, the convenience utility value \(\eta_2\). Therefore, the net utility values of holders are:

\[
U_i = v_1 + \theta_1 + p_i - TX + \eta f_1
\]

(1) When \(U_i = U_2\), namely: \(v_1 + \theta_1 + p_i - TX + \eta f_1 = v_2 + \theta_2 + p_i - T(1-X) + \eta f_2\)

(2)

Solving equation (3), it can be obtained that:

\[
X^* = \left[ (v_1 - v_2) + (\theta_1 - \theta_2) + (p_i - p_i) + (\eta f_1 - f_2) + T \right] / 2T
\]

holders at this position do not mind which channel is used for recycling. Holders in the range \([0, X^*]\) will choose formal recycling channels, while others will prefer personal recycling outlets.

3.3 The profit function

(1) (cooperation, active cooperation)

The formal dismantling companies and individual recycling outlets under this plan establish stable cooperative relations. The two profit functions are expressed as follows:

\[
\pi_1 = X^* (r - p_i) + (H(x) + k)(1-X^*) (r - a) - H(x)^2 / 2
\]

(4)

\[
\pi_2 = k + H(x) (1-X^*) (a - p_i) + (1-k - H(x)) (1-X^*) (s - p_i)
\]

(5)

Taking the derivative of equations (4) and (5) with respect to the recovery price, and solving the simultaneous equations, the profits of the two participants can be obtained as follows:

\[
\pi_1^* = \left[ 3T + (v_1 - v_2) + (\theta_1 - \theta_2) + (\eta f_1 - f_2) + T \right] / 2T
\]

\[
\pi_2^* = \left[ 3T - (v_1 - v_2) - (\theta_1 - \theta_2) - (\eta f_1 - f_2) - T \right] / 2T
\]

(2) (illegal dismantling, active cooperation)

Make decisions at the same time based on their own profit function maximization, and their profit functions are:

\[
\pi_1^* = X^* (r - p_i)
\]

(6)

\[
\pi_2^* = (1-X^*) (s - p_i)
\]

(7)

Similarly, the profits of the formal dismantling enterprise and personal recycling outlet are:

\[
\pi_1^* = \left[ 3T + (v_1 - v_2) + (\theta_1 - \theta_2) + (\eta f_1 - f_2) + T \right] / 2T
\]

(8)

\[
\pi_2^* = \left[ 3T - (v_1 - v_2) - (\theta_1 - \theta_2) - (\eta f_1 - f_2) - T \right] / 2T
\]

(9)

(3) (cooperation, negative cooperation)

The profit function at this time is:

\[
\pi_1^* = X^* (r - p_i) + (k - H(x)) (r - a)
\]

(10)

\[
\pi_2^* = k (1-X^*) (a - p_i) + (1-k - H(x)) (1-X^*) (s - p_i)
\]

(11)

The best levels of profit are:

\[
\pi_1^* = \left[ 3T + (v_1 - v_2) + (\theta_1 - \theta_2) + (\eta f_1 - f_2) - (1-k) (s - r) \right] / 2T
\]
\[ \pi^*_i = \frac{3T - (v_i - v_j) - \theta(e_i - e_j) - \eta(f_i - f_j) + (1 - k)(s - r)}{18T} \]

(4) (illegal dismantling, negative cooperation)
The profits under this plan are the same as those in the plan (illegal dismantling, active cooperation).

4. Model analysis

4.1 Analysis of Stability Strategy for Formal Dismantling Enterprises

The income matrix between formal enterprises and individual recycling outlets is shown in table 2.

| Formal dismantling companies | individual recycling outlets | Cooperation \( x \) | Illegal dismantling \( 1-x \) |
|------------------------------|-------------------------------|----------------|------------------|
| Active cooperation \( y \)   | \( \pi^*_i - C_i \)          | \( \pi^*_i - (1-k - H(x))F \) | \( \pi^*_i - C_i - C_2 \) |
| Negative cooperation \( 1-y \) | \( \pi^*_i - H(x)C_2 \)      | \( \pi^*_i - C_2 \)      | \( \pi^*_i - C_2 \)      |

The income matrix \( M \) of a formal dismantling enterprise is:

\[ M = \begin{pmatrix} \pi^*_i - C_i & \pi^*_i - C_i - C_2 \\ \pi^*_i - H(x)C_2 & \pi^*_i - C_2 \end{pmatrix} \]

When a formal dismantling company chooses to actively participate in dismantling activities, the expected benefits are:

\[ F(y) = dy/dt = y(U_{\text{active}} - \bar{U}) = y(1-y)[(\pi^*_i - \pi^*_2 + H(x)C_2)x - C_1] \]

To obtain the derivative of \( F(y) \):

\[ dF(y)/dy = (1-2y)[(\pi^*_i - \pi^*_2 + H(x)C_2)x - C_1] \]

From observation of equation (10), it is found that when \( y = 0 \), \( y = 1 \) and \( x = C_1(\pi^*_i - \pi^*_2 + H(x)C_2)x^{-2} \), then \( F(y) = dy/dt = 0 \). According to the stability theory, when \( F(y) = dy/dt = 0 \) and \( dF(y)/dy \leq 0 \), \( y \) is a stable evolution strategy.

If \( x = C_1(\pi^*_i - \pi^*_2 + H(x)C_2)x^{-1} \), no matter the value of \( y \), the conditions \( F(y) = 0 \) and \( dF(y)/dy = 0 \) are satisfied, then the number of actively cooperation formal dismantling enterprises is stable.

If \( x \neq C_1(\pi^*_i - \pi^*_2 + H(x)C_2)x^{-1} \), there are three cases and the evolution path is shown in figure 1:

1. If \( \pi^*_i - \pi^*_2 + H(x)C_2 < 0 \), \( x < 0 < x' < 1 \), when \( dF(y)/dy \big|_{y=0} > 0 \); \( dF(y)/dy \big|_{y=0} < 0 \). In this case, \( y = 0 \) is the stable evolution strategy (ESS).

2. If \( C_1(\pi^*_i - \pi^*_2 + H(x)C_2)x^{-1} < 0 \), there are two conditions.

   When \( 0 < x < x' \), \( dF(y)/dy \big|_{y=0} < 0 \), \( dF(y)/dy \big|_{y=0} > 0 \), \( y = 0 \) is ESS. When \( x' < x < 1 \), \( dF(y)/dy \big|_{y=0} > 0 \), \( dF(y)/dy \big|_{y=0} < 0 \), \( y = 1 \) is ESS.

3. If \( 0 < \pi^*_2 - \pi^*_i + H(x)C_2 < C_1 \), \( 0 < x < 1 < x' \), then \( dF(y)/dy \big|_{y=0} > 0 \); \( dF(y)/dy \big|_{y=0} < 0 \). So, \( y = 0 \) is the stable evolution strategy (ESS).
Proposition 1. 1) If $s > r$, higher convenience $f_1$ allows companies to make decisions in a shorter time; if $s < r$, it will hinder the speed of evolution.

2) If $s > r$, $e_1 > e_2$ or $s < r$, $e_1 < e_2$, increasing the holder’s recognition of the environmental protection of recycling will speed up the decision-making speed of the company; otherwise, the evolution speed will slow down as the recognition increases.

Proof. $F = \frac{\partial F}{\partial y} = \left[2h^2 y (1-y)(s-r)\eta \right](18T)^1$, if $s > r$, $e_1 > e_2$ or $s < r$, $e_1 < e_2$, $\frac{\partial F}{\partial \theta} > 0$; otherwise $\frac{\partial F}{\partial \theta} < 0$.

Proposition 2. 1) The increase in the proportion $k$ of EOF products voluntarily sold by individual recycling outlets is beneficial for formal dismantling companies to choose active cooperation strategies.

2) When the idle cost $L = \pi_1^* - \pi_2^* + H(x)C_x$, note that the larger $L$ is, the more beneficial it is to satisfy the condition (2). Let $L$ take the derivative of $k$ and $x$ respectively, then we can get: $\frac{\partial L}{\partial k} = hx(s-r)^3(18T)^1 > 0$; $\frac{\partial L}{\partial x} = \left[2h(s-r)[3T+(v_1-v_2)+\vartheta(e_2-e_1)+\eta(f_1-f_2)+k(r-a)+hx]\right](18T)^1$. When $\frac{\partial L}{\partial k} > 0$, proposition 2 can be proof.

Proposition 3. When the cost of idleness is small, formal dismantling companies will choose to take free-ride of the government. The government bears the cost of supervision, and encourages individual recycling outlets to seek cooperation through the implementation of a punishment mechanism without making additional efforts.

4.2 Analysis of Stability Strategy of Personal Recycling outlets

The replication dynamic equation of personal recycling outlets is:

$$F(x) = \frac{dx}{dt} = x(U_0 - G) = x(1-x)[(\pi_1^* - \pi_2^* + H(x)F) + \pi_1^* - \pi_2^* + kF]$$

To obtain the derivative of $F(x)$:

$$\frac{dF(x)}{dx} = (1-2x)[(\pi_1^* - \pi_2^* + H(x)F) + \pi_1^* - \pi_2^* + kF]$$

From observation of (12), it is found that when $x = 0$, $x = 1$ and $y = (\pi_1^* - \pi_2^* - kF)[\pi_1^* - \pi_2^* + H(x)F]^2$, then $F(x) = \frac{dx}{dt} = 0$.

If $y = (\pi_1^* - \pi_2^* - kF)[\pi_1^* - \pi_2^* + H(x)F]^2$, the number of actively cooperation enterprises ($y$) is stable.

If $y = (\pi_1^* - \pi_2^* - kF)[\pi_1^* - \pi_2^* + H(x)F]^2$, the discussion can be divided into two situations:

Situation (1): $\pi_1^* - \pi_2^* + kF < 0$, and the evolution path is shown in figure 2.

(4) If $\pi_1^* - \pi_2^* + H(x)F < 0$, $x = 0$ is ESS.

(5) If $0 < \pi_1^* - \pi_2^* - kF < \pi_1^* - \pi_2^* + H(x)F$: When $0 < y < y^*$, $x = 0$ is ESS; when $y^* < y < 1$, $x = 1$ is ESS.
(6) If \(0 < \pi_i^w - \pi_i^c + H(x)F < \pi_i^w - \pi_i^c - kF\), \(x = 0\) is ESS.

\[
\begin{align*}
F(x) & \quad y \\
0 & \quad 1 \\
(a) \pi_i^w - \pi_i^c + H(x)F < 0 & \quad (b) 0 < \pi_i^w - \pi_i^c - kF < \pi_i^w - \pi_i^c + H(x)F \\
0 & \quad (c) 0 < \pi_i^w - \pi_i^c + H(x)F < \pi_i^w - \pi_i^c - kF
\end{align*}
\]

Figure 2. The evolution path of individual recycling outlets under situation ①

It can be seen from situation ① that when the profit under negative cooperation is less than the profit under illegal dismantling, and \(F\) is small, it is difficult for individual recycling outlets to generate cooperative ideas.

Situation ②: \(\pi_i^w - \pi_i^c + kF > 0\), and the evolution path is shown in figure 3.

(7) If \(\pi_i^w - \pi_i^c + H(x)F > 0\), then \(y^* < y < 1\), \(x = 1\) is the ESS for individual recycling outlets.

(8) If \(\pi_i^w - \pi_i^c + H(x)F < \pi_i^w - \pi_i^c - kF < 0\): When \(0 < y < y^*\), \(x = 1\) is the ESS; when \(y^* < y < 1\), \(x = 0\) is the ESS.

(9) If \(\pi_i^w - \pi_i^c - kF < \pi_i^w - \pi_i^c + H(x)F < 0\), \(x = 1\) is the ESS for individual recycling outlets.

\[
\begin{align*}
F(x) & \quad y \\
0 & \quad 1 \\
(a) \pi_i^w - \pi_i^c + H(x)F > 0 & \quad (b) \pi_i^w - \pi_i^c - kF < 0 \\
0 & \quad (c) \pi_i^w - \pi_i^c + H(x)F < \pi_i^w - \pi_i^c - kF
\end{align*}
\]

Figure 3. The evolution path of individual recycling outlets under situation ②

Situation ② shows that when the profit under the active cooperation is greater than the profit under the passive cooperation or the profit of the former is smaller than the latter but the \(F\) is larger, the cooperation has a greater possibility of becoming a stable evolutionary strategy for individual recycling outlets. The greater the voluntary sales ratio \(k\) when individual recycling outlets are willing to cooperate, the better the effect of the penalty mechanism.

4.3 Stable evolution strategy (ESS) analysis

The analysis shows that if and only when the number of formal dismantling enterprise satisfies case (2) and the number of individual recycling outlets under case (7) or (9), the two participants can reach a stable state of (cooperation, active cooperation).

When the formal dismantling enterprises’ number is in case (2) and the number of individual recycling outlets is in case (5) or (8), the final state cannot be directly observed. Let situation I be that the two players in cases (2) and (5); situation II is that they are in cases (2) and (8) respectively. This section examines the stability of the system in situations I and II. Namely, Situation I satisfies \(\pi_i^w - \pi_i^c + kF < 0\), \(C_i - H(x)C_i < \pi_i^w - \pi_i^c\) and \(0 < \pi_i^w - \pi_i^c - kF < \pi_i^w - \pi_i^c + H(x)F\); situation II satisfies \(\pi_i^w - \pi_i^c + kF > 0\), \(C_i - H(x)C_i < \pi_i^w - \pi_i^c\) and \(\pi_i^w - \pi_i^c + H(x)F < \pi_i^w - \pi_i^c - kF < 0\).

Five equilibrium points can be obtained from the replication dynamic equation, namely: (0,0), (0.4),
\[(1,0), (1,1), (x', y')\], where \(x' = C_1 (\pi_1' - \pi_2') + H(x) C_1\) and \(y' = (\pi_3' - \pi_2') - kF (\pi_3' - \pi_2' + H(x'))\).

Regarding the game process is a non-linear system. The Jacobian matrix \([9]\) is used to approximate this system to a linear model to study its stability at the equilibrium point. From equation (10) and equation (12), the Jacobian matrix can be obtained as follows:

\[
J = \begin{bmatrix}
\frac{df(x)/dx}{df(y)/dy}
\end{bmatrix}
\begin{bmatrix}
(1-2\alpha) \left[ (\pi_1' - \pi_2') + H(x') + kF \right] + x(1-x) \left[ \frac{\partial H(x)}{\partial x} \right] + (1-x) \left[ (\pi_1' - \pi_2') + R + H(x') \right] \\
(1-2\alpha) \left[ (\pi_1' - \pi_2') + H(x) C_1 \right] + x(1-x) \left[ \frac{\partial H(x)}{\partial x} \right] C_1
\end{bmatrix}
\]

According to Lyapunov's first law principle, when and only when all the eigenvalues of the Jacobian matrix have negative real parts, the system is asymptotically stable at the equilibrium point. Bringing the equilibrium points into the Jacobian matrix, and finding the eigenvalues, the results are as shown in Table 3 and Table 4.

**Table 3. Analysis of equilibrium point in situation I**

| equilibrium point | eigenvalue \(\lambda_1\) | eigenvalue \(\lambda_2\) | stability |
|-------------------|--------------------------|--------------------------|-----------|
| \((0,0)\)         | \(\lambda_1 = \pi_1' - \pi_2' + kF < 0\) | \(\lambda_2 = -C_1 < 0\) | ESS       |
| \((0,1)\)         | \(\lambda_1 = \pi_1' - \pi_2' + H(\dot{x}) > 0\) | \(\lambda_2 = C_1 > 0\) | Unstable  |
| \((1,0)\)         | \(\lambda_1 = -[\pi_1' - \pi_2' + kF] > 0\) | \(\lambda_2 = \pi_1' - \pi_2' + H(\dot{x}) C_1 > 0\) | Unstable  |
| \((1,1)\)         | \(\lambda_1 = -[\pi_1' - \pi_2' + H(\dot{x}) + kF] < 0\) | \(\lambda_2 = -[\pi_1' - \pi_2' + H(\dot{x}) C_1] < 0\) | ESS       |
| \((x', y')\)      | \(\lambda' = (\alpha_1 + \sqrt{\alpha_1^2 + 4\alpha_2\alpha_3}) 2^i\) | \(\lambda' = (\alpha_1 - \sqrt{\alpha_1^2 + 4\alpha_2\alpha_3}) 2^i\) | Unstable  |

**Table 4. Analysis of equilibrium point in situation II**

| equilibrium point | eigenvalue \(\lambda_1\) | eigenvalue \(\lambda_2\) | stability |
|-------------------|--------------------------|--------------------------|-----------|
| \((0,0)\)         | \(\lambda_1 = \pi_1' - \pi_2' + kF > 0\) | \(\lambda_2 = -C_1 < 0\) | Unstable  |
| \((0,1)\)         | \(\lambda_1 = \pi_1' - \pi_2' + H(\dot{x}) > 0\) | \(\lambda_2 = C_1 > 0\) | Unstable  |
| \((1,0)\)         | \(\lambda_1 = -[\pi_1' - \pi_2' + kF] < 0\) | \(\lambda_2 = \pi_1' - \pi_2' + H(\dot{x}) C_1 > 0\) | Unstable  |
| \((1,1)\)         | \(\lambda_1 = -[\pi_1' - \pi_2' + H(\dot{x}) + kF] < 0\) | \(\lambda_2 = -[\pi_1' - \pi_2' + H(\dot{x}) C_1] < 0\) | Unstable  |
| \((x', y')\)      | \(\lambda' = (\alpha_1 + \sqrt{\alpha_1^2 + 4\alpha_2\alpha_3}) 2^i\) | \(\lambda' = (\alpha_1 - \sqrt{\alpha_1^2 + 4\alpha_2\alpha_3}) 2^i\) | \(\alpha_1 > 0\), Unstable; \(\alpha_1 < 0\), ESS |

**Proposition 4**

In situation I, the conclusion can be drawn:

(1) \((0,1)\) and \((1,0)\) are unstable points, \((0,0)\) and \((1,1)\) are stable evolution strategies;

(2) The center point \((x', y')\) is an unstable point.

In situation II, we know:

(1) \((0,0)\), \((0,1), (1,0)\) and \((1,1)\) are all unstable points;

(2) If \(\alpha_4 > 0\), then the central point \((x', y')\) is not the stable evolution point; if \(\alpha_4 < 0\), then it is ESS.

**Proof.** Taking the central point \((x', y')\) as an example to explain the proof of the stability of the equilibrium point. Take the center point into the Jacobian matrix J to get: \(J' = \begin{bmatrix} 0 & \alpha_3 \\ \alpha_2 + \alpha_4 & 0 \end{bmatrix}\). Among them \(\alpha_2 \geq \alpha_4 \geq (1-x') (\pi_1' - \pi_2') + H(\dot{x'})\), \(\alpha_3 \geq (1-x') (\pi_1' - \pi_2') + H(\dot{x'}) C_1\), \(\alpha_4 \geq (1-x') (\pi_1' - \pi_2') + H(\dot{x'}) C_1\), \(4h(r-a) \left[ (r-a) (1-x') x + h(1-x') \right] \geq 18F R \geq 986\), \(\alpha_2 \geq \alpha_4 \geq (1-x') (\pi_1' - \pi_2') + H(\dot{x'})\). Due to \(\partial \pi_1' / \partial x > 0\), \(\alpha_4 > 0\) can be obtained.

Solving \(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - J = \begin{bmatrix} \lambda - \alpha_3 \\ \alpha_2 + \alpha_4 \end{bmatrix} = \lambda (\lambda - \alpha_1) - \alpha_1 (\alpha_2 + \alpha_3) = 0\), the eigenvalues can be obtained:
\( \lambda_{1,2} = \left( \alpha_1 \pm \sqrt{\alpha_1^2 + 4a_1(\alpha_2 + \alpha_3)} \right)^{1/2}. \)

Situation I: \( \alpha_1 > 0 \) and \( \alpha_2 > 0 \), at this time \( \sqrt{\alpha_1^2 + 4a_1(\alpha_2 + \alpha_3)} > \alpha_1 \), no matter whether \( \alpha_1 \) is greater than 0, existing \( \lambda_1^* \) or \( \lambda_2^* \) is a positive real number. According to Lyapunov principle, \((x^*, y^*)\) is an unstable point.

Situation II: \( \alpha_1 < 0 \) and \( \alpha_2 > 0 \), at this time there is \( \sqrt{\alpha_1^2 + 4a_1(\alpha_2 + \alpha_3)} < \alpha_1 \). The discussion can be divided into two cases:

If \( \alpha_1 > 0 \), then \( \lambda_1^* \) and \( \lambda_2^* \) both have positive real parts, so \((x^*, y^*)\) is unstable points. This situation can be inferred that when \( s > r \), that is, when the profit per unit of illegal dismantling is greater than that of formal dismantling, the number of cooperative individual recycling outlets and active cooperative formal dismantling enterprises always change.

If \( \alpha_1 < 0 \) and \( \alpha_2 > 0 \), the eigenvalues are both negative real numbers, so \((x^*, y^*)\) is a stable point; if \( \alpha_1 < 0 \) and \( \alpha_1^2 + 4a_1(\alpha_2 + \alpha_3) < 0 \), the replicator dynamic nonlinear system has two conjugate complex roots with negative real parts, according to the Lyapunov first law judges that the point is asymptotically stable. Overall, as long as \( \alpha_1 < 0 \), \((x^*, y^*)\) is ESS.

The above is the proof of proposition 4.

5. Sensitivity Analysis

5.1 Data and parameter settings

Let the initial value of \( x \) equals to 0.3 and \( y \) equals to 0.5. Taking EOF refrigerators as an example, according to the survey [10], the recycling price ranges from 80 to 135 yuan, assuming \( p^r = 85 \) yuan, \( p^s = 100 \) yuan, \( a = 110 \) yuan. Referring to a CCTV’s report in 2017 [11], the average dismantling unit profit of a refrigerator is 120 yuan. The illegal dismantling costs extremely low labor and electricity costs, assuming \( s = 120 \) yuan/unit. The income of formal dismantling is about 85 yuan per unit. The “Administrative Measures for the Collection and Use of EOF Electrical and Electronic Products Treatment Funds” pointed out that refrigerators’ subsidy is 80 yuan/unit, assuming that \( r = 85 + 80 = 165 \) yuan/unit. Referring to the “Solid Waste Pollution Prevention and Control Law”, we set \( F = 150,000 \). Nearly 57% of residents do not know the existence of the current recycling platform [2], supposing \( f_1 = 0.43 \) and \( f_2 = 0.8 \). Considering that the weak public awareness of dismantling \( (\theta = 30 \) yuan), but the convenience is very important for holders \( (\eta = 80 \) yuan). Take the month as the evolution unit.

5.2 Result analysis

5.2.1 Convenience of formal dismantling companies for holders f
Assuming three different situations, \( f_1 = 0.3 \), \( f_2 = 0.6 \), and \( f_3 = 0.9 \), the dynamic evolution diagrams shown in figure 4 and figure 5. The construction of convenient recycling channels by formal dismantling companies has increased the difficulty of recycling at individual recycling outlets. With the improvement of their recycling network system, the dependence of formal dismantling enterprises on individual recycling outlets decreases, so the desire for active cooperation is gradually reduced, and the evolution time is prolonged. In addition, the larger \( f \) is, the more time it takes.

From the perspective of formal dismantling companies, it is important to improve the convenience of front-end recycling and enhance their competitiveness in recycling channels. On the one hand, it
promotes the probability of waste electronic and electrical products flowing into formal channels. On the other hand, it also pressures the outlets to seek cooperation with formal dismantling companies.

5.2.2 Penalty amount F. In order to examine the impact of the fines on the dynamic evolution system, the fines $F_1=50,000$, $F_2=150,000$, $F_3=250,000$, respectively, charged by the government on the illegally dismantled personal recycling outlets, the results are shown in figure 6-7. When $F$ is small, the number of cooperative participants in individual recycling outlets will gradually decrease to zero, and the number of dismantling companies willing to actively cooperate will quickly drop to zero at a faster rate.

5.2.3 The effort coefficient $h$. Let the value of the effort coefficient be 0.3, 0.5 and 0.7, as shown in figure 8-9. With the increase in the effort coefficient $h$, the evolution of cooperative personal recycling outlets and active cooperative formal dismantling companies will accelerate. Personal recycling outlets and formal dismantling companies will format a stable cooperative relationship in a shorter time which is beneficial to the standardization of the recycling industry. When this coefficient is very low, it will take a long time for dismantling companies to actively participate in the construction of stable cooperative relations with individual recycling outlets.
6. Conclusions
This article uses evolutionary game theory as an analysis tool to study the decision-making changes on cooperation between formal dismantling companies and individual recycling outlets when the government is involved. The main conclusions and management significance are as follows:

Firstly, the government should reasonably set the amount of punishment and strengthen the supervision of illegal dismantling activities, so that the punishment mechanism can encourage individual recycling outlets to seek cooperation with formal dismantling companies, such as issue licenses to compliant individual recycling outlets to regulate the industry. Once illegal dismantling activities are discovered, their licenses can be cancelled, increasing the penalty cost of illegal dismantling.

Secondly, formal dismantling companies have improved their recycling network by carrying out Internet + recycling, dealer recycling, and trade-in measures to reduce the inconvenience of disposal of EOF product for holders. Governments and enterprises can also take methods to raise citizens' awareness of the environmental hazards of illegal dismantling and reduce the amount of EOF products flowing into individual recycling outlets.

Thirdly, the government should strengthen the construction of illegal dismantling supervision mechanism, improve the supporting policies for recycling and dismantling enterprises, so as to enhance the confidence of formal dismantling enterprises in the recycling and processing industry, and make formal dismantling company willing to make greater efforts to deepen cooperation.

It is assumed that all formal dismantling companies and individual recycling outlets have the same profitability, but there are also differences within these two groups.

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