1 Introduction

The experimental result on the lifetime ratio of the \( \Lambda_b \) baryon and \( B \) meson, \( \tau(\Lambda_b)/\tau(B^0) = 0.795 \pm 0.053 \), still needs theoretical understanding. It can be calculated systematically by heavy quark expansion if we do not assume the failure of the local duality assumption. To the order of \( 1/m_b^2 \), the calculated ratio is still close to unity. The potential importance of the \( O(1/m_b^3) \) effect has been pointed out. The lifetime ratio was calculated as follows,

\[
\frac{\tau(\Lambda_b)}{\tau(B^0)} \simeq 0.98 + \xi \left\{ p_1 B_1(m_b) + p_2 B_2(m_b) + p_3 \epsilon_1(m_b) + p_4 \epsilon_2(m_b) \right\} + [p_5 + p_6 \bar{B}(m_b) r(m_b)],
\]

where the term proportional to \( \xi \equiv (f_B/200\text{MeV})^2 \) arises from the \( 1/m_b^3 \) contributions. At the scale \( m_b \), the values of the perturbative coefficients \( p_i \)'s are \( p_1 = -0.003, p_2 = 0.004, p_3 = -0.173, p_4 = -0.195, p_5 = -0.012, p_6 = -0.021 \). \( B_1, B_2, \epsilon_1, \epsilon_2, r \) and \( \bar{B} \) are the parameterization of the hadronic matrix elements of the following four-quark operators,

\[
\langle \bar{B} | \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma^\mu (1 - \gamma_5) b | \bar{B} \rangle \equiv B_1 f_B^2 m_B^2, \\
\langle \bar{B} | \bar{b} (1 - \gamma_5) q \bar{q} (1 + \gamma_5) b | \bar{B} \rangle \equiv B_2 f_B^2 m_B^2, \\
\langle \bar{B} | \bar{b} \gamma_\mu (1 - \gamma_5) t_a q \bar{q} \gamma^\mu (1 - \gamma_5) t_a b | \bar{B} \rangle \equiv \epsilon_1 f_B^2 m_B^2, \\
\langle \bar{B} | \bar{b} (1 - \gamma_5) t_a q \bar{q} (1 + \gamma_5) t_a b | \bar{B} \rangle \equiv \epsilon_2 f_B^2 m_B^2,
\]

and

\[
\frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma^\mu (1 - \gamma_5) b | \Lambda_b \rangle \equiv -\frac{f_B^2 m_B}{12} r, \\
\frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | \bar{b} (1 - \gamma_5) q \bar{q} (1 + \gamma_5) b | \Lambda_b \rangle \equiv -\frac{f_B^2 m_B}{24} r.
\]
These parameters have been calculated by QCD sum rules. In Refs. 6, 7, the mesonic parameters $B_i$ and $\epsilon_i$ were calculated within the framework of heavy quark effective theory (HQET). The baryonic parameters $r$ and $\tilde{B}$ were calculated in Refs. 8 and 9. Here we report our result of 9.

2 The Calculation

The new ingredients of our analysis compared to Ref. 8 is the following. (1) Gluon condensate and six-quark condensate are included. (2) A different duality assumption is adopted. The result of $\tilde{B} = 1$ does not change in the valence quark approximation.

To calculate $r$, the following three-point Green's function is constructed,

$$\Pi(\omega, \omega') = i^2 \int dx dy e^{ik' \cdot x - i{k \cdot y}} \langle 0 | \tilde{T} j^\nu(x) \tilde{O}(0) j^\nu(y) | 0 \rangle ,$$

where $\omega = v \cdot k$ and $\omega' = v \cdot k'$. The $\Lambda_Q$ baryonic current $\tilde{j}^\nu$ is

$$\tilde{j}^\nu = \epsilon^{abc} q^T a 1 C\gamma_5 (a + b \not{v}) \tau q^b h^c_v ,$$

where $a$ and $b$ are certain constants, $h_v$ is the heavy quark field in the HQET with velocity $v$, $C$ is the charge conjugate matrix, $\tau$ is the flavor matrix for $\Lambda_Q$. In Eq. (4), $\tilde{O}$ denotes the four-quark operator

$$\tilde{O} = \tilde{h}_v \gamma_\mu \frac{1 - \gamma_5}{2} h_v \bar{q}_1 \gamma_\mu \frac{1 - \gamma_5}{2} \bar{q}_2 .$$

Note $\langle \Lambda_b | \tilde{O} | \Lambda_b \rangle = - \langle \Lambda_b | O | \Lambda_b \rangle$ where

$$O = h_v \gamma_\mu \frac{1 - \gamma_5}{2} \bar{q}_1 \gamma_\mu \frac{1 - \gamma_5}{2} h_v .$$

In terms of the hadronic expression, the parameter $r$ appears in the ground state contribution of $\Pi(\omega, \omega')$,

$$\Pi(\omega, \omega') = \frac{1}{\Lambda - \omega} \frac{f^2 \langle \Lambda_Q | O | \Lambda_Q \rangle}{2 (\Lambda - \omega')(\Lambda - \omega')} \frac{1 + \not{k}}{2} + \text{higher states} .$$

$\Lambda = m_{\Lambda_Q} - m_Q$ and the quantity $f_\Lambda$ is defined as $\langle 0 | j^\nu | \Lambda_Q \rangle = f_\Lambda u$ with $u$ being the unit spinor in the HQET. The QCD sum rule calculations for $f_\Lambda$ were given in Refs. 10, 11, 12, 13.

On the other hand, this Green’s function $\Pi(\omega, \omega')$ can be calculated in terms of quark and gluon language with vacuum condensate straightforwardly.
The fixed point gauge is used in the calculation. The tadpole diagrams in which the light quark lines from the four-quark vertex are contracted have been subtracted. While the calculation can be justified if \((\omega)\) and \((\omega')\) are large, however the hadron ground state property should be obtained at small \((\omega)\) and \((\omega')\). These contradictory requirements are achieved by introducing double Borel transformation for \(\omega\) and \(\omega'\).

### 3 Duality Assumption

Generally the duality is to simulate the higher states by the whole quark and gluon contribution above some threshold energy \(\omega_c\). The whole contribution of the three-point correlator \(\Pi(\omega, \omega')\) can be expressed by the dispersion relation,

\[
\Pi(\omega, \omega') = \frac{1}{\pi} \int_0^\infty d\nu \int_0^\infty d\nu' \frac{\text{Im}\Pi(\nu, \nu')}{(\nu - \omega)(\nu' - \omega')}. \tag{9}
\]

With the redefinition of the integral variables \(\nu_+ = \frac{\nu + \nu'}{2}\), \(\nu_- = \frac{\nu - \nu'}{2}\),

\[
\int_0^\infty d\nu \int_0^\infty d\nu' \ldots = 2 \int_0^{\omega_c} d\nu_+ \int_{-\nu_+}^{\nu_+} d\nu_- \ldots. \tag{11}
\]

It is in \(\nu_+\) that the quark-hadron duality is assumed,

\[
\text{higher states} = \frac{2}{\pi} \int_{\omega_c}^\infty d\nu_+ \int_{-\nu_+}^{\nu_+} d\nu_- \frac{\text{Im}\Pi(\nu, \nu')}{(\nu - \omega)(\nu' - \omega')}. \tag{12}
\]

This kind of assumption was suggested in calculating the Isgur-Wise function in Ref. and was argued for in Ref. The sum rule for \(\langle \Lambda_Q | \hat{O} | \Lambda_Q \rangle\) after the integration with the variable \(\nu_-\) is

\[
\frac{(a + b)^2}{2} f_A^2 \exp \left( -\frac{\Lambda}{T} \right) \langle \Lambda_Q | \hat{O} | \Lambda_Q \rangle = \int_0^{\omega_c} d\nu \exp \left( -\frac{\nu}{T} \right) \frac{a^2 + b^2}{840\pi^6} \nu^8 - \frac{ab}{6\pi^3} \nu^5 \langle \bar{q}q \rangle + 3(a^2 + b^2) \nu^4 \langle g_s^2 G^2 \rangle + \frac{5ab}{48\pi^4} m_0^2 \langle \bar{q}q \rangle \nu^3 + \kappa_1 \frac{17(a^2 + b^2)}{96\pi^2} \langle \bar{q}q \rangle^2 \nu^2 - \kappa_2 \frac{ab}{144} \langle \bar{q}q \rangle^3 \langle \bar{q}q \rangle^3. \tag{13}
\]
Where $\kappa_1$, $\kappa_2$ are the parameters used to indicate the deviation from the factorization assumption for the four- and six-quark condensates. $\kappa_{1,2} = 1$ corresponds to the vacuum saturation approximation. $\kappa_1 = (3 \sim 8)$ is introduced in order to include the nonfactorizable contribution and to fit the data. There is no discussion of $\kappa_2$ in literature so we use $\kappa_2 = 1$. We shall adopt $a = b = 1$ in our numerical analysis. The parameters $f_{\Lambda}$ and $\bar{f}_{\Lambda}$ were obtained by the HQET sum rule analysis of two-point correlator.

Our final sum rule is obtained from Eq. (13) by dividing that for $f_{\Lambda}$. The value of $\omega_c$ is $(1.2 \pm 0.1)$ GeV. The sum rule window is $T = (0.15 - 0.35)$ GeV.

We obtain for $\kappa_1 = 4$

$$\langle \Lambda_Q | \bar{O}|\Lambda_Q \rangle = (1.6 \pm 0.4) \times 10^{-2} \text{GeV}^3 \quad \text{or} \quad r = (3.6 \pm 0.9) \quad . \quad (14)$$

By taking $f_B = 200$ MeV. If we use $\kappa_1 = 1$, we get

$$\langle \Lambda_Q | \bar{O}|\Lambda_Q \rangle = (5.5 \pm 1.0) \times 10^{-3} \text{GeV}^3 \quad \text{or} \quad r = (1.3 \pm 0.3) \quad . \quad (15)$$

Note that our results depend on $\omega_c$ weakly.

The value of $r$ we have obtained above is at some hadronic scale, because we have been working in the HQET. By choosing $\alpha_s(\mu_{\text{had}}) = 0.5$ (corresponding to $\mu_{\text{had}} \sim 0.67$ GeV), we obtain $\bar{B}(m_b) \simeq 0.58$ and

$$r(m_b) \simeq (6.2 \pm 1.6) \quad \text{for} \quad \kappa_1 = 4 \quad , \quad \text{and} \quad r(m_b) \simeq (2.3 \pm 0.6) \quad \text{for} \quad \kappa_1 = 1 \quad . \quad (16)$$

The $\Lambda_b$ and $B^0$ lifetime ratio given in Eq. (1) is expressed specifically as

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} \simeq 0.83 \pm 0.04 \quad \text{for} \quad \kappa_1 = 4 \quad ,$$

$$\simeq 0.93 \pm 0.02 \quad \text{for} \quad \kappa_1 = 1 \quad . \quad (17)$$

Where the values $\epsilon_1(m_b) = -0.08$ and $\epsilon_2(m_b) = -0.01$ have been taken from the QCD sum rules. We see that with the vacuum saturation ($\kappa_1 = 1$), although $r$ is enhanced by about six times compared to that in Ref. [3], it is still not large enough to account for the data. The lifetime ratio between $\Lambda_b$ and $B$ mesons can be explained if we also take into account the nonfactorizable contribution of the four-quark condensate.

4 Conclusion

In summary, we have reanalyzed the QCD sum rule for the $\Lambda_b$ matrix element of the four-quark operator relevant to the lifetime of $\Lambda_b$. The difference between Ref. [3] and ours is mainly because of duality assumptions. While a large nonfactorizable effect in the four-quark condensate can make the theoretical result consistent with the experiment, our main conclusion is that the lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ can be as low as 0.91.
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