Covariant quark model for the electromagnetic structure of light nucleon resonances

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(Dated: June 7, 2018)

We present estimates from the covariant spectator quark model for the electromagnetic transition form factors for the resonances $N(1440)_{\frac{1}{2}^+}$, $N(1535)_{\frac{3}{2}^-}$, $N(1520)_{\frac{1}{2}^-}$ and $\Delta(1232)_{\frac{3}{2}^+}$, at intermediate and large square momentum transfer ($Q^2$). The calculations associated to the $N(1440)_{\frac{1}{2}^+}$, $N(1535)_{\frac{1}{2}^-}$ and $N(1520)_{\frac{1}{2}^-}$ states are based exclusively on the parametrizations derived for the nucleon. In the case of the $\Delta(1232)_{\frac{3}{2}^+}$ (isospin 3/2), we use lattice QCD data to estimate the radial structure, and take into account the well known effects associated with the pion cloud dressing of the baryons. Our estimates are based mainly on the valence quark degrees of freedom and are in good agreement with the data for $Q^2 > 2$ GeV$^2$, with a few exceptions. The present predictions can be tested in a near future for large $Q^2$ at the Jefferson Lab – 12 GeV upgrade.

BACKGROUND

With the construction of the modern accelerators a significant amount of data associated with the electromagnetic structure of the nucleon ($N$) and the nucleon resonances ($N^*$) has been collected, at intermediate and large square momentum transfer ($Q^2$). The new data, parametrized in terms of $\gamma^*N \rightarrow N^*$ transition form factors, call for the development of relativistic theoretical models that can be applied to the description of the present data, as well as the expected results from the Jefferson Lab –12 GeV upgrade \[1, 2\].

In the large-$Q^2$ region, the $\gamma^*N \rightarrow N^*$ transitions are expected to be dominated by the valence quark degrees of freedom. One of the quark models that includes relativity is the covariant spectator quark model \[2, 3\]. The model was originally developed to study the electromagnetic structure of the nucleon \[3\]. The motivation for the model was to test if the new results from Jefferson Lab (JLab) \[4\] for the ratio between the electric and the magnetic form factors of the nucleon \[3\]. The results for the ratio of the electric to magnetic form factors were \[1, 2\], suggesting a difference between the electric charge and the magnetic dipole density distributions \[3, 4\].

The model has three basic ingredients:

(i) the wave function of the baryon (including the nucleon) can be represented in terms of the spin-isospin structure of the individual quarks based on the $SU_S(2) \times SU_F(3)$ spin-flavor symmetry, rearranged as an active quark and a spectator quark-pair \[3, 4\];

(ii) the three-quark system can be reduced to a quark-diquark system, parametrized by a radial wave function $\psi_B$, integrating into the quark-pair degrees of freedom \[3, 5\];

(iii) the electromagnetic structure of the quark is parametrized by quark isoscalar/isovector form factors $f_{i\pm}(Q^2)$ ($i = 1$ for Dirac, and $i = 2$ for Pauli), which simulate the substructure associated with the gluons and quark-antiquark effects, and it is parametrized using the vector meson mechanism \[2, 7, 8\].

A very good description of the new JLab results as well as the neutron data is obtained when we calibrate the two building blocks of the model: the quark form factors and the nucleon radial wave function, $\psi_N$, by the proton and neutron form factor data \[3\]. The model can then be extended to nucleon resonances, in particular to the lightest $N^*$ states $J^P$ (spin $J$ and parity $P$).

RESULTS

The covariant spectator quark model has been in the recent years extended to the negative parity states $N(1535)_{\frac{1}{2}^-}$ and $N(1520)_{\frac{1}{2}^-}$ \[3, 11\]. The analytic expressions for the transition form factors became simpler when we consider the semirelativistic approximation \[9\]. In that approximation the radial wave function of the resonance ($\psi_R$) has the same form as the wave function of the nucleon ($\psi_N$) and the difference of masses between the nucleon ($M$) and the resonance ($M_R$) is neglected in the overlap of the radial functions. In these conditions the transition form factors can be determined with a few additional assumptions. The only input are the parametrization of the quark form factors and the nucleon radial wave function, both determined in the study of the nucleon \[3\]. The results for the $\gamma^*N \rightarrow N(1535)$ and $\gamma^*N \rightarrow N(1520)$ form factors are presented in Fig. in comparison with the data from JLab \[12\] and MAID \[13\].
In general, we obtain a good overall description of the data, particularly for \(Q^2 > 2\text{ GeV}^2\). The exceptions are the Pauli form factor \(F_2^*\) in the case of \(N(1535)\) and the electric form factor \(G_E\) in the case of \(N(1520)\) at low \(Q^2\) (not shown here) \[2\]. These deviations may be interpreted as an indication that the meson cloud effects, not included in the present framework, may be significant at low \(Q^2\). For \(N(1535)\) it was shown that the calculations from valence quark contributions and meson cloud contributions to \(F_2^*\) have different signs, which may lead to significant cancellation between those effects \[1\,14\,15\,16\].

The consequence of that cancellation is the correlation between the transverse and scalar amplitudes for large \(Q^2\):

\[
S_{1/2} = -\frac{\sqrt{1 + \tau}}{\sqrt{2}} \frac{M^2_R - M^2}{2M_RQ} A_{1/2},
\]

where \(\tau = \frac{Q^2}{(M+M_R)^2}\). The previous relation agrees remarkably well with the available data \[14\]. In the case of \(N(1520)\), the model fails at small \(Q^2\), for \(G_E\), because it predicts that the longitudinal amplitude \(A_{3/2} \propto (G_E + G_M)\) vanishes, contrary to the experimental evidences \((A_{3/2} \neq 0)\) \[1\,12\]. This discrepancy can be understood admitting that \(A_{3/2}\) is dominated by meson cloud effects, as discussed in detail in Refs. \[2\,10\,11\,17\].

The model has also been applied to the description of the \(N(1440)\), traditionally interpreted as the first radial excitation of the nucleon \[1\,20\]. In this case the radial wave function of the resonance is written in the form \(\psi_R(\kappa) = g(\kappa) \psi_N(\kappa)\) where \(\kappa\) is a variable defined in terms of the quark-diquark relative momentum \[3\,18\,19\]. The function \(g(\kappa)\) is expressed in a form compatible with the expected asymptotic behavior for the transition form factors at large \(Q^2\) and includes one adjustable parameter. This parameter is determined by the condition that the nucleon and the \(N(1440)\) are orthogonal states. Once defined \(\psi_R(\kappa)\), the transition form factors are determined without the inclusion of any extra parameters, except for the ones included in the parametrization for \(\psi_N\) \[3\].

The results for \(N(1440)\) are present in the Fig. \[2\] (solid line) in comparison with the CLAS/JLab data \[12\]. In the figure, we can notice that the model describes very well the \(Q^2 > 2\text{ GeV}^2\) data, corroborating the idea that \(N(1440)\) is in fact the radial excitation of the nu-
The deviations at small $Q^2$ can be interpreted as a consequence of the meson cloud effects, omitted in the present model, or as a consequence of the approximated form considered for the orthogonality condition, since the orthogonality was imposed only in the first order of $(M_R - M)^2.$ More details can be found in Appendix B from Ref. [18]. In a recent work, the valence quark contributions to the $\gamma^* N \rightarrow N(1440)$ form factors have been calculated within a holographic model [21, 22]. The results are also presented in Fig. 2 by the red band. Those results suggest that holographic methods can be used to estimate the valence quark effects, even at small $Q^2$, and that, in the case of the Pauli form factor, the effects of the meson cloud may be small [21].

The covariant spectator quark have been also applied to the study of the $\Delta(1232)^3$ resonance. The wave function associated with the $\Delta(1232)$ can be written as a combination of three angular momentum states for the quark-diquark system: a $S$ state and two different $D$ states, labeled as $D_1$ and $D_3$ (core spin 1/2 and 3/2, respectively) [23, 24]. The spin-isospin structure is determined by the respective symmetries. For the radial wave functions, $\psi_{S,D_1,D_3}^\Delta$, one needs, however, to use some ansatz. In this case we cannot relate the $\Delta$ radial wave functions with the nucleon radial wave function. There are two main reasons for that: i) the nucleon and the $\Delta$ are based on very different spin-isospin states; ii) in the case of the $\Delta$, we cannot parametrize the radial wave function using directly the elastic data ($\gamma^* \Delta \rightarrow \Delta$), neither by the $\gamma^* N \rightarrow \Delta$ data, since there are evidences that the data are strongly contaminated by meson cloud effects [23, 27]. One uses, therefore, lattice QCD data [28] to determine $\Delta$ radial wave functions $\psi_{S,D_1,D_3}^\Delta$.

The extension of the model to the lattice QCD regime takes advantage of two properties of the model: the representation of the quark form factors $f_{i\pm}$ in terms of the vector meson dominance mechanism (implicit dependence on the rho mass $m_\rho$: $f_{i\pm}(Q^2) \equiv f_{i\pm}(Q^2; m_\rho, M)$), and the representation of the radial wave functions in terms of the baryon mass, $M_B$. In the lattice QCD regime, we can then replace the dependence of the masses $(m_\rho, M$ and $M_B$) in the physical regime by the lattice QCD masses. More details can be found in Refs. [5, 8]. Once determined the radial wave functions by the lattice QCD data, the model is extrapolated to the physical regime, and used to calculate the valence quark contributions for each form factor. The results are presented in Fig. 3 for the magnetic dipole $G_M$ (dashed-line) and for the electric $G_E$ and Coulomb $G_C$ quadrupole form factors (thin-lines near the horizontal axis).

For the magnetic dipole form factor, $G_M$, in the left panel of Fig. 3 one can observe that the valence quark contribution (Bare), estimated with the assistance of the lattice QCD data (dashed-line), underestimate the data below $Q^2 = 2$ GeV$^2$. Only for larger values of $Q^2$ there is the convergence with the full result (solid-line) [7, 24]. In the present study the full result (Bare + Pion cloud) is obtained using a phenomenological parametrization to the pion cloud component $G_M^\pi \propto \left( \frac{\Lambda_\pi^2}{\Lambda_\pi^2 + Q^2} \right)^2$, with an adjustable strength coefficient and a cutoff $\Lambda_\pi$ [7, 24]. The estimates for the bare and meson cloud contributions are in good agreement with the estimates from the EBAC/JLab dynamical coupled-channel model for the baryon-meson reactions [7, 27].

Our estimates for the electric and Coulomb quadrupole form factors are presented in the left panel of Fig. 3. For convenience, we present the results for $G_C$ multiplied by the factor $\kappa = \frac{M_\Delta - M_B}{2M_\Delta}$, where $M_\Delta$ is the $\Delta(1232)$ mass. In the figure, one can notice the very good agreement between the final results (tick-lines), which include the bare and the pion cloud contributions, and the overall data [24, 30]. Only at low $Q^2$, there are some discrep-
ancy with the $G_C$ data, which has been interpreted as a consequence of errors in the analysis of the data. The old low-$Q^2$ data have replaced by a more recent and reliable analysis [30, 31]. The most recent results are represented by the solid circles and diamonds. For the good agreement between theory and data contribute the combination of the small valence contributions ($\approx 10\%$, thin lines) and the pion cloud contributions. The pion cloud contributions are estimated by large $N_c$ parameter-free relations [30, 32, 33]. Since the valence quark contribution is fixed by the lattice QCD data, the final results are true predictions [30]. The convergence between the results for $G_E$ and $\kappa G_C$ at the pseudothreshold, when $Q^2 = -(M_\Delta - M_N)^2$ is a consequence of Siegert's theorem [30, 31].

**SUMMARY AND OUTLOOK**

The covariant spectator quark model, successful in the description of the nucleon electromagnetic form factors revealed by the JLab polarization transfer experiments, has been extended to the calculation of transition form factors associated with several light nucleon resonances $N^*$. Combining the electromagnetic structure of the constituent quarks, calibrated by the nucleon data, with appropriate ansatz for the radial wave functions of the resonances, we are able to estimate the transition for factors for the resonances $N(1440)^{1+}, N(1535)^{1-}$ and $N(1520)^{3-}$. The estimates are based on the parametrization of the nucleon radial wave function (nucleon shape) with no adjustable parameters. The calculations take into account, exclusively, the effect of the valence quarks, and are in good agreement with the data for momentum transfer $Q^2 > 2$ GeV$^2$, with a few exceptions. The exceptions may be explained by meson cloud effects. All the estimates are based on the parametrization of the nucleon structure, and can be tested in a near future for large transfer momentum in the Jefferson Lab – $12$ GeV upgrade. The model has been also extended to the $\Delta(1232)^{3+}$, with the assistance of the lattice QCD, to estimate the radial wave functions of the $\Delta(1232)$ valence quark core. The model estimates are in agreement with the empirical data when theoretical and phenomenological parametrizations of the pion cloud are taken into account.

**Acknowledgments**

This work was supported by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP): project no. 2017/02684-5, grant no. 2017/17020-BCO-JP.

[1] I. G. Aznauryan *et al.*, Int. J. Mod. Phys. E 22, 1330015 (2013).
[2] G. Ramalho, Phys. Rev. D 95, 054008 (2017).
[3] F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. C 77, 015202 (2008).
[4] M. K. Jones *et al.* [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. 84, 1398 (2000).
[5] G. Ramalho, K. Tsushima and F. Gross, Phys. Rev. D 80, 033004 (2009).
[6] F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. D 85, 093005 (2012).
[7] G. Ramalho and M. T. Peña, Phys. Rev. D 80, 013008 (2009).
[8] G. Ramalho and M. T. Peña, J. Phys. G 36, 115011 (2009).
[9] G. Ramalho and M. T. Peña, Phys. Rev. D 84, 033007 (2011).
[10] G. Ramalho and M. T. Peña, Phys. Rev. D 89, 094016 (2014).
[11] G. Ramalho, Phys. Rev. D 90, 033010 (2014).
[12] V. I. Mokeev, https://userweb.jlab.org/~mokeev/resonance_electrocouplings/.
[13] D. Drechsel, S. S. Kamalov and L. Tiator, Eur. Phys. J. A 34, 69 (2007).
[14] G. Ramalho and K. Tsushima, Phys. Rev. D 84, 051301 (2011).
[15] G. Ramalho, D. Jido and K. Tsushima, Phys. Rev. D 85, 093014 (2012).
[16] D. Jido, M. Doering and E. Oset, Phys. Rev. C 77, 065207 (2008).
[17] G. Ramalho and M. T. Peña, Phys. Rev. D 95, 014003 (2017).
[18] G. Ramalho and K. Tsushima, Phys. Rev. D 81, 074020 (2010).
[19] G. Ramalho and K. Tsushima, Phys. Rev. D 89, 073010 (2014).
[20] I. G. Aznauryan, Phys. Rev. C 76, 025212 (2007).
[21] G. Ramalho and D. Melnikov, Phys. Rev. D 97, 034037 (2018).
[22] G. Ramalho, Phys. Rev. D 96, 054021 (2017).
[23] G. Ramalho, M. T. Peña and F. Gross, Phys. Rev. D 81, 113011 (2010).
[24] G. Ramalho, M. T. Peña and F. Gross, Phys. Rev. D 78, 114017 (2008).
[25] G. Ramalho, M. T. Peña and F. Gross, Eur. Phys. J. A 36, 329 (2008).
[26] G. Ramalho, M. T. Peña, J. Weil, H. van Hees and U. Mosel, Phys. Rev. D 93, 033004 (2016).
[27] B. Julia-Diaz, T.-S. H. Lee, T. Sato and L. C. Smith, Phys. Rev. C 75, 015205 (2007).
[28] C. Alexandrou, G. Koutsou, H. Neff, J. W. Negele, W. Schroers and A. Tsapalis, Phys. Rev. D 77, 085012 (2008).
[29] G. Ramalho, Phys. Rev. D 94, 114001 (2016).
[30] G. Ramalho, Eur. Phys. J. A 54, 75 (2018).
[31] A. Blomberg *et al.*, Phys. Lett. B 760, 267 (2016).
[32] V. Pascalutsa and M. Vanderhaeghen, Phys. Rev. D 76, 115001 (2007).
[33] A. J. Buchmann, Phys. Rev. Lett. 93, 212301 (2004).
[34] G. Ramalho, Phys. Rev. D 93, 113012 (2016).
I. DETAILS OF THE MODEL

The radial wave function of the nucleon, which encode the dependence on the nucleon ($P$) and diquark ($k$) momentum, can be represented in terms of the dimensionless variable

$$\chi = \frac{(M_N - m_s)^2 - (P - k)^2}{M m_s}, \quad (2)$$

where $M_N$ is the mass of the nucleon and $m_s$ is the mass of the diquark. This particular dependence is possible because in the covariant spectator quark model the nucleon and the diquark are both on mass shell (more details can be found in Ref. [3]).

The explicit form of the radial wave function is

$$\psi_N(\chi) = \frac{N_0}{m_s(\beta_1 + \chi)(\beta_2 + \chi)}, \quad (3)$$

where $N_0$ is a normalization constant and $\beta_i$ are dimensionless parameters in units $M_N m_s$, which can be converted into square momentum scales.

The representation of the quark form factors is motivated by the vector dominance model

$$f_{1\pm}(Q^2) = \lambda + (1 - \lambda) \frac{m_v^2}{m_v^2 + Q^2} + c_{\pm} \frac{M_h^2 Q^2}{(M_h^2 + Q^2)^2},$$

$$f_{2\pm}(Q^2) = \kappa_{\pm} \left[ d_{\pm} \frac{m_v^2}{m_v^2 + Q^2} + (1 - d_{\pm}) \frac{M_h^2}{M_h^2 + Q^2} \right], \quad (4)$$

where $\kappa_{\pm}$ are the quark isoscalar/isovector anomalous magnetic moments, $m_v$ is a vector meson mass and $M_h$ is fixed heavy mass parameter (short range scale). In the model those are fixed as $m_v = m_{\rho} \simeq m_{\omega}$. The quark anomalous magnetic moments are determined by the proton and neutron magnetic moments ($Q^2 = 0$). $\lambda$ is a parameter fixed in the study of the deep inelastic scattering. The remaining parameters $c_{\pm}$ and $d_{\pm}$ are determined by the phenomenology, more specifically by the fit to the nucleon elastic form factors.

The minimal model (model II from Ref. [2]) fixes $M_h = 2M_N$ and $d_{+} = d_{-}$, leaving only 3 free parameters for the quark form factors ($c_{+}$, $c_{-}$ and $d_{+}$).