Space-Time-Polarization Adaptive Antenna Arrays for GNSS Receivers

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Abstract. This paper discusses about space-time-polarization adaptive antenna arrays (STPA) for global navigation satellite system (GNSS) receivers. The conventional right-handed circularly polarized (RHCP) antenna is in-situ replaced by the proposed dual-polarized antenna, which combines space, time, and polarization for high anti-interference performance. By signal modelling, we suggest an efficient generalized sidelobe canceller (GSC) structure of STPA to equivalent the conventional linearly constrained minimum variance (LCMV) beamformer. We demonstrate by simulations that 4-element dual-polarized STPA can suppress 7 broadband interferences with different direction and polarization. However, if RHCP characteristic of GNSS signal is constrained, 7 broadband RHCP interferences with the same polarization characteristic cannot be suppressed effectively. The performance of this scenario is related to the normalized bandwidth of interferences. At last, we demonstrate that 4-element STPA and 7-element conventional spatial adaptive array have comparable anti-interference performances.

1. Introduction

Adaptive antenna array is widely used in global navigation satellite system (GNSS) anti-interference receivers for the military application. These conventional spatial adaptive arrays focus the mainlobe on the relatively susceptible desired signal and null the interferences from other directions. Space-time adaptive processing (STAP) is paid extensive attention recently as it naturally extends array processor for wideband interference cancelling [1][2]. However, with the decrease of array aperture, its resolution will also decrease, and the performance will deteriorate seriously.

Polarization is another important information about electromagnetic wave, beside the amplitude, phase, and frequency[3][4]. In order to use polarization information, the traditional right-handed circularly polarized (RHCP) antenna in GNSS receiver can be in-situ replaced by the dual-polarized antenna for compact application. An M-element dual-polarized antenna array with space-polarization adaptive processing has $2M-1$ degrees of freedom in theory [5]. In addition, it’s exciting that through space-polarization processing dual-polarized antenna array can distinguish the signal and interference from almost the same direction but with different polarization[6]. Although the implementation complexity spends double, this is still acceptable.

Therefore, space-time-polarization adaptive antenna arrays (STPA)[7]–[9] are proposed in this paper for a compact and high performance GNSS anti-interference antenna. The well-known linearly constrained minimum variance (LCMV) beamformer is introduced to STPA to preserve the desired signal while minimize the output power of processor. The LCMV beamformer of STPA can be equivalent to the unconstrained and efficient generalized sidelobe canceller (GSC) structure [10]. The
GSC beamformer orthogonally decomposes the weight of LCMV beamformer into upper and lower branches, where the upper branch is the quiescent vector to satisfy constraint conditions, and the lower branch minimizes the output power. The GNSS signal may come from all upper half-space, and interferences are unknown, thus only polarization characteristic of the signal should be constrained, direction of arrival (DOA) and frequency response unconstrained [11].

2. Signal Module

In this section, STPA signal module will be established by polarized steering vector, spatial steering vector, and temporal frequency steering vector. Typical 4-element uniform circular array (UCA) is discussed in this paper. Each element in conventional UCA on xoY plane is in-situ replaced by a dual-polarized antenna, which includes a horizontal and a vertical linearly polarized antenna. In the spherical coordinate system, the electric vector $e$ of incident signal at arbitrary position $P(r, \theta, \phi)$ is perpendicular to the propagation direction $-r$, and has only $e_\theta$ and $e_\phi$ components. Ignoring the absolutely phases, $e$ in geometric mode is given by $e = A\mathbf{Q}(\tau)\mathbf{h}(e)$, where $A$ denotes the amplitude,

$$\mathbf{Q}(\tau) = \begin{bmatrix} \cos \tau & \sin \tau \\ -\sin \tau & \cos \tau \end{bmatrix}$$

(1)

denotes the rotation matrix, and $\mathbf{h}(e) = [\cos e \ j\sin e]^T$ denotes the ellipticity vector, $[\tau]^T$ denotes the transpose of a vector or matrix, $\tau$ denotes the rotation angle, $\tau \in [-\pi/2, \pi/2]$. Polarization of the electric vector can also be interpreted by phase mode $\mathbf{e} = A[\cos \gamma \ \sin \gamma e^{j\phi}]$, where $\tan \gamma$ denotes the axis ratio, $\gamma \in [0, \pi/2]$, $\delta$ denotes the phase difference of $e_\theta$ and $e_\phi$, and $\delta \in [-\pi, \pi]$. The polarized steering vector can be expressed in Cartesian coordinate system [6]

$$\mathbf{v}_p = (\sin \gamma \cos \theta \cos \phi e^{j\delta} - \cos \gamma \sin \phi)\hat{x} + (\sin \gamma \cos \theta \sin \phi e^{j\delta} + \cos \gamma \cos \phi)\hat{y} - (\sin \gamma \sin \theta e^{j\delta})\hat{z}$$

(2)

The array only senses the $x$ and $y$ components, therefore, the polarization vector can be simplified to $\mathbf{v}_p = [v_x \ v_y]^T$. Spatial steering vector of UCA is

$$\mathbf{v}_s = \begin{bmatrix} \exp(-j\phi_0) \exp(-j\phi_1) \cdots \exp(-j\phi_{M-1}) \end{bmatrix}$$

(3)

where $j = 2\pi / \lambda$ is the spatial phase constant, $\lambda$ is the wavelength, $R$ is the radius of UCA, and $\phi_i$ is the phase about element position. STPA has a number of tapped delay lines (TDLs) at each channel. The temporal frequency steering vector is $\mathbf{v}_t = [1 \ \exp(-2\pi f_s T) \cdots \exp(-2\pi f_s (L-1) T)]^T$, where $T_s$ denotes the sample period, $f_s \in [-B_w / 2, B_w / 2]$ denotes the centre frequency in baseband, and $B_w$ denotes the bandwidth of signal. STPA steering vector can be expressed by

$$\mathbf{v}_{sp} = \mathbf{v}_s \otimes \mathbf{v}_x \otimes \mathbf{v}_p$$

(4)

where $\otimes$ is the Kronecker product. Using the STPA steering vector, we can model the signal as

$$\mathbf{x} = A_0 \mathbf{v}_s \mathbf{s}_0(k) + \sum_{i=1}^{K} A_i \mathbf{v}_s \mathbf{s}_i(k) + \mathbf{n}(k)$$

(5)

where $A$, $\mathbf{v}$, $\mathbf{s}(k)$, $i = 0, \ldots, K$ represent the amplitude, the STPA steering vector, and the $k$ th instant waveform of signal or interferences respectively. $\mathbf{n}(k)$ is the zero-mean white noise, which is uncorrelated in space, time and polarization.

3. GSC of STPA

Instead of the conventional LCMV beamformer in STPA, an efficient implementation GSC structure is introduced to this section, which transforms the constraint problem of LCMV into a non-constrained issue. In a space-time processing structure with $M$ channels and $L$ TDLs, the $l$ th tap ($l = 0, 1, \ldots, L-1$) of inputs can be defined as $\mathbf{x}_l = [x_{l,0} \ x_{l,1} \cdots \ x_{l,M-1}]$, and the corresponding
weights have the similar form \( w_i = [w_{i,0}, w_{i,1}, \ldots, w_{i,M-1}] \), where \( x_{i,j} \) and \( w_{i,j} \) denote the \( l \) th tap and the \( i \) th element of input and weight respectively. By above definition, inputs and weights of space-time processor are then constructed by \( x = [x_0, x_1, \ldots, x_L]^T \), \( w = [w_0, w_1, \ldots, w_{L-1}]^T \). The \( k \) th instant output of the space-time processing is \( y(k) = w'^H x \), where \([\cdot]^T\) denotes the conjugate transpose of a vector or matrix. The LCMV beamformer can be described as an optimal problem of finding a \( \mathbf{w} \in \mathbb{C}^{M \times 1} \) that minimizes the power of the output among all weights that satisfy some constraints

\[
\mathbf{w}_{\text{opt}} = \arg \min_{\mathbf{w}} E[|y(k)|^2] = \arg \min_{\mathbf{w}} \mathbf{w}' \mathbf{R}_{xx} \mathbf{w} \quad \text{subject to} \quad \mathbf{C}' \mathbf{w} = \mathbf{f}
\]

(6)

where \( \mathbf{C} \in \mathbb{C}^{S \times L} \) denotes the constraint matrix, \( \mathbf{f} \in \mathbb{C}^{S \times 1} \) denotes the constraint response vector, \( S \) denotes the number of linearly independent constraints, \( S < M \), \( \mathbf{C} \) denotes complex space, and \( \mathbf{R}_{xx} = E[\mathbf{x} \mathbf{x}'^H] \) denotes the correlation matrix of the inputs. Using the method of Lagrange multipliers, the LCMV optimal weight is given by

\[
\mathbf{w}_{\text{opt}} = \mathbf{R}_{xx}^{-1} \mathbf{C} (\mathbf{C}' \mathbf{R}_{xx}^{-1} \mathbf{C})^{-1} \mathbf{f}
\]

(7)

The LCMV beamformer can be alternatively substituted by the unconstrained GSC structure[12]. The GSC beamformer orthogonally decomposes \( \mathbf{w} \) of LCMV into upper branch \( \mathbf{w}_u \) and lower branch \( \mathbf{w}_l \), that is \( \mathbf{w} = \mathbf{w}_u - \mathbf{w}_l \), where \( \mathbf{w}_u \in \text{range}(\mathbf{C}) \) is the quiescent vector to satisfy constraint conditions, and \( \mathbf{v} \in \text{null}(\mathbf{C}) \) minimizes the output power. The constraint conditions \( \mathbf{C}' \mathbf{w}_u = \mathbf{f} \) can be easily obtained from LCMV beamformer. \( \mathbf{w}_q \) is given through pseudo-inverse with the symbol (·)'\n
\[
\mathbf{w}_q = (\mathbf{C}' \mathbf{w})' = \mathbf{C} (\mathbf{C}' \mathbf{C})^{-1} \mathbf{f}
\]

(8)

The lower branch \( \mathbf{v} \) can be expressed by linear combination through the basis vectors of \( \text{null}(\mathbf{C}) \), i.e., once a block matrix \( \mathbf{B} \in \mathbb{C}^{ML \times (SL-1)} \) is constructed, such that \( \mathbf{C}' \mathbf{B} = \mathbf{0} \), then \( \mathbf{v} \) can be obtained from \( \mathbf{v} = \mathbf{B} \mathbf{w}_u \), where \( \mathbf{w}_u \in \mathbb{C}^{(ML \times SL)} \) is a redefined optimal vector. The output of the lower branch is given by \( \hat{y}(k) = (\mathbf{B} \mathbf{w}_u)' \mathbf{x} = \mathbf{w}_u'^H \mathbf{u}(k) \), where \( \mathbf{u}(k) = \mathbf{B}' \mathbf{x} \). The weight of LCMV can be replaced by \( \mathbf{w} = \mathbf{w}_q - \mathbf{B} \mathbf{w}_u \), we can get \( \mathbf{w}_{\text{opt}} \) of GSC through a new optimal weight

\[
\mathbf{w}_{a,\text{opt}} = (\mathbf{B}' \mathbf{R}_{xx} \mathbf{B})^{-1} \mathbf{B}' \mathbf{R}_{xx} \mathbf{w}_q
\]

(9)

The constraint matrix with \( S \) DOAs and \( L \) frequency bins is given by

\[
\mathbf{C} = [\mathbf{C}_0, \mathbf{C}_1, \ldots, \mathbf{C}_{S-1}]
\]

(10)

where \( \mathbf{C}_i \in \mathbb{C}^{M \times 1}, (i = 0, \ldots, S-1) \) means spatial and frequency bin constraints of the \( i \) th DOA

\[
\mathbf{C}_i = \begin{bmatrix}
\mathbf{c}_i \\
\vdots \\
0 \mathbf{c}_i
\end{bmatrix}
\]

(11)

where \( \mathbf{c}_i \in \mathbb{C}^{M \times 1} \) indicates the constrained spatial steering vector. The constrained response vector \( \mathbf{f} \in \mathbb{C}^{S \times 1} \) is given by \( \mathbf{f} = [\mathbf{a}_0 \mathbf{\bar{f}}', \ldots, \mathbf{a}_{S-1} \mathbf{\bar{f}}']' \), where \( \mathbf{a}_0, \ldots, \mathbf{a}_{S-1} \) are the coefficients about DOAs gain and \( \mathbf{\bar{f}}' = [\mathbf{f}_0', \ldots, \mathbf{f}_{L-1}']' \in \mathbb{C}^{L \times 1} \) is the frequency response vector which is equivalent to the coefficients of FIR filer of each channel. In space-time processing, the constraint matrix \( \mathbf{C} \) is obviously sparse, thus the block matrix \( \mathbf{B} \) is also sparse

\[
\mathbf{B} = \begin{bmatrix}
\tilde{\mathbf{B}} & 0 & \cdots & 0 \\
0 & \tilde{\mathbf{B}} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \tilde{\mathbf{B}}
\end{bmatrix}
\]

(12)
A smaller matrix $\mathbf{B} \in \mathbb{C}^{M \times (M-S)}$ can be obtained from $\tilde{\mathbf{C}}^H \mathbf{B} = 0$, where $\tilde{\mathbf{C}} = [\mathbf{c}_0 \cdots \mathbf{c}_{S-1}]$. The spatial quiescent vector is given by $\tilde{\mathbf{w}}_q = \tilde{\mathbf{C}} (\tilde{\mathbf{C}}^H \tilde{\mathbf{C}})^{-1} \mathbf{g}$, where $\mathbf{g} = [a_0 \cdots a_{S-1}]^T$. Then the upper branch quiescent vector is given by

$$
\mathbf{w}_q = \tilde{\mathbf{f}} \otimes \tilde{\mathbf{w}}_q = [f_0 \tilde{\mathbf{w}}_q^T \cdots f_{L-1} \tilde{\mathbf{w}}_q^T]^T
$$

(13)

The output of upper branch can be interpreted as $d(k) = \mathbf{w}_q^H \mathbf{x} = \sum_{i=0}^{L-1} f_i \hat{d}(k)$, where $\hat{d}(k) = \tilde{\mathbf{w}}_q^H \mathbf{x}_i(k)$, Figure 1 shows the efficient GSC structure of STPA. The number of channels extend from $M$ to $N = 2M$. The spatial-polarized input signals $\mathbf{x} = [x_{1H} \ x_{1V} \ \cdots \ x_{M_H} \ x_{M_V}]^T$ are decomposed into upper and lower branches. The upper branch constraints the DOAs, spatial frequency responses, and polarization of signal or interferences, the lower branch minimizes the output power. The widely used method to construct $\tilde{\mathbf{B}}$ is

$$
\tilde{\mathbf{B}} = \mathbf{I}_{sp} - \mathbf{v}_p \mathbf{v}_p^H
$$

(14)

where $\mathbf{I}_{sp}$ is an $N \times N$ identity matrix, and $\mathbf{v}_p = \mathbf{v}_s \otimes \mathbf{v}_p$ is spatial-polarized steering vector.

![Figure 1. GSC structure of STPA: $\tilde{\mathbf{w}}_q$ and $\tilde{\mathbf{B}}$ indicate spatial quiescent vector and block matrix, $S$ indicates the number of constrained DOAs.](image)

4. Simulation

In this section, we provide some simulations for STPA based on 4-element UCA, which is widely used in GNSS anti-interference adaptive antenna. We consider the polarization wavenumber response and the frequency wavenumber response of the GSC-STPA to show the results of different scenarios, and then compare the STPA to the conventional 7-element spatial adaptive array. There are $L = 10$ TDLs for each channel, and the distance between adjacent elements is $\lambda/2$. User received signal may come from all upper half-space, therefore, we only constrain the RHCP characteristic of GNSS signal, that is $\gamma = 45^\circ$, $\delta = -90^\circ$, $\zeta = -45^\circ$, $\mathbf{v}_p = \sqrt{2} [1 \ -j]^T$, $\mathbf{v}_s = [1 \ 0 \ \cdots \ 0]^T$, and $\tilde{\mathbf{f}} = [1 \ 0 \ \cdots \ 0]^T$. There are no constraints for unknown interferences. In order to simplify simulations and show the results clearly, we set $\phi = 30^\circ$ and interference-noise-ratio (INR) is 40dB for each interference.
4.1. Polarization wavenumber response

In this scenario, there are 7 broadband interferences with the normalized bandwidth 1 and $\tau_i = \tau_s = 30^\circ$. Figure 2 shows the contour of polarization wavenumber response. The parameters are specialized as $(\theta, \varepsilon) = \{(-73^\circ, -23^\circ), (-49^\circ, 35^\circ), (-25^\circ, -29^\circ), (10^\circ, 32^\circ), (31^\circ, 36^\circ), (55^\circ, 24^\circ), (76^\circ, -30^\circ)\}$.

We can see that 7 nulls are formed to suppress all broadband interferences, meanwhile other directions and ellipticity angles are not attenuated.

![Figure 2. Polarization wavenumber response of STPA.](image)

4.2. Frequency wavenumber responses

In this scenario, frequency wavenumber responses of the STPA are considered. 5 to 7 interferences are issued, which are relative to different center frequency, bandwidth, and DOA. All interferences are RHCP for worst consideration. In Figure 3 (a), there are 5 interferences with the normalized bandwidth 1, and $\theta = \{-72^\circ, -36^\circ, -3^\circ, 33^\circ, 69^\circ\}$, the result shows that nulls in whole bandwidth are generated at different DOA of interferences. Figure 3 (b) shows the result for 6 interferences with $\theta = \{-65^\circ, -51^\circ, -15^\circ, 24^\circ, 45^\circ, 63^\circ\}$, normalized center frequency $f_0 = [0.2, -0.25, 0, 0.25, -0.25, 0.1]$, and normalized bandwidth $B_w = [0.5, 0.5, 1, 0.5, 0.5, 0.6]$, that is about 60% frequency bandwidths of all DOA of interferences are occupied. It can be seen that six frequency bands at different DOA of interferences are formed to nulls, but these null depths decrease, this means if frequency bandwidths of interferences are occupied more, it will be unacceptable. Figure 3 (c) shows the simulation for 7 interferences with $\theta = \{-55^\circ, -44^\circ, -21^\circ, 0^\circ, 23^\circ, 45^\circ, 53^\circ\}$, $f_0 = [0.3, -0.3, 0.3, -0.3, 0.3, -0.3, 0.3]$. Each interference occupies the normalized bandwidth $B_w = 0.3$. The result is like previous, these null depths of different interference decrease, if the whole band of frequency are occupied, it will be unacceptable. These simulations indicate that $M$-element STPA cannot suppress $2M-1$ broadband RHCP interferences with normalized bandwidth 1. Its degrees of freedom to this situation are associated with its bandwidth.

![Figure 3. Resultants of frequency wavenumber response: (a) 5 interferences with 100% bandwidth, (b) 6 interferences with 60% bandwidth, (c) 7 interferences with 30% bandwidth.](image)
4.3. **Comparing to 7-element spatial adaptive array**

By averaging 100 independent results, Figure 4 compares the null depths of 4-element STPA and 7-element conventional spatial adaptive array as the number of interferences varies. All interferences are broadband with normalized bandwidth 1, and the DOAs and characteristics of polarization are selected randomly. The results show that as the number of interferences increasing, two kinds of adaptive array have comparable anti-interference performances.

![Figure 4. Null depth performance comparation of two kinds of adaptive antenna array.](image)

5. **Conclusion**

This paper suggested a high anti-interference performance STPA for GNSS receivers, which combined the space, time, and polarization. The proposed efficient implementation GSC-STPA structure transformed the constraint problem of LCMV into a non-constrained issue. In GNSS application, an $M$-element STPA could suppress $2M - 1$ interferences with different DOA and polarization, but it could not suppress $2M - 1$ RHCP interferences, which had the same polarization characteristics to GNSS signal. The performance of this scenario was relative to the normalized bandwidth. Lastly, we demonstrated 4-element STPA and 7-element conventional spatial array had comparable anti-interference performances. These methods could also be used for radar, MIMO, etc.

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