USE OF DYNAMIC REGRESSION MODEL FOR REDUCTION OF SHORTAGES IN DRUG SUPPLY

Aurelija BURINSKIENĖ*

Vilnius Gediminas Technical University, Vilnius, Lithuania

Received 2 October 2019; accepted 19 November 2019

Abstract. The study is given to the use of dynamic regression model for reduction of shortages in drug supply:

Purpose – the use of a dynamic regression model to identify the influence of lead-time on the reduction of time delays in drugs supply. To reach the goal, the author focuses on the improvement of drugs availability and the minimisation of time delays in drugs supply.

Research methodology – the application of dynamic regression method to minimise shortage. The author suggests a dynamic regression model and accompanies it with autocorrelation and heteroskedasticity tests: Breush-Godfrey Serial Correlation LM Test for autocorrelation and ARCH test for heteroskedasticity.

Findings – during analysis author identifies the relationship between lead-time and time delays in drugs supply. The author delivers a specific regression model to estimate the effect of deterministic lead-time on shortage. Probability F and Probability Chi-Square of this testing show that there is no significant autocorrelation and heteroskedasticity.

Research limitations – the research is delivered for a one-month time frame. For the future, the study could review other periods. The author has incorporated the lead-time component in shortage reduction study by leaving capacity uncertainty component unresearched. The future studies could incorporate both elements into shortage reduction case analysis.

Practical implications – presented framework could be useful for practitioners, which analyse drug shortage reduction cases. The revision of supply time table is recommended for pharmacies aiming to minimise the shortage level.

Originality/Value – the analysis of deterministic lead-time and identification that the periodicity of shortage is evident each eight days. The study contributes to lead-time uncertainty studies where most of the authors analyse the stochastic lead-time impact on shortages.

Keywords: supply, shortage, drugs, delays, regression, causes.

JEL Classification: C20, L65.

*Corresponding author. E-mail: aurelija.burinskiene@vgtu.lt

Copyright © 2019 The Author(s). Published by VGTU Press

This is an Open Access article distributed under the terms of the Creative Commons Attribution License (https://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.
Introduction

Shortages of medicines put patients at risk to get the most efficient health improvement. It is crucial to ensure the rights of patients and patients’ accessibility to health care, the right of access to preventive health care and the right to benefit from medical treatment. Each of the national health systems of the EU countries manifests quite different realities concerning patients’ rights.

However, there are political, economic, historical, environmental and other reasons causing shortage. The disruption of supply has negative impact for all supply chain actors. Many pharmaceutical manufacturers import raw components from India, China and Europe. If some of these foreign suppliers have supply disruption (Newman, 2016) due to political, economic, historical (Coomber, Moyle, & South, 2016), environmental issues, this could cause shortage. Delays in the supply of raw materials lead to overcrowding on the production when raw materials are received. For economic reasons, manufacturers may reduce their production volumes or cease the production. There are more cases where some manufacturers withdraw drugs from the market because they are less than profitable, or demand is higher than the available capabilities of manufacturer. By stating above mentioned reasons in studies, authors usually face the problem that it is not easy to provide solution which could help to minimize the impact on patients.

Author selects single component for the study and proposes solution for shortage reduction in paper below, which could result into better drug’s accessibility for patients.

The study consists of three parts. The first part is dedicated to the literature review and criticality of lead-time as supply parameter. The second part of the paper presents the methodology. It constructs a dynamic regression model. Finally, the author applied the proposed regression model. A case study is presented here and options for shortage reduction.

The results showed the tendencies of shortage cases appearance and bring a solution for their minimization.

1. Literature review

There are many inventory policies, but these do not consider supply constraints. In the literature, shortage causes are analyzed under the models dedicated to various activities: collection, production, inventory and delivery. A review of contemporary literature in the area of Operations research and Management Science was presented by Snyder et al. (2016). Their study focuses on supply chain disruptions and models present in the literature.

Authors Snyder et al. (2016) review 180 papers on the topic and identify five categories affecting supply disruptions: inventory, flexibility, sourcing, facility location, and interaction with partners. Some of these causes are presented below (see Table 1). Most of them are linked with distribution factors or human work aspects.

The models presented in the literature seek to plan an inventory considering internal supply parameters such as capacity and lead-time. Several forms of supply uncertainty are discussed by researchers. There are models with supply uncertainty, which include different period versions: single-period and multi-period.
Use of dynamic regression model for reduction of shortages in drug supply

There are models considering uncertainty, where demand is stochastic but is having continuous distribution. Other models assume probability that a supplier delivers the order with the highest reliability. When a supplier is unreliable, the disruption affects the lead-time – if disruption occurs with a fixed probability, the standard lead-time is increased by a stochastic delay. When likelihood is random, i.e. the quantity delivered or produced is a random variable, i.e. the supply quantity depends on order quantity. Researchers focus on probability, and they identify that the current period is influenced by supplier performance in the previous period.

Models with capacity uncertainty are treating capacity as a random variable, which is independent of order quantity.

For supply uncertainty analysis, authors, which are identified in Table 1, use linear regression, simulation techniques, Markov process, Bayesian model and other methods (Azghandi, Griffin, & Jalali, 2018). In these models, regression results are often transmitted directly into causal analysis or causal implications (e.g., searching for improvement of overall outcome). The authors point the attention that some studies with multiple regression analysis, had

| Shortage causes                              | Models                                                                 | Authors                                                                 |
|----------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------|
| Production lead-time                         | The stochastic aggregate production planning model                     | Mirzapour Al-e-Hashem, Baboli, and Sazvar (2013)                       |
| Nonconforming items                          | Stochastic integrated manufacturing and remanufacturing model with a shortage | Moshtagh and Taleizadeh (2017)                                        |
| Supplier fault                              | A deteriorating item inventory model with a shortage                  | Rau, Wu, and Wee (2004)                                               |
| Lack of integration in planning              | Integrated production and maintenance planning model with time windows and shortage | Najid, Alaoui-Selsouli, and Mohafid (2011)                             |
| A decision was taken by a single supply chain partner | An integrated inventory model for deteriorating items under a multi-echelon supply chain | Rau, Wu, and Wee (2003)                                               |
| Incoordination                               | Three-echelon supply chain model                                       | Heydari, Mahmoodi, and Taleizadeh (2016)                              |
| Inventory management                         | Vendor managed inventory of multi-item economic order quantity model under shortage | Nia, Far, and Niaki (2014)                                            |
| Bullwhip effect                             | Information hub model                                                  | Lee and Whang (2000)                                                  |
| Expiration of products                       | An inventory model for deteriorating items with expiration dates       | Tiwari, Cárdenas-Barrón, Goh, and Shaikh (2018)                       |
| Complexity in perishable supply chain        | Quantitative models in the blood supply chain                          | Osorio, Brailsford, and Smith (2015)                                 |
| Limited warehouse space                      | Optimal lot-sizing model of integrated multi-level multi-wholesaler supply chain | Hoseini Shekarabi, Gharei, and Karimi (2019)                          |
multicollinearity which appeared due to high correlations among independent variables and led to unreliable results (Schmitt, Kumar, Stecke, Glover, & Ehlen, 2017).

Among supply uncertainty topic, researchers investigate lead-time uncertainty and capacity uncertainty due to the modelling and managerial differences between them, as first one focus on period aspects and the second one – the lack of resources.

Below author is presenting the analysis of lead-time component for drug shortage reduction. Author has selected single component for empirical research as very important component to be studied.

1.1. A lead-time component in shortage studies

Lead-time plays a vital role in many areas, including the drug supply chain. The time component is investigated by authors Lacerda, Xambre, and Alvelos (2015), Rivera and Chen (2007), Dinis-Carvalho et al. (2015), Cuatrecasas-Arbo, Fortuny-Santos, and Vintrotro-Sanchez (2011) as a critical operational profile.

Lead-time uncertainty represents stochasticity in order processing. These authors analyse lead-time (or time from order-to-delivery) component of different nature: as stochastic, flexible, random, and randomly interruptible lead-time. Paul and Venkateswaran (2017) analysis lead-time component in drug production. There are studies, which focus on lead-time fluctuations, for example, those, which pay attention to Ripple effect (Sawik, 2017).

The lead-time component could be classified into two types:
1. Stochastic lead-time means that lead-time is a random variable;
2. Deterministic lead-time means that lead-time is fixed.

Many authors study the drug inventory problem with stochastic lead-time. The authors incorporating deterministic lead-time into their studies, assuming the lead-time is short-lasting. In the study below, the author focuses on the deterministic lead-time case, as the one to which was not given enough attention in previous studies.

Let’s assume that order is placed at the beginning of the planning horizon (t), and under normal conditions, the supply from the supplier is expected on (t+1), where (t+1) represents lead-time required for production and transportation. This lead-time is the minimum delivery time from the supplier. In unnatural conditions, the maximum delivery lead-time is given for supply. This case of supply is disbalancing products’ demand and leads to a shortage. When a product runs out of stock shortage appears.

It is evident that lead-time is strong component in inventory management and ordering. It is vital for ordering cycle time T, which is a period from one ordering point to another ordering point (i.e. time from order-to-order). And in case ordering and delivering points are fixed and specified by concrete weekdays, it consists time table.

The author in the study is giving attention to existing ordering and delivering time table and its effect on drug shortages.

2. Methodology

Aiming to identify if order and the delivering point is selected correctly, the author investigates the occurrence of shortage. The author performs systematic shortage analysis aiming to
identify shortage causes. The shortage of trendline for various products is analyzed, aiming to identify a shortage of days.

The author uses period analysis, where each period has n number of order days, m number of delivery days, and z number of drugs availability days.

Aiming to investigate if supply time table is directly linked with shortage, a dynamic regression method is used.

Shortly about the method. Let’s say that we need to predict $X(t+1)$ given $X(t)$. Then the source and target variables will look like as follows (see Table 2):

Table 2. Dynamic regression database for t+1 period

| X(t) | X(t+1) |
|------|--------|
| 1    | 2      |
| 2    | 3      |
| 3    | 4      |
| 4    | 5      |

Dataset would look like rolling windows of variables that follow a precedent one in succession (see Table 3).

Table 3. Dynamic regression database for periods interval between t-2 and t+1

| X(t-2) | X(t-1) | X(t) | X(t+1) |
|--------|--------|------|--------|
| 1      | 2      | 3    | 4      |
| 2      | 3      | 4    | 5      |

Then, the author uses the transformed dataset to figure out the autocorrelation coefficients from $X(t-2)$ to $X(t)$.

The author delivers a regression model, which general formulation is as follows:

$$sh_t = \beta_0 + \theta_{sh} L_{sh} + \beta_{dot} dot_t + \theta_{dot} L_{dot} + \varepsilon_t.$$  \hspace{1cm} (1)

Variables:

- $sh_t$ – a shortage of period $t$;
- $L_{sh}$ – lags operator for shortages;
- $dot_t$ – delivery on time of period $t$;
- $L_{dot}$ – lags operator for delivery on time;
- $\theta_{sh}$ – a matrix of coefficients for the lag operator of shortages;
- $\theta_{dot}$ – a matrix of coefficients for the lag operator of shortages;
- $\varepsilon_t$ – an error term (iid).

The dynamic regression has testing statistics: Breush-Godfrey Serial Correlation LM Test for autocorrelation and ARCH test for heteroskedasticity.
The application of constructed regression model is revised in the case study analysis, where the ordering cycle time is of 7 days.

For the analyse non-prescription drugs are selected. In study case the data is collected from pharma enterprise database and is analysed by calendar days and weekdays. The main data about 235 non-prescription drugs and 10 pharmacies is identified in the study, in such presentation: product ID, pharmacy ID, supplier ID, order day, order quantity, delivery day stated in order, actual delivery day, delivery quantity and quantity in stock at the end of particular day. For the statistical analysis eViews software is applied.

3. Results of the research

According time table, most of the deliveries are on Thursday and Friday, and most of the shortage is on Monday and Wednesday. This inforces the revision of suppliers time table and slight reschedule of delivery days. The average lead-time for products is 4.1 day (see Table 4). The number of generated orders per week is 1444 orders (or 6.1 orders per product), and the reliability of suppliers is 97 percentage, i.e. the supply performance of these orders. This shows that the orders are generated continuously, and that the reliability of suppliers is quite high.

Table 4. Lead-time parameters (4 weeks' time period)

| Number of products | Generated orders a week | Average-lead time (from order day to delivery day) | Reliability of suppliers (quantity delivered vs quantity ordered) |
|--------------------|-------------------------|--------------------------------------------------|---------------------------------------------------------------|
| 235                | 6.14                    | 4.1                                              | 97 percentage                                                 |

Statistical analysis shows that the number of pharmacies facing drug shortage is almost constant. In part of pharmacies which do not receive delivery from a supplier and has no buffer stock, the shortage is evident, and in another part of pharmacies, which have the buffer stock the shortage is not be present. Moller’s Junior 45 pieces, Neuromed 15 tablets (e.g. Figure 1), Carbon 300 mg 20 tablets (e.g. Figure 2), A+E 30 tablets and Super Validol 60 mg 10 tablets have the same shortage trend at the beginning of the month, and it is evident at the 2nd part of the month, such is not common only for Magnis+B6 complex and Humer 150 ml cases.

There is also systematic shortage, which appears at 2nd and 3rd day of the month for sample products.

There is also systematic shortage which increases at 10th day of the month for some products (following are evident in Figure 1 and Figure 2). Moreover, the supplier could revise delivery schedules from the producer as at the same time; various pharmacies struggle with a high level of shortage if they do not have a buffer for problematic products.

The author has investigated that ordering cycle time is directly linked with shortages. From Table 4 parameters analysis, we could see that minimisation of lead-time, especially at the beginning of the month, is the priority. The highest shortage occurs on weekend days when there are no deliveries from suppliers. In case the supplier is delivering less than ordered, a new order is processed only after one day.
A. Burinskienė. Use of dynamic regression model for reduction of shortages in drug supply

The author finds out the probabilities of events: the likelihood that shortage lasts two days is 38.7 percentage. The possibility of delivery is 82 percent on the day of shortage. The probability of facing shortage next day after delivery is 23.4 percentage. Below the application of dynamic regression model is provided.

The application of dynamic regression techniques gives interesting results. Below is the chart presenting values of normalised variables (e.g. Figure 3) and main statistics attributes. The correlation coefficient is equal to 0.65, and the R squared of the regression is 0.6.

The author delivered a specific regression model, which formulation is:

\[ sh_t = \beta_0 + \beta_1 sh_{t-1} + \beta_2 sh_{t-8} + \beta_3 dot_t + \beta_4 dot_{t-1} + \epsilon_t. \]  

(2)

This dynamic regression delivered results as follows:

\[ sh_t = 0.095 + 0.583 sh_{t-1} + 0.056 sh_{t-8} + 0.148 dot_t - 0.07 dot_{t-1}. \]

\[ (5.69) \quad (19.72) \quad (2.54) \quad (15.61) \quad (-6.7) \]
The equation is presenting t-statistic. The equation shows that if the shortage appears, it lasts for one period more (t+1). The periodicity of shortage is each eight days. Having the data starting from Monday, the statistical significance of 8th lagged value, which means that shortages occur after each weekend. This is the evidence of not enough supplies that occur periodically. Such a period cycle is very closely linked with a delivery time table.

The dynamic regression model is presented graphically in Figure 4.

Figure 3. Dynamic regression results
where: on the x-axis – number of time series, on the primary y-axis (on the left) – delivery in calendar days, on the secondary y-axis (on the right) – the appearance of normalised shortage event

Figure 4. Dynamic regression results
where: on the x-axis – number of time series, on the primary y-axis (on the left) – output of regression model, on the secondary y-axis (on the right) – the appearance of shortage event
In Figure 4, the red line represents the original shortage, while the green line demonstrates the modelled shortage according to the estimated equation; the blue line shows residuals of the dynamic regression model.

The author provides also testing statistics for autocorrelation and heteroskedasticity. Main tests are provided for autocorrelation analysis – Breusch-Godfrey, for heteroskedasticity analysis – ARCH test and ML ARCH – Normal distribution (BFGS / Marquardt steps) test. Probability F and Probability Chi-Square of these testing statistics show that there is no significant autocorrelation and heteroskedasticity. The more detailed results of the dynamic regression analyses are presented in the Annex of this paper.

The author obtains the evidence that ordering cycle time is directly linked with the number of shortage cases. According to the research results, it is possible to shorten ordering cycle time by one day aiming to reduce the number of shortage cases. Author suggests the reduction of ordering cycle time to 6 days, instead of the existing one with 7 days.

The provided solution formulation could be treated as shortage research framework evaluating shortage cases.

Conclusions

The case study shows that lead-time component must be revised in drug supply. Also, the drug supply chain must be tightened up, as in most of the cases. According to the literature time constrain is main attribute for shortage avoidance.

The author researches the lead-time component for shortage reduction purposes. To respond to studies with unsuccessful multi-regression analysis, the author selects dynamic regression technique and constructed the model. The model is applied for pharmaceutical supply chain case study.

The case study shows that if shortage appears a day before, then is the probability of 38.7% that it will occur on the next day. The equation shows that on the day of shortage the likelihood of delivery is 82%. Which means that those deliveries come at the end of a day not at the beginning thereof and the lead-time between ordering and delivering is too long. If delivery occurs, it diminishes probability on shortage next day by 23.4%. The empirical part of the study confirms a dynamic regression model and proves that time table improvement could help to minimize drug shortages. The performed practical assessment shows that the suggested framework is applicable for the delivery of drug shortage reduction.

The findings suggest future research directions. The study results also give insights on the necessity to have more frequent deliveries for products. The action to be taken for supplier time table revision aiming to increase the number of deliveries and minimize shortage at the beginning of the month. Also, future studies may include capacity attribute as playing important role for shortage minimization, into these studies.

References

Azghandi, R., Griffin, J., & Jalali, M. S. (2018). Minimisation of drug shortages in pharmaceutical supply chains: A simulation-based analysis of drug recall patterns and inventory policies. *Complexity, 2018*, 6348413. https://doi.org/10.1155/2018/6348413
Cuatrecasas-Arbo, L., Fortuny-Santos, J., & Vintro-Sanchez, C. (2011). The operations-time chart: A graphical tool to evaluate the performance of production systems – from batch-and-queue to Lean Manufacturing. Computers & Industrial Engineering, 61(3), 663-675. https://doi.org/10.1016/j.cie.2011.04.022

Coomber, R., Moyle, L., & South, N. (2016). The normalisation of drug supply: The social supply of drugs as the “other side” of the history of normalisation. Drugs: Education, Prevention and Policy, 23(3), 255-263. https://doi.org/10.1016/j.drugpo.2017.01.016

Dinis-Carvalho, J., Moreira, F., Braganca, S., Costa, E., Alves, A., & Sousa, R. (2015). Waste identification diagrams. Production Planning & Control: The Management of Operations, 20(3), 235-247. https://doi.org/10.1111/jtc.v76.3659

Heydari, J., Mahmoodi, M., & Taleizadeh, A. A. (2016). Lead time aggregation: A three-echelon supply chain model. Transportation Research Part E: Logistics and Transportation Review, 89, 215-233. https://doi.org/10.1016/j.tre.2016.03.006

Hoseini Shekarabi, S. A., Gharaei, A., & Karimi, M. (2019). Modelling and optimal lot-sizing of integrated multi-level multi-wholesaler supply chains under the shortage and limited warehouse space: generalised outer approximation. International Journal of System Science: Operations & Logistics, 6(3), 237-257. https://doi.org/10.1080/23302674.2018.1435835

Lacerda, A. P., Xambre A. R., & Alvelos, H. M. (2015). Applying value stream mapping to eliminate waste. International Journal of Production Research, 54, 1708-1720. https://doi.org/10.1080/00207543.2015.1055349

Lee, H. L., & Whang, S. (2000). Information sharing in a supply chain. International Journal of Technology Management, 20(3/4), 79-93. https://doi.org/10.1504/IJMTM.2000.001329

Mirzapour Al-e-Hashem, S. M. J., Baboli, A., & Sazvar, Z. (2013). A stochastic aggregate production planning model in a green supply chain: Considering flexible lead times, nonlinear purchase and shortage cost functions. European Journal of Operational Research, 230(1), 26-41. https://doi.org/10.1016/j.ejor.2013.03.033

Moshtagh, M. S., & Taleizadeh, A. A. (2017). Stochastic integrated manufacturing and remanufacturing model with shortage, rework and quality based return rate in a closed loop supply chain. Journal of Cleaner Production, 141, 1548-1573. https://doi.org/10.1016/j.jclepro.2016.09.173

Najid, N. M., Alaoui-Selsouli, M., & Mohafid, A. (2011). An integrated production and maintenance planning model with time windows and shortage cost. International Journal of Production Research, 49(8), 2265-2283. https://doi.org/10.1080/00207541003620386

Nia, A. R., Far, M. H., & Niaki, S. T. A. (2014). A fuzzy vendor managed inventory of multi-item economic order quantity model under shortage: An ant colony optimization algorithm. International Journal of Production Economics, 155, 259-271. https://doi.org/10.1016/j.ijpe.2013.07.017

Newman, D. J. (2016). Developing natural product drugs: Supply problems and how they have been overcome. Pharmacology and Therapeutics, 162, 1-9. https://doi.org/10.1016/j.pharmthera.2015.12.002

Osorio, A. F., Brailsford, S. C., & Smith, H. K. (2015). A structured review of quantitative models in the blood supply chain: a taxonomic framework for decision-making. International Journal of Production Research, 53(24), 7191-7212. https://doi.org/10.1080/00207543.2015.1005766

Paul, S., & Venkateswaran, J. (2017). Impact of drug supply chain on the dynamics of infectious diseases. System Dynamics Review, 33(3-4), 280-310. https://doi.org/10.1002/sdr.1592

Rau, H., Wu, M. Y., & Wee, H. M. (2004). Deteriorating item inventory model with shortage due to supplier in an integrated supply chain. International Journal of Systems Science, 35(5), 293-303. https://doi.org/10.1080/0020772041000171483

Rau, H., Wu, M. Y., & Wee, H. M. (2003). Integrated inventory model for deteriorating items under a multi-echelon supply chain environment. International Journal of Production Economics, 86(2), 155-168. https://doi.org/10.1016/S0925-5273(03)00048-3
Rivera, L., & Chen, F. F. (2007). Measuring the impact of lean tools on the cost–time investment of a product using cost–time profiles. *Robotics and Computer-Integrated Manufacturing, 23*(6), 684-689. https://doi.org/10.1016/j.rcim.2007.02.013

Sawik, T. (2017). A portfolio approach to supply chain disruption management. *International Journal of Production Research, 55*(7), 1970-1991. https://doi.org/10.1080/00207543.2016.1249432

Schmitt, T. G., Kumar, S., Stecke, K. E., Glover, F. W., & Ehlen, M. A. (2017). Mitigating disruptions in a multi-echelon supply chain using adaptive ordering. *Omega, 68*, 185-198. https://doi.org/10.1016/j.omega.2016.07.004

Snyder, L. V., Atan, Z., Peng, P., Rong, Y., Schmitt, A., & Sinsoysal, B. (2016). OR/MS Models for supply chain disruptions: a review. *IIE Transactions, 48*(2), 89-109. https://doi.org/10.2139/ssrn.1689882

Tiwari, S., Cárdenas-Barrón, L. E., Goh, M., & Shaikh, A. A. (2018). Joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade credits in supply chain. *International Journal of Production Economics, 200*, 16-36. https://doi.org/10.1016/j.ijpe.2018.03.006

Appendix

Formation of Equation (1)

| Dependent Variable: SHORTG_YES | Method: Least Squares |
|-------------------------------|-----------------------|
| Date: 04/26/19 Time: 08:53   | Sample (adjusted): 9852 |
| Included observations: 944 after adjustments |

| Variable       | Coefficient | Std. Error | t-Statistic | Prob.  |
|----------------|-------------|------------|-------------|--------|
| C              | 0.095433    | 0.018744   | 5.899641    | 0.0000 |
| SHORTG_YES(-1) | 0.582895    | 0.029529   | 19.73968    | 0.0000 |
| SHORTG_YES(-8) | 0.056079    | 0.022067   | 2.541297    | 0.0112 |
| DEL_ON_TIME   | 0.147911    | 0.009473   | 15.61323    | 0.0000 |
| DEL_ON_TIME(-1) | -0.069225  | 0.010333   | -6.889287   | 0.0000 |

| R-squared      | 0.599322 | Mean dependent var | 0.438389 |
| Adjusted R-squared | 0.597412 | S.D. dependent var | 0.498484 |
| S.E. of regression | 0.315018 | Akaike info criterion | 0.533534 |
| Sum squared resid | 83.25938 | Schwarz criterion | 0.581603 |
| Log likelihood  | -220.1512 | Hannan-Quinn crit. | 0.544289 |
| F-statistic     | 313.7378 | Durbin-Watson stat | 1.934439 |
| Prob(F-statistic)| 0.00000 |                    |          |
Analysis of autocorrelation (AC and PAC values are lower than 0.1)

![Correlogram of Residuals](image)

**Correlogram of Residuals**

- Date: 04/20/19  Time: 08:54
- Sample: 1852
- Included observations: 844
- Q-statistic probabilities adjusted for 2 dynamic regressors

| Autocorrelation | Partial Correlation | AC   | PAC   | Q-Stat | Prob* |
|-----------------|---------------------|------|-------|--------|-------|
| 0.032           | 0.032               | 0.8425| 0.359  |        |       |
| -0.066          | -0.067              | 4.5575| 0.102  |        |       |
| -0.004          | 0.001               | 4.5687| 0.206  |        |       |
| 0.021           | 0.017               | 4.9490| 0.293  |        |       |
| 0.008           | 0.007               | 5.0039| 0.415  |        |       |
| 0.027           | 0.030               | 5.6417| 0.464  |        |       |
| -0.015          | -0.016              | 5.8227| 0.561  |        |       |
| -0.003          | 0.002               | 5.8299| 0.668  |        |       |
| -0.004          | -0.006              | 5.8442| 0.755  |        |       |
| -0.002          | -0.003              | 5.8462| 0.828  |        |       |
| 0.002           | 0.001               | 5.8489| 0.883  |        |       |
| 0.008           | 0.008               | 5.9102| 0.921  |        |       |
| -0.029          | -0.028              | 6.6223| 0.921  |        |       |
| -0.028          | -0.026              | 7.3092| 0.922  |        |       |
| 0.012           | 0.010               | 7.4312| 0.945  |        |       |
| 0.015           | 0.010               | 7.8230| 0.959  |        |       |
| -0.042          | -0.041              | 9.1228| 0.936  |        |       |
| -0.024          | -0.019              | 9.6200| 0.944  |        |       |
| -0.003          | -0.006              | 9.6295| 0.962  |        |       |
| 0.049           | 0.047               | 11.684| 0.927  |        |       |
| -0.043          | -0.048              | 13.310| 0.897  |        |       |
| -0.025          | -0.015              | 13.834| 0.907  |        |       |
| -0.007          | -0.009              | 13.882| 0.930  |        |       |
| -0.058          | -0.063              | 16.799| 0.857  |        |       |
| 0.015           | 0.020               | 16.993| 0.882  |        |       |
| 0.012           | 0.001               | 17.129| 0.905  |        |       |
| 0.018           | 0.023               | 17.417| 0.920  |        |       |
| 0.005           | 0.008               | 17.438| 0.939  |        |       |
| -0.001          | 0.003               | 17.436| 0.955  |        |       |
| 0.050           | 0.053               | 19.658| 0.925  |        |       |
| 0.005           | -0.008              | 19.680| 0.942  |        |       |
| -0.029          | -0.025              | 20.438| 0.943  |        |       |
| -0.072          | -0.069              | 25.035| 0.839  |        |       |
| 0.003           | 0.002               | 25.043| 0.868  |        |       |
| -0.008          | -0.023              | 25.093| 0.892  |        |       |
| 0.047           | 0.046               | 27.022| 0.860  |        |       |

*Probabilities may not be valid for this equation specification.
Autocorrelation analysis: Breusch-Godfrey test

| Breusch-Godfrey Serial Correlation LM Test |
|-------------------------------------------|
| Null hypothesis: No serial correlation at up to 9 lags |
| F-statistic | R-squared | Prob. F(9, 830) | Prob. Chi-Square(9) |
|------------|-----------|-----------------|-------------------|
| 0.668200   | 0.071249  | 0.7382          | 0.7328            |

Test Equation:
- Dependent Variable: RESID
- Method: Least Squares
- Date: 04/26/19 Time: 08:54
- Sample: 9,852
- Included observations: 844
- Presample missing value lagged residuals set to zero.

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|----------|-------------|------------|-------------|-------|
| C        | 0.001735    | 0.053303   | 0.032547    | 0.9740 |
| SHORTG  | -0.028065   | 0.202753   | -0.136418   | 0.8999 |
| DEL ON TIME | 0.010765  | 0.046505   | 0.231917    | 0.8167 |
| RESID(-1) | 0.008802   | 0.009574   | 0.083791    | 0.9332 |
| RESID(5)  | 0.006575    | 0.033778   | 0.194643    | 0.8457 |
| RESID(8)  | 0.051239    | 0.206975   | 0.297164    | 0.7664 |
| RESID(2)  | -0.033456   | 0.120834   | -0.442772   | 0.6530 |
| RESID(4)  | 0.007793    | 0.076716   | 0.101290    | 0.9193 |
| RESID(3)  | 0.023031    | 0.056944   | 0.404450    | 0.6860 |
| RESID(5)  | 0.007827    | 0.045765   | 0.171029    | 0.8642 |
| RESID(6)  | 0.033034    | 0.040417   | 0.822829    | 0.4108 |
| RESID(-7) | -0.015132   | 0.036560   | -0.413891   | 0.6791 |
| RESID(-8) | -0.008265   | 0.004204   | -0.151479   | 0.8796 |
| RESID(-9) | -0.012116   | 0.043559   | -0.278158   | 0.7810 |

R-squared: 0.007183 Mean dependent var: 5.20E-17
Adjusted R-squared: -0.000857 S.D. dependent var: 0.314270
S.E. of regression: 0.315580 Akaike info criterion: 0.547641
Sum squared resid: 82.66046 Schwarz criterion: 0.625236
Log likelihood: -217.1046 Hannan-Quinn criter: 0.577757
F-statistic: 0.462600 Durbin-Watson stat: 1.996361
Prob(F-statistic): 0.944990

Heteroskedasticity analysis: ARCH test

| Heteroskedasticity Test: ARCH |
|------------------------------|
| F-statistic | R-squared | Prob. F(1, 841) | Prob. Chi-Square(1) |
|-------------|-----------|-----------------|-------------------|
| 0.003210    | 0.003218  | 0.9548          | 0.9548            |

Test Equation:
- Dependent Variable: RESID^2
- Method: Least Squares
- Date: 04/26/19 Time: 08:55
- Sample (adjusted): 10,852
- Included observations: 843 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|----------|-------------|------------|-------------|-------|
| C        | 0.008450    | 0.007079   | 13.90700    | 0.0000 |
| RESID^2(-1) | 0.001954  | 0.004483   | 0.056660    | 0.9548 |

R-squared: 0.000034 Mean dependent var: 0.000043
Adjusted R-squared: -0.000185 S.D. dependent var: 0.180132
S.E. of regression: 0.180259 Akaike info criterion: -0.566478
Sum squared resid: 27.32679 Schwarz criterion: -0.575239
Log likelihood: 249.2003 Hannan-Quinn criter: -0.582171
F-statistic: 0.003210 Durbin-Watson stat: 2.000224
Prob(F-statistic): 0.954829
Heteroskedasticity analysis: ML ARCH – Normal distribution test

| Variable         | Coefficient | Std. Error | z-Statistic | Prob.  |
|------------------|-------------|------------|-------------|--------|
| C                | 0.091819    | 0.037765   | 2.431324    | 0.0150 |
| SHORTG_YES(-1)   | 0.583175    | 0.051614   | 11.29875    | 0.0000 |
| SHORTG_YES(b)    | 0.051871    | 0.022904   | 2.274666    | 0.0229 |
| DEL ON TIME      | 0.150258    | 0.011621   | 12.92983    | 0.0000 |
| DEL_ON_TIME(-1)  | -0.069213   | 0.009375   | -7.382485   | 0.0000 |

| Variance Equation |
|-------------------|
| C                 | 0.032383     | 0.006430   | -5.036148   | 0.0000 |
| RESID(-1)^2       | -0.011394    | 0.018700   | -0.605379   | 0.5443 |
| RESID(-2)^2       | 0.075015     | 0.024421   | 3.071751    | 0.0021 |
| GARCH(-1)         | 1.209086     | 0.134534   | 8.987239    | 0.0000 |
| GARCH(-2)         | -0.601848    | 0.116540   | -5.164293   | 0.0000 |

| Statistics        | Value       |
|-------------------|-------------|
| R-squared         | 0.599212    |
| Mean dependent var| 0.438339    |
| S.E. of regression| 0.315061    |
| Akaike info criterion| 0.520295 |
| S.D. dependent var | 0.497301    |
| Schwarz criterion | 0.576434    |
| Log likelihood    | -209.5645   |
| Hannan-Quinn criter. | 0.541807  |

Durbin-Watson stat 1.938894