Generalized Form Factors, Generalized Parton Distributions and Spin Contents of the Nucleon

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Abstract.
A model independent prediction is given for the nucleon spin contents, based only upon a reasonable theoretical postulate on the isosinglet anomalous gravitomagnetic moment of the nucleon.

Keywords: spin contents of the nucleon, generalized parton distributions, Ji’s sum rule

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INTRODUCTION

If the intrinsic quark spin carries little of the total nucleon spin, what carries the rest of the nucleon spin? That is the question we must answer. Quark orbital angular momentum (OAM) $L^Q$? Gluon OAM $L^g$? Or gluon polarization $\Delta g$? Several remarks are in order here. The recent COMPASS measurement of the quasi-real photoproduction of high-$p_T$ hadron pairs indicates that $\Delta g$ is likely to be small [1]. There also appeared an interesting paper by Brodsky and Gardner, in which, based on the conjecture on the relation between the Sivers mechanism and the quark and gluon OAM, it was argued that small single-spin asymmetry observed by the COMPASS collaboration on the deuteron target is an indication of small gluon OAM [2].

On the other hand, the importance of quark OAM was pointed out many years ago based on the chiral soliton picture of the nucleon : first within the Skyrme model [3], second within the Chiral Quark Soliton Model [4]. According the latter, the dominance of quark OAM is intimately connected with collective motion of quarks in the rotating hedgehog mean field. The CQSM predicts at the model energy scale around 600 MeV that $\Delta \Sigma$ is around 0.35, while $2L_q$ is around 0.65. The CQSM also well reproduces the spin structure functions for the proton, neutron and the deuteron [5],[6]. Very recently, new measurement of the deuteron spin structure function $g_1^d(x,Q^2)$ was reported by the COMPASS group [7]. The precise measurement of $g_1^d(x,Q^2)$ is very important, since, aside from a small effects of $s$-quark polarization as well as the nuclear effects, it is just proportional to the iso-singlet spin distribution, the integral of which gives the intrinsic quark-spin contribution to the nucleon spin. The left panel of Fig.1 show the comparison between our predictions for $xg_1^d(x,Q^2)$ given several years ago and the new COMPASS data as well as the old SMC data, while the right panel of Fig.1 gives a similar comparison for the quantity $g_1^{N}(x,Q^2) = g_1^d(x,Q^2)/(1-1.5\omega_D)$ with $\omega_D = 0.05 \pm 0.01$. One can clearly see that the new COMPASS data became closer to our prediction especially at the smaller $x$ region. This means that the CQSM reproduces not only the magnitude of $\Delta \Sigma$ but also its Feynman $x$-space distribution as well.
FIGURE 1. The predictions of the SU(2) and SU(3) CQSM in comparison with the new COMPASS data for $xg_1^d(x)$ (left panel) and $g_1^N(x)$ (right panel). The old SMC data are also shown.

The recent noteworthy development is the possibility of direct measurement of $J_q$ and $L_q$, through the generalized parton distribution functions appearing in high-energy deeply virtual Compton scatterings and deeply virtual meson productions. What plays a crucial role here is the celebrated Ji’s angular momentum sum rule.

GENERALIZED FORM FACTORS AND JI’S ANGULAR MOMENTUM SUM RULE

The generalized form factors $A_{20}(t)$ and $B_{20}(t)$ for the quarks and gluons are defined as the nucleon nonforward matrix elements of QCD energy momentum tensor $T^{\mu\nu}$. The famous Ji’s sum rule relates the total angular momentum carried by quarks and gluons to the forward limits of these generalized form factors:

$$J_q = \frac{1}{2} \left[ A_{20}^{u+d}(0) + B_{20}^{u+d}(0) \right], \quad J_g = \frac{1}{2} \left[ A_{20}^g(0) + B_{20}^g(0) \right].$$

Here, the first $A$ part reduces to the total momentum fractions of quarks and gluons, $A_{20}^{u+d}(0) = \langle x \rangle^{u+d}$, $A_{20}^g(0) = \langle x \rangle^g$, while the second $B$ part represents the quark and gluon contribution to the anomalous gravitomagnetic moment (AGM) of the nucleon. An important observation is that the total nucleon AGM identically vanishes: $B_{20}^{u+d}(0) + B_{20}^g(0) = 0$. This follows from the two sound identities, i.e. the total momentum sum rule $\langle x \rangle^{u+d} + \langle x \rangle^g = 1$ and the total angular momentum sum rule $J_q + J_g = 1/2$ of the nucleon. Here, there are two possibilities. In the 1st case, the quark and gluon contribution to the nucleon AGM vanishes separately. In the 2nd case, both have sizable magnitude with opposite signs. Fig.2 shows the theoretical predictions for the relevant AGM form factors [8]. As one sees, both of LHPC lattice QCD simulation [9] and the CQSM predicts very small AGM consistent with zero.
The nucleon AGM is a measurable quantity, since it is given as the Mellin moment of unpolarized GPD $E(x, \xi, t)$. For model calculation, the spin decomposition is most conveniently done in the Breit frame. In this frame, the following combinations of GPDs $H$ and $E$ appear:

$$H_E(x, \xi, t) = H(x, \xi, t) + \frac{1}{4M_N^2} E(x, \xi, t), \quad E_M(x, \xi, t) = H(x, \xi, t) + E(x, \xi, t),$$

which corresponds to the famous Sachs decomposition of the nucleon electromagnetic form factors. Mellin moment sum rules relevant to our discussion are as follows. The 1st moment of GPD $E_M$ reduces to the isoscalar combination of magnetic moment, which consists of canonical part and anomalous part.

$$\int_{-1}^{1} E_M^{u+d}(x, 0, 0) dx = \int_{-1}^{1} H^{u+d}(x, 0, 0) dx + \int_{-1}^{1} E^{u+d}(x, 0, 0) dx = (N^u + N^d) + (\kappa^u + \kappa^d) = \mu^u + \mu^d.$$

On the other hand, the 2nd moment gives the total angular momentum carried by quarks, which is also made up of canonical part and anomalous part.

$$\int_{-1}^{1} x E_M^{u+d}(x, 0, 0) dx = \int_{-1}^{1} x H^{u+d}(x, 0, 0) dx + \int_{-1}^{1} x E^{u+d}(x, 0, 0) dx = (\langle x \rangle^u + \langle x \rangle^d) + B_{20}^{u+d}(0) = 2(\langle J^u + J^d \rangle).$$

An important fact to remember here is that the isoscalar anomalous magnetic moment of the nucleon is very small, which means that the $x$ integral of $E^{u+d}(x, 0, 0)$ is small. This also indicates that $B_{20}^{u+d}(0)$, obtained as a $x$-weighted integral of $E^{u+d}(x, 0, 0)$, would be even smaller, assuming a smooth behavior of $E^{u+d}(x, 0, 0)$. Curiously, the smoothness assumption of $E^{u+d}(x, 0, 0)$ is not respected by the CQSM, due to the effects of polarized Dirac sea. Nevertheless, the above conjecture, i.e. the smallness or the absence of the net quark contribution to the nucleon anomalous AGM is realized in a nontrivial manner. (See ref.[8], for more detail.)

In view of the observation above, let us assume smallness of $B_{20}^d(0)$ and $B_{20}^s(0)$, and set them to 0, for simplicity. We are then led to surprisingly simple relations, i.e. the
proportionality of the linear and angular momentum fractions for both of quarks and gluons:

\[ J_Q = \frac{1}{2} \langle x \rangle_Q, \quad J_g = \frac{1}{2} \langle x \rangle_g. \]

Important facts to remember here are that the quark and gluon momentum fractions are already well determined through the empirical PDF fits. Furthermore, this proportionality relation holds scale-independently, since \( J_Q \) and \( \langle x \rangle_Q \) and also \( J_g \) and \( \langle x \rangle_g \) obey exactly the same evolution equations. There also exist phenomenological fits for the longitudinally polarized PDFs, which contains the information for \( \Delta \Sigma \) and \( \Delta g \), although with larger uncertainties, compared with the unpolarized case. These information are enough to determine the quark and gluon OAM as functions of energy scale. Imagine now that the answers obtained in this way at \( Q^2 = 4 \text{GeV}^2 \) are evolved down to low energy model scale. Around the energy scale of 600MeV, one would then obtain the following estimate for the nucleon spin contents:

\[ 2J_{u+d} = \langle x \rangle_{u+d} \simeq (0.75 \sim 0.8), \quad 2J_g = \langle x \rangle_g \simeq (0.2 \sim 0.25), \]

\[ \Delta \Sigma \simeq (0.2 \sim 0.3), \quad 2L_{u+d} \simeq (0.45 \sim 0.6). \]

which indicates that nearly half of the nucleon spin comes from the quark OAM.

**SUMMARY AND CONCLUSION**

\( L_Q \) or \( \Delta g \)? There has been long-lasting dispute over this issue. Here, we have shown that, based only upon a reasonable theoretical postulate, i.e. the smallness of the net quark contribution to the nucleon AGM, we are naturally led to the following conclusion for the nucleon spin contents. At the low energy scale around 600MeV, about 30% of the nucleon spin is due to the intrinsic quark spin, whereas about (40 \sim 50) % comes from the quark OAM. The remaining (20 \sim 30) % of the nucleon spin is likely to be carried by the gluon fields. The last statement is not inconsistent with the recent observation by Brodsky and Gardener, since what would be related to the Sivers mechanism is the anomalous part of the gluon OAM. On the other hand, our postulated identity \( 2J_g \simeq \langle x \rangle_g \) implies that this gluon angular momentum comes totally from its canonical orbital motion, not from the anomalous contribution related to the GPD \( E(x, \xi, t) \).

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