Broad Spiral-Bandwidth of Orbital Angular Momentum Interface between Photon and Memory

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The complex interactions between orbital angular momentum (OAM) light and atoms are particularly intriguing in the areas of quantum optics and quantum information science. Building a versatile high-dimensional quantum network needs broad spiral-bandwidth for preparing high-quaanta OAM mode and resolving the bandwidth mismatch in spatial space and etc. Here, we experimentally demonstrate a broad spiral-bandwidth quantum interface between photon and memory. Through twisted fields of the writing and reading, the correlated OAM distribution between photon and memory is significantly broadened. This broad spiral-bandwidth quantum interface could be spanned in multiplexing regime and could work in high-quaanta scenario with capability of $|l| = 30$, and we demonstrate the entanglement within 2-D subspace with a fidelity of 80.5±4.8% for high $l$.

Such state-of-the-art technology to freely control the spatial distribution of OAM memory is very helpful to construct high-dimensional quantum networks and provides a benchmark in the field of actively developing methods to engineer OAM single photon from matters.

Building a high-D OAM quantum interface could be based on the protocol of Duan-Lukin-Cirac-Zoller (DLCZ) [21] where the probabilistically generated OAM photon is entangled with memory [12, 22]. There are many parameters to characterize the performance of the interface between photon and memory [23, 24], such as lifetime, efficiency and fidelity and etc [24–26]. The most unique parameter of high-D OAM quantum interface could be spiral-bandwidth $\delta l$, which characterizes the mode-matching bandwidth window [27]. The adjacent nodes in high-dimensional quantum networks may be diverse and different in spatial mode, spiral-bandwidth and etc, for example, one is encoded in $\pm l$ OAM spaces and the other one is in $\pm (l + m)$ OAM spaces; or one has the spatial-bandwidth of $\delta l$ and another is $\delta (l + m)$, thus needing a technology to make the quantum interface be more flexible and controllable and then people can connect them freely [28].

In this letter, we experimentally demonstrate a high-D OAM interface between photon and memory in delayed spontaneously four-wave mixing process. In this configuration, the write- and read-laser beams are individually encoded, thus making the joint of correlation against $l$ modes broadened because the interaction length is increased in transverse azimuthal direction. This offers the ability to control the spatial distribution, including entangled OAM eigenmode $\pm l$ and the spiral-bandwidth $\delta l$. Based on that, we demonstrate the potential applications for OAM multiplexing, and give an obvious contrast data with inputting $\Delta l = 10$. We also have achieved high-D entanglement with $l$ up to 16 and high-quaanta 2-dimensional OAM entanglement with $l$ up to 30, all of which obey the entanglement properties. The reported results are useful for realizing broad-spiral-bandwidth and high-D quantum memory and increasing the capacity of quantum communication, and also is a benchmark of searching ways to explore versatile quantum interfaces.

The experimental media is an optically thick atomic ensemble of Rubidium 85 ($^{85}$Rb) that is trapped in two-dimensional magneto-optic trap (MOT). The involved schematic of the energy levels and the experimental setup are shown in Fig. 1(a) and Fig. 1(b). We firstly establish the correlation between a collective spin excited state (spin wave, also called as atomic memory) and a single photon (Signal 1) through spontaneous Raman scattering (SRS) in atomic ensemble. In this process, the write-laser is set to blue-detuned with atomic transition $|2\rangle \rightarrow |3\rangle$. After reflecting from SLM 1 as depicted in Fig. 1 (b), the write-laser has carried on the OAM phase message loaded by a computer. Then, a 4-f image system with unequal arms, which is consisted of two lenses L1 and L2 with focal length of 300 mm and 500 mm respectively, is utilized to map the OAM phase of the write-laser to the center of atomic ensemble accurately. The Signal 1 photon emit-
Write have created OAM entanglement between Signal 1 and
fact that SRS process conserves angular momentum, we
beams to modulate the light-atom interaction length. We
correlated coincidences of photons decay very quickly against
maximum dark count rate of 25/s) respectively. 
diode, PerkinElmer SPCM-AQR-16-FC, 60% efficiency,
fibers, which are detected by two detectors (avalanche
curately, see supplementary information. The reflected
to map the OAM phase of signal photons to SLM ac-
two couples of 4-f systems with unequal arms are used
both of SLM 2 and 4 are loaded for measurement. Here,
1 and Signal 2 by projecting them onto SLM 2 and SLM
wave, we measure the coincidence counts between Signal
quantum correlation between Signal 1 and atomic spin
spin wave out to Signal 2, the Signal 2 is also mapped
from atomic ensemble is mapped onto another SLM
for detecting the OAM modes. Due to the angular
momentum is conserved in SRS process, hence the spatial
modes of the spin wave and Signal 1 are entangled
in OAM degree of freedom. This OAM correlation can be
flexibly demonstrated by mapping and checking the
OAM modes on SLM 1 and SLM 2 respectively.
The OAM-based DLCZ quantum memory is built
when the entanglement between the spin wave and Signal
1 photon is created. After a storage time of Δt, we use
another SLM 3 to load OAM structured light to read the
spin wave out to Signal 2, the Signal 2 is also mapped
onto another SLM 4. Ultimately, in order to check the
quantum correlation between Signal 1 and atomic spin
wave, we measure the coincidence counts between Signal
1 and Signal 2 by projecting them onto SLM 2 and SLM
4 respectively, in which the different phase structures on
both of SLM 2 and 4 are loaded for measurement. Here,
two couples of 4-f systems with unequal arms are used
to map the OAM phase of signal photons to SLM ac-
curately, see supplementary information. The reflected
photons from SLMs are collected into two single-mode
fibers, which are detected by two detectors (avalanche
diode, PerkinElmer SPCM-AQR-16-FC, 60% efficiency,
maximum dark count rate of 25/s) respectively.
In previous work [17] for demonstrating high-D OAM
quantum interface with Gaussian mode input, it is hard to
generate higher-D entanglement because the corre-
related coincidences of photons decay very quickly against l.
Here, we turn the OAM quanta of write- and read-
beams to modulate the light-atom interaction length. We
input the write-laser with OAM quanta of l_W. Due to the
fact that SRS process conserves angular momentum, we
have created OAM entanglement between Signal 1 and
atomic memory, which can be specified by the formula

$$|\psi\rangle_{\text{photon-atom}}^{l_W} = \sum_{l=-\infty}^{l=\infty} c_l |l\rangle_{S1} \otimes |l_W - l\rangle_a$$

here, $|c_l|^2$ represents excitation probability, $|l\rangle_{S1}$ is the
OAM eigenmode of Signal 1 with quanta of $l$. $|l_W - l\rangle_a$
is the OAM eigenmode of atomic memory with quanta of
$l_W - l$. Through this method, the atomic memory
could carry the arbitrary OAM topological charge with
the term of $l_W - l$, thus resulting in the redistributed
quantum interface. We also check the conservation of
OAM modes at two situations $l_W = 2$, $l_R = 0$ and $l_W = 1$
and $l_R = 2$, which can be found in the supplement. Most
importantly, the spiral-bandwidth of generated photons
is broadened when we increase the OAM quanta of write-
and read-beams.

Due to the broadening effect of spiral-bandwidth with
larger l laser beam input, the distribution of generated
OAM signal 1 and memory would be redistributed
in more flat range. This is because the generated OAM
modes are dependent on the interaction length and the
waist of the write- and read-beams [29]. The vector
mismatching $\Delta k$ from transverse azimuthal phase would
increase the value of $\Delta k \cdot L$, where L is the interaction
length. This effect is very promising because it
is regarded as a concentration operation. In order to
achieve high-D OAM quantum memory, we utilize the
above method to extend the quanta of write-laser, we
set $l_W = 10$. In addition, we set $l_R = -10$ for reading
process. The writing and reading process of DLCZ
quantum memory is essentially a delayed spontaneously
four-wave mixing process. Based on the unique advan-
tages of individually modulating write- and read-beams
of four-wave mixing process (not like a single pump field used in spontaneous parametric down conversion), the write- and read-laser beams can be individually loaded OAM modes with opposite signs whilst the input total angular momentum can be zero, thus making the joint spectrum of correlation broadened. We map different OAM phases onto SLM 2 and SLM 4, and record the coincidence between Signal 1 and Signal 2 photons. The angular momentum can be zero, thus making the joint property along radial direction, see Fig. 3(d). Since the nonlinearity of interleaved OAM modes (for example \(l_W = 0 \) and \(l_R = 10\)) in the center of ensemble can be distinguished by inputting distinct OAM modes (Fig. 3(e)), this may result in multiplexing along radial direction. We map the different OAM modes with \(\Delta l = 1, 2 \ldots 10\) in inner and outer rings (\(\Delta l = l_{out} - l_{in}\)) and detect the correlation given in Fig. 3(e). The crosstalk between different OAM modes is detected by setting same phase structure or the different phase structure. The contrast of coincidence counts is increased against with \(\Delta l\) because the large difference \(\Delta l\) means that the mismatch between write- and read-beams decreases the correlation (Fig. 3(f)), agreeing with the above analysis. This whole process could be regarded as the multiplexing of two OAM spectra, which are created by inputting two distinct OAM writing and reading \((l_{out}, l_{in})\), thus

Figure 3. Multiplexing OAM modes. (a) The multiplexing with different OAM modes of write- and read-lasers. The cross and check marks in up/down equation shows the weak/strong correlation under the different \((|l_W| \neq |l_R|) / \)same OAM orders \((|l_W| = |l_R|)\). (b) The coincidence counts for the different situations of \(|l_W| = |0|/|10|\), \(|l_S| = |0|/|10|\), \(|l_R| = |0|/|10|\), \(|l_S| = |0|/|10|\). The memory decays exponentially with coherence time of \(\tau = 1655\) ns. (d) The multiplexing in radial direction with different OAM modes in inner and outer rings. When the OAM orders in the inner and outer rings are different (up equation), the correlation is weak; On the contrary, the correlation is strong when the OAM orders are same (down equation). (e) The normalized coincidence counts with \(|l_W| = |0| + |10|\), \(|l_S| = |−10| + |0|\) and \(|l_W| = |0| + |10|\), \(|l_R| = |0| + |10|\) in the left two bars. The right two bars are the corresponding OAM mode of \(|l_W| = |0| + |9|\), \(|l_R| = |−9| + |0|\) and \(|l_W| = |0| + |9|\), \(|l_R| = |0| + |9|\). (f) The detected contrast of coincidence counts against different \(|\Delta l|\). The storage time is set to be 500 ns. The different signs of OAM quanta set in (a) and (d) are required to maintain OAM conservation.
satisfying some quantum information protocols with one OAM spectra docking to another OAM spectra.

At last, broad spiral-bandwidth offers an ability for demonstrating high-quanta OAM quantum interface in 2-D subspace. For this, we set \( l_W = 30 \), \( l_R = -30 \), the storage time is set to be twice of the width of write pulse, the decoherence from the transverse azimuthal momentum mismatch between write-laser and the Signal 1 photons is ignored. The nonlinearity of the DLCZ process at large quanta \( l \) is relatively small, we then only consider the two OAM modes for verifying entanglement. We choose the OAM modes of \( l = 28, 32 \) to verify the high-quanta OAM entanglement. The photonic entangled state is expressed as:

\[
|\psi\rangle^{30,-30}_{\text{photon-photon}} = \frac{1}{\sqrt{2}} (|\theta_{S1}\rangle |28\rangle |S2\rangle + |\theta_{S2}\rangle |32\rangle |S1\rangle).
\]

Through quantum state tomography, we obtain the reconstructed density matrix as shown in Fig. 4(a) and (b). The fidelity of reconstructed density matrix is calculated as 80.5 ± 4.8\% by comparing with the ideal density matrix. We also check the violation of Clauser-Horne-Shimony-Holt (CHSH) inequality [30–32] to demonstrate the nonlocality of the entangled state. The CHSH parameter \( S \) is represented as following: \( S = E(\theta_{S2},\theta_{S1}) - E(\theta_{S2},\theta'_{S1}) + E(\theta'_{S2},\theta_{S1}) + E(\theta'_{S2},\theta'_{S1}) \). Here, the correlation function \( E(\theta_{S2},\theta_{S1}) \) can be calculated from the rates of coincidence at several particular orientations, \( \theta / \theta' \) represents the angle of phase distribution on the surface of SLM 2 and SLM 4. The calculated \( S \) is 2.22 ± 0.07 which is larger than 2 violating the CHSH inequality, thus it demonstrates the real entanglement between Signal 1 and Signal 2 photons.

In this work, we demonstrate a broad spiral-bandwidth OAM DLCZ memory, the OAM distribution and the quanta of OAM quantum interface are freely manipulated. In this state-of-the-art quantum interface, we have achieved high-D OAM entanglement with OAM modes difference \( \Delta l \) up to 16, the quanta of OAM 2-D subspace can be accessible to \( l = \pm 30 \). The experiment reported here would be very promising to demonstrate high-quanta OAM quantum interface and study the fundamental physics in OAM-based light and matter interaction.

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SUPPLEMENTARY

Experimental time sequence.

The repetition rate of our experiment is 100 Hz, and the MOT trapping time is 8.7 ms. Besides, the operation window of 1.3 ms consists of 2600 cycles with a cycle time of 500 ns. Write-laser and read-laser are pulsed by acousto-optic modulator with pulse width of 50 ns and 200 ns respectively in each cycle. The optical depth in MOT is about 40. The storage time is controlled by changing the delay time between write- and read-laser through an arbitrary function generator. The magnetic field for trapping is shut down in the experiment window.

4-F image system for four SLMs.

The SLM 1 acts as a mask plane, and the center of atomic ensemble in MOT is the image plane. Two lenses L1 and L2 with focal length of 300 mm and 500 mm are utilized to map the phase message of SLM 1 to the atomic ensemble. Due to the phase matching condition \( k_W - k_{S1} = k_R - k_{S2} \), the imaging system can be easily optically aligned. The Signal 1 and Signal 2 fields are collinear, the Signal 1 beam is completely overlapped by the write beam through demonstrating electromagnetically induced transparency effect. Here, the write-laser carrying high OAM quanta diffracts very strongly and results in the waist of laser beam too large in the center of atomic ensemble, which results in weak interaction between write-laser and atomic ensemble. Through the 4-f image system with unequal arms, we can not only map the OAM phase message to the center of atomic ensemble accurately but also decrease the waist of write-laser

Figure 4. The reconstructed density matrix for photonic entangled state |\psi\rangle^{30,-30}_{\text{photon-photon}}. (a) and (b) are the real and imaginary parts of density matrix respectively. Each data for reconstructing density matrices are recorded in 3000 s.
with high OAM quanta. Similarly, the single photon carried with OAM phase message from the center of atomic ensemble is retrieved to project on SLM 1 via the other 4-f image system, and ultimately we collect photons by single-mode fibers.

**Theoretical analysis.**

In the interaction picture, despite the decay of spin wave, the effective Hamiltonian for the delayed four-wave mixing process is written as [34]

$$\hat{H}_I = \frac{\varepsilon_0}{4} \int_{-L/2}^{L/2} dz \chi^{(3)}(z) \hat{E}_W \hat{E}_R \hat{E}_{S_1} \hat{E}_{S_2} + H.c$$

where $H.c.$ means the Hermitian conjugate. $\chi^{(3)}$ is the third-order nonlinear susceptibility for resonant signal 2 photon, which is given [35]:

$$\chi^{(3)} = \frac{N \mu_{13} \mu_{24} \mu_{41}}{(\Delta_W + i\gamma_{23})[|\Omega_R|^2 - 4(\omega + i\gamma_{24})(\omega + i\gamma_{21})]}$$

(4)

here, $\mu_{ij}$ are the electric dipole matrix elements. $\gamma_{ij}$ are the dephasing rates. $\Omega_R$ is the Rabi frequency of read laser. The probability to generate the Signal 1 and Signal 2 in modes $|l_{S1}\rangle$, $|l_{S2}\rangle$ is given by the overlap with write- and read-laser beam profiles:

$$c_{l_W l_R l_{S1} l_{S2}} \sim \int_{-L/2}^{L/2} \int_0^{2\pi} \varepsilon_0 \chi^{(3)}(z) R_{l_W}(r, \phi) L_{l_R}(r, \phi) L_{l_{S1}}(r, \phi) L_{l_{S2}}(r, \phi) d\phi dr dz$$

(5)

The integral over the azimuthal coordinate is

$$\int_0^{2\pi} d\phi \exp[i(l_W + l_R - l_{S1} - l_{S2})\phi] = 2\pi \delta_{l_W + l_R, l_{S1} + l_{S2}}$$

(6)

From which, we can obtain the topological charge conservation law in OAM space is $l_W + l_R = l_{S1} + l_{S2}$. According to Eq. (4), we can find the probability of $l_{S1}$-Signal 1 and $l_{S2}$-Signal 2 photons with $l_W$-write and $l_R$-read lasers, which strongly depends on the profiles match between the four fields.

In order to illustrate the topological charge conservation law in our OAM quantum interface in DLCZ memory, we input the write-laser with OAM quanta of $l_W$. Due to the fact that SRS process conserves angular momentum, we have created OAM entanglement between Signal 1 and atomic spin wave, which can be specified by the formula $|\psi\rangle_{\text{photon-atom}} = \sum_{l=0}^{\infty} c_l |l_{S1}\rangle \otimes |l_W - l\rangle_a$, here, $|c_l|^2$ represents excitation probability, $|l_{S1}\rangle$ is the OAM eigenmode of Signal 1 with quanta of $l$. $|l_W - l\rangle_a$ is the OAM eigenmode of atomic spin wave with quanta of $l_W - l$. Through this method, the atomic spin wave could carry the arbitrary OAM topological charge with the term of $l_W - l$, thus resulting in the redistributed quantum interface.

Figure 5. Reconstructed density matrices for Modulated OAM entanglement. The real (a, c) and imaginary (b, d) parts of density matrices for photonic OAM entangled state $|\psi\rangle_{\text{photon-photon}}$ and $|\psi\rangle_{\text{photon-photon}}$. Each data for reconstructing density matrices are recorded in 1000 s.

After a period of storage, we check photon-atom entanglement by inputting read-laser with OAM quanta of $l_R$, and check the entanglement between Signal 1 and Signal 2. The entanglement between Signal 1 and Signal 2 can be written as $|\psi\rangle_{\text{photon-photon}} = \sum_{l=0}^{\infty} c_l |l_{S1}\rangle \otimes |l_W + l_R - l\rangle_{S2}$. At first, we set $l_W = 2$ and $l_R = 0$, it means using OAM quanta of 2 and 0 to write and read respectively. Thus, the photonic entangled state is a sum of $|l_{S1}\rangle \otimes |2 - l\rangle_{S2}$ with different $l$, this is a modulated asymmetric OAM entangled state. Here, we only post-select the OAM mode of entangled state into two-dimensional subspace $|0\rangle_{S1} |2\rangle_{S2}$ and $|2\rangle_{S1} |0\rangle_{S2}$, that is $|\psi\rangle_{\text{photon-photon}} = 1/\sqrt{2} (|0\rangle_{S1} |2\rangle_{S2} + |2\rangle_{S1} |0\rangle_{S2})$. To characterize the OAM entanglement between Signal 1 and Signal 2, we reconstruct the density matrices by projecting Signal 1 and Signal 2 onto OAM bases of $|0\rangle$, $|2\rangle$, $(|0\rangle - i |2\rangle)/\sqrt{2}$, $(|0\rangle + |2\rangle)/\sqrt{2}$ for demonstrating quantum tomography. Then we use the obtained 16 coincidence rates to reconstruct the density matrix of state as shown in Fig. 5 (a) and (b). According to the formula $F = \text{Tr} \left( \sqrt{\rho} \rho_{\text{ideal}} \sqrt{\rho} \right)^2$, which
compares the constructed density matrix $\rho$ with the ideal density matrix $\rho_{\text{ideal}}$, we obtain the fidelity of $83.3 \pm 3.5\%$. We also try another data set of $m_1 = 1$ and $m_2 = 2$, and obtain the photonic entangled state $|\psi\rangle_{\text{photon-photon}}^{1,2} = \sqrt{\gamma} (|0\rangle_{S1}|3\rangle_{S2} + |3\rangle_{S1}|0\rangle_{S2})$. Similarly, we reconstruct the density matrix of this state, the real and imaginary parts of reconstructed density matrix are shown in Fig. 5 (c) and (d), with fidelity of $81.1 \pm 4.2\%$. In this process, although the fidelity is not very high, but it reveals that in DLCZ quantum memory, the OAM modes are conserved in the whole writing and reading process.

The entanglement dimensionality witness.

In order to demonstrate the high-D entanglement between Signal 1 and atomic memory, we avoid the crosstalk between neighboring OAM modes. We select the modes of $l = 0, 4, 8, 12, 16$ in which three modes between adjacent terms are removed for better isolation. We read the photon-atom entanglement out to photon-photon entanglement for verification. So, the entangled state is $|\psi\rangle_{\text{photon-photon}}^{1,2} = c_1|0\rangle_{S1}|0\rangle_{S2} + c_2|4\rangle_{S1}|4\rangle_{S2} + c_3|8\rangle_{S1}|8\rangle_{S2} + c_4|12\rangle_{S1}|12\rangle_{S2} + c_5|16\rangle_{S1}|16\rangle_{S2}$, here, $c_5 \approx c_3$ are the corresponding amplitudes of different terms $|0\rangle_{S1}|0\rangle_{S2} \sim |16\rangle_{S1}|16\rangle_{S2}$. For verifying the high-D state, it is very promising to use high-D entanglement dimensionality witness Krenn et al. [11], Agnew et al. [36] to characterize the entanglement existed in our system. The entanglement dimensionality witness is expressed as $W_d = 3\frac{D(D-1)}{2} - D(D - d)$, where, $D$ is the number of measured OAM modes, and $d$ is associated with dimensions of entanglement. If $W > W_d$, the two photons entangled in at least $d + 1$ dimensions, where $W$ is obtained from calculating the sum of visibilities $N = V_x + V_y + V_z$ in total two dimensional subspace. The $V_x$, $V_y$ and $V_z$ represent the visibilities of two-photon interference in the diagonal/anti-diagonal, left-circular/right-circular and horizontal/vertical bases respectively in each OAM mode of $a$ and $b$, here $a$ and $b$ are selected from $l = 0, 4, 8, 12, 16$. A disadvantage of quantum tomography for high-D entanglement is that the needed measurement data is the order of $d^4$, which is a large challenge in its realization and is impractical while $d$ is set to 5 in our experiment. Therefore, we adopt the method of dimensionality witness to certificate the existence of high-D entanglement and characterize the dimensionality. We calculate the value $W$ is 21.93$\pm$0.55, which violates the bound $W_d$ of 20 (the number of measured OAM mode $D$ is 5 and $d$ is 3) by 3 s.d.’s, thus there is at least a 4-D OAM entanglement between Signal 1 and Signal 2 photons. In these measurements, the atom-photon entangled states are both detected in photonic regime, we assume the fidelity of reading out from ensembles is near unit. Although there are definitely some noise or inefficient elements from reading process, making the degree of the measured entanglement lower than that existed in ensembles.

![Figure 6](image.png)

(a) The post-selected correlated OAM matrix between Signal 1 and Signal 2 photons with OAM modes difference $|\Delta l|$ up to 16. (b) The each sum of visibilities for 2-D subspaces for detecting the high-D entanglement dimensionality witness.

2-D high-$l$ Entanglement and state tomography

If we considered the OAM modes of $a$ and $b$ with $l=32$ and 28, the Signal 1 and Signal 2 are entangled in OAM space and entangled state is expressed as $|\Psi\rangle = \sqrt{\gamma} (|28\rangle_{S1}|28\rangle_{S2} + |32\rangle_{S1}|32\rangle_{S2})$ (7)

Here, $|\pm 28\rangle_{S1}$ represents the Signal 1 carrying with OAM quanta of $-28$. By using two computers, we project two photons onto two SLMs respectively and four state of $|\varphi_{1-4}\rangle (|28\rangle, |32\rangle, (|28\rangle - i|32\rangle)/\sqrt{2})/2$ are programmed onto SLM 2 and $|\varphi_{1-4}\rangle (|28\rangle, |32\rangle, (|28\rangle - i|32\rangle)/\sqrt{2})/2$ are programmed onto SLM 4. Then, we obtain a set of 16 data for reconstructing the density matrix given in the main text. The error bars in our experiment are estimated by Poisson statistics and using Monte Carlo simulations with the aid of Mathematica software.

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