Quantum Magnetoresistive \((hc/2e)/m\)-Periodic Oscillations in a Superconducting Ring

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It has been found experimentally that quantum magnetoresistive \((hc/2e)\)-periodic Little–Parks oscillations in a mesoscopic superconducting ring with a decrease in the temperature and an increase in the applied direct current are modified to the sum of harmonic \((hc/2e)/m\)-periodic oscillations. A possible reason for this effect can be multiple Andreev reflection.

To describe a multiply connected superconductor penetrated by the magnetic flux \(\Phi\), Fritz London [1] proposed a concept of a superconducting fluxoid \(\Phi^*\) defined as \(\Phi^* = \Phi + (m^*c/e^*) \int \lambda_0 J_s ds = \Phi + (m^*c/e^*) \int v_s ds\), where \(c\) is the speed of light, \(\lambda_0\) is the London field penetration depth, \(J_s\) is the circulating superconducting current density, \(v_s\) is the supervelocity, and \(m^*\) and \(e^*\) are the effective mass and charge of a superconducting pair, respectively. A remarkable property of superconductivity is the quantization of the superconducting fluxoid \(\Phi^*\), i.e., \(\Phi^* = n(hc/e^*) = n\Phi_0\) (where \(n\) is an integer, \(h\) is the Planck constant, \(e^* = 2e\), \(e\) is the electron charge, and \(\Phi_0 = hc/e^* = hc/2e\) is the superconducting magnetic flux quantum). Furthermore, the found experimental value \(e^* = 2e\) confirms that the elementary charge of the superconductor is \(2e\). In particular, the quantization of the fluxoid results in quantum oscillations of the circulating superconducting current and superconducting critical temperature \(T_c\) (the Little–Parks effect [2]) as a function of the axial magnetic field \(B\) in a thin-wall superconducting cylinder that is penetrated by the flux and is displaced by a very small direct current \(I_0\) at temperatures \(T\) very close to \(T_c\). The periods of these oscillations correspond to the superconducting magnetic flux quantum \(\Phi_0 = hc/2e\) through the middle cross section of the cylinder \(S\).

The Little–Parks effect was observed in cylinders with a small radius \(r \approx \xi(T)/2\) (where \(\xi(T)\) is the temperature-dependent superconducting coherent length) under conditions very close to the equilibrium state. Under conditions close to a nonequilibrium state (at low temperatures and high currents), some violations of \(hc/2e\)-periodicity of quantum magnetoresistive oscillations as functions of the axial magnetic field were found in inhomogeneous doubly connected structures with a small cross area such as superconducting loops [3, 4], \(Au_0.7In_{0.3}\) cylinders [5], and hybrid normal metal Ag rings with superconducting Al mirrors [6]. An anomalous negative magnetoresistance (NMR) and the absence of quantum \(hc/2e\)-periodic \(R(B)\) oscillations were detected in low fields at low temperatures and high currents [3]. Deviations from the \(hc/2e\) periodicity of Little–Parks oscillations in low \(B\) fields were found in [4]. Furthermore, NMR and \(hc/4e\)-periodic \(R(B)\) oscillations were observed in superconducting cylinders [5] and hybrid SNS structures [6] at low temperatures. The period \(hc/4e\) was explained by the appearance of a new minimum of the free energy at \(\Phi = \Phi_0(n + 1/2)\) because of the formation of a \(\pi\) contact in the \(Au_{0.7}In_{0.3}\) cylinder [5] and multiple Andreev reflection [6].

Quantum oscillations in superconducting loops with a larger area under obviously nonequilibrium conditions (high currents and temperatures below \(T_c\)) are almost unstudied. It should be expected that the amplitude of quantum oscillations in rings with a larger radius \(r = 2\xi(T)\) will be low because the circulating superconducting current is lower and only a part of the ring can be switched from the superconducting state to the normal one and back with the variation of the magnetic field.

The study of quantum magnetoresistive oscillations in mesoscopic superconducting rings without weak links (or without tunnel junctions) is relevant because such rings can be used as highly sensitive superconducting quantum interference devices
We found that the periods of quantum magnetoresistive oscillations under strongly nonequilibrium conditions (at a high direct current $I_{dc}$ and temperatures below $T_c$) can be smaller than a usual value by a factor of $m$ and correspond to the flux $\Phi/m = (hc/2e)/m$ ($m$ is an integer) through the effective area of the superconducting ring with a larger radius ($r = 2 \mu m > \xi(T)$).

We measured the voltage $V(B)$ at different currents $I_{dc} > I_c$ and temperatures $T$ slightly below $T_c$ in mesoscopic superconducting structures (with similar and different shapes) penetrated by the magnetic flux. The structure studied in this work (Fig. 1) was obtained by the thermal deposition of an aluminum film with a thickness of $d = 51$ nm on a silicon substrate using lift-off electron beam lithography. The central part of the structure consists of a ring with wall thickness $w_r = 0.27 \mu m$ and the middle radius $r_m = 1.94 \mu m$, narrow current inputs $I$ with the width $w_n = 0.27 \mu m$, wide current inputs $I$ with the width $w_w = 2 \mu m$, and potential inputs $V$. The voltage $V$ was taken from a segment including the ring, narrow current inputs, and a part of wide current inputs (Fig. 1).

The parameters of the structure were as follows: the total resistance at $T = 4.2$ K $R_{4.2\,K} = 52.7 \Omega$, the resistance per square of the thickness of the film $R_{sq} = \rho/d = 1.97 \Omega$, and the ratio of resistances at temperatures of 300 and 4.2 K $R_{300\,K}/R_{4.2\,K} = 1.8$. The mean free path of the electron $l = 10$ nm is found from the refined theoretical relation [10] $pl = 5.1 \times 10^{-16} \Omega$ m², where $p$ is the resistivity of the wire. The structure is a dirty superconductor because $l < \xi(0)$ (where $\xi(0) = 1.6 \mu m$ is the superconducting coherent length of pure aluminum at $T = 0$ K). Near $T_c$ for the dirty case [11], $\xi(T) = \xi(0)(1 - T/T_c)^{-1/2}$ (where $\xi(0) = 0.85\xi(0)/l^{1/2} = 0.11 \mu m$). The condition of quasi-one-dimensional superconductivity ($w_n, w_w < 2\xi(T)$) is satisfied near $T_c$. The nonequilibrium diffusion length of quasiparticles [3, 12] $\lambda_0(T, I_{dc}, B) = 6-9 \mu m$ near $T_c$. The structure has the length $L = 10 \mu m$ (the distance between $V$ inputs), which satisfies the condition $\xi(T) < L < 2\lambda_0(T, I_{dc}, B)$.

The resistive $R(T)$ transition of the structure from the normal (N) to superconducting (S) state was recorded at the direct current $I_{dc} = 0.06 \mu A$ (left inset of Fig. 1). The transition is fairly long, indicating the inhomogeneity of the structure. The abrupt drop of $R(T)$ begins at $T_{ch} = 1.403$ K, and the resistivity vanishes at $T_{ch} = 1.318$ K. The superconducting critical temperature $T_c = 1.339$ K is determined at the middle of the $R(T)$ transition. The contribution of the ring expected from the geometry to the total resistance of the structure is 23 $\Omega$. We assume that the upper part of the $R(T)$ transition (from 52 to 18 $\Omega$) and the lower part of $R(T)$ (from 18 to 0 $\Omega$) correspond to NS transitions in the current inputs and ring, respectively. The function $V(I)$ recorded at $T = 1.290$ K demonstrates the phase current separation into segments with different resistances (right inset of Fig. 1). The initial part of the function $V(I)$ at low currents including an almost linear segment with a resistance of 22 $\Omega$ characterizes SN (NS) transitions in the ring.

When measuring the functions $V(B)$, the magnetic field was varied from a conditionally negative value of $-20$ G to a conditionally positive value of $+20$ G and back. In the field interval from $-20$ to $+20$ G, only a part of the structure corresponding to the ring transitioned to a resistive state with a resistance below 20 $\Omega$ (Fig. 2 and inset).

Experimental functions $V(B)$ demonstrate an anomalous hysteresis depending on the direction of field variation (Fig. 2 and inset). The ring has two states—the more dissipative state (line $1a$ and the upper line in the inset of Fig. 2) and the less dissipative state (lines $1b$ and $2$ in Fig. 2 and the lower line in the inset of Fig. 2). The reasons for the existence of two states will be analyzed elsewhere. A large difference between the two states is seen in the inset of Fig. 2. The inset of Fig. 2 shows the field dependence of the resistance $R(B)$ measured at a low current $I_{dc} = 0.06 \mu A$ and the temperature $T = 1.312$ K very close to the bottom of the NS transition. The feature of the upper line (inset of Fig. 2) is an anomalous NMR, which reaches at $B = 0$ a maximum equal to the resistance of the ring.
in the normal state. The anomalous dissipative state occurs because of thermodynamic fluctuations of the superconducting order parameter, which lead to the formation of a phase-slip center [12] in the ring in spite of a low current. Quantum magnetoresistive $\frac{hc}{2e}$-periodic Little—Parks oscillations are seen on the lower line in the inset of Fig. 2. The main field period of oscillations is $dB_0 = \Phi_0/S_{\text{eff}} = (hc/2e)/S_{\text{eff}} = 1.76$ G and corresponds to the superconducting magnetic flux quantum $\Phi_0 = hc/2e$ through the effective area of the ring $S_{\text{eff}}$. The area $S_{\text{eff}}$ almost coincides with the middle geometric area of the ring. The fundamental frequency $f_0 = dB_0^{-1} = 0.569$ G$^{-1}$ has the meaning of the inverse to the main period of oscillations $dB_0$.

Functions $V(B)$ recorded at lower temperatures $T = 1.280–1.284$ K and higher currents $I_{dc} = 7.5–11$ μA also have a hysteresis decreasing with an increase in $I_{dc}$. In addition, lines 1a, 1b, and 2 in Fig. 2 show the anomalous NMR in two field intervals: nearly zero fields and low fields. Figure 2 shows two of such unusual functions $V(B)$ measured at $I_{dc} = 8.2$ μA and $T = 1.280$ K (lines 1a and 1b) and at $I_{dc} = 9.8$ μA and $T = 1.282$ K (line 2). The extreme right part of line 1b and the upper line close to line 2, which was recorded for the other direction of field variation, are not shown in Fig. 2.

The negative magnetoresistance occurs stepwise at a certain current $I_{dc} > I_c$. Here, $I_c$ is the return superconducting critical current at which the voltage $V(I)$ across the structure vanishes at a decrease in $I_{dc}$. Furthermore, we detected unusual oscillations $V(B)$ against the background of NMR near $B = 0$ at currents $I_{dc} = 1–3$ μA (not shown here) and at low fields $6–12$ G at high currents $I_{dc} = 7.5–11$ μA (Fig. 2). Figure 3 shows the repeatedly measured right part of line 2 (Fig. 2) demonstrating unusual oscillations. These oscillations are not noise, are almost reproducible at repeated recording, and are slightly different at different directions of field variation.

A detailed analysis is usually based on the Fourier spectrum of oscillations. The Fourier spectrum of any oscillations $x(t)$ existing in a limited time interval from $t_1$ to $t_2$ includes not only physical frequencies but also fictitious frequencies: zero frequency and frequencies $f_k = kd_{t_2}^{-1}$ (where $k$ is an integer and $d_{t_1} = t_2 – t_1$ is the length of the interval). Moreover, physical frequencies can be shifted by $kd_{t_2}^{-1}$. Fictitious low frequencies were observed in the spectra of the function $V(B)$ in [9, 13]. We found that the fundamental frequency $f_0$ for oscillations $V(B)$ (Fig. 3) is close in order of magnitude to $dB_{1,2}^{-1}$ (here, $dB_{1,2} = B_2 – B_1$ is the length of the range of existence of oscillations); consequently, distortions can be expected in the Fourier spectrum.

To reliably reveal distortions in the spectra, we obtained the Fourier transform of oscillations of $V(B)$ (Figs. 4–6) with given interval lengths $dB_{1,2} = jdB_0 = jf_0^{-1}$ (where $j = 3$ and 1). We expected to find zero and fictitious frequencies $f_{k,j} = kdB_{1,2}^{-1} = (k/j)dB_0^{-1} = (k/j)f_0^{-1}$ in the spectrum in addition to physical frequencies whose shift by $f_{k,j}$ was also expected.

The fast Fourier transform (FFT) spectra were obtained using 16384 uniformly distributed points in given field ranges. In particular, the condition $dB_{1,2} = 0.002$ G
$B_2 - B_1 = 11.36 - 6.09 = 3d_B$ is satisfied for the FFT spectrum of the function $V_1(B)$ (line 3 in Fig. 3) taken in the field range from 6.09 to 11.36 G. This spectrum (Fig. 4) contains the fundamental frequency $f_0 = dB - S_{df}/\Phi_0 = 0.569 G^{-1}$. The $f_0$ value found from the period of Little–Parks oscillations (inset of Fig. 2) coincides with that found from the spectrum shown in Fig. 4. In addition to $f_0$, the spectrum contains numerous higher harmonics $f_m = mf_0$ (where $m = 2–20$) of the fundamental frequency $f_0$. Some frequency peaks (Fig. 4) are shifted by 1/3 because the condition $dB_{1,2} = 11.36 - 6.09 = 3d_B = 3f_0^{-1}$ is imposed. Furthermore, all FFT spectra, including this spectrum (Fig. 4), contain fictitious zero frequency. The contributions of the fundamental frequency and many higher harmonics to the spectrum are close to each other. This indicates the presence of various fractional periods ($\Phi_0 = (hc/2e)m$) of oscillations of $V_1(B)$ that are not due to anharmonicity of $hc/2e$ oscillations. Some fractional magnetic flux periods $(hc/2e)m$ corresponding to the magnetic field periods $dB_0/m$ are clearly identified on the function $V_1(B)$ (line 3 in Fig. 3). We believe that the anharmonicity of oscillations makes a very small contribution to higher harmonics of the fundamental frequency $f_0$.

For the detailed analysis, the experimental function $V_1(B)$ (line 3 in Fig. 3) is separated into two functions—$V_{1a}(B)$ in the field range of 7.6–9.4 G (Fig. 5) and $V_{1b}(B)$ in the field range of 9.24–10.6 G (Fig. 6). The FFT spectra of both functions $V_{1a}(B)$ and $V_{1b}(B)$ were obtained in two field ranges of 7.61–9.37 G and 9.26–11.02 G (insets of Figs. 5 and 6). Unlike the spectrum shown in Fig. 4, the spectra shown in Figs. 5 and 6 do not contain peaks shifted in frequency because the conditions $dB_{1,2} = B_2 - B_1 = 9.37 - 7.61 = d_B = f_0^{-1}$ and $dB_{1,2} = 11.02 - 9.26 = dB_0 = f_0^{-1}$ were imposed. Frequencies with certain numbers $m$ prevail. The seeming absence of frequencies with other $m$ values including $m = 1$ in the spectra is due to a low spectral resolution broadening the spectral peaks.

In order to qualitatively demonstrate that the function $V_1(B)$ is indeed characterized by a certain set of different periods of oscillations, we approximated some segments of the function $V_1(B)$ by fitting functions. This approximation does not mean the theoretical description of oscillations of $V_1(B)$. The fitting functions were taken in the form $s + p\sum(a_k \sin(2\pi mf_0(B + \Phi_0)))$, including the constant shift $s$ and the product of the coefficient $p = 1$ and the sum of sinusoidal oscillations with different amplitudes $a_k$, frequencies $mf_0$, and phases $\Phi_k$ multiple of $\pi/4$. Integer $m$ corresponds to the number of a harmonic. We took into account the spectra shown in Figs. 5 and 6; for this reason, the amplitudes $a_k$ are close to the Fourier amplitudes obtained from the spectra. Other approximations of the function $V_1(B)$ are also possible.

The functions $V_{1a}(B)$ (Fig. 5) and $V_{1b}(B)$ (Fig. 6), which are parts of the experimental function $V_1(B)$, are
The functions $V_{3(B)}$ and $V_{5(B)}$ dominate in the spectra (insets of Figs. 5 and 6). The functions $V_{2(B)}$ and $V_{4(B)}$ have the form

$$V_{2(B)} = 14.05 + 1.30(0.12\sin(2\pi f_0 B - \pi/2)$$

$$+ 0.18\sin(2\pi 4f_0 B) + 0.17\sin(2\pi 7f_0 B + \pi/4)$$

$$+ 0.1\sin(2\pi 10f_0 B - \pi/4) + 0.1\sin(2\pi 12f_0 B + \pi/2)$$

$$+ 0.2\sin(2\pi 14f_0 B - \pi/2) + 0.1\sin(2\pi 20f_0 B)$$

$$+ 0.08\sin(2\pi 22f_0 B + 3\pi/4),$$

$$V_{4(B)} = 14.3 + 1.25(0.245\sin(2\pi 3f_0 B + \pi)$$

$$+ 0.2\sin(2\pi 5f_0 B - 3\pi/4) + 0.17\sin(2\pi 7f_0 B + \pi/4)$$

$$+ 0.14\sin(2\pi 9f_0 B + \pi/4) + 0.18\sin(2\pi 1f_0 B)$$

$$+ 0.24\sin(2\pi 3f_0 B) + 0.22\sin(2\pi 6f_0 B + \pi/2)$$

$$+ 0.15\sin(2\pi 8f_0 B + 3\pi/4).$$

Figures 7 and 8 show (solid lines) the functions $V_{1c(B)}$ and $V_{1d(B)}$, respectively, that are parts of the experimental function $V_1(B)$ (line 3 in Fig. 3) and (dashed lines) approximations of short segments of these functions. Short segments 1–11 of the function $V_{1c(B)}$ (Fig. 7) and segments 1–10 of the function $V_{1d(B)}$ (Fig. 8) are fitted by one sinusoidal cycle or by a sum of several oscillations with different amplitudes, frequencies $f_m = m f_0$, and phases.

Fitting functions $F_2(B)$ (Fig. 7) for lines 1 (7.0–7.5 G), 2 (7.41–7.67 G), 3 (7.68–7.89 G), 4 (7.84–8.13 G), 5 (8.09–8.38 G), 6 (8.25–8.57 G), 7 (8.45–8.69 G), 8 (8.62–8.89 G), 9 (8.79–9.03 G), 10 (9.08–9.3 G), and 11 (9.08–9.3 G) have the form $F_2(B) = 13.95 + 0.14\sin(2\pi 9f_0 B - \pi/2), F_2(B) = 14 + 0.14\sin(2\pi 7f_0 B + \pi/4) + 0.06\sin(2\pi 14f_0 B - 3\pi/4), F_2(B) = 13.8 + 0.1\sin(2\pi 7f_0 B + \pi/2) + 0.29\sin(2\pi 14f_0 B - \pi/2), F_2(B) = 13.8 + 0.45\sin(2\pi 7f_0 B + \pi/4) + 0.15\sin(2\pi 14f_0 B), F_2(B) = 14 + 0.16\sin(2\pi 7f_0 B + \pi/4) + 0.34\sin(2\pi 14f_0 B - \pi/4), F_2(B) = 14.03 + 0.39\sin(2\pi 10f_0 B - \pi/4), F_2(B) = 14.0 + 0.1\sin(2\pi 4f_0 B - \pi/2) + 0.39\sin(2\pi 14f_0 B - \pi), F_2(B) = 13.9 + 0.47\sin(2\pi 7f_0 B) + 0.25\sin(2\pi 14f_0 B - \pi/2), F_2(B) = 14.25 + 0.25\sin(2\pi 7f_0 B - 3\pi/4) + 0.37\sin(2\pi 14f_0 B - \pi/2), F_2(B) = 14.15 + 0.17\sin(2\pi 7f_0 B + \pi/2) + 0.56\sin(2\pi 120f_0 B + 3\pi/4), F_2(B) = 14.05 + 0.25\sin(2\pi 7f_0 B + 3\pi/4) + 0.36\sin(2\pi 12f_0 B + \pi/2).

It is seen that short segments 1–11 of the function $V_{1c(B)}$ (Fig. 7) are approximated with the following frequencies of oscillations:$f = 9 f_0$ (segment 1); $7f_0$ and $14 f_0$ (segments 2, 3, 4, 5, 8, 9); $10 f_0$ (segment 6); $4 f_0$ (segment 7); and $7 f_0$ and $12 f_0$ (segments 10, 11).

Fitting functions $A_2(B)$ (Fig. 8) for lines 1 ($B = 9.25–9.4$ G), 2 (9.38–9.58 G), 3 (9.58–9.83 G), 4
(9.73–9.83 G), 5 (9.8–10.3 G), 6 (9.9–10.2 G), 7 (10.3–10.6 G), 8 (10.6–10.77 G), 9 (10.81–11.06 G),
and 10 (11.12–11.44 G) have the form $A_0(B) = 14.3 + 0.92 sin(2\pi f_{16}B + \pi/2), A_1(B) = 13.8 + 0.19 sin(2\pi f_5B – \pi/4) + 0.17 sin(2\pi f_7B + 3\pi/4) + 0.22 sin(2\pi f_6B + \pi/2), A_2(B) = 13.7 + 1.1(0.2 sin(2\pi f_5B + 3\pi/4) + 0.3 sin(2\pi f_7B + 3\pi/4)), A_3(B) = 14.6 + 1.2(0.3 sin(2\pi f_7B – \pi/2) + 0.65 sin(2\pi f_6B + \pi/2), A_4(B) = 14.05 + 1.1(0.16 sin(2\pi f_11f_0B + \pi) + 0.4 sin(2\pi f_6B + \pi)), A_5(B) = 13.9 + 1.2(0.077 sin(2\pi f_9B – \pi/4) + 0.097 sin(2\pi f_7B – \pi/4)), A_6(B) = 13.83 + 1.2(0.009 sin(2\pi f_7B - \pi/2) + 0.01 sin(2\pi f_6B - 3\pi/4)).$

Short segments $1–10$ of the function $V_{1d}(B)$ (Fig. 8) are approximated with the following frequencies of oscillations: $16f_0$ (segment 1); $5f_0$, $16f_0$, and $18f_0$ (segment 2); $13f_0$ (segment 3); $16f_0$ (segment 4); $3f_0$, $5f_0$, $7f_0$, and $16f_0$ (segment 5); $5f_0$ and $7f_0$ (segment 6); $7f_0$ and $16f_0$ (segment 7); $11f_0$ and $16f_0$ (segment 8); $9f_0$ and $16f_0$ (segment 9); and $7f_0$ and $16f_0$ (segment 10).

Thus, we performed a detailed analysis, including the fast Fourier transform, of oscillations of $V(B)$ measured at $I_{dc} = 9.8 \mu A$ and $T = 1.282$ K (line 2 in Fig. 2) in the field range of 6–12 G. Figure 4 demonstrates that the contributions of the fundamental frequency $f_0$ and most of the higher harmonics $f_m = mf_0 (m = 2–20)$ to the FFT spectrum are close to each other. The contribution from each of the remaining higher harmonics is about half the contribution from the fundamental frequency $f_0$.

In addition, other unusual experimental oscillations of $V(B)$ in low fields of 6–12 G at $T = 1.280–1.1284$ K were considered (they are not presented here). The maximum amplitude of oscillations decreases from 25 $\mu$V to 0 with an increase in the current $I_{dc}$ from 7.5 to 11 $\mu$A. In particular, this amplitude at currents of 8.2 and 9.8 $\mu$A reaches 20 and 2 $\mu$V, respectively (Fig. 2). At currents $I_{dc} = 7.7–8.6$ $\mu$A, the contributions of higher harmonics $f_m = mf_0 (m = 3–20)$ to the spectra (are not shown here) are almost equal to each other but are about half the contributions of the frequencies $f_0$ and $2f_0$.

The negative magnetoresistance near $B = 0$ was previously measured in quasi-one-dimensional superconducting wires with a transverse contraction and in small rings with inhomogeneities [3]. Quantum magnetoresistive oscillations in the anomalous NMR region were not detected in [3]. We found two NMR regions (near zero field and in low fields).

Any commonly accepted mechanism of NMR has not yet been proposed. The negative magnetoresistance in our structure can be due to a decrease in the resistance of the nonequilibrium SN interface $R_q = \lambda Q(T, I_{dc}, B)$ with increasing field [3]. Other reasons for NMR can be an increase in the return superconducting critical current $I_c$ and a decrease in the resistance of the structure (recovery of superconductivity) at a given current with increasing field because of a decrease in the “effective temperature” of hot quasiparticles in the nonequilibrium region of the structure [14]. The effective temperature decreases because of the enhancement of diffusion of overheated quasiparticles to the neighboring superconducting shores with a slightly higher superconducting order parameter $\Delta(T, B)$, when the parameter $\Delta(T, B)$ in superconducting shores decreases (or is completely suppressed) with increasing field [14].

The nonequilibrium region is a phase-slip center or an SNS junction, in the center of which the parameter $\Delta(T, B)$ vanishes periodically and has a nonzero time-average value lower than that in neighboring superconducting regions [12]. Oscillations of the order parameter produce a large number of hot quasiparticles, strongly heating the nonequilibrium region. Multiple Andreev reflection [15–17] occurring in the phase-slip center or SNS junction at voltages lower than the superconducting gap increases quasiparticle heating. Heating is enhanced because of a large electron–phonon relaxation time in the aluminum structure. Superconductivity and dissipation coexist in the nonequilibrium [18].

The theory developed in [14] is applicable only for short samples where the distance between potential
inputs is \( L = 5 \xi^2(T) < 2 \lambda_\phi(T, I_{dc}, B) \). Nevertheless, it can qualitatively explain NMR manifested in experimental functions \( V(B) \) in our structure with the average length \( L = 20 \xi^2(T) = 10 \mu m \) satisfying the condition \( 2 \xi^2(T) \ll L < 2 \lambda_\phi(T, I_{dc}, B) \). We suppose that both NMR regions near zero field and in low fields exist because a superconducting thermal barrier is formed at the place of the transition of a narrow current input into a wide current input and prevents the diffusion of hot quasiparticles.

This superconducting thermal barrier is significantly suppressed (or disappears) in a low field \( B_0(T, I_{dc}) \), which depends on the temperature \( T \) and \( I_{dc} \). At low currents, the field \( B_0(T, I_{dc}) \) is close to the third critical field \( B_{c3}(T) \). The formation of superconductivity along the edge of a wide current input becomes impossible in fields above the third critical field \( B_{c3}(T) \). For places of the transition of narrow current inputs to wide current inputs, we estimated the critical field as \( B_{c3}(T) = 2 B_{c2}(T) \). Here, a factor of 2 is introduced because of the wedge shape of the transition [19]. The critical field \( B_{c3}(T = 1.312 K) \approx 2 \Phi_0/2 \pi \xi^2(T) \approx 12 G \) calculated for the function \( V(B) \) shown in the inset of Fig. 2 is in agreement with the experimental field of disappearance of NMR (\( B = 14 G \)). The critical field \( B_{c3}(T = 1.282 K) \approx 22–26 G \) for the function \( V(B) \) shown by line 2 in Fig. 2 is higher than the field at which NMR disappears (\( B = 12 G \)), because the effect of a high direct current \( I_{dc} \) is disregarded. We believe that two NMR regions exist because of several transitions between the diamagnetic and paramagnetic states of wide current inputs with the variation of the field. In our measurements, we did not reach the maximum critical field \( B_{max}(T) \) at which superconductivity in the narrow current inputs and rings is completely suppressed [11]. At the temperature \( T = 1.282 K \), this field is \( B_{max}(T) = 77 G \).

Fractional quantum magnetoresistive \((hc/2e)/m\)-periodic oscillations (where \( m > 2 \)) have not yet been observed. We suppose that oscillations with periods corresponding to \( \Phi_0/m = (hc/2e)/m \) (where \( m = 1–20 \)) and approximately equal amplitudes (except for the amplitude corresponding to \( m = 1 \)) indicate that the circulating superconducting current has the effective charge \( e^* = 2em \). This property can be due to multiple Andreev reflection [15–18] occurring in non-equilibrium regions of the structure (SNS junction or phase-slip center formed in the ring) at voltages \( V(I_{dc}) < 2 \Delta(T, B)/e \), where \( \Delta(T, B) = \Delta(T)(1 – B^2/B^2_{max}(T))^{1/2} \) is the superconducting gap in the magnetic field \( B \) at temperatures \( T \) slightly below \( T_c \). Here, \( \Delta(T) = 3.07k_B(T)\Delta(0)(1 – T/T_c)^{1/2} \) is the gap at \( B = 0 \) [11]. For the function \( V(B) \) shown by line 2 in Fig. 2, \( \Delta(T = 1.282 K, B = 6–12 G) \approx 74–73 \mu V \).

It is known that multiple Andreev reflection can occur in SNS junctions both in the ballistic case where the mean free path of quasiparticles \( l \) is larger than the length \( l_n \) of the normal region of the junction [15] and in the diffusion case \( (l \ll l_n) [16–18] \). In our work, a diffusive phase-slip center or an SNS junction \((l \ll l_n = 2 \xi(T)) \) is formed. To observe a large number \( n \) of multiple Andreev reflections in diffusive SNS junctions, the inelastic scattering length \( l_n [16] \) (in our case, \( l_n = 2 \lambda_\phi(T, I_{dc}, B) = 12–18 \mu m \)) should be much larger than \( l_n \). In our work, this condition written in the form \( l_n = 2 \lambda_\phi(T, I_{dc}, B) \gg l_n = \xi(T) \) is fulfilled.

In the process of multiple Andreev reflections, the quasielectron (quasihole) that has the energy lower than the gap and is located between two NS interfaces undergoes Andreev reflection as a quasihole (quasielectron) successively from both NS interfaces until it reaches the energy \( V(I_{dc}) = 2 \Delta(T, B) \). After \( n \) reflections, \( m = n/2 \) or \( m = n/2 – 1 \) superconducting pairs enter the S region of the SNS junction or phase-slip center. As a result, the effective superconducting charge increases by a factor of \( m \). The probability of observing multiple Andreev reflections usually decreases with an increase in the number of reflections \( n \), but the experiment reported in [18] demonstrated that multiple Andreev reflections can be observed for a large number of reflections (up to \( n = 32 \)). In [18], it was found that multiple Andreev reflections and very strong quasiparticle heating in the core of the phase-slip center or SNS junction formed in the quasi-one-dimensional superconducting wire are responsible for the appearance of current plateau singularities on \( V(I) \) curves at voltages \( V_{pl.\,n}(T, B) = 2 \Delta(T, B)/ne \) corresponding to the subharmonics of the superconducting gap. Here, \( n \) is an integer depending on \( T, B, \) and \( V \).

The ratio \( 2 \Delta(T, B)/eV_{pl.\,n}(I_{dc}) \approx 146/144 \approx 10 \) for the function \( V(B) \) represented by line 2 in Fig. 3 shows the possible average number of multiple Andreev reflections. We believe that the maximum value of this ratio can be larger. In particular, it is expected that the instantaneous voltage \( V(I_{dc}) \) will vary from nearly zero to a value close to the maximum voltage \( V_{n}(I_{dc}) = 20 \Omega I_{dc} = 160–200 \mu V \) because of the presence of two different (less and more dissipative) states at currents \( I_{dc} = 8–10 \mu A \). Therefore, the number of reflections can vary from one to the maximum possible value \( 2 \lambda_\phi(T, I_{dc}, B)/\xi(T) \approx 18–36 \).

To summarize, we have found that the mesoscopic superconducting aluminum ring that is penetrated by the magnetic flux and is displaced by the direct current \( I_{dc} \) at temperatures \( T \) slightly below \( T_c \) can be in two different (less and more dissipative) states.

The implementation of a certain state depends on the direction of field variation. Quantum magnetoresistive \((hc/2e)/2\)-periodic Little–Parks oscillations are observed in the less dissipative state at temperatures \( T \) corresponding to the extreme bottom of the \( R(T) \) transition and low currents \( I_{dc} \). Meanwhile, in the more dissipative state, \( hc/2e \) oscillations have not been
detected and the function $V(B)$ demonstrates negative magnetoresistance. At lower temperatures $T$ and higher currents $I_{dc}$, both parts of the function $V(B)$ that correspond to both states demonstrate two NMR regions (near $B = 0$ and in low fields). These NMR regions appear because a superconducting thermal barrier is formed in wide current inputs, prevents the cooling of the overheated part of the structure, and nonmonotonically depends on the field. Unusual quantum oscillations of $V(B)$ can be observed in both NMR regions.

We have studied for the first time fractional ($hc/2e$)/$m$-periodic ($m = 2–20$) oscillations of $V(B)$ in low fields. These oscillations have neither yet been described theoretically nor been measured experimentally. A decrease in the period of oscillations by a factor of $m$ can be treated as an increase in the effective charge of Cooper pairs by a factor of $m$ because of multiple Andreev reflections in the phase-slip center or SNS junction formed in the ring.

The functions $V(B)$ measured in other structures also demonstrate two dissipative states, NMR, and fractional ($hc/2e$)/$m$-periodic oscillations. In this case, the amplitude corresponding to the main period of oscillations was much higher than the amplitudes corresponding to fractional periods.

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