An analysis of soft terms in Calabi-Yau compactifications

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Abstract

We perform an analysis of the soft supersymmetry-breaking terms arising in Calabi-Yau compactifications. The sigma-model contribution and the instanton correction to the Kähler potential are included in the computation. The existence of off-diagonal moduli and matter metrics gives rise to specific features as the possibility of having scalars heavier than gauginos or the presence of tachyons. Although non-universal soft terms is a natural situation, we point out that there is an interesting limit where universality is achieved. Finally, we compare these results with those of orbifold compactifications. Although they are qualitatively similar some features indeed change. For example, sum rules found in orbifold models which imply that on average the scalars are lighter than gauginos can be violated in Calabi-Yau manifolds.
1 Introduction

The computation of the soft Supersymmetry (SUSY)-breaking terms in effective N=1 theories coming from four-dimensional (4-D) Superstrings is crucial in order to connect these theories with the low-energy phenomena [1]. The soft terms not only contribute to the Higgs potential generating the radiative breakdown of the electroweak symmetry, but also determine the SUSY spectrum, like gaugino, squark or slepton masses.

The most extensive studies of soft terms, working at the perturbative level\textsuperscript{1}, have been carried out in the context of (0,2) symmetric Abelian orbifolds \textsuperscript{2} assuming that the seed of SUSY breaking is located in the dilaton/moduli sectors \textsuperscript{3}. The latter implies that the Goldstino field is a linear combination of the fermionic partners of the dilaton $S$, the field whose vacuum expectation value (VEV) determines the gauge coupling constant, and the moduli $T_i$, $U_j$, the fields whose VEVs parametrize the size and shape of the compactified space. In particular, in refs.\textsuperscript{6, 9}, where no special assumption was made about the possible origin of SUSY breaking, the soft terms depend on the gravitino mass $m_{3/2}$ and the parameters specifying the Goldstino direction. This simplify the analysis of soft terms and leads to some interesting relationships among themselves which could perhaps be experimentally tested \textsuperscript{6, 8, 9, 13}.

It would be interesting to extend this approach for more complicated compactifications as Calabi-Yau (CY) manifolds \textsuperscript{4}. This is in fact the aim of the present paper\textsuperscript{3}. Unfortunately the state-of-the-art CY technology will limit our computation of soft terms. First, the analysis of (0,2) vacua is an open problem in CY compactifications and therefore we are forced to restrict our study to (2,2) theories. In this type of theories the gauge group is $E_6 \times E_8$ and several families of $27$ and $\overline{27}$ matter fields can be present. May be these are not the most realistic low-energy theories since usually one likes gauge groups smaller than $E_6$. In any case we hope, as occurs in orbifolds, they give us an insight into features of (0,2) models. On the other hand, if the suggestion that all SUSY (2,2) vacua with $E_6 \times E_8$ gauge symmetry are compactifications on CY manifolds \textsuperscript{14} is correct, our results would be of interest for any compactification scheme with those properties. Finally, we will focus on the region of the moduli space where all the $\text{Re}T_i$ are large, often called the large-radius limit. The reason is that this limit provides a good approximation to the 4-D effective theory. The extension to small manifolds is quite involved and again an open problem. Large $T_i$ in practice, does not really mean $T \rightarrow \infty$, since the world-sheet instanton corrections are exponentially suppressed. For values $|T| \geq 2 – 3$ these world-sheet instanton contributions can often be neglected and, in this sense these $T$-values are already large. On the other hand, the VEV of the moduli are not necessarily of order one. As studied in specific orbifold examples in ref.\textsuperscript{15}, $\text{Re}T \simeq 5 – 10$ is mandatory if the discrepancy between the unification scale of the gauge couplings and the string unification scale is

\textsuperscript{1}\textsuperscript{1}For possible non-perturbative string effects on soft terms see ref.\textsuperscript{2}.

\textsuperscript{2}\textsuperscript{2}The limit where only the dilaton contributes to SUSY breaking is compactification-scheme independent and was analyzed in [10, 6, 11]. Its phenomenology was studied in refs.[12, 6].

\textsuperscript{3}Some computations of soft terms in CY compactifications were also carried out in refs.\textsuperscript{3, 7}. In particular, in \textsuperscript{3} only the “overall modulus” case was studied whereas in \textsuperscript{7} non-perturbative effects breaking SUSY were considered in order to cancel the tree-level and one-loop cosmological constant.
explained by the effect of string threshold corrections. This might also be the case of CY compactifications.

The structure of the paper is as follows. In section 2 we discuss some formulae for the computation of soft terms in effective Supergravity (SUGRA) theories from Strings, in the most general case of off-diagonal metrics. In the spirit of refs.\[3, 4\], i.e. just assuming that SUSY is broken by dilaton/moduli fields, we parametrize the SUSY breaking in terms of “Goldstino angles” generalizing the ones used there to the case of off-diagonal moduli metric. This is precisely the case of CY compactifications. Unlike orbifolds, where off-diagonal metrics are relatively rare, these are present in general in CY manifolds. Thus in section 3 we apply the formulae obtained in section 2 to the computation of the soft terms. After summarizing some generic features of effective SUGRAs coming from CY compactifications, we obtain general formulae for the soft terms which are independent of the size of the manifold. An interesting limit where universality is achieved is studied in 3.2. Then, in order to obtain more concrete features we work in the large-radius limit, where the prepotential is known, computing the Kähler potential including the sigma-model and instanton contributions. First we discuss the simple cases of CY manifolds with only one Kähler modulus. These will be good guiding examples in order to study in detail the modifications produced on soft terms by the above-mentioned contributions\[\text{4}\]. Second, we consider the general case of CY compactifications with several moduli studying what new features appear. Finally, in section 4, we compare the results on soft terms obtained in the previous sections with those of orbifold compactifications. We leave the conclusions for section 5.

2 General parametrization of soft terms

We are going to consider $N = 1$ SUSY 4-D Strings with $m$ moduli $T_i$, $i = 1, .., m$. Such notation refers to both $T$-type and $U$-type (Kähler class and complex structure in the Calabi-Yau language) fields. In addition there will be charged matter fields $C_\alpha$ and the dilaton field $S$. The associated effective $N = 1$ SUGRA Kähler potentials (to first order in the matter fields) are of the type:

$$K(S, S^*, T_i, T_i^*, C_\alpha, C_\alpha^*) = -\log(S + S^*) + \tilde{K}(T_i, T_i^*) + \tilde{K}_{\alpha\beta}(T_i, T_i^*)C_\alpha^*\overline{C_\beta}$$

$$+ (Z_{\alpha\beta}(T_i, T_i^*)C_\alpha\overline{C_\beta} + \text{h.c.}).$$

(1)

The first piece is the usual term corresponding to the dilaton $S$ which is present for any compactification whereas the second is the Kähler potential of the moduli fields. The greek indices label the matter fields and their kinetic term functions are given by $\tilde{K}_{\alpha\beta}$ and $Z_{\alpha\beta}$ to lowest order in the matter fields. The last piece is often forbidden by gauge invariance in specific models although it may be relevant in order to solve the $\mu$ problem \[13, 14\]. The complete $N = 1$ SUGRA Lagrangian is determined by the Kähler potential $K(\phi_M, \phi_M^*)$, the superpotential $W(\phi_M)$ and the gauge kinetic functions $f_a(\phi_M)$, where $\phi_M$ generically denotes the chiral fields $S, T_i, C_\alpha$. As is well known, $K$ and $W$ appear in the Lagrangian only in the combination $G = K + \log |W|^2$.

\[4\] The importance of considering this type of contributions was first pointed out in ref.\[5\].
In particular, the \((F\) part of the\) scalar potential is given by

\[
V(\phi_M, \phi_M^*) = e^G \left( G_M K^{MN} G_N - 3 \right) = F^N K_{NM} F^M - 3e^G ,
\]

where \(G_M \equiv \partial_M G \equiv \partial G / \partial \phi_M\) and \(K^{MN}\) is the inverse of the Kähler metric \(K_{NM} \equiv \partial_N \partial M K\). We have also written \(V\) as a function of the \(\phi_M\) auxiliary fields, \(F^M = e^{G/2} K^M \bar{P}_G \bar{P} G\).

Following the spirit of refs.\[6, 9\] the crucial assumption now is to locate the origin of SUSY breaking in the dilaton/moduli sector. Then, applying the standard SUGRA formulae [19, 10] to the most general case when the moduli and matter metrics are not diagonal we obtain:

\[
m^2_{\pi \beta} = m_{3/2}^2 \bar{K}_{\pi \beta} - F^i (\partial_i \partial_j \bar{K}_{\pi \beta} - \partial_i \bar{K}_{\pi \gamma} \bar{K}^{\gamma \delta} \partial_j \bar{K}_{\beta \delta}) F^j ,\]

\[
A'_{\alpha \beta \gamma} = F^S K_S Y'_{\beta \gamma} + F_i \left[ \frac{1}{2} \bar{K}_i Y'_{\alpha \beta \gamma} + \partial_i Y'_{\alpha \beta \gamma} - \left\{ \bar{K}^{\beta} \partial_i \bar{K}_{\pi \alpha} Y'_{\delta \beta \gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right\} \right],
\]

where \(m^2_{\pi \beta}\) and \(A'_{\alpha \beta \gamma}\) are the soft mass matrix and the soft trilinear parameters respectively (corresponding to un-normalized charged fields), \(m_{3/2}^2 = e^G\) is the gravitino mass-squared, \(Y'_{\alpha \beta \gamma} \equiv e^{K/2} Y_{\alpha \beta \gamma}\) with \(Y_{\alpha \beta \gamma}\) a renormalizable Yukawa coupling involving three charged chiral fields and finally, \(F^S = e^{G/2} K_{S S}^{-1} G_S, F^i = e^{G/2} \bar{K}^{\gamma \delta} G_{\gamma \delta}\) are the dilaton and moduli auxiliary fields respectively. Notice that, as discussed in [9], after normalizing the fields to get canonical kinetic terms, the first piece in eq.(3) will lead to universal diagonal soft masses but the second piece will generically induce off-diagonal contributions. Concerning the \(A\)-parameters, notice that we have not factored out the Yukawa couplings as usual, since proportionality is not guaranteed. Indeed, although the first term in \(A'_{\alpha \beta \gamma}\) is always proportional in flavour space to the corresponding Yukawa coupling, the same thing is not necessarily true for the other terms. We will see below that this is precisely what occurs in general with scalar masses and trilinear parameters in CY compactifications. However, there is an interesting limit where universality is achieved.

If the VEV of the scalar potential eq.(2), \(V_0\), is not assumed to be zero one just has to replace \(m^2_{3/2} \to m^2_{3/2} + V_0\) in the expression of \(m^2_{\pi \beta}\).

Physical gaugino masses \(M_a\) for the canonically normalized gaugino fields are given by

\[
M_a = \frac{1}{2} (\text{Re} f_a)^{-1} \left[ F^S \partial_S f_a + F^i \partial_i f_a \right].
\]

Let us take the following parametrization for the VEVs of the dilaton and moduli auxiliary fields

\[
F^S = \sqrt{3} m_{3/2} \sin \theta (S + S^*) e^{-i \gamma_S} ,
\]

\[
F^i = \sqrt{3} m_{3/2} \cos \theta P^i \Theta_j ,
\]
which generalize the one used in ref.[1] to the case of off-diagonal moduli metric.\footnote{We note for completeness that in ref.[1] some orbifold models with off-diagonal metrics were also studied. The moduli and matter metrics were $\tilde{K}_{ij} = t_i^{-1}t_j^{-1}$ ($i \equiv \alpha \overline{\beta}$, $j \equiv \gamma \overline{\delta}$) and $\tilde{K}_{\alpha \beta} = t_{\alpha \beta}$, respectively, where $t \equiv t^{\alpha \overline{\beta}} = (T + T^\dagger)^{\alpha \overline{\beta}}$ is a hermitian matrix. In our general notation this implies that the matrix $P$ is given by $P^{ij} = (t^{1/2})^a_i (t^{1/2})^a_j$ and the SUSY-breaking parametrization becomes $F^S = \sqrt{3}m_{3/2}^2 \sin \theta (S + S^*) e^{-i \gamma_8}$, $F = \sqrt{3}m_{3/2}^2 \cos \theta t^{1/2} \Theta t^{1/2}$, where $\Theta$ is a matrix satisfying $\text{Tr} \Theta^2 = 1$.} $P$ is a matrix canonically normalizing the moduli fields, i.e. $P^i \tilde{K}_i = 1$ where $1$ stands for the unit matrix, the angle $\theta$ and the complex parameters $\Theta_i$ just parametrize the direction of the Goldstino in the $S, T_i$ field space and $\sum_j \Theta_i^* \Theta_j = 1$. We have also allowed for the possibility of some complex phases which could be relevant for the CP structure of the theory. This parametrization has the virtue that when we plug it in the general form of the SUGRA scalar potential eq.(2), the cosmological constant

$$V_0 = (S + S^*)^{-2} |F^S|^2 + F^i \hat{K}_{ij} F^j - 3m_{3/2}^2,$$

vanishes by construction. Notice that such a phenomenological approach allows us to `reabsorb' (or circumvent) our ignorance about the (nonperturbative) $S$- and $T_i$- dependent part of the superpotential, which is responsible for SUSY breaking \footnote{$P$ can be written as $P = U \hat{K}_d^{-1/2}$, where $U$ is a unitary matrix which diagonalizes $\hat{K} \equiv K_{ij}$, $U^\dagger \hat{K} U = \hat{K}_d$.}. Plugging eq.(3) into eqs.(4,5,6) one finds the following results

$$m_{\alpha \beta}^2 = m_{3/2}^2 \left[ \tilde{K}_{\alpha \beta} + 3 \cos^2 \theta \Theta_i^* K^{ij} \left( \partial_i \tilde{K}_{\alpha \gamma} \tilde{K}^\gamma \beta \partial_j \tilde{K}_{\beta \delta} - \partial_i \partial_j \tilde{K}_{\alpha \delta} \right) P^{ji} \Theta_j \right], \quad (8)$$

$$A'_{\alpha \beta \gamma} = -\sqrt{3}m_{3/2}^2 e^{K/2} \left[ \sin \theta e^{-i \gamma_8} Y_{\alpha \beta \gamma} + \cos \theta P^{ij} \Theta_j \times \left\{ \left( \tilde{K}^{\alpha \gamma} \partial_i \tilde{K}_{\alpha \gamma} Y_{\delta \beta \gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right) - \hat{K}_i Y_{\alpha \beta \gamma} - \partial_i Y_{\alpha \beta \gamma} \right\} \right], \quad (9)$$

$$M_a = \sqrt{3}m_{3/2}^2 \sin \theta e^{-i \gamma_8}, \quad (10)$$

where the tree-level gaugino masses are independent of the moduli sector due to the fact that the tree-level gauge kinetic function is given for any 4-D String by $f_a = k_a S$ with $k_a$ the Kac-Moody level of the gauge factor.

As we mentioned above, the parametrization of the auxiliary field VEVs was chosen in such a way to guarantee the automatic vanishing of the VEV of the scalar potential ($V_0 = 0$). If the value of $V_0$ is not assumed to be zero the above formulae (8,9,10) are modified in the following simple way. One just has to replace $m_{3/2}^2 \rightarrow C m_{3/2}^2$, where $|C|^2 = 1 + V_0/3m_{3/2}^2$. In addition, the formula for $m_{\alpha \beta}^2$ gets an additional contribution given by $2m_{3/2}^2 (|C|^2 - 1) \tilde{K}_{\alpha \beta} = (2V_0/3) \tilde{K}_{\alpha \beta}$.

The soft term formulae above (8,9,10) are valid for any compactification scheme. In addition one is tacitally assuming that the tree-level Kähler potential and $f_a$-functions constitute a good approximation. In fact, the effects of the one-loop corrections will in general be negligible except for those corners of the Goldstino directions in which the tree-level soft terms vanish. As discussed in ref.[9] this situation would be a sort of fine-tuning.
In order to obtain more concrete expressions for the bosonic soft terms (we recall that gaugino masses \((10)\) are compactification-scheme dependent) one needs some information about the Kähler potential \(K\). In the following we will concentrate on CY compactifications where this type of information is known.

3 Calabi-Yau compactifications

3.1 General characteristics

Let us summarize some generic features of effective SUGRAs coming from (2,2) CY compactifications\(^7\). The gauge group is \(E_6 \times E_8\) and the matter fields in \(27\) representations of \(E_6\) are in one-to-one correspondence with the \(T_i\) moduli \((i=1,...,m)\), whereas the \(\bar{27}\) representations of \(E_6\) are in one-to-one correspondence with the \(U_k\) moduli \((k=1,...,n)\). The tree-level Kähler potential is

\[
K = -\log(S + S^*) + \hat{K}_1(T_i, T_i^*) + \hat{K}_2(U_k, U_k^*) \\
+ \hat{K}_{1ij} \bar{27}^i \bar{27}^j + \hat{K}_{2kl} \bar{27}^k \bar{27}^l + (Z_{ik} \bar{27}^i \bar{27}^k + \text{h.c.}) ,
\]

where we have included the last term for completeness since it has recently been realized that it do appears in some CY compactifications \([10, 7]\). Since its relation with the phenomenologically required \(B\) term of the MSSM is very model dependent we will not study it in what follows. The matter metrics are related to moduli Kähler potentials as \([20]\)

\[
\hat{K}_{1ij} = \frac{\partial^2 \hat{K}_1}{\partial T_i \partial T_j} \exp \frac{1}{3}(\hat{K}_2 - \hat{K}_1) ; \quad \hat{K}_{2kl} = \frac{\partial^2 \hat{K}_2}{\partial U_k \partial U_l} \exp \frac{1}{3}(\hat{K}_1 - \hat{K}_2) .
\]

On the other hand, \(\hat{K}_1\) and \(\hat{K}_2\) are completely determined in terms of two holomorphic functions, the prepotentials \(\mathcal{F}_1(T)\) and \(\mathcal{F}_2(U)\), one for each type of moduli, as \([21, 20]\)

\[
\hat{K}_{1,2} = -\log \hat{K}_{1,2} ,
\]

\[
\hat{K}_1' = (t_i - t_i^*) (\partial_i \mathcal{F}_1 + \partial_i \mathcal{F}_1^*) - 2(\mathcal{F}_1 - \mathcal{F}_1^*) ,
\]

\[
\hat{K}_2' = (u_k - u_k^*) (\partial_k \mathcal{F}_2 + \partial_k \mathcal{F}_2^*) - 2(\mathcal{F}_2 - \mathcal{F}_2^*) .
\]

where the variables \(t = iT, u = iU\) are used. The same property follows for \(\bar{27}\) and \(\bar{27}\) Yukawa couplings

\[
Y_{1ijk} = \partial_i \partial_j \partial_k \mathcal{F}_1 ,
\]

\[
Y_{2ijk} = \partial_i \partial_j \partial_k \mathcal{F}_2 .
\]

Finally, using eqs.\([13, 12]\), the moduli and matter metrics associated with \(27\) representations can be written as

\[
\hat{K}_{1ij} = \hat{K}_1'^{-2} X_{1ij} ,
\]

\[
\hat{K}_{1ij} = \hat{K}_2'^{-5/3} \hat{K}_1'^{-1/3} X_{1ij} ,
\]

\(^7\)For a review see \([20]\) and references therein.
where

\[ X_{1ij} = \partial_i \tilde{K}_1' \partial_j \tilde{K}_1' - \tilde{K}_1' \partial_i \partial_j \tilde{K}_1' . \]  \hfill (20)

The metrics of 27 representations have exactly the same form with the obvious replacement 1 ↔ 2.

### 3.2 Soft terms

Let us apply the previous generic CY formulae \([18,19,20]\) and the parametrization introduced above \([8,9]\) to the computation of the soft terms. We will concentrate first on those which are associated with 27 representations. The results are

\[
m_{kl}^2 = m_{3/2}^2 \left[ \tilde{K}_{1kl} + \cos^2 \theta \left\{ - \left( 1 + 4 \sum_i \Theta_{1i}^* \Theta_{1i} \right) \tilde{K}_{1kl} \\
+ 3 \tilde{K}_1^{r-5/3} \tilde{K}_2^{r-1/3} \Theta_{1m}^* P_1^{m} \right\} \right] \]  \hfill (21)

\[
A_{jkl}' = - \sqrt{3} m_{3/2} e^{K/2} \left\{ \sin \theta e^{-\gamma_S} Y_{1jkl} + \cos \theta P_{1m}^* \Theta_{1m} \right\}
\times \left\{ (X_1^{n\alpha} \partial_i X_{1\alpha j} Y_{1nk} + (j \leftrightarrow k) + (j \leftrightarrow l)) - 4 \tilde{K}_1^{r-1} \tilde{K}_1' Y_{1jkl} - \partial_i Y_{1jkl} \right\} \]  \hfill (22)

where we use latin indices for both matter and moduli fields reflecting the one-to-one correspondence between them. Notice that, since the two types of moduli form separate moduli spaces and the Kähler function is \( \hat{K} = \hat{K}_1(T_i, T_i^*) + \hat{K}_2(U_k, U_k^*) \) we have taken the matrices \( P \) and \( \Theta \) of eq.\([8]\) as

\[
P = \begin{pmatrix} P_1^{ij} & 0 \\ 0 & P_2^{kl} \end{pmatrix} ; \quad \Theta = \begin{pmatrix} \Theta_{ij} \\ \Theta_{kl} \end{pmatrix} \]  \hfill (23)

where \( P_{1,2} \) are the matrices canonically normalizing the moduli fields, i.e. \( P_{1,2}^{i,2} P_{1,2} = 1 \), and \( \sum_j \Theta_{1j}^* \Theta_{1j} + \sum_i \Theta_{2i}^* \Theta_{2i} = 1 \).

Before canonically normalizing (through a “rotation”) the matter fields we already see that the expression for CY trilinear soft terms is in general very complicated. On the one hand, as we will see below, the instanton contribution to \( Y_{1jkl} \) implies that \( \partial_i Y_{1jkl} \neq 0 \). On the other hand, all Yukawa couplings between 27’s are allowed\([8]\). As a consequence, it is not possible in general to factorize out the Yukawa coupling \( Y_{1jkl} \) in (22) since the terms contained in parenthesis are not proportional to it. In addition, universality of trilinear parameters is lost.

Let us focus now on soft scalar masses eq.\( (21) \). If the matter fields are canonically normalized with the matrix \( Q_1, Q_1^* \tilde{K}_1 Q_1 = 1 \) where \( \tilde{K}_1 = \tilde{K}_{1ij} \), and the matrix \( \tilde{R}_1 \) is defined such that \( R_1^{i} X_i R_1 = 1 \) where \( X_i \equiv X_{1ij} \) (see eq.\( (20) \)), then the relations \( Q_1 = K_1^{r/6} K_2^{n/6} R_1, P_i = K_1^{r/6} R_i \) are fulfilled. Now, taking into account these relations, the normalized soft mass matrix can be written as

\[
m_{\delta}^2 = m_{3/2}^2 \left[ \delta_{\delta} + \cos^2 \theta \left( - \delta_{\delta} + \Delta_{\delta} (T_i, T_i^*, \Theta_i, \Theta_i^*) \right) \right] , \]  \hfill (24)

\(^*\)For example, in CY compactifications with two moduli, the four possible different Yukawa couplings \( Y_{jkl} \) with \( j, k, l = 1, 2 \) are in general non-vanishing \([22, 23]\).
Given eqs. (16, 27), it is clear that only $F$ and the non-perturbative instanton correction $\Delta m^2_{1\bar{2}}$ enter the $F$-Yukawa couplings while those for $F_{R}$ are complicated. The soft terms of degeneracy among the eigenvalues of the matrix $m^2_{1\bar{2}}$. In general, $\Delta m^2_{1\bar{2}}$ depends on $\Theta_i$ and the moduli $T_i$ and therefore will have a generic matrix structure with non-degenerate eigenvalues. On the other hand, tachyons may appear. We will study in more detail these issues in specific multimoduli examples below.

However, it is worth noticing here that there is an interesting situation where universality of soft masses and trilinear parameters is achieved. Notice that, although the metric $\tilde{K}_1$ associated with $27$'s has $U$-moduli dependence through the $\tilde{K}^{r-1/3}_2$ factor, soft terms in (22, 24) are independent of $\Theta_{2i}$. The latter are absorbed by the relation $\sum_j \Theta^i_{1j} \Theta_{1j} + \sum_i \Theta^i_{2i} \Theta_{2i} = 1$. The related consequence is that SUSY breaking by dilaton and/or $U$-moduli sector (but not by the $T$-moduli sector), that is $\sum_j \Theta^i_{1j} \Theta_{1j} = 0$ and $\sum_i \Theta^i_{2i} \Theta_{2i} = 1$, leads to universal soft-breaking parameters for $27$'s

$$\sqrt{3}m = |M| = |A| = \sqrt{3}m_{3/2} \sin \theta,$$

while those for $27$'s are complicated. The soft terms of $27$ representations have exactly the same form as above (22, 24) with the replacement $1 \leftrightarrow 2$. Obviously, the opposite case (SUSY breaking by $T$ moduli) leads to universal soft-breaking parameters for $27$'s.

Finally, let us remark that the above results are completely general. They are valid for any expression of the prepotentials since they were obtained just using property (13).

### 3.3 The large-radius limit

In order to study more concrete features of the soft terms one needs some information about the prepotentials $\mathcal{F}_{1,2}$ (see eqs. (14, 15)). As explained in the introduction, this information is only known in the large-radius limit. In particular, whereas $\mathcal{F}_2(U)$ is a complicated function of the complex structure moduli, $\mathcal{F}_1(T)$ is simply (barring instanton corrections) a cubic polynomial. From now on we will concentrate on $T$ moduli associated with $27$ representations. We know that one of them is the “overall radius $R$” of the manifold and therefore is always present. A formal large-radius expansion of $\mathcal{F}_1(T)$ is

$$\mathcal{F}_1 = k_{ijk} t_i t_j t_k + a_{ij} t_i t_j + b_i t_i + c + \mathcal{F}_{\text{inst}},$$

where the imaginary constant $c$ can be identified with a $\sigma$-model loop contribution [27] and the non-perturbative instanton correction $\mathcal{F}_{\text{inst}}$ is a power series in $q_i = e^{2\pi t_i} (= e^{-2\pi T_1})$

$$\mathcal{F}_{\text{inst}} = d_i q_i + e_{ij} q_i q_j + \cdots .$$

Given eqs. (14, 27), it is clear that only $\mathcal{F}_{\text{inst}}$ and the real constant $k_{ijk}$ enter the $27$'s Yukawa couplings

$$Y_{1ijk} = 6k_{ijk} + \partial_i \partial_j \partial_k \mathcal{F}_{\text{inst}} .$$
With respect to the moduli Kähler potential, plugging (27) into eq.(14) one can show that the real constants $a_{ij}$, $b_j$ are irrelevant for it (and therefore irrelevant for physical quantities). The final result (to first order in instanton correction) is

$$
\dot{K}'_i = k_{ijk}(T_i + T'_i)(T_j + T'_j)(T_k + T'_k) - 4ic \\
+ 4d_i[\pi(T_i + T'_i) + 1]e^{-\pi(T_i + T'_i)} \sin i\pi(T_i - T'_i). \quad (30)
$$

**One-modulus case**

Several examples of CY manifolds which possess only one Kähler modulus can be found in the literature [28]. In these cases the previous computation of soft terms is simplified since the moduli and matter metrics are trivially diagonal. The study of these examples, although do not lead to any realistic model, is highly interesting because it will allow us to analyze in detail the modifications produced on soft terms by the σ-model contribution and the instanton correction to the Kähler potential (30). In more realistic models the same type of modification will be present. Now eqs.(30,24) transform in

$$
\dot{K}'_1 = k(T + T^*)^3 - 4ic + 4d[\pi(T + T^*) + 1]e^{-\pi(T + T^*)} \sin i\pi(T - T^*), \quad (31) \\
X_1 = \bar{\partial}\hat{K}_1\bar{\partial}\hat{K}_1 - \hat{K}_{1}\bar{\partial}\hat{K}_1, \quad (32)
$$

and the soft breaking parameters, using eqs.(24,25,22) with $R_1 = X_1^{-1/2}$, $P_1 = \hat{K}'_1X_1^{-1/2}$ and $\Theta = e^{-i\gamma_T}$, are given by

$$
m^2 = m_{3/2}^2 \left[1 + \cos^2 \theta (-1 + \Delta(T, T^*))\right], \quad (33) \\
A = -\sqrt{3}m_{3/2} \left[\sin \theta e^{-i\gamma_S} + \cos \theta e^{-i\gamma_T} \omega(T, T^*)\right], \quad (34)
$$

where

$$
\Delta(T, T^*) = -4 + 3\hat{K}'_1X_1^{-2}(\bar{\partial}X_1X_1^{-1}\partial X_1 - \bar{\partial}X_1), \quad (35) \\
\omega(T, T^*) = \hat{K}'_1X_1^{-1/2}(3X_1^{-1}\partial X_1 - 4\hat{K}'_1^{-1}\partial \hat{K}'_1 - Y_1^{-1}\partial Y_1). \quad (36)
$$

Note that we are assuming for simplicity vanishing $F$ terms associated with $U$ moduli, i.e. $\sum_i \Theta_{ii}^*\Theta_{ii} = 1$ in eq.(25) This will be enough to give us an insight into features of soft terms. In any case, the inclusion of the $F$ terms associated with $U$ moduli is straightforward. Since we have only one Yukawa coupling, it is possible to factorize it out trivially in (22). This has been carried out in eq.(34) where $e^{K/2}$ has also been factorized out.

In ref. [6] the large-radius limit of CY compactifications was studied in the approximation that only the first term in eq.(31) contributes to the Kähler potential. The final soft terms were

$$
m^2 = m_{3/2}^2 \left[1 - \cos^2 \theta\right], \quad (37) \\
A = -\sqrt{3}m_{3/2} \left[\sin \theta e^{-i\gamma_S} - \cos \theta e^{-i\gamma_T} \frac{(T + T^*)}{\sqrt{3}} \partial_T Y\right], \quad (38)
$$

It is worth noticing here that, as we will see in the multimoduli case, the “overall-modulus” limit, where the assumption that all $T_i$ moduli contribute exactly the same to SUSY breaking is made, is equivalent to the one-modulus case.
It is easy to see that this result is recovered with our formulae since $\Delta = 0$ and $\omega = -\frac{1}{(\sqrt{3})^{-1}(T + T^*)}Y^{-1}\partial_T Y$ when $K = -\log k(T + T^*)^3$. It can be further argued that, in the large-radius limit, the Yukawa couplings tend exponentially to constants (see eqs.(28,29)). Then one can take $\omega \to 0$ and therefore

$$A \simeq -\sqrt{3}m_{3/2} \sin \theta e^{-i\gamma S}. \quad (39)$$

Using eq.(10) the following relations between soft terms are fulfilled

$$\sqrt{3}m = |M| \simeq |A|. \quad (40)$$

Let us now study the departure from the previous results (37,38) when the most general case, including the terms proportional to $c$ and $d$ in (31), is considered. Due to these $\sigma$-model and instanton contributions the analytic calculation is more involved.

Using again eqs.(35,36) and expanding the quantities in terms of $1/\pi D$ we obtain at the leading order

$$\Delta(T, T^*) = \frac{40C}{(T + T^*)^3} + \frac{8D}{3\pi}e^{-\pi(T + T^*)} \sin i\pi(T - T^*), \quad (41)$$

$$\omega(T, T^*) = \frac{10\sqrt{3}C}{(T + T^*)^3} + \frac{4D}{\sqrt{3}}e^{-2\pi T^*} - \frac{(T + T^*)}{\sqrt{3}} \partial_T Y, \quad (42)$$

where $C \equiv -4ic/k$ and $D \equiv 4\pi^3d/k$. As discussed above, the third term in (12) is negligible. These are the moduli-dependent modifications to soft terms arising from $\sigma$-model and instanton contributions. They are also model dependent due to the factors $k$, $c$ and $d$. For the four one-modulus models classified in ref.[28] these factors have been computed. In particular

$$(k, c) = (5/6, 25i\zeta(3)/\pi^3); (1/2, 51i\zeta(3)/2\pi^3); (1/3, 37i\zeta(3)/\pi^3); (1/6, 36i\zeta(3)/\pi^3) \quad (43)$$

and $d$ is of order 100 [24]. Using these numbers we can see that the orders of magnitude of $C$ and $D$ are $10$ and $10^4$ respectively, and therefore, depending on the value of $\Re T$, the corrections (12) might be sizeable. For instance, taking $\Re T = 5$, we obtain (using the average values $C = 10$ and $D = 10^4$) that the first term in eqs.(41) and (42) acquires the values 0.4 and 0.17 respectively whereas the second one acquires the values $-2 \times 10^{-10} \sin 2\pi \Im T$ and $5 \times 10^{-10} e^{2\pi \Im T}$ respectively. Clearly the latter, which is due to instanton corrections, is negligible as was to be expected since it is exponentially suppressed in the large-radius limit. However, the $\sigma$-model contribution in this “average” model is large and produces the following soft terms

$$m^2 \simeq m_{3/2}^2 \left[ 1 - 0.6 \cos^2 \theta \right], \quad (44)$$

$$A \simeq -\sqrt{3}m_{3/2} \left[ \sin \theta e^{-i\gamma S} + 0.17 \cos \theta e^{-i\gamma T} \right], \quad (45)$$

\[10\] We note for completeness that the matter metric (13) is $\hat{K}_1 = k^{1/3}K_2^{1/3} \hat{3}(T + T^*)^{-1} - 11C(T + T^*)^{-4} + 4\pi D(\pi(T + T^*)^{-1} - e^{-\pi(T + T^*)} \sin \pi(T - T^*)].$

\[11\] Notice that in the modulus-dominated SUSY-breaking scenario, $\sin \theta \to 0$, these corrections may be very important in general since the tree-level soft terms are vanishing. This was first pointed out in the context of SUSY breaking by gaugino condensation in ref.[17].
to be compared with those of eqs.(37,39) in the $\Delta = \omega = 0$ limit. Unlike what happens in that limit (see eq.(40)) now there is, in principle, the possibility of having scalars heavier than gauginos. In this “average” model the necessary condition is $\cos^2 \theta > 0.83$, and in general it will be

$$\cos^2 \theta > \frac{2}{2 + \Delta}. \quad (46)$$

In Figure 1 we show the behavior of $\Delta$ and $|\omega|$ as functions of $\text{Re} \, T$ for the last model of eq.(43). The solid lines correspond to the exact results eqs.(35,36) whereas the dashed lines correspond to their approximations eqs.(41,42). These approximations are quite good and for large enough values of $\text{Re} \, T$ both lines coincide. Let us remark that $\omega$, although we neglect the Yukawa coupling contribution, is a complex quantity due to the instanton correction. In the figure we take the particular value $\text{Im} \, T = 1/4$ for which $\Delta$ in (11) is maximum, but different values will not modify the figure due to the smallness of the instanton correction. In this CY model the departure from the $\Delta = \omega = 0$ limit is larger than in the “average” model studied above. In particular, for $\text{Re} \, T = 5$ as above we get $\Delta = 1.62$ and $|\omega| = 0.64$. For instance, $m^2 \simeq m^2_{3/2}[1 + 0.62 \cos^2 \theta]$ and scalars will be heavier than gauginos if $\cos^2 \theta > 0.55$. Finally, the dotted lines represent the absolute value of instanton contributions inside eqs.(35,36). As mentioned above they are essentially negligible.

**Multimoduli case**

As we already mentioned above, in the case of CY compactifications with several moduli, the metrics are not diagonal and therefore a general analysis becomes very involved. We will try to study what new features can appear in this situation. Since we are mainly interested in the consequences of off-diagonal metrics, in order to simplify the analysis, we will consider in (30) only those terms which are relevant for this issue. Thus we are left with the cubic terms. The inclusion of $\sigma$-model and instanton contributions is straightforward following the lines of the previous section.

Let us first consider the two illustrative two-moduli CY manifolds of ref.[24] whose prepotentials are

$$F = \frac{5}{6}(t_1^3 + t_2^3) + 5(t_1^2 t_2 + t_1 t_2^2), \quad (47)$$

$$F' = \frac{1}{3}t_1^3 + 4t_1^2 t_2 + 3t_1 t_2^2. \quad (48)$$

Taking into account the general formulae of section 3.1 and eq.(30), this information is enough in order to discuss the soft scalar masses (24). The matrix $\Delta_{ij}$ (24) with $i, j = 1, 2$ is extremely involved and depends on moduli $T_i$ as well as $\Theta_i$. In fact, the moduli dependence of soft breaking parameters appears through the ratio $T_2/T_1$ due to our parametrization. In order to get some conclusions we examine $\Delta_{ij}$ for specific values of $T_2/T_1$. In the case of $F$ with $T_2/T_1 = 1$ and $T_2/T_1 = 2$, $\Delta_{ij}$ is simplified to

$$\begin{pmatrix}
0 & -4\Theta_1 \Theta_2 \\
-4\Theta_1 \Theta_2 & -3\Theta_2^2
\end{pmatrix} \quad (49)$$
and
\[
\begin{pmatrix}
-0.7281 \Theta_1^2 + 2.262 \Theta_1 \Theta_2 + 0.1866 \Theta_2^2 \\
1.131 \Theta_1^2 - 3.627 \Theta_1 \Theta_2 + 0.07544 \Theta_2^2 \\
1.131 \Theta_1^2 - 3.627 \Theta_1 \Theta_2 + 0.07544 \Theta_2^2
\end{pmatrix}
\]
respectively, where the parameters $\Theta_j$ and the $F$ terms associated with $U$ moduli have been taken real and vanishing respectively for simplicity. $F$ possesses a symmetry under the exchange of $t_1$ and $t_2$. For example, $T_2/T_1 = 2$ and $T_2/T_1 = 1/2$ yield the same result. In the case of $F'$, e.g. with $T_2/T_1 = 1$ we obtain
\[
\begin{pmatrix}
-0.1712 \Theta_1^2 + 1.127 \Theta_1 \Theta_2 + 0.1267 \Theta_2^2 \\
0.5637 \Theta_1^2 - 3.747 \Theta_1 \Theta_2 - 0.1945 \Theta_2^2 \\
0.5637 \Theta_1^2 - 3.747 \Theta_1 \Theta_2 - 0.1945 \Theta_2^2
\end{pmatrix}
\]
As mentioned below (25), $\Delta_{ij}$ has a generic matrix structure with non-vanishing eigenvalues and therefore universality is lost in general, unless $\cos^2 \theta \ll 1$, i.e. the dilaton-dominated SUSY-breaking scenario \[10, 13\], or SUSY is broken by $U$-moduli sector. E.g., For $\Delta_{ij}$ given by (49) the two squared-mass eigenvalues are
\[
m_{1,2}^2 = m_{3/2}^2 \left[ 1 + \cos^2 \theta \left( -1 + \Delta_{1,2} \right) \right]
\]
where
\[
\Delta_{1,2} = -\frac{3}{2} \Theta_2^2 \pm \sqrt{\frac{9}{4} \Theta_2^4 + 16 \Theta_1^2 \Theta_2^2}.
\]
For instance, $\Theta_1 = \Theta_2 = 1/\sqrt{2}$ implies $\Delta_{1,2} = 1.39, -2.89$, i.e. $m_1^2 = m_{3/2}^2 (1 + 0.39 \cos^2 \theta)$ and $m_2^2 = m_{3/2}^2 (1 - 3.89 \cos^2 \theta)$. We can find the specific $\Theta_i$ at which the eigenvalues become degenerate (this is of course a sort of fine-tuning). In this case $\Theta_1 = 1$, $\Theta_2 = 0$. For $\Delta_{ij}$ given by eq.(50) (eq.(51)), $\Theta_1 = 0.9541$, $\Theta_2 = 0.2994$ ($\Theta_1 = 0.9890$, $\Theta_2 = 0.1477$). This requires $\Delta_{ij} = 0$, i.e. one is pushed towards the “overall-modulus” limit\[11\], $|F_{T_1}| = |F_{T_2}| = \sqrt{3} m_{3/2} \cos \theta / \sqrt{2}$ (see eq.(53)).

This is a general result, the universality of soft breaking parameters in multimoduli cases is achieved by vanishing $\Delta_{ij}$. The equivalent result for trilinear parameters is that the braces in (22) must be vanishing.

Since some of the eigenvalues $\Delta_i$ can be positive (see e.g. [53]), this open the possibility of having scalars heavier than gauginos depending on the Goldstino direction. The necessary condition is the same than in (49) with $\Delta = \Delta_i$.

Finally, for negative eigenvalues, depending again on the Goldstino direction, tachyons may appear unless\[13\]
\[
\cos^2 \theta < \frac{1}{1 - \Delta_i}.
\]

\[12\] For a discussion of how to ameliorate the situation concerning FCNC, taking into account the flavour independent contribution from gauginos to low-energy scalar masses, see [1, 24].

\[13\] Notice that this limit is equivalent to the one-modulus case, see [17]. We recall that we are not considering in this computation, for the sake of simplicity, the $\sigma$-model and instanton contributions.

\[14\] It is worth pointing out here that their possible existence is not necessarily a problem, but in some cases may be an interesting advantage in order to break extra gauge symmetries [1].

11
4 A comparison with orbifold compactifications

It is interesting to compare the previous results on soft terms with those of orbifold compactifications [6, 9]. We recall that orbifolds are a special limit of particular CY manifolds. Let us focus first on the “overall-modulus” case. For this class of orbifold models the Kähler potential has the form

\[ K = - \log(S + S^*) - 3 \log(T + T^*) + \sum_{\alpha} |C_{\alpha}|^2 (T + T^*)^{n_{\alpha}}. \]  

(55)

It is important to remark that, unlike the case of smooth CY models, the above \( T \) dependence does not get corrections from world-sheet instantons and is equally valid for small and large \( T \). Notice that, with matter fields in the untwisted sector, i.e. modular weight \( n_{\alpha} = -1 \), the resulting Kähler potential is analogous to the one obtained in the large-radius limit of CY models neglecting the \( \sigma \)-model and instanton corrections (see eqs.(13,31) and footnote 10). As a consequence, in the untwisted sector of orbifolds the soft terms are also given by (54) with gaugino masses always bigger than scalar masses (53). Unlike CY models now eq.(39) is exact since Yukawa couplings involving untwisted fields are constants and therefore the last term in (53) is exactly zero.

As discussed below eq.(54), the situation is qualitatively different when \( \sigma \)-model and instanton corrections are included in CY models. Then, the soft terms are modified due to the non-vanishing values of \( \Delta \) and \( \omega \) in eqs.(11,12) allowing e.g. the possibility of scalars heavier than gauginos.

Relaxing the “overall-modulus” assumption in orbifolds, some qualitative changes appear\(^{15}\). In the multimoduli case, non-universal soft scalar masses for untwisted particles are allowed and in fact this will be the most general situation. Tachyons may also be present. Finally, there may be scalars with mass bigger than gauginos. However, on average the scalars are lighter than gauginos since three particles linked via a renormalizable untwisted Yukawa coupling always fulfill the sum rule

\[ m_1^2 + m_2^2 + m_3^2 = |M|^2, \]  

(56)

as in the “overall-modulus” case (3\( m^2 = |M|^2 \)). For trilinear parameters one also has

\[ A_{123} = -M. \]  

(57)

In ref.[4] was shown that these results (54,55) are satisfied even in the presence of off-diagonal metrics. Let us recall that although diagonal metrics is the generic case in most of orbifolds, off-diagonal ones appear for fields in the untwisted sector of the orbifolds \( Z_3, Z_4, Z_6' \) (see footnote 4).

In CY compactifications where off-diagonal metrics are always present, some of the above orbifold properties are qualitatively similar. In the previous section we showed that non-universality is a natural situation due to the generic matrix structure of the soft masses. Likewise, the presence of tachyons or scalars heavier than gauginos was

\(^{15}\) For an extensive discussion see [5].

\(^{16}\) However, as showed in section 3.2, in the special situation where only the \( U(T) \) moduli contribute to SUSY breaking, the 27 (27) soft masses and trilinear terms are universal. This is not true in the case of orbifolds.
allowed. However, the sum rule (56) is violated in general. As an example, we can consider the two-moduli cases studied in the previous section, where \( Y_{111}, Y_{122}, Y_{211}, Y_{222} \) are the allowed Yukawa couplings. From (52) we obtain

\[
\begin{align*}
3m_1^2 &= |M|^2 + 3m_{3/2}^2 \Delta_1 \cos^2 \theta, \\
3m_2^2 &= |M|^2 + 3m_{3/2}^2 \Delta_2 \cos^2 \theta, \\
m_1^2 + 2m_2^2 &= |M|^2 + 3m_{3/2}^2 \frac{\Delta_1 + 2\Delta_2}{3} \cos^2 \theta, \\
m_2^2 + 2m_1^2 &= |M|^2 + 3m_{3/2}^2 \frac{\Delta_2 + 2\Delta_1}{3} \cos^2 \theta,
\end{align*}
\]

(58)

where, taking into account (53), is easy to see that \( 3m_1^2 > |M|^2; 3m_2^2, m_1^2 + 2m_2^2 < |M|^2 \) and both possibilities are allowed for \( m_2^2 + 2m_1^2 \) depending on the \( \Theta_{1,2} \) values. Due to these two possibilities, we cannot say as in the case of orbifolds that on average the scalars are lighter than gauginos. This is an interesting novel fact.

Likewise (57), which is fulfilled in orbifolds due to the antisymmetric property of Yukawa couplings, no longer is true in CY compactifications as discussed below eq. (22).

## 5 Conclusions

The study of the soft SUSY-breaking terms coming from string compactifications is highly important since they determine the low-energy phenomenology of these theories. We have carried out this study for some specially interesting and complicated spaces. In particular, Calabi-Yau manifolds have been analyzed with all the detail allowed by the present state-of-the-art technology. As discussed in the introduction only the large-radius limit of (2,2) theories is really available.

After providing a parametrization of the soft terms in terms of “Goldstino angles” for the most general case of off-diagonal metrics, we have computed them for Calabi-Yau spaces. Although these results are general in the sense that they do not depend on the size of the manifold, in order to obtain more concrete features we have also worked in the large-radius limit where the prepotential is known. As mentioned above, the soft terms show formidable complexity mainly due to the \( \sigma \)-model and instanton contributions to the Kähler potential as well as the existence of off-diagonal moduli and matter metrics. Although in the large-radius limit instanton contributions are negligible since they are exponentially suppressed, \( \sigma \)-model corrections to the soft terms may be important depending on the value of \( \text{Re} T \). On the other hand, very specific features appear due to the presence of off-diagonal metrics. For example, soft scalar masses have a generic matrix structure giving rise to non-universality in general. However, there is an interesting situation, SUSY breaking by \( U(T) \)-moduli sector, where universality for \( 27 \) (27) representations is achieved. Tachyons may also appear. Other features are related with the possibility of having scalars lighter or heavier than gauginos depending on the Goldstino direction. In this sense, sum rules found in orbifold models, which imply that on average the scalars are lighter than gauginos, can be violated in Calabi-Yau manifolds.
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Figure Captions

Figure 1 $\Delta(T, T^*)$ and $|\omega(T, T^*)|$ as functions of Re $T$. The solid lines correspond to the exact results. The dashed lines correspond to the approximate results. Finally, the dotted lines represent the absolute value of instanton corrections.