An Enhanced Mathematical Model in Sports Events Timetabling

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Abstract

Events such as Olympics, Football World Cup and Golf and Tennis huge tournaments have always drawn the attention from fans and TV viewers. To obtain this attention many factors are involved. One the major reasons to attract fans is the time order of matches. Another factor is that prominent matches must be distributed over a time interval of the tournament. These make a problem which is called "Sport Events Timetabling". This article develops a MIP model to scheduling sport events. In this model, intrinsic constraints of sport events timetabling are considered. For example, it is possible to schedule group matches and matches which plays between two teams, simultaneously. At the end a case study is introduced and solved with the GAMS 9.32 software.

1. Introduction

Sports tournaments have global attraction, and there are significant investments on players in the professional leagues. Events such as the Olympic Games, the FIFA World Cup, and the great Golf and Tennis tournaments have very TV viewers, and many sports have been changed into multi-million dollar industry [1]. One key aspect of sports events is the ability to create a timetable in such a way that the logistical aspects are optimized and fair. This area has a history of 40 years, and in recent years, the number of articles written for this subject has increased dramatically, indicating that scientific interest in this field is increasing [2].

Sports have become a big business. In a global economy, many cities and countries compete for the right to host major events such as the Olympic Games and the Football World Cup, as they bring thousands of jobs, urban rebuilding, and economic opportunities to the host. The tournaments are followed by millions of people around the world who are eager to know about the progress of their team in any competition [3]. Fans check newspapers, radio, television and the internet for searching information. Timetabling and managing the sports events have attracted the attention of a large number of researchers in multidisciplinary areas such as research on operations, timetabling theory, constraint planning, graph theory, combine optimization and applied mathematics [4]. As a result of this extent and importance, there is always a need for sport timetabling for sports events. Particularly when the various constraints are imposed by the custodian organizations, we face a very difficult optimization problem. These constraints guarantee the minimum of fairness of the timetabling, the attractiveness of the game, support issues, television broadcasting, quality of holding, support for national sport and ...[5].

The aim of this study was to provide a mathematical model for the timetabling of sports Olympiads consisting of several teams in the different sports field. In the next section, the literature review will be addressed using the research records. In section 3, the base model of this research is discussed and in section 4, the enhanced model of timetabling is presented. Section 5 reports the way of implementation of the model on a case study (Sports Olympiad of the Khorasan Gas Company), and section 6 summarizes the conclusion of the research.
2. Content

In this section the content of this paper is explained.

2.1. Literature review

Recalde et al. [1] improved Ecuadorian professional football schedule, which was provided manually before the year of 2011, they indicated by providing evidence to the Ecuadorian Football Federation that the use of mathematical planning makes the timetable more flexible and with more benefits. In Briskorn and Drexel research [7], various types of problems are described based on periodic competitions. In addition, many real-world constraints are discussed. For example, some teams cannot play at certain times because the stadium is available to another team or event at that time. Trick [8] presented a two-phase method for timetabling of tournaments. In the first phase, a program is created that ignores any requirement (guest-host) by using the constraint programming method. In the second phase, the pattern of guest-host is determined by using a mixed integer programming (MID) model through minimizing undesired structures (such as consecutive guest and host competitions). Elf et al. [9] showed that the problem of minimizing the stop for an opponent's program turns into the maximum cutting problem in an undirected graph which is solved by the branch and bound algorithm. Miyashiro and Matsui [61] proved that for an opponent program with n team, it can be checked by reduction into the Boolean satisfiability problem (SAT) in "polynomial time" whether there is a guest-host pattern (home-away) with n-2 stops or not. Miyashiro and Matsui [11] formulated the problem of minimizing the stop for a program. Post and Woeginger [19] in terms of stop minimizing problem indicated that there is a host-guest pattern with a maximum stop for each opponent program with n team. In Briskorn's research [1], the problem of optimizing the pattern set is studied and essential condition is given for the formulation of the integer. De Werra et al. [61] argued that the applications of the tournament can be modeled both by constraint programming and integer programming. Constraint programming is often faster but depends on the limits and the objective function. Drexel and Knust [61] presented a hybrid algorithm of integer and constraint programming which was designed to find the timetable of the tournament with at least number of stops and also in order to consider the local constraints. Suzaka et al. [61] presented a solid vision of the guest-host allocation problem (such as minimizing distance) and the problem of minimizing stops in tournaments. Computer computational results indicate that these methods provide solutions with a good approximation and at the quick computational time. The basic model of this research is described below. In 2016, Akkan et al. [11] presented a timetable model to determine the time of university students' presentation. The objective function of this model consists of two parts and seeks to simultaneously achieve the two objectives of diversity and homogeneity. Here, diversity means that different teams with the same learning characteristics are scattered as much as possible during the timetabling period in order to preserve the attractiveness of project presentation.

2.2. The proposed model for the timetabling of sports events

In this model, we seek to put different competitions from various sports fields in the timetable so that competitions, which are held at the same time, select from one field of sport; and also competitions with different levels of sensitivity are held at a different time as much as possible.

For this purpose, by considering the model presented by Akkan et al. [11], the model is presented which able to consider sports tournaments with high flexibility in terms of the number of participating teams in addition to the benefits in relation to previous models. So that solitary games like track and field and face-to-face matches like football, both are definable in the model. Also, in this model, the difference in time of holding different competitions is also considered in the timetabling. The new parameters, which are required for this model, are given in table 1.

| Table 1. Definition of the parameters for the enhanced model |
|-------------------------------------------------------------|
| Model parameters                                            |
| If match i is the type of l, the parameter set to 1 otherwise 0 | \( p_{il} \) |
| If time t is allowed for the match i, the parameter set to 1 otherwise 0 | \( f_{it} \) |
| If matchi is at level k, the parameter set to 1 otherwise 0 | \( a_{ik} \) |
| Match time is the type of l | \( h_{it} \) |
| Maximum available time in a range | \( m_{ik} \) |
| Minimum available time in a range | \( m_{ik} \) |
| If team j is participated to match i, the parameter set to 1 otherwise 0 | \( a_{ij} \) |
| The standard time range between matches of a team according to the type of matches of that team | \( M_{ik} \) |
| The average number of matches from a sensitivity type k that can be set at time range which will be obtained by dividing the number of matches of type k into the number of the available time range | \( q_{ik} = \frac{\sum a_{ik}}{T} \) |
It is also necessary to change the variables for sports game timetabling problem and define a number of new variables.

| Table 2. Definition of variables for the enhanced model |
|--------------------------------------------------------|
| Model variable                                        |
| If time $t$ is used, it is set to 1 otherwise 0        |
| $Y_t$                                                  |
| If match $i$ is allocated to time $t$, it is set to 1 otherwise 0 |
| $X_{it}$                                               |
| Homology deficiency variable of matches type at time $t$ and for type 1 |
| $R_{it}$                                               |
| Homology deficiency variable of matches type at time $t$ |
| $\overline{R}_t$                                      |
| Deficiency and surplus variables for a variety of tournament sensitivity in the timetable |
| $S_{tk}, S^+_{tk}$                                    |
| If match type 1 is suited to time $t$, it is set to 1 otherwise 0 |
| $T_{it}$                                               |
| If match type 1 is not suited to time $t$, it is set to 1 otherwise 0 |
| $S^-_{it}$                                             |

The objective function of this model consists of two parts. The first part is to increase diversity in the sensitivity of the tournament and the second part is to perform similar matches in terms of the type of game in the same time range. The new model will be changed in the form below.

\[
\begin{align*}
\min & \sum_{t=1}^{T} \sum_{k=1}^{K} (S_{tk}^- + S_{tk}^+) + \overline{R}_t \\
\sum_{t=1}^{T} X_{it} &= 1 \quad \forall i \\
X_{it} &\leq f_{it} \quad \forall i, t \\
\sum_{i=1}^{I} \left( X_{it} \sum_{l=1}^{L} p_l h_l \right) &\leq h_{\max} Y_t \quad \forall t \\
\sum_{i=1}^{I} \left( X_{it} \sum_{l=1}^{L} p_l h_l \right) &\geq h_{\min} Y_t \quad \forall t \\
\sum_{l=1}^{L} p_l = Y_t \quad \forall t \\
\sum_{l=1}^{L} p_l X_{lt} h_l + R_{lt} &= \sum_{l=1}^{L} \left( X_{lt} \sum_{l=1}^{L} p_l h_l \right) \quad \forall t, l \\
R_{lt} &\leq \overline{R}_t + h_{\max} (1 - P_{lt}) \quad \forall t, l \\
R_{lt} - \overline{R}_t &\geq Y_t - T_{lt} \quad \forall t, l \\
R_{lt} - \overline{R}_t &\leq h_{\max} S_{lt} \quad \forall t, l \\
T_{lt} + S_{lt} &= Y_t \quad \forall l, t \\
T_{lt} + T_{kt} - 1 &\leq P_{kt} \quad \forall t, \forall k < l \\
\sum_{i=1}^{I} a_{ik} X_{it} + S_{tk}^- - S_{tk}^+ &\geq Y_t q_k \quad \forall k, t \\
c_{ji} X_{it} t' - c_{ji} x_{it} t &\geq -M (1 - c_{ji} x_{it} t') \\
+ c_{ji} x_{it} \sum_{l=1}^{L} p_l M_l &\quad \forall j, i, \forall t', \forall i' \neq i
\end{align*}
\]
Constraint (1) indicates that each match must be held in only one-time range. Constraint (2) ensures that a time can be only assigned to time range which is allowed for that match. Constraints (3) and (4) cause a time range to be used only if the minimum and maximum time of that range are observed. Constraint (5) states that a time range should have a major issue between sports fields if it used. Constraints (6) and (7) are intended to set one type of sport in a time range as much as possible. Constraints (8), (9), (10) and (11) indicate that if at the one-time range, the two sports field jointly assigned the highest amount of time, then the sports field will be the subject of that time range, which is ahead in terms of the number. Constraint (12) also tries to disperse the various competitions in terms of sensitivity in different time range depending on the objective function. The ultimate Constraint (13) follows to set the standard gap between the games of a team.

2.3. Case study

In order to validate the model in a sports event timetabling, a case study was selected and then modeled and solved using GAMS 9.32 software. The data is related to the sports Olympiad of Khorasan Razavi Gas Company.

| Table 3. Olympiad of Khorasan Razavi Gas Company |
|-----------------------------------------------|

| Competition  | Number of teams | Explanations                          |
|--------------|-----------------|---------------------------------------|
| Swimming     | 16              | 2 matches with 8 teams (game number 1 and 2) |
| Football     | 4               | 6 matches as a league (game number 3 to 8) |
| Badminton    | 4               | 6 matches as a league (game number 9 to 14) |

In the table 4, the way of teams’ allocation to different games is presented. For example, two teams of 1 and 2, and also the two teams of 1 and 3 will participate in the game number 3 and 4, respectively. Obviously, due to the presence of team 1 in both matches, these games cannot be played consecutively in the tournament.

After solving the model using the data of the tournament, the following timetabling was obtained in a 7-day period.

| Table 4. Allocation of the subject to the days of the tournament |
|---------------------------------------------------------------|

| Game number | Day |
|-------------|-----|
| 1           | Swimming | Football | Badminton | Football | Badminton | Badminton | Football |
| 2           |       |       |       |       |       |       |       |
| 3           |       |       |       |       |       |       |       |
| 4           |       |       |       |       |       |       |       |
| 5           |       |       |       |       |       |       |       |
| 6           |       |       |       |       |       |       |       |
| 7           |       |       |       |       |       |       |       |

This model works in a way that the minimum allowed gap is observed between the games of a team in two consecutive matches. For example, given that the football team 1 participates in game number 3, 4 and 5, these matches are assigned to day 2, 7 and 4 respectively. In the table below, the allocation of football teams to the tournament is shown, which will be the same for other sports.

| Table 5. The matrix indicating the presence of football teams in matches |
|---------------------------------------------------------------------|

| Team number | Match number |
|-------------|--------------|
| 1           | 3 4 5 6 7 8 |
| 2           | 1 1 1 1     |
| 3           | 1 1 1 1     |
| 4           | 1 1 1 1     |

In the table below, the allocation of football matches to match days is shown, which will be the same for other sports.

| Table 6. the matrix indicating football matches allocation |
|----------------------------------------------------------|

| Match number | Day |
|--------------|-----|
| 3            | 1   |
| 4            | 1   |
| 5            | 1   |
| 6            | 1   |
| 7            | 1   |
| 8            | 1   |
3. Conclusion

This research has been designed to provide a timetable method for sports events. The aim is to determine a timetable for a tournament in such a way that both objective functions are improved simultaneously.

The first goal is to increase diversity in the sensitivity of tournament so that sensitive matches are dispersed as much as possible during the tournament and the second goal is to hold similar competitions in terms of the type of game in a time range. This model is able to consider sports competitions with a high degree of flexibility in terms of the number of teams participating in a tournament so that group games such as the track and field and face-to-face games like football can both be defined in the model. In this model, the difference in holding times of various competitions is also considered in the timetabling. This model was used to timetable the sports Olympiad of Khorasan Gas Company. This model is capable to be used in larger scale due to its high flexibility in the definition of group and face-to-face competitions. For this reason, it is suggested that future researchers, after eliminating this research certain limitations, use the meta-heuristic methods to solve the problem.

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