On equilibrium magnetoplasma configurations in “Galatea-Belt” magnetic traps

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Abstract. The computation results of equilibrium magnetoplasma configurations in magnetic Galatea-traps developed for solution of controlled thermonuclear fusion problem are presented in the paper. In traps of this type, electric current conductors creating a confining magnetic field are immersed into the plasma volume but do not come in contact with it. In numerical solution of two-dimensional boundary value problems with the elliptic Grad-Shafranov equation, distributions of the main parameters of the plasma and the magnetic field in the trap “Galatea-Belt” with two electric current conductors are obtained. In the series of numerical experiments, the research on how the problem parameters such as the maximum plasma pressure measured in magnetic units, the relative radius of the electric current conductors and the configuration transverse size influence on the properties and characteristics of the configurations is provided.

1. Introduction

Clean next generation energy and transition to recourse efficient technologies are vital modern challenges for our world. Thermonuclear fusion is one of perspective solution of these problems [1]. It can be one of the safe and cheap energy sources for the steadily increasing energy need of humanity. In thermonuclear reactions, nuclei of the periodic table light elements fuse in the heavier ones with energy generation. The most energy effective reaction is fusion of hydrogen isotopes deuterium and tritium with creation of helium nucleus and fast neutron with the energy release of 17.6 MeV (figure 1).

To realize the fusion reaction, it is necessary to bring participating nuclei closer together enough to overcome the long-range forces of Coulomb repulsion and involve the short-range strong interaction with a heavier element formation. Such reactions are constantly taking place in stars due to gravitational compression, but it is technologically impossible to do this in Earth conditions. The other way is to give the nuclei a sufficient amount of kinetic energy in modern accelerators, but this approach is energy ineffective: the necessary energy consumed for the acceleration is far more than the energy released in the fusion reaction.
In the present time, the most preferred and technologically possible way is to heat a substance to a very high temperature under which the energy of chaotic movement of the particles is enough to launch the fusion reaction. Under such a high temperature, the heated matter can exist only in the state of the quasi-neutral ionized gas – plasma. Obviously, construction elements of a thermonuclear reactor being in the contact with the plasma in which thermonuclear reactions are constantly taking place would be destroyed. But this problem has a technological solution based on the plasma opportunity to conduct an electric current and thereby to be influenced by a magnetic field. This solution is the traps for magnetic plasma confinement. In these plants, the magnetic field has a configuration which does not allow the heated plasma to be in contact with the reactor construction elements.

The most famous magnetic traps are tokamaks in which confining magnetic field is created mainly by the current flowing in the plasma and stellarators in which the plasma is maintained in the equilibrium state only by external conductors with complex geometry.

The magnetic traps “Galatea” [2] are the promising class of the plants for plasma magnetic confinement. The feature of these traps is that electric current conductors creating the confining magnetic field are immersed into the plasma volume but do not contact with it. Such conductors were called “mikkins”. The spectrum of possible magnetic configurations in Galatea-traps is wider than in tokamaks and stellarators, and this fact allows to expect more effective plasma confinement.

In the present paper, the magnetic traps “Galatea-Belt” with two electric current conductors are considered. The study is provided in terms of numerical experiment and computations of corresponding mathematical modelling problems [3]. This approach allows to reduce cost of expensive natural experiments. The mathematical model of the problem is outlined (for more details, see [4]) and the results of the calculation series of equilibrium plasma configurations are presented. Their dependencies on the main problem parameters were analyzed.

2. Mathematical model of equilibrium magnetoplasma configurations in “Galatea-Belt” traps

The object of mathematical modelling is equilibrium magnetoplasma configurations in a toroidal magnetic trap “Galatea-Belt” [5] straightened to a cylinder in the quadratic area \((x, y)\) perpendicular to the cylinder axis. Originally, the model of the “Galatea-Belt” trap discussed below, but for the ring section of the cylinder, was introduced in the paper [6].

The plasma is considered as a continuous electric conductivity medium with common for ions and electrons macroparameters. The dissipative effects (thermal conductivity, gas and magnetic viscosities) are not relevant and therefore are overlooked.

In these approximations, an equilibrium magnetoplasma configuration is described in terms of the differential equations of magnetic hydrodynamics (MHD) under \(\partial / \partial t \equiv 0\), which in the dimensionless variables have the form:

\[
\nabla p = j \times H, \quad j = \text{rot } H, \quad \text{div } H = 0, \quad j = j^{pl} + j^{ex}
\]
The plasmastatic equations (1) connect with each other the plasma pressure $p$, the electric current density $\mathbf{j} = (0,0,j)$ and the magnetic field $\mathbf{H} = (H_x,H_y,0)$.

The units of measurement are the dimension problem parameters – the distance from the cylinder axis to the conductors $x_0$ and the value of the electric current in them $j$ – and their combinations:

$$x_u = y_u = x_0, \quad H_u = \frac{2j}{c x_u}, \quad j_u = \frac{c}{4\pi x_u}, \quad p_u = \frac{H_u^2}{4\pi}$$

(2)

The electric current in the system $\mathbf{j} = (0,0,j)$ is consisted from the plasma current $j_{\text{pl}}$ and the current in the conductors $j_{\text{ex}}$ artificially introduced in the model. Such an approximation allows to solve the problem in the simply connected area without the necessity to pose boundary conditions on the conductor area. In doing so, the current $j_{\text{ex}}(x,y)$ can be approximated by any function which fast decrease outside the conductors. In the present work, this function is the following:

$$j_{\text{ex}}(x,y) = j_0 \sum_{k=1}^{2} \exp\left\{\frac{-|(x-x_k)^2 + y^2|}{r_c^2}\right\}$$

(3)

where $x_1 = -1$, $x_2 = 1$ – abscissa of the miksin centers and $r_c$ – their relative radius. The factor $j_0$ is determined from the requirement that the integral of the function $j_{\text{ex}}(x,y)$ taken over the neighborhood of the current is equal to the full current $j$. In the dimensionless variables, this parameter is determined by the equality $j_0 = 2/r_c^2$.

The considered magnetoplasma configurations have a plane symmetry. This fact seriously simplifies the mathematical apparatus used for their further numerical study. The plasmastatics equations (1) are reduced to the one equation – the plane variety of the Grad-Shafranov equation [7, 8] for the magnetic flux function $\psi(x,y)$ – the z-component of the vector potential $\Psi$ of the magnetic field $\mathbf{H} = \text{rot} \, \Psi$:

$$\Delta \psi + \frac{dp}{d\psi} + j_{\text{ex}} = 0$$

(4)

It is assumed that external boundaries of the trap are opaque for the magnetic field, therefore $\psi|_\Gamma = \text{const}$ on them. Without loss of generality, we can set $\psi|_\Gamma = 0$.

The plasma pressure functionally depends on the magnetic flux function. This dependence $p(\psi)$ is determined in a specific problem according to technological requirements for the plant. The plasma volume is distributed along the magnetic field lines $\psi = \text{const}$ (see figure 2 below). The plasma pressure has a maximum on the separatrix of the magnetic field which passes through the trap center. As applied to galateas, the main requirement is to prevent the heat plasma from being in contact with the conductors. In the considered model of the “Galatea-Belt” trap, the function $p(\psi)$ has the form:

$$p(\psi) = p_0 \exp\left\{-\frac{(\psi - \psi_0)^2}{q^2}\right\}$$

(5)

where the parameter $\psi_0 = \psi(0,0)$.

The equation (5) contains two more important problem parameters which describe the equilibrium magnetoplasma configurations observed in the trap. The value $p_0$ corresponds to the maximum plasma pressure measured in the magnetic units (2). This maximum value is achieved on the magnetic field separatrix $\psi(x,y) = \text{const} = \psi(0,0)$. The parameter $q$ characterizes speed of the pressure function $p(\psi)$ change relative to the maximum value $p_0$. Obviously, smaller this value, faster the plasma pressure decreases relative to the maximum value $p_0$.

The problem about the equilibrium magnetoplasma configurations in the “Galatea-Belt” trap with the equation (4) presented above and the defined functions $j_{\text{ex}}(x,y)$ and $p(\psi)$ is the main research object of the present paper. The aim of the study is to determine the dependence of the configuration properties and characteristics on the main problem parameters — the maximum plasma pressure $p_0$, the radius of the current conductors $r_c$, and the parameter $q$ regulating a transverse size of the configuration.
The corresponding mathematical modelling problems are solved numerically using the iteration relaxation method. In doing so, the elliptic equation (4) is changed on the parabolic one

$$\frac{\partial \psi}{\partial t} = \Delta \psi + \frac{dp}{d\psi} + j^{\text{ex}},$$

and during the numerical solving of its difference analog in the transition from the \(n\)-th time level to the \((n + 1)\)-st level, the longitudinal-transverse Thomas algorithm is used [3].

3. Computation results

On the figure 2, the typical variant of the equilibrium magnetoplasma configuration in the “Galatea-Belt” trap under the parameter values \(p_0 = 0.75, r_c = 0.2\) \(u q = 0.2\) is demonstrated. The computation results are presented by the distributions of the magnetic flux function \(\psi(x, y)\), or rather the difference \(\psi(x, y) - \psi(0,0)\), and the plasma pressure \(p(x, y)\) in the quadratic area \(-2 \leq (x, y) \leq 2\). In the present paper, this computation variant is basic. In the series of the further numerical experiments, the influence of the model parameters on properties of the configurations realized in the trap is determined by comparison of the corresponding calculations with the basic variant.

The plasma is concentrated near the magnetic field separatrix \(\psi(x, y) = \text{const} = \psi(0,0)\) forming the concave quadrangle with the petals circumflexing the mixins. The graphics show that the obtained configuration meets the main engineering requirement for all plants developed for magnetic plasma confinement: the plasma doesn’t contact with the conductors and the external boundary of the trap. The plasma isolation is provided by interaction of the electric current in the system and the configuration magnetic field: the Ampere force on the plasma is directed towards the magnetic field separatrix on either side of it.

![Figure 2. Magnetic flux \(\psi(x, y) - \psi(0,0)\) (left graphic) and plasma pressure \(p(x, y)\) (right graphic) in basic variant of configuration under \(p_0 = 0.75, r_c = 0.2\) \(u q = 0.2\).](image)

The computation results with different values of the relative radius of the electric current conductors immersed into the plasma showed that the magnetoplasma configurations formed in the trap qualitatively and even quantitively almost do not depend on \(r_c\) under sufficiently small values of it. Under unnatural increase of this parameter, the plasma approaches to the conductors first and then begins to fill the space occupied by them. The observed result is connected with the approximation (3) of the current in the mixins. In this case, the artificially defined current \(j^{\text{ex}}(x, y)\) blurs over the trap area.
Figure 3. Plasma pressure under $r_c = 0.1$ (left graphic), $r_c = 0.3$ (center graphic), $r_c = 0.5$ (right graphic) and fixed $p_0 = 0.75$, $q = 0.2$.

The influence of the parameter $q$ characterizing a transverse size of the plasma configuration is presented on the figure 4. Under its increase, the plasma amount in the trap also increases, the pressure decreasing from $p_0$ becomes smoother. The configuration transforms into convex towards the external trap boundary one and approaches to the conductors. Under decrease of this parameter, the configuration becomes denser, and the plasma petals circumflecting the conductors thin.

Figure 4. Plasma pressure under $q = 0.1$ (left graphic), $q = 0.3$ (center graphic), $q = 0.5$ (right graphic) and fixed $p_0 = 0.75$, $r_c = 0.2$.

The influence of the maximum plasma pressure $p_0$ on the characteristics of the magnetoplasma configurations is demonstrated on the figure 5. Decrease of this parameter leads to compression of the configuration towards to the trap center, and the petals circumflecting electric current conductors becomes thinner. The plasma amount in the system decreases. Conversely, increase of the parameter $p_0$ leads to increase of the volume occupied by the configuration. The configuration becomes convex towards the external trap boundary.

The common results of all numerical experiments here is that the maximum plasma pressure is bounded by some critical value. If we try to define $p_0$ higher than this value, the stationary problem solution does not relax in calculations [3, 4]. The physical meaning of this results is that a fixed current in the conductors can confine only the finite volume of the plasma.
Development of plants for magnetic plasma confinement is closely connected with the questions about stability of the equilibrium configurations relative to the perturbations linked with various plasma instabilities. The numerical studies of stability of “Galatea-Belt” magnetic traps are carried out in the present time. Their first results can be found in the paper [4].

4. Conclusion
The paper focuses on mathematical modelling and numerical studies of the “Galatea-Belt” magnetic traps with two electric current conductors immersed into the plasma volume. In the series of numerical experiments, the influence of the main problem parameters on the properties and characteristics of the equilibrium magnetoplasma configurations in these traps was determined. These results have the applied importance and can be used in further development of the plants created for magnetic plasma confinement.

References
[1] Artsimovich L A 1964 Controlled Thermonuclear Reactions ed by A C Kolb and K S Pease (New York: Gordon & Breach)
[2] Morozov A I 1992 Sov. J. Plasma Phys. 18 (3) 159–65
[3] Brushlinskii K V 2009 Mathematical and Computational Problems in Magnetohydrodynamics (Moscow: Binom) [in Russian]
[4] Brushlinskii K V and Stepin E V 2021 Diff. Eq. 57 (7) 835–47
[5] Morozov A I and Frank A G 1994 Plasma Phys. Reports 20 879–86
[6] Brushlinskii K V and Ignatov P A 2010 Comp. Math. and Math. Phys. 50 (12) 2071–81
[7] Shafranov V D 1958 Sov. Phys. JETP 6 545–54
[8] Grad H and Rubin H 1959 Proc. 2-nd UN Int. Conf. on the Peaceful Uses of Atomic Energy Geneva 31 (NY: Columbia Univ. Press) p 190