Chiral Symmetry Breaking and Restoration in (2+1) Dimensions from Holography: Magnetic and Inverse Magnetic Catalysis

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ABSTRACT: We study the chiral symmetry breaking and restoration in (2 + 1)-dimensional gauge theories from the holographic hard and softwall models. We describe the behavior of the chiral condensate in the presence of an external magnetic field for both models at finite temperature. For the hardwall model we find Magnetic Catalysis (MC) in different set ups. For the softwall model we find Inverse Magnetic Catalysis (IMC) and MC in different situations. We also find for the softwall model a crossover transition from IMC to MC at a pseudo-critical magnetic field. This study also shows spontaneous symmetry breaking for both models. Interestingly, for $B = 0$ in the softwall model we found a non-trivial expectation value for the chiral condensate.

KEYWORDS: Holographic Models, Chiral Phase Transition, Restoration, (2+1) Gauge Theories.
1 Introduction

Much theoretical progress on the non-perturbative physics of relativistic quantum field theories (RQFT) has been recently achieved. In particular, the role played by an external magnetic field in these RQFT has been investigated in many works, especially in connection with QCD [1–12]. In the context of lattice QCD it has been observed, for the chiral phase transition, an inverse magnetic catalysis (IMC), i.e., the decreasing of the critical temperature ($T_c$) with increasing magnetic field ($B$) for $eB \sim 1\text{ GeV}^2$ [13] and more recently for $eB \sim 3\text{ GeV}^2$ [14]. This is in contrast with what would be expected: a magnetic catalysis (MC), meaning the increasing of the critical temperature with increasing magnetic field [4].

A promising approach to investigate the phenomena of IMC and MC is based on the AdS/CFT correspondence, or Holographic Duality [15–18]. Such duality has become very useful to address strongly coupled gauge theories, including the nonperturbative regime of QCD. However, in order to reproduce the nonperturbative physics of QCD, one must break the conformal invariance in the original AdS/CFT correspondence. There are two well-known models which do this symmetry breaking: the hardwall [19–25] and softwall [26–28] models. In particular, some works have been done using these models discussing the IMC for as a perturbation in powers of the magnetic field [9–12], and can only be trusted for weak fields $eB < 1\text{GeV}^2$. Since their approach is perturbative in the magnetic field $B$, they could not predict what would happen for $eB > 1\text{GeV}^2$.

Concerning free fermions in $(2 + 1)$ dimensions in the presence of a magnetic field we know that, perturbatively, the phenomenon of magnetic catalysis is due to
the effect of the magnetic field itself, which is a strong catalyst of dynamical chiral symmetry breaking, leading to the generation of a nonzero chiral condensate in the chiral limit \((m \to 0)\) given by [3]:

\[
\langle \tilde{\psi}\psi \rangle \propto \frac{eB}{2\pi}.
\]  

(1.1)

At finite temperature, also in the perturbative regime, it was shown in [29] that the chiral condensate is extremely unstable, meaning that it vanishes as soon as we introduce a heat bath with and without chemical potential.

In this work we investigate, nonperturbatively, the chiral phase transition and symmetry restoration in \((2 + 1)\)-dimensional holographic gauge theories in an external magnetic field from the hard and softwall models and we confirm the picture mentioned before, but we also find IMC, in the softwall model, for small magnetic fields (in units of the string tension squared), in agreement with the lattice results in \((3 + 1)\) dimensions, while the traditional MC behavior happens for large magnetic fields in this nonperturbative approach. In the hardwall model we only find MC. At finite temperature our approach predicts a very fast decreasing in the chiral condensate as we increase the temperature, which means that we do not observe the instability found in the perturbative thermal field theory computation presented in [29].

2 The Chiral Condensate and Chiral Symmetry Breaking in the Holographic Framework

2.1 Background Geometry

Via Kaluza-Klein dimensional reduction, the supergravity theory on \(AdS_4 \times S^7\) may be consistently truncated to Einstein-Maxwell Theory on \(AdS_4\) [30]. The action for this theory, in Euclidean signature, is given by

\[
S = -\frac{1}{2\kappa_4^2} \int d^4x \sqrt{g} \left( \mathcal{R} - 2\Lambda - L^2 F_{MN} F^{MN} \right),
\]  

(2.1)

where \(\kappa_4^2\) is the 4-dimensional coupling constant, which is proportional to the 4-dimensional Newton’s constant \((\kappa_4^2 \equiv 8\pi G_4)\), \(d^4x \equiv d\tau dx_1 dx_2 dz\), \(\mathcal{R}\) is the Ricci scalar and \(\Lambda\) is the negative cosmological constant.

The field equations coming from the bulk action (2.1) together with the Bianchi identity are [30]

\[
R_{MN} = 2L^2 \left( F^P_M F^{NP} - \frac{1}{4} g_{MN} F^2 \right) - \frac{3}{L^2} g_{MN},
\]  

(2.2)

\[
\nabla_M F^{MN} = 0.
\]  

(2.3)
Our Ansatz for the metric and the background magnetic field to solve (2.2) are given by

\[ ds^2 = \frac{L^2}{z^2} \left( f(z) dt^2 + \frac{dz^2}{f(z)} + dx_1^2 + dx_2^2 \right), \]
\[ F = B \, dx_1 \wedge dx_2. \]

Using this Ansatz the field equations (2.2) are simplified and given by

\[ z^2 f''(z) - 4zf'(z) + 6f(z) - 2B^2z^4 - 6 = 0, \]  
\[ zf'(z) - 3f(z) - B^2z^4 + 3 = 0, \]

with \( F^2 \) in (2.1) given by

\[ F^2 = \frac{2B^2z^4}{L^4}. \]

The general solution for Eqs. (2.6) and (2.7) is given by:

\[ f(z) = 1 + B^2z^4 + bx^3. \]

If we impose the horizon condition \( f(z = z_H) = 0 \) one finds:

\[ f_{BT}(z) = 1 + B^2z^4(z - z_H) - \frac{z^3}{z_H^3}. \]

This solution was found recently in [31, 32]. The Eq. (2.10), corresponds to the AdS in the presence of a background magnetic field at finite temperature, \( T \), where \( z_H \) is the horizon position, such that \( f_{BT}(z = z_H) = 0 \). One can note that this solution indeed satisfy both differential equations (2.6) and (2.7). Note that the solution Eq. (2.10) implies the existence of an inner and outer horizons. The outer horizon satisfies \( f'_{BT}(z = z_H) < 0 \) and is the physically relevant one. The temperature is given by the Hawking formula

\[ T = \frac{|f'_{BT}(z = z_H)|}{4\pi}. \]

Using (2.10) and the condition \( f'_{BT}(z = z_H) < 0 \) we have

\[ T(z_H, B) = \frac{1}{4\pi} \left( \frac{3}{z_H} - B^2z_H^3 \right), \quad z_H^4 < \frac{3}{B^2}. \]

In what follows and in the rest of this work we set the AdS radius \( L = 1 \).

2.2 Holographic Setup for Chiral Symmetry Breaking

Here we describe how to realize the chiral symmetry breaking of \( SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_{\text{diag}} \) in the hardwall and softwall models. For both models we consider the action:

\[ S = -\frac{1}{2\kappa_4^2} \int d^3x \, dz \sqrt{g} \, e^{-\Phi} \text{Tr} \left( D_M X^\dagger \, D^M X + V_X - (F_L^2 + F_R^2) \right), \]
where $X$ is a complex scalar field, $D_M$ is the covariant derivative defined as $D_M X = \partial_M X + iA^L_M X - iX A^R_M$, with $A^L_R$ being the chiral left- and right-handed gauge fields, $F_{MN}$ the field strength defined as $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$ and $V_X = M_4^2 X^\dagger X + a(X^\dagger X)^2 + ...$ is the potential for the complex scalar field, where $M_4$ is the mass of the complex scalar field $X$. From the AdS$_4$/CFT$_3$ correspondence we have

$$M_4^2 = \Delta(\Delta - 3).$$

(2.14)

Since the complex scalar field in 4 spacetime dimensions is supposed to be dual to the chiral condensate $\sigma \equiv \langle \bar{\psi} \psi \rangle$ in 3 spacetime dimensions, whose dimension is $\Delta = 2$, we have therefore $M_4^2 = -2$. So, we have the potential $V_X = -2X^2 + aX^4$, where the first term is just the mass term and the second is the term needed to realize the spontaneous symmetry-breaking mechanism [33, 34].

The field equations coming from (2.13) are given by

$$D_M \left[ \sqrt{g} e^{-\Phi(z)} g^{MN} D_N X \right] - \sqrt{g} e^{-\Phi(z)} \partial_X V_X = 0. \quad (2.15)$$

Here, we also assume that the expectation value for $X$ takes a diagonal form $X = \frac{1}{4} \chi I_2$ for the SU(2) case [9, 11], where $I_2$ is the $2 \times 2$ identity matrix. For the general case of SU($N_f$) we would have $X = \frac{1}{\sqrt{2N_f}} \chi I_{N_f}$, where the factor $\sqrt{2N_f}$ is introduced to maintain the kinetic term of $\chi$ canonical [33, 34]. In addition, we assume that $\chi(x^\mu, z) \equiv \chi(z)$. With all these assumptions we can write (2.15) as

$$\chi''(z) + \left( -\frac{2}{z} - 2kz + \frac{f'(z)}{f(z)} \right) \chi'(z) - \frac{1}{z^2 f(z)} \partial_z V(\chi) = 0, \quad (2.16)$$

where $'$ means derivative with respect to $z$. For our purposes the dilatonic field is $\Phi(z) = k z^2$ with $k$ a dimensional constant. For the hardwall model we set $k = 0$. On the other hand for the softwall model $k \neq 0$. Finally, we take $V(\chi) \equiv \text{Tr} V_X = -\chi^2 + \lambda \chi^4$, in both models.

First, let us consider only the quadratic term in the potential $V(\chi)$, i.e., $V(\chi) = -\chi^2$, which implies a linear and analytically solvable equation of motion. In this case we can clearly see that the UV behavior ($z \to 0$) of (2.16) takes the form

$$\chi''(z) - \frac{2}{z} \chi'(z) + \frac{2}{z^2} \chi(z) = 0, \quad (2.17)$$

whose solution is given by

$$\chi(z \to 0) = c_1 z + c_2 z^2, \quad (2.18)$$

where $c_1$ and $c_2$ are integration constants. Since the complex scalar field $\chi(z)$ is dual to the operator $\langle \bar{\psi} \psi \rangle$, we can identify these two integration constants as proportional to:

$$c_1 \propto m_q, \quad (2.19)$$

$$c_2 \propto \sigma, \quad (2.20)$$
with $m_q$ being the fermion mass in $(2+1)$ dimensions, and $\sigma = \langle \bar{\psi} \psi \rangle$ is the chiral condensate.

In this work we solve numerically Eq. (2.16) using the quartic potential $V(\chi) = -2\chi^2 + \lambda \chi^4$, with $\lambda = 1$. The boundary conditions used to solve (2.16) were the UV ($z \to 0$) behavior of $\chi(z)$, that is, $\chi(z) = m_q z + \sigma z^2$. The solution for $\chi(z)$ is regular at the horizon, $z_H$, meaning $\chi(z_H) < \infty$.

In the following sections we present our results for IMC and MC in the context of the chiral phase transition for the hardwall and softwall models.

### 3 Results for the Hardwall model

In this section we present our results for the hardwall model concerning the dependence of the chiral condensate, $\sigma$, with the external magnetic field, $B$, for some finite temperatures, and also the dependence of $\sigma$ with temperature, $T$, for some values of magnetic field. This analysis allow us to investigate whether there is IMC or MC and the role played by the thermal effects. We will also investigate whether the chiral symmetry is spontaneous or explicitly broken at the non-pertubative level. It is worthy to remember that we are working in $(2+1)$ dimensions so that all observables quantities like $\sigma$, $B$, $T$, and $m_q$ will always be measured in units of the string tension ($\sqrt{\sigma_s}$). For instance, the temperature and mass will be measured in units of $\sqrt{\sigma_s}$. On the other hand, the magnetic field and the chiral condensate will be measured in units of the string tension squared ($\sqrt{\sigma_s}$)². One should note that the hardwall model can be defined setting $k = 0$ in Eq. (2.13) for the dilaton field, given by $\Phi(z) = k z^2$ and introducing a hard cutoff $z_{\text{max}}$ such that the holographic coordinate $z$ obeys $0 \leq z \leq z_{\text{max}}$ [19–25]. Note that the temperature of the black hole is related to the horizon position $z_H$, subjected to $z_H < z_{\text{max}}$.

Let us start the discussion concerning the hardwall model presenting the Figure 1 where one can see MC for very low temperature ($T/\sqrt{\sigma_s} = 0.005$) and three fermion masses $m_q/\sqrt{\sigma_s} = 0.001, 0.01, 0.1$, where $\sqrt{\sigma_s}$ is the string tension. Also in this picture one can note that for very weak magnetic fields the chiral condensate is almost independent of the fermion masses implying that, in the chiral limit ($m_q \to 0$), the chiral symmetry is spontaneously broken due to presence of the magnetic field. As a last comment, for weak fields one can see an approximate linear behaviour for the chiral condensate consistent with the perturbative result found in [3], as can be seen in Eq. (1.1).

In Figure 2 we can see that the increasing of the temperature still provides MC in the hardwall model. In this case the temperature is now 40 times greater ($T/\sqrt{\sigma_s} = 0.200$) than in Figure 1, and we still use the same values for the three fermion masses.

In Figures 3 and 4 we present the dependence of the chiral condensate $\sigma$ with respect to the temperature $T$. These figures show the restoration of the chiral sym-
Figure 1. The chiral condensate $\sigma$ normalized by the fermion mass $m_q$ in units of $\sqrt{\sigma_s}$ versus the magnetic field $B$ in units of the string tension squared $\sigma_s$. This plot shows MC for low temperature $\frac{T}{\sqrt{\sigma_s}} = 0.005$ for three values of the fermion mass in the hardwall model.

Figure 2. The chiral condensate $\sigma$ normalized by the fermion mass $m_q$ in units of $\sqrt{\sigma_s}$ versus the magnetic field $B$ in units of $\sigma_s$. This plot shows MC for the temperature $\frac{T}{\sqrt{\sigma_s}} = 0.200$ for three values of the fermion mass in the hardwall model.

metry ($\sigma \to 0$) for different values of the external magnetic field and of the fermion mass $m_q$. 
Figure 3. The chiral condensate $\sigma$ normalized by the fermion mass $m_q$ versus the temperature $T$, both in units of $\sqrt{\sigma_s}$. This plot shows the restoration of the chiral symmetry for the hardwall model for a fixed and weak external magnetic field $\frac{B}{\sqrt{\sigma_s}} = 2.0$ and different values of the fermion mass $m_q$ and temperatures in the interval $[0.030, 0.045] \sqrt{\sigma_s}$.

Figure 4. The chiral condensate $\sigma$ normalized by the fermion mass $m_q$ versus the temperature $T$, both in units of $\sqrt{\sigma_s}$. This plot shows the restoration of the chiral symmetry for the hardwall model for a fixed and strong external magnetic field $\frac{B}{\sqrt{\sigma_s}} = 20$ and different values of the fermion mass $m_q$ and temperature of the order of $\frac{T}{\sqrt{\sigma_s}} \sim 0.14$.

One can also note that by comparison between Figures 3 and 4 the MC behavior since the values of the chiral condensate increases when the magnetic field is also
increased as shown explicitly in Figures 1 and 2.

4 Results for the Softwall Model

In this section we do all the analysis of the previous section this time for the softwall model [26–28], characterized by a dilaton-like field \( \Phi(z) \) whose profile we assumed to be of the form \( \Phi(z) = k z^2 \), where the dilaton constant \( k \) was set equal \( k = \frac{1}{\sqrt{\sigma_s}} \). In the softwall framework, we are assuming that there is no backreaction due to the dilaton-like field so that the equations of motions for this model are still given by (2.2), whose solution corresponds to (2.10). In the following we present our results.

Figures 5 and 6 show IMC and MC for the softwall model for both low and high temperatures and three fermion masses \( m_q \sqrt{\sigma_s} = 0.001, 0.01, 0.1 \). In Figure 5 the temperature is \( T \sqrt{\sigma_s} = 0.005 \), while in Figure 6 the temperature is \( T \sqrt{\sigma_s} = 0.200 \). One should note that these two figures present a crossover between the two phases IMC and MC around a pseudo-critical magnetic field \( B_c \) which depends on the temperature. Note that the comparison between Figures 5 and 6 also shows implicitly a chiral symmetry restoration since \( \sigma \) decreases for an increasing temperature, for \( B < B_c \).

Figure 5. The chiral condensate \( \sigma \) versus the magnetic field \( B \), both in units of \( \sigma_s \). This plot shows IMC for \( B < B_c \) and MC for \( B > B_c \) for the softwall model with fixed temperature \( T \sqrt{\sigma_s} = 0.005 \) and three fermion masses. The dot in the lines represent the pseudo-critical magnetic field \( B_c \), which characterizes a crossover between the IMC and MC phases. In particular, one can read \( \frac{B_c}{\sigma_s} = 10.44 \) for \( \frac{m_q}{\sqrt{\sigma_s}} = 0.001 \); \( \frac{B_c}{\sigma_s} = 9.67 \) for \( \frac{m_q}{\sqrt{\sigma_s}} = 0.01 \); and \( \frac{B_c}{\sigma_s} = 5.59 \) for \( \frac{m_q}{\sqrt{\sigma_s}} = 0.1 \).

The Figures 7, 8 and 9 represent closer looks for the chiral condensate behavior in the region of weak magnetic fields. In Figure 7 for very low temperature, one can note that the chiral condensate’s behavior is approximately the same for the three values of the fermion mass, indicating a spontaneous chiral symmetry breaking. On the
Figure 6. The chiral condensate $\sigma$ versus the magnetic field $B$, both in units of $\sigma_s$. This plot shows IMC for $B < B_c$ and MC for $B > B_c$ for the softwall model with fixed temperature $\frac{T}{\sqrt{\sigma_s}} = 0.200$ and three fermion masses. The dot in the lines represent the pseudo-critical magnetic field $B_c$, which characterizes a crossover between the IMC and MC phases. In particular, one can read $B_c\sigma_s = 15.78$ for $\frac{m_0}{\sqrt{\sigma_s}} = 0.001$; $B_c\sigma_s = 15.77$ for $\frac{m_0}{\sqrt{\sigma_s}} = 0.01$; and $B_c\sigma_s = 15.52$ for $\frac{m_0}{\sqrt{\sigma_s}} = 0.1$.

On the other hand, as one increases the temperature as shown in Figures 8 and 9, its effect on the chiral condensate is more visible for the lowest fermion masses ($\frac{m_0}{\sqrt{\sigma_s}} \leq 0.01$), close to the chiral limit. In this case we still have an approximate chiral symmetry breaking. Comparing Figures 8 and 9, one can see that a small increasing of the temperature from $\frac{T}{\sqrt{\sigma_s}} = 0.150$ to $\frac{T}{\sqrt{\sigma_s}} = 0.200$ affects more the lighter fermions masses.

Figures 10, 11 and 12 show the behavior of the chiral condensate with respect to the temperature for different values of the fermion mass and different values of the magnetic field. All three figures show the restoration of the chiral symmetry as one increases the temperature, so that the chiral condensate goes to zero. In particular, in Fig. 10, one notes that even at zero external magnetic field this model provides a non-zero value for the chiral condensate at low temperatures. This is in contrast with the perturbative result, Eq. (1.1), so one could associate this behavior with a non-perturbative effect.
Figure 7. The chiral condensate $\sigma$ versus the magnetic field $B$, both in units of $\sigma_s$ for the softwall model. This plot shows IMC for weak magnetic fields and low temperature. This plot also shows spontaneous breaking of the chiral symmetry. The independence of the fermion mass for the chiral condensate is more evident for $\frac{m_q}{\sqrt{\sigma}} \leq 0.01$.

Figure 8. The chiral condensate $\sigma$ versus the magnetic field $B$, both in units of $\sigma_s$ for the softwall model with temperature $\sqrt{\sigma}$. This plot shows IMC for weak magnetic fields and high temperature and spontaneous breaking of the chiral symmetry. The approximate independence of the fermion mass for the chiral condensate occurs for $\frac{m_q}{\sqrt{\sigma}} \leq 0.01$. 
Figure 9. The chiral condensate $\sigma$ versus the magnetic field $B$, both in units of $\sigma_s$, for the softwall model with temperature $T/\sqrt{\sigma_s} = 0.200$. This plot shows IMC for weak magnetic fields and high temperature and spontaneous breaking of the chiral symmetry. The approximate independence of the fermion mass for the chiral condensate occurs for $m_q/\sqrt{\sigma_s} \leq 0.01$.

Figure 10. The chiral condensate $\sigma$, in units of $\sigma_s$, versus the temperature $T$, in units of $\sqrt{\sigma_s}$ for the softwall model. This plot shows the chiral symmetry restoration for different values of the fermion mass without external magnetic field. The temperature of the restoration corresponds approximately to the interval $[0.2, 0.4]$, in units of the string tension. This picture also shows an approximate independence of the chiral condensate with the fermion mass, in particular for $m_q \leq 0.01 \sqrt{\sigma_s}$. 
Figure 11. The chiral condensate \( \sigma \), in units of \( \sqrt{\sigma_s} \), versus the temperature \( T \), in units of \( \sqrt{\sigma_s} \) for the softwall model. This plot shows the chiral symmetry restoration for different values of the fermion mass with external magnetic field \( \frac{B}{\sigma_s} = 2.0 \). The temperature of the restoration corresponds approximately to the interval \([0.02, 0.4]\), in units of the string tension. This picture also shows an approximate independence of the chiral condensate with the fermion mass, in particular for \( \frac{m_q}{\sqrt{\sigma_s}} \leq 0.01 \).

Figure 12. The chiral condensate \( \sigma \), in units of \( \sigma_s \), versus the temperature \( T \), in units of \( \sqrt{\sigma_s} \) for the softwall model. This plot shows the chiral symmetry restoration for different values of the fermion mass with external magnetic field \( \frac{B}{\sigma_s} = 20 \). The temperature of the restoration corresponds approximately to the interval \([0.01, 0.2]\), in units of the string tension. This picture also shows an approximate independence of the chiral condensate with the fermion mass, in particular for \( \frac{m_q}{\sqrt{\sigma_s}} \leq 0.01 \).
5 Discussion and Conclusion

In this work we have described holographically the chiral phase transition in \((2 + 1)\) dimensions in the presence of an external magnetic field \(B\) at finite temperature \(T\). We have used two well-known holographic models: the hardwall and the softwall. In the first one we have observed just magnetic catalysis (MC), that is, the chiral condensate \(\sigma\) is enhanced due to the presence of the magnetic field, and is zero in the absence of magnetic field, in qualitative agreement with the perturbative analysis in [3–5]. Moreover, in the region of small magnetic field, the behavior of the chiral condensate as function of the magnetic field seems to be approximately linear with \(B\). This can be seen in Fig. 1 for very low \(T\). This is also in agreement with Eq. (1.1), derived from a perturbative quantum field theory approach for free fermions in a magnetic field in \((2 + 1)\) dimensions at \(T = 0\). Although these results for the hardwall model look very similar to the perturbative one, we considered in this work a non-perturbative approach, since we are in the framework of the AdS/CFT correspondence. Moreover, our results for the hardwall model agrees with the one found in [9], in the context of the holographic QCD in \((3 + 1)\) dimensions. There the authors concluded that there is no IMC for the hardwall, but instead what they obtained was just MC.

Our results within the softwall model, on the other hand, reveal some new interesting features. First of all, we obtained chiral symmetry breaking even for zero magnetic field \((B = 0)\), as can be seen in Fig. 10, completely in contrast with the perturbative result described by Eq. (1.1). This fact seems to be associated with the non-pertubative feature of our holographic approach. Also, we have concluded, for zero and non zero \(B\), that the breaking of the chiral symmetry is a spontaneous one, since the dependence on the chiral condensate \(\sigma\) with the magnetic field \(B\) for different fermion masses \(m_q\) is approximately the same, as can be seen in the plots of \(\sigma \times B\), represented by Figures 7, 8 and 9. Remarkably, the softwall model provides a crossover between the IMC/MC phases as shown in the plots of \(\sigma \times B\), Figures 5 and 6, characterized by a pseudocritical magnetic field \(B_c\). Therefore, in this sense, for \(B > B_c\) we recover approximately the linear behaviour \(\sigma \sim B\) as in the pertubative result given by (1.1). On the other hand, for \(B < B_c\), we found IMC which is in qualitative agreement with those found in lattice QCD [13, 14] and the holographic approaches dealing with QCD in \((3 + 1)\) dimensions [11, 33, 34]. Note that all these features that our holographic model present in \((2 + 1)\) dimensions were not observed in the other holographic approaches for QCD because their results are perturbative in \(B\). Therefore they could trust the numerical results only for small \(B\), while ours is exact in the magnetic field, meaning that it is valid for any \(B\).

Now, concerning the thermal effects on the chiral condensate \(\sigma\), as can be seen in the plots of \(\sigma \times T\), Figures 3 and 4 for the hardwall and Figures 10, 11 and 12 for the softwall, we have observed that the restoration of the chiral symmetry
(σ = 0) happens at some critical temperature $T_c$ for each model, depending on the fermion mass. This phenomenon is consistent with lattice QCD results and with the holographic approaches for QCD previously mentioned.

Finally, in comparison with the perturbative counterpart at finite temperature $T$ in (2 + 1) dimensions [29], in which the authors showed that the chiral condensate $\sigma$ is extremely unstable (it vanishes as soon as one introduces a heat bath, with or without a chemical potential), we have seen that our results show that the chiral condensate persists up to some critical temperature, and that instability does not happen here, perhaps due to the non-perturbative nature of our approach.

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