Spin Collective Modes of Two-Species Fermi Liquids: Helium-3 and Atomic Gases near the Feshbach Resonance

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We present theoretical findings on the spin collective modes of a two-species Fermi liquid, prepared alternatively in a polarized equilibrium or a polarized non-equilibrium state. We explore the effects on these modes of a diverging s-wave scattering length, as occurs near a Feshbach resonance in a Fermionic atomic gas. We compare these atomic gas modes with those of the conventional Helium-3 system, and we find that they differ from the conventional systems, and that the gap and spin stiffness are tunable via the Feshbach resonance.

While the BCS and BEC states in atomic gases garner wide interest across many fields \cite{1, 2, 3, 4}, investigations into the normal state of atomic gases, i.e. above the superfluid phase transition, can also provide interesting results and important insights into the properties of these gases and other related systems. For instance, theoretical studies directed towards the density excitations of atomic gases in the hydrodynamic, collisionless, and intermediate regimes \cite{5, 6, 7} paved the way for experimental investigations into the excitation spectra \cite{8, 9} and the discovery of surprising features. Application of Fermi liquid theory to density fluctuations of atomic gases has also led to interesting predictions and results \cite{10, 11}.

The inhomogeneity of a confined atomic gas is not expected to affect the Fermi liquid results significantly \cite{12}. In lower dimensions, however, the confining potential restricts the motion of the atoms in certain directions, thus Fermi liquid results have been shown to change significantly for cigar-shaped traps \cite{13}. Therefore, all of the following calculations are done for the three-dimensional, homogeneous case.

We begin by defining the FLT parameters that are used throughout this paper. For a complete reference, we refer the reader to the literature \cite{14, 15, 16}, which we will follow closely in form and notation. In FLT the variation of the energy $\delta E$ due to a variation of the quasi-particle $(q,p)$ distribution function $\delta n_{pq}$ from its ground state can be written as

$$\delta E = \frac{1}{V} \sum_{p\sigma} \epsilon^0_{pq\sigma} \delta n_{pq\sigma} + \frac{1}{V^2 \pi^2} \sum_{p\sigma,p'\sigma'} f_{pq\sigma p'\sigma'} \delta n_{pq\sigma} \delta n_{p'\sigma'} + \ldots ,$$

(1)

where $\epsilon^0_{pq\sigma}$ is the single-particle excitation spectrum, and the q-p interaction energy $f$ is a second functional derivative of the total energy

$$f_{pq\sigma p'\sigma'} = V^2 \frac{\delta^2 E}{\delta n_{pq\sigma} \delta n_{p'\sigma'}}$$

(2)

which can be separated into symmetric and antisymmetric parts, $f_{pp'} = f_{pp'}^{\uparrow\uparrow} + f_{pp'}^{\uparrow\downarrow} \sigma \cdot \sigma'$, where $\sigma$ denotes the spin state of the q-p, which are in turn related in a two-spin-component system to $f_{11}$ and $f_{11}$. Furthermore, $f$ can be written in the usual way in terms of the Legendre expansion of the angle $\theta$ between $p$ and $p'$, $f_{pp'}^{\alpha,\beta} = \sum_{i=0}^{\infty} f_i^{\alpha,\beta} P_i(\cos(\theta))$. The dimensionless Landau parameters (LP) are obtained by the relation $F_i^{\alpha,\beta} = N(0) f_i^{\alpha,\beta}$, where $N(0)$ is the q-p density of states at the Fermi surface. The definition of the spin polarization density $m$ in a two-component Fermi system is $m = \delta n_{\uparrow} - \delta n_{\downarrow}$.

A PEQ system is a spin-polarized system that has a net polarization that arises from, and is in equilibrium with, a polarizing external magnetic field. Furthermore, the polarization is simply related to the external magnetic field strength and the LP’s, and is given by $m_0 = \delta n_{\uparrow} - \delta n_{\downarrow}$.
\[ \delta n_1 = H(N(0)/(1 + F_0^a)) \], where \( H \) is the magnitude of the applied external magnetic field, and \( \hbar, \gamma \equiv 1 \).

A PNEQ system is one in which the system is polarized, but is not in equilibrium with an external magnetic field. The system is instead kept in the polarized state by external means other than a magnetic field, for instance, by constantly pumping a certain spin species into the system in order to maintain a finite polarization, or by using laser-induced transitions to convert one spin species to the other. We will denote the PNEQ polarization density by \( m' \).

With the parameters of the system suitably defined, we now briefly review the transverse spin collective mode calculation as derived in [18] and then discuss the behavior of the modes in Helium-3 and in an atomic gas near an FBR. We begin with the familiar Landau kinetic equation (LKE), which governs the temporal and spatial evolution of a local spin polarization density,

\[
\frac{\partial m_p}{\partial t} + \frac{\partial}{\partial r_i} \left( \frac{\partial E_m}{\partial p_i} m_p + \frac{\partial h_p}{\partial p_i} n_p \right) = - \frac{\partial}{\partial r_i} \frac{\partial \epsilon_m}{\partial r_i} m_p + \frac{\partial h_p}{\partial t} n_p = \left( \frac{\partial m_p}{\partial t} \right)_{\text{prec.}} + \left( \frac{\partial m_p}{\partial t} \right)_{\text{coll.}} \tag{3}
\]

where \( m_p \equiv m_p(r, t) = \frac{1}{2} \sum_{\alpha, \alpha'} \tau_{\alpha \alpha'} [n_p(r, t)]_{\alpha \alpha'} \) is the local spin polarization, and \( h_p = - \frac{1}{2} H_0 + 2 \int \frac{\partial^2}{\partial r_i \partial r_i'} f_{p \sigma} f_{p \sigma'} \) is the effective internal field. Eq. (3) describes the evolution of a spin perturbation in an interacting Fermi system. From this equation the expression for the evolution of a transverse spin perturbation can be derived. The right hand side contains two terms, the precession term and the collision term. For the case of cold atomic gases we assume that the collision term can be taken to be zero, and we retain the precession term. A solution to this equation is achieved through a Fourier transform and spherical harmonic expansion of the Fermi surface deformation. We truncate the harmonic expansion at \( l = 1 \), since the \( l = 2 \) term gives a small correction and does not change the structure of the solution in the limit \( q \to 0 \).[19, 20]. From [19], the two PNEQ solutions are

\[
\omega_{0,\text{PNEQ}}^\pm = \mp \omega_L \pm \frac{1}{2} \left( 1 + \frac{F_0^a}{3} \right) \left( 1 + \frac{F_0^a}{3} \right) (q v F)^2 \tag{4}
\]

\[
\omega_{1,\text{PNEQ}}^\pm = \mp \frac{2 m' F_0^a}{N(0)} \left( 1 + \frac{F_0^a}{3} \right) \left( 1 + \frac{F_0^a}{3} \right) (q v F)^2 \tag{5}
\]

The PEQ dispersion is given by the addition of the Larmor frequency \( \omega_L \) to the PNEQ results (note that an accompanying change of notation \( m' \to m_0 \) is also required).

\[
\omega_{0,\text{PEQ}}^\pm = \pm \omega_L \pm \frac{1}{2} \left( 1 + \frac{F_0^a}{3} \right) (q v F)^2 \tag{6}
\]

\[
\omega_{1,\text{PEQ}}^\pm = \pm \frac{2 m_0 F_0^a}{N(0)} \left( 1 + \frac{F_0^a}{3} \right) (q v F)^2 \tag{7}
\]

The p-h continuum dispersion is given by the relation

\[
\omega_{0,\text{PHE}}^\pm = \pm \frac{2 m' F_0^a}{N(0)} + q \cdot v \tag{8}
\]

\[
\omega_{1,\text{PHE}}^\pm = \pm \frac{2 m_0 F_0^a}{N(0)} + q \cdot v \tag{9}
\]

It is instructive to apply these solutions first to the He-3 system, since this system has been studied extensively (e.g. see [18] and references therein). In He-3, the anti-symmetric Landau parameters at zero pressure are \( F_0^a = -0.7 \), \( F_1^a = -0.55 \)[21], and the LP’s for different pressures are shown in Fig. [1]. As seen in the figure, large changes in pressure in the He-3 system lead to only small variation in the anti-symmetric LP’s (it is the symmetric LP’s that vary greatly with pressure in He-3).

Theoretical calculations for the transverse spin wave dispersion for Helium-3 at \( P = 3 \) bar is shown in Fig. [2]. These modes were theoretically predicted by Abrikosov
FIG. 2: The PEQ spin collective mode dispersion (theory) for Helium-3 at a pressure of 3 bar. The non-zero $F_1^a$ brings the current mode out of the particle-hole continuum for very small values of $q$.

FIG. 3: Calculated Landau parameters, $F_0^a$ and $F_1^a$, for a gas of $^6$Li atoms in the appropriate high-field seeking spin states near the 834-Gauss Feshbach resonance. The horizontal axis is the inverse of the bare scattering length $a_s$ times the fermi wave vector $k_F$.

and Dzyaloshinski in 1959, and the modes have been experimentally observed in $^3$He and $^3$He-$^4$He mixtures.

In order to evaluate the collective mode solutions for an atomic gas near an FBR, values for the pertinent LP’s must be calculated. We calculate the LP’s using the induced interaction model, which provides a formal relation between the scattering length and the LP’s, and our results are shown in Fig. 3. The figure shows the parameter $F_0^a$ diverging towards $+\infty$ as the Feshbach resonance is approached from the attractive side ($a_s < 0$). The parameter $F_1^a$ is less than zero and remains small in magnitude near the resonance.

With the behavior of the LP’s near the FBR determined, the dispersion relations can be evaluated for the atomic gas system. The spin modes in the PNEQ system are characterized by a gapless spin precession mode and a gapped spin current mode. The gap in the current mode is given by $\omega_{i_{PNEQ}}(q = 0) = 2m'F_0^a/N(0) - 2m'F_1^a/3N(0)$. The first part arises from the effective internal field produced by the $q$-p’s, equal to $2m'F_0^a/N(0)$, and the second part from the modification of the field due to the Fermi surface distortion, equal to $-2m'F_1^a/3N(0)$.

The qualitative behavior of the PNEQ modes far from the FBR in an atomic gas is shown in Fig. 4 (a). The current mode gap could be tuned by manipulating the polarization $m'$ of the system. For instance, increased injection of spin “up” $^6$Li atoms ($F = 1/2, m_F = +1/2$) would result in an increase in the gap of the current mode.

The spin stiffness of the PNEQ modes can be seen to be inversely proportional to the polarization density $m'$. Thus for small polarization, the spin stiffness is very large. Therefore, the emergence of the current mode from the p-h continuum, as indicated in Fig. 4 (b) by $q_{prop}$, could be tuned to occur at low values of the wave number $q$ by manipulation of the polarization density.

In Fig. 4 (c) we plot the qualitative behavior of the PEQ modes far from the FBR. The gap of the PEQ spin precessional mode is given simply by the Larmor frequency. However, the gap of the PEQ spin current mode is inversely proportional to $F_0^a$, as can be seen in the expression for the gap after substitution for the equilibrium value of the polarization, $\omega_{i_{PEQ}}(q = 0) = +\omega_F + 1 + F_1^a/1 + F_0^a$. Thus if our calculations for the LP’s are correct, this mode could be made nearly gapless near an FBR due to the divergence of $F_0^a$, and thus be brought below the spin precessional mode. This scenario is shown in Fig. 4 (d).

Due to the equilibrium relation $m_0 = H_0N(0)/(1 + F_0^a)$, the spin stiffness in a PEQ system behaves differently than in a PNEQ. In the limit of $F_0^a \gg 1$, the spin stiffness $D$ is approximately proportional to $D \sim 1/m_0$, and $m_0 \sim 1/F_0^a$. Thus the spin stiffness increases linearly with increasing $F_0^a$. This would mean that as the FBR is approached from the attractive side, the spin stiffness would increase, and the current mode would exit the p-h continuum at lower and lower values of $q$.

These collective spin mode effects are unique to the atomic gas system near an FBR above the superfluid transition. No other Fermi liquid system offers such easily tunable effects. An experimental investigation into these collective mode effects could yield the observation of a variety of new physical phenomena, and lead to a better understanding of collective mode phenomena in other systems and temperature regimes, as well.

In conclusion, we have presented our findings on the collective spin modes of a three-dimensional, homogeneous Fermionic atomic gas in the normal phase. We have discussed the general mode behavior for PEQ and PNEQ systems, as well as specific behavior for $^3$He and atomic gases. In contrast to $^3$He, we have shown that...
the gap and spin stiffness of the atomic gas modes can be tuned near the Feshbach resonance by manipulation of the s-wave scattering length. We postulate that these modes and effects could be experimentally detected and confirmed in atomic gases, and thus lead to a better understanding of dilute atomic gases near the Feshbach resonance, as well as the collective spin modes of a variety of Fermi liquid systems. We are currently investigating further theoretical implications of the results presented above, including the thermodynamic repercussions of coupling the density and spin excitations.

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