Decoupling behaviour of $\mathcal{O}(m_t^4)$ corrections to the $h^0$ self-couplings

Wolfgang Hollik$^a$ and Siannah Peñaranda$^{a,b,*}$

$^a$Max-Planck-Institut für Physik,
Föhringer Ring 6, D-80805 München, Germany

$^b$Institut für Theoretische Physik, Universität Karlsruhe
Kaiserstraße 12, D–76128 Karlsruhe, Germany

The decoupling behaviour of the leading one-loop Yukawa-coupling contributions of $\mathcal{O}(m_t^4)$ to the lightest MSSM Higgs boson self-couplings, when the top-squarks are heavy as compared to the electroweak scale, is discussed. As shown analytically and numerically, the large corrections can almost completely be absorbed into the $h^0$-boson mass and therefore, the $h^0$ self-couplings remain similar to the coupling of the SM Higgs boson for a heavy top-squark sector.

MPI-PhT/2002-54, KA-TP-14-2002, hep-ph/0210108.

1. Introduction

To establish the Higgs mechanism experimentally, the characteristic self-interaction potential must be reconstructed once the Higgs particle will be discovered. This task requires the measurement of the trilinear and quartic Higgs boson self-couplings, as predicted in the Standard Model (SM) or in supersymmetric theories. It is known that relevant radiative corrections, dominated by top-quark/squark loops, affect the Higgs-boson masses and the self-couplings of the neutral Higgs particles in the Minimal Supersymmetric Standard Model (MSSM) [1–5]. In this context, the investigation of the decoupling behaviour of quantum effects in the Higgs self-interaction could play a crucial role to distinguish between a SM and a MSSM light Higgs boson. Here we are concerned with the one-loop corrections to the self-couplings of the lightest CP-even MSSM Higgs boson $h^0$. As a first step, the leading one-loop Yukawa contributions of $\mathcal{O}(m_t^2)$ to the $h^0$ one-particle irreducible (1PI) Green functions were analyzed in details in [6] studying, both numerically and analytically, the asymptotic behaviour of these corrections in the limit of heavy top squarks, with masses large as compared to the electroweak scale. This talk summarizes results of [6]. The corresponding analysis of the one-loop contributions to the $h^0$ self-couplings originating from the Higgs sector itself has been presented recently in [7].

2. Tree-level Higgs boson self-couplings

The trilinear and quartic vertices of the Higgs field $H$ in the SM are given by

$$\lambda_{HHH} = 3 g M_Z^2 / 2 M_Z c_W$$

and

$$\lambda_{HHHH} = 3 g^2 M_Z^2 / 4 M_Z^2 c_W^2 (H \equiv H_{SM})$$

with the SU(2)$_L$ gauge coupling $g$ and $c_W = \cos \theta_W$.

In the MSSM, two parameters, conveniently chosen to be the CP-odd Higgs-boson mass $M_A$ and the ratio of the vacuum expectation values of each doublet, $\tan \beta = v_2 / v_1$, are sufficient to fix all the other parameters of the tree-level Higgs sector [8]. Other masses and the mixing angle $\alpha$ in the CP-even Higgs sector are then fixed, and the Higgs boson self-couplings can be predicted. The tree-level trilinear and quartic self-couplings of the lightest MSSM Higgs boson can be written as follows,

$$\lambda_{hhh}^0 = 3 g M_Z \cos 2 \alpha \sin (\beta + \alpha) / 2 c_w$$

and

$$\lambda_{hhhh}^0 = 3 g^2 \cos^2 2 \alpha / 4 c_w^2$$.

Obviously, for arbitrary values of $\tan \beta$ and $M_A$, these couplings are different from the couplings of the SM Higgs boson. However,
the situation changes in the so-called decoupling limit of the Higgs sector [9], defined by considering $M_A^0 \gg M_Z$ yielding a particular spectrum with very heavy $H^0$, $H^{\pm}$, $A^0$ bosons of similar masses and a light $h^0$ boson with a tree-level mass of $M_{h^0}^{\text{tree}} \simeq M_Z |\cos 2\beta|$. This limit also implies $\alpha \to \beta - \pi/2$, and one obtains that the $h^0$ self-couplings tend towards $\lambda^0_{hhh} \simeq 3gM^2_{h^0} / 2M_Z c_W$, $\lambda_{hh}^0 \simeq 3g^2M^2_{h^0} / 4M_Z^2 c_W^2$. Thus, the tree-level couplings of the light MSSM Higgs boson approach the couplings of a SM Higgs boson with the same mass.

3. $\mathcal{O}(m_t^4)$ one-loop contributions

The one-loop leading Yukawa corrections from top and stop loop contributions to the $h^0$ vertex functions were derived in [6] by the diagrammatic method using FeynArts 3 and FormCalc [10]. To obtain the UV-finite renormalized vertex functions, renormalization has to be performed by adding appropriate counter-terms. The standard procedure [12,13] yields the counterterms for the $n$-point $(n = 1, \ldots, 4)$ vertex functions expressed in terms of the renormalization constants for fields and parameters, $\delta Z_{\mu_1 \ldots \mu_n}$, $\delta v$, $\delta g_1 \delta g_2$, $\delta m^2_1 \delta m^2_2$, $\delta m_{12}$, which are fixed by imposing the on-shell renormalization conditions. Explicit results for these renormalization constants, with restriction to the dominant $\mathcal{O}(m_t^4)$ contributions, are listed in [6]. Here we summarize the results for the MSSM renormalized vertex functions.

The discussion of decoupling requires the asymptotic limit in which the $t$ masses are very large as compared to the external momenta and to the electroweak scale, $m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2} \gg M_Z^2, M^2_{h^0}$. We consider two scenarios.

(i) In the first case we assume that $\tilde{t}_1$ and $\tilde{t}_2$ are both heavy but with masses close to each other [11], i.e.

$$|m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}| \ll |m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}|.$$  \(1\)

Under this condition the analytical results for the MSSM renormalized vertex functions $\Delta \Gamma_{h^0}^{\ell,n}(n = 1, \ldots, 4)$, become rather simple. The renormalized one-point function vanishes, according to the corresponding renormalization condition: $\Delta \Gamma_{h^0}^{\ell,n}(1) + \delta \Gamma_{h^0}^{\ell,n}(1) = 0$. So, by adding the one-loop and counterterm contributions, we are able to write the results for $\Delta \Gamma_{h^0}^{\ell,n}$ as follows,

$$\Delta \Gamma_{h^0}^{\ell,1} = \Delta M^2_{h^0},$$

$$\Delta \Gamma_{h^0}^{\ell,2} = -\frac{3}{8\pi^2} \frac{g}{M_W} m_t^4,$$

$$\Delta \Gamma_{h^0}^{\ell,3} = -\frac{3}{v^2} \Delta M^2_{h^0} - \frac{3}{4\pi^2} \frac{g^2}{M_W m_t^4} m_t^4,$$

$$\Delta \Gamma_{h^0}^{\ell,4} = -\frac{3}{v^2} \Delta M^2_{h^0} - \frac{3}{4\pi^2} \frac{g^4}{M_W m_t^4} m_t^4,$$  \(2\)

where $v = 2M_W/g$. $\Delta M^2_{h^0}$ represents the (leading) one-loop correction to the $h^0$ mass,

$$\Delta M^2_{h^0} = -\frac{3}{8\pi^2} \frac{g^2}{M_W m_t^4} m_t^4 \log \frac{m^2_{\tilde{t}_1} m^2_{\tilde{t}_2}}{m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}},$$  \(3\)

corresponding to the fact that the renormalized two-point function is responsible for a shift in the pole of the $h^0$ propagator.

The UV-divergences cancel between the one-loop and the counterterm contributions. Moreover, a logarithmic heavy mass term, which looks like a non-decoupling effect of the heavy particles in the renormalized vertices, disappears when the vertices are expressed in terms of the Higgs-boson mass (see eqs. (2) and (3)) and, therefore, they do not appear directly in related observables, i.e. they decouple.

Notice that, without the non-logarithmic top-mass terms in the trilinear and quartic $h^0$ self-couplings in (2), the $h^0$ self-couplings at the one-loop level have the same form as the tree-level couplings, with the tree-level Higgs-boson mass replaced by the corresponding one-loop mass.

To interpret the non-logarithmic top-mass terms $\mathcal{O}(m_t^4)$ in (2) in a correct way, we have to calculate the equivalent one-loop $\mathcal{O}(m_t^4)$ contributions in the SM. After the on-shell renormalization of the trilinear and quartic couplings in the SM, one finds that the SM results correspond precisely to the two non-logarithmic terms in (2). Hence, the non-logarithmic top-mass terms are common to both $h^0$ and $H_{SM}$.

Therefore, we conclude that the $\mathcal{O}(m_t^4)$ one-loop contributions to the MSSM $h^0$ vertices either represent a shift in the $h^0$ mass and in the
$h^0$ triple and quartic self-couplings, which can be absorbed in $M_{h^0}$, or reproduce the SM top-loop corrections. The triple and quartic $h^0$ couplings thereby acquire the structure of the SM Higgs-boson self-couplings. Heavy top squarks with masses close to each other thus decouple from the low energy theory when the self-couplings are expressed in terms of the Higgs-boson mass in the $M_{h^0} \gg M_Z$ limit.

To illustrate these results also quantitatively, we plot in Fig. 1 the ratio $\Delta \lambda_{hhh}/\lambda_{hhh}^0$ and $\Delta M^2/M_h^2$ ($h \equiv h^0$) as function of $M_{A^0}$, for different values of $\tan \beta$, in the limit of heavy stop masses but very close to each other.

![Figure 1](image1.png)

**Figure 1.** $\mathcal{O}(m_t^4)$ results for $\Delta \lambda_{hhh}/\lambda_{hhh}^0$ and $\Delta M^2_{hhh}/M_h^2$ ($h \equiv h^0$) as function of $M_{A^0}$, for different values of $\tan \beta$, in the limit of heavy stop masses but very close to each other.

For the second scenario, we consider a squark sector where the stop mass splitting is of the order of the SUSY mass scale, i.e.,

$$|m_{t_1}^2 - m_{t_2}^2| \approx |m_{t_1}^2 + m_{t_2}^2|.$$

The analysis has been done numerically, based on the exact results for $\mathcal{O}(m_t^4)$ corrections to the triple and quartic self-couplings. The set of SUSY parameters has been specified as follows: $M_Q \sim 1$ TeV, $M_G \sim \mu \sim |A_t| \sim 500$ GeV [6]. With this choice of SUSY parameters, the top-squark masses are large but their difference is of $\mathcal{O}(M_G)$, such that $|m_{t_1}^2 - m_{t_2}^2|/|m_{t_1}^2 + m_{t_2}^2| \approx 0.6$. In Fig. 2 we present numerical results for the variation of the trilinear coupling and for the $\mathcal{O}(m_t^4)$ $h^0$ mass correction as functions of $M_{A^0}$, for different values of $\tan \beta$. The radiative correction to the angle $\alpha$ is also taken into account. The non-logarithmic finite contributions to the three-point function owing to the top-triangle diagrams is not taken into account in the figures since it converges always to the SM term.

![Figure 2](image2.png)

**Figure 2.** $\mathcal{O}(m_t^4)$ radiative corrections to the trilinear $h^0$ self-coupling and to the $h^0$ mass as a function of $M_{A^0}$, when the stop mass splitting is of $\mathcal{O}(M_G)$.

(ii) For the second scenario, we consider a squark sector where the stop mass splitting is of the order of the SUSY mass scale, i.e,

$$|m_{t_1}^2 - m_{t_2}^2| \approx |m_{t_1}^2 + m_{t_2}^2|.$$

The analysis has been done numerically, based on the exact results for $\mathcal{O}(m_t^4)$ corrections to the triple and quartic self-couplings. The set of SUSY parameters has been specified as follows: $M_Q \sim 1$ TeV, $M_G \sim \mu \sim |A_t| \sim 500$ GeV [6]. With this choice of SUSY parameters, the top-squark masses are large but their difference is of $\mathcal{O}(M_G)$, such that $|m_{t_1}^2 - m_{t_2}^2|/|m_{t_1}^2 + m_{t_2}^2| \approx 0.6$. In Fig. 2 we present numerical results for the variation of the trilinear coupling and for the $\mathcal{O}(m_t^4)$ $h^0$ mass correction as functions of $M_{A^0}$, for different values of $\tan \beta$. The radiative correction to the angle $\alpha$ is also taken into account. The non-logarithmic finite contributions to the three-point function owing to the top-triangle diagrams is not taken into account in the figures since it converges always to the SM term.

We can see in Fig. 2 that the relation $\Delta \lambda_{hhh}/\lambda_{hhh}^0 \approx \Delta M^2_{hhh}/M_h^2$ is only fulfilled up to a small difference which remains also for large $M_{A^0}$. But even in the most unfavorable cases, namely low $\tan \beta$ and $M_{A^0}$ values, the difference...
between the $h^0$ mass and self-coupling at one-loop does not exceed 6% (for $\tan \beta = 5$ and $M_{A^0} = 200$ GeV, it is about 5%). For large $M_{A^0}$, i.e. in the decoupling limit of the MSSM Higgs sector, the difference decreases to the level of 1%. By taking into account that experimental studies indicate that for a SM-like Higgs boson with $m_h = 120$ GeV at $1000 \text{fb}^{-1}$ a precision of $\delta \lambda_{hhh}/\lambda_{hhh} = 23\%$ can be reached [15,16], it will not be possible to measure this difference e.g. at TESLA.

For very large SUSY scales this small difference, observed in Fig. 2, also vanishes. To show this, we choose the SUSY parameters to be in accordance with the condition (4), as follows: $M_{\tilde{Q}} \sim 15 \text{ TeV}$, $M_{\tilde{U}} \sim \mu \sim |A_U| \sim 1.5 \text{ TeV}$. In this case, one gets $|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2|/|m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2| \simeq 0.97$. The results are given in Fig. 3. Here we see that the difference between the vertex corrections and the Higgs-boson mass disappears. Quantitatively, one finds that for $\tan \beta = 5$ and $M_{A^0} = 2$ TeV it decreases to $\sim 0.03\%$, and for the most unfavorable case, i.e $\tan \beta = 5$ and $M_{A^0} = 200$ GeV, it is about 0.2%.

Therefore, from the numerical analysis one can conclude that also for the case of a heavy stop system with large mass splitting, of the order of the typical SUSY scale, the $O(m^4_t)$ corrections to the trilinear $h^0$ self-couplings are absorbed to the largest extent in the loop-induced shift of the $h^0$ mass, leaving a small difference of at most a few per cent, which can be interpreted as the genuine one-loop corrections when $\lambda_{hhh}$ is expressed in terms of $M_{h^0}$. Similar results have been obtained also for the quartic $h^0$ self-coupling.

4. Conclusions

We showed analytically that, in the limit of large $M_{A^0}$ and heavy top squarks, with $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ close to each other, all the apparent non-decoupling one-loop effects, which constitute large corrections to the $h^0$ self-couplings, are absorbed in the Higgs-boson mass $M_{h^0}$, and the $h^0$ self-couplings get the same form as the couplings of the SM Higgs boson. Therefore, such a heavy top-squark system decouples from the low energy theory, at the electroweak scale, and leaves behind the SM Higgs sector also in the Higgs self-interactions.

The limiting situations where the $\tilde{t}$-mass difference is of the order of the SUSY mass scale have been also analyzed numerically. Similarly to the previous limit, the radiative corrections to the $h^0$ self-couplings are large, but their main part can again be absorbed in the mass $M_{h^0}$. For large $M_{A^0}$, i.e. in the decoupling limit, the differences between the vertex corrections and the mass results are insignificant.

Therefore, the $h^0$ self-interactions are very close to those of the SM Higgs boson for the heavy stop sector and would need high-precision experiments for their experimental verification.

Acknowledgments

The work of S.P. has been partially supported by the Fundación Ramón Areces. Support by the European Union under HPRN-CT-2000-00149 is greatfully acknowledged. We thank H. Haber for valuable discussions.

REFERENCES

1. J. Ellis, G. Ridolfi, F. Zwirner, Phys. Lett. B257 (1991) 83; ibid. B262 (1991) 477;
Y. Okada, M. Yamaguchi, T. Yanagida, Prog. Theor. Phys. 85 (1991) 1; H. E. Haber, R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815.

2. V. Barger, M. S. Berger, A. L. Stange, R. J. Phillips, Phys. Rev. D45 (1992) 4128.

3. P. Osland, P. N. Pandita, Phys. Rev. D59 (1999) 055013, hep-ph/9806351; hep-ph/9911295; hep-ph/9902270.

4. A. Djouadi, H.E. Haber, P.M. Zerwas, Phys. Lett. B375 (1996) 203, hep-ph/9602234; A. Djouadi, W. Kilian, M. Mühleitner, P.M. Zerwas, Eur. Phys. J. C10 (1999) 27, hep-ph/9903229; C10 (1999) 45, hep-ph/9904287; hep-ph/0001169.

5. T. Plehn, M. Spira, P. M. Zerwas, Nucl. Phys. B479 (1996) 46, Erratum-ibid. B531 (1996) 655, hep-ph/9603205; R. Lafaye, D. J. Miller, M. Mühleitner, S. Moretti, hep-ph/0002238.

6. W. Hollik, S. Peñaranda, Eur. Phys. J. C23 (2002) 163, hep-ph/0108245.

7. A. Dobado, M. J. Herrero, W. Hollik, S. Peñaranda, Phys. Rev. D66 (2002) 095016, hep-ph/0208014.

8. J. F. Gunion, H. E. Haber, G. Kane, S. Dawson, *The Higgs Hunter’s Guide* (Addison-Wesley, 1990), Erratum hep-ph/9302272.

9. H. E. Haber, Y. Nir, Nucl. Phys. B335 (1990) 363; H. E. Haber, hep-ph/9305248.

10. T. Hahn, M. Pérez-Victoria, Comput. Phys. Commun. 118 (1999) 153, hep-ph/9807565; T. Hahn, hep-ph/0012260; T. Hahn, C. Schappacher, hep-ph/0105349.

11. A. Dobado, M. J. Herrero, S. Peñaranda, Eur. Phys. J. C7 (1999) 313, hep-ph/9710313; C12 (2000) 673, hep-ph/9903211; C17 (2000) 487, hep-ph/0002134; hep-ph/9711441; hep-ph/9806488.

12. A. Dabelstein, Z. Phys. C67 (1995) 495, hep-ph/9409375; Nucl. Phys. B456 (1995) 25, hep-ph/9503443.

13. M. Böhm, H. Spiesberger, W. Hollik, Fortsch. Phys. 34 (1986) 687; W. Hollik, Fortsch. Phys. 38 (1990) 165; P. H. Chankowski et al., Nucl. Phys. B417 (1994) 101; P. Chankowski, S. Pokorski and J. Rosiek, Nucl. Phys. B423 (1994) 437, hep-ph/9303309.

14. D. E. Groom et al. [Particle Data Group Collaboration], Eur. Phys. J. C15 (2000) 1.

15. D. J. Miller and S. Moretti, hep-ph/0001194; C. Castanier, P. Gay, P. Lutz and J. Orloff, hep-ex/0101028.

16. J. A. Aguilar-Saavedra et al., ECFA/DESY LC Physics Working Group Collaboration, TESLA Technical Design Report, DESY 2001-011, ECFA-2001-209, hep-ph/0106315; T. Abe et al., American Linear Collider Working Group Collaboration, SLAC-R-570, hep-ex/0106056.