The method of calculating inelastic elements of rod structures under loading, unloading and reloading regimes

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Abstract. This article describes a variational method for calculating the rod structures in cases where the deformations are elastoplastic, and external forces change in the loading-unloading-repeated loading mode. The basic design ratios for three stages of external load changes are presented. At the first stage, elastoplastic deformations in the elements in the rod structure from the acting loads are determined. Then, at the second stage, the stress-strain state of the structure is determined after unloading. At the third stage, the structure is calculated after repeated loading under the effect of the resulting load. The presented calculation technique is based on the separation of longitudinal deformation of the rod into two parts: elastic and plastic. Advantages of the proposed approach lie in the fact that it is possible to carry out calculations for rod structures from a reinforced and ideally elastic-plastic material according to a single scheme. On the basis of calculations using the method described above, it is possible to obtain graphs that, in particular, determine the ultimate load for rod structures from an ideally elastoplastic material. The results of calculation of some rod systems by the proposed variational method are presented. The calculated data obtained are consistent with the known analytical solutions.

1. Introduction

As is known, in many cases, when reconstructing existing structures that are in a stressed state, are make unloading some elements of structures subjected to repair or strengthening, and after reconstructing they are additionally loaded. The deformations arising in this case can be elastoplastic [1–2]. In connection with this, when strengthening structures in a stressed state, it becomes necessary to estimate the initial stress-strain state for elastoplastic deformation, determine the residual deformations and stresses arising after unloading, and calculate the stress-strain state under repeated loading [3–4].

It should be pointed out that the problems of determining and recording residual stresses during unloading of strengthened rod structures, as well as methods for determining stress-strain state of structures under elastoplastic deformation, are presented in classical works. However, at the present time methods for calculating of rod structures elastoplastic deformation also develop intensively and are presented in a number of scientific publications. In particular, the problems of calculating the stress-strain state of structures under various load regimes are discussed in [5–6].

Methods for estimating the stress-strain state of strengthened rod structures are described in publications [7–8]. In these studies, the problems of taking residual stresses into account when unloading strengthened structures were not considered, since there was not a sufficiently effective and universal variational method for stress-strain state calculating with due account for residual...
deformations and stresses arising after unloading and stress-strain state calculating upon repeated loading.

A distinctive feature of this work is that the calculation method presented is based on the separation of the rod deformation into two parts: elastic and plastic. The application of this approach makes it possible to calculate the rod structures from the hardening and elastic-perfectly-plastic material according to a single scheme.

2. Methods

This article describes a variational method for calculating the rod structures in cases where the deformations are elastoplastic, and external forces change in the loading-unloading-repeated loading mode. The basic design ratios for three stages of external load changes are as follows:

- at the first stage, elastoplastic deformations in the rod structure occur when external forces act

\[ F = F^*; \]

- at the second stage, external forces reduce, unloading occurs

\[ F = F^* - \Delta F^\text{unl}; \]

- at the third stage, repeated loading is performed

\[ F = F^* + \Delta F^d. \]

The results of calculation of some rod systems by the proposed variational method are presented.

The diagram of a linearly strengthening body is shown in Fig. 1.

We denote by the modulus of elasticity of the material under elastic deformations (section BD of the diagram, fig. 1), and through - the tangential elastic modulus for elastoplastic deformations (the section DN of the diagram).

We represent the longitudinal deformation of the rod \( \varepsilon_x \) as a sum of elastic \( \varepsilon_x^{el} \) and plastic \( \varepsilon_x^{pl} \) deformations:

\[ \varepsilon_x = \varepsilon_x^{el} + \varepsilon_x^{pl}, \]

We assume

\[ \varepsilon_x^{el} = \alpha \varepsilon_x, \quad \varepsilon_x^{pl} = (1 - \alpha) \varepsilon_x, \]

where \( 0 < \alpha \leq 1 \) - is the coefficient, determining the values of the elastic and plastic deformations.

As can be seen from Fig. 1, if \( \sigma_x \leq \sigma_Y \), then the deformations are elastic (\( \varepsilon_x^{pl} = 0 \)) and in formulas (2) it should be assumed that \( \alpha = 0 \). In this case \( \sigma_x = E \varepsilon_x. \) For elastoplastic deformations (\( \sigma_x > \sigma_Y \)), taking into account that, \( \sigma_Y = E \varepsilon_x^{el} = E \alpha \varepsilon_x \), we obtain

\[ \alpha = \frac{\sigma_Y}{E \varepsilon_x}. \]

Consequently,

\[ \alpha = \begin{cases} 
1, & \text{if} \quad \sigma_x \leq \sigma_Y; \\
\frac{\sigma_Y}{E \varepsilon_x}, & \text{if} \quad \sigma_x > \sigma_Y.
\end{cases} \]

For elastoplastic deformations (\( \alpha < 1 \)),

\[ \sigma_x = \sigma_Y + E \varepsilon_x (\varepsilon_x - \varepsilon_x^{el}). \]

Taking into account (2), (4), we obtain

\[ \sigma_x = E^* \varepsilon_x, \]

where

\[ E^* = E \alpha + E_e (1 - \alpha). \]
The value $E'$ depends on the value $\varepsilon_x$. For elastic deformations ($\sigma_x \leq \sigma_Y$), one should assume that the dependence between normal stresses and deformations is expressed with Hooke's law $\sigma_x = E' \varepsilon_x$. For elastoplastic deformations, $\alpha$ is determined by formula (3). In cases where the value $\varepsilon_x$ is not known in advance, an iterative method should be used, sequentially determining $\varepsilon_x$ and $\alpha$.

If $E_t = 0$, then the deformation of the material is described by the diagram of an ideally elastic-plastic body. In this case $\sigma_x = E_t \varepsilon_x$.

We describe a variational method for determining the stress-strain state of a rod system.

In accordance with the tension diagram of the material (Fig. 1), the specific potential energy of deformation of the rod under tension and compression is equal to the area of the figure BDNL:

$$u_0^a = \frac{1}{2} \sigma_x^d \varepsilon_x^d + \sigma_Y \varepsilon_Y^{pl} + \frac{1}{2} \sigma_Y \varepsilon_Y^{pl} = \frac{1}{2} E(\varepsilon_x^d)^2 + \sigma_Y \varepsilon_Y^{pl} + \frac{1}{2} E_t (\varepsilon_Y^{pl})^2.$$

Using (2), we obtain

$$u_0^a = \frac{1}{2} E \alpha^2 \varepsilon_x^2 + \sigma_Y (1-\alpha) \varepsilon_x + \frac{1}{2} E_t (1-\alpha)^2 \varepsilon_x^2.$$

Note, that when calculating the variation of the potential energy of deformations varied $\varepsilon_x$ and $\alpha$.

Taking into account (4),

$$\delta \alpha = -\frac{\sigma_Y}{E_t \varepsilon_x} \delta \varepsilon_x = -\frac{\alpha}{\varepsilon_x} \delta \varepsilon_x.$$

Consequently,

$$\delta u_0^a = E(\varepsilon_x^d \delta \alpha + \alpha \varepsilon_x \delta \varepsilon_x) + \sigma_Y \left[ -\varepsilon_x \delta \alpha + (1-\alpha) \delta \varepsilon_x \right] + E_t \left[ -\varepsilon_x^d (1-\alpha) \delta \alpha + (1-\alpha)^2 \varepsilon_x \delta \varepsilon_x \right].$$

Substituting the expression (7) into this equation, we find

$$\delta u_0^a = \sigma_Y \delta \varepsilon_x + E_t (1-\alpha) \varepsilon_x \delta \varepsilon_x.$$

Using equalities (3), (6), the last relation can be represented in the form

$$\delta u_0^a = E^2 \varepsilon_x \delta \varepsilon_x.$$

As is known, the influence of shear stresses on the stress-strain state of a rod structure under tension, compression and bending are insignificant. However, when implementing the variational method, it is desirable to take into account shear stresses, since it is necessary for joining of various elements of the rod system.

In elastic deformations, shear stresses are related to the angular deformation by Hooke's law under shear: $\tau^{yd} = G Y^{yd}$, where $G$ - is the shear modulus. As is known $G = E / (2(1 + \nu))$. This dependence between $G$ and $E$ is obtained geometrically and does not depend on whether the deformations are elastic or elastoplastic. Proceeding from this, we assume that for elastoplastic deformations $\tau = G^* \gamma$, where, taking into account (6),

$$G^* = \frac{E}{2(1+\nu)} \alpha + \frac{E_t}{2(1+\nu^*)} (1-\alpha).$$

Here $\nu^*$ is the modulus of cross deformation of the material under elastoplastic deformations.

When the rod is bent in two planes, deformations $\gamma_{xy}, \gamma_{xz}$ and stresses $\tau_{xy}, \tau_{yz}$ occur in mutually perpendicular planes. In this case,

$$\tau_{xy} = G^* \gamma_{xy}, \quad \tau_{yz} = G^* \gamma_{yz}.$$

The variation of the specific potential shear strain has the form
\[
\delta u_0^* = \frac{1}{2} \delta \left( \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz} \right) = G^* \gamma_{xy} \delta \tau_{xy} + G^* \gamma_{xz} \delta \tau_{xz}.
\]

When determining the displacements in the rod system, the technique described in [9] is used. The deformed state of the rod structure is determined using the Lagrange variational principle:

\[
\delta U - \delta W = 0.
\]

Here

\[
\delta U = \int \left( \int_\Omega \left( \sum_i \delta u_i^0 + \delta u_i^* \right) dx \right) dx
\]

- variation of the potential energy of the deformation of the rod system; \( \delta W \) – variation of the work of external forces [9].

The expressions for the deformations \( \varepsilon_x, \gamma_{xy}, \gamma_{xz} \) entering into (12) are written using the components of the displacement vector \( \mathbf{a}(a_1, a_2, a_3) \) and the rotation angles \( \mathbf{\phi}(\phi_1, \phi_2, \phi_3) \) [9].

When distributed and concentrated forces act on a rod structure

\[
\delta W = \int \left( \delta q_1 \delta u_1 + \delta q_2 \delta u_2 + \delta q_3 \delta u_3 \right) dl + \sum_{i=1}^{3} \left( F_{i1} \delta u_1(x_i) + F_{i2} \delta u_2(x_i) + F_{i3} \delta u_3(x_i) \right)
\]

Here \( q_1 = q_1^* \), \( q_2 = q_2^* \), \( q_3 = q_3^* \), \( F_{i1} = F_{i1}^* \), \( F_{i2} = F_{i2}^* \), \( F_{i3} = F_{i3}^* \) are effective loads.

With the use of condition (11), the displacements and rotation angles are determined, on the basis of these quantities the deformations are calculated and the stresses are found using formulas (5) and (10).

Let us assume that when the point N (fig. 2) is reached, there is an unloading caused by the fact that external forces change:

\[
q_1 = q_1^* - \Delta q_1^{\text{unl}}, \quad q_2 = q_2^* - \Delta q_2^{\text{unl}}, \quad q_3 = q_3^* - \Delta q_3^{\text{unl}},
\]

\[
F_{i1} = F_{i1}^* - \Delta F_{i1}^{\text{unl}}, \quad F_{i2} = F_{i2}^* - \Delta F_{i2}^{\text{unl}}, \quad F_{i3} = F_{i3}^* - \Delta F_{i3}^{\text{unl}},
\]

where \( \Delta q_1^{\text{unl}}, \Delta q_2^{\text{unl}}, \Delta q_3^{\text{unl}}, \Delta F_{i1}^{\text{unl}}, \Delta F_{i2}^{\text{unl}}, \Delta F_{i3}^{\text{unl}} \) - are additional loads, that cause unloading of the structure.

Unloading corresponds to the section NN_1 of the diagram. At point N, the deformation is equal to \( \varepsilon_x^* \), and after unloading, at point NN_1, its value is \( \varepsilon_x^{\text{res}} \). Consequently, during unloading a deformation occurs

\[
\varepsilon_x = \varepsilon_x^{\text{res}} - \varepsilon_x^*.
\]

Residual deformations and stresses after unloading

\[
\varepsilon_x^{\text{res}} = \varepsilon_x^* + \varepsilon_x, \quad \sigma_x^{\text{res}} = \sigma_x^* - E(\varepsilon_x^* - \varepsilon_x^{\text{res}}).
\]

When the load corresponding to the section NN_1 of the diagram changes

\[
\sigma_x^{\text{res}} = \sigma_x^* + E \varepsilon_x
\]

\[
\tau_{xy} = \tau_{xy}^* + G^* \gamma_{xy}, \quad \tau_{xz} = \tau_{xz}^* + G^* \gamma_{xz},
\]

where \( \sigma_x^*, \tau_{xy}^*, \tau_{xz}^* \) - are the normal and shear stresses, acting in the structural elements before the unloading.

The specific potential energy of deformation associated with normal stresses is equal to the area of the figure BDNN_1L_1 (fig. 2):
\[ u_0^\sigma = \frac{1}{2} \sigma_Y e_x^{ed} + \sigma_Y e_x^{pl} + \left( \frac{1}{2} \sigma_Y^* - \sigma_Y \right) e_x^{pl} - \sigma_x^{es} \left( e_x^{\text{ed}} - e_x^{\text{pl}} \right) - \frac{1}{2} E \left( e_x^{\text{ed}} - e_x^{\text{pl}} \right)^2. \]

Using formulas (14), (15), the last equality may be represented in the form of
\[ u_0^\sigma = \frac{1}{2} \sigma_Y e_x^{ed} + \sigma_Y e_x^{pl} + \frac{1}{2} \left( \sigma_Y^* - \sigma_Y \right) e_x^{pl} + \sigma_x^{es} e_x + \frac{1}{2} E e_x^{pl}. \]

Considering that when unloading the corresponding quantities \( \sigma_Y^*, \sigma_x^*, e_x^{pl}, e_x^{pl} \) are not variable values, we get
\[ \delta u_x = E \varepsilon_x^{\text{pl}} + \sigma_x^{es} \delta e_x. \quad (17) \]

To determine the displacement vectors \( \pmb{u} = (u_1, u_2, u_3) \) and rotation angles \( \pmb{\varepsilon} = (\phi_1, \phi_2, \phi_3) \), the variational principle of Lagrange (11) is also used. Taking into account (16), (17),
\[ \delta U = \left[ \int_A \left( E \varepsilon_x^{\text{pl}} + G \varepsilon_y^{xy} + G \varepsilon_z^{xz} \right) dA + \delta U \right] dA \quad (18) \]

When the rod structure is reloaded, external forces increase:
\[ \sigma_1 = \sigma_1^{\text{pl}} + \Delta \sigma_{1,2}^{\text{unl}} + \Delta \sigma_{1,3}^{\text{pl}} \]
\[ F_1 = F_1^{\text{pl}} + \Delta F_1^{\text{unl}} + \Delta F_1^{\text{pl}} \]
\[ F_2 = F_2^{\text{pl}} + \Delta F_2^{\text{unl}} + \Delta F_2^{\text{pl}} \]
\[ F_3 = F_3^{\text{pl}} + \Delta F_3^{\text{unl}} + \Delta F_3^{\text{pl}} \]

In these equalities \( \Delta \sigma_1^{\text{unl}}, \Delta \sigma_2^{\text{pl}}, \Delta \sigma_3^{\text{unl}}, \Delta \sigma_3^{\text{unl}}, \Delta F_1^{\text{unl}}, \Delta F_2^{\text{unl}}, \Delta F_3^{\text{unl}} \) are the additional loads, acting upon repeated loading of the structure.

In the case of an increase in external forces to a certain value, the deformations will be elastic \( (\sigma_x < \sigma_x^*) \), and deformation is described by a line \( N_1N \). With a further increase in the load \( (\sigma_x \geq \sigma_x^*) \), plastic deformations arise (line NC in fig. 3). In these cases, the deformations are determined by the formula
\[ e_x^{\text{pl}} = e_x^{\text{res}} + e_x^{\text{pl}}, \]

where \( e_x^{\text{pl}} \) - deformation upon repeated loading; \( e_x^{\text{res}} \) - residual deformation.

Let us represent \( e_x \) it in the form of (2), (3). When unloading, hardening of the material (slander phenomenon) takes place, the limit of proportionality changes. As can be seen from fig. 3, under repeated loading the Hooke's law will be valid if \( \sigma_x \leq \sigma_x^* \). Therefore, to determine the coefficient \( \alpha \) that determines the values of elastic and plastic deformations, the following relations should be used instead of (4):
\[ \alpha = \begin{cases} 1, & \text{if } \sigma_x \leq \sigma_x^*; \\ \frac{\sigma_x^*}{E e_x}, & \text{if } \sigma_x > \sigma_x^*. \end{cases} \quad (19) \]

As can be seen from fig. 3, when repeatedly loaded, the stress is at point C of the diagram
\[ \sigma_x = \sigma_x^{\text{res}} + E e_x^{\text{pl}} + E (e_x - e_x^{\text{pl}}). \]

Taking into account formulas (1), (2), (11), the last equality can be written in the form
\[ \sigma_x = \sigma_x^{\text{res}} + E e_x^{\text{pl}}. \quad (20) \]

To calculate the tangential stresses, one can use the formulas

\[ \text{Figure 3. Diagram of the material at unloading and repeated loading} \]
Under repeated loading, the specific potential energy of deformation \( u_0 \), caused by normal stresses, is equal to the area of the figure BDNN, \( S_L \) (Fig. 3). It can be shown that in this case the variation of the specific potential energy of deformation associated with normal stresses

\[
\delta u_0^0 = E^* \varepsilon \delta \varepsilon + \sigma_{x}^{res} \delta \sigma_x.
\]

The deformed state of the structure under repeated loading is determined from the condition (11), in which it is necessary to assume

\[
\delta U = \int \left[ \int E^* \varepsilon \delta \varepsilon + G^* \gamma \delta \gamma + G^* \gamma_{xy} \delta \gamma_{xy} \right] dA + \int \left[ \int \left( \sigma_{x}^{res} \delta \sigma_x + \tau_{xy}^{res} \delta \tau_{xy} + \tau_{x}^{res} \delta \tau_{x} \right) dA \right].
\]

When solving a problem, the values of the coefficients \( E^*, G^*, \alpha \) are determined by the iterative method, depending on the value \( \varepsilon \). To do this, a series of points in the rod structure is selected (these can be, for example, the integration points used to calculate the integrals in (18)). At the first iteration it is assumed that \( E^* = E, G^* = G, \alpha = 1 \) and deformations \( \varepsilon_x \) and stresses \( \sigma_x \) are determined at the selected points of the rod structure. Then using the formula (19) the value of the coefficient \( \alpha \) is refined. When \( \alpha = 1 \), deformations at the considered point are elastic, and in the case if \( \alpha < 1 \), the deformations are elastoplastic. The iterative process is completed only when the condition

\[
\left( \varepsilon^{(n)}_{max} - \varepsilon^{(n-1)}_{max} \right) \cdot 100\% \leq \Delta
\]

is satisfied, where \( \Delta \) is the specified error value, \( \varepsilon^{(n)}_{max}, \varepsilon^{(n-1)}_{max} \) - the maximum relative deformations at the points of the elements of the rod structure at the next two iterations.

3. Results and Discussion

Let’s look at the elastoplastic deformation of a statically indeterminate system of three rods (Fig. 4).

We assume that the deformation of the rods is described by the Prandtl diagram, \( E = 2 \cdot 10^5 \text{ MPa} \), \( \sigma_y = 240 \text{ MPa} \), \( \varphi = 30^\circ \). Cross-sectional areas of rods \( A = 1 \text{ cm}^2 \). The rod system is loaded with force \( F \) until the limit state is reached.

The value of the force \( F \), until it has reached the deformation in the system is elastic

\[
F = F_{el} = \sigma_y A (1 + 2 \cos^3 \varphi).
\]

When \( F > F_{el} \) there will be plastic deformations in the first rod, the normal stresses in the section become equal to \( \sigma_y \), the force \( N_1 = \sigma_y A \). In the case when \( F = F_a < F_{ult} \), the deformations in the second and third rods will be elastic. Here \( F_{ult} \) – the value of the load influencing the system and is within \( F_{el} < F_a < F_{ult} \).

If plastic deformations occur in all three rods, \( \sigma = \sigma_y \), then a limiting state will be reached in the rod system. In this case \( F = F_{ult} = \sigma_y A (1 + 2 \cos \varphi) \),

\[
N_1 = N_2 = N_3 = \sigma_y A.
\]

When unloading the rod system, we assume that the force \( F \) decreases from the value \( F = F_a \) to \( F = 0 \). In this case, in all three rods the deformations will be elastic.

After unloading (\( F = 0 \)) there are residual deformations and forces in the rods, which are obtained in the form:
\[
\begin{align*}
\Delta l_{1\text{res}} &= \left[ F_a - \sigma \sqrt{\frac{A(1+2\cos^3\varphi)}{2EA\cos^3\varphi(1+2\cos^3\varphi)}} \right] l_1, \\
\Delta l_{2\text{res}} &= \Delta l_{3\text{res}} = \Delta l_{1\text{res}} \cos \varphi, \\
N_{1\text{res}} &= -\left[ F_a - \sigma \sqrt{\frac{A(1+2\cos^3\varphi)}{2EA\cos^3\varphi(1+2\cos^3\varphi)}} \right] / (1+2\cos^3\varphi), \\
N_{2\text{res}} &= N_{3\text{res}} = -N_{1\text{res}} / (2\cos \varphi).
\end{align*}
\]

When the rod system is repeatedly loaded, as the force \( F \) is increased to a certain value, the forces and deformations are determined by the following formulas:

\[
\Delta l_{1\text{repl}} = \Delta l_{1\text{res}} + F l_1 / EA(1+2\cos^3 \varphi), \\
\Delta l_{2\text{repl}} = \Delta l_{3\text{repl}} = \Delta l_{1\text{repl}} \cos \varphi, \\
N_{1\text{repl}} = \left[ F - F_a + \sigma \sqrt{A(1+2\cos^3 \varphi)} \right] / (1+2\cos^3 \varphi), \\
N_{2\text{repl}} = N_{3\text{repl}} = (F - F_a^*) / 2 \cos \varphi.
\]

The results of this structure calculation using the described variational method are shown in Fig. 5 and in the Table. It was assumed that \( F_a = 60 \text{kN}, \) \( \Delta = 0.001. \)

In Fig. 5 it is a graph showing the variation of the vertical displacement of the node \( B \) of the rod system depending on the amount of the force applied. It should be noted that the specifics of the methodology for performing calculations under the active loading of the system are set forth in [8]. The solid line shows the dependence of the displacement on the force in the case of increasing of \( F \) from zero to the ultimate load \( F = F_{\text{ult}} = 65.5 \text{kN}. \) The dashed lines show the plot of the graph during unloading and repeated loading.

The table shows the calculation data for unloading and repeated loading, obtained by analytical and variational methods. It can be seen that the results of calculations using different methods are in good agreement. The solid line of the graph in Fig. 5 shows, that when the ultimate load is reached, the movement of the node \( B \) increases and can be arbitrarily large. The graph is constructed on the basis of calculations, during which for the maximum load \( F = F_{\text{ult}} = 65.5 \text{kN} \) at each iteration the value \( w_B \) increased and, as a result, a horizontal line of the graph was obtained. This result clearly illustrates one of the advantages of the calculation technique described in the article.

### Table. Calculation results for loading, unloading and repeated loading

| Loading regimes | Calculation method | Forces in the rods | Deformation of rods |
|-----------------|-------------------|-------------------|---------------------|
|                 | \( N_1, \text{kN} \) | \( N_2, \text{kN} \) | \( N_3, \text{kN} \) |
| Active loading for \( F = 60 \text{kN} \) | Analytical method | 24.0 | 20.785 | 20.785 | 0.001386 | 0.0012 | 0.0012 |
|                 | Variational method | 24.0 | 20.784 | 20.784 | 0.00139 | 0.0012 | 0.0012 |
| Unloading for \( F = 0 \text{kN} \) | Analytical method | -2.097 | 1.21 | 1.21 | 0.8075 \times 10^{-4} | 0.6993 \times 10^{-4} | 0.6993 \times 10^{-4} |
|                 | Variational method | -2.097 | 1.21 | 1.21 | 0.807 \times 10^{-4} | 0.699 \times 10^{-4} | 0.699 \times 10^{-4} |
| Repeated loading for \( F = 60 \text{kN} \) | Analytical method | 24.0 | 20.785 | 20.785 | 0.00139 | 0.0012 | 0.0012 |
|                 | Variational method | 24.0 | 20.784 | 20.784 | 0.00139 | 0.0012 | 0.0012 |
which assumes the introduction of a coefficient $\alpha$ for the separation of longitudinal deformation into the elastic and plastic parts (1), (2). As a result, on the basis of calculations, graphs are obtained, which allow, in particular, to determine the maximum load for rod structures made of elastic-perfectly-plastic material. In the case of using traditional calculation methods, information on the magnitude of the maximum load is obtained on the basis of an emergency shutdown of the computer count. Data on the deformation of the system in the form of graphs are more informative and have greater certainty than an emergency stop of the computer count.

3. Conclusions
1. A variational method for rod structures calculating is proposed for cases when the deformations are elastoplastic, and external forces change in the loading-unloading-repeated loading mode. The presented calculation technique is based on the separation of longitudinal deformation of the rod into two parts: elastic and plastic one.
2. Advantages of the proposed approach consist in the fact that it is possible to carry out calculations for rod structures made of work-hardening and elastic-perfect-plastic material according to a single scheme.
3. On the basis of calculations using the method described above, it is possible to obtain graphs that, in particular, determine the ultimate load for rod structures made of elastic-perfect-plastic material.

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