Rotating AdS black hole stealth solution in $D = 3$

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We show that the rotating asymptotically anti de Sitter black hole solution of new massive gravity in three dimensions can support a static stealth configuration given by a conformally coupled scalar field. By static stealth configuration, we mean a nontrivial time independent scalar field whose energy-momentum tensor vanishes identically on the rotating black hole metric solution of new massive gravity. The existence of this configuration is rendered possible because of the presence of a gravitational hair in the black hole metric that prevents the scalar field to be trivial. In the extremal case, the stealth scalar field diverges at the horizon as it occurs for the conformal scalar field of the Bocharova-Bronnikov-Melnikov-Bekenstein solution in four dimensions.

I. INTRODUCTION

The fundamental tenet of general relativity is the manifestation of the curvature of spacetime produced by the presence of matter. This phenomena is encoded through the Einstein equations that relate the Einstein tensor or any other gravity tensor $G_{\mu\nu}$ to the energy-momentum tensor $T_{\mu\nu}$ that arises from the variation of the matter, i.e.

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}. \quad (1)$$

Since the energy-momentum tensor depends explicitly on the metric, both sides of the equations must be solved simultaneously. However, one can ask if, for a fixed geometry solving the vacuum gravity equations, is it possible to find a matter source coupled to this spacetime that does not affect the geometry. Concretely, this problem consists in examining a particular solution of the Einstein equations (1) where both sides of the equations vanish, i.e.

$$G_{\mu\nu} = 0 = 8\pi G T_{\mu\nu}. \quad (2)$$

In three dimensions, such gravitationally undetectable solutions called stealth configurations have been obtained in [1] for a nonminimally coupled scalar field with the BTZ (Banados-Teitelboim-Zanelli) metric [2]. That is, the gravity action is described by the standard Einstein-Hilbert action with a negative cosmological constant while the matter source is given by

$$S_M = - \int d^3x \sqrt{-g} \left[ \frac{1}{2} \partial_\mu \Phi \partial^{\mu} \Phi + \frac{\xi}{2} R \Phi^2 + U(\Phi) \right], \quad (3)$$

where $\xi$ denotes the nonminimal coupling parameter, $R$ the scalar curvature and $U(\Phi)$ is a potential term. As shown in [1], a stealth configuration on the BTZ metric can be obtained for any value of $\xi$ provided the scalar field being nonstatic (time dependent) and the angular momentum of the BTZ metric being switched off. The same problem has also been considered in higher dimensions for the same stealth matter source but with a flat geometry [3]. Recently, black hole stealth configurations have been obtained with a nonminimal scalar field with a mass term, $U(\Phi) \propto \Phi^2$, and where the gravity side is described by the Einstein-Gauss-Bonnet gravity [4] or its Lovelock generalization [5].

Here, we will consider the problem of three-dimensional stealth configuration for which the gravity side of the stealth equations (2) is given by the so-called new massive gravity [6]. This alternative three-dimensional gravity theory has raised a lot of interest in the last five years due to interesting properties. Indeed, this theory presents the advantage of being at the linearized level equivalent to the unitary Fierz-Pauli theory for free massive 2 spin gravitons and hence it is a good candidate for a quantum theory of gravity. Up to now, there only exist two black hole solutions for the new massive gravity equations: an asymptotically AdS spacetime [7] and a solution with a Lifshitz dynamical exponent $z = 3$, [8]. The rotating version of the AdS black hole solution can easily be obtained by operating an improper boost in the $(t - \varphi)$-plane where $t$ (resp. $\varphi$) stands for the time (resp. the angular) coordinate, and the resulting geometry turns out to be a rotating asymptotically AdS solution of new massive gravity [7]. In the present work, we show that this rotating solution of new massive gravity can support a conformal static stealth configuration given by a nonminimally and conformally coupled scalar field. By conformal stealth configuration, we mean that the solution only exists for $\xi = 1/8$ and for a potential $U \propto \Phi^6$, which are the two conditions that ensure that the matter action (3) enjoys the conformal symmetry. We will clearly establish that the existence of this stealth configuration is rendered possible because of the presence of a gravitational hair in the black hole metric. Interestingly enough, we will also show that in the extremal case, the stealth scalar field diverges at the horizon as it occurs for the conformal scalar field of the BBMB (Bocharova-Bronnikov-Melnikov-Bekenstein) solution in four dimensions. [2] [10].

The plan of this letter is as follows. In the next section, we will give the stealth field equations (2) where the gravity side is described by the equations of new massive gravity.
and, where the function $H$ is defined by

$$H(r) = \left[ r^2 - 2GMl^2\alpha - \frac{b^2 l^4}{16}\alpha^2 \right]^{1/2}.$$ 

In these expressions, the constant $\alpha$ is related to the rotation parameter $a$ through

$$\alpha = 1 - (1 - a^2/l^2)^{1/2}. \quad (9)$$

Hence, this solution is described by two constants related to the mass $M$ and the angular momentum $J = Ma$, while the constant $b$ which contributes to the expression of the mass can be viewed as a short of gravitational hair \cite{11, 12}.

We now investigate whether the black hole spacetime geometry \cite{8} may accommodate a stealth configuration given by a nonminimally coupled scalar field. To be more precise, we are interested on finding a static nontrivial scalar field $\Phi = \Phi(r)$, and eventually a potential term such that the following equations

$$G_{\mu\nu} + \lambda g_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu} = 0 = T_{\mu\nu}, \quad (10a)$$

are satisfied on the rotating black hole background given by \cite{8} at the special point \cite{7}. Here, $T_{\mu\nu}$ is the stress tensor associated to the variation of the matter action \cite{8}, and is given by

$$T_{\mu\nu} = \frac{\partial}{\partial \sigma} \Phi \partial_{\sigma} \Phi - g_{\mu\nu} \left( \frac{1}{2} \partial_{\sigma} \Phi \partial^{\sigma} \Phi + U(\Phi) \right) + \xi (g_{\mu\nu} \Box - \nabla_{\mu} \nabla_{\nu} + G_{\mu\nu}) \Phi^2. \quad (11)$$

We already know that the gravity part of the stealth equations \cite{11} are satisfied on the background \cite{8} at the point \cite{7}, and so it only remains to solve the equations $T_{\mu\nu} = 0$.

In order to gain in clarity, we first present the details of the computations in the nonrotating case $a = 0$ (or equivalently $\alpha = 0$) and then we switch to the rotating case. For a vanishing rotation parameter $a = 0$, the following combination $T^r_r - T^\varphi_\varphi = 0$ yields to a first-order differential equation for the scalar field whose solution is given by

$$\Phi_{\pm}(r) = \frac{C}{\sqrt{\pm(b \tau + 8GM)}},$$

where $C$ is an integration constant. Injecting this expression into the combination $T^r_r - T^\varphi_\varphi = 0$, one obtains the following constraint

$$\frac{1}{4} \left( -r^2 - 2bl^2 + 4GMl^2 \right)^2 C^2 b^2 (8\xi - 1) = 0,$

which is solved for $C = 0$, or $b = 0$, or $\xi = 1/8$. However, the first two options implies that the scalar field...
becomes constant and, hence in order to satisfy the constraint with a nontrivial scalar field, the nonminimal coupling parameter must take its three-dimensional conformal value \( \xi = 1/8 \). Finally, the remaining independent equation given by the combination \( T^t_t - T^r_r - T^\varphi_\varphi = 0 \) allows to express the potential term \( U \) as a local expression of the scalar field as

\[
U(\Phi) = \beta \Phi^6. \tag{12}
\]

It is interesting to note that this form of the potential together with the coupling \( \xi = 1/8 \) are precisely those that ensure the matter action \( [3] \) to be conformally invariant. We then conclude that the static solutions of the stealth equations \([10]\) on the nonrotating background \([3]\) with \( \alpha = 0 \) requires a conformal scalar field source, and are given by

\[
ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2d\varphi^2,
\]

\[F(r) = \frac{r^2}{l^2} + br - 4GM,
\]

\[\Phi_{\pm}(r) = \left[ \frac{16G^2M}{2\beta l^2} \left( M + \frac{b^2l^2}{16G} \right) \right]^{\frac{1}{4}} \frac{1}{\sqrt{\pm(-br + 8GM)}}. \tag{13}\]

It is evident that the existence of this stealth configuration is indicative to the presence of the gravitational hair \( b \) since in the limit \( b = 0 \), the scalar field becomes constant. This result is not in contradiction with the one of Ref. \([1]\) where the authors showed that in the BTZ case, \( b = 0 \), the scalar field must be nonstatic. In fact, the results obtained in Ref. \([1]\) together with the solution \([13]\) clearly establish the correlation between the presence of the gravitational hair \( b \) and the possibility of having a nontrivial static stealth scalar field.

In the extremal case, that is for \( M = -b^2l^2/(16G) \), the derivation of the stealth solution along the same line as before implies again that \( \xi = 1/8 \) but the potential must be zero. The resulting extremal stealth solution reads

\[
ds^2 = -\left( \frac{r}{l} + \frac{bl}{2} \right)^2 dt^2 + \frac{dr^2}{\left( \frac{r}{l} + \frac{bl}{2} \right)^2} + r^2d\varphi^2, \tag{14a}\]

\[\Phi_{\pm}(r) = \frac{A}{\sqrt{\pm(2r + bl^2)}}, \tag{14b}\]

where now the scalar field depends on an arbitrary constant \( A \). Various comments can be made concerning this extremal solution. Firstly, this latter can be obtained from the non extremal one \([13]\) by taking the limit \( M \to -b^2l^2/(16G) \) but at the same time \( \beta \to 0 \) such that \( (M + b^2l^2/(16G)) = O(\beta) \). In doing so, the scalar field depends on an arbitrary constant. The occurrence of this arbitrary constant can be easily explained since, in the absence of the potential term, the energy-momentum tensor has a scaling symmetry, \( \Phi \to \Omega \Phi \), where \( \Omega \) is an arbitrary constant. Hence, the presence of the arbitrary constant \( A \) is just a consequence of this symmetry. We also stress that the scalar field diverges at the horizon \( r_+ = -\frac{l^2}{2b} \) as it occurs for the BBMB solution in four dimensions. This is intriguing in the sense that the BBMB solution shares some features with this extremal stealth configuration. Indeed, the BBMB solution is a solution of a static conformal scalar field in four dimensions without potential, and whose metric is also extremal (the extremal Reissner-Nordstrom spacetime). The divergence of the BBMB scalar field at the horizon makes its physical interpretation and the problem of its stability a subject of debate \([12, 14]\). A way of circumventing this problem is to introduce a cosmological constant, whose effect is to precisely push this singularity behind the horizon, as it has been done in Ref. \([15]\). Because of these two examples (the BBMB solution and the extremal stealth configuration), one is tempted to associate this pathology to the extremal character of the metric together with the conformal symmetry of the source.

Let us now consider the rotating case, \( a \neq 0 \), for which the different steps to obtain the stealth solution turn on to be analogous to those operated in the nonrotating case. Indeed, as before, the combination \( T^t_t - T^r_r - T^\varphi_\varphi = 0 \) permits to obtain the expression of the scalar field while injecting this form into the combination \( T^r_r - T^\varphi_\varphi = 0 \) yields to a rather complicated constraint. This latter is satisfied and yields to a nontrivial solution only in the case \( \xi = 1/8 \). Finally, the combination \( T^t_t - T^r_r - T^\varphi_\varphi = 0 \) allows to express the potential term \( U \), and one obtains again the conformal potential \([12]\) while the remaining independent Einstein equations \( T^\varphi_\varphi = 0 \), being proportional to the combination \( T^t_t - T^\varphi_\varphi = 0 \) is also satisfied. We end with the following rotating asymptotically anti de Sitter stealth configuration given by the metric \([3]\) together with the scalar field

\[
\Phi_{\pm}(r) = \left( \frac{256M^2G^2 + 16MGb^2l^2(\alpha + 1) + b^4l^4\alpha}{2\beta l^2} \right)^{\frac{1}{4}} \frac{1}{\sqrt{\pm(-b\sqrt{16r^2 - 32\alpha MG l^2 - b^4l^2\alpha^2 + b^2l^2\alpha + 32MG})}}, \tag{15}\]

where \( \alpha \) is defined in \([3]\). It is evident that in the vanishing rotation limit \( a \to 0 \) (or equivalently \( \alpha \to 0 \), this
solution reduces to [13]. Moreover, as in the nonrotating case, we emphasize again that this is the presence of the gravitational hair $b$ that prevents the scalar field to be trivial. The extremal version for $M = -b^2 l^2 / (16G)$ requires the nonminimal coupling $\xi$ to be the conformal one $\xi = 1/8$ as well as a vanishing potential $U = 0$, and the scalar field is given by

$$\Phi(r) = \frac{A}{\sqrt{\pm (-b \sqrt{16r^2 + b^2 l^4 (2 - \alpha)} + l^2 b^2 (\alpha - 2))}},$$

where $A$ is an arbitrary constant. As in the nonrotating case, the scalar field diverges at the horizon and this configuration can be obtained by taking the limits $M \to -b^2 l^2 / (16G)$ and $\beta \to 0$ such that $(M + b^2 l^2 / (16G)) = O(\beta)$.

III. COMMENTS AND CONCLUSIONS

Here, we have shown that the rotating asymptotically anti de Sitter black hole solution of new massive gravity in three dimensions can support a nontrivial static stealth configuration given by a nonminimally and conformally coupled scalar field. We have clearly established that this is the presence of the gravitational hair $b$ that prevents the scalar field to be trivial. The extremal version of this stealth configuration presents the same pathology (namely the divergence of the scalar field at the horizon) that the BBMB solution in four dimensions.

There are many issues related to the present work that will be interesting to explore but we would like to emphasize the thermodynamics issue. Indeed, since we have obtained a black hole solution, it is natural to wonder about the thermodynamics. However, in order to compute the mass, the temperature and the entropy of this solution, we are faced with the following problem. In fact, one may note that the stealth equations [10] can be viewed as a particular solution of the field equations arising from the variation of the action

$$S = S_G + S_M,$$

where $S_G$ is the new massive gravity action [2] and $S_M$ is the source action [3]. For simplicity, let us consider the nonrotating case. The temperature is given as in the free source case [11, 12] by

$$T = \frac{1}{\pi l} \sqrt{GM + \frac{b^2 l^2}{16}}$$

while the entropy $S$ computed with the help of the Wald formula [16] yields

$$S = \frac{2 \pi l}{\sqrt{G}} \sqrt{M + \frac{b^2 l^2}{16G} - \frac{\pi}{8} \sqrt{2GM} \beta}.$$ 

However, a simple computation shows that the product $T dS$ is not a total derivative, and hence we are faced with the problem that the first law is not satisfied unless there is some additional charge to be considered. It will be interesting to further explore the thermodynamic issue of the stealth solutions found here.

Other questions can be asked related to this work as for example what is the precise role of the gravitational hair in the emergence of such configuration. Also, we have been interested on looking only for static stealth configuration that can be supported by the rotating solution of new massive gravity. From Ref. [1], we learn that in the BTZ case, the stealth scalar field configuration must be nonstatic and the angular momentum must be zero. We may ask whether there exists a nonstatic stealth configuration in the case of the rotating solution of new massive gravity. Finally, it is also natural to explore if our results can be extended in higher dimensions.

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References

[1] E. Ayon-Beato, C. Martinez and J. Zanelli, “Stealth scalar field overflying a (2+1) black hole,” Gen. Rel. Grav. 38, 145 (2006) [hep-th/0403228].
[2] M. Banados, C. Teitelboim and J. Zanelli, “The Black hole in three-dimensional space-time,” Phys. Rev. Lett. 69, 1849 (1992) [hep-th/9204099].
[3] E. Ayon-Beato, C. Martinez, R. Troncoso and J. Zanelli, “Gravitational Cheshire effect: Nonminimally coupled scalar fields may not curve spacetime,” Phys. Rev. D 71, 104037 (2005) [hep-th/0505086].
[4] M. B. Gaete and M. Hassaine, “Topological black holes for Einstein-Gauss-Bonnet gravity with a nonminimal scalar field,” arXiv:1308.3076 [hep-th].
[5] M. B. Gaete and M. Hassaine, “Planar AdS black holes in Lovelock gravity with a nonminimal scalar field,” arXiv:1309.3338 [hep-th].
[6] E. A. Bergshoeff, O. Hohm and P. K. Townsend, “Massive Gravity in Three Dimensions,” Phys. Rev. Lett. 102,
[7] J. Oliva, D. Tempo and R. Troncoso, “Three-dimensional black holes, gravitational solitons, kinks and wormholes for BHT massive gravity,” JHEP 0907, 011 (2009) [arXiv:0901.1766 [hep-th]].
[8] E. Ayon-Beato, A. Garbarz, G. Giribet and M. Hassaine, “Lifshitz Black Hole in Three Dimensions,” Phys. Rev. D 80, 104029 (2009) [arXiv:0905.1545 [hep-th]].
[9] J. D. Bekenstein, “Exact Solutions Of Einstein Conformal Scalar Equations,” Annals Phys. 82, 535 (1974).
[10] N. M. Bocharova, K. A. Bronnikov and V. N. Melnikov, “An exact solution of the system of Einstein equations and mass-free scalar field,” Vestn. Mosk. Univ. Fiz. Astron. 6 (1970) 706 [Moscow Univ. Phys. Bull. 25 (1970) 80].
[11] G. Giribet, J. Oliva, D. Tempo and R. Troncoso, “Microscopic entropy of the three-dimensional rotating black hole of BHT massive gravity,” Phys. Rev. D 80, 124046 (2009) [arXiv:0909.2564 [hep-th]].
[12] A. Perez, D. Tempo and R. Troncoso, JHEP 1107, 093 (2011) [arXiv:1106.4849 [hep-th]].
[13] K. A. Bronnikov and Y. A. Kireev, “Instability of Black Holes with Scalar Charge,” Phys. Lett. A 67 (1978) 95.
[14] P. L. McFadden and N. G. Turok, “Effective theory approach to brane world black holes,” Phys. Rev. D 71 (2005) 086004 [hep-th/0412109].
[15] C. Martinez, R. Troncoso and J. Zanelli, “De Sitter black hole with a conformally coupled scalar field in four dimensions,” Phys. Rev. D 67, 024008 (2003) [arXiv:hep-th/0205319].
[16] R. M. Wald, “Black hole entropy is the Noether charge,” Phys. Rev. D 48, 3427 (1993) [gr-qc/9307038].