Modeling and neural sliding mode control of mems triaxial gyroscope

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Abstract
In this paper, a neural sliding mode control approach is developed to adjust the sliding gain using a radial basis function (RBF) neural network (NN) for the tracking control of Microelectromechanical Systems (MEMS) triaxial vibratory gyroscope. First a sliding mode control with a fixed sliding gain is proposed to assure the asymptotic stability of the closed loop system. Then a RBF neural network is derived to adjust the sliding gain using a gradient method in a switching control law. With the adaptive sliding gain using the learning function of RBF neural network, the chattering phenomenon is eliminated. Numerical simulation is investigated to verify the effectiveness of the proposed neural sliding mode control scheme.

Keywords
Adaptive control, backstepping approach, tracking performance, microgyroscope

Introduction
As an important measuring element and inertial navigation instrument, micro gyroscope is commonly used to measure angular velocity in navigation system, automobile, etc. Because of the design and manufacture errors and the influence of temperature and operating environment, the resonant frequency of micro gyroscope will change and the vibration amplitude of micro gyroscope will be unstable. The difference between the original characteristics and design will decrease the quality of the gyroscope. The measurement of angular speed and the compensation of manufacturing error have become the main control problem in the micro gyroscope. However, the traditional control method mainly solves the stability control of oscillation amplitude and frequency of driving shaft and the matching problem of two-axis frequency. It cannot effectively solve the shortcomings and defects of micro gyroscopes. Many advanced intelligent control scheme have been applied to the control of micro gyroscopes for improving the accuracy and sensitivity. Recently, researchers developed advanced control approaches to control the MEMS gyroscopes.1–5

Sliding mode controller (SMC) is a nonlinear control strategy with strong robustness.6–8 Terminal sliding mode controllers and super-twisting sliding mode controller have developed for PMSM drives and microgyroscope.9–11 Recently, intelligent control approaches such as neural network and fuzzy control have been investigated.12–14 The universal approximation theorem indicates that the fuzzy system is a new universal approximator in addition to polynomial function approximators and neural network approximators.
Because the universal approximation theory of fuzzy system and neural network can approximate any nonlinear model and realize arbitrary nonlinear control law, they are widely used in the control systems. An adaptive fuzzy sliding mode control scheme was developed for robot manipulator. Lin and Wai proposed a sliding-mode-controlled slider-crank mechanism with a fuzzy neural network controller. Lin et al. employed a neural network based robust nonlinear control for a magnetic levitation system. Some neural network sliding mode control approaches have been developed for robot manipulators. Baric et al. presented a neural network-based sliding mode controller for an electronic throttle. Javadi Moghaddam et al. developed a neural network-based sliding-mode control for rotating stall and surge in axial compressors. Duwaish and Hamou proposed a neural network based adaptive sliding mode controller for a power system stabilizer. Neural controller and fuzzy controller have been widely used in identification and control for several dynamic systems.

In the sliding mode control (SMC), the bound of uncertainties must be available in advance. Lin et al. proposed an adaptive fuzzy sliding mode control to estimate the upper bound of uncertainties for a permanent-magnet synchronous motor drive. This paper proposed a sliding mode controller whose sliding gain can be updated online using a RBF neural network scheme.

The contribution of this paper can be highlighted as:

1. The requirement of the upper bound of model uncertainties and external disturbances must be known is relaxed by RBF sliding gain adjustment, therefore the information of such upper bound does not need to be known in advance. A RBF neural network is used to adaptively learn such unknown upper bound, thereby effectively eliminating the chattering phenomenon. A key property of this scheme is that the prior knowledge of the upper bound of the system uncertainties is not required but estimated using a RBF neural estimator.

2. The control scheme integrates the sliding mode control and the nonlinear mapping of neural network. The adaptive neural network control with gradient method is proposed to adjust the sliding gain to attenuate the chattering. The sliding mode technique has been combined with adaptive control algorithm and neural network approximation method to achieve the desired attenuation of chattering and maintain good tracking in the presence of model uncertainties and external disturbances.

The paper is organized as follows. In Section 2, dynamics of triaxial MEMS gyroscope is introduced. In Section 3, a sliding mode control with a fixed sliding gain is proposed to guarantee the asymptotic stability of the closed loop system. In Section 4, a neural network sliding mode control is developed to adjust the sliding gain in a switching control law. Simulation results are presented in Section 5 to verify the effectiveness of the proposed control. Conclusions are provided in Section 6.

Dynamics of MEMS triaxial gyroscope

A MEMS vibratory gyroscope mainly contains three parts: one is the sensitive element of single mass suspended by spring beams, another is electrostatic actuations and sensing mechanisms for forcing an oscillatory motion and sensing the position and velocity of the sensitive element, the other is the rigid frame which is rotated along the rotation axis. A schematic diagram of a MEMS triaxial gyroscope in $xy$ plane is shown in Figure 1.

Dynamics of a MEMS gyroscope is derived from Newton’s law in the rotating frame.

As we know, Newton’s law in the rotating frame becomes

$$F_r = F_{phy} + F_{centri} + F_{Corilis} + F_{Euler} = ma,$$

where $F_r$ is the total applied force to the proof mass in the gyro frame, $F_{phy}$ the total physical force to the proof mass in the inertial frame, $F_{centri}$ the centrifugal force,
the Coriolis force, \( F_{\text{Euler}} \) the Euler force, \( a_r \) the acceleration of the proof mass with respect to the gyro frame. \( F_{\text{phy}}, F_{\text{Coriolis}}, F_{\text{Euler}} \) are inertial forces caused by the rotation of the gyro frame.

With the definition of \( r_x, r_y \) as the position and velocity vectors relative to the rotating gyroscope frame, and \( \Omega \) as the angular velocity vector of the gyroscope frame, the expressions for the inertial forces reduce to

\[
F_{\text{Coriolis}} = -2m\Omega \times v_r, F_{\text{centr}} = -m\Omega \times (\Omega \times r_r),
\]

\[
F_{\text{Euler}} = -m\frac{d\Omega}{dt} \times r_r
\]

(2)

then

\[
ma_r + m\Omega \times (\Omega \times r_r) + 2m\Omega \times v_r + m\Omega \times r_r = F_{\text{phy}}
\]

(3)

where \( F_{\text{phy}} \) contains spring, damping, and control forces applied to the proof mass.

In a z-axis MEMS gyroscope, by supposing the stiffness of spring in z direction much larger than that in the x, y directions, the motion of the proof is constrained to only along the x-y plane. Referring to Park et al.,\(^3\) assuming that the gyroscope is almost rotating at a constant angular velocity \( \Omega_z \) over a sufficiently long time interval, the dynamics of gyroscope based on equation (3) is simplified as

\[
\begin{align*}
mx' + dx'x + [k_x - m(\Omega_x^2 + \Omega_y^2)]x + m\Omega_x\Omega_y + \\
y' = u_x + 2m\Omega_yy
\end{align*}
\]

\[
my' + dy'y + [k_y - m(\Omega_x^2 + \Omega_y^2)]y + m\Omega_x\Omega_y + \\
x' = u_y - 2m\Omega_xx
\]

(4)

where \( x \) and \( y \) are the coordinates of the proof mass with respect to the gyro frame in a Cartesian coordinate system; \( dx, dy, k_x, k_y \) are damping and spring coefficients in the \( x \) and \( y \) directions; \( \Omega_x, \Omega_y, \Omega_z \) are the angular rate components along each axis of the gyro frame, \( u_x, u_y \) are the control forces in the \( x \) and \( y \) directions. In this design, the control forces are the electrostatic forces in parallel plate actuator which can be expressed as the gradient of the potential energy stored on the capacitor. The last two terms in equation (4), \( 2m\Omega_yy, 2m\Omega_xx \) are the Coriolis forces used to reconstruct the unknown input angular rate \( \Omega_i \). Under typical assumptions \( \Omega_x \approx \Omega_y \approx 0 \), only the component of the angular rate \( \Omega_z \) causes a dynamic coupling between the \( x \) and \( y \) axes.

Now we extends the discussion above from \( x-y \) plane to triaxial system. Assume that the gyroscope is moving with a constant linear speed; the gyroscope is rotating at a constant angular velocity; centrifugal forces are assumed negligible; the gyroscope undergoes rotations along \( x, y \), and \( z \) axis. The dynamics of triaxial gyroscope system can be derived as

\[
\begin{align*}
m\dddot{x} + d_x\dddot{x} + d_y\dddot{y} + d_z\dddot{z} + k_{xx}x + k_{xy}y + k_{xz}z = u_x + 2m\Omega_z\dddot{y} - 2m\Omega_y\dddot{z} \\
m\dddot{y} + d_y\dddot{y} + d_z\dddot{z} + k_{xy}x + k_{yy}y + k_{yz}z = u_y - 2m\Omega_x\dddot{x} + 2m\Omega_y\dddot{z} \\
m\dddot{z} + d_z\dddot{z} + k_{xz}x + k_{yz}y + k_{zz}z = u_z + 2m\Omega_x\dddot{x} - 2m\Omega_y\dddot{y}
\end{align*}
\]

(5)

where \( m \) is the mass of proof mass. Fabrication imperfections contribute mainly to asymmetric spring terms \( k_{xy}, k_{xz}, \) and \( k_{yz} \) and asymmetric damping terms \( d_{xy}, d_{xz}, \) and \( d_{yz} \) are spring terms; \( d_{xx}, d_{yy}, \) and \( d_{zz} \) are damping terms in the \( x, y, \) and \( z \) directions; \( \Omega_x, \Omega_y, \) and \( \Omega_z \) are angular velocities; \( u_x, u_y, \) and \( u_z \) are control forces in the \( x, y, \) and \( z \) directions.

Because of the non-dimensional time \( t' = w_0t \), dividing both sides of equation by reference frequency \( w_0^2 \), reference length \( q_0 \), and reference mass, then rewriting the dynamics in vector forms as

\[
\begin{align*}
\ddot{q} + D\ddot{q} + K_q q = & \quad \frac{u}{m w_0^2 q_0} - 2 \Omega \frac{\dot{q}}{w_0 q_0}.
\end{align*}
\]

(6)

where

\[
\begin{align*}
q &= \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \\
u &= \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}, \\
\Omega &= \begin{bmatrix} 0 & -\Omega_y & \Omega_x \\ \Omega_y & 0 & -\Omega_x \\ -\Omega_x & \Omega_y & 0 \end{bmatrix}, \\
D &= \begin{bmatrix} d_{xx} & d_{xy} & d_{xz} \\ d_{yx} & d_{yy} & d_{yz} \\ d_{zx} & d_{zy} & d_{zz} \end{bmatrix}, \\
K_q &= \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}.
\end{align*}
\]

Define new parameters as:

\[
q^* = \frac{q}{q_0}, D^* = \frac{D}{m w_0^2}, \Omega^* = \frac{\Omega}{w_0}, u_{x^*} = \frac{u_x}{m w_0^2 q_0},
\]

\[
u_{y^*} = \frac{u_y}{m w_0^2 q_0}, u_{z^*} = \frac{u_z}{m w_0^2 q_0},
\]

\[
w_x = \sqrt{\frac{k_{xx}}{m w_0^2}}, w_y = \sqrt{\frac{k_{yy}}{m w_0^2}}, w_z = \sqrt{\frac{k_{zz}}{m w_0^2}},
\]

\[
w_{xy} = \frac{k_{xy}}{m w_0^2}, w_{xz} = \frac{k_{xz}}{m w_0^2}, w_{yz} = \frac{k_{yz}}{m w_0^2}.
\]

For the convenience of notation, ignoring the superscript, then the final form of the non-dimensional motion equation of gyroscope can be obtained

\[
\ddot{q} + D^* \ddot{q} + K_q^* q = u_{x^*} - 2\Omega^* \frac{\dot{q}^*}{w_0 q_0}
\]

(7)
where $K_b = \begin{bmatrix} w_x^2 & w_{xy} & w_{xz} \\ w_{xy} & w_y^2 & w_{yz} \\ w_{xz} & w_{yz} & w_z^2 \end{bmatrix}$.

The control target for MEMS gyroscope is to find a control law so that the proof mass position can track the desired trajectory. In addition, in the presence of model uncertainties and external disturbances, a robust control design is required to improve the robustness and tracking resolution of the control system.

**Design of sliding mode control**

Sliding mode control has many attractive features such as robustness to parameter variations and insensitivity to disturbances. In this section, a sliding mode control with fixed sliding gain is proposed to guarantee the asymptotic stability of the closed loop system.

The control objective for MEMS gyroscope is to keep the proof mass oscillating in $x, y,$ and $z$ direction at given frequencies $\omega_i$ and amplitudes $A_i, i = 1, 2, 3$ as:

$$\dot{x}_m = A_1 \sin(\omega_1 t), \quad \dot{y}_m = A_2 \sin(\omega_2 t), \quad \dot{z}_m = A_3 \sin(\omega_3 t).$$

Therefore the reference model can be written as:

$$\ddot{q}_m + K_b q_m = 0 \quad (8)$$

where $q_m = [x_m, y_m, z_m]^T$, $K_b = \text{diag}\{\omega_1^2, \omega_2^2, \omega_3^2\}$.

Considering the parametric uncertainties and external disturbance, the dynamics of MEMS gyroscope (7) can be expressed as

$$\ddot{q} + (D + 2\Omega + \Delta D)\dot{q} + (K_b + \Delta K_b)q = u + d \quad (9)$$

where $\Delta D$ is unknown parameter uncertainties of matrix $D + 2\Omega, \Delta K_b$ is unknown parameter uncertainties of matrix $K_b, d$ is an uncertain external disturbances in the gyroscope.

Rewriting equation (9) as

$$\ddot{q} = -(D + 2\Omega)\dot{q} - K_b \dot{q} - \Delta D \dot{q} - \Delta K_b q + u + d \quad (10)$$

Defining the right side of equation (10) as $f(q, \dot{q}, t)$

$$f(q, \dot{q}, t) = -(D + 2\Omega)\dot{q} - K_b \dot{q} - \Delta D \dot{q} - \Delta K_b q + d \quad (11)$$

Then equation (10) can be expressed as

$$\ddot{q} = f(q, \dot{q}, t) + u \quad (12)$$

Define

$$f = f_N + \Delta f \quad (13)$$

where $f_N = -(D + 2\Omega)\dot{q} - K_b \dot{q}$ is nominal part of the system, $\Delta f = -\Delta D \dot{q} - \Delta K_b q + d$ is the model uncertainties and external disturbances.

Assuming system satisfies $\|f - f_N\| \leq F$ and $\|d\| \leq M$, where $F$ and $M$ are positive constants.

Defining tracking error $e$ and sliding surface $s$ as

$$e = q - q_m \quad (14)$$

$$s = Ce + \dot{e} \quad (15)$$

where $C > 0$ is a constant matrix.

The derivative of sliding surface becomes

$$\dot{s} = C \dot{e} + \dot{e} = C \dot{e} + q - q_m = f + u - q_m + Ce \quad (16)$$

Setting $\dot{s} = 0$ and assuming $\Delta f = 0$, equivalent control $u_{eq}$ can be solved as

$$u_{eq} = -f_N - C e + \dot{q}_m \quad (17)$$

Based on (17), considering a robust compensator, then a comprehensive controller is designed as

$$u = u_{eq} + u_n = -f_N - C \dot{e} + \dot{q}_m - K \text{sgn}(s) \quad (18)$$

where $u_n = -K \text{sgn}(s)$, and the sliding gain $K$ satisfying

$$K \geq \alpha(F + M + W + \eta) + (\alpha - 1)u_{eq} \quad (19)$$

where $|\dot{q}_m| \leq W, \alpha \geq 1, \eta$ is a positive constant.

**Theorem 1.** The controller (18), applied to the system (9) guarantees that all closed-loop signals are bounded, if $K \geq \alpha(F + M + W + \eta) + (\alpha - 1)u_{eq}$, the tracking error and sliding surface go to zero asymptotically.

**Proof:** Define a Lyapunov function candidate

$$V = \frac{1}{2} s^T s \quad (20)$$

Differentiating $V$ with respect to time yields

$$\dot{V} = s^T (f + u + d - \ddot{q}_m + C \dot{e})$$

$$= s^T (f + u_{eq} - K \text{sgn}(s) + d - \ddot{q}_m + C \dot{e}) \quad (21)$$

From (17), the following can be obtained

$$C \dot{e} = -f_N - u_{eq} + \dot{q}_m \quad (22)$$

Substituting (22) into (21) yields

$$\dot{V} = s^T (f + u_{eq} - K \text{sgn}(s) + d - \ddot{q}_m - f_N - u_{eq} + \dot{q}_m)$$

$$= s^T (f - f_N) + s^T d - K ||s|| \leq ||s|| (F + D) - K ||s|| \quad (23)$$

Since $K \geq \alpha(F + M + W + \eta) + (\alpha - 1)u_{eq}$, choose $\alpha = 1, K \geq F + D + W + \eta$

$$\dot{V} \leq ||s|| (F + M) - (F + M + W + \eta) ||s||$$

$$\leq - (W + \eta) ||s|| \leq - \eta ||s|| \leq 0 \quad (24)$$

$\dot{V}$ is negative definite implies that $s$ converge to zero. $\dot{V}$ is negative semi-definite ensures that $V, s$ are all bounded. From (16) it is concluded that $\dot{s}$ is also
bounded, equation (20) implies that $s$ is integrable as $\int_0^t \|s\| \, dt \leq \frac{1}{2} |V(0) - V(t)|$. Since $V(0)$ is bounded and $V(t)$ is nonincreasing and bounded, it can be concluded that $\lim_{t \to \infty} \int_0^t \|s\| \, dt$ is bounded. Since $\lim_{t \to \infty} \int_0^t \|s\| \, dt$ is bounded and $\dot{s}$ is also bounded, according to Barbalat lemma, $s(t)$ will asymptotically converge to zero, $\lim_{t \to \infty} s(t) = 0$, then, $e(t)$ will asymptotically converge to zero, $\lim_{t \to \infty} e(t) = 0$.

**Design of neural network sliding mode control**

Neural network has the capability of approximating any nonlinear continuous function over the compact input space. Therefore neural network’s learning ability to approximate arbitrary nonlinear continuous function makes it a useful tool for adaptive application. RBF NN has the universal approximation property which states that any sufficiently smooth function can be approximated by a suitable large network for all inputs in a compact set and the resulting function reconstruction error is bounded. RBF NN has a faster convergence property and a simpler architecture than BP (back propagation) NN. The RBF NN structure employed in our work is shown as Figure 2.

We will address the design of RBF network based sliding mode control problem. Because of the great advantages of neural networks in dealing with the nonlinear system, neural sliding mode controller is designed and analyzed in this section. Since sliding mode has some limitation such as chattering or high frequency oscillation in practical applications, neural network sliding mode control is adopted to facilitate adaptive gain adjustment and reduce the chattering. The block diagram of sliding mode control using RBF neural network is shown in Figure 3. Replacing $K$ by $K_N$ yields the new controller\(^{11}\) as

$$u = u_{eq} + u_n = -f_N - C\dot{\varphi} + \ddot{q}_m - K_N sgn(s) \quad (25)$$

If the upper bound value $K_N$ is unknown and can not be measured properly, RBF neural network can be used to adaptively learn the upper bound $K_N$. The structure of RBF neural network is a three-layer feedforward network shown as in Figure 3. The input layer is a set of source nodes. The second layer is a hidden layer of high dimension. The output layer gives the response of the network to the activation patterns applied to the input layer.

The estimate of the upper bound $K_N$ is

$$K_N(x, \omega) = \omega^T \phi(x) \quad (26)$$

where $x = [\varphi, \dot{\varphi}]$ is the input of RBF neural network, $\omega = [w_1, w_2, \ldots, w_L]$ are weights of RBF neural network and $\phi(x)$ is Gaussian function,

$$\phi_i(x) = \exp\left(-\frac{\|x - m_i\|^2}{\sigma_i^2}\right), \quad i = 1, 2, \ldots, L$$

where $L$ is number of output node, $\phi(x) = [\phi_1, \phi_2, \ldots, \phi_L]^T$, $m_i$ is $i$th center vector, and $\sigma_i$ is $i$th standard deviation.

*Theorem 2.* The neural network controller (26), applied to the system (4) guarantees that weight learning is $\omega(t) = \omega(t-1) + \Delta \omega(t) + \alpha_1(\omega(t) - \omega(t-1))$.

*Proof:* The performance index of energy function is defined as

$$E = \frac{1}{2} e^2 \quad (28)$$

where $e$ is a position tracking error.

The tracking error $e \to 0$ as $t \to \infty$ implies $s \to 0$ and $\dot{s} \to 0$ as $t \to \infty$.

The learning algorithm of network weight is derived as

$$\Delta \omega = -\eta_1 \frac{\partial E}{\partial \omega} = -\eta_1 \frac{\partial e}{\partial \omega} = -\eta_1 \frac{\partial q}{\partial \omega}$$

$$= -\eta_1 \frac{\partial q}{\partial u} \frac{\partial u}{\partial K_N} \sim -\eta_1 \delta_{\text{sign}} \left(\frac{T q}{\partial u} \frac{\partial u}{\partial K_N} \frac{\partial K_N}{\partial \omega}\right) \quad (29)$$

Since the value of $\frac{\partial q}{\partial u}$ can be adjusted by parameter $\eta_1$, therefore the sign determines its value.

$$\frac{\partial u}{\partial K_N} = \frac{\partial(u_{eq} + u_n)}{\partial K_N} = \frac{\partial(-f_m - C\dot{\varphi} + \ddot{q}_m - K_N sgn(s))}{\partial K_N} = -sgn(s) \quad (30)$$

$$\frac{\partial K_N}{\partial \omega} = \frac{\partial(\omega^T \phi(x))}{\partial \omega} = \phi(x) sgn(\omega^T \phi(x)) \quad (31)$$
Therefore the weight is updated by the amount
\[ \Delta w(t) = - \eta_1 \text{sgn} \left( \frac{\partial q}{\partial w} \right) (s)(\phi(x)\text{sgn}(\omega^T \phi(x))) \]

The weight learning algorithm is expressed as
\[ \omega(t) = \omega(t-1) + \Delta w(t) + \alpha_1 (\omega(t) - \omega(t-1)) \]

where \( \eta_1 \) is learning rate of the weight, \( \eta_1 \in (0, 1) \), \( \alpha_1 \in (0, 1) \).

The mean and standard deviation of the hidden layer are updated in the similar ways.

The learning law of \( \text{ith} \) center vector \( m_i \) is obtained as
\[ \Delta m_i = - \eta_m \frac{\partial E}{\partial m_i} = - \eta_m \left( \frac{1}{2} \frac{e^2}{C_0} \right) = - \eta_m \frac{\partial e}{\partial m_i} \]
\[ = - \eta_m \frac{\partial (q - m_i)}{\partial m_i} = - \eta_m \frac{\partial q}{\partial m_i} \]
\[ = - \eta_m \frac{\partial q}{\partial m_i} \frac{\partial K_i}{\partial m_i} \]

where:
\[ \frac{\partial K_i}{\partial m_i} = \frac{\partial (w^T \phi(x))}{\partial m_i} \text{sgn}(w^T \phi(x)) = w_i \frac{\partial \phi_i(x)}{\partial m_i} \text{sgn}(w^T \phi(x)) \]

\[ = w_i \exp \left( - \frac{||x - m_i||^2}{\sigma_i^2} \right) \text{sgn}(w^T \phi(x)) \]
\[ = w_i \exp \left( - \frac{||x - m_i||^2}{\sigma_i^2} \right) \cdot \frac{2||x - m_i||}{\sigma_i^2} \cdot \text{sgn}(w^T \phi(x)) \]

Then the updating algorithm of \( m_i \) becomes
\[ \Delta m_i(t) = \eta_m \text{sgn} \left( \frac{\partial q}{\partial u} \right) \text{sgn}(s)w_i \exp \left( - \frac{||x - m_i||^2}{\sigma_i^2} \right) \]
\[ \left( 2 \frac{||x - m_i||}{\sigma_i^2} \right) \text{sgn}(w^T \phi(x)) \]

Therefore the learning algorithm of \( m_i \) becomes
\[ m_i(t) = m_i(t-1) + \Delta m_i(t) + \alpha (m_i(t) - m_i(t-1)) \]

The learning law of the \( \text{ith} \) standard deviation \( \sigma_i \) is obtained as
\[ \Delta \sigma_i = - \eta_\sigma \frac{\partial E}{\partial \sigma_i} = - \eta_\sigma \left( \frac{1}{2} \frac{e^2}{C_0} \right) = - \eta_\sigma \frac{\partial (q - m_i)}{\partial \sigma_i} \]
\[ = - \eta_\sigma \frac{\partial q}{\partial \sigma_i} \frac{\partial K_i}{\partial \sigma_i} \]
\[ = - \eta_\sigma \frac{\partial q}{\partial \sigma_i} \frac{\partial K_i}{\partial \sigma_i} \]

where
\[ \frac{\partial K_i}{\partial \sigma_i} = \frac{\partial (w^T \phi(x))}{\partial \sigma_i} \text{sgn}(w^T \phi(x)) \]
\[ = w_i \exp \left( - \frac{||x - m_i||^2}{\sigma_i^2} \right) \text{sgn}(w^T \phi(x)) \]

Then its updating algorithm is derived as
\[ \Delta \sigma_i(t) = \eta_\sigma \text{sgn} \left( \frac{\partial q}{\partial u} \right) \text{sgn}(s)w_i \exp \left( - \frac{||x - m_i||^2}{\sigma_i^2} \right) \]
\[ \left( \frac{2||x - m_i||}{\sigma_i^2} \right) \text{sgn}(w^T \phi(x)) \]

The learning algorithm of \( \sigma_i \) is expressed as
\[ \sigma_i(t) = \sigma_i(t-1) + \Delta \sigma_i(t) + \alpha (\sigma_i(t) - \sigma_i(t-1)) \]

Remark 1. The weight error not only has relationship with current time, but also depends on previous time. \( \alpha_1 \) is the forgetting factor establishing the previous weight error with current weight error. Selection of learning rate has a significant effect on the network performance.

The effectiveness of the online training of neural network to adjust the control gain will be demonstrated by the following simulation results.

**Simulation study**

In this section, we will evaluate the proposed neural network sliding mode approach on the lumped MEMS gyroscope sensor model by using MATLAB/SIMULINK. Parameters of the MEMS gyroscope sensor are as follows:

- \( m = 0.57e - 8kg, \omega_0 = 3kHz, q_0 = 10^{-6}m, \)
- \( d_{xx} = 0.429e - 6Ns/m, d_{yy} = 0.0429e - 6Ns/m, \)
- \( d_{zz} = 0.895e - 6Ns/m, d_{xyz} = 0.0429e - 6Ns/m, \)
- \( k_{xx} = 80.98N/m, k_{xy} = 5N/m, k_{yx} = 71.62N/m, \)
- \( k_{xz} = 60.97N/m, k_{kz} = 6N/m, k_{xz} = 7N/m. \)
In order to make the numerical simulation more easily realized and simplify the design of the controller, the dimensionless procedure for the micro gyroscope is implemented. Since the general displacement range of the micro gyroscope sensor in each axis is sub-micrometer level, it is reasonable to choose $1 \mu m$ as the reference length $q_0$. Given that the usual natural frequency of a vibratory MEMS gyroscope sensor is in the KHz range, $v_0$ is chosen as $1\text{kHz}$. For example, the dimensionless parameters of the micro gyroscope system in $x$-$y$ axis are listed as in Table 1:

| Parameters | Values |
|------------|--------|
| $\omega_x$ | 355.3  |
| $\omega_y$ | 532.9  |
| $\omega_{xy}$ | 70.99 |
| $d_{xx}$ | 0.01 |
| $d_{yy}$ | 0.01 |
| $d_{xy}$ | 0.002 |
| $\Omega_z$ | 0.1 |

Table 1. Dimensionless parameters of micro gyroscope.

The unknown angular velocity is assumed $\Omega_z = 5.0 \text{rad/s}$, $\Omega_x = 3.0 \text{rad/s}$.

The desired motion trajectories are $x_m = \sin (\omega_1 t)$, $y_m = 1.2 \sin (\omega_2 t)$, $z_m = 1.5 \sin (\omega_3 t)$, where $\omega_1 = 6.71 \text{kHz}$, $\omega_2 = 5.11 \text{kHz}$, $\omega_3 = 4.17 \text{kHz}$.

Initial value of $w(0) = [200 200 200]$, external disturbance random noise, $C = 300$, $\alpha = 2$. In the Gaussian function (27), $\sigma = [1 1 1]$, $m = [0.6 0.6 0.6]$, $\eta_1$ and $\alpha_1$ are chosen as $\eta_1 = 0.8$ and $\alpha_1 = 0.05$ respectively.

Figure 4 is the position tracking in the $x$, $y$, and $z$ direction using the sliding mode control with fixed sliding gain. Figure 5 is the position tracking using the neural sliding mode control with RBF sliding gain adjustment. It can be observed from Figures 4 and 5 that the position trajectory can track that of reference model in very short time. Figures 6 and 7 depict the convergence of the tracking errors. It can be seen that tracking errors converge to zero asymptotically. The MEMS gyroscope can maintain the proof mass oscillating in the $x$, $y$, and $z$ direction at given frequency and amplitude both with sliding mode control and with neural sliding mode control.

Figure 8 depicts the control inputs using the sliding mode control with fixed sliding gain, Figure 9 draws the control inputs using the neural sliding mode control with RBF sliding gain adjustment. The performance of neural sliding mode control is better than that of sliding mode control using fixed sliding gain because RBF neural network can adjust the sliding gain to eliminate the chattering phenomenon. Figure 10 shows the adaptation of the sliding gain adjustment using neural sliding mode control.

It it noted that the dimensionless procedure for the micro gyroscope system is carried out, there are no unit of magnitude from Figures 4 to 10. It is concluded that the system response is as expected, and the simulation results demonstrated that both neural sliding mode control and sliding mode control have good tracking performance and tracking errors converge to zero asymptotically. Neural network-based adaptive sliding mode controller with adaptive sliding gain has better performance in eliminating the chattering than sliding mode control using fixed switching gain.

A universal standard is adopted to quantify tracking error by calculating root mean square error (RMSE) to observe the tracking performance under different controller. The RMSE reflects how much the measured value deviates from its true value and the smaller the
RMSE is, the higher the measurement accuracy is. Meanwhile, based on the statistics of RMSE, we carry out quantitative comparison between the SMC with fixed sliding gain and SMC with NN sliding gain under the same reference signal excitation.

It is evident from the Table 2 that the performance of the proposed control scheme is remarkably superior to that of the standard SMC in tracking accuracy since the corresponding steady-state errors of the proposed method are significantly smaller than those of the standard SMC.

**Conclusion**

A neural network based sliding mode control is proposed for a triaxial angular velocity sensor. Neural network compensator is used to approximate the upper bound of model uncertainties and external disturbances and adjust the sliding gain, making the system react to the changes of the model uncertainties and external disturbances with better adaptability. Simulation results show that neural network-based adaptive sliding mode controller has satisfactory performance in reducing the chattering.
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