Towards a Formalisation of Aristotle’s Topics

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Abstract

The Topics are of considerable interest not only as part of the early development of Aristotelic logic but for the information they provide regarding the practice of argument and proof in the Platonic Academy and other contemporary schools of ancient philosophy. This paper contributes to the project of carrying out a modern formal analysis of ancient logic (for Stoic logic see Bobzien[7]) with the ultimate aim of being able to formalise ancient philosophical arguments (either as deployed in the ‘game’ of debate or in formal proof) and axiomatically reconstruct ancient philosophical systems in a style similar to Zalta[15], Meixner and Van Lambalgen and his students. Its specific contribution to Aristotelic scholarship is in showing to what extent the theories of the Analytics are already present in the Topics and in reassessing the consistency, sophistication and relevance of this traditionally neglected work. There is also material of interest to linguistics and cognitive science.

We present a multi-sorted first-order approach to formalising Aristotle’s Topics based on a simple ontology involving individuals and predicates connected by two binary relations $K$ and $E$ expressing Aristotle’s ‘being said of a subject’ and ‘being in a subject’. The latter is further decomposed into several predicates expressing the degree of ‘essentiality’ of $E$-predication, thus allowing us to capture the nuances in Aristotle’s (not always entirely consistent) definitions of ‘accident’, ‘property’ and ‘difference’. Following the lead of Slomkowski [14] we interpret a topic to be a (universal) sentence which may include binary predicates (anticipating the relational syllogism featured in Galen’s Introduction to Logic and which we plan to show present in Aristotle’s mature works.). Our approach is based on a system $T$ loosely inspired by Bealer’s logics $T1, T2$ and $T3$ as presented in Quality and Concept[5, 6]. The language of $T$ is that of classical first-order modal logic with equality extended with a Bealer-style intensional operator $[\phi]_{x_1...x_n}$. Even if a strong system such as $T$ is justified by the ontologically richness of the Topics, weaker subsystems are adequate for formalising interesting fragments. Using $T$ we set up a system of axioms and definitions and show how a fair portion of the topics are either direct expressions of the axioms or else can be derived from them. We make use of the enumeration of topics in Owen[3]. Finally we prove the soundness of $T$.

Keywords: Aristotle, Non-classical Logics; Semantics of Natural Language, Algebraic Logic, History of Logic.

1 Introduction

The Topics are of considerable interest not only as part of the early development of Aristotelic logic but for the information they provide regarding the practice of argument and proof in the Platonic Academy and other contemporary schools of ancient philosophy.
Indeed in this early work Aristotle attempts to codify in a proto-formal way all the rules of philosophical argument and debate commonly employed in his time. The specifically modern interest of this text lies in that it presents us with a conception of 'logic' not geared specifically to mathematics but having a general scope tied to natural language discourse and its associated semantic, ontological and epistemic peculiarities.

In this paper we adopt the conclusions of [14] regarding the nature of a topic. A topic is considered a general proposition which is agreed upon by both parties in a dialectical exercise. The approach to formalisation is partially inspired by [12] and the first-order approach in [10]. The *Topics* and the *Sophistical Elenchi* are an early work of Aristotle and many fundamental concepts (logical, ontological and semantical) were further refined in the mature works of the *Organon* and the *Metaphysics* and in commentaries down through the ages. Of particular interest is Porphyry's *eisagogē*. The establishment of the original manuscript of the *Topics* and the correct interpretation of numerous passages is notoriously difficult. Thus our proposal will be necessarily incomplete and tentative.

Our proposal for formalisation is based on many-sorted first-order logic with intensional abstracts. We have two basic sorts $I$ and $P$ representing individual substances *ousias* and terms *oroi* (or concepts *logoi*). Following the *Categories* we take as fundamental binary relations that of 'being said of a subject' and that of 'being in a subject'. Faithful to Aristotle's ontology we take these relations as being defined between individuals and terms and attempt to derive all the other *predicabilia* from them, that is, we define the relations of genus and species, accident, property, difference, definition, all of which can also hold between terms.

The logic and ontology (not to mention semantic and grammatical consideration) of the *Topics* is very rich. In our proposal we will not attempt to treat those topics which involve grammatical considerations such as 'flections' and 'tenses'. Also we will assume that the terms, our elements of sort $P$, are all clearly defined 'concepts' or 'lexemes' so that we need not consider those topics involving problems of ambiguity, vagueness, obscurity or metaphorical usage (which is an interesting subject in its own right).

Our formalisation also abstracts completely from the problem of generation and corruption of individual substances. As we shall see, time is reduced to a minimal role, that involving 'non-essential temporary accidents' of a substance. For our take on Aristotle’s modal syllogistic see [13]. The formal approach in [13] seems to us to be an interesting way to extend our present systems to include a more thorough detailed treatment of temporality and modality involving possible 'states-of-affairs'.

In section 2 we present $T$ and show how we can define other key concepts and how many topics correspond either to our axioms or can be directly proven. We attempt to formalise relation-terms, ‘more-and-less’ and probabilistic reasoning in other ontological categories.

In this work we use [1, 2] and [3] for the Greek text and translations. We make use of the list of the topics in [3][pp.677-713]. For example, by topic 5.3.1 we mean topic number 1 of Chapter III of Book V as listed by Owen.

2 The System $T$

Our approach is based on a system $T$ loosely inspired by Bealer's logics $T1, T2$ and $T3$ as presented in *Quality and Concept* [5, 6]. The language of $T$ is that of classical first-order modal logic with equality extended with a Bealer-style intensional operator $[\phi]_{x_1...x_n}$.

**Definition 2.1** The language of $T$ consists of a countable collection of variables $x, y, z, ...$, $n$-ary function symbols $f, g, h, ...$, $n$-ary predicate symbols ($n \geq 1$) $A, B, C, ...$, with distinguished binary predicates $=, K, E_1, E_0$, logical connectives $\&$, $\neg$, $\exists$, and the intensional abstraction operator $[\ ]_{x_1...x_n}$ where $x_1...x_n$ is a possibly empty sequence of distinct variables.
We use the notation $\bar{x}$ for a (possibly empty) sequence of distinct variables.

**Definition 2.2** Formulas and terms of $L^\omega$ are defined by simultaneous induction:

- Variables are terms.
- If $t_1, ..., t_n$ are terms and $f$ is $n$-ary predicate, then $f(t_1, ..., t_n)$ is a term.
- If $\phi$ and $\psi$ are formulas, $x$ a variable, then $(\phi \& \psi)$, $\sim \phi$ and $\exists x.\phi$ are formulas.
- If $\phi$ is a formula $\bar{x}$ is a sequence of distinct variables, then $[\phi]_{\bar{x}}$ is a term.
- If $\phi$ is a formula then $\Box \phi$ is a formula.

A term of the form $[A]_{\bar{x}}$ where $\bar{x}$ has length $n$ is called an $n$-intensional abstract. Intensional abstracts of the form $[A(x_1, ..., x_n)]_{x_1...x_n}$ are called elementary. $\forall$, $\exists$, $\rightarrow$ and $\Diamond$ are defined as usual. $[\phi]_{\bar{x}}$ binds the variables in $\bar{x}$ in $\phi$. We sometimes write $tKs$ and $tE_i s$ for $K(t,s)$ and $E_i(t,s)(i = 0, 1)$ respectively.

If we interpret ‘sameness’ as first-order equality 7.1.1 is just an instance of Leibniz’s rule.

The predicate $K$ represents the most basic instance of essential predication, *kat’ hupokeimenou* (that which applies only to individual substances and tells us what they are) whilst $E_0$ and $E_1$ represent *en hupokeimenô(i)* predication, things which reside within individual substances. These primitive relations allow us in fact to define individual substance through a unary predicate $I(x)$ and ‘term’ (or concept) through a unary predicate $P(x)$. In what follows we will take certain mutually exclusive unary predicates as defining sorts and use a special notation for variables ranging over these sorts. Thus for instance $\forall i.\phi$ is short for $\forall x.I(x) \rightarrow \phi$.

Variables ranging over sort $I$ will be denoted by $i, j, k, ...$ and those ranging oversort $P$ by $p, q, r, ...$. We will later divide $P$ into various subsorts.

We use $E_0$ and the modal operator $\Box$ to define two relations $E_2$ and $E_3$ which together with $E_1$ express degrees of the ‘essentiality’ of the containment in a substance. This is justified for instance by 5.4.4 in which the non-essentiality of ‘property’ is clearly stated. We will take the weakest reading of ‘accident’ from 1.5.4 and thus 5.3.3. shows that it also differs from non-essential property. $E_1(i,a)$ expresses that $a$ is an essential attribute of $i$ (like ‘featherless biped’ or ‘rational’), $E_2(i,a)$ that it is non-essential but permanent (like the ability to laugh), $E_3(i,a)$ expresses that it is contingent (there are situations in which it holds and situations in which it does not hold).

It is contingent that a man walks but it is a permanent natural quality that a man has the ability to walk. Thus we distinguish between the term ‘walking’ and ‘has the natural ability to walk’.

In what follows we use the abbreviation

$$\Diamond \phi \equiv \Box \phi \& \Diamond \sim \phi$$

Our first definitions are

(Def1)  
$$E_2(x,y) \equiv \Box E_0(x,y)$$

(Def2)  
$$E_3(x,y) \equiv \Diamond E_0(x,y)$$
(Def3) \[ E(x, y) \equiv E_1(x, y) \lor E_2(x, y) \lor E_3(x, y) \]

Of first axioms are

(Ax1) \[ \sim (E_0(x, y) \land E_1(x, y)) \]

If we impose standard S5 axioms on \( \Box \) then it seems likely that we can prove the mutual exclusivity of \( E_1, E_2 \) and \( E_3 \) from this axiom.

(Ax2) \[ K(x, y) \rightarrow \forall y. \sim E(x, y) \land E(x, y) \rightarrow \forall y. \sim K(x, y) \]

(Ax3) \[ (K(x, y) \lor E(x, y)) \rightarrow \sim K(y, x) \land \sim E(y, x) \]

Thus we have the fundamental division between individual substances and terms/concepts:

(Def4) \[ I(x) \equiv \exists y. K(x, y) \lor E(x, y) \]

(Def5) \[ P(x) \equiv \exists y. K(y, x) \lor E(y, x) \]

This introduces a fundamental division of \( P \) into two (sub)sorts \( S \) (secondary substances) and \( A \) (attributes). We define

(Def6) \[ S(x) \leftrightarrow \exists y. K(x, y) \]

(Def7) \[ A(x) \leftrightarrow \exists y. E(x, y) \]

Hence the axioms yield that the sort \( P \) has two distinct types of element

(Prop1) \[ S(p) \lor A(p) \& \sim (S(p) \& A(p)) \]

Also we require that the elements of \( P \) must be predicated of some substance and each substance must have some \( E \)-predicate and some \( S \)-predicate.

(Ax4) \[ \forall p. \exists i. K(i, p) \lor E(i, p) \]

(Ax5) \[ \forall i. \exists s, a. K(i, s) \land E(i, a) \]

A formula \( \phi(x) \) with one free variable is said to belong to sort \( S \) (resp. \( A \)) if \( S([\phi(x)]_x) \) (resp. \( A([\phi(x)]_x) \)). We will consider that \( T \) possess a finite number of constants \( a, b, c, \ldots \) and that if a constant \( a \) is of sort \( P \) then there is either a unary predicate symbol \( P_a \) such that \( a = [P_a(x)]_x \) or else an \( n \)-ary predicate symbol \( R_a \) (for \( n > 1 \)) such that \( a = [R_a(x_1, \ldots, x_n)]_{x_1 \ldots x_n} \).
In Aristotle we have a non-reflexive transitive relation \( p_1 \prec p_2 \) defined on \( P \) which expresses that \( p_2 \) is a genus of \( p_1 \) or equivalently that \( p_1 \) is a species of \( p_2 \). There is the special case in which \( p_2 \) is the \textit{proximate genus} of \( p_1 \), that is, there is no \( q \) such that \( p_1 \prec q \prec p_3 \).

In 4.2.1 and other places there is stated the fundamental property of \( \prec \):

\[(Ax6) \quad q \prec p_1 \land q \prec p_2 \rightarrow (p_1 \prec p_2 \lor p_2 \prec p_1)\]

This says that \( \prec \) has the structure of a disjoint union of trees.

Aristotle also states in 4.1.3 that if \( p \prec q \) then \( p \) and \( q \) must belong to the same category.

The question is: how do we define \( \prec \)? We do this by cases. We first consider the case of \( S \). Given \( s, t \) of sort \( S \) we define

\[(Def8) \quad s \prec t \leftrightarrow (\forall i.K(i, s) \rightarrow K(i, t)) \land \exists j.K(j, t) \land \sim K(j, s))\]

Likewise we define for \( a, b \) of sort \( A \)

\[(Def9) \quad a \prec b \leftrightarrow (\forall i.E(i, a) \rightarrow E(i, b)) \land \exists j.E(j, a) \land \sim E(j, b))\]

These definitions are reflected directly in 4.1.7.

Clearly we have

\[(Prop2) \quad S(p) \land A(q) \rightarrow p \not< q\]

We get transitivity easily

\[(Prop3) \quad p \prec q \land q \prec r \rightarrow p \prec r\]

This is 4.1.1 and 4.2.2.

And also

\[(Prop4) \quad p \not< p\]

We get 4.2.7 as a corollary, using Prop3:

\[(Prop5) \quad p \prec q \rightarrow q \not< p\]

and also 4.6.4

\[(Prop6) \quad p \prec q \rightarrow p \neq q\]

Topic 4.6.2 suggests that \( S \) and \( A \) have their maximal elements. We introduce constants \( \top_S \) and \( \top_A \) with axiom

\[(Ax7) \quad S(p) \rightarrow p \prec \top_S \land A(p) \rightarrow p \prec \top_A\]

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We define what it is to be a 'proximate genus'

(Def10) \[ p \triangleleft q \iff p \prec q \land \forall r. p \prec r \land r \prec q \rightarrow (r = p \lor r = q) \]

In 4.1.6 Aristotle states that if \( p \triangleleft q \) then there is another \( r \) different from \( p \) such that \( r \triangleleft q \). No genus has only one species.

(Ax8) \[ p \triangleleft q \rightarrow \exists r. r \neq p \land r \triangleleft q \]

Aristotle also seems to imply that each genus has only a finite number of species and that \( \prec \)-chains are finite, ending in the \textit{infima species}. Unfortunately this cannot be expressed in a first-order context. This also introduces limitation on the intensional abstracts we can form that will belong to \( S \) or \( A \). We note that there is a constant preoccupation with non-redundancy throughout the Topics and in the treatment of definition in particular.

A possible first-order treatment would be what we call \textit{Davidson’s axioms} (D1, D2). Consider the finite list of constants \( c_1, ..., c_m \) of sort \( P \) and \( n_1, ..., n_l \) of sort \( I \) representing our 'vocabulary' and the axioms

\[
p = c_1 \lor p = c_2 \lor ... \lor p = c_m \]

\[
i = n_1 \lor p = n_2 \lor ... \lor p = n_l \]

We assume standard first-order axioms for equality including Leibniz’s rule. We will not go into the subject of Aristotelic 'sameness' here but note that topics 7.1.2, 7.1.3 and 7.1.5 can be easily construed as expressing modern equality axioms.

We could extend \( \prec \) to a relation \( \prec^* \) to include sort \( I \) by defining \( i \prec^* p \) if \( K(i, p) \). Then Topics 4.6.1, 4.7.1 follow immediately.

Consider the binary relation \( E^* \) defined between sort \( S \) and \( A \) as follows

(De10) \[ E^*(s, a) \iff \forall i. K(i, s) \rightarrow E(i, a) \]

With this definition 2.2.2 is immediate as are 4.1.2 and 4.2.3 are because of the sorts. We also get 4.1.5

(Prop7) \[ a \prec b \land E^*(s, a) \rightarrow E^*(s, b) \]

From now on where there is no danger of confusion we denote \( E^* \) by \( E \). We can also define a particular version \( \Sigma(s, a) \)

(Def12) \[ \Sigma(s, a) \iff \exists i. K(i, s) \rightarrow E(i, a) \]

A treatment of \textit{property} would be given by a binary relation \( P(p, a) \) between sort \( P \) and sort \( A \) (read \( a \) is the property of \( p \)) defined as follows:

(Def13) \[ P(p, a) \iff p \neq a \land ((S(p) \land \forall i.K(i, p) \rightarrow E_2(i, a)) \lor (A(p) \land \forall i.E_2(i, p) \leftrightarrow E_2(i, a))) \]

5.4.1 and 5.7.4 follow immediately from this definition.
It is easy to prove 5.3.7

(Prop8) \[ P(p, a) \rightarrow p \not\equiv a \]

and 5.5.4 follows immediately

(Prop9) \[ \neg P(p, p) \]
as well as 5.5.1 (and 5.4.4)

(Prop10) \[ E_1(p, a) \rightarrow \neg P(p, a) \]

and 5.3.3

(Prop11) \[ E_3(p, a) \rightarrow \neg P(p, a) \]

Aristotle’s notion of property and the way it is to be distinguished from difference poses however numerous difficulties (the text of the Topics is not entirely consistent). In fact Barnes [11] states that Porphyry held that only infima species could have properties.

The notion of opposition plays a major role in the Topics. This is formalised in \( T \) be a unary function denoted \( x^\circ \).

For Aristotle there are many species of opposites: contraries, correlatives, privations and contradictories (our internal negation). Correlatives apply to relation terms which will be discussed further ahead. We formalise negation and opposition (excluding negation) in entirely separate ways. Opposes excluding negation correspond to our function \( x^\circ \).

The negation of term \( a \) corresponding to \([\phi(x)]_x\) is the term \([\sim \phi(x)]_x\).

Individual substances have no opposites

(Ax9) \[ I(x) \lor S(x) \rightarrow x^\circ = x \]

A basic property is

(Ax10) \[ x^{\circ \circ} = x \]

and

(Ax11) \[ p < q \rightarrow p^\circ < q^\circ \]

(see 4.3.5).

We can derive 5.6.1

(Prop12) \[ P(p, a) \rightarrow P(p^\circ, a^\circ) \]

and 5.6.3

(Prop13) \[ P(p, a) \rightarrow \sim P(p, a^\circ) \]
We have a division of \( P \) into terms admitting opposites and those that do not (i.e. \( p^o = p \)).

In order to deal with definitions and with complex terms in general (such as 'featherless biped') it is absolutely essential that \( T \) has some form of expressing 'internal' conjunction. Here is were the intensional abstracts play a central role. We introduce the axioms

(Ax12) \[ I(y) & S([\phi(x)]_x) \rightarrow (K(y, [\phi(x)]_x) \leftrightarrow \phi(y)) \]

(Ax13) \[ I(y) & A([\phi(x)]_x) \rightarrow (E(y, [\phi(x)]_x) \leftrightarrow \phi(y)) \]

Thus we can express the internal conjunction of intensional abstracts \([\phi(x)]_x\) and \([\psi(y)]_y\) by \([\phi(x) \& \psi(x)]_x\) and we get

(Prop14) \[ K(i, [\phi(x) \& \psi(x)]_x) \leftrightarrow K(i, [\phi(x)]_x) \& K(i, [\psi(y)]_y) \]

(Prop15) \[ E(i, [\phi(x) \& \psi(x)]_x) \leftrightarrow E(i, [\phi(x)]_x) \& E(i, [\psi(y)]_y) \]

If \( a \) and \( b \) are constants equal to intensional abstracts involving formulas \( \phi \) and \( \psi \) then we denote by \( a \bullet b \) the term given by the intensional abstract involving the formula \( \phi \& \psi \). We use the variable \( t \) to range over intensional abstracts \([\phi]_x\).

Now we can give a precise formalisation to topics such as 5.2.5, for any constant \( a \):

(Prop16) \[ \sim P(p, T_A \bullet a) \]

We now come to the formalisation of difference and definition. Let us start with definitions of terms of sort \( S \).

(Def14) \[ D(s, t \bullet a) \leftrightarrow s \triangleleft t \& \forall i. K(i, s) \leftrightarrow (K(i, t) \& E_1(i, a)) \]

Then we define difference for the \( S \) sort

(Def15) \[ \Delta(s, a) \leftrightarrow \exists t. D(s, t \bullet a) \]

We get immediately 4.2.6 for \( S \)-terms

(Prop17) \[ s \triangleleft t \rightarrow \sim \Delta(s, t) \]

and also 5.4.4

(Prop18) \[ P(s, a) \rightarrow \sim \Delta(s, a) \]

We have instances of 4.2.5 and 4.1.4:

(Prop19) \[ s \triangleleft t \& D(t, q) \rightarrow s \triangleleft q \]
Aristotle seems to require a sort of minimality or non-redundancy as in 6.3.2. which would rule out definitions having the form \( s \cdot T_A \cdot a \). We can express (partially) this non-redundancy condition through the axiom

\[(Ax14)\]  
\[D(s, p) \rightarrow \sim D(s, p \cdot a)\]

which is 6.3.2.

What is more tricky is to define definition between \( A \)-terms.

We might try for example

\[(Def16)\]  
\[D(b, c \cdot a) \leftrightarrow b \triangleleft c \& \forall i. E(i, b) \leftrightarrow (E(i, c) \& E_1(i, a))\]

The question then arises if 4.2.6 holds

\[(Quest1)\]  
\[p \triangleleft q \rightarrow \sim \Delta(p, q)\]

It is crucial to be able to deal with relation-terms. For example the relation-term 'knowledge' can be modified to 'knowledge of the good'. The immediate species of \( T_A \) will be the Aristotelic categories except for 'substance' which corresponds to \( T_S \). 4.1.3 suggests that if \( p \triangleleft q \) then \( p \) and \( q \) are in the same 'category'. However this is not so for the category of 'relation' as Aristotle's own example of the relation 'knowledge' having non-relation species 'grammar'. This poses problems for our definition of \( \triangleleft \) on \( A \).

Note that not all constants of sort \( A \) correspond to unary intensional abstracts (but those of sort \( S \) do). Let us focus on binary intensional abstracts \([R(x, y)]_{xy}\).

We have syntactic operations of \textit{inversion} \( \text{Inv}[R(x, y)]_{xy} = [R(y, x)]_{xy} \) and \textit{instantiation} \( [R(x, y)]_{xy} \circ t = [R(x, t)]_{x} \). Note that we can compose these to obtain \([R(t, x)]_{x}\) as well.

If \( r \) is a relation term then \( r^o \) is the inverse relation such as in 'being double' and 'being half'.

It is natural to impose

\[(Ax15)\]  
\[[\phi(x, y)]^o_{xy} = [\phi(y, x)]_{xy}\]

One reading of difficult topics such as 5.7.3 is

\[(Quest2)\]  
\[E(a, r \circ b) \& E(a, r \circ c) \& \neg P(a, r \circ b) \rightarrow \neg P(a, r \circ c)\]

Prudence is the science of the fair and prudence is the science of the foul. And since being the science of the fair is not a property of prudence neither is being the science of the foul.

There is also a characteristic of terms: 'admitting more and less'. We can introduce a unary predicate to distinguish theis \( \triangleleft a \) and use 4.6.6 as an axiom

\[(Ax16)\]  
\[a \triangleleft b \& \triangleleft b \rightarrow \triangleleft a\]

This predicate will only hold for elements of sort \( A \).

Aristotle frequently makes use of the relation of a term being 'better known' than another (and he distinguishes between 'in itself' and 'to us'). This could be formalised by a binary
relation \( a \succ b \) which is to be read ‘\( a \) is better known than \( b \)’. One axiom would be given by topic 5.2.1

\[(Ax17) \quad P(p, a) \rightarrow a \succ p \]

Aristotle’s notion of ‘more and less’ seems to correspond to various distinct notions. In one sense it corresponds to a binary ‘modal’ operator expressing that a given sentence \( \phi \) is more probable than a sentence \( \phi' \). We can employ a series of quaternary predicates \( P_X(a, b, c, d) \) where \( X \) is either \( K \) or \( E \). This is to be read ‘it is more probable that \( a \) is \( X \)-predicated of \( b \) than \( c \) is \( X \)-predicated of \( d' \). Topics 5.8.4 and 5.8.5 would read

\[(Ax18) \quad P_E(a, b, a', b') \& E(a', b') \rightarrow E(a, b) \]

\[(Ax19) \quad P_E(a, b, a', b') \& \neg E(a, b) \rightarrow E(a', b') \]

On other occasions ‘more and less’ seems to suggest an order relation defined on the species of a genus as is the case for quantities, where different numbers are considered species of the genus ‘number’. And in another sense it corresponds to an order relation which expresses how close a term is or how well a term ‘participates’ in the essence of another term, as in Aristotle’s example ‘a flame is more fire than light’. We formalise the last two senses by a trinary function \( M_X(a, b, c) \) which is read ‘\( b \) is more \( X \)-predicated of \( a \) than \( X \)-predicated of \( c \)’.

The ‘more and less’ is a fascinating aspect of the logic of the \textit{Topics} because it suggests a sort of semantic metric and that terms (or genuses) are ‘spaces’ endowed with internal symmetries expressed by opposition. Aristotle even considers a further operation of \textit{mixture} on a genus, for instance combining the opposites white and black to produce grey.

Then there is the relation between the part and the whole. How do we deal the sort \( I \)? We could consider a relation \( P(i, j) \) expressing that \( i \) is a part of \( j \) and extend it in the obvious way to \( S \). Consider topic 5.5.5. Here we seem to meet with the famous concept of ‘homeomere’. A tentative definition would be

\[(Def17) \quad H(s) \leftrightarrow \forall t. P(t, s) \rightarrow (\forall a. E(s, a) \leftrightarrow E(t, a)) \]

Then 5.5.5 reads

\[(Quest3) \quad H(s) \rightarrow P(s, a) \& P(t, s) \leftrightarrow P(t, a) \]

There are many other aspects of the \textit{Topics} that we have not discussed.

The \textit{Topics} (chapters 1 and 8) give us very precise rules regarding how the ancient Greek game of dialectical debate was carried out at the time this work was written. For a detailed discussion see [14].

Consider the axiomatic system \( T \) (which we assume consistent) with given choice of constants. Let there be two debaters \( A \) and \( B \). \( B \) starts by proposing a sentence \( \phi \) such as a definition \( D(s, t) \). \( A \) can propose any first-order sentence in the language of \( T \) and \( B \) must either accept or reject it. Both \( A \) and \( B \) must accept the axioms of \( T \) and inference rules for first-order (modal) logic. The goal of \( A \)’s choices is to lead \( B \) to admit \( \neg \phi \).
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