Unified solutions on axial bearing capacity of T-shaped concrete-filled steel tube short columns with binding bars

Qingyun Ge\textsuperscript{1,a}, Fulian Yang\textsuperscript{1,b}, Wenxiu Dong\textsuperscript{1,c}, Zhu Bian\textsuperscript{1,d}

\textsuperscript{1}West Anhui University, Lu’an Anhui China
\textsuperscript{a}email: 05000050@wxc.edu.cn, \textsuperscript{b}email: 05000069@wxc.edu.cn, \textsuperscript{c}email: 1018690878@qq.com, \textsuperscript{d}email: 450927314@qq.com

*Corresponding author’s e-mail: 05000069@wxc.edu.cn

Abstract: Based on the unified strength theory, according to the characteristics of axial bearing capacity of T-shaped concrete-filled steel tubular stub short columns with binding bars, the cross shaped section was divided into 3 rectangular sections which have binding bars and one rectangular section which has no bars. Considering intermediate principal stress, the ration of width to thickness of pipe to the bearing capacity of steel tube was in consideration too, the paper analyzed force mechanism. In the end, the mathematical formulation of bearing capacity for the T-shaped concrete-filled steel tube stub column with binding bars was obtained. The formula calculation values are in good agreement with the test value of the relevant literature.

1. Summary

With the development of high-rise buildings, the use of special-shaped columns can avoid angular column exposure, increase the use of space, and can meet the requirements of architectural graphic design. Concrete-filled steel tube (CFST) with special section with binding bars is an improvement on the general special section CFST.

At present, a little research has been done on special-shaped concrete-filled steel tube at home and abroad [1-6]. Research shows that, for square, rectangular, L-shaped and T-shaped concrete-filled steel tube short columns, the buckling mode of the steel tube is changed and the local buckling of the steel pipe is delayed after the binding bars are set. And it is helpful to improve the constraint effect of the middle steel tube around the cross-section on the core concrete, so as to improve the bearing capacity and ductility of the short axial compression column of concrete-filled steel tube [7-11]. In this paper, based on the unified strength theory [12], the T-shaped section of CFST is divided into three rectangular areas with binding bars and one rectangular area without binding bar (Fig.1). The constraint of binding bars and outer steel tube to concrete is equivalent to effective lateral stress. At the same time, the rectangular core concrete area is reasonably equivalent to the circular area, and the formula for T-shaped concrete-filled steel tube short columns with binding bars is established.
2. Unified Strength Theory

In 1991, based on his twin shear strength theory, Yu Mao-hong established twin shear unified strength theory, which considering the impact of the intermediate principal stress $\sigma_2$ and the different effects of material between tensile and compressive [12].

For materials whose tensile strength and compressive strength are different, in the field of strength calculation standards, it need two material strength coefficients, namely the tensile ultimate strength $f_t$ and the compressive strength limit $f_c$. Then the formula is

$$ F = \sigma_1 \frac{\alpha}{1+b} (b\sigma_2 + \sigma_3) = \sigma_1, \quad \sigma_2 \leq \frac{\sigma_1 + \alpha \sigma_3}{1 + \alpha} $$

(1a)

$$ F' = \frac{1}{1+b} (\sigma_1 + b\sigma_2) \cdot \alpha \sigma_3 = \sigma_1, \quad \sigma_2 \geq \frac{\sigma_1 + \alpha \sigma_3}{1 + \alpha} $$

(1b)

Where, $\sigma_1$, $\sigma_2$ and $\sigma_3$ are the three principal stress; $\alpha=f_t/f_c$ is material strength ratio of tension and pressure; $b$ is not only a coefficient which can reflect the effect of intermediate principal stress and the corresponding surface of positive stress on material damage influence degree, but also a parameter which reflects the different theory of strength.

3. Ultimate Bearing Capacity Analysis

3.1. Bearing Capacity Mechanism

Previous studies have shown that [10], under the axial pressure, the binding bars restrict the protruding deformation in the middle of the steel tube wall, which provides transverse elastic support for the steel tube and improves the bending stiffness of the steel tube wall. Accordingly, the steel tube can provide greater average lateral binding force for core concrete, so that the compressive strength of core concrete can be greatly improved.

3.2. Steel Tube Bearing Capacity

Under the action of axial pressure, the outer steel tube of the concrete-filled rectangular steel tube is in the state of three-direction stress under longitudinal and radial compression and annular tension. However, because the radial stress is far less than the longitudinal and annular stress, the influence of radial stress is not considered [7].

The width-thickness ratio parameters of each side of T-shaped steel tube ($R'_i$) are defined as,

$$ R'_i = \frac{a_i}{t} \sqrt{\frac{12(1-\nu^2)}{4\pi^2}} \sqrt{\frac{f_{ty}}{E_a}} $$

(2a)

$$ R'_3 = \frac{b_i}{t} \sqrt{\frac{12(1-\nu^2)}{4\pi^2}} \sqrt{\frac{f_{ty}}{E_a}} $$

(2b)
$$R'_i = \frac{a}{t} \sqrt{\frac{12(1-\nu^2)}{4\pi^2} \frac{f_{sy}}{E_s}}$$

(2c)

Where, $f_{sy}$ and $E_s$ are respectively the yield strength and elastic modulus of the steel tube; $\nu$ is the Poisson's ratio of the steel tube, take 0.3; $t$ is the thickness of the steel tube.

The buckling modes of the steel tube are mainly related to the width-thickness ratio parameters of steel tubes ($R'_i$) [13]. When $R'_i > 0.85$, local buckling will occur in the specimen, $R'_i \leq 0.85$, local buckling can be not considered. According to the literature [11], the reduction of the longitudinal strength of the rectangular steel tube can be achieved by the reduction coefficient of the longitudinal strength of the rectangular steel tube ($\phi_i$). $\phi_i$ is as follows:

$$\phi_i = \begin{cases} 0.89 & R'_i \leq 0.85 \\ \frac{1.2}{R'_i} - \frac{0.3}{R_i^2} & R'_i > 0.85 \end{cases}$$

(3)

When $R'_i > 0.85$, if $\phi > 0.89$, $\phi_i = 0.89$.

Then the axial bearing capacity of the steel tube is,

$$N_i = \sum_{j=1}^{4} \phi_i A_{si} f_{sy}$$

(4)

Where, $A_{si}$ is the steel tube area of each rectangular area.

3.3 Core Concrete Bearing Capacity

3.3.1. Lateral Effective Constraint Stress of Core Concrete

According to literature [11], the effective constraint coefficient of concrete in each area ($k_{ei}$) is the product of the effective constraint coefficient of the cross-section ($k_{ehi}$) and the effective constraint coefficient of the side ($k_{eli}$).

$$k_{ei} = k_{ehi} k_{eli} \geq 0 (i=1-4)$$

(5a)

$$k_{e1} = \frac{2}{5n'_i(b_i - 2t)} (b_i - 2t) \tan \theta - \frac{b_i - 2t}{5(a_i - t)} \times \frac{1}{3(b_i - 2t)}$$

(5b)

$$k_{e3} = \frac{2}{5n'_i(a_i - 2t)} (a_i - 2t) \tan \theta - \frac{b_i - 2t}{5(b_i - t)} \times \frac{1}{3(a_i - 2t)}$$

(5c)

$$k_{es} = 1$$

(5d)

Where, $n_{sij}$ is the number of segments separated by a binding bar on each edge $j$ of a section in a region $i$; $\theta$ is the parabolic angle of the unconstrained region. If there is no unconstrained region on the boundary, $\theta = 0$. $n'_i$ is the total number of parabolas on the long side of the rectangular section, which is related to the number of binding bar ($n_{sa}$). If there is no binding bar, $n'_i = 1$.

The parabolic angle of the unconstrained region ($\theta$) can be obtained from literature [11]:

$$\theta = \frac{\pi}{180} (A + B C) \leq 45 \left( \frac{\pi}{180} \right)$$

(6)

$$A = 68.58 - 16.13 \frac{a_i}{b_i} , B = -58.42 + 64.73 \xi , C = 0.0126 - 4.151 \zeta + 17.10 \zeta^2$$

Where, the constraint coefficient of the binding bar ($\zeta$), $\tilde{\zeta} = \sqrt{\zeta + 3}$, $\zeta_1 = A_b f_{sy} / (f_{ak} a_k b_k)$, $\zeta_2 = A_b f_{sy} / (f_{ak} a_k b_k)$. When there is no binding bar in either direction, $\tilde{\zeta} = 0$. $\tilde{\zeta}$ is the constraint coefficient of T-shaped steel tube, $\tilde{\zeta} = A_b f_{sy} / (A_k f_{ak})$. $A_b$ represents the calculated area of
a single binding bar. $f_{ck}$ is the axial compressive strength of concrete, and $b_i$ is the longitudinal spacing of binding bars.

In area 1 and area 2 (Fig.2), according to the formula $k_{e1}(f_{ck}A_{s1}) = f_{ck1} \times (b_i - 2t)b_i$, the lateral mean constrained stress in the direction of the short side can be obtained as follows: $f_{ck1} = k_{e1} \frac{2f_{ck}t}{b_i - 2t}$.

According to the formula $k_{e2}(n_{f_{ck}}A_{s2} + f_{ck}b_i) = f_{ck2} \times (a_i - t)b_i$, the lateral mean constrained stress in the direction of the long side can be obtained as follows: $f_{ck2} = k_{e2} \frac{n_{f_{ck}}A_{s2} + f_{ck}b_i(a_i - t)b_i}{(b_i - 2t)b_i}$.

In area 3 (Fig.2), it can be deduced by the same method of reasoning,

In area 4 (Fig.3), according to the balance of forces, we can get $f'_{ck1} = k_{e1} \frac{2f_{ck}t}{b_i - 2t}$, $f'_{ck2} = k_{e2} \frac{2f_{ck}t}{b_i - 2t}$.

Where, $n_1$ and $n_2$ are respectively the number of the binding bars in each region parallel to the direction of the short side; $A_{s1}$ is the area of the binding bars parallel to the direction of the short side in each region.

### 3.3.2. Axial Compressive Strength of Core Concrete

There are effective constraint area and weak constraint area for T-shaped concrete-filled steel tube short columns with binding bars. This paper will take different restraint in counting method for equivalent average, regardless of binding bars of core concrete constraints. And it will be uniform rectangular steel tube equivalent for circular steel tube side pressure calculation [14].

The effective lateral stress of the equivalent round steel tube in each area is:

$$
\sigma_{ei} = \frac{t(\frac{f_{ck1}}{R_i} + \frac{f_{ck2}}{R_i})}{R_i + r_i}
$$

Where, $R_i$ and $r_i$ are the outer diameter and inner diameter of the equivalent round steel tube in each area respectively, and $i$ is taken as 1-4.

In area 1 and area 2, according to the formula $a_i b_i = \pi R_i^2 = \pi R_i^2$, $(a_i - t)(b_i - 2t) = \pi r_i^2 = \pi r_i^2$, we can get $R_i = R_i = \sqrt{a_i b_i / \pi}$, $r_i = \sqrt{(a_i - t)(b_i - 2t) / \pi}$.

In area 3, $R_3$ and $r_3$ can be obtained as follows $R_3 = \sqrt{a_2 b_2 / \pi}$, $r_3 = \sqrt{(a_2 - 2t)(b_2 - 2t) / \pi}$.
In area 4, \( R_4 \) and \( r_4 \) can be obtained as follows:

\[
R_4 = \sqrt{a_2 b_1 / \pi}, \quad r_4 = \sqrt{a_2 (b_1 - t) / \pi}.
\]

The effective side stress of the equivalent round steel tube in each area can be obtained:

\[
\sigma_{11} = k_{e1} = \frac{\sqrt{\pi t} \left( \frac{2 f_{s1t} t + n_i f_{s1} A_{s1} + f_{s1} t b_s}{b_1 - 2t} \right)}{a_1 b_1 + \sqrt{(a_1 - t)(b_1 - 2t)}},
\]

\[
\sigma_{22} = k_{e2} = \frac{\sqrt{\pi t} \left( \frac{2 f_{s2t} t + n_i f_{s1} A_{s1} + f_{s2} t b_s}{b_1 - 2t} \right)}{a_1 b_1 + \sqrt{(a_1 - t)(b_1 - 2t)}},
\]

\[
\sigma_{33} = k_{e3} = \frac{\sqrt{\pi t} \left( \frac{2 f_{s3t} t + n_i f_{s1} A_{s1} + f_{s3} t b_s}{a_2 - 2t} \right)}{a_2 b_2 + \sqrt{(a_2 - 2t)(b_2 - t)}},
\]

\[
\sigma_{44} = \frac{\sqrt{\pi t} \left( k_{e4} \right)}{a_1 b_1 + \sqrt{a_1 (b_1 - t)}},
\]

According to literature [15], the compressive strength of core concrete area \( i \) meets the following requirements:

\[
\sigma_{ii} = f_c + k \gamma_{ui} \sigma_{ii}.
\]

Where, \( f_c \) is the axial compressive strength of concrete; \( k \) is the pressure measurement coefficient, \( k = (1 + \sin \phi)/(1 - \sin \phi) \), \( \phi \) is the concrete internal friction angle, the specific value can be determined by the test; \( \gamma_{ui} \) is the reduction coefficient of concrete strength, \( \gamma_{ui} = R_i^{-0.112} \) and \( R_i \) is the outer diameter of the equivalent round steel tube.

By substituting into the relevant formula of each area, the compressive strength of core concrete in each area can be obtained as follows:

\[
\sigma_{31} = f_c + k \gamma_{u1} \sigma_{31},
\]

\[
\sigma_{32} = f_c + k \gamma_{u2} \sigma_{32},
\]

\[
\sigma_{33} = f_c + k \gamma_{u3} \sigma_{33},
\]

\[
\sigma_{34} = f_c + k \gamma_{u4} \sigma_{34}.
\]
\[ R' \leq 0.85, \quad f_{sj} = 0.19f_{sy}, \quad f_{sj} = -0.89f_{sy} \]
\[ R' > 0.85, \quad f_{sj} = \left( \frac{1.2}{R' - 0.3} \right) f_{sy}\left| f_{sy} \right| \leq 0.89f_{sy}, \quad f_{sj} = \frac{f_{sy}^2}{4.169f_{sy}} \]

3.3.3. Bearing Capacity of Core Concrete

\[ N_c = \sum_{i=1}^{4} A_{\alpha} \sigma_{\gamma j} \]  

(11)

3.4. Formula for Calculating the Axial Bearing Capacity of T-shaped Concrete-filled Steel Tube Short Columns with Binding Bars

The axial bearing capacity of T-shaped concrete-filled steel tube short column with binding bars is the sum of the steel tube bearing capacity and the concrete bearing capacity of area "1" to area "4", namely:

\[ N = N_1 + N_2 \]  

(12)

4. Calculation of Bearing Capacity

The pressure measurement coefficient \( k \) is related to the lateral pressure of concrete and the strength of concrete itself. When the concrete strength is certain, the value \( k \) decreases with the increase of the lateral pressure. When the lateral pressure is certain, the value \( k \) increases with the increase of the strength of concrete. The specific value can be measured by experiments, and generally \( k = 1.5 \sim 7 \). When \( \frac{\tau_s}{\sigma_s} = 0.5, \quad b = 0 \), that is corresponding to the Tresca criterion, namely the lower yield criterion of metals. When \( \frac{\tau_s}{\sigma_s} = 0.577, \quad b = 0.364 \), corresponding to the Mises criterion for the linear approximation. Where \( \tau_s \) is shearing yield limit and \( \sigma_s \) is tensile yield strength limit of the material. Unable to get the \( \tau_s \) and \( \sigma_s \) and \( \phi \) by the real test, this paper let \( b = 0.364, \quad k = 7 \), use Eq. 12 to calculate the literature data [11], results are listed in Table 1.

The comparison results show that the calculation results in this paper are in good agreement with the experimental results. The average ratio of the experimental results to the theoretical calculation results is 0.984. The maximum error is not more than 9%, and the error is small. Based on the unified strength theory, the bearing capacity of each part is calculated separately, and the axial bearing capacity of the column is finally obtained by superposition. Moreover, when there is no binding bar, the error between the test bearing capacity and the calculated value of the formula in this paper is still within 12%. That shows that the formula in this paper is still applicable when there are no binding bars.

Table 1 Comparisons Between Calculating Results and Literature Data

| No. | \( a_1/a_2/b_1/b_2/t \) | \( a/b_1/d_1 \) | \( n_b \) | \( f_{as}/f_{sy}/f_{yt} \) | \( N_{ue} \) | \( N_c \) | \( N_{ue}/N_c \) |
|-----|----------------|-------------|--------|-----------------|-----------|------|-----------|
| 1   | 78/78/78/78/3.75 | -           | 1      | 34.84/374/-     | 1654      | 1707 | 1.03      |
| 2   | 78/78/78/78/3.75 | 50/50/6.75  | 1      | 34.84/374/493  | 1932      | 1911 | 0.99      |
| 3   | 78/78/78/5.73   | -           | 1      | 34.84/347/-     | 2225      | 2045 | 0.92      |
| 4   | 78/78/78/78/5.73| 50/50/6.75  | 1      | 34.84/347/493  | 2567      | 2553 | 0.99      |
| 5   | 78/78/78/3.75   | 50/50/6.75  | 1      | 34.84/374/493  | 1796      | 1710 | 0.95      |
| 6   | 78/78/78/78/3.75| 50/50/5     | 1      | 34.84/374/489  | 1781      | 1712 | 0.96      |
| 7   | 78/78/78/78/3.75| 50/50/8.5   | 1      | 34.84/374/372  | 1863      | 1710 | 0.92      |
| 8   | 78/78/78/78/7.8 | -           | 1      | 34.84/285/-     | 2294      | 2139 | 0.93      |
| 9   | 78/78/78/78/7.8 | 50/50/6.75  | 1      | 34.84/285/493  | 2513      | 2550 | 1.01      |
5. Conclusions

(1) The influence of intermediate principal stress is considered in this paper. \( k \) value reflects the internal friction angle of core concrete. When \( k \) value is not the same, bearing capacity values of different accuracy can be obtained.

(2) Based on the unified strength theory, the section of T-shaped concrete-filled steel tube short columns with binding bars is divided into four areas. The mechanical mechanism of each area is analyzed and the formula for calculating the axial bearing capacity of T-shaped concrete-filled steel tube short columns with binding bars is derived. The formula obtained is of high precision and simple expression, and has good engineering applicability.

Acknowledgements

This work was financially supported by Anhui Provincial scientific research projects in Universities of Department of Education (KJ2018A412, KJ2018A415). Science and Technology Project of Housing Urban-rural Development of Anhui Provincial Department of Housing and Construction (2014 YF-15). University-level quality engineering project of West Anhui University (wxxy2019023, wxxy2019038, wxxy2019056). University-level scientific research project of West Anhui University (WXZR201908).

References

[1] Chen X., Ye Q.Y., et al. (2000) Structural design of Guangzhou New China Mansion. Journal of Building Structures, 21(3): 2-9.
[2] Cai J., Sun G. (2008) Experimental investigation on L-shaped concrete-filled steel tube stub columns with binding bars under axial load. China Civil Engineering Journal, 41(9): 14-20.
[3] Chen Deming. Foundational study on mechanic behavior of abnomal-shaped CFT columns with binding bars. Guangzhou: South China University of Technology, 2000.
[4] Yang Y.L., Yang H., Zhang S.M. (2010) Compressive behavior of T-shaped concrete filled steel tubular columns. International Journal of Steel Structures, 10(4): 419-430.
[5] Du G.F., Xu L.H., Xu H.R. (2008) Test study on behavior of T-shaped concrete filled steel tubular short columns under axial compression. Journal of Huazhong University of Science and Technology: Urban Science Edition, 25 (3): 188-194.
[6] Xu L.H., Du G.F., Wen F., et al. (2009) Experimental study on normal section compression bearing capacity of composite t-shaped concrete-filled steel tubular columns. China Civil Engineering Journal, 42(6): 14-21.
[7] CAI J., LONG Y.L. (2009) Axial Bearing Capacity of Square and Rectangular CFT Stub Columns with Binding Bars. Journal of Building Structures, 30(1): 7-14.
[8] CAI J., HE Z.Q., CHEN X. (2007) Experimental Study on Behavior of Rectangle CFT Stub Column with Binding Bars Subjected to Axially Loading. Industrial Construction, 37(3): 75-80.
[9] Liu Z. (2018) Research on Compressive Behavior of Concrete-filled Round-ended Steel Tubular Stub Columns with Binding Bars. Wuhan University of Technology.
[10] Li Y.B. Research on Compressive Behavior of T-shaped Concrete-filled Steel Tubular Stub Columns with Binding Bars. South China University of Technology Guangzhou, China.
the degree of master.

[11] Zuo Z.L. (2010) Research on Compressive Behavior of Special-shaped Concrete filled Steel Tubular Stub Columns with Binding Bars. South China University of Technology Guangzhou, China. For the degree of doctor.

[12] YU M.H. (2002) Concrete Strength Theory and Application. HIGHER EDUCATION PRESS.

[13] GE H B, USAMI T. (1994) Strength analysis of concrete-filled thin-walled steel box columns. Journal of Constructional Steel Research, 30(3): 259-281.

[14] GUO H.X., ZHAO J.H., WEI X.Y. (2008) Analysis of Bearing Capacity of Concrete-filled Square Steel Tube Column Under Axial Load. Industrial Construction, 38(3): 9-11.

[15] ZHAO J.H., MA K.K., ZHANG D.F., et al. (2019) Unified solutions on bearing capacity of cross-shaped concrete filled steel tube stub column with binding bars. Journal of Guangxi University (Nat Sci Ed), 44(1): 1-11.