Self-Interacting Dark Matter with Naturally Light Mediator

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Abstract

A promising proposal for resolving the cusp-core anomaly in the density profile of dwarf galaxies is to allow dark matter to interact with itself through a light mediator of mass much less than a GeV. The theoretical challenge is to have a complete renormalizable theory where this happens naturally even though dark matter itself may be of the electroweak scale, i.e. 100 GeV to 1 TeV. I propose here such a model, with just two neutral complex scalar singlets under a softly broken dark global U(1) symmetry.


**Introduction:**

The nature of dark matter is an open question. If it interacts only weakly with visible matter, then there are certain astrophysical observations which are not consistent with numerical simulations based on this simple hypothesis. One such discrepancy, i.e. that the density profile of dark matter in dwarf galaxies is much flatter near the center (core) than predicted (cusp) [1], has prompted the idea [2, 3, 4, 5] that dark matter interacts with itself through a mediator, much lighter than the dark matter itself. Many phenomenological studies have been made, but the theoretical challenge is to understand why the mediator is light, and what other properties it may have, all within a complete renormalizable extension of the standard model (SM).

In this paper, I propose such a model. It assumes a global $U(1)_D$ symmetry which is softly and spontaneously broken to $(-1)^D$. It has just two neutral complex scalars: $\zeta$ which has $D = 1$, and $\eta$ which has $D = 2$. The $U(1)_D$ symmetry is broken spontaneously by the vacuum expectation value $\langle \eta^0 \rangle = u$. The dark particles are $\zeta_{R,I}$ which have odd $(-1)^D$ and they interact with $\eta_R$ which is heavy and $\eta_I$ which is naturally light, because it would be massless if $U(1)_D$ is not broken also by an explicit dimension-two soft term. Now $\eta_{R,I}$ are even under $(-1)^D$. Whereas $\eta_R$ mixes with the SM Higgs boson $h$ at tree level as usual, $\eta_I$ does so only in one loop. This radiative mixing is finite and calculable, a phenomenon discovered only recently [6]. It is very important because it allows the light $\eta_I$ to decay quickly to $e^-e^+$ even if its mass is only 35 MeV, thereby not disturbing the success of big bang nucleosynthesis in the SM.

**Model:**

Under the assumed $U(1)_D$, the new scalar singlets are

$$\zeta \sim 1, \quad \eta \sim 2,$$

and all SM particles are trivial. The scalar potential consisting of $\zeta$, $\eta$, and the SM Higgs
doublet $\Phi = (\phi^+, \phi^0)$ which becomes $(0, v + h/\sqrt{2})$ in the unitarity gauge, is given by

$$V = m_0^2 \Phi^\dagger \Phi + m_1^2 \zeta \zeta + m_2^2 \eta \bar{\eta} - \frac{1}{2} m_3^2 (\zeta \zeta + \bar{\zeta} \bar{\zeta}) - \frac{1}{2} m_4^2 (\eta \bar{\eta} + \bar{\eta} \eta) + \mu \bar{\eta} \zeta + \mu^* \eta \bar{\zeta}^2$$

(2)

Note that $V$ respects $U(1)_D$ in all its dimension-four and dimension-three terms, whereas the dimension-two $m_3^2 (m_4^2)$ terms break $U(1)_D$ to $Z_2(\mathbb{Z}_4)$. Without the $m_3^2, m_4^2$ terms, the spontaneous breaking of $V$ by $\langle \eta \rangle = u$ would imply that $\eta_f$ is a massless Goldstone boson.

Let $\zeta = (\zeta_R + i \zeta_I)/\sqrt{2}$ and $\eta = u + (\eta_R + i \eta_I)/\sqrt{2}$, then the minimum of $V$ is determined by

$$0 = m_0^2 + \lambda_0 v^2 + \lambda_{02} u^2,$$

(3)

$$0 = m_2^2 - m_4^2 + \lambda_2 u^2 + \lambda_{02} v^2.$$  

(4)

The mass of $\eta_f$ is then naturally small because it comes from a soft term which breaks $U(1)_D$ explicitly, i.e.

$$m_{\eta_f}^2 = 2 m_4^2.$$  

(5)

The mass-squared matrix spanning $(h, \eta_R)$ is

$$M_h^2 = \begin{pmatrix} 2 \lambda_0 v^2 & 2 \lambda_{02} u v \\ 2 \lambda_{02} u v & 2 \lambda_2 u^2 \end{pmatrix},$$

(6)

and that spanning $(\zeta_R, \zeta_I)$ is

$$M_\zeta^2 = \begin{pmatrix} m_1^2 + \lambda_{01} v^2 + \lambda_{12} u^2 + 2 \mu_R u - m_3^2 & -2 \mu_R u \\ -2 \mu_R u & m_1^2 + \lambda_{01} v^2 + \lambda_{12} u^2 - 2 \mu_R u + m_3^2 \end{pmatrix},$$

$$= \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} m_{\chi_1}^2 & 0 \\ 0 & m_{\chi_2}^2 \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix},$$

(7)

where $\mu = \mu_R + i \mu_I$, and

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \zeta_R \\ \zeta_I \end{pmatrix}.$$  

(8)
Hence $\zeta_R^2 + \zeta_I^2 = \chi_1^2 + \chi_2^2$, whereas $\zeta_R^2 - \zeta_I^2 = \cos 2\theta(\chi_1^2 - \chi_2^2) - 2\sin 2\theta\chi_1\chi_2$, and $2\zeta_R\zeta_I = \sin 2\theta(\chi_1^2 - \chi_2^2) + 2\cos 2\theta\chi_1\chi_2$.

**Dark matter self-interactions**:

The relevant trilinear couplings involving the physical $\chi_{1,2}$ dark-matter mass eigenstates are

\[
\mathcal{L}_3 = \sqrt{2}\lambda_{01}v h(\chi_1^2 + \chi_2^2) + \sqrt{2}\lambda_{12}u\eta_R(\chi_1^2 + \chi_2^2) + \eta_R\sqrt{2}[\mu_R\cos 2\theta(\chi_1^2 - \chi_2^2) - 2\mu_R\sin 2\theta\chi_1\chi_2 - \mu_I\sin 2\theta(\chi_1^2 - \chi_2^2) - 2\mu_I\cos 2\theta\chi_1\chi_2] + \eta_I\sqrt{2}[\mu_I\cos 2\theta(\chi_1^2 - \chi_2^2) - 2\mu_I\sin 2\theta\chi_1\chi_2 + \mu_R\sin 2\theta(\chi_1^2 - \chi_2^2) + 2\mu_R\cos 2\theta\chi_1\chi_2].
\]

Let $m_{\chi_1} < m_{\chi_2}$, then $\chi_1$ is dark matter and interacts with itself through $h, \eta_R$, and $\eta_I$. In particular, $\eta_I$ may be naturally light ($m_4 << v, u$), say 35 MeV, and be an excellent candidate for solving the cusp-core problem [4]. As shown in Fig. 1, the elastic scattering cross section of $\chi_1$ by exchanging $\eta_I$ is proportional to $m_{\eta_I}^{-4}$, whereas the annihilation cross section (as shown in Fig. 2) of $\chi_1\chi_1$ to $\eta_I\eta_I$ (and $hh, \eta_R\eta_R$ if kinematically allowed) is proportional to

\[
\begin{align*}
\chi_1 &\rightarrow \chi_1 \\
\chi_1 &\rightarrow \eta_I \\
\eta_I &\rightarrow \chi_1 \\
\chi_1 &\rightarrow \eta_I \\
\end{align*}
\]

Figure 1: Dark matter $\chi_1$ scattering by exchanging $\eta_I$.

\[
\begin{align*}
\chi_1 &\rightarrow \eta_I \\
\eta_I &\rightarrow \chi_1 \\
\end{align*}
\]

Figure 2: Dark matter $\chi_1\chi_1$ annihilation to $\eta_I\eta_I$. 

shown in Fig. 2) of $\chi_1\chi_1$ to $\eta_I\eta_I$ (and $hh, \eta_R\eta_R$ if kinematically allowed) is proportional to
$m_{\chi_1}^{-4}$. Actually there is also the quartic coupling $\lambda_{12}$ which has been assumed negligible here for simplicity. To solve the cusp-core discrepancy, the condition is then roughly

$$
\left( \frac{m_{\chi_1}}{m_{\eta_I}} \right)^4 \sim 10^{12} \left( \frac{m_{\chi_1}}{\text{GeV}} \right). 
$$

This may be satisfied with $m_{\chi_1} \sim 100$ GeV, and $m_{\eta_I} \sim 35$ MeV for example.

**Linkage to the standard model**:

There are three linkages between the new particles and the standard model, all coming from the SM Higgs boson $h$.

- The dark scalars $\chi_{1,2}$ are odd under $(-1)^D$. They cannot mix with $h$, but they do interact through their trilinear couplings $\sqrt{2}\lambda_{01} v h (\chi_1^2 + \chi_2^2)$ as shown in Eq. (9). This means that $\chi_1\chi_1$ annihilation through $h$ to SM particles is possible as shown in Fig. 3 for relic abundance, together with $\chi_1$ elastic scattering off nuclei as shown in Fig. 4 for its direct detection in underground experiments. From the severe LUX limit [7] on $\lambda_{01} < 0.01$ is implied [8] [9] [10] which in turn gives much too small an
annihilation cross section from Fig. 3 for obtaining the correct relic abundance unless there is a resonance effect, i.e. \( m_{\chi_1} \) just below \( m_h/2 \). However, there is also Fig. 2 in this model, which involves a different coupling, i.e. \( (\mu_I \cos 2\theta + \mu_R \sin 2\theta)/\sqrt{2} \), thus evading this stringent constraint without any difficulty.

- The heavy particle \( \eta_R \) mixes with \( h \) as shown in Eq. (6). It will decay to SM particles through \( h \).

- The light particle \( \eta_I \) does not mix with \( h \) at tree level, but does so in one loop as shown in Fig. 5. Note that if \( \zeta \) is replaced by a Majorana fermion, then this mixing is forbidden, because \( \eta_I \) would be odd under the \( \gamma_5 \to -\gamma_5 \) tranformation, whereas \( h \) and \( \eta_R \) are even. Since most models assume that dark matter is a fermion, this mechanism is not applicable in those cases. For a stable light \( \eta_I \), it could only annihilate to \( e^-e^+ \) through \( h \), but then its cross section would be so small that it would overclose the Universe. The phenomenon of radiative Higgs mixing has only been discovered recently, and the effective quadratic \( \eta_I h \) term is easily calculated to be

\[
m^{2}_{\eta_I h} = 2i\lambda_{01} v(\mu_I \cos 2\theta + \mu_R \sin 2\theta) \int \frac{d^4k}{(2\pi)^4} \left[ \frac{1}{(k^2 - m^2_{\chi_1})^2} - \frac{1}{(k^2 - m^2_{\chi_2})^2} \right]
\]

Figure 5: One-loop finite mixing of \( \eta_I \) with \( h \).
Note that $\mu_I \neq 0$ is crucial in obtaining this finite result. If $\mu_I = 0$, then $\theta = 0$ also [see Eq. (7)]. This would be the case if the dimension-two $m_3^2$ term were absent, because then $\mu$ may always be redefined as real. The residual symmetry with $m_3^2 = 0$ would then become $Z_2 \times Z_2$, with $\eta_R \sim (+, +)$, $\eta_I \sim (+, -)$, $\zeta_R \sim (-, +)$, $\zeta_I \sim (-, -)$. There would then be at least two stable dark-matter particles, say $\zeta_I$ and $\eta_I$. It is an interesting model in its own right, but not the subject of this paper. With only the $Z_2$ residual symmetry considered here, $\eta_I$ is not stable. In order not to disturb the success of big bang nucleosynthesis, its lifetime should be less than about 1 s [14]. For $m_{\eta_I} = 35$ MeV so that it decays mainly to $e^-e^+$, and $m_h = 125$ GeV, this translates to

$$|\lambda_{01}(\mu_I \cos 2\theta + \mu_R \sin 2\theta) \ln(m_{\chi_1}^2/m_{\chi_2}^2)| > 0.05 \text{ GeV.}$$

(12)

Since $\lambda_{01} < 0.01$ from direct detection, this requires $\mu_{R,I}$ to be greater than about 5 GeV. For comparison, the annihilation cross section of about 1 pb (suitable for the correct relic abundance) is obtained for a value of about 20 GeV. Note that $\eta_I$ also mixes radiatively with $\eta_R$, with mixing proportional to $(\mu_I \cos 2\theta + \mu_R \sin 2\theta)(\mu_R \cos 2\theta - \mu_I \sin 2\theta)$. Note also that unlike most other proposals of a light mediator [15, 16, 17], $\eta_I$ does not contribute to the direct detection of dark matter, i.e. $\chi_1$, which is dominated here by $h$ exchange as shown in Fig. 4.

Some numerical examples:

Let $\mu_{eff} = \mu_I \cos 2\theta + \mu_R \sin 2\theta$, then the cross section for $\chi_1\chi_1 \to \eta_I\eta_I$ (see Fig. 2) $\times$ their relative velocity is given by

$$\sigma \times v_{rel} = \frac{\mu_{eff}^2}{16\pi m_{\chi_1}^6}. \quad (13)$$

Setting this equal to $3 \times 10^{-26}$ cm$^3$/s for the correct dark-matter relic abundance, a value of
\( \mu_{\text{eff}} = 19 \text{ GeV} \) is obtained for \( m_{\chi_1} = 100 \text{ GeV} \).

For the elastic self-scattering of \( \chi_1 \) through \( \eta_I \) exchange (see Fig. 1), the cross section is given by

\[
\sigma = \frac{\mu_{\text{eff}}^4}{4 \pi m_{\eta_I}^4 m_{\chi_1}^2}.
\]

(14)

For the benchmark value of \( \sigma/m_{\chi_1} = 1 \text{ cm}^2/\text{g} \) in self-interacting dark matter, a value of \( m_{\eta_I} = 39 \text{ MeV} \) is obtained, using \( m_{\chi_1} = 100 \text{ GeV} \) and \( \mu_{\text{eff}} = 19 \text{ GeV} \) as before.

For the decay of the light scalar mediator \( \eta_I \) to \( e^-e^+ \), its rate is given by

\[
\Gamma = \frac{m_{\eta_I} m_e^2}{16\pi} \left[ \frac{\lambda_{01} \mu_{\text{eff}}}{8 \pi^2 m_h^2} \ln \frac{m_{\chi_1}^2}{m_{\chi_2}^2} \right]^2.
\]

(15)

Using \( m_{\eta_I} = 39 \text{ MeV} \), \( m_e = 0.511 \text{ MeV} \), \( m_h = 125 \text{ GeV} \), \( \mu_{\text{eff}} = 19 \text{ GeV} \), \( m_{\chi_1} = 100 \text{ GeV} \), \( m_{\chi_2} = 200 \text{ GeV} \), and \( \lambda_{01} = 0.01 \), the decay lifetime \( \Gamma^{-1} = 0.07 \text{ s} \) is obtained. This is short enough so that big bang nucleosynthesis may proceed without being disturbed.

**Production of the light pseudoscalar mediator**:

The decay rate of the SM Higgs boson \( h \) to a pair of \( \eta_I \) is given by

\[
\Gamma(h \to \eta_I \eta_I) = \frac{\lambda_{02}^2 v^2}{4 \pi m_h}.
\]

(16)

Compared to the decay rate of \( h \to \tau^-\tau^+ \), i.e.

\[
\Gamma(h \to \tau^-\tau^+) = \frac{m_h m_{\tau}^2}{16\pi v^2},
\]

(17)

the two are equal if \( \lambda_{02} = 3.7 \times 10^{-3} \). Hence the decay \( h \to \eta_I \eta_I \) may occur readily. However, the lifetime of \( \eta_I \), i.e. 0.07 s, is far too long for its decay product \( e^-e^+ \) to be observed within the Large Hadron Collider. As for production by annihilation of dark matter at present, Sommerfeld enhancement may be possible [18, 19], in which case \( e^-e^+ \) production from \( \eta_I \) decay may be observed. On the other hand, this does not affect the fluctuations of the cosmic microwave background [20] because \( \chi_1 \) is assumed to be significantly heavier than 10 GeV.
Discussion and synopsis:

Because the only connection between the dark sector and the standard model is through the one Higgs boson $h$, this model belongs to a general class considered in Ref. [21]. However it is the first model which explains why a light mediator should occur, and why its mixing with $h$ is suppressed, both in terms of a symmetry and the details of how it is broken. Note that in models of an $U(1)_D$ gauge boson with arbitrary kinetic mixing to the SM $U(1)_Y$, there is no fundamental understanding of why this mixing is so small.

To summarize, two complex scalars are introduced beyond the standard model. They are singlets of the SM, but transform under a dark $U(1)_D$ symmetry, with $\zeta \sim 1$ and $\eta \sim 2$. The complete renormalizable Lagrangian containing them and the SM Higgs doublet is given in Eq. (2). The $U(1)_D$ symmetry is respected by all dimension-four and dimension-three terms, but are explicitly broken to $Z_2$ and $Z_4$ respectively by the dimension-two $\zeta^2 + \bar{\zeta}^2$ and $\eta^2 + \bar{\eta}^2$ terms. In addition $\eta_R$ acquires a nonzero vacuum expectation value, so that the residual dark symmetry becomes $Z_2$. Under this $Z_2$, the two mass eigenstates formed out of $\zeta_{R,I}$ are odd, the lighter one becoming dark matter, whereas $\eta_{R,I}$ are even, with $\eta_I$ much lighter naturally, corresponding to a would-be massless Goldstone boson from the spontaneous breaking of $U(1)_D$. Hence $\eta_I$ acts as a naturally light mediator for the self-interacting dark matter. Furthermore, it mixes radiatively with $h$ (a phenomenon discovered only recently) and decays fast enough to avoid disturbing big bang nucleosynthesis. The correct relic abundance is obtained without conflicting with direct-search limits. This is thus a minimal model of self-interacting dark matter with all the desirable theoretical and phenomenological properties.

$U(1)_D$ as lepton number:

The global $U(1)_D$ considered in the above may be taken to be lepton number, under which neutrinos and charged leptons have $D = 1$. To connect $\zeta$ and $\eta$ to the SM leptons, the singlet
right-handed neutrinos $N_R$ are added, with the allowed Yukawa interaction $\bar{\eta}N_RN_R$. With the spontaneous and soft breaking of $U(1)_D$, $N_R$ acquires a large Majorana mass from $\langle \eta \rangle = u$ and neutrinos obtain small seesaw Majorana masses. The residual symmetry is $(-1)^D$, i.e. lepton parity, from which dark parity, i.e. $(-1)^D(-1)^2j$, may be derived [22]. Hence $\zeta$ has odd dark parity, and all other particles have even dark parity. The light mediator $\eta_I$ now decays also to two neutrinos through the $\bar{N}_R\nu_L\phi^0$ term. This scenario is not as minimal, but it links the existence of neutrino mass to the dark sector, and offers a possible answer to the question: where does $U(1)_D$ come from?

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