Effect of vertical velocity profile approximations on estimates of dam breach discharge using surface velocities

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Abstract
In order to investigate the relationship between water surface velocity and breach hydrograph, a series of dam breach experiments with a generalised landslide dam were conducted in an open channel of 50 m × 4 m × 2 m. The large-scale particle image velocimetry (LSPIV) technique was applied to measure the time history of water surface velocity during the dam breach process, and the hydrography was obtained by integrating the surface velocity along the water depth. The influence of different vertical velocity profile approximation and suspended sediment concentration on the peak breach discharge were analysed and discussed. The results showed that the water depth over the breach crest can be described as a function of water surface velocity using

\[ \frac{d(t)}{g} = \frac{k_1^2 u_{surf}^2}{k_1 g}. \]

A simple formula based on surface velocity and breach width for the estimation of breach discharge was further proposed and verified.

KEYWORDS
breach crest, breach hydrograph, dam breach, LSPIV, surface velocity

1 | INTRODUCTION

The 2008 Wenchuan Earthquake (8.0 Mₛ) and 2013 Lushan Earthquake (7.0 Mₛ) induced a series of landslide dams in the mountainous watersheds of China. The failure of these landslide dams in mountainous rivers can cause serious damages to the life and property in the downstream. In order to evaluate the potential risks of these landslide dams, detailed information on the dam breach hydrograph was critical. However, studies by different researchers (e.g., Al-Riffai, 2014; O’Connor & Costa, 2004; Rifai et al., 2017) found that the existing prediction methods for dam breach hydrographs can produce a wide range of results, and an uncertainty (error) of 50% was expected in the results of these prediction models.

In addition to the analytical or empirical prediction methods, previous researchers conducted a large number of laboratory experiments to study the dam breach processes, which focused mainly on the breach formation and the water level variations. For example, Coleman, Andrews, and Webby (2002) studied the overtopping breach geometries, topographies, and flows with homogeneous small-scale embankments in a 2.4 m wide by 12 m long flume. Based on the experimental data, Coleman et al. (2002) proposed equations for the breach geometry...
and correlated the breach hydrograph with the breach geometry using the broad-crested weir flow assumption. However, the water level in the upstream was kept constant for all of the tests that were reported in Coleman et al. (2002), and the effects of the reservoir volume were ignored. Hunt, Hanson, Cook, and Kadavy (2005) found that the rates of the breach channel growth were strongly influenced by the compaction water content based on the earthen embankment tests. Gregoretti, Maltauro, and Lanzoni (2010) carried out a series of dam breach experiments with three different sediment materials in a sloping flume (the slope ranging from 0 to 10%). Based on the experimental results, Gregoretti et al. (2010) examined the critical conditions for the failure of landslide dams. Morris (2011) used field data, including video footage from the EU IMPACT project, and large-scale test data to improve approaches for predicting breach initiation and growth and hydrograph. Kamalzare et al. (2012) studied the erosion of dam embankments using a geotechnical centrifuge facility. Farina et al. (2017) described two methods for estimating the discharge in an open channel, which represent valid alternatives to the Velocity–Area method. Wu, Yu, Wei, Liang, and Zeng (2018) conducted a series of experiments to reveal the overtopping process of non-cohesive and cohesive levees in a U-bend flume in which the levee breaching process was divided into four stages.

Although many experimental studies are available in the literature on dam breach formation, little has been reported on the dam breach discharge using surface velocity during the landslide dam failure process. One possible reason was that direct experimental measurements of water surface velocity within breach channels are very difficult due to the rapidly changing bathymetry during the dam failure process. For example, Orendorff (2009) attempted to use ADV to measure the velocity in a breach channel, but only a limited data set was obtained. Nevertheless, there are a limited number of experimental studies that indirectly measured the velocity fields of dam break flows. These indirect measurement techniques included particle tracking velocimetry (PTV) and particle image velocimetry (PIV). PTV involved determining the velocity of tracer particles by tracking individual particles’ trajectories, whereas particle image velocimetry (PIV) utilised the cross-correlation between subsequent images in order to determine average particle displacement in individual areas and thus local velocities (Orendorff, Rennie, & Nistor, 2011). For example, Tauro, Piscopia, and Grimaldi (2017) compared the PTV and PIV-type methods on the estimation of the surface velocity of stream flows.

LSPIV is an extension of PIV, which aims at provide velocity fields spanning large flow area in laboratory or field conditions. While the image process and data processing algorithms of LSPIV are similar to conventional PIV, adjustments are required for illumination, seeding procedures, and pre-processing of the recorded images. By using LSPIV-type techniques, Eaket et al. (2005) measured the velocity of unsteady open channel flows using video stereoscopy. Aleixo et al. (2011) obtained velocity profiles in two-dimensional dam-break flows using a particle tracking velocimetry method. Lewis and Rhoads (2018) used LSPIV to map surficial patterns of mean flow and turbulent structures at two small stream confluences. Li, Liao, and Ran (2019) applied the LSPIV for stream flow and flood measurements, and the three-dimensional topography and water surface distribution were reconstructed. Naves, Anta, Jernimo, Regueiro-Picallo, and Surez (2019) applied LSPIV method in the measurement of runoff velocities in urban drainage experiment. However, these studies only considered steady-state flows or instantaneous dam failure scenarios where dam-break flow was generated by suddenly lifting a water gate with fixed channel configuration and bathymetry, while the dynamic discharge through an ever-changing breach channel in case of earthen dam failure was rarely discussed or studied. The accurate estimation of dam breach hydrograph is more challenging, especially considering the rapid change nature of the flow and the bathymetry through the breach channel in the laboratory environment.

The primary goal of this article is to address the issue of how to estimate the dynamic discharge through a rapidly evolving dam breach channel based on the measurement of transient surface velocity, as the latter is relatively easy to obtain in case of a dam breach event. Using LSPIV to measure the surface velocities has certain advantages. For example, LSPIV is a non-intrusive technique that can instantaneously measures the velocities of the water surface, capable of measuring fast flow through the breach section, as well as the surface velocity at low water depth during the dam breach process. Liu et al. (2019) carried out two dam breach experimental trials, and LSPIV was used to measure the surface flow velocities of dam breaches. The accuracy and reliability of the LSPIV technique for breach hydrograph was validated. However, it was still unclear to what extent the choices of the vertical velocity approximations and sediment concentrations affect the estimated dam breach hydrographs.

In the present study, laboratory experiments were conducted to investigate the relationship between water surface velocity and breach discharge. The LSPIV method was used to obtain the surface velocities. The dam breach discharges that were integrated from the surface velocities under different vertical velocity approximations were compared with each other. The effect of the sediment concentration on the peak breach discharge was
evaluated. A simple formula based on surface velocity and breach width for rapid estimation of breach discharge was further discussed and verified.

2 | LANDSLIDE DAM BREACH EXPERIMENT SETUPS

The generalised model experiments of landslide dam failures due to overtopping were conducted in the State Key Laboratory of Hydraulics and Mountain River Engineering in Sichuan University of China. The flume that was employed in the experiments had a length of 50 m, a width of 4 m, and a height of 2 m. A generalised landslide dam which was 50 cm high, 50 cm wide at the top, and 150 cm wide at the bottom was installed in the centre of the flume. The experimental setups and material particle size distribution were illustrated in Figure 1.

The idealised landslide dam was constructed in a layer-by-layer manner. In the first step, the material (sand) was filled carefully into the flume at the dam foundation position in dry and loose form with the initial

F I G U R E  1  Diagrams of the experimental setups for the dam breach experiments
depth 10 cm, using rakes and shovels. In the second step, a vibratory compactor was used to compact the layer for three times, and then sprinkled with water. These two steps were repeated until the total dam height exceeded the designed height about 5 cm. In the third step, a wooden board was used to smooth the dam crest and slope, and let it stands for 48 hr. Dams built in this manner had a moderate degree of variability in water content and dry bulk density.

Two electrical pressure transducers (EPTs) were used to measure the time history of the surface elevation (water depth) at the front toe of the dam. Based on the concept of mass conservation of the total water volume in the reservoir, the breach hydrograph can be calculated using the temporal variation of water depth. The relationship between the discharge and water volume of the upstream reservoir was as follows:

\[
Q_{in} - Q_{out} = \frac{dV}{dt} \tag{1}
\]

\[
V = Bh^2 + LBh \tag{2}
\]

\[
\frac{dh}{dt} = \frac{\Delta h}{\Delta t} = \frac{h(n+1) - h(n)}{t(n+1) - t(n)} \tag{3}
\]

where, \( Q_{in} (= 0) \) is the inlet discharge, which was kept zero in the present study; \( Q_{out} \) is the breach discharge; \( V \) is the water volume of the upstream reservoir; \( L \) is the length of the flume; \( B \) is the width of the flume, and \( h \) is the upstream water depth; The subscripts \( n \) and \( n+1 \) denote the variable values at two consecutive measurements.

Combining Equations (1)–(3), the outflow discharge can be expressed in terms of water depth and reservoir geometry as Equation (4).

\[
Q_{out} = (2Bh + LB) \frac{h(n+1) - h(n)}{t(n+1) - t(n)} \tag{4}
\]

The dam breach hydrograph estimated using Equation (4) was used as the benchmark data in the following sections to evaluate the performance of different vertical velocity approximations in the estimation of dam breach hydrograph.

In addition to time history of water depth, the velocity field at the upstream toe of the landslide dam were also measured using the LSPIV technique. Polypropylene balls with a diameter of 5 mm were used as tracers for visualisation of the dam breach process. Two digital video recorders (DV) and an industry camera (IC) were employed to capture the image pairs of the dam breach process, as shown in Figure 1. Based on the snapshots captured, the LSPIV method was applied to estimate the surface velocity filed at the upstream dam toe. Once the surface velocity distribution was obtained, the dam breach hydrograph can be obtained by integrating the surface velocities along the water depth under a certain velocity profile approximation, such as the classic logarithmic distribution. The dam breach hydrograph computed using different vertical velocity profile approximations were compared with the benchmark data that was estimated using the water depth.

An initial breach was created in the centre of the dam crest so that the dam breach processes in the generalised model experiments were symmetric. When the experiment started, the water was pumped into the reservoir and the depth of the water increased. When the water elevation exceeded the elevation of the dam crest, the dam breach process began and the time was recorded as \( t = 0 \). From this moment \((t = 0)\), all instruments were triggered, and the water level variations and snapshots of the dam breach process were recorded.

3 | EXPERIMENT RESULTS

3.1 | The landslide dam breach process

Figure 2 shows the snapshots of the dam breach processes. The dam breach phenomena were similar to those reported by the previous researchers, such as Coleman et al. (2002), Morris (2011), Cao, Yue, and Pender (2011a), and Cao, Yue, and Pender (2011b). Firstly, a narrow channel was formed on the dam surface and deepened to the bottom (Figure 2a). Secondly, this narrow channel expanded in the transverse direction and the dam breach discharge increased accordingly (Figure 2b). Thirdly, the width of the breach channel continued expanding and the dam breach discharge gradually decreased (Figure 2c,d). Finally, a residual dam was formed in the flume and the discharge reached zero.

3.2 | Measurement of surface velocity using LSPIV

Following the classification of Fujita, Muste, and Kruger (1998), the application of LSPIV technique involves four procedures: flow visualisation, illumination, image recording, and image processing. The LSPIV image processing algorithm is similar to conventional PIV. In this study, a business software Insight 3G (https://insight-3g.software.informer.com/) was used for image processing to obtain the surface velocity during dam...
3.3 The history of water depth and breach discharge during dam breach process

In addition to the snapshots of the dam breach process, the time history of the water depth as recorded by EPT1 and EPT2, and the dam breach discharge that was estimated using the concept of mass conservation were presented in Figure 4. The curves labelled Trail 1 and Trail 2 correspond to data measured by EPT1 and EPT2, respectively.

The snapshot pairs that were captured by the industry camera (IC) were employed in order to estimate the surface velocity at the dam toe (“Integral section,” as shown in Figure 1b) using the LSPIV method. The time interval between the two images of a snapshot pair was 0.02 s. As the industry camera (IC) that was selected for this procedure was located perpendicular to the horizontal water surface, no distortions of images were observed in the snapshots that were captured. Therefore, orthorectification of the images was unnecessary, and the image pairs were imported directly into the INSIGHT 3G program for the velocity calculation. It was mentioned in Section 2 that polypropylene balls were used as tracers in the experiments. Using the LSPIV method that was described above, the time history of the surface velocity (averaged transversely) at the dam toe was obtained and presented in Figure 5. It suggested that the surface velocity was strongly dependent on the evolution of the breach channel.

4 DATA ANALYSIS

4.1 Effect of velocity profile approximation on dam breach discharge

Once the time history of the surface velocity at the cross-section over the front dam toe was obtained, it was straightforward to estimate the dam breach discharge by integrating the velocity along the water depth under a certain profile approximation. There were a few vertical velocity profile approximation methods for free surface flows that were available in the literature, and choosing a different velocity profile may result in a different breach hydrograph.

In this section, three vertical velocity profiles were selected for measuring the discharge, namely, the classic logarithmic velocity profile, the velocity profile for the clear water flow by Zhang (1995), and the third-order Boussinesq-type velocity profile by Castro-Orgaz and Hager (2013). The calculated discharge for each velocity profile was compared with the one that was estimated using the measured reservoir water depth (Figure 2), and the influence of the different velocity profile approximations on the dam breach discharge was examined.

It should be noted that the three velocity profile approximations that are chosen are valid for clear water flows only, and the effect of the suspended sediment concentration of sediment-laden and hyperconcentrated
flows will be further discussed in the following section. Moreover, the cross-section over the dam toe was chosen for measuring the discharge instead of those in the breach channel because the underwater width and depth of the breach channel changes too rapidly in order to accurately measure the discharge during the dam breach process.

The first velocity profile that was selected was the universal logarithmic law of the wall with a smooth surface, which was applicable to the fully turbulent region outside the viscous sublayer.

\[
\frac{u}{u_*} = \frac{1}{\kappa_0} \ln \frac{u_z \nu}{\nu} + C_0
\]

where, \( u \) is the resulting velocity parallel to the wall (flume bottom), \( u_* \) is the friction velocity, \( \kappa_0 = 0.41 \) is the von Karman constant for clean water, \( z \) is the normal distance to the wall boundary, \( \nu \) is the kinetic viscosity of water, and \( C_0 \) is an empirical constant that is related to the thickness of the viscous sub-layer. The suggested value of \( C_0 \) in a boundary layer over a smooth surface was 5.0, while a smaller value of \( C_0 \) is desirable for a rough wall. The friction velocity \( u_* \) is calculated by substituting the measured velocity at the free surface into Equation (6), where \( z = h \) and \( u = u_{\text{surface}} \). The discharge per unit length \( q \) is then obtained by measuring \( u \) along the water depth and, in turn, the discharge \( Q \) in the flume.

The second velocity profile that was chosen was the one that was proposed by Zhang (1995), which relates the cross-sectional average velocity \( \bar{u} \) to the surface velocity \( u_{\text{surface}} \) as follows:

\[
\bar{u} = u_{\text{surf}} \left( 1 + 2.62 \frac{\sqrt{g}}{C} \right)^{-1}
\]

where, \( C = \frac{1}{n} R^\frac{n}{2} \) is the Chezy coefficient, \( R \) is the hydraulic radius of the flume flow, and \( n \) is the Manning coefficient (or roughness coefficient). In this study, \( n = 0.011 \) is used for the flume bed with a smooth concrete surface, and the discharge \( Q \) is calculated as the product of the cross-sectional area and the average velocity \( \bar{u} \).

The third velocity profile that was used was the third-order Boussinesq-type velocity profile by Castro-Orgaz and Hager (2013), which is as follows:
where, $\beta_1$ to $\beta_4$ are the analytical coefficients, $u$ is the horizontal velocity at a specific location, $\eta$ is the vertical distance (from the location of interest) to the flume bottom, and $h$ is the depth of the water. By substituting $u = u_{surf}$ and $\eta = h$ into Equation (7), the discharge per unit length $q$ can be obtained.

The dam breach discharges that were calculated using the three velocity profile approximations presented above were plotted in Figure 6, and compared against the one that was estimated using water depth. It can be seen in Figure 6 that in the first few seconds of dam breach process, the dam discharge was almost negligible because the initial breach channel on the dam crest was small, and only a small amount of water runs off the rear slope of the dam. For the discharges that were calculated using the velocity profiles, the discharge starts to increase with a large temporal gradient from around $T = 150$ s, reaches a peak value at $T = 230$ s, and then gradually drops to zero. Figure 6 also indicated that the peak discharge that was computed using Castro-Orgaz and Hager (2013) velocity profile was the largest at $Q_{max} = 0.29$ m$^3$/s, the peak discharges using the logarithmic and Zhang (1995)'s velocity profiles were quite close at $Q_{max} = 0.26$ m$^3$/s, and the peak discharge that was estimated using the water depth was the smallest at $Q_{max} = 0.22$ m$^3$/s.

Figure 6 suggested that the discharges that were computed using the four methods agree reasonably well with each other, and the velocity profile approximations have limited effects on the dam discharge calculation for clear water conditions. This was because the three velocity profiles that were chosen (Equations 5–7) all accurately reflected the velocity distribution in clear-water, open channel flows, despite their respective forms.

### Figure 6

**Comparison of the dam breach discharges that were estimated using the different vertical velocity profile approximations.** The three velocity profiles considered are the classic logarithmic velocity profile (Velocity Profile I), the velocity profile for the clear water flow by Zhang (1995) (Velocity Profile II), and the third-order Boussinesq-type velocity profile by Castro-Orgaz and Hager (2013) (Velocity Profile III).

### 4.2 Influence of the suspended sediment concentration on the peak dam breach discharge

In the dam breach process, the flow in the breach channel carries a certain amount of suspended sediment, and can be classified as a sediment-laden or even hyper-concentrated flow. Previous studies indicated that the vertical velocity profiles in sediment-laden and hyper-concentrated flows differed from those in clear-water flows, and many theoretical and experimental equations were proposed (Einstein & Chien, 1995; Guo & Julien, 2001; Muste & Patel, 1997; Zhang, 1995). For example, Zhang (1995) proposed a velocity distribution formula based on the turbulence eddy model, and the predicted velocity profile agreed reasonably well with the measured data. Zhang (1995) also demonstrated that the formula was applicable for both ordinary sediment-laden and hyper-concentrated sediment flows. In this section, the velocity distribution formula for sediment-laden and hyperconcentrated flows (Equation (8)) that was proposed by Zhang (1995) was selected to calculate the discharges under the representative flow conditions with varying sediment concentrations, and their effects on the discharge were examined.

$$\frac{u_{max} - u}{u_s} = \frac{\pi}{2c_n} - \frac{1}{c_n} \left[ \sqrt{\left(1 - \frac{y}{h}\right)^2 + \frac{1}{h} \left(1 - \frac{y}{h}\right)^2} \arcsin \frac{y}{h} \right]$$

where, $u_{max}$ is the maximum velocity in the vertical direction, $y$ is the water depth, and $c_n$ is the eddy coefficient. For a flume flow, the velocity at the free surface can be regarded as the maximum velocity such that $u_{max} \approx u_{surf}$.

The value of the eddy coefficient $c_n$ in Equation (8) is strongly dependent on the volume concentration of the suspended sediment in the flow, and Zhang (1995) formulated the following equation based on measured data in natural rivers:

$$c_n = 0.15 \left[ 1 - 4.2\sqrt{S_v(0.365 - S_v)} \right]$$

where, $S_v$ is the volume concentration of the suspended sediment.

Since the surface velocity $u_{surf}$ has been obtained using the LSPIV in the previous section, the average vertical
velocity $\bar{u}$ can be estimated by vertically integrating Equation (8) from the flume bottom to the free surface:

$$\bar{u} = \frac{1}{h} \int_0^h u dy = u_{surf} - \frac{\pi}{8c_n} u'$$ (10)

The next step was to cancel out $u'$ in Equation (10). For flume flows, the friction velocity $u'$ can be estimated using the average velocity $\bar{u}$:

$$u' = \bar{u} \frac{\sqrt{g}}{C}$$ (11)

where, $C$ is the Chezy coefficient.

By combining Equation (10) and (11), the relationship between the mean and surface velocities for sediment-laden and hyperconcentrated flows can be obtained as follows:

$$\bar{u} = \frac{u_{surf}}{1 + \frac{\pi}{8c_n} \frac{\sqrt{g}}{C}}$$ (12)

where, the Chezy coefficient $C$ is estimated using the Manning equation $C = R^{1/2}$, $R$ is the hydraulic radius of the flume flow, and the Manning coefficient (or roughness coefficient) $n = 0.011$ is chosen for the flume bottom with a smooth concrete surface. It should be noted that if the sediment concentration $S_v$ equals 0, then $c_n = 0.15$ and $\frac{\pi}{8c_n} = 2.62$ (using Equation (9)), and Equation (12) is reduced to Equation (6), which is the velocity profile for clear water.

To investigate the effect of the suspended sediment concentration on the dam breach discharge, volume concentrations ranging from 0.0 to 0.3 were considered, which covered the sediment-laden and hyperconcentrated flow conditions. For each sediment concentration, the average velocity was calculated from the surface velocity using Zhang (1995)'s vertical profile approximation (Equation (12)), and the discharge was then computed as a product of the average velocity and cross-sectional area. Consequently, the dam breach discharge and the peak discharge for each of the sediment concentrations were obtained.

In order to better understand the effect of the suspended sediment concentration on the dam breach discharge, the peak dam breach discharges were plotted against the sediment concentrations and presented in Figure 7. By comparing the results shown in Figure 7, one can see the effect of the suspended sediment concentration on the peak discharge during the dam breach process. The curve in Figure 7 indicates that the peak discharge $Q_{\text{max}}$ is strongly dependent on the sediment concentration $S_v$. As the volume concentration of the suspended sediment increases, the peak discharge decreases to its lowest value and then gradually rises. As shown in Figure 7, the peak discharge for the clear water condition ($S_v = 0$) is $Q_{\text{max}0} = 0.26$ m$^3$/s. When the volume concentration of the sediment increases from $S_v = 0$ to 0.1, the peak discharge decreases from 100% to 94.20% of $Q_{\text{max}0}$. At about $S_v = 0.12$, the peak discharge reaches its minimum (which amounts to 94.1% of the clear water case), and then starts to rise with the sediment concentration. As was evident from Figure 7, with the increase of the sediment concentration from $S_v = 0.15$–0.30, the peak discharge increased from 94.2 to 98.0% of $Q_{\text{max}0}$.

5 | DISCUSSION

5.1 | Water depth over the breach crest

It was found the development of the breach crest (head of breach channel), a “curved section” on the embankment (confined within dashed and solid lines in Figure 8b) that functions hydraulically as a weir, was crucial to the breach hydrograph. This curved section was defined by connecting the highest points on the dam that lie along the streamlines passing through the breach channel (Andrews, 1998; Coleman et al., 2002). Following the definition of Al-Riffai (2014), the curved line $\bar{y}$ was assumed to be a circular arc in shape, and the straight-line $B$ was the distance between the ends of the breach crest (as shown in Figure 8b).

Photogrammetric method was employed to estimate the geometries of this flow controlling section, and the evolution of the breach crest was obtained and shown in Figure 9. It was found that $\bar{y}$/B was approximately 1.05 in the present study.

In addition to the breach crest length, the breach cross-section shape is also a key factor that influences the
estimation of breach hydrograph. Walder, Iverson, Godt, Logan, and Solovitz (2015) used the pebble-motion analyses to estimate the sharp cross-section of the breach, and found the vertical cross section at the breach crest resembled a parabolic shape. Since the breach crest length was obtained (in Figure 9), the determination of the parabolic shape of the breach crest cross-section (head of breach channel) can be reduced to the estimation of flow depth at the breach crest. The flow depth at the breach crest is defined as follows:

$$h = z_w - z_d$$

where, $z_w$ is the water free surface elevation over the breach crest, and $z_d$ is the breach crest elevation. In the present study, $z_w$ is equal to the water depth of the reservoir (Figure 4), and $z_d$ can be estimated using the temporal variation of the breach crest (Figure 9).

To better illustrate the procedure for estimation of water depth over the breach crest, a typical breach channel shape during the dam breach process is present in Figure 10. In the present study, $X_s$ can be obtained using the breach crest length presented in Figure 9b, and $\varphi$ is a constant value ($\pi/4$ in the present study). Therefore,

$$h_b(t) = X_s(t)\tan\varphi$$

$$z_d(t) = H_s - h_b(t)$$

By combining Equations (13)–(15), the water depth over the breach crest can be expressed as:
The bar “−” is used to denote an average over the breach crest, that is,

\[ h(t) = z_w(t) + h_b(t) - H_s \]  

(16)

The average water depth along the breach crest can be expressed as,

\[ \bar{h}(t) = \left( \frac{1}{B} \right) \int_0^B h(t) \, ds \]  

(17)

for an arbitrary function \( F \).

Therefore, the average water depth along the breach crest can be expressed as,

\[ \bar{h}(t) = \left( \frac{1}{B} \right) \int_0^B h(t) \, ds \]  

(18)

Using the procedure described above, the history of surface elevation (\( z_w \)), breach crest elevation (\( z_d \)), and the water depth over the breach crest (\( d \)) during the dam breach process are obtained and presented in Figure 11.

It should be noted that Equations (14)–(17) are invalid when the breach crest is relatively incomplete in shape, for example, in the early stage of the dam breach when the headcut is yet to form (0–100 s in the present experiment), or in the final stage when the breach crest is nearly vanished (after 130 s).

In addition to the procedure described above, the water depth over the breach crest can also be estimated from the critical Froude number. As the breach crest functions hydraulically as a weir during the dam breach process, the outflow through the breach becomes critical near the breach crest. However, the critical Froude number at the breach crest is not equal to 1, due to the strong curvature of streamlines (O’Connor & Costa, 2004; Walder et al., 2015). For flows over a weir with a trapezoidal profile and narrow crest, the critical Froude number is \( F_r = 0.75 - 0.85 \) (Al-Riffai, 2014; Al-Riffai & Nistor, 2013), while for flows over a triangular-profile weir the critical Froude number is \( F_r = 0.78 \) (Kirkgoz, Akoz, & Oner, 2008). In the present experiments, the critical Froude number at the breach crest is found to be \( F_r = 0.85 \) during the dam breach process.

The critical Froude number \( F_r \) can be expressed as,

\[ F_r = \frac{\bar{u}}{\sqrt{gB}} = \frac{\bar{u}}{\sqrt{gh}} \]  

(19)

where, \( \bar{u} \) is the average velocity of the cross-section over the breach crest.

The water depth over the breach crest can be written as,

\[ \bar{h}(t) = \frac{\bar{u}^2}{k_1 g} \]  

(20)

where, \( k_1 = F_r^2 \) is a constant (0.72 in the present experiment).

It is assumed in hydraulic engineering that the relationship between surface velocity and average velocity is linear,

\[ \bar{u} = k_2 u_{surf} \]  

(21)

where, \( k_2 \) is defined as the velocity coefficient. In the present study, surface velocity is obtained using the LSPIV method (Figure 5), and average velocity is calculated using the hydrograph (Figure 4), and consequently \( k_2 \) is found to be \( k_2 = \frac{\bar{u}}{u_{surf}} = 0.65 \) during the dam breach process.

Therefore, the breach water depth can be expressed in terms of surface velocity as,
where coefficients $k_1 = 0.72$, and $k_2 = 0.65$.

Using Equation (22) the water depth over the breach crest is calculated, and compared with the results by Equation (16), as shown in Figure 12. The results by the two different methods agree reasonably well with each with minor discrepancy after 200 s.

5.2 | A new formula for breach discharge estimation

Based on the analysis above, the breach outflow can be regarded as flow over a broad-crest weir with changing crest. The equation for calculating broad crested weir flow rate is,

$$Q_{\text{out}} = C_d \frac{2}{3} B \left( \frac{2}{3} g \frac{h^2}{k_1 g} \right)^{1/2} \tilde{h}^{3/2}$$  \hspace{1cm} (23)

where, $C_d$ is the weir discharge coefficient, $B$ is the width of the weir crest, and $d$ is the upstream water head over the weir.

Substituting Equation (22) in to Equation (23),

$$Q_{\text{out}} = C_d g^{-1} B u_{\text{surf}}$$  \hspace{1cm} (24)

where, $C_d = C_d \frac{2}{3} B \left( \frac{2}{3} g \frac{h^2}{k_1 g} \right)^{1/2} \tilde{h}^{3/2}$ is defined as the discharge coefficient for weir with changing crest.

Equation (24) includes two main variables, namely the width of the breach crest $B$ and the breach surface velocity $u_{\text{surf}}$, which can be readily measured using photogrammetric method during the dam-breach process.

5.3 | Verification of the new breach discharge formula

In this section, the new breach discharge formula Equation (24) is used to estimate the hydrograph of the present dam breach experiment, and compared with the benchmark data using mass-conservation method (Figure 4).

The surface velocity in the breach channel was measured using the LSPIV method, and presented in Figure 13a. The breach crest width data was presented in Figure 9a. The weir discharge coefficient was calculated using
The comparison between the new formula result and benchmark data is given in the Figure 13b. It can be seen from Figure 13b that the discharge estimated by the new formula and benchmark data agreed with each other reasonably well.

6 | CONCLUSION

In this study, laboratory experiments were conducted to investigate the relationship between water surface velocity and breach hydrograph. The effects of different vertical velocity approximations on the estimation of dam breach discharges using the LSPIV method were analysed. The largest peak discharge, at $Q_{\text{max}} = 0.29 \text{ m}^3/\text{s}$, was found using Castro-Orgaz and Hager (2013) velocity profile, while the lowest one, at $Q_{\text{max}} = 0.22 \text{ m}^3/\text{s}$, was obtained using the mass-conservation method. Moreover, the peak discharge that was estimated using water depth occurred slightly later than those that were estimated using the surface velocity. The velocity profile approximations have limited effects on the dam discharge calculation for clear water conditions. For dam breaches with sediment-laden or hyperconcentrated flows, it was found that the peak discharge was strongly dependent on the sediment concentration ($S_v$). With the increase of the sediment concentration in the flume flow, the peak discharge of the dam breach decreased to its lowest value and then gradually increased.

Based on the experimental data, a new formula was proposed for the rapid estimation of dam breach hydrograph. The input data required includes the breach width and the surface velocity in the breach channel, which are relatively easy to obtain using photogrammetric method (such as LSPIV) during a dam breach event. The hydrograph by the new formula was verified against the benchmark data by the mass-conservation method. The formulas can be readily applied to estimate the dam hydrograph for real dam breach cases.

Although the rapid estimation of dam breach hydrograph based on surface velocity measurement in case of a landslide dam breach event has many advantages, the method still has its limitations. One of such limitations is that the shape of the breach cross-section (e.g., parabolic, triangular, rectangular, etc.) needs to be pre-determined or in some occasion guessed. To analyse the influence of the breach cross-section shape on the estimation of hydrograph, further laboratory experiments are planned to consider more dam breach scenarios, with different earth dam configurations and materials. Based on the experimental data, the influence of the breach cross-section shape will be incorporated into the hydrograph formula.

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DATA AVAILABILITY STATEMENT

I confirm that my article contains a Data Availability Statement even if no data is available (list of sample statements) unless my article type does not require one. I confirm that I have included a citation for available data in my references section, unless my article type is exempt.

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REFERENCE

Andrews, D. (1998). Embankment failure due to overtopping flow (ME Thesis). University of Auckland, Auckland, New Zealand.

Al-Riffai, M., & Nistor, I. (2013). Influence of boundary seepage on the erodibility of overtopped embankments: A novel measurement and experimental technique. In: Proceedings of the 35th IAHR World Congress, Vols I and II, 5447–5458. https://doi.org/10.13140/2.1.2354.3686

Al-Riffai, M. (2014). Experimental study of breach mechanics in overtopped noncohesive earthen embankments (PhD Dissertation). University of Ottawa, Ottawa, Canada.

Cao, Z., Yue, Z., & Pender, G. (2011a). Flood hydraulics due to cascade landslide dam failure. Journal of Flood Risk Management, 4(2), 104–114. https://doi.org/10.1111/j.1753-318X.2011.01098.x

Cao, Z., Yue, Z., & Pender, G. (2011b). Landslide dam failure and flood hydraulics. Part I: Experimental investigation. Natural Hazards, 59(2), 1003–1019. https://doi.org/10.1007/s11069-011-9814-8

Castro-Orgaz, O., & Hager, W. H. (2013). Velocity profile approximations for two-dimensional potential open channel flow. Journal of Hydraulic Research, 51(6), 645–655. https://doi.org/10.1080/00221686.2013.809387

Coleman, S. E., Andrews, D. P., & Webby, M. G. (2002). Overtopping breaching of noncohesive homogeneous embankments. Journal of Hydraulic Engineering-ASCE, 128(9), 829–838. https://doi.org/10.1061/(asce)0733-9429(2002)128:9(829)

Einstein, H. A., & Chien, N. (1995). Effects of heavy sediment concentration near the bed on the velocity and sediment distribution. Report. No. 8, University of California, Berkeley, CA.

Eaket, J., Hicks, F. E., & Peterson, A. E. (2005). Use of stereoscopy for dam break flow measurement. Journal of Hydraulic Engineering-ASCE, 131(2), 120–126. https://doi.org/10.1061/(asce)0733-9429(2005)131:2(120)

Eaket, J., Hicks, F. E., & Peterson, A. E. (2005). Use of stereoscopy for dam break flow measurement. Journal of Hydraulic Engineering-ASCE, 131(2), 120–126. https://doi.org/10.1061/(asce)0733-9429(2005)131:2(120)
