Disk and Washer Coupled Cavity Linac Design and Cold-Model for Muon Linac

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Abstract. A disk and washer (DAW) coupled cavity linac (CCL) was developed for a medium-velocity part in a muon linac to measure muon anomalous magnetic moment and search for electric dipole moment. Muons were accelerated from \( \frac{v}{c} = \beta = 0.3 \) to 0.7 at an operational frequency of 1.3 GHz. In this paper, the cavity design, beam dynamics design, and cold-model measurements are presented.

1. Introduction

A disk and washer (DAW) coupled cavity linac (CCL) was first proposed in the early 1970s [1] and the first DAW was used in a proton and H⁺ linac at the Moscow meson factory [2]. A DAW CCL is a cylindrical cavity with conducting washers interspersed with disks. There are several types of supporting structures for the washer [3] and Fig. 1 shows a biperiodic L-support type [4]. The DAW CCL has many advantages such as high shunt impedance and high coupling between the acceleration and coupling cells. Its disadvantages include the mode overlapping and the difficulties in analyzing its mode structure. Owing to recent enhancements in computational power, a satisfactory cavity design including washer support can be achieved; thus, three-dimensional electromagnetic fields can be considered. This paper describes the design of a DAW CCL for a low-emittance muon beam.

Low-emittance muon beams exhibit potential advantages in various branches of science. After muons are cooled to thermal energy [5, 6], they are accelerated to the specific energy required by an application. One such application of accelerated muon beams is the transmission muon microscope [7]. If the muons are accelerated up to 10 MeV, three-dimensional imaging of living cells can be achieved, which is impossible with the use of a transmission electron microscope. Muon linacs have also been used for the precise measurement of muon anomalous magnetic moment \((g_\mu - 2)\) and electric dipole moment (EDM) at the Japan Proton Accelerator Research Complex (J-PARC E34) [8]. The J-PARC E34 experiment aims to accelerate muons to 212 MeV. Recently, a muon acceleration was demonstrated using, for the first time, using a radio-frequency quadrupole linac (RFQ) [9]. The muons will be accelerated to few megaelectronvolts after the...
RFQ using a drift-tube linac (DTL) [10, 11, 12]. In order to reach the higher energy levels required for various applications, it is necessary to perform additional acceleration with following cavities. The DAW CCL is one of the most promising candidates for this.

In this paper, the DAW CCL designs for the J-PARC E34 experiment and the prototype status are presented. The DAW CCL accelerates muons from $v/c = \beta = 0.27$ to 0.7 at an operational frequency of 1.3 GHz. First, we present designs for cavity and beam dynamics. Then, we describe the measurement for an Al cold-model of the DAW cavity. Finally, we summarize our results.

2. Cavity design

In order to maximize the high-acceleration efficiency, the cavity design is optimized using several electro-magnetic field solver software. First, a two-dimensional model without washer supports, as shown in Fig. 2 is optimized by calculating the acceleration and coupling mode with SUPERFISH [13]. The variable parameters are disk radius ($T_d$), disk thickness ($T_d$), washer radius ($R_w$), and the gap between the washer (G) as shown in red characters in Fig. 2. The optimization process is performed using the SIMPLEX algorithm [14] and the optimization function is constructed with the confluent condition ($f_a = f_b$, where $f_a$ and $f_b$ are the resonant frequencies of the acceleration and coupling modes, respectively), high shunt impedance (ZTT), and small peak-to-average ratio ($E_0/E_{max}$).

Then, a three-dimensional model with washer supports is constructed in CST MICROWAVE STUDIO [15] based on the optimized two-dimensional model. Here, the connection radius of the supports is selected to be the zero-electric point to minimize perturbation to the accelerating mode. The three-dimensional model is also optimized by using the same optimization function as that used in the two-dimensional one. Finally, the dispersion curve is investigated to check whether an unfavorable mode exists around the operation frequency. All the steps are repeated for several cavity lengths of $\beta \lambda/4$ ($\beta = 0.3, 0.4, 0.5, 0.6$).

Figure 3 shows the three-dimensional model (left) and the dispersion curve (right) of the optimized model for $\beta = 0.3$. Because of the bi-periodic structure, some stop bands appear in $\pi/2$. Although the TM11 mode is near the operational frequency, the cavity is tuned in the optimization process so that the operational frequencies sit in the stop band at $\pi/2$. Although
3. Beam dynamics design
A DAW module consists of two DAW tanks connected with a bridge tank and quadrupole doublets placed at inter-tank spacing. The inter-tank spacing is set to 4.5$\beta\lambda$ considering the feasibility of magnet installation. The average acceleration field ($E_0$) is considered according to the Kilpatrick limit and the peak-to-average ratio calculated in the previous section; $E_0$ is set to be 5.6 MV/m that is lower than Kilpatrick limit. Then the longitudinal dynamics is designed by TRACE3D [16]. Table 2 lists the parameters for the dynamics design including the transit time factor ($T$).

The upper limit on the number of DAW cells, or the length of the tank, is determined by the limitation of the quadrupole strength due to transverse instabilities. There may be instabilities or resonances when the zero current phase advance ($\sigma_0$) is greater than 90 degrees [17]. Although the muon beam intensity is expected to be much smaller than that in the region of such instabilities, the number of cells is chosen to match this criterion. The maximum $\sigma_0$ is 83 degrees.
Figure 3. Dispersion curve with optimized cavity in several $\beta$ calculated by CST MICROWAVE STUDIO.

Table 1. Parameters of the Optimized DAW Cavity.

| $\beta$ | 0.6 | 0.5 | 0.4 | 0.3 |
|---------|-----|-----|-----|-----|
| $L$     | $\beta \lambda / 4$ |     |     |     |
| $R_b$ [mm] | 12  | 2.6 |     |     |
| $R_n$ [mm] | 2.6 | 3.5 |     |     |
| $T_w$ [mm] | 30  |     |     |     |
| $\theta$ [deg.] |     |     |     |     |
| $R_c$ [mm]  | 155 | 157 | 154 | 151 |
| $R_d$ [mm]  | 111.3 | 108.352 | 104.52 | 103.221 |
| $T_d$ [mm]  | 16.014 | 14.790 | 10.97 | 9.630 |
| $R_m$ [mm]  | 105.969 | 105.63 | 108.14 | 110.391 |
| $G$ [mm]    | 15.975 | 11.285 | 7.8976 | 6.148 |
| $f_a$ [GHz] | 1.300 | 1.300 | 1.299 | 1.301 |
| $f_c$ [GHz] | 1.299 | 1.301 | 1.302 | 1.301 |
| $Z_{TT}$ [MΩ/m] | 57.8 | 46.3 | 33.8 | 18.0 |

at the first tank when the number of cells is ten. The number of cells in all tanks is chosen to be ten based on this result.

The beam transport line between the previous accelerator (inter-digital drift-tube linac [10]) and the DAW is then designed to match the injection beam to the beam parameters estimated by TRACE3D. The transport line consists of four quadrupole magnets and two RF cavities. The field strength of the quadrupoles and the RF cavities are fitted to obtain a matched beam.

Finally, the beam dynamics in the entire modules including the transport line is simulated using PARMILA [18]. Figure 4 shows the phase-space distributions at the DAW exit. Figure 5 shows the emittance growth along the DAW cells. The growth is estimated to be less than a
Table 2. Parameters for the dynamics design estimated by SUPERFISH and CST MW Studio. T (S) is the transit-time factor, integral of $E_z \cos(kz)$ ($E_z \sin(kz)$), and $T'$ ($S'$) is first derivative of T with respect to wave number. $Z$ and $ZTT$ are the shunt impedance and effective shunt impedance in $\text{M}\Omega/\text{m}$, respectively.

| $\beta$ | T  | T' | S  | S' | $g_z$ | Z   | ZTT | $E_0/E_{\text{max}}$ |
|--------|----|----|----|----|------|-----|-----|---------------------|
| 0.70   | 0.85 | 0.045 | 0.21 | 0.028 | 0.23 | 87  | 63   | 4.6                 |
| 0.65   | 0.84 | 0.047 | 0.22 | 0.028 | 0.24 | 83  | 59   | 4.5                 |
| 0.60   | 0.84 | 0.047 | 0.22 | 0.028 | 0.23 | 78  | 55   | 4.4                 |
| 0.55   | 0.84 | 0.047 | 0.22 | 0.028 | 0.22 | 70  | 50   | 4.4                 |
| 0.50   | 0.84 | 0.045 | 0.21 | 0.027 | 0.20 | 63  | 45   | 4.8                 |
| 0.45   | 0.84 | 0.047 | 0.22 | 0.027 | 0.19 | 55  | 39   | 4.7                 |
| 0.40   | 0.83 | 0.048 | 0.22 | 0.027 | 0.17 | 47  | 33   | 5.1                 |
| 0.35   | 0.82 | 0.051 | 0.23 | 0.028 | 0.18 | 38  | 26   | 4.9                 |
| 0.30   | 0.81 | 0.055 | 0.23 | 0.028 | 0.18 | 29  | 19   | 4.9                 |
| 0.27   | 0.80 | 0.056 | 0.24 | 0.028 | 0.16 | 22  | 14   | 5.2                 |

A few percent, and the output beam emittance is $0.32 \pi \text{ mm mrad}$ and $0.21 \pi \text{ mm mrad}$ for $x$ and $y$, respectively.

Table 3 summarizes the beam dynamics results. The total length is 16.3 m with 15 modules, which corresponds to $138\beta\lambda$, and the beam transit time is calculated to be $\frac{138\beta\lambda}{\beta c} = 106.0 \text{ ns}$. The survival probability is calculated to be $\exp(-\frac{106.0}{\gamma\tau}) = 0.961$, where $\gamma = (\gamma_{\text{in}} + \gamma_{\text{out}})/2 = 1.212$. The estimated power dissipation is 4.5 MW, which is comparable to the expected value.

Figure 4. Phase-space distributions in the last cell estimated by PARMILA. (A) the horizontal divergence angle $x'$ vs $x$, (B) the vertical divergence angle $y'$ vs $y$, (C) $y$ vs $x$, and (D) $\Delta E$ vs $\Delta\phi$.
Figure 5. Emittance growth along the DAW cells. The growth is less than a few percent.

Table 3. Summary of Simulation Output Beam Parameters in DAW.

| Parameter                  | Input | Output |
|----------------------------|-------|--------|
| $\beta$                    | 0.28  | 0.68   |
| Energy (MeV)               | 4.49  | 40.3   |
| $\varepsilon_x$ [$\pi$ mm mrad] | 0.315 | 0.322  |
| $\varepsilon_y$ [$\pi$ mm mrad] | 0.195 | 0.216  |
| $\varepsilon_z$ [$\pi$ MeV deg] | 0.121 | 0.466  |
| Transmission [%]           | 99.5  |        |
| Beam transit time [ns]     | 106   |        |
| Survival rate [%]          | 96.1  |        |
| Transmission total [%]     | 95.6  |        |

4. Cold model measurements
Based on the optimized cavity design, an Al cold model with $\beta = 0.3$ were fabricated. A unit includes four washers, and corresponds to three accelerating cells. In order to measure the accelerating and coupling modes, two types of the end plate were fabricated.

Figure 6 shows the measurement results for the coupling mode. The resonant frequency is measured to be 1.315 GHz and it is consistent to the simulation result within approximately 0.1%. The difference from 1.3 GHz is owing to the effect of the end plate. Further measurements with different numbers of cells and for the acceleration mode are being conducted.

5. Conclusion
We designed a DAW CCL cavity and beam dynamics for muon linac to measure muon $g - 2$ and search for EDM. The required power is comparable to the expected value and the emittance growth satisfies the requirement. An Al cold-model was fabricated and measurements are being conducted to confirm the design.
Figure 6. Resonant frequency measurement with the Al cold model.

6. Acknowledgment
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References
[1] Andrev V G et al. 1972 Proc. 1972 Linear Acel. Conf. (Los Alamos)
[2] Esin S K et al. 1988 Proc. 1988 Linear Acel. Conf. (Virginia USA)
[3] Iwashita Y et al. 1992 Proc. 1992 Linear Acel. Conf. (Ottawa, Canada)
[4] Ao H et al., 2000 Jpn. J. Appl. Phys. 39 651
[5] Bakule P et al. 2008 Nucl. Instru. Meth. B266 335
[6] Beer G A et al. 2014 Prog. Theor. Exp. Phys. 091 C01
[7] http://slowmuon.kek.jp/MuonMicroscopy_e.html
[8] http://g-2.kek.jp/portal/index.html
[9] Bae S et al., 2018 Phys. Rev. Accel. Beams 21 050101
[10] Otani M et al., 2016 Phys. Rev. Accel. Beams 19 040101
[11] Otani M et al., 2017 J. Phys. :Conf. Ser. 875 021038
[12] Otani M et al., 2017 Proceedings of the 14th Annual Meeting of PASJ (Sapporo, Japan) TUP024
[13] Billen J Billen H and Young L M “Poisson Superfish” 1996 LA-UR-96-1834
[14] James F and Roos M “Minuit: A System for Function Minimization and Analysis of the Parameter Errors and Correlations” 1975 Comput. Phys. Commun. bf 10 343
[15] https://www.cst.com/products/CSTMWS
[16] Crandall K R and Rustoi D P 1997 Los Alamos Report No. LA-UR-97-886
[17] Reiser M ”Theory and Design of Charged Particle Beams”1994 JOHN WILEY & SONS, INC.
[18] http://www.laacg.lanl.gov