Chapter

Superconductivity and Microwaves

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Abstract

Superconductivity conforms to a quantum, thermal, and electrodynamic set of physical phenomena of great interest by themselves. They have, also, the potential to be one clean energy source that technology is looking for. Superconductors do not allow static magnetic fields to penetrate them below a critical field, that is, Meissner effect. However, microwave magnetic fields do penetrate them already, and their energy is readily absorbed by the superconductor. High-temperature, perovskite superconductors do absorb microwave energy the most due to the presence of unpaired electron spins, fluxoid dynamics, and quasiparticle motion. We describe the fundamental physics of the interaction of the superconductors with microwaves. Experimental techniques to measure microwave absorption are presented. Experimental setups for absorption of energy are described in terms of the central quantity, Q. The measurements are analyzed in terms of irreversible energy exchange processes. The knowledge gained can inform the design of superconducting devices operating in microwave environments.

Keywords: superconductivity, microwaves, absorption, resonant cavity, superconductors type I, superconductors type II, HTSC

1. Introduction

Superconductivity (SC) conforms to an electrodynamic, quantum, and thermal set of physical phenomena of great interest by themselves [1–4]. A great amount of new fundamental physics has been extracted experimentally and theoretically since its discovery in 1911 by Onnes [4–17]. They have, also, the potential to be one clean energy source that technology is looking for. Superconducting solenoids could store clean magnetic energy to provide electricity to an entire house or set of houses for weeks, months, or years (Figure 1A). The actual electricity supply is shown in Figure 1B, SC solenoids installed at the house or set of houses, or factories with a simple system as shown in Figure 1B could provide an equivalent electrical energy to cover the domestic and industrial needs. The challenge is to charge the solenoid with a simple mechanism and control the discharge of very little amounts of energy, for their use, at a time. Leakage should be avoided at all times [18–20].

One of the most popular potential applications of superconductivity (SC) is the levitated train that can carry hundreds of people through hundreds of kilometers from one city to another at speeds in excess of 550 kms/h. in a very safe trip. A prototype is shown in Figure 1D and follows the same principle of the levitated magnet (Figure 1C). Several countries are developing real-scale prototypes, and engineering problems are being faced one by one [21, 22]. The above applications rely on just
two properties of superconductors, namely, the Meissner effect and the transport of electrical current with no resistance. The problem of interfacing with the conventional conductors that connect to machines, motors, etc. is not considered here.

2. Other technological uses of superconductors include electromagnetic cavities and microwaves

Pippard in 1947 reported an article [7]: “the first in a series in which the behaviour of the electrical impedance of metals at low temperatures and very high frequencies will be considered from experimental and theoretical standpoints. The technique of resonator measurements at 1200 Mcyc./s. is described in detail, and experimental curves are given showing the variation with temperature of the r. f. resistivities of superconducting tin and mercury. In contrast to the behaviour of superconductors in static fields, a finite resistance is present at all temperatures, tending as the absolute zero is approached to a very low value, which is probably zero for mercury but not for tin. The experimental results are in good agreement with London’s measurements on tin by a different method.”

Since then, spectacular discoveries and applications of superconductivity have been at work for decades in different areas of engineering, high-energy particle physics and microwave technology, communications, and scientific and medical (nuclear magnetic resonance) equipment [23–27].

At radio frequency and microwave range, SC has found applications in areas such as satellite communications, particle accelerators, microwave technology applied to precision instruments, and metrology and qubit formation [26, 27]. In particular, high-energy physics laboratories use both, SC solenoids and electromagnetic cavities [28, 29]. In one case, the production of dc strong magnetic fields, in the other high electric field at GHz is required.

Superconducting niobium cavities, the heart of linear accelerators: more specifically, to discover, experimentally, new elementary particles, or to investigate what are they made of, it is necessary to smash them against other high-energy particles. They are made to collide head-on at the highest attainable energies possible, ~500 Gev. The electron-positron colliders [30] need to work with particles at very high kinetic energy; hence their velocity must be as high as possible. To reach such high velocities, it is mandatory to accelerate the charged particles with extremely high electric fields, so that the force \( F = qE \) acts and \( F = ma \) makes its work accelerating the charged beam of particles (Figure 2A). The most efficient way (cost and physics) to accelerate electron and positron beams is to produce very high electric, \( E \), fields, inside a connected series of SC niobium resonant cavities and to expose serially the charged particle beams to the electric field and let Newton’s second Law to operate for as long as possible, as shown inside the red oval in Figure 2A. This part is the heart of the accelerating process and is supported

Figure 1.
(A) A house or a set of houses electrically powered conventionally could be powered with a superconducting reservoir of energy, kind of a cylinder as if it were a dynamo or a car alternator; (B) a sealed SC solenoid storing thousands of KWTTS of usable electrical energy [18–20], (C) a levitating magnet on top of a SC disc, and (D) a levitated train running on a bed of air because it has been levitated as the magnet at the left [21, 22].
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DOI: http://dx.doi.org/10.5772/intechopen.86239

by other fundamental parts as is the liquid helium bath to keep the temperature at 4.2 K, below the critical temperature for niobium to become SC.

Superconducting microwave cavities can sustain very concentrated patterns of electromagnetic fields with very, very small losses at the walls of the cavity; the surface impedance, $Z_s$, is extremely low; the quality factor ($Q$) of a SC cavity is very high, $>10^5$; and the losses, proportional to $1/Q$, are very small. Recalling the definition of $Q = (\omega_0)$ stored electrodynamic energy/power loss [31, 32], where $\omega_0$ is the operating frequency (~GHz), we see that the losses are just one part in $10^5$ (or less) of the electrodynamic energy inside the cavity. The photo in Figure 3 shows the work on a tandem of four superconducting cavities to mount detectors, and so on, inside a 100 clean room. The conversion of energy from the electromagnetic to the charged particle acceleration is the highest technologically possible at present [29, 30]. Acceleration of charged particles inside conventional conducting (copper) cavities presents much more losses at the cavity walls, $Q_{\text{cu}} \ll Q_{\text{sc}}$ and $Z_{\text{sc}} \gg Z_{\text{cu}}$, and the heat produced can melt the cavity. Accelerating charged particles in open space can disperse, and they cannot, by far, reach the concentrated fields and intensities that can be tailored inside SC cavities as the ones shown in Figure 2. The heart of the accelerating process is inside the superconducting cavity and inside it, in the red oval. The central piece in a linear particle accelerator is a tandem of these SC accelerating microwave cavities (Figure 2B).
Other salient superconducting microwave cavity applications and uses are (A) the measurement of fundamental constants through the electrodynamic response of a superconductor inside a cavity resonator [33] and (B) the measurement of superconductor losses at microwave frequencies dependent on applied magnetic fields and/or on temperature [7–8, 34–37]. The actual and the potential applications, just briefly mentioned here, are a sample of many propositions that are treated at length in Refs. [38–40]. The world of applications is so vast and promising that pushes the field of discovering new superconducting materials at higher and higher temperatures. Figure 4 shows a compilation of several families of superconductors discovered at different times from 1911 up to the present (≈110 years).

SC comprises a large family of materials with a wide range of complex properties, behaviors, and types. They are classified in different ways: conventional, non-conventional, type I, type II, gapless, s-wave, d-wave, and so forth. To understand more precisely the interactions of superconductors with microwaves, we, next, highlight seven of the most fundamental characteristics of superconductors.

3. Eight of the most fundamental characteristics of superconductors

A. Superconducting critical temperature, Tc: a superconductor becomes so below a critical temperature, Tc. The superconducting state is ordered, and so, it is destroyed by thermal agitation energy, k_B T. But, as temperature is lowered, the thermal energy goes down and the atomic and molecular giggling becomes quieter, and the superconducting state produced by the pairing of electrons is not so disturbed by the thermal energy k_B T below k_B Tc. We could say that the two energies, the superconducting, ordering energy, Δ, compete with k_B T. In a broad picture, when k_B T > ordering energy (Δ), the material ceases to be superconductor, and when k_B T < (Δ), the material becomes superconducting.
B. The formation of Cooper pairs. The ordering consists of the forming of electron-bound pairs, called Cooper pairs, by means of an attractive interaction mediated by the lattice vibrations (phonons) that overcomes the Coulomb repulsion that experience these electrons. Figure 5 depicts in an exaggerated manner the lattice distortion when the couple of electrons move through this part of the material. Through the vibrations of the lattice, these two electrons form a bound system with zero linear momentum $p$ and zero spin, a singlet Cooper pair.

C. The macroscopic quantum state $\psi(r, t)$, Cooper pair, coherence length, and penetration depth. These Cooper pairs are bosons (spin zero), and they all condensate in the lowest energy state, and all has exactly the same wave function, $\psi(r, t) = \psi_0 \exp(i\varphi)$, where $\varphi$ is now the macroscopic phase of the wave function. The whole macroscopic quantum state is represented by $\psi(r, t) = \psi_0 \exp(i\varphi)$. The square of the wave function gives the probability of finding a Cooper pair, let say, at $r^*$, at time $t^*$, but such probability is exactly the same to find another Cooper pair at such $r^*$ and $t^*$ and another Cooper pair at $r^*$ at $t^*$. If there are $n_s$ bosons in the superconductor, after normalization, we have $|\psi(r^*, t^*)|^2 = n_s|\exp(i\varphi)|^2 = n_s$, the total number of Cooper pairs in the superconductor [1–3, 17]. The probability of finding a Cooper pair at $r^*$, $t^*$ is the same as finding another Cooper pair at any other location at any other time inside the superconductor. $\psi(r, t)$ is also the complex order parameter first introduced by Ginsburg-Landau in their 1950s superconductivity theory [10] in which the superconducting electrons were envisioned as to enter (a second-order transition) or form a superfluid state, $\psi$, which is an ordered state.

The Ginzburg-Landau equations [43] predicted two new characteristic lengths in a superconductor. The first characteristic length was termed coherence length, $\xi$. For $T > T_c$ (normal phase), it is given by:

$$\xi = \sqrt{\frac{h^2}{2m|\alpha|}} \quad \text{(1)}$$

where $\alpha$ is a parameter in the GL Equations [43]. While for $T < T_c$ (superconducting phase), where it is more relevant, it is given by

$$\xi = \sqrt{\frac{h^2}{4m|\alpha|}} \quad \text{(2)}$$

Figure 5. A lattice of atoms is distorted by the passage motion of two electrons, possibly tracing time reversal trajectories [42]. The center of mass does not move and the pair is in a singlet state. The passing electrons attract the positive-charged nuclei of the lattice, causing a slight ripple in its wake.
It sets the exponential law according to which small perturbations of density of superconducting electrons recover their equilibrium value $\psi_0$. Thus this theory characterized all superconductors by two length scales. The second one is the penetration depth, $\lambda$. It was previously introduced by the London brothers in their London theory [6]. Expressed in terms of the parameters of Ginzburg-Landau model, it is.

$$\lambda = \sqrt{\frac{m}{4\mu_0 e^2 \psi_0^2}}$$

where $\psi_0$ is the equilibrium value of the order parameter in the absence of an electromagnetic field. The penetration depth sets the exponential law according to which an external magnetic field decays inside the superconductor. The ratio $\kappa = \lambda/\xi$ is known as the Ginzburg-Landau parameter. It has been proposed by Landau that Type I superconductors are those with $0 < \kappa < 1/\sqrt{2}$ and Type II superconductors those with $\kappa > 1/\sqrt{2}$.

D. Zero electrical resistance. Supercurrents, $J_s$, of Cooper pairs. Superconducting conductivity, $\sigma_s$. No losses versus normal conductivity, $\sigma_n$. Losses and heating. When these Cooper pairs move in the material, they do so in phase with the lattice vibrations, and no scattering of electron pairs occurs and no electrical resistance develops, and very importantly, no dissipation of energy develops due to this charge transport (see blue portion of the temperature axis and coupled blue dots (Cooper pairs) in Figure 6A and B). In contrast, when single conduction electrons move in a normal metal or in YBaCuO perovskites, at temperatures above $T_c$ (red portion and red single dots in Figure 6A and B), they scatter many times from the nuclei, imperfections, impurities. They lose energy in each one of these processes, giving rise to the well-known and measurable electric resistance and/or normal conductivity, $\sigma_n$. The material develops Joule heat, and for high currents (hundreds to thousands of amperes), the metal can even melt. But, in the SC state, the electrical resistance is zero, the whole conductivity is $\sigma_s$, and thousands of amperes can flow without losses nor Joule heating.

E. Meissner effect: magnetic fields get expelled. When a superconductor is placed in an external magnetic field, or a material becomes SC in the presence of such external magnetic field, the SC material expels the magnetic field from its interior, as shown in Figure 7A. Irrespective of the state and history of the material.
prior to the superconducting state, once in it, all magnetic lines of the external magnetic field get expelled from its interior. This experimental result was discovered by Meissner and Ochsenfeld in 1933 and is known as the Meissner effect. By comparison, a perfect conductor is penetrated by the magnetic field lines irrespective of the temperature. In great contrast, the superconducting material expels the field lines while in the superconducting state, $T < T_c$. Above $T_c$, the material behaves as a normal conductor, and so, the magnetic field lines go through it.

F. Type I superconductor, one critical magnetic field. Type II superconductors. Two critical magnetic fields, $H_{c1}$ and $H_{c2}$. The mixed state. The SC that shows the behavior shown in Figure 7A up to a field $H_c$, before the SC state breaks down into the normal state, is said to be of the first kind or type I SC. But not all superconductors are of the first kind. A second kind of SC materials presents the expulsion of magnetic field lines’ behavior up to a critical field, $H_{c1}$, which is much smaller than the critical field that withstand the superconductors of the first kind, $H_{c1} \ll H_c$. For fields above $H_{c1}$, the SC of the second kind admits the entrance of magnetic field lines but in an ordered and quantized manner. These flux lines penetrate and cross the whole material in “ramilletes” or bundles of very small radii $\xi$ as shown in Figure 7B. These lines define approximate cylinders that can be bent and twisted. Inside these flux cylinders or fluxoids, the material is in the normal state, and outside the fluxoids, the material is superconducting. This configuration is allowed in the mixed state, $H_{c1} < H < H_{c2}$. The cylinders are surrounded by supercurrents of Cooper pairs (gray discs in Figure 6) that limit the expansion of the radius of the cylinder. The flux is quantized, $\Phi_n = n \Phi_0 = n(\hbar/2e)$.

Hence, coexistence of both normal and SC states is allowed in the mixed state. Inside these fluxoids, $\xi$, there are no Cooper pairs, but only normal electrons that if moved, they will do it in the normal way, $\sigma_n$, dissipating energy because inside the vortices, the medium is normal-viscous for the motion of the electrons. If alternating electromagnetic fields are applied to such superconductor in the mixed state, the Lorentz force on the normal electrons will force them into periodic damped motion; in addition, the inside of these vortices will radiate since it contains electric charges in accelerated motion [16, 31]. If microwaves are applied, again, the electrons are subjected to a Lorentz force that will make the normal electrons to move with friction and would be accelerated and radiate microwaves. This result was established more than 70 years ago [7, 8].
G. When two superconductors, SC1 and SC2, are put in proximity, the non-intuitive Josephson effects emerge. When the surfaces of SC1 and SC2 are a few microns, or tenths, or hundredths of microns apart, several non-intuitive quantum phenomena happen. Josephson discovered them theoretically in 1962, and they are known as Josephson effects, for which he received the Nobel Prize in 1973. The main effects are J1 at the junction of two superconductors separated by a thin insulator, without any voltage applied (Figure 8), Cooper pairs can tunnel the barrier and a supercurrent is established between both superconductors [15, 16, 46], and the current is governed by the equation:

\[ J_{sc} = J_{cs} \sin(\varphi_2 - \varphi_1) \]  \( (4) \)

where \( \varphi_1 \) and \( \varphi_2 \) are the corresponding phases of the superconducting wave functions of each superconductor and \( J_c \) is the maxim current that the structure can withhold without producing NEW phenomenology and depends on material properties. Given that \( \varphi_1 \) and \( \varphi_2 \) are constant, then in Eq. (4) \( J_{sc} \) is constant and is usually termed the dc Josephson effect. The equation was first derived fully under theoretical considerations, for the possibility of tunneling Cooper pairs from one superconductor to the other as shown in Figure 8. Few months later, it was corroborated experimentally by Anderson and Rowell [45].

The materials depicted in Figure 8 can be small enough to be point-contact electrodes.

The second main effect is known as the ac Josephson effect and is realized when a constant voltage is applied to the junction. Such applied voltage, \( V \), causes an oscillating supercurrent, \( J_{2e} \), to flow across the junction and is given by

\[ J_{2e}(\omega t) = J_{c} \cos(\omega t) \]  \( (5) \)

Where \( \omega = 2eV/\hbar = E/\hbar \) and \( E = hv = 2eV \). This equation tells us that the application of a CONSTANT voltage across the junction produces that the cooper pairs...
flow through the junction, $J_2e$, as a periodic function given by $\cos(\omega t)$. This is an exact transformation of a direct voltage (macoscopic) to a periodic response of the superconducting electrons, in which they move with the quantized energy: $E = \hbar \omega$. The frequency of the oscillating current depends only on the voltage applied, the charge of the Cooper pair ($2e$), and Plank's constant. The relation between voltage and frequency of produced current is exact and has served as a very precise voltage standard. This effect is fully quantum macroscopic, first deduced by Josephson and soon afterwards was confirmed experimentally. The energy $\hbar \nu$ equals the energy change of a Cooper pair transferred across the junction. The voltage applied to a Josephson junction is typically on the order of a few microvolts. Thus, Josephson junctions (JJ) and superconducting quantum circuits (SQCs) are usually operating at frequencies in the microwave regime, from a few GHz up to THz [46, 47]. Hence, a JJ or a SQC becomes a microwave source when injected a few dc microvolts.

When, in general, the phases of the wave functions are functions of time, then the Josephson equations are

$$V(t) = \frac{\hbar}{2e} \frac{\partial \phi}{\partial t} \quad \text{(superconducting phase evolution equation)} \quad (6)$$

$$J_{sc} = J_c \sin(\phi_2(t) - \phi_1(t)) \quad \text{(Josephson or weak-\link current – phase relation)} \quad (7)$$

The rate of change of the phase difference $\phi$ directly proportional to the voltage $V$ across the junction through the relation $\partial \phi / \partial t = 2 \text{eV}/\hbar$. From these two equations, it can be shown that the Josephson junction has an energy $-E_J \cos \phi$, where $E_J = \hbar c/(2e)$ is called the Josephson energy. Furthermore, the current-voltage relations reveal that the Josephson junction functions like a nonlinear inductance with $L = \hbar/(2ec \cos \phi)$. It is this nonlinearity of the Josephson junction that brings about the anharmonicity of JJJs and SQCs. In a given SQC, we can thus select the two lowest energy levels from the non-equally spaced energy spectrum. These two levels form a quantum bit (qubit) for quantum-information processing. When an ac voltage $V$ is applied to the two electrodes of the JJ, the supercurrent $I$ is periodically modulated as $I = I_c \sin(\omega t + \phi)$ with the Josephson frequency $\nu = \omega/(2\pi) = 2 \text{eV}/\hbar$, and the energy $\hbar \nu$ equals the energy change of a Cooper pair transferred across the junction.

H. Low-temperature superconductors (LTS) and high-temperature superconductors (HTSC). The discovery of HTSC in 1986 by Bednorz and Müller [48] and the following higher critical temperature superconductor discovery by Wu et al. [49] in 1987 and the following discoveries of even higher critical temperature superconductors (up to approx. 150 K) marked a great contrast with the already known low-temperature superconductors (LTSC). Figure 4 shows a graph of the critical temperature of many superconductors as a function of the year they were discovered. It is apparent in the graph the great number of superconductors already known at present. Many of them belong to families of compounds, as cuprates, the actinides, the pure SC elements, and so forth. The SC that are well described by the BCS theory are called conventional superconductors (cSC), and the ones that resist the BCS theory are called non-conventional superconductors (ncSC). It is generally believed that the HTSC cuprates are not conventional, because their energy gap is characterized as d-wave in contrast to the s-wave energy gap of conventional SC [50]. However, such belief has been challenged very recently [51] and the debate follows. The point is that we do not know fully the whole SC state of these HTSC cuprates.
4. Superconductors generate and respond to microwaves: the effect of microwave fields on superconducting tunneling in JJs and on SQCs

Beyond theoretical issues, given the great potential for their technological applications, the characterization of their properties is, and had been, a very intense activity worldwide for many years producing thousands and thousands of research reports. Here we concentrate on the interactions of microwaves with superconductors as single samples and as Josephson junctions (structures). We pay attention to systems where weak links are present. We mention here that the dissipation processes are plenty. It could be a type I or type II cSC, or an ncSC, or JJs and SQCs being bathed by microwaves.

Shapiro \[47\] introduced several types of JJs at X-band and K-band microwave cavities and determined experimentally, for the first time, the effects of microwaves of 9.3 GHz and 24.85 GHz on them. He found the, now well-known, Shapiro steps in the I-V characteristic curve. The effect of microwave power in modifying the I-V curve, especially the zero-voltage currents, is evident in the trace in Figure 9. The samples were mounted in a microwave cavity resonant, at low temperatures. All the observations were independent of sweep frequency and were also seen when dc was passed through the sample. The interval in voltage from one zero-slope region to the next is not always $h/e$; sometimes a step is missing so that the voltage interval is $h/e$.

Shapiro states “Josephson has already discussed briefly the effect rf should have on the pair-tunneling supercurrent. He predicted the occurrence of zero-slope regions separated by $h/e$ in the I-V characteristic in the presence of the rf field. This prediction was based on the frequency modulation by the rf of the ac supercurrents previously referred to. Our experiments have confirmed this prediction and represent indirect proof of the reality of Josephson’s ac supercurrent.”

If a Josephson junction (JJ) is microwave irradiated, it absorbs part of its energy (which can be used as a detection mechanism), and it excites periodic dynamics that again produce microwaves. These processes have been already in use.

Figure 9.
Microwave power at 9.3 GHz (a) and 24.85 GHz (B) produces many zero-slope regions spaced at $h/2e$ or $h/e$. for a, $h/e = 38.5$ pV, and for B, 103 pV. For a, vertical scale is 58.8 pV/cm and horizontal scale is 67 nA/cm; for B, vertical scale is 50 pV/cm and horizontal scale is 50 pA/cm. Similar effects occur at X-band and at K-band \[47\].
to produce microwaves with superconducting junctions [46, 52]. The so-called weak link junctions are appropriate for the development of ultra-low-power microwave sources [52, 53].

Wertham and others [46, 52] have studied the nonlinear self-coupling of Josephson radiation in diverse superconducting tunnel junctions. Since the original prediction by Josephson of an ac supercurrent generated in a superconducting tunnel junction by a dc bias voltage, considerable work has developed in the electromagnetic radiation associated with the ac current. Detection of the produced radiation is complicated by the high degree of nonlinearity of the device, JJ, SQC-detector, together with the large amount of feedback intrinsic to its configuration, networks of weak links, or tandems of JJs. The structures radiate into the insulator (conductors) and the surrounding superconductor islands-grains, and the resulting electromagnetic fields will feed back to influence significantly the currents which generate them. The starting point would be Maxwell’s equations for the electric and magnetic fields radiated from an oscillating charge distribution and then finding the radiation fields in the near zone [31]. Wertham shows detailed calculations and finds the ac tunneling current contains components at all harmonics of the fundamental frequency, ω₀ [52]. One would like to impose the requirement of very low loss in the radiating junctions, but that is inconsistent with the large amount of the external battery power being supplied by the dc current. The more power radiated, the more power supplied and more losses.

Very recently Tikhonov, Skvortsov, and Klapwijk have addressed the problem of nonequilibrium superconductivity in the presence of microwave irradiation [54]. Using contemporary analytical methods, they refine the old Eliashberg theory and generalize it to arbitrary temperature T and frequencies ω. Microwave radiation is shown to stimulate superconductivity in a bounded region in the (ω, T) plane. In particular, for T < 0.47Tc and for ω > 3.3kBTc, superconductivity is always suppressed by a weak ac driving. They also study the supercurrent in the presence of microwave irradiation and establish the criterion for the critical current enhancement. They interpret qualitatively in terms of the interplay between the kinetic (“stimulation” vs. “heating”) and spectral (“depairing”) effects of the microwaves [54].

Losses by heat, radiation, dissipative fluxoid dynamics, quasiparticle scattering, and other dissipating processes can limit considerably the SC performance as detector elements and in SC devices, or SQC, or as quantum bits (qubits). The microwave absorption profile of any given superconductor or Josephson junction is a fundamental measurement of performance, and it brings rich information on the SC electrons, the normal electrons, and the moving or stationary vortices and on their capacity to absorb and to produce microwaves in the range from GHz to THz.

Hence the response of the superconductors as bulk, as films, and as single arrays, or networks, of Josephson junctions to these electromagnetic excitations needs to be characterized experimentally. We focus on the measurement of losses in the laboratory using one of the most sensitive methods: perturbation cavity, the determination of the Q of the cavity, and its change due to material properties, or as a function of temperature.

5. Measurement of absorption of microwaves by superconductors. The experiment, the Q of electromagnetic cavities, the perturbation cavity method

One of the most sensitive measurement techniques to determine absorption of microwaves is the so-called perturbation cavity method. An electromagnetic resonant cavity made of very good conductor, or even superconductor (v.gr: niobium as for particle accelerators), reflects from its walls the electromagnetic fields inside
it. Absorption by the walls is very low. Two are the fundamental characteristics of a cavity that have in its interior a small SC piece as a sample to measure how much microwave power it absorbs [7–8, 32, 34–37, 47]: the quality factor, Q, and the frequency shift of the resonance, Δf. These changes can be very small, and it is convenient to include a very selective, filtered, amplification stage, such as the technique of lock-in amplification. What is measured is the quality factor change, \( \Delta Q = Q_S - Q_0 \), from the quality factor, \( Q_0 \), from the empty cavity, and the frequency shift produced by inductive changes in the electromagnetic energy inside the cavity [35–37]. Remembering, the quality factor is defined as [31]:

\[
Q = \omega_0 \left( \frac{\text{stored electrodynamic energy}}{\text{power loss}} \right) \tag{8}
\]

Its inverse is \( 1/Q = \frac{\text{[electromagnetic energy loss (dissipated) by the sample]}}{\omega_0 \text{[total electromagnetic energy inside the empty cavity]}} \).

Thus, effectively, the inverse of Q is a direct measurement of electromagnetic energy inside the cavity, other than the wall losses. Hence, \( \Delta(1/Q) \) is a direct measurement of the electromagnetic absorption by the superconducting sample inside the cavity. Knowing the eight fundamental properties of superconductors briefly described above will allow an informed interpretation of the measurements and disentangle several contributions to the microwave losses. The type of superconductor, the state it is in, the level of impurities, the presence of JJs, and in particular, the presence of weak links would determine the superconducting processes taking place and which one of them can generate microwave absorption. A typical granular HTSC, or LTSC, conventional, or unconventional, superconductor with many processes pictorially represented (but not all that could operate) is shown in Figure 10. It should be noticed how crowded it looks. This indicates partly its complexity and how many things are going on inside the sample.

An electron paramagnetic resonance equipment where absorption of microwaves is measured very accurately is an adequate way to measure microwave absorption of SCs. Such measurement is realized by capturing the reflected

![Image](image-url)
microwaves (not the absorbed microwaves) from the superconducting specimen and recombining (superposing) them with part of the incident waves (about 10% of these) within the microwave bridge, to reach the crystal detector. The crystal detector, in turn, generates a crystal current that is converted to voltage and then sent to the electronic filtering, amplification, and display units of the equipment. All these experimental stages are represented by blocks in part B of Figure 11.

Typically, an oscilloscope trace shows the mode of the unloaded cavity and of the loaded cavity. From the display, the shifted, $\Delta f$, and width, $\Delta Q$, and the reduced high with respect to the empty cavity modes, $f_0$ and $Q_0$, are obtained (see red oval in Figure 11B). The shift and the broadening are, respectively, proportional to the inductive conductivity (more generally, lossless processes that alter the eigenresonance of the cavity), $\sigma_I$. The differences in the width, $\Delta w$, are proportional to the

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**Figure 11.**

The combined theory and experiment involved in microwave losses by superconductors. Theory includes Maxwell electrodynamics, with complex impedance and complex conductivity, and radiation terms inside resonant cavities. SC theory is large and complex, and few characteristics are represented in the SC blocks. The connection is the quality factor $Q$. 

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loss of electromagnetic energy inside the cavity and due to the SC sample inside it, and these losses are quantified by $1/Q_s - 1/Q_o$. Now, how does the measurement of $Q$ connects with the whole of the superconductivity theory and Maxwell equations?

### 6. The combined superconducting theory and experiment of microwave losses

The electrodynamics theory of E and B fields and losses, $Q$, in resonant cavities is well developed. Jackson [31], Csősz et al. [55], Kwon et al. [56], and other books [1, 2] have explicit equations for losses due to dielectric, metallic, or superconducting materials. Following Kwon et al., here we present in compact form (block diagram) (Figure 11A) a summary of the electrodynamics and losses of superconductors in resonant cavities. We CONNECT theoretical expressions with the experimental block diagram that measures $Q$ (Figure 11B). The quantity that articulates both parts is the quality factor, $Q$, of the loaded cavity. On one hand, it can be arrived at from theory (BLUE OVAL), taking into account dissipative processes that develop in a superconducting sample or a Josephson junction. The SC theory represented by the leftmost blocks of the diagram feed the electromagnetic theory of $Q$ with fundamental SC parameters, such as the coherence length, $\xi$; the London penetration depth, $\lambda$; critical temperatures; energy gap, $\Delta$; values and critical fields, $H_c$, $H_1$, and $H_2$; their temperature and field behaviors; and so forth. On the other hand, we obtain $Q$ by measurement (RED OVALS) with the equipment shown in Figure 11B or any other version of measurement with the perturbation cavity method.

Figure 11 shows two block diagrams connected by the quantity $Q$. Block A includes the fundamental theory of superconductivity and $t$, the main phenomena, effects, and fundamental parameters, and block B shows a commonly used experimental arrangement for measuring microwave losses by means of the technique of cavity perturbation. Here, we include a field modulation stage that can bring much more sensitivity to details of the recordings [35–37]. But, the modulation stage can be bypassed and record directly the microwave absorption (see orange arrows in Figure 11). If temperature is slowly varied, then one obtains $Q(T)$ (from $P_{\text{ABS}}(T)$). Likewise, if magnetic field is varied (and excite a lot of vortex dynamics and quasiparticle dynamics), then one obtains $Q(H)$ (from $P_{\text{ABS}}(H)$).

Experimentally, it is possible to measure the $Q$ of the cavity directly as an output of the amplifier or directly from the detector used. If the output is being modulated, then the measurement is the derivative with respect to varying magnetic field, $H_o < H_1$, or $H_{c1} < H_o < H_{c2}$, if the superconductor is in the Meissner state or the SC sample is a type II superconductor in the mixed state. Our block diagram includes low-temperature equipment coupled to the resonant cavity, so the sample could be set at any temperature between 4.2 K and 77 K, or 300 K, or any desired temperature. Temperature can also be gradually changed in order to register $Q(T)$. By field modulation, the derivative of $Q(T)$ with respect to magnetic field can also be recorded. Much more structure is captured in this way. SC losses, as measured inside resonant cavities, continue to be vigorous due to the wealth of novel information that continues to be discovered.

Next, we take a SC system, granular, crystalline JJ, with strong or weak links, JJ networks, SQC, and so forth, that is bathed with microwaves coming from a particular direction; several portions of the sample will respond to microwaves differently. Take the JJ represented in Figure 10; quasiparticles and electrons and holes in the sandwiched (the “middle”) material will execute dissipative accelerated motions due to the Lorentz force they experience. Vortices will also execute...
translational viscous-dissipative motions because, again, the Lorentz force (or Magnus force) drives them [1–3, 26]. Pinned vortices will execute damped oscillations, and this will absorb electromagnetic energy. The normal electrons inside the fluxoid cores will move along with the fluxoids themselves; their motion is, again, dissipative. If we, abstractly, enumerate the different loss processes we have mentioned and some others that could operate (as the electrons and the holes produced in the multiple Andreev reflections), then we can write: total loss = Lt = L1 + L2 + L3 + L4 + ..., and 1/Qt = 1/Q1 + ....

We notice that several of these terms are of the same form with respect to the expression of the dissipated electromagnetic energy that enters into Poynting’s theorem and in consequence into the Q expression:

The higher the impurities and/or defects, the higher the pinning centers, the higher the microwave losses due to both fluxoid dissipative dynamics, (1/Q)impure. The higher the normal conductivity, σn, the lower the normal resistivity, and the larger the local mean free path that the normal charge carriers experience, and in consequence, the less charge scattering and less energy loss. Conversely, the lower the normal conductivity, the higher the normal resistivity and the higher the losses (1/Q). The higher the applied static field in the Hc1 < Ho < Hc2 range, the larger the number of fluxoids present or forming at the edge of the SC, Φ = nΦo, the more elements executing dissipative dynamics and more overall microwave loss.

Varying temperature: given a superconductor, either HTCS or LTSC, the lower the temperature with respect to its critical temperature (Tc), let us say 22%, the more electrons form Cooper pairs and they will move around without dissipation. The electrons that remain normal are less and less as T departs from Tc; these electrons will produce less losses, (1/Q)T, than the same material at, let us say, 77% of Tc. Hence, for identical materials, all other parameters fixed, we would expect (1/Q)T = 22% < (1/Q)T = 77%.

From the theory, the integrals of electromagnetic energy are related to the complex conductivity, \( \sigma = \sigma_n - i\sigma_{SC} \), and with the Q of the cavity. These integrals are a deduction of the electrodynamics of the superconductor based in Maxwell equations and the London equations. The superconducting parameters, \( \xi, \lambda, \Delta, \kappa, \sigma_s, \sigma \), are involved in these equations, and hence they determine also the microwave absorption. From the experimental side, we have represented by blocks the main constituents of the generation of microwaves, the resonant cavity, and the detection system. The quantity measured is the Q of the cavity. Contrasting the theoretical formulas with the measured absorption and knowing the type of SC sample and/or if it is Josephson junction, an assignment of the losses can be made.

As a manner of example, we take the case of magnetic field dependent microwave losses in superconducting niobium microstrip resonators reported by Kwon et al. [56]. They find that quasiparticle generation is the dominant loss mechanism for parallel magnetic fields. For perpendicular fields, the dominant loss mechanism is vortex motion or switches from quasiparticle generation to vortex motion, depending on the cooling procedures. They calculate the expected resonance frequency and the quality factor as a function of the magnetic field by modeling the complex resistivity. Key parameters characterizing microwave loss are estimated from comparisons of the observed and expected resonator properties. Based on these key parameters, they find that a niobium resonator whose thickness is similar to its penetration depth is the best choice for X-band electron spin resonance applications. They also detect partial release of the Meissner current at the vortex penetration field, suggesting that the interaction between vortices and the Meissner current near the edges is essential to understand the magnetic field dependence of the resonator properties.

Most recent results: Very recently the absorption of microwaves by granular, fine powders of MgB2 and K3C60 type II superconductors was measured and found to
be giant, much larger than in the normal state, for magnetic fields as small as a few % of the upper critical field, a quite unexpected result [55]. The authors had the care to experimentally have the grains separated in large distances, so no weak links could be formed and no microwave absorption at the Josephson junctions could take place since no JJ were present.

Yet the microwave absorption is very large, and it represent the case in which \( \sigma_1 \gg \sigma_2 \) [46]. We need to remember that \( \sigma_1 \) is due to the normal conductivity and \( \sigma_2 \) is the superconducting conductivity.

The effect is predicted by the theory of vortex motion in type II superconductors; however, its direct observation had been elusive due to skin-depth limitations; conventional microwave absorption studies employ larger samples where the microwave magnetic field exclusion significantly lowers the absorption. The authors show that the enhancement is observable in grains smaller than the penetration depth. A quantitative analysis on K3C60 in the framework of the Coffey-Clem (CC) theory (see the connection theory experiment in Figure 11) explains well the temperature dependence of the microwave absorption and also allows to determine the vortex pinning force constant \( P \).

Dissipation and radiation of microwaves of the HTSCs have been measured since their discovery. Bednorz and Muller (the very same discoverers) and Blazey and many others [34–38] reported the modulated microwave absorption of YBaCuO and of several other HTSC families. The basic equipment used in these studies is pretty much like the one shown in Figure 11. The electron paramagnetic resonance equipments are well suited to carry on these \( Q \) and \( \Delta f \) determinations provided that a zero magnetic field unit that could cross the zero value of the magnetic field is incorporated to a EPR spectrometer, in addition to a low-temperature (helium and/or liquid nitrogen) system. The cuprates, with their 3d\({ }^{9}\) unpaired electron, were expected to elicit EPR signals, and in consequence, some microwave absorption by this means was also expected. And certainly, these signals and absorptions have been detected and reported many times [34–38]. But in addition, the cuprates show an important non-resonance absorption at zero magnetic field or near-zero magnetic field [34–37]. Now it is clear that the Cu unpaired, bounded electron is not responsible of such microwave absorption. Bednorz and Muller and other authors have identified as the main absorbers, in the cuprates, the fluxoids, their dissipative dynamics and the loss processes in the weak links.

A strong microwave absorption at low magnetic fields is observed for a variety of metallic cuprates below their critical superconducting transition temperatures (\( T \)); using conventional electron paramagnetic resonance (EPR), instrumentation has been recorded [35]. The low-field microwave signal was investigated in the following high-temperature superconductors, YBaCuO (\( T = 95 \text{ K} \)), GdBaCu (\( T = 95 \text{ K} \)), etcetera. It has been investigated the usefulness of the technique as a practical noninvasive method for screening potentially superconducting samples, with particular emphasis on sensitivity, the effect of microwave power, and the inherent problems of studying signals at small magnetic fields [32, 35].

### 7. Conclusions

The fundamental property of superconductors to absorb microwave energy in spite of the expulsion of static magnetic fields (Meissner effect) has been explored for various SC, and its use in superconducting microwave technology has been indicated. The experimental measurements of the absorption of microwaves in the form of ESR-like measurements were described, and emphasis was put on the quality factor as one central quantity to measure, and we stressed its great theoretical
understanding. $Q(T)$ and $Q(H)$ magnetically modulated (or not) from microwave absorption were described, and few examples, spanning about 70 years of this field of research, were given. The knowledge gained on microwave losses can inform the design of new superconducting devices, SQC, JJs, SC qubits, microwave detectors, and/or radiators, operating in heavy microwave environments and/or as microwave devices themselves.

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