THE LARGE SCALE ORGANIZATION OF TURBULENT CHANNELS

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I. OBJECTIVES

We have investigated the organization and dynamics of the large turbulent structures that develop in the logarithmic and outer layers of high-Reynolds-number wall flows. These structures have sizes comparable to the flow thickness and contain most of the turbulent kinetic energy. They produce a substantial fraction of the skin friction and play a key role in turbulent transport.

In spite of their significance, there is much less information about the large structures far from the wall than about the small ones of the near-wall region. The main reason for this is the joint requirements of large measurement records and high Reynolds numbers for their experimental analysis. Their theoretical analysis has been hampered by the lack of successful models for their interaction with the background small-scale turbulence.

II. RESEARCH ROUTE AND METHODS

We have performed new direct numerical simulations of turbulent channels at higher Reynolds numbers, \(180 \lesssim Re_\tau \lesssim 1900\), and in larger computational domains than the ones previously available \([1, 2]\). They have been the first numerical experiments of wall turbulence where the dynamics of the large scales of the logarithmic region are properly captured. Here, \(Re_\tau = u_\tau h/\nu\) is the friction Reynolds number, based on the friction velocity \(u_\tau\) and the channel half height, \(h\). The space discretization is spectral, with Fourier expansions in the streamwise and spanwise directions \((x\) and \(z\), and Chebyshev polynomials in the wall-normal direction \((y)\). The time integration scheme is a third-order Runge-Kutta. The resulting numerical problem, whose size is \(O(10^{14})\) space-time nodes, has been solved in supercomputer centers of Spain and the USA. For this purpose we have developed a parallel code that has run efficiently on up to 384 processors.

The results from our simulations are increasing becoming the reference numerical data base for wall turbulence researchers. Part of them can be downloaded from our servers [9]

http://torroja.dmt.upm.es/ftp/channels
and
http://davinci.tam.uiuc.edu/data/channels

The post-processing of the simulation results has adopted two complementary approaches. The first one has been based on statistical analysis. We have studied the scaling properties of the three-dimensional two-point correlation functions of the flow variables \([1, 2]\). Previous experimental and numerical studies were restricted to only one dimension. The second approach has been based on the analysis of turbulent structures extracted from instantaneous flow fields. We have developed a novel method to isolate turbulent eddies by using a thresholding operation with variable thresholds, which can be determined from the analogy of a percolation transition \([1, 2]\). This method has allowed us to extract \(O(10^6)\) eddies from our simulations and to analyse them in a well-defined systematic manner, free from the subjective interpretations that are usually inherent to structural analysis.

Finally, we have proposed a simplified model for the dynamics of the large structures based on the linear stability equations for the turbulent mean profile \([1, 2]\). The model includes an eddy viscosity to represent the dissipation felt at the large scales because of the smaller ones. Because the turbulent mean profile was shown to be stable decades ago, this problem had been abandoned by the research community. However, we will show below that these equations have transiently-amplified solutions that reproduce the dominant structures in channels.

III. OUTCOME

This thesis has characterized for the first time the dynamics and structure of the self-similar range of wall turbulence. It has given place to four papers in the Journal of Fluid Mechanics \([1, 2, 3]\) and one in Physics of Fluids \([2]\). Another paper is in preparation \([4]\).

The streamwise turbulent velocity \((u)\) fluctuations are organized forming very large structures whose lengths and widths, \(\lambda_x = 5 - 10 h\) and \(\lambda_z = 2 - 3 h\), scale with the channel half height \(h\) \([2]\). These structures span the whole flow thickness in the wall-normal direction, from the near-wall region to the center of the channel, and for this reason we have called them global modes \([1, 2]\).

The influence of the global modes in the near-wall region leads to an incomplete scaling of the turbulent energy spectra that has important implications on the \(Re_\tau\) dependence of the turbulence intensity. While the classical theory suggests that the turbulence intensity in the near-wall region does not depend on \(Re_\tau\) when expressed in wall units \([2]\), the observed incomplete scaling of the spectra implies that it should increase as \(\log(Re_\tau)\).
This logarithmic correction explains for the first time the $Re_\tau$ behaviour of the existing near-wall measurements of the turbulent intensity up to atmospheric boundary layer Reynolds numbers, $Re_\tau = O(10^6)$. See figure 1a.

We have also found and explained that, far from the wall, the intensity of the global modes scales with the mean stream velocity, $U_c$ [1]. This introduces a mixed scaling for the turbulent kinetic energy in the outer region which tends to $U_c^2$ for $Re_\tau \to \infty$. This new scaling leads to a log$^2(Re_\tau)$ correction to the classical one in the outer region which is consistent with the available laboratory one-point measurements, shown in figure 1b.

In the logarithmic layer, the energy spectra from our simulations reveal important anomalies in the relation between the lengths $\lambda_x$ and widths $\lambda_y$ of the largest turbulent structures. Figure 2 shows that the form of the anomalous scaling is $\lambda_x \sim (\lambda_x y)^{1/2}$. This suggests that the $u$ structures are “wakes” generated by the stirring of the mean profile caused by compact eddies of the wall-normal ($v$) and spanwise ($w$) velocity fluctuations [1, 4]. The value $1/2$ of the spreading exponent suggests that the large structures are dispersed passively by the incoherent background turbulence. The self-similarity failure introduces further logarithmic corrections to several ranges of the classical $k^{-1}$ spectrum of the logarithmic region, which have been verified using data from our simulations and from laboratory experiments at very high Reynolds numbers [1].

The compact eddies that generate the $u$ wakes have been isolated in instantaneous flow realizations. They are associated to vortex clusters which are rooted in the near-wall region and that reach very far from the wall. On average, these eddies consist of a wall-normal ejection surrounded by two inclined counter-rotating vortices, as shown in figure 3. Although this average structure is consistent with a single large-scale vortex loop, most of the individual clusters are more complex. Their lengths and widths ($\Delta_x, \Delta_y$) are proportional to their heights ($\Delta_z$) and grow self-similarly with time after originating at different wall-normal positions in the logarithmic region [1]. They agree with the dominant scales of the $v$ spectrum in that region, as shown in figure 2.

Figure 3 indicates that the clusters are in fact associated to larger structures of $u$ that appear in their wakes. The average geometry of these structures is a cone tangent to the wall along the $x$ axis. The clusters form groups of a few members within each cone, with the larger individuals in front of the smaller ones. This behaviour has been proven consistent with the $\lambda_x \sim (\lambda_x y)^{1/2}$ scaling of the energy spectrum in the logarithmic layer [1].

The clusters themselves are triggered by the wakes left by yet larger clusters in front of them. The whole process repeats self-similarly in a disorganized version of the well-known vortex-streak regeneration cycle of the near-
Figure 3: Three-dimensional plot of the average velocity field conditioned to the vortex clusters, $\langle \vec{u} \rangle$, in a reference frame fixed to their centers. The spatial coordinates, $\vec{r}$, are normalized with the wall distances of the cluster centers, $y_c$. The black mesh is an isosurface of the p.d.f. of the vortex positions and contains 57% of the data. The blue volume surrounding the cluster is the isosurface $\langle u' \rangle^+ = 0.3$. The red volume downstream of the cluster is the isosurface $\langle u' \rangle^+ = -0.1$. The green objects are vortices of the average velocity field conditioned to the clusters. They have been obtained plotting an isosurface of the discriminant of the velocity gradient tensor. The arrow plots represent $(\langle v \rangle, \langle w \rangle)$ in the planes $r_x/y_c = 10, 20$. The upper left corner shows a magnification of the surroundings of the average position of the clusters, including a vector plot of $(\langle v \rangle, \langle w \rangle)$ in the plane $r_x = 0$. The longest arrow measures $0.5u_\tau$. The data come from our numerical turbulent channel at $Re_\tau = 950$.

This linear spreading can be modelled by the Orr-Sommerfeld-Squire’s equations for the mean turbulent profile, using the eddy viscosity required to maintain that profile [3, 4]. The dominant structures of $u$ in turbulent channels are well described by the solutions with the largest transient growth. Two maxima are found (see figure 4). The two peaks separate well when $Re_\tau$ is large enough, and scale respectively in inner and outer units. One corresponds to the sublayer streaks $(\lambda_z^+ = 100)$ and the other one to the global modes $(\lambda_z = 3h)$. The intermediate minimum is not very pronounced, and describes self-similar modes that agree well with the observed structures of the logarithmic layer [5].

The structures for the transverse velocity also agree well with the highest-growth solutions, although they decay soon both in the linear model and in direct simulations. They act mainly as 'seeds' for the longer-lived and stronger structures of the streamwise velocity [5].

[1] J. C. del Álamo, J. Jiménez, P. Zandonade, and R. D. Moser. Scaling of the energy spectra of turbulent channels. *J. Fluid Mech.*, 500:135–144, 2004.
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[3] J. Jiménez, J. C. del Álamo, and O. Flores. The large-scale dynamics of near-wall turbulence. *J. Fluid Mech.*, 505:179 – 199, 2004.
[4] J. C. del Álamo, J. Jiménez, P. Zandonade, and R. D. Moser. Self-similar vortex clusters in the turbulent logarithmic region. *J. Fluid Mech.*, 2006. In press.
[5] J. C. del Álamo and J. Jiménez. Linear energy amplification in turbulent channels. *J. Fluid Mech.*, 559:205–213, 2006.
[6] J. C. del Álamo, O. Flores, J. Jiménez, P. Zandonade, and R. D. Moser. The linear dynamics of the turbulent logarithmic region. *In preparation*, 2006.
[7] A variable is expressed in wall units when normalized with $u_\tau$ and $\nu$. This is indicated with a $^+$ superscript.
Figure 4: Maximum energy amplifications $G(\lambda_\tau)$ obtained from the linear stability equations of the mean turbulent profile for fixed disturbance length $\lambda_x = 60h$ and different Reynolds numbers. 

- $\ldots$, $Re_\tau = 200$; 
- $\ldots$, $Re_\tau = 500$; 
- $\blacktriangle$, $Re_\tau = 10^3$; 
- $\ldots$, $Re_\tau = 2 \times 10^3$; 
- $\ldots$, $Re_\tau = 5 \times 10^3$; 
- $\blacktriangle$, $Re_\tau = 10^4$; 
- $\ldots$, $Re_\tau = 2 \times 10^4$. 

(a) $G(\lambda_x)$; the solid vertical line is $\lambda_x = 100$. 
(b) $G(\lambda_x/h)$; the solid vertical line is $\lambda_x = 3h$. 