Predictor-Based Control for Nonlinear Mechanical Systems with Measurement Delay.*

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INTRODUCTION

Industry 4.0 is the forth industrial revolution, by integrating the Internet of Things into factories they will interact with other factories, suppliers, consumers, transportation and smart grids. Even more, inside the factory different cells and production stages will be interconnected, making requests and giving notice of the production status, all of these interactions supported by wireless sensor-actuator networks (WSAN) Kagermann et al. (2013).

Nevertheless, one of the main withdraws that holds back the implementation of these industrial paradigms is the requirements of the wireless networks. According to Kagermann et al. (2016), the minimum required latency for WSAN intended to perform motion control is 1 ms with a reliability of 99.9999999% , however, current wireless industrial networks such as ISA100, wirelessHART and ZigBee are unable to fulfill these requirements.

Having that in mind, it is important to develop control strategies that enables the use wireless technologies to be implemented. In this paper it is proposed a strategy to deal with delays in the measurement of the state process, which are caused mainly by the communication delay between the sensors and the controller Fridman (2014).

Nonlinear mechanical systems constitute a good case of study since most of the industrial robots belong to this class of systems. Also, robots are popular because they serve various practical purposes. Decker et al. (2017) study since most of the industrial robots belong to this class of systems. Also, robots are popular because they serve various practical purposes. Decker et al. (2017)

The control of systems with delay is a current challenge for the control community Richard (2003). Delays complicate the direct implementation of control techniques, because the introduction of a delay in the output can down-perform the controller or even destabilize the system when delays are not taken into account.

The strategy proposed in this work is a predictor-based approach which enables to compensate for the time delay resulting in the delay-free closed-loop system, can be applied Léchappé et al. (2015); Loukianov et al. (2017); Caballero-Barragán et al. (2018, 2016).

Nevertheless, the predictors proposed in Léchappé et al. (2015); Loukianov et al. (2017); Caballero-Barragán et al. (2018, 2016) are designed for linear systems, and the systems considered in this work are nonlinear, since they are mechanical systems modeled using the Euler-Lagrange formulation. So, for the proposed scenario, the consideration of a delay in the measurement of a nonlinear system complicates the design of a controller Richard (2003).

Still, there are in the literature predictors designed for nonlinear systems such as Bresch-Pietri et al. (2015);
Bekiaris-Liberis and Krstic (2016, 2017) that can be used in order to design a controller for a robotic mechanical system.

The rest of the paper is organized as follows. In Section 2 the main mathematical concepts used in this paper are reviewed to give the reader a general idea of the applied techniques. In Section 3 the statement of the problem is made, lightening out the things aimed to solve in this work. In Section 4, it is applied a predictor for mechanical nonlinear systems and the tracking PD controller is designed using the predictor to compensate the delay effect, two examples are presented. A tracking controller for time-varying references is designed for mechanical systems with measurement delay, and it is presented in the Section 5.

Then, the case for disturbed system with time delay is considered proposing a way to estimate and compensate the external disturbances. Finally, in Section 7 the conclusions for this work are given.

2. MATHEMATICAL BACKGROUND

2.1 Predictor for Nonlinear Systems

Consider the following nonlinear system:

\[
\dot{x}(t) = f(x(t), u(t-D))
\]

where \( x \in \mathbb{R}^{2n} \) is the state, \( u \in \mathbb{R}^{n} \) is the control input, \( D \) is a known scalar constant and \( f : \mathbb{R}^{2n} \times \mathbb{R}^{n} \to \mathbb{R}^{2n} \) is a locally Lipschitz vector field that satisfies \( f(0,0) = 0 \).

The predictor \( \xi(t) = x(t+D) \) for the system (1) is designed as follows Bekiaris-Liberis and Krstic (2017)

\[
\xi(t) = x(t) + \int_{t-D}^{t} f(\xi(\theta), u(\theta))d\theta
\]

where \( \xi \in \mathbb{R}^{2n} \) is the predictor state. Now, taking the time derivative of (2) along trajectories of (1), the delay-free system reads as

\[
\dot{\xi}(t) = f(\xi(t), u(t)).
\]

The controller \( u \) can be designed using system (3).

3. STATEMENT OF PROBLEM

Consider the mechanical model using Euler-Lagrange formulation

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= M(x_1)^{-1}(-C(x_1, x_2)x_2(t) - G(x_1) + \tau(t)), \\
\dot{y}(t) &= x(t-D) = \begin{bmatrix} x_1(t-D) \\ x_2(t-D) \end{bmatrix},
\end{align*}
\]

where \( x_1(t) \in \mathbb{R}^{n} \) is the angular position vector, \( x_2(t) \in \mathbb{R}^{n} \) is the angular velocity vector, \( M(x_1) \in \mathbb{R}^{n \times n} \) is the inertia matrix, \( C(x_1, x_2) \in \mathbb{R}^{n \times n} \) is the matrix of Coriolis and centrifugal forces, \( G(x_1) \in \mathbb{R}^{n} \) is the vector of gravitational forces, \( \tau(t) \in \mathbb{R}^{n} \) is the vector of control input, \( y(t) \) is the available measurement state and \( D \) is the time-delay.

Assumption 1. The state vector \( x(t-D) = \begin{bmatrix} x_1(t-D) \\ x_2(t-D) \end{bmatrix} \) is available.

4. PREDICTOR AND CONTROL DESIGN

\[
\xi(t) = \dot{x}(t+D) = \dot{x}(t) + \int_{t-D}^{t} f(\xi(\theta), \tau(\theta))d\theta
\]

where

\[
f(\xi(\theta), \tau(\theta)) = \begin{bmatrix} M(\xi_1(\theta))^{-1}(\xi_2(\theta), \xi_2(\theta))\xi_1(\theta) - G(\xi_1(\theta)) + \tau(\theta) \end{bmatrix},
\]

with \( \xi(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} \).

4.1 Example 1.

The predictor’s performance is shown using the following model of a two-link planar manipulator (see Craig (2005) and Fig. 2)

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= M(x_1)^{-1}(-C(x_1, x_2)x_2(t) - G(x_1) + \tau(t)), \\
x(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \\
x(0) &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } \tau(t) = 0.1[\sin(0.5t) \sin(0.7t)]^T.
\end{align*}
\]

\[
M(x_1) = \begin{bmatrix} l_2^2 m_2 + 2l_1 l_2 m_2 c_2 + l_2^2 (m_1 + m_2) & l_1 l_2 m_2 c_2 \\ l_1 l_2 m_2 c_2 & l_1^2 m_2 
\end{bmatrix},
\]

\[
C(x_1, x_2) = \begin{bmatrix} 0 & -2m_2 l_1 l_2 s_2 q_1 - m_2 l_2 s_2 q_2 \\ m_2 l_2 s_2 q_1 & m_2 l_2 s_2 q_1 + m_2 l_2 c_2
\end{bmatrix}
\]

and

\[
G(x_1) = \begin{bmatrix} m_2 l_2 g c_1 + (m_1 + m_2) l_1 g c_1 \\ m_2 l_2 g c_1
\end{bmatrix},
\]

with \( c_1 = \cos(q_1), c_2 = \cos(q_2), s_2 = \sin(q_2), c_{12} = \cos(q_1 + q_2), g = -9.81 \text{ m/s}^2, l_1 = 0.9 \text{ m}, l_2 = 0.7 \text{ m}, m_1 = 0.4 \text{ kg}, m_2 = 0.3 \text{ kg} \) and \( D = 100 \text{ ms} \).

Fig. 1. The control scheme.

Assumption 2. The system (4) is fully actuated and the \( M(x_1) \) matrix has full rank. Also, its inverse \( M(x_1)^{-1} \) exists \( \forall x_1 \).

The objective of this work is to design a controller using a predictor for nonlinear systems and reduce the time-delay effect. Fig. 1 shows the general control scheme proposed in this work. Having a desired reference for the state of the system and considering a delay \( D \) in the measurement, the controller has to be designed in order to track said reference by means of a predictor.
To illustrate the performance of the proposed controller (7), the closed-loop system is simulated using MATLAB.

Fig. 5 shows the behavior of $\tilde{x}(t)$ and $\xi(t - D)$.

The predictive error $e_p(t) = \bar{x}(t) - x(t - D)$.

or

$$\dot{\xi}(t) = \xi_2(t)$$
$$\ddot{\xi}_2(t) = M(\xi_1)^{-1}(-C(\xi_1, \xi_2)\xi_2(t) - G(\xi_1) + \tau(t))$$

To make the tracking of the reference $x_{ref}$ by the system (4) output $x_1$ the control law $\tau$ is chosen of the form

$$\tau(t) = -K_p e(t) - K_v \xi_2(t) + G(\xi_1),$$

where $e(t) \in \mathbb{R}^n$ is the vector of angular position error, and it is defined as

$$e(t) = \xi_1(t) - x_{ref},$$

with the positive defined matrices $K_p \in \mathbb{R}^{n \times n}$ and $K_v \in \mathbb{R}^{n \times n}$.

To prove the convergence of the closed-loop system (4) and (7), the following positive definite candidate Lyapunov function is proposed:

$$V = \frac{1}{2} e^T K_p e + \frac{1}{2} \xi_2^T M(\xi_1) \xi_2.$$  

(8)

Taking the time derivative of (8) along the trajectories of the closed-loop system yields

$$\dot{V} = \xi_2^T K_p e + \xi_2^T M(\xi_1) \dot{\xi}_2 + \frac{1}{2} \xi_2^T M \ddot{\xi}_2,$$

$$= \xi_2^T K_p e + \xi_2^T M(\xi_1)(M(\xi_1))^{-1}(-C(\xi_1, \xi_2)\xi_2 - G(\xi_1))$$
$$- K_p e - K_v \dot{\xi} + G(\xi_1)) + \frac{1}{2} \xi_2^T M \ddot{\xi}_2,$$

$$= -\xi_2^T C(\xi_1, \xi_2)\xi_2 + \frac{1}{2} \xi_2^T M \ddot{\xi}_2 - \xi_2^T K_v \dot{\xi}_2.$$  

(9)

Making use of the property for articular robots stated in Kelly and Santibañez (2003) $\ddot{q}^T (\frac{1}{2} M - C) \ddot{q} = 0$, the derivative (9) becomes

$$\dot{V} = -\xi_2^T K_v \dot{\xi}_2.$$  

(10)

Since the function (10) is semi negative definite the LaSalle invariance principle can be easily applied to show that the equilibrium point $e = 0$ and $\dot{\xi}_2 = 0$ of the closed-loop system is asymptotically stable.

4.3 Example 2.

To illustrate the performance of the proposed controller (7), the closed-loop system is simulated using MATLAB.
Simulink. The matrices for the system (4) are defined accordingly to the robot structure depicted in Fig. 2. Consider the parameters described in example 1 4.1.

The control gains are described by

\[ K_p = \begin{bmatrix} 15 & 0 \\ 0 & 13 \end{bmatrix} \quad \text{and} \quad K_v = \begin{bmatrix} 9 & 0 \\ 0 & 7 \end{bmatrix}. \]

The constant reference is described as follows

\[ x_{ref} = \begin{bmatrix} \pi/4 \\ 0 \end{bmatrix}. \]

The control is defined as (7), reads as

\[ \tau(t) = - \begin{bmatrix} 15 & 0 \\ 0 & 13 \end{bmatrix} e(t) - \begin{bmatrix} 9 & 0 \\ 0 & 7 \end{bmatrix} \dot{\xi}_2(t) + G(\xi_1), \]

where \( e(t) = \dot{x}_1(t) - x_{ref} \).

Simulations results are presented in Figs. 6-8. Fig. 6 presents the behavior of the system state \( \ddot{x}(t) \). Fig. 7 shows the tracking of the constant reference. The torque control is presented in Fig. 8. The results without predictor are not presented because the result is foreseeable and well-known (instability or best-case scenario a bad transient behavior in close-loop).

5. TRACKING OF TIME-VARYING REFERENCE

Consider the system (4) under Assumption 1 and using the predictor (5), the delay-free system is obtained as (6)

\[
\begin{align*}
\dot{\xi}_1(t) &= \dot{\xi}_2(t) \\
\dot{\xi}_2(t) &= M(\xi_1)\dot{\xi}_2(t) - G(\dot{\xi}_1) + \tau(t),
\end{align*}
\]

(11)

To track time-varying references the controller \( \tau(t) \) is designed as follows. The time-varying references are obtained by following exosystem:

\[ x_{ref}(t) = p(g, \dot{g}) = d(g). \]

(12)

Now, using the reference \( x_{ref}(t) \), the tracking error can be defined as follows

\[ e(t) = \ddot{x}_1(t) - x_{ref}(t). \]

(13)

Using the backstepping technique Krstic et al. (1995) the tracking control is designed in two steps.

**Step 1.**
Taking the time derivative of (13), the dynamic of \( e(t) \) is obtained

\[ \dot{e}(t) = \ddot{\xi}_1(t) - \ddot{x}_{ref}(t). \]

(14)

Now, using the virtual control \( \ddot{\xi}_2 \), the following change variable is proposed

\[ \dot{z}_1(t) = \ddot{\xi}_2(t) + K_1 e(t) - \ddot{x}_{ref}(t), \]

(15)

where \( K_1 > 0 \) is a positive and symmetric matrix. A candidate Lyapunov function is proposed and it reads as

\[ V_1(t) = \frac{1}{2} e^T(t) e(t). \]

(16)

Taking the time derivative of (16) and using (14) and (15), results

\[
\begin{align*}
\dot{V}_1(t) &= e^T(t) \dot{e}(t) = e^T(t)(-K_1 e(t) + z_1(t)) \\
&= -e^T(t)K_1 e(t) + e^T(t)z_1(t).
\end{align*}
\]

(17)

**Step 2.**
Now, taking the time derivative of (15) along the trajectories (11) and (14), it read as

\[
\dot{z}_1(t) = M^{-1}(\ddot{\xi}_1)(-C(\xi_1, \dot{\xi}_2)\dddot{\xi}_2(t) - G(\dot{\xi}_1) + \tau(t)) + K_1 \dot{\xi}_2(t) - K_1 \ddot{x}_{ref}(t) - \ddot{x}_{ref}(t).
\]

(18)

The control \( \tau(t) \) is designed as follows

\[
\begin{align*}
\tau(t) &= C(\ddot{\xi}_1, \dot{\xi}_2)\dddot{\xi}_2(t) + G(\dot{\xi}_1) + M(\dddot{\xi}_1)(-K_1 \dot{\xi}_2(t) + K_1 \ddot{x}_{ref}(t) + \dddot{x}_{ref}(t) - K_2 z_1(t) - e(t).
\end{align*}
\]

(19)
where $K_2 > 0$ is a positive and symmetric matrix. The closed-loop system by the control \(19\) results
\[
\dot{e} = -k_1 e(t) + z_1(t) \\
z_1 = -k_2 z_1(t) - e(t).
\] (20)

A candidate Lyapunov function is proposed as follows
\[
V_2(t) = V_1(t) + \frac{1}{2} z_1^T(t) z_1(t).
\] (21)

Now, taking the time derivative of (21) and using (17) and (20), it reads as
\[
\dot{V}_2(t) = -e^T(t) K_1 e(t) - z_1^T(t) K_2 z_1(t) \\
\leq -\lambda_{\text{min}}(K_1) \|e(t)\| - \lambda_{\text{min}}(K_2) \|z_1(t)\|.
\] (22)

The time derivative of (21) is negative defined then the tracking control $\lim_{t \to \infty} e(t) \to 0$.

5.1 Example 3.

To test the effectiveness of the controller described in this section the following model of two-link planar manipulator (see Craig (2005) and Fig. 2) described in section 4.1 is used:
\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= M(x_1)^{-1}(-C(x_1, x_2)x_2(t) - G(x_1) + \tau(t)),
\end{align*}
\]
with the same matrices description and parameters.

The exosystem that describe the references to track is described by following system:
\[
\begin{align*}
\dot{g}(t) &= \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix} g(t) \\
x_{\text{ref}}(t) &= g(t) + \begin{bmatrix} \pi/3 \\ \pi/7 \end{bmatrix},
\end{align*}
\] (23)

where $g(0) = [0.2 \ 0]^T$ and $\alpha = 2$. The state available is $x(t - D) = \bar{x}(t)$. The control gains are $K_1 = K_2 = \begin{bmatrix} 8 & 0 \\ 0 & 7 \end{bmatrix}$.

The simulation results are presented in Fig. 9. The predictor tracks the reference $x_{\text{ref}}(t - D)$ and the measurement state $\bar{x}(t)$ tracks the retarded reference $x_{\text{ref}}(t - D)$ as Fig. 9 shows. This is a natural situation when working with systems with delay in the input and/or in the output.

6. TRACKING OF REFERENCE UNDER DISTURBANCES

Consider now the system (1) under external disturbances, such that the description of the dynamics result as
\[
\dot{x}(t) = f(x(t), u(t - D), \Delta(t))
\] (24)
where $\Delta(t)$ is a disturbance vector of appropriate dimensions and $x$ is the state vector under disturbance.

Using as example the model described in section 4.1 under disturbance:
\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= M(x_1)^{-1}(-C(x_1, x_2)x_2(t) - G(x_1) + \tau(t)) + \\
& \quad + \Delta(t),
\end{align*}
\]
with the same matrices description and parameters and setting the disturbance as $\Delta(t) = M(x_1)^{-1} \begin{bmatrix} 2.5 \\ 1 \end{bmatrix}$. Using the controller described in (7) the tracking performance can be seen in Fig 10, with the references set at $2\pi/3$ and $-2\pi/3$ for each angle. The blue and brown lines are the tracking performance under disturbance.

The error in the tracking is present because predictors are highly sensitive to external disturbances, as is reflected in the predictor performance under disturbance $\Delta(t)$.

In order to deal with these undesired effects, it is necessary to mitigate the effect of the disturbances in the predictor so it does not affect the tracking performance. To do so, an approximation of the disturbance $\Delta(t)$ is proposed as:
\[
\begin{align*}
\bar{x} &= \begin{bmatrix} x_1(t - D) \\ x_2(t - D) \end{bmatrix}, \\
\hat{\Delta}(t) &= \frac{\bar{x}(t) - \bar{x}(t - \lambda)}{\lambda} - f(\bar{x}, u(t - D)),
\end{align*}
\] (25)
with $\lambda$ a design constant. Using this $\hat{\Delta}(t)$ approximation in the predictor design
\[
\xi(t) = \bar{x}(t) + \int_{t - D}^{t} [f(\bar{x}(\theta), u(\theta)) + \hat{\Delta}(\theta)] d\theta.
\] (26)
And, since the prediction improves, the tracking error also diminishes as shown in Fig 11, where the purple and green lines are the tracking performance of each joint and the black lines are the references to be reached. In order to compare the effect of the use of disturbance estimation on the prediction, Fig 12 shows the mean squared error of the tracking performance with and without the use of the disturbance estimation.

7. CONCLUSIONS

In this work a predictor-based tracking control for mechanical systems with measurement delay is presented. The predictor is applied in simulation to show the behavior in open-loop and near to an equilibrium point. The predictor and tracking PD controller is designed to control the mechanical system with measurement delay and to track constant references. Simulation results are presented. A tracking control for time-varying references and a predictor for mechanical systems with measurement delay is applied using an exosystem to get the time-varying reference and the simulations results are shown as well. Also, it is considered the case of the tracking performance under disturbances. In order to diminish the effect of these disturbances on the prediction error (and consequently on the tracking error) an estimation of the disturbances is proposed and the simulation results are shown. As a future work is proposed to considered parametric variation in the systems and to consider time-varying delays. This is interesting because in this case there exist prediction error and as a result the analysis is more challenging and complex.

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