Unified Power Conservative Equivalent Circuit for DC Networks

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ABSTRACT  
This article introduces a new equivalent circuit for linear direct current networks consisting of independent voltage and current sources, and resistors, which represents all the power dissipated internally in the resistors. It is demonstrated that the internal losses of any network have two components. One is variable and dependent on the internal resistances of the actual circuit and the power transferred to the pair of accessible terminals. The other is constant and dependent only on the internal voltage and current sources and the resistances of the actual network. It is also demonstrated theoretically and validated by numerical simulation that the traditional Thevenin and Norton equivalent circuits are particular cases of the proposed equivalent circuit in this article. The proposed equivalent circuit can be used to analyze power and efficiencies of the actual network.

INDEX TERMS  
Equivalent circuit, power conservation, Thévenin’s theorem, Thévenin equivalent circuit, Norton’s theorem, Norton equivalent circuit, efficiency of equivalent circuits, general network theorem.

I. INTRODUCTION

The Thévenin [1] and Norton [2] equivalent circuits are important to the theory of DC networks and are commonly used in the analysis of electrical circuits basically because they provide simplicity in many practical applications.

However, as it is well known, they are limited in representing the phenomena that occur in the actual circuit, since they can only be used to determine voltage and current at a pair of accessible terminals of the network.

Since they do not represent the totality of the internal losses of the actual circuit, they cannot be used to determine efficiency and for power analysis.

As presented in [3], a review of publications since 1883 presenting the DC network classical theorems reveals that none of them address the analysis of the internal losses and how to include them in an equivalent circuit.

In [3] it is demonstrated that any DC network formed by independent voltage sources and resistors, with a pair of accessible terminals \(ab\), can be represented by the equivalent circuit shown in Fig. 1.

In the equivalent circuit shown in Fig. 1(b), \(V_T\) and \(R_T\) are the voltage and resistance of the traditional Thévenin equivalent circuit. The resistor \(R_X\) is constant, dependent only on the network current internal parameters, and represents the internal constant power dissipation that is independent of the power transferred to the terminals \(ab\).

This article extends the analysis presented in [3], for networks formed by independent voltage and current sources, and resistors. A unified equivalent circuit is introduced. It represents and can be used to determine not only the power transferred to the external terminals, but also the internal losses and the actual network efficiency.

II. THE EQUIVALENT CIRCUIT FOR DC NETWORKS FORMED BY CURRENT SOURCES AND RESISTORS

A linear resistive network formed by independent current sources and resistors, represented by \(N\), is shown in Fig. 2. There are two accessible external terminals, designated by \(ab\), to which an external voltage source \(V_a\) is connected.
The node voltage equations in matrix form are given by

\[
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n \\
I_a
\end{bmatrix} = \begin{bmatrix}
G_{11} & G_{12} & \cdots & G_{1n} & G_{1a} \\
G_{12} & G_{22} & \cdots & G_{2n} & G_{2a} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
G_{n1} & G_{n2} & \cdots & G_{nn} & G_{na} \\
G_{a1} & G_{a2} & \cdots & G_{an} & G_{aa}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n \\
V_a
\end{bmatrix}
\]

(1)

where

\( I_1, I_2, \ldots, I_n, I_a \) are the algebraic sums of all the source currents at principal nodes 1, 2, \ldots, \( n \), \( a \) of the network. The node \( b \) is the reference node.

\( V_1, V_2, \ldots, V_n \) are voltages between the nodes 1, 2, \ldots, \( n \), \( a \), and the reference node \( b \).

\( G_{ij} \) is the sum of all the conductances connecting nodes \( i \) and \( j \). Note that \( G_{ij} = G_{ji} \).

In order to reduce the size of the equations, without loss of generality, we will assume that the original network contains only three principal nodes, designated by 1, 2 and \( a \).

From (1) we can write (2).

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_a
\end{bmatrix} = G
\begin{bmatrix}
V_1 \\
V_2 \\
V_a
\end{bmatrix} + G_{1a}
\begin{bmatrix}
V_1 \\
V_2 \\
V_a
\end{bmatrix} V_a
\]

(2)

where

\[
G = \begin{bmatrix}
G_{11} & G_{12} \\
G_{12} & G_{22}
\end{bmatrix}
\]

(3)

From (2) we find

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = G^{-1}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} - G^{-1} G_{1a}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} V_a
\]

(4)

From (1) we can obtain

\[
I_β = \begin{bmatrix}
G_{1a} \\
G_{2a}
\end{bmatrix} G^{-1}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
+ G_{aa} V_a
\]

(5)

Substituting (4) in (5) gives

\[
I_β = \begin{bmatrix}
G_{1a} \\
G_{2a}
\end{bmatrix} G^{-1}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
- \begin{bmatrix}
G_{1a} \\
G_{2a}
\end{bmatrix} G^{-1} G_{1a}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} + G_{aa} V_a
\]

(6)

A. NORTON EQUIVALENT CIRCUIT

Let us analyze the case where the terminals \( ab \) are connected in short circuit causing \( V_{ab} = 0 \) and \( I_a = -I_N \), as shown in Fig. 3.

From (6) we obtain

\[
I_β = \begin{bmatrix}
G_{1a} \\
G_{2a}
\end{bmatrix} G^{-1}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

(7)

which makes

\[
I_N = -\begin{bmatrix}
G_{1a} \\
G_{2a}
\end{bmatrix} G^{-1}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

(8)

where \( I_N \) represents the current value of the current source of the Norton equivalent circuit.

Next, we will set equal to zero all currents from the internal current sources of the network and apply a voltage \( V_a \) between terminals \( ab \), as shown in Fig. 4.

When \( I_1 = I_2 = 0 \), from (6) we find

\[
I_β = G_{aa} V_a - \begin{bmatrix}
G_{1a} \\
G_{2a}
\end{bmatrix} G^{-1} G_{1a}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} V_a
\]

(9)

The conductance of the Norton equivalent circuit is defined by

\[
G_N = \frac{I_a}{V_a}
\]

(10)

Therefore, from (9) we find

\[
G_N = G_{aa} - \begin{bmatrix}
G_{1a} \\
G_{2a}
\end{bmatrix} G^{-1} G_{1a}
\]

(11)

Substitution of (8) and (11) into (6) yields

\[
I_β = G_N V_a + \begin{bmatrix}
G_{1a} \\
G_{2a}
\end{bmatrix} G^{-1}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

(12)

or yet

\[
I_β = G_N V_a - I_N
\]

(13)

Equation (13) represents the Norton equivalent circuit shown in Fig. 5.
B. INTERNAL DISSIPATED POWERS IN THE NETWORK WHEN THE TERMINALS AB ARE CONNECTED IN SHORT CIRCUIT (V_a = 0)

The power dissipated internally in the network when the external terminals ab are connected in short circuit is defined by

\[ P_y = [I_1 \ I_2] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \Delta P \] (14)

where \( \Delta P \) is the sum of the dissipated powers in the resistors associated in series with the internal independent current source.

For this condition, in which \( V_a = 0 \), from equation (2) we obtain

\[ \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \] (15)

Substitution of (15) in (14) gives

\[ P_y = [I_1 \ I_2] G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \Delta P \] (16)

C. INTERNAL POWER DISSIPATED IN THE NETWORK WHEN V_a \( \neq \) 0

The power dissipated internally in the original network when \( V_a \neq 0 \) is given by

\[ P = [I_1 \ I_2] G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + V_a I_a + \Delta P \] (17)

Substituting (4) in (17) we find

\[ P = [I_1 \ I_2] G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} - [I_1 \ I_2] G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} V_a + V_a I_a + \Delta P \] (18)

Multiplying (6) by \( V_a \) we obtain

\[ I_a V_a = \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} V_a \]

\[ - \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} V_a^2 + G_{aa} V_a^2 \] (19)

Substitution of (19) in (18) gives

\[ P = [I_1 \ I_2] G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} - [I_1 \ I_2] G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} V_a \]

\[ + \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} V_a \]

\[ - \begin{bmatrix} G_{aa} - \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} \end{bmatrix} V_a^2 + \Delta P \] (20)

Since

\[ \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [I_1 \ I_2] G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} \] (21)

from (20) we find

\[ P = [I_1 \ I_2] G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} G_{aa} - \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} \end{bmatrix} V_a^2 + \Delta P \] (22)

Substituting (11) and (16) in (22), we find

\[ P = P_y + G_N V_a^2 \] (23)

Equation (23) indicates that there are two components of internal losses in the actual network, one of which is constant and represented by \( P_y \), which depends only on the internal parameters of the actual network, and another variable given by the term \( G_N V_a^2 \) that, in addition to depending on the internal parameters represented by \( G_N \), also depends on the voltage (or current) at the external terminals ab. The invariant losses are not represented in the traditional Norton equivalent circuit, shown in Fig. 5. Equation (23) represents the equivalent circuit shown in Fig. 6.

\[ \begin{bmatrix} I_N \\ I_\beta \end{bmatrix} \]

\[ \begin{bmatrix} R_y \end{bmatrix} \]

\[ \left( R_N \right) \]

\[ \begin{bmatrix} V_a \end{bmatrix} \]

\[ I_N \]

\[ I_\beta \]

\[ R_N \]

\[ V_a \]

\[ I_N \]

\[ I_\beta \]

\[ R_N \]

\[ V_a \]

\[ I_N \]

\[ I_\beta \]

\[ R_N \]

\[ V_a \]

\[ I_N \]

\[ I_\beta \]

\[ R_N \]

\[ V_a \]
Norton equivalent circuit shown in Fig. 5 is a particular case of the equivalent circuit shown in Fig. 6, when \( R_y = 0 \).

The power transferred to the terminals \( ab \) is independent of \( R_y \). However, the power transmitted to the circuit by the current source \( I_N \) depends on it.

We can also conclude that the traditional Norton equivalent circuit efficiency is always less than the original DC network. Thus, its equivalence is limited, and it cannot be used for analysis of powers, losses and efficiency.

**D. NUMERICAL EXAMPLE I**

Let us consider the circuit shown in Fig. 7, which will be used to illustrate the use of the analysis results presented in the previous sections, to determine the parameters of the proposed equivalent circuit.

![FIGURE 7. Electric circuit for the numerical example I.](image)

The parameters of the circuit are \( I_p = 10 \, A \), \( I_q = 4 \, A \), \( I_\theta = 3 \, A \), \( R_1 = 10 \, \Omega \), \( R_2 = 5 \, \Omega \), \( R_3 = 2 \, \Omega \), \( R_4 = 7 \, \Omega \) and \( R_y = 1 \, \Omega \).

The node voltage equations in matrix form are

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_\theta
\end{bmatrix} =
\begin{bmatrix}
G_{11} + G_3 & -G_3 & 0 \\
-G_3 & G_2 + G_3 + G_4 & -G_4 \\
0 & -G_4 & G_{aa}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_\theta
\end{bmatrix}
\]

Thus,

\[
G_{11} = G_1 + G_3 = \frac{1}{R_1} + \frac{1}{R_3} = 0.6 \, S
\]

\[
G_{22} = G_2 + G_3 + G_4 = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = 0.843 \, S
\]

\[
G_{12} = -G_3 = -\frac{1}{R_3} = -0.5 \, S
\]

\[
G_{21} = -G_3 = -\frac{1}{R_3} = -0.5 \, S
\]

\[
G_{1a} = 0 \, S
\]

\[
G_{2a} = -G_4 = -\frac{1}{R_4} = -0.143 \, S
\]

\[
G_{aa} = G_4 = \frac{1}{R_4} = 0.143 \, S
\]

\[
I_1 = I_p = 10 \, A
\]

\[
I_2 = I_\theta - I_\phi = -1 \, A
\]

\[
I_\theta = I_\phi + I_\beta
\]

The current \( I_N \) of the Norton equivalent circuit is given by

\[
I_N = I_\phi - \left[ G_{1a} \quad G_{2a} \right] \left[ G_{11} \quad G_{12} \right]^{-1} \left[ I_1 \quad I_2 \right] \quad (28)
\]

Substituting the values of the circuit parameters in (28) we obtain \( I_N = 6.128 \, A \).

The admittance value \( G_N \) of the Norton equivalent circuit is given by

\[
G_N = G_{aa} - \left[ G_{1a} \quad G_{2a} \right] \left[ G_{11} \quad G_{12} \right]^{-1} \left[ G_{1a} \quad G_{2a} \right] \quad (29)
\]

Substitution of network parameters in (29) yields \( G_N = 0.095 \, S \) and \( R_N = 10.529 \, \Omega \). The power dissipated internally with the terminals \( ab \) connected in short circuit is given by

\[
P_N = \left[ I_1 \quad I_2 \right] \left[ G_{11} \quad G_{12} \right]^{-1} \left[ I_1 \quad I_2 \right] + R_p I_p^2 \quad (30)
\]

Substituting in (30) the parameters of the network of Fig. 7, we find \( P_N = 471.061 \, W \) and substituting the values of \( P_N \) and \( I_N \) in (25) we obtain \( R_y = 12.542 \, \Omega \). The resulting equivalent circuit with the obtained parameters is shown in Fig. 8.

![FIGURE 8. Power conservative equivalent circuit for the network shown in Fig. 7.](image)

The internal losses as a function of the voltage \( V_a \) for the actual circuit, the proposed equivalent circuit and the Norton equivalent circuit are given respectively by equations (31), (32) and (33).

\[
\Delta P_1(V_a) = V_1 I_p + V_2 I_q + (V_a - V_2) I_\phi + V_a I_\phi + R_y I_p^2 \quad (31)
\]

\[
\Delta P_2(V_a) = R_y I_N^2 + \frac{V_a^2}{R_N} \quad (32)
\]

\[
\Delta P_N(V_a) = \frac{V_a^2}{R_N} \quad (33)
\]

In Fig. 9 the three curves are shown as a function of \( V_a \). It can be seen that the proposed equivalent circuit internal losses are equal to the actual network internal losses. In turn, the Norton equivalent circuit internal losses are less than the losses of the actual network losses. For this reason, the Norton equivalent circuit efficiency is always greater than the original circuit one.
III. THE EQUIVALENT CIRCUIT FOR DC NETWORKS FORMED BY INDEPENDENT VOLTAGE AND CURRENT SOURCES, AND RESISTORS

Let us consider a network $N$ formed by independent voltage and current sources, and resistors shown in Fig. 10, with a pair of accessible terminals $ab$, to which a voltage source $V_a$ is connected.

The voltage $V_o$ when $I_o = 0$ is the voltage of the Thévenin’s equivalent circuit, denoted by $V_T$. Hence,

$$V_T = -\left[ \begin{array}{c} g_{o1} \\ g_{oo} \\ g_{po} \end{array} \right] \left[ \begin{array}{c} V_1 \\ I_p \end{array} \right]$$

Let us define

$$V_{T1} = -\frac{g_{o1}}{g_{oo}} V_1$$

and

$$V_{T2} = -\frac{g_{op}}{g_{oo}} I_p$$

Substitution of (39) and (40) in (38) gives

$$V_T = V_{T1} + V_{T2}$$

Making $V_1 = I_p = 0$ in (36) we find the resistance of the Thévenin’s equivalent circuit, given by

$$R_T = \frac{1}{g_{oo}}$$

The power dissipated in the resistors of the network when $I_o = 0$ is

$$P = \left[ V_1 \quad I_p \right] \left[ \begin{array}{c} V_1 \\ I_p \end{array} \right]$$

Substitution of (35) in (43) yields

$$P = \left[ V_1 \quad I_p \right] \left[ \begin{array}{c} g_{o1} \\ g_{oo} \\ g_{po} \end{array} \right] \left[ \begin{array}{c} V_1 \\ I_p \end{array} \right] + [V_1 \quad I_p] \left[ \begin{array}{c} g_{o1} \\ g_{oo} \end{array} \right] V_o$$

Substituting (37) in (44) we find

$$P = \left[ V_1 \quad I_p \right] \left[ \begin{array}{c} g_{o1} \\ g_{oo} \\ g_{po} \end{array} \right] \left[ \begin{array}{c} V_1 \\ I_p \end{array} \right] - [V_1 \quad I_p] \left[ \begin{array}{c} g_{o1} \\ g_{oo} \\ g_{po} \end{array} \right] \left[ \begin{array}{c} V_1 \\ I_p \end{array} \right]$$

where

$$g_{p1} = -g_{1p}$$

and

$$g_{o1} = g_{o1}$$

From (39) we can write

$$V_1 = \frac{g_{oo}}{g_{o1}} V_{T1}$$

Making $V_1 = V_o = 0$ in (36) we find

$$I_{N1} = g_{op} I_p$$

Hence,

$$I_p = \frac{I_{N1}}{g_{op}}$$
Substitution of (46), (47), (48), (49) and (51) in (45), and appropriate algebraic manipulation gives

\[ P = \left( \frac{1}{g_{oo}} + \frac{g_{pp}}{g_{op}^2} \right) I_{N1}^2 + \left( \frac{g_{11}g_{oo}^2}{g_{10}^2} - g_{oo} \right) V_{T1}^2 \]  (52)

Let us define

\[ R_y = \frac{g_{pp}}{g_{op}^2} \]  (53)

and

\[ R_x = \frac{g_{11}g_{oo}^2}{g_{10}^2} - g_{oo} \]  (54)

Substituting (41), (53), and (54) in (52) we find

\[ P = (R_T + R_y) I_{N1}^2 + \frac{V_{T1}^2}{R_y} \]  (55)

Substitution of (39) and (50) in (37) yields

\[ V_o = V_{T1} + R_T I_{N1} \]  (56)

Equations (55) and (56) describe the equivalent circuit shown in Fig. 11.

**A. THEVENIN AND NORTON EQUIVALENT CIRCUITS DERIVED FROM THE UNIFIED EQUIVALENT CIRCUIT**

In the equivalent circuit shown in Fig. 11, the resistors \( R_x \) and \( R_y \) do not interfere in the phenomena observable from the external terminals \( ab \).

In applications where the power dissipated internally can be ignored, the resistors \( R_x \) and \( R_y \) can be removed, which results in the equivalent circuit shown in Fig. 12.

From the circuit shown in Fig. 12, the traditional Thevenin equivalent circuit shown in Fig. 13 is obtained, where

\[ V_T = V_{T1} + R_T I_{N1} \]  (57)

Likewise, from the unified equivalent circuit shown in Fig. 11, the Norton equivalent circuit shown in Fig. 14 is obtained, where

\[ I_N = I_{N1} + \frac{V_{T1}}{R_T} \]  (58)

**B. NUMERICAL EXAMPLE II**

To illustrate the determination of the proposed equivalent circuit parameters, we will use the circuit shown in Fig. 15, with \( V_1 = 100 \text{ V}, V_2 = 35 \text{ V}, I_g = 10 \text{ A}, I_k = 3.5 \text{ A}, R_1 = 5\Omega, R_2 = 7\Omega, R_3 = 3\Omega, R_4 = 2\Omega, R_5 = 2\Omega \) and \( R_6 = 21\Omega \).

1) **MEASUREMENT OF RESISTANCE \( R_T \)**

To determine \( R_T \) (or \( R_N \)), the voltage sources \( V_1 \) and \( V_2 \) are replaced by short circuit and the internal current sources \( I_g \) and \( I_k \) are replaced by open circuit. The resistance \( R_T \) is measured from the terminals \( ab \), as shown in Fig. 16.

The value of \( R_T \) found by simulation and given by

\[ R_T = \frac{V_T}{I_N} \]  is \( R_T = 7.177\Omega \).
2) MEASUREMENT OF RESISTANCE \( R_X \) AND VOLTAGE \( V_{T1} \)
To determine \( R_X \), the power \( P_X \) consumed by the actual circuit with the terminals \( ab \) open and the current sources \( I_g \) and \( I_k \) set equal to zero is determined. The corresponding equivalent circuit is shown in Fig. 17.

The values of currents obtained by simulation are \( I_1 = 9.577 \) A and \( I_2 = 3.787 \) A. The power \( P_X \) is given by

\[
P_X = V_1 I_1 + V_2 I_2 \tag{59}
\]

Substituting the obtained values for \( V_1, V_2, I_1 \) and \( I_2 \) we obtain \( P_X = 1090.63 \) W. The voltage at the terminals \( ab \) obtained by simulation is \( V_{T1} = 79.54 \) V. The resistance \( R_X \) is determined by

\[
R_X = \frac{V_{T1}^2}{P_X} \tag{60}
\]

The values substitution for \( V_{T1} \) and \( P_X \) into (40) gives \( R_X = 5.804 \) Ω.

3) MEASUREMENT OF RESISTANCE \( R_Y \) AND CURRENT \( I_{N1} \)
To determine \( R_Y \), the voltage sources \( V_1 \) and \( V_2 \) are set equal to zero and the terminals \( ab \) are connected in short circuit, as shown in Fig. 18.

The dissipated power in the circuit resistors shown in Fig. 18 is given by

\[
P_Y = V_g I_g + V_k I_k \tag{61}
\]

The voltages value obtained by simulation are \( V_g = 30 \) V and \( V_k = 155 \) V. Substituting the values of \( I_g, I_k, V_g \) and \( V_k \) into (41) we find \( P_Y = 354.25 \) W. The simulation also yields \( I_{N1} = 4.827 \) A. The resistance \( R_Y \) is determined by

\[
R_Y = \frac{P_Y}{I_{N1}^2} \tag{62}
\]

Substitution of \( P_Y \) and \( I_{N1} \) values in (62) yields \( R_Y = 15.20 \) Ω.

4) INTERNALLY DISSIPATED POWER
The equivalent circuit with the obtained parameters is shown in Fig. 11, with \( V_{T1} = 79.54 \) V, \( R_X = 5.804 \) Ω, \( R_Y = 15.20 \) Ω, \( I_{N1} = 4.827 \) A and \( R_T = 7.177 \) Ω.

Fig. 19 shows the plot of the power dissipated internally in the equivalent circuit shown in Fig. 11 and calculated using equation (55), and the power dissipated internally in the original network shown in Fig. 15 obtained by numerical simulation, against voltage \( V_a \).
IV. ON THE VALUE OF \( R_Y \) AND \( R_X \)

It was demonstrated in [3] that the value of \( R_X \) for networks formed by independent voltage sources and resistors, depends on the voltage sources and the actual circuit internal resistances. For the particular case where the network has only one voltage source, the value of \( R_X \) depends only on the internal resistances and is independent of the voltage source.

In Section II of this article it was shown that for networks consisting of independent current sources and resistors, the value of \( R_I \) is determined by (26). When all internal current sources are set equal to zero except \( I_1 \), the resistance \( R_y \) is determined by

\[
R_y = \frac{G_{22} (G_{11} G_{22} - G_{12}^2)}{(G_{12} G_{2a} - G_{22} G_{1a})^2}
\]

(63)

This result then shows that the value of \( R_y \), for networks containing only one current source and resistors, is also dependent only on the internal resistances and independent of the current source.

V. A GENERAL NETWORK THEOREM

In the previous sections, the theorem proof formulated below was presented, which unifies the extension of Thévenin theorem presented in [3], with the extension of the Norton theorem presented in section II of this article.

\[
\begin{align*}
& \text{Actual Network} \\
& \text{(a)} \quad \hat{a} \\
& V_{T1} \quad R_s \\
& I_{N1} \quad R_T \\
& \text{(b)} \quad \hat{b}
\end{align*}
\]

FIGURE 20. The power conservative unified equivalent circuit of DC networks consisting of independent voltage and current sources, and resistors.

Theorem: Any linear DC network consisting of independent voltage and current sources, and resistors, with two accessible terminals (Fig. 20(a)) can be replaced by an equivalent circuit with a DC voltage source \( V_{T1} \), a DC current source \( I_{N1} \) and three resistors \( R_X \), \( R_y \) and \( R_T \) (Fig. 20(b)). The voltage \( V_{T1} \) is measured at the open terminals, with all internal current sources of the network set equal to zero. The current \( I_{N1} \) is the current through a short circuit applied to the external terminals, with all network internal voltage sources set equal to zero. The resistance \( R_X \) associated in parallel with the voltage source \( V_{T1} \) is given by the equation

\[
R_X = \frac{V_{T1}^2}{P_X}, \quad \text{where } P_X \text{ is the power supplied by the internal voltage sources and dissipated internally in the circuit with the external terminals open and all internal current sources set equal to zero. The resistance } R_y \text{ associated in series with the current source } I_{N1} \text{ is given by } R_y = \frac{P_y}{I_{N1}^2}, \quad \text{where } P_y \text{ is the power supplied by the internal current sources and dissipated in the network, with a short circuit applied to the external terminals and all internal voltage sources set equal to zero. The resistance } R_T \text{ is that measured at the terminals with all internal voltage replaced by a short circuit and all internal current sources replaced by an open circuit.}
\]

VI. CONCLUSION

This article proposes a new equivalent circuit for DC networks formed by independent voltage and current sources, and resistors, that is power conservative in the sense that its internal losses are equal to the actual network internal losses.

It is demonstrated that the traditional Thévenin and Norton equivalent circuits are particular cases of the proposed circuit and can obtained from it by ignoring the internal resistors responsible for the dissipated power components that are independent of the power transferred to the component, device or system connected at the accessible external terminals.

The proposed equivalent circuit is universal for DC networks consisting of independent voltage and current sources, and resistors and can be used to determine the actual network power and efficiency.

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VOLUME 8, 2020 178237