Diverse PBGS Patterns and Superbranes

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This is a brief account of the approach to superbranes based upon the concept of Partial Breaking of Global Supersymmetry (PBGS).

1. Introduction. The view of superbranes as theories of partial spontaneous breaking of global SUSY (PBGS) received a considerable attention (see, e.g., [1] - [12]). The salient feature of this approach is that it deals with the Goldstone superfields living on the worldvolume superspace of unbroken SUSY and accommodating the superbrane physical degrees of freedom.

It is well known that the superbranes in the Green-Schwarz (GS) formulation break half of the target SUSY (see, e.g., [13]). Choosing the static gauge with respect to the worldvolume diffeomorphisms and killing half of the target $\theta$-coordinates by $\kappa$-symmetry, one ends up with the physical multiplet which comprises transverse target bosonic coordinates and the rest of $\theta$-coordinates (in the case of D-branes and their generalizations, the physical multiplets can include additional fields). The remaining $\theta$'s are shifted under half of the target SUSY, which suggests to interpret them as the relevant Goldstone fermions. Thus one half of the original SUSY, with respect to which the physical degrees of freedom form a supermultiplet, is unbroken, while the other is spontaneously broken, with the physical fermions as the Goldstone ones.

One can reverse the argument: adopt PBGS as the guiding principle and deduce superbranes just from it. In doing so, one can take advantage of the nonlinear realizations method [14]-[16]
which provides the universal framework for treating spontaneously broken symmetries. In this method, the Goldstone (super)fields are identified with the parameters of the coset space of the full (super)symmetry group over its unbroken symmetry subgroup. The invariant actions of the Goldstone superfields constructed as integrals over the worldvolume superspace are reduced to the corresponding GS-type actions after passing to components and eliminating the auxiliary fields.

The nonlinear realizations method was worked out for constructing nonlinear sigma models of internal symmetries. It is less known that the bosonic $p$-branes in the physical (or “static”) gauge can also be treated in the nonlinear realizations language [17]. To describe in this way some $p$-brane moving in $D$-dimensional Minkowski space, one should consider a nonlinear realization of the relevant Poincaré group $\mathcal{P}_D = ISO(1, D - 1)$, such that the residual unbroken (vacuum stability) symmetry group is the product of the Poincaré group of $p + 1$-dimensional brane worldvolume and the rotation group of the transverse brane coordinates. The relevant coset manifold is $\mathcal{P}_D / SO(1, p) \otimes SO(D - p - 1)$. One splits the full $D$ space translation generator $P_M$, $M = 0, 1, \ldots D - 1$, as

\[ P_M \Rightarrow (P_m, P_\mu), \quad m = 0, 1, \ldots p; \quad \mu = p + 1, \ldots D, \quad (1) \]

and associates with $P_m$, $P_\mu$ the worldvolume coordinate $x^m$ and the Goldstone field $X^\mu(x)$ as the coset parameters ($X^\mu$ becomes the transverse $p$-brane coordinate). Also, one introduces the Goldstone fields $\Lambda_m^\mu(x)$ parametrizing the spontaneously broken part of the Lorentz group $SO(1, D - 1)$ (with the generators $L_\mu^m$):

\[ P_m \Rightarrow x^m, \quad P_\mu \Rightarrow X^\mu(x), \quad L_\mu^m \Rightarrow \Lambda_m^\mu(x). \quad (2) \]

The application of the Cartan forms techniques augmented with some extra covariant constraints (the inverse Higgs [18]) gives rise to the following minimal invariant action

\[ S_{br} \sim \int d^{p+1}x \left( \sqrt{(-1)^p g - 1} \right), \quad g = \det (\eta_{mn} - \partial_m X^\nu \partial_n X^\mu). \quad (3) \]
It is the static gauge form of the $p$-brane Nambu-Goto (NG) action.

As an illustration, let us consider the simplest example of the massive particle (0-brane) in $D = 2$. The $D = 2$ Poincaré group $\mathcal{P}(2)$ involves two translation generators $P_0, P_1$ and the $SO(1,1)$ Lorentz generator $L$, with the only non-vanishing commutators
\[ [L, P_0] = iP_1, \quad [L, P_1] = iP_0. \quad (4) \]

In accord with the said above, we should construct a nonlinear realization of $\mathcal{P}(2)$, with the one-dimensional “Poincaré group” generated by the generator $P_0$ as the stability subgroup. Thus we are led to place all generators into the coset (in this particular case it coincides with the full group):
\[ G = e^{itP_0} e^{iX(t)P_1} e^{i\Lambda(t)L}. \quad (5) \]

Here the wordline evolution parameter $t$ (time) is the coset coordinate associated with $P_0$, and the Goldstone fields are associated with the rest of generators. The group $\mathcal{P}(2)$ acts as left shifts of $G$. The Cartan forms
\[ G^{-1} dG = i \omega_1 P_0 + i \omega_1 P_1 + i \omega_L L, \quad (6) \]

\[ \omega_1 = \sqrt{1 + \Sigma^2} dt + \Sigma dX, \quad \omega_1 = \sqrt{1 + \Sigma^2} dX + \Sigma dt, \]
\[ \omega_L = \frac{1}{\sqrt{1 + \Sigma^2}} d\Sigma, \quad \Sigma \equiv \text{sh} \Lambda \quad (7) \]

by construction are invariant under this left group action. Next, we observe that the Lorentz Goldstone field $\Sigma(t)$ can be traded for $\dot{X}(t)$ by the inverse Higgs [18] constraint
\[ \omega_1 = 0 \quad \Rightarrow \quad \Sigma = -\frac{\dot{X}}{\sqrt{1 - X^2}}. \quad (8) \]

This constraint is covariant since $\omega_1$ is the group invariant (in the generic case, the coset Cartan forms undergo homogeneous rotation.
in their stability subgroup indices). Thus the obtained expression for \( \Sigma \) possesses correct transformation properties. Substituting it into the remaining Cartan forms we find

\[
\omega_0 = \sqrt{1 - \dot{X}^2} \, dt , \quad \omega_L = \sqrt{1 - \dot{X}^2} \, \frac{d}{dt} \left( \frac{\dot{X}}{\sqrt{1 - \dot{X}^2}} \right) \, dt .
\] (9)

The simplest invariant action, the covariant length

\[
S = \int \omega_0 = \int dt \sqrt{1 - \dot{X}^2} ,
\] (10)

is recognized, up to a renormalization factor of the dimension of mass, as the action of \( D = 2 \) massive particle in the static gauge \( X^0(t) = t \). The equation of motion for \( X(t) \) can also be given the manifestly covariant form

\[
\omega_L = 0 .
\] (11)

Actually, we could start from the action (10) with the original expression (7) for \( \omega_0 \) and reproduce (8) as the algebraic equation of motion for \( \Sigma(t) \). It is important to realize that the correct form of the action is recovered just because we have included the Lorentz Goldstone field into the coset.

In the generic case of \( p \)-brane in \( D \)-dimensional Minkowski space the construction is analogous. One equates to zero the Cartan forms \( \omega^\mu \) associated with the transverse translation generators, which is once again the manifestly covariant constraint, and in this way eliminates the Lorentz Goldstone fields \( \Lambda^{\mu m}(x) \):

\[
\Lambda^\mu_m \sim \partial_m X^\mu .
\] (12)

Substituting these expressions into the Cartan forms \( \omega^m \) which are covariant differentials of \( x^m \), one finds that the invariant volume of \( x \)-space, i.e. the integral of the external product of these 1-forms, is just the \( p \)-brane static gauge NG action (3).
2. Superbranes from PBGS. The PBGS approach is the generalization of the above nonlinear realization view of branes to the superbranes. Though originally [1] the phenomenon of partial breaking of global SUSY ($N = 2$ down to $N = 1$ in $D = 4$) was studied without any reference to superbranes, the subsequent study [2]-[12] revealed the profound relationship between both concepts. It was shown in [2] that $N = 1, D = 4$ superstring in the static gauge can be understood as the theory of partial breaking of $N = 1, D = 4$ SUSY to its $N = (2,0), d = 2$ subgroup. The very existence of self-consistent GS type actions for superbranes, with the appropriate fermionic $\kappa$-symmetry, was inferred in the pioneering paper [3] from the study of the partial breaking $N = (1,0), D = 6 \Rightarrow N = 1, d = 4$. The same PBGS pattern was treated in [4] from the $D = 4$ perspective as the partial breaking of $N = 2$ SUSY with two central charges down to $N = 1$ one, with the systematic use of the nonlinear realizations and inverse Higgs effect techniques. It was shown that the self-consistent theory can be constructed in terms of the (covariantly) chiral and antichiral bosonic $N = 1$ superfields which are the Goldstone ones corresponding to the central charges (that is, to the translation operators in fifth and sixth directions from the $D = 6$ viewpoint). The fermionic Goldstone superfields associated with the spontaneously broken supertranslation generators are covariantly expressed in terms of these basic Goldstone superfields by the covariant constraints of the type (8). The relevant Goldstone superfields action, a nonlinear generalization of the standard free action of $N = 1$ chiral superfields, was constructed in [7, 8]. In components, after eliminating the auxiliary fields, it yields just the static gauge form of the GS action for the $N = (1,0), D = 6$ 3-brane in a flat background. An interesting new phenomenon was discovered in [6]. It turned out that the same SUSY admits several different PBGS options depending on into which multiplet of unbroken SUSY one embeds the Goldstone fermion. In the $N = 2 \Rightarrow N = 1, D = 4$ case, instead of placing this field into the chiral $N = 1$ multiplet, one can place it into the abelian vector $N = 1$ multiplet as a “photino”, by imposing
the appropriate covariant constraints on the Goldstone fermionic
superfield which generalize the Bianchi identities for the flat $N = 1$
Maxwell superfield strength. The relevant invariant action, on the
one hand, is $N = 2$ extension of the Born-Infeld (BI) action with
the hidden, nonlinearly realized half of supersymmetry; and, on the
other, is (in components) a gauge-fixed form of the GS action of
the “space-time filling” D3-brane. One more option is to embed
the Goldstone fermion into the $N = 1$ tensor (or linear) multiplet
$[7, 8, 9]$. The emerging brane is the super “L3-brane” in $D = 5$
in terminology of $[19]$, it is related to the $N = (1, 0), D = 6$
3-brane via the familiar duality between $N = 1$ tensor and chiral
superfields.

The study of the PBGS patterns corresponding to partial break-
ing of SUSY with 16 supercharges ($N = 1, D = 10; N = 2, D = 6;
N = 4, D = 4; \ldots$) was initiated in our works $[10, 11]$. Let us dwell
on this and related subjects in some detail.

3. Hypermultiplet as a Goldstone superfield. To describe the
$1/2$ breaking of Poincaré SUSY with 16 supercharges, it is natural
to start from the maximally symmetric situation where the broken
and unbroken SUSY “live” as simple ones. This amounts to con-
sidering the PBGS option $N = 1, D = 10 \Rightarrow N = (1, 0), d = 6$
(we could equally choose $N = (0, 1)$). From the $d = 6$ perspective,
$N = 1, D = 10$ is a central extension of $N = (1, 1)$:

$$N = 1, D = 10 \propto \{Q^i_{\alpha}, P_{\alpha\beta}, S^{\beta a}, Z^{ia}\}, \quad (13)$$

where $\alpha, \beta = 1, \ldots, 4$ ; $i = 1, 2$ ; $a = 1, 2$ are, respectively, the
$Spin(1,5)$ indices and the $SU(2)$ doublet indices. The basic anti-
commutation relations read

$$\{Q^i_{\alpha}, Q^j_{\beta}\} = \epsilon^{ij} P_{\alpha\beta}, \quad \{Q^i_{\alpha}, S^{a\beta}\} = \delta^i_a Z^{ia}, \quad \{S^{a\alpha}, S^{b\beta}\} = \epsilon^{ab} P^{\alpha\beta}. \quad (14)$$

One also must add generators of the $D = 10$ Lorentz group $SO(1,9) \propto
\{M_{\alpha\beta\gamma\delta}, T^{ij}, T^{ab}, K^\alpha_i\}$, with $M$ and $T$ generating the $d = 6$
Lorentz group $SO(1,5)$ and $R$-symmetry $SU(2) \times SU(2)$.  

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We wish \( N = (1, 0), d = 6 \) SUSY \( \propto \{ Q^i_\alpha, P_{\alpha\beta}, T^{ij}, T^{a\beta}, M_{\alpha\beta\gamma\delta} \} \) to be unbroken. Like in the bosonic case of Sect.1, we are led to place the generators \( Q^i_\alpha, P_{\alpha\beta}, S^{\alpha a}, Z^{ia}, K^{ia}_{\alpha\beta} \) into the coset and to treat the coset parameters associated with two first generators as the coordinates of \( N = (1, 0), d = 6 \) superspace, while the remaining ones as the Goldstone superfields:

\[
\begin{align*}
Q^i_\alpha &\Rightarrow \theta^i_\alpha, & P_{\alpha\beta} &\Rightarrow x^{\alpha\beta}.
S^{\alpha a} &\Rightarrow \Psi^{\alpha a}(x, \theta), & Z^{ia} &\Rightarrow q^{ia}(x, \theta), & K^{ia}_{\alpha\beta} &\Rightarrow \Lambda^{ia}_{\alpha\beta}(x, \theta).
\end{align*}
\] (15)

The coset element \( G \) in the exponential parametrization is constructed like in eq. (5). The Cartan forms are defined by

\[
G^{-1}dG = \Omega_Q + \Omega_P + \Omega_Z + \Omega_S + \Omega_K + \ldots ,
\] (16)

where the subscripts denote the relevant generators. The forms \( \Omega_Q, P \) are the covariant differentials of the superspace coordinates, \( \Omega_Z, S, K \) are those of Goldstone superfields, they are homogeneously transformed under the supergroup left shifts. We shall be interested in the linearized structure of \( \Omega_Z = \Omega_Z^{ia} Z_{ia} \) [3], [4], [5], [6]:

\[
\Omega_Z^{ia} = dq^{ia} + 2\Lambda^{ia}_{\alpha\beta} dx^{\alpha\beta} + \psi^{ia}_\alpha d\theta^i_\alpha + \ldots .
\] (17)

Comparing it with the form \( \omega_1 \), eq. (7) of the toy example of Sect. 1, we observe that both forms have a similar structure. Hence, the superfields \( \Lambda, \Psi \) can be expressed through the basic Goldstone superfield \( q^{ia} \) from the inverse Higgs constraint analogous to (8)

\[
\Omega_Z = 0 \Rightarrow \Lambda^{ia}_{\alpha\beta} = \nabla_{\alpha\beta} q^{ia} = \partial_{\alpha\beta} q^{ia} + \ldots ,
\] (18)

\[
\Psi^{ia}_\alpha = \frac{1}{2} \nabla^k q^{ia}_k = \frac{1}{2} D^k q^{ia}_k + \ldots .
\]

Here \( D^i_\alpha = \partial/\partial \theta^i_\alpha - \frac{1}{2} \theta^j_\beta \partial_{\beta\alpha} \) and dots stand for nonlinear terms. Thus, \( q^{ia} \) is the essential Goldstone superfield, analogue of \( X(t) \) in the \( D = 2 \) example. Its first bosonic component \( q^{ia}(x) \) parametrizes the transverse directions in the \( D = 10 \) Minkowski space, while
the physical fermionic component is the Goldstino related to the spontaneously broken supertranslations (with the generator $S^{a a}$). This field content is that of the scalar $N = 1, D = 10$ 5-brane.

The constraint (18) differs in a few important aspects from (8) and the similar inverse Higgs constraint considered in ref. [5]. It not only eliminates the redundant Goldstone superfields, but also implies the differential constraint for $q^{ia}(x, \theta)$

$$\nabla^{(k} q^{i)a} = D^{(k} q^{i)a} + \ldots = 0 .$$

(19)

It is just the nonlinear, “brane” generalization of the standard hypermultiplet constraint [20]. The latter reduces the field content of $q^{ia}$ to the set of $(8+8)$ components and puts them on shell:

$$q^{ia}(x, \theta) \Rightarrow \phi^{ia}(x) + \theta^{ai} \psi^{a}_{\alpha}(x) + x\text{-derivatives} , \quad \Box \phi^{ia}(x) = 0 , \quad \partial^{\alpha \beta} \psi^{a}_{\beta} = 0 .$$

(20)  (21)

Thus eq. (19) describes the on-shell dynamics of $N = 1, D = 10$ 5-brane as the natural generalization of the free hypermultiplet dynamics. Since the off-shell superfield action for the latter can be constructed only in harmonic superspace [21], it is reasonable to expect that there exists a brane generalization of the hypermultiplet harmonic superspace action. It would be $N = (1, 0), d = 6$ (or $N = 2, d = 4$ after dimensional reduction) analogue of the $N = 1$ Goldstone superfield action of refs. [7, 8]. No systematic recipes are known so far for constructing PBGS actions. At present we are aware only of the appropriate fourth-order correction to the free hypermultiplet action. Even for the dimensionally-reduced cases, including the simplest case of $N = 2, D = 5$ superparticle, the full action is rather difficult to find. For the superparticle case it is known up to the sixth order in fields [11]:

$$S_{br}^{q} = \int d\zeta^{(-4)} q^{+} D^{+} q^{+a} + \alpha \int dZ (A_{2} + 2 \alpha A B^{++} B^{--} + \ldots)$$

(22)

where

$$A = q^{+} D^{--} q^{+a} , \quad B^{++} = q^{+} \partial_{\mu} q^{+a} ,$$
$q^{+a}, D^{\pm \pm}$ are the $d = 1$ reduction of the analytic hypermultiplet superfield and harmonic derivatives $[21]$, $B^{--}$ is defined by

$$D^{++}B^{--} - D^{--}B^{++} = 0$$

and $\alpha$ is a coupling constant. It would be tempting to find out the geometric principle allowing to restore the whole action. Let us now apply to a simpler PBGS case where the complete off-shell action can be constructed $[12]$.

4. **N=1, D=4 supermembrane.** This case corresponds to the partial breaking of $N = 1, D = 4$ SUSY to $N = 1, d = 3$ one. The $N = 1, D = 4$ superalgebra in the $d = 3$ notation reads

$$\{Q_a, Q_b\} = P_{ab}, \{Q_a, S_b\} = \epsilon_{ab}Z, \{S_a, S_b\} = P_{ab}, a, b = 1, 2. \quad (23)$$

The $d = 3$ momentum operator $P_{ab}$ together with the central charge $Z$ form the $D = 4$ translation operator. The vacuum stability subalgebra consists of the $N = 1, d = 3$ superalgebra (generators $Q_a$ and $P_{ab}$) and that of the $d = 3$ Lorentz group $SO(1,2)$. The second SUSY, the central charge $Z$ and the generators $K_{ab}$ of the coset $SO(1,3)/SO(1,2)$ define spontaneously broken symmetries. The appropriate coset parameters are introduced as

$$Q_a \Rightarrow \theta^a, P_{ab} \Rightarrow x^{ab}, S_a \Rightarrow \psi^a(x, \theta), Z \Rightarrow q(x, \theta), K_{ab} \Rightarrow \lambda^{ab}(x, \theta). \quad (24)$$

After construction of the relevant Cartan forms and covariant elimination of the Goldstone superfields $\psi^a(x, \theta), \lambda^{ab}(x, \theta)$ by the appropriate inverse Higgs constraints, one is left with $q(x, \theta)$ as the only unremovable Goldstone superfield. Its physical fields (one boson and two fermions) parametrize one transverse bosonic and two fermionic directions in $N = 1, D = 4$ superspace, the auxiliary bosonic field also admits a nice geometric interpretation of the Goldstone field for the spontaneously broken $U(1)$ automorphism group of $N = 1, D = 4$ superalgebra ("$\gamma^5$ invariance"). The physical field content of $q$ coincides with that of $N = 1, D = 4$ supermembrane.
The inverse Higgs conditions in this case do not imply the equation of motion for $q$. However, the dynamical equation for $q$ turns out to admit, like in the $D = 2$ particle example, a covariant representation as the vanishing of the covariant $d\theta$ projection of the coset Cartan form $\omega^a_\theta$ associated with the spontaneously broken SUSY generator $S_a$ \[1\]. We have no place to present details. Let us explain how to construct the off-shell action for this case.

Like in other PBGS cases, the nonlinear realizations approach on its own provides no clear recipe how to construct such an action. This becomes possible using the trick similar to the one exploited in \[4\]. Namely, let us start from a linear realization of $N = 1, D = 4$ SUSY in terms of $N = 1, d = 3$ superfields $\Phi, \xi_a \equiv D_a \rho$ with the following transformation rules under the second SUSY:

$$
\delta \rho = \theta^a \eta_a - 2 D^a \Phi \eta_a, \quad \delta \Phi = \frac{1}{2} \eta^a D_a \rho,
$$

where $\eta_a$ is the transformation parameter and $D_a$ is the flat $N = 1, d = 3$ spinor covariant derivative, $\{D_a, D_b\} = \partial_{ab}$. It is easy to check that the closure of these transformations and those of manifest $N = 1, d = 3$ SUSY is just the superalgebra \[23\], with $Z$ realized as a pure shift of $\rho$. The transformation of $\xi_a = D_a \rho$ starts with $\eta_a$, suggesting the interpretation of this superfield as the linear realization Goldstone fermion. After some work one finds that $\Phi$ can be covariantly expressed in terms of $\xi_a$ as follows

$$
\Phi = \frac{1}{2} \frac{\xi^2}{1 + \sqrt{1 + D^2 \xi^2}}.
$$

Recalling the transformation law of $\Phi$, one finds that the integral

$$
S = \int d^3xd^2\theta \Phi \equiv \frac{1}{2} \int d^3xd^2\theta \frac{\xi^2}{1 + \sqrt{1 + D^2 \xi^2}}, \quad \xi^a = D^a \rho,
$$

is invariant under the hidden SUSY transformations as well as $D = 4$ Poincaré translations and so it can be identified with the sought $d = 3$ worldvolume superspace action of $N = 1, D = 4$.
supermembrane. Indeed, it is easy to find that the bosonic core of this action is just the static gauge membrane NG action:

\[ S = \int d^3x \left( 1 - \sqrt{1 - \frac{1}{2} \partial q \cdot \partial q} \right). \]  

(27)

One can find the equivalence field redefinition relating \( \xi_a \) to the nonlinear realization Goldstone fermion \( \psi_a \) and \( \rho \) to \( q \). Also, the equations of motion following from the action (26) can be shown to be equivalent to those conjectured at the level of the Cartan forms. This implies the presence of hidden \( SO(1,4) \) Lorentz symmetry in the action. It still remains to prove the precise equivalence of this action to the GS one (e.g., along the lines of ref. [4]) and the action proposed in the superembedding approach [19]. Note an interesting peculiarity: one can add to the lagrangian in (26) the “cosmological” term \( \sim \rho \). This term is invariant under the second SUSY (up to surface terms) and the shifts \( \rho \rightarrow \rho + \text{const} \). Adding it changes the equation of motion for the auxiliary field and so can influence the structure of the component action (without this term, the auxiliary field is vanishing on shell). The pure physical boson part of the action is always given by (27).

Besides a scalar multiplet \( \rho \) (or \( q \)), we can choose a vector \( N = 1, d = 3 \) multiplet as the Goldstone one (like in [4]). It is represented by \( N = 1 \) spinor superfield strength \( \mu_a \) obeying the constraint:

\[ D^a \mu_a = 0. \]  

(28)

It leaves in \( \mu_a \) the first fermionic component together with the divergenceless vector \( F_{ab} \equiv D_a \mu_b |_{\theta=0} \) (just the gauge field strength). Due to the vector-scalar \( d = 3 \) duality, the superfield \( \mu_a \) is expected to describe a D2-brane which is dual to the supermembrane.

The relevant action can be found using the previous trick. One can extend \( \mu^a \) to the \( N = 1, D = 4 \) multiplet \( (\mu^a, \phi) \) with the following transformation rules under the second SUSY:

\[ \delta \mu_a = \eta_a - D^2 \phi \eta_a + \partial_{ab} \phi \eta^b, \quad \delta \phi = \frac{1}{2} \eta^a \mu_a. \]  

(29)
Thus \( \mu^a \) can also be interpreted as the linear realization Goldstone fermionic superfield. The following expression for \( \phi \)

\[
\phi = \frac{1}{2} \frac{\mu^2}{1 + \sqrt{1 - D^2 \mu^2}}
\]

(30)
can be checked to be consistent with (29). Then the action

\[
S = - \int d^3 x d^2 \theta \phi = - \frac{1}{2} \int d^3 x d^2 \theta \frac{\mu^2}{1 + \sqrt{1 - D^2 \mu^2}}
\]

(31)
is invariant under the second SUSY in virtue of the transformation rule of \( \phi \) (29) and the constraint (28). This nonlinear generalization of the standard \( N = 1, d = 3 \) abelian vector multiplet action \( \sim \mu^2 \) involves as its bosonic core the \( d = 3 \) BI action

\[
S = \int d^3 x \left( \sqrt{1 + 2 F^2} - 1 \right),
\]

\[
\partial^{ab} F_{ab} = 0 \quad \rightarrow \quad F_{ab} = \partial_{ab} G^c_c + \partial_{bc} G^c_a.
\]

(33)

So it is \( N = 2 \) extension of the \( d = 3 \) BI action with nonlinearly realized second SUSY. It can be regarded as the worldvolume superfield action of the space-time filling D2-superbrane in a flat background.

It is easy to prove its dual equivalence to the action (26) by inserting the constraint (28) in it with the Lagrange multiplier \( \rho \) and integrating out \( \mu^a \) [12].

Finally, let us show how the duality between the bosonic NG and BI actions (27), (32) can be recovered in the nonlinear realizations approach of Sect.1. It is easy to find that in the case of nonlinear realization of \( \mathcal{P}_{(4)} \) in the coset \( \mathcal{P}_{(4)}/SO(1,2) \), which corresponds just to membrane in \( D = 4 \), the covariant differentials of \( x^m \) read

\[
\omega^m = dx^m + 2 \frac{\lambda^m}{1 - 2 \lambda^2} (2 \lambda_n + \partial_n q) dx^n \equiv \omega^m_n dx^n,
\]

(34)

where \( \lambda^m(x) \) is the Lorentz \( SO(1,3)/SO(1,2) \) Goldstone field in the appropriate parametrization and \( q(x) \) is the transverse coordinate of membrane. We could eliminate \( \lambda^m \) by the inverse Higgs
constraint but in the present case it is instructive to reproduce it as the equation of motion for \( \lambda^m \). The minimal invariant action constructed as the covariant worldvolume

\[
S_{\text{mem}} = \int d^3 x \det \omega^m_n ,
\]

(35)

up to a normalization factor and constant shift, is

\[
S_{\text{mem}} = \int d^3 x \frac{1}{1 - 2\lambda^2} \left( 2\lambda^2 + \lambda \partial q \right) .
\]

(36)

Varying \( \lambda^m \) yields the inverse Higgs expression for it

\[
\lambda^m = -\frac{1}{2} \frac{\partial_m q}{1 + \sqrt{1 - \frac{1}{2}(\partial q)^2}} .
\]

(37)

After this (36) takes the standard NG form (27). On the other hand, one can treat \( q \) in (36) as the Lagrange multiplier for the differential constraint on \( \lambda^m \):

\[
\partial_m F^m = 0 , \quad F^m \equiv 2 \frac{\lambda^m}{1 - 2\lambda^2} .
\]

(38)

Expressing \( \lambda^2 \) through \( F^m \)

\[
\lambda^2 = \frac{1}{4F^2} \left( 1 - \sqrt{1 + 2F^2} \right)^2 ,
\]

one reduces (36) just to the BI form (32)

\[
S_{\text{mem}} \sim \int d^3 x \left( \sqrt{1 + 2F^2} - 1 \right) .
\]

(39)

This simple consideration shows that within the nonlinear realization approach the \( d = 3 \) Maxwell field strength entering the \( d = 3 \) BI action acquires the nice geometric interpretation as the Goldstone field representing the Lorentz coset \( SO(1,3)/SO(1,2) \), while the BI action itself can be algorithmically derived as the action dual to the static gauge membrane NG action.
5. Concluding remarks. The geometric PBGS approach can be thought of as a useful and viable alternative to the standard GS description of superbranes. Its main merit is that it gives manifestly worldsurface supersymmetric off-shell superfield actions. Leaving aside such important conceptual questions as whether it can be helpful for quantization, etc, we list here a few more modest problems solving which could extend its range of applicability.

First of all, it is desirable to develop convenient general recipes of constructing superfield PBGS actions similar to those provided by the nonlinear realizations for the internal symmetries sigma models. At present, constructing such actions is an art to some extent.

It is important to learn how to construct self-consistent PBGS actions on non-trivial backgrounds involving the worldvolume and target supergravity and super Yang-Mills fields.

At last, it seems interesting to set up PBGS actions for the systems with $1/4$ and other exotic partial breaking options. Some progress in this direction is reported in the contribution by Delduc, Krivonos and myself in this Volume.

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