SUSY CONTRIBUTIONS TO THE $Z \to 3\gamma$ DECAY*

HEINZ KÖNIG
Département de Physique
Université du Québec à Montréal
C.P. 8888, Succ. Centre Ville, Montréal
Québec, Canada H3C 3P8

ABSTRACT

In this talk I comment on the contributions of all three types of particles (scalars, fermions and bosons) to the three-photon $Z$ decay within the standard model (SM) and its extensions to the 2 Higgs doublet model (2 HDM) and the minimal supersymmetric extension of the standard model (MSSM). I also comment briefly on the charginos contribution to this decay rate. Finally I comment on the fermions and scalars contribution to the three-gluon $Z$ decay.

1. Fermionic contribution

The fermionic contribution was considered a while ago in [1–4] and complete analytical results were presented in [5]. There is only one type of diagram with four internal fermions within the loop. The divergencies as well as the $\gamma_5$ terms drop out when summed over all six possible permutations of the photons (or, what is topologically equivalent, when summed over diagrams where the fermions circulate in one direction and in the opposite direction). The result is written as:

$$\Gamma_F(Z \to 3\gamma) = \frac{\alpha^4}{\sin^2\Theta_W \cos^2\Theta_W} (3 \sum_q e_q^3 V_q + \sum_L e_L^3 V_L)^2 \frac{m_Z^3}{3 \cdot 3! \cdot 4 \pi^3} X_F$$

$$\approx 1.05 \times 10^{-9} \text{ GeV}$$

with $m_Z = 91.1$, $\alpha = 1/128$, $\sin^2\Theta_W = 0.23$ and $X_F \approx 15$. $V_F = (T_3F - 2e_F \sin^2\Theta_W)/2$. The result is given for five flavours of light quarks since the top quark decouples rapidly for $m_{\text{top}} \geq m_Z/2$ as was shown in [5], where $\Gamma(Z \to 3\gamma)$ was plotted as a function of the top quark mass. A small top quark mass enhances the result by approximately 50% [5].

2. Bosonic contribution

The bosonic contribution were presented in [6–8]. Here we have four different kind of diagrams. Each diagram is divergent, but the sum is finite. In [6] the authors made extensive use of the transversality of the amplitude (that is $q_\mu M^{\mu\alpha\beta\gamma} = 0 = p_1^\mu M^{\mu\alpha\beta\gamma} = p_2^\mu M^{\mu\alpha\beta\gamma} = p_3^\mu M^{\mu\alpha\beta\gamma}$, where $q$ is the momenta of the $Z$ boson and $p_{1,2,3}$ the momentas of the photons and $\mu, \alpha, \beta$ and $\gamma$ their Lorentz indices respectively), gauge invariance and the masslessness of the photons, which sets many terms in the amplitude identical to 0. After expansion in the $W$ boson mass, which was

* Talk presented at DPF-94, U. of New Mexico, Albuquerque, New Mexico, USA, August 2-6, UQAM-PHE-94/11.
kinematically valid, (as explained in [6]) the result is:

$$\Gamma_W(Z \to 3\gamma) = \frac{\alpha^4}{\sin^2 \Theta_W \cos^2 \Theta_W} \frac{m_Z}{3 \cdot 34 \pi^3} X_W$$

with $X_W \approx 0.16$. Which gives us a number about 50 times smaller than the fermionic contribution.

In [5] the authors present the full analytical W bosons and fermions contribution to the three–photon Z decay, including the interference term of the W bosons with the larger fermion loop contribution, which enhances the width by about 27% from $1.05 \times 10^{-9}$ to $1.35 \times 10^{-9}$. Furthermore the authors show that there are strong relations between the fermionic and bosonic amplitudes.

3. Scalar contribution

Since there are no charged scalar particles in the SM this kind of contribution only occurs in models beyond the standard model such as the 2 HDM or the MSSM. In [9] I present a detailed analysis. The diagrams we have to consider are exactly the same as in the bosonic case with the W bosons replaced by the charged Higgs bosons. As in the bosonic case the divergencies cancel when summed over all diagrams in a non trivial way. In [9] I have shown through explicit calculation that the quadratic terms $(m_{H^+}/m_Z)^2$ are identical to 0 after expansion with respect to the Higgs mass squared. This is a necessary consequence of gauge invariance, as was shown. It is also a direct consequence of the effective gauge invariant Lagrangian for the four vector interaction. This term has to be of the form $\mathcal{L} = GB_{\mu\nu}B^{\mu\nu}W_{\rho\sigma}^\dagger W_{\rho\sigma}^\dagger$, where $B_{\mu\nu}$ and $W_{\rho\sigma}^\dagger$ are the tensor fields of the U(1) and SU(2) gauge fields respectively and therefore gives G a dimension of $1/m^4$. As was done in the bosonic case I made use of the masslessness of the photons and showed the transversality of the amplitude. As a final result I have:

$$\Gamma_{H^+}(Z \to 3\gamma) = \alpha^4 \cot^2 2\Theta_W \frac{m_Z}{3 \cdot 34 \pi^3} X_{H^+} \left(\frac{m_Z}{m_{H^+}}\right)^8$$

$$\approx 3.75 \times 10^{-16} \left(\frac{m_Z}{m_{H^+}}\right)^8 \text{GeV}$$

with $X_{H^+} \approx 6 \times 10^{-6}$. This result is more than a factor of $5.4 \times 10^4$ smaller than the W boson contribution and even a factor of $2.8 \times 10^6$ smaller than the fermionic contribution.

In [10] a more general analysis of the contribution of all three types of particles to the three–photon Z decay using supergraph techniques was presented. The authors there obtain strong relations between the fermionic, bosonic and scalar amplitudes. In [11] a summary of the contribution of all three types of particles was given.

4. MSSM contribution

In the MSSM there are besides the charged Higgs bosons, charged scalar leptons and scalar quarks. The calculation is exactly the same as for the charged Higgs. We only have to replace $\cot \Theta_W$ in Eq.(3) by $e_{3l,R} (T_{3l,R} - e_{3l,R} \sin^2 \Theta_W) / \sin \Theta_W \cos \Theta_W$. The summation over all generations (including the colour factor) and the scalar partner of the left and right handed fermions leads to $[3(1 - 88 \sin^2 \Theta_W/27)/\sin \Theta_W \cos \Theta_W]^2 \approx$
3.19, which is only a factor 8 larger than \( \cot^2 \Theta_W \approx 0.41 \) and therefore gives us still a result far below the SM contribution. Mixing of the scalar fermions, which is proportional to the fermion masses and therefore relevant only for the top quark, does not change the result to the three–photon \( Z \) decay rate since no new couplings are introduced. The scalar top quark contribution is only divided in different terms suppressed by the mixing angles.

In the MSSM there are also new fermions which contribute to the three–photon \( Z \) decay: the charginos, the mass eigenstates of the fermionic partners of the \( W \) bosons (winos) and the charged Higgses (Higgsinos). The mass eigenvalues, diagonalizing angles and couplings are well defined and in the notation of Eq.(1) given by \( V_{\tilde{\chi}i} = [2 \cos^2 \Theta_W - (U^2_{i\tilde{\chi}} + V^2_{i\tilde{\chi}})/2]/2 \). \( U \) and \( V \) are the diagonalizing angles of the charginos. With the parameters given in Eq.(1) I have \( 3e_{\tilde{\chi}1}V_L \approx 6 \times 10^{-2}, 9e_{\tilde{\chi}3}V_D \approx 5.8 \times 10^{-2} \) and \( 9e_{\tilde{\chi}3}V_u \approx 0.26 \) (0.17 without the top quark). For a wide range of the gaugino mass \((0 \leq m_{\tilde{g}} \leq 400 \text{ GeV})\), the mixing parameter \( \mu \) \((-500 \text{ GeV} \leq \mu \leq +500 \text{ GeV})\) and the ratio of the Higgs vacuum expectation values \((1 \leq \tan \beta \leq 10)\) I obtain \( 0.52 \leq e_{\tilde{\chi}1}V_{\tilde{\chi}1} \leq 0.77 \) and \( 0.27 \leq e_{\tilde{\chi}2}V_{\tilde{\chi}2} \leq 0.52 \). That is the couplings of the charginos are even higher than those of the up quarks. Unfortunately the couplings of the lighter chargino eigenstate is smaller than the heavier one and from experiment we know that the lighter chargino mass eigenstate has to be larger than half of the \( Z \) mass and therefore decouples like the top quark from the three–photon \( Z \) decay.

5. Final remarks

As a final remark I want to mention that the results given here can also be used for the three–gluon \( Z \) decay. In [3] it was shown that from the three–gluon self couplings there are also axial vector couplings contributions to this decay rate, although much smaller than the vector couplings. As a result they have \( \Gamma_F(Z \rightarrow 3g) \approx 6.5 \times 10^{-6} \text{ GeV} \), which is a factor of 6200 larger than the three–photon \( Z \) decay rate. When including scalar quarks we are lead to four different new types of diagrams with three–gluon self couplings. The calculations shows that when using gauge invariance, after Feynman integration and after summation of all permutation that each diagram itself leads to a null result. To obtain the three–gluon \( Z \) decay from the three–photon \( Z \) decay we therefore only have to replace \( e^3 \cos 2\Theta_W \) by \( g_312d_{abc}(T_{3qL,R} - e_{qL,R}\sin^2 \Theta_W) \) which leads to a factor of about 7400 larger than the scalar quark contribution to the three–photon \( Z \) decay. Unfortunately, experimentally this decay mode is indistinguishable from the similar \( Z \rightarrow q\bar{q}g \) decay.

6. Conclusions

We have seen that fermions lead to the largest contribution to the three–photon \( Z \) decay within the SM, followed by the \( W \) boson contribution, which is about a factor of 50 smaller. When extending the SM to the 2 HDM or the MSSM we have to include the scalars, which lead to a contribution several orders of magnitude smaller than the SM contribution.

Considering that the experimental upper limit is \( 8.1 \times 10^{-6} \) [12] and therefore even a factor of almost \( 10^4 \) smaller than the highest contribution of the SM this decay mode turns out not to be a very good one to test neither the SM nor the 2 HDM nor the MSSM. Decay rates of this order can only be obtained by composite
models [13–14] or, as was shown recently, in monopole models [15].

References

[1] V.N. Baier, E.A. Kurayev and V.S. Fadin, *Sov.J.Nucl.Phys.* **31**(1980)364.
[2] M.L. Laurson, K.O. Mikaelin and A. Samuel, *Phys.Rev.* **D21**(1981)2795.
[3] J.J. van der Bij and E.W.N. Glover, *Nucl.Phys.* **B313** (1989)237.
[4] J.J. van der Bij and E.W.N. Glover, ”Rare decays”, in Z physics at LEP 1, CERN 89-08 Vol.2 p.30.
[5] E.W.N. Glover and A.G. Morgan, *Z.Phys.-Particles and Fields* **C60**(1993)175.
[6] M. Baillargeon and F. Boudjema, *Phys.Lett.* **B272**(1991)158.
[7] X.Y. Pham, *Phys.Lett.* **B272**(1991)373
[8] F. Dong, X. Jiang and X. Zhou, *Phys.Rev.* **D47**(1993)214, **D46**(1992)5074.
[9] H. König, *Phys.Rev.* **D50** (1994)602.
[10] Z. Bern and A.G. Morgan, *Phys.Rev.* **D49**(1994)6155.
[11] J. Hořejší and M. Stöhr, ”One–Loop induced effective on–shell \(Z\gamma\gamma\) interactions”, PRA–HEP–94/4 to be published in *Z.Phys.-Particles and Fields* **C**.
[12] M. Sarakinos, ”Search for \(Z\to\gamma\gamma\) Decays at L3” these Proceedings.
[13] D. Treille et al.,”*Compositeness at LEP II*”, CERN 87-08.
[14] F. Boudjema and F.M. Renard, ”*Compositeness*”, in Z physics at LEP 1, CERN 89-08 Vol.2 p.205.
[15] A. de Rújula, ”Effects of Virtual Monopoles”, [hep-th/9405191](http://arxiv.org/abs/hep-th/9405191).