Electron paths and double-slit interference in the scanning gate microscopy

K Kolasiński and B Szafran
AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, al. Mickiewicza 30, 30-059 Kraków, Poland
E-mail: krzysztof.kolasinski@fis.agh.edu.pl

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Abstract

We analyze electron paths in a solid-state double-slit interferometer based on two-dimensional electron gas and mapping by scanning gate microscopy (SGM). A device with a quantum point source contact of a split exit and a drain contact for electron detection is considered. We study the SGM maps of source-drain conductance (G) as functions of the probe position, and we find that for a narrow drain, the classical electron paths are clearly resolved without any trace of double-slit interference. The latter is only present in the SGM maps of backscattering (R) probability. Double-slit interference is found in the G maps for a wider drain contact, but at the expense of a loss of information on the electron trajectories. We discuss the interplay of Young’s interference and interference effects between various electron paths introduced by the tip and the electron detector. The stability of the G and R maps versus the geometry parameters of the scattering device is also discussed.

1. Introduction

Scanning gate microscopy (SGM) is an experimental technique [1, 2] that uses the charged tip of the atomic force microscope to probe the transport properties of devices with two-dimensional electron gas buried shallow beneath the surface of the sample. The technique has been used to investigate quantum point contacts, which are the most elementary quantum transport devices [3, 4]. The conductance maps gathered with the SGM contain the characteristic oscillation of the period of half the Fermi wavelength, which appears due to the interference of electron wave functions coming from the quantum point contact and backscattered by the tip [5–13]. In our recent paper [14] we proposed a system with a split source channel to observe of the double-slit interference. Young’s interference should be present in the SGM maps, provided that the transport in the channel that feeds the split source occurs in the lowest sub-band lateral to the quantization [14]. The proposal of [14] and most previous experimental studies [5–8] dealt with systems in which the electron, after passing through the constriction defining the quantum contact, enters an infinite half-plane. In this work we consider the possibility of observing of electron paths in the context of double-slit interference. According to the which-path thought experiment by Feynman [15], determination of the slit that the electron goes through destroys the double-slit interference. Here, we demonstrate that SGM can be used to detect classical electron paths—although without indicating the one taken by the electron—with a simultaneous resolution of the double-slit interference effects. SGM has been frequently used to detect the semiclassical electron trajectories as deflected by the Lorentz force [16], including observation of the skipping orbits [17, 18] for systems with an additional confinement. Observation of electron paths requires placing an electron detector in the system—usually another quantum point contact (QPC) serving as the drain channel [16–18]. In this paper we consider a system with a split QPC serving as the electron source and a second QPC used to detect the electron passage. We demonstrate that the source-drain conductance maps for a narrow drain detector indicate the classical paths, but miss the double-slit interference. For an enlarged drain width, the double-slit interference appears in the map but the image of the paths is lost. The simultaneous observation of both the semiclassical paths and the Young’s interference is possible when one considers both the map of source-drain conductance and the SGM maps for backscattered electrons, or equivalently, the maps for conductance between the source and the rest of the system, excluding the
The effective width of the source channel, \( w_{\text{qpc1}} \), which splits the incoming electron wave function into two beams. The width of the QPC1 slits is kept at 40 nm throughout this work. The width of the feeding channel is \( w_{\text{qpc1}} = 40 \) nm throughout this work, with the exception of figure 10(a). The slits of QPC1 in this figure are separated a distance of \( d_{\text{qpc1}} = 160 \) nm. In the rest of the paper, we mainly use \( d_{\text{qpc1}} = 400 \) nm. The outgoing current is gathered by the drain channel–QPC2 at the right of width \( w_{\text{qpc2}} \) to determine source–drain conductance, \( G \). Additionally, the current between the source and the rest of the system, excluding the drain detector, is measured to evaluate the \( G_{\text{rest}} \) leakage conductance. The sample is assumed to be large and the blue dashed line indicates the ends of the computational box where transparent boundary conditions are applied. The distance between the computational box and the edge of the sample (\( L_d \)) is assumed to be much bigger than the coherence length, \( L_d \gg \lambda_{\text{coherence}}. \) The blue rectangle between the left and right QPCs shows the area of calculated SGM conductance maps. The distance between QPC1 and QPC2 is 600 nm. (b) The absolute value of the scattering wave function, \(|\psi|\). (c) The real part of \( \psi \). (d) The real part of the wave function calculated from equation (9) as the superposition of three point sources in the centers of the slits.

2. Model

We consider the experimental situation depicted in figure 1(a). The current is fed by the source contact to the channel that is filtered by the split quantum point contact, QPC1. The second QPC (QPC2) serves as an entrance to the drain contact at the right side of the figure and will be used for electron detection, as in [16–18], and more recently in [19]. The effective width of the source channel, \( w_{\text{qpc1}} \), and the width of both slits at the QPC1 side is taken to equal 40 nm, which for the considered Fermi energy transmits the current in the lowest sub-band only. According to a previous paper [14], the lowest sub-band transport on the input part is necessary to observe the Young’s interference. For a larger number of incident sub-bands, the conductance map for the double-slit system is a simple sum of maps for separate QPCs [14] since the double-slit interference disappears in the Landauer summation over the incident sub-bands. The electron leaving QPC1 enters a two-dimensional electron gas (2DEG). By ‘large,’ we mean that the \( L_d \) distance between the region of interest for the SGM and the edges of the sample is larger than the coherence length, so that the interference with the edges can be neglected. The source-drain voltage is considered to be low enough for the linear transport conditions to occur. The current passing to the drain is measured to evaluate the \( G \) conductance. The sample is grounded by a large, reflectionless contact, and this current is measured to evaluate the leakage conductance, \( G_{\text{rest}} \). From conservation of the current and for \( M_{\text{in}} \) sub-band in the input channel (we consider mostly \( M_{\text{in}} = 1 \) ), we have

\[
\frac{2e^2}{h} M_{\text{in}} = R + G + G_{\text{rest}},
\]

where \( R = \frac{2e^2}{h} P_{\text{bs}} \), where \( P_{\text{bs}} \) is the backscattering probability. Thus, \( R \) can be determined when \( G \) and \( G_{\text{rest}} \) are measured. In the following, we discuss numerical results for \( R \) and \( G \).

In the calculations, we focus on the region marked by the dashed lines in figure 1(a). We assume that the transport is coherent within the computational box. At the dashed lines, we apply transparent boundary conditions so that the region of interest is effectively open, in contrast to systems with a pair of QPCs used as source and drain for a closed stadium studied by SGM in [20, 21]. In this paper we set the distance between the input slits at \( d_{\text{qpc1}} = 400 \) nm, which is large enough to distinguish the trajectories of the electrons arriving at QPC2 from one of the input slits. In figure 1, a smaller value of \( d_{\text{qpc1}} = 160 \) nm is used for illustration. The total computational box covers the region where the transparent boundary conditions of size 800 nm × 1000 nm are applied, and the region where the scans by the SGM are taken is significantly smaller: 400 nm × 600 nm, as seen in figure 1(a).
To simulate the propagation of the Fermi level electrons within the region inside the computational box (the dashed lines in figure 1(a)), we consider coherent transport, as described by the effective-mass Schrödinger equation,

\[
\left\{-\frac{\hbar^2}{2m_{\text{eff}}} \nabla^2 + V_{\text{tip}}(x, y)\right\} \psi(x, y) = E_F \psi(x, y), \tag{2}
\]

where \(m_{\text{eff}} = 0.067m_0\) is the GaAs electron effective mass, \(E_F\) is the Fermi level energy, and the potential

\[
V_{\text{tip}}(x, y) = \frac{U_{\text{tip}}}{1 + \left(\frac{x - x_{\text{tip}}}{d_{\text{tip}}}\right)^2 + \left(\frac{y - y_{\text{tip}}}{d_{\text{tip}}}\right)^2 / d_{\text{tip}}^2} \tag{3}
\]

describes the effective perturbation induced by the atomic force microscope (AFM) tip, with amplitude \(U_{\text{tip}} = 15\) meV and width \(d_{\text{tip}}\), localized above point \(x_{\text{tip}}, y_{\text{tip}}\). This type of effective Lorentzian-shaped perturbation, which results from the screening of the charge on the tip by the 2DEG electron gas located under the sample surface, was obtained previously in our self-consistent Schrödinger-Poisson calculations [22, 23]. The width of the potential is of the order of the distance between the tip and the 2DEG [23]. In the SGM experiments, the 2DEG is buried at least 25 nm below the surface [26], and the minimal distance from the tip to the surface applied in SGM is 20 nm [1], so a minimal realistic value of \(d_{\text{tip}}\) is about 50 nm. In this paper we consider two widths of the tip: a small one, \(d_{\text{tip}} = 10\) nm, which is useful for the initial discussion since it sets the precision of the determination of electron paths, and a realistic one, \(d_{\text{tip}} = 50\) nm.

For the sake of simplicity, to define the contacts we assume hard-wall boundary conditions on QPC1 and QPC2. In experimental setups, the QPCs are usually defined electrostatically by the potential applied between the gates, so that the QPC potential has a saddle point profile. Nevertheless, we do not discuss conductance quantization as a function of the QPC width, and for a fixed number of transmitting sub-bands, the hard-wall potential gives qualitatively similar results to a saddle point potential, which was applied in a previous work [27]. The transparent boundary conditions at the blue dashed line of figure 1(a) were introduced by the method described in [27].

We work within the Landauer approach for a zero temperature in the finite difference implementation [28] of the quantum transmitting boundary method [29, 30]. Within the input channel, before the splitting end with the two QPC1 slits, the wave function at the Fermi level is given by the superposition of incoming and backscattered transverse modes

\[
\psi_{\text{qpc1}}(x, y) = \sum_{k=1}^{M_{\text{qpc1}}} \left\{ a_k e^{ikx} \chi_k^{\text{qpc1}}(y) + b_k e^{-ikx} \chi_\ast_k^{\text{qpc1}}(y) \right\} + \sum_{k=M_{\text{qpc1}}+1}^{+\infty} b_k e^{ikx} \chi_k^{\text{qpc1}}(y), \tag{4}
\]

where the last term corresponds to the summation over the evanescent modes [29–31] and \(M_{\text{qpc1}}\) is the number of current propagating transverse modes, \(\chi_k^{\text{qpc1}}\), in QPC1. We consider a single sub-band in the QPC1 channel and an arbitrary number of sub-bands in the QPC2 channel. The summation runs over the Fermi level wave vectors, \(k\), at subsequent lateral sub-bands. The transport from the input channel to the two slits is nonadiabatic, with a pronounced backscattering. Note, that the central island in figure 1(a) used as a beam splitter in the presence of a strong external magnetic field is likely to form a quantum Hall interferometer, as discussed in [24, 25].

For QPC2, we assume the boundary conditions of form

\[
\psi_{\text{qpc2}}(x, y) = \sum_{k=1}^{M_{\text{qpc2}}} d_k e^{ikx} \chi_k^{\text{qpc2}}(y) + \sum_{k=M_{\text{qpc2}}+1}^{+\infty} d_k e^{-ikx} \chi_k^{\text{qpc2}}(y), \tag{5}
\]

with \(M_{\text{qpc2}}\) being the number of conducting transverse modes in QPC2. The transverse modes, \(\chi_k\), for both QPCs were calculated using the method from [32]. One established, the scattering amplitudes, \(b_k, d_k\), and the amplitudes of the incoming modes, \(a_k\), of equation (4) are used to calculate the transmission probabilities. Throughout this paper we use \(E_F = 8\) meV, which gives the value of \(\lambda_F = 2\pi/k_F = 2\pi\sqrt{2m_{\text{eff}}E_F} \approx 53\) nm. We choose discretization grid spacing of \(\Delta x = 4\) nm, which is small compared to \(\lambda_F\).

After the solution of equation (2), for each incoming mode the source-drain conductance is evaluated by the Landauer formula

\[
G = G_0 \sum_{i=1}^{M_{\text{qpc1}}} T_i, \tag{6}
\]

where \(T_i\) is the transmission probability of the \(i\)th mode incoming from the QPC1 to the QPC2 and \(G_0 = 2e^2/h\). We refer to the sum of backscattering probabilities as the ‘resistance’, which is given by the formula
where \( R_i \) is the backscattering probability of the \( i \)th incoming mode to the QPC1.

We discuss below the maps of backscattering probability (SGM-R) and conductance maps (SGM-G) for the current flow between QPC1 and QPC2. To calculate the SGM-R/G images, we evaluated the 'resistance', \( R \), and conductance, \( G \), by scanning the system with the tip potential given by equation (3) inside the rectangle shown in figure 1(b). The spacing between the two subsequent positions of the tip was 4 nm, so to obtain the SGM image presented later in paper, for the scan area 400 nm × 600 nm, one has to solve the scattering problem 15000 times.

3. Results and discussion

3.1. In the absence of the tip: Young interference and interference due to the QPC2 detector

We begin the discussion by setting equal widths of both input and output channels within QPCs, \( w_{\text{QPC1}} = w_{\text{QPC2}} = 40 \) nm. In both the input and the output channels we have a single sub-band transport for the applied Fermi energy of 8 meV. According to our previous paper [14], the single sub-band transport on the input part allows us to use the SGM technique to resolve the Young’s interference pattern.

The amplitude of the scattering wave function in the absence of the tip is shown in figure 1(b). In addition to the well-resolved Young interference pattern with five essentially straight lines of constructive and destructive interference for the waves passing through both the slits (see figure 2(c)), one also notices additional vertical interference fringes. This feature is caused by the presence of the output QPC2, which reflects the incoming wave function back to the double-slit device.

The role of QPC2 in the scattering is more clearly visible in the real part of the wave function, displayed in figure 1(c). We found that a very similar wave function can be reproduced by a superposition of three point sources positioned at the center of each slit of the system, following the Huygens–Fresnel principle for the circular waves propagating from the input slits and QPC2; the latter is due to backscattering of the incoming wave [10]. In the limit of the thin slit, the Fermi level wave function at position \( r \) traveling from the slit located at position \( r_{\text{source}} \) can be well described by the angle-modulated Henkel function

\[
R = G_0 \sum_{i=1}^{M_{\text{QPC1}}} R_i, \quad (7)
\]
\[
\psi_{\text{Henkel}}(r, r_{\text{source}}, \alpha_{\text{source}}) = \alpha_{\text{source}} \cos(\theta) \frac{e^{i k_f |r - r_{\text{source}}|}}{\sqrt{k_f |r - r_{\text{source}}|}},
\]

where \(\theta\) is the angle between the \(r - r_{\text{source}}\) vector and the x-axis (i.e., \(\cos(\theta) = |x - x_{\text{source}}|/r\) and \(\alpha_{\text{source}}\) is the scattering amplitude). For the superposition of the three point sources, one has

\[
\psi(r) \approx \psi_{\text{Henkel}}(r, \eta_1, 1) + \psi_{\text{Henkel}}(r, \eta_2, 1) + \psi_{\text{Henkel}}(r, \eta_3, \alpha_{\text{reflection}}),
\]

where \(\eta_1 = (110 \text{ nm}, -80 \text{ nm})\), \(\eta_2 = (110 \text{ nm}, +80 \text{ nm})\), and \(\eta_3 = (670 \text{ nm}, 0 \text{ nm})\) are the positions of slits in the system in figure 1(b). We set \(\alpha_{\text{reflection}} = \alpha_{\text{source}}/2\), for which we get the best agreement with the exact solution obtained from equation (2). The real part of the wave function obtained from equation (9) is plotted in figure 1(d). Note that the fitted value of \(\alpha_{\text{reflection}}\) is quite large, which suggests that the backscattering from QPC2 will have a substantial influence on the SGM images discussed below.

The Young interference (figure 2(d)) and the interference due to backscattering by the QPC2 detector (figures 1(b) and (c)) are present already without the tip. The Young’s interference involves the superposition of waves passing through each of the slits with the modulation of the scattering density dependent on the relative phase

\[
\rho = \cos(k_f (\eta_1 - \eta_2)),
\]

where \(\eta_1, \eta_2\), stand for the distance from the lower and upper slit of the input QPC1. The density modulation in the Young’s interference pattern is plotted in figure 2(g), with the constant \(\rho\) values \(\eta_1 - \eta_2 = \text{const}\) forming hyperbolas with the slits of QPC1 as the focal points.

### 3.2. Interference mechanisms due to the tip

The backscattering by the tip introduces additional interference effects with the electrons reflected to the source QPC (figure 2(a)), the tip–induced double slit interference (figure 2(b)), and the interference of the direct wave with the scattered wave (figure 2(c)). These interference effects will be discussed later in the paper. The conductance map fringes due to the scattering of the type given by figure 2(a) are well known and have been discussed in a number of papers [5–12]. The backscattering \(R\) is enhanced when the phase shift of the wave going to the tip and back (the red arrows in figure 2(a)) produces a constructive interference with the incident wave (the blue arrow in figure 2(a)) at the input slit. The \(R\) signal is then proportional to

\[
I = \cos(k_f (\eta_1 + \eta_1)),
\]

where in the argument, \(k_f \times 2\eta_1\) is the phase acquired by the electron wave function on its way from a slit of QPC1 to the tip and back from the tip to the same slit. The interference of figure 2(a) leads to the angle-independent modulation of the \(R\) maps, which oscillate as a function of distance from the slit with the period of half the Fermi wavelength, \(\lambda_f/2\) [9, 10].

The tip induces a double-slit interference of the wave passing through one of the slits of QPC1, which is then scattered by the tip with the wave incident from the other QPC1 slit (figure 2(b)). An enhanced backscattering can be expected when the interference of the incident and the returning path is positive, or the \(R\) signal will be proportional to

\[
I = \cos(k_f (\eta_1 + \eta_2)).
\]

The result of formula (12) is plotted in figure 2(b). The fringes corresponding to a constant argument of the cosine function in equation (12) are elliptic, with both the slits of QPC1 as the focal points. The double-slit interference according to the mechanism of figure 2(b) and the resulting SGM effects were never discussed before. Equations (11) and (12) produce the same periodicity of \(\lambda_f/2\) at a large distance from QPC1.

The tip induces interference of the waves incident directly from QPC1 to QPC2, with the waves scattered by the tip depicted in figure 2(c). The electron current passing to QPC2, and thus the conductance \(G\), is then proportional to

\[
I = \cos(k_f (\eta_1 + \eta_3)),
\]

where \(\eta_3\) is the distance from the tip to QPC2. The results of equation (13) are displayed in the lower panel of figure 2(d). The interference fringes resulting from this mechanism are again elliptic, but with one slit of QPC1 and the QPC2 detector as the focal points. This type of interference leads to lateral fringe patterns in SGM-G images, which will be discussed below.

### 3.3. Resistance maps

In figures 3(a)–(f) we show the SGM maps of resistance (SGM-R) as functions of the position of the AFM tip. The area of the scan is shown by the blue rectangle in figures 1(a) and (b). We consider a single or both QPC1 slits to be open, as illustrated schematically in the insets in the top right corner. For each system in figures 3(h)–
we plotted the corresponding probability current distribution in the second row. In figures 3(a) and (b) we show the results of SGM-R images obtained for a single open (lower) input slit. In the absence of QPC2 (figure 3(b)), we observe circular fringes in SGM map due to interference of the type given in figure 2(a) between the wave incoming from the slit and the wave backscattered by the AFM tip (see the arrow in figures 3(d) and 2(a), and equation (11)).

Figure 3(d) presents the sum of the SGM-R maps of figures 3(a) and (c) obtained for a single open slit. The sum is quite different from the image obtained for the system where both QPC1 slits are open (see figure 3(e)), which is a signature of the double-slit interference effects and involves both the Young’s interference (figure 2(d)) and the tip-induced interference (figure 2(b)). The Young’s interference is more clearly resolved in the absence of QPC2 (figure 3(f)), although it is also present when the QPC2 detector is a part of the setup (figure 3(e)). To demonstrate this more clearly, we extracted the enlarged fragments of figures 3(e) and (f) in figures 4(b) and (c). For a simple sum of SGM images for separate slits (figure 3(d)), instead of the elliptic fringes and Young’s pattern, we observe a checkerboard pattern (figures 3(d) and 4(a)). This checkerboard pattern should be observed in the experiments for a large number of incident sub-bands (i.e., at a higher Fermi energy or a wider input channel, as discussed in [14]).

In figures 3(h)–(m) we plotted the current density within the system in the absence of the tip. In the absence of QPC2 (figure 3(l)), the current distribution is very similar with or without QPC2. The deformation of the Young’s interference for the system with QPC2 is due to the lateral interference pattern involving the paths marked in figure 2(b), which are introduced by QPC2.

The Young’s interference pattern given by equation (10) and calculated for the Fermi wave vector is displayed in figure 3(n), and it has a good agreement with the probability current distribution of figures 3(l) and (m) and the features of the SGM map of figure 3(f). A resemblance to figure 3(e) with QPC2 present can also be
spotted, although the lines of flat R in figure 3(e) are curved. The curvature and the presence of the lateral fringes can be reduced by placing the QPC2 further from QPC1, thus reducing the backscattering by QPC2, but at the expense of the reduced G map contrast (see below).

3.4. Source-drain conductance maps

The SGM maps of source-drain conductance (SGM-G) in figure 5 exhibit a valley of minimal values along the shortest path from the source channel to the drain. The pattern of the image obtained for both slits open (figure 5(d)) is exactly the same as the one given by the sum of images in figures 5(a) and (b) (see figure 5(c)). This shows that in SGM-G images, the double-slit interference is absent, which may be quite surprising, given that the SGM-R image clearly resolved the interference.

In figure 5(a), a lateral fringe pattern is observed along the classical trajectory. This valley of minimal G values coincides with the line of maximal backscattering R observed in figure 3(a). Figure 6(a) shows that the tip separates the electron wave into beams, which arise from the interference of waves incoming from QPC1 and scattered by the tip. The variation of the electron density depends on the relative phase of the wave reaching point r from QPC1 directly, and the one that first gets to the tip and then is scattered to r, which results in the conductance around the classical path.

\[ \rho(r) = \cos \left( k \left( r - (n_1 + n_2) \right) \right), \]  

where \( r_1 \) is displayed in figure 2 and \( r_2 = |r - r_1| \). The results of formula (14) are plotted in figure 6(b) with a very good agreement with the numerical result of figure 6(a), which also contains the fringes due to backscattering by QPC2. Figure 6(a) corresponds to the position of the tip above the point A and explains the valley of low conductance R observed in figure 3(a). Figure 6(a) shows that the tip separates the electron wave into beams, which arise from the interference of waves incoming from QPC1 and scattered by the tip. The variation of the electron density depends on the relative phase of the wave reaching point r from QPC1 directly, and the one that first gets to the tip and then is scattered to r, which results in the conductance around the classical path.

The points A, B, C in figure 3(a) are the same as in figure 5(a), and they correspond to plots in figures 6(a)–(d). Note that point B corresponds to a minimum of R and a maximum of G, while point C corresponds to a
maximum of both quantities. The lateral pattern in the $R$ map appears only in presence of the QPC2 detector (cf figures 3(a) and (b)), and is a result of scattering first by the tip, and next by QPC2. The effect of the interference of the type in figure 2(c) depends on both the position of the tip with respect to the QPC2-QCP1 return path for the backscattered electrons and the incidence angle of the waves deflected by the tip on the edges of the QPC2 electron.

Let us now consider both slits open with the AFM tip at a position where the electron flow from the upper slit is totally blocked (see figure 6(e)). The current flux from the lower slit will be the only one reaching QPC2. In this manner, the tip turns off one of the slits of the source channel. Using the symmetry arguments for the considered device, we will get the same conclusion with second slit blocked and the first transmitting current.

Such an argument can explain why there is no visible interference at certain points in the obtained SGM-G maps. Nevertheless, there are points where the potential of the tip does not totally block the current from any slit, so interference could be expected. Such a case is presented in figure 6(f), with the tip located at a position where the current distribution is almost the same as in the unperturbed system (for comparison, see figure 3(l)). We conclude that the double-slit interference is clearly visible in the current distribution, but not in the SGM-G images.

The lack of Young’s interference in the source-drain conductance images can be explained using a reversed bias and the current flowing in the reverse direction (i.e., from QPC2 to QPC1). The upper row of figure 7 shows the SGM-G, SGM-R, and probability density for the current flowing from the double slit to QPC2, while the lower row presents the quantities for the opposite current direction. We can see that the results for SGM-G are identical for both current directions (figures 7(a) and (d)) in spite of the fact that the electron scattering is very different in both setups, with a clear Young’s interference in figure 7(c) and no similar counterpart in figure 7(f).

The SGM-G images for both the cases (figures 7(a) and (d)) are bound to be identical due to the Onsager microreversibility relation for the single sub-band transport, $T_{qpc2,qpc1} = T_{qpc1,qpc2}$, which in terms of the Landauer approach implies no current flow for zero bias. For the electron incident from the right, there is no reason to expect the presence of Young’s interference, which is indeed missing in figure 7(f). The absence of the double-slit interference in SGM-G follows from this observation and the microreversibility relation. Note that the SGM-R image for the reversed current orientation (figure 7(e)) clearly indicates the classical paths that the electron can follow from QPC2 to QPC1. Also, note that QPC2 is 100 nm further to the right of the end of the figure, and that two bright beams are emitted from QPC2, which is only 40 nm wide.

There is a way to restore the interference in the SGM-G images. Each of the $M_{in} \text{sub-bands}$ is with a certain probability backscattered, transferred to QPC2, or exits the computational box through the transparent

![Figure 7](image-url)

Figure 7. (a) The SGM-G image, (b) SGM-R image, and (c) the electron density, $|\psi|^2$, obtained for current flow to the right direction in the absence of the tip. (d)–(f) The same as in the top row, but with current flow in the opposite direction. The results were obtained for $U_{tip} = 15 \text{meV}$ and $d_{tip} = 10 \text{nm}$. 
boundary conditions to the rest of the system (see equation (1)); \( M_{in} \) is independent of the tip position. Now if we consider that \( w_{QPC2} \) becomes wider, the ratio \( G_{rest} / G \) decreases, since QPC2 will be able to transfer more current, and the probability of the electron transfer form QPC1 to QPC2 increases with the width of the latter. Thus, for a large width, \( w_{QPC2} \), the value of \( G_{rest} \) in equation (1) will be small, and hence \( MR_{G} \) in \( 2e^2 \approx + \). We can express the conductance of the system \( G_{MR} \), \( \propto - \) for large values of \( w_{QPC2} \), the \( SG_{MG} - G \) should start to exhibit the interference pattern as the SGM-R images do. In figures 8(a)–(e), we show the results obtained for a large value of QPC2 width, \( w_{QPC2} = 800 \text{ nm} \). Figure 8(a) shows the SGM-G image for the having the bottom slits open, and figure 8(b) is a sum of the images obtained for bottom and upper slit open separately. Now, the SGM-G image for both slits open (figure 8(c)) is clearly different from figure 8(b), with distinct stripes due to the Young’s interference. We can compare this result with the SGM-R image of figure 8(d), which is now quite similar to the SGM-G image, particularly in the center and close to QPC2. In this region, the approximated relation, \( G \propto -R \), is easily visible and the SGM-G image is indeed a negative of the SGM-R image. To illustrate this further, in figure 9 we plotted a cross section of these images along the axis of the device. In figure 9 we find that the \( G \) and \( R \) change almost in antiphase for \( x_{tip} > 450 \text{ nm} \), which is not exactly the case for lower values of \( x_{tip} \). In figure 9 we also plotted the leakage conductance, \( G_{rest} \), which corresponds to electrons going out of the device by the region marked by the blue dashed line in figure 1. Note that \( R + G_{rest} + G = G_{0} \) and \( G_{rest} \) was multiplied by 3 for the presentation in figure 9. For the tip close to QPC1, \( x_{tip} < 250 \text{ nm} \), \( G_{rest} \) and \( R \) are almost in phase. On the other hand, for a large \( x_{tip} \), \( G_{rest} \) gets in phase with \( G \). The change of the correlation can be explained when we consider that the tip is a source of a circular scattered wave, which produces a larger electron flux to the closer QPC. Scattering by the tip increases \( G_{rest} \) independent of \( x_{tip} \) and one of two quantities, \( R \) or \( G \), depending whether QPC1 or QPC2 is closer to the tip. Varying correlation \( G_{rest} \) and the other conductance probabilities seems responsible for the variation of the phase between \( G \) and \( R \) in figure 9.

Note that the increased width of QPC2 allows us to restore the interference in the SGM-G images, but at the expense of the lost information on the electron trajectories from QPC2 to QPC1. The double-slit interference is
present as long as one does not interfere with the measurement by trying to determine through which slit the particle passes [15]. Here, if we set the detector (QCP2) for mapping the electron trajectories with a small value of $w_{qpc2}$, we gain the information about electron trajectories, but we lose the interference pattern. Increasing the width, $w_{qpc2}$, leads to a reduction of the spatial resolution of the detector, so we lose the paths in the images, but restore the interference pattern.

### 3.5. Stability of classical trajectories in SGM-G images

The classical trajectories as extracted from the SGM-G images should be stable against the geometrical parameters the system, the width of the QPC1 channel, $w_{qpc1}$ (see figure 1 (a)), or the distance between the QPC1 slits, $d_{qpc1}$ in particular. For each value of the width of the QPC1 channel, $w_{qpc1}$, we calculated the SGM-G ($w_{qpc1}$) and SGM-R ($w_{qpc1}$) images. The calculations were performed for $d_{qpc1} = 400$ nm. Both slits had the same width equal 40 nm and unchanged position as the width of the feeding channel, $w_{qpc1}$, is changed. In figure 10 (a), we plotted the Pearson correlation coefficient [22] between SGM-G ($w_{qpc1}$) and the last image of SGM-G (400 nm), which is the $r(G)$ curve, and the correlation between SGM-R ($w_{qpc1}$) and SGM-R (400 nm), which is the $r(R)$ curve. We chose the SGM image for $w_{qpc1} = 400$ nm as a reference. One can see in figure 10 (a) that lines $r(G)$ and $r(R)$ quickly stabilize around 1 after $w_{qpc1} = 150$ nm, which means that through all the values from around $w_{qpc1} = 150$ nm to 400 nm, both images stay almost unchanged.

We found that the SGM-G images are generally stable in the function of $d_{qpc1}$, which means that paths are always visible, except when the distance between slits is small enough that both trajectories (from each QPC) overlap, which makes the calculated SGM-G maps difficult to interpret. Sample images of SGM-G and SGM-R for small values of $d_{qpc1} = 160$ nm are displayed in figures 10 (b) and (e), respectively. For larger values of $d_{qpc1}$, the classical trajectories are restored (see figures 10 (c) and (d)). Note that for figures 10 (c) and (d), the difference between the values of $d_{qpc1}$ in each case is equal to 16 nm. The results for SGM-G are nearly identical. However, the SGM-R images, which are sensitive to the interference effects, very strongly depend on a specific value of $d_{qpc1}$ (see figures 10 (f) and (g)). The wave function passing through both the input slits interferes with the AFM tip, as well as with QPC2. The result of the interference in terms of the backscattering depends on the variation of the distance between the slits of the order of the period of the waves formed by interference at the Fermi level, which is equal to $\lambda_F/2$.

### 3.6. Wide tip potential

Let us now consider the wider tip potential, $d_{tip} = 50$ nm. The results for conductance $G$ for a single slit in figure 11 (a) and both slits in figure 11 (b) still indicate the classical current paths, although naturally the widths of the G minima are significantly increased. The G map pattern is still very similar to the one obtained by a sum of maps for separate slits (figure 11), indicating a lack of double-slit interference features in the source-drain conductance maps for $M_{out} = 1$, as discussed above.

The ‘resistance’ map (figure 11 (e)) for both QPC1 slits open contains the circular fringes near the input slits (single-slit interference of figure 2 (a)), elliptical fringes (double-slit interference of figure 2 (d)), and the lateral fringes (figure 2 (b)). All these effects are tip related and in this case dominate the Young’s interference, which is
weaker but still detectable. The Young’s interference is restored when the QPC2 detector is placed further from the QPC1 (see figure 11(h)), but naturally at the expense of the amplitude of the G signal (figure 11(g)). The reduction of the lateral fringe pattern is observed for R since it involves backscattering by QPC2 (figure 11(h) and (i)).

4. Conclusions

We have considered using the SGM technique to image the electron trajectories for the double-slit experiment. Several interference mechanisms induced by the tip and the drain contact as the electron detector have been found, and the paths leading to the interference have been identified. We studied the SGM source-drain conductance maps and demonstrated that the classical electron paths are clearly resolved, but only for a narrow drain contact, for which the double-slit interference features are absent. The double-slit interference pattern is present in the conductance maps, but only for a wider drain contact, when the electron paths are no longer resolved.

We have indicated that a way to observe both the trajectories and the interference pattern is to look simultaneously at two different SGM maps for the backscattering, R (which contain double-slit interference signal) and for the conductance, G (that reveal the paths). The latter allows one to map all the equivalent classical trajectories, but without indicating the specific path the electron took on its way to the drain channel.
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