BARYONS WITH HOLOGRAPHY

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We perform the first study of baryons in holographic QCD with D4/D8/D8 multi-D-brane system in type IIA superstring theory. The baryon is described as a chiral soliton solution in the four-dimensional meson effective action derived from holographic QCD. We also present a brief review of the holographic model from the viewpoints of recent hadron physics and QCD phenomenologies.

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1. Review of Holography

Holography is a new concept of the superstring theory based on the duality between gauge and supergravity theories, proposed by Maldacena in 1997. The superstring theory with conformal and Lorentz invariance has ten-dimensional space-time to avoid anomalies. Its elemental degrees of freedom are one-dimensional open or closed strings. The fluctuation modes of these strings are expected to give the elementary particle degrees of freedom as in the standard model. One can also find a soliton of strings as the (p+1)-dimensional membrane composed by the condensed strings, called Dp-brane. In particular, Dp-brane has two important aspects as follows.

First, a (p+1)-dimensional gauge theory appears on Dp-brane. Here, an open string with the two edges on the Dp-brane has ten independent fluctuation modes; scalar fields Φ_{i=(p+1)−9} and the other vector-like fields A_{μ=0−p}. A_{μ=0−p} have an index μ on Dp-brane extending to (p+1)-dimensional space-time, and A_{μ} fields can be regarded as gauge fields on the Dp-brane. In this sense, (p+1)-dimensional gauge theory appears on Dp-brane.

Second, (p+1)+1)-dimensional supergravity appears around the Dp-brane. In fact, Dp-brane is found to be a strong gravitational system in ten-dimensional space-time as a black hole or a black brane giving nontrivial curvatures around it. Due to the geometrical symmetry, the angle coordinates around Dp-brane become trivial and can be integrated out in the action of Dp-brane. Then, there eventually
remains just one radial dimension with nontrivial curvature, indicating the existence of gravity. In this sense, \((p+1)+1\)-dimensional supergravity appears *around* \(D_p\)-brane. (The italic "1" denotes the radial dimension around \(D_p\)-brane.)

Now we discuss the holography. Holography indicates the equivalence between the \((p+1)\)-dimensional gauge theory without gravity on \(D_p\)-brane and the \((p+1)+1\)-dimensional supergravity *around* \(D_p\)-brane. Dimensions are different with each other to give the concept of holography. In particular, the \((p+1)\)-dimensional gauge theory projected on the surface of \(D_p\)-brane is often called as the "hologram" of the \((p+1)+1\)-dimensional supergravity.

The most essential property of holography is the existence of strong-weak duality (S-duality) between the gauge theory and the supergravity: couplings are transversely related with each other. Therefore, if one wants to analyze the non-perturbative (strong-coupling) aspects of one side, one can attack from the other dual side only with the tree-level (weak-coupling) calculations. Consequently, this new concept of holography is expected to be a powerful tool for both sides of the gauge theory in hadron physics and the gravitational theory in astrophysics.3,4

2. Holographic QCD

If one succeeds in constructing QCD with quarks and gluons on a special configuration of D-branes, one can attack the non-perturbative aspects of QCD from the tree-level dual supergravity side. This is the strategy of holographic QCD.

In 2005, Sakai and Sugimoto succeeded in constructing massless QCD on \(D4/D8/\overline{D8}\)-brane system.\(^5\) \(D8\)-brane has opposite chirality relative to \(D8\)-brane to introduce the concept of chiral symmetry in this model. Then, by using the concept of holography, non-perturbative aspects of massless QCD can be analyzed by the tree-level calculation in the dual \((4+1)+1\)-dimensional supergravity side, called holographic QCD. In fact, this theory describes mesons as color-singlet low-energy objects, and reproduces meson mass spectra in coincidence with experimental data, and also other many traditional meson phenomenologies like vector-meson dominance, KSRF relation, GSW model and so on.\(^5\) In this sense, holographic QCD can be regarded as the "unified meson theory" based on QCD.

In this construction, however, \((4+1)+1\)-dimensional classical supergravity is dual with large-\(N_c\) QCD. Then, as a general property of large-\(N_c\) QCD, another important color-singlet objects as *baryons* do not directly appear in the large-\(N_c\) limit.\(^6\) In fact, it is non-trivial to describe baryons in the large-\(N_c\) holographic QCD.

3. Baryons as Brane-induced Skyrmions in Holographic QCD

In order to describe baryons in holographic QCD, we introduce the concept of chiral solitons (Skyrmions) in the large-\(N_c\) holographic model. The Skyrm model is first proposed in 1961,\(^7\) describing a baryon as a topological chiral soliton of the pion field, which is the Nambu-Goldstone boson relating to spontaneous chiral-symmetry breaking. Here, the stability of the chiral soliton is known to be sensitive
to the four-derivative terms of pion fields. Chiral and Lorentz symmetries allows three candidates as four-derivative terms:

$$\text{tr}[L_\mu, L_\nu]^2, \quad \text{tr}[L_\mu, L_\nu]^2, \quad \text{tr}(\partial_\mu L_\nu)^2, \quad (1)$$

where $L_\mu$ is one-form of pion fields, $L_\mu = (1/i) U^\dagger \partial_\mu U$. The first term in Eq. (1) is the Skyrme term. The other two candidates in Eq. (1) are known to give instability of the Skyrmion but these two cannot be excluded by the symmetry arguments.

As a remarkable fact, in the leading order of holographic QCD, only the Skyrme term in Eq. (1) appears. This can be understood from the fact that the leading order of the effective action (DBI action) of D8-brane includes only “two time-derivatives” at most, which gives a severe restriction on the possible terms in the effective meson theory. Thus, the topological picture for baryons is derived from QCD and seems to be justified ingeniously in this framework with the superstring theory.

We investigate the hedgehog configuration for pion fields $U(x)$ and $\rho$ meson fields $\rho_\mu(x)$ in the meson effective action derived from holographic QCD as

$$U^*(x) = e^{i\tau_a \hat{x}_a F(r)}, \quad \rho^a_0(x) = 0, \quad \rho^a_i(x) = \rho^{i a}(x) \tau_a/2 = \left\{ \varepsilon_{iab} \hat{x}_b \hat{G}(r) \right\} \tau_a. \quad (2)$$

$F(r)$ ($r \equiv |x|$) is a dimensionless function with boundary conditions $F(0) = \pi$ and $F(\infty) = 0$, giving topological charge equal to unity as a unit baryon number. With the hedgehog configuration Ansatz (2), we uniquely derive the energy density $\varepsilon[F(r), \hat{G}(r)]$ for the brane-induced Skyrmion including interaction terms of pions and $\rho$ mesons from holographic QCD as follows:

$$r^2 \varepsilon[F(r), \hat{G}(r)] = \frac{f_\pi^2}{4} \left[ 2 \left( r^2 F'^2 + 2 \sin^2 F \right) \right] + \frac{1}{32 \epsilon^2} \left[ 16 \sin^2 F \left( 2 F'^2 + \frac{\sin^2 F}{r^2} \right) \right] + \frac{1}{2} \left[ \left\{ 3 \hat{G}'^2 + 2 r \hat{G}' \left( \hat{G} + r \hat{G}' \right) \right\} + m^2_{\rho} \left[ 4 r^2 \hat{G}' \right] - g_{3\rho} \left[ 16 r \hat{G}^3 \right] + \frac{1}{4} g_{4\rho} \left[ 16 r^2 \hat{G}'^4 \right] - g_1 \left[ 16 \left( F' \sin F \cdot \left( \hat{G} + r \hat{G}' \right) + \sin^2 F \cdot \hat{G}/r \right) \right] - g_2 \left[ 16 \sin^2 F \cdot \hat{G}' \right] - g_3 \left[ 16 \sin^2 F \cdot (1 - \cos F) \hat{G}/r \right] - g_4 \left[ 16 (1 - \cos F) \hat{G}' \right] + g_5 \left[ 16 r (1 - \cos F) \hat{G}^3 \right] + g_6 \left[ 16 r^2 F'^2 \hat{G}' \right] + g_7 \left[ 8 (1 - \cos F)^2 \hat{G}' \right] \right]. \quad (3)$$

All the coupling constants ($\epsilon$, $g_{3\rho}$, $g_{4\rho}$, $g_{1-7}$) in Eq. (3) are uniquely determined by just two experimental inputs as $f_\pi = 92.4\text{MeV}$ and $m_\rho = 776\text{MeV}$, which is a remarkable consequence of holographic framework. We numerically obtain the stable chiral soliton solution as a baryon in holographic QCD, which basically supports the “Skyrme soliton picture” for baryons with corrections by $\rho$ mesons. Figure (1) shows the total energy density of a baryon and each contribution of $\rho$ meson interaction terms derived from holographic QCD. This result indicates the active contribution of $\rho$ mesons in the deeper interior region of baryons, and such a “$\rho$ mesonic parton” in the soliton may lead to a new picture for baryons from holographic framework.
4. Summary and Concluding Remarks

We have performed the first study of baryons as chiral solitons in holographic QCD with D4/D8/D8 multi-D-brane system.\textsuperscript{9,10} We have found that the holographic QCD has a stable hedgehog soliton solution as a baryon, which is basically described as the Skyrme soliton of pions including $\rho$ mesons in the interior region.

Just after our study, similar studies of baryons were done using instantons in the five-dimensional gauge theory in holographic framework.\textsuperscript{11,12} It is also interesting to compare chiral solitonic baryons in the holographic QCD with other-type construction of stringy hadrons in a holographic framework.\textsuperscript{13}

From more general point of view, "holography" is a powerful concept which links QCD-hadron physics and blackhole-astrophysics by virtue of its S-duality. Large applications of this new concept are much expected in near future.

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