Probabilistic Assessment on Stability of Slopes Reinforced with Piles Considering Spatial Variability of Soil Properties

Fuyong Chen¹, Longfei Cheng¹, Tingqiang Zhou⁴, Xiang Chen⁴ and Wengang Zhang ¹,²,³*

¹Key Laboratory of New Technology for Construction of Cities in Mountain Area, Chongqing University, Chongqing 400045, China
²School of Civil Engineering, Chongqing University, Chongqing 400045, China
³National Joint Engineering Research Center of Geohazards Prevention in the Reservoir Areas, Chongqing University, Chongqing 400045, China
⁴School of Civil Engineering, Chongqing Three Gorges University, Chongqing 404100, China

*Corresponding author’s e-mail: cheungwg@126.com

Abstract. The spatial variability is an inherent property of soil materials, which heavily affected the failure probability of slope. In the present study, deterministic analysis with factor of safety and reliability analysis in uniform soil grounds and spatial variable soil conditions are successively carried out to investigate the effect of uncertainties and the spatial variability of soil on the stability of slope reinforced with piles using random limit equilibrium method (RLEM). The optimal location and length of pile are also analysed based on the calculation results of RLEM, which is inconsistent with the results of traditional deterministic method. The obtained results demonstrate the significant effect of the spatial variability on the failure probability of slope reinforced with pile.

1. Introduction

In practice, the slope failure often leads to serious disasters. Therefore, slope stability analysis is always a research hotspot in geotechnical engineering. Stabilizing piles are widely used to reinforce slope in engineering[1]. The deterministic analysis with the factor of safety (FS) is traditionally adopted to evaluate the stability of reinforced slope in the support design. The traditional method always selects a series of representative values of soil parameters to calculate the factor of safety, which has ignored the variance of soil properties[2]. Many studies have shown that the factor of safety cannot fully evaluate the stability of slope since some slopes with a high factor of safety still ultimately fail[1-3].

Therefore, the variability of soil parameters has received considerable attention by researchers in recent years. However, few literature has reported the reliability analysis on the stability of slope reinforced with pile. Li and Liang[4] investigated the failure probability of a slope reinforced with piles along a given sliding surface. Zhang et al.[1] extended an existing method for system reliability analysis of unreinforced slopes to slopes reinforced with piles. But these two articles only considered the variance of soil properties, while the inherent spatial variability of soil was omitted. Different methods including random limit equilibrium method (RLEM), random finite element method (RFEM),
random finite difference method (RFDM) are commonly used to calculate the failure probability of slope considering the spatial variability of soil. The stability of slope reinforced with the geogrid in spatial variable soils is only investigated by Luo et al.\cite{5} and Luo and Bathurst\cite{6}. There is almost no literature about the random field simulation of slope with the reinforcement of pile.

In this study, the effect of spatial variability of soil properties (cohesion $c$ and friction $\phi$) on the slope reinforced with pile was studied by using the random limit equilibrium method. Deterministic analysis with factor of safety and reliability analysis of slope in uniform soil condition and spatial variable soil were carried out successively to compare the optimal reinforcement scheme considering different pile location and pile length. Parametric analyses were also conducted to investigate the influence of horizontal and vertical fluctuation of soil properties on the failure probability of slope reinforced with pile.

2. Numerical model of slope reinforced with pile

Numerical Package Slide2\cite{7} is adopted to carry out the reliability analysis of slope considering the spatial variability of soil. A typical slope model is shown in figure 1\cite{8}. The slope height is 10m. The slope angle is 45°. The ideal slope contains only one soil material. Following Li et al.\cite{8}, cohesion $c$ and friction angle $\phi$ are assumed as cross-correlated lognormal random fields. The statistical properties of soil parameters are presented in table 1. The unreinforced slope in figure 1 is calculated by Bishop’s simplified method through the mean value of soil parameters in table 1 to obtain the factor of safety which is equal to 1.206. The value of safety factor of the unreinforced slope is close to 1.208 as reported in Li et al.\cite{8}. Therefore, the established model of the soil slope in figure 1 is validated to be appropriate to conduct the following analysis. In this paper, only the pile length $L$ and the location of single pile are included to study the failure probability of slopes reinforced with pile in spatial variable soils. As shown in figure 1, there are five locations P1, P2, P3, P4, P5 which are horizontal spaced by 2m from the top to the toe. The shear strength of pile is set to be large enough to 12000kN, which is not assumed to fail. In figure 1, the soil slope is reinforced by a single pile with a length of 6m at the location of P2.

![Figure 1. Slope reinforced with stabilizing pile.](image)

| Soil parameters       | Mean value | COV | Distribution | Scale of fluctuation | Correlation efficient |
|-----------------------|------------|-----|--------------|----------------------|-----------------------|
| Cohesion $c$(kPa)     | 10         | 0.3 | Lognormal    | $\delta_h = 40m$    | $\rho_{c,\phi} = -0.5$ |
| Friction angle $\phi$(°) | 30     | 0.2 | Lognormal    | $\delta_\phi = 4m$  |                       |
| Unit weight $\gamma$(kN/m$^3$) | 20    |     |              |                      |                       |
3. Safety Factor of slopes reinforced with pile

The factor of safety is often used in the conventional stability assessment of slope. Pile length and pile location are considered to calculate the factor of safety of the reinforced slope. Figure 2 shows the effect of pile location and pile length on the safety factor of slope based on Bishop’s simplified method. As the pile length \( L \) increases at locations P1, P2, P3 and P4, the factor of safety does not change at first, then increases, finally stabilizes. The reason why the factor of safety at first is equal to the safety factor of unreinforced slope is that the pile bottom does not exceed the sliding surface of unreinforced slope. Therefore, the pile has little effect on the safety factor of slope at this point. When the pile passes through the sliding surface of unreinforced slope, the pile exerts a supporting force on the slope so that the safety factor increases with the increasing of pile length as shown in figure 3. Due to the transformation of the failure surface of the slope, the factor of safety gradually stabilizes when the length of the pile increases to a certain value. The distance from the point P5 to the sliding surface is the shortest among the five locations of pile. Therefore, when the pile length is 2m at P5, the safety factor is significantly increased. When it comes to getting the maximum safety factor, the slope should be reinforced with the pile length of 7m at the location P3.

![Figure 2. Effect of pile location and pile length on the safety factor (FS) of the slope.](image1)

![Figure 3. Vertical distance D between pile top and sliding surface of unreinforced slope.](image2)
4. Reliability analysis of slope reinforced with pile

Since the deterministic factor of safety method considered in the previous sections does not explicitly reflect the inherent and spatial uncertainties in the soil properties, a reliability-based approach is considered in this section to assess the failure probability of soil slope reinforced with pile considering the spatial variability using Bishop’s simplified method. Also shown is the case without considering the spatial variability (i.e., uniform soil conditions). In reliability analysis of slope reinforced with pile, the Latin Hypercube Sampling (LHS) with sufficient samples (the maximum number of simulation is 200000) is performed to ensure the steady that the calculated failure probability reaches convergence.

4.1. Random field theory

Many researchers have found that the soil properties exist considerable variation in different locations and directions[9-10]. Vanmarcke[11] put forward the random field method to simulate the spatial variability of soil properties. The scale of fluctuation is used to describe the spatial variability. Some theoretical functions including exponential, Gaussian, Spherical and Cubic functions, are usually used to characterize the spatial correlation of soil properties, which have avoided to develop the spatial correlation relationship based on large amount of statistical data. The Markov autocorrelation function in 2D form are selected to characterize the spatial correlation as below:

\[ \rho(h, v) = \exp \left\{ -\frac{2}{\delta_h^2} \left( \frac{h}{\delta_h} \right)^2 - \frac{2}{\delta_v^2} \left( \frac{v}{\delta_v} \right)^2 \right\}^{1/2} \]  

(1)

where \( \rho(h, v) \) is the autocorrelation function, \( h \) and \( v \) are the horizontal and vertical distances between two points. Parameters \( \delta_h \) and \( \delta_v \) are the horizontal and vertical scale of fluctuation, respectively. The scale of fluctuation represents the maximum distance, beyond which the spatially random parameters are uncorrelated. The soil properties become more uniform with the increasing of the scale of fluctuation. In this study, the horizontal and vertical scale of fluctuation of cohesion \( c \) and friction \( \phi \) are \( \delta_h = 40m \) and \( \delta_v = 4m \) as shown in table 1. Two example of the random field of cohesion \( c \) and friction angle \( \phi \) are illustrated in figure 4 and figure 5. As shown in figure 4 and figure 5, blue elements represent strong soil while red elements correspond to weak soil.

![Figure 4. Random field of cohesion c.](image)

![Figure 5. Random field of friction angle \( \phi \).](image)

4.2. Considerations of spatial variability of soil properties

Figure 6 shows the comparison of the failure probability \( P_f \) of the pile reinforced slope between uniform soil condition and spatial variability soil. As shown in figure 6(a), the maximum failure probability \( P_f \) of slope with uniform soil condition is about 8.7%, while figure 6(b) shows that the maximum failure probability \( P_f \) of slope with spatial variability is about 0.96%. No matter where the pile is and how long the pile is, the failure probability \( P_f \) of slope considering the spatial variability is significantly less than the uniform soil condition from figure 6. It is obvious that the spatial variability of soil has a significant effect on the failure probability of soil. The calculated failure probability of slope tends to be conservative without considering the spatial variability of soil.
In figure 6, the failure probability $P_f$ remains constant at first with the increasing of pile length $L$. The failure probability $P_f$ gradually increases as pile length $L$ increases when the pile crosses through the sliding surface of unreinforced slope. Finally, the failure probability $P_f$ eventually reaches a minimum value and tends to stabilize because of the transformation of sliding surface.

Compared with figure 2 and figure 6(a), the safest reinforcement solution is to install the pile at the location P3 with the length $L=8m$ for maximum safety factor $FS=1.587$ and minimum failure probability $P_f=0.07\%$. But in figure 6(b), the failure probability $P_f$ is 0.009% when the pile is located in P5 with the length $L=3m$ in spatial variable soils. The value 0.009% of failure probability is satisfactory for general engineering requirements to ensure the safety of slope. At the same time, because the pile length $L=3m$ is small at the location P5, it is more economical in the slope reinforcement under the premise of ensuring safety.

Figure 7 shows the influence of horizontal and vertical fluctuation on the failure probability of slope reinforced with pile. The increase of vertical fluctuation would result in a relatively small increase of failure probability compared with the increase of horizontal fluctuation in figure 7. Therefore, the vertical fluctuation of soil properties has a greater effect on the stability of slope reinforced with pile.

**Figure 6.** Comparison of (a) uniform soil conditions and (b) spatial variability soil.

5. Conclusion
In the present paper, the failure probability of soil slope reinforced with pile in spatial variable soil is calculated by random limit equilibrium method (RLEM). The results of RLEM is compared with factor of safety using deterministic analysis and probabilistic analysis in uniform soil ground condition. The following conclusions are obtained:
(1) Deterministic analysis with factor of safety and probabilistic analysis of slope in uniform condition may produce a conservative assessment. The calculated failure probability of slope considering spatial variability is less than uniform soil condition.

(2) The optimal location of reinforcement in spatial variable soil is different from the result of traditionally deterministic analysis and probabilistic analysis in uniform soil condition under the condition of meeting the safety and economy. In this paper, the optimal location of pile in spatial variable soil is P5 with length of 3m, while the results of deterministic analysis and probabilistic analysis in uniform soil condition are P3 with length of 8m.

(3) The vertical and horizontal fluctuation of soil parameters also have influence on the failure probability of slope reinforced with pile. The effect of vertical fluctuation of soil is greater than the effect of horizontal fluctuation.

Acknowledgments
The authors are grateful to the financial support from the National Natural Science Foundation of China (51608071), the sponsorship by Natural Science Foundation of Chongqing, China (cstc2018jcyjAX0632), the Venture & Innovation Support Program for Chongqing Overseas Returnees (cx2017123), as well as Chongqing Engineering Research Center of Disaster Prevention & Control for Banks and Structures in Three Gorges Reservoir Area (SXAPGC18ZD01).

References
[1] Zhang, J., Wang, H., Huang, H. W., & Chen, L. H. (2017). System reliability analysis of soil slopes stabilized with piles. Engineering geology, 229, 45-52.
[2] Oguz, E. A., Yalcin, Y., & Huvaj, N. (2017). Probabilistic Slope Stability Analyses: Effects of the Coefficient of Variation and the Cross-Correlation of Shear Strength Parameters. In Geotechnical Frontiers 2017 (pp. 363-371).
[3] Liu, L. L., & Cheng, Y. M. (2018). System Reliability Analysis of Soil Slopes Using an Advanced Kriging Metamodel and Quasi–Monte Carlo Simulation. International Journal of Geomechanics, 18(8), 06018019.
[4] Li, L., & Liang, R. Y. (2014). Reliability-based design for slopes reinforced with a row of drilled shafts. International Journal for Numerical and Analytical Methods in Geomechanics, 38(2), 202-220.
[5] Luo, N., Bathurst, R. J., & Javankhoshdel, S. (2016). Probabilistic stability analysis of simple reinforced slopes by finite element method. Computers and Geotechnics, 77, 45-55.
[6] Luo, N., & Bathurst, R. J. (2018). Probabilistic analysis of reinforced slopes using RFEM and considering spatial variability of frictional soil properties due to compaction. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 12(2), 87-108.
[7] Rocscience Inc. 2018. Slide2 Version 2018.0.21 – 2D Limit Equilibrium Slope Stability Analysis. www.rocscience.com, Toronto, Ontario, Canada.
[8] Li, D. Q., Jiang, S. H., Cao, Z. J., Zhou, W., Zhou, C. B., & Zhang, L. M. (2015). A multiple response-surface method for slope reliability analysis considering spatial variability of soil properties. Engineering Geology, 187, 60-72.
[9] Phoon, K. K., & Kulhawy, F. H. (1999). Characterization of geotechnical variability. Canadian geotechnical journal, 36(4), 612-624.
[10] El-Ramly, H., Morgenstern, N. R., & Cruden, D. M. (2003). Probabilistic stability analysis of a tailings dyke on presheared clay shale. Canadian Geotechnical Journal, 40(1), 192-208.
[11] Vanmarcke, E. H. (1977). Probabilistic modeling of soil profiles. Journal of the geotechnical engineering division, 103(11), 1227-1246.