Deep Learning Inversion of Electrical Resistivity Data

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Abstract—The inverse problem of electrical resistivity surveys (ERS) is difficult because of its nonlinear and ill-posed nature. For this task, traditional linear inversion methods still face challenges such as sub-optimal approximation and initial model selection. Inspired by the remarkable non-linear mapping ability of deep learning approaches, in this paper we propose to build the mapping from apparent resistivity data (input) to resistivity model (output) directly by convolutional neural networks (CNNs). However, the vertically varying characteristic of patterns in the apparent resistivity data may cause ambiguity when using CNNs with the weight sharing and effective receptive field properties. To address the potential issue, we supply an additional tier feature map to CNNs to help it get aware of the relationship between input and output. Based on the prevalent U-Net architecture, we design our network (ERSInvNet) which can be trained end-to-end and reach real-time inference during testing. We further introduce depth weighting function and smooth constraint into loss function to improve inversion accuracy for the deep region and suppress false anomalies. Four groups of experiments are considered to demonstrate the feasibility and efficiency of the proposed methods. According to the comprehensive qualitative analysis and quantitative comparison, ERSInvNet with tier feature map, smooth constraints and depth weighting function together achieve the best performance.

Index Terms—Electrical resistivity inversion, Deep learning.

I. INTRODUCTION

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sequently, people tend to refer to deep Artificial Neural Networks (ANNs) as Deep Neural Networks (DNNs). Methods based on DNNs are usually called deep learning which has shown superior performance in many problems that require the perception and decision abilities of machines (LeCun et al. [27]). Originally succeeding in computer vision for the tasks of image perception (e.g., Krizhevsky et al. [23], Simonyan and Zisserman [29], He et al. [24], Jiang et al. [30], etc.), now DNNs have been prevalent in many fields such as computer graphics, natural language processing (NLP).
Besides, at present, DNNs have developed many variants with different computational logic, such as Multilayer Perceptrons (MLPs, also known as fully connected networks), convolutional neural networks (CNNs) and recurrent neural networks (RNNs), which further extends the application scenarios.

Many attempts have already verify that DNNs could approximate very complex nonlinear mapping functions, especially for ill-posed inverse problems, such as image super-resolution (Dong et al. [31]), medical imaging (Jin et al. [32]), and 3D model reconstruction (Choy et al. [33]). The successes have also inspired many approaches for geophysical problems, particularly in the field of seismic inversion. Araya-Polo et al. [34] use a velocity related feature cube transferred from raw seismic data to generate velocity model by CNNs, while Wu, Lin, and Zhou [35] treat seismic inversion as image mapping and build the mapping from seismic profiles to velocity model directly. Further, Li et al. [36] figure out the weak spatial correspondence and the uncertain reflection-reception relationship problems between seismic data and velocity model, and propose to generate spatially aligned features by MLPs at first. The latter two works could build the mapping from raw seismic data to velocity model directly without data pre-processing. All of these works demonstrate promising performance in result accuracy and computation speed, which bring new perspectives for ERS inversion. Besides, by utilizing big data, DNNs based inversion methods are more robust and have the potential of the practical application.

In this paper, deep learning inversion of ERS is to learn the mapping from input (apparent resistivity data) to output (resistivity model) directly by CNNs, which is illustrated in Fig. 1. Typically, the existence of resistivity anomalies in the model will cause responses in apparent resistivity data. Meanwhile, as can be seen from Fig. 1, the responses of observed data caused by different resistivity anomalies also show different patterns, and the patterns demonstrate certain spatial correspondence to resistivity anomalies in the model. Since the patterns are local existing and spatial corresponding to the resistivity anomalies, the input and output can be considered as the natural image, and the task can be treated as the common mapping between images. In this case, CNNs are preferred among a variety of DNNs variants, because they are much powerful in extracting local patterns and more efficiency regards to the number of parameters. And the performance of CNNs has been widely concerned in the research of remote sensing scene (Maggiori et al. [37], Cheng et al. [38], Zhang et al. [39]). Apart from the similarities with the natural images, apparent resistivity data has its own characteristic. As demonstrated in Fig. 2, when the same anomalous body is located at the different vertical positions, the apparent resistivity data show patterns with large difference, namely patterns have the vertically varying characteristic. This characteristic poses a great challenge for CNNs and may make outputs of CNNs ambiguity. It is because that, with the local spatial and weight sharing convolutional kernels in CNNs, CNNs would have certain receptive field and effective area, and when applying CNNs to the data with vertically varying characteristic, there may be situations where CNNs are requested to give different outputs from similar patterns within effective area. The details will be discussed in Sec. III. This may be the main difficulty for applying CNNs to inverse electrical resistivity data.

We adopt the prevalent CNNs based U-Net architecture [40] to design our networks (ERSInvNet). To capture the potential global resistivity distribution change caused by resistivity anomalies, we set ERSInvNet with 30 layers to have enough receptive fields. Then to reduce the potential ambiguity caused by vertically varying characteristic of apparent resistivity data, we supply ERSInvNet with vertical position information by concatenating an additional tier feature map to the input data. To address the common problem that deep anomalies are difficult to inverse well, we introduce depth weight function in loss function to let network pay more attention to the deep region of resistivity models. Besides, we apply smooth constraints in the loss function to suppress potential false anomalies. In experiments, synthetic examples in our proposed ERSInv dataset are used to verify the feasibility and efficiency, and ERSInvNet is trained end-to-end without any data processing. Through comprehensive qualitative analysis and quantitative comparison, the proposed ERSInvNet consistently achieves promising performance regarding the real-time inference and high inversion accuracy.

II. BACKGROUNDS

The basic measurements of ERS are made by injecting current into the ground through two current electrodes and measuring the potential difference between other pairs of electrodes. In a typical scenario of ERS, the potential difference data are acquired at the Earth’s surface as observed data $d$. The inverse problem involves inferring a set of parameters in model

Fig. 2. Two apparent resistivity data and resistivity model pairs. The two resistivity models have the same anomalous bodies but in the different vertical positions. The corresponding patterns of the responses in observed data demonstrate a large difference.
Multilayer perceptrons (MLPs) are the most basic type of neural networks and have been studied for decades that they are sometimes colloquially referred to as "vanilla" neural networks. It consists of many layers of neurons which weight and non-linearly map all inputs from the previous layer to outputs in the current layer. MLPs are a stack of fully connected layers which defined as:

\[ y_l^i = f(w_l^i y_{l-1}^i + b_l^i), \quad l = 2 \ldots L, \]

where \( w_l^i \) denotes the weights in the links from all the neurons in layer \( l-1 \) (\( y_{l-1}^i \)) to the \( i \)-th neuron in layer \( l \) (\( y_l^i \)) while \( b_l^i \) is the corresponding bias. \( f \) represents used nonlinear activation function, such as Sigmoid and ReLU (Maas et al. [23]) that:

\[
\text{Sigmoid}(x) = \frac{1}{1 + \exp^{-x}},
\]

\[
\text{ReLU}(x) = \max(0, x).
\]

Sigmoid is the most common non-linear activation function. However, it may cause gradient vanish problem as layer increase. Thus, most recent works prefer to use rectified linear activation functions such as ReLU which are also more biologically plausible. Nowadays, dozens of non-linear activation functions have been proposed with properties to handle different tasks. According to Hornik [41], even the simplest MLPs that contain three layers of neurons (an input layer, a hidden layer, and an output layer) could approximate continuous functions on compact subsets of \( \mathbb{R}^n \) under mild assumptions on the activation function, which is known as Universal approximation theorem. To optimize parameters of networks more efficiently, the back-propagation (BP) algorithm is proposed, it computes the error gradients layer by layer according to the chain rule. Using MLPs is the most direct way to construct the mapping between data of uncertain relationship or with global dependency.

Though MLPs have shown promising performance for many tasks, they are inefficient and impractical to process input with large dimensions, such as images. Usually, the patterns in the images are locally correlated that pixels form the patterns are spatially nearby. Moreover, the patterns in the images are spatially irrelevant that the same pattern may appear in any position. Given these characteristics, CNNs with local spatial and weight sharing convolutional kernels are proposed, which are more efficient and could make the best use of characteristics of natural images. The most basic 2D convolutional operation with 3D inputs in CNNs is defined as:

\[
x_{k,i,j}^l = \sum_{c=1}^{C} \sum_{m=1}^{M} \sum_{n=1}^{N} w_{c,m,n}^{l,k} y_{c,m,n}^{l-1} + b_{l,k} + b_{l,k}^{l,j,} \quad (3)
\]

where \( w_{c,m,n}^{l,k} \) is the convolution kernel of size \([C, M, N]\) for layer \( l \) and channel \( k \) while \( b_{l,k}^{l,j,} \) is the corresponding bias, and \( y_{c,m,n}^{l-1} \) is the 3D feature map of layer \( l-1 \) while \( x_{k,i,j}^l \) is the element of position \([k, i, j]\) in convolution results of layer \( l \).

It is easy to get that the feature map in layer \( l \) is \( y_l^i = f(x_l^i) \) where \( f \) is the non-linear activation function. Please note that Eq. (3) is the most basic 2D convolutional operation with 3D inputs without stride and dilation choice.

![Fig. 3. Illustration of convolutional operation and receptive field concept. The kernel and bias in each layer are weight sharing. The gray area in the input indicates the receptive field caused by two convolutional operations with the 5×5 kernel for elements in the output. The center parts of receptive field (effective area [42]) usually have more influence than the surrounding parts and are indicated with light gray color.](image-url)

A three layers CNNs with only one channel each layer is shown in Fig. 3 the kernel \( w_{c,m,n}^{l,k} \) and the bias \( b_{l,k} \) in each layer share the same weight that \( \tilde{w}_{c,m,n}^{l,k} = \tilde{w}_{c,m,n}^{l,k} \) and \( \tilde{b}_{l,k} = \tilde{b}_{l,k} \), which is referred to as weight sharing property of CNNs. Modern CNNs are with many convolutional layers, and each layer includes convolution, non-linear activation and some optional operations, such as batchnorm and pooling. Usually, more layers the CNNs have, much stronger the capacity of CNNs is. One rule to decide the number of CNNs’ layers is making it at least have enough size of receptive field to cover the whole pattern of the object and meanwhile have enough nonlinear expressiveness. Receptive field is the total number of neighborhood pixels or grids in the image the CNNs consumed to give output. As illustrated in Fig. 3 to classify an image with pentagrams, we concatenate two convolutional layers with 5×5 kernels (stride 1 and dilate 1). In this way, each final output element could have the receptive filed of size 7×7 in the input image and thus get aware of the existence of pentagram nearby. Besides, according to Luo et al. [42], the center parts of receptive field usually have more influence than the surrounding parts, which is commonly referred to as effective area. Certainly, there are many choices to satisfy the requirement of receptive field, and the size of receptive field will be affected by many operations such as pooling and upsampling. As for nonlinear expressiveness of CNNs, it is hard to know whether it enough for the task as the mechanism of CNNs has not been studied comprehensively, so people usually use excessive layers to guarantee the nonlinearity.

### III. METHODOLOGY

#### A. Approach

In this work, we intend to learn the mapping function \( F \) from apparent resistivity data \( d \) to resistivity model \( m \) directly by DNNs that:

\[ m = F(d) \]
As stated in Sec. I, because of patterns shown in apparent resistivity data, our task can be treated as the common image to image mapping where CNNs are much powerful and efficient because of CNNs’ weight sharing local convolution kernels. However, with patterns of vertically varying characteristic, CNNs may encounter ambiguous situations. In the following, we will demonstrate and discuss the potential problem when using CNNs on apparent resistivity data.

Firstly, as illustrated in Fig. 4(a) and proved in Luo et al. [42], even using deep CNNs with receptive field cover the whole apparent resistivity data, the patterns within center parts of receptive field usually have much more influence on the corresponding output model value. Thus, the center parts of receptive field are usually referred to as effective area. Secondly, due to the definition of apparent resistivity, the patterns within effective area in apparent resistivity data have vertically varying characteristic. There may be situations as shown in the top two figures in Fig. 4(b). The similar patterns within effective area appear at different tier positions, but correspond to different anomalous bodies and model values. Consequently, during training, CNNs may get ambiguous when requested to give different model values from similar input patterns within effective area. As a comparison, for natural images, as the bottom figure in Fig. 4(b) shows, the patterns of cats correspond to the same semantic meaning in the output no matter where they are located, which means natural images do not have position varying characteristic.

Characteristics of natural images and weight-sharing property of CNNs make CNNs powerful and efficient in dealing with natural images. To make the best use of CNNs in ERS and reduce potential ambiguity, we should supplement input data with more distinguishable information which related to input data, thus reducing the potential ambiguity of CNNs when giving output. In surface ERS, when the distance between two injecting electrodes enlarged, the apparent resistivity data with deeper tier positions in vertical direction could be calculated to achieve the electrical sounding purpose. The data with deeper tier position has the stronger correlation with the deeper anomalous body, which means that the tier position information is helpful for CNNs to distinguish the data patterns caused by anomalous bodies with different depths. Therefore, adding the tier position information of the data to the input would benefit CNNs for building the mapping. As shown in Fig. 5, CNNs could be easier to determine the model values from both patterns and location information together than only rely on possible indistinguishable patterns. Finally, we let CNNs learn the mapping from data and location to model value:

$$m = F(d, t)$$

where $t$ denotes the tier positions in vertical directions of apparent resistivity data. In the following section, we will detail the architecture of CNNs and how we introduce $t$.

### B. Networks

We design our networks based on prevalently used U-Net architecture (Ronneberger et al. [40]) as shown in Fig. 7. U-Net is well known for its shortcut operation which concatenates feature maps from the shallow layer (low-level feature maps) to feature maps from the deep layer (high-level feature maps). Normally, high-level features contain knowledge more related to final result value, while low-level features have knowledge related to some general concepts such as position, shape, etc. In this way, the shortcut would make the last several layers give...
outputs based on high-level and low-level knowledge together, so as to help get final results with both accurate value and anomalous morphology. Moreover, the shortcut will help back propagate gradients and accelerate parameter optimization in shallow layers. We also add several residual blocks (He et al. [24]) at the end of U-Net to enhance the capacity. Finally, there are 26 layers with convolution operation (3 x 3 kernel) and nonlinear activation function, and also 4 layers with 2 x 2 max-pooling operation, which results in enough large receptive field and nonlinearity for our data and task.

C. Loss Function

D. Basic Metric and Weighting Function

For value regression problem, we apply prevalently used \( \text{MSE} \) metric for data value term \( v \) in loss function. For classical ERS inversion, it is usually more difficult to obtain accurate inversion results for deep anomalies. Li and Oldenburg [43], [44] proposed a weighting function to counteract the natural decay of the static field to overcome the tendency of putting structure at the surface. Its effectiveness has been demonstrated (Kang and Oldenburg [45], Qin et al. [46]). In this case, we also take the idea of depth weighting function in loss function to let the network invest more capability in the deep area. In this way, the inversion accuracy and resolution of deep anomalous bodies will be improved. The depth weighting function \( d_w \) is defined as:

\[
d_w(\hat{m}_{i,j}) = (i + \lambda)^{0/2},
\]

where \( \hat{m}_{i,j} \) is the predicted value at position \( (i,j) \) of the resistivity model. \( \lambda \) is the constant parameter related to the grid.
size and the location of the current electrodes. The parameter $\beta$ is a constant for controlling depth weight distribution.

Finally, we design our data value term $v$ as

$$v(\hat{m}, m) = \sum_{i,j} d u(\hat{m}_{i,j}) \cdot (\hat{m}_{i,j} - m_{i,j})^2,$$

where $\hat{m}$ is the inverse result by our networks and $m$ is the corresponding ground truth.

### E. Smooth Constraints

Inversion tasks are often mathematically ill-posed that the solutions are usually non-unique and unstable. One way to solve this problem is by adopting the well-tested smoothness constrained least-squares approach (Tikhonov et al. [47]). Restricted by the smooth constraints, sudden changes between adjacent grids in the resistivity model will be reduced. We carry out the smooth constraints by introducing smooth term:

$$s(\hat{m}) = \sum_{i,j} |\hat{m}_{i+1,j} - \hat{m}_{i,j}| + |\hat{m}_{i,j+1} - \hat{m}_{i,j}|.$$

Smooth term plays the role of regularization and is also known as total variation loss.

### F. Final Formulation

Consequently, our final loss function $L$ is defined as

$$L = v(\hat{m}, m) + \alpha \cdot s(\hat{m})$$

where $\alpha$ is the smoothness factor. All the operations and losses are derivable and result in our end-to-end networks which we call ERSInvNet.

### IV. EXPERIMENTS

#### A. Dataset Preparation

For deep learning based geophysical inversion, data set generally should reach a sufficient amount and guarantee the diversity. As such, in our work, we collect a dataset with 36,214 pairs of different resistivity model and corresponding apparent resistivity data, which is called ERSInv Dataset. Resistivity model is designed by referring to real 2-D ERS scenarios. We generate synthetic data by predefining a few anomalous bodies with different resistivity value, and then embedding them to the different positions of homogeneous medium (500 $\Omega \cdot m$). The resistivity anomalous bodies consist of 5 subsets as follows. **Type I:** Single rectangular body (5236 sample pairs), **Type II:** Two rectangular bodies (7,560 sample pairs), **Type III:** Three rectangular bodies (7,920 sample pairs), **Type IV:** Single declining bodies (6,426 sample pairs) and **Type V:** Two declining bodies (9,072 sample pairs). For each type, the resistivity anomalous bodies may have different resistivity values, in our dataset low resistivity anomaly is the one with body value from $[10 \Omega \cdot m, 20 \Omega \cdot m, 50 \Omega \cdot m]$ and high resistivity anomaly is the one with body value from $[1000 \Omega \cdot m, 1500 \Omega \cdot m, 2000 \Omega \cdot m]$. Accordingly, we have total five different types of resistivity models and the corresponding apparent resistivity data are generated by forward modeling. Schematic diagram and parameters of anomalous bodies are shown in Tab. 1.

The selection of electrode configuration for the ERS is crucial in acquiring the response of the observed target because different electrode configurations have different horizontal and vertical resolution [49]. The apparent resistivity data of Wenner and Wenner-Schlumberger arrays are adopted in this work since these two configurations will have good vertical resolution and appropriate horizontal resolution (Sasaki [49], Szalai et al. [50]) when they used together. Our observation data are generated through forward modeling on resistivity models.

The simulated electrical fields are usually generated by finite-element methods based on anomalous potential method. The values of both input and output during training are normalized to the range of $[0,1]$ since standardizing either input or target variables tend to make the training process better behaved (El-Qady and Ushijima [19]). With tier feature map proposed in the last section, each input data will have three channels with two channels of apparent resistivity data and one channel of tier feature map. The dataset is randomly divided into training set, validation set and test set in a ratio of 10 : 1 : 1 (training set: 30,180 pairs; validation set: 3,017 pairs; test set: 3,017 pairs).

#### B. Implementation

The neural networks in this work are built using PyTorch. SGD optimizer with batchsize 5, learning rate 0.1, momentum 0.9 and weight decay 1e-4 is used to optimize networks. During training, we carry out 500 epochs of optimization in total, and also perform one time of validation after each training epoch to verify training effect. The parameters $\beta$ and $\lambda$ of depth weighting function are set to be 1 and 8 respectively, and the smoothness factor $\alpha$ is set to be 0.2. In this work, the hyper-parameters such as $\beta$, $\lambda$ and $\alpha$ are chosen according to the evaluation on the validation set. All computations are carried out with the machine of single NVIDIA TITAN Xp. It is worth to note that in this environment, our ERSInvNet could reach real-time inference during testing with 0.013s/data.

In order to verify the proposed ERSInvNet, four experiments are arranged as follows: **Experiment 1**, ERSInvNet performance analysis; **Experiment 2**, the ablation study of tier feature map; **Experiment 3**, the ablation study of depth weighting function and smooth regularization; **Experiment 4**, comparison of ERSInvNet and linear least squares inversion. In addition to qualitative evaluation through visual judgment, weighted mean square error (WMSE) and weighted correlation coefficient (WR) are also used to measure the performance quantitatively, which is given as follows:

$$WMSE = \frac{1}{N} \sum_{n=1}^{N} |u^n(\hat{m}^n - m^n)|^2 |u^n(\hat{m}^n - m^n)|$$

$$WR = \frac{1}{N} \sum_{n=1}^{N} |u^n(\hat{m}^n - m^n)|^2 |u^n(\hat{m}^n - m^n)|$$

where $m^n$ and $\hat{m}^n$ are vectorized actual and predicted model values respectively while $u^n$ is the vectorized weight, and we use $\bar{m}$ to denote the average values of $m$. $u^n$ is designed to make the region far from anomalies in the resistivity model has large
TABLE I
SCHEMATIC DIAGRAM AND PARAMETERS OF ANOMALOUS BODIES.

| Type | Description | Model Quantity | Anomalous Bodies Size | Anomalous Bodies Types |
|------|-------------|----------------|-----------------------|------------------------|
| I    | Single rectangular body | 5236 pairs | 4x4; 6x6; 8x8; 10x10; 12x12; 14x14; 16x16; 18x18; 20x20; 8x30; 20x10 | 1 low resistivity body; 1 high resistivity body |
| II   | Two rectangular body | 7560 pairs | 8x8; 10x10; 8x30; 20x10 | 2 low resistivity bodies; 2 high resistivity bodies; 1 low resistivity body & 1 high resistivity body |
| III  | Three rectangular body | 7920 pairs | 8x8; 10x10; 8x30; 20x10 | 3 low resistivity bodies; 3 high resistivity bodies; 1 low resistivity body & 2 high resistivity body; 2 low resistivity body & 1 high resistivity body |
| IV   | Single declining body | 6426 pairs | 8x4; 10x5; 12x6 layer: 3; 4; 5 | 1 low resistivity body; 1 high resistivity body |
| V    | Two declining body | 9072 pairs | 8x4; 12x6 layer: 4; 5 | 2 low resistivity bodies; 2 high resistivity bodies; 1 low resistivity body & 1 high resistivity body |

weight, because false anomalies far from true anomalies are not preferred, while false anomalies close to true anomalies are usually acceptable. \( N \) is the number of samples. \( \text{WMSE} \) measures the value fitting between prediction and groundtruth with the value the lower the better, while \( \text{WR} \) measures the statistical relationship between prediction and groundtruth with the value the larger the better and value range in \([0, 1]\).

V. RESULTS AND DISCUSSION

A. Results of Experiment 1

In Experiment 1, some examples will be shown to demonstrate the inversion performance of the proposed method. The misfit degree of locations and shapes of anomalous bodies as well as resistivity values are the major factors considered during evaluation. In Fig. 8 we randomly select five inversion results which correspond to five model types respectively in the test set. The images arranged from left to right are the corresponding ground truth, apparent resistivity data, ERSInvNet results and the vertical resistivity profiles, respectively.

From the first two columns, we can notice that spatial correspondence extensively exists between the apparent resistivity data and the resistivity models. From the overall observation of Fig. 8 ERSInvNet could predict model value accurately and also get good localization of anomalous bodies, which demonstrates its promising inversion ability. In order to visualize the positions, shapes and resistivity values of anomalous bodies in inversion results more intuitively, the resistivity change along anomalous body profile (shown by the black line in Fig. 8) is given in the form of curves on the fourth column. We can see that the resistivity curves of inversion results and models are almost aligned (with the error within 0.4 %) and change synchronously in most places except regions near the boundary of the anomalous bodies. This is because that the smooth constraints restrict the mutation of resistivity value near abnormal body boundary. The effect of smooth constraints will be discussed in the Sec. V-C.

In the third example, three anomalous bodies with different depths can be clearly and accurately reflected in the inversion results. Among them, the high resistivity bodies at the depth of 15 \( m \) and 25 \( m \) are closed to the model value, while the resistivity of the deepest one (with value 400 \( \Omega \cdot m \)) is lower than the background (with value 2000 \( \Omega \cdot m \)). The reason is that when powered on the surface, the field responses caused by deep anomaly have not shown obvious patterns in the apparent resistivity data. The lack of obvious pattern makes ERSInvNet hard to give accurate predictions. Besides, the inversion results show interesting phenomena that boundary description of high resistivity body is more accurate than that of low resistivity body. More examples from the validation and test sets are shown in the supplementary for further comparison.
In Fig. 8, we show the loss curves of our ERSInvNet on the training and validation set respectively. Both loss curves decrease gradually with the increase of epochs, which indicates the non-existence of over-fitting during training. When the epochs reached 500 times, the loss was reduced below 0.001 and the trend of decline seems will continue.

**B. Results of Experiment 2**

In Experiment 2, to verify the role of tier feature map, we compare the ERSInvNet results with/without the tier feature map when having both depth weighting function and smooth constraints in the loss function. Examples with five different types of anomalous bodies are randomly selected. Inversion results with/without tier feature map are shown in Fig. 10. From example 1, example 2 and low resistivity body on the right side of example 5, we can see the supplement of tier information helps improve the morphological accuracy when inverse resistivity anomalies. Also, the delineation of the boundaries of anomalous bodies is also improved. In example 2 and 3, for multiple anomalous bodies, the tier feature map helps suppress the obvious false anomalies near true ones. Specifically, in example 2, the false anomalies around the low resistivity anomalous body are removed after introducing tier feature map, meanwhile the shape of the anomalous body is more accurate. Similarly, three obvious high resistivity false anomalies in example 3 are completely removed. Without tier feature map, using CNNs on data with vertically varying characteristic will cause ambiguity as discussed in Sec. III and give many assumptions to make loss function lower which finally results in false anomalies. (It is worth to note that what the possible results the networks guessed look like depends on what samples the networks learn during training.)

In summary, the tier feature map can suppress false anomalies. Such rules generally exist in other results. To check the overall performance on validation and test sets, we quantitatively compare results by MAE and $R^2$ metrics in Table II. It is easy to get that quantitative evaluation also supports the positive effect of the tier feature map. Certainly, besides the effects of tier feature map, the inversion performance also depends on the contribution of our smooth constraints and depth weighting function, which will be discussed in the next subsection.

**C. Results of Experiment 3**

In experiment 3, we compare the results with/without smooth constraints and depth weighting function when having tier feature map in the input data. Thus, we have four configurations in total that with smooth constraint and depth weighting function together (SD), with only smooth constraint (OS), with only depth weighting function (OD) and with nothing (NA).
In Fig. 10, ERSInvNet inversion results with/without tier feature map on the test set. Rows from top to bottom exhibit examples with the anomalous bodies from type I-V, respectively. The comparison in this figure indicates that the tier feature map has positive effects on the inversion accuracy.

**TABLE II**

|          | Test       | Valid      |
|----------|------------|------------|
|          | MSE        | $R^2$ ↑    | MSE        | $R^2$ ↑    |
| With     | 0.000335   | 0.540852   | 0.000341   | 0.538876   |
| tier map |            |            |            |            |
| Without  | 0.000731   | 0.387227   | 0.000721   | 0.385944   |
| tier map |            |            |            |            |

In Fig. 11, resistivity models and inversion results of NA, OS, OD and SD are given from left to right.

By comparing the results of NA and OS, we see the results with smooth constraints have fewer false anomalies but poor boundary accuracy. (See the second and third columns). By comparing NA and OD, we found results with depth weighting function have more accurate anomaly morphology as well as anomaly value, especially in the deep area. That is to say, the main contribution of smooth constraints is to suppress false anomalies, while depth weighting function will benefit inversion accuracy. The overall comparison indicates that ERSinvNet with all the tier feature map, smooth constraints and depth weighting function (SD) has the best performance.

**TABLE III**

|          | Test       | Valid      |
|----------|------------|------------|
|          | MSE        | $R^2$ ↑    | MSE        | $R^2$ ↑    |
| SD       | 0.000335   | 0.540852   | 0.000341   | 0.538876   |
| OD       | 0.000608   | 0.387756   | 0.000592   | 0.387921   |
| OS       | 0.000959   | 0.335904   | 0.000971   | 0.335029   |
| NA       | 0.000549   | 0.425744   | 0.000555   | 0.421256   |

**D. Results of Experiment 4**

In Experiment 4, we benchmark ERSInvNet against the well-known iterated linear inversion using RES2DINV software which is widely applied in ERS inversion. For a fair comparison, we use synthetic model with position, size and resistivity value of anomalies that unprecedented during the training of our ERSInvNet. And the same configuration are used to generate corresponding resistivity data for both methods. Fig. 12 (b) and (c) show the results of the linear method and our proposed ERSInvNet respectively. The results of both methods can depict the existence of the anomalous bodies, but of applying depth weighting function, we guess it may be caused by the overproduced false anomalies in the deep area. With depth weighting function, to avoid missing detection of anomalies in the deep area and causing high loss, networks may make many assumptions which result in false anomalies. After introducing smooth constraints and depth weighting function together (SD), we got the best performance which indicates that smooth constraints and depth weighting function can mutually benefit and restrain the negative effects.
the ERSInvNet predicts the location, shape of the conductive block and the resistivity value more precisely than the linear method. Compared with traditional methods, DNNs based methods could utilize data prior learned from the training set as well as human-introduced priors such as smoothness, meanwhile they have more powerful nonlinear approximation abilities. With all these advantages, DNNs reach these promising results in our task.

To train, validate and test the proposed ERSInvNet, we collect a dataset that contains 36,214 pairs of apparent resistivity data and resistivity model. Comparative experiments show that including the tier feature map helps to obtain more accurate inversion results and suppress false anomalies. The individually use of smooth constraints and depth weighting function can reduce false anomalies or improve prediction accuracy for the deep region. However it will sacrifice performance in other aspects. Through comprehensive qualitative analysis and quantitative comparison, simultaneously use of both them achieves the best results. Moreover, comparing with traditional methods, ERSInvNet could reach real-time inference during testing. In future research, we will focus on the establishment of a general dataset covering typical geological conditions and the application of field data.

VI. CONCLUSIONS

In this paper, we propose a CNNs based network called ERSInvNet for inverse problems on resistivity data. Though some attempts of CNNs based tomography have been made, ERS inversion is different from the previous studies because of the vertically varying characteristic inherent in apparent resistivity data. This characteristic will lead to ambiguity when using CNNs directly. To address this issue, we supplement a tier feature map to the input data. Besides, to further reduce the false anomalies and improve the prediction accuracy for the deep region, smooth constraints and depth weighting function are introduced into loss function during training.

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