Mode Selection and Spectrum Partition for D2D Inband Communications: A Physical Layer Security Perspective

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Abstract

This paper investigates from the physical layer security (PLS) perspective the fundamental issues of mode selection and spectrum partition in cellular networks with inband device-to-device (D2D) communication. We consider a mode selection scheme allowing each D2D pair to probabilistically switch between the underlay and overlay modes, and also a spectrum partition scheme where the system spectrum is orthogonally partitioned between cellular and overlay D2D communications. We first develop a general theoretical framework to model both the secrecy outage/secrecy capacity performance of cellular users and outage/capacity performance of D2D pairs, and to conduct performance optimization to identify the optimal mode selection and spectrum partition for secrecy capacity maximization and secrecy outage probability minimization. A case study is then provided to demonstrate the application of our theoretical framework for performance modeling and optimization, and also to illustrate the impacts of mode selection and spectrum partition on the PLS performances of inband D2D communications.

Index Terms

Device-to-device, physical layer security, mode selection, spectrum partition.

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I. INTRODUCTION

Device-to-device (D2D) communication, which enables nearby users to communicate directly without traversing the base station, has been recognized as one of the key technologies in 5G cellular communications [1]. D2D communication can bring many benefits like increased spectrum efficiency, enlarged cellular coverage and reduced transmission latency, and also open up new opportunities for various proximity-based services such as social networking, content sharing, multi-player gaming, etc. [2]–[4].

Depending on the spectrum used by D2D pairs, the D2D communication can be classified into inband D2D on cellular spectrum and outband D2D on unlicensed spectrum (e.g., Bluetooth [5] and WiFi Direct [6]). This paper focuses on the inband D2D, which improves the overall spectrum efficiency compared to the outband one [3], [4]. In the inband D2D, a potential D2D pair can operate either in cellular mode relaying their messages through the base station or in D2D mode communicating directly. The D2D mode can be further divided into overlay D2D using dedicated spectrum resources and underlay D2D reusing the spectrum resources of cellular users (CUEs). As a result, two fundamental and interrelated issues arise in cellular networks with inband D2D communication. The first one is mode selection, i.e., the process of determining which mode each D2D pair should operate in, and the second one is spectrum partition, i.e., how to partition the system spectrum between cellular and overlay D2D communications. This paper jointly considers these two issues in cellular networks with inband D2D communication. In particular, we aim to investigate the optimal mode selection and spectrum partition there from the physical layer security perspective.

Physical layer security (PLS), which exploits the inherent randomness of wireless channels and noise to provide a strong form of security guarantee, has been identified as a highly promising security approach for 5G cellular communications [7], [8]. One typical PLS technique is cooperative jamming, which utilizes the artificial noise from helping nodes (also known as friendly jammers) to create a relatively better legitimate channel than the eavesdropping channel [9]–[12]. In cellular networks with inband D2D communication, underlay D2D pairs are allowed to reuse the spectrum resource of CUEs, causing interference to CUEs. Such interference is conventionally regarded as an obstacle that degrades the cellular performances and hinders the application of D2D communication in 5G cellular systems [13]–[15]. However, from the PLS perspective, the interference from underlay D2D pairs can play the similar role as the artificial
noise in cooperative jamming to protect cellular communications from being intercepted by eavesdroppers.

Motivated by this observation, extensive research efforts have been devoted to the study of inband D2D communication from the PLS perspective, such as security-oriented resource sharing between underlay D2D pairs and CUEs [16]–[21], PLS performance evaluation of D2D underlaying cellular networks [22]–[24], security-constrained interference management [25], etc. These works demonstrate the potentials of inband D2D communication in enhancing the PLS performances of cellular networks, but the fundamental mode selection and spectrum partition issues were largely ignored therein. Actually, these two issues have significant impacts on the system PLS performances. For example, different settings of mode selection lead to different densities of underlay D2D pairs (i.e., jammers) and different settings of spectrum partition result in different amount of spectrum available to CUEs and overlay D2D pairs, which greatly affects the PLS performances of CUEs and D2D pairs. However, these impacts remain largely unexplored due to the lack of a joint study on the mode selection and spectrum partition issues from the PLS perspective. Although there have been joint studies on these two issues in [26]–[32] with the objective of maximizing the system overall rate or energy efficiency (Please refer to Section II for related works), the security issue was not considered therein. Consequently, their results cannot be readily applied to model and optimize the system PLS performances. Therefore, a new and dedicated research is deserved to investigate the impacts of mode selection and spectrum partition on the PLS performances of cellular networks with inband D2D communication, and to further determine the optimal mode selection and spectrum partition settings from the PLS perspective.

To address this issue, this paper develops a general theoretical framework to model both the PLS performances of CUEs and reliable communication performances of D2D pairs, and to identify the optimal mode selection and spectrum partition for performance optimization. To the best of our knowledge, this is the first work that jointly studies the mode selection and spectrum partition issues from the PLS perspective. The main contributions are summarized as follows.

- This paper considers a cellular network with one base station, multiple D2D pairs and multiple CUEs whose transmissions are overheard by an eavesdropper. We adopt the probabilistic mode selection scheme, where each D2D pair independently selects with certain probability to operate in either the underlay mode or overlay mode, and the orthogonal spectrum partition scheme, where the system spectrum is orthogonally divided into two
fractions and each fraction is equally shared among the CUEs and among the overlay D2D pairs, respectively. The underlay D2D pairs reuse the spectrum of the CUEs and simultaneously act as friendly jammers to protect the cellular communications. For this network scenario, we derive the theoretical models for the secrecy outage probability (SOP) and average secrecy capacity (ASC) of the CUEs as well as the outage probability (OP) and average capacity (AC) of the D2D pairs.

- With the help of above theoretical models, we conduct performance optimization to identify the optimal mode selection probabilities and spectrum partition factors to maximize a weighted proportional fair function in terms of the sum ASC of CUEs and the sum AC of D2D pairs, and to minimize a weighted proportional fair function in terms of the sum SOP of CUEs and the sum OP of D2D pairs, respectively.

- A case study under the scenario with one CUE and one D2D pair is provided to demonstrate the application of our theoretical framework for performance modeling and optimization, and numerical results for the case study are further presented to illustrate the impacts of mode selection probability and spectrum partition factor on the system performances.

The remainder of the paper is organized as follows. Section II presents the related works and Section III introduces the system model, mode selection/spectrum partition scheme, and performance metrics in this paper. Section IV presents our theoretical framework for performance modeling and optimization. Section V provides the case study to illustrate the application of our theoretical framework. Numerical results and the corresponding discussions are presented in Section VI. Finally, Section VII concludes this paper.

II. RELATED WORK

For the inband D2D-enabled cellular networks, available works on the joint study of mode selection and spectrum partition issues without the consideration of security can be roughly classified into two categories. In the first category, mode selection is performed between cellular and overlay D2D modes, while in the second category mode selection is performed among cellular, overlay D2D and underlay D2D modes simultaneously.

Regarding the works in the first category, Ye et al. [26] investigated the optimal probabilistic mode selection and orthogonal spectrum partition to jointly maximize the total rate of potential D2D pairs and CUEs per unit area in a cellular network, where CUEs, potential D2D pairs and base stations are distributed according to Poisson Point Processes (PPPs). Based on the same
orthogonal spectrum partition scheme in [26] and a new D2D distance-based mode selection scheme, Lin et al. [27] studied the optimal D2D distance threshold and spectrum partition factor to maximize a weighted proportional fair function in terms of the average rates of potential D2D pairs and CUEs. Later, the authors in [28] extended [27] by considering a more practical path loss model and studied the cellular and D2D coverage probabilities as well as the average network throughput. The authors in [29] also extended [27] by adopting two generalized fading models and evaluated the spectrum efficiency and outage probability performances of potential D2D pairs and CUEs, respectively.

Regarding the works in the second category, Zhu et al. [30] proposed a dynamic Stackelberg game framework to identify the optimal mode selection and orthogonal spectrum partition in a finite cellular network, where potential D2D pairs and CUEs are distributed according to PPPs. For a two-tier cellular network with a potential D2D pair, a macro base station and a femto access point, the authors in [31] proposed a mode selection scheme based on the D2D distance, received interference of the D2D pair and the availability of orthogonal spectrum resource, and further explored the optimal spectrum partition issues under both the overlay and cellular D2D modes. Different from [26]–[31], which explored the mode selection and spectrum partition issues from the perspective of sum rate maximization, the authors in [32] investigated the optimal mode selection and spectrum partition to maximize the overall energy efficiency in a network with one base station, one CUE and one potential D2D pair. It is notable that this paper differs from the above works by jointly investigating the mode selection and spectrum partition issues from the PLS perspective.

III. Preliminaries

A. System Model

As illustrated in Fig. 1, we consider a cellular network consisting of one base station $B$, one eavesdropper $E$, $n$ cellular users (CUEs) $\mathcal{A} = \{A_1, A_2, \cdots, A_n\}$ and $m$ D2D pairs $\mathcal{D} = \{D_1, D_2, \cdots, D_m\}$. We focus on the uplink transmissions of CUEs, as sharing the uplink resource with D2D pairs offers several benefits like improved spectrum utilization and better interference management [33]. We use $D_j^t$ and $D_j^r$ to denote the transmitter and receiver of the $j$-th D2D pair, respectively. We assume that the eavesdropper $E$ overhears the transmissions of

\footnote{Although we focus on the uplink scenario in this paper, our results also apply to the downlink scenario.}
all CUEs and only its statistical channel state information (CSI) is known. Each node has a single omnidirectional antenna, and the CUE $A_i$ ($i \in \{1, 2, \cdots, n\}$) and D2D transmitter $D^t_j$ ($j \in 1, 2, \cdots, m$) transmit with power $P_{A_i}$ and $P_{D^t_j}$, respectively. We assume all CUEs (resp. D2D transmitters) adopt a common transmit power, i.e., $P_{A_i} = P_A$ (resp. $P_{D^t_j} = P_D$). We consider a time-slotted system and a quasi-static Rayleigh fading channel model where each channel remains static for one slot but changes randomly and independently from slot to slot. The channel coefficient between nodes $i$ and $j$ is denoted as $h_{i,j}$, which is modeled as a complex zero mean Gaussian random variable with variance $\sigma^2_{i,j} = d_{i,j}^{-\alpha}$, where $\alpha$ is the path-loss exponent and $d_{i,j}$ is the Euclidean distance between $i$ and $j$. Thus, the corresponding channel gain $|h_{i,j}|^2$ is an exponentially distributed random variable with mean $d_{i,j}^{-\alpha}$. In addition, all wireless channels are impaired by additive white Gaussian noise with variance $\sigma^2$. We assume that the available system spectrum has a total bandwidth of $W$ MHz. Without loss of generality, we assume $W = 1$ throughout this paper.

B. Mode Selection and Spectrum Partition

This paper considers two communication modes for D2D pairs, i.e., overlay mode where D2D pairs use dedicated spectrum resource, and underlay mode where D2D pairs reuse the spectrum resource of CUEs. We consider a probabilistic mode selection scheme where each D2D pair
independently and randomly selects to operate in the underlay mode with probability $p$ (and thus in the overlay mode with probability $1 - p$) in each time slot. We use $D_u$ and $D_o$ to represent the set of D2D pairs operating in the underlay mode and in the overlay mode, respectively. We adopt the orthogonal spectrum partition scheme, which partitions the system spectrum into two fractions: a fraction $\beta$ of the spectrum is orthogonally and equally shared among the CUEs and the remaining fraction is orthogonally and equally shared among the overlay D2D pairs in $D_o^2$. The underlay D2D pairs in $D_u$ reuse the spectrum resource allocated to the CUEs and simultaneously act as friendly jammers to protect the CUEs from the eavesdropping attack of the eavesdropper $E$. We assume that each underlay D2D pair in $D_u$ is allowed to independently and randomly reuse the resource of only one CUE with equal probability $1/n$.

C. Performance Metrics

Consider the uplink transmission of the $i$-th CUE $A_i$, the instantaneous signal-to-interference-plus-noise ratio (SINR) at the base station $B$ is given by

$$\text{SINR}_{A_i,B} = \frac{P_{A_i} |h_{A_i,B}|^2}{\sum_{D_k \in D_u} P_{D_k} |h_{D_k,B}|^2 + \sigma^2} = \frac{\text{SNR}_{A_i,B}}{\sum_{D_k \in D_u} \text{SNR}_{D_k,B} + 1}, \quad (1)$$

where $D_u^i \subseteq D_u$ denotes the set of D2D pairs reusing the spectrum of CUE $A_i$, $\text{SNR}_{a,b} = P_a |h_{a,b}|^2 / \sigma^2 (a \in \{A_i, D_k^i\} \text{ and } b \in \{B\})$ denotes the signal-to-noise ratio (SNR) from nodes $a$ to $b$. Note that $\text{SNR}_{a,b}$ is an exponentially distributed random variable with mean $\gamma_{a,b} = P_a d_{a,b}^{-\alpha} / \sigma^2$.

Similarly, the instantaneous SINR at the eavesdropper $E$ is given by

$$\text{SINR}_{A_i,E} = \frac{\text{SNR}_{A_i,E}}{\sum_{D_k \in D_u^i} \text{SNR}_{D_k,E} + 1}. \quad (2)$$

According to [7], [9], the instantaneous secrecy capacity $C_s^i$ of CUE $A_i$ is determined as

$$C_s^i = \frac{\beta}{n} \left[ \log \left( \frac{1 + \text{SINR}_{A_i,B}}{1 + \text{SINR}_{A_i,E}} \right) \right]^+, \quad (3)$$

where $[x]^+ = \max\{x, 0\}$ and log is to the base of 2.

To model the security performance of CUE $A_i$, we adopt the metric of secrecy outage probability (SOP) to characterize the probability that the instantaneous secrecy capacity of $A_i$ falls below a target secrecy rate $r_s$. Formally, we formulate the SOP $P_{so}^i$ of CUE $A_i$ as

$$P_{so}^i = \mathbb{P} \left( C_s^i < r_s \right), \quad (4)$$

$^2\beta = 1$ if no D2D pair operates in the overlay mode (i.e., $D_o = \emptyset$); otherwise, $\beta \in [0, 1]$. 
where \( P(\cdot) \) represents the probability operator. We also adopt the metric of **average secrecy capacity** (ASC) to depict the expected maximum achievable secrecy rate of \( A_i \), which is denoted as \( C_s^i \) and given by

\[
C_s^i = \mathbb{E}[C_s^i],
\]  

(5)

where \( \mathbb{E}[\cdot] \) represents the expectation operator.

For the \( j \)-th D2D pair \( D_j \), when it operates in the underlay mode and reuses the spectrum of CUE \( A_i \), the instantaneous SINR from the transmitter \( D_j^t \) to the receiver \( D_j^r \) is given by

\[
\text{SINR}_{D_j^t,D_j^r} = \frac{\text{SNR}_{D_j^t,D_j^r}}{\text{SNR}_{A_i,D_j^r} + \sum_{D_k \in D_{iu} \setminus \{D_j\}} \text{SNR}_{D_k^t,D_j^r} + 1}.
\]  

(6)

Thus, the instantaneous capacity \( R_{u,j}^i \) of \( D_j \) in the underlay mode is given by

\[
R_{u,j}^i = \frac{\beta}{n} \log \left( 1 + \text{SINR}_{D_j^t,D_j^r} \right).
\]  

(7)

When \( D_j \) operates in the overlay mode, it orthogonally and equally shares the allocated spectrum with other overlay D2D pairs in \( D_o \), so no interference from other D2D pairs will exist during its transmission. The instantaneous capacity \( R_o^j \) of \( D_j \) in the overlay mode is given by

\[
R_o^j = \frac{(1 - \beta)}{|D_o|} \log \left( 1 + \text{SNR}_{D_j^t,D_j^r} \right).
\]  

(8)

To model the performance of D2D pair \( D_j \), we adopt the metric of **outage probability** (OP), which is defined as the probability that the instantaneous capacity of \( D_j \) falls below a target rate \( r_t \). Formally, we formulate the OP \( P_o^j \) of D2D pair \( D_j \) as

\[
P_o^j = \frac{p}{n} \sum_{i=1}^{n} \mathbb{P}(R_{u,j}^i < r_t) + (1 - p) \mathbb{P}(R_o^j < r_t),
\]  

(9)

where \( \mathbb{P}(R_{u,j}^i < r_t) \) denotes the OP when \( D_j \) operates in the underlay mode and reuses the resource of CUE \( A_i \), and \( \mathbb{P}(R_o^j < r_t) \) denotes the OP when \( D_j \) operates in the overlay mode.

To characterize the expected maximum achievable rate of \( D_j \), we adopt the metric of **average capacity** (AC), which is denoted as \( R_j \) and given by

\[
R_j = \frac{p}{n} \sum_{i=1}^{n} \mathbb{E}[R_{u,j}^i] + (1 - p) \mathbb{E}[R_o^j],
\]  

(10)

where \( \mathbb{E}[R_{u,j}^i] \) denotes the AC when \( D_j \) operates in the underlay mode and reuses the resource of CUE \( A_i \), and \( \mathbb{E}[R_o^j] \) denotes the AC when \( D_j \) operates in the overlay mode.
IV. PERFORMANCE MODELING AND OPTIMIZATION

This section presents our general theoretical framework for the performance modeling and optimization. In this framework, we first derive the theoretical models for the SOP and ASC of CUEs as well as the OP and AC of D2D pairs, based on which we then study the optimal settings of mode selection probability and spectrum partition factor for performance optimization.

A. SOP and ASC of CUEs

For a CUE $A_i$ ($i \in \{1, 2, \ldots, n\}$), we define $I_B = \sum_{D_k \in D_u} \text{SNR}_{D_k,B}$ and $I_E = \sum_{D_k \in D_u} \text{SNR}_{D_k,E}$ as the total interference at the base station and at the eavesdropper $E$, respectively. We can see that both $I_B$ and $I_E$ are the sums of a random number of independent random variables and their pdfs vary with different realizations of $D_u$. We use $D_u$ to denote a particular realization of $D_i$. It is easy to see that $D_u \in 2^D$, where $2^D$ denotes the power set of $D$.

Before giving the main results, we first introduce two basic pdfs $f_{D_u}^i(x)$ and $g_{D_u}^i(x)$ regarding the $I_B$ and $I_E$ for a particular $D_u$, respectively. For a given $D_u$, $I_B$ and $I_E$ are the sum of multiple independent random variables. In general, the analytical expressions of $f_{D_u}^i(x)$ and $g_{D_u}^i(x)$ are difficult to obtain, but they can be determined by the following multi-fold convolutions

\[
\begin{align*}
    f_{D_u}^i(x) &= (f_k * \cdots * f_l)(x), \\
    g_{D_u}^i(x) &= (g_k * \cdots * g_l)(x), \\
\end{align*}
\]

where $f_k$ and $f_l$ are the pdfs of $\text{SNR}_{D_k,B}$ and $\text{SNR}_{D_l,B}$, and $g_k$ and $g_l$ are the pdfs of $\text{SNR}_{D_k,E}$ and $\text{SNR}_{D_l,E}$ for $D_k, D_l \in D_u$. For the special case where all $\text{SNR}_{D_k,B}$ (resp. $\text{SNR}_{D_k,E}$) are independently and identically distributed with parameter $\lambda$, $f_{D_u}^i(x)$ (resp. $g_{D_u}^i(x)$) can be analytically given by the pdf of an Erlang distribution with parameters $|D_u|$ and $\lambda$. Based on $f_{D_u}^i(x)$ and $g_{D_u}^i(x)$, we are now ready to give the main results for the SOP and ASC of CUE $A_i$ in the following theorem.

**Theorem 1.** Consider the network scenario as shown in Fig. 1 with one base station, $n$ CUEs, $m$ D2D pairs and one eavesdropper, the SOP of CUE $A_i$ ($i \in \{1, 2, \ldots, n\}$) under the mode selection and spectrum partition schemes introduced in Section III-B is given by

\[
    P_{so}^i = 1 - \sum_{D_u \in 2^D} \varepsilon^{|D_u|} \left[ \theta_1(D_u, 1) + (1 - \varepsilon)^{m - |D_u|} \left( (1 - \varepsilon)^{m - |D_u|} - \theta_1(D_u, \beta) \right) \right],
\]

where $\theta_1(D_u, \beta)$ is the CDF of the sum of $|D_u|$ independent random variables each with parameter $\beta$. The ASC of CUE $A_i$ is then given by

\[
    \phi_{so}^i = 1 - \sum_{D_u \in 2^D} \varepsilon^{|D_u|} \left[ \theta_1(D_u, 1) + (1 - \varepsilon)^{m - |D_u|} \left( (1 - \varepsilon)^{m - |D_u|} - \theta_1(D_u, \beta) \right) \right].
\]

These expressions allow for the optimization of mode selection and spectrum partitioning to maximize the SOP and ASC of CUEs, respectively.
where \( \varepsilon = \frac{p}{n} \) denotes the probability that an underlay D2D pair reuses the resource of CUE \( A_i \), \( \vartheta = p \left(1 - \frac{1}{n}\right) \) denotes the probability that an underlay D2D pair reuses the resource of other CUEs except for \( A_i \) and

\[
\Theta_i(D_u, \beta) = e^{-\frac{2}{\gamma_{A_i,B} n_{rs}} \frac{\beta m - |D_u|}{1 - \varepsilon} + 1},
\]

(13)

if \( D_u = \emptyset \); otherwise,

\[
\Theta_i(D_u, \beta) = \int_0^\infty \int_0^\infty e^{-\frac{2}{\gamma_{A_i,E} n_{rs}} \frac{(x+1)(y+1)}{1 - \varepsilon} + 1} f_{D_u}^i(x) g_{D_u}^i(y) dx dy.
\]

(14)

The ASC of CUE \( A_i \) is given by

\[
C_i = \frac{1}{n \ln 2} \sum_{D_u \in 2^{D'}} \varepsilon^{|D_u|} \Lambda_i(D_u) \left( \beta \left(1 - \varepsilon\right)^m - |D_u| + (1 - \beta) \vartheta^{m - |D_u|} \right),
\]

(15)

where

\[
\Lambda_i(D_u) = \Psi\left(\frac{1}{\gamma_{A_i,E}} + \frac{1}{\gamma_{A_i,B}}\right) - \Psi\left(\frac{1}{\gamma_{A_i,B}}\right),
\]

(16)

if \( D_u = \emptyset \); otherwise,

\[
\Lambda_i(D_u) = \int_0^\infty \int_0^\infty \left[ \Psi\left(\frac{x+1}{\gamma_{A_i,B}} + \frac{y+1}{\gamma_{A_i,E}}\right) - \Psi\left(\frac{x+1}{\gamma_{A_i,B}}\right) \right] f_{D_u}^i(x) g_{D_u}^i(y) dx dy,
\]

(17)

where \( \Psi(x) = e^x \text{Ei}(-x) \) and \( \text{Ei}(x) = -\int_x^\infty \frac{e^{-t}}{t} dt \) denotes the exponential integral.

**Proof:** See Appendix A.

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**B. OP and AC of D2D Pairs**

This subsection provides the general expressions for the OP and AC of D2D pairs. For a D2D pair \( D_j \ (j \in \{1,2,\cdots,m\}) \) reusing the spectrum of CUE \( A_i \), we define \( I_{D_j}^i = \sum_{D_k \in D_u \setminus \{D_j\}} \text{SNR}_{D_k,D_j} \) as the interference at the D2D receiver \( D_j \). We use \( D_u^i \) to denote a particular realization of \( D_u \setminus \{D_j\} \). It is easy to see that \( D_u^i \in 2^{D'} \), where \( 2^{D'} \) is the power set of \( D' = D \setminus \{D_j\} \). We can see that, for a given realization of \( D_u^i \), \( I_{D_j}^i \) is the sum of \( |D_u^i| \) independent random variables. We use \( h_{D_u^i}^j(x) \) to denote the pdf of \( I_{D_j}^i \) for a given \( D_u^i \). Similar to \( f_{D_u}^i(x) \) and \( g_{D_u}^i(x) \), \( h_{D_u^i}^j(x) \) can be determined by the following multi-fold convolution,

\[
h_{D_u^i}^j(x) = \left( h_k \ast \cdots \ast h_l \right)(x),
\]

(18)
where \( h_k \) and \( h_l \) denote the pdfs of \( \text{SNR}_{D_k^t,D_j^r} \) and \( \text{SNR}_{D_l^t,D_j^r} \) for \( D_k, D_l \in D_i^u \), respectively. Based on \( h^j_u(x) \), we give the following theorem regarding the OP and AC of D2D pair \( D_j \).

**Theorem 2.** Consider the network scenario as shown in Fig. 1 with one base station, \( n \) CUEs, \( m \) D2D pairs and one eavesdropper, the OP of D2D pair \( D_j \) \((j \in \{1, 2, \cdots, m\}) \) under the mode selection and spectrum partition schemes introduced in Section III-B is given by

\[
P_{o_j} = 1 - \sum_{l=1}^{m} \left( \frac{m-1}{l-1} \right) p^{m-l}(1-p)^l e^{-\frac{\Delta_{D_j^t,D_j^r}^t}{\Delta_{D_j^t,D_j^r}^r}} \tag{19}
\]

\[
-\sum_{i=1}^{n} \left[ \sum_{D_i^u \in 2^D^u} \left( \begin{array}{c} |D_i^u|+1 \\ \beta \end{array} \right) \Omega_{i,j}(D_i^u, 1) + \left( \begin{array}{c} (1-\varepsilon)(m-1-|D_i^u|) \end{array} \right) \right] \Omega_{i,j}(D_i^u, \beta) \right],
\]

where

\[
\Omega_{i,j}(D_i^u, \beta) = \frac{e^{-\frac{\Delta_{A_i,D_j^r}^t}{\Delta_{D_j^t,D_j^r}^r}}}{\frac{\gamma_{A_i,D_j^r}}{\gamma_{D_j^t,D_j^r}^r}(2^{\frac{n_r}{n_t}} - 1) + 1}, \tag{20}
\]

if \( D_i^u = \emptyset \); otherwise,

\[
\Omega_{i,j}(D_i^u, \beta) = \int_0^\infty e^{-\frac{\Delta_{D_j^t,D_j^r}^t}{\Delta_{D_j^t,D_j^r}^r}} h^j_u(x) \left[ \frac{\Omega_{i,j}(D_i^u, 1) + \left( \begin{array}{c} (1-\varepsilon) \end{array} \right) \Omega_{i,j}(D_i^u, 1) \right]}{\Omega_{i,j}(D_i^u, \beta) \Omega_{i,j}(D_i^u, \beta)} \right], \tag{21}
\]

The AC of D2D pair \( D_j \) is given by

\[
R_j = \sum_{i=1}^{n} \sum_{D_i^u \in 2^D^u} \left[ \begin{array}{c} |D_i^u|+1 \\ \beta \end{array} \right] \Delta_{i,j}(D_i^u) \beta \left( \begin{array}{c} 1-\varepsilon \end{array} \right)^{m-1-|D_i^u|} \left( \begin{array}{c} 1-\beta \end{array} \right)^{m-1-|D_i^u|} \right] \tag{22}
\]

\[
-\left( \begin{array}{c} |D_i^u|+1 \\ \beta \end{array} \right) \Omega_{i,j}(D_i^u, 1) + \left( \begin{array}{c} (1-\varepsilon) \end{array} \right) \Omega_{i,j}(D_i^u, 1) \right] \Omega_{i,j}(D_i^u, \beta) \right],
\]

where

\[
\Delta_{i,j}(D_i^u) = \psi \left( \frac{1}{\gamma_{D_j^t,D_j^r}^t} \right) - \psi \left( \frac{1}{\gamma_{A_i,D_j^r}^t} \right), \tag{23}
\]

if \( D_i^u = \emptyset \); otherwise,

\[
\Delta_{i,j}(D_i^u) = \int_0^\infty h^j_u(x) \left[ \psi \left( \frac{x+1}{\gamma_{D_j^t,D_j^r}^t} \right) - \psi \left( \frac{x+1}{\gamma_{A_i,D_j^r}^t} \right) \right] dx. \tag{24}
\]

**Proof:** See Appendix B.
Remark 1. Notice that $f_{D_u}^i(x)$, $g_{D_u}^i(x)$ and $h_{D_u}^j(x)$ serve as three fundamental pdfs in our theoretical framework. Once they are determined, the analytical expressions for $P_{so}^i$, $C_s^i$, $P_o^j$ and $R_j$ can be determined accordingly.

C. Performance Optimization

Based on the results in Theorems 1 and 2, this subsection investigates the optimal settings of mode selection probability and spectrum partition factor for system performance optimization. In general, optimization problems in D2D-enabled cellular networks need to consider the fairness between the performances of CUEs and D2D pairs. To do this, we adopt the following weighted proportional fair function \[27\]

$$U(U_c, U_d) = w_c \ln U_c + w_d \ln U_d,$$

where $w_c + w_d = 1$, and $U_c$ and $U_d$ represent the utilities of CUEs and D2D pairs, respectively.

We first explore the optimal settings from the perspective of sum rate maximization, for which we consider the following optimization problem (referred to as Problem P1)

$$\mathbf{P1} : (p_1^*, \beta_1^*) = \arg \max_{p, \beta \in [0, 1]} \left( \sum_{i=1}^{n} C_s^i, \sum_{j=1}^{m} R_j \right),$$

where $p_1^*$ and $\beta_1^*$ denote the optimal values of mode selection probability and spectrum partition factor for Problem P1, respectively. To make this problem more explicit, we rewrite $C_s^i$ as

$$C_s^i = \frac{a_i \beta + b_i}{n \ln 2},$$

where

$$a_i = \sum_{D_u \in 2^D} \varepsilon^{|D_u|} \Lambda_i(D_u) \left( (1 - \varepsilon)^{m - |D_u|} - \vartheta^{m - |D_u|} \right),$$

$$b_i = \sum_{D_u \in 2^D} \varepsilon^{|D_u|} \Lambda_i(D_u) \vartheta^{m - |D_u|}. \quad (28)$$

Notice that $\varepsilon > 0$, $1 - \varepsilon > \vartheta > 0$ and $\Lambda_i(D_u) > 0$, so we have $a_i > 0$ and $b_i > 0$. We also rewrite $R_j$ as

$$R_j = \frac{(u_j + s_j) \beta + v_j - s_j}{\ln 2},$$

$$U(U_c, U_d) = w_c \ln U_c + w_d \ln U_d,$$
Lemma 1. The optimal spectrum partition factor $\beta_1^*$ for Problem P1 is given by

$$\beta_1^* = \min \left\{ \max \left\{ 0, -w_d \frac{\sum_{i=1}^{n} b_i}{\sum_{i=1}^{n} a_i} - w_c \frac{\sum_{j=1}^{m} (v_j - s_j)}{\sum_{j=1}^{m} (u_j + s_j)} \right\} , 1 \right\}$$

if $\sum_{j=1}^{m} (u_j + s_j) < 0$, otherwise, $\beta_1^* = 1$. In particular, $\lim_{p \to 0} \beta_1^* = w_c$.

Proof: We first take the derivative of $\mathcal{U}$ with respect to $\beta$, which is given by

$$\frac{\partial \mathcal{U}}{\partial \beta} = \frac{w_c}{\beta + \sum_{i=1}^{n} b_i \sum_{i=1}^{n} a_i} + \frac{w_d}{\beta + \sum_{j=1}^{m} (v_j - s_j) \sum_{j=1}^{m} (u_j + s_j)}.$$  

Recall that $a_i > 0$, $b_i > 0$, $u_j > 0$, $v_j > 0$, $s_j < 0$, so we have $\sum_{i=1}^{n} a_i > 0$, $\sum_{i=1}^{n} b_i > 0$ and $\sum_{j=1}^{m} (v_j - s_j) > 0$. We can see that the monotonicity of $\mathcal{U}$ depends on the sign of $\sum_{j=1}^{m} (u_j + s_j)$. If $\sum_{j=1}^{m} (u_j + s_j) \geq 0$, we have $\frac{\partial \mathcal{U}}{\partial \beta} > 0$ and $\mathcal{U}$ is increasing with $\beta$. Thus, the optimal $\beta$ is $\beta_1^* = 1$. If $\sum_{j=1}^{m} (u_j + s_j) < 0$, we have $\sum_{j=1}^{m} (v_j - s_j) + \sum_{j=1}^{m} (u_j + s_j) = \sum_{j=1}^{m} (v_j + u_j) > 0$ and thus $\frac{\sum_{j=1}^{m} (v_j - s_j)}{\sum_{j=1}^{m} (u_j + s_j)} < -1$. So, we have $\beta + \frac{\sum_{j=1}^{m} (v_j - s_j)}{\sum_{j=1}^{m} (u_j + s_j)} < 0$. Defining $\beta'$ as the solution of $\frac{\partial \mathcal{U}}{\partial \beta} = 0$, we have

$$\beta' = -w_d \frac{\sum_{i=1}^{n} b_i}{\sum_{i=1}^{n} a_i} - w_c \frac{\sum_{j=1}^{m} (v_j - s_j)}{\sum_{j=1}^{m} (u_j + s_j)}.$$  

We can see that $\frac{\partial \mathcal{U}}{\partial \beta} < 0$ for $\beta > \beta'$ and $\frac{\partial \mathcal{U}}{\partial \beta} > 0$ for $\beta < \beta'$. Thus, the optimal $\beta$ is $\beta_1^* = \min \{ \max \{ 0, \beta' \} , 1 \}$. In particular, as $p \to 0$, we have $a_i \to 0$, $v_j \to 0$, $u_j \to 0$ and thus $\beta_1^* \to w_c$. 


From Lemma 1, we can see that as \( p \to 0 \), i.e., no D2D pair chooses the underlay mode, the \( \beta_1^* \) converges to \( w_c \), which is the weight assigned to the utility of CUEs. It is notable that \( \beta_1^* \) is a function of \( p \). Substituting \( \beta_1^* \) back into (36) reduces \( U \) to a function of only \( p \). Thus, the optimal \( p_1^* \) can be found efficiently, which in return determines the value of \( \beta_1^* \).

Next, we explore the optimal settings of mode selection probability and spectrum partition factor from the perspective of outage probability minimization. For this purpose, we consider the following optimization problem (referred to as Problem P2)

\[
P2 : (p_2^*, \beta_2^*) = \arg \max_{p, \beta \in [0, 1]} -U \left( \sum_{i=1}^{n} P_{so}^i, \sum_{j=1}^{m} P_{o}^j \right),
\]

where \( p_2^* \) and \( \beta_2^* \) denote the optimal values of mode selection probability and spectrum partition factor for Problem P2, respectively. From the expressions of \( P_{so}^i \) and \( P_{o}^j \), we can see that closed-form solutions for \( p_2^* \) and \( \beta_2^* \) are usually difficult to obtain, so a two-dimensional search over \( (p, \beta) \) can be used to find the \( p_2^* \) and \( \beta_2^* \).

V. Case Study

In this section, we provide a case study to illustrate the application of our theoretical framework for performance modeling and optimization. We consider a simple system with one CUE \( A \) and one D2D pair \( D \) (i.e., \( n = 1 \) and \( m = 1 \)), as considered in [32], [34], [35]. We first give analytical expressions for the SOP and ASC of CUE \( A \) as well as the OP and AC of D2D pair \( D \). Based on the analytical expressions, we then solve the related optimization problems to find the optimal settings of mode selection probability and spectrum partition factor.

A. Performance Modeling

Based on Theorem 1 we first provide the analytical expressions for the SOP and ASC of CUE \( A \) in the following corollary.

**Corollary 1.** Consider a cellular network with one base station \( B \), one eavesdropper \( E \), one CUE \( A \) and one D2D pair \( D \), the SOP \( P_{so} \) of CUE \( A \) under the mode selection and spectrum partition schemes introduced in Section III-B is given by

\[
P_{so} = 1 - (1 - p) \Theta(\emptyset, \beta) - p \Theta(D, 1),
\]

(37)
where $\Theta(\emptyset, \beta) = \frac{e^{-\frac{\bar{x}_D}{\gamma A,B}}}{\frac{1}{\gamma A,E} e^{\frac{\bar{x}_D}{\gamma A,B}} + 1}$ and

$$\Theta(D, 1) = \frac{e^{-\tau}}{\gamma D', B, \eta} + \frac{e^{-\tau}}{\gamma D', B, \eta^2} \left( \kappa (\eta + 1) \Psi \left( \frac{\kappa + 1}{\gamma D', E} \right) + (\eta - \kappa) \Psi \left( \frac{\tau + \frac{1}{\gamma D', B}}{\gamma D', B} \right) \right), \tag{38}$$

where $\tau = \frac{2^{s-1}}{\gamma A,B}$, $\kappa = \frac{2^s \gamma A,E}{\gamma A,B}$ and $\eta = \tau + \frac{1}{\gamma D', B} - \frac{\kappa}{\gamma D', E}$. The ASC $C_s$ of CUE $A$ is given by

$$C_s = \frac{1}{\ln 2} \left( p \Lambda(\emptyset) + (1 - p) \beta \Lambda(\emptyset) \right), \tag{39}$$

where

$$\Lambda(\emptyset) = \Psi \left( \frac{1}{\gamma A,B} + \frac{1}{\gamma A,E} \right) - \Psi \left( \frac{1}{\gamma A,B} \right)$$

and

$$\Lambda(D) = \frac{1}{\gamma D', B} - \frac{\gamma A,E}{\gamma D', B} - 1 \left[ \frac{\gamma A,E}{\gamma A,E} - 1 \right] + \frac{\gamma A,E}{\gamma A,E} - 1 \right] + \Psi \left( \frac{1}{\gamma A,B} \right) - \Psi \left( \frac{1}{\gamma A,B} \right). \tag{40}$$

Proof: The results follow from (12) and (15) in Theorem 1 by letting $n = 1$, $m = 1$, $\vartheta = 0$, $\varepsilon = p$, $f_D(x) = e^{-\frac{x}{\gamma D', B}}$ and $g_D(x) = e^{-\frac{x}{\gamma D', E}}$, and then computing the involved integrals.

Next, we give the following corollary regarding the analytical expressions for the OP and AC of D2D pair $D$.

**Corollary 2.** Consider a cellular network with one base station $B$, one eavesdropper $E$, one CUE $A$ and one D2D pair $D$, the OP $P_o$ of $D$ under the mode selection and spectrum partition schemes introduced in Section III-B is given by

$$P_o = 1 - p \Omega(\emptyset, 1) - (1 - p) e^{-\frac{\bar{x}_D}{\gamma D', D}} \tag{41},$$

where $\Omega(\emptyset, 1) = \frac{e^{-\frac{\bar{x}_D}{\gamma D', D}}}{\frac{1}{\gamma D', D} \left( 2^{s-1} + 1 \right)}$. The AC $R$ of D2D pair $D$ is given by

$$R = \ln 2 \left( \frac{p \Delta(\emptyset)}{\gamma A,D} - 1 \right) - (1 - p)(1 - \beta) \Psi \left( \frac{1}{\gamma D', D} \right), \tag{42}$$

where $\Delta(\emptyset) = \Psi \left( \frac{1}{\gamma D', D} \right) - \Psi \left( \frac{1}{\gamma A,D} \right)$. Proof: The results follow from (19) and (22) in Theorem 2 by letting $n = 1$, $m = 1$, $\vartheta = 0$, $\varepsilon = p$ and $D_d = \emptyset$. □
B. Optimal Mode Selection and Spectrum Partition

Based on the results in Corollaries 1 and 2, we proceed to find the optimal settings of mode selection probability and spectrum partition factor by solving the optimization problems in Section IV-C. We first consider Problem P1, which is reduced to

\[
(p_1^*, \beta_1^*) = \arg \max_{p, \beta \in [0,1]} \mathcal{U}(C_s, R),
\]

where \(C_s\) and \(R\) are given in (39) and (42), respectively. By applying Lemma 1, we can determine the \(p_1^*\) and \(\beta_1^*\) in the following lemma.

**Lemma 2.** For the considered case, the optimal mode selection probability \(p_1^*\) and spectrum partition factor \(\beta_1^*\) for Problem P1 are given as follows:

1) for \(\mu + \nu > 1\) and \(w_c/\mu < w_d/\nu\), where \(\mu = \Lambda(D)/\Lambda(\emptyset)\) and \(\nu = -\left(\frac{\gamma_{A,D} \gamma_{D,D} - 1}{\gamma_{D,D}^2}\right)\psi\left(\frac{1}{\gamma_{D,D}^2}\right)\),

\[
(p_1^*, \beta_1^*) = \left(\min\left\{\frac{w_c}{1-\mu}, 1\right\}, 0\right);
\]

2) for \(\mu + \nu > 1\) and \(w_c/\mu \geq w_d/\nu\), \((p_1^*, \beta_1^*) = \left(\min\left\{\frac{w_d}{1-\mu}, 1\right\}, 1\right)\), if \(\mu < 1\); otherwise \(p_1^* = 1\) and \(\beta_1^*\) can be any value in \([0,1]\);

3) for \(\mu + \nu \leq 1\), \((p_1^*, \beta_1^*) = (0, w_c)\).

**Proof:** See Appendix C.

Notice that the \(\mu\) (resp. \(\nu\)) in Lemma 2 is the ratio of the ASC for CUE A (resp. the AC for D2D pair D) when D operates in the underlay mode to that when D operates in the overlay mode with full spectrum usage, i.e., \(\beta = 1\) (resp. \(\beta = 0\)). We can interpret \(\mu\) and \(\nu\) as the underlay rate gains of the CUE and the D2D pair, and their reciprocals \(1/\mu\) and \(1/\nu\) as the overlay rate gains. From cases 1) and 2), we can see that when the system underlay gain (i.e., \(\mu + \nu\)) is greater than 1, to achieve the optimal system rate performance, on the one hand, the D2D pair is encouraged to reuse the spectrum resource of the CUE with a certain probability. On the other hand, when the D2D pair chooses the overlay mode, the base station allocates all its spectrum resource to the D2D pair (i.e., \(\beta_1^* = 0\)) if the weighted overlay gain of the CUE is less than that of the D2D pair; otherwise, the base station allocates all its spectrum to the CUE (i.e., \(\beta_1^* = 1\)). From case 3), we can see that when the system underlay gain is less than 1, to optimize the optimal system rate performance, the base station disables the spectrum reuse between the CUE and the D2D pair, and sets the spectrum partition factor as the weight assigned to the utility of the CUE. Special attention needs to be paid to the optimal solutions with \(p_1^* = 1\). In this case,
the D2D pair reuses the spectrum of the CUE and all the spectrum is allocated to the CUE, so no spectrum partition is needed and the optimal spectrum partition factor $\beta_1^*$ can be any value in $[0, 1]$.

Next, we consider Problem P2, which is reduced to

$$(p_2^*, \beta_2^*) = \arg \max_{p, \beta \in [0, 1]} -\mathcal{U}(P_{so}, P_o),$$

(44)

where $P_{so}$ and $P_o$ are given by (37) and (41), respectively. According to the discussions in Section [IV-C], a two-dimensional search over $(p, \beta)$ is usually required to find the $p_2^*$ and $\beta_2^*$. However, for this simple case, we find that $p_2^*$ is either $p_2^* = 0$ or $p_2^* = 1$ due to the convexity of the objective function in terms of $p$. Based on this property, we can find the $p_2^*$ and $\beta_2^*$ for this problem. Before giving the main results, we first define two functions in terms of $\beta$, i.e.,

$$\hat{\mu}(\beta) = (1 - \Theta(\emptyset, \beta))/(1 - \Theta(D, 1)) \text{ and } \hat{\nu}(\beta) = (1 - e^{-2\frac{\gamma^2}{\gamma D^1 D^r}})/(1 - \Omega(\emptyset, 1)).$$

In addition, we define a set $\varrho = \{\beta|\hat{\mu}(\beta)^w \hat{\nu}(\beta)^w < 1\}$. With the help of these definitions, we can give the following lemma about $p_2^*$ and $\beta_2^*$.

**Lemma 3.** For the considered case, the optimal mode selection probability $p_2^*$ and spectrum partition factor $\beta_2^*$ for Problem P2 are given by $(p_2^*, \beta_2^*) = (0, \hat{\beta}_2)$, where

$$\hat{\beta}_2 = \arg \max_{\beta \in \varrho} -\mathcal{U}(1 - \Theta(\emptyset, \beta), 1 - e^{-2\frac{\gamma^2}{\gamma D^1 D^r}}),$$

(45)

if $\varrho \neq \emptyset$, otherwise $p_2^* = 1$ and $\beta_2^*$ can be any value in $[0, 1]$.

**Proof:** We first take the second partial derivative of the objective function with respect to $p$, which is,

$$-\frac{\partial^2 \mathcal{U}}{\partial p^2} = \frac{w_c}{\left(p + \frac{1}{1/\hat{\mu}(\beta)-1}\right)^2} + \frac{w_d}{\left(p + \frac{1}{1/\hat{\nu}(\beta)-1}\right)^2} > 0.$$  

(46)

We can see that the objective function is a convex function of $p$. Thus, the maximum can only be achieved at either $p = 0$ or $p = 1$. For $\beta \in \varrho$, we have $-\mathcal{U}_{p=0} > -\mathcal{U}_{p=1}$ and the optimal $p$ is $p_2^* = 0$. The optimal $\beta$ in this case is given by $\beta_2^* = \hat{\beta}_2$. For $\beta \notin \varrho$, the optimal $p$ is $p_2^* = 1$. The optimal $\beta$ can be any $\beta$ in the complement of $\varrho$, as the objective function is independent of $\beta$ for $p = 1$. If $\varrho \neq \emptyset$, we have $-\mathcal{U}_{p=0, \beta=\hat{\beta}_2} > -\mathcal{U}_{p=1, \beta=\hat{\beta}_2} = -\mathcal{U}_{p=1, \beta \notin \varrho}$, and thus $\beta_2^* = \hat{\beta}_2$ and $p_2^* = 0$. Otherwise, $p_2^* = 1$ and $\beta_2^*$ can be any value in $[0, 1]$.

Notice that the $\hat{\mu}(\beta)$ (resp. $\hat{\nu}(\beta)$) in Lemma 3 is the ratio of the SOP for CUE $A$ (resp. the OP for D2D pair $D$) when $D$ operates in the overlay mode to that when $D$ operates in the underlay.
| Parameter                           | Value     |
|------------------------------------|-----------|
| Cell radius                        | 500 m     |
| Location of base station           | (0 m, 0 m)|
| Total Bandwidth $W$                | 1 MHz     |
| Noise spectral density             | -174 dBm/Hz|
| Path loss exponent $\alpha$        | 4         |
| Small-scale fading                 | Rayleigh fading |
| Transmit power of cellular user $P_A$ | 23 dBm  |
| Transmit power of D2D user $P_D$   | 20 dBm    |

mode with a fixed $\beta$. Thus, the $\hat{\mu}(\beta)$ (resp. $\hat{\nu}(\beta)$) can be interpreted as the underlay security gain of the CUE (resp. the underlay reliability gain of the D2D pair), and its reciprocal $1/\hat{\mu}(\beta)$ (resp. $1/\hat{\nu}(\beta)$) as the corresponding overlay gains. We can see from Lemma 3 that if there exists at least one spectrum partition factor $\beta$ such that $\hat{\mu}^{uc}(\beta)\hat{\nu}^{ud}(\beta) < 1$, the optimal mode selection probability is $p^*_2 = 0$. In this case, the D2D pair should choose the overlay mode to minimize the system outage probability.

VI. NUMERICAL RESULTS FOR CASE STUDY

In this section, we first provide simulation results to validate the analytical expressions of SOP, ASC, OP and AC for the case study. We then explore how these performances vary with the mode selection probability $p$, spectrum partition factor $\beta$ and other system settings. Finally, we demonstrate the feasibility of our approach to find the optimal settings of mode selection probability and spectrum partition factor.

A. Simulation Settings and Model Validation

We developed a dedicated simulator in C++ for the case study to simulate the message transmission processes of both the CUE and D2D pair, which is now available at [36]. We consider an isolated cellular cell with a radius of 500 m. The base station $B$ is located at the center (0 m, 0 m). The simulation parameters are summarized in Table I. To verify the accuracy of our theoretical analysis, we compare the simulated and theoretical values of the SOP, ASC, OP and AC. Each simulated value is calculated as the average value of $10^2$ batches of simulation results. In each batch, $10^6$ random and independent simulations are conducted and
TABLE II
SOP and ASC validation for CUE at (100 m, 100 m), $r_s = 0.1 \text{Mbits/s}$, $p = 0.5$, $\beta = 0.5$, Simulated/Theoretical

| $E$     | $D'$  | SOP           | ASC            |
|---------|-------|---------------|----------------|
| (0, 100)| (100, 0) | 0.851151 ± 0.004/0.851236 | 0.161597 ± 0.004/0.1614 |
|         | (0, 200) | 0.559092 ± 0.004/0.559389 | 1.03312 ± 0.006/1.03242 |
|         | (50, 100) | 0.554693 ± 0.003/0.55475 | 0.582294 ± 0.008/0.58243 |
| (0, 200)| (0, 300) | 0.290782 ± 0.007/0.290804 | 2.51383 ± 0.02/2.51353 |
|         | (50, 200) | 0.279723 ± 0.004/0.279718 | 1.96078 ± 0.02/1.95986 |
|         | (100, 200) | 0.321149 ± 0.003/0.320976 | 1.78978 ± 0.01/1.79134 |
| (0, 300)| (0, 100) | 0.421326 ± 0.006/0.421194 | 0.915345 ± 0.02/0.91561 |
|         | (0, 200) | 0.101184 ± 0.002/0.101194 | 2.28495 ± 0.02/2.28545 |
|         | (100, 300) | 0.0830492 ± 0.002/0.0829538 | 3.43883 ± 0.02/3.43743 |

TABLE III
OP and AC validation for CUE at (100 m, 100 m), $r_t = 0.5 \text{Mbits/s}$, $p = 0.5$, $\beta = 0.5$, Simulated/Theoretical

| $D'$ | $D''$ | OP           | AC             |
|------|-------|---------------|----------------|
| (100, 0) | (100, 50) | 0.226646 ± 0.004/0.226541 | 5.77726 ± 0.05/5.77642 |
|      | (150, 50) | 0.226472 ± 0.004/0.226542 | 5.27576 ± 0.04/5.27642 |
|      | (150, 0) | 0.0159588 ± 0.001/0.0160373 | 7.25613 ± 0.03/7.25675 |
| (200, 0) | (200, 50) | 0.0160605 ± 0.001/0.0160373 | 7.25671 ± 0.02/7.25675 |
|      | (150, 100) | 0.4769606 ± 0.005/0.476973 | 4.1746 ± 0.04/4.17307 |
|      | (250, 0) | 0.0024383 ± 0.0004/0.00243919 | 8.51556 ± 0.03/8.51502 |
| (300, 0) | (300, 50) | 0.0014218 ± 0.0003/0.00142934 | 8.8886 ± 0.03/8.8886 |
|      | (300, 100) | 0.024502 ± 0.001/0.0246167 | 5.99154 ± 0.02/5.99048 |
|      | (350, 0) | 0.0005022 ± 0.0002/0.000492216 | 9.94441 ± 0.02/9.64406 |

the corresponding SOP (resp. OP) is calculated as the ratio of the number of simulations with secrecy outage (resp. outage) to the total number of simulations $10^6$. Similarly, the ASC and AC in each patch are calculated as the average value of the ASC and AC of $10^6$ simulations, respectively.

For the validation of SOP and ASC, we place the CUE $A$ at (100 m, 100 m) and set the target secrecy rate as $r_s = 0.1 \text{Mbits/s}$, the mode selection probability as $p = 0.5$ and the spectrum partition factor as $\beta = 0.5$. We consider three different cases of the location of the eavesdropper $E$, i.e., (0 m, 100 m), (0 m, 200 m) and (0 m, 300 m). For each case, three different locations of the D2D transmitter $D'$ have been considered. The simulated and theoretical values are summarized in Table II. For the validation of OP and AC, we place $A$ at (100 m, 100 m) and set $p = 0.5$, $\beta = 0.5$ and the target rate as $r_t = 0.5 \text{Mbits/s}$. We consider three different
cases of the location of \( D_t \), i.e., (100 m, 0 m), (200 m, 0 m) and (300 m, 0 m). For each case, three different locations of the D2D receiver \( D_r \) have been examined. The simulated and theoretical values are summarized in Table III. We can see from Table III and Table III that the simulated values match proficiently with the theoretical ones, which indicates that our theoretical framework is efficient to model the SOP and ASC of CUEs as well as the OP and AC of D2D pairs under the considered mode selection and spectrum partition schemes.

B. Performance Evaluation

We now investigate how the mode selection probability \( p \), spectrum partition factor \( \beta \) and the location of the D2D transmitter \( D_t \) affect the SOP and ASC performances of the CUE \( A \). We consider a scenario where \( A \) is located at (100 m, 100 m) and \( E \) is located at (0 m, 200 m). We fix the \( x \) coordinate of \( D_t \) as 0 m and vary its \( y \) coordinate from 1 m to 199 m. For this scenario, we show in Fig. 2 how the SOP and ASC of the CUE vary with the \( y \) coordinate of \( D_t \) under different settings of \( p \) (i.e., 0.1 and 0.5) and \( \beta \) (i.e., 0 and 0.5) for \( r_s = 0.1 \text{ Mbits/s} \). We can see from Fig. 2a that the SOP decreases as the \( y \) coordinate of \( D_t \) increases. This implies that a larger ratio of the distances \( d_{D_r,B} \) to \( d_{D_r,E} \) (i.e., \( d_{D_r,B}/d_{D_r,E} \)) can achieve a lower SOP. We can also see from Fig. 2a that the SOP always decreases as \( \beta \) increases and this trend is independent of both \( p \) and the location of \( D_t \). On the contrary, however, the behavior of SOP versus \( p \) depends on both \( \beta \) and the location of \( D_t \). For example, for the case of \( \beta = 0 \), the SOP decreases as \( p \) increases for all \( y \) coordinates of \( D_t \), while for the case of \( \beta = 0.5 \), the SOP increases as \( p \) increases when the \( y \) coordinate of \( D_t \) is less than a threshold (about 110 m in Fig. 2a) but decreases as \( p \) increases when the \( y \) coordinate of \( D_t \) is larger than the threshold. This is because that, as can be seen from (37), the condition for SOP decreasing as \( p \) increases is \( \hat{\mu}(<\beta) > 1 \), i.e., the SOP achieved when the D2D pair chooses the underlay mode is less than that when the D2D pair chooses the overlay mode.

Regarding the impacts of \( p \), \( \beta \) and the location \( D_t \) on the ASC of the CUE, we can see from Fig. 2b that the ASC increases as the \( y \) coordinate of \( D_t \) increases, which implies that a larger distance ratio \( d_{D_r,B}/d_{D_r,E} \) is also beneficial for increasing the ASC. Similar to the behaviors of the SOP versus \( p \) and \( \beta \), it can also be observed from Fig. 2b that the ASC always increases as \( \beta \) increases, while the ASC increases with \( p \) only if the ASC achieved when the D2D pair chooses the underlay mode is larger than that when the D2D pair chooses the overlay mode (i.e., \( \mu > \beta \)).
We now examine the impacts of $p$, $\beta$ and the ratio of the average SNR $\gamma_{A,D^r}$ to the average SNR $\gamma_{D^t,D^r}$ (i.e., $\gamma_{A,D^r}/\gamma_{D^t,D^r}$) on the performances of the D2D pair. For the scenario with $r_t = 1$ Mbits/s and $d_{D^t,D^r} = 100$ m, Fig. 3 illustrates the OP and AC of the D2D pair versus $\gamma_{A,D^r}/\gamma_{D^t,D^r}$ under different settings of $p$ and $\beta$. We can see from Fig. 3a that for a given SNR $\gamma_{D^t,D^r}$, the OP of the D2D pair increases as the SNR $\gamma_{A,D^r}$ increases. This is because that, when the D2D pair selects the underlay mode, more interference is generated by the CUE at the D2D receiver $D^r$, resulting in a lower communication rate and thus a larger outage probability. We can also see from Fig. 3a that the OP always increases as $\beta$ increases, which is regardless of the values of $p$ and the SNR ratio $\gamma_{A,D^r}/\gamma_{D^t,D^r}$. In contrast, we can see that the behavior of OP versus $p$ depends on both $\beta$ and $\gamma_{A,D^r}/\gamma_{D^t,D^r}$. For example, the OP increases as $p$ increases for $\beta = 0.5$, while it decreases as $p$ increases for $\beta = 1$. This is due to the reason that the OP decreases as $p$ increases if the OP achieved when the D2D pair operates in the underlay mode is less than that when the D2D pair operates in the overlay mode, i.e., $\hat{\nu}(\beta) > 1$ as can be inferred from (41).

Similar behaviors of the AC versus $p$, $\beta$ and $\gamma_{A,D^r}/\gamma_{D^t,D^r}$ can also be observed from Fig. 3b. These observations are: 1) the AC decreases as the ratio $\gamma_{A,D^r}/\gamma_{D^t,D^r}$ increases for a given $\gamma_{D^t,D^r}$ due to the more interference at $D^r$ generated by operating in the underlay mode; 2) the AC always decreases as $\beta$ increases, while it decreases as $p$ increases only if the AC achieved when the D2D pair operates in the underlay mode is less than that when the D2D pair operates...
in the overlay mode (i.e., $\nu < 1 - \beta$).

C. Optimal $p$ and $\beta$

We first study the optimal mode selection probability $p_1^*$ and spectrum partition factor $\beta_1^*$ of Problem P1. We show in Fig. 4 the objective function of Problem P1 versus $p$ and $\beta$ for $A = (100 \text{ m}, 100 \text{ m})$ and $E = (0 \text{ m}, 300 \text{ m})$. Fig. 4 includes four sub-figures with different settings of weights $w_c$ and $w_d$ and locations of $D^t$ and $D^r$, which correspond to different cases in Lemma 2. Fig. 4a corresponds to case 1) with $\mu = 1.09974$, $\nu = 0.187658$, $w_c = 0.4$ and $w_d = 0.6$ under the scenario of $D^t = (0 \text{ m}, 200 \text{ m})$ and $D^r = (50 \text{ m}, 200 \text{ m})$. We can see from Fig. 4a that the maximum of the objective function is achieved at $(p_1^* = 0.492404, \beta_1^* = 0)$, which matches the solution of $(p_1^* = w_c/(1 - \nu), \beta_1^* = 0)$ in case 1) of Lemma 2. Fig. 4b corresponds to case 2) with $\mu = 0.770184$, $\nu = 0.28467$, $w_c = 0.9$ and $w_d = 0.1$ under the scenario of $D^t = (0 \text{ m}, 170 \text{ m})$ and $D^r = (-50 \text{ m}, 170 \text{ m})$. It can be seen from Fig. 4b that the maximum of the objective function is achieved at $(p_1^* = 0.435131, \beta_1^* = 1)$, which matches the optimal solution of $(p_1^* = w_d/(1 - \mu), \beta_1^* = 1)$ for case 2) with $\mu < 1$ in Lemma 2. To illustrate the solutions of case 2) with $\mu > 1$, we consider the same locations of $D^t$ and $D^r$ (i.e., $\mu = 1.09974$, $\nu = 0.187658$) as for case 1) and show the results in Fig. 4c for $w_c = 0.9$ and $w_d = 0.1$. As can be seen from Fig. 4c, the optimal $p_1^*$ is $p_1^* = 1$ and optimal $\beta_1^*$ can be any value in $[0, 1]$, which verifies the solution for case 2) with $\mu > 1$. Finally, Fig. 4d illustrates the solution in case 3) with $\mu = 0.552902$, $\nu = 0.126527$, $w_c = 0.4$ and $w_d = 0.6$ for the scenario of $D^t = (0 \text{ m}, 150 \text{ m})$ and
(a) Case 1): $\mu = 1.09974$, $\nu = 0.187658$, $w_c = 0.4$ and $w_d = 0.6$.

(b) Case 2): $\mu = 0.770184$, $\nu = 0.284676$, $w_c = 0.9$ and $w_d = 0.1$.

(c) Case 2): $\mu = 1.09974$, $\nu = 0.187658$, $w_c = 0.9$ and $w_d = 0.1$.

(d) Case 3): $\mu = 0.552902$, $\nu = 0.126527$, $w_c = 0.4$ and $w_d = 0.6$.

Fig. 4. Optimal mode selection probability $p_1^*$ and spectrum partition factor $\beta_1^*$ of Problem P1. Spatial distribution of network users: $A = (100 \text{ m}, 100 \text{ m})$ and $E = (0 \text{ m}, 300 \text{ m})$; $D_t = (0 \text{ m}, 200 \text{ m})$ and $D_r = (50 \text{ m}, 200 \text{ m})$ for (a) and (c), $D_t = (0 \text{ m}, 170 \text{ m})$ and $D_r = (-50 \text{ m}, 170 \text{ m})$ for (b), $D_t = (0 \text{ m}, 150 \text{ m})$ and $D_r = (50 \text{ m}, 200 \text{ m})$ for (d).

Fig. 4d shows that the optimal solution is $(p_1^* = 0, \beta_1^* = 0.4)$, which agrees with the solution of $(p_1^* = 0, \beta_1^* = w_c)$ for case 3) in Lemma 2. Therefore, the above figures demonstrate the feasibility of our approach for determining the $p_1^*$ and $\beta_1^*$ of Problem P1.
Next, we explore the optimal mode selection probability $p^*_2$ and spectrum partition factor $\beta^*_2$ of Problem P2. For the scenario of $A = (100 \text{ m}, 100 \text{ m})$, $E = (0 \text{ m}, 300 \text{ m})$, $D^t = (0 \text{ m}, 250 \text{ m})$ and $D^r = (50 \text{ m}, 250 \text{ m})$, Fig. 5a and Fig. 5b illustrate the objective function of Problem P2 versus $p$ and $\beta$ for the settings of $w_c = 0.4$, $w_d = 0.6$ and $w_c = 0.9$, $w_d = 0.1$, respectively. We can see from Fig. 5a and Fig. 5b that for a given $\beta$, the objective function of Problem P2 is a convex function of $p$, and thus the optimal $p^*_2$ that maximizes the objective function can only be either $p^*_2 = 0$ or $p^*_2 = 1$. For the settings of $w_c = 0.4$ and $w_d = 0.6$, we can see from Fig. 5a that there exist at least one $\beta$ such that the value of the objective function at $p = 0$ is greater than that at $p = 1$ (i.e., the set $\varrho$ is not empty). Thus, the optimal $p^*_2$ in this case is $p^*_2 = 0$. We can also verify that the optimal $\beta^*_2$ shown in Fig. 5a agrees with the optimal $\beta^*_2$ that maximizes the objective function for $p^*_2 = 0$. However, for the settings of $w_c = 0.9$ and $w_d = 0.1$, we can see from Fig. 5b that the value of the objective function at $p = 0$ is always smaller than that at $p = 1$ (i.e., the set $\varrho$ is empty). Thus, the optimal $p^*_2$ in this case is $p^*_2 = 1$ and the optimal $\beta^*_2$ can be any value in $[0, 1]$. These observations from Fig. 5a and Fig. 5b match the results in Lemma 3 and demonstrate the feasibility of our approach for determining the $p^*_2$ and $\beta^*_2$ of Problem P2.
VII. CONCLUSIONS

This paper investigated the mode selection and spectrum partition issues in cellular networks with inband D2D communication from the physical layer security (PLS) perspective. In particular, we developed a theoretical framework for the performance modeling and optimization of both cellular users (CUEs) and D2D pairs. The results based on a case study indicated that the PLS performances of the CUE and D2D pair can be flexibly controlled by mode selection and spectrum partition. For example, we can reduce (resp. increase) the secrecy outage probability (resp. average secrecy capacity) of the CUE by allocating more spectrum to the CUE, but such PLS performance improvement of CUE usually comes with the cost of an increased outage probability (resp. reduced average capacity) of the D2D pair. On the other hand, the mode selection of D2D pair has a complicated impact on the PLS performance of CUE and reliable communication performance of D2D pair, which is heavily dependent on the setting of spectrum partition and the spatial distribution of network users.

APPENDIX A

PROOF OF THEOREM 1

We first prove the SOP result in (12). Following from the definition of SOP in (4), we have

\[ P_{so}^i = \mathbb{P} \left[ \frac{1 + \frac{\text{SNR}_{A_i,B}}{I_{B}+1}}{1 + \frac{\text{SNR}_{A_i,E}}{I_{E}+1}} < 2^{\frac{\alpha}{\beta}} \right]. \]  

(47)

As \( \text{SNR}_{A_i,B} \) and \( \text{SNR}_{A_i,E} \) are independent exponentially distributed random variables, we have

\[ P_{so}^i = 1 - \mathbb{E} \left[ e^{-\frac{(2^{\frac{\alpha}{\beta}}-1)(I_{B}+1)}{\gamma_{A_i,B}}} \right], \]  

(48)

where the expectation is with respect to \( I_B \) and \( I_E \). As both \( I_B \) and \( I_E \) depend on \( D_u \), we applying the law of total expectation in terms of \( D_u \) to change (48) to

\[ P_{so}^i = 1 - \sum_{D_u \in 2^D} \mathbb{E} \left[ e^{-\frac{(2^{\frac{\alpha}{\beta}}-1)(I_{B}+1)}{\gamma_{A_i,B}}} \right] \mathbb{P}(D_u), \]  

(49)

where \( \mathbb{P}(D_u) \) denotes the probability \( \mathbb{P}(D_d = D_u) \). For the special case where all the other \( m - |D_u| \) pairs also select the underlay mode (i.e., \( D_u = D \) and \( D_o = \emptyset \)), we have \( \beta = 1 \) and \( \mathbb{P}(D_u) = \varepsilon^{|D_u|} \beta^{m-|D_u|} \). After calculating the expectation in \( T \), we can determine the \( T \) in this case as \( T = \)
\( \varepsilon^{\|D_u\|} \beta^{m-\|D_u\|} \Theta_i(D_u, 1) \). On the contrary, for the case where at least one of the \( m-\|D_u\| \) D2D pairs selects the overlay mode, we have \( \beta \in [0, 1] \) and \( \mathbb{P}(D_u) = \varepsilon^{\|D_u\|} \left( (1 - \varepsilon)^{m-\|D_u\|} - \beta^{m-\|D_u\|} \right) \).

Calculating the expectation in \( T \) yields \( T = \varepsilon^{\|D_u\|} \left( (1 - \varepsilon)^{m-\|D_u\|} - \beta^{m-\|D_u\|} \right) \Theta_i(D_u, \beta) \) in this case. Finally, substituting \( T \) into (49) yields (12).

Next, we proceed to prove the ASC in (15). According to the definition in (5), we have

\[
\text{We first prove the OP in (19). It can be seen from (9) that we need to derive}
\[
\mathbb{P}(R_{ij}^u < r_t) \quad \text{and} \quad \mathbb{P}(R^d_j < r_t) \quad \text{separately. First, we have}
\[
\mathbb{P}(R_{ij}^u < r_t) = \mathbb{P} \left( \frac{\text{SNR}_{D_i^u, D_j^u}}{\text{SNR}_{A_i, D_j^u} + I_{D_j^u}^1 + 1} < 2^{\frac{m^2}{2}} - 1 \right) = 1 - \mathbb{P} \left[ \frac{\varepsilon^{\|D_i^u\|} \left( \sum_{d_j^u \in 2^{D_j^u}} \varepsilon^{\|D_i^u\|} \right) - 1}{\gamma_{A_i, D_j^u}^d} \left( \frac{2^{m^2}}{2^{m^2} - 1} + 1 \right)^{-1} \right].
\]

Following the idea of the proof in Theorem 1, we obtain

\[
\mathbb{P}(R_{ij}^u < r_t) = 1 - \sum_{D_i^u \in 2^{D_i^u}} \varepsilon^{\|D_i^u\|} \beta^{m-\|D_i^u\|} \Theta_i(D_i^u, 1)
- \sum_{D_i^u \in 2^{D_i^u}} \varepsilon^{\|D_i^u\|} \Theta_i(D_i^u, \beta) \left( (1 - \varepsilon)^{m-\|D_i^u\|} - \beta^{m-\|D_i^u\|} \right).
\]
Next, we calculate \( \mathbb{P}(R_o^j < r_t) \), which can be given by

\[
\mathbb{P}(R_o^j < r_t) = 1 - \sum_{l=1}^{m} \left( \frac{m-1}{l-1} \right) p^{m-l}(1-p)^{l-1} \frac{\nu}{2^{(l-1)}}.
\]

Finally, substituting (54) and (55) into (9) completes the proof of OP.

We now proceed to prove the AC in (22). According to (10), we first calculate \( \mathbb{E}[R_u^{j,i}] \) and then calculate \( \mathbb{E}[R_o^j] \). Following the same idea in calculating the OP, \( \mathbb{E}[R_u^{j,i}] \) can be given by

\[
\mathbb{E}[R_u^{j,i}] = \sum_{D_u \in 2^{D'}} \frac{\mathbb{E}[\mathbb{L}_u]}{1+\mathbb{E}[\mathbb{L}_u]} \left[ \beta(1-\varepsilon)^{m-1-|D_u|} + (1-\beta) \psi_{m-1-|D_u|} \right] n \ln 2 \left( \frac{\gamma_{D',D_r}}{\gamma_{D_u,D_r}} - 1 \right),
\]

Next, \( \mathbb{E}[R_o^j] \) can be given by

\[
\mathbb{E}[R_o^j] = -\frac{(1-\beta)\psi_{\frac{1}{\gamma_{D',D_r}}}}{\ln 2} \sum_{l=1}^{m} \left( \frac{m-1}{l-1} \right) p^{m-l}(1-p)^{l-1}.
\]

Finally, substituting (56) and (57) into (10) completes the proof.

APPENDIX C

PROOF OF LEMMA [2]

According to Lemma [1] the \( \beta^*_c \) is given by \( \beta^*_c = \max \left\{ 0, -w_d\mu - w_c\nu \right\} \). We can easily prove that \( \nu \in (0, 1) \), using the fact that \( x\Psi(x) \) is an increasing function of \( x \).

Now, we continue to find the optimal \( p^*_t \). First, consider case 1) in Lemma [2] where we have

\( 0 < \tilde{p}_1 < \min\{1, w_c/(1-\nu)\} \) due to \( \nu \in (0, 1) \), where \( \tilde{p}_1 = (w_d/w_c\mu + (1-\nu))^{-1} \). For the sub-region \( p \in [0, \tilde{p}_1] \), the optimal \( \beta \) is \( \beta^*_t = -w_d\mu - w_c\nu \). \( \beta^*_1 = -w_d\mu - w_c\nu \) \( \frac{p}{1-p} + w_c \in [0, w_c] \), which reduces \( C_s \) and \( R \) to \( C_s = \frac{w_c\Lambda(\theta)}{\ln 2} \left[ (\mu - (1-\nu))p + 1 \right] \) and \( R = -\frac{w_c\psi_{\frac{1}{\gamma_{D',D_r}}}}{\ln 2} \left[ (\mu - (1-\nu))p + 1 \right] \). As \( \mu \geq 1-\nu \), \( U \) is an increasing function of \( p \) in this sub-region. For the sub-region \( p \in (\tilde{p}_1, 1] \), the optimal \( \beta \) is \( \beta^*_t = 0 \), which reduces \( C_s \) and \( R \) to \( C_s = \frac{\Lambda(D)p}{\ln 2} \) and \( R = -\frac{\psi_{\frac{1}{\gamma_{D',D_r}}}}{\ln 2} \left[ (1-\nu)p - 1 \right] \). The derivative of \( U \) in this case is \( \frac{\partial U}{\partial p} = \frac{w_c}{p} + \frac{w_d}{1-p} \). Thus, \( U \) increases with \( p \) for \( p \in (\tilde{p}_1, \frac{w_c}{1-p}) \) and decreases with \( p \) for \( p \in (\frac{w_c}{1-p}, 1] \). Based on the above two sub-regions, the optimal \( p \) in this case is thus \( p^*_1 = \min\{\frac{w_c}{1-p}, 1\} \) and the optimal \( \beta \) is \( \beta^*_1 = 0 \).

Next, we consider case 2), where we have \( \tilde{p}_1 \geq 1 \). Thus, for \( p \in [0, 1] \), the optimal \( \beta \) is \( \beta^*_t = \min\left\{ -w_d\mu - w_c\nu \frac{p}{1-p} + w_c, 1 \right\} \). For the sub-region \( p \in [0, \tilde{p}_2] \), where \( \tilde{p}_2 = (1-\mu + w_c/w_d\nu)^{-1} \in (\frac{w_c}{\nu}, 1] \), \( \beta^*_t = -w_d\mu - w_c\nu \frac{p}{1-p} + w_c \). As \( \mu \geq 1-\nu \), \( U \) is an increasing function
of \( p \) in this sub-region. For the sub-region \( p \in (\tilde{p}_2, 1] \), \( \beta^*_1 = 1 \), which reduces \( \mathcal{C}_s \) and \( \mathbf{R} \) to \( \mathcal{C}_s = \frac{\Lambda(\emptyset)}{\ln 2} [(\mu - 1)p + 1] \) and \( \mathbf{R} = \frac{1}{\ln 2} \left( \frac{p\Delta(\emptyset)}{\gamma d_1^2 d_2^2} - 1 \right) \). The derivative of \( \mathcal{U} \) is \( \frac{\partial \mathcal{U}}{\partial p} = \frac{w_c}{p + \mu} + \frac{w_d}{p} \). Thus, if \( u < 1 \), we have \( \tilde{p}_2 < \frac{w_d}{1 - \mu} \) and \( \mathcal{U} \) is increasing in \( p \in (\tilde{p}_2, \frac{w_d}{1 - \mu}) \) but decreasing in \( p \in (\frac{w_d}{1 - \mu}, 1] \).

Combining the above two sub-regions, we can see that if \( u < 1 \), \( (\beta^*_1, p^*_1) = (1, \min\{\frac{w_d}{1 - \mu}, 1\}) \). If \( u \geq 1 \), we have \( \frac{\partial \mathcal{U}}{\partial p} > 0 \) and \( \mathcal{U} \) is also increasing in \( p \in (\tilde{p}_2, 1] \). Thus, the optimal \( p^*_1 = 1 \) and the optimal \( \beta^*_1 \) can be any value in \([0, 1]\), since \( \mathcal{U} \) is independent of \( \beta \) for \( p = 0 \).

Finally, for case 3) (i.e., \( \mu + \nu < 1 \)), we have \( \frac{w_c}{1 - \mu} < \tilde{p}_1 \) if \( w_c/\mu < w_d/\nu \) and \( \frac{w_d}{1 - \nu} < \tilde{p}_2 \) if \( w_c/\mu \geq w_d/\nu \). So, the objective function \( \mathcal{U} \) always decreases with \( p \) for \( p \in [0, 1] \). Thus, the optimal \( p \) is \( p^*_1 = 0 \) and hence \( \beta^*_1 = w_c \) in this case.

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