Abstract

Differential privacy (DP) has arisen as the state-of-the-art metric for quantifying individual privacy when sensitive data are analyzed, and it is starting to see practical deployment in organizations such as the US Census Bureau, Apple, Google, Facebook, and Microsoft. There are two popular models for deploying differential privacy – standard differential privacy (SDP), where a trusted server aggregates all the data and runs the DP mechanisms, and local differential privacy (LDP), where each user perturbs their own data and perturbed data is analyzed. Due to security concerns arising from aggregating raw data at a single server, several real world deployments in industry have embraced the LDP model [17, 1, 2, 3]. However, systems based on the LDP model tend to have poor utility – “a gap” in the utility achieved as compared to systems based on the SDP model.

In this work, we survey and synthesize emerging directions of research at the intersection of differential privacy and cryptography. First, we survey solutions that combine cryptographic primitives like secure computation, anonymous communication and oblivious computation with differential privacy to give alternatives to the LDP model that avoid a trusted server as in SDP but close the gap in accuracy. These cryptographic primitives introduce performance bottlenecks and necessitate efficient alternatives. Second, we synthesize work in an area that we call “DP-Cryptography” – cryptographic primitives that are allowed to leak differentially private outputs. These primitives have orders of magnitude better performance than standard cryptographic primitives. DP-cryptography primitives are perfectly suited for implementing alternatives to LDP, but are also applicable to scenarios where standard cryptographic primitives do not have practical implementations. Through this unique lens of research taxonomy, we survey the landscape of ongoing research in these directions while also providing novel directions for future research.

1. INTRODUCTION

On Feb 15, 2019, John Abowd, chief scientist at the US Census Bureau, announced the results of a reconstruction attack that they proactively launched using data released under the 2010 Decennial Census [18]. The decennial census released billions of statistics about people like “how many people of the age 10-20 live in New York City” or “how many people live in 4 person households”. Using only the data publicly released in 2010, an internal team was able to (a) correctly reconstruct records of address (by census block), age, gender, race and ethnicity for 142 million people (about 46% of the US population), and (b) correctly match these data to commercial datasets circa 2010 to associate personal-identifying information such as names for 52 million persons (17% of the population). This is not specific to the US Census Bureau – such attacks can occur in any setting where statistical information in the form of deidentified data, statistics or even machine learning models are released. That such attacks are possible was predicted over 15 years ago by a seminal paper by Irit Dinur and Kobbi Nissim [12] – releasing a sufficiently large number of aggregate statistics with sufficiently high accuracy provides sufficient information to reconstruct the underlying database with high accuracy. The practicality of such a large scale reconstruction by the US Census Bureau underscores the grand challenge that public organizations, industry, and scientific research faces: how can we safely disseminate results of data analysis on sensitive databases?

An emerging answer is differential privacy. An algorithm satisfies differential privacy (DP) if its output is insensitive to adding, removing or changing one record in its input database. Differential privacy is considered the “gold standard” for privacy for a number of reasons. It provides a persuasive mathematical proof of privacy to individuals with several rigorous interpretations [24, 23]. The differential privacy guarantee is composable and repeating invocations of differential private algorithms lead to a graceful degradation of privacy. The US Census Bureau was the first big organization to adopt differential privacy in 2008 for a product called OnTheMap [27], and subsequently there have been deployments by Google, Apple, Microsoft, Facebook, and Uber.

Differential privacy is typically implemented by collecting data from individuals in the clear at a trusted data curator, then applying one or more differentially private algorithms, and finally releasing the outputs. This approach, which we call standard differential privacy (SDP), works in cases like the US Census Bureau where there is a natural trusted data curator. However, when Google wanted to monitor and analyze the Chrome browser properties of its user base to detect...
security vulnerabilities, they chose a different model called \textit{local differential privacy (LDP)}. In LDP, individuals perturb their records before sending it to the server, obviating the need for a trusted data curator. Since the server only sees perturbed records, there is no centralized database of sensitive information that is susceptible to an attack or subpoena requests from governments. The data that Google was collecting – browser fingerprints – uniquely identify individuals. By using LDP, Google was not liable to storing these highly identifying user properties. Due to these attractive security properties a number of real world applications of differential privacy in the industry – Google’s RAPPOR \cite{17}, Apple Diagnostics \cite{1} and Microsoft Telemetry \cite{11} – embrace the LDP model.

However, the improved security properties of LDP come at a cost in terms of utility. Differentially private algorithms hide the presence or absence of an individual by adding noise. Under the SDP model, counts over the sensitive data, e.g., “number of individuals who use the \textit{bing.com} search engine”, can be released by adding a constant amount of noise. In the LDP model, noise is added to each individual record. Thus, answering the same count query requires adding $O(\sqrt{N})$ error (Theorem 2.1 from \cite{10}) for the same level of privacy, where $N$ is the number of individuals participating in the statistic. In other words, under the LDP model, for a database of a billion people, one can only learn properties that are common to at least 30000 people ($O(\sqrt{N})$). In contrast, under SDP, one can learn properties that are shared by as few as a 100 people ($O(1)$ including constants; cf \cite{13}). Thus, the LDP model operates under more practical trust assumptions than SDP, but as a result incurs a significant loss in data utility. In this work, we review literature in this domain under two categories:

- **Cryptography for DP**: We review a growing line of research that aims to use cryptographic primitives to bridge the gap between SDP and LDP. In these solutions, the trusted data curator in SDP is replaced by cryptographic primitives that result in (a) more practical trust assumptions than the SDP model, and (b) better utility than under the LDP model. Cryptographic primitives such as anonymous communication and secure computation have shown significant promise in improving the utility of differentially private implementations while continuing to operate under the practical trust assumptions that are accepted by the security community.

- **DP for Cryptography**: Differential privacy is typically applied to settings that involve complex analytics over large datasets. Introducing cryptographic primitives results in concerns about the feasibility of practical implementations at that scale. This has given rise to a second line of work that employs differential privacy as a tool to speed up cryptographic primitives, thereby pushing the frontiers of their practical deployments. While the original cryptographic primitives are defined with respect to perfect privacy, under differential privacy, it is ok to learn distributional information about the underlying dataset. We explore in depth the following cryptographic primitives (a) secure computation (b) secure communication, and show how in the context of differential privacy one can build “leaky” but efficient implementations of these primitives.

These lines of work both reflect exciting directions for the computer science community. We begin by giving a brief technical introduction to differential privacy in Section 2. We discuss the “Cryptography for DP” paradigm in Section 3 and “DP for cryptography” in Section 4. Section 5 provides concrete ideas for future work as well as open problems in the field through the lens of combining differential privacy and cryptography.

### Key Insights
- **Local Differential Privacy** is increasingly being embraced as the primary model of deployment of differential privacy, albeit at a heavy accuracy cost.
- **Cryptographic primitives** can help bridge the utility gap between systems deployed in the local differential privacy model and standard differential privacy model but the increased utility may come at the cost of performance.
- **DP-cryptographic primitives**, that are relaxed notions of cryptographic primitives that leak differentially private outputs, permit implementations that are orders of magnitude faster than the regular primitives.

## 2. DIFFERENTIAL PRIVACY

Differential privacy \cite{13} is a state-of-the-art privacy metric for answering queries from statistical databases while protecting individual privacy. Since its inception, there has been considerable research in both the theoretical foundations \cite{12} \cite{14} as well as some real world deployments \cite{17} \cite{1} of differential privacy. The rigorous mathematical foundation and the useful properties of differential privacy have led to an emerging consensus about its use among the security and privacy community.

### 2.1 Definition of Differential Privacy

Informally, the privacy guarantees of differential privacy can be understood as follows: Given any two databases, otherwise identical except one of them contains random data in place of data corresponding to any single user, differential privacy ensures that the response mechanism will behave approximately the same on the two databases. Formally,

**Definition 1.** Let $M$ be a randomized mechanism that takes a database instance $D$ and has a range $O$. We say $M$ is $(\epsilon, \delta)$-differentially private, if for any neighboring databases $(D_1, D_2)$ that differ in the data of a single user, and for any $S \subseteq O$, we have

$$\Pr[M(D_1) \in S] \leq e^\epsilon \Pr[M(D_2) \in S] + \delta$$

(Differential privacy enjoys some important properties that make it a useful privacy metric. First, the privacy guarantees of differential privacy have been thoroughly studied using various metrics from statistics and information theory such as hypothesis testing and Bayesian inference \cite{22} \cite{23}. Thus, the semantic meaning of its privacy guarantees is well understood. Differential privacy also has a number of composition properties which enable the analysis of privacy leakage for complex algorithms. In particular, sequential composition addresses the impossibility result by Dinur and Nissim \cite{12} and quantifies the degradation of privacy as the number of sequential accesses to the data increases. The post-processing theorem (a special case of sequential composition) ensures that the adversary cannot weaken the
privacy guarantees of a mechanism by transforming the received response. The end-to-end privacy guarantee of an algorithm over the entire database can thus be established using the above composition theorems and more advanced theorems [15].

2.2 Differentially Private Mechanisms

Next, we review two classic differentially private mechanisms, the Laplace mechanism and the Randomized Response mechanism, with the following scenario: a data analyst would like to find out how many users use drugs illegally. Should a question not elicit any truthful answers from users and hence we require a mechanism that guarantees (a) response privacy for the users and (b) good utility extraction for the data analyst.

Laplace Mechanism: The Laplace mechanism [13] considers a trusted data curator (SDP model) who owns a table $N$ of truthful records of users, for example, each record indicates whether a user uses drugs illegally. If a data analyst would like to learn how many users use drugs illegally, the data curator (trusted) computes the true answer of this query and then perturbs it with a random (Laplace distributed) noise that is sufficient to provide privacy. The magnitude of this noise depends on the largest possible change on the query output – also known as the sensitivity of the query – if the data corresponding to a single user is changed.

Randomized Response Mechanism: Randomized response was first introduced by Warner in 1965 as a research technique for survey interviews. It enabled respondents to answer sensitive questions (about topics such as sexuality, drug consumption) while maintaining the confidentiality of their responses. An analyst interested in learning aggregate information about sensitive user behavior would like to query this function on a database that is distributed across $N$ clients with each client having its own private response $x_1, ..., x_N$. Instead of releasing $x_i$ directly, the clients release a perturbed version of their response $y_i$, thus maintaining response privacy. The analyst collects these perturbed responses and recovers meaningful statistics using reconstruction techniques.

Both these approaches have gained popularity in many applications of differential privacy due to their simplicity as well as the rigorous privacy guarantee on user data. Fig. 1 shows the behavior of differentially private mechanisms for two different privacy values in reference to the true statistic. A less private response results in a more accurate query result while a more private response results in a less accurate query result.

3. CRYPTOGRAPHY FOR DIFFERENTIAL PRIVACY

By itself, differential privacy is a guarantee on a mechanism and hence is “independent” of the deployment scenario. However, when used in practice, practical trust assumptions are made that enable the deployment of differential privacy based systems. In this section, we consider two popular deployment scenarios for differential privacy – Standard Differential Privacy (SDP, graphically represented in Fig. 2A) and Local Differential Privacy (LDP, graphically represented in Fig. 2A). SDP relies on the need for a trusted data aggregator who follows the protocol. However, in practice, a trusted data aggregator may not always exist. LDP on the other hand does not require a trusted data aggregator. With privacy regulations such as GDPR and FERPA, large organizations such as Google increasingly embrace the LDP model thereby avoiding the liability of storing such sensitive user data. This approach also insures data collectors from potential theft or subpoenas from the government. For these reasons, LDP is frequently a more attractive deployment scenario. However, the utility of the statistics released in LDP is poorer than that in SDP. Consequently, there is a gap in the trust assumptions and the utility achieved by mechanisms in SDP and LDP: high trust assumptions, high utility in SDP and lower trust assumptions, lower utility in LDP. We ask the following question:

Can cryptographic primitives help in bridging the gap that exists between mechanisms in the standard differential privacy model and the local differential privacy model?

An emerging direction of research has been to explore the use of cryptography to bridge the trust-accuracy gap and obtain the best of both worlds: high accuracy without assuming trusted data aggregator. In this section, we explore in depth two concrete examples of the role of cryptography in bridging this gap (a) anonymous communication (b) secure computation and encryption.

Key Challenges: There exists a big gap in the accuracy and trust achieved by known mechanisms in the standard differential privacy setting with a trusted data curator (Fig. 2D) and local differential privacy without such a trusted curator (Fig. 2A). Achieving the utility as in the SDP setting while operating under practical trust assumptions such as those in LDP has proven to be a tough challenge. Cryptographic primitives show promise in solving this challenge.

1Differentially private federated learning is simply a special case of the LDP deployment scenario.
Figure 2: This figure shows various deployment scenarios of differential privacy and the underlying trust assumptions in each of them. Standard Differential Privacy (SDP, Fig. 2D) assumes a trusted database, and is thus able to achieve high accuracy i.e., $O(\sqrt{N})$ error. Local Differential Privacy (LDP, Fig. 2A) on the other hand, does not rely on the use of a trusted database but achieves lower accuracy i.e., $O(\sqrt{N})$ error. The goal is to achieve utility of the SDP setting while operating under more practical assumptions such as the LDP setting (i.e., no trusted database). Fig 2B and Fig 2C show how different cryptographic primitives can be used to improve the utility of DP deployments under such practical assumptions.

Key Insights of using Cryptography for DP

- Increasing privacy regulations such as GDPR and FERPA have pushed organizations such as Google to embrace the LDP model for deployment of differential privacy applications.
- Cryptographic primitives show promise in enabling practical differentially private applications without a trusted server, while bridging the utility gap between LDP and SDP.

3.1 Improve Accuracy via Anonymous Communication

In LDP, each data owner independently perturbs their own input (e.g., using the randomized response mechanism) before the aggregation on an untrusted server. This results in a large noise in the final output, $O(\sqrt{N})$ for the case of statistical counting queries. Applications such as Google’s RAPPOR, Apple Diagnostics, and Microsoft Telemetry which use this LDP deployment model operate under more practical trust assumptions yet suffer from poor accuracy/utility. Recent works show that the use of an anonymous communication channel can help improve the accuracy of statistical counting query for LDP and thereby eliminate the need for a trusted data curator. We will use one of these systems called Prochlo to illustrate the key idea of how anonymity can help improve the accuracy of such applications.

3.1.1 Case Study: Prochlo

Anonymous communication channels, first proposed by Chaum in 1981 are systems that enable a user to remain identifiable from a set of other users (called the anonymity set). A larger anonymity set corresponds to a greater privacy guarantee. Examples of such systems include Mixnets, which use proxies to mix communications from various users. In order to circumvent the limitations of LDP, Google explored the use of an anonymous communication channel to improve the accuracy of queries under differential privacy. The proposed technique is called Prochlo. This technique consists of three steps as shown in Fig. 2B: Encode, Shuffle, and Analyze (ESA). The first encoding step is similar to LDP where data owners randomize their input data in independent. The second step uses an anonymous communication channel to collect encoded data into batches during a lengthy time interval and shuffles this data to remove the linkability between the output of the communication channel and the data owners. Last, the anonymous, shuffled data is analyzed by a data analyst.

The shuffling step is the crucial link in achieving anonymous communication by breaking linkability between the user and their data. This step strips user-specific metadata such as time stamps or source IP addresses, and batches a large number of reports before forwarding them to data analysts. Additional thresholding in this step will discard highly unique reports (e.g. a long API bit-vector) to prevent attackers with sufficient background information from linking a report with its data owner. Hence, attacks based on traffic analysis and longitudinal analysis can be prevented, even if a user contributes to multiple reports. Prochlo implements this shuffling step using trusted hardware as proxies to eliminate the need for a trusted third party. Furthermore, this shuffling step can amplify the privacy guarantee of LDP and hence improves the accuracy of the analysis, even when there is a single invocation from a user. We will next show the intuition for this base case.

3.1.2 Accuracy Improvement

To illustrate how anonymous communication can help improve accuracy, let us look at a simple example of computing the sum of boolean values from $N$ data owners, $f \cdot \sum_{i=1}^{N} x_i$, where $x_i \in \{0, 1\}$. In LDP, each data owner reports a random bit with probability $p$ or reports the true bit with probability $1-p$ to achieve $\epsilon$-LDP. When using additional anonymous communication channels, the data owners can enhance their privacy by hiding in a larger set of $N$ values, since the attack-
ers (aggregator and analyst) see only the anonymized set of reports \( \{\tilde{x}_1, \ldots, \tilde{x}_N\} \). The improved privacy guarantee can be shown equivalent to a simulated algorithm that (a) first samples a value \( s \) from a binomial distribution \( B(N, p) \) to simulate the number of data owners who report a random bit, and then (b) samples a subset of responses for these \( s \) data owners from \( \{\tilde{x}_1, \ldots, \tilde{x}_N\} \). The randomness of these sampling processes can amplify the privacy parameter based on a well studied sub-sampling argument \cite{21, 5}. Therefore, given the value of the privacy parameter, the required noise parameter can be scaled down and hence the corresponding error can be reduced to \( O(\sqrt{\log(N)}) \). For general bounded real-valued linear statistics, the error is established to be \( O(\log(N)) \) \cite{16, 10}. Note that these accuracy improvements assume that there is no collusion between the analyst and the anonymous communication, otherwise, the privacy guarantee will fall back to the same as LDP.

In reference to Fig. 2 these works demonstrate the improvement in going from Fig. 2A to Fig. 2B showing a trade-off between accuracy and trust assumptions.

### 3.2 Improve Trust via Encryption & Secure Computation

SDP requires the use of a trusted data aggregator to achieve high accuracy. A number of works have explored the use of encryption and secure computation to eliminate the need for this trusted data aggregator \cite{21, 31}. The key challenge here is to maintain the same level of accuracy as in SDP. We will use one of these proposed systems called DJoin to demonstrate the use of secure computation to enable high accuracy computation without the need for a trusted data aggregator. There is a complementary synergy between secure computation and differential privacy and thus their combination achieves a strong privacy protection. For instance, secure computation ensures all parties learn only the output of the computation but nothing else while differential privacy bounds the information leakage of individuals in the output of the computation, resulting in a system that is better than the use of secure computation or DP alone.

#### 3.2.1 Case Study: DJoin

Consider a simple setting where two parties would like to compute the intersection size of their data while preserving differential privacy for both datasets. If each party does not trust each other, how can we ensure a constant additive error as if they trust each other? It is well known that the lower bound for this query is \( \sqrt{N} \), where \( N \) is the data size of each party \cite{28}, if we want to ensure the view of each party satisfies differential privacy. However, if we assume both parties are computationally bounded, a constant additive error can be achieved.

DJoin \cite{31} offers a concrete protocol for achieving DP under this assumption. This protocol applies private set-intersection cardinality technique to privately compute the noisy intersection set of the two datasets. First, party A defines a polynomial over a finite field whose roots are the elements owned by A. Party A then sends the homomorphic encryptions of the coefficients to party B, along with its public key. Then the encrypted polynomial is evaluated at each of Party B’s inputs, followed by a multiplication with a fresh random number. The number of zeros in the results is the true intersection size between A and B. To provide differential privacy, party B adds a number of zeros (differentially-private noise of \( O(1) \) independent of data size) to the results and sends the randomly permuted results back to party A. Party A decrypts the results and counts the number of zeros. Party A also adds another copy of differentially private noise to the count and sends the result back to party B. In other words, both parties add noise to their inputs to achieve privacy. However, the final protocol output has only an error of \( O(1) \), which is the same as the SDP setting.

#### 3.2.2 Trust Improvement

Using secure computation and encryptions achieves a constant additive error like SDP and prevents any party from seeing the other party’s input in the clear. However, this requires an additional assumption of all parties being computationally bounded in the protocol. Hence, the type of differential privacy guarantee achieved in DJoin is known as computational differential privacy \cite{30}. In addition, most of the existing protocols consider honest-but-curious adversaries who follow the protocol specification or consider malicious adversaries with an additional overhead to enforce honest behaviour i.e., verify that the computation was performed correctly.

In reference to Fig. 2 these works demonstrate the improvement in going from Fig. 2D to Fig. 2C eliminating the need for a trusted data aggregator.

### 4. DIFFERENTIAL PRIVACY FOR CRYPTOGRAPHY

We have seen in Section 3 that cryptographic primitives show promise in bridging the utility gap between SDP and LDP. However, the large overhead of implementing these conventional cryptographic primitives forms a bottleneck for the deployment of such systems. This motivates the need to enhance the performance of such cryptographic primitives. We ask the following question:

Can we formulate leaky versions of cryptographic primitives for enhancing system performance while rigorously quantifying the privacy loss using differential privacy?

DP-cryptographic primitives are significant for two reasons. First, since the final privacy guarantees of such systems are differential privacy, it is natural to relax the building blocks such as cryptographic primitives to provide differentially private guarantees. Secondly, the composability properties of differential privacy allow for rigorous quantification of the privacy of the end-to-end system. We showcase benefits of “DP-cryptographic” systems through two detailed case studies on (a) secure computation and (b) secure communication.

**Key Challenges:** Cryptographic primitives provide strong privacy guarantees. However, deployment of certain cryptographic primitives in practical systems is limited due to the large overhead of these primitives. Relaxing the privacy guarantees in a manner that is amenable to rigorous quantification is difficult and differential privacy can be well utilized to provide a solution to this problem to improve performance overhead.
Key Insights of using DP for Cryptography

- We can obtain practical cryptographic implementations that are efficient, while bounding the privacy leakage using differential privacy.
- In the context of the end goal of differentially private systems, it is natural to relax the privacy of cryptographic primitives to provide differentially private guarantees.

4.1 Improve Performance of Cryptographic Computation Primitives

Cryptographic computation primitives such as Fully Homomorphic Encryption (FHE) and secure Multi-Party Computation (MPC) enable private computation over data. Over the past few years, there has been tremendous progress in making these primitives practical – most promising of which has been Multi-Party Computation. MPC allows a group of data owners to jointly compute a function while keeping their inputs secret. In this section, we show the performance improvement on MPC based private computation, in particular, differentially private query processing.

4.1.1 Case Study: Shrinkwrap

Shrinkwrap [7] is a system that applies differential privacy throughout an SQL query execution to improve performance. In secure computation, the computation overheads depend on the largest possible data size so that no additional information is leaked. For example, two parties would like to securely compute the answer for the SQL query shown in Figure 3A. This query asks for the number of patients with heart disease who have taken a dosage of “aspirin.” Figure 3A expresses this query as a directed acyclic graph of database operators. For example, the first filter operator takes \( N \) records from the two parties and outputs an intermediate result which has patients with heart disease (hd). To hide the selectivity (fraction of records selected) of this operator, the baseline system needs to pad the intermediate result to its maximum possible size, which is the same as the input size. Exhaustive padding will also be applied to the intermediate output of the two joins and result in an intermediate result cardinality of \( N^3 \) and a high performance overhead. However, if the selectivity of the filter is \( 10^{-3} \), cryptographic padding adds a 1000× overhead. Is there a way to pad fewer dummies to the intermediate result while ensuring a provable privacy guarantee?

Shrinkwrap helps reduce this overhead by padding each intermediate output of the query plan to a differentially private cardinality rather than to the worst case. As shown in Figure 3B, without Shrinkwrap, the output of a join operator with two inputs, each of size \( N \) is padded to a size of \( N^3 \). With Shrinkwrap, the output is first padded to the worst size and the output is sorted such that all the dummies are at the end of the storage. This entire process is executed obliviously. Then Shrinkwrap draws a non-negative integer value with a general Laplace mechanism [2] and truncates the storage at the end. This approach reduces the input size of the subsequent operators and thereby their I/O cost. We can see from Figure 3C that Shrinkwrap provides a significant improvement in performance over the baseline without DP padding for increasing database sizes.

The relaxed privacy in the secure computation of Shrinkwrap can be quantified rigorously using computational differential privacy. Assuming all parties are computationally bounded and work in the semi-honest setting, it can be shown that data owners have a computational differentially private view over the input of other data owners; when noisy answers are returned to the data analyst, the data analyst has a computational differentially private view over the input data of all the data owners.

4.2 Improve Performance of Cryptographic Communication Primitives

Anonymous communication systems aim to protect user identity from the communication recipient and third parties. Despite considerable research efforts in the domain, practical anonymous communication over current internet architecture is proving to be a challenge. Even if the message contents are encrypted, the packet metadata is difficult to hide. On one end, systems such as Dissent [35] offer strong privacy guarantees yet can scale only to a limited number of participants. On the other end, practical deployed systems such as Tor are vulnerable to traffic analysis and other attacks, limiting their use due to the non-rigorous nature of their privacy guarantees. We will show a case study that uses differential privacy to reduce the communication cost while offering rigorous privacy guarantee. We denote this primitive differentially private anonymous communication.

4.2.1 Case Study: Vuvuzela

Vuvuzela [34] is an anonymous communication system that uses differential privacy to enable a highly scalable system with relaxed yet rigorously quantified privacy guaran-
Vuvuzela employs a number of servers $S_1, \ldots, S_n$ where at least one of the servers is assumed to be honest. Clients send (and receive) messages to (and from) the first server, which in turn is connected to the second server and so on. The client creates a layered encryption of its message $m$ i.e., $\text{Enc}_{S_1}(\ldots \text{Enc}_{S_n}(m))$, where $\text{Enc}_S(\cdot)$ is the encryption under the key of server $S$. The clients leave messages at virtual locations in a large space of final destinations (called dead drops), where the other legitimate client can receive it. To hide if a client is communicating or not, a client not in an active conversation makes fake requests to appear indistinguishable from a client in an active conversation. If two clients are in active conversation, they exchange messages via the same random dead drop.

Vuvuzela’s threat model assumes that at least one server is honest and the adversary is a powerful network level adversary (observing all network traffic) potentially corrupting all other servers. The only computation hidden from the adversary is the local computation performed by the honest server which unlinks users’ identifiers from the dead drops and adds cover (dummy) traffic. As a consequence, the adversary can only observe the number of single or double exchange requests at the dead drop locations. Each Vuvuzela server adds cover traffic using a Laplace distribution to randomize the (a) number of single dead drops and (b) number of double dead drops, which is observable by the adversary. Such random cover traffic addition along with the assumption of at least one honest server provides differentially private guarantees for the observed variables. In other words, Vuvuzela adds noise (cover network traffic) to the two observables (by the adversary) viz. the number of dead drops with one exchange request, and the number of dead drops with two exchange requests, thereby providing communication privacy to clients. This privacy relaxation enables Vuvuzela to scale to a large number of users – it can achieve a throughput of 68,000 messages per second for a million users scaling linearly with number of users.

Figure 4: Vuvuzela is a secure messaging system. An adversary who can observe and tamper with all network traffic cannot distinguish whether Alice is messaging Bob, Charlie, or is simply not communicating. Vuvuzela uses differential privacy to add noise and mask the privacy invasive metadata, thereby provably hiding information about user communication patterns. Vuvuzela achieves a throughput of 68,000 messages per second for a million users scaling linearly with number of users.
trust assumptions, and other DP-cryptographic primitives. Finally, we caution readers against callous combinations of differential privacy and cryptography.

**Differential Privacy Frameworks – SDP, LDP, and Beyond:** Over the past decade, there has been significant progress in enabling applications in the standard differential privacy model. For instance, there have been research efforts in attaining differential privacy to handle realistic challenges such as multi-dimensional and complex data – involving graphs, time series, correlated data [26][22]. Similarly, there has been work in designing a tailored differential privacy mechanism that is optimized for particular application settings to achieve good accuracy [29][20]. Prior work has explored combinations of sequential and parallel composition, dimensionality reduction, and sensitivity bound approximations to achieve good accuracy in the SDP model. However, much work needs to be done in adapting state-of-the-art techniques in SDP to more complex deployment scenarios such as LDP. For instance, an open question is the following:

Is there an algorithm that can efficiently search the space of differentially private algorithms in the LDP setting for the one that answers the input query with the best accuracy?

Research advances have demonstrated such mechanisms for the SDP model [29][20], however, the discovery of such mechanisms in the LDP setting remains an open question. On a similar note, it is unclear how nuanced variants of differential privacy that have been proposed to handle these more complex databases [26][22] in the SDP setting translate into LDP or more complex deployment settings.

**Differential Privacy in Practice – Trust Assumptions vs Accuracy Gap:** We have seen how deployments of differential privacy that differ in the trust assumptions provide roughly the same privacy guarantee, but with varying levels of accuracy. In particular, we looked at a two popular deployment scenarios viz., SDP and LDP. There exist other trust assumptions that we have not covered in this article in detail. For instance, Google’s recently proposed Prochlo system [8] uses trusted hardware assumptions to optimize utility of data analytics. On a similar note, Groce et al. [10] consider yet another model – where the users participating are malicious. This is the first work to explore a malicious adversarial model in the context of differential privacy and the development of better accuracy mechanisms for such a model is an open research question. More concretely, we can ask:

What other models of deployment of differential privacy exist and how do we design mechanisms for them? Can other technologies such as MPC, FHE, trusted hardware open up new opportunities in mechanism design?

An interesting theoretical question is to characterize the separation between different trust models in terms of the best accuracy achievable by a differential privacy algorithm under that model. For instance, McGregor et al. [28] provide separation theorems i.e., gaps in achievable accuracy between (information-theoretic) differential privacy and computational differential privacy for two-party protocols. In reference to Section 3.2, we can ask the following concrete question:

In the Mixnets model (Fig. 2B), what is the lower bound on the error for aggregate queries over relational transformations (like joins and groupby) over the data records? An example of such an aggregate is the degree distribution of a graph that reports the number of nodes with a certain degree.

**Relaxing Cryptographic Security via Differential Privacy:** The emerging paradigm of leaky yet differentially-private cryptography leads to a number of open questions for the research community. So far, the research community has explored the intersection of differential privacy and cryptographic primitives in limited contexts such as ORAM, MPC, and anonymous communication. However, there exists a broader opportunity to explore the trade-offs of DP-cryptographic primitives in contexts such as program obfuscation, zero-knowledge proofs, encrypted databases, and even traffic/protocol morphing. As described in Section 4, we can ask:

What other cryptographic primitives can benefit in performance from a privacy relaxation quantified rigorously using differential privacy? How can we design such relaxed primitives?

In the context of differentially-private data analysis, there is a trade-off between privacy and utility. In the context of differentially-private cryptographic primitives and resulting applications, there is a broader trade-off space between privacy, utility, and performance. Another open question is the following:

What lower bounds exist for overhead of cryptographic primitives when the privacy guarantees are relaxed using differential privacy?

Another challenge is how to design optimized protocols that achieve desired trade-offs in the new design space of differentially-private cryptography. The trade-off space between privacy, utility, and performance is non-trivial, especially for complex systems. An interesting research question is:

How to correctly model the trade-off space of real systems so that system designers can decide whether it is worth sacrificing some privacy or utility for a better performance?

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