Research on Sliding Mode Control of Underwater Vehicle-Manipulator System Based on an Exponential Approach Law

Qirong Tang, Yang Hong, Zhenqiang Deng, Daopeng Jin, and Yinghao Li

Laboratory of Robotics and Multibody System, School of Mechanical Engineering, Tongji University, Shanghai 201804, People’s Republic of China
qirong.tang@outlook.com

Abstract. To improve the performance of underwater vehicle-manipulator system (UVMS), which is subject to system uncertainties and time-varying external disturbances in trajectory tracking control, a sliding mode controller is proposed in this paper. Firstly, in order to reduce a influence of system uncertainties and external disturbances, a sliding mode controller is designed based on an exponential approach law. Then the error asymptotic convergence of the trajectory tracking control is proven by the Lyapunov-like function. Finally, the effectiveness of the sliding mode controller is verified by rich simulation. Results show that the designed controller can not only realize the coordination control of UVMS accurately, but also can eliminate the chattering of control signal.

Keywords: UVMS · Trajectory tracking · Sliding mode controller · Exponential approach law

1 Introduction

With the development of marine exploration in the world, autonomous underwater vehicle (AUV) has been widely used in the ocean. However, its application has certain limitations. It usually can only perform tasks such as underwater search and monitoring, rather than operation. The underwater vehicle-manipulator system (UVMS) is then attracting more attentions than AUVs because it carries one or even several manipulators to work more flexible. Therefore, more and more countries have carried out systematic studies on UVMS in recent years.

In order to complete various complex tasks, high-precision trajectory tracking controller is then demanded. However, UVMS is strongly coupled and with high nonlinearity [1]. When the manipulator is working, it will disturb the vehicle and even cause interference to the entire system [2]. Therefore, it is urgent to design a reasonable control system for underwater vehicle’s trajectory tracking.
In recent years, various controllers have been proposed for UVMS control, including PID control \cite{3,4}, fuzzy control \cite{5,6}, adaptive control \cite{7,8}, robust control \cite{9,10}, sliding mode control \cite{11,12} and neural network based control \cite{13}. The response PID control system is slow, and the control precision is limited. Fuzzy control belongs to a kind of experience control. Its control accuracy depends on the perfection of the summarized experience. Although it can have strong interference ability, it is rarely applied to the actual control system alone, and most of them are combined with other control methods. In addition, adaptive control is not very stable. The robust controller synthesis process is time-consuming and cannot be completed online, usually. The initial weight of the neural network based methods have high randomness and a long learning process, so they are difficult to be applied in practice. In comparison, sliding mode control has the advantages of simple implementation, fast response, small perturbation of model parameters and strong robustness, so it is very suitable for the trajectory tracking control of underwater robots. However, its discontinuous switching characteristics will cause chattering in the system. Therefore, it is necessary to tackle of the chattering problem of sliding mode control through certain methods.

In this study, a new control scheme based on an exponential approach law is proposed. The approach law can reduce arrival time and eliminate chattering. Based on this, a sliding mode control scheme for UVMS trajectory tracking control is designed.

The remainder of this paper is organized as follows. Section 2 presents the dynamic model of the concerned UVMS. In Sect. 3, presents the design of sliding mode controller based on the exponential approach law (SMC-EAL). In Sect. 4, the effectiveness of the proposed the SMC-EAL is evaluated through adequate simulations. While Sect. 5 concludes the paper.

2 Dynamic Modelling of UVMS

2.1 Lagrangian Dynamics Modelling of UVMS

The coordinate system of our study object which is equipped with a 2-DOFs manipulator is shown in Fig. 1. And the 3D model of UVMS is shown in Fig. 2. Here $\sum E$, $\sum V$ represent the earth-fixed frame and the vehicle frame, respectively. The position and orientation of UVMS are considered in the inertial frame, i.e., the earth-fixed frame. The linear velocities and angular velocities of UVMS, as well as the external forces applied to UVMS are considered in the vehicle frame.

The UVMS system consists of two parts: the underwater vehicle and the manipulator, and its generalized coordinates and generalized control force are defined as follows,

$$\xi = [x \ y \ z \ \varphi \ \psi \ q_1 \ \cdots \ q_n]^T,$$

$$\tau = [F_x \ F_y \ F_z \ W_x \ W_y \ W_z \ \tau_1 \ \cdots \ \tau_n]^T.$$
Research on SMC-EAL of UVMS

where $\xi = [\eta^T \ q^T]^T \in \mathbb{R}^{6+n}$ is the generalized position vector of UVMS, including the position, attitude and joint variables of the underwater vehicle. Define vector $\eta = [x \ y \ z \ \varphi \ \theta \ \psi]^T$, $x$, $y$ and $z$ represent the three dimension position of the vehicle, $\varphi$, $\theta$ and $\psi$ represent the roll angle, longitudinal inclination angle and heading angle of underwater vehicle, respectively. And vector $q = [q_1 \ \cdots \ q_n]^T \in \mathbb{R}^n$ is the joint angle variable of manipulator. Meanwhile, $\tau \in \mathbb{R}^n$ is the generalized control force term of the system, including the generalized thrust forces of the underwater vehicle and the joint driving forces of the manipulator.

The dynamic equation of UVMS is established according to the second type of Lagrange equation as follow,

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\xi}} \right) - \frac{\partial L}{\partial \xi} = Q,$$

where $L$ is the Lagrange function, $L = T - V$, $T$ is system kinetic energy and $V$ is system potential energy, $Q$ is the generalized force corresponding to the conservative active force, which mainly includes the generalized control force, the restoring torque and the water resistance of the system. Through derivation, the whole dynamic equation of the system is defined as follow,

$$M(\xi)\ddot{\xi} + C(\xi, \dot{\xi})\dot{\xi} + D(\dot{\xi})\dot{\xi} + G(\xi) = \tau,$$

where $M(\xi) \in \mathbb{R}^{(6+n) \times (6+n)}$ is the inertia matrix of UVMS, $C(\xi, \dot{\xi})\dot{\xi} \in \mathbb{R}^{6+n}$ is the centripetal force and corioli term, and $D(\dot{\xi})\dot{\xi} \in \mathbb{R}^{6+n}$ is the resistance term. Since the center of gravity of UVMS does not coincide with buoyancy, the restoring torque will be generated, and $G(\xi) \in \mathbb{R}^{6+n}$ is the restoring force term generated by the combined torque of gravity and buoyancy of UVMS.

2.2 Determination of Hydrodynamic Coefficient

It is difficult to calculate the hydrodynamic coefficient fully in underwater hydrodynamics, so it is necessary to determine which coefficients are important. Inertial hydrodynamics are only related to the acceleration of each moving part of

---

**Fig. 1.** Coordinate system of UVMS

**Fig. 2.** Three dimensional model of UVMS
the UVMS. It is linearly related to acceleration, and in the opposite direction of acceleration. Similar to the conventional mass concept, the inertial hydrodynamic force and acceleration are defined as the additional mass and moment of inertia, which are only related to the structure of UVMS, and have the form of

\[ R_i = -\lambda_i \cdot U_i, \]  

where \( i = 1, 2, ..., 6 \), and \( R_i \) is the inertial hydrodynamic coordinate component in the moving coordinate system, \( U_i \) is the acceleration of UVMS, \( \lambda_i \) is the additional inertia mass or inertia moment of UVMS, as shown in Table 1.

| \( \lambda_i \) | \( \lambda_1 \) (kg) | \( \lambda_2 \) (kg) | \( \lambda_3 \) (kg) | \( \lambda_4 \) (kg \( \cdot \) m\(^2\)) | \( \lambda_5 \) (kg \( \cdot \) m\(^2\)) | \( \lambda_6 \) (kg \( \cdot \) m\(^2\)) |
|---|---|---|---|---|---|---|
| Calculation results | 29.13 | 247.04 | 462.66 | 2.01 | 48.6 | 23.455 |

Viscous hydrodynamic force is mainly the water resistance applied to UVMS. It is mainly affected by the velocity and the square term of the moving system, so the higher order term of the velocity is usually ignored. In the system dynamics model, the viscous flow resistance is defined as follow,

\[ D(\dot{\xi}) = -\text{diag}\{X_u, Y_v, Z_w, K_p, M_q, N_r\} \]

\[ = -\text{diag}\{X_u|u|, Y_v|v|, Z_w|w|, K_p|p|, M_q|q|, N_r|r|\}, \]  

where \( X_u, Y_v, Z_w, K_p, M_q, N_r \) represent the hydrodynamic coefficient terms in different directions, respectively. By introducing the force and torque of UVMS measured in the ship model test into the fitting formula of hydrodynamic coefficient, the hydrodynamic coefficients are obtained as shown in the Table 2.

| Hydrodynamic item | \( X_u \) | \( Y_v \) | \( Z_w \) | \( M_q \) | \( N_r \) |
|---|---|---|---|---|---|
| Fitting results | 0.00735 | -0.1765 | -0.1940 | -0.0655 | -0.0148 |

3 Design of Sliding Mode Controller

3.1 Sliding Mode Variable Structure Control

The basic principle of sliding mode variable structure control is that the controller limits the current state or state error of the system to the sliding mode surface and makes it stable on the sliding mode surface. Since the sliding mode
motion characteristics are designed in advance according to the requirements, and the trajectory given at the same time has nothing to do with the control object parameters and external disturbance changes. Therefore, sliding mode control is insensitive to parameter changes and disturbances, and the system is extremely robust. However, in the actual control system, due to the mass inertia and time delay and other factors, the sliding mode variable structure control will have chattering in the sliding mode.

The process of implementing sliding mode variable structure control mainly includes determining the sliding mode arrival condition of the system, selecting the sliding mode surface and obtaining the system controller law, then establishing the controller.

3.2 Design of UVMS Sliding Mode Controller

The designed exponential approach law is applied to the sliding mode controller of UVMS, and compensation is carried out in the controller to improve the control accuracy, reduce the gain of sliding mode control, and thus reduce the chattering of the system. The proposed sliding mode control system is shown in Fig. 3.

![Fig. 3. The implementation structure of the sliding mode controller](image)

Firstly, the expected trajectory of UVMS is set as follow $\xi_d(t), t \in [0, T]$. The corresponding velocity is $\dot{\xi}(t)$, exists and continues throughout the time period. The trajectory tracking error and trajectory tracking error change rate are set as follow,

$$\begin{align*}
    e &= \xi_d - \xi \\
    \dot{e} &= \frac{de}{dt} = \dot{\xi}_d - \dot{\xi},
\end{align*}$$

meanwhile, the linear sliding mode surface is selected, as shown in Eq. (8),

$$s = ce + \dot{e},$$

and

$$\begin{align*}
    s &= [s_1\ s_2\ \cdots\ s_N]^T \\
    c &= \text{diag}\{c_1\ c_2\ \cdots\ c_N\},
\end{align*}$$
where $\mathbf{s}$ is the sliding mode surface vector of the control system, and $\mathbf{c}$ is the parameter matrix of the sliding mode surface of the system. In order to improve the dynamic quality of the sliding mode surface and reduce the chattering, the controller is designed based on the exponential approach law, which is shown in Eq. (10),

$$\dot{s} = ds/dt = -\varepsilon \text{sgn}(s) - ks, \quad (\varepsilon > 0, k > 0),$$

where

$$\text{sgn}(s) = \begin{cases} 1, & s > 0 \\ 0, & s = 0 \\ -1, & s < 0 \end{cases},$$

combining Eqs. (6)–(8) and (10), the sliding mode controller law is designed as follows,

$$\mathbf{u} = \mathbf{r} = M(\mathbf{e}) (\mathbf{c}\dot{\mathbf{e}} + \varepsilon \text{sgn}(s) + ks + \ddot{\mathbf{s}}_d) + C(\mathbf{e}, \dot{\mathbf{e}}) (\mathbf{c}\mathbf{e} - s + \ddot{s}_d) + D(\dot{\mathbf{e}}) (\mathbf{c}\mathbf{e} - s + \ddot{s}_d) + G(\mathbf{e}).$$

In order to satisfy the sliding mode arrival condition and have certain stability, Lyapunov stability condition needs to be satisfied. Lyapunov function of the system is selected as

$$\dot{V} = -\varepsilon |s| - ks^2,$$

since $\varepsilon > 0, k > 0, \dot{V} < 0$, if and only if $s = 0$. The system satisfies Lyapunov stability condition, so the system is gradually stable.

4 Simulation Studies

In order to verify the effectiveness of the designed sliding mode control strategy, the expected trajectory is set under the simulation condition, and the actual trajectory is compared with the expected trajectory.

4.1 Simulation Object

To illustrate the effectiveness of the proposed SMC, comparative simulations in presence of environmental disturbances have been performed on a UVMS, which consists of a 6-DOFs vehicle and a 2-DOFs rotary joint manipulator. The specific parameters of the system are listed in Table 3 and Table 4.

In the simulation, expected trajectory of UVMS is set as $\mathbf{e}_d = [x_d \ y_d \ z_d \ \varphi_d \ \theta_d \ \psi_d \ q_1d \ q_2d]^T$, where $x_d = 4\cos(0.04\pi t), \ y_d = 4\sin(0.04\pi t), \ z_d = 0.5t, \ \varphi_d = 0, \ \theta_d = 0, \ \psi_d = 0, \ q_1d = 10^\circ, \ q_2d = 20^\circ$.

The design trajectory is a three-dimensional spiral with a turning period of 50s. The UVMS adjusts its attitude during the dive, and the manipulator is deployed during the dive. The initial pose vector of UVMS is $\mathbf{e}_0 = [2201 0 -20 0 0 0]^T$. 

### Table 3. Main parameters of UVMS

| Items         | Vehicle | Link 1 | Link 2 |
|---------------|---------|--------|--------|
| $m$/kg        | 85.2    | 2.603  | 3.159  |
| $I_{xx}$/kg·m$^2$ | 35.040  | 0.0062 | 0.177  |
| $I_{yy}$/kg·m$^2$ | 2.140   | 0.0061 | 0.177  |
| $I_{zz}$/kg·m$^2$ | 35.667  | 0.0071 | 0.0047 |
| $I_{xy}$/kg·m$^2$ | −0.0063 | 0      | 0      |
| $I_{xz}$/kg·m$^2$ | 0       | 0      | 0      |
| $I_{yz}$/kg·m$^2$ | −0.0233 | 0      | 0.00559|

### Table 4. D-H parameters of manipulator in UVMS

| joint | $\theta_i$ (rad) | $\alpha_i$ (rad) | $\alpha_{i-1}$ (rad) | $d_i$ (mm) |
|-------|------------------|------------------|----------------------|-------------|
| 1     | 0                | $\pi/2$          | 0                    | 0           |
| 2     | $\pi$            | $\pi/2$          | 0                    | 0.312       |

**Fig. 4.** 3D trajectory tracking result of UVMS

**Fig. 5.** Position control inputs of UVMS in separate dimension

### 4.2 Analysis of Simulation Results

In the simulation, SMC’s gain is set as $k = 20$, and trajectory tracking result shown in Fig. 4. The desired trajectory is well stacked under SMC-ELA. And Figs. 5, 6 and 7 represent the position and attitude of UVMS, and the control inputs of each joint of the manipulator in each single degree of freedom, respectively. It can be seen from Figs. 4, 5, 6 and 7 that the sliding mode controller with exponential approach law is feasible for the control of UVMS with a relatively acceptable accuracy. The control switching gain in the sliding mode control simulation process is small, and the chattering of the control signal can be significantly suppressed.
5 Conclusion

This study presents a sliding mode controller. It is validated in this study by the trajectory tracking control of UVMS, which is subject to system uncertainties and environment disturbances. In the simulation, the effectiveness of the proposed method has been demonstrated. Moreover, the proposed controller is featured with higher tracking accuracy of UVMS with regard to environmental disturbances and provides a reference for the following UVMS control.

Acknowledgements. This work is supported by the projects of National Natural Science Foundation of China (No. 61603277, No. 61873192, No. 51579053).

References

1. Dai, Y., Yu, S.: Design of an indirect adaptive controller for the trajectory tracking of UVMS. Ocean Eng. 151, 234–245 (2018)
2. Liu, H., Li, M., Liu, X.: Control of underwater robot attitude in wave based on fuzzy sliding mode method. J. Donghua Univ. 27(2), 143–147 (2010)
3. Lashin, M., Fanni, M., Magdy, M., Mohamed, A.: PD type of fuzzy controller for a new 3DOF fully decoupled translational manipulator. In: Proceedings of the 2016 International Conference on Control, Automation and Robotics (ICCAR), Hong Kong, China, 28–30 April, pp. 263–267 (2016)
4. Han, J., Chung, W.: Active use of restoring moments for motion control of an underwater vehicle-manipulator system. IEEE J. Oceanic Eng. 39(1), 100–109 (2014)
5. Xu, B., Pandian, S., Sakagami, N., Petry, F.: Neuro-fuzzy control of underwater vehicle-manipulator systems. J. Franklin Inst. 27(3), 1125–1138 (2012)
6. Kazuo, T., Hua, W.: Fuzzy control systems design and analysis: a linear matrix inequality approach. Automatica 39(11), 2011–2013 (2001)
7. Yong, C., Junku, Y.: A unified adaptive force control of underwater vehicle-manipulator systems (UVMS). In: Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 27–31 October Las Vegas, USA, pp. 553–558 (2003)
8. Mai, T.-L., Wang, Y.: Adaptive-backstepping force/motion control for mobile-manipulator robot based on fuzzy CMAC neural networks. Control Theory Technol. 12(4), 368–382 (2014). https://doi.org/10.1007/s11768-014-3181-4

9. Xu, B., Abe, S., Sakagami, N., Pandian, S.: Robust nonlinear controller for underwater vehicle-manipulator systems. In: Proceedings of the 2005 International Conference on Advanced Intelligent Mechatronics, 24–28 July, Monterey, USA, pp. 711–716 (2005)

10. Taira, Y., Sagara, S., Oya, M.: A robust controller with integral action for underwater vehicle-manipulator systems including thruster dynamics. In: Proceedings of the 2014 International Conference on Advanced Mechatronic Systems (ICAMechS), 10–12 August Kumamoto, Japan, pp. 415–420 (2014)

11. Yang, Q., Su, H., Tang, G., Gao, D.: Robust optimal sliding mode control for AUV system with uncertainties. Inf. Control 47(2), 176–183 (2018)

12. Kim, D., Choi, H.-S., Kim, J.-Y., Park, J.-H., Tran, N.-H.: Trajectory generation and sliding-mode controller design of an underwater vehicle-manipulator system with redundancy. Int. J. Precis. Eng. Manuf. 16(7), 1561–1570 (2015). https://doi.org/10.1007/s12541-015-0206-y

13. Ge, S., Hong, F., Lee, T.: Adaptive neural network control of nonlinear systems with unknown time delays. In: Proceedings of the 2003 American Control Conference, 4–6 June, Denver, USA, pp. 4524–4529 (2003)