Is the newly reported $X(5568)$ a $B\bar{K}$ molecular state?

Rui Chen$^{1,2,*}$ and Xiang Liu$^{1,2,**}$

$^1$School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
$^2$Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China

(Dated: July 29, 2016)

In this work, we perform a dynamical study of the $B^{(*)}$ and $\bar{K}$ interaction and show that the newly reported $X(5568)$ or $X(5616)$ cannot be assigned to be an isovector $B\bar{K}$ or $B^{*}\bar{K}$ molecular state. We continue to investigate the isoscalar $B^{0}\bar{K}$ systems, and the $B^{*}\bar{K}$ systems with isospin $I = 0, 1$, and predict the existence of several isoscalar $B^{0}\bar{K}^*$ molecular states. A new task of exploring open-bottom molecular states will be created for future experiments.

PACS numbers: 14.40.Rt, 12.39.Pn

I. INTRODUCTION

In a recent experimental analysis [1], the DØ Collaboration reported a new enhancement structure $X(5568)$ in the $B^0\pi^+$ invariant mass spectrum, which has mass $m = 5567.8 \pm 2.9 \text{(stat)} + 0.9 \text{(syst)}$ MeV and width $\Gamma = 21.9 \pm 6.4 \text{(stat)} + 5.2 \text{(syst)}$ MeV [1]. Due to its observed decay mode, we conclude that the $X(5568)$ must contain four different valence quark components, which makes the $X(5568)$ a good candidate for a tetraquark state. Experimental and theoretical exploration of exotic multiquark states has become an intriguing issue, especially with the experimental progress on charmonium-like tetraquark state. By making a calibration by the mass of the $X(5568)$ or the $X(5568)$ as a tetraquark state composed of a diquark and an-$\bar{s}$-$d$ tetraquark was studied in Ref. [15]. However, some groups hold opposite view. In a relativized quark model, the mass spectra of open-bottom tetraquark states were obtained [16]. They found that the $X(5568)$ disfavors the assignment of the $s\bar{s}q\bar{q}$ tetraquark state since the theoretical result is higher than the data. In Ref. [17], Esposito et al. calculated the mass of the $X_0 = [b\bar{d}]_{s=0}[s\bar{q}]_{L=0}$ state using the constituent quark model, which has the same quantum number as that of $X(5568)$. The mass of the $X(5568)$ is below the obtained mass of $X_0$. Besides these tetraquark studies of the $X(5568)$, there were some discussions of the $X(5568)$ as the $B\bar{K}$ molecular state [18, 19]. In Ref. [18], the $B^{0}\pi^+$ decay width of the $X(5568)$ as the $B\bar{K}$ molecular state was estimated, which is comparable with the experimental data on $X(5568)$. A QCD sum rule study in Ref. [19] showed that a diquark-antidiquark configuration for the $X(5568)$ is more favorable than the $B\bar{K}$ molecular state picture.

In addition, the $X(5568)$ was explained to be the threshold effect [23]. We also noticed an investigation of the production of the $X(5568)$ in high-energy multiproduction process [24], where the authors indicated that it is hard to understand the large production rate of the $X(5568)$ using various general hadronization mechanisms. In recent work [25, 26], the difficulty of explaining the $X(5568)$ as the $B\bar{K}$ molecular state was indicated. The authors of Ref. [27] further found that the $X(5568)$ signal can be reproduced by using $B_s\pi - B\bar{K}$ coupled channel analysis, if the corresponding cutoff value is larger than a natural value $\Lambda \sim 1$ GeV. Thus, they concluded that it is difficult to explain the properties of the $X(5568)$. Later, a further study along this line was given in Ref. [28].

When facing different proposals for the $X(5568)$, a crucial task is to find the evidence to distinguish these different explanations from the $X(5568)$. In this work, we perform a serious dynamical study of the interaction between $B^{(*)}$ and $\bar{K}$ using the one-boson exchange (OBE) model. In this investigation, we check whether $B^{(*)}$ and $\bar{K}$ can be bound together to form a hadronic molecular state corresponding to the $X(5568)$ or the $X(5616)$.

This paper is organized as follows. We illustrate why the

---

$^*$ Corresponding author
$^*$ Electronic address: chenr2012@lzu.edu.cn
$^*$ Electronic address: xiangliu@lzu.edu.cn

---

$^\dagger$ There were some theoretical studies of the interactions between bottom-strange meson and kaon in Refs. [20–22].
X(5568) or the X(5616) cannot be a \( \bar{B}^0 K \) molecular state in Sec. II and Sec. III. In Sec. IV, we present the prediction of the possible \( \bar{B}^0 K^0 \) molecular states. Finally, the paper ends with a short summary.

II. THE X(5568) CANNOT BE AN S-WAVE B\( \bar{K} \) MOLECULAR STATE

The quantum number \( I(J^P) \) for the \( X(5568) \) is constrained as \( 1(0^+) \), since it has the decay channel \( B^0 \pi^0 \). The flavor wave functions \( |I, J \rangle \) of the \( B \bar{K} \) system are defined as \( |1, 1 \rangle = |B^+ K^0 \rangle, |1, 0 \rangle = |B^0 K^- \rangle, \) and \( |1, -1 \rangle = |B^0 K^- \rangle \). For the isoscalar \( B \bar{K} \) system, its flavor wave function is \( |0, 0 \rangle = \frac{1}{\sqrt{2}} \left( |B^0 K^- \rangle + |B^0 K^0 \rangle \right) \). Here, we consider the S-wave \( B \bar{K} \) molecular state \([29-33]\), which has the same quantum number as that of the \( X(5568) \). Thus, the spin-orbit wave function of the \( B \bar{K} \) system corresponds to \(|S_0 \rangle \) with spin \( S = 0 \) and orbital \( L = 0 \). In fact, we notice that the mass of the \( X(5568) \) is about 206 MeV lower than the \( B \bar{K} \) threshold. This means that the \( X(5568) \) should be a deeply bound state composed of \( B \) and \( \bar{K} \) if the \( X(5568) \) is a \( B \bar{K} \) molecular state. In the following, we need to carry out a quantitative dynamical calculation to test this scenario.

In the OBE model, the interaction between \( B \) and \( \bar{K} \) can be due to the light vector-meson (\( \rho \) and \( \omega \)) exchanges. The corresponding effective Lagrangians describing the couplings of \( B^{(*)} B^{(*)}\rho(\omega) \) \([34, 35]\) and \( \bar{K}^{(*)} \bar{K}^{(*)}\rho(\omega) \) \([36]\) are

\begin{equation}
\mathcal{L}_{\rho_{B\bar{K}_K}^{(*)\pi_\rho}} = \sqrt{2} g_{\rho_{B\bar{K}_K}^{(*)\pi_\rho}} \bar{\rho}_a \rho_b \vec{V}_{ab} \cdot \vec{V}_{ab} - \sqrt{2} g_{\rho_{B\bar{K}_K}^{(*)\pi_\rho}} \bar{\rho}_a \rho_b \vec{V}_{ab} \cdot \vec{V}_{ab} - i 2 \sqrt{2} g_{\rho_{B\bar{K}_K}^{(*)\pi_\rho}} \bar{\rho}_a \rho_b \partial_\mu \vec{V}_{ab} \cdot \vec{V}_{ab} \partial_\nu \end{equation}

\begin{equation}
\mathcal{L}_{\rho_{B\bar{K}_K}^{(*)\rho}} = i g_{\rho_{B\bar{K}_K}^{(*)\rho}} \left( \vec{K}_\mp \cdot \vec{K}_\mp \right) \rho_\mu \rho_\nu \end{equation}

where the pseudoscalar \( \vec{P} \) and vector \( \vec{P}' \) have the definition \( \vec{P}^{(*)T} = (B^{(*)0}, B^{(*)-}, \bar{B}^{(*)+}) \). The vector matrix \( \mathcal{V} \) has the form

\begin{equation}
\mathcal{V} = \begin{pmatrix}
\rho^+ & \omega & \rho^+
\omega & \rho^- & K^{0+}
\rho^- & K^{0-} & \phi
\end{pmatrix}
\end{equation}

In addition, the coupling constants involved in Eq. (1) are taken as \( \beta = 0.9, g_{\rho_{B\bar{K}_K}^{(*)\rho}} = 5.8 \), and \( \lambda = 0.56 \text{GeV}^{-1} \) \([35]\), while the \( KK\rho(\omega) \) constants \( g_{\rho(\omega) B\bar{K}_K^{(*)}} \) are

\begin{equation}
g_{\rho B\bar{K}_K^{(*)}} = -\frac{1}{4} g_1 = -3.425,
\end{equation}

which were given in Ref. \([37]\).

The effective potential of the isovector \( B \bar{K} \) system is deduced as

\begin{equation}
V_{B\bar{K}}^{(1)}(r) = -\frac{\beta g_{\rho_{B\bar{K}_K}^{(*)\pi_\rho}}}{2} \left[ g_{\rho_{B\bar{K}_K}^{(*)\pi_\rho}} Y(\Lambda, m; r) - g_{\rho_{B\bar{K}_K}^{(*)\pi_\rho}} Y(\Lambda, m; r) \right]
\end{equation}

In the above expression, the cutoff factor \( \Lambda \) denotes the phenomenological parameter around 1 GeV \([29, 30]\), which is introduced in the monopole form factor \( f(q^2, m_2^2) = (\Lambda^2 - m_2^2)/(\Lambda^2 - q^2) \) when writing out the scattering amplitude of \( B \bar{K} \rightarrow B \bar{K} \). Here, the function \( Y(\Lambda, m; r) \) reads as

\begin{equation}
Y(\Lambda, m; r) = \frac{1}{4\pi\rho} (e^{-mr} - e^{-\Lambda r}) - \frac{\Lambda^2 - m^2}{8\pi\Lambda} e^{-\Lambda r}.
\end{equation}

FIG. 1: The dependence of the OBE effective potential for the isovector S-wave \( B \bar{K} \) system on \( r \) and typical \( \Lambda \) values. Here, we also show the variations of the subpotentials from the \( \rho \) and \( \omega \) meson exchanges to \( r \).

In Fig. 1, we first present the \( r \) dependence of effective potentials for the isovector \( B \bar{K} \) system, where we take several typical values of the cutoff \( \Lambda \). As shown in Fig. 1, the total OBE effective potentials corresponding to \( \Lambda = 1 \sim 4 \) GeV are attractive. As the values of \( \Lambda \) increases, the attraction between \( B \) and \( \bar{K} \) becomes stronger. Furthermore, we numerically solved the Schrödinger equation with the obtained effective potential, and could not find the corresponding bound-state solution for this S-wave isovector \( B \bar{K} \) system when taking \( \Lambda = 1 \sim 5 \) GeV \([29, 30]\), which means that the \( B \) and \( \bar{K} \) cannot be bound together to form an S-wave \( B \bar{K} \) molecular state with isospin \( I = 1 \).

Since the \( X(5568) \) was observed in the \( B^+ \pi^0 \) channel, which is close to the mass of the \( X(5568) \), we further consider the coupled-channel effect due to the mixing between the \( B^+ \pi^0 \) and \( B^0 \bar{K}^0 \) channels. In our calculation, we adopt the effective potential \([36]\)

\begin{equation}
\mathcal{L}_{B\bar{K}_K} = i g_{\pi B\bar{K}_K^{(*)}} \left[ \vec{K}_\mp \cdot \vec{K}_\mp \partial_\mu \partial_\nu \vec{K}_\mp \cdot \vec{K}_\mp \partial_\mu \partial_\nu + H.c. \right]
\end{equation}
where $g_{\pi KK} = \frac{1}{2} g_1$ [37]. Then, the obtained total effective potentials corresponding to the discussed $X(5568)$ can be written as

$$V(r) = \left( \langle B, \pi | V | B, \pi \rangle \langle B, \pi | V | B \bar{K} \rangle \langle B \bar{K} | V | B, \pi \rangle \langle B \bar{K} | V | B \bar{K} \rangle \right)^{(8)}$$

with

$$\langle B, \pi | V | B, \pi \rangle = 0,$$

$$\langle B, \pi | V | B \bar{K} \rangle = 0,$$

$$\langle B \bar{K} | V | B, \pi \rangle = \frac{\sqrt{7}}{4} \beta g_v g_{\pi KK} (m_\pi + m_K) Y(\Lambda, m_K, r),$$

$$\langle B \bar{K} | V | B \bar{K} \rangle = \frac{\beta g_v}{2} \left[ g_{\rho KK} Y(\Lambda, m_\rho, r) - g_{\omega KK} Y(\Lambda, m_\omega, r) \right].$$

With this deduced effective potential, we solve the coupled-channel Schrödinger equation. Unfortunately, we still cannot find the bound-state solutions when scanning the range $\Lambda = 1 \sim 5$ GeV.

According to our study, we can fully exclude the $X(5568)$ as an isovector $S$-wave $B \bar{K}$ molecular state with $J^P = 0^+$, which is consistent with the conclusion made in Refs. [38, 39].

### III. THE $X(5616)$ CANNOT BE AN $S$-WAVE $B \bar{K}$ MOLECULAR STATE

Since the quantum number $I(J^P)$ of the $X(5616)$ is $1(1^+)$ [1], the $S$-wave $B \bar{K}$ molecular state is possible assignment for the $X(5616)$. If we only consider the $S$-wave interaction between $B^*$ and $\bar{K}$ mesons, the obtained OBE effective potential is

$$\langle V_{\rho KK}^\pm (r) = -\frac{\beta g_v}{2} \left[ g_{\rho KK} Y(\Lambda, m_\rho, r) - g_{\omega KK} Y(\Lambda, m_\omega, r) \right],$$

which is the same as the expression in Eq. (5). The difference between $B \bar{K}$ and $B^* \bar{K}$ with $I = 1$ can be seen in the difference of their reduced masses. Although the total effective potential of an $S$-wave $B^* \bar{K}$ system with isospin $I = 1$ is attractive, we cannot find the corresponding bound-state solution.

When further considering the S-D mixing effect on the $B^* \bar{K}$ system since there exists mixing of the $B^* \bar{K}$ systems with spin-orbit wave functions $|3/2S\rangle$ and $|3/2D\rangle$, the effective potential in Eq. (9) should be modified as

$$\langle V_{\rho KK}^\pm (r) = -\frac{\beta g_v}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \left[ g_{\rho KK} Y(\Lambda, m_\rho, r) - g_{\omega KK} Y(\Lambda, m_\omega, r) \right],$$

which is a $2 \times 2$ matrix, where the matrix $\text{diag}(1, 1)$ is deduced from

$$\begin{pmatrix} \langle 3/2S \mid \epsilon_1 \cdot \epsilon_1 \rangle \langle 3/2S \rangle \\ \langle 3/2D \mid \epsilon_1 \cdot \epsilon_1 \rangle \langle 3/2S \rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Here, $\epsilon_1$ and $\epsilon_1$ correspond to the operators of the polarization vectors of the initial and finial $B^*$ meson, respectively.

To search for the bound-state solution, we solve the coupled-channel Schrödinger equation with Eq. (10). The bound-state solution is still absent when we scan the range $\Lambda = 1 \sim 5$ GeV in our numerical analysis.

In our calculation, we further consider the coupled-channel effect with the $B^* \pi$ and $B^* \bar{K}$ channels. However, the bound solutions cannot obtained.

Thus, our study does not support the $X(5616)$ as an isovector $S$-wave $B^* \bar{K}$ molecular state.

### IV. THE PREDICTION OF POSSIBLE $B^{(*)} \bar{K}^{(*)}$ MOLECULAR STATES

#### A. Isoscalar $B \bar{K}$ and $B^* \bar{K}$ systems

In the above sections, we discussed isovector $B \bar{K}$ and $B^* \bar{K}$ systems, which also stimulates our interest in further studying other $B^{(*)} \bar{K}^{(*)}$ systems. First, we focus on the isoscalar $B \bar{K}$ and $B^* \bar{K}$ systems. Their OBE effective potentials are

$$\langle V_{\rho KK}^{J=0} (r) = \frac{\beta g_v}{2} \left[ 3 g_{\rho KK} Y(\Lambda, m_\rho, r) + g_{\omega KK} Y(\Lambda, m_\omega, r) \right] \right),$$

$$\langle V_{\rho KK}^{J=0} (r) = \frac{\beta g_v}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \left[ 3 g_{\rho KK} Y(\Lambda, m_\rho, r) + g_{\omega KK} Y(\Lambda, m_\omega, r) \right] \right).$$

When comparing the OBE effective potentials of the isoscalar and isovector $B^{(*)} \bar{K}^{(*)}$ systems, we find that an isospin factor $-3$ is introduced in the $\rho$-exchange potentials for these isoscalar systems, while the isoscalar and isovector $B^{(*)} \bar{K}^{(*)}$ systems have the same $\omega$-exchange potential. The behaviors of the effective potentials of the isoscalar $B^{(*)} \bar{K}^{(*)}$ systems make it easier to form the isoscalar $B^{(*)} \bar{K}^{(*)}$ molecular states. By solving the Schrödinger equation, we confirm the above speculation, namely that we can find the bound-state solutions for the isoscalar $B^{(*)} \bar{K}^{(*)}$ systems. In Table I, we list the obtained binding energy, root-mean-square radius and the corresponding $\Lambda$ values. When taking $\Lambda = 1.9$ GeV, there exist shallow isoscalar $B^{(*)} \bar{K}^{(*)}$ molecular states. As the value of $\Lambda$ increases, the binding energies of these two systems become deeper. Here, the input of $\Lambda$ is not far away from 1 GeV, which come from studying the nuclear force [29, 30]. Thus, we may conclude that there probably exist isoscalar $B \bar{K}$ and $B^* \bar{K}$ molecular states, which have the quantum numbers $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$, respectively.

In fact, the above formula can be extended to the discussion of the $DK$ system with $(I = 0, J = 0)$ and the $D^* K$ system with $(I = 0, J = 1)$. Our calculation shows that the masses of the $D_{q0}(2317)$ and the $D_{s0}(2460)$ [40] can be reproduced when the cutoff $\Lambda$ is taken around 3.5 GeV, where the $D_{q0}(2317)$ and the $D_{s0}(2460)$ correspond to the $DK$ system with $(I = 0, J = 0)$ and the $D^* K$ system with $(I = 0, J = 1)$, respectively, since the reduced masses of the $BK$ and $B^* \bar{K}$ systems are heavier than those of the $DK$ and $D^* K$ systems, respectively. Thus, we can conclude that the cutoff $\Lambda$ for $BK/B^* \bar{K}$ should be smaller than that of $DK/D^* K$. The numerical results listed in Table I indeed can reflect this point.
TABLE I: The $\Lambda$ dependence of the obtained bound-state solutions (binding energy $E$ and root-mean-square radius $r_{\text{RMS}}$) for isoscalar $B\bar{K}$ systems. Here, $E$, $r_{\text{RMS}}$, and $\Lambda$ are in units of MeV, fm, and GeV, respectively.

| State | $\Lambda$ | $E$ | $r_{\text{RMS}}$ | State | $\Lambda$ | $E$ | $r_{\text{RMS}}$ |
|-------|-----------|-----|------------------|-------|-----------|-----|------------------|
| $[B\bar{K}]_{I=0}^{[1]}$ | 1.90 | -0.29 | 5.66 | $[B^*\bar{K}]_{I=0}^{[1]}$ | 1.90 | -0.30 | 5.64 |
| | 2.10 | -4.36 | 2.45 | | 2.10 | -4.40 | 2.44 |
| | 2.30 | -11.69 | 1.58 | | 2.30 | -11.76 | 1.57 |

If isoscalar $B\bar{K}$ and $B^*\bar{K}$ molecular states exist, finding them becomes a crucial task. For an isoscalar $B\bar{K}$ molecular state, its two-body and three-body Okubo-Zweig-lizuka-allowed decay channels are forbidden. Thus, experimental searches for this isoscalar $B\bar{K}$ are very difficult. For an isoscalar $B^*\bar{K}$ molecular state, we suggest an experiment to further analyze its $B,\pi\pi$ final state, by which this isoscalar $B^*\bar{K}$ molecular state can be discovered.

B. The $B\bar{K}^*$ and $B^*\bar{K}^*$ systems

Besides the systems discussed in Sec. II and IV A, in this work we also investigate the $B\bar{K}^*$ and $B^*\bar{K}^*$ systems. For the $B^*\bar{K}^*$ systems, there also exist $\pi$ and $\eta$ meson-exchange contributions to the effective potentials. In deducing the effective potentials, we need to adopt the following effective Lagrangians:

$$
\mathcal{L}_{\pi\bar{K}^*} = \frac{2g}{f_\pi} g_{\pi\bar{K}^* \bar{K}} \bar{\psi}_d \gamma^\mu \psi_u \partial_\mu \bar{K}^* \gamma_5, \\
\mathcal{L}_{\eta\bar{K}^*} = -g_{\eta\bar{K}^* \bar{K}} \bar{\psi}_d \gamma^\mu \gamma_5 \partial_\mu \bar{K}^* \gamma_5, \\
\mathcal{L}_{\eta^0\bar{K}^*} = g_{\eta^0\bar{K}^* \bar{K}} \bar{\psi}_d \gamma^\mu \gamma_5 \partial_\mu \bar{K}^* \gamma_5
$$

with

$$
\mathcal{P} = \begin{pmatrix}
\frac{g_{\eta^0\bar{K}^* \bar{K}}}{\sqrt{2}} & \frac{g_{\eta\bar{K}^* \bar{K}}}{\sqrt{2}} & \frac{g}{\sqrt{2}} \\
\frac{g_{\pi\bar{K}^* \bar{K}}}{\sqrt{2}} & \frac{g_{\eta\bar{K}^* \bar{K}}}{\sqrt{2}} & \frac{g_{\eta^0\bar{K}^* \bar{K}}}{\sqrt{2}} \\
\frac{g_{\pi\bar{K}^* \bar{K}}}{\sqrt{2}} & \frac{g_{\eta\bar{K}^* \bar{K}}}{\sqrt{2}} & \frac{g_{\eta^0\bar{K}^* \bar{K}}}{\sqrt{2}}
\end{pmatrix}.
$$

The obtained general expressions of the $B\bar{K}^*$ and $B^*\bar{K}^*$ systems when considering the S-D mixing effect read

$$
\mathcal{W}^J_{B\bar{K}^*}(\rho) = \frac{1}{2} g(I) \left[ g_{\pi\bar{K}^* \bar{K}} \left( \begin{array}{cc} 0 & 1 \\
0 & 0 \end{array} \right) \right] Y(\Lambda, m_\rho, \rho) \\
+ \frac{1}{2} g_{\eta^0\bar{K}^* \bar{K}} \left( \begin{array}{cc} 0 & 1 \\
0 & 0 \end{array} \right) Y(\Lambda, m_\eta, \rho), \\
\mathcal{W}^J_{B^*\bar{K}^*}(\rho) = \frac{1}{2} g(I) \left[ \begin{array}{c} E_1(J) \langle \frac{1}{2} & 0 \rangle \\
E_2(J) \langle \frac{1}{2} & 0 \rangle \end{array} \right] Y(\Lambda, m_\rho, \rho) \\
+ \frac{1}{2} g_{\eta^0\bar{K}^* \bar{K}} \left( \begin{array}{cc} 0 & 1 \\
0 & 0 \end{array} \right) Y(\Lambda, m_\eta, \rho),
$$

where the superscripts $I$ and $J$ denote the isospin and total angular momentum of these discussed systems. $g(I)$ is the isospin factor, which is taken as $-3$ for the isoscalar system, and 1 for the isovector system. The concrete forms of $E_1(J)$, $E_2(J)$, and $S(J)$ are $E_1(0) = \text{diag}(2,-1)$, $E_2(1) = \text{diag}(1,1,1)$, $E_2(2) = \text{diag}(1,1,1)$, $S(0) = \left( \begin{array}{cc} 0 & \sqrt{2} \\
-\sqrt{2} & 0 \end{array} \right)$, and $S(2) = \left( \begin{array}{cc} 0 & \sqrt{2} \\
-\sqrt{2} & 0 \end{array} \right)$.

With the above preparation, we try to search for the bound solutions by solving the Schrödinger equation. In Table II, the obtained results are collected. Among the discussed isovector $B\bar{K}^*$ and $B^*\bar{K}^*$ systems, only the $B^*\bar{K}^*$ system with $J = 0$ has a bound-state solution when $\Lambda$ is around 3 GeV, which is obviously different from 1 GeV [29, 30]. Thus, if strictly considering this criterion of the $\Lambda$ value, we conclude that there do not exist isovector $B^0\bar{K}^*$ molecular states. Different from the isovector case, the isoscalar $B^0\bar{K}^*$ systems may exist, as shown in Table II. In the following, we further discuss their allowed decay modes:

1. The $B\bar{K}^*$ molecular state with $(I = 0, J = 1)$ can decay into $B^*\bar{K}$, $B_\omega\bar{K}$, and $B_1\bar{K}$.

2. $B_1\bar{K}$ is an allowed decay mode of the $B^*\bar{K}^*$ molecular state with $(I = 0, J = 2)$.

3. The allowed decay channels of the $B^*\bar{K}^*$ molecular state with $(I = 0, J = 1)$ include $B^*\bar{K}$, $B\bar{K}^*$, $B_\omega\bar{K}^*$, and $B_1^*\omega$.

4. $B\bar{K}$, $B_1\eta$, and $B_1^*\omega$ are the allowed two-body decay channels for the $B^*\bar{K}^*$ state with $(I = 0, J = 0)$.
and $B$ (binding energy isoscalar sector. The relevant numerical results for the $D$ and $B^*K$ systems. Here, $E$, $r_{RMS}$, and $\Lambda$ are in units of MeV, fm, and GeV, respectively.

| State $[B^*K^+]_{J^P}$ | $\Lambda$ | $E$ | $r_{RMS}$ |
|------------------------|----------|-----|----------|
| $[B^*K^+]_{J^P=0}^{I^G=0}$ | 1.40 | -0.32 | 5.16 |
| $[B^*K^+]_{J^P=1}^{I^G=1}$ | 1.60 | -10.30 | 1.37 |
| $[B^*K^+]_{J^P=1}^{I^G=1}$ | 1.80 | -30.20 | 0.88 |
| $[B^*K^+]_{J^P=0}^{I^G=0}$ | 0.88 | -0.60 | 4.91 |
| $[B^*K^+]_{J^P=1}^{I^G=1}$ | 1.08 | -6.06 | 2.04 |
| $[B^*K^+]_{J^P=2}^{I^G=2}$ | 1.28 | -20.97 | 1.24 |

In our calculation, we also extend our study to the charm sector. The relevant numerical results for the $DK^*$ and $D^*K^*$ systems are collected in Table III.

| State $[D^*K^+]_{J^P}$ | $\Lambda$ | $E$ | $r_{RMS}$ |
|------------------------|----------|-----|----------|
| $[D^*K^+]_{J^P=0}^{I^G=0}$ | 1.60 | -0.90 | 4.18 |
| $[D^*K^+]_{J^P=1}^{I^G=1}$ | 1.80 | -9.30 | 1.56 |
| $[D^*K^+]_{J^P=2}^{I^G=2}$ | 2.00 | -23.87 | 1.05 |

These numerical results shown in Table III indicate that the isoscalar $DK^*$ and $D^*K^*$ states are very promising molecular candidates. Their decay behaviors are

$[D^*K^+]_{J^P=0}^{I^G=0} \rightarrow DK, D_s\eta, D_s^*\omega,$

$[D^*K^+]_{J^P=1}^{I^G=1} \rightarrow D^*K, DK^*, D_s\omega, D_s^*\omega,$

$[D^*K^+]_{J^P=2}^{I^G=2} \rightarrow D_s^*\omega.$

It is obvious that experimental searches for these predicted isoscalar $B^*\bar{K}$ and $D^*\bar{K}$ molecular states will be an intriguing issue. The above information is valuable to further study them experimentally.

V. SUMMARY

Stimulated by the recent evidence of a new enhancement structure $X(5568)$ or $X(5616)$ [1], we carried out a study of the interactions of isovector $B\bar{K}$ and $B^*\bar{K}$ systems via the OBE model. This dynamical study makes us exclude the $X(5568)$ or the $X(5616)$ as the isovector $B\bar{K}$ or $B^*\bar{K}$ molecular state. In Refs. [25, 26], the difficulty of assigning the X(5568) to be the $B\bar{K}$ molecular state was discussed. Obviously, we reach the same conclusion using different approaches.

In this work, we also studied isoscalar $B\bar{K}$ and $B^*\bar{K}$ systems; we predicted that there isoscalar $B\bar{K}$ and $B^*\bar{K}$ molecular states may exist, and their decay behaviors were discussed. In addition, we also focused on the $B^*\bar{K}$ systems. Our calculation illustrates that $B^*\bar{K}$ and $\bar{K}$ cannot form isovector molecular states, but they can be bound together to construct isoscalar $B^*\bar{K}$ molecular states. The allowed decay modes of these possible isoscalar $B^*\bar{K}$ molecular states show that it is possible to find them in experiments. Thus, we suggest future experimental exploration of these isoscalar open-bottom molecular states.

Acknowledgments

This project is supported by the National Natural Science Foundation of China under Grants No. 11225474 and No. 11175073 and the Fundamental Research Funds for the Central Universities. X. L. is also supported by the National Youth Top-notch Talent Support Program (Thousands-of-Talents Scheme).

Note added—When preparing the manuscript, we noticed the preliminary result from the LHCb experiment [43], where the signal of $X(5568)$ was not observed. In Ref. [43], the LHCb’s analysis also shows that the cone cut selection criterion can generate broad peaking structures. The DØ Collaboration performed an analysis of the $B_{\pi\eta}^0$ data with and without the cone cut, which indicates that there exists a structure with and without the cone cut. Here, the cone cut clearly enhances the resonance state as analyzed in Ref. [1]. According to our present study, we can deny the possibility of the $X(5568)$ or $X(5616)$ as an isoscalar $B\bar{K}$ or $B^*\bar{K}$ hadronic molecular state.
[1] V. M. Abazov et al. [D0 Collaboration], Observation of a new $B^0\pi^\pm$ state, Phys. Rev. Lett. 117, 022003(2016)

[2] X. Liu, An overview of XYZ new particles, Chin. Sci. Bull. 59, 3815 (2014)

[3] H. X. Chen, W. Chen, X. Liu and S. L. Zhu, The hidden-charm pentaquark and tetraquark states, Phys. Rept. 639, 1-121 (2016).

[4] W. Chen, H. X. Chen, X. Liu, T. G. Steele and S. L. Zhu, Decoding the $X(5568)$ as a fully open-flavor $sar{u}ar{d}$ tetraquark state, Phys. Rev. Lett. 117, 022002 (2016).

[5] S. S. Agaev, K. Azizi and H. Sundu, Mass and decay constant of the newly observed exotic $X(5568)$ state, Phys. Rev. D 93, 074024 (2016).

[6] W. Wang and R. Zhu, Can $X(5568)$ be a tetraquark state?, arXiv:1602.08806.

[7] C. M. Zanetti, M. Nielsen and K. P. Khemchandani, A QCD sum rule study for a charged bottom-strange scalar meson, Phys. Rev. D 93, 096011 (2016).

[8] Z. G. Wang, Analysis of the $X(5568)$ as scalar tetraquark state in the diquark-antidiquark model with QCD sum rules, arXiv:1602.08711.

[9] L. Tang and C. F. Qiao, $X(5568)$ as tetraquark state with Open flavors and its charmed partners, arXiv:1603.04761.

[10] S. S. Agaev, K. Azizi and H. Sundu, Width of the exotic $X_0(5568)$ state through its strong decay to $B^0\pi^\pm$, Phys. Rev. D 93, 114007 (2016).

[11] J. M. Dias, K. P. Khemchandani, A. M. Torres, M. Nielsen and C. M. Zanetti, A QCD sum rule calculation of the $X^+(5568) \rightarrow B^0\pi^+$ decay width, Phys. Lett. B 758, 235 (2016).

[12] Z. G. Wang, Analysis of the strong decay $X(5568) \rightarrow B^0\pi^+$ with QCD sum rules, Eur. Phys. J. C 76, 279 (2016).

[13] Y. R. Liu, X. Liu and S. L. Zhu, $X(5568)$ and its partner states, Phys. Rev. D 93, 074023 (2016).

[14] X. G. He and P. Ko, Flavour $SU(3)$ symmetry for the $X(5568)$ state, arXiv:1603.06230.

[15] F. Stancu, $X(5568)$ as a $sar{u}ar{d}$ tetraquark in a simple quark model, arXiv:1603.03325.

[16] Q. F. Lv and Y. B. Dong, Masses of open charm and bottom tetraquark states in relativized quark model, arXiv:1603.06417.

[17] A. Esposito, A. Pilloni and A. D. Polosa, Hybridized Tetraquarks, Phys. Lett. B 758, 292 (2016).

[18] C. J. Xiao and D. Y. Chen, Possible $B^0\gamma\bar{K}$ hadronic molecule state, arXiv:1603.00228.

[19] S. S. Agaev, K. Azizi and H. Sundu, Exploring $X(5568)$ as a meson molecule, arXiv:1603.02708.

[20] F. K. Guo, P. N. Shen, H. C. Chiang, R. G. Ping and B. S. Zou, Dynamically generated $0^+$ heavy mesons in a heavy chiral unitary approach, Phys. Lett. B 641, 278 (2006).

[21] F. K. Guo, P. N. Shen and H. C. Chiang, Dynamically generated $1^+$ heavy mesons, Phys. Lett. B 647, 133 (2007).

[22] M. Cleven, F. K. Guo, C. Hanhart and U. G. Meissner, Light meson mass dependence of the positive parity heavy-strange mesons, Eur. Phys. J. A 47, 19 (2011).

[23] X. H. Liu and G. Li, Could the observation of $X(5568)$ be resulted by the near threshold rescattering effects?, arXiv:1603.00708.

[24] Y. Jin and S. Y. Li, Large production rate of new $B^0\pi^\pm$ and $D^+_s\pi^+$ states in high energy multi-production process, arXiv:1603.03250 [hep-ph].

[25] T. J. Burns and E. S. Swanson, Interpreting the $X(5568)$ challenges our understanding of QCD, Commun. Theor. Phys. 65, 593 (2016).

[26] M. Albaladejo, J. Nieves, E. Oset, Z. F. Sun and X. Liu, Can $X(5568)$ be described as a $B_s\pi, B\bar{K}$ resonant state?, Phys. Lett. B 757, 515 (2016).

[27] X. W. Kang and J. A. Oller, $P$-wave coupled-channel scattering of $B_s\pi, B^0\pi, B^0\bar{K}, B^0\bar{K}$ and the puzzling $X(5568)$, arXiv:1606.06665.

[28] N. A. Tornqvist, From the deuteron to deusons, an analysis of deuteron-like meson meson bound states, Z. Phys. C 61, 525 (1994).

[29] N. A. Tornqvist, On deusons or deuteron-like meson meson bound states, Nuovo Cim. A 107, 2471 (1994).

[30] N. Li and S. L. Zhu, Isospin breaking, Coupled-channel effects and Diagnosis of $X(3872)$, Phys. Rev. D 86, 074022 (2012).

[31] R. Chen, Z. F. Sun, X. Liu and S. M. Gerasyuta, Predicting exotic molecular states composed of nucleon and $P$-wave charged meson, Phys. Rev. D 90, 034011 (2014).

[32] R. Chen, X. Liu and S. L. Zhu, Hidden-charm molecular pentaquarks and their charm-strange partners, Nucl. Phys. A (to be published), arXiv:1601.03232.

[33] G. J. Ding, Are $Y(4260)$ and $Z_{45}(4250)$ are $D_1D$ or $D_2D^*$ hadronic molecules?, Phys. Rev. D 79, 014001 (2009) citeSun:2011uh

[34] Z. F. Sun, J. He, X. Liu, Z. G. Luo and S. L. Zhu, $Z_0(10610)^{\pm}$ and $Z_0(10650)^{\pm}$ as the $B\bar{B}$ and $B^0\bar{B}^*$ molecular states, Phys. Rev. D 84, 054002 (2011).

[35] Z. W. Lin and C. M. Ko, A model for $J/\psi$ absorption in hadronic matter, Phys. Rev. C 62, 034903 (2000).

[36] D. Y. Chen, X. Liu and T. Matsuki, Two charged strangeonium-like structures observable in the $Y(2175) \rightarrow \phi(1020)\pi^+\pi^-$ process, Eur. Phys. J. C 72, 2008 (2012).

[37] Y. J. Zhang, H. C. Chiang, P. N. Shen and B. S. Zou, Possible $S$-wave bound-states of two pseudoscalar mesons, Phys. Rev. D 74, 014013 (2006).

[38] Y. R. Liu, X. Liu and S. L. Zhu, Light Pseudoscalar Meson and Heavy Meson Scattering Lengths, Phys. Rev. D 79, 094026 (2009).

[39] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).

[40] C. Isola, M. Ladisa, G. Nardulli, and P. Santorelli, Charming penguins in $B \rightarrow K^\pm\pi, K(\rho,\omega,\phi)$ decays, Phys. Rev. D 68, 114001 (2003)

[41] O. Kaymakcalan, S. Rajeev, and J. Schechter, Nonabelian anomaly and vector meson decays, Phys. Rev. D 30, 594 (1984).

[42] LHCb Collaboration, in 51st Rencontres de Moriond, La Thuile, Italy, 18 March-1 April, 2016 (to be published), http://lhcproject.web.cern.ch/lhcproject/Publications/LHCbProjectPublic/LHCb-CONF-2016-004.html.