Matter induced charge symmetry violating NN potential

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We construct density dependent Class III charge symmetry violating (CSV) potential due to mixing of $\rho$-$\omega$ meson with off-shell corrections. Here in addition to the usual vacuum contribution, the matter induced mixing of $\rho$-$\omega$ is also included. It is observed that the contribution of density dependent CSV potential is comparable to that of the vacuum contribution.

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I. INTRODUCTION

The exploration of symmetries and their breaking have always been an active and interesting area of research in nuclear physics. One of the well known examples, that can be cited here, is the nuclear $\beta$ decay which violates parity that led to the discovery of the weak interaction. Our present concern, however, is the strong interaction where, in particular, we focus attention to the charge symmetry violation (CSV) in nucleon-nucleon (NN) interaction.

Charge symmetry implies invariance of the NN interaction under rotation in isospin space, which in nature, is violated. The CSV, at the fundamental level is caused by the finite mass difference between up ($u$) and down ($d$) quarks \[ \frac{m_u - m_d}{2} \]. As a consequence, at the hadronic level, charge symmetry (CS) is violated due to non-degenerate mass of hadrons of the same isospin multiplet. The general goal of the research in this area is to find small but observable effects of CSV which might provide significant insight into the strong interaction dynamics.

There are several experimental data which indicate CSV in NN interaction. For instance, the difference between $pp$ and $nn$ scattering lengths at $^1S_0$ state is non-zero \[ \frac{m_u - m_d}{2} \]. Other convincing evidence of CSV comes from the binding energy difference of mirror nuclei which is known as Okamoto-Nolen-Schifer (ONS) anomaly \[ \frac{m_u - m_d}{2} \]. The modern manifestation of CSV includes difference of neutron-proton form factors, hadronic correction to $g-2$ \[ \frac{m_u - m_d}{2} \], the observation of the decay of $\Psi'(3686) \rightarrow (J/\Psi)\eta^{\prime}$ etc \[ \frac{m_u - m_d}{2} \].

In nuclear physics, one constructs CSV potential to see its consequences on various observables. The construction of CSV potential involves evaluation of the $NN$ scattering diagrams with intermediate states that include mixing of various isospin states like $\rho$-$\omega$ or $\pi$-$\eta$ mesons. The former is found to be most dominant \[ \frac{m_u - m_d}{2} \]. This success has been called into question \[ \frac{m_u - m_d}{2} \] on the ground of the use of on-shell mixing amplitude for the construction of CSV potential. First in \[ \frac{m_u - m_d}{2} \] and then in \[ \frac{m_u - m_d}{2} \], it is shown that the $\rho$-$\omega$ mixing has strong momentum dependence which even changes its sign as one moves away from the $\rho$ (or $\omega$) pole to the space-like region which is relevant for the construction of the CSV potential. Therefore inclusion of off-shell corrections are necessary for the calculation of CSV potential. We here deal with such mixing amplitude induced by the $N$-$N$ loop incorporating off-shell corrections.

In vacuum, the charge symmetry is broken explicitly due to the non-degenerate nucleon masses. In matter, there can be another source of symmetry breaking if the ground state contains unequal number of neutrons ($n$) and protons ($p$) giving rise to ground state induced mixing of various charged states like $\rho$-$\omega$ meson even in the limit $M_n = M_p$. This additional source of symmetry breaking for the construction of CSV potential has, to the best of our knowledge, not been considered before.

The possibility of such matter induced mixing was first studied in \[ \frac{m_u - m_d}{2} \] and was subsequently studied in \[ \frac{m_u - m_d}{2} \]. For the case of $\pi$-$\eta$ meson also such asymmetry driven mixing is studied in \[ \frac{m_u - m_d}{2} \]. But none of these deal with the construction of two-body potential and the calculations are mostly confined to the time-like region where the main motivation is to investigate the role of such matter induced mixing on the dilepton spectrum observed in heavy ion collisions, pion form factor, meson dispersion relations etc. \[ \frac{m_u - m_d}{2} \]. In Ref.\[ \frac{m_u - m_d}{2} \], attempt has been made to calculate the density dependent CSV potential where only the effect of the scalar mean field potential on the nucleon mass is considered excluding the possibility of matter driven mixing. All existing matter induced mixing calculations, however, suggest that, at least in the $\rho$-$\omega$ sector, the inclusion of such a matter induced mixing amplitude into the two body $NN$ interaction potential can significantly change the results both qualitatively and quantitatively. It is also to be noted that such mixing amplitudes, in asymmetric nuclear matter (ANM), have non-zero contribution even if the quark or nucleon masses are taken to be equal \[ \frac{m_u - m_d}{2} \]. We consider both of these mechanisms to construct the CSV potential.

Physically, in dense system, intermediate mesons might be absorbed and re-emitted from the Fermi spheres. In symmetric nuclear matter (SNM) the emission and absorption involving different isospin states like...
ρ and ω cancel when the contributions of both the proton and neutron Fermi spheres are added provided the nucleon masses are taken to be equal. In ANM, on the other hand, the unbalanced contributions coming from the scattering of neutron and proton Fermi spheres, lead to the mixing which depends both on the density (ρB) and the asymmetry parameter [α = (ρn - ρp)/ρB]. Inclusion of this process is depicted by the second diagram in Fig. 1 represented by VNN med which is non-zero even in symmetric nuclear matter if explicit mass differences of nucleons are retained. In the first diagram, VNN vac involves NN loop denoted by the circle. The other important element which we include here is the contribution coming from the external legs. This is another source of explicit symmetry violation which significantly modify the CSV potential in vacuum as has been shown only recently by the present authors. 

This paper is organized as follows. In Sec.II we present the formalism where the three momentum dependent ρω-ω mixing amplitude is calculated to construct the CSV potential in matter. The numerical results are discussed in Sec.III. Finally, we summarize in Sec.IV.

II. FORMALISM

We start with the following effective Lagrangians to describe ωNN and ρNN interactions:

\[
\mathcal{L}_{\omega NN} = g_\omega \bar{\Psi}_\mu \Phi^\mu \Psi, \tag{1a}
\]

\[
\mathcal{L}_{\rho NN} = g_\rho \bar{\Psi} \left[ \gamma_\mu + \frac{C_\rho}{2M} \sigma_{\mu\nu} \partial^\nu \right] \vec{\tau} \cdot \Phi^\mu \Psi, \tag{1b}
\]

where \( C_\rho = f_\rho/g_\rho \) is the ratio of vector to tensor coupling, \( M \) is the average nucleon mass and \( \vec{\tau} \) is the isospin operator. \( \Psi \) and \( \Phi \) represent the nucleon and meson fields, respectively, and \( g \)'s stand for the meson-nucleon coupling constants. The tensor coupling of \( \omega \) is not included in the present calculation as it is negligible compared to the vector coupling.

The matrix element, which is required for the construction of CSV NN potential is obtained from the relevant Feynman diagram:

\[
\mathcal{M}_{\rho\omega}^{NN}(q) = \left[ \bar{u}_N(p_3) \Gamma_\rho^\mu(q) u_N(p_1) \right] \Delta_{\rho\omega}^\mu(q) \left[ \bar{u}_N(p_4) \Gamma_\omega^\nu(-q) u_N(p_2) \right]. \tag{2}
\]

In the limit \( q_0 \to 0 \), Eq. (2) gives the momentum space CSV NN potential, \( V_{\omega NN}^{CSV}(q) \). Here \( \Gamma_\rho^\mu(q) = g_\omega \gamma^\mu \), \( \Gamma_\omega^\nu(q) = g_\rho \left[ \gamma^\nu - C_\rho \frac{i}{2\sqrt{2}} q^\nu q_0 \right] \) denote the vertex factors, \( u_N \) is the Dirac spinor and \( \Delta_{\rho\omega}^{\mu\nu} \) (\( i = \rho, \omega \)) is the meson propagator. \( p_j \) and \( q \) are the four momenta of nucleon and meson, respectively.

In the present calculation, ρ-ω mixing amplitude (i.e. polarization tensor) \( \Pi_{\rho\omega}^{\mu\nu}(q^2) \) is generated by the difference between proton and neutron loop contributions:

\[
\Pi_{\rho\omega}^{\mu\nu}(q^2) = \Pi_{\rho\omega}^{\mu\nu}(p^2) - \Pi_{\rho\omega}^{\mu\nu}(n^2). \tag{3}
\]

Explicitly, the polarization tensor is given by

\[
i\Pi_{\rho\omega}^{\mu\nu}(q^2) = \int \frac{d^d k}{(2\pi)^d} \text{Tr} \left[ \frac{\bar{\Psi} \gamma^\mu G_N(k) \gamma^\nu \bar{\Psi}}{\rho_\omega(k + q)} \right], \tag{4}
\]

where \( k = (k_0, \mathbf{k}) \) denotes the fourth momentum of the nucleon in the loops (see Fig. 1 and \( G_N \) is the in-medium nucleon propagator consisting of free \( G_N^0 \) and density dependent \( G_N^D \) parts). 

\[
G_N^0(k) = \frac{k^0 + M_N}{k^2 - M_N^2 + i\epsilon}, \tag{5a}
\]

\[
G_N^D(k) = \frac{i\pi}{E_N}(k^0 + M_N)\delta(k_0 - E_N)\theta(k_0 - |\mathbf{k}|) \tag{5b}
\]

The subscript \( N \) stands for nucleon index (i.e. \( N = p \) or \( n \), \( k_N \) denotes the Fermi momentum of nucleon, nucleon energy \( E_N = \sqrt{M_N^2 + k_N^2} \) and nucleon mass is denoted by \( M_N \). \( \theta(k_0 - |\mathbf{k}|) \) is the Fermi distribution function at zero temperature.

The origin of \( G_N^D(k) \), in addition to the free propagator resides in the fact that one deals with vacuum containing real particles which when acted upon the annihilation operator does not vanish (see Appendix A for details). The appearance of the delta function in Eq. (5b) indicates the nucleons are on-shell while \( \theta(k_0 - |\mathbf{k}|) \) ensures that propagating nucleons have momentum less than \( k_N \).

Likewise, the polarization tensor of Eq. (4) also contains a vacuum \( [\Pi_{\rho\omega}^{\mu\nu}(q^2)] \) and a density dependent \( [\Pi_{\rho\omega}^{\mu\nu}(q^2)] \) parts as shown in Fig. 1. It is to be noted that the density dependent part given by the combination of \( G_N^0 G_N^D + G_N^D G_N^0 \) corresponds to scattering that we have discussed already, whereas the term proportional to \( G_N^0 G_N^D \) vanishes for low energy excitation. The vacuum part, \( \Pi_{\rho\omega}^{\mu\nu}(q^2) \) on the other hand involves \( G_N^0 G_N^D \) which gives rise to usual CSV part of the potential due to the splitting of the neutron and proton mass.
It might be worthwhile to mention here that Eq. 5(b) can induce charge symmetry breaking in asymmetric nuclear matter due to the appearance of the Fermi distribution function in the propagator itself which can distinguish between neutron and proton, even if their mass are taken to be degenerate. Evidently, this is an exclusive medium driven effect where, as mentioned already in the introduction, the charge symmetry is broken by the ground state. The total charge symmetry breaking would involve both the contributions where, it is clear that even for $\alpha = 0$, the medium dependent term can contribute if the non-degenerate nucleon masses are considered.

Note that the polarization tensor $\Pi_{\nu\sigma}^{\mu}(q^2)$ can be expressed as the sum of longitudinal component $[\Pi_{\nu\sigma}^{L}(q^2)]$ and transverse component $[\Pi_{\nu\sigma}^{T}(q^2)]$ which will be useful to simplify the matrix element given in Eq.(2).

$$\Pi_{\nu\sigma}^{\mu}(q^2) = \Pi_{\nu\sigma}^{L}(q^2)A^{\mu\nu} + \Pi_{\nu\sigma}^{T}(q^2)B^{\mu\nu}, \quad (6)$$

where $A^{\mu\nu}$ and $B^{\mu\nu}$ are the longitudinal and transverse projection operators [36]. We define $\Pi_{\nu\sigma}^{L} = -\Pi_{\rho\sigma}^{00} + \Pi_{\rho\sigma}^{33}$ and $\Pi_{\nu\sigma}^{T} = \Pi_{\rho\sigma}^{11} = \Pi_{\rho\sigma}^{22}$.

We, in the present calculation, use the average of longitudinal and transverse components of the polarization tensor instead of $\Pi_{\nu\sigma}^{L}$ and $\Pi_{\nu\sigma}^{T}$. The average mixing amplitude is denoted by

$$\Pi(q^2) = \frac{1}{3} \left[ \Pi_{\nu\sigma}^{L}(q^2) + 2\Pi_{\nu\sigma}^{T}(q^2) \right] = \Pi_{\text{vac}}(q^2) + \Pi_{\text{med}}(q^2). \quad (7)$$

In the last line of Eq. 7, $\Pi_{\text{vac}}(q^2)$ and $\Pi_{\text{med}}(q^2)$ denote the average mixing amplitudes of vacuum and density dependent parts, respectively.

To obtain $\Pi_{\text{vac}}(q^2)$ and $\Pi_{\text{med}}(q^2)$ one would calculate the total polarization tensor given in Eq.(4). After evaluating the trace of Eq. [3], we find the following vacuum and density dependent parts of the polarization tensor.

$$\Pi_{\text{vac}}^{\mu\nu}(N)(q^2) = Q^{\mu\nu} \left[ \Pi_{\text{vac}}^{\nu\nu}(q^2) + \Pi_{\text{vac}}^{\text{tt}}(q^2) \right], \quad (8)$$

and

$$\Pi_{\text{med}}^{\mu\nu}(N)(q^2) = 16g_{\rho\sigma}g_{\omega} \int \frac{d^3k}{(2\pi)^3 2E_N} \theta(k_N - |k|) \times \left[ \frac{q^2 K^{\mu\nu} - (q \cdot k)^2 Q^{\mu\nu}}{q^4 - 4(q \cdot k)^2} \right], \quad (9)$$

$$\Pi_{\text{med}}^{\mu\nu}(N)(q^2) = 4g_{\rho\sigma}g_{\omega} C_{\rho} \int \frac{d^3k}{(2\pi)^3 2E_N} \theta(k_N - |k|) \times \left[ \frac{q^4 Q^{\mu\nu}}{q^4 - 4(q \cdot k)^2} \right], \quad (10)$$

where $Q^{\mu\nu} = \left(-q^{\mu\nu} + q^{\mu}q^{\nu}/q^2\right)$ and $K^{\mu\nu} = \left(k^{\mu} - (q \cdot k)q^{\nu}/q^2\right)$.

It is to be mentioned that both $\Pi_{\text{vac}}^{\mu\nu}(q^2)$ and $\Pi_{\text{med}}^{\mu\nu}(q^2)$ obey the current conservation as $q_{\mu}Q^{\mu\nu} = q_{\mu}Q^{\nu\mu} = 0$ and $q_{\mu}K^{\mu\nu} = q_{\mu}K^{\nu\mu} = 0$. The superscripts $vv$ and $tv$ in Eqs.(8) - (10) indicate the vector-vector and tensor-vector interactions, respectively. The dimensional counting shows that vacuum part of the polarization tensor $[\Pi_{\text{vac}}^{\nu\nu}(N)(q^2)]$ is ultraviolet divergent and dimensional regularization [37, 38, 39] is used to isolate the divergent parts. Since the mixing amplitude is generated by the difference between the proton and neutron loop contributions, the divergent parts cancel out yielding the vacuum amplitude finite.

$$\Pi_{\text{vac}}^{\mu\nu}(q^2) = \Pi_{\text{vac}}^{\mu\nu} - \Pi_{\text{vac}}^{\nu\mu}(q^2)$$

$$= \frac{g_{\rho\omega}g_{\omega}}{2\pi^2} q^{\mu\nu} \int_0^1 dx \left[ (1-x)x + C_{\rho} \right] \times \ln \left( \frac{M_p^2 - x(1-x)q^2}{M_n^2 - x(1-x)q^2} \right). \quad (11)$$

Eq.(11) shows the four-momentum dependent vacuum polarization tensor. From the above equation one can calculate longitudinal ($\Pi_{\text{vac}}^{L}(N)$) and transverse ($\Pi_{\text{vac}}^{T}(N)$) components of the vacuum mixing amplitude and in the limit $q_0 \to 0$, $\Pi_{\text{vac}}^{L}(q^2) = \Pi_{\text{vac}}^{T}(q^2)$. Therefore, the average vacuum mixing amplitude is

$$\Pi_{\text{vac}}(q^2) = \frac{1}{3} \left[ \Pi_{\text{vac}}^{L}(q^2) + 2\Pi_{\text{vac}}^{T}(q^2) \right]$$

$$= -\frac{g_{\rho\omega}g_{\omega}}{12\pi^2} (2 + 3C_{\rho}) \ln \left( \frac{M_p}{M_n} \right) q^2$$

$$= -4q^2. \quad (12)$$

Eq.(12) represents the three momentum dependent vacuum mixing amplitude. This mixing amplitude vanishes for $M_n = M_p$ and then no CSV potential in vacuum will exist.

To calculate density dependent mixing amplitude from Eq.(3) and (10) we consider $E_N \approx M_N$. In the limit $q_0 \to 0$, one finds following expressions:
\[ \Pi_{med}^{100(N)}(q^2) = - \frac{g_\rho g_\omega}{4\pi^2 M_N} \left[ \left\{ \frac{4}{3} k_N^3 - \frac{1}{2} k_N q^2 + 2 k_N M_N^2 \right\} \right. \\
- \left. \left\{ \frac{q^3}{8} - \frac{q k_N}{2} + \frac{q M_N^2}{2} + 2 M_N^2 k_N \right\} \right\} \right] \\
\times \ln \left( \frac{q - 2 k_N}{q + 2 k_N} \right) + \left\{ \frac{C_\rho}{2} \{ q^2 k_N \} \right\} \\
+ \left\{ \frac{q^3}{4} - q k_N \right\} \ln \left( \frac{q - 2 k_N}{q + 2 k_N} \right) \right) \right) . \tag{13} \]

\[ \Pi_{med}^{111(N)}(q^2) = \frac{g_\rho g_\omega}{4\pi^2 M_N} \left[ \left\{ \frac{1}{3} k_N^3 - \frac{3}{8} q^2 k_N \right\} \right. \\
- \left. \left\{ \frac{3}{32} q^3 + \frac{k^4}{2q} + \frac{q^2 k_N}{4} \right\} \right\} \right] \\
\times \ln \left( \frac{q - 2 k_N}{q + 2 k_N} \right) + \left\{ \frac{C_\rho}{2} \{ q^2 k_N \} \right\} \\
+ \left\{ \frac{q^3}{4} - q k_N \right\} \ln \left( \frac{q - 2 k_N}{q + 2 k_N} \right) \right) \right) . \tag{14} \]

To construct CSV potential we take non-relativistic (NR) limit of the Dirac spinors in which case we obtain

\[ u_N(p) \approx \left( 1 - \frac{\mathbf{P}^2}{8M_N^2} - \frac{q^2}{32M_N^2} \right) \left( \frac{1}{2M_N} \right), \tag{21} \]

where \( \sigma \) is the spin of nucleon. \( \mathbf{P} \) denotes the average three momentum of the interacting nucleon pair and \( \mathbf{q} \) stands for the three momentum of the meson.

The explicit expression of full CSV potential in momentum space can be obtained from Eq.(13) of ref.\[32\] by replacing the mixing amplitude \( \Pi_{med}(q) \) with \( \Pi(q^2) \).

\[ V_{CSV}^{NN}(q) = - \frac{g_\rho g_\omega \Pi(q^2)}{(q^2 + m^2_N)(q^2 + m^2_\sigma)} \times \left[ T_3^+ \left\{ \left( 1 + \frac{3\mathbf{P}^2}{2M_N^2} - \frac{q^2}{4M_N} - \frac{q^2}{4M_N} \right)(\sigma_1 \cdot \sigma_2) \right\} \\
+ \frac{3i}{2M_N} \mathbf{S} \cdot (q \times \mathbf{P}) + \frac{1}{4M_N} (\sigma_1 \cdot q)(\sigma_2 \cdot q) \right\} \\
+ \frac{1}{M_N} \left( (\overline{q} \cdot \mathbf{P})^2 - \frac{C_\rho}{2M} \left( \frac{q^2}{2M_N} + \frac{q^2}{2M_N} \right)(\sigma_1 \cdot \sigma_2) \right) \\
- \frac{2i}{M_N} \mathbf{S} \cdot (q \times \mathbf{P}) - \frac{1}{2M_N} (\sigma_1 \cdot q)(\sigma_2 \cdot q) \right\} \\
- \frac{1}{2M_N} (\sigma_1 \cdot q)(\sigma_2 \cdot q) \left( \Delta M(1,2) - \frac{1}{M} \right) \left( \frac{1}{M} \right) \right) \right) \right) . \tag{22} \]

Eq.\[22\] presents the full CSV NN potential in momentum space in matter. Here \( T_3^+ = \tau_3(1) \pm \tau_3(2) \) and \( \mathbf{S} = \frac{1}{2}(\sigma_1 + \sigma_2) \) is the total spin of the interacting nucleon pair. We define \( M = (M_N + M_p)/2 \), \( \Delta M = (M_N - M_p)/2 \). For \( \Delta M(1,2) = -\Delta M(2,1) = \Delta M \). It is be mentioned that the spin dependent parts of the potential appear because of the contribution of the external nucleon legs shown in Fig.\[1\]. On the other hand, \( 3\mathbf{P}^2/2M_N^2 \) and \(-\mathbf{q}^2/8M_N^2 \) arise due to expansion of the relativistic energy \( E_N \) of the spinors.

In matter, \( V_{CSV}^{NN}(q) \), consists of two parts, one contains the vacuum mixing amplitude and other contains the density dependent mixing amplitude. The former is denoted by \( V_{vac}^{NN}(q) \) and later by \( V_{med}^{NN}(q) \). Thus, \( V_{CSV}^{NN}(q) = V_{vac}^{NN}(q) + V_{med}^{NN}(q) \).

From Eq.\[22\] we extract the following term which, in coordinate space gives rise to the \( \delta \)-function potential.

\[ \delta V_{CSV}^{NN} = \frac{g_\rho g_\omega }{8M_N^2} \left( A + A' \right) \times \left[ \left( 1 + 2C_\rho \right) \frac{8M_N^2}{8M_N^2} + \left( 1 + C_\rho \right) \frac{8M_N^2}{4M_N^2} \right] \left( \sigma_1 \cdot \sigma_2 \right) \right) . \tag{23} \]

To avoid the appearance of \( \delta \)-function potential in coordinate space, one should introduce form factors.
Here we use the following form factor

$$F_i(q^2) = \left( \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 + q^2} \right),$$

(24)

where $\Lambda_i$ is the cut-off parameter governing the range of the suppression and $m_i$ denotes the mass of exchanged meson.

The full CSV potential presented in Eq. (22) contains both Class III and Class IV potentials, and both break the charge symmetry in $NN$ interactions. The terms within the first and the second curly braces represent Class (III) and Class IV potentials, respectively. Class (III) potential differentiates between $nn$ and $pp$ systems while Class (IV) $NN$ potential exists in the $np$ system only. We, in this paper, restrict ourselves to Class (III) potential only.

The coordinate space potential can be easily obtained by Fourier transformation of $V_{CSV}^{NN}(q)$ i.e.

$$V_{CSV}^{NN}(r) = \int \frac{d^3q}{(2\pi)^3} V_{CSV}^{NN}(q) e^{-iq\cdot r}$$

(25)

We drop the term $3P^2/2M_N^2$ from Eq. (22) while taking the Fourier transform as it is not important in the present context. However, this term becomes important to fit $^1S_0$ and $^3P_2$ phase shifts simultaneously.

Now the CSV potential in coordinate space without $\delta V_{CSV}^{NN}$ reduces to

$$V_{vac}^{NN}(r) = -\frac{g_{\rho g_\omega}^2}{4\pi} A T_3^+ \left[ \left( \frac{m_\rho^3 Y_0(x_\rho) - m_\omega^3 Y_0(x_\omega)}{m_\omega^2 - m_\rho^2} \right) \right]$$

$$+ \left( \frac{m_\rho^5 V_{\rho\rho}(x_\rho) - m_\omega^5 V_{\rho\omega}(x_\omega)}{m_\rho^2 - m_\omega^2} \right)$$

$$+ \left( \frac{m_\rho^5 V_{\omega\rho}(x_\rho) - m_\omega^5 V_{\omega\omega}(x_\omega)}{m_\rho^2 - m_\omega^2} \right),$$

(26)

and

$$V_{med}^{NN}(r) = -\frac{g_{\rho g_\omega}^2}{4\pi} A T_3^+ \left[ \left( \frac{m_\rho^3 Y_0(x_\rho) - m_\omega^3 Y_0(x_\omega)}{m_\omega^2 - m_\rho^2} \right) \right]$$

$$+ \frac{1}{M_N^2} \left( \frac{m_\rho^5 V_{\rho\rho}(x_\rho) - m_\omega^5 V_{\rho\omega}(x_\omega)}{m_\rho^2 - m_\omega^2} \right)$$

$$+ \frac{C_\rho}{2M_N^2} \left( \frac{m_\rho^5 V_{\omega\rho}(x_\rho) - m_\omega^5 V_{\omega\omega}(x_\omega)}{m_\rho^2 - m_\omega^2} \right)$$

(27)

where $x_i = m_i r$. The explicit expressions of $V_{\rho\omega}(x_i)$ and $V_{\omega\rho}(x_i)$ are given in Ref. [33]. In Eqs. (26) and (27), the $M_N^{-2}$ independent terms represent central parts without contributions of external nucleon legs.

The potentials presented in Eqs. (26) and (27) do not include the form factors so that these potentials diverge near the core. The problem of divergence near the core can be removed by incorporating form factors as discussed before. With the inclusion of form factors, $V_{vac}^{NN}(r)$ and $V_{med}^{NN}(r)$ take the following form:

$$V_{vac}^{NN}(r) = -\frac{g_{\rho g_\omega}^2}{4\pi} A T_3^+ \left[ \left( \frac{a_\rho^3 m^3 Y_0(x_\rho) - a_\omega m^3 Y_0(x_\omega)}{m_\omega^2 - m_\rho^2} \right) \right]$$

$$+ \frac{1}{M_N^2} \left( \frac{a_\rho^5 m^5 V_{\rho\rho}(x_\rho) - a_\omega m^5 V_{\rho\omega}(x_\omega)}{m_\rho^2 - m_\omega^2} \right)$$

$$+ \frac{C_\rho}{2M_N^2} \left( \frac{a_\rho^5 m^5 V_{\omega\rho}(x_\rho) - a_\omega m^5 V_{\omega\omega}(x_\omega)}{m_\rho^2 - m_\omega^2} \right)$$

$$+ \frac{b_\rho A_\rho^3 Y_0(X_\rho) - b_\omega A_\omega^3 Y_0(X_\omega)}{m_\omega^2 - m_\rho^2}$$

$$+ \frac{1}{M_N^2} \left( \frac{b_\rho A_\rho^5 V_{\rho\rho}(X_\rho) - b_\omega A_\omega^5 V_{\rho\omega}(X_\omega)}{m_\rho^2 - m_\omega^2} \right)$$

$$+ \frac{C_\rho}{2M_N^2} \left( \frac{b_\rho A_\rho^5 V_{\omega\rho}(X_\rho) - b_\omega A_\omega^5 V_{\omega\omega}(X_\omega)}{m_\rho^2 - m_\omega^2} \right)$$

(28)
and

\[
V_{med}^N(r) = -\frac{g_{\omega N} T_3}{4\pi} \left[ \delta V \left( \frac{a_p m_p Y_0(x_p) - a_s m_s Y_0(x_s)}{m_s^2 - m_p^2} \right) + \frac{1}{M_N^2} \left( \frac{a_p m_p^3 V_{\rho \rho}(x_p) - a_s m_s^3 V_{\rho \rho}(x_s)}{m_s^2 - m_p^2} \right) + \frac{C_\rho}{2M_N^2} \left( \frac{a_p m_p^3 V_{\omega \omega}(x_p) - a_s m_s^3 V_{\omega \omega}(x_s)}{m_s^2 - m_p^2} \right) \right] + A \left( \frac{a_p m_p^3 Y_0(x_p) - a_s m_s^3 Y_0(x_s)}{m_s^2 - m_p^2} \right) \]

In Eqs. (28) and (29), \( X = 0 \) and \( \rho = 1/3 \). The dashed and dotted curves show \( \Delta V_{vac} \) and \( \Delta V_{med} \) respectively. The total contribution (i.e., \( \Delta V_{vac} + \Delta V_{med} \)) is shown by the solid curve. It is observed that the density dependent CSV potential can not be neglected while estimating CSV observables such as binding energy difference of mirror nuclei.

\[
\lambda = \left( \frac{m_s^2 - m_p^2}{A_s^2 - A_p^2} \right), \quad a_i = \left( \frac{A_j^2 - m_s^2}{A_j^2 - m_p^2} \right), \quad b_i = \left( \frac{A_j^2 - m_s^2}{A_j^2 - m_p^2} \right), \quad (i \neq j).
\]

where \( i, j = p, \omega \). Note that Eqs. (28) and (29) contain the contribution of \( \Delta V_{CSV} \) and the problem of divergence near the core is removed.

### III. RESULTS

Using Eqs. (26) and (27) we show the difference of CSV potentials between \( nn \) and \( pp \) systems i.e., \( \Delta V = V_{CSV}^{nn} - V_{CSV}^{pp} \) in Fig. 2 for \(^1S_0\) state at nuclear matter density \( \rho_B = 0.148 \text{ fm}^{-3} \) with asymmetry parameter \( \omega = 1/3 \). The dashed and dotted curves show \( \Delta V_{vac} \) and \( \Delta V_{med} \), respectively. The total contribution (i.e., \( \Delta V_{vac} + \Delta V_{med} \)) is shown by the solid curve. It is observed that the density dependent CSV potential can not be neglected while estimating CSV observables such as binding energy difference of mirror nuclei.

![Graph showing \( \Delta V \) for \(^1S_0\) state including the Fourier transform of \( \Delta V_{CSV} \) and form factors.](image)

**Fig. 2:** \( \Delta V \) for \(^1S_0\) state (without \( \Delta V_{CSV} \) and form factors) at \( \rho_B = 0.148 \text{ fm}^{-3} \) and \( \omega = 1/3 \).

\[
\Delta V_{vac} + \Delta V_{med}
\]

\[
\Delta V_{vac}
\]

\[
\Delta V_{med}
\]

**Fig. 3:** \( \Delta V \) for (with form factors including the Fourier transform of \( \Delta V_{CSV} \) \(^1S_0\) state at \( \rho_B = 0.148 \text{ fm}^{-3} \) and \( \omega = 1/3 \).

Note that Eqs. (28) and (29) contain the contribution of \( \Delta V_{CSV} \) and the problem of divergence near the core is removed.

### IV. SUMMARY AND DISCUSSION

In this work we have constructed the CSV \( NN \) potential in dense matter using asymmetry driven momen-
tum dependent $\rho^{3d}\omega$ mixing amplitude within the framework of one-boson exchange model. Furthermore, the correction to the central part of the CSV potential due to external nucleon legs are also considered. The closed-form analytic expressions both for vacuum and density dependent CSV NN potentials in coordinate space are presented.

We have shown that the vacuum mixing amplitude and the density dependent mixing amplitude are of similar order of magnitude and both contribute with the same sign to the CSV potential. The contribution of density dependent CSV potential is not negligible in comparison to the vacuum CSV potential.

\section*{APPENDIX A}

The position space Fermion propagator in vacuum is given by the vacuum expectation value of the time ordered product of Fermion fields.

\[ i\tilde{G}_N(x-x') = \langle 0|T[\bar{\psi}(x)\psi(x')]|0\rangle. \]

In medium, the vacuum $|0\rangle$ is replaced by the ground state $|\Psi_0\rangle$ which contains positive-energy particles with same Fermi momentum $k_N$ and no antiparticles. Thus, the time-ordered product in Eq. (A2) involves negative sign for Fermions. The Fermion field contains both the positive- and negative-energy solutions. The modal expansion for the Fermion fields are,

\[ \psi(x) = \int \frac{d^3k}{\sqrt{(2\pi)^32E_k}} \sum_{s} (a_{ks}u_{ks}e^{-ik\cdot x} + b_{ks}^\dagger v_{ks}e^{ik\cdot x}), \]

\[ \bar{\psi}(x) = \int \frac{d^3k}{\sqrt{(2\pi)^32E_k}} \sum_{s} (a_{ks}^\dagger \bar{u}_{ks}e^{ik\cdot x} + b_{ks}u_{ks}e^{-ik\cdot x}). \]

Here $a_{ks}^\dagger$ and $a_{ks}$ are the creation and annihilation operators for particles and likewise $b_{ks}^\dagger$ and $b_{ks}$ are the creation and annihilation operators for antiparticles. The only nonvanishing anticommutation relations are

\[ \{a_{ks}, a_{ks'}^\dagger\} = \{b_{ks}, b_{ks'}^\dagger\} = \delta_{ss'}\delta^3(k-k'). \]

Since $|\Psi_0\rangle$ contains only positive-energy particles, we have the following relations:

\[ b_{ks}|\Psi_0\rangle = 0 \quad \text{for all } k \]

\[ a_{ks}|\Psi_0\rangle = 0 \quad \text{for } |k| > k_N \]

\[ a_{ks}^\dagger|\Psi_0\rangle = 0 \quad \text{for } |k| < k_N \]

\[ a_{ks}a_{ks}^\dagger|\Psi_0\rangle = n(k)|\Psi_0\rangle \]

$n(k)$ is either 0 or 1 and this can be accomplished with the step function $\theta(k_N - |k|)$. Using Eqs. (A3)-(A6) one obtains

\[ \langle \Psi_0|\bar{\psi}(x')\psi(x')|\Psi_0\rangle = \int \frac{d^3k}{\sqrt{(2\pi)^32E_k}} \int \frac{d^3k'}{\sqrt{(2\pi)^32E_{k'}}} \times \sum_{ss'} \left( \langle a_{ks}u_{ks}|\Psi_0\rangle u_{ks'}v_{ks'}^\dagger \right) e^{-i(k-k'-x')} \]

\[ = \int \frac{d^3k}{(2\pi)^32E_k} (k + M_N)e^{-i(k-x')} \theta(k_N - |k|), \]

and

\[ \langle \Psi_0|\bar{\psi}(x')\psi(x')|\Psi_0\rangle = \int \frac{d^3k}{\sqrt{(2\pi)^32E_k}} \int \frac{d^3k'}{\sqrt{(2\pi)^32E_{k'}}} \times \sum_{ss'} \left[ \langle a_{ks}^\dagger u_{ks}|\Psi_0\rangle \bar{u}_{ks'}v_{ks'}^\dagger \right] e^{i(k-k'+x')} \]

\[ + \langle \Psi_0|b_{ks}^\dagger b_{ks}|\Psi_0\rangle \bar{u}_{ks'}v_{ks'}^\dagger e^{i(k-k'+x')} \]

\[ = \int \frac{d^3k}{(2\pi)^32E_k} [(k + M_N)e^{-i(k-x')} \theta(k_N - |k|) \]

\[ + (k - M_N)e^{i(k-x')}]. \]

Now,

\[ \theta(t-t')e^{-i(k-x')} = i \int \frac{d k_0}{2\pi} \frac{e^{-i(k-x')}}{k_0 - E_k + i\epsilon}. \]

\[ \theta(t-t')e^{-i(k-x')} = -i \int \frac{d k_0}{2\pi} \frac{e^{-i(k-x')}}{k_0 - E_k - i\epsilon}. \]

\[ \theta(t-t')e^{i(k-x')} = i \int \frac{d k_0}{2\pi} \frac{e^{i(k-x')}}{k_0 - E_k + i\epsilon}. \]

From Eqs. (A8), (A10) and (A11) we have

\[ \langle \Psi_0|\bar{\psi}(x')\psi(x')|\Psi_0\rangle \theta(t-t') \]

\[ = -i \int \frac{d^4k}{(2\pi)^32E_k} e^{-i(k-x')} (k + M_N) \frac{\theta(k_N - |k|)}{k_0 - E_k - i\epsilon} \]

\[ + i \int \frac{d^4k}{(2\pi)^32E_k} (k - M_N) \frac{e^{i(k-x')}}{k_0 - E_k + i\epsilon}. \]

Now changing $k \to -k$ in the last integral of Eq. (A12) and substituting Eqs. (A4), (A9) and (A12) in Eq. (A2) we get,
\[ i\tilde{G}_N(x - x') = i\int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot (x - x')} (\tilde{G} + M_N) \]
\times \left[ \frac{1 - \theta(k_N - |k|)}{k_0 - E_N + i\epsilon} + \frac{\theta(k_N - |k|)}{k_0 - E_N - i\epsilon} - \frac{1}{k_0 + E_N - i\epsilon} \right]. \quad (A13)

In Eq. (A13), \( E_k \) has been replaced by \( E_N \). The first term of Eq. (A13) represents particle propagation above the Fermi sea and the second term indicates the propagation of holes inside the Fermi sea. The last term shows the propagation of holes in the infinite Dirac sea. Now,

\[ \frac{1}{k_0 - E_N + i\epsilon} - \frac{1}{k_0 + E_N - i\epsilon} = \frac{2E_N}{k^2 - M_N^2 + i\epsilon}, \quad (A14) \]

\[ \frac{1}{k_0 - E_N - i\epsilon} - \frac{1}{k_0 - E_N + i\epsilon} = 2i\pi\delta(k_0 - E_N). \quad (A15) \]

From Eqs. (A13)-(A15),

\[ i\tilde{G}_N(x - x') = i\int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot (x - x')} G_N(k) \quad (A16) \]

where \( G_N(k) \) is the in-medium Fermion propagator in momentum space.

\[ G_N(k) = \frac{\tilde{G} + M_N}{k^2 - M_N^2 + i\epsilon} + \frac{i\pi}{E_N} (\tilde{G} + M_N) \delta(k_0 - E_N)\theta(k_N - |k|) = G_N^P(k) + G_N^D(k). \quad (A17) \]