On the first derivative probe method for electron energy
distribution function measurements in tokamak edge plasma

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Abstract. The applicability of the first derivative Langmuir probe method in strongly
magnetized tokamak plasmas is discussed. The method is used for processing the electron part
of the current-voltage (IV) probe characteristics measured in the CASTOR tokamak edge
plasma (Institute of Plasma Physics, Association EURATOM-IPP, Prague, Czech Republic).
The comparison of the results obtained with the results given by the Stangeby method yields a
satisfactory agreement.

1. Introduction
Langmuir probes have been widely used in the study of the Electron Energy Distribution Function
(EEDF), $f_\varepsilon$, beginning with Druyvesteyn’s work [1]. The measurements have been mainly carried
out in unmagnetized, low-pressure (< 100 Pa) plasma, when

$$\lambda \gg R + r_s.$$

Here $\lambda$ is the free path of the electrons, $R$ and $r_s$ are the probe radius and the radius of the probe
sheath. The recently developed kinetic theory in a non-local approach extend the applicability of the
probe method yielding an extended Druyvesteyn formula at intermediate gas pressures (100 – 1000
Pa) [2], or the first derivative electron probe method at high gas pressures (>1000 Pa) [3].

In this work we will discuss the applicability of the first derivative Langmuir probe method to
measurements in strongly magnetized tokamak plasma. The method is used for processing the electron
part of the current-voltage (IV) probe characteristics measured in CASTOR tokamak edge plasma
(Institute of Plasma Physics, Association EURATOM-IPP, Prague, Czech Republic). Results of the
first measurements of the EEDF will be presented and discussed.

2. First derivative Langmuir probe method for tokamak plasma
A kinetic theory in a non-local approach for processing the electron probe current in the presence of
collisions in the probe sheath was published recently [4] and widely discussed in [5]. The theory was
developed in the case when the linear size of the region disturbed by the probe $L_p$ is smaller than the
electron energy relaxation length $\lambda_e$:

$$\lambda_e \approx 2 \left( \frac{D}{\nu_{ee} + \delta \nu_{ee} + \nu'} \right)^{1/2},$$  \hspace{1cm} (1)

where the diffusion coefficient is $D = \nu \lambda(\varepsilon)/3$, $\lambda(\varepsilon)$ is the free path of the electrons and $\nu_{ee}, \nu', \nu''$ are the frequencies of electron-electron, electron-heavy particles elastic and inelastic collisions. $\delta = 2m/M$, where $m$ and $M$ are the electron mass and the heavy particles mass, respectively.

It was shown that the density of the electron probe current for a cylindrical probe with radius $R$ and length $L$ at negative probe potentials is

$$j_e(U) = -C \int K(W,U)f(W)dW,$$  \hspace{1cm} (2)

where $C = \frac{8\pi e}{3m^2\gamma}$, $K(W,U) = \frac{(W - eU)}{1 + (W - eU)/\psi(W)}$, $e$ and $n$ are the electron charge and density, $U$ is the probe potential with respect to the plasma potential $U_{pl}$. The geometric factor $\gamma$ assumes values in the range $4/3 \leq \gamma \leq 0.71$ [5]; $f(W)$ is the isotropic electron distribution function, normalized by

$$\frac{4\pi \sqrt{2}}{m^{3/2}} \int f(W)\sqrt{W}dW = \int f(\varepsilon)\sqrt{\varepsilon}d\varepsilon = n.$$  

The diffusion parameter $\psi(W)$ for a cylindrical probe is

$$\psi(W) = \frac{1}{\gamma \lambda(W)} \int_{r/R}^{\infty} \frac{D(W)dr}{(r/R)D(W - e\phi(r))}.$$  \hspace{1cm} (3)

In the presence of a magnetic field $B$, the diffusion coefficient $D$ is a tensor with two components [5]:

$$D_\parallel = D \quad \text{and} \quad D_\perp = D / \rho,$$  \hspace{1cm} (4)

where $\rho = \left[1 + \frac{\lambda(W)^2}{R(W,B)}\right]^{-1/2}$, $R_L(W,B)$ - Larmor radius.

Thus the value of the diffusion parameter will depend on the probe orientation with respect to the magnetic field lines. To calculate the diffusion parameter one has to take also into account that, although the local diffusion near the probe is classical [6], the global transport is anomalous (Bohm diffusion) due to the turbulence in tokamak plasma [7]. Then the diffusion parameter for a cylindrical probe oriented perpendicularly to the magnetic field is

$$\psi(W) = \frac{R \ln L_P}{16\gamma R_L}.$$  \hspace{1cm} (5)

For small tokamaks the typical magnetic field $B$ is in the range of 1-5 T and the diffusion parameter is much larger than 1 ($\psi >> 1$).

The first derivative of the electron probe current density is

$$j_e'(U) = -\text{const} \int K'(W,U)f(W)dW,$$  \hspace{1cm} (6)

where $K'(W,U) = \frac{W^2}{[W(1+\psi) - \psi eU]^2}$.
One can present $K'(W,U)$ as a sum of two terms $K_1'(W,U)$ and $K_2'(W,U)$:

$$K'(W,U) = \frac{\psi eUe}{W} \left[ \frac{eU + (1 + \psi)(W - eU)}{1 + \psi} \right]^2 + \frac{\psi eUe}{W} \left[ \frac{\psi eU + (1 + \psi)(W - eU)}{1 + \psi} \right]^2. \quad (7)$$

It can be shown that the first term

$$K_1'(W,U) = \frac{\psi eUW}{(1 + \psi)(W - eU)^2} = \frac{\psi eU}{1 + \psi} \cdot \frac{1}{W} \frac{eU}{eU + (1 + \psi)(W - eU)} = \frac{\psi eU}{1 + \psi} \cdot \frac{1}{W} \left[ \frac{eU}{W + (1 + \psi)(1 - eU/W)} \right]^2 = \frac{\psi eU}{1 + \psi} \cdot \frac{1}{W} \left[ 1 + \frac{eU}{1 - eU/W} \right]^2. \quad (8)$$

Then at $\psi >> 1$

$$j_{cl}(U) = -\text{const} \int_{eU}^{\psi} K_1'(W,U)f(W)dW = -\text{const} \int_{eU}^{\psi} \frac{eU}{W} \frac{f(W)dW}{1 + \psi \left( 1 - \frac{eU}{W} \right)^2}. \quad (8)$$

If one substitutes $t = eU/W$, then $W = \frac{eU}{t}$; $dW = -\frac{eU}{t^2}dt$ and the first part of the integral (6) becomes

$$j_{cl}(U) \approx -\text{const} \int_0^{\psi} \frac{eU}{\psi} \frac{1}{t \left[ \psi (1 - t) \right]} f(eU/t)dt = -\text{const} \int_0^{\psi} \frac{eU}{\psi} \frac{1}{P(t,\psi) f(eU/t)}dt. \quad (9)$$

**Figure 1.** Plot of the function $P(t,\psi)$ at different values of diffusion parameter $\psi$. 
We should point out that our substitution for $t$ differs from the substitution in [5]. The plot of the function $P(t, \psi)$ at different $\psi$ is presented in figure 1. It is seen that at $\psi \gg 1$ the function $P(t, \psi)$ may be considered as $P(t, \psi) \rightarrow \delta(1-t)$ within the interval $[0,1]$. Then

$$j_{e1}(U) = -\text{const} \frac{eU}{\psi} \int_0^1 \delta(1-t)f(eU/t)dt = -\text{const} \frac{eU}{\psi} f(eU).$$

(10)

The second part of the integral (6) is

$$j_{e2}(U) = -\text{const} \int_0^\infty \frac{Wf(W)dW}{eU(1+\psi)^2[1+\psi^2W-\psi eU]}.$$

(11)

It can be presented as

$$j_{e2}(U) = -\text{const} \int_0^\infty \frac{Wf(W)dW}{eU(1+\psi)^2[1+\psi^2W-\psi eU]}.$$

(12)

This equation differ from that obtained in [5] at constant $\psi$ for increased gas pressures.

As a result, for the first derivative of the electron probe current density, taking into account $\psi \gg 1$, one obtains

$$j_e(U) = -\text{const} \left[ \frac{eU}{\psi} f(eU) + \int \frac{Wf(W)}{eU^2(1+\psi)^2W-\psi eU}dW \right].$$

(13)

It can be seen that the contribution of the integral in this equation at large values of $\psi$ is negligible. Then for EEDF one arrives at

$$f(eU) = -\text{const} \frac{\psi}{eU} j_e(U).$$

(14)

3. Langmuir probe measurements in the CASTOR tokamak edge plasma

The IV characteristics measurements in the CASTOR tokamak edge plasma were carried out by using an array of 16 single Langmuir probes, the Rake probe being oriented perpendicularly to the magnetic field lines [7]. Each cylindrical probe tip has a length of $L=2$ mm and a radius of $R=0.35$ mm. The probes are spaced by 2.5 mm in the radial direction. All probe tips are biased simultaneously by a triangular voltage $U(t)$ with respect to the tokamak chamber, which serves as a reference electrode. The time necessary to measure a single IV characteristic is typically ~ 1 ms.

Figure 2 shows an example of the EEDF in the middle of the current pulse of shot #26402 for probe radial position $r = 86$ mm from the centre of the tokamak poloidal cross section (14 mm distance from the tokamak chamber wall).

The black curve is EEDF in absolute scale obtained using equation (14) from the experimental data:

$$f(e) = \frac{3\sqrt{2mR}}{32e^2SRU} \ln \left( \frac{4L}{R} \right) \frac{dI(U)}{dU}.$$

(15)

The blue curve is the best fit using a Maxwellian energy distribution obtained with an accuracy of 5%. The discrepancy in the energy interval 0-8 eV is a shortcoming of the method. It is clearly seen,
however, that the result derived by means of the first derivative probe method describe well the experimental result.

Figure 2. EEDF from shot #26402 at probe position 14 mm from the tokamak chamber wall. Experimental EEDF – black, best fit with Maxwellian EEDF – blue.

Figure 3. EEDF from shot #26402 at probe position 44 mm from the tokamak chamber wall. Experimental EEDF – black; the best fit by a bi-Maxwellian EEDF – green as the sum of blue and red curves.

The results presented in figure 2 are obtained by a probe moved to the limiter shadow. Figure 3 presents the EEDF in magnetic confined plasma (probe position at 44 mm distance from the tokamak chamber wall). Here the energy distribution may be approximated successfully by a bi-Maxwellian function. The total electron density acquired is \( n = 2.9 \times 10^{18} \text{ m}^{-3} \). The results for the electron temperature and electron density are in good agreement with those obtained by Stangeby method \[8\] - \( n = 3.3 \times 10^{18} \text{ m}^{-3} \) and electron temperature \( T = 37 \text{ eV} \). We must point out that the Stangeby method assumes a Maxwellian EDF of the electrons so that the temperature of the high-energy electron fraction of the real EEDF can only be evaluated.

4. Conclusion
The applicability is discussed of the first derivative Langmuir probe method to measurements in strongly magnetized tokamak plasmas.

The method is used for processing the electron part of the current-voltage (IV) probe characteristics measured in the CASTOR tokamak edge plasma (Institute of Plasma Physics, Association EURATOM-IPP, Prague, Czech Republic). The results of the first measurements of the EEDF are presented and discussed.

The comparison of the results obtained with the results given by the Stangeby method yields a satisfactory agreement.

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