Analysis of equivalent thickness of geological media for lab-scale study of radon exhalation

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Abstract. Geological media are widely distributed in nature. Lab-scale tests are frequently employed in radon studies for these media. Thus, it is critical to find the thickness of the medium at an experimental scale that is equivalent to the medium thickness in a real geological system. Based on the diffusion–advection transport of radon, theoretical models of the surface radon exhalation rate for homogeneous semi-infinite and finite-thickness systems were derived (denoted as $J_{se}$ and $J_{fi}$, respectively). Analysis of the equivalency of $J_{se}$ and $J_{fi}$ was subsequently carried out by introducing several dimensionless parameters, including the ratio of the exhalation rates for the semi-infinite and finite-thickness models, $\varepsilon$, and the number of diffusion lengths required to achieve a desired $\varepsilon$ value, $n$. The results showed that when radon transport in geological media is dominantly driven by diffusion, if $n > 3.6626$, then $\varepsilon > 95\%$ (and if $n > 5.9790$, then $\varepsilon > 99.5\%$). When radon migration is dominantly driven by advection, if $n > 2.5002$, then $\varepsilon > 95\%$ (and if $n > 4.0152$, then $\varepsilon > 99.5\%$). Therefore, if the thickness of the geological media ($x_0$) is greater than a certain $n$ times the radon diffusion length of the media ($L$), the media can be modeled as semi-infinite. To validate the model, a pure radon diffusion experiment (no advection) was developed using uranium mill tailings, laterite, and radium-bearing rocklike material with different thicknesses ($x_0$). The theoretical model was demonstrated to be reliable and valid. This study provides a basis for determining the appropriate thickness of geological media in lab-scale experiments of radon exhalation.

Keywords: geological media; radon exhalation; lab-scale study; diffusion–advection mechanism; equivalent thickness
1. Introduction

\(^{222}\)Rn (hereafter referred to as radon) is a major contributor to ionizing radiation, corresponding to about half of the total radiation exposure received by the human population. It is the second most frequent cause of lung cancer (3–14% of all lung cancer is attributable to radon) after smoking (Baskaran 2016). \(^{226}\)Ra (hereafter referred to as radium) is considered a radon source and is intrinsically present in geological media, such as the ground (e.g., soil, sand, or bedrock), uranium ore stockpiles, and mill tailings piles, in various amounts (Girault et al. 2010). Such geological media are called radium-bearing porous media.

Radon transport in geological media includes two modes: diffusion and advection. The diffusive transport of radon is driven by a concentration gradient, as governed by Fick’s law. The advective transport of radon is driven by pressure-induced flow or convection, as governed by Darcy’s law (Fleischer 1997; Tanner 1980). To understand the radon diffusion–advection transport mechanism, lab-scale studies of radon release from geological media have been extensively carried out. The most widely studied subjects are the radon profiles in various media for geophysical purposes, indoor radon concentrations, radon exhalation from uranium ore stockpiles and uranium mill tailings (UMTs) piles, and the radioactive impacts of radon on the surrounding environment. For instance, van der Graaf et al. (1992, 1994) and van der Spoel et al. (1997, 1998a, 1998b, 1999) studied the diffusive and advective radon transport in soil and sand using laboratory experiments with a stainless-steel vessel (height and diameter of 2 m). Jong and van Dijk (2005) tested radon-reducing measures by constructing a laboratory-scale house (dimensions of about 3.5 m × 2 m) with a living area (height 2.6 m) and a crawlspace beneath it (height 1 m) made from phosphogypsum blocks (\(^{222}\)Rn source strength of approximately 800 Bq·h\(^{-1}\)), where the two spaces were separated by a concrete floor (thickness 15 cm). Catalano et al. (2015a, 2015b) carried out a laboratory study of radon transport
through porous soil in a large cylindrical stainless-steel vessel (diameter 0.5 m, height 1.25 m). The height was on the order of the diffusion length of radon in dry soil (1–2 m). Wang et al. (2014) developed a columnar experimental device for exploring radon seepage migration and exhalation in uranium ore piles. The total height of this device was 2.76 m, and it was mainly composed of five identical cylinders filled with sample material, each with a height of 0.5 m and an internal diameter 0.1 m. Ye et al. (2016) analyzed the effects of the water level on the radon exhalation from fragmented uranium ore using a self-made apparatus with an ore loading barrel (diameter of 0.1 m, height of 0.5 m) and a water loading barrel (height of 0.70 m). Ferry et al. (2002) conducted an experimental study of the effect of a cover layer on the transient radon flux from UMTs. This study was conducted using a small artificial pond with dimensions of 14 m × 15 m and total height of 2.1 m (namely, a draining layer of 0.3 m, UMTs layer of 0.8 m, and cover layer of 1 m).

In addition to experimental studies, various theoretical models have been proposed for the study of radon transport. The key difference between these approaches is that geological media is modeled using semi-infinite models, whereas uranium ore stockpiles and mill tailings piles are modeled using finite-thickness models with thicknesses of several meters or even greater than 20 meters. Jönsson (1997) presented a model for radon transport in a homogeneous, semi-infinite medium. Yakovleva (2005) extended this model to examine the transport and equilibrium concentration of radon as well as its flux from the Earth’s surface. Catalano et al. (2015a) presented a numerical model to examine the radon transport through porous media and compared the model with experimental results. Yakovleva and Parovik (2010) solved the diffusion–advection equations to examine radon transport in geological media with multiple layers using an integro-interpolation approach. Hafez and Awad (2016) used a numerical model to examine the transport of radon from deep soil to the Earth’s surface. Several authors have examined the diffusion of radon through building materials using numerical models (Muhammad et al. 2020; Sabbarese et al. 2020; Szajerski and
The abovementioned radon studies of geological media based on lab-scale experiments should achieved significant conclusions. The equipment adopted for these studies was relatively well-designed and well-constructed, and the experimental parameters could be set and strictly controlled in the laboratory. However, the bases for selecting the dimensions (especially the height or thickness) of the sample materials were vaguely described or not specified. In general, due to the vertical migration of radon in geological media, the influence of a lab-scale sample’s thickness on the comparability and reliability of the experimental results cannot be ignored. A sample material with a certain thickness at the lab-scale would be representative of a semi-infinite or finite-thickness medium in an actual environment in terms of radon exhalation. Although numerous models have been presented, a theoretical analysis of the sample thickness at the lab scale and the full-scale equivalent thickness has not been performed. Thus, analysis of the equivalent thickness of geological media for the lab-scale study of radon transport is presented in this paper.

In Section 2, the theoretical equations for the advection–diffusion transport of radon through semi-infinite and finite-thickness models are presented, and the conditions under which the surface fluxes of radon predicted by the models are equivalent are derived. The experiments used to validate the models are presented in Section 3. The results are presented in Section 4, and the paper is concluded in Section 5.

2. Derivation of theoretical formulas

For simplicity, geological media are typically modeled using with two types of homogeneous porous media models: (1) homogeneous semi-infinite models and (2) homogeneous finite-thickness models. In a semi-infinite model, the medium has only a single boundary surface, and
the opposite boundary is considered to be infinitely far away. In a finite-thickness model, the medium has two boundary surfaces separated by a finite distance. For a homogeneous porous medium, the values of the density, porosity, tortuosity, and radium content are constant. Therefore, the emanation coefficient, diffusion coefficient, and permeability coefficient of radon are determinable if the temperature and moisture content of the medium are known. In this case, the free radon production rate of geological media can be expressed as follows (IAEA 2013):

\[ A = \lambda C_{Ra} \rho f, \]  

where \( A \) is the free radon production rate (Bq·m\(^{-3}\)·s\(^{-1}\)), \( \lambda \) is the radon decay constant (s\(^{-1}\)), \( C_{Ra} \) is the radium content (Bq·kg\(^{-1}\)), \( f \) is the radon emanation coefficient, and \( \rho \) is the density of the medium (kg·m\(^{-3}\)). The density and other parameters (namely the radon concentration, radon diffusion coefficient, and radon seepage velocity) used in the subsequent analysis are the bulk values.

It is assumed that the radium, radon and its progenies reach a state of radioactive equilibrium. The flux of radon is assumed to be steady and one-dimensional exhalation from inside the geological medium to the exposed surface. The radon concentration in the medium is a time-invariant function of the thickness from the exposed surface, and radon advection occurs with a stable seepage velocity and direction. In this study, we consider the case in which the direction of radon advection is opposite from its diffusion direction. In a Cartesian coordinate system (Fig. 1), the radon flux from the surface of the homogeneous semi-infinite or finite-thickness model can be treated as a one-dimensional migration process. Based on the theory of the radon diffusion–advection transport mechanism, the radon migration process for the semi-infinite and finite-thickness models can be described as follows (Antonopoulos-Domis et al. 2009; Rogers and Nielson 1991; Tanner 1964):

\[ D \frac{d^2C}{dx^2} - v \frac{dC}{dx} - \lambda C + A = 0, \]  

where \( D \) is the diffusion coefficient, \( v \) is the seepage velocity, and \( A \) is the radon production rate.

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where \( x \) is the depth from exposed surface to inside the medium (m), \( C \) is radon concentration at the depth \( x \) (Bq \cdot m^{-3}), \( D \) is radon diffusion coefficient in the medium (m\(^2\) \cdot s\(^{-1}\)), and \( v \) is the radon seepage velocity (m\( \cdot \)s\(^{-1}\)). Substituting Eq. (1) into Eq. (2) yields the following:

\[
D \frac{d^2 C}{dx^2} - v \frac{dC}{dx} - \lambda C + \lambda C_{Ra} \rho f = 0.
\]  

(3)

For the convenience of deriving theoretical formulas of the radon exhalation rate, it is assumed that the studied geological medium is placed in open environment with a very low background radon concentration. Moreover, in a state of radioactive equilibrium, the pore radon concentrations inside the geological medium are much higher than the radon concentration of the surrounding environment close to the exposed surface of the medium. Therefore, the environmental radon concentration is set to zero.

2.1. Homogeneous semi-infinite model

The radon migration and exhalation processes from a homogeneous semi-infinite medium are depicted in Fig. 1. The upper surface is the only free surface for radon exhalation. The boundary conditions are as follows: (1) \( x = 0, \ C = 0; \ x \rightarrow \infty, \ C \) is bounded. The solution of Eq. (3) is

\[
C = C_{Ra} \rho f - C_{Ra} \rho f \cdot \exp \left\{ -\frac{\sqrt{v^2 + 4\lambda D} - v}{2D} x \right\}.
\]  

(4)

The surface radon exhalation rate (at \( x = 0, \ J_{se} \) Bq \( \cdot \) m\(^{-2}\) \( \cdot \) s\(^{-1}\)) can be expressed as follows:

\[
J_{se} = \left( D \frac{dC}{dx} - vC \right) |_{x=0}.
\]  

(5)

Substitution of Eq. (4) into Eq. (5) yields the following:
2.2. Homogeneous finite-thickness model

The radon migration and exhalation processes from a homogeneous finite-thickness medium are depicted in Fig. 2. Radon exhalation occurs at two free surfaces, namely the upper and lower surfaces. Therefore, the boundary conditions are as follows: $x = 0, C = 0$; $x = x_0, C = 0$. The solution of Eq. (3) is

$$
C = \alpha \exp(\zeta_1 x) + \beta \exp(-\zeta_2 x) + C_{Ra} \rho f,
$$

where

$$
\zeta_1 = \frac{\sqrt{v^2 + 4\lambda D} + v}{2D}, \text{ m}^{-1},
$$

$$
\zeta_2 = \frac{\sqrt{v^2 + 4\lambda D} - v}{2D}, \text{ m}^{-1},
$$

$$
\alpha = \frac{C_{Ra} \rho f \cdot [1 - \exp(-\zeta_2 x_0)]}{\exp(-\zeta_2 x_0) - \exp(\zeta_1 x_0)}, \text{ Bq/m}^3,
$$

$$
\beta = -\frac{C_{Ra} \rho f \cdot [1 - \exp(\zeta_1 x_0)]}{\exp(-\zeta_2 x_0) - \exp(\zeta_1 x_0)}, \text{ Bq/m}^3.
$$

The surface radon exhalation rate (at $x = 0, J_{in}, \text{ Bq} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$) can also be expressed by Eq. (5). Substituting Eq. (7) into Eq. (5) yields the following expression for the surface radon exhalation:

$$
J_{in} = \frac{1}{2} C_{Ra} \rho f \left( \sqrt{v^2 + 4\lambda D} \cdot \frac{\exp(\zeta_1 x_0) + \exp(-\zeta_2 x_0) - 2}{\exp(\zeta_1 x_0) - \exp(-\zeta_2 x_0)} - v \right).
$$
2.3 Derivation of equivalency conditions

In this section, the conditions for which the surface exhalation of radon from the semi-infinite and finite-thickness models are equivalent are derived. The surface exhalation expressions given by Eq. (6) for the semi-infinite model and Eq. (8) for the finite-thickness model are highly similar, differing only by the coefficient of the $\sqrt{v^2 + 4\lambda D}$ term. In both models, the direction of radon advection is opposite to the diffusion direction. We introduce two dimensionless parameters, $\psi$ and $\zeta$, which are defined as follows:

$$\psi = \frac{\exp(\zeta_1 x_0) + \exp(-\zeta_2 x_0) - 2}{\exp(\zeta_1 x_0) - \exp(-\zeta_2 x_0)},$$  \hspace{1cm} (9)$$

$$\zeta = \sqrt{\frac{\zeta_1}{\zeta_2}} = \sqrt{1 + \frac{\left(1 + \sqrt{1 + 4\lambda D} \right)}{2\lambda D}}. \hspace{1cm} (10)$$

As $\zeta_1$ and $\zeta_2$ are greater than zero, and $\zeta_1 \geq \zeta_2$, we can conclude the following: (1) $\exp(\zeta_1 x_0) > 1$, and $0 < \exp(-\zeta_2 x_0) < 1$; (2) $\exp(\zeta_1 x_0) + \exp(-\zeta_2 x_0) - 2 \geq \exp(\zeta_1 x_0) + \exp(-\zeta_1 x_0) - 2 \geq 0$. Thus, $0 < \psi < 1$. In addition, $\zeta$ is not less than 1. We define a dimensionless parameter ($m$) as the ratio of the radon seepage velocity to the radon diffusion velocity in the geological medium. Based on Eq. (10), $m$ can be expressed as follows:

$$m = \frac{v}{\sqrt{\lambda D}} = \frac{1}{\zeta}. \hspace{1cm} (11)$$

Using Eq. (11), we can determine the following criteria:
Furthermore, 

\[ \zeta_1 \cdot \zeta_2 = \frac{\lambda}{D} = \frac{1}{L}, \]

where \( L \) is the radon diffusion length in the geological medium (m).

We assume that

\[ x_0 = n \cdot L, \]

where \( n \) is a real number greater than zero. Specifically, it is the number of radon diffusion lengths in the geological medium.

By substituting Eqs. (10), (13), and (14) into Eq. (9), we arrive at the following:

\[ \psi = \frac{\exp(n\zeta) + \exp(-n/\zeta) - 2}{\exp(n\zeta) - \exp(-n/\zeta)}. \]

Finally, to facilitate the equivalency analysis, \( \varepsilon \) is defined as the ratio of \( J_0 \) to \( J_{\infty} \). Based on the equations above, we determined that

\[ \varepsilon = J_0 / J_{\infty} = \left[ (\zeta^2 + 1)\psi - (\zeta^2 - 1) \right] / 2 \]

and \( 0 < \varepsilon < 1 \).

3. Experiments
An experimental setup was developed, as shown in Fig. 3. The radon exhalation rates for various media with different thicknesses were measured. In particular, the radon exhalation rates were measured for UMTs (height 0.1–2.0 m), laterite (height 0.1–2.5 m), and radium-bearing rocklike material (height 0.1–1.0 m) (Hong et al. 2017). The stainless-steel vessel of the experimental setup had a diameter of 0.2 m, and the height was adjustable as required by a flange connection. The top of the vessel was a radon-collecting hood, and the bottom was unrestricted. The sample of the geological medium in the vessel was separated from the experimental table by gauze screen (100 mesh size). The parameters of the abovementioned geological media are summarized in Table 1.

4. Results and discussion

To intuitively show the dependence of $\Psi$ on $\zeta$ and $n$ from Section 2, seven typical cases are listed in Table 2, and plots $\Psi$ versus $n$ and $\epsilon$ are shown in Figs. 4 and 5, respectively. Based on Eq. (15), Table 1, and Fig. 4, we determined the following: (1) $\Psi$ first increases rapidly, then slowly, and eventually approaches 1; (2) when $n > 0.5018$, $\Psi$ is greater than 0.25 for all cases defined in Table 1; (3) when $n$ is 1.0987, 0.5560, and 0.1205 (cases 1–3, respectively), the value of $\Psi$ is greater than 0.50, while $\Psi > 0.50$ for the remaining four cases; (4) when $n$ is 1.9437, 1.3170, 0.8239, and 0.6886 (cases 1–4, respectively), the value of $\Psi$ is greater than 0.75, while $\Psi > 0.75$ for the remaining three cases; (5) when $n$ is 3.6626, 2.7623, 2.0860, 1.9022, and 0.9675 (cases 1–5, respectively), the value of $\Psi$ is greater than 0.95, while $\Psi > 0.95$ for the remaining two cases; (6) when $n$ is 5.9790, 4.6113, 3.6276, 3.3644, 2.0252, and 0.5719 (cases 1–6, respectively), the value of $\Psi$ is greater than 0.995, while $\Psi > 0.995$ for the remaining case.

As stated in Section 2.2, $\epsilon$ is defined as the ratio of the surface fluxes. Based on Eq. (16), Table 1, and Fig. 5, we determined the following: (1) $\epsilon$ is linearly proportional to $\Psi$, and the slope increases gradually from case 1 to case 7; (2) the value interval of $\epsilon$ is 0–1; (3) when $\Psi$ is 0.9500, 0.9625, 0.9724, 0.9750, 0.9875, 0.9975, and 0.9998 (the corresponding values of $n$ are 3.6626, 3.0011, 2.5002, 2.3607, 1.6271, 0.7158, and
0.2257, respectively) for cases 1–7, respectively, the value of $\varepsilon$ is 0.95; (4) when $\Psi$ is taken as 0.995, 0.9963, 0.9972, 0.9975, 0.9988, 0.9998 and 0.999975 (the corresponding values of $n$ are 5.9790, 4.8445, 4.0152, 3.7789, 2.6109, 1.1507, and 0.3632, respectively) as for cases 1–7, respectively, the value of $\varepsilon$ is 0.995.

In case 3, where the radon migration is jointly driven by diffusion and advection effects, the appropriate thickness of the medium ($x_0$) for a lab-scale experiment to achieve an equivalent percentage ($\varepsilon$) of 95% or greater is at least 2.5002$L$. For an $\varepsilon$ of 99.5% or greater, $x_0$ needs to be at least 4.0152$L$. For the cases 4–7, where radon migration is dominantly driven by advection, to achieve an $\varepsilon > 95\%$, $x_0$ must be at least 2.3607$L$, and to achieve an $\varepsilon > 99.5\%$, $x_0$ must be at least 3.7789$L$. Most previous studies of radon exhalation from geological media considered cases in which the radon migration was dominantly driven by diffusion. In this scenario (cases 1 and 2), to achieve an $\varepsilon > 95\%$, $x_0$ must be at least 3.6626$L$; to achieve an $\varepsilon > 99.5\%$, $x_0$ must be at least 5.9790$L$.

Case 1 was considered for three geological media (i.e., UMTs, laterite, and radium-bearing rocklike material) for different thicknesses ($x_0$). Experiments were performed to measure the surface radon exhalation rates from these media, as described in Section 3. As shown in Fig. 6–8, the models derived above were fit to the data for each medium ($J_i = J_w \varepsilon$) with a high goodness of fit (adjusted coefficients of determination $R^2 = 0.97563$, 0.95803, and 0.89125 for UMTs, laterite, and radium-bearing rocklike material, respectively, and $J_w = 5.49, 0.0185, and 0.079$ Bq·m$^{-2}$·s$^{-1}$, respectively). The high fitting degrees indicated the reliability of the theoretical models.

5. Conclusions

The issue of radon exhalation from radium-bearing geological media (e.g., soil, sand, bedrock, uranium ore, and its mill tailings) are of worldwide concern, and lab-scale tests are frequently employed to study these media. However, the influence of a lab-scale sample’s thickness on the comparability to a real
environment and the reliability of the experimental results cannot be ignored. Based on the theory of radon diffusion–advection transport, theoretical and experimental analyses of the equivalent thicknesses of geological media for lab-scale studies of radon exhalation were conducted. The following were concluded:

(1) Geological media can be modeled by semi-infinite and finite-thickness models, and the former can be equivalent to the latter in terms of the rate of radon exhalation from the media’s exposed surface (namely $J_\alpha = \varepsilon J_{se}$, where $0 < \varepsilon < 1$).

(2) By introducing several dimensionless parameters (i.e., $\varepsilon$, $m$, $n$, $\Psi$, and $\zeta$), dimensionless models of homogeneous semi-infinite and finite-thickness media were derived, and the equivalency of $J_{se}$ and $J_\alpha$ was subsequently analyzed.

(3) For cases in which radon migration is dominantly driven by diffusion, when $n > 3.6626$, $\varepsilon > 95\%$, and when $n > 5.9790$, $\varepsilon > 99.5\%$. For cases in which radon migration is dominantly driven by advection, when $n > 2.5002$, $\varepsilon > 95\%$, and when $n > 4.0152$, $\varepsilon > 99.5\%$.

(4) The experimental results for case 1 (pure diffusion) with UMTs, laterite, and radium-bearing rocklike material with different thicknesses ($x_0$) proved that the theoretical model is reliable and valid, with $R^2$ values of 0.97563, 0.95803 and 0.89125, respectively.

Only case 1 was examined experimentally using three geological media to validate the theoretical model, and the maximum thicknesses of the UMTs, laterite, and radium-bearing rocklike material were restricted to 2.0, 2.5, and 1.0 m, respectively. Moreover, a finite-thickness model with exhalation from a single free surface (i.e., the upper surface) should be analyzed in the future.

Declarations

Ethics approval and consent to participate
Consent for publication

Not applicable.

Availability of data and materials

The data from this study is available upon request from the corresponding author.

Competing interests

The authors declare that they have no competing interests.

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Authors’ contributions

Changshou Hong contributed to the conception of the study, Hong Wang designed the experiments, Ming Lan and Bo Lei developed the experimental apparatus, Yifan Chen and Xingwang Dai performed the experiments, and Yong Liu and Xiangyang Li conducted the theoretical analysis. Changshou Hong and Yini Yang contributed significantly to the writing and editing of this manuscript.

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### Table captions

**Table 1** Basic parameters of uranium mill tailings, laterite, and radium-bearing rocklike material

**Table 2** Parameter details for several typical cases

### Table 1

| Geological media                      | Bulk density (kg·m⁻³) | Water saturation | Porosity ratio | Radium content (Bq·kg⁻¹) | Radon emanation coefficient (%) | Radon diffusion coefficient (m²·s⁻¹) | Radon diffusion length (m) |
|---------------------------------------|-----------------------|------------------|----------------|--------------------------|---------------------------------|--------------------------------------|---------------------------|
| Uranium mill tailings                 | 1.76 × 10³            | 0.65             | 0.854          | 8.512 × 10³              | 14.38                           | 3.994 × 10⁻⁶                        | 1.379                     |
| Laterite                              | 1.28 × 10³            | 0.36             | 0.872          | 62.524                   | 20.53                           | 0.812 × 10⁻⁶                        | 0.622                     |
| Radium-bearing rocklike material      | 2.39 × 10³            | 0.48             | 0.105          | 1.880 × 10³              | 9.11                            | 0.023 × 10⁻⁶                        | 0.105                     |

### Table 2

| Case | ζ       | m     | Ψ     | ε     | Implication                 |
|------|---------|-------|-------|-------|-----------------------------|
| 1    | 1       | 0     | >0    | Ψ     | Pure diffusion              |
| 2    | √15/3   | 0.5164| >0.25 | (4Ψ−1)/3| Diffusion > Advection      |
| 3    | (√5+1)/2| 1     | >1/√5 | (√5+5)Ψ−(√5+1)/4| Diffusion = Advection      |
| 4    | √3      | 1.1547| >0.50 | (2Ψ−1) | Diffusion < Advection      |
|   |   |   |   |
|---|---|---|---|
| 5 | $\sqrt{7}$ | 2.2678 | $>0.75$ | $(4\psi-3)$ |
| 6 | $\sqrt{39}$ | 6.0849 | $>0.95$ | $(20\psi-19)$ |
| 7 | $\sqrt{399}$ | 19.9249 | $>0.995$ | $(200\psi-199)$ |
Figure captions

**Fig. 1** Schematic diagram for homogeneous semi-infinite model

**Fig. 2** Schematic diagram for homogeneous finite-thickness model

**Fig. 3** Experimental setup for measurement of radon exhalation rate from geological media. 1: sample (geological medium); 2: stainless-steel vessel; 3: RAD7 radon detector; 4: drying tube with anhydrous calcium sulfate; 5: airflow tube (vinyl ester resins); 6: experimental table; 7: tiny hole (diameter of 0.005 m); 8: supports.

**Fig. 4** $\Psi$ versus $n$ for several typical cases: case 1: pure diffusion; case 2: diffusion > advection; case 3: diffusion = advection; case 4–7: diffusion < advection.

**Fig. 5** $\varepsilon$ versus $\Psi$ for several typical cases: case 1: pure diffusion; case 2: diffusion > advection; case 3: diffusion = advection; case 4–7: diffusion < advection.

**Fig. 6** $J_i$ versus $x_0$ for uranium mill tailings

**Fig. 7** $J_i$ versus $x_0$ for laterite

**Fig. 8** $J_i$ versus $x_0$ for radium-bearing rocklike material
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6

\[
J_f (Bq \cdot m^{-2} \cdot s^{-1}) = 2.549 \exp(1.379x_0) + 2.549 \exp(1.379x_0 - 2)
\]

\[
\sum (J_f - J_{f, \text{fit}})^2 = 0.97563
\]

Measured value

Fitting curve

\(x_0 (m)\)

\(J_f (Bq \cdot m^{-2} \cdot s^{-1})\)
\[
R = \frac{\exp(0.622) - \exp(-0.622)}{\exp(0.622) + \exp(-0.622)}
\]

Fig. 7
Fig. 8