1. Introduction

Mechanical energy is often either introduced as a derived physical quantity, for instance by relating mechanical energy to work and force, or it is defined as a ‘new’ quantity unrelated to other concepts [2]. However, it is known that students have difficulty understanding both approaches because one assumes that students have understood work and force while the other is not tied to prior content knowledge [2, 3].

In this paper we outline an alternative approach to teaching energy that does not face the problems stated above. The basic idea is to define energy as a basic observable that can be measured directly. Hence energy can be seen like other basic observables students are familiar with (e.g. length and weight) and for which well defined measurement processes exist. Such a view on energy has not been introduced to physics education so far. We have described what we understand by basic observables and by direct measuring in the context of gravitational potential energy [1]. Roughly summarized, potential energy is measured by lifting standard weights. In this paper we extend the general approach. We define an analog operation for measuring kinetic energy. To do this, we begin with everyday-life examples and then develop a direct comparison method for kinetic energy. By defining standard processes (compressing springs and ejecting objects) kinetic energy can be quantified. Finally, we determine kinetic energy with mass and velocity.

2. Measuring energy

According to Papadouris, Constantinou [2] (p 209) one measures the kinetic energy possessed by a moving object by exploring ‘the extent of damage that it could cause’ by colliding with certain other objects: a badminton ball, a bullet, a cannonball may leave a shallow or deeper imprint in the sand. If they are hitting a nail, that nail can be pushed less or deeper into a block of styrofoam. Thus, one explores the amount of work done by decelerating objects. When they stop moving this possibility to do work is exhausted.
Most students know the observation that the heavier an object is and the faster, the more work it can conduct, e.g. push a nail deeper into a block of styrofoam. By varying individual influencing factors one even finds a qualitatively different behavior, that doubling the velocity gives more work than doubling the mass. However, so far it remains open, how exactly this work increases if one doubles or triples the mass or the velocity?

The amount of work done by an object can be quantified by counting the number of well defined springs—called standard springs—that are compressed until the object comes to a stand-still (figure 1). Consider the bullet B and the cannonball C that are decelerating from a certain speed. We discuss both objects solely with regard to their capability to do work: let B and C conduct work on the same test system. Following Leibniz [8] we compare the work as follows: test whether cannonball C can compress a larger, the same or a smaller number of standard springs than the bullet B

**Definition 1.** Energy is the capability of one source or system to work against another system. One source C has more energy than another source B if, until complete exhaustion, the work of C exceeds the work of B on the same test system. We symbolize \( E[C] > E[B] \).

In this analysis we assume that the processes are frictionless, reversible, and that the total capability to do work is conserved [8].

**2.1. Reference units**

One defines measuring a quantity by the direct or indirect comparison with a measurement unit. Following Leibniz we use the compression of equivalent springs up to a fixed mark\(^1\). We define standard springs and standard bodies as sufficiently constant reference objects for ‘capability to do work’ and ‘mass’.

Consider a reservoir of (i) standard bodies with the same mass \( m \), Note that one can test this operationally: according to Galilei [9] object a has ‘more inertial mass than’ object b if in an head-on collision test a overruns b (they collide with the same initial velocity, stick together and move in a’s initial direction). In a special case they can come to rest; then both objects have the same resistance against changes of their motion, their inertial mass is the same \( m[a] = m[b] \). Further, we have (ii) standard springs \( S \) with the same capability to do work. We can test this by catapulting standard particles. For our measurements we refer to two elementary standard processes:

1. Let a standard spring catapult two initially resting standard objects with the same mass \( m_1 = m_2 = m \), in opposite directions (figure 2). Hence, the standard spring \( S \) turns standard particles with a standard mass into carriers of kinetic energy. Both objects have the same speed, \( v_1 = -v \), and \( v_2 = v \), \( v \) the standard velocity). Thus, we can say, that two springs \( S' \) and \( S'' \) contain the same energy, when both catapult standard particles in the same way.

2. Let a particle pair of two standard bodies with the same mass \( m_1 = m_2 = m \) and standard velocities \( \pm v \), in opposing directions compress a standard spring that stays at rest (figure 3). Hence, the standard particles with a standard velocity in opposite direction compress a standard spring.

The initial and final states of both inelastic collision processes are well-defined. Therefore, there is an equivalence between the associated capability of the spring to work against the standard particles and the ability of the particle pair to transfer energy to the spring. We call the unit, to compress one standard spring up to a fixed mark, the unit of energy ‘1 J’.

**2.2. The calorimeter model**

In the following, we construct a device—called calorimeter—that measures the kinetic energy of an object. Any object that enters our calorimeter gets absorbed until it stops and must generate only standard energy carriers. The goal is to divide up all work in identical portions [7]. We construct such a device in the following way:

To begin with we consider only objects with standard masses \( m \). Later it will become obvious how the process works with objects

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\(^1\) This is a very instructive approach, Schlaudt remarks [8]. The effect is quantified—not by the depth of compression (in one spring) but instead - by the number of springs which are compressed by a fixed distance. In this way one can disregard completely the inner dynamics of the compression process.
Teaching kinetic energy as an observable quantity

that have multiple standard masses. We collide the incident object $m_s$ with calorimeter elements, so that the (absolute value of the) initial velocity, e.g. $v(m_s) = 5 \cdot v_s$, gets reduced by one unit velocity $v_s$. After five such successive deceleration kicks the incident particle stops $v(m_s) = 0$ (see figure 7). In that process it kicks initially resting standard elements of the calorimeter with the same standard velocity out to the left and right side. By counting the number of these standard elements and—thus—energy units produced the kinetic energy becomes measurable.

2.2.1. One deceleration step. To describe one step of the deceleration process, we begin with the easiest case: the elastic head-on collision between an object from the left and a set of objects coming from the right. In figure 4 Bob collides one standard particle $O$ with mass $m_s$ (the general case follows later) and velocity $v$ with a rigid packet of $z$ standard objects $O_1, \ldots, O_z$ with equal masses $m_1 = \ldots = m_z = m_s$ and equal velocities $w_1 = \ldots = w_z = w$. They collide and rebound off of one another with reversed velocities. The velocity changes of the standard particles can be written as: $v(O) = v, \quad v(O_1) = v(O_2) = \ldots = v(O_z) = w \Rightarrow v(O) = -v, \quad v(O_1) = v(O_2) = \ldots = v(O_z) = -w$. Then the initial velocities must satisfy the relation $1 \cdot v = -z \cdot w$ which is determined by the numbers $1$ and $z$ of standard balls.

To illustrate this process we chose the example $z = 3, v = 3 \cdot \frac{1}{2} v_s$ and $w = -1 \cdot \frac{1}{2} v_s$. One can verify that the condition $v = -z \cdot w$ is satisfied. In figure 5 Bob collides one particle (from left) with a rigid composite of $z = 3$ standard objects (from right). Now we assume that Bob’s collision happens on a raft floating down a river. The velocity changes are independent of the background (Bob’s raft). We want to shift to a frame of reference (Alice on the shore) from where a standard object from the left runs into a three-packet of standard elements, that is initially at rest, kicks this three-packet into motion with standard velocity $v_s$ and rebounds with reduced velocity to the left. This is the situation we need to describe how

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2 Bob views this collision process from the so-called center of mass frame. One can demonstrate the relation with sliders on an air track. A derivation from one elementary reference process (figure 3) and symmetry principles (isotropy under rotations and Galilei covariance for observers in relative motion) can be found in [11].
our calorimeter works. An object hits resting standard objects that leave the calorimeter with standard velocities. For this we choose Bob’s raft to float on the river with velocity $v_{\text{float}} = \frac{1}{2} \cdot v_s$.

From the perspective of Alice, who is standing on the shore all objects get an additional velocity component $+\frac{1}{2} \cdot v_s$ (figure 6). In her view the single particle kicks the three-packet into motion with standard velocity $v_s$. After the rebound that particle slows down by one unit of velocity $v_s$.

This trick—the Galilei transformation—allows us to view Bob’s collision as one step to decelerate that particle.

Students can construct analogous deceleration examples for other initial velocities (see e.g. the other collision frames of figure 7). They use the given relation between the masses and the velocities (momentum expression $1 \cdot v = z \cdot w$) on Bob’s raft and transform the velocity changes to Alice view from the shore (Galilei covariance).

Exercise questions are (solutions are given in brackets):

- How many mass units $z$ will you need for incident velocities $v(O) = 3v_s, 4v_s, 5v_s, 7v_s, \ldots, n \cdot v_s$ to slow down the object $O$ by one velocity unit? ($z = 5, 7, 9, 13, \ldots, (2n - 1)$)

From analyzing the numbers one finds the result:

**Lemma 1.** For decelerating one particle with initial velocity $n \cdot v_s$ by one velocity unit, requires an initially resting bundle of $z = 2n - 1$ elements. In each case the bundle is kicked out with the standard velocity $v_s$.

2.2.2. Complete deceleration series. Finally, in our last exercise students operate a series of deceleration steps, that reduce the velocity by one unit of velocity, after the other, until the particle stops. From those deceleration kicks we build our calorimeter model. The following example highlights the process.

**Example.** One fast moving standard particle with $v = 4 \cdot v_s$ kicks a resting composite of 7 elements into motion and rebounds with reduced velocity to the right (see second frame in figure 7). On the right (third frame) we place again a suitable number of 5 reservoir elements into the way, such that after the next collision they get kicked out with the same standard velocity $v_s$. The incident particle successively rebounds with reduced velocity, until (after frame four and five) it stops inside the calorimeter. Thus, for a particle $m_i$ with velocity $v_{\text{impact}} = 4 \cdot v_s$ we eject a total of $1 + 3 + 5 + 7 = 16$ initially resting reservoir elements $m_1, \ldots, m_{16}$ with the same standard velocity $v_s$ out of both sides of the calorimeter.

Students can analyze more examples of the calorimeter machinery (solutions in brackets):

- How many calorimeter elements do you eject in total for stopping an incident particle
Teaching kinetic energy as an observable quantity

Figure 7. An incident standard particle with mass $m_s$ and velocity $5 \cdot v_s$ ($v_s$ the standard velocity) comes to rest after a series of deceleration collisions $v(m_s) = 0$. In the process it kicks $9 + 7 + 5 + 3 + 1 = 25$ initially resting calorimeter elements with the mass $m_s$ and velocity $v_i = v_s$ resp. $v_j = -v_s$ out to the left and right side of the calorimeter.

with velocity $v_{\text{impact}} = 2v_s, 3v_s, 5v_s, 7v_s$? (see figure 8)

The correlating numbers in table 8 indicate: the absorption process for stopping a particle with velocity $n \cdot v_s$ produces the number $k = n^2$ calorimeter elements. The list of examples may already convince students (for completion teachers find the exact proof in appendix with Gauß trick for adding the odd natural numbers). This leads to:
Proposition 1. We did construct a physical model for absorbing a standard object with impact velocity $n \cdot v_s$ (where $v_s$ is a standard velocity) in an external calorimeter, where it comes to rest. The absorption process produces the number $k = n^2$ standard particles with velocity $\pm v_s$ from a reservoir with resting standard elements $v_i = 0$.

2.3. Quantification

For every absorption process, it is possible to count the portions of work (ejecting standard elements from the calorimeter reservoir). It is important to note, that we eject all calorimeter elements with the same standard velocity. In our calorimeter output each reference object functions as a measurement unit: our standard spring $S$ represents the unit energy $E[S]$. Then, the standard particle $m_s$ with $v(m_s) = v_s$ has the energy $E[m_s] = \frac{1}{2} \cdot E[S]$ because it takes two standard elements with $v_s$ to compress one spring (see figure 3). Thus, by operating with the mechanical calorimeter we provide a vivid way to quantify energy.

The calorimeter extract has the same capability to do work as the incident particle. We essentially assume, that our calorimeter model is reversible$^3$. The kinetic energy $E[O]$ of the incident object $O$ with a certain velocity is depleted, transformed into potential energy of the absorber material, and divided into identical portions.

$$E[O] \overset{\text{Def. 1}}{=} E[S + \ldots + S] \overset{\text{Congr.}}{=} k \cdot E[S]$$

We measure ‘how many times’ larger its kinetic energy is than the potential energy of one standard spring $S$. We quantify the observable as a certain number $k$ (of reference units) times the dimension of our reference device $E[S]$ (reference energy of the spring).

3. Basic equation

With the calorimeter model one can directly count the activated standard springs, mass units, and velocity units. From absorbing one standard particle $O$ with the mass $m_s$ and the velocity $n = 5$ times the standard velocity $v_s$ (figure 7) we generate 25 initially resting reservoir elements with velocity $\pm v_s$ and with the energy $\frac{1}{2} \cdot E[S]$. In total one can compress $25 \cdot \frac{1}{2}$ springs and thus measure the energy $E[O] = 25 \cdot \frac{1}{2} \cdot E[S]$ (see figure 8). This process works analogous if the incoming particle has a multiple of the standard

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$^3$ We steer the absorption by a series of elastic head-on collisions between the incident particle and elements of the calorimeter reservoir. Every step of the deceleration cascade is reversible, because it is build up from solely congruent standard actions (figure 2) and because one can steer these processes both ways.
Teaching kinetic energy as an observable quantity

mass \( z \cdot m_2 \). One can absorb every element of the group one by one. If e.g. \( z = 3 \) and \( v = 5 \cdot v_1 \), we get \( E(O) = E(3 \cdot m_1) = 3 \cdot \frac{1}{2} \cdot 25 \cdot E[S] \). In general this leads to:

**Theorem 1.** An object \( O \) with mass \( m_O = z \cdot m_1 \) (\( m_1 \), a standard mass) and velocity \( v_O = n \cdot v_1 \) (\( v_1 \), the standard velocity) has a kinetic energy

\[
E[O] = \frac{1}{2} \cdot z \cdot n^2 \cdot E[S].
\]

The physical equation looks more familiar in the numerical form

\[
\frac{E[O]}{E[S]} = \frac{1}{2} \cdot \frac{m_O}{m_1} \cdot \left( \frac{v_O}{v_1} \right)^2
\]

(1)

\( (\text{kg}) \ (\text{m s}^{-1})^2 \)

in which all numerical values for energy \( k =: \frac{E[O]}{E[S]} \), mass \( z =: \frac{m_O}{m_1} \), and velocity \( n =: \frac{v_O}{v_1} \) occur in the form measure/unit measure.

4. Discussion

Introducing physical quantities like length or weight by measurement processes is a straight forward and familiar approach. What makes our approach novel is that we use the same principle to measure energy. In doing so, we start with students every-day experiences [4] which we develop into a physicist’s view of measurements.

Thus, our approach is related to a phenomenological view of physics: we focus on observable entities and do not use hypothetical concepts or formal expressions. This is in line with other phenomenological approaches which are described in Ostergaard, Dahlin, & Hugo [6]. Some work focusing on phenomenology in physics education has already been done in optics [5]. Mechanics has received much less attention in phenomenology despite its general relevance for many topics in physics and despite its closeness to experiences in every-day life. Thus, our work contributes to advancing a phenomenological view of mechanics but the general approach is not restricted to energy or mechanics. It can, for example, be extended to relativistic energy-momentum, mass, kinematics and geometry [10].

One of the authors teaches energy with this approach in a high school with students of grade 9 and 11 (aged 15 resp. 17 years). The teaching unit—that covers the content described in this paper—lasted 90 min. The biggest challenge is to motivate the Galilei transformation—here students raised most of their questions. We conclude from this that it is important to put emphasis on the idea that the Galilei transformation helps to derive the deceleration process from easy to handle symmetrical head-on collisions. The transformation simply leads to a reference frame in which the standard elements of the calorimeter are initially at rest. It is interesting to note that the derivations of the lemma and the proposition caused no problems even though the proof was not discussed. Students accepted the general formulas \( z = 2n - 1 \) and \( k = n^2 \) by simply looking at the number series given by the examples. In a feedback session after the unit students remarked that they experienced the concept as ‘tangible’ because they found it easy to work with the ideas of moving objects (with masses and velocities) and compressed and uncompressed springs.

We have outlined an approach to introduce energy that starts from every-day experiences and that develops a measurement principle. By doing this, energy is not an abstract entity or remains hidden behind formulas. The approach was successfully tested with high school students. Future research will show, if our approach addresses the indicated learning difficulties in a long run.

Appendix. Calorimeter counting

**Lemma A.1.** For stopping a particle with initial velocity \( n \cdot v_1 \), the calorimeter ejects a series of odd natural numbers \( 1 + 3 + 5 + \ldots + (2n - 1) = n^2 \) of calorimeter elements.

**Proof.** According to lemma 1 the first deceleration step ejects \( (2n - 1) \) calorimeter elements.

The next step ejects two elements less etc until a single element from the last step. For adding these odd natural numbers beginning with 1 and ending with 99 see figure A1. By combining the first and the last number 1 + 99, the second first and the second last 3 + 97 etc gives a total of 25 pairs with the same value 100. The total sum is \( 25 \cdot (1 + 99) = 2500 \). Analogously for adding the odd natural numbers starting from 1 up to \( (2n - 1) \) gives \( n/2 \) pairs with a total value \( n/2 \cdot \{1 + (2n - 1)\} = n^2 \).
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