Modelling the Tensile Softening Behaviour of Concrete in LS-Dyna Software

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Abstract. The problematic of the paper focuses on the numerical modelling of tensile softening behaviour of concrete. For this purpose, the explicit finite element software LS-Dyna contains several nonlinear material models. The tensile softening behaviour of a total of four selected explicit nonlinear concrete models (namely the Concrete Damage models Release 1 and Release 3, the Winfrith Concrete model and the Continuous Surface Cap model) is tested within numerical simulations of this paper. The task used within the numerical simulations is represented by so-called UUT (Unconfined Uniaxial Tension) test. For this test, specific concrete specimens are used. The numerical simulations are performed with using explicit finite elements. Results of numerical simulations are shown by l-d (load-displacement) diagrams of the UUT test for individual nonlinear concrete models. These results are compared with results obtained from the laboratory measurements for the purpose of validation of the numerical models. The comparisons enable to obtain information about differences between individual nonlinear concrete material models in terms of the tensile softening behaviour. They also show the accuracy of approximations of experimental results by numerical simulations. Based on the comparisons, it can be concluded which nonlinear material models are best for the description of behaviour of concrete in uniaxial tension. Based on the obtained results, it can further be discussed the suitability of nonlinear concrete models in terms of other tests for measuring the tensile mechanical properties of concrete.

1. Introduction

Nowadays, in terms of material behaviour, we have two options to how numerically analyse the response of building constructions from concrete for the purpose of their design. The first option is to consider only the linear behaviour of the material within numerical simulation with the subsequent design of the construction according to the design codes. The second option is to consider the real nonlinear behaviour of concrete within numerical simulation. This option is used in cases when more information about crushing or cracking (crack patterns) of concrete is needed during the design of the construction [1, 2]. For this purpose, quite many nonlinear material models are implemented in current commercial computational programmes [3, 4]. Usually, these programmes are primarily based on the finite element method which can have the explicit, implicit or combined formulation.
History contained several papers or conference contributions which have been focused on nonlinear numerical or experimental analysis of concrete constructions \[5-8\]. The contents of papers or contributions differ especially in terms of the type of load (static or dynamic) and loading rate (slow or fast) to which the concrete construction is subjected. However, performed studies in such publications show the fact that the nonlinear analysis with using material models of concrete has its place between current methods for soft computing. For this reason, verification or validation of nonlinear concrete material models is desirable.

The problematic of the paper focuses on the numerical modelling of tensile softening behaviour of concrete. For this purpose, the tensile softening behaviour of a total of four selected nonlinear concrete material models implemented in LS-Dyna software \[9\] is tested within numerical simulations of this paper. The task used within the numerical simulations is represented by so-called UUT (Unconfined Uniaxial Tension) test. For this test, specific concrete specimens are used. The numerical simulations are performed with using explicit finite elements. Results of the numerical simulations are compared with results obtained from the laboratory measurements for the purpose of validation of the numerical models. Based on these comparisons, it can be concluded which nonlinear material models are best for the description of behaviour of concrete in uniaxial tension. In terms of obtained results, this paper also contains a discussion about the suitability of nonlinear concrete models for performing the computer simulations of other tests intended for measuring the tensile mechanical properties of concrete.

2. Material models

The brief description of used explicit nonlinear concrete material models is given below.

2.1. The Winfrith Concrete model

Mathematical formulation of the Winfrith Concrete model is based on the concept of smeared cracks and rebars, and on the plasticity theory \[10\]. The plasticity part of the theory is represented by the Ottosen’s shear failure function which is given by the expression \[11\]:

\[
F(I_1, J_2, \cos 3\theta) = a - \frac{J_2}{(f_c^0)^2} + \lambda \frac{J_2}{f_c^0} + b \frac{I_1}{f_c^0} - 1
\]

(1)

together with expressions:

\[
\cos 3\theta = \frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}}
\]

(2)

\[
\lambda = k_1 \cos \left[ \frac{1}{3} \cos^{-1}(k_2 \cos(3\theta)) \right] \text{ for } \cos 3\theta \geq 0
\]

(3)

\[
\lambda = k_1 \cos \left[ \frac{\pi}{3} - \frac{1}{3} \cos^{-1}(-k_2 \cos(3\theta)) \right] \text{ for } \cos 3\theta \leq 0
\]

(4)

in which \(I_1, J_2\) and \(J_3\) are stress invariants (first, second and third invariant of standard or deviatoric part of the stress tensor). Parameter \(f_c^0\) in equation (1) represents the uniaxial compressive strength, variable \(\theta\) represents the lode angle, and functions of the strength ratio \(f_t^0 / f_c^0\) are considered via variables \(a, b, k_1, \) and \(k_2\). Parameter \(f_t^0\) in the strength ratio is the uniaxial tensile strength.

Constitutive equations of the model consider not only the dependency of stress on strain, but also the dependency of stress on strain rate. This means that the effect of strain rate can be modelled with
using the Winfrith model. Also, reinforcement can be directly modelled due to the concept of smeared rebars.

2.2. The Concrete Damage model Release 3

The Concrete Damage material model Release 3 is based on the three-invariant concept which uses three shear failure functions [12]. Within the mathematical formulation of the model, the failure functions are mutually independent and can be expressed as [13]:

\[ F_i(p) = a_{i0} + \frac{p}{a_{i1} + a_{i2}p} \]  \hspace{1cm} (5)

where index \( i \) can be either \( y \) (for the initial yield function) or \( m \) (for the maximum shear failure function) or \( r \) (for the residual failure function). Parameters \( a_{ij} (j = 0, 1, 2) \) in equation (5) can be calibrated based on results of experimental measurements, and variable \( p \) represents the pressure \( (p = -\sigma/\gamma) \).

The resulting failure function of the model is then obtained by the interpolation formulation which can be described via equations:

\[ F(I_1, J_2, J_3) = r(J_3)[\eta(\lambda)(F_y(p) - F_y(p)) + F_y(p)] \quad \text{for} \quad \lambda \leq \lambda_m \] \hspace{1cm} (6)

\[ F(I_1, J_2, J_3) = r(J_3)[\eta(\lambda)(F_m(p) - F_y(p)) + F_y(p)] \quad \text{for} \quad \lambda > \lambda_m \] \hspace{1cm} (7)

in which \( I_1, J_2 \) and \( J_3 \) are stress invariants (first, second and third invariant of standard or deviatoric part of the stress tensor). Variable \( \lambda \) in equations (6) and (7) represents the so-called internal damage parameter, \( \eta(\lambda) \) in the same equations is naturally its function, and parameter \( r(J_3) \) represents the William-Warnke scale factor.

Constitutive equations of the model consider not only the dependency of stress on strain, but also the dependency of stress on strain rate. This means that the effect of strain rate can be modelled with using the Concrete Damage model Release 3. Also, it is good to mention that the model includes the input parameter generation ability.

2.3. The Concrete Damage Model Release 1

The Concrete Damage material model Release 1 represents the original unmodified version of the Concrete Damage model Release 3. This version of the model does not include the input parameter generation ability. Just like in the case of Release 3, constitutive equations of the model also consider the dependency of stress on strain rate. This means that the effect of strain rate can be modelled with using the Concrete Damage model Release 1 too.

2.4. The Continuous Surface Cap model

Mathematical formulation of the model is based on the plasticity theory and the continuous damage mechanics [14]. The plasticity theory is represented by the three-invariant yield function which can be formulated as [15]:

\[ Y(I_1, J_2, J_3) = J_2 - 9\mathcal{R}(J_3)^2 F_f^2(I_1)F_c(I_1, \kappa) \] \hspace{1cm} (8)

where \( I_1, J_2 \) and \( J_3 \) are stress invariants (first, second and third invariant of standard or deviatoric part of the stress tensor). In equation (8), variable \( \mathcal{R}(J_3) \), respectively \( \kappa \), is the Rubin scale factor, respectively the hardening parameter. As can be seen in equation (8), the shear failure function \( F_f(I_1) \) is multiplied by the hardening cap function \( F_c(I_1, \kappa) \). This multiplication ensures smooth and continuous combination of both functions.
The shear failure function is given by the following equation:

\[ F_f(I_1) = \alpha - \lambda \exp^{-\beta I_1} + \theta I_1 \]  

(9)

within which material parameters \( \alpha, \beta, \lambda, \) and \( \theta \) can be calibrated based on the triaxial compression test data. The mathematical formulation of the hardening cap function is as follows:

\[ F_c(I_1, \kappa) = 1 - \frac{(I_1 - L(\kappa))^2}{(X(\kappa) - L(\kappa))^2} \quad \text{for} \quad I_1 > L(\kappa) \]  

(10)

\[ F_c(I_1, \kappa) = 1 \quad \text{for} \quad I_1 \leq L(\kappa) \]  

(11)

together with following equations:

\[ L(\kappa) = \kappa \quad \text{for} \quad \kappa > \kappa_0 \]  

(12)

\[ L(\kappa) = \kappa_0 \quad \text{for} \quad \kappa \leq \kappa_0 \]  

(13)

\[ X(\kappa) = L(\kappa) + RF_f(I_1) \]  

(14)

Parameter \( R \) in equation (14) represents the cap ellipticity ratio.

Constitutive equations of the model consider not only the dependency of stress on strain, but also the dependency of stress on strain rate. This means that the effect of strain rate can be modelled with using the Continuous Surface Cap model. Also, it is good to mention that the model includes the input parameter generation ability.

3. Quasi-static explicit simulations

Quasi-static explicit simulations were executed with using the LS-Dyna computational system.

3.1. The computational model used within explicit simulations

Within this paper, the simulated task was the UUT test during which specific concrete specimens are used. The computational model and explicit numerical simulations were set up to correspond to the laboratory measurements with quasi-static loading rate for which the results are given in [16]. However, the computational model was subsequently simplified.

Figure 1. The used computational model for explicit simulations
Within the computational model, only the middle part of the concrete specimen with length of 85 mm was modelled (see figure 1) because the measurements were just set up only on this part of the specimens. The numerical modelling had three steps. First, the geometrical model was created. Second, the discretization of the geometrical model was conducted by explicit finite elements. Third, boundary conditions were defined in the form of linearly increasing displacements over time. These displacements were applied on model bases in the X-axis direction, see figure 1. No zero displacements (supports) were necessary to define because the explicit algorithm of finite element method did not exhibit any instability without using of supports.

3.2. Results and discussion

Results of explicit simulations are shown by l-d (load-displacement) diagrams of the UUT test for individual nonlinear concrete models, see figure 2 (a)-(d). It can be seen from the diagrams that the tensile behaviour of the specimen can be divided into two phases for all used material models. These are pre-peak and post-peak behaviour. In terms of the pre-peak tensile behaviour, it can be seen in figure 2 (a) and (d) that for the Continuous Surface Cap material model and the Winfrith Concrete model, only linearly elastic response occurred. For the Concrete Damage models, it can be seen in figure 2 (b) and (c) that elasto-plastic response occurred, but only with minor plasticity of the material before reaching the peak of the diagram. In terms of the post-peak tensile behaviour, it is obvious that the softening response occurred for all material models used. This means that due to the damage caused by exhausting the ultimate load capacity of the specimen, the tensile force was decreasing with increasing the subsequent deformation.

![Figure 2. Comparisons for: (a) the Winfrith Concrete model, (b) the Concrete Damage model Release 3, (c) the Concrete Damage model Release 1, (d) the Continuous Surface Cap model](image-url)
Figure 2 (a)-(d) also compares the explicit numerical simulations for all used material models with the result of laboratory measurement [16]. It is obvious from the comparisons that the numerical results are practically identical to the measured result regarding the initial stiffness and the maximum attained load (except the Continuous Surface Cap model for which it could not be possible to obtain greater tensile strength). Differences in the tensile strain softening phase between results are caused by the different tensile strain softening functions implemented within the individual concrete material models. It can be seen from the 1-d diagrams that the Winfrith Concrete model exhibited the linear tensile softening behaviour, the Concrete Damage model Release 3 and the Continuous Surface Cap material model exhibited the nonlinear tensile softening behaviour, and the Concrete Damage model Release 1 exhibited no softening behaviour but a very brittle response. In terms of the best approximation of the measured result, it is clear from figure 2 (a)-(d) that the Continuous Surface Cap material model best describes the behaviour of the tested concrete in uniaxial tension, although it exhibits less tensile strength than in the case of the measured data.

In terms of conducting the computer simulations of other tests intended for measuring the tensile mechanical properties of concrete, the Continuous Surface Cap material model does not have to be the one which will provide the best approximation of the measured experimental data. The reason is that the softening behaviour of other models can be for other types of tensile tests also nonlinear similarly as in the case of the Continuous Surface Cap model. It follows that for the satisfying mathematical approximation of the tensile softening behaviour of the real concrete, the strain softening function of the used material model in the uniaxial tension does not have to be important if the satisfying value of the global fracture energy in tension is defined.

4. Conclusions
Within this paper, the tensile softening behaviour of concrete was numerically modelled by four different nonlinear concrete models which are implemented in the material model library of LS-Dyna computational system. The task used within the numerical simulations was represented by UUT test. The comparisons of results of the explicit simulations with the experimentally measured data were conducted. It was obvious from the comparisons that the numerical results were practically identical to the measured result in some respects. The comparisons also showed that the differences in the tensile strain softening phase between results were caused by the different tensile strain softening functions implemented within the individual concrete material models used. However, it could be seen that the Continuous Surface Cap material model described best the behaviour of the tested concrete in uniaxial tension, although it exhibited less tensile strength than in the case of the measured data.

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