The Way of Students’ Reasoning of Elementary Students in Solving Integer Problem

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Abstract. Students’ reasoning may implement to investigate how students learn to think mathematically in solving mathematical problems. This study investigates the way of students’ reasoning in solving some integer comparisons. The sample were the 5th grade students of Labschool Unesa, consisting 11 girls and 8 boys. Considering the existing students’ portfolio, all students completed both maths ability and integer comparisons tests, and were ranked the result based on the results. Three students were voluntarily chosen from each mathematic ability level to assess their reasoning way in solving integer comparison problem. The data was obtained from the reasoning test about integer comparison and a semi-structured interview based on reasoning test. It is underlined that students used a variety of ways to show their conclusions. Students presented appropriately ways and reasons in solving the problem. The student with high mathematics ability often used the number line to illustrate the problem, its mean used order. Meanwhile the student with medium ability also utilized the number line. The student with low mathematics ability applied commonly the other context to solve the problem, its mean used magnitude reasoning. This result suggest that teachers should exercises the students to more explore their reasoning way in any problem.

1. Introduction

One of material in elementary school is integer. Integers are an important topic in arithmetic. Children in elementary school learn about integer, because they will use it to counting of items in the world[1]. In many everyday contexts or activities, such as the numbers of balls, pencils, books, and so on. But sometimes, students feel confused to illustrate negative number. When students are asked to compare two integers with negative signs, they are often answered incorrectly, for example –5 > 2[2]. In comparing integers, 3 students of the 5th grade gave differently ways of comparing between five pencils and the number of pencils. While another student made a sequence of numbers 1 through 5 to show that the number five is greater than the number one. There are various ways of students in comparing integers[3]. The way of students’ reasoning is very rich and interesting, especially the solving of integers problems. Reasoning is an important element that is not only used to solve problems but is also used during classroom learning activities. It has long been agreed that reasoning is important in mathematics, that is in learning and its use[4].

In reasoning, students have different way. Similarly, the use of reasoning in comparing integer. First-grade students who play with the number line argue that the location of numbers on the number lines gives the possibility that the numbers on the left of zero can be compared with debt. Thus, reasoning by sequence allows the number to be on the left of zero[5]. In the other hand, the students’
reasoning of the 2nd and 4th grade tend to use the sign of integer as operation in non-negative integers. Meanwhile when involving negative integers, students are more likely to use the number line to show the position of number because it make easier to compare integer[6]. When the first grade students had to order integers and compare their values, there were 71% of students removed automatically negative signs, 23% knew the concept of negative numbers so that they can compare negative numbers with positive numbers, and only 11% can sort and compare numbers consistently[7]. Students are said to reason when doing a logical thinking process related to the withdrawal of conclusions. Logical thinking here means mental activity derived from behaviors that arise based on statements and problem-solving results in accordance with previously known knowledge. The students' mental behavior that appears will show how students are reasoning[8]. The ability of one's math is one aspect that influences one's reasoning in solving problems related to mathematics[9]. Basically every student has different abilities, including mathematic ability. Many factors that affect it. The differences in students' ability will affect the differences in behavior, development, and cognitive processing of each student [10]. Based on the explanation above, we may answer the following questions how do elementary students reason in solving integer problem, especially integer comparison.

2. Method
2.1 Participant
This study investigates the way of students’ reasoning in solving integer problem especially integer comparison. The sample were the 5th grade of elementary students in Labschool Unesa, consists of 11 girls and 8 boys. All students were given mathematics test to know students’ mathematics ability. Based on both the tests and students’ portfolio were ranked. Three students were chosen as participants from each group to assess their way of reasoning in solving integers comparison problem and will be analyzed based on the way of reasoning indicator in Table 1. The given problem in reasoning test about integer comparison: In February 2018, there was an uncertain weather change in Indonesia. The last temperature change was recorded in Batu –4°C and Semarang 1°C. Will the temperature change in Semarang be the same as Batu if there is an increase of 3°C?

2.2 Analysis
The data was obtained from the reasoning test about comparison temperature and a semi-structured interview based on reasoning’s test. In this paper, we focus on three students’ reasoning way to the integer comparison problem. The problem was printed on a sheet and posed to students one at time. First, the students were asked to give justification then give the reason for their justification. After the data were identified their justification, the justification within each way of reasoning were combined to determine how the ways of reasoning were used. The interviewer posed some questions to know the way of students’ reasoning, such as “Why do you agree with that statement? How did you think about that?”. Then posed additional question if necessary to know deeply about the way of students’ reasoning in comparison integer. Each student way was based on their justification. In particular, we present the way of reasoning [6] below.

2.2.1 Ordering-based reasoning
This way of reasoning is based on the sequence of integers in the count or its position on the number line. This method focuses on numbers that lie before or after other numbers when counting, or that are located more to the right, more to the left, or near zero on the number line.

2.2.2 Magnitude-based reasoning
Students who use reason-based reasoning tend to compare the magnitude or the cardinality of a number. In accordance with this reasoning, students argue that zero and negative numbers are meaningless, as well as some integers associated with a specific context.

2.2.3 Developmental-based reasoning
In this reasoning, students are not familiar with negative numbers. This indicates that the students are more knowledgeable about the number of counts and their operations. As a result of that, students...
prefer to remove the minus sign, such as replacing \(-7\) as 7. There is also a sign indicating minus as a reduction and there are also assume that there is no negative number.

Table 1. The way of reasoning

| The way of reasoning categories | Students’ activities |
|---------------------------------|----------------------|
| Ordering-based reasoning        | • Comparing two integers, a negative integer is always less than a positive integer or zero, or a positive integer is always greater than a negative or zero integer.  
• Comparing two integers, integers close to zero or close to positive integers will be larger.  
• Comparing two integers, more to right integers are greater than, or more to left integers are less than.  
• Comparing two integers, the more left integer is greater (assuming a negative integer of the positive integer) |
| Magnitude-based reasoning        | • Comparing two integers, negative integers will be more or less negative than other negative numbers.  
• Compare non-zero integers to zeros, on the ground that zero has no meaning.  
• Determine that all positive integers are larger than negative integers, since negative integers do not have any magnitudes.  
• Describes the magnitude of an integer in a context. |
| Developmental-based reasoning    | • Removess the minus sign so that it becomes a natural number.  
• Treats a minus sign as an operation. |

3. Result
About 80% of all 19 students in the sample answered correctly, but only few students give reasonable argument about their answer. At the way of reasoning, After the sample is classified based on the both test (mathematics test and reasoning test) and students’ portfolio, there were 2 high students’ ability, 4 medium students’ ability, and 2 low students’ ability. Three students were voluntarily chosen from each group to assess their reasoning in solving integer comparison problem deeply. The selection of three volunteer based on consistency and uniqueness in solving integer comparison problem, fluent communication, and willingness. Here, we present three description of students’ reasoning way based on different ability as follows:

3.1. Reasoning Way of High Mathematics Ability Student (HMAs)
When the subject was asked to compare the temperature changes of Semarang and Batu, the subject stated that the given statement was wrong because “Semarang is \(1\)° C, while Batu is \(-4\)° C. If Semarang increased \(3\)° C equal to Semarang \(4\)° C, while Batu is \(-4\)° C”. From the subject’s worksheet, she also describes the number line by stating that \(-4\)° C for Batu. Meanwhile \(1\)° C for Semarang. The subject also added a sum operation with number 3 to obtain a new temperature change of \(4\)° C.
Figure 1. The way of students reasoning of high mathematics ability

In the interview, the subject also said

R: Is the temperature changes in Semarang equal to Batu?
S: No. Batu has a temperature changes negative 4, while Semarang positive 4 so they are not same.
R: Whereas the two cities have the same value of 4, how can they not be the same?
S: Different, because Semarang is to the right of zero, if Batu is to the left of zero. So if the number is getting to the left will be the smaller. If the number is more to the right will be the greater.
R: When viewed from the number line, both of Semarang and Batu jump as much as 4 away from zero. Are they not the same?
S: Not the same, the temperature change in Batu is smaller because of the temperature decreasing. While Semarang is greater because of the temperature rising.

Based on interview data, the subject explains that the temperature change for Batu is on the left side of zero so it is marked with a negative sign. While the temperature change for Semarang is on the right zero so it will be positive sign. If the temperature changes are located increasingly to the left then the smaller. So the subject states the temperature changes Batu is smaller than Semarang.

3.2. Reasoning Way of Medium Mathematics Ability Student (MMAs)

When subject was asked to compare temperature changes from two cities, the subject stated disagree that the temperature change in Semarang and Batu is same. The subject gave the reason “if Semarang added by 3°C it will be +4°C”. The subject also made a number line. The subject gave a description of “Bat” on the number −4, “Se” on +1 and +4. The description written on number line indicates that –4 for Batu, while +1 for Semarang, and +4 is also for Semarang after experiencing temperature rise 3°C.

Figure 2. The way of students reasoning of medium mathematics ability

In the interview, the subject also said

R: Is it true that the temperature change in Semarang will be the same as Batu if it increases 3°C?
S: Em .. wrong
R: Why is it wrong?
S: Because at first Semarang is 1°C, then up 3°C become 4°C. Differences with Batu –4°C.
R: How can that be?
S: If we drawn a line of numbers like this (make number line), –4 is for Bat (its mean Batu) and this is +4 for Se (its mean Semarang)
R: Are these two cities not the same when both are worth 4?
S: Not the same, Batu has decreased means on the left. If Semarang increased means on the right.
R: Which cities have smaller or greater temperature changes?
S: The city that experienced a bigger temperature change is Semarang, because Semarang is located in the right of Batu. The sign of Batu is negative and Semarang is positive. Positive definitely greater than negative.
Based on interview data, the subject explained that the temperature change in Semarang will not be equal to Batu because Semarang has positive sign while Batu has negative sign. Which means positive definitely greater than negative. It means Semarang experienced a greater temperature change compared to Batu.

3.3. Reasoning Way of Low Mathematics Ability Student (LMA)

When the subject was asked to compare the temperature changes of Semarang and Batu, the subject stated that the statement given was wrong with the reason of “far difference”. Subject does not give any information or reason on the worksheet provided.

Figure 3. The way of students reasoning of low mathematics ability

In the interview, the subject also said
R: Is the temperature change in Semarang the same as Batu after increased 3°C?
S: Wrong, because it is much different.
R: How can it be different?
S: If Semarang 4, while the city of Batu –4. Means Batu is below and Semarang is above.
R: What does it mean below and above?
S: So it is like a stone that is arranged. Semarang means 4 pieces of stone arranged on the ground for positive, if Batu, the stone is arranged underground for negative.
R: So which temperature change is greater or smaller between Batu and Semarang?
S: The smaller one is Batu because it is underground in its stone. If Semarang is on the ground so it will be greater.

Based on interview data, the subject explained that Semarang experiencing temperature rise is illustrated by arranging stone on the ground as many as 4 pieces of stone because it is positive, while Batu is illustrated with 4 pieces of stone arranged in underground because of negative value.

4. Discussion

Our aim in this study was to investigate the way of students’ reasoning with various mathematics ability about integer problem, especially integer comparison. In the results reported here, the students compared integers : positive versus negative. We discuss findings in students’ ways of reasoning.

4.1. Reasoning Way of High Mathematics Ability Student (HMA)

In comparing integers, the way in which the subject is used is to draw the number line but not to write the numbers in sequence on the number line, only on the number required. Next, the subject gives information on the number to indicate the location of the city in question. The subject also states that the number located on the left of zero is smaller than the number located to the right of zero. As in the case, Batu is located on the left of zero and Semarang is located on the right side of zero. This means that the temperature change in Batu is smaller than Semarang. In addition, the subject also states the numbers are getting to the left will be smaller [7]. This is shown by the subject that the location of Batu is more to the left than Semarang. So it can be concluded that the temperature changes in the city of Batu is smaller than the city of Semarang. The way of reasoning that subject used include in order-based.

4.2. Reasoning Way of Medium Mathematics Ability Student (MMA)

In comparing the integers, almost the same as the subject of high-ability that is making a number line to show the temperature changes experienced by the two cities of Batu and Semarang. However, the
subject only gives an explanation by writing down the initials of the city. As in pointing out of Batu, the subject only wrote “Bat” just below the number –4. As for showing the city of Semarang, the subject wrote “Se” under +1 and +4. The reason the subject wrote under +1 and +4 because Semarang experience temperature rise 3°C. In addition, the subject also states that the city located on the left of the other city will be smaller. So if it is seen on the number line the subject has made, he explains that “Batu is located on the left side of Semarang, meaning that the temperature of Batu city is smaller than Semarang”. It shows that numbers decrease in value further to the left on the mental number line and increase in value further to the right [7]. Based on the explanation above, this subject used order-based way of reasoning.

4.3. Reasoning Way of Low Mathematics Ability Student (LMAs)
In comparing integers, subjects with low mathematics ability provide true justification. But the reason given in writing is very brief and does not provide any information about the way the subject is used. When the subject explains the idea used, he says that the two comparable cities are illustrated by arranging several stones. Semarang experiences positive temperature changes is illustrated with 4 pieces of stone arranged on the ground. While Batu with a negative value is illustrated with 4 pieces of stone arranged underground. Then the subject explains that the stone underground will be worth less than the stone on the ground. So it can be concluded that the temperature change in Batu is smaller than Semarang. This indicates that the subject uses another context contained in his environment in comparing integers. It means that integer reasoning can be referred to contexts such as lending and owing money, traveling forward and backward, elevation (above and below some relative zero), temperature (above and below zero degrees), happy-sad day, and so on [11]. Its mean that the subject used magnitude-based way of reasoning.

We relate our research to the literature bases that informed this study. We focus on the way of students’ reasoning in solving integer problem. In the relevant mathematics literature order and magnitude reasoning have been particular ways that students may use when reasoning about integer comparison[6].

5. Conclusion
This study had demonstrated the way of students reasoning in solving the integer problem. It is underlined that students used a variety of way to draw their conclusions. Students presented appropriately ways and reasons in solving the problem. Subjects with high mathematics ability use a number line to show where the temperature changes experienced by both cities. Besides the subject gives the reason that the location of the number that is on the left of the zero will be smaller than the number located on the right of zero. The subject also says that on the number line, the numbers that are located increasingly to the left will be worth less. The way of reasoning that subject used include in order-based reasoning. Subjects with medium mathematics ability also use a number line to indicate the temperature changes experienced by both cities. The subject explains that on the number line, the number is located more to the left of the other number then the number will be smaller, and vice versa. This subject used order-based way of reasoning. Meanwhile, subjects with low mathematics ability, using a different context from those given. He uses an illustration of the arrangement of stones on the ground. Stones that are arranged on the ground (positive value) will be greater in value than stones arranged underground (negative value). Its mean that the subject used magnitude-based way of reasoning. We offer this result as a tool for mathematics teachers to inform students’ reasoning about integer comparison more deeply [2] and explore the way of students’ reasoning in integer problem.

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