Stochastic Propagation of Solar Energetic Particles in Coronal and Interplanetary Magnetic Fields

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Abstract. This paper describes a method of simulating solar energetic particle propagation through the magnetic fields of the solar corona and interplanetary medium. The simulation code is based on the focus transport equation of energetic particles in 3-d magnetic fields, which contains all the particle transport mechanisms, including streaming, convection, gradient/curvature drift, adiabatic focusing, pitch angle scattering by Alfvénic magnetic field fluctuations and perpendicular diffusion due to the random walk of field lines. In the simulation, particles are injected at their source in the corona, and their guiding center trajectories are calculated using stochastic differential equations. Because of the vastly different time scales of particle transport mechanisms included in the equation, we use the 4-th order Runge-Kutta method to integrate the particle streaming and adiabatic focusing terms, while the stochastic terms of pitch angle scattering and perpendicular diffusion are integrated with the Euler scheme. The model is applied to the 2017 September 10 solar energetic particle event. With perpendicular diffusion, we are able to explain SEP observations from Earth and STEREO-A. A pattern of SEP precipitation on the solar surface is also predicted.

1. Introduction
When a solar flare or a coronal mass ejection (CME) occurs in the corona, solar energetic particles (SEPs) up to many GeV can be produced and released into interplanetary space. The high-energy radiations travel to Earth or anywhere in the solar system, becoming a hazard to electronics on spacecraft or astronauts working in space. Therefore, we want to know how they are produced in the corona and how they propagate through the magnetic fields of the corona and interplanetary medium to Earth. We hope that, in future, we can reliably predict radiation dose using remote observations of solar events ahead of the arrival of radiation hazard.

Below the energy of a few GeV, the motion of SEPs in the coronal or interplanetary magnetic field is expected to strongly bound along magnetic field lines due to their small gyroradii compared to the domain size. Thus, we would expect that radiation exposure associated with SEPs only occur when one is directly connected by a magnetic field line to SEP sources near the sun. The size of a solar flare is small and even a large CME only covers a limit area of the solar surface, so SEPs should appear only at places magnetically connected to the solar event. However, multi-spacecraft measurements show that SEPs are often observed simultaneously over wide ranges of longitude, latitude, and radial distance. Wiedenbeck et al. (2013) reported that $^3$He ions were detected by three spacecraft, ACE, STEREO-A (ST-A) and STEREO-B, spanning over 130° in longitude during the 2010 February 7 SEP event. Since $^3$He is produced by...
a stochastic particle acceleration process associated with magnetic reconnection in solar flares, the observation demonstrates that SEPs can spread widely across the longitude during their transport to 1 AU.

A number of theories have been proposed to explain the observation of widespread of SEPs. An extended or moving source of SEPs accelerated by a large CME shock could explain the widespread of SEPs observed in interplanetary space (Cliver et al. 1995). EUV waves are often seen to propagate very far on the solar surface from solar flare site (Klassen et al. 2000; Kienreich et al. 2009; Thompson & Myers 2009). If EUV waves are the footpoint of coronal CME shockwaves on the solar surface, one could argue that the CME shock can cover a much larger solar surface area than its interplanetary counterpart. Rouillard et al. (2012) hypothesized that the EUV waves can be used to track the expansion of a coronal shock, which is responsible for particle acceleration. Sometimes, a solar flare can trigger another or multiple eruptions in remote active regions that are linked by large-scale coronal magnetic fields or sympathetic solar flares (Wang et al. 2001; Schrijver & Title 2011; Shen et al. 2012; Schrijver et al. 2013). SEPs from multiple solar flares may be simultaneously observed by widely separated spacecraft. If the interplanetary magnetic field deviates significantly from the Parker spirals, locations of widely separated longitudes may be directly connected through those severely distorted interplanetary field lines (e.g., Richardson et al. 1991; Richardson & Cane 1996; Torsti et al. 2004; Chollet et al. 2010), but such an explanation only works for locations with a large separation in radial distance as well. The magnetic field in the solar coronal itself can connect over a wide range of longitudes and latitudes, so SEPs can spread laterally just by propagating along field lines in the corona (Schrijver & De Rosa 2003). Furthermore, magnetic field structures in the corona are extremely complex. In some places, magnetic reconnection can occur. This makes field lines from different regions get connected. SEPs following the reconnected field lines can easily access to a large volume of space. Lastly, energetic particles do not have to follow the magnetic field line all the time. Driven by magnetic field inhomogeneity and fluctuations, cross-field diffusion of particles can occur in both the corona and interplanetary medium. Marsh et al. (2013) and Dalla et al. (2013) suggest that drifts due to curvature and gradient of Parker spiral could be a considerable source of asymmetric cross-field transport for high-energy SEPs. Even in the absence of cross-field diffusion of the particles from the initial field lines, large-scale field line meandering can make an effective contribution to cross-field transport in the average magnetic field (Giacalone & Jokipii 2012; Kelly et al. 2012; Laitinen et al. 2013). The inclusion of particle cross-field transport is necessary to explain the SEP reservoir phenomenon. At the beginning of an SEP event, particle intensities at different locations could be very different depending on the magnetic connectivity to the SEP source near the sun, but after some time (typically one to several days), they reach a uniform level at all locations in the inner heliosphere up to a few AU radial distance (McKibben, 1972; Roelof et al. 1992; McKibben et al. 2001, Dalla et al. 2003; Zhang et al. 2003). This so-called SEP reservoir effect can only be explained by particle transport perpendicular to large-scale (averaged) magnetic fields in the corona or interplanetary medium.

Driven by the need to incorporate all the above possibilities of SEP transport modes, we have developed a computation code to calculate SEP propagation through the corona and interplanetary medium using a realistic magnetic field model. As the coronal magnetic field varies drastically over a time scale, from minutes to solar cycles, it is best to use a data-driven coronal magnetic field. In this paper, we present some calculation results of SEP transport in a potential field source surface coronal magnetic field model, which is derived from measurements of the photospheric magnetic field.
2. Model and computer code
We use the focus transport equation in the 3d spatial domain to describe SEP propagation from its source near the sun to anywhere in interplanetary space. The particle distribution function \( f \) is gyrotropic and can be written as a time-dependent diffusion-drift equation in the 5-d phase space of position \( \mathbf{r} \), momentum \( p \) and pitch-angle cosine \( \mu \). The equation is formulated as follows:

\[
\frac{\partial f}{\partial t} = \nabla \cdot \mathbf{\kappa}_\perp \cdot \nabla f - (v\mu \mathbf{b} + \mathbf{V} + \mathbf{V}_d) \cdot \nabla f + \frac{\partial}{\partial \mu} \frac{D_{\mu\mu}}{d\mu} \frac{\partial f}{\partial \mu} - \frac{dp}{dt} \frac{\partial f}{\partial p}.
\]

(1)

The right-hand side of the equation includes the cross-field diffusion tensor \( \mathbf{\kappa}_\perp \) and particle pitch-angle diffusion coefficient \( D_{\mu\mu} \). These two diffusion terms contain the effect of magnetic field fluctuations. The terms of the first-order derivatives come from particle adiabatic motion in the ambient regular (averaged) large-scale magnetic field \( \mathbf{B} \). These terms contain convection with the solar wind plasma velocity \( \mathbf{V} \), streaming \( v\mu \mathbf{b} \) with particle speed \( v \) times pitch-angle cosine \( \mu \) along the magnetic field direction \( \mathbf{b} \), gradient/curvature drift velocity \( \mathbf{V}_d \), focusing \( \frac{d\mu}{dt} \), and adiabatic cooling \( \frac{dp}{dt} \), which can be written in terms of \( \mathbf{B} \) and \( \mathbf{V} \) as follows (e.g., Zhang 2006):

\[
\mathbf{V}_d = \frac{cv}{qB} \left\{ \frac{1 - \mu^2}{2} \mathbf{B} \times \nabla \mathbf{B} + \mu \mathbf{B} \times (\mathbf{B} \cdot \nabla \mathbf{B}) + \frac{1 - \mu^2}{2} \frac{\mathbf{B} (\mathbf{B} \cdot \nabla \times \mathbf{B})}{B^3} \right\},
\]

(2)

\[
\frac{d\mu}{dt} = -\frac{1 - \mu^2}{2} \mathbf{b} \cdot \nabla \ln B + \frac{\mu^2(1 - \mu^2)}{2} \nabla \cdot (\mathbf{V} - 3 \mathbf{b} \mathbf{b} \cdot \nabla \mathbf{V}) - \frac{(1 - \mu^2)p}{v} (\mathbf{V} \cdot \nabla \mathbf{V}) \cdot \mathbf{b},
\]

\[
\frac{dp}{dt} = \left[ \frac{1 - \mu^2}{2} (\nabla \cdot \mathbf{V} - \mathbf{b} \mathbf{b} \cdot \nabla \mathbf{V}) + \mu^2 \mathbf{b} \mathbf{b} \cdot \nabla \mathbf{V} \right] p - \frac{\mu p}{v} (\mathbf{V} \cdot \nabla \mathbf{V}) \cdot \mathbf{b}.
\]

2.1. Magnetic field model
We choose to use the potential field source surface (PFSS) model for the coronal magnetic field. The model assumes that there is no electric current. It could be a good approximation on the large scale, which is established after the plasma in the corona achieves mechanical equilibrium under the magnetic force. The PFSS magnetic field \( \mathbf{B} \) can be calculated from a scalar potential \( \Phi \) through \( \mathbf{B} = -\nabla \Phi \). In a current free environment, the scalar potential obeys the Laplace equation with a radial field boundary condition at the solar wind source surface \( R_{ss} \). The solution to the PFSS field model in the polar coordinates can be written as a summation of spherical harmonics (Altschuler & Newkirk, 1969):

\[
\Phi = R_0 \sum_{l=0}^{\infty} \sum_{m=-l}^{l} P_l^m (\cos \theta) (g_{lm} \cos m\phi + h_{lm} \sin m\phi)
\]

\[
\times \left[ l + 1 + l \left( \frac{R_0}{R_{ss}} \right)^{2l+1} \right]^{-1} \left[ \left( \frac{R_0}{r} \right)^{l+1} - \left( \frac{R_0}{R_{ss}} \right)^{l+1} \left( \frac{r}{R_{ss}} \right)^l \right].
\]

(3)

In the above equation, \( P_l^m \) is the associated Legendre polynomial. The inner boundary is set at the solar surface \( R_0 = 1R_s \) (photosphere), and the outer boundary is typically at \( R_{ss} = 2.5R_s \) (the solar wind source surface). As we have measurements of the photospheric magnetic field at \( R_0 \), the spherical harmonic coefficients \( g_{lm} \) and \( h_{lm} \) can be determined from fits to photospheric magnetogram measurements. These coefficients along with photospheric magnetogram measurements are available at a few solar telescope observatories. We choose to use the products that are published on the National Solar Observatory (NSO) Global Oscillation...
Network Group (GONG) website. The large-scale magnetic field structure in the corona does not change dramatically in a few days. It rigidly corotates with the sun. For this reason, in this paper, we use a stationary magnetic field model to study SEP propagation. The computation is performed in the corotating frame for convenience. The speed of plasma in the corona below $R_{ss}$ is low compared to other particle transport mechanisms, so we assume $V = 0$.

2.2. Solar wind model
Outside of $R_{ss}$, the effect of solar wind flow is not negligible. We use the model by LeBlanc (1998), in which the radial solar wind speed is prescribed as:

$$V_r = V_\infty \left[ 1 + k_4 \left( \frac{r}{R_0} \right)^{-2} + k_6 \left( \frac{r}{R_0} \right)^{-4} \right]^{-1},$$

and the tangential speed in the corotating frame is

$$V_\phi = -\Omega_s (r - R_{ss}) \sin \theta,$$

where $\Omega_s$ is the sun’s angular rotation speed. Note that $V_\phi$ is just a velocity component relative to a rotating coordinate, and it does not represent the true speed of the particles. The meridional plasma speed is set to $V_\theta = 0$ everywhere. The magnetic field in the solar wind can be calculated from the Parker model and the solar wind speed given its boundary condition at the solar wind source surface $B_r(R_{ss})$.

$$B_r = B_r(R_{ss}) \left( \frac{R_{ss}}{r} \right)^2, \quad B_\theta = 0, \quad \text{and} \quad B_\phi = B_r \frac{V_\phi}{V_r}.$$ (6)

2.3. Diffusion coefficients
We use the following form for the pitch-angle diffusion coefficient:

$$D_{\mu\mu} = D_0(r) \rho^{-2}(1 - \mu^2)(|\mu|^{q-1} + h_0).$$ (7)

It mimics the result of particle scattering by an Alfvénic turbulence with a power-law spectrum of spectral index $q$. An additional parameter $h_0$ is used to phenomenologically describe the enhancement of scattering through $\mu = 0$ by either non-resonant scattering or non-linear effects. Unless $h_0$ is very small $\ll 0.05$ or very large $\gg 0.5$, it cannot affect the calculation result significantly. The level of particle scattering is mainly controlled by the value of $D_0(r)$. It is left as a free parameter in our model because it is sensitive to solar activities and can change drastically from one event to another or from one region to another. We typically use the parallel mean free path to specify the level of particle pitch angle scattering, as they are related by the following equation (e.g., Hasselmann and Wibberenz, 1970):

$$\lambda_\parallel = \frac{3v}{8} \int_{-1}^{1} d\mu \frac{(1 - \mu^2)^2}{D_{\mu\mu}}.$$ (8)

The spatial dependence of $D_0(r)$ reflects the spatial distribution of the magnetic field turbulence level. Following Bieber (1994), we set the radial mean free path to be constant so that $\lambda_\parallel$ is proportional to $\cos^{-2} \psi$, where $\psi$ is the spiral angle of the Parker magnetic field to the radial direction. In the corona, we take $\psi = 0$ for this part of the calculation although the magnetic field in the PFSS model is not radial everywhere.
The spatial diffusion perpendicular to the ambient magnetic field $\kappa_\perp$ mainly comes from the motion of particles that follow meandering or random walking magnetic field lines. The random walk of field line is not included in the large-scale PFSS field model, so we include its effect on particle transport in the diffusion coefficient. For a population of particles moving along the field lines with an average parallel speed of $v/2$, their perpendicular diffusion coefficient is:

$$\kappa_\perp = \frac{v^2}{2}D_{FLRW}, \quad (9)$$

where $D_{FLRW}$ is the diffusion coefficient of field line random walk per unit length of the field line. The granular motion of plasma in the photosphere is the ultimate driver of field line random walk in the corona and interplanetary space. The footpoint of the field line in the photosphere is assumed to undergo a diffusive motion. As the field line is drawn out by the solar wind with speed $V$, $D_{FLRW}$ can be estimated using the following equation:

$$D_{FLRW} = \frac{\langle \delta x_0^2 \rangle}{\langle \delta z \rangle} = k_{gd} B_0 \frac{\langle \delta x_0^2 \rangle}{V \langle \delta t_0 \rangle}. \quad (10)$$

The typical size for supergranules is about $\delta x_0 = 3 \times 10^4$ km within a lifetime of $\delta t_0 = 24$ hours, and $k_{gd} < 1$ is a free parameter, indicating the coupling of the field line motion between the ionized corona and partially ionized photosphere. The ratio of magnetic field strength to the surface value $B/B_0$ describes the divergence effect of magnetic field flux tubes, which amplifies the extent of footpoint motion by that ratio because of the frozen-in magnetic field lines.

### 2.4. Stochastic differential equation solver

The focus transport equation (1) is a 5-dimensional diffusion-drift equation. Due to the complexity of magnetic field geometry and diffusion coefficients, the only way to obtain its solution is through numerical simulation. Regular ways to solve the partial differential equation, i.e., finite difference or finite element methods, requires too many computer resources. Therefore, we resort to using stochastic differential equations to solve the focus transport equation. The Fokker-Planck diffusion equation (1) can be recast into 5 stochastic differential equations, which describe the time evolution of the guiding center position, pitch-angle cosine, and momentum of a stochastic process, which can be considered as a representation of real particles in this formulation of stochastic processes (e.g. Gardiner 1985):

$$d\mathbf{x}(t) = \sqrt{2 \kappa_\perp} \cdot d\mathbf{w}(t) + \langle \nabla \cdot \kappa_\perp + v_\mu \mathbf{b} + \mathbf{v} + \mathbf{V} \rangle dt, \quad (11)$$

$$d\mu(t) = \sqrt{2 \max(D_{\mu \mu},0)} dw(t) + \left[ \frac{\partial D_{\mu \mu}}{\partial \mu} + \frac{d\mu}{dt} \right] dt,$$

$$dp(t) = \frac{dp}{dt} dt.$$ 

In the above equation, $dw(t)$ is the Wiener process as a function of time $t$, and it can be generated by random numbers with a Gaussian distribution. The function max in (11) takes the maximum value of its arguments.

The stochastic differential equation is like a first-order ordinary differential equation. We solve the stochastic differential equations using an Euler scheme (Kloeden and Platen, 1992). As particle streaming along the magnetic field line and the focusing term in Equation (2), both of which are proportional to the particle speed $v$, are much faster than the other transport terms, we use a 4th-order Range-Kutta scheme (Press et al. 1991) to integrate those parts. In this way, particle trapping is correctly captured in our calculation. The high-order Range-Kutta integration accuracy is only necessary when the particle pitch-angle diffusion and perpendicular
spatial different are very weak, but normally with the typical values of the diffusion coefficients for most SEP events, accuracy higher than the 1-st order Euler scheme does not make much difference because, in the diffusion terms, \( dw(t) \propto \sqrt{dt} \).

Equations (11) are time-forward stochastic differential equations that correspond to the focus transport equation (1). It is most suitable for the simulations in which particles are injected with a specific pattern. We typically inject particles at a solar flare site slightly above the solar surface or at a CME shock in the corona. As it is unlikely for particles to be effectively accelerated on closed field lines due to heavy loss in the solar atmosphere, and as it is also difficult to get the particles from closed field lines to interplanetary space, we only inject particles on open lines in the corona. The stochastic differential equations are integrated to follow the random trajectory of particles. Particle distribution, after some time of propagation, can be built by properly binning a large number of simulated particles.

3. An example of model results

The sun emits an intense X8.2 solar flare that peaks at 12:06 UT on 2017 September 10. It is a capstone event from Active Region 12673, which has been making solar flares since August. Viewed from Earth, the solar flare is located at S08W88. The solar flare triggers a fast halo CME. The center of the CME propagates out in the direction roughly perpendicular to the line of sight from Earth, so its speed and size can be more accurately determined from coronographs. The CME speed is greater than 3000 km/s, and its angular size is approximately 180° in latitude. Rapid increases in energetic particle fluxes can be seen at Earth immediately following the solar flare. At the time of the solar flare, the solar wind speed at Earth is 512 km/s. No shock is observed until the end of September 12, but the shock is not fast, being only less than 650 km/s. This suggests that the fast CME observed in the corona does not reach Earth either due to its size or strength.

ST-A spacecraft is located 128° west of Earth in longitude. The solar flare occurs on the opposite side of ST-A. In-situ measurements on ST-A show a slow wind of \( \sim 400 \) km/s and do not detect any shock for several days. SEP intensities at ST-A increase gradually and they do not reach their peak intensity until 3 days later. This suggests that the observed particles could have reached ST-A through perpendicular transport in interplanetary space. ST-A gets into an SEP reservoir after observing a peak intensity 3 days after the solar flare.

We inject 30 MeV protons at the CME shock when it is at 1.5 \( R_s \). The top panel of Figure 1 shows in the loci of open field lines at 1.5 \( R_s \). We assume that the CME shock area is circular, extending 90° in each direction from the location of the solar flare. Particles are uniformly injected with an isotropic pitch-angle distribution over the open field lines within the CME shock. The radial mean free path is set to 5 \( R_s \) in the corona below 20 \( R_s \) and 20 \( R_s \) in interplanetary space outside of 20 \( R_s \). Without perpendicular diffusion (\( k_{pd} = 0 \)), a small fraction (1.7%) of particles enter into the loss cone and precipitate on the solar surface (Figure 1 middle panel). The bottom panel of Figure 1 shows the latitude-longitude distribution of escaped (\( r > 2.5 R_s \)) SEPs on the solar wind source surface. Once the particles reach 2.5 \( R_s \), the chance to return to the solar surface is minimal in this simulation. The longitude and latitude of the particles are mapped along the Parker spirals on to the solar wind source surface at 2.5 \( R_s \). The area coverage is very much the same as the CME shock although particles are only injected on open field lines with a smaller area at 1.5 \( R_s \). The result is just a coincidence after mapping the field lines from 1.5 \( R_s \) to the solar wind source surface using the PFSS magnetic field of this particular event. On the other hand, this CME is so large that any distortion of its location during the field-line mapping could become less visible. The intensity distribution of the escaped particles also shows an effect of particle gradient/curvature drift, which is particularly clear near the neutral sheet. The footpoint of the magnetic field line to Earth (symbol \( E_m \)) is located directly within the SEP source regions. This explains why we see an immediate and fast
Figure 1. (Top) Loci of outward (green) and inward (red) open field lines as a function of solar latitude and longitude at 1.5 $R_s$. The black curve with the shaded area indicates an assumed CME shock size. (Middle) Loci of precipitating SEPs on the solar surface, and (Bottom) distribution of escaped SEPs projected on the solar wind source surface. The letter F shows the location of the solar flare. Letters E and A show the projection of Earth and ST-A. Letters $E_m$ and $A_m$ show the footpoint of magnetic field lines to Earth and ST-A. The blue and white curves are the neutral sheet from the PFSS coronal magnetic field model. Perpendicular diffusion is turned off in the calculation.

Figure 2. Loci of precipitating SEPs on the solar surface (Top), and (Middle at $t = 12$ min and Bottom at $t = 48$ min) the latitude-longitude intensity distribution of escaped SEPs on the solar wind source surface at 2.5 $R_s$ when SEP transport has a perpendicular diffusion equal to 10% the rate derived from supergranular diffusion. The contours are in the log scale with an arbitrary intensity unit. The white curve shows the neutral sheet of the PFSS magnetic field model. Letters $E_m$ and $A_m$ indicate the footpoint of magnetic field lines to Earth and ST-A.
Figure 3. Distribution of escaped SEPs projected latitudinally on the solar equatorial plane at two time intervals \( t = 12 \) min and \( t = 48 \) min. The contours are in the \( \log_{10} \) scale with an arbitrary intensity unit. The top two panels include particles from all latitudes, while the bottom two panels only include particles at low latitudes between \(-30^\circ\) and \(30^\circ\). Increase of SEPs after the solar flare. There are no SEP source regions on magnetic field lines that connect to ST-A, which is consistent with ST-A observations.

Figure 2 shows calculation results of the latitude-longitude distribution of precipitating (top) and escaped (middle and bottom) SEPs with a finite \( k_{gd} = 0.1 \) level of perpendicular diffusion. The percentage of SEPs precipitating on the solar surface has increased to 4.0%, and the precipitation covers more solar surface area. This is because particles can diffuse across the magnetic field into close field lines. The longitudinal spread of escaped SEPs can be seen by comparing the distribution at 12 min and 48 min in the middle and bottom panels of Figure 2. The footpoint of the magnetic field line to ST-A does not lie within the source initially. The
SEPs spread in longitude, which occurs particularly more easily near the neutral sheet due to the combined effects of enhanced particle drift and perpendicular diffusion in weak field regions. This could be used to explain the gradual increase of SEP fluxes at ST-A. Figure 3 shows the intensity distributions of SEPs projected on the solar equatorial plane at 12 min and 48 min. It demonstrates that Earth is inside the regions of immediate radiation exposure, whereas the radiation will take a much longer time to reach ST-A. The bottom two panels only include the particles at low latitudes between $-30^\circ$ and $30^\circ$. Particle distribution spreads less in radial distance, mostly due to the tighter winding of Parker magnetic field lines at low latitudes.

4. Summary
We have described a computer code for conducting simulations of SEP propagation in the coronal and interplanetary magnetic fields. We use stochastic differential equations to solve the focus transport equation in 3-d. The transport equation essentially includes all the particle transport mechanisms in regular and turbulent magnetic fields. Because of the flexibility and simplicity of solving stochastic differential equations, our computer model can incorporate any complicated magnetic field geometry derived from magnetogram measurements of the solar photospheric magnetic field.

We have used the computer model to simulate the 2017 September 10 solar flare SEP event. Its results show our capability to predict SEP escape from the corona and precipitation on to the solar surface. A small fraction of SEP produced by the CME shock in the high corona can precipitate on the solar surface. These precipitating particles may produce high-energy, non-thermal gamma-rays or neutrons, some of which could be detected at Earth. The emission can be far away from the site of the solar flare that emits mostly in soft x-rays. Currently, we are conducting further simulations to investigate whether SEPs can be confined in the corona for many hours so that we can use the model to explain long-duration gamma-ray emissions detected by the Fermi telescope (Omodei et al. 2018). This situation is highly plausible, given the high levels of magnetic field turbulence left behind by the CME shock. The majority of SEPs produced and injected at the CME shock can escape to interplanetary space. Particle mirroring by the magnetic field is the major reason for the asymmetry of particle intensity between precipitation and escape. Although particles are only injected on open field lines with a limited latitude-longitude coverage, the distribution of escaped SEPs is essentially the entire range of latitude and longitude that is covered by the CME. Perpendicular diffusion can help spread the SEPs in longitude and latitude a little bit, particularly into weaker field regions near the neutral sheet. However, compared to the size of the CME, the role of perpendicular diffusion transport in the spread of particles within an hour is not that important. Subsequent particle transport in interplanetary space is needed to further spread the particles to locations where magnetic field lines are far away from the CME. The gradual increase of SEPs observed by ST-A is consistent with perpendicular particle transport in the interplanetary space.

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