A stable flat universe with variable cosmological constant in \( f(R, T) \) gravity

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Abstract In this paper, a general FRW cosmological model has been constructed in \( f(R, T) \) gravity reconstruction with variable cosmological constant. A number of solutions to the field equations has been generated by utilizing a form for the Hubble parameter that leads to Berman’s law of constant deceleration parameter \( q = m - 1 \). The possible decelerating and accelerating solutions have been investigated. For \( (q > 0) \) we get a stable flat decelerating radiation-dominated universe at \( q = 1 \). For \( (q < 0) \) we get a stable accelerating solution describing a flat universe with positive energy density and negative cosmological constant. Nonconventional mechanisms that are expected to address the late-time acceleration with negative cosmological constant have been discussed.

Key words: modified gravity — cosmology — deceleration parameter

1 INTRODUCTION

Recent observations suggest that the present universe is flat (de Bernardis et al. 2000; Bennett et al. 2003; Spergel et al. 2003) and expanding with acceleration (Perlmutter et al. 1999; Riess et al. 1998; Percival et al. 2001; Jimenez et al. 2003). This accelerating expansion cannot be explained in the framework of general relativity and the reason behind it is assumed to be an exotic form of energy with negative pressure known as “dark energy” (DE). Other observations (Spergel et al. 2003; Planck Collaboration 2014; Eisenstein et al. 2005) confirmed that about 70\% of the universe is made of DE. Several dynamical DE models with rolling scalar fields have been proposed to explain the nature of DE and the accelerated expansion, including for example quintessence (Fujii 1982; Ratra & Peebles 1988; Chiba et al. 1997; Ferreira & Joyce 1997, 1998; Copeland et al. 1998; Caldwell et al. 1998; Zlatev et al. 1999), Chaplygin gas (Kamenshchik et al. 2001), phantom energy (Caldwell 2002), k-essence (Chiba et al. 2000; Armendariz-Picon et al. 2000, 2001), tachyon (Sen 2002) and ghost condensate (Arkani-Hamed et al. 2004; Piazza & Tsujikawa 2004; Ahmed & Moss 2008).

Detecting the value and evolution of the equation of state parameter \( \omega = \frac{p}{\rho} \) is essential for understanding the nature of DE: \( \omega = 0 \) for dust, \( 1/3 \) for radiation and \(-1\) for vacuum energy (cosmological constant). The varying \( \omega \) that can become less than \(-1\) appears in scalar field models such as phantom \( \omega \leq -1 \), quintessence \(-1 \leq \omega \leq 1 \) and quintom, which is able to evolve across the cosmological constant boundary \( \omega = -1 \). The largest value of \( \omega \) consistent with causality is \( \omega = 1 \) for some exotic type of matter called stiff matter (stiff fluid or Zel’’dovich fluid), where the speed of sound is equal to the speed of light (Zeldovich 1972). A matter fluid with \( \omega = \frac{p}{\rho} \gg 1 \) is assumed to exist in Ekpyrotic cosmology (Khouri et al. 2001; Cai et al. 2012). Although recent observations show the consistency of the cosmological constant DE scenario (a perfect fluid with \( \omega_\Lambda = -1 \)), dynamical DE models are still allowed, especially those with equation of state across \(-1\) (Cai et al. 2010). The phenomenological dynamics associated with the \( \omega = -1 \) crossing behavior, dubbed as the quintom model, were first proposed in Feng et al. (2005). They give rise to
the equation of state $> -1$ in the past and $< -1$ today, satisfying recent observations. The bouncing solution in a universe dominated by quintom matter has been studied in Cai et al. (2007) where it has been found that the Big Bang singularity can be avoided in the presence of quintom matter (see Cai et al. 2010 for a comprehensive review on quintom cosmology). The simplest quintom model can be constructed by introducing two scalar fields with one being quintessence and the other phantom. Quintom-like behavior has also been found in the context of holographic DE (Zhang 2006).

Modified gravity theories represent another explanation based on modifying the geometrical part of the Einstein-Hilbert action. These theories can successfully explain the galactic rotation curves with no need for the DE assumption or cosmological constant (Nojiri & Odintsov 2006; Nojiri et al. 2008; Capozziello et al. 2009b,a; De Felice & Tsujikawa 2010). Several modified gravity theories have been proposed such as $f(R)$ gravity (Carroll et al. 2004; Capozziello et al. 2006; Nojiri & Odintsov 2006) where the Lagrangian is a function, $f$, of the Ricci scalar $R$, Gauss-Bonnet gravity (Nojiri & Odintsov 2005, 2011; Cognola et al. 2008; Nojiri et al. 2008) and torsional-based modified gravity $f(T)$ (Ferraro & Fiorini 2007; Bengochea & Ferraro 2009; Linder 2010) where $f(T)$ is an arbitrary function of the torsion scalar $T$. In the framework of $f(T)$ gravity and driven by the torsion effects, the accelerated cosmic expansion can be explained without the need to introduce a new form of energy (Bengochea & Ferraro 2009; Linder 2010). It can also easily be accommodated with regular thermal expansion history (Cai et al. 2016) and solve the problem of inflation with no inflaton (Ferraro & Fiorini 2007). While $f(R)$ gravity has fourth-order equations, an important advantage of $f(T)$ gravity is the second-order field equations which have attracted increasing interest in the literature.

Harko et al. (2011) generalized $f(R)$ gravity by introducing an arbitrary function $f(R, T)$ where $R$ is the curvature scalar and $T$ is the trace of the energy momentum tensor. $f(R, T)$ gravity is an interesting version of modified gravity theories as it can reproduce the unification of $F(R)$ and $F(T)$ gravity theories. Some cosmological aspects of $f(R, T)$ gravity have been already explored such as the thermodynamics of Friedmann-Robertson-Walker (FRW) spacetimes (Jiménez et al. 2012), the reconstruction of cosmological solutions where the late-time acceleration was obtained (Harko et al. 2011), future singularities (Houndjo et al. 2013), anisotropic cosmology (Ahmed & Pradhan 2014; Pradhan et al. 2015) and scalar perturbations (Alvarenga et al. 2013). For the case of ultra-relativistic fluids, the trace of the energy momentum tensor vanishes and so it is not included in the function $f(R, T)$. To correct this lack, a generalization has been suggested by including a new invariant, $R_{\mu\nu}T^{\mu\nu}$, in the function $f(R, T)$. This generalized model is called $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity (Odintsov & Sáez-Gómez 2013; Haghani et al. 2013).

In Ahmed & Pradhan (2014), a specific reconstruction of $f(R, T)$ gravity has been obtained with a time varying cosmological constant given by

$$\Lambda(t) = -\frac{1}{2} \left( \rho(t) - p(t) \right).$$

This variable cosmological constant has the same expression for the thermodynamical work density

$$W = \frac{1}{2} \left( \rho(t) - p(t) \right),$$

where $\rho$ and $p$ are the energy density and pressure of cosmic matter respectively (Cai & Kim 2005; Akbar & Cai 2007), which gives a thermodynamical meaning to the cosmological constant in this $f(R, T)$ gravity reconstruction. This specific $f(R, T)$ gravity model with varying cosmological constant has been used by many authors to study Bianchi cosmological models where a good agreement with cosmological observations has been obtained (Sahoo & Sivakumar 2015; Bishi et al. 2017; Mahanta 2014; Chaubey & Shukla 2017; Sharma & Pradhan 2018; Shaikh & Wankhade 2017). Using this $f(R, T)$ gravity model, Sahoo et al. investigated a class of Kaluza-Klein cosmological models (Sahoo et al. 2016). Ram & Chandel (2015) used this gravity model to investigate Bianchi type V cosmological solutions of a magnetized massive string. In this paper, we investigate the FRW cosmology of this $f(R, T)$ gravity model.

There is strong motivation in the literature to use variable cosmological constant which has been proposed as an attempt to solve some $\Lambda$CDM model problems. In spite of its good agreement with observations, the $\Lambda$CDM model suffers from several difficulties such as fine-tuning and cosmic coincidence. The fine-tuning problem refers to the huge discrepancy between the predicted vacuum energy and the observed one (old cosmological constant problem), while cosmic coincidence means that the observed cosmological constant in the present day universe is very close to the matter density (Perlmutter et al.
where negative cosmological constant have been investigated in the literature where it has been shown that a repulsive force (repulsive gravity) that accelerates the expansion. Alternative scenarios for accelerated expansion with negative cosmological constant have been investigated in the literature where it has been shown that a negative $\Lambda$ can halt eternal acceleration. This will be discussed in detail in Subsection 4.3.

The paper is organized as follows: Section 1 is an introduction. In Section 2, we review the derivation of modified field equations with variable cosmological constant from $f(R, T)$ gravity action. In Section 3, we derive the cosmological equations and utilize a form for the Hubble parameter that leads to Berman’s law of constant deceleration parameter $q = m - 1$ to generate a number of solutions. In Section 4, we perform a detailed analysis of the solutions and investigate all possible cosmological scenarios corresponding to different values of $q$. The final conclusion is included in Section 5.

2 DERIVATION OF FIELD EQUATIONS

The $f(R, T)$ gravity action is given by (Harko et al. 2011)

$$ S = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x, $$

(1)

where $L_m$ is the matter Lagrangian density. By varying the action $S$ with respect to $g^{\mu\nu}$, we obtain the field equations of $f(R, T)$ gravity as

$$ f_R(R, T) R_{\mu\nu} - \frac{1}{2} f(R, T) g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f_T(R, T) = 8\pi T_{\mu\nu} - f_T(R, T) T_{\mu\nu} - f_R(R, T) \Theta_{\mu\nu}, $$

(2)

where $\Box = \nabla^\mu \nabla_\mu$, $f_T(R, T) = \frac{\partial f(R, T)}{\partial T}$ and $\nabla_\mu$ denotes the covariant derivative. $\Theta_{\mu\nu}$ and the stress-energy $T_{\mu\nu}$ are given by

$$ \Theta_{\mu\nu} = -2T_{\mu\nu} - p g_{\mu\nu}, $$

$$ T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}, $$

(3)

where $u^\mu = (0, 0, 0, 1)$ is the four velocity which satisfies $u^\mu u_\mu = 1$ and $u^\mu \nabla_\nu u_\mu = 0$. $\rho$ and $p$ are the energy density and pressure of the fluid respectively. Different theoretical models can be obtained for each choice of $f$. Taking

$$ f(R, T) = f_1(R) + f_2(T), $$

the gravitational field described by Equation (2) becomes

$$ f_1'(R) R_{\mu\nu} - \frac{1}{2} f_1(R) g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f_1'(R) = 8\pi T_{\mu\nu} + f_2'(T) T_{\mu\nu} + \left(f_2'(T) p + \frac{1}{2} f_2(T)\right) g_{\mu\nu}. $$

(4)

We choose a specific form of the functions as

$$ f_1(R) = \lambda_1 R, $$

and

$$ f_2(T) = \lambda_2 T, $$

where $\lambda_1$ and $\lambda_2$ are arbitrary parameters. Taking

$$ \lambda_1 = \lambda_2 = \lambda $$

leads to

$$ f(R, T) = \lambda(R + T) $$

and then Equation (4) can be written as

$$ \lambda R_{\mu\nu} - \frac{1}{2} \lambda (R + T) g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) \lambda = 8\pi T_{\mu\nu} - \lambda T_{\mu\nu} + \lambda (2T_{\mu\nu} + p g_{\mu\nu}). $$

(5)

Setting

$$ (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) \lambda = 0 $$

and after simple rearrangement we get

$$ G_{\mu\nu} = \left( p + \frac{1}{2} T \right) g_{\mu\nu} = \frac{8\pi + \lambda}{\lambda} T_{\mu\nu}, $$

(6)

where

$$ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R $$

is the Einstein tensor. Recalling Einstein equations with cosmological constant

$$ G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}. $$

(7)
To ensure the same sign of the right hand side for (7) and (6), the inequality \( \frac{2\pi^2 - \lambda}{\lambda} > 0 \) must be satisfied which implies that \( \lambda > 0 \) or \( \lambda < -8\pi \). The term \( (p + \frac{1}{2} T) \) can now be considered as the negative of the cosmological constant. i.e., \( p + \frac{1}{2} T = -\Lambda(T) \). So, for the energy momentum tensor (3) we get

\[
\Lambda(t) = -\frac{1}{2}(\rho(t) - p(t)).
\]  

Taking into account the general equation of state \( p = \omega \rho \), then \( \Lambda(t) \) could also be expressed as

\[
\Lambda(t) = \frac{1}{2} \rho(t)(1 - \omega).
\]

3 COSMOLOGICAL EQUATIONS

The metric that describes a homogeneous and isotropic universe is the FRW metric given by

\[
ds^2 = -dt^2 + a^2(t) \times \left[ \frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right],
\]  

where \( r, \theta, \phi \) are comoving spatial coordinates, \( a(t) \) is the cosmic scale factor, \( t \) is time, and \( K \) is either 0, -1 or +1 for a flat, open or closed universe respectively. Applying Equation (6) to the metric (9) we get the following cosmological equations

\[
H^2 = \frac{1}{3} \left( \frac{8\pi + \lambda}{\lambda} - \frac{1}{2} \right) \rho + \frac{1}{6} p - \frac{K}{a^2},
\]

\[
\frac{\ddot{a}}{a} = \left( -\frac{8\pi + \lambda}{2\lambda} - \frac{1}{6} \right) \rho + \left( -\frac{8\pi + \lambda}{2\lambda} + \frac{1}{6} \right) p.
\]

The field Equations (10) and (11) contain three unknown parameters \( a, p \) and \( \rho \). In Maharaj & Naidoo (1993), the generalized Einstein equations with variable cosmological constant \( \Lambda(t) \) have been considered for the FRW metric in which the following variation of the Hubble parameter has been assumed

\[
H = Da^{-m},
\]

where \( D \) and \( m \) are constants. This Hubble parameter form (12) was first utilized in Berman (1983); Berman & de Mello Gomide (1988) and it leads to Berman’s law of constant deceleration parameter \( q = m - 1, m \geq 0 \). In Maharaj & Naidoo (1993), a number of classes of new solutions for variable cosmological constant \( \Lambda(t) \) has been introduced

\[
a = \begin{cases} 
(C + mDt)^\frac{m}{4} & \text{for } m \neq 0, \\
Ee^{Dt} & \text{for } m = 0.
\end{cases}
\]

In order to solve the modified field Equations (6) with their time varying cosmological constant

\[
\Lambda(t) = -\frac{1}{2}(\rho(t) - p(t)),
\]

we will make use of (13) and investigate different values of \( m \) corresponding to a decelerating expansion where \( q = m - 1 > 0 \) (i.e. \( m > 1 \)) and an accelerating expansion where \( q = m - 1 < 0 \) (i.e. \( m < 1 \)).

4 SOLUTIONS

4.1 The Case for \( q > 0 \) (\( m > 1 \)): A Decelerating Radiation-dominated Flat Universe

For \( m \neq 0 \), the metric (9) has the form

\[
ds^2 = -dt^2 + \left( C + mDt \right)^\frac{m}{4} \times \left[ \frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right].
\]

Substituting (13) in (10) and (11) we get the pressure \( p \) and energy density \( \rho \) as

\[
p = 72\lambda \times \left[ \frac{K(\pi + \frac{A}{8})}{(192\pi^2 + 40\pi \lambda + 3\lambda^2)(AmC + C)^2} \right] - \frac{2m\lambda(\pi + \frac{A}{8})}{(192\pi^2 + 40\pi \lambda + 3\lambda^2)(AmC + C)^2}
\]

\[
+ \frac{5}{4} \lambda(\pi + \frac{A}{8}) \left[ \frac{K(\pi + \frac{A}{8})}{(192\pi^2 + 40\pi \lambda + 3\lambda^2)(AmC + C)^2} \right].
\]

\[
\]
\[
\rho = 3\lambda \times \left[ \frac{24K(\pi + \frac{\lambda}{12})(Am\tau + C)^{2m-2}}{(192\pi^2 + 40\pi\lambda + 3\lambda^2)(Am\tau + C)^2} + \frac{(m + 1)\lambda + 24\pi A^2}{(192\pi^2 + 40\pi\lambda + 3\lambda^2)(Am\tau + C)^2} \right].
\]

Using (15) and (16) we obtain the equation of state parameter \(\omega\) as

\[
\omega = -\frac{16A^2((\pi + \frac{\lambda}{12})m - \frac{8}{3} - \frac{5}{12}\lambda)(Am\tau + C)^{\frac{m}{2}}}{24K(\pi + \frac{\lambda}{12})(Am\tau + C)^2} + \frac{24K(\pi + \frac{\lambda}{12})(Am\tau + C)^2}{24K(\pi + \frac{\lambda}{12})(Am\tau + C)^2} + ((m + 1)\lambda + 24\pi A^2).
\]

The cosmological constant (8) is given by

\[
\Lambda = -3\lambda \times \left[ \frac{K\lambda (Am\tau + C)^{2m-2}}{(192\pi^2 + 40\pi\lambda + 3\lambda^2)(Am\tau + C)^2} + \frac{(8\pi + 2\lambda - m(8\pi + \lambda)) A^2}{(192\pi^2 + 40\pi\lambda + 3\lambda^2)(Am\tau + C)^2} \right].
\]

Figure 1 (a), (b) and (c) shows variation of pressure \(p\), density \(\rho\) and cosmological constant \(\Lambda\) respectively versus time. The pressure is a positive decreasing function for flat and closed universes, i.e. it starts from a large positive value and then approaches a small positive value near zero. The energy density is negative for \(K = -1\) and hence the open universe is not possible. For flat and closed universes, the energy density is positive and tends to zero as the cosmic time \(t \to \infty\). The cosmological constant \(\Lambda\) (Fig. 1(c)) is positive, and it reaches a very small positive value at the current epoch which agrees with observations (Perlmutter et al. 1999; Riess et al. 1998; Tonry et al. 2003; Clocchiatti et al. 2006). The equation of state parameter (Fig. 1(i)) \(\omega = 0.3335984859 \approx \frac{1}{5}\) for \(K = 0\) which means a universe very close to the radiation-dominated era where the cosmic expansion was decelerating (Liddle 2003). It is generally believed that the universe is decelerating during the radiation and dark matter dominated eras, and is accelerating during the early and late-time inflations eras (Perico et al. 2013). The decelerated radiation-dominated era in this model arises with \(q = 1\), in good agreement with the complete cosmic history investigated in Perico et al. (2013). For \(K = 1\), \(\omega \to 1\) which means a decelerating universe approaching a stiff matter-dominated era where \(\omega = 1\).

Table 1 shows the behavior of \(\rho(t), \rho(t), \Lambda(t), c_s^2\) and energy conditions verses cosmic time for different values of \(m\).

### 4.2 Stability of Solutions

The physical acceptability of the model can be checked through testing the energy conditions and the sound speed. The null, weak, strong and dominant energy conditions are, respectively as marked by semicolons, given by (Hawking & Ellis 1973; Wald 1984)

\[
\rho + p \geq 0; \quad \rho \geq 0; \quad \rho + p \geq 0; \quad \rho + 3p \geq 0
\]

and

\[
\rho \geq |p|.
\]

The energy conditions have been plotted in Figure 1(d), (e) and (f). They are all satisfied for flat and closed universes. Now, the adiabatic square sound speed for any fluid is defined as: \(c_s^2 = \frac{dp}{d\rho}\). In addition to the positivity of \(c_s^2\), the causality condition must also be satisfied. Causality implies that the sound speed must not exceed the speed of light. Since we are using relativistic units in which \(c = G = 1\), the sound speed should exist within the range \(0 \leq \frac{dp}{d\rho} \leq 1\). For the current model, we get the sound speed as

\[
c_s^2 = \frac{-16mA^2((\pi + \frac{\lambda}{12})m - \frac{8}{3} - \frac{5}{12}\lambda)(Am\tau + C)^{\frac{m}{2}}}{mA^2(m\lambda + 24\pi + \lambda)(Am\tau + C)^{\frac{m}{2}} + 24K(\pi + \frac{\lambda}{12})(Am\tau + C)^2} + \frac{24K(\pi + \frac{\lambda}{12})(Am\tau + C)^2}{mA^2(m\lambda + 24\pi + \lambda)(Am\tau + C)^{\frac{m}{2}} + 24K(\pi + \frac{\lambda}{12})(Am\tau + C)^2},
\]

\[
= \frac{-16mA^2((\pi + \frac{\lambda}{12})m - \frac{8}{3} - \frac{5}{12}\lambda)(Am\tau + C)^{\frac{m}{2}}}{mA^2(m\lambda + 24\pi + \lambda)(Am\tau + C)^{\frac{m}{2}} + 24K(\pi + \frac{\lambda}{12})(Am\tau + C)^2} + \frac{24K(\pi + \frac{\lambda}{12})(Am\tau + C)^2}{mA^2(m\lambda + 24\pi + \lambda)(Am\tau + C)^{\frac{m}{2}} + 24K(\pi + \frac{\lambda}{12})(Am\tau + C)^2}.
\]
Fig. 1 The behavior of $p$, $\rho$, $\Lambda$, $\omega$, $c_s^2$ and energy conditions verses cosmic time for $m = 2$. Here $A = C = 2$ and $\lambda = 0.01$. $\omega \approx \frac{1}{3}$ for a flat universe.

Figure 1(g) shows that this quantity is always less than one for $K = 0$ and tends to 1 for $K = 1$. Again, for the stiff matter dominated era the sound speed is equal to the speed of light. So, in summary, $K = 0$ describes a stable flat decelerating universe very close to the radiation dominated era ($\omega \approx \frac{1}{3}$), while $K = 1$ corresponds to a stable closed decelerating universe approaching the stiff matter dominated era ($\omega \approx 1$ and $c_s^2 \approx 1$). Although all figures have been plotted using $\lambda = 0.01$, we have tried other values for $\lambda$ in Figure 1(h) for the flat $K = 0$ case supported by observations. We have found that $c_s^2$ is always $\leq 1$ for any positive value of $\lambda$. As $\lambda$ increases,
4.3.1 For \( m = 0 \) (\( q = -1 \))

For \( m = 0 \), metric (9) is

\[
ds^2 = -dt^2 + Ee^{2Mt}[\frac{dr^2}{1 - Kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2].
\]

The expressions for \( p, \rho, \Lambda \) and \( \omega \) are

\[
p = \frac{120\lambda \left(\frac{4}{9}(\lambda + \frac{4}{3})Ke^{-2M} + E^2M^2\left(\pi + \frac{4}{3}\right)\right)}{E^2\left(192\pi^2 + 40\pi\lambda + 3\lambda^2\right)},
\]

(20)

\[
\rho = \frac{72\lambda \left(K\left(\pi + \frac{4}{3}\right)Ke^{-2M} + E^2M^2\left(\pi + \frac{4}{3}\right)\right)}{E^2\left(192\pi^2 + 40\pi\lambda + 3\lambda^2\right)},
\]

(21)

\[
\Lambda = -\frac{24\lambda \left(\frac{4}{9}\lambda Ke^{-2M} + E^2M^2\left(\pi + \frac{4}{3}\right)\right)}{E^2\left(192\pi^2 + 40\pi\lambda + 3\lambda^2\right)},
\]

(22)

\[
\omega = \frac{24K\left(\pi + \frac{4}{3}\right)e^{-2M} + 40E^2M^2\left(\pi + \frac{4}{3}\right)}{24K\left(\pi + \frac{4}{3}\right)e^{-2M} + 24E^2M^2\left(\pi + \frac{4}{3}\right)}.
\]

(23)

In this case, we have a stable flat universe with a negative cosmological constant and positive energy density (Table 1). Only the spatially flat universe is allowed (as observations tell us) where the closed and open universes have negative energy density. Although recent observations suggest a positive cosmological constant \( \Lambda > 0 \), a negative cosmological constant which can fit a large data set quite well is also possible and can solve the eternal acceleration problem (Cardone et al. 2008). The flat and accelerating universe, confirmed by observations, does not have to keep accelerating forever and this eternal acceleration is a consequence of assuming a positive cosmological constant to explain the accelerated expansion (Cardone et al. 2008). This radically different approach of a negative cosmological constant has been investigated by many authors (Cardenas et al. 2003; Grande et al. 2006; Gu & Hwang 2006; Cardone et al. 2008; Prokopec 2011; Landry et al. 2012, Kayll Lake, Maeda & Ohta 2014; Baier et al. 2015; Chruściel et al. 2018). The AdS/CFT correspondence presents a strong argument that the negative \( \Lambda \) case should also be examined (Aharony et al. 2000). Prokopec (2011) introduced modifications to Friedmann cosmology that can lead to observationally viable cosmologies with \( \Lambda < 0 \). Maeda and Ohta studied gravitational theories in 2014 with a cosmological constant and the Gauss-Bonnet curvature squared term, and found a stable de Sitter solution with negative \( \Lambda \). Landry et al. (2012) studied the McVittie solution with \( \Lambda < 0 \) and found that the negative \( \Lambda \) case ensures collapse to a Big Crunch as in the pure FRW case. Chruściel et al. (2018) proved existence of large families of solutions of Einstein-complex scalar field equations with \( \Lambda < 0 \). So, the accelerating solution we have obtained for \( m = 0 \) (\( q = -1 \)) with negative \( \Lambda \) is interesting and has been an active research point.

4.3.2 Other values for \( m < 1 \)

We have investigated other values for \( m < 1 \) as shown in Table 2. While we have \( \Lambda < 0 \) and \( \rho > 0 \) for all \( K \), the stability of the solutions is weaker than the \( m = 0 \) case in which only the flat universe is possible.
5 CONCLUSIONS

We have constructed a general FRW cosmological model in $f(R, T)$ gravity reconstruction with variable cosmological constant. A specific form for the Hubble parameter that leads to Berman’s law $q = m - 1$ has been utilized which generated a number of solutions to the modified Friedmann equations. The possibility and stability of decelerating ($q > 0$) and accelerating ($q < 0$) solutions have been investigated. For the decelerating case, we found the most stable solution is a decelerating radiation-dominated flat universe at $q = 1$, in good agreement with cosmic history in the literature. For the accelerating case, the most stable solution is a flat universe with positive energy density and a negative cosmological constant at $q = -1$. We have discussed the possibility of several stable accelerating solutions with $\Lambda < 0$ in the literature in which the eternal acceleration problem disappears. Nonconventional mechanisms that are expected to address the late-time acceleration have been an active research area and it is commonly acknowledged that application to the early universe would be nontrivial as well. For example, a nonsingular bouncing solution might arise in these models (Cai 2014). In the context of inflationary cosmology, there are several conceptual challenges without solutions which provide a good motivation to search for alternative proposals describing the very early universe beyond the standard inflationary $\Lambda$CDM model. In particular, we mention two interesting phenomenological scenarios alternative to the inflationary $\Lambda$CDM model which are the healthy nonsingular bouncing cosmology (Cai et al. 2012) and the matter-bounce in inflationary scenario (Cai et al. 2009). In the matter-bounce inflationary scenario, the inflationary cosmology is generalized by introducing a matter-like contracting phase before the inflationary phase. This scenario was found to be very powerful in reconciling the tension between Planck and BICEP2 observations when compared with $\Lambda$CDM (Xia et al. 2014). In bouncing cosmology, the expansion of the universe is preceded by an initial phase of contraction with a bouncing point connecting the contraction and expansion. A comprehensive introduction has been given in Cai (2014).

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| $m$ | $p$ | $\rho$ | $\Lambda$ | $c_s^2$ | $p + 3\rho$ | $\rho - p$ |
|---|---|---|---|---|---|---|
| $0.1$ | $+$ for all $K$ | $+$ for all $K$ | $-$ for all $K$ | invalid for all $K$ | $+$ for all $K$ | $-$ for all $K$ |
| $0.2$ | $+$ for all $K$ | $+$ for all $K$ | $-$ for all $K$ | invalid for all $K$ | $+$ for all $K$ | $-$ for all $K$ |
| $0.4$ | $+$ for all $K$ | $+$ for all $K$ | $-$ for all $K$ | invalid for all $K$ | $+$ for all $K$ | $-$ for all $K$ |
| $0.6$ | $+$ for all $K$ | $+$ for all $K$ | $-$ for all $K$ | invalid for all $K$ | $+$ for all $K$ | $-$ for all $K$ |
| $0.7$ | $+$ for all $K$ | $+$ for all $K$ | $-$ for all $K$ | invalid for all $K$ | $+$ for all $K$ | $-$ for all $K$ |
| $0.8$ | $+$ for all $K$ | $+$ for all $K$ | $-$ for all $K$ | invalid for all $K$ | $+$ for all $K$ | $-$ for all $K$ |
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