Strongly Asymmetric Tricriticality of Quenched Random-Field Systems

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In view of the recently seen dramatic effect of quenched randomness on tricritical systems, we have conducted a renormalization-group study on the effect of quenched random fields on the tricritical phase diagram of the spin-1 Ising model in \(d = 3\). We find that random fields convert first-order phase transitions into second-order, in fact more effectively than random bonds. The coexistence region is extremely flat, attesting to an unusually small tricritical exponent \(\beta_u\); moreover, an extreme asymmetry of the phase diagram is very striking. To accomodate this asymmetry, the second-order boundary exhibits reentrance.

Tricritical phase diagrams of three-dimensional \((d = 3)\) systems are strongly affected by quenched bond randomness: The first-order phase transitions are replaced, gradually as randomness is increased, by second-order phase transitions. The intervening random-bond tricritical point moves towards, and eventually reaches, zero-temperature, as the amount of randomness is increased. This behavior is an illustration of the general prediction that first-order phase transitions are converted to second-order by bond randomness \([1-7]\), in a thresholded manner in \(d = 3\). The randomness threshold increases from zero at the non-random tricritical point. The random-bond tricritical point maps, under renormalization-group transformations, onto a doubly unstable fixed distribution at strong coupling. Random-bond tricritical points exhibit a remarkably small value for the tricritical exponent \(\beta_u = 0.02\), reflected in the near-flat top of the coexistence region. In the conversion of the first-order phase transition to second-order, traced by the random-bond tricritical point, a strong violation of the empirical universality principle occurs, via a renormalization-group fixed-point mechanism. Thus, detailed information now exists on the effect of quenched bond randomness on tricritical points, revealing several qualitatively distinctive features. \([5,8]\)

No such information has existed on the effect of quenched field randomness on tricritical points. This topic is of interest also because renormalization-group studies have shown that quenched field randomness induces, under scale change, quenched bond randomness, as the presence of quenched field randomness continues. \([6,8]\) Accordingly, we have conducted a global renormalization-group study of a tricritical system in \(d = 3\) under quenched random fields. The results, presented below, show that these systems have their own distinctive behavior which is qualitatively different from that of non-random or random-bond tricritical systems. The latter distinction yields a microscopic physical intuition on the different effects of the two types of quenched randomness.
randomness. We have studied the Blume-Emery-Griffiths (i.e., spin-1 Ising) model under quenched field randomness. The Hamiltonian is

\[- \beta H = \sum_{<ij>} \left[ J s_i s_j + K s_i^2 s_j^2 - \Delta (s_i^2 + s_j^2) \right. \]

\[+ H_{ij} (s_i + s_j) + H_{ij}^\dagger (s_i - s_j) \] \hspace{1cm} (1)

where \(s_i = \pm 1, 0\) at each site \(i\) of a simple cubic \((d = 3)\) lattice and \(<ij>\) indicates summation over all nearest-neighbor pairs of sites. The quenched random fields \(H_{ij}, H_{ij}^\dagger\) are taken from a distribution

\[P(H, H^\dagger) = \frac{1}{4} \left[ \delta(H + \sigma_H) \delta(H^\dagger + \sigma_H) \right. \]

\[+ \delta(H + \sigma_H) \delta(H^\dagger - \sigma_H) + \delta(H - \sigma_H) \delta(H^\dagger + \sigma_H) + \delta(H - \sigma_H) \delta(H^\dagger - \sigma_H) \] \hspace{1cm} (2)

All other interactions in the initial Hamiltonian (1) are non-random. Under renormalization-group transformations, the Hamiltonian (1) maps onto a random-field random-bond Hamiltonian,

\[- \beta H = \sum_{<ij>} \left[ J_{ij} s_i s_j + K_{ij} s_i^2 s_j^2 - \Delta_{ij} (s_i^2 + s_j^2) - \Delta_{ij}^\dagger (s_i^2 - s_j^2) \right. \]

\[+ L_{ij} (s_i^2 s_j + s_i s_j^2) + L_{ij}^\dagger (s_i^2 s_j - s_i s_j^2) + H_{ij} (s_i + s_j) + H_{ij}^\dagger (s_i - s_j) \] \hspace{1cm} (3)

where all interactions are quenched random, with a distribution function \(P\left(J_{ij}, K_{ij}, \Delta_{ij}, \Delta_{ij}^\dagger, L_{ij}, L_{ij}^\dagger, H_{ij}, H_{ij}^\dagger\right)\) determined by the renormalization-group transformation. Specifically, the first four arguments here reflect the rescaling-induced bond randomness of the random-field system.

The renormalization-group transformation is contained in the mapping between the quenched probability distributions of the starting and rescaled systems,

\[P'\left(\overline{R}_{i'j'}\right) = \int \left[ \prod_{<ij>} d\overline{R}_{ij} P\left(\overline{R}_{ij}\right) \right] \delta\left(\overline{R}_{i'j'} - \overline{R}\left(\{R_{ij}\}\right)\right) \] \hspace{1cm} (4)

where \(\overline{R}_{ij} \equiv (J_{ij}, K_{ij}, \Delta_{ij}, \Delta_{ij}^\dagger, L_{ij}, L_{ij}^\dagger, H_{ij}, H_{ij}^\dagger)\) are the interactions at locality \(<ij>\), the primes refer to the renormalized system, the product is over all unrenormalized localities \(<ij>\) whose interactions \(\{\overline{R}_{ij}\}\) influence the renormalized interaction \(\overline{R}_{i'j'}\), and \(\overline{R}\left(\{\overline{R}_{ij}\}\right)\) is a local recursion relation that embodies the latter dependence. Simply said, Eq.(4) sums over the joint probabilities of the values of neighboring unrenormalized interactions that conspire to yield a given value of the renormalized interaction. The phenomena characteristic to quenched randomness should derive from the probability convolution shown in Eq.(4), rather than the precise form of the recursion \(\overline{R}\) that should be a smooth local function. In this work, we use the Migdal-Kadanoff recursion.
relation, given for this system in [10]. The convolution is
effected by representing \( P(K_{ij}) \) in terms of bins, the
degree of detail (i.e., the number of bins) reflecting the level
of approximation. In this work, we have used 531,441
bins, corresponding to renormalization-group flows in a
4,782,969-dimensional space. The application of Eq.(4)
via the binning procedure has been described elsewhere
[8,11].

Our main result, the striking difference between the
three types of \( d = 3 \) Ising tricriticality, is evident in
Fig.1, where the calculated random-field, random-bond,
and non-random phase diagrams are superimposed. The
same amount of quenched randomness \( \sigma_H = 0.2 = \sigma_\Delta \),
as in Eq.(2)] is used, for relevance of comparison. It is
seen that both bond randomness and field randomness
convert first-order phase transitions to second-order in a
thresholded manner, but that field randomness is more
effective than bond randomness in this conversion. Both
types of random tricritical points occur at remarkably
near-flat tops of coexistence regions, reflecting the unusually small values of the exponent \( \beta_u \), but the random-field
phase diagram is most strikingly asymmetrical. The tri-
critical point occurs at the high density \(< s^2_i > = 0.835
(as opposed to 0.613 and 0.665 in the random-bond and
non-random systems, respectively), essentially all of the
near-flat portion of the coexistence top occurring on the
dilute branch of the coexistence boundary. To accomo-
date this asymmetry, the randomness-extended line of
second-order phase transitions has to curve over and ex-
hibit reentrant behavior \[12\] as a function of tempera-
ture. This difference in behavior comes from the fact
that random bonds destroy order-disorder coexistence
without destroying order itself \[3,4\], whereas random
fields destroy both order-disorder coexistence, through
the rescale-generated bond randomness, and order per
se. The latter is more effective near the tricritical point,
where considerable vacancy fluctuations occur within the
ordered phase.

Random-field and random-bond tricritical points
renormalize onto obviously different doubly unstable
fixed distributions (the field variables \( L \) and \( H \) remain
at zero in the latter case).

It is seen in Fig.1 that the coexistence boundary of
either type of random system follows that of the non-
random system, until the temperature-lowered tricriti-
cal region sets in relatively abruptly. This is similar to
the magnetization of random-field systems following the
non-random curve until the critical region sets in quite
abruptly. \[13,11\]

On the high-temperature side of the tricritical point,
it has been known \[8\] that the break in slope of the criti-
cal line, in the random-bond system, is connected to the
strong violation of the empirical universality principle,
segments on each side of this point having their own crit-
ical exponents, respectively of the strong-coupling and
non-strong-coupling type. No such universality violation
occurs along the second-order line of the random-field
system, the entire line having random-field critical exponents that are governed by a strong-coupling fixed distribution, implying a modified hyperscaling relation [14,15].

The global phase diagrams of the random-field and random-bond systems are given in Figs.2(a,b). It is seen that all first-order phase transitions are completely converted to second-order, at a zero-temperature tricritical point, for \( \sigma_H \simeq 0.5 \) and \( \sigma_\Delta \simeq 0.7 \), respectively. Furthermore, the ordered phase is eliminated at \( \sigma_H \simeq 0.9 \) in the random-field case, but persists for all \( \sigma_\Delta \) in the random-bond case.

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Figure Captions

**Fig. 1:** Calculated tricritical phase diagrams for non-random (dotted), random-bond (full), and random-field (dashed) \( d = 3 \) systems for \( K = 0 \). In each phase diagram, a line of second-order phase transitions extending to high temperatures meets, at a tricritical point, a coexistence region extending to low temperatures. Note the
near-flat top of the coexistence regions in both quenched random systems, and the extreme asymmetry of the random-field system. Thus, field randomness is more effective than bond randomness in converting first-order transitions into second-order (i.e., the tricritical point is at lower temperature).

**Figs 2:** Calculated $d = 3$ global phase diagrams for $K = 0$: (a) Random-bond systems exhibit two universality classes of second-order phase transitions (thin and thick full lines) and first-order phase transitions (circles). (b) Random-field systems exhibit second-order (full lines) and first-order (circles) phase transitions. In both types of quenched randomness, the first-order transitions cede under increased randomness. The line of tricritical points (dashed) reaches zero temperature, as all transitions become second-order. In the random-field case, the ordered phase disappears under further randomness, whereas in the random-bond case, the ordered phase (and the strong violation of universality) persists for all randomness.
FIG. 1

A phase diagram illustrating the coexistence of ordered and disordered phases as a function of density and temperature. The diagram includes lines for different system parameters:
- Pure system
- $\sigma_\Delta = 0.2$
- $\sigma_H = 0.2$

The diagram shows the boundary between ordered and disordered phases at various temperatures and densities.
FIG. 2a
FIG. 2b