Dashen’s theorem and electromagnetic masses of the mesons

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Abstract

Employing $U(3)_L \times U(3)_R$ chiral field theory, we find that Dashen’s theorem, which holds for pseudoscalar mesons, can be generalized to the sector of axial-vector mesons, however, fails for vector mesons.

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1. Chiral symmetry plays an important role in describing the low energy hadronic physics. Based on chiral symmetry in three-flavor space, there are three multiplets due to spontaneous symmetry breaking: an octet of massless pseudoscalar mesons $0^- (\pi, K, \eta)$, and two massive multiplets with quantum number $1^- (\rho, \omega, K^* \text{ and } \phi)$ and $1^+ (a_1, K_1, f \text{ and } f_s)$. Significant SU(3) symmetry breaking will result in the mass splittings of these low-lying mesons in the same multiplet.

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Also, the energy of the virtual photon cloud around these mesons can cause the mass differences between charged particles and their corresponding neutral partners.

The earliest study on electromagnetic masses of mesons has been done by Dashen[1] nearly thirty years before. Dashen’s theorem, which states that the square electromagnetic mass difference between the charge pseudoscalar mesons and their corresponding neutral partners are equal in the chiral SU(3) limit, is expressed as

\[
(m_{\pi^\pm}^2 - m_{\pi^0}^2)_{EM} = (m_{K^\pm}^2 - m_{K^0}^2)_{EM},
(m_{\pi^0}^2)_{EM} = 0, \quad (m_{K^0}^2)_{EM} = 0.
\]

The subscript EM denotes electromagnetic mass.

The three multiplets(0−, 1− and 1+) correspond to the same quark constituent but different spin or parity in the quark model. The mass gaps between them are due to spontaneous chiral symmetry breaking. Thus a very natural question is to ask whether Dashen’s theorem(which holds for pseudoscalar mesons) can be generalized to be obeyed by vector and axial-vector mesons. In this paper, by employing \(U(3)_L \times U(3)_R\) chiral theory[2, 3], it will be shown that the generalization to axial-vector mesons is valid, or, in the chiral SU(3) limit,

\[
(m_{a^\pm}^2 - m_{a^0}^2)_{EM} = (m_{K_1^\pm}^2 - m_{K_1^0}^2)_{EM},
(m_{a^0}^2)_{EM} = 0, \quad (m_{K_1^0}^2)_{EM} = 0.
\]

However, similar generalization fails for vector mesons. The latter is the same conclusion as one given by Bijnens and Gosdzinsky[4] based on the heavy vector meson chiral Lagrangian[5, 6, 7], but Eq.(2) is new.

The formalism of \(U(3)_L \times U(3)_R\) chiral field theory of pseudoscalar, vector, and axial-vector mesons has been described in Refs.[2, 3]. This theory is of self-consistency and phenomenological success, and provides a unified description of meson physics at low energies. Because various meson-processes or properties related to strong, electromagnetic and weak interactions have been calculated in Refs.[2, 3] and the results are in good agreement with the experimental data, all chiral coefficients \(L_1, ..., L_{10}\) in the chiral perturbation theory(\(\chi PT\))[10, 11] corresponding to these
processes can be well determined. Actually, two of them ($\alpha_1 = 4L_1 + 2L_3, \alpha_2 = 4L_2$) have already been shown in\cite{2}. Therefore the present theory is believed to be consistent with both \chi PT and the chiral coupling theory of resonances\cite{12, 13}. A systematic study on this issue will be presented in detail elsewhere. In the present paper, we focus our attention on the investigation upon the electromagnetic masses of the mesons and the generalization of Dashen’s theorem in the framework of $U(3)_L \times U(3)_R$ chiral theory.

In the $U(3)_L \times U(3)_R$ chiral theory, vector meson dominance (VMD)\cite{8} in the meson physics is a natural result of this theory instead of an input. Namely, the dynamics of the electromagnetic interactions of mesons has been introduced and established naturally. Therefore, the present theory makes it possible to evaluate the electromagnetic self-energies of these low-lying mesons systematically. According to this pattern, for example, we can work out the well known old result of $(m_{\pi^\pm}^2 - m_{\pi^0}^2)_{EM}$ given by Das et al\cite{9}, which serves as the leading order in our more accurate evaluations (to see below). This indicates that our pattern of evaluating the electromagnetic self-energies of mesons is consistent with the known theories in the $\pi -$meson case under the lowest energies limit. Since the dynamics of mesons, including pseudoscalar, vector and axial-vector, is described in the present theory, the method of calculating electromagnetic masses in this paper is legitimate not only for $\pi$ and $K -$mesons, but also for $a_1$, $K_1$, $\rho$ and $K^* -$mesons. Thus, both checking the Dashen’s argument of Eq.(1) in the framework of the effective quantum fields theory and investigating its generalizations mentioned above become practical.

The basic Lagrangian of this chiral field theory is (hereafter we use the notations in Refs.\cite{2, 3})

$$\mathcal{L} = \bar{\psi}(x)(i\gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a\gamma_5 - mu(x))\psi(x) + \frac{1}{2}m_1^2(\rho_\mu^\mu \rho_\mu + \omega^\mu \omega_\mu + a_\mu^\mu a_\mu + f^\mu f_\mu) + \frac{1}{2}m_2^2(K^a_\mu K^{a\mu} + K_1^\mu K_1\mu) + \frac{1}{2}m_3^2(\phi_\mu^\mu + f_\mu^\mu s_\mu)$$

(3)

where $u(x) = \exp[i\gamma_5(\tau_i \pi_i + \lambda_i K^a + \eta + \eta')(i=1, 2, 3 \text{ and } a=4, 5, 6, 7)], a_\mu = \tau_\mu a_\mu^i + \lambda_i K_{1\mu}^a + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)f_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)f_{s\mu}, v_\mu = \tau_\mu \rho_\mu^i + \lambda_i K_{1\mu}^a + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)\omega_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)\phi_\mu$. The $\psi$ in Eq.(1) is
$u,d,s$ quark fields. $m$ is a parameter related to the quark condensate. Here, the mesons are bound states in QCD, and they are not fundamental fields. Therefore, in Eq.(1) there are no kinetic terms for these fields and the kinetic terms will be generated from quark loops.

Following Refs. [2, 3], the normal part and abnormal part of the effective Lagrangians, i.e. $L_{RE}$ and $L_{IM}$, can be obtained by performing path integration over quark fields. The dynamics of the mesons can be read off from $L_{RE}$ and $L_{IM}$. It is natural to obtain VMD in the present theory, which reads

$$L_{ργ} = -e\frac{f_ρ}{f_ρ} (\partial_μ A_γ^ρ - \partial_γ A_μ^ρ),$$

$$L_{ωγ} = -e\frac{f_ω}{f_ω} (\partial_μ A_γ^ω - \partial_γ A_μ^ω),$$

$$L_{φγ} = -e\frac{f_φ}{f_φ} (\partial_μ A_γ^φ - \partial_γ A_μ^φ).$$  (4)

The direct couplings of photon to other mesons are constructed through the substitutions

$$ρ_μ^0 \rightarrow e\frac{f_ρ}{f_ρ} A_μ, \quad ω_μ \rightarrow e\frac{f_ω}{f_ω} A_μ, \quad φ_μ \rightarrow e\frac{f_φ}{f_φ} A_μ.$$  (5)

where

$$\frac{1}{f_ρ} = \frac{1}{2} g, \quad \frac{1}{f_ω} = \frac{1}{6} g, \quad \frac{1}{f_φ} = -\frac{1}{3\sqrt{2}} g.$$  (6)

g is a universal coupling constant in this theory. It can be determined by taking the experimental value of $m_a$ as an input [2, 3, 14].

Using $L_i(Φ, γ, ...)|_{Φ=π,a,v,...}$ (which can be obtained from $L_{RE}$ or $L_{IM}$), we can evaluate the following S-matrix

$$S_Φ = \langle Φ | T \exp[i \int dx^4 L_i(Φ, γ, ...)] - 1 | Φ \rangle |_{Φ=π,a,v,...}.$$  (7)

On the other hand $S_Φ$ can also be expressed in terms of the effective Lagrangian of $Φ$ as

$$S_Φ = \langle Φ | i \int d^4x L_{eff}(Φ) | Φ \rangle.$$  (8)

Noting $L = \frac{1}{2} \partial_μ Φ \partial^μ Φ - \frac{1}{2} m_Φ^2 Φ^2$, then the electromagnetic interaction correction to the mass of $Φ$ reads

$$δm_Φ^2 = \frac{2iS_Φ}{\langle Φ | Φ^2 | Φ \rangle}.$$  (8)
where $\langle \Phi | \Phi^2 | \Phi \rangle = \langle \Phi | \int d^4x \Phi^2(x) | \Phi \rangle$. Thus, all of virtual photon contributions to mass of the mesons can be calculated systematically.

In the following, firstly, we shall re-examine Dashen’s theorem for pseudoscalar mesons in the framework of the present theory. Then the generalization of this theorem for axial-vector and vector mesons is studied. Finally, we give the summary and conclusions.

2. Due to VMD (Eq.(4)), the interaction Lagrangians which contribute to electromagnetic mass of the mesons have to contain the neutral vector meson fields which only include $\rho^0$, $\omega$ and $\phi$.

When the calculations are of $O(\alpha_{em})$ and one-loop, there are two sorts of vertices contributing to electromagnetic self-energies of pseudoscalar mesons: four points vertices and three points vertices. The former must be the coupling of two pseudoscalar fields and two neutral vector mesons fields, and the latter must be the interaction of a pseudoscalar field to a neutral vector meson plus another field.

Thus, from Refs.[2, 3], the interaction Lagrangians contributing to electromagnetic mass of pseudoscalar mesons($\pi$ and $K$) in the chiral limit can be easily obtained, which reads as follows

$$L_{\rho\rho\pi\pi} = \frac{4F^2}{g^2f_\pi} (\rho_\mu^0 \rho^0_{\mu} + \frac{1}{2\pi^2F^2} \partial_\mu \rho_\mu^0 \rho^0_{\mu}) \pi^+ \pi^-,$$

$$L_{\rho\pi a} = \frac{2i\gamma F^2}{f_\pi g^2} \rho_\mu^0 (a^{-\mu} - \frac{1}{2\pi^2F^2} \partial^2 a^{-\mu}) + h.c.,$$

$$L_{K^+K^-\nu\nu} = \frac{F^2}{f_\pi g^2} \{ (\rho^0_{\mu} + \nu^8_{\mu})^2 + \frac{1}{2\pi^2F^2} (\partial_\nu \rho^0_{\mu} + \partial^\nu \nu^8_{\mu})^2 \} K^+ K^-,$$

$$L_{K^{+\pm}K^{-\mp}} = \frac{i\gamma F^2}{g^2f_\pi} (\rho^0_{\mu} + v^8_{\mu}) K^- K^{+\mp} \left( K^{-\mu} - \frac{1}{2\pi^2F^2} \partial^2 K_{-\mu} \right) + h.c.,$$

$$L_{K^0\bar{K}^0\nu\nu} = \frac{F^2}{f_\pi g^2} \{ (\rho^0_{\mu} + \nu^8_{\mu})^2 + \frac{1}{2\pi^2F^2} (\partial_\mu \rho^0_{\mu} + \partial^\nu \nu^8_{\mu})^2 \} K^0 \bar{K}^0,$$

$$L_{K^0\bar{K}^0\nu\nu} = \frac{i\gamma F^2}{g^2f_\pi} (-\rho^0_{\mu} + \nu^8_{\mu}) \bar{K}^0 (\bar{K}^{0\mu} - \frac{1}{2\pi^2F^2} \partial^2 \bar{K}^{0\mu}) + h.c.$$

where

$$\pi^\pm = \frac{1}{\sqrt{2}} (\pi^1 \pm i\pi^2), \quad a^\pm = \frac{1}{\sqrt{2}} (a^1 \pm ia^2),$$

5
\[
K^\pm = \frac{1}{\sqrt{2}}(K^4 \pm iK^5), \quad K^0(K^0) = \frac{1}{\sqrt{2}}(K^6 \pm iK^7),
\]
\[
K^\pm_{1\mu} = \frac{1}{\sqrt{2}}(K^4_{1\mu} \pm iK^5_{1\mu}), \quad K^0_{1\mu}(K^0_{1\mu}) = \frac{1}{\sqrt{2}}(K^6_{1\mu} \pm iK^7_{1\mu}).
\]

with
\[
F^2 = \frac{f_\pi^2}{1 - \frac{c\gamma^2}{g^2}}, \quad c = \frac{f_\pi^2}{2gm_\rho^2},
\]
\[
\gamma = (1 - \frac{1}{2\pi^2 g^2})^{-1/2}.
\]

where \(v\) denotes the neutral vector mesons \(\rho^0, \omega\) and \(\phi\), \(v^8_\mu = \omega_\mu - \sqrt{2}\phi_\mu\). \(f_\pi\) is pion’s decay constant, and \(f_\pi = 0.186\text{GeV}\). Here, we neglect the contributions to electromagnetic mass of pions or kaons which are proportional to \(m_\pi^2\) or \(m_K^2\), because we are only interested in the case of chiral limit.

Note that there are no contributions to electromagnetic masses of \(\pi^0\) in the chiral limit. This means
\[
(m_{\pi^0})_{EM} = 0, \quad \text{for massless quark.}
\]

The major difference of the kaon and pion Lagrangians is that in the kaon case all the three nonet vector resonances \(\rho^0, \omega\) and \(\phi\) contribute, which results in a contribution to the neutral kaon electromagnetic self-energy. This contribution vanishes in the SU(3)-vector-meson symmetry limit that the three vector masses are equal, i.e. \(m_\rho = m_\omega = m_\phi\) (to see below). These results are in agreement with ones given by Donoghue and Perez in Ref.\[15\] in the framework of \(\chi\)PT.

The Lagrangians contributing to electromagnetic masses of \(K^\pm\) are different from ones to electromagnetic mass of \(K^0\). The difference comes from the structure constants of SU(3) group: \(f_{345} = -f_{367} = \frac{1}{2}, f_{458} = f_{678} = \frac{\sqrt{3}}{2}\). Note that the vector meson fields\((\rho, \omega\) and \(\phi)\) and axial-vector meson fields\((a_1\) and \(K_1(1400))\) in the above Lagrangians would always appear as propagators in the S-matrices which can contribute the electromagnetic self-energies of pseudoscalar mesons. Using \(\text{Eq. (4)}\) with SU(3) symmetry limit \(m_\rho = m_\omega = m_\phi\), we can easily obtain that \(v^8_\mu\) is equivalent to \(\rho^0_\mu\) in calculating the electromagnetic masses of the mesons. Thus, the Lagrangians contributing to
electromagnetic masses of $K^0$ will vanish, namely,

$$(m_{K^0}^2)_{EM} = 0, \quad \text{in the chiral SU}(3) \text{ limit.} \quad (17)$$

Likewise, we can conclude that the Lagrangian (11) and (12) are entirely equivalent to Lagrangians (9) and (10) respectively under the limit of $m_\rho = m_\omega = m_\phi$ and $m_a = m_{K_1}$. Then, according to Eqs.(7) and (8) we have

$$(m_{K^\pm}^2)_{EM} = (m_{\pi^\pm}^2)_{EM}, \quad \text{in the chiral SU}(3) \text{ limit.} \quad (18)$$

Above arguments and conclusions can also be checked by practical calculations which are standard in quantum fields theory. From Eqs.(9)-(14) and VMD, the electromagnetic masses of $\pi^\pm$, $K^\pm$ and $K^0$ can be derived (to see Ref.[14] for details), which are as follows,

$$(m_{\pi^\pm}^2)_{EM} = \frac{e^2}{f^2} \int \frac{d^4k}{(2\pi)^4} (D - 1) m_\rho \frac{(F^2 + \frac{k^2}{2\pi^2})}{k^2(k^2 - m_\rho^2)^2} \left[ 1 + \frac{\gamma^2 F^2 + \frac{k^2}{2\pi^2}}{g^2 k^2 - m_a^2} \right] \quad (19)$$

$$(m_{K^\pm}^2)_{EM} - (m_{K^0}^2)_{EM} = \frac{e^2}{f^2} \int \frac{d^4k}{(2\pi)^4} (D - 1) \left( F^2 + \frac{k^2}{2\pi^2} \right) \left( 1 + \frac{\gamma^2 F^2 + \frac{k^2}{2\pi^2}}{g^2 k^2 - m_{K_1}^2} \right) \times \left[ \frac{1}{3} k^2(m_\rho^2 - m_\omega^2)(k^2 - m_\omega^2) + \frac{2}{3} k^2(m_\rho^2 - m_\omega^2)(k^2 - m_\phi^2) \right] \quad (20)$$

$$(m_{K^0}^2)_{EM} = \frac{e^2}{4f^2} \int \frac{d^4k}{(2\pi)^4} (D - 1) \left( F^2 + \frac{k^2}{2\pi^2} \right) \left( 1 + \frac{\gamma^2 F^2 + \frac{k^2}{2\pi^2}}{g^2 k^2 - m_{K_1}^2} \right) k^2 \times \left[ \frac{1}{k^2 - m_\rho^2} - \frac{1}{3 k^2 - m_\omega^2} - \frac{2}{3 k^2 - m_\phi^2} \right] \quad (21)$$

where $D = 4 - \varepsilon$. Obviously, taking $m_\rho = m_\omega = m_\phi$, and $m_a = m_{K_1}$, Eq.(20) is exactly Eq.(19). Also, the contribution of Eq.(21) is zero with $m_\rho = m_\omega = m_\phi$. Thus, Eq.(18) holds, and Dashen’s theorem (Eq.(1)) is automatically obeyed in the chiral SU(3) limit.

In the above, the calculations on electromagnetic masses are up to $O(p^4)$ (or up to 4-order derivative operators in effective Lagrangians). In the remainder of this Section, we would like to
back to $O(p^2)$ in order to show the leading order of electromagnetic mass difference of $\pi-$meson, $(m_{\pi^\pm} - m_{\pi^0})_{EM}^{(0)}$, and to see if it is just the old result of Das et al\cite{9} or not. To $O(p^2)$, the interaction Lagrangians $L_{\rho\rho\pi\pi}$ and $L_{\rho\pi a}$(Eqs.(9)(10)) will be simplified as follows

$$L_{\rho\rho\pi\pi} = \frac{4F^2}{g^2f^2_\pi}\rho_\mu^0\rho_\mu^0\pi^+\pi^-,$$

$$L_{\rho\pi a} = \frac{2iF^2}{f^2g^2}\rho_\mu^0\pi^a_{\mu} + h.c..$$

Thus, in the chiral limit, the electromagnetic self-energy of pions is

$$(m_{\pi^\pm} - m_{\pi^0})_{EM}^{(0)} = (m_{\pi^\pm})_{EM}^{(0)} = \frac{3e^2}{f^2_\pi} \int \frac{d^4k}{(2\pi)^4} m^4_\rho \frac{F^2}{k^2(k^2 - m^2_\rho)^2}(1 + \frac{F^2}{g^2(k^2 - m^2_\rho)}) (22)$$

The Feynman integration in Eq.(22) is finite. So it is straightforward to get the result of $(m_{\pi^\pm} - m_{\pi^0})_{EM}^{(0)}$ after performing this integration, which is

$$(m_{\pi^\pm} - m_{\pi^0})_{EM}^{(0)} = \frac{3\alpha_{em}}{4\pi} \frac{m^4_\rho}{2m^2_\rho} \left\{ \frac{2F^2}{g^2(m^2_a - m^2_\rho)} \left( \frac{1}{m^2_a} + \frac{1}{m^2_a - m^2_\rho \log(m^2_a/m^2_\rho)} \right) \right\} (23)$$

where $\alpha_{em} = \frac{e^2}{4\pi}$. Because we only consider the second order derivative terms in the Lagrangian, the relation between $m_a$ and $m_\rho$ is $m^2_a = \frac{F^2}{g^2} + m^2_\rho$ instead of Eq.(27) in Ref.\cite{2}. Combining this relation with Eq.(15), Eq.(23) can be simplified

$$(m_{\pi^\pm} - m_{\pi^0})_{EM}^{(0)} = \frac{3\alpha_{em}}{4\pi} \frac{m^2_a m^2_\rho}{m^2_a - m^2_\rho} \log \frac{m^2_a}{m^2_\rho} (24)$$

It has been pointed out that Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin(KSRF) sum rule\cite{16} is satisfied reasonably well in the present theory\cite{2}. Thus, using Eq.(53)$\left(2f^2_\pi = g^2m^2_\rho\right)$ in Ref.\cite{2}, we can get

$$m^2_a = 2m^2_\rho$$

This relation is consistent with the second Weinberg sum rule\cite{17}. Now, Eq.(24) is

$$(m_{\pi^\pm} - m_{\pi^0})_{EM}^{(0)} = \frac{3\log2}{2\pi} \alpha_{em} m^2_\rho \frac{\alpha_{em}}{m^2_a} \alpha_{em} m^2_\rho (25)$$

which is exactly the result given by Das et al\cite{9}, and serves as the leading term of Eq.(19) in the lowest energies limit.
3. It is straightforward to extend the present study to the case of axial-vector mesons. In the same way as in the case of pions and kaons, the Lagrangians contributing to electromagnetic masses of axial-vector mesons ($a_1$ and $K_1$) read

\begin{align}
\mathcal{L}_{a_1 \rho} &= -\frac{4}{g^2}[\rho_\mu \rho^0 a_1^+ a^{-}\nu - \frac{\gamma^2}{2} \rho_\mu \rho^0 (a^{\mu} a^{-\nu} + a^{-\mu} a^{\nu})], \\
\mathcal{L}_{a_2 \rho} &= \frac{2i}{g} \left(1 - \frac{\gamma^2}{\pi^2 g_\pi^2}\right) \partial^\nu \rho_\mu a^{\mu} a^{-\nu} - \frac{2i}{g} \rho_\mu a^{\mu} (a^{-\mu} - \gamma^2 \partial_\mu a^{-\nu}) + h.c., \\
\mathcal{L}_{a_2 \rho} &= \frac{2i}{g} (\beta_1 \rho_\mu \pi a^{-\mu} + \beta_2 \rho_\mu \partial^{\mu} a^{\mu} + \beta_3 \rho_\mu a^{\mu} \partial^{\nu} a^{-\mu} - \beta_4 \rho_\mu \pi^2 a^{\mu} + h.c.,
\end{align}

\begin{align}
\mathcal{L}_{a_1 K_1} &= -\frac{1}{g^2} \left(\rho_0 + v_\mu K_1^+ K_1^- \right) \\
\mathcal{L}_{a_1 K_1} &= \frac{\gamma^2}{2} \left(\rho_0 + v_\mu (K_1^{+\mu} K_1^{-\nu} + K_1^{-\mu} K_1^{+\nu})\right), \\
\mathcal{L}_{a_1 K_1} &= \frac{i}{g} \left(1 - \frac{\gamma^2}{\pi^2 g_\pi^2}\right) \partial^\nu \rho_\mu a^{\mu} a^{-\nu} - \frac{i}{g} \partial_\mu a^{-\nu} + h.c., \\
\mathcal{L}_{a_1 K_1} &= \frac{i}{g} \beta_1 (\rho_0 + v_\mu) K_1^{+\mu} K_1^{-\nu} + \frac{i}{g} \beta_2 (\rho_0 + v_\mu) a^{\mu} a^{-\nu} K_1^{+\mu} K_1^{-\nu} + \frac{i}{g} \beta_3 (\rho_0 + v_\mu) a^{\mu} \partial^{\nu} K_1^{+\mu} a^{-\nu} + h.c.,
\end{align}

\begin{align}
\mathcal{L}_{K_1^0 K_1^0} &= \mathcal{L}_{a_1 K_1} \{\rho^0 \leftrightarrow -\rho, K_1^{+} \leftrightarrow K_1^{-}\}, \\
\mathcal{L}_{K_1^0 K_1^0} &= \mathcal{L}_{a_1 K_1} \{\rho^0 \leftrightarrow -\rho, K_1^{+} \leftrightarrow K_1^{-}\}, \\
\mathcal{L}_{K_1^0 K_1^0} &= \mathcal{L}_{a_1 K_1} \{\rho^0 \leftrightarrow -\rho, K_1^{+} \leftrightarrow K_1^{-}\},
\end{align}

with

\begin{align}
\beta_1 &= \frac{\gamma F_\pi^2}{g_\pi}, \quad \beta_2 = \frac{\gamma}{2\pi^2 g_\pi^2},
\end{align}
\[ \beta_3 = \frac{3\gamma}{2\pi^2 g f_\pi} (1 - \frac{2c}{g}) + \frac{2\gamma c}{f_\pi}, \quad \beta_4 = \frac{\gamma}{2\pi^2 g f_\pi}, \]
\[ \beta_5 = \frac{3\gamma}{2\pi^2 g f_\pi} (1 - \frac{2c}{g}) + \frac{4\gamma c}{f_\pi}. \]

From the above Lagrangians (Eqs. (26)-(34)), we can find the case of axial-vector mesons is very like that of pseudoscalar mesons. Therefore, similar analyses can yield the similar results for axial-vector mesons, i.e. in the chiral SU(3) limit,

\[
(m_{a^\pm}^2 - m_{a^0}^2)_{EM} = (m_{K^\pm}^2 - m_{K^0}^2)_{EM},
\]
\[
(m_{a^0}^2)_{EM} = 0, \quad (m_{K_i^0}^2)_{EM} = 0. \tag{35}
\]

Here, it is necessary to check Eq. (35) by explicit calculations which can be done in the same way as in the previous section. Using Eqs. (26)-(34) together with VMD, we can get the expressions for the electromagnetic masses of axial-vector mesons, which are as follows,

\[
(m_{a^0}^2)_{EM} = 0 \tag{36}
\]
\[
(m_{a^\pm}^2)_{EM} = (m_{a^\pm}^2)_{EM}(1) + (m_{a^\pm}^2)_{EM}(2) + (m_{a^\pm}^2)_{EM}(3)
\]

with

\[
(m_{a^\pm}^2)_{EM}(1) = \frac{i e^2 \gamma^2}{\langle a | \int d^4x \bar{a}^\mu a^\mu | a \rangle} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_{a^\pm}^2} \frac{m^4_{a^0}}{k^2}, \tag{37}
\]
\[
(m_{a^\pm}^2)_{EM}(2) = \frac{i e^2}{\langle a | \int d^4x \bar{a}^\mu a^\mu | a \rangle} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_{a^\pm}^2} \frac{m^4_{a^0}}{k^2} \times \{ \langle a | \int d^4x \bar{a}^\mu a^\mu | a \rangle [4m_{a^0}^2 + (b^2 + 2b^2)k^2 + 2\gamma^4 p \cdot k - \frac{4(p \cdot k)}{k^2}]
\]
\[ - \frac{1}{m_{a^0}^2} (bk^2 - (b - \gamma^2) p \cdot k)^2 \} + \langle a | \int d^4x \bar{a}^\mu a^\mu | a \rangle [k^\mu k^\nu - (3b^2 - 4b + 4)
\]
\[ + D(b + \gamma^2)^2 + 4\gamma^2 - 6b\gamma^2 - 2\gamma^4 - \frac{2\gamma^4 p \cdot k}{k^2} + \frac{1}{m_{a^0}^2 k^2} (bk^2 - 2(1 - \gamma^2) p \cdot k)^2 \}, \tag{38}
\]
\[
(m_{a^\pm}^2)_{EM}(3) = \frac{-i e^2}{\langle a | \int d^4x \bar{a}^\mu a^\mu | a \rangle} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(p - k)^2} \frac{m^4_{a^0}}{k^2 (k^2 - m_{a^0}^2)^2}
\]
\[ - i e^2 \frac{\gamma^2}{\langle a | \int d^4x \bar{a}^\mu a^\mu | a \rangle} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_{a^\pm}^2} \frac{m^4_{a^0}}{k^2}. \]
\[ \times \{ \langle a | \int d^4x a^\mu_1 a^\nu_1 | a \rangle \langle \beta_1' - 3 \beta_2 p \cdot k + \beta_3 k^2 \rangle^2 + \langle a | \int d^4x a^\mu_1 a^\nu_1 | a \rangle k^\mu k^\nu \left[ \beta_2 m_k^2 - \frac{(\beta_1' - 2 \beta_2 p \cdot k + \beta_3 k^2)^2}{k^2} \right] \}. \]  

(39)

where \( i = 1, 2 \).

\[
\left( m_{K_1}^2 \right)_{EM} - \left( m_{K_1}^2 \right)_{EM} = \left[ \left( m_{K_1^\pm} \right)_{EM}(1) - \left( m_{K_1^\pm} \right)_{EM}(1) \right] + \left[ \left( m_{K_1^\pm} \right)_{EM}(2) - \left( m_{K_1^\pm} \right)_{EM}(2) \right] + \left[ \left( m_{K_1^\pm} \right)_{EM}(3) - \left( m_{K_1^\pm} \right)_{EM}(3) \right]
\]

with

\[
\left( m_{K_1^\pm} \right)_{EM}(1) - \left( m_{K_1^\pm} \right)_{EM}(1) = \frac{ie^2}{2} \gamma^2 \langle K_1 | \int d^4x K_1^{\mu+} K_1^{\mu-} | K_1 \rangle - \langle K_1 | \int d^4x K_1^{\lambda+} K_1^{\lambda-} | K_1 \rangle g^{\mu\nu}
\]

\[
\times \int \frac{d^4k}{(2\pi)^4} \left( g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} \right) \left[ \frac{1}{3} k^2 (k^2 - m_p^2)(k^2 - m_\phi^2) + \frac{2}{3} k^2 (k^2 - m_p^2)(k^2 - m_\phi^2) \right],
\]

(40)

\[
\left( m_{K_1^\pm} \right)_{EM}(2) - \left( m_{K_1^\pm} \right)_{EM}(2) = \frac{ie^2}{2} \gamma^2 \langle K_1 | \int d^4x K_1^{\mu+} K_1^{\mu-} | K_1 \rangle \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - 2p \cdot k}
\]

\[
\times \left\{ \langle K_1 | \int d^4x K_1^{\mu+} K_1^{\mu-} | K_1 \rangle \right\} \left[ \frac{1}{3} k^2 (k^2 - m_p^2)(k^2 - m_\phi^2) + \frac{2}{3} k^2 (k^2 - m_p^2)(k^2 - m_\phi^2) \right]
\]

\[
\left( m_{K_1^\pm} \right)_{EM}(3) - \left( m_{K_1^\pm} \right)_{EM}(3) = \frac{-ie^2}{2} \gamma^2 \langle K_1 | \int d^4x K_1^{\mu+} K_1^{\mu-} | K_1 \rangle \int \frac{d^4k}{(2\pi)^4} \frac{1}{(p - k)^2 - m_K^2}
\]

\[
\times \left\{ \langle K_1 | \int d^4x K_1^{\mu+} K_1^{\mu-} | K_1 \rangle \right\} \left[ \frac{1}{3} k^2 (k^2 - m_p^2)(k^2 - m_\phi^2) + \frac{2}{3} k^2 (k^2 - m_p^2)(k^2 - m_\phi^2) \right]
\]

(41)
\[
(m_{K_1}^2)_{EM} = (m_{K_1}^2)_{EM}(1) + (m_{K_1}^2)_{EM}(2) + (m_{K_1}^2)_{EM}(3)
\]

with

\[
(m_{K_1}^2)_{EM}(1) = i e^2 \frac{\gamma^2}{4} \frac{\mathbf{g}_{\mu\nu} \left[ K_1 \mid \int d^4x K_1^{0\mu} \bar{K}_1^{0\nu} | K_1 \right] - \langle K_1 \mid \int d^4x K_1^{0\mu} \bar{K}_1^{0\nu} | K_1 \rangle g^{\mu\nu}}{4 \langle K_1 \mid \int d^4x K_1^{0\mu} \bar{K}_1^{0\nu} | K_1 \rangle \int \frac{d^4k}{(2\pi)^4} \left( k^2 g_{\mu\nu} - k_\mu k_\nu \right) \left[ \frac{1}{k^2 - m_\rho^2} - \frac{1}{3} \frac{k^2}{k^2 - m_\omega^2} - \frac{2}{3} \frac{k^2}{k^2 - m_\phi^2} \right]^2},
\]

\[
(m_{K_1}^2)_{EM}(2) = \frac{i e^2}{4 \langle K_1 \mid \int d^4x K_1^{0\mu} \bar{K}_1^{0\nu} | K_1 \rangle \int \frac{d^4k}{(2\pi)^4} \left( k^2 - 2p \cdot k \right) \left[ \frac{1}{k^2 - m_\rho^2} - \frac{1}{3} \frac{k^2}{k^2 - m_\omega^2} - \frac{2}{3} \frac{k^2}{k^2 - m_\phi^2} \right]^2,
\]

\[
(m_{K_1}^2)_{EM}(3) = \frac{-i e^2}{4 \langle K_1 \mid \int d^4x K_1^{0\mu} \bar{K}_1^{0\nu} | K_1 \rangle \int \frac{d^4k}{(2\pi)^4} \left( k^2 - 2p \cdot k \right) - m_K^2 \left[ \frac{1}{k^2 - m_\rho^2} - \frac{1}{3} \frac{k^2}{k^2 - m_\omega^2} - \frac{2}{3} \frac{k^2}{k^2 - m_\phi^2} \right]^2},
\]

(43)

(44)

(45)

where \( p \) is the external momentum of \( a_1 \) or \( K_1 \)-fields \( (p^2 = m_{a_1}^2 \text{ or } m_{K_1}^2 \text{ on mass-shell}) \), and

\[
b = 1 - \frac{\gamma^2}{\pi^2 g^2},
\]

\[
\beta'_1 = \beta_1 + (\beta_3 + \beta_4 - \beta_5) m_{K_1}^2.
\]

Taking \( m_\rho = m_\omega = m_\phi \), and \( m_a = m_{K_1} \), we obtain

\[
(m_{a_1}^2)_{EM}(i) = (m_{K_1}^2)_{EM}(i), \quad (m_{K_1}^2)_{EM}(i) = 0, \quad i = 1, 2, 3.
\]

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Then Eq.(35) or (2) holds, which indicates that Dashen’s theorem can be generalized to SU(3) sector of axial-vector mesons in the present theory.

Similarly, according to Refs.[2, 3], the Lagrangians which contribute to the electromagnetic masses of $\rho^\pm$ and $K^{\ast \pm}$ read

$$\mathcal{L}_{\rho\rho\rho} = -\frac{4}{g^2} \rho^0_\mu \rho^0_\nu \rho^{-\nu} + \frac{2}{g^2} \rho^0_\mu \rho^0_\nu (\rho^{+\mu} \rho^{-\nu} + \rho^{-\mu} \rho^{+\nu}),$$

$$\mathcal{L}_{\rho\rho\rho} = \frac{2i}{g} \partial_\nu \rho^0_\mu \rho^{+\mu} \rho^{-\nu} - \frac{2i}{g} \rho^0_\mu \rho^{+\nu} (\partial_\mu \rho^{-\nu} - \partial_\nu \rho^{-\mu}) + h.c.,$$

$$\mathcal{L}_{\rho\omega\pi} = -\frac{3}{\pi^2 g^2 f_\pi} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \rho_\alpha^0 \partial_\beta \pi^- + h.c.$$ (46)

$$\mathcal{L}_{K^{\ast \pm K^{\ast -}\nu\nu}} = -\frac{1}{g^2} (\rho^0_\mu + v^8_\mu)^2 K^+ K^-$$

$$+ \frac{1}{2g^2} (\rho^0_\mu + v^8_\mu)(\rho^0_\mu + v^8_\mu)(K^{+\mu} K^- - K^{-\mu} K^+),$$

$$\mathcal{L}_{K^{\ast +} K^{\ast -}\nu} = \frac{i}{g} (\partial_\nu \rho^0_\mu + \partial_\nu v^8_\mu) K^{+\mu} K^-$$

$$- \frac{i}{g} (\rho^0_\nu + v^8_\mu) K^+_\mu (\partial_\nu K^{-\mu} - \partial^\mu K^-) + h.c.,$$

$$\mathcal{L}_{K^{\ast \pm} K^{\ast \pm}\nu} = -\frac{3}{\pi^2 g^2 f_\pi} \epsilon^{\mu\nu\alpha\beta} K^+_\mu \partial_\nu K^- \left( \frac{1}{2} \partial_\nu \rho^0_\alpha + \frac{1}{2} \partial_\nu \omega_\alpha + \frac{\sqrt{2}}{2} \partial_\nu \phi_\alpha \right) + h.c.$$ (49)

Eq.(48) and Eq.(51) come from the abnormal part of the effective Lagrangian $\mathcal{L}_{1M}$ (to see Refs.[2, 3]).

Thus, similar to the above, it is not difficult to conclude that $\rho^\pm$ and $K^{\ast \pm}$ can receive the same electromagnetic self-energies in the chiral SU(3) limit,

$$(m_{\rho^\pm}^2)_{EM} = (m_{K^{\ast \pm}}^2)_{EM}$$ (52)

However, $\rho^0$ and $K^{\ast 0}$ can also obtain electromagnetic masses even in the chiral SU(3) limit, which is different from the case of neutral pseudoscalar and axial-vector mesons. The Lagrangian contributing to the electromagnetic masses of $K^{\ast 0}$ is

$$\mathcal{L}_{K^{\ast 0} K^{\ast 0}\nu\nu} = \mathcal{L}_{K^{\ast +} K^{\ast -}\nu\nu} \{ \rho^0 \leftrightarrow -\rho^0, K^{\ast \pm} \leftrightarrow K^{\ast 0}(\bar{K}^{\ast 0}) \},$$

$$\mathcal{L}_{K^{\ast 0} K^{\ast 0}\nu} = \mathcal{L}_{K^{\ast +} K^{\ast -}\nu} \{ \rho^0 \leftrightarrow -\rho^0, K^{\ast \pm} \leftrightarrow K^{\ast 0}(\bar{K}^{\ast 0}) \},$$

$$\mathcal{L}_{K^{\ast 0} K^{\ast 0}\nu} = \mathcal{L}_{K^{\ast \pm} K^{\ast \pm}\nu} \{ \rho^0 \leftrightarrow -\rho^0, K^{\ast \pm} \leftrightarrow K^{\ast 0}(\bar{K}^{\ast 0}), K^{\pm} \leftrightarrow K^{0}(\bar{K}^{0}) \}.$$ (53)
Note that in Eq.(55), the combination of the neutral vector mesons is \(-\rho_\mu + \omega_\mu + \sqrt{2} \phi_\mu\) instead of \(-\rho_\mu + \omega_\mu - \sqrt{2} \phi_\mu\) emerging in Eqs.(53)(54) and the Lagrangians contributing to the electromagnetic masses of \(K^0\) and \(K_1^0\). Therefore, even in the chiral SU(3) limit, the electromagnetic masses of \(K^{*0}\) is nonzero due to the contribution coming from Eq.(55)(the contributions of Eqs.(53) and (54) vanish in the limit of \(m_\rho = m_\omega = m_\phi\)).

As to electromagnetic masses of \(\rho^0\)-mesons, the things are more complicated. The contribution to \((m_{\rho^0}^2)_{EM}\) from \(L_{IM}\) is

\[
L_{\rho\omega\pi} = -\frac{3}{\pi^2 g^2 f_\pi} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \rho_\alpha^0 \partial_\beta \pi^0
\]  

(56)

Distinguishing from the case of \(K^{*0}\), the direct \(\rho^0\)-photon coupling which comes from VMD(Eq.(4)) can bring both the tree and one-loop diagrams contributing to the electromagnetic masses of \(\rho^0\) in the chiral limit. Thus, \((m_{\rho^0}^2)_{EM}\) is nonzero. Furthermore, from the point of view of large-\(N_c\) expansion[18], the tree diagrams are \(O(N_C)\) while the one-loop diagrams are \(O(1)\)[2], in general, we can not expect that \((m_{\rho^0}^2)_{EM} = (m_{K^{*0}}^2)_{EM}\) in the chiral SU(3) limit(only loop diagrams can contribute to \((m_{K^{*0}}^2)_{EM}\)). So the generalization of Dashen’s theorem fails for the vector mesons. This result is in correspondence with one given by Bijnens and Gosdzinsky[4].

4. In summary, employing \(U(3)_L \times U(3)_R\) chiral theory of mesons, we obtain(in the chiral SU(3) limit)

\[
(m_{\pi^\pm}^2)_{EM} = (m_{K^\pm}^2)_{EM}, \quad (m_{a^0}^2)_{EM} = (m_{K_1^0}^2)_{EM}, \quad (m_{\rho^0}^2)_{EM} = (m_{K^{*0}}^2)_{EM}.
\]

As pointed out in Ref.[2], because \(\pi^\pm\) and \(K^\pm\)(or \(a_1^\pm\) and \(K_1^\pm\), \(\rho^\pm\) and \(K^{*\pm}\)) belong to the same U-spin multiplet of the SU(3) group, their electromagnetic self-energies must be equal in the chiral SU(3) limit. The electromagnetic masses of \(\pi^0\), \(K^0\), \(a_1^0\) and \(K_1^0\) vanish in the chiral SU(3) limit. Therefore, Dashen’s theorem(Eqs.(1) and (2)) holds for both pseudoscalar and axial-vector mesons. However, the contributions from the abnormal part of the effective Lagrangian result in the nonzero electromagnetic masses of \(\rho^0\) and \(K^{*0}\) even in the chiral SU(3) limit, and VMD produces the direct coupling of \(\rho^0\) and photon(Eq.(4)), which provides the another contributions to electromagnetic
masses of $\rho^0$. Generally, $(m^2_{\rho})_{EM} \neq (m^2_{K^*0})_{EM}$. Hence, the generalization of Dashen’s theorem fails for vector-mesons.

Dashen’s theorem is valid only in the chiral SU(3) limit. The violation of this theorem at the leading order in quark mass expansion has been investigated extensively [19, 20, 21, 22, 14], and a large violation has been revealed in Refs. [19, 20, 14]. In particular, a rather large violation of Eq.(2) has been obtained in Ref. [14].

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