On the Berkovits covariant quantization of GS superstring

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Abstract

We study the covariant quantization of the Green–Schwarz (GS) superstrings proposed recently by Berkovits. In particular, we reformulate the Berkovits approach in a way that clarifies its relation with the GS approach and allows to derive in a straightforward way its extension to curved spacetime background. We explain the procedure working explicitly in the case of the heterotic string.

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1. Introduction

Notably with the advent of the Green–Schwarz (GS) superstring action with a manifest space–time supersymmetry [1], there have been a lot of efforts to quantize the action in a Lorentz-covariant manner. However, no one has succeeded in making a fully covariant quantization of the GS superstring action. The source of the difficulty is well known, that is, it is impossible to achieve the desired separation of fermionic first class and second class constraints associated with local \( \kappa \) symmetry in a manifestly covariant way. As in ten dimensions the smallest covariant spinor corresponding to a Majorana–Weyl spinor has 16 real components, 8 first class and 8 second class constraints that arise in heterotic or type I GS superstrings do not fit into such covariant spinor representation separately. For type II GS superstrings the same happens in each of the two, left-handed or right-handed, sectors.

If one tries to perform the quantization following the standard BRST-BV recipe, one ends with an infinite set of ghosts and ghosts of ghosts, that is, \( \kappa \) symmetry is infinitely reducible. All attempts [2–4], to extract from this situation a consistent quantization scheme failed, leading to a BRS charge with the wrong cohomology.

Recently, Berkovits has proposed an interesting approach to covariant quantization of superstrings, using pure spinors [5–8]. The starting point of this approach is the BRS charge

\[
Q_{\text{BRS}} = \oint \lambda^\alpha d^\alpha
\]

where \( \lambda^\alpha \) are pure spinors satisfying the equation

\[
\lambda^\alpha \Gamma^m_{\alpha\beta} \lambda^\beta = 0
\]

and \( d^\alpha \approx 0 \) denote the GS fermionic constraints. The action is the free field action involving the superspace coordinates \( X^m \) and \( \theta^\mu \), the conjugate momenta of the Grassmann coordinate \( \theta \), the pure spinor ghost \( \lambda \) and its conjugate momentum. In this approach the central charge vanishes, the BRS charge is nilpotent and has the same cohomology as the BRS charge of the Neveu–Schwarz–Ramond (NSR) formalism [9]. Moreover, vertices can be constructed which, modulo...
a very plausible conjecture, give the correct tree amplitudes.

The Berkovits approach appears to be in the right direction for covariant quantization of the Green–Schwarz superstring action, but the method used there is not conventional. For instance, the BRS charge \( Q_{\text{BRS}} = \int \lambda^a d_\alpha \) contains both first class and second class constraints, whereas the conventional BRS charge involves only first class constraints. One of the motivations of this Letter is to fill the gap between the Berkovits approach and the conventional BRS approach in order to clarify the relation between this approach and the GS one. To be definite we shall consider only the case of the heterotic string. The other cases can be treated similarly.

This Letter is organized as follows. In Section 2, we review briefly the Green–Schwarz superstring action, pure spinors, the \( SO(1, 9)/U(5) \) coset formalism and the Berkovits approach. In Section 3, in a flat background we introduce a modification of the GS action to get a BRS-invariant action, from which the Berkovits action is derived by a standard BRS procedure. Moreover, in Section 4, the formulation used in Section 3 is generalized to the case of curved background. Section 5 is devoted to discussions.

2. Review

Before presenting our results, we shall review the salient points of the superspace formulation of the Green–Schwarz heterotic superstring action, pure spinors, the \( SO(1, 9)/U(5) \) coset formalism and the Berkovits approach, which will be fully utilized in later sections.

We start with the superspace formulation of the Green–Schwarz heterotic superstring action in a general curved space–time:

\[
I_{\text{GS}} = \frac{1}{2} \int_{\mathcal{M}_2} \det e \, e^\varphi E^a_+ E^-_a + \int_{\mathcal{M}_2} B_2 + \sum_i \int_{\mathcal{M}_2} \bar{\psi}^I D_\varphi \psi^I, \tag{1}
\]

where \( \mathcal{M}_2 \) denotes the two-dimensional world sheet, \( e^\varphi \) (with its inverse \( e^\varphi_\alpha \)) are world sheet vielbeins, \( E^a_\pm \) are the pullback of the superspace vielbeins, \( B_2 \) is the NS–NS two form potential and \( \varphi \) is the dilaton. Concretely, the pullback of the supervielbeins \( E^A_\pm \) can be expressed in terms of the superspace variables \( Z^M = (X^m, \theta^\mu) \) by \( E^A_\pm = e^\varphi_\alpha \partial Z^M E^A_\alpha (Z) \). The Latin letters are used for vectors, while the Greek ones are for spinors and the capital letters for both. Moreover, the letters from the beginning of the alphabet are tangent space indices, whereas the letters from the middle are target space indices. Finally, the last term in the right-hand side in Eq. (1) denotes a set of left-moving heterotic fermions where the covariant derivative is defined as \( D_\varphi = \partial_\varphi + \partial_\mu Z^M A_M \) with \( A = d\varphi Z^M A_M \) being the one-form gauge potentials.

It is well known that the Green–Schwarz action (1) is invariant under local \( \kappa \) symmetry only when the background satisfies the SUGRA-SYM background constraints [11]. Indeed, under the local \( \kappa \) symmetry

\[
\delta Z^M E^a_\pm = w^a = E^a_\alpha \Gamma^\alpha_\beta \kappa_\beta, \quad \delta Z^M E^a_\pm = 0, \tag{2}
\]

the action transforms as

\[
\delta I_{\text{GS}} = - \int_{\mathcal{M}_2} \det e \, w \Gamma_\alpha \bar{E}_+, \tag{3}
\]

where the SUGRA-SYM background constraints have been used and we have defined

\[
\bar{E}_+^a = \left( E^a_+ - \frac{1}{2} E^a_+ \Gamma_\alpha \Gamma^\beta \partial^2 \varphi \right) e^\varphi. \tag{4}
\]

Then, provided that the symmetry (2) is supplemented with \( \delta e_\pm = 2\kappa \bar{E}_+ e_\pm \) and \( \delta e_\pm = 0 \), the Green–Schwarz action becomes invariant, \( \delta I_{\text{GS}} = 0 \) under the local \( \kappa \) symmetry.

We now turn our attention to the case of a flat background in conformal gauge. Then, the action of (1) reduces to the form

\[
I_{\text{GS}} = \int d^2z \left[ \frac{1}{2} \Pi^m \bar{\Pi}_m + \frac{1}{4} \left( \Pi^m \theta \Gamma_m \bar{\theta} - \bar{\Pi}_m \theta \Gamma_m \theta \right) \right] + \sum_i \int \bar{\psi}^I \partial \psi^I. \tag{5}
\]

In this case, \( E^A_\pm = (E^A_+, E^A_-) \) are of form

\[
E^a_\pm \to \Pi^m = \partial X^m + \frac{1}{2} \theta \Gamma^m \partial \theta,
\]
$E^a_+ \to \tilde{\Pi}^m = \partial X^m + \frac{1}{2} \partial \Gamma^m \cdot \partial \theta$, \\
$E^a_- = \partial \lambda_a \theta^a$. (6)

As usual, $\Gamma^m$ are the Dirac matrices $\gamma^m$ times the charge conjugation matrix and are $16 \times 16$ matrices symmetric with respect to exchange of spinor indices, $\Gamma^m_{a\beta} = \Gamma^m_{\beta a}$. Moreover we shall use the notation $\Gamma^{m_1...m_p}$ to denote the antisymmetric product of $p \gamma$ times the charge conjugation.

This action (5) possesses the Virasoro constraint $\Pi^m \Pi_m \approx 0$ and fermionic constraints $d_a = p_a - \frac{1}{2}(\Pi^m - \frac{1}{2} \partial \Gamma^m \partial \theta)(\Gamma^m_\alpha)_a \approx 0$, where $p_a$ are the canonical momenta conjugate to $\theta^a$. The latter constraints include 8 first class constraints and 8 second class ones, a fact which is the source of the difficulty of covariant quantization as mentioned above. In what follows, the left-moving heterotic fermions play no role and therefore will be ignored for simplicity.

It is worthwhile to point out that there is an interesting identity by Siegel [12], which is given by

$$\int d^2 z \left[ \frac{1}{2} \partial X^m \partial X_m + p_a \partial \theta^a \right] = I_{GS} + \int d^2 z d_a \partial \theta^a.$$ (7)

With the OPEs

$$X^m(y)X^n(z) \to - \eta^{mn} \log |y - z|^2, \quad p_a(y)\theta^\beta(z) \to \frac{1}{y - z} \delta_a^\beta,$$ (8)

one can calculate the OPE among the fermionic constraints $d_a \approx 0$

$$d_a(y) d_p(z) \to - \frac{1}{y - z} \Pi^m (\Gamma^m_{a\beta}).$$ (9)

Here let us introduce the concept of the “pure spinors” which plays an important role in the Berkovits works [5–8]. (See also related works [13,14].) Pure spinors are simply defined as complex, commuting, Weyl spinors such that

$$\lambda^a \Gamma^m_{a\beta} = 0.$$ (10)

From this definition and Eq. (9), it turns out that the BRS charge

$$Q_{BRS} = \oint \lambda^a d_a,$$ (11)

becomes nilpotent $Q^2_{BRS} = 0$. At this stage, we wish to mention one important remark. The hermiticity condition on the BRS charge automatically leads to the hermiticity condition on the pure spinors $\lambda^a$

$$\lambda^\dagger = \lambda,$$ (12)

which must be imposed at the quantum level. On the other hand, as classical fields, the pure spinors $\lambda$ are complex, and using $\Gamma^0 = 1$ the time component of Eq. (10) gives

$$\lambda^2 = 0.$$ (13)

Then, Eqs. (12) and (13) are not inconsistent at the quantum level since the pure spinors $\lambda$ reside in a Hilbert space with indefinite metric.

As a final preparation for our purpose, let us explain the coset $SO(1,9)/U(5)$. $U(5)$ is a subgroup of $SO(1,9)$ which acts linearly on $X^r = X^{2r-2} + i X^{2r-1}$ (as well as $\bar{X}^r = X^{2r-2} - i X^{2r-1}$) as $X^r = \Lambda_r^s X^s$, where $\Lambda \in U(5)$ and $r,s = 1,2,\ldots,5$. A spinor can be expressed in a basis of eigenvectors of the 5 commuting $SO(1,9)$ generators $\frac{1}{2} \Gamma^{2r-2} \Gamma^{2r-1}$ as $\phi^a = | \pm \pm \pm \pm \pm \rangle$. Then, complex Weyl spinors have an even number of ‘+’ eigenvalues and are decomposed into irreducible representations of $U(5)$ as

$$|+++ + +\rangle \to \phi^0,$$ $$|++- + +\rangle \to \phi_{(rs)},$$ $$|++- - +\rangle \to \phi^0,$$ $$|+- + + +\rangle \to \phi_{(rs)},$$ where each representation transforms, respectively, as $(1, \bar{10}, 5)$. For pure spinors $\lambda^a$ we have the relation [5]

$$\lambda^a = \left( \lambda^0, \lambda_{(rs)}, \lambda^r = -\frac{1}{8 \lambda^0} \epsilon^{r_{xyz}} \lambda_{(s[tz]} \lambda_{s_{x]}t} \right).$$ (15)

and therefore a pure spinor has eleven degrees of freedom.

It is convenient to define the constant “harmonics” $(\nu^0_\alpha, \nu_{(rs)\gamma}, \nu^r_\alpha)$ that take out the $U(5)$ representations of an $SO(1,9)$ Weyl spinor, that is,

$$\phi^0 = \nu^0_\alpha \phi^\alpha, \quad \phi_{(rs)} = \nu_{(rs)\gamma} \phi^\gamma, \quad \phi^r = \nu^r_\alpha \phi^\alpha.$$ (16)

Of course, in a similar way, we can describe the ant Weyl spinor by means of $(\bar{\nu}^0_\alpha, \bar{\nu}^{(rs)\gamma}, \bar{\nu}^r_\alpha)$. Here let
and then adds to the action the gauge fixing term we mean that one starts with an invariant action and shown that the total central charge vanishes, pure spinor condition (10), i.e., \( \delta I \equiv 0 \). The projector is in the OPE (17) is needed in order that \( \omega \) should be consistent with the pure spinor condition (10), i.e., \( \omega(y) = \Gamma^m \lambda(z) \rightarrow 0 \). Moreover, it is useful to introduce the tensor operators

\[
N^{mn} = \frac{1}{2} \omega \Gamma^{mn} \lambda,
\]

which satisfy the OPE of the \( SO(1, 9) \) Lorentz generator densities up to a central charge. The total Lorentz generator densities \( M^{mn} = L^{mn} + N^{mn} \) have the same central charge as in NSR formalism.

With these facts in mind, Berkovits has considered the action in a flat background

\[
I_B = \int d^2 z \left[ \frac{1}{2} \bar{\omega} X^m \bar{\omega} X_m + p_{\alpha} \bar{\theta} \alpha \partial \gamma \lambda + \omega_0 \bar{\lambda} \alpha + \frac{1}{2} \omega \rho \Omega \bar{\lambda} \rho \right],
\]

and shown that the total central charge vanishes, \( I_{BR} \) has the same cohomology as the BRS charge of NSR formalism, and vertex operators yield the correct tree amplitudes [5–8].

### 3. New presentation of the Berkovits approach in flat background

In previous section we have discussed the Berkovits works briefly. Even if his formalism has many good properties as mentioned at the end of the section, it has some unusual features. In particular, the BRS charge \( I_{BR} \), (11) is composed of the constraints \( d_{\alpha} \approx 0 \), which contain not only first class but also second class constraints, whereas the conventional BRS charge is entirely composed of first class constraints. In addition and related to it, his action (19) cannot be obtained from the Green–Schwarz action by the “standard” BRS procedure. Here by “standard” BRS procedure we mean that one starts with an invariant action and then adds to the action the gauge fixing term plus the FP ghost term which are written together as \( \{ Q_{BR}, \Psi \} \) where \( \Psi \) is the so-called “gauge fermion” with ghost number \(-1\). In this section, we shall construct a BRS-invariant action starting from the GS one and derive the Berkovits action by adding to it the BRS transformation of a gauge fermion. We shall limit ourselves to the Green–Schwarz heterotic superstring action in a flat background. The case of a general curved background will be treated in next section.

In fact, the Green–Schwarz action \( I_{GS} \) in Eq. (5) in a flat background space–time is not invariant under the BRS transformation generated by \( Q_{BR} \), (11) and the variation takes the form

\[
\delta I_{GS} = \int d^2 z \lambda \Gamma^{m} \Pi_m \bar{\partial} \theta,
\]

where we have used the OPEs in Eq. (8). Note here that this result (20) precisely corresponds to Eq. (3) (an additional \(-1\) factor does not appear in (20) compared to (3) owing to the bosonic character of pure spinors \( \lambda \)).

The key idea is to add to \( I_{GS} \) a new term \( I_{new} \) so that \( I_0 \equiv I_{GS} + I_{new} \),

is invariant under the BRS transformation. Is it possible to find such a new term? We can see that the following expression works well. Actually, provided that we take

\[
I_{new} = - \frac{1}{2} \int d^2 z \left[ \bar{\omega} \partial X^m \bar{\omega} X_m + p_{\alpha} \bar{\theta} \alpha \partial \gamma \lambda + \omega_0 \bar{\lambda} \alpha + \frac{1}{2} \omega \rho \Omega \bar{\lambda} \rho \right],
\]

by means of Eqs. (9), (10) and the Fierz identity

\[
\Gamma^{m \alpha \beta} (\Gamma^{m \rho \sigma}) = 0,
\]

we find

\[
\delta I_{new} = - \int d^2 z \bar{\lambda} \gamma \Gamma^{m} \Pi_m \bar{\partial} \theta.
\]

As a result, theaction \( I_0 \) is BRS-invariant, \( \delta I_0 = 0 \).

Since we have constructed a BRS-invariant action, we are now ready to apply the “standard” BRS recipe. The appropriate choice of gauge fermion is given by

\[
\Psi = \int d^2 z \omega \bar{\partial} \theta \alpha.
\]

Then, adding this BRS variation to the BRS-invariant action \( I_0 \), we obtain a “gauge-fixed”, BRS-invariant action

\[
I = I_0 + \delta \Psi,
\]

\[
= \int d^2 z \left[ \frac{1}{2} \bar{\omega} X^m \bar{\omega} X_m + p_{\alpha} \bar{\theta} \alpha \partial \gamma \lambda + \omega_0 \bar{\lambda} \alpha \right].
\]
Here the last term in the integrand can be rewritten as
\[
\omega_a \tilde{\omega} \lambda^\alpha = \omega_0 \tilde{\omega} \lambda^0 + \frac{1}{2} \omega^{[r]} \tilde{\omega} \lambda_{[r]},
\]
\[
\omega_0 \tilde{\omega} \lambda^0 + \frac{1}{2} \omega^{[r]} \tilde{\omega} \lambda_{[r]},
\]
(26)
where
\[
\omega_0' = \omega_0 + \frac{1}{18(\lambda^0)^2} \varepsilon^{r_1 r_2 s_1 s_2} \omega_0 \lambda_{[r_1 r_2]} \lambda_{[s_1 s_2]},
\]
\[
\omega^{[r]} = \omega^{[r]} - \frac{1}{4(\lambda^0)^2} \varepsilon^{r_1 r_2 r_3 r_4} \omega_0 \lambda_{[r_1 r_2 r_3 r_4]},
\]
(27)
Thus, modulo the field redefinitions of \( \omega \), which is harmless, the “gauge-fixed”, BRS-invariant action \( I \) precisely coincides with the Berkovits action (19).

4. Generalization to curved background

In previous section, we have considered only the case of a flat background space–time. Now we move on to the construction of the Berkovits action in a curved background. Our presentation of the Berkovits approach allows to derive it in a quite straightforward and clean way.

As mentioned in section two, the Green–Schwarz action is invariant under local \( \kappa \) symmetry only when the background satisfies the SUGRA-SYM background constraints [11]. A standard set of constraints is given by [15,16]
\[
T^a_{\alpha \beta} = T^\alpha_{\alpha \beta} = T^\alpha_{\beta \gamma} = 0,
\]
\[
H_{\alpha \beta \gamma} = 0 = H_{\alpha \beta \alpha} - \frac{1}{2} \delta^\alpha (\Gamma^a)_{\alpha \beta},
\]
\[
F_{\alpha \beta} = 0,
\]
(28)
where \( T^A = \mathcal{D}E^A \) is the superspace torsion, and \( H = dB \) and \( F = dA + A^2 \) are, respectively, the curvatures of the \( B \) field and gauge fields. Note that at this level, SYM is completely decoupled from the \( B \) field, and the Chaplin–Manton coupling arises from \( \sigma \)-model loop corrections in order to cancel anomalies associated with the \( \kappa \) symmetry in the Green–Schwarz formulation [17,18]. The constraints (28) then lead to
\[
T^a_{\alpha \beta} = -\frac{1}{24} \left( \Gamma^a_{\alpha \beta} f_{12} f_3 \right)_\beta \gamma T_{f_1 f_2 f_3},
\]
\[
H_{\alpha \beta \gamma} = -\frac{1}{2} \epsilon^{\gamma} (\Gamma_{\alpha \beta}) \gamma \theta D_{\gamma} \phi,
\]
(29)
where
\[
D_{\gamma} D_{\gamma} \phi + D_{\gamma} \phi D_{\gamma} \phi + \frac{1}{2} \Gamma^a_{\alpha \beta} D_{\gamma} \phi
\]
\[
= -\frac{1}{12} \left( \Gamma^a_{f_1 f_2 f_3} \right)_\alpha \beta T_{f_1 f_2 f_3}.\]
(30)

Now the Green–Schwarz action is given by (1) taken in conformal gauge and the fermionic constraints are
\[
d = p_a - \frac{1}{2} \left( E^a B_{a\alpha} + E^\beta B_{a\beta} \right) \approx 0.
\]
(31)

Under the BRS transformation generated by the BRS charge (11), the Green–Schwarz action is transformed as
\[
\delta I_{GS} = \int d^2 z \lambda \Gamma^a E_{-a} \tilde{E}_+,\]
(32)

where \( \tilde{E}_+ \) is defined in Eq. (4).

Following the same procedure as in a flat background, it is easy to find a new term \( I_{\text{new}} \) such that a total action \( I_0 = I_{GS} + I_{\text{new}} \) is invariant under the BRS transformation. The new term takes the form
\[
I_{\text{new}} = \frac{1}{2} \int d^2 z \frac{(d \Gamma^b \lambda^0) (\lambda \Gamma^a \tilde{E}_+)}{\lambda^0 \lambda}.
\]
(33)

To show that this term transforms as
\[
\delta I_{\text{new}} = - \int d^2 z \left( \lambda \Gamma^a \tilde{E}_+ \right) E_{-a},
\]

it is necessary to make use of \( \delta \tilde{E}_+ \) which is given by
\[
\delta \tilde{E}_+ = \frac{1}{4} \epsilon^\beta E^b_{+ \beta \lambda} \lambda
\]
\[
\times \frac{1}{6} \left( \Gamma^a_{b\gamma} f_{12} f_3 \right)_\beta \gamma T_{f_1 f_2 f_3},
\]
\[
- \left( \Gamma^a \Gamma_{b\gamma} \right)_\beta \gamma D_{\gamma} \phi
\]
\[
- \epsilon^{\gamma} (\Gamma_{\alpha \beta}) \gamma \theta D_{\gamma} \phi
\]
\[
+ \epsilon^{\gamma} \partial_{\gamma} \lambda^a.
\]
(34)

(This equation is also needed to check the nilpotency of the BRS transformation, \( \delta^2 I_{GS} = 0 \).)
Since we have found an invariant action, we can perform the “gauge fixing” in a standard way. As gauge fermion we choose

$$\Psi = \int d^2 z \omega_\alpha \tilde{E}_+^\alpha. \quad (35)$$

Using Eqs. (17), (18), (34) as well as the identity

$$\delta^\alpha_\beta \delta^\gamma_\gamma = -\frac{1}{2} (\Gamma_b)_{\beta\gamma} (\Gamma^b)^{\alpha\gamma}$$

and we have rescaled the antighost \( \delta_\alpha^\alpha = \Psi \), we can evaluate the BRS transformation of the gauge fermion whose result is given by

$$\delta \Psi = \int d^2 z \left[ d_\alpha \left[ \delta^\alpha_\beta \frac{1}{2} (\Gamma^b)^{\lambda\beta} (\nu^b \Gamma_b)^{\alpha\gamma} \right] \tilde{E}_+^\beta + e^\psi \left[ e^{\psi/4} \omega_\beta (e^{-\psi/4} \lambda) \right] + \frac{1}{2} e^\psi N^{bc} \left[ E_+^a + E_{+b} + E_{+a} \right] \right]. \quad (37)$$

Then the “gauge-fixed”, BRS-invariant action \( I = I_{\text{GS}} + I_{\text{new}} + \delta \Psi \) takes the form

$$I = I_{\text{GS}} + \int d^2 z \left[ \tilde{\theta}_\alpha \tilde{\lambda}_\alpha + \tilde{\gamma}_\alpha \right] + d_\alpha \left[ E_+^\alpha \frac{1}{2} E_+^a \Gamma_a^\alpha \Gamma_b^\beta D_b \psi \right] + N_{\alpha} \beta \left[ D_\beta \psi E_+^\alpha - \frac{1}{2} E_+ \tilde{T}^\alpha_{\alpha\beta} \right]. \quad (38)$$

where we have defined

$$\tilde{T}^\alpha_{\alpha\beta} \equiv T^\alpha_{\alpha\beta} = \frac{1}{8} (\Gamma_a \Gamma_b)^{\alpha\beta} \Gamma^a \Gamma^b D_b \psi, \quad (39)$$

and we have rescaled the antighost \( \omega_\alpha \) and the ghost \( \lambda_\alpha \) as

$$\tilde{\omega}_\alpha = e^{\psi/4} \omega_\alpha, \quad \tilde{\lambda}_\alpha = e^{-\psi/4} \lambda_\alpha. \quad (40)$$

Eq. (38) is equivalent, modulo superfield redefinitions, to a \( \sigma \)-model action obtained by Berkovits (i.e., Eq. (5.2) in Ref. [5]) via a different procedure (and in the case of type II superstrings).

5. Discussions

In this Letter we have presented a reformulation of the Berkovits approach to the covariant quantization of the GS superstrings, which holds both in flat and in curved backgrounds. In particular, in curved background our formulation provides a straightforward way to write down the \( \sigma \)-model action.

The method consists of two steps. First, one adds to the GS action \( I_{\text{GS}} \) in conformal gauge a new action term \( I_{\text{new}} \) to get an action \( I_0 \) invariant under the BRS transformation generated by \( Q_{\text{BRS}}, (11) \). Then one adds to \( I_0 \) the BRS variation of a suitable gauge fermion, as in standard BRS formalism.

\( I_{\text{new}} \) contains the fields \( p_\alpha \) through \( d_\alpha \) and the variation of \( I_0 \) with respect to \( p_\alpha \) yields the field equation \( \mp E_+ = 0 \) (i.e., \( K \tilde{\theta} = 0 \) in the flat case). We recall that \( K \) is a projector and its trace is given by \( tr K = 5 \). Therefore, \( I_{\text{new}} \) can be considered as a sort of partial gauge fixing of \( \kappa \) symmetry which, however, has the, not obvious, virtue to yield an action \( I_0 \) invariant under a BRS symmetry involving a pure spinor of ghosts (eleven components).

A peculiar feature of this BRS symmetry, is that it is not related to a local gauge symmetry as usual (in this case with anticommuting parameters). Indeed, anticommuting pure spinors do not exist.

We stress the fact that the invariance under diffeomorphisms of the GS action has been gauge fixed in conformal gauge \textit{without} adding the corresponding \( b-c \) ghosts. This is justified by the fact that the central charge vanishes without these ghosts and that the cohomology of the BRS charge (11) is the correct one (see also [20], note 5 in page 9). However in our opinion this point requires a better understanding and deserves further investigation.

A possible problem in our formalism is that the action \( I_0 = I_{\text{GS}} + I_{\text{new}} \) is manifestly \textit{not} invariant under the Lorentz transformations. However, from Eq. (23) and the fact that \( Q_{\text{BRS}} \) commutes with the Lorentz generators, it follows that the Lorentz variation of \( I_{\text{new}} \) is BRS invariant. Even more, it is a trivial cocycle of the BRS cohomology. In fact, the total action \( I = I_0 + \delta \Psi \) is Lorentz invariant and the Lorentz variation of \( \delta \Psi \), a trivial cocycle, just compensates that of \( I_{\text{new}} \). The fact that the Lorentz variation of \( I_0 \) is a trivial cocycle assures us that, in the
physical sector, the theory remains Lorentz invariant despite the non invariance of $I_0$.

It is interesting to notice that, whereas the pure spinor $\lambda$ can be considered as a covariant object, its conjugate momentum $\omega$ is not so as a consequence of (17). However, the compound fields $\omega_\alpha \lambda^\alpha$, $\omega_\alpha \bar{\lambda}^\alpha$, $N^{ab} = \omega \Gamma^{ab} \lambda$ are covariant tensors unlike $N^{a_1...a_4} = \omega \Gamma^{a_1...a_4} \lambda$ that does not transform covariantly. It is gratifying that the SUGRA constraints prevent the presence of $N^{a_1...a_4}$ in the final action (38).

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