The Control Toolbox - An Open-Source C++ Library for Robotics, Optimal and Model Predictive Control

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Abstract—We introduce the Control Toolbox (CT), an open-source C++ library for efficient modelling, control, estimation, trajectory optimization and model predictive control. The CT is applicable to a broad class of dynamic systems, but features additional modelling tools specially designed for robotics. This paper outlines its general concept, its major building blocks and highlights selected application examples. The library contains several tools to design and evaluate controllers, model dynamical systems and solve optimal control problems. The CT was designed for intuitive modelling of systems governed by ordinary differential- or difference equations. It supports rapid prototyping of cost functions and constraints and provides common interfaces for different optimal control solvers. To date, we support Single Shooting, the iterative Linear-Quadratic Regulator, Gauss-Newton Multiple Shooting and classical Direct Multiple Shooting. We provide interfaces to different NLP and linear-quadratic solvers, such as IPOPT, SNOPT, HPIPM, or a custom Riccati solver. The CT was designed with performance for online control in mind and allows to solve large-scale optimal control problems highly efficiently. Some of the key features enabling fast run-time performance are full support for Automatic Differentiation, derivative code generation and thorough multi-threading. For robotics problems, the we offer an interface to a fully auto-differentiable rigid-body dynamics modelling engine. In combination with derivative code generation, this allows for an unprecedented performance in solving optimal control problems for complex articulated robotic systems. Therefore, one of the CT’s core strengths is nonlinear model predictive control for robotic systems. For example, on a fully dynamic model of a quadruped robot with 36 states and 12 control inputs, the CT achieves an MPC update frequency of 180 Hz for a time horizon of 125 steps using a standard desktop computer. The library is available as open-source software under the Apache v2 licence and can be retrieved from https://bitbucket.org/adrlab/ct.

I. INTRODUCTION

A. What is the Control Toolbox?

A common tasks for researchers and practitioners in both the control and the robotics communities is to model systems, implement equations of motion and design model-based controllers, estimators, planning algorithms, etc. Sooner or later, one is confronted with questions of efficient implementation, computing derivative information, formulating cost functions and constraints or running controllers in model-predictive control fashion.

The Control Toolbox is specifically designed for these tasks. It is written entirely in C++ and has a strong focus on highly efficient code that can be run online (in the loop) on robots or other actuated hardware. A major contribution of the CT is its implementations of optimal control algorithms, spanning a range from simple LQR reference implementations to constrained model predictive control. The CT supports Automatic Differentiation (Auto-Diff) and allows to generate derivative code for arbitrary scalar and vector-valued functions. We designed the toolbox with usability in mind, allowing users to apply advanced concepts such as nonlinear model predictive control (NMPC) or numerical optimal control easily and with minimal effort. While we provide an interface to a state-of-the art Auto-Diff compatible robot modelling software, all other modules are independent of the a particular modelling framework, allowing the code to be interfaced with existing C/C++ code or libraries.

The CT has been successfully used in a variety of different projects, including a large number of hardware experiments, demonstrations and academic publications. The project originated from research conducted at the Agile & Dexterous Robotics Lab at ETH Zurich, but is continuously extended to cover more fields of applications and algorithms.

B. Related Work

Software is one of the key building blocks for control systems and there is a great effort for creating solvers, tools and libraries for different aspects of optimization and control. To date, there are a few software packages that provide whole toolchains for optimal control in robotics, including cost and constraint modelling and optimal control problem solving, e.g. [1], [2]. Most other software is dedicated to selected parts of the toolchain, for example to transcribing and solving nonlinear optimal control problems [3], [4] or linear-quadratic optimal control problems [5], [6]. Very popular in robotics are general-purpose nonlinear-programming packages, such as IPOPT [7], SNOPT [8] or NLopt [9], which can tackle a broad variety of problems, but typically do not exploit sparsity well and lack efficiency in more structured, specialized problem settings. All of the latter options leave it up to the user to provide correct linearizations, quadratic cost function approximations, etc.

Many of the higher-level solvers, are still highly specialized, standalone tools, which require significant integration effort. Tools such as [4], which relieve the users from performing problem approximations themselves, require formulating the problem in a very specific syntax or modelling through third-party tools like MATLAB or Simulink. Complex articulated robots, however, are difficult to model and the kinematics and dynamics equations are not

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straight-forward to derive and implement efficiently. Existing, dedicated rigid body dynamics modelling software [10], [11] is often not compatible with high-level optimal control toolboxes.

Therefore, when it comes to control and especially numerical optimal control in a robotics context, there are not many open source tools available that are both easy to use for fast development as well as efficient enough for online usage. While it is possible and common for many researchers to implement all steps from rigid body dynamics modelling to transcription into a discrete-time optimal control problem and providing linearizations themselves, this is a tedious, time-consuming and very error-prone process.

C. Scope

Here is where the CT steps in. The CT has been designed from ground up to provide the tools needed for fast development and evaluation of control methods while being optimized for efficiency and allowing for online operation. While the emphasis lies on control, the tools provided can also be used for simulation, estimation or other optimization applications.

In contrast to other software, the CT is not a rigidly integrated application but can be seen quite literal as a ‘toolbox’: it offers a variety of tools which can be combined to solve a task at hand. While ease-of-use has been a major criterion during the design and many application examples help to get to know the library, employing the CT still requires programming and control knowledge. However, it frees the users from implementing standard methods that require in-depth experience with linear algebra or numerical methods.

Furthermore, by using common definitions and types, a seamless integration between different components such as systems, controllers or integrators is provided, which greatly helps for fast prototyping. As an example, this system of building blocks allows users to generate a nonlinear model predictive control setup for a fully dynamic robot model based on a semantic description of the robot and simple configuration files for costs and constraints.

D. Structure of this Paper

This paper is structured as follows. In Section II, we present an overview of the CT’s design and implementation and give an outline of its structure. The different main modules of the CT are highlighted in Sections III to VI. Selected application examples are given in Section VII. For real-time applications, optimizing runtime performance is an important issue, on which we comment in Section VIII. The paper is concluded by important links and licence information (Section IX) and acknowledgements to additional contributors in Section X.

II. OVERVIEW

A. Fundamental Dependencies

The CT is written in C++ and has been tested under Ubuntu 14.04 and 16.04 with library versions as provided in the package sources. Building the CT requires a C++ compiler with C++11 support. Since the CT is designed as a toolbox rather than an integrated application, we tried to provide maximum flexibility to the users. Therefore, it is not tied to a specific middleware such as ROS and dependencies are kept at a minimum. The two essential dependencies for CT are Eigen [12] and kindr [13] (which is based on Eigen). The Eigen dependency is intentional since Eigen is a defacto standard for linear algebra in C++, as it provides highly efficient implementations of standard matrix operations as well as more advanced linear algebra methods. Kindr is a header only kinematics library which builds on top of it and provides data types for different rotation representations such as quaternions, Euler angles or rotation matrices.

B. Structure and Modules of the CT

The Control Toolbox consists of three main modules. The core module (ct::core), the optimal control module (ct::optcon) and the rigid body dynamics module (ct::rbd). There is a clear hierarchy between the modules. That means, the modules depend on each other in this order. For example, one can use the core module without ct::optcon or ct::rbd.

- ct::core provides general type definitions and mathematical tools. For example, it contains most data type definitions, definitions for systems and controllers, as well as basic functionality such as numerical integrators for differential equations.
- ct::optcon builds on top of the ‘core’ module and adds infrastructure for defining and solving optimal control problems. It contains the functionality for defining cost functions, constraints, solver backends and a general NMPC wrapper.
- ct::rbd provides tools for modelling rigid body dynamics systems and interfaces with ct::core and ct::optcon data types.

For testing as well as for giving examples, we provide a fourth module: the ‘models’ module (ct::models) contains various robot models including a quadruped, a robotic arm, a normal quadrotor and a quadrotor with slung load. These four different modules are detailed in Sections III-VI.

III. CORE MODULE

A. Basic System Definitions

The core module defines basic data types, base classes and interfaces to describe non-linear system dynamics of the forms

\[ \dot{x} = f(x(t), t) \]  

\[ \dot{x} = f(x(t), u(t), t) \]

which is called a core::System if it only depends on the time and state \( x(t) \) (core::StateVector) and a core::ControlledSystem if it additionally depends on the control input \( u(t) \) (core::ControlVector). The system dynamics can be implemented by the user in any desired way, but is currently restricted to classical ODE’s and difference equations. For the remainder of this section, we limit the scope...
to a continuous-time perspective. Note that for modelling robotic systems in continuous-time, the rigid body dynamics module provides a variety of useful tools, which are detailed in Section V.

As the name suggests, the core::ControlledSystem provides the interface for closing a feedback control loop. Every controlled system can take a pointer to a control law deriving from core::Controller. Full flexibility for implementing a policy of general form \( u(x(t), t) \) is given to the user. This includes special cases where the control is simply constant, depending on neither \( x(t) \) nor \( t \), time-varying, only depending on \( t \), or a general feedback controller, depending on both \( x(t) \) and \( t \).

We provide a set of pre-defined control laws, which includes a core::ConstantController with fixed \( u \), a classical PID controller (core::PIDController), or a full time-varying core::StateFeedbackController with feedforward term of form \( u_f(t) + K(t)(x(t) - x_{ref}(t)) \).

### B. Integration and Simulation

The CT provides different numerical integrators (core::Integrator). We offer own implementations and integrators based on ‘boost odeint’ [14].

The CT currently features fixed-step integrators like Euler and fourth-order Runge-Kutta as well as different (error controlled) variable step integrators. Additionally, for symplectic systems (core::SymplecticSystem) a semi-implicit Euler integrator (core::SymplecticIntegrator) is available, which can help with stiff systems. All integrators take a pointer to a system and return trajectories (core::DiscreteTrajectory), i.e. timed series of states and control inputs (core::StateTrajectory and core::ControlTrajectory) respectively. These trajectories can be either equidistant in time or unevenly sampled. In both cases, an interpolation strategy can be applied to obtain states and inputs at a specific time which is not directly stored.

For rapid prototyping and testing of control loops, we provide a core::ControlSimulator which allows to run controllers and a system integration in parallel and in real-time. Please note, however, that the CT is not intended as high-fidelity physics simulator. For more advanced simulators, we refer for example to [15].

### C. Computing Derivatives

The CT can be used to compute derivatives of arbitrary vector-valued smooth nonlinear functions \( f(x) \). For computing first order derivatives (Jacobians) \( J = \frac{\partial f}{\partial x} \), the most-widespread methods are

| Derivative method | Numerical Accuracy | Computation Speed | Setup Time | Error Safety |
|-------------------|-------------------|-------------------|------------|-------------|
| Num-Diff          | –                 | –                 | ++ ++ ++   | ++ ++ ++    |
| Analytic Deriv.   | ++ ++             | ++                | –          | –           |
| Symbolic Engine   | ++ ++             | +                 | ++         | ++          |
| Auto-diff         | ++ ++             | +                 | ++         | ++          |
| Auto-diff Codegen | ++ ++             | ++ ++             | ++         | ++          |

1) Numerical differentiation, e.g. by the method of finite-differences,
2) Analytical derivation, e.g. performed manually,
3) Symbolic math engines,
4) Automatic Differentiation, also known as Algorithmic Differentiation, with an optional source code generation step.

The different approaches are compared in Table I. To this end, we note that Automatic Differentiation is the tool of choice for accurate, yet easy to setup derivatives. It relieves the user from deriving analytical derivatives manually, which is error-prone and tedious, or symbolically\(^1\), which may be intractable for complex systems. However, it is as accurate and fast as analytic derivatives and outperforms numerical differentiation in terms of accuracy and speed while providing a similar level of convenience. For best computational performance, Automatic Differentiation with code generation (Auto-Diff Codegen) should be used. For a detailed review, the interested reader is referred to [16]. For maximum flexibility, the CT natively supports analytical derivatives and implements both numerical differentiation by finite differences and Auto-diff with optional code generation.

### D. Linearizing Dynamic Systems

The CT defines the structure of a linear system (core::LinearSystem) as

\[
\dot{x}(t) = A(t)x(t) + B(t)u(t)
\]

where \( A \) and \( B \) are the Jacobians of a non-linear, time-varying system evaluated at desired setpoints for \( x \) and \( u \). In order to compute this linearization for a non-linear system, CT provides two different helper classes. The core::SystemLinearizer takes a core::ControlledSystem and applies numerical differentiation to compute the Jacobians. Alternatively, the core::AutoDiffLinearizer can be used to apply Auto-Differentiation for the Jacobians which is more accurate than numerical differentiation. Finally, Auto-Differentiation is combined with code generation in core::ADCodegenLinearizer which is as accurate as analytical derivatives and typically very fast to evaluate. The code-gen linearizer employs a technique called just-in-time compilation (JIT), which generates the derivative code at runtime. Since this can take a few seconds, the derivative code can be stored to file and compiled in separate libraries. Examples for this approach are given in ct.models, see Section VI.

### E. Computing Approximated and Exact Sensitivities

Many control algorithms, for example the direct approaches to optimal control shown in Section IV, require a discrete-time approximation of the nonlinear system dynamics of form

\[
x_{n+1} = A_n x_n + B_n u_n
\]

\(^1\)Auto-Diff uses graph structures to compute derivatives. Hence, it is inherently different from symbolic engines such as Matlab or Mathematica.
where we call $A_n$ and $B_n$ ‘sensitivities’. In many cases it may suffice to approximate these matrices based on the continuous-time counterparts $A(t)$, $B(t)$ and a simple Forward-Euler, Backward-Euler or Tustin discretization scheme. The CT provides the `SensitivityApproximation` class, which can be used to compute such low-order approximations in a straightforward way.

However, especially when aiming at a coarse time-discretization while dealing with a highly nonlinear dynamic systems, it can be beneficial to use higher-order integration schemes to compute $A_n$, $B_n$. The `SensitivityIntegrator` class solves the integrals

$$A_n = \int_0^{\Delta t} \frac{\partial f(x(t+\tau), u(t+\tau), t+\tau)}{\partial x(t)} d\tau$$

$$B_n = \int_0^{\Delta t} \frac{\partial f(x(t+\tau), u(t+\tau), t+\tau)}{\partial u(t)} d\tau$$

for a given starting time $t$ and time-step $\Delta t$ by means of integrating a Sensitivity ODE. Special cases for obtaining exact sensitivities for symplectic integration schemes are included, too. Exact sensitivities can help to robustify and improve the convergence behaviour of many optimal control algorithms in the CT, which are summarized in the following section.

IV. OPTIMAL CONTROL MODULE

A broad variety of model-based optimal control tasks can formulated as continuous-time optimal control problems. From a robotics perspective, this includes tasks such as agile flight, reaching an object in a cluttered scene, moving a mobile manipulator or quadrupedal locomotion. In direct optimal control, the continuous-time optimal control problem is first transcribed into a numerically tractable discrete problem. For this step, two main trends have emerged:

1) Transcribing the problem into a nonlinear program (NLP) using multiple-shooting, single shooting or direct collocation and subsequently solving it using standard NLP solvers such as IPOPT or SNOPT.

2) Using iterative Riccati-based shooting methods derived from the Principle of Optimality such as DDP [17], their Gauss-Newton counterparts, iLQR [18] or Gauss-Newton Multiple Shooting (GNMS) [19]. These methods are popular due to their overall efficiency and linear time complexity.

The package `optcon` covers both classical off-the-shelf NLP solvers and custom Riccati-based solutions, paired with different flavors of Single and Multiple Shooting. An important design feature is the CT’s modularity, which allows to combine different cost functions, dynamics, constraints and solvers in almost arbitrary way and therefore allows for rapid prototyping of optimal control setups, including nonlinear model predictive control.

A. Cost Functions

The cost function package provides means of quickly prototyping objective functions based on a highly modular approach. A CT cost function is assumed to consist of a sum of elementary cost function building blocks, which are called ‘terms’. Each term evaluates to a scalar as a function of the current time, control input and state and derives from `TermBase`.

The overall cost function is designed such that it holds intermediate terms and final terms, which can be assigned individually. The intermediate and final costs are then given as the sums over the evaluations of all intermediate and final terms. Equivalently, the intermediate and final derivatives result as the sums of the individual intermediate and final term gradients. The cost function package supports both analytic derivatives for terms as well as Automatic Differentiation and just-in-time compilation (JIT) up to second order derivatives.

We offer a selection of frequently used standard cost function terms, which penalize the deviations from given control and state reference points, including a purely quadratic term (`TermQuadratic`), a cross-term (`TermMixed`) and a purely linear term (`TermLinear`). Furthermore there are terms for tracking reference trajectories in state and control (`TermQuadTracking`) and terms which formulate soft constraints on state and control variables (`TermStateBarrier`).

All existing terms can be automatically constructed from text-files, in which the cost function weights and parameters can be structured in a simple manner. For custom terms, reading from file is simple to implement thanks to a pre-specified set of loading methods. As an additional feature, all terms can be made-time-varying using time-activation functions, which can for example be used to introduce waypoint costs.

B. Constraints

The constraint package generalizes the modular idea presented for cost functions in Section IV-A to vector-valued functions. The corresponding elementary building blocks derive from `ConstraintBase` and again support both analytic derivatives, Automatic-Differentiation and Auto-Diff with JIT. For constraints, the terms are not summarized but stacked in a so-called ‘constraint container’. Every container additionally features an upper and a lower bound. For constraints, we currently only support first-order derivatives (`LinearConstraintContainer`). To date, the predefined terms include simple linear path inequality constraints and box constraints on states and controls.

C. Optimal Control Problem Containers

A `OptConProblem` is a unified container for nonlinear controlled system dynamics, Equation (2), nonlinear cost functions, nonlinear constraints, a time horizon variable and an initial state. It serves as main interface between a user and the different implementations of optimal control algorithms and NMPC.

Similarly, the container `LQOCProblem` is dedicated to constrained linear-quadratic optimal control problems. However, this container is designed to directly store the linearized dynamics, the jacobians and Hessians of the
cost function and the constraint Jacobians in matrix representation.

D. LQR and Linear Quadratic Solvers

The CT provides a unit-tested C++ implementation for different variants of the classical Linear Quadratic Regulator. We provide direct and iterative solvers for the continuous-time Algebraic Riccati Equation \((optcon::CARE)\), and iterative solvers for the discrete-time Algebraic Riccati Equation \((optcon::DARE)\). Those can be used to design infinite-horizon LQR controllers and state- and disturbance estimators in both continuous- and discrete time. Furthermore, there is a time-varying, finite-horizon discrete-time LQR version available \((optcon::FHDTLQR)\).

For unconstrained linear-quadratic optimal control problems, the CT offers a custom Riccati solver \((optcon::GRiccatiSolver)\) which achieves high efficiency through several advanced options such as fixed hessian regularization.

For constrained LQ optimal control problems the CT includes an interface to the high-performance interior point solver HPIPM [20], which achieves an unprecedented performance for solving constrained problems with linear time complexity thanks to a Riccati factorization and using a highly optimized linear algebra package [21].

E. NLP Problems and Solvers

A unified, Eigen-based interface for formulating nonlinear programming problems \((optcon::Nlp)\) and solving them \((optcon::NlpSolver)\) is part of the CT. To date, we provide interfaces to the free interior-point solver IPOPT [7] and the commercial SQP-solver SNOPT [8].

F. Gauss-Newton Shooting Algorithms with Riccati solvers

The CT implements a family of Gauss-Newton Multiple Shooting algorithms in both unconstrained and constrained fashion [19]. Essentially, this family of algorithms performs Sequential Quadratic Programming on the original nonlinear optimal control problem, uses appropriate Riccati solvers to solve linear-quadratic sub-problems efficiently, and utilizes a line-search over a merit function for globalization. The algorithms employ a piece-wise constant control parameterization. A famous limit case of the family of algorithms is the iterative Linear Quadratic Regulator (iLQR). The details of these algorithms have been extensively covered elsewhere [18], [19]. However, we note that the CT shows how to integrate these algorithms in a single framework at almost identical computational cost. These algorithms are particularly powerful for unconstrained problems with long time horizons or very fine control discretizations. Additionally, at every iteration they design a time-varying state-feedback control law, which generalizes the policy in the vicinity of the optimal solution.

G. Classical Direct Multiple Shooting

Complementary to GNMS, the CT also implements the original Direct Multiple Shooting (DMS) technique by Bock and Plitt [22], which we solve using a classical NLP solver (see Section IV-E). We provide this method separately, since it complements the other algorithms in several aspects. While GNMS currently only supports a constant control parameterization, DMS also supports linear interpolation. DMS in the combination with IPOPT can furthermore leverage exact Hessians or different hessian approximations. DMS furthermore supports adaptive step-size integration. Lastly, DMS can make use of more advanced globalization techniques as employed by the NLP solvers, such as complex filter schemes [23]. However, note that for problems with long time horizons, the DMS implementation can not compete in runtime with with GNMS or iLQR, due to computational limitations of the currently available off-the shelf NLP solvers.

H. Nonlinear Model Predictive Control

Thanks to a dedicated design of interfaces between solvers and the optimal control problem definition, the CT optimal control problem solvers can be automatically run in nonlinear model predictive control fashion using the class \((optcon::MPC)\). The latter offers options like automatic warm-starting, pre-integration for delay-compensation, different modes to handle time horizons (e.g. receding horizon, fixed time horizon) and offers explicit support for real-time iteration schemes [24]. For a detailed example of NMPC using a GNMS nonlinear optimal control solver, the reader is referred to the \texttt{ct_optcon} online tutorial.

V. RIGID BODY DYNAMICS MODULE

Generally speaking, the main task of the rigid body dynamics module \texttt{rbd} is to provide wrappers that map specialized rigid-body dynamics code into a general ordinary differential equation of form (2), cost functions and constraints.

The rigid body dynamics module currently relies on RobCoGen [11], a code-generation framework for rigid body dynamics and kinematics. To include a new robot model in the Control Toolbox, an additional code-generation step is required, which creates the dynamics and kinematics equations based on a user-provided semantic robot description. To date, RobCoGen is the only rigid body dynamics engine that supports Automatic Differentiation, a major ingredient for fast optimal control [16].

Generating a new robot model based on the code-generation output of RobCoGen is straight-forward and boils down to creating a single header file with only a few lines of code. Essentially, one needs to specify kinematic branches and end-effector locations. In the background, \texttt{rbd} creates a number of containers and wrappers, which allow convenient access to the generated robot dynamics and kinematics functions as well as force-transforms and Jacobians. For fixed-base system, the final resulting dynamics container is the class \texttt{rbd::FixBaseFSys}, for floating-base system it is \texttt{rbd::FloatingBaseFSys}. We define the floating-base state as

\[
\mathbf{x} = [\mathbf{w}^\top \mathbf{q}^\top \mathbf{q}^\top] = [\mathbf{w}^\top \mathbf{\Omega}^\top \mathbf{w}^\top \mathbf{x}^\top \mathbf{\theta}^\top \mathbf{\omega}^\top \mathbf{\theta}^\top \mathbf{\dot{\theta}}^\top] 
\]
where $\mathbf{W} \Omega_B$ and $\mathbf{W} \mathbf{x}_B$ define base orientation and position expressed in the inertial (‘world’) frame. $\mathbf{B} \omega_B$ and $\mathbf{B} \mathbf{v}_B$ represent local angular and linear velocity expressed in a body fixed frame. Joint angles and velocities are represented by $\theta$ and $\dot{\theta}$, respectively.

For a straight-forward application of nonlinear optimal control to robotic systems, ct_rbd offers wrapper classes which allow to run nonlinear optimal control algorithms for any rigid-body dynamics model. As an easy to setup way to handle contacts on arbitrary reference frames, we currently support a soft spring-damper contact model, which is described in detail in [16]. Alternatively, contact forces can be chosen as additional control inputs.

As advanced features, the library also allows to:

- generate operational-space models from the generated dynamics equations,
- augment rigid-body dynamics systems with arbitrary user-defined actuator dynamics models,
- use a number of pre-defined standard controllers such as joint position controllers plus inverse dynamics,
- use pre-defined cost function terms that are specific to robotic systems, e.g. we define auto-differentiable cost function terms for end-effector task-space positioning.

Lastly, we support a basic interface to solving inverse kinematics problems using IKFast [25].

VI. Models Module

The models module, ct_rbd.models, contains a collection of fix- and floating base robot models which serve as examples of how to include systems in different ways:

- the quadrotor is a floating-base system which is modelled independent from ct_rbd and can serve as example of how to implement a system which derives directly from core::ControlledSystem.
- the inverted pendulum is the simplest system to be modelled using RobCoGen. It is a fix-base system with only 1 DoF.
- the robot ‘HyA’ models the fix-base, 6 DoF robot arm presented in [26].
- ‘HyQ’ [27] is a quadrupedal robot with 18 DoF.
- the quadrotor with slung-load is modelled using ct_rbd and RobCoGen gives an example of how to adapt or replace rbd::FloatingBaseFDSystem for robots with unusual actuation.

For systems modelled using RobCoGen, ct_rbd.models contains the generated, templated dynamics code. Furthermore, ct_rbd.models gives several examples of how to compile derivative code for forward and inverse dynamics into a separately loadable library.

VII. Application Examples

The CT has been successfully used for many projects, including a large number of hardware experiments, demonstrations and academic publications. The following presents a compact summary. For details we refer the interested reader to the referenced papers. CT application examples include:

- NMPC for a hexrotor flying through a window$^2$[28],
- a quadrotor with up to three failed rotors performing a go-to task$^3$,
- trajectory optimization and full-body nonlinear model predictive control on different quadruped robots, including performing agile squat jumps$^4$[29], [30],
- online trajectory optimization with collision avoidance [16] on a 6 DoF industrial robot arm,
- the computation of derivatives of constraints and cost functions including complex kinematic chains was demonstrated in hardware experiments where a mobile manipulator had to maintain a steady end-effector pose while repositioning$^5$[31],
- pick-and-place arm motions for mobile manipulator were demonstrated in [32].

In many of the above examples, our solvers reason about full rigid body dynamics models which are not simplified or altered by heuristics. Even for the most complex systems, the quadrupedal robots with 36 states and 12 control inputs, we are able to run our solvers in nonlinear MPC-fashion at rates higher than 150 Hz. These frequencies can be achieved even for long time horizons over 500 ms and for complicated locomotion tasks without pre-specified contact sequences, locations or timings.

VIII. Performance Optimization

The Control Toolbox is optimized for performance and, if used correctly, constitutes one of the fastest implementation for many state-of-the-art control approaches. This section gives an outline of important steps to achieve best performance. In order to achieve best runtime performance, the CT can make of two main techniques:

A. Multithreading

The performance of many optimal control algorithms can be increased by thorough multithreading. While some parts of the optimal control algorithms in the CT are strictly sequential, e.g. the backward propagation of the Riccati equations, other parts can be completely parallelized, e.g. the forward integration on separate multiple shooting intervals in DMS and GNMS, or computing linear-quadratic approximations about solution candidates. When employing multi-threading, the required computation time decreases approximately linearly with the number of available cores. In practice, a good trade-off needs to be achieved between single-core computation power (for the sequential algorithmic parts) and overall number of cores (for the simultaneous parts). The authors have made good experience with standard consumer PC processors with 4 physical cores.

B. Vectorization

In order to achieve best runtime in every core, one can employ the processor’s vectorization capabilities, which are

2https://youtu.be/Y7-1CBqa4x4
3https://youtu.be/5MbnM2Fij0M
4https://youtu.be/vuCSKtP67E4
5https://youtu.be/rVuL_tPCoM
Single Instruction Multiple Data (SIMD) implementations. SIMD is well-known to be particularly profitable an efficient execution of linear algebra operations, such as matrix-vector multiplications. To date, the authors recommend to employ AVX instructions [33], as the register sizes of AVX are continuously growing in modern CPUs.

IX. FURTHER INFORMATION

The Control Toolbox is released under the Apache Licence, version 2.0. A more detailed documentation as well as a step-by-step tutorial are available online, at https://adrlab.bitbucket.io/ct. The source-code is available at https://bitbucket.org/adrlab/ct.

X. CONTRIBUTORS

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