Super-renormalizable and Finite gravitational theories

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Finite Quantum Gravity

One of the simplest Lagrangian for finite QG theory in $d=4$

$$L = \lambda + \kappa^{-2} R + \bar{\kappa}^{-2} G_{\mu\nu} \frac{e^{H(z)}}{\Box} - \frac{1}{2} R_{\mu\nu} + s_1 R^2 \Box R^2 + s_2 R_{\mu\nu} R^\mu R^\nu \Box R_{\rho\sigma} R^{\rho\sigma} +$$

$$+ \sum_i c_i^3 (R^3)_i + \sum_i c_i^4 (R^4)_i + \sum_i c_i^5 (R^5)_i$$

with

$$H(z) = \frac{1}{2} \Gamma(0, p^2(z)) + \frac{1}{2} y_E + \frac{1}{2} \log p^2(z) \quad p(z) = z^4 \quad y = 3$$

Lagrangian in UV

$$L_{UV} = \lambda + \bar{\kappa}^{-2} R + \omega R_{\mu\nu} \Box^3 R_{\mu\nu} - \frac{\omega}{2} R \Box^3 R + s_1 R^2 \Box R^2 + s_2 R_{\mu\nu} \Box R^2_{\rho\sigma} +$$

$$+ \sum_i c_i^3 (R^3)_i + \sum_i c_i^4 (R^4)_i + \sum_i c_i^5 (R^5)_i \quad \omega = \frac{e^{y_E/2}}{\Lambda^8 \kappa^2}$$

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Non-local form-factors

\[ L = \lambda + \kappa^2 R + RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu} \]

- Extension of the quadratic in curvature terms Tomboulis, Krasnikov
- The most general theory describing gravitons' propagation around flat spacetime
- Intrinsically non-local due to non-polynomial functions \( F_1 \) and \( F_2 \)
- Example with one form-factor (multiplicative modification of the graviton propagator)

\[ L = \lambda + \kappa^2 R + \kappa^{-2} \Lambda^2 G_{\mu\nu} \frac{e^{H(\Box/\Lambda^2)}-1}{\Box} R^{\mu\nu} \]

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Non-local form-factors

Requirements:
- Propagator has only first single poles with real masses (no tachyons) and with positive residues (no ghosts)
- In the spectrum only physical massless transverse graviton (spin 2)

Demands on a form-factor $e^{H(z)}$:
- is real and positive on the real axis and has no zeros on the complex plane, is analytic on the whole complex plane
- has proper asymptotics for large $z$ (in UV) along and around real axis (nonpolynomial or polynomial with degree $\geq 1$)

Example:

\[ H(z) = \frac{1}{2} \Gamma(0, p^2(z)) + \frac{1}{2} \gamma_E + \frac{1}{2} \log p^2(z) \]
Non-local form-factors

\[ H(z) = \frac{1}{2} \Gamma(0, p^2(z)) + \frac{1}{2} \gamma_E + \frac{1}{2} \log p^2(z) \]

- If \( p(z) \) is a polynomial then UV behavior is asymptotically polynomial, so asymptotically in UV HDQG
- But in \( H(z) \) there are no poles of \( p(z) \) due to analytic properties of \( H(z) \)!

- Unitarity of the theory secured at the perturbative level

- If degree of \( p(z) \) greater than zero, then theory is automatically multiplicatively renormalizable in \( d=4 \)
- Define \( \text{deg } p(z) = \gamma + 1 \)
Super-renormalizability

In $d=4$ divergences are present:
- formally for $\gamma=-1$ at any loop order and $\Delta$ grows with growing $L \Rightarrow$ non-renormalizability of EH gravity
- for $\gamma=0$ at any loop order and $\Delta \leq 4 \Rightarrow$ renormalizability of $R^2$ gravity
- for $\gamma=1$ at loop order 1,2,3 $\Rightarrow$ 3-loop super-renormalizability
- for $\gamma=2$ at loop order 1,2 $\Rightarrow$ 2-loop super-renormalizability
- for $\gamma=3$ at loop order 1 $\Rightarrow$ 1-loop super-renormalizability

- Divergences remain only at 1-loop order for $\gamma \geq 3$

We achieved 1-loop super-renormalizability!
Finiteness

• No divergences at the quantum level

• Divergent part of the effective action ~ beta functions of the theory

\[ \beta_i = 0 \]

related to scale (conformal) invariance and FP of RG flow

• In 1-loop superrenormalizable theory perturbative contributions only at one loop only to four couplings

\[ \lambda \quad \kappa^{-2} \quad \omega_0^1 \quad \omega_0^2 \]

\[ L_{\text{div}} = \lambda + \kappa^{-2} R + \omega_0^1 R^2 + \omega_0^2 R_{\mu\nu} \]

• Contributions only from generally covariant terms, with \(2\gamma+4\) to \(2\gamma\) (partial) derivatives on the metric
Finiteness

- Contributions to beta functions

- For cosmological constant
  \[ \beta_{\lambda} \sim \frac{\omega_{y-2}}{\omega_y}, \left( \frac{\omega_{y-1}}{\omega_y} \right)^2 \]

- For Planck constant
  \[ \beta_{k^{-2}} \sim \frac{\omega_{y-1}}{\omega_y}, O(\text{Riem}^3) \]

- For quadratic in curvature terms
  \[ \beta_{\omega_0^{1,2}} \sim \frac{\omega_y}{\omega_y^2}, O(\text{Riem}^3), O(\text{Riem}^4) \]

- Set to zero all \( \omega_{y-2} \) and \( \omega_{y-1} \) and terms cubic, quartic in curvature

- Add two killers of beta functions \( \beta_{\omega_0^1} \) and \( \beta_{\omega_0^2} \)

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Killers

- Quadratic in curvature ("kinetic") part of the Lagrangian
  \[ L = \omega_1^y R \square^y R + \omega_2^y R_{\mu\nu} \square^y R^{\mu\nu} \]

- One of the simplest choice
  \[ s_1 R^2 \square^{y-2} R^2 + s_2 R_{\mu\nu} R^{\mu\nu} \square^{y-2} R^\rho R^\rho \]

- Contribution to beta functions from killers
  \[ \beta_{\omega_0^{1,2} \text{kill}} \sim \frac{s}{\omega_\gamma} \]

Finiteness if
\[ \beta_{\omega_0^{1,2}} + \beta_{\omega_0^{1,2} \text{kill}} = 0 \]

- Contribution of killers to be computed using Barvinsky-Vilkovisky technology for traces of covariant operators on any background and in Dimensional Regularization \((d=4-\varepsilon)\)
Computation

- 1-loop Quantum Effective Action
  \[ \Gamma = \frac{i}{2} \text{Tr} \ln \hat{H} \quad \Gamma_{\text{div}} = -\frac{1}{\varepsilon} \sum_i \beta_i X_i \]

- Kinetic operator for quantum fluctuations on any curved background
  \[ H_{\mu\nu,\rho\sigma} = \frac{\delta^2 S}{\delta g_{\mu\nu} \delta g_{\rho\sigma}} \]

- Contribution from killers we need only to quadratic in curvature order

- In BV trace technology killers contribute only to $U$ terms (with $2\gamma$ derivatives)
  \[
  \text{Tr} \ln \hat{H}_{K1} = s_1 \frac{i}{\varepsilon} \frac{24 R^2}{3 \omega^1_\gamma + \omega^2_\gamma} \\
  \text{Tr} \ln \hat{H}_{K2} = s_2 \frac{i}{\varepsilon} \left( \frac{(-10 \omega^1_\gamma + \omega^2_\gamma) R^2}{3 \omega^2_\gamma (3 \omega^1_\gamma + \omega^2_\gamma)} + \frac{2 (20 \omega^1_\gamma + 7 \omega^2_\gamma) R^2}{3 \omega^2_\gamma (3 \omega^1_\gamma + \omega^2_\gamma)} \right)
  \]
Finiteness

- Beta functions of quadratic in curvature couplings
  \[ \beta_{R^2} := a_1 s_1 + a_2 s_2 + c_1 \quad \beta_{R_{\mu \nu}^2} := b_2 s_2 + c_2 \]
  - \( c_1 \) and \( c_2 \) are contributions from terms in “kinetic” part of Lagrangian

- Coefficients of killers required to kill beta functions
  \[
  s_1 = \frac{-(3 \omega_y^1 + \omega_y^2)(40 c_1 \omega_y^1 + 10 c_2 \omega_y^1 + 14 c_1 \omega_y^2 - c_2 \omega_y^2)}{24 (20 \omega_y^1 + 7 \omega_y^2)}
  \]
  \[
  s_2 = \frac{-3 c_2 \omega_y^2 (3 \omega_y^1 + \omega_y^2)}{20 \omega_y^1 + 7 \omega_y^2}.
  \]
Conclusions

- E-H QG is valid non-renormalizable EFT below Planck scale
- HDQG is renormalizable, can be made even 1-loop super-renormalizable, has massive ghosts
- Nonlocality in formfactors solves unitarity problems, HDQG revival !!
- Still possible polynomial behaviors for propagation asymptotically in UV
- Divergences only at one-loop order
- Perturbative finiteness obtained by adding killers
- Easy generalizations to higher dimensions and for higher curvature terms in the action

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Conclusions

Finite Quantum Gravity Exists!!!
Thank you for attention!