Proof-of-principle demonstration of a translating coils-based method for measuring the magnetic field of axially-symmetric magnets

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ABSTRACT: In axially-symmetric magnets for particle accelerators, the magnetic field is usually surveyed by expensive and time-consuming 3D Hall-probe mappers. Problems arise for a coherent treatment among beam physics requirements, magnet design and manufacturing, and magnetic measurements. For example, when the longitudinal direction of the mapper is misaligned with respect to the magnet, the measured fringe fields will show spurious components. In this paper, an alternative measurement method, exploiting the inherent axial symmetry of the magnetic field, is proposed. The magnetic flux linked with a pair of sensing coils is measured as a function of the longitudinal position. An induction transducer, sensitive to the longitudinal and radial components of the magnetic field, and a measurement system have been designed and prototyped. The experimental proof-of-principle demonstration of the method in comparison with a Hall-probe mapper is presented for a solenoid magnet.

KEYWORDS: Accelerator Applications; Acceleration cavities and magnets superconducting (high-temperature superconductor; radiation hardened magnets; normal-conducting; permanent magnet devices; wigglers and undulators)

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1 Introduction

An axisymmetric (i.e., axially-symmetric) magnet is often used to focus charged particle beams in the low-energy section of accelerators [1] and other devices, such as electron microscopes [2], electron guns, or image intensifiers. Other potential applications are related to laser-accelerated protons or ions [3] — for example, in particle therapy or short-pulse radiographic diagnostics — where axisymmetric magnets are used for effective focusing and energy selection. A solenoidal magnetic lens consists of a region of cylindrically-symmetric, radial, and axial magnetic fields produced by axi-centered coils. Particles moving exactly along the magnetic axis of the magnet do not experience any force. Conversely, off-axis particles are azimuthally accelerated by the radial field components in the fringe field region of the magnet, correspondingly to the variation of the axial field component. The resulting helical particle motion in the longitudinal field region of the magnet yields a focusing effect due to the radial Lorentz force [4]. This approach allows a particle loss rate lower than using a pair of quadrupoles to focus the beam sequentially in the horizontal and vertical planes [5].

For a cylindrically-symmetric magnet, the focusing process is described according to a thin-lens approximation by the following expressions of the rotation $\varphi_z$ of the beam about the z-axis (the so-called Larmor angle):

$$\varphi_z = \frac{e}{2\gamma mv_z} \int_{z_1}^{z_2} B_z dz = \frac{e}{2\gamma mv_z} F_1,$$

and the focusing strength $1/f$ [6]:

$$\frac{1}{f} = \frac{e^2}{4\gamma^2 m^2 v_z^2} \int_{z_1}^{z_2} B_z^2 dz = \frac{e^2}{4\gamma^2 m^2 v_z^2} F_2$$

(1.1)
where \( m \) is the electron rest mass, \( e \) the elementary charge, \( \gamma = (1 - \beta^2)^{-1/2} \), \( \beta = v/c \) with \( v \) as particle velocity, \( c \) the speed of light in vacuum, and \( F_1 \) and \( F_2 \) the field integrals.

Employing axisymmetric lenses to provide the required focusing strength for charged particle beams imposes strict requirements on magnetic field measurement and axis determination. Therefore, a characterization of the magnetic field of axisymmetric magnets is clearly paramount for lattice design calculations.

Axisymmetric magnets are hardly compatible with the standard instrumentation optimized for accelerator multipole magnets and are routinely tested with general-purpose instruments such as 3D Hall probe mapping systems [9, 10]. However, these systems are poorly adapted to exploit the inherent symmetry. Analogously, available state-of-the-art stretched and vibrating-wire systems, only provide information about integral field properties [4, 7, 11].

In this paper, a measurement method based on translating sensing coils is proposed for measuring the integral field with high accuracy in axisymmetric magnets. The field integrals \( F_1 \) and \( F_2 \) are obtained more precisely and rapidly than by traditional techniques. Apart this intrinsic difference of integral measurement, translating coils have the following main advantages over standard Hall probes: 1) inherent linearity; 2) negligible temperature-related drift due the low expansion coefficient of the support (typically fiberglass); 3) easy absolute calibration with \( \pm 10^{-4} \) relative precision by the flip-coil method; and 4) arbitrary resolution in the transversal plane, since the field may be computed at any point of interest from the measured harmonic expansion.

In particular, in section 2, the working principle and the architecture of the translating coil method are presented, by highlighting also the design of the induction transducer. In section 3, the test setup and the main parts of the measurement system are presented. Then, experimental results about the calibration of the induction transducer and the validation tests in comparison to a reference method are reported.

2 Translating coil method

The basic design concept is to build a reliable and high-speed measurement system for characterizing the longitudinal and radial magnetic field components, and to determine the integrated field and the magnetic axis from the measurement data. In the following, (i) the working principle and (ii) the induction transducer design are highlighted.

2.1 Working principle

The basic idea of the method is to induce a voltage in a coil translating co-axially with the magnetic field of an axisymmetric magnet, according to Faraday’s induction law (generator convention):

\[
U = -\int_{\partial} \left( \frac{\partial \mathbf{B}}{\partial t} + \text{curl}(\mathbf{B} \times \mathbf{v}_p) \right) \, da \\
= -\frac{d}{dt} \int_{\partial} \mathbf{B} \cdot da ,
\]

where \( U \) is the induced voltage, \( \mathbf{B} \) the magnetic field and \( \mathbf{v}_p \) is the path velocity of the moving surface [8].
Figure 1. Architecture of the translating coil method.

Figure 1 shows the architecture of the measurement system. The induction transducer (translating coil) is moved by a linear actuator coaxially to the magnet. The position is measured by a linear laser interferometer, following a retroreflector rigidly connected to the cylindrical core of the transducer. The voltage signal induced at the coil terminals is digitized (data acquisition) and integrated between predefined positions along the longitudinal axis of the magnet. The position pulses are sent from the interferometer to the trigger generator, where they are counted and used to trigger the integrator of the voltage samples to calculate the flux increment between adjacent axial positions. In this way, the flux increments are obtained as a function of the linear position. Finally, an adder (Σ) computes the integral flux as a function of the longitudinal position.

This re-parametrization with respect to the axial position eliminates the time dependence and in particular the influence of linear speed variations, thus relaxing the requirements for the coil transport system. This is the same principle used for the rotating coil measurement method, which uses an angular encoder to obtain the flux as a function of the azimuthal position [12].

The integration can be performed (i) off-line, by acquiring the position pulses and the coil signals by an acquisition device and resorting afterwards to post-processing techniques for the integration, or (ii) on-line, by a digital integrator [13], releasing the value of a flux increment at each trigger signal.

2.2 Induction transducer design

The induction transducer consists of two ideally identical coils, slightly spaced longitudinally (figure 2). The coils are wound on an accurately-machined cylindrical core, have a diameter of 85 mm and are spaced by 3 mm. The coils are made of a 16-strand copper wire, wound in 16 layers for a total of 256 turns per coil. The total effective area is thus nominally 2.6 m$^2$.

When the two coils are connected in series, they operate like a single coil but with double number of turns. In this case, the transducer is sensitive to the longitudinal component $B_z$ of the field. On the other hand, when the coils are connected in opposite polarity, the longitudinal component is compensated and the radial component $B_r$ is measured.
The induction transducer with two co-axial sensing coils.

The coil diameter has to include the good field region of the magnet where the beam is passing, typically $2/3$ of the magnet bore. The voltage in the coil is given by:

$$-U_c = N_T \frac{\partial B_z}{\partial z} v_c A_c$$

where $N_T$ is the number of turns, $A_c$ is the area exposed to flux change for a single turn, $\frac{\partial B_z}{\partial z}$ is the longitudinal field gradient (typically up to $5\, \text{T/m}$ in the end region), and $v_c$ is the translating speed along the $z$-axis (in the range of a few centimeters per second). The number of coil turns is chosen to ensure a nominal peak coil signal about $2\, \text{V}$, which leaves some margin for the effects of vibrations and other mechanical imperfections with respect to the typical $\pm 5\, \text{V}$ input range of the acquisition system. The coil is wound using a flat multi-strand wire [17]. This technique, which requires a skillful manual operation, allows a much more regular cross-section with respect to common automated single-strand winding, leading to easier calibration of the coupling coefficient. A square cross-section, with as many layers as the number of parallel strands in the wire, minimizes non-ideal effects and improves the accuracy of the measurements [18].

The complete transducer is designed to minimize magnetic interferences; in particular, the electrical connections present the smallest area to the changing flux and the coil core is made of a mechanically rigid, non-conductive, and non-magnetic material (Delrin).

The measurement accuracy of the radial field depends strongly on the difference between the effective areas of the two coils. The difference is to be assessed more accurately by calibration, once nominal dimensions and number of turns are defined by design.

3 Experimental validation in a solenoid magnet

In the following, an experimental proof-of-principle demonstration of the translating coil method on a solenoid magnet is presented. In particular, (i) the measurement setup, (ii) the induction transducer calibration, and (iii) the validation results are illustrated.

3.1 Measurement setup

The solenoid under test is the spare magnet SNW 200 at CERN, with a coil of 280 turns of resistance $0.0841\, \Omega$, a length of 275 mm, a radius of 65 mm, and with a transfer function in the center of the solenoid $k = B_0/I = 0.0012\, \text{T/A}$. 

Figure 2. The induction transducer with two co-axial sensing coils.
The layout of the measurement setup is illustrated in figure 3. A linear actuator pushes and pulls the transducer inside an aluminum guide. The voltage signals induced at the terminals of the two coils are acquired by a data acquisition card (DAQ) NI PXI-6289 [15]. In particular, the voltage signal resulting from the series or anti-series combinations of the two coils is recorded. Synchronously, the coil position measured by a laser interferometer is acquired. The interferometer is fixed outside the aluminum guide and the retroreflector is attached to the coil core (figure 3). The relative linear motion between the optical components is detected. A digital A-Quad-B signal in 5 V CMOS standard is available on the output of the HPI-3D. The position pulses are acquired and counted by the Data Acquisition System Timing Controller (DAQ-STC), which integrates the counter/timer functionality of the DAQ. When the transducer hits the home position on the longitudinal axis (HP in figure 2), a LabVIEW software simultaneously stores the position, the inductive voltage, and the time duration within each acquisition step. The voltage samples, yielding the flux increment between adjacent linear positions, are integrated numerically in Matlab.

3.1.1 Translation and position measurement systems

In addition to the induction transducer, the two other fundamental parts of the measurement setup are: (i) the translation system, and (ii) the position measurement system. The translation system includes an aluminum guide, which can be centered in the magnet aperture, together with a linear actuator (figure 4a) for moving the transducer along the longitudinal axis. The transducer moves on non-magnetic rollers, mounted as shown in figure 4b. The actuator is an external moving system mechanically linked to the coil by a fiberglass stick. It pushes and pulls the transducer from an initial to a final position, such as to cover the entire field of the magnet, including the fringe field.

The linear position measurement has a direct impact on the overall system’s accuracy. Therefore, special attention was paid to its quality by using a high-performance, heterodyne laser interferometer HPI-3D from Lasertex [16] (figure 5).

The device operates according to the laser interferometer principle and is able to measure objects moving with speeds up to 7 m/s with a nominal resolution of 0.1 nm. The linear optics, con-
Figure 4. View of the transport system (a), and of the aluminum guide with the rollers for the displacement of the transducer along the longitudinal axis (b).

Figure 5. Laser head and linear interferometer (LI) and retroreflector (LR) for the position measurement.

sisting of a interferometer and a retroreflector, is employed for precise recording of the induction transducer displacement. The laser beam, generated in the laser head, consists of two polarizations: horizontal and vertical. The linear interferometer (LI in figure 5) splits the beam into two parts. The horizontally polarized beam is reflected back to the laser head and the vertical polarized beam is directed to the linear retroreflector (LR), which is attached to the translating transducer (figure 4b). The frequency of the vertical beam varies according to the Doppler effect when the retroreflector is in motion.

3.2 Induction transducer calibration

Another key factor for the proof-of-principle demonstration of the translating coil method is the transducer calibration. In the following, the calibration of (i) the magnetic equivalent area, and (ii) the distance are illustrated.
Table 1. Calibration results for the translating coils.

| Coil | $B_{\text{ref}}$ (T) | $\Delta \Phi$ (Vs) | Effective area $(m^2)$ | $R$ (Ω) |
|------|----------------------|------------------|---------------------|--------|
| 1    | 0.99991              | 2.66360          | 1.33171±0.00063     | 315.4  |
| 2    | 0.99991              | 2.66271          | 1.33126±0.00063     | 315.1  |

3.2.1 Magnetic equivalent area

The simplest method of calibration of induction coils is based on moving the coil in a known reference field $B_{\text{ref}}$ and directly measuring the magnetic flux $\Phi$ resulting from the integration of the coil voltage.

In this case, the total area of the coils is obtained by flipping them upside down in a reference dipole magnet. A second flip, back to the original position, is necessary to evaluate and correct the error due to the integrator drift [17]. Table 1 shows the calibration results for the two coils mounted on the induction transducer.

The measurement of the coil resistance $R$ allows (i) broken coils to be detected, (ii) the number of turns (1%) to be assessed in order to exclude inter-turn shorts, and (ii) load error and impedance mismatch to be corrected.

The difference between the two coils’ areas is also an important parameter for assessing the accuracy of the transducer based on the differential principle in radial field measurement. The coils’ areas differ by about 0.03%.

3.2.2 Longitudinal coil distance

An accurate calibration of the distance between the two coils of the induction transducer is essential for the correct measurement of the radial component of the magnetic field. Mechanical measurements are usually not possible, since the individual turns cannot be seen by optical instruments or touched by feelers.

Much better results can be achieved by an in-situ magnetic calibration [19]. The longitudinal distance between the two coils is assessed by directly exploiting the magnetic field measurements on the magnet under test. The longitudinal profile of the solenoid magnet is measured independently by the two coils mounted on the induction transducer. Simultaneously, the transducer position is recorded, by associating each element of the measurement arrays with a corresponding position along the longitudinal axis. When the resulting arrays $B_{z1}$ and $B_{z2}$ are plotted as a function of the longitudinal position $z$ (figure 6a), the two curves are slightly spaced by $\Delta z$, as highlighted by the zoom in figure 6b.

Considering that: (i) the solenoid magnet is powered with a stable DC current, (ii) the coils are designed to be identical and are coaxially mounted on the core of the induction transducer, and (iii) the measurements are carried out independently and simultaneously by the two coils, the offset $\Delta z$ between the longitudinal profiles measured by the two coils is assumed as due to the longitudinal distance between the two coils.

Ideally, the value $\Delta z$ should be the same for each pair of corresponding points $(B_1(j), B_2(j))$ of the two curves. Considering that the measurements are affected by several error sources, there is a
Figure 6. Longitudinal magnetic field measured independently and simultaneously by the two coils (a), with a zoom in \(z = [-0.15, 0.15]\) to highlight the coil distance effect (b).

Figure 7. Absolute difference for each pair of corresponding points \((B_1(j), B_2(j))\) of the two coils measurements (mean: green, 1-sigma: magenta).

The variation of \(\Delta z\) for each pair of points as shown in figure 7. Therefore, the calibration procedure for the distance consists in determining the value of \(\Delta z\) that minimizes the Euclidean distance between the vectors \(B_1\) and \(B_2\):

\[
\min_{\Delta z} \| B_2(z) - B_1(z) \| .
\] (3.1)

For the prototype, the calibrated longitudinal distance between the two coils is given by the mean difference of all the points, \(\Delta z = 3.723 \pm 0.021\) mm.

3.3 Validation results

The translating coil method was validated experimentally in comparison to a 3D Hall probe mapper available at CERN.

The magnetic field of the same cylindrical region inside the solenoid aperture was measured by both methods. This region is delimited longitudinally, by the maximum displacement of the mapping system along the axis of the solenoid, and transversally by the coils’ windings.
In figure 8a, the results of the translating coil method and Hall probe mapping are compared for the longitudinal magnetic field. In figure 8b, the absolute differences are plotted.

An analogous comparison is reported in figures 9a and 9b for the radial field.

These differences are compatible with the accuracy of the absolute calibration and the positioning of the Hall probe system. The impact of the mechanical precision of the movement of the translating coil in terms of tilt angle and offset, remains to be assessed.

The normalized root-mean-square error (NRMSE) is 0.14\% for the longitudinal component of the magnetic field \( B_z \) and 0.85\% for the radial component \( B_r \). The Larmor angle and the focusing strength for the solenoid under test can be easily evaluated from the measurement data. The field integrals provided by the solenoid are \( F_1 = 0.0359 \text{Tm} \) and \( F_2 = 0.0027 \text{T}^2 \text{m} \).
4 Conclusions

A method based on translating coils for the magnetic field measurement of axially-symmetric magnets has been proposed. An induction transducer, sensitive to the longitudinal and radial field components, was designed and constructed. The effective surface of the coils and the longitudinal distance between them were calibrated. The radial and longitudinal components of a CERN solenoid measured by the proposed method were compared to a reference measurement method with satisfying agreement. Furthermore, from the measurement data, the field integrals $F_1$ and $F_2$ were evaluated.

The required accuracy of the prototype still needs improvements on the mechanics and calibrations of the individual parts in order to achieve adequate metrological performance. A reference (calibrated) solenoid is required for the metrological characterization of the translating coil system. The uncertainty analysis of the measurement system and an optimization of the induction transducer are crucial as the technical realization of a better transport system. Theoretical studies for the reconstruction of 3D magnetic fields from the measurement data will have to be carried out to further advance the assessment of the local field. Additionally, a technique to determine the magnetic axis of a solenoid is under study.

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