Amplitude control of a quantum state in non-Hermitian Rice-Mele model driven by an external field

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In the Hermitian regime, a Berry phase is always the real number. It may be imaginary for a non-Hermitian system, which leads to amplitude amplification or attenuation of an evolved quantum state. We study the dynamics of the non-Hermitian Rice-Mele model driven by a time-dependent external field. The exact results show that it can have full real spectrum for any value of the field. Several rigorous results are presented for the Berry phase with respect to the varying field. We show that the Berry phase is the same complex constant for any initial state in a single sub-band. Numerical simulation indicates that the amplitude control of a state can be accomplished by a quasi-adiabatic process within a short time.

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I. INTRODUCTION

In the Hermitian regime, a Berry phase\textsuperscript{1,2} is always the real number. It may be imaginary for a non-Hermitian system, which leads to amplitude amplification or attenuation of an evolved quantum state. We study the dynamics of the non-Hermitian Rice-Mele model driven by a time-dependent external field. The exact results show that it can have full real spectrum for any value of the field. Several rigorous results are presented for the Berry phase with respect to the varying field. We show that the Berry phase is the same complex constant for any initial state in a single sub-band. Numerical simulation indicates that the amplitude control of a state can be accomplished by a quasi-adiabatic process within a short time.

In a Hermitian quantum system, geometric phase acquired during an adiabatic evolution is always real and can bring nothing to an evolved quantum state if only one eigenstate is involved. It has been shown that a non-Hermitian system can have real spectrum\textsuperscript{1} and possess peculiar phenomena. These include fast propagation\textsuperscript{3}, infinite reflection coefficient\textsuperscript{4,5}, unidirectional transmission\textsuperscript{6}, transmission phase lapse\textsuperscript{7}, maximum multiparticle entanglement associated with the phase transition\textsuperscript{8} as well as the complex Berry phase\textsuperscript{9}. The imaginary part of the Berry phase is significant for a propagating particle since it may be utilized to directly amplify or attenuate the particle probability. Very recently, the spectral and dynamical properties of a quantum particle constrained on a ring threaded by a time-varying magnetic flux in the presence of a complex (non-Hermitian) potential is investigated\textsuperscript{10}. It has been shown that several striking effects are observed in the non-Hermitian case in comparison with the Hermitian case.

In a previous work\textsuperscript{11}, the dynamical behavior has been investigated for a non-Hermitian Rice-Mele model in the absence of a flux. It has been shown that within the unbroken $PT$-symmetric region, the translational symmetry ensures the probability-preserving evolution of a state, which involves only one sub-band or two sub-bands with different $k$. In this paper we aim at investigating the dynamical behaviors in the same model but in the presence of a time-varying flux. We have determined that the law of probability preservation still holds in the presence of a constant flux. When the flux changes adiabatically, we will show rigorously that the Berry phases of all the eigenstates within a sub-band are identical complex number, which depends on the combination of the system parameters, including the lattice distortion, imaginary potential, and flux. The imaginary Berry phase leads to the amplification and attenuation of the amplitude of an evolved quantum state. We also provide some illustrative simulations to show that the amplitude control of a wave packet can be accomplished by a quasi-adiabatic process within a short time.

This paper is organized as follows. In Sec. II we present the non-Hermitian Rice-Mele model and its exact solution. Section III is devoted to show how the amplitude modulation is determined by the imaginary Berry phase under the time-dependent Hamiltonian. In Sec. IV we investigate wave packet dynamics with its accomplishment of amplitude control by a quasi-adiabatic process within a relatively short time. Finally, we give a summary and discussion in Sec. V.

II. MODEL AND SOLUTIONS

We consider a non-Hermitian Rice-Mele model\textsuperscript{12} with a flux, which can be described by the following Hamiltonian
FIG. 1. (Color online) Schematic illustration of the non-Hermitian Rice-Mele model driven by a time-dependent external field. It is $\mathcal{PT}$-symmetric with respect to the $OO'$ axis in the absence of the field. Constant field $\Phi$ breaks the $\mathcal{PT}$-symmetry, but keeps the translational symmetry. A time-varying field $\Phi(t)$ induces eddy field in a direction indicated by the red arrow. Together with the distortion, the imaginary potentials can also break the left-right chiral symmetry, inducing a direction of the system indicated by the blue arrow. In the case that two arrows are either the same or the opposite, the dynamics of a state exhibits different behaviors.

There are three elements in the structure of the model, lattice distortion, imaginary potentials, and flux. As shown in Fig. 1 the imaginary potentials and distortion can break the left-right chiral symmetry [14]. In addition, a time-varying field $\Phi(t)$ induces eddy field in another direction. The dynamics of a state should exhibit different behaviors with different configurations. It turns out that imaginary potentials can appear in open physical systems [16–20]. In experiment, effective magnetic flux threading a ring can be realized by rotating the lattice [21, 22].

In the absence of imaginary potential and flux, this model has been adequately studied in many perspectives [3]. For the case of purely imaginary potential and $\phi = 0$, the model has $\mathcal{PT}$ symmetry, and the dynamics has been systematically investigated [14] in the frameworks of biorthogonal and Dirac inner products. In this paper, we refer particle probability to Dirac probability. Moreover, we introduce the magnetic flux to the Rice-Mele model, which has been employed to control the dynamics of wave packet [23, 24]. Although non-zero $\phi$ breaks the $\mathcal{PT}$ symmetry, we will show that this model can have full real spectrum. We note that the Hamiltonian is invariant via a translational transformation, i.e., $[T_2, H] = 0$, where $T_2$ is the shift operator that defined as

$$T_2^{-1}c_{k}^{\dagger}T_2 = c_{k+2}^{\dagger}.$$ (2)

This allows invariant subspace spanned by the eigenvector of operator $T_2$. The single-particle eigenvector of $T_2$ can be expressed as $a_k^{\dagger}[0]$ and $b_k^{\dagger}[0]$, where

$$a_k^{\dagger} = \frac{1}{\sqrt{N}} \sum_{j} e^{ikj} c_{j-1}^{\dagger},$$ (3)

$$b_k^{\dagger} = \frac{1}{\sqrt{N}} \sum_{j} e^{ikj} c_{j}^{\dagger},$$ (4)

satisfying

$$T_2^{-1}a_k^{\dagger}T_2 = e^{-ik} a_k^{\dagger}, T_2^{-1}b_k^{\dagger}T_2 = e^{-ik} b_k^{\dagger}. $$ (5)

Here, $a_k^{\dagger}$ and $b_k^{\dagger}$ are two kinds of creation operators of bosons (or fermions), with $k = 2\pi n/N$ ($n \in [1, N]$), representing the particles in odd and even sublattices. Then the original Hamiltonian $H$ can be expressed as

$$H = \sum_{k} H_k, $$ (6)

where

$$H_k = \Lambda(k, \phi) J a_k^{\dagger} b_k + \text{H.c.} - (\mu + iv) J \left(a_k^{\dagger} a_k - b_k^{\dagger} b_k\right),$$ (7)

and

$$\Lambda(k, \phi) = -e^{-ik/2} \sum_{\lambda = \pm} (1 - \lambda \delta) e^{i\lambda(k/2 + \phi)}. $$ (8)

It is easy to check that

$$[H_k, H_{k'}] = 0, $$ (9)

which ensures us to arrive at the solution in each invariant subspace.

Considering the single-particle solution, we can introduce the pseudo-spin operators

$$s_{k}^{+} = (s_{k}^{-})^{\dagger} = a_{k}^{\dagger} b_{k}, s_{k}^{-} = \frac{1}{2} \left(a_{k}^{\dagger} a_{k} - b_{k}^{\dagger} b_{k}\right),$$ (10)

which obey

$$[s_{k}^{+}, s_{k}^{-}] = 2s_{k}^{z}, [s_{k}^{z}, s_{k}^{\pm}] = \pm s_{k}^{\pm}. $$ (11)
Accordingly, \( H_k \) has the form
\[
H_k = \tilde{B}_k \cdot \tilde{\sigma}_k,
\]
where \( \tilde{\sigma}_k \) is Pauli matrix. Components of the field \( \tilde{B}_k \) in the rectangular coordinates are
\[
\begin{align*}
B_k^x/J &= -(1 - \delta) \cos \phi - (1 + \delta) \cos (k + \phi), \\
B_k^y/J &= -(1 - \delta) \sin \phi - (1 + \delta) \sin (k + \phi), \\
B_k^z/J &= -(\mu + \nu),
\end{align*}
\]
where
\[
\cos \theta_k = \frac{B_k^z}{B_k}, \quad \tan \varphi_k = \frac{B_k^y}{B_k^x},
\]
and the field magnitude is
\[
B_k = \sqrt{\left( (B_k^x)^2 + (B_k^y)^2 + (B_k^z)^2 \right)^2 / 4}.
\]
Obviously, \( \theta_k \) can be a complex number even in the case with real \( B_k \).

The eigenvalues of \( H_k \) are
\[
\varepsilon_k^\pm = \pm B_k
\]
\[
= \pm 2J [(\mu + \nu)^2 / 4 + \delta^2 + (1 - \delta^2) \cos^2 (k/2 + \phi)]^{1/2},
\]
which give the spectrum of the whole system when all possible \( k \) are taken. Moreover, the eigenstates of a non-Hermitian Hamiltonian can construct a set of biorthogonal bases in association with its eigenstates of its Hermitian conjugate. For the present system, eigenstates \( |\psi^k_+\rangle, |\psi^k_\pm\rangle \) of \( H_k \) and \( |\eta^k_\pm\rangle, |\eta^k_\pm\rangle \) of \( H^T_k \) are the biorthogonal bases of the single-particle invariant subspace, which are explicitly expressed as,
\[
|\psi^k_+\rangle = \left( \begin{array}{c} \cos \frac{\theta_k}{2} \\ \sin \frac{\theta_k}{2} e^{i\varphi_k} \end{array} \right), \quad |\psi^k_\pm\rangle = \left( \begin{array}{c} -\sin \frac{\theta_k}{2} \\ \cos \frac{\theta_k}{2} e^{i\varphi_k} \end{array} \right),
\]
\[
|\eta^k_\pm\rangle = \left( \begin{array}{c} \cos \frac{\theta_k}{2} \\ \sin \frac{\theta_k}{2} e^{-i\varphi_k} \end{array} \right)^*, \quad |\eta^k_\pm\rangle = \left( \begin{array}{c} -\sin \frac{\theta_k}{2} \\ \cos \frac{\theta_k}{2} e^{-i\varphi_k} \end{array} \right)^*.
\]
It is ready to check that biorthogonal bases \( \{|\psi^k_+\rangle, |\eta^k_\pm\rangle \} \)
\( (\lambda = \pm) \) obey the biorthogonal and completeness conditions
\[
\begin{equation}
\langle \eta^k_\lambda | \psi^k_\mu \rangle = \delta_{\lambda\mu} \delta_{kk'}, \quad \sum_{\lambda, k} |\psi^k_\lambda\rangle \langle \eta^k_\lambda | = 1.
\end{equation}
\]
These properties are independent of the reality of the spectrum. In this paper, we focus on the case with full real spectrum. This happens when (i) \( \nu = 0 \), the Hamiltonian goes back to a Hermitian system, (ii) \( \nu = 0 \) and \( \nu < 2\delta \). The phase diagram is true for arbitrary value of constant \( \phi \), which is the basis for the subsequent investigation of the dynamics with time-dependent flux.

III. AMPLITUDE MODULATION BY IMAGINARY BERRY PHASE

The aim of this paper is to investigate the effect of the time-varying flux on the dynamics of the non-Hermitian system. We start with an adiabatic evolution, in which an initial eigenstate evolves into the instantaneous eigenstate of the time-dependent Hamiltonian.

From Eq. (11), we know that \( H \) is a periodic function of \( \phi \) with \( H(\phi) = H(\phi + 2\pi) \) and \( H_k(\phi) = H_k(\phi + 2\pi) \). Considering the time-dependent flux \( \phi(t) \), any eigenstate \( |\psi_k(0)\rangle \) will return back to \( |\psi_k(0)\rangle \) if \( \phi(t) \) varies adiabatically from 0 to \( 2\pi \), and the evolved state is the instantaneous eigenstate \( |\psi_k(\phi)\rangle \). More explicitly, we regard the flux as a linear function of time, that is, \( \phi = \beta t \). And the adiabatic evolution of the initial eigenstate \( |\psi_k(\phi)\rangle \) under the time-dependent Hamiltonian \( H(\phi(t)) \) can be expressed as
\[
|\Psi_k(\phi)\rangle = T \exp \left[ -i \int_0^t H(t) \, dt \right] |\psi_k(0)\rangle
\]
\[
= e^{i(\alpha_k + \gamma_k)} |\psi_k(\phi)\rangle.
\]
Here the dynamics phase defined by \( \alpha_k(\phi) \) and adiabatic phase \( \gamma_k(\phi) \) have the form
\[
\alpha_k(\phi) = -\frac{1}{\beta} \int_0^\phi \varepsilon_k(\phi') \, d\phi',
\]
\[
\gamma_k(\phi) = i \int_0^\phi \langle \eta^k_\lambda(\phi') | \partial_{\phi'} |\psi^k_\lambda(\phi')\rangle \, d\phi'
\]
\[
= -\frac{1}{\beta} \int_0^\phi \varepsilon_k(\phi') |J_k (\mu + i\nu)|. \]

In this paper, we focus on the system with full real spectrum. Then the dynamics phase depends only on the instantaneous dispersion relation and is always real. We are interested in the adiabatic phase since it may have an extra contribution to the evolved state. In the following, we will present several features of \( \gamma_k(\phi) \), based on Eq. (21).

It is obvious that we have \( \gamma_k(\phi) = 0 \), if the dimerization factor \( \delta = 0 \), which is a necessary condition for nonzero adiabatic phase. In contrast, we have nonzero \( \gamma_k(\phi) \) in the case of zero staggered potential, which indicates the significance of the Peierls distortion to the adiabatic phase. It is easy to check that
\[
\frac{\partial^2}{\partial \phi^2} \gamma_k(\phi) \propto \sin(k + 2\phi)
\]
which leads to \( \frac{\partial^2}{\partial \phi^2} \gamma_k(\phi) = 0 \) at \( \phi_n = \pi/2 - k/2 \). This fact indicates that both the real and imaginary parts of \( \gamma_k(\phi) \) experience a maximal (or minimal) change at this point. Furthermore, including Hermitian and non-Hermitian systems, we note that
\[
\frac{\partial \gamma_k(\pi n \nu)}{\partial k} = \frac{\partial \alpha_k(n \nu)}{\partial k} = 0.
\]
In view of the fact that for an arbitrary function 
\( g(\cos(k + 2\phi)) \), one can always obtain

\[
\frac{\partial}{\partial k} \int_0^{n\pi} g(\cos(k + 2\phi)) \, d\phi = \frac{1}{2} g(\cos(k + 2\phi)) \bigg|_{0}^{n\pi} = 0. \tag{25}
\]

This shows that the dynamic and Berry phases are \( k \)-

-independent after \( \phi \) varying \( n\pi \), which ensures that any

arbitrary initial state involved in the upper or lower band

solely can revive back exactly after \( \phi \) varying \( 2\pi \).

For a Hermitian system, the adiabatic phase is always

real, which ensures the probability preserving evolution.

However, the probability of the evolved state changes

due to the imaginary part of the adiabatic phase in a

non-Hermitian system. The gain or loss of probability

depends on the sign of the imaginary phase. There are

several rigorous results for the phase.

For given parameters \( \{\delta, \mu, \nu\} \), the adiabatic phase of

an eigenstate in \( \lambda \) band obeys the identity

\[
\gamma^\lambda_k(n\pi) = -\gamma^\lambda_k(-n\pi), \tag{26}
\]

which is obtained from Eq. (22), owing to the fact

\[
\varepsilon^\lambda_k(\phi - n\pi) = \varepsilon^\lambda_k(\phi). \tag{27}
\]

This means that the direction of \( \phi \) can determine the sign

of \( \Im\gamma^\lambda_k(n\pi) \), controlling the probability of an evolved

state.

In addition, the sign of \( \Re\gamma^\lambda_k(\phi) \) and \( \Im\gamma^\lambda_k(\phi) \)
could also depend on the sign of \( \delta \) and \( \nu \) via the Eqs. (28) and

(29), that is,

\[
\Re\gamma^\lambda_k(\phi) = \text{sgn}(\delta) \int_0^\phi \frac{-2|\delta|J^2 d\phi}{(B_k)^2 + J^2\nu^2}, \tag{28}
\]

\[
\Im\gamma^\lambda_k(\phi) = \text{sgn}(\delta\nu\lambda) \int_0^\phi \frac{-2J^3|\delta\nu| d\phi}{B_k \left[(B_k)^2 + J^2\nu^2\right]}, \tag{29}
\]
which yield
\[
[\gamma_k^\lambda (\phi)]_{\delta} = -[\gamma_k^\lambda (\phi)]_{-\delta}, \quad (30)
\]
\[
[\gamma_k^\lambda (\phi)]_{\nu} = [\gamma_k^\lambda (\phi)]^*_{-\nu}. \quad (31)
\]
Together with the Eq. (20), we show that the sign of \(\text{Im} (\gamma^\lambda_k)\) is determined by the following expression that
\[
\text{sgn} [\text{Im} \gamma^\lambda_k (\lambda' n \pi)] = -\text{sgn} (\nu \delta \lambda') \quad (\lambda' = \pm), \quad (32)
\]
which directly results in the amplification or attenuation of an evolved state.

Besides the exact results, it is useful to arrive at the whole profile of phases as a function of a group of parameters \{\phi, k, \delta, \mu, \nu, \lambda\}. Straightforward derivation obtains the approximate expressions of the phases for Hermitian and non-Hermitian systems as following, respectively.

For a Hermitian system (\(\nu = 0\)), we have
\[
\gamma^\lambda_k (\phi) \approx \frac{\text{sgn}(\delta/2)}{\sqrt{1 - \delta^2}} \{\tan^{-1} \Theta_k(0) - \tan^{-1} \Theta_k(\phi)\} \quad (33)
\]
\[
-\text{sgn}(\mu \lambda) \{\tan^{-1} (|\mu| \Gamma_k(\phi)) - \tan^{-1} (|\mu| \Gamma_k(0))\},
\]
and
\[
\alpha^\lambda_k (\phi) \approx \frac{\lambda}{4\delta} \{B_k(0) (k - \pi) - B_k(\phi) (k + 2\phi - \pi) + \frac{J (4\delta^2 + \mu^2)}{\sqrt{1 - \delta^2}} \ln \frac{B_k(\phi) - 2 |\delta| J \Theta_k(\phi)}{B_k(0) - 2 |\delta| J \Theta_k(0)}\}, \quad (34)
\]
where
\[
\Theta_k(\phi) = \sqrt{\delta^2 - 1} (k + 2\phi - \pi)/2, \quad (35)
\]
\[
\Gamma_k(\phi) = J \Theta_k(\phi)/B_k(\phi), \quad (36)
\]
are even functions of \(\delta, \mu, \) and \(\lambda\). It is shown that \(\gamma^\lambda_k (\phi)\) is a real number, preserving the probability.

While, for the non-Hermitian system (\(\mu = 0, \nu < 2\delta\)), we have
\[
\gamma^\lambda_k (\phi) \approx \frac{\text{sgn}(\delta/2)}{\sqrt{1 - \delta^2}} \{\tan^{-1} \Theta_k(0) - \tan^{-1} \Theta_k(\phi)\} \quad (37)
\]
\[
-\text{sgn}(\nu \lambda) \{\tanh^{-1} (|\nu| \Gamma_k(\phi)) - \tanh^{-1} (|\nu| \Gamma_k(0))\},
\]
and
\[
\alpha^\lambda_k (\phi) \approx \frac{\lambda}{4\delta} \{B_k(0) (k - \pi) - B_k(\phi) (k + 2\phi - \pi) + \frac{J (4\delta^2 - \nu^2)}{\sqrt{1 - \delta^2}} \ln \frac{B_k(\phi) - 2 |\delta| J \Theta_k(\phi)}{B_k(0) - 2 |\delta| J \Theta_k(0)}\}, \quad (38)
\]
where \(\Theta_k(\phi)\) and \(\Gamma_k(\phi)\) have the same form as above, but even functions of \(\delta, \nu, \) and \(\lambda\). The remarkable feature of \(\gamma^\lambda_k (\phi)\) is that it is a complex number. The approximate expression shows that both real and imaginary parts of \(\gamma^\lambda_k (\phi)\) are flat functions of \(\phi\) except for the region around the point \(\phi_{*}\), in which it experiences a drastic change. The key feature of an imaginary phase is its sign, which affects the amplitude of the evolved eigenstate directly, determining the gain or loss of the probability.

To verify and demonstrate the above analysis, numerical simulations are performed to investigate the dynamics behavior of a quasi-adiabatic process. We compute the time evolution of an eigenstate by using a uniform mesh in the time discretization for the time-dependent Hamiltonian \(H(t)\). The amplification factor (gain) is defined as
\[
A_k^\lambda (\phi) = \left| \langle \psi^k_\lambda (0) | \right|^{-1} \left| T \exp \left[ -i \int_0^t H(t) \, dt \right] | \psi^k_\lambda (0) \rangle \right|, \quad (39)
\]
which is the ratio of the output magnitude to the input magnitude of an eigenstate. We use the fidelity \(f^\lambda_k (\phi)\), which is defined as
\[
f^\lambda_k (\phi) = \left| \langle \Psi^k_\lambda (\phi) | T \exp \left[ -i \int_0^t H(t) \, dt \right] | \psi^k_\lambda (0) \rangle \right|, \quad (40)
\]
to describe derivation between adiabatic and quasi-adiabatic processes. For an adiabatic process (\(\beta \to 0\),

\[
\beta = 0.005, \quad \beta = 0.035, \quad \beta = 0.05
\]

\(\beta = 0.005, \quad \beta = 0.035, \quad \beta = 0.05\)
we have $A^k_A (\phi) = \exp [-\text{Im} (\gamma^k_A (\phi))]$ and $\gamma^k_A (\phi) = 1$. We can employ $A^k_A (\pi)$ to describe the amplification factor for an arbitrary quantum state. The computation is performed by using a uniform mesh in the time discretization for the time-dependent Hamiltonian $H(t)$. As an example, in Fig. 3 we show the evolution of $A^k_A (\phi)$ and $f^k_A (\phi)$ for different values of $\beta$. The plot in (a) and (b) shows the quasi-adiabatic process can be close to the adiabatic one.

IV. WAVE PACKET DYNAMICS

Before starting the investigation of the wave packet dynamics in the present system, we would like to give a brief review on the dynamics in a uniform ring. In the case of $\delta = \mu = \nu = 0$, it has been shown that the dynamics of a wave packet is the same as that driven by a linear field with strength $\beta$, according to the quantum Faraday’s law [25]. Furthermore, it is turned out that the center path of a wave packet driven by a linear field accords with the dispersion of the Hamiltonian in the absence of the field within the adiabatic regime [26].

\[
x_c (\phi) = x_c (0) + \frac{1}{\beta} \varepsilon^k_c (\phi) - \varepsilon^k_c (0),
\]

where $\varepsilon^k (\phi)$ is the dispersion relation and $k_c$ is the central momentum of the wave packet.

Now, we switch gears to the case of the present model. We note that the geometrical phase takes a role in the dynamics and the dynamics of the wave packet cannot be simply understood in terms of a semiclassical picture [27, 29]. Notice that the trajectory of a wave packet is essentially not only determined by the dispersion relation for the field-free system but also by the geometric phase. In other words, the dependence of the geometric phase on $k$ should also be considered.

However, in the adiabatic limit, $\beta$ is very small. The contribution of the geometric phase to the trajectory becomes negligible, that is,

\[
\text{Re} \gamma^k_A (\phi) \ll \alpha^k_A (\phi).
\]

The main effect of the geometric phase on the dynamics of wave packet is then just the modulation of the amplitude.

To verify and demonstrate the above analysis, numerical simulations are performed to investigate the dynamics behavior. We compute the time evolution of the wave packet by the same method as mentioned above. The initial Gaussian wave packet has the form

\[
\left| G_{k_N}^{N_A} (0) \right> = \frac{1}{\sqrt{\Omega_1}} \sum_{j=1}^{2N} e^{-\frac{(j-N_A)^2}{2}} e^{i k_0 j} |j\rangle
\]

with the velocity $k_0 \in [0, 2\pi]$, centered at the $N_A$th site, where $|j\rangle = c_j^\dagger |0\rangle$ and $\Omega_1$ is a normalization factor.

For one thing, we consider the trajectories of the wave packet with different $\beta$, as comparison to the dispersion relation. From the plots in Fig. 4 we find that for small $\beta$, the trajectory accords with the dispersion well, while as $\beta$ increases, the deviation becomes obvious.

For another thing, we investigate the flux-controlled probability of the wave packet. It can be rewritten in the form

\[
\left| G_{k_0}^{N_A} (2\pi n/\beta) \right> = \sum_k \left( g^k_+ |\psi^k_+\rangle + g^k_- |\psi^k_-\rangle \right).
\]

Here, we do not give the explicit expression of the coefficient $g^k_A$, since the following analysis is independent of $g^k_A$. Through an adiabatic evolution, we have

\[
\left| G_{k_0}^{N_A} (2\pi n/\beta) \right> \approx e^{i \Omega \zeta} e^{i \zeta (2\pi n/\beta)} \sum_k g^k_A |\psi^k_A\rangle.
\]
It is shown that only the component in either upper or lower sub-band survives. It also seems that the final state collapses to one of two sub-bands in the context of Dirac probability. The sign of $\zeta$ is crucial for the direction of the collapse. We compute the evolution for two cases with opposite flux. The result plotted in Fig. 5 shows that the evolved wave packet in upper or lower sub-band survives for two different varying fluxes, which is in agreement with our prediction.

In practice, the flux control can be implemented by a pulsed flux. We simulate this process by a Gaussian-shaped flux with the form

$$\phi(t) = 2\sqrt{\sigma} \int_0^t e^{-\sigma(t-\tau)^2} d\tau,$$  \hspace{1cm} (46)

which contributes $2\pi$ flux during the process. To characterize the feature, we introduce the reduced energy

$$E(t) = \frac{\sum_{k,\lambda} \epsilon_k^\lambda \left| \langle \eta_k^\lambda | G_{k0}(t) \rangle \right|^2}{\sum_{k,\lambda} \left| \langle \eta_k^\lambda | G_{k0}(t) \rangle \right|^2},$$  \hspace{1cm} (47)

in the adiabatic limit, we have $E(\infty) \approx \epsilon_k^+ \text{ or } \epsilon_k^-$, i.e., it converges to a positive (negative) constant if the upper (lower) sub-wave packet survives. Alternatively, we can replace $\epsilon_k^\lambda$ by $\lambda$ in Eq. (17) to redefine $E(t)$, which leads to $E(\infty) \approx +1$ or $-1$ even for a non-adiabatic process. Here we take the former, because $E(\infty)$ can indicate the deviation between the adiabatic and non-adiabatic processes. In Fig. 6 results are plotted for different values of $\sigma$, which determines the rate of the flux change. These results clearly demonstrate the finding of this paper that the external field can be utilized to control the quantum
state on demand via a quasi-adiabatic process within a relatively short time.

V. SUMMARY

In this paper, the particle dynamics of the non-Hermitian Rice-Mele model driven by a time-dependent external field has been theoretically investigated. The analysis shows that the Berry phase can be a complex number in the non-Hermitian regime. This results in the amplification and attenuation of the amplitude of an evolved quantum state. We found that it can have full real spectrum for any constant field, and the Berry phase with respect to the varying field has a constant imaginary part for an arbitrary initial state either in the upper or lower energy sub-band. The dependence of the imaginary part of the Berry phase on the parameters, such as lattice distortion, imaginary potential, and the direction of the flux, was explicitly presented. Numerical simulation indicates that the amplitude control of a wave packet can be accomplished by a quasi-adiabatic process within a relatively short time.

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