Random field effects in field-driven quantum critical points

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Abstract. We investigate the role of disorder for field-driven quantum phase transitions of metallic antiferromagnets. For systems with sufficiently low symmetry, the combination of a uniform external field and non-magnetic impurities leads effectively to a random magnetic field which strongly modifies the behavior close to the critical point. Using perturbative renormalization group, we investigate in which regime of the phase diagram the disorder affects critical properties. In heavy fermion systems where even weak disorder can lead to strong fluctuations of the local Kondo temperature, the random field effects are especially pronounced. We study possible manifestation of random field effects in experiments and discuss in this light neutron scattering results for the field driven quantum phase transition in CeCu$_{5.8}$Au$_{0.2}$.

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1 Introduction

Disorder effects can strongly modify critical properties close to phase transitions. When investigating the role of disorder, one usually distinguishes two cases: in random-field systems, the disorder couples linearly to the order parameter, while in so-called random mass systems, the coupling is to the square of the order parameter.

Random fields effects are by far more dramatic compared to the random-mass case. As has been shown in a seminal paper by Imry and Ma [1], weak random field even destroys completely long range order for magnets with $xy$ or Heisenberg symmetry as the energy costs to form domain walls are smaller than the energy gain when the magnetic structure adapts locally to the random field. For magnets with Ising symmetry, long range order is stable in three dimensions (3D) as long as the random fields are weak but the properties close to the phase transition are strongly modified.

Random field criticality, and especially the Random Field Ising Model (RFIM), has attracted considerable interests in the last decades. Despite this broad activity [2] including numerical, analytical and experimental investigations, a complete understanding of the RFIM is still lacking. It is established by now that the results of the perturbative RG calculation (the so-called “dimensional reduction” [3,4]) are incorrect and a consistent theoretical treatment should necessarily rely on some non-perturbative approach. Steps in this direction has been made recently with the help of the functional renormalization group [5–7] but the issue of the determination of the correct critical exponents is far from being settled. Moreover, the intrinsic “glassiness” of the RFIM renders the problem hard to be tackled numerically [8] and, to our knowledge, an unified view of the critical behavior is not yet available. All these complication also arise at a quantum-critical point in the presence of random fields as has been recently discussed by Senthil [9].

An important step towards the experimental realization of RFIM was made by Fishman and Ahrony [10] who suggested to study doped Ising antiferromagnets in uniform magnetic fields. Remarkably, they showed that the net effect of the random moments induced by the non-magnetic doping plus the applied field naturally leads to the same critical behavior as the RFIM. This setup has the unique advantage that one can easily tune the effective strength of the random field just by changing the external field. These observations paved the way for many interesting experiments [11]. For example, in a series of recent neutron scattering measures [12,13], the lightly-doped iron compound Fe$_{1-x}$Zn$_x$F$_2$ showed all the expected features of the RFIM in and out of equilibrium. For this material, an accurate experimental determination of critical exponents was possible.

In the present paper, we investigate the role of random fields on quantum phase transitions (QPTs) of metallic magnets. As in the setup proposed by Fishman and Ahrony [10] for classical systems, we consider field driven transition in weakly disordered magnets with Ising symmetry. In these systems the role of the magnetic field is
two-fold. On the one hand, coupled to the (non-magnetic) impurities, it generates the random field; on the other hand, it induces a QPT at a certain critical strength. Field-driven quantum phenomena are today a major topic of experimental investigation, both in the context of magnetic insulators, where the field typically induces a Bose-Einstein condensation of magnons, and in the one of magnetic metals, where the field instead leads to the suppression of long range order [14].

Our analysis addresses the latter case. As an example, we have in mind the heavy-fermion compound CeCu$_{6-x}$Au$_x$, for which many experimental data are already available in the literature [15–20]. This material is a prototypical heavy-fermion system governed by the competition of Kondo screening and RKKY interactions. For a finite concentration of gold and below a certain critical temperature, antiferromagnetic Ising order appears at a finite Néel temperature $T_N$. For a doping $x > 0.1$ the magnetic order can be suppressed by a uniform magnetic field which allows for a precise study of a field-driven quantum phase transition. The doping by Au atoms naturally induces disorder. For example, a doping $x = 0.2$ reduces [15] the effective Kondo temperature on average by approximately 50%. This implies that also the Kondo temperatures will strongly vary locally. Therefore one expects in the presence of a uniform magnetic field rather strong fluctuations in the local magnetization. These static spatial fluctuations play the role of effective random fields and are expected to modify dramatically the properties close to the quantum phase transition. For example, magnetic domains will start to nucleate even on the non-magnetic side of the phase diagram. As we will discuss, recent elastic neutron scattering results from Stockert et al. [18] appear to be consistent with this scenario.

In this paper, we will not try to describe the critical properties of field driven quantum phase transitions directly at or very close to the quantum critical point. In this regime one has to face all the (unsolved) problems well known from the classical RFIM as has been shown in reference [9]. Instead we will focus on the much more simple, but nevertheless experimentally relevant question, in what regime the quantum-critical properties of the clean system are modified by the random field. What are the signatures of the onset of random-field physics? To answer this question we use standard perturbative renormalization group (RG) methods to determine the properties of the phase diagram and the location of the crossover lines. The perturbative approach is combined with phenomenological considerations based on the Imry-Ma [1] argument.

In the following, we first review the derivation of the RG equations [4,21,22] and we obtain the general phase diagram with the different physical regimes and crossover lines. Then, we focus on equilibrium and out-of-equilibrium experimental quantities and we list possible smoking guns for the onset of random field physics.

### 2 Model and perturbative RG equations

The starting point is the usual Landau-Ginzburg-Wilson functional for the order parameter

$$S(m) = \frac{1}{2} \sum_{\lambda} \left[ \left( \delta + q^2 + \frac{\omega_n}{T} \right) m_{\lambda m_{\lambda}} - h - a_{m_{\lambda}} \delta_{\omega_n,0} \right]$$

$$+ \frac{\eta_0}{\beta} \sum_{\lambda,\lambda',\lambda'',\lambda'''} m_{\lambda m_{\lambda'}} m_{\lambda' m_{\lambda''}} m_{\lambda'' m_{\lambda'''}}$$

where $m$ is the Ising order parameter, $\lambda = (\omega_n, q)$, $\Gamma$ is a characteristic energy scale (set to $\Gamma = 1$ in the following) and $\delta$ is the critical tuning parameter (proportional to $B - B_c$ for a transition driven by a uniform field $B$). The momentum $q = k - Q$ is measured with respect to the ordering vector $Q$ of the antiferromagnet and the $|\omega_n|$ describes the damping of spin fluctuations by a coupling to particle-hole pairs in a metal. Due to its presence, typical energies scale as $q^2$ in the clean system and therefore the dynamical critical exponent is $z = 2$. For an insulator (or a metal with small Fermi surface, $2k_F < Q$) the $|\omega_n|$ is replaced by a $\omega_n^2$ term such that $z = 1$ in this case.

$h(x)$ is the static random field that is assumed to be Gaussian correlated,

$$\langle h(q) \rangle = 0 \quad \text{and} \quad \langle h(q) h(q') \rangle = h^2 \delta(q + q')$$

with tunable strength $h$ which is typically proportional to the strength of non-magnetic disorder and, more importantly, the external uniform field.

With the help of the standard replica trick, we can average over the disorder replicating the action

$$S(m^\alpha) = \frac{1}{2} \sum_{\lambda,\lambda',\lambda''} \left[ \left( \delta + q^2 + \frac{\omega_n}{T} \right) \delta_{\alpha,\alpha'} - \beta h^2 \delta_{\omega_n,0} \right] m^\alpha_{\lambda m_{\lambda}}$$

$$\times m^\alpha_{\lambda'} m^\alpha_{\lambda''} + \frac{\eta_0}{\beta} \sum_{\lambda,\lambda',\lambda'',\lambda'''} m^\alpha_{\lambda m_{\lambda'}} m^\alpha_{\lambda' m_{\lambda''}} m^\alpha_{\lambda'' m_{\lambda'''}}$$

where $\alpha = 1, 2, \ldots, n$ are the replica-indices. We have formally eliminated the random field and, in absence of the quartic interaction, the free propagator is given by

$$\langle m^\alpha_{\lambda m_{\lambda'}} \rangle = \delta_{\lambda,-\lambda'} \left( \delta + q^2 + \frac{\omega_n}{T} \right)^{-1} \left[ \delta_{\alpha,\alpha'} + \beta h^2 \delta_{\omega_n,0} \left( \delta + q^2 \right)^{-1} \right].$$

Observe that, already with the Gaussian approximation, the propagator at $\omega_n = 0$ is highly singular in the presence of the random field. Computing the diagrams in Figure 1 for the replicated action in equation (3), we obtain the following closed set of one-loop RG equations (similar to the one obtain by Micnas and Chao [21])

$$\frac{d\delta}{d\log b} = 2\delta + 6u f_1(T) + 6v f_2$$

$$\frac{dv}{d\log b} = (6-d)v - 36uv f_3(T) - 72v^2 f_4$$

$$\frac{du}{d\log b} = [4 - (d + z)]u - 36u^2 f_3(T) - 72uv f_4$$

$$\frac{dT}{d\log b} = z(T(b))$$