Numerical simulation of experiments on the high-speed impact of metal plates

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Abstract. The paper deals with modeling the high-speed impact of metal plates. In one-dimensional formulation, we numerically solve equations of continuum mechanics, supplemented by equations of dislocation plasticity, twinning and fracture models. The thermodynamic state of matter is described by means of interpolation equations of state. A comparison with experimental data in the form of velocity profiles of the free rear surface of a target is presented. Dynamics of shock waves in thin metal foils is numerically investigated.

1. Introduction

High-speed impact [1–3] and irradiation of metal plates by short [4,5] or ultrashort [6,7] powerful laser pulses are actively used for dynamic loading of metals. Experimental data characterizing the dynamic shear and spall strength of materials are important in such experiments. Velocity interferometry of loaded samples represents one of the most informative methods of studying fracture processes in materials. Theoretical models of dynamic fracture and plasticity of materials are built and tested on the basis of data of these experiments.

High-rate deformation of metals leads to the processes of nucleation and growth of defects in the material (dislocations, twins, voids). In this paper we investigate the behavior of shock waves in a metal sample at a high-speed impact of a thin impactor at the sample. In addition, we numerically investigate the shock-wave dynamics in thin metal films. Results of continuum modeling are verified by comparison with experimental data [1] obtained by velocity interferometry of the samples free surface.

2. Continuum model

We consider the one-dimensional case. Total deformation of the material is defined as the following sum: $u_{xx} + W_{xx} - (w_{xx}^D + w_{xx}^{TW})$, where $u_{xx}$ is the tensor of macroscopic deformation due to the motion of matter, $W_{xx}$ is the strain tensor describing material deformation due to the nucleation and growth of voids, $w_{xx}^D$ is the plastic deformation tensor consisting of two parts: $w_{xx}^{TW}$ is determined by the motion of dislocations, $w_{xx}^{TW}$ is determined by mechanical twinning.

Basic equations include the continuity equation:

$$\frac{d\rho}{dt} = -\rho \left( \frac{\partial v}{\partial x} + \frac{W_{xx}}{dt} \right),$$

(1)

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the equation of motion:

\[ \rho \frac{dv}{dt} = \frac{\partial (-P + S_{xx} + \sigma'_{xx})}{\partial x}, \]  
(2)

the equation for internal energy:

\[ \rho \frac{dU}{dt} = (-P + S_{xx} + \sigma'_{xx}) \left( \frac{\partial v}{\partial x} + \frac{dW_{xx}}{dt} \right) + \frac{3}{2} S_{xx} \frac{dw_{xx}}{dt}. \]  
(3)

In this equations, \( \sigma = -P + S_{xx} + \sigma'_{xx} \) is the component of the stress tensor along the x axis, \( P \) is the pressure, \( S_{xx} \) is the tensor component of the stress deviator, \( \sigma'_{xx} \) is the viscous stress, \( \rho \) is the density of matter, \( v \) is the material velocity along the axis, \( U \) is the specific internal energy. The second term in the equation for internal energy takes into account the heat release due to plastic deformation. The interpolation equation of state [8] is used to connect the specific energy, density, pressure and temperature.

The deviatoric part of stresses is calculated by the Hooke law:

\[ S_{xx} = 2G \left[ \frac{2}{3}(u_{xx} + W_{xx}) - (u_{xx}^D + w_{xx}^{TW}) \right], \]  
(4)

where \( G \) is the shear modulus. The tensor of macroscopic deformation is defined as:

\[ \frac{du_{xx}}{dt} = \frac{\partial v}{\partial x}, \]  
(5)

The plastic strain tensor is defined by the equation [9–11]:

\[ \frac{dw_{xx}^D}{dt} = \frac{b}{\sqrt{6}} V_D \rho_D, \]  
(6)

where \( b \) is the magnitude of the Burgers vector, \( V_D \) is the dislocation velocity relative to the matter:

\[ V_D = \frac{b}{2B} \left[ \sqrt{\frac{3}{2}} S_{xx} - Y \cdot \text{sign}(S_{xx}) \right] \left[ 1 - \left( \frac{V_D}{c_t} \right) \right]^{3/2}, \]  
(7)

where \( Y \) is the static yield stress, \( B \) is the coefficient of phonon friction linearly increasing with temperature; \( c_t \) is the transverse sound speed. Motion of dislocations and plastic deformation begin if \( |S_{xx}| > Y \sqrt{2/3} \). Scalar densities of mobile \( \rho_D \) and immobile \( \rho_I \) dislocations are calculated from the following kinetics equations:

\[ \frac{d\rho_D}{dt} = Q_D - Q_I - Q_{Da} + \frac{\rho_D}{\rho} \frac{d\rho}{dt}, \]  
(8)

\[ \frac{d\rho_I}{dt} = Q_I - Q_{Ia} + \frac{\rho_I}{\rho} \frac{d\rho}{dt}, \]  
(9)

where \( Q_D \) is the generation rate of mobile dislocations; \( Q_I \) is the rate of immobilization; \( Q_{Da} \) and \( Q_{Ia} \) are the rates of annihilation of mobile and immobilized dislocations respectively [10].

Twinning is an alternative mechanism of plastic deformation [12–14]:

\[ \frac{dw_{xx}^{TW}}{dt} = \varepsilon_T^{TW} \frac{d\alpha_{TW}}{dt}, \]  
(10)

where \( \varepsilon_T^{TW} = 1/\sqrt{2} \) is the deformation of twinned metal relative to the initial one, \( \alpha_T^{TW} = \pi R_T^{TW} r N_T^{TW} \) is the volume fraction of twins; \( R_T^{TW} \) is the twins radius; \( r \) is the aspect ratio;
$N_{TW}$ is the concentration of twins. The average radius of the twins is determined from the equation:

$$B \left[ 1 - \left( \frac{1}{c_t} \frac{dR_{TW}}{dt} \right)^2 \right]^{-3/2} \frac{dR_{TW}}{dt} = b \left( S_{xx} \varepsilon_{TW} - \frac{2 \gamma_{SF}}{r R_{TW}} \right),$$

where $\gamma_{SF}$ is the stacking fault energy. The rate of twinning is determined from the equation:

$$\frac{dN_{TW}}{dt} = \frac{\varepsilon_D (Q_{Pa} + Q_{Ta})}{2 \pi R_c^3 \gamma_{SF}},$$

which uses the energy release rate $\varepsilon_D (Q_{Pa} + Q_{Ta})$ due to annihilation of dislocations; $R_c = 2 \gamma_{SF} / (r S_{xx} \varepsilon_{TW})$ is the critical radius of the twin; $\varepsilon_D \approx 8 eV / b$ is the formation energy per unit length of dislocation line. If $R_{TW} > R_c$, the twin will grow under the action of shear stress. The nucleation and growth of twins influence the dislocation subsystem, increasing the static yield stress: $Y = 0.29 \text{GPa} + Gb(0.5 \sqrt{D_D} + \Delta^{-1})$, where $\Delta^{-1}$ is the inverse of the average distance between the twins [15]: $\Delta^{-1} = r R_{TW} (1 - \alpha_{TW}) / \alpha_{TW}$.

The strain tensor describing the material fracture is defined by the following equation [10, 16]:

$$\frac{dW_{xx}}{dt} = \frac{1}{1 - \beta} \frac{d\beta}{dt},$$

where $\beta = 2 \pi N \beta \delta^2 / G$, $N$ is the concentration of micro-cracks, $R$ is the radius of micro-cracks. The equation for determining the radius is as follows:

$$\rho \frac{\sigma}{G} \left( R^2 \frac{d^2 R}{dt^2} + \frac{3}{2} R \left( \frac{dR}{dt} \right)^2 \right) = -4(\gamma + \gamma') + 6 \frac{R \sigma^2}{G},$$

where $\gamma$ is the surface tension; $\gamma' = (1/2) \rho \beta D_c \beta R^2 (dR/dt)$. Microcracks will grow under tensile stress, if their radius $R > R_{cr} = (2/3) G \gamma / \sigma^2$. The equation for the concentration is the following:

$$\frac{dN}{dt} = f N_0 \left[ \exp \left( -\frac{2 \pi \gamma R_{cr}^2}{(3 k_B T)} \right) - \exp \left( -\gamma / \Delta \gamma \right) \right] \left[ 1 - \frac{(2 \pi \Delta \gamma R_{cr}^2)/(3 k_B T)}{1 - \exp \left( -\gamma / \Delta \gamma \right)} \right] (1 - \beta).$$

3. Results

Here we present the results of calculations by the above model of a high-speed impact of an aluminum impactor at an aluminum sample. The sample thickness is 0.9 mm; the impactor thickness is 0.2 mm; the impact velocity is 640 m/s; the initial temperature is 293 K.

Figure 1 shows the formation of a shock wave followed by a release wave. An elastic precursor is observed ahead of the shock wave. The amplitude of the precursor decreases due to the strain rate decrease at the shock front. With time, the release wave catches up with the shock front. The release wave also includes an elastic and a plastic part, which can be seen in figure 1.

Figure 2 shows the formation of a tensile wave due to the reflection of the shock wave from the free surface of the sample. The maximum value of tensile stress is about 1.8 GPa. At this value of stress, the process of fracture begins. One can see a spalled layer at the rear surface of the sample. In this layer, stress fluctuates around zero. After the beginning of fracture, the tension wave propagates deeper into the target but with a smaller amplitude.

Figure 3 shows a comparison of the simulation results with experimental data [1]. The elastic precursor followed by the plastic compression wave is observed in the velocity profile. The following decrease in the surface velocity is attributed to the arrival of the release wave. The spall pulse with the following reverberations indicates the material spallation. It can be seen that the results obtained by the model agree with the experimental data [1].
Figure 1. Pressure profiles in the aluminum plate before reflection from the free surface; elastic precursor is marked out.

Figure 2. Pressure profile in the aluminum plate after reflection from the free surface.

Figure 4 shows the shock profile during the first 10 ns in the thin-foil impact problem. The shock wave propagates symmetrically. At the beginning of the impact, the initial amplitude
of the elastic precursor is comparable with the shock amplitude, while the precursor amplitude decreases sharply with time.
4. Conclusions
A combined constitutive model for high-rate deformation and fracture of metals is described and numerically implemented for the one-dimensional case. The model includes dislocation plasticity, twinning and fracture due to the formation and growth of micro-cracks. Results of model verification against experimental data are presented. Further comparison with existing experimental data for a wide set of metals should be a continuation of the present work.

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