ON THE POSSIBLE SOLUTION OF THE DOUBLET-TRIPLET SPLITTING PROBLEM IN EXTENDED $SU(N \geq 8)$ SUSY GUTS

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Abstract

The generalization of the "custodial symmetry" mechanism which leads to the natural (without fine tuning) explanation of the doublet-triplet hierarchy is suggested in the frames of extended $SU(N \geq 8)$ SUSY GUTs. It is shown also that such type of SUSY GUT can predict the value of strong gauge coupling constant $\alpha_S(M_Z)$ consistent with present experimental data.

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1 Introduction

From the time of birth of Grand Unified Theories (GUTs) the gauge hierarchy problem remains as one of the challenging task of elementary particle physics. For the solution of this problem it is necessary to answer the following two questions:

1. Why the electroweak scale ($M_W \sim 10^2 GeV$) is so small in comparison with Grand ($M_G \sim 10^{16} GeV$) or Plank ($M_{Pl.} \sim 10^{19} GeV$) scales?
2. Why is the electroweak scale stable under the radiative corrections?

The second problem is solved by supersymmetry (SUSY), which ensures stability of any mass scale due to the "nonrenormalization" theorem [1]. This is the main motivation for introduction of the low-energy ($M_{SUSY} \sim 1 TeV$) broken SUSY. Moreover, some times ago, LEP precision measurements of the Standard Model (SM) gauge couplings $\alpha_S$, $\alpha_W$ and $\alpha_Y$ at $Z$-pick gives an evidence of the perfect unification of the running gauge couplings in the minimal SUSY extension of SU(5) GUT and finally ruled out non-SUSY SU(5) model [2]. This fact was considered by many physicists as a hint on the existence of low-energy broken SUSY in nature.\[2\]

However, the straightforward SUSY extension of the ordinary GUTs can not give an answer to the first question. In order to have phenomenologically desirable picture one must provide light ($\sim M_W$) masses for electroweak doublets and at the same time keep their colored triplet partners superheavy ($\sim M_G$). In the opposite case, colored triplets lead to unacceptable fast proton decay, except the case when they do not couple to light generation fermions [8], and heavy doublets give an unacceptable large electroweak symmetry breaking scale. This is the famous doublet-triplet splitting problem. The simplest solution of this problem is the appropriate fine tuning of the tree level superpotential parameters (so called "technical" solution) [9], but it seemed to be extremely unnatural.

Several attempts have been made to explain the doublet-triplet hierarchy problem in a natural (without fine tuning) way, such as: "sliding singlet" mechanism [10], which is either unstable under the SUSY breaking [11] or to be implemented in extended GUTs requires complicated Higgs sector [12]; "missing doublet" mechanism also requires Higgs superfields in a very complicate representations ($75 + 50 + \bar{50}$ in the SU(5) case) [13]; "missing VEV" mechanism [14]; GIFT mechanism, where Higgs doublets are identified with pseudogoldstone particles of the spontaneously broken global symmetry [15]; "custodial symmetry" mechanism [16]; "missing doublet representation" mechanism, which explore combined features of some above mentioned mechanisms [17].

In the present paper I suggest the generalization of the "custodial symmetry" mechanism for the explanation of the doublet-triplet hierarchy proposed some times ago by G.Dvali [16]. The main idea of this mechanism is the following: the Higgs doublet will be automatically light if it is related by a certain "custodial" symmetry to another doublet which after the GUT symmetry...
breaking becomes an eaten up goldstone particle. It is clear, that the necessary condition for the realization of this mechanism is that the GUT symmetry group being broken down to the \( G_{SM} \equiv SU(3)_C \otimes SU(2)_W \otimes U(1)_Y \) must produce an unphysical Higgs with quantum numbers of electroweak doublet. The minimal group satisfying this condition is \( SU(6) \). In [16] had been proved the following theorem:

Imagine an \( SU(6) \otimes SU(N)_c \) invariant theory with arbitrary renormalizable superpotential involving Higgs superfields: \( \Sigma^i_j \sim (35,1) \), \( H_{i\alpha} \sim (6,N) \), \( \overline{H}^{i\alpha} \sim (\overline{6},N) \), where \( SU(N)_c \) is an additional "custodial" symmetry (here latin indices stand for \( SU(6) \) group and those of greek for \( SU(N)_c \) one). If there exists a supersymmetric \( G_{SM} \) invariant minimum in which \( SU(3)_W \) is predominantly (up to the admixtures of the order of \( M_W \)) broken by vacuum expectation value (VEV) of the pair \( H_{i\alpha} + \overline{H}^{i\alpha} \), then \( N-1 \) pair of electroweak doublets are necessarily light with masses \( \sim M_W \).

So, in order to have one pair of light doublets in addition to the local \( SU(6) \) one needs \( SU(2)_c \) symmetry which relates light Higgs doublets with eaten up goldstone eigenstates.

However, the general analysis of the \( SU(6) \otimes SU(N)_c \) invariant superpotential shows that in the "Grand desert" region at the scale of the order of \( M_I \sim (M_W M_G)^{1/2} \) necessarily exists intermediate \( G_I \equiv SU(3)_C \otimes SU(3)_W \otimes U(1)_Y \) symmetry structure which spoils the standard unification picture.

As it will be shown below it is possible to generalize this mechanism by embedding the local "custodial symmetry" into higher GUTs and at the same time to resolve the unification problem arising in the simplest version of the model [16].

### 2 \( SU(6) \otimes SU(2)_c \) model

Before getting the generalization of the "custodial symmetry" mechanism let us briefly consider the original proposal [16]. As it was described in the previous section, the minimal model based on \( SU(6) \otimes SU(2)_c \) symmetry group, where \( SU(2)_c \) is global "custodial symmetry" under which one pair of \( SU(6) \) fundamental (antifundamental) Higgs superfields transforms as doublet (antidoublet): \( H_{i\alpha} + \overline{H}^{i\alpha} \). In addition one has \( SU(6) \) adjoint Higgs superfields \( \Sigma^i_j \). Then the most general renormalizable (cubic) superpotential involving these superfields

\[
W(\Sigma, H, \overline{H}) = \frac{1}{2} M_\Sigma T r \Sigma^2 + \frac{1}{3} \sigma T r \Sigma^3 + M_H H_{i\alpha} \overline{H}^{i\alpha} + h H_{i\alpha} \Sigma^i_j \overline{H}^{j\alpha}
\]

among the other SUSY degenerate vacua posses the following one:

\[
l < \Sigma > = diag \left[ 1 1 1 -1 -1 -1 \right] \cdot \frac{M_H}{h} + diag \left[ 2 2 2 -3 -3 -3 \right] \cdot \frac{M_\Sigma}{\sigma},
\]

\[
< H > = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ V & 0 \end{bmatrix}, \quad < \overline{H} > = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & V \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \text{where} \quad V = \left[ \frac{6M_\Sigma}{h} \left( \frac{M_\Sigma}{\sigma} + \frac{M_H}{h} \right) \right]^{1/2}
\]

(2)
Figure 1: The running of gauge couplings in $SU(6) \otimes SU(2)_c$ model of ref.[16] for $M_{SUSY} = M_Z$, $\alpha_S(M_Z) = 0.118$, $\sin^2\theta_W(M_Z) = 0.2312$ and $\alpha^{-1}_{EM} = 127.9$.

Suppose that $M_\Sigma$ is of the order of $M_W$ and $M_H \sim M_G$. In this case $SU(6)$ local symmetry is broken through $S(3)_C \otimes SU(3)_W \otimes U(1)$ channel and $SU(3)_W$ is predominantly broken by the VEV of $H_{i1} + \overline{H}^1_{i1}$, so the condition of the general theorem is holded. It is easy to verify that pair of Higgs doublets from $H_{i2} + \overline{H}^{i2}$ has mass $\sim \frac{3M_\Sigma}{h} \sim M_W$ while colored triplets are superheavy ($\sim M_H \sim M_G$). However one can see from Fig.1 that hierarchical $SU(6)$ breaking

$$SU(6) \xrightarrow{M_\Sigma} SU(3)_C \otimes SU(3)_W \otimes U(1) \xrightarrow{(M_G M_W)^{1/2}} G_{SM}$$

spoils the standard unification picture due to the influence of new gauge interactions on the running of gauge couplings above the intermediate symmetry scale. So, while achieving natural doublet-triplet splitting in a simple and elegant way at the same time we loose the gauge coupling unification. Thus model is inconsistent.

### 3 SU(8) generalization

Now let us consider the generalization of the model discussed above. Let me suppose that instead of the global $SU(2)_c$, dynamical origin of which is very hard to understand, this symmetry is global and moreover, it is embedded together with $SU(6)$ into the simple gauge group. The minimal unitary group of such type is SU(8). The minimal generalization of the Higgs superfield content responsible at the same time to the complete SU(8) breaking down to the
$G_{SM}$ is the following (latin indices now are those of SU(8)):

$$\Sigma^i_j \sim 63, \ H_{[ij]} \sim 28, \ \overline{H}^{[ij]} \sim 28, \ \Phi_{11} \sim 8, \ \Phi_{12} \sim 8, \ \overline{\Phi}_1 \sim \overline{8}, \ \overline{\Phi}_2 \sim \overline{8}$$  \hspace{1cm} (4)

In order to eliminate from the superpotential undesirable couplings such as $H \Phi_1 \Phi_2 + H \overline{\Phi}_1 \overline{\Phi}_2$ one can require additional discrete symmetry under which $\Phi_2$ and $\overline{\Phi}_2$ only change sign $^3$. Then the general superpotential looks like:

$$\frac{\sigma}{3} Tr \Sigma^3 + M_H H_{[ij]} \overline{H}^{[ij]} + h H_{[ij]} \Sigma^k_i \overline{H}^{[kj]} + a \Phi_{11} \Sigma^i_j \overline{\Phi}_1 + M_\Phi \Phi_{12} \overline{\Phi}_2 + b \overline{\Phi}_{12} \Sigma^j_i \overline{\Phi}_2$$  \hspace{1cm} (5)

where I have omitted the mass terms of $\Sigma^2$ and $\Phi_2 \overline{\Phi}_2$ assuming that they are small ($\sim M_W$) in comparison with $M_H$ and $M_\Phi$ and they can easily be reproduced in the scalar superpotential after the SUSY breaking, as a soft breaking parameters (in the minimal N=1 Supergravity version they are the universal gravitino mass $m_3/2$).

The superpotential (5) has a minimum:

$$< \Sigma > = diag \left[ 1 \ 1 \ 1 \ -1 \ -1 \ 1 \ 0 \ 0 \ 0 \right] \cdot \frac{M_H}{h} + diag \left[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \right] \cdot M_\Phi \over a,$$

$$< H > = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & V & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -V & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$< \overline{H} > = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$< \Phi_1 > = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},$$

$$< \overline{\Phi}_1 > = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & V_1 & 0
\end{bmatrix},$$

$$< \Phi_2 > = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},$$

$$< \overline{\Phi}_2 > = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & V_2
\end{bmatrix},$$

$^3$ This discrete symmetry is anomaly free and could be considered as a discrete gauge symmetry. Note that introduction of $\Phi_2$ and $\overline{\Phi}_2$ superfields and subsequently of the discrete symmetry is not necessary for the solution of the doublet-triplet splitting problem and complete SU(8) breaking down to the $G_{SM}$, but as will be seen below they play crucial role in the unification of gauge couplings
Figure 2: The value of $\alpha_S(M_Z)$ predicted from the gauge coupling unification requirement at two-loop level as a function of $M_\Phi$ (solid line) and $M_G$ (dotted line). The following parameters are used as input: $M_{SU SY} = M_Z$, $\sin^2 \theta_W(M_Z) = 0.2312$ and $\alpha_{EM}^{-1}(M_Z) = 127.9$

where $V = \left[ \frac{\sigma}{ah} M_\Phi \left( \frac{4M_H}{h} - \frac{M_\Phi}{a} \right) \right]^{1/2}$, $V_1 = \left[ \frac{4\sigma}{ah} M_H \left( \frac{M_H}{h} - \frac{M_\Phi}{a} \right) \right]^{1/2}$, $V_2 = \sqrt{\frac{4\sigma}{bh^2}} M_H$ (6)

As in the case considered in sec.2 one pair of doublets from $H + \Pi$, namely $H_{[w]\Phi} + \Pi_{[w]\Phi}$ (where $w=4,5$) again remain light (in order to ignore the small masses of $\Sigma^2$ and $\Phi_2\Pi_2$ exactly massless). Mass parameter $M_\Phi$ does not play any role in the mass formation of doublets $H_{[w]\Phi} + \Pi_{[w]\Phi}$ and could be as large as we need. In the case $M_\Phi > M_H$ the hierarchy of SU(8) breaking is the following:

$SU(8) \sim^{M_\Phi} SU(6) \sim^{(M_\Phi M_H)^{1/2}} SU(5) \sim^{M_H} G_{SM}$ (7)

and therefore the standard MSSM unification picture up to the GUT threshold corrections is retained. But there are solutions of RGE for gauge $M_\Phi < M_H$ compatible with unification condition. As one can see from Fig.2 the general tendency is the following: the decreasing of $M_\Phi$ leads to the decreasing of the strong coupling constant value $\alpha_S(M_Z)$, so the model in principle could predict the value of strong gauge coupling in agreement with those extracted from low-energy experiments.

The "custodial symmetry" mechanism demonstrated here explicitly on the SU(8) example can be implemented for any $SU(N \geq 8)$ SUSY GUTs by the appropriate choice of Higgs superfields and following the general prescriptions discussed above.
4 Discussion

I have shown that it is possible to solve the doublet-triplet splitting problem in a natural and economical way in the frames of extended $SU(N \geq 8)$ SUSY GUTs. While concentrating the Higgs sector of the model I said nothing about the quarks and leptons (matter sector). As it is well known, the family or, in general, flavour problem, which relates to the questions such as quark-lepton family replication, hierarchy of quark-lepton masses and mixings, suppression of the flavour changing neutral currents, side by side with the gauge hierarchy problem is one of the most difficult to explain. Very interesting framework for the solution of the flavour puzzle is the family (horizontal) symmetries, namely local chiral $SU(3)_H$ one [18]. So for flavour unified GUTs it is natural to consider $SU(N \geq 8)$ symmetries. This offers an intriguing possibility to solve the both gauge hierarchy and flavour problem in the frames of extended $SU(N \geq 8)$ SUSY GUTs [19].

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