Stability of Phase-modulated Quantum Key Distribution System

Zheng-Fu Han, Xiao-Fan Mo, You-Zhen Gui, and Guang-Can Guo

Key Laboratory of Quantum Information,
University of Science and Technology of China (CAS),
Hefei, Anhui 230026, People’s Republic of China.

Abstract

Phase drift and random fluctuation of interference visibility in double unbalanced M-Z QKD system are observed and distinguished. It has been found that the interference visibilities are influenced deeply by the disturbance of transmission fiber. Theory analysis shows that the fluctuation is derived from the environment disturbance on polarization characteristic of fiber, especially including transmission fiber. Finally, stability conditions of one-way anti-disturbed M-Z QKD system are given out, which provides a theoretical guide in pragmatic anti-disturbed QKD.

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Quantum cryptography (QC) or quantum cryptographic key distribution (QKD) has advanced for twenty years since Bennett and Brassard proposed the idea in 1984 [1]. It has approached practicability to date, and is applied in the field of information security [2, 3]. Many protocols for QC are proposed up to the present, including BB84 [1], B92 [4] and improved protocols [5, 6] based on single photons, and EPR protocols [7] based on entangled photon pairs. It has been proved that BB84 protocol and EPR protocol are consistent essence [8]. Considering current experimental technique, single-photon protocols are more practical for commercial QKD.

A general challenge for QKD is to choose a coding scheme free of disturbance by the dominant noise sources. In the case of single-photon protocols, quantum information is encoded by the states of single photons that travel from Alice to Bob, and two kinds of schemes are utilized: polarization coding and phase coding. Polarization coding is adopted in free-space QKD because atmospheric density fluctuations only reduce the collection efficiency but don’t degrade the contrast between the polarization basis states. C. Kurtsiefer et al. have realized the secure exchange of keys over a free space path of 23.4 kilometres between Zugspitze and karwendelspitze of the Alps [9]. This marks a pioneering step towards accomplishing key exchange with a near-Earth orbiting satellite and hence a global key-distribution system [10]. On the other hand, optical fibres are appropriate for communication on the ground without influence of weather and atmospheric pollution. However, it is a calamity to use polarization as basis for encoding quantum information due to significant birefringence. Hence, phase coding is widely used in fibre-optic QKD-prototype systems.

The most typical fiber QKD prototype was designed in Ref. [11]. In that prototype, two photon pulses pass through the same transmission fiber (quantum channel), and the same disturbance, from transmission processes, is expected to be counteracted on Bob’s side. But in fact, this prototype gives us a bad systemic stability. In order to improve the stability, two groups [12, 13] developed a new prototype independently, in which the two pulses transmit a round trip with a Faraday reflection in mid-course. Unfortunately, this prototype leaves a chance to Eavesdroppers in the key exchange. Eve can send Trojan-horse photons to tail signal photons thus pass in and out of Bob’s secure office, collecting Bob’s coded information without being discovered [14]. Furthermore, any go-and-return QKD protocol [15] is essentially unsecured, and true secure QKD should be based on photons propagating along one way. Therefore, for practical use of QKD, it is very important to investigate the stability of
double M-Z interferometers. This paper investigates the stability in experiments and establishes a theoretical model. Two kinds of perturbation motion, i.e., phase drift and random fluctuation of interference visibility, are observed and discussed. Formerly, it was ever believed that the dominant disturbance arises from different environments of interferometers on Alice and Bob’s sides, and the disturbance in quantum channel is equivalent to both pulses (with ns-scale time interval) which can be counteracted in the final interference. But our experiments show that interference visibility depends intensively on length of transmission fiber ($L$) and systemic stability is also influenced by the way disturbance. Theoretical analysis, here, deduces the conditions for systemic stabilization of double unbalanced M-Z interferometers. It will offer important reference for stable one-way QKD systems.

Typical QKD prototype is described in Fig. 1. Two uniform unbalanced M-Z interferometers were built with common single mode fiber (SMF-28), which is respectively the coder of Alice or Bob. Quantum channel between Alice and Bob and beamsplitters (50/50) are also made of the same fiber. Experimental result is depicted in Fig. 2. Fig. 2a shows the received optical power of single detector D1 for $L = 2$ m without any phase modulation. Apparently, the fluctuation is derived from phase drift, which is relevant to the environment of interferometers. The main reason is that environmental temperature has a small fluctuation, and is also influenced by other perturbation motion, for instance, fiber vibration. Fig. 2b describes the interference fringe of the QKD system when $L = 75$ km, and here the phase modulator is driven by long-periods saw-tooth wave. The fluctuation of envelope of interference fringe exceeds 50% and shows full randomness within six hours. The random fluctuation should be closely associated with the environment of the system.

Summarizing the results of Fig. 2, systemic instability resulted from environmental fluctuation behave as phase drift and random fluctuation of interference visibilities. The former leads to work-point destabilization for QKD system and can be corrected by instantaneous calibration. But the latter results in qubit-error-rate rising, which can not be artificially controlled due to randomness of environmental fluctuation. In principle, rigorous anti-disturbance methods can be used in Alice and Bob’s secure offices, so the effect of the disturbance can decrease to as weaker as possible. However, coupling between quantum channel and its environmental disturbance is uncontrolled. Therefore, disturbance from environment of transmission fiber is discussed as follows.

Fig. 3 shows practical interference visibilities for different transmission fiber length $L$. 
For $L = 0$, the fluctuation of the visibility does not exceed 5% within 280 hours. For $L = 25, 50, 75$ km, this fluctuation is more than 80%. It is most interesting, that the longer the transmission fiber is, the faster the visibilities fluctuate. These results indicate that visibilities are affirmatively influenced by way disturbance. Additional consideration is that, temperature fluctuation only leads to phase drift but not interference visibilities fluctuation. Therefore, fluctuation of interference visibilities should arise from polarization birefringence in fiber induced by perturbation motions, such as fiber bend and distortion. In order to explore the relationships between interference visibility and polarization characteristic in M-Z systems, a simple but reasonable theoretical model is built here.

For the sake of clarity and concision, we assume that there is no fiber nonlinearity and no polarization-dependent loss, and that the usual loss terms of the fiber have been factored out so that we can deal with unitary Jones matrix [16] to describe polarization-transport character of each part in the system (interferometers of Alice and Bob, transmission fiber). $A_1, A_2, C, B_1, B_2$ are respectively Jones matrixes to each part of fiber in the QKD apparatus (see Fig. 1). Two paths leading to single-photon interference on $BS_4$ are: $P_1 (A_1 \rightarrow C \rightarrow B_2 \rightarrow PM_B)$ and $P_2 (A_2 \rightarrow PM_A \rightarrow C \rightarrow B_1)$. Here $PM_A$ and $PM_B$ are two phase modulators. So single-photon optical transformation matrixes through two paths are respectively described by:

$$B_2 e^{i\beta_2} \cdot e^{i\varphi_B} \cdot C e^{i\phi} \cdot A_1 e^{i\alpha_1} = B_2 C A_1 e^{i(\alpha_1 + \beta_2 + \phi + \varphi_B)},$$

$$B_1 e^{i\beta_1} \cdot C e^{i\phi} \cdot A_2 e^{i\alpha_2} \cdot e^{i\varphi_A} = B_1 C A_2 e^{i(\alpha_2 + \beta_1 + \phi + \varphi_A)},$$

where $\alpha_i, \beta_i$ are common phases through $i$th fiber of Alice or Bob and $\varphi_A (\varphi_B)$ is modulated phase of $PM_A (PM_B)$.

Supposing input Jones vector of field [16] is $E_{in}$, and each effective Jones vector arriving at $BS_4$ is $E_{in}/4$. Hence input power $I_{in} = |E_{in}|^2$ and the output Jones vector can be written:

$$E_{out} = \left[ B_2 C A_1 e^{i(\alpha_1 + \beta_2 + \phi + \varphi_B)} + B_1 C A_2 e^{i(\alpha_2 + \beta_1 + \phi + \varphi_A)} \right] \frac{E_{in}}{4}.$$
On the consideration that $A_i, B_i, C$ are unitary, output power can be expressed as:

\[
P_{\text{out}} = E_{\text{out}}^+ \cdot E_{\text{out}} = \frac{P_{\text{in}}}{8} + \frac{1}{16} E_{\text{in}}^+ \left[ A_1^+ C^+ B_2^+ B_1 C A_2 e^{-i(\Delta\alpha + \Delta\beta + \Delta\varphi)} + A_2^+ C^+ B_1^+ B_2 C A_1 e^{i(\Delta\alpha + \Delta\beta + \Delta\varphi)} \right] E_{\text{in}},
\]

where $\Delta\alpha = \alpha_1 - \alpha_2$, $\Delta\beta = \beta_1 - \beta_2$, $\Delta\varphi = \varphi_B - \varphi_A$. Note that the polarized fluctuation, is random, and consequently the matrixes $A_i, B_i, C$ are not independent on time. Obviously, $P_{\text{out}}$ is a function of $A_i, B_i, C, \Delta\alpha, \Delta\beta, \Delta\varphi$, which means interferential output is dependent not only on both M-Z interferometers but also on transmission fiber. In fact, disturbance from M-Z interferometers and transmission fiber are not independent, which supports our experimental results.

In Eq. 4, if we make $B_1^+ B_2 = I$ on Bob’s side, then:

\[
P_{\text{out}} = \frac{P_{\text{in}}}{8} + \frac{1}{16} E_{\text{in}}^+ \left[ A_1^+ A_2 e^{-i(\Delta\alpha + \Delta\beta + \Delta\varphi)} + A_2^+ A_1 e^{i(\Delta\alpha + \Delta\beta + \Delta\varphi)} \right] E_{\text{in}},
\]

where we have considered $C$ is unitary and thus $C^+ C = I$. Now $P_{\text{out}}$ no longer depends on transmission fiber, i.e., $P_{\text{out}}$ is independent on any polarized disturbance in transmission fiber. Furthermore, if $A_1^+ A_2 = I$ on Alice’s side, Eq. 5 is simplified as

\[
P_{\text{out}} = P_{\text{in}} \frac{1 + \cos(\Delta\alpha + \Delta\beta + \Delta\varphi)}{8}.
\]

This means that interferential output power $P_{\text{out}}$ does not rely on any polarized perturbation motion in whole QKD system. In the ideal case, $\Delta\alpha, \Delta\beta$ are invariable, and hence interference fringes are only modulated by $\Delta\varphi$ brought by both phase modulators. However, in the actual case, the fluctuation of environmental temperature will bring some drift of $\Delta\alpha, \Delta\beta$, corresponding to Fig. 2a. In fact, the fluctuation of environmental temperature is so slow that one can calibrate it by instantaneous compensation.

Aforementioned anti-disturbance conditions $A_1^+ A_2 = I, B_1^+ B_2 = I$ can be equivalently written as:

\[
A_1 = A_2, B_1 = B_2,
\]

because of unitarity of matrixes $A_i, B_i$. These mean that if we can assure uniform polarization character for two arms of each interferometer, then the QKD system resists polarized
disturbance not only from transmission fiber (quantum channel) but also from the interferometers. A typical prototype to satisfy upper conditions is that the inner light paths of interferometers are polarization-maintaining. In this case, the Jones matrixes are

\[ A_1 = A_2 = I, \quad B_1 = B_2 = I. \]  

(8)

Obviously, Eq. 8 is a special case to Eq. 7, and it points out that each arm must maintain input photon’s polarization. In experiment, free space or polarization-maintaining fibers of interferometers can fulfill Eq. 8.

We have observed and distinguished two kinds of unstable phenomena in double unbalanced M-Z QKD system, i.e., phase drift and random fluctuation of interference visibilities. We have found that the interference visibilities can be influenced deeply by the disturbance of transmission fiber. Through theoretic analysis, it is pointed out that the influences are derived from the environmental disturbance on polarization characteristic of fiber, and give out the stability condition of anti-disturbance M-Z QKD system with one-way photons transmission. The conditions contribute to the researches on pragmatic no round trip anti-disturbed QKD system.

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FIG. 1: QKD system based on double unbalanced Mach-Zehnder interferometers.

FIG. 2: (a) Output power of single detector D1 without any phase modulation, here $L = 2\,\text{m}$, and shows phase excursion for different time. (b) Interference fringe using the QKD system for different time, here $L = 75\,\text{km}$. Phase modulator is controlled by long-periods saw-tooth wave.
FIG. 3: Interference visibilities based on double unbalanced M-Z interferometers at different time.

a: $L = 0$; b: $L = 25$ km; c: $L = 55$ km; d: $L = 75$ km.