Dominant energy condition and dissipative fluids in general relativity

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Abstract
Existing literature implements the Dominant Energy Condition for dissipative fluids in general relativity. It is pointed out that this condition fails to forbid superluminal flows, which is what it is ultimately supposed to do. Tilted perfect fluids, which formally have the stress-energy tensor of imperfect fluids, are discussed for comparison.

Keywords Dominant energy condition · Imperfect fluid · Superluminal motion · Tilted fluid

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1 Introduction
The study of fluids in general relativity (GR) is well-developed and perfect fluids play a dominant role in this literature [1, 2], but they do not describe situations in which dissipation becomes important such as, for example, the oscillation of neutron stars or the generation of gravitational waves from compact objects [3]. The most common
model for dissipative relativistic fluids exhibits a purely spatial heat current density $q^a$, which is obviously non-causal and is the subject [2, 4, 5] of Eckart’s first-order thermodynamics [6]. This causality problem is cured in the Israel-Stewart and in other versions of second-order (causal) thermodynamics [7–12] but, due to the inherent complication of these formalisms, the simplest non-causal model is still the most used in GR.

Relativistic forms of matter, including fluids, are supposed to satisfy energy conditions which forbid negative energy densities, superluminal mass and energy flows, and overly negative stresses. Indeed, without requiring any energy condition, one could write down any metric tensor $g_{ab}$ and, running the Einstein equations

$$R_{ab} - \frac{1}{2} g_{ab} R = 8\pi T_{ab} \tag{1}$$

from left to right, compute the effective stress-energy tensor $T_{ab}$ that sources such a metric (here $R_{ab}$ denotes the Ricci tensor of the metric $g_{ab}$ and $R \equiv R^{cc}$ is its trace). In general, this procedure produces senseless effective energy-momentum tensors $T_{ab}$ and completely unphysical solutions of the Einstein equations, hence the need to keep in check the physically admissible forms of matter described by $T_{ab}$ by imposing suitable energy conditions [13, 14]. Moreover, energy conditions are crucial in the proofs of the black hole and cosmological singularity theorems [15, 16] and of the positivity of mass [17]. The classical energy conditions are, however, doubted at least in the semiclassical context and are regarded as temporary requirements evolving as our knowledge of relativistic matter changes, usually on the scale of decades ([18], see [19] for a recent review).

Things become tricky when the energy conditions are contemplated for dissipative fluids. The literature on this subject is very limited: two articles [20, 21] address head-on the issue of energy conditions for imperfect fluids and they seem to have been quite influential. However, it is time to reconsider the energy conditions, and particularly the Dominant Energy Condition (DEC) for dissipative fluids. The main point here is that, while technically one can satisfy the energy conditions, the imperfect fluid model adopted unavoidably contains a purely spatial heat flux density vector. Specifically, this non-causal flow is not eliminated by imposing the usual DEC, which instead was originally supposed (and is naively believed) to eliminate superluminal flows completely. Thus, superficially it may seem reassuring that the DEC is satisfied but the usual dissipative fluid still, and unavoidably, contains non-causal heat propagation.

Another issue needs clarification. The DEC is universally taken to be meaningful for perfect fluids and understood as forbidding superluminal flows. Since perfect fluids are not dissipative, the problem mentioned above does not apply but, technically, the stress-energy tensor $T_{ab}$ of a perfect fluid that is tilted (i.e., seen from the frame of an observer not comoving with this fluid) has the imperfect fluid form. This happens because the relative motion of the two frames generates a convective current in any frame in which this fluid is not at rest, and this has the same form as the heat flux of an imperfect fluid.

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1 We use the notation of Ref. [13], in which the metric has signature $-+++$ and units are used in which the speed of light $c$ and Newton’s constant $G$ are unity.

2 At the time of writing, a cumulative citation count in Google Scholar returns 157 entries.
The DEC seems to take two different forms, the perfect fluid formulation in the comoving frame and the imperfect fluid DEC version in any non-comoving frame, leading to potential confusion, at least at first sight. We check explicitly that the two formulations of DEC, in fact, agree with each other for tilted perfect fluids.

**2 DEC and dissipative fluids**

In GR, the imperfect fluid is characterized by the stress-energy tensor

\[ T_{ab} = \rho u_a u_b + P h_{ab} + \pi_{ab} + q_a u_b + q_b u_a, \]  

where \( u^a \) is the fluid four-velocity satisfying \( u_c u^c = -1 \),

\[ h_{ab} \equiv u_a u_b + g_{ab} \]  

is the Riemannian metric on the 3-space orthogonal to \( u^a \), \( \rho \) is the energy density,

\[ P = \tilde{P} + P_{\text{viscous}} \]  

is the isotropic pressure, consisting of a non-viscous contribution \( \tilde{P} \) and of a viscous one \( P_{\text{viscous}} \), \( \pi_{ab} \) is the trace-free anisotropic stress-tensor, and \( q^a \) is the heat flux density. \( h_{ab}, \pi_{ab}, \) and \( q^a \) are purely spatial,

\[ h_{ab} u^a = h_{ab} u^b = \pi_{ab} u^a = \pi_{ab} u^b = q_a u^a = 0. \]  

The Dominant Energy Condition (DEC) requires \(-T_{ab} v^b \leq 0\) for all timelike vectors \( v^a \) and is supposed to forbid superluminal flows of mass-energy [14]. However, satisfying the DEC does not achieve this goal for imperfect fluids because the heat flux density vector, being purely spatial, will always describe a superluminal energy flow when it does not vanish.

Let us consider the special case in which the timelike vector \( v^b \) in the formulation of the DEC is the four-velocity of the imperfect fluid itself, \( v^a = u^a \). In this case, the corresponding energy flow is

\[ j_a \equiv -T_{ab} u^b = \rho u_a + q_a \]  

and consists of the material flow \( \rho u_a \) plus the heat flow \( q_a \). It is easy to see that

\[ j_a j^a = (\rho u_a + q_a) (\rho u^a + q^a) = -\rho^2 + q^2 \]  

is non-positive, and \( j^a \) is causal (i.e., timelike or null), if and only if \( \rho^2 \geq q^2 \equiv q_c q^c \), which is part of the formulation of the DEC reported in Ref. [21] (to guarantee that the vector \( S_a \equiv -T_{ab} v^b \) is future-oriented). However, the vector \( q^a \) remains purely spatial and non-causal. The DEC is enforced if, in addition [20, 21],

\[ \rho^2 \geq P^2_i + q^2 + 2(\rho + 3P) q, \quad i = 1, 2, 3, \]  

where \( P^2_i \) is the pressure in the \( i \)-th component.
where the $P_i$ are the eigenvalues of the total stress tensor $\Pi_{ab} = P h_{ab} + \pi_{ab}$ and

$$P \equiv \frac{1}{3} h^{ab} T_{ab}$$

(9)

is the (total) isotropic pressure. Even when (8) is satisfied, the generic energy flux $-T_{ab} v^b$ contemplated in the DEC cannot forbid the heat flux vector

$$q_a = -h_a^c u^b T_{cb} \neq -u^b T_{ab} \equiv j_a.$$

(10)

It seems that a more effective formulation of the DEC would require that

both $-T_{ab} v^b$ and $-h_a^c v^b T_{cb}$ are timelike or null for any timelike vector $v^a$.

but this is tautological because $-h_a^c v^b T_{cb}$, being the spatial component of $j_a$, is necessarily non-causal if it is non-vanishing. The point is that $-T_{ab} v^b$ includes two distinct fluxes of very different nature, the convective flow of mass-energy $\rho u_a$ and the purely diffusive heat flux $q_a$. Forbidding the first from being superluminal does nothing to restrict the second, which remains superluminal. Thus, although the “total” vector field $j_a = \rho u_a + q_a$ can be timelike, this is an artificial mathematical entity that does not catch the underlying physics and there remains a non-causal heat flux. In other words, the DEC restricts the magnitude $q$ of $q^c$ without changing its causal nature.

Similar conclusions are reached by considering the Landau frame of the dissipative fluid. The Landau (or energy, or Landau-Lifschitz) frame is the frame based on the direction of the total energy flux, hence the dissipation of energy does not appear explicitly in this frame,

$$q^{(L)}_a = -T_{cd} u^c_{(L)} h^d_{a(\L)} = 0$$

(11)

where

$$h^{(L)}_{ab} \equiv g_{ab} + u^a_{(L)} u^b_{(L)},$$

(12)

but the energy flux is traded with a particle flux $N^a$. The four-velocity $u^a_{(L)}$ defining the Landau frame, which is the direction of the total energy flux, is an eigenvector of the stress-energy tensor,

$$T_{ab} u^b_{(L)} = -\rho_{(L)} u^a_{(L)}.$$

(13)

In fact, in the Landau frame $q^{a}_{(L)} = 0$ and the stress-energy tensor of the imperfect fluid is decomposed (differently than in the Eckart frame) as

$$T_{ab} = \rho_{(L)} u^a_{(L)} u^b_{(L)} + P^{(L)} h^{ab}_{(L)} + \pi^{(L)}_{ab}.$$  

(14)
Since the diffusive energy flux is already contained within the direction of $u_a^{(L)}$, in this frame energy diffusion does not appear explicitly and the heat current is $q_a^{(L)} = 0$. Then $u_a^{(L)}$ is an eigenvector of $T_{ab}$:

$$
T_{ab} u_b^{(L)} = \left( \rho u_a^{(L)} + \frac{\nabla}{\Delta} h_a^{(L)} + \pi_{ab}^{(L)} \right) u_b^{(L)}
$$

$$
= \rho u_a^{(L)} \left( u_b^{(L)} u_b^{(L)} \right) + \pi_{ab}^{(L)} u_b^{(L)}
$$

$$
= -\rho u_a^{(L)}.
$$

When the strong energy condition holds, the timelike four-velocity eigenvector $u_a^{(L)}$ of the imperfect fluid stress-energy tensor is unique [23] (the uniqueness of the Landau frame has been derived also from the relativistic Boltzmann equation using renormalization group techniques [24]). The particle flux density is

$$
N^a = n(L) u_a^{(L)} + V_a^{(L)},
$$

where $V_a^{(L)}$ is the particle diffusion current density.

The requirement that the formal mass-energy current density $-T_{ab} u_b^{(L)}$ be timelike (respectively, null) is achieved for timelike (null) four-velocity $u_a^{(L)}$ since

$$
\left( -T_{ab} u_b^{(L)} \right) \left( -T^{c} u_{c(L)} \right) = \rho^2 u_a^{(L)} u_a^{(L)} \leq 0
$$

implies $u_a^{(L)} u_a^{(L)} \leq 0$. However, the non-causal nature of the heat flux is now traded with the non-causal nature of the particle flux since $V_a^{(L)}$ is spacelike. The relation between quantities in the Eckart and the Landau frames is

$$
\frac{u_a^{(E)}}{V_a^{(L)}} = \frac{n_a^{(E)}}{n_a^{(L)}} = \frac{n(L) u_a^{(L)} + V_a^{(L)}}{n_a^{(L)}}.
$$

Let $V_a^{(L)} V_a^{(L)} = \alpha^2$, then

$$
n_a^{(E)} = \sqrt{-N^c N_c} = \sqrt{n_a^{(L)^2} - \alpha^2} = n(L) \sqrt{1 - \left( \frac{\alpha}{n_a^{(L)}} \right)^2}
$$

$$
= n(L) \sqrt{1 - v^2}
$$

where $v = \alpha/n_a^{(L)}$ is the relative velocity between the Eckart and Landau frames (a purely spatial vector) [23]. One can now have $u_a^{(L)}$ and $N_a^{(L)} = n_a^{(E)} u_a^{(E)} - V_a^{(L)}$ timelike, but $V_a^{(L)}$ is spacelike.

The new definition of DEC requiring both $-T_{ab} v^b$ and its spatial projection to be causal for any timelike $v^b$ would simply prohibit one a priori from considering
imperfect fluids with spacelike heat flux density $q^a$. It seems rather pointless, therefore, to impose the usual DEC on imperfect fluids of the form (2) with $q^c q_c > 0$.

An alternative stress-energy tensor used in models of anisotropic spherical stars is given, in spherical coordinates, by [22]

$$T_{ab}^{(2)} = \rho u_a u_b + P_t h_{ab} + \sigma q_a q_b,$$

where

$$\sigma = P_t - P_r$$

is the difference between the tangential and radial pressures $P_t$ and $P_r$ and the purely spatial vector $q^a$ is interpreted as a radial velocity [22]. This interpretation is, however, questionable because the four-velocity of the fluid is $u^a$ and this fluid cannot have simultaneously two velocities, in addition to the fact that one would have a spacelike radial four-velocity. In this case, $-T_{ab} u^b = \rho u_a$ is a timelike vector. This fluid is anisotropic but apparently non-dissipative, so it really fails to address our problem of the DEC’s relation with dissipative fluids.

### 3 Tilted perfect fluid

There is another occurrence of imperfect fluid which, in reality, is only a perfect (non-dissipative) fluid in disguise. A perfect fluid with stress-energy tensor

$$T_{ab} = \rho^* u^*_a u^*_b + P^* h^*_{ab},$$

when seen from a non-comoving frame, i.e., from a frame based on observers with a (timelike) four-velocity $u^a$ different from the fluid four-velocity $u^*_a$, or “tilted”, appears as a dissipative fluid [23, 25]. The comoving frame is the unique frame in which the perfect fluid stress-energy tensor assumes the perfect fluid form (22) [23, 25]. In the frame of a different observer with timelike four-velocity $u^a$ related to $u^*_a$ by [23, 25]

$$u^*_a = \gamma (u^a + v^a),$$

$$\gamma = \frac{1}{\sqrt{1 - v^2}} = -u^c u_c,$$

$$v^2 \equiv v^c v_c, \quad u^c u_c = 0, \quad 0 \leq v^2 < 1,$$

this perfect fluid (now “tilted”) will appear as a dissipative fluid with the different stress-energy tensor decomposition [23, 25–27]

$$T_{ab} = \rho u_a u_b + P h_{ab} + q_a u_b + q_b u_a + \pi_{ab},$$
where $h_{ab} = g_{ab} + u_a u_b$ as usual and \[23, 25–27\]

\[
\rho = \rho^* + \gamma^2 v^2 (\rho^* + P^*) = \gamma^2 (\rho^* + v^2 P^*),
\]

(27)

\[
P = P^* + \frac{\gamma^2 v^2}{3} (\rho^* + P^*),
\]

(28)

\[
q^a = \left(1 + \gamma^2 v^2\right) (\rho^* + P^*) v^a
\]

\[
= \gamma^2 (\rho^* + P^*) v^a,
\]

(29)

\[
\pi^{ab} = \gamma^2 (\rho^* + P^*) \left(v^a v^b - \frac{v^2}{3} h^{ab}\right).
\]

(30)

That is, the same stress-energy $T_{ab}$ admits infinitely many decompositions: one based on the fluid four-velocity $u^a$ and others based on different timelike vectors $u^a$.

Equation (29) lends itself to a natural physical interpretation. The heat current density $q^a$ is the energy crossing the unit of normal area per unit time. In Newtonian physics this flux density would be $\rho^* v$: in GR also the pressure gravitates, hence $\rho^*$ is replaced by $(\rho^* + P^*)$. Then, the gravitating energy density is corrected by two Lorentz factors, one because of Lorentz contraction in the direction of motion (one of the three directions forming spatial volume, while the two directions perpendicular to the motion are not Lorentz-contracted). Then, this energy is blueshifted because of time dilation associated with the motion, which contributes the second Lorentz factor $\gamma$. It is clear that now the spatial vector $q^a$ arises solely due to the relative motion between the two frames, i.e., to the (spatial) vector $v^a$. In this context it would be incorrect to interpret this purely convective flux as due to heat conduction, which should instead be given by Eckart’s generalization of Fourier’s law [6]

\[
q_a = -Kh_{ab} \left(\nabla^b T + T \dot{u}^b\right),
\]

(31)

where $K$ is the thermal conductivity and $\dot{u}^a \equiv u^c \nabla_c u^a$ is the fluid’s four-acceleration. In other words, the non-dissipative nature of the fluid (22) remains in other (non-comoving) frames in spite of the form of the stress-energy tensor in these frames, which technically has the form (2) of a dissipative fluid but ultimately is non-dissipative. To corroborate this argument, one notes that in Eckart’s theory [6] (as well as in the ordinary description of three-dimensional non-relativistic Newtonian fluids), the viscous pressure is assumed to be linear in the velocity gradient,

\[
P_{\text{viscous}} = -\zeta \Theta = -\zeta \nabla_c u^c,
\]

(32)

where $\zeta$ is the bulk viscosity coefficient and $\Theta = \nabla_c u^c$ is the expansion scalar. Since

\[
\Theta = \nabla_c u^c = \frac{\Theta^*}{\gamma} - 2\gamma v^a u^c \nabla_c v_a - \nabla_c v^c,
\]

(33)

there is no natural way to relate the viscous pressure $P_{\text{viscous}}$ in the non-comoving frame with the expansion scalar $\Theta$ to satisfy the constitutive relation (32) for Newtonian
dissipative fluids in this frame, another pointer to the fact that there is no real heat flux. Similarly, there appears to be no way to satisfy the other constitutive relation of Eckart’s thermodynamics \[ \pi_{ab} = -2\eta\sigma_{ab} \] (i.e., the anisotropic stresses being proportional to the shear tensor \( \sigma_{ab} \) through a viscosity coefficient \( \eta \)) with a purely convective energy flux.

Imposing the DEC on the perfect fluid (22) guarantees that DEC is satified in any frame in which this fluid is tilted and in which, accordingly, the formulation of DEC should be different from that for perfect fluids [20, 21]. In fact, assume that DEC is satisfied in the comoving frame of the perfect fluid with four-velocity \( u^a \). Any other given timelike vector \( u^a \) can be related to \( u^a \) by Eqs. (23)-(25). Then consider the flux \( -T_{ab}u^b \) by decomposing the stress-energy tensor \( T_{ab} \) with respect to this vector \( u^a \). The DEC expressed using the arbitrary timelike vector \( u^a \) says that \( j^a \equiv -T_{ab}u^b = \rho u_a + q_a \) is causal and future-oriented, or \( q^2 - \rho^2 \leq 0 \) according to Eq. (7).

Using the transformation properties (27)-(29), one obtains

\[
q^2 \equiv q_aq^a = \gamma^4 \left( \rho^* + P^* \right) v^2
\]

and

\[
q^2 - \rho^2 = \gamma^4 \left( \rho^* + P^* \right) v^2 - (\rho^*)^2 - \gamma^4 v^4 \left( \rho^* + P^* \right)^2 - 2\gamma^2 v^2 \rho^* \left( \rho^* + P^* \right) \\
= \gamma^4 v^4 \left( \rho^* + P^* \right)^2 \left( 1 - v^2 \right) - (\rho^*)^2 - 2\gamma^2 v^2 \rho^* \left( \rho^* + P^* \right) \\
= -\left( \rho^* \right)^2 + \gamma^2 v^2 \left[ (P^*)^2 - (\rho^*)^2 \right] \\
\leq -\left( \rho^* \right)^2 < 0
\]

assuming \( P^* \geq -\rho^* \) for the perfect fluid in the comoving frame. In this case, there is no real heat flux but only a heat-like purely convective energy flux and the usual DEC requirement suffices to forbid non-causal flows of mass-energy.

### 4 Conclusions

As a conclusion, it seems rather pointless to impose the DEC on first-order dissipative fluids à la Eckart [6], in the sense that one can compute it for dissipative matter and formally satisfy it, but it loses its conventional physical meaning learned by studying non-dissipative matter. It still makes sense, though, to impose the DEC on fluids that appear dissipative because they are tilted perfect fluids observed in non-comoving frames. In this case, the DEC does a good job at forbidding non-causal flows of material and there is no diffusive (as opposed to convective) energy flow. This situation does not correspond to a true dissipative fluid because the energy flux is purely convective and not thermal, and this case is already covered by the DEC studied for perfect fluids. Thermal heat flow, however, has a different physical nature and eludes the usual DEC. Once a purely spatial heat flux is included in the stress-energy tensor \( T_{ab} \), it will haunt the dissipative fluid forever, except in very special situations in which the heat
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flux is forced to vanish due, for example, to symmetries (as in Friedmann-Lemaître-Robertson-Walker cosmology where $q^a \neq 0$ would violate spatial isotropy). This situation highlights that the DEC restricts flows of material but not all energy fluxes.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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