Measuring spin of a supermassive black hole at the Galactic centre – implications for a unique spin

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ABSTRACT

We determine the spin of a supermassive black hole in the context of disc-seismology by comparing newly detected quasi-periodic oscillations (QPOs) of radio emission in the Galactic centre, Sagittarius A∗ (Sgr A∗), as well as infrared and X-ray emissions with those of the Galactic black holes. We find that the spin parameters of black holes in Sgr A∗ and in Galactic X-ray sources have a unique value of ≈0.44 which is smaller than the generally accepted value for supermassive black holes, suggesting evidence for the angular momentum extraction of black holes during the growth of supermassive black holes. Our results demonstrate that the spin parameter approaches the equilibrium value where spin-up via accretion is balanced by spin-down via the Blandford–Znajek mechanism regardless of its initial spin. We anticipate that measuring the spin of black holes by using QPOs will open a new window for exploring the evolution of black holes in the Universe.

Key words: accretion, accretion discs – black hole physics – binaries: general – Galaxy: centre.

1 INTRODUCTION

The Galactic centre, Sagittarius A∗ (Sgr A∗), is a compact source of radio, infrared and X-ray emissions having variability in the range of a few tens of minutes to hours (Baganoff et al. 2001; Genzel et al. 2003; Yusef-Zadeh et al. 2006). These emissions seem to originate from a hot and low-density accreting gas plunging into a supermassive black hole (Yuan, Quataert & Narayan 2004; Kato, Umemura & Ohsuga 2009). A precise measurement of its mass and spin is a longstanding issue for astrophysics to investigate the mechanism of energy extraction from spinning black holes for astrophysical jet production as well as the evolution of supermassive black holes along the cosmic history (Bardeen 1970; Blandford & Znajek 1977; Wilson & Colbert 1995). Although the mass of Sgr A∗ has been constrained by using the stellar orbit method, a precise measurement of its spin for the best-estimated mass has been poorly conducted.

Recently, Miyoshi and colleagues have detected multiple quasi-periodic oscillations (QPOs) of radio emissions in Sgr A∗ (Miyoshi et al., in preparation) whose periods are close to the Keplerian period at the innermost stable circular orbit of a supermassive black hole with mass 4 × 10⁶ M⊙. Because of the excellent spatial resolution of the Very Long Baseline Array, the quasi-periodic radio emission certainly originates from within the central sub-mas scale, approximately 100rt around the central black hole at a distance of 7.6 kpc, where rt = GM/c² = 0.01(M/10⁶ M⊙) au is the gravitational radius (G, M and c are the gravitational constant, the mass of black hole and the speed of light, respectively). This is the first time that such multiple QPOs have been identified in the vicinity of a supermassive black hole. The spatial pattern of emission regions cannot be explained by the Keplerian rotation of a single emitting body at a given radius.

Four simultaneous QPOs (16.8, 22.2, 31.4 and 56.4 min) are detected and the first three periods are identical to QPOs in the near-infrared and X-ray observations during different observation epochs (see Table 1). Three identical periods in the different wavelength are stable at least for several years and the frequency ratio of last two periods is close to 3:2. Such a stable double peak QPO is a well-known feature for high-frequency QPOs (HF-QPOs) in Galactic X-ray sources (Remillard & McClintock 2006). The multiple periodicity and their coincidence between the different wavelengths, and also the different observation epochs, indicate that the origin of QPOs in the Galactic centre is closely related to the dynamics of an accretion disc feeding the black hole. Therefore, we measure the spin parameter of a black hole in Sgr A∗ by using the period of QPOs based on disc-seismology (e.g. Nowak & Wagoner 1993).

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2 METHOD AND MODEL

One promising mechanism of generating multiple QPOs is a global disc oscillation excited by the resonance between geodesic modes of the disc (the so-called resonant disc oscillation model; Abramowicz & Kluźniak 2001; Kato & Fukue 2006; Kato, Fukue & Mineshige 2008). The resonant frequency is the combination among geodesic frequencies at the radius where the resonance occurs. When the resonance condition is specified, both the resonant frequency and the resonant radius are determined uniquely in terms of the black hole mass \( M \) and the spin parameter \( a_\ast = J_c/GM^2 \), where \( J \) is the angular momentum of the black hole. Therefore, the metric of the black hole can be constrained by the frequency of the QPOs.

Resonance may occur at a radius where the frequency ratio of the geodesic modes is a ratio of small integers and resonant response can either spontaneously grow or damp the oscillation itself (Abramowicz & Kluźniak 2001). One of the most prominent resonances is a mode-coupling between acoustic waves and non-axisymmetric modes such as a warp in the disc, the so-called wave-warp resonance (Kato & Fukue 2006; Kato et al. 2008). For example, this resonance is excited at a radius \( r_{\text{res}} \) where \( \Omega_K = 2\kappa \). Here, \( \Omega_K \) and \( \kappa \) are the Kepler frequency and the epicyclic frequency, respectively (see fig. 1 of Kato & Fukue 2006 for the relation between \( a_\ast \) and \( r_{\text{res}} \)). \( \Omega_K \) and \( \kappa \) at the resonant radius \( \tilde{r}_{\text{res}} = r_{\text{res}}/r_g \) measured at infinity are expressed as

\[
\Omega_K = \frac{GM}{r_{\text{res}}^3} \left( 1 + \frac{a_\ast}{\tilde{r}_{\text{res}}} \right)^{-1}
\]

(1)

and

\[
\kappa = \sqrt{\frac{GM}{r_{\text{res}}^3}} \sqrt{1 - 3/r_{\text{res}}^3 + 8a_\ast (2\tilde{r}_{\text{res}})^{-3/2} - 3a_\ast^2 (2\tilde{r}_{\text{res}})^{-3}}
\]

(2)

as derived by Okazaki, Kato & Fukue (1987). The resulting frequencies of QPOs are \( m\Omega_K \pm \kappa \) and \( m\Omega_K \), where \( m \) is the azimuthal mode number, and some lower mode oscillations related to such resonances are reported by numerical studies (Kato 2004).

3 RESULTS

3.1 Unified model of QPOs

Fig. 1 shows the period of the observed QPO overlaid with lower mode \( m = 1, 2 \) resonant periods related to the wave-warp resonance as a function of the black hole mass ranging from a stellar mass black hole to a supermassive black hole (skip over the intermediate mass region). QPOs in the Galactic centre are selected with regard to the multiple detection among different wavelengths (Table 1). We found that such QPOs in Sgr A* detected at identical frequencies are consistent with a mass--period relation for the spin parameter \( a_\ast \sim 0.4 \) (see Fig. 1b). At the same time, HF-QPOs in the Galactic X-ray sources agree well with the resonant periods for the same spin parameter within the error of the estimated mass (Fig. 1a). Therefore, we identify the three identical periods (16.8, 22.2 and 31.4 min) with resonant modes \( 2\Omega_K, \Omega_K + \kappa \) and \( \Omega_K \), respectively.

3.2 Unique spin parameter

Now, we can determine the spin parameter of black holes by using the periods of QPOs corresponding to \( \Omega_K \). For instance, 31.4 min is used for Sgr A* and periods of lower HF-QPOs are used for the Galactic X-ray sources. Note that the frequency of single peak HF-QPOs are treated as \( \Omega_K \). In order to constrain the resultant spin parameter, the estimated mass of a supermassive black hole in Sgr A* is taken from recent measurements (Schödel et al. 2002; Ghez et al. 2008; Gillessen et al. 2009). Fig. 2 shows spin parameters of all samples evaluated by using the disc-seismic measurement. All spin parameters are relatively small (\( \leq 0.7 \)) in comparison with the equilibrium value of spinning black holes (\( \approx 0.95 \)) predicted by a numerical study (Gammie, Shapiro & McKinney 2004). When all samples are fitted by using a linear relation as a function of the black hole mass, the spin parameter becomes larger than 1 for black holes with \( M \geq 10^7 M_\odot \). Instead of a linear relation, we obtain a best-fitting unique spin parameter \( a_\ast = 0.44 \pm 0.08 \), which is depicted

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**Table 1. QPOs detected in Sgr A*.
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| Obs. epoch (UT) | K band | Period (min) | Ref. # |
|-----------------|--------|--------------|--------|
| 2003/06/15–16   | 1.60, 1.87, 1.90 (μm) | 16.8 ± 2, 28.0 | Genzel et al. (2003) |
| 2004/09         | 2–10 (keV) | 33 ± 2 | Yusef-Zadeh et al. (2006) |
| 2002/10, 2004/08| L band | 22.2 | Béclanger et al. (2006) |
| 2007/04/04      | L band | 22.6 | Hamaus et al. (2009) |
| 2007/07/22      | 45.4 | Hamaus et al. (2009) |
| 2004/03/08 09:30–16:30 | 43 (GHz) | 16.8 ± 1.4, 22.2 ± 1.4, 31.2 ± 1.5, 56.4 ± 6 | Miyoshi et al. (in preparation) |
by a grey shaded region, for 1σ uncertainty by linear least-square fitting.

3.3 Evolution of BH spin and mass

Next, we should ask why black holes have a unique spin parameter in spite of the fact that their age as well as mass accretion history may vary in general. Actually, our results contradict recent studies that predict extremely spinning black holes (Shapiro 2005; Volonteri et al. 2005). In order to test the feasibility of such a small unique spin parameter, we have to study the spin-up process by mass accretion and the spin-down process by the energy extraction as a result of the Blandford–Znajek mechanism, simultaneously.

Fig. 3 represents the equilibrium value of spin and also the time evolution of black holes surrounded by a relativistic standard accretion disc (Novikov & Thorne 1973; Page & Thorne 1974; see also Kato et al. 2008), assuming given disc parameters such as the viscosity parameter $\alpha$ (Shakura & Sunyaev 1973), the magnetized parameter $\beta$, the ratio of the gas pressure to the magnetic pressure and the mass accretion rate $\dot{m} = \dot{M}/M_{\text{EDD}}$ normalized by the Eddington mass accretion rate $M_{\text{EDD}} = 4\pi GM/c\kappa_{\text{es}}$, where $\kappa_{\text{es}}$ is the electron scattering opacity (e.g. Kato et al. 2008). In general, these parameters are not independent because magnetohydrodynamic (MHD) turbulence in the disc is thought to be the source of viscosity and their values can only be examined numerically. For instance, we employ $\alpha = 0.01$ on the basis of three-dimensional MHD simulations showing the total stress corresponds to $\alpha \approx 0.02-0.06$ (Hawley 2000; Machida, Hayashi & Matsumoto 2000) for $\dot{m} \ll 1$ and $\alpha \approx 0.01$ for $\dot{m} \sim 1$ (Hirose, Krolik & Stone 2006). Recent MHD simulations also exhibit the natural emergence of large-scale magnetic fields (the so-called magnetic tower) at the inner region of an accretion disc (Kato, Mineshige & Shibata 2004). The formation of a magnetic tower is key to the extraction of the energy and angular momentum of a spinning black hole by the Blandford–Znajek mechanism and it has been suggested that the necessary condition for the energy and angular momentum extraction at the innermost region of an accretion disc is $\beta \approx 1$ (McKinney & Gammie 2004).

The equations we solved in this study are the followings:

$$\frac{d\ln M}{dt} = \frac{\dot{M}}{M_{\text{in}}} - \frac{P}{M c^2}$$

(3)

$$\frac{dJ}{dt} = \dot{M}_{\text{in}} - \frac{P}{\Omega_{\text{in}}}$$

(4)

where $M_{\text{in}}$ and $\dot{M}_{\text{in}}$ are the mass accretion rate, the specific energy and the specific angular momentum at the inner edge of the accretion disc, respectively. The electromagnetic power loss $P$ from the black hole is assumed to be that of the Blandford–Znajek mechanism:

$$P = P_{\text{BZ}} \approx \frac{1}{8} B^2 r_H^4 c^3 \Omega_{\text{in}} \Omega_{\text{in}} (\Omega_{\text{H}} - \Omega_{\text{in}})$$

(5)

where $r_H$ is the radius of the event horizon and $\Omega_{\text{in}}$ and $\Omega_{\text{H}}$ are the angular velocity of the magnetic fields permeating the horizon and the angular velocity of the black hole, respectively (see Medalisky & Sikora 1996; Beskin et al. 2003). The strength of magnetic fields $B_1$, permeating the event horizon is assumed to be regulated by the pressure of accretion disc $P_{\text{disc}}$ so that $B_1^2 = \frac{8\pi \rho_{\text{disc}}}{\beta}$. Note that the electromagnetic power loss is not negligible when $\beta$ is less than the order of the unity.

The relativistic standard accretion disc model provides a complete set of equations for describing the pressure of accretion disc at the given radius as a function of the viscosity parameter $\alpha$, the black hole
mass $m = M/M_\odot$, the spin parameter $a_*$ and the mass accretion rate $\dot{m}$. For a given $\dot{m}$, the radiation pressure dominated region appears within the radius:

$$\tilde{r}_0 = \frac{r_0}{r_0}$$

$$= 36c^2/211 \times 16^{1/21} 16^{1/21} D^{1/21} H^{10/21} \Omega^{10/21},$$

where $B$, $D$, $H$ and $Q$ are the general relativistic correction factors (Page & Thorne 1974). To summarize, the pressure of the accretion disc can be described as follows:

$$p_{\text{disc}} = \left\{ \begin{array}{ll} p_{rad} & \tilde{r} \leq \tilde{r}_0, \\ p_{gas} & \tilde{r} > \tilde{r}_0, \end{array} \right.$$  (7)

and

$$p_{rad} = 1.4 \times 10^{16} (am)^{-1} R_1 \text{ dyne cm}^{-2},$$  (8)

$$p_{gas} = 3.0 \times 10^{17} (am)^{9/10} m^{3/5} R_2 \text{ dyne cm}^{-2},$$  (9)

where $R_1 = r^{-3} B^{-2} D^{-1} C$ and $R_2 = r^{-51/20} B^{-14/5} D^{-9/10} C H^{-1/2} Q^{9/5}$ are the radial dependence including the general relativistic correction factors at the Boyer–Lindquist coordinated radius $\tilde{r} = c^2 r/GM$. The radius for evaluating the strength of magnetic field is assumed to be $\tilde{r}_0 = 1.3 r_{200}$, where $r_{200}$ is the marginally stable circular orbit (Bardeen et al. 1972):

$$\tilde{r}_{200} = 3 + z_2 - [(3 - z_1)(3 + z_1 + 2 z_2)]^{1/2},$$  (10)

where

$$z_1 = 1 + \left(1 - a_*^2\right)^{1/3} \left[(1 + a_*)^{1/3} (1 - a_*)^{1/3}\right]$$  (11)

and

$$z_2 = \frac{3 a_*^2 + z_1^{1/2}}{}.$$  (12)

Finally, we rewrite equations (3) and (4) by using the normalized variables as

$$\frac{d \ln m}{dr} = \frac{1}{\tau_{\text{Edd}}} (\tilde{r}_m - \eta_{\text{BZ}}),$$  (13)

$$\frac{d a_*}{dr} = \frac{1}{\tau_{\text{Edd}}} \left(\tilde{l}_m - 2 \alpha_* \tilde{e}_m \right) - 2 \eta_{\text{BZ}} \left(\tilde{r}_m \tilde{e}_m - a_*\right).$$  (14)

where symbols are the Eddington time $\tau_{\text{Edd}} = M/M_{\text{Edd}}$, the specific energy input $\tilde{e}_m = \tilde{e}_m/c^2$, the efficiency of the Blandford–Znajek mechanism $\eta_{\text{BZ}} = \tilde{P}_{\text{BZ}}/M_{\text{Edd}} c^2$, the specific angular momentum input $\tilde{l}_m = \tilde{l}_m/GM$, the horizon radius $\tilde{r}_m = c^2 \tilde{r}_m/GM = 1 + (1 - a^2)^{1/2}$ and $k = \Omega_{\text{BZ}}/\Omega_{\text{eq}} = 1/2$ for the maximum efficiency of the Blandford–Znajek mechanism. Here, we assume that the inner boundary is at the marginally stable circular orbit and both the energy and the angular momentum of accreting matter at the boundary are advected into the black hole. The specific energy and the specific angular momentum at the boundary are

$$\tilde{e}_m = \tilde{e}_m = \sqrt{1 - \frac{2}{3\tilde{r}_m}},$$  (15)

$$\tilde{l}_m = \tilde{l}_m = 2 \sqrt{3} \left(1 - \frac{2 a_*}{3\tilde{r}_m}\right).$$  (16)

We numerically integrated equations (13) and (14) with given initial parameters and track the evolution of black hole mass and spin. We also determined the equilibrium spin for $m = 10, 10^6$ and $10^8$ by solving $da_*/dr = 0$ in equation (14) by using bisection method.

Fig. 3(a) shows the equilibrium value of spin as a function of $\alpha m$. The equilibrium spin becomes larger when either $\alpha$ or $m$ becomes larger. The best-fitting spin parameter determined by the disc-seismic method corresponds to an equilibrium value of $m \approx 1$.

**Measuring BH spin at the Galactic centre**

Fig. 3(b) shows the time evolution of spin, where the spin parameter of each model converges to a unique value regardless of the initial one. When the mass accretion rate is regulated by the Eddington value ($\dot{m} = 1$), the spin converges to the equilibrium value $\approx 0.55$ for stellar-mass black holes within the order of $10^4$ yr and then slowly approaches the equilibrium value $\approx 0.4$ for massive black holes. When $\dot{m} = 0.1$, the spin converges to a value $\approx 0.5$ within the Hubble time, but never actually reaches the equilibrium spin. Therefore, the resultant spin is consistent with the small unique spin $\approx 0.44$ when the mass accretion rate is regulated by the Eddington value $\dot{m} \approx 1$ with the appropriate disc parameters. On the other hand, when the accretion disc is somehow in a supercritically accreting phase, with $\dot{m} = 10$, the spin converges to the equilibrium value of $\approx 0.96$ within the order of $10^7$ yr. Although the equilibrium spin of the supercritical accretion phase is larger than the unique value, it could approach to this value during the subsequent subcritical accretion phase in less than $10^7$ yr. The evolution of the black hole mass is not affected by the initial spin parameter (see Fig. 3c). Note that the final mass becomes $10^8$ times larger than the initial mass for $m \geq 1$.

**4 CONCLUSIONS**

It has been suggested that the supermassive black hole in the Galactic centre used to be in the nearly critical mass accretion phase for more than the order of $10^8$ yr. A possible explanation for such a large mass accretion history is the massive star formations in the proximity of the Galactic centre region. During the critical accretion phase, the spin reaches the unique value and the mass becomes $\approx 10^6 M_\odot$, which is then maintained during the subsequent low accretion rate phase. Note that stochastic mass accretion history may also help to create the moderately spinning massive black hole (King & Pringle 2006). Similarly, black holes in Galactic X-ray sources have been in the nearly critical accretion rate phase for order of $10^8$ yr as well, suggesting their companion stars should be low-mass stars. Because they have reached the quasi-equilibrium state, the limit-cycle activities and also the emergence of jets does not alter their spin evolution. Thus, we conclude that the spin parameter of a supermassive black hole in the Galactic centre has a unique value of $a_* = 0.44 \pm 0.08$. Conversely, the mass of a black hole consistent with the unique spin is $M = (4.2 \pm 0.4) \times 10^6 M_\odot$.

Without detecting the event horizon, we have constrained the mass and spin of the supermassive black hole at the Galactic centre. The method we used here depends entirely on geodesic frequencies that are independent of the distance and viewing angle of a black hole. Once the unique spin parameter of the black hole in the Galactic centre has been confirmed by detection of the event horizon in the future observations (e.g. Takahashi 2004), studies of QPOs in other galaxies will open a new window to survey the growth history of massive black holes (Markowitz et al. 2007; Gierliński et al. 2008).

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REFERENCES

Abramowicz M. A., Kluźniak W., 2001, A&A, 374, L19
Baganoff F. K. et al., 2001, Nat, 413, 45
Bardeen J. M., 1970, Nat, 226, 64
Bardeen J. M., Press W. H., Teukolsky S. A., 1972, ApJ, 178, 347
Blandford R. D., Znajek R. L., 1977, MNRAS, 179, 433
Bélinger G., Terrier R., de Jager O. C., Goldwurm A., Melia F., 2006, J. Phys. Conf. Ser., 54, 420
Beskin V., Henri G., Menard F., Pelletier G., Dalibard J., 2003, in Beskin V., Henri G., Menard F., Pelletier G., Dalibard J., eds, Ecole d’ete de Physique des Houches, Session LXXVIII, Accretion Disks, Jets and High-Energy Phenomena in Astrophysics. NATO Advanced Study Institute, Euro Summer School, Ecole thematique du CNRS. Springer-Verlag, Heidelberg
Gammie C. F., Shapiro S. L., McKinney J. C., 2004, ApJ, 602, 312
Genzel R., Schödel R., Ott T., Eckart A., Alexander T., Lacombe F., Rouan D., Aschenbach B., 2003, Nat, 425, 934
Ghez A. M. et al., 2008, ApJ, 689, 1044
Gierliński M., Middleton M., Ward M., Done C., 2008, Nat, 455, 369
Gillessen S., Eisenhauer F., Trippe S., Alexander T., Genzel R., Martins F., Ott T., 2009, ApJ, 692, 1075
Hamana N., Paumard T., Müller T., Gillessen S., Eisenhauer F., Trippe S., Genzel R., 2009, ApJ, 692, 902
Hawley J. F., 2000, ApJ, 528, 462
Hirose S., Krolik J. H., Stone J. M., 2006, ApJ, 640, 901
Homan J., Klein-Wolt M., Rossi S., Miller J. M., Wijnands R., Belloni T., van der Klis M., Lewin W. H. G., 2003, ApJ, 586, 1262
Kato S., Fukue J., 2006, PASJ, 58, 909
Kato S., Fukue J., Mineshige S., 2008, Black-Hole Accretion Disks – Towards a New Paradigm. Kyoto Univ. Press, Kyoto
Kato Y., 2004, PASJ, 56, 931
Kato Y., Mineshige S., Shibata K., 2004, ApJ, 605, 307
Kato Y., Umemura M., Ohsuga K., 2009, MNRAS, 400, 1742
King A. R., Pringle J. E., 2006, MNRAS, 373, L90
Machida M., Hayashi M. R., Matsumoto R., 2000, ApJ, 532, L67
Markowitz A., Papadakis I., Arévalo P., Turner T. J., Miller L., Reeves J. N., 2007, ApJ, 656, 116
McKinney J. C., Gammie C. F., 2004, ApJ, 611, 977
Moderski R., Sikora M., 1996, MNRAS, 283, 854
Novikov I. D., Thorne K. S., 1973, in DeWitt C., DeWitt B., eds, Black Holes. Gordon & Breach, New York, p. 343
Nowak M. A., Wagoner R. V., 1993, ApJ, 418, 187
Okazaki A. T., Kato S., Fukue J., 1987, PASJ, 39, 457
Orosz J. A., McClintock J. E., Remillard R. A., Corbel S., 2004, ApJ, 616, 376
Page D. N., Thorne K. S., 1974, ApJ, 191, 499
Remillard R. A., McClintock J. E., 2006, ARA&A, 44, 49
Schödel et al., 2002, Nat, 419, 694
Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337
Shapiro S. L., 2005, ApJ, 620, 59
Takahashi R., 2004, ApJ, 611, 996
Volonteri M., Madau P., Quataert E., Rees M. J., 2005, ApJ, 620, 69
Wilson A. S., Colbert E. J. M., 1995, ApJ, 438, 62
Yusef-Zadeh F. et al., 2006, ApJ, 644, 198
Yuan F., Quataert E., Narayan R., 2004, ApJ, 606, 894

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