Spectrum Sharing in Cognitive Radio Using GSC with Suppressed Sidelobes

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1. Introduction

Adaptive beamforming is a spatial coding technique which is established in the current era for signal detection and estimation. This technique has applications in several fields such as wireless communications, radar, sonar, seismology, and medical imaging [1–4]. In the communication system, uplink and downlink beamforming utilizes the frequency division duplex (FDD) mode [5] from mobile to the base station and base station to mobile. However, uplink and the downlink established the channel vectors which are dissimilar from each other owing to the exercise of diverse frequency. Links from the cognitive transmitter to receiver or from primary transmitter to the respective receiver are random and attempt to exercise the reciprocity property in the time division duplex (TDD) mode.

Cognitive radios exploit the adaptive beamforming for efficient utilization of spectrum in spectrum sharing environment. Cognitive radio is an innovative form of software defined radio. A novel concept of cognitive radio is given in [6, 7]. Cognitive radio has the capability of operating the spectrum as soon as the primary spectrum is vacant and resolving the problem of scarcity due to the ample boost of wireless devices. Cognitive radio senses the radio environment and adjusts the communication parameters according to requirements [8]. In case of spectrum sharing, the cognitive user coexists with the licensed user with the condition that it will not cause any sort of disturbance for them. There is a predefined signal-to-interference-to-noise ratio (SINR) for primary users. The cognitive user has to keep the signal level below the noise margin of the primary users. With this constraint the cognitive user will be able to increase the throughput by using the concept of beamforming; that is, it focuses the beam toward the cognitive user, both at the transmitter and receiver side and places the nulls towards the primary users. Hence it will not disturb the primary users. In fact it will be hidden to them and will be visible only to the respective target of cognitive users. Cognitive radio utilizes...
beamforming techniques to generate main beam along the
direction of cognitive user and nulls towards primary users
as discussed in [9, 10]. The cognitive user will be effective
only when it cleverly occupied the vacant spectrum efficiently
[11, 12] in accordance with the latest spectrum sharing or
accessing technique. The spectral efficiency of the spectrum
drastically boosts whenever the primary and the cognitive
users concurrently share the spectrum via underlay or overlay
scenario. It is assumed that the location of the primary as
well as the cognitive user is available to the transmitter. Prior
knowledge of the location compel the transmitter to focus the
main beam toward the potential receiver and place the nulls
towards the primary users [13]. Same concept will be valid on
receiver side which will focus toward transmitting cognitive
radio user.

In case of abrupt movement of primary user in the
region of high sidelobes, the quality of service of cognitive
user will be degraded during the adaptation process as is
clarified in [14]. The cause of interference to the other users
is due to the high sidelobes levels in transmitting mode.
Moreover in receiving mode, the receiver picks up interfering
signals and increases the noise floor level in the receiver.

Spectrum sharing in cognitive radio badly exaggerated owing
to interference. The quality of service of the primary network
degraded due to the cognitive radio transmission power.
Conversely the cognitive radio receiver reject the interference
coming from the primary user consequently the cognitive
radio system successfully decodes the respective cognitive
user signal. In addition, cognitive system in the receiving
mode recognized the presence of the primary user in the
same locality. Therefore, the cognitive system intelligently
changes the direction, on the bases that the primary is
already in active mode in the spectrum and save the primary
user from interference. The other factor which can avoid
the interference of cognitive to the primary user is that
cognitive user has to limit their transmission power [15]. In
the transmission mode cognitive users use the orthogonal
beamforming and spawn no interference in the direction
of primary user. This is obligatory that the interference
power received at each PU should stay below its interference
temperature [16, 17]. The direction of arrival in the multipath
scenario is a difficult task in cognitive radio whenever the
cognitive system operates at low noise level. Therefore, it
is inflexible to estimate the precise DOAs of CR users at
base station. The rectil of adaptive beamerformer distorted
significantly owing to the imperfect knowledge of the array
response [18–25]. Consequently, we require robust adaptive
beam-formers for the cognitive radio system to direct main
beams towards CR users as DOA information of other CR
users are inaccurate.

Generalized sidelobe canceler (GSC) is a popular
method in this regard to suppress the sidelobes so that the
decodable process becomes efficient at the cognitive receiver.
There exist numerous approaches to suppress the sidelobes
including the iterative procedure for adaptive beamforming
with suppressed sidelobes. The different techniques such as
adaptive array processing are discussed in [26–28]. These
approaches use the quadratic beam pattern constraints and
try to limit the deviations from the desired pattern. Diagonal
loading method and penalty function technique [29] are also
the approaches to control sidelobes. The drawback of these
techniques is that they do not guarantee the sidelobe levels
below the desired limit. A second-order cone programming
approach was proposed in [30]. Here, in order to control
the sidelobe level, multiple quadratic inequality constraints,
outside the main beam pattern area, are used. This technique
is complicated with no standardized method to opt for steer-
ing vectors outside the main beam area. Moreover, bunch of
steering vectors has to be used from said area to guarantee
the sidelobe levels below the desired limit. Another technique
with sparse constraint on beam pattern is presented in [31].
This technique, just like [30], utilizes a lot of steering vectors
from the direction of arrival range $[-90^\circ, \theta_c, \theta_c, 90^\circ]$ to
cover all possible interferences from all directions excluding
$\theta_c$, the angle of desired source. The number of these steering
vectors depends on the sampling interval. However, there is
no criterion to decide that how many inequality constraints
(steering vectors) are to be used. Another problem is that
there is no standardized method to select the parameters that
determine the effectiveness of the sparse constraint on the
beam pattern.

In this work, we compare the algorithm [32] where the
efficiency of the array increases by not only suppressing the
noise and interference but also by suppressing the sidelobes.
The sidelobes in [32] are suppressed up to the level of $-30$ dB,
while in the proposed algorithm we suppress the sidelobe
up to the level of $-41$ dB. We propose two GSCs which
work in parallel for sidelobe suppression in cognitive radio
environment. One is meant for the cognitive user generating
the main beam in that direction and the nulls along the
directions of primary users. It has been assumed that the
direction of primary and the cognitive users is available to
both the transmitter and receiver. The second GSC is auxiliary
one that generates two main beams at the positions of the
highest amplitude sidelobes of the first one. The values of
these main beams are adjusted to be equal to these two
sidelobe peaks. The subtraction of the outputs of the two
GSCs causes the suppression of the highest pair of sidelobes.

The primary and the secondary user share the spectrum,
maneuvering the beam in the direction of the secondary user
and rejecting the signal coming from the primary user. Since,
this technique detects the presence of the primary user’s in
the spectrum; consequently, it changes the direction and set
aside the system from interference. In the transmitting
mode the nulls are generated in the direction of the primary user,
while in the receiving mode minimize the average receiving
power at the cognitive receiver. Hence, as a result the primary
and the cognitive user share the spectrum in response to the
suppression of sidelobes, steering the beam in the direction
of cognitive users and placing the nulls in the direction of
primary users. In the transmission mode, the efficiency of
the GSC increases at low SNR which is near to optimal level,
but with gradual increase of SNR it inter into the linear and
saturation region. As a result, in the cognitive radio scenario,
it shows the good result at the low SNR, while in the high
SNR it performance degraded. In other word, at low SNR the
nulls are deep and the sidelobe levels are low. However, when
the SNR move up, the Sidelobe level increases along with the
nulls in the linear and saturation region. In the linear and saturation region, the performances of the system degraded but at low SNR level the GSC show the optimum result in transmission mode. Consequently, the implementation of the proposed idea in spectrum sharing environment, the cognitive system will intelligently boost the throughput without negotiation on the permissible interference limit of the primary users.

The rest of the paper is organized as follows. In Section 2, the background contains mathematical modeling of the problem which is followed by the proposed method. Section 3 contains proposed method, Sections 3.1 and 3.2 contain N-problem which is followed by the proposed method. Section 4 is the simulation section, followed by Section 5 which contains conclusions. The following is the notation used throughout the document. $E[\cdot]$ indicates the expectation; $(\cdot)^T$ is the transpose.

2. Background

(a) Signal Model. Consider a uniform linear array of $M$ antenna elements with inter element spacing $\lambda/2$, where $\lambda$ is the wavelength of incoming signal of interest. The arrangement is shown in Figure 1.

This array is implicit to receive $K$ number of far field narrowband signals. The output of individual antenna element $\{y_i(n)\}_{i=1}^M$ is given by

$$y_i(n) = \sum_{l=1}^{K} e^{j(l-1)\pi \sin \theta_l} s_l(n) + v_i(n).$$

These outputs are combined together to form the output vector $y(n)$ given as

$$y(n) = [y_1(n) \ y_2(n) \ldots y_M(n)]^T.$$  

Similarly $s(n)$ is a vector of source signals having $K$ sources; that is,

$$s(n) = [s_1(n) \ s_2(n) \ldots s_K(n)]^T.$$  

These $K$ signals are assumed to be uncorrelated to each other and hence

$$R_s = E [s(n) s^H(n)] = \begin{bmatrix} \sigma_1^2 & 0 & \ldots & 0 \\ 0 & \ddots & \vdots \\ 0 & \ldots & \sigma_K^2 \end{bmatrix},$$

where $[\sigma_l^2]_{l=1}^K$ is the power of $l$th signal or users.

Consider the angle of arrival $\theta_l$, $l = 1, 2, \ldots, K$. A set of steering vectors $a_l : l = 1, \ldots, K$ can be defined as

$$a_l = [1 e^{j\phi_1} e^{j\phi_2} \ldots e^{j(M-1)\phi}]^T,$$

where $\phi_l = \pi \sin \theta_l$.

These vectors can be placed in a single matrix $A$ given as

$$A = [a_1 a_2 \ldots a_K].$$

$y(n)$ can be written as

$$y(n) = a_1 s_1(n) + a_2 s_2(n) + \ldots + a_K s_K(n) + v(n)$$

or equivalently as

$$y(n) = As(n) + v(n).$$

In the above equation $v(n)$ is the noise vector having uncorrelated components and hence its correlation matrix is given as

$$R_v = E [v(n) v^H(n)] = \sigma_v^2 I_M,$$

where $I_M$ is identity matrix of order $M$ and $\sigma_v^2$ is the variance of noise.

The correlation matrix $R_y$ of received signal is given by

$$R_y = E [y(n) y^H(n)] = E [As(n) s^H(n) A^H]$$

or equivalently by

$$R_y = AR_s A^H + \sigma_v^2 I_M.$$  

In realistic situation, the estimate of autocorrelation matrix $R_y$ is computed by taking the time average of $N$ snapshots and is given as

$$\hat{R}_y = \frac{1}{N} \sum_{n=1}^{N} y(n) y^H(n).$$  

(b) GSC. Figure 2 shows the way to find out the adaptive weights. For linearly constraint multiple variable (LCMV) beamformer, the adaptive weights are found by the constraint minimization of the term, $w_{LC}^H R_y w_{LC}$ [33]; that is,

$$\min_{w_{LC}} w_{LC}^H R_y w_{LC}$$

Subject to $C^H w_{LC} = f$,
where $C$ is matrix with $M$ rows and $L$ columns and has $L$ constraints given by

$$C = [a_1 a_2 \cdots a_L].$$  \hspace{1cm} (14)

$f$ is a vector with $L$ rows.

The LCMV beamformer generates a single beam with unit gain which is equivalent to minimum variance distortion less response (MVDR) beamformer. For this case $C = a_c$ in (14) and the constraint problem become

$$\min_w \; w^H R_y w$$

subject to

$$w^H a_c = 1.$$  \hspace{1cm} (15)

Here $a_c$ is the steering vector of the secondary user for the signal to be preserved.

The GSC converts constrained problem of LCMV beamformer into unconstrained one. The block diagram of GSC as given by [34] is being redrawn here in Figure 3, which consists of upper branch and lower branch. The weight vector $w_q$ in the upper branch is not adaptive and it preserves the signal coming from secondary user desired direction. The lower branch consists of blocking matrix $B$ and adaptive weight vector $w_a$. Matrix $B$ blocks the signal from desired direction and preserves the interferences and noise. The interference component preserved in the lower branch is subtracted from

the signal plus interference component of the upper branch. The output $z(n)$ of GSC is given as

$$z(n) = w_q^H y(n) - w_a^H B^H y(n) = (w_q - B w_a)^H y(n).$$  \hspace{1cm} (16)

The weight vector $w_a$ is used to minimize the effect of interference and noise power in the two branches. Thus nulls are generated in the direction of unwanted interferences by the combined effect of the two branches because the outputs of the two branches for these interferences are approximately equal. The combined effect of $w_q$, $B$, and $w_a$ is the main beam along direction of desired signal and nulls along the direction of interferences. The expressions for $w_q$, $B$, and $w_a$, as available in the literature, [34] are given by

$$w_q = C (C^H C)^{-1} f$$

$$B = \text{null} \{ C^H \}. \hspace{1cm} (17)$$

The optimized adaptive weight vector $w_a$ denoted by $w_{ao}$ is the one which minimizes the cost function

$$J(w_a) = (w_q - B w_a)^H R_y (w_q - B w_a).$$  \hspace{1cm} (18)

That is, $\min_{w_a} (w_q - B w_a)^H R_y (w_q - B w_a)$ and its solution is given as

$$w_{ao} = (B^H R_y B)^{-1} B^H R_y w_q$$

$$w_{GSC} = w_q - B w_{ao}. \hspace{1cm} (19)$$

3. Proposed Method

Our proposed approach consists of two GSCs connected in parallel, instead of a single one, as shown in Figure 4.

In order to limit the sidelobe at the specific level in (15), the contemporary approaches entail the following multiple quadratic inequality constraints outside the main beam pattern area

$$|w^H a(\theta_j)|^2 \leq \varepsilon, \; j = 1, \ldots, J,$$  \hspace{1cm} (20)

where $\varepsilon$ is the set sidelobe level and $a(\theta_j)$ is the steering vector and places the nulls outside the main beam for primary user.
So the constrained problem (15) becomes as given in [29]; that is,
\[
\min_w \ w^H R_w w \\
\text{Subject to } w^H a_\theta = 1, \\
|w^H a_\theta| \leq \epsilon, \\
j = 1, \ldots, J.
\] (21)

On the other hand, the proposed approach uses the same minimization problem with the same equality constraint as in (15) and (21); that is, \( \min_w w^H R_w w \) is subject to \( w^H a_\theta = 1 \)

But instead of a number of inequality constraints as in (21), it uses only as many equality constraints as the sidelobes to be suppressed by observing the sidelobes in (15). The proposed approach works in two steps. In the first step of the relation (15), where constrained one is converted into unconstrained. GSC-1 is used for this purpose. \( w_{\text{GSC1}} \) is obtained by using the following parameters:

\[
C_i = [a_i], \quad f_i = 1, \\
w_{q1} = C_i^{-1} f_i, \\
B_2 = \{C_i\}, \\
w_{\text{null}} = (\mathbf{B}_1^H R_w \mathbf{B}_1)^{-1} \mathbf{B}_1^H R_w w_{q1},
\] (22)

and hence \( w_{\text{GSC1}} = w_{q1} - B_2 w_{\text{null}} \).

In the second step, sidelobes of GSC-1 are suppressed. Steering vectors, at angles \( \theta_p \), where peaks of GSC-1 sidelobes exist, are selected. This is done by scanning the beam pattern of GSC-1. Let there be \( P \) sidelobes with \( P \) steering vectors corresponding to the peak positions of these sidelobes. These steering vectors are represented as \( [a(\theta_p)] \) for \( p = 1, 2, \ldots, P \). To suppress two nulls at a time, another beamformer, the auxiliary one, is used and the constrained problem for suppression of sidelobes with peaks at \( \theta_{i1} \) and \( \theta_{i2} \) is stated as

\[
\min_w \ w^H R_w w \\
\text{Subject to } w^H a_\theta = \ w_{\text{GSC1}}^H a(\theta_{i1}), \\
\ w^H a_\theta = \ w_{\text{GSC1}}^H a(\theta_{i2}),
\] (23)

where \( a(\theta_{i1}), a(\theta_{i2}) \in [a(\theta_p)] \) and \( w \) is the weight vector for auxiliary beamformer. In this way the main beams of auxiliary beamformer exist at \( \theta_{i1} \) and \( \theta_{i2} \). It is clear that this auxiliary beamformer and GSC-1 have the same outputs at \( \theta_{i1} \) and \( \theta_{i2} \) due to constraints in (23). So the outputs of the two beamformers are quite close to each other in these sidelobe regions. Due to common part \( \min_w w^H R_w w \) in (15) and (23) the auxiliary beamformer and GSC-1 have common nulls at interferences. The auxiliary beamformer, due to above mentioned minimization condition, has also a null overlapping with the main beam of GSC-1, caused by the equality constraint in (15). Since the outputs of the two beamformers are subtracted from each other as shown in Figure 4, so the main beam and nulls of GSC-1 are preserved and the sidelobes are suppressed. In order to convert the constraint problem (23) into unconstrained one, we use GSC-2 as shown in Figure 4. \( w_{\text{GSC2}} \) is obtained by using following parameters:

\[
C_2 = [a(\theta_{i1}), a(\theta_{i2})], \\
f_2 = [w_{\text{GSC1}}^H a(\theta_{i1}), w_{\text{GSC1}}^H a(\theta_{i2})]^H, \\
w_{q2} = C_2^{-1} f_2, \\
B_2 = \{C_2\}, \\
w_{\text{null}} = (B_2^H R_w B_2)^{-1} B_2^H R_w w_{q2}.
\] (24)

and hence \( w_{\text{GSC2}} = w_{q2} - B_2 w_{\text{null}} \).

The overall weight vector \( w_{\text{null}} \) for the beamformer with suppressed sidelobe is given as \( w_{\text{null}} = w_{\text{GSC1}} - w_{\text{GSC2}} \).

Now, in order to suppress the next pair of sidelobes while preserving the previous suppression, GSC-3 is used in the same manner as GSC-2 and weights of the proposed beamformer will be as given below; that is,

\[
w_{\text{null}} = w_{\text{GSC1}} - w_{\text{GSC2}} - w_{\text{GSC3}}.
\] (25)

In this way all the sidelobes are suppressed. In case of odd number of sidelobes, the auxiliary beamformer with single constraint, that is, auxiliary GSC with single beam corresponding to single sidelobe, can be used. After complete suppression of sidelobes, beamformer with weights \( w_{\text{null}} \) will have a new set of sidelobes. We call them 2nd generation sidelobes. These 2nd generation sidelobes can further be suppressed by scanning the positions (angles) where peaks of sidelobes of this beamformer exist and repeating the process from (23) to (25). In case of odd number of sidelobes, if we suppress only even number of them and leave the single one unsuppressed, then that will be suppressed as next generation sidelobe during further suppression process. The role of the two GSCs can be expressed as follows.

GSC-1. GSC-1 selects the main beam along the cognitive user direction. The role of this GSC is the same as given by the GSCs described in Section 2. That is, it works independent of the GSC-2. In fact it is unaware of the presence of the second one.

GSC-2. GSC-2 works in parallel to GSC-1; however, it is adaptive with the parameters taken from the output of GSC-1. Therefore, GSC-2 is totally dependent on the output of GSC-1 and remains in observe state until the output of first appears and is analyzed. It has nulls at the same positions as that of first one, that is, along primary users. It has an additional null at the main beam position of GSC-1. Apart from these nulls, the GSC-2 also produces two beams. The positions of the beams are kept exactly the same as that of the positions of the sidelobes of GSC-1, taken from the analysis of its beam pattern, and the heights of the beams are also kept the same.
as that of the heights of the sidelobes GSC-1. The simulated outputs of GSC-1 and GSC-2 for a pair of sidelobe suppression are given here in Figure 5 to visualize the concept.

The GSC-2 output, in this case, has to be taken ideally by considering GSC-1 output of a typical case. Next step is the subtraction of the output of GSC-2 from GSC-1 and in this case, the overall output power will look as in Figure 5(c). The two highest (selected) sidelobes are completely eliminated in this case. The position of nulls, sidelobes, and main beam for the case of GSC-1 and GSC-2 are also given in Table 1 for comparison.

The following remarks are highlighted corresponding to the proposed method.

Remark 1. The overall positions of the main lobe and nulls will not be disturbed in the new setup.

Remark 2. The main beams of GSC-2 have been placed at the same positions as that of the sidelobes of GSC-1 to be suppressed. Their heights are also kept the same as that of GSC-1 in the new setup to nullify their outcome. The overall result will be the suppression of sidelobes.

Remark 3. The exact position and peak value of sidelobe can be found by scanning the output power pattern of the GSC-1 which is used in next setup for main beams of GSC-2.

3.1. N-Sidelobe Suppression. The same idea can be extended for the suppression of N-sidelobes, where N is an arbitrary number. The suppression can be carried out in parallel as well as in serial mode.

3.1.1. Parallel Setup. In this case the output power pattern of GSC-1 is scanned for the N-pairs of sidelobes, instead of single one. In case of the number of sidelobe pairs greater than N, the largest N-pairs may be considered for suppression. The proposed setup in this case will be as shown in Figure 6. In this case, the cancellation of N-sidelobe pairs will be done parallel in single iteration.

3.1.2. Serial Setup. In this case, the initial setup presented in Figure 4 will be used. However, instead of single scan, it will be repeated for N-times. That is, the output power pattern of GSC-1 is scanned repeatedly in cycles. The largest sidelobes are located and suppressed in each cycle. The process is repeated for N-cycles to suppress N-pairs of sidelobes.

3.2. Cognitive Radio Model Using Adaptive Beamforming in Transmitting-Receiving Mode. A conceptual diagram of cognitive system in the paradigm of spectrum sharing is specified in Figure 7. Adaptive beamforming suppresses the interference at the cognitive base station (CBS); however, signal suppression of the primary takes place due to the
interference suppression techniques. Cognitive base station (CBS) is equipped with linear array. The linear array consists of multiple antenna elements and steers the beam in the direction of the cognitive user having the channel gain, while rejects the signal coming from the direction of the primary users. The linear array as discuss above is used in the cognitive radio system model at the base station. when Cognitive system used the interweave technique or opportunistically occupied the primary channel, the switch in the model closed and steer the beam in the direction of the secondary user while it generate the nulls in the direction of the primary users. The iterative procedure is an effective technique to suppress the sidelobe at the CBS. In the receiving mode, when the sidelobes are suppressed, the interference temperature limit [16] will not be increased and the decoding capability of the cognitive user will be increased. The capacity of the system increases due to the reduction of frequency reuse distance and increase of SINR. In the receiving beamforming as discussed in (21), the beams received at the receiver are measured by minimizing the mean output power of the array, subject to unit response of the desired cognitive radio user and generate nulls in the direction of primary as well as the other cognitive users. According to the Figure 7, the cognitive system consists of the multiple antennas at the cognitive transmitter. The primary base station and the primary user consist of single antenna element. Hence the channel matrix is developed from cognitive base station to cognitive user, while the channel vector developed from the cognitive to the primary user. As a result, this system acts at a time as the transceiver for primary as well as cognitive system. The steering vector of (5) in the paradigm of the cognitive radio is

$$
c(\theta) = \left[1 \ldots e^{j2\pi \sin(\theta)} e^{j(N-1)\pi \sin(\theta)} \right]^T.
$$

(26)

Therefore, the channel response [15] of the vector from cognitive base station to cognitive user is given as

$$
\mathbf{c}_c = \alpha^d \mathbf{c}(\theta),
$$

(27)

where $\alpha = \text{path loss}$, $\mathbf{c}(\theta) = \text{cognitive steering vector}$, and $d = \text{distance between the cognitive base station to the cognitive user and the primary user}$. According to the model the output signal received at the primary and secondary user is written as

$$
o_p = \sum_{m=1}^{K_p-1} \mathbf{c}_p \mathbf{w}_m x_m + p_p x_p + \eta_p
$$

(28)

$$
o_c = \mathbf{W}_c^H \mathbf{c}_c x_c + \sum_{k \neq c}^{K_p-1} \mathbf{w}_k^H \mathbf{c}_c w_k x_c + w_c^2 p_c x_p + \eta_c.
$$

where $\mathbf{w} = \text{weight vector}$, $x_c = \text{source signal of cognitive users}$, $x_p = \text{source signal of primary users}$, $x_m = \text{source signal of cognitive to primary users}$, $w_c = \text{receiving side weight vector} (1 \times N)^T$ of a cognitive user, $w_k = \text{transmitting source weight vector} C_{k \times N}$ of a cognitive user, $\mathbf{c}_c = \text{channel response vector} C_{1 \times N}$ of the cognitive to cognitive user, $\eta_c = \text{additive white Gaussian noise with mean zero of cognitive user}$, $\sigma_c^2 = \text{variance of cognitive users}$, $p_p = \text{gain in the direction of primary user}$, $p_{pc} = \text{gain vector in the direction of primary to cognitive user}$, $\mathbf{c}_{pc} = \text{gain vector in the direction of cognitive to primary user}$, $o_c = \text{the signal receive at the cognitive users}$, $o_p = \text{the signal receive at the primary users}$, and $\mathbf{C}_{cc} = \text{matrix between the cognitive to cognitive users}$. By applying the transmit beamforming technique as discussed in [15] to the (28) we get

$$
o_p = p_p x_p + \eta_p
$$

(29)

$$
o_c = \mathbf{w}_c^H \mathbf{c}_c x_c + \sum_{k \neq c}^{K_p-1} \mathbf{w}_k^H \mathbf{c}_c w_k x_c + \eta_c.
$$

The signal to interference plus noise ratio for the primary as well as cognitive user is written as

$$
(SINR)_p = \frac{|p_p|^2}{\sum_{c=1}^{K_p} |c_c w_c|^2 + \sigma_p^2},
$$

(30)

$$
(SINR)_c = \frac{|w_c^T c_c w_c|^2}{\sum_{k \neq c}^{Q} |w_k^T c_c w_k|^2 + |w_c^T p_{pc}|^2 + \sigma_c^2}.
$$

(31)

The weights of transmitted and received beamforming are distributed among cognitive radio users. Mean output
power minimization and maximization of SINR are the two accessible techniques which are used in the receive beamforming mode. By solving the (21) in the receiving beamforming mode, the weights are optimized to minimize the mean output power and as an outcome the self-interference of the cognitive user trims down. To maximize the SINR we solve (31) for optimizing the weights in the receiving mode keeping in view that the interference occurred due to the fact that the cognitive user is not consider, so both the receive beamforming scheme has the equal performance. In the transmission mode, at low SNR the performance of the GSC show optimal value, however, with gradual increase of SNR it inter into the linear and saturation region. Thus, in the cognitive radio scenario, it shows the good result at the low SNR, while in the high SNR it performance degraded. In other word, at low SNR the nulls are deep and the sidelobe levels are low. Whenever, the SNR increases, the sidelobe level increases along with the nulls in the linear and saturation region and the performance degrades. Therefore, the implementation of spectrum sharing in cognitive radio using GSC with suppressed side-lobes, will intelligently boost the throughput without negotiation on the permissible interference limit of the primary users.

4. Simulation Results

Simulations have been carried out in MATLAB for testing the performance of proposed method. We assumed a uniform linear array of 16 omnidirectional sensors with interelement spacing $\lambda/2$. Spatially white Gaussian noise is assumed with unit variance. Four examples are presented here to compare the performance of traditional GSC and the proposed method. In all examples, INR in a single sensor is equal to 30 dB, and the desired signal is always present in the training data cell. In Examples 4 and 5, 200 snapshots averaged over 200 independent trials are used. In Examples 6 and 7, 500 snapshots averaged over 100 independent trials are used. The sidelobe region is taken as $[\theta_c + 5^\circ, 90^\circ]$ and $[\theta_c - 5^\circ, 90^\circ]$, where $\theta_c$ is the direction of the cognitive user. In the figures primary GSC represents GSC-1 and secondary GSC represents GSC-2.

**Example 4.** In this example one cognitive user and two primary users are considered. The cognitive user is taken at $\theta_c = 0^\circ$. The primary users are placed at $\theta_{p1} = 30^\circ$ and $\theta_{p2} = 60^\circ$. SNR is kept at 10 dB. First of all the positions of two sidelobes from the output of GSC-1, with one side lobe on either side of the main beam, is selected. The peaks of these two sidelobes are at $\pm 10^\circ$ as observed from radiation pattern of GSC-1. Both of the sidelobes are then suppressed by placing corresponding beams in GSC-2 of same strength and at same positions, that is, at $\pm 10^\circ$. It is clear from Figure 8 that both GSCs have common nulls at $\theta_{p1} = 30^\circ$ and $\theta_{p2} = 60^\circ$. GSC-2 has an additional null overlapping with main beam of GSC-1, that is, at $0^\circ$. Thus main beam and desired nulls of GSC-1 are not disturbed after subtraction, that is, after suppression of these sidelobes. This suppression is immediate in single iteration. In next iteration, we may consider the next two sidelobes and repeat the same process as shown in Figure 9, where the peaks of 2nd pair of sidelobes in the pattern of GSC-1 is at $\pm 18^\circ$. Similarly by a complete suppression of first generation sidelobes the same process may be carried on for 2nd and 3rd generation's sidelobes. The process may be carried out for suppression below $-60$ dB with respect to the main beam. The simulation results for the suppression of first generation sidelobes obtained from GSC-1 are labeled as proposed beamformer-1 and suppression of ten higher generations of sidelobes obtained after suppression of previous generations are labeled as beamformer-2 and are shown in Figure 9.
Example 5. Here we have considered one source of interest at $\theta_c = 0^\circ$ with SNR = 10 dB and two interferences placed at $\theta_{p1} = 30^\circ$ and $\theta_{p2} = 60^\circ$. The comparison of [32] for suppression of 1 generation of sidelobes is represented as beamformer-1 and suppression of sidelobes of 10 generations is represented as beamformer-2 and are shown in Figures 10(a) and 10(b). Similarly, we have considered one source of interest at $\theta_c = 0^\circ$ with SNR = 10 dB and two interferences placed at $\theta_{p1} = 30^\circ$ and $\theta_{p1} = 60^\circ$. The suppression of 5 generation of sidelobes is represented as beamformer-1 and suppression of sidelobes of 15 generations is represented as beamformer-2 and are shown in Figures 11(a) and 11(b).

Example 6. The performance of two algorithms in terms of output SINR versus the SNR for the signals given in Example 4 is shown in Figure 12. The performance of [32] and the proposed algorithm is for suppression of 10 and 20 generation sidelobes and is represented as proposed beamformer-1 and proposed beamformer-2, respectively. The formula for output SINR is as given in [35] and [33] and is averaged for a set of fifty weight vectors.

Consider

$$\text{SINR} = \frac{(\sigma_d^2 w^H a_c a_c^H w)}{(w^H (\sum_{j=1}^J \sigma_j^2 a_j a_j^H + Q) w)}.$$  \hspace{1cm} (32)

Here $w$ is the weight vector, $\sigma_d^2$ is the desired signal power, $\sigma_j^2$ is the $j$th interference power, $a_c$ is the desired signal steering vector, $a_j$ is the $j$th interference steering vector, and $Q$ is the identity matrix of order $M$ (number of antenna elements).

Example 7. The performance of two algorithms in terms of output SINR versus the number of snapshots for the signals given in Example 4 is shown in Figure 13. SNR = 0 for this case. The performance of proposed algorithm for 10 generation sidelobe suppression is represented as “proposed beamformer-1” and for 20 generation sidelobe suppression is represented as “proposed beamformer-2” and is shown along with [32] and optimal performance in Figure 13.

In order to use GSC in transmission mode, cognitive direction is taken as $\theta_c$, while nulls are placed in primary user directions. These directions are mentioned as $\theta_{p1}, \theta_{p2}, \ldots, \theta_{pm}$ to generate $m$ nulls in these directions. The following parameters are used for this purpose:

$$R_p = a(\theta_{p1}) a^H(\theta_{p1}) + a(\theta_{p2}) a^H(\theta_{p2}) + \cdots + a(\theta_{pm}) a^H(\theta_{pm}) + I,$$

\hspace{1cm} (33)

where $a(\theta_{pm})$ is the steering vector along $\theta_{pm}$ and $I$ represents noise of unit variance.

The optimization problem for transmitter can be expressed as

$$\min_w w^H R_p w \hspace{1cm} (34)$$

Subject to $w^H a(\theta_c) = 1$.

In the Figure 14, we show the beam pattern for GSC in the transmitting mode using a ULA of 16 elements with CU while the primary users $\theta_c = 0^\circ$ and PUs along $\theta_{p1} = 15^\circ$ and $\theta_{p2} = -30^\circ$.

Output SINR versus SNR plot of GSC is shown in Figure 15. It can be seen from the figure that for SNR $\leq 15$ dB, the performance of GSC is close to optimal, while output SINR saturates for SNR $> 30$ dB. Three values of SNR have been selected; that is, one is close to optimal performance which is 15 dB, second one is 25 dB which is from the linear
Figure 10: Superdirective beamforming [32] and proposed beamformers suppressing one and ten sidelobe generations: Example 4.

Figure 11: Superdirective beamforming [32] and proposed beamformers suppressing 5 and 15 sidelobe generations: Example 5.

Figure 12: Output SINR versus SNR of proposed beamformers for 10 and 20 generation sidelobe suppressions: Example 6.
region but deviated from the optimal, while the last one is 35 dB and is from saturation region. The corresponding beam pattern for these values of SNR is shown in Figure 16. Figures 16(a), 16(b), and 16(c) show the beam pattern for SNR equal to 15, 25, and 35 dB, respectively. It can be seen that for Figure 16(a), the performance is very good in the context of main beam sidelobes and null depth because SINR versus SNR is close to optimal. According to Figure 16(b), side-lobe level increases, while the null depth decreases. SINR although lie in linear region but diverge from optimal. In Figure 16(c) the pattern is badly distorted because SINR versus SNR lies in saturation region. From the above discussion, it is clear that the performance of GSC is close to optimal for lower values of SNR. Since cognitive radio required low power compare to primary users. It is therefore evident that GSC will perform well for cognitive radio.

5. Conclusion

A new beamformer with suppressed sidelobes has been proposed by using transmit-receive beamforming in cognitive radio. The technique observes the sidelobes of a beamformer and then suppresses them. It is clear from simulation results that the proposed algorithm has excellent performance for sidelobe suppression. The sidelobes can further be suppressed iteratively which is the additional feature of this algorithm. The proposed algorithm shows better performance for SINR versus SNR and its curve is close to ideal for larger range of SNR. The SINR versus number of snapshots shows a little performance degradation and this price is small as compared to the achievements. Simple to implement and efficient to suppress the sidelobes are the main features of
this technique. Apart from simplicity, another important feature of this approach is that the GSC-2 does not require all the steering vectors outside the main beam pattern. In the future, this technique can be extended for a number of cognitive users simultaneously by generating multiple beams. In the transmission mode, the performance of the GSC at low SNR is near to optimal. Although the implementation of this technique is complicated, it is efficient and increases the data rate and range. In the future, with the development of the nanotechnology, adaptive beamforming with suppressed sidelobes will be used in the cognitive radio for sharing the spectrum with diverse channel.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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