Enhanced spin injection efficiency in a four-terminal double quantum dot system

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Abstract

Within the scheme of quantum rate equations, we investigate the spin-resolved transport through a double quantum dot system with four ferromagnetic terminals. It is found that the injection efficiency of spin-polarized electrons can be significantly improved compared with single dot case. When the magnetization in one of four ferromagnetic terminals is antiparallel with the other three, the polarization rate of the current through one dot can be greatly enhanced, accompanied by the drastic decrease of the current polarization rate through the other one. The mechanism is the exchange interaction between electrons in the two quantum dots, which can be a promising candidate for the improvement of the spin injection efficiency.

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I. INTRODUCTION

How to improve the injection efficiency of spin-polarized electrons from a ferromagnetic (FM) contact into a semiconductor microstructure has puzzled the researchers in the field of spintronics for many years.\textsuperscript{1} Due to the mismatch of conductivity between FM metal and semiconductor, spin polarization is almost lost at the interface\textsuperscript{2} and spin injection efficiency is very low.\textsuperscript{3,4,5,6} To now, various ideas have been proposed to solve this problem. Rashba\textsuperscript{7} suggested that tunnel contacts can dramatically increase spin injection efficiency, which was supported by subsequent theoretical works.\textsuperscript{8,9,10,11} Jiang et al.\textsuperscript{12} demonstrated that the spin injection efficiency could be improved dramatically by inserting a MgO tunnel barrier between the ferromagnetic contact and the semiconductor. Optical injection of spin-polarized carriers across a mismatched heterostructure is an effective method. By using circular polarized excitation and detection, it has been demonstrated that the injected spin-polarized carriers are quite robust and maintain their polarization memory even after passing through a dense array of misfit dislocations.\textsuperscript{13,14,15,16} However, it is still desirable to establish electrical, rather than optical, methods to achieve effective spin injection.

In strongly-correlated electron systems, spin dipole-dipole interactions between electrons play important roles, which determine the systems’ magnetism, specific heat, and other ground-state properties. In the weak coupling and strong Coulomb repulsion regime, the Heisenberg-type exchange interaction $J S_1 \cdot S_2$ can be derived through perturbation analysis (e.g., Schrieffer-Wolf transformation). For electronic transport in mesoscopic systems, electronic spin correlation drastically affects the conductance and the current correlation.\textsuperscript{17,18,19,20,21,22,23,24} For instance, the double quantum dot (QD) system enables the realization of the two-impurity Kondo problem, in which a competition between Kondo correlation and antiferromagnetic impurity-spin correlation leads to a quantum critical phenomenon.\textsuperscript{25} For the case of spin-polarized transport, the polarized spin in one dot behaves like an effective magnetic field and affects the spin transport in another dot through indirect spin-spin interaction between two dots.\textsuperscript{26} Therefore, it is expected that exchange interaction can induce efficient spin injection in QD systems.

In this work we propose an electrical and internal scheme to improve the spin injection efficiency based on a double quantum dot system, where each dot is connected with two FM electrodes. Two different configurations are examined, one is the magnetizations of four
FM electrodes are parallel with each other, and the other is one of them has antiparallel magnetization with other three ones. We find that in the latter case, due to the exchange interaction between electrons in the double dot, the spin-polarization rate of the current through one dot is greatly enhanced, while the spin-polarization rate through the other one is drastically suppressed. As for the case of two parallel and two antiparallel, spin-down electrons can hardly occupy the two dots, while the spin-up ones dominate in both of the two dots during transport processes, thus the exchange interaction cannot greatly enhance the current polarization.

II. MODEL AND FORMULA

The structure is depicted in Fig. 1. Dot $i$ ($i = 1, 2$) is connected to FM leads $iL$ and $iR$. The magnetizations of leads 1L, 2L, and 2R are parallel, while that of lead 1R can be parallel or antiparallel with the other three. We model this system with the Hamiltonian $H = H_{lead} + H_{dot} + H_T$. The FM leads are described by the Hamiltonian $H_{lead} = \sum_{i\alpha k} \varepsilon_{i\alpha k} a_{i\alpha k}^\dagger a_{i\alpha k}$, where $a_{i\alpha k}^\dagger$ ($a_{i\alpha k}$) is the creation (annihilation) operator for electrons with wave vector $k$ in lead $i\alpha$, $\alpha = L, R$. The isolated double dot are described by $H_{dot} = \sum_{i\sigma} \varepsilon_i d_{i\sigma}^\dagger d_{i\sigma} + \sum_i U_{ni_i} n_{i\downarrow} + J S_1 \cdot S_2$. Here $d_{i\sigma}^\dagger$ ($d_{i\sigma}$) is the creation (annihilation) operator for electrons with spin $\sigma$ in dot $i$, $n_{i\sigma} = d_{i\sigma}^\dagger d_{i\sigma}$ is the occupation operator, and $U_i$ stands for the intradot Coulomb repulsion. The last term denotes the Heisenberg exchange coupling with the exchange coupling parameter $J$ and the spin operator $S_i = (\hbar/2) \sum_{\sigma\sigma'} d_{i\sigma}^\dagger \sigma \sigma' d_{i\sigma'}^\dagger$. For simplicity, we neglect the direct interdot tunneling and interdot Coulomb repulsion. The tunneling Hamiltonian between dots and leads is $H_T = \sum_{i\alpha k} (V_{i\alpha k} a_{i\alpha k}^\dagger d_{i\sigma}^\dagger + H.c.)$. In the following, we assume the coupling coefficient $V_{i\alpha k}$ to be independent of $k$ and $U_1, U_2 \to \infty$, thus the double occupation of each dot is forbidden.

Since the exchange interaction is considered, it is natural to describe the double dot system by triplet and singlet states, which are defined as $|T_1\rangle = |\uparrow\rangle_1 |\downarrow\rangle_2$, $|T_2\rangle = |\downarrow\rangle_1 |\uparrow\rangle_2$, $|T_0\rangle = (1/\sqrt{2})(|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2)$ (triplet states), and $|S\rangle = (1/\sqrt{2})(|\uparrow\rangle_1 |\uparrow\rangle_2 - |\downarrow\rangle_1 |\downarrow\rangle_2)$ (singlet state). Following the procedure in previous works, we use nine slave-boson operators to represent these Dirac brackets: $e^\dagger = |0\rangle_1 |0\rangle_2$, $f^\dagger_{1\sigma} = |\sigma\rangle_1 |0\rangle_2$, $f^\dagger_{2\sigma} = |0\rangle_1 |\sigma\rangle_2$, $d_{T_\sigma}^\dagger = |T_\sigma\rangle$, $d_{T_0}^\dagger = |T_0\rangle$, and $d_{S}^\dagger = |S\rangle$. Thus, $d_{\sigma} = e^\dagger f_{i\sigma} + \sigma f_{i\sigma}^\dagger d_{T_\sigma} + (1/\sqrt{2}) \sigma f_{i\sigma}^\dagger [d_{T_0} + (-1)^\sigma d_{S}]$
and $H_{dot} = \sum_{\gamma=1,1/2} \mathcal{H}_{\gamma}^{d} + (\varepsilon_{1} + \varepsilon_{2} + J/4) \sum_{\gamma=1,1/2} d_{\gamma}^{d} d_{\gamma} + (\varepsilon_{1} + \varepsilon_{2} - 3J/4) d_{\gamma}^{d} d_{\gamma}$ with $\bar{\mathcal{I}}(2) = 2(1)$ and $\bar{\mathcal{I}}(1) = 1(1)$.

Using equation of motion, one can derive the dynamical equations of elements of the density matrix. Their statistical expectations involve the time-diagonal parts of the less Green’s functions, which can be calculated with the help of the Langreth analytic continuation rules and the Fourier transformation. Submitting the uncoupled dot’s Green’s function into the equations, the mater equations describe the electronic transport can be derived as

\begin{align}
\dot{\rho}_{0} &= \sum_{ia\sigma} \Gamma_{ia}^{\sigma} \{ [1 - f_{ia}(\varepsilon_{i})] \rho_{i\sigma} - f_{ia}(\varepsilon_{i}) \rho_{0} \}, \\
\dot{\rho}_{ia} &= \sum_{\alpha} \left\{ \Gamma_{ia}^{\sigma} f_{ia}(\varepsilon_{i}) \rho_{0} - \{ \Gamma_{ia}^{\sigma} [1 - f_{ia}(\varepsilon_{i})] + \Gamma_{ia}^{\sigma} f_{ia}(\varepsilon_{i} + J/4) \} \rho_{i\sigma} + \frac{1}{2} \Gamma_{ia}^{\sigma} [1 - f_{ia}(\varepsilon_{i} + J/4)] \rho_{T_{a}} \\
&+ \frac{1}{2} \Gamma_{ia}^{\sigma} [1 - f_{ia}(\varepsilon_{i} + J/4)] \rho_{T_{o}} + \frac{1}{2} \Gamma_{ia}^{\sigma} [1 - f_{ia}(\varepsilon_{i} - 3J/4)] \rho_{S} \\
&+ (-1) \frac{\sigma}{2} \Gamma_{ia}^{\sigma} [1 - f_{ia}(\varepsilon_{i} + J/4) - \frac{1}{2} f_{ia}(\varepsilon_{i} - 3J/4)] (\rho_{S T_{o} + \rho_{T_{o} S}}) \right\}, \\
\dot{\rho}_{T_{a}} &= \sum_{ia} \Gamma_{ia}^{\sigma} \{ f_{ia}(\varepsilon_{i} + J/4) \rho_{i\sigma} - [1 - f_{ia}(\varepsilon_{i} + J/4)] \rho_{T_{a}} \}, \\
\dot{\rho}_{T_{o}} &= \frac{1}{2} \sum_{ia\sigma} \Gamma_{ia}^{\sigma} \{ f_{ia}(\varepsilon_{i} + J/4) \rho_{i\sigma} - [1 - f_{ia}(\varepsilon_{i} + J/4)] \rho_{T_{o}} \\
&+ \frac{1}{4} (-1)^{\sigma} \{ [1 - \frac{1}{2} f_{ia}(\varepsilon_{i} + J/4) - \frac{1}{2} f_{ia}(\varepsilon_{i} - 3J/4)] (\rho_{S T_{o} + \rho_{T_{o} S}}) \}, \\
\dot{\rho}_{S} &= \frac{1}{2} \sum_{ia\sigma} \Gamma_{ia}^{\sigma} \{ f_{ia}(\varepsilon_{i} - 3J/4) \rho_{i\sigma} - [1 - f_{ia}(\varepsilon_{i} - 3J/4)] \rho_{S} \\
&+ \frac{1}{4} (-1)^{\sigma} \{ [1 - \frac{1}{2} f_{ia}(\varepsilon_{i} + J/4) - \frac{1}{2} f_{ia}(\varepsilon_{i} - 3J/4)] (\rho_{S T_{o} + \rho_{T_{o} S}}) \}, \\
\dot{\rho}_{T_{o} S} &= \frac{1}{4} \sum_{ia\sigma} (-1)^{\sigma} \Gamma_{ia}^{\sigma} \{ [1 - f_{ia}(\varepsilon_{i} + J/4)] \rho_{i\sigma} + [1 - f_{ia}(\varepsilon_{i} - 3J/4)] \rho_{S} \\
&- [f_{ia}(\varepsilon_{i} + J/4) + f_{ia}(\varepsilon_{i} - 3J/4)] \rho_{i\sigma} \} \\
&+ \{ i J - \frac{1}{2} \sum_{ia\sigma} \Gamma_{ia}^{\sigma} [1 - \frac{1}{2} f_{ia}(\varepsilon_{i} + J/4) - \frac{1}{2} f_{ia}(\varepsilon_{i} - 3J/4)] \rho_{T_{o} S} \},
\end{align}

where the elements of the density matrix are defined as $\rho_{0} = e^{1} e$, $\rho_{i\sigma} = f_{ia}^{d} f_{ia}$, $\rho_{T_{a}} = d_{\gamma}^{d} d_{\gamma}$, and $\rho_{S} = d_{S}^{d} d_{S}$. These elements represent the probability that both dots are empty, one electron with spin $\sigma$ occupies dot $i$, and two electrons form the triplet states and the singlet state, respectively. They satisfy the completeness relation $\rho_{0} + \sum_{\sigma} (\rho_{i\sigma} + \rho_{2\sigma} + \rho_{T_{a}}) + \rho_{T_{o}} + \rho_{S} = 1$. $\rho_{S,T_{0}}$ is induced by the exchange interaction. $f_{ia}(\omega) = [1 + e^{(\omega - \mu_{ia})/k_{B} T}]^{-1}$ is the Fermi
distribution function of lead $i\alpha$, and $\Gamma^\sigma_{i\alpha} = \sum_k 2\pi |V_{i\alpha k\sigma}|^2 \delta(\omega - \varepsilon_{i\alpha k\sigma})$ is the coupling strength between lead $i\alpha$ and dot $i$. In the stationary situation, the elements of the density matrix can be derived, and the spin component of current in lead $i\alpha$ can be obtained as

$$I^\sigma_{i\alpha} = \frac{e}{\hbar} \Gamma^\sigma_{i\alpha} \left\{ f_{i\alpha}(\varepsilon_i)\rho_0 - [1 - f_{i\alpha}(\varepsilon_i)]\rho_{\sigma \alpha} + f_{i\alpha}(\varepsilon_i + J/4)\rho_{\sigma \alpha} + \frac{1}{2} f_{i\alpha}(\varepsilon_i + J/4) \right.$$  

$$+ f_{i\alpha}(\varepsilon_i - 3J/4)\rho_{\sigma \alpha} - [1 - f_{i\alpha}(\varepsilon_i + J/4)]\rho_{T_0} - \frac{1}{2}[1 - f_{i\alpha}(\varepsilon_i + J/4)]\rho_{T_0}$$

$$- \frac{1}{2}[1 - f_{i\alpha}(\varepsilon_i - 3J/4)]\rho_T + (-1)^\sigma \frac{1}{2}[1 - f_{i\alpha}(\varepsilon_i + J/4)$$

$$- \frac{1}{2} f_{i\alpha}(\varepsilon_i - 3J/4)](\rho_{S,T_0} + \rho_{T_0,S}) \right\}. \quad (2)$$

When $J \to 0$, these quantum rate equations reduce to the equations describing two separate dots. For a single dot, interplay between Coulomb interaction and spin accumulation in the dot can result in a bias-dependent current polarization, which can be suppressed in the P alignment and enhanced in the AP case. Furthermore, the spin flip process make the occupations of spin-up and spin-down electrons in the dots tend to be equal, which can weaken the enhancement of current spin-polarization rate.

### III. NUMERICAL RESULTS AND DISCUSSIONS

For numerical calculations, we choose meV to be the energy unit and set $k_B T = 0.002$. The polarization rates of all leads are assumed to be $P = 0.4$, and the coupling strength is $\Gamma^\sigma_{i\alpha} = (1 + \sigma P)\Gamma$, except for lead 1R it becomes $(1 \pm \sigma P)\Gamma$, where $+$ for the parallel (P) configuration and $-$ for the antiparallel (AP) one. $\Gamma$ and $J$ are set to be 0.01 and 0.2, respectively, and the current are normalized to $e\Gamma/h$. The exchange coupling $J$ between two dots is the key interaction to improve the spin injection efficiency. Its strength sensitively depend on the e-e Coulomb interaction, interdot coupling, Bychkov-Rashba spin-orbit interaction, and magnetic field. $J$ can reach several hundreds eV and can be tuned to ferromagnetic ($J < 0$) type in the presence of magnetic field. Typical value of the dot-lead coupling strength $\Gamma$ is order of $1\mu$eV, therefore, $J/\Gamma \gg 1$, which makes sure that the quantum rate equations are valid in every bias region.

For clarity, first we show relevant results for single QD system connected to two FM leads. The spin components of the current are $I^\sigma = (e/h)(\Gamma_L^\sigma L^\dagger R^\dagger + \Gamma_R^\sigma R^\dagger L^\dagger + \Gamma_0^\sigma L^\dagger R^\dagger + \Gamma_R^\sigma R^\dagger L^\dagger)$. Thus, the spin-polarization rate is $\eta = (I^\dagger - I^\dagger)/(I^\dagger + I^\dagger) = P_L = P$, regardless of whether
the system is in P or AP configuration. However, for the four-terminal structure, when the exchange interaction is absent, \( n_{1\sigma} = n_{2\sigma} = 1/3 \) for the P configuration, while \( n_{1\uparrow} > n_{1\downarrow} \) and \( n_{2\uparrow} = n_{2\downarrow} \) for the AP one. Since the exchange interaction is sensitive to the spin-dependent occupation numbers in the two dots, we expect that in the P configuration the exchange interaction has little influence on the current polarization, while in the AP one it can affect the transport properties greatly. Further, we apply a large bias between leads 1L and 1R to make sure that \( \varepsilon_1 \) is deeply in the bias window.

Fig. 2(a) shows variations of \( I_{2\sigma} \) and \( n_{2\sigma} \) with the bias voltage in the P configuration. In the following, \( I_{2\sigma} \) is denoted by \( I_{\sigma} \), for convenience. As expected, both \( I_{\uparrow} \) and \( I_{\downarrow} \) increase monotonously with the bias, and three steps occur when \( \mu_{2L} \) crosses \( \varepsilon_2 - 3J/4, \varepsilon_2, \) and \( \varepsilon_2 + J/4 \), respectively. They correspond to the situations that electrons tunnel through dot 2 via the singlet state, the energy level \( \varepsilon_2 \), and the triplet states. Here we mark the bias regions \( \varepsilon_2 - 3J/4 < V/2 < \varepsilon_2, \varepsilon_2 < V/2 < \varepsilon_2 + J/4, \) and \( V/2 > \varepsilon_2 + J/4 \) as I, II, and III, respectively. In each region, \( I_{\uparrow} > I_{\downarrow} \). However, in region I, \( n_{2\downarrow} > n_{2\uparrow} \), which is different from the case of isolated single dot, where \( n_{\uparrow} = n_{\downarrow} \) and \( \eta = P = 0.4 \). Since \( n_{2\downarrow} > n_{2\uparrow} \), \( \eta_2 \) is suppressed from 0.4, accompanied by the increase of \( \eta_1 \). When the bias rises beyond region I, both \( \eta_1 \) and \( \eta_2 \) return to 0.4. So in the P configuration we can not enhance \( \eta_2 \) from its original value in single dot case.

In the AP configuration, \( \eta_2 \) can be strongly modified from the single dot case by the exchange interaction (see Fig. 3). Figs. 3(a) indicates both \( I_{\uparrow} \) and \( I_{\downarrow} \) increase monotonously with the bias, which is similar to that in the P configuration. However, from region I to region III, the discrepancy between \( I_{\uparrow} \) and \( I_{\downarrow} \) keeps increasing, resulting in the enhancement of \( \eta_2 \) in Fig. 3(b). In region III, \( \eta_2 \) approaches 0.7, which is much larger than its original value 0.4 in single dot system. At the same time, \( \eta_1 \) keeps decreasing when bias increases from region I to region III, and finally becomes smaller than 0.1. It is concluded that in the AP configuration one can greatly enhance the current polarization rate through one dot, accompanied by decrease of the current polarization rate through another dot. Such phenomenon looks as if the current polarization rate is “transferred” from one circuit to the other.

The enhancement of the current polarization rate can be understood with the aid of the expression of the current. Due to the absence of intradot spin flips, both the amplitude and spin polarization of the total current through dot 2 are conserved, i.e., \( I_{2L} = I_{2R} \). For
simplicity, the current $I_{2R}^\sigma$ is chosen in the calculation because it has an uniform expression in all three regions: $I^\sigma = (e/h)\Gamma_{2R}^\sigma/\rho_{2\sigma} + \rho_{T_\sigma} + (1/2)\rho_{T_0} + (1/2)\rho_S$. The first term denotes the process that one electron tunnels through dot 2 via the energy level $\epsilon_2$, and the second to fourth terms denote the processes that one electron with spin $\sigma$ transports through dot 2 via the triplet states and the singlet state. Because in $T_0$ and $S$ states, electrons with spin $\sigma$ or $\bar{\sigma}$ have the same probability to occupy dot 2, both the third and the fourth terms have a factor 1/2. From Fig. 3(b), in region I we can see $\eta_2$ is slightly larger than $P = 0.4$. In this region, only the energy level $\epsilon_2 - 3J/4$ enters the bias window, and electrons can only form the singlet state, which makes $\rho_S$ much larger than other elements [see Figs. 3(c) and 3(d)]. Thus, the forth term dominates in expression of the current, and we have $I^\sigma = (e/2h)\Gamma_{2R}^\sigma/\rho_S + \rho_{T_0})$, $\eta_2 = (I^\uparrow - I^\downarrow)/(I^\uparrow + I^\downarrow) = P = 0.4$. When the effects of $\rho_{2\sigma}$ and $\rho_{T_\sigma}$ are considered, the value of $\eta_2$ is slightly modified. From Eq. (1) we can obtain $\rho_{2\sigma} \approx \Gamma_{1R}^\sigma \rho_S/[2(\Gamma_{1L}^\sigma + \Gamma_{1L}^\bar{\sigma} + \Gamma_{2L}^\sigma + \Gamma_{2L}^\bar{\sigma})]$. Here we denote $(1 + \sigma P)\Gamma = \Gamma^\sigma$, then $\Gamma_{1\sigma}^\sigma = \Gamma^\sigma$, except for $\Gamma_{1R}^\sigma = \Gamma^\bar{\sigma}$. Thus, $\rho_{2\uparrow} \approx \rho_S/[2(3 + \Gamma^\bar{\sigma} / \Gamma^\sigma)] > \rho_{2\downarrow} \approx \rho_S/[2(3 + \Gamma^\sigma / \Gamma^\bar{\sigma})]$, and $\eta_2$ is enhanced from 0.4, as shown in Fig. 3(b). In region I, $\rho_S$ is much larger than other elements, which means that during most of the time electrons in the double dot form the singlet state. So $\rho_{2\sigma}$ is mainly contributed by the process that an electron in dot 1 tunnels to lead 1R and breaks the singlet state. Noticing that in such a configuration, $\Gamma_{1R}^\downarrow > \Gamma_{1R}^\uparrow$, electron with spin $\downarrow$ can tunnel to lead 1R more easily, and left an electron with spin $\uparrow$ in dot 2, which makes $\rho_{2\uparrow} > \rho_{2\downarrow}$.

When the bias locates in region II, the direct tunneling channel at $\epsilon_2$ opens. We can see the enhancement of $\rho_{2\uparrow}$ ($\rho_{T_\uparrow}$) is larger than $\rho_{2\downarrow}$ ($\rho_{T_\downarrow}$), which results in further increase of $\eta_2$. Here $\rho_{2\sigma} = [\Gamma_{2L}^\sigma \rho_0 + \Gamma_{1R}^\sigma \rho_{T_\sigma} + (1/2)\Gamma_{1R}^\sigma (\rho_{T_0} + \rho_S)]/(\Gamma_{1L}^\sigma + \Gamma_{1L}^\bar{\sigma} + \Gamma_{2L}^\bar{\sigma} + \Gamma_{2R}^\bar{\sigma})$. It is obvious that the increase of $\rho_{2\sigma}$ is mainly owing to the term $\Gamma_{2L}^\sigma \rho_0$ in the numerator, which is absent in region I. Following the same procedure, this term reads $\Gamma_{2L}^\sigma \rho_0 / (\Gamma_{1L}^\sigma + \Gamma_{1L}^\bar{\sigma} + \Gamma_{2R}^\sigma) = \rho_0 / (2 + \Gamma^\bar{\sigma} / \Gamma^\sigma)$, so the increase of $\rho_{2\uparrow}$ is larger than that of $\rho_{2\downarrow}$, and $\eta_2$ is enhanced from its value in region I.

When the bias enters region III, $\rho_{2\sigma}$ and $\rho_{T_\sigma}$ keep increasing, and the enhancement of $\rho_{T_\uparrow}$ is much more than other elements. This is because now the channel at $\epsilon_2 + J/4$ opens, and if dot 1 is occupied, electrons in lead 2L can directly tunnel into dot 2 and form the triplet state $T_\sigma$. Since lead 1R is in antiparallel with lead 1L, in most of the time, dot 1 is occupied by one electron with spin $\uparrow$. As a consequence, electrons with spin $\uparrow$ in lead 2L is more available to tunnel into dot 2 and form the triplet state $T_\uparrow$, which makes $\rho_{T_\uparrow} \gg \rho_{T_\downarrow}$. This
can also be seen in the formula \( \rho_{T_1} = \frac{(\Gamma_{1L}\sigma_1 + \Gamma_{1L}\rho_2)(\rho_1 + \Gamma_{2R})}{(\Gamma_{1L}\rho_1 + \Gamma_{2R})} \), where the first term in the numerator makes \( \rho_{T_1} \) increase intensively in region III. Thus, \( \eta_2 \) is greatly enhanced in region III.

In the case of \( J/\Gamma \gg 1 \), the analytical expressions in region I, II, and III are \( \eta_2 \sim 191P/(165 - 34P^2) \), \( 120P/(84 + 5P^2) \), and \( 51P/(27 + 6P^2) \), respectively. For \( P = 0.4 \), \( \eta_2 \sim 0.454, 0.542, \) and 0.673, which is consistent with our numerical results. As expected, when \( P \to 1 \), \( \eta_2 \to 1 \) in all regions. If we tune the bias into region III, the injection efficiency can be enhanced to almost twice of its original value. In the inset of Fig. 3(a), we present the variations of \( \eta_2 \) with \( P \) for different situations. It can be seen that when \( P \) is small, \( \eta_2 \) is greatly enhanced by the exchange interaction.

**IV. CONCLUSIONS**

In summary, we propose a scheme based on a four-terminal double quantum dot system to improve the spin injection efficiency greatly. We find that in the antiparallel configuration, the spin-polarization rate through one quantum dot can be dramatically enhanced, while the polarization rate through the other one is suppressed. The operating mechanism is the exchange interaction between the two quantum dots.

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FIG. 1: (color online) The system with two quantum dots coupled to four external FM leads. The magnetizations of three leads are parallel with each other, while the magnetization of lead 1R can be parallel (P) or antiparallel (AP) with the other three.

FIG. 2: (color online) The spin component of the current in dot 2 (a) and the spin-polarization rate (b) versus bias in the P configuration. The inset in (a) shows the variations of the occupation numbers in dot 2.

FIG. 3: (color online) The transport properties in the AP configuration. (a) The spin component of the current versus bias. The inset shows the variations of the spin-polarization rate with $P$ in different situations. The solid line corresponds to the single dot case, and the dashed, dotted, and dash-dotted lines correspond to the situations that the bias locates in region I, II, and III, respectively. (b) The spin-polarization versus bias. (c) and (d) The corresponding elements of the density matrix versus the bias.