Generalized Economic Order Quantity Inventory Model using Brownian Motion in Food Production Planning

K. Suganthi, G. Jayalalitha

Abstract: The goal of this paper is to propose a Generalized EOQ inventory model, which combines Inventory replenishment and pricing decisions for perishable product in an Inventory model. The Stochastic Demand requests for new and old items are derived via linear functions of their cost. The proposed solution obtains from the optimum system solution, such as food Procurement price, Shortage cost, Purchasing cost, Operational cost and Total cost reduced. A Mathematical model is introduced along with differential matrix to find optimal solutions for the Inventory method in food production planning. Numerical example and Sensitivity Analysis are explained in the proposed system. Brownian Motion are found from Cycle Length (L= 1.1 to 1.5). The Brownian Motion analyses all the costs and plays a major part to reduce it further.

Keywords: Inventory Model, EOQ, Shortage Price, Stochastic Demand, Brownian Motion.

AMS Classifications: 90B05, 90B36, 90B99, 60JXX, 60J65.

I. INTRODUCTION

Irregular movement of tiny elements suspended in a liquid or a gas, produced by the bombardment of the elements by molecules. First noticed by Robert Brown in 1827[1, 12]. Production planning also make sure that staffing is sufficient, that food is in-house, basic preparation steps are completed, foods are soft enough, and several other functions are done [11]. Stock affiliation ends up one fundamental thing that should be considered by the affiliation related to the remuneration of the alliance especially with the points of interest contained in the stock [1]. Having a lot of stock has an eventual outcome of the unit of central focuses for various parts in the relationship since far most by far of the points of interest are set assets into the stock. On the other hand, having less stock in hands will make the probability of the unfilled intrigue, and this will price the reputation of the association later on [2][3]. Hence, we need a reasonable model to find the perfect stock to help connection dealing with this issue. Starting late, predictable bits of knowledge have been used in different areas, everything considered, issues, particularly for controlling stock [4].

One of the most significant stresses of the affiliation is to pick when and the total to compose or to pass on so the unmitigated price related with the stock structure should be least [5]. This is somewhat reliably essential, when the stock experience decay or disintegrating. Isolating is portrayed as change, hurt, ruin, and crippling outdated quality. [6]. It is unprecedented that particular things, for instance, vegetable, prescription, fuel, blood and radioactive planned substances decay under disintegrating during their normal social affair period. As such, while picking the perfect stock technique of that kind of things, the disaster on account of rot can’t be rejected [7]. A stunning referencing procedure for upstream supply sort out, considering negative relationship of retailer forms between periods. This dynamic system may instigate astonishing execution improvements [8].

The perfect purchase and stock recuperation issue of temporary standard plant thing, in which the distributor needs to pick the aggregate to buy in the accumulate season and the entire to recoup from the forced moving away in each period to offer to the market [9]. Stock controlling issue (IRP) model [10] grants to separate the upsides of level joint exertion with related to a few chiefaction pointers, i.e., transmissions, dynamic time, full scale price included sorting out, stock and waste price given a scrappy intrigue. A mixed entire number nonlinear programming model [11] is used to restrict the full scale expenses using two counts and Lindo impacts of unequivocal components, for instance, the proportion of workplaces, perfect rates and deals on the unbending prices. A budgetary deals complete stock model [12] with screening price and pointless price at two area coordination plan pondered buyer in non coordination develop and no insufficiencies for coordination plot. In context on the upsetting materials, the sustenance business is looked with tangled stock affiliation issues because of the concise natures. Since green things ruin after some time, it is key for stock heads to use an affiliation procedure to neutralize some portion of the course of action time span [11]. The free show off movement of country and marine things impact in a general sense with climatic blends, conflicting changes and unquestionable social parts. Reclamation decisions for brief things are attempting an outcome of vulnerabilities in customer demand, achieved by bound thing time extent of ease of use and pricemarkdowns [7].

II. PROBLEM DEFINITION

The following assumptions and notations are used to develop the model

Notations:

- \( w_2 \) – Buyer setup price
- \( w_1 \) - Buyer holding price
- \( C \) – Production rate for manufactures
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R – Size of the back orders
V₁ – Manufacture setup price
V₂ – Manufacture holding price
L – Cycle length
Fₚ – Food Procurement Price
S – Operation price per growing period
Q – Storage quantity
Pₚ – Total buying price
Tₚ – Total price

An inventory scheme for standard things like books, set sustenance and different things, where the provider gets them and stock them for a level of time, x₁; by at that point, things are traded at a rate, i.e., request rate d, until the stock level gets in contact at zero. At last, during x₃, the stock structure faces need until the time that the level of need interfaces at Q.

\[ x₁ = \frac{W₁ - W₀}{uR} = \frac{v₁ - v₀}{R} \]  
\[ x₂ = \frac{W₂}{d} = \frac{w₁ - q}{d} \]  
\[ x₃ = \frac{q}{d} = \frac{q}{d} \]

Hence, cycle length is computed as

\[ L = x₂ + x₃ = \frac{w₁ - q}{d} + \frac{q}{d} = \frac{w₁}{d} \]

Hence

\[ u = \frac{dL}{v₁} \]  

The aim of the method is to minimize complete price per period (Tₚ) comprising buying price (Pₚ), holding price (Hₚ), operational prices (Oₚ), shortage price (Sₚ) and food acquisition price (Fₚ). This is represented as follows.

\[ Tₚ = Pₚ + Hₚ + Oₚ + Sₚ + Fₚ \]  

The next subdivision explains a complete analysis of computation of earlier discussed prices.

A. Buying Price For Each Time

The cost for every gramis pso the whole buying price for each time achieved by equation (7)

\[ Pₚ = pu₀v₀ \]

By substituting u from equation (5), we have

\[ Pₚ = pu₀\left(\frac{dL}{v₁}\right) \]

B. Holding Price For Each Time

Holding price for every gram per time period is additionally, holding price is computed at the final stage of growing period (x₂). Holding price per period is described as

\[ Hₚ = \frac{f}{2d}\left[(uv₁)^2 + (Q)^2 - 2uv₁Q\right] \]

\[ Hₚ = \frac{fQ^2}{2d} - fTS + \frac{fdL²}{2} \]

C. Operational Prices Per Time Interval

Operational price per rising time interval is S, so we have

\[ Oₚ = SLC \]

D. Shortage Price

Shortage price for every gram per time interval is g. Total shortage prices can be computed as

\[ Sₚ = g\left(\frac{x₁Q}{2}\right) = g\left(\frac{Q^2}{2d}\right) \]

E. Food Procurement Price For Each Time

Food Procurement price for every gram per time interval is c. Food Procurement prices per period, is calculated as

\[ Fₚ = \frac{dc(v₁ - v₀)^2L}{2Rv₁} \]

By substituting u from equation (5), we have

\[ Fₚ = \frac{dc(v₁ - v₀)^2L}{2Rv₁} \]

With respect to equation (8), (10-12) and (14), the inventory whole prices for each time unit are formulated as follows

\[ Tₚ = Pₚ + Hₚ + Oₚ + Sₚ + Fₚ \]

\[ Tₚ = dpv₀\left(\frac{dt}{v₁}\right) + \frac{fQ^2}{2d} - fTS + \frac{fdL²}{2} + S + g\left(\frac{Q^2}{2d}\right) + \frac{dc(v₁ - v₀)^2L}{2Rv₁} \]

Therefore, total inventory price for each unit time is calculated as follows,

\[ Tₚ = \frac{TₚU}{L} = \frac{dpv₀}{v₁} + \frac{dc(v₁ - v₀)^2L}{2Rv₁} + S\left(\frac{1}{2}\right) + \left(\frac{f + d}{2d}\right)\left(\frac{Q^2}{L}\right) + \frac{fd}{2}\left(L - f(Q)\right) \]

F. Constraint

Our inventory system assures that objects are available to utilize on time, the entire setup and development time should be less than or equal to usage and shortage time. Hence, the below mentioned parameter should be fulfilled:

\[ x₁ + x₃ \leq L \]

Substituting, x₁ from eq. 1, obtains the following constraint:

\[ x₁ + \frac{Q}{2} \leq L \]
By substituting $t$ from equation 5 transforms the parameter into the following for production time:

$$L \geq \left(\frac{v_1-v_0}{R} + x_s \right) L_{min}$$

(19)

G. Mathematical Expression Of The EOQ Inventory Method With Increasing Materials

As per the equation 16 and 19, the mathematical formation of the EOQ inventory method with developing object is mentioned as

$$\text{Min } T_c = \left(\frac{dp v_0}{v_1} + \frac{dc(v_1-v_0)}{2Rv_1} + S \left(\frac{1}{L} + \frac{(f+g)}{2d} \right) \left(\frac{Q}{L}\right) + \frac{f d 2 L - f Q}{L} \right) \tag{20}$$

Such that, $T \geq T_{min}$

$$S \geq 0$$

$$T > 0$$

III. SOLUTION PROCEDURE

The objective function of the proposed issue (equation 20) is calculated. Then again, the proposed numerical model has a straight essential, this coherent model is a twisted unsurprising non-direct programming. Bended goods imparts that if a reachable game-plan crushes its surroundings, i.e., close by optima, by then it is by and large optima also. Consequently, the ideal game-plan of the goal work (equation 20) is managed by utilizing fragmentary subordinates. Figuring the inadequate helper of the goal work (equation 20) as for the cycle-length ($L$) and making it proportionate to zero, the ideal game-plan of $L$ is as per the going with:

$$\frac{\partial T_c}{\partial L} = \frac{S}{L^2} - \frac{(f+g)}{2d} Q^2 + \frac{f d}{2}$$

$$= 0 \rightarrow L = \frac{2 d S + (f+g) Q^2}{f d^2} \tag{21}$$

Additionally, computing the partial derivative of objective function (equation20) with related to the deficiency amount ($Q$) and converting it near by zero, the minimal solution of $Q$ is as follows:

$$\frac{\partial T_c}{\partial Q} = \left(\frac{f+g}{d} \right) Q - f = 0 \rightarrow Q = \frac{f d c}{f + g}$$

(22)

Substituting $Q$ from equation 22 in equation 21, the optimum cycle length is computed as follows:

$$L = \sqrt{\frac{2 d S}{f d^2 (1 - \frac{f}{f + g})}} \tag{23}$$

Finally, as per the proposed ideas and formulas, solution stages of the optimization algorithm

For the suggested EOQ inventory system is

Step 1 Calculate $L_{min}$ from equation 19

Step 2 If $L_{min} \geq 0$ then question is possible and go to Step 3, else it is not possible and stop.

Step 3 If $1 - f/(f+g)$ is positive, then problem is infeasible and compute $L$, else problem is impossible and stop

Step 4 Compute $L$ from equation 23.

Step 5 If $L \geq L_{min}$, then $L' = L$, else $L' = L_{min}$.

Step 6 Calculate $L$ from equation 23.

Step 7 Calculate $T_c$ and $y^*$ through objective function of (equation 20) and equ 5, correspondingly, with respect to the achieved $L'$ and $Q^*$, and describe the optimal solution.

Step 8 End

IV. NUMERICAL EXAMPLE

Example:

Consider a production method with one item and the following values for the input factors. $D=1000, f=0.9, g=0.2, R=10, V_r=3, V_d=0.6, S=11, d=5, p=120$, and the index of the power demand pattern

$$F_c = 5.28, Q = 45, S_c = 40.5, O_c = 121$$

and $T_c = 299.39$

In Table 1, by fixing the replacement rate constraint, if the unit production price is then the total profit function and the best lot size and the value of decrease. In this subsection, the effect of factors on choice factors and the target capacity is examined. To do as such, the estimation of every parameter is changed by leaving different factors unaltered. Conversely, the most useful cycle length and the most efficient charge grows in the similar condition. In the equal Table 1, the unit production fee is fixed, the whole earnings function, somewhat, the best development time and the economic huge volume reduces as the production rate rises. Conversely, the foremost rated does not change in the similar situation. For the Brownian approximation is blocking base-stock and conditional threshold procedures, whereas considering to reactive procedures. It recognises the parameter management in which preventive or reactive insurance procedures are most excellent in the Brownian model with respective to the shortfall drifts and minute variance factors. The relative values of maintaining and backorder prices have a consequence on the border among the blocking and conditional areas changeability in production and demand performs no function. This border demonstration might also replicate the significance of managing inventory prices in this regime is obligatory to use the conditional supply to more quantity to make up for the lack of important capacity, and so inventory price is the last price that can be successfully handled.
When the foremost ability is sufficiently large, the conditional capability is used reactively to every so often resolve giant inventory shortfalls. The shortfall may every so often take excursions to excessive degrees due to variability in demand and manufacturing but, barring for the most serious shortfall spikes, the shortfall improves satisfactorily besides the aid of the conditional supply. The most important capability is larger such that the movement of the shortfall with only fundamental potential is powerful towards the origin, which involves a complete inventory. The Inventory of variability in the demand and the manufacturing tactics in finding out the most fulfilling policy in this system is focussed by means of the reactive vicinity border, which consists of no price factors. The Table 2 values are obtained from Table 1. For the cycle length of L (1.1 to 1.5), the mean of Fe, Sc, Q, Oc and Te are found and tabulated in table 2. The Brownian motion plot of cycle Length(L=1.1 to 1.5), is shown in Figure 1, 2, 3, 4 and 5 respectively.

V. CONCLUSION

An EOQ inventory model is proposed, which combines inventory replenishment and valuing choices for item in an inventory system. The stochastic Demand requests for new and old items passed on by means of direct elements of their costs. This model with differential matrix is utilized to get ideal answer for this inventory system. All these factors are analyzed with respective to the food production planning. The demand of a product is described by means of linear functions of their costs. Various cost factors involved to buy a product, maintenance, total and other costs are reduced. The Brownian Motion goes about as a significant job to decrease the cost further.

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Table 2: Procedure of Cycle Length in Brownian Motion

| Length Cycle | Fc  | Sc  | M  | Oc  | Tc  |
|--------------|-----|-----|----|-----|-----|
| L=1.1        | 10.56 | 182.25 | 90 | 242 | 864.96 |
| L=1.2        | 11.52 | 216.89 | 98.18 | 264 | 1029.05 |
| L=1.3        | 12.48 | 254.55 | 106.36 | 286 | 1122.82 |
| L=1.4        | 13.44 | 295.21 | 114.54 | 308 | 1289.73 |
| L=1.5        | 14.4 | 338.90 | 122.73 | 330 | 1269.85 |

(M= Mean)

Figure 1: L=1.1 in Brownian motion

Figure 2: L=1.2 in Brownian motion

Figure 3: L=1.3 in Brownian motion

Figure 4: L=1.4 in Brownian motion
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Figure 5: $L=1.5$ in Brownian motion