Is there any evidence for extra-dimensions or quantum gravity effects from the delayed MeV-GeV photons in GRB 940217?

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The discovery of X-ray afterglows of GRBs, and the identification of host galaxies of GRBs, confirm the cosmological origin of GRBs. However, the discovery of the delayed MeV-GeV photons in GRB 940217 imposes serious challenges for the standard emission model of GRB. Although the delayed MeV-GeV photons might be explained by some some radiation emission mechanisms, the mystery of detecting an 18 GeV photon still remains unsolved. We suggest that the detection of the 18-GeV photon ∼4500 s after the keV/MeV burst in GRB 940217 provides a strong evidence for the existence of extra-dimensions and/or quantum gravity effects. The delay scale of the 18-GeV photon leads to an estimation of the fundamental energy scale, associated with the linear energy dependence of the speed of light, of the order of $2 \times 10^{15}$ GeV, which is consistent with the results obtained by another independent analysis on the data of OSSE and BATSE.

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I. INTRODUCTION

Delayed high-energy photon emissions from gamma-ray burst (GRBs) sources can be considered a well-established fact. In 1994 Hurley et al. observed a particularly energetic burst, GRB 940217, with a duration of around 90 minutes. GRB 940217 is a very strong burst, with a total fluence above 20 keV of $(6.6 \pm 2.7) \times 10^{-4}$ erg cm$^{-2}$ and a duration of ∼180 s in the BATSE range. It has the third largest fluence of around 800 BATSE bursts up to the detection time of this burst. During the period of low energy emission, i.e., in the first ∼180 s, EGRET detected 10 photons, with energies ranging from a few tens of MeV to a few GeV, with the fluence in this range of ∼$2 \times 10^{-5}$ erg cm$^{-2}$. Most strikingly, after the low-energy emission has ended, an additional 18 high-energy photons were recorded after ∼5400 s following this event, including an 18 GeV energy photon ∼4500 s and 36 photons with 137 MeV energy. The fluence of the delayed emission was $7 \times 10^{-6}$ in the energy range 30 MeV-3 GeV. For comparison, typical GRBs emit photons in the energy range between a few keV and a few tens of MeV, and last a few tens of seconds.

There are several possible astrophysical sources and mechanisms, which could produce high energy gamma-ray photons. One such source could be the Galactic diffuse gamma ray radiation, also detected by EGRET. The photon spectrum can be represented by a broken power law, with a break energy at 1.9 GeV. Above the break energy the spectrum of the Galactic background radiation is

$$F_{\text{back}}(E_\gamma) \approx 2.2 \times 10^{-10} \left(\frac{E_\gamma}{1.9 \text{ GeV}}\right)^{-3.1}.$$  

(1)

The number of background photons with energy greater than 10 GeV is given by

$$N_\gamma(E_\gamma > 10 \text{ GeV}) = \int_{10\text{GeV}}^{\infty} F_\gamma(E_\gamma) A_{eff} \Delta t dE_\gamma,$$  

(2)

where $A_{eff}$ is the effective, energy dependent area of the gamma ray detector (700 cm$^2$ for $E_\gamma > 10$ GeV in the case of EGRET) and $\Delta t$ is the duration of the observation. For $\Delta t \approx 6000$ s we obtain $N_\gamma(E_\gamma > 10 \text{ GeV}) \approx 6 \times 10^{-5}$. Therefore the 18 GeV photon detected during GRB 940217 cannot be of galactic origin. An other possible source for the 18 GeV photon in the GRB 940217 observation is the collision between the high energy (in the TeV range) gamma-ray photons, produced in a GRB emission, and the diffuse background of microwave or infra-red photons, leading to electron-positron pair production. The pairs produced will scatter off the background photons, and since the number density of microwave background photons is significantly higher than that of infra-red photons, many 2.7 K microwave radiation photons will be boosted to a very high energy, in the (MeV-GeV) range.
The spectrum of the scattered cosmic microwave photons has a power law form \(E \leq (4/3)\gamma^2 \langle \varepsilon \rangle\), where \(\langle \varepsilon \rangle = 2.7k_BT_{CMB}\) is the mean energy of the CMB photons, with \(k_B\) the Boltzmann constant and \(T_{CMB}\) the temperature of the cosmic microwave background. \(\gamma_e\) is the Lorentz factor of the secondary pairs resulted from a primary photon with energy \(\varepsilon\). For \(\gamma_e \leq 10^6\), taking into account that \(\langle \varepsilon \rangle \approx 10^{-4}\), it follows that the energy of the scattered photons is of the order of \(E_\gamma \leq 200\) MeV. Therefore the 18 GeV photon from GRB 940217 cannot be a reprocessed primary TeV photon.

The redshift at which GRB 940217 was produced has been estimated in \([8]\), by considering the opacity of the intergalactic space to high energy gamma-rays. This depends upon the number density of soft target photons. \(\gamma\)-rays at above 20 GeV will be attenuated if they are emitted at a redshift of \(z \approx 3\). Therefore the highest energy photon in this burst is constrained to have originated at a redshift less than \(z \approx 2\) \([8]\).

Several physical mechanisms, which could explain the time delay between MeV-GeV photons in GRB 940217, have been proposed in the past years. The existence of an inter-galactic magnetic field (IGMF) could give a natural explanation of this delay \([8, 10]\). However, the IGMF has not been detected so far, Faraday rotation measures setting a limit of \(\sim 10^{-9}\) G for a field with 1 Mpc correlation length. Theoretical calculations show that these fields could be as low as \(10^{-20}\) or even \(10^{-29}\) G \([11]\). Some studies suggest that the IGMF could be generated from a much weaker primordial magnetic field, \(\sim 10^{-20}\) G, due to turbulence induced in the formation of large scale structure in the universe \([12]\). Therefore no convincing evidence for the existence of IGMFs exists. It has also been suggested that the delay could be the result of the collapse of a compact object \([13]\) or coalescence of a compact binary \([14]\).

The time delay between soft emission and high-energy emission is also suggested by some GRB models. Photon-pair electromagnetic cascades can produce delayed MeV-GeV photons \([4]\). In \([15]\) autocorrelation was used to determine the duration of substructures in different energy bands of selected bursts. Electron inverse Compton emission from afterglow shocks could produce a delayed GeV component \([16]\). The information provided by the delay time could give constraints on models for TeV gamma rays and could differentiate between mechanisms causing the time delay \([5]\). However, all these models mentioned above cannot give a convincing explanation for the detection of the 18-GeV delayed photon in GRB 940217.

It is the purpose of the present paper to suggest an other possibility for the explanation of the GeV-MeV photon delay in GRB 940217, namely, the possibility that this effect is due to the presence of extra space-time dimensions and/or to related quantum gravitational effects. The delay scale of the 18-GeV photon leads to an estimation of the fundamental energy scale, associated with the linear energy dependence of the speed of light, of the order of \(2.1 \times 10^{15}\) GeV, which is consistent with the results obtained by another independent analysis on the data of OSSE and BATSE \([17]\).

The present paper is organized as follows. In Section II we briefly review the possible effects of extra-dimensions on the propagation of gamma photons emitted in GRB explosions. The time delay for photons in multi-dimensional universes is considered in Section III. In Section IV we apply the derived results for the case of the 18 GeV photon in GRB 940217.

II. COULD EXTRA-DIMENSIONS BE DETECTED IN ASTROPHYSICAL PROCESSES?

One of the most challenging issues of modern physics is the existence of the extra-dimensions, idea proposed originally by Kaluza in 1921 \([18]\) and developed by Klein in 1926 \([19]\). Multi-dimensional geometries are the natural framework for the modern string/M theories \([20]\) or brane models \([21]\). String and Yang-Mills type models also provide a natural and self-consistent explanation for the possible variation of the fundamental constants \([22]\), as initially suggested by Dirac \([23]\). Hence the problem of the extra-dimensions of the space-time continuum is closely related to the problem of the variations of the fundamental constants, like, for example, the fine structure constant or the speed of light (for a recent review of the experimental and theoretical studies and the present status of these fields see \([24]\) and \([25]\)). Most of the theories with extra-dimensions contain a built-in mechanism, which allows the variation of the fundamental constants. Within the multi-dimensional approach the physical interactions are described by a theory formulated in \(4 + D\) dimensions, and the conventional four-dimensional theory appears as a result of a process of dimensional reduction. Couplings in four dimensions are determined by a set of constants of the multidimensional theory and the size \(A\) of the space of extra-dimensions. The multi-dimensional constants are assumed to be genuinely fundamental and, consequently, they do not vary with time. On the other hand it is natural to assume that in an astrophysical or cosmological context \(A\) varies with time, similarly to the scale factor \(a\) of our four-dimensional Universe. But this leads in four dimensions to the time variation of the fundamental constants, like fine structure constant \(\alpha\) or the gravitational coupling \(G\). Moreover, since their time dependence is given by the same factor \(A\), the time variations of \(\alpha\) and \(G\) could be correlated \([26]\).

The search for a unification of quantum mechanics and gravity is likely to require a drastic modification of the present day deterministic representation of the space-time properties. There is at present no complete mathematical
model for quantum gravity, and no one of the many different models proposed so far can give a satisfactory description of the physics on characteristic scales near the Planck length $l_P$. However, in several of the approaches used to find a theory of quantum gravity the vacuum can acquire non-trivial optical properties, because of the gravitational recoil effects induced by the motion of the energetic particles. The recoil effects may induce a non-trivial refractive index, with photons at different energies travelling at different velocities \cite{27}. Photon polarization in a quantum space-time may also induce birefringence \cite{28}, while stochastic effects in the vacuum could give rise to an energy dependent diffusive spread in the velocities of different photons \cite{29}.

Therefore a large class of physical models, incorporating quantum gravitation and/or multi-dimensional field theories predict that the propagation of particle in vacuum is modified, due to the supplementary effects induced by the modification of the standard general relativity. In particular, the possible violation of the Lorentz invariance or the existence of extra-dimensions can be investigated by studying the propagation of high energy photons emitted by distant astrophysical sources \cite{30}.

The confirmation that at least some gamma-ray bursts (GRBs) are indeed at cosmological distances raises the possibility that observations of these could provide interesting constraints on the fundamental laws of physics. The fine-scale time structure and hard spectra of GRB emissions are very sensitive to the possible dispersion of electromagnetic waves in vacuo, with velocity differences $\Delta u \sim E/E_{\text{QG}}$, as suggested in some approaches to quantum gravity. GRB measurements might be sensitive to a dispersion scale $E_{\text{QG}}$ comparable to the Planck energy scale $E_P \sim 10^{19}$ GeV, sufficient to test some of these theories \cite{30}. Hence the study of short-duration photon bursts propagating over cosmological distances is the most promising way to probe the quantum gravitational and/or the extra-dimensional effects. The modification of the group velocity of the photons by the quantum effects would affect the simultaneity of the arrival times of photons with different energies. Thus, given a distant, transient source of photons, one could measure the differences in the arrival times of sharp transitions in the signals in different energy bands. A key issue in such a probe is to distinguish the effects of the quantum-gravity/multi-dimensional medium from any intrinsic delay in the emission of particles of different energies by the source. The quantum-gravity effects should increase with the redshift of the source, whereas source effects would be independent of the redshift in the absence of any cosmological evolution effects. Therefore it is preferable to use transient sources with a known spread in the redshift $z$. The best way to probe the time lags that might arise from quantum gravity effects is to use GRBs with known redshifts, which range up to $z \sim 5$.

Data on GRBs may be used to set limits on variations in the velocity of light due to quantum gravitational effects. This has been done, by using BATSE and OSSE observations of the GRBs that have recently been identified optically, and for which precise redshifts are available, in \cite{31}. A regression analysis can be performed to look for an energy-dependent effect that should correlate with redshift. The analysis of GRBs data yield a limit $M_{\text{QG}} \sim 10^{15}$ GeV for the quantum gravity scale. The study of the the times of flight of radiation from gamma-ray bursts with known redshifts has been considerably improved by using a wavelet shrinkage procedure for noise removal and a wavelet ‘zoom’ technique to define with high accuracy the timings of sharp transitions in GRB light curves \cite{17}. This procedure optimizes the sensitivity of experimental probes of any energy dependence of the velocity of light. These wavelet techniques have been applied to 64 ms and TTE data from BATSE, and OSSE data. A search for time lags between sharp transients in GRBs light curves in different energy bands yields the lower limit $M_{\text{QG}} \geq 6.9 \times 10^{19}$ GeV on the quantum-gravity scale in any model with a linear dependence of the velocity of light, $c \sim E/M_{\text{QG}}$.

\section{Photon Delay in Multi-dimensional Universes}

The general expressions for the time delay of photons of different energies in the framework of multi-dimensional cosmological models was considered in \cite{32}. In models with compactified extra-dimensions (Kaluza-Klein type models), the main source of the photon time delay is the time variation of the electromagnetic coupling, due to dimensional reduction, which induces an energy-dependence of the speed of light. A similar relation between the fine structure constant and the multi-dimensional gauge couplings also appears in models with large (non-compactified) extra-dimensions. For photons of energies around 1 TeV, propagating on cosmological distances in an expanding Universe, the time delay could range from a few seconds in the case of Kaluza-Klein models to a few days for models with large extra-dimensions. As a consequence of the multi-dimensional effects, the intrinsic time profiles at the emitter rest frame differ from the detected time profiles. The formalism developed in \cite{32} also allows the transformation of the predicted light curves of various energy ranges of the emitter into the frame of the observer, for comparison with observations. Therefore the study of energy and redshift dependence of the time delay of photons, emitted by astrophysical sources at cosmological distances, could discriminate between the different multi-dimensional models and/or quantum gravity effects.

Let’s consider the propagation of gamma-rays from GRBs in the Kaluza-Klein type models. For simplicity we restrict our discussion to the five-dimensional case. Hence we assume a flat Friedmann-Robertson-Walker type background.
metric of the form
\[ ds^2 = c^2 dt^2 - a^2(t) \left[ dr^2 + r^2 \left( d\theta^2 + \sin^2 \vartheta d\varphi^2 \right) \right] + \varepsilon \Phi^2(t) dv^2, \] (3)
where \( a(t) \) is the scale factor of the Universe, \( \varepsilon = \pm 1 \) and \( \Phi(t) \) is the scale factor of the fifth dimension, denoted by \( v \). We also assume that the time variation of the fine-structure constant is entirely due to the change in the speed of light \( c \). Therefore we neglect any possible time variation of the electric charge or Planck’s constant. Then the time variation of the speed of light can be related to the size of the fifth dimension by means of the general equation
\[ \frac{\Delta c}{c} = \beta \frac{\Phi}{\Phi}, \] (4)
where \( \beta = 1 \) in the case of the Einstein-Yang-Mills model and \( \beta = 3 \) for the case of the pure Einstein gravity in five dimensions [29]. Hence for the variation of the speed of light we obtain
\[ c = c_0 (1 + \Phi^\beta), \] (5)
where \( c_0 \) is the four-dimensional speed of light.

In order to find a simple, and directly testable relation between the radius of the extra-dimension and the energy of the photon, we shall assume, following the initial proposal in [33], that the mass of a body (and the associated energy) corresponds to the length of a "line segment" of the fifth subspace. In a more general formulation, we shall assume that the variables parameters \( c, G \) and the photon energy \( E = h\nu \) are related to the metric tensor component of the fifth dimension by means of the equation [32, 34]
\[ \frac{G(t)E}{c^4} = \varepsilon \int_0^\nu \sqrt{|g_{44}|} dv = \varepsilon \int_0^\nu \Phi dv. \] (6)

Therefore the energy-dependence of the speed of light of the photon due to the presence of an extra-dimension is given by
\[ c = c_0 \left[ 1 + \varepsilon \left( \frac{E}{E_K} \right)^\beta \right], \] (7)
where we denoted \( E_K = c^4 \Delta \nu / G \), with \( \Delta \nu = \nu - \nu^0 \) describing the variation of the size of the fifth dimension between the moments of the emission and detection of a photon.

In the case of isotropic homogeneous cosmological models with large non-compact extra-dimensions [35], there is a non-zero contribution in the four-dimensional space-time (the brane) from the 5-dimensional Weyl tensor from the bulk, expressed by a scalar term \( U \), called dark radiation [34]. The “dark radiation" term is a pure bulk (five dimensional) effect, therefore we cannot determine its expression without solving the complete system of field equations in 5 dimensions. In the case of a Friedmann-Roberston-Walker type cosmological model the expression of the dark radiation is \( U = U_0/a^4 \) [35], with \( U_0 \) an arbitrary constant of integration.

For light propagating from cosmological distances the differential relation between time and redshift is [17],
\[ dt = -\frac{H_0^{-1} dz}{(1 + z) g(z)}, \] (8)
where
\[ g(z) = \sqrt{\Omega_\Lambda + \Omega_M (1 + z)^3 + \Omega_U (1 + z)^4}, \] (9)
and \( H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}, \Omega_M \approx 0.3 \) and \( \Omega_\Lambda \approx 0.7 \) are the Hubble constant, the mass density parameter and the dark energy parameter, respectively [35, 38]. \( \Omega_U \) is the dark radiation parameter.

By taking into account the expressions for \( \Delta c \) we obtain the following general equation describing the time delay of two photons:
\[ \Delta t = H_0^{-1} f^{(\beta)}(E_1, E_2) \int_0^z \frac{(1 + z)^{\beta - 1}}{g(z)} dz, \] (10)
where the functions \( f^{(\beta)}(E_1, E_2), \beta = 1, 2, 3 \), describe the different physical models incorporating extra-dimensional and/or quantum gravitational effects. For \( \beta = 1 \) we have the linear model, with
\[ f^{(1)}(E_1, E_2) = \frac{\Delta E}{E_K}, \] (11)
where we denoted $\Delta E = E_1 - E_2$. The linear model corresponds, from the point of view of the extra-dimensional interpretation, to an Einstein-Yang-Mills type model \[32\]. A linear energy dependence of the difference of the photon velocities has also been considered in \[17\] as a result of the dispersion-relation analysis of the Maxwell equations in the non-trivial background metric perturbed by the recoil of a massive space-time defect during the scattering of a low energy photon or neutrino. A quadratic model of the form

$$f^{(2)}(E_1, E_2) = \left( \frac{\Delta E}{E_K^{(2)}} \right)^2,$$

(12)

can also be considered in quantum gravitational models, in which selection rules, such as rotational invariance, forbids first order terms \[17\]. The function

$$f^{(3)}(E_1, E_2) = \frac{(E_3^1 - E_3^2)}{E_K^{(3)}},$$

(13)

corresponding to $\beta = 3$, describes the effect of a pure five-dimensional gravity on the propagation of light in four-dimensions \[32\]. In the above equations we denoted by $E_K^{(\beta)}$, $\beta = 1, 2, 3$ the energy scales associated with the different types of extra-dimensional and/or quantum gravity mechanisms.

In the case of non-compactified extra-dimensions, since the fifth dimension is large, the scale factor $\Phi$ can also be a function of $v$, and hence an explicit knowledge of the $v$-dependence of $\Phi$ is needed in order to derive the speed of light- photon energy dependence. However, taking into account that there is a linear dependence of $\alpha$ on the scale of the fifth dimension, we can assume again that the time delay between two different energy photons emitted during a gamma ray burst is given by the linear model \[32\].

IV. DISCUSSIONS AND FINAL REMARKS

By using Eq. (10) we can evaluate the extra-dimensional and/or quantum gravity energy scale, which follows from the time delay of the 18 GeV photon in GRB 940217. Assuming that the photon originated at $z \sim 2$ and taking $\Delta t \approx 4500 \pm 800$ s, where we have also included the uncertainty in the moment of photon emission during the burst, we obtain for the linear energy scale the value

$$E_K^{(1)} \approx (1.75 - 2.51) \times 10^{15} \text{GeV}.$$  

(14)

This value is very close to the value $E_K^{(1)} \approx 6.9 \times 10^{15} \text{GeV}$ obtained by using a wavelet technique analysis of BATSE and OSSE data \[17\]. For the quadratic model energy scale we have

$$E_K^{(2)} \approx (1.77 - 2.12) \times 10^6 \text{GeV}. $$  

(15)

In the case of quadratic quantum gravity corrections the corresponding energy scale, derived by using BATSE and OSSE data, is $E_K^{(2)} \geq 2.9 \times 10^6 \text{GeV}$ \[17\]. For the pure Einstein gravity five-dimensional model we obtain

$$E_K^{(3)} \approx (8.28 - 9.34) \times 10^5 \text{GeV}. $$  

(16)

From the point of view of multi-dimensional theories, the linear model could describe the effects on the propagation of light in Randall-Sundrum \[35\] or Einstein-Yang-Mills \[26, 32\] type models. The quadratic model is specific for quantum gravity effects, while the $\beta = 3$ case could describe the case of pure Einstein gravity in five dimensions. In all these cases the delay of the 18 GeV photon fixes the corresponding energy scales.

The remarkable concordance between the linear quantum gravity/extra-dimensional energy scale obtained from the present study of the time delay of the 18 GeV photon in GRB 940217 and from the independent study of the BATSE and OSSE data \[17\] strongly suggest that this time delay could be the signature of the extra-dimensions or and quantum gravitational effects.

Since the high energy photon in GRB 940217 is time delayed with respect to the low energy photons, it follows that in the five-dimensional line element $\varepsilon = -1$ and there is no reason to consider super-luminal speeds for gamma-ray photons. This result can also give some insights in the geometrical structure of the fifth-dimensional Universe.

There are also some general arguments which suggest that the signature of the five-dimensional metric corresponds to the choice $\varepsilon = -1$. The five-dimensional field equations define a cosmological constant $\Lambda = -3\varepsilon/L^2$, where $L$ is a universal length, which has been introduced for dimensional consistency \[34\]. Therefore a measurement of the
sign of the cosmological constant may thus also be a determination of the metric signature of the higher-dimensional world. Lower limits on the age of the Universe as well as recent observational data on high-redshift supernovae favor a positive $\Lambda$. An other constraint on $\varepsilon$ can be obtained from the study of the dynamics of a particle in five dimensions. In our model we have explicitly assumed that the fifth dimension $s^4 = v$ is functionally related to the inertial rest mass of the particle $m$ via Eq. 6, generally giving $m = m(s)$, where $s$ is the five-dimensional interval. The latter expression does not imply a violation of the usual condition $g_{\alpha\beta}u^\alpha u^\beta = 1$ for the four-velocities $u^\alpha = dx^\alpha/ds$, because it is a normalization condition on the velocities and not a coordinate condition on the metric. Multiplying by $m^2$ gives $p_\alpha p^\alpha = m^2$, with no restriction on $m = m(s)$. In other words, the energy $E^2 = m^2c^2u^0$ and the three-momentum $p^2 = m^2c^2\sum_{i=1}^3 u_i u^i$ can still satisfy an equation of the form $E^2 - p^2c^2 - M^2(v)c^4 = 0$, with $M^2(v) = m^2(s)\left[1 - p_4 p^4\right] = m^2(s)\left[1 - 4\Phi^2(t, v)\left(\frac{dv}{ds}\right)^2\right]$, even if the mass varies in space-time. In order that these equations describe real particles (with non-negative masses) it is necessary that the five-dimensional metric should have a signature corresponding to $\varepsilon = -1$, thus making the fifth dimension spacelike, in agreement with the conclusion obtained from the study of the time delay of the high energy photon in GRB 940217, and the observed sign of the cosmological constant.

An alternative way of observationally testing multi-dimensional models is the study of the gravitational Cherenkov radiation 39. In some theoretical models the gravity can propagate (in a preferred frame) with a maximum velocity $c_g$, which can differ from the speed of light $c$, $c_g \neq c$. If the speed of gravity is slower than the speed of light, particles moving faster than the speed of gravity would emit a "gravi-Cherenkov radiation", in analogy with the Cherenkov radiation emitted by particles moving faster than light in a medium. Accurate measurements of the speed of propagation and polarization of the gravitational waves can constrain brane world theories, in which gravity propagates in the fifth dimension only, while the matter is confined to four dimensions. The index of refraction $n$ for the gravitational Cherenkov effect can be estimated as $n - 1 \leq \left[\frac{m_{Pl}^2}{(0.1E)^3}t\right]^{1/2}$, where $m_{Pl}$ is the Planck mass and $t$ the travel time of the particle (for example, protons in cosmic rays), with energy $E$ 39. The existence of high energy cosmic rays which have travelled from astronomical distances without losing all their energy to the gravitational Cherenkov radiation places strong bounds on the speed of gravitational waves with very short wave lengths 39. Accurate determinations of the speed of the gravitational waves emitted by distant astrophysical objects would allow the study of the anomalous dispersion relation for gravitational waves, which is a consequence of the violation of the Poincare invariance in the higher dimension space-times, thus giving a better insight into the geometrical structure of the five-dimensional space-time. Therefore the analysis of the results obtained by using different observational techniques, like the study of the time delay of high energy photons from gamma ray bursts and the study of the gravitational waves, can strongly restrict energy scale of multi-dimensional and/or quantum gravity effects and the value of the coefficient $\beta$ in the generalized dispersion relation given by Eq. 7.

Since in the case of GRB 940217 only one high energy photon has been detected, much more tests must be performed in order to prove, via astrophysical observations, the existence of extra-dimensions and of quantum gravity effects. A single photon can also be generated as a result of an unusual fluctuation. Therefore the study of a much larger number of high energy photon delay events is necessary to firmly establish the astrophysical effects of extra-dimensions.

There are several proposed, satellite or ground based GRB research projects which can perform this task. The Gamma-ray Large Area Space Telescope (GLAST), a high energy (30 MeV-300 GeV) gamma-ray astronomy mission, is planned for launch at the end of 2006. GLAST should detect more than 200 GRBs per year, with sensitivity to a few tens of GeV for a few bursts 40. GLAST is expected to observe, within 1 - 2 years of observations, a large number of delayed photons, with energies of the order of $10^2$ GeV. Their detection could provide evidence for the energy and distance-dependent photon velocity dispersion predicted by quantum gravity and multi-dimensional models.

The possibility that the very high energy component of the gamma-ray bursts might be delayed makes the detection with the sensitive ground based atmospheric telescopes more feasible. Although TeV detectors already exist, their field of view is very narrow ($\sim 1^\circ$). The sensitivity that can be achieved is, for the current Whipple 10 m telescope of the order of $8 \times 10^{-8}$ ergs/cm$^2$ for a duration of the burst of 1 and 10 seconds, and of $2.4 \times 10^{-7}$ ergs/cm$^2$ for a 100 s duration burst 40. For the proposed VERITAS array of telescopes the minimum fluence is $10^{-8}$ ergs/cm$^2$ for 1 and 10 second bursts and $3 \times 10^{-8}$ ergs/cm$^2$ for 100 s bursts. These data can be regarded as representative of the present generation of operating telescopes and for those that will come on-line the next few years (CANGOROO III, HESS, MAGIC and VERITAS) 40. The coordination between satellite observatories and ground based gamma ray observatories observatories will further improve the accuracy and precision of the observations of the time delay.

Therefore, the detection of the time delay between Tev and GeV/MeV photons from GRBs could represent a new possibility for the study and understanding of some fundamental aspects of the physical laws governing our universe.
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[1] K. Hurley et al., *Nature* 371, 652 (1994).
[2] K. S. Cheng and T. Lu, *Chin. J. Astron. Astrophys.* 1, 1 (2001).
[3] S. D. Hunter et al., *Astrophys. J.* 481, 205 (1997).
[4] H. A. Mayer-Hasselwander et al., *Astron. Astrophys.* 335, 161 (1998).
[5] X. Y. Wang, K. S. Cheng, Z. G. Dai and T. Lu, *Astrophys. J.* 604, 306 (2004).
[6] L. X. Cheng and K. S. Cheng, *Astrophys. J.* 459, L79 (1996).
[7] Z. G. Dai and T. Lu, *Astrophys. J.* 580, 1013 (2002).
[8] M. H. Salamon and F. W. Stecker, *Astrophys. J.* 493, 547 (1998).
[9] R. Plaga, *Nature* 374, 430 (1995).
[10] E. Waxman and P. Coppi, *Astrophys. J.* 464, L75 (1996).
[11] P. F. Kronberg, *Rep. Progr. Phys.* 57, 325 (1994).
[12] R. Kulsrud, S. C. Cowley, A. V. Gruzinov and R. N. Sudan, *Phys. Repts.* 283, 213 (1997); R. Kulsrud, *Annu. Rev. Astron. Astrophys.* 37, 37 (1999).
[13] P. Meszaros and M. Rees, *Mon. Not. R. Acad. Sci.* 268, L41 (1994).
[14] J. Katz, *Astrophys. J.* 432, L27 (1994).
[15] L. X. Cheng, Y. Q. Ma, K. S. Cheng, T. Lu and Y. Y. Zhou, *Astron. Astrophys.* 300, 746 (1995).
[16] B. Zhang and P. Meszaros, *Astrophys. J.* 581, 1236 (2002).
[17] J. Ellis, N. E. Mavromatos, D. V. Nanopoulos and A. S. Sakharov, *Astron. Astrophys.* 402, 409 (2003).
[18] T. Kaluza, *Sitzungsberichte Preussische Akademie der Wissenschaften* 96, 69 (1921).
[19] O. Klein, *Zeitschrift fur Physik* 37, 895 (1926).
[20] E. Witten, *Nucl. Phys.* B 460, 335 (1996).
[21] P. Horava and E. Witten, *Nucl. Phys.* B 475, 94 (1996).
[22] P. Forgacs and Z. Horvath, *Gen. Rel. Grav.* 10, 931 (1979); P. Forgacs and Z. Horvath, *Gen. Rel. Grav.* 11, 205 (1979).
[23] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, *Phys. Rev.* D 61, 027503 (2000).
[24] J. P. Uzan, *Rev. Mod. Phys.* 75, 403 (2003).
[25] J. Magueijo, *Reports on Progress in Physics* 66, 2025 (2003).
[26] P. Loren-Aguilar, E. Garcia-Berro, J. Isern and Yu. A. Kubyshin, *Class. Quant. Grav.* 20, 3885 (2003).
[27] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, *Phys. Rev.* D 61, 027503 (2000).
[28] R. Gambini and J. Pullin, *Phys. Rev.* D 59, 124021 (1999).
[29] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, *Gen. Rel. Grav.* 32, 127 (2000); J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, *Phys. Rev.* D 62, 084019 (2000).
[30] G. Amelino-Camelia, J. Ellis, N. E. Mavromatos, D. V. Nanopoulos and S. Sarkar, *Nature* 393, 763 (1998).
[31] J. Ellis, K. Farakos, N. E. Mavromatos, V. Mitsou and D.V. Nanopoulos, *Astrophys. J.* 535, 139 (2000).
[32] T. Harko and K. S. Cheng, *Astrophys. J.*, to be published [astro-ph/0405074] (2004).
[33] G. W. Ma, *Phys. Lett.* A 146, 375 (1990); G. W. Ma, *Phys. Lett.* A 143, 183 (1990).
[34] M. K. Mak and T. Harko, *Class. Quantum Grav.* 16, 4085 (1999).
[35] L. Randall and R. Sundrum, *Phys. Rev. Lett.* 83, 3370 (1999); L. Randall and R. Sundrum, *Phys. Rev. Lett.* 83, 4690 (1999).
[36] T. Shiromizu, K. Maeda and M. Sasaki, *Phys.Rev.* D 62, 024012 (2000); C. M. Chen, T. Harko and M. K. Mak, *Phys. Rev.* D 64, 044013 (2001); C. M. Chen, T. Harko, W. F. Kao and M. K. Mak, *Nucl. Phys.* B 636, 159 (2002); T. Harko and M. K. Mak, *Class. Quantum Grav.* 20, 407 (2003); C. M. Chen, T. Harko, W. F. Kao and M. K. Mak, *JCAP* 0311, 005 (2003).
[37] W. L. Freedman et al., *Astrophys. J.* 553, 47 (2001).
[38] P. J. E. Peebles and Bharat Ratra, *Rev. Mod. Phys.* 75, 559 (2003).
[39] G. D. Moore and A. E. Nelson, *Journal of High Energy Physics*, 0109, 023 (2001).
[40] T. C. Weekes, *The next generation of ground-based TeV gamma-ray instruments and the identification of high-energy gamma-ray sources*, to appear in *Cosmic γ-ray sources*, Kluwer (2004).