Shadow features and shadow bands in the paramagnetic state of cuprate superconductors

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The conditions for the precursors of antiferromagnetic bands in cuprate superconductors are studied using weak-to-intermediate coupling approach. It is shown that there are, in fact, three different precursor effects due to the proximity to antiferromagnetic instability: i) the shadow band which associated with new pole in the Green’s function ii) the dispersive shadow feature due to the thermal enhancement of the scattering rate and iii) the non-dispersive shadow feature due to quantum spin fluctuation that exist only in \( \hat{k} \)-scan of the spectral function \( A(\omega_{\text{Fixed}}, \hat{k}) \). I found that dispersive shadow peaks in \( A(\omega, \hat{k}) \) can exist at finite temperature \( T \) in the renormalized classical regime, when \( T \gg \omega_{sf} \), \( \xi_{AFM} > \xi_{th} = v_F/T \) (\( \omega_{sf} \) is the characteristic energy of spin fluctuations, \( \xi_{th} \) is the thermal wave length of electron). In contrast at zero temperature, only non-dispersive shadow feature in \( A(\omega_{\text{Fixed}}, \hat{k}) \) has been found. I found, however, that the latter effect is always very small. The theory predict no shadow effects in the optimally doped materials. The conditions for which shadow peaks can be observed in photoemission are discussed.

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I. INTRODUCTION

Recent advances in the Angular Resolved Photoemission technique (ARPES) have allowed to determine electronic excitations of some High-\( T_c \) materials. Two different approaches to photoemission are currently used. In type I ARPES one measures \( A(\omega, \hat{k}_{\text{Fixed}}) \) the probability that an excitation of fixed momentum \( \hat{k}_{\text{Fixed}} \) has an energy \( \omega \). In the type II ARPES one measures the probability \( A(\omega_{\text{Fixed}}, \hat{k}) \) that an excitation of fixed energy \( \omega_{\text{Fixed}} \) has a momentum \( \hat{k} \). Using the latter technique on the high-temperature superconductor B2212, Aebi et al. have found, in addition to the main Fermi surface (F. S.), another F. S. that is identical to the first one but of much smaller intensity and displaced from the main F. S. by the antiferromagnetic (AFM) wave vector \( \vec{Q} = (\pi/a, \pi/a) \). This structure has been taken as evidence for the existence of the “shadow bands” that had been predicted much earlier by Kampf and Schrieffer. These authors had argued that precursors of the AFM bands should appear in the paramagnetic state close to the AFM instability. More recently, observation of dispersing shadow bands has been also reported in type I photoemission experiments. However, the AFM interpretation of the phenomenon observed in B2212 remains controversial because: i) the AFM correlation length \( \xi \) in optimally doped B2212 is rather short, ii) the effect has not been found yet in any other high-temperature superconductor or in the underdoped B2212 iii) it becomes more pronounced when one increase orthorombicity by lead doping. All these leave open the possibility of structural origin of the phenomenon observed in Refs. It is thus very important to understand under which general conditions shadow bands can occur in practice. Some aspects of this issue have been considered in a number of recent theoretical publications. In particular, the strong coupling numerical calculations for the t-J model show peaks outside non-interacting F. S. in the underdoped case for \( T = 0 \). However, these results have been obtained so far only for very small clusters and the results close to optimal doping are unclear.

In this paper, I study under which general conditions shadow bands can occur, using a well known method that can be justified up to intermediate coupling. I show that there are, in fact, a few different precursors effects due to the proximity to AFM instability depending on whether the effect is due to classical \( \omega_{sf} \ll T \) or quantum \( T \ll \omega_{sf} \) spin fluctuations (here \( \omega_{sf} \) is characteristic energy of spin fluctuations). I also show that shadow effects in type I and type II photoemission experiments may occur under different conditions. For type I ARPES experiments, I find that while classical spin fluctuations can produce dispersive shadow effects in \( A(\omega, \hat{k}) \), quantum spin fluctuations cannot. The key difference between both regimes is that classical thermal fluctuations lead to precursors of the magnetic Bragg peaks in the static spin structure factor, while the dynamical nature of quantum spin fluctuations does not allow sharp structures in \( S_{sp}(\vec{q}) \), no matter how large \( \xi/a \) is (a lattice constant). Consequently, shadow effects will appear in type I experiments at low temperature only if the AFM correlation length \( \xi \) has the appropriate temperature dependence, not just when \( \xi/a \) is large enough. This conclusion is qualitatively different from the results of numerical studies [1], in which it was tacitly assumed that the effect is determined by the parameter \( \xi/a \) alone. It is only in type II ARPES experiments that quantum spin fluctuations can lead to a non-dispersive shadow feature.
in \(A(\omega_{\text{fixed}}, \vec{k})\). I find, however, that the latter effect
is very small and exists only in a narrow range near zero
energy. More experimental implications of my results are
discussed in the conclusion.

II. MODEL AND CALCULATIONAL
PROCEDURE

I evaluate the effect of spin fluctuations on the elec-
tronic self-energy in the one-loop approximation. The
expression for the self-energy has the form:

\[
\Sigma(\omega, \vec{k}) = g\bar{g} \int \frac{d^2p}{(2\pi)^2} \frac{1}{\coth \left( \frac{\omega - \varepsilon(k + \vec{q})}{2T} \right) + i\pi} \]

where \(\chi_{sp}(\omega, \vec{q})\) is the spin susceptibility, \(g\bar{g}\)
is the effective coupling constant between electrons and spin
fluctuations and \(\varepsilon(\vec{k})\) is the energy dispersion relative to the
chemical potential. The constant vertex corrections are
included in \(\bar{g}\). It was shown within the Hubbard model
that by including vertex corrections in this way the above
self-energy expression \((1)\) remains a good approximation
up to intermediate coupling.

Close to AFM instability the low-energy asymptotic
form of the spin susceptibility \(\chi_{sp}\) can be written in the
Orstein-Zernike form:

\[
\chi_{sp}(\omega, \vec{q}) = \frac{\chi_{sp}(0, \vec{Q})}{1 + (\vec{q} - \vec{Q})^2 \xi^2 - i\omega/\omega_s f}
\]

where \(\xi\) is the AFM correlation length and \(\omega_s f\) is the charac-
teristic energy of the spin fluctuations. Any RPA-like
theory, whether it uses bare or renormalized vertices
or propagators, have the Orstein-Zernike form close to
\(\vec{q} = \vec{Q}\). In the context of High-\(T_c\) materials, the Orstein-
Zernike susceptibility with particular set of parameters
is often called MMP susceptibility. It has been used to fit
a number of experiments, that are sensitive to the strong
enhancement of \(\chi\) around \(\vec{q} = \vec{Q}\). Here, I will adopt
a more general approach and consider for which general
conditions on the parameters \(\xi\) and \(\omega_s f\) the precursors of
AFM bands can exist. Since shadow effects come from
the peak in \(\chi\) around \(\vec{q} = \vec{Q}\), the Orstein-Zernike form
of \(\chi\) is sufficient for my purposes and its simplicity will
allow me to obtain a number of results analytically. The
imaginary part of the susceptibility \(\chi''_{sp}(\omega, \vec{q})\) has to be
cut off at some high frequency \(\Gamma_{\text{cut}}\) in order to satisfy the
local moment sum rule. In electronic models this cutoff
is of the order of a few hopping integrals \(t\). For the low
energy physics the precise value of \(\Gamma_{\text{cut}}\) is not important.
I use \(\Gamma_{\text{cut}} = 5.5t\).

From a microscopic point of view (see, for example,[2]),
the parameters in the susceptibility Eq.\((2)\) scale with the
correlation length as: \(\chi_{sp}(\vec{Q}, 0) \propto \xi^2, \omega_s f \propto 1/\xi^2\).

Consequently the model is defined by only three pa-
rameters \(\xi, \omega_s f\) and \(g' = g\bar{g}\chi_{sp}(\vec{Q})/\xi^2\). Qualitatively,
our results mainly depend on \(\xi\) and on \(T/\omega_s f\) while the
parameter \(g'\) (in the Hubbard model \(g' \propto U\) determines
the quantitative scale of the effects. To model the
band-structure of the hole-doped high-\(T_c\) materi-
als I use the simplest tight-binding model that cor-
rectly reproduces the experimentally observed Fermi sur-
faces in YBCO and B2212 at optimal doping, namely\(\epsilon(\vec{k}) = -2t(\cos(k_x) + \cos(k_y)) - 4t' \cos(k_x) \cos(k_y)\) with
\(t' = -0.45t\) (I set \(a = 1\)). The band structure estimate for
\(t\) is about \(t \approx 250meV\). Using this approach, I first
present numerical results and then analytically find the
conditions for the existence of the various effects.

Both the self-energy and the spectral function
\(A(\omega, \vec{k}) \equiv -2\Sigma''(\omega, \vec{k})/(\omega - \epsilon(\vec{k}) + \mu - \Sigma'(\omega, \vec{k}))^2 + |\Sigma''(\omega, \vec{k})|^2\) are discussed. The chemical potential of the
interacting electrons is found from the usual condition
\[\int f(\omega)A(\omega, \vec{k})d\omega d^2k/(2\pi)^3 = n/2\]
I choose \(n = 0.83\) that corresponds to optimal doping in B2212.[2]

III. RESULTS

I start with numerical results and figures that illus-
trate various points I wish to make. Analytical results
for various regimes identified in the numerical examples
appear in a subsequent section.

A. Illustrative examples

The important aspect of critical AFM fluctuations is that
they lead to strongly \(\vec{k}\)-dependent self-energy. The
maximum of the scattering rate \(-2\Sigma''(0, \vec{k})\) at finite
temperature occurs on the so-called shadow Fermi surface
(Sh. F. S.). The latter is defined here as the set of points
obtained by the translation of the real F. S. by the wave
vector \(\vec{Q} = (\pi, \pi)\). The inset in Fig.\((a)\) shows the real
F.S. and the Sh. F. S. for the optimally doped materi-
als. The points at which the real F. S. and the Sh. F.
S. intersect are the points at which the spectral function
\(A(\omega, \vec{k}_F)\) is affected the most by spin fluctuations. I
start the analysis with these crossing points, \(\vec{k} = \vec{k}_c\), also
known as hot spots.

1. Shadow bands at the hot spots

The filled line on Fig.\[(a)\] shows the spectral function
\(A(\omega, \vec{k}_c)\) in the regime dominated by classical thermal
fluctuations (\(\omega_s f \ll T, T = 0.03t\) and \(\xi = 20a\)). We
see that instead of the usual quasi-particle peak at \(\omega = 0\)
there are two precursors of the antiferromagnetic bands
with a pseudogap between them. These bands are associ-
ated with two new quasiparticle solutions of the equation
ω − ε(⃗k) + μ − Σ′(ω, ⃗k) = 0. It is because these solutions exist that the term bands is used. I also find that in this case the scattering rate Σ″(ω, ⃗k) has maximum at the Fermi level ω = 0.

I will show now by contradiction, that this effect exists only if ξ(T) increases sufficiently rapidly with decreasing temperature. Indeed, suppose the correlation length does not increase when temperature decreases below the temperature T = 0.03 used for the filled line on Fig. 1. Taking a much smaller temperature T/t = 0.005 while keeping all other parameters fixed, the dashed line on Fig. 1 shows that the pseudogap in A(ω, ⃗k) disappears while the quasiparticle peak reappears. I find similar results when I keep the temperature and ξ constant but increase ωsf up to ωsf ≈ T. This means that the pseudogap in A(ω, ⃗k) can exist only if the correlation length ξ(T) increases sufficiently rapidly at low temperature. I show below that ξ(T) ∝ exp(const/T) is a sufficient condition. The latter condition is realized in the renormalized classical regime (R. C.) ωsf ∝ ξ−2 ≪ T that always precedes the T = 0 phase transition in two dimensions.

2. Shadow features in the diagonal direction (0,0) − (π, π)

In this direction, real and shadow F.S. are well separated and |ε(⃗k) − ε(⃗k + ⃗Q)| is large. The filled curve in Fig. 2 shows the spectral function at T = 0.03t for a wave vector ⃗k = (0.66, 0.66)π that is just a little bit inside the Sh. F. S. (⃗kSh.F. ≃ (0.63, 0.63)π). The quasiparticle peak in this case is at high positive energies and cannot be seen in the photoemission experiments since they measure f(ω)A(ω, ⃗k). There is no new quasiparticle solution to ω − ε(⃗k) + μ − Σ′(ω, ⃗k) = 0 in this case. Nevertheless, there is a local maximum at negative binding energies (shown by arrow) that coincides with the maximum in the imaginary part, Σ″(ω, ⃗k) in Fig. 2b. I call this a shadow feature. This maximum occurs because for large values of |ω − ε(⃗k)|, the quantity ω − ε(⃗k) + μ − Σ′(ω, ⃗k) is large and slowly varying in the region where Σ″(ω, ⃗k) acquires a maximum. Hence, this maximum is directly reflected in the spectral function that becomes proportional to Σ″(ω, ⃗k) rather than inversely proportional to it as in the case of quasiparticles. This maximum occurs at ω ≃ ε(⃗k + ⃗Q), a dispersion that looks like the AFM band but with the gap equal to zero. Nevertheless, this is definitely not a real AFM band since a) it is thermally excited (see below) b) it exists only in certain regions of ⃗k-space where |ε(⃗k) − ε(⃗k + ⃗Q)| ≫ Σ″(ω, ⃗k). Note that similar shadow features have been found in the Hubbard model with t = 0, using the FLEX approximation.

To show that this shadow feature is a thermally excited state I plot the dashed line in Fig. 3 for the case where the temperature is decreased to T = 0.003t with all other parameters the same as for T = 0.03t. Clearly the shadow feature (shown by arrow) disappears at lower temperature. It also disappears for ωsf > T. I note that the maximum in A(ω, ⃗k) around ωmax ≃ t is different from the “shadow band” observed in Ref. 8. It stays at ωmax ≃ −t < 0 even when ⃗k crosses the Sh. F. S.

3. Non-dispersive T → 0 shadow feature in type II configuration:

The above considerations lead to the conclusion that only classical thermal spin fluctuations that destroy long range order at T ≠ 0 in two dimensions can produce the above dispersive shadow peaks in both type I and type II experiments. Quantum spin fluctuations T ≪ ωsf alone cannot produce shadow effect in A(ω, ⃗k) measured in type I ARPES, no matter how large is ξ/a. However, in the type II configuration, (⃗k-dependence of A(ω, ⃗k)), quantum spin fluctuations can lead to a non-dispersive shadow feature at T = 0 when ξ/a ≫ 1 (Ref. 12). The effect is due to the enhancement of Σ″(ω, ⃗k) on the Sh.F.S. at finite ω. For small ω the maximum in Σ″(ω, ⃗k) translates into a maximum in A(ω, ⃗k) for ⃗k = ⃗kSh.F.S., since A(ω, ⃗k) ∝ Σ″(ω, ⃗k) for ⃗k far from the F.S.. Since A(0, ⃗k) ≫ ⃗kSh.F.S. = 0 at T = 0, one needs to increase ω to have a noticeable weight for this maximum in the incoherent background. However, when ω increases the width of the maximum in k-space rapidly increases and above a certain energy ω0 < T mainly determined by (ξ/a)2ωsf, the maximum disappears. This occurs because the effect from Σ″ is offset by the contribution from the denominator in A(ω, ⃗k) ≈ Σ″(ω, ⃗k)/(ω − ε(⃗k))2. For |ω| > ω0, the closer ⃗k is to the real F. S. the larger A(ω, ⃗k) is. This is illustrated in the inset of Fig. 3b. I found that the effect is always very small even for large values of ξ and ωsf, the maximum disappars. This because the effect from Σ″ is offset by the contribution from the denominator in A(ω, ⃗k) ≈ Σ″(ω, ⃗k)/(ω − ε(⃗k))2. For |ω| > ω0, the closer ⃗k is to the real F. S. the larger A(ω, ⃗k) is. This is illustrated in the inset of Fig. 3b. I found that the effect is always very small even for large values of ξ and ωsf.

B. Analytical results

1. Difference between classical and quantum regimes

I now explain analytically why the effect of classical and quantum AFM fluctuation on A(ω, ⃗k) are qualitatively different. When ωsf ≪ T the effect of spin fluctuations on single-particle properties can be considered quasistatic. Indeed, neglecting ω′ in the energy denominator of the self-energy formula Eq. (1) one obtains:

$$\Sigma(\omega, \vec{k}) = g\tilde{g} \int \frac{d^2q}{(2\pi)^2} \frac{S_{sp}(\vec{q})}{\omega - \varepsilon(\vec{k} + \vec{q}) + i0}$$  (3)

where S_{sp}(q) is the static structure factor S_{sp}(q) = ∫ χ_{sp}(ω, q) coth(ω/2T) dω/2π. The key point is that in
the RC regime, $S_{sp}(\bar{q})$ has the same singular behavior as the static susceptibility $\chi''_{sp}(0, \bar{q})$ because for $q \approx Q$, $S_{sp}(\bar{q}) \approx T \int (d\omega/\pi) \chi''(\omega, \bar{q})/\omega = T \chi''_{sp}(0, \bar{q})$. Since close to the AFM instability $\chi''_{sp}(0, \bar{q})$ is a Lorentzian peaked at $q = Q$, the static structure factor $S_{sp}(\bar{q})$ has a Lorentzian maximum in the regime $\omega_{sf} \ll T$:

$$S_{sp}(\bar{q}) \propto \frac{T}{\xi^{-2} + (\bar{q} - \bar{q})^2}$$

(4)

The presence of a sharp precursor of the Bragg peak in the RC regime leads to the shadow peak in $A(\omega, \vec{k}_{F,\text{fixed}})$. The situation is qualitatively different in the case $T \ll \omega_{sf}$ and $\xi/\alpha \gg 1$. In that case, $S_{sp}(\bar{q}) \approx \int_0^\infty \chi''(\omega, \bar{q})d\omega/\pi$ and using Eq. (3) I obtain:

$$S_{sp}(\bar{q})|_{T=0} \propto \ln[\xi^{-2} + (\bar{q} - \bar{q})^2]$$

(5)

The logarithmic singularity of $S_{sp}(\bar{q})$ in the case $T \ll \omega_{sf}$ is integrable and does not lead to any divergence in $\Sigma(\omega, \vec{k})$ when $\xi \rightarrow \infty$. Consequently, there is no shadow effects in $A(\omega, \vec{k}_{F,\text{fixed}})$, when electrons approach the AFM instability $\xi \gg a$ at zero temperature $T = 0$. This conclusion is different from the result of Kampf and Schrieffer who used the zero temperature formalism and found shadow bands for $\xi \approx 20a$. The difference is due to the fact that the phenomenological susceptibility used in Ref. 1 was separable in $\bar{q}$ and $\omega$, $\chi_{sp,KdH} = f(\bar{q})g(\omega)$. In that case, the $\bar{q}$-dependence of the static structure factor $S_{sp}(\bar{q})$ is always the same as that of the susceptibility $\chi''_{sp}(0, \bar{q})$. Consequently, the static structure factor $S_{sp,KdH}(\bar{q})$ has a Lorentzian-like peak even when $T \ll \omega_{sf}$. The separability in $\omega$ and $\bar{q}$ of the susceptibility is an oversimplification that can be partially justified only in the regime $\omega_{sf} \ll T$. Thus, the results of Ref. 1 should not be applied to the zero-temperature paramagnetic state.

2. Conditions on $\xi(T)$

The asymptotic form of $\Sigma(\omega, \vec{k})$ in the RC regime has been obtained in Ref. 1 for the case $n = 1, \nu' = 0$. The generalization to the present case is straightforward and for $\Sigma''$ I get, for $\xi > \xi_{th} = v_{F+Q}/T$:

$$\Sigma''(\omega, \vec{k}) = -\frac{g'T}{4\sqrt{[\omega - \bar{\epsilon}(\vec{k} + \vec{Q})]^2 + v_{k+Q}^2}}$$

(6)

We see that $\Sigma''(\omega, \vec{k})$ has a maximum when $\omega = \bar{\epsilon}(\vec{k} + \vec{Q})$ with $\Sigma''(\bar{\epsilon}(\vec{k} + \vec{Q}), \vec{k}) = -g'T \xi/4v_{k+Q}$. The effect is largest at the Van Hove point $v_k = 0$, in which case $\Sigma''(\bar{\epsilon}(\vec{k} + \vec{Q}), \vec{k}) \propto \xi^2$. Since away from $k = k_{F,S}$, we have $A(\omega, \vec{k}) \propto \Sigma''(\omega, \vec{k})$, we obtain for $\xi > \xi_{th}$, the shadow feature discussed above. On the other hand, at the crossing points $\bar{\epsilon}(\vec{k}_c + \vec{Q}) = \bar{\epsilon}(\vec{k}_c) = 0$, $A(0, \vec{k}) \propto 1/\Sigma''(0, \vec{k})$ and the enhancement in $\Sigma''$ leads to the AFM pseudogap with no quasiparticle peak inside. This is similar to what was found in the case of perfect nesting ($n = 1, \nu' = 0$) except that in the latter case the pseudogap opens up over all the F.S., since the real F.S. and the Sh.F.S coincide (all points on the F.S. are crossing points).

The real part of the self-energy can be obtained from Eq. (3) using Kramers-Kronig relation and has the form:

$$\Sigma'(\omega, \vec{k}) = \frac{g'T}{4\pi} \int \frac{d\omega'}{\omega'} \frac{\omega - \bar{\epsilon}(\vec{k} + \vec{Q}) + v_{k+Q} \xi^{-2}}{\sqrt{[\omega - \bar{\epsilon}(\vec{k} + \vec{Q})]^2 + v_{k+Q}^2 \xi^{-2}}}$$

(7)

To see how the shadow band solution appears, let us look at $\Sigma'(\omega, \vec{k})$ at frequencies $|\omega - \bar{\epsilon}(\vec{k} + \vec{Q})| \gg v_{F+Q}/\xi$. In this case Eq. (7) reduces to:

$$\Sigma'(\omega, \vec{k}) \approx \frac{g'T}{2\pi \omega - \bar{\epsilon}(\vec{k} + Q)} T \ln \frac{\xi}{\omega + \bar{\epsilon}(\vec{k} + \vec{Q})}$$

(8)

When $\omega \rightarrow const$ as $T → 0$, the large value of $|\Sigma'|$ for $|\omega - \bar{\epsilon}(\vec{k} + \vec{Q})| \sim v_{F+Q}/\xi$ leads to the appearance of two solutions for the quasiparticle equation. The effect exists in $D = 2$ in the R. C. regime $\xi \propto \exp(const/T)$. Since in the weak-to-intermediate coupling regime $T_X$ is of the order of the gap $\Delta$ in the ordered state, the condition $\xi \gg \xi_{th}$ can be also written as $\xi \gg \xi_{coh}$, where $\xi_{coh} = \Delta/\nu_F$ is the coherence length in the ordered state. In three dimensions precursors of AFM are very weak because the integration of Eq. (3) gives in 3D $\Sigma''(\omega, \vec{k}) \propto g'T \ln [\omega - \bar{\epsilon}(\vec{k} + \vec{Q})^2 + v_{F+Q}^2/\xi^{-2}]$, which is much less singular than in two-dimensional case Eq. (4). In the dimensions $D > 3$ the precursors of AFM bands do not appear even at the very point of the phase transition, because of the large phase space in Eqs. (1), (3). This result could be expected on general physical grounds, since in three and larger dimensions the gap is equal to zero at the Néel temperature. In contrast, in two-dimensions classical thermal fluctuations suppress long-range order at finite temperature and at the $T = 0$ phase transition the system goes directly into the ordered state with a finite gap. Finally, I note that the one-dimensional case is, as usual, very special. For the most recent studies of shadow bands in the 1D case see [3, 4, 5].

IV. CONCLUSIONS

I have shown that in 2D there are a few different antiferromagnetic precursor effects in the spectral function $A(\omega, \vec{k})$, which are realized under different physical conditions. The conditions for dispersive shadow features and shadow bands are given by $\omega_{sf} \ll T$ and $\xi \gg \xi_{th} = \nu_F/T$. The first conditions insures that the
short-range AFM order is quasi-static, which means that not only the susceptibility \( \chi''(q, \tilde{q}) \) has a Lorentzian form at \( q \approx Q \), but also the static structure factor \( S_p(q) \) has this form. The second condition \( \xi \gg \xi_{th} \equiv v_F / T \) implies that the AFM correlation length should be compared with the thermal wave length \( \xi_{th} = v_F / T \) of electrons, rather than with the lattice constant \( a \), as was assumed in some previous works on shadow bands. The above conditions for dispersive shadow bands can be realized in two dimensions when at \( T = 0 \) there is long-range AFM order. The situation is favorable in two dimension because this is the lower critical dimension for the AFM instability and the mean field phase transition is replaced by the crossover \( T_X \) to the regime with very rapidly growing correlation length \( \xi \propto \exp(\text{const}/T) \). These bands should develop first at the crossing points \( k = k_{cr} \) and only much later along the diagonal direction \((0,0) \rightarrow (\pi, \pi)\).

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FIG. 1. The $\omega$-dependence of $A(\omega, \vec{k})$ for $\vec{k} = k_{cr} = (0.825, 0.175)\pi$. The inset show the F. S. (filled line) and the shadow F. S. (dashed line).

FIG. 2. The $\omega$-dependence of $f(\omega)A(\omega, \vec{k})$ (upper panel) and $\Sigma''(\omega, \vec{k})$ (lower panel) for $\vec{k} = (0.66, 0.66)\pi$, that is slightly inside Sh. F. S. The inset shows the $\vec{k}$-dependence of $A(\omega, \vec{k})$ in the vicinity of $k_{Sh,F.S} = (0.63, 0.63)\pi$ for $\omega = -0.03t (\nabla)$ and $\omega = -0.125t (\square)$. 
\( \omega / t \)

\( \xi = 20 \)

\( g' = 10t \)

\( \omega_{si} = 10^{-3}t \)
\[ f(\omega)A \]

\[ -\Sigma'' \]

\[ \omega_s = 10^{-3}t \]

\[ \xi = 10 \]

\[ g' = 10t \]