Impact of eight-quark interactions in chiral phase transitions I: Secondary magnetic catalysis

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Abstract

The influence of a constant magnetic field on the order parameter of the four-dimensional Nambu and Jona-Lasinio model extended by the ’t Hooft six-quark term and eight-quark interactions is considered. It is shown that the multi-quark interactions cause the order parameter to increase sharply (secondary magnetic catalysis) with increasing strength of the field at the characteristic scale $H \sim 10^{14} A^2 \text{G/MeV}^2$.

It has been shown in a series of papers [1]-[3] that in 2+1 and 3+1 dimensions a constant magnetic field $H \neq 0$ catalyzes dynamical symmetry breaking leading to a fermion mass even at the weakest attractive four-fermion interaction between massless particles, and the symmetry is not restored at any arbitrarily large $H$.

It is known, however, that the QCD motivated effective lagrangian for the light quarks ($N_f = 3$) contains also the six-fermion term: the $U(1)_A$ breaking ’t Hooft interaction, and probably eight-quark terms. These extensions of the Nambu and Jona-Lasinio model are well-known, for instance, the four-quark $U(3)_L \times U(3)_R$ chiral symmetric lagrangian together with the ’t Hooft six-quark interactions has been extensively studied at the mean-field level [4]-[7]. Recently it has been also shown [8, 9] that the eight-quark interactions are of vital importance to stabilize the multi-quark vacuum.

The additional multi-quark forces can affect the result which is obtained when only four-fermion interactions are considered. We argue here that the ’t Hooft and eight-quark interactions can modify the theory in such a way that the local minimum, catalyzed by the constant magnetic field, is smoothed out by increasing the strength of the field. This is an alternative regime to the known one in which the strong magnetic field cannot change the ground state of the system. For the first scenario to become possible it is sufficient that the couplings of multi-quark interactions are chosen such that the system displays more than one solution of the gap equation at $H = 0$. However, the above condition is not a requirement. Even if the gap equation has only one nontrivial solution at small $H$, an increase in the magnetic field can induce the formation of a second minimum. Starting from some critical value $H_c$ the second minimum is becoming a new ground state. We call this phenomenon a secondary magnetic catalysis. To see the details we need the effective potential of the theory, $V(m, |QH|) = V_{st} + V_S$, which is the sum of two terms. The first contribution results from the many-fermion vertices, after reducing them to a bilinear form with help of bosonic auxiliary fields, and subsequent integration over these fields, using the stationary phase method. This part does not depend on the magnetic field. The specific details of these calculations are given in our recent work [8]. In the $SU(3)_f$ symmetric case the result is

$$V_{st} = \frac{1}{16} \left( 12Gh^2 + \kappa h^3 + \frac{27}{2} \lambda h^4 \right). \quad (1)$$
The function $h$ is a solution of the stationary phase equation $12\lambda h^3 + \kappa h^2 + 16(Gh + m) = 0$, where $G, \kappa, \lambda$ are couplings of four, six and eight-quark interactions correspondingly. This cubic equation has one real root, if $G/\lambda > (\kappa/24\lambda)^2$. Assuming that the couplings fulfill the inequality, one finds the single valued function $h(m, G, \kappa, \lambda)$.

The second term, $V_S$, derives from the integration over the quark bilinears in the functional integral of the theory in presence of a constant magnetic field $H$. As has been calculated by Schwinger a long time ago \cite{10} $V_S = \sum_{i=u,d,s} V_S(m_i, |Q_iH|)$, where

$$V_S(m, |QH|) = \frac{N_c}{8\pi^2} \int_0^\infty ds e^{-sm^2} \frac{\rho(s, \Lambda^2)|QH|}{\coth(s|QH|)} + \text{const.} \quad (2)$$

Here the cutoff $\Lambda$ has been introduced by subtracting off suitable counterterms to regularize the integral at the lower limit: $\rho(s, \Lambda^2) = 1 - (1 + s\Lambda^2)e^{-s\Lambda^2}$. The unessential constant is chosen to have $V_S(0, |QH|) = 0$. We ignore in the remaining the charge difference of $u$ and $d, s$ quarks: the averaged common charge $|Q| = |e/9|$ will be used.

One sees that the gap equation, $dV(m)/dm = 0$, has always a trivial solution $m = 0$, which corresponds to the point where the potential reaches its local maximum, if $H \neq 0$. This phenomenon is known as magnetic catalysis of dynamical chiral symmetry breaking. The nontrivial solution is contained in the equation

$$- \frac{2\pi^2h(m)}{\Lambda^2N_cm} = \psi\left(\frac{\Lambda^2 + m^2}{2|QH|}\right) - \frac{|QH|}{\Lambda^2} \left[ \ln\left(1 + \frac{m^2}{\Lambda^2}\right) - \frac{\Lambda^2}{\Lambda^2 + m^2} + 2\frac{\ln\left(\frac{\Lambda^2 + m^2}{2|QH|}\right)}{\Gamma\left(\frac{m^2}{2|QH|}\right)} \right], \quad (3)$$

where $\psi(x) = d\ln\Gamma(x)/dx$ is the Euler dilogarithmic function. Here the l.h.s. originates from $V_{st}$ and the r.h.s. from $V_S$.

Let us consider first the standard case with $\kappa, \lambda = 0$ and $h = -m/G$. Then the l.h.s. is a constant $\tau^{-1} = 2\pi^2/G\Lambda^2N_c$. Fig. 1 (left panel) illustrates this pattern. One sees that at $H = 0$ the system is in the subcritical regime of dynamical symmetry breaking. The introduction of a constant magnetic field, however small it might be, changes radically the dynamical symmetry breaking pattern: due to the singular behaviour of the r.h.s. of Eq. (3) close to the origin the curves corresponding to the r.h.s. and l.h.s. will always intersect and the value of $m$ where this happens is a minimum of $V(m)$. One concludes that in the theory with just four-fermion interactions the effective potential has only one minimum at $m > 0$, and this property does not depend on the strength of the field $H$.

In the theory with four-, six-, and eight-quark interactions one can find either one or two local minima at $m > 0$. We illustrate these two cases in the central panel of fig. 1. Namely, the upper full curve $f$ (r.h.s. of Eq. (3) for $|QH|\Lambda^{-2} = 0.5$) has only one intersection point with the bell-shaped curve $u$ (l.h.s. of Eq. (3) for $G\Lambda^2 = 3, \kappa\Lambda^5 = -10^3, \lambda\Lambda^8 = 3670$). This point corresponds to a single vacuum state of the theory. The other full curve $f$ for $|QH|\Lambda^{-2} = 0.1$ has three intersections with the same curve $u$. These intersections, successively, correspond to a local minimum, a local maximum and a further local minimum of the potential. The first minimum catalyzed by a constant magnetic field (that is, a slowly varying field) is then smoothed out with increasing $H$. It ceases to exist at some critical value of $|QH|\Lambda^{-2}$, from which on only the large $M_{\text{dyn}}$ solution survives. This is shown in the right panel of fig. 1. The new phenomenon might be a clear signature of eight-quark interactions.
Figure 1: **Left**: The l.h.s. (straight short-dashed line) and the r.h.s. of Eq. (3) at $\kappa, \lambda = 0$ and $GA^2 = 3$ as functions of $m/\Lambda$ for four different values of $H$: full curves (top to bottom) correspond to $|QH|\Lambda^{-2} = 0.5; 0.3; 0.1$, and the dashed curve to $H = 0$. Box insert: close-up of region around origin with solid lines for $|QH|\Lambda^{-2} = 0.2; 0.15; 0.1$. **Centre**: The l.h.s. (short-dashed line) and the r.h.s. of Eq. (3) at $\kappa \Lambda^5 = -10^3, \lambda \Lambda^8 = 3670$ (or $\lambda = 0$), and $|QH|\Lambda^{-2} = 0.5; 0.1; 0$. **Right**: The dimensionless dynamical mass $M_{\text{dyn}}/\Lambda$ as a function of the dimensionless magnetic field $|QH|\Lambda^{-2}$. The full lines are minima, the dashed line maxima. Up to $|QH|\Lambda^{-2} = 0.084$ the smaller $M_{\text{dyn}}/\Lambda$ corresponds to the deeper minimum of the potential; from this value on the larger solution becomes the stable configuration.

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