On 1/2 BPS Solutions in M-theory

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Abstract

We study singular 1/2 BPS solutions in M-theory using 11-dimensional superstar solutions. The superstar solutions and their corresponding plane wave limits could give an insight how one may deform the boundary conditions to get singular, but still physically acceptable, solutions. Starting from M-theory solutions with an isometry, we will also study 10-dimensional solutions coming from these M-theory solutions compactified on a circle.

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1 Introduction

The AdS/CFT correspondence [1–3] is an explicit example of holographic principle which might increase our knowledge about what a fundamental theory of quantum gravity could be. In this framework a quantum mechanical system which includes gravity can be described by a lower dimensional quantum mechanical system without gravity. Having had gravity on one side of the duality one may then wonder if we can learn about quantum gravity by studying a quantum field theory which by now we have more control on it.

We note, however, that this correspondence correspondence is a strong/weak duality in the sense that when the gauge theory is perturbative the string theory sigma model is strongly coupled and vice-versa. Hence usually one can only perform perturbative computations in one side of the duality which makes the comparison of two sides difficult. Nevertheless it has recently been shown that there is a certain large quantum numbers limit, the BMN sector [4] and the “semi-classical” strings [5,6], in which the gauge theory and the string theory sides are both perturbatively accessible and therefore might be used to study this duality better.

To be specific let us consider $AdS_5 \times S^5$ case in the global coordinates whose gauge theory dual is an $\mathcal{N} = 4$ SYM on $R^1 \times S^3$. Then one may consider 1/2 BPS states of the gauge theory which could be given by operators carrying R-charge $J$ with dimension $\Delta$ such that $\Delta = J$. From gravity dual point of view these BPS states correspond to the gravity modes for small energy; $J \ll N$, while for the energy of order of $N$ they correspond to the brane configurations which can in principle cause geometric changes in $AdS_5 \times S^5$ due to back reaction.

In order to find the geometric counterparts of this class of operators, the authors of [7] have established the general setting for obtaining the corresponding 1/2 BPS geometries. This is done by looking for those solutions of type IIB supergravity equations which have $SO(4) \times SO(4) \times R$ isometry and which solve the killing spinor equations. Doing so, one picks from all the excitations in $AdS_5 \times S^5$ in the global coordinates, those which constitute its 1/2 BPS sector. It turns out that these symmetry requirements, plus regularity, are very restrictive such that the whole solution is determined by a single function which should satisfy a linear differential equation subject to certain boundary conditions on a 2-plane.

This construction has also been generalized for M-theory in [7] where the authors have considered 1/2 BPS geometries of M-theory which have $SO(3) \times SO(6) \times R$ isometry. In this case the solution can be fixed by solving a three dimensional Toda equation. To get a non-singular solution one needs to put a specific boundary condition. In the next section we will review this construction.

In contrast to the 1/2 BPS geometries in type IIB string theory which can be obtained by solving the Laplace equation which is a linear differential equation, in M-theory we have to deal with a non-linear operator which is the three dimensional Toda equation. Thus it is very difficult to study 1/2 BPS states in M-theory, as we can not find a new solution by superposition of other known solutions such as
AdS$_{7,4} \times S^{4,7}$. Therefore it would be interesting to find any possible solutions of this Toda equation. Different aspects of 1/2 BPS in M-theory have also been studied in [8–14]. It is the aim of this paper to further study 1/2 BPS states in M-theory. We shall consider both singular and smooth solutions.

The organization of the paper is as follows. In section 2 we shall review the 1/2 BPS geometries in M-theory. We will also study well-known solutions of M-theory using different coordinates systems from that we could understand different aspects of M-theory 1/2 BPS solutions. In section 3 we will consider superstar solutions in M-theory in the limit where we get singular 1/2 BPS solutions. We will also study their Penrose limit using LLM parametrization. In section 4 using what we have learned from superstars solutions we shall study different possible boundary condition which could lead to either singular and smooth solutions. In section 5 we will study some 10-dimensional solution by making used by compactification of those M-theory solutions which have an isometry. The last section is devoted to conclusion.

## 2 1/2 BPS geometries in M-theory

In this section we fix our notation and review 1/2 BPS geometries in M-theory constructed in [7] where 1/2 BPS geometries with $SO(3) \times SO(6) \times R$ have been classified. These geometries can be thought of as 1/2 BPS geometries in AdS$_7 \times S^4$. These could be associated to the chiral primaries of the (2, 0) theory.

Using the Killing spinor equation the authors of [7] have been able to find M-theory supergravity solutions which preserve 16 supercharges and have $SO(3) \times SO(6)$ isometry

$$
\begin{align*}
\text{d}s^2_{11} &= -4e^{2\lambda}(1 + y^2 e^{-6\lambda})(dt + V_i dx^i)^2 + \frac{e^{-4\lambda}}{1 + y^2 e^{-6\lambda}}[dy^2 + e^D(dx_1^2 + dx_2^2)] \\
&\quad + 4e^{2\lambda}d\Omega_5^2 + y^2 e^{-4\lambda}d\Omega_2^2, \\
e^{-6\lambda} &= \frac{\partial_y D}{y(1 - y\partial_y D)}, \quad V_i = \frac{1}{2}\epsilon_{ij}\partial_j,
\end{align*}
$$

where $i, j = 1, 2$. There is also a four form which we have not written it (For detail see [7]). The solution is completely determined by a function $D(y, x_1, x_2)$ which satisfies the three dimensional Toda equation

$$
(\partial_1^2 + \partial_2^2) D + \partial_y^2 e^D = 0.
$$

The solution is fixed as soon as the boundary condition for this Toda equation is given. One may further put a boundary condition such that the obtained solution is non-singular. To do that we note that the radii of the five and two spheres are related to $y$ coordinate, $y = R_5 R_2^2 / 4$. This means that at $y = 0$ either the two sphere or five sphere shrinks to zero. Therefore the boundary condition must be such that
Figure 1: pp-wave and AdS$_{7,4}$ boundary conditions

the geometry become non-singular when either of the spheres shrink. One can see
that the geometry is non-singular if we impose the following boundary condition [7]

\[ \begin{align*}
&\text{At } y = 0, \quad \partial_y D = 0, \quad D = \text{finite}, \quad S^2 \text{ shrinks}, \\
&\text{For } y \to 0, \quad e^D \sim y, \quad S^5 \text{ shrinks}. 
\end{align*} \]  

(3)

This can separate the 12 plane into droplets where we have one or the other boundary
conditions.

Let us write down some specific examples. For simplicity we assume that the
solution has isometry in $x_1$ direction and therefore we will need to solve a two
dimensional Toda equation. The simplest example is the plane wave solution in
which the boundary condition is given by (see figure 1)

\[ \begin{align*}
\partial_y D &= 0, \quad D = \text{finite at } y = 0, \quad \text{in } x_2 > 0, \\
e^D &\sim y, \quad \text{for } y = 0, \quad \text{in } x_2 < 0. 
\end{align*} \]  

(4)

Since the solution has an isometry in $x_1$ direction it can be parametrized by two pa-
rameters $r_5$ and $r_2$ which are the radial coordinates in the first six transverse dimen-
sions and the last three transverse dimensions, respectively. In this parametrization
the plane wave solution is given by

\[ y = \frac{1}{4} r_5^2 r_2, \quad x_2 = \frac{r_5^2}{4} - \frac{r_2^2}{2}, \quad e^D = \frac{r_5^2}{4}. \]  

(5)

As another example let us consider $AdS_7 \times S^4$ solution in which going to polar
coordinates $(x, \phi)$ the solution has an isometry in $\phi$ direction and therefore can be
expressed by two parameters as follows [7]

\[ e^D = \frac{r^2 L^{-6}}{4 + r^2}, \quad x = (1 + \frac{r^2}{4}) \cos \theta, \quad 4y = L^{-3} r^2 \sin \theta, \]  

(6)
where $\theta$ is an angular direction on $S^4$ and $r$ is the radial coordinate in AdS$_7$ and $L$ is the radius of $S^4$. In terms of $x$ coordinate the corresponding boundary condition may be written as

\[
\partial_y D = 0, \quad D = \text{finite at } y = 0, \quad \text{for } y = 0, \quad \text{in } x > 1, \quad \text{in } x < 1,
\]

which could be thought of as a black disk in 12 plane (see figure 1). Similarly we can describe the solution for AdS$_4 \times S^7$ as follows

\[
e^D = 4L^{-6} \sqrt{1 + \frac{r^2}{4} \sin^2 \theta}, \quad x = \left(1 + \frac{r^2}{4}\right)^{1/4} \cos \theta, \quad 2y = L^{-3}r \sin^2 \theta, \quad (7)
\]

where $L$ is the radius of AdS$_4$. The boundary condition is given by

\[
\partial_y D = 0, \quad D = \text{finite at } y = 0, \quad \text{for } y = 0, \quad \text{in } x < 1, \quad \text{in } x > 1,
\]

which again represents a circular droplet in 12 plane, though in this case it is a white disk in a black plane (see figure 1).

We note that in both AdS $\times S$ cases we have circular droplets and the solutions are exchanged by exchanging the black white regions. In fact the situation is very similar to AdS$_5$ solution in type IIB where the corresponding boundary condition is given by droplet which is a black disk in 12 plane. In this case the theory has two $Z_2$ symmetries, both of which exchange black and white regions [15] (see also [12]). One of them which can be interpreted as the particle-hole symmetry is the symmetry of the whole supergravity solution though the second one is just the symmetry of the metric and changes the sign of five from flux.

On the other hand in M-theory we could still exchange the black and white regions, though we would not expect that this leads to any symmetry. In fact under this action, as shown in figure 1, AdS$_7$ maps to AdS$_4$. Actually, this can be interpreted as electric-magnetic duality in which M2-brane maps to M5-brane. One note, however, that the plane wave solution is invariant under this map which is acts as $x_2 \rightarrow -x_2$. This if of course expected because both AdS$_7$ and AdS$_4$ lead to the same plane wave solution by taking Penrose limit [16].

To further compare these two solutions, and in fact other possible solutions which we will study in the next section, it is useful to define new coordinates $x_2$ as $x = e^{x_2}$ and also consider a new parameter $\xi$ such that $\frac{x}{2} = \sinh \xi$. In this notation the AdS$_7$ reads

\[
y = L^{-3} \sin^2 \xi \sin \theta, \quad x_2 = 2 \ln \cosh \xi + \ln \cos \theta, \quad e^{\tilde{D}} = \frac{L^{-6}}{4} \sinh^2 2 \xi \cos^2 \theta. \quad (10)
\]

Here we have used the fact that the solution of the Toda equation is invariant under a conformal transformation which in our case we have $x = e^{x_2}, \quad D = \tilde{D} - 2x_2.$
Similarly for $AdS_4$ case one finds

$$y = L^{-3} \sinh \xi \sin^2 \theta, \quad x_2 = \ln \cosh \xi + 2 \ln \cos \theta, \quad e^{\tilde{D}} = \frac{L^6}{4} \sin^2 2\theta \cosh^2 \xi. \quad (11)$$

These two solutions are exchanged under $\xi \leftrightarrow i\theta$ which is expected as $\xi$ and $\theta$ represent $AdS_{7,4}$ in the global coordinates. Writing the solution in this coordinate is useful specially if one wants to study plane wave limit of the solutions.

We note also that writing $AdS_{7,4}$ in the $(x_2, y, \tilde{D})$ coordinates, the boundary condition we get is similar to that we have in plane wave solution (figure 1). This is of course expected as the conformal map $x = e^{x_2}$ maps a disk to a plane. This might look puzzling taking into account that both plane wave and $AdS_{7,4}$ seem to have the same boundary condition at $y = 0$. We note however that although the map $x = e^{x_2}$ maps a disk to plane, since the disk includes $x = 0$ point, which will be mapped to $x_2 = -\infty$ and therefore the boundary condition we get at the end of the day is defined on a plane which includes $x_2 = -\infty$ as well. This is the fact that distinguishes between $AdS$ and plane wave solutions. Therefore we conclude that the topology of the 12 plane is indeed important.

Let us now study the Penrose limit of the $AdS_{7,4} \times S^4$ using the LLM parametrization. Using the LLM parametrization is also useful to study the next order corrections to the plane wave background. These corrections for type IIB string they in the LLM language has been studied in [18]. The Penrose limit of different backgrounds in LLM context has also been studied in [19].

The procedure of taking the Penrose limit of a given solution in LLM context is as follows. In fact from LLM point of view this can be done by focusing on a small region around the edge of a droplet in the $(x_1, x_2)$ plane and then blowing up this region. The boundary condition we get corresponds to the plane-wave solution (see figure 2). We note, however, that, as we saw in the previous section, having a conformal symmetry the shape of the droplet is not really important. In particular for the cases we have considered, all of them have the same shape, though in the $AdS_{4,7}$ one needs to add to the plane the $x_2 = -\infty$ point. Therefore in order to find the plane wave solution in this parameterization, one need to look at a region which are far from $x_2 = -\infty$ point and therefore effectively we would see the plane wave boundary condition and thereby we will get plane wave solution.

Let us first study this procedure for the $AdS_7 \times S^4$ solution. To imply the above procedure we will consider the limit of $\xi \to 0, \theta \to 0$. To be precise we will consider a limit in which $L \to 0$ while keeping the following quantities fixed

$$r_5 = \frac{\xi}{L^3}, \quad r_2 = \frac{\theta}{L^3}, \quad \tilde{y} = \frac{y}{L^6}, \quad \tilde{x}_2 = \frac{x_2}{L^6}. \quad (12)$$

In order to make the equation similar to $AdS_7$ case we have also rescale $x_2 \to 2x_2$ and $e^{\tilde{D}} \to e^{\tilde{D}}/4$.

We would like thank M.M. Sheikh-Jabbari for a discussion on this point. The Penrose limit of these background which leads to the same plane wave in M-theory has been studied in [16, 17].
In this limit the solution (reflll) reads

\[
\begin{align*}
\tilde{y} &= r_5^2 r_2, \\
\tilde{x}_2 &= r_5^2 - \frac{1}{2} r_2^2, \\
\tilde{e}^D &= r_5^2,
\end{align*}
\] (13)

which is the plane wave solution (1) up to a rescaling. The procedure goes the same for AdS\(_4\) × S\(_7\) where upon defining \(r_2 = \frac{\tilde{x}}{L}, r_5 = \frac{\tilde{y}}{L}\) we get

\[
\begin{align*}
\tilde{y} &= r_5^2 r_2, \\
\tilde{x}_2 &= \frac{1}{2} r_2^2 - r_5^2, \\
\tilde{e}^D &= r_5^2,
\end{align*}
\] (14)

which is equivalent to the above plane wave solution taking into account the \(x_2 \rightarrow -x_2\) symmetry. It is of course well know that both AdS\(_4\) × S\(_7\) and AdS\(_7\) × S\(_4\) lead to the same plane wave solution under performing the Penrose limit.

### 3 Giant graviton deformation: Superstar

#### 3.1 Superstar solution

In the previous section we have studied the simplest examples of 1/2 BPS in M-theory, including plane wave, AdS\(_7\) × S\(_4\) and AdS\(_4\) × S\(_7\). From LLM point of view these solutions can completely be given as soon as the boundary condition on 12 plane is specified. For the solutions we have considered these boundary conditions are depicted in figure[1]. We have also seen that there is a “\(Z_2\)” transformation under which the boundary condition for AdS\(_7\) maps to that for AdS\(_4\) while the plane wave solution maps to itself. It is also expected because both AdS\(_7\) and AdS\(_4\) lead to the same 11-dimensional plane wave.

One may deform these solutions by adding spherical M5/M2 branes wrapped on S\(_{5,2}\) spheres. This corresponds to deform the theory by giant gravitons. Since the
“Z₂” map exchanges the role of S⁵ and S²; this corresponds to exchanging M5 and M2 branes or giant gravitons with dual giants. Following AdS₅ × S⁵ case in type IIB string theory, adding giant gravitons might lead to either singular or smooth solution. Of course since in the M-theory case we will have to deal with a three dimensional Toda equation and because it is not a linear equation, finding such a solution is very difficult.

Fortunately the singular deformation has already been studied in the literature and known as superstar [20, 21]. We can use this solution to study some feature of giant gravitons in M-theory using LLM construction.

Let start from supergravity solution representing giant gravitons on AdS₇ × S⁴ background. The corresponding solution is [20–22]

\[ ds_{11}^2 = \Delta^{1/3}\left(- (H_1 H_2)^{-1} f dt^2 + (f^{-1} dr^2 + r^2 d\Omega_5^2)\right) \]

\[ + \frac{1}{4} \Delta^{-2/3} \left[ L_7^2 d\mu_0^2 + \sum_{i=1}^2 H_i \left( L_7^2 d\mu_i^2 + \mu_i^2 (L_7 d\phi_i + \tilde{q}_i \frac{dt}{r^2 + q_i}) \right) \right], \]

where \( f = 1 - \frac{\mu}{r} + \frac{r^2}{L_7^2} (H_1 H_2), \) \( H_i = 1 + \frac{\mu}{r} \tilde{q}_i = \sqrt{q_i (\mu - q_i)} \) and

\[ \Delta \equiv (H_1 H_2) \left( \mu_0^2 + \sum_{i=1}^2 \frac{\mu_i^2}{H_i} \right), \quad \mu_0 \equiv \sin \theta_1 \sin \theta_2, \quad \mu_1 \equiv \cos \theta_1, \quad \mu_2 \equiv \sin \theta_1 \cos \theta_2, \]

with \( L_7^2 = 4l_p^2 (\pi N)^{2/3} \). This solution is asymptotically AdS₇ × S⁴ with radius \( L_7 \). \( N \) counts the number of M5-branes whose near-horizon limit is AdS₇ × S⁴. There is also a nonzero six form (dual to a three form) which we have not written it. Of course we are interested in the case where the solution is 1/2 BPS which can be obtained by setting \( q_2 = \mu = 0 \) and \( q_1 = Q \). In this case, setting \( \psi = \phi + t/L_7 \), the solution reads

\[ ds^2 = -\Delta^{1/3} H_1^{-1} f dt^2 + \frac{\Delta^{-2/3} H_1}{4} \cos^2 \theta_1 (L_7 d\psi - H_1^{-1} dt)^2 + \Delta^{1/3} f^{-1} dr^2 \]

\[ + \frac{L_7^2}{4} \Delta^{1/3} d\theta_1^2 + \frac{L_7^2}{4} \Delta^{1/3} r^2 d\Omega_5^2 + \frac{L_7^2}{4} \Delta^{-2/3} \sin^2 \theta_1 d\Omega_2^2, \]

which can be recast to the following form of 1/2 BPS solution of M-theory

\[ ds^2 = -4e^{2\lambda} (1 + y^2 e^{-6\lambda}) (dt + V_\psi d\psi)^2 + \frac{e^{-4\lambda}}{1 + y^2 e^{-6\lambda}} \left(dy^2 + e^D (dx^2 + x^2 dy^2)\right) \]

\[ + 4e^{2\lambda} d\Omega_5^2 + y^2 e^{-4\lambda} d\Omega_2^2, \]

with

\[ e^{2\lambda} = \frac{r^2}{4} \Delta^{1/3}, \quad V_\psi = \frac{-2L_7^2 \cos^2 \theta_1}{r^2 \Delta + 4L_7^2 \sin^2 \theta_1}, \]

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where
\[ x = g(r) \cos \theta_1, \quad y = \frac{L_7}{8} r^2 \sin \theta_1, \quad e^D = \frac{L_7^4 r^4 f}{64 g^2}. \] (21)

Note that the solution is given by two functions \( f \) and \( g \) such that \( g' = \frac{2r}{L_7 f} \), where in our notation \( f = 1 + r^2/L_7^2 + Q/r^2 L_7^2 \).

To summarize we saw that the superstar solution in M-theory is given by the metric (1) with the parametrization of (21). From LLM language this solution can be obtained by solving the three dimensional Toda equation with a boundary condition which should not be in the form of (3) and therefore the obtained solution can be obtained by solving the three dimensional Toda equation with a boundary condition, in the case of
\[ r = 0 : \quad x < x_0, \quad e^D \sim c_0(L_7^2 + \frac{L_7^4}{Q}y), \] (22)

where \( c_0 = (-4Q/(L_7^2 + \sqrt{L_7^4 - 4Q}^2)L_7^2/\sqrt{L_7^4 - 4Q}) \) and \( x_0 = \sqrt{-Q/c_0} \). We note that this boundary condition, in the case of \( r = 0 \), differs from that in (3) and therefore the solution is singular.

Let us now study a giant graviton deformation of \( AdS_4 \times S^7 \). The corresponding supergravity solution is given by [20–22]
\[
ds_{11}^2 = \Delta^{2/3} \left( -(H_1 H_2 H_3 H_4)^{-1} f dt^2 + (f^{-1} dr^2 + r^2 d\Omega_2^2) \right) + 4 \Delta^{-1/3} \sum_i H_i (L_4^2 d\mu_i^2 + \mu_i (L_4 d\phi_i + \frac{\bar{q}_i}{r + q_i} dt)^2),
\] (23)

where \( f = 1 - \frac{\mu}{r} + \frac{r^2}{L_4^2} (H_1 H_2 H_3 H_4) \), \( H_i = 1 + \frac{\mu_i}{r} \), \( \Delta = (H_1 H_2 H_3 H_4) \), \( \sum_{i=1}^4 \frac{\mu_i^2}{H_i} \), and \( \mu_1 = \cos \theta_1, \ \mu_2 = \sin \theta_1 \cos \theta_2, \ \mu_3 = \sin \theta_1 \sin \theta_2 \cos \theta_3, \ \text{and} \ \mu_4 = \sin \theta_1 \sin \theta_2 \sin \theta_3. \) (25)

There is also a nonzero three form. The solution is asymptotically \( AdS_4 \times S^7 \) where the AdS and sphere length scales are \( L_4 \) and \( 2L_4 \) respectively. We are particularly interested in the case where only one charge is non-zero, and the solution is in the extremal limit. This leads to a 1/2 BPS solution in 11-dimensions. In this case, setting \( q_2 = \mu = 0, q_1 = Q \) and \( \psi = \phi_1 + t/L_4 \), the metric reads
\[
ds_{11}^2 = -\Delta^{2/3} H_1^{-1} f dt^2 + 4 \Delta^{-1/3} \cos^2 \theta_1 H_1 (L_4 d\psi - H^{-1} dt)^2 + \Delta^{2/3} f^{-1} dr^2 + 4 \Delta^{2/3} L_4^2 d\theta_1^2 + \Delta^{2/3} r^2 d\Omega_2^2 + 4 \Delta^{-1/3} L_4^2 \sin^2 \theta_1 d\Omega_5^2,
\] (26)

which can be recast to the standard form as (19) with
\[ e^{2\lambda} = L_4^2 \Delta^{-1/3} \sin^2 \theta_1, \quad V_\psi = \frac{4L_4^2 \cos^2 \theta_1}{r^2 \Delta + 4L_4^2 \sin^2 \theta_1}, \] (27)
where
\[ x = g(r) \cos \theta, \quad y = L^2 r \sin^2 \theta, \quad e^D = \frac{4L_4^6 f \sin^2 \theta}{g^2}. \] (28)

The solution is given in terms of two functions \( f \) and \( g \) such that \( \frac{g'}{g} = \frac{r^2}{2X_4 f} \) with \( f = 1 + r^2/L_4^2 + qr/L_4^2 \).

To summarize we note that the giant graviton deformation of \( AdS_4 \times S^7 \) can be recast into the same form as (1) with the solution of three dimensional Toda equation given by (28) with a specific boundary condition at \( y = 0 \) which is
\[ \theta = 0 : \quad x > x_0, \quad e^D \sim y \]
\[ r = 0 : \quad x < x_0, \quad \partial_y D \sim q/\sin^2(\theta), \] (29)

where \( x_0 \) is a constant function of \( Q \). This means that the solution is singular as expected.

### 3.2 Penrose limit of superstar solution

In this subsection we will derive the plane wave limit of the superstar solutions we have studied using the procedure we have developed in the previous section. Let us start with giant graviton deformation of \( AdS_7 \times S^4 \) case. To compare the result with the \( AdS_7 \times S^4 \) case we first rescale the coordinates as \( L_7 = 2/L, \hat{r} = Lr, Q = 4qL^{-4} \) in which the solution is given by
\[ x = g(\hat{r}) \cos \theta, \quad 4y = L^{-3} \hat{r}^2 \sin \theta, \quad e^D = \frac{\hat{r}^2 f(\hat{r})}{4L_6^6 g^2}, \] (30)

where \( f = 1 + \hat{r}^2/4 + q/\hat{r}^2 \).

To get the Penrose limit of the solution we will consider a limit in which \( L \to 0 \) while keeping the following quantities fixed
\[ r_5 = \frac{\hat{r}}{L^3}, \quad r_2 = \frac{\theta}{L^3}, \quad \bar{y} = \frac{y}{L^6}. \] (31)

Of course one needs to scale \( q \) properly. Depending on how to rescale \( q \) we will get different plane wave solutions. In particular if we rescale \( q \) as \( q = \tilde{q}L^n \) for \( n > 6 \), the limit will remove the effect of the deformation such that the resulting solution is the standard plane wave solution in 11-dimensions given by (3). On the other hand for \( n < 6 \), the limit is not well defined. Finally for the case of \( q = \tilde{q}L^6 \) in the limit of \( L \to 0 \) we find
\[ \bar{x}_2 = \frac{r_5^2}{4} - \frac{r_2^2}{2} - \frac{1}{4} \tilde{q} (\ln \frac{r_5^2 + \tilde{q}}{4} - 1), \]
\[ \bar{y} = \frac{1}{4} r_5^2 r_2, \]
\[ e^D = \frac{\hat{q}}{4} + \frac{r_5^2}{4}, \]  
(32)

where \( \tilde{x} \) is fixed and defined by
\[ \tilde{x}_2 = \frac{x - 1 + \frac{3}{2}L^6 \ln L}{L^6}. \]  
(33)

Note that unlike the AdS_7 case one needs to regularize \( \tilde{x} \) as well.

Let us now study the Penrose limit of the deformation of AdS_4 \times S^7 solution. To do this we first rescale the coordinates as \( L_4 = 1/L \), \( \tilde{r} = 2rL \), \( Q = 2qL^{-1} \) in which the corresponding solution reads
\[ x = g(\tilde{r}) \cos \theta_1, \quad y = \frac{1}{2L^3} \tilde{r} \sin^2 \theta, \quad e^D = \frac{4f(\tilde{r}) \sin^2 \theta}{L^6 g^2(\tilde{r})}, \]  
(34)

where \( f = 1 + \tilde{r}^4/4 + q\tilde{r} \).

To find the corresponding plane wave solution we will consider a limit in which \( L \to 0 \) while keeping the following quantities fixed
\[ r_5 = \frac{2\theta}{\sqrt{2L^3}}, \quad r_2 = \frac{\sqrt{2r}}{4L^3}, \quad \tilde{y} = \frac{y}{L^6} \tilde{x}_2 = \frac{x - \sqrt{2}}{L^6}. \]  
(35)

Moreover in order to get a finite solution one also needs to rescale \( q \) appropriately. For example one may rescale \( q \) as \( \tilde{q} = q/L^4 \). Doing so we arrive at
\[ y = \frac{1}{\sqrt{2}} r_5^2 r_2, \quad \tilde{x}_2 = \sqrt{2} \left( \frac{r_2^2}{2} - \frac{r_5^2}{4} \right), \quad e^D = r_5^2, \]  
(36)

which leads to the same plane wave solution as that can be obtained from AdS_4 \times S^7. We note that as long as we require that the limit we are taking remains well defined, the results will be \( q \)-independent and it leads to the 11-dimensional non-singular plane wave solution. It is, of course, well-known, that Penrose limit can remove singularity [23].

4 Singular solutions and boundary condition

In the previous sections we have studied several solutions, both singular and smooth, of the three dimensional Toda equation. From what we have learned so far we would like to study any possible boundary condition one might have for both singular and smooth solutions.

The discussion of boundary condition for smooth 1/2 BPS solution in M-theory has recently been studied in [12] and the aim of this section is to further study the boundary conditions. To do that following [7] we introduce a new set of coordinates \( \eta \) and \( \rho \) such that
\[ e^D = \rho^2, \quad y = \rho \partial_\rho V, \quad x_2 = \partial_\eta V, \]  
(37)
for given function $V(\eta, \rho)$. Here we have assumed that the solution has translation Killing vector in $x_1$ direction. One may also consider the case where we have rotation Killing vector in $\phi$ direction in the $(r, \phi)$ plane. In this case the coordinate $x_2$ in (37) can be defined by $r = e^{x_2}$ in which we will also need to shift $D$ by $2 \ln x_2$.

In this new coordinates system the three dimensional Toda equation reduces to a Laplace equation with the cylindrically symmetry for function $V$

$$\frac{1}{\rho} \partial_\rho (\rho V) + \partial_\eta^2 V = 0. \tag{38}$$

One then needs to understand the corresponding boundary conditions in terms of these new coordinates which must be translated to boundary conditions for $V$ as a function of $\rho$ and $\eta$.

Actually it is shown [12] that finding a solution for the Toda equation reduces to an auxiliary three dimensional electrostatic system with conducting disks setting at positions $\eta_i$ with radii $\rho_i$. Let us first review the construction of the auxiliary system.

Starting from the definition of $\rho$ in terms of $D$ one finds

$$\rho^2 \partial_y D = \frac{-2 \partial_\eta^2 V}{(\partial_\eta^2 V)^2 + (\partial_\eta^2 V)^2}, \tag{39}$$

from which we can show that $\partial_\rho V \to 0$ whenever $\rho \to 0$. This can be seen as follows. For $\rho \to 0$ one finds $\partial_\eta^2 V \to 0$ which can be used to conclude $\frac{1}{\rho} \partial_\rho (\rho \partial_\rho V) \to 0$ which leads to $\partial_\rho V \sim \rho \to 0$. As the conclude we note that imposing boundary condition at $y \to 0$ can be expressed in the new coordinates system as the limit where $\rho \partial_\rho V \to 0$ which is possible if

$$\rho \neq 0, \quad \partial_\rho V \to 0, \quad \text{or} \quad \rho \to 0, \quad \partial_\rho V \to 0 \tag{40}$$

One can see that the first condition is consistent with the boundary condition in which $\partial_y D = 0$ while the second one is consistent with the boundary condition given by $e^D \sim y$ as $y \to 0$. In the first case we also get $\partial_\rho^2 V = 0$ and therefore the curve $y = 0, \rho \neq 0$ is at constant values of $\eta$. Moreover if we interpret $V$ as an electrostatic potential, then $-\partial_\rho V = 0$ is the electric field along $\rho$ and the condition that it is zero corresponds to the presence of a charged conducting surface. In general we could have $n$ conducting disks with radii $\rho_i$ sitting at $\eta_i$ for $i = 1, \cdots, n$ which could be in an external electric field. The second condition just tell us that the potential $V$ is regular at $\rho = 0$.

The superstar solutions we have studied in the previous section could give us an insight how to change the boundary condition if we want to relax the smoothness of the solution. One way to do this is just to remove the regularity condition at $\rho = 0$ which from three dimensional auxiliary electrostatic system it means we should have a charge distribution at $\rho = 0$. Let us now consider the superstar case studied in previous section in more detail.
Consider a boundary condition in which at \( y \to 0 \) we have \( e^D \sim a + by \) for some non-zero \( y \)-independent parameters \( a \) and \( b \). In the \((\rho, \eta)\) coordinates it means that we have the case where \( \rho \neq 0 \) while \( \partial_\rho V \to 0 \) for \( y \to 0 \). On the other hand for the other boundary condition we may still insist to get \( \partial_y D = 0 \) at \( y = 0 \) which again can be satisfied if we impose \( \rho \neq 0 \) while \( \partial_\rho V \to 0 \) boundary condition. As a conclusion we note that this boundary condition is enough to get whole supergravity solution though the solution would be singular. From there dimensional electrostatic system point of view, this corresponds to the case where the potential \( V \) is not regular at \( \rho = 0 \) which means that we should have a charge distribution at \( \rho = 0 \).

To see how this could happen it is instructive to study the singular plane wave solution we have obtained in the previous section which can be recast to the following form

\[
e^D = \rho^2, \quad y = 2\eta(\rho^2 - \tilde{q}), \quad x_2 = (\rho^2 - \frac{\tilde{q}}{2}\ln \rho) - 2\eta^2. \tag{41}\]

Note that to get the above expression we used a change of coordinates as \( r_2^2 + \tilde{q} = 4\rho^2, \ r_2 = 2\eta \). First of all we observe that in the limit of \( y \to 0 \) we get boundary conditions for \( D \) which which are

\[
e^D \sim \frac{\tilde{q}}{4}, \quad \text{for} \quad \rho^2 \rightarrow \frac{\tilde{q}}{4}, \quad \partial_y D = 0 \quad \text{at} \quad \eta = 0. \tag{42}\]

It is easy to see that this solution can be obtained from the following potential

\[
V = \rho^2 \eta - \frac{2}{3} \eta^3 - \frac{\tilde{q}}{2} \eta \ln \rho. \tag{43}\]

We see that in \( \rho \to 0 \) the potential blows up as \( \ln \rho \) which can be interpreted as a potential produced by a distribution of a linear charge setting at \( \rho = 0 \) and \( \tilde{q}\eta \) is the total electric charge.

One may also modify the other boundary condition to get a possible singular solution. More precisely we consider a boundary condition in which at \( y = 0 \), \( \partial_y D \neq 0 \), while the other condition remains unchanged, namely for \( y \to 0 \) one has \( e^D \sim y \).

To understand this boundary condition and the corresponding singular solution better let us study an explicit example exhibits such a singular boundary condition.

Consider a solution given by a potential corresponding to a charged ring located at the origin. Its electrostatic potential is given by

\[
V_0 = \frac{a}{\sqrt{\eta^2 + \rho^2}}. \tag{44}\]

We may consider a background potential which might present because of an infinite or finite disk at \( \eta = 0 \). For example in the case with an infinite disk the whole system is given by the following potential

\[
V = \rho^2 \eta - \frac{2}{3} \eta^3 + \frac{a}{\sqrt{\eta^2 + \rho^2}}, \tag{45}\]

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and therefore we arrive at

\[ e^D = \rho^2, \quad y = 2\rho^2 \eta - \frac{a\rho^2}{(\eta^2 + \rho^2)^{3/2}}, \quad x = \rho^2 - 2\eta^2 - \frac{a\eta}{(\eta^2 + \rho^2)^{3/2}}. \quad (46) \]

This leads to a singular solution such that \( e^D \sim y \) for \( y \to 0 \) while \( \partial_y D \neq 0 \) at \( y = 0 \) which is the singular boundary condition we were interested in. Therefore the solution is singular at \( y = 0 \). This solution has also been studied in [9].

Starting from this potential one may generate other solutions which could be either singular or smooth. In fact one can easily show that the following potential

\[ V = \rho^2 \eta - \frac{2}{3} \eta^3 + V_n, \quad (47) \]

with

\[ V_n = \frac{\partial^n}{\partial \eta^n} \left( \frac{a}{\sqrt{\eta^2 + \rho^2}} \right) \quad (48) \]

satisfies the three dimensional Laplace equation. Actually this is multipoles expansion for an electrostatic potential. Using this one finds

\[ e^D = \rho^2, \quad y = \rho^2 (2\eta - \partial^n_{\eta} \frac{a}{(\eta^2 + \rho^2)^{3/2}}), \quad x = \rho^2 - 2\eta^2 + \partial^n_{\eta} + 1 \frac{a}{(\eta^2 + \rho^2)^{1/2}}, \quad (49) \]

from which at \( y \to 0 \) one has \( e^D \sim y \). On the other hand we have

\[ \partial^n \eta V = -4\eta + \frac{a}{(\eta^2 + \rho^2)^{1/2}}, \quad (50) \]

which together with (49) shows that for odd \( n \) the \( y = 0 \) has a solution at \( \eta = 0 \) which also solves \( \partial^n \eta V = 0 \) and therefore one get \( \partial_y D = 0 \) at \( y = 0 \). Thus the potential (47) leads to smooth solutions for odd \( n \). In particular for \( n = 1 \) is the leading asymptotic form of the solution for the plane wave matrix model [12]. In general we can find a smooth solution by summing up \( 2^{2k+1} \)-multipoles

\[ V = \rho^2 \eta - \frac{2}{3} \eta^3 + \sum_{k=0}^{\infty} \partial_{\eta}^{2k+1} \frac{a}{(\eta^2 + \rho^2)^{1/2}}, \quad (51) \]

while for a potential coming from a summation over \( 2^2 \)-multipoles we get a singular solution such that at \( y = 0, \partial_y D \neq 0 \).

Another set of singular solutions have also been studied in [9] where the solution has null singularity. These solutions can simply be obtained by considering an ansatz such that \( V = \phi(\rho)T(\eta) \) which leads to the following solutions for the three dimensional Laplace equation

\[ K_0(\kappa \rho) e^{i\kappa \eta}, \quad I_0(\kappa \rho) e^{i\kappa \eta}, \quad J_0(\kappa \rho) e^{i\kappa \eta}, \quad N_0(\kappa \rho) e^{i\kappa \eta}. \quad (52) \]
All of these solutions which are given in terms of Bessel function are singular. Nevertheless one may construct new smooth solutions using the same procedure. For example consider the following solution of the three dimensional Laplace equation

\[ V = \rho^2 \eta - \frac{2}{3} \eta^3 + I_0(k\rho) \sin(k\eta), \]  

(53)

which leads to

\[ e^D = \rho^2, \quad y = \rho \left( 2\rho\eta - kI_1(k\rho) \sin(k\eta) \right), \quad x_2 = \rho^2 - 2\eta^2 + kI_0(k\rho) \cos(k\eta). \]  

(54)

The boundary condition of this solution is as follows. First of all we note that at \( y = 0 \) we have two solutions either \( \rho = 0 \) or \( \eta = 0 \) and in fact we find

\[ e^D \sim y \quad \text{for} \quad \rho \to 0, \quad \partial_y D = 0 \quad \text{at} \quad \eta = 0. \]  

(55)

This solution (without the plane wave part) and its compactification to type IIA string theory has been studied in [12] where in type IIA string theory this corresponds to wrapped NS5-brane on \( S^5 \). One could also consider a solution given by \( J_0(k\rho) \sinh(k\eta) \) which has the same behavior as above. If we had considered \( \cos(k\eta) \) or \( \cosh(k\eta) \) we would have gotten solutions which were singular. In fact in these cases we get \( e^D \sim y \) when \( y \to 0 \) while at \( y = 0 \) one finds \( \partial_y D \neq 0 \). On the other hand if we consider \( N_0(k\rho) \sinh(k\eta) \) or \( K_0(k\rho) \sin(k\eta) \) then solutions will also be singular with boundary condition given by \( e^D \sim a + by \) for \( y \to 0 \) while the other boundary condition remains the same as before. In the next section we will study compactification of solutions given by these potential following [12].

Finally we note that starting from two smooth solutions it is possible to find a singular solution one. For example consider a solution given by potential \( V = I_0(k\rho) \cos(k\eta) \). It is non-singular and in fact is equivalent to the case studied in [12]. But considering this potential to the background potential representing an infinite conducting disk at \( \eta = 0 \) leads to a singular solution such that for \( y \to 0 \) goes as \( e^D \sim \text{constant} \).

## 5 Ten dimensional solutions

In the previous section we have studied both singular and smooth 1/2 BPS solutions of M-theory. All solutions we have considered have an isometry which is translation invariant along \( x_1 \) and therefore can be compactified to find type IIA solutions. For a general background, doing so one finds [12]

\[
\begin{align*}
    ds^2 &= \left( \frac{\dot{V} - 2\dot{\bar{V}}}{-V''} \right)^{1/2} \left[ -4\frac{\dot{V}}{V - 2\bar{V}} dt^2 + \frac{2V''}{V}(d\rho^2 + d\eta^2) + 4d\Omega_3^2 + \frac{2V''\dot{V}}{\Delta} d\Omega_2^2 \right], \\
    e^{4\phi} &= \frac{4(\dot{V} - 2\dot{\bar{V}})^2}{-V''\bar{V}^2 \Delta^2}, \quad C_3 = -4\frac{\dot{V}V''}{\Delta} dt \wedge d^2 \Omega,
\end{align*}
\]

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For example if we start from a potential given by $V$ representing type IIA NS5-brane wrapping on $S^3$ that it satisfies the above conditions and in fact lead to a 10-dimensional solution. For example plugging section and plugging them into the above expression to get type IIA supergravity definite which is given by the following conditions [12].

In principle one can start from the potentials we have written in the previous section and plugging into the above expression to get type IIA supergravity solutions. For example plugging $V = I_0(k\rho) \sin(k\eta)$ we will get type IIA NS5-brane wrapped on $S^5$ [24]. Different aspects of this solution using semi-classical closed strings has been studied in [24].

As another example let us consider the solution corresponding to singular plane wave solution, given by the potential $V = \rho^2 - \frac{2}{3} \eta^3 - 2q^2 \eta \ln \rho$. The solution is

$$
C_1 = -\frac{2\dot{V}\ddot{V}}{V - 2\dot{V}} dt, \quad B_2 = 2 \left( \frac{\dot{V}\ddot{V}}{\Delta} + \eta \right) d^2 \Omega,
$$

where $\Delta = (\ddot{V} - 2\dot{V})V'' - \dot{V}^2$ and the dots denote derivatives with respect to $\log \rho$ and the primes denote derivatives with respect to $\eta$.

This solution and the corresponding boundary condition are very similar to that studied in [12] in case of $N = 4$ SY theory on $R \times S^3/Z_k$. In that case the potential was given by $V = \rho^2 - 2\eta^2 - 2q^2 \ln \rho$ leading to the following solution

$$
ds^2 = -\frac{4q}{\rho^2 - q^2} \rho^2 dt^2 + \frac{4q}{\rho^2 - q^2} (d\rho^2 + d\eta^2) + 4q d\Omega_3^2 + \frac{4q\eta^2 (\rho^2 - q^2)}{4q^2 \eta^2 + (\rho^2 - q^2)^2} d\Omega_2^2,
$$

$$
e^{4\phi} = \frac{q^6}{(\rho^2 - q^2)(4q^2 \eta^2 + (\rho^2 - q^2)^2)}, \quad C_3 = -\frac{16q^3 (\rho^2 - q^2)}{4q^2 \eta^2 + (\rho^2 - q^2)^2} dt \wedge d^2 \Omega,
$$

$$
B = 2 \left( -\frac{(\rho^2 - q^2)\eta}{4q^2 \eta^2 + (\rho^2 - q^2)^2} + \eta \right) d^2 \Omega, \quad C_1 = -\frac{2}{q^2} (\rho^2 - q^2)^2 dt.
$$

This solution and the corresponding boundary condition are very similar to that studied in [12] in case of $N = 4$ SY theory on $R \times S^3/Z_k$. In that case the potential was given by $V = \rho^2 - 2\eta^2 - 2q^2 \ln \rho$ leading to the following solution

$$
ds^2 = -\frac{4q}{\rho^2 - q^2} \rho^2 dt^2 + \frac{4q}{\rho^2 - q^2} (d\rho^2 + d\eta^2) + 4q d\Omega_3^2 + \frac{4q^2 - q^2}{q} d\Omega_2^2,
$$

$$
e^{2\phi} = \frac{q}{4(\rho^2 - q^2)}, \quad C_3 = -\frac{4(\rho^2 - q^2)}{q^2} dt \wedge d^2 \Omega, \quad B = 2\eta d^2 \Omega.
$$

It would be interesting to find the gauge theory dual of the solution [27]

One can also consider other potential we have presented in previous section to find type IIA solutions. We note, however, given a potential of the three dimensional auxiliary electrostatic system is not enough to get a 10-dimensional physical solution. One needs to check whether the different components of the metric are positive-definite which is given by the following conditions [12]

$$
\Delta \leq 0, \quad V'' \leq 0, \quad \ddot{V} - 2\dot{V} \geq 0, \quad \dot{V} \geq 0.
$$

For example if we start from a potential given by $V = I_0(k\rho) \sin(k\eta)$ one can see that it satisfies the above conditions and in fact lead to a 10-dimensional solution representing type IIA NS5-brane wrapping on $S^5$. On the other hand if we start from other solutions in this kind, such that $K_0(k\rho) \sin(k\eta)$, it does not satisfies the above conditions and therefore does not lead to a well-defined 10-dimensional
solution. Nevertheless it is possible to add a background potential to make the solution well-defined. For example we can consider the background potential given by

\[ V_B = \rho^2 \eta - \frac{2}{3} \eta^3. \]

Since the superstar solutions of M-theory have also an isometry along angular direction \( \psi \) it would also be interesting to compactify these solutions along this direction to find their type IIA string theory counterpart. Actually starting from superstar solution in \( AdS_7 \times S^4 \) one arrives at

\[ ds^2 = 2H_1^{1/2} \cos \theta_1 \left[ -4\frac{f}{H_1} dt^2 + 4f^{-1} dr^2 + 4r^2 d\Omega_5^2 + L_7^2 \left( d\theta_1^2 + \frac{\sin^2 \theta_1}{\Delta} d\Omega_2^2 \right) \right], \]

while for \( AdS_4 \times S^4 \) we get

\[ ds^2 = 2\Delta^{1/2} H_1^{1/2} \cos \theta_1 \left[ -\frac{f}{H_1} dt^2 + f^{-1} dr^2 + r^2 d\Omega_2^2 + 4L_4^2 \left( d\theta_1^2 + \frac{\sin^2 \theta_1}{\Delta} d\Omega_5^2 \right) \right], \]

6 Conclusions

In this paper we have studied different aspects of 1/2 BPS geometries in M-theory using some known M-theory supergravity solutions. These include both singular and smooth solutions. All of these solutions can fully be given by a function \( D(y, x_1, x_2) \) which satisfies a three dimensional Toda equation with specific boundary condition at \( y = 0 \). Depending on how the function \( D \) behaves at \( y = 0 \) one can assign a droplet in \( (x_1, x_2) \)-plane for any solution.

Since the solution is invariant under a conformal map acting as \( x_1 + ix_2 \rightarrow f(x_1 + ix_2) \) and \( D \rightarrow D - \ln |df|^2 \) the shape of the droplet does not play an essential role, though the topology of the plane is important. In particular we have seen for both \( AdS_{1,7} \times S^7,4 \) and their corresponding plane wave solution we get the same shape in the \((x_1, x_2)\)-plane using a conformal map. Of course we note that the topology of the plane for these cases are different and from that one may specify the correct solution.

We have also seen how one can define a boundary condition to get singular but physically acceptable solutions by making use of known M-theory superstar
solutions. We have also studied the penrose limit of these singular solutions using LLM parameterization.

It is also shown that for solutions with translation invariant one may map the Toda equation to a three dimensional Laplace equation with a function $V$ which could be interpreted as a potential of electrostatic system in three dimensions. Using this we have reformulated the singular solutions in this language and how to generalized to get new solutions. Since all of these solutions have an isometry one could also compactify them to get type IIA solutions which might provide the gravity dual for some gauge theories. It would be interesting if one could find and study these gauge theories.

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