Modeling of effects of technical errors on static and dynamic behavior of helical gear transmission

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Abstract. Materials of the present article are concerned with research related to identification of defects in a helical gearwheel of crossed helical gear pair, occurring as a result of certain operating modes and overloads, on such processes in a gearwheel as fatigue and wear of teeth. In the course of the research, using a modern computer program, a dynamic model of a helical gearwheel was developed, which made it possible to identify the degree of effect of engineering technical errors of mounting and operation on conditions of its operation.

1. Introduction

Gears are widely used in various mechanical systems and are the main components of most technical transmission devices. They are used in case of transmitting high torque, increasing rotation speed or changing the direction of rotation. On the other hand, they are considered to be one of the most functional parts in transmissions, where 60% of defects in mechanical systems occur in gears [1-4]. Gear defects are divided into the following: 1- Defects caused by operating conditions and excessive overload (fatigue, wear). 2- Shape defects (pitch errors - section errors). 3- Defects caused by improper manufacturing (mounting errors - alignment errors). Most of the research has focused on identifying defects resulting from operating conditions and overloading such as fatigue and wear. Although there are relatively little specialized research on the problems associated with shape defects and misuse and, as a rule, they are fragmented and do not provide a general approach that can be used to describe these defects.

In addition, these research were sometimes limited to only static or semi-static load conditions and largely neglected their effect on dynamic conditions [5-7], moreover they were focused on studying gears on an expedited basis, for example, when the driving and driven wheels were considered equivalent to two disks. At that, tooth contact patterns were considered to be constant and unchanged in time (8-10) at engagement, or a gear shift model with only one degree of freedom was used (11-13), for example, when determining values of displacement at contact points where the bending effect is ignored.

2. Goal of research

Study of technical errors caused by improper operation of gears of crossed helical gear pair and defects affecting their static and dynamic behavior, in particular, positioning errors, considered as the main causes and affecting their wear and vibration, to obtain necessary information to understand their nature, as well as interconnection between these defects and power transmitted by gears, through a dynamic model of gears having 36 degrees of freedom in tension, pressure, twisting and bending. This model includes pinions, wheels and transmission shafts, in which bodies of the driving and driven wheels will...
be considered to be deformable and divided into a great number of thin cuts to describe a contact point between teeth. Consequently, we will be able to determine values of displacement and deformation after being subjected to applied loads and to rely on the degree of freedom at any time and during the period of engagement.

3. Materials and methods

3.1 Dynamic model of a gear drive

Figure 1 shows the dynamic model used in this study for transmission of motion by means of helical gears, consisting of a transmission mechanism - shafts - a control mechanism - and a motor, which is located next to the input bearing assembly 1, as well as the output assembly 6. The complex of these elements provides us with a means to study static and dynamic behavior of gears and shafts. In this operational option, the electric motor is described only by its failure and torque, where its mass is introduced into the equation of full motion of the model system by means of an equivalent mass matrix.

3.1.1 Modeling of shafts. In order to take account of distortions caused by shafts, according to the studies [10], axles with a pinion and a wheel were considered to be divided into three circular objects with each of them having two nodes of tension, pressure, twisting and bending. Each node has six degrees of freedom, three degrees of linear movement along the axes (u, v, w) and three degrees of rotation around the axes (θ, φ, ψ). The total number of degrees of freedom on each control axis element is twelve, as shown in figure 2, with m symbol standing for the driving wheel called a pinion.

3.1.2 Modeling of gears. As it is known, a contact point of gear teeth will move along the active tooth profile. Reasoning from this fact, we will assume that the pinion and the wheel are equivalent to two
cylinders, and the contact between the active profiles of gears is represented by a strip, and that contact lines, which are inclined within the operating level, move along this plane with a linear velocity \( V \), as shown in figure 3. This velocity is calculated using the following ratio: \( V = R_{b1} \Omega_1 \), where \( R_{b1} \) - a pinion’s radius, and \( \Omega_1 \)– a pinion’s rotation velocity.

On the other hand, in order to determine the position of the contact point on the tooth profile, it is assumed that bodies of gears are divided into several thin cylindrical sliders, determined by the indicator \( j \) (figure 4). Thus, the position of any point of contact in the working plane will be determined by searching for \( ij \) by \( M_{ij} \), where \( i \) indicates a line of a contact line at the present sampling time, and \( j \) is a chip number.

By isolating a thin slice of driving and driven bodies of teeth, the functional behavior is described by 12 degrees of freedom, where six degrees of freedom contribute to each of the teeth of the driving and driven wheels, as shown in figure 5. From a physical point of view, these degrees of freedom contribute to identifying the displacement values: Stretching: \( u_1, u_2 \) (axial displacements), curvature \( v_1, v_2, w_1, w_2 \) (deviations in two vertical directions of centers of driving and driven gears), \( \psi_1, \psi_2, \Phi_1, \Phi_2 \) (rotation) and twisting \( \Theta_1, \Theta_2 \), where indicator 1 indicates where indicator 1 indicates a pinion and 2 indicates a wheel. Based on the above, load displacement of the contact point \( M_{ij} \) can be expressed by the following equation:
3.2. Modeling of effects of engineering errors

Engineering errors are considered to be actual errors in positioning of gears by reference to their ideal positioning, and they are divided into the following errors: alignment errors or mounting errors and decentralization errors.

3.2.1. Modeling of effects of alignment errors or mounting errors. Describes asymmetry of axes of gears’ bearing. These errors are determined by angles of inclination and deviation, which are corresponding to an angle error between the real axis of the drive shaft and its theoretical axis, as shown in figure 6.

Since these errors are regarded as deviations of angles against the initial position of each tooth, they can be defined as angles of rotation around the T and S axes, where the angle of rotation $\phi$ is magnitude of the deviation angle, and $\psi$ is magnitude of the inclination angle, as shown in figure 7.
Figure 7. Identification of alignment errors and errors of transmission decentralization.

Given this, when affected by these errors, a displacement field for each tooth can be expressed as follows:

$$\tau_{m_j} \left\{ \vec{W}_{m_j} = \phi_{m_j}^* S_j + \psi_{m_j}^* \vec{T}_j \right\}$$

where \( S, T, Z \) - coordinates related to junction lines. Thus, alignment errors \( e_d(M_{ij}) \) in every contact point \( M_{ij} \) are given as the sum of alignment errors on the pinion (\( M_{ij} \)) \( e_{d1} \) and the driven wheel (\( M_{ij} \)) \( e_{d2} \) in accordance with figure 7 above:

$$e_d(M_{ij}) = e_{d1}(M_{ij}) + e_{d2}(M_{ij})$$

where:

$$e_{d1}(M_{ij}) = \vec{U}_{1j} \cdot n_1 - \left( \vec{W}_{1j} \wedge \vec{O}_1 \vec{M}_{ij} \right) \cdot n_1$$

$$e_{d2}(M_{ij}) = \vec{U}_{2j} \cdot n_2 - \left( \vec{W}_{1j} \wedge \vec{O}_2 \vec{M}_{ij} \right) \cdot n_2$$

Then these errors can be expressed as follows:
\[ e_d(M_{ij}) = \begin{bmatrix} \sin \beta_i \cdot (R_{b1} \cdot \sin \alpha_i - l_1(M_{ij}) \cdot \cos \alpha_i) - \eta(M_{ij}) \cdot \cos \alpha_i \\ \sin \beta_i \cdot (R_{b1} \cdot \cos \alpha_i + l_1(M_{ij}) \cdot \sin \alpha_i) + \eta(M_{ij}) \cdot \sin \alpha_i \\ - \sin \beta_i \cdot (R_{b2} \cdot \sin \alpha_i - l_2(M_{ij}) \cdot \cos \alpha_i) + \eta(M_{ij}) \cdot \cos \alpha_i \\ - \sin \beta_i \cdot (R_{b2} \cdot \cos \alpha_i + l_2(M_{ij}) \cdot \sin \alpha_i) - \eta(M_{ij}) \cdot \sin \alpha_i \end{bmatrix} \begin{bmatrix} \phi_1^* \\ -\psi_1^* \\ \phi_2^* \\ -\psi_2^* \end{bmatrix} \]

- \( \alpha_i \) - pressure angle;
- \( \beta_i \) - spiral angle;
- \( \eta \) - horizontal axis of a contact point in the range of coordinates \((c_i, \eta)\) (figure 7);
- \( l_1 \) - distance between the center of a contact line, containing a contact point, and the initial point of contact T1;
- \( l_2 \) - distance between the center of a contact line, containing a contact point, and the end point of contact T2;
- \( R_{b1}, R_{b2} \) - the radii of roots of the driving and driven teeth respectively.

### 3.2.2 Modeling of effects of decentralization errors

Depending on the physical concept, decentralization of any gear is described by the parameter \( \epsilon \), which represents the distance between the axis of rotation of a tooth and the axis of inertia (gravity), as well as the parameter \( \Omega \), which represents the phase with respect to rotation of this tooth, as shown in figure 7, respectively. Displacement in coordinates \[ S, T, Z \] is specified as follows:

- for a pinion:
  \[ U_1(M_{ij}) = e_i \cos(\Omega_f - \lambda_1)S + e_i \sin(\Omega_f - \lambda_1)T^2; \]

- for a wheel:
  \[ U_2(M_{ij}) = e_2 \cos(\Omega_f - \lambda_2)S + e_2 \sin(\Omega_f - \lambda_2)T^2. \]

Thus, errors resulting from decentralization of the driving and driven wheels are defined by the following relationships:

\[ e_s(M_{ij}) = U_1(M_{ij}) \cdot n_1 + U_2(M_{ij}) \cdot n_2, \]

\[ e_s(M_{ij}) = \cos(\beta_b) \left[ e_1 \sin(\Omega_f + \alpha_i - \lambda_2) - e_2 \sin(\Omega_f + \alpha_i - \lambda_2) \right], \]

\[ e_s(M_{ij}) = \cos(\beta_b) \sum_{m=1}^{2} e_m \sin(\Omega_f + \alpha_i - \gamma_m). \]

### 3.2.3 Resultant errors

After alignment and decentralization errors are identified and calculated at the contact points on driving and driven lines, the total error \( M_{ij} \) at any point of contact will be the sum of alignment errors and decentralization errors at any contact point between teeth of the driving and driven wheels in accordance with the following relationship:

\[ e(M_{ij}) = e_s(M_{ij}) + e_s(M_{ij}). \]
3.3. Transmission errors
The concept of transport errors was first studied in (11), where the author considered the difference between the actual position of the driving wheel (a pinion) with respect to the axis of the driven wheel and the position it would be in, if the wheels had a regular geometric shape. This concept was then used in many studies (14–17), where transmission errors were considered the main source of induction for gears. These errors were divided into:

- transmission errors without load, which are symbolized by NLTE symbol and are errors resulting from the production process, such as alignment errors, decentralization and form errors, which are represented by the following relationship:

\[
NLTE = - \frac{E_{\text{max}}(t)}{\cos \beta_b} - \frac{\max_M \{e(M_{ij})\}}{\cos \beta_b},
\]

where \( E_{\text{max}}(t) \) - maximum value of errors resulting from the production process at the contact point in time;

- transport errors influenced by loads and symbolized by TE symbol, which are represented by the following relationship:

\[
TE = x + NLTE = Rb_1 \cdot \theta_1 + Rb_2 \cdot \theta_2 + NLTE.
\]

It is accepted that \( \theta_1, \theta_2 \) - are degrees of freedom in accordance with the position of a pinion and a wheel (symbol 1- for a pinion, symbol 2- for a wheel).

3.4. Equation of motion for a dynamic model of gears
Equation of motion for the executed dynamic model of gears is specified by presence of constant damping (\( c \)) according to the following differential form:

\[
m_{\text{tot}} \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = F_s + \frac{F_s}{\cos \beta_b},
\]

where: \( m_{\text{tot}} \) - dynamic model equivalent mass;

\( k \) - equivalent coefficient of engagement for the dynamic model, which is given on the basis of the ISO 6336 system [18]in accordance with the following relationship:

\[
k = k_m (1 + a \cdot g) \cos^2 \beta_b,
\]

where: \( k_m \) is a factor of average induction stiffness; \( a \) - relative change in contact conditions of pinion’s teeth (\( a < 1 \)); \( g \) - the coefficient correcting for direction of rotation and taking values: 1 or -1.

Static pinion load is determined from the formula:

\[
F_s = \frac{C_m}{Rb_1}
\]

where: \( F_s \) - static pinion load;

\( C_m \) - engine torque.

Effective load \( F_e \) involving geometric errors \( e(M_{ij}) \), is determined as follows:

\[
F_e = k \cdot e(M_{ij}) = k_m (1 + a \cdot g) \cdot e(M_{ij}) \cos^2 \beta_b.
\]

In using the relation of natural frequency \( \omega \), the final equation of motion is as follows:

\[
\ddot{x} + 2 \cdot \zeta \cdot \omega \cdot \dot{x} + \omega^2 (1 + a \cdot g) x = \frac{F_e}{m_{\text{tot}} \cdot \Omega^2} + \omega^2 (1 + a \cdot g) \cdot \frac{e(M_{ij})}{\cos \beta_b}.
\]
where:

\[ \omega = \frac{1}{\Omega} \cdot \cos \beta_p \cdot \sqrt{\frac{k_m}{m_m}}, \]

where \( k_m \) - the damping coefficient and its peak in the range 0.1-0.02 for a gear [16]. The previous equation of motion is a step-by-step time analysis using the New Mark method [17], which is considered to be one of the most commonly used analysis methods in the search for solutions of dynamic problems and analysis of equations of motion. When analyzing the equation of motion, ensure the lack of negative load and contact points outside the operating level defined by the points T1, T2 (figure 4). At each time step, contact lines are displaced within the working level, and therefore the contact point will move over time along the contact line and, therefore, engagement performance will be variable due to difference and location of the contact point on the tooth profile.

4. Discussion

Table 1 below shows technical and functional data for helical gears used in the present study.

| Numbers of teeth of a pinion and wheel | Z1 = 26, Z2 = 34 |
|-------------------------------|-----------------|
| Width of a pinion and wheel, mm | b1 = b2 = 20    |
| Module, mm                    | 4.5             |
| Inclination                   | 150             |
| Pressure angle                | 200             |
| Engine torque, Hm             | 183.4           |
| Weight of a pinion and wheel, kg | m1 = 0.95, m2 = 1.1 |
| Speed of a pinion, rpm        | n1 = 250        |

For the analysis, we used 48 engagement periods to obtain a stable gearshift pattern, and the total number of time increments was 2048. As the point of contact between the teeth of the driving and driven wheels is studied on the tooth profile over time, that is why the profile of the wheels’ teeth will be described in relation to \( \frac{t}{T_m} \), time t and engagement period tm. At that, the value of increment of time \( \Delta t \) was calculated in accordance with the relationship \( \Delta t = \frac{T_m}{tt} \), where \( tt \) is the number of time increments during one engagement period, which is calculated as the number of full increments in time divided by the number of engagement periods, on the other hand. Table 2 below shows limits of the operating area and engagement, calculated in accordance with technical and functional data of a transmission.

| Central measurement xe: | 0.04928 m |
|------------------------|-----------|
| Top circle radius re:  | 0.0319 m  |
| Main circle radius rb: | 0.0389 m  |
| 1- pinion:             | 0.01039 m |
| 2- wheel:              | 0.0215 m  |
| Number of time increments: Blocking period: | 42.66666 |
| Value of time increment: | 0.01 sec |
|                      | 0.0002344 sec |
4.1. Effect of alignment errors on load distribution

Figure 8 shows comparison of load distribution on a pair of teeth of helical gears at engagement with centering errors and without errors on the driving pinions, where we note the following:

- without errors: load distribution is uniform along the tooth profile, while its concentration is centered in the middle part of the tooth profile, that is, in the area of a point with a deviation along the length of the tooth width, and due to influence of its inclination (in the case of spur gears), load distribution has a straight form along the tooth width, because inclination is 00;
- in case of alignment errors: when we enter 0.001 radians as deviation of the driving tooth, we see a significant change in load distribution on one side and an increase in its value on the other. As the load is no longer uniform, it focuses on one part of the tooth width, while on the other part it is significantly reduced. An increase in the value of the load and its concentration on one part of the tooth width results in a decrease in engagement between the teeth of the gears and, thus, causes defects in fatigue and wear, since they are associated with defects in the value of the load and its distribution over the teeth and, as a result, a decrease in duration of the service life [17]. The same result will be in case when inclination of the front tooth is introduced by 0.001 radians, but to a lesser extent in terms of a change in load distribution and an increase in its value compared to the deviation effect. This result is explained in terms of the double effect of deviation and inclination on load distribution, however the effect of the deviation on load distribution is less than the effect of inclination.

It should be noted that the side of load focusing along the tooth width affected by deviation errors is related to the value of the deviation signal (the side of deviation with rotation of the driving tooth or opposite to it), as shown in figure 9. This effect is related to the direction of power flow from the motor.
4.2. Effect of alignment errors on transport errors (TE)

It is known that transport errors under load are considered as an important measure of the level of induction in gears and therefore for measuring vibration levels. The results of the effect of alignment errors, occurring on the master, on the average value of transmission errors over 48 engagement periods are shown in figure 10. It can be noted that the average value of transmission errors (the length of transmission errors) changes in case of errors with a very large increase in their values under the influence of inclination errors. Compared to the effect of deviation errors, and therefore the noise and vibration levels will increase significantly if these errors occur, and as a result, it can be said that transmission errors are highly dependent on alignment errors, and that is why transmission errors under load are considered to be an important measure for prediction of occurrence of alignment errors in gears.

![Figure 10. Effect of alignment errors on transport errors under load (TE).](image1)

On the other hand, alignment errors (e.g., inclination errors) will result in a significant increase in average value of transmission errors (TE) with increasing torque transmitted from the motor, and therefore the noise and vibration level will increase. As a result, the greater the torque transmitted by the electric motor, the greater the level of induction in gears with alignment errors, as shown in figure 11.

![Figure 11. The effect of the torque, transmitted by the engine, on the transmission errors (TE) when the drive wheel teeth are inclined (0.001 rad).](image2)

4.3. Effect of decentralization errors

As we have already said, gear decentralization errors are described by the parameter ε, which is the distance between the axis of rotation of a gear and its axis of inertia, and also by the parameter γ (phase angle). In our example we considered decentralization errors, which can occur on the driving pinion, in accordance with the following values: γ = 1 rad, ε = 25 μm. The results obtained with their influence on transmission errors are shown in figure 12. Based on the data obtained, it is possible to clearly trace the significant impact on distribution of transmission errors during blocking periods, on the one hand, and on their average values, on the other. This effect is more important than the effect of alignment errors shown in figure 10. Since alignment errors cause an increase in average values of transport errors, but without any change in the form of their distribution during blocking periods (transport errors are repeated in the same form and value during all blocking periods). The result causes decentralization errors - this is a much higher level of noise and vibration (induction level) than the one, which is caused...
by alignment errors due to a large change in the form of transport errors distribution during blocking periods and a significant increase in their values. It should be noted that decentralization errors are not related to the position of the blocking point, and therefore its influence on distribution of load on pinions’ teeth is negligible compared to alignment errors.

Figure 12. Major decentralization defects affecting transport errors under load (TE).

Figure 13 shows the degree of dynamic transport errors under load versus speed of the driving wheel when there are decentralization and alignment errors, where we note the following:

- decentralization errors significantly increase transmission error compared to alignment errors and for all speeds of the driving gear without any change in location of critical speed (725 rad / s). This leads to a high level of noise and vibration for a wheel, regardless of the value of a pinion’s rotation speed;
- alignment errors, in contrast to the effect of decentralization errors, change the value of critical speed of the mechanical system by 625 rad / s - in case of deflection errors, and 650 rad / s - in case of inclination errors with an increase in transmission errors at most rotation speeds.

On the other hand, the effect of deviation on the amplitude of transport errors becomes more important than the effect of the danger of inclination at high rotational speeds (over 400 rad / s).

Figure 13. Comparison of the effect of engineering errors on distribution of transport errors under load by the rotational speed of the driving tooth.

5. Results

Engineering (manufacturing) errors that have a negative effect on the operating conditions of a helical gear can be summarized as follows:

- alignment errors, especially deviation errors, lead to a significant increase in the value of the load on teeth of helical wheels and their concentration on one side of the tooth width. This effect
results in potential occurrence of other defects, such as fatigue defects, and, as a result, the service life of gears with such defects is much less than those without these defects;

• decentralization errors result in a significant change in distribution and value of transmission errors, and, therefore, their influence on the level of noise and vibration is very high compared to alignment errors, which caused only an increase in their average values without changing their distribution. Thus, the level of induction (noise and vibration) in gears is increased, and transmission errors are considered to be an important measure by means of which it is possible to predict the occurrence of manufacturing errors in the transmission.

The presented dynamic model can be used to study effects of other types of errors, such as, for example, shape errors, to study the effect of contact defects, such as fatigue and wear, on the static and dynamic behavior of gears.

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