Deformation analysis for geosynthetics in a landfill subjected to two adjacent local voids

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ABSTRACT

Studies for analysis of deformation behavior of geosynthetics suspending over voids in landfills are very limited, especially for situations in which several voids are closely distributed. Hence, the main objective of this paper is to study the behavior of geosynthetics over voids in a landfill. The coupling effect of two adjacent voids on geosynthetics and the simplified interface friction formula between geosynthetic and soil being considered, an analytical model is established to calculate the deformation of geosynthetics. Compared to the situation of two uncoupled cracks, the maximum strain of geosynthetics is greater and the maximum settlement is smaller when the two voids are coupled.

Keywords: geosynthetics, landfill, stress, deformation, adjacent local voids

1 INTRODUCTION

The existence of large cracks and sinkholes is the most common phenomenon in landfills resulting from the degradability, large compressibility and uneven distribution of garbage body (Giroud et al. 1990), which leads to the suspension of the geosynthetic over the subsidence area and large tensile stress and strain will be produced with in the geosynthetic, since the geosynthetic is subjected to gravity of the overlying garbage soil.

Methods and models for analyzing the geosynthetic behaviour proposed by the scholars are only suitable for a single underlying cavity (Giroud et al. 1990; Villard and Briançon 2008; Chen et al. 2008). However, there is more than one crack or cavity in a landfill commonly. If closely distributed, these voids will complicate the bearing condition of the geosynthetic, which obstructs a not only convenient but reliable evaluation of the geosynthetic’s serviceability. And analysis of the geosynthetic sheet will become complicated due to the coupling effect of these voids so that traditional method is no longer suitable in this case.

A simplified analytical model to evaluate the tension and strain of a landfill geosynthetic overlying two adjacent voids, is first proposed in this paper. The coupling effect of two adjacent voids and influence of several parameters associated with the voids’ distribution pattern on tensions, stresses, and strain of the geosynthetic are both investigated in this paper.

2 PLANE-STRAIN ANALYTICAL MODEL

2.1 Load imposed on the geosynthetic

Because of local subsidence or void, garbage soil overlying the subsidence area will deform along with deformation of geosynthetics (Fig. 1). Thus, soil arching effect (Terzaghi K. 1943) may occur, which leads to the decline of soil pressure on the geosynthetic. Load imposed on the geosynthetic sheet over the void is determined by Eq. (1) (Giroud et al., 1990). Qian et al. (2002) suggested a formula to calculate the coefficient of lateral soil pressure K, as $K = 1.06(1 - \sin \phi)$. 

$$p_i = q = \frac{(y - 2c / l_i)l_i}{2K \tan \phi} [1 - e^{-2K \tan \phi (l_i / l_1)}]$$

(1)

However, in the slide area, the load imposed on
geosynthetics can be calculated using Eq. (2) without considering the soil arching effect, as follows.

\[ p_2 = \gamma H \]  

\[ (2) \]

2.3 Interface friction between soils and geosynthetics

In order to simplify the formula being considered, value of the interface shear stress along the whole slide area is assumed to be uniformly distributed and taken as half of the peak shear stress. By adopting Coulomb’s friction law, interface friction force of any point in the slide area can be calculated using Eq. (3).

\[ f = \frac{\tau_1 + \tau_2}{2} = \frac{(c_1 + p_2 \tan \varphi_1) + (c_2 + p_2 \tan \varphi_2)}{2} \]  

\[ (3) \]

2.4 Behavior of geosynthetics in the slide area above two adjacent voids

In the subsidence area of two adjacent voids, behavior of the geosynthetic is almost the same as that of only a single void. However, since the two subsidence areas are closely distributed, geosynthetics in the slide area are likely to be coupled together, due to which neither of the subsidence geosynthetic sheet can be separately analyzed. On the contrary, when the two subsidence areas are far from each other, geosynthetics in the slide area are independent from each other, thereby can being regarded as uncoupled situation.

Deflection of geosynthetics resting over two adjacent subsidence areas is shown in Fig. 2.

![Fig. 2. Deflection of geosynthetics overlying two adjacent voids.](image)

To reasonably analyze behavior of the geosynthetic sheet in the slide area and to accurately evaluate the relation between displacement \( U_A \) and tensile force \( T_A \) on the edge of the voids, situation should be divided into two cases: uncoupled and coupled.

(1) Geosynthetics resting over two uncoupled voids

In this case, the two voids are located far away from each other so that they can be analyzed separately, which is similar to the research of Briançon and Villiard (2008). Only half of each single void is evaluated considering symmetry of the void.

In subsidence area, the following relation can be given.

\[ \frac{R_i}{2\beta_i}(\beta_i\sqrt{1+\beta_i^2} + \arcsin \beta_i) - R_i = U_A + 4p_iR_i^2 + \frac{\beta_i^2}{12\beta_i^2} \]  

\[ (4) \]

At point A, the change of the geosynthetic’s orientation taking place at the edge of the void will lead to a decrease in the geosynthetic sheet’s tensile force \((T_A < T_{\text{max}})\). The relation between \( T_A \) and \( T_{\text{max}} \) (algebraic values), based on the limit equilibrium of a geosynthetic resting on a circular arc (Villard and Briançon, 2008), is given by Eq. (5).

\[ T_A = T_{\text{max}} e^{A\sin \varphi} = p_i R_i / \beta_i \sqrt{1 + \beta_i^2} e^{\alpha \sin \beta_i \tan \varphi} \]  

\[ (5) \]

In slide area, as shown in Fig. 3, the geosynthetic’s displacement equals zero at point C. Considering the equilibrium equation of a unit element and constitutive relation, the following equation can be obtained:

\[ \frac{dU^2}{dx^2} = \frac{f}{J} \]  

\[ (6) \]

By introducing the boundary conditions:

\[ \begin{align*}
T_M &= -J \frac{dU_M}{dx} = -fx + T_r + R_f \\
U_M &= \frac{f}{2J} x^2 + \frac{(T_A + R_f)}{2J} x + \frac{T_r^2}{2J} + \frac{R_i R_f}{2J} \\
T_{co} &= T_A / f + R_i
\end{align*} \]

Combining Eqs. (4) and (5) with (7), the geosynthetic behavior in the situation of two uncoupled voids can be evaluated using the following equations.

\[ \begin{align*}
T_A &= T_{\text{max}} e^{A\sin \varphi} = \frac{p_i R_i}{\beta_i \sqrt{1 + \beta_i^2}} e^{\alpha \sin \beta_i \tan \varphi} \\
U_A &= \frac{T_A^2}{2J} \\
R_i &= \frac{R_i}{\frac{2\beta_i^2}{2J}} (\beta_i \sqrt{1 + \beta_i^2} + \arcsin \beta_i) - R_i = U_A + 4p_iR_i^2 + \frac{\beta_i^2}{12\beta_i^2}
\end{align*} \]

\[ (8) \]

\( \beta_i, U_A \) and \( T_A \) can be obtained by iterative calculation of Eq. (8) with an initial value of \( \beta_i \).

(2) Geosynthetic resting over two coupled voids

When the two voids are coupled, geosynthetic in the slide area (segment \( AB \) in Fig. 2) will relatively slip towards the nearer subsidence area. As a result, in the left and right side of segment \( AB \), displacement and tensile force of the geosynthetics will be in the opposite direction.
Similarly, C is assumed to be the certain point of geosynthetic where the relative displacement is zero. On the left side of point C, the geosynthetic moves towards the \( O_1 \) void, while the geosynthetic on the right side of point C moves towards the \( O_2 \) void, as shown in Fig. 4.

![Equilibrium of a unit element (two adjacent voids).](image)

Fig. 4. Equilibrium of a unit element (two adjacent voids).

As to segment \( O_1C \), Eqs. (4) and (5) can also be used to the subsidence area and the junction point between subsidence area and slide area. In the slide area, based on the equilibrium of a unit element of the geosynthetic sheet (Fig. 4), Eq. (9) is given as follows.

\[
\frac{dU^2}{dx^2} = \frac{f}{J} \quad \begin{cases} T_M = T_A & x = R_1 \\ U_M = 0 & x = L_{CO_2} \end{cases} \tag{9}
\]

As to segment \( O_1C \), similar equations can be obtained. In addition, at point C, the tension force at left and right side of the geosynthetic are equal, thus, \( T_M = T_N \).

Also, Eq. (10) shows the geometric relation within the geosynthetic.

\[
L_{CO_1} = L - L_{CO_2} \tag{10}
\]

Finally, equations evaluating behavior of the geosynthetic overlying two adjacent voids can be obtained as follows.

\[
\begin{align*}
\frac{R_1}{2\beta_1} (\beta_2^0 + \sqrt{1 + \beta_2^1 \beta_2^0} + \arg \sin \beta_2) - R_1 &= U_A + \frac{R_1}{2} \left[ \frac{3}{\beta_2^0} + \frac{\beta_2^1}{12\beta_2^0} \right] \\
\frac{R_2}{2\beta_2} (\beta_1^0 + \sqrt{1 + \beta_1^1 \beta_1^0} + \arg \sin \beta_1) - R_2 &= U_A + \frac{R_2}{2} \left[ \frac{3}{\beta_1^0} + \frac{\beta_1^1}{12\beta_1^0} \right] \\
T_A &= T_{AAA} \left( \frac{\pi \tan \beta_1}{2} \right) = \frac{R_1}{\beta_1^0} \left( \frac{3}{\beta_2^0} + \frac{\beta_2^1}{12\beta_2^0} \right) \\
T_B &= T_{BB} \left( \frac{\pi \tan \beta_2}{2} \right) = \frac{R_2}{\beta_2^0} \left( \frac{3}{\beta_1^0} + \frac{\beta_1^1}{12\beta_1^0} \right) \\
U_A &= \left( f(L_{CO_2} - R_1) / 2 - T_A (R_2 - L_{CO_2}) \right) / J \\
U_B &= \left( f(L_{CO_2} - R_2) / 2 - T_B (R_1 - L_{CO_2}) \right) / J \\
L_{CO_1} &= \left( T_A - T_B \right) / 2 + \left( L + R_2 - R_1 \right) / 2 \\
L_{CO_2} &= L - L_{CO_1} \tag{11}
\end{align*}
\]

(3) Procedure to analyze the behavior of geosynthetic above two adjacent voids

The general procedure required to analyze the behavior of geosynthetic above two adjacent voids in a landfill can be divided into three steps:

Step 1: Compute the length \( L_{CO_1} \) and \( L_{CO_2} \) from subsidence center \( O_1 \) and \( O_2 \) where the relative displacement of geosynthetic is zero respectively to point C, using the method mentioned in part (1);

Step 2: If \( L_{CO_1} + L_{CO_2} < L \), analyze the behavior of geosynthetic using the method proposed in part (1).

Step 3: If \( L_{CO_1} + L_{CO_2} > L \), analyze the behavior of geosynthetic using method proposed in part (2).

Given the complexity of solution process, a Matlab program is written to solve these equations.

3 GEOSYNTHETIC BEHAVIORS OVER ONE VOID AND TWO ADJACENT VOIDS

In this paper, comparison and analysis are carried out using geometric and material parameters coming from Chen et al. (2008).

3.1 Strain of geosynthetics

Fig. 5 shows the strain of geosynthetic resting on a local subsidence area and two adjacent subsidence areas respectively. Center distance between the two coupled subsidence centers is 3.0 m. Apparently, the maximum strain occurs at the junction points between subsidence area and slide area. In subsidence area, the strain of geosynthetic generally increases with the increase of distance to the subsidence center, while in slide area, the strain decreases with the increase of distance to the subsidence center. In addition, in the left subsidence area A, the geosynthetic’s strain in the situation of two coupled voids is larger than that of a local void due to the transformation of load from slide area to subsidence area.

![Strain of geosynthetics in two different situations.](image)

Fig. 5 Strain of geosynthetics in two different situations.

3.2 Settlement of geosynthetics

Fig. 6 shows the settlement of geosynthetics resting on a local subsidence area and two adjacent subsidence areas respectively. Also, in this case center distance between the two coupled subsidence areas centers is 3.0
m. The maximum settlement of geosynthetics appears in the subsidence center. However, compared to the single void situation, the geosynthetic’s settlement in subsidence area is smaller when resting on two adjacent voids, which is opposite to the geosynthetic’s strain.

Fig. 6. Settlement of geosynthetics in two different situations.

4 EFFECT OF SUBSIDENCE AREA PROPERTIES ON GEOSYNTHETIC BEHAVIOR

In the subsidence area, at points that have the same distance to the junction points, strain is larger when \( L = 3.0 \) m than that when \( L = 4.0 \) m. However, in the slide area, settlement of these points when \( L = 3.0 \) m is smaller than that when \( L = 4.0 \) m. As a result, the shorter the distance between the two subsidence centers is, the easier the geosynthetic above two voids being coupled with each other will be together with an increase in the maximum strain and a decrease in the maximum settlement.

Fig. 7. Effects of the width of two subsidence areas.

The effect of void \( B \)'s width on the maximum strain (Fig. 7) and maximum settlement of geosynthetics overlying void \( A \) when the width of void \( A \) remains 2.0 m \( (R_A = 1.0 \) m). \( R_1 \) and \( R_2 \) are the half width of void \( A \) and void \( B \) respectively and \( L \) is the distance between the two subsidence centers. It can be seen that with the increase of void \( B \)'s width, the maximum strain and maximum settlement of geosynthetics overlying void \( A \) increases and decreases respectively, which is triggered by the slip of geosynthetics toward void \( B \) resulting from the larger overlying load imposed on void \( B \) as its width increases. A conclusion deriving from this analysis can be drawn that geosynthetics will be uncoupled when the distance between two subsidence centers is more than 2.1 times the summation of the half width of two voids.

6 CONCLUSIONS

The findings from this study can be summarized as follows:

a). The proposed method is suitable for analyzing the behavior of geosynthetics overlying two adjacent voids.

b). In actual landfill projects, attention should be paid to the phenomenon that increased strain and decreased settlement of geosynthetics will occur if the two subsidence areas are coupled with each other.

c). Due to the complicated interaction mechanisms between soil and geosynthetic, further studies should be carried out to better understand the interface friction. In addition, the theoretical model should be validated further.

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