High order flow harmonics of identified hadrons in 2.76 A TeV Pb+Pb collisions

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(Dated: February 8, 2016)

Using iEBE-VISHNU hybrid model with the AMPT initial conditions, we study the higher order flow harmonics of identified hadrons in 2.76 A TeV Pb+Pb collisions. Comparison with the recent ALICE measurements at 20-30% centrality shows that our calculations nicely describe the data below 2 GeV, especially for the v2, v3 and v4 mass-orderings among pions, kaons and protons. We also extended the calculations to other centrality bins, which presents similar mass-ordering patterns for these flow harmonics as the ones observed at 20-30% centrality. In the later part of this article, we explore the development of vn mass ordering/splitting during the hadronic evolution through the comparison runs from iEBE-VISHNU hybrid model and pure hydrodynamics with different decoupling temperatures.

PACS numbers: 25.75.Ld, 25.75.Gz, 24.10.Nz

I. INTRODUCTION

It is widely believed that the quark-gluon plasma (QGP) has been created in relativistic heavy ion collisions at Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) [1–4]. The observation of strong collective flow and the successful description from hydrodynamics reveal that the QGP is strongly coupled and behaves as an almost perfect liquid [2–4]. With the development of viscous hydrodynamics and the related hybrid models, the elliptic flow, triangular flow and other higher order flow harmonics have been widely used to study and extract the transport properties of the QGP [5–22]. These higher order flow harmonics also reveal that the created QGP fireball strongly fluctuates event by event, which finally transforms into final state correlations of the produced hadrons after the collective expansion [20–31]. Meanwhile, other flow measurements, such as vn in ultra-central collisions [32], the distributions of event-by-event flow harmonics [33], the correlations between flow angles [34], and the correlations between flow harmonics of different order [35, 36] etc., provide more information on the initial state fluctuations, which help to constrain the initial conditions of hydrodynamic simulation for a precise extraction of the QGP transport properties.

Besides these flow data of all charged hadrons, the elliptic flow of identified hadrons have also been extensively measured and studied at RHIC and the LHC [12, 37–44]. As a typical feather of the collective expansion, the mass ordering of elliptic flow among various hadron species has been observed in different colliding systems [37–41], which reflects the information of the hadronic evolution after the phase transition. Recently, the ALICE collaboration has measured the higher order flow harmonics of identified hadrons in ultra-central and semi-central Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \) [45], which showed that \( v_3(p_T) \) and \( v_4(p_T) \) present similar mass ordering patterns among pions, kaons and protons as observed in the elliptic flow. It is thus the right time to systematically study these new flow measurements with sophisticated model calculations. In this paper, we will implement iEBE-VISHNU hybrid model that couples 2+1-d viscous hydrodynamics to the hadron cascade model, together with the fluctuating AMPT initial conditions, to calculate \( v_2(p_T) \), \( v_3(p_T) \) and \( v_4(p_T) \) of pions, kaons and protons in 2.76 A TeV Pb+Pb collisions. We will compare our model calculations with the recent ALICE measurements at 20-30% centrality and make predictions for other centrality bins. In the later part of this article, we will explore the development of \( v_n \) mass orderings/splittings during the hadronic evolution through the comparison runs from iEBE-VISHNU and from pure hydrodynamics decoupled at different temperatures.

II. SETUP

iEBE-VISHNU [46] is an event-by-event version of VISHNU hybrid model [47] which combines (2+1)-dimensional viscous hydrodynamics (VISH2+1) [10, 48] to describe the QGP expansion with the hadron cascade model (UrQMD) [49, 50] to simulate the evolution of the hadronic matter. The transition between the macroscopic and microscopic approaches is determined by a switching temperature \( T_{sw} \), which is set to 165 MeV as used in Ref. [12, 42, 43]. For the hydrodynamic simulations, we input an equation of state (EoS) \( s95p-PCE \) constructed from combing the lattice EoS for the QGP phase with a partially chemical equilibrium EoS for the hadron resonance gas phase [51, 52].

In the published version of iEBE-VISHNU [46], the initial entropy density (energy density) profiles are generated from either MC-Glauber or MC-KLN model [53, 54].
Although, with these two initial conditions, the hybrid model simulations successfully describe the elliptic flow of all charged and identified hadrons at RHIC and the LHC [12, 42–44], they fail to simultaneously fit all the flow harmonics $v_n$ (n=2-5) with one constant $\eta/s$ and other fine tuned parameters [55, 56]. Recently, Ref. [57] has shown that, with the fluctuating initial conditions generated from AMPT, event-by-event viscous hydrodynamics (VISH2+1) could nicely describe the integrated and differential flow harmonics $v_2$, $v_3$, $v_4$ and $v_5$ of all charged hadrons in 2.76 A TeV Pb+Pb collisions. Following that paper [57], we implement a string melting AMPT model (version 2.21) 1 to generate the needed initial conditions. We assume the system reaches thermalization at the hydrodynamic starting time $\tau_0$. The corresponding initial energy density profiles in the transverse plane are constructed from the deposed energies of produced partons from AMPT (within space-time rapidity $|\eta_s| < 1$) with a Gaussian smearing [57, 59]:

$$\epsilon(x, y) = K \sum_i \frac{E_i^*}{2\pi \sigma^2 \tau_0 \Delta \eta_s} \exp \left( -\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma^2} \right),$$  \hspace{1cm} (1)$$

where $\sigma$ is the Gaussian smearing factor, $E_i^*$ is the Lorentz invariant energy of the produced partons and $K$ is an additional normalization factor. Here, we neglect the initial flow 2 because one can not precisely construct both initial flow and initial energy density for (2+1)-d hydrodynamics with an exact boost invariance from the (3+1)-d AMPT simulations without such boost invariance. We thus truncate the total produced partons of AMPT within $|\eta_s| < 1$ ($\Delta \eta_s = 2$) and then construct the needed boost invariant initial energy density profiles in the transverse plane according to Eq. (1).

In the following calculations, the hydrodynamic starting time $\tau_0$ is set to 0.4 fm/c with the normalization factor $K$ slightly tuned around 1 to fit the multiplicity and $p_T$ spectra of all charged hadrons in the most central Pb+Pb collisions. We also find that, with the Gaussian smearing parameter and the specific shear viscosity respectively tuned to $\sigma = 0.6$ fm and $\eta/s = 0.08$, iEBE-VISHNU with AMPT initial conditions could simultaneously fit the differential flow harmonics $v_2(p_T)$, $v_3(p_T)$ and $v_4(p_T)$ of all charged hadrons at various centralities, especially from semi-central to semi peripheral Pb+Pb collisions. For simplicity, we neglect the bulk viscosity, net baryon density, and heat conductivity as most of the hydrodynamics and hybrid model calculations at the LHC energies [18, 20–22, 42, 43]. Following Ref. [60], the differential flow harmonics of identified hadrons are calculated with the Q-cumulant methods [60–62] with a pseudo-rapidity gap $|\Delta \eta| > 0$.

1 Following [58], the related parameters in AMPT are set as: Lund string fragmentation parameters $a = 0.5$ and $b = 0.9$, strong coupling constant $\alpha = 0.33$, and the screening mass $\mu = 3.2$ fm$^{-2}$.

2 Using (3+1)-d ideal hydrodynamics, Pang and his collaborators have demonstrated that initial flow from AMPT does not significantly influence the soft hadron data like the $p_T$-spectra and the elliptic flow [59].
**III. RESULTS AND DISCUSSIONS**

Fig. 1 shows the $p_T$-spectra of pions, kaons and protons in 2.76 A TeV Pb-Pb collisions at 10–20%, 20–30%, 30–40% and 40–50% centralities. After tuning the starting time $\tau_0$ to 0.4 fm/c, iEBE-VISHNU with AMPT initial conditions nicely fits the ALICE data, especially for the slope of the $p_T$-spectra of various hadron species, which indicates that iEBE-VISHNU generates a proper amount of radial flow during its QGP and hadronic expansion. We have also noticed that the $p_T$-spectra of pions in our model calculations are slightly below the experiment data at 30-40% and 40-50% centralities because iEBE-VISHNU under-predicts the multiplicities of all charged hadrons in semi-peripheral and peripheral collisions.

Instead of using the empirical formula of AMPT to cut the centrality as used in [57, 58] (which gives better descriptions of the centrality-dependent soft hadron data with the typical AMPT parameter sets), we define the centrality bins from the distributions of initial entropy, which leads to the slight under-predictions of multiplicity, spectra of all charged hadrons for semi-peripheral and peripheral collisions. Since the related effects on the differential flow are pretty small, we do not further fine-tuning the typical parameters inside AMPT [58] to achieve better descriptions for the centrality dependent multiplicity and spectra, but leave it to the future study.

Fig. 2 shows the differential flow harmonics $v_n(p_T)$ of pions, kaons and protons in 2.76 A TeV Pb-Pb collisions. The ALICE $v_2(p_T)$ data at 10–20%, 20–30% and 30–40% and 40–50% centralities are measured with the scalar product method with a pseudorapidity cut $|\Delta \eta| > 0.9$ [41]. The $v_3(p_T)$ and $v_4(p_T)$ data at 20–30% centrality bin are also from ALICE, but measured with the Q-cumulant method ($|\Delta \eta| > 0$). The theoretical curves are calculated from iEBE-VISHNU with AMPT initial conditions and the parameter sets described in Sec. II. Since this paper aims to study and predict the higher order flow harmonics of identified hadrons for the ALICE experiment, we calculate $v_n(p_T)$ from iEBE-VISHNU simulations with the Q-cumulant method.
with a zero pseudorapidity cut ($|\Delta \eta| > 0$) following Ref. [45].

For 20-30% centrality, these ALICE data in Fig. 2 show that $v_3(p_T)$ and $v_4(p_T)$ present similar mass orderings among pions, kaons and protons as the $v_2$ mass ordering once observed in different colliding systems at RHIC and the LHC [4]. With $\eta/s$ and $\sigma$ fine tuned to fit $v_\pi(p_T)$ of all charged hadrons, \textit{iEBE-VISHNU} with AMPT initial conditions nicely describes $v_2(p_T)$, $v_3(p_T)$, and $v_4(p_T)$ of identified hadrons at 20-30% centrality, especially for the mass ordering among pions, kaons and protons. In Fig. 2, we also predict $v_3(p_T)$ and $v_4(p_T)$ of protons, kaons and protons at 10 – 20%, 30 – 40%, and 40 – 50% centrality bins, which show similar mass orderings patterns as calculated/measured at 20-30% centrality [5].

To explore in depth the development of flow anisotropy and its distribution among various hadron species during the hadronic evolution, we further calculate the $p_T$ spectra and differential flow harmonics of identified hadrons at 20-30% centrality with three comparison runs, using the same AMPT initial conditions and parameter sets: (1) / (2) viscous hydrodynamics simulations + resonance decays, with the decoupling temperature set to $T_{dec} = 165$ MeV / $T_{dec} = 120$ MeV, (3) \textit{iEBE-VISHNU} simulations that combines viscous hydrodynamics with the \textit{UrQMD} hadronic afterburner, which are the same simulations as presented in Fig. 1 and Fig. 2 for 20-30% centrality. Please note that $T_{dec} = 165$ MeV is also the switching temperature between the macroscopic and microscopic approaches in the \textit{iEBE-VISHNU} simulations. With $T_{dec} = 165$ MeV, we therefore eliminate the hadronic evolution in the hydrodynamic simulations and concentrate on the flow developed in the QGP phase. For the hydrodynamic simulations with $T_{dec} = 120$ MeV, the hadronic matter below $T_c$ expands like a fluid, which keeps the local thermal equilibrium till the kinetic freeze-out. Meanwhile, the specially constructed equation of state s95p-PCE maintains the partially chemical equilibrium of the system during the hadronic evolution. In contrast, \textit{iEBE-VISHNU} below $T_{sw}$ simulates the microscopic expansion of a hadron resonance gas with both scatterings and decays, which quickly evolves out of equilibrium and finally becomes free-streaming hadrons in the very late stage of evolution.

FIG. 3. (Color online) $p_T$-spectra of pions, kaons and protons in 2.76 A TeV Pb+Pb collisions at 20 – 30% centrality, calculated from \textit{iEBE-VISHNU} and from \textit{VISH2+1} with $T_{dec} = 165$ MeV and $T_{dec} = 120$ MeV.

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4 The ALICE collaboration has also measured $v_n(p_T)$ of identified hadrons in ultra-central collisions at 0-1% centrality, which show similar mass ordering patterns as 20-30% centrality.

5 The flow harmonics in ultra-central collisions are very computationally expensive for \textit{iEBE-VISHNU} simulations. Previous studies have also shown that integrated $v_n$ ($n=2, 3, 4, 5, 6$) of all charged hadrons can not be simultaneously fitted without further considering the effects from bulk viscosity, nuclear-nucleon correlations in the initial state, etc. [64]. Therefore, we will not further explore $v_n(p_T)$ of identified hadron at 0-1% centrality as already measured by ALICE in our current investigations.

Fig. 3 presents the differential flow harmonics $v_2(p_T)$, $v_3(p_T)$ and $v_4(p_T)$ of identified hadrons in 20-30% Pb+Pb collisions, calculated from viscous hydrodynamics with $T_{dec} = 165$ MeV and $T_{dec} = 120$ MeV, and from \textit{iEBE-VISHNU}. In each panel, the theoretical calculations are respectively compared with the ALICE data. The \textit{iEBE-VISHNU} results in the right panels are exactly the same as the corresponding ones shown in Fig.2, which nicely fit the ALICE data of pions, kaons and protons at 20-30% centrality. In contrast, pure hydrodynamics simulations without hadronic evolution ($T_{dec} = 165$ MeV, left panels) under-predict the pion’s $v_2$, $v_3$ $v_4$ due to the insufficient flow anisotropy development in the QGP phase. Meanwhile, the smaller radial flow there (indicated by Fig. 3) also leads to smaller mass-splittings between pions and protons for different flow harmonics. With $T_{dec}$ decreased to 120 MeV, the hydrodynamics simulations (middle panels) roughly fit the ALICE data for pions and show enhanced the mass-splittings between
FIG. 4. (Color online) $v_n(\pT)$ ($n = 2, 3, 4$) of pions, kaons and protons in 2.76 A TeV Pb+Pb collisions at 20–30% centrality, calculated from VISH2+1 with $T_{\text{dec}} = 165$ MeV (left panels) and $T_{\text{dec}} = 120$ MeV (middle panels) and from iEBE-VISHNU (right panels).

pions and protons for $v_2$, $v_3$ and $v_4$ due to the additional collective flow developed in the hadronic stage. However, the mass-splittings are still under-predicted when compared with the ALICE data since $v_2(\pT)$, $v_3(\pT)$ and $v_4(\pT)$ of protons are still well above the experimental data. Such situation is similar to the case of elliptic flow as found in Ref. [65], which showed that, although pure hydrodynamics could nicely fit many soft hadron data of all charged hadrons and pions, the elliptic flow of protons below 2 GeV are still over predicted in central and semi-central Pb+Pb collisions. A later calculations from VISHNU hybrid model [42] pointed out that the hadronic scatterings in the UrQMD evolution rebalance the generation of radial and elliptic flow, leading to improved descriptions of the proton $v_2$ as well as the mass-splitting of $v_2$ between pions and protons. The comparison runs in Fig. 4 here demonstrate that microscopic hadronic scatterings in iEBE-VISHNU are also important for the nice fit of $v_3(\pT)$ and $v_4(\pT)$ of protons, which leads to an improved description of the mass-splitting between pions and protons for these higher order flow harmonics.

IV. SUMMARY

Using the event-by-event VISHNU hybrid model iEBE-VISHNU that couples 2+1-d viscous hydrodynamics to the hadron cascade model UrQMD, together with the fluctuating AMPT initial conditions, we studied the $\pT$-spectra, differential flow harmonics $v_2(\pT)$, $v_3(\pT)$ and $v_4(\pT)$ of identified hadrons in 2.76 A TeV Pb+Pb collisions. With fine tuned parameters, our model calculations nicely described the ALICE data at 20–30% centrality, especially for the mass-orderings among pions, kaons and protons. We also predicted the higher order flow harmonics of identified hadrons at other centrality bins, which showed a similar mass-ordering pattern as measured/calculated at 20–30% centrality.

We explore the development of $v_n$ mass orderings/splittings during the hadronic evolution through the comparison runs from iEBE-VISHNU simulations and from pure hydrodynamic simulations decoupled at different temperatures. We found that, in pure hydrodynamics, the additional radial flow developed in the fluid-like hadronic phase could enhance mass splitting between pi-
ons and protons for all $v_n$ ($n=2, 3, 4$). However, the protons data are still over-predicted there. In contrast, the microscopic hadronic scatterings in iEBE-VISHNU rebalance the generation of radial and anisotropy flow, leading to nice descriptions of the $v_2$, $v_3$ and $v_4$ data for pions, kaons and protons.

At last, we would like to remind the interested readers that the past research on elliptic flow [42] has showed, although VISHNU hybrid model improves the description of elliptic flow for pions, kaons and protons from most central to semi-central collisions, it still slightly under-predicts the proton $v_2$ in semi-peripheral collisions. Further investigations on strange and multi-strange hadrons [43, 44, 66] also showed that VISHNU simulations can not correctly reproduce all the $v_2$ mass orderings among various stable hadrons, especially for the mass ordering between protons and Lambdas. Currently, the ALICE collaboration only release the measurement of higher order flow harmonics of pions, kaons and protons in ultra-central (0-1%) and semi-central (20-30%) Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ A TeV. In fact, extending the current measurements to various centrality bins and collision energies, and even to rarely produced strange and multi-strange hadrons will further test our model calculations, broadening our knowledge on the hot and evolving hadronic matter after the QCD phase transition.

ACKNOWLEDGMENTS

We thank R. S. Bhalerao, L. Pang and X. Zhu for valuable discussions. This work is supported by the NSF and the MOST under grant Nos.11435001 and 2015CB556900. H. X. is partially supported by China Postdoctoral Science Foundation with grant No. 2015M580908. We gratefully acknowledge the extensive computing resources provided to us by Supercomputing Center of Chinese Academy of Science (SCCAS) and Tianhe-1A from the National Supercomputing Center in Tianjin, China.

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