Nonlinear model and simulation of a rolling bearing

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Abstract. This paper presents an analysis of a nonlinear model of a rolling bearing with two degrees of freedom. The acceleration response provides information regarding the relationship between vibration and a different number of balls, values of internal clearance and rotary shaft speeds. The acceleration response is higher when the radial internal clearance and the shaft speed increase. When the number of balls increases, the acceleration decreases for the entire range of radial internal clearance. These results are useful for understanding the vibration response mechanism of ball bearings under variable parameters.

1. Introduction

The rolling bearings are one of the most important components in the entire machine construction domain. They are widely used in various mechanical equipment across industries, including the aerospace, automotive, construction, mining, steel, paper, textile, railways, and renewable energy [1]. The bearings are frequently implemented due to their low friction characteristics and their load-carrying capacity. Rolling bearings work under different conditions and are frequently subject to heavy loads generated in the machinery, as well as to varying time and space dynamic loads, not to mention numerous other unknown factors that could affect the bearing life. Rolling bearings affect the running accuracy, reliability and service life of mechanical equipment. The complexity of bearing internal geometry and the loading mechanism will influence the vibration expression of its behaviour [2]. To prevent machine breakdown it is often necessary to diagnose and detect defects in rolling bearings so as to replace it before the fault reaches a certain degree. The dynamic behaviour of the bearing plays a pivotal role especially in high-rotation systems where large dynamical forces occur that can cause its deformation [3].

When the bearing is running and its defective component, either a rolling element, an outer raceway or an inner raceway, interacts with its corresponding mating component, either defective or non-defective, abrupt changes in the contact stresses occur. It can be a root cause of undesirable vibration, excited by the changes in the bearing structure, which can be also observed in other structural components connected to the bearing and consequently may lead to the emergence of acoustic signals. These signals can be used to detect the presence of a defect with the help of appropriate condition-based diagnostic techniques [4]. Theoretical ball bearing models help understand the mechanisms of vibration signal generation, thus enabling the study of the impact of different parameters, such as load and force transmission paths, to better understand the vibration generated at the beginning of an incipient defect. Despite the long history of the use of bearings the vast experience accumulated with regard to their
fatigue-life prediction or load capacity calculation, relatively few models have been proposed to explain the dynamic behaviour of bearings [5]. Most of these models proposed by many researchers refer to the simulation of the ball bearing movement in the presence of a localised defect. As a first, Sunnersjo [6] proposed the mathematical model of bearing vibrations in which a two degree of freedom (2DOF) system was built, and which provided the load-deflection according to Hertzian contact theory. In the model, the mass and inertia of the rolling elements were neglected. A decade later, Sunnersjo [7] explained in detail the relation between surface irregularities and the vibration characteristics of the radial bearing with positive clearance and subjected to radial loads. Another study focused on the influence of inner ring waviness and the non-uniform diameter of the rolling elements on vibratory response [8]. At about the same time, McFadden and Smith [9] proposed a mathematical model of bearing vibration for a single-point and multi-point defect under radial load, where the defect-induced vibrations are modelled as a result of a series of impulses as the bearing ran at a constant frequency and radial load. In that model, it was assumed that the excitation was represented as a series of repeated impulses when the rolling element strikes the point of defect. These impulses may excite resonances in the bearing and machine. The bearing rotation exhibits a periodical characteristic of the frequency specified by the location of the defect, whether it is on the inner raceway, outer raceway or one of the rolling element [8],[10]. Feng et al. [11] were developing the model with 2DOF further considering the influence of bearing cage and ball slide and also the impact of vibration response for the defect in outer and inner raceway.

In the early 1990s, more refined bearing models were introduced. Lim and Singh [12] proposed the 5DOF model of bearing, which assumed the rigid inner and outer rings and deformable balls. Su et al. [13] extended the model proposed by McFadden for the determination of periodic characteristics of various loading and transmission path effects and their impact on vibration. These effects are associated with the axial and radial loads, misalignment and dynamic unbalance of the shaft [14]. In the model, the bearing vibration was modelled as the system output subjected to excitations from surface waviness and roughness through the lubrication film. It was shown that the vibration spectrum of a normal bearing under a preloaded condition has a pattern of equal frequency spacing distribution that is similar to that of a defective bearing [8]. In the late 1990s, Tandor and Choudry [15] proposed an analytical model for predicting the vibration frequencies of rolling bearings and the amplitudes of significant frequency bearing components caused by a defect on the inner/outer raceway or on the rolling elements. The model considered the effect of loading and pulse shape on the vibration amplitude [8]. Randal and Antoni proposed the vibration fault signal model in which the slip between the rolling elements was introduced. This slip may cause random fluctuation between the impulses due to the defect in the bearing. In the model, the vibration signal with random fluctuation will have a spectrum where the defect frequency components are smeared into each other [16]. Patil et al. [14] presented the 2DOF analytical model of ball bearing for predicting the effect of a localised defect on the bearing vibration and developed a computer program to simulate the defect on the raceway with the results presented in the time and frequency domains. In the model, contacts between the ball and the raceway are considered as nonlinear springs and Hertzian contact deformation theory is used to calculate the contact forces. Grajale et al. presented in their work [8] the 2DOF mathematical ball bearing model with localised outer raceway defect. In the paper, the model of the bearing was considered as mass-spring-damper system considering each rolling element as a contact spring-damper pair, based on Hertzian contact deformation theory. The paper made use of Peak Value analysis to reveal the characteristic frequencies of the studied type of the defects. In Sadok’s work [8], the dynamic behaviour of bearing as a coupled three-degree-of-freedom system was presented. In the paper, the generation of vibration by a point defect in bearing was modelled as a function of rotation, the distribution of the load, the bearing structure elasticity, the oil film characteristics and the transfer path between the bearing and the transducer. Cui et al. presented the 5DOF nonlinear vibration model for fault severity assessment of ball bearings [17]. In our study, the outer raceway defect size parameter was introduced into the dynamic model. The vibration response signal of rolling bearing was simulated under different fault size. The results of the study may help understand the vibration response mechanism of rolling bearing under a various degree of defects.
This work is focused on bearing behaviour with numerical 2DOF model. The vibration response signal is simulated under different number of balls, different radial internal clearance and different bearing rotary speed. The 2DOF model is implemented in MatLab-Simulink environment. This model accounts for the effect of normal force due to ball race contact and ignores the effects resulting from the traction force.

2. Model of bearing

In regular rolling bearing systems, the inner ring is seated on the shaft and rotates together with the rotating shaft, whereas the outer ring is mounted into the housing and the balls undergo pure rolling movement on the raceways. Usually, every ball bearing should be manufactured with more or less positive radial clearance. Hence, during the operation of the bearing, the raceway is divided into the loaded and unloaded zones, based on the range of radial load. The balls which enter the loaded zone are deformed and produce varying flexibility vibration. The rolling element bearing is the most critical element of rotating machines, therefore its proper description of the bearing is of high importance from the viewpoint of the dynamic behaviour of the entire machine. The key components of any vibratory system are the mass, stiffness, damping and the external forces. Therefore, the deep groove ball bearing with a single row of balls is simplified and modelled as a nonlinear spring-damper system (figure 1).

![Figure 1. 2DOF model of the rolling bearing.](image)

In the model, the outer ring was fixed in a rigid, non-deformable housing (the model considers the case of a non-rotating outer race), and the mass of the balls is neglected. The inner ring rotates together with the shaft. The elastic deformation between the raceways and each point of contact with the ball is assumed to be Hertzian. The assumptions and simplifications of the model are:

- the ball bearing model has equi-spaced balls rolling on the surface of the inner and outer raceway and there is no interaction between them,
- the slipping effect of the balls is neglected,
- the motion of race and balls occur in the bearing’s plane,
- the deformations at the contacts are described by the Hertzian contact theory.

The inner raceway is fixed rigidly to the rotor shaft. Therefore, it rotates with a constant angular speed. The equations of motion of the inner race can be written using the Newton equations, and have the following form:

\[ m_s \ddot{x}_s + c_s \dot{x}_s + k_s x_s + F_x = 0, \]

\[ m_s \ddot{y}_s + c_s \dot{y}_s + k_s y_s + F_y = F, \]
where $m_i$ is the mass of the shaft together with the inner ring, $c_i, k_i$ are the damping and stiffness of the shaft with inner ring, the $F_x, F_y$ are nonlinear forces that describe the contact between the inner raceway and the rolling elements. 

The complete deformation of any rolling element ($j$) is the result of relative displacement between the inner and outer rings ($x_j, y_j$), the angular position ($\phi_j$) of the rolling element, and the radial internal clearance ($RIC_j$). The $RIC$ parameter can be used for modelling of the lubricant film thickness. The deformation $\delta_j$ is obtained from [18]:

$$\delta_j = (x_j) \cos \phi_j + (y_j) \sin \phi_j - RIC,$$

The nonlinear contact forces of any rolling element with the raceway are described by the Hertzian theory. These forces have the following formulations [17]:

$$F_x = k_b \sum_{j=1}^{Z} H(\delta_j) \delta_j^{1.5} \cos \phi_j, \quad F_y = k_b \sum_{j=1}^{Z} H(\delta_j) \delta_j^{1.5} \sin \phi_j,$$

$H(\delta_j)$ is the Heaviside function that defines the condition of deformability (the rolling element is in contact with the raceway or lack of contact -- switch function), $k_b$ means the stiffness of the rolling elements, and $Z$ denotes the number of the rolling elements (balls).

The angular position of the $j$-th ball $\phi_j$ is a function of the period $dt$, the initial angular position of the bearing cage $\phi_0$ and the angular speed of the bearing cage $\omega_c$. The $\phi_j$ and $\omega_c$ are defined as follows:

$$\phi_j = \frac{2\pi(j-1)}{Z} + \omega_c dt + \phi_0, \quad \omega_c = (1 - \frac{D}{d_m}) \frac{\omega_\mu}{2}$$

where $\omega_\mu$ is the angular speed of the shaft, $D$ is the diameter of the ball, and $d_m$ is the pitch diameter of the bearing.

3. Numerical results and discussion

The equations of the motion are solved numerically in MatLab-Simulink environment using ODE45 Runge-Kutta solver with the fixed time step 1·10-5. The parameters of the 2DOF model of a bearing, based on the commercial bearing 6009, are given in table 1.

| Parameter | Value |
|-----------|-------|
| Number of balls, $Z$ | 11, 13, 15 |
| Ball diameter, $D$ | 8.731 mm |
| Pitch diameter, $d_m$ | 60 mm |
| Contact stiffness, $k_b$ | 1.8978·10^10 |
| Stiffness of the inner race, $k$ | 4.241·104 N/m |
| Damping of the inner race, $c$ | 1376.8 Ns/m |
| Mass of inner race, $m_c$ | 0.48 kg |

The calculations were conducted for different numbers of balls ($Z$), various values of the radial internal clearance ($RIC$) and different rotary speeds of the shaft ($n$). The paper presents an analysis of the vibration for different combinations of these parameters.

Figure 2 and Figure 3 show acceleration time response of the simulated bearing with thirteen balls corresponding to the x-direction and the y-direction. The $RIC$ was fixed and equalled 10 $\mu$m. The rotary shaft speed was $n=1500$ rpm (blue line) and $n=6000$ rpm (orange line).

The amplitude of acceleration response in the y-direction is twice higher than the amplitude in the x-direction. This result is probably caused by the gravity force. Of course, the amplitude of acceleration has a higher level for the high rotary speed. A fourfold increase in the rotary speed causes an increase in the acceleration of even eight times (in the y-direction). Moreover, the rotary speed influences the time period vibration, which is clearly visible in figure 2 and 3.
Figure 4 and figure 5 show acceleration time responses for the bearing with thirteen balls and the rotary speed of shaft $n=3000$ rpm. The two values of the RIC parameter were compared. The small radial clearance was assumed $RIC=5\, \mu m$ (blue line), and high-level $RIC=20\, \mu m$ (orange line). The greater $RIC$ causes an increase in the acceleration values. This effect can be explained by lower stiffness of the bearing system. The maximal level of acceleration obtained in the y-direction was about $a_y=4\, m/s^2$ (figure 5).

Figures 6-9 present the root mean square (RMS) acceleration vs. the number of balls for the different rotary speed $n$ and a different $RIC$ value. The specified plot shows RMS acceleration in the $x$ and $y$-direction with the $y$-axes on both left and right sides. The continuous line presents results for $n=1500$ rpm, the dotted line for $n=3000$ rpm, the dot for $n=4500$ rpm, and the dashed-dot line for $n=6000$ rpm. For all rotary speeds, the acceleration is smaller for the bearing with the larger number of balls. This is particularly evident for higher shaft speeds. Moreover, this relationship can be described by a simple linear function, which is very useful from the practical point of view.
Figure 6. RMS value of acceleration in the x and y-directions v the number of balls, for $RIC=5\mu m$.

Figure 7. RMS value of acceleration in the x and y-directions v the number of balls, for $RIC=10\mu m$.

Figure 8. RMS value of acceleration in the x and y-directions v the number of balls, for $RIC=20\mu m$.

Figure 9. RMS value of acceleration in the x and y-directions v the number of balls, for $RIC=30\mu m$.

Figure 10. RMS value of acceleration in the x and y-directions v radial internal clearance $RIC$, for $Z=11$.

Figure 11. RMS value of acceleration in the x and y-directions v radial internal clearance $RIC$, for $Z=13$. 
Figure 12. RMS value of acceleration in the \(x\) and \(y\)-directions vs radial internal clearance \(RIC\), for \(Z=15\).

Figures 10, 11 and 12 show the influence of \(RIC\) parameter on the accelerations for different values of the shaft speed. The investigations have been performed for bearings with the number of balls: \(Z=11\) (figure 10), \(Z=13\) (figure 11) and \(Z=15\) (figure 12). For shaft speeds \(n=1500\) rpm, the acceleration responses are almost on the same level for the different number of balls. When the shaft speed increases the acceleration responses also increase in both directions. The increase of radial internal clearance has more impact on the acceleration response for the \(y\)-direction. The acceleration responds strongly in the function of radial internal clearance, especially at higher shaft speeds. The number of balls is a crucial parameter from the practical point of view.

4. Conclusions
In this paper, the 2DOF nonlinear numerical model of deep groove ball bearings was studied. The bearing vibration response for a different number of balls, value of radial internal clearance and shaft speed were analysed.

The numerical investigations show that the radial internal clearance and the shaft speed have a significant influence on the bearing vibration response in both directions. The bearing dynamic behaviour is similar in both directions. However, higher acceleration was obtained in the vertical direction. When the number of balls increases, the acceleration values are lower. The radial clearance is of great importance: its increase causes a marked rise in acceleration.

The next step of our research will be to include the temperature in the bearing model. Moreover, the radial internal clearance model will be modified.

5. References
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