Kinematics analysis of a new parallel robotics

Shi Baoyu\textsuperscript{1,2} and Wu Hongtao\textsuperscript{1}

Abstract
A new type of parallel robot ROBO_003 is presented. Its mechanisms, kinematics, and virtual prototype technology are introduced. The research of degrees of freedom (DOF) is based on screw theory, a set of screw is separated as a branch, which named as constrain screw. The type of three DOF gained by counting constrain screw, the moving platform’s frame, and base platform’s frame is set, respectively, a complete kinematic research including closed-form solutions for direct kinematic problem. The 3-D model of ROBO_003 is established using SOLIDWORKS; position and orientation of motion platform can be gained using ADMS, which is a type of virtual prototype technology. The resultant shows that the structure of ROBO_003 is reasonable, three DOF of motion platform can be operated in a reasonable range, the solutions to the direct kinematics are right, and robot ROBO_003 can be used in many industrial fields. The research of this article provides a basis for the practical application of parallel robotics ROBO_003.

Keywords
ADAMS, ROBO_003, screw theory, virtual prototype technology

Introduction
Since the advent of robots, its development prospects have become wider and wider, especially in many industrial fields, which greatly improve labor productivity and product quality, reduce production costs, reduce labor intensity, and improve labor conditions.

Most of the traditional robotic manipulators are made up of joints in series, which are base, waist, upper arm, lower arm, wrist, and end-effector, so they are called serial robots. This kind of robot has the advantages of wide working range and flexible movement. However, if degree of freedom (DOF) is more and the arm is longer, the stiffness is poor and the bearing capacity is low, so there is a phenomenon of error accumulation and amplification. Therefore, people need to study a new type of robot with high position accuracy, good stiffness, strong bearing capacity, and simple structure.

Parallel robot was first proposed by Stewart in 1965, it is called Stewart mechanism.\textsuperscript{1} It consists of a moving platform, a fixed platform, and six parallel branches with two platforms. Each branch can be retracted. The two ends of the branch are connected with two platforms by spherical or Hooke hinges. When the length of each branch changes, the moving platform can obtain three DOFs (Figure 1).

\textsuperscript{1}College of Mechanical and Electrical Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, China
\textsuperscript{2}College of Mechanical Engineering, Anhui University of Technology, Ma’anshan, China

Corresponding author:
Wu Hongtao, College of Mechanical and Electrical Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China.
Email: mehtwu@126.com
In 1998, Professor Joachim in Germany designed a six-DOF parallel robot named triplanar (Figure 2). Triplanar is driven by three 2-DOF planar motors, ROBO_003 is the first model derived from triplanar since its birth (Figure 3). Its design conception comes from triplanar parallel robot and has its own obvious characteristics.

Three 2-DOF planar motors of triplanar parallel robot are replaced by three rotating discs around fixed axes in ROBO_003, which is equivalent to restricting two-DOF free motion of three planar motors to one-DOF along fixed circumference.

ROBO_003 is a new type of parallel robot with three DOF, so it can also be called pure rotation parallel robot. With its unique structural advantages, ROBO_003 parallel mechanism has great application prospects in many fields, such as satellite antenna, radar, photovoltaic tracker, and so on. However, there is almost no systematic research on the mechanism of this kind of parallel robot, so its mathematical modeling and simulation is a very innovative and challenging work, as well as laying a solid theoretical foundation for its practical application, which is of great significance.

In this article, the screw method is used to analyze the number and form of the DOF of the mechanism, the virtual prototype technology of SOLIDWORKS and ADMAS is used to carry out kinematics modeling and simulation. The theoretical analysis and software simulation of ROBO_003 parallel robot are also an innovative work, which has important practical significance.

### Analysis of DoF form

#### Screw theory

Among many mathematical methods for analyzing space mechanisms, screw is a very effective tool.\(^1\) Firstly, it integrates six scalars, or two 3-D vectors, so that a screw can represent the direction and position of vectors at the same time, the velocity and angular velocity in rigid body motion, and the force and torque in rigid body mechanics. Therefore, screw with six scales is very useful for kinematics and dynamics analysis of spatial mechanisms. A vector in space is constrained on a line whose position is determined, the vector constrained by the line is called a line vector, which is expressed by a dual vector \((s; s_0)\), where \(s\) is the primitive part of the dual vector and \(s_0\) is the dual part of the dual vector. \(s\) is a space vector, \(s_0 = r \times s\) is the line distance of the space vector, having a unit of length, \(r\) is the radius from the origin to any point on the line in space; when \(s \cdot s = 1\), it is a unit line vector, \(s_0\) represents the position of the line vector in space, as \(s \cdot s_0 = 0\), it is the characteristic of a line vector\(^4\) according to the definition of \(s_0\). If \(s\) is the angular velocity that an object rotates around an axis, \(s_0\) is defined as line velocity. The primitive and dual parts of dual vectors are not orthogonal in most cases. The nonorthogonal dual vectors are called screw, it is defined as \((s; s_0)\), \(s \cdot s_0 \neq 0\) if \(s \cdot s = 1\), it is a unit screw, \(s \cdot s_0 / s \cdot s = h\) is the pitch of screw. It has a unit of length if \(h = 0\), it is called line vector; if \(h = \infty\), it is called dual vector, which is defined as \((0; s)\); a screw can be transformed as follows: \((s; s_0) = (s; s_0 - hs + hs) = (s; s + hs)\); a line position can be gained as \(r \times s =\)
According to the above definition, any form of motion can be regarded as screw motion. If a screw motion is pure rotation, we call it velocity screw, whose pitch is zero, and the axis coincides with the rotating shaft of the rotating pair. A pure moving motion screw is a velocity screw with infinite pitch and straight line direction along the moving pair. Compound motion is a linear combination of the above two forms. Motion screw, also known as unit velocity screw, written as $\dot{s}$, its Plucker coordinate, has the form as following

$$\dot{s} = [L \ M \ N \ : \ P \ Q \ R]$$

(1)

where $L$, $M$, and $N$ can be expressed as unit angular velocity; $P$, $Q$, and $R$ can be expressed as unit line velocity. Corresponding to $r$, $\dot{s}$ is defined as constrain Screw as following

$$r \ \dot{s} = [rL \ rM \ rN \ : \ rP \ rQ \ rR]$$

(2)

The first three components represent the principal vector of the force screw, and the last three components represent the principal moment of the force screw. For two screws, if the following formula is satisfied, they are called reciprocal screws

$$\dot{s} \circ \ \dot{s}' = 0$$

(3)

According to the definition of reciprocal screw, motion screw and force screw are a pair of reciprocal screw, and the sum of virtual power of external force system to stable rigid body is constant to zero at any time.

For an object connected to a fixed base by a pure series kinematic chain with $n$ ($n > 0$) DOFs, its motion screw is the linear combination of the motion screw of all the motion pairs in the kinematic chain, which is expressed as follows

$$\dot{s} = \sum_{i=1}^{n} k_{i} \ \dot{s}_{i}$$

(4)

where $\dot{s}_{i}$ is the number $i$ motion screw of the kinematic chain. Plucker coordinate can be expressed as follows

$$\dot{s}_{i} = (L_{i} \ M_{i} \ N_{i} ; P_{i} \ Q_{i} ; R_{i}) ;$$

$$\dot{s} = K \ J ;$$

$$K = [k_{1} \ k_{2} \ldots \ k_{n}] ;$$

$$J = \begin{bmatrix} L_{1} & M_{1} & N_{1} & P_{1} & Q_{1} & R_{1} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ L_{n} & M_{n} & N_{n} & P_{n} & Q_{n} & R_{n} \end{bmatrix}$$

(5)

where $J$ is motion screw system of the kinematic chain, $K$ is motion screw system of the kinematic chain, and $\dot{s}$ is constraint anti-Screw $\dot{s}$ of the kinematic chain on the object connected to its terminal, which is called constraint terminal. As $\dot{s} \circ \ \dot{s}' = 0$, so $KJ \circ \ \dot{s}' = 0$

$K$ can take any value. Therefore, the necessary and sufficient condition for the establishment of the above formula is

$$J \circ \ \dot{s}' = 0$$

(6)

In formula (6)

$$\dot{s}' = [P' ; Q' ; R' ; L' ; M' ; N']$$

Therefore, for any given pure series kinematic chain, the terminal constraint (anti-Screw) can be obtained by this formula, and the terminal constraint is known. The unconstrained motion is the DOF of the terminal.

**DOF analysis of ROBO_003**

Structural diagram of ROBO_003, as shown in Figure 4, one of branched chain $P_{1}G_{1}Q_{1}$ of ROBO_003 was chosen, in this branched chain, $P_{1}$ is a rotating pair, $G_{1}$ is a spherical pair, $Q_{1}$ is a rotating pair, coordinate system $P_{1} - x_{1}y_{1}z_{1}$ established, as shown in Figure 5, the rotating joint in $P_{1}$, its screw:

$$\dot{s}_{1} = (0 \ 0 \ 1 \ 0 \ 0 \ 0)$$

(7)

Spherical pairs in $G_{1}$ can be decomposed into three orthogonal rotating pairs. And they intersect the center of rotation of the spherical pair, the coordinate of the center is $(a \ b \ 0)$, the screw of the three rotating pairs, receptively, separately is

$$\dot{s}_{2} = (1 \ 0 \ 0 \ 0 \ 0 \ b)$$

(8)

$$\dot{s}_{3} = (0 \ 1 \ 0 \ 0 \ 0 \ a)$$

(9)

$$\dot{s}_{4} = (0 \ 0 \ 1 \ b \ a \ 0)$$

(10)
The direction and vector diameter of the axis of the rotating pair in $Q_1$ cannot be determined; so, its Screw can be set as follows:

$$ \mathbf{s}_5 = (d \ e \ f; g \ h \ i) $$

(11)

The motion Screw of $P_1G_1Q_1$ in the parallel robot ROBO_003 can be set as follows:

$$ \mathbf{s} = (\mathbf{s}_1 \ \mathbf{s}_2 \ \mathbf{s}_3 \ \mathbf{s}_4 \ \mathbf{s}_5) $$

It is expressed as the matrix:

$$ \mathbf{s} =
\begin{bmatrix}
0 & 1 & 0 & 0 & d \\
0 & 0 & 1 & 0 & e \\
1 & 0 & 0 & 1 & f \\
0 & 0 & 0 & b & g \\
0 & 0 & 0 & -a & h \\
0 & -b & a & 0 & i \\
\end{bmatrix} $$

(12)

The rank of the matrix $\mathbf{s}$ is 5, the five motion screw of the first branched chain is linearly independent. According to the reciprocal screw theory, when the rank of the screw system composed of all the kinematic pairs in the kinematic chain is less than 6, there exists an anti-screw, which is reciprocal to each screw in the screw system.\textsuperscript{11,12} If the rank of the screw is $W$, the number of anti-screw is $6-W$, it is an anti-screw system that has 6- $W$ screws, there is only one basic screw in the constrained screw system of the parallel robot ROBO_003, it can be set as follows

$$ \mathbf{s}^r = [x_1 \ x_2 \ x_3; x_4 \ x_5 \ x_6] $$

(13)

According to the relation between the screw system, it is an anti-screw system, five equations are obtained as follows

$$ \mathbf{s}_1 \circ \mathbf{s}^r = 0 $$

(14)

$$ \mathbf{s}_2 \circ \mathbf{s}^r = 0 $$

(15)

$$ \mathbf{s}_3 \circ \mathbf{s}^r = 0 $$

(16)

$$ \mathbf{s}_4 \circ \mathbf{s}^r = 0 $$

(17)

$$ \mathbf{s}_5 \circ \mathbf{s}^r = 0 $$

(18)

By solving the above equations, we finally get a group of equation solutions

$$ x_1 = \frac{b}{a} $$

$$ x_2 = \frac{g + \frac{b}{a} h}{\frac{b^2}{a} h} $$

$$ x_3 = \frac{ae - bd - i}{ae - bd - i} $$

$$ x_4 = \frac{bg + \frac{b^2}{a} h}{ae - bd - i} $$

$$ x_5 = \frac{-ag + bh}{ae - bd - i} $$

$$ x_6 = 0 $$

After calculation, we get the equation:

$$ x_1 \cdot x_4 + x_2 \cdot x_5 + x_3 \cdot x_6 = 0 $$

(19)

We can conclude from the resultant that the anti-screw of the motion screw is a line vector, the anti-screw has a meaning of force, and its direction vector is $(x_1, x_2, x_3)$. Similarly, the constraints of the other two branched chains of ROBO_003 on the moving platform are force vectors, it is proved that the three force vectors are linearly independent. Therefore, the three branches restrict the three DOF of the moving platform. For the spatial mechanism, only three DOF of rotation are unconstrained. Therefore,
ROBO_003 is a pure rotating mechanism with three DOF, that is, rotating along the x, y, and z axes, respectively.

**Kinematics**

Kinematics research only considers motion condition, not force condition. Position analysis is the most basic task of ROBO_003 kinematics analysis. Its essence is to solve the position relationship between input and output components of mechanism. It can be divided into two subproblems: the forward and inverse position solutions.\(^6\)\(^7\) The former involves the known joint variables to solve the operation variables, and the latter involves the known operation variables to solve the joint variables. For parallel mechanisms, the process of calculating all possible motion parameters of the driving joint by giving the position and attitude parameters of the moving platform relative to the static platform is the inverse position solution; and the known driving joint is the inverse position solution. The forward position solution is the process of solving the position and posture of all possible moving platforms relative to the basic platforms.

**Establishment of coordinate system**

As shown in Figure 5(a) and (b), two equilateral triangles represent the base and the moving platform, respectively. \(O_1(i = 1, 2, 3)\) represent three vertices of the base triangle, \(Q_1(i = 1, 2, 3)\) represent the midpoint of the three sides of the moving platform triangle, the axis of the rotating pair between the moving platform, and the connecting rod coincides with the three sides of the triangle of the moving platform. \(P_1(i = 1, 2, 3)\) is on the drive turntable shaft, \(G_1(i = 1, 2, 3)\) represent the center of spherical pair, \(P_1, P_2, P_3, G_1, G_2, G_3\) locate in the same plane, inertial frame \(O - xyz\) is established in the center point \(O\) of base platform, it is also called generalized frame, x-axis points \(O\) to \(O_1\), the \(\xi\)-axis is perpendicular to the base platform. The \(Y\)-axis conforms to the right-hand rule. Inertial frame \(P_i - x_iy_iz_i\) is fixed on the input joint, the \(x_i\)-axis is in the same direction as the \(O_1Q_1\)-axis, \(z_i\)-axis is rotating along revolving shaft, the \(y_i\)-axis conforms to the right-hand rule to describe the orientation of moving platform, moving frame \(C -uvw\) is established, moving frame \(C -uvw\) is fixed on moving platform, the origin point \(C\) is in the center, \(u\)-axis points from \(C\) to \(Q_1\), \(w\)-axis is perpendicular to the moving platform, and \(v\)-axis conforms to the right-hand rule. Moving frame \(Q_1 - u_1v_1w_1\) is fixed on the output joint, the \(u_1\)-axis is in the same direction as \(Q_1C\), \(v_1\)-axis is rotating along revolving shaft, the \(w_1\)-axis conforms to the right-hand rule.

The input joint refers to the rotating pair between the base and the turntable, and the output joint refers to the rotating pair between the rod and the moving platform. The input and output refer to the input and output of motion. From the point of view of motion, the input joint is the driving joint and the output joint is the guiding joint, while the joint between the two, for example, the spherical hinges or pairs between the turntable and the rods are all transmission joints.

Quantitative analysis of motion requires setting motion parameters.\(^8\) Each moving joint transmits the corresponding motion parameters, in parallel mechanism ROBO_003, \(\beta_1(i = 1, 2, 3)\) represents the input angle of no. \(i\) drive turntable, that is, quantitative number of input joint, \(\alpha_1(i = 1, 2, 3)\), is counterclockwise angle from \(OQ_1\) to \(CQ_1\), that is, output angle, that is, quantitative parameters representing output joint motion.

**Position and orientation describing of moving platform**

**Position of moving platform.** The position of the moving platform can be represented by the position vector of the center point \(C\) of the moving platform, the coordinate of point \(C\), which relative to the base frame \(O -xyz\) is \((X_C, Y_C, Z_C)\), the vector of \(C\) is expressed as follows

\[
C = [X_C, Y_C, Z_C]^T
\]

**Orientation of moving platform.** For orientation of moving platform, we describe it by rotational transformation of the moving frame \(C -uvw\) relative to the inertial frame \(O -xyz\). Rotation transformation of moving frame relative to inertial frame: firstly, the direction of moving frame is in the coincidence of inertial frame, the moving frame rotates around \(x\)-axis, the rotating angle is \(\psi\); then rotates around \(y\)-axis, the rotating angle is \(\theta\); then rotates around \(z\)-axis, the rotating angle is \(\gamma\), we get the final frame \(C -uvw\). Let us multiply the basic rotation matrix \(R_x(\psi)\), \(R_y(\theta)\), \(R_z(\gamma)\) by left, in turn, the transformation matrix \(^{\text{O}}_C R\) can be obtained by moving frame \(C -uvw\) rotating relative to inertial frame \(O -xyz\):

\[
^{\text{O}}_C R = R_z(\gamma) \ast R_y(\theta) \ast R_x(\psi)
\]

For orientation of moving platform, we describe it by rotational transformation of the moving frame \(C -uvw\) relative to the inertial frame \(O -xyz\). Rotation transformation of moving frame relative to inertial frame: firstly, the direction of moving frame is in the coincidence of inertial frame, the moving frame rotates around \(x\)-axis, the rotating angle is \(\psi\); then rotates around \(y\)-axis, the rotating angle is \(\theta\); then rotates around \(z\)-axis, the rotating angle is \(\gamma\), we get the final frame \(C -uvw\). Let us multiply the basic rotation matrix \(R_x(\psi)\), \(R_y(\theta)\), \(R_z(\gamma)\) by left, in turn, the transformation matrix \(^{\text{O}}_C R\) can be obtained by moving frame \(C -uvw\) rotating relative to inertial frame \(O -xyz\):

\[
^{\text{O}}_C R = R_z(\gamma) \ast R_y(\theta) \ast R_x(\psi)
\]

The three angles \(\psi, \theta, \gamma\) are called RPY angle, they are called roll, pitch, yaw angle, respectively,\(^9\) which is the
rotating angle of moving frame rotating around x-axis, y-axis, and z-axis.

Solutions for direct kinematic problem

Analytic method for close-form solutions. The mechanism parameters and input parameters of ROBO_003 parallel robot can be used to get the solution equation of moving frame, the closed solution is obtained, the mechanism parameters of ROBO_003 are provided in Table 1.

The kinematics of ROBO_003 studies the relationship between the input angle and the position and orientation of the moving platform. The input angle is the input parameter $\beta_i$ ($i = 1, 2, 3$), and the position and orientation of the moving platform are the output parameter $(X_C, Y_C, Z_C)^T, O_0 R$, as provided in Table 2.

The selection of reference frame will not affect the distance between two points. Obviously, the distance between the center of the spherical hinge $G_i$ and $G_{i+1}$ is equal in the base frame and the moving frame, so there are the following constraint equations:

$$|G_{i+1} - G_i| = |C_{G_{i+1}} - C_{G_i}| (i = 1, 2, 3)$$

The position vector of $G_i$, that is, in frame $C -uvw$ is $C_{G_i}$; $\lambda_i$ is the angle between $\overrightarrow{G_i G_{i+1}}$ and $\overrightarrow{G_i X}$, $\lambda_i$ reflects the position of the frame $P_i - x_i y_i z_i$ relative to the base frame $O -xyz$, it can be obtained from the structure:

$$\lambda_i = (i - 1) \frac{2\pi}{3}$$

Setting the initial state of ROBO_003 parallel robot motion, point $G_i$ is on the $x_i$-axis of inertial frame $P_i - x_i y_i z_i$, $P_i T$ is the transformation matrix from frame $P_i - x_i y_i z_i$ to frame:

$$P_i T = \begin{bmatrix}
\cos\lambda_i & -\sin\lambda_i & 0 & Rcos\lambda_i \\
\sin\lambda_i & \cos\lambda_i & 0 & Rsin\lambda_i \\
0 & 0 & 1 & d \\
0 & 0 & 0 & 1 \\
\end{bmatrix}, (i = 1, 2, 3)$$

Setting $\phi_i$ is the angle between $\overrightarrow{P_i G_i}$ and $\overrightarrow{P_i x_i}$. That is, the input angle of the drive turntable, $\alpha_i$, is the counterclockwise from $\overrightarrow{Q_i G_i}$ to $\overrightarrow{Q_i Q_i}$, that is, the output angle of platform position. The position vector of $G_i$ point in the frame $C -uvw$ is $P_i G_i$:

$$P_i G_i = [\cos\phi_i \ Isin\phi_i \ 0 \ 1]^T$$

The position vector of $G_i [X_{G_i}, Y_{G_i}, Z_{G_i}]^T$, that is, in base frame can be obtained as in the following equation

$$G_i = O_i T^{\beta_i} G_i = [\cos(\phi_i + \lambda_i) + Rcos\lambda_i \ Isin(\phi_i + \lambda_i) + Rsin\lambda_i \ d_i]^T$$

where $\varphi_i$ is the intersection angle of $\overrightarrow{O_i G_i}$ and $\overrightarrow{u_i w_i}$, $\varphi_i$ reflects the position of the frame $Q_i - u_i v_i w_i$ relative to the moving frame $C -uvw$, we get

$$\varphi_i = (i - 1) \frac{2\pi}{3}$$

$O_i T$ is the transformation matrix from frame $Q_i - u_i v_i w_i$ to frame $C -uvw$, we get

$$O_i T = \begin{bmatrix}
\cos\varphi_i & -\sin\varphi_i & 0 & r \cos\varphi_i \\
\sin\varphi_i & \cos\varphi_i & 0 & r \sin\varphi_i \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}$$

The position vector of $G_i$ in the frame $Q_i - u_i v_i w_i$ is $O_i G_i$, we get

$$O_i G_i = [L \cos\alpha_i \ 0 \ -L \sin\alpha_i \ 1]^T$$

The position vector of the $G_i$ point in the frame $C -uvw$ can be obtained by transformation of the frame as follows

$$C_{G_i} = O_i T^{\alpha_i} G_i = [(r + L \cos\alpha_i) \cos\varphi_i \ (r + L \cos\alpha_i) \sin\varphi_i \ -L \sin\alpha_i]^T$$

(30)

Setting $m_i$ is the distance from $G_i$ to $G_{i+1}$, that is, $m_i = |G_i G_{i+1}|$, so

$$m_i^2 = (X_{G_i} - X_{G_{i+1}})^2 + (Y_{G_i} - Y_{G_{i+1}})^2 + (Z_{G_i} - Z_{G_{i+1}})^2$$

when $\phi_i$ ($i = 1, 2, 3$) is known, the coordinates of point $G_i$ and $G_{i+1}$ can be obtained from the formulas, so $m_i^2$ can be obtained; the formula $|G_{i+1} - G_i| = |C_{G_{i+1}} - C_{G_i}| (i = 1, 2, 3)$ is square, the left of the formula is $m_i^2$. By substituting the coordinate expression of $C_{G_i}$ into the above formula, the following results can be obtained:

| Parameter | $|O_0|$ | $|O_P|$ | $|P_G|$ | $|G_Q|$ | $|C_Q|$ |
|-----------|--------|--------|--------|--------|--------|
| Length    | $R$    | $D$    | $I$    | $L$    | $r$    |

| Parameter | $\beta_i (i = 1, 2, 3)$ | $[X_C, Y_C, Z_C]^T$ |
|-----------|-----------------|-------------------|
| $\beta_1$ | $1$             | $[X_1, Y_1, Z_1]^T$ |
| $\beta_2$ | $2$             | $[X_2, Y_2, Z_2]^T$ |
| $\beta_3$ | $3$             | $[X_3, Y_3, Z_3]^T$ |
\[ A \cdot \sin \alpha \cdot \sin \alpha + B \cdot \cos \alpha \cdot \cos \alpha + C \cdot \cos \alpha + C \cdot \cos \alpha + D = 0 \]
\[ A_t = -2L^2; B_t = L^2; C_t = 3Lr; \]
\[ D_t = 2L^2 + 3r^2 - m_r^2 \]
\[ (31) \]

Therefore, the forward position analysis of ROBO_003 parallel mechanism can be reduced to the closed solution of the upper form.

Let \( t_i = \tan \frac{\alpha_i}{2}, \cos \alpha_i = \frac{1-t_i^2}{1+t_i^2}, \text{and } \sin \alpha_i = \frac{2t_i}{1+t_i^2}. \)

Substitute the upper form and get it:
\[ E_t t_i^2 t_{i+1}^2 + F_t t_i^2 + G_t t_{i+1}^2 + I_t t_i t_{i+1} + J_i = 0 \text{ in the formula:} \]
\[ E_i = 3L^2 + 3r^2 - 6Lr - m_r^2 \]
\[ F_i = L^2 + 3r^2 - m_r^2 \]
\[ I_i = -8L^2 \]
\[ J_i = 3L^2 + 3r^2 + 6Lr - m_r^2 \]
\[ (32) \]

When \( i = 1, 2, 3, \) the above formula is equivalent to the following equations:
\[ (E_1 t_1^2 + F_1 t_1^2 + (I_1 t_1) t_2 + (F_1 t_1^2 + J_1) = 0 \]
\[ (E_2 t_2^2 + F_2 t_2^2 + (I_2 t_2) t_3 + (F_2 t_2^2 + J_2) = 0 \]
\[ (E_3 t_3^2 + F_3 t_3^2 + (I_3 t_3) t_4 + (F_3 t_3^2 + J_3) = 0 \]
\[ (33) \]

Let
\[ K_1 = E_1 t_1^2 + F_1; M_1 = I_1 t_1; N_1 = F_1 t_1^2 + J_1 \]
\[ K_2 = E_2 t_2^2 + F_2; M_2 = I_2 t_2; N_2 = F_2 t_2^2 + J_2 \]
\[ K_3 = E_3 t_3^2 + F_3; M_3 = I_3 t_3; N_3 = F_3 t_3^2 + J_3 \]
\[ (34) \]

After simplification
\[ K_1 t_1^2 + M_1 t_2 + N_1 = 0 \]
\[ K_2 t_2^2 + M_2 t_3 + N_2 = 0 \]
\[ K_3 t_3^2 + M_3 t_4 + N_3 = 0 \]
\[ (35) \]

To eliminate variables \( t_2, \) the first two equations are multiplied by \( t_3, \) two new equations are obtained. Consider the following two equations
\[ K_1 t_1^2 + M_1 t_2 + N_1 = 0 \]
\[ K_2 t_2^2 + M_2 t_3 + N_2 = 0 \]
\[ K_3 t_3^2 + M_3 t_4 + N_3 = 0 \]
\[ (36) \]

The above equations are written in the form of matrices:
\[ \begin{bmatrix} 0 & K_1 & M_1 & N_1 & 1 \\ K_1 & M_1 & N_1 & 0 & t_1^2 \\ 0 & K_2 & M_2 & N_2 & t_2 \\ K_2 & M_2 & N_2 & 0 & 1 \end{bmatrix} = 0 \]
\[ (37) \]

Only if the determinant of the fourth-order coefficient matrix is equal to 0, as following:

\[ \text{Table 3. The value of structure parameter.} \]
\[
\begin{array}{ccccccc}
R & R & L & L & d & \lambda_1 & \lambda_2 \\
0.7 & 0.3109 & 0.3 & 0.9290 & 0.36 & 0 & 2\pi/3 \quad 4\pi/3 \\
\end{array}
\]

\[ \text{Table 4. Real number solutions of moving platform.} \]
\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 \\
X_c & 0.2365084 & 0.23650842 & 0.18229494 & 0.18229494 \\
Y_c & 0.02301114 & 0.02301114 & -0.17201958 & -0.17201958 \\
Z_c & 0.06823364 & 0.65176635 & 0.34051953 & 0.37940846 \\
\end{array}
\]

| \( X_c \) | 0.09371023 | 0.09371023 | -0.05540199 | -0.05540199 |
| \( Y_c \) | 0.54190801 | 0.54190801 | 0.16787604 | 0.16787604 |
| \( Z_c \) | 0.17393507 | 0.54606492 | -0.36534417 | 1.08534417 |

\[ \text{Table 5. Parameters of structure parameter.} \]
\[
\begin{array}{cccc}
K_1 & M_1 & N_1 & 0 \\
0 & K_2 & M_2 & N_2 \\
K_2 & M_2 & N_2 & 0 \\
\end{array}
\]
\[ (38) \]

Expanding
\[ -K_1 M_1 M_2 N_1 + K_1 M_2^2 N_1 + K_2^2 N_1^2 + K_2 M_1^2 N_2 - K_1 M_1 M_2 N_2 - 2K_1 K_2 N_1 N_2 + K_1^2 N_2^2 = 0 \]
\[ (39) \]

Substitute \( K_1, M_1, N_1, K_2, M_2, N_2 \) into the upper form and simplify it
\[ U_1 t_1^4 + U_2 t_1^3 + U_3 t_1^2 + U_4 t_1 + U_5 = 0 \]
\[ (40) \]

To eliminate the variables \( t_1, \) combine the above two formulas. Let the following six-order coefficient row number equals 0
\[ \begin{bmatrix} 0 & U_1 & U_2 & U_3 & U_4 & U_5 \\
U_1 & U_2 & U_3 & U_4 & U_5 & 0 \\
0 & 0 & K_3 & M_3 & N_3 & 0 \\
0 & 0 & K_3 & M_3 & N_3 & 0 \\
0 & K_3 & M_3 & N_3 & 0 & 0 \\
K_3 & M_3 & N_3 & 0 & 0 & 0 \\
\end{bmatrix} = 0 \]
\[ (41) \]

The 16-degree polynomial equation of \( t_1 \) obtained by expanding and merging the same terms
\[ T_1 t_1^{16} + T_2 t_1^{14} + T_3 t_1^{12} + T_4 t_1^{10} + T_5 t_1^8 + T_6 t_1^6 + T_7 t_1^4 + T_8 t_1^2 + T_9 = 0 \]
\[ (42) \]

where \( T_1 \sim T_9 \) are functions of \( \phi, \lambda_i \) and structural parameters \( R, l, r, L, \) formula (42) is an eighth-order equation of \( t_1. \)

There are up to 16 real solutions of \( t_1, \) all of which appear in pairs \( t_1. \)
Numerical examples. The structural parameters of parallel mechanism ROBO_003 are provided in Table 3 (unit: meter). Set input angle
\[ \varphi_1 = \frac{\pi}{4}, \varphi_2 = \frac{\pi}{6}; \varphi_3 = -\frac{\pi}{2} \]

Calculating forward kinematics solution based on MATLAB, obtaining the three points’ coordinate of \( G_1, G_2, \) and \( G_3 \) respectively
\[
G_1 = [0.912132034359, 0.212132034359, 0.36]^T \\
G_2 = [-0.60980762113, 0.7562177826491, 0.36]^T \\
G_3 = [-0.609807621135, -0.4562177826491, 0.36]^T
\]

The 16th order polynomial equation is as follows:
\[
-3.941311223 \times 10^4 t_1^6 + 1.395344728 \times 10^5 t_1^4 + 6.725185981 \times 10^2 t_1^2 - 1.359223050 \times 10^6 t_1^{10} \\
-3.824110543 \times 10^8 t_1^8 + 2.010143645 \times 10^9 t_1^6 \\
+ 6.462243414 \times 10^9 t_1^4 + 1.870048120 \times 10^9 t_1^2 \\
-1.564682648 \times 10^5 = 0
\]

The eight real roots of \( t_1 \) are obtained by solving the above equation. The corresponding value of \( \alpha_1 \) is
\[
\pm 2.294124417; \pm 2.075031481; \\
\pm 1.827233337; \pm 0.5087138979
\]

Two sets of solutions of \( \{X_C, Y_C, Z_C\} \) can be obtained from each group of \( \{\alpha_1, \alpha_2, \alpha_3\} \), eight sets of solutions of \( \{X_C, Y_C, Z_C\} \) can be obtained from four group of \( \{\alpha_1, \alpha_2, \alpha_3\} \), the eight sets of solutions of \( \{X_C, Y_C, Z_C\} \) are corresponding to eight platform positions, and eight sets of real number solutions of \( \{X_C, Y_C, Z_C\} \) are provided in Table 4.

The corresponding four rotation matrices are obtained as follows
\[
^O R_C(1) = \begin{bmatrix}
-0.7633 & -0.3473 & -0.9765 \\
-0.2814 & 0.9455 & -0.1818 \\
-1.2072 & -0.2544 & 0.0101
\end{bmatrix}
\]
The above results are the forward kinematics solutions of parallel mechanisms of ROBO_003.
ADMAS simulation

Introduction of ADMAS

Automatic dynamic analysis of mechanical systems (ADAMAS) is a virtual prototype analysis software developed by American MDI Company. The start-up interface of ADMAS is shown in Figure 7. The simulation steps of ADAMS are provided in Table 5.

Kinematics simulation of ROBO_003 based on ADMAS

Before simulation, CAD software Solidworks is used to build 3-D assembly drawing of ROBO_003, as shown in Figure 6. The assembly drawings are established in Solidworks, the parts of the 3-D drawings are saved in the form of suffix x_t. Import ADMAS and reassemble in ADMAS. The assembled model establishes the simulation model in ADMAS interface according to the simulation steps in Table 5. The simulation time and steps are set up. The results are shown in Figure 7.

Simulation results

Centroid position change curve of moving platform

Results and discussion

In this article, a new type of parallel mechanism ROBO_003 was presented, it is DOF form,
kinematics, and virtual prototype technology have been studied.

The DOF of parallel mechanism is based on screw theory by choosing the joint motion screw of three branches of parallel mechanism, the screw system of each joint is obtained, and the constraint screw of each branch chain is obtained by matrix calculation. The constraint screw of the screw system consisting of the constraint screw of each branch chain is the motion screw of the moving platform. From the calculation results of formulas (1) to (10), it can be concluded that the motion form of ROBO_003 parallel mechanism moving platform is three DOF rotation, that is, rotation around X, Y, and Z axes, respectively (Figures 8 and 9).

For the kinematics research, first, we established the frame of the moving platform and the base platform, then the frame of each joint was established, choosing the rotation angle of the moving platform frame relative to the base platform frame respectively, they are roll, pitch, yaw angle, the rotation matrix is obtained using homogeneous matrix transform to obtain position and orientation of moving platform relative to base platform, choosing key points on moving platform and base platform, respectively, and setting constraint equation. After eliminating, the 16-degree equation is simplified, the closed solution is obtained by analytic method, the 16-degree equation is solved by programming, and 8 pairs of positive position solutions are obtained. After choosing or rejecting, the final position and
orientation of the moving platform relative to the base platform are obtained.

ADMAS virtual prototyping technology is a simulation technology for kinematics and dynamics of robots. It can correctly analyze the kinematics of ROBO_003 parallel mechanism. According to theoretical calculation and ADMAS virtual prototyping technology, Figures 10 to 12 are displacement curves of the center of mass of moving platform in three directions of $X$, $Y$, and $Z$ axes. Figures 13 to 15 are velocity curves of moving platform centroid in three directions of $X$, $Y$, and $Z$ axes. Figures 14 and 15 are angular velocity and acceleration curves of moving platform relative to base platform orientation. The results show that the simulation results coincide with the actual theoretical calculation.

**Conclusions**

In this article, the screw theory is used to study the DOF form of ROBO_003 parallel mechanism, the closed solution of forward kinematics is studied by analytical method, and the forward kinematics solution is obtained by simulation with ADMAS software. The results accord with the theoretical calculation value. The research results provide a theoretical basis for the practical application of ROBO_003 parallel mechanism.

For complex parallel mechanisms, the screw analysis method proposed in this article can easily solve the analysis of the DOF form of parallel mechanisms. It is concluded that the DOF form of ROBO_003 parallel mechanisms is
three DOF rotation. The ADMAS virtual prototype can easily obtain the forward kinematics solution and the process of pose change.

The research of parallel mechanism has broad prospects. In the future, there will be more mathematical theories applied in mechanics, kinematics, and dynamics. ROBO_003 parallel mechanism can be applied to various industrial aerospace and aerospace fields, and its application prospect and values are very great.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) disclosed following financial support for the research, authorship, and/or publication of this article: This work was supported by the College Natural Science Foundation of Jiangsu Province (no. 17KJB460003), National Key Research Planning Project(No.2018YFC0309100), National Natural Science Foundation of China (no. 51375230), and Key Construction Subject Project of Jiangsu Province (no. (2016)9)

ORCID iD
Shi Baoyu https://orcid.org/0000-0003-1330-0427

References
1. Stewart D. A platform with six degrees of freedom. Proc Institute of Mechanical Engr, Lond Eng 1965; 180: 371–381.
2. Featherstone R. The calculation of robot dynamics using articulated-bodyinertias. Int J Robot Res 1983; 2(1): 13–30, Spring.
3. Shili C, Hongtao WU, Chaoqun W, et al. Analytical method for the forward kinematics analysis of the general 6-SPS parallel mechanisms [in Chinese]. China Mech Eng 2010; 21(11): 1261–1264.
4. Featherstone R. Robot dynamics algorithms. Dordrecht: Kluwer Academic Publishers, 1987.
5. Naudet J, Lefèber D, Daerden F, et al. Forward dynamics of open-loop multibody mechanisms using an efficient canonical momenta, Berlin: Kluwer Academic Publishers, 10(2003), pp. 45–59.
6. Fu KS, Gonzalez RC, and Lee CSG. Robotics: control, sensing, vision, and intelligence, New York, McGraw-Hill Book Company, 1987.
7. Gourdeau R. Object-oriented programming for robotic manipulator simulation. IEEE Robot Auto Mag 1997; 4(3): 21–29.
8. Mitrić BV. Impulse-based dynamic simulation of rigid body systems, PhD thesis, University of California, Berkeley, 1996.
9. Schmitt A, Bender J, and Prautzsch H. Impulse-based dynamic simulation of higher order and numerical results. Technical Report 2005-21.
10. Fangyuan H, Dingxuan Z, and Tianyu L. A fast forward algorithm for 3-RPS parallel mechanism [in Chinese]. Tran Chin Soc Agric Mach 2011; 4(2): 229–233.
11. Zhen H, Jingfang L, and Daxing Z. General method formability analysis of mechanisms based on constrained screw theory [in Chinese]. Sci China Press 2009; 39(1): 84–93.
12. Zhen H, Ping X, and Feng DH. Screw analytical of bennett structure DOF. JYAN Shan Univ 2004; 28(3): 189–191.
13. Yang XL, Wu HT, Li Y, et al. A dual quaternion solution to the forward kinematics of a class of six-DOF parallel robots with full or reductant actuation. Mech Mach Theor 2017; 107: 27–36.
14. Shili C, Hongtao W, Chaoqun W, et al. Analytical method for the forward kinematics analysis of the general 6-SPS parallel mechanisms [in Chinese]. China Mech Eng 2018; 21(11): 1261–1264.
15. Xiaolong Y, Hongtao W, Bai C, et al. Fast numerical solution to forward kinematics of general stewart mechanism using quaternion. Tran Nanj Univer Aeronaut Astron 2018; 31(4): 377–385.
16. Ju Li, Qingfei Z, Jiaming D, et al. Screening process analysis for multidimensional parallel vibrating screen and optimization of screen surface movement [in Chinese]. *Tran Chin Soc Agricul Mach* 2016; 47(11): 399–407.

17. Li Q and Chai X. Mobility analysis of limited degrees of freedom parallel mechanisms in the framework of geometric algebra. *J Mech Robot* 2016; 8(4): 041005.

18. Tingli Y, Anxin L, Huiping S, et al. *Topology design of robot mechanisms*. Berlin: Springer, 2018.

19. Huiping S, Xiaorong Z, Dan Z, et al. The design methodology for less input-more output parallel mechanisms. *Mech Mach Theor* 2016; 104(10): 43–58.

20. Huiping S, Liangjie Y, and Jiaming D. A one-input three-rotation output parallel mechanism used for shoulder rehabilitation and its kinematics design [in Chinese]. *China Mech Eng* 2015; 26(22): 2983–2988.

21. Jiaming D, Lifang D, Huiping S, et al. A novel parallel foot massage mechanism and its kinematics design [in Chinese]. *J Mach Des* 2016; 33(2): 78–82.

22. Pingsong L, Gang G, and Haining Z. Workspace analysis of the I4R parallel robot [in Chinese]. *Mach Build Automat* 2012; 41(4): 156–157.

23. Xiguang H, Qizheng L, Shimin W, et al. Forward kinematics analysis of the general 6-6 platform parallel mechanism based on algebraic elimination [in Chinese]. *Chin J Mech Eng* 2009; 45 (1): 56–61.

24. Yanli L, Shili C, Surong J, et al. Forward kinematics of 6-UPS parallel manipulators with one displacement sensor [in Chinese]. *J Mech Eng* 2018; 54(5): 1–7.

25. Cheng S, Liu Y, and Wu H. A new approach for the forward kinematics of nearly general Stewart platform with an extra sensor. *J Adv Mech Des Syst Manuf* 2017; 11(2): JAMDSM0032JAMDSM0032.