New Physics effects on decay $B_s \to \gamma \gamma$ in Technicolor Model

Qin XiuMei, Wujun Huo, and Xiaofang Yang

Department of Physics Department,
Southeast University, Nanjing, Jiangsu 211189, China

Abstract

In this paper we calculate the contributions to the branching ratio of $B_s \to \gamma \gamma$ from the charged Pseudo-Goldstone bosons appeared in one generation Technicolor model. We find that the theoretical values of the branching ratio, $BR(B_s \to \gamma \gamma)$, including the contributions of PGBs, $P^\pm$ and $P^\pm_8$, are much different from the SM prediction. The new physics effects can be enhance 2-3 levels to SM result. It is shown that the decay $B_s \to \gamma \gamma$ can give the test the new physics signals from the technicolor model.
I. INTRODUCTION

As is well known, the rare radiative decays of $B$ mesons is in particular sensitive to contributions from those new physics beyond the standard model (SM). Both inclusive and exclusive processes, such as the decays $B_s \rightarrow X\gamma$, $B_s \rightarrow \gamma\gamma$ and $B \rightarrow X_s\gamma$ have been received some attention in the literature\[1−14]. In this paper, we will present our results in Technicolor theories.

The one generation Technicolor model (OGTM)\[15−16] is the simplest and most frequently studied model which contained the parameters are less than SM. Same as other models, the OGTM has its defects such as the S parameter large and positive\[17]. But we can relax the constraints on the OGTM form the $S$ parameter by introducing three additional parameters $(V, W, X)$\[18]. The basic idea of the OGTM is: we introduce a new set of asymptotically free gauge interactions and the Technicolor force act on Technifermions. The Technicolor interaction at $1Tev$ become strong and cause a spontaneous breaking of the global flavor symmetry $SU(8)_L \times SU(8)_R \times U(1)_Y$. The result is $8^2 - 1 = 63$ massless Goldstone bosons. Three of the these objects replace the Higgs field and induce a mass of $W^\pm$ and $Z^0$ gauge bosons. And at the new strong interaction other Goldstone bosons acquire masses. As for the $B_s \rightarrow \gamma\gamma$, only the charged color single and color octets have contributions. The gauge couplings of the PGBs are determined by their quantum numbers. In Table 1 we listed the relevant couplings\[19] needed in our calculation, where the $V_{ud}$ is the corresponding element of $Kobayashi – Maskawa$ matrix. The Goldstone boson decay constant $F_\pi$\[20] should be $F_\pi = v/2 = 123GeV$, which corresponds to the vacuum expectation of an elementary Higgs field.

| $P^+ P^- \gamma_\mu$ | $-ie(p_+ - p_-)_\mu$ |
| $P^+_{sa} P^-_{sb} \gamma_\mu$ | $-ie(p_+ - p_-)_\mu \delta_{ab}$ |
| $P^+ u d$ | $i\frac{V_{ud}}{2F_\pi} \sqrt{\frac{2}{3}}[M_u(1 - \gamma_5) - M_d(1 + \gamma_5)]$ |
| $P^+_{sa} u d$ | $i\frac{V_{ud}}{2F_\pi} \lambda_a[M_u(1 - \gamma_5) - M_d(1 + \gamma_5)]$ |
| $P^+_{sa} P^-_{sb} g_{c\mu}$ | $-gf_{abc}(p_a - p_b)_\mu$ |

TABLE I: The relevant gauge couplings and Effective Yukawa couplings for the OGTM.
At the LO in QCD the effective Hamiltonian is

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(M_W) O_i(M_W).$$ (1)

Where, as usual, $G_F$ denotes the Fermi coupling constant and $V_{tb} V_{ts}^*$ indicates the Cabibbo-Kobayashi-Maskawa matrix element. And the current-current, QCD penguin, electromagnetic and chromomagnetic dipole operators are of the form

$$O_1 = (\bar{\tau}_{L\alpha} \gamma^{\mu} b_{L\alpha})(\bar{\tau}_{L\alpha} \gamma_\mu c_{L\beta})$$ (2)

$$O_2 = (\bar{\tau}_{L\alpha} \gamma^{\mu} b_{L\alpha})(\bar{\tau}_{L\beta} \gamma_\mu c_{L\beta})$$ (3)

$$O_3 = (\bar{\tau}_{L\alpha} \gamma^{\mu} b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{\tau}_{L\beta} \gamma_\mu q_{L\beta})$$ (4)

$$O_4 = (\bar{\tau}_{L\alpha} \gamma^{\mu} b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{\tau}_{L\beta} \gamma_\mu q_{L\alpha})$$ (5)

$$O_5 = (\bar{\tau}_{L\alpha} \gamma^{\mu} b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{\tau}_{R\beta} \gamma_\mu q_{R\beta})$$ (6)

$$O_6 = (\bar{\tau}_{L\alpha} \gamma^{\mu} b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{\tau}_{R\beta} \gamma_\mu q_{R\alpha})$$ (7)

$$O_7 = (e/16\pi^2) m_b \bar{\tau}_{L} \gamma^{\mu \nu} b_{R} F_{\mu \nu}$$ (8)

$$O_8 = (g/16\pi^2) m_b \bar{\tau}_{L} \sigma^{\mu \nu} T^a b_{R} G_{\mu \nu}^a$$ (9)

where $\alpha$ and $\beta$ are color indices, $\alpha = 1, \ldots, 8$ labels SU(3)$_c$ generators, $e$ and $g$ refer to the electromagnetic and strong coupling constants, while $F_{\mu \nu}$ and $G_{\mu \nu}^a$ denote the QED and QCD field strength tensors, respectively.

The Feynman diagrams that contribute to the matrix element as the following. In Fig.2

![Feynman Diagrams](image)

**FIG. 1: Examples of Feynman diagrams that contribute to the matrix element.**

the shot-dash lines represent the charged PGBs $P^\pm$ and $P_8^\pm$ of OGTM. We at first integrate out the top quark and the weak $W$ bosons at $\mu = M_W$ scale, generating an effective five-quark theory and run the effective field theory down to b-quark scale to give the leading
log QCD corrections by using the renormalization group equation. The Wilson coefficients are process independent and the coefficients $C_i(\mu)$ of 8 operators are calculated from the Fig. 2. The Wilson coefficients are read\cite{21}

$$C_i(M_W) = 0, \quad i = 1, 3, 4, 5, 6, \quad C_2(M_W) = 1, \quad (10)$$

$$C_7(M_W) = -A(\delta) + \frac{B(x)}{3\sqrt{2}G_F F^2_\pi} + \frac{8B(y)}{3\sqrt{2}G_F F^2_\pi} \quad (11)$$

$$C_8(M_W) = -C(\delta) + \frac{D(x)}{3\sqrt{2}G_F F^2_\pi} + \frac{8D(y) + E(y)}{3\sqrt{2}G_F F^2_\pi} \quad (12)$$

with $\delta = M_W^2/m_t^2$, $x = (m(P^\pm)/m_t)^2$ and $y = (m(P^\pm_8)/m_t)^2$. From the Eq(11), (12), we can see the situation of the color-octet charged PGBs is more complex than that of the color-singlet charged PGBs, because of the involvement of the color interactions. Where

$$A(\delta) = \frac{1}{3} + \frac{5}{22}\delta - \frac{7}{22}\delta^2 + \frac{3}{2}\delta - \frac{1}{2}\delta^2 \log[\delta] \quad (13)$$

$$B(y) = -\frac{11}{36} + \frac{53}{72}y - \frac{25}{72}y^2 \quad (1 - \delta)^3$$

$$+ \frac{1}{3}y^3 - \frac{1}{3}y^3 \log[y] \quad (14)$$

$$C(\delta) = \frac{1}{8} - \frac{5}{8}\delta - \frac{1}{4}\delta^2 - \frac{3}{4}\delta^2 \log[\delta] \quad (15)$$

$$D(y) = -\frac{5}{24} + \frac{19}{24}y - \frac{5}{6}y^2 \quad (1 - \delta)^3$$

$$+ \frac{1}{4}y^2 - \frac{1}{2}y^3 \log[y] \quad (16)$$

$$E(y) = \frac{3}{2} - \frac{15}{8}y - \frac{15}{8}y^2 \quad (1 - \delta)^3$$

$$+ \frac{3}{4}y - \frac{9}{2}y^2 \log[y] \quad (17)$$

By calculate the graphs of the exchanged $W$ boson in the SM we gained the function $A$ and $C$; And by calculate the graphs of the exchanged color-singlet and color-octet charged PGBs in OGTM we gained the function $B$, $D$ and $E$. When $\delta < 1$, $x, y >> 1$, the OGTM contribution $B$, $D$ and $E$ have always a relative minus sign with the SM contribution $A$. 

FIG. 2: The Feynman diagrams that contribute to the Wilson coefficients $C_7, C_8$. 

![Feynman diagrams](image-url)
and $C$. As a result, the OGTM contribution always destructively interferes with the SM contribution.

The leading-order results for the Wilson coefficients of all operators entering the effective Hamiltonian in Eq.(1) can be written in an analytic form. They are

$$C_7^{\text{eff}}(m_b) = \eta^{16/23}C_7(M_W) + \frac{8}{3}(\eta^{14/23} - \eta^{16/23}) \times$$

$$C_8(M_W) + C_2(M_W) \sum_{i=1}^{8} h_i \eta^{a_i}.$$ (18)

With $\eta = \alpha_s(M_W)/\alpha_s(m_b)$,

$$h_i = \left( \frac{626126}{272277}, \frac{-56281}{51730}, \frac{-3}{7}, \frac{-1}{14}, -0.6494, \right.$$

$$\left. -0.0380, -0.0186, -0.0057 \right).$$ (19)

$$a_i = \left( \frac{14}{23}, \frac{16}{23}, \frac{6}{23}, \frac{-12}{23}, \right.$$

$$\left. 0.4086, -0.4230, -0.8994, 0.1456 \right).$$ (20)

To calculate $B_s \rightarrow \gamma \gamma$, one may follow a perturbative QCD approach which includes a proof of factorization, showing that soft gluon effects can be factorized into $B_s$ meson wave function; and a systematic way of resumming large logarithms due to hard gluons with energies between 1 GeV and $m_b$. In order to calculate the matrix element of Eq.(1) for the $B_s \rightarrow \gamma \gamma$, we can work in the weak binding approximation and assume that both the $b$ and $s$ quarks are at rest in the $B_s$ meson, and the $b$ quarks carries most of the meson energy, and its four velocity can be treated as equal to that of $B_s$. Hence one may write $b$ quark momentum as $p_b = m_b v$ where is the common four velocity of $b$ and $B_s$. We have

$$p_b \cdot k_1 = m_b v \cdot k_1 = \frac{1}{2} m_b m_{B_s} = p_b \cdot k_2,$$

$$p_s \cdot k_1 = (p - k_1 - k_2) \cdot k_1 =$$

$$-\frac{1}{2} m_{B_s} (m_{B_s} - m_b) = p_s \cdot k_2.$$ (21)

We compute the amplitude of $B_s \rightarrow \gamma \gamma$ using the following relations

$$\langle 0 | \bar{s} \gamma_\mu \gamma_5 b | B_s(P) \rangle = -i f_{B_s} P_\mu,$$

$$\langle 0 | \bar{s} \gamma_5 b | B_s(P) \rangle = i f_{B_s} M_B,$$ (22)

where $f_{B_s}$ is the $B_s$ meson decay constant which is about 200 MeV.
The total amplitude is now separated into a CP-even and a CP-odd part

\[ T(B_s \rightarrow \gamma\gamma) = M^+ F_{\mu\nu} F^{\mu\nu} + iM^- F_{\mu\nu} F^{\mu\nu}. \] (23)

We find that

\[ M^+ = -\frac{4\sqrt{2} \alpha G_F}{9\pi} f_{B_s} m_b V_{ts} V_{tb} \times \left( \frac{m_b}{m_{B_s}} B K(m_b^2) + \frac{3C_7}{8\Lambda} \right). \] (24)

with \( B = -(3C_6 + C_5)/4, \) \( \Lambda = m_{B_s} - m_b, \) and

\[ M^- = \frac{4\sqrt{2} \alpha G_F}{9\pi} f_{B_s} m_b V_{ts} V_{tb} \times \left( \sum_q A_q J(m_q^2) + \frac{m_b}{m_{B_s}} B L(m_b^2) + \frac{3C_7}{8\Lambda} \right). \] (25)

where

\[ A_u = (C_3 - C_5) N_c + (C_4 - C_6) \]
\[ A_d = \frac{1}{4} [(C_3 - C_5) N_c + (C_4 - C_6)] \]
\[ A_c = (C_1 + C_3 - C_5) N_c + (C_2 + C_4 - C_6) \]
\[ A_s = \frac{1}{4} [(C_3 + C_4 - C_5) N_c + (C_3 + C_4 - C_6)] \]
\[ A_s = \frac{1}{4} [(C_3 + C_4 - C_5) N_c + (C_3 + C_4 - C_6)]. \] (26)

The functions \( J(m^2), K(m^2) \) and \( L(m^2) \) are defined by

\[ J(m^2) = I_{11}(m^2), \]
\[ K(m^2) = 4(I_{11}(m^2) - I_{00}(m^2)), \]
\[ L(m^2) = I_{00}(m^2), \] (28)

with

\[ I_{pq}(m^2) = \int_0^1 dx \int_0^{1-x} dy \frac{x^p y^q}{m^2 - 2xy k_1 \cdot k_2 - i\varepsilon}. \] (29)

The decay width for \( B_s \rightarrow \gamma\gamma \) is simply

\[ \Gamma(B_s \rightarrow \gamma\gamma) = \frac{m_{B_s}^3}{16\pi} (|M^+|^2 + |M^-|^2). \] (30)

In SM, with \( C_2 = C_2(M_W) = 1 \), and the other Wilson coefficients are zero, we find

\[ \Gamma(B_s \rightarrow \gamma\gamma) = 1.3 \times 10^{-10} \text{ eV} \] which amounts to a branching ratio \( Br(B_s \rightarrow \gamma\gamma) = 3.5 \times 10^{-7}, \)
for the given $\Gamma_{B_s}^{total} = 4 \times 10^{-4}$ eV. In numerical calculations we use the corresponding input parameters $M_W = 80.22$ GeV, $\alpha_s(m_Z) = 0.117$, $m_c = 1.5$ GeV, $m_b = 4.8$ GeV and $|V_{tb}V_{ts}^*|^2/|V_{cb}|^2 = 0.95$, respectively. The present experimental limit$^{[22]}$ on the decay $B_s \to \gamma\gamma$ is

$$\text{Br}(B_s \to \gamma\gamma) \leq 8.6 \times 10^{-6},$$

(31)

which is far from the theoretical results. So, we can not put the constraint to the masses of PGBs. The constraints of the masses of $P^\pm$ and $P^\pm_8$ can be from the decay$^{[24]}$ $B \to s\gamma$: $m_{P^\pm} > 400$ GeV.

![Br(Bs -> γγ) vs m_pb8(GeV)](image)

**FIG. 3:** the $\text{Br}(B_s \to \gamma\gamma)$ about the mass of $P^\pm_8$ under different values of $m_{P^\pm}$.

Fig.3(4) denotes the $\text{Br}(B_s \to \gamma\gamma)$ about the mass of $P^\pm_8$ ($P^\pm$) under different values of $m_{P^\pm}$ ($m_{P^\pm_8}$). From Fig.3 and 4, we find the the curves are much different from the the SM one. It can be enhanced about 1-2 levels to the SM prediction in the reasonable region of the masses of PGBs. This gives the strong new physics signals from the Technicolor Model. The branching ratio of $B_s \to \gamma\gamma$ decrease along with the mass of $P^\pm_8$ and $P^\pm$ reduce. This is from the decoupling theorem that for heavy enough nonstandard boson. When $m(P^\pm)$ and $m(P^\pm_8)$ have large values, the contributions from OGTG is small. From the $Eq(16), (17), (18)$, we can see the functions $B, D$ and $E$ go to zero, as $x, y \to \infty$. The branching ratio in the
FIG. 4: the $Br(B_s \to \gamma\gamma)$ about the mass of $P^\pm$ under different values of $P^\pm_8$.

Fig.(3) is changed much faster than that in the Fig.(4). This is because the contribution to $B_s \to \gamma\gamma$ from the color octet $P^\pm_8$ is large when compared with the contribution from color singlet $P^\pm$.

As a conclusion, the size of contribution to the rare decay of $B_s \to \gamma\gamma$ from the PGBs strongly depends on the values of the masses of the charged PGBs. This is quite different from the SM case. By the comparison of the theoretical prediction with the current data one can derived out the the contributions of the PGBs: $P^\pm$ and $P^\pm_8$ to $B_s \to \gamma\gamma$ and give the new physics signals of new physics.

[1] A N Mitra. Phys. Lett., 2000, B473: 297-304
[2] Junjie Cao, Zhenjun Xiao, Gongru Lu. Phys. Rev., 2001, D64: 014012
[3] Z.J. Xiao, C.D.L, W.J. Huo. Phys. Rev., 2003, D67: 094021
[4] L. Reina, G. Ricciardi, A. Soni. Phys. Rev., 1997, D56: 5805
[5] G. Hiller, E.O. Iltan. Phys. Lett., 1997, B409: 425
[6] C.-H. Chang, G.-L. Lin, Y.-P. Yao. Phys. Lett., 1997, B415: 395-401
[7] M.R. Ahmady, E. Kou. hep-ph/9708347
[8] G. Hiller, E.O. Iltan, Mod.Phys.Lett., 1997. A12: 2837-2846
[9] S. Choudhury, J. Ellis. hep-ph/9804300
[10] T.M. Aliev, G. Turan. Phys. Rev., 1993, D48: 1176
[11] T.M. Aliev, G. Hiller, E.O. Iltan. Nucl. Phys., 1998, B515: 321-341
[12] T.M. Aliev, E.O. Iltan. Phys. Rev., 1998, D58: 095014
[13] S. Bertolini, J. Matias. Phys. Rev., 1998, D57: 4197-4204
[14] P. Singer, D.-X. Zhang. Phys. Rev., 1997, D56: 4274
[15] S. Dimopoulos. Nucl. Phys., 1980, B168: 69
[16] E. Farhi, L. Susskind. Phys. Rev., 1979, D20: 3404
[17] M.E. Peskin, T. Takeuchi. Phys. Rev. Lett., 1990, 65: 964
[18] I. Maksymyk, C.P. Burgess. Phys. Rev., 1994, D50: 529
[19] Z.J. Xiao, L.D. Wan, J.M. Yang, et al. Phys. Rev., 1994, D49: 5949
[20] E. Eichten, I. Hinchcliffe, K. Lane, et al. Phys. Rev., 1986, D34: 1547
[21] C.D. Lu, Z.J. Xiao. Phys. Rev., 1996, D53: 2529
[22] J. Wicht, I. Adachi, H. Aihara, et al. Phys. Rev. Lett., 2008, 100: 121801