Spectroscopic Bogoliubov features near the unitary limit

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We analyze the single particle excitation spectrum of the ultracold fermion atom system close to the unitary limit where there has been found experimental evidence for the Bogoliubov quasiparticles below as well as above the transition temperature $T_c$. We consider the short-range correlations originating from the preformed pairs and try to reproduce the experimental data adapting phenomenological selfenergy previously used for description of the anti-nodal spectra of the underdoped cuprate superconductors. We show that it fairly fits the lineshapes obtained by the momentum resolved RF spectroscopy for $^{40}$K atoms.

I. INTRODUCTION

Spectroscopic tools such as the Bragg scattering technique [1], the RF-pulse spectrometry [2] and its $k$-resolved improvement [3] were able to provide a clear-cut evidence for the superfluid nature of the ultracold fermion atom systems. Especially intriguing among the obtained data is the quasiparticle back-bending dispersion near the Fermi momentum observed below and above the superfluid transition temperature $T_c$ [4]. This fact indicates that the Bogoliubov-type quasiparticles survive even in a normal state where the long-range coherence between fermion pairs does no longer exist.

Similar fingerprints of the dispersive Bogoliubov quasiparticles have been previously detected above $T_c$ also in the underdoped cuprate superconductors by the measurements of ARPES [5] and the Fourier-transformed STM [6]. They confirmed expectations motivated by the Uemura scaling below $T_c$, and later on supported by the residual Meissner rigidity seen above $T_c$ in the tera-Hertz [5] and the torque magnetometry [6] experiments. Superconducting transition of the underdoped cuprates seems however to be controlled not by the pair-formation but rather by onset of the phase coherence. This point is however still a controversial issue.

In the present work we consider the spectroscopic Bogoliubov features common above $T_c$ for the ultracold fermion gases and underdoped cuprate materials taking into account the short-range correlations driven by preformed fermion pairs. Such problem is currently widely discussed in the literature [10–14] (for a comprehensive discussion see e.g. [18] and other references cited therein). We shall present the results obtained for the single particle excitations using the selfenergy motivated by the local solution of the Feshbach coupling and also suggested by perturbative studies of the pairing fluctuations [15–17].

We start with analysis of the exact solution for the local Feshbach coupling problem. We next discuss how this result can be cast on the itinerant case. Introducing the phenomenological scattering rate we then try to reproduce the single particle spectra for temperatures corresponding to the experiment of the Boulder group [3]. Summarizing our results we point out the problems relevant for future studies.

II. LOCAL SCATTERING ON PAIRS

The momentum-resolved spectroscopic measurements of the Boulder group [4] have been done using $^{40}$K atoms equally populated in the hyperfine states $|9/2, −9/2⟩$ and $|9/2, −7/2⟩$ (we shall denote them symbolically as $σ = ↑$ and $σ = ↓$). By applying magnetic field the atoms were adiabatically brought to vicinity of the unitary limit, slightly on the BEC side ($k_Fa)^{-1} = 0.15$. Under such conditions energies of the atoms are nearly degenerate with the weakly bound molecular configurations. The single atoms and molecules are there strongly scattered from each other through the conversion processes.

At a given position $r$ in the magneto-optical trap such Feshbach resonant interactions can be described by the following local Hamiltonian [18]

$$\hat{H}_{loc}(r) = \sum_{\sigma=\uparrow, \downarrow} \varepsilon(r) \hat{c}_{\sigma}^{\dagger}(r)\hat{c}_{\sigma}(r) + E(r) \hat{b}_{\uparrow}^{\dagger}(r)\hat{b}_{\uparrow}(r) + g \left( \hat{b}_{\uparrow}^{\dagger}(r)\hat{c}_{\uparrow}(r)\hat{c}_{\downarrow}(r) + \hat{c}_{\uparrow}^{\dagger}(r)\hat{c}_{\downarrow}^{\dagger}(r)\hat{b}_{\downarrow}(r) \right)$$

(1)

where $g$ denotes the s-wave channel scattering strength, $\hat{c}_{\sigma}^{\dagger}(r)$ are fermion operators of the single atoms in two hyperfine states $\sigma = \uparrow, \downarrow$ and operators $\hat{b}_{\sigma}^{\dagger}(r)$ correspond to the molecular state. Spatial variation of the energies $\varepsilon(r)$, $E(r)$ comes from the trapping potential and usually take the parabolic dependence with some characteristic radial and axial frequencies.

Hilbert space of the local Hamiltonian (1) is spanned by four fermion configurations $|F⟩ = |0⟩$, $|↑⟩$, $|↓⟩$, $|↑↓⟩$ and two molecular ones $|B⟩ = |0⟩$, $|1⟩$ - altogether 8 states. Six of these states $|F⟩ \otimes |B⟩$ are eigenfunctions of (1) and two vectors ($|↑↓⟩ \otimes |0⟩$ and $|0⟩ \otimes |1⟩$) get mixed by the Feshbach interaction. With the suitable unitary transformation we can determine the true eigenfunctions

$$|\psi_{A}⟩ = u |0⟩ \otimes |1⟩ + v |↑↓⟩ \otimes |0⟩$$

$$|\psi_{B}⟩ = -v |0⟩ \otimes |1⟩ + u |↑↓⟩ \otimes |0⟩$$

(2)

(3)

where $u^2$, $v^2 = \frac{1}{2} \left[ 1 ± (\varepsilon - E/2)^2/\sqrt{(\varepsilon - E/2)^2 + g^2} \right]$ and the eigenvalues are given by $\varepsilon - E/2 ± \sqrt{(\varepsilon - E/2)^2 + g^2}$. The equations (2) are reminiscent of the Bogoliubov - Valatin transformation of the standard BCS problem.
where true quasiparticles are represented by linear combinations of the particle and hole states. In our present case the additional degree of freedom related to the molecular state causes qualitative differences discussed below.

Using the spectral Lehmann representation we can exactly determine the single particle Green’s function $\mathcal{G}_{\text{loc}}(\tau) = -\langle T_\tau \hat{c}_\sigma^\dagger(\tau) \hat{c}_\sigma(0) \rangle$. Its Fourier transform takes the three-pole structure

$$
\mathcal{G}_{\text{loc}}(i\omega_n) = \left[ 1 - Z(T) \right] \left( \frac{u^2}{i\omega_n - \varepsilon_+} + \frac{v^2}{i\omega_n - \varepsilon_-} \right) + \frac{Z(T)}{i\omega_n - \varepsilon}, \tag{4}
$$

where $\varepsilon_{\pm} = E/2 \pm \sqrt{(E-E/2)^2 + g^2}$ and explicit form of $Z(T)$ was given by us in Ref. [19]. Let us now focus on $E = 0$, i.e. the unitary limit case. The single particle spectral function $\frac{-1}{\pi} \text{Im} \{ \mathcal{G}_{\text{loc}}(\omega + i0^+) \}$ consists then of:

- a remnant of the free particle state at $\omega = \varepsilon$ with the temperature dependent residue $Z(T)$, and
- Bogoliubov-type quasiparticles at $\omega = \pm \sqrt{\varepsilon^2 + g^2}$ whose spectral weights are $[1 - Z(T)]u^2$ and correspondingly $[1 - Z(T)]v^2$.

The free particle residue $Z(T)$ is sensitive to temperature. For instance, at $\varepsilon = 0$ we have $Z(T) = \frac{g^2}{3 + \cosh (g/k_B T)}$ which vanishes exponentially when $T \to 0$. It means that at low temperatures only the Bogoliubov-type quasiparticles are present. Upon increasing temperature an amount $Z(T)$ of the spectral weight is transferred from the Bogoliubov quasiparticles to the free fermion state, effectively filling-in the gaped spectrum.

### III. SIMILARITY TO OTHER STUDIES

Our exact solution of the local Feshbach scattering problem [11] coincides with physical conclusions obtained by T. Senthil and P.A. Lee [17] who have explored influence of the incoherent pairs (preformed above $T_c$) on the single particle spectrum. The local pair operator $\hat{F}(r, t) \equiv \hat{c}_\uparrow(\mathbf{r}, t)\hat{c}_\downarrow^\dagger(\mathbf{r}, t)$ can be formally represented through the amplitude and phase

$$
\hat{F}(r, t) = \hat{\chi}(r, t) e^{i\phi(r, t)}. \tag{5}
$$

Since above $T_c$ the pairs need not be dissociated $\chi \neq 0$ their phase $\phi(\mathbf{r}, t)$ must be randomly oriented, precluding any off-diagonal long-range order (ODLRO) $\langle \hat{F}(\mathbf{r}, t) \rangle = 0$.

To account for superconducting fluctuations the authors assumed certain temporal $\tau_\phi$ and spacial $\xi_\phi$ scales, over which the pairs are short-range correlated

$$
\langle \hat{F}^\dagger(\mathbf{r}, t)\hat{F}(0, 0) \rangle \propto |\chi|^2 \exp \left( -\frac{|\mathbf{r}|}{\tau_\phi} - \frac{|\mathbf{r}|}{\xi_\phi} \right). \tag{6}
$$

Taking into account the pairing field $\varphi(\mathbf{r}, t)$ by means of the lowest order perturbative scheme they have determined the single particle Green’s function $\mathcal{G}(\mathbf{k}, i\omega_n) = [i\omega_n - \varepsilon_\mathbf{k} - \Sigma(\mathbf{k}, i\omega_n)]^{-1}$ interpolating it by

$$
\Sigma(\mathbf{k}, i\omega_n) = -\Delta^2 \frac{i\omega_n - \varepsilon_\mathbf{k}}{\omega_n^2 + \varepsilon_\mathbf{k}^2 + \pi \Gamma^2}, \tag{7}
$$

where $\Delta \propto |\chi|$ is a magnitude of the energy gap due to pairing and parameter $\Gamma$ is related to the in-gap states. At low energies (i.e. for $|\omega| \ll \Delta$) a dominant contribution of the spectrum comes from the in-gap quasiparticle with residue $Z \equiv \left( 1 + \frac{\Delta^2}{\pi^2 g^2} \right)^{-1}$ whereas at higher energies the BCS-type quasiparticles are formed. All these features are present in the exact solution [11] of the local Feshbach scattering problem [11] for which we have

$$
\Sigma_{\text{loc}}(i\omega_n) = -\left[ 1 - Z(T) \right] g^2 \frac{i\omega_n - \varepsilon}{\omega_n^2 + \varepsilon^2 + Z(T)g^2}. \tag{8}
$$
IV. PHENOMENOLOGICAL PAIRING ANSATZ

Combining the local physics (1) with the itinerancy $\hat{T}_{\text{kin}}(r)$ of fermions and molecules is a highly non-trivial task. Certain aspects of the complete Hamiltonian

$$\hat{H} = \int dr \left( \hat{T}_{\text{kin}}(r) + \hat{H}_{\text{loc}}(r) \right)$$

have been so far addressed by: the selfconsistent perturbative treatment [20], dynamical mean field theory [21], self-consistent T-matrix approach [22], conserving diagrammatic approximations [15], RG-like scheme [23], path integral formulation for the bond operators [24] and several other techniques. Some of these studies directly [23, 24] or indirectly [15] pointed at the Bogoliubov quasiparticles surviving above $T_c$.

Here we would like to focus on the main physical outcomes which could be relevant to the experimental situation of the Boulder group [4]. For this purpose we apply the phenomenological selfenergy

$$\Sigma(k, \omega) = \frac{\Delta^2}{\omega + \varepsilon_k + i \gamma(T)} - i \Sigma_0$$

which, according to the argumentation outlined in section III of the Ref. [17], originates from (7) and similarly [8]. The particular structure (10) has been also suggested by previous studies of the precursor pairing in the cuprates [16, 25] and ultracold gasses [15]. The essential effects are here provided by temperature dependent parameter $\gamma(T)$ related to scattering caused by the preformed pairs and responsible for filling-in the low energy states (instead of closing the energy gap as in classical superconductors). Its role is hence similar to $Z(T)$ of the local solution [4]. Another parameter $\Sigma_0$ merely controls the line broaden-ing so we simply take it as a structureless constant.

For specific numerical computations we used $\Sigma_0 = \Delta$, assuming $\Delta = \text{const}$. Such assumption seems reasonable for temperature region exceeding $T_c$ as long as the binding energy of preformed pairs stays constant [26] (this constraint can be modified if necessary). We obtained fairly good fitting of the experimental data [4] using the following empirical temperature dependence

$$\gamma(T) = 4k_BT Z(T).$$

At low temperatures $\gamma(T)$ is predominantly governed by the exponential decay of $Z(T)$, whereas for higher temperatures acquires the linear relation $\lim_{T \to \infty} \gamma(T) \propto T$.

FIG. 3: (Color online) Momentum and energy dependence of the spectral function $A(k, \omega)$ for the set of temperatures reported experimentally by the Boulder group [4]. We can notice that the Bogoliubov quasiparticle features (bending-down dispersion) is preserved to nearly $1.5T_c$. 
suggested by various studies [16, 17]. To establish correspondence with the temperature scale we have imposed the ratio $2\Delta/k_B T_c = 4$ realistic for the cuprate superconductors and hopefully valid for ultracold superfluids near the unitarity. Variation of $\gamma(T)$ is illustrated in figure [1].

The k-resolved RF measurements provide information on the occupied part of the single particle excitation spectrum $A(k, \omega) = -\frac{1}{\pi} \text{Im} [\omega - (\varepsilon_k - \mu) - \Sigma(k, \omega)]^{-1}$. In figure [2] we show $\omega$-dependence of $A(k, \omega)$ at the Fermi momentum $k_F$. With constant pairing energy $\Delta$ we obtained the gaped spectrum (at low temperatures) which gradually evolved into the singly peaked structure for $T \geq 1.5T_c$.

In figure [3] we show the momentum dependence of the spectral function $A(k, \omega)$ at temperatures $T/T_c = 0.76$, 1.24, 1.47 and 2.06. Two-peak shape of the EDC-curve $A(k_F, \omega)$ versus $\omega$ is always accompanied by presence of the Bogoliubov quasiparticle branches with their characteristic bending-down (for $\omega < 0$) and bending-up features (for $\omega > 0$), the latter unfortunately hardly accessible by ARPES and k-resolved RF measurements. This is driven by the preexisting pairs above $T_c$ correlated over the short- range scales. The Bogoliubov-type quasiparticles represent admixtures of the particle and hole states. In the underdoped cuprates their presence has been manifested indirectly through the residual diamagnetism response [3] or the large Nernst coefficient and also directly in the single particle spectroscopy [2, 3].

We would like to stress that presence of the Bogoliubov quasiparticles does not go hand in hand with a superposition of the local density of states $\rho(\omega) = \sum_k A(k, \omega)$. This is illustrated in figure [4]. For temperatures above $1.5T_c$ when we have the singly peaked EDC (figure [2] and MDC (figure [3]) the local density of states is still clearly depleted around $\omega \sim 0$, even for temperatures as high as $3T_c$. Such property might reflect the known discrepancy between large values of $T^*$ (signaling the opening of pseudogap) and the actual estimations of $T^*_c$ at which the short-range superconducting correlations establish [4].

V. SUMMARY

Transition to the superconducting/superfluid state at the unitary limit [26] can be accompanied by a number of pre-pairing signatures [27] showing up above $T_c$. Among the known hallmarks of the symmetry broken BCS state are the Bogoliubov-type quasiparticles representing the mixed particle and hole entities. Recent experimental data [4, 5] clearly indicate that such Bogoliubov quasiparticles survive even beyond the superconducting/superfluid state. Near the unitary limit this is caused by the preformed pairs correlated over short distances which strongly affect the single particle spectra through the interconversion processes [1].

We have examined the impact of preformed pairs on the single particle excitation spectrum. In the instructive solution [1] of the local Feshbach scattering problem [11] we have shown how the free fermion states emerge out of the Bogoliubov quasiparticles [strictly speaking the bonding and antibonding states [29]] upon increasing temperature. Guided by the perturbative studies [16, 17] we next employed the pairing ansatz [10] incorporating the local correlations and itinerancy of atoms/molecules. Introducing the phenomenological scattering rate [11] we were able to reproduce the data of the momentum resolved RF spectroscopy obtained for $^{40}$K atoms [3].

For future studies we suggest use of the Andreev tunneling [28] as a useful and complementary technique to the k-resolved RF spectroscopy [3]. If it were feasible for the ultracold fermion systems the Andreev reflections could unambiguously establish the temperature extent of the superconducting/superfluid correlations above $T_c$.

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