Color dipoles, PCAC and Adler’s theorem

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Abstract

Being reformulated in the color dipole basis of small-$x$ QCD Adler’s theorem establishes a connection between perturbative and non-perturbative descriptions of DIS and quantifies the effect of non-perturbative dynamics on would-be-perturbative observables. In particular, it provides a quantitative measure of the non-perturbative influence on the longitudinal structure function in charged current DIS and imposes stringent constraints on non-perturbative parameters of color dipole models. Our analysis calls for new experimental tests of Adler’s theorem in diffractive neutrino scattering.

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1 Introduction

Adler’s theorem [1] connects charged current DIS (deep inelastic scattering) with soft hadronic physics. Here we study its efficiency as a constraint on parameters of color dipole models intended to describe phenomenologically both soft and hard dynamics of DIS processes. We focus on the vacuum exchange dominated leading $\log(1/x)$ region of large Regge parameter $x^{-1} \gtrsim 10^2$ which is

$$x^{-1} = \frac{2m_N\nu}{m_A^2 + Q^2}. \quad (1)$$

In (1) $\nu$ and $Q^2$ are the laboratory frame energy and virtuality of the probe, respectively. The parameter $m_A \simeq 1 \text{ GeV}$ serves to define the mass scale in the light-flavor axial channel. We base our consideration on the color dipole (CD) approach to the BFKL [2], evolution of small-$x$ DIS [3, 4]. In this approach the interaction of high-energy neutrino with the target nucleon, viewed in the laboratory frame, derives from the coherent interaction of $q\bar{q}, q\bar{q}g, \ldots$ states in the light-cone electro-weak (EW) boson. At small $x$ the color dipole size, $r$, of the constituent quark-antiquark pair is conserved in the interaction process and the absorption cross section for the EW boson in the helicity state $\lambda$ is calculated as the quantum mechanical expectation value of the flavor independent CD cross section $\sigma(x,r)$,

$$\sigma(\lambda, x, Q^2) = \langle \Psi(\lambda, z, r) | \sigma(x, r) | \Psi(\lambda, z, r) \rangle. \quad (2)$$

The CD structure of the W boson is described by the light-cone wave function (LCWF) of the quark-antiquark state $\Psi(\lambda, z, r)$ [5, 6]. An interesting possibility to gain deeper insight into the dynamics of small and large color dipoles is offered by the neutrino DIS in the axial channel. In the axial channel at $Q^2 \rightarrow 0$ the light-cone wave function of the longitudinal W boson is proportional of the divergence of the axial-vector current, $\Psi_L \propto \partial_\mu A_\mu$. The PCAC (partially conserved axial-vector current) relation [7], $\partial_\mu A_\mu = m_\pi^2 f_\pi \varphi$, connects via Adler’s theorem [1] the longitudinal cross section $\sigma_L$ defined by Eq.(2) and the on-shell pion-nucleon total cross section $\sigma_\pi = \langle \Psi_\pi(z, r) | \sigma(x, r) | \Psi_\pi(z, r) \rangle$,

$$\lim_{Q^2 \rightarrow 0} Q^2 \sigma_L(x, Q^2) = g^2 f_\pi^2 \sigma_\pi(\nu). \quad (3)$$
In Eq. (3) $\nu$ and $x$ are linked by the Eq.(1). The weak charge $g$ in (3) is related to the Fermi coupling constant $G_F$,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{m_W^2},$$

(4)

and $f_\pi \simeq 131$ MeV is the pion decay constant. The Eq.(3) connects absorption cross sections of two high-energy projectiles having very different CD structure, the pointlike $W$ and the non-pointlike $\pi$. While the light-cone wave function of the pion $\Psi_\pi(z, r)$ is smooth and finite at small $r$\(^1\), the EW boson wave function is singular, $\Psi_L \sim r^{-1}$. This singularity is a legitimate pQCD effect and it makes evaluation of hard contributions to $Q^2 \sigma_L(x, Q^2)|_{Q^2 \to 0}$ more reliable. In particular, this singularity uniquely predicts that the small dipole (hard) contribution to $\sigma_L$ is much stronger than to $\sigma_\pi$.

Since the pioneering paper [10], where the axial-vector meson dominance (AVMD) was suggested, Adler’s theorem has been considered as a relation between the higher mass contributions to the axial current and the pion-nucleon cross section. The AVMD was successfully applied to the analysis of the coupled-channel problem of neutrino-nucleus interactions in Refs. [11, 12, 13] where the mechanism of the $\rho\pi$ dominance was analyzed in detail. In Ref. [14] the PCAC component of the cross section was identified with the longitudinal part of the $a_1$-meson and the constraint for the longitudinal AVMD DIS structure function was obtained. Thus, in the AVMD representation Adler’s theorem relates so to say soft physics to soft physics, the pion to higher axial-vector excitations. Only in the CD basis Adler’s theorem gains its rightful heuristic power. It establishes a connection between perturbative and non-perturbative processes and shows the effect, if not reveals a mechanism, of non-perturbative dynamics on would-be-perturbative observables. In particular, it provides a quantitative measure of the non-perturbative influence on the longitudinal structure function of the light-flavor charged current (CC) DIS. Below we discuss the origin and consequences of this observation.

\(^1\)for a review of the dominance of the soft LCWF and references see [8, 9]
2 Adler’s theorem and the axial mass scale

The color dipole (CD) approach [15, 16] proved to be very successful in describing of inclusive and diffractive electroproduction DIS data in the vector channel down to $Q^2 \sim m_q^2$, where $m_q$ is the constituent quark mass (for the review see [17] and also [18]). The mass scale in the vector channel is fixed by the mass of lightest vector mesons, $m_V \sim 1$ GeV. In the axial channel the spectrum of hadronic excitations starts with the nearly massless pion. To get an idea of the characteristic axial mass scale let us turn to Adler’s theorem. Following Adler [1], consider the particular case of forward lepton production in the reaction of neutrino-nucleon scattering

$$\nu(k) + N(p) \rightarrow l(k') + X(p_X)$$

in the limit $Q^2 = -q^2 \rightarrow 0$ and suppose that $k'^2 = m_l^2 = 0$. In (5) $p_X$ is the 4-momentum of the final hadronic state $X$, $k$ and $k'$ are the 4-momenta of the neutrino and final lepton and $q = k - k'$ is the 4-momentum carried by W-boson. The amplitude of the process (5) is

$$M = \frac{G_F}{\sqrt{2}} l_\mu \langle X | J_\mu | p \rangle,$$

(6)

The massless leptonic current $l_\mu$ is conserved and at $Q^2 \rightarrow 0$ is proportional to $q_\mu$,

$$l_\mu = \bar{u}_{l}(k')\gamma_\mu(1 - \gamma_5)u_{\nu}(k) = \frac{4\sqrt{1 - y}}{y} q_\mu$$

(7)

where $y = pq/kp = \nu/E$. The divergence of the vector component of the hadronic current $J_\mu = V_\mu - A_\mu$ is supposed to be zero. Let $\langle X | A_\mu | p \rangle = M_\mu$ be the sum of the pion pole term and amplitudes of higher axial-vector hadronic states, $|a\rangle$,

$$M_\mu = M_\mu^\pi + \sum_{a\neq\pi} M_\mu^a.$$  

(8)

In (8)

$$M_\mu^\pi = iq_\mu f_\pi \frac{1}{Q^2 + m_\pi^2} M_\pi.$$  

(9)

Since $q_\mu l_\mu = 0$, the pion pole does not contribute to $l_\mu M_\mu$ and $M$ is saturated by higher axial-vector states [19]

$$M = \frac{G_F}{\sqrt{2}} \sum_{a\neq\pi} l_\mu M_\mu^a.$$  

(10)
So, the mass scale in the axial channel has been determined, in fact, in experiments on the diffraction dissociation of high-energy pions, where the mass spectrum of final $a_1, \rho \pi, \pi \pi \pi \ldots$ states was measured (see [20] and also the discussion in [11]).

Adler’s amplitude is linear in the divergence of the axial current. To this accuracy, making use of $q_\mu M_\mu = 0$ and

$$\sum_{a \neq \pi} q_\mu M^a_\mu = i f_\pi M_\pi$$

- the Goldberger-Treiman conspiracy [21], - yields the neutrino amplitude which is multiple of the pion amplitude

$$M = \frac{i G_F f_\pi}{\sqrt{2}} \frac{4 \sqrt{1 - y}}{y} M_\pi.$$ (12)

Summing over all final hadronic states one arrives at Adler’s relation between $\sigma_\pi(\nu)$ and the double differential cross section of the process (5),

$$\left. \frac{d\sigma}{dy dQ^2} \right|_{Q^2=0} = \frac{\pi}{(pk)(4\pi)^3} \sum_X |M|^2 (2\pi)^4 \delta^{(4)}(p + q - p_X)$$

$$= \frac{G_F^2}{2\pi^2} \frac{1 - y}{y} f_\pi^2 \sigma_\pi(\nu).$$ (13)

3 CD models and the axial mass scale again

CD models rely upon the small-$x$ flux-cross section factorization,

$$y Q^2 \frac{d\sigma}{dy dQ^2} = f_\lambda \sigma_\lambda.$$ (14)

The fluxes $f_\lambda$ and cross sections $\sigma_\lambda$ depend on the polarization state $\lambda$ of the EW boson. In the $W$-proton collision frame $q_\mu = (\nu, 0, 0, q_z)$, and the 4-vector of the so called longitudinal polarization, which we are interested in, is

$$s_\mu = \frac{1}{\sqrt{Q^2}} (q_z, 0, 0, \nu) = \frac{\sqrt{Q^2}}{\sqrt{(pq)^2 + m_\lambda^2 Q^2}} \left( p_\mu + \frac{pq}{Q^2} q_\mu \right).$$ (15)

Throughout this paper we use, following tradition, the name “longitudinal” for the time-like vector $s_\mu$ and provide corresponding variables with the label $L$. Then, in close similarity with the QED flux of longitudinal photons, the flux of $W_L$ bosons is

$$f_L = \frac{4 \alpha_W}{\pi} \frac{Q^4}{m_W^4} (1 - y).$$ (16)
where \( \alpha_W = g^2 / 4\pi \). Applying the optical theorem to the amplitude for Compton scattering of the \( W_L \) yields

\[
\left. \frac{d\sigma}{dydQ^2} \right|_{Q^2=0} = \frac{G_F^2}{2\pi^2} \frac{(1-y)Q^2}{g^2} \sigma_L(x, Q^2) \bigg|_{Q^2=0},
\]

where, in the fixed-\( r \) representation,

\[
\sigma_L(x, Q^2) = \langle W | r | \hat{\sigma} | r \rangle \langle r | W \rangle \\
= \int dz d^2r |\Psi_L(z, r)|^2 \sigma(x, r).
\]

The Eq.(3) then follows from comparison of Eqs. (13) and (17). In Eq. (18) \( \langle r | \hat{\sigma} | r \rangle = \sigma(x, r) \), \( \hat{\sigma} \) is the cross section operator and \( \langle r | W \rangle = |\Psi_L(z, r)\rangle \) is the LCWF of the \( |ud\rangle \) state with the \( u \) quark carrying fraction \( z \) of the \( W^+ \) light-cone momentum and \( \bar{d} \) with momentum fraction \( 1-z \) (see [5] for details). In [5] we used for \( \Psi_L \) the notation \( \Psi_0 \). The expansion (8) is written in the basis of physical hadrons (mass operator eigenstates) \( |a\rangle \) related to fixed-\( r \) states via \( |r\rangle = \sum_a \langle a | r \rangle |a\rangle \). In this basis the diagonal matrix elements of \( \hat{\sigma} \) give the total cross section of \( a-N \) scattering, \( \sigma_a = \langle a | \hat{\sigma} | a \rangle \) [22], and Eq.(18) turns into

\[
Q^2 \sigma_L \bigg|_{Q^2=0} = Q^2 \langle W | r | \hat{\sigma} | r \rangle \langle r | W \rangle \\
= Q^2 \sum_{a, a' \neq \pi} \langle W | a \rangle \langle a | \hat{\sigma} | a' \rangle \langle a' | W \rangle \\
= Q^2 \langle W | \pi \rangle \langle \pi | \hat{\sigma} | \pi \rangle \langle \pi | W \rangle = g^2 f_\pi^2 \sigma_\pi.
\]

In (19) the Goldberger-Treiman conspiracy, Eq.(11), was used to come from the second line to the third one.

Notice that the sum over hadronic states \( |a\rangle \) in (19) does not include the pion, the \( W \)-boson in the polarization state \( s_\mu \) (see Eq.(15)) does not mix with the pion, \( s_\mu q_\mu = 0 \). Consequently, the CD states described by the light-cone wave function \( \Psi_L(z, r) \) in Eq.(18) are dual not to the nearly massless \( \pi \)-meson but to “normal” axial-vector hadronic states \( (a_1, \rho \pi, ...) \) of a mass \( \sim 1 \) GeV. This observation justifies, in particular, the choice of the mass scale \( m_A = 1 \) GeV in Eq.(1) and lends support to the CD description of small-\( x \) phenomena in the axial channel.
4 $F_L f_\pi \sigma_\pi$-correlation in color dipole basis.

In terms of the longitudinal structure function

$$F_L(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_W} \sigma_L(x, Q^2),$$

(20)

Eq. (19) can be rewritten as

$$F_L(x, 0) = \frac{f_\pi^2}{\pi} \sigma_\pi(\nu),$$

(21)

where $\nu$ is related to $x$ by Eq. (1). In (20) $\sigma_L(x, Q^2)$ is defined by the CD factorization equation (18) and the light-cone density of CD states $u\bar{d}, c\bar{s}, ...$ is

$$|\Psi_L(z, r)|^2 = |V_L(z, r)|^2 + |A_L(z, r)|^2$$

with

$$|V_L(z, r)|^2 = \frac{2\alpha_W N_c}{(2\pi)^2 Q^2} \left\{ [2Q^2 z(1 - z) + (m - \mu)[(1 - z)m - z\mu]]^2 K_0^2(\varepsilon r) + (m - \mu)^2 \varepsilon K_1^2(\varepsilon r) \right\}$$

(22)

$$|A_L(z, r)|^2 = \frac{2\alpha_W N_c}{(2\pi)^2 Q^2} \left\{ [2Q^2 z(1 - z) + (m + \mu)[(1 - z)m + z\mu]]^2 K_0^2(\varepsilon r) + (m + \mu)^2 \varepsilon K_1^2(\varepsilon r) \right\},$$

(23)

where $m$ and $\mu$ are the quark and antiquark masses and $\varepsilon^2 = z(1 - z)Q^2 + (1 - z)m^2 + z\mu^2$ [5, 6]. At $Q^2 \to 0$ and for equal masses of constituent quarks $m = \mu = m_q$, the axial-vector light-cone density of $u\bar{d}$ states does not depend on $z$ and is as follows

$$|\Psi_L(z, r)|^2 = \frac{2\alpha_W N_c m_q^2}{\pi^2 Q^2} \left[ m_q K_0^2(m_q r) + m_q^2 K_1^2(m_q r) \right]$$

(24)

At $y \lesssim 1$, $K_0(y) \sim \log(1/y)$ and $K_1(y) \sim 1/y$. Then, from Eqs. (20) and (24)

$$F_L(x, 0) \sim \frac{N_c m_q^2}{2\pi^3} \int_{m_q^{-2}}^0 dr^2 \frac{dr^2}{r^2} \sigma(x, r).$$

(25)

The CD cross section is $\sigma(x, r) = r^2 C(x, r)$ with $C(x, r)$ slowly varying with $r$. For small dipoles $C(x, r)$ depends on $r$ only logarithmically. For large dipoles, such that $r > r_s$, $\sigma(x, r)$ saturates and $C(x, r) = \sigma_s(x)/r^2$ [15]. Therefore $F_L$ depends on several non-perturbative
parameters, \( m_q, r_s \) and \( \sigma_s \), and, because of the axial current non-conservation, it is sensitive to the value of the constituent quark mass,

\[
F_L \propto m_q^2 \sigma_s \log[1 + 1/(m_q r_s)^2].
\]  

(26)

The sensitivity is lost, however, for \( m_q^2 \gg r_s^{-2} \).

The rhs of Eq.(21) is known experimentally with high accuracy and, in view of all theoretical uncertainties with non-perturbative effects, Eq.(21) could be considered as a very useful constraint on parameters of CD models. Let us start, however, with a self-consistency check and before imposing experimental bounds on \( f_\pi \) and \( \sigma_\pi \) let us evaluate \( F_L, f_\pi \) and \( \sigma_\pi \) within the LCWF CD technique. Then, the deviation of the ratio

\[
R_{PCAC} = \frac{\pi F_L(x,0)}{f_\pi^2 \sigma_\pi(\nu)}.
\]  

(27)

from unity may serve as the measure of accuracy of the approach. Because of (26) the dependence of \( f_\pi \) and \( \sigma_\pi \) on the constituent quark mass is of prime importance here.

Notice first, that contrary to the pointlike probe, the dipole size of the non-pointlike pion and, consequently, the value of \( \sigma_\pi \) is determined not by \( m_q \) but by an additional parameter \( R \) which introduces the intrinsic momentum cut-off. \(^3\) The latter removes the small-\( r \) singularity from \( \Psi_\pi(z, r) \) and, simultaneously, ensures the correct value of the charge radius of the pion. The dependence of \( \sigma_\pi \) on \( m_q \) appears to be marginal and

\[
\sigma_\pi = \langle \Psi_\pi(z, r)|\sigma(x, r)|\Psi_\pi(z, r) \rangle \sim r_\pi^2 C(x, r_\pi).
\]  

(28)

The quantity which is very sensitive to \( m_q \) is the pion decay constant [23, 24],

\[
f_\pi = \frac{m_q N_c}{4\pi^3} \int \frac{dz d^2k}{z(1-z)} \Psi_\pi(M^2).
\]  

(29)

To a crude approximation, \( f_\pi \propto m_q \) and for \( m_q R \ll 1 \) (in [23] \( R = 2.2 \text{ GeV}^{-1} \))

\[
f_\pi \propto m_q \sqrt{\log(2/m_q R)}.
\]

This gives a chance to satisfy (21) adjusting properly non-perturbative parameters.

\(^2\)There is, of course, one more (hidden) non-perturbative parameter - the axial charge \( g_A \). The renormalization of \( g_A \) is neglected here and the ratio \( g_A/g_V \) for constituent quarks is assumed to be the same as for current quarks, \( g_A/g_V = 1 \).

\(^3\)The radial part of \( \Psi_\pi \) in momentum space is \( \Psi_\pi(M^2) \propto M^{-2} \exp[-\frac{1}{2} R^2 M^2] \) [23] (see also [24]), where \( M \) stands for the invariant mass of the light-cone \( q\bar{q} \) state and \( M^2 = (m_q^2 + k^2)/z(1-z) \).
Figure 1: Left panel: the ratio $R_{PCAC}$ as a function of $\nu$ evaluated with the pion LCWF of Ref.[23] for $m_q = 150$ MeV - thin solid line; the ratio $R_{PCAC}$ with the pion LCWF of Ref.[24] and $m_q = 250$ MeV - dotted line. Right panel: the CD BFKL evaluation of $\sigma_\pi(\nu)$ - dashed line. Data points (triangles and circles) are measurements of total $\pi^+p$ and $\pi^-p$ cross sections, respectively [28]. Also shown by the thick solid line is the quantity $\sigma_{PCAC}(\nu)$ for the empirical value of $f_\pi$ and with $F_L$ evaluated for $m_q = 150$ MeV.

Invoking the CD factorization, which is valid for soft as well as for hard diffractive interactions, we evaluate the vacuum exchange contribution to $F_L(x,0)$ and $\sigma_\pi(\nu)$. The structure function $F_L(x,0)$ comes from Eqs.(18,20,24) with $m_q = 150$ MeV, the value commonly used now in CD models successfully tested against DIS data. The $\log(1/x)$-evolution of $\sigma(x,r)$ is described by the CD BFKL equation [25, 26]. Corresponding boundary condition is found in [27]. With the pion LCWF of Ref.[23] we obtain $\sigma_\pi$ shown in Fig.1 (right panel) by the dashed line. The $\nu$-dependence of $R_{PCAC}$ is shown in Fig. 1 (left panel) by the thin solid line. With certain reservations about the slope of $R_{PCAC}(\nu)$ (see below) we conclude that our CD model successfully passed the consistency test. However, this “purely theoretical” approach to Adler’s theorem is not quite satisfactory. The point is that the constituent quark with $m_q = 150$ MeV amounts to $f_\pi = 96$ MeV vs. the empirical value $f_\pi = 130.7 \pm 0.46$ MeV [28]: not quite bad for the model evaluation of the soft parameter $f_\pi$, although not satisfactory either. Within the model [23] $f_\pi = 131$ MeV corresponds to $m_q = 245$ MeV, the value which is very close to $m_q = 250$ MeV of Ref.[24]. In [24] an oscillator type ansatz for the pion LCWF was used and good agreement of predictions of the model with both the empirical value of the pion decay constant and the charge radius of the pion was found. The
ratio $R_{PCAC}$ evaluated with the pion LCWF of Ref.[24] is shown in Fig.1 (left panel) by the dotted line. Evidently, making the light quark heavier affects the distribution of color dipoles in the light-cone $W_L$ boson in such a way that the characteristic dipole sizes contributing to $F_L$, $r^2 \lesssim m_q^{-2}$ (mind also the small-$r$ singularity in $\Psi_L$), becomes much smaller than those contributing to $\sigma_\pi$, $r^2 \sim r_\pi^2 \simeq 1.2$ fm$^2$. The BFKL log$(1/x)$-evolution of dipole cross sections is characterized by the exponent $\Delta(x,r)$ of the local $x$-dependence of $\sigma(x,r)$

$$
\sigma(x,r) \propto \exp[-\Delta(x,r) \log(1/x)].
$$

$\Delta(x,r)$ varies with $r$ and $\Delta(x,r_1) > \Delta(x,r_2)$ for $r_1 < r_2$ [25]. Hence, the structure function $F_L(x,0)$ growing with $\nu$ faster than $\sigma_\pi(\nu)$ in conflict with the requirement of Eq.(21).

As we noted above Eq.(21) can be considered as a condition on parameters of CD models. To see its restrictive power in action one can construct the quantity

$$
\sigma_{PCAC}(\nu) = \pi F_L(x,0)/f_\pi^2
$$

with the empirical value of $f_\pi = 130.7$ MeV and compare its $\nu$-dependence with experimental data on $\sigma_\pi(\nu)$. Our $\sigma_{PCAC}(\nu)$ is shown by the thick solid line in Fig.1 (right panel). It strongly undershoots $\sigma_\pi(\nu)$ thus indicating that $F_L(x,0)$ evaluated with $m_q = 150$ MeV fails to satisfy the Eq.(21). We tried also the CD cross sections of Refs.[18]. Corresponding $F_L$ proved to be close to ours. Notice that, the discrepancy observed may have the same origin as the deficit of the differential cross section of diffractive vector meson production found in [29]. That calls for better understanding of the infrared properties of the CD cross section.

From a different point of view, good agreement with data of both $\sigma_\pi(\nu)$ and $f_\pi$ spoiled, however, by the under-predicted $F_L(x,0)$ implies that non-perturbative interactions in the axial channel blow-up the dipole size and, in the spirit of PCAC, $\sqrt{Q^2}\Psi_L(z,r) \rightarrow g f_\pi \Psi_\pi(z,r)$ at $Q^2 \rightarrow 0.$
Figure 2: Left panel: three lower lines represent the ud-current contribution to $F_L(x, Q^2)$ as a function of Bjorken variable denoted by $x_{Bj}$ for different values of $Q^2$. Three upper lines correspond to the sum of ud- and cs-current contributions to $F_L$. Right panel: $F_L$ as a function of $\nu$ for $Q^2 = 0$.

5 Weak current non-conservation and charm-strange dominance.

One more concluding remark is on the role of charm-strange current. The structure function $F_L(x, 0)$ shown in Fig.1 does not contain the cs-current contribution. The latter is presented in Fig.2 were the ud-term and the sum ud + cs are shown separately for different virtualities of the probe. For the cs-current $x = x_{Bj}(1 + M^2_{cs}/Q^2)$ with $M^2_{cs} = 4 \text{ GeV}^2$. Eqs.(22,23) make it clear that it is the non-conservation of both axial-vector and vector currents what leads to the charm-strange dominance in $F_L(x, Q^2)$ at small $x$.

6 Summary and conclusions

Summarizing, we considered the PCAC hypothesis in a specific domain of small Bjorken $x$, where the relevant degrees of freedom are the QCD color dipoles. We reformulated Adler’s theorem in the CD basis and analyzed its efficiency as a constraint requiring identical cross sections for scattering processes with pointlike and non-pointlike probes. This requirement, with certain reservations about absorption/unitarity corrections, was found hard to meet within the color dipole models successfully tested against HERA data. Corresponding non-perturbative parameters including $m_q$ were adjusted to pave the way from the region $Q^2 \simeq m_q^2$
to high-$Q^2$ DIS. The analysis of diffractive vector mesons [29] shows that the adjustment is not perfect. The discrepancy found in our paper can be understood as the non-perturbative effect of increasing dipole size of the light-cone $W_L$ boson at $Q^2 \to 0$. Adler’s theorem provides its quantitative measure. This observation makes topical new experimental tests of Adler’s theorem in the diffraction region of $x \lesssim 0.01$. In view of the charm-strange dominance discussed above (which also should be tested experimentally) the $cs$-current contribution to the differential cross section $d\sigma/dydQ^2$ of the reaction (5) has to be isolated properly to separate the PCAC term.

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