Testing the CP-violating MSSM in stau decays at the LHC and ILC

Herbi Dreiner$^{1}$, Olaf Kittel$^{2}$, Suchita Kulkarni$^{1}$, Anja Marold$^{1}$

$^{1}$ Bethe Center for Theoretical Physics & Physikalisches Institut, Universität Bonn, D-53115 Bonn, Germany
$^{2}$ Departamento de Física Teórica y del Cosmos and CAFPE, Universidad de Granada, E-18071 Granada, Spain & II. Institut für Theoretische Physik, Universität Hamburg, D-22761 Hamburg, Germany

We study CP violation in the two-body decay of a scalar tau into a neutralino and a tau, which should be probed at the LHC and ILC. From the normal tau polarization, a CP asymmetry is defined which is sensitive to the CP phases of the trilinear scalar coupling parameter $A_f$, the gaugino mass parameter $M_1$, and the higgsino mass parameter $\mu$ in the stau-neutralino sector of the Minimal Supersymmetric Standard Model. Asymmetries of more than 70\% are obtained in scenarios with strong stau mixing. As a result, detectable CP asymmetries in stau decays at the LHC are found, motivating further detailed experimental studies for probing the SUSY CP phases.

I. INTRODUCTION

The surplus of matter over anti-matter within the universe can only be explained with a thorough understanding of CP violation. The CP phase in the quark mixing matrix of the Standard Model, which has been confirmed by B-meson experiments, is not sufficient to understand the baryon asymmetry of the Universe. However, the Minimal Supersymmetric Standard Model (MSSM) provides new physical phases that are manifestly CP-sensitive. After absorbing non-physical phases, we chose the complex parameters to be the higgsino mass parameter $\mu$, the U(1), and SU(3) gaugino mass parameters $M_1$ and $M_2$, and the trilinear scalar coupling parameters $A_f$ of the third generation sfermions ($f = h, t, \tau$). The corresponding phases violate CP and are generally constrained by experimental bounds on electric dipole moments (EDMs). However, these restrictions are strongly model dependent, such that additional measurements outside the low energy EDM sector are required.

Many CP observables have been proposed and studied in order to measure CP violation. Total cross sections, masses, and branching ratios are CP-even quantities. For a direct evidence of CP violation, however, CP-odd (T-odd) observables are required. Examples are rate asymmetries of either branching ratios, cross sections, or angular distributions. Since these rate asymmetries require the presence of absorptive phases, they are typically small, of the order of $< 10\%$, if they are not resonantly enhanced. Larger CP-odd observables which already appear at tree-level are desirable. These are T-odd triple products of momenta and/or spins, from which CP-odd asymmetries can be constructed. Such triple product asymmetries are highly CP-sensitive, and have been intensively studied both at lepton and hadron colliders.

Third generation sfermions have a rich phenomenology at high energy colliders like the LHC or ILC due to a sizable mixing of left and right states. In addition, the CP phases of the trilinear coupling parameters $A_f$ are rather unconstrained by the EDMs. The phases of $A_b$ and $A_t$ have been studied in stop and sbottom $\tilde{\chi}_0^0 \rightarrow \tilde{\nu}_\ell \tau^\pm$, and also in chargino decays $\tilde{\chi}_1^\pm \rightarrow \nu_\ell \tau^\mp$. It was shown that the normal tau polarization itself is CP-sensitive, and that the asymmetries are large and of the order of 60\% to 70\%.

We are thus motivated to study CP violation, including the tau polarization, in the two-body decay of a stau

$$\tilde{\tau}_m \rightarrow \tau + \tilde{\chi}_1^0, \quad m = 1, 2, \quad i = 2, 3, 4,$$ 

followed by the subsequent chain of two-body decays

$$\tilde{\chi}_1^0 \rightarrow \ell_1 + \tilde{\ell}_n; \quad (2a)$$
$$\tilde{\ell}_n \rightarrow \tilde{\ell}_2; \quad n = L, R, \quad \ell = e, \mu.$$ 

See Fig. 1 for a schematic picture of the entire stau decay. This process is kinematically open for a mass hierarchy

$$m_\tau > m_{\tilde{\chi}_1^0} > m_{\tilde{\ell}} = m_{\tilde{\mu}},$$

where the staus are heavier than the smuons and selectrons. We thus work in MSSM scenarios with heavier stau soft SUSY breaking parameters

$$M_{\tilde{E}_\tau} > M_{\tilde{E}_L} = M_{\tilde{E}_\mu},$$
$$M_{\tilde{L}_\tau} > M_{\tilde{L}_L} = M_{\tilde{L}_\mu}.$$ 

We show that the normal tau polarization, with respect to the plane spanned by the $\tau$ and $\ell_1$ momentum, is a triple product asymmetry which is sensitive to the
phases of $A_\tau$, $M_1$, and $\mu$ in the stau-neutralino sector. For nearly degenerate stau masses, $M_{\tilde{\tau}_L} \approx M_{\tilde{\tau}_R}$, a strong stau mixing is obtained which results in tau polarization asymmetries of more than 70%. This should be measurable at colliders\textsuperscript{1}. Since the stau is a scalar particle, its particular production does not contribute to CP-sensitive spin-spin correlations, and can thus be considered separately. This allows a collider-independent study, where we only discuss the boost dependence of the CP asymmetries.

The paper is organized as follows. In Section II we review stau mixing and the stau-neutralino Lagrangian with complex couplings. We calculate the amplitude squared for the entire stau decay in the spin-density matrix formalism \[30\]. We construct the CP asymmetry matrix formalism \[30\]. We construct the CP asymmetry matrix in the (\(\tilde{\tau}_L, \tilde{\tau}_R\))-basis as \[31\]

\[
M_\tilde{\tau} = \begin{pmatrix}
m^2_{\tilde{\tau}_L} & e^{-i\phi_\tau} m_{\tilde{\tau}} |\Lambda_\tau| \\
e^{i\phi_\tau} m_{\tilde{\tau}} |\Lambda_\tau| & m^2_{\tilde{\tau}_R}
\end{pmatrix}.
\]

CP violation is parameterized by the physical phase

\[
\phi_\tau = \arg[\Lambda_\tau],
\]

\[
\Lambda_\tau = A_\tau - \mu^* \cot \beta,
\]

with the complex trilinear scalar coupling parameter $A_\tau$, the complex higgsino mass parameter $\mu$, and $\tan \beta = v_1/v_2$, the ratio of the vacuum expectation values of the two neutral Higgs fields. The left and right stau masses are

\[
m^2_{\tilde{\tau}_L} = M^2_{\tilde{\tau}_L} + \left(-\frac{1}{2} + \sin^2 \theta_W\right)m^2_Z \cos(2\beta) + m^2_{\tau},
\]

\[
m^2_{\tilde{\tau}_R} = M^2_{\tilde{\tau}_L} - \sin^2 \theta_W m^2_Z \cos(2\beta) + m^2_{\tau},
\]

with the real soft SUSY breaking parameters $M^2_{\tilde{\tau}_L, \tilde{\tau}_R}$, the electroweak mixing angle $\theta_W$, and the masses of the $Z$ boson $m_Z$, and of the tau lepton, $m_{\tau}$.

In the mass basis, the stop eigenstates are \[3, 31\]

\[
\begin{pmatrix}
\tilde{\tau}_1 \\
\tilde{\tau}_2
\end{pmatrix} = \mathcal{R}^T \begin{pmatrix}
\tilde{\tau}_L \\
\tilde{\tau}_R
\end{pmatrix},
\]

with the diagonalization matrix

\[
\mathcal{R}^T = \begin{pmatrix}
e^{i\phi_\tau} \cos \theta_\tilde{\tau} & \sin \theta_\tilde{\tau} \\
-\sin \theta_\tilde{\tau} & e^{-i\phi_\tau} \cos \theta_\tilde{\tau}
\end{pmatrix},
\]

and the stau mixing angle

\[
\cos \theta_\tilde{\tau} = \frac{-m_{\tau} |\Lambda_\tau|}{\sqrt{m^2_\tau |\Lambda_\tau|^2 + (m^2_{\tilde{\tau}_L} - m^2_{\tilde{\tau}_R})^2}},
\]

\[
\sin \theta_\tilde{\tau} = \frac{m^2_{\tilde{\tau}_L} - m^2_{\tilde{\tau}_R}}{\sqrt{m^2_\tau |\Lambda_\tau|^2 + (m^2_{\tilde{\tau}_L} - m^2_{\tilde{\tau}_R})^2}}.
\]

The stau mass eigenvalues are

\[
m^2_{\tilde{\tau}_{1,2}} = \frac{1}{2} \left[ (m^2_{\tilde{\tau}_L} + m^2_{\tilde{\tau}_R}) \mp \sqrt{(m^2_{\tilde{\tau}_L} - m^2_{\tilde{\tau}_R})^2 + 4m^2_\tau |\Lambda_\tau|^2} \right].
\]

\textsuperscript{1} Note that we do not include the tau decay in our calculations. However, some of the decay products of the tau have to be reconstructed in order to measure the tau spin. The main goal of our work is to motivate such an experimental study, to address the feasibility of measuring the CP phases at the LHC or ILC.
B. Lagrangian and complex couplings

The relevant Lagrangian terms for the stau decay \( \tilde{\tau}_m \to \tau \chi_1^0 \) are \[ \mathcal{L}_{\tau \tilde{\chi}_1^0} = g \tilde{\tau} (a_{mi} P_R + b_{mi} P_L) \tilde{\chi}_1^0 \tilde{\tau}_m + \text{h.c.}, \] (16)
with \( P_{L,R} = (1 \mp \gamma_5)/2 \), and the weak coupling constant \( g = e/\sin \theta_w \), \( e > 0 \). The couplings are defined as \[ a_{mi} = \sum_{n=1}^4 (R_{mn}^*) A^*_n, \quad b_{mi} = \sum_{n=1}^4 (R_{mn}^*) B^*_n. \] (17)
The stau diagonalization matrix \( R^i \) is given in Eq. (2), and
\[ A^i = \begin{pmatrix} f_{ri}^L \\ h_{ri}^R \end{pmatrix}, \quad B^i = \begin{pmatrix} h_{ri}^L \\ f_{ri}^R \end{pmatrix}. \] (18)
In the photino, zino, higgsino basis \( (\tilde{\gamma}, \tilde{Z}, \tilde{H}_0^a, \tilde{H}_0^b) \), we have
\[ f_{ri} = \sqrt{2} \left[ \frac{1}{\cos \theta_w} \left( \frac{1}{2} - \sin^2 \theta_w \right) N_{i2} + \sin \theta_w N_{i1} \right], \] (19)
\[ f_{ri} = \sqrt{2} \sin \theta_w \left( \tan \theta_w N_{i2} - N_{i1} \right), \] (20)
\[ h_{ri} = (h_{ri}^*)^* = -Y_{\tau} (N_{i3}^* \cos \beta + N_{i4}^* \sin \beta), \] (21)
\[ Y_{\tau} = \frac{m_{W}}{\sqrt{2} m_{W} \cos \beta}. \] (22)
with \( m_W \) the mass of the W boson, and \( N \) the complex, unitary 4 \( \times \) 4 matrix that diagonalizes the neutralino mass matrix
\[ N^* \cdot M_{\chi^0} \cdot N = \text{diag}(m_{\chi_1^0}, \ldots, m_{\chi_4^0}). \] (23)
The interaction Lagrangian relevant for the neutralino decay \( \tilde{\chi}_i^0 \to \ell_{R,L} \tilde{\ell} \tilde{\tau} \), for \( \ell = e, \mu \) is \[ \mathcal{L}_{\ell \tilde{\chi}_1^0} = g f_{ri} P_R \tilde{\chi}_1^0 \tilde{\ell} R + g f_{ri} P_L \tilde{\chi}_1^0 \tilde{\ell} L + \text{h.c.}, \] (24)
with the couplings \( f_{ri} \) given in Eqs. (19) and (20).

C. Tau spin density matrix

The unnormalized, 2 \( \times \) 2, hermitian, \( \tau \) spin density matrix for stau decay, Eqs. (1) and (2), reads
\[ \rho_{\lambda' \lambda} \equiv \int (|\mathcal{M}|^2)^{\lambda' \lambda} d\mathcal{L}_{\text{rips}}, \] (25)
with the amplitude \( \mathcal{M} \), and the Lorentz invariant phase space element \( d\mathcal{L}_{\text{rips}} \), for details see Appendix [3]. The \( \tau \) helicities are denoted by \( \lambda_\tau \) and \( \lambda'_\tau \). In the spin density matrix formalism [30], the amplitude squared is given by
\[ (|\mathcal{M}|^2)^{\lambda' \lambda} = |\Delta(\chi_1^0)|^2 |\Delta(\tilde{\ell})|^2 \times \sum_{\lambda, \lambda'} \rho_D(\tilde{\ell})^{\lambda' \lambda} \rho_D(\chi_1^0)^{\lambda \lambda'} D_2(\tilde{\ell}), \] (26)
with the neutralino helicities \( \lambda_i, \lambda'_i \). The amplitude squared decomposes into the remnants of the propagators
\[ \Delta(j) = \frac{i}{s_j - m_j^2 + i m_j \Gamma_j}, \] (27)
with mass \( m_j \), and width \( \Gamma_j \) of particle \( j = \chi_1^0 \) or \( \tilde{\ell} \), and the unnormalized spin density matrices for stau decay \( \rho_{DD} \), and neutralino decay \( \rho_{D_1} \). The decay matrix of the spinless slepton is a factor since the polarizations of the final lepton and LSP are not accessible. The corresponding amplitude is denoted by \( D_2(\tilde{\ell}) \). Defining a set of spin basis vectors \( s^a_\ell \) for the tau, see Eqs. (A10) in Appendix [A], the spin density matrices can be expanded in terms of the Pauli matrices \( \sigma \)
\[ \rho_{D}(\tilde{\ell})^{\lambda' \lambda} = D \delta^{\lambda' \lambda} + \Sigma_D (\sigma^a)^{\lambda' \lambda} \delta_{\lambda' \lambda} + \Sigma_D^b (\sigma^a)^{\lambda' \lambda} (\sigma^b)^{\lambda' \lambda}, \] (28)
\[ \rho_{D_1}(\chi_1^0)^{\lambda' \lambda} = D_1 \delta^{\lambda' \lambda} + \Sigma_D (\sigma^b)^{\lambda' \lambda}, \] (29)
with an implicit sum over \( a, b = 1, 2, 3 \), respectively. The real expansion coefficients \( D, D_1, \Sigma_D, \Sigma_D^a, \Sigma_D^b \) and \( \Sigma_D^h \) contain the physical information of the process. \( D \) denotes the unpolarized part of the amplitude for stau decay \( \tilde{\tau}_m \to \chi_{i}^0 \tau \), \( D_1 \) denotes the unpolarized part for neutralino decay \( \chi_{i}^0 \to \tilde{\ell}_i \ell_1 \), respectively. \( \Sigma_D^a \) gives the tau polarization, \( \Sigma_D^b, \Sigma_D^h \) and \( \Sigma_D^b \) describe the contributions from the neutralino polarization, and \( \Sigma_D^h \) is the spin-spin correlation term, which contains the CP-sensitive parts. We give the expansion coefficients explicitly in Appendix [C].

Inserting the density matrices, Eqs. (28) and (29), into Eq. (26), we get for the amplitude squared
\[ (|\mathcal{M}|^2)^{\lambda' \lambda} = 2 |\Delta(\chi_1^0)|^2 |\Delta(\tilde{\ell})|^2 \times \left[ (DD_D + \Sigma_D^a \Sigma_D^b) \delta^{\lambda' \lambda} + (\Sigma_D^a D_1 + \Sigma_D^b \Sigma_D^h) (\sigma^a)^{\lambda' \lambda} \right] D_2, \] (30)
with an implicit sum over \( a, b = 1, 2, 3 \). The amplitude squared \((|\mathcal{M}|^2)^{\lambda' \lambda}\) is now decomposed into an unpolarized part (first summand), and into the part for the tau polarization (second summand), in Eq. (30). By using the completeness relations for the neutralino spin vectors,
Eq. (A12), the products in Eq. (50) can be written\textsuperscript{2},
\[
\Sigma_D \Sigma_D^b = \frac{g^4}{2} \left( |s_{mi}^*|^2 - |b_{mi}^*|^2 \right) |f_{1\ell}^R|^2 \times \\
\left[ m_{\tilde{\chi}_i}^2 (p_\ell \cdot p_\ell) - (p_{\tilde{\chi}_i} \cdot p_\ell)(p_\ell \cdot p_{\tilde{\chi}_i}) \right], \quad (31)
\]
\[
\Sigma_D^a \Sigma_D^b = \frac{g^4}{2} \left( |s_{mi}^*|^2 + |b_{mi}^*|^2 \right) |f_{1\ell}^R|^2 m_\tau \times \\
\left[ (s_{\tau}^\mu p_{\tilde{\chi}_i} - p_{\tilde{\chi}_i}^\mu)(p_{\tilde{\chi}_i}^\mu p_\ell) - m_{\tilde{\chi}_i}^2 (s_{\tau}^\mu p_\ell) \right] \times \\
\left[ (s_{\tau}^\mu p_{\tilde{\chi}_i} - p_{\tilde{\chi}_i}^\mu)(s_{\tau}^\mu p_\ell) - (p_\ell - p_\ell)(s_{\tau}^\mu p_{\tilde{\chi}_i}) \right] \times \\
\mathfrak{Im} \{ a_{mi}^* b_{mi}^* \} |p_\ell p_\ell p_\tau s_{\tau}^\mu |. \quad (32)
\]

The spin-spin correlation term $\Sigma_D^a \Sigma_D^b$, Eq. (32), explicitly depends on the imaginary part $\mathfrak{Im} \{ a_{mi}^* b_{mi}^* \}$. The stau-neutralino couplings, Eq. (10). This term is manifestly CP-sensitive, i.e., it depends on the phases $\phi_{A_\tau}, \phi_1, \phi_\mu$ of the stau-neutralino sector. The imaginary part is multiplied by the totally anti-symmetric (episolon) product,
\[
\mathcal{E}^a \equiv [p_{\tilde{\tau}}, p_{\ell i}, p_\tau, s_{\tau}^\mu] = \epsilon_{\mu \nu \rho \sigma} p_\tau^\mu p_\ell^\nu p_{\ell i}^\rho s_{\tau}^{\sigma}, \quad (33)
\]
with the convention $\epsilon_{1234} = 1$. Since each of the spatial components of the four-momenta $p$, or the spin vectors $s_{\tau}^\mu$, changes sign under a time transformation, $t \rightarrow -t$, the epsilon product $\mathcal{E}^a$ is T-odd. In the stau rest frame, $p_\ell^\mu = (m_\tau, 0)$, the epsilon product reduces to the T-odd triple product $\mathcal{T}^a$
\[
[p_{\tilde{\tau}}, p_{\ell i}, p_\tau, s_{\tau}^\mu] = m_\tau (p_{\ell i} \times p_\tau) \cdot s_{\tau}^\mu \equiv m_\tau \mathcal{T}^a. \quad (34)
\]
The task in the next section is to define an observable that projects out from the amplitude squared the part proportional to $\mathcal{E}^a$ (or $\mathcal{T}^a$), in order to probe the CP-sensitive coupling combination $\mathfrak{Im} \{ a_{mi}^* b_{mi}^* \}$.

\textbf{D. Normal tau polarization and CP asymmetry}

The $\tau$ polarization is given by the expectation value of the Pauli matrices $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ \textsuperscript{33}
\[
\mathcal{P} = \frac{\text{Tr} \{ p \sigma \} }{\text{Tr} \{ p \} }, \quad (35)
\]
with the $\tau$ spin density matrix $\rho$, as given in Eq. (25).

In our convention for the polarization vector $\mathcal{P} = (P_1, P_2, P_3)$, the components $P_1$ and $P_3$ are the transverse and longitudinal polarizations in the plane spanned by $p_\ell$ and $p_\tau$, respectively, and $P_2$ is the polarization normal to that plane. See our definition of the tau spin basis vectors $s_{\tau}^\mu$ in Appendix A.

The normal $\tau$ polarization is equivalently defined as
\[
\mathcal{P}_2 = \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}, \quad (36)
\]
with the number of events $N$ with the $\tau$ spin up ($\uparrow$) or down ($\downarrow$), with respect to the quantization axis $p_\ell \times p_\tau$, see Eq. (A10). The normal $\tau$ polarization can thus also be regarded as an asymmetry
\[
\mathcal{P}_2 = \frac{\sigma(\mathcal{T} > 0) - \sigma(\mathcal{T} < 0)}{\sigma(\mathcal{T} > 0) + \sigma(\mathcal{T} < 0)}. \quad (37)
\]
of the triple product
\[
\mathcal{T} = (p_{\ell i} \times p_\tau) \cdot \xi_\tau, \quad (38)
\]
where $\xi_\tau$ is the direction of the $\tau$ spin vector for each event.

The triple product $\mathcal{T}$ is included in the spin-spin correlation term $\Sigma_D^a \Sigma_D^b$, Eq. (32), cf. Eq. (51), and the asymmetry thus probes the term which contains the CP-sensitive coupling combination $\mathfrak{Im} \{ a_{mi}^* b_{mi}^* \}$.

Since under naive time reversal, $t \rightarrow -t$, the triple product $\mathcal{T}$ changes sign, the tau polarization $\mathcal{P}_2$, Eq. (37), is T-odd. Due to CPT invariance $\mathcal{P}_2$ would thus be CP-odd at tree level. In general, $\mathcal{P}_2$ also has contributions from absorptive phases, e.g. from intermediate $s$-state resonances or final-state interactions, which do not signal CP violation. Although such absorptive contributions are a higher order effect, and thus expected to be small, they can be eliminated in the true CP asymmetry $\mathcal{A}_\tau^\text{CP}$
\[
\mathcal{A}_\tau^\text{CP} = \frac{1}{2}(\mathcal{P}_2 - \overline{\mathcal{P}}_2), \quad (39)
\]
where $\overline{\mathcal{P}}_2$ is the normal tau polarization for the charged conjugated process $\tau^i_m \rightarrow \tau^\mp \tilde{\chi}_0^i$. For our analysis at tree level, where no absorptive phases are present, we find $\overline{\mathcal{P}}_2 = -\mathcal{P}_2$, see the sign change in Eqs. (51) and (52), and thus $\mathcal{A}_\tau^\text{CP} = \mathcal{P}_2$. We study $\mathcal{A}_\tau^\text{CP}$ in the following, which is, however, equivalent to $\overline{\mathcal{P}}_2$ at tree level.
Inserting now the explicit form of the density matrix $\rho$, Eq. (25), into Eq. (35), together with Eq. (36), we obtain the CP asymmetry

$$A^\text{CP}_\tau = \mathcal{P}_2 = \frac{\int \Sigma b \Sigma d \mathcal{Z} \mathcal{I} \mathcal{P}_s}{\int DD_1 d \mathcal{Z} \mathcal{I} \mathcal{P}_s}, \quad (40)$$

where we have used the narrow width approximation for the propagators in the phase space element $d \mathcal{Z} \mathcal{I} \mathcal{P}_s$, see Eq. (19). Note that in the denominator of $A^\text{CP}_\tau$, Eq. (40), the spin correlation terms vanish, $\int \Sigma D_2 \Sigma D_1 d \mathcal{Z} \mathcal{I} \mathcal{P}_s = 0$, see Eq. (31), when integrated over phase space. In the numerator only the spin-spin correlation term $\Sigma b D_1 \Sigma a$ for $a = 2$ contributes, which contains the T-odd epsilon product $\mathcal{E}_1$, see Eq. (33).

### E. Parameter dependence of the CP asymmetry

To qualitatively understand the dependence of the asymmetry $A^\text{CP}_\tau$, Eq. (19), on the MSSM parameters, we study in some detail its dependence on the $\tau_{\text{m}}-\tau_{\chi_i^0}$ couplings, $a_{m i}^\tau$ and $b_{m i}^\tau$, see Eq. (13). From the explicit form of the decay terms $\Sigma b \Sigma d_1$, Eq. (31), and $D_1$, $D_2$, Eqs. (31), (35), respectively, we find that the asymmetry

$$A^\text{CP}_\tau = \eta_{m i} \int \mathcal{Z} \mathcal{I} \mathcal{P}_s \mathcal{P}_s (p_i \cdot p_\tau) (p_i \cdot p_\tau) \int d \mathcal{Z} \mathcal{I} \mathcal{P}_s, \quad (41)$$

with $(p_i \cdot p_\tau) = (m_i^2 - m_{\chi_i}^2)/2$, and $(p_i \cdot p_\tau) = (m_{\chi_i}^2 - m_{\tilde{\tau}}^2)/2$, is proportional to the decay coupling factor

$$\eta_{m i} = \frac{\mathcal{I} \mathcal{F} \mathcal{P}_s (a_{m i}^\tau)^* (b_{m i}^\tau)^*}{\sqrt{2 |a_{m i}^\tau|^2 + |b_{m i}^\tau|^2}}, \quad (42)$$

with $\eta_{m i} \in [-1, 1]$. We thus expect maximal asymmetries for equal moduli of left and right couplings, $|a_{m i}^\tau| \approx |b_{m i}^\tau|$, which have a phase difference of about $\pi/2$, where the coupling factor can be maximal $\eta_{m i} = \pm 1$, see Eq. (42).

To study the dependence of $\eta$ on the CP phase $\phi_\tau$ of the stau sector, and the stau mixing angle $\theta_\tau$, we expand the imaginary part of the product of $\tau_{\text{m}}-\tau_{\chi_i^0}$ couplings

$$\mathcal{I} \mathcal{F} \mathcal{P}_s (a_{m i}^\tau)^* (b_{m i}^\tau)^* = \mathcal{I} \mathcal{F} \mathcal{P}_s \left\{ |\mathcal{R}_{m \tau_i}|^2 f_{\tau_i h_{\tau_i}^R}^L + |\mathcal{R}_{m \tau_i}|^2 f_{\tau_i h_{\tau_i}^R}^L + \mathcal{R}_{m \tau_i}^{\tau R} \left( (h_{\tau_i}^R)^2 - f_{\tau_i h_{\tau_i}^R}^L \right) \right\}, \quad (43)$$

in terms of the stau mixing matrix $\mathcal{R}$, the gauge couplings $f_{\tau_i h_{\tau_i}^R}^L$ and the higgs couplings $h_{\tau_i h_{\tau_i}^R}^L$. In particular, for a CP-conserving neutralino sector, $\phi_1 = \phi_\mu = 0$, we have

$$\mathcal{I} \mathcal{F} \mathcal{P}_s (a_{m i}^\tau)^* (b_{m i}^\tau)^* = \frac{1}{\tau^2} \sin \phi_\tau \sin(2\theta_\tau) \frac{1}{2} \left( (h_{\tau_i}^R)^2 - f_{\tau_i h_{\tau_i}^R}^L \right), \quad (44)$$

for $m = 1$, and the sign in parentheses holds for $m = 2$.

### F. Boost dependence

The triple product asymmetry $A^\text{CP}_\tau$, Eq. (40), is not Lorentz invariant but depends on the boost of the decaying stau,

$$\beta_\tau = \frac{|P_{\tau_i}|}{\mathcal{E}_\tau}. \quad (45)$$

In Fig. 2, we show the boost dependence of the asymmetry $A^\text{CP}_\tau(\beta_\tau)$, normalized by $A^\text{CP}_\tau(\beta_\tau = 0)$, for three different sets of stau masses, $m_{\tilde{\tau}_{1,2}} \approx 200$ GeV (solid, red), 400 GeV (dashed, green), and 1000 GeV (dotted, blue), see text, for stau decay $\tilde{\tau}_1 \rightarrow \tau_{\chi_i^0}$ followed by $\chi_i^0 \rightarrow \ell_1 \ell_2$, and $\ell_2 \rightarrow \ell_1 \ell_2$ ($\ell = e$ or $\mu$), see Fig. 1. The SUSY parameters are given in Table II.

---

3 Note that a maximal mixing is naturally achieved for nearly degenerate staus. However then the asymmetries for $\tilde{\tau}_1$ and $\tilde{\tau}_2$ decay typically have similar magnitude but opposite sign, and thus might cancel. See the discussion at the end of the numerics in Section III D.
The boost distribution \( \mathcal{A}_{\tau \rightarrow \ell} \) of the staus depends on their mass, which typically peaks at high values \( \beta \approx 0.9 \) for stau masses of the order of a few hundred GeV up to a 1 TeV, see e.g. Refs. [21][22]. Then the normal tau polarization in the laboratory frame is obtained by folding the boost dependent polarization \( \mathcal{A}_{\tau \rightarrow \ell} \) with the normalized stau boost distribution \( \mathcal{A}_{\tau \rightarrow \ell} \).}

\[
\mathcal{A}_{\tau \rightarrow \ell} = \frac{1}{\sigma_P} \int_0^1 \frac{d \sigma_P}{d \beta} \mathcal{A}_{\tau \rightarrow \ell}(\beta) d \beta, \tag{46}
\]

with the production cross section \( \sigma_P = \sigma(pp \rightarrow \tilde{\tau}^+ \tilde{\tau}^-) \).

The typical reduction of the normal tau polarization \( \mathcal{A}_{\tau \rightarrow \ell} \) is of the order of two thirds of the asymmetry compared to that in the stau rest frame \( \mathcal{A}_{\tau \rightarrow \ell}(0) \). However, it has been recently shown (for similar asymmetries in stop decays at the LHC), that the rest frame can be partly reconstructed event by event using on-shell mass conditions, see Refs. [22]. The LHC cross section for stau pair production, \( \sigma(pp \rightarrow \tilde{\tau}^1 \tilde{\tau}^-_1) \), also sensitively depends on the stau masses, e.g., for our benchmark scenario in Table II we find cross sections up to 10 fb at \( \sqrt{s} = 14 \text{ TeV} \)

### III. Numerical Results

We quantitatively study the tau polarization asymmetry, and the branching ratios for the two-body decay chain

\[
\tilde{\tau}_1 \rightarrow \tau + \chi^0_2, \quad \chi^0_2 \rightarrow \ell^+_R + \ell^-_R, \quad \ell^+_R \rightarrow \chi^0_1 + \ell^-_2, \tag{47}
\]

for \( \ell = e, \mu \). The asymmetry probes the MSSM phases \( \phi_1, \phi_\mu, \phi_A \), of the neutralino and stau sector. We center our numerical discussion around a general MSSM benchmark scenario, see Table II. We choose heavier soft breaking parameters in the stau sector than in the \( \tilde{e}, \tilde{\mu} \) sector, to enable the mass hierarchy

\[
m_{\tilde{t}_m} > m_{\tilde{\chi}^0_1} > m_{\tilde{\mu}_R} > m_{\tilde{\chi}^0_3}. \tag{48}
\]

Further we choose almost degenerate staus which enhances their mixing, leading to maximal asymmetries. We choose a large value of the trilinear scalar coupling parameter, \( |A_t| > |\mu| \tan \beta \), to enhance the impact of \( \phi_A \) in the stau sector. Finally, to reduce the number of MSSM parameters, we use the (GUT inspired) relation \( |M_1| = 5/3 M_2 \tan^2 \theta_w \) for the gaugino mass parameters. The resulting masses of the staus, neutralinos and charginos are summarized in Table II.

#### A. Phase dependence

For the benchmark scenario given in Table II we study the phase dependence of the asymmetry \( \mathcal{A}_{\tau \rightarrow \ell} \) in the stau rest frame. In Fig. 3(a) we show the dependence on the CP phases in the neutralino sector, \( \phi_1, \phi_\mu, \phi_A \), and \( \phi_{\tilde{\mu}} \). In Fig. 3(b) we show the dependence on the phases in the stau sector \( \phi_A \). The asymmetry strongly depends on \( \phi_A, \phi_{\tilde{\mu}} \), which we expect for \( |A_t| \gg |\mu| \tan \beta \) as in our benchmark scenario, see Table II. In particular for \( \phi_\mu = 0 \) in Fig. 3(b) the asymmetry follows the approximation formula Eq. (44), and attains its maximal values at \( \phi_e \approx \phi_{\tilde{\mu}} \approx \pm \pi/2 \).

#### B. \( |A_t| \)-\tan \beta dependence and stau mixing

In Fig. 4(a) we show the \( |A_t| \) and \( \tan \beta \) dependence of the asymmetry \( \mathcal{A}_{\tau \rightarrow \ell} \) in the stau rest frame. We can observe that the asymmetry obtains its maximum,
\[ A_{CP} \approx -77\%, \] where also the coupling factor is maximal, \( \eta \approx 0.95 \), see Fig. 4(b). As discussed in Subsection III E, the imaginary part of the product of the stau couplings \( 3m(a_{\mu i} \bar{b}_{i\mu})^* \) is maximal for a maximal CP phase \( \phi_\tau = \pi/2 \) in the stau sector, which we show in Fig. 4(c). Note that the location of the maximum of \( A_{CP} \) is not at maximal stau mixing, \( \sin(\theta_\tau) = 1/\sqrt{2} \approx 0.7 \), since \( \eta \propto \sin(2\theta_\tau)/(|a^2|^2 + |\bar{b}|^2) \) starts to decrease for increasing \( A_\tau \) and \( \tan \beta \).

To study the stau mixing, we show the \( M_{\tilde{E}} - M_{\tilde{L}} \) dependence of the asymmetry \( A_{CP} \) in Fig. 5(a). In the entire \( M_{\tilde{E}} - M_{\tilde{L}} \) plane, the CP phase in the stau sector is almost maximal, \( \phi_\tau = 0.61\pi \). However, the asymmetry obtains its maxima in the small corridor \( M_{\tilde{E}} \approx M_{\tilde{L}} \), where the stau mixing is maximal, \( \theta_\tau = \pi/4 \).

C. \(|\mu|-M_2\) dependence and branching ratios

We show the \(|\mu|-M_2\) dependence of the asymmetry \( A_{CP} \) in Fig. 5(b). The maxima of \( A_{CP} \) are obtained where the coupling factor \( \eta \) is also maximal, see Eq. 42.

In Fig. 6(a) we show the corresponding stau branching ratio, \( BR(\tilde{\tau}_1 \rightarrow \tau \tilde{\chi}_2^0) \), which can be as large as 40%. Other competing channels can reach \( BR(\tilde{\tau}_1 \rightarrow \tau \tilde{\chi}_1^\pm) \approx 65\% \), and \( BR(\tilde{\tau}_1 \rightarrow \nu_\tau \tilde{\chi}_1^0) \approx 20(10)\% \). The stau decay into the chargino \( \tilde{\chi}_1^\pm \) is always open since typically the second lightest neutralino and the lightest chargino are almost degenerate, \( m_{\tilde{\chi}_2^0} \approx m_{\tilde{\chi}_1^0} \). The neutralino branching ratio \( BR(\tilde{\chi}_2^0 \rightarrow \ell \tilde{\ell}_R) \), summed over \( \ell = e, \mu \), is shown in Fig. 6(b) which reaches up to 100%. The other important competing decay channels are \( BR(\tilde{\chi}_2^0 \rightarrow \nu_\mu \tilde{\nu}_\mu) \), and \( BR(\tilde{\chi}_2^0 \rightarrow \ell \ell_L) \), which open around \( \mu \approx 250 \text{ GeV} \) and \( \mu \approx 300 \text{ GeV} \), respectively, for \( M_2 = 250 \text{ GeV} \). Note that in our benchmark scenario, see Table II we have \( BR(\tilde{\ell}_R \rightarrow \tilde{\chi}_1^0 \ell) = 1 \).

D. Impact of \( \tilde{\tau}_2 \) decay

As we discussed in Section III B, we find large asymmetries for nearly degenerate staus, where we naturally obtain a maximal stau mixing. However, then typically the asymmetries for \( \tilde{\tau}_1 \) and \( \tilde{\tau}_2 \) decay are similar in magnitude, but opposite in sign. For example in our benchmark scenario we find \( A_{CP} = -71\% \) for \( \tilde{\tau}_1 \) decay, but \( A_{CP} = +32\% \) for the decay of \( \tilde{\tau}_2 \). If the production and decay process of \( \tilde{\tau}_1 \) cannot be experimentally disentangled from that of \( \tilde{\tau}_2 \) properly, the two asymmetries might cancel. We show their sum in Fig. 7(a) in the \( M_{\tilde{E}} - M_{\tilde{L}} \) plane. In Fig. 7(b) we show the corresponding stau mass splitting.

Note that also the stau branching ratios are similar in size; for example in our benchmark scenario we have \( BR(\tilde{\tau}_1 \rightarrow \tau \tilde{\chi}_2^0) = 18\% \), and \( BR(\tilde{\tau}_2 \rightarrow \tau \tilde{\chi}_2^0) = 30\% \). For the \( M_{\tilde{E}} - M_{\tilde{L}} \) plane shown in Fig. 3 the decay branching ratio \( BR(\tilde{\tau}_1 \rightarrow \tau \tilde{\chi}_2^0) \) is at least 10\%, and that of \( \tilde{\tau}_2 \) is larger by roughly a factor of 2 to 4.
FIG. 4: $|A_{\tau}|$–tan $\beta$ dependence of (a) the $\tau$ polarization asymmetry $A_{\tau}^{\text{CP}}$, Eq. (39), in percent, in the stau rest frame (for the decay $\tilde{\tau}_1 \to \tau \tilde{\chi}^0_2$, followed by $\tilde{\chi}^0_2 \to \ell R$ and $\ell R \to \tilde{\chi}^0_2 \ell$ for $\ell = e$ or $\mu$, cf. Fig. 1), (b) the coupling factor $\eta$, Eq. (42), (c) the phase $\phi_{\tilde{\tau}}$ in the stau sector, Eq. (7), and (d) $\sin(2\theta_{\tilde{\tau}})$, with $\theta_{\tilde{\tau}}$ the stau mixing angle, Eqs. (13), (14). The plots are for $\phi_{A_{\tau}} = \pi/4$, the other MSSM parameters are given in Table 1.
FIG. 5: Dependence of the τ polarization asymmetry $A_{\text{CP}}^{\tau}$, Eq. (39), in percent, in the stau rest frame (for the decay $\tilde{\tau}_1 \to \tau \tilde{\chi}_0^0$, followed by $\tilde{\chi}_0^0 \to \ell_1^+ \bar{\ell}_R^-$ and $\ell_R^- \to \tilde{\chi}_1^0 \ell_1^+$ for $\ell = e$ or $\mu$, see Fig. 1), on (a) the soft breaking parameters in the stau sector $M_{\tilde{E}_\tau}$, $M_{\tilde{L}_\tau}$, Eqs. (9), Eqs. (10). In (b) the dependence of $A_{\text{CP}}^{\tau}$ on the gaugino and higgsino parameters $|\mu|$, $M_2$. Below the contour $m_{\tilde{e}_R} = m_{\tilde{\chi}_0^0}$, the two-body decay $\tilde{\chi}_0^0 \to \ell \bar{\ell}_R$ is kinematically forbidden, above the contour $m_{\tilde{e}_R} = m_{\tilde{\chi}_0^0}$ the lightest neutralino is no longer the LSP since $m_{\tilde{e}_R} < m_{\tilde{\chi}_0^0}$. Below the contour $m_{\chi_1^0} = 100$ GeV the lightest chargino is lighter than 100 GeV. The MSSM parameters are given in Table I.

FIG. 6: Contour lines in the $|\mu|$–$M_2$ plane of (a) the stau branching ratio BR($\tilde{\tau}_1 \to \tau \tilde{\chi}_0^0$) in percent, and (b) the neutralino branching ratio BR($\tilde{\chi}_0^0 \to \ell \bar{\ell}_R$), in percent, summed over both lepton flavors $\ell = e$, $\mu$ and charges, for the MSSM parameters as given in Table I. Below the contours $m_{\tilde{e}_R} = m_{\tilde{\chi}_0^0}$ in Figs. 6(a) 6(b) the two-body decay $\tilde{\chi}_0^0 \to \ell \bar{\ell}_R$ is kinematically forbidden, above the contours $m_{\tilde{e}_R} = m_{\tilde{\chi}_0^0}$ the lightest neutralino is no longer the LSP since $m_{\tilde{e}_R} < m_{\tilde{\chi}_0^0}$. Below the contours $m_{\chi_1^0} = 100$ GeV the lightest chargino is lighter than 100 GeV.
IV. SUMMARY AND CONCLUSIONS

We have analyzed the normal tau polarization and the corresponding CP asymmetry in the two-body decay chain of a stau

\[ \tilde{\tau}_1 \rightarrow \tau + \tilde{\chi}_2^0. \]  

(49)

The CP-sensitive parts appear only in the spin-spin correlations, which can be probed by the subsequent neutralino decay

\[ \tilde{\chi}_2^0 \rightarrow \ell_1 + \tilde{\ell}_R; \quad \tilde{\ell}_R \rightarrow \chi_1^0 + \ell_2, \]  

(50)

for \( \ell = e, \mu \). The T-odd tau polarization normal to the plane spanned by the \( \tau \) and \( \ell_1 \) momenta, can then be used to define a CP-odd tau polarization asymmetry. It is based on a triple product, which probes the CP phases of the trilinear scalar coupling parameter \( A_\tau \), the higgsino mass parameter \( \mu \), and the U(1) gaugino mass parameter \( M_1 \).

We have analyzed the analytical and numerical dependence of the asymmetry on these parameters in detail. In particular, for nearly degenerate staus where the stau mixing is strong, the asymmetry obtains its maxima and can be larger than 70%. The normal tau polarization can thus be considered as an ideal CP observable to probe the CP phases in the stau and neutralino sector of the MSSM.

Since the CP-sensitive parts appear only in the subsequent stau decay products the stau production process can be separated. Thus both, ILC, and LHC collider studies are possible. Concerning the kinematical dependence, the asymmetry is not Lorentz invariant, since it is based on a triple product. At the LHC, staus are produced with a distinct boost distribution. Evaluated in the laboratory frame, the resulting tau polarization asymmetries get typically reduced by a factor of two thirds, compared to the stau rest frame.

We want to stress that a thorough experimental analysis, addressing background processes, detector properties, and event rate reconstruction efficiencies, will be needed in order to explore the measurability of CP phases in the stau sector at the LHC or ILC. We hope that our work motivates such a study.

Acknowledgements

We thank M. Drees and F. von der Pahlen for enlightening discussions and helpful comments. This work has been supported by MICINN project FPA.2006-05294. AM was supported by the Konrad Adenauer Stiftung, BCGS, and a fellowship of Bonn University. HD was supported by the Hemholtz Alliance “Physics at the Terascale” and BMBF “Verbundprojekt HEP-Theorie” under the contract 0509PDE. SK was supported by BCGS. OK acknowledges support from CPAN.

Appendix A: Momenta and spin vectors

For the stau decay \( \tilde{\tau}_m \rightarrow \tau \tilde{\chi}_1^0 \), we choose the coordinate frame in the laboratory (lab) system, such that the
m_{\tau} \rightarrow 0$. The momenta of the leptons from the subsequent neutralino decay $\chi_2^0 \rightarrow \ell_1 \bar{\ell}; \bar{\ell} \rightarrow \chi_1^0 \ell_2$ (11), can be parameterized by

$$p_{\ell_1}^\mu = E_{\ell_1}(1, \sin \theta_{\ell_1} \cos \phi_{\ell_1}, \sin \theta_{\ell_1} \sin \phi_{\ell_1}, \cos \theta_{\ell_1}),$$

$$p_{\ell_2}^\mu = E_{\ell_2}(1, \sin \theta_{\ell_2} \cos \phi_{\ell_2}, \sin \theta_{\ell_2} \sin \phi_{\ell_2}, \cos \theta_{\ell_2}),$$

with the energies

$$E_{\ell_1} = \frac{m_{\chi_2^0}^2 - m_{\ell_1}^2}{2(E_{\chi_2^0} - |p_{\ell_1}| \cos \theta_{D_1})},$$

$$E_{\ell_2} = \frac{m_{\chi_2^0}^2 - m_{\ell_2}^2}{2(E_{\chi_2^0} - |p_{\ell_2}| \cos \theta_{D_2})},$$

and the decay angles $\theta_{D_1} = \angle(p_{\ell_1}^\mu, p_{\ell_1}), \theta_{D_2} = \angle(p_{\ell_2}^\mu, p_{\ell_2})$, that is,

$$\cos \theta_{D_1} = \frac{(p_{\ell}^\mu - p_{\tau}^\mu) \cdot \hat{P}_{\ell_1}}{|p_{\ell}^\mu - p_{\tau}^\mu|},$$

$$\cos \theta_{D_2} = \frac{(p_{\ell}^\mu - p_{\tau}^\mu - p_{\ell_1}^\mu) \cdot \hat{P}_{\ell_2}}{|p_{\ell}^\mu - p_{\tau}^\mu - p_{\ell_1}^\mu|},$$

with the unit momentum vector $\hat{P} = p/|p|$. We define the tau spin vectors by

$$s_{\tau}^{1,\mu} = \left(0, \frac{1}{\sin \theta_{\tau}} \times \frac{1}{\sin \theta_{\tau}} \right),$$

$$s_{\tau}^{2,\mu} = \left(0, \frac{p_{\ell_1} \times p_{\tau}}{|p_{\ell_1} \times p_{\tau}|}, \right),$$

$$s_{\tau}^{3,\mu} = \frac{1}{m_{\tau}} \left( |p_{\tau}|, \frac{E_{\tau}}{|p_{\tau}|^2} p_{\tau} \right).$$

The spin vectors $s_{\alpha}^a$, $\alpha = 1, 2, 3$, for the tau, and $s_{\beta}^b$, $b = 1, 2, 3$, for the neutralino $\chi_2^0$, fulfill completeness relations

$$\sum_a s_{\tau}^{a,\mu} s_{\tau}^{a,\nu} = -g^{\mu \nu} + \frac{p_{\mu}^\mu p_{\nu}^\nu}{m_{\tau}^2},$$

$$\sum_b s_{\alpha}^{\mu} s_{\beta}^{\nu} = -g^{\mu \nu} + \frac{p_{\mu}^\mu p_{\nu}^\nu}{m_{\alpha}^2},$$

and they form orthonormal sets

$$s_{\tau}^a \cdot s_{\tau}^c = -\delta^{ac},$$

$$s_{\alpha}^a \cdot s_{\beta}^b = -\delta^{ab},$$

$$s_{\alpha}^a \cdot \hat{P}_{\alpha} = 0,$$

with $\hat{P}_{\alpha} = p_{\alpha}/m_{\alpha}$. Note that the asymmetry $A_{CP}$, Eq. (10), does not depend on the explicit form of the neutralino spin vectors, since they are summed in the amplitude squared, see Eq. (11), using the completeness relation.

**Appendix B: Phase space**

The Lorentz invariant phase-space element for the tau decay chain, see Eqs. (1) - (2), can be decomposed into two-body phase-space elements [37]

$$d\mathcal{Z}(s_{\tau}; p_{\ell_1}, p_{\ell_2}, p_{\chi_i}) = \frac{1}{4\pi} \frac{1}{m_{\chi_i}^2 - m_{\tau}^2} d\mathcal{Z}(s_{\tau}; p_{\tau}, p_{\chi_i})\times d\mathcal{Z}(s_{\tau}; p_{\ell_1}, p_{\ell_2}) d\mathcal{Z}(s_{\tau}; p_{\ell_2}, p_{\chi_i}).$$

The different contributions are

$$d\mathcal{Z}(s_{\tau}; p_{\tau}, p_{\chi_i}) = \frac{1}{4\pi} \frac{|p_{\tau}|^2}{m_{\chi_i}^2 - m_{\tau}^2} \sin \theta_{\tau} \, d\theta_{\tau},$$

$$d\mathcal{Z}(s_{\tau}; p_{\ell_1}, p_{\ell_2}) = \frac{1}{2(2\pi)^2} \frac{|p_{\ell_1}|^2}{m_{\ell_2}^2 - m_{\tau}^2} d\Omega_1,$$

$$d\mathcal{Z}(s_{\tau}; p_{\ell_2}, p_{\chi_i}) = \frac{1}{2(2\pi)^2} \frac{|p_{\ell_2}|^2}{m_{\ell_1}^2 - m_{\tau}^2} d\Omega_2,$$

with $s_j = p_j^2$ and $d\Omega_j = \sin \theta_j \, d\theta_j \, d\phi_j$.

**Appendix C: Density matrix formalism**

The coefficients of the tau decay matrix, Eq. (26), are

$$D = \frac{g^2}{2} \left( |a_{mi}^\tau|^2 + |b_{mi}^\tau|^2 \right) (p_{\chi_i}^\tau \cdot p_{\tau}),$$

$$-g^2 \Re \{a_{mi}^\tau (b_{mi}^\tau)^* \} m_{\chi_i} m_{\tau},$$

$$\Sigma_a^D = -\frac{g^2}{2} \left( |a_{mi}^\tau|^2 - |b_{mi}^\tau|^2 \right) m_{\tau} (p_{\chi_i}^\tau \cdot s_{\tau}^a),$$

$$\Sigma_b^D = -\frac{g^2}{2} \left( |a_{mi}^\tau|^2 - |b_{mi}^\tau|^2 \right) m_{\tau} (p_{\chi_i}^\tau \cdot s_{\tau}^b),$$

$$\Sigma_{ab}^D = \frac{g^2}{2} \left( |a_{mi}^\tau|^2 + |b_{mi}^\tau|^2 \right) (s_{\tau}^a \cdot s_{\tau}^b) m_{\tau} m_{\chi_i}^0 + g^2 \Re \{a_{mi}^\tau (b_{mi}^\tau)^* \} \times$$

$$\left[ (s_{\tau}^a \cdot p_{\tau}) (s_{\tau}^b \cdot p_{\tau}) - (s_{\tau}^a \cdot s_{\tau}^b) (p_{\chi_i}^\tau \cdot p_{\tau}) \right]$$

$$-g^2 \Im \{a_{mi}^\tau (b_{mi}^\tau)^* \} [s_{\tau}^a \cdot p_{\tau}, s_{\tau}^b \cdot p_{\tau}, p_{\chi_i}^\tau].$$

The formulas are given for the decay of a negatively charged tau, $\tau_m^\tau \rightarrow \tau^- \chi_2^0$. The signs in parentheses hold for the charge conjugated decay $\tau_m^\tau \rightarrow \tau^+ \chi_2^0$.

Note that the terms proportional to $m_{\tau}$ in Eqs. (C1), (C2), and (C4), are negligible at high particle energies $E \gg m_{\tau}$, in particular $\Sigma_{ab}^D$ can be neglected.

The coefficients of the $\chi_2^0$ decay matrix, Eq. (29), are

$$D_1 = \frac{g^2}{2} |f_{\ell_1}^R|^2 (m_{\chi_i}^2 - m_{\tau}^2),$$

$$\Sigma_{D_1}^a = (-g^2 |f_{\ell_1}^R|^2 m_{\chi_i}^0 s_{\tau}^a \cdot p_{\ell_1}).$$
and the selectron decay factor is
\[ D_2 = g^2 |f_{\ell i}|^2 (m_2^2 - m_{\chi_i^0}^2). \]  
(C7)
The signs in parentheses hold for the charge conjugated processes, that is \( \chi_i^{0} \to \ell_i^+ \bar{\ell}_i^L \) in Eq. (C9).

For the decay into a left slepton \( \chi_i^{0} \to \ell_i^+ \bar{\ell}_i^L \), Eqs. (C6), (C8), and (C7) read
\[ D_1 = \frac{g^2}{2} |f_{\ell i}|^2 (m_{\chi_i^0}^2 - m_1^2), \]  
(C8)
\[ \Sigma^b \Delta = (-g^2 |f_{\ell i}|^2 m_{\chi_i^0} s_{\chi_i^0} \cdot p_{\ell_i}), \]  
(C9)
\[ D_2 = g^2 |f_{\ell i}|^2 (m_2^2 - m_{\chi_i^0}^2), \]  
(C10)
respectively. The expressions for Eqs. (31) and (32) have to be changed accordingly. The sign in parenthesis in Eq. (C9) holds for the charge conjugated process \( \chi_i^{0} \to \ell_i^+ \bar{\ell}_i^L \).

Appendix D: Stau decay widths

The partial decay width for the decay \( \tilde{\tau}_m \to \tau \chi_i^{0} \) in the stau rest frame is
\[ \Gamma(\tilde{\tau}_m \to \tau \chi_i^{0}) = \frac{m_2^2 - m_{\chi_i^0}^2}{4 \pi m_{\tau}^2} D, \]  
(D1)
with the decay function \( D \) given in Eqs. (C1), and the approximation \( m_\tau = 0 \). For the decay \( \tilde{\tau}_m \to \nu \chi_j^{\pm} \) the width is
\[ \Gamma(\tilde{\tau}_m \to \nu \chi_j^{\pm}) = \frac{(m_2^2 - m_{\chi_j^0}^2)^2}{16 \pi m_{\tau}^2} g^2 |t_{m_j}|^2, \]  
(D2)
with the stau-chargino-neutrino coupling (31)
\[ t_{m_j} = - (R_{m_1}^\tau)^* U_{j1} + Y_\tau (R_{m_2}^\tau)^* U_{j2}, \]  
(D3)
and the stau diagonalization matrix \( R^\tau \), Eq. (12), the Yukawa coupling \( Y_\tau \), Eq. (22), and the matrix \( U \), that diagonalizes the chargino matrix (3),
\[ U^* \cdot \mathcal{M}^{\pm} \cdot U^\dagger = \text{diag}(m_{\chi_1^\pm}, m_{\chi_2^\pm}). \]  
(D4)
The stau decay width for the entire decay chain, Eqs. (1) - (2), is then given by
\[ \Gamma(\tilde{\tau} \to \tau \ell_1 \nu_\chi_1^0) = \frac{1}{2 m_{\tau}} \int |\mathcal{M}|^2 d\mathcal{L} \text{ips}(s_{\tau}; p_{\tau}, \ell_{1}, p_{\nu_\chi_1^0}) \]  
(D5)
\[ = \Gamma(\tilde{\tau}) \times BR(\tilde{\tau} \to \tau \chi_1^0) \times BR(\chi_1^0 \to \ell_{1} \nu_\chi), \]  
(D6)
with the phase-space element \( d\mathcal{L} \) for the process, as given in the Appendix A in the amplitude squared
\[ |\mathcal{M}|^2 = 4 |\Delta(\chi_1^0)|^2 |\Delta(\ell)|^2 D_1 D_2, \]  
(D8)
which is justified for \( \Gamma_{\ell} / m_{\chi_1} \approx 1 \), which holds in our case with \( \Gamma_{\ell} \lesssim \mathcal{O}(1 \text{ GeV}) \). Note, however, that in principle the naive \( \mathcal{O}(\Gamma / m) \)-expectation of the error can easily receive large off-shell corrections of an order of magnitude, and more, in particular at threshold, or due to interferences with other resonant, or non-resonant processes (38), obtained from Eqs. (60) by summing the tau helicities \( \lambda_{\tau}, \lambda'_{\tau} \). The neutralino branching ratios are given, for example, in Ref. (32), and we assume \( \text{BR}(\tilde{\tau} \to \ell_{1} \nu_\chi_1^0) = 1 \). We use the narrow width approximation for the propagators
\[ \int |\Delta(j)|^2 ds_j = \frac{\pi}{m_j^2 \Gamma_j}, \]  
(D9)
for a recent review of the CP-violation constraints within the MSSM, see

[1] BELLE Collab., A. Abashian et al., Phys. Rev. Lett. 86, 2509 (2001), [hep-ex/0102018].
[2] BABAR Collab., B. Aubert et al., Phys. Rev. Lett. 86, 2515 (2001), [hep-ex/0102030].
[3] A. D. Sakharov, Zh. Eksp. Teor. Fiz. Pis'ma 5, 32 (1967), JETP Lett. 91B, 24 (1967).
[4] H. E. Haber and G. L. Kane, Phys. Rept. 117 (1985) 75; H. P. Nilles, Phys. Rept. 110 (1984) 1; M. Dress, R. Godbole and P. Roy, “Theory and phenomenology of sparticles: An account of four-dimensional N=1 supersymmetry in high energy physics,” Hackensack, USA: World Scientific (2004).
[5] T. Ibrahim and P. Nath, Phys. Rev. D 57 (1998) 478 [Erratum-ibid. D 58 (1998) 019901], Erratum-ibid. D 60, 019901 (1999) [Erratum-ibid. D 60, 119901 (1999)] [arXiv:hep-ph/9708456].
[6] see, e.g., A. Bartl, T. Gajdosik, W. Porod, P. Stockinger. Y. Li, S. Profumo and M. Ramsey-Musolf, JHEP 1008, 062 (2010) [arXiv:1006.1440 [hep-ph]].
For studies with neutralino 3-body decays at the ILC, see
Y. Kizukuri and N. Oshimo, Phys. Lett. B 249 (1990) 449;
S. Y. Choi, H. S. Song and W. Y. Song, Phys. Rev. D 61 (2000) 075004 [arXiv:hep-ph/9907474] ;
For further studies with neutralino 2-body and 3-body decays at the ILC, see
A. Bartl, H. Fraas, O. Kittel and W. Majerotto, Phys. Rev. D 69 (2004) 035007 [arXiv:hep-ph/0308141];
A. Bartl, H. Fraas, O. Kittel and W. Majerotto, Eur. Phys. J. C 36 (2004) 233 [arXiv:hep-ph/0402016];
J. A. Aguilar-Saavedra, Nucl. Phys. B 697 (2004) 207 [arXiv:hep-ph/0404104];
A. Bartl, H. Fraas, S. Hesselbach, K. Hohenwarter-Sodek and G. A. Moortgat-Pick, JHEP 0408 (2004) 038 [arXiv:hep-ph/0406190];
S. Y. Choi, B. C. Chung, J. Kalinowski, Y. G. Kim and K. Rolbiecki, Eur. Phys. J. C 46 (2006) 511 [arXiv:hep-ph/0504122];
For studies with chargino 2-body decays at the ILC, see
A. Bartl, H. Fraas, O. Kittel and W. Majerotto, Phys. Lett. B 598 (2004) 76 [arXiv:hep-ph/0406309];
O. Kittel, A. Bartl, H. Fraas and W. Majerotto, Phys. Rev. D 70 (2004) 115005 [arXiv:hep-ph/0410054];
For studies with chargino 3-body decays at the ILC, see
Y. Kizukuri and N. Oshimo, arXiv:hep-ph/0310224;
A. Bartl, H. Fraas, S. Hesselbach, K. Hohenwarter-Sodek, T. Kernreiter and G. Moortgat-Pick, Eur. Phys. J. C 51 (2007) 149 [arXiv:hep-ph/0608065];
For recent reviews see, for example,
G. Moortgat-Pick, K. Rolbiecki, J. Tattersall and P. Wienemann, arXiv:0910.1371 [hep-ph];
O. Kittel, arXiv:0904.3241 [hep-ph].

S. Abdullin et al. [CMS Collaboration], J. Phys. G 28 (2002) 469 [arXiv:hep-ph/9806366];
ATLAS collabor., ATLAS detector and physics performance. Technical design report. Vol. 2, CERN-LHCC-99-15;
G. Weiglein et al. [LHC/LC Study Group], arXiv:hep-ph/0410364.

J. Brau et al. [ILC Collaboration], [arXiv:0712.1950 [physics.acc-ph]]; J. A. Aguilar-Saavedra et al. [ECFA/DESY LC Physics Working Group], [arXiv:hep-ph/0106315];
T. Abe et al. [American Linear Collider Working Group], [arXiv:hep-ex/0106055];
K. Abe et al. [ACFA Linear Collider Working Group], [arXiv:hep-ph/0109166];
J. A. Aguilar-Saavedra et al., Eur. Phys. J. C 46 (2006) 43 [arXiv:hep-ph/0511344].

Y. K. Somertzidis, Nucl. Phys. Proc. Suppl. 131 (2004) 244 [arXiv:hep-ex/0401016];
J. R. Ellis, S. Ferrara and D. V. Nanopoulos, Phys. Lett. B 114 (1982) 231;
W. Buchmuller and D. Wyler, Phys. Lett. B 121 (1983) 321;
P. del Aguila, M. B. Gavela, J. A. Grifols and A. Mendez, Phys. Lett. B 126 (1983) 71 [Erratum-ibid. B 129 (1983) 473];
D. V. Nanopoulos and M. Srednicki, Phys. Lett. B 128 (1983) 61;
M. Dugan, B. Grinstein and L. J. Hall, Nucl. Phys. B 255 (1985) 413;
C. S. Huang and W. Liao, Phys. Rev. D 62 (2000) 016008.
[20] S. Y. Choi, M. Drees and B. Gaissmaier, Phys. Rev. D 70 (2004) 014010 [arXiv:hep-ph/0403054].
[21] F. Deppisch and O. Kittel, JHEP 0909, 110 (2009) [Erratum-ibid. 1003, 091 (2010)] [arXiv:0905.3088 [hep-ph]].
[22] G. Moortgat-Pick, K. Rolbiecki, J. Tattersall and P. Wienemann, JHEP 1001, 004 (2010) [arXiv:0908.2631 [hep-ph]].
[23] A. Bartl, E. Christova, K. Hohenwarter-Sodek and T. Kernreiter, Phys. Rev. D 70 (2004) 095007 [arXiv:hep-ph/0409060].
[24] A. Bartl, T. Kernreiter and O. Kittel, Phys. Lett. B 578 (2004) 137 Phys. Rev. D 66 (2002) 115009 [arXiv:hep-ph/0207186].