Transverse patterns in broad-area lasers with anisotropy

A A Krents$^{1,2}$ and N E Molevich$^{1,2}$

$^1$Lebedev Physical Institute, Novo-Sadovaya street 221, Samara, Russia, 443011
$^2$Samara National Research University, Moskovskoe Shosse 34A, Samara, Russia, 443086

e-mail: krenz86@mail.ru

Abstract. The spatiotemporal dynamics of broad lasers with anisotropy is studied. Anisotropy in broad-area lasers leads to different thresholds for x- and y-polarized modes. In addition, the transverse wave numbers for the x- and y-polarized modes can also differ. The investigated regimes were observed experimentally in semiconductor vertical cavity surface emitting laser with cryogenic cooling. Our theoretical results agree well with the experimental data.

1. Introduction
The transverse spatio-temporal dynamics of broad-area lasers and other nonlinear resonators is an area of active research in the last three decades [1-5]. There are three universal mechanisms leading to the formation of transverse structures in broad-area lasers. The first one is related to off-axis generation, which is observed in the case when the central frequency of the gain line is higher than the longitudinal mode frequency (positive detuning). The tilted waves and various patterns formed as a result of their interference were first predicted theoretically [6], and then found experimentally [7]. The second mechanism is related to the instability of the spatially homogeneous on-axis generation when the laser was pumped above the second threshold [8]. The third mechanism is associated with an external action on a spatially distributed optical system [9].

This paper is devoted to the study of the effect of anisotropy inherent in semiconductor lasers on the dynamics of off-axis generation and the formation of transverse patterns with positive detuning. It was shown experimentally in [10] that at high temperatures corresponding to the axial generation, optical frequencies are separated for x and y polarized radiation $\omega_x \neq \omega_y$. However, upon cooling of the active medium of the vertical cavity surface emitting laser (VCSEL) and the development of an off-axis generation regime, optical frequencies are captured for x- and y-polarized radiation. For off-axis generation, the frequencies are equal $\omega_x = \omega_y$. It was shown experimentally in [11] that the transverse structure of optical radiation for x and y polarized radiation has a different characteristic spatial dimension.

2. Mathematical model and analysis
In this paper, we investigate the spatiotemporal dynamics of broad-area laser with the polarization degree of freedom. We consider a standart situation, transitions from states $J_z = \pm 1$ in the upper level to $J = 0$ in the lower level, which produce the circularly polarized components of the electric field. The standard Maxwell-Bloch equations are extended to [12]:

$$\frac{\partial E_x}{\partial t} = i\alpha V^2 E_x - \sigma E_x + \sigma P_x - (\gamma_x + i\gamma \rho) E_x,$$
\[
\frac{\partial P_i}{\partial t} = -(1 + i\delta)P_i + rE_i - N_iE_i - ME_i,
\]
\[
\frac{\partial P_\pm}{\partial t} = -(1 + i\delta)P_\pm + rE_\mp - N_\mp E_\mp - M^*E_\mp,
\]
\[
\frac{\partial N_\pm}{\partial t} = -\gamma N_\pm + \frac{1}{2}(E_\mp^*P_\mp + E_\pm P_\pm^*) + \frac{1}{4}(E_\pm^*P_\mp + E_\mp P_\pm^*),
\]
\[
\frac{\partial M}{\partial t} = -\mu M + \frac{1}{4}(E_\pm^*P_\mp + E_\mp P_\pm^*),
\]
where \( E_\pm = (E_\pm \pm iE_r)/\sqrt{2} \) are the right and left circularly polarized components of the electric field, \( P_\pm \) are the complex material polarizations, \( N_\pm \) are the population differences between levels (\( J = 1, J_\pm = \pm 1 \)) and (\( J = 0 \)), and \( M \) describes interaction between upper level states (\( J_\pm = 1 \)) and (\( J_\mp = -1 \)).

Parameters \( \gamma = \gamma_\parallel/\gamma_\perp, \sigma = k/\gamma_\perp \), where \( \gamma_\parallel, \gamma_\perp \), and \( k \) are the polarization relaxation rate, population inversion relaxation rate, and electric field decay rate, respectively. \( \delta = (\omega - \omega)/\gamma_\perp \) is the normalized detuning of the atomic resonant frequency from the laser field frequency. \( \nabla^2 \) is the transverse Laplacian, \( a = c^2/(2\omega_0d^2) \) is the diffraction coefficient, where \( d \) is the transverse spatial scale of the laser, \( r \) is the pumping level normalized versus its threshold value. Time is rescaled for \( \gamma_\perp^{-1} \), that is inverse half-width of a homogeneously broadened gain line, \( \gamma_\parallel \) and \( \gamma_\perp \) are coefficients of dichroism and birefringence respectively.

Equations (1) have the trivial solution:
\[
E_\pm = P_\pm = N_\pm = M = 0.
\]

Solution (2) corresponds to the nonradiative steady-state solution. Let us investigate stability of this solution. Linearized around solution (2) equations (1) have the form:
\[
\frac{\partial e_\pm}{\partial t} = ia\nabla^2 e_\pm - \sigma e_\pm + \sigma p_\pm - (\gamma_\parallel + i\gamma_\perp)e_\pm,
\]
\[
\frac{\partial p_\pm}{\partial t} = -(1 + i\delta)p_\pm + re_\pm,
\]
\[
\frac{\partial n_\pm}{\partial t} = -\gamma n_\pm,
\]
\[
\frac{\partial m}{\partial t} = -\mu m,
\]
where \( e_\pm, p_\pm, n_\pm \) and \( m \) are small perturbations. Solutions of linearized system (3) have the forms:
\[
\begin{bmatrix}
  e_\pm \\
  p_\pm \\
  n_\pm \\
  m
\end{bmatrix}
= 
\begin{bmatrix}
  e_{0\pm} \\
  p_{0\pm} \\
  n_{0\pm} \\
  m_0
\end{bmatrix}
\cdot e^{i\omega t + ikx}.
\]

Nonradiative steady-state solution (2) is stable if
\[
r \leq 1 + \left(\frac{ak^2 - \delta^2}{1 + \sigma}\right)^2.
\]
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\[ E_x = (E_x + E_y) / \sqrt{2}, \quad E_y = i(E_x - E_y) / \sqrt{2}, \]
\[ P_x = (P_x + P_y) / \sqrt{2}, \quad P_y = i(P_x - P_y) / \sqrt{2}, \]
\[ N_x = (N_x + N_y) / \sqrt{2}, \quad N_y = i(N_x - N_y) / \sqrt{2}, \quad M = M. \]

(6)

The transition to variables with linear polarization is justified by the fact that, as a rule, linearly polarized components are measured at the experiment. After substituting variables (6) into the system of equations (1), we obtained a system of equations for components with linear polarization:

\[ \frac{\partial E_x}{\partial t} = \frac{i\alpha}{\gamma_\sigma} E_x - \sigma E_x + \sigma P_x - (\gamma_a + i\gamma_p) E_x, \]
\[ \frac{\partial E_y}{\partial t} = \frac{i\alpha}{\gamma_\sigma} E_y - \sigma E_y + \sigma P_y + (\gamma_a + i\gamma_p) E_y, \]
\[ \frac{\partial P_x}{\partial t} = -\left(1 + i\delta\right) P_x + rE_x - \frac{(E_x N_x - E_y N_y)}{\sqrt{2}} - \frac{M^*}{2} \left(E_x - iE_y\right) - \frac{M}{2} \left(E_x - iE_y\right), \]
\[ \frac{\partial P_y}{\partial t} = -\left(1 + i\delta\right) P_y + rE_y - \frac{(E_x N_x + E_y N_y)}{\sqrt{2}} + \frac{M^*}{2} \left(iE_x + E_y\right) - \frac{M}{2} \left(iE_x + E_y\right), \]
\[ \frac{\partial N_x}{\partial t} = -\gamma N_x + \frac{3 \sqrt{2}}{8} \left(E_x P_x + E_x P_x^* + E_y P_y + E_y P_y^*\right), \]
\[ \frac{\partial N_y}{\partial t} = -\gamma N_y + \frac{3 \sqrt{2}}{8} \left(E_y P_y + E_y P_y^* - E_x P_x - E_x P_x^*\right), \]
\[ \frac{\partial M}{\partial t} = -\mu M + \frac{1}{8} \left(E_x + iE_y\right) \left(P_x + iP_y\right) + \frac{1}{8} \left(E_x + iE_y\right) \left(P_x^* + iP_y^*\right). \]

(7)

The system of equations (7) has two simple solutions, corresponding to single linearly polarized components. The first of these solutions has the form:

\[ E_{x0} = E_0 e^{i(kx - \omega t)}, \quad E_{y0} = 0. \]

(8)

This solution in the form of a traveling wave with a nonzero X-component exists under the condition \( \gamma_a > -\sigma \). After substituting (8) into (7), we define the other variables:

\[ P_{x0} = E_0 e^{i(kx - \omega t)} \frac{\gamma_a + i\gamma_p + i\alpha k^2 + \sigma - i\omega}{\sigma}, \quad P_{y0} = 0, \]
\[ N_{x0} = 3 \sqrt{2} |E_0|^2 \frac{\sigma + \gamma_a}{4 \gamma \sigma}, \quad N_{y0} = 0, \]
\[ M_0 = |E_0|^2 \frac{\gamma_a + \sigma}{4 \mu \sigma}, \]
\[ I_0 = |E_0|^2 = \frac{4 \gamma \mu}{\gamma + 3 \mu} \left(\frac{r \sigma}{\sigma + \gamma_a} - \frac{\left(\gamma_p + ak^2 - \delta\right)^2}{\left(\sigma + \gamma_a + 1\right)^2} - 1\right). \]

(9)

From (9), we can obtain the threshold pump required for the generation of the X-polarized traveling wave, the wave number, and frequency of a traveling wave with a minimum threshold:

\[ r_{th} = \frac{\sigma + \gamma_a}{\sigma} - \frac{\left(\gamma_p + ak^2 - \delta\right)^2}{\sigma \left(\sigma + \gamma_a + 1\right)^2}, \]
\[ \omega_{th} = \delta + \frac{ak^2 + \gamma_p - \delta}{\sigma + \gamma_a + 1}. \]

(10)
for \( \delta \geq \gamma_p : k_{x0} = \sqrt{\frac{\delta - \gamma_p}{a}} \), for \( \delta < \gamma_p : k_{x0} = 0 \).

An analogous solution in the form of the traveling wave with the nonzero Y-linear component:

\[
E_{x0} = 0, \quad E_{y0} = E_0 e^{(kx-\omega t)}.
\]

It exists under the condition \( \gamma_a < \sigma \). After substituting (11) into (7), we define the other variables:

\[
P_{x0} = 0, \quad P_{y0} = -E_0 e^{(kx-\omega t)} \gamma_a + i\gamma_p - i\omega \frac{k^2 - \sigma}{\sigma},
\]

\[
N_{x0} = 3\sqrt{2}|E_0|^2 \frac{\sigma - \gamma_a}{4\gamma \sigma}, \quad N_{y0} = 0,
\]

\[
M_0 = |E_0|^2 \frac{\gamma_a - \sigma}{4\mu \sigma},
\]

\[
I_0 = |E_0|^2 = \frac{4\gamma \mu}{(\gamma + 3\mu)} \left( \frac{r\sigma}{\sigma - \gamma_a} - \frac{(\gamma_a - ak^2 + \delta)^2}{(\sigma - \gamma_a + 1)^2} \right) - 1.
\]

From (12), we can obtain the value of the threshold pump required for the generation of the Y-polarized traveling wave, the wave number, and frequency of a traveling wave with a minimum threshold:

\[
r_{y0} = \frac{\sigma - \gamma_a}{\sigma} + \frac{(\gamma_a - ak^2 + \delta)^2}{\sigma(\sigma - \gamma_a + 1)^2},
\]

\[
\omega_{y0} = \delta + \frac{ak^2 - \gamma_a - \delta}{\sigma - \gamma_a + 1},
\]

for \( \delta \leq -\gamma_p : k_{y0} = \sqrt{\frac{\delta + \gamma_p}{a}} \), for \( \delta < -\gamma_p : k_{y0} = 0 \).

The reversal of the sign \( \gamma_a \) leads to a replacement of the threshold pump values in (10) and (13), as well as the values of the stationary values of the intensity of the radiation. Moreover, with a positive detuning \( \delta \geq \gamma_p \) the replacement is accurate, for \( \delta < \gamma_p \) it is with approximate accuracy. In addition, the change of sign affects the time frequency for the detuning \( \delta \gamma_p \). When the detuning \( \delta \geq \gamma_p \), the frequencies for any \( \gamma_a \) are equal to each other for two linearly polarized components. The values of the characteristic wave numbers depend only on the birefringence coefficient \( \gamma_p \). In addition, in this case solution (9) with the Y-component becomes unstable, and solution (12), on the contrary, acquires stability.

The change in sign \( \gamma_p \) leads to a change in the values of the characteristic wave numbers and frequencies in (10) and (13) to each other. And with the positive detuning, the wave numbers change exactly, and the frequencies either change to close to each other for \( \delta < \gamma_p \) or are exactly equal to each other when \( \delta \geq \gamma_p \). With the negative detuning, the wave numbers are zero for both components of the field, so change the sign of \( \gamma_p \) does not affect the spatial spectrum. The values of the frequencies also vary among themselves only approximately. Simultaneous change of signs \( \gamma_a \) and \( \gamma_p \) to the opposite for any parameters leads to the fact that formulas (10) and (13) are symmetrically completely transformed into each other; the threshold pumping, the characteristic wave number and frequency change places, and also the values of the stationary intensity change places.
3. Numerical method and results

For numerical modeling of the vector system of Maxwell-Bloch equations (1), a pseudospectral method of exponential differentiation with respect to the Runge-Kutta was used [13]. Taking into account spatially periodic boundary conditions, the partial differential equations solution is naturally written as a sum of Fourier modes with time-dependent coefficients. As a result, the original system of partial differential equations is replaced by a system of ordinary differential equations (ODE). Nevertheless, the problem of solving the resulting set of ODEs for mode amplitudes is rigid. This means that higher spatial modes develop on shorter time scales. Indeed, the time scale associated with the scale of the n-th mode is of the order \( O(n^{-m}) \) for large n, where m is the order of the higher spatial derivative.

Let us consider in detail the formation of optical structures with parameters \( \delta = 1, \ r = 2, \ \sigma = 2, \ \gamma = 0.1, \ a = 0.01, \ \gamma_a = 0.1, \ \gamma_p = 0.1 \). Transverse instabilities leads to irregular spatitemporal patterns (Figure 1 (a)), but the characteristic spatial scale is in a good agreement with formula (13) (Figure 1 (b)).

![Figure 1](image1.png)

**Figure 1.** Results of numerical simulation for \( \delta = 1, \ r = 2, \ \sigma = 2, \ \gamma = 0.1, \ a = 0.01, \ \gamma_a = 0.1, \ \gamma_p = 0.1 \) (a) irregular spatiotemporal pattern and (b) far field.

Spatiotemporal dynamics for parameters \( \delta = 1, \ r = 2, \ \sigma = 2, \ \gamma = 0.1, \ a = 0.01, \ \gamma_a = 0, \ \gamma_p = 0.1 \) is presented in Figure 2. The change in the dichroism \( \gamma_a \) parameter significantly changes the spatial structure of the radiation. The ordered spatiotemporal structure is obtained.

![Figure 2](image2.png)

**Figure 2.** Results of numerical simulation for \( \delta = 1, \ r = 2, \ \sigma = 2, \ \gamma = 0.1, \ a = 0.01, \ \gamma_a = 0, \ \gamma_p = 0.1 \) (a) ordered spatiotemporal pattern and (b) far field.

Spatiotemporal dynamics for parameters \( \delta = 1, \ r = 2, \ \sigma = 2, \ \gamma = 0.01, \ a = 0.01, \ \gamma_a = 0.1, \ \gamma_p = 0 \) is presented in Figure 3. A slightly disordered spatial structure was obtained numerically for \( \gamma_p = 0 \).
Figure 3. Results of numerical simulation for $\delta = 1$, $r = 2$, $\sigma = 2$, $\gamma = 0.1$, $a = 0.01$, $\gamma_p = 0.1$ (a) slightly disordered spatiotemporal pattern and (b) far field.

One of the basic structures formed by four anticollinear tilted waves is a vortex lattice. Figure 4 shows the distribution of the intensity in the near field obtained numerically. The similar distribution was observed experimentally [14]. Parameters of the model: $\delta = 3$, $r = 2$, $\sigma = 2$, $\gamma = 0.01$, $a = 0.01$, $\gamma_a = 0.1$, $\gamma_p = 0.1$.

Figure 4. Results of numerical simulation for $\delta = 3$, $r = 2$, $\sigma = 2$, $\gamma = 0.1$, $a = 0.01$, $\gamma_a = 0.1$, $\gamma_p = 0.1$ (a) optical vortex lattice and (b) far field.

4. Conclusion

In the present paper, the anisotropy of the active medium is investigated for the spatiotemporal dynamics of broad-area lasers. It is shown that with the negative sign of the detuning (corresponding to heating of the active medium) x- and y- polarized modes have different frequencies, the approximate value found for the frequency difference. With the positive detuning (corresponding to cryogenic cooling of the active medium) x- and y-polarized modes have the same frequency. The threshold values of the pumping parameter for x- and y- polarized modes are determined. The analytical results obtained in this paper are in good agreement with the results of experimental work. Numerical simulations of the spatiotemporal dynamics of a broad-area laser have also been carried out, and pictures in the near field have been obtained, qualitatively consistent with the experimental results.

5. References

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