Applying Popper’s Probability

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Abstract

Professor Sir Karl Popper (1902-1994) was one of the most influential philosophers of science of the twentieth century, best known for his doctrine of falsifiability. His axiomatic formulation of probability, however, is unknown to current scientists, though it is championed by several current philosophers of science as superior to the familiar version. Applying his system to problems identified by himself and his supporters, it is shown that it does not have some features he intended and does not solve the problems they have identified.

1 Probability and the philosophers

Professor Sir Karl Popper (1902-1994) is known to scientists as the author of the ‘doctrine of falsifiability,’ in which a statement is only admitted to be scientific if it can, in principle, be falsified. Although not strictly the originator of the idea, he can be credited with emphasizing it and it is a useful test for pseudoscientific statements. His clearest statement of this is from the Postscript to The Logic of Scientific Discovery:

...we adopt, as our criterion of demarcation, the criterion of falsifiability, i.e. of an (at least) unilateral or asymmetrical or one-sided decidability. According to this criterion, statements, or systems of statements, convey information about the empirical world only if they are capable of clashing with experience; or more precisely, only if they can be systematically tested, that is to say, if they can be subjected (in accordance with a ‘methodological decision’) to tests which might result in their refutation. \[1\] §8*i, pp. 313-4, p. 315. [Here I use his italics, as I will henceforth.]

1The work is available in at least two editions, either of which may be conveniently available but which differ slightly in pagination. In what follows I will cite the section; then the page numbers in [1]; then those in [2].
There is a great deal of other material in this, his most famous work, however; a previous investigation [3] has shown that it demonstrates Popper’s understanding of science and especially mathematics to be often inadequate or erroneous. Here I will examine one part of the work that has been singled out as useful.

A significant part of *The Logic of Scientific Discovery* is given over to an axiomatic formulation of probability. It is unknown among practicing scientists, but a number of philosophers cite it with approval, considering it superior to the familiar version.

Fitelson, Hajek and Hall [4] consider Popper’s formulation ‘the most general and elegant’ of proposed axiomatizations of two-place probability (that is, formulations in which conditional probability is taken as elementary, and absolute or one-place probability is a derived form). McGee [5] agrees that ‘Popper’s axioms constitute a true generalization of ordinary probability theory.’ Roeper and Leblanc [6] note that autonomous formulations of probability, that is those constructed independent of semantic constraints, ‘pioneered by Popper, are preferred by us.’ (p. 5). A clue as to what the generalization consists of is given by the same authors when they state

It is advantageous to turn Carnap’s partial functions into total ones, this by stipulating that $P(A,B)=1$ when $B$ is logically false, an option discussed by Carnap in (1950) and widely adopted in studies of the subject. (p. 11)

That is, Roeper and Leblanc desire a formulation of probability in which any statement ($A$) is true (probability unity), if a false statement (zero probability, $B$) is given as a condition.

Perhaps the clearest statement championing Popper, however, is given by Alan Hajek [7]. His overall thesis is that conditional probability is basic and absolute probability a derived notion, something Popper provides. He also identifies some problems with the conventional formulation that Popper’s version is claimed to solve. These are centered on situations with zero probability.

\[^2\text{Notation among the references is almost uniform, but not quite. Letters, upper or lower case, are events or hypotheses, things that might be true or not. P(A,B), with the letter ‘P’ upper or lower case, is the probability of event A given that B is true. The probability of a combination of events, that is the probability of both A and B happening given C, is P(AB,C). Converting to the form more usual for scientists, } p(a \mid b), \text{ is straightforward. In my own expositions I will follow that used by Popper, } p(a, b), \text{ to make comparison easier.}\]
Using the normal formulation, the probability of two things $A$ and $B$ both occurring is

$$p(AB) = p(A, B)p(B)$$

(1)

which is quickly rearranged to form the conditional probability, that of $A$ given $B$:

$$p(A, B) = \frac{p(AB)}{p(B)}$$

(2)

Hajék finds this inadequate in two types of problems. First, the sphere:

A point is chosen at random from the surface of the earth (thought of as a perfect sphere); what is the probability that it lies in the Western hemisphere given that it lies on the equator? 1/2, surely. Yet the probability of the condition is 0, since a uniform probability measure over a sphere must award probabilities to regions in proportion to their area, and the equator has area 0. The ratio analysis thus cannot deliver the intuitively correct answer. (pp. 111-112)

Next, there is the situation of all possible sequences of infinite tosses of a coin.

Any particular sequence has probability zero (assuming that the trials are independent and identically distributed with intermediate probability for heads). Yet surely various corresponding conditional probabilities are defined—e. g., the probability that a fair coin lands heads on every toss, given that it lands heads on tosses 3, 4, 5, . . . , is 1/4. (p. 112)

(In other places Hajék makes more extensive attacks on conventional probability, but this is where they are most clearly connected with Popper.) Here Hajék seeks a system that gives a defined probability conditional upon an event with zero probability, something he finds in Popper.

So Popper’s formulation of probability is currently considered a viable and even superior version among at least a group of philosophers.

2 Popper’s motivation

It is instructive to gain a flavor of Popper’s reasoning by examining his motivation for developing a new formulation of probability.
In §80 of *The Logic of Scientific Discovery* Popper is concerned with constructing the probability of a hypothesis from a series of events, or rather with refuting Richenbach’s attempt to do so. He notes that, if one makes a ratio of events confirming the hypothesis to total events, one finds a ‘probability’ of 1/2 for a hypothesis that is refuted by half the events (p. 257, p. 255). After trying several variations on this theme, he concludes, ‘This seems to me to exhaust the possibilities of basing the concept of the probability of a hypothesis on that of the frequency of true statements (or the frequency of false ones), and thereby on the frequency theory of the probability of events (p. 260, p. 258).’

In the *Postscript*, §vii, Popper modifies this conclusion, stating

> We may to this end interpret the universal statement $a$ as entailing an infinite product of singular statements, each endowed with a probability which of course must be less than unity. (p. 364, p. 376)

Popper does not explicitly connect these ‘singular statements’ to the more familiar language of a theory and its predictions. However, from this and other sections it seems clear that he is asserting that the probability of a theory is the product of the probabilities of all its predictions. He realises that this implies that all the ‘singular statements’ must be independent, if considered as probabilities of events, but justifies his formulation with the opaque assertion that any other situation is ‘non-logical’ (pp. 367-8, p. 379). Popper’s conclusion, in an infinite universe, is that *all* theories are of zero probability (p. 364, p. 376).

A separate route leading to the same conclusion starts from a development in Jeffreys’ *Theory of Probability* ([9], pp. 38-9). Jeffreys starts with the hypothesis $q$, previous information $H$ (which is not, as Popper notes, vital to the argument) and some experimental fact $p_1$. Writing $P(qp_1, H)$ two ways and rearranging,

$$P(q, p_1H) = \frac{P(q, H)P(p_1, qH)}{P(p_1, H)}$$

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[^3]: It has been pointed out to the author that an infinite product of factors all less than one is not necessarily zero. If $a_n = (1 - 1/n^\frac{1}{2})$, for instance, and one begins at $n = 2$, the product converges to 1/2. This is less important in the present context than the fact that Popper’s whole approach is erroneous.
Now if $p_1$ is a consequence of $q$, $P(p_1, qH) = 1$ so

$$P(q, p_1H) = \frac{P(q, H)}{P(p_1, H)}.$$  

(4)

If $p_2, p_3, \ldots, p_n$ are further consequences of $q$ which are each in turn found to be true,

$$P(q, p_1p_2 \ldots H) = \frac{P(q, H)}{P(p_1, H)P(p_2, p_1H) \cdots P(p_n, p_1p_2 \cdots p_{n-1}H)}$$  

(5)

in which we are dividing $P(q, H)$ by a growing product of probabilities. Jeffreys notes three possibilities for $P(q, p_1p_2 \ldots H)$: (1) it can grow without limit, as it will if the (unbounded in number) series of $P(p_n, \cdots)$ has a significant population less than unity; (2) $P(q, H)$ may be identically zero, in which case $P(q, p_1p_2 \ldots H)$ will also be identically zero; or (3) the $P(p_n, \cdots)$ become arbitrarily close to unity. The first option is ruled out by the definition of probability. Jeffreys chooses option (3), interpreting it as a growing confidence in the predictions of a theory based on a growing number of successful predictions.

In §*vii (pp. 370-1, pp. 383-4), Popper asserts that (3) leads to a paradox. He adduces two hypotheses (call them $q_1$ and $q_2$), each of which predict $p_1, p_2, \ldots, p_{n-1}$. But while $q_1$ predicts $p_n$, $q_2$ predicts its contradiction, $\bar{p}_n$. Then Jeffreys’ formulae predict both $p_n$ and $\bar{p}_n$ with near-unity probability. Hence the only choice is (2), all theories have zero probability.

Of course there is no paradox in $P(p_n, p_1 \cdots Hq_1) \simeq 1$ at the same time as $P(\bar{p}_n, p_1 \cdots Hq_2) \simeq 1$; the probabilities have different conditions. (It is interesting that, in a section devoted to the probability of hypotheses, Popper ignores them.) In fact this is the classical decisive experiment, which actually happens much more rarely than the tidy-minded would like.

There are things about Jeffreys’ calculation to make one uneasy. It is rather circular, for instance, to stipulate that the $p_n$ all actually happen (so that $P(p_n, \cdots) = 1$), then conclude that their probabilities are high. But one cannot use it to conclude that the probabilities of all theories vanish.

In these three sections, then, we have examples of Popper using the conventional theory of probability to calculate the probability of a theory, based on its predictions and events, and getting it wrong. It is worth noting that he has seen it done properly in at least one instance, and accepts it as correct. In §*ix (pp. 407-8, p. 425) he asserts that the probability of a coin landing...
heads is 1/2, given that it has landed heads in 500,000 ± 1,350 tosses out of a million previously. The tacit hypothesis ‘this is a fair coin’ has been given a high probability based on the probability of its predicted events, and not by forming a product of a million factors of 1/2.

As a consequence of Popper’s reasoning, he asserts, ‘...there is a need for a probability calculus in which we may operate with second arguments of zero absolute probability’ (§*vi, p. 330, p. 335). Note that his motivation, based on errors in applying the conventional system of probability, is not the same as that of Hajé’s problems [7] or the ‘total functions’ of Fitelson, Hajé and Hall [4], though they lead to the same requirement.

We will add one further problem Popper identifies with conventional probability as applied to the confirmation of hypotheses. It is not directly related to the zero-probability matter, but forms another aspect of Popper’s attack and we may apply his system to it. In §*ix (pp. 390-1, pp. 406-7) he presents the situation of a standard six-sided die. Hypothesis $x$ is that it rolls a six, and $\bar{x}$ that it rolls some other number. Given a fair die (tacitly assumed by Popper) and no other information, $P(x) = 1/6$, $P(\bar{x}) = 5/6$. Then given the information $z$ that the die roll was even, we have $P(x, z) = 1/3$, $P(\bar{x}, z) = 2/3$. The added information has increased the probability of $x$ and decreased that of $\bar{x}$, but still $P(x, z) < P(\bar{x}, z)$. Popper finds this ‘clearly self-contradictory’ if probability is to be used to judge the corroborations of a theory. Popper is requiring that any evidence that supports a theory makes it more likely than its contradiction. Let us call this the requirement for absolute support.

3 Developing Popper’s formulation

It would be very useful to have a demonstration of exactly how Popper’s system of probability is used in a given instance, and especially one showing its advantages over the conventional one. Unfortunately, in the references at hand the authors do not set out any details about how Popper’s formulation solves the problems they identify. Nor does Popper show, in any specific problem, where his formulation solves or even addresses the flaws he identifies with conventional probability. We must work that out from the axioms themselves, an approach that also permits us to see what the formulation says and does not say. After some development, we will then apply the system to the several problems.

Popper sets out his axioms in slightly different ways in several places in
[1],[2], but the development here (as well as the exposition in [7]) generally follows §iv, pp. 332ff, pp. 336ff; §v, pp. 349-353, pp. 356-361. Although everything in the formulation can be immediately interpreted in terms of conditional probability, we will refrain from any such interpretation until after a number of results have been obtained formally.

We begin with \( S \), a collection (one would say *set*, but Popper is mistaken on important parts of set theory [3] so we avoid the term) of otherwise undefined objects \( a, b, \ldots \). There is a unary operation of complementation, with \( \bar{a} \) also being in \( S \), and two binary operations: conjunction, with \( ab \) also a member of \( S \), and \( p(a, b) \) being a real number.

The axioms, with \( a, b, c, d, \ldots \) representing any member of \( S \), are:

**A1.** There are elements \( a, b, c \) and \( d \) in \( S \) such that \( p(a, b) \neq p(c, d) \).

**A2.** If \( p(a, c) = p(b, c) \) for every \( c \) in \( S \), then \( p(d, a) = p(d, b) \) for every \( d \) in \( S \).

**A3.** \( p(a, a) = p(b, b) \)

**B1.** \( p(ab, c) \leq p(a, c) \)

**B2.** \( p(ab, c) = p(a, bc)p(b, c) \)

**Ci.** \( p(a, b) + p(\bar{a}, b) = p(b, b) \), if there is some \( c \) such that \( p(c, b) \neq p(b, b) \).

**Cii.** \( p(\bar{a}, b) = p(a, a) \) if there is no such \( c \).

Note in particular the exclusive nature of Ci and Cii.

First, we set bounds on \( p(a, a) \), invoking A3, B1 and B2:

\[
\begin{align*}
p(a, a) &= p(b, b) = k \\
p((aa)a, a) &\leq p(aa, a) \\
p(aa, a) &\leq p(a, a) = k \\
p((aa)a, a) &= p(aa, aa)p(a, a) = k^2 \\
k^2 &\leq k \\
0 &\leq k \leq 1
\end{align*}
\]

Next, some bounds on the general \( p(a, b) \) and a useful result on arguments in both positions. Invoking B1 and B2:

\[
\begin{align*}
p(ab, ab) &\leq p(a, ab) \\
k &\leq p(a, ab) \\
p(aa, b) &= p(a, ab)p(a, b)
\end{align*}
\]
\[ p(a, b) \geq p(a, ab)p(a, b) \]
\[ 1 \geq p(a, ab), p(a, b) \neq 0 \]

Now using B1 and B2 again,
\[ p(ab, c) = p(a, bc)p(b, c) \]
\[ p(b, c) \geq p(a, bc)p(b, c) \]
\[ 1 \geq p(a, bc), p(b, c) \neq 0 \]

Popper derives \( k = 1 \) using Ci. To save time and because we want to avoid depending on a particular branch of C, we shall simply assume \( p(a, a) = 1 \), which of course must be true if we are to interpret these axioms as a formulation of probability. With the above development, that means
\[ p(a, ab) = 1 \] (6)

It is convenient to have a version of eq. 6 with the conditional reversed (note we have not yet shown that \( p(\cdot, ab) = p(\cdot, ba) \)). Starting with B2,
\[ p(ab, c) = p(a, bc)p(b, c) \] (7)

but since \( p(a, bc) \) is bounded by unity,
\[ p(ab, c) \leq p(b, c) \] (8)

and so
\[ p(ab, ab) \leq p(b, ab) \] (9)

and, with \( k = 1 \),
\[ p(b, ab) = 1. \] (10)

Now using Ci (and employing \( b \) as a general member of \( S \) to avoid potential confusion in what follows),
\[ p(b, b) + p(\bar{b}, b) = p(b, b) \]
\[ p(\bar{b}, b) = 0 \]

Note that in using Ci we have made an assumption about the behaviour of \( b \); the following shows it isn’t necessarily true.

We again write Ci,
\[ p(a, a\bar{a}) + p(\bar{a}, a\bar{a}) = p(a\bar{a}, a\bar{a}) \] (11)
which, in light of equations 6 and 10, gives $2 = 1$; so Ci cannot hold and we are forced into Cii:

$$p(b, a\bar{a}) = 1$$  \hspace{1cm} (12)

for any $b$ in $S$.

Note that the elements of $S$ have been divided into two classes by Popper: those for which Ci is true and $p(\bar{a}, a) = 0$, and those for which Cii is true and $p(\bar{b}, b) = 1$. All elements of $S$ of the form $a\bar{a}$ are in the latter class.

At this point we may impose the obvious interpretation of Popper’s system as a formulation of probability. The elements of $S$ are events, things that happen or not; they may be composite, $ab$ meaning that both $a$ and $b$ happen; and $p(a, b)$ is the probability that $a$ occurs, given that $b$ does. A Ci event and its complement exhaust all possibilities, and either $b$ or $\bar{b}$ is true. Cii events, however, do unconventional things, which we shall examine in a moment.

### 4 Applying Popper

As a warm-up, we will apply Popper’s system to his requirement of absolute support. That is the requirement that any event or information making an hypothesis more probable than before must make it more probable than its contradiction. In our notation, we require

$$p(a, bc) > p(a, c) \Rightarrow p(a, bc) > p(\bar{a}, bc)$$  \hspace{1cm} (13)

(note that the inequalities preclude $bc$ or $c$ being Cii events). This means

$$p(a, c) \geq p(\bar{a}, bc)$$  \hspace{1cm} (14)

since if $p(a, c) < p(\bar{a}, bc)$ there are possible values for $p(a, bc)$ that would allow

$$p(a, c) < p(a, bc) < p(\bar{a}, bc)$$  \hspace{1cm} (15)

and absolute support is not satisfied. Applying B2 to Eq. 14 and rearranging,

$$p(b, c)p(a, c) \geq p(\bar{a}b, c)$$  \hspace{1cm} (16)

Next we write a version of Ci:

$$p(b, c) + p(\bar{b}, c) = 1$$  \hspace{1cm} (17)
and multiply the left-hand term by unity, again using Ci:

$$[p(a, bc) + p(\bar{a}, bc)]p(b, c) + p(\bar{b}, c) = 1$$  \hspace{1cm} (18)

and applying B2,

$$p(ab, c) + p(\bar{a}b, c) + p(\bar{b}, c) = 1$$  \hspace{1cm} (19)

which rearranges (again using Ci) to give

$$p(\bar{a}b, c) = -p(ab, c) + p(b, c)$$  \hspace{1cm} (20)

which we insert in Eq. (16) obtaining

$$p(b, c)p(a, c) \geq -p(ab, c) + p(b, c)$$  \hspace{1cm} (21)

which rearranges to give

$$p(b, c) - p(b, c)p(a, c) \leq p(ab, c)$$  \hspace{1cm} (22)

and, invoking B1,

$$p(b, c) - p(b, c)p(a, c) \leq p(a, c)$$  \hspace{1cm} (23)

which rearranges to give

$$p(b, c) \leq \frac{p(a, c)}{1 - p(a, c)}.$$  \hspace{1cm} (24)

For \(p(a, c) \geq 1/2\), that is an original event at least as probable as its contradiction, this is no restriction on the supporting information \(b\) at all; which is what one would expect. If \(p(a, c)\) is small, however, condition (24) is very limiting. For very improbable original events \(p(b, c) \approx p(a, c)\), which (recalling that \(b\) must support \(a\)) essentially makes \(b\) identical with \(a\). Popper has not succeeded in implementing his requirement of absolute support.

To be fair, there is no indication that he developed his formulation of probability with this requirement in mind; he does not refer to it in his exposition of the system. We may say, however, that any success he may have in remedying what he sees as the flaws in conventional probability is less than total.

We now turn to the situations of zero-probability conditionals. Unfortunately, we cannot yet apply Popper’s system to either Hajek’s problems or scientific hypotheses. It requires, as a conditional, \(a\bar{a}\); that is, a statement
and its complement together, which not only have probability zero of occurring simultaneously, but exhaust all possibilities between them. ‘A point is chosen at random on the surface of a sphere’ and ‘the point lands on the equator,’ while having probability zero of happening together, do not exhaust all possibilities. ‘The point lands on the equator’ and ‘the point does not land on the equator’ would satisfy the requirement, but that’s not the problem Hajek sets. Similarly, ‘an hypothesis is true’ together with ‘the hypothesis is false’ would satisfy the condition, but again that’s not what Popper was aiming for.

(One might conceivably postulate that all zero-probability statements in Popper’s system take the form of $\bar{a}a$. It’s not clear that it could be done consistently, and in any case makes any interpretation as a system of probability problematic.)

What Popper was clearly aiming for, and what Hajek’s problems and ‘total functions’ require, is a definite value for $p(a, bc)$ whenever $p(b, c) = 0$, and not only when $b = \bar{c}$. Popper desires a value of unity; he asserts in §23, p. 91, p. 71, ‘From a self-contradictory statement, any statement whatsoever can be validly deduced.’ (In the accompanying footnote he comments on others’ treatment of the idea.) It is very similar to Roeper and Leblanc’s ‘total functions’ region (above), and thus is ‘widely adopted.’ Fitelson and Hawthorne [8] agree that ‘everything follows from any inconsistent set of statements.’ The difference between contradictory statements and merely inconsistent ones may seem trivial, but is important in applying Popper’s mathematics.

As Fitelson, Hajek and Hall [4] noted, Carnap ([10] pp. 295-6) considered the more general prospect. For statements conditional on logically false (L-false) evidence, he notes that if they are not explicitly excluded, systems such as Jeffreys’ contain contradictions (though he allows that Jeffreys may have tacitly excluded such statements). The value $P = 1$ ‘seems most natural,’ though some of Carnap’s theorems must retain the exclusion of L-false conditionals even if this is chosen. If one assumes $P = 0$ there are also problems, though different ones.$^4$ Carnap seems inclined to simply exclude L-false conditionals.

Given the clear intention of Popper and his supporters, let us enumerate

$^4$Everyone among the authorities consulted has made the tacit assumption that the probabilities of all statements conditional on zero-probability events are identical. This is remarkable. It does not hold for conditionals of any other value. But further discussion would take us too far from the matter at hand.
the possibilities. The desired statement is

\( D \). For any \( c \) in \( S \), and any \( a \) and \( b \) in \( S \) such that \( p(a, b) = 0, p(c, ab) = 1 \).

Three things could happen:
(1) It could be that \( D \) is inconsistent with Popper’s formulation.
(2) It could be that \( D \) is derivable in Popper’s system, and that the derivation simply doesn’t appear in the references at hand or has not yet been accomplished.
(3) It could be that \( D \) is consistent with Popper’s formulation, but must be added as a separate axiom.

If (1) holds, Popper has accomplished the precise opposite of his intention and what his supporters understand him to have done. It is also unclear why it shouldn’t hold for \( \bar{a}a \) conditionals. We shall disregard this possibility.

If (2) or (3) holds the effect is the same. Rather than spend what might be a lot of time and effort attempting to determine which is true, we will proceed to add \( D \) as an additional axiom, realizing that it might be unnecessary.

Applying it now to Hajék’s sphere, \( c = \text{‘the point lands in the Western Hemisphere’,} \ a = \text{‘the point lands on the Equator’} \) and \( b = \text{‘a point is chosen at random on a sphere.’} \) We find that the probability of the point landing in the Western Hemisphere is 1, which does not accord with Hajék’s intuition. But we also find, with probability 1, \( c = \text{‘the point lands in the Eastern Hemisphere’,} \ c = \text{‘the point lands on the Arctic Circle,’} \ c = \text{‘the point does not land on the Equator.’} \) All statements are true; Hajék’s intuitive answers are there among them, but so are their direct contradictions. A similar situation holds with his restricted series of coin flips.

(It might be possible to restrict the allowed \( c \) to those Hajék desires, but to do so contradicts Popper’s system as formulated, and so is beyond the scope of this paper.)

Next we apply \( D \) to a scientific theory, of (as Popper asserts) zero probability, which was the original motivation for Popper’s formulation. We find that \textit{everything} is true, given \textit{any} theory. All theories predict every event. Thus, in an enormous irony, Popper the exponent of falsification has produced a system in which no theory can ever be falsified.

Of course Popper’s derivations that the probabilities of all theories vanish are erroneous. That sends us back to the very beginning, with nothing in particular accomplished.
5 Conclusions

Popper’s formulation of probability, motivated by his own errors in applying the conventional theory, does not accomplish what he intended and what his present supporters claim. Like a great deal of The Logic of Scientific Discovery, it has nothing to do with the logic of scientific discovery.

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