New constraint on the tensor-to-scalar ratio from the Planck and BICEP/Keck Array data using the profile likelihood

Paolo Campeti 1,2* and Eiichiro Komatsu1,3
1Max Planck Institute for Astrophysics, Karl-Schwarzschild-Str.1, 85741 Garching, Germany
2Excellence Cluster ORIGINS, Boltzmannstr. 2, 85748 Garching, Germany
3Kavli Institute for the Physics and Mathematics of the Universe (Kavli IPMU, WPI), UTIAS, The University of Tokyo, Chiba, 277-8583, Japan

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ABSTRACT

We derive a new upper bound on the tensor-to-scalar ratio parameter \( r \) using the frequentist profile likelihood method. We vary all the relevant cosmological parameters of the \( \Lambda \)CDM model, as well as the nuisance parameters. Unlike the Bayesian analysis using Markov Chain Monte Carlo (MCMC), our analysis is independent of the choice of priors. Using Planck Public Release 4, BICEP/Keck Array 2018, Planck CMB lensing, and BAO data, we find an upper limit of \( r < 0.037 \) at 95\% C.L., similar to the Bayesian MCMC result of \( r < 0.038 \) for a flat prior on \( r \) and a conditioned Planck low\( l_{\text{EB}} \) covariance matrix.

Key words: cosmology: cosmic background radiation – methods: data analysis – cosmology: cosmological parameters

1 INTRODUCTION

Detecting the stochastic background of primordial gravitational waves predicted within the inflationary paradigm (Grishchuk 1974; Starobinsky 1979) represents one of the principal objectives of the current cosmological research, as it would provide the definitive evidence for cosmic inflation (Guth 1981; Sato 1981; Linde 1982; Albrecht & Steinhardt 1982).

Whereas inflation produces gravitational waves (i.e. tensor modes) over a wide range in frequency measurable by several different probes (see e.g. Campeti et al. 2021, for a review), the most promising route to detect the B-mode polarization of the cosmic microwave background (CMB) (Kamionkowski et al. 1997; Seljak & Zaldarriaga 1997).

The current datasets only provide upper bounds on the tensor-to-scalar ratio \( r \) (i.e. the ratio of the amplitudes of the tensor and scalar modes power spectra). To date, the tightest limit on \( r \) (customarily measured at the pivot scale \( k_0 = 0.05 \text{ Mpc}^{-1} \)) is \( r < 0.032 \) at 95\% C.L. (Tristram et al. 2022), coming from the Planck latest CMB temperature and \( E \) and \( B \)-mode polarization data (Tristram et al. 2021), the BICEP/Keck Array \( B \)-mode data (BICEP/Keck Collaboration 2021, hereafter BK18), the baryon acoustic oscillations (BAO) of the large-scale structure (Alam et al. 2021), and the CMB lensing data (Planck Collaboration VIII 2020). This upper limit is derived using a standard Bayesian Monte Carlo Markov Chain (MCMC) procedure, varying the relevant cosmological parameters of a flat \( \Lambda \) cold dark matter (\( \Lambda \)CDM) model and adopting the Sellentin & Hennan (2016) correction (hereafter SH) to the Hamimeche & Lewis (2008) likelihood (hereafter HL) for the Planck large-scale \( EE \), \( BB \) and \( EB \) power spectra (the “low\( l_{\text{EB}} \)” likelihood). The SH correction is needed to account for the increased uncertainty in parameter estimation due to the limited number of simulations used to estimate the covariance matrix. This is obtained by analytically marginalizing over the unknown true covariance matrix.

Most of the constraining power on \( r \) at the pivot scale comes from BK18’s \( B \)-mode data. An upper limit of \( r < 0.036 \) at 95\% C.L. (BICEP/Keck Collaboration 2021) is obtained just from the BK18 data, provided that we fix the \( \Lambda \)CDM parameters to their best-fitting values given in Planck Collaboration VI (2020). The Planck satellite provides, on the other hand, the tightest constraints to date on the \( B \) modes at the largest angular scales, which are not accessible from the ground. Exploiting the latest NP1PE-processed Public Release 4 (PR4) of temperature and polarization maps, the Planck collaboration reported a limit of \( r < 0.056 \) at 95\% C.L. (Tristram et al. 2021), which is relaxed to \( r < 0.075 \) when properly accounting for the SH correction in the low\( l_{\text{EB}} \) likelihood (Beck et al. 2022).

While the SH correction accounts for the Monte Carlo noise in the estimated covariance matrix, it does not correct for the additional scatter in the best-fitting maximum a posteriori parameter (MAP) estimate, which can lead to a misestimation of confidence limits (Beck et al. 2022). This effect is especially relevant near the physical boundary of a given parameter (i.e. \( r \geq 0 \) in our case of interest) and can produce a significant underestimation of the upper limit. The issue can be corrected by increasing the number of (computationally expensive) time-ordered data simulations used in the covariance matrix estimation or by properly conditioning the covariance matrix. The latter method has been applied in Beck et al. (2022) to the low\( l_{\text{EB}} \) Planck likelihood (which we will refer to as “conditioned HL” in the following), resulting in a much weaker Planck-only upper limit of \( r < 0.13 \) at 95\% C.L., associated to a large shift of the peak of the marginalized distribution to larger \( r \) values than in the SH case. Similarly, for the Planck + BK18 + BAO + lensing combination, the conditioning results in a more conservative upper limit of \( r < 0.038 \).

In this paper, we present constraints on \( r \) using the frequentist profile likelihood method, and compare them to the standard Bayesian

* E-mail: pcampeti@mpa-garching.mpg.de

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MCMC procedure adopted throughout the literature. While the profile likelihood is a standard data analysis tool in particle physics (see e.g. Zyla et al. 2020; ATLAS Collaboration 2013), it has been seldom used in cosmology, notable cases of use being the application to ΛCDM parameters estimation from the Planck data (Planck Collaboration Int. XVI 2014), to the Early Dark Energy fraction (Herold et al. 2022), to coupled dark energy and Brans-Dicke models (Gómez-Valent 2022) and to the estimation of r from the SPI- DER data (Ade et al. 2022). Nonetheless, this approach bears several potentially interesting differences with Bayesian methods (Cousins 1995). First, the profile likelihood does not require priors, which may have an impact on the final constraints. Second, while in Bayesian methods the choice of a specific set of parameters to sample might represent an implicit prior choice, the maximum likelihood estimate (MLE) is invariant under model reparameterization. Third, the parameter estimates obtained from the profile likelihood are not affected by “volume effects” which can arise during marginalization in the MCMC approach (Hamann et al. 2007). Moreover, the profile likelihood formalism allows to conveniently include the effect of the parameter’s physical boundary in the confidence intervals via the Feldman-Cousins prescription (Feldman & Cousins 1998).

Our work aims to deconstruct the current constraints on r and scrutinise their robustness. Similarly to the profile likelihood analysis performed on the ΛCDM parameters (Planck Collaboration Int. XVI 2014), we study the effect of priors and marginalization on the inference of r from the Planck and BK18 data. We also explore the effect of conditioning the Planck lowE likelihood (Planck Collaboration V2020) for the Planck likelihood, on the profile likelihood.

The structure of the paper is the following. We describe the data and likelihood used in our analysis in Section 2. We review the profile likelihood formalism and the Feldman-Cousins prescription in Section 3. We discuss the new constraints on r from our frequentist analysis and compare them to the Bayesian credible intervals in Section 4. We conclude in Section 5.

2 DATA AND LIKELIHOODS

We use the latest Planck NPIPE-processed PR4 maps (Tristram et al. 2021) and the BK18 dataset (BICEP/Keck Collaboration 2021). We use the data and likelihoods publicly available for the Cobaya solver (Lewis et al. 2022), as done in Tristram et al. (2022). We also use Cobaya as an interface with the CAMB Boltzmann solver (Lewis et al. 2000).

2.1 Planck likelihoods

The Planck likelihood consists of three parts: the low-ℓ TT Commander likelihood (Planck Collaboration V 2020) for ℓ = 2–30, the high-ℓ TT + TE + EE HiLLiPoP likelihood (Planck collaboration XV 2014; Planck Collaboration XI 2016; Couchot et al. 2017) for ℓ = 30–2500, and the low-ℓ EE + BB + EB LoLLiPoP or the lowE likelihood (Tristram et al. 2021) for ℓ = 2 – 150.

The low-ℓ TT likelihood is the same as in PR3, since no improvement is expected with the PR4 update for the high signal-to-noise temperature data. The HiLLiPoP likelihood is instead a Gaussian likelihood for cross-power spectra of the Planck 100, 143, and 217-GHz data.

The LoLLiPoP likelihood for large-scale EE, BB and EB power spectra implements the HL approximation for a non-Gaussian likelihood (Hamimeche & Lewis 2008), adapted specifically for cross-power spectra (Mangilli et al. 2015). In this case, an offset term is needed to make the distribution of cross-power spectra similar to that of auto-power spectra, as required by the HL approximation. The covariance matrix for this likelihood is estimated from 400 Monte Carlo simulations of PR4, which include Planck noise, systematic effects and foreground residuals. The Planck lowE likelihood implements the SH correction to the HL likelihood to account for the Monte Carlo noise in the covariance matrix estimate. This is not sufficient to amend the additional scatter in the MAP estimate: a possible solution indicated in Beck et al. (2022) involves using the HL likelihood (without the SH correction) with a conditioned covariance matrix. The conditioning strategy removes all off-diagonal elements beyond the next-to-nearest neighbour for unbinned multipoles (ℓ ≤ 35) and all off-diagonal elements beyond the nearest neighbour for binned multipoles (ℓ > 35). We will refer to this specific choice as “cond. HL” in the following.

2.2 BICEP/Keck Array 2018 likelihood

The BK18 likelihood, which includes only B modes at ℓ = 30 – 300, also applies the HL approximation to auto- and cross-power spectra in conjunction with the WMAP data at 23 and 33 GHz and Planck NPIPE-processed data at 30, 44, 143, 217 an 353 GHZ. The bandpower covariance matrix is estimated from 499 simulations. The default BK18 likelihood already incorporates conditioning to reduce the Monte Carlo noise.

2.3 Likelihood combination and priors in the default analysis

We combine the Planck and BK18 likelihoods neglecting correlations between them. This is a good approximation because the current B-mode data are noise-dominated, the two CMB surveys have uncorrelated noises, and they observe very different fractions of the sky (i.e. 50% for Planck and 1% for BK18, see Tristram et al. 2022, 2021). In the following, whenever we use the Planck likelihood, we will also include the BAO data (Alam et al. 2021) and the Planck CMB lensing data (Planck Collaboration VIII 2020).

There are in total 33 free parameters in the default Planck + BK18 analysis, including r, 6 parameters of a flat ΛCDM model (Ω_bh^2, Ω_ch^2, τ, A_s, n_s, θ_M), and the nuisance parameters. The tensor spectral index n_t is fixed via the inflationary consistency relation n_t = −r/8, similarly to previous analyses (Tristram et al. 2022, 2021). We also checked that fixing n_t = 0 as in the BICEP/Keck Collaboration (2021) analysis does not impact our results.

The Planck likelihoods introduce 19 nuisance parameters, accounting for map and absolute calibration and foreground modeling (for a description see Appendix B in Tristram et al. 2021). Of these, 8 parameters have a Gaussian prior in the default MCMC analysis, whereas the others have uniform priors. The BK18 likelihood has 7 nuisance parameters accounting for Galactic dust and synchrotron foreground modeling. Of these, 6 parameters have uniform priors in the default BK18 analysis (BICEP/Keck Collaboration 2021), whereas the synchrotron spectral index β_s has a Gaussian prior β_s = −3.1±0.3 (motivated by the WMAP 23 and 33 GHz data, Fuskeland et al. 2014). As shown in BICEP/Keck Collaboration (2021), the constraint on β_s from the BK18 data is prior-dominated; therefore,
for a more direct comparison with the Bayesian results in the literature, we also explore the possibility of fixing $\beta_s = -3.1$ in the profile likelihood, since frequentist analyses do not incorporate priors. We indicate such choice as “fixed $\beta_s$” in the following.

3 PROFILE LIKELIHOOD

We use the profile likelihood to investigate the effects of priors and marginalization on the current Bayesian constraints on $r$. The profile likelihood is a staple in the frequentist’s toolbox. As it does not incorporate priors, explicitly or implicitly via the model parameterization, it is immune to volume effects which may appear during marginalization in MCMC.

The profile likelihood for a parameter of interest $\mu$ (in our case $\mu = r$) is obtained by fixing $\mu$ to multiple values within the range of interest and minimsing the $\chi^2(\mu) = -2 \log L(\mu)$ with respect to all the remaining cosmological and nuisance parameters for each fixed value of $\mu$. Here, $L$ is the likelihood. By construction the minimum $\chi^2_{\text{min}}$ coincides with the global MLE (also called “best-fit”).

We use $\Delta \chi^2(\mu) = \chi^2(\mu) - \chi^2_{\text{min}}$ to construct frequentist confidence intervals on $\mu$. If $\mu$ is far away from its physical boundary, a confidence interval at a C.L. can be obtained by cutting $\Delta \chi^2(\mu)$ at a fixed threshold $\Delta \chi^2_{\text{th}}$ such that the cumulative distribution function of the $\chi^2$ distribution with one degree of freedom is equal to $\alpha$ (e.g. cutting at $\Delta \chi^2_{1} = 1$ and $\Delta \chi^2_{2} = 3.84$ for 68% or 95% C.L., respectively, see e.g. Trotta 2017). We can use this procedure for both parabolic (associated to a Gaussian-distributed parameter) and nonparabolic $\Delta \chi^2(\mu)$ thanks to invariance of the MLE under reparameterization.

3.1 The Feldman-Cousins prescription

If the parameter estimate is instead close to its physical boundary, the classical Neyman’s construction of frequentist confidence intervals is unsatisfactory. It can lead to empty intervals (as in our case of interest, the classical Neyman’s construction of frequentist confidence intervals on $\mu$ (associated to a Gaussian-distributed parameter) and nonparabolic $\Delta \chi^2(\mu)$ is obtained by fixing $\mu$ to multiple values within the range of interest and minimsing the $\chi^2(\mu)$ with respect to all the remaining cosmological and nuisance parameters for each fixed value of $\mu$. Here, $L$ is the likelihood. By construction the minimum $\chi^2_{\text{min}}$ coincides with the global MLE (also called “best-fit”).

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3.2 Minimization algorithm

We minimise $\chi^2(r)$ with the MIGRAD algorithm implemented in the iminuit package, a python interface for the popular Minuit multidimensional minimiser. We scan the parameter space, fixing $r$ to values over a wide range and minimising $\chi^2(\mu)$ with respect to the remaining 32 free parameters for each fixed $r$. Each point in the profile likelihood typically requires around $O(10^4)$ evaluations of the likelihood, with each evaluation taking $O(1)$ s (using 10 logical CPUs on a computer cluster node), almost exclusively absorbed by the evaluation of the CAMB Boltzmann code. To increase the chance of the minimiser reaching the global minimum, each minimization is started from ten different random initial parameter sets. We take the point with the lowest $\chi^2$ as the final result. We checked that increasing the accuracy settings of the CAMB code does not change our results.

4 RESULTS AND COMPARISON TO THE BAYESIAN ANALYSIS

In Fig. 1 and Table 1 we report 95% C.L. upper limits on $r$ obtained from the profile likelihood (darker shaded bars) and compare them to their Bayesian MCMC counterparts (lighter shaded bars). We also show the best-fitting $r$, that is, the global MLE values found in the profile likelihood analysis as the black dots, and indicate negative (unphysical) values of $r$ by the hatched dark grey region.

We found significantly better performances using iminuit compared to other common minimising algorithms (e.g. the scipy minimize module (Virtanen et al. 2020) and PyBOBYQA (numericalalgorithmgroup.github.io) which are already implemented in the Cobaya sampler).

The profile likelihood is highly competitive with the more traditional MCMC approach, which requires $O(10^7)$ points to reach convergence, due to inefficient sampling of the Metropolis-Hasting algorithm near the boundary of a parameter with a uniform positive prior.
Figure 1. Summary of 95% C.L. upper limits on $r$ for datasets considered in this work. The darker shaded bars indicate the upper limit from the profile likelihood, whereas the lighter shaded bars the MCMC one. The best-fitting $r$ values from the profile likelihood analysis are shown as the black dots. Negative (unphysical) values of $r$ are indicated by the hatched dark grey region. The baseline result of this work is highlighted in bold.
| Data | Likelihood | Fixed $\beta_s$ | Profile (95% C.L.) | MCMC (95% C.L.) | $r_{\text{MLE}}$ | Colour |
|------|------------|----------------|---------------------|------------------|-----------------|--------|
| Planck + BAO + lensing | SH | - | $r < 0.068$ | $r < 0.075$ | -0.027 |  |
| | cond. HL | - | $r < 0.15$ | $r < 0.13$ | 0.053 |  |
| BK18 | fix $\Lambda$CDM params. | ✓ | $r < 0.033$ | $r < 0.036$ | -0.015 |  |
| Planck + BK18 + BAO + lensing | SH | ✓ | $r < 0.032$ | $r < 0.032$ | 0.01 |  |
| | cond. HL | ✓ | $r < 0.042$ | $r < 0.038$ | -0.0021 |  |

Table 1. Upper limits on the tensor-to-scalar ratio parameter $r$ (95% C.L.) from the profile likelihood method with the FC prescription and the MCMC. We also report the MLE for $r$ obtained from the profile likelihood method. For “cond. HL” we adopt the conditioning prescription defined in Beck et al. (2022) for the Planck lowEB likelihood. For “SH” we marginalise the likelihood over the covariance matrix (Sellentin & Heavens 2016). Note that for BK18-only data we fix all 6 $\Lambda$CDM parameters to the best-fitting values given in Planck Collaboration VI (2020). For each case involving the BK18 likelihood, we indicate whether we are fixing the synchrotron spectral index to $\beta_s = 3 \pm 1$ (see Section 2 for details). The baseline result of this work is highlighted in bold. The colours shown in the rightmost column match those in Figures 1 and 2.

Figure 2. Profile likelihoods for $r$ from the datasets combinations considered in this work. The points are the $\chi^2$ values obtained from the likelihood maximization, whereas the parabolic fits are shown as the solid lines. The dashed lines indicate the upper limits at 95% C.L. according to the FC prescription. Unphysical (negative) values of $r$ are shown as the dark grey hatched area. The baseline result is shown in the thick red line.

prior-dominated nuisance parameters such as $\beta_s$ are fixed in the profile likelihood analysis. This suggests that volume effects do not play a prominent role in the Bayesian constraints. We note also that, because of the inefficiency of the Metropolis-Hastings algorithm in sampling near the boundary when a uniform positive prior is imposed on $r$, an apparent lower limit $r > 0$, which is entirely caused by the prior-dominated posterior, appears (Hergt et al. 2021). This issue can be addressed for instance with the adoption of a logarithmic prior on $r$ (introducing however a dependence of the constraints on the choice of the prior lower edge) as well as with the profile likelihood approach we adopt in this paper.

5 CONCLUSIONS

In this paper, we derived confidence intervals on $r$ from the state-of-the-art CMB datasets Planck and BK18 via the frequentist profile likelihood method, and compared with the Bayesian MCMC procedure typically adopted in the literature. This is a useful robustness test for a potential future detection of $r$ or for putting robust upper limits on this parameter, checking simultaneously for the dependence on priors and the volume effect upon marginalization in the Bayesian constraints. The profile likelihood is not affected by the inefficiency of the MCMC sampling near the boundary when a uniform prior is imposed on $r \geq 0$.

We confirmed that the profile likelihood method can provide upper limits comparable with the MCMC ones. Specifically, we reported an upper limit of $r < 0.042$ at 95% C.L. for the combination of Planck, BK18, BAO and lensing with a conditioned Planck lowEB covariance matrix as suggested in Beck et al. (2022). This limit is slightly more conservative than the corresponding MCMC limit of $r < 0.038$. We find that the Bayesian constraint is driven by the Gaussian prior adopted for the synchrotron spectral index $\beta_s$ in the BK18 likelihood. Fixing this nuisance parameter to the central value of the prior, $\beta_s = -3.1$, we obtained an upper limit of $r < 0.037$ from the profile likelihood, slightly tighter than the MCMC limit because of the well-known overcoverage of Bayesian intervals near the parameter boundary.
We also confirmed the findings in Beck et al. (2022) regarding the conditioning of the Planck lowlEB covariance matrix: the additional scatter due to the limited number of simulations used in the covariance matrix construction moves the MLE of the lowlEB likelihood towards lower values, deceptively tightening the resulting upper limit.

Given the consistency of the limits from the profile likelihood and the MCMC approach (provided that we fix $\beta_s$), we confirmed that the Bayesian limits set in Tristram et al. (2022) and Beck et al. (2022) are not significantly affected by volume effects arising during marginalization or by differences due to the choice of model parameterization (i.e. implicit priors).

The profile likelihood method is computationally more efficient than the MCMC, providing a useful alternative for a fast and robust evaluation of confidence limits near the physical boundary of a parameter.

Although we do not find substantial differences with respect to the standard Bayesian approach using the current data, we anticipate that the profile likelihood will represent a useful sanity check for prior effects in future and increasingly sensitive surveys, such as the BICEP array (Moncelsi et al. 2020), the Simons Observatory (Ade et al. 2019), the LiteBIRD satellite (LiteBIRD Collaboration 2022) and the CMB-S4 (Abazajian et al. 2016) experiments.

APPENDIX A: BEST-FITTING PARAMETERS

In Table A1 we compare the best-fitting parameters for the Planck+BK18+BcO+lensing combination and conditioned lowlEB covariance matrix with and without fixing the synchrotron spectral index $\beta_s$ in the BK18 likelihood. See Section 2 and references therein for details on likelihoods and parameters used here.

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DATA AVAILABILITY

The data underlying this article will be shared on request to the corresponding author.

REFERENCES

ATLAS Collaboration 2013, Report ATLAS-CONF-2013-014, Combined measurements of the mass and signal strength of the Higgs-like boson with the ATLAS detector using up to 25 fb$^{-1}$ of proton-proton collision data

Abazajian K. N., et al., 2016, preprint (arXiv:1610.02743)

Ade P., et al., 2019, JCAP, 02, 056

Ade P. A. R., et al., 2022, Astrophys. J., 927, 174

Alam S., et al., 2021, Phys. Rev. D, 103, 083533

Albrecht A., Steinhardt P. J., 1982, Phys. Rev. Lett., 48, 1220

BICEP/Keck Collaboration 2021, Phys. Rev. Lett., 127, 151301

Beck D., Cukierman A., Wu W. L. K., 2022, preprint (arXiv:2202.05949)

Campeti P., Komatsu E., Poletti D., Baccigalupi C., 2021, JCAP, 01, 012

Couchot F., Henrot-Versillé S., Perdereau O., Plaszczynski S., Rouiléd’Orfeuil B., Spinelli M., Tristram M., 2017, Astron. Astrophys., 602, A41

Cousins R. D., 1995, Am. J. Phys., 63, 398

Cousins R. D., 2018, preprint (arXiv:1807.05966)

Cousins R. D., 2019, Phys. Rev. D, 100, 083513

Feldman G. J., Cousins R. D., 1998, Phys. Rev. D, 57, 3873

Fuskeland U., Wehus I. K., Eriksen H. K., Ness S. K., 2014, Astrophys. J., 790, 104

Gómez-Valent A., 2022, preprint (arXiv:2203.16285)

Guth A. H., 1981, Phys. Rev. D, 23, 347

Hamann J., Hannestad S., Rafelt G. G., Wong Y. Y. Y., 2007, JCAP, 08, 021

Hamimeche S., Lewis A., 2008, Phys. Rev. D, 77, 103013

Herdt L. T., Handley W. J., Hobson M. P., Lasenby A. N., 2021, Phys. Rev. D, 103, 123511

Herold L., Ferreira E. G. M., Komatsu E., 2022, Astrophys. J. Lett., 929, L16

Kamionkowski M., Kosowsky A., Stebbins A., 1997, Phys. Rev. Lett., 78, 2054

Lewis A., Challinor A., Lasenby A., 2000, ApJ, 538, 473

Linde A. D., 1982, Phys. Lett. B, 108, 389

Lieb A. S., 2016, Phys. Lett. B, 755, 185

LiteBIRD Collaboration 2022, preprint (arXiv:2202.02773)

Mangilli A., Plaszczynski S., Tristram M., 2015, Mon. Not. Roy. Astron. Soc., 453, 3174

Moncelsi L., et al., 2020, Proc. SPIE Int. Soc. Opt. Eng., 11453, 1145314

Planck Collaboration Int. XVI 2014, Astron. Astrophys., 566, A54

Planck Collaboration V 2020, Astron. Astrophys., 641, A5

Planck Collaboration VI 2020, Astron. Astrophys., 641, A6

Planck Collaboration VIII 2020, Astron. Astrophys., 641, A8

Planck Collaboration XI 2016, Astron. Astrophys., 594, A11

Planck collaboration XV 2014, Astron. Astrophys., 571, A15

Sato K., 1981, Mon. Not. Roy. Astron. Soc., 195, 467

Seljak U., Zaldarriaga M., 1997, Phys. Rev. Lett., 78, 2054

Sellentin E., Heavens A. F., 2016, Mon. Not. Roy. Astron. Soc., 456, L132

Starobinsky A. A., 1979, JETP Lett., 30, 682

Torrado J., Lewis A., 2021, JCAP, 05, 057

Tristram M., et al., 2021, Astron. Astrophys., 647, A128

Tristram M., et al., 2022, Phys. Rev. D, 105, 083524

Trotta R., 2017, preprint (arXiv:1701.01467)

Virtanen P., et al., 2020, Nature Methods, 17, 261

Zyla P. A., et al., 2020, PTEP, 2020, 083C01

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| Parameter                  | Type                  | Best-fit (free $\beta_i$) | Best-fit (fixed $\beta_i$) |
|----------------------------|-----------------------|---------------------------|----------------------------|
| Cosmological parameters    |                       |                           |                            |
| $r$                       | profile MLE           | –0.0021                   | 0.015                      |
| $\theta_{BC}$             | free                  | 0.104062                  | 0.0104062                  |
| $\log(10^{10}A_s)$        | free                  | 3.054                     | 3.054                      |
| $n_s$                     | free                  | 0.966                     | 0.967                      |
| $\Omega_b h^2$            | free                  | 0.0223                    | 0.0223                     |
| $\Omega_c h^2$            | free                  | 0.119                     | 0.119                      |
| $\tau$                    | free                  | 0.0603                    | 0.0604                     |
| $A_s$                     | derived               | $2.12 \times 10^{-9}$    | $2.12 \times 10^{-9}$      |
| $H_0$                     | derived               | 67.43                     | 67.43                      |
| $\sigma_8$                | derived               | 0.813                     | 0.813                      |
| BK18 nuisance parameters  |                       |                           |                            |
| $\beta_6$                 | free or fixed         | –2.0 (free)               | –3.1 (fixed)               |
| $A_d$                     | free                  | 4.433                     | 4.397                      |
| $A_{sync}$                | free                  | 0.17                      | 0.517                      |
| $\alpha_d$                | free                  | –0.641                    | –0.657                     |
| $\alpha_s$                | free                  | –1.9 $\times 10^{-7}$    | –4.3 $\times 10^{-6}$      |
| $\beta_d$                 | free                  | 1.500                     | 1.484                      |
| $e$                       | free                  | 0.03                      | –0.131                     |
| Planck nuisance parameters |                       |                           |                            |
| $A_{pl}$                  | free (HiLLiPoP, lowlTT, lensing) | 1.00189                   | 1.00192                    |
| $c_{10}(100A)$            | free (HiLLiPoP)       | $3.85 \times 10^{-3}$    | $3.86 \times 10^{-3}$      |
| $c_{10}(100B)$            | free (HiLLiPoP)       | –$1.0053 \times 10^{-2}$ | –$1.0053 \times 10^{-2}$   |
| $c_{14}(143B)$            | free (HiLLiPoP)       | –$1.0031 \times 10^{-2}$ | –$1.0026 \times 10^{-2}$   |
| $c_{21}(217A)$            | free (HiLLiPoP)       | –$1.0053 \times 10^{-2}$ | –$1.0053 \times 10^{-2}$   |
| $c_{21}(217B)$            | free (HiLLiPoP)       | –$4.419 \times 10^{-3}$  | –$4.417 \times 10^{-3}$    |
| $A^P_{PS}(100 \times 100)$ | free (HiLLiPoP)       | $2.620 \times 10^{10}$   | $2.619 \times 10^{10}$     |
| $A^P_{PS}(100 \times 143)$ | free (HiLLiPoP)       | $1.245 \times 10^{10}$   | $1.244 \times 10^{10}$     |
| $A^P_{PS}(100 \times 217)$ | free (HiLLiPoP)       | 84.71                     | 84.65                      |
| $A^P_{PS}(143 \times 143)$ | free (HiLLiPoP)       | 53.09                     | 53.05                      |
| $A^P_{PS}(143 \times 217)$ | free (HiLLiPoP)       | 37.70                     | 37.67                      |
| $A^P_{PS}(217 \times 217)$ | free (HiLLiPoP)       | 74.39                     | 74.41                      |
| $A_{100}$                 | free (HiLLiPoP)       | 1.694 $\times 10^{-2}$   | 1.688 $\times 10^{-2}$     |
| $A_{143}$                 | free (HiLLiPoP)       | 3.966 $\times 10^{-2}$   | 3.963 $\times 10^{-2}$     |
| $A_{217}$                 | free (HiLLiPoP)       | 0.1322                    | 0.1322                     |
| $A_{SZ}$                  | free (HiLLiPoP)       | 1.059                     | 1.048                      |
| $A_{CIB}$                 | free                  | 1.056                     | 1.055                      |
| $A_{SZ\mathrm{CIB}}$      | free (HiLLiPoP)       | 7.722 $\times 10^{-5}$   | 2.723 $\times 10^{-5}$     |
| $A_{SZ\mathrm{CIB}}$      | free (HiLLiPoP)       | 3.961 $\times 10^{-5}$   | 3.822 $\times 10^{-6}$     |
| $c_{14}(143A)$            | fixed (HiLLiPoP)      | 0.0                       | 0.0                        |
| $A^P_{\mathrm{radio}}$    | fixed (HiLLiPoP)      | 0.0                       | 0.0                        |
| $A_{\mathrm{M}^2T}$       | fixed (HiLLiPoP)      | 0.0                       | 0.0                        |
| $\chi^2_{\text{tot}}$    | derived (HiLLiPoP)    | 1.694 $\times 10^{-2}$   | 1.688 $\times 10^{-2}$     |
| $\chi^2_{\text{M}^2T}$   | derived (HiLLiPoP)    | 3.966 $\times 10^{-2}$   | 3.963 $\times 10^{-2}$     |
| $\chi^2_{\text{tot}}$    | derived (HiLLiPoP)    | 0.1322                    | 0.1322                     |
| $\chi^2_{\text{tot}}$    | derived (HiLLiPoP)    | 1.694 $\times 10^{-2}$   | 1.688 $\times 10^{-2}$     |
| $\chi^2_{\text{tot}}$    | derived (HiLLiPoP)    | 3.966 $\times 10^{-2}$   | 3.963 $\times 10^{-2}$     |
| $\chi^2_{\text{tot}}$    | derived (HiLLiPoP)    | 0.1322                    | 0.1322                     |

Table A1. Best-fitting parameters obtained from the combination Planck + BK18 + BAO + lensing with the conditioned HL covariance matrix, fitting (“free $\beta_i$” column) or fixing (“fixed $\beta_i$” column) $\beta_i$ in the BK18 likelihood.