As a consequence of the theory, while exhibiting an action of this result is extremely useful: it is in general very difficult to directly equip and proves that it is an equivalence, where A categorified operators on an affine DG scheme. The right hand side of this equivalence consists of the category of and proves that it is an equivalence, where J. Bernstein "categorified adjunction The main theorem of this paper is a 1-affineness result that constructs a (localisation, global sections)

Theorem via a restriction of scalars argument along the coefficient systems

The author then proves smooth descent for ShvCat

The 1-affineness theorem is much broader in scope than its “0-affine” analogue: for instance, all schemes are 1-affine.

The main theorem of this paper is a 1-affineness result that constructs a (localisation, global sections) adjunction

\[
\text{Loc}^H_Y : \mathbb{H}(Y) \cdot \text{-mod} \leftrightarrows \text{ShvCat}^H_Y : \Gamma_Y^H
\]

and proves that it is an equivalence, where \(\mathbb{H}(Y)\) is the categorical counterpart of the algebra of differential operators on an affine DG scheme. The right hand side of this equivalence consists of the category of categorified D-modules on \(Y\), or sheaves of categories over \(Y\) with local actions of Hochschild cochains. This “\(H\)-affineness theorem” can be considered a categorical version of a scheme being D-affine [A. Beilinson and J. Bernstein, C. R. Acad. Sci., Paris, Sér. I 292, 15–18 (1981; Zbl 0476.14019)], and much as with the usual 1-affineness theorem there are many more \(H\)-affine objects than D-affine objects.

As a consequence of the \(H\)-affineness theorem, the author is further able to prove that if \(C\) is a left \(\mathbb{H}(Y)\)-module over a quasi-smooth stack \(Y\), then \(C\) is equipped with a singular support theory relative to Sing \(\mathbb{H}(Y)\) [D. Arinkin and D. Gaitsgory, Sel. Math., New Ser. 21, No. 1, 1–199 (2015; Zbl 1423.14085)]. This result is extremely useful: it is in general very difficult to directly equip \(C\) with a singular support theory, while exhibiting an action of \(\mathbb{H}(Y)\) on \(C\) can be much easier in practice.

To provide a concrete application of this result to the geometric Langlands program the author lays out a proposal for how to prove that the action of the renormalised spherical Hecke category on \(D\cdot\text{-mod}(\text{Bun}_G)\) via Hecke functors factors through an action of \(\mathbb{H}(\text{LocSys}_G)\). This then equips \(D\cdot\text{-mod}(\text{Bun}_G)\) with a singular support theory relative to \(\text{Sing}(\text{LocSys}_G)\), independently of the geometric Langlands conjecture.

In order to prove the results of the paper, the author introduces the notion of a coefficient system. This is a functor \(A : \text{Alf} \to \text{Alg}^{\text{bimod}}(\text{DGCat})\), where the target category is the \(\infty\)-category whose objects are monoidal DG-categories and whose morphisms are bimodules. Such a functor allows one to define sheaves of categories with \(A\)-coefficients, i.e. a functor \(\text{ShvCat}^A : \text{PreStk}_{\text{Aff}} \to \text{Cat}_{\infty}\). Moreover, the collection of coefficient systems itself forms an \(\infty\)-category, and a map of coefficient systems \(A \to B\) induces a restriction of scalars natural transformation \(\text{ShvCat}^B \Rightarrow \text{ShvCat}^A\). Examples of coefficient systems are provided: in particular the assignment \(Y \rightsquigarrow \mathbb{H}(Y)\) is shown to enhance to a coefficient system \(\mathbb{H}\), as does the assignment \(Y \rightsquigarrow \text{Qcoh}(Y)\), which the author denotes by \(Q\).

The author then proves smooth descent for \(\text{ShvCat}^H_Y\), and uses this to construct the adjunction \(\text{Loc}^H_Y : \mathbb{H}(Y) \cdot \text{-mod} \leftrightarrows \text{ShvCat}^H_Y : \Gamma_Y^H\). Proof of the \(H\)-affineness theorem is reduced to Gaitsgory’s 1-affineness theorem via a restriction of scalars argument along the coefficient systems \(Q \to H\). The singular support statement then follows by considering an \(\mathbb{H}(Y)\)-module as a sheaf of \(\mathbb{H}\)-categories on \(Y\), and computing singular support smooth locally, where we are simply studying right modules for Hochschild cochains.

For those who are interested, further applications which are discussed include applications to the derived enhancement of the category of Harish-Chandra bimodules, and applications to the gluing theorems of geometric Langlands.

Reviewer: Richard Derryberry (Waterloo)
MSC:

14D24 Geometric Langlands program (algebro-geometric aspects)
13D03 (Co)homology of commutative rings and algebras (e.g., Hochschild, André-Quillen, cyclic, dihedral, etc.)
18F99 Categories in geometry and topology

Keywords:
Hochschild cochains; quasi-smooth stacks; derived algebraic geometry; ind-coherent sheaves; singular support; formal completions; Hecke functors; derived Satake

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