On some new soliton solutions of \((3 + 1)\)-dimensional Boiti–Leon–Manna–Pempinelli equation using two different methods

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ABSTRACT
In this article, a new solution for the \((3 + 1)\) dimensions Boiti–Leon–Manna–Pempinelli (BLMP) equation using the sine Gordon expansion method and the extended tanh function method is given. The methods were chosen very carefully due to the precision of their solutions. Also, some of the two- and three-dimensional figures and some of the contours plots of the obtained solutions were presented. Finally, a discussion of the results was given.

1. Introduction
Nonlinear partial differential equations (NPDEs) appear in modeling many phenomena in physics, engineering, biology, archeology, hydrodynamics, plasma physics, molecular biology, quantum mechanics, nonlinear optics, surface water waves and so on. Many equations of mathematical physics have the solutions of soliton type. Solitons exhibit the particle-like properties because the energy is at any instant confined to a limited region of space (Ghanbari & Nisar, 2020; Nisar et al., 2021; Zafar, Ali, Raheel, Jafar, & Nisar, 2020). With the development of soliton theory, many powerful methods for obtaining exact solutions of NLPDEs have been presented, such as homotopy perturbation method, nonperturbative method, homogeneous balance method, Backlund transformation, Darboux transformation, extended tanh-function method, extended F-expansion method, \(\varphi\) function method, exp-function method, sine–cosine method, Jacob elliptic function method, extended Riccati equation rational expansion method, extended auxiliary function method and other methods (Achab et al., 2020; Akbar, Alam, & Hafez, 2016; Alam, Hafez, Akbar, & Roshid, 2015; Ali Akbar, Ali, & Tarikul Islam, 2019; Duan & Lu, 2021; Hong & Lu, 2013; Islam, Akbar, & Khan, 2018; Kazi Sazzad Hossain & Ali Akbar, 2017; Khan & Akbar, 2014; Lu, 2018; Mahmud, Samshuzzoha, & Akbar, 2017; Mohyud-Din, Nawaz, Azhar, & Akbar, 2017; Nur Alam1, Ali Akbar, & Fazlul Hoque, 2014; Shafiqul Islam, Khan, Ali Akbar, & Mastroberardino, 2014). In this article, we used the sine Gordon expansion method and extended tanh function method to the \((3 + 1)\) dimensional Boiti–Leon–Manna–Pempinelli equation (BLMP) which is used to describe incompressible liquid in fluid mechanics (Wazwaz, 2019). The equation is given as:
\[
(u_x + u_y + u_z)_{t} + (u_x u_y + u_z)_{xx} + (u_x(u_y + u_z))_{x} = 0
\] (1)
which was derived by Boiti et al. when they researched a Korteweg–de Vries (KdV) equation through weak Lax pairs relations (Boiti, Leon, Manna, & Pempinelli, 1986). Many researchers discussed the BLMP equation (Darvishi, Najafi, Kavitha, & Venkatesh, 2012; Li & Ma, 2018; Liu, Du, Zeng, & Nie, 2017; Liu, Tian, & Hu, 2018; Mabrouk & Rashid, 2017; Osman & Wazwaz, 2019; Peng, Tian, & Zhang, 2019; Tang & Zai, 2015; Wu, Liu, Piao, Zhuang, & Wang, 2020; Xu, 2019; Zuo, Gao, Yu, Sun, & Xue, 2015).

The modified form of Eq. (1) is given by Wazwaz (see Liu & Wazwaz, 2021; Wazwaz, 2019). In this article, we will study the \((3 + 1)\) BLMP equation that was proposed by Wazwaz in this form:
\[
(u_x + u_y + u_z)_{t} + (u_x(u_y + u_z))_{xx} + (u_x(u_y + u_z))_{x} = 0.
\] (2)
This article is organized as follows: In Section 2, description of the extended tanh function method (Nuruddeen, Aboodh, & Ali, 2020; Shukri & Al-Khaled,
We consider a partial differential equation:

\[ P(u, u_t, u_x, u_y, u_{xx}, u_{xy}, u_{yy}, \ldots) = 0. \quad (3) \]

**Step 1.** Introduce the wave transformation:

\[ u(x, y, z, t) = U(\xi), \xi = ax + by + dz - ct, \quad (4) \]

where \( a, b, d \) are constants and \( c \) is the velocity of the traveling wave. By using Eq. (4) in Eq. (2), we get an ordinary differential equation of the form:

\[ F(U', U'', U''', \ldots) = 0. \quad (5) \]

**Step 2.** The modified extended tanh method present the wave solution of Eq. (5) in the form of the finite series:

\[ U(\xi) = a_0 + \sum_{i=1}^{N} (a_i \phi(\xi)^i + b_i \phi(\xi)^{-i}), \quad (6) \]

where \( \phi = \phi(\xi) \) is a solution of the Riccati equation of the form

\[ \frac{d\phi}{d\xi} = w + \phi^2. \quad (7) \]

The Riccati Eq. (7) has the general solutions:

If \( w < 0 \) then

\[ \phi(\xi) = -\sqrt{-w} \tanh(\sqrt{-w} \xi), \quad \phi(\xi) = -\sqrt{-w} \coth(\sqrt{-w} \xi). \quad (8) \]

If \( w = 0 \) then

\[ \phi(\xi) = -\frac{1}{\xi}. \quad (9) \]

If \( w > 0 \) then

\[ \phi(\xi) = \sqrt{w} \tan(\sqrt{w} \xi), \quad \phi(\xi) = -\sqrt{w} \cot(\sqrt{w} \xi). \quad (10) \]

**Step 3.** Obtaining \( N \) by balancing the highest order derivative term with the highest power nonlinear term in Eq. (5). Substituting Eqs. (7) and (6) into (5) and then set the coefficients of \( \phi(\xi)^i \), we get a system of algebraic equations for \( w, a_0, \ldots, a_N, b_1, \ldots, b_N \) and we solve this system to find all constants.

### 2.2. Sine Gordon expansion method

Consider the sine-Gordon equation

\[ u_{xx} - u_{tt} = m^2 \sin(u), \quad (11) \]

where \( u = u(x, t) \) and \( m \) is a constant. Use the wave transformation \( u(x, t) = U(\xi), \xi = x - ct \) in Eq. (11), we get the nonlinear ordinary differential equation:

\[ U'' = \frac{m^2}{(1 - c^2)} \sin(U), \quad (12) \]

where \( U = U(\xi) \), \( \xi \) and \( c \) are the amplitude and velocity of the traveling waves. By integrating once and put the constant of integral equal to zero, we get:

\[ \left[ \frac{U'}{2} \right]^2 = \frac{m^2}{(1 - c^2)} \sin^2 \left( \frac{U}{2} \right), \quad (13) \]

let \( w(\xi) = \frac{U}{2} \) and \( a^2 = \frac{m^2}{(1 - c^2)} \), so Eq. (13) becomes:

\[ w' = a \sin(w). \quad (14) \]

Set \( a = 1 \) in Eq. (14), we get:

\[ w' = \sin(w). \quad (15) \]

Solving Eq. (15), we obtain the two significant equations as:

\[ \sin(w) = \sin(w(\xi)) = \frac{2pe^{i\xi}}{p^2e^{2i\xi} + 1} \bigg|_{p=1} = \text{sech}(\xi), \quad (16) \]

\[ \cos(w) = \cos(w(\xi)) = \frac{p^2e^{2i\xi} - 1}{p^2e^{2i\xi} + 1} \bigg|_{p=1} = -\tanh(\xi), \quad (17) \]

where \( p \) is the integral constant and non-zero.

We consider the solution of Eq. (5), which can be expressed as:

\[ U(\xi) = A_0 + \sum_{i=1}^{N} ((-\tanh(\xi))^{-i} (B_i \text{sech}(\xi) - A_i \tanh(\xi)). \quad (18) \]

Using Eqs. (16) and (17), we get:

\[ U(w) = A_0 + \sum_{i=1}^{N} (\cos i^{-1}(w)) (B_i \sin(w) + A_i \cos(w)). \quad (19) \]

We applied the balance principle to determine the value of \( N \) as we did in the previous method. We put the summation of coefficients of \( \sin^i(w) \cos^i(w) \) with the same power equal to zero, we get a system of algebraic equations, which can be solved using Mathematica program.

### 3. Mathematical analysis of the model and its solutions

Applying the transformation (4) in Eq. (2), then the partial differential equation is Blackced to the
following ordinary differential equation:
\[(a + b + d)((-c + 2a^2U'(\xi))U'(\xi) + a^3U''(\xi)) = 0. \tag{20}\]

Integrating it once and set the integration constant equal zero, we get:
\[-CU'(\xi) + a^2(U'(\xi))^2 + a^3U''(\xi) = 0. \tag{21}\]

Balancing \(U''(\xi)\) with \((U'(\xi))^2\) in Eq. (21) we get \(N + 3 = 2N + 2\), then \(N = 1\).

### 3.1. Solution with the extended tanh function method

For the value \(N = 1\), the solution of (21) can be written in the form:
\[U(\xi) = a_0 + a_1\phi(\xi) + \frac{b_1}{\phi(\xi)}. \tag{22}\]

Substituting Eq. (22) into (21) and using Eq. (7), collecting the coefficients of \(\phi'(\xi)\), we obtain the following system:
- Coefficients of \(\phi^k\): 
  - \(6a^3a_1 + a^2a_1^2\),
  - \(-ca_1 + 8a^3wa_1 + 2a^2wa_1^2 - 2a^2a_1b_1\),
  - \(-ca_1 + 2a^3wa_1 + a^2wa_1^2 + ca_1 - 2a^3wb_1 + 4a^2wa_1b_1 + a^2b_1^2\),
  - \(cw_1 - 8a^3wb_1^2 - 2a^2wa_1b_1 + 2a^2wb_1^2\),
  - \(-6a^3wb_1 + a^2wa_1^2b_1^2\).

Put these coefficients equal to zero, and solving the system by the aid of Mathematica with Eqs. (22) and (4) we get more than one solution of Eq. (2) as follows:

**Case 1:**
\[a_1 = 0, b_1 = -\frac{3c}{2a^3}, w = -\frac{c}{4a^3}. \tag{24}\]
\[U(\xi) = (a_0 + a_1(-\sqrt{-c}coth(\sqrt{-c}\xi))) + b_1(-\sqrt{-c}coth(\sqrt{-c}\xi))^{-1}\]
\[u(x, y, z, t) = a_0 + 3a\sqrt{\frac{c}{a^3}}\tanh\left(\frac{1}{2}\sqrt{\frac{c}{a^3}}(ax + by + dz - ct)\right). \tag{25}\]

**Case 2:**
\[a_1 = -6a, b_1 = 0, w = -\frac{c}{4a^3}. \tag{26}\]
\[U(\xi) = (a_0 + a_1(-\sqrt{-c}tanh(\sqrt{-c}\xi))) + b_1(-\sqrt{-c}tanh(\sqrt{-c}\xi))^{-1}\]
\[u(x, y, z, t) = a_0 + 3a\sqrt{\frac{c}{a^3}}\tanh\left(\frac{1}{2}\sqrt{\frac{c}{a^3}}(ax + by + dz - ct)\right). \tag{27}\]

**Case 3:**
\[a_1 = -6a, b_1 = -\frac{3c}{8a^2}, w = -\frac{c}{16a^2}. \tag{28}\]
\[U(\xi) = (a_0 + a_1(-\sqrt{-c}coth(\sqrt{-c}\xi))) + b_1(-\sqrt{-c}coth(\sqrt{-c}\xi))^{-1}\]
\[u(x, y, z, t) = a_0 + \frac{3a}{2}\sqrt{\frac{c}{a^3}}\coth\left(\frac{1}{2}\sqrt{\frac{c}{a^3}}(ax + by + dz - ct)\right) + \frac{3a}{2}\sqrt{\frac{c}{a^3}}\tanh\left(\frac{1}{4}\sqrt{\frac{c}{a^3}}(ax + by + dz - ct)\right). \tag{29}\]

**Case 4:**
\[a_1 = 0, b_1 = -\frac{c}{a^2}, w = 0. \tag{30}\]
\[U(\xi) = a_0 + a_1\left\{-\frac{1}{\xi}\right\} - b_1(\xi)\]
\[u(x, y, z, t) = a_0 + \frac{c(ax + by + dz - ct)}{a^2}. \tag{31}\]

### 3.2. Solution with sine Gordon expansion method

For the value \(N = 1\), Eq. (19) takes the form,
\[U(w) = A_0 + B_1 \sin(w) + A_1 \cos(w), \tag{32}\]
substituting from Eq. (32) into the ordinary differential equation [Eq. (21)], we equate to zero the coefficients of the same power of the trigonometric functions, so we get the following algebraic system of equations:
\[cA_1 - 4a^3 + a^2B_1^2 = 0, \]
\[2a^1A_1 + a^2A_1^2 + 4a^1A_1 - a^2B_1^2 = 0, \]
\[-cB_1 + a^2B_1 = 0, \]
\[-5a^3B_1 - 2a^2A_1B_1 - a^2B_1 = 0. \tag{33}\]

By solving above system by the aid of Mathematica with Eqs. (32), (18) and (4), we get more than one solution of Eq. (2) as follows:

**Case 1:**
\[A_1 = 3(-1)^\frac{1}{2}c^1, B_1 = -3(-1)^\frac{1}{2}c^1, a = (-1)^\frac{1}{2}c^1. \tag{34}\]
\[u(x, y, z, t) = A_0 - 3(-1)^\frac{1}{2}c^1\text{sech}\left((-1)^\frac{1}{2}c^1x - by - dz + ct\right) + 3(-1)^\frac{1}{2}c^1 \times \tanh\left((-1)^\frac{1}{2}c^1x - by - dz + ct\right). \tag{35}\]

**Case 2:**
\[A_1 = 3(-1)^\frac{1}{2}c^1, B_1 = 3(-1)^\frac{1}{2}c^1, a = (-1)^\frac{1}{2}c^1. \tag{36}\]
Figure 1. Graph of case 1 using the extended tanh method at $c = 0.05, a = 0.1, b = 0.01, d = 0.1, a_0 = 0.1, y = 0.5, z = 0.5.$

\[
\begin{align*}
  u(x, y, z, t) & = A_0 + 3(-1)^{3/2}c\text{sech}\left((-1)^{3/2}c x - by - dz + ct\right) \\
  & + 3(-1)^{1/2}c^2 \times \tanh\left((-1)^{3/2}c x - by - dz + ct\right). \\
\end{align*}
\]

**Case 3:**
\[
\begin{align*}
  A_1 & = -3(-1)^{3/2}c, B_1 = -3(-1)^{3/2}c, a = (-1)^{3/2}c^2. \\
  u(x, y, z, t) & = A_0 - 3(-1)^{3/2}c\text{sech}\left((-1)^{3/2}c x - by - dz + ct\right) \\
  & - 3(-1)^{3/2}c^2 \times \tanh\left((-1)^{3/2}c x - by - dz + ct\right). \\
\end{align*}
\]

**Case 4:**
\[
\begin{align*}
  A_1 & = -3(-1)^{3/2}c, B_1 = 3(-1)^{3/2}c, a = (-1)^{3/2}c^2. \\
  u(x, y, z, t) & = A_0 + 3(-1)^{3/2}c\text{sech}\left((-1)^{3/2}c x - by - dz + ct\right) \\
  & - 3(-1)^{3/2}c^2 \times \tanh\left((-1)^{3/2}c x - by - dz + ct\right). \\
\end{align*}
\]

**Case 5:**
\[
\begin{align*}
  A_1 & = -3(2)^{1/2}c, B_1 = 0, a = \frac{c^1}{2^3}. \\
  u(x, y, z, t) & = A_0 - 3(2)^{1/2}c\text{tanh}\left(\frac{c^1}{2^3} x - by - dz + ct\right). \\
\end{align*}
\]

**Case 6:**
\[
\begin{align*}
  A_1 & = 3(-2)^{1/2}c, B_1 = 0, a = -\frac{(-1)^{1/2}c^1}{2^3}. \\
  u(x, y, z, t) & = A_0 + 3(-2)^{1/2}c\text{tanh}\left(\frac{(-1)^{1/2}c^1}{2^3} x - by - dz + ct\right). \\
\end{align*}
\]

**Case 7:**
\[
\begin{align*}
  A_1 & = -3(-1)^{3/2}c, B_1 = 0, a = \frac{(-1)^{3/2}c^1}{2^3}. \\
  u(x, y, z, t) & = A_0 - 3(-1)^{3/2}c\text{tanh}\left(\frac{(-1)^{3/2}c^1}{2^3} x - by - dz + ct\right). \\
\end{align*}
\]
Figure 2. Graph of case 2 using the extended tanh method at $c = 0.1, a = 0.3, b = 0.5, d = 0.5, a_0 = 0.1, y = 0.5, z = 0.5$.

Figure 3. Graph of case 4 using the extended tanh method at $c = 0.2, a = 0.2, b = 2, d = 0.1, a_0 = 0.1, y = 0.5, z = 0.5$. 
Figure 4. Graph of case 1 using the sine Gordon expansion method at $c = 0.1, b = 0.4, d = 2, A_0 = 1, y = 0.5, z = 0.5$.

Figure 5. Graph of case 2 using the sine Gordon expansion method at $c = 0.1, b = 0.1, d = 0.1, A_0 = 1.5, y = 0.5, z = 0.5$. 
Figure 6. Graph of case 3 using the sine Gordon expansion method at $c = 0.1, b = 0.01, d = 0.01, A_0 = 2, y = 0.5, z = 0.5$.

Figure 7. Graph of case 4 using the sine Gordon expansion method at $c = 0.1, b = 2, d = 0.1, A_0 = 1.5, y = 0.5, z = 0.5$. 
4. Some graphical illustrations

Herein, we present some figures in the two-dimensional, three-dimensional and contours to illustrate the solutions that we have got. Some of the analytical solutions are presented in Figures 1–8, while the accuracy of the methods was compared to the solution given by Duan & Lu, (2021) and Lu (2018) as shown in Figures 1–8, respectively. In Figure 1, we introduce the graph of case 1 for Eq. (25) using the extended tanh method at $c = 0.05, a = 0.1, b = 0.01, d = 0.1, A_0 = 0.1, y = 0.5, z = 0.5$. Graph of case 2 for Eq. (27) using the extended tanh method at $c = 0.1, a = 0.3, b = 0.3, d = 0.5, A_0 = 0.1, y = 0.5, z = 0.5$ is presented in Figure 2, we introduce the graph of case 4 for Eq. (31) using the extended tanh method at $c = 0.2, a = 0.2, b = 2, d = 0.1, A_0 = 0.1, y = 0.5, z = 0.5$. In Figure 3, we present the graph of case 1 for Eq. (35) using the sine Gordon expansion method at $c = 0.1, b = 0.01, d = 0.02, A_0 = 1.5, y = 0.5, z = 0.5$ in Figure 4. Graph of case 2 for Eq. (37) using the sine Gordon expansion method at $c = 0.1, b = 0.01, d = 0.01, A_0 = 2, y = 0.5, z = 0.5$ is presented in Figure 5, we present the graph of case 3 for Eq. (39) using the sine Gordon expansion method at $c = 0.1, b = 2, d = 0.01, A_0 = 1.5, y = 0.5, z = 0.5$ in Figure 6. Graph of case 5 for Eq. (41) using the sine Gordon expansion method at $c = 0.05, b = 0.01, d = 0.4, A_0 = 1.5, y = 0.5, z = 0.5$ is presented in Figure 7. Finally, we give the graph of case 5 for Eq. (43) using the sine Gordon expansion method at $c = 0.05, b = 0.01, d = 0.4, A_0 = 1.5, y = 0.5, z = 0.5$ in Figure 8.

5. Conclusion

A new solution for the $(3 + 1)$ dimensions of BLMP equation by using the sine Gordon expansion method and the extended tanh function method have been presented. These methods have been chosen very carefully due to their accuracy and ease of application. The accuracy has tested from presented some figures in two- and three dimensions and some of the contours for solutions that we have obtained. In the end, we can say that we have made a clear contribution to finding solutions to the proposed equation, and these solutions are satisfactory. Note that the solutions we obtained have soliton waves characteristics, and this is evident in Figures 1–8 in that it keeps its shape over time.

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