MESON-PHOTON TRANSITION FORM FACTORS IN THE CHARMONIUM ENERGY RANGE

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The study of electron-positron collisions at center-of-mass energies corresponding to charmonium production has reached new levels of sensitivity thanks to experiments by the BES and CLEO Collaborations. Final states $\gamma P$, where $P$ is a pseudoscalar meson such as $\pi^0$, $\eta$, and $\eta'$ can arise either from charmonium decays or in the continuum through a virtual photon: $e^+e^- \rightarrow \gamma^* \rightarrow \gamma P$. Estimates of this latter process are given at center-of-mass (c.m.) energies corresponding to the $J/\psi(1S)$, $\psi(2S)$, and $\psi(3770)$ resonances.

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I. INTRODUCTION

The decays of charmonium states to a photon and a neutral pseudoscalar meson $P = (\pi^0, \eta, \eta')$ can shed light on a number of different mechanisms, including two-gluon couplings to $q \bar{q}$ states, vector meson dominance, mixing of heavy quarkonium with bound states of light quarks and antiquarks, and final-state radiation by light quarks [1]. In these final states there can be direct contributions from the continuum process

$$e^+e^- \rightarrow \gamma^* \rightarrow \gamma P,$$

(1)

where $P$ is a neutral pseudoscalar meson. The $\gamma^*\gamma P$ vertex was shown [2] to be characterized by a form factor $F(Q^2)$, where $Q^2 \equiv -q^2$ and $q$ is the four-momentum of the virtual photon $\gamma^*$, behaving as $F(Q^2) \rightarrow 2f_\pi/Q^2$. Here $f_\pi = 93$ MeV is the neutral-pion decay constant. In the present article we evaluate the cross sections for $P = \pi^0$, $\eta$, and $\eta'$ at $\sqrt{s} = 3097$, 3696, and 3773 MeV. These results are relevant to possible continuum backgrounds in searches for decays of $J/\psi$, $\psi(3686)$, and $\psi(3770)$ to $\gamma P$, currently undertaken by the CLEO Collaboration [1].

In Section II we calculate the cross section for the process (1), given the form factor estimate in Ref. [2]. We then estimate bounds on the actual cross section in Section III by comparing the expected result for $e^+e^- \rightarrow J/\psi \rightarrow \gamma \pi^0$ using vector dominance with the observed (slightly larger) cross section. We summarize in Section IV.

II. CONTINUUM $\gamma P$ PRODUCTION

The $\gamma^*\gamma P$ vertex was discussed in Ref. [2], where a proposal was made to test it in photon-photon collisions with one photon highly off-shell. In that case, the off-shell photon will have a spacelike $Q^2 > 0$. Experimental tests in this regime were indeed
The four-momentum of the pseudoscalar meson (here, $\pi^0$) and the polarization vector of the outgoing on-shell photon $e^0$ makes with the beam axis in the center-of-mass system, is

$$\Gamma_\mu = -ie^2 F_{\pi^0}(Q^2) \epsilon_{\mu\nu\rho\sigma} p^\nu e^\rho q^\sigma.$$  \hspace{1cm} (2)

Here the form factor $F_{\pi^0}(Q^2)$ is expected to behave for large $Q^2$ as $\pi^0$ is the polarization vector of the outgoing on-shell photon.

The differential cross section with respect to $\cos \theta$, where $\theta$ is the angle the outgoing photon makes with the beam axis in the $e^+e^-$ center-of-mass system, is

$$\frac{d\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma\pi^0)}{d(\cos \theta)} = \frac{\alpha^3}{3} \left( \frac{\pi f_\pi}{s} \right)^2 K_{\pi^0}^3 (1 + \cos^2 \theta), \hspace{1cm} K_F = 1 - \frac{M_P^2}{s},$$  \hspace{1cm} (4)

where $s = q^2$ is the square of the center-of-mass energy and $M_P$ is the mass of the pseudoscalar meson (here, $\pi^0$). Integrating with respect to $\cos \theta$, we find

$$\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma\pi^0) = \frac{8\alpha^3}{3} \left( \frac{\pi f_\pi}{s} \right)^2 K_{\pi^0}^3.$$

This may be compared with the cross section for muon pair production (neglecting $m_\mu$),

$$\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} = \frac{86.8 \text{ nb}}{s \text{ (GeV)}^2},$$  \hspace{1cm} (6)

which is 9.05 nb at $\sqrt{s} = 3.097 \text{ GeV}$. Thus

$$R_{\sigma}^{\pi^0}(s) = \frac{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma\pi^0)}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)} = \frac{2\pi\alpha f_\pi^2}{s} K_{\pi^0}^3$$

which is $4.11 \times 10^{-5}$ at $\sqrt{s} = 3.097 \text{ GeV}$. At this energy we thus have

$$\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma\pi^0) = 372 \text{ fb}.$$  \hspace{1cm} (8)

The ratio (7) can also be used to predict the virtual-photon contribution to the branching fraction $B(J/\psi \rightarrow \gamma\pi^0)$ in terms of $B(J/\psi \rightarrow \mu^+\mu^-)$:

$$R_{B(J/\psi)}^{\pi^0} = \frac{B(J/\psi \rightarrow \gamma^* \rightarrow \gamma\pi^0)}{B(J/\psi \rightarrow \gamma^* \rightarrow \mu^+\mu^-)} = R_{\sigma}^{\pi^0}(M_{J/\psi}^2).$$  \hspace{1cm} (9)

With $B(J/\psi \rightarrow \mu^+\mu^-)$ assumed to be equal to $B(J/\psi \rightarrow e^+e^-) = 5.94\%$, this implies $B(J/\psi \rightarrow \gamma^* \rightarrow \gamma\pi^0) = 2.44 \times 10^{-6}$, far below the observed value of $(3.3^{+0.6}_{-0.4}) \times 10^{-5}$. We shall see in the next section that the vector-dominance process $J/\psi \rightarrow \rho^0\pi^0 \rightarrow \gamma\pi^0$ accounts for most if not all of the observed branching fraction.
Whereas the asymptotic form factor \( F_{\pi^0}(Q^2) = 2f_{\pi}/Q^2 \) of Ref. \[2\] implies \( R_{B(J/\psi)}^{\pi^0} = 4.11 \times 10^{-5} \), a slightly larger value of \( R_{B(J/\psi)}^{\eta^0} = 10^{-4} \) is implied by the calculation of Ref. \[6\]. This would imply
\[
\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma\pi^0) = 905 \text{ fb} .
\]
for the continuum contribution at \( \sqrt{s} = 3.097 \text{ GeV} \).

We shall now scale the cross section estimates (8) and (10) to other energies and particle types. Let the quark model wave function of a neutral particle \( P \) be characterized by the sum of pairs \( q_i\bar{q}_i \) with coefficients \( c_i^P \):
\[
|P\rangle = \sum_i c_i^P |q_i\bar{q}_i\rangle .
\]
Then the ratio of the \( \gamma^* \rightarrow \gamma P \) form factor to that for \( \pi^0 \) is just
\[
F_P(Q^2)/F_{\pi^0}(Q^2) = \sum_i c_i^P Q_i^2 / \sum_i c_i^{\pi^0} Q_i^2 .
\]
The \( \eta \) and \( \eta' \) may be represented as octet-singlet mixtures,
\[
\eta = \cos \theta \eta_8 + \sin \theta \eta_1 , \quad \eta' = -\sin \theta \eta_8 + \cos \theta \eta_1 ,
\]
\[
\eta_8 \equiv (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6} , \quad \eta_1 \equiv (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{3} .
\]
An approximate form which fits the data well \[7\], and which we shall take in what follows, is
\[
\eta = (u\bar{u} + d\bar{d} - s\bar{s})/\sqrt{3} , \quad \eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6} ,
\]
corresponding to a mixing angle \( \theta = -\arcsin(1/3) = -19.5^\circ \). This implies
\[
|F_\eta(Q^2)/F_{\pi^0}(Q^2)|^2 = 32/27 , \quad |F_{\eta'}(Q^2)/F_{\pi^0}(Q^2)|^2 = 49/32 .
\]
(For comparison, the unmixed octet and singlet states give \(|F_{\eta_8}(Q^2)/F_{\pi^0}(Q^2)|^2 = 1/3 \), \(|F_{\eta_1}(Q^2)/F_{\pi^0}(Q^2)|^2 = 8/3 \).)

The above mixing ansatz implies
\[
\lim_{q^2 \rightarrow \infty} |q^2 F_{\eta}(Q^2)| = 2f_{\pi} \sqrt{\frac{32}{27}} = 202 \text{ MeV} ,
\]
\[
\lim_{q^2 \rightarrow \infty} |q^2 F_{\eta'}(Q^2)| = 2f_{\pi} \sqrt{\frac{49}{27}} = 251 \text{ MeV} ,
\]
in satisfactory agreement with the results from BaBar \[4\] at \( q^2 = 112 \text{ GeV}^2 \):
\[
|q^2 F_{\eta}(Q^2)| = 229 \pm 30 \pm 8 \text{ MeV} ,
\]
\[
|q^2 F_{\eta'}(Q^2)| = 251 \pm 19 \pm 8 \text{ MeV} .
\]
Other ansätze for \( \eta-\eta' \) mixing have been explored, for example, in Ref. \[8\]. The two-gluon components of \( \eta, \eta' \) are not considered here, but are treated in Ref. \[9\].

The kinematic factors \( K^3 \) are summarized for \( P = \pi^0 \), \( \eta, \eta' \) and c.m. energies corresponding to \( J/\psi, \psi(2S), \) and \( \psi(3770) \) in Table \[1\]. Also shown are the continuum cross sections at these c.m. energies. At 3.77 GeV, the predicted continuum cross sections for \( e^+e^- \rightarrow \gamma\eta(0) \) are \((0.19,0.25) \text{ pb} \), consistent with the values of \((0.17^{+0.05}_{-0.04} \pm 0.03, 0.21^{+0.07}_{-0.05} \pm 0.03) \text{ pb} \) observed by CLEO \[11\]. Thus, it is consistent to assume that the \( \gamma\eta \) and \( \gamma\eta' \) signals at 3.77 GeV come entirely from continuum. Upper bounds on \( B[\psi(3770) \rightarrow \eta\eta(0)] \)
Table I: Kinematic suppression factors $K^3 \equiv [1 - (M_P^2/s)]^3$ and continuum cross sections $\sigma(e^+e^- \to \gamma^* \to \gamma P)$ for neutral pseudoscalar mesons $P$.

|       | $J/\psi$ | $\psi(2S)$ | $\psi(3770)$ | $J/\psi$ | $\psi(2S)$ | $\psi(3770)$ | $\sigma$ (fb) |
|-------|----------|----------|-----------|---------|----------|-----------|------------|
| $\pi^0$ | 0.994    | 0.996    | 0.996     | 372     | 186      | 169       |            |
| $\eta$  | 0.909    | 0.935    | 0.938     | 403     | 207      | 189       |            |
| $\eta'$ | 0.740    | 0.811    | 0.819     | 502     | 274      | 252       |            |

Table II: Predicted ratios $R_B^P$ and $\gamma P$ branching ratios for quarkonium states decaying to $\gamma P$ via a virtual photon $\gamma^*$.

|       | $R_B^P$ ($10^{-5}$) | $B(\gamma P)$ |
|-------|---------------------|---------------|
| $P$   | $J/\psi$ | $\psi(2S)$ | $\psi(3770)$ | $J/\psi$ | $\psi(2S)$ | $\psi(3770)$ |
| $\pi^0$ | 4.11   | 2.91    | 2.78     | 2.44 x 10^{-6} | 2.19 x 10^{-7} | 2.7 x 10^{-10} |
| $\eta$  | 4.45   | 3.24    | 3.10     | 2.64 x 10^{-6} | 2.43 x 10^{-7} | 3.0 x 10^{-10} |
| $\eta'$ | 4.68   | 3.62    | 3.49     | 2.78 x 10^{-6} | 2.73 x 10^{-7} | 3.4 x 10^{-10} |

under various scenarios of interference between direct decay and interference are quoted in Ref. [1].

The ratios $R_B^P$ characterize the branching ratios of charmonium states to $\gamma P$ via the virtual photon $\gamma^*$. We summarize these ratios and the corresponding predicted contributions to quarkonium branching ratios in Table III. Here we have taken $[5]$ $B(J/\psi \to \mu^+\mu^-) = 5.94 \times 10^{-2}$, $B(\psi(2S) \to \mu^+\mu^-) = 7.52 \times 10^{-3}$, and $B(\psi(3770) \to \mu^+\mu^-) = 9.71 \times 10^{-7}$. Except for the case of $J/\psi \to \gamma\pi^0$, which we shall discuss further in the next section, these contributions are negligible.

All the estimates presented above were for the form factor $F_{\pi^0}(Q^2)$ behaving as $2 f_{\pi}/Q^2$ [2]. The calculation of Ref. [6] would give $R_B^{\pi^0} = 10^{-4}$ at the $J/\psi$, and hence all values approximately 2.43 times as large.

### III. COMPARISON WITH DATA AND VECTOR DOMINANCE

The observed branching ratios $[5]$ for $J/\psi$ and $\psi(2S)$ decaying to $\gamma P$ are summarized in Table III. The Particle Data Group averages $[5]$ include determinations by the BES Collaboration at the $J/\psi$ [11] but not the recent CLEO results [1]. As mentioned earlier, the single-virtual-photon process $J/\psi \to \gamma^* \to \gamma\pi^0$ is insufficient to account for the observed branching ratio.

We now show that most of not all of the observed $B(J/\psi \to \gamma\pi^0)$ can be accounted for by the vector dominance model (VDM). We follow the calculation of Ref. [6], in which

$$\frac{\Gamma(J/\psi \to \gamma\pi^0)_{\text{VDM}}}{\Gamma(J/\psi \to \rho^0\pi^0)} = \left(\frac{p_{\pi^0}^*}{p_{\rho^0\pi^0}^*}\right)^3 \left(\frac{ef_{\rho}}{m_{\rho}}\right)^2$$

(20)

Here the c.m. 3-momenta in $J/\psi \to \gamma\pi^0$ and $J/\psi \to \rho^0\pi^0$ are $p_{\pi^0}^* = 1546$ MeV/$c^2$ and
A very similar result is obtained from the expression

\[
\langle 2f_\rho \rangle = \sqrt{2} f_\pi \left[ \frac{B(\tau^- \to \nu_\tau \rho^-)}{B(\tau^- \to \nu_\tau \pi^-)} \right] \frac{m_\rho^2 - m_\pi^2}{m_\tau^2 - m_\rho^2} \frac{m_\tau}{\sqrt{m_\tau^2 + 2m_\rho^2}} \tag{21}
\]

where particle masses and branching fractions are taken from \[5\]. Then

\[
f_{\rho^0} = f_{\rho^\pm}/\sqrt{2} = 147.8 \pm 1.1 \text{ MeV} \tag{23}
\]

A very similar result is obtained from the expression

\[
\Gamma(\rho^0 \to e^+e^-) = \frac{4\pi\alpha^2}{3m_\rho} f_{\rho^0}^2 \tag{24}
\]

based on the matrix element \( \langle 0|J_{\mu}^{em}|\rho^0(q,e)\rangle = \epsilon_{\mu}m_\rho f_{\rho^0} \). With \( B(\rho^0 \to e^+e^-) = (4.71 \pm 0.5) \times 10^{-5} \), \( m_\rho = (775.49 \pm 0.34) \text{ MeV/c}^2 \), and \( \Gamma_\rho = (149.4 \pm 1.0) \text{ MeV} \), one finds \( f_{\rho^0} = (156 \pm 8) \text{ MeV} \).

We choose not to use the Particle Data Group average for \( B(J/\psi \to \rho\pi) \), but to average the three most recent determinations of this branching fraction, summarized in Table \[IV\]. Taking 1/3 of this average for the \( \rho^0\pi^0 \) final state, we have \( B(J/\psi \to \rho^0\pi^0) = (7.10 \pm 0.03) \times 10^{-3} \). Eq. \( (20) \) then implies

\[
B(J/\psi \to \gamma\pi^0)_{\text{VDM}} = (28.8 \pm 1.3) \times 10^{-6} \tag{25}
\]

This value is consistent with the observed branching fraction quoted in Table \[III\] but also allows for an additional contribution.

In Ref. \[6\], the relative signs of the VDM and virtual-photon (\( \gamma^* \)) contributions are predicted to be positive, so that we may place rather restricted upper bounds on the latter. We take a 90\% confidence level (c.l.) bound of \( B(J/\psi \to \gamma\pi^0) < 41 \times 10^{-6} \). The prediction of Ref. \[6\] was that \( B_{\gamma^*}(J/\psi \to \gamma\pi^0)/B(J/\psi \to e^+e^-) = 10^{-4} \), or (using \( B(J/\psi \to e^+e^-) = 5.94\% \)),

\[
B_{\gamma^*}(J/\psi \to \gamma\pi^0)_{\text{CZ}} = 5.94 \times 10^{-6} \tag{26}
\]

Let \( \lambda \) be the maximum allowed fraction of the CZ branching ratio. Then the CZ assumption of constructive interference between the VDM and virtual-photon contributions implies

\[
(\sqrt{28.8 \pm 1.3} + \sqrt{5.94 \lambda})^2 \leq 41 \tag{27}
\]
Table IV: Values of $\mathcal{B}(J/\psi \to \rho\pi)$ used in computing average, in percent.

| Source Reference | Value          |
|------------------|----------------|
| BES direct $J/\psi$ [12] | $2.184 \pm 0.005 \pm 0.201$ |
| BES $J/\psi$ from $\psi(2S)$ [12] | $2.091 \pm 0.021 \pm 0.116$ |
| BaBar radiative return [13] | $2.18 \pm 0.19$ |
| Average          | $2.13 \pm 0.09$ |

or, taking the lower limit of the VDM theoretical error, $\lambda < 0.23$. The CZ continuum cross section for $e^+e^- \to \gamma^* \to \gamma\pi^0$ at the $J/\psi$ c.m. energy, corresponding to $10^{-4}$ of that for muon pair production, is 905 fb, so the assumption of constructive interference between VDM and virtual photon contributions in $J/\psi \to \gamma\pi^0$ implies

$$\sigma(e^+e^- \to \gamma^* \to \gamma\pi^0) \leq 210 \text{ fb},$$

(28)
a bit more than half the value predicted in Table I from the form factor in Ref. [2]. The agreement between prediction and data for $\gamma\eta^{(')}$ production at 3.77 GeV suggests that not all of the predictions of Table I are subject to the same suppression.

It is quite possible that as $|q^2| \to \infty$, the asymptotic values of the relevant form factors are approached from below. This is indeed what is found for spacelike $q^2 < 0$ in singly-tagged photon-photon collisions where one photon is highly virtual [3]. Moreover, the corrections to the form factors in perturbative QCD are of the form $1 - [5\alpha_s(q^2)]/(3\pi)$ [14], easily entailing a suppression of the cross section by 25% or more.

IV. CONCLUSIONS

We have presented continuum cross sections for $e^+e^- \to \gamma^* \to \gamma P$, where $P$ is a neutral pseudoscalar meson $\pi^0$, $\eta$, or $\eta'$. Calculations are presented at c.m. energies corresponding to the masses of $J/\psi$, $\psi(2S)$, and $\psi(3770)$ for the $\gamma^*\gamma\pi^0$ form factor advocated by Brodsky and Lepage [2], and rescaled to the $\eta$ and $\eta'$ using their anticipated quark content and known kinematic factors.

It is shown that the contributions of the $\gamma^*\gamma P$ vertex to the branching fractions for the above-mentioned charmonium states to $\gamma P$ are negligible except in the case of $J/\psi \to \gamma\pi^0$, where the assumption of constructive interference with the VDM contribution permits rather stringent bounds to be placed, equivalent to continuum cross sections for $e^+e^- \to \gamma^* \to \gamma P$ a bit more than half those based on the asymptotic form factor behavior predicted in Ref. [2].

The cleanliness of the CLEO-c detector environment has permitted new studies of $\gamma P$ final states at the $J/\psi$, $\psi(2S)$, and $\psi(3770)$, with $\gamma\eta^{(')}$ signals at 3.77 GeV consistent with continuum production.

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