Signals of a Critical Behavior in Peripheral Au + Au Collisions at 35 MeV/nucleon

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(MULTICS - MINIBALL Collaboration)

Abstract

Multifragment events resulting from peripheral Au + Au collisions at 35 MeV/nucleon are analysed in terms of critical behavior. The analysis of most of criticality signals proposed so far (conditional moments of charge distributions, Campi scatter plot, fluctuations of the size of the largest fragment, intermittency analysis) is consistent with the occurrence of a critical behavior of the system.
Initiated by the observation of fragments in the final stages of the reaction exhibiting a power law in fragment charge distributions \cite{1}, and stimulated by the similarity of nuclear matter equation of state with that of a van der Waals gas \cite{2}, the possibility of observing a liquid-gas phase transition in nuclear systems has been the subject of intensive investigations (experimental and theoretical) for more than a decade \cite{3,4}. This interest increased recently with the attempt by the EOS Collaboration to extract the critical exponents of fragmenting nuclear systems produced in the collision of 1 GeV/nucleon Au nuclei with a carbon target \cite{5}, and with the extraction by the ALADIN Collaboration of the caloric curve resulting from the fragmentation of the quasiprojectile formed in the collision Au + Au at 600 MeV/nucleon which exhibits a behavior suggested for a liquid-gas phase transition \cite{6}.

In this contribution, we report the results of a recent experiment conducted by the MULTICS-MINIBALL Collaboration in which we studied the fragmentation (determined on an event-by-event basis) resulting from peripheral Au + Au collisions at E = 35 MeV/nucleon. The study of this reaction within the framework of Classical Molecular Dynamics (CMD) model indicates the possible occurrence of a critical behavior in peripheral collisions as reported in Ref. \cite{8,9}. Following this investigation the experimental data are analyzed in terms of critical behavior.

The experiment has been performed at the National Superconducting Cyclotron Laboratory of the Michigan State University using the coupled Multics-Miniball apparatus which has a geometric acceptance greater than 87% of $4\pi$. The Multics apparatus covered angles between 3 and 23 degrees in the laboratory frame and detected fragments with charge up to $Z = 83$, with an energy threshold of about 1.5 MeV/nucleon, independently of fragment charge \cite{10}. Charged particles with charge up to $Z = 20$ were detected at $23 \leq \theta_{\text{lab}} \leq 160$ by 159 phoswich detector elements of the MSU Miniball \cite{11} with energy thresholds of about 2, 3, and 4 MeV/nucleon for $Z = 3, 10, 18$, respectively.

Guided by the previously cited CMD calculations, semi-peripheral and peripheral events are identified selecting the events where the largest fragment has the velocity component along the beam direction greater than 75% of the beam velocity and the total detected charge between 70 and 90.

Figure 1 shows a scatter plot of $\ln(Z_{\text{max}}^{i})$ versus $\ln(M_{2}^{j})$ ("Campi scatter plot" \cite{12}) where $Z_{\text{max}}^{i}$ is the charge of the heaviest fragment and $M_{2}^{j}$ is the second conditional moment of the charge distribution detected in the $j$-th event,

$$M_{2}^{(j)} = \sum_{Z} Z^{2}n_{j}(Z)$$\hspace{1cm}(1)

Here, $n_{j}(Z)$ denotes the number of fragments of charge $Z$ detected in the $j$-th event, and the summation is over all fragments but the heaviest detected one. Theoretical investigations suggest that such plots may be useful in characterizing near-critical behavior of finite systems \cite{12}. The calculated Campi scatter plots typically exhibit two branches: an upper branch with a negative slope containing largely undercritical events (e.g. $T < T_{\text{crit}}$ in a liquid-gas phase transition or $p > p_{\text{crit}}$ in a percolation
phase transition) and a lower branch with a positive slope containing largely over-critical events \((T > T_{\text{crit}} \text{ or } p < p_{\text{crit}})\). The two branches were shown to meet close to the critical point of the phase transition \([3, 12, 13]\).

The data shown in Fig. 1, display two branches similar to the ones predicted for undercritical and overcritical events. In the top-right part, close to the intersection of these two branches (potentially containing near-critical events), a separate island is observed which is due to fission events, as first noted by Ref. \([13]\). By appropriate gates in the Campi plot, these fission events are removed from the following analysis.

To further investigate the two branches observed in Fig. 1 and the region where they intersect, we employ three cuts selecting the upper branch (cut 1), the lower branch (cut 3) and the intersection region (cut 2); these cuts are indicated in Fig. 1. The charged particle multiplicity distributions observed for these three cuts are shown as dashed histograms in the upper part of Fig. 2 together with the multiplicity distribution obtained for the totality of the selected events (solid histogram). Cuts 1 and 3 largely select low and high multiplicity events corresponding to very peripheral and more central collisions (assuming on the average a monotonic relation between \(N_c\) and impact parameter); cut 2 represents a wide range of charged particle multiplicities and thus may involve a wide range of intermediate impact parameters. Thus emission from a unique source cannot be ascertained for cut 2, and it is likely that this cut contains contributions from projectile and target-like sources and from the neck \([14]\) which emits lighter fragments with enhanced probability as compared to the projectile residue \([14]\). However, one cannot exclude that this large multiplicity distribution be related to the occurrence of large fluctuations as expected at the critical point. Moreover reducing the size of the region 2 does not change the large multiplicity distribution. In the lower part of Fig. 2 the multiplicity distribution given by cut 2 is reported as a solid line, while the dashed lines represents the multiplicity distribution obtained splitting this cut 2 in two equal parts in the vertical direction. No substantial change is noticed.

Fragment charge distributions, not corrected for detection efficiency, are presented \([15]\) for the three cuts in the upper panels of Fig. 3 (cut 1: left panel; cut 2: central panel and cut 3: right panel). Cut 1 contains light fragments and heavy residues and thus resembles the “U”-shaped distributions predicted by percolation calculations in the sub-critical region \([16]\). For cut 3, one observes an unusually flat charge distribution similar to the one previously reported \([17]\) for central collisions which were selected without the specific constraints employed in this paper and attributed to a Coulomb-driven breakup of a very heavy composite system \([17]\) (The steep fall-off at large \(Z\) is an artifact of the selection of events with \(Z = 70–90\) used in this paper). For cut 2, a fragment charge distribution is observed which resembles a power-law distribution, \(P(Z) \propto Z^{-\tau}\), with \(\tau \approx 2.2\). For macroscopic systems exhibiting a liquid-gas phase transition, such a power-law distribution is predicted to occur near the critical point \([18]\). However, it is not yet known by how much the final fragment distributions differ from the primary ones after the sequential decays of particle unstable primary fragments.
The lower panels of Fig. 3 show the logarithm of the Scaled Factorial Moments (SFM), defined as

\[
F_i(\delta s) = \frac{\sum_{k=1}^{Z_{tot}/\delta s} < n_k \cdot (n_k - 1) \cdot ... \cdot (n_k - i + 1) >}{\sum_{k=1}^{Z_{tot}/\delta s} < n_k >^i}
\]  
(2)

\(i = 2, ..., 5\), as a function of the logarithm of the bin size \(\delta s\). In the above definition of the SFM, \(Z_{tot} = 158\) and \(i\) is the order of the moment. The total interval \([1, Z_{tot}]\) is divided into \(M = Z_{tot}/\delta s\) bins of size \(\delta s\), \(n_k\) is the number of particles in the \(k\)-th bin for an event, and the brackets \(<\cdot\>\) denote the average over many events. A linear rise of the logarithm of the SFM versus \(\delta s\) (i.e. \(F_i \propto \delta s^{-\lambda_i}\)) indicates an intermittent pattern of fluctuations [13,19,20]. Even though this quantity is ill defined for fragment distributions [21–23], several theoretical studies have indicated that critical events give a power law for the SFM versus the bin size [5,20]. For cut 3 (right part of the figure), the logarithm of the scaled factorial moments \(ln(F_i)\) is always negative (i.e. the variances are smaller than Poissonian [20]) and almost independent on \(\delta s\); there is no intermittency signal. The situation is different for cut 2 (central part). The logarithm of the scaled factorial moments are positive and almost linearly increasing as a function of \(-ln(\delta s)\), and an intermittency signal is observed. Region 1, corresponding to evaporation, gives zero slope. Increasing or reducing the size of the three cuts in the respective regions does not change significantly these results. The interpretation of experimentally observed intermittency signals may, however, be problematic due to ensemble averaging effects [22], even though calculations show that impact parameter averaging only increases the absolute value of the SFM [23]. Since cut 2 may involve a large range of impact parameters, the observed intermittency signal could be an artifact of ensemble averaging and can, therefore, not be taken as a definitive proof of unusually large fluctuations in a sharply defined class of events.

In the upper part of Fig. 4, we have plotted the second moment \(M_2\) versus the multiplicity of charged particles \(N_c\). In a macroscopic thermal system, \(M_2\) is proportional to the isothermal compressibility which diverges at the critical temperature \([24,25]\). Of course, in finite systems, the moments remain finite due to finite size effects. The moment \(M_2(N_c)\) is averaged over all the events having the multiplicity \(N_c\), except those corresponding to fission (see above). One sees that the second moment shows a "peak" for a multiplicity \(N_c\) around 20-25. The EOS Collaboration has found a "critical" multiplicity of 26, somewhat higher than our result [3]. We stress, however, that, because for this experiment the Multics apparatus was set not to well detect \(Z<3\) charges, some particles (protons and alphas) are lost. Thus our results are not in contrast with those of EOS.

Another quantity introduced by Campi to characterize the critical behavior of a system is the relative variance \(\gamma_2\) defined as [21,20]:

\[
\gamma_2 = \frac{M_2M_0}{M_1^2}
\]  
(3)
It was shown by Campi that this quantity presents a peak around the critical point which means that the fluctuations in the fragment size distributions are large near the critical point \[21\]. The lower part of Fig. 4 shows the plot of \(\gamma_2\) versus charged particle multiplicity \(N_c\). This quantity shows a peak for \(N_c \approx 20 - 25\), consistent with that observed for \(M_2\).

The upper panel of Fig. 5 shows the second moment of charge distribution \(M_2\) calculated according to Eq. (1) by excluding the largest fragment (solid line) and including it as suggested by Stauffer and Aharony \[20\]. Physically, \(Z_{\text{max}}\) corresponds to the bulk in the liquid region and thus should be subtracted from the calculation of the moments, but not in the gas region where the bulk no more exists. This plot must be compared with the lower panel of the figure which reports the results of simulations done by Bauer and Botvina \[27\] for the reaction Au + C at 1 GeV/nucleon based on INC+SMM model and on INC+Percolation models. This figure reports also the experimental data of the EOS collaboration (open circles). See Ref. \[27\] for details. The shape of this quantity resembles that of our experimental data. One notes that our data show the trend very similar to that of the EOS data and the simulations.

Another quantity recently proposed so far as a signal for criticality is the normalized variance of the charge of the largest fragment \(\sigma_{NV}\). This quantity, defined by:

\[
\sigma_{NV} = \frac{\sigma_{Z_{\text{max}}}^2}{<Z_{\text{max}}^2>}
\]  

(4)

where

\[
\sigma_{Z_{\text{max}}}^2 = <Z_{\text{max}}^2> - <Z_{\text{max}}^2>
\]  

(5)

(the brackets \(<\cdot\>\) indicate an ensemble averaging) shows a peak at the critical point, where charge distributions are expected to show the largest fluctuations. Figure 6 shows in the upper part the size of the largest fragment, and in the lower part the \(\sigma_{NV}\) for our data versus charged particle multiplicity. This plot shows a clear peak for multiplicities around \(N_c = 20\) as expected around the critical point, indicating that charge distributions show the largest of fluctuations at this multiplicity.

In conclusion, we have analyzed fragment production in Au + Au collisions at \(E = 35\) MeV/nucleon. Events were selected by requiring a total detected charge between 70 and 90 and the velocity of the largest detected fragment larger than 75% of the projectile velocity. A Campi scatter plot of these events displays two branches similar to the sub- and overcritical branches observed in theoretical studies. The selection of events from the intersection of these two branches (which has been associated with critical events in theoretical studies) shows a power law charge distribution with an exponent of \(\tau \approx 2.2\) similar to that characterizing the mass distribution near the critical point of a liquid-gas transition. These events, further, display an intermittent behavior similar to that expected for near-critical events. Moreover, the second moment of fragment charge distribution \(M_2\), the relative variance \(\gamma_2\) and the normalized variance of the size of the largest fragment show peaks
indicating large fluctuations for multiplicities around $N_c = 20$, as expected near the critical point. While these signatures have been associated with near-critical events, we must caution that the effects of finite experimental acceptance and event mixing with possible contributions from the decay of projectile-like fragments and the neck-region are not yet sufficiently well understood to allow an unambiguous conclusion of critical behavior in the present reaction. Our work does, however, show that different regions of the nuclear phase diagram can be probed at one incident beam energy by selecting events according to different impact parameters and/or energy depositions.
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FIGURES

FIG. 1. Campi scatter plot, $ln(Z_{max})$ versus $ln(M_2)$. The three different regions are discussed in the text. Fission events are to the right of region 2.

FIG. 2. Upper panel: Multiplicity distribution of the events selected for the analysis (solid histogram). The three dashed histograms (1, 2 and 3) represent the multiplicity distributions of the events falling in the three cuts drawn in Fig. 1. Lower panel: Multiplicity distribution for cut 2 in Fig. 1 (solid histogram). Dashed histograms show $N_c$-distributions when this cut 2 is split vertically in two equal parts.

FIG. 3. Charge distributions and the corresponding scaled factorial moments $ln(F_i)$ versus $-ln(\delta s)$ for the events falling in the three cuts in the Campi scatter plot; the left part of the figure corresponds to cut 1, the central part to cut 2 and the right part to cut 3. The line in the upper central part represents the power law $Z^{-\tau}$ with $\tau = 2.2$. In the lower part, solid points represent the SFM of order $i = 2$, open circles $i = 3$, open squares $i = 4$, and open triangles $i = 5$.

FIG. 4. Upper panel: Second moment $M_2$ of charge distribution versus multiplicity $N_c$. Lower panel: Relative variance $\gamma_2$.

FIG. 5. Upper panel: Second moment $M_2$ of charge distribution versus multiplicity distribution calculated omitting the biggest fragment (lower curve) and taking it into account (upper curve). Lower panel: Same as upper panel. This figure is taken from Ref. [27]. Open circles represent experimental data of the EOS Collaboration [6]. For the description of solid lines and histograms in the lower panel see Ref. [27].

FIG. 6. Size of the largest fragment (upper panel), and Normalized variance of the charge of the biggest fragment $\sigma_{NV}$ (lower panel) versus multiplicity of charge particles $N_c$. 