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GRACE’s Implication of Temporal Inertia Moment and Length of Day

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Abstract  This paper aims to study the Earth’s temporal inertia moment and its influence on length of day (LOD). First, the GRACE data are processed by wavelet analysis to remove abnormal jumps and noises. Then the theoretical impacts of the second-order potential coefficients on the inertia moment and LOD are studied. Finally, the processed GRACE data are applied and results show that mass redistribution has led to decreasing tendencies of inertia moment and LOD as well as some unexpected seasonal oscillations in the recent 6 years.

Keywords  GRACE; second-order potential coefficients; wavelet analysis; Earth’s inertia moment; variations in LOD

Introduction

Due to the mass migration and coupling effects within the Earth as well as the interactions between the Earth and astronomical objects, both the spin rate and the orientation of the spin axis of the Earth vary in multiple time-scales. In other words, the Earth’s rotation is a process of great complexity. Theoretical studies of the Earth’s rotation are based on the application of the Liouville equations, and all the causes perturbing the rotation state, in general, falls into three categories, namely mass redistribution, relative motions, and torques. Mass redistribution, or sometimes called mass term, is directly related with the Earth’s gravity field and was once difficult to be obtained[1,2]. However, the development of high-accuracy satellite gravimetry, such as CHAMP and GRACE, has provided us the possibility to access the Earth’s temporal gravity and thus the variations of the inertia moment tensor which is relevant with the second-order potential coefficients[3-5]. Shen et al.[4] studied the secular variation in the Earth’s rotation and concluded that the tilt of the rotation axis increases $2.1 \times 10^{-8}$ mas/a while the rotational rate has a decrease of $1.0 \times 10^{-22}$ rad/s$^2$ based on the static gravity models EGM96 and EIGEN GL-04C.

Launched in March of 2002, the GRACE mission has accurately mapped variations in the Earth's gravity field and yielded crucial information about the distribution and flow of mass within the Earth and its surroundings[6]. One of the GRACE products is the serial gravity model with monthly interval (such as the GSM2-RL04 products), and thus we could estimate the variations of the temporal inertia tensor and length of day (LOD) with higher frequencies while only low frequency variations are concerned in the study of Shen et al.[4].
1 GRACE data

We adopt the GSM2-RL04 products, ranging from Aug. 2002 to Dec. 2007, provided by JPL (Jet Propulsion Laboratory)\[7\] to obtain the mass variations of the Earth system.

Considering the data noises caused by orbit adjustment and contaminations from other geophysical signals\[8-10\], errors in the data should be removed to obtain results with geophysical meanings. We process the fully-normalized potential coefficients $\overline{C}_{2,0}, \overline{C}_{2,1}, \overline{C}_{2,2}, \overline{S}_{2,2}$ of the monthly gravity models using wavelet analysis to obtain the processed coefficients $\overline{R}_{2,0}, \overline{R}_{2,1}, \overline{R}_{2,2}, \overline{S}_{2,2}$, and then compare our results with those of Cheng & Tapley\[8\], who had published a more accurate time serial of $\overline{C}_{2,0}$ (denoted by $\overline{C}_{2,0}^\ast$) from SLR (Satellite Laser Ranging) observations. These results and comparisons are shown in Fig.1, where the original GRACE data, the processed coefficients and the SLR data are plotted with green, blue and red lines respectively. One can see that the jumps and errors are greatly weakened and the processed data coincide with the SLR data much better after wavelet processing. It is noted that the jumps might be the noise caused by orbit- adjusting of the GRACE twin satellites, we can conclude that the processed data are more reliable to reflect the actual variations of gravity and mass distributions of the Earth (see the discussion in section 2).

![Fig.1 The second-order potential coefficients before (green line) and after (blue line) procession](image)

2 Earth’s temporal inertia moment tensor

The Earth’s inertia moment tensor is closely related with the second-order potential coefficients\[3,4\]. The solution of the principal moments of inertia can be expressed as\[3,4\]

$$\begin{align*}
A &= K(\varepsilon_1 + 2H\varepsilon_2) \\
B &= K[(1 - 2H)\varepsilon_1 + 2H\varepsilon_2] \\
C &= 3K\varepsilon_1
\end{align*}$$

where

$$K = -\frac{\sqrt{15}Ma^2}{6H}$$

$$\begin{align*}
\varepsilon_1 &= 2\sqrt{\frac{P}{3}\sin\left(\frac{\varphi + \pi}{3}\right)} \\
\varepsilon_2 &= -2\sqrt{\frac{P}{3}\sin\frac{\varphi}{3}} \\
\varepsilon_3 &= 2\sqrt{\frac{P}{3}\sin\left(\frac{\varphi - \pi}{3}\right)}
\end{align*}$$
\[
\begin{align*}
\mathbf{p} &= \mathbf{C}_{2,0} - \mathbf{C}_{2,1} + \mathbf{C}_{2,2} + \mathbf{S}_{2,2} \\
\mathbf{q} &= \frac{2\mathbf{C}_{2,0}}{3\sqrt{3}} + \mathbf{C}_{2,0} - 2\mathbf{C}_{2,2} - 2\mathbf{S}_{2,2} \\
&\quad + \mathbf{C}_{2,1}(\mathbf{C}_{2,1} - \mathbf{S}_{2,1}) + 2\mathbf{C}_{2,1}\mathbf{C}_{2,1} - \mathbf{C}_{2,2} - \mathbf{C}_{2,2} \\
\varphi &= \arcsin\left(\frac{3\sqrt{3}}{2} p^{-\frac{3}{2}} q\right) \\
&\quad \left(-\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}\right)
\end{align*}
\]

In Eqs. (1)-(4), \(M\) and \(a\) denote the Earth’s mass and semi-major axis, respectively, and \(H = \frac{C - (A + B)/2}{C}\) is the dynamic flattening of the Earth.

The orientations of the principal axes can be expressed as:
\[
\begin{align*}
\lambda_i &= \arctan\frac{m_i}{l_i}, \quad (i = A, B, C) \\
\theta_i &= \arctan\frac{n_i}{\sqrt{l_i^2 + m_i^2}}
\end{align*}
\]

where
\[
\begin{align*}
l_i &= \frac{n_i}{\mathbf{C}_{2,1}} \left(\mathbf{C}_{2,1} - \mathbf{S}_{2,1} u_i\right) \\
m_i &= u_i n_i \\
n_i &= \left[1 + u_i^2 + \left(\mathbf{S}_{2,1} u_i - \mathbf{S}_{2,1}^2 \right)\right]^{\frac{3}{2}}
\end{align*}
\]

In the principal axial coordinate system\(^{[4,5]}\), the Earth’s inertia moment tensor could be written as
\[
\mathbf{I} = \begin{bmatrix}
I_{11} & I_{12} & I_{13} \\
I_{21} & I_{22} & I_{23} \\
I_{31} & I_{32} & I_{33}
\end{bmatrix}
\]

while in the conventional Greenwich system it is
\[
\mathbf{I} = \begin{bmatrix}
c_{11} & c_{12} & c_{13} \\
c_{21} & c_{22} & c_{23} \\
c_{31} & c_{32} & c_{33}
\end{bmatrix}
\]

where \(\mathbf{I}\) is a symmetric tensor with \(I_{ij} = I_{ji}, c_{ij} = c_{ji}\) (\(i, j = 1, 2, 3\) and \(i \neq j\)). \(I_{ij}\) and \(c_{ij}\) (\(i, j = 1, 2, 3\)) are the constant and variable part of \(\mathbf{I}\), respectively. The transformation matrix between the two systems is
\[
\mathbf{P} = \begin{bmatrix}
\cos \lambda_i \cos \theta_i & \cos \lambda_i \sin \theta_i & -\sin \lambda_i \\
-\sin \lambda_i \cos \theta_i & \sin \lambda_i \sin \theta_i & \cos \lambda_i \\
\sin \lambda_i \sin \theta_i & -\cos \lambda_i \sin \theta_i & \sin \lambda_i \\
\end{bmatrix}
\]

Then according to the law of tensor transformation, \(\mathbf{I}\) might be accessed by
\[
\mathbf{I} = \mathbf{P}^{-1} \mathbf{I} = \mathbf{P}^{-1} \mathbf{I} 
\]

Based on the parameters listed in Table 1, the Earth’s principal axes and moments of inertia could be obtained (see Fig. 2 for their temporal values and Table 2 for their mean values). Then \(I_{ij}\) and \(c_{ij}\) can be obtained by Eqs. (10) and (11).

Cheng & Tapley\(^{[8]}\) concluded that the Earth’s actual mass redistributions cannot give rise to gravity variations as large as the original GRACE data shows, and the data must contain abnormal jumps and errors caused by orbit-adjusting and orbit-determination errors. However, the GRACE data can be ‘purified’ by wavelet analysis to match the high-quantity SLR observations (see section 1). Based on the processed data, the amplitude of variations became smaller and thus might be more reliable to reflect the Earth’s actual mass redistributions\(^{[8]}\).

### Table 1 Relevant geophysical parameters

| Parameter                      | Value                        | Variance ratio |
|--------------------------------|------------------------------|----------------|
| Geogravitational constant \(GM\)\(^{[7]}\) | \(3.986 004 415 \times 10^{14} \text{ m}^3 \cdot \text{s}^{-2}\) |                |
| Earth’s semi-major axis \(a\)\(^{[7]}\)       | \(6 378 136.46 \text{ m}\)   |                |
| Earth’s dynamic flattening \(H\) (Epoch 1997)\(^{[11]}\) | \((3 273 763.447 \pm 3.2) \times 10^{-5}\) | \((-7.864 \pm 0.270) \times 10^{-11}/\text{yr}\) |
| Gravitational constant \(G\)\(^{[11]}\)      | \((6 672.59 \pm 0.30) \times 10^{14} \text{ m}^3 \cdot \text{s}^{-2} \cdot \text{kg}^{-1}\) |                |
Table 2  The mean values of the principal axes and moments of inertia

|          | Mean(A) = 8.010 014×10^{37} kg•m² | Mean(θ_A) = 4°.318 008×10^{-5} | Mean(λ_A) = 14°.928 734 W |
|----------|-----------------------------------|---------------------------------|---------------------------|
|          | Mean(B) = 8.0101 90×10^{37} kg•m² | Mean(θ_B) = 9°.064 362×10^{-5}  | Mean(λ_B) = 75°.071 266 E  |
|          | Mean(C) = 8.0364 11×10^{37} kg•m² | Mean(θ_C) = 89°.999 899         | Mean(λ_C) = 78°.948 495 W  |

3  Associated fluctuation of LOD

The Earth’s rotation rate can be expressed as \( \omega = (1 + m_3) \Omega \), where \( \omega \) and \( \Omega \) are the instantaneous and mean rotation rates respectively. According to Lambeck (1980)[1]

\[
\dot{m}_3 = \psi_3
\]

where \( h_3, L_3 \) and

\[
\psi_3 = \frac{C_3}{I_{33}} - \frac{h_3}{I_{33}} + \Omega \int_0^t L_3 dt
\]

are the \( z \) components of relative angular moment, torque and excitation function, respectively. Only the first term of Eq.(13) is needed when concerning the effect of mass redistribution. Integrating Eq.(12) \( m_3^0 \) is the initial value of \( m_3 \), one gets

\[
m_3 = m_3^0 - \frac{C_3}{I_{33}}
\]

\( m_3 \) is directly related with variation of LOD[1]

\[
m_3 = -\frac{\Delta LOD}{LOD}
\]

where LOD = \( 2\pi/\Omega \) is the mean value of LOD. Thus

\[
\Delta LOD = -m_3 \text{ LOD}
\]

On the other hand, from Eqs.(8)–(11), one gets

\[
I_{33} + c_{33} = A \sin^2 \theta_c + C \cos^2 \theta_c
\]

Then one might get

\[
I_{33} = \text{mean}(A \sin^2 \theta_c + C \cos^2 \theta_c)
\]

\[
c_{33} = A \sin^2 \theta_c + C \cos^2 \theta_c - I_{33}
\]

Using the temporal principal axes and moments of inertia obtained in Section 3 and substituting Eq.(18) for Eqs.(14) and (16), the variation of length of day, \( \Delta LOD \), could be determined (see Fig.3).

In Fig.3, the obtained total \( \Delta LOD \) is given by the black line which owns complex variations. We adopt the wavelet ‘coif5’ to analyze the \( \Delta LOD \) data and obtain a 0.02 s decrease of LOD, which implies that recent mass redistributions force the Earth to rotate faster, though the Earth is slowing down at much longer time scale. Besides, fluctuations with periods of seven months (light blue line), one year (green line), two years (purple line) and four years (blue line)
are also very clear. Among these fluctuations, the maximal amplitude of variation is around 0.04 s, which can be observed by present LOD observations.

**Fig.3** Components of $\Delta$LOD obtained by wavelet analysis

However, why does $\Delta$LOD own the same frequencies with the tides (in fact, some solid tides and atmosphere tides own periods of seven months, one year, two years) even when the tides are removed (note that the GRACE data used in this paper are tidal free)? Two reasons may explain this. One is that there are some other geo-phenomena causing mass flow with the same frequencies with the tides (e.g., the core motions). The other is that tides are not completely removed from the GRACE data and thus the tide model needs to be improved.

## 4 Discussion and conclusion

The observed LOD varies at multiple time scales and these variations are principally caused by the core, mantle, ocean, and atmosphere, etc. Generally, core and mantle contribute to seasonal and decadal fluctuations while ocean and atmosphere are mainly related with seasonal and high frequency variations\(^\text{[1-2]}\). The present study, showing that mass redistribution will give rise to multiple time scale variations in LOD, does not conflict with the traditional conclusion but provides a wider space for further study.

In this study, the time-serials of Earth’s principal axes and moments of inertia, as well as associated time-serials of LOD variations are obtained. The results show that the principal moments, $A$, $B$, $C$, and length of day all have decreasing tendencies as well as some seasonal fluctuations in the recent 6 years. These seasonal fluctuations, originated in the fluctuations of the second-order potential coefficients, unexpectedly coincide well with the tidal frequencies. It can be predicted that a better understanding of the mass redistribution of the Earth would be obtained if we get to know why these coefficients experience such complex fluctuations (so do the moment inertia tensor and LOD) even when the tides are removed. The Earth’s core or/and mantle might explain it but the imperfection in the tide model is also possible. Thus, many further studies are needed.

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