Comment on ”Giant Nernst Effect due to Fluctuating Cooper Pairs in Superconductors” In a recent Letter, Serbyn et al. [1] microscopically and phenomenologically studied thermomagnetic effects above the superconducting transition and generalized results of [2,3] for arbitrary magnetic fields. In our opinion, the results of [1] disagree with basic physical principles.

(i) In the Gaussian model, using the Kubo method the authors of [1] calculated the bulk heat current, \( j^h = T \beta (E \times H) / H \), and found that the coefficient \( \beta \) diverges at \( T \to 0 \). To get rid of the contradiction with the third law of thermodynamics, they amend the heat current by the ”circular magnetization heat current, \( j^c_M = c(\mathbf{M} \times \mathbf{E}) \),” where \( \mathbf{M} \) is the magnetization. Below we will show that dissipationless magnetization currents do not transfer the heat and, therefore, the Kubo method provides an exact expression for the thermomagnetic coefficient \( \beta \).

It is known that in a finite sample, besides the bulk currents given by the Kubo formulas, charge and energy are also transferred by surface magnetization currents [4]. Circular electric and energy magnetization currents, 

\[
j^c_M = c \nabla \times \mathbf{M}, \quad j^e_M = \nabla \times (c \phi \mathbf{M}) = \phi j^c_M + c \mathbf{M} \times \mathbf{E} \tag{1}
\]

(\( \phi \) is the electric potential), are divergence-free and corresponding net magnetization currents are always zero (see Eqs. 3, 4, 7 and 39 in [4]). Therefore, instead of adding the surface magnetization current, one can subtract its bulk counter-flux [4]. Let us consider the surface and bulk magnetization energy currents in the direction of \( \mathbf{M} \times \mathbf{E} \) (Fig.1). According to Eqs. 1, the surface electric magnetization current \( j^s = c \mathbf{M} \times \mathbf{n} \) (\( \mathbf{n} \) is the unit vector normal to the surface) leads to the surface energy current

\[
J^s = \phi_A j^s_A + \phi_B j^s_B, \tag{2}
\]

which may be presented as \( J^s = j^s (\phi_B - \phi_A) = -cM E w \), where \( w \) is the width of the sample. Certainly, the surface energy current \( J^s \) and its bulk counter-flux,

\[
J^b = c(\mathbf{M} \times \mathbf{E}) w, \tag{3}
\]

are equal and have opposite directions, \( J^s = -J^b \), as it is shown in Fig. 1.

Note, that Eqs. 2 and 3 do not contain any transport characteristics. Therefore, they can be derived directly from the Maxwell equations. To do it, we remind that without magnetization currents the transformation of electromagnetic energy into the heat is described by equation

\[
- \text{div} \left( \frac{c \mathbf{E} \times \mathbf{H}}{4 \pi} \right) = j^h - \mathbf{E}, \tag{4}
\]

where \( j^h = c \nabla \times \mathbf{H} / 4 \pi \) is the transport electric current. Integrating Eq. 4 over the sample volume, \( V \), we get the well-known result: the power dissipated in the sample, \( j^h \cdot \mathbf{E} \), is equal to the electromagnetic power given by the Pointing vector integrated over the sample surface.

Now let us add to the above consideration dissipationless magnetization currents. Taking into account that \( j^c_M = c \nabla \times \mathbf{M} \) and \( \nabla \times \mathbf{E} = 0 \), we have

\[
- \text{div} (c \mathbf{E} \times \mathbf{M}) = j^c_M \cdot \mathbf{E}. \tag{5}
\]

While formally Eq. 5 looks analogous to Eq. 4, it has completely different physical sense due to the dissipationless nature of the magnetization currents. To see it, let us consider a sample, where the magnetization \( \mathbf{M} \) changes in the direction of \( \mathbf{M} \times \mathbf{E} \) as it is shown in Fig. 2. We will analyze the energy balance in a small volume, which is formed by two close cross-sections, \((A,B)\) and \((A',B')\), shifted by \( \Delta \mathbf{R} \) in the direction \( \mathbf{E} \times \mathbf{M} \). Integrating the l.h. side of Eq. 5 over the volume between \((A,B)\) and \((A',B')\), we get the net power carried to the volume by the bulk energy currents, which are expressed in terms of the magnetization-related part of the Pointing vector,

\[
-cw \{ (\mathbf{E} \times (\mathbf{M} + \Delta \mathbf{M})) \cdot \mathbf{n}_{AB} + (\mathbf{E} \times \mathbf{M}) \cdot \mathbf{n}_{AB} \} = cw E \Delta M = J^c_s(AB) - J^c_s(A'B') = \Delta J^c_s. \tag{6}
\]

To show that this power is removed by the surface energy currents, we should present the r.h. side of Eq. 5 in terms of the surface currents. First, let us note that the change of the magnetization \( \Delta \mathbf{M} \) between \((A,B)\) and \((A',B')\) is created by the magnetization current \( \Delta j^s = \Delta j^c_M \), which flows from the one side of the sample \((A,A')\) to another side \((B,B')\) between the two cross-sections \((A,B)\) and \((A',B')\) as it is shown in Fig. 2. Because of the charge conservation, the current \( \Delta j^s \) across the sample is exactly equal to the changes in the surface magnetization currents between \( A \) and \( A' \), and \( B \) and \( B' \): \( \Delta j^s = j^s_B - j^s_B = -(j^s_A - j^s_A) \). Finally, integrating the the r.h. side of Eq. 5 over the volume, we get

\[
E \Delta j^s w = (\phi_A - \phi_B) \Delta J^s = \Delta J^s. \tag{7}
\]

Eqs. 6 and 7 is nothing more than the integral representation of l.h. and r.h. sides of Eq. 5. Compare Eq. 6 and 7, we see that in any volume the bulk energy magnetization currents are compensated by the surface energy magnetization currents. Obviously, Eqs. 6 and 7 are the finite-difference form of Eqs. 3 and 2 correspondingly.

Thus, in the magnetic field the important part of the energy is transferred by the surface magnetization currents. To get the net energy current through the sample, the electromagnetic flux \( J^s = -J^c_s = c(\mathbf{E} \times \mathbf{M}) w \) should be added to the Kubo’s energy current [4]. But, as we will see, for the heat current no such corrections to the Kubo method are required.

The thermal energy is counted from the electrochemical potential \( \mu + e \phi \) and the heat current is defined as \( [5] j^h = j^s - \mu j^c / e \). The surface energy current \( J^s = \phi_A j^s_A + \phi_B j^s_B \) does not have a heat component, because every \( \phi j^s \) term in the energy current is canceled by \( \phi j^c \) in the definition of \( j^h \). Naturally, its bulk counter-flux 

\[
\Delta j^c_M = \Delta j^c_s = \Delta j^c_M = j^c_M - j^c_M = \Delta j^c_M. \tag{8}
\]

Consequently, the Kubo method correctly describe the energy current distribution in the sample.
also transfers only electromagnetic energy \((c\mathbf{M}\times\mathbf{E})\) is a magnetization part of the Poynting vector), which represents reversible work and can be entirely used. Thus, magnetization heat currents are absent. This statement also follows from the fact that circular temperature gradients do not exist (see discussion of Fig. 4 in [6]) and circular heat currents requires permanent energy supply. Finally, contrary to [1] the Kubo method gives an exact expression for the thermomagnetic heat current [6].

(ii) While for noninteracting electrons, thermomagnetic effects are proportional to the square of the particle-hole asymmetry (PHA) and very small, according to [1-3] the fluctuation thermomagnetic effects do not require PHA and, therefore, huge. The Gaussian model is fully applicable to ordinary superconductors, for which the works [1-3] predict the fluctuation correction to \(\beta\) to be at least \(e_F/T\sim 10^5\) times bigger than \(\beta\) in the normal state. Certainly, such huge effects are not known for ordinary superconductors [7]. Also, the calculations of [1] for superconductors with the negative interaction constant in the Cooper channel being generalized for nonsuperconducting metals with a positive constant would also lead to giant thermomagnetic effects even in ordinary metals.

(iii) The authors of [1] also proposed the phenomenological theory, where \(\nabla T\) was introduced via \(\nabla \mu(T(r))\). The authors claim that in this way they derived a general Einstein-type relation: \(\nu_N \equiv \beta/(\sigma H) = (\sigma/\nu e^2c)(\partial \mu/\partial T)\), where \(\sigma\) is the electrical conductivity and \(n\) is the electron concentration. However, according to textbooks [5], \(\nabla \mu\) should always be included in the effective electric field and such relation does not exist. Even with the relation above, to get the giant effect from the Cooper pairs, the authors of [1] introduce the thermodynamic chemical potential of pairs in the form: \(\mu_{c.p.}(T) = Tc + T\) [1]. It is known that \(\mu_{c.p.}\) is always zero, because a number of pairs is not conserved.

We note in conclusion, that technically speaking the authors of [1] calculated \(\beta\) for the Aslamazov - Larkin diagram. Previous works predicted huge thermal and thermoelectric effects originating from this diagram have been found to be wrong [3]. "The reason is that this diagram corresponds to the contribution of the superfluid flow to the current. Since the superfluid carriers no entropy, it does not contribute to the thermal current" [8].

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Fig. 1. Bulk and surface energy currents.
Fig. 1. The energy transfer related to the magnetization: the surface flux cancels the bulk flux from the magnetization-related Pointing vector. Both surface and bulk energy currents do not transfer the heat.

\[ \phi_A J^S_A + \phi_B J^S_B = -w(M \times E) \]

Fig. 2. Change in the magnetization, \( \Delta M \), requires the transverse magnetization current \( \Delta J^s \), which leads to the changes in surface magnetization currents.

\[ \Delta J^s = c \Delta M \]