A linear magnetic flux-to-voltage transfer function of a differential DC SQUID

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Abstract
A superconducting quantum interference device with differential output or ‘DSQUID’ was previously proposed for operation in the presence of large common-mode signals. The DSQUID is the differential connection of two identical SQUIDs. Here we show that besides suppression of electromagnetic interference this device provides effective linearization of the DC SQUID voltage response. In the frame of the resistive shunted junction model with zero capacitance, we demonstrate that spur-free dynamic range of the DSQUID magnetic flux-to-voltage transfer function is higher than 100 dB while the total harmonic distortion of a signal is less than $10^{-3}\%$ with a peak-to-peak amplitude of a signal being a quarter of half flux quantum, $2\Phi_0/8$.

Analysis of the DSQUID voltage response stability to a variation of circuit parameters shows that DSQUID implementation allows for highly linear magnetic flux-to-voltage transformation at the cost of a high identity of Josephson junctions and high-precision current supply.

Keywords: DC SQUID, voltage response, linearity, working margins, DSQUID

1. Introduction

Modern Josephson junction fabrication technology [1] allows for the development of complex circuits [2] with high-precision control of their parameters. Both low-temperature and high-temperature superconductor (LTS and HTS) technologies provide the potential to fabricate SQUID arrays with about a million Josephson junctions [3, 4]. This expands the area of SQUID applications to include SQUID-based structures that should ideally act as linear magnetic flux-to-voltage transformers [4–10]: from electrically small antennas to analog-to-digital converter circuits and from susceptometers to SQUID-based multiplexers.

SQUID-based structures with high dynamic range and highly linear voltage responses obtained without a feedback loop are named ‘superconducting quantum arrays’ (SQA) [11–13]. Two types of cells have been proposed as the basic blocks of SQA. These are the bi-SQUID [14–21] and the so-called differential quantum cell (DQC) [5, 11–13, 22–26]. Unfortunately, despite several attempts to realize bi-SQUID-based structures [16, 27, 28], no outstanding results have been reported [21]. DQCs seem to deliver better performance for SQAs at present [5]. However, since a DQC is a differential connection of identical parallel SQUID arrays, it usually occupies a large area which is not convenient in some cases.

In this paper, we consider the simplest version of DQC—two identical DC SQUIDs with a differential output which we call a ‘DSQUID’, see figure 1(a). It has been demonstrated that the DSQUID allows one to obtain a high common-mode rejection ratio [29]. This feature is especially useful where the SQUID-based system contains long wiring. It was also noted that the effects of background magnetic fields and of temperature fluctuations are also suppressed due to this differential configuration [29].
The system of equations describing the SQUID in terms of the Josephson junction critical current $I_c$ is obtained by subtracting the high linearity of its voltage response as well as analysis of the linearity decrease with deviations of the circuit parameters from their optimal values.

2. Model

The DSQUID voltage response is obtained by the subtraction of voltage responses of its parts: $u_{DS} = u_a - u_s$. The Josephson junctions of DSQUID ought to be overdumped to accomplish the high linearity of DQCs [5]. Equality of DSQUID parts naturally suggest the use of LTS technology where the technological spread of parameters can be minimized.

For the temperature $T = 4.2 \text{K}$ the effective current noise value is $I_f = (2\pi / \phi_0)k_BT \approx 0.18 \mu A$, where $\phi_0$ is the magnetic flux quantum and $k_B$ is the Boltzmann constant. The choice of Josephson junction critical current $I_c \geq 180 \mu A$ leads to dimensionless noise intensity $\gamma = I_f / I_c \leq 10^{-3}$ which makes the noise impact to DQC characteristics insignificant [5]. The transfer functions of each SQUID of the DSQUID are calculated in the frame of the well-known resistive shunted junction (RSJ) model with zero capacitance.

The system of equations describing the SQUID in terms of Josephson phase sum and difference $\varphi_{\pm} = (\varphi_1 \pm \varphi_2) / 2$ (where $\varphi_{1,2}$ are the Josephson phases of SQUID junctions) is as follows:

$$\frac{d\varphi_+}{d\tau} = i_b - \sin \varphi_+ \cos \varphi_-, \quad (1a)$$

$$\frac{d\varphi_-}{d\tau} = - (\varphi_+ - \varphi_-) / \beta_0 - \sin \varphi_+ \cos \varphi_-, \quad (1b)$$

where time $\tau = \tau_{\varphi}$ is normalized to the characteristic frequency, $\omega_\varphi = 2\pi I_c / \beta_0$, $\beta_0$ is the junction shunt resistance, $i_b = b_b / 2I_c$ is the normalized bias current, $\beta_0 = \pi L_b / \phi_0$ is the normalized SQUID inductance, and $\phi_0 = \pi \phi_0 / \phi_0$ is the normalized applied magnetic flux. For each SQUID the DSQUID $\varphi_\alpha$ is the sum of the signal flux, $\varphi_\alpha$, and the magnetic bias flux setting the working point and the width of the working region, $\pm \phi_b / 2$, as it is shown in figure 1.

We use two approaches for the estimation of voltage response linearity. The first one is calculation from the so-called spur-free dynamic range (SFDR). In this approach we apply the external magnetic flux to each SQUID of the DSQUID in the form,

$$\varphi_b = (\phi_b / 2) \sin \omega_1 \tau + (\phi_b / 2) \sin \omega_2 \tau \pm \phi_b / 2,$$  \quad \text{(2)}$$

where frequencies $\omega_b = 1.1 \omega_1$ are much smaller than Josephson oscillation frequency, $\omega_{1,2} \ll \omega_b$, $\phi_b$ is the amplitude of the signal. SFDR is calculated as a ratio of one of the signal tones to maximum amplitude of distortions arising in the spectrum of output signal due to nonlinearity of the magnetic flux-to-voltage transfer function.

The second approach is the calculation of total harmonic distortion (THD). In this case the applied signal contains only one harmonic component:

$$\varphi_b = \phi_b \sin \omega_1 \tau \pm \phi_b / 2.$$ \quad \text{(3)}$$

THD is calculated as $\text{THD} = \sqrt{\sum_{n=1}^N A_n^2} / A_1$, where $A_n$ are amplitudes of output signal spectral harmonics.

SFDR and THD calculations requires finding the accurate shape of the DSQUID voltage response by numerical solutions of system (1) for each SQUID. For this purpose we define the dependence $\varphi_\alpha(\varphi_\alpha)$ combining equations (1a), (1b),

$$\frac{d\varphi_\alpha}{d\tau} = - (\varphi_+ - \varphi_-) / \beta_0 - \sin \varphi_+ \cos \varphi_-, \quad (4)$$

and then calculate the period of Josephson oscillations using (1a):

$$T = \int_0^{2\pi} \frac{d\varphi_\alpha}{ib - \sin \varphi_+ \cos \varphi_-(\varphi_\alpha)}.$$ \quad \text{(5)}$$

This gives us the Josephson oscillation frequency, $\omega_{\text{J}} = 2\pi / T$, which is equal to the time-averaged voltage, $u$, normalized to the $I_R$ product.

Calculation of the voltage response shape for the estimation of the magnetic flux-to-voltage transfer coefficient is completed much faster by using analytical expressions presented in [30, 31].

3. Linearization

Differential connection of two identical SQUIDs in DSQUID with their mutual flux bias allows subtraction of some part of the SQUID voltage response from its mirrored image, see figure 1(b), due to the symmetry and periodicity of the SQUID voltage response. This subtraction leads to partial compensation of nonlinear terms in the DSQUID magnetic flux-to-voltage transfer function for the two regions of SQUID voltage response marked by numbers I and II in figure 1(b).

The first region (I in figure 1(b)) is in the vicinity of zero external magnetic flux ($\phi_b \approx 0$) [30]. In the limit of zero
SQUID inductance, $\beta_l = 0$, and for the bias current equal to the critical current, $i_b = 1$, the SQUID voltage response shape is described by the function: $u = |\sin \phi_b|$. For the bias flux $\phi_b \leq \pi/2$ and inside the region $\phi_b \in [1-\phi_b/2, \phi_b/2]$ the voltage responses of DSQUID arms can be written as $u_x = \pm \sin(\phi_b \pm \phi_b/2)$. Thus, in the range where sine can be approximated by a linear function the total response becomes linear:

$$u_x \approx 2\phi_b \cos \phi_b/2.$$  

(6)

It is seen that the most linear part of the voltage response (at $\phi_b = 0$) is moved from the boundary of the SQUID working region ($0 \leq \phi_b \leq \pi/2$) to its center in the DSQUID, making its utilization possible. At the same time, the bias flux providing the maximum transfer coefficient, $\phi_b = 0$, simultaneously makes the width of the working (and linearized) region decrease.

The second region (II in figure 1(b)) suitable for linearization in the DSQUID is located near the opposite boundary of the SQUID working region ($\phi_b \approx \pi/2$). Analytical approximation of the SQUID voltage response found in [30, 31] is

$$u = u_0 - a[1 + (\beta_1^2u_0)^{-2}]^{-1}(i_b - u_0)\tan^2 \phi_b,$$  

(7)

where $u_0 = \sqrt{\beta_1^2 - \cos^2 \phi_b}$ is the voltage response in the limit of vanishing inductance, $\beta_l = 0$, and $a$, $\beta_1^2$ are parameters depended on $i_b$, $\beta_l$ (see the appendix).

In the vicinity of $\phi_b = \pi/2$ expression (7) can be accurately represented by a Taylor series limited to the quadratic term:

$$\lim_{\phi_b \to \pi/2} u \approx i_b - \frac{1}{2i_b}(Q + (1 - QZ)\left[\phi_b - \frac{\pi}{2}\right]^2).$$  

(8)

Dependencies of $Q$ and $Z$ on $i_b$, $\beta_l$ are presented in the appendix.

For the bias flux close to $\phi_b = \pi$, the voltage response of DSQUID arms can be written as $u_x = u(\phi_b + \pi/2 \pm \delta)$, where $\delta = \pi/2 - \phi_b/2$, due to symmetry of the SQUID voltage response. According to (8) this leads to the linearized total response:

$$u_x = \frac{\phi_b - \phi_b}{i_b}(1 - QZ).$$  

(9)

While the width of the region II is defined by the range of validity of approximation (8), the width of the linearized region of DSQUID voltage response is determined by overlapping of regions I i.e. by the bias flux $\phi_b$. Deviation of the bias flux from $\phi_b = \pi$ increases the transfer coefficient but decreases the linearized region width.

Therefore, in both considered cases we face a tradeoff between the transfer coefficient and the width of the linearized region of DSQUID voltage response. Below we show that it is possible to satisfy this tradeoff for the bias flux values in the vicinity of $\phi_b = 0$ or $\pi$. Corresponding examples shown in figure 2 are discussed in more detail below.

4. Optimization of parameters

An optimization procedure is performed in the range of the bias current, $i_b \in [1, 1.2]$, and the inductance, $\beta_i/\pi \in [0.05, 1]$. Using the standard Nelder-Mead simplex algorithm [32] we numerically find the optimal bias flux providing the highest SFDR of DSQUID voltage response.

We use two different conditions to consider linearization of the two SQUID voltage response regions (I and II in figure 1(b)). The first one is limitation of the peak-to-peak signal to $30\%$ of the width of the DSQUID working region, $2\phi_a = 0.35\phi_b$. It is used to consider utilization of the region I ($\phi_b \approx 0$). The usage of region II ($\phi_b \approx \pi/2$) is considered with limitation of the peak-to-peak signal to a fixed value equal to a quarter of the SQUID working region, $2\phi_a = \pi/8$.

Results of optimization obtained with utilization of the first condition is presented in figures 3(a), (b), (c). It is seen that the highest SFDR $> 100$ dB is obtained with the bias current close to the critical current, $i_b \to 1$, see figure 3(a). However, while the transfer coefficient for this bias current is also high (figure 3(c)), the bias flux is quite small (figure 3(b)), and thus the width of the DSQUID working region is negligible.

Fortunately, there is a kind of plateau on the $i_b$, $\beta_i$ plane of parameters where $i_b > 1$ and linearity is still rather high, SFDR $> 90$ dB, see figure 3(a). The example shown in figure 2(a) corresponds to the values of parameters: $i_b = 1.02$, $\beta_i = 0.35\pi$ at the boundary of this plateau providing both high enough bias flux, $\phi_b = 0.3895\pi$, and transfer coefficient, $du/d\phi_b = 1.04$, while linearity is SFDR $= 92$ dB and THD $= 6 \times 10^{-4}\%$.

The chosen optimal bias flux corresponds to the width of the linear range equal to approximately a quarter of the width of the SQUID working region, $2\phi_a \approx \pi/8$. The linearity increase is possible with decreases to the signal amplitude as shown in figure 4 with the solid line. However, the bias flux should additionally be slightly tuned (see inset in figure 4). SFDR reaches 130 dB with the peak-to-peak signal equal to
Figure 3. Results of the bias flux optimization for obtaining the highest SFDR of DSQUID voltage response under different conditions: (i) $2\phi_b = 0.3\beta_b$ for (a), (b), (c) and (ii) $2\phi_a = \pi/8$ for (d), (e), (f). SFDR is presented in (a), (d) planes, the optimal bias flux $\phi_b/\pi$ is in (b), (e) planes, and the transfer coefficient $du/d\phi_b$ is in (c), (f).

Figure 4. SFDR of DSQUID voltage response versus peak-to-peak signal amplitude for chosen values of parameters, $i_b = 1.02$, $\beta = 0.35\pi$, and optimal bias flux presented in inset (solid line). SFDR for the constant bias flux, $\phi_b = 0.389 \pi$, is shown by dotted line.

10% of the bias flux. The SFDR obtained with constant bias flux is presented by the dotted line.

While limitation of signal amplitude to a percent of the DSQUID working region width makes utilization of region I preferable, setting a fixed signal amplitude and vice versa allows the usage of region II. Optimization results obtained under utilization of the second condition ($2\phi_a = \pi/8$) is presented in figures 3(d), (e), (f). Figure 3(d) shows that for inductance values higher than $\beta_l = 0.15\pi$ there are some values of the bias current $i_b > 1$ providing high linearity of the voltage response, SFDR $> 100$ dB (figure 3(d)), with the bias flux $\phi_b$ close to $\pi$ (figure 3(e)). Unfortunately, the transfer coefficient decreases with increase of SFDR, see figures 3(d), (f). For the previously chosen inductance, $\beta_l = 0.35\pi$, the optimal bias current is $i_b = 1.108$ and with the bias flux equal to $\phi_b = 0.8337\pi$ the transfer coefficient is $du/d\phi_b = 0.22$. The corresponding linearity is SFDR $= 112$ dB, THD $= 6 \cdot 10^{-4}$%. The DSQUID voltage response for this set of parameters is presented in figure 2(b).

Figure 5 shows the SFDR dependence with deviation of DSQUID parameters from their optimal values, $\delta i_b/i_b$, $\delta \phi_b/\phi_b$, $\delta \beta_l/\beta_l$, for both examples shown in figure 2. It is seen that the linearity strongly depends on the bias current, see figure 5(a). One should set $i_b$ with precision of $\pm 0.1 - 0.2\%$ to keep SFDR not less than 10 dB from its maximum. At the same time for the same margin for SFDR the bias flux $\phi_b$ can be set with an order less precision, $\pm 1 - 2\%$, see figure 5(b). The requirements for the inductance value are even weaker (figure 5(c)).

5. Discussion

The data shown in figure 5 indicates high requirements for the identity of SQUID parameters in the DSQUID. The statistics of Josephson junctions critical currents fabricated by modern LTS fabrication processes can be described approximately as Gaussian with standard deviation strongly dependant on the junction size [33]. For junctions with critical currents greater
than 180 μA (corresponding size is greater than 1500 nm) the standard deviation is less than 1% for the process using 248-nm photolithography, see equation 5 in [1]. Based on the data presented in figure 5(a), we can estimate the real attainable linearity (SFDR) above 80 dB, accordingly.

Using the conventional SQUID configuration, one needs to restrict the width of the working range below 0.1 Φ0 to make THD less than 1% [34]. With a similar width of the working region, 2Φin = Φ0/8, DSQUID provides THD up to three orders better. By assuming the root mean square flux noise of Φn = 10−6 Φ0/Hz−1/2, one can obtain the DSQUID dynamic range of about 100dB/Hz−1/2. This means that with SFDR of an order of 100 dB, DSQUID allows truly linear magnetic flux-to-voltage transformation since the distortions can be made lower than the noise floor.

SQA can be based on a DSQUID as far as the DSQUID is the DQC. The SQA formation can serve for the increase in dynamic range and transfer coeficient of the RFBR grant 16-29-09515 of the Russian Science Foundation. Section 1 was written with the support of the RFBR grant 17-12-01079 of the Russian Science Foundation. V I R acknowledges the Basis Foundation scholarship.

Appendix

Expression (7) is the approximation of the DC SQUID voltage response shape. Analytical dependencies of its parameters a, βf, on ib, β0 are as follows [30, 31]:

\[ a = 2\kappa\nu(2\xi^2 - 1)/\xi, \quad \beta_f = \sqrt{\xi/\chi}, \]  

where

\[ \kappa = \beta_0^{1.66}/(2.154\beta_1^{0.48} + 2.285), \]  

\[ \nu = \beta_1^{0.92}/(4.28\beta_1^{0.625} + 5.06), \]  

\[ \xi = -0.586\kappa + 2\nu + 4\kappa\nu(i_0^2 - 1), \]  

\[ \chi = 0.586\kappa^2 - \kappa - 2\kappa\nu(i_0^2 - 1). \]

Parameters Q, Z of the Taylor series (8) are combinations of a, βf, and ib:

\[ Q = a[1 + (\beta_f^2ib)^2]^{-1}, \]

\[ Z = 1 - [((\beta_f^2ib)^2 - 3(4i_0^2[1 + (\beta_f^2ib)^2])^{-1}. \]

6. Conclusion

We considered a differential SQUID—a ‘DSQUID’ possessing a highly linear voltage response. The DSQUID is the differential connection of two identical SQUIDs. Two regions of SQUID voltage response suitable for linearization in the DSQUID are identified. A rising tradeoff between the transfer coefficient and the width of the linearized region is revealed. Optimal values of the circuit parameters providing high linearity of the voltage response (SFDR > 100 dB and THD < 10−3%) are found. The linearity dependence on deviation of the circuit parameters from their optimal values is studied. It is shown that the ultimate linearity comes at the cost of high identity of Josephson junctions (at the level of tenths of a percent) and high-precision current supply (up to the third decimal place).

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Figure 5. SFDR of DSQUID voltage response versus deviation of (a) the bias current, δib/ib, (b) the bias flux, δΦib/Φ0, and (c) the inductance, δβ0/β0, for two chosen sets of parameters: (i) ib = 1.02, β0 = 0.35π, φ0 = 0.389 5π and (ii) ib = 1.108, β0 = 0.35π, φ0 = 0.833 7π (corresponding lines are marked by ib values).
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