Floquet topological transition by unpolarized light

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We study Floquet topological transition in irradiated graphene when the polarization of incident light changes randomly with time. We numerically confirm that the noise averaged time evolution operator approaches a steady value in the limit of exact Trotter decomposition of the whole period where incident light has different polarization at each interval of the decomposition. This steady limit is found to coincide with time-evolution operator calculated from the noise-averaged Hamiltonian. We observe that at the six corners (Dirac(K) point) of the hexagonal Brillouin zone of graphene random Gaussian noise strongly modifies the phaseband structure induced by circularly polarized light whereas in zone-center (Γ point) even a strong noise isn’t able to do the same. This can be understood by analyzing the deterministic noise averaged Hamiltonian which has a different Fourier structure as well as lesser no of symmetries compared to the noise-free one. In 1D systems noise is found to renormalize the drive amplitude only.

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I. INTRODUCTION

Realizing topological phenomenon in solid state system has been one of the major topics in condensed matter physics since the discovery of IQHE in 2D semiconductor device. These materials are model system for 2D non-interacting electron gas which under the application of strong magnetic field forms highly gapped Landau levels at low temperature. This results in very precise quantization of Hall conductance and supports robust conducting chiral states at the edges. Later it was shown that the magnetic field is not necessary and one can also observe such phenomena in systems described by tight-binding Hamiltonians. The so called “Haldane Model” describe electrons hopping in a honeycomb lattice threaded by periodic magnetic flux with zero net flux. The resulting complex hopping is difficult to implement experimentally and it is only recently that the advancement in ultra-cold atomic systems have made such experiments possible. To avoid such complicated implementation of the Haldane model and thus realize Chern insulating states more easily, a possible alternative way, namely “irradiation of electromagnetic wave on graphene”, is proposed recently to achieve the essential goal of time reversal symmetry breaking.

Graphene is a gapless 2D Dirac system which can open up a gap at the Dirac Point under irradiation of circularly polarized light. This resulting new state, termed as Floquet topological insulator was found later in many other systems. It is also detectable by various transport signatures. These are steady states of periodically driven non-equilibrium systems which recently gained tremendous attention because of it’s potential to create new phases. These phases can hardly be found in their equilibrium counterparts. Traditional bulk-boundary correspondence was extended to Floquet topological systems taking into account the periodicity of the Floquet spectrum. Experimental verification of such states has already been achieved using both time and angle resolved photoemission spectroscopy (PES) and also in photonic systems.

Throughout the last decade a large number of studies of real time dynamics in closed quantum systems have extended the notions of universality from equilibrium to non-equilibrium via Kibble-Zurek scaling. Further studies show that the qualitative nature of these scalings can be completely reversed by introducing noise in the drive. In these studies the Heisenberg equation of motion picks up a dephasing term due to averaging over different noise realizations which leads to non-unitary dynamics. Recently in equilibrium systems it has been shown that periodicity in space (i.e the crystal structure) is not necessary to get topological behavior and one can also see it in amorphous systems. Analogously one can ask at this point that what would happen in Floquet systems if time periodicity of the Hamiltonian is broken due to the presence of noise in the drive. Several studies in this direction in models decomposable in free fermions have already revealed that the nature of the asymptotic steady state depends on the type of aperiodic protocol. Further some analytical studies show that disorder-averaging can be avoided for a special class of protocols.

Influenced by this kind of works we plan to study the fate of the Floquet topological systems when the smooth time variation of incident electromagnetic wave is broken by the insertion of a random phase in one of the component of vector potential. This kind of noise is always there in a typical experiment if the setup to produce polarized light isn’t calibrated properly. Moreover such noise can also be generated artificially using synthetic gauge fields. We term this kind of monochromatic wave as unpolarized light in the sense that the associated Lissajous figures keeps on changing with time. The central results of this work can be summarized as follows. We show that depending on the spatial dimension of the problem Floquet topological transitions can be influenced by the random change in polarization of inci-
dent light. For graphene we find that the transitions at Dirac (K) point are significantly modified compared to Γ point. The origin of this effect can be understood to be due to a fundamental change in Fourier structure of the noise-averaged time-dependent Hamiltonian at K point. At low frequencies of the incident radiation, it is well known that symmetries of the underlying Hamiltonian is crucial for topological transition. In the presence of noise, we find such symmetries to be broken. Interestingly, in contrast to standard expectation, we find that few of these symmetries are restored in the noise-averaged Hamiltonian. This symmetry restoration has impact on the self-averaging limit in this parameter regime. Finally for a 1D model (p-wave superconducting wire), using a non-trivial drive protocol, we show that even a strong noise (large standard deviation) can’t prohibit the transition.

The rest of the paper is planned as follows. In Sec.[II] we introduce our protocol for irradiated graphene and plot the results (phasebands) for numerical disorder averaging. In Sec.[II A] we establish the existence of self-averaging limit which suggests numerical averaging is meaningful and can be mimicked by the ensemble averaged Hamiltonian. This is followed by possible explanation of the deviation from noise free (circularly polarized case) behavior separately in high and low frequency regime in Sec.[II B] and Sec.[II C] respectively. Next, in Sec.[III] we shows results for 1D systems. Finally we conclude and discuss possible experimental scenarios in Sec.[IV]

II. IRRADIATED GRAPHENE

We consider graphene irradiated by electromagnetic wave defined by the vector potential \( A = A_0(\cos(\omega t + \phi(t)), \sin(\omega t)) \). One have to further assume it to be space independent in graphene plane to keep the integrability of the problem intact. The \( \phi = 0 \) (circularly polarized) case is well studied in the literature[31]. We allow \( \phi \) to be a normally distributed random variable with mean \( \mu \) and standard deviation \( \sigma \) at each instant of time which gives rise to its unpolarized nature. If one wish to produce this vector potential in lab then this kind of noise will be inherently present as random experimental error. The normalized probability distribution of \( \phi \) at each time instant \( t \) is given by

\[
P(\phi) = \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{(\phi-\mu)^2}{2\sigma^2}}
\]

where \( \mu \) can be any real number within the interval \( -\pi \leq \mu \leq \pi \). Here we will concentrate on the special value \( \mu = 0 \) (i.e. this is the value of \( \mu \) in all plot). This will allow us to directly compare the result with circularly polarized case.

The time-dependent graphene Hamiltonian (for each k-mode) after Peierls’s substitution with this protocol becomes

\[
H(k, t) = \begin{pmatrix}
0 & Z(k, t) \\
Z^*(k, t) & 0
\end{pmatrix}
\]

where \( Z(k, t) = -\gamma(2e^{i\frac{\varphi}{2}} \cos(\sqrt{\frac{3}{2}}x) + e^{-i\hat{k}x}) \) and \( \hat{k} = k + eA \).

Next we calculate the time-evolution operator over one time period \( T \) for each k-mode by dividing the period in \( N \) parts

\[
U_k(T, 0) = T_e^{-i \int_0^T H_k(t')dt'} = e^{-iH_k(T-\delta t)\delta t}e^{-iH_k(T-2\delta t)\delta t}...e^{-iH_k(2\delta t)\delta t}
\]

\[
\text{(2)}
\]

where \( T_e \) denotes time-ordered product and \( \delta t = T/N \) is a very small but fixed time interval. Such decomposition introduces Trotter error which gets reduced with increasing \( N \) and reproduces the exact \( U \) for the chosen continuous drive in the \( N \to \infty \) limit. We calculate the time-dependent Hamiltonian at each partition by drawing \( \phi \) from a normal distribution and using Eq[2] get \( U(T, 0) \) for one particular noise realization. We then average over several such realizations numerically and get the noise averaged time evolution operator

\[
\langle U_k(T, 0) \rangle = \langle T_e^{-i \int_0^T H_k(t')dt'} \rangle
\]

\[
\text{(3)}
\]

Eq[3] has a self-averaging limit[32], in the sense that all four elements of \( \langle U(T, 0) \rangle \) goes to some steady value with increasing no of partitions (\( N \)). We shall discuss this in more details in the next sub-section.

In Fig.1 we plot the phasebands \( \Phi(T) \) obtained using \( \cos(\Phi(T)) = Re\langle U(T) \rangle_{11} \). One can see with increasing magnitude of random noise the phasebands gets modified but we recover the results for pure circularly polarized light in \( \sigma \to 0 \) limit as expected. We find that the phasebands remain almost unchanged for Γ point for a broad range of parameter values; however at K point, they are strongly modified by the noise. We calculate Chern number of the lower Floquet band using the eigenfunctions of \( \langle U(T) \rangle \) in a discretized Brillouin zone. The plot is shown in Fig.2. We find that the transitions (position of

FIG. 1: Noise averaged phasebands vs T for Γ point (left) (at \( \alpha = 1.5 \)) and for K point (right) (at \( \alpha = 2.0 \)) for various values of standard deviation(\( \sigma \)). N=1000, no of sample=1000 and \( \alpha = eA_0/c \)
and thus helps to achieve the thermodynamic result fast.

celled when averaged out over several disorder realizations
the finite size of the system but these deviations get can-
the mean) is introduced in all physical observable due to
each disorder realization some amount of deviation (from
close analogy to equilibrium disordered systems where for
limit exists here. It is only in this limit that the disorder
of partitions(N)(see Fig.3) which suggest self-averaging
uations. We see power law fall of both
evolution operator constructed using it resembles the
noise averaged time-evolution operator. In a recent work Lobejko et al have showed rigorously that the
difference of ensemble averaged time-evolution operator
and the time-evolution operator constructed by the en-
semble averaged Hamiltonian scales as $O(\frac{1}{N})$ for a cer-
tain class of protocols. For these protocols the en-
semble averaged Hamiltonian at two different time commutes
which they have termed as “commutation in statistical
sense”. They further extends the applicability of above
to some simple non-commuting Hamiltonian by
numerical simulations. But unlike those cases irradiated
graphene contains the noise term within the argument
of complicated trigonometric functions. Hence the en-
semble averaged Hamiltonian can not be obtained here
simply by substituting $\phi$ by it’s mean value. Therefore
we explicitly calculate the ensemble-averaged Hamilto-
nian for irradiated graphene at time $t$

$$
\langle H_k(t) \rangle = \int_{-\infty}^{\infty} P(\phi) H_k(\phi, t) d\phi
$$

with $P(\phi)$ in Eq. we get using Jacobi-Anger relations.

Using this we numerically calculate the Frobenius norm
of the distance between $\langle U(H(t)) \rangle$ and $\langle U(H(t)) \rangle$

$$
D_N = \| (T e^{-i \int_0^T H(t') dt'} - T e^{-i \int_0^T (H(t')) dt'}) \| \tag{6}
$$

and the same norm for the corresponding variance matrix

$$
S_N = \| \langle (T e^{-i \int_0^T H(t') dt'} - T e^{-i \int_0^T (H(t')) dt'})^2 \rangle \| \tag{7}
$$

where $N$ is the no of partitions used to calculate (us-
ing Eq. and this) each quantities inside the norm. These
are two appropriate quantities to measure the deviation
of the time-evolution operator in different noise realiza-
tions. We see power law fall of both $D_N$ and $S_N$ in no
of partitions(N)(see Fig.3) which suggest self-averaging
limit exists here. It is only in this limit that the disorder
averaging is meaningful in dynamical systems. This is in
close analogy to equilibrium disordered systems where for
each disorder realization some amount of deviation (from
the mean) is introduced in all physical observable due to
the finite size of the system but these deviations get can-
celed when averaged out over several disorder realiza-
tions and thus helps to achieve the thermodynamic result fast.

\[\langle Z(k, t) \rangle = -\gamma (2 e^{\frac{i k_x T}{2}} \cos(\frac{\sqrt{3}(k_y + \alpha \sin(\omega t))}{2}) [J_0(\frac{\alpha}{2}) + 2 \sum_{n=1}^{\infty} (-i)^n J_n(\alpha) e^{-\frac{n^2 x^2}{2}} \cos(\mu t + \mu)]) \]

\[\langle H_k(t) \rangle = \int_{-\infty}^{\infty} P(\phi) H_k(\phi, t) d\phi \]

Here in dynamical system finite no of partition(N) play
the role of finite system size and the thermodynamic limit
corresponds to the continuous drive ($N \to \infty$). Vanish-
ing of $S_N$ in large N also implies the equivalence

$$
\cos(\Phi(T)) \equiv \langle \cos(\Phi(T)) \rangle \tag{8}
$$

which we have used throughout the paper. In Fig.3 note
that $D_N$ and $S_N$ have larger values at $K$ point compared
to $\Gamma$ point for small N. This is related to the fact that
time dependent Hamiltonian of irradiated graphene at
$K$ point is more complicated than at $\Gamma$ point due to the
presence of lesser no of symmetries. Larger the com-
plexity larger N one need to use to reduce these errors.
This power law fall suggests that the time consuming
numerical disorder averaging can be avoided by the use
of ensemble averaged Hamiltonian to calculate $U(T, 0)$
with a sufficiently large no of partitions of whole period.
We further demonstrate this by explicitly comparing the
phasebands from both this way in Fig.4. Our next tar-
get is to understand better why in some cases a weak
noise is sufficient to abolish all the transition (as in $K$
point) where as in some other cases(as in $\Gamma$ point) even a
where \((m, n)\) is row and column index of different square blocks each of size \((H_1 \times H_1)\) where \(H_1\) is the Hilbert space dimension of the equilibrium problem (2 for each k-mode in our case) and \((i, j)\) denotes position of each matrix element within one such block. For numerical purposes one can truncate this matrix after some order which depends on details of the problem especially the absolute value of maximum order of the Fourier components (of time-dependent Hamiltonian) with non-vanishing coefficient. One also needs to increase the truncation dimension with decreasing frequency. Following this prescription one can safely truncate the Floquet Hamiltonian in zero-th order at \(\Gamma\) point (where one has a \(2 \times 2\) \(H_F\)) and in 1st order at \(K\) point (where one has a \(6 \times 6\) \(H_F\)) for high frequencies and low Amplitude of radiation\[8,31\]. Thus one gets expressions of Floquet conduction band \((\Phi(T))\) in 1st quasi-energy BZ for the noise free (circularly polarized) case with hopping-amplitude(\(\gamma\)) set to unity

\[
\Phi(\Gamma, T) = 3J_0(\alpha)T
\]

\[
\Phi(K, T) = \frac{\sqrt{4\pi^2 + 36J_F^2(\alpha)T^2} - 2\pi}{2}
\]

Next we aim to calculate some simplified expression of phaseband for the unpolarized light using the ensemble averaged Hamiltonian in some suitable parameter regime. We can sufficiently simplify Eq.5 for strong noise. Note that though \(\phi\) appears as argument of trigonometric functions due to its random nature at each instant of time \(\phi[\mu, \sigma]\) and \(\phi[\mu + 2n\pi, \sigma + 2n\pi]\) will not give same time evolution operator. Using \(e^{-\frac{\alpha^2x^2}{\sigma^2}}\approx 0\) for large \(\sigma\) in Eq.5 we get

\[
\langle Z(k, t) | _{\sigma > 0} \approx -\gamma (2J_0(\frac{\alpha}{2})e^{ikx} \cos(\frac{\sqrt{3}}{2}(ky + \alpha \sin(\omega t))) + J_0(\alpha)e^{-ikx})
\]

for \(\Gamma\) point this gives a Hamiltonian proportional to \(\sigma_x\) only and hence one simply gets the phaseband

\[
\Phi(\Gamma, T) = \int_0^T \langle Z(\Gamma, t') \rangle dt'
\]

the integrand is difficult but again using Jacobi-Anger relations we get (taking \(\gamma = 1\))

\[
\Phi(\Gamma, T) = \left(2J_0(\frac{\alpha}{2})J_0(\frac{\sqrt{3}\alpha}{2}) + J_0(\alpha)\right)T + 4J_0(\frac{\alpha}{2})\sum_{n=1}^{\infty} J_{2n}(\frac{\sqrt{3}\alpha}{2}) \int_0^T \cos(2n\omega t') dt'
\]

\[
= \left(2J_0(\frac{\alpha}{2})J_0(\frac{\sqrt{3}\alpha}{2}) + J_0(\alpha)\right)T
\]

similarly for \(K\) point we get

\[
\Phi(K, T) = (J_0(\alpha) - J_0(\frac{\alpha}{2})J_0(\frac{\sqrt{3}\alpha}{2}))T
\]
we compare cosines of Floquet bands for circularly polarized ($\sigma = 0$) and unpolarized ($\sigma \gg 0$) case in Fig.5.

The functional behavior of these two bands do not change much for $\Gamma$ point whereas for $K$ point they show drastically different behavior. This huge change for $K$ point is due to the fact that strong noise (highly unpolarized light) changes the lowest non-vanishing Fourier component of $(H_K(t))$ from 1 to 0 and thus reduces the effective Sambe space dimension from 6 to 2. These changes make the Floquet band at $K$ point to depend on $J_0$ only abolishing $J_1$ s. Note that $J_0$ and $J_1$ has completely different behavior when the argument is small, the former is a decreasing function but the later is an increasing function of the argument.

C. Low frequency

At low frequencies (and also at high radiation amplitudes) one need to take into account the higher Fourier components of the time-dependent Hamiltonian and consequently the truncation dimension of the Floquet Hamiltonian increases. This is why at low frequencies one can’t have simple analytical expression of Floquet bands in terms of Bessel functions and one needs to consider other methods like the adiabatic-impulse which gives good matching with numerics in low to moderate frequencies and high amplitude. Symmetries of $H(t)$ also play a crucial role in predicting the existence of phase-band crossings at different high symmetry points. But before going into the details of that we investigate the behavior of $D_N$ and $S_N$ as a function of $N$ at low frequencies. Generally low $\omega$ and hence a high period ($T$) necessitates a proportional increase of no of partitions but numerics suggests that the convergence of these quantities to zero is much slower than that in this parameter regime. In Fig.6(a)-(c) we demonstrate this. We see for a typical high $\sigma$ one needs to increase $N$ nearly quadratically (instead of linearly) with $T$ to make the value of $D_N$ go below some particular threshold. We, therefore, to reduce the numerical cost, keep our all calculations confined within small $\sigma$ values at low frequencies.

It was shown in ref[33] that there exists 6 fold symmetries at $\Gamma$ point of graphene irradiated by circularly polarized light. This was shown to be responsible for phaseband crossing simultaneously at $T/3$, $2T/3$ and $T$. But here for unpolarized light typically all these symmetries are absent for any disorder-realization. Consequently, disorder averaging also leads to avoided crossing. Here also the ensemble averaged Hamiltonian can capture the essential physics but interestingly two of the symmetries get restored in it. We chart out the symmetries of $\Gamma$ point under the irradiation of CP and unpolarized (ensemble averaged $H(t)$) light in detail in Table.1. This kind of symmetry mismatch between the two quantities inside the norm of Eq[6] has significant impact on fall of $D_N$ at low frequencies. We find that $D_N$ falls very slowly with $N$ (see Fig.6) here.

In Fig.7 we show this symmetry mismatch between CP
and unpolarized light pictorially (a large $\sigma$ is used for this purpose in Fig.7(a)) and its consequences. Fig.7(b) shows for exact numerical disorder averaging a small $\sigma$ is sufficient to abolish the crossing at $T/3$. Fig.7(c)-(d) shows for ensemble averaged Hamiltonian the crossings at $T/3$ and $2T/3$ gets increasingly avoided with increasing $\sigma$.

III. 1D SYSTEMS

One-dimensional interacting spin chains whose Hamiltonian can be expressed in terms of free fermions via Jordan-Wigner transformation have attracted a lot of theoretical attention in last decades due to their integrable structure, existence of topological transition as well as possibility of experimental realization using ion-traps and ultracold atom systems. Non-equilibrium dynamics in these models is equally interesting because non-trivial topology can be induced by periodic drive of different terms in the Hamiltonian. This can be independently done using multiple lasers with different amplitudes and frequency. In these experiments phase differences between different drive terms can be randomly changed in a time scale $t_0 \ll 1/\omega$ where $\omega$ is the frequency of drive. This constitutes a 1D platform to study similar physics as studied in previous section for 2D systems using unpolarized light. The survival of the topological transition under such noisy drive is the key issue we would like to address. To this end, we consider a p-wave superconductor described by the following Hamiltonian \cite{36,37}

$$H = \sum_{i=1}^{L-1} [(\gamma c_i^+ c_i + H.c.) + \Delta (c_i c_{i+1} + H.c.)] - \mu \sum_{i=1}^{L} (2c_i^+ c_i - 1)$$

(16)

This model is equivalent to a spin-$1/2$ XY chain in perpendicular magnetic field via Jordan-Wigner transformation \cite{38}. After a Fourier transformation defined by $c_k = \frac{1}{L} \sum_{j=1}^{L} \psi_k \psi_k e^{ikj}$ we can write this as

$$H = 2 \sum_{0 \leq k \leq \pi} \psi_k^+ H_k \psi_k$$

(17)

where $\psi_k = (c_k, c_{-k}^\dagger)^T$ is a two component vector. Thus each k-mode of such systems can be described by the following Hamiltonian (where we scale everything by $\gamma$)

$$H(k, t) = (\mu - \cos(k)) \sigma_z + \Delta \sin(k) \sigma_x$$

(18)

and we use the following drive protocol $\mu = A \cos(\omega t + \phi(t))$ and $\Delta = \cos(r \omega t)$ where $r$ is an integer and $\phi$ is as usual a random variable at each time $t$. The dynamics of this model is non-trivial for $r > 1$ due to the non-removable time dependence in both diagonal and off-diagonal element.\cite{39,40}. This model (with $\phi(t) = 0$) has a phaseband crossing for $k = \pi/2$ at $t = T/2$ which exists at all frequencies. We study here what happens to this crossing if at each instant of time $\phi$ is a random Gaussian variable with zero mean. Below we mention the scheme for partitioning a full period to calculate the noise averaged $U(t, 0)$ now at any time $t \leq T$

$$\delta t = \frac{t}{N} = \text{const}$$

(19)

i.e we increase no of partitions proportionally as the time $t$ gets closer to $T$ keeping the duration of constant time evolution($\delta t$) fixed. Thus we calculate noise averaged phaseband at all time $t$ within a period for different noise strength ($\sigma$) and compare it with noise free case in Figure 8(a). Interestingly noise modifies the phaseband at all times except at $t = T/2$ which is the phaseband crossing point for noise free drive. This shows that the transition at $t = T/2$ is immune to any amount of temporal disorder. As a routine task we calculate the noise averaged instantaneous Hamiltonian for the chosen protocol

$$\langle H(k = \frac{\pi}{2}, t) \rangle = A \cos(\omega t) e^{-\sigma^2/2} \sigma_z + \cos(r \omega t) \sigma_x$$

(20)

In Fig.8(left panel) we see time evolution governed by this averaged $H$ mimics the numerically disorder averaged $U$ operator as like before. We note that this numerical agreement leads to the following statement “The effect of random noise is just to renormalize the laser amplitude”

$$\tilde{A} = Ae^{-\sigma^2/2}$$

(21)
The robustness of the transition at $t = T/2$ also follows from the symmetry of Eq.\[18\]. Note that the symmetry of the noise free Hamiltonian for $k = \pi/2$ and for odd $r$ (namely $H(T/2 - t) = -H(t)$) is not destroyed by the insertion of noise here (see Fig.8(right panel)). This can be used together with the Trotter like decomposition of $U$ operator (as in Eq.\[2\]) to show $U^{-1}(T/2) = U^{(T/2)} = U(T/2)$ signifying that a crossing through Floquet zone-center will always be there at $t = T/2$ for all parameter values ($A$, $\omega$, $\phi$ etc). Further right panel of Fig.8 demands that the same adiabatic-impulse method (as done for the noise free case in ref.\[33\]) can be used to show the existence of the crossing at $t = T/2$ in spite of the change in sizes of different adiabatic regions.

IV. DISCUSSION

In this work we have studied the existence of self-averaging limit in graphene irradiated by unpolarized light. We see the limit holds in high-frequency regime and can be captured by the noise-averaged Hamiltonian. In low frequencies the limit is achieved very slowly as a possible consequence of retaining two of the symmetries in noise-averaged Hamiltonian. This opens up an opportunity to search for some other deterministic Hamiltonian for speeding up the convergence to asymptotic limit. We hardly found any steady limit at extremely low frequencies to the best of our numerical ability. Floquet topological transitions are found to be modified by the insertion of noise to various degrees depending on the $k$-point in BZ. These range from a small shift in crossing positions to complete abolition of the transition depending on the amount of disorder. We find that certain $k$-points are more affected as a consequence of a change in Fourier structure of their time-dependent Hamiltonian induced by the noise. The presence of a 6-fold symmetry at $\Gamma$ point plays a crucial role for the existence of a special type of crossings which simultaneously happens at $T/3, 2T/3, T$\[33\]. This kind of crossings are ubiquitous in low frequencies but ceases to exist in high frequency (scanning the whole parameter regime as much as possible we found they are absent below $T \approx 1$). Now breaking of 4 out of those 6-symmetries by the noise abolishes these transitions confirming again the importance of symmetries in low frequencies. In 1D systems due to the simplicity of the BZ, noise obeys all symmetries of the clean time-dependent Hamiltonian and as a consequence crossings persist at all noise strengths. It merely renormalizes the drive amplitude.

In typical experiments one needs to keep the optical axis of a quarter wave plate exactly at 45° with the plane of vibration of the incident plane polarized light to extract pure circularly polarized light. Now if this angle changes randomly (which is always present in small amount if the experiment is not performed carefully such as a small vibration of the table on which the set up lies may cause it) then the polarization of the outgoing light will also fluctuate. One can also use synthetic gauge fields to produce such noisy vector potential. This kind of perturbation is very common in an interference experiment if incoherent sources are used. The quantitatively different noise-response from various $k$-points can be experimentally verified by measuring the photoinduced gap in a momentum resolved manner using pump-probe spectroscopy as done in ref\[22\]. The abolition of transition and hence a change in topological structure of the Floquet bands can be detected by analyzing the intensity and angular dependence of ARPES spectra\[23\].

In conclusion we have shown random noise in the vector potential of incident light has significant impact on Floquet topological transition in graphene. One can analyze the symmetries and Fourier structure of the noise-averaged Hamiltonian to understand the modifications done by the noise. In 1D systems such noisy drives has no effect on the transitions.

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