Supernovae constraints on models of dark energy reexamined

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We use the Type Ia Supernova gold sample data of Rieess et al in order to constrain three models of dark energy. We study the Cardassian model, the Dvali-Turner gravity modified model and the generalized Chaplygin gas model of dark energy - dark matter unification. In our best fit analysis for these three dark energy proposals we consider flat model and the non-flat model priors. We also discuss the degeneracy of the models with the XCDM model through the computation of the so-called jerk parameter.

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I. INTRODUCTION

The surprising discovery of the present late-time acceleration of the Universe [1] and the related fact that most of its energy is in the form of a mysterious dark energy is possibly one of the most puzzling issues of modern cosmology. Several scenarios have been put forward as a possible explanation. A positive cosmological constant, although the simplest candidate, is not particularly attractive given the extreme fine tuning that is required to account for the observed accelerated expansion. This fact has led to models where the dark energy component varies with time, such as quintessence models [1]. In these models, the required negative pressure is achieved through the dynamics of a single (light) scalar field [2] or, in some cases, two coupled scalar fields [3]. Despite some pleasing features, these models are not entirely satisfactory, since in order to achieve $\Omega_X \sim \Omega_m$ (where $\Omega_X$ and $\Omega_m$ are the dark energy and matter energy densities at present, respectively) some fine tuning is required. Many other possibilities have been considered for the origin of this dark energy component such as a scalar field with a non-standard kinetic term and k-essence models [4]; it is also possible to construct models which have $w_X = p/\rho < -1$, the so-called phantom energy models [5].

Recently, it has been proposed that the evidence for a dark energy component might be explained by a change in the equation of state of the background fluid, with an exotic equation of state, the generalized Chaplygin gas (GCG) model [6, 7, 8]. The striking feature of this model is that it allows for an unification of dark energy and dark matter [8].

Another possible explanation for the accelerated expansion of the Universe could be the infrared modification of gravity one should expect from, for instance, extra dimensional physics, which would lead to a modification of the effective Friedmann equation at late times [9, 10, 11, 12]. An interesting variation of this proposal has been suggested by Dvali and Turner [10] (hereafter referred to as DT model). Another possibility, also originally motivated by extra dimensions physics, is the modification of the Friedmann equation by the introduction of an additional nonlinear term proportional to $\rho^n$, the so-called Cardassian model [13].

Currently type Ia supernovae (SNe Ia) observations provide the most direct way to probe the dark energy component at low to medium redshifts. This is due to the fact that supernova data allows for a direct measurement of the luminosity distance, which is directly related to the expansion of the Universe, which, in turn, is the physical quantity that is directly related with dark energy or that is modified by extra dimensional physics. This approach has been explored by various groups in order to obtain insight into the nature of dark energy. Indeed, recently supernova data with 194 data points has been analysed [14] and it was shown that it yields relevant constraints on some cosmological parameters. In particular, it is possible to conclude that, when one considers the full supernova data set, the decelerating model is ruled out with a significant confidence level. It is also shown that one can measure the current value of the dark energy equation of state with higher accuracy and the data prefers the phantom kind of equation of state, $w_X < -1$. Furthermore, the most significant result of that analysis is that, without a flat prior, that supernova data does not favor a flat ΛCDM model at least up to 68% confidence level, which is consistent with other cosmological observations. In what concerns the equation of state of the dark energy component, it has been shown in Ref. [15], using the same set of supernovae data, that the best fit equation of state of dark energy evolves rapidly from $w_X \simeq 0$ in the past to $w_X \lesssim -1$ in the present, which suggests that a time varying dark energy is better fitted with the data than the ΛCDM model. This result is also robust to changes of $\Omega_m$ and remains valid for the inter-

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5 An evolving vacuum energy was discussed somewhat earlier, see e.g. Refs. [2].
Supernova data has also been used in the context of different cosmological models for dark energy \[16, 17, 18, 19, 20, 21\]. In this paper, we analyze the Cardassian, the DT and the GCG models in light of the Riess et al. SNe Ia compilation of data \[22\]. We consider both flat and non-flat priors.

Our result is that the analysis is restricted to the very late history of the Universe and does not address dark energy effects on the cosmic microwave background fluctuations or on structure formation.

Section II is devoted to the discussion of the degeneracy of the discussed models with the XCDM model through the introduction of the so-called jerk parameter. In section VII we present our conclusions.
TABLE II: Best fit parameters for the Cardassian, DT and GCG models, for the two data samples of Riess et al., considering flat and non-flat priors. The best fit value used for $M^*$ is 43.3.

FIG. 1: Confidence contours in the $\Omega_m - n$ parameter space for the flat Cardassian model. The solid and dashed lines represent the 68% and 95% confidence regions, respectively; in the left panel are shown the results for the gold sample without the HST SNe Ia, whereas in the right panel, the full gold sample is taken into account.

### III. CARDASSIAN MODEL

We first consider the so Cardassian model, which justifies the late time accelerating universe by invoking an additional term in the Friedmann equation proportional to $\rho^n$. In this model, the universe is composed only of radiation and matter (including baryon and cold dark matter) and the increasing expansion rate is given by

$$H^2 = \frac{8\pi}{3M_{Pl}^2} (\rho + b\rho^n) - \frac{k}{a^2}$$  \hspace{1cm} (8)$$

where $M_{Pl} = 1.22 \times 10^{19}$GeV is the 4-dimensional Planck mass, $b$ and $n$ are constants, and we have added a curvature term to the original Cardassian model. At present, the universe is matter dominated, i.e. $\rho_m \gg \rho_{rad}$, hence $\rho \approx \rho_m$. The new term dominates only recently, at about $z \sim 1$, hence in order to get the recent acceleration in the expansion rate, $n < 2/3$ is required.

The theoretical motivation for the Cardassian term is fairly speculative. It can be argued that its origin may arise as a consequence of embedding our (3+1)-dimensional brane universe in extra-dimensions, or from some unknown interactions between matter particles.

In a matter dominated universe, equation (8) can be
rewritten as

\[
\left( \frac{H}{H_0} \right)^2 = \Omega_m (1 + z)^3 + \Omega_k (1 + z)^2 + (1 - \Omega_m - \Omega_k)(1 + z)^{3n},
\]  

where \( H_0 \) is the present day value of the Hubble constant and \( \Omega_k = -\frac{k}{a^2 H_0^2} \) is the present curvature parameter.

Notice that the only parameter of the model is \( n \) and

\( \Omega_k = -0.75 \)

\( \Omega_k = 0.47 \)

\( n = -0.93 \)

\( n = -3.1 \)

\( \Omega_m = 0.97 \)

\( \Omega_m = 0.21 \)
that the case $n = 0$ corresponds to the $\Lambda$CDM model.

The best fit results for this model, for the two data samples we are considering, and taking into account flat and non-flat priors, are summarized in Table II.

For the flat case, we have only two parameters and the best fitting results we get are $\{\Omega_m, n\} = \{0.53, -2.0\}$, without HST, and $\{\Omega_m, n\} = \{0.49, -1.4\}$, with HST SNe Ia. In Fig. 1 we show the 68% and 95% confidence regions, respectively; in the left panel are shown the results for the gold sample without the HST SNe Ia, whereas in the right the full gold sample is taken into account.

The contour plots are presented in Fig. 2.

**IV. DVALI-TURNER MODEL**

The second model we will consider is the one proposed by Dvali & Turner [10], where the Friedmann equation is modified by the addition of the term $H^2/r^{2-\beta}$, which can arise in theories with extra dimensions [11]; $r_c$ is a crossover scale which sets the scale beyond which the laws of the 4-dimensional gravity breakdown and become 5-dimensional. In this case, in contrast with theories with infinite volume extra dimensions, the laws of gravity are modified in the far infrared and the cosmological evolution gets modified at late times; the short distance gravitational dynamics is very close to that of the 4-dimensional Einstein gravity, hence the early times cosmological evolution is very close to the Friedmann-Robertson-Walker picture.

As a motivation for this type of modification of gravity, consider the model with a single extra dimension [11, 12], with the effective low-energy action given by

$$S = \frac{M_P^2}{r_c} \int d^4 x \, dy \sqrt{g^{(5)}} \mathcal{R} + \int d^4 x \sqrt{g} \left( M_P^2 \mathcal{R} + L_{SM} \right),$$

where $y$ is the extra spatial coordinate, $g^{(5)}$ is the trace of the 5-dimensional metric $g^{(5)}_{AB}$ ($A, B = 0, 1, 2, ..., 4$), $g$ the trace of the 4-dimensional metric induced in the brane, $g_{\mu\nu}(x) \equiv g^{(5)}_{\mu\nu}(x, y = 0)$. The first term in the action is the bulk 5-dimensional Einstein action, where $\mathcal{R}$ is the 5-dimensional Ricci scalar, and the second one is the 4-dimensional Einstein term, localized on
the brane (at $y = 0$), where $R$ is the 4-dimensional Ricci scalar and $\mathcal{L}_{\text{SM}}$ is the Lagrangian of the fields in the Standard Model. For the maximally symmetric FRW ansatz, $ds_5^2 = f(y, H)ds_4^2 - dy^2$, where $ds_4^2$ is the 4-dimensional maximally-symmetric metric, one gets the modified Friedmann equation on the brane

$$H^2 \pm \frac{H}{r_c} = \frac{8\pi}{3M_{Pl}^2} \rho ,$$

where $\rho$ is the total energy density in the brane.

Inspired in this construction, Dvali & Turner considered a more generic (and radical) modification of the Friedmann equation [10]

$$H^2 - \frac{H^\beta}{r_c^{2-\beta}} = \frac{8\pi}{3M_{Pl}^2} \rho - \frac{k}{a^2} ,$$

where we have also introduced the curvature term in the brane. The crossover scale $r_c$ is fixed in order to eliminate the need for dark energy,

$$r_c = H_0^{-1} (1 - \Omega_m - \Omega_k)^{1/(\beta - 2)} ,$$

where we assume again that the universe is matter dominated. The Friedmann expansion law can then be written as

$$\left( \frac{H}{H_0} \right)^2 = \Omega_m (1 + z)^3 + (1 - \Omega_m - \Omega_k) \left( \frac{H}{H_0} \right)^\beta + \Omega_k (1 + z)^2 .$$

Notice that $\beta$ is the only parameter of the model: for $\beta = 0$ the new term behaves like a cosmological constant, and for $\beta = 2$ it corresponds to a “renormalization” of the Friedmann equation. Note also that the case $\beta = 1$ corresponds to the model in Eq. [11] hereafter called Dvali-Gabadadze-Porrati (DGP) model [11]. The successful predictions of the Big-Bang nucleosynthesis impose a limit on $\beta$, namely, $\beta \leq 1.95$; a more stringent bound follows from requiring that the new term does

FIG. 4: Confidence contours in the $\Omega_m - \beta$ and $\Omega_k - \Omega_m$ parameter space for the non-flat DT model using the gold sample without HST SNe Ia (left panel) and in the $\Omega_m - \beta$ and $\Omega_k - \beta$ plane using the full gold sample (right panel). As in Figure 3, the solid and dashed lines represent the 68% and 95% confidence regions, respectively.
not interfere with the formation of large-scale structure: \( \beta \leq 1 \). Moreover, it can be shown that, in the recent past \((10^4 > z > 1)\), this correction behaves like dark energy with equation of state \( w_{eff} = -1 + \beta/2 \), and \( w = -1 \) in the distant future and, moreover, it can mimic \( w < -1 \) without violating the weak-energy condition \([10]\).

For the flat DT model, the best-fitting parameters are \( \{\Omega_m, \beta\} = (0.55, -60.0) \) without HST and \( \{\Omega_m, \beta\} = (0.51, -19.2) \) with HST supernovae, as is summarized in Table II. From Fig. 3 where the 68% and 95% confidence contour plots are shown, it is clear that \( \beta \) is very weakly constrained by both supernovae data sets, and can become arbitrarily large and negative. Moreover, the cosmological constant, corresponding to \( \beta = 0 \), seems to be disfavoured; also \( \beta = 1 \), the DGP model, is strongly disfavoured. These results are consistent with those of Elgaroy et al. \([13]\) obtained with another data set.

If we fix \( \beta = 1 \) (DGP model), and allow only \( \Omega_m \) to vary, the best fit value is \( \Omega_m = 0.17 \) and \( \Omega_m = 0.16 \) for the gold sample and gold sample without HST SNe Ia, respectively, which are consistent with the results of Gong & Duan \([13]\).

Now, if we relax the flat prior, we find that \( \{\Omega_k, \beta\} = (-0.71, -15.6) \) are the best fit values for the gold sample without HST SNe Ia (Table II); the best fit value for \( \Omega_m \) is in the end of the variation range we considered for this parameter, \( \Omega_m = 1 \). As we can see from the contour plots shown in Fig. 3, \( \Omega_m \) cannot be constrained at 95% confidence level, however the 68% confidence contours show that data prefer \( \Omega_m > 0.25 \). In what concerns \( \beta \), it can become arbitrarily large and negative, if we allow \( \Omega_m \) to be large. Moreover, both \( \Lambda \)CDM and DGP models are disfavored with 68% confidence.

Once the HST data brings the amount of matter to lower values and pushes curvature to positive values. The best fit results are \( \{\Omega_m, \Omega_k, \beta\} = (0.24, 0.43, -60, 0) \). Also in this case both \( \Lambda \)CDM and DGP models are disfavored.

V. GENERALIZED CHAPLYGIN GAS MODEL

Finally, we consider the generalized Chaplygin gas model, which is characterized by the equation of state

\[
\rho_{ch} = \frac{A}{\rho_{ch0}^\alpha},
\]

(15)

where \( A \) and \( \alpha \) are positive constants. For \( \alpha = 1 \), the equation of state is reduced to the Chaplygin gas scenario \([7]\).

Integrating the energy conservation equation with the equation of state \([16]\), one gets \([2]\)

\[
\rho_{ch} = \rho_{ch0} \left[ A s + \frac{(1 - A s)}{a^{3(1+\alpha)}} \right]^{1/(1+\alpha)},
\]

(16)

where \( \rho_{ch0} \) is the present energy density of GCG and \( A_s \equiv A/\rho_{ch0}^{1+\alpha} \). One of the most striking features of this expression is that the energy density of this GCG, \( \rho_{ch} \), interpolates between a dust dominated phase, \( \rho_{ch} \propto a^{-3} \), in the past and a de-Sitter phase, \( \rho_{ch} = -p_{ch} \), at late times. This property makes the GCG model an interesting candidate for the unification of dark matter and dark energy. Moreover, one can see from the above equation that \( A_s \) must lie in the range \( 0 \leq A_s \leq 1 \): for \( A_s = 0 \), GCG behaves always as matter whereas for \( A_s = 1 \), it behaves always as a cosmological constant. We should point out, however, that if we want to unify dark matter and dark energy, one has to exclude these two possibilities resulting the range for \( A_s \) as \( 0 < A_s < 1 \). Notice also that \( \alpha = 0 \) corresponds to the \( \Lambda \)CDM model.

The Friedmann equation for a non-flat unified GCG model in general is given by

\[
\left( \frac{H}{H_0} \right)^2 = \left[ 1 - \Omega_k \right] \left[ A_s + \left( 1 - A_s \right) \left( 1 + z \right)^{3(1+\alpha)} \right]^{1/(1+\alpha)} + \Omega_k \left( 1 + z \right)^2.
\]

(17)

This model has been thoroughly scrutinized from the observational point of view; indeed, its compatibility with the Cosmic Microwave Background Radiation (CMBR) peak location and amplitudes \([28, 29]\), with SNe Ia data \([17, 20]\), and gravitational lensing statistics \([31, 32]\) has been analysed by various groups. The issue of structure formation \([5, 6, 17]\) and its difficulties \([34]\) have been recently addressed \([33]\). More recent analysis, based on the 194 SNe Ia data points from Ref. \([22]\), has yielded interesting results in what concerns the allowed parameter space of the model \(\Lambda\)CDM:

1) Data favors \( \alpha > 1 \), although there is a strong degeneracy on \( \alpha \). At 68% confidence level the minimal allowed values for \( \alpha \) and \( A_s \) are 0.78 and 0.778, which rules out the \( \Lambda \)CDM model \(\alpha = 0 \) case. However, at 95% confidence level it is found that there is no constraint on \( \alpha \).

2) Dropping the assumption of flat prior, it is found that GCG is consistent with data for values of \( \alpha \) sufficiently different from zero. Allowing for some small curvature, positive or negative, one finds that the GCG model is a more suitable description than the \( \Lambda \)CDM model.

These results are similar to the ones obtained in Refs. \([14, 15]\), where it is concluded that the supernova data of Ref. \([22]\) favors “phantom”-like matter with an equation of state of the form \( p = \omega \rho \) with \( \omega < -1 \).

In our present analysis, we have new results both with and without flat prior. For the flat case, the 68% and 95% confidence level contours are quite similar both with and without HST data with that obtained by \(\Lambda\)CDM model as shown in Fig. 6. But still there is one interesting feature to note. It appears now that without the HST data, \(\Lambda\)CDM model \(\alpha = 0 \) is ruled out even at 95% confidence level which was not the case in the previous analysis. But incorporating the HST data, again makes \(\Lambda\)CDM model consistent at 95% C.L although it is still ruled out at 68% C.L.
When we relax the condition of flat prior, one can have some interesting features (see Fig. 6). Without the HST
data, supernova data prefers a nonflat model, but flat case is consistent both at 68% and 95% C.L if one choose a value for \( \alpha \) sufficiently different from zero. This means without flat prior, and without HST data, data prefers a curved model for GCG but flat case is still consistent which is similar to the conclusion in [17].

But now if one includes the HST data, the best fit model itself becomes a very close to flat case (\( \Omega_k = 0.02 \), Table [I]) which is completely a new feature. This shows that with the new HST data, the current gold sample of supernova data prefers a flat GCG model which is consistent with CMBR observation. It also shows that GCG is a better choice among the three possible choices of alternative models that we have considered here as far the gold sample of supernova data including those from HST measurements are concerned.

VI. DEGENERACY WITH XCDM MODEL

To illustrate the degeneracy between our models and the XCDM model, with constant equation of state \( p/\rho = w_X \) and dark energy density \( \Omega_X = 1 - \Omega_m \), let us consider the Taylor expansion of the luminosity distance as

\[
d_L = \frac{c}{H_0} \left[ z + \frac{(1 - q_0)}{2} z^2 - \frac{1}{6} (1 - q_0 - 3q_0 + j_0) z^3 + \ldots \right],
\]

where \( q_0 \) is the deceleration parameter, related with the second derivative of the scale-factor, and \( j_0 \) is the so-called statefinder or jerk parameter [36], related with the third derivative of the scale-factor. The subscript “0” means that quantities are evaluated at present. The jerk parameter is related with the deceleration parameter \( q_0 \) as

\[
j_0 = q_0 + 2q_0^2 + \left. \frac{dq}{dz} \right|_{z=0}.
\]

Notice that the jerk parameter is related to the geometry of the Universe (see Ref. [37], where it is shown that the measurement of the cubic correction to the Hubble law via high-redshift supernovae is the first cosmological measurement, after the CMBR, that probes directly the effects of spatial curvature). We have calculated \( q_0 \) and \( \frac{dq}{dz} \bigg|_{z=0} \) for our models and obtained the results shown in Table [II]. We have considered only the flat case and, for simplicity, we study the DGP model instead of the more generic DT model; \( \Omega_m \) is the only parameter of this model.

For low redshifts, it is sufficient to consider the first two terms in the series expansion of the luminosity distance, Eq. (13). From the expression of \( q_0 \) for the GCG, we can see that in this case the SNe Ia can only constrain \( A_s \), as \( q_0 \) is independent of \( \alpha \). However for the Cardassian model the \( q_0 \) dependence is both on \( \Omega_m \) and \( n \), allowing low redshifts SNe Ia to put constraints in the model.

Moreover, in order to have degeneracy between the models we are analyzing and XCDM model, the \( q_0 \) parameter of these models must be equal, \( q_0^{XCDM} = q_0^{\text{Model } i} \), which results that:

\[
w_X = (n - 1) \frac{1 - \Omega_m}{\Omega_X} \quad \text{for Cardassian model;}
\]

\[
w_X = -\frac{1 - \Omega_m}{(1 + \Omega_m)\Omega_X} \quad \text{for DGP model;}
\]

\[
w_X = -\frac{A_s}{\Omega_X} \quad \text{for GCG model}.
\]

If one goes to higher redshifts, one also has to consider the higher order terms in the expansion of \( d_L(z) \); hence, in this case also the jerk parameters have to be equal, which means that \( \frac{d^2q}{dz^2} \big|_{z=0} \) have to be the same. We get that for Cardassian model

\[
w_X = n - 1 \ ,
\]

\[
\Omega_X = 1 - \Omega_m \ ,
\]

meaning that the dynamical evolution of this model is equivalent to a dark energy model with the same matter density and a constant equation of state given by \( w_{\text{eff}} = n - 1 \); the equivalent dark energy potential can be written as \( V(\phi) = A \left[ \sinh k(\phi/\sigma + C) \right]^{-\sigma} \) with \( \sigma = -2 - 2/(n - 1) \) (see also Ref. [38]). We have seen that negative values of \( n \) are preferred which is consistent with the fact that phantom equation of state, \( w_X < -1 \), is favored by the data [14, 15, 17].

For DGP model one finds

\[
w_X = \frac{\Omega_m^2 - 2\Omega_m - 1}{(1 + \Omega_m)^2} \ ,
\]

\[
\Omega_X = \frac{\Omega_m^2 - 1}{\Omega_m^2 - 2\Omega_m - 1} \ .
\]

We see that for \( \Omega_m < 0.6 \) the the equation of state is phantom-like and, as \( \Omega_m \to 0 \), \( w_X \to -1 \). We saw that the SNe Ia prefer \( \Omega_m \sim 0.2 \), consistent with the fact that phantom equation of state is favored. Moreover, the amount of matter needed in a XCDM model to be equivalent to a given DGP model is larger, \( \Omega_m^{XCDM} = 1 - \Omega_X \geq \Omega_m \).

For the GCG model we find

\[
w_X = -\alpha (1 - A_s) - 1 \ ,
\]

\[
\Omega_X = \frac{A_s}{1 + \alpha (1 - A_s)} \ .
\]

We see that for any GCG model, the corresponding dark energy model equation of state has to be always phantom type [17], as \( \alpha > 0 \) and \( 0 < A_s < 1 \). Nevertheless, for low redshift data, GCG is degenerate with all kinds of constant equation of state dark energy model, including \( \Lambda \)CDM model, as can be inferred from Eq. (22).
VII. DISCUSSION AND CONCLUSIONS

In this paper we have performed likelihood analysis of the latest type Ia supernova data for three distinct dark matter models. We have considered the Cardassian model, the modified gravity Dvali-Turner model and the generalized Chaplygin gas model of unification of dark energy and dark matter. We find that SNe Ia most recent data allows, in all cases, for non-trivial constraints on model parameters as summarized in Table II. We find that for all models relaxing the flatness condition implies that data favors then a considerable negative curvature for the gold without HST dataset. For the gold dataset the resulting best fit value for the curvature is positive (the GCG model is nearly flat in this case). For all models we have found, in what concerns the deceleration and jerk parameters, the conditions under which they are degenerate to the XCDM model. Thus, SNe Ia data clearly favors phantom-like equivalent equations of state.

Finally, in what concerns the gold sample of supernova data including those from HST measurements, our analysis reveals that that the GCG flat model is the better choice among the three possible alternative models that we have considered in this paper.

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\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Model & \(q_0\) & \(\frac{\Delta \chi^2}{2} \) \\
\hline
XCDM & \( \frac{1}{3} + \frac{2}{3} w_X \Omega_X \) & \( \frac{3}{2} w_X \Omega_X (1 - \Omega_X) \) \\
Cardassian & \( \frac{1}{3} + \frac{2}{3} (n - 1)(1 - \Omega_m) \) & \( \frac{3}{2} (n - 1)^2 \Omega_m (1 - \Omega_m) \) \\
DGP & \( \frac{1}{3} + \frac{2}{3} \Omega_m \) & \( \Omega_m (1 - \Omega_m) \) \\
GCG & \( \frac{1}{3} (1 - A_X) - 1 \) & \( \frac{3}{2} A_X (1 - A_X) (1 + \alpha) \) \\
\hline
\end{tabular}
\caption{Deceleration and jerk parameters for XCDM, Cardassian, DGP and GCG models.}
\end{table}
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