E6 Unification with Bi-Large Neutrino Mixing

Masako Bando and Nobuhiro Maekawa

Physics Division, Aichi University, Aichi 470-0296, Japan
Department of Physics, Kyoto University, Kyoto 606-8502, Japan

(Received )

The scenario of SO(10) grand unified theory (GUT) proposed by one of the authors is extended to E6 unification. This gives realistic quark and lepton mass matrices. In the neutrino sector, the model reproduces the large mixing angle (LMA) MSW solution as well as the large mixing angle for the atmospheric neutrino anomaly. In this model, the right-handed down quark and the left-handed lepton of the first and second generations belong to a single multiplet 27. This causes natural suppression of the flavour changing neutral current (FCNC).

§1. Introduction

There are strong reasons to believe in the validity of grand unified theories (GUT), in which the quarks and leptons are unified in several multiplets in a simple gauge group. They explain various matters that cannot be understood within the standard model: the ‘miracle’ of anomaly cancellation between quarks and leptons, the hierarchy of gauge couplings, charge quantization, etc. The three gauge groups in the standard model are unified into a simple gauge group at a GUT scale, which is considered to be just below the Planck scale. On the other hand, the GUT scale destabilizes the weak scale. One of the most promising ways to avoid this problem is to introduce supersymmetry (SUSY). However, it is not easy to obtain a realistic SUSY GUT. First, it is difficult to obtain realistic fermion mass matrices. Also unification of quarks and leptons puts strong constraints on the Yukawa couplings. Finally, one of the most difficult obstacles is the “doublet-triplet (DT) splitting problem.”

There have been several attempts to solve the DT splitting problem. Among them, the Dimopoulos-Wilczek mechanism is a promising way to realize DT splitting in the SO(10) SUSY GUT.

Concerning the fermion masses, recent progress in neutrino experiments provides important information on family structure. There are several impressing works in which the large neutrino mixing angles are realized within GUT framework. It is now natural to examine SO(10) and higher gauge groups because they allow for every quark and lepton, including the right-handed neutrino, to be unified in a single multiplet, which is important in addressing neutrino masses.

Recently, one of the authors (N. M.) proposed a scenario of SO(10) grand unified theory (GUT) with anomalous U(1)A gauge symmetry, which has the following

* E-mail: bando@aichi-u.ac.jp
** E-mail: maekawa@gauge.scphys.kyoto-u.ac.jp
interesting features:
1. The doublet-triplet (DT) splitting is realized using the Dimopoulos-Wilczek mechanism.
2. Proton decay via the dimension-five operator is suppressed.
3. Realistic quark and lepton mass matrices can be obtained in a simple way. In particular, in the neutrino sector, bi-large neutrino mixing is realized.
4. The symmetry breaking scales are determined by the anomalous $U(1)_{A}$ charges.
5. The mass spectrum of the super heavy particles is fixed by the anomalous $U(1)_{A}$ charges.

As a consequence of the above features, the fact that the GUT scale is smaller than the Planck scale leads to modification of the undesirable GUT relation between the Yukawa couplings $y_{\mu} = y_{s}$ (and also $y_{e} = y_{d}$) while preserving the relation $y_{\tau} = y_{b}$. Moreover, it is remarkable that the interaction is generic; all the interactions that are allowed by the symmetry are taken into account. Therefore, once we fix the field contents with their quantum numbers, all the interactions are determined, except coefficients of order 1.

The anomalous $U(1)_{A}$ gauge symmetry, whose anomaly is cancelled by the Green-Schwarz mechanism, plays an essential role in explaining the DT splitting mechanism at the unification scale as well as in reproducing Yukawa hierarchies. Also, bi-large neutrino mixing is naturally obtained by choosing the $10$ representation with an appropriate $U(1)_{A}$ charge, in addition to the three family $16$ representations. This anomalous $U(1)_{A}$ is a powerful tool not only to reproduce DT splitting but also to determine the GUT breaking scales.

This paper aims to show further that the above $SO(10)$ model is naturally extended to $E_{6}$ GUT, in which the additional field $10$ of $SO(10)$ is included in a chiral multiplet $27$ of $E_{6}$. In order to realize this scenario, it is important to introduce the concept of “twisting family structure” in the $E_{6}$ unified model.

Under the $SO(10)$ group, we know that $10$ and $5$ of $SU(5)$ are combined into $16$ of $SO(10)$. Usually, each family belongs to $16$. In this framework, however, it is not easy to reproduce the large Maki-Nakagawa-Sakata (MNS) mixing and small Cabbibo-Kobayashi-Maskawa (CKM) mixing. A promising way to reproduce this is to introduce other multiplets, $10$ of $SO(10)$, in addition to the usual $3 \times 16$ multiplets. Since $10$ of $SO(10)$ is decomposed into $5(10)$ and $\overline{5}(10)$ of $SU(5)$, one of the fields $\overline{5}(16)$ can be replaced by this $\overline{5}(10)$. Such a replacement is essential to reproduce large MNS mixing, preserving small CKM mixing. In the case of $E_{6}$, $16$ and $10$ of $SO(10)$ are naturally included in a single multiplet $27$ of $E_{6}$. The $E_{6}$ model automatically prepares such a replacement, as we see in the next section. We call the mechanism responsible for this the “twisting mechanism.” This gives us a strong motivation to examine $E_{6}$ unification.

It is interesting that the above desired scenario in $SO(10)$ unification can be extended to $E_{6}$ unification while keeping the desirable features of $SO(10)$ unification. In this paper we focus on the extension of the matter sector to $E_{6}$ unification, leaving discussion of DT splitting to a separate paper. The extension of DT splitting to $E_{6}$ unification is non-trivial. Moreover, we show that the condition for the suppression of the flavour changing neutral current (FCNC) is automatically satisfied. This is
essentially caused by the twisting mechanism and the unification of the matter fields into a single multiplet \(27\), which guarantees that \(\mathbf{5}(16)\) and \(\mathbf{10}(10)\) have the same anomalous \(U(1)_A\) charge. Then it can happen that the charge of the first generation of \(\mathbf{5}\) becomes equivalent to that of the second generation of \(\mathbf{5}\). This weakens the severe constraint resulting from the FCNC. It is interesting that the selection of the anomalous \(U(1)_A\) charge to realize bi-large neutrino mixing angles automatically causes the above mentioned FCNC suppression.

In section 2, we briefly review the twisting mechanism and classify the patterns of the massless modes of the \(\mathbf{5}\) fields. In section 3 and the Appendix, we discuss how the SUSY vacua are determined in the anomalous \(U(1)_A\) framework. In section 4 and 5, we study the realistic quark and lepton mass matrices in \(E_6\) unification, where in the neutrino sector, bi-large neutrino mixing angles are naturally realized.

In section 6, we examine the effect of SUSY breaking. Specifically, we are able to automatically obtain a condition for suppression of flavour changing neutral current process \(K^0\bar{K}^0\) mixing) in \(E_6\) unification.

§2. Twisting in \(E_6\) unification

Let us first recall the twisting mechanism, which has been proposed by one of the authors (M. B.).\(^5\) The twisting family structure arising through this mechanism is peculiar to the \(E_6\) unification model, and here we explain how it arises. In the case of \(E_6\), \(16\) and \(10\) of \(SO(10)\) are naturally included in a single multiplet \(27\) of \(E_6\). The fundamental representation of \(E_6\) contains \(16\) and \(10\) of \(SO(10)\) automatically: Under \(E_6 \supset SO(10) \supset SU(5)\), we have

\[
27 \to \begin{bmatrix} (16, 10) + (16, \bar{5}) + (16, 1) \end{bmatrix} + \begin{bmatrix} (10, 5) + (10, 5) \end{bmatrix} + \begin{bmatrix} (1, 1) \end{bmatrix},
\]

(2.1)

where the representations of \(SO(10)\) and \(SU(5)\) are explicitly denoted. As we have already seen, the \(E_6\) model naturally possesses the freedom to replace matter fields \((16, \bar{5})\) by \((10, \bar{5})\). So here let us explain how the twisting family structure arises in the \(E_6\) unification. In order to do this, it is enough to introduce the following Higgs fields\(^6\), which are necessary to determine the mass matrices of matter multiplets \(\Psi_i(27)\), whose \(U(1)_A\) charges are denoted as \(\psi_i^{(x)}\) \((i = 1, 2, 3)\):

1. A Higgs field that breaks \(E_6\) into \(SO(10)\): \(\Phi(27)\) \(\langle \Phi(1, 1) \rangle = v\).
2. A Higgs field that breaks \(SO(10)\) into \(SU(5)\): \(C(27)\) \(\langle C(16, 1) \rangle = v'\).
3. A Higgs field that includes the Higgs doublets: \(H(27)\).

Throughout this paper we denote all the superfields with uppercase letters and their anomalous \(U(1)_A\) charges with the corresponding lowercase letters. Assigning negative R-parity to the ordinary matter \(\Psi_i(27)\), as usual, and using a field \(\Theta\) with charge

---

\(^5\) Note that the additional Higgs fields \(\Phi(27)\) and \(C(27)\) are required to satisfy the \(D\)-flatness condition of \(E_6\) gauge theory, and an adjoint field \(A(78)\) is required to break the GUT gauge group into the standard gauge group. In order to realize doublet-triplet splitting, the actual breaking pattern might be \(E_6 \to SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \to SU(3)_C \times SU(2)_L \times U(1)_{Y} [\text{[4]}]\).

\(^6\) We assume that \(\psi_1 > \psi_2 > \psi_3\)
–1, the $U(1)_A$ invariant superpotential for low energy Yukawa terms becomes

$$W_Y = \left( \frac{\Theta}{M_P} \right)^{\psi_i + \psi_j + h} \Psi_i \Psi_j H,$$

(2.2)

where we omit coefficients of order 1, and for the above we assume that $\psi_i + \psi_j + h \geq 0$ for each $i, j$ pair so that there appears no SUSY zero. After obtaining non-zero VEV $\langle \Theta \rangle = \lambda M_P (\lambda \sim 0.2)$ through the $D$-flatness condition of the anomalous $U(1)_A$ gauge symmetry, a hierarchical structure of the Yukawa couplings is realized. Since we need $3 \times [10 + \bar{5} + 1]$ in $SU(5)$ representations for three families, among the above three $27$ fields, three pairs of $(5, \bar{5})$ must become heavy. Indeed, the Higgs fields $\Phi$ and $C$ can yield such masses: The superpotentials, which give large masses for $(5, \bar{5})$ pairs, are

$$W = \left( \frac{\Theta}{M_P} \right)^{\psi_i + \psi_j + c} \Psi_i \Psi_j C + \left( \frac{\Theta}{M_P} \right)^{\psi_i + \psi_j + \phi} \Psi_i \Psi_j \Phi,$$

(2.3)

which we analyse here to see how $\bar{5}$ fields acquire large masses. The VEV $\langle \Phi(1, 1) \rangle = v$ gives the $3 \times 3$ mass matrix of $\Psi_i(10, 5)\Psi_j(10, 5)$ pairs,

$$(M_{ij}) = \begin{pmatrix} \lambda_{\psi_1}^{2} & \lambda_{\psi_1 + \psi_2} & \lambda_{\psi_1 + \psi_3} \\ \lambda_{\psi_1 + \psi_2} & \lambda_{\psi_2} & \lambda_{\psi_2 + \psi_3} \\ \lambda_{\psi_1 + \psi_3} & \lambda_{\psi_2 + \psi_3} & \lambda_{\psi_3} \end{pmatrix} \lambda^\phi v,$$

(2.4)

while the VEV $\langle C(16, 1) \rangle = v'$ gives the mass terms of $\Psi_i(16, \bar{5})$ and $\Psi_j(10, 5)$,

$$(M'_{ij}) = \begin{pmatrix} \lambda_{\psi_1}^{2} & \lambda_{\psi_1 + \psi_2} & \lambda_{\psi_1 + \psi_3} \\ \lambda_{\psi_1 + \psi_2} & \lambda_{\psi_2} & \lambda_{\psi_2 + \psi_3} \\ \lambda_{\psi_1 + \psi_3} & \lambda_{\psi_2 + \psi_3} & \lambda_{\psi_3} \end{pmatrix} \lambda^{c'} v'.$$

(2.5)

Then, the full mass matrix is

$$\begin{pmatrix} \psi_1(16, 5) & \psi_2(16, 5) & \psi_3(16, 5) & \psi_1(10, 5) & \psi_2(10, 5) & \psi_3(10, 5) \\ \psi_1(10, 5) & \lambda_{\psi_1 + \psi_2 + \psi_3} & \lambda_{\psi_1 + \psi_2 + \psi_3} & \lambda_{\psi_1 + \psi_2 + \psi_3} & \lambda_{\psi_1 + \psi_2 + \psi_3} \end{pmatrix} \lambda^\phi v,$$

(2.6)

where we have defined the parameter $r$ as

$$\lambda^r \equiv \frac{\lambda^c v'}{\lambda^0 v},$$

(2.7)

Note that if the total charge of an operator is negative, the $U(1)_A$ invariance forbids the existence of operators in the action, since the field $\Theta$ with negative charge cannot compensate for the negative total charge of the operator (the SUSY zero mechanism).

$^{**}$ The possible right-handed neutrino modes $\Psi_i(16, 1)$ and $\Psi_i(1, 1)$ also acquire large masses, but here we concentrate on the family structure of $\bar{5}$. 

---

$^*$ Note that if the total charge of an operator is negative, the $U(1)_A$ invariance forbids the existence of operators in the action, since the field $\Theta$ with negative charge cannot compensate for the negative total charge of the operator (the SUSY zero mechanism). 

$^{**}$ The possible right-handed neutrino modes $\Psi_i(16, 1)$ and $\Psi_i(1, 1)$ also acquire large masses, but here we concentrate on the family structure of $\bar{5}$.
which we use frequently in the following discussion. Note that some of the matrix elements become zero if the index becomes negative (the SUSY zero). For the moment we assume that no such zero appears in the superpotential. In general, it is seen that we have three massless modes out of the six $5$ fields by solving the above $3 \times 6$ matrix. However, since the matrix has hierarchical structure, we can easily classify the cases.

1. Under the condition that we have no SUSY zeros, it is evident that the largest mass is either $M_{33}$ or $M'_{33}$, whose ratio is $M'_{33}/M_{33} = \lambda^r$. 
2. $0 < r$ case: In this case $M'_{33}$ is larger than $M'_{33}$, and the pair $(\Psi_3(10, 5), \Psi_3(10, 5))$ is heavy. Next, compare $M'_{23}$ and $M_{22}$, whose ratio is $M'_{23}/M_{22} = \lambda^{r+\psi_3-\psi_2}$. Thus, there are several cases, depending on $r$ and $\psi_3 - \psi_2$.
3. $0 < r < \psi_2 - \psi_3$: In this case, $M'_{23} > M_{22}$, so that the pair $\Psi_3(16, 5)\Psi_2(10, 5)$ becomes heavy, and at the same time the pair $\Psi_2(10, 5)\Psi_1(10, 5)$ obtains a large mass, because $M'_{12}/M_{12} = \lambda^r < 1$. $\Psi_1(10, 5)$, $\Psi_1(16, 5)$ and $\Psi_2(16, 5)$ are left massless. This case is denoted $(1, 1', 2)$. [In this paper, massless mode whose dominant component is $\Psi_i(16, 5)$ ($\Psi_i(10, 5)$) is simply denoted by $i(i')].$
4. $(0 <) \psi_2 - \psi_3 < r$: The pair $\Psi_2(10, 5)\Psi_2(10, 5)$ becomes heavy. Further, this case is divided into two cases, according to the sign of $r + \psi_3 - \psi_1$.
5. $\psi_2 - \psi_3 < r < \psi_1 - \psi_3$: In this case, $M'_{13}$ is larger than $M_{11}$. Thus $\Psi_3(16, 5)\Psi_1(10, 5)$ becomes heavy, and this case also becomes the case $(1, 1', 2)$.
6. $(\psi_2 - \psi_3 <) \psi_1 - \psi_3 < r$: In this extreme case, $M'_{13} < M_{11}$, and thus $\Psi_1(10, 5)\Psi_1(10, 5)$ becomes heavy. Hence all the $\Psi_i(10, 5)$ are heavy states and the $\Psi_i(16, 5)$ are massless modes. This corresponds to the situation in which the three massless $5$ fields (quarks and leptons) belong to $\Psi_i(16, 5)$. This is just the case usually adopted in the $SO(10)$ model. We call this case that of “parallel family structure.” We denote this case simply as $(1, 2, 3)$.
7. $r < 0$ case: This case is easily classified just replacing the $10$ representation with the $16$ representation.

Thus we can classify all the cases as follows:
1. $\psi_1 - \psi_3 < r : (1, 2, 3)$ type.
2. $0 < r < \psi_1 - \psi_3 : (1, 1', 2)$ type.
3. $\psi_3 - \psi_1 < r < 0 : (1, 1', 2')$ type.
4. $r < \psi_3 - \psi_1 : (1', 2', 3')$ type.

If we use SUSY zero coefficients, various types of massless modes can be realized. For example, if $\psi_1 + \psi_3 + \phi < 0$, SUSY zeros appear, and the Yukawa terms $\Psi_3\Psi_i\phi$ ($i = 1, 2, 3$) are forbidden. Hence the mass matrix $M$ becomes

$$
M \rightarrow \begin{pmatrix}
\Psi_1(10, 5) & \Psi_2(10, 5) & \Psi_3(10, 5) \\
\lambda^{\psi_1} & \lambda^{\psi_1+\psi_2} & \lambda^{\psi_1+\psi_2} \\
\lambda^{\psi_1+\psi_2} & (\lambda^{2\psi_2} & 0 \\
0 & 0 & 0 \\
\end{pmatrix} \lambda^{\phi\psi_i}.
$$

and the massless mode $\Psi_3(10, 5)$ does not mix through non-diagonal mass matrix elements with any other $5$ field. We call such a massless field an “isolated” field. There are various different patterns of massless modes containing the “isolated”
fields. For example, if the conditions $2\psi_2 + \phi \geq 0$, $2\psi_3 + c \geq 0$ and $\lambda^{\psi_1+\psi_2+c'}v' \geq 0$, we have the pattern $(1, 2, 3')$, i.e.,

$$
\begin{pmatrix}
\mathbf{3}_1 \\
\mathbf{3}_2 \\
\mathbf{3}_3
\end{pmatrix} = 
\begin{pmatrix}
\psi_1(16, \mathbf{5}) + \cdots \\
\psi_2(16, \mathbf{5}) + \cdots \\
\psi_3(10, \mathbf{5})
\end{pmatrix},
$$

(2.9)

which has been adopted in Ref. [11]. Note that $\mathbf{3}_3$ does not mix with any other states (an isolated field).

In addition to the mixing of the matter content, the massless Higgs doublets itself can in principle be mixed as

$$
H(\mathbf{5}) = H(10, \mathbf{5}) \cos \theta + H(16, \mathbf{5}) \sin \theta,
$$

(2.10)

which is also determined by obtaining a whole mass matrix of the doublet Higgs fields. Note that the Yukawa couplings $\psi_i(16, 10)\psi_j(16, 5)H(\bar{\mathbf{5}})$ $(\psi_i(16, 10)\psi_j(10, \mathbf{5})H(\bar{\mathbf{5}}))$ are proportional to $\cos \theta(\sin \theta)$.

§3. Features of the vacua in the $U(1)_A$ framework

In this section, we explain how the vacua of the Higgs fields are determined by the anomalous $U(1)_A$ quantum numbers.

First, the VEV of a gauge invariant operator with positive anomalous $U(1)_A$ charge must vanish. Otherwise, the mechanism of the SUSY zero does not work, since such a VEV can compensate for the negative $U(1)_A$ charge of the term. Generically, such an undesired vacuum is allowed, but as is shown in the Appendix, in such a vacuum, the Froggatt-Nielsen mechanism does not operate. Therefore we are not interested in such a vacuum. Here, we simply assume that we are in the vacuum where the SUSY zero and Froggatt-Nielsen mechanism operate, namely any VEV of a gauge invariant operator with positive anomalous $U(1)_A$ charge vanishes.

Next, we show that the VEV of a gauge invariant operator $O$ is determined by its $U(1)_A$ charge $o$ as $\langle O \rangle = \lambda^{-o}$ if the $F$-flatness condition determines the VEV. For simplicity, we examine this relation using singlet fields $Z_i$ with anomalous $U(1)_A$ charge $z_i$. The general superpotential is written

$$
W = \sum_i \lambda^{z_i} Z_i + \sum_{i,j} \lambda^{z_i+z_j} Z_i Z_j + \cdots
$$

(3.1)

$$
= \sum_i \tilde{Z}_i + \sum_{i,j} \tilde{Z}_i \tilde{Z}_j + \cdots,
$$

(3.2)

where $\tilde{Z}_i \equiv \lambda^{z_i} Z_i$. The equations for the $F$-flatness of the $Z_i$ fields require

$$
\lambda^{z_i}(1 + \sum_j \tilde{Z}_j + \cdots) = 0,
$$

(3.3)

which generically lead to solutions $\tilde{Z}_j \sim O(1)$ so that $\langle Z_i \rangle \sim \lambda^{-z_i}$, as stated above. Note that the Froggatt-Nielsen structure of Yukawa couplings, $\lambda^{\psi_i+\psi_j+h} \psi_i \psi_j H$, is not changed by the interactions $W = \lambda^{\psi_i+\psi_j+h+z_k} Z_k \psi_i \psi_j H$ with the VEVs $\langle Z_k \rangle \sim \lambda^{-z_k}$. 
If an adjoint field $A$ possesses a non-zero VEV by the $F$-flatness condition, this VEV is determined as $(A) \sim \lambda^{-a}$, because $A^2$ can be gauge invariant. Suppose that, in addition to $\Phi$ and $C$, there are $\Phi^{(27)}$ and $C^{(27)}$. Since $\Phi \Phi$ is also gauge invariant, the VEV of the operator is given by $\langle \Phi \Phi \rangle \sim \lambda^{-(\hat{\phi} + \phi)}$ if it is determined by the $F$-flatness condition. The $D$-flatness condition of the $E_6$ gauge group requires

$$\langle \Phi \rangle = | \langle \Phi \rangle | \sim \lambda^{-(\hat{\phi} + \phi)/2}. \quad (3.4)$$

Note that these VEVs are also determined by the anomalous $U(1)_A$ charges, but they are different from the naive expectation $\langle \Phi \rangle \sim \lambda^{-\phi}$. This is because the $D$-flatness condition plays an important role in fixing the VEVs. The VEVs of $C$ and $\bar{C}$ are also determined by the anomalous $U(1)_A$ charges as

$$| \langle C \rangle | = | \langle \bar{C} \rangle | \sim \lambda^{-(c+\bar{c})/2}. \quad (3.5)$$

By the above argument, it is found that $v$ and $v'$ are determined by the anomalous $U(1)_A$ charges. Therefore, the massless modes of $\bar{5}$, which are determined by the twisting mechanism, are also determined by the anomalous $U(1)_A$ charges.

### §4. Quark and lepton masses in $E_6$ unification

Now let us consider a simple model in which realistic mass matrices of quark and lepton are obtained. Consider the following minimal matter content and Higgs chiral fields. Here, in addition to R-parity, we introduce $Z_2$ parity, which plays an important role in solving the DT splitting problem, as explained in separate papers.\footnote{See Refs. [4, 5].}

1. Matter multiplet (odd R-parity): $\Psi_i^{(27), +}$, $i = 1, 2, 3$.
2. Higgs field which breaks $E_6$ into $SO(10)$: $\Phi^{(27), +}$, $\Phi^{(27), +}$, $\langle \Phi \rangle = \langle \bar{\Phi} \rangle$.
3. Adjoint Higgs field $A^{(78), -}$, whose $SO(10)$ component $A^{(45)}$ breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ by the VEV $\langle A^{(45)} \rangle_{B-L} = i \tau_2 \times \text{diag}(V, V, V, 0, 0)$. (This Dimopoulos-Wilczek form of the VEV plays an important role in solving the DT splitting problem.)
4. Higgs field which breaks $SU(2)_R \times U(1)_{B-L}$ into $U(1)_Y$: $C^{(27), +}$, $\bar{C}^{(27), +}$ by developing $\langle C \rangle = \langle \bar{C} \rangle$.
5. Higgs field which contains usual $SU(2)_L$ doublet: $H^{(27), +}$.

In the above, the signature $\pm$ indicates the $Z_2$ parity of the fields. Here we have introduced Higgs fields $H$ in addition to the other Higgs fields $\Phi$, $\bar{\Phi}$, $C$ and $\bar{C}$, but it might be the case that the Higgs doublet can be a part of a component of the other Higgs fields $\Phi$ and/or $C$. Even in that case, the following argument can be applied by taking $h = \phi$ or $h = c$.

In the following, we take the $U(1)_A$ charges of the matter fields $\Psi_i$ as $\psi_1 = 3 + n$, $\psi_2 = 2 + n$ and $\psi_3 = n$, which have been determined in previous papers to be consistent with the up-type quark masses and mixings. Then the top Yukawa coupling of order 1 determines the anomalous $U(1)_A$ charge of the Higgs field $H$ as $h = -2n$.

Also, in this paper, we assume that the mixing angle $\sin \theta$ [defined in Eq. (2.10)] is zero, i.e., the down-type Higgs is purely $H^{(10,5)}$. This assumption makes the
following analysis much simpler. Of course, once we determine the model that realizes the DT splitting, the Higgs mixing angle is also determined by the anomalous $U(1)_A$ charges. We shall discuss this point in a separate paper. Actually, we find various DT splitting models that give $\sin \theta = 0$.

Now the Yukawa couplings are obtained by Froggatt-Nielsen mechanism as

$$W = \lambda^{\psi_i + \psi_j + h} \Psi_i \Psi_j H, \quad (4.1)$$

where the mass matrix of the up quark sector is uniquely determined, since we have already fixed the $U(1)$ charges of the fields $\Psi_i(27)$:

$$M_u = \begin{pmatrix} \Psi_1(16, 10) & \Psi_2(16, 10) & \Psi_3(16, 10) \\ \Psi_2(16, 10) & \lambda^6 & \lambda^5 & \lambda^3 \\ \Psi_3(16, 10) & \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \langle H(10, 5) \rangle. \quad (4.2)$$

The twisting mechanism discussed in the section 2 causes the down quark mass matrix to differ from that of the up quark.

We examine the massless modes of $\mathbf{5}$ in the following under the assumption $\sin \theta = 0$. Note that in such a situation, component fields $\Psi_i(10, \mathbf{5})$, have no Yukawa couplings, because the Yukawa terms $\Psi_i(10, \mathbf{5}) \Psi(16, 10) H(10, \mathbf{5})$ are forbidden by $SO(10)$ gauge symmetry. This excludes the cases that include isolated $\Psi_i(10, \mathbf{5})$ fields. Moreover, the cases that include isolated $\Psi_i(16, \mathbf{5})$ fields should be discarded, since we cannot obtain large neutrino mixing angles in such cases. With the charge assignment $\psi_1 = n + 3, \psi_2 = n + 2$ and $\psi_3 = n$, the classification discussed in section 2 applies even to cases in which SUSY zeros appear, provided that there are no isolated fields. Among the options $(1, 1', 2), (1, 1', 2), (1, 1', 2')$ and $(1', 2, 3')$, only the option $(1, 1', 2)$ gives realistic quark and lepton mass matrices.

Let us examine an case of $(1, 1', 2)$, i.e., $0 < r < 3$, with the constraints that forbid the existence of an isolated state, $0 \leq \psi_1 + \psi_3 + c$ and $0 \leq \psi_1 + \psi_3 + \phi$. Here, the parameter $r$, which has been defined by $\lambda^r = \lambda^c v'/(\lambda^0 v)$, is given by

$$r = \frac{1}{2} \left[ c - \bar{c} - (\phi - \bar{\phi}) \right], \quad (4.3)$$

because the VEVs $v$ and $v'$ are fixed by the anomalous $U(1)_A$ charges. Note that even if we take the anomalous $U(1)_A$ charges as integers, $r$ can be a half-integer. This fact plays an important role in realizing the bi-large neutrino mixing angles, as we see in the next section. With this case, we investigate which type of mixing pattern of $\mathbf{5}$ fields can reproduce the bi-large neutrino mixing. In order to see this, let us consider the cases $(\mathbf{5}_1, \mathbf{5}_2, \mathbf{5}_3) = (1, 1', 2)$ and $(\mathbf{5}_1, \mathbf{5}_2, \mathbf{5}_3) = (1, 2, 1')$ as phenomenologically viable patterns of the massless three fields $(\mathbf{5}_1, \mathbf{5}_2, \mathbf{5}_3)$. Note that the correct expression of the massless states $\mathbf{5}_i$ at low energy are obtained as mixed states of $\Psi_j$ by solving the mass matrix of Eq.(2.6). It should be remarked that $\Psi_i(10, \mathbf{5})$ itself does not have a Yukawa coupling, and therefore the field $1'$ really can have a Yukawa coupling only through the mixing with $\Psi_i(16, \mathbf{5})$. In order to obtain the exact mass matrix for down quarks as well as leptons, we should take account of
the mixing effects from the non-dominant states. We first fix the three bases of the massless modes \((\bar{\Psi}_1, \bar{\Psi}_2, \bar{\Psi}_3)\) to \((\bar{\Psi}_1(16,\bar{5}), \bar{\Psi}_1(10,\bar{5}), \bar{\Psi}_2(16,\bar{5}))\). On this basis, we can estimate the order of mixing parameters with the heavy states \(\Psi_3(16,\bar{5}), \Psi_2(10,\bar{5})\) and \(\Psi_3(10,\bar{5})\) as

\[
\bar{\Psi}_1 = \Psi_1(16,\bar{5}) + \lambda^{\psi_1-\psi_3}\Psi_3(16,\bar{5}) + \lambda^{\psi_1-\psi_2+r}\Psi_2(10,\bar{5}) + \lambda^{\psi_1-\psi_3+r}\Psi_3(10,\bar{5}) \tag{4.5}
\]

where the first terms on the right-hand sides are the main components of these massless modes, and the other terms are mixing terms with heavy states, \(\psi_2(16,\bar{5})\), \(\psi_2(10,\bar{5})\) and \(\psi_3(10,\bar{5})\). The order of these mixing parameters can be estimated by the ratios of the relevant mass matrix elements. For example, the ratio of the mass matrix element \(M_{k1} = \lambda^{\psi_1+\psi_k+r}\) to \(M'_{k3} = \lambda^{\psi_3+\psi_k+r}\) becomes \(M_{k1}/M'_{k3} = \lambda^{\psi_1-\psi_3}\), which appears in the coefficient of the second term of Eq. (4.4). Note that this ratio is independent of the parameter \(\psi_k\). Similarly, the ratio of \(M'_{k1} = \lambda^{\psi_1+\psi_k+r}\) to \(M_{k3} = \lambda^{\psi_3+\psi_k+r}\) becomes \(M'_{k1}/M_{k3} = \lambda^{\psi_1-\psi_3+r}\), which appears in the coefficient of the third term in Eq. (4.4).

The mass matrices of the down-type quark and charged lepton can be obtained from the above mixing pattern by introducing the RGE factor \(\eta^{-1} \sim 2-3\). We then have

\[
M_D = M_D^T \eta^{-1} = \begin{pmatrix} \Psi_1(16,10) & \bar{\Psi}_2(16,10) & \bar{\Psi}_3(16,10) \end{pmatrix} \begin{pmatrix} \lambda^6 & \lambda^6-r & \lambda^5 \\ \lambda^5 & \lambda^5-r & \lambda^4 \\ \lambda^4 & \lambda^4-r & \lambda^2 \end{pmatrix} \langle H(10,\bar{5}) \rangle, \tag{4.7}
\]

which corresponds to the case \((1,1',2)\), for which \(3-r > 2 \rightarrow 1 > r\). Note that in Eq. (4.5), the main mode of \(\bar{\Psi}_2\) is \(\Psi_1(10,\bar{5})\), which has no Yukawa coupling to \(H(10,\bar{5})\). Therefore the contribution from the mixing term \(\lambda^{\psi_1-\psi_3-r}\Psi_3(16,\bar{5})\) determines the order of the Yukawa couplings. On the other hand, the main modes of \(\bar{\Psi}_1\) and \(\bar{\Psi}_3\) determine the order of the Yukawa couplings, while the contribution of the mixing terms is of the same order.

Now that we have the mass matrices for up and down quarks, we can estimate the CKM matrix as

\[
U_{\text{CKM}} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \tag{4.8}
\]

which is consistent with the experimental value if we take \(\lambda \sim 0.2\). Since the ratio of the Yukawa couplings of top and bottom quarks is \(\lambda^2\), a small value of \(\tan \beta \equiv \frac{\lambda_{\text{top}}}{\lambda_{\text{bottom}}}\)
\[ \langle H(10,5) \rangle / \langle H(10,\bar{5}) \rangle \sim m_t/m_b \cdot \lambda^2 \] is predicted by these mass matrices. The Yukawa matrix for the charged lepton sector is the same as the transpose of \( M_d \) at this stage, except for an overall factor \( \eta \) induced by the renormalization group effect.

\section*{5. Bi-large neutrino mixing in \( E_6 \) unification}

Now we treat the neutrino masses and mixing. In order to do this, we must estimate the mixings in the neutrino mass matrix, since the Maki-Nakagawa-Sakata (MNS) matrix \([24]\) is given by

\[ U_{\text{MNS}} = U_l U_\nu^\dagger , \] (5.1)

with the unitary matrices \( U_l \) and \( U_\nu \) that make the matrices \( U_E (M_E^1 M_E) U_E^\dagger \) and \( U_\nu^a M_\nu U_\nu^b \) diagonal. The matrix \( M_\nu \) is the Majorana mass matrix of the light (almost) left-handed neutrinos, which is obtained from the Dirac masses and right-handed Majorana masses. First, the Dirac neutrino mass matrix is given by the \( 3 \times 6 \) matrix

\[
\begin{pmatrix}
\Psi_1(1,1) & \Psi_2(1,1) & \Psi_3(1,1) & \Psi_1(16,1) & \Psi_2(16,1) & \Psi_3(16,1) \\
5_1 & 5_2 & 5_3 & 10 & 10 & \langle H(10,5) \rangle \eta,
\end{pmatrix}
\]

or we simply express it as

\[ M_N = \left( \begin{array}{cc} \lambda^{r+2} & \lambda^2 \end{array} \right) \otimes \left( \begin{array}{ccc} \lambda^4 & \lambda^3 & \lambda \\ \lambda^{4-r} & \lambda^{3-r} & \lambda^{1-r} \\ \lambda^3 & \lambda^2 & 1 \end{array} \right) \langle H(10,5) \rangle \eta. \] (5.2)

The right-handed Majorana masses come from the interaction

\[ \lambda^{\psi_i+\psi_j+2\bar{\psi}_i \tilde{\psi}_j \tilde{\Phi} \Phi} + \lambda^{\psi_i+\psi_j+\bar{\psi}_i \tilde{\psi}_j \tilde{\Phi} \Phi} + \lambda^{\psi_i+\psi_j+2\bar{\psi}_i \tilde{\psi}_j \tilde{\Phi} \Phi}. \] (5.4)

Then, the \( 6 \times 6 \) matrix for \( \Psi_i(1,1), i = 1, 2, 3 \), and \( \Psi_k(16,1), k = 1, 2, 3 \), the right-handed neutrinos, is expressed as

\[ M_R = \lambda^{\psi_i+\psi_j+2\bar{\psi}_i \tilde{\psi}_j} \Psi_i(1,1) \Psi_j(1,1) \langle \Phi \rangle^2 + \lambda^{\psi_i+\psi_j+\bar{\psi}_i \tilde{\psi}_j} \Psi_i(1,1) \Psi_j(1,1) \langle \Phi \rangle \langle \tilde{C} \rangle \]
\[ + \lambda^{\psi_i+\psi_j+2\bar{\psi}_i \tilde{\psi}_j} \Psi_i(16,1) \Psi_j(16,1) \langle \tilde{C} \rangle^2 \]
\[ = \lambda^{2n} \left( \begin{array}{c} \lambda^{\delta-\phi} \\ \lambda^{(\delta-\phi+\bar{\delta}+\bar{\psi})/2} \\ \lambda^{\delta-\bar{\phi}+c} \end{array} \right) \otimes \left( \begin{array}{ccc} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{array} \right), \] (5.5)

from which the neutrino mass matrix is found using the seesaw mechanism \([24]\) to be

\[ M_\nu = M_N M_R^{-1} M_N^T = \lambda^{1-2n+c-\bar{c}} \left( \begin{array}{ccc} \lambda^2 & \lambda^{2-r} & \lambda \\ \lambda^{2-r} & \lambda^{2-r} & \lambda^{1-r} \\ \lambda & \lambda^{1-r} & 1 \end{array} \right) \langle H(10,5) \rangle^2 \eta^2, \] (5.7)

where we have used the relation \([43]\).
Combining the charged lepton sector from the previous section and neutrino sector from above, we finally obtain the Maki-Nakagawa-Sakata matrix:

\[
U_{\text{MNS}} = \begin{pmatrix}
1 & \lambda^r & \lambda \\
\lambda^r & 1 & \lambda^{1-r} \\
\lambda & \lambda^{1-r} & 1
\end{pmatrix}.
\] (5.9)

Recent experiments on atmospheric neutrinos have suggested a very large mixing angle between second and third generations, and thus \( r = 1/2, 1 \) may be realistic [for the case of \((1, 1', 2)\), i.e., \( r \leq 1\)]\(^*\). It turns out that \( r = 1/2 \) actually leads to bi-large neutrino mixing angles, which are examined within the \( SO(10) \) model in Ref. 14\(^{**}\). Indeed if we take \( r = 1/2 \), namely,

\[
c - \bar{c} = \phi - \bar{\phi} + 1,
\] (5.10)

the MNS matrix is given by

\[
U_{\text{MNS}} = \begin{pmatrix}
1 & \lambda^{1/2} & \lambda \\
\lambda^{1/2} & 1 & \lambda^{1/2} \\
\lambda & \lambda^{1/2} & 1
\end{pmatrix},
\] (5.11)

which gives bi-large mixing angles for the neutrino sector, since \( \lambda^{1/2} \approx 0.5 \). At the same time it predicts \( V_{e3} \approx \lambda \). It will be interesting to see if future experiments find evidence just below the CHOOZ upper limit \( V_{e3} \leq 0.15 \). For the neutrino masses, the model predicts \( m_{\nu_{\mu}}/m_{\nu_\tau} \sim \lambda \), which is consistent with the experimental data: \( 1.6 \times 10^{-3} \text{eV}^2 \leq \Delta m^2_{\text{atm}} \leq 4 \times 10^{-3} \text{eV}^2 \) and \( 2 \times 10^{-5} \text{eV}^2 \leq \Delta m^2_{\text{solar}} \leq 1 \times 10^{-4} \text{eV}^2 \), which is the allowed region for the most probable MSW solution for the solar neutrino (LMA)\(^8\).

If we enforce the condition

\[
\phi - \bar{\phi} = 2n - 10 - l,
\] (5.12)

the neutrino mass matrix is obtained as

\[
M_\nu = \lambda^{-(5+l)} \begin{pmatrix}
\lambda^2 & \lambda^{2-r} & \lambda \\
\lambda^{2-r} & \lambda^{2-2r} & \lambda^{1-r} \\
\lambda & \lambda^{1-r} & 1
\end{pmatrix} \langle H(10, 5) \rangle^2 \eta^2,
\] (5.13)

\(^*\) In the case \( r > 1 \) \((1, 2, 1')\), we obtain

\[
U_{\text{MNS}} = \begin{pmatrix}
1 & \lambda & \lambda^r \\
\lambda & 1 & \lambda^{r-1} \\
\lambda^r & \lambda^{r-1} & 1
\end{pmatrix}.
\] (5.8)

\(^{**}\) In the case of \((1, 2, 1')\), the parameter value \( r = 3/2 \) may yield a prediction consistent with the large mixing indicated by atmospheric neutrino experiments.

\(^{***}\) When \( r = 1 \), the fermion mass matrices become of the “lopsided” type. This would seem to give a small mixing angle solution for the solar neutrino problem. However, recently it has been pointed out that taking account \( O(1) \) coefficients, lopsided-type mass matrices can give even large mixing angle solutions for solar neutrino problem.
where we have used the relation (5.10). From the above equation, we obtain
\[
\lambda = \lambda - 5 \frac{\langle H(10, 5) \rangle^2 \eta^2}{m_{\nu_r} M_P}.
\]

We are supposing that the cutoff scale \( M_P \) is in the range \( 10^{16} \text{GeV} < M_P < 10^{20} \text{GeV} \), which allows \(-2 \leq l \leq 2\). If we choose \( l = 0 \), the neutrino masses are given by
\[
m_{\nu_r} \sim 3 \times 10^{-2} \text{eV}, \quad n_{\nu_\mu} \sim 6 \times 10^{-3} \text{eV} \quad \text{and} \quad n_{\nu_e} \sim 1 \times 10^{-3} \text{eV}.
\]
From such a rough estimation, we can obtain values that are nearly consistent with the experimental data for atmospheric neutrinos and we can also obtain a large mixing angle (LMA) MSW solution for the solar neutrino problem.\(^{26)}\ This LMA solution for the solar neutrino problem gives the best to the present experimental data.\(^{27)}\)

Finally, we would like to make a comment on an interesting feature of this scenario, which is also seen in the \( SO(10) \) model.\(^{14)}\ In addition to Eq.(4.1), the interactions
\[
\lambda \psi_i + \psi_j + 2a + h \Psi_i A^2 \psi_j H
\]
also contribute to the Yukawa couplings after \( A \) develops a non-vanishing VEV. Here, only \( A^2 \) appears because of its odd \( Z_2 \) parity. Since \( \langle A \rangle \) is proportional to the generator of \( B - L \), the contribution to the lepton Yukawa coupling is nine times larger than that to the quark Yukawa couplings. If we set \( a = -2 \), the additional matrices are
\[
\frac{\Delta M_u}{\langle H(10, 5) \rangle} = \frac{V^2}{4} \begin{pmatrix}
\lambda^2 & \lambda & 0 \\
\lambda & 1 & 0 \\
0 & 0 & 0
\end{pmatrix}, \quad \frac{\Delta M_d}{\langle H(10, 5) \rangle} = \frac{V^2}{4} \begin{pmatrix}
\lambda^2 & 0 & \lambda \\
\lambda & 0 & 1 \\
0 & 0 & 0
\end{pmatrix},
\]
\[
\frac{\Delta M_s}{\langle H(10, 5) \rangle} = \frac{9V^2}{4} \begin{pmatrix}
\lambda^2 & \lambda & 0 \\
0 & 0 & 0 \\
\lambda & 1 & 0
\end{pmatrix}.
\]

Note that the additional terms contribute mainly to the lepton sector. It is interesting that this modification essentially changes the mass eigenvalues of only the first and second generation. Hence it is natural to expect that a realistic mass pattern can be obtained by this modification: It changes the unrealistic prediction \( m_\mu = m_s \) at the GUT scale while preserving the beautiful prediction \( m_b = m_\tau \) at the GUT scale (the GUT relation).\(^{\ast)}\ This enhancement factor of \( 2 - 3 \) of \( m_\mu \) can be enough to improve the unwanted situation of the lepton quark relation in the second family.

Remarkably enough, this charge assignment of \( A \) determines the scale of \( \langle A \rangle \) as \( \sim \lambda^2 \). This strong correlation of the unification scale, which is a bit smaller than the Planck scale, and the improvement of the undesired GUT prediction \( m_\mu = m_s \) is indeed a consequence of \( U(1)_A \). It is also interesting that the SUSY zero

\(^{\ast)}\) Strictly speaking that are forbidden by the SUSY zero mechanism are generically induced by integrating out heavy fields that are introduced to solve the DT splitting problem. These terms may give a small correction to the GUT relation \( m_b = m_\tau \).
plays an essential role again. When $z, \bar{z} \geq -4$, the terms $\lambda \psi_i + \psi_j + a + z + h Z \Psi_i A \Psi_j H + \lambda \psi_i + \psi_j + 2z + h Z^2 \Psi_i \Psi_j H$ also contribute to the fermion mass matrices, though only to the first generation.

§6. SUSY breaking and FCNC

Finally, we discuss SUSY breaking. Since we should choose the anomalous $U(1)_A$ charges dependent on the flavour to produce the hierarchy of Yukawa couplings, generically non-degenerate scalar fermion masses are induced through the anomalous $U(1)_A D$-term. Various experiments on FCNC processes give strong constraints on the off-diagonal terms $\Delta$ in the sfermion mass matrices due to the fact that the flavour-changing terms appear only in the off-diagonal parts of the sfermion propagators, as seen in Ref. 30). The sfermion propagators can be expanded in terms of $\delta = \Delta/\tilde{m}^2$, where $\tilde{m}$ is the average sfermion mass. As long as $\Delta$ is sufficiently smaller than $\tilde{m}^2$, it is enough to take the first term of this expansion, and then the experimental information concerning FCNC and CP violating phenomena is translated into the upper bounds on these $\delta_{F_{ij}XY}$, where $F = U, D, N, E$, the chirality index is $X,Y = L, R$, and the generation index is $i,j = 1, 2, 3$. For example, the experimental value of $K^0 - \bar{K}^0$ mixing gives

$$\sqrt{\text{Re}(\delta_{12}^D)_{LL}(\delta_{12}^D)_{RR}} \leq 2.8 \times 10^{-3} \left( \frac{\tilde{m}_q (\text{GeV})}{500} \right),$$

(6.1)

$$|\text{Re}(\delta_{12}^D)_{LL}|, |\text{Re}(\delta_{12}^D)_{RR}| \leq 4.0 \times 10^{-2} \left( \frac{\tilde{m}_q (\text{GeV})}{500} \right),$$

(6.2)

with $\tilde{m}_q$ the average value of the squark masses. The $\mu \rightarrow e\gamma$ process gives

$$|\langle \delta_{12}^E \rangle_{LL}|, |\langle \delta_{12}^E \rangle_{RR}| \leq 3.8 \times 10^{-3} \left( \frac{\tilde{m}_l (\text{GeV})}{100} \right)^2,$$

(6.3)

where $\tilde{m}_l$ is the average mass of the scalar leptons. In the usual anomalous $U(1)_A$ scenario, $\Delta$ can be estimated as

$$(\Delta_{ij}^F)_{XX} \sim \lambda |f_i - f_j| ( |f_i - f_j| \langle D_A \rangle),$$

(6.4)

since the mass difference is given by $(f_i - f_j) \langle D_A \rangle$, where $f_i$ is the anomalous $U(1)_A$ charge of $F_i$. Here, the reason for the appearance of the coefficient $\lambda |f_i - f_j|$ is that

*) The large SUSY breaking scale can make it possible to avoid the flavour changing neutral current (FCNC) problem, but in our scenario this is not the case, because the anomalous $U(1)_A$ charge of the Higgs $H$ is inevitably negative, which forbids the Higgs mass term at the tree level.

**) The CP violation parameter $\epsilon_K$ gives constraints on the imaginary part of $(\delta_{12}^D)_{XY}$ that are approximately one order more severe than those it places on the real part. Here we concentrate on the constraints from the real part of $K^0 - \bar{K}^0$ mixing, since under the other experimental constraints on the CP phase originating from SUSY breaking sector, which are mainly given by the electric dipole moment, we may expect that the CP phases are small enough to satisfy the constraints from the imaginary part of the $K^0 - \bar{K}^0$ mixing.
the unitary diagonalizing matrices are given by
\[
\begin{pmatrix}
1 & \lambda |f_1-f_2| \\
-\lambda |f_1-f_2| & 1
\end{pmatrix}.
\]

In our scenario, the anomalous $U(1)_A$ charge of $\bar{5}_1$ is the same as that of $\bar{5}_2$; i.e., the sfermion masses of $\bar{5}_1$ and $\bar{5}_2$ are almost equal. This weakens the constraints from these FCNC processes. This is because the constraints from $K^0 - \bar{K}^0$ mixing and CP violation on the product $(\delta_{12})_{LL} \times (\delta_{12})_{RR}$ are much stronger than those on $(\delta_{12})^2_{LL}$ or $(\delta_{12})^2_{RR}$, as shown in Eqs. (6.1) and (6.2). Therefore, suppression of $(\Delta D^2)_{RR}$ makes the constraints much weaker. Because the constraints from the $K^0 - \bar{K}^0$ mixing (and the CP violation) become weaker, as discussed above, we have a larger region in the parameter space where lepton flavour violating processes like $\mu \to e\gamma$ are appreciable. Actually, if the ratio of the VEV of $D_A$ to the gaugino mass squared at the GUT scale is given by
\[
R \equiv \frac{\langle D_A \rangle}{M^2_{1/2}},
\]
then the scalar fermion mass square at low energy scales is estimated as
\[
\bar{m}^2_{\bar{F_i}} \sim f_i R M^2_{1/2} + \eta_F M^2_{1/2},
\]
where $\eta_F$ is a renormalization group factor. Therefore, in our scenario, Eq. (6.2) for $(\delta_{12}^{D})_{LL}$ becomes
\[
(\delta_{12}^{D})_{LL} \sim \lambda \frac{(\psi_1 - \psi_2)(\psi_1 + \psi_2) R M^2_{1/2}}{(\eta_{DL} + \frac{\psi_1 + \psi_2}{2} R)} = \lambda \frac{(\psi_1 - \psi_2)(\psi_1 + \psi_2) R}{(\eta_{DL} + \frac{\psi_1 + \psi_2}{2} R)} \leq 4.0 \times 10^{-2} \left( \frac{(\eta_{DL} + \frac{\psi_1 + \psi_2}{2} R)^{1/2} M_{1/2} (\text{GeV})}{500} \right),
\]
which can be rewritten
\[
M_{1/2} \geq 1.25 \times 10^4 \lambda \frac{(\psi_1 - \psi_2) R}{(\eta_{DL} + \frac{\psi_1 + \psi_2}{2} R)^{3/2} (\text{GeV})}.
\]

Though the main contribution to $(\Delta D^2)_{RR}$ vanishes, through the mixing in Eqs. (4.4) and (4.5), $(\delta_{12}^{D})_{RR}$ is estimated as
\[
(\delta_{12}^{D})_{RR} \sim \lambda^2 \frac{\psi_1 - \psi_2}{\eta_{DR} + \psi_1 R},
\]
where the mixing $\lambda^2$ is different from the naively expected value $1 = \lambda^{\psi_1 - \psi_1}$. From Eq. (6.1) for $\sqrt{(\delta_{12}^{D})_{LL} (\delta_{12}^{D})_{RR}}$, the constraint on the gaugino mass $M_{1/2}$ is given by
\[
M_{1/2} \geq 1.8 \times 10^5 \frac{\lambda^{1.75} R (\psi_1 - \psi_2)}{(\eta_D + \psi_1 R)^{1.5}}.
\]

\(^{a)}\) We thank S. Yamashita for pointing out the contribution from the normalization factor of the main mode.
On the other hand, Eq. (6.3) for $\delta_{12}^{RR}$ leads to

$$M_{1/2} \geq 1.6 \times 10^3 \frac{\lambda (\psi_1 - \psi_2) R^{1/2}}{\eta_{ER} + \frac{\psi_1 + \psi_2}{2} R} \text{GeV}. \quad (6.13)$$

Taking the reasonable values $\psi_1 = 5$, $\psi_2 = 4$, $\eta_{DL} \sim \eta_{DR} \sim 6$ and $\eta_{ER} \sim 0.15$, the lower limits of the gaugino mass are roughly given as in Table I.

Table I. Lower bound of gaugino mass $M_{1/2}$ at the GUT scale (in GeV).

| $R$  | 0.1 | 0.3 | 0.5 | 1   | 2   |
|------|-----|-----|-----|-----|-----|
| $(\delta_{12}^{RR})_{LL}$ | 15  | 38  | 53  | 73  | 86  |
| $\sqrt{(\delta_{12}^{RR})_{LL} (\delta_{12}^{RR})_{RR}}$ | 60  | 150 | 210 | 280 | 350 |
| $|\delta_{12}^{RR}|$ | 370 | 260 | 210 | 150 | 110 |

Note that when $R < 0.5$, the $\mu \rightarrow e\gamma$ process gives the severest constraint in these FCNC processes. We conclude that the lepton flavour violating processes might be seen in the near future.

The reason for the suppression of $(\Delta_{12}^{RR})_{RR}$ is that the anomalous $U(1)_A$ charge of $\bar{5}_2$ becomes the same as that of $\bar{5}_1$, because the fields $\bar{5}_1$ and $\bar{5}_2$ originate from a single field, $\Psi_1$. This is a non-trivial situation. The massless mode of the second generation $\bar{5}_2 = \Psi_1(10, \bar{5}) + \lambda^{5/2}\Psi_3(16, \bar{5})$ has Yukawa couplings through the second term $\lambda^{5/2}\Psi_3(16, \bar{5})$. However, for the SUSY breaking term, which is proportional to the anomalous $U(1)_A$ charge, the contribution from the first term dominates that from the second term. This results in degenerate SUSY breaking terms between the first and the second generation. It is obvious that the twisting mechanism in $E_6$ unification plays an essential role in realizing this non-trivial structure. Note that such a structure is realized only when $(5_1, \bar{5}_2) = (1, 1')$ in which bi-large neutrino mixing angles are also realized. It is suggestive that the requirement to reproduce the bi-large mixing angles in the neutrino sector leads to this non-trivial structure, which suppresses the FCNC processes. In this way, such a non-trivial structure is automatically obtained in the $E_6$ model, which is much different from the situation for the $SO(10)$ model, in which the condition can be satisfied only by hand.

---

*) The case $(1, 1', 2')$ cannot realistically yield the large mixing angle indicated by atmospheric neutrino experiments.

**) We should comment on the $D$-term contribution to the scalar fermion masses. Generically, such a $D$-term has non-vanishing VEV when the rank of the gauge group is reduced by the symmetry breaking and SUSY breaking terms are non-universal. In our scenario, when the $E_6$ gauge group is broken to the $SO(10)$ gauge group, the $D$-term contribution gives different values to the fermion masses of $16$ and $10$ of $SO(10)$. This destroys the natural suppression of FCNC in the $E_6$ unification. However, if SUSY breaking parameters become universal for some reason, the VEV of $D$ can become negligible. Actually, the condition $m_0^2 = m_3^2$ causes the VEV of the $D$ to be greatly suppressed. Therefore, in principle, we can control the $D$-term contribution, though it is dependent on the SUSY breaking mechanism.
§7. Discussions and summary

In this paper, we examined an $E_6$ unified model in which bi-large neutrino mixing angles are realized. A noteworthy fact of such GUT model with the anomalous $U(1)_A$ framework, is that once we fix the charges of all the fields of the model, all the hierarchical scales, the symmetry breaking scales at high energy and also the hierarchical structure of the Yukawa couplings, are determined without any ambiguity. The only exceptions are the SUSY breaking scale and the electroweak breaking scale, which we here adjusted from the experimental W masses. Even if the SUSY breaking scale is introduced by hand, we have to explain why the SUSY Higgs mass parameter $\mu$ is around the SUSY breaking scale (the $\mu$ problem). One possible solution of the $\mu$ problem has recently been examined in Ref. [34]. Here we summarize the essence of the finding there. The SUSY Higgs mass, which is forbidden by the SUSY zero mechanism, can be induced when SUSY is broken. Thus the parameter $\mu$ must be proportional to a SUSY breaking parameter, and its coefficient is determined by anomalous $U(1)_A$ charges. Let us introduce the GUT gauge singlets $S$, with positive $s$, and $Z$, with negative $z$, with the mass term $\lambda s + zSZ$ in the superpotential. Since $S$ has positive charge, it has vanishing VEV in SUSY vacua (see the Appendix). When SUSY is broken, generically a tadpole term $\lambda s A$ ($A$ is a SUSY breaking parameter) is induced in the SUSY breaking potential $V_{SB}$. As a result, the $S$ field develops a non-vanishing VEV as

$$\langle S \rangle = \lambda^{-s-2z}A.$$ (7.1)

Using this VEV shift, we generically find the $\mu$ term to be proportional to the SUSY breaking parameter $A$. In our $E_6$ scenario, introducing the superpotential

$$W = \lambda^{s+\phi+2h} S\Phi H^2,$$ (7.2)

the SUSY Higgs mass $\mu$ is obtained as

$$\mu \sim \lambda^{2(h-z)+1/2(\phi-\bar{\phi})} A.$$ (7.3)

Therefore, if

$$-1 \leq 2(h-z) + \frac{1}{2}(\phi-\bar{\phi}) \leq 1,$$ (7.4)

the $\mu$ parameter becomes naturally around the SUSY breaking scale. Moreover, the $E_6$ gauge singlet fields $S$ and $Z$ can be identified with composite operators, for example, we can take $Z \sim \bar{C}C$ or $Z \sim \bar{\Phi}\Phi$.

In our $E_6$ case, the minimal field contents are, in addition to $\Theta$, three matter multiplets, $\Psi_i(27)$, the pair of Higgs fields $\Phi(27)\bar{\Phi}(27)$ and the pair of Higgs fields $C(27)$ and $\bar{C}(\bar{27})$, which are needed for the breaking $E_6 \to SO(10) \to SU(5) \to$ the standard gauge groups, together with an adjoint field $A(78)$, which also provides a natural D-T splitting mechanism, as explained in the context of the $SO(10)$ model in separate papers, and leaves light Higgs doublets $H$. Among those minimal contents of matter and Higgs fields, we have nine charges, $(\psi_1, \psi_2, \psi_3), (\phi, \bar{\phi}), (c, \bar{c}), a$ and $h$, which determine the main features of the mass matrices of quarks and leptons. First, the CKM mixing angle almost fixes the charges of the matter fields $\psi_i$ =
Unification with Bi-Large Neutrino Mixing

\[ (n + 3, n + 2, n), \text{ and the doublet Higgs } h = -2n, \]

and in order to get bi-large mixing angles for the neutrino sector, we need a constraint on the charges of the Higgs fields, \( r = 1/2 \), i.e., \( c - \bar{c} = \phi - \bar{\phi} + 1 \), and also we have the constraint \( \phi - \bar{\phi} = 2n - 10 - l \) \((-2 \leq l \leq 2)\) in order to give the proper neutrino masses. If we choose the charge as \( a = -2 \) in order to insure that the GUT relation between the masses of down-type quarks and charged leptons acts only for third generation, there remain only three degrees of freedom. Moreover, it may be possible to build DT splitting models in which the light Higgs can be identified with the components of \( \Phi \) or \( C \). Actually, it is naturally realized that \( \Phi \) can play the role of \( H \), so in that case, \( \phi = h \).

This implies that there are now two degrees of freedom. If we further impose the condition of solving the \( \mu \) problem, there remains just one degree of freedom. It is quite suggestive that there is a set of charge assignments that can satisfy all the above conditions. Actually, if we take \( n = 2, \phi = h = -4, \bar{\phi} = 0, c = -4, \bar{c} = -1 \), all the above conditions are satisfied for \( l = -2 \). The charge assignment \( n = 2, \phi = h = -4, \bar{\phi} = 3, c = -6, \bar{c} = 0, l = 1 \) is quite interesting, because the composite operator \( \Phi \Phi \) can even play the same role as the \( \Theta \) field when \( \xi^2 \sim \lambda \); that is, \( \langle \Phi \Phi \rangle \sim \lambda \equiv \xi^2 \).

In this case, we have the minimum model in which there are \( \Psi_1, \Psi_2, \Psi_3, \Phi, \bar{\Phi}, C, \bar{C} \) and \( A \), where all the charges are uniquely determined.

What is interesting in the \( E_6 \) unified model is that the condition for suppression of FCNC is automatically satisfied. The essential point is that the first and second generation fields of \( \bar{5} \) have the same anomalous \( U(1)_A \) charge because these fields originate from a single field \( \Psi_1 \).

The aspect of the family structure that has recently been made clear by the neutrino experiments gives a guide to investigate the origin of the family. The scenario discussed here is quite impressive, and it leads us to expect that we may find “the real GUT” in the near future.

Acknowledgements

We would like to express our sincere thanks to T. Kugo for his collaboration in the early stage of this work, reading this manuscript and useful comments. Thanks are also due to H. Nakano, K. Kurosawa, M. Yamaguchi and S. Yamashita for instructive and interesting discussions. We were very much stimulated by discussions with the members who attended to a series of research meetings [supported by Grants-in-Aid for Scientific Research on Priority Area A “Neutrinos” (Y. Suzuki)]. One of the authors (M. B.) is supported in part by Grants-in-Aid for Scientific Research Nos. 12047225(A2) and 12640295(C2) from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

Appendix

In this appendix, we explain how the vacua of the Higgs fields are determined by the anomalous \( U(1)_A \) quantum numbers.

First, we show that none of the fields with positive anomalous \( U(1)_A \) charge acquire nonzero VEV if the Froggatt-Nielsen (FN) mechanism acts effectively in the
vacuum. Let the gauge singlet fields be $Z_i^\pm \ (i = 1, 2, \cdots, n_\pm)$ with charges $z_i^\pm$, when $z_i^+ > 0$ and $z_i^- < 0$. From the $F$ flatness conditions of the superpotential, we get $n = n_+ + n_-$ equations plus one $D$-flatness condition,

$$\frac{\delta W}{\delta Z_i} = 0, \quad D_A = g_A \left( \sum_i z_i |Z_i|^2 + \xi^2 \right) = 0, \quad (A.1)$$

where $\xi^2 = \frac{g_A^2 g}{16 \pi^2} (\equiv \lambda^2 M_P^2)$. At first glance, these look to be over-determined. However, the $F$ flatness conditions are not independent, because the gauge invariance of the superpotential $W$ leads to the relation

$$\frac{\delta W}{\delta Z_i} z_i Z_i = 0. \quad (A.2)$$

Therefore, generically a SUSY vacuum with $\langle Z_i \rangle \sim M_P$ exists (Vacuum a), because the coefficients of the above conditions are generally of order 1. However, if $n_+ \leq n_-$, we can choose another vacuum (Vacuum b) with $\langle Z_i^- \rangle = 0$, which automatically satisfies the $F$-flatness conditions $\frac{\delta W}{\delta Z_i} = 0$. Then the $\langle Z_i^- \rangle$ are determined by the $F$-flatness conditions $\frac{\delta W}{\delta Z_i} = 0$ with the constraint $(A.2)$ and the $D$-flatness condition $D_A = 0$. Note that if $\lambda < 1$ (i.e., $\xi < 1$), the VEVs of $Z_i^-$ are less than the Planck scale. This can lead to the Froggatt-Nielsen mechanism. If we fix the normalization of $U(1)_A$ gauge symmetry so that the largest value $z_1^-$ in the negative charges $z_i^-$ equals -1, then the VEV of the field $Z_1^-$ is determined from $D_A = 0$ as $\langle Z_1^- \rangle \sim \lambda$, which breaks $U(1)_A$ gauge symmetry. (The field $Z_1^-$ was introduced in the previous section as $\Theta$.) Other VEVs are determined by the $F$-flatness conditions of $Z_i^+$ as $\langle Z_i^- \rangle \sim \lambda^{-z_i}$, which is shown below. Since $\langle Z_i^+ \rangle = 0$, it is sufficient to examine the terms linear in $Z_i^+$ in the superpotential in order to determine $\langle Z_i^- \rangle$. Therefore, in general the superpotential can be written

$$W = \sum_{i}^{n_+} W_{Z_i^+}, \quad (A.3)$$

$$W_{Z_i^+} = \lambda^{z_i^+} Z_i^+ \left( \sum_j^{n_-} \lambda^{z_j^-} Z_j^- + \sum_{j,k}^{n_-} \lambda^{z_j^- + z_k^-} Z_j^- Z_k^- + \cdots \right) \quad (A.4)$$

$$= \sum_{i}^{n_+} \tilde{Z}_i^+ \left( \sum_j^{n_-} \tilde{Z}_j^- + \sum_{j,k} \tilde{Z}_j^- \tilde{Z}_k^- + \cdots \right), \quad (A.5)$$

where $\tilde{Z}_i \equiv \lambda^{z_i} Z_i$. The $F$-flatness conditions of the $Z_i^+$ fields require

$$\lambda^{z_i^+} \left( 1 + \sum_j \tilde{Z}_j^- + \cdots \right) = 0, \quad (A.6)$$

$^*)$ We thank H. Nakano for pointing out this relation.
which generally lead to solutions $\tilde{Z}_j \sim O(1)$ if these $F$-flatness conditions determine the VEVs. Thus the $F$-flatness condition requires

$$\langle Z_j \rangle \sim O(\lambda^{-2}).$$

(A.7)

Here we have examined the VEVs of singlets fields, but generally the gauge invariant operator $O$ with negative charge $o$ has non-vanishing VEV $\langle O \rangle \sim \lambda^{-o}$ if the $F$-flatness conditions determine the VEV.

If Vacuum a is selected, the anomalous $U(1)_A$ gauge symmetry is broken at the Planck scale, and the FN mechanism does not act. Therefore, we cannot know the existence of the $U(1)_A$ gauge symmetry from the low energy physics. On the other hand, if Vacuum b is selected, the FN mechanism acts effectively, and we can understand the signature of the $U(1)_A$ gauge symmetry from the low energy physics. Therefore, it is natural to assume that Vacuum b is selected in our scenario, in which the $U(1)_A$ gauge symmetry plays an important role for the FN mechanism. This amounts to assuming that the VEVs of the fields $Z_i^+$ vanish, which guarantees that the SUSY zero mechanism acts effectively.

References

[1] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974), 438.
[2] E. Witten, Nucl. Phys. B188 (1981), 513.
[3] E. Witten, Phys. Lett. B105 (1981), 267.
[4] A. Masiero, D. V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. 115 (1982), 380.
[5] K. Inoue, A. Kakuto and T. Takano, Prog. Theor. Phys. 75 (1986), 664.
[6] E. Witten, Nucl. Phys. B258 (1985), 75.
[7] E. Witten, Nucl. Phys. B344 (1990), 211.
[8] Y. Kawamura, Prog. Theor. Phys. 105 (2001), 999.
[9] S. Dimopoulos and F. Wilczek, NSF-ITP-82-07.
[10] S. Dimopoulos and F. Wilczek, Nucl. Phys. B202 (1982), 327.
[11] S. M. Barr and S. Raby, Phys. Rev. Lett. 79 (1997), 4748.
[12] Z. Chacko and R. N. Mohapatra, Phys. Rev. D59 (1999), 011702; Phys. Rev. Lett. 82 (1999), 2836.
[13] K. S. Babu and S. M. Barr, Phys. Rev. D48 (1993), 5354; D50 (1994), 3529.
[14] The Super-Kamiokande Collaboration, Phys. Lett. B436 (1998), 33; Phys. Rev. Lett. 81 (1998), 1562; 86 (2001), 5656.
[15] J. Sato and T. Yanagida, Phys. Lett. B430 (1998), 127.
[16] Y. Nomura and T. Yanagida, Phys. Rev. D59 (1999), 017303.
[17] K.-I. Izawa, K. Kurosawa, Y. Nomura, T. Yanagida, Phys. Rev. D60 (1999), 115016.
[18] M. Bando and T. Kugo, Prog. Theor. Phys. 101 (1999), 1313.
[19] M. Bando, T. Kugo and K. Yoshioka, Prog. Theor. Phys. 104 (2000), 211.
[20] C. H. Albright, K. S. Babu and S. M. Barr, Phys. Rev. Lett. 81 (1998), 1167.
[21] C.H. Albright and S. M. Barr, Phys. Rev. Lett. 85 (2000), 244; Phys. Rev. D62 (2000), 093008; Phys. Lett. B461 (1999), 218; B452 (1999), 287; Phys. Rev. D58 (1998), 013002.
[22] Q. Shafi and Z. Tavartkiladze, Phys. Lett. B148 (2000), 145.
[23] N. Maekawa, Prog. Theor. Phys. 106 (2001), 408 (hep-ph/0104203).
[24] E. Witten, Phys. Lett. B149 (1984), 351.
[25] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B289 (1987), 589.
[26] J. J. Atick, L. J. Dixon and A. Sen, Nucl. Phys. B292 (1987), 109.
[27] M. Dine, I. Ichinose and N. Seiberg, Nucl. Phys. B293 (1987), 253.
[16] M. Green and J. Schwarz, Phys. Lett. B149 (1984), 117.
[17] L. Ibáñez and G.G. Ross, Phys. Lett. B332 (1994), 100.
P. Binétruy and P. Ramond, Phys. Lett. B350 (1995), 49.
E. Dudas, S. Pokorski and C.A. Savoy, Phys. Lett. B356 (1995), 45.
P. Binétruy, S. Lavignac and P. Ramond, Nucl. Phys. B477 (1996), 353.
[18] P. Binétruy, S. Lavignac, S. Petcov and P. Ramond, Nucl. Phys. B496 (1997), 3.
[19] H. Dreiner, G.K. Leontaris, S. Lola, G.G. Ross and C. Scheich, Nucl. Phys. B436 (1995), 461.
[20] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962), 870.
[21] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973), 652.
[22] M. Bando and N. Maekawa, in preparation.
[23] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B147 (1979), 277.
[24] T. Yanagida, in Proceedings of the Workshop on the Unified Theory and Baryon Number in the Universe, ed. O. Sawada and A. Sugamoto (KEK report 79-18,1979).
M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, ed. P. van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdam, 1979).
[25] The CHOOZ Collaboration, Phys. Lett. B420 (1998), 397.
[26] L. Wolfenstein, Phys. Rev. D17 (1978), 2369.
S. P. Mikheev and A. Smirnov, Yad. Fiz. 42 (1985), 1441; Nuovo Cim. 9C (1986), 17.
[27] J.W.F. Valle, astro-ph/0104085.
M. C. Gonzalez-Garcia, M. Maltoni, C. Pena-Garay and J. W. F. Valle, Phys. Rev. D63 (2001), 033005.
[28] S. Dimopoulos and G. F. Giudice, Phys. Lett. B357 (1995), 573.
[29] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Lett. B388 (1996), 588.
[30] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B477 (1996), 321.
[31] K. Kurosawa and N. Maekawa, Prog. Theor. Phys. 102 (1999), 121.
[32] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57 (1986), 961.
R. Barbieri and L. Hall, Phys. Lett. B338 (1994), 212.
J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B357 (1995), 579.
J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D53 (1996), 2442.
J. Sato and K. Tobe, Phys. Rev. D63 (2001), 116010.
[33] M. Drees, Phys. Lett. B181 (1986), 279.
Y. Kawamura, H. Murayama and M. Yamaguchi, Phys. Lett. B324 (1994), 52; Phys. Rev. D51 (1995), 1337.
[34] N. Maekawa, Phys. Lett. B521 (2001), 42 [hep-ph/0107313].