Modified Higgs Boson Phenomenology from Gauge or Gaugino Mediation in the NMSSM

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Abstract

In the Next-to-Minimal Supersymmetric Standard Model (NMSSM), the presence of light pseudoscalars can have a dramatic effect on the decays of the Standard Model-like Higgs boson. These pseudoscalars are naturally light if supersymmetry breaking preserves an approximate $U(1)_R$ symmetry, spontaneously broken when the Higgs bosons take on their expectation values. We investigate two classes of theories that possess such an approximate $U(1)_R$ at the mediation scale: deformations of gauge and gaugino mediation. In the models we consider, we find two disjoint classes of phenomenologically allowed parameter regions. One of these regions corresponds to a limit where the singlet of the NMSSM largely decouples. The other can give rise to a Standard Model-like Higgs boson with a dominant branching into light pseudoscalars.
1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) is a leading theory for new physics at the weak scale, providing for a stabilization of the weak hierarchy, the unification of gauge couplings, and an excellent dark matter candidate [1]. In contrast to the Standard Model (SM), the MSSM possesses a pair of electroweak Higgs doublets. These give rise to five physical degrees of freedom instead of the single SM Higgs boson. Despite this complication, throughout much of the allowed MSSM parameter space one of the MSSM Higgs bosons behaves in very much the same way as the SM Higgs [2].

The MSSM is not without its puzzles. In particular, it possesses a dimensionful parameter in the superpotential that marries the two Higgs multiplets together: \( W \supset \mu H_u H_d \). This parameter must be of order the weak scale in order to achieve proper electroweak symmetry breaking. One approach to this problem is the Next-to-Minimal Supersymmetric Standard Model (NMSSM), which consists of the MSSM augmented by a singlet chiral superfield [3, 4]. This singlet superfield leads to two new physical Higgs states: a neutral scalar and a neutral pseudoscalar. The appearance of these states, along with their mixing with the MSSM Higgs states, can significantly modify the Higgs phenomenology at the Large Hadron Collider (LHC) and other colliders [4, 5, 6, 7, 8].

The LHC Higgs boson signatures of the NMSSM are particularly different from the SM (and the MSSM) when the light SM-like Higgs boson can decay into pairs of very light mostly singlet Higgs pseudoscalars. (For a recent review of these decays see Ref. [9].) It was observed in Ref. [10] that if this decay mode dominates over the standard SM modes such as \( h \rightarrow b \bar{b} \), the LEP-II bound on the SM-like Higgs mass is lowered below 114 GeV. Masses of 110 GeV or below are possible if the pseudoscalars decay primarily into bottom quarks [9, 11], and Higgs bosons as light as 90 GeV can be consistent with the LEP data if the pseudoscalars decay primarily into tau leptons. Since the dominant contribution to fine-tuning within the MSSM comes from the Higgs boson mass bound, reducing the bound on the SM-like Higgs mass in this way can conceivably ameliorate the MSSM Higgs-sector fine-tuning problem. Recent discussions of this point have appeared in Refs. [10, 12, 13, 14].

Often, the Higgs pseudoscalars of the NMSSM are too heavy for the SM-like Higgs to decay into them. An important exception occurs when the theory has an approximate continuous symmetry under which at least one of the Higgs states is charged. In this case, a light pseudoscalar arises as the pseudo-Nambu-Goldstone boson (pNGB) of the approximate symmetry when it is broken in the course of electroweak symmetry breaking. A promising candidate for this symmetry is a \( U(1)_R \) [13, 14, 15], which is exact in the limit where the trilinear \( A \) terms of the Higgs sector vanish.

This symmetry is realized approximately in a natural way if the mediation of supersymmetry breaking is dominated by gauge interactions, and the Higgs sector \( A \) terms vanish at the mediation scale. In this case the dominant source of \( U(1)_R \) violation comes from the (Majorana) gaugino masses. This breaking is communicated to the Higgs fields in the course of renormalization group (RG) running between the mediation scale and the electroweak scale.
Two ways to mediate supersymmetry breaking through gauge interactions with vanishing $A$ terms are gauge mediation (GMSB) \[16, 17, 18\] and gaugino mediation (gMSB) \[19, 20\]. In both of these mechanisms of SUSY breaking, it is challenging to generate both the $\mu$ and the $B\mu$ terms with the correct relative size in the MSSM. One way to address this challenge is to add a singlet superfield whose vacuum expectation value (VEV) generates these terms, as is done in the NMSSM. Unfortunately, previous work on combining gauge or gaugino mediation with the NMSSM has found that the effective shielding of the singlet sector from the supersymmetry breaking also makes it difficult to obtain an acceptable pattern of electroweak symmetry breaking \[21\]. A number of works have investigated extensions of minimal gauge mediation in the hope of relieving this tension \[22, 23, 24, 25\].

In the present work, we study the LHC Higgs boson phenomenology of two simple extensions of minimal gauge and gaugino mediation within the NMSSM. The first extension consists of relaxing some of the GMSB boundary conditions on the singlet-sector soft terms. This might arise from a direct coupling of the singlet sector to the messenger states, or if there is additional source of supersymmetry breaking coupled solely to the singlet \[26\]. Such a modification of gauge mediation was considered recently in Ref. \[27\] while this work was in preparation. We extend and expand upon their results within gauge mediation, and apply the same modification to gaugino mediation. The second extension we consider involves adding new charged vector-like states to the theory \[22, 21\]. These modify the low-energy soft parameters of the NMSSM through their effects on the renormalization group (RG) running. Before discussing these modifications in detail, we introduce notation and review why a modification of the minimal gauge mediation scenario is needed.

### 1.1 The NMSSM Higgs Sector

The Higgs sector of the NMSSM consists of the $H_u$ and $H_d$ doublets of the MSSM along with a singlet $S$. The corresponding superpotential is given by

$$W \supset \lambda S H_u \cdot H_d + \frac{1}{3} \kappa S^3,$$

(1.1)

where $A \cdot B := \epsilon^{ab} A_a B_b$ with $\epsilon^{12} = +1$. The soft supersymmetry breaking operators within the Higgs sector are

$$-\mathcal{L}_{soft} \supset m^2_{H_u}|H_u|^2 + m^2_{H_d}|H_d|^2 + m^2_S|S|^2 + \left( \lambda A \lambda H_u \cdot H_d S + \frac{1}{3} \kappa A \kappa S^3 + h.c. \right)$$

(1.2)

The NMSSM Lagrangian as shown has a $Z_3$ discrete symmetry that forbids the appearance of linear and quadratic singlet operators. When the singlet obtains a VEV in the early universe, this symmetry is broken spontaneously in one of three degenerate vacua, and dangerous domain walls can form \[20\]. These can be avoided by including a relevant operator that softly breaks the $Z_3$ well below the electroweak scale. Such operators can arise in a natural way \[30, 31\] such that they eliminate domain walls, but are too small to have a significant effect on the Higgs boson phenomenology.

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1 We follow the notation and sign conventions of NMHDECY \[28\].
It is useful to introduce a complete basis for the set of all $U(1)$ transformations on $S, H_u, H_d$.

|  | $S$ | $H_u$ | $H_d$ | $Q$ | $U^c$ | $D^c$ | $L$ | $E^c$ |
|---|---|---|---|---|---|---|---|---|
| PQ | $-2$ | $1$ | $1$ | $-1$ | $0$ | $0$ | $-1$ | $0$ |
| R  | $2/3$ | $2/3$ | $2/3$ | $2/3$ | $2/3$ | $2/3$ | $2/3$ | $2/3$ |
| PQ' | $0$ | $1$ | $-2$ | $0$ | $-1$ | $2$ | $0$ | $-1$ |

All of these couplings are broken by terms in the superpotential or the soft SUSY-breaking Lagrangian. $U(1)_{PQ}$ is broken explicitly by $\kappa$ and $\kappa A_\kappa$, $U(1)_R$ is broken by $\lambda A_\lambda$, $\kappa A_\kappa$, and the other trilinear soft terms and gaugino masses, while $U(1)_{PQ}'$ is broken explicitly by $\lambda$ and $\lambda A_\lambda$. Throughout this paper, we will implicitly make a $U(1)_{PQ}'$ field redefinition such that $\lambda$ is real and positive, as well as a $U(1)_R$ transformation such that $\kappa$ is real. With a $U(1)_R$ rotation, we can take gaugino masses to all be real and positive provided they have no relative phases. The $A$ terms subsequently generated by RG running will then be (mostly) real as well. Having fixed $\kappa$ to be real, but not necessarily positive, there is still a residual $Z_2$ subgroup of $U(1)_{PQ}$ that can be used (in conjunction with $U(1)_Y$), to make the VEVs $\langle H_u^0 \rangle = v_u$ and $\langle H_d^0 \rangle = v_d$ real and positive. The values of $\langle S \rangle = v_s$ and $\kappa$ can then take either sign.

1.2 Minimization Conditions and a Challenge

The Higgs-sector parameters in the NMSSM are $\{\lambda, \kappa, A_\lambda, A_\kappa, m_{H_u}^2, m_{H_d}^2, m_S^2\}$. Electroweak symmetry breaking provides three minimization conditions that allow three of these parameters to be exchanged for three VEVs. While it is a common procedure to solve for the soft masses $m_{H_u}^2$ and $m_{H_d}^2$ in terms of $v$ and tan $\beta$, we find that it is instructive to instead solve for $m_S^2$, $\mu = \mu_{eff} = \lambda v_s$, and $\kappa$ in terms of the other parameters.

Demanding that $v_u$, $v_d$, and $v_s$ are all non-zero and real, we find

$$\mu = \operatorname{sgn}(\mu) \sqrt{\frac{m_{H_u}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1}} - M_Z^2 / 2,$$

$$\frac{\kappa}{\lambda} = \frac{\sin 2\beta}{2} \left( 2 + \frac{\lambda^2 v^2}{\mu^2} + \frac{m_{H_u}^2 + m_{H_d}^2}{\mu^2} \right) - \frac{A_\lambda}{\mu},$$

$$\frac{m_S^2}{\mu^2} = -2 \frac{\kappa^2}{\lambda^2} + \frac{\sin 2\beta}{2} \left( \frac{A_\lambda}{\mu} + \frac{\kappa}{\lambda} \right) \frac{\lambda^2 v^2}{\mu^2} - \frac{\kappa}{\lambda} A_\kappa.$$

These relations imply a difficulty for the NMSSM in gauge mediation [21]. We now review this argument. To leading order, the trilinear singlet soft terms and $m_S^2$ vanish at the messenger scale and are generated primarily from RG running to the electroweak scale. For lower mediation scales, the logarithm does not compensate the loop factor suppression, and these terms are expected to be somewhat smaller than the other soft terms. On the other hand under the condition that the lightest slepton is sufficiently heavy, the soft terms generated by minimal GMSB force $|m_{H_u}^2|$ to be larger than about $(250 \text{ GeV})^2$. Eq. (1.4) then forces $\mu^2$ to be about the same size as $|m_{H_u}^2|$ (for moderate tan $\beta$). These large values of $\mu^2$ lead to
the hierarchies $\lambda^2 v^2/\mu^2 \ll 1$ and $|A_{\lambda,\kappa}/\mu| \ll 1$. To satisfy Eq. \( \text{(1.6)} \) given the small values of $m_S^2/\mu^2$ expected from minimal gauge mediation, it is necessary to have $|\kappa/\lambda| \ll 1$. As a result, $\sin 2\beta \ll 1$ is needed for Eq. \( \text{(1.5)} \) to be satisfied.

With both $|\kappa/\lambda|, \sin 2\beta \ll 1$, it is difficult to obtain an acceptable pattern of symmetry breaking. In this limit, the phenomenologically interesting local extremum (corresponding to the solutions of Eqs. \( \text{(1.4,1.5,1.6)} \) where $v_u, v_d, v_\kappa$ are all non-zero) tends to be a saddle point rather than a minimum. This can be seen in the determinant of the $CP$-even Higgs boson mass-squared matrix,

$$\det M_S^2 \simeq 4\mu^4 \lambda^2 v^2 \frac{\kappa}{\lambda} \frac{2}{\sin 2\beta} \left( \frac{\kappa^2 g'^2 + g^2}{2\lambda^2} - 1 \right). \quad \text{(1.7)}$$

With $|\kappa/\lambda| \ll 1$, this expression is negative unless $\lambda^2 \ll 1$ is also tiny. Very small values of $\lambda$ and $\kappa$, with $\mu$ fixed, correspond to a decoupling limit of the NMSSM in which the singlet states couple only very weakly to the rest of the theory, and the Higgs phenomenology reduces to that of the MSSM. The argument presented here applies to minimal gaugino mediation as well.

Eqs. \( \text{(1.4,1.6)} \) also indicate a way out of this difficulty: a deformation of minimal gauge or gaugino mediation that generates a large negative value of $m_S^2$ near the electroweak scale. In the present work, we investigate the phenomenology of two of the simplest possibilities for modifying the low-scale value of $m_S^2$ in gauge and gaugino mediation. The first modification we consider consists of treating $m_S^2$ as a free parameter at the mediation scale of supersymmetry breaking. The second consists of adding charged vector-like states to the theory with superpotential couplings to the singlet $[17, 22, 21]$. These couplings drive the singlet soft mass negative in the course of RG running, in a manner analogous to the effect of the top Yukawa coupling in the MSSM. The goal of the present work is to determine the viability of these modifications and to investigate the LHC Higgs signatures they predict. We do not dwell on a reduction of fine-tuning relative to the MSSM. Rather, our focus is to better understand under what conditions one might expect novel Higgs boson signatures. Then, if such signatures are indeed observed at colliders, we will have an important clue about the identity of the underlying theory.

The plan of the remainder of this paper is as follows. In Section 2 we derive analytic expressions for the Higgs boson masses and couplings in the approximate $U(1)_R$ limit. In Section 3 we study the NMSSM Higgs sector within gauge mediation with an additional boundary contribution to $m_S^2$ at the gauge messenger scale. Next, in Section 4 we investigate the effect of adding charged vector-like states on the NMSSM Higgs states within minimal (unmodified) gauge mediation. We study analogous deformations of gaugino mediation within the NMSSM in Section 5. Section 6 is reserved for our conclusions. Some of our technical results are collected in Appendices A and B.
2 NMSSM Higgs Bosons with an Approximate $U(1)_R$

We begin by deriving approximate analytic expressions for the Higgs boson masses, mixing angles, and couplings in the NMSSM when the theory has an approximate $U(1)_R$ symmetry in the singlet sector [15]. This approximate symmetry is realized numerically as the hierarchies $|A_\lambda/\mu| \ll 1$ and $|A_\kappa/\mu| \ll 1$. We shall also assume $|\lambda v/\mu| \ll 1$, which is valid throughout most of the parameter space of the models we study in the coming sections. Further details about this expansion are listed in Appendix A.

To leading non-trivial order in the small ratios, and assuming $\kappa/\lambda$ and $\sin 2\beta$ are not too terribly small, the tree-level $CP$-odd masses are

$$m_{a_s}^2 = \frac{\kappa}{\lambda} \mu \left( -\frac{3\lambda s_{2\beta} A_\lambda - A_\kappa}{\mu^2} \right), \quad (2.1)$$

$$m_{A^0}^2 = \left( 1 + \frac{\lambda A_\lambda}{\kappa \mu} \right) \frac{2 \lambda}{s_{2\beta} \kappa} \left( \frac{\kappa}{\lambda} \right)^2 + \frac{2 \kappa}{\lambda} s_{2\beta} \left( 1 - \frac{2 \lambda A_\lambda}{\kappa \mu} \right) \lambda^2 v^2. \quad (2.2)$$

The expression for $m_{a_s}^2$ vanishes in the limit $A_{\lambda,\kappa} \to 0$ showing that this state is the pNGB of the approximate $U(1)_R$. This state is primarily singlet, while the $A^0$ state is similar to the Higgs pseudoscalar in the MSSM. We are able to make this identification because the mixing among the singlet and the non-singlet pseudoscalars is suppressed by a factor of $\lambda v/\mu \ll 1$. Full expressions for the mass and mixing matrices in this expansion are listed in Appendix A.

Applying this expansion to the $CP$-even masses yields

$$m_{h^0}^2 = \lambda^2 v^2 \left[ \frac{g^2}{2 \lambda^2} - s_{2\beta}^2 - \left( \frac{\lambda}{\kappa} - s_{2\beta} \right)^2 \right], \quad (2.3)$$

$$m_{H^0}^2 = \frac{2 \lambda}{s_{2\beta} \kappa} \left( \frac{\kappa}{\lambda} \right)^2, \quad (2.4)$$

$$m_{h_s}^2 = 4 \left( \frac{\kappa}{\lambda} \right)^2. \quad (2.5)$$

Among these states, $h^0$ is SM-like, $H^0$ is similar to the corresponding state in the MSSM, and $h_s$ is predominantly singlet. Again, this identification is possible because the mixing between the MSSM states and the singlet is suppressed by factors of $\lambda v/\mu \ll 1$. The $CP$-even sector mixing matrices are also listed in Appendix A.

The (exact) mass of the charged Higgs is

$$m_{H^{\pm}}^2 = \frac{2}{s_{2\beta} \kappa} \frac{\lambda}{\kappa} \left( \frac{\kappa}{\lambda} \right)^2 \left( 1 + \frac{\lambda}{\kappa} A_\lambda \right) + \lambda^2 v^2 \left( \frac{g^2}{2 \lambda^2} - 1 \right). \quad (2.6)$$

In the limit of $\mu^2 \gg \lambda^2 v^2$, $A_\lambda^2$, $A_\kappa^2$ this coincides closely with the masses of the $A^0$ and $H^0$ states.

The coupling between the SM-like $h^0$ state and pairs of the light $a_s$ pseudoscalars is particularly important for the phenomenology of this scenario. This coupling corresponds
to the operator

$$\mathcal{L} \supset \frac{c}{\sqrt{2}} v h^0 a_1 a_1. \quad (2.7)$$

Within the expansion, the coefficient $c$ is given by

$$c = \left( \frac{1}{2} \lambda^2 \right) \left( \frac{\lambda}{\kappa} - 2 s_{2\beta} \right) \left( \frac{\lambda}{\kappa} + s_{2\beta} \right) \frac{m_{h^0}^2}{2\mu^2} + \left( \frac{1}{2} \lambda^2 \right) \left[ \frac{1}{2} \frac{A_\kappa}{\mu} \left( 1 - \frac{\kappa}{\lambda} s_{2\beta} - 12 \frac{\kappa^2}{\lambda^2} s_{2\beta}^2 \right) - 9 \left( \frac{\kappa}{\lambda} s_{2\beta} \right) \frac{A_\lambda}{\kappa} \frac{A_\lambda}{\mu} \left( 1 - \frac{\lambda A_\lambda}{\kappa} \right) \right]. \quad (2.8)$$

The first line in this expression coincides with the result of Ref. [15], and corresponds to the axion-like derivative coupling of the light pNGB pseudoscalar. The second line is new to our calculation. It is useful because it captures the contributions to the coupling that arise from the explicit breaking of the $U(1)_R$ symmetry by the $A$-terms. In terms of the coefficient $c$, the decay width for $h^0 \to a_s a_s$ is

$$\Gamma(h^0 \to a_s a_s) = \frac{c^2 v^2}{16\pi m_{h^0}} \left( 1 - \frac{4 m_{a_s}^2}{m_{h^0}^2} \right)^{1/2}. \quad (2.9)$$

The approximate expression in Eq. (2.8) agrees well with the full numerical result from NMHDECAY [28] in the appropriate limit.

3 Gauge Mediation in the NMSSM with a Free $m_S^2$

The first scenario we consider is a deformation of minimal gauge mediation in which the value of the singlet soft mass $m_S^2$ taken to be a free parameter at the messenger scale $M$. All other soft terms are set to their standard gauge mediated values at scale $M$. Without this deformation, $m_S^2(M)$ vanishes at the leading order. This feature is the primary obstruction to merging gauge mediation with the NMSSM [21]. By liberating $m_S^2$ from its minimal GMSB boundary condition we avoid this obstacle by fiat. The same deformation was considered in Ref. [27]. We expand upon and extend their results.

We do not have a particular model in mind for how this deformation of minimal gauge mediation could arise. However, as a gauge singlet, one might imagine that $S$ is on a special footing and might feel the mediation of supersymmetry differently from the rest of the MSSM. A number of recent works have considered related modifications of gauge mediation within the NMSSM with this fact in mind [23, 25, 27]. Prior explicit constructions typically generate new contributions to $A_\lambda$ and $A_\kappa$. This spoils the approximate $U(1)_R$ symmetry, and prevents the interesting decays to pseudoscalars. Modifying $m_S^2$ without altering $A_\lambda$ and $A_\kappa$ requires coupling $S$ directly to a source that breaks supersymmetry but not the $U(1)_R$.

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2This philosophy is similar in spirit to [32, 33], where supersymmetry boundary conditions for the Higgs fields were chosen to be different from the matter fields.
We do not explicitly write a full model of supersymmetry breaking for the singlet field. However, we point out that supersymmetry breaking without spontaneous $U(1)_R$ breaking is a generic feature of simple O’Raifertaigh models \cite{34}, and that some care and complication is often required to break the $U(1)_R$ in this context \cite{34, 35}. It is a model-building challenge to ensure that the additional SUSY breaking felt by the singlet field is of the same order as the rest of the SUSY breaking.

### 3.1 Allowed Parameter Regions

We begin by searching for phenomenologically acceptable regions of the NMSSM parameter space subject to GMSB boundary conditions for all soft terms other than $m^2_S$. We specify the superpotential coupling $\lambda$, as well as the supersymmetry breaking scale $F/M$, at the GMSB messenger scale, $M$. We specify $\tan \beta$ near the electroweak scale. With these inputs set, we compute the resulting low-energy spectrum. The soft supersymmetry breaking parameters at scale $M$, other than $m^2_S$, are set to their gauge-mediated values assuming a minimal GMSB sector with a single set of $5 \oplus \bar{5}$ messengers. We use a modified version of NMSPEC/NMHDECAY \cite{28} to perform the RG evolution of the model parameters, to find the low-scale values of $\kappa$, $|\mu|$, and $m^2_S$, and to compute the low-energy spectrum and constraints. In general, the value of $m^2_S$ obtained in this way does not agree with the GMSB boundary condition of $m^2_S(M) \approx 0$. Our scan encompasses the parameter ranges

$$\lambda \in [0.001, 0.7], \quad \tan \beta \in [1, 50], \quad M \in [10^5, 10^{14}] \text{ GeV}, \quad F/M \in [2, 40] \times 10^4 \text{ GeV}. \quad (3.1)$$

These ranges lead to low-scale gluino masses between about 350 GeV and 2500 GeV.

The results of our parameter scans are shown in Fig. 1 within the $\lambda - \frac{\kappa}{\lambda}$ and $\lambda - \tan \beta$ planes. The red (dark) points in these plots are consistent with all relevant phenomenological bounds except for the collider constraints on the neutral $CP$-even and $CP$-odd Higgs bosons, while the green (light) points satisfy the Higgs constraints as well. We do not demand that the lightest neutralino be the LSP since for gauge mediation the gravitino is usually expected to be the true LSP. However, we do require that the couplings remain perturbative up to the scale $M$, but not $M_{\text{GUT}}$. This eliminates points with large $\lambda$ and $\kappa$ for larger mediation scales. In the limit $M \to M_{\text{GUT}}$, we find that perturbativity requires $(\lambda^2 + \kappa^2) \lesssim 0.45$ near the electroweak scale \cite{7, 25}. When $M < M_{\text{GUT}}$, additional charged states that enter the running at $M$, such as the messengers themselves, can help to slow down the growth of $\lambda$ and $\kappa$ above the messenger scale \cite{36}.

In the left panel of Fig. 1 the lower limit on the red (dark) region corresponds to the condition $\det M^2_S > 0$. For smaller values of $\lambda$, this cutoff agrees with the relation given in Eq. (1.7), while more generally, it coincides with $m^2_{h^0} > 0$ using the expression in Eq. (2.3). The upper boundary of the allowed red (dark) region in Fig. 1 can be understood from an examination of Eq. (1.5). There is a close relationship between $\kappa/\lambda$ and $\sin 2\beta$ when $|\mu|$ is much larger than $\lambda v$ and the singlet trilinear couplings. Since $\sin 2\beta$ is necessarily bounded by 1, so is $\kappa/\lambda$.

It is clear from Fig. 1 that there are two disjoint populations of phenomenologically consistent (light/green) parameter points. The first population, which we call Region I,
has larger values of $\lambda \gtrsim 0.4$, $\kappa/\lambda$ on the order of unity, and smaller values of $\tan \beta \lesssim 2.5$ ($\sin 2\beta \gtrsim 0.7$). The second population, Region II, has $\lambda \lesssim 0.08$, $\kappa/\lambda$ well below unity, and larger values of $\tan \beta \gtrsim 5$ ($\sin 2\beta \lesssim 0.38$). For values of $\lambda$ between the large and small values taken on within the disjoint green regions, we find that the SM-like $h^0$ Higgs boson is too light to satisfy the LEP bounds. In the larger $\lambda$ portion, Region I, the singlet $F$-term becomes important for smaller $\tan \beta$ and is responsible for increasing the mass of the $h^0$. At smaller $\lambda$, in Region II, $m_{h^0}$ is very close to the value it would have in the MSSM, and larger values of $\tan \beta$ are necessary to push it above the LEP II bound. We discuss the phenomenology of Regions I and II below.

3.2 Region I: Higgs Decays to Pseudoscalars

Region I consists of points with $\lambda \sim \kappa \gtrsim 0.4$ and $\tan \beta \lesssim 2.5$. The Higgs phenomenology in this region can be very different from the SM when $m_{a^0} < m_{h^0}/2$ and the $h^0$ state decays predominantly into pairs of $a^0$ pseudoscalars. Evidently this requires a light $a^0$ pseudoscalar and a sizeable effective coupling $c$. When this is not the case, the Higgs phenomenology turns out to be very similar to the MSSM at large $m_{A^0}$.

As discussed in Section 2, a light $a^0$ pseudoscalar can emerge from the spontaneous breakdown of an approximate $U(1)_R$ symmetry in the singlet sector. This $U(1)_R$ is broken explicitly by the singlet trilinear $A$ terms, but remains a good approximate symmetry provided they are much smaller than $\mu$. In Fig. 2 we plot the values of $A_\lambda$ against $A_\kappa$, as well as $A_\lambda$ against $\mu$ for points within Region I. The green (light) points are consistent with all phenomenological bounds, while the blue (dark) points also have $m_{a^0} < m_{h^0}/2$. From the right panel of this figure we see clearly a hierarchy between the singlet $A$ terms.
Figure 2: Values of $A_\lambda$, $A_\kappa$, and $\mu$ among phenomenologically acceptable parameter points in Region I described in the text. The green (light gray) points satisfy all collider phenomenological constraints, while the blue (dark gray) points also have a light pseudoscalar with $m_{a_s} < m_{h^0}/2$.

and $\mu$.

It is not hard to understand how the approximate $U(1)_R$ can give rise to $m_{a_s} < m_{h^0}/2$ by comparing Fig. 2 with the results of Eq. (2.1) and Eq. (2.3). To obtain such a light pseudoscalar, very small values of $A_\kappa$ are necessary. There is a contribution to $m_{a_s}$ that goes as $\mu A_\kappa$, whereas the mass of the light $h^0$ Higgs boson is largely controlled by $\lambda v$ in this region. Recall that $\lambda^2 v^2 / \mu^2 \ll 1$. It is also necessary to have $A_\lambda / \mu$ reasonably small, although this requirement is much less severe due to the additional suppression by $\lambda^2 v^2 / \mu^2 \ll 1$ in Eq. (2.1).

Among the parameter points with a sufficiently light $a_s$, the dominant contribution to the coupling $c$ in Eq. (2.8) comes from the term involving $A_\lambda$. Thus, the small amount of $U(1)_R$ breaking induced by the gaugino masses and transmitted to $A_\lambda$ in the course of RG running plays a dual role. It must be small enough to keep the pseudoscalars light, but still large enough to facilitate $h^0 \rightarrow a_s a_s$ decays. As shown here, and previously observed in [14] (there with breaking at the GUT scale), the size of the $A$ terms derived from running can provide the right amount of $U(1)_R$ breaking to accomplish both of these tasks.

The low-scale values of $A_\lambda$ and $A_\kappa$ are generated in the course of RG running down from the messenger scale. They are sourced indirectly by the gaugino masses. Thus, the precise values of $A_\lambda$ and $A_\kappa$ near the electroweak scale are sensitive to the GMSB parameters $F/M$ and $M$. In Fig. 3 we show the values of $A_\lambda$ and $A_\kappa$ among allowed parameter points in Region I as functions of both $F/M$ and $M$. We exhibit allowed points both with and without

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3 Numerically, we find the tree-level expression of Eq. (2.1) to be accurate only for larger values of $m_{a_s}$ due to additional quantum corrections. However, Eq. (2.1) remains useful for determining when the $a_s$ state will be light.
Figure 3: Dependence of the singlet trilinear $A_\lambda$ and $A_\kappa$ terms on the messenger scale $M$ and the scale of supersymmetry breaking $F/M$. We exhibit phenomenologically allowed parameter points, as well as points satisfying the additional condition of $m_{a_s} < m_{h^0}/2$.

$m_{a_s} < m_{h^0}/2$. Not surprisingly, smaller values of the trilinear $A$ terms are obtained from lower values of $F/M$, corresponding to a lighter superpartner spectrum. For higher messenger scales $M$, the logarithmic enhancement of the singlet $A$ terms becomes stronger, leading to larger values of these couplings. This tends to push up the mass of the $a_s$ pseudoscalar, but it also helps to enhance the decay width of $h^0 \rightarrow a_s a_s$ allowing this mode to dominate over decays to bottom quarks.

In Fig. 4 we show the masses of the lightest scalar $h^0$ and the lightest pseudoscalar $a_s$ for allowed parameter points in Region I in the left panel. In the right panel we show the dominant branching fractions for these states. We see that the decay properties of the $h^0$ Higgs differ markedly from those of a SM Higgs when the branching fraction for $h^0 \rightarrow a_s a_s$ is close to unity. When this is the case, the $h^0$ state can be significantly lighter than 114 GeV and still be consistent with the bounds from LEP II, as discussed in Ref. [10]. For pseudoscalar masses larger than about 10 GeV, the $a_s$ decays primarily into $b\bar{b}$, weakening the bound on the $h^0$ mass to roughly 110 GeV. Very light pseudoscalars, below about 10 GeV in mass, tend to decay mostly into $\tau \bar{\tau}$ (unless they are extremely light), allowing for $h^0$ masses as low as about 90 GeV to be consistent with the bounds from LEP II [11]. In the present context, it is challenging to make $A_\kappa$ small enough to get such a light $a_s$. We find that $m_{a_s} < 10$ GeV generally requires a cancellation between the $A_\kappa$ and $A_\lambda$ terms in Eq. (2.1), implying a degree of fine-tuning. This possibility is constrained by and will be further probed by searches for $\Upsilon \rightarrow a_s \gamma$ [37, 38] as well as measurements of $(g - 2)_\mu$ [39].

While the decays of the SM-like $h^0$ Higgs boson can be significantly modified by the presence of a light singlet pseudoscalar, the other properties of the $h^0$ state such as its production rates and decay widths into SM final states generally remain nearly unchanged relative to the SM Higgs. For example, the mixing between the SM-like combination of the $CP$-even gauge eigenstates (i.e. the combination that has the same couplings with the gauge
bosons as in the SM) with the singlet in the $h^0$ is given by the matrix element

$$U_{12} \simeq -\frac{\lambda}{2\kappa} \frac{\lambda v}{\mu} \left(\frac{\lambda}{\kappa} - s_{2\beta}\right).$$

(3.2)

Here we have kept only the leading terms in the expansion in $\lambda v/\mu$ and $A_{\lambda,\kappa}/\mu$. More details about the mixing matrix are given in Appendix A. Numerically, we find $|U_{12}| \lesssim 0.10$, along with $|U_{11}| \gtrsim 0.995$. In particular, the dominant Higgs production channels via loop-induced coupling to gluons or direct couplings to gauge bosons will only be reduced by a factor of $U_{11}^2$, or less than about 1%.

The LHC signatures of the $h^0$ Higgs depend strongly on its branching fraction into pairs of $a_s$. If these decays are suppressed, either by kinematics or a small coupling, the Higgs signatures will be very similar to the MSSM. When $BR(h^0 \rightarrow a_s a_s)$ is non-trivial, the branching fractions of the $h^0$ into MSSM final states will be reduced according to

$$BR_i = BR_{i_i}^{\text{MSSM}} \left[1 - BR(h^0 \rightarrow a_s a_s)\right].$$

(3.3)

Despite this suppression, some of the more promising SM Higgs channels such as $h^0 \rightarrow \gamma\gamma$ ($m_{h^0} \lesssim 125$ GeV) and $h^0 \rightarrow WW^*$ ($m_{h^0} \gtrsim 125$ GeV) could still be visible in their own right even with $BR(h^0 \rightarrow a_s a_s)$ close to its maximal value of about 0.9.

It may also be possible to search for the $h^0$ Higgs boson through its decays to light pseudoscalars if both $BR(h^0 \rightarrow a_s a_s)$ and $BR(a_s \rightarrow bb)$ are close to unity [40, 41]. This search relies on Higgs production in association with a weak gauge boson to reduce the background. It is challenging because it requires multiple $b$-tags to reduce the background, and therefore requires a high $b$-tag efficiency along with a good understanding of the mis-tagging rate. Even so, Ref. [41] finds that it should be possible to discover the $h^0$ Higgs through this channel with at least 30 fb$^{-1}$ of integrated luminosity. When the $a_s$ pseudoscalar is so light
that it decays predominantly into $\tau \bar{\tau}$, which we find to be fairly unlikely in Region I, Ref. \[42\] proposes a LHC search strategy using forward proton tagging (central exclusive production) with the proposed FP420 detectors \[43\]. The recent study of Ref. \[44\] suggests that this decay channel might also be observable at the LHC in the weak boson fusion production channel.

We have concentrated so far on the phenomenology of the $h^0$ Higgs boson and the $a_s$ pseudoscalar. The other Higgs states, the $CP$-even $H^0$ and $h_s$ along with the $CP$-odd $A^0$ and the charged $H^\pm$, could also turn up at the LHC. However, among the phenomenologically allowed points in Region I these states are all heavier than about 500 GeV and mix only very weakly with the SM-like $h^0$. This makes them difficult to detect at the LHC. Since $\tan \beta$ is small in Region I, these states have significant branching fractions for decays into $t \bar{t}$. The $h_s$ scalar can also decay efficiently into pairs of $a_s$ pseudoscalars, while the $A^0$ pseudoscalar can also decay appreciably to the lighter superpartners. Given the large masses of these states along with their weak production cross-sections, we do not expect that they will produce a significant signal at the LHC \[45, 46\]. The charged $H^+$ Higgs decays primarily into $t \bar{b}$, as well as $h^0W^+$, and may possibly be visible through the former mode \[45, 46\].

Besides the new Higgs bosons, the singlet superfield $S$ also gives rise to an additional fifth neutralino state. For the parameter values in Region I, this state consists primarily of the fermion component of $S$ with a small higgsino admixture, and has a mass close to $2k\mu/\lambda$. As $\kappa/\lambda \sim 1$ and $|\mu| > M_{1,2}$ in Region I, this state is typically the heaviest neutralino. Moreover, its small mixing with the MSSM neutralinos implies that it will have a low production rate at the LHC, either by direct creation or through SUSY cascade decays. Cascade decays could also provide an interesting source of $h^0$ Higgs bosons, although we do not pursue this possibility here.

### 3.3 Region II: (Partial) Singlet Decoupling

This region corresponds to the phenomenologically allowed points in Fig. 1 with $\lambda \lesssim 0.1$, $\kappa/\lambda \lesssim 0.4$, and $\tan \beta \gtrsim 5$ ($\sin 2\beta \lesssim 0.4$). As in Region I, the minimal GMSB boundary conditions lead to values of $|\mu|$ that are much larger than $\lambda v$ and the singlet trilinear $A_{\lambda,\kappa}$ couplings. This signals a certain amount of fine-tuning in the electroweak symmetry breaking conditions. From the above values and the minimization conditions of Eqs. \(1.4,1.5,1.6\), we see that this region has $|m_S^2/\mu^2| \ll 1$ near the electroweak scale. The small values of $\lambda \ll 1$ that arise in Region II are needed to satisfy the LEP bound on the SM-like Higgs boson mass. Expanding Eq. \(2.3\) in small $k/\lambda$ and $\sin 2\beta$, we find that

$$m_{h^0}^2 \simeq \lambda^2 v^2 \left( \frac{g^2 + g_s^2}{2\lambda^2} - \frac{\lambda^2}{\kappa^2} \right).$$

(3.4)

To obtain a sufficiently large Higgs mass, it is then necessary that $g^2/2\lambda^2 \gg \lambda^2/\kappa^2$, forcing $\lambda^2 \ll 1$.

The Higgs sector in Region II always contains a very light pseudoscalar $a_s$ as well as a SM-like $h^0$ Higgs boson. The pseudoscalar mass is protected by an approximate $U(1)_R$ symmetry
in the singlet sector due to the GMSB boundary conditions, in addition to an approximate $U(1)_{PQ}$ arising from the small value of $\kappa/\lambda$. On account of this double-protection, the electroweak scale value of $A_\kappa$ is particularly small among the allowed parameter points. This is clearly shown in the left panel of Fig. 5. From the right panel of Fig. 5 we also see that the value of $|\mu|$ is much larger than $A_\kappa$, $A_\lambda$, and $\lambda v$ within the allowed parameter space.

With these very small values of $A_\lambda$ and $A_\kappa$, the mass of the pseudoscalar $a_s$ in Region II is always much less than half the mass of the SM-like $h^0$ Higgs boson. This feature is illustrated in the left panel of Fig. 6. In the right panel of Fig. 6 we show the dominant branching fractions of the $h^0$ Higgs boson. Despite being kinematically allowed, we find that $h^0 \rightarrow a_s a_s$ almost never occurs, with $BR(h^0 \rightarrow a_s a_s) < 3 \times 10^{-4}$. Instead, the decay fractions of the $h^0$ Higgs are nearly identical to those of a SM Higgs. The reason for this suppression of $h^0 \rightarrow a_s a_s$ can be seen in Eq. (2.8); the coupling $c$ is proportional to $\lambda^2 \ll 1$.

Collider production of the $h^0$ Higgs in Region II is also nearly identical to the SM. The singlet component of this mass eigenstate is quite small, corresponding to the mixing element $|U_{12}| \lesssim 0.25$, while the suppression of Higgs production through the usual channels is down by only $U_{11}^2 \gtrsim 0.94$. (See Appendix A for a full account of the mixing matrices.) Thus, the LHC signatures of the $h^0$ in Region II will be nearly identical to those of a SM Higgs (or an MSSM Higgs at large $M_A^0$). The mixing among the CP-odd states is even more suppressed, with the light $a_s$ being almost exclusively singlet. This provides another way to understand why the branching fraction of the $h^0$ into pseudoscalar pairs is so small in Region II.

Among the heavier Higgs bosons, the mostly-singlet $h_s$ can be relatively light since its mass is suppressed by a factor of $\kappa/\lambda \ll 1$. We find masses as low as 170 GeV, and as high as 1500 GeV. While this state is mostly singlet, a mixing with the SM-like Higgs as large as $|U_{21}| \simeq 0.25$ is possible. The primary decay modes are $WW$, $ZZ$, and $h^0 h^0$. For larger mixings and moderate masses, in the range $200 \text{ GeV} \lesssim m_{h_s} \lesssim 500 \text{ GeV}$, the $h_s$ may be
visible at the LHC through its $ZZ$ final state. All the other Higgs bosons, the $H^0$, $A^0$, and $H^\pm$, have masses in excess of 800 GeV, and are challenging to find at the LHC.

The rest of the particle spectrum within Region II is similar to the MSSM, but with an additional neutralino from the fermion component of $S$. To push $m_{h^0}$ above the LEP-II bound, the superpartner spectrum must be somewhat heavy with $M_3 \gtrsim 800$ GeV. On the other hand, the corresponding mostly-singlet neutralino state can be as light as 170 GeV in Region II, and can even be the lightest superpartner aside from the gravitino. Due to the very small values of $\lambda$ and $\kappa$, this state is almost pure singlet with a tiny higgsino component, and couples only very weakly to the rest of the MSSM. It will therefore almost completely decouple from the LHC phenomenology unless it is the NLSP (with a gravitino LSP). In this case, the presence of an additional state can modify cascade decay chains [47, 48, 49] and possibly also give rise to neutral displaced vertices from decays of the NNLSP to the NLSP [47, 50]. A mostly singlet NLSP could also be problematic if it decays after the onset of nucleosynthesis [51, 52, 53]. While a full analysis of this issue is beyond the scope of the present work, we expect that lighter gravitino masses (leading to shorter decay times) might be necessary to ensure that these decays occur before the light elements are formed.

4 Gauge Mediation in the NMSSM with Exotics

As a second variation on minimal gauge mediation in the NMSSM, we consider adding charged vector-like exotics to the theory [17, 21, 22]. We study exotics in the form of $\tilde{D} \oplus \tilde{D}^c$, and $\tilde{L} \oplus \tilde{L}^c$, with $\tilde{D} = (3, 1, -1/3)$ and $\tilde{L} = (1, 2, -1/2)$. Taken together, these exotics have
the quantum numbers of a $5 \oplus \bar{5}$ of $SU(5)$. We include the trilinear superpotential couplings

$$W \supset \xi_D S \bar{D} \bar{c} + \xi_L S \bar{L} \bar{c},$$

(4.1)
as well as a corresponding set of soft supersymmetry breaking operators. These couplings tend to drive $m_S^2$ negative in the course of RG running from the gauge messenger scale down to near the electroweak scale, thereby facilitating singlet condensation and electroweak symmetry breaking. The relevant RG equations for this evolution with $N_D$ sets of $\bar{D} \oplus \bar{D}^c$ and $N_L$ sets of $\bar{L} \oplus \bar{L}^c$ are listed in Appendix B. Note that we assume the exotics do not act as gauge messengers, which can be enforced with the approximate $\mathbb{Z}_3$ symmetry discussed in the introduction.

### 4.1 Allowed Parameter Regions

We have searched for regions of the NMSSM parameter space that lead to an acceptable phenomenology by scanning over the model parameters. Our strategy consists of specifying $\lambda$, $\xi_D$, $\xi_L$, and $F/M$ at the GMSB messenger scale $M$, along with $\tan \beta$ near the electroweak scale, and computing the low-energy spectrum that results from these input parameters. In doing so, we use a modified version of NMSPEC/NMHDECAY [28] to perform the RG evolution of the model parameters, to find the low-scale values of $\kappa$, $|\mu|$, and $m_S^2$, as in Eqns. (1.4, 1.5, 1.6), and to determine the phenomenological constraints. The value of $m_S^2$ computed in this way will not usually agree with the GMSB boundary condition of $m_S^2(M) \simeq 0$. To correct for this, we adjust the value of $\xi_D$ (or $\xi_L$), repeat the running, and iterate until the value of $|m_S^2|$ at the input scale $M$ lies below a small cutoff value. The parameter ranges covered in our scans are

$$\lambda \in [0.001, 0.7], \quad \xi_L \in [0, 1], \quad \xi_D \in [0, 1],$$

$$\tan \beta \in [1, 50], \quad M \in [10^5, 10^{14}] \text{ GeV}, \quad F/M \in [2, 40] \times 10^4 \text{ GeV}. \quad (4.2)$$

We assume minimal GMSB boundary conditions for all the soft terms with a single set of $5 \oplus \bar{5}$ messengers.

In Fig. 7 we show the allowed parameter points obtained by scanning with a single set $N_5 = 1$ of $5 \oplus \bar{5}$ (non-messenger) exotics. In the left panel of this figure we show points in the $\lambda$-$\kappa$ plane, while in the right panel we exhibit points in the $\lambda$-$\tan \beta$ plane. The red (dark) points agree with all relevant phenomenological bounds except for the mass constraints on the neutral $CP$-even and $CP$-odd Higgs bosons, while the green (light) points are also consistent with the Higgs constraints. As in the previous section, we do not demand the lightest neutralino be the LSP: in gauge mediation the gravitino is usually expected to be the true LSP. We do require that the couplings remain perturbatively small up to the scale $M$ (but not $M_{GUT}$).

The phenomenologically acceptable parameter points in Fig. 7 are similar to the parameter points found in Region II discussed in the previous section. These points all have very small values of $\lambda$, and relatively small values of $\kappa/\lambda$ and $\sin 2\beta$. Small values of $\lambda$ in
this region are needed to make the SM-like Higgs boson sufficiently heavy. Unlike in the previous section, however, we do not find any points at larger values of $\lambda$ and $\kappa$, analogous to Region I. For points of this type to lead to an acceptable pattern of symmetry breaking, Eqs. (1.4, 1.5, 1.6) require $|m_S^2|/\mu^2 \sim 1$ (or large $A$ terms). While the new Yukawa couplings in Eq. (4.1) help to drive $m_S^2$ to more negative values, their effect is not strong enough to open a region of parameter space with $\lambda \sim \kappa \sim 1$. With a single set of non-universal exotics, such as a lone $\tilde{L} \oplus \tilde{L}^c$ or $\tilde{D} \oplus \tilde{D}^c$, we find qualitatively similar results.

To magnify the effect of the exotics on the running of $m_S^2$, we have also performed scans with multiple sets of $5 \oplus \bar{5}$ (non-messenger) exotics. For simplicity, we assume universal values of the couplings $\xi_D$ and $\xi_L$ for all flavors of exotics. Adding even only a second set of $5 \oplus \bar{5}$ exotics opens a new region of phenomenologically consistent points, similar to Region I discussed in Section 3. These points have $\kappa \sim \lambda \gtrsim 0.4$ and $\tan \beta \lesssim 2.5$. A similar region can emerge with more than two sets of $5 \oplus \bar{5}$ exotics. The allowed region with $\lambda \ll 1$ also remains. With several sets of exotics, a tension arises between larger values of $\lambda$ and $\kappa$ at the low scale and perturbativity up to the messenger scale $M$ since the exotic Yukawa couplings of Eq. (4.1) speed up to the running of $\lambda$.

The NMSSM Higgs boson spectrum depends sensitively on the values of $A_\lambda$, $A_\kappa$, and $\mu$ near the electroweak scale. In Fig. 8 we plot these parameters for $N_5 = 1$ and $N_5 = 2$ sets of $5 \oplus \bar{5}$ exotics. For the case of $N_5 = 2$, we split up the points according to whether they fall into the $\lambda < 0.1$ region, or the $\lambda > 0.4$ region. These plots indicate that $A_\lambda$ and $A_\kappa$ remain smaller than $|\mu|$ (larger values of $A_{\lambda,\kappa}$ correspond to very large values of $\mu$), but are enhanced relative to the electroweak scale. This enhancement comes from the contributions from the new Yukawa couplings $\xi_D$ and $\xi_L$ in Eq. (4.1) to $A_\lambda$ and $A_\kappa$ in the course of RG running.
4.2 LHC Higgs Signatures with Exotics

Much like in Section 3, the Higgs phenomenology with additional vector-like exotics can be split into two classes, depending on whether $\lambda$ is small ($\lambda \lesssim 0.1$) or $\lambda$ is larger ($\lambda \gtrsim 0.4$). This second possibility only occurs with at least two sets of $5 \oplus \bar{5}$ exotics. We will discuss both of these cases in turn.

In the allowed parameter region with $\lambda \ll 1$ (along with $|\kappa/\lambda| \ll 1$ and $\sin 2\beta \ll 1$) the Higgs boson phenomenology is quite similar to what we found in Region II discussed in Section 3. We show the masses of the neutral $CP$-even and $CP$-odd Higgs bosons in Fig. 9 for phenomenologically consistent points obtained in our scan with a single set of $5 \oplus \bar{5}$ exotics. The lighter Higgses consist of the SM-like $h^0$, the mostly-singlet $h_s$, and a light mostly-singlet pseudoscalar $a_s$. The $A^0$, $H^0$, and $H^\pm$ states are similar to their MSSM counterparts, and are generally very heavy. There are some slight but important differences compared to Region II of Section 3 however.

The singlet $A$ terms that arise in the course of RG running are now somewhat larger, due to the new exotic Yukawa couplings (see Fig. 8). The result is the mostly singlet pseudoscalar $a_s$ can be considerably heavier than in Region II discussed above. Depending on the values of these trilinear couplings, $m_{a_s}$ can range from below 1 GeV all the way up to over 250 GeV. This state is nearly pure singlet with only a tiny admixture of the $A^0$ as a result of the very small values of $\lambda$ and $\kappa$. We expect it to be almost completely decoupled from the rest of the theory, and consequently invisible at the LHC. Similar considerations minimize the ability to probe this state through upsilon decays, if kinematically accessible. The dominant decay modes of the $a_s$ are into $b\bar{b}$, $\tau\bar{\tau}$, and so on, depending on which channels are kinematically accessible.
The LHC phenomenology of the $h^0$ and $h_s$ states is more interesting. As in Region II of the previous section, the branching fractions of the $h^0$ Higgs are similar to those of a SM Higgs, and thus the LEP bound of about 114 GeV applies throughout most of the parameter region. An exception occurs when the mixing between the $h^0$ and $h_s$ states is enhanced when they become nearly degenerate. Even in this case, the reduction in the mass bound on the SM-like state is usually less than a few GeV. The mostly-singlet $h_s$ Higgs can be very light in some cases, with masses as low as 20 GeV, but also as heavy as about 800 GeV. This state inherits its couplings to the MSSM through its mixing with the SM-like Higgs. Thus, the $h_s$ decay modes mirror those of a SM Higgs boson. The chief exception to this occurs when the channel $h_s \to h^0h^0$ opens up. The corresponding branching fraction can be as large as $BR(h_s \to h^0h^0) \approx 0.4$. For heavier $h_s$ masses, this state might be visible at the LHC through its decays to $ZZ$, even though its production cross-section is suppressed relative to a SM Higgs by its large singlet component. All the other Higgs boson states are heavier than about 500 GeV, making them challenging to find at the LHC unless $\tan \beta$ is very large.

The rest of the particle spectrum within this small $\lambda$ region is similar to that of the MSSM. Compared to Region II, the overall superpartner scale can be somewhat lower, with gluino masses as small as $M_3 \simeq 600$ GeV now possible, and the additional mostly-singlet neutralino can be very light. The mass of this singlet neutralino is approximately $2\kappa \mu/\lambda$, which can be as light as about 20 GeV for particularly small values of $\kappa/\lambda$, but also as large as several hundred GeV in other portions of the allowed parameter space. Mixing between the singlet fermion and the other neutralinos is always very small. The dominant contribution comes from the higgsinos with a mixing angle $|O_{15}| \lesssim 0.01$. Thus, this fifth neutralino state is nearly invisible at colliders unless it is the NLSP (with a gravitino LSP). As discussed in Section 3, a mostly-singlet NLSP can modify sparticle cascade chains \cite{47,48,49}. In this case, lighter gravitino masses may be required to ensure that the NLSP decays safely before nucleosynthesis.
The second allowed region of NMSSM parameter space with heavy exotics and minimal GMSB boundary conditions has $\lambda \gtrsim 0.4$ and smaller values of $\tan \beta \lesssim 2.5$. These parameter ranges are similar to those encountered in Region I of Section 3. Despite the similarity of these values, the Higgs boson phenomenology does not mirror that of Region I. In fact, it does not significantly differ from that of the MSSM. To populate this large $\lambda$ region, at least two sets of $5 \oplus \bar{5}$ exotics are required in order to drive $m_S^2$ sufficiently negative over the course of the RG evolution. The exotic Yukawa couplings, $\xi_D$ and $\xi_L$, responsible for doing so also contribute to the low-scale values of $A_\lambda$ and $A_\kappa$. This ruins the approximate $U(1)_R$ symmetry in the singlet sector. As a result, the singlet pseudoscalar is no longer very light, with masses above $m_{a_s} \gtrsim 350$ GeV. The SM-like $h^0$ state is therefore very SM-like, both in its production and decay modes, since it is no longer able to decay into pairs of the $a_s$ pseudoscalar. All the other Higgs boson states are heavier than about 500 GeV, and will be difficult to find at the LHC, particularly with the smaller values of $\tan \beta$ that occur in this region. The additional neutralino state that arises from the fermion component of the singlet $S$ is also quite heavy, with a mass of $2 \kappa \mu / \lambda \sim 2\mu$, and has only a small mixing with the higgsinos. It too will be essentially invisible at the LHC.

4.3 Phenomenology of the Exotics

The vector-like exotic states we have added to the theory to facilitate electroweak symmetry breaking can themselves be a source of new signatures at the LHC. These charged exotics must certainly be heavy enough to have avoided detection already. Beyond this, the exotics are problematic for cosmology if they are overly long-lived (or stable). We briefly consider the additional bounds and potential signatures that arise from the exotics. These should inform any model building attempt.

In Fig. 10 we plot the masses of the charged $\tilde{D}$- and $\tilde{L}$-type exotics originating from the superpotential couplings of Eq. (4.1). As before, the red (dark) points in this plot are consistent with all phenomenological bounds other than the constraints on the Higgs sector (and on the exotics themselves), while the green (light) points satisfy all relevant Higgs bounds as well. Nearly all the points in this figure have exotic masses well in excess of the current limits.

The precise collider bounds on the exotics depend on whether they are long- or short-lived on collider timescales. In the case of long-lived charged leptons, the best bound comes from searches by OPAL for (effectively) stable charged particles, and is $m_\ell \gtrsim 100$ GeV for a heavy spin-1/2 particle with electric charge $\pm 1$ and no color [54]. The bound for decaying charged leptons is about the same. The most stringent bound on long-lived heavy quarks comes from Tevatron searches for charged massive particles. A preliminary analysis with 1 $fb^{-1}$ of Tevatron Run II data suggests a limit on the cross section for such states of about 0.1 pb [55], corresponding to quark masses up to nearly 300 GeV. For short-lived heavy quarks, the precise bound depends on how they decay. CDF has performed a preliminary search for a heavy exotic top quark decaying through $t' \rightarrow Wq$, and find $m_{t'} > 258$ GeV [56]. No $b$-tag is used in the analysis so we expect this result to apply for a short-lived exotic $\tilde{D}$ as well [57].
Figure 10: Masses of the charged $D$ and $L$ exotics among the phenomenologically allowed parameter points found for the NMSSM with a single set of $\mathbf{5} \oplus \bar{\mathbf{5}}$ exotics and a minimal GMSB soft mass spectrum.

To be cosmologically acceptable, the heavy exotics cannot be too long-lived. If stable on the lifetime of the universe, heavy charged exotics are very stringently constrained by searches for anomalously heavy atoms. These bounds are so severe that even the tiny density of heavy exotics created by cosmic rays is unacceptably large \cite{58}. Long-lived heavy exotics are also dangerous if they decay after nucleosynthesis as their decay products can modify the light element abundances, distort the CMB blackbody spectrum, or contribute to cosmic rays \cite{59}. On the other hand, a significant coupling between the exotics and the MSSM matter fields can give rise to too much flavor mixing.

In the NMSSM with an approximate $\mathbb{Z}_3$ discrete symmetry, the exotic $\tilde{D}^{(c)}$ and $\tilde{L}^{(c)}$ states can decay through the $d = 4$ superpotential operators

$$L\tilde{L}^c H_u H_d, \quad L\tilde{L}^c SS, \quad D^c\tilde{D} H_u H_d, \quad D^c\tilde{D} SS.$$  \hspace{1cm} (4.3)

Such operators can be consistent with the approximate $\mathbb{Z}_3$ symmetry while still allowing all the standard NMSSM operators, as well as the neutrino mass operator $(LH_u)^2$, and forbidding mixing between the exotics and the MSSM matter states at the renormalizable level. Decays through the operators of Eq. (4.3) occur safely before nucleosynthesis provided the heavy mass scale suppressing them is less than about the GUT scale, $M_{\text{GUT}} \approx 10^{16}$ GeV \cite{59, 60}.

If the charged and colored exotics are not too heavy, they might lead to observable signatures at the LHC. Stable (on collider times) heavy quarks were studied recently in
Ref. [59]. These can form charged exotic hadrons that punch through to the muon chamber. By measuring the time of flight, they can be distinguished from ordinary muons. A significant signal of ten events with almost no background will be generated with $10\, fb^{-1}$ for heavy quark masses below $m_D \lesssim 1700$ GeV, and $m_\tilde{D} \lesssim 1450$ GeV for the scalar superpartner. The precise limits on unstable heavy quarks depend on how they decay, but are generally of the similar size. Heavy stable charged leptons can also be detected at the LHC through time-of-flight techniques [61, 62]. Masses below about 950 GeV can be detected in this way with 100 fb$^{-1}$ of integrated luminosity.

5 Gaugino Mediation in the NMSSM

Gaugino mediation of supersymmetry breaking shares many of the attractive features of gauge-mediated supersymmetry breaking [19, 20]. In the context of obtaining a light pseudoscalar in the NMSSM, minimal gaugino mediation has the helpful property of vanishing trilinear $A$ terms in the singlet sector at the mediation scale. Unfortunately, like minimal gauge mediation, this mediation mechanism in its minimal form has trouble generating a sufficiently negative low-scale value of the singlet soft mass $m_S^2$ to obtain an acceptable pattern of electroweak symmetry breaking. In this section we study gaugino mediation in the NMSSM with the same modifications of the previous two sections: we augment the theory by additional contributions to the input value of $m_S^2$, or by adding new vector-like exotics coupled to the $S$ field.

5.1 Higgs Phenomenology with $m_S^2$ Free

We study first minimal gaugino mediation within the NMSSM with the singlet soft mass $m_S^2$ taken to be a free parameter. One might motivate this scenario by putting the $S$ field in the bulk, so that it could feel the SUSY breaking directly. However, to avoid generating singlet $A$ terms that would ruin the approximate $U(1)_R$ symmetry in the singlet sector, such a bulk singlet should couple to a SUSY breaking source that also preserves the $U(1)_R$. It is a model-building challenge to obtain such a source of supersymmetry breaking with the correct mass scale.

As in Section 3, we search for phenomenologically acceptable parameter regions by scanning over the model parameters. Our scan encompasses the ranges

$$\lambda \in [0.001, 0.7], \quad \tan \beta \in [1, 50] \text{ GeV}, \quad M_{1/2} \in [100, 1500] \text{ GeV}, \quad M_c \in [10^5, 10^{17}] \text{ GeV}. \quad (5.1)$$

The parameter $M_{1/2}$ defines the gaugino mass at the compactification scale $M_c$ through the relation

$$M_a(M_c) = 2 g_a^2(M_c) M_{1/2}. \quad (5.2)$$

This choice corresponds to universal gaugino masses equal to $M_{1/2}$ when $M_c = M_{\text{GUT}}$ ($g_a^2(M_{\text{GUT}}) \simeq 1/2$). The input values of all the soft scalar masses (save $m_S^2$) and all the
A terms are taken to vanish at the input scale $M_c$. Over the course of the RG running, we require that all couplings remain perturbatively small up to $M_c$.

The allowed regions found in our scans for minimal gaugino mediation with $m^2_S$ free are nearly identical to what we found in Section 3 for minimal GMSB, and illustrated in Fig. 7. As before, there are two disjoint phenomenologically consistent regions: one at small values of $\lambda$, $\kappa/\lambda$, and $\sin 2\beta$ much like in Region II; and the other with larger values of $\lambda$ and $\sin 2\beta \sim \kappa/\lambda \sim 1$ similar to Region I. In both allowed regions, there exists an approximate $U(1)_R$ symmetry in the singlet sector that is realized as a hierarchy between $A_\lambda$ and $A_\kappa$ relative to $\mu$. The Higgs boson phenomenology within these two regions is qualitatively the same as in Regions I and II discussed in Section 3.

Let us also mention that in the parameter scans, we do not demand that the lightest NMSSM superpartner be neutral. For compactification scales below about $M_c \lesssim 10^{16}$ GeV, the lightest MSSM superpartner in minimal gaugino mediation is often a mostly-right-handed stau [19, 20, 63]. The true LSP in this case is typically the gravitino [19, 64]. At larger values of the compactification scale, the lightest MSSM superpartner in minimal gaugino mediation is usually a mostly-Bino neutralino. Within the NMSSM, there can also arise a very light mostly-singlet neutralino state when $\kappa/\lambda$ is small, as we discussed in Sections 3 and 4. Such a state can supplant the stau as the lightest NMSSM superpartner in gaugino mediation, providing another way to get around the problem of a charged LSP. This also leads to new possibilities for the production and identity of the dark matter, such as by the decoupling of a singlet neutralino LSP [49], or through the superWIMP scenario [51] with a singlet neutralino NLSP. This latter possibility, however, is likely constrained by the overproduction of hadronic debris after the onset of nucleosynthesis [51, 53].

### 5.2 Higgs Phenomenology with Vector-Like Exotics

A second modification of minimal gaugino mediation that can improve the prospects for electroweak symmetry breaking in the NMSSM is to add vector-like exotics to the theory. As in Section 4 we consider $5 \oplus \bar{5}$ exotics with superpotential couplings to the singlet given by Eq. (4.1). We again search for phenomenologically acceptable parameter regions by scanning over the model parameters. Our scans encompass the same ranges of input values as listed above, along with the exotic couplings

$$
\xi_D \in [0, 1], \quad \xi_L \in [0, 1].
$$

(5.3)

When we include more than one set of exotics, we assume that the values of $\xi_D$ and $\xi_L$ are the same for all exotic flavors. Gaugino mediated boundary conditions are imposed at the compactification scale $M_c$ for all the soft terms, including $m^2_S$.

Our search strategy is similar to Section 4 and consists of specifying $\lambda$, $\xi_D$, $\xi_L$, and $M_{1/2}$ at the compactification scale $M_c$, along with $\tan \beta$ near the electroweak scale, and computing the low-energy spectrum that results from these input parameters. In doing so, we use a modified version of NMSPEC/NMHDECAY [28] to perform the RG evolution of the model parameters and to find the low-scale values of $\kappa$, $|\mu|$, and $m^2_S$. The value of $m^2_S$
computed in this way will not usually agree with the gaugino mediation boundary condition of \( m_S^2(M_c) \simeq 0 \). To correct for this, we adjust the value of \( \xi_D \) (or \( \xi_L \)), repeat the running, and iterate until the value of \( |m_S^2| \) at the input scale \( M_c \) lies below a specified small cutoff value. Again, we demand that all couplings remain perturbative up to \( M_c \), but we do not require the lightest NMSSM superpartner to be neutral.

The results of our scans are very similar to what we found Section 4, and illustrated in Fig. 7. With a single set of \( 5 \oplus \bar{5} \) exotics, the phenomenologically allowed region of the parameter space is nearly identical to the small \( \lambda \) region discussed in Section 4 as well as Region II studied in Section 3. With two or more sets of \( 5 \oplus \bar{5} \) exotics, we find a second disjoint allowed parameter region with \( \lambda \sim \kappa > 0.4 \) and \( \tan \beta \lesssim 2.5 \). This region of the parameter space is much the same as the large \( \lambda \) region discussed in Section 4. In particular, the new exotic Yukawa couplings \( \xi_D \) and \( \xi_L \) help to transmit the \( U(1)_R \) breaking from the gauginos to the singlet sector, leading to somewhat larger values for the trilinear \( A \) terms at the low scale. The mostly-singlet pseudoscalar \( a_s \) is always heavier than the \( h^0 \) Higgs as a result, leading to a Higgs collider phenomenology nearly identical to the MSSM with a pseudoscalar mass in excess of 500 GeV.

6 Conclusions

In the present work we have studied two simple deformations of gauge and gaugino mediation within the NMSSM. The first deformation consists of allowing the singlet soft mass \( m_S^2 \) to be a free parameter at the mediation scale, such as might arise if the singlet couples to a \( U(1)_R \) preserving source of supersymmetry breaking. The second deformation involves adding vector-like exotics with superpotential couplings to the singlet superfield. Both deformations facilitate electroweak symmetry breaking by driving the singlet soft mass \( m_S^2 \) to negative values in the infrared.

Near the electroweak scale, for either deformation of both minimal gauge and gaugino mediation, we find a hierarchy between the value of the effective \( \mu \) parameter, \( \mu = \mu_{\text{eff}} = \lambda v_s \), and the \( A_\lambda \) and \( A_\kappa \) soft trilinear couplings, as well as \( \lambda v \). The relative smallness of the singlet \( A \) terms leads to an approximate \( U(1)_R \) symmetry in the singlet sector, and consequently to a light pNGB pseudoscalar when this would-be symmetry is spontaneously broken. The presence of this light pseudoscalar can have a large effect on the Higgs signatures of the theory.

We find two distinct ways in which these deformations can lead to consistent electroweak symmetry with a phenomenologically acceptable Higgs boson spectrum. The first and more interesting case requires \( |m_S^2/\mu^2| \) as well as \( \kappa/\lambda \) and \( \sin 2\beta \) to be on the order of unity near the electroweak scale, along with \( \lambda \gtrsim 0.4 \). With these parameter values, there is an additional \( F \)-term contribution to the mass of the lightest \( CP \)-even Higgs boson. If, in addition, \( A_\kappa \) and \( A_\lambda \) remain small near the electroweak scale, there is a SM-like Higgs boson state in the spectrum that can decay predominantly into pairs of the light pNGB \( a_s \) pseudoscalar leading to new Higgs boson signatures at the LHC. These pseudoscalars usually decay primarily into \( b\bar{b} \), but can have a dominant branching into \( \tau \bar{\tau} \) if they are particularly light (\( m_{a_s} < 10 \) GeV).
This scenario is realized most easily in both gauge and gaugino mediation when \( m^S_2 \) is allowed to be free, with no additional contributions to the singlet trilinear couplings at the mediation scale.

The second way to obtain consistent electroweak symmetry breaking within the deformations considered here is to have \( \kappa/\lambda \sim \sin 2\beta \ll 1 \) near the electroweak scale. This occurs when \( |m^S_2|/\mu^2 \ll 1 \) with small singlet \( A \) terms. For the electroweak symmetry breaking extremum to be stable, it is then necessary to have \( \lambda \ll 1 \). In this case, the singlet sector couples only very weakly to the MSSM states, and therefore mostly decouples. It is still possible to have a very light pNGB pseudoscalar, but since it interacts only feebly with the SM-like Higgs state in the spectrum, the decays of this Higgs boson into pseudoscalar pairs are extremely rare. In general, the Higgs phenomenology in this case is very similar to the MSSM with a relatively heavy pseudoscalar Higgs boson. Despite the decoupling property of this region, the supersymmetric phenomenology can still be modified if the lightest NMSSM superpartner state is a mostly-singlet neutralino.

Let us emphasize that we have only considered simple deformations of minimal gauge and gaugino mediation. In these minimal versions, we always find relatively large values for the effective \( \mu \) parameter near the electroweak scale. By allowing for non-minimal versions of these mediation mechanisms, it may be possible for much smaller values of \( \mu \) to emerge. In a scenario with \( \mu \sim \lambda v \), there may be new ways to obtain a consistent pattern of electroweak symmetry breaking, possibly also with decays of a SM-like Higgs bosons into pairs of light pseudoscalars. While not all models of supersymmetry breaking will give rise to modified Higgs boson phenomenology, it is exciting that very minor modifications to models as simple and well-motivated as gauge mediation and gaugino mediation can. This emphasizes the need to search for Higgs bosons at the LHC with as broad a net as possible.

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**Appendix**

**A Higgs Boson Masses and Mixings**

In this Appendix we present analytic expressions for the Higgs boson mass eigenvalues and mixing matrices valid in the limit of an approximate \( U(1)_R \) symmetry in the singlet sector and \( \lambda v \ll |\mu| \). Our results extend the findings of Ref. [15]. A related expansion valid in the limit of an approximate \( U(1)_{PQ} \) symmetry can be found in Ref. [7].
The CP-odd Higgs bosons of the NMSSM are made up of \( \text{Im}(S)/\sqrt{2} \) and the non-Goldstone combination \( A_v^0 \) of \( \text{Im}(H_u^0)/\sqrt{2} \) and \( \text{Im}(H_d^0)/\sqrt{2} \). In the \( \{\text{Im}(S)/\sqrt{2}, A_v^0\} \) basis the mass matrix is

\[
\mathcal{M}_A^2 = \left( \frac{\mu}{\lambda} \right)^2 \begin{pmatrix} \frac{s_{2\beta}}{2\lambda} (4 + \alpha)\gamma^2 - 3\delta & \gamma(\alpha - 2) \\ \gamma(\alpha - 2)^{-1} (1 + \alpha) \end{pmatrix}.
\]  

(A.1)

In writing this expression, we have defined the variables

\[
\bar{\lambda} = \frac{\lambda}{\kappa}, \quad \gamma = \frac{\lambda v}{\mu}, \quad \alpha = \frac{\lambda A_\lambda}{\kappa \mu}, \quad \delta = \frac{\lambda A_\kappa}{\kappa \mu}.
\]  

(A.2)

We will assume that \( \gamma, \alpha, \) and \( \delta \) are all much less than unity, and treat \( \bar{\lambda} \) and \( s_{2\beta} \) as being on the order of unity. Under these assumptions, the \( M_{A_{11}}^2 \) element is much larger than the \( M_{A_{11}}^2 \) and \( M_{A_{12}}^2 \) elements. Thus, the rotation angle to diagonalize the mass matrix will be small and the mass eigenstates will be close to \( \text{Im}(S)/\sqrt{2} \) and \( A_v^0 \). Labelling these mass eigenstates by \( \{a_s, A_0^0\} \), we find the mixing matrix to be

\[
\begin{pmatrix} a_s \\ A_0^0 \end{pmatrix} = \mathcal{O} \begin{pmatrix} \text{Im}(S)/\sqrt{2} \\ A_v^0 \end{pmatrix},
\]

(A.3)

with the approximate rotation matrix given by

\[
\mathcal{O}_{11} = 1 - \frac{s_{2\beta}}{2\lambda^2} (1 - 3\alpha)\gamma^2 = \mathcal{O}_{22}
\]

(A.4)

\[
\mathcal{O}_{12} = \frac{s_{2\beta}}{2\lambda} \gamma \left( 2 - 3\alpha + 3\alpha^2 - 3\frac{s_{2\beta}}{\lambda} \delta \right) - \frac{s_{2\beta}^3}{2\lambda^3} \gamma^3 = -\mathcal{O}_{21}.
\]

(A.5)

In deriving these expressions we have treated \( \alpha, \gamma = \mathcal{O}(\epsilon) \) and \( \delta = \mathcal{O}(\epsilon^2) \) with \( \epsilon \ll 1 \), and we have kept terms only up to \( \mathcal{O}(\epsilon^3) \).

The corresponding mass eigenvalues to this level of approximation are

\[
m_{a_s}^2 \left( \frac{\lambda}{\mu} \right)^2 = -3\delta + 9\alpha \frac{s_{2\beta}}{2\lambda} \gamma^2,
\]

(A.6)

\[
m_{A_0^0}^2 \left( \frac{\lambda}{\mu} \right)^2 = \frac{2\bar{\lambda}}{s_{2\beta}} (1 + \alpha) + 4\frac{s_{2\beta}}{2\lambda} (1 - 2\alpha)\gamma^2.
\]

(A.7)

To discuss the NMSSM CP-even Higgs mass matrices and their eigenvalues, it is convenient to work in the basis \( \{h_u^0, h_s^0, H_v^0\} \), where \( h_u^0 \) is the combination of \( \text{Re}(H_u^0)/\sqrt{2} \) and \( \text{Re}(H_d^0)/\sqrt{2} \) that has the same tree-level couplings to the electroweak gauge bosons as the

\footnote{This expansion also works quite well for small \( \kappa/\lambda \) and \( s_{2\beta} \) up to higher-order terms in \( \alpha \).}
SM Higgs, $H_0^0$ is the orthogonal combination, and $h_s^0 = Re(S)/\sqrt{2}$. The transformation to this basis is simply

$$
\begin{pmatrix}
Re(H_u^0)/\sqrt{2} \\
Re(H_d^0)/\sqrt{2} \\
Re(S)/\sqrt{2}
\end{pmatrix}
= \begin{pmatrix}
\cos \beta & \sin \beta & 0 \\
-\sin \beta & \cos \beta & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
H_v^0 \\
h_v^0 \\
h_s^0
\end{pmatrix}.
$$  (A.8)

The $CP$-even symmetric mass matrix in the basis $\{h_v^0, h_s^0, H_v^0\}$ reads

$$
\mathcal{M}^2_H = \left( \frac{\mu}{\lambda} \right)^2 \begin{pmatrix}
\gamma^2 (s_{2\beta}^2 + \tilde{g}^2 c_{2\beta}) & 2\tilde{\lambda}\gamma - \gamma(2 + \alpha)s_{2\beta} & \gamma^2 (1 - \tilde{g}^2)s_{2\beta}c_{2\beta} \\
4 + \delta + \frac{s_{2\beta}}{2\lambda}\alpha\gamma^2 & -\gamma(2 + \alpha)c_{2\beta} & (1 + \alpha)\frac{2\tilde{\lambda}}{s_{2\beta}} - \gamma^2 (1 - \tilde{g}^2)s_{2\beta}^2
\end{pmatrix}.
$$  (A.9)

Here, we have defined

$$
\tilde{g}^2 = \frac{(g^2 + g'^2)}{2\lambda^2}.
$$  (A.10)

The $\{h_v^0, h_s^0, H_v^0\}$ basis is useful because all the mixing elements in this matrix are suppressed by at least a factor of $\gamma$, while the lower two diagonal elements are of order unity. Since this mixing is small, we will designate the mass eigenvalues by $\{h^0, h_s, H^0\}$ in analogy with the corresponding MSSM states.

The transformation to the mass eigenbasis is

$$
\begin{pmatrix}
h^0 \\
h_s \\
H^0
\end{pmatrix}
= U
\begin{pmatrix}
h_v^0 \\
h_v^0 \\
H_v^0
\end{pmatrix},
$$  (A.11)

with the unitary matrix $U$ given by
\[ U_{11} = 1 - \frac{\gamma^2}{32} \left[ 2\bar{\lambda} - (2 + \alpha)s_{2\beta} \right]^2 \]  
\tag{A.12}

\[ U_{12} = \frac{\gamma}{16} \left[ (-4 + \delta)(2\bar{\lambda} - (2 + \alpha)s_{2\beta}) \right] \]  
\tag{A.13}
\[ + \frac{\gamma^3}{16} \left( (\bar{\lambda} - s_{2\beta})(3\bar{\lambda}^2 - 6\bar{\lambda}s_{2\beta} + s_{2\beta}^2) \right. \]
\[ \left. - 2\frac{c_{2\beta}}{\lambda}[2\bar{\lambda}s_{2\beta} + \bar{g}^2(\bar{\lambda} - 2s_{2\beta})(\bar{\lambda} + s_{2\beta})] \right) \]

\[ U_{21} = -\frac{\gamma}{16} \left[ (-4 + \delta)(2\bar{\lambda} - (2 + \alpha)s_{2\beta}) \right] \]  
\tag{A.14}
\[ - \frac{\gamma^3}{16} \left( (\bar{\lambda} - s_{2\beta})(3\bar{\lambda}^2 - 6\bar{\lambda}s_{2\beta} + s_{2\beta}^2) \right) \]
\[ - \frac{2c_{2\beta}}{(\lambda - 2s_{2\beta})^2} \left[ 2s_{2\beta}(\bar{\lambda}^2 - 3\bar{\lambda}s_{2\beta} + s_{2\beta}^2) + \bar{g}^2\bar{\lambda}(\bar{\lambda} - 3s_{2\beta})(\bar{\lambda} - 2s_{2\beta}) \right] \]

\[ U_{22} = 1 - \frac{1}{8}\gamma^2 \left( (\bar{\lambda} - s_{2\beta})[\bar{\lambda} - (1 + \alpha)s_{2\beta}] \right) \]  
\tag{A.15}
\[ - \frac{4s_{2\beta}^2c_{2\beta}^2}{(\lambda - 2s_{2\beta})^3} \left( (-1 + \alpha)\bar{\lambda} + 2(1 + \alpha)s_{2\beta} \right) \]

\[ U_{13} = \frac{\gamma^2s_{2\beta}c_{2\beta}}{4} \left[ (2 + \alpha)\bar{\lambda} + 2(\alpha + \bar{g}^2 - \alpha\bar{g}^2)s_{2\beta} \right] \]  
\tag{A.16}

\[ U_{31} = -\frac{\gamma^2}{2} \left( \frac{c_{2\beta}s_{2\beta}^2}{\lambda(\lambda - 2s_{2\beta})}\left[ \bar{\lambda} + \bar{g}^2\bar{\lambda} - 2\bar{g}s_{2\beta} \right] \right) \]  
\tag{A.17}
\[ - \alpha\frac{s_{2\beta}^2c_{2\beta}}{\lambda(\lambda - 2s_{2\beta})^2}\left[ (2 + \bar{g}^2)\bar{\lambda}^2 - 4\bar{g}^2\bar{\lambda}s_{2\beta} - 4(1 - \bar{g}^2)s_{2\beta}^2 \right] \]

\[ U_{23} = \gamma \left( \frac{c_{2\beta}s_{2\beta}}{2(\lambda - 2s_{2\beta})^2}\left[ (2 - \alpha)\bar{\lambda} - (4 + 2\alpha - \delta)s_{2\beta} \right] + \alpha^2\frac{s_{2\beta}^2c_{2\beta}\bar{\lambda}}{2(\lambda - 2s_{2\beta})^3}(\bar{\lambda} + 2s_{2\beta}) \right) \]  
\tag{A.18}
\[ - \gamma^3 \frac{1}{8(\lambda - 2s_{2\beta})^2}c_{2\beta}s_{2\beta}\left( \bar{\lambda}^4 - 2(4 + \bar{g}^2)\bar{\lambda}^3s_{2\beta} + (19 + 10\bar{g}^2)\bar{\lambda}^2s_{2\beta}^2 - 4(5 + 3\bar{g}^2)\bar{\lambda}s_{2\beta}^3 + 12s_{2\beta}^4 \right) \]

\[ U_{32} = -\gamma \left( \frac{s_{2\beta}^2c_{2\beta}}{2(\lambda - 2s_{2\beta})^2}\left[ (2 - \alpha)\bar{\lambda} - (4 + 2\alpha - \delta)s_{2\beta} \right] + \alpha^2\frac{s_{2\beta}^2c_{2\beta}\bar{\lambda}}{2(\lambda - 2s_{2\beta})^3}(\bar{\lambda} + 2s_{2\beta}) \right) \]  
\tag{A.19}
\[ + \gamma^3 \frac{c_{2\beta}s_{2\beta}^2}{2(\lambda - 2s_{2\beta})^3}\left( 3c_{2\beta}\bar{\lambda} - (\bar{\lambda} - 2s_{2\beta})[(1 + \bar{g}^2)\bar{\lambda}^2 - 4\bar{g}^2\bar{\lambda}s_{2\beta} + 2\bar{g}^2s_{2\beta}^2] \right) \]

\[ U_{33} = 1 + \gamma^2 \frac{s_{2\beta}^2c_{2\beta}^2}{2(\lambda - 2s_{2\beta})^3} \left[ (1 + \alpha)\bar{\lambda} + 2(1 + \alpha)s_{2\beta} \right] . \]  
\tag{A.20}
The CP-even Higgs mass eigenvalues are

\[ m_{H^0}^2 \left( \frac{\tilde{\lambda}}{\mu} \right)^2 = \left[ g^2 c_{2\beta}^2 - \tilde{\lambda}^2 + (2 + \alpha)s_{2\beta}\tilde{\lambda} - \alpha s_{2\beta}^2 \right], \]  
(A.21)

\[ m_{h_s}^2 \left( \frac{\tilde{\lambda}}{\mu} \right)^2 = 4 + \delta \]  
(A.22)

\[ m_{H^0}^2 \left( \frac{\tilde{\lambda}}{\mu} \right)^2 = \frac{2\tilde{\lambda}(1 + \alpha)}{s_{2\beta}} \]  
(A.23)

\[ + \gamma^2 \left( -1 + g^2 \right)s_{2\beta}^2 + \frac{2c_{2\beta}s_{2\beta}}{(\tilde{\lambda} - 2s_{2\beta})^2}[\tilde{\lambda} - 2(1 + \alpha)s_{2\beta}] \].

\[ + \gamma^2 \left( \tilde{\lambda} + \frac{\alpha s_{2\beta}}{2\tilde{\lambda}} - (2 + \alpha)\tilde{\lambda}s_{2\beta} + (1 + \alpha)s_{2\beta}^2 - \frac{2c_{2\beta}s_{2\beta}}{(\tilde{\lambda} - 2s_{2\beta})^2}[\tilde{\lambda} - 2(1 + \alpha)s_{2\beta}] \right), \]

\[ + \gamma^2 \left( \tilde{\lambda} + \frac{\alpha s_{2\beta}}{2\tilde{\lambda}} - (2 + \alpha)\tilde{\lambda}s_{2\beta} + (1 + \alpha)s_{2\beta}^2 - \frac{2c_{2\beta}s_{2\beta}}{(\tilde{\lambda} - 2s_{2\beta})^2}[\tilde{\lambda} - 2(1 + \alpha)s_{2\beta}] \right). \]

B RG Equations with Exotics

We collect here the modifications to the RG equations that arise when multiple sets of vector-like exotics are added to the NMSSM. Our notation conventions follow NMHDECA Y. The additional exotics we consider consist of \( \tilde{D} \oplus \tilde{D}^c \) and \( \tilde{L} \oplus \tilde{L}^c \), with \( \tilde{D} = (3, 1, -1/3) \) and \( \tilde{L} = (1, 2, -1/2) \). Taken together, these exotics have the quantum numbers of a \( 5 \oplus 5 \) of \( SU(5) \). We include the trilinear exotic superpotential couplings

\[ W \supset \xi_{D_i} S \tilde{D}_i \tilde{D}_i^c + \xi_{L_j} S \tilde{L}_j \tilde{L}_j^c. \]  
(B.1)

We also add the soft terms

\[ - \mathcal{L}_{soft} \subset m_{D_i}^2 |\tilde{D}_i|^2 + m_{\tilde{D}_i}^2 |\tilde{D}_i^c|^2 + m_{L_j}^2 |\tilde{L}_j|^2 + m_{\tilde{L}_j}^2 |\tilde{L}_j^c|^2 \]  
(B.2)

\[ + \xi_{D_i} A_{D_i} S \tilde{D}_i \tilde{D}_i^c + \xi_{L_j} A_{L_j} S \tilde{L}_j \tilde{L}_j^c. \]
The RG equations for the superpotential couplings are
\[
(16\pi^2) \frac{d\ln \lambda}{dt} = (N_w + 2) \lambda^2 + 2\kappa^2 + N_c (\lambda_f^2 + \lambda^2) + \lambda^2 \\
-2 [(Y_{H_F}^2 + Y_{H_D}^2)g^2 + 2C_2g^2] \\
+ N_c \sum_i \xi_{D_i}^2 + N_w \sum_j \xi_{L_j}^2, \quad (B.3)
\]
\[
(16\pi^2) \frac{d\ln \kappa}{dt} = 3N_w \lambda^2 + 6\kappa^2 + 3N_c \sum_i \xi_{D_i}^2 + 3N_w \sum_j \xi_{L_j}^2, \quad (B.4)
\]
\[
(16\pi^2) \frac{d\ln \xi_{D_k}}{dt} = N_w \lambda^2 + 2\kappa^2 - 2 [(Y_{D_k}^2 + Y_{D_k}^2)g^2 + 2C_2g^2] \\
+ 2\xi_{D_k}^2 + N_c \sum_i \xi_{D_i}^2 + N_w \sum_j \xi_{L_j}^2, \quad (B.5)
\]
\[
(16\pi^2) \frac{d\ln \xi_{L_k}}{dt} = N_w \lambda^2 + 2\kappa^2 - 2 [(Y_{L_k}^2 + Y_{L_k}^2)g^2 + 2C_2g^2] \\
+ 2\xi_{L_k}^2 + N_c \sum_i \xi_{D_i}^2 + N_w \sum_j \xi_{L_j}^2. \quad (B.6)
\]
Here $N_c = 3$ is the number of colors and $N_w = 2$ is the number of "weak" colors.

For the trilinear $A$ terms, we have
\[
(16\pi^2) \frac{dA_\lambda}{dt} = 2(N_w + 2) \lambda^2 A_\lambda + 4\kappa^2 A_\kappa + 2N_c (\lambda_f^2 A_f + \lambda^2 A_b) + 2\lambda^2 A_r \\
+ 4 [(Y_{H_F}^2 + Y_{H_D}^2)g^2 M_1 + 2C_2g^2 M_2] \\
+ 2N_c \sum_i \xi_{D_i}^2 A_{D_i} + 2N_w \sum_j \xi_{L_j}^2 A_{L_i}, \quad (B.7)
\]
\[
(16\pi^2) \frac{dA_\kappa}{dt} = 12\kappa^2 A_\kappa + 6N_w \lambda^2 A_\lambda + 6(N_c \sum_i \xi_{D_i}^2 A_{D_i} + N_w \sum_j \xi_{L_j}^2 A_{L_i}), \quad (B.8)
\]
\[
(16\pi^2) \frac{dA_{D_k}}{dt} = 2N_w \lambda^2 A_\lambda + 4\kappa^2 A_\kappa + 4 [(Y_{D_k}^2 + Y_{D_k}^2)g^2 M_1 + 2C_3g^2 M_3] \\
+ 4\xi_{D_k}^2 A_{D_k} + 2N_c \sum_i \xi_{D_i}^2 A_{D_i} + 2N_w \sum_j \xi_{L_j}^2 A_{L_j}, \quad (B.9)
\]
\[
(16\pi^2) \frac{dA_{L_k}}{dt} = 2N_w \lambda^2 A_\lambda + 4\kappa^2 A_\kappa + 4 [(Y_{L_k}^2 + Y_{L_k}^2)g^2 M_1 + 2C_2g^2 M_2] \\
+ 4\xi_{L_k}^2 A_{L_k} + 2N_c \sum_i \xi_{D_i}^2 A_{D_i} + 2N_w \sum_j \xi_{L_j}^2 A_{L_j}. \quad (B.10)
\]
Finally, the modifications to the running of the scalar soft masses is

\[
(16\pi)^2 \frac{dm_S^2}{dt} = 2N_w \lambda^2 (m_{H_u}^2 + m_{H_d}^2 + m_S^2 + A_k^2) + 4\kappa^2 (3m_S^2 + A_k^2) \\
+ 2N_c \sum_i \xi_{D_i}^2 (m_{D_i}^2 + m_{D_i}^2 + m_S^2 + A_{D_i}^2) \\
+ 2N_w \sum_j \xi_{L_j}^2 (m_{L_j}^2 + m_{L_j}^2 + m_S^2 + A_{L_j}^2),
\]

(B.11)

\[
(16\pi)^2 \frac{dm_{D_i}^2}{dt} = 2\xi_{D_i}^2 (m_{D_i}^2 + m_{D_i}^2 + m_S^2 + A_{D_i}^2) \\
- 8(Y_{D_i} g'^2 M_1^2 + C_3 g_3^2 M_3^2) + 2Y_D \zeta,
\]

(B.12)

\[
(16\pi)^2 \frac{dm_{L_j}^2}{dt} = 2\xi_{L_j}^2 (m_{L_j}^2 + m_{L_j}^2 + m_S^2 + A_{L_j}^2) \\
- 8(Y_{L_j} g'^2 M_1^2 + C_2 g_2^2 M_2^2) + 2Y_L \zeta.
\]

(B.13)

The quantity \( \zeta \) in these expressions is the hypercharge \( D \)-term which vanishes in many simple models of gauge mediation \[18\].

References

[1] For general reviews, see:
  S. P. Martin, hep-ph/9709356; D. J. H. Chung, L. L. Everett, G. L. Kane, S. F. King, J. D. Lykken and L. T. Wang, Phys. Rept. 407, 1 (2005) [hep-ph/0312378].

[2] M. S. Carena and H. E. Haber, Prog. Part. Nucl. Phys. 50, 63 (2003) [hep-ph/0208209]; A. Djouadi, Phys. Rept. 459, 1 (2008) [hep-ph/0503173].

[3] J. M. Frere, D. R. T. Jones and S. Raby, Nucl. Phys. B 222, 11 (1983); J. P. Derendinger and C. A. Savoy, Nucl. Phys. B 237, 307 (1984); H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B 120, 346 (1983).

[4] J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwieber, Phys. Rev. D 39, 844 (1989);

[5] U. Ellwanger, M. Rausch de Traubenberg and C. A. Savoy, Phys. Lett. B 315, 331 (1993) [hep-ph/9307322]; T. Elliott, S. F. King and P. L. White, Phys. Rev. D 49, 2435 (1994) [hep-ph/9308309]; J. i. Kamoshita, Y. Okada and M. Tanaka, Phys. Lett. B 328, 67 (1994) [hep-ph/9402278]; U. Ellwanger, M. Rausch de Traubenberg and C. A. Savoy, Z. Phys. C 67, 655 (1995) [hep-ph/9502206]; S. F. King and P. L. White, Phys. Rev. D 52, 4183 (1995) [hep-ph/9505326]; S. F. King and P. L. White, Phys. Rev. D 53, 4049 (1996) [hep-ph/9508346]. F. Franke and H. Fraas, Int. J. Mod. Phys. A 12, 479 (1997) [hep-ph/9512366]; U. Ellwanger, M. Rausch de Traubenberg and C. A. Savoy, Nucl. Phys. B 492, 21 (1997) [hep-ph/9611251].
[6] U. Ellwanger and C. Hugonie, Eur. Phys. J. C 25, 297 (2002) [hep-ph/9909260]; M. Bastero-Gil, C. Hugonie, S. F. King, D. P. Roy and S. Vempati, Phys. Lett. B 489, 359 (2000) [hep-ph/0006198]; U. Ellwanger, J. F. Gunion, C. Hugonie and S. Moretti, hep-ph/0305109; U. Ellwanger, J. F. Gunion and C. Hugonie, JHEP 0507, 041 (2005) [hep-ph/0503203].

[7] D. J. Miller, R. Nevzorov and P. M. Zerwas, Nucl. Phys. B 681, 3 (2004) [hep-ph/0304049].

[8] V. Barger, P. Langacker, H. S. Lee and G. Shaughnessy, Phys. Rev. D 73, 115010 (2006) [hep-ph/0603247].

[9] S. Chang, R. Dermisek, J. F. Gunion and N. Weiner, 0801.4554 [hep-ph].

[10] R. Dermisek and J. F. Gunion, Phys. Rev. Lett. 95, 041801 (2005) [hep-ph/0502105]; R. Dermisek and J. F. Gunion, Phys. Rev. D 73, 111701 (2006) [hep-ph/0510322].

[11] S. Schael et al. [ALEPH Collaboration], Eur. Phys. J. C 47, 547 (2006) [hep-ex/0602042].

[12] S. Chang, P. J. Fox and N. Weiner, JHEP 0608, 068 (2006) [hep-ph/0511250].

[13] P. C. Schuster and N. Toro, [hep-ph/0512189].

[14] R. Dermisek and J. F. Gunion, Phys. Rev. D 75, 075019 (2007) [hep-ph/0611142]; R. Dermisek and J. F. Gunion, Phys. Rev. D 76, 095006 (2007) [0705.4387 [hep-ph]].

[15] B. A. Dobrescu, G. L. Landsberg and K. T. Matchev, Phys. Rev. D 63, 075003 (2001) [hep-ph/0005308]; B. A. Dobrescu and K. T. Matchev, JHEP 0009, 031 (2000) [hep-ph/0008192].

[16] M. Dine and W. Fischler, Nucl. Phys. B 204, 346 (1982). L. Alvarez-Gaume, M. Claudson and M. B. Wise, Nucl. Phys. B 207, 96 (1982);

[17] M. Dine and A. E. Nelson, Phys. Rev. D 48, 1277 (1993) [hep-ph/9303230]; M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D 51, 1362 (1995) [hep-ph/9408384]; M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D 53, 2658 (1996) [hep-ph/9507378].

[18] For a review, see: G. F. Giudice and R. Rattazzi, Phys. Rept. 322, 419 (1999) [hep-ph/9801271].

[19] D. E. Kaplan, G. D. Kribs and M. Schmaltz, Phys. Rev. D 62, 035010 (2000) [hep-ph/9911293].

[20] Z. Chacko, M. A. Luty, A. E. Nelson and E. Ponton, JHEP 0001, 003 (2000) [hep-ph/9911323].

[21] A. de Gouvea, A. Friedland and H. Murayama, Phys. Rev. D 57, 5676 (1998) [hep-ph/9711264].
[22] K. Agashe and M. Graesser, Nucl. Phys. B 507, 3 (1997) [hep-ph/9704206].

[23] A. Delgado, G. F. Giudice and P. Slavich, Phys. Lett. B 653, 424 (2007) [0706.3873 [hep-ph]].

[24] G. F. Giudice, H. D. Kim and R. Rattazzi, Phys. Lett. B 660, 545 (2008) [0711.4448 [hep-ph]].

[25] T. Liu and C. E. M. Wagner, JHEP 0806, 073 (2008) [0803.2895 [hep-ph]].

[26] A. Djouadi et al., JHEP 0807, 002 (2008) [0801.4321 [hep-ph]]. A. Djouadi, U. Ellwanger and A. M. Teixeira, [0803.0253 [hep-ph]].

[27] U. Ellwanger, C. C. Jean-Louis and A. M. Teixeira, JHEP 0805, 044 (2008) [0803.2962 [hep-ph]].

[28] U. Ellwanger, J. F. Gunion and C. Hugonie, JHEP 0502, 066 (2005) [hep-ph/0406215]; U. Ellwanger and C. Hugonie, Comput. Phys. Commun. 175, 290 (2006) [hep-ph/0508022]; U. Ellwanger and C. Hugonie, Comput. Phys. Commun. 177, 399 (2007) [hep-ph/0612134].

[29] S. A. Abel, S. Sarkar and P. L. White, Nucl. Phys. B 454, 663 (1995) [hep-ph/9506359].

[30] S. A. Abel, Nucl. Phys. B 480, 55 (1996) [hep-ph/9609323].

[31] C. Panagiotakopoulos and K. Tamvakis, Phys. Lett. B 446, 224 (1999) [hep-ph/9809475]; C. Panagiotakopoulos and K. Tamvakis, Phys. Lett. B 469, 145 (1999) [hep-ph/9908351].

[32] H. Baer, A. Mustafayev, S. Profumo, A. Belyaev and X. Tata, Phys. Rev. D 71, 095008 (2005) [hep-ph/0412059]; H. Baer, A. Mustafayev, S. Profumo, A. Belyaev and X. Tata, JHEP 0507, 065 (2005) [hep-ph/0504001].

[33] J. L. Evans, D. E. Morrissey and J. D. Wells, Phys. Rev. D 75, 055017 (2007) [hep-ph/0611185].

[34] D. Shih, JHEP 0802, 091 (2008) [hep-th/0703196].

[35] K. Intriligator, N. Seiberg and D. Shih, JHEP 0707, 017 (2007) [hep-th/0703281].

[36] R. Barbieri, L. J. Hall, A. Y. Papaioannou, D. Pappadopulo and V. S. Rychkov, JHEP 0803, 005 (2008) [0712.2903 [hep-ph]].

[37] R. Dermisek, J. F. Gunion and B. McElrath, Phys. Rev. D 76, 051105 (2007) [hep-ph/0612031]; R. N. Hodgkinson, [0802.3197 [hep-ph]].

[38] S. W. Love [CLEO Collaboration], [0807.1427 [hep-ex]].

[39] F. Domingo and U. Ellwanger, [0806.0733 [hep-ph]].

[40] K. Cheung, J. Song and Q. S. Yan, Phys. Rev. Lett. 99, 031801 (2007) [hep-ph/0703149].
[41] M. Carena, T. Han, G. Y. Huang and C. E. M. Wagner, JHEP 0804, 092 (2008) [0712.2466 [hep-ph]].

[42] J. R. Forshaw, J. F. Gunion, L. Hodgkinson, A. Papaefstathiou and A. D. Pilkington, JHEP 0804, 090 (2008) [0712.3510 [hep-ph]].

[43] M. G. Albrow et al. [FP420 R&D Collaboration], [0806.0302 [hep-ex]].

[44] A. Belyaev, S. Hesselbach, S. Lehti, S. Moretti, A. Nikitenko and C. H. Shepherd-Themistocleous, [0805.3505 [hep-ph]].

[45] ATLAS Collaboration, Detector and Physics Performance Technical Design Report, Vols. 1 and 2, CERN-LHCC-99-14 and CERN-LHCC-99-15.

[46] G. L. Bayatian et al. [CMS Collaboration], J. Phys. G 34, 995 (2007).

[47] U. Ellwanger and C. Hugonie, Eur. Phys. J. C 5, 723 (1998) [hep-ph/9712300]; U. Ellwanger and C. Hugonie, Eur. Phys. J. C 13, 681 (2000) [hep-ph/9812427].

[48] S. Kraml and W. Porod, Phys. Lett. B 626, 175 (2005) [hep-ph/0507055].

[49] V. Barger, P. Langacker and G. Shaughnessy, Phys. Lett. B 644, 361 (2007) [hep-ph/0609068].

[50] S. Hesselbach, F. Franke and H. Fraas, Phys. Lett. B 492, 140 (2000) [hep-ph/0007310].

[51] J. L. Feng, A. Rajaraman and F. Takayama, Phys. Rev. D 68, 063504 (2003) [hep-ph/0306024]; J. L. Feng, S. f. Su and F. Takayama, Phys. Rev. D 70, 063514 (2004) [hep-ph/0404198]; J. L. Feng, S. Su and F. Takayama, Phys. Rev. D 70, 075019 (2004) [hep-ph/0404231].

[52] J. R. Ellis, K. A. Olive, Y. Santosos and V. C. Spanos, Phys. Lett. B 588, 7 (2004) [hep-ph/0312262]; L. Roszkowski, R. Ruiz de Austri and K. Y. Choi, JHEP 0508, 080 (2005) [hep-ph/0408227]; D. G. Cerdeno, K. Y. Choi, K. Jedamzik, L. Roszkowski and R. Ruiz de Austri, JCAP 0606, 005 (2006) [hep-ph/0509275].

[53] M. Kawasaki, K. Kohri and T. Moroi, Phys. Rev. D 71, 083502 (2005) [astro-ph/0408426]; M. Kawasaki, K. Kohri, T. Moroi and A. Yotsuyanagi, [0804.3745 [hep-ph]].

[54] G. Abbiendi et al. [OPAL Collaboration], Phys. Lett. B 572, 8 (2003) [hep-ex/0305031].

[55] http://www-cdf.fnal.gov/physics/exotic/r2a/20070208.champ/ http://www-d0.fnal.gov/Run2Physics/WWW/results,np.htm

[56] http://www-cdf.fnal.gov/physics/new/top/2005/ljets/tprime/gen6/public.html

[57] G. D. Kribs, T. Plehn, M. Spannowsky and T. M. P. Tait, Phys. Rev. D 76, 075016 (2007) [0706.3718 [hep-ph]].
[58] M. Byrne, C. F. Kolda and P. Regan, Phys. Rev. D 66, 075007 (2002) [hep-ph/0202252].

[59] J. Kang, P. Langacker and B. D. Nelson, Phys. Rev. D 77, 035003 (2008) [0708.2701 [hep-ph]].

[60] A. Arvanitaki, C. Davis, P. W. Graham, A. Pierce and J. G. Wacker, Phys. Rev. D 72, 075011 (2005) [hep-ph/0504210].

[61] B. C. Allanach, C. M. Harris, M. A. Parker, P. Richardson and B. R. Webber, JHEP 0108, 051 (2001) [hep-ph/0108097].

[62] M. Fairbairn, A. C. Kraan, D. A. Milstead, T. Sjostrand, P. Skands and T. Sloan, Phys. Rept. 438, 1 (2007) [hep-ph/0611040].

[63] M. Schmaltz and W. Skiba, Phys. Rev. D 62, 095005 (2000) [hep-ph/0001172]; M. Schmaltz and W. Skiba, Phys. Rev. D 62, 095004 (2000) [hep-ph/0004210].

[64] W. Buchmuller, K. Hamaguchi and J. Kersten, Phys. Lett. B 632, 366 (2006) [hep-ph/0506105]. W. Buchmuller, J. Kersten and K. Schmidt-Hoberg, JHEP 0602, 069 (2006) [hep-ph/0512152]; W. Buchmuller, L. Covi, J. Kersten and K. Schmidt-Hoberg, JCAP 0611, 007 (2006) [hep-ph/0609142].