Joint Inversion of Rayleigh and Love Dispersion Curves Extracted from Ambient Seismic Noise Based on Secular Function

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Abstract. Traditional multi-mode dispersion curve inversion requires correct mode discrimination. However, when the stratum contains complex structures such as low-speed soft interlayer or high-speed hard interlayer, the dispersion curve may show phenomena such as "mode kissing" and "mode jumping", which can easily cause mode misjudgment and lead to erroneous inversion results. Based on the "secular function", this paper constructs a new type of objective function applied to the inversion of dispersion curve. This objective function does not require prior mode discrimination, which effectively solves the "mode misjudgment" problem of multi-mode dispersion curve inversion. The joint inversion of Rayleigh and Love dispersion curves extracted from ambient seismic noise is used to improve the constraint of the inversion and avoid the inversion falling into a local minimum in the case of a large-scale search of parameters. Finally, a numerical simulation was performed to verify the feasibility of the new inversion method.

1. Introduction
The ambient seismic noise exploration is an important method in passive source exploration. This method does not require seismic source, without any damage to the environment, and data collection is extremely simple. The flow is to collect the ambient seismic noise of the surrounding environment and extract the dispersion curve from it. The existing noise tomography is mainly for Rayleigh wave, and there are relatively few ambient seismic noise data tomography about Love wave. The advantage of Love wave over Rayleigh is that it is more sensitive to the shallow shear wave velocity structure and can better constrain the shallow velocity structure.

In 2019, Wang proposed the Frequency-Bessel transform method for extracting the Rayleigh wave dispersion curve from the ambient seismic noise data. Compared with the traditional method, it has a clearer image of the higher-order mode [1]. In 2020, Hu used the multi-component cross-correlation coefficient to extract the Love wave dispersion curve from the ambient seismic noise data [3]. These
two research results have greatly promoted the development of ambient seismic noise exploration, and the dispersion curves of Love waves and Rayleigh waves can be extracted from seismic records at the same time.

In the inversion of multi-mode dispersion curve, the high-order mode dispersion curve is prone to mode misjudgment. If it is not handled correctly, the inversion result will be greatly deviated from the real situation. In 2010, Maraschini proposed to use the dispersion function as the objective function to avoid the mode misjudgment problem in the inversion of the multi-mode dispersion curve [4]. This new objective function also greatly accelerates the calculation speed, but at the same time, compared with the traditional objective function, the new inversion objective function lacks the constraints caused by the corresponding order mode, and has more local minima. The joint inversion of Love wave and Rayleigh wave will greatly reduce the ambiguity of the inversion.

2. Secular Function of Generalized E/R Coefficients Method

2.1. Basic Theory

In 1993, Chen proposed the generalized reflection and transmission coefficient method to calculate the dispersion curve of surface waves. This method subtly eliminates the e-exponential growth term that causes the instability of numerical calculations, and completely solve the problem of high frequency instability in the calculation of the theoretical dispersion curve [5]. He proposed to improve the original algorithm of the secular function family in 2006, which can still accurately and efficiently give the dispersion curves of Love waves and Rayleigh waves can be extracted from seismic records at the same time.

The calculation formula of the modified reflection-transmission coefficient is as follows:

\[
\begin{bmatrix}
T_d^{(j)} \\
R_{du}^{(j)} \\
T_u^{(j)} \\
R_{ud}^{(j)}
\end{bmatrix} =
\begin{bmatrix}
E_{11}^{(j+1)} & -E_{12}^{(j+1)} & E_{11}^{(j)} & -E_{12}^{(j)} \\
E_{21}^{(j+1)} & -E_{22}^{(j+1)} & E_{21}^{(j)} & -E_{22}^{(j)}
\end{bmatrix}^{-1}
\begin{bmatrix}
\Lambda_d^{(j)}(\gamma^{(j)}) \\
\Lambda_u^{(j)}(\gamma^{(j)})
\end{bmatrix}
\]

\[j = 1, 2, \ldots, N = 1 \]

From the free boundary conditions of the ground surface, we get:

\[R_{ud}^{(0)} = - \left(E_{21}^{(1)}\right)^{-1} E_{22}^{(1)} \Lambda_u^{(1)}(0) \]  

For P-SV wave:
$$E^{(j)} = \frac{1}{\omega} \begin{bmatrix} \alpha^{(j)}k & \beta^{(j)}v^{(j)} \\ \alpha^{(j)}y^{(j)} & \beta^{(j)}k \\ -2\alpha^{(j)}\mu^{(j)}ky^{(j)} & -\beta^{(j)}\mu^{(j)}\chi^{(j)} \\ -\alpha^{(j)}\mu^{(j)}\chi^{(j)} & -2\beta^{(j)}\mu^{(j)}kv^{(j)} \\ -\alpha^{(j)}\mu^{(j)}y^{(j)} & -\beta^{(j)}\mu^{(j)}k \end{bmatrix}$$

(5)

For SH wave:

$$E^{(j)} = \begin{bmatrix} 1 \\ -\mu^{(j)}v^{(j)} \\ 1 \\ \mu^{(j)}v^{(j)} \end{bmatrix}$$

(6)

For the Rayleigh roots whose modal velocity is close to the Rayleigh wave velocity in half space, it is difficult to obtain using equation (1). To solve this problem, the following surface secular function is obtained from the free boundary conditions of the surface:

$$det \left\{ E^{(1)}_{21} + E^{(1)}_{22} \Lambda^{(1)}_{u} (0) \hat{R}^{(1)}_{du} \right\} = 0$$

(7)

Equations (1) and (8) are collectively called the secular function family.

2.2. Inversion Objective Function Based on Secular Function

When only using the surface secular function to search the root of the dispersion curve, some modes with strong energy in the low-velocity layer will be missed. However, since the seismic source of the existing surface wave exploration is on the surface and the signal reception is also on the surface, so the dispersion curves extracted from the measured data is also unable to visualize certain modes that only have strong energy in the low-velocity layer. Therefore, in this paper, the objective function is constructed based on the surface secular function. Since the surface secular function has different orders of magnitude at different frequencies, its maximum value is proportional to the square of the frequency. Therefore, when summing the different dispersion points, it needs to be divided by corresponding value, the basic form of the inversion objective function is shown in equation (8):

$$obf(m) = \left\{ \sum_{i=1}^{n_r} [S_r(m, f_{ri}, c_{ri}) w_i] + \sum_{i=1}^{n_l} [S_l(m, f_{li}, c_{li}) w_i] \right\}^{1/k}$$

(8)

where: $m$ is the stratum model; $(f_{ri}, c_{ri})$ and $(f_{li}, c_{li})$ are the dispersion curves points of Rayleigh wave and Love wave; $S_r(m, f_{ri}, c_{ri}), S_l(m, f_{li}, c_{li})$ are the surface secular function values, $S(m, f_{i}, c_{i}) = \frac{det\left\{ E^{(1)}_{21} + E^{(1)}_{22} \Lambda^{(1)}_{u} (0) \hat{R}^{(1)}_{du} \right\}}{f_i^2}$; $w_i$ is the weight of different dispersion points; $k$ is the norm, which is 2 in this paper.

3. Tests With Synthetic Data

This paper is based on the discrete wavenumber method to synthesize theoretical seismic records of ambient seismic noise data [7]. A total of 1000 seismic sources are randomly distributed in an annular area ranging from 500 m to 1500 m. The Ricker wavelet is selected as the source time function, the center frequency is randomly distributed between 6 Hz and 10 Hz, the onset direction is (1, 1, 1), the takeoff time is randomly distributed from -5 s to 60 s, and the intensity is randomly distributed between 0.001 and 1. A total of 100 observation stations are randomly distributed within a 100 m center range. Three-component background noise theoretical seismic records are synthesized, the frequency range is between 0.5 Hz and 28 Hz, the time length is 60 s, and the sampling rate is 68.23 Hz. The observation array is shown in figure 1 and three-component noise data is shown in figure 2.
Figure 1. Distribution of source and stations. Figure 2. Synthesized ambient seismic noise data.

In practical applications, the appearance of low-velocity layers, underground cavities or silt layers stratum will lead to "mode jumping" and "mode kissing" of the dispersion curve, which can easily lead to misjudgment of the model in the inversion of multi-order modes. This paper uses four-layer model with low-velocity layer to perform numerical simulation, and the parameters and search range are shown in table 1 and figure 3. In this paper, a joint inversion of the multi-mode dispersion curve will be performed, the density and longitudinal wave velocity are set as the real values of the theoretical model, and the shear wave velocity and layer thickness are used as the target parameters of the inversion. A total of 50 inversion calculations are carried out using the particle swarm algorithm, and the anomalous solutions are eliminated and then the numerical average is performed to obtain final result.

The dispersion curve extracted from the synthetic seismic record through Frequency-Bessel transform method and based on the inversion result are shown in figure 5[10]. The fundamental-order mode of the Rayleigh wave dispersion curve is missing at 5~8 Hz, where the energy of the first higher-order mode dominates. A mode kissing occurs between the first high-order mode and the second high-order mode at 20 Hz. In this case, it is extremely difficult for the traditional inversion objective function to make a correct mode discrimination of the dispersion curve, resulting in wrong inversion results. The new type of inversion objective function based on secular function does not require model judgment, each order mode is automatically matched, and the dispersion curve fits well. It can still obtain more accurate inversion results under a wide range of parameters search. The Inversion results for shear wave structure is shown in Figure 4, and the layering error is less than 10%, the shear wave velocity error of the first three layers is less than 4%, and the error of the fourth layer increases to 10%. The main reason for error is that there are fewer dispersion points extracted at low frequencies, and the phase velocity has a certain deviation from the theoretical dispersion curve.

Table 1. The parameters and inversion search range for four-layer stratum model with low velocity interlayer.

| Layer number | Model parameters | Search range |
|--------------|------------------|--------------|
|              | Z(m) | ρ(g·cm⁻³) | Vs(m·s⁻¹) | Vp(m·s⁻¹) | Vs(m·s⁻¹) | Z(m) |
| 1            | 10.0 | 1.78   | 180      | 1500     | 90~400   | 5~15 |
| 2            | 20.0 | 1.85   | 350      | 1700     | 150~700  | 10~30 |
| 3            | 40.0 | 1.80   | 250      | 1600     | 125~800  | 25~55 |
| half space   | ∞    | 1.94   | 600      | 2000     | 200~1000 | ∞    |
Table 2. The inversion results of four-layer stratum model.

| Parameters   | Actual value | Rayleigh wave inversion | Joint inversion |
|--------------|--------------|-------------------------|-----------------|
| Z1(m)        | 10.00        | 7.70                    | 10.60           |
| Z2(m)        | 10.00        | 8.40                    | 9.60            |
| Z3(m)        | 20.00        | 22.60                   | 21.90           |
| Vs1(m/s)     | 180.00       | 90.00                   | 182.70          |
| Vs2(m/s)     | 350.00       | 333.50                  | 361.00          |
| Vs3(m/s)     | 250.00       | 293.40                  | 248.60          |
| Vs4(m/s)     | 600.00       | 392.90                  | 422.05          |

Figure 3. Horizontally layered stratum model.

Figure 4. The Inversion results for Vs structure.

4. Conclusion

In this paper, a new type of inversion objective function is constructed based on the secular function. There is no need to judge the mode of the dispersion curve before the inversion calculation, which effectively solves the problem of pattern misjudgment of the traditional inversion objective function. The joint inversion of Rayleigh wave and Love wave is used to improve the constraint of the inversion and avoid the inversion falling into a local minimum in the case of a large-scale search of parameters. Finally, the ambient seismic noise data is synthesized by the vector wavenumber method and the dispersion curve is extracted by the Frequency-Bessel transform for inversion. There is a good fit for the dispersion curves and a more accurate shear wave velocity structure in the inversion result, which proves the new inversion method feasible.

Figure 5. The Inversion results for dispersion curve, Inversion results (solid white line); Dispersion curve for inversion (black plus)
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References
[1] Aki K. 1957 Space and Time Spectra of Stationary Stochastic Waves, with Special Reference to Microtremors [J] Bull.earthq.res.inst.
[2] Wang J, Wu G, Chen X. 2019 Frequency-Bessel Transform Method for Effective Imaging of Higher-Mode Rayleigh Dispersion Curves From Ambient Seismic Noise Data [J] Journal of Geophysical Research: Solid Earth 124 (4) 3708-3723.
[3] Hu S, Luo S, Yao H. 2020 The frequency - Bessel spectrograms of multi - component cross - correlation functions from seismic ambient noise[J] Journal of Geophysical Research: Solid Earth 125 (8) 1271-1280.
[4] Margherita M, Fabian E, Sebastiano F et al. 2010 A new misfit function for multimodal inversion of surface waves [J] GEOPHYSICS 75 (4) 31–43.
[5] Chen X F. 1993 A Systematic and Efficient Method of Computing Normal Modes for Multi-Layered Half-Space [J] Geophysical Journal International (115) 391-409.
[6] He Y F, Chen W T, Chen X F. 2006 Normal mode computation by the generalized reflection_transmission coefficient method in planar layered half space. Chinese J .Gophys. (In Chinese) 49 (4) 1074-1081.
[7] Bonnefoy C S, Cornou C, Kristek J, et al. 2004 Simulation of seismic ambient noise: I. Results of H/V and array techniques on canonical models 1120.
[8] Zhang H M, Chen X F, Chang S. 2003 An Efficient Numerical Method for Computing Synthetic Seismograms for a Layered Half-space with Sources and Receivers at Close or Same Depths [J] 160 (3-4) 467-486.
[9] Chen X F. 1999 Seismogram synthesis in multi-layered half-space Part I. Theoretical formulations. Earthquake Research in China 13 (2) 149–174.
[10] Wu B, Chen. X. 2016 Stable, accurate and efficient computation of normal modes for horizontal stratified models [J] Geophysical Journal International 206 (2) 1281-1300.