An Empirical Evaluation of Probabilistic Lexicalized Tree Insertion Grammars

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Abstract
We present an empirical study of the applicability of Probabilistic Lexicalized Tree Insertion Grammars (PLTIG), a lexicalized counterpart to Probabilistic Context-Free Grammars (PCFG), to problems in stochastic natural-language processing. Comparing the performance of PLTIGs with non-hierarchical \( N \)-gram models and PCFGs, we show that PLTIG combines the best aspects of both, with language modeling capability comparable to \( N \)-grams, and improved parsing performance over its non-lexicalized counterpart. Furthermore, training of PLTIGs displays faster convergence than PCFGs.

1 Introduction
There are many advantages to expressing a grammar in a lexicalized form, where an observable word of the language is encoded in each grammar rule. First, the lexical words help to clarify ambiguities that cannot be resolved by the sentence structures alone. For example, to correctly attach a prepositional phrase, it is often necessary to consider the lexical relationships between the head word of the prepositional phrase and those of the phrases it might modify. Second, lexicalizing the grammar rules increases computational efficiency because those rules that do not contain any observed words can be pruned away immediately. The Lexicalized Tree Insertion Grammar formalism (LTIG) has been proposed as a way to lexicalize context-free grammars (Schabes and Waters, 1994). We now apply a probabilistic variant of this formalism, Probabilistic Tree Insertion Grammars (PLTIGs), to natural language processing problems of stochastic parsing and language modeling. This paper presents two sets of experiments, comparing PLTIGs with non-lexicalized Probabilistic Context-Free Grammars (PCFGs) (Pereira and Schabes, 1992) and non-hierarchical \( N \)-gram models that use the right branching bracketing heuristics (period attaches high) as their parsing strategy. We show that PLTIGs can be induced from partially bracketed data, and that the resulting trained grammars can parse unseen sentences and estimate the likelihood of their occurrences in the language. The experiments are run on two corpora: the Air Travel Information System (ATIS) corpus and a subset of the Wall Street Journal TreeBank corpus. The results show that the lexicalized nature of the formalism helps our induced PLTIGs to converge faster and provide a better language model than PCFGs while maintaining comparable parsing qualities. Although \( N \)-gram models still slightly out-perform PLTIGs on language modeling, they lack high level structures needed for parsing. Therefore, PLTIGs have combined the best of two worlds: the language modeling capability of \( N \)-grams and the parse quality of context-free grammars.

The rest of the paper is organized as follows: first, we present an overview of the PLTIG formalism; then we describe the experimental setup; next, we interpret and discuss the results of the experiments; finally, we outline future directions of the research.

2 PLTIG and Related Work
The inspiration for the PLTIG formalism stems from the desire to lexicalize a context-free grammar. There are three ways in which one might do so. First, one can modify the tree structures so that all context-free productions contain lexical items. Greibach normal form provides a well-known example of such a lexicalized context-free formalism. This method is
not practical because altering the structures of the grammar damages the linguistic information stored in the original grammar (Schabes and Waters, 1994). Second, one might propagate lexical information upward through the productions. Examples of formalisms using this approach include the work of Magerman (1993), Charniak (1997), Collins (1997), and Goodman (1997). A more linguistically motivated approach is to expand the domain of productions downward to incorporate more tree structures. The Lexicalized Tree-Adjoining Grammar (LTAG) formalism (Schabes et al., 1988), (Schabes, 1990), although not context-free, is the most well-known instance in this category. PLTIGs belong to this third category and generate only context-free languages.

LTAGs (and LTIGs) are tree-rewriting systems, consisting of a set of elementary trees combined by tree operations. We distinguish two types of trees in the set of elementary trees: the initial trees and the auxiliary trees. Unlike full parse trees but reminiscent of the productions of a context-free grammar, both types of trees may have nonterminal leaf nodes. Auxiliary trees have, in addition, a distinguished nonterminal leaf node, labeled with the same nonterminal as the root node of the tree, called the foot node. Two types of operations are used to construct derived trees, or parse trees: substitution and adjunction. An initial tree can be substituted into the nonterminal leaf node of another tree in a way similar to the substitution of nonterminals in the production rules of CFGs. An auxiliary tree is inserted into another tree through the adjunction operation, which splices the auxiliary tree into the target tree at a node labeled with the same nonterminal as the foot and root of the auxiliary tree. By using a tree representation, LTAGs extend the domain of locality of a grammatical primitive, so that they capture both lexical features and hierarchical structure. Moreover, the adjunction operation elegantly models intuitive linguistic concepts such as long distance dependencies between words. Unlike the N-gram model, which only offers dependencies between neighboring words, these trees can model the interaction of structurally related words that occur far apart.

Like LTAGs, LTIGs are tree-rewriting systems, but they differ from LTAGs in their generative power. LTAGs can generate some strictly context-sensitive languages. They do so by using wrapping auxiliary trees, which allow non-empty frontier nodes (i.e., leaf nodes whose labels are not the empty terminal symbol) on both sides of the foot node. A wrapping auxiliary tree makes the formalism context-sensitive because it coordinates the string to the left of its foot with the string to the right of its foot while allowing a third string to be inserted into the foot. Just as the ability to recursively center-embed moves the required parsing time from $O(n)$ for regular grammars to $O(n^3)$ for context-free grammars, so the ability to wrap auxiliary trees moves the required parsing time further, to $O(n^6)$ for tree-adjoining grammars. This level of complexity is far too computationally expensive for current technologies. The complexity of LTAGs can be moderated by eliminating just the wrapping auxiliary trees. LTIGs prevent wrapping by restricting auxiliary tree structures to be in one of two forms: the left auxiliary tree, whose non-empty frontier nodes are all to the left of the foot node; or the right auxiliary tree, whose non-empty frontier nodes are all to the right of the foot node. Auxiliary trees of different types cannot adjoint into each other if the adjunction would result in a wrapping auxiliary tree. The resulting system is strongly equivalent to CFGs, yet is fully lexicalized and still $O(n^3)$ parsable, as shown by Schabes and Waters (1994).

Furthermore, LTIGs can be parameterized to form probabilistic models (Schabes and Waters, 1993b). Appendix A describes the parameters in detail. Informally speaking, a parameter is associated with each possible adjunction or substitution operation between a tree and a node. For instance, suppose there are $V$ left auxiliary trees that might adjoin into node $\eta$. Then there are $V + 1$ parameters associated with node $\eta$ that describe the distribution of the likelihood of any left auxiliary tree adjoining into node $\eta$. (We need one extra parameter for the case of no left adjunction.) A similar set of parameters is constructed for the right adjunction and

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1The best theoretical upper bound on time complexity for the recognition of Tree Adjoining Languages is $O(M(n^3))$, where $M(k)$ is the time needed to multiply two $k \times k$ boolean matrices. (Rajasekaran and Yooseph, 1993)
3 Experiments

In the following experiments we show that PLTIGs of varying sizes and configurations can be induced by processing a large training corpus, and that the trained PLTIGs can provide parses on unseen test data of comparable quality to the parses produced by PCFGs. Moreover, we show that PLTIGs have significantly lower entropy values than PCFGs, suggesting that they make better language models. We describe the induction process of the PLTIGs in Section 3.1. Two corpora of very different nature are used for training and testing. The first set of experiments uses the Air Travel Information System (ATIS) corpus. Section 3.2 presents the complete results of this set of experiments. To determine if PLTIGs can scale up well, we have also begun another study that uses a larger and more complex corpus, the Wall Street Journal TreeBank corpus. The initial results are discussed in Section 3.3. To reduce the effect of the data sparsity problem, we back off from lexical words to using the part of speech tags as the anchoring lexical items in all the experiments. Moreover, we use the deleted-interpolation smoothing technique for the N-gram models and PLTIGs. PCFGs do not require smoothing in these experiments.

3.1 Grammar Induction

The technique used to induce a grammar is a subtractive process. Starting from a universal grammar (i.e., one that can generate any string made up of the alphabet set), the parameters are iteratively refined until the grammar generates, hopefully, all and only the sentences in the target language, for which the training data provides an adequate sampling. In the case of a PCFG, the initial grammar production rule set contains all possible rules in Chomsky Normal Form constructed by the nonterminal and terminal symbols. The initial parameters associated with each rule are randomly generated subject to an admissibility constraint. As long as all the rules have a non-zero probability, any string has a non-zero chance of being generated. To train the grammar, we follow the Inside-Outside re-estimation algorithm described by Lari and Young (1990). The Inside-Outside re-estimation algorithm can also be extended to train PLTIGs. The equations calculating the inside and outside probabilities for PLTIGs can be found in Appendix B.

As with PCFGs, the initial grammar must be able to generate any string. A simple PLTIG that fits the requirement is one that simulates a bigram model. It is represented by a tree set that contains a right auxiliary tree for each lexical item as depicted in Figure 1. Each tree has one adjunction site into which other right auxiliary trees can adjoin. The tree set has only one initial tree, which is anchored by an empty lexical item. The initial tree represents the start of the sentence. Any string can be constructed by right adjoining the words together in order. Training the parameters of this grammar yields the same result as a bigram model: the parameters reflect close correlations between words that are frequently seen together, but the model cannot provide any high-level linguistic structure. (See example in Figure 2.)

To generate non-linear structures, we need to allow adjunction in both left and right directions. The expanded LTIG tree set includes a
Elementary Tree Sets:

\[ \begin{array}{c}
\alpha & \beta & \gamma & \delta \\
\epsilon & \zeta & \eta & \theta \\
\end{array} \]

Figure 3: An LTIG elementary tree set that allow both left and right adjunctions.

Example sentence:

The cat chases the mouse.

Corresponding derivation tree:

\[ \begin{array}{c}
t_{\text{cat}} \rightarrow \text{rt\text{-}adj.} \\
t_{\text{chases}} \rightarrow \text{lt\text{-}adj.} \\
t_{\text{mouse}} \rightarrow \text{lt\text{-}adj.} \\
t_{\text{the}} \rightarrow \text{rt\text{-}adj.} \\
\end{array} \]

Figure 4: With both left and right adjunctions possible, the sentences can be parsed in a more linguistically plausible way.

left auxiliary tree representation as well as right for each lexical item. Moreover, we must modify the topology of the auxiliary trees so that adjunction in both directions can occur. We insert an intermediary node between the root and the lexical word. At this internal node, at most one adjunction of each direction may take place. The introduction of this node is necessary because the definition of the formalism disallows right adjunction into the root node of a left auxiliary tree and vice versa. For the sake of uniformity, we shall disallow adjunction into the root nodes of the auxiliary trees from now on. Figure 5 shows an LTIG that allows at most one left and one right adjunction for each elementary tree. This enhanced LTIG can produce hierarchical structures that the bigram model could not (See Figure 4).

It is, however, still too limiting to allow only one adjunction from each direction. Many words often require more than one modifier. For example, a transitive verb such as "give" takes at least two adjunctions: a direct object noun phrase, an indirect object noun phrase, and possibly other adverbial modifiers. To create more adjunction sites for each word, we introduce yet more intermediary nodes between the root and the lexical word. Our empirical studies show that each lexicalized auxiliary tree requires at least 3 adjunction sites to parse all the sentences in the corpora. Figure 5(a) and (b) show two examples of auxiliary trees with 3 adjunction sites. The number of parameters in a PLTIG is dependent on the number of adjunction sites just as the size of a PCFG is dependent on the number of nonterminals. For a language with \( V \) vocabulary items, the number of parameters for the type of PLTIGs used in this paper is \( 2(V+1)+2V(K)(V+1) \), where \( K \) is the number of adjunction sites per tree. The first term of the equation is the number of parameters contributed by the initial tree, which always has two adjunction sites in our experiments. The second term is the contribution from the auxiliary trees. There are \( 2V \) auxiliary trees, each tree has \( K \) adjunction sites; and \( V+1 \) parameters describe the distribution of adjunction at each site. The number of parameters of a PCFG with \( M \) nonterminals is \( M^3 + MV \). For the experiments, we try to choose values of \( K \) and \( M \) for the PLTIGs and PCFGs such that

\[ 2(V+1) + 2V(K)(V+1) \approx M^3 + MV \]

3.2 ATIS

To reproduce the results of PCFGs reported by Pereira and Schabes, we use the ATIS corpus
for our first experiment. This corpus contains 577 sentences with 32 part-of-speech tags. To ensure statistical significance, we generate ten random train-test splits on the corpus. Each set randomly partitions the corpus into three sections according to the following distribution: 80% training, 10% held-out, and 10% testing. This gives us, on average, 406 training sentences, 83 testing sentences, and 88 sentences for held-out testing. The results reported here are the averages of ten runs.

We have trained three types of PLTIGs, varying the number of left and right adjunction sites. The L2R1 version has two left adjunction sites and one right adjunction site; L1R2 has one left adjunction site and two right adjunction sites; L2R2 has two of each. The prototypical auxiliary trees for these three grammars are shown in Figure 5. At the end of every training iteration, the updated grammars are used to parse sentences in the held-out test sets $D$, and the new language modeling scores (by measuring the cross-entropy estimates $\hat{H}(D, L2R1)$, $\hat{H}(D, L1R2)$, and $\hat{H}(D, L2R2)$) are calculated. The rate of improvement of the language modeling scores determines convergence. The PLTIGs are compared with two PCFGs: one with 15-nonterminals, as Pereira and Schabes have done, and one with 20-nonterminals, which has comparable number of parameters to L2R2, the larger PLTIG.

In Figure 6 we plot the average iterative improvements of the training process for each grammar. All training processes of the PLTIGs converge much faster (both in numbers of iterations and in real time) than those of the PCFGs, even when the PCFG has fewer parameters to estimate, as shown in Table 1. From Figure 6 we see that both PCFGs take many more iterations to converge and that the cross-entropy value they converge on is much higher than the PLTIGs.

During the testing phase, the trained grammars are used to produce bracketed constituents on unmarked sentences from the testing sets $T$. We use the crossing bracket metric to evaluate the parsing quality of each grammar. We also measure the cross-entropy estimates $\hat{H}(T, L2R1)$, $\hat{H}(T, L1R2)$, $\hat{H}(T, L2R2)$, $\hat{H}(T, PCFG_{15})$, and $\hat{H}(T, PCFG_{20})$ to determine the quality of the language model. For a baseline comparison, we consider bigram and trigram models with simple right branching bracketing heuristics. Our findings are summarized in Table 2. The three types of PLTIGs generate roughly the same number of bracketed constituent errors as that of the trained PCFGs, but they achieve a much lower entropy score. While the average entropy value of the trigram model is the lowest, there is no statistical significance between it and any of the three PLTIGs. The relative statistical significance between the various types of models is presented in Table 2. In any case, the slight language modeling advantage of the trigram model is offset by its inability to handle parsing.

Our ATIS results agree with the findings of Pereira and Schabes that concluded that the performances of the PCFGs do not seem to depend heavily on the number of parameters once a certain threshold is crossed. Even though $PCFG_{20}$ has about as many number of param-

| PLTIGs | bigram | trigram | PCFGs | PLTIGs | bigram |
|--------|--------|--------|-------|--------|--------|
| better | –      | better |

Table 2: Summary of pair-wise t-test for all grammars. If “better” appears at cell $(i,j)$, then the model in row $i$ has an entropy value lower than that of the model in column $j$ in a statistically significant way. The symbol “–” denotes that the difference of scores between the models bears no statistical significance.
eters as the larger PLTIG (L2R2), its language modeling score is still significantly worse than that of any of the PLTIGs.

### 3.3 WSJ

Because the sentences in ATIS are short with simple and similar structures, the difference in performance between the formalisms may not be as apparent. For the second experiment, we use the Wall Street Journal (WSJ) corpus, whose sentences are longer and have more varied and complex structures. We use sections 02 to 09 of the WSJ corpus for training, section 00 for held-out data $D$, and section 23 for test $T$. We consider sentences of length 40 or less. There are 13242 training sentences, 1780 sentences for the held-out data, and 2245 sentences in the test. The vocabulary set consists of the 48 part-of-speech tags. We compare three variants of PCFGs (15 nonterminals, 20 nonterminals, and 23 nonterminals) with three variants of PLTIGs (L1R2, L2R1, L2R2). A PCFG with 23 nonterminals is included because its size approximates that of the two smaller PLTIGs. We did not generate random train-test splits for the WSJ corpus because it is large enough to provide adequate sampling. Table 3 presents our findings. From Table 3, we see several similarities to the results from the ATIS corpus. All three variants of the PLTIG formalism have converged at a faster rate and have far better language modeling scores than any of the PCFGs. Differing from the previous experiment, the PLTIGs produce slightly better crossing bracket rates than the PCFGs on the more complex WSJ corpus. At least 20 nonterminals are needed for a PCFG to perform in league with the PLTIGs. Although the PCFGs have fewer parameters, the rate seems to be indifferent to the size of the grammars after a threshold has been reached. While upping the number of nonterminal symbols from 15 to 20 led to a 22.4% gain, the improvement from PCFG$_{20}$ to PCFG$_{23}$ is only 0.5%. Similarly for PLTIGs, L2R2 performs worse than L2R1 even though it has more parameters. The baseline comparison for this experiment results in more extreme outcomes. The right branching heuristic receives a crossing bracket rate of 49.44%, worse than even that of PCFG$_{15}$. However, the $N$-gram models have better cross-entropy measurements than PCFGs and PLTIGs; bigram has a score of 3.39 bits per word, and trigram has a score of 3.20 bits per word. Because the lexical relationship modeled by the PLTIGs presented in this paper is limited to those between two words, their scores are close to that of the bigram model.

### 4 Conclusion and Future Work

In this paper, we have presented the results of two empirical experiments using Probabilistic Lexicalized Tree Insertion Grammars. Comparing PLTIGs with PCFGs and $N$-grams, our studies show that a lexicalized tree representation drastically improves the quality of language modeling of a context-free grammar to the level of $N$-grams without degrading the parsing accuracy. In the future, we hope to continue to improve on the quality of parsing and language modeling by making more use of the lexical information. For example, currently, the initial untrained PLTIGs consist of elementary trees that have uniform configurations (i.e., every auxiliary tree has the same number of adjunction sites) to mirror the CNF representation of PCFGs. We hypothesize that a grammar consisting of a set of elementary trees whose number of adjunction sites depend on their lexical anchors would make a closer approximation to the “true” grammar. We also hope to apply PLTIGs to natural language tasks that may benefit from a good language model, such as speech recognition, machine translation, message understanding, and keyword and topic
## Parameters of PLTIG

Each elementary tree of a Probabilistic Tree Insertion Grammar, denoted as $\rho$, has the following parameters:

- $P_I(\rho)$: the probability that tree $\rho$ is the start of a derivation (i.e., tree $\rho$ does not adjoin or substitute into other trees). If $\rho$ is an auxiliary tree, $P_I(\rho) = 0$. The grammars used in our experiments have exactly one empty initial tree with $P_I(\rho_\epsilon) = 1$.

The parameters for adjunction and substitution are associated with each node of an elementary tree, denoted as $\eta$.

- $P_L(\eta, \rho_L)$: the probability of adjunction between left auxiliary tree $\rho_L$ and node $\eta$.
$P_{NL}(\eta)$: the probability that no tree left joins into node $\eta$ such that

$$P_{NL}(\eta) + \sum_{\rho_L} P_L(\eta, \rho_L) = 1$$

$P_R(\eta, \rho_R)$: the probability of adjunction between right auxiliary tree $\rho_R$ and node $\eta$

$P_{NR}(\eta)$: the probability that no tree right joins into node $\eta$ such that

$$P_{NR}(\eta) + \sum_{\rho_R} P_R(\eta, \rho_R) = 1$$

$P_S(\eta, \rho_S)$: the probability that an initial tree $\rho_S$ can substitute into node $\eta$. The grammars we used for our experiments have no substitution nodes, so this parameter is not used.

**B Inside-Outside Probabilities**

Let $O = O_1, O_2, \ldots, O_T$ be the observed sequence we wish to parse with a PLTIG. To estimate the likelihood of observing this sequence in the grammar and to maximize the parameters of the grammar to reflect the observations, we compute the inside and outside probabilities.

**B.1 Inside Probabilities**

The inside probability of a node $\eta$ between positions $s$ and $t$, is the probability that node $\eta$ can generate the partial observations between $s$ and $t$ (i.e., $O_{s+1}, \ldots, O_t$). This probability is denoted as $\tilde{e}(s, t, \eta)$. We calculate $\tilde{e}$ recursively in a bottom-up manner. The base cases are when a node is an empty node or a foot node, which does not cover anything; and when a node covers a single lexical item (e.g., $O_{s+1}$).

$$\tilde{e}(s, s, \eta) = \begin{cases} 1 & \text{: } Foot(\eta) \text{ or } Label(\eta)=\epsilon \\ 0 & \text{otherwise} \end{cases}$$

$$\tilde{e}(s, s + 1, \eta) = \begin{cases} 1 & \text{: } Label(\eta)=O_{s+1} \\ 0 & \text{otherwise} \end{cases}$$

We now show the general case of computing the inside probability for a node $\eta$ generating the sub-sequence of observations between positions $s$ and $t$. Following the model outlined in Schabes and Waters (1993), we enforce the restriction that a node cannot have more than one left or right adjunction. More specifically, there are four ways that node $\eta$ can generate the sub-sequence between positions $s$ and $t$:

1. $e(s, t, \eta, \emptyset)$: the probability that $\eta$ covers $O_{s+1}, \ldots, O_t$ without any adjunction at $\eta$.
2. $e(s, t, \eta, L)$: exactly one left adjunction at $\eta$.
3. $e(s, t, \eta, R)$: exactly one right adjunction at $\eta$.
4. $e(s, t, \eta, LR)$ a simultaneous left and right adjunction at $\eta$.

The final value of $\tilde{e}(s, t, \eta)$ is the normalized sum of the four parts.

$$\tilde{e}(s, t, \eta) = P_{NL}(\eta)P_{NR}(\eta)e(s, t, \eta, \emptyset) + P_{NR}(\eta)e(s, t, \eta, L) + P_{NL}(\eta)e(s, t, \eta, R) + e(s, t, \eta, LR)/2$$

For a node $\eta$ to cover the substring between positions $s$ and $t$ without any adjunction, it must be the case that its children jointly generate $O_{s+1}, \ldots, O_t$. If node $\eta$ has only one child $\eta_1$, then

$$e(s, t, \eta, \emptyset) = \tilde{e}(s, t, \eta_1)$$

If node $\eta$ has two children such that $\eta_1$ is the left child and $\eta_2$ is the right child, then

$$e(s, t, \eta, \emptyset) = \sum_{r=s}^{t} \tilde{e}(s, r, \eta_1)\tilde{e}(r, t, \eta_2)$$

Next, we consider the case when $\eta$ generates $O_{s+1}, \ldots, O_t$ by letting a left auxiliary tree adjoin into it. The auxiliary tree generates the front of the substring, $O_{s+1}, \ldots, O_t$, and $\eta$ generates the rest of the substring, $O_{t+1}, \ldots, O_t$, without adjunctions. The breaking position $r$ can be anywhere between $s$ and $t$.

$$e(s, t, \eta, L) = \sum_{\rho_L} \sum_{r=s+1}^{t} \left( \frac{\tilde{e}(s, r, \rho_L) \times \epsilon(r, t, \eta, \emptyset)}{P_L(\eta, \rho_L)} \right)$$

Similarly, $e(s, t, \eta, R)$ represents the case when a right auxiliary tree, $\rho_R$, is adjoined into node $\eta$ to generate $O_{s+1}, \ldots, O_t$.

$$e(s, t, \eta, R) = \sum_{\rho_R} \sum_{r=s}^{t-1} \left( \frac{\tilde{e}(s, r, \rho_R) \times \epsilon(r, t, \eta, \emptyset)}{P_R(\eta, \rho_R)} \right)$$

$^2$Or, if $\eta$ were a substitution node, then there must be a tree that generate the substring and can be substituted into $\eta$. We did not include the equations for this case because our grammars have no substitution nodes.
Finally, if both a left auxiliary tree and a right auxiliary tree adjoin into node \( \eta \) simultaneously to generate \( O_{s+1}, \ldots, O_t \), then we have to consider two breaking positions \( r_1 \) and \( r_2 \). The left auxiliary tree, \( \rho_L \), generates \( O_{s+1}, \ldots, O_{r_1} \); the node \( \eta \) generates \( O_{r_1+1}, \ldots, O_{r_2} \) (without adjunction); and the right auxiliary tree, \( \rho_R \), generates the remaining observations \( O_{r_2+1}, \ldots, O_t \).

\[
e(s, t, \eta, LR) = \sum_{\rho_L, \rho_R} \sum_{r_1=s+1}^{t-1} \sum_{r_2=r_1}^{t-1} \left( \begin{array}{c} \tilde{e}(s, r_1, \rho_L) \\ \times e(r_1, r_2, \eta, \emptyset) \\ \times \tilde{e}(r_2, t, \rho_R) \\ \times P_{R}(\eta, \rho_R) \\ \times P_{L}(\eta, \rho_L) \end{array} \right)
\]

The remaining cases are similar, and we do not consider the sixth case in which the node is the root of an initial tree that might substitute into a substitution node.

### B.2 Outside Probabilities

The outside probability of a node \( \eta \) between positions \( s \) and \( t \), denoted as \( \tilde{f}(s, t, \eta) \), is the probability that the derived tree will generate \( \eta \) and the two partial observations outside of \( s \) and \( t \) (i.e., the two sub-sequences \( O_1, \ldots, O_s \) and \( O_{t+1}, \ldots, O_T \)). The outside probabilities complement the inside probabilities: the product of the matching inside and outside probabilities is the total probability of the observation sequence being generated by the grammar. Similar to the constructs of the inside probabilities, we define four types of outside probabilities:

1. \( f(s, t, \eta, \emptyset) \): the probability that \( \eta \) is generated without having any tree adjoining into it.

2. \( f(s, t, \eta, L) \): \( \eta \) is generated and a left auxiliary tree has adjoined into it. Moreover, the auxiliary tree does not cover any part of the substring between \( s \) and \( t \).

3. \( f(s, t, \eta, R) \): \( \eta \) is generated and a right auxiliary tree has adjoined into it. Moreover, the auxiliary tree does not cover any part of the substring between \( s \) and \( t \).

4. \( f(s, t, \eta, LR) \): \( \eta \) is generated and a left auxiliary tree and a right auxiliary tree have simultaneously adjoined into it. Neither auxiliary tree can cover any part of the substring between \( s \) and \( t \).

Finally, \( \tilde{f}(s, t, \eta) \) is the normalized sum of its four parts.

\[
\tilde{f}(s, t, \eta) = \sum_{r=t}^{T} \tilde{f}(s, r, \eta_0) \tilde{e}(r, t, \eta_2)
\]

- node \( \eta \) is the left child of node \( \eta_0 \) and \( \eta \) has a sibling \( \eta_2 \), the right child of \( \eta_0 \). The outside probability of \( \eta \) between positions \( s \) and \( t \) is the product of the outside probability of its parent node between positions \( s \) and \( r \), where \( t < r \leq T \) and the normalized inside probability of its sibling node \( \eta_2 \) deriving the substring between \( t \) and \( r \).

\[
f(s, t, \eta, \emptyset) = \sum_{r=t}^{T} \tilde{f}(s, r, \eta_0) \tilde{e}(r, t, \eta_2)
\]

- node \( \eta \) is the right child of node \( \eta_0 \) and \( \eta \) has a sibling \( \eta_1 \), the left child of \( \eta_0 \).

\[
f(s, t, \eta, \emptyset) = \sum_{r=0}^{s} \tilde{f}(r, t, \eta_0) \tilde{e}(r, s, \eta_1)
\]

- node \( \eta \) is the root node of a left auxiliary tree \( \rho_L \) that left adjoins into a node \( \eta_0 \).

\[\text{For the work presented here, we do not consider the sixth case in which the node is the root of an initial tree that might substitute into a substitution node.}\]
Suppose that node $\eta_0$ derives the substring between positions $t$ and $r$, where $t < r \leq T$. Then the outside probability of $\eta$ between $s$ and $t$ is the product of the outside probability of $\eta_0$ between $s$ and $r$ and the inside probability of $\eta_0$ deriving the observations between $t$ and $r$ without left adjunction. In order for $\rho_L$ to left adjoint into $\eta_0$, $\eta_0$ must not have previously left adjoined with any tree. Therefore, the inside probability of $\eta_0$ between $t$ and $r$ cannot include $e(t, r, \eta_0, L)$ or $e(t, r, \eta_0, LR)$.

\[
f(s, t, \eta, \emptyset) = \sum_{\eta_0} \sum_{r=0}^{s} \sum_{t}^{T} P_L(\eta_0, \rho_L) f(s, r, \eta_0, \emptyset) \times \left[ e(t, r, \eta_0, \emptyset) P_N(\eta_0) \right. \\
\left. + e(t, r, \eta_0, R) / 2 \right]
\]

\[
f(s, t, \eta, \emptyset) = \sum_{\eta_0} \sum_{r=0}^{s} \sum_{t}^{T} P_R(\eta_0, \rho_R) f(r, t, \eta_0, \emptyset) \times \left[ e(r, s, \eta_0, \emptyset) P_N(\eta_0) \right. \\
\left. + e(r, s, \eta_0, L) / 2 \right]
\]

The remaining three types of outside probabilities are the cases in which auxiliary trees are adjoined into node $\eta$. First, we consider the case of left adjunction. Let tree $\rho_L$ be an auxiliary tree that is to be adjoined into $\eta$. $\rho_L$ must derive the partial observation immediately before position $s$ (i.e., $O_r, \ldots, O_s$, where $0 \leq r < s$).

\[
f(s, t, \eta, L) = \sum_{\rho_L} \sum_{r=0}^{s} \left( \tilde{e}(r, s, \rho_L) \times f(s, r, \eta, \emptyset) \times P_L(\eta, \rho_L) \right)
\]

Similarly, if a right auxiliary tree, $\rho_R$, is to be adjoined into node $\eta$, then it must derive the partial observation immediately after position $t$ (i.e., $O_{t+1}, \ldots, O_r$, where $t < r \leq T$).

\[
f(s, t, \eta, R) = \sum_{\rho_R} \sum_{r=1}^{T} \left( \tilde{e}(t, r, \rho_R) \times f(s, r, \eta, \emptyset) \times P_R(\eta, \rho_R) \right)
\]

Finally, in the case of simultaneous adjunction, both a left auxiliary tree $\rho_L$ and a right auxiliary