A Monte Carlo Study of the Dynamical-Fluctuation Property of the Hadronic System Inside Jets

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ABSTRACT

A study of the dynamical fluctuation property of jets is carried out using Monte Carlo method. The results suggest that, unlike the average properties of the hadronic system inside jets, the anisotropy of dynamical fluctuations in these systems changes abruptly with the variation of the cut parameter $y_{cut}$. A transition point exists, where the dynamical fluctuations in the hadronic system inside jet behave like those in soft hadronic collisions, i.e. being circular in the transverse plan with respect to dynamical fluctuations. This finding obtained from Jetset and Herwig Monte Carlo is encouraged to be checked by experiments.

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The presently most promising theory of strong interaction — Quantum Chromo-Dynamics (QCD) has the special property of both asymptotic freedom and colour confinement. For this reason, in any process, even though the energy scale, $Q^2$, is large enough for perturbative QCD (pQCD) to be applicable, there must be a non-perturbative hadronization phase before the final state particles can be observed. Therefore, the transition or interplay between hard and soft processes is a very important problem.

Theoretically, the transition between perturbative and non-perturbative QCD is at a scale $Q_0 \sim 1–2$ GeV. Experimentally, the transition between hard and soft processes is determined by the identification of jets through some jet-finding processes, e.g. Jade [1] or Durham [2] algorithm. In these processes there is a parameter — $y_{\text{cut}}$, which, in case of Durham algorithm, is essentially the relative transverse momentum $k_t$ squared [3],

$$k_t = \sqrt{y_{\text{cut}}} \cdot \sqrt{s},$$

where $\sqrt{s}$ is the center-of-mass energy of the collision. From the experimental point of view, $k_t$ can be taken as the transition scale between hard and soft. Its value depends on the definition of “jet”.

Historically, the discovery in 1975 [4] of a two-jet structure in $e^+e^-$ annihilation at c.m. energies $\geq 6$ GeV has been taken as an experimental confirmation of the parton model [5], and the observation in 1979 of a third jet in $e^+e^-$ collisions at 17 – 30 GeV has been recognised as the first experimental evidence of gluon [6] – [9]. These jets, being directly observable in experiments as “jets of particles”, will be called in the following as “visible jets”. The aim of this paper is to find out the scale corresponding to these visible jets and discuss its meaning.

For this purpose, let us remind that the qualitative difference between the typical soft process — moderate energy hadron-hadron collisions and the typical hard process — high energy $e^+e^-$ collisions can be observed most clearly in the property of dynamical fluctuations therein. It is found recently [10] that the dynamical fluctuations in the hadronic systems from these two processes are qualitatively different — the former is anisotropic in the longitudinal-transverse plane and isotropic in the transverse planes while the latter is isotropic in three dimensional phase space. This observation inspired us to think that the dynamical-fluctuation property may provide a hint for the determination of the scale of the appearance of visible jets.

The dynamical fluctuations can be characterized as usual by the anomalous scaling of normalized factorial moments (NFM) [11]:

$$F_q(M) = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m-1)\cdots(n_m-q+1) \rangle}{\langle n_m \rangle^q} \propto (M)^{\phi_q} \quad (M \to \infty),$$

where a region $\Delta$ in 1-, 2- or 3-dimensional phase space is divided into $M$ cells, $n_m$ is the multiplicity in the $m$th cell, and $\langle \cdots \rangle$ denotes vertically averaging over the event sample.
Note that when the fluctuations exist in higher-dimensional (2-D or 3-D) space, the projection effect [12] will cause the second-order 1-D NFM goes to saturation according to the rule:

\[ F_2^{(a)}(M_a) = A_a - B_a M_a^{-\gamma_a}, \tag{3} \]

where \( a = 1, 2, 3 \) denotes the different 1-D variables. The parameter \( \gamma_a \) describes the rate of going to saturation of the NFM in direction \( a \) and is the most important characteristic for the higher-dimensional dynamical fluctuations. If \( \gamma_a = \gamma_b \) the fluctuations are isotropic in the \( a, b \) plane; while when \( \gamma_a \neq \gamma_b \) the fluctuations are anisotropic in this plane. The degree of anisotropy is characterized by the Hurst exponent \( H_{ab} \), which can be obtained from the values of \( \gamma_a \) and \( \gamma_b \) as [14]

\[ H_{ab} = \frac{1 + \gamma_b}{1 + \gamma_a}. \tag{4} \]

The dynamical fluctuations are isotropic when \( H_{ab} = 1 \), and anisotropic when \( H_{ab} \neq 1 \).

For the 250 GeV/c \( \pi(K) \)-p collisions from NA22 the Hurst exponents are found to be [15]:

\[ H_{pt\phi} = 0.99 \pm 0.01, \quad H_{yp\phi} = 0.48 \pm 0.06, \quad H_{y\phi} = 0.47 \pm 0.06, \tag{5} \]

which means that the dynamical fluctuations in this moderate energy hadron-hadron collisions are isotropic in the transverse plane and anisotropic in the longitudinal-transverse planes. This is what should be [16], because there is almost no hard collisions at this energy and the direction of motion of the incident hadrons (longitudinal direction) should be privileged. Note that the special role of longitudinal direction in these soft processes is present both in the magnitude of average momentum and in the dynamical fluctuations in phase space.

In high energy \( e^+e^- \) collisions, the longitudinal direction is chosen along the thrust axis, which is the direction of motion of the primary quark-antiquark pair. Since this pair of quark and antiquark move back to back with very high momenta, the magnitude of average momentum of final state hadrons is also anisotropic due to momentum conservation. However, the dynamical fluctuations in this case come from the QCD branching of partons [17], which is isotropic in nature. Therefore, although the momentum distribution still has an elongated shape, the dynamical fluctuations in this case should be isotropic in 3-D phase space.

A Monte Carlo study for \( e^+e^- \) collisions at 91.2 GeV confirms this assertion [10]. The dynamical fluctuations are approximately isotropic in the 3-D phase space, the corresponding Hurst exponents being

\[ H_{pt\phi} = 1.18 \pm 0.03, \quad H_{yp\phi} = 0.95 \pm 0.02, \quad H_{y\phi} = 1.11 \pm 0.02. \tag{6} \]

The present available experimental data for \( e^+e^- \) collisions at 91.2 GeV also show isotropic dynamical fluctuations in 3-D [18].

\(^3\)In order to eliminate the influence of momentum conservation [13], the first few points (\( M = 1, 2 \) or 3)
Now we apply this technique to the “2-jet” sub-sample of \( e^+e^- \) collision obtained from a certain, e.g. Durham, jet-algorithm with some definite value of \( y_{\text{cut}} \). Doing the analysis for different values of \( y_{\text{cut}} \), the dependence of dynamical-fluctuation property of the “2-jet” sample on the value of \( y_{\text{cut}} \) can be investigated.

Two event samples are constructed from Jetset7.4 and Herwig5.9 generators, respectively, each consists of 400 000 \( e^+e^- \) collision events at c.m. energy 91.2 GeV. The results of 3-D \( F_2 \) for the full samples are shown in Fig.1 and the variation of \( \gamma \)'s of the 2-jet sample with \( y_{\text{cut}} (k_t) \) are shown in Fig’s 2.

![Fig.1 The log-log plot of 3-D NFM of the full event sample as function of partition number M](image)

![Fig.2 The parameter \( \gamma \) of 2-jet sample as function of \( y_{\text{cut}} (k_t) \)](image)
It can be seen from Fig.1 that after neglecting the first point to eliminate the influence of momentum conservation [13] the results from both Jetset and Herwig fit very well to straight lines with only slightly different slope. This means that the results from these two generators are qualitatively the same, showing that the full sample is self-similar (isotropic) fractal. This is just as expected [10]. Quantitatively, they have slightly different fractal dimension.

The variation of the three γ’s of “2-jet” sample with the parameter $y_{\text{cut}} (k_t)$, plotted in Fig.’s 2, show an interesting pattern. When $y_{\text{cut}} (k_t)$ is very small, the three γ’s are separate. As the increasing of $y_{\text{cut}} (k_t)$, $\gamma_{p_t}$ and $\gamma_{\phi}$ approach each other and cross over each other sharply at a certain point. After that the three γ’s approach to a common value due to the fact that when $y_{\text{cut}}$ is very large the “2-jet” sample coincides with the full sample and the dynamical fluctuations in the full sample is isotropic, cf. Eq.(6).

We will call the point where $\gamma_{p_t}$ crosses $\gamma_{\phi}$ as transition point. It has the unique property: $\gamma_{p_t} = \gamma_{\phi} \neq \gamma_y$, i.e. the jets at this point are circular in the transverse plan with respect to dynamical fluctuations. These jets will, therefore, be called circular jets.

The above-mentioned results are qualitatively the same for the two event generators, cf. Fig.2 (a) and (b), only the $y_{\text{cut}} (k_t)$ values at the transition point are somewhat different. It is $y_{\text{cut}} \approx 0.0048$ ($k_t \approx 6.3$ GeV) for Jetset and $y_{\text{cut}} \approx 0.0022$ ($k_t \approx 4.3$ GeV) for Herwig. The values of γ’s and the corresponding Hurst exponents $H$, cut parameter $y_{\text{cut}}$ and relative transverse momentum $k_t$ at the transition point are listed in Table I.

Table I Parameters γ, Hurst exponents $H$, cut-parameters $y_{\text{cut}}$ and $k_t$ at the transition point

| Generator (GeV) | $y_{\text{cut}}$ (Durham) $\gamma_y$ | $\gamma_{p_t}$ | $\gamma_{\phi}$ | $H_{y_{p_t}}$ | $H_{y_{\phi}}$ | $H_{y_{\psi \phi}}$ | $y_{\text{cut}}$ | $k_t$ GeV |
|----------------|--------------------------------------|----------------|----------------|---------------|---------------|-------------------|---------------|----------|
| Jetset7.4      | 0.0048                               | 1.074          | 0.514          | 0.461         | 0.73          | 0.70              | 0.0048        | 6.32     |
|                | ±0.0007                              | ±0.037         | ±0.080         | ±0.021        | ±0.06         | ±0.10             | ±0.0007       | ±0.03    |
| Herwig5.9      | 0.0022                               | 1.237          | 0.633          | 0.637         | 0.73          | 0.73              | 0.0022        | 4.28     |
|                | ±0.0008                              | ±0.066         | ±0.064         | ±0.051        | ±0.05         | ±0.07             | ±0.0008       | ±0.02    |

It is natural to ask the question: Is there any relation between the circular jets determined by the condition $\gamma_{p_t} = \gamma_{\phi} \neq \gamma_y$ and the visible jets directly observable in experiments as “jets of particles”?

In order to answer this question, we plot in Fig.’s 3 the ratios $R_2$ and $R_3$ of “2-jet” and “3-jet” events as functions of the relative transverse momentum $k_t$ at different c.m. energies.
Let us consider the point where a third jet starts to appear. Historically, a third visible jet was firstly observed in $e^+e^-$ collisions at c.m. energy 17 GeV. It can be seen from Fig.3 that, for $\sqrt{s} = 17$ GeV, $R_3$ starts to appear at around $k_t = 8–10$ GeV, cf. the dashed vertical lines in Fig.'s 3. This value of $k_t$ is consistent with the $k_t$ value (4.3–6.3 GeV) of a circular jet within a factor of 2, cf. Table I. Thus we see that the circular jet, defined as a kind of jet circular in the transverse plan with respect to dynamical fluctuations, and the visible jet, defined as a kind of jet directly observable in experiments as “jets of particles”, have about the same scale — $k_t \sim 5–10$ GeV.

This scale is to be compared with the scale $k_t \sim 1–2$ GeV, which is the scale for the transition between perturbative and non-perturbative. It is interesting also to see what happens in the results of jet-algorithm at the latter scale.

It can be seen from Fig.3a (Jetset7.4) that, at this scale ($k_t \sim 1–2$ GeV) the ratio $R_2$ of “2-jet” events tends to vanish almost independent of energy, provided the latter is not too low. This can be explained as the following. Consider, for example, an event with only two hard partons, having no perturbative branching at all. Even in this case, the two partons will still undergo non-perturbative hadronization to produce final-state particles. If the $k_t$ is chosen to be less than 1–2 GeV then the non-perturbative hadronization with small transverse momentum will also be considered as the production of new “jets” and this “should-be” 2-jet event will be taken as a “multi-jet” (more than two jets) ones too. This means that, when $k_t < 1–2$ GeV, events with small transverse momentum will also become “multi-jet” ones, and $R_2$ vanishes.

However, even when $k_t < 1–2$ GeV, a few 2-jet events may still survive if the hadronization is almost collinear. This effect becomes observable when the energy is very low, see, e.g., the $R_2$ curve for $\sqrt{s} = 6$ GeV in Fig.3.
Similar picture holds also for the results from Herwig5.9, cf. Fig.3b, but the almost-colinear hadronization appears earlier.

Before closing the paper, let us give some comments on the physical picture behind the above-mentioned two scales. A circular (or visible) jet is originated from a hard parton. The production of this parton is a hard process. Its evolution into final state particles includes a perturbative branching and subsequent hadronization. The hadronization is a soft process. The perturbative branching (sometimes called parton shower) in between the hard production and soft hadronization connects these two processes. The isotropic property of dynamical fluctuations provides a criterion for the discrimination of the hard production of circular jets and the parton shower inside these jets.

In this paper we found through Monte Carlo study that unlike the average properties of the hadronic systems inside jets, the anisotropy of dynamical fluctuations in these systems changes abruptly with the variation of the cut parameter $y_{\text{cut}}(k_t)$. A transition point exists, where the dynamical fluctuations in the hadronic system inside jets behave like those in soft hadronic collisions, i.e. being circular in the transverse plane with respect to dynamical fluctuations. The scale of these circular jets is about $k_t \sim 5$–10 GeV in contrast to the scale of pQCD which is $Q_0 \sim 1$–2 GeV. It is encouraged to check this observation using real experimental data.

This observation is not only meaningful in the study of jets in $e^+e^-$ collisions but also enlightening in the jet-physics in relativistic heavy ion collisions, which will become important [19] after the operation of the new generation of heavy ion colliders at BNL (RHIC) [20] and CERN (LHC-ALICE)[21].

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References

[1] W. Bartel et al. (JADE Coll.), Phys. Lett. B 123 (1983) 460; Z. Phys. C 33 (1986) 23.

[2] Yu. L. Dokshitzer, J. Phys. G 17 (1991) 1537.

[3] Yu. L. Dokshitzer, G. D. Leder, S. Moretti and B. R. Webber, JHEP 08 (1997) 001.

[4] G. Hanson et al., Phys. Rev. Lett. 35 (1975) 1609.

[5] J. Ellis et al., Nucl. Phys. B111 (1976) 253.

[6] R. Brandelik et al. (TASSO Coll.), Phys. Lett. 86 B (1979) 243.

[7] D. P. Barber (Mark J Coll.), Phys. Rev. Lett. 43 (1979) 830.

[8] Ch. Berger et al. (PLUTO Coll.), Phys. Lett. 86 B (1979) 418.

[9] W. Bartel et al. (JADE Coll.), Phys. Lett. 91 B (1980) 142.

[10] Liu Feng, Liu Fuming and Liu Lianshou, Phys. Rev. D 59 (1999) 114020.

[11] A. Bia/su pest and R. Peschanski, Nucl. Phys. B 273 (1986) 703; ibid 308 (1988) 857.

[12] W. Ochs, Phys. Lett. B 347 (1990) 101.

[13] Liu Lianshou, Zhang Yang and Deng Yue, Z. Phys. C 73 (1997) 535.

[14] Wu Yuanfang and Liu Lianshou, Science in China (series A) 38 (1995) 435.

[15] N. M. Agabayan et al. (NA22), Phys. Lett. B 382 (1996) 305; N. M. Agabayan et al. (NA22), Phys. Lett. B 431 (1998) 451.

[16] Wu Yuanfang and Liu Lianshou, Phys. Rev. Lett. 21 (1993) 3197.

[17] G. Veneziano, Momentum and colour structure of jet in QCD, talk given at the 3rd Workshop on Current Problems in High Energy Particle Theory, Florence, 1979.

[18] P. Abreu (DELPHI), Nucl. Phys. B386 (1992) 471.

[19] X.-N. Wang, Phys. Reports 280 (1997) 287.

[20] John Harris, Relativistic Heavy Ion Physics and Relativistic Heavy Ion Collider, in Proc. of the Lake Louise Winter Institute on Quantum Chromodynamics, Feb. 1998, World Scientific, Singapore.

[21] Letter of Intent for A Large Ion Collider Experiment, CERN/LHCC/93-16, 1993.