Dilaton Stabilization in Effective Type I String Models

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Abstract

We show that the dilaton and $T$-moduli can be stabilized by a single gaugino condensation mechanism in the four-dimensional effective field theory derived from Type IIB orientifolds. A crucial role is played by the mixing of the blowing-up modes $M_k$ with the $T$-moduli in the Kähler metric, and by the presence of the $M_k$ in the gauge kinetic functions. Supersymmetry breaking in these models is dominated by the auxiliary fields of the $T$ moduli, and phenomenologically interesting patterns can emerge.
1 Introduction

Understanding how the dilaton gets a phenomenologically consistent expectation value is one of the major problems of string-derived effective field theories. Perhaps the most promising approach to dilaton stabilization is gaugino condensation in some hidden gauge group, leading to the dynamical generation of a non-perturbative dilaton-dependent superpotential. In heterotic string theories, however, the simplest resulting scalar potentials do not stabilize the dilaton. Instead it runs away either to infinite values where the coupling is weak, or to zero where the coupling becomes strong and perturbative control is lost.

Attempts have been made to circumvent this problem by having a gauge group with several factors, and multiple gaugino condensation. In this case, several exponential terms have to conspire to produce a minimum in the potential at finite dilaton values. These are the so-called ‘race-track’ models. To be realistic, race-track models require that the gauge coupling at the string scale be compatible with estimates based on renormalization group evolution of the Standard Model gauge couplings. The vacuum expectation value of the dilaton is then constrained to be $\langle \text{Re}(S) \rangle \sim 2 \sim g_{\text{GUT}}^{-2}$, which requires some degree of fine-tuning \[1, 2\].

In this paper we examine dilaton stabilization from gaugino condensation in effective theories of type I strings derived from type IIB orientifolds. We find a picture that is radically different from heterotic strings and in particular find that the dilaton can be stabilized with only one condensing gauge group. The novel feature of type I strings which allows us to do this is the existence of twisted moduli, $M_k$, associated with fixed points. These not only modify the Kähler metric but also appear in the gauge kinetic functions, and consequently in the superpotential that is generated by gaugino condensation. As we shall see, it is the mixing of these new fields with the moduli in the Kähler metric which generically leads to a simple stabilization.

After briefly presenting relevant aspects of Type I models, we discuss in section 3 gaugino condensation and the dynamical superpotential that we will use in our study \[1\]. Section 4 previews the general features of the resulting scalar potentials in heterotic and type I models in order to explain why dilaton stabilization is considerably easier in the latter. In section 5 we give an explicit computation of the scalar potential in type I and describe a local minimum where the dilaton may be trapped. In section 6 we discuss the resulting soft breaking terms.

2 Preliminaries; Structure of Type I Models

Type I string theories have interesting phenomenological properties which have been investigated (using type IIB orientifolds) in refs. \[3–6\]. For example, their brane structure allows the fundamental scale to be essentially a free parameter, and in addition the visible

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\[1\]We understand that gaugino condensation in type I theories has been considered by Aldazabal, Font, Ibáñez and Quevedo (unpublished).
gauge couplings are no longer tied to the vacuum expectation value of the dilaton but can instead be determined by the twisted moduli fields. (Consequently the problem of stabilizing the dilaton and moduli is more democratic than in the heterotic case.)

In this paper we will be concerned with the effective scalar potential of type I models and the important aspects are therefore the gauge couplings and the Kähler potential which we now review. The reader is referred to ref. [7] for details on the construction; ref. [4] for a broad phenomenological outline, including the effect of choosing different fundamental scales; ref. [8,9] for discussions of supersymmetry breaking and phenomenology with an intermediate fundamental scale; ref. [10] for some other aspects of type I models.

Type I models constructed from type IIB orientifolds include different types of D-branes on which open strings can be attached in various ways. Supersymmetric models either have just D9 branes or D9 and D5 branes (by T-dualizing with respect to the three complex dimensions it is sometimes useful to exchange D9-branes with D3-branes and D5-branes with D7-branes). There are three classes of moduli fields that we need to consider: the complex dilaton $S$, the untwisted moduli $T_i$ associated with the size and shape of the extra dimensions and the twisted moduli $M_k$ associated with the fixed points of the underlying orbifold. In contrast with the Green–Schwarz mechanism of heterotic compactifications, the complex dilaton does not generically play any role in $U(1)$ anomaly cancellation of $D = 4, N = 1$ type IIB orientifolds. Instead, only the twisted moduli $M_k$ participate in the generalized Green–Schwarz mechanism [11, 12]. Moreover, they induce a Fayet–Iliopoulos term which is determined by the VEVs of the $M_k$ fields and which can therefore be zero. (In the heterotic case the FI term is given by the complex dilaton and is therefore constrained by the gauge couplings.)

In the gauge sector, gauge groups and charged chiral fields will depend on the type and location of D-branes present in the vacuum. One can generally consider the case with one set of 9-branes and three sets of 5$_i$-branes ($i = 1, 2, 3$). There are gauge groups $G_9$, $G_{5i}$ associated with each, and four types of charged matter fields; $C^9_i$ ($i$ labels the three complex dimensions) comes from open strings starting and ending on the 9-branes; $C^{5_i}$ from open strings starting and ending on the 9-branes; $C^{5,5_j}$ from open strings starting and ending on the same 5$_i$-branes; $C^{9,5_j}$ from open strings starting and ending on different sets of 5$_i$-branes; $C^{9s}$ from open strings with one end on the 9-branes and the other end on the 5$_i$-branes.

The gauge kinetic functions for a $Z_N$ orientifold model differ from the heterotic case. Firstly there is no Kac–Moody coefficient multiplying the $S$-field dependence. In addition the blowing-up modes appear linearly, and for $G_{5i}$, the $S$-field is replaced by the $T_i$-fields. For the D9-branes, the gauge kinetic function is [12, 13]

$$f_{9a} = S + \sum_k \sigma_{a}^{k} M_k,$$

(1)

whereas for the D5-branes

$$f_{5i a} = T_i + \sum_k \sigma_{ia}^{k} M_k,$$

(2)

where $\sigma_{a}^{k}$ are model dependent coefficients and $k$ runs over the different twisted sectors. The gauge coupling is given by Re$f_a = 1/g_a^2$. 

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To describe the Kähler potential, we will henceforth work with the overall modulus, taking $T_i = T$. At one-loop level the Kähler potential for arbitrary numbers of $M_k$ fields has the form [5],

$$K = - \ln s - 3 \ln \tau + \hat{K}(m_k),$$  \hspace{1cm} (3)

where

$$s = S + \mathcal{S}; \quad \tau = T + \mathcal{T} - \sum_n |\phi_n|^2; \quad m_k = M_k + \mathcal{M}_k - \delta_k \ln \tau,$$  \hspace{1cm} (4)

We have introduced generic fields $\phi$ to represent some linear combination of the $C^9_i$ or $C^9_i \tilde{j}$ fields which will later condense. (Formally, our choice of putting the $\phi$ fields in a single $\tau$ corresponds to the linear combination $\phi_n = \frac{1}{\sqrt{3}}(C^9_1 n + C^9_2 n + C^9_3 n)$.) However it turns out that the minimization of the potential is independent of the particular linear combination to high order – see later.) They are singlets under the gauge group of the visible sectors but charged under the anomalous $U(1)$ and will appear in the dynamical superpotential. The first two terms are similar to the usual no-scale models [14] where the $T$ and $\phi$-dependence appears in the combination $\tau$ only. Giving a VEV to $m_k$ takes us away from the orientifold point. The $\delta_k \ln \tau$ term is a correction whose general form can be deduced from the one loop expression for the gauge coupling [13, 15] (and $\delta_k$ may be related to the Green–Schwarz coefficients associated with $SL(2, R)$ anomaly cancellation [15].)

There may also be dilaton dependent corrections to $m_k$ but their precise form is unclear (although various symmetry arguments have been put forward for them [15]) so we shall omit them, assuming that they are negligible. (Note that the corrections in $m_k$ depend on the tree-level expression for $K$. In contrast with previous work, we do not expand it by assuming small $\phi_n$, but instead retain the full no-scale structure, $\tau$.) For the moment we will also omit the various charged visible matter fields because they do not condense but will return to them later when we compute their soft masses.

All that we currently know about the form of $\hat{K}$ is that it is an even function of $m_k$ thanks to the orbifold symmetry, and that the leading term in an expansion about the orientifold point, $m_k = 0$, is quadratic, $\frac{1}{2} \sum_k m_k^2$. Later we accommodate our ignorance by working with the parameter $x_k = \partial \hat{K}/\partial M_k$ where near the orientifold point $x_k \approx m_k$.

### 3 Gaugino condensation in heterotic and type I

In the heterotic string, a non-perturbative superpotential for the fields $S$ and $T$ can be generated by hidden sector gaugino condensation with gauge group $SU(N_c)$ and with extra ‘matter’ in fundamental representations. We shall consider only one flavour of quarks $Q$ in the fundamental of $SU(N_c)$ and antiquarks $\tilde{Q}$ in the antifundamental of $SU(N_c)$. Below the scale $\Lambda = e^{-1/2\beta}$, where $\beta$ is the one-loop beta function coefficient of the hidden gauge group, the appropriate degree of freedom is the meson $Q\tilde{Q}$. It is usual to treat the composite superfield, $\phi_2 = \sqrt{Q\tilde{Q}}$, as the relevant superfield appearing in the Kähler potential, and (for convenience) we will include it in $\tau$. In addition to $\phi_2$ it will be necessary in both the heterotic and type I cases to include a field $\phi_1$ of charge $q_1$ in order to generate a perturbative mass term for $\phi_2$. 

3
The non-perturbative contribution to the superpotential can be fixed uniquely by considering global symmetries and reads \[ W_{np} = \left( \frac{\Lambda^{3N_c-1}}{\phi_2^2 h(T)} \right)^{\frac{1}{N_c-1}} \] (5)

where \( h(T) \) is a product of Dedekind eta functions resulting from a one-loop correction to \( f \) (which gives \( W \) the required modular weight, -3) and \( \Lambda \sim e^{-k_N S/2\beta} \). (This is in the so-called ‘truncated’ approximation; see ref. [17] for recent developments.) Here \( \beta = (3N_c - 1)/16\pi^2 \) and \( k_N \) is the Kac–Moody level of the hidden gauge group. Note that we have not yet ‘integrated out’ any fields except the gaugino condensate.

In the heterotic string, the mixed \( U(1)_X \times [SU(N_c)]^2 \) anomaly under the transformation

\[ A^X_\mu (x) \rightarrow A^X_\mu (x) + \partial_\mu \alpha \] (6)

is cancelled by the transformation

\[ S \rightarrow S + \frac{i}{2} \delta_{GS} \alpha . \] (7)

With one flavour, the anomaly is given by

\[ C_{N_c} = \frac{q_2}{2\pi^2} = k_N \delta_{GS} , \] (8)

where \( q_2 = \frac{q + \delta}{2} \), is the \( U(1)_X \) charge of \( \phi_2 \), and one can check that the total \( W_{np} \) is invariant.

The extension to type I models is straightforward. Again we consider the gauge group \( SU(N_c) \) with one flavour of quarks \( Q \) in the fundamental of \( SU(N_c) \) and antiquarks \( \bar{Q} \) in the antifundamental of \( SU(N_c) \), which together form a composite meson field, \( \phi_2 \). Assuming that the \( SU(N_c) \) resides on a D9-brane we now have \( \Lambda = e^{-f_9/2\beta} \), where \( f_9 \) is given by eq.(1). \( W_{np} \) is fixed uniquely by global symmetries and reads

\[ W_{np} = \left( \frac{\Lambda^{3N_c-1}}{\phi_2^2} \right)^{\frac{1}{N_c-1}} . \] (9)

There is no \( T \)-dependence in this expression since there is no \( T \)-dependence in the one-loop expression for the gauge kinetic function \( f \) in the type I case. (If there exists a modular symmetry, the requisite modular weight of \( W \) must therefore come entirely from transformations of \( M \).) The mixed anomaly under the \( U(1)_X \) gauge transformation is cancelled by a transformation of the \( M_k \). Assuming only one \( M_k = M \) we have

\[ M \rightarrow M + i \frac{\delta_{GS}}{2\alpha} \] (10)

where \( \delta_{GS} = \frac{C_{N_c}}{\sigma_{N_c}} = \frac{C_X}{\sigma_X} \). The \( C_N \)’s are the mixed anomaly \( U(1)_X \times [G_N]^2 \) coefficients. Under \( U(1)_X \), \( \Lambda \) has charge \( q_\Lambda = \frac{\sigma_{N_c} \delta_{GS}}{2\beta} = \frac{C_{N_c}}{4\beta} \) and in our case \( C_{N_c} = \frac{q_2}{2\pi^2} \) so that \( q_\Lambda = \frac{2q_2}{(3N_c-1)} \) and again we see that \( W_{np} \) is \( U(1)_X \) invariant as required.
4 Preview; heterotic versus Type I

Before presenting our results in detail, let us discuss in general terms why dilaton stabilization is difficult in the heterotic string, but can work in type I theories. We first review the situation for heterotic strings in the case where there is one condensing gauge group. We then preview the results (to be derived in later sections) for the scalar potential of effective type I theories, and highlight the new features that make a stabilization with a single gaugino condensate possible.

The heterotic case

Consider the effective theory for heterotic strings with \( \delta_{GS} = 0 \) (so that \( q_2 = 0 \));

\[
K = -\ln s - 3\ln \tau ,
\]

where \( \tau \) is as defined above, and includes hidden sector fields, \( \phi_n \). The \( F \)-part of the supergravity scalar potential is given by

\[
V_F = e^G(-3 + G_\alpha \bar{G}^\alpha \bar{G})
\]

\[
= \frac{1}{s\tau^3} \left( |sW_s - W|^2 + \frac{\tau}{3}|W_n + \bar{\phi}_n W_T|^2 + \frac{1}{3}|\tau W_T - 3W|^2 - 3|W|^2 \right) ,
\]

where

\[
G = K + \ln |W|^2 ,
\]

and subscripts indicate differentiation. If \( W \) does not depend on \( S \) or \( T \) then

\[
V_F \sim \frac{|W|^2}{s\tau^3}
\]

and obviously neither \( S \) nor \( T \) are stabilized.

In order to attempt a stabilization we invoke a non-perturbative superpotential as described in the previous section. We also add an additional field, \( \phi_1 \) (which for this example we take to have zero charge under \( U(1)_X \)), which generates a mass term for the mesons. The effective superpotential contains a perturbative piece, so that we can write

\[
W = W_p + W_{np},
\]

where \( W_p \) includes a mass term for \( \phi_2 \); for example

\[
W_p = a\phi_1\phi_2^2 + b\phi_1^3 .
\]

(Note that more general functions of these invariants are possible but we restrict ourselves to the linear case here.) We also assume that the fields are uncharged under all other symmetries so that we can ignore the \( D \)-terms for this example.
Now let us look at the minimization of the scalar potential in eq.(12). The usual assumption to make is that at the minimum the VEVs of all the $\phi_n$ are much smaller than any of the moduli. The potential is therefore dominated by the $|W_n|^2$ terms and setting $W_n = 0$ determines $\phi_1$ and $\phi_2$ in terms of $\Lambda$. For any reasonable value of $s$, the third term (involving $W_T$) fixes the VEV of the $T$ modulus to a value close to $T = 1.2$, up to modular transformations. After these minimizations the effective superpotential is

$$W_{\text{eff}} \sim \frac{e^{-3kN_S}}{\eta(T)^6}.$$  \hspace{1cm} (17)

This is the effective potential after ‘integrating out’ the mesons and the $\phi_1$ field, and is often the starting point for studies of dilaton stabilization.

The remaining dilaton dependence in the scalar potential can then be written,

$$V_F \sim \frac{e^{-2\Delta s}}{s}(g + (1 + \Delta s)^2)$$  \hspace{1cm} (18)

where $\Delta = -W_T/W = \frac{3kN_S}{2\beta}$ is a positive constant, and $g$ is independent of $s$. The point to appreciate here (because it will contrast with the type I case) is that $\Delta$ is fixed as soon as we eliminate the $\phi_n$ fields using the $W_n = 0$ constraint.

Defining $y = \Delta s$, the minimization condition is

$$(1 + y)(1 + g) + y^2 + y^3 = 0.$$  \hspace{1cm} (19)

This leads to the following situations (see figure [1]): If $g \geq -1$ there are no positive solutions. If $g < -1$, there is one positive solution to this equation which is a maximum, with the potential running to zero at infinite $s$ and $-\infty$ at $s = 0$. In all cases there is no minimum at positive and finite $s$.

As mentioned in the introduction, ‘race-track’ models get around this problem by considered several asymptotically free gauge groups (see for example reference ref. [1]). $W$ can then be a sum of exponentials which can conspire to give rise to a local minimum with non vanishing gauge coupling. There are two other approaches that have been taken in the context of heterotic string theory. The first also requires several group factors, but assumes that one of them is not asymptotically free; i.e. it has negative $\beta$. This contribution to the superpotential removes the minimum at $s \rightarrow \infty$, and the stabilization occurs rather more naturally [18]. The second approach [19] is to assume that the Kähler potential receives non-perturbative corrections of the form $e^{-1/g} \sim e^{-\sqrt{Re S}}$ as first conjectured by Shenker [20]. We should mention at this point a completely different way of generating a non perturbative superpotential for $S$ which was noted in ref. [22].

\footnote{Note that this “Kähler stabilization” would not be possible in type I models where the divergence of perturbation theory in open strings indicates only $e^{-1/g^2}$ terms [21].}
compactification of M-theory using a Scherk–Schwarz mechanism, the authors obtained a superpotential linear in $S$ whose minimization in the absence of matter gives $S = 1$.

\textit{The type I case}

We now contrast the above with the general situation that we will find in type I theories in the following sections. We will consider the scalar potential with a Kähler potential given by eq. (3) and with only one $m_k$ which we call $m$. We will assume a single gaugino condensate in a hidden sector living on a D9-brane which generates a $W_{np}$ as described in the previous section. The superpotential may be written

$$W = W_p(\phi_1, \phi_2) + W_{np}(\phi_2, S, M).$$

(20)

There are two important differences with respect to the heterotic case. The first is that there are no factors of $\eta(T)$ appearing in the superpotential, and therefore $W$ does not necessarily depend on $T$ if the condensing group lives on the D9-brane. The second difference is that $W_{np}$ depends on the gauge kinetic function $f_9 = S + \sigma M$ and therefore includes both $S$ and $M$.

We will find $F$-term contributions of the form

$$V_F = e^G B$$

(21)

where

$$B = g(m) + |1 + s\Delta|^2 + \frac{\tau}{3} \left| \frac{W_n}{W} \right|^2 + |B_0(m) - \sigma \Delta|^2,$$

(22)
$g$ and $B_0$ are functions of $m$ only, and where we have assumed $W$ is independent of $T$.

The most important aspect of this expression is that $\Delta$ appears twice because $W_{np}$ is a function of $f_\theta$ and hence

$$\frac{W_M}{W} = \sigma \frac{W_S}{W} = -\sigma \Delta.$$  \hspace{1cm} (23)

Now let us demonstrate the existence of a minimum when both $g(m)$ and $B_0/\sigma$ are small and negative. Provided $g$ is small enough (which we check is always the case) or zero, we can neglect the $e^G$ prefactor and discuss the minimization of $B$. For a given $m$, the minimum of $B$ is close to where the squared terms all vanish. However, gauge invariance of $W$ tells us that

$$q_n \phi_n \frac{W_n}{W} - \frac{\delta_{GS}}{2} \sigma \Delta = 0,$$  \hspace{1cm} (24)

and so $W_n = 0$ cannot be satisfied at the same time as $B_0(m) - \sigma \Delta = 0$ if $B_0 \neq 0$.

We therefore have a different option to the heterotic case. To simplify the discussion in this section, suppose that

$$\sqrt{\frac{\tau}{3}} \left| \frac{1}{W} \frac{\partial W_{np}}{\partial \phi_2} \right| \ll |\sigma \Delta|, |s \Delta|.$$  \hspace{1cm} (25)

In this case it is clear that the minimum will be near

$$\frac{\partial W_2}{\partial \phi_n} = 0$$

$$B_0(m) - \sigma \Delta = 0.$$ \hspace{1cm} (26)

Eq. (25) is easily satisfied if the additional $D_X$ terms in the potential generate a large VEV for $\phi_2/\sqrt{\tau}$ whilst the VEVs of the $\phi_n \neq 2$ remain small. If the $\phi_n$ fields are charged under $U(1)_X$, setting $D_X = 0$ gives

$$\frac{q_2 |\phi_2|^2}{\tau} \approx \frac{|\delta_{GS} K'|}{6} \approx \frac{|\delta_{GS} m|}{6}.$$  \hspace{1cm} (27)

The nett result of eq.(25) is a lower bound on the Fayet-Illiopoulos term, so that $m$ cannot be zero.

We stress that the main difference with the heterotic case is that here $\Delta$ is fixed by eq.(26) rather than being fixed (implicitly) by $W_2 = 0$. Indeed, we find

$$\Delta = \frac{B_0}{\sigma} ; s = -\frac{\sigma}{B_0},$$  \hspace{1cm} (28)

provided that $B_0/\sigma$ is negative, thereby fixing both the $\phi_2$ condensate and the dilaton. Note that the minimum at $s \to \infty$ still exists, but if for example $g(m)$ is small and negative the minimum at finite $s$ is lower. Indeed we can eliminate $\Delta$ to find

$$B = g + \frac{(\sigma + B_0 s)^2}{(\sigma^2 + s^2)}.$$  \hspace{1cm} (29)
A typical potential (with \( g(m) = 0 \)) is plotted in fig.(2) (including the \( 1/s \) prefactor from \( e^K \)). (When \( g < 0 \) the minimum is at negative values of \( V \).)

Since \( \phi_2/\sqrt{\tau} \) is already fixed by the \( D_X \) term the remaining task is of course to show that \( g(m) \) can actually have a minimum at small negative values, and that at this point \( B_0/\sigma \) is indeed negative. One of the main results of the explicit discussion in the sections which follow is the condition on \( \hat{K} \) required to form a minimum for \( m \) close to \( m = 0 \) with small and negative cosmological constant. We also discuss a different possibility which is reminiscent of the ‘no-scale’ models. We can tune the cosmological constant to be exactly zero by fine-tuning \( \sigma \). In this case the value of \( m \) is undetermined and instead negative \( \delta_{GS} m \) values parameterize a flat direction with stabilized dilaton VEV.

The detailed discussion in the following sections will demonstrate that the behaviour we have outlined above is very general. Indeed we find that the dilaton is stabilized even when the inequality in eq.(25) is not satisfied, and hence the only requirement is that \( W \) is a function of \( f_9 \) and that \( B_0/\sigma < 0 \). Finally we consider the pattern of supersymmetry breaking which emerges.

5 Minimization of the SUGRA Scalar Potential

We now compute the scalar potential using the Kähler (3) and the superpotential (20). We get the following for the first derivative of the Kähler potential:

\[
K_\alpha = \left( -\frac{1}{s}, \frac{(3 + \delta.x)}{\tau}, x_k, \frac{\Phi_n (3 + \delta.x)}{\tau} \right).
\]  

(30)

Here we have introduced \( x_k = \partial \hat{K}/\partial M_k \), and have defined a dot product, \( \delta.x \equiv \sum \delta_k x_k \). In the small \( m_k \) limit we have \( x_k \approx m_k \). The fact that the \( K_n \) terms are \( -\phi_n \times K_T \) (which is really a result of the ‘no-scale’ structure) will make the scalar potential simplify dramatically. Differentiating again we get

\[
K_{\alpha \beta} = \left( \frac{1}{s^2} \begin{matrix} 0 \\ 0 \end{matrix} \right)
\]  

(31)

To express \( K_{\alpha \beta} \) we define

\[
J_{kk'} = \frac{\partial^2 \hat{K}}{\partial M_k \partial M_{k'}}
\]

\[
A_{kk'} = (J^{-1})_{kk'} (3 + \delta.x) + \delta_k \delta_{k'}
\]

\[
C = (3 + \delta.x) + \delta.J \delta
\]

\[
\delta.J \delta = \delta_k J_{kk'} \delta_{k'}
\]  

(32)

and then have

\[
K_{\alpha \beta} = \left( \begin{array}{ccc}
-\frac{C}{\tau} & -\frac{(\delta.J)_k}{\tau} & -\frac{\phi_m C}{\tau} \\
-\frac{(\delta.J)_{k'}}{\tau} & J_{kk'} & \frac{\phi_m (\delta.J)_{k'}}{\tau} \\
-\frac{\phi_m C}{\tau} & \frac{(3 + \delta.x) \delta_{mm}}{\tau}
\end{array} \right).
\]  

(33)
The inverse of this matrix is
\[
K^{βα} = \begin{pmatrix}
  s^2 & 0 \\
  0 & K^{βα}
\end{pmatrix},
\]
where
\[
K^{βα} = \frac{1}{(3 + \delta.x)} \begin{pmatrix}
  \tau^2 + \tau \sum_n |φ_n|^2 & δ_k τ & φ_m τ \\
  δ_k τ & A_{kk'} & 0 \\
  φ_m τ & 0 & δ_{nm} τ
\end{pmatrix}.
\]
(34)

We have made no approximations to get this result. Notice that there is no mixing between the $M_k$ and the $φ_n$ fields.

The $F$-part of the scalar potential is given by
\[
V_F = e^G (-3 + G_α K^{αβ} G_β) = e^G B,
\]
where the reduced Planck mass is set to one, and where, assuming that the superpotential does not depend on $T$ (i.e. the hidden sector group is on a D9-brane),
\[
B = -3 + s^2 |G_S|^2 + K_T K^{TT} K_T \\
+ 2Re \left( K_T K^{TT} G_T + K_T K^{Tπ} G_π \right) \\
+ G_{k'} K^{k_T} G_T + G_n K^{nm} G_m.
\]
(35)

Inserting the expressions for $K^{αβ}$ and completing the squares gives
\[
B = \delta.x - (\delta.A^{-1})_{kk'} (3 + \delta.x) + s^2 |G_S|^2 \\
+ \sum_n |φ_n| \sqrt{(3 + \delta.x)} - G_π \sqrt{\frac{τ}{(3 + \delta.x)}} \\
+ \sum_k |(A^{-\frac{1}{2}})_k \sqrt{(3 + \delta.x)} - (G.A)^{\frac{1}{2}}_k |^2
\]
(38)

where $(G.A^\frac{1}{2})_k = G_{k'} (A^\frac{1}{2})_{kk'}$ and $(\delta.A^{-\frac{1}{2}})_k = δ_{kk'} (A^{-\frac{1}{2}})_{kk'}$. Substituting for $K_T$, $G_n = K_n + W_n/W$, and $G_k = x_k + W_k/W$ we finally get
\[
B = \delta.B_0 + s^2 |G_S|^2 + \frac{τ}{(3 + \delta.x)} \sum_n \left| \frac{W_n}{W} \right|^2 + \sum_{k,k'} \left( B_{0k} + \frac{W_k}{W} \right) A_{kk'} \left( B_{0k'} + \frac{W_{k'}}{W} \right),
\]
(39)

where we have defined
\[
B_{0k} = x_k - (A^{-1})_{kk'} (3 + \delta.x).
\]
(40)

This form of potential is obviously similar to the no-scale result, but there is a difference. Here part of the contribution to $B$ (i.e. the $B_{0k}$ functions) can be negative and we can (at least formally) have unbroken supersymmetry at finite values of parameters. To find the global minimum we set all the squares to zero which gives us
\[
B = \delta_k x_k - \frac{δ_k δ_{k'}}{(J^{-1})_{kk'} + \frac{δ_k δ_{k'}}{(3 + \delta.x)}}.
\]
(41)
This function always has a minimum of $B = -3$ at $\delta.x = -3$. Such large values of $x_k$ are almost certainly unphysical because $x_k$ (or $m_k$) describe the blowing up of the fixed points, and hence the orbifold ‘approximation’ must break down.

We will mostly consider (for simplicity) only one $M_k$ field which we call $M$ (it is easy to generalize to any number) in which case

$$B = \delta B_0 + \left| 1 - s \frac{W_S}{W} \right|^2 + \frac{\tau}{(3 + \delta.x)} \left| W_n \right|^2 + \frac{A}{(3 + \delta.x)} \left| B_0 + \frac{W_M}{W} \right|^2. \quad (42)$$

As well as this $F$-term contribution to the potential we have the $D$-term contribution

$$V_D = \frac{g_X^2}{2} |D_X|^2, \quad (43)$$

where

$$D_X = \sum_n q_n K_n \phi_n + \delta_{GS_k} x_k - \frac{(3 + \delta.x)}{\tau} q_n |\phi_n|^2 + \delta_{GS_k} \frac{x_k}{2}. \quad (44)$$

The Fayet–Iliopoulos term,

$$\xi^2 = \frac{1}{2} \delta_{GS_k} x_k, \quad (45)$$

is given by $m_k$ not $S$ and so can be zero.

We now minimize the potential assuming that the final cosmological constant is small or zero (we will show this is possible later on) so that we can ignore the first term in

$$V_F' = G' e^G B + e^G B'. \quad$$

Assume that $\langle W \rangle \sim m_W$ (as phenomenology demands) so that we can impose the constraint $\langle D_X \rangle \approx 0$

$$q_2 |\phi_2|^2 + \sum_{n \neq 2} q_n |\phi_n|^2 = \left| \delta_{GS} x \right| 2 \left(3 + \delta.x\right) M_P^2. \quad (46)$$

For definiteness we will take $q_2 > 0$, $q_{n \neq 2} \leq 0$, and here we are anticipating the fact that $\langle \delta_{GS} x \rangle$ will eventually be negative (see the end of this section). On the other hand, the perturbative part of the superpotential involves $\phi_{n \neq 2}$ and in order to make $\langle W \rangle \sim m_W$ we typically require $|\phi_{n \neq 2}|^2 \ll |\phi_2|^2$ and hence

$$q_2 \frac{|\phi_2|^2}{\tau} \simeq \frac{|\delta_{GS} x|}{2(3 + \delta.x)} M_P^2. \quad (47)$$

As in the heterotic case, because of the smallness of $\phi_{n \neq 2}$ we can impose $W_{n \neq 2} = 0$, and since $W_{np}$ only involves $\phi_2$ gauge invariance of $W_p$ then tells us that

$$\sum_n q_n W_{np} \phi_n = 0 = W_{p2}. \quad (48)$$
Let us briefly digress to discuss an explicit example of superpotential. Consider a perturbative superpotential \( W_p \) containing \( \phi_2 \) and two other fields \( \phi_{1,3} \):

\[
W = a\phi_3(\phi_1\phi_2) + b(\phi_1\phi_2)^2 + \frac{c}{3}\phi_3^3 + W_{np}
\]

Here we have taken \( \phi_3 \) to be a singlet, whilst \( q_1 = -q_2 \). Then we have

\[
W_1 = a\phi_2\phi_3 + 2b\phi_1\phi_2^2 \tag{50}
\]

\[
W_2 = a\phi_1\phi_3 + 2b\phi_1^2\phi_2 + \partial_2 W_{np} = \frac{\phi_1}{\phi_2}W_1 - \frac{2}{N_c - 1}\frac{W_{np}}{\phi_2} \tag{51}
\]

\[
W_3 = a\phi_1\phi_2 + c\phi_3^2 \tag{52}
\]

and the solution to \( W_1 = W_{p2} = W_3 = 0 \) is

\[
\langle \phi_1\phi_2 \rangle = -\frac{a^2}{4b^2c} \tag{53}
\]

\[
\langle \phi_3 \rangle = \frac{a^2}{2bc} \tag{54}
\]

so that

\[
\langle W_p \rangle = -\frac{a^6}{48b^3c^2} \tag{55}
\]

The condition \( m_{3/2} \sim m_W \) implies \( \frac{a^6}{38b^3c^2} \sim 10^{-16} \). Since \( b \) is dimensionful, it seems natural to associated the suppression with a large non-renormalizable term (coming from a low fundamental scale) giving a large \( b \); i.e. \( b \sim 10^5/M_P \). As promised, since \( q_1 \) is negative, imposing the \( D_X = 0 \) condition then requires \( \phi_2 \) to be many orders of magnitude larger than \( \phi_1 \), for any reasonable value of \( \tau \).

Returning now to our general discussion, we introduce the variable \( \Delta = -\frac{W_S}{W} \) and rewrite \( B \) in the \( \partial_nW_p = 0 \) directions:

\[
B = \delta B_0 + |1 + s\Delta|^2 + \frac{A}{(3 + \delta x)}|B_0 - \sigma\Delta|^2 + \frac{\tau}{(3 + \delta x)} \left| \frac{\partial_2 W_{np}}{W} \right|^2
\]

\[
= \delta B_0 + |1 + s\Delta|^2 + \frac{A}{(3 + \delta x)}|B_0 - \sigma\Delta|^2 + \frac{q_2}{8\pi^4|\delta GSx|}\Delta^2, \tag{56}
\]

where we have used

\[
\frac{\partial_2 W_{np}}{W} = -\frac{\Delta}{4\pi^2\phi_2}. \tag{57}
\]

Note that \( B_0 \) only depends on \( x \). Minimizing with respect to \( s \) and \( \Delta \) gives

\[
s = -\frac{1}{\Delta} \frac{\sigma B_0}{\sigma^2 + \lambda^2}; \ 0, \tag{58}
\]

\[
\Delta = \frac{\sigma B_0}{\sigma^2 + \lambda^2}; \ 0.
\]
where we have defined
\[ \chi^2 = \frac{(3 + \delta x)}{A} \frac{q_2}{8\pi^4|\delta_{GS}x|}. \]  
(59)

Note that the first solution always requires \( B_0\sigma < 0 \). The VEVs of \( \phi_2 \) and \( \tau \) are determined by eqs.\((\ref{eq:phi2})\) and \((\ref{eq:tau})\) to be
\[ \phi_2^2 \approx \left( \frac{8\pi^2}{\Delta(N_c - 1)} \right)^{N_c-1} e^{-8\pi^2S} \quad \text{and} \quad \tau \approx \frac{2q_2|\phi_2|^2(3 + \delta x)}{|\delta_{GS}x|}, \]  
(60)
so that, at this stage, \( x \) is the only parameter remaining unfixed. Since \( \langle W \rangle \sim e^{-4\pi^2} \) in natural units, we can have virtually any value of \( \phi_2 \), with \( \phi_2 \sim 1 \) corresponding to \( N_c - 1 \approx s \).

The remaining potential is given by
\[ B = B_0 \left( \delta + \frac{A}{(3 + \delta x)\sigma^2 + \lambda^2} \right) \]
\[ = B_0 \frac{\delta}{\sigma^2 + \lambda^2} \left( \sigma^2 - \frac{q_2}{8\pi^4\delta_{GS}\delta} \right). \]  
(61)

There are now two options that one can consider in treating the remaining \( x \) degree of freedom, and we now describe each of them in turn.

**The ‘no-scale’ case**

The first option is to set the cosmological constant to be exactly zero. As we have seen, \( B_0 \) must be non-zero to stabilize the dilaton, so instead we tune the parameter \( \sigma \);
\[ \sigma^2 = \frac{q_2}{8\pi^4\delta_{GS}}. \]  
(62)

We should bear in mind that, since \( \sigma \) depends on the particle content, it is not a continuous parameter, and the constraint can only be approximately satisfied by for example choosing \( N_c \). For the particular value of \( \sigma \) in eq.\((\ref{eq:sigma})\), the \( m \) dependent VEVs we have found for \( \phi_1, \phi_2, \tau \) and \( s \) correspond to a flat direction in the parameter \( m \) with zero cosmological constant. For generic small values of \( x \) with \( \delta_{GS}x < 0 \) the dilaton is stabilized at
\[ B_0(0) \approx -\delta^2, \]
\[ s \approx \frac{\sigma^2 + \lambda^2}{\sigma^2}. \]  
(63)

As \( m \to 0 \), \( \lambda \) dominates and the dilaton VEV diverges. In figure \((\ref{fig:potential})\), we plot the potential including the 1/s prefactor from \( e^G \), with \( \Delta = -1/s \) and imposing the cosmological constant condition. A natural possibility (which we will not explore here) is that \( x \) and therefore \( m \) can be fixed (with \( \delta_{GS}x < 0 \)), as in the conventional ‘no-scale’ models, by minimizing the potential after radiatively induced electroweak symmetry breaking.
The minimized $B_0(x)$ case

The second option is to tolerate a small but negative cosmological constant which as we shall now see allows a suitable local minimum in $x$. If we assume that $\lambda^2 \ll \sigma^2$ then we may simply minimize $\delta B_0$ which is given by

$$
\delta B_0 = \delta x - \frac{(3 + \delta x)}{(3 + \delta x)^2 + 1}.
$$  \hfill (64)

In the case that $\sigma = \beta / 2$ the assumption is true for large $N_c$. $B_0$ is minimized where either $(3 + \delta x) = 0$ (i.e. the unbroken supersymmetry minimum) or

$$
1 + \frac{2\delta^2 J}{(3 + \delta x)} - \delta J_x = 0.
$$  \hfill (65)

This can be satisfied for small $x$ by functions $J$ that vary sufficiently fast close to $x = 0$. For small $x$ the extremum is always at

$$
x \approx \frac{1}{a\delta},
$$  \hfill (66)

where $J = 1 + a \frac{x^2}{2} + \ldots$. The sign of $B_{xx}$ is the same as that of $-J_{xx}$. In other words for positive $a$ we get a maximum at small values of $x$ with $\delta_{GS}x > 0$ and for negative $a$ we get a minimum at small values of $x$ with $\delta_{GS}x < 0$. 

Figure 2: Effective potential $B(s) / s$ with the cosmological constant set to zero; $s = S + \bar{S}$
Figure 3: The function $\delta B_0(x)$ for $J = e^{ax^2/2}$ where $a = -60$.

We plot $B_0(x)$ in figure (3) for $J = e^{ax^2/2}$ where $a = -60$ and $\delta = 0.3$ (a large but realistic value according to ref. [13]). Any function $J$ that falls off sufficiently fast will form a minimum, the important feature being the coefficient of $x^2$ in $J$ or, since we are at small $x \sim m$, the value of $a = K'''$ at $m = 0$ (where primes denotes differentiation with respect to $m$). A perhaps more realistic example is

$$\hat{K}(m) = \frac{3}{a} \ln \left(1 + \frac{a}{6}m^2\right).$$  \hspace{1cm} (67)

The minimum in $B_0$ is found at

$$\delta B_0|_{\text{min}} \approx \frac{1}{2a} - \delta^2.$$  \hspace{1cm} (68)

In other words, since $a$ is negative to form a minimum, the condition that $B_0\sigma < 0$ to stabilize the dilaton requires $\sigma\delta > 0$.

All fields, including the $m$ field, can be stabilized with small negative cosmological constant provided we have a large negative value for $K'''(0)$. Plugging our expressions for $\sigma$ and $B_0$ back into the solutions for $s$ in eq.(58), we have

$$s \approx \frac{\sigma}{\delta^2}.$$  \hspace{1cm} (69)

To summarize, in both cases the dilaton is stabilized primarily because of the $M/T$ mixing. Without this mixing (i.e. setting $\delta = 0$) we find only the runaway solution. The second important factor is that the gauge kinetic functions involve a linear combination of $S$ and $M$, described by the $\sigma$ coefficient which, like $\delta$, is model-dependent. We need a particular sign of $\sigma$ in order to have a minimum other than the usual runaway minimum.
We also note that in the minimization there was an interesting interplay between the $D$-terms and the $F$-terms. This is similar to the $D$-term mediation of supersymmetry breaking described in ref. [23], except here different terms are set to zero at the minimum. (In the present case we have $W_1 \approx 0$ and $W_2 \neq 0$ whereas in ref. [23] the minimization is at $W_2 \approx 0$ and $W_1 \neq 0$.) In order to achieve this we a priori need to choose a perturbative superpotential, $W_p$, which gets a non-zero expectation value.

6 Supersymmetry breaking terms

Since we have control over the VEVs of all the fields, it is now possible to write the complete expressions for supersymmetry breaking without having to define an arbitrary goldstino angle. Again we will consider only one $M_k$ field for simplicity. The supersymmetry breaking effects are carried by the auxiliary fields $F^\alpha$

$$F^\alpha = e^{G/2}G^{\alpha \beta}G_{\beta}. \quad (70)$$

At the minimum,

$$G_S = 0 \quad G_M = x - B_0 \frac{\sigma^2}{\sigma^2 + \lambda^2} \quad G_{\pi} = \frac{(3 + \delta x)}{\tau} \phi_n \quad n = 1, 3 \quad G_{\bar{\tau}} = \frac{(3 + \delta x)}{\tau} \phi_2 - \frac{\Delta}{4\pi^2 \phi_2}. \quad (71)$$

Defining

$$\omega = \frac{\Delta}{4\pi^2 (3 + \delta x)}, \quad (72)$$

this gives

$$F^\alpha = \left(0, F^T, F^M, F^i\right) \quad (73)$$

where

$$F^T = e^{G/2} \left(\frac{\delta^2}{A} - 1 - \omega\right) \quad F^M = e^{G/2} \frac{\lambda^2}{\sigma^2 + \lambda^2} \left(\frac{A}{(3 + \delta x)x - \delta}\right) \quad F^1 = F^3 = 0 \quad F^2 = -e^{G/2} \frac{\tau}{\phi_2} \omega. \quad (74)$$

The gaugino masses are given by

$$M_a = \frac{1}{2} \left(Re(f_a)\right)^{-1} F^{\alpha} \partial_{\alpha} f_a \quad (75)$$
where \( f_a \) is the gauge kinetic function for the gauge group. For gauge groups that live on the D9-brane we have \( f_a = S + \sigma_a M \) so that

\[
M_9 = \frac{1}{2}m_{3/2} \frac{\sigma_a \lambda^2}{\sigma^2 + \lambda^2} \left( \frac{A}{\delta x + A} x - \delta \right)
\]  
(76)

where the VEVs of \( S \) and \( M \) can be deduced from the expressions above. These relatively small D9 gaugino mass terms arise solely due to the non-zero value \( F^M \) in a manner suggested in ref. [8].

We shall present the remaining supersymmetry breaking in the limit where \( \lambda^2 \ll \sigma^2 \), allowing us to drop terms of order \( \lambda^2/\sigma^2 \) and to set \( F^M = 0 \). Consider the supersymmetry breaking for visible sector fields, \( C_\alpha \), which occur in the same no-scale structure as the \( \phi_1, \phi_2 \) fields (i.e. they are \( C_9 \) or \( C_{5}\) fields). As usual, we expand the Kähler potential around \( C_\alpha = 0 \) in a basis in which the Kähler metric is diagonal in the matter fields;

\[
K = K_0 + Z_\alpha |C_\alpha|^2 + \ldots
\]  
(77)

The expressions for the scalar masses (where \( m_{3/2} = e^{G/2} \)) are

\[
m^2_\alpha = m^2_{3/2} + V_0 - F^T F^\rho \partial_\rho \ln Z_\alpha.
\]  
(78)

Substituting the Kähler potential we get

\[
|F^T|^2 \partial_\tau \partial_T \ln Z_\alpha \approx \frac{A^2 (1 + \omega - \delta^2)^2}{(A - \delta^2)^2} m^2_{3/2}
\]
\[
F^T F^2 \partial_\tau \partial_2 \ln Z_\alpha \approx -\frac{A^2 \omega}{(A - \delta^2)^2} (1 + \omega - \delta^2) m^2_{3/2}
\]
\[
|F^2|^2 \partial_2 \partial_T \ln Z_\alpha \approx \frac{A^2 \omega^2}{(A - \delta^2)^2} m^2_{3/2},
\]  
(79)

where we have used eq.(65). Hence we find

\[
m^2_{\alpha,9} = V_0 \approx 0.
\]  
(80)

This is a small negative mass-squared term of order \(-\delta^2\). However eq.(63) is not true for the ‘no-scale’ case, and also relies on our assumption that \( \lambda^2 \ll \sigma^2 \). If either of these conditions are not satisfied then we can get nett positive mass squared terms of order \( \delta^2 m^2_{3/2} \).

Finally the \( A \)-terms for a Yukawa coupling \( C_\alpha C_\beta C_\sigma \) are given by

\[
A_{\alpha \beta \gamma} = F^\rho \left( K_\rho + \partial_\rho \ln Y_{\alpha \beta \gamma} - \partial_\rho \ln (Z_\alpha Z_\beta Z_\gamma) \right)
\]
\[
= m_{3/2} \left( -\delta x + 3 \frac{\delta^2 J}{3 + \delta x} \right) (-1 + \omega)
\]
\[
\approx 0.
\]  
(81)
where we have assumed that $\partial_T Y_{\alpha\beta\gamma} = \partial_2 Y_{\alpha\beta\gamma} = 0$.

So, for the fields and gauge groups associated with the D9-branes, the soft breaking terms are suppressed by powers of $\delta^2$. However, supersymmetry breaking can be more interesting for the fields living on the D5-branes. In general the Kähler potential is of the form

$$K = -\ln \left( s - \sum_i |C_{5i}|^2 \right) - \sum_i \ln \left( \tau - |C_9^{(i)}|^2 - \sum_{j \neq k \neq i}^3 |C_{5j}^{(i)}|^2 \right) + \frac{1}{2} \sum_{j \neq k \neq i}^3 \frac{|C_{5j}^{(i)}|^2}{s^{1/2} \tau^{1/2}} + \sum_{i=1}^3 \frac{|C_{95i}|^2}{\tau}, \quad (82)$$

where again we have assumed degenerate moduli fields ($T_i = T_j = T_k = T$). (As we mentioned in the introduction, our choice of putting the $\phi_n$ fields in a single $\tau$ formally corresponds to the linear combination $\phi_n = \frac{1}{\sqrt{3}}(C_9^{(1)n} + C_9^{(2)n} + C_9^{(3)n})$. However, once we have imposed $D_X = 0$, the VEVs of $\phi_n$ and $\tau$ indicate that the Kähler potential we have been using throughout is equivalent to the above up to order $(\delta_{GS} x)^2$, independently of the particular linear combination). This gives us the following supersymmetry breaking pattern.

**Mass-squareds:**

$$C_i^9, C_{j\neq i}^{5i}, C_{95i}^{5i} : m_\alpha^2 = m_{\alpha,9}^2 \approx 0$$

$$C_{5j}^{5i} : m_\alpha^2 \approx \frac{1}{2} m_{3/2}^2$$

$$C_{5i}^{5i} : m_\alpha^2 \approx m_{3/2}^2. \quad (83)$$

**Gaugino Masses:**

$$M_9 \approx 0$$

$$M_{5_1 a} \approx -\frac{1}{2} \frac{m_{3/2} \tau}{Re(T + \sigma_a M)}. \quad (84)$$

**$A$-terms:**

$$A_{\alpha\beta\gamma} \approx m_{3/2} (3 - (\alpha + \beta + \gamma)) \quad (85)$$

where

$$\alpha = -\tau \partial_T \ln Z_\alpha = \left( 1, \frac{1}{2}, 0 \right) \quad \text{for} \quad \left( C_i^9, C_{j\neq i}^{5i}, C_{5j}^{5i}, C_{5i}^{5i} \right) \quad \text{respectively,} \quad (86)$$

so that, for example, a coupling between $C_i^9 C_{j\neq i}^{5i} C_{5i}^{5i}$ would give $A = 3/2 m_{3/2}$. Note the usual sum-rule relating $A$-terms and mass-squareds

$$m_\alpha^2 + m_\beta^2 + m_\gamma^2 = A_{\alpha\beta\gamma}. \quad (87)$$
To conclude, while the auxiliary fields $F^2$ can be larger than $F^T$, under the simplifying assumptions we have made, the soft masses and $A$-terms are independent of $F_2$. In addition the supersymmetry breaking shows a rather interesting structure which may allow us to realise of a number of suggestions that have been put forward as solutions to the supersymmetric flavour and CP problems. For example it might be possible to make the first and second generations of squarks heavy [24] (of order a few TeV) if they are $C_{5i}^i$ fields whilst the 3rd generation is a $C_{j\neq i}^j$ field. Alternatively, if the higgs plus first generation particles are $C_{i\neq j}^j$ fields, one would have an interesting non-universal structure for the $A$-terms [25], and a suppression of contributions to electric dipole moments, in a manner similar to that described in ref. [26].

7 Conclusion

In this paper we considered gaugino condensation in 4D effective theories of type I strings, and discussed its effect on dilaton stabilization and the structure of soft breaking terms. Our main observation was that dilaton stabilization is possible with only one gaugino condensate. An important role is played by the twisted ($\mathcal{M}$) moduli fields which are a novel feature of these models. These fields enter in two important ways:

- First they contribute a new term to the Kähler potential, which includes, at the one-loop level, a mixing with the $T$-moduli. This mixing is one essential feature preventing the dilaton running away to infinity as in the heterotic case.

- The other crucial ingredient comes from the tree level $\mathcal{M}$-dependance in the gauge kinetic functions. This leads to some unusual properties of the non-perturbative dynamics of these gauge theories. In particular the $\mathcal{M}$ field appears in the condensation scale and it is this, combined with the $\mathcal{M}/T$ mixing in the Kähler metric, that can stabilize the dilaton.

We found that dilaton stabilization occurs quite generally with two possible outcomes. In the first we set the cosmological constant to be exactly zero and all fields except the $\mathcal{M}$ field are fixed. The $\mathcal{M}$ field then parameterizes the supersymmetry breaking in a way which is reminiscent of ‘no-scale’ models. The second possibility is to tolerate a small negative cosmological constant. In this case we showed that certain types of $\mathcal{M}$ dependent terms in the Kähler potential can lead to a stabilization of all fields including the $\mathcal{M}$ field itself. (The latter is stabilized at values close to the orbifold point.) Although there is still some ignorance about the precise form of the $\mathcal{M}$ dependence in the Kähler potential, we were able to derive the general conditions required to develop a minimum for $\mathcal{M}$; namely that $\partial^4 K/\partial M^4$ must be large and negative.

The issues we have presented here certainly deserve further investigation. For example, the phenomenological possibilities of the resulting supersymmetry breaking patterns seem promising and we briefly mentioned some potential avenues of exploration. Furthermore, in this paper we have made only the simplest (in a sense, minimal) assumption, that the condensing gauge group lives on a D9-brane. It would be interesting to consider cases
in which the gauge groups and mesons are assigned differently. In addition we have not touched upon the question of fundamental scales; it may be interesting to re-examine ideas such as ‘mirage unification’, in which the apparent unification of couplings is partly explained by the VEV of $M$ \cite{27}. Consistent mirage unification would directly relate the unification scale to the parameters in the theory (\textit{i.e.} $\delta$ and $\sigma$) that determine $\langle M \rangle$.

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