Photonic realization of nonlocal memory effects and non-Markovian quantum probes

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The study of open quantum systems is important for fundamental issues of quantum physics as well as for technological applications such as quantum information processing. Recent developments in this field have increased our basic understanding on how non-Markovian effects influence the dynamics of an open quantum system, paving the way to exploit memory effects for various quantum control tasks. Most often, the environment of an open system is thought to act as a sink for the system information. However, here we demonstrate experimentally that a photonic open system can exploit the information initially held by its environment. Correlations in the environmental degrees of freedom induce nonlocal memory effects where the bipartite open system displays, counterintuitively, local Markovian and global non-Markovian character. Our results also provide novel methods to protect and distribute entanglement, and to experimentally quantify correlations in photonic environments.

Results

The physical system and experimental setup. Our experimental open system consists of the polarization degrees of freedom of a pair of entangled photons. The photon pair is created by spontaneous parametric down-conversion after which the photons travel along different arms i = 1, 2 and move through different quartz plates of variable thickness (see Fig. 1). The quartz plates act as birefringent media which lead to a local coupling between the polarization degrees of freedom of the photon in arm i and its frequency (mode) degrees of freedom.
forming its local environment. This local interaction can be described by a local unitary operator which is defined by

\( U(t_i)|\lambda \rangle \otimes |\omega_i \rangle = e^{i \Delta n \sin \lambda \cdot \hat{\Delta} \otimes |\omega_i \rangle} \), where \( |\lambda \rangle \otimes |\omega \rangle \) denotes the state of the photon in arm \( i \) with polarization \( \lambda = H, V \) (horizontal or vertical) and frequency \( \omega_i \). The refraction index for photons with polarization \( \lambda \) is denoted by \( n_\lambda \), and \( t_i \) represents the interaction time.

Any pure initial state for the polarization degrees of freedom of the photon pair, which forms our bipartite open system, can be written as

\[ |\psi_{12} \rangle = a |HH \rangle + b |HV \rangle + c |VH \rangle + d |VV \rangle. \]

The total system initial states are assumed to be product states

\[ |\Psi(0) \rangle = |\psi_{12} \rangle \otimes \int d\omega_1 d\omega_2 g(\omega_1, \omega_2) |\omega_1, \omega_2 \rangle, \]

where \( g(\omega_1, \omega_2) \) is the probability amplitude for the photon in arm 1 to have frequency \( \omega_1 \) and for the photon in arm 2 to have frequency \( \omega_2 \), with the corresponding joint probability distribution \( P(\omega_1, \omega_2) = |g(\omega_1, \omega_2)|^2 \).

**Open system dynamics.** The time evolution of the open system can be represented by means of the dynamical map \( \Phi_{12}^{t_i} \)** which maps the initial polarization state \( \rho_{12}(0) = |\psi_{12} \rangle \langle \psi_{12} | \) to the polarization state \( \rho_{12}(t) = \Phi_{12}^{t_i}(\rho_{12}(0)) \) at time \( t \). Calculating the total system time evolution and taking the trace over the environment yields\(^{14}\)

\[ \rho_{12}(t) = \begin{pmatrix}
|a|^2 & ab^* \kappa_1(t) & ac^* \kappa_1(t) & ad^* \kappa_1(t) \\
ba^* \kappa_2(t) & |b|^2 & bc^* \Lambda_{12}(t) & bd^* \kappa_1(t) \\
cb^* \kappa_2(t) & eb^* \Lambda_{12}(t) & |c|^2 & cd^* \kappa_2(t) \\
dc^* \kappa_2(t) & db^* \kappa_1(t) & ec^* \kappa_1(t) & |d|^2
\end{pmatrix}. \tag{1}
\]

The dynamics represents a pure dephasing process whose decoherence functions can be expressed completely in terms of the Fourier transform of the joint probability distribution

\[ G(t_1,t_2) = \int d\omega_1 d\omega_2 P(\omega_1, \omega_2) e^{-i \Delta \Delta \omega (\omega_1, \omega_2)} \]

as \( \kappa_1(t) = G(t_1, 0), \kappa_2(t) = G(0, t_2), \kappa_{12}(t) = G(t_1, t_2) \) and \( \Lambda_{12}(t) = G(t_1, - t_2) \), where \( \Delta \Delta \omega = n_\phi - n_\lambda \) denotes the birefringence. The function \( \kappa(t) \) describes the decoherence of the polarization state of photon \( i \) induced by the local coupling to its frequency environment caused by the quartz plate in arm \( i \). This can be seen by taking the trace over the polarization state of photon 2 to obtain the polarization state of photon 1:

\[ \rho_1(t) = \begin{pmatrix}
|a|^2 + |b|^2 & (ac^* + bd^*) \kappa_1(t) \\
(ac^* + bd^*) \kappa_1^*(t) & |c|^2 + |d|^2
\end{pmatrix}, \]

or by tracing over photon 1 to find the state of photon 2:

\[ \rho_2(t) = \begin{pmatrix}
|a|^2 + |c|^2 & (ab^* + cd^*) \kappa_2(t) \\
(ab^* + cd^*) \kappa_2^*(t) & |b|^2 + |d|^2
\end{pmatrix}. \]

**Nonlocal dynamical map.** The nonlocal character of the dynamical map \( \Phi_{12}^{t_i} \) is controlled by the decoherence functions \( \kappa_{12}(t) \) and \( \Lambda_{12}(t) \) in Eq. (1). As a matter of fact, the map \( \Phi_{12}^{t_i} \) is a product of local dynamical maps, i.e., \( \Phi_{12}^{t_i} = \Phi_1^{t_i} \otimes \Phi_2^{t_i} \) if and only if \( \kappa_{12}(t) = \kappa_1(t) \kappa_2(t) \) and \( \Lambda_{12}(t) = \Lambda_1(t) \Lambda_2(t) \). In particular, this implies that frequency correlations in the initial state do not influence the local dynamics of each photon and cannot be detected by observing the local polarization states. However, the evolution of the composite two-photon polarization state is determined by the frequency correlations in the environment which lead to memory effects and global non-Markovian dynamics. This also means that, counterintuitively, by adding degrees of freedom to an open system one can change its dynamics from the Markovian to the non-Markovian regime.

**Trace distance dynamics.** To quantify non-Markovianity we use the trace distance based measure\(^{17}\) which is defined by

\[ N(\Phi) = \max_{\rho_A, \rho_B} \int_{t > 0} dt \, \sigma(t, \rho_A, \rho_B(0)) = \frac{1}{2} \text{tr} |\rho_A - \rho_B|. \]

The amount of non-Markovianity is therefore equal to the total increase of the trace distance during the time evolution and quantifies the total information flow from the environment to the system. This measure has been used recently in several theoretical and experimental contexts, see, e.g. Refs. 6,15,18,19.

For anticorrelated frequencies \( \omega_i \) the pair of the Bell-states

\[ |\psi_{12} \rangle = (|HH \rangle \pm |VV \rangle) / \sqrt{2} \]

maximizes the increase of the trace dis-
tance and thus determines the non-Markovianity measure $N$. These states are created by using spontaneous parametric down conversion as is illustrated in Fig. 1. A femtosecond pulse (the duration is about 150 fs and the operation wavelength is at 780 nm, with a repetition rate of about 76 MHz) generated from a Ti: sapphire laser is frequency doubled to pump two 1 mm-thick beamlike cut beta barium borate (BBO) crystals creating the two-photon entangled state. With the help of QWP1 we can easily tune the entangled state between $|\psi_{12}\rangle$ (QWP1 is set at 0°) and $|\psi_{12}\rangle$ (QWP1 is set at 90°). The size of the anticorrelations between the photon frequencies is controlled in the experiment by the spectrum of the UV pump pulse. Since energy is conserved in the down conversion process, decreasing the spectral width of the pump pulse decreases the uncertainty of the sum of the frequencies of the photons, and hence increases the anticorrelation between the frequencies. We use four different pulse widths as described in more detail in the Methods Section. It is also worth noting that even though our experimental setup is based on downconversion, which is a commonly used tool to prepare specific two-photon states in quantum optical experiments, the physical phenomena that we describe and realize experimentally was theoretically predicted only very recently[16].

Assuming a Gaussian joint frequency distribution $P(\omega_1, \omega_2)$ with identical single frequency variance $C$ and correlation coefficient $K$, the time evolution of the trace distance corresponding to the optimal Bell-state pair is found to be

$$D(t) = \exp \left( -\frac{1}{2} \Delta n^2 C(t_1 + t_2^2 - 2|K|t_1t_2) \right).$$

For uncorrelated photon frequencies, i.e. for uncorrelated local environments we have $K = 0$ and the trace distance decreases monotonically, corresponding to Markovian dynamics. However, as soon as the frequencies are anticorrelated, $K \neq 0$, the trace distance is non-monotonic which signifies quantum memory effects and non-Markovian behavior.

**Consecutive interactions with local environments.** In our first experiment the local decoherence induced by the quartz plates in arm 1 and arm 2 act consecutively, and we change the size of the initial correlations between the local reservoirs by tuning the spectral width of the pump pulse. The results for the open system’s trace distance dynamics are displayed in Fig. 2, which shows how the initial environmental correlations induce and influence the

![Figure 2](image-url)
quantum non-Markovianity. First, when only the quartz plate in arm 1 is active, the trace distance decreases monotonically demonstrating that information flows continuously from the system to the environment. When subsequently the quartz plate in arm 1 becomes inactive and the quartz plate in arm 2 active, the trace distance increases again highlighting non-Markovian behavior and a reversed flow of information from the environment back to the open system. It is important to note that this increase of the trace distance implies a revival of the entanglement between the photon polarizations. Whilst there exists earlier literature, e.g., on how the distance implies a revival of the entanglement between the photon open system. It is important to note that this increase of the trace distance dynamics remains the same irrespective of the character of the correlations. See the Methods Section for further details.

Figure 3 | Trace distance dynamics of the open system with different dispersions of the pump pulse. The quartz plate on the path of the pump pulse has thickness (a) 0 (b) 13.19 mm (c) 39.57 mm. In all cases $K = -0.55$ and the measured values of non-Markovianity are $\mathcal{N} = 0.14, 0.13, 0.13$. The trace distance dynamics remains the same irrespective of the character of the correlations. See the Methods Section for further details.

Figure 4 | Trace distance dynamics for five different quartz plate configurations and a fixed correlation coefficient of $K = -0.92$. For all curves the maximal length of the quartz plates in both arms is $199\lambda_0$, giving a maximal total effective path difference of $398\lambda_0$. From top to bottom, the curves show the results for the cases, when we start to add quartz plates to arm 2 when the length of the plate in arm 1 has reached the length of 0, 75, 100, 150, 199$\lambda_0$ respectively; see the Methods Section for details. Hence, in the bottom curve the quartz plates are added simultaneously to both arms while in the bottom curve they act consecutively. The results demonstrate that the final state is the same while in terms of the information flow the behaviour is completely different. For the top (red) curve information flows slowly out of the system with a small rate and, in fact, the decoherence would completely halt in the ideal ideal case of fully anticorrelated frequencies ($K = -1.0$). In the other extreme, for the bottom (violet) curve information first flows out of the system with a high rate and later on returns back nearly completely indicating strong memory effects. The solid lines are fits on the basis of Eq. (3), see the Methods Section for details.
the quartz plates allow to engineer the information flow between local systems while keeping fixed the final state. It is also important to note that when the local dephasing channels act simultaneously on our open bipartite system (the top red curve in Fig. 4), the initial correlations between the composite environments protect the entanglement between the qubits. One can see this also from Eq. (3) which shows that for $t_1 = t_2$ and $K = -1$ the trace distance remains constant and that there is thus no outflow of information from the system for the considered Bell-states.

**Discussion**

We have realized nonlocal quantum dynamical maps of a photonic system and demonstrated experimentally how initial quantum or even classical correlations between the local environments of a composite open system induce nonlocal memory effects. While the global dynamics of the open system is non-Markovian, its local subsystems behave perfectly Markovian. The observation of the open system dynamics yields also important information about the environment. In fact, we have seen that a measurement of the non-Markovian evolution of the two-photon polarization state leads to a novel method for the determination of the correlation coefficient of the photon frequencies. Thus, the open system dynamics can be viewed as a non-Markovian quantum probe which enables to extract nontrivial information on characteristic features of the environment. This fact as well as our demonstration that initial correlations in the environment can protect the entanglement within an open system, could find important applications in quantum technologies and control strategies, and in the development of novel diagnostics tools for complex quantum systems.

**Methods**

**Controlling the widths of the pump pulse.** The width of the pump spectrum controls the frequency anticorrelations between the pair of photons created in the down conversion process. By using filters and fused silica plates, we can control the width of the pump pulses which are depicted in Fig. 5. The laser source is filtered to 3 nm (FWHM) and then passes through the frequency doubler (1.5 mm thick BiB3O4). Hence, the bandwidth of the UV pulse is about 0.52 nm as shown in Fig. 5 (b). To obtain a sharper spectrum, as shown in Fig. 5 (a), we insert a thin fused silica plate, which is 0.05 mm thick and coated with a partial reflecting coating on each side, with approximately 75% at 390 nm. For the spectra displayed in Fig. 5 (c) and (d), we have no filter before the doubler and no fused silica plate after the doubler, and for Fig. 5 (d) we also replace the doubler by a 0.3 mm-thick BBO.

**Counting statistics.** The details of the counting statistics and pump power for the experimental results of Fig. 2 are as follows. For Fig. 2 (a) the pump power is about 2.4 mW and the total coincident count rate on the basis vectors HH, HV, VH, VV is about 18,000 in 10 seconds, (b) 10 mW and the rate is about 35,000 in 4 seconds, (c) 20 mW and the rate is about 35,000 in 2 seconds, and (d) 20 mW and rate is about 36,000 in 4 seconds. Please note that the coincident count rate always depends also on the bandwidth of UV pump source.

**Fitting the experimental data.** Employing Eq. (3), the theoretical fits of the experimental data presented in Figs. 2 and 3 have been carried out as follows. We first use the function $f(x) = A \exp\left(-Bx^2\right)$ to make a fit for the process when the quartz

![Figure 5](https://example.com/figure5.png)

**Figure 5** | The frequency spectra of the ultra-violet (UV) pump pulses used for the down conversion process. Full widths at half maximum (FWHM) of the pulse spectra are (a) $\delta = 0.18$ nm, (c) $\delta = 0.52$ nm, (c) $\delta = 0.73$ nm, and (d) $\delta = 1.89$ nm. The solid lines represent Gaussian fits of the experimental data.
we use the function experimental data. Then in the second part, when the quartz plate in arm 2 is active, of Fig. 4 from top to bottom. For all cases the maximal length of the configurations from (a) to (e) correspond to the experimental results of Fig. 3, the FWHM of the pump pulse is about 0.73 nm, the total coincident count rate on the basis vectors HH, HV, VH, VV is about 35,000 in 2 seconds.

Trace distance dynamics with different dispersions of the pump pulse. For Fig. 4, again keeping in mind Eq. (3), we can do the fitting in the following way. First, we use the functions f(x) and g(x) mentioned above to make a fit of the lowest curve and determine $A_1$, $B_1$, and $K_1$. Then we use a function $G(x) = A \exp \left[ -B \left( x^2/2 - |K|^2/2 \right) \right]$ to fit the top curve of the figure. From here, we determine $A_2$, $B_2$, and $K_2$. Then we determine $A_0 = \left( A_1 + A_2 \right)/2$, $B_0 = \left( B_1 + B_2 \right)/2$, and $K_0 = \left( K_1 + K_2 \right)/2$ which are then used to plot the functions \( f(t_1, t_2) = A_0 \exp \left[ -B_0 \left( t_1^2 + t_2^2 - 2K_0 t_1 t_2 \right) \right] \) in the figure.

Quartz plate configurations. Figure 6 shows the five different quartz plate configurations used for Fig. 4. To obtain the data of Fig. 4, the FWHM of the UV pulse is about 0.18 nm, the pump power is about 2.4 mW, and the total coincident count rate on the basis vectors HH, HV, VH, VV is about 18,000 in 10 seconds.

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Author contributions
B.-H.L., D.-Y.C., Y.-F.H., C.-F.L. and G.-C.G. planned, designed and implemented the experiments. C.-F.L., G.-C.G., E.-M.L., H.-P.B. and J.P. carried out the theoretical analysis and developed the interpretation. B.-H.L., C.-F.L., E.-M.L., H.-P.B. and J.P. wrote the paper.

Additional information
Competing financial interests: The authors declare no competing financial interests.

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