The Lefkovitch Matrix of *Aedes Aegypti* with Rainfall Dependent Model for Eggs Hatching

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Abstract. Dengue is known as borne disease that was spread by a species of mosquito which is *Aedes aegypti*. This disease is considered as a dangerous disease since it can threat human life. Shah Alam city has the high reported dengue cases in Malaysia. Based on this situation, this research was done in order to simulate the population of the mosquito in the city which involved in dengue transmission. A stage-structured of the mosquito life cycle was interpreted into Lefkovitch matrix model which consist of possible rate relating to the mosquito life cycle. The egg hatching rate was set as the main factor which influence the number of mosquito population and it depends on water availability due to rainfall. In this research, the temperature was set as constant because the temperature in Malaysia does not show any significant different compared to the rainfall distribution. The validation of the result was made by doing comparison between the simulations of eggs number with the amount of rainfall in order to see the relationship of the trends for both variables. As a result, the dynamic of dengue cases might be possible related to the trend of rainfall which provide breeding site for the female mosquito to lay eggs which contribute to the increase in the number of mosquito population. However, for the further studies, the researcher should consider other factor such as humidity since it might influence the mosquito life cycle and thus can give more accurate results.

1. Introduction
Mosquito represent the significant arthropod vector in tropical region that has been very serious contributing to many human diseases [1]. There are many species of mosquito in the world. However, there is a species which contribute to the dengue fever disease namely *Aedes aegypti* [2]. The life cycle of mosquito is definitely varied from other species but most hatch from eggs. There are four stage of life cycle of mosquito which is egg, larva, pupa and adult.

The mosquito is a small and dark with lyre shaped and branded legs. Their breed primarily can be found at the human environment and fly at low distance with is only about 200 yard [3]. Moreover, in the other fact this species is not only bite human but also bites dog and domestic animal which is mostly mammal. This species is commonly active during day and night. Normally, they used natural location such artificial containers with water in area piped system to lay their eggs [4]. Female *Aedes aegypti* need blood not only to produce eggs but used to be active in daytime. During the day they laid their eggs in the water that mixed with organic material in wide opening containers and choose dark coloured containers located in resident area.

There are many ways to describe the population for each stage of *Aedes aegypti*. However, to obtain the accurate result is obviously difficult but by using mathematical model we can forecast or predict the population of *Aedes aegypti* based on their life cycle. However in this research, the projection matrix model was preferred because the mathematical model can bring us to understand more about the mosquito life cycle. Besides that, the projection matrix model is popular model that is used for ecology and demographic such as Leslie matrix model and
Lefkovitch matrix model [5]. Leslie matrix model is discrete age structured model of population [6] meanwhile Lefkovitch matrix model enable to describe the population of each stage of the life cycle obviously. In this research, the simulation of the population of *Aedes aegypti* will be done by considering that the rainfall distribution will associate and influence the egg hatching rate of the mosquito. The Lefkovitch matrix is more appropriate model to use for the simulation because it can be associated with the stage-structured of mosquito life cycle which are egg, larva, pupa and adult [7].

2. Material and Methodology
A basic aim of matrix population models is frequently used by researchers in studying the demography of a population. There are various methods that can be used in a wide range of situations. It is a specific type of population model that uses matrix algebra. The demographic matrix models allow for a detailed analysis that comes from the life cycle history, such as the schedule of survival and reproductions across the life span of a certain species. Hence, the matrix model commonly offered the powerful means one exploring the interspecific differences of species. In this section, the detail of the simulation and the matrix model will be discussed in details.

2.1 The Equation of Aedes Aegypti Population
In this section, each stage of the *Aedes Aegypti* cycle will be represented by some parameters respect to time \( t \) [8], [9], [10]. The parameters are,

\[
E(t) = \text{number of mosquito eggs at time } t, \\
L(t) = \text{number of mosquito larva at time } t, \\
P(t) = \text{number of mosquito pupae at time } t, \\
A1(t) = \text{number of Adult 1 mosquito at time } t, \\
A2(t) = \text{number of Adult 2 mosquito at time } t.
\]

The parameters can be written into matrix \( X_t \) which consists of the whole growth stage of mosquito population.

\[
X_t = \begin{pmatrix}
E(t) \\
L(t) \\
P(t) \\
A1(t) \\
A2(t)
\end{pmatrix},
\]

(1)

The classification for \( A1 \) refers to an adult mosquito which laying for the first time and meanwhile the classification of \( A2 \) is an adult which had been laying for more than one time. The calculation of mosquito population at time \( t \) can be obtained by doing multiplication matrix \( A \) with mosquito population \( X \) at time \( t - 1 \). The population growth equation of the *Aedes aegypti* mosquito is illustrated as follows

\[
X_t = AX_{t-1}.
\]

(2)

Where \( X_t \) was considered as the number of *Aedes aegypti* population at time \( t \), while \( X_{t-1} \) represented as initial or current number of *Aedes aegypti* population.
2.2 The Stage-structured Matrix of Aedes Aegypti

The flow of the stage-structured matrix for Aedes aegypti can be described as follows.

The parameters of $F_i$ denote the fertility of stage $i$. In this cycle, during stage 2 and 3, the value of fertility $F_1$ and $F_2$ considered zero since mosquito in both stage larva and pupa do not able to produce eggs. Therefore, only $F_4$ and $F_5$ have values since both represented the number of female eggs for Adult 1 and Adult 2. The number of female eggs is equal the average number of eggs laid per oviposition divided by duration of the mosquito that remain in the different stages. So, there will be one oviposition in stage Adult 1 and up to 5 oviposition in stage Adult 2. The next parameter is $P_i$ which represents the probability of surviving and remaining in stage $i$. Meanwhile, the parameter $G_i$ represents the probability of growing of each stage in the mosquito life cycle from one stage to another stage. However, a manipulation has been made to the parameter $G_1$ which egg hatching process is changed into a function associated or influenced with rainfall distribution, $E_1(t)$.

The figure 1 can be expressed into a following matrix which call as the Lefkovitch matrix model for Aedes aegypti population shown in equation (3),

$$
A = \begin{pmatrix}
    P_1 & F_2 & F_3 & F_4 & F_5 \\
    E_1(t) & P_2 & 0 & 0 & 0 \\
    0 & G_2 & P_3 & 0 & 0 \\
    0 & 0 & G_3 & P_4 & 0 \\
    0 & 0 & 0 & G_4 & P_5
\end{pmatrix}
$$

2.3 The Element of Lefkovitch Matrix

Suppose that $G_i$ is probability of survival from stage $i$ pass into stage $(i+1)$. As for $P_i$ and $G_i$ contains $s_i$ and $d_i$ where $s_i$ refers to the survival rate of the mosquito for stage $i$ and $d_i$ is the period or duration stage $i$. In next paragraph, the derivation of the parameter $P_i$ and $G_i$ are shown in details.

Begin with the sum of geometric series for $P_i$ in term of $s_i$ and $d_i$,

$$
P_i = \frac{1 + s_i + s_i^2 + \ldots + s_i^{d_i-2}}{1 + s_i + s_i^2 + \ldots + s_i^{d_i-1}}, \quad (4)
$$

The Equation (4) can be simplified further by letting $k_{d_i}$ as the sum of the first $d_i$ terms,
\[ k_{d_i} = 1 + s_i + s_i^2 + \ldots + s_i^{d_i - 1}, \]  
\[ \text{and the second equation is the first equation multiplied by } s_i, \]
\[ s_i k_{d_i} = s_i + s_i^2 + s_i^3 + \ldots + s_i^{d_i - 1} + s_i^{d_i}. \]

Subtracting these two equations yields
\[ k_{d_i} - s_i k_{d_i} = 1 - s_i^{d_i}, \]
and manipulated Equation (7), then we get
\[ (1 - s_i) k_{d_i} = 1 - s_i^{d_i}. \]

The \( k_{d_i} \) can be expressed as follows
\[ k_{d_i} = \frac{1 - s_i^{d_i}}{1 - s_i}, \]

If refer to the Equation (5), then the equation (9) can written as
\[ 1 + s_i + s_i^2 + \ldots + s_i^{d_i - 1} = \frac{1 - s_i^{d_i}}{1 - s_i} \]

In the next step, the same procedure we get Equation (10) was applied to get Equation (11).
\[ 1 + s_i + s_i^2 + \ldots + s_i^{d_i - 2} = \frac{1 - s_i^{d_i - 1}}{1 - s_i}, \]

The transition rate \( P_i \) can be obtained by substituting Equation (10) (11) into Equation (4) and some basic division concepts were applied,
\[ \left( \frac{1 - s_i^{d_i - 1}}{1 - s_i} \times \frac{1 - s_i}{1 - s_i^{d_i}} \right) s_i, \]
\[ \text{and hence} \]
\[ P_i = \frac{s_i (1 - s_i^{d_i - 1})}{1 - s_i^{d_i}}. \]

Equation (13) illustrates the number of \textit{Aedes aegypti} populations which consist of egg, larva, pupa, Adult 1 and Adult 2 will decrease depending on the probability of survival rate and the duration of \textit{Aedes aegypti} survive and spend in each stage of its life cycle.
The transition rate of $G_i$ can be calculated approximately by considering the stage proportion of *Aedes aegypti* population in its oldest group of the stage $i$ and multiplying by the survival rate for that stage $i$ which $s_i$. The transition rate can be firmly described as below

$$G_i = \frac{s_i^{d_i-1}}{1 + s_i + s_i^2 + \cdots + s_i^{d_i-1}} s_i . \quad (14)$$

Since the geometric series $1 + s_i + s_i^2 + \cdots + s_i^{d_i-1}$ can be written as

$$1 + s_i + s_i^2 + \cdots + s_i^{d_i-1} = \frac{1 - s_i^{d_i}}{1 - s_i} \quad (15)$$

Thus, it can be simplified into this equation

$$G_i = \left( \frac{s_i^{d_i-1}}{1} \times \frac{1 - s_i}{1 - s_i^{d_i}} \right) s_i \quad (16)$$

and the simplest form is

$$G_i = \frac{s_i^{d_i}(1 - s_i)}{1 - s_i^{d_i}} \quad (17)$$

### 2.4 Water Availability of Dependent Model

For the transition rate of egg hatching $E_i(t)$ here, can be estimated by doing multiplication between the survival rate of egg stage, $s_1$ with a parameter $c_1$ which represents the probability of egg hatching. Another multiplication was done toward $s_1$ and $c_1$ with a division of the water availability in an opening container at time $t$, $R_d(t-1)$, with the maximum height of water depth in the opening container, $R_{d_{\text{max}}}$. The division was expressed with power $c_2$ which represents as the pattern of egg hatching [8][9][10]. Therefore, the manipulated transition rate of egg hatching can be defined by,

$$E_i(t) = s_1c_1 \left( \frac{R_d(t-1)}{R_{d_{\text{max}}}} \right)^{c_2} \quad (18)$$

The transition was called as water availability of dependent model because the element of water from daily rainfall will be directly influence the transition rate of egg hatching. The water availability was considered because it is the main element or factor that enables the egg hatching occurred.

### 3. Implementation

To get the simulation of the *Aedes aegypti* population, matrix $A$ needs to be completed first by substituting the data in Table 1 into the equation (13), (17) and (18).
Table 1. The survival rate and duration of each stage in mosquito life cycle

| \( i \) | Stage | Survival rate, \( s_i \) | Age (days), \( d_i \) |
|-------|-------|----------------------|---------------------|
| 1     | Egg   | 0.9890               | < 4                 |
| 2     | Larvae| 0.9898               | 4 – 8               |
| 3     | Pupae | 0.9898               | 9 – 10              |
| 4     | Adult 1 | 0.9100            | 11 – 14             |
| 5     | Adult 2 | 0.9100            | 15 - 24             |

In this case, the parameters \( c_1 \) and \( c_2 \) are set 0.27901 and 5 respectively [8][10][12]. The matrix \( A \) will be multiplied with an initial population, \( X_0 \) and an assumption had made which the initial population matrix consists of only 1000 eggs at stage egg and the other stages are none or zero. The element of the matrix namely transition rate of egg hatching, \( E_i(t) \) will be different because it depends on the factor of rain. Every time multiplication is done, the daily simulation of \textit{Aedes aegypti} will be obtained. In the next section, the simulation will be discussed further.

4. Result and Discussion

In order to see the fluctuation of egg hatching process, the daily simulation of \textit{Aedes} egg was made and shown in the graphs below. Daily rainfall (mm) data of Subang region in 2017 obtained from the Malaysian Meteorological Department (MET) will be used in obtaining the simulation. Referring to our previous studies, [8][9] the maximum of water depth stagnant in the open containers was estimated at 20 mm. If the water exceeds the level, the hatching process will not occur because the eggs and water flow out of the container.

![Graph showing rainfall and egg numbers in January](image)

Figure 2. The relationship between daily rainfall (mm) and number of \textit{Aedes aegypti} in January

Figure 2 shows the numbers of eggs are decreasing due to no rainfall from 4\textsuperscript{th} until 14\textsuperscript{th} January. The amount of rain received with small volume during period 15\textsuperscript{th} until 19\textsuperscript{th} January causing the number of eggs decrease until January 19. However, this number continuously does not increase until the end of the month. This might be happened due to high amount of rainfall that influences the frequent of egg hatching process of \textit{Aedes aegypti} mosquito.
Figure 3. The relationship between daily rainfall (mm) and number of egg of *Aedes aegypti* in February

Figure 3 shows the number of eggs is start to increase from 1st day until 8th day even though the amount of rainfall is low. Meanwhile, from 9th until 14th, the number of eggs started increase slightly even the amounts of rainfall is still low. Then, the amounts of eggs are rapidly decreased from 17th to 28th day of February since the amount of rainfall was quite high. Here, the assumption that can make is the eggs enable to remain dry and remain survive longer until they get water.

Figure 4. The relationship between daily rainfall (mm) and number of egg of *Aedes aegypti* in March

Figure 4 show the number of egg does not increase from 1st day until 4th day since there is no rainfall within that period. From day 5 until day 14, the numbers of eggs are slowly increased due to small amount of rainfall. From 22nd until 31st March the number of eggs increased rapidly. It was observed that population of eggs was decreased during heavy rainfall. The research done [12] could be supported this situation by revealing that an overflowing of water in certain areas can lead to flooding and this would be worse if this continuously occurs for a long period.
5. Conclusion
In this paper, the temperature was set as a constant at 27.2º Celsius because there were no much change toward the Malaysian temperature and meanwhile the daily rainfall as a main factor that contributes and influences the egg hatching process. As a result, we found that the medium amount of rainfall in certain period will give sufficient condition to hatching egg. Besides, when there was a heavy rain, the process of egg hatching cannot be occurred a lot due to the water in the container is overflowing and hence bring the eggs out of the container. If this happened continuously, the number of adult mosquito decrease highly is possible to occur. However, the eggs also do not able hatch and survive as the amount of rainfall received is low. Besides, the acceleration of Aedes Aegypti life cycle to complete is not only depends on the day of the rainfall occurs but it is depends on the previous days amount of receiving rain. The suggestion can be given is to recommend to the other researchers to look significantly at others aspects or factors such as environmental humidity in order to get more accurate result that could be useful to give better simulation of the mosquito population.

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