Construction and Analysis of Three Types of Portfolio
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ABSTRACT
This paper constructs three portfolios with different risk preferences and return levels, namely, aggressive, balanced and defensive, for the investment capital of 10 million yuan. Based on the related principles of portfolio theory and the calculation formula of variance and covariance of Markowitz model, the optimal investment portfolio is given for some stocks and bonds selected from 2016 to 2018. Firstly, the weekly closing price and the LN function of each asset collected are used to get the weekly logarithm return rate, and then the expected return rate of each asset in the portfolio is calculated. Secondly, the variance-covariance moment is calculated, the programming solution model is built, three kinds of constraints are formulated, and the investment proportion of each asset in the three kinds of portfolio is obtained. According to the monthly return rate of each asset and the proportion of investment in 2018, the monthly return rate and annual return rate of the three investment portfolios are obtained. According to this work, it can be seen that the aggressive portfolio performs best in the bull market, and the defensive portfolio performs best in the bear market.

Keywords: Portfolio proportion, Variance covariance matrix, Markowitz model, Planning to solve.

1. INTRODUCTION

1.1. Overview of research Background

In real life, investors often build their own portfolios according to different risk preferences, that is, different securities are constructed into a package of assets for investment, to achieve the purpose of risk diversification.[1]

To construct an investor, the first thing we should decide is how many kinds of stocks to buy. The more securities there are in the portfolio, the easier the risk is to disperse, and the less the risk investors will bear. The construction of this investment portfolio is mainly based on 10 million yuan of investment funds. [2]

According to three asset allocation situations: aggressive, balanced and defensive, the assets are invested in different quantities and different proportions of stocks (Shanghai and Shenzhen A-shares) and bonds. The relevant data from 2016 to 2018 were selected to construct the portfolio, and the programming solution model was constructed by solving variance and covariance between assets to obtain the optimal portfolio. Finally, the actual portfolio data in 2019 are used to verify the profit and loss performance of the portfolio.[3]

1.2. Research Object

The object of this study is to select stocks and bonds of different proportions for the aggressive, balanced and defensive portfolios according to the asset allocation of the three portfolios. It is assumed that a total of 10 assets are selected for the construction of three types of investment portfolios, among which 7 are stocks (Shanghai and Shenzhen A-shares) and 3 are bonds[4].

1. The seven stocks were selected respectively:
   ①Kweichow Moutai [600519] Industry: Liquor
   ② China Coastal defense [600764] Industry: Ground soldier outfit
   ③ Zhongke Dawning [603019] Industry: Computer equipment
   ④ Huatai Securities [601688] Industry: securities
   ⑤ Wentai Technology [600745] Industry: Electronic parts manufacturing
   ⑥ Hengrui Pharmaceutical [600276] Industry: Chemical pharmaceutical
   ⑦ Yili Shares [600887] Industry: Video processing and manufacturing, dairy
II The three bonds are selected respectively:
①Treasury bonds 02 [019530] Five-year Savings Bonds, fixed rate, annual interest rate: 4.1%
②Oceanwide 02 [122765] 10-year corporate bond, fixed rate, annual rate: 8.90%
③Fuxing 01 [112220] Five-year corporate bond, fixed interest rate, annual interest rate: 9.20%

Alternative bond is 16 Treasury bonds 10, annual interest rate: 3.1%

The relevant data of each asset during 2016-2018 are selected, and the return and risk levels of the three portfolios are measured by capital asset theory and Markowitz model, and the optimal asset portfolio allocation of each type of asset is obtained through programming solution.

2. PORTFOLIO MODEL THEORY

2.1. Theory of Markowitz model

Markowitz model is also called mean - variance model[5]. According to the assumptions of Markowitz model, the formula of expected return rate of investment portfolio, the calculation formula of risk, and the efficient boundary theory are used to establish the model of mean-variance model of optimal asset allocation. The calculation formula is as follows:

Objective function: \( \min \sigma_p^2 = \sum \sum w_i w_j \text{Cov}(r_i, r_j) \)  
(1)

Constraint of the function: \( \sum w_i = 1 \) and \( \sum w_i \bar{r}_i = \bar{r}_p \) (Allow short-selling) \( \sum w_i = 1 \) and \( \sum w_i \bar{r}_i = \bar{r}_p \) and \( w_i \geq 0 \) (Short-selling is not allowed) \( \sum w_i = 1 \)

2.2. Construct programming model theory

Before solving the program, the weekly historical net value of stocks and bonds from 2016 to 2018 should be collected. Then the weekly return rate of each asset can be calculated. The Average function is used to calculate the expected return rate of each asset. Finally, the variance-covariance matrix of each asset in the portfolio is obtained. In addition, the original model of programming solution is formulated by using the calculated data combined with the following programming solution principle, and the limiting conditions are set to realize the programming solution.

The solution model is as follows:

\[
\begin{align*}
\text{Max } S_p &= \frac{E_R - \bar{r}_p}{\sigma_p} \\
\text{s.t. } \sum_{i=1}^{n} x_i &= 1
\end{align*}
\]
\( x_i \geq 0, i = 1, 2, \ldots, 10 \)
\( x_i \leq 0.20, i = 1, 2, \ldots, 10 \)

① Aggressive portfolios, 75% equities and 25% bonds:
\[
\sum_{i=3}^{7} x_i = 0.75, \quad \sum_{i=9}^{10} x_i = 0.25
\]
(5)

The final restriction is the upper limit on the proportion of individual assets invested.

② A balanced portfolio, 50% equities and 50% bonds:
\[
\begin{align*}
\text{s.t. } \sum_{i=1}^{5} x_i &= 0.5, \quad \sum_{i=6}^{10} x_i &= 0.5
\end{align*}
\]
(6)

③ A defensive portfolio, 25% equities and 50% bonds:
\[
\begin{align*}
\text{s.t. } \sum_{i=1}^{3} x_i &= 0.25, \quad \sum_{i=4}^{10} x_i &= 0.75
\end{align*}
\]
(7)

3. VARIANCE COVARIANCE CALCULATION

3.1. Calculation of expected yield of stocks and bonds

3.1.1. Calculation of expected stock return

The expected rate of return is the expected rate of return in the future. Before that, it collected the historical weekly return rate of each asset from 2016 to 2018, and calculated the expected return rate of each asset according to the formula below:

\[
\begin{align*}
\text{Average function is } \bar{r}_p &= \frac{\sum r_t}{n} \quad (\text{Expected return rate}) \\
\text{Variance function is } \sigma_p^2 &= \frac{\sum (r_t - \bar{r}_p)^2}{n-1} \quad (\text{Risk})
\end{align*}
\]

According to the weekly logarithmic return of each stock that has been calculated, the expected return is estimated by the arithmetic average of the return. Therefore, the arithmetic average of the weekly return rate of all assets during 2016-2018 is calculated. The expected returns of 7 stocks in 2019 are Guizhou maotai in 0.70%, China's coastal defense in 0.28%, Zhongke dawn in -0.41%, Huatai securities in -0.01%, WenTai technology in -0.25%, Hengrui pharmaceutical in -0.07%, and Erie shares in 0.28%.

3.1.2 Calculation of expected bond yield

Since the three bonds selected in this paper are all calculated with fixed interest rates, the expected yields of the bonds in 2019 are their respective fixed interest rates. The expected yields are 16National debt02 in 4.10%, 11Fanhai02 in 8.90%, and 14Fuxing01 in 9.20%. The average return rate is 7.40%.
3.2. Calculation of variance and standard deviation

Generally, investment risk is defined as the deviation between the actual return and the expected return, which can be measured mathematically by the variance $\sigma$ of the expected return.

The variance or standard deviation of an asset is greater, the deviation between the random variable and the mathematical expectation is also greater, that is, the greater the deviation between the actual return and the expected return, and the greater the risk of the asset. As for bonds, the three bonds selected this time are all calculated with fixed interest rate and are risk-free bonds, so their variance standard deviation is 0. For stocks, the calculation results of variance and standard deviation are shown in Table 4.

Table 1 Variance and standard deviation of 7 stocks

| Stock Name        | Guizhou M.T. | China's C.D. | Zhongke D. | Huatai S. | Wen tai S&T | Hengrui P. | Erie shares |
|-------------------|--------------|--------------|------------|-----------|-------------|------------|-------------|
| E(X)              | 0.70%        | 0.28%        | -0.41%     | -0.01%    | -0.25%      | 0.07%      | 0.28%       |
| Variance          | 0.0014       | 0.0034       | 0.0073     | 0.0022    | 0.0035      | 0.0026     | 0.0019      |
| $\sigma$          | 0.0380       | 0.0585       | 0.0854     | 0.0466    | 0.0590      | 0.0515     | 0.0436      |

3.3. Implementation of covariance matrix

Covariance is the interdependence between two or more random variables. Let $x_1$ and $x_2$ be two random variables, and their mean values are $E(x_1)$ and $E(x_2)$ respectively, then the covariance between the two variables can be calculated.

According to $\sigma^2 = D(R) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}$, if the row vectors of each asset are viewed as vector $W$, and $D$ is used to represent the covariance matrix of the return rate of these assets, then the equation can be obtained $\sigma^2 = WDW^T$.

In general, if the calculated covariance between two assets is equal to 0, the two assets are not correlated; If the covariance is greater than 0, then the two assets are positively correlated. In this case, if one random variable is higher than the mean, the other random variable is also higher than the mean. If the covariance is less than 0, the two assets are negatively correlated.

It can be seen from the calculation results in the above table that the covariances of the 7 stocks selected in this paper are all greater than 0, indicating that there was a positive correlation between the 7 stocks before, but the absolute value of the correlation coefficient is low, indicating that the correlation between the two stocks is not significant. For the bond asset, its variance standard deviation is 0, its risk measure is 0, and the average yield is 7.40%.

4. CONSTRUCT THE PROGRAMMING SOLUTION MODEL AND GAIN THE RESULT

4.1. Construct the programming solution mode

Find the annual risk-free rate of return from 2016 to 2018, calculate the average risk-free rate of return, $R_f = 0.0076554$, and then process the data according to the function $\max S_p = \frac{E_r - R_f}{\sigma_p}$, as a figure to prepare for the solution of the programming (optimal combination weight).

4.2. Planning model portfolio results

According to the constraint conditions, Table 2 shows that the aggressive portfolio is:

Table 2 The proportion of assets invested in the aggressive portfolio

|              | Guizhou M.T. | China's C.D. | Zhongke D. | Huatai securities | Wen tai S&T | Hengrui P. | Erie shares |
|--------------|--------------|--------------|------------|-------------------|-------------|------------|-------------|
| Aggressive portfolio | 0.1           | 0.11         | 0.11       | 0.11              | 0.11        | 0.01       | 0.12        |

Table 3 shows that the balanced portfolio is:

Table 3 Proportion of each asset in balanced portfolio

|              | Guizhou M.T. | China's C.D. | Zhongke D. | Huatai securities | Wen tai S&T | Hengrui P. | Erie shares |
|--------------|--------------|--------------|------------|-------------------|-------------|------------|-------------|
| Balanced portfolio | 0.1           | 0.09         | 0.1        | 0.09              | 0.09        | 0.12       | 0.12        |

Table 4 shows that the defensive portfolio is:
5. EMPIRICAL ANALYSIS OF PORTFOLIO RETURN RATE IN 2019

5.1. Portfolio yield calculation in 2019

Portfolio theory is built on the basis of the expected utility maximization of wealth in a single period and focus.

In order to investigate the performance of the new investment portfolio in the expected return, 2019 must be set as the inspection period, and the monthly return rate of all assets in 2019 should be queried. And calculate the monthly returns of different portfolios in each month.

By multiplying the monthly return rate of each asset in 2019 by the corresponding weight of the three portfolio types, the monthly return rate of the three portfolios and the total return rate of 2019 are calculated and analyzed.

5.2. Empirical Analysis

According to the Figure 1, it can be seen that before April 2019 and from June to October 2019, the return rate of the aggressive portfolio is higher, followed by the return rate of the balanced portfolio and the last is the return rate of the defensive portfolio.

This is mainly related to the volatility of the market and the changes in the returns of each stock. The stock market has experienced ups and downs, but with different investment proportions of various assets in the portfolio, investment risks can be well dispersed in different industries and different types of assets, thus maintaining relatively stable returns. In the end, the aggressive portfolio returned a combined 49.11% in 2019, the balanced portfolio returned 27.93%, and the defensive portfolio returned a combined 20.25%.

6. CONCLUSIONS AND SUGGESTIONS FOR FOLLOW-UP RESEARCH

In this portfolio construction, according to three different risk levels, the paper designs the portfolio construction process of aggressive, balanced and defensive portfolios, and uses Markowitz model to model and calculate the optimal portfolio proportion of each type.

The yield curve chart of the three types of monthly returns calculated in 2019 fully proves that when the stock market is in a bull market, the order of the return rate of the portfolio is: aggressive, balanced and defensive. In a bear market, the return order is defensive, balanced and aggressive. As it turns out, the different proportions of each asset in the portfolio are not limited to the future development of each asset and the changes in the rate of return. To a large extent, the higher the proportion of a portfolio's funds invested in risky assets, the more volatile the return rate of the portfolio will be subject to the fluctuations of the stock market. The higher the proportion of investment in risk-free assets, the more stable the return rate of asset portfolio will be.

Therefore, when making asset investment allocation, different investors should select the proportion of risk-free asset portfolio that they want to adapt according to their personal risk preference and risk level that they can bear, so as to build a suitable
portfolio and obtain corresponding returns.

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