Cosmology with Light Axions from Technicolor

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We consider cosmological consequences of the spontaneous breaking of a global symmetry that is anomalous under technicolor interactions, leading to the emergence of a light axion-like particle. Avoiding overclosure of the universe by such axions yields the upper bound $f_a < 10^{10}$ GeV on the symmetry breaking scale, corresponding to keV-scale axions. However, diffuse x-ray background data typically require larger values of $f_a$. The overclosure and x-ray bounds can be reconciled if the axion initial amplitude of oscillations $A_i \sim f_a/10$. In this case, a viable axionic dark matter candidate with a mass in the $50 - 100$ eV range emerges. The detection of this type of dark matter may pose a challenge.

A main goal of future experiments at high energies is to uncover the mechanism for electroweak symmetry breaking (EWSB). The most economic proposal employs the Higgs doublet of the Standard Model (SM). However, this simple picture is unstable against quantum corrections.

There are only a few theoretical frameworks for stable EWSB. One such framework postulates the existence of a new interaction that results in EWSB via confinement near the weak scale $M_W \sim 100$ GeV. This mechanism is a higher energy analogue of QCD and, therefore, often called Technicolor. Here, the smallness of the weak scale compared to, say, the gravity scale $M_P \sim 10^{19}$ GeV, is a consequence of dynamics, like the proton mass in QCD, and no longer a puzzle. Since QCD is the only known mechanism for generation of microscopic mass scales in Nature, technicolor is a well-motivated construct.

Confinement in QCD can have other interesting consequences besides generating the hadronic scale $\Lambda_{QCD} \sim 200$ MeV. For example, if a spontaneously broken global symmetry is anomalous under QCD, as in the Peccei-Quinn (PQ) mechanism, the Goldstone boson, generally called an axion, associated with the broken symmetry acquires a mass through confinement at $\Lambda_{QCD}$. The PQ mechanism was originally devised to explain the tiny size of the CP violating angle $\theta_{QCD} < 10^{-10}$ in QCD, as suggested by data. QCD dynamics generates a mass $m \sim \Lambda_{QCD}^2/f$ for the axion, where $f$ is the scale of PQ symmetry breaking. The coupling of the axion to the SM is set by $1/f$. Astrophysical data suggest $f \sim 10^{9}$ GeV. Hence, the PQ axion must be very light and extremely weakly coupled.

Remarkably, cosmological overclosure arguments also yield an upper bound $f \lesssim 10^{12}$ GeV near the upper bound, the axion can be a good dark matter candidate. In this paper, we consider the analogous question within a technicolor-like framework. That is, we investigate the cosmological implications of breaking a techni-anomalous global symmetry (henceforth generically referred to as a PQ symmetry, but unrelated to the one relevant for $\theta_{QCD}$), at a high scale $f_a$. The resulting axion $a$ can pick up a mass $m_a \sim M_W^2/f_a$. Therefore, $a$ can be very light and weakly coupled if $f_a \gg M_W$.

The problem of cosmological domain walls from axion dynamics is not addressed here. This problem can be avoided if, for example, inflation takes place after PQ symmetry breaking, resulting in a constant axion background throughout the visible universe. We also note that light particles that could in principle affect cosmology have been discussed in other contexts, such as supersymmetry. Our work suggests that technicolor can in principle be another context for a cosmologically important light particle.

Before we begin our analysis, we make a few remarks. First of all, we do not address model-building issues related to a specific techni-anomalous PQ construct. However, as PQ symmetry breaking usually involves a non-zero scalar vacuum expectation value (vev) and technicolor is invoked to eliminate the need for scalars, one may object that our assumptions are conceptually inconsistent. Here we note that this is not the case, since we will show that various considerations typically require $f_a$ to be in excess of $10^{10}$ GeV. If the cutoff of a theory is near such scales, as may be the case with extra dimensions and the quantum gravity scale $M_F \sim 10^{10}$ GeV, then PQ symmetry breaking near $M_F$ is quite natural. In the absence of low energy supersymmetry, technicolor-like strong dynamics is still well-motivated in order to keep $M_W$ hierarchically below $f_a \sim M_F \gtrsim 10^{10}$ GeV. Hence, our setup can be consistently embedded within a large class of models.

Next, we will introduce the framework that will be used to study the effects of technicolor strong dynamics and high scale PQ symmetry breaking. Later, we will consider this setup in the early universe, at temperatures relevant to EWSB. We will only focus on key theoretical features of technicolor. Our conclusions will then be applicable to a wide range of models with acceptable low energy physics. We find that the upper and lower bounds on $f_a$, from overclosure and diffuse x-ray background con-

1 We will only consider technicolor and ignore QCD. However, the original strong CP problem may be addressed by postulating a second PQ symmetry.
siderations, respectively, can be reconciled if the initial amplitude of primordial axion oscillations is an order of magnitude below \( f_a \). Hence, with a mild suppression of the initial amplitude, a dark matter candidate in the mass range \( 50 - 100 \) eV can emerge.

We will assume that EWSB is realized by the condensation of a chiral fermion bilinear \( \langle Q_L Q_R \rangle \approx \Lambda_T^2 \), with the correct \( SU(2) \times U(1) \) quantum numbers to result in the observed SM pattern. The condensation is caused by strong dynamics of a technicolor gauge group \( SU(N_T) \) at the scale \( \Lambda_T \sim M_W \). Further, we assume that \( N_D \) left-handed weak doublet and \( 2 N_D \) right-handed weak singlet techni-quarks are in the \( N_T \) fundamental representation. Thus, the theory is endowed with a \( SU(2N_D)_L \times SU(2N_D)_R \) chiral symmetry. Upon EWSB, \( (2N_D)^2 - 1 \) Pseudo-Goldstone Bosons (PGB’s) are generated. These particles are the analogues of pions in QCD.

We will assume that the techni-sector PGB’s have a decay constant \( F_T \).

The scale at which strong dynamics sets in is given by

\[
\Lambda_T \approx \Lambda_{\text{QCD}}(v_0/f_\pi) \left( \frac{3}{N_D N_T} \right)^{1/2},
\]

where \( \Lambda_{\text{QCD}} = 200 \) MeV, \( v_0 = 246 \) GeV is the scale of EWSB in the SM, and \( f_\pi = 93 \) MeV is the pion decay constant. The PGB decay constant \( F_T \) is given by

\[
F_T \approx v_0 N_D^{-1/2}.
\]

We assume that techni-quarks have a PQ chiral symmetry that is spontaneously broken at a high scale \( f_a \gg \Lambda_T \). The massless axion corresponding to the broken symmetry will acquire a mass \( m_a \) due to the \( SU(N_T) \) techni-anomaly. We will derive a low energy expression for \( m_a \) by analogy with the results from QCD.

The QCD related axion mass is given by

\[
m_a^{(\text{QCD})} = \frac{m_0^2 v_0^2}{f_\pi^2 F_T^2 M^{-1}},
\]

where \( M = \text{diag}(m_u, m_d, m_s) \) is the diagonal light quark mass matrix. Let \( m_0 \) be the typical techni-pion mass, generated by explicit chiral symmetry breaking. In QCD, this is due to the masses of the light quarks. However, in technicolor, there could be several sources of chiral symmetry breaking. We will parameterize this effect by assuming a common techni-quark mass \( m_Q \). Even though this will not encode all aspects of chiral symmetry breaking in technicolor, our approach will capture the relevant physics and yield a good estimate for the size of the effects. Hence, in Eq. (3), we make the replacements \( m_u + m_d \rightarrow 2m_Q, \ m_\pi \rightarrow m_a, \ f_\pi \rightarrow F_T, \) and \( \text{Tr}(M^{-1}) \rightarrow 2N_D/m_Q \). Using Eq. (2), we find

\[
m_a \approx \frac{m_0 v_0}{2 N_D F_a} \quad \text{(Technicolor)}.
\]

Current experimental bounds require \( m_0 \gtrsim 100 \) GeV [4].

In order to study the cosmological implications of the above setup, we will need to consider what happens to the axion when strong techni-dynamics sets in around the weak-scale in the early universe. Generically, due to the techni-anomaly of the PQ symmetry, the axion will be endowed with a potential and pick up a temperature dependent mass \( m_a(T) \). Once \( m_a(T) \approx 3H(T) \), where \( H \) is the Hubble constant, the axion will start to oscillate with an amplitude \( A_a \). Assuming that the zero-mode axion field does not decay on the time scales we will consider, its energy gets red-shifted according to [5]

\[
\frac{[m_a A^2]}{m_a A^2} = \left( \frac{R_i}{R_f} \right)^3,
\]

where \( R \) is the scale factor and the subscripts \( i \) and \( f \) refer to initial and final values, respectively. In deriving Eq. (5), it has been assumed that \( m_a^{-1}(d m_a/dt) \), \( H \ll m_a \).

Conservation of entropy in equilibrium then yields

\[
[m_a A^2] = [m_a A^2]_i \left( \frac{g_s}{g_{s_i}} \right) \left( \frac{T_f}{T_i} \right)^3,
\]

where \( g_s \) refers to the number of relativistic degrees of freedom in equilibrium and \( T_f \) is the temperature at which the axion starts to oscillate.

The energy density stored in the axion zero mode is given by \( \rho_a = (1/2) m_a^2 A^2 \). Using Eq. (6), we can write

\[
\rho_{a} = \frac{1}{2} m_a f_a \left( g_s / g_{s_i} \right) \left( T_f / T_i \right)^3 A_i^2.
\]

The Hubble scale during radiation domination is given by \( H = 1.66 g_i^{1/2} (T^2 / M_P) \), with \( M_P \approx 1.2 \times 10^{19} \) GeV; \( g_{s_i} = g_i \) for \( T_i \gg m_e \). We get

\[
m_{a_i} \equiv m_a(T_i) \approx 5 \frac{g_{s_i}^{1/2} T_i^2}{M_P}.
\]

Eqs. (7) and (8) together yield

\[
\rho_a(T_f) = \left( \frac{5 g_s}{2 g_{s_i}^{1/2}} \right) \left( \frac{f_a}{M_P} \right) \left( \frac{m_a f_a A_i}{T_i} \right)^2 T_f^3.
\]

The above equation can be used to obtain a bound on the PQ scale \( f_a \) by requiring that axions do not overclose the universe [5]. Here, we implicitly assume that the axion has a sufficiently long lifetime to alter the late evolution of the cosmos. We will see below that this assumption can be justified given our results. In order to proceed, we need to solve Eq. (8) for \( T_i \) and hence we will need an expression for \( m_a(T_i) \).
Following the treatment in Ref. [8], \( m_a(T_i) \) can be obtained from
\[
m_a^2(T) = \left( \frac{1}{f_a^2} \right) \left[ \frac{\partial^2 F(T, \Theta)}{\partial \Theta^2} \right]_{\tilde{\Theta}=0},
\]
where \( \Theta \) is the “strong” CP violating angle and \( F(T, \Theta) \) is the free energy density in the technicolor model. We note that \( \Theta \) is assumed to be physically non-zero in the absence of a PQ mechanism. Therefore, we require all chiral symmetries to be broken at the weak scale. This will ensure that there are no massless PGB’s left in the low energy spectrum, which is a desired feature in any realistic model.

We have [14]
\[
\left[ \frac{\partial^2 F(T, \Theta)}{\partial \Theta^2} \right]_{\tilde{\Theta}=0} \simeq \int_0^{\infty} d\rho n(\rho, T = 0),
\]
where instanton density \( n(\rho, T = 0) \), as a function of instanton size \( \rho \), is given by [14]:
\[
n(\rho, 0) = \frac{C_{N_T}}{\rho^3} \left( \frac{4\pi^2}{g^2} \right)^{2N_T} \left( \prod_{i=1}^{N_D} \xi_m \right) e^{-8\pi^2/\rho^2},
\]
with \( 4\pi^2/\rho^2 = (1/6)(11N_T - 4N_D) \ln[1/(\rho\Lambda_T)], C_{N_T} \simeq 0.26\xi^{-3(N_T-2)}[(N_T - 1)!(N_T - 2)!]^{-1}, \) and \( \xi \simeq 1.34; m_i \) are the techniquark masses.

We have used the high temperature dilute instanton gas approximation in writing Eq. (11) [14], corresponding to a sum of \( T = 0 \) solutions. However, to regulate the infrared divergence of the integral, we use the thermal cutoff \( \rho_c = (\pi T)^{-1} \) [14]. Hence, we have
\[
m_a(T) \simeq \frac{2\pi T}{\kappa} \frac{C_{N_T}}{\rho^2} \left( \frac{4\pi^2}{g^2} \right)^{2N_T} \left( \prod_{i=1}^{N_D} \xi_m \right) e^{-8\pi^2/\rho^2},
\]
with \( 4\pi^2/\rho^2 = (1/6)(11N_T - 4N_D) \ln[1/(\rho\Lambda_T)], C_{N_T} \simeq 0.26\xi^{-3(N_T-2)}[(N_T - 1)!(N_T - 2)!]^{-1}, \) and \( \xi \simeq 1.34; m_i \) are the techniquark masses.

We assume \( g_s \) to be given by the degrees of freedom contained in SM−\{H\} plus technicolor, as a function of \( (N_T, N_D) \)
\[
g_{si} \simeq 102.75 + 7N_TN_D/2 + (N_T^2 - 1). \tag{17}
\]
Here, the value of \( g_{si} \) is set by the degrees of freedom at late times, after \( e^+e^- \) annihilation; \( g_{si} \simeq 4.13 \).

We can now use the above results to study the effects of axions on cosmology. We will assume \( m_0 = 100 \) GeV, hereafter. Then, given \( (N_T, N_D) \) and \( f_a/f_0 \), Eq. (15) implies that \( \Omega_a \) is only a function of \( \omega \). Thus, bounds on \( \Omega_a \) translate into bounds on \( f_a/M_p \), via Eq. (13).

Avoiding the overclosure of the universe requires \( \Omega_a < 1 \). For some choices of technicolor model parameters, the overclosure upper bounds on \( f_a/M_p \) are given in Table I, where we have set \( f_a/f_0 = 1 \). The first set of parameters is motivated by minimality [12] and the other sets show the effects of modest variations in \( N_D \) and \( N_T \).

In Table I, we also present \( m_a \) from Eq. (4), and the lifetime \( \tau_a \) of the axion for maximal \( f_a/M_p \). Here, \( \tau_a \) is given by the width \( \Gamma(a \rightarrow \gamma\gamma) \) for decay into photons [13]
\[
\tau_a^{-1} = \Gamma(a \rightarrow \gamma\gamma) = \frac{\alpha^2 m_a^3}{64\pi^3 f_a^2},
\]
where, \( \alpha \simeq 1/137 \) is the fine structure constant. We see that, in all cases considered in Table I, \( \tau_a \) is much larger than the age of the universe (\( \sim 10^{10} \) yr), justifying the implementation of the overclosure bound at late times.

Given our results, an interesting possibility is that the axions of Table I, with values of \( f_a/M_p \) just below the upper bound, can be good dark matter candidates provided that \( \Omega_a \simeq 0.2 \) [13]. Note that sub-critical values of \( f_a/M_p \) imply shorter lifetimes than listed in Table I.

The lifetime \( \tau_x \) of a particle, with mass \( m_x \sim 1 \) keV and relic density \( \Omega_x = \rho_x/\rho_c \), is constrained by diffuse x-ray background data [4]:
\[
\tau_x \gtrsim 3.3 \times 10^{19} \Omega_x \left( \frac{m_x}{\text{keV}} \right)^{-0.6} \text{yr}. \tag{19}
\]
The bound in [19] can be satisfied for $m_a \sim 1$ keV if dark matter axions have $\tau_a \gtrsim 10^{19}$ yr. This means that the cases presented in Table I cannot be dark matter. Nonetheless, a somewhat different choice of initial conditions can change these conclusions.

| $f_a/M_P$ | $m_a$ (keV) | $\tau_a$ (yr) | $\tau_\alpha$ (yr) |
|------------|-------------|---------------|------------------|
| (4, 1)     | $1.4 \times 10^{-8}$ | $0.071$ | $6.3 \times 10^{19}$ | $3.2 \times 10^{19}$ |
| (4, 2)     | $1.1 \times 10^{-9}$ | $0.048$ | $1.2 \times 10^{20}$ | $4.1 \times 10^{19}$ |
| (6, 2)     | $9.4 \times 10^{-9}$ | $0.054$ | $6.2 \times 10^{19}$ | $3.8 \times 10^{19}$ |

TABLE II: Axion parameters resulting in dark matter density $\Omega = 0.2$. For these results, $A_i/f_a = 0.1$ and $m_0 = 100$ GeV.

The results in Table II have been obtained with $A_i/f_a = 1$. However, $A_i/f_a = 0.1$ is also a plausible choice. Using this value for the initial amplitude, we have listed the parameters of axions with dark matter energy density $\Omega = 0.2$, in Table I. The results show that these axions can satisfy the x-ray background bound [19] and be good dark matter candidates, within a typical range of parameters. Note that the parameters of the above dark matter axions are very different from those associated with the strong CP problem, for $A_i/f_a \approx 0.1$: $f_a(Q_{CD})/M_P \sim 10^{-5}$ and $m_a(Q_{CD}) \sim 10^{-8}$ eV. The discovery of the technicolor related axions we have studied may therefore pose an experimental challenge.

Various astrophysical constraints, such as supernova over-cooling, place a lower bound on $f_a/M_P$, where the precise bound has model-dependence. However, we may take $f_a/M_P \gtrsim 8 \times 10^{-11}$ for light axion-like particles [20]. In principle, values of $f_a/M_P$ larger than this lower bound but below the value resulting in dark matter density $\Omega = 0.2$ could be allowed. However, the axion lifetime $\tau_a \sim f_a^2$ and the bound in [19] is generally a stronger constraint than astrophysical considerations.

Nonetheless, for small enough relic densities, values of $f_a/M_P$ near the astrophysical lower bound could still be allowed. For example, if $A_i/f_a \sim 10^{-2}$, the (4, 1) case with $f_a/M_P \approx 8 \times 10^{-11}$ gives $m_a \approx 12.8$ keV, $\tau_a \approx 3.3 \times 10^8$ yr, and $\Omega = 5 \times 10^{-6}$. This set of values is roughly consistent with the x-ray background bounds and may have an observable impact on the early reionization history of the universe [21, 22]. Definitive bounds and predictions for the effect of such axions on early reionization require specific models and a more detailed study and is hence outside the scope of this work.

Given the high scales involved in PQ symmetry breaking, it is interesting to ask whether such scales can be important for other reasons in Nature. Generation of small neutrino masses typically requires operators suppressed by high scales and is thus a good candidate. Assuming that neutrinos are Dirac particles, the operator

$$O_a = \frac{\langle Q_L Q_R \rangle (Q

\text{generates a small neutrino mass } m_\nu \text{ upon EWSB via the techni-condensate } \langle Q_L Q_R \rangle \approx \Delta^2$. In Eq. (20), $L$ is the SM lepton doublet and $\nu_R$ is a right handed neutrino; the coefficient of $O_a$ is assumed to be $O(1)$. For example, the (4, 1) case, with $f_a$ near the astrophysical lower bound, yields $m_\nu \approx 0.1$ eV, which can accommodate the neutrino oscillation data. As discussed before, this set of parameters can be allowed for $A_i/f_a \lesssim 10^{-2}$ and may lead to observable early reionization effects.

In summary, we have studied the possibility that technicolor dynamics is the source of mass for axion-like particles. This assumes a techni-anomalous PQ symmetry. Given existing experimental constraints and the potentially important role of high scales in Nature, the PQ scale may be large: $f_a \gtrsim 10^{19}$ GeV. Hence, the axions we have studied are light, with $m_a \lesssim 0.1$ keV, and ‘invisible’. Assuming a generic parametrization of technicolor models based on the size of their gauge groups and number of weak doublets, we derived overclosure bounds on the scale $f_a$. However, bounds from diffuse x-ray background data further constrain $f_a$. We showed that for typical sets of parameters, these axions can be suitable dark matter and consistent with the x-ray data, if the initial amplitude of axion primordial oscillations is mildly suppressed. Detection of this type of dark matter could be difficult. For smaller values of $f_a$, the axion decay into photons may impact early reionization history of the universe. Such values of $f_a$ can lead to acceptable neutrino masses through a seesaw involving the technicolor condensate.

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