Disentangling genuine from matter-induced CP violation in neutrino oscillations

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Abstract. We prove [1] that, in any flavor transition, neutrino oscillation CP violating asymmetries in matter have two disentangled components: (a) a CPT-odd T-invariant term, non-vanishing iff there are interactions with matter; (b) a T-odd CPT-invariant term, non-vanishing iff there is genuine CP violation. As function of the baseline, these two terms are distinct $L$-even and $L$-odd observables, respectively. In the experimental region of terrestrial accelerator neutrinos, we calculate [2] their approximate expressions from which we prove that, at medium baselines, the CPT-odd component is small and nearly $\delta$-independent, so it can be subtracted from the experimental CP asymmetry as a theoretical background, provided the hierarchy is known. At long baselines, on the other hand, we find that (i) a Hierarchy-odd term in the CPT-odd component dominates the CP asymmetry for energies above the first oscillation node, and (ii) the CPT-odd term vanishes, independent of the CP phase $\delta$, at $E = 0.92$ GeV($L/1300$ km) near the second oscillation maximum, where the T-odd term is almost maximal and proportional to $\sin \delta$. A measurement of the CP asymmetry in these energy regions would thus provide separate information on (i) the neutrino mass ordering, and (ii) direct evidence of genuine CP violation in the lepton sector.

1. Introduction

Neutrino oscillations arise from the mismatch between the three interaction (flavor) eigenstates and the propagation (mass) eigenstates, described by the PMNS mixing matrix, $\nu_\alpha = \sum_k U_{\alpha k} \nu_k$. In the standard parametrization, this matrix is written as

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

in terms of three mixing angles ($c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$) and a CP-violating phase $\delta$. Its effects in the flavor evolution Hamiltonian

$$H = \frac{1}{2E} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger,$$

lead to the appearance of energy ($E$) and baseline ($L$) dependent oscillations with probability

$$P_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{j<i} \text{Re} \left( J^{ij}_{\alpha\beta} \right) \sin^2 \Delta_{ij} - 2 \sum_{j<i} \text{Im} \left( J^{ij}_{\alpha\beta} \right) \sin 2\Delta_{ij},$$
where \( J_{ij}^{\alpha \beta} \equiv U_{\alpha i} U_{\beta j}^{*} \) are the rephasing-invariant mixings, \( \Delta_{ij} \equiv \frac{\Delta m_{ij}^{2}}{4E} \) are the oscillation phases, and \( \Delta m_{ij}^{2} = m_{i}^{2} - m_{j}^{2} \) are the neutrino mass-squared differences.

Even though the absolute scale of the neutrino masses is still unknown, both mass differences \( \Delta m_{21}^{2} \) and \( |\Delta m_{31}^{2}| \), as well as all three mixing angles, have reached the precision era \([3]\),

\[
\begin{align*}
\Delta m_{21}^{2} &= 7.55(20) \times 10^{-5} \text{ eV}^{2} \\
|\Delta m_{31}^{2}| &= 2.50(3) \times 10^{-3} \text{ eV}^{2}
\end{align*}
\]

The main goals of the next generation experiments, such as DUNE \([4]\) and T2HK \([5]\), will be the measurement of the CP phase \( \delta \) and the sign(\( \Delta m_{31}^{2} \)), which will determine whether the mass Hierarchy is Normal \((m_{1} < m_{2} < m_{3})\) or Inverted \((m_{3} < m_{1} < m_{2})\).

Notice that there are terms in the oscillation probability (3) that are \( \delta \)-dependent through the CP-conserving \( \cos \delta \), coming from \( \text{Re} \ J_{ij}^{\alpha \beta} \). Although they allow to extract the value of the parameter \( \delta \), their measurement cannot be considered as observation of CP violation. In this work, we study the feasibility of a direct observation of CP violation in the lepton sector, in as much a model-independent manner as possible. Such a probe must come from the measurement of a non-vanishing value of a CP-odd observable like the asymmetry \( \mathcal{A}_{\text{CP}} \equiv P(\nu) - P(\bar{\nu}) \). The complication in this measurement stems from the propagation of neutrinos through the Earth. Since matter is CP asymmetric, it induces a fake contribution to the CP asymmetry, contaminating the test of CP via \( \mathcal{A}_{\text{CP}} \).

In the next Section, we exploit the different behavior of the different terms in the oscillation probability under the discrete symmetries CP, T and CPT to cleanly separate genuine from matter-induced terms in the CP asymmetry. This disentanglement will lead to peculiar dependencies of the separate components of \( \mathcal{A}_{\text{CP}} \) in the mixing parameters. In Section 3, we show the signatures induced at future accelerator experiments by these ideas, focusing especially on the determination of the Hierarchy and the direct observation of CP violation. Our conclusions are presented in Section 4.

### 2. Asymmetry Disentanglement Theorem

The matter effects in neutrino oscillations are described by the Hamiltonian \([6–11]\)

\[
H = \frac{1}{2E} \left\{ \begin{pmatrix} \begin{array}{ccc} m_{1}^{2} & 0 & 0 \\ 0 & m_{2}^{2} & 0 \\ 0 & 0 & m_{3}^{2} \end{array} \end{pmatrix} U^{\dagger} + \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\} = \frac{1}{2E} \tilde{U} \tilde{M}^{2} U^{\dagger},
\]

where \( a = 2EV \) is the energy-dependent matter parameter, proportional to the matter potential \( V \). The same Hamiltonian applies to antineutrinos changing \( \delta \leftrightarrow -\delta \) in \( U \), and \( a \leftrightarrow -a \). In practice, the same analytical expressions as in vacuum can be used to describe neutrino oscillations in matter, if one writes them in terms of the energy-dependent masses \( \tilde{M}^{2} \) and mixings \( \tilde{U} \) obtained from the diagonalization of the Hamiltonian (5).

In general, let’s assume there are different masses and mixings for neutrinos \( (\tilde{M}^{2}, \tilde{U}) \) and antineutrinos \( (\bar{M}^{2}, \bar{U}) \), given by an arbitrary number (at least 3) of eigenstates. Using the probability in Eq. (3) for both of them we can compute the CP asymmetry,

\[
\mathcal{A}_{\text{CP}}^{\alpha \beta} \equiv P(\nu_{\alpha} \rightarrow \nu_{\beta}) - P(\bar{\nu}_{\alpha} \rightarrow \bar{\nu}_{\beta}) = -4 \sum_{j<i} \left[ \text{Re} \ J_{ij}^{\alpha \beta} \sin^{2} \Delta_{ij} - \text{Re} \ J_{ij}^{\alpha \beta} \sin^{2} \tilde{\Delta}_{ij} \right] - 2 \sum_{j<i} \left[ \text{Im} \ J_{ij}^{\alpha \beta} \sin 2\Delta_{ij} - \text{Im} \ J_{ij}^{\alpha \beta} \sin 2\tilde{\Delta}_{ij} \right].
\]
In the CPT-invariant limit, as happens in vacuum, the first term of the CP asymmetry vanishes due to $\tilde{M}^2 = \tilde{M}^2$ and $\tilde{U} = \tilde{U}^*$. Conversely, the second one vanishes in the T-invariant limit, since the real mixing matrices ensure $\text{Im } \tilde{J}^{ij}_{\alpha \beta} = \text{Im } \tilde{J}^{ij}_{\alpha \beta} = 0$. Therefore, the first term quantifies CP and CPT violation, whereas the second one is CP and T violating.

Looking at the behavior of each of them under T and CPT, we find that the components

$$A^{\text{CPT}}_{\alpha \beta} \equiv -4 \sum_{j<i} \left[ \text{Re } \tilde{J}^{ij}_{\alpha \beta} \sin^2 \tilde{\Delta}_{ij} - \text{Re } \tilde{J}^{ij}_{\alpha \beta} \sin^2 \tilde{\Delta}_{ij} \right],$$

$$A^T_{\alpha \beta} \equiv -2 \sum_{j<i} \left[ \text{Im } \tilde{J}^{ij}_{\alpha \beta} \sin 2\tilde{\Delta}_{ij} - \text{Im } \tilde{J}^{ij}_{\alpha \beta} \sin 2\tilde{\Delta}_{ij} \right]$$

have definite parities on both symmetries: $A^{\text{CPT}}_{\alpha \beta}$ is CPT-odd and T-invariant, whereas $A^T_{\alpha \beta}$ is CPT-invariant and T-odd. Also, by construction, both of them are CP-odd. In the sense of these symmetry principles, we call these quantities the disentangled components of $A_{\alpha \beta}$, since the way in which each of them violates CP is fundamentally different. The fact that CPT holds in vacuum means that effects of genuine CP violation affect the CP asymmetry as the T-odd component, so a non-vanishing measurement of $A^T_{\alpha \beta}$ is a proof of CP violation in the lepton sector. On the other hand, the CPT-violating and T-invariant matter effects, taking into account in the Hamiltonian (5) via the real parameter $a$, contribute to the CP asymmetry as $A^{\text{CPT}}_{\alpha \beta}$.

Without explicit expressions for the masses and mixings, the way in which this disentanglement translates into experimental measurements is through the definite T parity of both components, which ensures that the matter-induced component—the T-invariant $A^{\text{CPT}}_{\alpha \beta}$—is an even function of the baseline, whereas the genuine component—the T-odd $A^T_{\alpha \beta}$—is an odd function (notice the different functions of Eqs.(7, 8) in the oscillation phases, which are proportional to $L$ at a given energy). Therefore, the existence of an $L$-odd term in a measurement of the CP asymmetry as a function of the baseline would be a direct test of CP violation in the lepton sector. Since this kind of measurement is not feasible, we study how these symmetry behaviors affect the dependence of each component on the parameters in the Hamiltonian (5).

3. Signatures at Future Accelerator Experiments

The difference between neutrinos and antineutrinos in the Hamiltonian lies in the signs of the complex phase $\delta$ and the matter parameter $a$, which are associated to T ($\delta \rightarrow -\delta$) and CPT ($a \rightarrow -a$) transformations. The definite parities of both components imply that $A^{\text{CPT}}_{\alpha \beta}$ must be an even function of $\delta$ (typically cos $\delta$) and an odd function of $a$, and so it vanishes in vacuum, whereas $A^T_{\alpha \beta}$ must be an even function of $a$ and an odd function of $\delta$, so it vanishes in the absence of genuine CP violation.

We calculate approximated expressions for both components at energies between the two MSW resonances $[6, 12] \Delta m^2_{21} \ll a \ll |\Delta m^2_{31}|$, for the case of the golden $\nu_\mu \rightarrow \nu_e$ transitions. The definite parity in $a$ ensures that higher-order corrections will be quadratic, so our perturbation parameters are

$$\frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sim 0.030, \quad |U_{e3}|^2 \sim 0.022,$$

$$\left[ \frac{\Delta m^2_{21}}{a} \right]^2 \sim 0.12 \left( \frac{E}{\text{GeV}} \right)^2, \quad \left[ \frac{a}{\Delta m^2_{31}} \right]^2 \sim 0.008 \left( \frac{E}{\text{GeV}} \right)^2, \quad \left[ \frac{aL}{4E} \right]^2 \sim 0.084 \left( \frac{L}{1000 \text{km}} \right)^2,$$

where we used the mean value of the Earth mantle density [13] in $a$.

We perturbatively solve for the eigenvalues ($\tilde{M}^2$) and eigenstates ($\tilde{U}$) of the Hamiltonian (5) assuming constant matter density, which lead to the approximated expressions for the
The fact that the $\delta$-dependent term $A_{\mu e}^{\text{CPT}}$ is proportional to $\Delta_{21}$, i.e. to $1/E$, means that it is negligible at high enough energy, ensuring that the whole matter-induced component $A_{\mu e}^{\text{CPT}}$ is Hierarchy-odd. Our expressions show that the condition $|A_{-}^{\text{CPT}}| > |A_{+}^{\text{CPT}}|$ holds for energies $E > 1.1 E_{1\text{st node}}$ above the first node of the vacuum oscillation. At the T2HK baseline, this includes the whole energy spectrum; however, as seen in Fig. 1, the dominance of the genuine
component over the matter-induced one forbids the exploitation of this fact in a measurement of the observable CP asymmetry to test the Hierarchy.

At DUNE baseline, the matter-induced component is Hierarchy-odd at energies above 1.4 GeV, which correspond to the region in which it is larger than the genuine component. The measurement of the (sign of the) CP asymmetry at any of these points would thus determine the Hierarchy.

3.2. Direct observation of CP violation
The measurement of a non-vanishing value of the genuine component $A_{\mu e}^T$ (11) is a proof of CP violation in the lepton sector. To extract this information from the observable CP asymmetry, we suggest different strategies depending on the baseline.

At the medium baseline of T2HK, the matter-induced component is smaller than the genuine one. It is also hierarchy-odd and nearly $\delta$-independent. Therefore, once the Hierarchy is measured, this component can be subtracted from the experimental CP asymmetry as a theoretical background in order to obtain the value of the genuine component.

Due to its longer baseline, the matter-induced component dominates the CP asymmetry at DUNE in general. A measurement of the genuine component is thus only possible at those points where the matter-induced one vanishes. From the expression (10) for $A_{\mu e}^{\text{CPT}}$ we find a family of $\delta$-independent zeros given by

$$\tan \Delta_{31} = \Delta_{31},$$

at energies close to the oscillation maxima $\sin^2 \Delta_{31} = 1$ and the maximal values of the genuine component

$$\tan \Delta_{31} = -2\Delta_{31}.$$ (15)

The highest-energy solution of this family of zeros happens at $L/E = 1420$ km/GeV, i.e. $E = 0.92$ GeV at DUNE baseline. Notice that this result is Hierarchy-independent, as well as independent of all mixing angles; it comes from the condition (14) for $\Delta_{31}$, so it only depends on the value of $|\Delta m^2_{31}|$. We show in Figure 2 (left) a zoom of the DUNE spectrum in Figure 1 around this energy, where one can clearly see that the zero of the matter-induced component $A_{\mu e}^{\text{CPT}}$ is $\delta$-independent (the bands collapse to a line) and close to a maximal value of the genuine component $A_{\mu e}^T$.

The matter-induced component changes sign around the zero, so the average value of the component in a finite energy bin around the zero is still free from matter effects. Since the genuine component is close to maximal, this averaging has little effect on its value. We show these properties in Figure 2 (right), from which we find that a measurement of the CP asymmetry $A_{\mu e}^{\text{CPT}}$ around $E = 0.92$ GeV at DUNE baseline, with a bind width up to 200 MeV is free from matter effects: it is as clean a test of genuine CP violation as it would be in vacuum.

4. Conclusions
We present a Disentanglement Theorem able to separate, from symmetry principles, the genuine and matter-induced components of the CP asymmetry $A_{\alpha\beta}^{\text{CP}}$. From the definite parities of these components under T and CPT we find that the genuine component $A_{\alpha\beta}^T$ is an even function of the matter parameter $a$ and an odd function of $L$ and $\delta$, whereas the matter-induced component $A_{\alpha\beta}^{\text{CPT}}$ is odd in $a$ and even in $L$ and $\delta$.

We exploit these parities to build approximate analytical expressions of these components in the $\nu_\mu \rightarrow \nu_e$ flavor channel. A non-vanishing genuine component tests CP violation in the lepton sector, whereas only the matter-induced component changes sign under a change of Hierarchy. This Hierarchy dependence allows us to show only plots for Normal Hierarchy; the case of
Figure 2. Left: zoom of Figure 1 at DUNE baseline. Right: Averaged value of the disentangled components around \( E = 0.92 \text{ GeV} \) in an energy bin width \( \Delta E_{\text{bin}} \).

Inverted Hierarchy would show the same genuine component and a matter-induced component with opposite sign. We analyze the signatures of the disentangled components at the baselines of T2HK and DUNE.

At the medium baseline of T2HK, the matter-induced component is smaller than the genuine one. At the \( L/E \) reachable by the experiment, \( A_{\mu e}^{\text{CPT}} \) is nearly \( \delta \)-independent, so it can be subtracted from the experimental CP asymmetry provided the hierarchy is known. This would allow to measure the genuine component \( A_{\mu e}^{\text{T}} = A_{\mu e}^{\text{CP}} - A_{\mu e}^{\text{CPT}} \).

At the longer baseline of DUNE, both the Hierarchy and the genuine component are accessible. The whole CP asymmetry changes its sign under a change of Hierarchy at energies above 1.4 GeV, so it can be used to measure the mass ordering. On the other hand, the matter-induced component vanishes \( \delta \)-independently at \( E = 0.92 \text{ GeV} \)

\[
E = 0.92 \text{ GeV} \frac{L}{1300 \text{ km}} \frac{|\Delta m_{31}^2|}{2.5 \times 10^{-3} \text{ eV}^2},
\]

so the measurement of the CP asymmetry at this point is a direct test of CP violation in the lepton sector, uncontaminated by matter effects.

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