SUSY-QCD Corrections to Dark Matter Annihilation in the Higgs Funnel

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We compute the full \(\mathcal{O}(\alpha_s)\) SUSY-QCD corrections to dark matter annihilation in the Higgs-funnel, resuming potentially large \(\mu \tan\beta\) and \(A_b\) contributions and keeping all finite \(\mathcal{O}(m_b, s/1+\tan^2\beta)\) terms. We demonstrate numerically that these corrections strongly influence the extraction of SUSY mass parameters from cosmological data and must therefore be included in common analysis tools such as DarkSUSY or micrOMEGAs.

INTRODUCTION

Thanks to the recent mission of the WMAP satellite and other cosmological observations, the matter and energy decomposition of our Universe is known today with unprecedented precision [1]. Direct evidence for the existence of Cold Dark Matter (CDM) is accumulating [2][3], and its relic density \(\Omega_{\text{CDM}}\) can now be constrained to the rather narrow range [4]

\[
0.094 < \Omega_{\text{CDM}} h^2 < 0.136 \quad (1)
\]

at 95% (2\sigma) confidence level. Here, \(h\) denotes the present Hubble expansion rate \(H_0\) in units of 100 km s\(^{-1}\) Mpc\(^{-1}\).

Although the nature of Cold Dark Matter still remains unknown, it is likely to be composed of Weakly Interacting Massive Particles (WIMPs), as proposed by various extensions of the Standard Model (SM) of particle physics. In Supersymmetry (SUSY), a natural candidate is the Lightest Supersymmetric Particle (LSP), which is stable, if \(R\)-parity is conserved. It is usually the lightest of the four neutralinos, denoted \(\chi^0\) or shortly \(\chi\).

The Minimal Supersymmetric Standard Model (MSSM) depends a priori on 124 soft SUSY-breaking parameters, which are often restricted to five universal parameters that are imposed at the unification scale and can be constrained using data from high-energy colliders. As the lightest neutralino relic density depends also on these parameters, its computation is another powerful tool to put constraints on the parameter space and provide complementary information, in particular at high energies or masses that would otherwise not be accessible at colliders.

To evaluate the number density \(n\) of the relic particle with velocity \(v\), one has to solve the Boltzmann equation

\[
\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2) \quad (2)
\]

with the Hubble rate \(H\) and the thermal equilibrium density \(n_{\text{eq}}\). The present number density \(n_0\) is directly related to the relic density \(\Omega_{\text{CDM}} h^2 = m_\chi n_0 / \rho_c \propto \langle \sigma_{\text{eff}} v \rangle^{-1}\), where \(m_\chi\) is the LSP mass, \(\rho_c = 3H_0^2/(8\pi G_N)\) is the critical density of our Universe, and \(G_N\) is the gravitational constant [5]. The effective cross section \(\sigma_{\text{eff}}\) involves all annihilation and co-annihilation processes of the relic particle \(\chi\) into SM particles, and \(\langle \sigma_{\text{eff}} v \rangle\) in Eq. (2) signifies the thermal average of its non-relativistic expansion \((v \ll c)\).

The most important processes contributing to \(\sigma_{\text{eff}}\) are those that have two-particle final states and that occur at the tree level [6]. Possible final states include those with fermion-antifermion pairs as well as with combinations of gauge \((W^\pm, Z^0)\) and Higgs \((h^0, H^0, A^0, H^\pm)\) bosons, depending on the region of the parameter space. Processes producing fermions or antifermions may be detectable either directly or through their annihilation into photons. In addition, these channels are always open (for \(b\)-quarks if \(m_\chi \geq 4.5\) GeV) in contrast to the other channels, which may be suppressed or even closed [5][6].

Several public codes perform a calculation of the dark matter relic density within supersymmetric models. The most developed and most popular ones are DarkSUSY [7] and micrOMEGAs [8]. All relevant processes are implemented in these codes, but for most of them no (or at least not the full) higher order corrections are included. However, due to the large magnitude of the strong coupling constant, QCD and SUSY-QCD corrections are bound to affect the annihilation cross section in a significant way. They may even be enhanced logarithmically by kinematics or in certain regions of the parameter space.

In this Letter, we compute these corrections for neutralino-pair annihilation into a bottom quark-antiquark pair through the s-channel exchange of a pseudoscalar Higgs-boson \(A^0\). This process dominates in the so-called A-funnel region of minimal supergravity (mSUGRA) parameter space at large \(\tan\beta\), which is theoretically favored by the unification of Yukawa couplings in Grand Unified Theories (GUTs) [9]. Supposing a WIMP mass of 50-70 GeV, this process has also been claimed to be compatible with the gamma-ray excess observed in all sky directions by the EGRET satellite [10]. However, the corresponding scenarios may lead to antiproton overproduction, so that they would not be compatible with the observed antiproton flux [11].
ANALYTICAL RESULTS

Denoting the neutralino and b-quark velocities by

\[ \beta_h = \frac{v}{2} = \sqrt{1 - \frac{4m^2_h}{s}} \quad \text{and} \quad \beta_b = \sqrt{1 - \frac{4m^2_b}{s}} \]  

(3)

and the squared total center-of-mass energy by s, the properly antisymmetrized neutralino annihilation cross section can be written at leading order (LO) of perturbation theory as

\[ \sigma_{LO} v = \frac{1}{2} \frac{\beta_b}{8\pi s} \frac{N_C g^2 T_{A11}^2 h_{bb}^2 s^2}{(s - m_h^2 + i m_A \Gamma_A)^2}. \]  

(4)

It is proportional to the inverse of the flux factor sv, the integrated two-particle phase space sβ/s(8πs), the number of quark colors NC = 3 and the squares of the weak coupling constant g, a neutralino mixing factor

\[ T_{Aij} = \frac{1}{2} \left( N_{2j} - \tan \theta_W N_{1j} \right) \left( N_{4i} \cos \beta - N_{3j} \sin \beta \right) \]

(5)

the bottom-quark mass mb through the Yukawa coupling h_{bb} = g m_b tan β/(2mw), and the Higgs-boson propagator. Expanding in powers of v^2, we obtain \( \sigma_{LO} v \approx a_{LO} + b_{LO} v^2 + O(v^4) \) with

\[ a_{LO} = 2 b_{LO} = \frac{N_C g^2 T_{A11}^2 h_{bb}^2}{4\pi m_h^2} \left( \frac{m_b^2}{m_h^2} + \frac{1}{m_h^2} \right)^2 \left( 1 - \frac{m_b^2}{m_h^2} \right) \]  

(6)

in agreement with Ref. [4].

Using standard methods for the virtual one-loop and real emission contributions shown on the left-hand side of Fig. 1, we compute the O(\( \alpha_s \)) QCD correction in the on-shell scheme

\[ \Delta_{QCD}^{(1)} = \left( \frac{\alpha_s(s)}{\pi} \right) C_F \left[ 1 + \frac{\beta_b^2}{\beta_b} \left( 4 \log \frac{1 - \beta_b}{1 - \beta_b} (1 + \beta_b) - 2 \log \beta_b \log \frac{1 - \beta_b}{1 - \beta_b} \right) \right] \]

\[ -3 \log \frac{2}{1 + \beta_b} \log \frac{1 + \beta_b}{1 - \beta_b} - 2 \log \beta_b \log \frac{1 + \beta_b}{1 - \beta_b} \]

\[ -3 \log \frac{4}{1 - \beta_b^2} - 4 \log \beta_b + \frac{3}{8} (7 - \beta_b^2) \]

\[ + \frac{1}{16 \beta_b} \left( 19 + 2 \beta_b^2 + 3 \beta_b \right) \log \frac{1 + \beta_b}{1 - \beta_b} \]  

(7)

which agrees with the known result for pseudoscalar Higgs-boson decays [12] and contributes to the total correction

\[ \sigma = \sigma_{LO} \left[ 1 + \Delta_{QCD} + \Delta_{top} + \Delta_{SUSY} \right] \]  

(8)

and equivalently for \( \Gamma_A \). Here, \( \Delta_{QCD} = \left( \frac{\alpha_s}{\pi} \right) \Delta_{QCD}^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 \Delta_{QCD}^{(2)} + \left( \frac{\alpha_s}{\pi} \right)^3 \Delta_{QCD}^{(3)} + \ldots \)

In the limit \( m^2_b \ll s, (\beta_b \rightarrow 1) \), the correction in Eq. (7) develops a logarithmic mass singularity

\[ \Delta_{QCD}^{(1)} \approx \left( \frac{\alpha_s(s)}{\pi} \right) C_F \left[ -\frac{3}{2} \log \frac{s}{m_b^2} + \frac{9}{4} \right] \]  

(9)

which can be resummed to all orders using the renormalization group, i.e. by replacing \( m_b \) with the running mass \( m_b(s) \) in the Yukawa coupling \( h_{bb} \). The remaining finite QCD-corrections in the \( \overline{\text{MS}} \)-scheme are known up to \( O(\alpha_s^3) \) [14]

\[ \Delta_{QCD} = \left( \frac{\alpha_s(s)}{\pi} \right) C_F \left( 17 + \frac{9}{4} \right) (35.94 - 1.36 n_f) \]

\[ + \left( \frac{\alpha_s(s)}{\pi} \right)^3 (164.14 - 25.76 n_f + 0.259 n_f^2). \]  

(10)

A separately gauge-independent \( O(\alpha_s^2) \) correction is induced by the top-quark loop diagram shown on the upper right-hand side of Fig. 1. Its contribution [15]

\[ \Delta_{top} = \left( \frac{\alpha_s(s)}{\pi} \right)^2 \left( \frac{23}{6} \log \frac{s}{m_t} + \frac{1}{6} \log^2 \frac{m_b^2(s)}{s} \right) \]  

(11)

can be important at small values of tan β, but is largely suppressed in the Higgs-funnel region considered here.

In SUSY, additional \( O(\alpha_s) \) corrections arise through the sbottom-gluino exchanges shown in the central and lower right-hand side diagrams of Fig. 1. The self-energy diagram leads to the mass renormalization [9]

\[ \Delta m_b = \left( \frac{\alpha_s(s)}{\pi} \right) C_F \left( 17 + \frac{9}{4} \right) (35.94 - 1.36 n_f) \]

\[ \times \left( A_b - \mu \tan \beta \right) I(m^2_{b_1}, m^2_{b_2}, m^2_{b_3}), \]  

(12)

which is for \( m_b \ll m_{SUSY} \) proportional to

\[ \sin 2\theta_b = \frac{2 m_b (A_b - \mu \tan \beta)}{m^2_{b_1} - m^2_{b_2}}, \]  

(13)
i.e. the off-diagonal component of the sbottom mass matrix, and the 3-point function at zero external momentum

\[ I(a, b, c) = \frac{ab \log \frac{a}{b} + bc \log \frac{b}{c} + ca \log \frac{c}{a}}{(a - b)(b - c)(c - a)}. \]  

(14)

In this low-energy (LE) limit, and neglecting \( A_b \) with respect to the \( \tan \beta \)-enhanced \( \mu \), the vertex correction equals the mass renormalization \[16\] up to a factor \( 1/\tan^2 \beta \), so that the total SUSY correction becomes

\[ \Delta_{\text{SUSY}}^{(\text{LE})} = \left( \frac{\alpha_s(s)}{\pi} \right) C_F \left( 1 + \frac{1}{\tan^2 \beta} \right) m_3 \mu \tan \beta \]

\[ \times I(m_{b_1}^2, m_{b_2}^2, m_{b_3}^2). \]  

(15)

It has long been known that for large \( \tan \beta \), \( \Delta m_b \) can be significant and must be resummed by replacing \( m_b \to m_b/(1 + \lim_{A_b \to 0} \Delta m_b) \) in the Yukawa coupling \( h_{A_b} \). More recently it has been observed that \( A_b \) may be of similar size as \( \mu \tan \beta \), e.g., in no-mixing scenarios, so that its contribution must also be resummed by replacing \( \lim_{A_b \to 0} \Delta m_b \to (\lim_{A_b \to 0} \Delta m_b)/(1 + \lim_{\mu \tan \beta \to 0} \Delta m_b) \). Our result for the full SUSY-QCD correction \( \Delta_{\text{SUSY}} \) agrees with those in \[18, 19\], and we implement the finite \( O(m_b, s, 1/\tan^2 \beta) \) remainder as described in \[17\].

**NUMERICAL EVALUATION**

For our numerical study of the impact of QCD, top-quark loop, and SUSY-QCD corrections on dark matter annihilation in the Higgs-funnel, we place ourselves in a minimal supergravity (mSUGRA) scenario with \( A_0 = 0 \) and large \( \tan \beta = 44.5 \) (54) for \( \mu < 0 \) (\( \mu > 0 \)), which still allows for electroweak symmetry breaking (EWSB) in a large region of the scanned \( m_{1/2} - m_0 \) plane. The weak-scale MSSM parameters are then determined with **SPheno** \[20\], which includes resummed \( \lim_{A_b \to 0} \Delta m_b \) corrections, and the physical Higgs and SUSY masses with **FeynHiggs** \[21\] after imposing the current SM masses (in particular \( m_t(m_t) = 4.2 \text{ GeV} \) and \( m_t = 174.2 \text{ GeV} \)), gauge couplings (\( \alpha \) and \( \sin^2 \theta_W \) in the improved Born approximation, \( \alpha_4(M_Z) = 0.1176 \)), and direct and indirect SUSY mass limits \[22\]. For the impact of SUSY spectrum calculations on dark matter annihilation see \[23\].

Comparing the observed CDM relic density in Eq. 1 to the one calculated with **DarkSUSY** \[3\], which includes the QCD corrections up to \( O(\alpha_s^3) \) and where we have added the \( O(\alpha_s^3) \) QCD and \( O(\alpha_s) \) SUSY-QCD corrections described above, we determine the allowed regions in the \( m_{1/2} - m_0 \) plane shown in Fig. 2. The Higgs-funnel contribution to \( \sigma \text{eff} \) rises from 40% for low values of \( m_{1/2} \) (or \( m_0 \) for \( \mu < 0 \)) to more than 95%, when \( m_{1/2} \) (and \( m_0 \)) is (are) large. For \( m_0 = 1200 \text{ GeV} \) (dashed line), the condition \( (m_A - 2m_\chi)/\Gamma_A = 0 \) is, e.g., satisfied when \( m_{1/2} = 840 \) (570) GeV for \( \mu < 0 \) (\( \mu > 0 \)), where \( m_\chi \simeq 360 \) (420) GeV. It is obvious from Fig. 2 that the LO allowed regions (light) are dramatically changed by the \( O(\alpha_s^2) \) (medium) and \( O(\alpha_s^3) \) QCD and \( O(\alpha_s) \) SUSY-QCD corrections (dark), which reduce \( \sigma \text{eff} \) by more than a factor of two. The increase in \( \Omega_{\text{CDM}} h^2 \) must therefore be compensated by smaller masses. However, \( \Gamma_A \) is reduced by approximately the same amount, so that on the Higgs pole, where \( \Gamma_A \) is of particular importance, the effect is reversed. As expected, the effect is negligible in the focus point (very low \( m_{1/2} \)) and co-annihilation (very low \( m_0 \), see also \[24\]) regions.

In Fig. 3 we plot the relic density for \( m_0 = 1200 \text{ GeV} \) as a function of \( m_{1/2} \). Since \( \Gamma_A \) increases with \( m_{1/2} \), in particular for \( \mu > 0 \), \( \sigma \text{eff} \) (\( \Omega_{\text{CDM}} h^2 \)) reaches its maximum (minimum) at some distance from the pole (vertical dashed line). Further away, the (SUSY-)QCD corrections decrease (increase) \( \sigma \text{eff} \) (\( \Omega_{\text{CDM}} h^2 \)) as expected. Closer to the pole, the reduced width becomes important, so that the maximum (minimum) approaches the pole. The effect of the \( O(\alpha_s^2) \) QCD corrections already included in DarkSUSY and microMegas \[25, 26\] is considerably en-
hanced by our $\mathcal{O}(\alpha_s^3)$ QCD and SUSY-QCD corrections, while the top-quark loop contributes less than 0.01% for the large values of $\tan\beta$ under consideration here. Note that the dip at $m_{1/2} = 420$ GeV occurs when $m_\chi = m_t$, where the $tt$ annihilation channel is opened [27].

CONCLUSION

In summary, we have computed the full $\mathcal{O}(\alpha_s)$ SUSY-QCD corrections to dark matter annihilation in the Higgs-funnel, resumming potentially large $\mu \tan\beta$ and $A_0$ contributions and keeping all finite $\mathcal{O}(m_b, \mu, 1/\tan^2\beta)$ terms. We have demonstrated numerically that these corrections strongly influence the extraction of SUSY mass parameters from cosmological data and must therefore be included in common analysis tools like DarkSUSY or micrOMEGAs.

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