Promote cooperation by localised small-world communication

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The emergence and maintenance of cooperation within sizable groups of unrelated humans offer many challenges for our understanding. We propose that the humans’ capacity of communication, such as how many and how far away the fellows one can build up mutual communication, may affect the evolution of cooperation. We study this issue by means of the public goods game (PGG) with a two-layered network of contacts. Players obtain payoffs from five-person public goods interactions on a square lattice (the interaction layer). Also, they update strategies after communicating with neighbours in learning layer, where two players build up mutual communication with a power law probability depending on their spatial distance. Our simulation results indicate that the evolution of cooperation is indeed sensitive to how players choose others to communicate with, including the amount as well as the locations. The tendency of localised communication is proved to be a new mechanism to promote cooperation.

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I. INTRODUCTION

In behavioral sciences, evolutionary biology and more recently in economics, understanding conditions for the emergence and maintenance of cooperative behavior among unrelated and selfish individuals becomes a central issue 1 2. In the investigation of this problem, the most popular framework is game theory together with its extensions involving evolutionary context 3 4. The public goods game (PGG), which attracted much attention from economists, is a general paradigm to explain cooperative behavior through group interactions 5. The PGG model is characterised by groups of cooperators doing better than groups of defectors, but defectors always outperforming the cooperators in their group. In typical examples, the individual contributions are multiplied by a factor r and then divided equally among all players. With r smaller than the group size, this is an example of a social dilemma: every individual player is better off defecting than cooperating, no matter what the other players do. Groups would therefore consist of defectors only and forego the public good.

Considerable efforts have been concentrated on exploration of the origin and persistence of cooperation. During the last decades, five rules, namely, kin selection 6, direct reciprocity 7 8, indirect reciprocity 9 10, network (or spatial) reciprocity 11 12 13 14 15, and group selection 17, have been found to benefit the evolution of cooperation in biological and ecological systems as well as within human societies (for a review, see 18). In realistic systems, most interactions among elements are spatially localised, which makes spatial or graph models more meaningful. Unlike the other four rules, spatial games (i.e., network reciprocity) can lead to cooperative behavior in the absence of any strategic complexity 11 12 13 19. In spatial evolutionary PGG, the cooperators can survive by forming compact clusters, which minimizes the exploitation by defectors 13. Furthermore, Szabó and Hauert et al. have recently discussed the effects of compulsory and voluntary interactions of players in evolutionary PGG, with the structured populations bound to regular lattices 15 16 20, as well as with the well-mixed population 16 21 22 23. The factors such as the voluntary participation 20, and small density of population 22, are found to be capable of boosting cooperation. More recently, Huang et al. have proposed an extended public goods interaction model to study the evolution of cooperation in heterogeneous population, and proved that scale-free networks of contacts can lead to more competitive cooperation 24.

In real world, people always wish to make decisions based on a comprehensive knowledge of the pertinent background, such as the historical performance of each alternative choice 25, which they may obtain by learning from people they are regarding. However, the limited eyereach or capacity of humans actually exists and induces the unperfect communication. One can merely learn from a small set of people (corresponding to the so-called “role models” in the context of cultural evolution 16 26). Furthermore, these role models rarely can be extended over the whole population, but localised to the learner’s “vicinity”, which abstractly implies the people having similar characteristics as the learner (such as religious background, age, education, lifestyle, or social class) that in favour of building up mutual communication.

As a natural extension of those aforementioned factors, an new intriguing task is to understand how the limitation of communication capacity of people influences the cooperative behavior in real world. In this letter, we will study a spatially extended PGG on two-layered graphs, where one layer especially depicts the communi-
culation among players, with the aim to find out how the evolution of cooperation depends on the players' communications.

II. THE MODEL

In spatial game models, the players occupying the sites of a graph can follow one of the two pure strategies, cooperation (C) or defection (D). There are two types of contacts among players in the evolutionary process: players collect payoffs from their neighbours by playing games with them [12, 13, 14, 15, 19, 20], and then update strategies by learning from neighbours. Thus, the graph occupied by the players can be split into two layers, an interaction layer and a learning (communication) layer [26, 27, 28]. The former one defines the interaction neighbourhood (IN), i.e., who plays game with whom. The latter one specifies learning neighbourhood (LN) for evolutionary updating or, in other words, defines who-is-the-role-model-of-whom.

In our model, the IN layer where players have public goods interactions is a \( L \times L \) square lattice with periodic boundary conditions. The group size of the interaction neighbours in the PGG therefore is \( N_I = 5 \). The score achieved in PGG interactions denotes the reproductive success, i.e., the probability that other players will adopt the player’s strategy. This score is assumed to be determined merely by a single, typical PGG involving the player and its four nearest interaction neighbours [15]. Thus the payoff of one player \( i \) is,

\[
P(i) = \begin{cases} 
\frac{r}{n_C + n_D} - 1 & \text{if } s(i) = C, \\
\frac{r}{n_C} & \text{if } s(i) = D,
\end{cases}
\]

where \( n_C \) and \( n_D \) (with \( n_C + n_D = N_I = 5 \)) denote, respectively, the number of participants choosing \( C \), and \( D \), and \( s(i) \) denotes the strategy of player \( i \). The cooperative investments are normalised to unity and \( r \) specifies the multiplication factor on the public goods.

After each round of the game interactions, for the reference of the strategy update, each player communicates with its role models (neighbours in the LN layer), inquiring the individual information such as the payoff and the strategy. The player’s capacity of communication can be measured by how many and how far away the role models it can selected. In order to regulate this capacity, we introduce the LN layer as a variant of the two-dimensional small-world network [23], in which connections to further neighbours occur with a tunable power law probability. This network is constructed by adding shortcuts among the sites on a \( L \times L \) square lattice. With periodic boundary condition, the lattice distance between two sites \( (x, y) \) and \((x', y')\) can be written in a two-dimensional fashion as

\[
r_{(x,y),(x',y')} = |\Delta x| + |\Delta y|,
\]

with

\[
\Delta x = L/2 - \dfrac{1}{2}(|x - x'| - L/2) \\
\Delta y = L/2 - \dfrac{1}{2}(|y - y'| - L/2)
\]

This value is actually the length of the shortest path connecting these two sites through only lattice links. Each site is additionally linked to \( q \) other sites [30] by shortcuts (excluding its original nearest neighbours). Following the idea of Kleinberg [29], those other sites are selected in a biased manner: the probability that site \( j(x_j, y_j) \) is selected to be linked to site \( i(x_i, y_i) \) by one shortcut depends on the lattice distance between them in the following way,

\[
P(r_{(x_i, y_i),(x_j, y_j)} = 1) = \frac{1}{A} r_{(x_i, y_i),(x_j, y_j)}^{-\alpha},
\]

where \( \alpha \) is a positive exponent and

\[
A = \sum_{(x', y') \neq (x_i, y_i), (x_i, y_i) \neq (x_j, y_j)} r_{(x, y),(x', y')}^{-\alpha},
\]

is a normalization factor. Obviously, the probability that \( i \) and \( j \) are connected is

\[
R_{i \leftrightarrow j} = 1 - \left(1 - \frac{r_{(x_i, y_i),(x_j, y_j)}^{-\alpha}}{A}\right)^{2q}.
\]

For this network, when \( \alpha = 0 \), it reduces to the small-world model with random shortcuts [31]. As \( \alpha \) increases, one site’s shortcuts will be clustered in its vicinity and two distant sites are less likely to be connected. Here, the multiple or self-connected edges are not allowed. Such a network has \( qN \) shortcuts and an average degree \( \langle k \rangle \approx 4 + 2q \). Figure 1 shows the average lattice length of the shortcuts \( l \), and the clustering coefficient \( E \) of this kind graph. We clearly notice the increase of \( E \) with \( \alpha \) by reason of the clustered shortcuts.

In this paper, we just consider this kind network as the LN of the game players where merely the information flow takes place. In this LN network, the neighbours linked by one edge act as the role model of each other via mutual communication, and the degree of each site corresponds to the amount of role models each player has. For the sake of convenience, we call those role models
FIG. 2: (a1), (a2), and (a3) show the stationary densities of cooperators \( \rho_c \) as a function of multiplication factor \( r \) for \( \alpha = 0, 1, 2, \) and 5 denoted by different symbols. (b1), (b2), and (b3) plot the corresponding phase diagrams. The two bold solid lines shows the average \( r_c \) (lower) and \( r_d \) (upper). The straight dot line in the middle is \( r = 5.0 \). The three dash lines from bottom to top present the contours \( \rho_c = 0.25, 0.5, \) and 0.75, respectively. The red dash-dot lines in (b1) and (b2) show the cross points of those \( \rho_c \) curves of \( \alpha \neq 0 \) with the curves of \( \alpha = 0 \). (c1), (c2), and (c3) show the strategy update rate \( R \) in the dynamical equilibrium state as a function of \( \rho_c \). The results are averaged over 10 realizations of the \( N = 51^2 \) systems with \( \alpha = 0 \) (left), 2 (middle), and 5 (right).

introduced by shortcuts the additional role models (shortly ARMs). Thus, each player has 2\( q \) ARMs on average. Also, the biased effect of nonzero \( \alpha \), giving rise to the clustering of shortcuts, can be understood as the “localization” of players’ communication.

Following previous studies \cite{15, 19}, the evolution of the present system is governed by random sequential strategy adoptions, that is, the randomly chosen player \( i \) adopts one of its (randomly chosen) role model \( j \)'s strategy with a probability depending on the payoff difference

\[
W[s(j) \rightarrow s(i)] = \frac{1}{1 + \exp\{[P(i) - P(j) + \tau]/\kappa\}}, \quad (5)
\]

where \( \tau > 0 \) denotes the cost of strategy change, and \( \kappa \) characterises the noise introduced to permit irrational choices. For \( \kappa = 0 \) the neighbouring strategy \( s(j) \) is adopted deterministically provided the payoff difference exceeds the cost of strategy change, i.e., \( P(j) - P(i) > \tau \). For \( \kappa > 0 \), strategies performing worse are also adopted with a certain probability, e.g., due to imperfect information. It is proved that the dynamics remains unaffected qualitatively when changing \( \kappa \) and \( \tau \) within realistic limits. Following the previous work \cite{15}, we simply fix the value of \( \kappa \) and \( \tau \) to be 0.1, and concentrate on the general dynamical properties affected by the structure of LN.

III. SIMULATION RESULTS

We study above model by Monte-Carlo (MC) simulations started from a random initial distribution of C and D strategies. After appropriate relaxation times, the system can converge to a dynamical equilibrium state. We characterise this state by the stationary density of cooperators \( \rho_c \), averaged over the last 5000 MC steps of the 25000 total sampling steps. Figures\[2\] (a1), (a2), and (a3) show the dependence of stationary density of cooperators \( \rho_c \) on the multiplication factor \( r \) for the systems with \( \alpha = 0, 2, \) and 5, respectively. The simulation data result from an average over either ten realizations of independent initial strategy distributions or ten realizations of the LN networks. For \( q = 0 \), i.e., when the IN and LN are identical, we recover the results of ref. \cite{15}: below the threshold value \( r < r_c = 4.526(1) \) cooperators quickly vanish (the absorbing homogenous state with all defectors), whereas for high \( r > r_d \) defectors go extinct (the absorbing homogenous state with all cooperators); for intermediate \( r \), strategies C and D coexist in dynamical equilibrium. The \( r_c \) (\( r_d \)) indicates the value of \( r \) where cooperators (defectors) vanish. In the case that \( q \neq 0 \), i.e., when each player can communicate with more ARMs via shortcuts, the quantitative properties of the stationary density \( \rho_c \) are different [see the curves with different \( q \) in figs.\[2\] (a1), (a2) and (a3)].
One can find from figs. [2](a1), and (a2) that, each of the $\rho_c$ curve with $q \neq 0$ has a cross point (denoted by $r_{\text{cross}}$) with the curve of $q = 0$. The density of cooperators $\rho_c$ for the system with larger $q$ is comparatively smaller at $r < r_{\text{cross}}$ region, but larger $\rho_c$ at $r > r_{\text{cross}}$ region. That is to say, for the systems with $\alpha = 0$ and 2, the more available role models will favour cooperators when $r$ is large, and favour defectors in contrast when $r$ is small. However, the system with $\alpha = 5$ [figs. 2(a3)] is obviously different from that of $\alpha = 0$ and 2, that is, the ARMs can favour cooperators in the whole range of $r$ with respect to the $q = 0$ system, moreover, defectors vanish at much lower $r$.

We plot the phase diagrams of the corresponding systems in figs. [2](b1), (b2), and (b3), respectively. The two bold solid curves respectively show the $r_c$ and $r_d$ averaged over 10 realizations, which divide the region into absorbing homogenous states of C (upper) and D (lower), as well as the coexistence regime of the two strategies (intermediate). That is, for fixed number of ARMs, when varying the factor $r$, two phase transitions occur between the coexistence regime and homogeneous states of C or D. In the coexistence regime, the three dash lines present the contours $\rho_c = 0.25, 0.5,$ and 0.75, respectively. The straight dot line in the middle corresponds to $r = 5.0$, larger than which the social dilemma raised by the PGG will be relaxed in the sense that each unity investment has a positive net return. As $q = 0$ defectors may exist in the system and exploit cooperators until $r$ is very larger. However, from the rapid decline of $r_d$ with increasing $q$ we know that, with the aid of the ARMs, defectors vanish at much smaller $r$. More, interestingly, even if $r < 5.0$, defectors can be eliminated as long as $q > 3.49$ for $\alpha = 0$, $q > 2.42$ for $\alpha = 2$, or $q > 1.41$ for $\alpha = 5$. Also, the region of homogenous C becomes wider as the value of $\alpha$ increases.

The cross points $r_{\text{cross}}$ of those $\rho_c$ curves of $q \neq 0$ with the curve of $q = 0$ in fig. [2](a1) [and (a2)] are marked by the (red) dash-dot line in the phase diagram. The coexistence regime of C and D thus is separated into two regions by this line, with ARMs favouring cooperation in the upper region, but favouring defection in the lower region. It can be seen that, the cross points $r_{\text{cross}}$ is not sensitive to the parameter $q$. For the system with $\alpha = 2$ the upper region favouring cooperation is comparatively wider than that with $\alpha = 0$. Moreover, for the system with $\alpha = 5$ [see figs. 2(a3) and (b3)], those $\rho_c$ curves are not intersectant, and the ARMs favour cooperators in the whole range of $r$. The curves of $r_c$, $r_d$, and other contours of $\rho_c$ for various $\alpha$ systems are proved to asymptotically approach $r = 5.0$ with increasing $q$, and finally collapse into one there ($r = 5.0$) for $q \rightarrow \infty$, which implies that each player can learn from the whole population. That is to say, as a result of the dynamical process the system with $q \rightarrow \infty$ will end up in the absorbing D state for any $r$ smaller than 5.0, where the first-order phase transition from D to C takes place.

We also notice that the ARMs may affect the severity of the competition between C and D, which is characterised by the rate of the effective strategy update $R$ (the fraction of players who adopt the opposite strategy in each generation averaged over 1000 MC steps). For the sake of comparison, the rate $R$ are plotted as a function of $\rho_c$ in figs. [2](c1), (c2), and (c3) for the systems with $\alpha = 0$, 2, and 5, respectively. For a given stationary density $\rho_c$, the system with larger $q$ results in larger rate $R$, which implies that more ARMs induce more intense competition. This is due to the additional mutual contacts between C and D, or, in other words the larger surface between C and D in the LN network, induced by the additional shortcuts. Also, $R$ exhibits a bell-like form, because the surface between C and D will shrink with the density difference of the two strategies. Additionally, for a given value of $q$, the system with larger $\alpha$ are found to result in smaller rate $R$.

Figure 3 shows the snapshots of the system with identical IN and LN (left hand), as well as the system with $q = 2$ and $\alpha = 2$ LN (right hand), respectively. For the case of identical IN and LN [fig. 3(a)(c)], it is known from PGG and other cooperation games that, cooperators persist through forming compact C clusters and thereby reducing exploitation by defectors [12, 13, 19, 24, 25]. However, for the availability of ARMs [fig. 3(b)(d)], the C clusters break into smaller pieces. Furthermore, one interesting phenomenon distinctly exhibited in fig. 3(d) is, some rare cooperators may take place at the sites totally surrounded by defectors, where it is extremely difficult for C to survive because of the very low payoffs. These sites are found to be connected by shortcuts to the well-paid C role models who are located in the C clusters. Therefore, we can say that, the C strategy is adopted there when players blindly imitate the successful role models without regard to their own actual
cuts are clustered by larger and survive by forming clusters. Thus, when the short-W e have known that, the cooperators protect each other ation is favoured more compared to the case with which implies that the LN is distinctly localised, cooper-

ation. Furthermore, the best communication strategy for-"i.e. corresponds to the extremely localised selection of ARMs, 

sion of D, although the rare C therein are unstable and changing slowly with.

FIG. 4: Stationary densities of cooperators ρc as a function of α with r ranges from 4.5 to 5.05. The results are averaged over 10 realizations of the system with N = 201² and q = 2.

‘habitats’. This may correspond to the phenomenon in society that, some people follow like sheep to the manner of others ignoring the question whether it fits into their own social surrounding. Interestingly, this unreasonable behavior simply introduces mutations to the population of D. We therefore suggest that, it may provide one mechanism to favour cooperation, for the invasions of C may form mutual protection and thus survive in the population of D, although the rare C therein are unstable and hard to propagate by themselves. In addition, these sites exhibit one path to put up the fluctuating individuals [? ], who alternatively spend some time as cooperators and some time as defectors.

Finally, we focus our attention on how the localised communication affects the evolution of cooperation. The stationary density of cooperators ρc at different values of r are plotted as a function of α in fig. [1]. We can see that the values of ρc keep almost invariable in the region 0 ≤ α < 1. This can be naturally understood from the results in fig. [1] as well as the analysis in the ref. [32, 33, 34] that, in this region the LN network behaves as a small world with the topological properties changing slowly with α. However, in the large α region, which implies that the LN is distinctly localised, cooperation is favoured more compared to the case with α = 0.

We have known that, the cooperators protect each other and survive by forming clusters. Thus, when the shortcuts are clustered by larger α, one cooperator is more likely to select role models within its own C cluster via shortcuts. That is to say, the mutual protection among cooperators is enhanced by large α, and thereby cooperation is favoured. This mechanism is obviously reflected by the monotonic increase of ρc with α at large α region. Furthermore, the best communication strategy for players to promote cooperation is α → ∞, which corresponds to the extremely localised selection of ARMs, i.e., the LN network with the shortcuts merely connects the next-nearest neighbours through lattice links.

It is also notable in fig. [1] that, the ρc monotonically increases with α when r is comparatively large (about r > 4.6), however, the nonmonotonic behaviors of ρc occur for the cases of smaller r. This result is proved to remain unaffected qualitatively for different values of q within realistic limits. As it is well known, the cooperators located along the boundary of the C clusters (the so-called boundary C in the following text) outweigh the losses against defectors by gains from interactions within the C cluster [14, 19]. These boundary C gain low payoffs and thus would be sensitive to the introduction of the ARMs, i.e., what kind of role models they meet with via shortcuts really matters. The system at the coexistence state is composed of cooperators as well as two kinds of defectors, the defector located right at the boundary of C cluster which we named D1, and the defec-

tor surrounded by the same-strategy interaction neighbours which we named D2. One D1 would accumulate comparatively higher payoff by exploiting the boundary C, and then act as an attractive role model of its learning neighbours. However, one D2 gains zero payoff, and thus will not affect the stability of others’ strategy. They put up fluctuating individuals if they are linked to well-paid C by shortcuts, as that discussed above. In our model, the players will choose ARMs nearer and nearer to its vicinity as α increases from 0. When α is around 2.5, the lattice lengths of these shortcuts (see fig. [1]) approximately reach the general size of the C clusters, then the probability for the boundary C to meet with the well-paid D1 players becomes larger, which would pose a high risk to the stability of the boundary C. The C cluster becomes constricted and thus cooperation is depressed when the boundary C imitates D1. This effect can be clearly observed from the comparatively smaller ρc around α = 2.5 at small r (see fig. [3]). However, when the value of r is large, this effect is weakened by the C clusters highly crowded [see fig. [3]b].

IV. CONCLUSION

In summary, we have shown how the communication among players may affect the evolution of cooperation by means of a two-layered PGG model. The learning layer is constructed as a Kleinberg small-world network, where two players can communicate with probability depending on their spatial Euclidean lattice distance in the power-law form controlled by an exponent α. The players’ capacity of communication is characterised by the number and the distance of the role models from whom they can learn the individual information. The biased effect of nonzero α, which gives rise to the preferential selection of role models near the vicinity, corresponds to the localization of players’ communication induced by limited capacity. Our simulation results indicate that, the communication among players plays a highly important role in the evolution of cooperation: the density of C is crucially influenced by the number (q) and the location of the ARMs; the coexistence region of C and D is reduced and rc (r_d) is expected to tend to 5 (the group
size $N_f$) if $q$ increases; in addition, for certain density of C, more available ARMs result in more intense competition between C and D. Moreover, the communication via shortcuts (i.e. the ARMs) are found to introduce mutation to the population of D, which might be helpful to the emergence of C. It is also notable that, the localised communication with large $\alpha$ can favour cooperators.

Our model gives a crude simulation of real social behavior. However, it does catch a few features of potential interest. The most interesting feature is the localised communication favouring cooperation. We suggest that, the limitation of humans’ capacity to build up communication system, which results in the localised selection of role models, would be a new mechanism supporting the emergence and persistence of cooperation in society. The other feature is that a population of cheaters would be invaded by cooperators by the presence of long-range social connections.

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