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Quartic monoid surfaces with maximum number of lines. (English) [Zbl 1470.14070]
J. Symb. Comput. 109, 250-258 (2022).

Summary: In 1884 the German mathematician K. Rohn published a substantial paper [Math. Ann. 24, 55–152 (1883; JFM 16.0705.01)] on the properties of quartic surfaces with triple points, proving (among many other things) that the maximum number of lines contained in a quartic monoid surface is 31. In this paper we study in details this class of surfaces. We prove that there exists an open subset \( A \subseteq \mathbb{P}^4_K \) (\( K \) is a characteristic zero field) that parametrizes (up to a projectivity) all the quartic monoid surfaces with 31 lines; then we study the action of \( \text{PGL}(4,K) \) on these surfaces, we show that the stabiliser of each of them is a group isomorphic to \( S_3 \) except for one surface of the family, whose stabiliser is a group isomorphic to \( S_3 \times C_3 \). Finally, given two quartic surfaces \( Q(a) \) and \( Q(b) \), with \( a, b \in A \), we show that \( Q(a) \) and \( Q(b) \) are projectively equivalent if and only if \( j(a) = j(b) \), where \( j \) is the \( j \)-function.

To get our results, several computational tools, available in computer algebra systems, are used.

MSC:
14J28 K3 surfaces and Enriques surfaces
14J26 Rational and ruled surfaces
14J17 Singularities of surfaces or higher-dimensional varieties
14Q10 Computational aspects of algebraic surfaces

Keywords:
quartic monoid surface; algebraic surface; incidence structure; constructive geometry; \( j \)-function

Software:
SageMath; CoCoA

Full Text: DOI arXiv

References:
[1] Abbott, J.; Bigatti, A. M.; Robbiano, L., CoCoA: a system for doing computations in commutative algebra, available at
[2] Beltrametti, M. C.; Carletti, E.; Gallarati, D.; Monti Bragadin, D., Lectures on Curves, Surfaces and Projective Varieties—a Classical View of Algebraic Geometry, Textbooks in Mathematics, vol. 9 (2009), European Mathematical Society: European Mathematical Society Zurich, Translated by F. Sullivan · Zbl 1180.14001
[3] Conforto, F., Le superficie razionali (1939), Zanichelli · Zbl 65.0714.03
[4] Degtyarev, A. I., Classification of quartic surfaces that have a nonsimple singular point, Math. USSR, Izv., 35, 3, 607-627 (1990) · Zbl 0722.14019
[5] Essop, C. M., Quartic Surfaces with Singular Points (1916), Cambridge Univ., Press
[6] Gonzalez-Alonso, V.; Rams, S., Counting lines on quartic surfaces, Taiwan. J. Math., 20, 4, 769-785 (2016) · Zbl 1357.14052
[7] Harris, J., Algebraic Geometry, a First Course (1992), Springer-Verlag · Zbl 0779.14001
[8] Johansen, P. H.; Loberg, M.; Piene, R., Monoid Hypersurfaces, 55-77 (2008), Springer, ISBN 978-3-540-72184-0/bbk · Zbl 1140.14049
[9] Logar, A., Torrente, M., 2020. Complete intersection of cubic and quartic curves and a classification of the configurations of lines of quartic monoid surfaces, in preparation.
[10] Polo-Blanco, I.; van der Put, M.; Top, J., Ruled quartic surfaces, models and classification, Geom. Dedic., 150, 151-180 (2011) · Zbl 1226.14048
[11] Rohn, K., Ueber die flächen vierter Ordnung mit dreifachem Punkte, Math. Ann., 24, 55-151 (1884) · Zbl 16.0705.01
[12] Stein, W. A., Sage Mathematics Software (Version 6.7). The Sage Development Team (2015)
[13] Takahashi, T.; Watanabe, K.; Higuchi, T., On the classification of quartic surfaces with triple point. I, Sci. Rep. Yokohama Natl. Univ., 29, 47-70 (1982)
[14] Takahashi, T.; Watanabe, K.; Higuchi, T., On the classification of quartic surfaces with triple point. II, Sci. Rep. Yokohama Natl. Univ., 29, 71-94 (1982)
