Gauged $B - x_iL$ origin of $R$ parity and its implications

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Gauged $B - L$ is a popular candidate for the origin of the conservation of $R$ parity, i.e. $R = (-1)^{3B+L+2j}$, in supersymmetry, but it fails to forbid the effective dimension-five terms arising from the superfield combinations $QQQL$, $u^c d^c e^c$, and $u^c d^c N^c$, which allow the proton to decay. Changing it to $B - x_i L$, where $x_e + x_\mu + x_\tau = 3$ (with $x_i \neq 1$) for the three families, would forbid these terms while still serving as a gauge origin of $R$ parity. We show how this is achieved in two minimal models with realistic neutrino mass matrices, and discuss their phenomenological implications.

Introduction

In the Minimal Supersymmetric Standard Model (MSSM) of particle interactions, the imposition of $R$ parity, i.e. $R = (-1)^{3B+L+2j}$, where $B$, $L$, and $j$ stand for baryon number, lepton number, and spin, respectively, serves two purposes. One is to establish a candidate for the dark-matter of the Universe, because the lightest supersymmetric particle (LSP) is absolutely stable, being odd under $R$. The other is to forbid the otherwise allowed renormalizable superfield terms $L H_u$, $L L e^c$, $L Q d^c$, and $u^c d^c e^c$, so that the proton does not decay as a result of these interactions. However, the higher-dimensional quadrilinear terms $QQQL$ and $u^c d^c e^c$ are still allowed, giving rise thus to effective dimension-five terms in the Lagrangian which would also induce fast proton decay. This is a serious problem for grand unification, even at a scale as high as $10^{16}$ GeV.

With the addition of three neutral singlet superfields $N^c$, desirable for neutrino masses, gauged $B - L$ becomes possible. In that case, $R$ parity is better understood theoretically as a discrete remnant of $B - L$ breaking. However, the offending quadrilinear terms remain, together with the new term $u^c d^c e^c$. To get rid of these terms, we propose a simple solution. Instead of gauged $B - L$, we adopt a flavor-dependent variation, i.e. $B - x_i L$, where the usual leptons still have $L = 1$, but $x_{e,\mu,\tau}$ for the three families are not equal to 1. In fact, the idea that there is just one $N^c$ and that $x_{e,\mu,\tau} = (3, 0, 0)$ or $(0, 3, 0)$ or $(0, 0, 3)$ has already been explored in the past. If $x_i \neq 1$, then all the unwanted quadrilinear terms are forbidden by gauged $B - x_i L$ so that proton stability is assured as shown for $(0, 0, 3)$ case in Ref. 3.

Here we assume that there are three $N^c$, with each $x_i$ nonzero. Unlike the previous $(0, 0, 3)$ type of models, this opens up a new possibility that the spontaneous breaking of $B - x_i L$ by suitably chosen Higgs singlet superfields $S_{1, 2}$ will result in an exact discrete residual symmetry which is just the usual $R$ parity. In the following, we will discuss the conditions on $x_i$ and construct two minimal models with realistic neutrino mass matrices. We will examine their phenomenological constraints and the prognosis of their verification at the Large Hadron Collider (LHC).

Model

Our minimal model consists of the usual particle content of the MSSM plus three singlet superfields $N^c$ with $L = -1$ and two singlet superfields $S_{1, 2}$ with $L = \pm 2$. Under the gauged $U(1)_X$ symmetry of $B - x_i L$, quarks have charges $1/3$ and leptons have charges $-x_i$. The two usual Higgs doublet superfields have charges 0 and the Higgs singlets $S_{1, 2}$ have charges $\mp 2 x_S$. The various $x_i$ and $x_S$ will be determined by the requirements of anomaly cancellation and a realistic neutrino mass matrix.

The anomaly-free conditions for the addition of $U(1)_X$ to the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group are easily written down. The $[SU(3)_C]^2 U(1)_X$ and $[U(1)_Y]^3$ anomalies are automatically zero because of the vectorial nature of $SU(3)_C$ and $U(1)_X$. The $[\text{gravity}]^2 U(1)_Y$ anomaly also vanishes for the same reason. The $[U(1)_X]^2 U(1)_Y$ anomaly is zero because the sum of $Y$ charges is zero separately for each family of quarks and for each family of leptons. The remaining two conditions, i.e. $[SU(2)_L]^2 U(1)_X$ and $[U(1)_Y]^2 U(1)_X$, are given respectively by

\begin{align}
(3)(3)(1/3) - x_e - x_\mu - x_\tau &= 0, \\
(3)(3)[2(1/6)^2 - (2/3)^2 - (-1/3)^2](1/3) + [2(-1/2)^2 - (-1)^2](-x_e - x_\mu - x_\tau) &= 0.
\end{align}

Both are satisfied if

\begin{equation}
x_e + x_\mu + x_\tau = 3.
\end{equation}

The usual $B - L$ is recovered for $x_e = x_\mu = x_\tau = 1$, and $B - 3L_e$ is obtained for $x_e = x_\mu = 0$ and $x_\tau = 3$. 

As $S_{1,2}$ acquire vacuum expectation values, $U(1)_X$ is broken. $(x_e, x_\mu, x_\tau)$ as well as $x_S$ will be chosen in such a way that a residual symmetry of $B - x_i L$ will remain which is exactly $R$ parity, and that realistic neutrino masses and mixing are obtained via the canonical seesaw mechanism. We note that since the Higgs doublets do not transform under $U(1)_X$ and $S_{1,2}$ do not transform under $SU(3)_C \times SU(2)_L \times U(1)_Y$, the resulting $Z$ and $Z'_X$ bosons do not mix. This avoids the stringent constraint from precision electroweak measurements at the $Z$ resonance. (See Ref. [7] and references therein.)

To obtain $R$ parity (equivalently, matter parity) as a total residual symmetry from the breaking of $B - x_i L$ using $S_{1,2}$, these new singlet superfields (with $L = \pm 2$ and $B = 0$) should satisfy $|x_S| = 1/3$. Since $B = 1/3$ for quarks and $L = 1$ for leptons, we obtain the usual definition of $R$ parity for all particles if $3x_{e,\mu,\tau}$ are odd integers. For a detailed discussion about conditions to get a $Z_N$ out of a $U(1)$ gauge symmetry, see Ref. [8] and references therein.

Flavor-dependent $U(1)$ models have been widely studied to address many issues (e.g., see Ref. [9]). Our idea of having a particular discrete symmetry out of a flavor-dependent $U(1)$ gauge symmetry can be viewed as a useful guide in constraining such models.

**Neutrino sector**

The general requirements discussed above would still allow an infinite number of possible models, until realistic neutrino masses and mixing are considered.

If all $x_i$ are different for the three families of leptons, the charged lepton mass matrix and the Dirac mass matrix linking $\nu_i$ with $N_j^c$ are both constrained to be diagonal. To obtain mixing among the three neutrinos, the $3 \times 3$ Majorana matrix spanning $N_j^c$ must have enough nonzero entries. However, the only sources of such terms are $S_{1,2}N_j^cN_k^c$ and $N_j^cN_k^c$ if $x_j + x_k = 0$. With three different $x_{e,\mu,\tau}$ and just $\pm x_S$ to work with, this is clearly impossible.

We now assume that $x_\mu = x_\tau$, then some simple algebra will show that there are only two solutions (recalling that $S_{1,2}$ have $L = \pm 2$):

(I)  \[ x_S = -x_e = x_\mu, \]  \hspace{1cm} (4)

(II) \[ x_S = -x_\mu = (x_e + x_\mu)/2. \]  \hspace{1cm} (5)

Together with Eq. (3), this means that

(I) \[ x_{e,\mu,\tau} = (3, 3, 3), \hspace{0.5cm} x_S = 3, \]  \hspace{1cm} (6)

(II) \[ x_{e,\mu,\tau} = (9, -3, -3), \hspace{0.5cm} x_S = 3. \]  \hspace{1cm} (7)

In model (I), the $3 \times 3$ Majorana mass matrix spanning $N_j^c$ has all nonzero entries: $N_{e,\mu,\tau}^cN_{e,\mu,\tau}^c$ are invariant mass terms, $N_{e,\mu,\tau}^cN_{e,\mu,\tau}^c$ come from $\langle S_2 \rangle$, and $N_{e,\mu,\tau}^cN_{e,\mu,\tau}^c$ come from $\langle S_1 \rangle$. In model (II), the $N_{\mu,\tau}^cN_{\mu,\tau}^c$ entry is zero, but all others are nonzero: $N_{\mu,\tau}^cN_{e,\mu,\tau}^c$ come from $\langle S_2 \rangle$, and $N_{e,\tau}^cN_{\mu,\tau}^c$ from $\langle S_1 \rangle$. Both are general enough for obtaining a realistic neutrino mass matrix, with mixing among all three lepton families.

**Baryon triality**

As discussed above, the requirement of a realistic neutrino mass matrix using only $S_{1,2}$ demands $|x_S| = 3$ instead of $|x_S| = 1/3$. This means that the total discrete symmetry from $B - x_i L$ is not just $R$ parity any more. It has been extended to a larger symmetry. Following the general arguments in Ref. [8], we find that our total discrete symmetry is now $Z_6$, which is a direct product of $R$ parity and baryon triality ($Z_6 = R_2 \times B_3$).

Under baryon triality ($B_3 = Z_3$) \[ 10 \], baryons transform as $\omega = \exp(2\pi i/3)$, so that the proton is absolutely stable, being the lightest particle with that charge. This result came as a pleasant surprise, because it was not our intention to construct a model which contains $B_3$. It points to a possible deep connection between neutrino mass and proton stability.

In regard to baryon stability, there are other relevant anomaly-free $U(1)$ gauge symmetry models \[ 8 \] \[ 11 \] \[ 12 \], as well as discrete gauge symmetry models \[ 10 \] \[ 13 \] \[ 14 \]. Especially, we note that when model (II) is shifted by some hypercharge, it can reach the equivalent form of a model in Ref. \[ 10 \].

\[ e - \mu - \tau \] nonuniversality

The salient prediction of our proposal is the existence of a new neutral gauge boson $Z'_X$. It does not mix with the electroweak $Z$ at tree level; but it couples to all quarks and leptons in a specified way. In particular, it breaks $e - \mu - \tau$ universality. Thus it may be important as a one-loop effect \[ 4 \] in the precision measurements of $Z \rightarrow \ell^+\ell^-$. However, this effect is proportional to $x_e^2$, and its contribution to nonuniversality is zero for model (I). As for model (II), the prediction is that $\Gamma(Z \rightarrow e^+e^-)$ should be bigger than $\Gamma(Z \rightarrow \mu^+\mu^-)$ and $\Gamma(Z \rightarrow \tau^+\tau^-)$, and it is proportional to $(81 - 9)g_Z^2$. The present world averages are \[ 15 \]

\[ \Gamma_e = 83.91 \pm 0.12 \text{ MeV,} \]  \hspace{1cm} (8)

\[ \Gamma_\mu = 83.99 \pm 0.18 \text{ MeV,} \]  \hspace{1cm} (9)

\[ \Gamma_\tau = 84.08 \pm 0.22 \text{ MeV.} \]  \hspace{1cm} (10)

After adding a kinematical correction of $0.19 \text{ MeV}$ to $\Gamma_\tau$, we find the deviation of $\Gamma_e$ from the average of $\Gamma_\mu$ and $\Gamma_\tau$ to be bounded at $95\%$ C.L. by

\[ \Delta \Gamma_e/\Gamma_{\mu,\tau} < 0.002. \]  \hspace{1cm} (11)
Let \( r \equiv M_{Z_X}^2/M_Z^2 \), then the one-loop radiative correction to \( Z \to \ell^+\ell^- \) from \( Z_X \) exchange in model (II) is given by \[ \Delta \Gamma_e = \frac{(81 - 9)\alpha_e^2}{8\pi^2} F_2(r), \] where \[ F_2(r) = \frac{7}{4} - 2r - (2r + 3) \ln r + 2(1 + r)^2 \times \left[ \frac{\pi^2}{6} - \text{Li}_2 \left( \frac{r}{1 + r} \right) - \frac{1}{2} \ln^2 \left( \frac{r}{1 + r} \right) \right]. \] In the above, \( \text{Li}_2(x) = \int_0^x \frac{t \ln(1-t)}{t} \, dt \) is the Spence function. For \( r \gg 1 \) (i.e. \( M_{Z_X}^2 \gg M_Z^2 \)) which will be required by Tevatron data in any case (as shown below), \( F_2 \simeq r^{-1}[11/9 + (2/3) \ln r] \) and the resulting numerical bound is \[ g_X^2/M_{Z_X}^2 \lesssim 0.05 \text{ TeV}^{-2}, \] to a good approximation. There is also an overall correction to \( \Gamma(Z \to \text{hadrons})/\Gamma(Z \to \text{leptons}) \), but that effect may be absorbed into the value of \( g_S \) in QCD.

**LEP2 contact interaction**

The agreement of the SM and the LEP2 data provides the indirect bounds on the \( Z_X^\prime \) mass. The strongest one for our models (I) and (II), where \( \mu \) and \( \tau \) have the same charges and \( e \) has the charge of opposite sign, comes from the \( e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^- \) channel with \( \Lambda_{VV} = 16 \) TeV \[ 17. \] (See Ref. \[ 18 \] for some useful discussions.) The \( Z_X^\prime \) mass is bounded by \[ M_{Z_X^\prime}^2 \gtrsim \frac{g_e^2}{4\pi} |x_e x_\mu| (\Lambda_{VV})^2 \] for sufficiently large \( M_{Z_X} \) compared to the LEP2 energy. It results \[ M_{Z_X^\prime} \gtrsim 1.4 \text{ TeV} \quad \text{in model (I)}, \] \[ M_{Z_X^\prime} \gtrsim 2.3 \text{ TeV} \quad \text{in model (II)}, \] for \( g_X = 0.1 \).

**Phenomenology of \( Z_X^\prime \)**

The direct production of \( Z_X^\prime \) is possible at hadron colliders through its coupling to quarks. Its decay to leptons is then a clear signature. At present, there is a Tevatron limit on the cross section \( \sigma(p\bar{p} \to Z_X^\prime \rightarrow e^+e^-) \) at a center-of-mass energy \( E_{cm} = 1.96 \text{ TeV} \), based on an integrated luminosity of \( L = 2.5 \text{ fb}^{-1} \) \[ 19, \] and similarly for dimuons \[ 20. \] To compare against these results, we take

![FIG. 1: Tevatron bounds on \( Z_X^\prime \) mass in models (I) and (II).](image)

For the numerical analysis, we use CompHEP/CalcHEP \[ 21, 22 \] and the parton distribution functions of CTEQ6L \[ 23 \]. Since the Tevatron bounds from dielectron and dimuon data are similar for the same coupling, we show only the dielectron result. We find \( M_{Z_X^\prime} \gtrsim 830 \) GeV for model (I) and \( M_{Z_X^\prime} \gtrsim 940 \) GeV for model (II) as shown in Fig. 1. (Since \( |x_\mu| < |x_e| \) in model (II), the bound from the dimuon data is less stringent.) We note also that the \( e - \mu - \tau \) nonuniversality constraint of Eq. \[ 14 \] is easily satisfied.

For the LHC discovery reach (with the design energy \( E_{cm} = 14 \) TeV) through the dilepton \( Z_X^\prime \) resonance, we use cuts \( p_T > 20 \) GeV, \( |y| < 2.4 \) (for each lepton) and \( |m_{inv}(\ell^+\ell^-)| < 3 \Gamma_Z^\prime \). SM background at the LHC with these cuts is negligible, and we just require 10 signal events to claim its discovery at the LHC for a fixed flavor.

Figs. 2(a) and 2(b) show the LHC discovery reach using dileptons for models (I) and (II) respectively. From the dilepton resonance only, model (I) cannot be distinguished from the flavor-independent case of \( B-L \). In model (I), \( Z_X^\prime \) will be revealed by both \( e^+e^- \) and \( \mu^+\mu^- \) channels at the same luminosity (\( L \approx 1 \text{ fb}^{-1} \) for \( M_{Z_X^\prime} = 1.5 \) TeV). In model (II), the \( \mu^+\mu^- \) resonance will need a luminosity about an order of magnitude larger than that for the \( e^+e^- \) resonance because \[ \sigma(\mu^+\mu^-)/\sigma(e^+e^-) \approx (3)^{-2}/(9)^2. \]

Since Higgs doublets have zero charges under \( B-x; L \), the channels which require nonzero charges such as the 6-lepton resonance discussed in Ref. \[ 24 \] will be absent.

In the presence of \( Z_X^\prime \) at the TeV scale, a (predominantly right-handed) sneutrino as well as the usual neutralino can be a good LSP dark-matter candidate \[ 25. \]
FIG. 2: The LHC discovery reach for (a) model (I) and (b) model (II). The required luminosity is the same for both $e^+e^-$ and $\mu^+\mu^-$ resonances in model (I). The required luminosity for the $e^+e^-$ (solid line) resonance is about an order of magnitude smaller than that for the $\mu^+\mu^-$ (dashed line) resonance in model (II).

One-loop charged lepton processes

There are two sources of lepton flavor violation. One comes from explicit interactions linking $N_c$ through $S_{1,2}$; the other through the mismatch of lepton and slepton mass matrices (which is common to all supersymmetric models). Since we choose $x_e \neq x_\mu = x_\tau$, the latter applies only to the $\mu - \tau$ sector, whereas the former applies to all leptons, but for one-loop processes such as $\mu \rightarrow e\gamma$, they are negligible because of the smallness of neutrino masses.

The muon anomalous magnetic moment ($a_\mu$) does not violate lepton flavor, so it has a contribution from $Z'_X$, i.e. \[ \Delta a_\mu = \frac{3g_X^2}{4\pi^2} \frac{m_\mu^2}{M_{Z'_X}^2} \] for both models (I) and (II). Using the Tevatron bounds on $M_{Z'_X}$ with $g_X = 0.1$, we find $M_{Z'_X}$ to be greater than 830 GeV in model (I), and 940 GeV in model (II). The indirect bounds from LEP2 are 1.4 TeV in model (I), and 2.3 TeV in model (II). It should be accessible at the LHC, possibly at a very early stage. To have a background-free signal of $10$ dilepton events at the LHC, the $Z'_X$ with $M_{Z'_X} = 1.5$ TeV in model (I) may be discovered for an integrated luminosity of only about $1 fb^{-1}$. In model (II), because of the flavor-dependent charges, the event rate of the $e^+e^-$ resonance is predicted to be nine times that of $\mu^+\mu^-$. This work was supported by U. S. Department of Energy Grants No. DE-AC02-98CH10886 (HL) and No. DE-FG03-94ER40837 (EM).

Conclusion

We have proposed a new $U(1)_X$ gauge symmetry $B - x_iL$, with $x_{e,\mu,\tau} = (-3,3,3)$ [model (I)] or $(9,-3,-3)$ [model (II)], in the context of supersymmetry with three neutral singlets $N_{e,\mu,\tau}$. The spontaneous breaking of this $U(1)_X$ by the addition of singlets $S_{1,2}$ with $L = \pm 2$ and $x_S = 3$ accomplishes three objectives. (i) The conventional $R$ parity survives as an exact discrete symmetry, desirable for having the LSP as a good dark-matter candidate. (ii) Realistic neutrino masses and mixing are obtained. (iii) An exact residual $Z_3$ symmetry, i.e. baryon triality $B_3$, emerges which makes the proton absolutely stable.

The neutral gauge boson of this new $U(1)_X$ has large couplings to leptons over quarks: three or nine times larger than in the case of $B - L$. Using present Tevatron bounds with $g_X = 0.1$, we find $M_{Z'_X}$ to be greater than 830 GeV in model (I), and 940 GeV in model (II). The indirect bounds from LEP2 are 1.4 TeV in model (I), and 2.3 TeV in model (II). It should be accessible at the LHC, possibly at a very early stage. To have a background-free signal of 10 dilepton events at the LHC, the $Z'_X$ with $M_{Z'_X} = 1.5$ TeV in model (I) may be discovered for an integrated luminosity of only about 1 fb$^{-1}$. In model (II), because of the flavor-dependent charges, the event rate of the $e^+e^-$ resonance is predicted to be nine times that of $\mu^+\mu^-$. This work was supported by U. S. Department of Energy Grants No. DE-AC02-98CH10886 (HL) and No. DE-FG03-94ER40837 (EM).

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