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Large eddy simulation study of extended wind farms with large inter turbine spacing

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Abstract. We study the performance of extended wind farms with very large inter turbine spacings, i.e. with an inter turbine spacing of more than 10 turbine diameters, using large-eddy simulations (LES) for the special case of neutral conditions. We compare the results with predictions from different analytical modeling approaches and discuss in which parameter regimes the different model predictions agree with the LES predictions. We find in LES that the normalized power output variance increases further downstream in the wind farm. Besides, we analyze the power output correlation between the subsequent downstream turbines. We find that the correlations decrease with increasing turbine spacing due to the increased mixing induced by the wind turbine wakes. For all cases considered here, the power output correlation with downstream turbines becomes negligible after about 40 turbine diameters.

1. Introduction

To optimize wind farm performance, it is crucial to have accurate predictions for the expected power production for different wind farm designs. For this purpose, the industry uses computationally efficient wind farm design tools based on the ideas put forward in simple analytical wake models [1, 2]. Wake models focus on capturing the wake profile accurately. However, the wake model approach in isolation is less suitable for large wind farms in which it is difficult to correctly capture the wake-wake interactions and the interaction with the atmosphere. The so-called top-down model approach, on the other hand [3], aims at capturing the mean interaction between the wind farm and the atmospheric boundary layer (ABL). An important drawback of the top-down model is that the relative turbine positioning is not accounted for.

Frandsen [4] made essential contributions to combine the wake and the top-down model approaches. Later, Peña and Rathmann [5] and Yang and Sotiropoulos [6] introduced a one-way coupling between the Frandsen [4] and Jensen model [7]. In previous work, we developed the coupled wake boundary layer (CWBL) model [8,9] in which a two-way coupling between the Jensen and the Calaf et al. [3] top-down model is introduced. It was shown that the CWBL model gives improved predictions over its constitutive models [8,9] when the wind is aligned with the farm layout. For example, Gaumond et al. [10] show that the Jensen model does not accurately predict the measurement data for a wind direction of 270° in Horns Rev, while the results for other wind directions are captured well. The CWBL model shows a better prediction...
for the 270° direction, while for other wind directions, the Jensen and CWBL models provide similar levels of accuracy predicting the mean power observed in LES [11] and field data [12].

Here we use LES to analyze wind farm dynamics in a regime not considered before in LES and for which no measurement data are available, namely wind farms with very large turbine spacings, i.e. when the spacing between turbines in the downstream direction is more than ten turbine diameters. This large inter turbine spacing limit is relevant as a test case that allows us to verify model predictions in this regime. Obtaining accurate predictions for the large spacing limit is relevant as previous studies have shown that the “optimal” wind turbine spacing depends on the size of the wind farm and may be as large as 15 turbine diameters in infinitely large wind farms [13–15]. These predictions are based on predictions obtained by top-down models. Considering that larger and larger wind farms are envisioned it is crucial to study the performance of wind farms in this large scale limit, which may become relevant for the design of future wind farms. We also wish to point out that we believe that good wind farm models should be built so that they provide accurate predictions in asymptotic limits and not limited to conditions of present-day industry relevance. In this study we will compare the LES data against predictions from the Jensen [7], the Calaf et al. top-down [3], and the CWBL [8, 9] model. We emphasize that more analytical models are available in the literature, e.g. a recent dynamical extension of the CWBL approach [16]. Other models are described in reviews, see e.g. Ref. [1,2]

Many wind farm studies focus on the mean performance of the wind farm. However, there are large temporal fluctuations in the flow speeds, which lead to substantial variations of the power output over time for individual wind turbines and the entire farm. Detailed knowledge about the power output variations of wind farms enables better predictions of the power required to enable the provision of a constant level of power to the power-grid [17]. This is relevant information for grid operators who prefer smooth power signals rather than rapidly varying ones. In this work, we will also analyze the variance of the turbine power output, i.e. the level of its fluctuations and temporal variability, and the correlation between the power output of turbines in the farm.

In section 2 and section 3 we introduce the LES and analytical models considered in this study. In section 4 we first compare the power output data obtained from LES with the analytical model predictions before we present an analysis of the power fluctuations and temporal correlations between the turbine power outputs in the wind farm.

2. LES modeling

Here we complement our previous LES study [18] in which we considered smaller wind farms. In short, we consider wind farms with turbines that have a diameter of \( D = 100 \) m and a hub-height of \( z_h = 100 \) m. An actuator disk model is used to represent the turbines using a constant thrust coefficient of \( C_T = 0.75 \). The used turbine is the same as the one we used in the previous study. In section 3.4 we discuss various assumptions we considered. The roughness length of the ground is chosen to be \( z_{0,lo} = 0.1 \) meters, while the height of the computational domain is \( H = 2000 \) meters. The corresponding turbulence intensity at hub-height is about 14%. The roughness height was selected to match the value used in the previous study [18] and so our analysis is more directly applicable to on-shore conditions over heavily vegetated surfaces. We consider wind farms with ten or more rows in the streamwise direction to study the fully developed wind farm regime in which the power output as a function of the downstream distance becomes nearly constant. The mean inflow is in the streamwise \((x)\) direction for all cases. The distances between wind turbines are \( s_x D \) and \( s_y D \) in the streamwise and spanwise directions, respectively. We describe the overall dimensionless turbine spacing by the geometric mean \( s = \sqrt{s_x s_y} \). We vary the streamwise spacing \( s_x \) in the range \( [3.49, 35.90] \) and spanwise spacing \( s_y \) in the range \( [3.49, 11.78] \) and use aligned and staggered wind farm geometries. A sketch of the aligned and staggered wind farm configurations is shown in figure 1. To resolve the domains, we have used simulations with up to \( 4096 \times 128 \times 256 \) nodes in the streamwise, spanwise,
Table 1. The considered wind farm simulations in this study, which complement the cases considered by Stevens et al. [18], see table 1 of that study. The columns from left to right indicate the combined length of the precursor and the wind farm simulation domain in the streamwise, spanwise, and vertical direction ($L_x \times L_y \times L_z$), the used grid resolution ($N_x \times N_y \times N_z$), the dimensionless spacing in streamwise and spanwise direction ($s_x \times s_y$) in turbine diameters, the geometric mean turbine density $s = \sqrt{s_x s_y}$, and the number of turbines in streamwise and spanwise direction. For all cases an aligned and a staggered wind farm is simulated.

| $L_x \times L_y \times L_z$ | $N_x \times N_y \times N_z$ | $s_x \times s_y$ | $\sqrt{s_x s_y}$ | $N_T$ |
|---------------------------|-----------------------------|-----------------|------------------|-------|
| $8\pi \times 1.5\pi \times 2$ | $1024 \times 192 \times 256$ | $5.24 \times 11.78$ | $7.85$ | $18 \times 4$ |
| $8\pi \times 1.5\pi \times 2$ | $1024 \times 192 \times 256$ | $7.85 \times 11.78$ | $9.62$ | $12 \times 4$ |
| $12\pi \times 1.0\pi \times 2$ | $1536 \times 128 \times 256$ | $11.78 \times 3.49$ | $6.41$ | $12 \times 9$ |
| $12\pi \times 1.0\pi \times 2$ | $1536 \times 128 \times 256$ | $11.78 \times 5.24$ | $7.85$ | $12 \times 6$ |
| $12\pi \times 1.5\pi \times 2$ | $1536 \times 128 \times 256$ | $11.78 \times 7.85$ | $9.62$ | $12 \times 4$ |
| $16\pi \times 1.0\pi \times 2$ | $2048 \times 128 \times 256$ | $17.95 \times 3.49$ | $7.92$ | $10 \times 9$ |
| $16\pi \times 1.0\pi \times 2$ | $2048 \times 128 \times 256$ | $17.95 \times 5.24$ | $9.70$ | $10 \times 6$ |
| $16\pi \times 1.0\pi \times 2$ | $2048 \times 128 \times 256$ | $17.95 \times 7.85$ | $11.87$ | $10 \times 4$ |
| $24\pi \times 1.0\pi \times 2$ | $3072 \times 128 \times 256$ | $26.93 \times 3.49$ | $9.70$ | $10 \times 9$ |
| $24\pi \times 1.0\pi \times 2$ | $3072 \times 128 \times 256$ | $26.93 \times 5.24$ | $11.87$ | $10 \times 6$ |
| $24\pi \times 1.0\pi \times 2$ | $3072 \times 128 \times 256$ | $26.93 \times 7.85$ | $14.54$ | $10 \times 4$ |
| $32\pi \times 1.0\pi \times 2$ | $4096 \times 128 \times 256$ | $35.90 \times 3.49$ | $11.20$ | $11 \times 9$ |
| $32\pi \times 1.0\pi \times 2$ | $4096 \times 128 \times 256$ | $35.90 \times 5.24$ | $13.71$ | $11 \times 6$ |
| $32\pi \times 1.0\pi \times 2$ | $4096 \times 128 \times 256$ | $35.90 \times 7.85$ | $16.79$ | $11 \times 4$ |

Figure 1. A sketch of the aligned and staggered configurations considered in this study indicating the definition of the streamwise $s_x$ and spanwise $s_y$ turbine spacings.

and vertical direction, respectively. The resolution for each simulation is scaled according to the used computational domain size such that the resolution compared to the wind turbines remains constant. The cases presented here, see table 1, complement our earlier results from Ref. [18] in which we considered smaller wind farms. All figures presented here show the simulation results of the present and the prior [18] study, thus combining the results of 46 wind farm simulations.

3. Considered analytical models

Here we summarize the three analytical models that will be considered. We emphasize that more analytical models are available in the literature, and for complete overviews we refer to Refs. [1,2].
3.1. Jensen wake model

Here we use a standard version of the Jensen wake model in which the wind turbine wakes grow linearly with downstream distance and the wind velocity in the wakes is given by

$$\frac{u(x)}{u_0} = 1 - \frac{\Delta U_{hub}(x)}{u_0} = \left(1 - \frac{1 - \sqrt{1 - C_T}}{(1 + k_w x/R)^2}\right) = u_0 \left(1 - \frac{2a}{(1 + 2k_w x/D)^2}\right). \tag{1}$$

Here $k_w$ is the wake expansion coefficient, $u_0$ is the incoming free stream velocity, $R$ is the turbine radius and $D$ the turbine diameter, $C_T = 4a(1 - a)$ is the thrust coefficient where $a$ is the axial induction factor, and $x$ is the downstream distance with respect to the turbine. Wake interactions are taken into account by superposition of squared velocity deficits according to $u(x) = u_0 - \sum_i (u_0 - u_{ki})^2$. The effect of the ground is taken into account by using ghost turbines \cite{20}. The normalized power of each turbine is calculated by comparing the averaged cubed velocity at the location of each turbine with the corresponding value obtained for a turbine in the free stream. We use $k_w = \kappa/\ln(z_h/z_{0,lo}) = 0.0579$ (for our case $\kappa = 0.4$ and $z_h/z_{0,lo} = 1000$) to set the wake decay parameter \cite{20}.

3.2. The top-down model

The top-down model approach from Calaf et al. \cite{3} assumes the presence of two constant stress layers, one above and one below the turbine hub-height, and a wake layer in between. This model can be used to obtain the ratio of the mean velocity at hub-height in the fully developed region of the wind farm to the incoming reference velocity as

$$\frac{\langle u \rangle(z_h)}{\langle u \rangle(z_{0,hi})} = \ln\left(\frac{z_h}{z_{0,hi}}\right) \ln \left[1 + D \frac{2}{2z_h} \right] \left[\ln \left(\frac{z_h}{z_{0,lo}}\right)\right]^{-1}, \tag{2}$$

Here $\delta_H$ indicates the height of the ABL and the roughness height of the wind farm $z_{0,hi}$ is

$$z_{0,hi} = z_h \left(1 + \frac{D}{2z_h}\right)^{-\beta} \exp \left[-\frac{\pi C_T}{8w_f s_x s_y \kappa^2} + \left(\ln \left(\frac{z_h}{z_{0,lo}}\right) \left(1 - \frac{D}{2z_h}\right)\right)^{-1/\beta} \right], \tag{3}$$

with $\beta = \nu^*_w/(1+\nu^*_w)$ and $\nu^*_w \approx 28\sqrt{\pi C_T/(8w_f s_x s_y)}$. Here, $w_f$ is the effective wake area coverage. For the top-down model $w_f = 1$. The relative performance of turbines in the fully developed regime is given by the cubed velocity ratio defined in equation (2).

3.3. CWBL model

The CWBL model \cite{8} combines the Jensen \cite{5, 7, 21} and the Calaf et al. \cite{3, 22} model. A two-way coupling between these modules is enforced in the fully developed wind farm regime. The iteration procedure determines the effective wake area coverage $w_f$ in equation (3) using the Jensen model and obtains the wake expansion in the fully developed wind farm regime $k_w,\infty$ by matching the velocity at hub-height in the Jensen and top-down models. The effective wake area coverage $w_f$ is calculated as the fraction of the wind farm area where the velocity is below 95% of the free stream velocity in the Jensen model, see details in Ref. \cite{8}. By definition $w_f \leq 1$, where $w_f = 1$ gives the Calaf et al. \cite{3, 22} top-down limit described above. In the CWBL model the entrance effects are modeled by assigning a wake expansion coefficient to the wake originating from each turbine $k_w,T$ by using interpolation between $k_w,0$ and $k_w,\infty$ using

$$k_{w,T} = k_{w,\infty} + (k_{w,0} - k_{w,\infty}) \exp(-\zeta m), \tag{4}$$

where $m$ is the number of turbine wakes (not including ghost wakes) that overlap with the turbine of interest and $\zeta = 1$. The value of $\zeta$ is an empirical choice based on comparison with LES data \cite{8, 9}. This approach converges to the Jensen model in the entrance region and the top-down model in the fully developed regime.
3.4. Modeling assumptions
Due to the computational requirements for LES, several simplifying assumptions have to be made. In the present work, we restrict ourselves to neutral atmospheric thermal stability conditions, nor do we include the effect of the Coriolis force. As is explained in the appendix of Ref. [3], this is equivalent to assuming a given value for the transverse geostrophic wind. This approach is expected to give meaningful results as the flow in the inner region (up to $0.15 - 0.20H$) is not influenced much by external effects. These assumptions are widely used in the wind farm literature [1, 2] and these conditions allow for more straightforward comparison with the considered wind farm models. Besides, we assume in the LES and analytical model that all turbines operate in regime II [23], in which the thrust coefficient $C_T$ is independent of the wind speed, and we neglect the variation of the power coefficient $C_P$ with wind speed. According to Porté-Agel et al. [11] the turbines operate most of the time in regime II. These are reasonable assumptions since LES [11] and field measurement [12] generally report data for narrow wind speed bins, typically $\approx 8 \pm 0.5$ m/s, which corresponds to turbines operating in regime II. We follow this approach to focus on the main physical effects that influence the model performance, such that specific turbine designs or site conditions do not obscure the results.

4. Results
In section 4.1 we start with a description of the mean power production in LES and the corresponding comparison to the model predictions. We conclude with an analysis of the power fluctuations and correlations in section 4.2 and section 4.3. The uncertainties are discussed in section 4.4.

4.1. Power development in wind farms
Figure 2 shows the turbine power output as a function of the downstream position in the wind farm, normalized by the power output of turbines in the first row. The vertical bars indicate the uncertainty in the data based on the difference in the turbine power outputs obtained over the first and second half of the considered simulation time interval (after removing an initial transient period). The considered simulation interval for the data analysis varies between approximately 2 to 20 flow-through times. For the smaller cases, we find that the uncertainties in the data are small. As discussed in section 4.4, the uncertainty for some of the largest cases is larger than one would ideally aim for due to limitations in available computational time.

Figure 2 shows that for aligned wind farms, the power degradation as a function of the downstream position is similar in the entrance region of the wind farm for the different spanwise spacings considered here. Around 35 to 45 turbine diameters downstream, the turbine power output in aligned wind farms with a spanwise spacing of $3.49D$ becomes lower than for the other spanwise spacings. This is most pronounced in the wind farms with a streamwise spacing between $s_x = 5.24$ and $s_x = 11.78$. This observation is in agreement with a first-order approximation of the wake expansion, according to which wake effects for an aligned wind farm with a spanwise spacing of $3.49D$ should set in at $x = (3.49D - D)/k_w, 0 \approx 43D$. A similar estimate gives that differences between aligned wind farms with a spanwise spacing of $s_y = 5.24$ and $s_y = 7.85$ should become visible at $\approx 73D$. However, the data in figure 2 do not show a visible performance difference between aligned wind farms with $s_y = 5.24$ and $s_y = 7.85$ at this location. Instead, the performance of aligned wind farm with a spanwise spacing of $5.24D$ to $11.78D$ is, within the obtained accuracy, very similar. The reason is that the secondary sideways wake effects are negligible compared to the wake effects of directly upstream turbines for such large distances. This implies that increasing the spanwise spacing beyond a specific spanwise value ($s_y \sim 4 - 5$) does not increase the total power output of the wind farm and therefore reduces the average power output density normalized by the area occupied by each turbine, i.e. $s_x s_y D^2$.

Comparing the results for aligned and staggered wind farms confirms the expected trend
Figure 2. Normalized power output $P_i/P_1$ in aligned (black) and staggered (red) wind farms for different streamwise and spanwise turbine spacings. Each panel shows the results for wind farms with the same streamwise turbine spacing and different spanwise spacings; see the legend. The vertical error bars indicate the uncertainty based on the power output of the turbines obtained from the second part of the simulation with the full simulation interval. For a detailed discussion on the error bars we refer to section 4.4.

that a staggered configuration leads to a significantly better performance in the entrance region of the wind farm because more turbines in the entrance region do not experience wake effects from an upstream turbine and the effective distance between consecutive downstream turbines is doubled. In agreement with the results in Ref. [18] figure 2 reveals that the benefit of a staggered configuration becomes less further downstream in the wind farm, in particular when the spanwise spacing is relatively small. For $s_y = 3.49$ the performance for aligned and staggered wind farms is almost identical in the fully developed wind farm regime, while for $s_y = 5.24$ the turbines in a staggered wind farm perform slightly better in the fully developed regime than in an aligned wind farm. For $s_y = 7.85$, a staggered wind farm configuration offers a substantial performance benefit compared to an aligned wind farm, even in the fully developed regime. Increasing the spanwise spacing from $s_y = 7.85$ to $s_y = 11.78$ in staggered wind farms results in a minor increase in the performance of the turbines in the fully developed wind farm regime.

Figure 3 compares the performance of the Jensen, top-down, and CWBL model with the LES
Figure 3. Comparison between the LES observations with the predictions by the Jensen [7,21], the Calaf et al. top-down [3], and the CWBL model [8,9] as function of the mean geometrical mean turbine spacing $s = \sqrt{s_x s_y}$. Panels (a-l) show the results for fixed streamwise spacing and panels m-x for fixed spanwise spacing. Panels a-f and m-r show the absolute values, while panels g-l and s-x show the difference between the LES observations and model predictions, see details in text. For a detailed discussion on the error bars we refer to section 4.4.
observations in the fully developed wind farm regime. This figure presents the performance of the turbines in the fully developed regime compared with the performance of the turbines in the first row, i.e. $P_\infty/P_1$ as a function of the turbine spacing (not downstream distance). $P_\infty$, the power production data in the fully developed regime is determined over the last 25% of the wind farm. In this study, we analyze the data of 46 simulations, see table 1 and the cases presented in Ref. [18]. So each data point in figure 3 presents the performance in the fully developed regime of a different wind farm simulation, i.e. with different turbine spacing $s$, different combinations of streamwise and spanwise spacing, and aligned and staggered configuration. To quantify the ability of the different models to predict the LES data, we determined the relative difference between the LES and model predictions for all available cases as

$$\text{Difference} = \left( \frac{\text{Model}}{\text{LES}} - 1 \right) \cdot 100\%$$

For staggered wind farms, we see that the performance of the turbines in the fully developed regime measured in the LES reasonably collapses when plotted against the mean geometric turbine spacing for most spanwise spacings, except for the very large spanwise spacings of $s_y = 11.78$. For the Jensen model, we observe the largest deviations with respect to the LES results for relatively small inter turbine spacings, while the large inter turbine spacings limit is satisfactory predicted. For the top-down model, the predictions for the aligned configuration are rather unsatisfactory, especially for small streamwise spacings. The reason is that the top-down model assumes horizontal homogeneity to compute the interaction between the wind farm and the ABL. As this assumption is reasonably satisfied for most staggered wind farm configurations considered here, the corresponding top-down model predictions are quite accurate. The presented model uncertainties in figure 3 see equation 5 shows that the CWBL predictions capture the general LES trends better than the Jensen and the Calaf et al. [3] top-down model. In particular, figure 5 shows that for small inter turbine spacings, i.e. $s \lesssim 7$, the difference between the CWBL model and LES predictions is consistently below 15%, while differences up to 35% between the LES and wake and top-down model results are observed for some cases.

We note that the CWBL model builds on the Jensen model and adjusts the wake expansion coefficient to account for changes in turbulence intensity by using information from the top-down model. It was already suggested by, for example, Nygaard [24] that appropriate adjustment of the wake decay exponent can improve the Jensen model predictions. Nygaard [24] also showed that the Jensen model, in its basic form, already predicts the performance of several wind farms well. We emphasize that the results in figure 3 do not contradict these observations as we show that each model has a parameter regime in which satisfactory model predictions are obtained. In figure 5 we find that for many cases the Jensen model predictions are within 15% of the LES results. However, figure 3 shows that for most aligned wind farms with $s_x = 5.24$ and $s_x = 3.49$, the difference between LES and the Jensen model results is more significant. The same holds for staggered wind farms with $s_y = 3.49$ and $s_x \lesssim 18$, see figure 3. This is in agreement with observations that the Jensen model in general struggles most to predict the performance of the aligned wind direction [1][10]. Here we show that the Jensen and CWBL model predictions are good for the large spacing limit, while the top-down model predictions for this limit are less accurate.

4.2. Power fluctuations

Figure 4 shows the normalized power output variations, which are determined by calculating the variance of the turbine power output normalized with its mean value as a function of the downstream position for all cases considered in this study. It should be pointed out that the variability computed from the LES is due to the ABL turbulence under steady-state mesoscale conditions. In real wind farms, additional variability at longer timescales occurs due to the atmospheric mesoscale phenomena. At the entrance of the wind farm, the normalized power
Figure 4. Normalized turbine power variations output $\sqrt{P_{\text{var}}/P_i}$ in aligned (black) and staggered (red) wind farms as function of the downstream position for different streamwise and spanwise turbine spacings. Each panel shows the results for wind farms with the same streamwise turbine spacing and different spanwise spacings; see the legend. The vertical error bars indicate the uncertainty based on the power output of the turbines obtained from the second part of the simulation with the full simulation interval. For a detailed discussion on the error bars we refer to section 4.4.

variance is approximately 0.3. The normalized power variance increases further downstream in the wind farm. The power variations increase more quickly as a function of the downstream position in aligned wind farms than in staggered wind farms. The power variations in the fully developed wind farm regime are higher when the average turbine density is higher. This is more clearly visible in figure 5 which shows the normalized power variance as a function of the geometrical mean turbine spacing $s = \sqrt{s_x s_y}$ in the fully developed regime. From figure 5 it is clear that a higher normalized power variance is observed for wind farms with a smaller geometrical mean turbine spacing. Looking at the results for the aligned wind farms, it appears that the normalized power variance depends more on the streamwise spacing than on the spanwise spacing. However, such a trend is not visible in the staggered wind farm data. As discussed in section 4.4, the uncertainty in the power fluctuation data is most likely larger than indicated by the represented error bars.
Figure 5. Normalized power variations in the fully developed regime $\sqrt{P_{\text{var}}}/P_{\infty}$ as function of the mean geometrical mean turbine spacing $s = \sqrt{s_x s_y}$. Left panels show the results for fixed streamwise spacing and right panels for fixed spanwise spacing. For a detailed discussion on the error bars we refer to section 4.4.

Figure 6. Correlation of the power output of turbines on the first and second row. Panel (b) and (c) shows the correlation between the first turbine row and subsequent downstream turbines as function of (b) row number and (c) distance. For a detailed discussion on the error bars we refer to section 4.4.

4.3. Power correlations

Figure 6a shows the correlation between the power output of turbines in the first and second row, which reveals a small decrease with increasing streamwise distance between the turbines. The reason for this weak dependence is the very long elongated flow structures that are formed in ABL flow, which tends to preserve the correlations in the streamwise direction. Figure 6b,c show the power correlation between turbines in the first row and subsequent downstream rows plotted as a function of row number and the physical distance. These panels reveal that the power output of turbines at the end of the wind farm is uncorrelated with the power output of turbines in the first row. We note that in previous work [25], we had already shown that there are no significant correlations between turbines placed in different columns (spanwise direction). In agreement with figure 6a, we find that the correlation in the turbine power outputs decreases with increasing turbine distance. Interestingly, figure 6b,c shows that the wind turbine wakes help mixing in the flow due to which the flow decorrelates sooner. Figure 6c shows that we find a higher correlation at the same physical distance from the entrance of the farm when the streamwise turbine spacing is smaller. However, the correlation is negligible after about 40 turbine diameters. We note that the power output between subsequent downstream rows is highest. In addition, we emphasize that the obtained correlation values will depend on, for example, the background atmospheric turbulence intensity and the used turbine model.
4.4. Uncertainties

We note that there is some uncertainty in the data for the very large wind farms due to the presence of a very persistent pattern of high and low-velocity wind speed streaks that are formed in ABL simulations, which are very difficult to average out in the current method. Improved results can be obtained by periodically shifting the inflow field to average away these streak effects \[26\]. However, such a method was not used in the present simulations to avoid introducing additional changes compared to the methods used in the previous simulations with smaller spacings that were done without shifting boundary conditions. This means that the uncertainty in the average power (figure 2 and 3) and the power variance (figures 4 and 5), which are based on the difference obtained from the second part of the simulation with the full simulation interval, can be larger than represented by the indicated uncertainties. Unfortunately, it is not possible to directly determine the uncertainty caused by some of the effects described above. As a result the presented error bars do not include all possible uncertainties and should be seen as an indication of the uncertainty.

However, some general trends can be observed. The uncertainty increases when the inter turbine spacing is increased. The simulations for wind farms with a larger streamwise turbine spacing \( s_x \) have been performed in larger domains, see table 1. For the larger domains it is more challenging, due to limitations in computational time, to obtain statistically converged data. In most simulations the spanwise domain length was fixed. For \( s_y = 3.49 \), \( s_y = 5.23 \), and \( s_y = 3.49 \) there are, respectively, 9, 6, and 4 turbine columns in the spanwise direction. As there are fewer turbines on each row for larger \( s_y \), the statistical uncertainty increases with increasing \( s_y \). In agreement with these trends we see in figure 2 that the statistical uncertainty is largest for \( s_x = 35.90 \) and \( s_y = 7.85 \). Also, the uncertainty in the power production (figure 2) and its variance (figure 4) as a function of the downstream position is larger for staggered farms than for aligned farms. The reason is that the location of the high and low-velocity streaks with respect to the turbine locations has a larger influence on the performance of a given row of a staggered farm than of an aligned farm. For some staggered wind farms a small wiggle pattern in the turbine power output between even and odd turbine rows, which is also an indication of the uncertainty in the data. For several staggered wind farms, the relative placement of the turbines with respect to the location of the high and low-speed streaks results in slightly higher power output in the second turbine row although the higher power output in the second row can also be caused by a Venturi effect \[27,28\], which leads to a speedup of the wind in between turbines in the first row. The power output and its variance for the fully developed wind farm regime have been averaged over the last 25% of the wind farm to reduce uncertainties. For the power production this is an effective method; see figure 3. However, the power variance, which is a second-order statistic, convergences slower with downstream direction than the power production. Therefore the uncertainties in the power variance in the fully developed regime, see figure 5, are larger than for the corresponding power production values, see figure 3.

5. Conclusions

We discussed the result of large scale LES focusing on the performance of extended wind farms with large inter turbine distances, i.e. wind farms in which the spacing between subsequent downstream turbines is more than 10 turbine diameters. The LES results are compared with predictions obtained from the Jensen, top-down, and CWBL models. In the fully developed regime, we find that the CWBL model predictions are closest to the LES results over the extensive range of wind farm configurations considered here. Besides, the simulations show that the power output variance increases with downstream distance in the wind farm due to the added turbulence intensity by the wakes. In connection to this trend, we find that the power output correlation between the subsequent downstream turbines decreases when turbines are placed closer together, and we find no correlation between the power output of turbines at the begin-
ning and end of the wind farm. The present set of LES results may also be useful to provide data to test new wake models as they are being developed.

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