Strong coupling of a qubit to shot noise

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We perform a nonperturbative analysis of a charge qubit in a double quantum dot structure coupled to its detector. We show that strong detector-dot interaction tends to slow down and halt coherent oscillations. The transitions to a classical and a low-temperature quantum overdamping (Zeno) regime are studied. In the latter, the physics of the dissipative phase transition competes with the effective shot noise.

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The study of fluctuations and noise provide deep insights into quantum processes in systems with many degrees of freedom. If coupled to a few-level system such as a qubit, fluctuations usually lead to destabilization of general qubit states and induce decoherence and energy relaxation. One important manifestation is the back-action of detection on qubits. This topic has been extensively studied in the regime of weak coupling between qubit and noise source. It has been shown that the qubit dephases into a mixture of qubit eigenstates (dephasing), whose classical probabilities thermalize to the noise temperature at a longer time scale. Mesoscopic charge detectors such as quantum point contacts (QPCs) and radio-frequency single electron transistors (rf-SETs), whose low-temperature noise is shot noise, are particular powerful detectors as they provide high resolution and potentially reach the quantum limit. A particular attractive regime for qubit applications is the QND regime, realized if the qubit Hamiltonian and the qubit-detector coupling commute.

We study a quantum point contact potentially strongly coupled to the coordinate (left/right) of a double quantum dot charge qubit by a nonperturbative approach involving the Gaussian and noninteracting blip approximations. We analyze the qubit at the charge degeneracy point, where the two lowest energy eigenstates are delocalized between the qubits. In the weak coupling regime, low-temperature relaxation would thus always delocalize charge. We show that, in strong coupling, the qubit state gets localized in one of the dots. Localization is manifest by a suppression of both the coherent oscillations and the incoherent tunneling rate. This “freezing” of the state also applies a high bias and can e.g. lock an excited state. Thus, in the strong coupling regime, the environment naturally pushes the physics to the QND limit even if the bare Hamiltonian is not QND. We point out the analogy of this physics to the case of the dissipative phase transition in oscillator bath models, which in the QPC competes with the nonequilibrium induced by the voltage driving the shot noise.

We consider the case of a degenerate two-state system (TSS), realized by the charge states in a double quantum dot structure (see Figure 1). These charge can be read out by the current through a nearby quantum point contact. The Hamiltonian for the TSS with time-dependent fluctuation \( \tilde{\varepsilon}(t) \) reads

\[
H_{\text{sys}} = \frac{\hbar}{2} \left( \begin{array}{cc} \tilde{\varepsilon}(t) & \Delta \\ \Delta & -\tilde{\varepsilon}(t) \end{array} \right) \rightarrow \tilde{H}_{\text{sys}} = \frac{\hbar}{2} \left( \begin{array}{cc} 0 & e^{i\phi} \\ -e^{-i\phi} & 0 \end{array} \right).
\]

In the last step of eq. (1), we applied a Polaron transformation introducing the fluctuating phase \( \phi = \int_0^t dt' \tilde{\varepsilon}(t') \), with \( \tilde{\varepsilon}(t) = \varepsilon + \delta \varepsilon(t) \), for the tunneling matrix elements in the qubit. The microscopic foundation of the noise term \( \delta \varepsilon(t) \) for a QPC is given in Refs. and an SET in Refs.

Without loss of generality, we assume \( \langle \hat{\sigma}_z(0) \rangle = 1 \). We can now formally solve the Liouville equation. The expectation value of \( \hat{\sigma}_z \), the difference of occupation probabilities of the dots, satisfies a closed equation

\[
\langle \hat{\sigma}_z(t) \rangle = -\Delta^2 \int_0^t dt' \cos[\varepsilon(t - t')] \langle e^{i\phi(t')} e^{-i\phi(t')} \rangle \langle \hat{\sigma}_z(t') \rangle \\
= -\Delta^2 \int_0^t dt' \cos[\varepsilon(t - t')] e^{i(t-t')} \langle \hat{\sigma}_z(t') \rangle,
\]

where the second line of eq. (2) has been derived by assuming that the noise represented by \( J(t-t') \) is stationary. This procedure is analogous to the noninteracting blip approximation (NIBA) of the path-integral solution of the Spin-Boson model. This automatically includes a Gaussian approximation to the shot noise. This approach is nonperturbative in \( \phi \) and a good approximation in the two cases \( \varepsilon = 0 \) and \( |\varepsilon| \gg |\Delta| \).

FIG. 1: Schematic view of the double dot system analyzed in the paper.
We start with the charge-degeneracy case \( \varepsilon = 0 \). Here, we can solve eq. (2) in Laplace space and find

\[
\mathcal{L} \{ \langle \hat{\sigma}_z(t) \rangle \} = \frac{1}{s + \Xi(s)},
\]

with the Laplace-transformed self-energy \( \Xi(s) = \Delta^2 \int_0^\infty dt e^{-st}e^{J(t)} \). The phase correlation function \( J(t) \) as seen by the dots reads \[\text{[3]}\]

\[
J(t) = \frac{2\pi}{\hbar R_K} \int_{-\infty}^{\infty} d\omega \frac{|Z(\omega)|^2}{\omega^2} S_f(\omega) (e^{i\omega t} - 1),
\]

where \( S_f(\omega) \) is the full current noise in the QPC that for sufficient environmental impedance is given \[\text{[3]}\] by

\[
S_f(\omega) = \frac{4}{R_K} \sum_m^N D_m (1 - D_m) \left\{ \frac{\hbar \omega + eV}{1 - e^{\beta(\hbar \omega + eV)}} + \frac{\hbar \omega - eV}{1 - e^{\beta(\hbar \omega - eV)}} \right\} + \frac{4}{R_K} \sum_m^N D_m^2 \frac{2\hbar \omega}{1 - e^{-\beta \hbar \omega}}
\]

and the transimpedance \( Z(\omega) \) between qubit and point contact. In eq. (3), \( V \) is the bias voltage of the QPC, \( R_K \) is the quantum resistance, and \( D_m \) is the transmission eigenvalue of the \( m \)th conductance channel.

**Semiclassical limit:** We now discuss the resulting dynamics in a number of limiting cases. We start by first taking the limit \( \omega \to 0 \). This corresponds to \( \hbar \Delta, \hbar \varepsilon \ll eV, k_BT \), i.e. the qubit probes the shot noise at energy scales much lower than its internal ones. Here, the noise expression [eq. (3)] becomes frequency independent \[\text{[3]}\]. We can then compute the semiclassical spectral function \( J_c(t) = -\gamma_c t \). Here, we have assumed a frequency-independent transimpedance controlled by a dimensionless parameter \( \kappa, |Z(\omega)|^2 \approx \kappa^2 R_K^2 \) and \( \gamma_c = 2\pi^2 \kappa^2 R_K S_f(0) \) with \( S_f(0) = \frac{4}{R_K} \sum_m^N D_m (1 - D_m) eV \coth \left( \frac{\hbar \omega}{2k_B T} \right) + \frac{4}{R_K} \sum_m^N D_m^2 \frac{2\hbar \omega}{1 - e^{-\beta \hbar \omega}} \). The self-energy is then readily calculated and analytical, so we can go back from Laplace to real time and obtain

\[
\langle \hat{\sigma}_z(t) \rangle = \left[ \cos (\omega_{eff,c} t) + \frac{\gamma_c}{2\omega_{eff,c}} \sin (\omega_{eff,c} t) \right] e^{-\frac{\omega_{eff,c}}{2} t},
\]

where \( \omega_{eff,c} = \sqrt{\Delta^2 - \frac{\gamma_c}{2}} \). We observe that the coherent oscillations of the qubit decay on a scale \( \gamma_c^{-1} \) and get slowed down. At \( \gamma_c = 2\Delta \), the damping becomes critical and the oscillations disappear, ending up with a purely exponential overdamped regime at \( \gamma_c > 2\Delta \). This crossover corresponds to the classical overdamping of a harmonic oscillator. Even in the overdamped regime, the qubit decays exponentially to \( \langle \hat{\sigma}_z(t) \rangle \to 0 \) at long times, e.g. it gets completely mixed by the shot noise, whose noise temperature is high \( k_BT_{\text{noise}} \approx \max \{ eV, k_BT \} \gg \hbar \Delta \). Note that it is possible to discuss the overdamped regime, where \( \gamma_c \) is *not* a small parameter and our theory is also non-Markovian, see eq. (3), capturing the necessary time-correlations arising in strong coupling.

Figure 2 shows the resulting dynamics in the one-channel case. With increasing bias voltage \( V \) over the QPC, the expectation value \( \langle \hat{\sigma}_z(t) \rangle \) drops down quite fast. The transmission \( D \) of the QPC has also an important impact on the stability of the oscillations of \( \langle \hat{\sigma}_z(t) \rangle \) (see inset of Figure 2). At \( D = 0.5 \), the expression for \( S_f(0) \) has a maximum, therefore the oscillations are there maximally suppressed. \( S_f(0) \) represents the shot noise of the QPC in the low frequency regime \[\text{[14]}\]. The more noise the QPC provides, the quicker the oscillations decay. Note that changes in the QPC transmissions (and therefore the Fano factor) do not play any role other than entering the total noise level.

**Quantum limit:** Now, we let \( T \to 0 \) and leave \( \omega \) arbitrary. \( S_f(\omega) \) reads in this limit

\[
S_f(\omega) = \frac{4}{R_K} \sum_m^N D_m (1 - D_m) \left\{ \left( \frac{\hbar \omega}{1 + e^{\beta \hbar \omega}} \right) \theta(\hbar \omega + eV) + \left( \frac{\hbar \omega}{1 - e^{\beta \hbar \omega}} \right) \theta(\hbar \omega - eV) \right\} + \sum_m^N D_m^2 2\hbar \omega \theta(\hbar \omega).
\]

This shape is dominated by two terms, which resemble the Ohmic spectrum at low \( T \), \( S_f \propto \omega \theta(\omega) \) with shifted origins of energy. For computing the quantum correlation function \( J_q(t) \), an ultraviolet cutoff \( \omega_c \) has
to be introduced, which physically originates either from the finite bandwidth of the electronic bands in the microscopic Hamiltonian or from the high-frequency limitations of the transimpedance $Z(\omega)$. We end up with the long-time limit for $J_q(t)$ applicable at $\hbar \Delta \ll eV$

$$J_q(t) = -\alpha_1 + \alpha_2 \ln \left[ \left( \frac{eV}{\hbar} \right)^F \frac{1}{\omega_c} I^{F-1} \right] - \gamma_q t + i\alpha_3 \omega, \quad (8)$$

This holds for any number of channels, for simplicity we concentrate on the single-channel case with a Fano factor then is given by $F = 1 - D$, which we use from now on. Here, we can introduce $\alpha_2 = g = 16\pi \kappa^2 D$, the dimensionless conductance as seen by the qubit, $\alpha_1 = g \gamma D$, $\alpha_3 = \pi g/2$ and $\gamma_q = \pi g(1 - D)eV/2\hbar$. The resulting self-energy is now non-analytical

$$\Xi(s) = \Delta^2_\text{eff} \frac{(s + \gamma_q)^{gD-1}}{(eV/\hbar)^D} e^{i\pi g/2}, \quad (9)$$

where we have introduced the effective tunnel splitting $\Delta^2_\text{eff} = \Delta^2 e^{-\gamma g D} \left( \frac{eV}{\hbar \omega_c} \right)^g \Gamma(-gD + 1)$. In our regime, $\omega_c \gg eV/\hbar \gg 1/t \simeq \Delta$, this expression resembles the renormalized $\Delta$ of the Spin-Boson model and we have $\Delta_{\text{eff}} \ll \Delta$. This is a sign of massive entanglement between system and detector. Note that similar to the adiabatic scaling treatment in Ref. [11], the NIBA is compatible with forming entangled states between system and detector. This has been numerically confirmed, for the Spin-Boson model, in Ref. [22].

The self-energy is analytically only at $F = 1$, which corresponds to the no-noise case $D = 0$. Due to the generally non-analytic self-energy, it is difficult to compute the full real-time dynamics by back-transformation to the time domain. The structure of the result will be $\left\langle \Delta \sigma_\varepsilon(t) \right\rangle = P_\text{cut}(t) + P_\text{coh}(t) + P_\text{incoh}(t)$. For our case of $\varepsilon = 0$, there is no incoherent exponential decay $P_\text{incoh}$. $P_\text{cut}$ is a nonexponential branch cut contribution. In the following, we concentrate on the coherent part $P_\text{coh}(t)$, given through the poles $s_t = -\gamma_{\text{eff}} \pm i\omega_{\text{eff}}$ of $\Xi$ with finite imaginary part, and hence this leads to damped harmonic oscillations with frequency $\omega_{\text{eff}}$ and decay rate $\gamma_{\text{eff}}$.

To $D = 0$, we can characterize these poles perturbatively. We find a renormalized oscillation frequency $\omega_{\text{eff}}$, namely $\omega_{\text{eff}} = \text{Re} \left( \sqrt{\Delta^2_p (1 + \frac{\varepsilon}{2} g) - \gamma_r^2/4} \right)$ whereas $\gamma_{\text{eff}} = \frac{\gamma_r}{2} \pm \text{Im} \left( \sqrt{\Delta^2_p (1 + \frac{\varepsilon}{2} g) - \gamma_r^2/4} \right)$. Here, $\Delta^2_p$ is defined as $\Delta^2 = \Delta^2 \left( 1 + g \ln \left( \frac{eV}{\hbar \omega_c} \right) \right)$. For arbitrary $F$ or $D$, we can solve the pole equation numerically, see Fig. 3. With the numerical results from Figure 3 one can again calculate the Laplace back-transformation, where the two residues of the kind $\alpha_1 = \frac{i\varepsilon}{\pi g(\omega_c - \omega_q) + \gamma_{\text{eff}}}$ have to be summed up. This leads finally again to decaying oscillations as already mentioned above.

We see that at sufficiently strong coupling to the detector, a finite Fano factor can lead to a complete suppression of the coherent oscillations, whereas the decay rate increases. Both these tendencies together show that a finite Fano factor brings the system closer to charge localization. In fact, for sufficient damping, we can tune the tunneling frequency all the way to zero by increasing $D$. On the other hand, also $\gamma_{\text{eff}}$ can become very small — in these points the detector completely localizes the particle up to nonexponential contributions. At other values of $D$, unlike the dissipative phase transition in the Spin-Boson model, the hot electrons driving the shot noise again drive the relaxation rate close to its bare value, and thus this resembles the classical overdamping case.

This scenario is not limited to $\varepsilon = 0$. NIBA permits to reliably study the opposite regime $\varepsilon \gg \Delta$ as well. As already shown in Refs. [11, 23], the resulting dynamics is dominated by incoherent exponential relaxation dominating over $P_\text{coh}$ and $P_\text{cut}$. The relaxation rate is

$$\Gamma_r = 2\text{Re} \left[ \Xi(i\varepsilon + 0) \right] = 2\Delta^2 \text{Re} \left[ \frac{(i\varepsilon + \gamma_q)^{2D-1}}{(eV)^D} e^{i\pi g/2} \right]. \quad (10)$$

This again demonstrates the slowdown (through $\Delta_{\text{eff}}$) of the decay to the other dot due to the interaction with the detector. Notably, this rate does not display standard detailed balance at $T = 0$, rather, around $\varepsilon = 0$, the rate...
is smeared out on a scale of $\gamma_q$, reflecting the role of the nonequilibrium shot noise temperature. We have plotted this result in Figure 4.

![Figure 4: Quantum limit: relaxation rate $\Gamma_r$ as a function of the QPC transmission $D$. The other parameters are $\varepsilon = 10 \Delta$, $\Delta = 1.524 \times 10^2$ 1/s, $eV = 10^2 \hbar \Delta$, $\omega_c = 10^{12} \Delta$. Inset: relaxation rate $\Gamma_r$ as a function of the qubit bias $\varepsilon$. Other parameters as above, but with $D = 0.5$.](image)

Another view on this is that the effective size of the noncommuting term between qubit and detector, given by $\Delta_{\text{eff}}$, is reduced, hence the strong interaction brings the effective Hamiltonian closer to a QND situation.

On the other hand, such dynamics is known as the quantum Zeno effect. Note that unlike standard derivations [1, 8, 22], this has been derived in a nonperturbative way, which is consistent with the necessary strong coupling and which retains the non-Markovian structure.

Summarizing the QPC results, we can observe that, on the one hand, the system shows traces of the physics of environment-induced localization, which competes with classical overdamping by effectively ”hot” electrons at finite voltage and somewhat reinforced at finite Fano factor. This can be understood as follows: the dissipative phase transition occurs when the environmental noise is highly asymmetric in frequency and when the full bandwidth plays a role. At high voltage, the asymmetry of the shot noise spectrum is reduced [8]. In fact, the $\gamma_q t$ contribution in the correlation function $J_q(t)$ resembles the finite temperature term in the correlation function of the Ohmic Spin-Boson model — both terms originate from the zero-frequency part of the noise.

A similar analysis on back-action by strong coupling of a QPC to a quantum device — there an Aharonov-Bohm experiment [22] — has been done in Ref. 22. That work concentrates on a stationary situation and weak hopping into the dot, whereas in our case the dots are not connected to leads. The inter-dot interaction however is strong and we concentrate on the real-time dynamics.

These results can be extended to shot noise sources other than QPCs. In fact, it may today be quite challenging to reach $\kappa$-values high enough, such that slowdown and localization can be observed, when the noise source has only a few open channels. An alternative approach is given by readout using metallic SETs fabricated on another sample layer [8], see Fig. 4. In these devices, there is a number of rather opaque conductance channels.

In that case, we use the expression [14, 15, 16, 17] of the voltage noise of the SET (only valid for small frequencies)

$$S_V(\omega, \omega_I) = \sqrt{\frac{2 e^2}{\hbar \omega I} \frac{2 \omega_I}{\omega^2 + 16 \omega_I^2}},$$

where $E_{\text{SET}} = \frac{e^2}{2C_{\text{SET}}}$ is the charging energy of the SET and $\omega_I = I/e$ is the tunneling rate through the SET. Then the final result for $\langle \sigma_z(t) \rangle$ is again the same as in eq. 16. The difference, of course, is that $\gamma_c$ is now defined as $\gamma_c = \frac{2 e^2}{\hbar \omega I} E_{\text{SET}} \omega$. The full quantum mechanical analysis in the low-temperature regime works along the same lines as the QPC case but goes beyond the scope of this Letter.

We performed a nonperturbative analysis of the quantum dynamics of a double quantum dot coupled to shot noise. We analyze the crossover from under- to overdamped oscillations in the classical case. In the quantum case, we show that at strong coupling the oscillations show the same behavior, competing with a critical slowdown similar to the dissipative phase transition. This can be interpreted as the onset of a Zeno effect.

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