Ultra-high energy neutrino scattering

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Estimates are made of ultra-high energy neutrino cross sections based on an extrapolation to very small Bjorken $x$ of the logarithmic Froissart dependence in $x$ shown previously to provide an excellent fit to the measured proton structure function $F_2^p(x,Q^2)$ over a broad range of the virtuality $Q^2$. Expressions are obtained for both the neutral current and the charged current cross sections. Comparison with an extrapolation based on perturbative QCD shows good agreement for energies where both fit data, but our rates are as much as a factor of 10 smaller for neutrino energies above $10^9$ GeV, with important implications for experiments searching for extra-galactic neutrinos.

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Introduction: The experimental effort to detect extra-galactic, ultra-high-energy (UHE) neutrinos has grown rapidly in the past decade. Optical and radio telescopes and cosmic ray air shower arrays are now rapidly in the past decade. Optical [1] and radio [2] telescopes and cosmic ray air shower arrays [3] are now搜索 for evidence of point and diffuse neutrino sources up to and beyond EeV energies. Proposals have been made and others are in preparation [4] for new telescopes or expansions of ones currently deployed, and ambitious satellite-born telescopes have been proposed [5]. The highest energies proposed reach beyond $10^{12}$ GeV.

Critical to all of this effort are accurate estimates of event rates, based on the extrapolation of measured neutrino deep-inelastic scattering (DIS) cross sections to energies far beyond currently available data [6, 7, 8]. The estimates are only as reliable as the extrapolations, and determination of fluxes and extraction of signals of new physics at UHE depend on them. Most existing extrapolations are done within the framework of perturbative quantum chromodynamics (pQCD), and they involve extending fitted parton distribution functions (PDFs) into domains in Bjorken $x$ much below those now accessible experimentally, and into domains in which linear pQCD evolution [9] is of questionable applicability. Other physical phenomena are expected to alter the $x$ dependence in this very small $x$ region [10], although a complete analytic solution does not yet exist.

New, alternative methods of extrapolation in $x$ are of significant interest, both theoretically and for phenomenological applications. Imposition of the Froissart [11] unitarity and analyticity constraints on inclusive deep-inelastic cross sections [12] leads to the expectation that the $x$ dependence of the proton structure function $F_2^p(x,Q^2)$ should grow no more rapidly at very small $x$ than $\ln^2(1/x)$. This relatively slow growth may be contrasted with the more rapid inverse power dependence characteristic of PDFs. Excellent fits to data were obtained [12] for $x < 0.1$ with an assumed logarithmic expansion, for a wide range of virtuality $Q^2$. We explore in this Letter the consequences of the Froissart logarithmic form for UHE neutrino phenomena, computing both neutral and charged current cross sections. In doing so, rather than working with parton distribution functions for the decomposition into quark and antiquark contributions, we devise and test a procedure based directly on experimental $F_2^p$ data. We obtain excellent agreement with extrapolations based on the CTEQ4-DIS parton densities in the neutrino energy range less than $10^8$ GeV. However, we predict an important departure for larger energies, with our neutrino cross sections being about a decade smaller at the highest energies. At the very least, our results suggest that estimates that fall between ours and those obtained from PDF extrapolations be used for guidance in the consideration of new experiments.

Neutrino-isoscalar nucleon cross sections: In the standard parton model the inclusive differential cross section for the charged current (CC) reaction $\nu + N \to \ell^- + X$ on an isoscalar nucleon $N = (n + p)/2$ and the neutral current (NC) cross section $\nu + N \to \nu + X$, where in both cases, $\ell = e, \mu, \tau$, is

$$\frac{d^2\sigma}{dx dy}(E_\nu) = \frac{2G_F^2mE_\nu}{\pi} \left(\frac{M_V^2}{Q^2 + M_V^2}\right)^2 \times \left[xq_i(x, Q^2) + x\bar{q}_i(x, Q^2)(1 - y)^2\right],$$

(1)

where $-Q^2$ is the invariant squared momentum transfer between the incoming neutrino and the outgoing muon, $m$ is the proton mass, and $G_F$ is the Fermi coupling constant. The intermediate vector boson mass, $M_V$, is $M_W = 80.4$ GeV for CC and $M_Z = 91.2$ GeV for NC. Symbols $q_i$ and $\bar{q}_i$, $i = CC, NC$, are linear combinations of quark and antiquark PDFs. The Bjorken scaling variables, where $\nu = E_\nu - E_\ell$ is the energy loss in the laboratory frame, are given by

$$x \equiv \frac{Q^2}{2mv}, \quad y \equiv \frac{\nu}{E_\nu}, \quad 0 \leq x, y \leq 1.$$  

(2)

Charged current cross section: With valence and sea
quark distributions denoted by subscripts $v$ and $s$, respectively, the relevant PDFs in Eq. (1) are

$$q_{CC}(x, Q^2) = \frac{u_v(x, Q^2) + d_v(x, Q^2)}{2} + \frac{u_s(x, Q^2) + d_s(x, Q^2)}{2} + s_s(x, Q^2) + b_s(x, Q^2),$$

and

$$\bar{q}_{CC}(x, Q^2) = \frac{u_s(x, Q^2) + d_s(x, Q^2)}{2} + c_s(x, Q^2) + t_s(x, Q^2),$$

where $u, d, c, s, t$, and $b$ represent the contributions from the up, down, charm, strange, top, and bottom flavors.

**Neutral current cross section:** The relevant PDFs in Eq. (1) involve chiral couplings $L_u = 1 - \frac{1}{2} \sin^2 \theta_W, L_d = -1 + \frac{3}{2} \sin^2 \theta_W, R_u = -\frac{3}{2} \sin^2 \theta_W, R_d = \frac{3}{2} \sin^2 \theta_W$, where $\sin^2 \theta_W = 0.226$ is the weak mixing parameter. For details, see Ref. [7].

**Kinematics:** Replacing $Q^2$ in Eq. (1) by $Q^2 = 2m_E x y$, we obtain an expression in terms of $E_v, x$ and $y$. We choose to integrate first over $y$. To avoid singularities in the integration, we introduce $Q_{\text{min}}^2 = 0.01 \text{ GeV}^2$, such that $Q^2 = 2m_E x y \geq Q_{\text{min}}^2$. This defines $x_{\text{min}}$, the $x$-integration minimum, as $x_{\text{min}} = Q_{\text{min}}^2 / (2m_E v)$. Thus, for $x_{\text{min}} \leq x \leq 1$, our integration limits for $y$ are $y_{\text{min}} = x_{\text{min}} / x \leq y \leq 1$.

The vector boson propagator, $(M_V^2 / (Q^2 + M_V^2))^2$, essentially fixes an “effective” $x$ at $x_{\text{eff}} \sim M_V^2 / (2m_E v)$. For $E_v = 10^{12} \text{ GeV}$, this means we must explore quark distributions having $x_{\text{eff}} \sim 5 \times 10^{-9}$, at $Q^2 \sim M_V^2 \sim 10,000 \text{ GeV}^2$, both of which involve enormous extrapolations from currently available structure function data. At these energies, the propagator also serves to make the calculation insensitive to the choice of $Q_{\text{min}}^2$.

**Analytic expression for the structure function:** In prior work [12], it was shown that an excellent fit to the DIS structure function for $x \leq x_P$, is given by

$$F_2^p(x, Q^2) = (1 - x) \left( \frac{F_P}{1 - x P} + A(Q^2) \ln \left[ \frac{x P}{1 - x} \right] \right) + B(Q^2) \ln^2 \left[ \frac{x P}{1 - x} \right],$$

where

$$A(Q^2) = a_0 + a_1 \ln Q^2 + a_2 \ln^2 Q^2,$$

$$B(Q^2) = b_0 + b_1 \ln Q^2 + b_2 \ln^2 Q^2.$$ (6)

The fitted numerical values of $a_j$ and $b_k$ and their uncertainties may be found in Ref. [12]; $F_P = 0.41$, and $x_P = 0.09$.

The bulk of the neutrino cross section comes from exceedingly small $x$. For large $x$, where $x_P \leq x \leq 1$, it suffices to approximate the proton structure function by

$$F_2^p(x, Q^2) = \frac{F_P}{x P (1 - x P)} x^{\alpha(Q^2)} (1 - x)^3,$$ (7)

where the exponent $\alpha(Q^2)$ is chosen so that the first derivatives of Eq. (6) and Eq. (7) are equal at $x = x_P$.

This choice satisfies the spectator valence quark counting rule $F_2^p(x) \to 0$ as $(1 - x)^3$ as $x \to 1$. Numerical analysis shows that this choice has the important consequence that the integral of the proton structure function over $x$ is nearly constant over an enormous $Q^2$ range, i.e.,

$$\int_0^1 F_2^p(x, Q^2) \, dx \approx 0.16, \quad 0.1 \leq Q^2 \lesssim 10^5 \text{ GeV}^2.$$ (8)

The constant 0.16 is compatible with results that show that quarks carry $\sim 50\%$ of the momentum in a proton.

The description of $F_2^p(x, Q^2)$ by Eqs. (5) - (8) yields a high quality fit to the HERA inclusive deep-inelastic data for all $x$ and $Q^2$.

**“Wee parton” picture:** We obtain the quark distribution functions in Eq. (1) from a wee parton model for very small Bjorken $x$, having the following features:

- there are essentially only sea quarks at small enough $x$, with negligible valence quark contributions (for earlier use, see Ref. [8]), i.e., we set $u_v(x, Q^2) = d_v(x, Q^2) = 0$.
- all sea quarks give equal contribution (i.e., equipartition), $U(x, Q^2) = u_s(x, Q^2) = u_s(x, Q^2) = d_s(x, Q^2) = d_s(x, Q^2) = c_s(x, Q^2) = c_s(x, Q^2) = e_s(x, Q^2)$.

If only two families contribute ($u, d, c, s$),

$$F_2^p(x, Q^2) = \sum_i c_i^2 x \bar{q}_i(x, Q^2) = \bar{q}_i(x, Q^2), \quad i = 1, \ldots, 4,$$ (9)

or, alternatively,

$$x U(x, Q^2) = \frac{9}{20} F_2^p(x, Q^2),$$ (10)

for $x < x_{\text{max}}$, where $x_{\text{max}} \sim 10^{-3} - 10^{-4}$. If we had used only one family of quarks—$u, d$—or three families—$u, d, c, s, t, b$—instead two families—$u, d, c, s$—we would also find that $x q(x, Q^2) = x \bar{q}(x, Q^2) = \frac{9}{10} F_2^p(x, Q^2)$, so that Eq. (11) for charged currents is independent of the number of families. A similar result is true for the neutral current cross section. Employing this picture, we find that accurate knowledge of $F_2^p(x, Q^2)$ at small $x$ and large $Q^2$ provides the ingredients necessary to calculate the charged and neutral current neutrino cross sections. The fitted form of Eq. (6) is sufficiently accurate to furnish us with quark distribution functions having the needed precision. Using the full squared error matrix for
Eq. (7) for $x_U \leq G$ and the proton structure function that varies as $\ln F$ statistically for $x_U \leq G$.

Also shown, for comparison, are the neutral current cross sections, respectively, for $10^{10} \leq E_\nu \leq 10^{14}$ GeV, based on the CTEQ4-DIS quark distributions.

The solid and dash-dot-dot curves are our CC and NC cross sections, respectively, for $10 \leq E_\nu \leq 10^{14}$ GeV, based on a proton structure function that varies as $\ln F$.

The long dash curve and the dash-dash-dot curve are the Gandhi et al. CC and NC cross sections, respectively, for $10 \leq E_\nu \leq 10^{14}$ GeV, based on the CTEQ4-DIS quark distributions.

the structure function determination [12], we find that $F_2^p(x = 10^{-8}, Q^2 = 6400 \text{ GeV}^2) = 24.84 \pm 0.17$, a fractional statistical accuracy of only $\sim 0.7\%$. This very small uncertainty due to parameter errors assumes, of course, the validity of our $\ln^2(1/x)$ model at very small $x$.

**Charged current cross section evaluation:** For our model, $x\sigma_{CC}(x, Q^2) = x\sigma_{CC}(x, Q^2) = 2xU(x, Q^2)$. Thus $xU(x, Q^2) = \frac{9}{20}F_2^p(x, Q^2)$ and Eq. (11) simplifies to

$$\frac{d^2\sigma_{CC}}{dx dy}(E_\nu) = \frac{2G_F^2mE_\nu}{\pi} \left( M_W^2 \right)^2 \times \left[ \frac{9}{10}F_2^p(x, Q^2) \right] \left( 2 - 2y + y^2 \right).$$

with $F_2^p(x, Q^2)$ given by Eq. (16) for $0 \leq x \leq x_P$ and Eq. (17) for $x_P < x \leq 1$.

Results of a direct double integration of Eq. (11), with $Q_{\text{min}}^2 = 0.01 \text{ GeV}^2$, for the neutrino energy range $10 \leq E_\nu \leq 10^{14}$ GeV, are given in Table I and shown in Fig. 1 as the solid curve. Also shown, for comparison, are the results of Gandhi et al. [7] for the CC cross section with the quark distributions from CTEQ4-DIS [14]. The Gandhi et al. curve—the long dash curve—covers the energy range $10 \leq E_\nu \leq 10^{12}$ GeV. The agreement up to neutrino energies $\lesssim 10^8 \text{ GeV}$ is striking.

**Neutral current cross section evaluation:** For our model, the NC quark distributions in Eq. (11) are

$$xq_{NC}(x, Q^2) = x\bar{q}_{NC}(x, Q^2) = 2xU(x, Q^2) \times \left( L_u^2 + L_d^2 + R_u^2 + R_d^2 \right)$$

$$= 4(1 - 2\sin^2 \theta_w + \frac{20}{9} \sin^4 \theta_w)U(x, Q^2)$$

$$= 2.65xU(x, Q^2) = 1.19F_2^p(x, Q^2),$$

where Eq. (10) is used in the last step. The neutral current cross section simplifies considerably. For direct comparison with the charged current cross section of Eq. (11), it can be rewritten as

$$\frac{d^2\sigma_{NC}}{dx dy}(E_\nu) = \frac{2G_F^2mE_\nu}{\pi} \left( M_W^2 \right)^2 \times \left[ 0.298F_2^p(x, Q^2) \right] \left( 2 - 2y + y^2 \right).$$

To the extent that the $Z$ propagator is somewhat less restrictive as a cutoff than the $W$ propagator, comparison of Eq. (13) and Eq. (11) shows that the ratio of the NC cross section to the CC cross section is $\gtrsim 0.298/0.9 = 0.33$, independent of energy. Numerical evaluation gives 0.40 at $E_\nu = 10^7 \text{ GeV}$, slightly higher because of the $Z$ propagator. Our NC cross section for isoscalar nucleons is given in Table I and shown in Fig. 1 as the dash-dot-dot curve, plotted in the energy interval $10 \leq E_\nu \leq 10^{14}$ GeV. The Gandhi et al. CC NC cross section, for $10 \leq E_\nu \leq 10^{12}$ GeV, is the dash-dash-dot curve. Again, the agreement is excellent up to $E_\nu \sim 10^8 \text{ GeV}$.

**Table I: Neutrino CC and NC total cross sections, with neutrino energy $E_\nu$ in GeV and cross sections in cm$^{-2}$.**

| $E_\nu$ (GeV) | $\sigma_{CC}$ | $\sigma_{NC}$ | $E_\nu$ (GeV) | $\sigma_{CC}$ | $\sigma_{NC}$ |
|--------------|---------------|---------------|--------------|---------------|---------------|
| $10^7$       | $5.93 \times 10^{-38}$ | $1.96 \times 10^{-38}$ | $10^8$       | $4.49 \times 10^{-33}$ | $1.83 \times 10^{-33}$ |
| $10^8$       | $5.51 \times 10^{-37}$ | $1.82 \times 10^{-37}$ | $10^9$       | $8.90 \times 10^{-33}$ | $3.70 \times 10^{-33}$ |
| $10^9$       | $5.01 \times 10^{-36}$ | $1.67 \times 10^{-36}$ | $10^{10}$    | $1.58 \times 10^{-32}$ | $6.63 \times 10^{-33}$ |
| $10^{10}$    | $3.80 \times 10^{-35}$ | $1.32 \times 10^{-35}$ | $10^{11}$    | $2.57 \times 10^{-32}$ | $1.09 \times 10^{-32}$ |
| $10^{11}$    | $1.91 \times 10^{-34}$ | $7.03 \times 10^{-35}$ | $10^{12}$    | $3.92 \times 10^{-32}$ | $1.67 \times 10^{-32}$ |
| $10^{12}$    | $6.87 \times 10^{-34}$ | $2.65 \times 10^{-34}$ | $10^{13}$    | $5.68 \times 10^{-32}$ | $2.44 \times 10^{-32}$ |
| $10^{13}$    | $1.94 \times 10^{-33}$ | $7.74 \times 10^{-34}$ | $10^{14}$    | $7.92 \times 10^{-32}$ | $3.40 \times 10^{-32}$ |

**Robustness of cross sections:** The differential cross sections were evaluated numerically in Mathematica and found to be numerically stable, essentially independent of $Q_{\text{min}}^2$ and the methods of integration. The dependence of the cross sections on the functional form of $F_2^p(Q^2, x)$ for $1 \geq x \geq x_P$ was tested by setting $F_2^p(Q^2, x) \sim x(1 - x)^3$ for large $x$, and the change was found to be $\sim 2\%$ at $E_\nu = 10^8$ and $\sim 0$ at $E_\nu = 10^{12}$ GeV. If we set $F_2^p(Q^2, x) = 0$ for $1 \geq x \geq x_P$, an extreme case, we find the changes to be $6\%$ at $E_\nu = 10^{10}$ GeV and $\sim 0$ at $E_\nu = 10^{12}$ GeV. We tested our equipartition hypothesis by changing the strengths of the heavy sea quark distributions such that

$s_u(x, Q^2) = \bar{s}_u(x, Q^2) = 0.96U(x, Q^2)$

c s(x, Q^2) = \bar{c}_s(x, Q^2) = 0.80U(x, Q^2),$ (14)

similar to the distributions used by CTEQ. This change gives us cross sections that are $\sim 6\%$ greater at $E_\nu = 10^8$.
GeV and \( \sim 3\% \) greater at \( E_\nu = 10^{12} \) GeV. These variations are negligible compared to the very large differences with respect to the cross sections of Gandhi et al. at the highest neutrino energies. Our calculations are numerically stable with regard to our choice of \( x_{\text{min}} \) in the integration, and thus, insensitive to our choice of \( Q_{\text{min}} = 0.01 \) GeV

Conclusions: We compute ultra-high energy neutrino cross sections based on an extrapolation to very small Bjorken \( x \) of the logarithmic Froissart dependence in \( x \) shown previously to provide an excellent fit to the measured proton structure function \( F_2^p(x, Q^2) \) over a broad range of the virtuality \( Q^2 \). In order to devise expressions for the neutral current and the charged current cross sections, we first extract quark and antiquark contributions based on a simple equi-partition wee parton picture valid for \( x_{\text{max}} \lesssim 10^{-3} - 10^{-4} \) or \( E_{\nu} \lesssim 3 \times 10^6 - 3 \times 10^7 \) GeV. However, it is gratifying to see in Fig. [1] that we are in excellent agreement with calculations based on CTEQ4-DIS parton densities over the much larger energy range 10 \( \leq \ E_\nu \leq \ 10^8 \) GeV. The two sets of expectations diverge for \( E_\nu \gtrsim 10^8 \) GeV, as may be expected since our proton structure functions agree with those from CTEQ only for \( x \)-values greater than \( 10^{-3}[12] \). The increasing differences for \( x \lesssim 10^{-3} \) reflect the fundamental difference in the assumed functional forms for the \( x \) dependence, in our case a form that is constrained to increase no more rapidly than \( \ln^2(1/x) \), in contrast to the inverse power growth in the CTEQ case. For large neutrino energies—above \( 10^9 \) GeV—where much smaller \( x \) is sampled, our Froissart-bound-model neutrino cross sections are as much as a decade smaller than those based on a pQCD extrapolation, a consequence of the fact that our structure function \( F_2^p(x, Q^2) \) is significantly smaller at small \( x \). The very small \( x \) region is also the region where our wee parton picture is most robust.

The region of very small \( x \) is a region of growing interest theoretically. It is a region in which non-perturbative physics is expected to set in [10] and in which linear DGLAP pQCD evolution is not expected to hold. While we cannot claim that logarithmic dependence on \( x \) will result from a first-principles solution to small \( x \) dynamics, neither can we expect an inverse power form to survive. The logarithmic form we provide offers a fit to data over the range in \( x \) and \( Q^2 \) where it has been tested. It extrapolation to energies relevant in UHE neutrino studies provides estimates for event rates that should be taken into serious consideration for the planning and data analysis of new experiments.

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[1] A. Achterberg et al. [IceCube Collaboration], Phys. Rev. D 76, 027101 (2007); 507 (2004); A. Achterberg et al. [IceCube Collaboration], arXiv:astro-ph/0705.1315; G. Anassontzis et al. [NESTOR Collaboration], Nucl. Inst. Meth. 479, 439 (2002); J. A. Aguilar et al. [ANTARES Collaboration], Nucl. Inst. Met. A 570, 107 (2007); C. Spiering et al. [Baldaak collaboration], Nucl. Phys. B Proc. Suppl. 138, 175 (2005).

[2] I. Kravchenko et al. [RICE Collaboration], Phys. Rev. D 73, 082002 (2006); S.W. Barwick et al. [ANITA Collaboration], Phys. Rev. Lett. 96, 171101 (2006); N. Letween et al. [FORTE Collaboration], Phys. Rev. D 69, 013008 (2004); P. Gorham et al. [GLUE Collaboration], Phys. Rev. Lett. 93, 041101 (2004).

[3] "Limits on the Diffuse Neutrino Flux from the HiRes Data", W. Deng et al [HiRes Collaboration], Proceedings of the 29th International Cosmic Ray Conference; X. Berton et al., Astropart. Phys. 17, 183 (2002).

[4] P. Gorham et al., Phys. Rev. D 72, 23002 (2005); H. Landsman [AURA Collaboration], Nucl. Phys. B Proceedings Suppl. 168, 268 (2007).

[5] S. Bottai [EUSO Collaboration], Proceedings of the 27th International Cosmic Ray Conference, Aug. 2001, Hamburg, p. 848 (2001).

[6] Yu. Andreev, v. Berezinsky and A. Smirnov, Phys. Lett. B 84, 247 (1979); M. H. Reno and C. Quigg, Phys. Rev. D 37, 657 (1987); R. Gandhi, C. Quigg, M. H. Reno and I. Sarcevic, Astropart. Phys. 5, 81 (1996).

[7] R. Gandhi, C. Quigg, M. H. Reno and I. Sarcevic, Phys. Rev. D 58, 093009 (1998).

[8] D. W. McKay and J. P. Ralston, Phys. Lett. B, 187, 103, (1986); G. M. Frichter, D.W. McKay and J. P. Ralston, Phys. Rev. Lett. 74, 1508, (1994).

[9] V. N. Gribkov and L. N. Lipatov, Sov. J. Nucl. Phys. 15, 438 (1972), 15, 675 (1972); Yu. L. Dokshitzer, Sov. Phys. JETP 46, 641 (1977); G. Altarelli and G. Parisi, Nucl. Phys. B 126, 298 (1977).

[10] L.V. Gribov et al., Phys. Rep. 100, 1 (1983); A. Capella et al., Phys. Rev. D 63, 054010 (2001); G. Sotey, Phys. Rev. D 71, 076001 (2005); R. C. Brower et al. [hep-th/0603115, arXiv:0702.2148] [hep-th]; Y. Hatta, E. Iancu, and A. H. Mueller, arXiv:0710.2148 [hep-th].

[11] M. Froissart, Phys. Rev. D 123, 1053 (1961).

[12] M. M. Block, E. L. Berger, and C-II Tan, Phys. Rev. Lett. 97, 252003 (2006); E. L. Berger, M. M. Block, and C-II Tan, Phys. Rev. Lett. 98, 242001 (2007).

[13] R. Blankenbecler and S.J. Brodsky, Phys. Rev. D 10, 2973 (1974); J.F. Gunion, Phys. Rev. D 10, 242 (1974); S.J. Brodsky and G.P. Lepage, in Proc. 1979 Summer Inst. on Particle Physics, SLAC (1979).

[14] CTEQ Collaboration, H. Lai et al., Phys. Rev. D 55, 1280 (1997), arXiv:hep-ph/0611254.