Teleportation of Accelerated Information

N. Metwally
Math. Dept., Faculty of Science, Aswan University, Aswan, Egypt.
Math. Dept., College of Science, University of Bahrain, Bahrain

Abstract

A theoretical quantum teleportation protocol is suggested to teleport accelerated and non-accelerated information over different classes of accelerated quantum channels. For the accelerated information, it is shown that the fidelity of the teleported state increases as the entanglement of the initial quantum channel increases. However as the difference between the accelerated channel and the accelerated information decreases the fidelity increases. The fidelity of the non accelerated information increases as the entanglement of the initial quantum channel increases, while the accelerations of the quantum channel has a little effect. The possibility of sending quantum information over accelerated quantum channels is much better than sending classical information.

1 Introduction

Dynamics of entangled qubits in non-inertial frames is investigated in the literature, from different points of view. For example, Unruh and Wald [1] have investigated the nature of the interaction between a quantum field and accelerating particle detector. The entanglement dynamics of Dirac fields in non-inertial frames is studied by Alsing et al. [2]. Generating entangled photons from the vacuum by accelerated measurementshas been investigated by Han et. al [3]. In [4], the effect of quantum decoherence generated by the Unruh effect on the dynamics of a maximum entangled state is discussed. Said and Adami [5] have investigated the Einstein-Podolsky-Rosen correlation in Kerr-Newman space time. Montero et al. [6] have studied the case of families of fermionic field states in non inertial frames and the dynamics of multipartite entanglement of fermionic systems in non-inertial frames is considered by Wang and Jing [7]. The classical and quantum correlations of scalar field in the inflationary universe have been discussed by Nambu and Ohsumi [8]. The analogy between static quantum emitters coupled to a single mode of quantum field and accelerated Unruh-DeWitt detectors is shown by Rey et al. [9]. Demonstrating entanglement generation between mode pairs of a quantum field in a single, rigid cavity that moves nonuniformly in Minkowski space-time has been investigated by Friis et al. [10]. The correlations between the accelerated field modes and the modes in an inertial reference cavity are discussed in [11].

Implementation of quantum information tasks between accelerated partners represents one of the most important topics in the context of quantum communication. One of these tasks is quantum teleportation, which enables to send information between two users without sending the source itself which carries this information [12]. Some efforts have been achieved to perform quantum teleportation in non-inertial frames. For example, the possibility of performing some quantum information protocols in non-inertial is investigated by Enk et al. [13]. Teleportation by using three parties with an accelerated receiver is investigated by Jin et. al [14]. Quantum teleportation in the presence of Unruh effect is investigated by Landulfo et al. [15].

Recently, Metwally [16] investigated the dynamics of a general two qubits system in non inertial frame analytically, by assuming that both of its subsystems are differently accelerated. The most useful classes of travelling entangled channels in non-inertial frames are
classified. The conclusions was, the maximum entangled channels and a pure state with large
degree of entanglement could be used as a quantum channel to perform quantum teleporta-
tion. This motivated to suggest a theoretical quantum teleportation protocol between two
users share an accelerated channels. Our protocol is different from the other in two aspects:
firstly we assume that the subsystems of the quantum channel are differently accelerated and
two classes of useful quantum channels for quantum teleportation are considered, namely
maximum and partial entangled channels [16]. Second, the desired teleported state is as-
umumed to be accelerated in the same frame of the sender. Also, the possibility of teleporting
non-accelerated information by using accelerated channels is discussed.

This paper is organized as follows: in Sec.2, we review the dynamics of quantum channels
between two users in non-inertial frame, where two classes of initial quantum channels are
considered: generic pure state and maximum entangled state. In Sec.3, the generated entan-
gled channels between the users are employed as quantum channels to perform the original
quantum teleportation protocol. Two classes are considered for the teleported state: one is
accelerated with a uniform acceleration while the second is non accelerated. The fidelity of
the teleported state is quantified for different accelerated channels. Finally, the results are
summarized in Sec.4.

2 Accelerated channels

Let’s assume that two users Alice and Rob that share a two-qubit state, where each qubit of
this state (Alice and Rob’s qubit) travels in non-inertial frames with a uniform acceleration.
In this context, it is assumed that the source supplies the users with two types of initially
entangled state: maximum or partial entangled pure state.

Let the partners initially share pure state of the form,

\[ \rho_p = \frac{1}{4} \begin{pmatrix}
(1-q) & -p & p & -(1-q) \\
p & (1+q) & -(1+q) & p \\
p & -(1+q) & (1+q) & -p \\
-(1-q) & p & -p & (1-q)
\end{pmatrix}, \]

(1)

where \( q = \sqrt{1-q^2} \). This channel is represented by one parameter, \( p \). For \( p = 0 \) one gets a
maximum entangled state of Bell’s type. However for larger values of \( p \), the pure state turns
into a partial entangled state and completely separable for \( p = 1 \) [17] [18]. To investigate
the teleportation by using accelerated channels, we find the new channel in the non-inertial
frame by using the Unruh modes transformation [19] [20],

\[ |0\rangle_i = C_i |0\rangle_I |0\rangle_{II} + S_i |1\rangle_I |1\rangle_{II}, \]
\[ |1\rangle_i = |1\rangle_I |0\rangle_{II}, \]

(2)

where \( C_i = \cos r_i, S_i = \sin r_i, \) with \( tan r_i = e^{-\pi \omega_i t_i}, \) \( a_i \) is the acceleration, \( \omega_i \) is the frequency
of the travelling qubits, \( c \) is the speed of light and \( i = A \) (Alice), \( B \) (Bob). The accelerated
channel in the region \( I \) is given by [16],
The elements $\rho_{ij}$ and $i, j = 1..4$ are given by,

$$
\begin{align*}
\rho_{11} &= \frac{1}{4} \left[ 1 - q \right] \left( 1 + 2 \cos 2r_1 \right) + \frac{3 - q}{4} \cos 2r_2, \\
\rho_{13} &= \frac{p}{8} \cos r_1 \left[ 1 + \cos 2r_2 \right], \\
\rho_{21} &= -\frac{p}{8} \cos 2r_2 \left[ 3 - \cos 2r_1 \right], \\
\rho_{23} &= -\frac{1 + q}{4} \cos r_1 \cos r_2, \\
\rho_{31} &= \rho_{13}, \\
\rho_{33} &= \frac{1}{4} \left[ 1 + q \right] \cos 2r_1 + \frac{3 + q}{4} \left( 1 + \cos 2r_2 \right), \\
\rho_{41} &= \rho_{14}, \\
\rho_{44} &= \frac{1}{4} \left[ 9 - q - \frac{1 + q}{2} \cos 2r_1 - \frac{3 + q}{4} \cos 2r_2 \right].
\end{align*}
$$

(4)

### 3 Teleportation

In this section, it is assumed that the partners share the accelerated state (3) as quantum channel to perform quantum teleportation [12]. A source supplies Alice with unknown information coded in Rob’s state as:

$$
|\psi\rangle_R = \alpha |0\rangle + \beta |1\rangle, \tag{5}
$$

where $|\alpha|^2 + |\beta|^2 = 1$. The description of the suggested protocol is described in Fig. (1). It is assumed that Alice’s qubit and Rob’s qubit are accelerated in the same frame with different accelerations. In this context we consider two situations:

1. The teleported information is accelerated

   In this case, we assume that the coded information in the state (5) is accelerated according to Eq.(2), with an acceleration $r_3$. The density operator in the region I, which carries the unknown information is given by,

   $$
   \rho_R = |\alpha|^2 \cos^2 r_3 |0\rangle \langle 0| + |\alpha \sin r_3 + \beta|^2 |1\rangle \langle 1|. \tag{6}
   $$

   To teleport the unknown information which is coded on the state (6), Alice and Bob will use the accelerated state (3) in the region I as an quantum channel. For this purpose the partners perform the original quantum teleportation protocol [12] as follows:

   (a) Alice performs CNOT gate on her qubit and the given qubit state (5). After performing this operation, the final state of the system is given by,

   $$
   \rho_s = CNOT \rho_R \otimes \hat{\rho}_p CNOT \tag{7}
   $$

   where the elements $\hat{\rho}_p$ and $i, j = 1..4$ are given by,

   $$
   \hat{\rho}_p = 
   \begin{pmatrix}
   \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\
   \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\
   \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\
   \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44}
   \end{pmatrix}, \tag{3}
   $$

   and

   $$
   \rho_{11} = \rho_{22} = \frac{1}{4} \left[ 1 - q \right] \left( 1 + 2 \cos 2r_1 \right) + \frac{3 - q}{4} \cos 2r_2, \\
   \rho_{12} = -\frac{p}{8} \cos r_2 \left[ 3 - \cos 2r_1 \right], \\
   \rho_{13} = \frac{p}{8} \cos r_1 \left[ 1 + \cos 2r_2 \right], \\
   \rho_{14} = -\frac{1 - q}{4} \cos r_1 \cos r_2, \\
   \rho_{21} = -\frac{p}{8} \cos 2r_2 \left[ 3 - \cos 2r_1 \right], \\
   \rho_{22} = \frac{1}{4} \left[ q - 1 + 2(q + 1) \cos 2r_1 + (q + 3) \cos 2r_2 \right], \\
   \rho_{23} = -\frac{1 + q}{4} \cos r_1 \cos r_2, \\
   \rho_{24} = \frac{p}{8} \cos r_1 \left[ 3 - \cos 2r_2 \right], \\
   \rho_{31} = \rho_{13}, \\
   \rho_{32} = \rho_{23}, \\
   \rho_{33} = \frac{1}{4} \left[ 1 + q \right] \cos 2r_1 + \frac{3 + q}{4} \left( 1 + \cos 2r_2 \right), \\
   \rho_{34} = -\frac{p}{8} \cos r_1 \cos r_2 \left[ 1 + \cos 2r_1 \right], \\
   \rho_{41} = \rho_{14}, \\
   \rho_{42} = \rho_{24}, \\
   \rho_{44} = \frac{1}{4} \left[ 9 - q - \frac{1 + q}{2} \cos 2r_1 - \frac{3 + q}{4} \cos 2r_2 \right].
   $$

(4)
Figure 1: Mainkowski space time diagram for Alice, Bob and Rob who has the unknown information. An accelerated users Alice and Bob travel with uniform accelerations $r_1$ and $r_2$ respectively in the region I. Theses users are causally disconnected from their Anti-users in the region II. The unknown information is coded in Rob’s qubit which is accelerated with another different acceleration $r_3$. Quantum teleportation is achieved in the region I, where Alice’s aim is teleporting Rob’s state to Bob.

(b) Alice applies Hadamard gate on the given qubit follows by performing Bell measurements on the given qubit and her own qubit. Then, she transmits her results to Bob via classical channel.

(c) Depending on the results of Alice’s measurements, Bob performs a suitable unitary operations on his own qubit to get the original message. If Alice measures the state $|\phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, then Bob will get the state,

$$\rho_B = \mu_{00}|0\rangle\langle 0| + \mu_{01}|0\rangle\langle 1| + \mu_{10}|1\rangle\langle 0| + \mu_{11}|1\rangle\langle 1|,$$

(8)

where,

$$\mu_{00} = \frac{1}{4}\left((B_1 + B_2)(g_{11} + g_{33}) + (B_1 - B_2)(g_{13} + g_{31})\right),$$

$$\mu_{01} = \frac{1}{4}\left((B_1 + B_2)(g_{12} + g_{34}) + (B_1 - B_2)(g_{14} + g_{32})\right),$$

$$\mu_{10} = \frac{1}{4}\left((B_1 + B_2)(g_{21} + g_{43}) + (B_1 - B_2)(g_{23} + g_{41})\right),$$

$$\mu_{11} = \frac{1}{4}\left((B_1 + B_2)(g_{22} + g_{44}) + (B_1 - B_2)(g_{24} + g_{42})\right),$$

(9)

and $B_1 = |\alpha|^2 \cos^2 r_3$, $B_2 = |\alpha \sin r_3 + \beta|^2$. The fidelity of the teleported information which is coded in the state(6) is given by,

$$F_{Pa} = B_1\mu_{00} + B_2\mu_{11}.$$

(10)
Figure 2: The fidelity $F_{P_a}$ of the accelerated teleported state, where the partners used a quantum channel initially prepared in maximum entangled state, i.e. $p = 0$. The dot, dash and solid curves are plotted for $r_3 = 0.1, 0.3$ and $0.8$ respectively while i.e $r_1 = r_2 = 0.7$. The most lower curve represents the fidelity of the non accelerated information $F_{P_s}$.

2. The teleported information is non-accelerated

In this case the teleported information is coded in the state,

$$\rho_{Rs} = |\alpha|^2 |0\rangle\langle 0| + \alpha^*\beta |0\rangle\langle 1| + \alpha^*\beta |1\rangle\langle 0| + |\beta|^2 |1\rangle\langle 1|,$$

(11)

Alice and Bob perform the original teleportation protocol as described above. At the end of the protocol Bob will get the state

$$\rho_B = \nu_{00}|0\rangle\langle 0| + \nu_{01}|0\rangle\langle 1| + \nu_{10}|1\rangle\langle 0| + \nu_{11}|1\rangle\langle 1|,$$

(12)

where,

$$\nu_{00} = \frac{1}{4}\left\{ \varrho_{11} + \varrho_{33} + (|\alpha|^2 - |\beta|^2)(\varrho_{13} + \varrho_{31}) - \lambda^+\varrho_{12} - \lambda^-\varrho_{13} - \varrho_{33} \right\},$$

$$\nu_{01} = \frac{1}{4}\left\{ \varrho_{22} + \varrho_{44} + (|\alpha|^2 - |\beta|^2)(\varrho_{23} + \varrho_{41}) - \lambda^+\varrho_{21} - \lambda^-\varrho_{23} - \varrho_{41} \right\},$$

$$\nu_{10} = \frac{1}{4}\left\{ \varrho_{21} + \varrho_{43} + (|\alpha|^2 - |\beta|^2)(\varrho_{23} + \varrho_{41}) - \lambda^+\varrho_{22} - \lambda^-\varrho_{23} - \varrho_{41} \right\},$$

$$\nu_{11} = \frac{1}{4}\left\{ \varrho_{22} + \varrho_{44} + (|\alpha|^2 - |\beta|^2)(\varrho_{24} + \varrho_{42}) - \lambda^+\varrho_{22} - \lambda^-\varrho_{24} - \varrho_{42} \right\},$$

(13)

with $\lambda^+ = \alpha^*\beta + \alpha^+\beta^*$ and $\lambda^- = \alpha\beta^* - \alpha^*\beta$. In this case the fidelity of the teleported state is given by,

$$F_{P_s} = |\alpha|^2 \nu_{00} + \alpha^*\beta \nu_{01} + \alpha^*\beta \nu_{10} + |\beta|^2 \nu_{11}.$$

(14)

The fidelity $F_{P_a}$ of the accelerated teleported state and the non accelerated teleported state $F_{P_s}$ are described in Fig.(1). In this case, we assume that the partners share a quantum channel initially prepared in maximum entangled states (MES). The two qubits are accelerated with an equal acceleration i.e. $r_a = r_b = 0.7$, while the teleported state is accelerated with different accelerations. The fidelities are plotted against the parameter $\alpha$ in Fig. 2.
Figure 3: The same as Fig. (1) but the partners share an initially a pure state. The most upper three curves represent the fidelity of the accelerated teleported state $F_{Pa}$, while the most lower curves represent the fidelity of non-accelerated teleported state $F_{Ps}$. The solid, dash dot are for $p = 0.1, 0.2, 0.3$ respectively, where $r_1 = r_2 = r_3 = 0.7$.

which represents the structure of the teleported state. The three upper curves represent the fidelity of the accelerated teleported state $F_{Pa}$, while the lower one represents the fidelity of the non-accelerated teleported state $F_{Ps}$. From this figure, it is clear that, $F_{Pa}$ increases as the acceleration $r_3$ of the teleported state increases. However the fidelity $F_{Pa}$ decreases if the difference between the channel’s accelerations and the telepo rted state’s acceleration increases. The fidelity $F_{Ps}$ of the non-accelerated teleported state( the most lower curves) decreases as $\alpha$ increases. Also, it is clear that all the fidelities coincide. This shows that the behavior of $F_{Ps}$ depends on the entanglement of the accelerated channel.

In Fig.(3), the behavior of $F_{Pa}$ and $F_{Ps}$ for different entangled pure states is displayed. The fidelity $F_{Pa}$ of the accelerated information behaves similar to that depicted for MES in Fig.(2). In this figure, it is assumed that the accelerations of the channel and the teleported state are equal i.e., $r_1 = r_2 = r_3 = 0.7$. The fidelity increases for small values of $p$, namely, the partners starting with entangled channels have a large degree of entanglement. However for larger values of $p$, the degree of entanglement of the quantum channel decreases and consequently the fidelity of the teleported state decreases for both accelerated and non-accelerated teleported states.

From Figs.(2&3), we conclude that, for the accelerated information the fidelity $F_{Pa}$ depends on the difference between the acceleration of the quantum channel and the acceleration of the teleported state as well as the degree of entanglement of the quantum channel between the partners. However for the non-accelerated teleported state, the fidelity $F_{Ps}$, depends only on the degree of entanglement. The accelerated maximum entangled channel teleports the accelerated and non-accelerated information with larger fidelity compared with that teleported by partial entangled channels.

The behavior of the teleported information depends on the parameter $\alpha$, where the initial information is coded with a probability $\alpha^2$ in the state $|0\rangle$ and with probability $\beta = \sqrt{1 - \alpha^2}$ in the state $|1\rangle$. For $\alpha$ or $\beta$ equals ”1”, this is equivalent to coding classical information, while for $0 < \alpha < 1$ and $0 < \beta < 1$, this corresponds to coding quantum information. It is clear that, teleportating quantum information is much better than teleportating classical information over an accelerated channel.
Figure 4: The fidelity of the accelerated information which is initially coded in the state $|\psi_u\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, where the used accelerated channel is prepared initially in (a) maximum entangled state and (b) pure state (Partial entangled state with $p = 0.5$). The solid, dash and dot for $r_1 = r_2 = 0.8$, 0.5 and 0.0001 respectively.

Fig.4, displays the behavior of the fidelity $F_{Pa}$ for different accelerated channels, for $|\alpha|^2 = 0.5$. In Fig.(4a), we assume that the partners share initially a maximum entangled state. For larger values of $r_1$ and $r_2$, the fidelity $F_{Pa}$ increases as $r_3$ increases. However, the fidelity of the teleported state decreases as the acceleration of the teleported state decreases. As shown in Fig.(4b), starting from a partial entangled channel between the partners, the fidelity of the teleported state is smaller than that depicted for its corresponding one in Fig.(4a), where the partners share a MES. Also at $r_3 = 0$, the fidelity of the teleported state $F_{Pa}$ depends on the degree of the quantum channel between the partners. This explain why the initial fidelities at $r_3 = 0$ have different values.

4 Conclusion

Quantum teleportation in non-inertial frames, is investigated by using different classes of accelerated channels between the users: maximum and pure entangled channels. It is shown that, these accelerated quantum channels represent useful classes for quantum teleportation. In this protocol we consider the desired information to be coded either in an accelerated or non-accelerated state. Our results show that the fidelity of the teleported state depends on the initial degree of entanglement of the used quantum channel, the accelerations of the quantum channel, the acceleration of teleported state and the structure of the coded information.

Performing the original quantum teleportation protocol by using a maximum entangled channel is much better than using a partial entangled state. It is shown that, the fidelities of the accelerated and non accelerated teleported state using an accelerated channel prepared in a maximum entangled state is much larger than that depicted for accelerated partial entangled channels. The fidelity of the non-accelerated state depends only on the initial entanglement of the accelerated quantum channels between the two users.

On the other hand, the fidelity of the teleported information increases when the difference between the channel’s acceleration and the teleported state’s acceleration is small. The possibility of sending classical information via quantum accelerated channel is much better
than teleporting quantum information.

In conclusion, one can teleport information between two moving users sharing a maximum or partial accelerated entangled channels. If we can control the accelerations (for channel and teleported state) to be the same, the accelerated information can then be teleported with high fidelity. Sending classical information via accelerated channel is much better than sending quantum information by using same channels.

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