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Threshold condition and non pharmaceutical interventions’s control strategies for elimination of COVID-19

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\textbf{ABSTRACT}

In this work we focus on the eradication of the COVID-19 infection with the help of almost Non Pharmaceutical Interventions(NPIs), using mathematical modelling. First the basic reproduction number $R_0$ is investigated. Then, on the basis of sensitivity test of $R_0$, the most active/sensitive parameters are presented in detail. Non Pharmaceutical Interventions(NPIs) are applied to control the sensitive parameters. The major NPIs are, stay home (isolation), sanitizers (wash hands), Treatment of side effects of infection, like throat infection etc and face mask. These NPIs helps in mitigation and reducing the size of outbreak of the disease. Threshold condition for global stability of the disease free state is investigated. The NPI’s are used in different ratios to formulate a strategy. The results of these strategies are validated using Matlab software.

\textbf{Introduction}

COVID-19 is a highly infectious disease, which is caused by a virus called Severe Acute Respiratory Syndrome coronavirus 2, or SARS-CoV-2. The virus transmission among human population, from person to person is very rapid specially those in close contact (within about 6 feet, or 2 meters). The virus spreads by respiratory droplets released when an infected individual coughs, sneezes or talks. These droplets can be inhaled or directly reach the mouth or nose of a nearby person with medium of air. A person can catch infection from contaminated surfaces, however this isn’t considered to be a main way it spreads through [1].

Coronaviruses infect represents a big family. This family causes different types of infections. The infection ranges from common cold/flu to the most severe infection like severe acute respiratory syndrome and MERS; middle east respiratory syndrome [2]. The novel COVID-19 first emerged in December, 2019, Wuhan, China, in the form of severe cases of pneumonia and respiratory problems. The correct etiology of the infection could not be traced that time. WHO reported the virus as a novel coronavirus (2019-nCoV). The disease was named as severe acute respiratory syndrome coronavirus-2 (SARS-CoV-2). The virus was first identified from a single individual. Subsequently the virus was verified in sixteen more cases [3,4].

It is expected that the virus might be bat origin [5], and the infection transmission might be initiated from a seafood market (Huanan Seafood Wholesale Market) of China [6,9]. Currently 7,597,304 cases of the disease as been confirmed and 423,844 deaths has occurred as of June 12, 2020, world wide [10].

About 75% of the victims of COVID-19 don’t develop symptoms of the disease and recovered naturally [29], 20% of the exposed individuals develop symptoms. The most common symptoms of COVID-19 are tiredness, fever and dry cough. Some patients may have aches and pains, runny nose, nasal congestion, Muscle aches, Chills, Loss of taste or smell or both, Headache, Chest pain and sore throat. Other less common symptoms have been reported, such as rash, nausea, vomiting and diarrhea. These symptoms are usually mild and starts slowly and gradually. Most symptomatic individuals (about 80%) recover from the disease without needing special treatment. In children and young adults, COVID-19 is generally minor. However, for some people it can cause serious illness. This type of severe attack of the virus may cause death. In some cases the attack may result SARS (severe acute respiratory syndrome) or pneumonia. The symptoms of infection appears in 2–14 days [7,15]. The recovery time of mild cases is approximately 2 weeks and in severe/critical cases the recovery may take 3–6 weeks [16]. The individuals who developed severe form of disease, the medium time to
The recovered individuals of disease can have antibodies mainly focused the transmission/spreading of coronavirus or basic reproduction number of coronavirus, $R_0$. The genetic features and some clinical findings of the infection have been reported recently [9,22,23]. International air travel contributed the international spread of the infection. The infection has got global attention regarding its elimination and control [24].

The world whole is highly concerned with drastic future forecast of the disease. The scientists and researchers, therefore focus the development of mathematical model. The model not only helps estimating dynamics of the transmission of the virus but other important forecasts. Recent mathematical modeling includes [6,25–27] and the forecast of the transmission of the virus but other important forecasts. The authors followed an incrimination growth rate and the serial intervals. Wu et al. in their study focus the forecasting and Newscasting of the novel coronavirus both nationally and internationally. The authors used Markov Chain Monte Carlo methods in their study [25].

Some novel corona models discuss the origin (bat), the route of transmission/spreading (seafood market) and reservoir class [14,28]. Aranganga in his recent work focus the importance of lockdown and Vaccination [11]. For more study on COVID-19 the reader is referred to [12,13,30,31].

We, in this study focus three dynamics of the disease:

- The formulation of mathematical model.
- To investigate the ratio of different interventions to formulate a control strategy.
- To find a rule to stop re-attack of the disease to the community

From the proposed model we find the initial transmission rate of the disease. With help of sensitivity test, we select the parameters playing most important role in transmission of the disease. The sign of the sensitivity indexes help in deciding an increase or decrease in the concerned parameters. We combine different interventions in particular ratio and formulate a strategy. The effect of different strategies on disease control is shown graphically to facilitate strategy selection for the agencies fighting against COVID-19.

The re-attack of COVID-19 is really heartbroken issue of the scientists. To address the issue we find a threshold condition. If the magnitude of interventions is capable of disease elimination and satisfy the threshold condition. Then there is guarantee for global stability of the disease free state in the community.

Model formulation

The population concerned with the disease is divided into the following six compartments:

- $S$: The susceptible human class.
- $E$: The Exposed/latency class.
- $I_1$: The infectious class with disease symptoms.
- $I_2$: The vent Bol class, critically infected individuals kept on ventilator.
- $I_3$: The Asymptomatic infectious class.
- $R$: The Recovered class.
- $W$: The class of contaminated stuff or surfaces.

The susceptible human after catching infection from the infectious humans (both symptomatic and asymptomatic) or contaminated surfaces/stuff moves to the exposed class ($E$). The 20% of the exposed individuals move to symptomatic infectious class ($I_1$) and the rest are placed in asymptomatic infectious class ($I_3$), after completing the transition period $\eta_1$ at $E$. About 5% percent of the infectious (both symptomatic and asymptomatic) individuals face severe form of infection and are put on ventilator and are placed in ($I_2$). 49% of the vent bols dies due to disease. The symptomatic infectious individuals are isolated. Some of the infectious individuals dies due to disease and the rest moves to the recovered class $R$. The recovered individuals loose immunity at the rate $\beta_3$ and rejoin the susceptible class. $N$ denotes the density of human population.

The Mathematical model of the disease is given by the following set of coupled differential equation:

$$
\begin{align*}
\dot{S} &= \Gamma_1 + \beta I_1 R - \left( \frac{\beta_3 (I_1 + I_2)}{N} S + \beta_3 WS \right) - \mu S \\
\dot{E} &= \left( \frac{\beta_3 (I_1 + I_2)}{N} S + \beta_3 WS \right) - \eta_1 E - \mu E \\
\dot{I}_1 &= \left( 1 - \delta_1 \right) \eta_1 E - \left( K + \mu \right) I_1 \\
\dot{I}_2 &= \delta_1 \eta_1 E - \left( K + \mu \right) I_2 \\
\dot{I}_3 &= K \beta I_1 + \kappa I_1 I_2 - \left( \alpha_1 + D_2 + \mu \right) I_2 \\
\dot{R} &= \left( 1 - \beta \right) K I_1 + \alpha_1 I_2 + \left( 1 - \alpha_2 \right) K I_2 - \mu R - \beta_3 R \\
\dot{W} &= \theta_1 I_1 + \theta_2 I_2 - \varepsilon W.
\end{align*}
$$

The following table (1) contains the values of the different parameters used in the model (1). Table 2.

Model analysis

Here, in this Section 3 properties of the model; Disease-Free-Equilibrium, Invariant region and the Basic Reproduction Number would be discussed.

Invariant region

The state variables and parameters used in the model are all non-negative because the model is concerned with the living population.

$$
\dot{N} = \Gamma_1 - \mu N - D_1 I_1. 
$$

From Eq. (2) we have

$$
\dot{N} \leq \Gamma_1 - \mu N. 
$$

Solving this equation, we have

$$
N \leq \frac{\Gamma_1}{\mu} \left( 1 - e^{-\mu t} \right) \Rightarrow N \leq \frac{\Gamma_1}{\mu} \text{ when } t \to \infty
$$

So we claim the following proposition:

Proposition 0.1. [34] The region $\Omega$, given by:

$$
\Omega = \left\{ \left( S, E, I_1, I_2, R, W \right) \in \mathbb{R}^6_+ \right. \Big| L_2 \left( I_1 + I_2 \right) \leq \left[ \frac{\Gamma_1}{\mu} \right] \left( 1 - e^{-\mu t} \right) \Big\}
$$

is positively invariant domain, and the model is epidemiologically and mathematically well posed and all the trajectories are forward bounded.

Disease reproduction number

The number of secondary infections caused by a single primary infection in completely susceptible population is called $R_0$ or the disease reproduction rate. The Reproduction number is find by next generation matrix [35,33].

$$
R_0 = \rho \left( \frac{-FV}{\mu} \right), \text{ where } \rho \text{ is spectral radius.}
$$
For simplicity we write $F$ as.

$$F = \begin{pmatrix}
    0 & n_1 & 0 & n_2 & n_3 \\
    0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}$$

and

Table 1

| Notation | Parameter definition | Value | Source |
|----------|----------------------|-------|--------|
| $\Gamma_h$ | Humans recruitment rate | $0.0015875 \times N$ day$^{-1}$ | [33] |
| $\mu$ | Humans natural mortality rate | $0.00004$ day$^{-1}$ | [33,35] |
| $1/\epsilon$ | $\epsilon$ is the life time of virus on W | $0.1$ day$^{-1}$ | [28] |
| $1/\eta_1$ | $\eta_1$ is the incubation period of human | $0.192$ day$^{-1}$ | [44,28] |
| $K$ | The transition period at $I_f$ | $2$ – weeks | [32] |
| $a_1$ | The ratio of recovery of critical class | $51\%$ | [37] |
| $a_2$ | The ratio of asymptomatic moving to vent bol | $2\%$ | [39,40] |
| $\delta$ | The of ratio exposed moving to asymptomatic | $75\%$ | [29] |
| $\beta_1$ | The transmission rate of infection from $I_f$ to $S$ | $0.65$ day$^{-1}$ | [28] |
| $\beta_2$ | The transmission rate of infection from stuff | $0.165$ day$^{-1}$ | [28] |
| $\theta_1$ | The shedding coefficient of $I_f$ on W | $0.5$ | [28] |
| $\theta_2$ | The shedding coefficient of $I_f$ on W | $0.5$ | [28] |
| $\gamma$ | The multiple of the transmissibility of $I_f$ to that of $I_f$ | $0.5$ | [28] |
| $D_2$ | Disease induced death ratio of vent bol | $49\%$ | [38] |
| $\rho$ | The ratio of asymptomatic moving to vent bol | $5\%$ | [36] |
| $\beta_3$ | The Immunity loosing rate of recovered individuals | $0.066$ day$^{-1}$ | [21] |

The column in matrix $f$ denotes the individuals who get infected.

$$f = \begin{pmatrix}
    f_1 \\
    f_2 \\
    f_3 \\
    f_4 \\
    f_5 \\
\end{pmatrix} = \begin{pmatrix}
    \frac{\beta_1 (I_f + I_s) S + \beta_2 W S}{N} \\
    0 \\
    0 \\
    0 \\
    0 \\
\end{pmatrix}$$

The column of matrix $V$ denotes the individuals that enter the infected class or leave the infected class, excluding those coming from susceptible class.

$$V = \begin{pmatrix}
    \delta v_1 & \delta v_2 & \delta v_3 & \delta v_4 & \delta v_5 \\
    \delta E & \delta I_1 & \delta I_2 & \delta I_3 & \delta W \\
\end{pmatrix} = \begin{pmatrix}
    -(\eta_1 + \mu) & 0 & 0 & 0 & 0 \\
    (1 - \delta)\eta_1 & -K + \mu & -\alpha_1 + D_2 + \mu & -\alpha_2 K & 0 \\
    0 & \beta K & -\alpha_1 + D_2 + \mu & -\alpha_2 K & 0 \\
    0 & 0 & 0 & -\theta_1 & 0 \\
    0 & 0 & 0 & \theta_2 & -\theta_3 \\
\end{pmatrix}$$

For simplicity we write $V$ as.

$$V = \begin{pmatrix}
    -\eta_1 E - \mu E \\
    (1 - \delta)\eta_1 E - (K + \mu)I_1 \\
    \beta K I_1 + \alpha_2 K I_2 - (\alpha_1 + D_2 + \mu)I_2 \\
    \delta_1 E - (K + \mu)I_2 \\
\end{pmatrix}$$

The column of matrix $V$ denotes the individuals that enter the infected class or leave the infected class, excluding those coming from susceptible class.

$$F = \begin{pmatrix}
    \beta_1 S \\
    \beta_2 S \\
    0 \\
    0 \\
    0 \\
\end{pmatrix}$$

For simplicity we write $F$ as.

$$F = \begin{pmatrix}
    0 & n_1 & 0 & n_2 & n_3 \\
    0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}$$

Table 2

| Parameter | value | index | Parameter | value | index |
|-----------|-------|-------|-----------|-------|-------|
| $\delta$ | $0.75$ | $-0.0074$ | $\mu$ | $0.00004$ | $-0.9963$ |
| $\beta_1$ | $0.65$ | $0.123$ | $\eta_1$ | $0.1923$ | $0.0020797$ |
| $\beta_2$ | $0.165$ | $0.9877$ | $\Gamma_h$ | $0.0015875$ | $0.9877$ |
| $\theta_1$ | $0.5$ | $0.2469$ | $\theta_2$ | $0.5$ | $0.7408$ |
| $K$ | $0.044129$ | $-0.999$ | $\tau$ | $0.5$ | $0.0074$ |
| $\epsilon$ | $0.1$ | $-0.9877$ | |

Stability analysis

Here we discuss the global stability of the system (1). We use the following theorem, stated here for convenience:

**Theorem 0.1.** (42) For the system

$$\frac{dX}{dt} = D(X) (Y - Y_0) + \mathbb{X}_{21} (Y - Y_1) \mathbb{Y} = D_1 (Y - Y_0) \mathbb{Y}$$

the DFS (Disease free state) is GAS (globally asymptotically stable) if the following conditions hold.

$(a_1)$: All the populations involved in the model are forward bounded and hence the system is mathematically well posed.
(a2): The sub-system of non-infected classes \( \mathcal{Y}_1 = \mathcal{B}_1(\mathcal{Y}_1, 0)(\mathcal{Y}_1 - \mathcal{Y}_1) \) is globally asymptotically stable at the origin.

(a3): The matrix of infected compartments denoted by \( \mathcal{B}_2(\mathcal{Y}) \) is both metzler and irreducible.

(a4): The matrix of infected classes, \( \mathcal{B}_2 \) is bounded by some matrix \( \mathcal{B}_2(\mathcal{Y}) \) and \( \mathcal{Y} = (\mathcal{B}_2(\mathcal{Y}), \mathcal{Y} \in \Omega) \). Then \( \mathcal{B}_2 \) may or may not belong to \( \mathcal{N} \). However if \( \mathcal{B}_2 \in \mathcal{N} \) then for any \( \mathcal{Y} \in \Omega \) such that \( \mathcal{B}_2 = \mathcal{B}_2(\mathcal{Y}), \mathcal{Y} \in \mathbb{R}^{n \times \{0\}} \)

(a5): The spectral radius of \( \mathcal{B}_2(\mathcal{Y}) \) is less than or equal to zero. For simplicity we divide the model in infected and non-infected subclasses as:

\[
Y = (S, I_1, I_2, I_3, R, W)^{T}, \quad Y_i = (E, I_1, I_2, W)^{T}
\]

Theorem 0.2. Given the sub-system:

\[
\begin{align*}
\dot{S} &= \Gamma_S + \beta R - \left( \frac{\beta_1 (I_1 + I_2)}{N} \right) S + \beta_2 WS - \mu S \\
\dot{R} &= \left( 1 - \beta \right) K I_1 + \alpha I_1 + \left( 1 - \alpha_1 \right) K I_2 - \left( \mu + \beta_1 \right) R
\end{align*}
\]

The above sub-system is GAS at the sub-domain Q, where \( Q = \{ \mathcal{Y} \in \Omega; \mathcal{Y}_1 = 0, \mathcal{Y}_2 \neq 0 \} \).

**Proof.** If \( Y_1 = 0 \), the system;

\[
\begin{align*}
\dot{S} &= \Gamma_S + \beta R - \left( \frac{\beta_1 (I_1 + I_2)}{N} \right) S + \beta_2 WS - \mu S \\
\dot{R} &= \left( 1 - \beta \right) K I_1 + \alpha I_1 + \left( 1 - \alpha_1 \right) K I_2 - \left( \mu + \beta_1 \right) R
\end{align*}
\]

reduces to the form:

\[
\begin{align*}
\dot{S} &= \Gamma_S + \beta R - \mu S \\
\dot{R} &= -\left( \mu + \beta_1 \right) R.
\end{align*}
\]

OR

\[
\begin{align*}
\dot{S} &= \Gamma_S + \beta R - \mu S \\
\dot{R} &= -\left( \mu + \beta_1 \right) R.
\end{align*}
\]

Since all the eigenvalues of the matrix \( \mathcal{E}_2 \) are -ve. Therefore the Disease Free Equilibrium \( \mathcal{E}_2 \) is globally stable.

The sub-system:

\[
\begin{align*}
\dot{E} &= \left( \frac{\beta_1 (I_1 + I_2)}{N} \right) S + \beta_2 WS - \eta E - \mu E \\
\dot{I}_1 &= \left( 1 - \delta \right) \eta_1 E - \left( K + \mu \right) I_1 \\
\dot{I}_2 &= \delta_1 \eta_1 E - \left( K + \mu \right) I_2 \\
\dot{W} &= \delta_1 \eta_1 E - \left( K + \mu \right) I_2 - eW
\end{align*}
\]

can also be written as:

\[
\begin{align*}
\dot{E} &= \left( \frac{\beta_1 (I_1 + I_2)}{N} \right) S + \beta_2 WS - \eta E - \mu E \\
\dot{I}_1 &= \left( 1 - \delta \right) \eta_1 E - \left( K + \mu \right) I_1 \\
\dot{I}_2 &= \delta_1 \eta_1 E - \left( K + \mu \right) I_2 \\
\dot{W} &= \delta_1 \eta_1 E - \left( K + \mu \right) I_2 - eW
\end{align*}
\]

\[
\mathcal{Y}_i = \mathcal{B}_1(\mathcal{Y}) \mathcal{Y}_i
\]

The maximum (upper bound matrix) is obtained if \( S = N \). And the model attains \( S = N \) at DFE.

The jacobian of the system (6) is

\[
\mathcal{J}_i = \begin{pmatrix}
-(\eta_1 + \mu) & \beta_1 S_N & (1 - \delta) \eta_1 & 0 & 0 \\
\beta K & -(\alpha_1 + D_2 + \mu) & \alpha_2 K & 0 & 0 \\
0 & 0 & -\theta_1 & 0 & \theta_2 - \epsilon
\end{pmatrix}
\]
Matrix $J_i$ at disease free equilibrium is given by:

$$
\begin{pmatrix}
-p_1 & \beta_1 & 0 & \gamma \beta_1 & \beta_2 \\
-d_1 & -p_2 & 0 & 0 & 0 \\
0 & \beta K & -p_3 & \alpha K & 0 \\
0 & 0 & 0 & -p_4 & 0 \\
0 & \theta_1 & 0 & \theta_2 & -p_5
\end{pmatrix}
$$

The matrix $\mathcal{A}_i(J)$ is equal to $J_i$; the block of the Jacobian at the DFE, corresponding to the matrix $\mathcal{A}_i(J)$.

This proves (9) and (8).

Next we prove $a_8$ or (10).

**Theorem 0.5.** Given the matrix

$$
\begin{pmatrix}
-p_1 & \beta_1 & 0 & \gamma \beta_1 & \beta_2 \\
-d_1 & -p_2 & 0 & 0 & 0 \\
0 & \beta K & -p_3 & \alpha K & 0 \\
0 & 0 & 0 & -p_4 & 0 \\
0 & \theta_1 & 0 & \theta_2 & -p_5
\end{pmatrix}
$$

Then

$$
\begin{pmatrix}
-p_1 & \beta_1 & 0 & \gamma \beta_1 & \beta_2 \\
-d_1 & -p_2 & 0 & 0 & 0 \\
0 & \beta K & -p_3 & \alpha K & 0 \\
0 & 0 & 0 & -p_4 & 0 \\
0 & \theta_1 & 0 & \theta_2 & -p_5
\end{pmatrix}
$$

The change that occur in some phenomenon $Z$ due to change in parameter $K$ is called sensitivity of $Z$, denoted by $\gamma_Z^K$ and is given by [41,33,45]:

$$
\gamma_Z^K = \frac{\partial Z}{\partial K}
$$

All the parameters appearing in $R_0$ have impact on the initial rate of corona transmission. The effect of parameter on the transmission of infection is called the index of the parameter. Positive and negative signs of the index indicates that the change of parameter is directly or inversely proportional to the initial rate of transmission, $R_0$. The magnitude of the index of a parameter shows the degree of sensitivity of the parameter. Therefore we often address the parameters with high sensitivity index. However some parameters have high sensitivity index but the control of some of these parameters is beyond human control, like the natural death rate of human population, $\mu$, the birth rate of human population, $\Gamma_0$ etc. Therefore we address easily controllable parameters $K$, the transition period at $I_1$, $\beta_1$, the transmission probability of infection from $I_1$, $\beta_2$, the transmission probability of infection from contaminated materials and the shedding coefficients $\theta_1$ and $\theta_2$ of $I_1$ and $I_2$. $K$ has got sensitivity index of $-0.99$. So an increase of 10% in $K$ would cause a decrease of 9.9% in the transmission rate of COVID-19. A decrease of 10% in contact rates $\beta_2$ would cause a decrease of 9.8% in transmission rate of disease. Similar are the cases of $\theta_1$, $\theta_2$ and $\beta_1$.

We combine the above mentioned five interventions in particular ratios and formulate three control strategies. Since the transmission rate of Coronavirus is high. Therefore the density of infected cases increases too rapidly to be accommodated by the hospitals. The propose three strategies would help in elimination and control flattening the curve of infection) of COVID-19 and reduction of burden on hospitals.

**Strategy 1**

| $K$   | $\beta_1$ | $\beta_2$ | $\theta_1$ | $\theta_2$ |
|-------|-----------|-----------|------------|------------|
| 0.041429 | 0.65    | 0.165    | 0.5        | 0.5        |
| 0.0625   | 0.1     | 0.015    | 0.3        | 0.3        |
| 0.211    | 0.15    | 0.005    | 0.1        | 0.1        |

The following Figs. (1) (2), (3), (4), (5), (6) and (7) represent the results and comparison of the proposed control strategies.

**Conclusions**

In this work, a mathematical model of COVID-19 transmission and control was presented. On the basis of sensitivity indices of parameters
we propose a particular combination of interventions in some particular
ratio, called a strategy. We use five interventions each in transition
period at $I_1$, $\beta_1$, the transmission probability of infection due to $I_1$, $\beta_2$ the
transmission probability of infection due to $I_2$, $\theta_1$, the shedding coeffi-
cient of $I_1$, and $\theta_2$, the shedding coefficient of $I_2$. We formulate three
control strategies. strategy 1 uses the actual values of the parameters
without intervention. strategy 2 and strategy 3 use the intervened
values of the parameters.

The transition period of $I_1$ is intervened by giving proper treatment
to the side effects of the coronavirus like throat infection, vomiting,
gastrointestinal problems, nausea, diarrhea and pneumonia etc. $\beta_1$
is intervened by opting stay home most the time. The shedding coefficients
are intervened by using face mask. $\beta_2$ is intervened by using sanitizer.

Fig (1) shows that implementing strategy 1 or strategy 2, the density
of susceptible human population reduces to zero. However the
proposed strategy 3, maintain the density of susceptible human popu-
lation constant after an initial decrease of 15 days. The exposed class
plays the role of gateway in the transmission of the disease and as result
of proposed strategy 3, the density of this class reduces to zero with the
period of 400 days, as shown in Fig. (2). Fig (3) shows that as result of
strategy 3 the density of symptomatic infectious class reduces to zero
with in a period of 450 days. Fig (4) shows that using strategy 3, we can
reduce the density of vent boll class to zero with in 500 days. Increase is
observed in vent bol class in the initial period of 30 days. The reason of
this increase is the decrease of transition period of symptomatic infec-
tious individuals at $I_1$. The density of asymptomatic infectious class re-
duces to zero with the period of 490 days, as shown in Fig (5). As a result
of strategy 3, we see in Fig. (6) that recovery increases initially for 30
days, start decreasing and reduces to zero after a period of 530 days. The
reason of decrease of recovery is not the failure of strategy but the
decrease in densities of the infected classes due to interventions. Greater
the number of infected individuals, greater would be ratio of recovery.

No doubt contaminated materials of the market play significant role in
the transmission of the infection. Using strategy 3, we can eliminate this
class with in period of 400 days, as shown in Fig. (7).

Strategy 3 is recommended for complete elimination of COVID-19.
Since the value of threshold, $\xi$, is less than one for the used values of
parameters, so there is no chance of new outbreaks of the disease. The
disease free state so obtained would be globally asymptotically stable.

![Fig. 1. The comparison of the strategies regarding maintaining the density of susceptible human population.](image1)

![Fig. 2. The graph represents the comparison of the strategies regarding exposed human population.](image2)
Fig. 3. The graph represents the comparison of the strategies regarding infectious human population.

Fig. 4. The graph represents the comparison of the strategies regarding the density of vent bol human population.

Fig. 5. The graph represents the comparison of the strategies regarding asymptomatic infectious human population.
Credit authorship contribution statement

Muhammad Zamir: Conceptualization, Methodology, Investigation, Visualization. Writing original draft, review, supervision & editing; F. Nadeem: Conceptualization, Methodology, Investigation, Visualization. Writing original draft, review & editing; T. Abdeljawad: Conceptualization, Methodology. Investigation Visualization, Writing original draft, review & editing; Zakia Hammouch: Conceptualization, Methodology, Investigation, Visualization, review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] Causes of COVID-19: https://www.mayoclinic.org/diseases-conditions/coronavirus/symptoms-causes/syc-20479963.
[2] Myth busters: https://www.who.int novel-coronavirus-2019 advice-for-public.
[3] WHO statement regarding cluster of pneumonia cases in Wuhan, China, Available: https://www.who.int/china/news-detail/09-01-2020.
[4] World Health Organization. Coronavirus. World Health Organization, January 19, 2020. Available: https://www.who.int/health-topics/coronavirus.
[5] Zhou P, Yang XL, Wang XG, Hu B, Zhang L, Zhang W. A pneumonia outbreak associated with a new coronavirus of probable bat origin. Nature 2020. https://doi.org/10.1038/s41586-020-2012-7.
[6] Li Q, Guan X, Wu P, Wang X, Zhou L, Tong Y. Early transmission dynamics in Wuhan, China, of novel coronavirus-infected pneumonia. N Engl J Med 2020. https://doi.org/10.1056/NEJMoa2001316.
[7] Coronavirus poster https://www.who.int/docs corona-virus-poster-english-srilanka.
[8] Q and A on coronaviruses (COVID-19) https://www.who.int news-room-q-a-deta ilq-a-coronaviruses.
[9] Huang C, Wang Y, Li X, Ren L, Zhao J, Hu Y. Clinical features of patients infected with 2019 novel coronavirus in Wuhan, China. Lancet 2019;2020. https://doi.org/10.1016/S0140-6736(20)30183-5.
[10] COVID-19 CORONAVIRUS/CASES Available at: https://www.worldometers.info/coronavirus/coronavirus-cases/.
[11] Atangana A. Modelling the spread of COVID-19 with new fractal-fractional operators: Can the lockdown save mankind before vaccination? Chaos Solitons Fractals 2020;136. 109860.
[12] Atangana A, Aratz S. Nonlinear equations with global differential and integral operators: Existence, uniqueness with application to epidemiology. Results Phys 2020;103593.
[13] Gao W, Baskonus HM, Shi L. New investigation of bats-hosts-reservoir-people coronavirus model and application to 2019-nCoV system. Adv Difference Equ 2020; 1–11.
[14] Veeresh P, Prakash DG, Malagi NS, Baskonus HM, Gao W. New dynamical behaviour of the coronavirus (COVID-19) infection system with nonlocal operator from reservoirs to people; 2020.

[15] Media Statement: Knowing the risks for COVID-19: Available at: https://www.who.int/08-03-2020-knowing-the-risk-for-covid-19,

[16] Report of the WHO-China Joint Mission on Coronavirus Disease 2019 (COVID-19) Available at: https://www.who.int/docs/default-source/who-china-joint-mission-on-covid-19-final-report.pdf.

[17] Huang C, Wang Y, Li X. Clinical features of patients infected with 2019 novel coronavirus in Wuhan, China. Lancet 2020;395:497–506.

[18] Wang D, Hu B, Hu C. Clinical Characteristics of 138 Hospitalized Patients With 2019 Novel Coronavirus-Infected Pneumonia in Wuhan, China, JAMA; 2020.

[19] Zhou F, Yu T, Du R. Clinical course and risk factors for mortality of adult inpatients with COVID-19 in Wuhan, China: a retrospective cohort study. Lancet 2020.

[20] Pan L, Mu M, Yang P. Clinical characteristics of COVID-19 patients with digestive symptoms in Hubei, China: a descriptive, cross-sectional, multicenter study. Am J Gastroenterol 2020.

[21] https://www.sciencemag.org/article/what-immunity-to-covid-19-really

[22] Pan L, Mu M, Yang P. Clinical characteristics of COVID-19 patients with digestive symptoms in Hubei, China: a descriptive, cross-sectional, multicenter study. Am J Gastroenterol 2020.

[23] Zhu N, Zhang D, Wang W, Li X, Yang B, Song J. A novel coronavirus from patients with pneumonia in Wuhan, China, 2019. N Engl J Med 2020. https://doi.org/10.1056/NEJMoA2001017.

[24] Bogoch II, Watts A, Thomas-Bachli A, Hober C, Kraemer MuG, Khan K. Pneumonia of unknown etiology in Wuhan, China: potential for international spread via commercial air travel. J Travel Med 2020. https://doi.org/10.1093/jtm/taaa008.

[25] Wu JT, Leung K, Leung GM. Nowcasting and forecasting the potential domestic and international spread of the 2019-nCoV outbreak originating in Wuhan, China: a modelling study. Lancet 2020. https://doi.org/10.1016/S0140-6736(20)30260-9.

[26] Zhao S, Lin Q, Ran J, Musa SS, Yang G, Wang W. Preliminary estimation of the unreported number of novel coronavirus (2019-nCoV) cases in China, from 2019 to 2020: a data-driven analysis in the early phase of the outbreak. Int J Infect Dis 2020. https://doi.org/10.1016/j.ijid.2020.01.050.

[27] Zhao S, Musa SS, Lin Q, Ran J, Yang G, Wang W. Estimating the unreported number of novel coronavirus (2019-nCoV) cases in China in the first half of January 2020: a data-driven Modelling analysis of the early outbreak. J Clin Med January 2020;2020. https://doi.org/10.3390/jcm9020388.

[28] Chen TM, Rui J, Wang QP, Zhao ZY, Cui JA, Yin L. A mathematical model for simulating the phase-based transmissibility of a novel coronavirus. Infectious Diseases Poverty 2020;9(1). https://doi.org/10.1186/s13662-020-02979-1.

[29] Ivorra B, Fernandez MR, Vela-Perez M, Ramos AM. Mathematical modeling of the spread of the coronavirus disease 2019, (COVID-19) considering its particular characteristics. The case of China 2020;MOMAT,1–23. https://doi.org/10.13140/ RG.2.2.21543.29604.

[30] Yang C, Wang J. A mathematical model for the novel coronavirus epidemic in Wuhan, China. Math Biosci Eng 2020;17(3):2708–24.

[31] Spencer JA, Shutt DP, Moser SK, Clegg H, Wearing HJ, Mukundan H, et al. Epidemiological parameter review and comparative dynamics of influenza, respiratory syncytial virus, influenza, human coronavirus, and adenovirus, medRxiv; 2020.

[32] Report of the WHO-China Joint Mission on Coronavirus Disease 2019 (COVID-19) https://www.who.int/docs/default-source/who-china-joint-mission-on-covid-19-final-report.pdf.

[33] Nadeem F, Zamir M, Tridane A, Khan Y. Modeling and Control of Zoonotic Leishmaniasis: current situation and new perspective. Comparative Immunology Microbiology and Infectious Disease 2004;27(5):305–18.

[34] Zamir M, Sultana R, Ali R, Panhwar WA, Kumar S. Study on the threshold condition for infection of visceral leishmaniasis. Sindh Univ Res J (Sci Ser) 2015;47(3): 619–22.

[35] https://www.webmd.com/lung/qa/what-percentage-of-covid19-cases-are-mild.

[36] https://www.theguardian.com/society/2020/mar/28/coronavirus-intensive-care -uk-patients-50-per-sent-survival-rate.

[37] Wu Z, McGoogan JM. Characteristics of and Important Lessons From the Coronavirus Disease 2019 (COVID-19) Outbreak in China: Summary of a Report of 72314 Cases From the Chinese Center for Disease Control and Prevention. JAMA 2020.

[38] Lavezzo E, Franchin E, Ciavarella C. Suppression of COVID-19 outbreak in the municipality of Vo’, Italy. available herehttps://www.medrxiv.org/content/10.1101/ 2020.04.17.20053157v1.full.pdf.

[39] Covid-19: four fifths of cases are asymptomatic, China figures indicate. BMJ2020; 369 doi: 10.1136/bmj.m1375.

[40] Covid-19: four fifths of cases are asymptomatic, China figures indicate. BMJ2020; 369 doi: 10.1136/bmj.m1375.

[41] Zamir M, Shah Z, Nadeem F, Memood A, Alrabaiah H, Kumam P. Non pharmaceutical interventions for optimal control of COVID-19. Comput Methods Appl Comput Math 2015;4(5):363–8.

[42] Kamgang JC, Sallet G, Computation of threshold conditions for epidemiological models and global stability of the disease-free equilibrium. Math Biosci 2008;213: 1–12.

[43] Sundar S, Agnwal G, Rai M, Mukharia MK, Murray HW. Treatment of Indian visceral leishmaniasis with single or daily infusion of low dose liposomal amphotericin, Randomised trial. BMJ 2001;323:419–22.

[44] Zamir M, Shah Z, Nadeem F, Memood A, Alrabaiah H, Kumam P. Non pharmaceutical interventions for optimal control of COVID-19. Comput Methods Programs Biomed 2020;196:105642. https://doi.org/10.1016/j. cmpb.2020.105642. ISSN 0169–2607.

[45] Zamir M, Nadeem F, Zaman G. Optimal control of visceral, cutaneous and post kala azar leishmaniasis. Adv Difference Eqs 2020:548. https://doi.org/10.1186/ s13662-020-02979-1.