Brightness Fluctuation Spectra of Sun-like Stars. I. The Mid-frequency Continuum

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Abstract

We analyze space-based time-series photometry of Sun-like stars, mostly in the Pleiades, but also field stars and the Sun itself. We focus on timescales between roughly 1 hr and 1 day. In the corresponding frequency band these stars display brightness fluctuations with a decreasing power-law continuous spectrum. K2 and Kepler observations show that the rms flicker due to this mid-frequency continuum (MFC) can reach almost 1%, approaching the modulation amplitude from active regions. The MFC amplitude varies by a factor up to 40 among Pleiades members with similar Teff, depending mainly on the stellar Rossby number Ro. For Ro ≤ 0.04, the mean amplitude is roughly constant at about 0.4%; at larger Ro the amplitude decreases rapidly, shrinking by about two orders of magnitude for Ro ≈ 1. Among stars, the MFC amplitude correlates poorly with that of modulation from rotating active regions. Among field stars observed for 3 yr by Kepler, the quarterly average modulation amplitudes from active regions are much more time variable than the quarterly MFC amplitudes. We argue that the process causing the MFC is largely magnetic in nature and that its power-law spectrum comes from magnetic processes distinct from the star’s global dynamo, with shorter timescales. By analogy with solar phenomena, we hypothesize that the MFC arises from a (sometimes energetic) variant of the solar magnetic network, perhaps combined with rotation-related changes in the morphology of supergranules.

Unified Astronomy Thesaurus concepts: Stellar activity (1580); Stellar rotation (1629); Solar dynamo (2001); Supergranulation (1662)

Supporting material: machine-readable table

1. Introduction

It has long been known that some stars have surface magnetic active regions (spots and faculae) analogous to those seen on the Sun (e.g., Kron 1947, 1952; Chugainov 1966; Bopp & Evans 1973). The principal evidence for such starspots is the periodic variation of stellar brightness as the active regions across the star’s visible hemisphere. With the advent of spaceborne time-series photometry, it became possible to detect active regions even on stars with roughly solar activity levels. In the past 15 yr, missions such as Microvariability & Oscillations of STars (Matthews et al. 2004), Convection, Rotation, and Transits (Baglin et al. 2006), Kepler (Borucki et al. 2010), K2 (Howell et al. 2014), and Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2014) have produced stellar photometry with precision and temporal coverage that is not possible from the ground. In particular, the Kepler and K2 missions generated prolonged and precise time series of hundreds of thousands of stars, allowing us to characterize in detail the brightness signatures of stellar activity modulated by rotation (García et al. 2014a; McQuillan et al. 2014; Santos et al. 2019; Gordon et al. 2021). Among the available mission results, those from K2 stand out because some K2 campaigns pointed at nearby open star clusters. Exploiting the data from these campaigns gives all of the usual dividends expected from open cluster data, allowing unambiguous comparisons among member stars.

Here we discuss mostly observations of the Pleiades obtained by the K2 mission, to investigate brightness fluctuations with timescales between roughly a day and the K2 long-cadence (LC) Nyquist period of about an hour. These timescales are longer than granulation lifetimes but shorter than the great majority of stellar rotation periods, or the lifetimes of magnetic active regions. The corresponding frequencies lie between roughly 20 and 300 μHz. To align with the K2 LC frequency coverage, we will usually confine our interest to a slightly smaller frequency range, viz., 20 μHz ≤ ν ≤ 285 μHz. Rebull et al. (2016a, 2016b) and Stauffer et al. (2016) studied the K2 time series of the Pleiades in great detail, producing (among many other things) a list of 759 cluster members with reliably measured rotation periods, covering all spectral types from B to M. Here we study mostly stars drawn from this list.

Figure 1 shows power spectra of relative brightness fluctuations for two Pleiades members (EPIC211113202, Teff = 4843 K, mass = 0.764 M⊙, EPIC210946764, Teff = 5547 K, mass = 0.974 M⊙) and the Sun (Teff = 5777 K, mass = 1 M⊙). The upper spectrum comes from K2 short-cadence (SC) data (SC = 1-minute sampling), and the middle one from K2 LC data (LC = 1/2 hr sampling). The solar spectrum is from a 23 yr time series of data from VIRGO/SPM (Fröhlich et al. 1995, 1997; Jiménez et al. 2002) taken with 1-minute observing cadence, starting near solar minimum in 1996.

A striking feature of Figure 1 is the wide range of variability amplitude observed on otherwise fairly similar stars. The three shown here are all dwarfs with vigorous surface convection zones and masses that differ by at most 25%. Yet among the
three stars plotted here, the photometric variability amplitude ranges over a factor of about 200. The property that chiefly distinguishes these three stars from one another is their rotation period $P_{\text{rot}}$, which varies among these stars by a factor of about 100. As we shall see below, rotation is key to understanding the variability of our stellar sample.

It is useful to decompose the stellar relative brightness power spectra into two components. First, there are periodic variations that remain coherent over time spans that cover at least a sizable fraction of the total duration of the data set. Physically, these arise from long-lived localized brightness features (presumably spots and faculae) that persist for a few to many rotation periods. These cause narrow peaks in the temporal power spectrum, at or near integer multiples of the star’s rotation frequency.

Second, there are variations that are incoherent over time spans of a few rotation periods or less. These lead to continuous power spectra. For a large majority of the stars we have examined, the signal from the incoherent variations appears in the spectra as power laws, with the power spectral density (PSD) given by $PSD = A \nu^{-\alpha}$, valid in different stars over various ranges of $\nu$. We call particular attention to the dominant role of such continua in all the stars shown in Figure 1, for frequencies between the LC observations’ Nyquist frequency (about 285 $\mu$Hz) and some minimum frequency $\nu_{\text{min}}$, where for the purposes of this paper we will always take $\nu_{\text{min}} = 20$ $\mu$Hz. This frequency range is indicated by the bar labeled “MFC” at the bottom of the plot.

Note that for main-sequence stars, oscillations, granulation, and related flows described in the $p$-mode literature (Kallinger et al. 2014; Santos et al. 2018, and references therein) have timescales that are short compared to the K2 LC sampling time. Hence, within the frequency range marked “MFC” in Figure 1, the high-frequency processes contribute only a near-constant power background.

Although the mid-frequency power-law continua are prominent in the brightness fluctuation spectra of most dwarf stars, they have not yet been studied in a systematic way, except as a noise source with which planetary transit observations must contend (Sulis et al. 2020). The current paper is intended as the first in a series that will study their properties and physics.

The rest of this paper is laid out as follows: Section 2 turns briefly to the spectrum of the Sun’s photometric fluctuations, to provide context for the following discussion of distant stars. Section 3 describes the K2 observations, how we chose our sample of Pleiades stars, the model that we use to describe their power spectra, and our techniques for parameter fitting. Section 4 examines the relationships that emerge among the stellar structure properties and the various fitted spectrum model parameters for our sample. Section 5 considers whether the MFC is merely an artifact of the stellar global dynamo, by appealing to data from sources other than K2, and from stars that are not members of the Pleiades cluster. Section 6 attempts a coherent description of all these results, ending with a consistent but speculative interpretation of the power-law continua in terms of a variant of the solar magnetic network—one that brightens dramatically with faster stellar rotation and that may involve rotation-dependent changes in supergranule morphology.
ranges 1.0
inset shows the relative rms for each block, constrained to the limited frequency
fl
the time intervals
Figure 2. The Astrophysical Journal, 916:66 (14pp), 2021 August 1
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2. The Sun’s Photometric Fluctuations

Our knowledge of the photometric variability spectrum of the Sun (Figure 1) can help us to understand the variability spectra of other stars. As others have noted (Ulrich 1970; Harvey 1985; García et al. 2009; Karoff 2012), in this spectrum one can identify frequency ranges in which one or two physical processes are the dominant sources of variability; several of these are shown in Figure 1. At high frequencies (above 1000 µHz, say), the dominant sources of variability are almost purely dynamical (granulation and p-modes). At the lowest frequencies (below roughly 20 µHz), the variations either are essentially magnetic (growth and decay of magnetic active regions powered by a global dynamo) or else arise from rotation of active regions across the solar disk.

At intermediate frequencies, there is evidence that the fluctuating sources have both dynamical and magnetic properties. For instance, facular elements and photospheric bright points seem to be created when granular and supergranular flows, permeated and constrained by magnetic fields, generate evacuated flux tubes having strong radiative signatures. But at the applicable spatial and temporal scales for these short-lived flows, it is doubtful that the processes at work connect in any direct way to the Sun’s global dynamo.

These considerations raise questions about the MFC frequency range identified with the dashed bar in Figure 1, spanning roughly 20 µHz ≤ ν ≤ 300 µHz. Is the power in this region a symptom of the global dynamo? Is the underlying mechanism essentially fluid dynamic, or do magnetic fields play an important role? Is the power in the MFC related to known near-surface phenomena on the Sun, such as facular regions, granulation, or supergranulation?

To address some of these questions, it is instructive to examine how the power spectrum of the Sun’s fluctuations varies with the solar cycle. The VIRGO/SPM time series whose spectrum appears in Figure 1 spans two full solar cycles. We have broken the complete time series into 13 almost-contiguous blocks, each 625 days in length, and computed the power spectra of these blocks individually. In Figure 2 we overplot the smoothed power spectra of these blocks, illustrating that the block-to-block solar photometric variability is much larger at very low frequencies (below 20 µHz)—identified with “Active Regions” in Figure 1)—than at higher frequencies. The typical PSD values at the lowest frequencies vary by up to 1.5 orders of magnitude during the cycle. Moreover, the variability is modulated in phase with the solar sunspot cycle. We infer that the low-frequency intensity fluctuations arise from processes that participate in the global dynamo cycle, e.g., evolution of magnetic active regions, and modulation of their photometric signature by rotation.

For frequencies above about 20 µHz, however, the fluctuating brightness power shows no such obvious cycle dependence; at these frequencies the cyclic variation of the observed power is much smaller, with block-to-block rms variability of only a few percent. This suggests that the mechanism driving photometric fluctuations in the intermediate frequency range is not closely connected to the global dynamo. The relevant mechanisms may or may not involve magnetic fields in an essential way (we will argue later that they do, but in any case they do not arise directly from the conventional manifestations of the Sun’s magnetic cycle.

At higher frequencies (ν ∼ 300 µHz, approximately) solar photometric fluctuations have generally been modeled using Harvey functions (Harvey 1985)

\[
PSD_{\text{Harvey}} \propto \frac{1}{(1 + (\nu/\nu_0)^a)},
\]

where \(\nu_0 = (2\pi\tau)^{-1}\), and \(\tau\) is a characteristic timescale (e.g., Mathur et al. 2011; Kallinger et al. 2014; Santos et al. 2018, their Figure 2). Thus, for frequencies below \(\nu_0\), the Harvey functions are nearly constant, independent of \(\nu\). Two such Harvey functions combine in the solar spectrum to describe the solar granulation. These have fitted timescales \(\tau \approx \{80, 190\}\) s and make roughly equal contributions of about 35 ppm rms to the solar variability. There is little consensus in the literature as to what these components should be called, so in Figure 1 we term them both “surface convection.”

If we take the Harvey function models seriously, we conclude that the solar power spectrum shows unexplained excess power, largest at the low frequencies, in 20 µHz ≤ ν ≤ 300 µHz. Since we do not yet understand the physical nature of this excess, for naming purposes we revert to observational morphology, and for now we term it the mid-frequency continuum (MFC). If needed, a more physics-related name can be applied in due course.

The solar MFC contributes about 25 ppm rms to the Sun’s photometric variability in a broad wavelength bandpass (green plus red, similar to the 400–850 nm bandpass used by the Kepler mission)—an amount that is similar in magnitude to the disk-averaged signals from p-modes or granulation, but much smaller than that caused by solar magnetic active regions. In what follows, we investigate the behavior of this feature as seen in distant stars, notably in the Pleiades cluster. We will show that in younger, more rapidly rotating stars the amplitude of the MFC can be much greater than it is in the Sun.

3. Target Sample, Data Preparation, Derived Indices

NASA’s K2 mission targeted M45 (the Pleiades open cluster) during K2 Campaign 4, running from 2015 February 7 to April 23 (Howell et al. 2014).“
3.1. Star Sample and Stellar Properties

We selected 101 stars to be analyzed from the list of 759 Pleiades stars with reliable rotation periods from Rebull et al. (2016a, hereafter R16). Our aims were to select a subsample of the R16 stars (to keep the manual labor associated with our data analysis methods within bounds), while achieving adequate statistics for stable conclusions, and also providing fairly constant numbers of stars in color bins 0.5 mag wide, spanning $0.5 \leq V - K \leq 5.0$. This selection is reasonably complete for stars with $V - K \leq 2.5$, becoming increasingly incomplete for redder colors. For example, in R16 the magnitude bin $4.0 \leq V - K \leq 4.5$ contains 34 stars, whereas our sample contains only four.

For our sample of 101 stars, we took rotation periods $P_{\text{rot}}$ and membership assessments (“not member,” “good,” “best”) from R16. We obtained stellar classification estimates ($T_{\text{eff}}, \log(g), v \sin(i)$) from the APOGEE DR16 ASPCAP pipeline (Ahumada et al. 2020; Jönsson et al. 2020). Three of our selected stars did not have APOGEE data; for those we used parameter values from the MAST EPIC catalog (Huber et al. 2016). Stellar magnitudes $V$ and $K$, as well as $V - K$ colors, came from R16. We estimated stellar radii by combining measured values of $K$ magnitude, $T_{\text{eff}}$, and Gaia parallax. For 24 stars, not all of these properties were available; in these cases we used radius estimates from the MAST EPIC catalog (Huber et al. 2016). To identify binary or multiple stars, we compared the position of each star on a $(V$ vs. $V - K)$ color–magnitude diagram to the position of the single-star locus, which we fitted using photometry from R16. We identified stars more than $0.3, 0.45$ mag brighter than the locus as possible or probable binaries, respectively. For the Pleiades members in our star sample, the rms scatter in parallax is about 2% of the mean parallax. Thus, we do not expect the finite cluster depth to have a significant effect on our photometric binary classifications. The stellar power spectra also sometimes contained evidence for binarity. In these cases the spectra displayed two or more harmonic sequences of coherent periodic fluctuations. If the ratio of the fundamental periods of these sequences was less than 1.4, we considered them to be examples of single-star differential rotation (Donahue et al. 1996); if greater than 1.4, we deemed them probable binaries.

In the final list, six stars are identified as nonmembers of the Pleiades. We retained those stars to allow a preliminary assessment of differences, if any, between members and (presumably older) field stars. A total of 24 of our stars are classified as probable binaries, and eight are possibly binary.

We obtained EVEREST (EPIC Variability Extraction and Removal for Exoplanet Science Targets; Luger et al. 2016) LC time series for each of our sample stars from the Mikulski Archive for Space Telescopes (MAST)\(^9\). For each time series we post-processed the data following the procedures explained in García et al. (2011) and interpolated the gaps following inpainting techniques using a multiscale discrete cosine transform (García et al. 2014b; Pires et al. 2015). The gap-filled time series spanned 71.8 days with a mean sampling cadence of 29.42 minutes, for a total of typically 3465 samples per star.

3.2. Short-cadence Observations from Kepler

To extend our star sample to older, more slowly rotating stars, to see how their photometric power spectra varied on few-year timescales, and to verify that Kepler SC observations yielded results similar to those from LC observations, we analyzed a selected group of 11 stars from the Kepler field. For this purpose, we chose SC stars that had been analyzed with asteroseismology (Lund et al. 2017) and that also have been the subject of careful studies of their rotation and magnetic activity (García et al. 2014a; Santos et al. 2018, 2019). Hence, these stars have accurate estimates of mass, $T_{\text{eff}}$, radius, and $P_{\text{rot}}$. In hopes of learning something about stellar activity cycles, we preferred stars with evidence of time-varying magnetic activity (Mathur et al. 2014; Santos et al. 2018). As a consequence of our selection strategy, this group of stars is hotter on average than the Pleiades sample, and also older, with slightly lower surface gravity.

Structural parameters for these stars come from Silva Aguirre et al. (2017), except for KIC3733735, which is from Mathur et al. (2014). We processed Kepler SC data following the methods described in García et al. (2011; the so-called KEPSEISMIC series\(^11\)). As for the K2 time series, the gaps were filled with the inpainting algorithm mentioned earlier. The time-series coverage shows small differences from star to star, but typically each contains about $1.7 \times 10^5$ samples spanning 3.14 yr (usually Kepler quarters Q5 to Q16), with a mean time between samples of 58.85 s.

3.3. Power Spectrum Fitting—Definition of the MFC

From the time series of relative brightness for each star, we computed the PSD (PSD($\nu$), where PSD has units of ppm$^2$ $\mu$Hz$^{-1}$). We then fit each power spectrum with a model

$$\text{PSD}(\nu) = P_{\text{cont}}(\nu) + P_{\text{harm}}(\nu).$$

Here $P_{\text{cont}}$ is a two-part broken power-law function of $\nu$, and $P_{\text{harm}}$ is a sum of generalized Lorentzian line profiles with frequencies that are integer multiples of the star’s rotation frequency. The power-law index for $P_{\text{cont}}$ is allowed to be discontinuous across the break frequency $\nu_0$ but constant on either side of it, and the two power-law amplitudes are constrained such that $P_{\text{cont}}$ is continuous across the break. For all of the fits discussed in this work, the break frequency $\nu_0$ was arbitrarily fixed at 20 $\mu$Hz. The parameters describing the model PSD($\nu$) are then the amplitude $A$ of the continuum model at the break frequency $\nu_0$, the power-law indices $\alpha_1$, $\alpha_2$ that apply, respectively, above and below $\nu_0$; and the list of Lorentzian frequencies, amplitudes, and widths that define $P_{\text{harm}}(\nu)$.

For a more detailed description of how we carried out the power spectrum computation and model fitting, see Appendix A.

3.4. Fitted Parameters

Our fitting process yields three derived quantities that are important in the following discussion, as well as a number of others that we treat as nuisance parameters. The important quantities are as follows.

\(^9\) https://archive.stsci.edu/k2/epic/search.php
\(^10\) https://archive.stsci.edu/hsps/everest
\(^11\) https://archive.stsci.edu/prepds/kepseismic/
The total harmonic rms, denoted \( \sigma_{\mathrm{H}} \), is the rms photometric variability (ppm) attributable to all frequencies less than \( \nu_0 \), plus any portion of \( P_{\mathrm{harm}} \), the fitted harmonic power, that lies at \( \nu \geq \nu_0 \). It represents the variability due to rotating active regions, and from any other sources with periods longer than about half a day. This parameter is closely similar to the photometric magnetic activity index \( S_{\mathrm{M}} \), defined by Mathur et al. (2014). \( \sigma_{\mathrm{H}} \) is essentially a measure of the spottedness of the stellar photosphere and hence is one index of the general level of magnetic activity. Other activity indices exist (e.g., \( R \)-values computed from equivalent widths of \( \text{H}_\alpha \) or the \( \text{Ca II} \) infrared triplet), which typically show behavior that is similar to that of spottedness but differs in detail. It would be interesting to trace the connections between these various indices and the broadband photometric parameters we describe below, but this is beyond the scope of the present work.

The continuum rms, denoted \( \sigma_{\mathrm{C}} \), is the rms photometric variability (ppm) attributable to the MFC. We calculate it from the model PSD, excluding from the integral frequencies below \( \nu_0 \) and above \( \nu_{\max} \):

\[
\sigma_{\mathrm{C}} = \left( \int_{\nu_0}^{\nu_{\max}} P_{\mathrm{cont}}(\nu) d\nu \right)^{1/2}
\]

The MFC slope \( \alpha_1 \) is \( d \log(P_{\mathrm{cont}})/d \log \nu \) in the power-law domain of the photometric fluctuating power spectrum.

The lowest harmonic frequency \( \nu_{H1} \) is less important than the other parameters, but it is occasionally useful. It is the fitted frequency of the fundamental harmonic peak in the \( P_{\mathrm{harm}} \) spectrum. Absent a high-quality estimate from the literature, we sometimes use this value to compute a star’s rotation period as \( P_{\mathrm{rot}} = 11.574 \mu \text{Hz}/\nu_{H1} \) days.

We have applied our fitting procedure to the list of 101 selected K2 Campaign 4 (“Pleiades”) stars, the 11 Kepler SC stars, and the Sun. Table 1 contains the physical characteristics of all of these stars from sources in the literature as described above, as well as our measured values for the three main fitted parameters just described. Other parameters that emerge from the fits (e.g., the slope of the low-frequency continuum, the periods, and line width parameters of the higher-order harmonic peaks) may perhaps contain useful information, but they do not enter into our current discussion.

4. Dependence of MFC on Stellar Parameters

The MFC amplitude \( \sigma_{\mathrm{C}} \) is highly variable across stars; observed values in our sample range over almost 3 orders of magnitude. The same is true of the global activity index \( \sigma_{\mathrm{H}} \). And yet, importantly, these two quantities are only modestly well correlated: across our full star sample, the Spearman rank correlation between \( \log(\sigma_{\mathrm{H}}) \) and \( \log(\sigma_{\mathrm{C}}) \) is only 0.57. Though this correlation is highly significant, it is not good enough to allow using one of these amplitudes to make a meaningful prediction of the other. Thus, near any given value of \( \log(\sigma_{\mathrm{H}}) \), the range of \( \log(\sigma_{\mathrm{C}}) \) is typically 2 dex. What, then, do these amplitudes depend on? We first address \( \sigma_{\mathrm{C}} \), because in this case the answer is fairly clear. In a later section we turn to the question of whether \( \sigma_{\mathrm{C}} \) and \( \sigma_{\mathrm{H}} \) may be considered as different manifestations of a single process.

4.1. MFC Amplitude versus Rossby Number

To understand the physical nature of the MFC amplitude \( \sigma_{\mathrm{C}} \), we have examined how it varies with the known global parameters of our stars, namely, \( (T_{\text{eff}}, \log(g), \text{mass}, \text{radius}, P_{\text{bol}}) \), and various combinations of these. As one might guess from the behavior of magnetic indices in stars generally (Noyes et al. 1984), the parameter that predicts observations of \( \sigma_{\mathrm{C}} \) with the least scatter turns out to be the Rossby number, \( R_\circ = P_{\text{rot}}/\tau_{\text{C}} \), where \( \tau_{\text{C}} \) (days) is the turnover time for the stellar convection zone, a timescale that decreases with increasing stellar mass; for the purposes of this paper we compute \( R_\circ \) from \( P_{\text{rot}} \) and the estimated mass using Equation (11) of Wright et al. (2011).

Figure 3 shows \( \sigma_{\mathrm{C}} \) versus \( R_\circ \) for all stars described in this paper, including Pleiades members, nonmembers, likely binaries, Kepler SC stars, and the Sun. From the figure, we see that for a range spanning 2 dex at large \( R_\circ \), \( \log(\sigma_{\mathrm{C}}) \) decreases linearly with increasing \( \log(R_\circ) \). For smaller \( \log(R_\circ) \), the amplitude \( \sigma_{\mathrm{C}} \) saturates at what appears to be a constant value, at least for the range of \( R_\circ \) spanned by this data set. The dashed line in Figure 3 represents a simple fit to these data, defined on a saturated or “fast” domain in which \( R_\circ \leq R_{\text{sat}} \) and an unsaturated or “slow” domain in which \( R_\circ > R_{\text{sat}} \). We estimated the parameters of this model from a minimum-\( \chi^2 \) fit to the observed \( \sigma_{\mathrm{C}} \) values, obtaining \( \log(R_{\text{sat}}) = -1.65 \), with \( \log(\sigma_{\mathrm{C}}) \) modeled by

\[
\text{fast: } \log(\sigma_{\mathrm{C}}) = 3.59
\]

\[
\text{slow: } \log(\sigma_{\mathrm{C}}) = 3.59 - 1.24 \log \left( \frac{R_\circ}{R_{\text{sat}}} \right).
\]

Almost all of the stars in our sample fall close to the best-fit \( \sigma_{\mathrm{C}} \) versus \( R_\circ \) relation. Cluster nonmembers follow the same relation as members, except that the Kepler SC stars extend the relation to larger \( R_\circ \) values than are found in the Pleiades. Likewise, single stars and binaries follow the mean relation about equally well. A convenient implication is that our conclusions are unlikely to be significantly biased by errors in binary or membership identification.

We identify five outlier stars in the plot that we exclude from the functional fit; these are flagged in Figure 3 with enclosing parentheses. Two of these are binaries with fairly small \( R_\circ \) (EPIC211072441 and EPIC211106344), one a cluster member and one not. These both have unusual power spectra that are not fit well by the broken power-law model that works well for most stars. The other excluded stars are three of the four hottest members of the Kepler SC group of stars, all with \( T_{\text{eff}} > 6300 \text{ K} \). In Section 5.1 we will show that for these stars leakage of power from rotating active regions is unusually important. More details about these and a few other stars with unusual properties are given in Appendix B. With those exclusions, the rms scatter of \( \log(\sigma_{\mathrm{C}}) \) around the fit is 0.22 dex.

The functional relation in Figure 3 is strikingly similar to that shown by Wright et al. (2011) in their Figure 2, which displays the ratio of X-ray to bolometric luminosity as a function of \( R_\circ \) for a sample of 824 Sun-like stars. These two relations are similar enough that it seems likely that they arise from analogous processes. This congruent morphology should not, however, obscure the significant differences between the two cases. Wright et al. (2011) deal with X-rays from stellar coronal plasma, whereas \( \sigma_{\mathrm{C}} \) is observed at visible-light wavelengths, and its color dependence in the solar case identifies it as a photospheric phenomenon. Also the X-ray luminosity plotted by Wright et al. (2011) represents an energy loss from the star, while \( \sigma_{\mathrm{C}} \) is the amplitude of a zero-mean fluctuation, without an obvious connection to the star’s energy budget. In the
| Star Name       | Hz     | R.A.     | Decl.     | V    | V – K | Mass | T eff | log(g) | P rot | Ro | Bin | Memb | Cdnc | log σH | log σC | α1  |
|----------------|--------|----------|-----------|------|-------|------|-------|--------|-------|----|-----|------|------|--------|--------|-----|
| EPIC211153286  | 8508   | 58.57837 | 25.49526  | 12.87| 3.07  | 0.73 | 4226  | 4.76   | 7.74  | 0.312| b   | nm   | LC   | 4.22  | 2.36  | 1.655|
| EPIC211149600  | 8544   | 57.64885 | 25.42650  | 11.96| 2.38  | 0.85 | 5018  | 4.57   | 0.31  | 0.017| pb  | ok   | LC   | 4.59  | 3.76  | 1.333|
| EPIC211147822  | 8545   | 57.66375 | 25.39375  | 12.33| 2.89  | 0.87 | 4763  | 3.42   | 0.58  | 0.033| b   | nm   | LC   | 4.11  | 3.72  | 1.701|
| EPIC211129308  | 56.42688| 25.05709 | 13.60     | 3.09  | 0.68  | 4542 | 4.67  | 2.84   | 0.115 | s   | best| LC   | 4.04  | 2.69  | 1.710|
| EPIC211129308  | 57.75964| 25.05546 | 13.37     | 2.99  | 0.82  | 4903 | 4.58  | 5.79   | 0.299 | s   | best| LC   | 3.75  | 2.26  | 2.054|
| EPIC211111414  | 0451   | 56.20907 | 24.91113  | 13.57| 3.28  | 0.71 | 4642  | 4.64   | 5.68  | 0.243| s   | best| LC   | 3.81  | 2.31  | 2.204|
| EPIC21118542   | 0885   | 56.53240 | 24.86687  | 12.17| 2.81  | 0.90 | 5043  | 3.11   | 6.95  | 0.414| b   | nm   | LC   | 3.71  | 1.98  | 2.839|
| EPIC21117077   | 0191   | 55.96730 | 24.84162  | 14.42| 3.82  | 0.51 | 3950  | 4.84   | 3.00  | 0.085| s   | best| LC   | 4.31  | 2.76  | 1.964|
| EPIC21114317   | 0314   | 56.08369 | 24.79618  | 10.43| 1.52  | 1.10 | 5574  | 3.89   | 1.47  | 0.118| s   | best| LC   | 4.05  | 2.74  | 1.585|
| EPIC21113345   | 56.05799| 24.77939 | 10.84     | 1.77  | 1.04  | 5814 | 4.44  | 4.03   | 0.294 | s   | best| LC   | 3.65  | 1.82  | 1.814|

**Note.** Column (1): star name. Column (2) Hertzsprung Catalog number from WEBDA (Paunzen 2008). Columns (3) and (4): right ascension and declination (degrees). Column (5): Johnson V magnitude. Column (6): V – K color index (magnitudes). Column (7): stellar mass ($M_\odot$). Column (8): effective temperature (K). Column (9): log surface gravity (cgs). Column (10): rotation period (days). Column (11): Rossby number. Column (12): binarity (s = single, pb = possible binary, b = probable binary). Column (13) Pleiades membership class (best = very likely member, ok = likely member, nm = not member). Column (14): observing cadence (LC = long cadence ≥30 minutes, SC = short cadence ≥1 minute); Column (15): log total harmonic rms (ppm). Column (16): log continuum rms (ppm). Column (17): MFC power-law index.

(This table is available in its entirety in machine-readable form.)
unsaturated regimes, the two relations have slopes that differ by about a factor of 2: for X-rays, \( \frac{d \log \text{flux}}{d \log \text{Ro}} = -2.7 \), but \( \frac{d \log (\sigma_C)}{d \log \text{Ro}} = -1.24 \). For \( \sigma_C \), perhaps the analogous signal to consider is not the rms fluctuation, but rather the variance. In this case, the slopes in the unsaturated regimes would be more nearly equal. Still, the most substantial difference between these cases is that the transition between saturated and unsaturated behavior occurs at different Rossby numbers. For the X-ray fluxes, Wright et al. (2011) find the break between regimes at \( \text{Ro} = 0.13 \), but the break in \( \sigma_C \) falls at \( \text{Ro} = 0.022 \), more than 5 times smaller.

### 4.2. Power-law Index \( \alpha_1 \) versus Rossby Number

The MFC is characterized not only by its amplitude but also by its power-law index \( \alpha_1 \). Figure 4 shows \( \alpha_1 \) as a function of \( \log(\sigma_C) \). It suggests that, apart from showing considerable star-to-star scatter, \( \alpha_1 \) displays a rather complex dependence on \( \sigma_C \). Plotting \( \alpha_1 \) against \( \text{Ro} \) or \( P_{\text{rot}} \) gives diagrams showing behavior that is similar but not so clear, especially for small \( \sigma_C \), corresponding to large \( \text{Ro} \). Apart from three apparent outliers, the steepest slopes \( \alpha_1 \) occur only within approximately \( 1.6 \lesssim \log \sigma_C \lesssim 2.4 \), which from Figure 3 corresponds roughly to \( -0.6 \lesssim \log \text{Ro} \lesssim -0.1 \). It is not obvious what to make of this dependence, but the behavior is distinctive, so that reproducing it may prove to be a powerful test of future physics-based models. One worrisome feature of Figure 4 is that the rapid rise in \( \alpha_1 \) for \( 1 \lesssim \log(\sigma_C) \lesssim 1.8 \) is apparent only for the Kepler SC stars. A goal for future work is to measure the properties of the MFC in Kepler SC stars with shorter \( P_{\text{rot}} \), and conversely for K2 LC stars.

### 5. The MFC in Relation to the Global Dynamo

Having established some of the properties of the MFC, we now must address more fully whether we should think of this phenomenon as an artifact of the stellar global dynamo, or rather as distinct from it. Is it a process that derives mostly from the existence of a global dynamo, or does it simply coexist, proceeding (as the granulation does) in parallel and governed by its own rules?

The above question is too open-ended to elicit definite answers. To do better, we consider a few specific ways in which global dynamo processes might generate effects that resemble the observed MFC. First, the MFC may be simply numerical leakage of large-amplitude, narrowband rotation features into the computed power in surrounding frequencies. This might arise from poorly constructed window functions, or nonlinear effects in the time series, or other more complex data
that for \( \sigma \) to the rms values measured by Kepler for each of between 9 and 13 quarters, to the Sun. Each star is represented by a series of connected dots, corresponding to the rms values measured by Kepler for each of between 9 and 13 “quarters,” as explained in the text. Plotted points are color-coded according to \( T_{\text{eff}} \). Note that for \( \sigma \) values above 800, rightward of the vertical dashed line, the horizontal scale is compressed by a factor of 2.

processing errors. Second, the MFC might arise from diffuse magnetic regions on the target stars that themselves are born of stellar active regions. For example, one might imagine long-lived facular regions being shredded from the boundaries of starspot groups and eventually becoming global-scale diffuse magnetic features. Last, the MFC might come from some small-scale but widespread dynamo process that requires a seed field (generated by the global dynamo) for its long-term survival (e.g., Fletcher et al. 2010). As we proceed down this list, the line dividing side effects of the global dynamo from separate, self-sustaining processes becomes increasingly unclear. We might nonetheless gain some insight into the plausibility of scenarios such as these by investigating how the amplitude of the MFC behaves in comparison with various indices of global magnetic activity.

5.1. Time-dependent MFC Amplitude versus an Activity Metric for Kepler Stars

The surface manifestations of stellar global magnetic activity vary erratically with time as active regions form and disappear; in stars with discernible activity cycles, they also vary more systematically and on longer timescales. To search for connections between activity-based variability and the strength of the MFC signal in the same star, we subdivided the time series of photometric variability for each of our sample of 11 stars observed by the Kepler mission. We then broke these time series into nominal “quarters,” similar but not identical to Kepler quarters—the differences involve small shifts in quarter boundaries to place data gaps between the new quarters, rather than within them. Partitioned in this way, the time series for all stars have between 11 and 13 valid quarters, and the quarters have average durations of 84 days. Given these partitions, we computed the PSD(\( \nu \)) for each star \( i \) and each nominal quarter \( j \) and integrated appropriately over frequency to yield quarter-dependent harmonic and MFC indices \( \sigma_{ij} \) and \( \sigma_{ij} \).

Figure 5 shows the measured \( \sigma_{ij} \) as a function of \( \sigma_{ij} \), plotted for all stars \( i \) and quarters \( j \). The quarters for each star are plotted in order of increasing \( \sigma_{ij} \) and connected with solid lines. Our aim in this figure is to display explicitly any functional connection between \( \sigma \) and \( \sigma \).

The stars in this sample show \( \sigma \) variability across quarters that typically spans a factor of 4, whereas the variability range in \( \sigma \) is typically only 20%. Further, most of the stars in Figure 5 show little or no correlation between changes in \( \sigma \) and changes in \( \sigma \).

This lack of correlation is good evidence that the processes that we measure with \( \sigma \) and \( \sigma \) are, in most stars, physically distinct. For the same reason, we conclude that numerical power leakage between the frequency regimes below and above \( \nu_{\text{eff}} \) is not important. For the hot stars where significant correlation is present, we think it most likely that basic differences in the structure of near-surface convection have shortened the timescales associated with global magnetic activity, moving power from low \( \nu \) into the \( \nu \) range that we identify with \( \sigma \). How this might happen is an interesting question, but one that is beyond the scope of this paper.

5.2. Inclination Angle

One expects variability of rotating stars to depend to some degree on the inclination \( i \) of the rotation axis to the line of sight. This may occur for two different reasons. First, a portion of the star’s photometric variability arises from long-lived brightness structures being carried across the stellar limb by rotation. At small inclination, these transitions slow down, causing the power spectrum of their variability to compress toward smaller temporal frequencies. In the extreme pole-on case (\( i = 0 \)), one sees no rotation-driven variability at all, and brightness fluctuations result only from the intrinsic time evolution of the features. Second, when rotation is fast enough to affect the internal dynamics of brightness features, their morphology and time evolution may depend on the stellar latitude. In this case, the observed brightness power spectrum may depend on whether one is observing mostly polar regions (for small \( i \) or equatorial ones (for \( i \approx \pi/2 \)).

We have therefore searched for inclination-dependent variation of the observed power spectra. This is feasible because, for the bulk of our observed sample, there are observations in R16 of both \( P_{\text{rot}} \) and the projected rotational speed \( \nu \sin i \). We thus estimate for each star \( \sin i = \nu \sin i / \nu_{\text{eq}} \), where \( \nu_{\text{eq}} \) is the estimated equatorial rotation speed \( \nu_{\text{eq}} = 2\pi R_{*} / P_{\text{rot}} \) and \( R_{*} \) is the stellar radius. We have data to compute this estimate for 77 of our sample stars, all but one of these being Pleiades cluster members.

The inclination values we derived are distributed as expected for a sample that has randomly oriented rotation axes: about half of the sample shows \( \sin i \geq 0.8 \), while only six stars have \( \sin i \leq 0.5 \), and three of these have \( \sin i \leq 0.4 \); because of errors in the estimates of \( \nu \sin i \) and \( \nu_{\text{eq}} \) about a quarter have \( \sin i > 1 \).

The results of comparing \( \sin i \) with our various photometric indices (\( \sigma_{ij}, \sigma_{ij}, \alpha_{ij} \), as well as the measures of integrated power and spectrum slope for frequencies below \( \nu_{\text{eff}} \)) are, however, entirely inconclusive: none of these parameters show a credible dependence on \( \sin i \). We have also examined the power spectra of the six sample stars with \( \sin i < 0.5 \) and compared them with the spectra of other stars having similar \( P_{\text{rot}} \), finding no obvious differences.

We conclude that evidence suggests that the photometric fluctuations at frequencies above \( \nu_{\text{eff}} \) arise mostly from the intrinsic time evolution of brightness features, rather than from rotational modulation of small, essentially static features. Quantifying this conclusion in a meaningful way will likely...
require significantly better estimates of $v \sin i$ and of stellar radii.

5.3. Dependence on the Rossby Number

If the MFC is merely an artifact of the global dynamo, then we would expect the observables $\sigma_C$ and $\sigma_H$ to vary in similar ways with variations in controlling parameters (notably the Rossby number). Real stellar behavior may confound this expectation if one or both observables are unreliable indicators of some more fundamental internal state of the star. Nevertheless, it is informative to ask whether $\sigma_C$ and $\sigma_H$ respond in similar ways to changes in Ro. The answer to this question is implicit in Figures 3 and 5: $\sigma_C$ is a fairly tight function of Ro, but $\sigma_H$ is poorly correlated with $\sigma_C$, so $\sigma_H$ must not depend on Ro in the same way that $\sigma_C$ does.

Figure 6 makes this inference explicit, by showing both $\sigma_H$ and $\sigma_C$ versus Ro, for the entire sample of stars. Evidently $\sigma_C$ is strongly dependent on Ro, whereas $\sigma_H$ is roughly independent of it, except perhaps near the upper limit of Ro represented here.

Indeed, from Figure 6 one sees that the relation $\sigma_H$ (Ro) follows the same generic form as does $\sigma_C$ (Ro), except that the saturated regime extends to $\log(Ro) \approx -0.5$, and the scatter of $\sigma_H$ is much larger than for $\sigma_C$. Also, note that the stars that fall far (sometimes 2 dex) below the upper envelope of $\sigma_C$ are not preferentially binaries. Thus, $\sigma_H$ may be saturated for almost all the stars reported here, with only the very slow-rotating stars occupying the unsaturated regime. Other work (van Saders et al. 2016; Brandenburg et al. 2017; Metcalfe & van Saders 2017) suggests that stellar dynamos show qualitatively different behavior at long rotation periods, starting at Ro near the solar value.

6. Discussion

Up to this point, our discussion of the MFC phenomenon has been almost entirely empirical and descriptive. We now turn to a more physics-based discussion, necessarily conjectural because of our incomplete knowledge of the processes that create the MFC. We will proceed in three steps. First, we describe a few relatively assumption-free inferences that we may derive from the observations described above. Next, we attempt to relate the MFC as observed in the Sun to known solar processes. Last, we consider how the surface structures of rapidly rotating stars might differ from those seen on the Sun, in order to produce the large observed values of $\sigma_C$ that are seen in rapid rotators.

6.1. Physical Inferences

From the blue/green/red color dependence of the solar $\sigma_C$ as observed by VIRGO/SPM, the process generating MFC photometric variability must be almost entirely thermal, with a temperature corresponding roughly to that of the stellar photosphere.

On the other hand, the large variation in $\sigma_C$ across stars with similar structural parameters (mass, age, composition, $T_{\text{eff}}$) suggests, by the absence of alternative explanations, that the underlying process is magnetic. This idea is reinforced by the tight relation between the MFC amplitude $\sigma_C$ and rotation, specifically the Rossby number Ro. Since Ro is a measure of a convection zone’s ability to generate magnetic helicity, and since it is connected to other magnetic processes in stars (e.g., Noyes et al. 1984; Wright et al. 2011), it appears likely that the MFC is connected to some process involving a magnetic dynamo.

The temporal power spectrum of the MFC is a continuum, by definition lacking notable modulation of the photometric fluctuations by the stellar rotation. This implies that whatever process generates the MFC signal, it must be distributed almost uniformly in stellar longitude, and possibly uniformly over the entire surface of the star. Therefore, large-amplitude signals from a few localized spots or active regions do not yield a viable model of the observed fluctuations.

If we suppose that the MFC process presents a cellular morphology, like granulation, present almost everywhere on the stellar surface all the time, then we can place rough limits on the properties of the cells.

Thus, from the low-frequency limit of the MFC ($\nu_0 \approx 20 \mu$Hz), we infer a cell lifetime $\tau_{\text{cell}}$ of about $1/\nu_0$, or roughly half a day. If we equate $\tau_{\text{cell}}$ to the cell turnover time and assume that flow speeds $v_{\text{cell}}$ within the cell are no larger than typical macroturbulent velocities of about $3 \text{ km s}^{-1}$, then we estimate that $2\pi r_{\text{cell}}/v_{\text{cell}} \leq \tau_{\text{cell}}$, so $r_{\text{cell}} \leq 1.6 \times 10^4 \text{ km}$. On the Sun, this is within a factor of 1.5 of the scale of solar supergranules (Rincon & Rieutord 2018).

Assuming that the MFC results from the hemispheric average of independent cellular structures, the net photometric signal $\sigma_C$ from a star should be given by $\sigma_C = \sigma_{\text{cell}}/\sqrt{N_{\text{cell}}}$, where $\sigma_{\text{cell}}$ is the rms relative brightness variation of one cell and $N_{\text{cell}}$ is the number of distinct cells in a hemisphere. Thus, to achieve a large value of $\sigma_C$, we require $\sigma_{\text{cell}}$ to be large or $N_{\text{cell}}$ to be small, or both.

To summarize, observations suggest that the MFC observed in the photometric fluctuation spectra of Sun-like stars is associated with a photospheric process that has a significant magnetic component and a timescale similar to that of solar supergranulation. Constraints on the dominant spatial scale of the process are weak but are consistent with a ubiquitous cellular spatial structure with typical cell sizes similar to those of solar supergranules.
6.2. Back to the Sun

We would like to establish a tentative connection between the structures generating the MFC and known phenomena on the Sun. To conform with our estimated MFC lifetimes and length scales, to satisfy the requirement of near-uniform presence on the whole star surface, and to place the main source of radiation in the photosphere, we are led to consider supergranulation and features related to it.

Since the supergranulation phenomenon is not commonly encountered outside of solar physics, we draw here on the recent review by Rincon & Rieutord (2018) to summarize its relevant properties. Supergranulation is the largest cellular circulation pattern visible at the solar surface. Many of the properties of supergranulation are well known, although controversies remain about its underlying nature. For instance, there is little consensus about what sets the size scale of supergranules, or even whether one should think of them as flows driven by thermal convection. Solar supergranules have typical diameters of about 35 Mm, or about 5% of the solar radius. Estimates of their lifetimes vary, depending on definitions, but the most commonly cited values are between 1 and 2 days. Because of their large horizontal scale relative to the photospheric scale height, their velocity fields are mostly horizontal, with flows diverging from the cell centers at typically 300–400 m s\(^{-1}\).

Supergranular flows sweep magnetic fields with them to the cell boundaries, where they become entrained in downflows that compress the fields into small (100–500 km) flux tubes with field strengths of one to a few kG. Collectively, these tubes make up the magnetic network (Bellot Rubio & Orozco Suárez 2019). Flux tubes often appear in broadband emission as photospheric bright points—the result of lowered opacity in the partially evacuated tubes allowing radiation from deep, hot layers of the solar atmosphere to escape (Spruit 1976; Libbrecht & Kuhn 1984). If one excludes strongly magnetized regions (the network downdrafts), then supergranules display only very little brightness structure (with typical contrast of \(7 \times 10^{-3}\); Goldbaum et al. 2009). Including the magnetized regions, however, supergranules show broadband relative flux variations with spatial rms of \((2–3) \times 10^{-3}\), depending on wavelength (Lin & Kuhn 1992).

We wish to estimate the supergranular contribution to the temporal rms of the disk-integrated solar flux, so we can compare this to the VIRGO/SPM measurements described above. To do this, we assume that the Lin & Kuhn (1992) numbers for the spatial rms of the supergranular contrast also describe the temporal rms, and we take the number of supergranules per hemisphere to be \(N_{\text{cell}} = 2500\). Then, we extrapolate the Lin & Kuhn (1992) red and green measurements to the VIRGO/SPM blue wavelength, assuming that these scale according to the temperature derivative of the Planck function. This process gives estimated supergranule rms for {blue, green, red} colors equal to \([60, 40, 25] \text{ ppm}\). These agree within better than a factor of 2 with the VIRGO/SPM measurements of \(\sigma_C\), namely, \(\{\text{blue}, \text{green}, \text{red}\} = \{46.0, 23.4, 13.7\}\) ppm.

We conclude that the VIRGO/SPM observations are broadly consistent with the idea that the solar MFC arises from supergranulation and its associated magnetic phenomena, in particular photospheric bright points. We warn, however, that this apparent consistency is by no means a proof of identity. Such a proof will require a reliable quantitative theory of supergranulation and the magnetic network and also more comprehensive and directed observations.

These uncertainties notwithstanding, we now hypothesize that the MFC observed in the Sun’s disk-integrated flux arises from photospheric bright points located in the quiet-Sun magnetic network that delineates supergranule boundaries. Individual bright points have lifetimes of tens of minutes to a few hours (Giannattasio et al. 2018), but the network structures to which they belong may survive for considerably longer. We then suggest that it is the formation and decay of these tubes (with their corresponding radiative signatures) that lead to the power-law temporal spectrum defining the MFC. The time-scales and inferred spatial scales of the MFC then arise partly from the lifetimes and peculiar motions of the individual bright points, but mostly from the lifetimes and sizes of the supergranules’ organizing flows. More work is needed, of course, to see whether this picture is accurate.

6.3. Extension to More Active Stars

If the Sun’s MFC results from photospheric bright points, then what of the MFC seen in rapidly rotating Pleiades members, in the saturated regime of Figure 3? If we adopt the idea that the MFC results from a hemispheric average of signals from uncorrelated cellular structures, then attaining the typical saturated photometric rms of 0.4% means that \(\sigma_C = \sigma_{\text{cell}} / \sqrt{N_{\text{cell}}}\) for such stars must be larger than the solar \(\sigma_C\) by a factor of about 50.

It is plausible that both \(\sigma_{\text{cell}}\) and the size or shape of overturning cells (hence \(N_{\text{cell}}\)) might depend on the Rossby number. Thus, in dynamos, Ro governs the creation rate of magnetic helicity; in nonmagnetic convection it measures the relative strength of Coriolis and buoyancy forces. But our present knowledge, both observational and theoretical, is not sufficient to go much beyond these simple statements. For now, we restrict ourselves to four comments.

First, since the filling factor of bright points in the quiet Sun is roughly 1% (Giannattasio et al. 2018), it seems implausible that fast rotation (or any other process) can increase their number density on the stellar surface by the required factor of 50. Put differently, if we ascribe all of the Ro-related variation in \(\sigma_C\) to variations in \(\sigma_{\text{cell}}\), then the required \(\sigma_{\text{cell}}\) at saturation is about 10%. This is comparable to the contrast seen in solar granulation, but on spatial and temporal scales that are about an order of magnitude larger. It is not obvious how the fields and flows might organize themselves to achieve this. We therefore speculate that as Ro decreases, both \(\sigma_{\text{cell}}\) and \(N_{\text{cell}}\) are involved in the growth of \(\sigma_C\). That is, the flows and fields responsible for the MFC not only become more intense but likely also undergo significant changes in the size and shape of their organizing cells. Alternatively, with the stronger nonglobal fields at low Ro, the fields may start to interfere with the convective energy flow from the interior and produce short-lived pores and small starspots. This could produce the observed photometric variability without involving as much surface area as would bright points, because spot brightness contrasts are larger.

Second, in the solar literature one finds discussion as to whether the supergranular intranetwork magnetic fields result from a near-surface magnetic dynamo that operates independently from the global one that is responsible for sunspots and the solar magnetic cycle (Vögler & Schüssler 2007; Lites et al. 2008; Lites 2011; Bellot Rubio & Orozco Suárez 2019). We observe (e.g., in Figure 5) a general lack of correlation,
pronounced in the solar case, between the strength $\sigma_C$ of the MFC phenomenon and $\sigma_{11}$, the corresponding index of global magnetic activity. This poor correlation is evidence for two or more different dynamos.

Third, Figure 3 suggests that there is a small-scale dynamo that at small Ro grows in strength to compete with the global one, or perhaps that the rotation-dominated global dynamo extends to smaller spatial scales at low Ro. In either case, for rapidly rotating stars one should expect to see stronger fields at small spatial scales, hence increased magnetic complexity, and hence weakened braking torque exerted by the stellar wind (Garaﬀo et al. 2015, 2016, 2018; Réville et al. 2015). Such a torque reduction may provide a mechanism to explain the bimodal $P_{\text{rot}}$ distributions seen in young star clusters such as the Pleiades (Soderblom et al. 1993; Barnes 2010; Hartman et al. 2010; Brown 2014).

Lastly, we ﬁnd it curious that the relation between Ro and $\sigma_C$, shown in Figure 3, shows so little scatter. The MFC range of timescales suggests turnover times and cell depths that are considerably smaller than those for the observed stars’ entire convection zones. But naively, we expect that the relevant Rossby number for these ﬂows should relate to the turnover times for the shallow ﬂows, while the Rossby numbers used in Figure 3 are computed using $\tau_C$, the turnover time for the entire convection zone. Thus, to get the fairly tight relation between $\sigma_C$ and Ro that is shown in Figure 3, it must be that the MFC ﬂow turnover times scale with $T_{\text{eff}}$ in the same way as does $\tau_C$. It is not obvious why this should be so. Moreover, at least three other stellar magnetic phenomena show amplitude versus Ro relationships that resemble the one we plot in Figure 3. These are the harmonic amplitude $\sigma_{11}$ shown in Figure 6, the coronal X-ray emission described by Wright et al. (2011), and the unsigned magnetic ﬂux measured by See et al. (2019). Some of these processes appear to be physically related, in that the photospheric magnetic network, the chromospheric network, and the footpoints of X-ray loops are often cospatial. This makes sense if the different heights at which these phenomena occur are linked by strong, more or less vertical magnetic ﬁelds. But if these various phenomena reﬂect a single underlying dynamo process, then why are their governing parameters (break point in Ro, slope of unsaturated regime) so different from one to another? If they represent physically different dynamos, why are their morphologies so similar? Also, why is the scatter in these relationships so much smaller for $\sigma_C$ and for the X-ray ﬂux than for the other two cases? To answer these and related questions, we will likely need better observational statistics, new kinds of observational diagnostics, and improved quantitative models of magnetized ﬂows in stellar atmospheres.

We expect that the MFC will prove important in understanding stellar rotation and magnetism, both because MFC properties are relatively easy to observe and because, once understood, the MFC may well provide powerful diagnostic tools with which to probe stellar convective dynamics and magnetic activity. With respect to observability, note that the MFC observables $\sigma_C$, $\alpha_1$ may be recovered from data sets of only a few days’ duration, needing a sampling cadence of only one or two samples per hour, and having (by the standards that apply to space photometry) relatively poor photometric precision. For example, for almost the whole sky down to $V$ magnitudes of 12 or so, the TESS mission (Ricker et al. 2014) provides a 27-day-long near-continuous stellar time series with a 2-minute sampling cadence (for targeted stars) and signal-to-noise ratio (S/N) adequate for this purpose. Thus, MFC parameters may be obtained for vast samples of stars. As for diagnostic power, the dependencies shown in Figures 3 and 4 are fairly well deﬁned and can be improved with larger samples of stars. Also, they are complex enough that the ability to reproduce them, even in part, would provide a strong validation of future numerical models.

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A.1. Preprocessing

The EVEREST time series from which we worked already consisted of weighted sums of Kepler pixels, corrected for various instrumental effects as described by Luger et al. (2016). We did several comparisons of the EVEREST time series with other methods (VARCAT, Armstrong et al. 2016; K2SFF, Vanderburg & Johnson 2014). For the latter two time series we followed the same preprocessing as for the EVEREST and Kepler light curves as described in Section 3.1. It was found that the EVEREST data best served our purposes, because they gave the highest S/N near peaks in the power spectrum.

Appendix A

Data Reduction and Fitting Procedures

For the Pleiades LC time series, our data analysis proceeded through four steps: (1) preprocessing, (2) power spectrum computation, (3) ﬁtting the coherent (harmonic) part of the spectrum, and (4) ﬁtting the continuous spectrum.
The inputs to our Pleiades pipeline were zero-mean, high-pass-filtered, and apodized time series, sampled once per 29.42 minutes, in units of parts per million (ppm) of the mean stellar brightness.

A.2. Power Spectrum

For LC data, we used the Scargle (1982) algorithm to compute the power spectrum on a grid with resolution 0.1 μHz. Given the duration of the LC time series, this oversampled the true resolving power of the data strings by a factor of about 1.3. For the SC data, we computed the power directly from the discrete Fourier transform of the time series. The resulting resolution was about 0.011 μHz. We normalized the power spectrum to give the PSD, in units of ppm²/μHz⁻¹.

It is computationally convenient to frame the spectrum fitting as a least-squares problem. But to avoid biased results, the data being fitted should have errors that are more or less normally distributed, unlike the (χ² with 2 degrees of freedom) distribution that emerges from the power spectrum computation. To get unbiased fits, we therefore smoothed the spectra in frequency, using pseudo-Gaussian kernels with widths of about 5 frequency elements (for LC spectra) or 19 elements (for SC spectra). This smoothing is acceptable, because in fitting the continuum power we do not require even the full frequency resolution of the LC data.

The fitting procedure we used is a sequential one that first fits the coherent (line) part of the spectrum and then uses the residuals from this fit to model the continuum part. In the future we hope to develop a more elaborate fitting code, which will fit the entire spectrum simultaneously and consistently, and moreover will be completely automated, removing the biases and lack of repeatability that go with human intervention. Such a code would also make it practical to analyze many more stars than we are now capable of doing. For the present, however, we find it most expedient to proceed with the current analysis tools, bearing their shortcomings in mind.

A.3. Fitting Rotational Harmonic Components

Rotating spotted stars produce observed photometric time series that are often dominated by the star’s rotation frequency and its harmonics. Accounting for these multiple large-amplitude narrowband signals is the most complicated part of our spectrum-fitting process, but for our immediate purposes (fitting the continuum spectrum) its results are largely irrelevant.

We represent the coherent part of each power spectrum as

\[
P_{\text{harm}}(\nu) = \sum_{i=1}^{N_i} \left( \sum_{j=1}^{N_j} A_{ij} W_i(\nu - j\nu_i) \right).
\] (A1)

Here the sum over harmonic sequences i is itself a sum of terms related to different stellar rotation frequencies (e.g., the various members of a multiple star system, each with its own rotation frequency, or different latitudes on a single differentially rotating star). For most stars, this sum over distinct rotating systems involves only a single system. Each of the Nᵢ rotating systems is taken to have its own line profile Wᵢ, which we take to be shared by all of the associated rotation harmonics, having frequencies ν = j νᵢ for j = 1, 2, ..., Nᵢ. Finally, the various harmonics j composing the rotating system i have amplitudes A_ij, each taken to be independent of all the others. The parameters required for a fit are the number of rotation systems Nᵢ, and for each of these the corresponding fundamental frequency νᵢ, the parameters of the line profile Wᵢ, the number of harmonics N_j, and the amplitudes of all the harmonics A_ij.

We parameterize the shape of the line profile Wᵢ as a generalized Lorentzian

\[
W_i(\nu) = 1/(1 + |\nu/\nu_0|^m),
\] (A2)

where s is a width parameter and m is a power-law index, which would be 2 for a true Lorentzian. The code takes m = 4.5 for all lines but fits s independently for each rotation system, fitting to the observed profile of the fundamental frequency νᵢ for the system. (This is usually, though not always, the largest peak in the spectrum.)

We fit the model parameters to the smoothed observed power spectrum via χ² minimization, assuming uncertainties σ = PSD/N_σ, where N_σ is the effective smoothing width.

Figure 7 shows in orange the result of fitting the coherent part of the power spectrum of a typical star (EPIC210905362) in the way just described. Evidently the fit succeeds in capturing the broad features of the coherent spectrum, in particular the total power integrated across each line profile. The fit does not, however, accurately reproduce the line shapes outside the line core regions, leaving substantial residuals in the near wings of the lines.

A.4. Continuum Fitting

We proceed in the continuum-fitting step simply by subtracting the fitted coherent spectra as best we can and assigning small or zero fitting weights to the residuals within seven line widths of the modeled line-center frequencies.

After subtracting the coherent signal from rotating active regions, we model the remaining power spectra over the frequency range 1 μHz ≤ ν ≤ 285 μHz as a continuous piecewise power law, using only two frequency domains, with a break point at the frequency ν₀:

\[
\nu \geq \nu_0: \quad P_{\text{cont}} = A \left(\nu/\nu_0\right)^{\alpha_1}
\]

\[
\nu \leq \nu_0: \quad P_{\text{cont}} = A \left(\nu/\nu_0\right)^{\alpha_2}.
\] (A3)

By performing the fit in log(ν), log(PSD) coordinates, this reduces to fitting for the P_cont amplitude A and the two straight line slopes α₁, α₂. We perform this fit via χ² minimization, using as input the log of the PSD residuals from the coherent harmonic spectrum fit described above. We compute weights based on the smoothed χ² distribution also described above. Figure 7 shows in black the result of such a fit applied to the same star, using ν₀ = 20 μHz. This is a typical case, giving a fit that represents the observed spectrum in a way that is satisfactory, though not perfect.

In the fits described in the main text, we use ν₀ = 20 μHz for fitting all stars. Since we do not yet understand the physics involved, there is no a priori reason to believe that ν₀ should be the same for all stars. But neither do we have justification for any particular dependence on stellar parameters. So for the current study we adopt ν₀ = 20 μHz for all stars, justified by simplicity and by the fact that almost always this value delivers sensible fits.
Appendix B

Exceptional Stars

A few stars are notable exceptions to the systematic behaviors that characterize most stars. In the upper left corner of Figure 5 one finds three stars (KIC 11081729, KIC 7103006, KIC 3733735) that stand out for large values of $\sigma_C$ and for relatively strong correlation between $\sigma_C$ and $\sigma_H$. As the color-coding suggests, these are the hottest stars in our Kepler SC sample; all have $T_{\text{eff}} \geq 6300$ K. Only five stars in our Pleiades sample have such large $T_{\text{eff}}$. In addition, these stars have relatively short rotation periods, and they fall about 0.5 dex below the mean log Ro–log $C_s$ relation. All of these characteristics suggest that these three stars lie near the boundary of the Kraft break (Kraft 1967), having shallow and inefficient convection zones that cause them to be unrepresentative of our larger star sample. Also anomalous is KIC 8379927, with $\sigma_H$ both unusually large and unusually variable. This star is one of the LEGACY sample of stars with high-quality Kepler asteroseismic measurements (Lund et al. 2017). Its time series shows what appear to be beats between fundamental rotation harmonics separated by about 0.05 $\mu$Hz. Speckle (Horch et al. 2012) and radial velocity (Griffin 2007) observations show this star to be binary, with a companion that is about 1.4 mag fainter than the primary. It is not clear whether its peculiar activity parameters arise from its fairly bright companion, from its large apparent differential rotation, or from some other cause.

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Figure 7. PSD of brightness fluctuations of a typical Pleiades member, having $P_{\text{rot}} = 2.379$ days. The thin blue trace is the observed spectrum. The thick orange trace is the fitted model of the harmonic part of the power spectrum. The thick black trace is the full model, i.e., the sum of the harmonic and continuum models.
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