Right-handed Currents Searches and Parity Doubling

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The extraction of right-handed currents, beyond the Standard Model, faces theoretical challenges from long-distance contributions. We show that these effects can be controlled by combining, for example, studies of $B \to V(1^-)\gamma$ and $B \to A(1^+)\gamma$ observables. The sum of the long-distance contributions can be extracted without compromise, and the individual pieces follow from a ratio predicted by theory. This leads to significant reduction in the uncertainty of long-distance contributions. The ideas extend to charm decays and the low $q^2$-region of $B \to V\ell\bar{\ell}$, and open the prospect of checking input affecting the angular $B \to K^{*}\mu\mu$-anomaly.

1 Introduction

The Standard Model (SM) is a highly successful, yet peculiar, theory. One of its peculiarities is that the weak interactions are of the V-A type. It is intuitively clear that this leaves traces in the polarisation (or the angular distribution) of weak decays. Such traces would be perfect probes for right-handed currents (RHC) searches were it not for non-perturbative effects of QCD diluting the purity of the signal.

A particularly good setting to test the chirality of interactions is when there is a photon in the final state, as the photon helicity is then in direct correspondence with the handedness of the interaction. In particular, in the limit of no (quark) masses, chirality and handedness are the same. For example, the QED interaction reads $\bar{q}Aq = \bar{q}_L A q_L + \bar{q}_R A q_R$. Thus, the reaction $\bar{q}_L + q_L \to \gamma_L$ is an on-shell process where the two half-helicities of the quarks add up to match the ±1 helicity of the photon (termed left- and right-handed respectively). If the interactions were chiral, $H^{\text{int}} \sim \bar{q}_L A q_L$, then the resulting photon polarisation would always be left-handed. Denoting the amplitude of left- and right-handed photons by $A_{L,R}$ respectively, this reads $A_R/A_L = 0$.

Such transitions are not present in the flavour-changing neutral currents (FCNCs) in the SM. The next best possibility is to couple the photon to two quarks of opposite chirality in a directly gauge-invariant way, at the expense of a quark mass term. This is realised by the so-called electric dipole operator (known as the $O^{(5)}_7$-term in the effective Hamiltonian)

$$H^{\text{eff}} \supset C^{(5)}_7 O^{(5)}_7 \sim m_b(m_s)\bar{s}_L(R)\sigma_{\mu\nu}F^{\mu\nu}b_{R(L)} ,$$

where $F^{\mu\nu}$ is the photon field strength tensor. The concrete appearance of $m_b(m_s)$ can be understood from a spurion analysis. The dimension-six effective Hamiltonian is written as $H^{\text{eff}} \supset C^{(7)}_7 \bar{s}_\gamma \Gamma b O_{\gamma}$, with flavour-neutral $O_{\gamma}$ and $C^{(7)}_7/C^{(5)}_7 \lesssim 1$ similar to $C^{(7)}_7/C^{(5)}_7 \sim m_s/m_b$. This hierarchy, and therefore RHC searches, is affected by the non-perturbative QCD matrix elements

$$A_{\pm(5)} = \langle X_\pm \gamma^* | \bar{s}(\gamma_5) \Gamma b O_{\gamma} | B \rangle .$$

More concretely, one of the simplest processes testing the helicity is the $B \to V\gamma$ decay, where $V$ is a $J = 1$ vector meson. The amplitude of left- and right-handed polarised photon is

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the signal. Schematically, \( RHC \) in this context. As hinted at, long-distance (LD) contributions \( \epsilon \) Beyond the SM (BSM) shifts to axial flavour symmetries are restored: \( SU(2) \to SU(1) \pm SU(1) \). It is conceptually beneficial to consider the chiral restoration limit uncertainty (in the real world), which of course indirectly benefits from the closeness to the \( R \) at this point that the crucial practical question is not the actual value of \( \epsilon_{L,R} \) but rather its \( R \) arises through an \( \bar{s}_L \Gamma q \bar{q}' \gamma \)-interaction, and is sensitive to the parity quantum number of the \( V \)-state. Hence, if for every vector state there were a partner of opposite parity then one could discern \( \Delta_{RHC} \) from \( \epsilon_{R} \). This is the idea of our work, and we advocate to combine decay channels of nearly-degenerate parity partners.

In the chiral symmetry restoration limit, the following exact relation will be shown to hold:

\[
A^{B \to V \gamma}_{\chi}(C,C') = A^{B \to A \gamma}_{\chi}(-C,C') ,
\]

(6)

with \( \chi = L, R \), and \( A \) an opposite-parity partner of the \( V \) meson. This is the solution to our problem, since we are concerned with mixing up \( C' \) (i.e. \( \Delta_{RHC} \)) with LD-effects \( \epsilon_{R} \) induced by \( C \)-type operators cf. Eq. (3). As we shall see, the problem is then shifted from estimating \( A_2 - A_5 \) to estimating

\[
R_{A,V} \equiv \frac{\text{Re}[\epsilon_{R}^{B \to A \gamma}]}{\text{Re}[\epsilon_{R}^{B \to V \gamma}]} = 1 + O(m_{q}, \langle \bar{q}q \rangle) .
\]

(7)

To some extent it is the operator state correspondence of quantum field theory, expressed by the LSZ formalism, that allows for this shift in perspective. We would like to stress already at this point that the crucial practical question is not the actual value of \( R_{A,V} \), but rather its uncertainty (in the real world), which of course indirectly benefits from the closeness to the symmetry limit.

2 Relating axial and vector meson matrix elements in the chiral symmetry limit

It is conceptually beneficial to consider the chiral restoration limit \( \{m_{q}, \langle \bar{q}q \rangle, \ldots \} \to 0 \), where the axial flavour symmetries are restored: \( SU(N_{F})_{V} \to SU(N_{F})_{V} \times SU(N_{F})_{A} \times U(1)_{A} \). The relation we are to use is that in the restoration limit the quark propagator in the gluon background field, \( S_{G}^{q}(w,z) = \langle w|(\not\! p + im_{q})^{-1}|z) \), obeys

\[
\gamma_{5}S_{G}^{q}(w,z) = -S_{G}^{q}(w,z)\gamma_{5} ,
\]

(8)

To what extent the \( U(1)_{A} \) is restored due to the axial anomaly is an interesting question, but is not relevant for our purposes. Finite-temperature lattice computations above the chiral phase transition give evidence of \( U(1)_{A} \)-restoration \( \gamma \).
for which the vanishing of the SU($N_F)_A \times U(1)_A$-violating condensates is a necessary condition, as can be understood from the Banks-Casher relation \(^6\).

The starting point is that any information of the matrix element $\langle V\gamma^s|h_{\text{eff}}|B\rangle$, where $\gamma^s$ is a potentially off-shell photon, can be extracted from the correlation function

$$M^{[V]}_{(v,a)} \equiv \langle 0|\mathcal{T}\{J_B(x)V^I_\mu(y)h_{\text{eff}}(0)\}|0\rangle \; , \; h_{\text{eff}} = \bar{q}(v + a\gamma_5)\Gamma b O_r \; ,$$

by analysing its dispersion relation, as $J_B = \bar{b}\gamma_5 q$ and $V^I_\mu \rightarrow \rho(a_1)^I_\mu = \bar{q}\gamma_\mu T^I(\gamma_5)q$ are interpolating operators for the $B$-meson and the vector (axial) mesons respectively. In Eq. (9), $\Gamma$ is a Dirac structure, while $O_r$ stands for the remaining part of the operator. For example, $O_r = \overline{c}\Gamma' c, \overline{u}\Gamma' u, \ldots$ distinguish between short-distance (SD) and LD (e.g. four-quark) operators. Contracting the quark lines and focusing on the $\rho$ meson final state, the matrix element assumes the form

$$M^{[\rho]}_{(v,a)} \sim \int D\mu_G \text{Tr}[(v + a\gamma_5)S_G^{(b)}(0,x)\gamma_5 S_G^{(d)}(x,y)\gamma_\mu S_G^{(d)}(y,0)] \; ,$$

where the path-integral measure is given by $D\mu_G = D\mu_G \det(\partial - igG)\mu \; , \; D\mu = (\partial - igG)\mu$.

Now comes the main trick. Substituting $\gamma_\mu \rightarrow \gamma_\mu(\gamma_5)^2$ and using Eq. (8) leads to an expression, $M_{(a,v)}^{[a_1]} = -M_{(v,a)}^{[\rho]}$, for which the $a_1$ meson matrix element is the same up to a sign with the variables $a$ and $v$ interchanged (Fig. 1). From this expression, Eq. (6) follows, which is our main formal result. In the last equation it is understood that $V$ and $A$ become degenerate in the chiral restoration limit \(^6\), and are referred to as parity doublets: \(^c\).

3 Phenomenological implications

In effect, Eq. (6) means that the ratio Eq. (5), amended to include an axial meson, is then given by

$$\frac{\mathcal{A}_R}{\mathcal{A}_L}_{B\rightarrow V(A)\gamma} = \epsilon_R \pm (\hat{m}_s + \Delta_{\text{RHC}}) \left(1 + \epsilon_L \right) \simeq \epsilon_R \pm (\hat{m}_s + \Delta_{\text{RHC}}) \; ,$$

\(^c\)Parity doubling has a long history in particle physics \(^8\), and has recently been investigated on the lattice \(^9,10\), with the additional surprise of an emergent symmetry. For a table of relevant opposite parity states, we refer the reader to Tab. 1 in Ref. \(^6\).
which is the equation from which our phenomenological results are derived. Beyond the symmetry limit, the only relevant change to Eq. (11) is that the LD contributions $\epsilon_R, \epsilon_L \rightarrow \epsilon_V(A), R, \epsilon_V(A), L$ are dependent on the final state to a degree that needs to be estimated by analytical methods.

The crucial question for RHC searches is how the hierarchy and the sign change in Eq. (11) can be exploited. Concerning the hierarchy, the rate itself is not promising, since $\Gamma_{\text{tot}} \sim |A_L|^2 + |A_R|^2$ and the effect of the RHCs might be too small to be seen in experiment. A more promising route is to consider angular distributions, e.g. $B \rightarrow V \ell \bar{\ell}$, or time-dependent decay rates in $B \rightarrow V\gamma$, as originally proposed by the authors of Ref. 11.

The time-dependent rate of a neutral $B_D$-meson ($D = d,s$), under general and valid assumptions, reads\(^d\)

$$B(B_D[|D_D] \rightarrow V\gamma) = B_0 e^{-\Gamma_D t}[\text{ch}(\frac{\Delta\Gamma_D}{2} t) - H\text{sh}(\frac{\Delta\Gamma_D}{2} t) \mp C \cos(\Delta m_D t) \pm \mp \text{S} \sin(\Delta m_D t) ], \quad (12)$$

where $\Delta\Gamma_D \equiv \Gamma_D^{(H)} - \Gamma_D^{(L)}$ is the width difference, and $\Delta m_D \equiv m_D^{(H)} - m_D^{(L)}$ the mass difference, of the heavy $(H)$ and light $(L)$ mass eigenstates. $S$ and $C$ are related to indirect and direct CP violation respectively.\(^d\) The quantities $S$ and $H$ are linear in $A_R$, and given in terms of the amplitudes by

$$S(H) = 2\text{Im}(\text{Re})\left[\frac{q}{p} (\bar{A}_L A_L^* + \bar{A}_R A_R^*)\right] N^{-1} , \quad (13)$$

with $N = |A_L|^2 + |\bar{A}_L|^2 + |A_R|^2 + |\bar{A}_R|^2$. We choose to illustrate the approach by the mode $B_s \rightarrow \phi\gamma$ and $B_s \rightarrow f_1(1420)\gamma$, with other modes discussed in Ref. 6, as this mode is not sensitive to CKM factors. The observables $H$ and $S$ are well-approximated by

$$H_{B_s \rightarrow \phi(f_1)\gamma} \simeq 2\{ \pm (\Delta_R \cos(\phi_R) + \hat{m}_s) - \text{Re}[\epsilon_{\phi(f_1)\gamma}] \}, \quad S_{B_s \rightarrow \phi(f_1)\gamma} \simeq 2\{ \pm \Delta_R \sin(\phi_R) \} . \quad (14)$$

The vanishing of $S_{B_s \rightarrow \phi(f_1)\gamma} \simeq 0$ in the SM comes from the cancelation of all weak phases involved, and this quantity is therefore a null test for weak phases of RHC. From Eq. (14), we obtain the remarkable equation

$$H_{\phi\gamma} + H_{f_1\gamma} \simeq -2\text{Re}[\epsilon_{\phi,R} + \epsilon_{f_1,R}] = -2\text{Re}[\epsilon_{\phi,R}] \frac{1}{2} \left[ 1 + \text{Re}[\epsilon_{f_1,\phi}] \right] , \quad (15)$$

where the SD physics drops out. Its SD-sensitive counterpart is

$$\Delta_R \cos(\phi_{\Delta_R}) = \frac{1}{4} (H_{\phi\gamma} - H_{f_1\gamma}) + \frac{1}{2} \text{Re}[\epsilon_{\phi,R} - \epsilon_{f_1,R}] - \hat{m}_s . \quad (16)$$

In Eq. (15), $F^i_{A,V} \equiv \text{Re}[\epsilon^i_{A,R}] / |\epsilon^i_{V,R}|$ is the more refined version of Eq. (7), in that it includes the information on the flavour of the four-quark operator from which it derives. The main points are as follows:

- Eq. (15) shows that one can measure the sum of the LD contributions without compromise from RHC or SM SD physics, owing to the previously-mentioned exact form factor relation $T_1(0) = T_2(0)$.

- One can extract the LD parts of the individual modes, entering Eq. (16), by an analytic prediction of $F^i_{f_1,\phi}$. We again stress that it is not the value (deviation from unity) but the error on $F^i_{f_1,\phi}$ which is important. By flavour symmetries it is clear that the measurement of a single axial vector meson can reveal valuable information on the size of LD contributions.

- Making the last point more concrete, an error of 20% on $F^i_{f_1,\phi}$, assuming a perfect measurement, allows us to extract $\text{Re}[\epsilon_{\phi,R}]$ to 10%. This is a much-improved situation as compared to an a priori computation\(^{13}\).

\(^d\) $H \equiv \Delta_\Sigma$ in the Particle Data Group (PDG) notation.
• These methods apply straightforwardly to charm physics, and can be extended to $B \to V\ell\bar{\ell}$ at low $q^2$, although this will require taking into account that the exact form-factor relation $T_1(0) = T_2(0)$ no longer holds. In particular, the real and imaginary parts of the angular moment $G_2 = P_1 = A_2^{(2)}$ and $P_3$ respectively, also exhibit the required linear dependence on the right-handed amplitude. Measuring the analogues of $\epsilon_R$ in this channel allows to cross-check the LD theory input into the anomalous angular $B \to K^*\mu^+\mu^-$ measurement e.g. $P_5$.  

4 Conclusions

In this work, we have advocated that long-distance effects contaminating searches for right-handed currents in $B \to V\gamma(\ell\bar{\ell})$ decays can be controlled by considering the corresponding parity-doubler decay mode $B \to A\gamma(\ell\bar{\ell})$. In the limit where the chiral symmetry is restored, the V-A contributions to the right-handed amplitude come with the opposite sign between these two channels. This can be applied phenomenologically by combining observables, as shown explicitly for example in measurements of time-dependent CP asymmetry in $B_s \to \phi(f_1)\gamma$ in Eqs. (15) and (16), to extract and measure ratios of long-distance contributions. In turn this can lead to a cleaner extraction of Beyond the SM contributions to right-handed currents.

It is again important to stress that the main benefit is not in the prediction of the long-distance ratio itself, but the reduced theoretical uncertainty that results. This is also useful in resolving the current tension between predictions of the size of long-distance charm loop contaminations in exclusive and inclusive $B \to X_s\gamma$ decays, where the inclusive contamination was estimated to be roughly an order of magnitude larger.

Corrections to the results in this work, applying beyond the symmetry limit, can still be understood systematically by investigating the symmetry relations between vector and axial mesons, such as the parameters entering their light-cone distribution amplitudes. This allows the approach we advocate above to be applied in real-world experimental searches, with good prospects at Belle II and LHCb.

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