A Novel Thermal Response Factor Method for the Dynamic Load Calculation of Buildings

Chunlu Zhang¹ and Guoliang Ding²

¹Associate Professor, Institute of Refrigeration and Cryogenics, Shanghai Jiaotong University
1954 Huashan Road, Shanghai 200030 P.R. China (clzhang@mail.sjtu.edu.cn)
²Professor, Institute of Refrigeration and Cryogenics, Shanghai Jiaotong University
(glding@mail.sjtu.edu.cn)

Abstract
The traditional thermal response factor method for load calculation employs invariable time-step. To ensure accuracy with the less possible computing quantity, the method with variable time steps is expected to deal with the complicated dynamic thermal loads of buildings. In this paper, the traditional method of excitation disintegration is analyzed first because the time step is initially fixed here. Two new functions for excitation disintegration are presented, namely the step function and the ramp-step function. Based on the new excitation method, a novel thermal response factor method with variable time steps is developed. In an example, the thermal response to a sine-wave input is evaluated by the traditional and present methods. It illustrates that about 36% computing quantity is decreased with the new method at the same precision.

Keywords: dynamic load; thermal response factor; variable time step

1. Introduction
Since Stephenson and Mitalas (1967, 1971) presented the thermal response factor method and the z-transfer function method for calculating the dynamic cooling or heating loads of buildings, the two methods have become the key calculation tools in the field. Researches and applications using the two methods have deepened for the past years (Haghighat and Liang, 1992), in which many researchers studied how to obtain the accurate thermal response factors and z-transfer function coefficients with different approaches (Hittle and Bishop, 1983; Ouyang and Haghighat, 1991; Davies, 1995a, 1995b).

To the best of the authors’ knowledge, the available studies on the thermal response factor method and the z-transfer function method in open literature are carried out under conditions of invariable time-step. When this method is used, time-step should be appointed. The thermal response factors or z-transfer function coefficients can then be determined. Finally, the dynamic loads at different instants can be calculated with these factors or coefficients. In calculating the load of buildings, the time-step is normally set as 1 hour. For example, ASHRAE (1993) provided the z-transfer function coefficients of some typical walls, which were calculated with the time step of 1 hour.

In modern buildings, the variations of excitations that control cooling or heating loads are more complicated than before. Therefore the invariable time-step method will not balance the computing quantity and accuracy. For instance, one is ready to calculate the dynamic loads of a building over a longer span of time. For a majority of the time span, the variations of excitations are slow and only hourly loads need be calculated, while for the rest, the input variations are sharp and the loads had better be calculated by the minute. If one uses the method with time step of 1 hour to calculate the loads, large deviation will occur in the region of sharp input variations. On the contrary, if one adopts 1 minute as the time step, the accuracy can be guaranteed, but the computing quantity will increase 60 times.

In order to ensure sufficient accuracy and in the meantime reduce the computing quantity as much as possible, it had better develop a load calculation method with variable time steps. In this work, the authors present a novel thermal response factor method with variable time steps.

2. Method description
2.1 Disintegration of excitations
In the traditional thermal response factor method, the excitation curves are approximately disintegrated by rectangular or triangular waves. As shown in Figs. 1 and 2, the abscissa is time \( t \) and the ordinate is excitation \( u(t) \). The widths of the rectangles in Fig. 1 or those of the triangles in Fig. 2 are identical and known as the time steps. If the response to the input of unit rectangular wave or unit triangular wave has been
saved in discrete series, namely the thermal response factors, one can apply the superposition principle of the linear system to calculate the system response to the practical excitation $u(t)$. This is the fundamental principle of the traditional thermal response factor method. Therefore, the time step of the thermal response factor method depends on the width of rectangular waves or triangular waves used in the disintegration of excitations.

If one uses rectangular waves or triangular waves of different widths to disintegrate the excitations, not only will more memory space for storing thermal response factors of different time steps be needed, but also the calculation process could become more complicated, especially for the triangular waves. Therefore, on the basis of the traditional method of excitation disintegration, it is hard to expand the traditional thermal response factor method with invariable time steps to that with variable time steps. In other words, the traditional method of excitation disintegration should be modified to meet requirements of the variable time-step.

In Fig. 1, the excitation curve is disintegrated by rectangular waves and in Fig. 2, the excitation curve is disintegrated by triangular waves.

Fig.1. Excitation curve disintegrated by rectangular waves

Fig.2. Excitation curve disintegrated by triangular waves

To analyze the traditional excitation disintegration method shown in Figs. 1 and 2, one could consider the relationship between time $t$ and excitation $u(t)$ as a function. In this function, $t$ is the independent variable and excitation $u(t)$ is the dependent variable. By comparison, when one thinks of a new method with variable time steps, his actual purpose is to control the amplitude of excitation variation in the excitation disintegration process, which is the common means for accuracy control. Thus in the new method, excitation $u(t)$ should be considered as the independent variable controlled by the user while time $t$ becomes the dependent variable.

This was realized in the new excitation disintegration method shown in Figs. 3 and 4. The basic functions used to disintegrate the excitation are different from those in the traditional method. Their heights are determined by the controlled amplitude of excitation variation and their widths are semi-infinite. Consequently, the time steps will be adaptively generated by excitation information in the new method.

In Fig. 3, the basic function used to disintegrate the excitation is the step function. If the adaptively generated time steps are constant, for example if the excitation curve is linear, the disintegration effect using either the new method or the traditional one shown in Fig. 1 is identical.

Fig.3. Excitation curve disintegrated by step functions

In Fig. 4, the basic function used to disintegrate the excitation is a kind of special function. It could be regarded as the combination of the ramp function and the step function. Thus we call it the ramp-step function. In theory, however, it is the superposition of two ramp functions seen in Fig.5. One can calculate the thermal response to the ramp-step function with the superposition principle. If the adaptively generated time steps are constant, the disintegration effect using either the new method or the traditional one shown in Fig. 2 is equivalent, too.

Fig.4. Excitation curve disintegrated by ramp-step functions

In Fig. 4, the basic function used to disintegrate the excitation is a kind of special function. It could be regarded as the combination of the ramp function and the step function. Thus we call it the ramp-step function. In theory, however, it is the superposition of two ramp functions seen in Fig.5. One can calculate the thermal response to the ramp-step function with the superposition principle. If the adaptively generated time steps are constant, the disintegration effect using either the new method or the traditional one shown in Fig. 2 is equivalent, too.
2.2 New thermal response factor method

On the basis of the new method for excitation disintegration, the traditional thermal response factor method should be correspondingly modified. Two necessary modifications are described as follows.

First, the thermal response to the new disintegration functions will be recalculated. The calculation method used is the traditional method introduced in classical textbooks, and so is not further interpreted here. The traditional rectangular or triangular thermal response factors are related to the time step. By comparison, in the new method, the thermal response to the unit step function or the unit ramp-step function is independent of the time step. Therefore, the thermal response function can be flexibly stored in one of two forms.

One storage form is the discrete series, which is similar to the traditional thermal response factors. A series of time steps are necessary, but may not remain constant. Since the variation of thermal response to the unit excitation is sharper at the beginning and lapses into slowness, the time step series could begin with smaller values and then be enlarged step-by-step. If the time steps in load calculation are different from those in the stored series, interpolation between the two thermal response factors is recommended.

The other is the continuous function. As a matter of experience, the proper function used to fit the thermal response curve of buildings is the polynomial of the exponential function $A_i\exp(t/T_i)$. In general, a few coefficients will be needed to reach satisfactory precision of the curve fitting.

Comparing the two storage forms of the thermal response, one could find that the discrete one needs a larger memory space for the factors, while the continuous one is somewhat lower in computing speed because the exponential function is used.

Below is the basic formulae for dynamic load calculation. For convenient comparison between the traditional method and the new one, the following equations are written in discrete form. In fact, the continuous form is similar.

When the excitation curve is disintegrated by step functions shown in Fig. 3, the output can be calculated as follows.

$$Y(t_n) = u(t_0)S(t_n - t_0) + \sum_{t_0 \leq t_k < t_{k+1} \leq t_n} [u(t_k) - u(t_{k-1})]S(t_n - t_k)$$  \hspace{1cm} (1)

where, $u(t_k)$ ($k = 0, 1, 2, \ldots$) is the excitation at the $k^{th}$ instant. $S(t)$ is the thermal response to unit step input. $Y(t_n)$ ($n = 0, 1, 2, \ldots$) is the output at the $n^{th}$ instant. Commonly, the initial instant $t_0 = 0$.

In Eq. (1), the time instant $t_k$ is determined by the control precision of excitation disintegration. For example, if the local precision of excitation disintegration is controlled as a constant, namely the increment of excitation is invariant, one has

$$u(t_k) - u(t_{k-1}) = u(t_{k+1}) - u(t_k) = \delta u$$  \hspace{1cm} (2)

where, the increment of excitation $\delta u$ is constant.

If necessary, the increment of excitation could also be time-variant. It is not difficult for Eq. (1) to deal with this change.

When the excitation curve is disintegrated by ramp-step functions shown in Fig. 4, the output can be calculated as follows.

$$Y(t_n) = u(t_0)T(t_n - t_0) + \sum_{t_0 \leq t_k < t_{k+1} \leq t_n} [u(t_k) - u(t_{k-1})]T(t_n - t_k)$$  \hspace{1cm} (3)

where, $T(t)$ is the thermal response to unit ramp-step input. As shown in Fig. 5, $T(t)$ can be calculated by

$$T(t_n - t_0) = R(t_n - t_0 + \delta t) - R(t_n - t_0)$$
$$T(t_n - t_k) = R(t_n - t_{k-1}) - R(t_n - t_k)$$  \hspace{1cm} (4)

where, $R(t)$ is the thermal response to unit ramp input. The positive parameter $\delta t$ is the leading time. This can also be seen in the triangular disintegration as well. However, here the value of leading time is not invariant. If the leading response is not expected, one could set $\delta t = 0$. Eq. (3) could be rewritten as follows.

$$Y(t_n) = u(t_0)S(t_n - t_0) + \sum_{t_0 \leq t_k < t_{k+1} \leq t_n} [u(t_k) - u(t_{k-1})]S(t_n - t_k)$$  \hspace{1cm} (5)

where, the first item on the right side of equal sign is the same as that in Eq. (1). Therefore, the new method without limitation of time step is more flexible than the traditional one.

3. Case study

A simple example is given to compare the new
method with the traditional one. A flat roof consisting of a concrete slab with a thickness of 150mm, density 2500kg/m³, heat conductivity 1.63W/(m·K) and specific heat capacity 840J/(kg·K). The heat transfer coefficients of the outside and inside surface are 7.0W/(m²·K) and 23.3W/(m²K) respectively. The temperature excitation is a sine-wave \( u(t) = \sin(\pi t/12) \) on the outside surface. Here the unit of time \( t \) is hour. The thermal response of heat gain through the inside surface within a full day is illustrated as follows.

At first, the traditional thermal response factor method with invariable time step is used for evaluation. Rectangular waves are used to disintegrate the excitation. The time step is 0.25 hour. When one day is divided by the time step, 97 (= 24/0.25 + 1) instants are needed in the calculation. The thermal response curve and the excitation curve are shown in Fig. 6.

![Graph showing thermal response and excitation](image)

The present thermal response factor method with variable time steps is then used for contrast. Step functions are used to disintegrate the excitation, while time steps are automatically determined by the controlled variation of excitation. Here, the increment of excitation \( \Delta u \) is set as a constant. For comparison of the two methods at the same precision, \( \Delta u \) should be equal to the maximum of excitation variation in the traditional excitation disintegration, namely

\[
\Delta u = \max_{1 \leq k \leq 96} \{ u(t_k) - u(t_{k-1}) \} = 6.54 \times 10^{-2} \quad (6)
\]

where, \( t_k (k = 0 \sim 96) \) is the 97 instants given in the traditional thermal response factor method.

According to the value of \( \Delta u \) given by Eq. (5), 62 instants are chosen by the program. The thermal response curve is also shown in Fig. 6.

The two thermal response curves from the traditional and the present method almost overlap each other in Fig. 6. This proves that the present and traditional excitation disintegration methods are of the same precision. If one pays attention to the distribution of small triangles on the thermal response curve, it will be found that the density of small triangles corresponds to the slope of the excitation curve \( u(t) \). This reflects the original intention of the method with variable time steps. As a result, 36% instants used in the calculation is decreased with the new method while about the same percent computing quantity is reduced.

4. Discussions

Here, we discuss the \( z \)-transfer function method with variable time steps that has not been established. Among the traditional load calculating methods, the \( z \)-transfer function method is more practical than the thermal response factor method because less memory space and computing quantity are needed. Therefore, following development of the thermal response factor method with variable time steps, the authors are ready to establish the \( z \)-transfer function method with variable time steps. However, when the memory space and the computing quantity of the new thermal response factor method are evaluated, the authors find it hard to develop a more efficient method. Even if the \( z \)-transfer function method with variable time steps could be established, its memory space and computing quantity would not always be less than the new thermal response factor method. There are two main reasons for this. At first, if we use the continuous method to store the thermal response to the unit step and ramp-step inputs, a few coefficients need to be stored. Even the number of these coefficients could be less than that of the \( z \)-transfer function coefficients. For the \( z \)-transfer function method, the predominance of the computing quantity is based on the number of coefficients. If there is no predominance in the number of coefficients, its computing quantity will also not be in the ascendant. Consequently, the authors do not try to develop the \( z \)-transfer function method with variable time steps.

5. Conclusions

In this work, the authors find that the traditional method of excitation disintegration is the main reason to limit variation of the time step in the thermal response factor method. Consequently, two new functions for excitation disintegration are presented, namely the step function and the ramp-step function. Based on the new method of excitation disintegration, the traditional thermal response factor method has been rebuilt. In the new thermal response factor method, the time step is adaptive to the variation of excitation. Sinusoidal excitation is used in a simple example to compare the thermal response evaluated by the traditional and the new methods. This illustrates that about 36% computing quantity is decreased with the new method.

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