A Adaptive Stochastic Resonance Method Based on Two-Dimensional Tristable Controllable System and Its Application in Bearing Fault Diagnosis

GANG ZHANG, (Member, IEEE), JIANG CHUAN, AND TIANQI ZHANG
School of Communication and Information Engineering, Chongqing University of Posts and Telecommunications, Chongqing 400065, China
Corresponding author: Jiang Chuan (1513943272@qq.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61771085 and Grant 61371164, and in part by the Research Project of Chongqing Educational Commission under Grant KJQN201900601.

ABSTRACT At present, most systems with good performance have more parameters. However, increasing the number of parameters lead to increasing the difficulties of parameter optimization, thus reducing the system’s feasibility. To solve this problem, a model of controllable coupled stochastic resonance system is proposed. Firstly, the formula of the output signal-to-noise ratio of the model is derived and analyzed. Then the accuracy of the formula’s derivation and results is proved by numerical simulation. It provides a theoretical basis for adjusting the parameters and coupled coefficients of the control system to induce a stochastic resonance in the controlled system or make it much stronger. Finally, better system parameters are obtained by genetic algorithm for the controlled system (tristable system), and then better system performance is obtained by adjusting the parameters and coupled coefficient of the control system (monostable system). The model is applied into the bearing fault detection and the results show that the model achieves better performance without increasing the complexity of parameter optimization, and has great practical value in weak signal detection.

INDEX TERMS Stochastic resonance, linear coupled, signal detection, genetic algorithms.

I. INTRODUCTION

Weak signal detection have become a hot spot currently because weak signal processing is widely used in medical, biology, geology, materials science, physical mechanics and other disciplines. In general, detecting or extracting target signals from background noise use some noise suppression techniques. However, conventional denoising methods usually suppress noise that can damage target signals at the same time, such as wavelet transform, empirical mode decomposition and other methods [1]. The concept of “stochastic resonance” was firstly proposed by Benzi et al in 1981 [2], [3]. When the periodical driving force, noise, and the parameters of the stochastic resonance system reach a synergy, the phenomena of stochastic resonance will be generated, so that the noise energy is transferred to target signals to improve the signal-to-noise ratio, which is conducive to signal detection and extraction [4]. Therefore, stochastic resonance has become a typical method of noise-assisted signal processing. It has the characteristics of using noise to enhance periodic signals, and has been widely used in weak signal detection.

Rotating machinery is widely used in many important fields, such as aircraft engines and locomotive engines. As the core component of rotating machinery, rolling bearings are prone to failure under harsh environments and complex working conditions. Early detection of failures can effectively prevent safety accidents. However, the fault-induced feature information contained in the vibration signal is relatively weak, and it is often submerged in strong background noise and difficult to identify. Due to the strong noise, it is difficult for other noise suppression methods to extract the fault characteristic signals, and the stochastic resonance system can convert the noise’s energy into the target signal’s energy, so that the target signal can be extracted [5]–[7].

Today, stochastic resonance technologies have been attracting more attentions. Many scholars are focusing on how to obtain a system with excellent performance that can be applied to various practical engineering applications.
Review of the previous literatures reveal that most engineering applications are single stochastic resonance system with different potential functions. In fact, the performance can be further improved by coupled stochastic resonance systems [8], [9] or cascade structure [10]. However, the cascaded system reprocesses a signal to increase its performance, ignoring the correlation parameter between the two systems. The coupling system formed by coupling two stochastic resonance systems is a two-dimensional stochastic resonance system when it is regarded as a stochastic resonance system. If two complex systems are coupled, a two-dimensional stochastic resonance system is obtained. The resulting system will be very complex with multiple parameters. Because the shape of potential function is affected by parameters, the correlation between parameters is very strong. When one parameter is adjusted, several potential wells may be added or reduced, resulting in great performance changes [11], [12]. Therefore, how to find system parameters that can obtain satisfactory system performance will be a key issue [13]–[15]. But because different application scenarios need to correspond to different parameters to be optimized [16] which makes some systems perform well, but the practical value of stochastic resonance systems with a large number of parameters becomes low.

Based on this, a controllable stochastic resonance coupled system, which obtains better performance and make the system more convenient in various application scenarios, is proposed. It is consisted of a simple monostable system linearly coupled with a tristable system which has high performance but a large number of parameters. The controlled system (a constructed tristable system) improves performance as much as possible, ignoring the number of parameters. The control system (monostable system) will reduce the number of parameters as much as possible. It is known from references [6], [17]–[19] that the linear segmentation method can be used to overcome the saturation of the system and improve the performance. It is known from reference [20]–[22] that a tristable system can obtain better system performance than a bistable system. The controlled system is proposed by using these two methods to improve the performance. SNR (signal-to-noise ratio) has always been a simple and effective evaluation index in the field of signal processing which is used to measure the system’s performance. The theoretical expression of SNR is deduced through the formula, and the influence of the parameters and coupled coefficient of the control system on the performance of the controlled system is analyzed. Due to the large number of controlled system parameters, and in order to obtain a better set of system parameters of the controlled system, an adaptive algorithm (genetic algorithm) suitable for multi-parameter optimization is adopted [23]. Then, the derivation of the formula is verified by numerical simulation, and the correctness of the derivation is proved. Finally, it is applied to bearing fault detection based on the conclusion of theoretical analysis. It compared with other coupled systems [16].

The theoretical formula and numerical simulation show that the system parameters can be optimized in sections, which greatly reduces the time complexity of adaptive parameter optimization algorithm. In practice, it is also found that the system can detect faults faster and better than other coupling systems. The coupling system is considered as the relationship between controlling and controlled. When the parameters of the controlled system are not ideal, the system performance can be improved by adjusting the parameters of the control system. The system can be applied to all kinds of practical application scenarios and ensure the excellent performance of the system.

II. THEORETICAL ANALYSIS OF SYSTEM
A. COUPLED SYSTEM MODEL

This model is proposed to improve system performance and minimize the complexity of parameter optimization. The systems are linearly coupled and constructed whose principle diagram of feedback coupled control shown in Figure 1.

The Langevin function of the coupled systems can be expressed in the following Eq. (1):

$$\begin{align*}
\frac{dx}{dt} &= -\frac{du(x,t)}{dx} + r(y-x) + \varepsilon(t) \\
\frac{dy}{dt} &= -\frac{dv(y)}{dy} + r(x-y)
\end{align*}$$

where \(u(x,t) = u(x) - x\delta(t)\), \(s(t) = A\cos(\omega t)\) represents the periodic driving force. \(A\) is its amplitude, \(\omega\) is the input signal frequency. \(\varepsilon(t)\) is noise, whose mean value is zero whose autocorrelation function is \(\langle \varepsilon(t)\varepsilon(t+\tau) \rangle = 2D\delta(t-\tau)\). \(D\) is the noise intensity, \(\tau\) is the delay time. \(u(x)\) represents the potential function of the controlled system, and its expression is as follows:

$$u(x) = \begin{cases} 
\frac{-k_3}{k_4}(x + k_1) + (k_3k_2^2/2 - k_3) & x < -k_1 \\
\frac{-k_3}{k_2 - k_1}(x + k_2) + \frac{k_3k_2^2}{2} & -k_1 \leq x < -k_2 \\
\frac{k_3x^2}{2} & -k_2 \leq x \leq k_2 \\
\frac{k_3}{k_4}(x - k_2) + \frac{k_3k_2^2}{2} & k_2 < x \leq k_1 \\
\frac{k_3(k_4 - k_1) + (k_3k_2^2/2 - k_3)}{k_4} & k_1 < x
\end{cases}$$

\(k_i (i = 1, 2, 3, 4, 5)\) is a system parameter, and there are \(k_i > 0\), \(k_1 > k_2\) to ensure that the system is a tristable system.\(v(x)\) is the potential function of the control system, and its expression is \(v(x) = 0.5w^2\) of which \(w > 0\). The potential function of the coupled system is expressed by the following Eq. (3):

$$V(x, y) = u(x) + \left(\frac{w}{2} + \frac{r}{2}\right)y^2 + \frac{r}{2}x^2 - rxy$$

where, \(w, r\) are system parameters and coupled coefficients of the control system. The potential function graph is shown in Figure 2. It can be seen from Figure 2 that the constructed
system has three steady-state points, and the Brownian particles jump in three potential wells, indicating that the system is a tristable system.

In order to find the positions of potential well and barrier in the potential function, \( \partial V(x,y)/\partial x = 0 \), \( \partial V(x,y)/\partial y = 0 \) because the control system changes the positions of the potential function through parameter \( w, r \). They need to be classified and discussed. \((r + w)/(rw) = 0\), the system will become uncoupled, beyond discussion.

When \((r + w)/(rw) > 0\), the coordinate of the three potential well bottom’s \( u_1, u_2, u_3 \) are as follows:

\[
(x_{u1}, y_{u1}) = (((w+r)k_3)/(wr(k_2-k_1))), k_3/(w(k_2-k_1))) \], 
\[
(x_{u2}, y_{u2}) = (0, 0) (x_{u3}, y_{u3}) \]

\[
= (((w+r)k_3)/(w(k_1-k_2))), k_3/(w(k_1-k_2))) .
\]

To ensure the existence of well bottom coordinates \(-k_1 \leq x_{u1} \leq -k_2, k_2 \leq x_{u3} \leq k_1\), the coordinates of the two barriers \( s_1, s_2 \) are \((x_{s1}, y_{s1}) = (k_2, rk_2/(w + r))\) \((x_{s2}, y_{s2}) = (-k_2, -rk_2)/(w + r)\) whose position distribution is shown in Figure 3.

When \((r + w)/(rw) < 0\), the coordinate of the bottom of the three-potential well \( u_1, u_2, u_3 \) are as follows:

\[
(x_{u1}, y_{u1}) = (((w+r)k_3)/(wrk_4)), k_3/(wk_4)) \], 
\[
(x_{u2}, y_{u2}) = (0, 0) \], 
\[
(x_{u3}, y_{u3}) = (((w+r)k_3)/(wk_4)), -k_3/(wk_4)) .
\]

To ensure the existence of well bottom coordinates \(-k_1 \leq x_{u1} \leq -k_2, k_2 \leq x_{u3} \leq k_1\), the coordinates of the two barriers \( s_1, s_2 \) are \((x_{s1}, y_{s1}) = (k_2, rk_2/(w + r))\) \((x_{s2}, y_{s2}) = (-k_2, -rk_2)/(w + r)\) whose position distribution is shown in Figure 4.

**FIGURE 1.** Coupled control principle block diagram.

**FIGURE 2.** Coupled system potential function.

**FIGURE 3.** Schematic diagram of potential well location distribution.

**FIGURE 4.** Schematic diagram of potential well location distribution.

**B. OUTPUT SNR**

From the analysis above, the coupled system has three steady state points and two non-steady state points. Let \( p_1(t), p_2(t), p_3(t) \), be the probability that the system is in three steady states at time \( t \), and define \( R_{12}(t), R_{21}(t), R_{23}(t), R_{32}(t) \) as the escape rates of the transition from three steady states to other steady states at time \( t \). Under adiabatic approximation conditions, Taylor series expansion is used when appropriately, taking the first term as an approximation, and then the escape rate according to Eq. (4) is shown in Eq. (5) and Eq. (6).

\[
R_{ij} = \frac{1}{2\pi} \left( \frac{\sigma_{ij}^{+}}{\sigma_{ij}^{-}} \right)^{\frac{1}{2}} \left( \frac{\sigma_{ij}^{+}}{\sigma_{ij}^{-}} \right) \exp \left( -\frac{\Delta V(x,y)}{D} \right) \tag{4}
\]

\[
R_{12}(t) = \frac{r}{2\pi (w + r)} \left( \frac{r^2}{4} - \frac{16r^2 + w^2}{16} \right)^{\frac{1}{2}} \left( 1 + \left( \frac{x_{u1} - y_{u1}^2 + y_{u1}^2}{2D} \right)^{\frac{1}{2}} \right) \cos(\omega t) \left( \frac{V(x_{u1}, y_{u1}) - V(x_{s1}, y_{s1})}{D} \right) \tag{5}
\]
where $\sigma_1^{(1)}$, $\sigma_2^{(1)}$, $\sigma_2^{(2)}$, and $\sigma_2^{(3)}$ are the two eigenvalues of the Hessian matrix of $V(x, y)$. $\sigma_2^{(1)}$, $\sigma_2^{(2)}$, and $\sigma_2^{(3)}$ are the values of the second-order partial derivatives at the non-stable point $V(x, y)$. $\Delta V(x, y)$ is the height difference between the stable point and the unstable point. $(x_{01}^+, y_{01}^+)$ and $(x_{01}^-, y_{01}^-)$ are the coordinates of the steady state points when $r + w > 0$, $r + w < 0$. In order to facilitate the calculation, the following Eq. (5) and (6) are written as the following Eq. (7):

$$R_{12}(t) = R_{32}(t) = a + b \cos(\omega t)$$
$$R_{21}(t) = R_{23}(t) = c + d \cos(\omega t)$$

$$a = \frac{r}{\pi (w + r) \left(\frac{w + k_5 + r}{4} + \frac{r}{2}\right)} \exp \left(-\frac{V(x_{01}^+, y_{01}^+)}{D}\right)$$
$$b = \frac{\left(x_{01}^+ + x_{01}^-\right)^2 + \left(y_{01}^+ + y_{01}^-\right)^2 - \left(x_{01}^+ + y_{01}^-\right)^2}{D}$$
$$c = \frac{k_5 + r}{\pi (w + r) \left(\frac{w + k_5 + r}{4} + \frac{r}{2}\right)} \exp \left(-\frac{V(x_{01}^+, y_{01}^+) - V(x_{01}^-, y_{01}^-)}{D}\right)$$
$$d = \frac{c \left(x_{01}^2 + y_{01}^2\right)^{\frac{1}{2}}}{D}$$

The two-dimensional Fokker-Planck equation has the form shown in Eq. (8).

$$\frac{\partial \rho(x, y, t)}{\partial t} = -\frac{\partial}{\partial x} \left[ \frac{\partial V(x, y)}{\partial x} \rho(x, y, t) \right] - \frac{\partial}{\partial y} \left[ \frac{\partial V(x, y)}{\partial y} \rho(x, y, t) \right] + D \left( \frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} \right)$$

According to Eq. (8), the state probability equations of Eq. (9) can be obtained. Eq. (7) is substituted into Eq. (9) and linear ordinary differential method is used to solve the ternary equations of Eq. (9), and the solution is shown in Eq. (10).

$$\begin{align*}
\frac{dp_1(t)}{dt} &= -R_{12}(t) p_1(t) + R_{21}(t) p_2(t) \\
\frac{dp_2(t)}{dt} &= R_{12}(t) p_1(t) - R_{21}(t) p_2(t) + R_{23}(t) p_3(t) \\
\frac{dp_3(t)}{dt} &= R_{23}(t) p_2(t) - R_{32}(t) p_3(t)
\end{align*}$$

$$\begin{align*}
\frac{p_1(t)}{G_1} &= Q_1(cQ_2 + dQ_3) \\
\frac{p_2(t)}{G_1} &= -Q_1\left[cG_2 + dG_3 - p_1(t)\right] \\
\frac{p_3(t)}{G_1} &= -Q_1\left[\left[cG_2 + dG_3\right] - p_3(t)\right]
\end{align*}$$

$$\begin{align*}
Q_1 &= \exp \left[-\frac{n}{\omega} \sin(\omega t)\right] \\
Q_2 &= \frac{1}{f} + \frac{mv}{\omega} \left[\theta_2 \sin(\omega t) - \theta_1 \cos(\omega t)\right] \\
G_1 &= \exp \left[-\frac{n}{\omega} \sin(\omega t)\right] \\
G_2 &= \frac{1}{f} + \frac{mv}{\omega} \left[\theta_2 \sin(\omega t) - \theta_1 \cos(\omega t)\right] \\
Q_3 &= v \left[\theta_2 \cos(\omega t) + \theta_1 \sin(\omega t)\right] + \frac{nu}{2}\left[\theta_2 \sin(2\omega t) - \theta_1 \cos(2\omega t)\right] \\
G_3 &= v \left[\theta_2 \cos(\omega t) + \theta_1 \sin(\omega t)\right] + \frac{nu}{2}\left[\theta_2 \sin(\omega t) - \theta_1 \cos(\omega t)\right] \\
f &= a + 2c, n = b + 2d, v = \frac{1}{\sqrt{f^2 + \omega^2}}, \theta_1 = v\omega, \theta_2 = v\omega, \alpha_1 = 2\omega, \alpha_2 = uf
\end{align*}$$

When the time is long enough, the initial moment can be ignored. Therefore, the state probability can be reduced to the Eq. (11).

$$\begin{align*}
p_1^n(t) &= p_3^n(t) = Q_1(cQ_2 + dQ_3) \\
p_2^n(t) &= Q_1(aQ_2 + bQ_3)
\end{align*}$$

According to the state probability given in Eq. (10), the conditional probability of state transition can be obtained as
follows Eq. (12):

\[
\begin{align*}
  &\left\{ p (u_1, t+\tau | u_1, t) \\
  &= M_1 [(cM_2 + dM_3) - (cQ_2 + dQ_3) \exp (-f \tau)] \\
  &+ \frac{n}{\omega} \sin (\omega t + \omega \tau) \\
  &- \frac{M_1}{f} \left[ x_1 \{p(u_1, t+\tau | u_1, t) + p(u_2, t+\tau | u_2, t) \} + p(u_1, t+\tau | u_2, t) + p(u_2, t+\tau | u_1, t) \}
\end{align*}
\]

(12)

\[
\begin{align*}
  &\langle x(t+\tau) x(t) \rangle_{\text{average}} \\
  &= \int_{-\infty}^{\infty} \langle x(t+\tau) x(t) \rangle_{t_{\text{st}}} \, dt \\
  &= 2x_1^2 M_1 (cQ_2 + dQ_3) [Q_1 (cM_2 + dM_3) - (cQ_1 Q_2 + dQ_1 Q_3 - 1) \exp (-f \tau)] \\
  &= 2x_1^2 [M_1 (cQ_2 + dQ_3) [Q_1 (cM_2 + dM_3) - (cQ_1 Q_2 + dQ_1 Q_3 - 1) \exp (-f \tau)] \\
  \end{align*}
\]

(13)

Both the noise and the periodic driving force act in the direction \( x \) of the controlled system, the generated stochastic resonance motion can only be carried out along the direction \( x \), while the control system can change the shape and height of the potential function. Therefore, according to the definition of autocorrelation function, we can substitute Eq. (11) and (12) to obtain the autocorrelation function expression of system response (13). When the autocorrelation function is averaged over a period, Eq. (14) is obtained. Moreover, the power spectrum of the system response can be obtained by its autocorrelation function through Fourier transform, as shown in Eq. (15).

Where \( S_1 (W) \), \( S_2 (W) \) represents the energy of the output signal and the energy of the output noise. The total output energy of the system represented by the sum of the signal energy and the noise energy is obtained from (15), and then the SNR formula shown in Eq. (16) is obtained according to Eq. (15).

\[
\text{SNR} = \frac{\int_{-\infty}^{\infty} S_1 (W) \, dW}{S_2 (W = \omega)}
\]

(15)
Since the controlled system also has three steady states, the analysis method is the same as above. The difference is that its potential function is no longer controlled by the control system, so its escape rate changes. Their escape rate become as follows Eq. (17) and Eq. (18):

\[
R_{12}(t) = R_{32}(t) = \frac{k_2^2}{k_4(k_1-k_2)} e^{-\frac{i_3 t}{2}} \left(1 + \frac{A(k_1+k_4-k_2)}{2D} \cos(\omega t)\right)
\]

\[
R_{21}(t) = R_{23}(t) = \frac{1}{20\pi} e^{\frac{-i_3 t}{2}} \left(1 + \frac{A k_2}{D} \cos(\omega t)\right)
\]

The comparison diagram shown in Figure 4 can be obtained based on the above analysis. The parameter values of the controlled system (tectonic tristable system) are \(k_1 = 0.8, k_2 = 0.1, k_3 = 0.1, k_4 = 0.1, k_5 = 0.1\), the amplitude and frequency of the periodic driving force are \(A = 0.05, \omega = 0.01\). Control system parameters and coupled coefficients \(w = 0.2, r = 1.1\).

As shown in Figure 4, it can be seen that the SNR for the construction of the tristable system does not increase first with the increase in noise intensity, indicating that this set of system parameters cannot generate stochastic resonance. However, it can be seen that by adjusting the coupled coefficient and the parameters of the monostable system, the SNR increases first and then decreases with the increase of the noise intensity, indicating that stochastic resonance phenomena are induced. It proves the correctness of the formula derivation, and also provides a theoretical basis for making the system performance better by adjusting the control system and the coupled coefficient. By adjusting the system parameters and coupling coefficients of the control system, the performance of the coupled system is better. Therefore, it is necessary to analyze these two parameters. As shown in Figure 5 and Figure 6:

From Figure 5 and Figure 6, it can be seen that by adjusting the parameters of the control system and the coupled coefficient, the peak value of the signal-to-noise ratio can be increased and the noise intensity \(D\) corresponding to the peak value can be changed. However, because there is a coordination effect between the parameters, it can only be explained here that better performance can be obtained by adjusting these two parameters, and the impact on it cannot be described qualitatively as shown in Figure 5 and Figure 6.

As shown in Figure 7, the influence of these two parameters on system performance is nonlinear. Therefore, intelligent optimization algorithm is needed to optimize the parameters. As the adaptive parameter optimization algorithm is adopted, the larger the range of parameter optimization is, the more time is spent. Through theoretical analysis, it can effectively eliminate the range of impossible optimal parameters, thus...
greatly reducing the time spent on Optimization and improving the efficiency. Therefore, it is of great value to analyze the parameters.

III. NUMERICAL SIMULATION

A. GENETIC ALGORITHM

Although the stochastic resonance can be induced by adjusting the controlled system’s coupled coefficient and the parameters which indicating that a set of optimal parameters are critical to induce stochastic resonance. The genetic algorithm is used on coupled systems for parameter optimization. There are 5 parameters in the controlled system which needs to be optimized. Although the commonly used adaptive iterative algorithm has the advantages of simple principle and easy implementation, but with high time complexity \( o(n^N) \) (\( N \) is the number of parameters, \( n \) is also greatly affected by accuracy). Therefore, it is not suitable to use an adaptive iterative algorithm when there are many parameters which make the time complexity too large and the parameter’s accuracy not high enough. The adaptive genetic algorithm simulates the biological genetic process which does not make the time complexity increase exponentially with the optimization parameters increasing. At the same time, genetic algorithm is also used to obtain higher accuracy. In this paper, genetic algorithm is used to optimize the parameters of control system and controlled system respectively.

As required, the real number encoding method is used for conversion. The specific flowchart is shown in Figure 8, and the specific steps are as follows:

1. Construct a suitable fitness function, and select signal-to-noise ratio gain or other indexs that can directly reflect the system’s advantages and disadvantages as the fitness function.

2. Initialize the number and range of optimization parameter. Determine the number of population and the number of hereditary offspring. The number of parameters is set as required, and the parameter range can be roughly determined according to the previous analysis. The population size and number of iterations are set according to demand.

3. Calculate the values of all individuals according to the fitness function constructed in step (1), and find the individual that is most suitable for the environment in the evaluation process. Different from natural selection, the algorithm does not allow the most adaptive individual to die. Therefore, after each evaluation, the most adaptive individual is directly placed in the next generation.

4. Individual genes within the population are crossed. If the mutation conditions meet during the crossing process, (5) is executed. Because the choice of crossover operator is different for different situations, the following crossover operator (19) is adopted according to the requirements of the paper.

\[
\begin{align*}
X'_1 &= \lambda_1 X_1 + (1 - \lambda_2) X_2 \\
X'_2 &= \lambda_1 X_2 + (1 - \lambda_2) X_1 \\
\end{align*}
\]

(19)

5. Mutation is a random process whose probability is set to determine how frequently it occurs. The mutation operator is constructed as (20):

\[
X' = X + \Delta
\]

(20)

6. Generate offspring and replace any random individual in them with the optimal solution individual in step (3). According to whether the termination condition reaches the maximum number of iterations, the branch flow of the algorithm is determined.

B. VERIFICATION FORMULA ANALYSIS

In order to analyze and verify the above formula and prove the correctness of its derivation, numerical simulations are performed and analyzed by using the 4th-order Runge-Kutta method. The signal-to-noise ratio gain can also directly reflect the signal-to-noise ratio of the output signal, and can also avoid the situation that the output signal-to-noise ratio is also large, which is induced by the high input signal-to-noise ratio. The average signal-to-noise ratio gain (MSNRI) is used
FIGURE 9. Comparison of the change of MSNRI with noise intensity $D$ ($w = 0.1$, $r = 0.1$, $A = 0.01$, $\omega = 0.01$ Hz, $k_1 = 2.244695055323959$, $k_2 = 2.11$, $k_3 = 4.63493002001412$, $k_4 = 3.329276651815723$, $k_5 = 0.05$).

FIGURE 10. The effect of $w$ on MSNRI.

as a measurement which is shown in Eq. (21) ($M = 500$).

$$MSNRI = \frac{1}{M} \sum_{i=1}^{M} (SNR_{out} - SNR_{in})$$

Figure 9 is simulation verification for Figure 4, a set of parameters for generating stochastic resonance phenomena in the controlled system is obtained through a genetic algorithm. Then, change one of the parameters of them to make it unable to produce stochastic resonance phenomena. Finally, apply this parameter to the coupled system and adjust the coupled coefficient and control system parameters to induce stochastic resonance. The results are shown in Figure 8 which verifies the correctness of the formula derivation. Figure 10 is a simulation diagram made for Figure 5. $w$ is increased under the condition that other parameters are not changed. It can be seen from the figure that the $MSNRI$ peak shifts to the right and increases. Similar to the figure in Figure 5, the correctness of the formula derivation is further verified.

C. COMPARATIVE ANALYSIS OF NUMERICAL SIMULATION

In order to verify that the constructed tristable system (controlled system) has good system performance under appropriate parameters, the controlled system and the classical tristable stochastic resonance system are simultaneously applied with a genetic algorithm to obtain better system parameters.

The Langevin equation for a classical tristable stochastic resonance system is shown in Eq. (22) below:

$$\dot{x} = -ax + bx^3 - cx^5 + A\cos(\omega t) + \epsilon(t)$$

The periodic driving force and noise are the same as in Eq. (1). $a, b, c$ are system parameters. Simulations show that the time complexity increases significantly with the population and the number of offspring increasing when the genetic algorithm is used due to the existence of higher-order terms in the classical tristable system. Since there is no higher-order term to construct the tristable system, the time complexity does not increase sharply compared with the classical tristable system. Figure 12(b) and (d) are the amplitude-frequency characteristics of the input signal and the output signal after passing through the classical tristable system, and Figure 12(a) and (c) are the time-domain waveforms of the input and output signals.

Figure 12(e) and (f) show the time-domain waveform and frequency-domain characteristics of the output signal of the tristable stochastic resonance system. The input signal is the same as the input signal of the classical tristable system. Figure 12(g) and (h) are the images obtained by adjusting the coupled coefficient and control system parameters through genetic algorithms with other parameters fixed.

When the genetic algorithm is used to optimize the parameters of the classical tristable system and the constructed tristable system, the time complexity is the same. Comparing Figure 12(d) and (f), if the period is set the same for tow system, it can be seen that the performance of the constructed tristable system is much better than the classical tristable system obviously. Comparing Figure 12(f) and (h), it shows that the coupled system achieves better performance when the controlled system’s parameters fixed, thus further
proving the correctness of the formula derivation. It also shows that the coupled system model can obtain better performance, indicating that the system has strong application value.

IV. BEARING FAULT DETECTION

A. FAULT CHARACTERISTICS EXTRACTION OF THE BEARING TYPE 6205-2RS JEM SKF

Because the above analysis is under adiabatic condition, in order to apply the model in practical engineering area. It compared with the published similar method [16]. The coupled differential formula of the model is shown in the following Eq. (23):

\[
\begin{align*}
\frac{dx(t)}{dt} &= -u_1(x) + r_1(y - x) + \varepsilon(t) \\
\frac{dy(t)}{dt} &= -v_1(y) + r_1(x - y)
\end{align*}
\]

(23)

where

\[ u_1(x) = -\frac{a_0}{2}x^2 + \frac{b_0}{4}x^4 \]

\[ v_1(y) = \frac{c_1}{6}y^6 - \frac{a_1}{4}y^4 - \frac{b_1}{2}y^2 \]

\( u_1(x), v_1(y) \) are the system potential function, \( r_1 \) is the coupling coefficient. \( \varepsilon(t) \) same as above.

The bearing failure data used are derived from deep groove ball bearings of type 6205-2RS JEM SKF as shown in Figure 13. It can be seen from the waveform of the signal to be detected in Figure 14 and Figure 15 that the data to be measured already contains strong noise. Its main parameters are shown in Table 1 [24]. Because it does not meet the conditions of small parameters, the method of subsampling produces resonance. Sampling frequency \( f_s = 12000Hz \), number of sampling points \( N = 10000 \), 5Hz is selected as the second sampling frequency. The fault signals are periodic pulses, which can be diagnosed by a single frequency. By comparing the consistency of the characteristic frequency and the detection frequency, it can be decided whether the fault occurs and determine the fault type.

The characteristic frequency calculation is as follows Eq. (24):

\[ f_{BPFI} = \frac{n_f r}{2} \left( 1 + \frac{D_1}{D_2 \cos \alpha} \right) \]

\[ f_{BPFO} = \frac{n_f r}{2} \left( 1 - \frac{D_1}{D_2 \cos \alpha} \right) \]

(24)
TABLE 1. The main structure parameters of the rolling element.

| Inside Diameter (inch) | Outside Diameter (inch) | Thickness (inch) | Ball Diameter (inch) | Pitch Diameter (inch) | Ball number |
|------------------------|-------------------------|-----------------|---------------------|----------------------|-------------|
| 0.9843                 | 2.0472                  | 0.5906          | 0.3126              | 1.537                | 9           |

outer rings of the bearing. Outer rings fault characteristic frequency is 107.28 Hz, and the inner ring characteristic frequency is 162.11 Hz.

1) OUTER RING FAULT DETECTION

Because the number and position of the potential wells of the coupled bistable system are not convenient for theoretical analysis, it is impossible to use one system as the control system and the other system as the controlled system to perform segmented optimization of parameters. It can only be regarded as a whole, and the parameters of this entire system can be optimized. However, the controllable coupled system proposed in this paper solved this problem, so it can be optimized by segmentation.

Figure 14 (c) and (d) are the waveforms obtained by the genetic algorithm of the bistable coupled system. Compared with the original signal, it can be seen from Figure 14 (d) that the fault signal has been detected. Figures 14 (e) and (f) are the waveforms obtained by constructed tristable system using a genetic algorithm (with optimization time as the termination condition, and the time used is 3/4 of the bistable coupled system time).

It can be seen from Figure 14 (f) that the signal is detected, and the accuracy and amplitude values are higher than the bistable coupled.

That is because there is one less optimization parameter for the construction of a tristable system. At the same time, the correlation between the parameters is small, and the optimization range can be reduced by the above formula analysis.

Figure 14 (g) and (h) are obtained by optimizing the coupled coefficient and the parameters of the control system using the remaining 1/4 time without changing the parameters of the controlled system. It can be seen that compared with Figure 14 (d), the accuracy is improved, and the amplitude is also greatly improved. The correctness of the above analysis is proved, and the coupled system model is also very suitable for practical engineering applications.

2) INNER RING FAULT DETECTION

Similar to the detection method of the outer ring fault, the bistable coupled system (using genetic algorithm genetic algorithm) is firstly used for fault detection. When a fault is detected, the optimization time is recorded. The waveforms shown in Figure 15 (c) and (d) are obtained. Then the constructed tristable system is used for fault detection (the parameters are the same as those used for outer ring fault detection), and the waveform diagrams shown in Figure 15 (e) and (f) are obtained.

FIGURE 14. Comparison of time-domain and frequency-domain for outer ring fault detection of different systems (c)(d) $a_0 = 0.9801, b_0 = 0.8992, c_1 = 1.1248, a_1 = -0.0497, b_1 = 2.695433, c_1 = -2.566$, (e) $f_1 = 1.244695055, f_2 = 2.11, k_5 = 4.637490020014124, k_4 = 5.332976651815723, k_5 = 0.1$ (g) $w = 2, r = 0.012$. 

The number of rolling elements $n_r$, diameter $D_1$, bearing pitch $D_2$, rotational frequency $f_r$, contact angles $\alpha$, $f_{BPFI}$ and $f_{BPFO}$ represent the characteristic frequencies of the inner and
The coupled coefficient and control system parameters ($w = 0.193827r = 0.2305171$) are optimized with quarter of the same time as the bistable coupled system, and the waveforms shown in Figure 15 (g) and (h) are obtained.
Comparing Fig. 15 (f) and (h), it can be seen that the unsuitable parameters can be adjusted for the detection by adjusting the coupled coefficient and the parameters of the control system. At the same time, comparing Fig. 15 (d) and (h), we can see that the same optimization time can get better optimization performance.

**B. FAULT DIAGNOSIS OF ID-25/30 BEARING HEALTH TEST BENCH**

ID-25 / 30 bearing health test bench is suitable for single point bearing fault signal acquisition and bearing health assessment experiment. The entity diagram of ID-25 / 30 is shown in Figure 16. As above, the data to be measured in this part is also collected under the background of strong noise. According to equation (23), the theoretical inner raceway frequencies \( f_{\text{inner}} = 117.14 \text{ Hz} \) can be obtained. Using the system as shown in Figure 17(c) and (d) are an effect diagram of checking the fault signal with the obtained parameters. Genetic algorithm is used to optimize the parameters of the system. The system proposed in this paper is used for detection. It is not stopped until a better result is obtained than that in Figure 17(c) and (d) for checking the fault signal, and the time is recorded. Check the same fault signal with the found parameters to get the renderings as shown in Figure 17(e) and Figure 17(f).

Compared with Figure 17 (d) and Figure 17 (f), it can be seen that Figure 17 (f) not only has higher amplitude at the detection frequency, but also has a double frequency, which shows that the detection effect is better. However, comparing the time spent by the two systems, it is found that the system in this paper only spends half of the time shown in equation (22).

The reason is that the system in this paper can be divided into two optimizations, which greatly reduces the time complexity of the algorithm.

**V. CONCLUSION**

In this paper, a special coupling method is proposed, which does not have the disadvantage of high cost in parameter optimization of coupled system. The controllable coupled stochastic resonance system model has excellent system performance. Through theoretical derivation and analysis, it is found that the performance of the system can be adjusted and improved by the parameters and coupled coefficients of the control system, or induce the system that cannot generate stochastic resonance by itself to produce the phenomena of stochastic resonance. Applying it to fault detection, it is found that a monostable system with fewer parameters is used to control a tristable system with more parameters, so that a set of parameters of the controlled system can be adapted into a variety of practical scenarios, showing great values. And compared with other coupled systems, it is found that the system has the characteristics of low time complexity in parameter optimization.

Next, we will change the system to underdamped or delayed system. Study whether under damping coefficient can improve its performance. More extensive analysis of the system in the presence of time delay system performance will decline.

**ACKNOWLEDGMENT**

The authors would like to thank the anonymous reviewers for their valuable comments and suggestions, and help us to improve the work comparatively and fundamentally.

**REFERENCES**

[1] Y. Wang, F. Liu, Z. Jiang, S. He, and Q. Mo, “Complex variational mode decomposition for signal processing applications,” *Mech. Syst. Signal Process.*, vol. 86, pp. 75–85, Mar. 2017, doi: 10.1016/j.ymssp.2016.09.032.

[2] R. Benzi, G. Parisi, A. Sutera, and A. Vulpiani, “Stochastic resonance in climatic change,” *Tellus*, vol. 34, no. 1, pp. 10–16, Feb. 1982, doi: 10.1111/j.2153-3490.1982.tb01787.x.

[3] R. Benzi, G. Parisi, A. Sutera, and A. Vulpiani, “A theory of stochastic resonance in climatic change,” *SIAM J. Appl. Math.*, vol. 43, no. 3, pp. 565–578, Jun. 1983, doi: 10.1137/0143037.

[4] S. Lu, Q. He, and J. Wang, “A review of stochastic resonance in rotating machine fault detection,” *Mech. Syst. Signal Process.*, vol. 116, pp. 230–260, Feb. 2019, doi: 10.1016/j.ymssp.2018.06.032.

[5] Y. Lei, D. Han, J. Lin, and Z. He, “Planetary gearbox fault diagnosis using an adaptive stochastic resonance method,” *Mech. Syst. Signal Process.*, vol. 38, no. 1, pp. 113–124, Jul. 2013, doi: 10.1016/j.ymssp.2012.06.021.

[6] Z. Huo, Y. Zhang, P. Franço, L. Shu, and J. Huang, “Incipient fault diagnosis of roller bearing using optimized wavelet transform based multi-speed vibration signatures,” *IEEE Access*, vol. 5, pp. 19442–19456, Mar. 2017, doi: 10.1109/ACCESS.2017.2661967.

[7] X. Chen, G. Cheng, X. Shan, X. Hu, Q. Guo, and H. Liu, “Research of weak fault feature information extraction of planetary gear based on ensemble empirical mode decomposition and adaptive stochastic resonance,” *Measurement*, vol. 73, pp. 55–67, Sep. 2015, doi: 10.1016/j.measurement.2015.05.007.

[8] M. J. He, Z. Sun, and W. Jia, “Characterizing stochastic resonance in coupled bistable system with Poisson white noises via statistical complexity measures,” *Nonlinear Dyn.*, vol. 88, no. 2, pp. 1163–1171, Jan. 2017, doi: 10.1007/s11071-016-3302-3.

[9] W. Zou, D. V. Senthilkumar, Y. Tang, Y. Wu, J. Lu, and J. Kurths, “Amplitude death in nonlinear oscillators with mixed time-delayed coupling,” *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 89, no. 3, Sep. 2013, Art. no. 032916, doi: 10.1103/PhysRevE.89.032916.

[10] Z. Guang-Lu and W. Fu-Zhong, “Research of stochastic resonance in cascaded bistable system,” *J. Comput. Theor. Nanosci.*, vol. 6, no. 3, pp. 676–681, Mar. 2009, doi: 10.1166/jctn.2009.1092.

[11] T.-T. Yang, H.-Q. Zhang, Y. Xu, and W. Xu, “Stochastic resonance in coupled undamped bistable systems driven by symmetric trichotomous noises,” *Int. J. Non-Linear Mech.*, vol. 67, pp. 42–47, Dec. 2014, doi: 10.1016/j.ijnonlinmech.2014.07.008.

[12] A. Kenfack and K. P. Singh, “Stochastic resonance in coupled undamped bistable systems,” *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 82, no. 4, Oct. 2010, Art. no. 046224, doi: 10.1103/PhysRevE.82.046224.

[13] J. Wang, Q. He, and F. Kong, “Adaptive multiscale noise tuning stochastic resonance for health diagnosis of rolling element bearings,” *IEEE Trans. Instrum. Meas.*, vol. 64, no. 2, pp. 564–577, Feb. 2015, doi: 10.1109/TIM.2014.2347217.

[14] G. Zhang, Y. Zhang, T. Zhang, and J. Xiao, “Stochastic resonance in second-order underdamped system with exponential bistable potential for bearing fault diagnosis,” *IEEE Access*, vol. 6, pp. 42431–42444, Jul. 2018, doi: 10.1109/ACCESS.2018.2856620.

[15] G. Zhang, D. Y. Hu, and T. Q. Zhang, “The analysis of stochastic resonance and bearing fault detection based on linear coupled bistable system under Lévy noise,” *Chin. J. Phys.*, vol. 56, no. 6, pp. 2718–2730, Dec. 2018, doi: 10.1166/j.cjph.2018.10.010.

[16] J. Li, J. Zhang, M. Li, and Y. Zhang, “A novel adaptive stochastic resonance method based on coupled bistable systems and its application in rolling bearing fault diagnosis,” *Mech. Syst. Signal Process.*, vol. 114, pp. 128–145, Jan. 2019.
[17] M. Gosak, M. Perc, and S. Kralj, “Stochastic resonance in a locally excited system of bistable oscillators,” Eur. Phys. J. B, vol. 80, no. 4, pp. 519–528, Apr. 2011, doi: 10.1140/epjb/e2011-10573-8.

[18] Z. Qiao, Y. Lei, and N. Li, “Applications of stochastic resonance to machinery fault detection: A review and tutorial,” Mech. Syst. Signal Process., vol. 122, pp. 502–536, May 2019, doi: 10.1016/j.ymssp.2018.12.032.

[19] B. Jiao, C. Ren, P. H. Li, Q. Zhang, and G. Xie, “Stochastic resonance in an overdamped monostable system with multiplicative and additive α-stable noise,” Acta. Phys. Sin., vol. 63, no. 7, 2014, Art. no. 070501, doi: 10.7498/aps.63.070501.

[20] G. Zhang, G. Junpeng, and H. Li, “Characteristic research and application of cascade tristable stochastic resonance,” Comput. Sci., vol. 45, no. 9, pp. 146–151, 2018.

[21] J. Shangbin, L. Shuang, Z. Qing, and Q. Xiaoxue, “Signal detection and recovery of tristable stochastic resonance system under a stable noise,” in Proc. 14th IEEE Conf. Ind. Electron. Appl. (ICIEA), Jun. 2019, pp. 745–750.

[22] Y. Liu, F. Wang, L. Liu, and Y. Zhu, “Symmetry tristable stochastic resonance induced by parameter under Levy noise background,” Eur. Phys. J. B, vol. 92, no. 8, Aug. 2019, doi: 10.1140/epjb/e2019-90759-8.

[23] C. Zhang and Y. He, “Bearing fault diagnosis method based on self-adaptive stochastic resonance of genetic algorithm and VMD,” J. Mech.Transmiss., vol. 42, no. 4, pp. 156–163, 2018.

[24] Fault Characteristics Extraction of the Bearings Type 6205-2RS JEM SKF. Accessed: Sep. 10, 2018. [Online]. Available: http://csegroups.case.edu/bearingdatacenter/pages/download-data-file

GANG ZHANG (Member, IEEE) was born in 1976. He received the Ph.D. degree from the College of Communication Engineering, Chongqing University, Chongqing, in 2009. He is currently an Associate Professor with the Chongqing University of Posts and Telecommunications. His research interests include chaotic synchronization, chaotic secure communication, and weak signal detection.

JIANG CHUAN received the bachelor’s degree from the Chongqing University of Posts and Telecommunications, in 2018, where he is currently pursuing the master’s degree. His main research interest includes weak signal detection.

TIANQI ZHANG was born in 1971. He received the B.S. degree in physics from Southwest University, China, in 1994, and the M.S. degree in communication and electronic system and the Ph.D. degree in circuits and systems from the University of Electronic Science and Technology of China, in 1997 and 2003, respectively. From 2003 to 2005, he was a Postdoctoral Fellow in communication and information system with Tsinghua University. Since August 2005, he has been a Professor with the School of Communication and Information Engineering, Chongqing University of Posts and Telecommunications. His research interests include the areas of communication, and image and speech signal processing.

* * *