Singularity-Free Two Dimensional Cosmologies

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Abstract

We present a class of theories of two dimensional gravity which admits homogeneous and isotropic solutions that are nonsingular and asymptotically approach a FRW matter dominated universe at late times. These models are generalizations of two dimensional dilaton gravity and both vacuum solutions and those including conformally coupled matter are investigated. In each case our construction leads to an inflationary stage driven by the gravitational sector. Our work comprises a simple example of the ‘Nonsingular Universe’ constructions of ref. [1].

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1 Introduction

The singularity theorems of Penrose and Hawking\cite{2} pose serious physical and philosophical problems concerning the behaviour of physics in certain regions of spacetime. Taken at face value the theorems tell us that the manifolds described by General Relativity (GR) are generically geodesically incomplete - there are always geodesics that cannot be continued for arbitrary values of their affine parameter. This conclusion is highly unsatisfactory for a number of reasons.

Firstly, the existence of singularities limits the predictive power of any physical theory for which the manifold is a background. Particles reaching the singularity cease to exist, carrying with them vital information about the particle’s history.

A second problem involves the fact that, as experience with ‘physical’ solutions to GR shows, the existence of a singularity is often accompanied by the unbounded increase of one of the curvature invariants of the theory. Examples of this include the infinite Weyl tensor at the centre of a Schwarzschild black hole and the ‘big bang’ singularity encountered in cosmological models. In fact it can be shown that if the singularity is reached on a timelike curve in a globally hyperbolic spacetime then the Riemann tensor $R_{\mu\nu\rho\sigma}$ is unbounded\cite{3}.

Finally, in connection with both of the above there is the problem of the quantum information loss paradox. This is associated with the evolution of pure quantum states into mixed states during the evaporation of black holes.

It is a common belief that when we have a complete quantum theory of gravity the singularities predicted for GR will not occur. The appearance of singularities should be seen as a signal of the breakdown of GR and an indication of the need for a new theory.

Since there is at present no theory of quantum gravity we shall attempt a more modest goal. We shall implement the ‘Limiting Curvature Construction’ (LCH)\cite{4} in a class of two dimensional theories of gravity. By doing so we shall obtain theories to which all solutions are singularity free and the only curvature invariant, the Ricci scalar $R$, is bounded. Thus, our key assumption is that those processes that may be possible for curing the ‘sickness’ of GR are representable at the classical level in a modified theory of gravity.
The models we will consider are generalizations of two-dimensional Dilaton Gravity which is obtained from the low energy limit of string theory and is a theory of gravity coupled to a scalar field, the dilaton. We generalize these models by allowing an arbitrary coupling between the fields and a general potential for the scalar (which we still refer to as the dilaton). In a theory derived from string theory such a scalar field would be dynamical and the analysis of the model would be quite complicated. Here, however, we treat the scalar as a Lagrange multiplier.

Nonsingular black hole solutions to this theory were studied in and so, in this paper, we shall concentrate on cosmological solutions to the theory. Cosmological solutions to 1+1 dimensional dilaton gravity have been studied by other authors but our goals differ from theirs in that we wish to study generalizations of that theory with emphasis on the possible removal of spacetime singularities.

The outline of this paper is as follows. In section 2 we review the model of generalized dilaton gravity and obtain the field equations. In section 3 we explicitly search for homogeneous cosmological solutions and show that our construction yields theories with a ‘natural’ initial inflationary stage evolving into a FRW matter dominated universe. All manifolds described by this theory are singularity-free. In section 4 we investigate the effect of including a conformally coupled scalar field into the theory and show that this does not affect our ability to construct nonsingular cosmological theories. Finally in section 5 we conclude and speculate on the implications of our results.

2 Two Dimensional Dilaton Gravity

Here we shall describe the two dimensional models that we have investigated. Given that the model of dilaton gravity discussed in the previous section appears to be important for physics we shall study general two dimensional models describing a massless scalar field (which we shall still refer to as the dilaton) coupled to gravity. Our aim is to identify classes of theories to which all solutions are nonsingular. In this way we hope to pinpoint those features of the models which may be important for gravitational physics if nonsingular
behaviour is manifested at the classical level. We shall present the model and derive the general field equations in preparation for the implementation of our construction in the next section.

Our starting point is the most general renormalizable lagrangian for gravity coupled to a scalar field in 1+1 spacetime dimensions.

\[
\mathcal{L} = \sqrt{-g} \left[ D(\phi) R + G(\phi) (\nabla \phi)^2 + H(\phi) \right] 
\]

with action

\[
S[g, \phi] = \int \mathcal{L} \, d^2x
\]

where \(D(\phi)\) is the coupling of the dilaton to gravity, \(G(\phi)\) is just a conformal prefactor for the dynamical terms for the dilaton and \(H(\phi)\) is a potential for the dilaton.

We are free to perform a conformal or Brans-Dicke transformation on this Lagrangian\[8\][9] via

\[
g_{\mu\nu}^{\text{old}} = e^{2\tau(\phi)} g_{\mu\nu}^{\text{new}}
\]

If we now require that

\[
\frac{d\tau}{d\phi} \frac{dD}{d\phi} = -G(\phi)
\]

then we may express the Lagrangian in the form

\[
\mathcal{L} = \sqrt{-g} \left( V(\phi) + D(\phi) R \right)
\]

where the new potential is given by \(V(\phi) = H(\phi) e^{2\tau(\phi)}\). This is the fundamental Lagrangian which we shall consider. In particular note that it is the metric appearing in this expression that governs the motion of test particles. Note that in principle we may eliminate \(\phi\) from this Lagrangian and thus obtain a higher derivative theory describing the gravitational field

\[
\mathcal{L}^{\text{new}} = \mathcal{F}(R)
\]
Now consider the equations of motion arising from the variation of the action with Lagrangian as in equation (5). Variations with respect to the scalar field yield

\[ \frac{\partial V}{\partial \phi}(\phi) = -\frac{\partial D}{\partial \phi}(\phi)R \]  

(7)

Now consider variations with respect to the metric

\[
\delta S = \int d^2x \left[ \left( V(\phi) + D(\phi)R \right) \frac{1}{2} g_{\alpha \beta} \sqrt{-g} \delta g^{\alpha \beta} + \right. \\
\left. \sqrt{-g} \left( -D(\phi)R_{\alpha \beta} \delta g^{\alpha \beta} - D(\phi)\nabla^2 g_{\alpha \beta} \delta g^{\alpha \beta} + D(\phi)\nabla_\alpha \nabla_\beta \delta g^{\alpha \beta} \right) \right] 
\]

(8)

Integrating by parts twice, ignoring surface terms and requiring \( \delta S = 0 \) gives

\[ -D(\phi)R_{\alpha \beta} + (\nabla_\alpha \nabla_\beta - g_{\alpha \beta} \nabla^2)D(\phi) + \frac{1}{2} V(\phi) g_{\alpha \beta} + \frac{1}{2} D(\phi) R_{\alpha \beta} = 0 \]  

(9)

Finally we note that in two dimensions \( R_{\alpha \beta} = g_{\alpha \beta} R/2 \) which puts our field equations in the form

\[ V(\phi) g_{\alpha \beta} = 2(\nabla^2 g_{\alpha \beta} - \nabla_\alpha \nabla_\beta)D(\phi) \]  

(10)

Equations (7) and (10) are our basic equations. In particular, (7) is the constraint equation due to the dilaton and it is through this that we shall force our theories to have limiting curvature and to be singularity free. In ref. 6 nonsingular static black hole solutions to this model were constructed. Here we seek to address the case when the spacetime is homogeneous and isotropic. To be exact we shall search for singularity free cosmological models.

3 Cosmologies in Two Dimensions

In this section we shall construct a vacuum model for which all homogeneous and isotropic solutions are nonsingular and approach a matter dominated compactified FRW spacetime at late times. Note that in particular these spacetimes exhibit no big-bang singularity.
We begin by making a redefinition of $\phi$ so that $D(\phi) = \phi^{-1}$. We are perfectly free to do this and the choice is made for calculational simplicity. Since we are looking for spatially homogeneous solutions we shall use the ansatz

$$g_{\mu\nu} = \text{diag}(-1, a^2(t))$$

for the metric. With these choices the field equations become

$$V(\phi) - 2\frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi^2} = 0$$

and

$$V(\phi) - 2\frac{\ddot{\phi}}{\phi^2} + 4\frac{\dot{\phi}^2}{\phi^3} = 0$$

where dot denotes differentiation with respect to time. Also our constraint equation (7) becomes

$$\phi^2 \frac{\partial V}{\partial \phi}(\phi) = R$$

We may combine (12) and (13) and, for $d\phi^{-1}/dt \neq 0$, integrate to obtain the scale factor $a(t)$

$$a(t) = -\gamma \frac{\dot{\phi}}{\phi^2}$$

where $\gamma > 0$ is an arbitrary integration constant. Substituting into (12) we have

$$V(\phi) = -\gamma^{-1}2\dot{a}$$

Now, our construction consists of the following steps. Firstly we shall require that the scale factor approach that of a matter dominated FRW spacetime at late times. We shall then solve for the asymptotic behaviour of the potential in this region. We then demand that the curvature be bounded at early times. This yields a particular form for the scale factor which allows us also to solve for the asymptotic form of the potential in the early universe. Finally we shall choose a well-behaved interpolating potential and use this to solve for the general scale factor for our model.

At late times, $t \to \infty$, we shall impose $a(t) = \nu t^{2/3}$, where $\nu > 0$ is a constant. Integration of (15) yields

$$\phi(t) = \frac{5}{3} \frac{\gamma t^{-5/3}}{\nu}$$
up to an integration constant. From (16) the late time form of our potential is given by

\[ V(\phi) = -\frac{4\nu}{3\gamma} t^{-1/3} = -\alpha \phi^{1/5} \]  

(18)

where \( \alpha \) is a constant constructed from the other constants we have introduced.

Now, at early times, \( t \to -\infty \), we require the curvature, \( R \), to be bounded, \( R = 2\beta^2 \) say. By (14) this implies

\[ V(\phi) = -\frac{2\beta^2}{\phi} \]  

(19)

where \( \beta > 0 \) is a constant. Then (13) becomes a harmonic oscillator equation for \( \phi^{-1} \) giving

\[ \phi(t) = \frac{1}{a_1 e^{\beta t} + a_2 e^{-\beta t}} \]  

(20)

with \( a_1 \) and \( a_2 \) arbitrary constants. Further, (15) can be rewritten as

\[ a(t) = \gamma \frac{d}{dt}(\phi^{-1}) \]  

(21)

So for various choices of \( a_1 \) and \( a_2 \) we can obtain an expanding de Sitter solution with Hubble constant \( H = 2\beta^2 \), \( (a_2 = 0) \), a contracting de Sitter solution \( (a_1 = 0) \) or a de Sitter bounce \( (a_1 = -a_2) \).

Note that we have obtained an initial inflationary phase by bounding \( R \), the only curvature invariant in 2 dimensions, and thus our solutions are nonsingular (both in the sense of avoiding infinite curvature and having a geodesically complete manifold). It is an attractive feature of this and other higher derivative theories of gravity that inflation\[1\], which provides a solution to the homogeneity, horizon and flatness problems in cosmology, can be the result of the action of gravity alone and needs no matter fields to drive it.

Now to find a model close to a compactified version of our real Universe, we shall search for a solution which has a de Sitter expansion at early times and evolves into a ‘matter-dominated’ FRW universe at late times. To this end we choose

\[ V(\phi) \to -\frac{2\beta^2}{\phi} \quad \text{as} \quad \phi \to \infty \]

\[ V(\phi) \to -\alpha \phi^{1/5} \quad \text{as} \quad \phi \to 0 \]
A possible choice for the interpolating potential is
\[ V(\phi) = -\frac{\alpha \phi^{1/5}}{1 + \alpha (2\beta^2)^{-1} \phi^{6/5}} \] (22)

Explicit solution for the scale factor is not possible but it is only necessary to show that our metric is well behaved as it interpolates between its asymptotic limits. From (13) we have (in terms of \( D \equiv \phi^{-1} \))
\[ 2 \ddot{D} = \frac{\alpha D}{D^{6/5} + b} \] (23)
where \( b = \alpha/2\beta^2 \). This may be integrated to give
\[ \dot{D}^2 = \alpha f(D) \] (24)
where
\[
\begin{align*}
f(x) &= \frac{5}{4} \frac{x^{4/5}}{x^{4/5} - x^{2/5} + b^{2/3}} - \frac{5}{12} b^{2/3} \ln(x^{4/5} - x^{2/5} + b^{2/3}) \\
&\quad - \frac{5}{2\sqrt{3}} b^{2/3} \arctan \left( \frac{2x^{2/5} - b^{1/3}}{\sqrt{3}b^{1/3}} \right) \\
&\quad + \frac{5}{6} b^{2/3} \ln(x^{2/5} + b^{1/3}) - \frac{5}{4\sqrt{3}} b^{2/3} \arctan(\sqrt{3})
\end{align*}
\] (25)

The scale factor, \( a(t) \), may be expressed implicitly using (15) as
\[ a(D) = \gamma \dot{D} = \gamma \sqrt{\alpha f(D)} \] (26)
where we have chosen the positive square root for consistency. In Figure 1 we plot \( a(D) \) for \( \alpha = 2\beta^2 = \gamma = 1 \). The analysis is not qualitatively sensitive to the value of these positive constants.

It can be seen that \( a(D) \) is a real, positive, monotonically increasing function of \( D \) for \( D > 0 \). Further, when the universe emerges from an expanding de Sitter phase, \( \frac{\partial}{\partial D} \dot{D} = \ddot{D}/\dot{D} > 0 \) for all \( D > 0 \) and \( D \) is positive for all time because \( D \to 0^+ \) as \( t \to -\infty \) and \( \dot{D}(D) > 0 \) for all \( D \) and so we can use the fact that \( \dot{D} > 0 \ \forall \ t \) to assert that the scale factor is also a monotonically increasing function of time. Our construction is therefore complete.

In order to check the above results we also studied numerically the case of a contracting universe to ensure that the model made a smooth transition between the two asymptotic
regions. Beginning with final conditions in the $t \to \infty$, matter dominated region we evolved the equations backwards to the initial de Sitter solution at $t \to -\infty$. Our numerical results confirmed the above analysis.

Note that, although $\phi \to \infty$ as $t \to -\infty$ this is in fact a coordinate singularity. The coordinates $t, x$ which we have chosen only cover half of de-Sitter space. If we consider de-Sitter space as a hyperboloid embedded in a flat three-dimensional space then we may define new coordinates which describe the whole of the space\[11\]. In these coordinates it is clear that the point $t = -\infty$ is a coordinate singularity and that the space is geodesically complete since geodesics may be smoothly continued through the point $t = -\infty$.

To summarize, we have shown that the class of theories with potentials satisfying the asymptotic conditions (18), (19) have all homogeneous isotropic solutions which are non-singular. Further, in bounding the curvature at early times our solutions have a ‘natural’ inflationary stage driven by the gravitational sector. Figure 2 shows a log-log plot of the scale factor demonstrating the interpolation between the behaviours of the asymptotic regions.

We shall now turn our attention to modifications to the vacuum theory.

4 Inclusion of Conformal Matter

Now that we have found a class of theories for which all vacuum cosmological solutions are singularity free we wish to extend our analysis to consider the effect of matter on our construction.

In these two dimensional models the requirement of quantum consistency of the theory is equivalent to the conformal invariance of the Lagrangian. We plan to include matter using a scalar field and since the dynamical terms imply that a scalar field must transform trivially under a conformal transformation, we may not include a simple, polynomial potential in the Lagrangian. Thus, by far the most simple choice is to conformally couple a scalar field into the theory using just the dynamical terms. We shall represent this matter by $\sigma$ and shall demonstrate that the presence of this field does not prevent us from constructing singularity
free theories. We shall again restrict ourselves to search for theories with homogeneous, isotropic cosmological solutions.

Our action is now

$$S[g, \phi; \sigma] = \int (L_g + L_m) \, d^2x$$

where $L_g$ represents the gravitational sector of the theory given by the previous Lagrangian (26) and $L_m$ represents the Lagrangian for the matter described by the conformally coupled scalar field $\sigma$.

$$L_m = \lambda^2 (\partial_\mu \sigma \partial^\mu \sigma) \sqrt{-g}$$

The equations of motion derived from this action are the constraint equation (14), the equation of motion for $\sigma$

$$\nabla^2 \sigma = 0$$

and the modified field equations for the metric which, with the ansatz (11) and $D(\phi) = \phi^{-1}$, take the form

$$\frac{\dddot{\phi}}{\dot{\phi}^2} - 2 \frac{\ddot{\phi}^2}{\dot{\phi}^3} - \frac{1}{2} V(\phi) - \frac{\lambda^2}{2} \sigma^2 = 0$$

and

$$\frac{\dddot{\phi} \dot{a}}{\dot{\phi}^2 \dot{a}} - \frac{1}{2} V(\phi) + \frac{\lambda^2}{2} \sigma^2 = 0$$

Since we seek solutions that depend only on time, we may solve (29) exactly to give

$$\dot{\sigma} = \frac{\alpha}{a}$$

Substituting this into our field equations and combining we obtain

$$\left( \frac{\dot{\phi}}{\dot{\phi}^2} \right) - \frac{\dddot{\phi} \dot{a}}{\dot{\phi}^2 \dot{a}} - \frac{\lambda^2 \alpha^2}{a^2} = 0$$

and
\[
\left( \frac{\dot{\phi}}{\phi^2} \right) + \frac{\dot{\phi} \, \dot{a}}{\phi \, a} - V(\phi) = 0
\]  
(34)

Now, as before, let us search for solutions that asymptotically approach a matter-dominated FRW universe with scale factor given by \( a(t) = \nu t^{2/3} \). In this region it is straightforward to solve for the evolution of the dilaton and we obtain

\[
\phi(t) = \left[ \frac{3 \lambda^2 \alpha^2}{2 \nu^2} t^{2/3} - \frac{3}{5} \delta t^{5/3} \right]^{-1} \]
(35)

where \( \delta \) is a constant of integration. This expression is valid for late times, \( t \to \infty \).

For \( \delta \neq 0 \) the second term in eq.(35) dominates. Eq.(34) can then be solved for the potential at late times as

\[ V(\phi) \to -\mu \phi^{1/5} \text{ as } t \to \infty \quad (\phi \to 0) \]
(36)

where \( \mu = (3\delta/5)^{1/5}(4\delta/3) \).

At early times, \( t \to 0 \), we shall impose the condition that the curvature remain bounded, \( R = 2\beta^2 \), where \( 2\beta^2 \) is a constant representing the limiting curvature. Equation (14) then implies

\[ V(\phi) = \frac{-2\beta^2}{\phi} \text{ as } t \to 0 \]
(37)

For the purposes of this paper we shall concentrate on the solution where the universe emerges from a de Sitter bounce at \( t = 0 \) since this case turns out to be the easiest to treat analytically. Thus we write the scale factor at early times as

\[ a(t) = \cosh(\beta t) \]
(38)

where we have chosen \( a(0) = 1 \) without loss of generality. We can combine (33) and (34) into the following two equations, written in terms of \( D = \phi^{-1} \)

\[ 2\dot{D} + V(D) + \frac{\kappa^2}{a^2} = 0 \]
(39)

\[ 2\dot{\phi} + V(D) - \frac{\kappa^2}{a^2} = 0 \]
(40)
where $\kappa^2 = \lambda^2 \alpha^2$. Equation (39) with $a(t)$ from (38), linearized about $t = 0$, admits the solution

$$D(t) = \frac{\kappa^2}{2\beta^2} - 2c \cosh(\beta t)$$

(41)

where $c$ is an integration constant. Provided that

$$c = \frac{\kappa^2}{2\beta^2}$$

(42)

(40) is also satisfied by this solution to linear order in $t$. For $c \ll 1$, $D(t)$ tends to the small constant $-c$ as $t \to 0$, so that the limit $t \to 0$ corresponds to $|\phi| \gg 1$. To connect the two asymptotic regions of the de Sitter bounce and matter domination, we can then find a form of the interpolating potential similar to the one used in the vacuum model

$$V(\phi) = -\frac{\mu \phi^{1/5}}{(1 + c\phi)^{6/5} + \mu (2\beta^2)^{-1} \phi^{6/5}}$$

(43)

It is an attractive feature of our theory that similar potentials can describe the nonsingular cosmological evolution of a universe with and without matter. We cannot solve for the time evolution analytically with this potential, so we evolve (14) and (39) numerically with a Runge-Kutta algorithm for $a(t)$ and $D(t)$, using the fact that in two dimensions with our ansatz for the metric, the Ricci scalar takes the simple form $R = 2\ddot{a}/a$. In Figures 5, 6 and 7 we plot the scale factor $a(t)$, the Ricci scalar $R(t)$ and the dilaton $\phi(t)$ for $\mu = 2\beta^2 = 1$ and $\kappa^2 = 0.1$. The evolution from the de Sitter bounce into a matter dominated FRW universe can clearly be seen.

Our spacetime has finite curvature everywhere for $t \geq 0$. It can be made geodesically complete by patching on the contracting part of the de Sitter bounce for negative times. This is possible since the matching conditions for the metric are satisfied at $t = 0$, namely $a(t)$ and $\dot{a}(t)$ are continuous. Also, $\phi(t)$ is finite at $t = 0$ and does not cause any problems. Therefore we have indeed obtained a nonsingular two-dimensional cosmological model.

5 Conclusion

Since quantum gravity is non-renormalizable\(^{14}\) and at present we can extract little information from string theory concerning the quantum nature of spacetime, we have used a toy
model in 1+1 dimensions to investigate what conclusions may be reached at the classical level concerning singularities. The key assumption in our work is that the quantum processes responsible for the removal of singularities may be represented in an effective, classical theory of gravity, valid at curvature scales intermediate between those of any future theory of everything and GR.

Motivated by the low energy limit of string theory and attempts to apply these ideas in four spacetime dimensions we have constructed a class of two dimensional models for gravity for which all homogeneous and isotropic solutions are nonsingular and approach a FRW matter dominated universe at late times. We have also demonstrated that our construction is valid for both the vacuum theory and, for a large class of initial conditions, when a conformally coupled scalar field is included in the model.

Our construction is based on explicitly limiting the Ricci scalar (the only curvature invariant in two dimensions) by means of a non-dynamical scalar field \( \phi \) which we refer to as the dilaton. The particular choice of potential for the dilaton defines the theory and ensures that the solutions remain singularity free as regions of high curvature are approached.

If the type of behaviour that we reveal here is realized by nature, there may be very interesting implications for cosmology. The existence of a de Sitter phase arising purely from the gravitational sector at early times (also seen in the four dimensional case) may provide us with a natural inflationary epoch. Although at present inflation appears to be the only viable solution to the horizon problem, there are difficulties associated with the fine tuning of parameters necessary to implement this scenario using elementary scalar fields. Thus, an alternative formulation such as ours may be very attractive for cosmology.

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Figure Captions

Figure 1: The scale factor of the universe, as a function of the dilaton coupling, for the vacuum theory with an expanding de Sitter solution.

Figure 2: An example of the scale factor, as a function of cosmological time, in the vacuum theory with an expanding de Sitter solution.

Figure 3: The Ricci scalar, as a function of time, for the vacuum solution with the same parameters as those used in Figure 2.

Figure 4: The dilaton, as a function of time, for the vacuum solution of Figure 2.

Figure 5: An example of the scale factor, as a function of cosmological time, when a conformally coupled scalar field has been included. This example shows a de Sitter bounce.

Figure 6: The Ricci scalar, as a function of time, for the matter solution of Figure 5.

Figure 7: The dilaton, as a function of time, for the matter solution of Figure 5.
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