Research Article

Semi-Active Pulse-Switching Vibration Suppression Using Sliding Time Window

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The performance of pulse-switching vibration control technique is investigated using a new method for switching sequence, in order to enhance the vibration damping. The control law in this method which was developed in the field of piezoelectric damping is based on triggering the inverting switch on each extremum of the produced voltage (or displacement); however, its efficiency in the case of random excitation is arguable because of the local extremum detection process. The new proposed method for switching sequence is only based on the fact that the triggering voltage level was determined using windowed statistical examination of the deflection signal. Results for a cantilever beam excited by different excitation forces, such as stationary and nonstationary random samples, and pulse forces are presented. A significant decrease in vibration energy and also the robustness of this method are demonstrated.

1. Introduction

The ability to reduce the vibration amplitude over a wide frequency band is essential in the vibration control. The dependency of dynamic stiffness and damping properties of special materials such as viscoelastic materials on the excitation frequency or temperature variations cause other methods or materials to be considered [1, 2]. Several methods have been investigated for semi-active vibration control, using piezoelectric elements [3–10]. These methods are interesting because they do not rely on any operating energy as in active control. They consist of driving by a few solid-state switches requiring very little power. The common strategy of these methods is the electric boundary conditions modification of the piezoelectric elements. Synchronized Switch Damping (SSD) or pulse-switching technique which is implemented in this study consists of leaving the piezoelectric elements in open circuit, except during a very brief period of time, where the electric charge is either suppressed in a short circuit (SSDS) or inverted with a resonant network (SSDI). Corr and Clark experimented with SSDI method in the case of multimodal vibrations [5]. Also, they showed that the original SSD control law is not optimal in the case of wide band excitations.

In the case of wide band excitations, the optimization of the piezoelectric elements (size and location) as well as the switching network is not sufficient. It is therefore necessary to establish methods to define the accurate switch triggering time that operates exclusively on certain selected extrema, which would maximize the electric power produced by the piezoelectric elements [3]. In a similar way, this paper utilizes statistical analysis of the structural deflection to predict the optimum instants for the switching sequence in order to maximize the extracted energy and vibration damping. This new developed strategy allows an easy implementation of this damping technique for any type of excitation forces. In order to analyze this method, a beam equipped with piezoelements wired on a pulse-switching cell was experimented. The response of a cantilever beam subjected to pulse, stationary, and nonstationary random excitations has been considered. The stationary random excitation is the white noise and the nonstationary random excitations are the pulses of white noise shaped in time as a rectangle [11–14]. The following section describes the multimodal model used for simulation, as well as the pulse-switching device model. It is followed by a discussion on the strategy of semi-active vibration control using sliding time window and the proposed statistical method.
where \( q_i(t) \) can be determined from these equations. \( \alpha_i \) is the \( i \)th generalized damping coefficient and \( \overline{Q}_i \) is the generalized excitation force [12] given by

\[
\overline{Q}_i = \int_0^l p(x) \phi_i(x) \, dx.
\]

For the cantilever beam problem represented in Figure 1, the concentrated external force \( f(t) \) is applied at the free end of the beam and the piezoelectric patches are bonded on the structure, and then (5) becomes

\[
M_i \ddot{q}_i + c_i \dot{q}_i + K_i q_i = \overline{Q}_i f(t) - \alpha_i v_i(t),
\]

where \( K_{ci} \) is the short circuit generalized stiffness. The term \( \alpha_i v_i(t) \) corresponds to the force due to the \( i \)th component of the voltage according to the macroscopic piezoelectric coefficient \( \alpha_i \). In the open circuit condition, the piezoelectric voltage \( v_i \) due to the \( i \)th mode of vibration and the global piezoelectric voltage \( v \) resulting from the modes superposition are

\[
v_i = \frac{\alpha_i}{C_0} \overline{Q}_i(t); \quad v = \frac{1}{C_0} \sum_{i=1}^N \alpha_i q_i(t),
\]

\[
I = \sum_{i=1}^N \alpha_i i_q_i - C_0 v.
\]

In this equation, \( C_0 \) is the capacitance of the piezoelectric elements, \( v \) is the total piezoelectric voltage, and \( I \) is the total outgoing current from the piezoelectric patches. In general, the piezoelectric patches are wired together in parallel. The open circuit stiffness \( K_{di} \) of the structure is related to \( K_{ci}, \alpha_i \), and \( C_0 \) by

\[
K_{di} = K_{ci} + \frac{\alpha_i^2}{C_0}.
\]

The eigenmode angular frequencies \( \omega_{di} \) in open circuit and \( \omega_{ci} \) in short circuit are defined as

\[
\omega_{di} = \sqrt{\frac{K_{di}}{M_i}}; \quad \omega_{ci} = \sqrt{\frac{K_{ci}}{M_i}}.
\]

The coupling coefficients \( k_i \) and mechanical quality factor \( Q_{ma} \) are defined as

\[
k_i = \frac{\alpha_i}{K_{di} C_0} = \frac{\omega_{di} - \omega_{ci}^2}{\omega_{di}^2}; \quad Q_{ma} = \frac{K_{di}}{c_i \omega_{di}}.
\]

The various parameters previously defined can be identified (Table 1). In the proposed control technique, an electric circuit is connected to the piezoelectric elements in order to perform vibration damping. The goal is to maximize transferred energy that corresponds to the part converted into electrical energy. Multiplying each term of (9) by voltage and integrating over time shows that the transferred energy is sum of the electrostatic energy stored on the piezoelectric 

![Figure 1: Schematic diagram of cantilever beam with pulse switching circuit.](image-url)
Table 1: Modal parameters of the electromechanical structure.

| $\omega_{di}$ | $i$th open circuit angular frequency | Measured |
| $\omega_{ei}$ | $i$th short circuit angular frequency | Measured |
| $Q_{mi}$ | $i$th mechanical quality factor | Measured |
| $C_0$ | Piezoelectric capacitance | Measured |
| $M_i$ | $i$th modal mass | Computed from (4) |
| $K_{di}$ | $i$th open circuit stiffness | Computed from (11) |
| $K_{ei}$ | $i$th short circuit stiffness | Computed from (11) |
| $c_i$ | $i$th damping coefficient | Computed from (12) |
| $\alpha_i$ | $i$th macroscopic piezoelectric coefficient | Computed from (12) |

Elements and the energy absorbed or dissipated by the electrical device as

$$E_i = \sum_{i=1}^{N} \alpha_i \int_{0}^{T} q_i v_i \, dt = \frac{1}{2} C_0 v^2 + \int_{0}^{T} v I \, dt.$$  

(13)

Also, this nonlinear damping technique consists of adding a switching device in parallel with the piezoelectric elements. The current in the switching device is always zero except during the voltage inversion that takes place at each switch trigger. At each inversion, the energy extracted from the piezoelectric elements is equal to the difference in the electrostatic energy on the piezoelectric elements before and after the voltage inversion jump. The energy dissipated in the switching device is then given by

$$\int_{0}^{T} v I \, dt = \frac{1}{2} C_0 \sum_{k} v_k^2 (1 - \gamma^2).$$

(14)

where $v_k$ is the piezoelectric voltage just before the $k$th inversion, and $\gamma$ is the inversion coefficient. In this case, the voltage inversion (switch) is not perfect, because one part of the energy stored on the piezoelectric capacitance is lost in the switching network (electronic switch and inductance). These losses are modeled by an electrical quality factor $Q_i$. The relationship between $Q_i$ and the voltage of the piezoelectric element before and after the inversion process is given by

$$v_{after} = -\gamma v_{before} = -v_{before} e^{-\pi/2Q_i}.$$  

(15)

In the following numerical simulations, the switching sequence is simply modeled by (15). It is evident that optimization of this technique (maximizing the energy extracted by switching device) is obtained by maximizing the sum of the piezoelectric voltage squared before each switch (14).

3. Strategy Enhancement of Pulse-Switching Vibration Control Using Sliding Time Window

In this case, by using pulse-switching strategy of control, which consists of triggering the inverting switch on each voltage extremum or strain extremum, the switching sequence is implemented based on statistical analysis of the deflection signal on a sliding time window. The statistical strategy is based on the idea that it allows the piezoelectric deformation to reach a significant value before allowing voltage inversion in which the energy stored on the piezoelements is higher. Recent developments have been aimed at the optimal switching strategy to have maximum damping. This strategy amplifies the amplitude of piezoelectric voltage. Consequently, the switching sequence or switching strategy allows maximizing $\sum v_k^2$. Therefore, it is not the number of switching sequences which is important, but much more the instant of the switch. This point has been illustrated in Figure 2. In this case, in each instant the deflection signal $u(t)$ is studied during a given time window $T_{es}$ just before the present time (a moving sliding window on the signal), and statistically probable deflection threshold $u_m$ is determined from both the temporal average $\mu_u$ and standard deviation $\sigma_u$ of the signal during the observation period $T_{es}$. Then, it is compared to the observation signal, to define the instant of the next switch. $\mu_u$ and $\sigma_u$ of the signal $u(t)$ on $T_{es}$ are defined, respectively, as

$$\mu_u = \frac{1}{T_{es}} \int_{T_{es}} u(t) \, dt,$$

$$\sigma_u = \left( \frac{1}{T_{es}} \int_{T_{es}} (u(t) - \mu_u)^2 \, dt \right)^{1/2}.$$  

(16)

The positive deflection threshold $u_m$ will be derived as

$$u_m = |\mu_u| + \beta \sigma_u,$$

(17)

where $\beta$ is an arbitrary coefficient. In fact, the calculations in each instant of time on the sliding time window are to make a decision for the next switch. According to the general principle aimed at triggering the switch, once the voltage has reached a significant and statistically probable value, switching will occur when the absolute value of the deflection reaches the threshold $u_m$ (switching condition $|u(t)| > u_m$). This method in the case of random vibration is very efficient. However, in this case the most obvious characteristic is being nonperiodic. The knowledge of the past history of random motion is adequate to predict the probability of occurrence of various displacement magnitudes, but it is not sufficient to predict the precise magnitude at a specific instant [16]. In this method, the history of signal in the past time window $T_{es}$ is studied two times by average and standard deviation of the signal. Therefore, extremes estimation is better especially in the case of random vibration. Also, it amplifies the piezoelectric voltage amplitude (Figure 2).

4. Results in the Case of Vibration Damping

4.1. Vibration Damping Performance Analysis. In order to compare this control approach with uncontrolled case, either for simulation or experimental data, it is necessary to define quantities that will be evaluated for this comparison. The quantity used is related to the deflection of the structure. This quantity $I_u$, which is a summation on the time variable for
the considered modes, is the mean squared response of $u(x,t)$ \cite{17} and is defined as

$$I_u = u(x,t)^2 = \lim_{\tau \to \infty} \left( \frac{1}{\tau} \int_0^\tau u^2(x,t) dt \right) = \sum_k \sum_i \left[ \psi_i(x) \phi_k(x) \lim_{\tau \to \infty} \left( \frac{1}{\tau} \int_0^\tau \psi_i(t) \phi_k(t) dt \right) \right].$$

(18)

This quantity will be computed in this form for the simulation results. The displacement damping $A_u$ is then evaluated from these quantities, computed with the uncontrolled case and then with the piezoelement driven with the desired control process, and is defined as

$$A_u = 10 \log \left( \frac{I_u(\text{controlled})}{I_u(\text{uncontrolled})} \right).$$

(19)

4.2. Numerical Results. The numerical sample is a cantilever steel beam equipped with piezoelectric patches. A sample force is applied to the free end of the clamped beam. The model ((7) and (9) principally) is simulated by numerical integration using the fourth-order Runge-Kutta algorithm. The simulations are carried out using the statistical method described previously. The signal considered for the definition of the threshold is $u(t)$. The observation time window $T_{es}$ has to be twice the period of the lowest resonance frequency in order to give satisfactory results. This time should be sufficiently long to obtain a realistic image of the deflection especially for the lowest frequency mode and sufficiently short to maintain a good frequency response of the control.

The controlled responses of the cantilever beam when the applied excitations are pulse, stationary, and nonstationary random forces have been studied. The stationary random excitation is Gaussian white noise, and the nonstationary random excitation is a Gaussian white noise shaped in time by a rectangular envelope function. This appears as a pulse of white noise shaped as a rectangle (random shock loadings). These vibrations with stop and start driving are clearly non-stationary; however, the system experiences a series of transient vibration events. Transient vibration is nonstationary \cite{18}. A simple model for a nonstationary random process with a time varying mean value is given by a process where each sample record is of the form $f(t) = a(t)x(t)$. Here, $a(t)$ is a deterministic function (known physical process) and $x(t)$ is a sample record from a stationary random process. The considered structure and the corresponding numerical data calculated using the expressions detailed in Section 2 are gathered in Tables 2 and 3, respectively.

Figure 3 shows the variations of displacement damping $A_u$ versus the $\beta$ coefficient under the same random shocks. The applicable interval of $\beta$, for which the displacement damping $A_u$ stays minimum, is very large. This indicates that the extremum estimation in the case of statistical study is better and more reliable. The curve shows a sharp decrease in the displacement damping for $\beta$ values larger than $\sqrt{2}$, which would correspond to the extremum for a pure sinusoidal signal. This is very understandable, because in this simulation, the mechanical quality factor is equal to $Q_m = 500$, and consequently the excitation signal is very well filtered by the system and locally close to a sinusoid. The result of this filtering and pseudosinusoidal response, especially in the structures with high values of $Q_m$, or small damping ratio, results in better extremum detection by pulse switching technique and therefore higher damping. In this method, the sinusoidal signal is the ideal signal to have an optimum switch. The optimum displacement damping is about $-13$ dB. But, by damping ratio increasing, the filtering decreases; consequently the signal is more complicated and the statistical switching is not optimum; thus damping decreases. The weak sensitivity of this control technique to the $\beta$ coefficient according to

| Table 2: Characteristics of the numerical structure. |
|---|---|---|
| Plate material | Steel |
| Plate dimensions | $180 \times 90 \times 2$ mm |
| Piezoelectric material | P189 |
| Piezoelements capacitance ($C_p$) | $148$ mF |
| Inversion coefficient ($\gamma$) | $0.6$ |
| Open circuit resonance frequencies | $57$ Hz; $324$ Hz; $904$ Hz |
| Coupling coefficients ($\kappa$) | $0.0959; 0.0663; 0.0265$ |
| Young’s modulus $E$ | $200$ Gpa |

| Table 3: Modal parameters of the considered system. |
|---|---|---|
| First mode | Second mode | Third mode |
| $M_i$ (g) | $71.6$ | $72.2$ | $71.5$ |
| $K_{dx}$ (N/m) | $7800$ | $319000$ | $2398800$ |
| $K_{dx'}$ (N/m) | $7700$ | $317600$ | $2397100$ |
| $\alpha_i$ (N/V) | $0.0023$ | $0.0102$ | $0.0111$ |
| $c_j$ (N/ms) | $0.0591$ | $0.3794$ | $1.0357$ |
Figure 3: Variations of the displacement damping ($A_u$) in dB versus the $\beta$ coefficients. The observed signal is the beam tip deflection $u(L,t)$.

Figure 4: The rectangular pulse of white noise excitation.

the large range of $\beta$ (approximately $0.1 < \beta < 1.4$) allows nearly optimal displacement damping for this method. In these calculations, $\tau = 300T$, where $T = 0.0571$ s is the period of first resonant frequency.

Figure 4 shows the white noise excitation shaped in time by a rectangular envelope function (rectangular pulse of white noise). The numerical constant (the effective time interval $\Delta T$) of rectangular pulse of white noise is equal to the actual time duration of the pulse and is 15 times the longest natural period of the mechanical system.

Figure 5 illustrates the beam tip displacement with and without the statistical pulse-switching control, for rectangular pulse of white noise sample of Figure 4. The damping achieved for a wide band semi-active method is remarkable; it is equal to $-12.96$ dB for $Q_m = 500$ and $\beta = 0.8$.

Figure 6 shows the beam tip displacement with and without statistical SSDI control for a pulse force. The pulse duration is $0.006$ s, and the displacement damping for $Q_m = 500$ and $\beta = 0.8$ is more than $-10.4$ dB. The vibration damping using piezoelectric is related to mechanical strain induced in piezoelectric elements. Therefore, in the case of free vibration, at the beginning, the damping is well, but by vibration amplitude decreasing the piezoelectric strain decreases and consequently the damping performance decreases as well.

Figure 7 shows the values of displacement damping for some excitation samples with different behaviors (harmonic, stationary and nonstationary random, random shock, and pulse forces). The results show that this method of control is not very sensitive to the type of excitation force. Since the sliding window is always moving with the signal and with the variations of signal during the time, the statistical values ($\mu$ and $\sigma$) change proportionally, as well. Therefore, the decided moments for the switch triggering and extremums detection are approximately optimum. This is a good advantage, because for a high range of excitation signals, it is not necessary to adjust $\beta$ for each force type to have optimum damping.

5. Conclusion

The pulse-switching semi-active nonlinear control technique is interesting for structural damping applications, because it provides simultaneously good damping performance, good
robustness, and very low power requirements. It is important to consider that this technique is simple enough to be self-powered. The main limitations are related to the case of complex or random excitation where the synchronization on the strain extremum is not trivial. The proposed work demonstrates improved performance over nearly the entire anticipated operational range. The results show that this method of control is not too sensitive to all of the types of excitation behaviors. The reason is that the sliding time window is always moving with the signal, and with the variations of signal during the time, the statistical values change proportionally as well. Therefore, the moments of switch triggering are approximately optimum. It can be said that the sliding time window is independent of the type of excitation signal. Then, the statistical analysis of the strain could allow defining the criteria to identify more accurate switching instants. However, this method requires a little knowledge of the signal which results in the definition of a tuning parameter $\beta$. The results show that this adjustment can be made very coarse. Also, because of the excitation signal filtering by the system, the extremum detection (switching on extremums is a characteristic of this method) becomes easier especially for the systems with higher values of $Q_m$. In general, the main idea of this paper deals with complex vibrations and focuses on development of the new strategy based on a simple statistical analysis of the displacement signal, allowing an easy implementation of this damping technique for any excitation force.

**References**

[1] A. L. Webster and W. H. Semke, "Broad-band viscoelastic rotational vibration control for remote sensing applications," *Journal of Vibration and Control*, vol. 11, no. 11, pp. 1339–1356, 2005.

[2] M. D. Rao, "Recent applications of viscoelastic damping for noise control in automobiles and commercial airplanes," *Journal of Sound and Vibration*, vol. 262, no. 3, pp. 457–474, 2003.

[3] D. Guyomar, C. Richard, and S. Mohammadi, "Semi-passive random vibration control based on statistics," *Journal of Sound and Vibration*, vol. 307, no. 3–5, pp. 818–833, 2007.

[4] A. Badel, G. Sebald, D. Guyomar et al., "Piezoelectric vibration control by synchronized switching on adaptive voltage sources: towards wideband semi-active damping," *Journal of the Acoustical Society of America*, vol. 119, no. 5, pp. 2815–2825, 2006.

[5] L. R. Corr and W. W. Clark, "A novel semi-active multi-modal vibration control law for a piezoceramic actuator," *Journal of Vibration and Acoustics*, vol. 125, no. 2, pp. 214–222, 2003.

[6] D. Guyomar and A. Badel, "Nonlinear semi-passive multimodal vibration damping: an efficient probabilistic approach," *Journal of Sound and Vibration*, vol. 294, no. 1-2, pp. 249–268, 2006.

[7] W. Q. Liu, Z. H. Feng, J. He, and R. B. Liu, "Maximum mechanical energy harvesting strategy for a piezoelement," *Smart Materials and Structures*, vol. 16, no. 6, pp. 2130–2136, 2007.

[8] H. Ji, J. Qiu, K. Zhu, Y. Chen, and A. Badel, "Multi-modal vibration control using a synchronized switch based on a displacement switching threshold," *Smart Materials and Structures*, vol. 18, no. 3, Article ID 035016, 8 pages, 2009.

[9] H. Ji, J. Qiu, K. Zhu, and A. Badel, "Two-mode vibration control of a beam using nonlinear synchronized switching damping
based on the maximization of converted energy," *Journal of Sound and Vibration*, vol. 329, no. 14, pp. 2751–2767, 2010.

[10] H. Ji, J. Qiu, and P. Xia, "Analysis of energy conversion in two-mode vibration control using synchronized switch damping approach," *Journal of Sound and Vibration*, vol. 330, no. 15, pp. 3539–3560, 2011.

[11] A. Steinwolf, N. S. Ferguson, and R. G. White, "Variations in steepness of the probability density function of beam random vibration," *European Journal of Mechanics A*, vol. 19, no. 2, pp. 319–341, 2000.

[12] L. D. Lutes and S. J. Hu, "Non-normal stochastic response of linear system," *Journal of Engineering Mechanics (ASCE)*, vol. 112, no. 2, pp. 127–141, 1986.

[13] C. G. Bucher and G. I. Schueller, "Non-Gaussian response of linear systems," in *Structural Dynamics. Recent Advances*, pp. 103–127, Springer, Berlin, Germany, 1991.

[14] J. B. Roberts, "On the response of a simple oscillator to random impulses," *Journal of Sound and Vibration*, vol. 4, no. 1, pp. 51–61, 1966.

[15] K. G. McConnell, *Vibration Testing Theory and Practice*, John Wiley & Sons, New York, NY, USA, 1995.

[16] D. Steinberg, *Vibration Analysis for Electronic Equipment*, Wiley-Interscience, New York, NY, USA, 1988.

[17] W. T. Thomson, *Theory of Vibration With Applications*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1981.

[18] M. Sakata and K. Kimura, "Calculation of the non-stationary mean square response of a non-linear system subjected to non-white excitation," *Journal of Sound and Vibration*, vol. 73, no. 3, pp. 333–343, 1980.
