A Note on (i,j)-πgβ Closed Sets in Intuitionistic Fuzzy Bitopological Spaces

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Abstract: In this paper we introduce the concept of (i,j)-πgβ-closed set in intuitionistic fuzzy bitopological spaces as a generalization of πgβ-closed set in fuzzy bitopological space and study their related notions in bitopological spaces. Next, we introduce (i,j)-πgβ- open sets in intuitionistic fuzzy bitopological spaces, and investigate some of their basic properties. Using these concepts, the characterizations for the intuitionistic fuzzy pairwise (i,j)-πgβ continuous mappings are obtained. The relationships between intuitionistic fuzzy pairwise (i,j)-πgβ continuous mappings are discussed. Finally, we prove the irresoluteness in (i,j)-πgβ intuitionistic fuzzy bitopological spaces.

Keywords: IF Bitopological Spaces, IF (i, j)-Open Sets, IF(i, j)-Closed Sets, IF (i, j) πgβ-Open Sets, IF (i, j) πgβ-Closed Sets, IF (i, j) πgβ-Pairwise Continuous Function and IF (i, j) πgβ-Irresolute Function

1. Introduction

The notion of β-open set was introduced by Abd El-Monsef et al. [1] and Andrijevic [2]. Later on, as a generalization of the above mentioned set, ngβ sets have been introduced by Caldas and Jafari [4]. The concept of bitopological spaces \((X, τ_i, τ_j)\) was introduced by Kelly J. C in 1963 [8] where \(X\) is a nonempty set and the bitopological spaces are equipped with two arbitrary topologies \(τ_i\) and \(τ_j\). Where \(τ_i\) and \(τ_j\) are two topologies on \(X\). After that several authors turned their attention towards generalizations of various concepts of topology for bitopological spaces. In 2005, the concept of \((i,j)\)-β-open sets was defined and investigated by Raja Rajeswari and Lellis Thivagar [9]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy set was introduced by Atanassov [3]. Recently, Coker [5] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. On the other hand, Kandil [7] introduced the concept of fuzzy bitopological spaces as a natural generalization of Chang’s fuzzy topological spaces.

In 2012, the notion of bitopological space was introduced in intuitionistic fuzzy topology by - Jin Tae Kim, Seok Jong Lee [6]. In this paper, the concepts of ngβ-closed set have been extended to the bitopological spaces in intuitionistic fuzzy topology and we introduce a new form of closed set called Intuitionistic fuzzy (IF) \((i,j)\)-ngβ-closed set. The notion of IF \((i,j)\)-ngβ -continuous function and irresolute function is introduced and studied.

2. Preliminaries

The interior and the closure of a subset \(A\) of an intuitionistic fuzzy bi topological space (IFBTS) \((X, τ_i, τ_j)\) are denoted by \(\text{Int}(A)\) and \(\text{Cl}(A)\), respectively.

In the following sections by \(X, Y\) and \(Z\), we mean an intuitionistic fuzzy bi topological space \((X, τ_i, τ_j)\), \((Y, σ_i, σ_j)\) and \((Z, η_i, η_j)\), respectively.

Throughout the paper the triplet \((X, τ_i, τ_j)\) denotes an intuitionistic fuzzy bi topological space (IFBTS) and \((X, τ_i, τ_j)\) be the intuitionistic fuzzy bi topological space, where \(i,j \in \{1,2\}, \text{ and } i \neq j\).

For, a subset \(A\) of a bi topological space \((X, τ_i, τ_j)\), we denote the closure of \(A\) and the interior of \(A\) with respect to \(τ_i\) by \(\text{Cl}(A)\) and \(\text{Int}(A)\), respectively.

Definition 2.1. Let \(A\) be an intuitionistic fuzzy set in an IFBTS \((X, τ_i, τ_j)\). Then \(A\) is said to be an

(i) IF \((i, j)\)-semi open [6] if there exists an IF, \(τ_i\)-open set \(U\) in \(X\) such that \(U \subseteq A \subseteq \text{Cl}(U)\) and IF \((i, j)\)-semi closed [6] if
there exists an IF, -closed set U in X such that j-Int(U) ⊆ A ⊆ U.

(ii) If (i, j)-preopen [6], A ⊆ τ \ Int(τ - Cl(A)), and if (i, j)-pre closed if
\[ τ \setminus Cl(τ - Cl(A)). \]

(iii) If (i, j)-open [7] if A ⊆ τ \ Cl(τ \ Int(τ \ Cl(A))), where i, j = 1, 2 and i ≠ j, and
(i, j)-closed [7] if τ \ (Int(τ \ Cl(A)) \ ⊆ A.

Definition 2.2. A subset A of an intuitionistic fuzzy bitopological space (X, τ, τ) is said to be (i, j)-πgβ-Cl(A).

Definition 2.3. A subset A of an intuitionistic fuzzy bitopological space (X, τ, τ) is said to be IF (i, j)-β-closed (j,i) if for every IF(i,j)-πgβ-open set U where \( A \subseteq \cup \) and \( U \subseteq \tau \), where i, j = 1, 2 and i ≠ j.

Definition 2.4. [6] Let A be an intuitionistic fuzzy subset of X, Then A is said to be (i, j)-regular open if
\[ A = (\tau \setminus \Int(\tau \setminus Cl(\tau \setminus Cl(A)))) \cup \{F \subseteq X : F \subseteq (i,j)-\pi-g-beta-cl(A) \}. \]

The union of all IF(i,j)-regular open sets is known as IF(i,j)-π-g-beta-set.

3. Intuitionistic Fuzzy (i,j)-πgβ-Closed Set

Definition 3.1. A subset A of an intuitionistic fuzzy bitopological space (X, τ, τ) is said to be IF(i,j)-πgβ-closed [6] if IF(i,j)-πgβ-Cl(A) ⊆ U whenever A ⊆ U and U is IF(i,j)-π-open.

Definition 3.2 A subset A of a bitopological space (X, τ, τ) is said to be

(i) IF(i,j)-πgβ-closure of A, is defined by the intersection of all IF(i,j)-πgβ-closed sets containing A. That is IF(i,j)-πgβ-Cl(A) = \( \bigcap \{F \subseteq X : F \subseteq (i,j)-\pi-g-beta-cl(A) \} \) & IF(i,j)-πgβ-int(A) = \( \bigcup \{F \subseteq X : F \subseteq (i,j)-\pi-g-beta-cl(A) \} \).

Theorem 3.1 Let (X, τ, τ) be an intuitionistic fuzzy bitopological space and A be a subset of X. Then \( x \in \text{IF}(i,j)-\pi-g-beta-cl(A) \) if and only if for every IF(i,j)-πgβ-open set U containing x, such that \( U \cap A \neq \phi \).

Proof: Suppose that \( x \in \text{IF}(i,j)-\pi-g-beta-cl(A) \), we shall show that \( U \cap A \neq \phi \) for every \( U \subseteq IF(i,j)-\pi-g-beta-cl(A) \). Suppose that there exists \( U \subseteq IF(i,j)-\pi-g-beta-cl(A) \) such that \( U \cap A = \phi \).

Then \( A \subseteq X \setminus U \) and \( X \setminus U \) is IF(i,j)-πgβ-closed.

Since \( A \subseteq X \setminus U \), IF(i,j)-πgβ-Cl(A) ⊆ IF(i,j)-πgβ-Cl(X\U).

Since \( x \in \text{IF}(i,j)-\pi-g-beta-cl(A) \), we have \( x \in \text{IF}(i,j)-\pi-g-beta-cl(X\U) \).

Since \( X \setminus U \equiv IF(i,j)-\pi-g-beta-cl \), then \( x \in X \setminus U \); hence \( x \in X \setminus U \), which is a contradiction that \( x \notin U \). Therefore, \( U \cap A \neq \phi \).

Conversely, suppose that \( U \cap A \neq \phi \) for every \( U \subseteq IF(i,j)-\pi-g-beta-cl(A) \).

We can show that \( x \in \text{IF}(i,j)-\pi-g-beta-cl(A) \). Suppose that \( x \notin \text{IF}(i,j)-\pi-g-beta-cl(A) \).

Then there exists U \( \subseteq IF(i,j)-\pi-g-beta-cl(A) \). Suppose that \( x \notin IF(i,j)-\pi-g-beta-cl(A) \). Then there exists \( U \subseteq IF(i,j)-\pi-g-beta-cl(A) \) such that \( U \cap A = \phi \).

This is a contradiction to \( U \cap A \neq \phi \). Hence \( x \in \text{IF}(i,j)-\pi-g-beta-cl(A) \).

Lemma 3.1 Let (X, τ, τ) be a bitopological space and A be a subset of X. Then

(i) \( X \setminus IF(i,j)-\pi-g-beta-cl(A) \subseteq IF(i,j)-\pi-g-beta-cl(X \setminus A) \).

(ii) \( X \setminus IF(i,j)-\pi-g-beta-cl(A) \subseteq IF(i,j)-\pi-g-beta-cl(X \setminus A) \).

Proof: (i) Let \( x \in IF(i,j)-\pi-g-beta-cl(A) \). There exists \( V \subseteq IF(i,j)-\pi-g-beta-cl(X \setminus A) \) such that \( V \cap A = \phi \).

Hence we obtain \( x \in IF(i,j)-\pi-g-beta-cl(X \setminus A) \). This shows that \( X \setminus IF(i,j)-\pi-g-beta-cl(A) \subseteq IF(i,j)-\pi-g-beta-cl(X \setminus A) \).

Definition 3.3 A space X is said to be IF(i,j)-πgβT \(_\alpha\) if for any two distinct points \( x, y \in X \), there exists IF(i,j)-πgβ-open sets U, V such that \( x \in \text{Int}(U) \) and \( y \notin \text{Int}(V) \).

Theorem 3.2 An intuitionistic fuzzy bitopological space X is IF(i,j)-πgβT \(_\alpha\) if and only if \( x \in X \) is IF(i,j)-πgβ-closed in X for every \( x \in X \).

Proof: If \( x \in X \) is IF(i,j)-πgβ-closed in X for every \( x \in X \), then \( x \notin \text{Int}(X \setminus \{x\}) \).

Conversely, if \( x \notin \text{Int}(X \setminus \{x\}) \), then \( x \notin \text{Int}(X \setminus \{x\}) \).

Therefore, \( x \notin \text{Int}(X \setminus \{x\}) \). Hence \( x \notin \text{Int}(X \setminus \{x\}) \).

Let G be the union of all such V\(_j\). Then G is an IF(i,j)-πgβ-open set and G \( \subseteq X \setminus \{x\} \).

Therefore, \( X \setminus \{x\} \) is an IF(i,j)-πgβ-open set in X.

Theorem 3.3: IF A is IF(i,j)-πgβ closed and \( A \subseteq B \subseteq IF(i,j)-\pi-g-beta-cl(A) \), then B is also IF(i,j)-πgβ closed.

Proof: Let U be an IF(i,j)-π-open set in X such that \( B \subseteq U \).

Since A is IF(i,j)-πgβ closed, \( A \subseteq B \subseteq IF(i,j)-\pi-g-beta-cl(A) \).

Since \( x \in \text{IF}(i,j)-\pi-g-beta-cl(A) \), we have \( x \in \text{IF}(i,j)-\pi-g-beta-cl(B) \).

Since \( X \setminus U \equiv IF(i,j)-\pi-g-beta-cl \), then \( x \in X \setminus U \); hence \( x \in X \setminus U \), which is a contradiction that \( x \notin U \). Therefore, \( U \cap A \neq \phi \).

Conversely, suppose that \( U \cap A \neq \phi \) for every \( U \subseteq IF(i,j)-\pi-g-beta-cl(A) \).

We can show that \( x \in IF(i,j)-\pi-g-beta-cl(A) \). Suppose that \( x \notin IF(i,j)-\pi-g-beta-cl(A) \).

Then there exists \( U \subseteq IF(i,j)-\pi-g-beta-cl(A) \) such that \( U \cap A = \phi \).

This is a contradiction to \( U \cap A \neq \phi \). Hence \( x \in IF(i,j)-\pi-g-beta-cl(A) \).
Proof. Since A is IF (i,j) π-open and IF(j,i) πgβ-closed, IF(j,i) β-Cl(A) ⊆ A, but A ⊈ IF(j,i) β-Cl(A), So A = IF(j,i) β-Cl(A). Hence, A is IF (i,j) β-closed.

**Theorem 3.6.** Let A be an IF(i,j)-πgβ-closed in intuitionistic fuzzy bitopological space X. Then IF(j,i) β-Cl(A) does not contain any nonempty IF(i,j) π-closed set.

Proof. Let U be a nonempty IF (j,i) π-closed subset of IF(j,i) β-Cl(A). Then A ⊈ X \ U, where A is IF (i,j)-πgβ-closed and X \ U is IF (i,j)-π-open. Thus IF(j,i) β-Cl(A) ⊆ X \ U, or U ⊈ IF(j,i) β-Cl(A). Since by assumption U ⊈ IF(j,i) β-Cl(A), we get a contradiction.

**Corollary 3.2.** Let A be IF(i,j)- πgβ-closed in X. Then A is IF(i,j)- β-closed if and only if IF(j,i)-βCl(A)/A is IF(i,j)- β-closed.

Proof. Necessity: Let A be an IF(i,j)- πgβ-closed. By hypothesis IF(i,j)- β-Cl(A) = A and so IF(j,i)- β-Cl(A)/A = A = φ which is IF(i,j)- β-closed.

Sufficiency. Suppose IF(j,i)- β-Cl(A)/A = A, then by theorem 3.6,

IF(j,i)- β-Cl(A)/A = φ, that is, IF(j,i)- β-Cl(A) = A. Hence, A is IF(i,j)- β-closed.

**Definition 3.4.**

In a bitopological space (X, τ1, τ2), let B ⊆ A ⊆ X. Then we say that B is IF(i,j)- πgβ -closed relative to A if IF(i,j)- πgβ-Cl(B) ⊆ A.

**Theorem 3.7.** Let B ⊆ A ⊆ X where A is a IF(i,j)- πgβ -closed and IF(i,j)- π-open set. Then B is IF(i,j)- πgβ -closed relative to A if and only if B is IF(i,j)- πgβ -closed in X.

Proof. Here, B ⊆ A and A is both a IF(i,j)- πgβ -closed and IF(i,j)- π-open set, then IF(i,j)- πgβ -closed relative to A if and only if B is IF(i,j)- πgβ -closed in X.

**Theorem 3.8.** Let B ⊆ A ⊆ X where A is a IF(i,j)- πgβ -closed and IF(i,j)- π-open set. Then B is IF(i,j)- πgβ -closed relative to A if and only if B is IF(i,j)- πgβ -closed in X.

Proof. Here, B ⊆ A and A is both a IF(i,j)- πgβ -closed and IF(i,j)- π-open set, then IF(i,j)- πgβ -closed relative to A if and only if B is IF(i,j)- πgβ -closed in X.

**Definition 3.3.**

An intuitionistic fuzzy bitopological space is the intersection of all IF(i,j)- π-open sets containing A and the intersection of all the IF(i,j)- π-open sets containing A which is by the usual notation, IF(i,j)- πgβ-ker(A).

**Lemma 3.2.** Let X be an intuitionistic fuzzy bi topological space and x ∈ X. The following are equivalent.

(i) x ∈ IF (i,j)-πgβker{y}
(ii) y ∈ IF (i,j)-πgβker{z}

Proof. (i) ⇒ (ii): Let IF (i,j)-πgβker{y} ≠ IF (i,j)-πgβker{z}. Then there exists an IF (i,j)-π-open set U containing x such that y ∈ IF (i,j)-πgβker{y} but z ∉ IF (i,j)-πgβker{y}.

(ii) ⇒ (i): If y ∈ IF (i,j)-πgβker{z}, then there exists an IF (i,j)-π-open set U containing y such that x ∈ IF (i,j)-πgβker{y}.

**Lemma 3.3.** The following statements are equivalent for any two points x, y in an intuitionistic fuzzy bi topological space X.

(i) IF (i,j)-πgβker{y} ≠ IF (i,j)-πgβker{z}
(ii) IF (i,j)-πgβ-Cl{y} ≠ IF (i,j)-πgβ-Cl{z}

Proof. (i) ⇒ (ii): Let IF (i,j)-πgβker{y} ≠ IF (i,j)-πgβker{z}. Then there exists a point z in X such that z ∈ IF (i,j)-πgβker{y} but z ∉ IF (i,j)-πgβker{z}.

(ii) ⇒ (i): If IF (i,j)-πgβ-Cl{y} ≠ IF (i,j)-πgβ-Cl{z}, then there exists a point z in X such that z ∈ IF (i,j)-πgβ-Cl{y} but z ∉ IF (i,j)-πgβ-Cl{z}.
A space \( X \) is said to be \((i,j)\)-\(\pi g\beta R_0\) if every \((i,j)\)-\(\pi g\beta\)-open set contains the \((i,j)\)-\(\pi g\beta\)-closure of each of its singletons.

**Lemma 3.4** A space \( X \) is \((i,j)\)-\(\pi g\beta R\) if and only if for any \( x \) and \( y \) in \( X \) and \( \text{IF}(i,j)\)-\(\pi g\beta\)-open set \( V \) such that \( (i,j)\)-\(\pi g\beta\)-Cl\(\{x\}\) \(\neq\) \( (i,j)\)-\(\pi g\beta\)-Cl\(\{y\}\), and therefore, \( x \neq y \) and \( x \notin V \), Hence \( y \) and \( x \) but not \( y \). Therefore, \( y \notin \text{IF}(i,j)\)-\(\pi g\beta\)-Cl\(\{x\}\) and \( \text{IF}(i,j)\)-\(\pi g\beta\)-Cl\(\{y\}\) implies and \( (i,j)\)-\(\pi g\beta\)-Cl\(\{x\}\) \(\cap\) \( (i,j)\)-\(\pi g\beta\)-Cl\(\{y\}\) = \(\phi\).

**Proof.** Let \( U \) be \((i,j)\)-\(\pi g\beta\) open and \( x \notin U \). Then \( x \notin \text{IF}(i,j)\)-\(\pi g\beta\)-Cl\(\{y\}\). This implies that \((i,j)\)-\(\pi g\beta\)-Cl\(\{x\}\) \(\cap\) \( (i,j)\)-\(\pi g\beta\)-Cl\(\{y\}\) = \(\phi\). Hence \( x \notin U \) and \( y \in U \). Thus we get \( x \notin (i,j)\)-\(\pi g\beta\)-Cl\(\{x\}\) and \( x \notin \text{IF}(i,j)\)-\(\pi g\beta\)-Cl\(\{x\}\).

**4. IF (i,j)-\(\pi g\beta\)-Continuous Functions and Irresoluteness**

**Definition 4.1** Let \( f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2) \) be a mapping from an IFBTS \( X \) to an IFBTS \( Y \). Then \( f \) is said to be IF pairwise continuous if the inverse image of every \( \sigma_1 \)-open set of \( Y \) is \((i,j)\)-open in \( X \), where \( i \neq j \).

**Definition 4.2** Let \( f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2) \) be a mapping from an IFBTS \( X \) to an IFBTS \( Y \). Then \( f \) is said to be IF pairwise \( \pi g\beta \)-continuous if \( f^{-1}(A) \) is \((i,j)\)-\(\pi g\beta\)-open in \( X \) for each \( \sigma_1 \)-open set \( A \) in \( Y \) and \( f^{-1}(B) \) is \((i,j)\)-\(\pi g\beta\)-open in \( X \) for each \( \sigma_1 \)-open set \( B \) in \( Y \).

**Theorem 4.1** For a function \( f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2) \), the following statements are equivalent:

(i) \( f \) is IF pairwise \( \pi g\beta \)-continuous.

(ii) For each point \( x \) in \( X \) and each \( \sigma_1 \)-open set \( F \) in \( Y \) such that \( f(x) \in F \), there exists an IF \((i,j)\)-\(\pi g\beta\)-open set \( A \) in \( X \) such that \( x \in A \), \( f(A) \subset F \).

(iii) The inverse image of each \( \sigma_1 \)-closed set in \( Y \) is \((i,j)\)-\(\pi g\beta\)-closed in \( X \).

(iv) For each subset \( A \) of \( X \), \( f(\text{IF}(i,j)\)-\(\pi g\beta\)-Cl\(\{A\}\)) \(\subset\) \(\text{IF}(i,j)\)-\(\pi g\beta\)-Cl\(\{f^{-1}(A)\}\).

(v) For each subset \( B \) of \( Y \), \( f(\text{IF}(i,j)\)-\(\pi g\beta\)-Cl\(\{f^{-1}(B)\}\)) \(\subset\) \(\text{IF}(i,j)\)-\(\pi g\beta\)-Cl\(\{f^{-1}(B)\}\).
Proof: (i)⇒(ii): Let x∈X and U be a σi–open set of Y containing f(x). By (i), f−1(U) is an IF(i,j)-πgβ-open in X, let A′ = f−1(U), then x∈A and f(A)⊂U.

(ii)⇒(i): Let f be a function and X⊂Y. By (i), f contains f−1(Y \ B) = Xf−1(B).

(iii)⇒(iv): Let A be a subset of X. Hence f is IF(i,j)-πgβ-continuous in Y.

(iv)⇒(v): Let B be any subset of Y. Then f−1(B) = IF(i,j)-πgβ-continuous subset of Y.

(v)⇒(vi): Let V be a set in Y. Hence f is IF(i,j)-πgβ-continuous.

Proof: (i)⇒(ii): Let B be any subset of Y. Suppose that x∉f−1(IF(i,j)-πgβ-C(B)). Then f(x)∉IF(i,j)-πgβ-C(B) and there exists U∈IF(i,j)-πgβ-O(X,f(x)) such that IF(i,j)-πgβ-Cl(V \ B) = φ. Since f is IF(i,j)-πgβ-irresolute, there exists U∈IF(i,j)-πgβ-O(X,x) such that f(IF(i,j)-πgβ-C(U))⊂IF(i,j)-πgβ-C(V).

(iii)⇒(iv): Let B be any subset of Y. Then f−1(B) is IF(i,j)-πgβ-continuous.

(iv)⇒(v): Let B be any subset of Y. Then IF(i,j)-πgβ-Cl(f−1(B))⊂Cl(f−1(U))⊂σi-Cl(U).

(v)⇒(vi): Let B be any subset of Y. Then f−1(B) is IF(i,j)-πgβ-continuous.

Proof: (i)⇒(ii): Let B be any subset of Y. Suppose that x∉f−1(IF(i,j)-πgβ-C(B)). Then f(x)∉IF(i,j)-πgβ-C(B) and there exists U∈IF(i,j)-πgβ-O(X,f(x)) such that IF(i,j)-πgβ-Cl(V \ B) = φ. Since f is IF(i,j)-πgβ-irresolute, there exists U∈IF(i,j)-πgβ-O(X,x) such that f(IF(i,j)-πgβ-C(U))⊂IF(i,j)-πgβ-C(V).

(iii)⇒(iv): Let B be any subset of Y. Then f−1(B) is IF(i,j)-πgβ-continuous.

(iv)⇒(v): Let B be any subset of Y. Then f−1(B) is IF(i,j)-πgβ-continuous.

(v)⇒(vi): Let B be any subset of Y. Then f−1(B) is IF(i,j)-πgβ-continuous.

5. IF (i,j)-πgβ–Irresolute Functions

Definition 5.1 A map f:(X,τ1,τ2)→(Y,σ1,σ2) is called IF(i,j)-πgβ–irresolute if for each x∈X and each V∈IF(i,j)-πgβ-O(Y,f(x)), there exists U∈IF(i,j)-πgβ-O(X,x) such that if f(IF(i,j)-πgβ-C(U))⊂IF(i,j)-πgβ-C(B).

Theorem 5.1 For a function f:(X,τ1,τ2)→(Y,σ1,σ2) the following are equivalent:

(i) f is IF(i,j)-πgβ-irresolute.

(ii) IF(i,j)-πgβ-Cl(f−1(B))⊂f−1(IF(i,j)-πgβ-Cl(B)) for every subset B of Y.

(iii) f(IF(i,j)-πgβ-Cl(A))⊂IF(i,j)-πgβ-Cl(f(A)) for every subset A of X.
\( V \cap (Y \setminus \pi g^\beta \cdot \text{Cl}(V)) = \emptyset \)

and \( f(x) \in (Y \setminus \pi g^\beta \cdot \text{Cl}(Y \setminus \pi g^\beta \cdot \text{Cl}(V))) \).

Therefore, \( x \notin f^{-1}(\pi g^\beta \cdot \text{Cl}(Y \setminus \pi g^\beta \cdot \text{Cl}(V))) \) and by (iii),

\[ x \notin (\pi g^\beta \cdot \text{Cl}(f^{-1}(Y \setminus (\pi g^\beta \cdot \text{Cl}(V)))). \]

There exists \( U \subseteq (\pi g^\beta \cdot \text{Cl}(O(X, x)) \) such that \( \pi g^\beta \cdot \text{Cl}(U) \cap f^{-1}(Y \setminus (\pi g^\beta \cdot \text{Cl}(V))) \neq \emptyset \).

Hence \( f^{-1}(\pi g^\beta \cdot \text{Cl}(U)) \subseteq (\pi g^\beta \cdot \text{Cl}(V)) \) and hence \( f^{-1}(\pi g^\beta \cdot \text{Cl}(V)) \)

is \( \pi g^\beta \cdot \text{Cl}(V) \).

Theorem 5.3 Let \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) and \( g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2) \) be any two functions. Then

(i) \( g \circ f \) is \( \pi g^\beta \cdot \text{continuous}, \) if \( g \) is \( \pi g^\beta \cdot \text{continuous} \) and \( f \) is \( \pi g^\beta \cdot \text{continuous}, \)

(ii) \( g \circ f \) is \( \pi g^\beta \cdot \text{continuous}, \) if \( g \) is \( \pi g^\beta \cdot \text{irregular} \) and \( f \) is \( \pi g^\beta \cdot \text{irregular} \),

(iii) \( g \circ f \) is \( \pi g^\beta \cdot \text{irregular}, \) if \( g \) is \( \pi g^\beta \cdot \text{irregular} \) and \( f \) is \( \pi g^\beta \cdot \text{irregular} \).

Proof (i) Let \( V \subseteq \pi g^\beta \cdot \text{Cl}(V) \).

Hence \( f^{-1}(\pi g^\beta \cdot \text{Cl}(V)) \)

is \( \pi g^\beta \cdot \text{Cl}(V) \).

Then \( g^{-1}(\pi g^\beta \cdot \text{Cl}(V)) \) is \( \pi g^\beta \cdot \text{Cl}(V) \).

Hence \( f^{-1}(\pi g^\beta \cdot \text{Cl}(V)) \subseteq (\pi g^\beta \cdot \text{Cl}(V)) \).

Theorem 5.4: \( \pi g^\beta \cdot \text{Cl}((Y \setminus (\pi g^\beta \cdot \text{Cl}(V))) \subseteq (\pi g^\beta \cdot \text{Cl}(V)) \).

Proof: Define a binary operation \( * : (\pi g^\beta \cdot \text{Cl}(X) \to (\pi g^\beta \cdot \text{Cl}(X)) \) by \( * = f \circ g \).

Then \( * \) is well-defined and it is easily proved that under this binary operation \( \pi g^\beta \cdot \text{Cl}(X) \) is a group.

6. Conclusion

In this paper, we introduce the concept of \( \pi g^\beta \cdot \text{closed set} \) in intuitionistic fuzzy bitopological spaces and study some of their properties. We also introduce the concept of \( \pi g^\beta \cdot \text{continuous functions} \) in bitopological spaces and some of their properties have been established. We hope that the findings in this paper are just the beginning of a new structure and not only will form the theoretical basis for further applications of topology on intuitionistic fuzzy bitopological sets but also will lead to the development of information system and various fields in engineering.

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