Performance Analysis and Optimization of Spaceborne Multi-baseline InSAR Systems

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Abstract. The spaceborne interferometric synthetic aperture radar (InSAR) technique has become a useful tool to obtain the surface elevation or deformation. The multi-baseline InSAR technique is capable of fusing data from multiple channels for interferometric processing and improving the accuracy of elevation estimation of the observed scenes. In this paper, we study the joint distribution to statistically characterize the interferometric phases of multi-baseline InSAR. The baseline parameters are sought based on the second-order moment of the interferometric phase joint probability density function (PDF). The study done in this thesis provides a basis for the design of parameters for multi-baseline interferometric systems.

1. Introduction

Recently, the multi-baseline interferometric synthetic aperture radar (InSAR), one of the hotspots in the technical research field, is widely adopted in acquiring high-precision statistical digital elevation model (DEM) estimation. The usage of multi-baseline takes the advantages of easy unwrapping from short baselines and high inversion accuracy from long baseline, and it also improves the accuracy of InSAR measurements. In the process of multi-baseline InSAR data processing, the interferometric phase is a very important factor. Considering the statistical dependence assumption, the literature [1] proposed using the second-order joint probability density function (PDF) for the interferometric phase as the statistical characterization and estimating the second statistical moment by two methods: one based on the numerical Monte-Carlo integration and the other based on an analytical approximation using nonlinear error propagation. Also, both the length of the baseline and the ratio between baselines are important to the process of altimetry. Literature [2] estimates the interferometric phase joint PDF based on maximum posterior probability, and then carries out the baseline design and simulation. Literature [3] performs multi-baseline phase statistical analysis based on the maximum likelihood algorithm under the assumption of independent interferograms. Multi-baseline lengths are designed optimally using the peak of the sidelobe ratio of the likelihood function as a criterion. However, there are few studies on the optimal design of baseline parameters for multi-baseline InSAR under the statistical dependence assumption in recent literatures.

In this paper, we estimate the second-order phase statistics of multi-baseline InSAR phase interferograms from the statistical characteristics of interferometric phase noise, derive the standard deviation of elevation from the second-order moment estimation of interference phase, analyze the correlation between baseline parameters and height measurement errors, and finally optimize the baseline parameters of multi-baseline InSAR. The article is structured as follows. Firstly, we give a second statistical moment estimation of the interferometric phase based on the flow of multi-baseline interferometric processing and derive the standard deviation of the elevation from the second-order
moment in (Section 2). Secondly, the effect of baseline parameters on height measurement errors is then analyzed in (Section 3). Then, the optimal baseline configuration based on the standard deviation of elevation is given in (Section 4). Finally, the conclusions and summary are given in (Section 5).

2. Multi-baseline interferometric phase statistics

To calculate the second-order moment of the interferometric phase, we need to define a random vector consisting of multiple interferometric phases. Taking three interferometric phases for example, a multivariate stochastic vector \( \mathbf{y} \) can be easily used to calculate the mean and the variance of multiple interferometric phases. The mean can be derived as follows.

\[
E\{y\} = E\left[\begin{bmatrix} \psi_{12} \\ \psi_{13} \\ \psi_{23} \end{bmatrix}\right] = \begin{bmatrix} E[\psi_{12}] \\ E[\psi_{13}] \\ E[\psi_{23}] \end{bmatrix} = \begin{bmatrix} \psi_{0,12} \\ \psi_{0,13} \\ \psi_{0,23} \end{bmatrix}
\]

(1)

And the variance is a 3x3 Hermite covariance matrix with the diagonal elements being the variances of interferometric phases and the off-diagonal elements being the covariances among interferometric phases.

\[
Q_y = D\left[\begin{bmatrix} \psi_{12} \\ \psi_{13} \\ \psi_{23} \end{bmatrix}\right] = \begin{bmatrix} \sigma_{\psi_{12}}^2 & \sigma_{\psi_{12},\psi_{13}} & \sigma_{\psi_{12},\psi_{23}} \\ \sigma_{\psi_{13},\psi_{12}} & \sigma_{\psi_{13}}^2 & \sigma_{\psi_{13},\psi_{23}} \\ \sigma_{\psi_{23},\psi_{12}} & \sigma_{\psi_{23},\psi_{13}} & \sigma_{\psi_{23}}^2 \end{bmatrix} = \int (y - E\{y\})(y - E\{y\})^T f_y(y)dy
\]

(2)

Where \( f_y(y) \) represents the multivariate PDF of \( y \), which is unknown. This full covariance matrix can be simulated based on the numerical Monte-Carlo integration [1].

\[
Q_y = \frac{1}{M} \sum_{i=1}^{M} (y_i - E\{y\})(y_i - E\{y\})^T
\]

(3)

Where \( y_i, \quad i = 1, ..., M \) are the \( M \) random realizations of vector specified by a coherence matrix, and \( M \) should be chosen as a large number. Another approach is to use the concept of error propagation to propagate the coherence matrix to the dispersion of interferometric phases. So the general equation for the coherence matrix of interferometric phases is expressed as follows.

\[
\sigma_{\psi_{ij}}^2 = \frac{1 - y_{ij}^2}{2Ly_{ij}^2}, \quad \sigma_{\psi_{ij},\psi_{kl}} = \frac{\gamma_{ik}y_{ij} - \gamma_{ij}y_{ik}}{2Ly_{ij}y_{kl}}
\]

(4)

Compared with the first method, the second method is more efficient and less time-consuming. Both methods require the coherence matrix \( \Gamma \) between complex SAR images, whose elements are as follows.

\[
\gamma_{ij} = \gamma_0 \cdot \gamma_t \cdot \gamma_v
\]

(5)

Where \( \gamma_0 \) represents the decorrelation due to the thermal noise and is set to 0.9 in this paper, \( \gamma_t \) is the temporal decorrelation factor, and \( \gamma_v \) is the spatial correlation factor. Constructing a matrix equation for phase and elevation as follows.

\[
y = Ah \quad , \quad A = [\alpha_{12}, \alpha_{13}, \alpha_{23}]
\]

(6)

Where \( h \) is the parameter to be estimated and \( \alpha_{ij} \) is the corresponding interferometric phase elevation conversion factor. An estimate of \( h \) can be obtained by least squares estimation.

\[
\hat{h} = (A^T A)^{-1} A^T \cdot y
\]

(7)

The variance and standard deviation of elevation are estimated as follows.
3. Baseline parameters and height measurement errors

In a multi-baseline system, the baseline parameters are very important in determining the system's accuracy and performance. An investigation of (8) reveals that the error propagation from interferometric phase (and other error sources such as baseline measurement) to the height estimation is in inverse proportion to the baseline length. However, there is a critical baseline beyond which the interferogram from two SLCs cannot be formed. On the other side, a greater decoration coefficient requires a smaller baseline. At the same time, the ratio between the long and short baselines will also greatly affect the accuracy of the elevation measurement. Therefore, when designing the baseline length in a multi-baseline system, the influence of coherence value, error propagation factor and baseline ratio should be all considered comprehensively to determine the optimal choice of the baselines combination.

3.1. Baseline parameters and coherence coefficient

The coherence coefficient refers to the normalized complex correlation between SAR image pairs. The higher the coherence value is, the better the interference quality will be, as well as the higher the evaluation accuracy will be. We denote the factors that cause a decrease in the coherence of the system as decorrelation, and the decrease in coherence brought about by the baseline as spatial decorrelation, which can be calculated according to the first-order spatial decorrelation model [4].

\[
\gamma_v = 1 - \frac{B_{\perp}}{B_c} \quad B_c = \frac{\lambda R_0}{2 \cos \theta \rho_y} \quad \rho_y = c/(2\Delta f \sin \theta) \quad (9)
\]

Where \(B_c\) is the orthogonal critical baseline, \(\rho_y\) is the distance direction resolution, and \(\Delta f\) is the pulse bandwidth.

When the system parameters are determined, the critical baseline is a constant. Intuitively, the coherence value decreases linearly with increasing baseline length. Fig.1 reflects the relationship between baseline length and spatial decoherence, using the Envisat-ASAR system parameters with a critical baseline of 1089m. Therefore, it can be concluded that when considering the decoherence factor, a better performance requires a smaller length.

3.2. Baseline parameters and error transmission coefficient

The vertical effective baseline \(B_{\perp}\) has a very significant effect on the error propagation coefficient. There is a conversion factor \(\alpha\) between the elevation of the target point and the interferometric phases, so the propagation coefficient formula of the interference phase error can be derived. It can be seen that the longer the vertical effective baseline is, the smaller the error propagation of the interference phase error \(\Delta \psi\) will be.

\[
\alpha = \frac{4\pi B_{\perp}}{\lambda R_0 \sin \theta} \quad \Delta h = \frac{\lambda R_0 \sin \theta}{4\pi B_{\perp}} \Delta \psi \quad (10)
\]

The height measurement error propagation coefficient of the vertical effective baseline \(B_{\perp}\) can be expressed as follows.

\[
\frac{\partial h}{\partial B_{\perp}} = -\frac{\lambda R \sin \theta \psi}{2\pi B_{\perp}^2} = -\frac{h}{B_{\perp}^2}, \quad \Delta h = -\frac{h}{B_{\perp}} \cdot \Delta B_{\perp} \quad (11)
\]

That is, the error propagation coefficient is related to the ratio of the elevation of the target point to the vertical baseline. Similarly, the error propagation increases with the increase of \(B_{\perp}\). In summary, when the error propagation coefficient is considered individually, the longer the vertical effective baseline is, the smaller the error propagation coefficient will be, and the higher the evaluation accuracy will be.
3.3. Long and short baseline configuration

In multi-baseline systems, in addition to the baseline length that affects the elevation estimation, the ratio of long and short baselines also affects the systems’ high-measurement accuracy to a certain extent. On the one hand, the setting of long baseline should meet the requirement of high-precision measurement, and the assistance of short baselines can solve the complicated difficulties of phase unwrapping. On the other hand, length of short and long baselines will also affect the shape of the joint PDF of interferometric phase and ultimately affect the height measurement errors.

Therefore, the two principles should be followed in determining the baseline configuration:

- With the constant short baseline, the optimal range of the long baseline is judged by the criterion of high-measurement accuracy (or standard deviation of elevation). As a reference, the coherence is better when the baseline length is about 1/3 of the limit baseline.
- The ratios of the short baselines to the long baseline are optimized to obtain the length range of the short baseline while fixing the long baseline.

4. A Numerical Design of baseline parameters

The multi-baseline length design optimization method is based on the optimization of the long baseline, as well as the proportions of the remaining short baselines to the long baseline. Based on the multi-baseline height measurement standard deviation derived in section 3, the short baseline proportion is optimized by changing within a certain value range. In order to make the result simple and clear, it is not advisable to introduce too many variables in the analysis. To exclude the influence of multi-views on the baseline design, a pre-experiment with multi-views $L = 30, 50$ is conducted. The experimental results are shown in Fig.2. It shows that the change of multi-view number does not affect the optimal baseline range. It also demonstrates that as the baseline increases, the elevation standard deviation will lower it to a critical value and raise it again. Next, the parameters are set for the simulation test. The multi-baseline elevation estimation process is simulated for four SAR images with the parameter settings in Table 1. The effect of the baseline parameters on the height measurement errors is studied, where $B_c = 1089$, $L = 50$ and $\gamma_t = 1 - \frac{c}{r} = 0.7$.

| Parameter            | Value     | Parameter          | Value     |
|----------------------|-----------|--------------------|-----------|
| Wavelength           | 0.0566 m  | Pulse bandwidth    | 16 Mhz    |
| Angle of incidence   | 23°       | Time interval      | 6 days    |

Table 1. Simulation based on the Envisat-ASAR system parameters
According to the baseline design principles, the length of the long baseline is firstly designed based on the measurement accuracy. In the test, all the baseline parameters vary in the range of $B=100:1000$, $p=0.1:1$, $q=0.1:1$ to obtain the full spatial slice of elevation standard deviation in Fig.3, where $B$ is the length of long baseline, $p$ and $q$ are the proportions of short baselines to long baseline. Splitting Fig.3 into ten sub-slice plots (as in Fig.4), it can be seen that the long baseline range that makes the standard deviation of elevation smaller is located at 600m around, and the baseline ratios are 0.2-0.4 and 0.6-0.8.

Similarly, the results of this test can be used as a reference for the next fine test. The results obtained by performing the test with $B=500:700$, $p=0.2:0.4$, $q=0.6:0.8$ are shown in Fig.5. It can be demonstrated that the best high-measurement accuracy is achieved when the optimal long baseline range is 560-580m and the short baseline ratios are about 0.3 and 0.7.

| Distance | 850 km | Time constant | 40 days |
|----------|--------|---------------|---------|

Fig.3 a full-space slice of elevation standard deviation for baseline parameters $B=100:1000$, $p=0.1:1$, $q=0.1:1$
Fig. 4 Elevation standard deviation sub-slice graph
(a) to (j): B=100:1000m

(a) a full-space slice of altimetric standard deviation
( B=500:600m, p=0.2:0.4, q=0.6:0.8)

(b) the 8th sub-slice of (a) graph with minimum standard deviation (B=570 m)

Fig. 5 Further tests based on preliminary scope

5. Conclusion
In this paper we present the second-order moment of the complex joint pdf of statistical dependent interferograms. We also derive the estimation of the elevation standard deviation from the second-order moment by least squares estimation. After that, the baseline parameters are designed based on
the elevation standard deviation with the Envisat-ASAR system parameters. Studies have shown that the best high-measurement accuracy is achieved when the optimal long baseline range is between 560-580m and the short baseline ratios are about 0.3 and 0.7. Then, the optimal baseline configuration we proposed promises achieving a more significant DEM enhancement. Actually, this research route is also applicable to other multi-baseline InSAR systems.

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