2-LOCAL DERIVATIONS ON ALGEBRAS OF LOCALLY MEASURABLE OPERATORS

SH. A. AYUPOV, K. K. KUDAYBERGENOV, AND A. K. ALAUADINOV

ABSTRACT. The paper is devoted to 2-local derivations on the algebra $LS(M)$ of all locally measurable operators affiliated with a type $I_{\infty}$ von Neumann algebra $M$. We prove that every 2-local derivation on $LS(M)$ is a derivation.

1. Introduction

Given an algebra $A$, a linear operator $D : A \to A$ is called a derivation, if $D(xy) = D(x)y + xD(y)$ for all $x, y \in A$ (the Leibniz rule). Each element $a \in A$ implements a derivation $D_a$ on $A$ defined as $D_a(x) = [a, x] = ax - xa, x \in A$. Such derivations $D_a$ are said to be inner derivations. If the element $a$, implementing the derivation $D_a$, belongs to a larger algebra $B$ containing $A$, then $D_a$ is called a spatial derivation on $A$.

There exist various types of linear operators which are close to derivations [9,10,17]. In particular R. Kadison [9] has introduced and investigated so-called local derivations on von Neumann algebras and some polynomial algebras.

A linear operator $\Delta$ on an algebra $A$ is called a local derivation if given any $x \in A$ there exists a derivation $\tilde{D}$ (depending on $x$) such that $\Delta(x) = \tilde{D}(x)$. The main problems concerning this notion are to find conditions under which local derivations become derivations and to present examples of algebras with local derivations that are not derivations [9]. In particular Kadison [9] has proved that each continuous local derivation from a von Neumann algebra $M$ into a dual $M$-bimodule is a derivation.

In 1997, P. Semrl [17] introduced the concept of 2-local derivations and automorphisms. A map $\Delta : A \to A$ (not linear in general) is called a 2-local derivation if for every $x, y \in A$, there exists a derivation $D_{x,y} : A \to A$ such that $\Delta(x) = D_{x,y}(x)$ and $\Delta(y) = D_{x,y}(y)$. A map $\Theta : A \to A$ (not linear in general) is called a 2-local automorphism if for every $x, y \in A$, there exists an automorphism $\Phi_{x,y} : A \to A$ such that $\Theta(x) = \Phi_{x,y}(x)$ and $\Theta(y) = \Phi_{x,y}(y)$. Local and 2-local maps have been studied on different operator algebras by many authors [2–5,7,9–14,17,18].

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In [17], P. Semrl described 2-local derivations and automorphisms on the algebra $B(H)$ of all bounded linear operators on the infinite-dimensional separable Hilbert space $H$. A similar description for the finite-dimensional case appeared later in [10], [14]. In the paper [12] 2-local derivations and automorphisms have been described on matrix algebras over finite-dimensional division rings. J. H. Zhang and H. X. Li [21] described 2-local derivations on symmetric digraph algebras and constructed a 2-local derivation on the algebra of all upper triangular complex $2 \times 2$-matrices which is not a derivation. In [3] first two authors considered 2-local derivations on the algebra $B(H)$ of all linear bounded operators on an arbitrary (no separability is assumed) Hilbert space $H$ and proved that every 2-local derivation on $B(H)$ is a derivation.

The present paper is devoted to study 2-local derivations on $*$-subalgebras of the von Neumann algebra $A$ of all locally measurable operators with respect to the type $I_\infty$ von Neumann algebra $M$. We prove that every 2-local derivations on every $*$-subalgebra $A$ in $L_S(M)$, such that $M \subseteq A$, is a derivation.

2. Algebra of locally measurable operators

Let $B(H)$ be the $*$-algebra of all bounded linear operators on a Hilbert space $H$, and let $1$ be the identity operator on $H$. Consider a von Neumann algebra $M \subseteq B(H)$. Denote by $P(M) = \{p \in M : p = p^2 = p^*\}$ the lattice of all projections in $M$ and by $P_{fin}(M)$ the set of all finite projections in $P(M)$.

A linear subspace $D$ in $H$ is said to be affiliated with $M$ (denoted as $D\eta M$), if $u(D) \subseteq D$ for every unitary $u$ from the commutant
\[ M' = \{y \in B(H) : xy = yx, \forall x \in M\} \]
of the von Neumann algebra $M$.

A linear operator $x : D(x) \to H$, where the domain $D(x)$ of $x$ is a linear subspace of $H$, is said to be affiliated with $M$ (denoted as $x\eta M$) if $D(x)\eta M$ and $u(x(\xi)) = x(u(\xi))$ for all $\xi \in D(x)$ and for every unitary $u \in M'$.

A linear subspace $D$ in $H$ is said to be strongly dense in $H$ with respect to the von Neumann algebra $M$, if

- $D\eta M$;
- there exists a sequence of projections $\{p_n\}_{n=1}^\infty$ in $P(M)$ such that $p_n \uparrow 1$, $p_n(H) \subseteq D$ and $p_n^* = 1 - p_n$ is finite in $M$ for all $n \in \mathbb{N}$.

A closed linear operator $x$ acting in the Hilbert space $H$ is said to be measurable with respect to the von Neumann algebra $M$, if $x\eta M$ and $D(x)$ is strongly dense in $H$.

Denote by $S(M)$ the set of all linear operators on $H$, measurable with respect to the von Neumann algebra $M$. If $x \in S(M)$, $\lambda \in \mathbb{C}$, where $\mathbb{C}$ is the field of complex numbers, then $\lambda x \in S(M)$ and the operator $x^*$, adjoint to $x$, is also measurable with respect to $M$ (see [16]). Moreover, if $x, y \in S(M)$, then the operators $x + y$ and $xy$ are defined on dense subspaces and admit closures that are called, correspondingly, the strong sum and the strong product of the operators $x$ and $y$, and are denoted by $x + y$ and $x \ast y$. It was shown in [16] that $x + y$ and $x \ast y$ belong to $S(M)$ and these algebraic operations make $S(M)$ a $*$-algebra with
the identity 1 over the field C. It is clear that, M is a *-subalgebra of S(M). In
what follows, the strong sum and the strong product of operators x and y will be
denoted in the same way as the usual operations, by x + y and xy.

A closed linear operator x in H is said to be locally measurable with respect
to the von Neumann algebra M, if xηM and there exists a sequence \( \{z_n\}_{n=1}^{\infty} \)
of central projections in M such that \( z_n \uparrow 1 \) and \( z_n x \in S(M) \) for all \( n \in \mathbb{N} \) (see [20]).

Denote by LS(M) the set of all linear operators that are locally measurable
with respect to M. It was proved in [20] that LS(M) is a *-algebra over the field
C with the identity 1, the operations of strong addition, strong multiplication,
and passing to the adjoint. In such a case, S(M) is a *-subalgebra in LS(M).
In the case where M is a finite von Neumann algebra or a factor, the algebras
S(M) and LS(M) coincide. This is not true in the general case. In [15] the class
of von Neumann algebras M has been described for which the algebras LS(M)
and S(M) coincide.

We say that a measure \( \mu \) on a measure space \((\Omega, \Sigma, \mu)\) has the direct sum
property if there is a family \( \{\Omega_i\}_{i \in I} \subset \Sigma, 0 < \mu(\Omega_i) < \infty, i \in I \), such that for
any \( A \in \Sigma, \mu(A) < \infty \), there exist a countable subset \( J_0 \subset J \) and a set \( B \) with
zero measure such that \( A = \bigcup_{i \in J_0} \left( A \cap \Omega_i \right) \cup B \).

It is well-known (see e.g. [16]) that for each commutative von Neumann algebra
M there exists a measure space \((\Omega, \Sigma, \mu)\) with \( \mu \) having the direct sum property
such that M is *-isomorphic to the algebra \( L^\infty(\Omega, \Sigma, \mu) \) of all (equivalence classes
of) complex essentially bounded measurable functions on \((\Omega, \Sigma, \mu)\) and in this case
LS(M) = S(M) \( \cong L^0(\Omega, \Sigma, \mu) \), where \( L^0(\Omega, \Sigma, \mu) \) the algebra of all (equivalence
classes of) complex measurable functions on \((\Omega, \Sigma, \mu)\).

Further we consider the algebra \( S(Z(M)) \) of operators which are measurable
with respect to the center \( Z(M) \) of the von Neumann algebra M. Since \( Z(M) \) is
an abelian von Neumann algebra it is *-isomorphic to \( L^\infty(\Omega, \Sigma, \mu) \) for an appro-
priate measure space \((\Omega, \Sigma, \mu)\). Therefore the algebra \( S(Z(M)) \) coincides with
\( Z(LS(M)) \) and can be identified with the algebra \( L^0(\Omega, \Sigma, \mu) \).

Let M be a von Neumann algebra. Given an element \( x \in LS(M) \) the smallest
projection \( p \) in M with \( xp = x \) is called the right support of \( x \) and denoted by
\( r(x) \). The left support \( l(x) \) is smallest projection \( p \) in M with \( px = x \). For a
*-subalgebra \( A \subset LS(M) \) denote
\[
\mathcal{F}(A) = \{ x \in A : l(x) \in P_{fin}(M) \}.
\]

From the definition of the algebra \( \mathcal{F}(A) \) we have that the following properties
are equivalent:

1. \( x \in \mathcal{F}(A) \);
2. \( \exists p \in P_{fin}(M) \) such that \( px = x \);
3. \( \exists p \in P_{fin}(M) \) such that \( xp = x \);
4. \( \exists p \in P_{fin}(M) \) such that \( pxp = x \).

Note that \( \mathcal{F}(A) \) is an *-ideal in \( A \). Moreover the algebra \( \mathcal{F}(A) \) is semi-prime,
i.e. if \( a \in \mathcal{F}(A) \) and \( a\mathcal{F}(A)a = \{0\} \) then \( a = 0 \). Indeed, let \( a \in \mathcal{F}(A) \) and
\( a\mathcal{F}(A)a = \{0\} \), i.e. \( axa = 0 \) for all \( x \in \mathcal{F}(A) \). In particular for \( x = a^* \) we have
\( aa^*a = 0 \) and hence \( a^*aa^*a = 0 \), i.e. \( |a|^4 = 0 \). Therefore \( a = 0 \).
Recall the definition of the faithful normal semifinite extended center valued trace on the algebra $M$ (see [19]).

Let $M$ be an arbitrary von Neumann algebra with the center $Z(M) \equiv L^\infty({\Omega, \Sigma, \mu})$. By $L_+$ we denote the set of all measurable functions $f : ({\Omega, \Sigma, \mu}) \to [0, \infty]$ (modulo functions equal to zero $\mu$-almost everywhere). Then there exists a map $\Phi : M_+ \to L_+$ with the following properties:

\begin{enumerate}
  \item $\Phi(x + y) = \Phi(x) + \Phi(y)$ for $x, y \in M_+$;
  \item $\Phi(ax) = a\Phi(x)$ for $a \in Z(M)_+, x \in M_+$;
  \item $\Phi(xx^*) = \Phi(x^*x)$;
  \item $\Phi(x^*x) = 0 \Rightarrow x = 0$;
  \item $\Phi\left(\sup_{i \in J} x_i\right) = \sup_{i \in J} \Phi(x_i)$ for any bounded increasing net $\{x_i\}$ in $M_+$.
\end{enumerate}

This map $\Phi : M_+ \to L_+$, is a called the extended center valued trace on $M$.

The set $\{x \in M : \Phi(x^*x) \in Z(M)\}$ is an ideal $M$. If this ideal is $\sigma$-weakly dense in $M$, then $\Phi$ is said to be semifinite.

It is well-known (see e.g. [19]) that a von Neumann algebra $M$ is semifinite if and only if $M$ admits a faithful, semifinite, normal extended center valued trace.

Let us remark that a projection $p \in M$ is finite if and only if $\Phi(p) \in S(Z(M))$.

Hence for any $x \in {\mathcal F}(LS(M)) \cap M_+$ we have that $\Phi(x) \in S(Z(M))$.

Note that the algebra $LS(M)$ has the following remarkable property: given any family $\{z_i\}_{i \in I}$ of mutually orthogonal central projections in $M$ with $\bigvee_{i \in I} z_i = 1$ and a family of elements $\{x_i\}_{i \in I}$ in $LS(M)$ there exists a unique element $x \in LS(M)$ such that $z_i x = z_i x_i$ for all $i \in I$. This element is denoted by $x = \sum_{i \in I} z_i x_i$ (see [15]). Conversely if $M$ is a type I von Neumann algebra then for an arbitrary element $x \in LS(M)$ there exists a sequence $\{z_n\}$ of mutually orthogonal central projections with $\bigvee_{n \in \mathbb{N}} z_n = 1$ such that $z_n x \in M$ for all $n \in \mathbb{N}$ (see [1]). For $0 \leq x \in {\mathcal F}(LS(M))$ set

$$\Phi(x) = \sum_{n \in \mathbb{N}} z_n \Phi(z_n x).$$

(2.1)

Since the trace $\Phi$ is $Z(M)$-homogeneous, the equality (2.1) gives a well-defined map from $F(LS(M))_+$ into $S(Z(M))$.

Since each element of $F(LS(M))$ is a finite linear combinations of positive elements from $F(LS(M))$ we can naturally extend $\Phi$ to a $S(Z(M))$-valued trace on $F(LS(M))$.

Now let $\mu$ be an arbitrary faithful normal semifinite trace on $Z(M)$. Put $\tau = \mu \circ \Phi$. Then by [19, Lemma 2.16] we have that

$$\tau(xy) = \tau(yx)$$

for all $x \in M, y \in F(LS(M)) \cap M$. Therefore

$$\Phi(xy) = \Phi(yx)$$
for all \(x \in LS(M),\ y \in \mathcal{F}(LS(M)).\) Since the trace \(\Phi\) maps the set \(\mathcal{F}(LS(M))\) into \(S(Z(M))\) and \(\mathcal{F}(LS(M))\) is an ideal in \(LS(M)\) we have
\[
\Phi(axy) = \Phi((ax)y) = \Phi((ya)x) = \Phi(xya),
\]
i.e.
\[
\Phi(axy) = \Phi(xya)
\]
for all \(a, x \in LS(M),\ y \in \mathcal{F}(LS(M)).\)

3. Main results

Let \(D\) be a derivation on \(LS(M)\). Then \(D\) maps the ideal \(\mathcal{F}(LS(M))\) into itself. Indeed, for any \(x \in \mathcal{F}(LS(M))\) there exists a finite projection \(p \in M\) such that \(x = xp\). Then
\[
D(x) = D(xp) = D(x)p + xD(p),
\]
and therefore \(D(x) \in \mathcal{F}(LS(M))\). Hence any 2-local derivation on \(LS(M)\) also maps \(\mathcal{F}(LS(M))\) into itself.

Lemma 3.1. Let \(b \in LS(M)\) be an arbitrary element. If \(\Phi(xb) = 0\) for all \(x \in \mathcal{F}(LS(M))\) then \(b = 0\).

Proof. Let \(b \in LS(M)\). For any finite projection \(e \in LS(M)\) we have \(eb^* \in \mathcal{F}(LS(M))\) and therefore by the assumption of the lemma it follows that \(\Phi(eb^*b) = 0\). Thus
\[
0 = \Phi(eb^*b) = \Phi(e^2b^*b) = \Phi(eb^*be) = \Phi((be)^*(be)),
\]
i.e.
\[
\Phi((be)^*(be)) = 0.
\]
Since the trace \(\Phi\) is faithful, we obtain \((be)^*(be) = 0\), i.e. \(be = 0\).

Now take a family of finite projections \(\{e_\alpha\}_{\alpha \in I}\) in \(M\) such that \(e_\alpha \uparrow 1\). Then
\[
0 = be_\alpha b^* \uparrow bb^*,
\]
i.e. \(bb^* = 0\). Thus \(b = 0\). The proof is complete. \(\square\)

Lemma 3.2. Let \(M\) be an arbitrary von Neumann algebra of type \(I_\infty\) and let \(\Delta : LS(M) \to LS(M)\) be a 2-local derivation. Then

(1) \(\Delta\) is \(S(Z(M))\)-homogenous, i.e. \(\Delta(cx) = c\Delta(x)\) for all \(c \in S(Z(M)),\ x \in LS(M)\);

(2) \(\Delta(x^2) = \Delta(x)x + x\Delta(x)\) for all \(x \in LS(M)\).

Proof. (1) For each \(x \in LS(M)\), and for \(c \in S(Z(M))\) there exists a derivation \(D_{x,cx}\) such that \(\Delta(x) = D_{x,cx}(x)\) and \(\Delta(cx) = D_{x,cx}(cx)\). Since \(M\) is a type \(I_\infty\) then by [1, Theorem 2.7] every derivation on \(LS(M)\) is inner, in particular, \(S(Z(M))\)-linear. Therefore
\[
\Delta(cx) = D_{x,cx}(cx) = cD_{x,cx}(x) = c\Delta(x).
\]
Hence, \(\Delta\) is \(S(Z(M))\)-homogenous.

(2) For each \(x \in LS(M)\), there exists a derivation \(D_{x,x^2}\) such that \(\Delta(x) = D_{x,x^2}(x)\) and \(\Delta(x^2) = D_{x,x^2}(x^2)\). Then
\[
\Delta(x^2) = D_{x,x^2}(x^2) = D_{x,x^2}(x)x + xD_{x,x^2}(x) = \Delta(x)x + x\Delta(x)
\]

for all $x \in LS(M)$. The proof is complete. \hfill $\Box$

**Lemma 3.3.** Let $M$ be an arbitrary von Neumann algebra of type $I_\infty$. If $\Delta : LS(M) \to LS(M)$ is a 2-local derivation such that $\Delta|_{\mathcal{F}(LS(M))} \equiv 0$, then $\Delta \equiv 0$.

**Proof.** Let $\Delta : LS(M) \to LS(M)$ be a 2-local derivation such that $\Delta|_{\mathcal{F}(LS(M))} \equiv 0$. For arbitrary $x \in LS(M)$ and $y \in \mathcal{F}(LS(M))$ there exists a derivation $D_{x,y}$ on $LS(M)$ such that $\Delta(x) = D_{x,y}(x)$ and $\Delta(y) = D_{x,y}(y)$. By [1, Theorem 2.7] there exists an element $a \in LS(M)$ such that

$$[a, xy] = D_{x,y}(xy) = D_{x,y}(x)y +xD_{x,y}(y) = \Delta(x)y + x\Delta(y),$$

i.e.

$$[a, xy] = \Delta(x)y + x\Delta(y).$$

Since $y \in \mathcal{F}(LS(M))$ we have $\Delta(y) = 0$, and therefore $[a, xy] = \Delta(x)y$. By the equality (2.2) we obtain that

$$0 = \Phi(axy - yxa) = \Phi ([a, xy]) = \Phi (\Delta(x)y),$$

i.e. $\Phi(\Delta(x)y) = 0$ for all $y \in \mathcal{F}(LS(M))$. By Lemma 3.1 we have that $\Delta(x) = 0$. The proof is complete. \hfill $\Box$

**Lemma 3.4.** Let $M$ be an arbitrary von Neumann algebra of type $I_\infty$ and let $\Delta : LS(M) \to LS(M)$ be a 2-local derivation. Then the restriction $\Delta|_{\mathcal{F}(LS(M))}$ of the operator $\Delta$ on $\mathcal{F}(LS(M))$ is additive.

**Proof.** Let $\Delta : LS(M) \to LS(M)$ be a 2-local derivation. For each $x, y \in \mathcal{F}(LS(M))$ there exists a derivation $D_{x,y}$ on $LS(M)$ such that $\Delta(x) = D_{x,y}(x)$ and $\Delta(y) = D_{x,y}(y)$. By [1, Theorem 2.7] there exists an element $a \in LS(M)$ such that

$$[a, xy] = D_{x,y}(xy) = D_{x,y}(x)y +xD_{x,y}(y) = \Delta(x)y + x\Delta(y),$$

i.e.

$$[a, xy] = \Delta(x)y + x\Delta(y).$$

Similarly as in Lemma 3.3 we have

$$0 = \Phi(axy - yxa) = \Phi ([a, xy]) = \Phi (\Delta(x)y + x\Delta(y)), $$

i.e. $\Phi(\Delta(x)y) = -\Phi(x\Delta(y))$. For arbitrary $u, v, w \in \mathcal{F}(LS(M))$, set $x = u + v$, $y = w$. Then from above we obtain

$$\Phi(\Delta(u + v)w) = -\Phi((u + v)\Delta(w)) =$$

$$= -\Phi(u\Delta(w)) - \Phi(v\Delta(w)) = \Phi(\Delta(u)w) + \Phi(\Delta(v)w) = \Phi((\Delta(u) + \Delta(v))w),$$

and so

$$\Phi((\Delta(u + v) - \Delta(u) - \Delta(v))w) = 0$$

for all $u, v, w \in \mathcal{F}(LS(M))$. Denote $b = \Delta(u + v) - \Delta(u) - \Delta(v)$ and put $w = b^*$. Then $\Phi(b^*) = 0$. Since the trace $\Phi$ is faithful it follows that $bb^* = 0$, i.e. $b = 0$. Therefore

$$\Delta(u + v) = \Delta(u) + \Delta(v),$$

i.e. $\Delta$ is an additive map on $\mathcal{F}(LS(M))$. The proof is complete. \hfill $\Box$

The following theorem is the main result of this paper.
**Theorem 3.5.** Let $M$ be an arbitrary von Neumann algebra of type $I_\infty$ and let $\mathcal{A}$ be a $*$-subalgebra of $LS(M)$ such that $M \subseteq \mathcal{A}$. Then every 2-local derivation $\Delta : \mathcal{A} \rightarrow \mathcal{A}$ is a derivation.

**Proof.** First we consider the case $\mathcal{A} = LS(M)$. By Lemma 3.4 the restriction $\Delta|_{\mathcal{F}(LS(M))}$ of the operator $\Delta$ on $\mathcal{F}(LS(M))$ is additive. Further by Lemma 3.2 $\Delta$ is a homogeneous. Therefore, the map $\Delta|_{\mathcal{F}(LS(M))}$ is a linear Jordan derivation on $\mathcal{F}(LS(M))$ in the sense of [6]. In [6, Theorem 1] it is proved that any Jordan derivation on a semi-prime algebra is a derivation. Since $\mathcal{F}(LS(M))$ is semiprime, therefore the linear operator $\Delta|_{\mathcal{F}(LS(M))}$ is a derivation on $\mathcal{F}(LS(M))$.

Since by Lemma 3.2 $\Delta$ is $S(Z(M))$-homogeneous then by [4, Corollary 3] the derivation $\Delta|_{\mathcal{F}(LS(M))} : \mathcal{F}(LS(M)) \rightarrow \mathcal{F}(LS(M))$ is spatial, i.e.

$$\Delta(x) = ax - xa, x \in \mathcal{F}(LS(M))$$

(3.1)

for an appropriate $a \in LS(M)$.

Let us show that $\Delta(x) = ax - xa$ for all $x \in LS(M)$. Consider the 2-local derivation $\Delta_0 = \Delta - D_0$. Then from the equality (3.1) we obtain that $\Delta_0|_{\mathcal{F}(LS(M))} \equiv 0$. Now by Lemma 3.3 it follows that $\Delta_0 \equiv 0$. This means that $\Delta = D_a$.

Now let $\mathcal{A}$ be an arbitrary $*$-subalgebra of $LS(M)$ such that $M \subseteq \mathcal{A}$. Since $M$ is a type I von Neumann algebra for any element $x \in LS(M)$ there exists a sequence $\{z_n\}$ of mutually orthogonal central projections with $\bigvee_{n \in \mathbb{N}} z_n = 1$ such that $z_n x \in M$ for all $n \in \mathbb{N}$. Set

$$\tilde{\Delta}(x) = \sum_{n \in \mathbb{N}} z_n \Delta(z_n x).$$

(3.2)

Since the map $\Delta$ is $Z(M)$-homogeneous, the equality (3.2) gives a well-defined 2-local derivation on $LS(M)$. From above we have that $\tilde{\Delta}$ is a derivation. Therefore $\Delta$ is a derivation. The proof is complete. \(\square\)

**Corollary 3.6.** Let $M$ be an arbitrary von Neumann algebra of type $I_\infty$. Then every 2-local derivation $\Delta : LS(M) \rightarrow LS(M)$ is a derivation.

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Institute of Mathematics National University of Uzbekistan, 100125 Tashkent, Uzbekistan and the Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy

E-mail address: sh_ayupov@mail.ru

Department of Mathematics, Karakalpak state university, Ch. Abdirov 1, 230113, Nukus, Uzbekistan

E-mail address: karim2006@mail.ru

Institute of Mathematics National University of Uzbekistan, 100125 Tashkent, Uzbekistan

E-mail address: amir_t85@mail.ru