Quantum criticality in two dimensions and Marginal Fermi Liquid

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Kinetic properties of a two dimensional model of fermions interacting with antiferromagnetic spin excitations near the quantum critical point (QCP) are considered. The temperature or doping are assumed to be sufficiently high, such that the pseudogap does not appear. In contrast to standard spin-fermion models, it is assumed that there are intrinsic inhomogeneities in the system suppressing space correlations of the antiferromagnetic excitations. It is argued that the inhomogeneities in the spin excitations in the “strange metal” phase can be a consequence of existence of “π-shifted” domain walls in the doped antiferromagnetic phase. Averaging over the inhomogeneities and calculating physical quantities like resistivity and some others one can explain unusual properties of cuprates unified under the name “Marginal Fermi Liquid” (MFL). The dependence of the slope of the linear temperature dependence of the resistivity on doping is compared with experimental data.

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Properties of the normal state of high $T_c$ superconducting cuprates in the vicinity of the quantum critical point (QCP) are not consistent with the Landau Fermi liquid theory. Such unusual effects as the linear dependence of the resistivity on temperature, the linear tunnelling conductivity as a function of voltage, almost frequency and temperature independent backgrounds in the Raman-scattering intensity, constant thermal conductivity, and a very large nuclear relaxation time are similar in all CuO based high-$T_c$ compounds. This region is usually referred to as “strange metal”.

In the pioneering work Varma et al [1, 2] have proposed a “marginal Fermi liquid” (MFL) phenomenology that allowed them to describe the unusual experimental findings surprisingly well. The theory is based on the assumptions that 1) electrons are scattered by unknown bosonic excitations characterized by a retarded propagator $\chi^R(q,\omega,T)$, where $q$ is momentum, $\omega$ is frequency and $T$ is temperature, 2) the imaginary part of this propagator has the form

$$\text{Im}\chi^R(q,\omega,T) = \begin{cases} \nu(\omega/T), & \omega \ll T \\ \nu(\text{sgn}\omega), & T \ll \omega \ll \omega_c \end{cases}, \quad (1)$$

where $\nu$ is the density of states per unit volume and per spin direction, and $\omega_c$ is a high energy cutoff.

Later, Abrahams and Varma [3] have demonstrated that the marginal Fermi liquid (MFL) assumption described results of angle-resolved photoemission (ARPES) [4, 5] very well, too (see, also [6]).

In spite of the evident success in describing the experiments [7–11], the final agreement on the origin of the bosonic mode specified by Eq. (1) seems to be lacking so far. The strange metal behavior is attributed to quite different phenomena like, e.g., existence of spontaneous orbital currents [12], quantum criticality near antiferromagnetic transition [13] and many others.

A recent discovery of the charge modulation in cuprates [14–21] signals a competition between the superconductivity and a charge density wave (CDW) in the pseudogap region of the phase diagram of cuprates. Many important experimental findings of these works can be explained in the framework of the so-called spin-fermion (SF) model introduced earlier for description of electron-electron interaction in the vicinity of QCP. In particular, it has been proposed in Ref. [22] that the pseudogap (PG) state arises as a consequence of the competition between the superconducting and a charge modulated state.

Experimentally, increasing the temperature and doping one passes from the pseudogap state to a strange metal state described by the MFL phenomenology. Assuming that the pseudogap state can be understood in terms of the SF model it is natural to use this model also for description of the “neighboring” strange metal state. However, the correlation function of antiferromagnetic spin fluctuations used in the SF model is definitely different from the one given by Eq. (1), and new ideas are necessary to overcome this inconsistency.

In this paper we show that the MFL with the bosonic mode, Eq. (1), can nevertheless be derived from the SF model for the antiferromagnet-normal metal quantum phase transition in 2D. However, in order to achieve this goal one should introduce into the model a disorder reducing the antiferromagnetic correlations at large distances. It is argued that such a disorder is intrinsically present due to doping and, being sufficiently smooth, does not contribute to the residual resistivity.

Following this idea we assume that the CuO planes consist of domains $f$, such that the antiferromagnetic (AF) field $\phi_f$ varies almost periodically with the modulation vector $Q = (\pi/b,\pi/b)$ inside the domains but sharply changes the sign when crossing the boarder between them. In other words, the fluctuating field $\phi_f$ is...
shifted on the boarder by the lattice period \( b \) (the phase of the oscillations is shifted by \( \pi \)) and we write it as
\[
\tilde{\phi}_t(r) = I_t(r) \tilde{\phi}(r), \quad I_t(r) = \begin{cases} 1, & f \in \text{"pink"} \\ -1, & f \in \text{"white"} \end{cases}
\]
In Eq. (2) the field \( \tilde{\phi} \) is almost periodic everywhere in space, and “pink” and “white” domains are represented in Fig. 1.

FIG. 1. (Color online.) Domains separated by \( \pi \)-shifted domain walls.

The size and the form of the domains is not critical at the transition between the antiferromagnet and paramagnet, and Eq. (2) is assumed to be applicable on both sides of it. We write the Lagrangian \( L \) of the model as
\[
L = L_0 + L_\psi + L_\phi + L_\theta
\]
In Eq. (3), \( L_0 \) stands for the Lagrangian of non-interacting fermions (holes)
\[
L_0 = \int d^3 r \langle \tilde{\psi}(\tau, \mathbf{r}) [\partial_\tau + \varepsilon (-i \nabla_r) - \mu] \psi(\tau, \mathbf{r}) \rangle \, d\mathbf{r},
\]
while
\[
L_{\psi\phi} = \lambda \sum_{\sigma} \int d^3 r \tilde{\sigma} \tilde{\phi}_t(\tau, \mathbf{r}) \psi(\tau, \mathbf{r}) \, d\mathbf{r}
\]
describes interaction of the fermions with the effective exchange field \( \tilde{\phi}_t(\tau, \mathbf{r}) \) of the antiferromagnet. In Eqs. (4-6), \( \psi \) is the anticommuting fermionic field, \( \tilde{\sigma} \) is the vector of Pauli matrices, and \( \tau \) is the imaginary time. The second term in Eq. (4) stands for the electron energy operator, and \( \mu \) is the chemical potential.

The Lagrangian of \( L_\phi \) for the exchange field \( \phi \) is written near QCP as
\[
L_\phi = \frac{1}{2} \int [\tilde{\phi}(\tau, \mathbf{r}) \left( \tilde{D}_0^{-1} + \frac{g \tilde{\phi}_t^2(\tau, \mathbf{r})}{2} \right) \tilde{\phi}(\tau, \mathbf{r})] \, d\mathbf{r},
\]
where the Fourier transform of \( \tilde{D}_0 \) has the form
\[
\tilde{D}_0(\omega_n, \mathbf{q}) = \left( v_s^2 \omega_n^2 + (Q - \mathbf{q})^2 + a \right)^{-1},
\]
and \( \omega_n \) is the bosonic Matsubara frequency.

In Eq. (7), \( v_s \) is the velocity of the spin waves, \( a \) characterizes the distance from QCP \((a > 0 \text{ on the metallic side and } a < 0 \text{ in the AF region})\).

Actually, domain walls (DW) separating domains with opposite directions of the staggered magnetization have been found in 2D using a Hartree-Fock approximation for a CuO lattice and for the \( t-J \) model, as well as using a mean field approximation for the Hubbard model. Similar DW (stripes) have been obtained later within the \( t-J \) model numerically using the Density Matrix Renormalization Group (DMRG) method.

The DW derived in these works separate regions with opposite direction of the AF ordering (\( \pi \)-shifted DW). They contain chains of holes in the middle of DW, while the magnetization vanishes there. According to this picture, the doped holes are not distributed homogeneously in the AF but are located inside the DW implying that the doped AF is intrinsically inhomogeneous. The typical distance between the DW is proportional to \( p^{-1} \), where \( p \) is the number of doped holes per Cu atom.

A stripe correlation of spins and holes is evident in cuprates from neutron diffraction. As the DW contain holes, their shape and locations are affected also by an inhomogeneous electrostatic field of doping ions located outside the CuO planes. This interaction should make the shape and size of the domains rather irregular and we assume that Fig. 1 together with Eqs. (2-6) can properly describe the antiferromagnet doped with holes.

On the metallic side, \( a > 0 \), field \( \phi \) can be finite only as a result of fluctuations. Although the AF order parameter \( I_t \phi_0 \) vanishes at QCP, the distance between DW determined by the hole density remains finite at \( a = 0 \).

In the limit of a weak doping \( p \sim 0.1 - 0.2 \), the typical size of the domains \( Q_D^{-1} \sim (Qp)^{-1} \) is considerably larger than the atomic length \( Q^{-1} \), while the length \( l_T \) can be even larger than \( Q_D^{-1} \) for relevant temperatures.

Neglecting the quartic term in \( L_\phi \), Eq. (4), we integrate out the field \( \phi \) and come with help of Eq. (2) to action \( S_{\text{eff}} [\psi] \)
\[
S_{\text{eff}} [\psi] = \int_0^\beta L_0 [\psi] \, d\tau + S_{\text{int}} [\psi],
\]
where
\[
S_{\text{int}} [\psi] = -\frac{\lambda^2}{2} \sum_{t,\tau, f=\pm, k=x,y,z} \sum_{\mathbf{r}} \int d\mathbf{r}' d\tau' \psi^*(\tau, \mathbf{r}) \sigma^k \psi(\tau, \mathbf{r}) \times I_t(\mathbf{r}) I_{\tau'}(\mathbf{r}') D_0(\tau - \tau', \mathbf{r} - \mathbf{r}') \psi^*(\tau', \mathbf{r}') \sigma^k \psi(\tau', \mathbf{r}').
\]
As the DW can randomly be distorted by the potential of the \( O \) atoms located outside the CuO planes, averaging over random \( I_t(\mathbf{r}) \) looks a reasonable method of calculation. The propagator \( D_0 \) varies on distances of order \( l_T \), and in the limit \( l_T Q_D \gg 1 \), one can simply replace the product \( I_t I_{\tau'} \) in Eq. (9) by its average. Writing correlations as
\[
\langle I(\mathbf{r}) \rangle = 0, \quad \langle I(\mathbf{r}) I(\mathbf{r}') \rangle = U(Q_D |\mathbf{r} - \mathbf{r}'|),
\]
where the function \( U(x) \) decays sufficiently fast at \( x \to \infty \) and \( U(0) = 1 \), and averaging in Eq. (10) we immediately come to an effective fermion-fermion interaction \( \lambda^2 D_0(\tau - \tau', r - r') \) with the propagator

\[
\tilde{D}_0(\tau - \tau', r - r') = U(Q_D |r - r'|) D_0(\tau - \tau', r - r').
\]

(11)

Eq. (11) shows that the presence of the \( \pi \)-shifted DW destroys the spin correlations at distances exceeding the typical domain size \( Q_D^{-1} \).

In the homogeneous case, the bare propagator \( D_0 \) is modified due to the Landau damping \([35]\). This effect can be obtained in the random phase approximation (RPA). The polarization function \( \Pi(\omega_n, q) \) does not depend on \( q \) and is short ranged in the real space. The function \( \Pi(\omega_n, r - r') \) is essentially non-zero only when both \( r \) and \( r' \) are located in the same domain. Then, as in the homogeneous case, one comes to the following relation

\[
D^{-1}(\omega_n, q) = D^{-1}_0(\omega_n, q) - \Pi(\omega_n, q),
\]

(12)

where

\[
\Pi(\omega_n, q) = C + \gamma |\omega_n|, \quad \gamma = \frac{4\lambda^2}{\pi v^2 \sin \delta}
\]

(13)

and \( C \) is a constant renormalizing the position of the QCP. In Eq. (13), \( v \) is the Fermi velocity at the hotspots and \( \delta \) is the angle between the velocities of the neighboring hot spots (see, e.g., Refs. [27, 28], and SI of Ref. [22]). As usual [22, 27, 28], we neglect the \( \omega^2 \)-term in the propagator \( D \), Eqs. (7, 12).

Formally, the parameter \( a \) entering the propagator \( D(\omega_n, q) \), Eqs. (7, 12), should vanish at the transition point. However, the transition is smeared in 2D at any finite temperature by thermal fluctuations. One can estimate the characteristic width of the transition considering corrections to the coupling constant \( g \) within the perturbation theory and keeping only the most divergent static contributions (SI of [22]). This gives in the first order

\[
g \to g - T g^2 \int \frac{d^2k}{(k^2 + a)^2},
\]

(14)

which leads in the limit \( a \to 0 \) to a divergency. Since the transition is smeared, we conclude that \( a \) cannot be effectively smaller than some minimal value \( a_0(T) \) at which the correction in Eq. (13) is of the same order as the bare coupling \( g \). This gives an estimate for \( a_0(T) \)

\[
a_0(T) = cgT,
\]

(15)

where \( c \) is a numerical coefficient.

Then, one should replace parameter \( a \) in Eq. (11) by

\[
a(T) = a_0(T) + \tilde{a},
\]

(16)

where \( \tilde{a} \) characterizes the distance from the critical line, to obtain

\[
D(\omega_n, q) = \left( |\omega_n| + (Q - q)^2 + a(T) \right)^{-1}
\]

(17)

Replacing the function \( D_0 \) in Eq. (11) by \( D \) from Eq. (17) one obtains an effective propagator \( \tilde{D} \) instead of \( D_0 \)

\[
\tilde{D}(\omega_n, q) = Q_D^{-2} \int \tilde{U} \frac{Q_D}{|Q_D|} D(\omega_n, k) \frac{d^2k}{(2\pi)^2},
\]

(18)

where \( \tilde{U} \) is the Fourier transform of \( U \).

The integration over \( k \) makes the propagator \( \tilde{D} \) weakly dependent on \( q \) for \( |Q - q| \lesssim Q_D \). Performing analytical continuation from positive Matsubara frequencies to the real axis, \( \omega_n \to \omega + i0 \), and calculating the integral over two-dimensional momenta \( k \) one comes in the limit \( |\omega| \lesssim Q_D^2 \) to the following expression for the imaginary part of the retarded propagator \( \tilde{D}^R \)

\[
\text{Im} \tilde{D}^R(\omega, q, T) = \frac{1}{4Q_D^2} \tilde{U} \frac{|q - Q|}{Q_D} \arctan \frac{\gamma |\omega|}{a(T)}
\]

(19)

which is in accord with the hypothesis of MFL, Eq. (1), for temperatures exceeding the distance from the critical line, when \( a_0(T) \gtrsim \tilde{a} \). Provided this inequality is fulfilled, and \( g \) and \( \gamma \) are of the same order (as they should) one obtains the asymptotics of Eq. (1) in the limits of high \( \omega \gtrsim T \) and low \( \omega \lesssim T \) frequencies. The temperature \( T \) should also be higher than the coupling energy between the layers, which guarantees that the spin fluctuations are effectively two-dimensional.

The function \( \text{Im} \tilde{D}^R(\omega, q) \), Eq. (19), is peaked at \( q = Q \) and thus differs from \( \text{Im} \chi^R(q, \omega) \), Eq. (1). At the same time, the momentum dependence of \( \text{Im} \tilde{D}^R(\omega, q) \), Eq. (19), is rather weak for a small size of the domains (sharp decay with the distance of the function \( U \)) and the difference between the functions \( \text{Im} \tilde{D}^R(\omega, q) \) and \( \text{Im} \chi_R(q, \omega) \) is not very important. One can see from Eq. (19) that the Lorentzian dependence of the propagator \( D \) on the momentum \( Q - k \) is smeared due to the random shapes of the domains. As a result, the paramagnon peaks in the neutron scattering should be broad, which agrees with experimental observations [34, 38].

The structure of the DW containing not only magnetic moments but also holes should result in a coupling of the mode \( \tilde{D}^R \) not only to spin but also to charge excitations.

As an example, we present here calculation of the resistivity using the simplest Born approximation. Within this approach one should calculate the imaginary part of the self-energy \( \Sigma^R \) of the one particle retarded Green function. The standard representation of this quantity has the form

\[
\text{Im} \Sigma^R(\varepsilon, p) = -\frac{\lambda^2}{(2\pi)^3} \int dp_1 \int_{-\infty}^{\infty} d\omega \text{Im} G^R(\varepsilon - \omega, p_1) \\
\times \text{Im} \tilde{D}^R(\omega, p - p_1) \left( \frac{\sinh \frac{\varepsilon - \omega}{2T} + \sinh \frac{\omega}{2T}}{\frac{\varepsilon - \omega}{2T}} \right),
\]

(20)
where

\[ G^R (\varepsilon, p) = (\varepsilon - \varepsilon (p) + \mu + i/ (2\tau_{el}))^{-1} \]  

(21)

and \( \tau_{el} \) is the elastic scattering time. The elastic mean free path \( l_{el} = v \tau_{el} \) is assumed to exceed the domain size \( Q_D^{-1} \). Integrating over \( p \), we use the fact that, in this limit, \( \text{Im} G^R (\varepsilon - \omega, p) \) is a sharp function of the component \( p \) perpendicular to the Fermi surface, while the function \( \text{Im} D^R (\omega, p - p_i) \) is a smooth function of the latter. Then, we obtain

\[ \text{Im} \Sigma^R (\varepsilon, p) = \frac{\lambda^2 A (p) T}{(4\pi)^2} f \left( \frac{\varepsilon}{2T} \right), \]  

(22)

where

\[ A (p) = Q_D^{-2} \int_{FS} \tilde{U} \left\langle |p - Q - \tilde{p}_1|/Q_D \right\rangle \frac{d\tilde{p}_1}{v (\tilde{p}_1)}. \]  

(23)

\( v (\tilde{p}_1) \) is the velocity at the point \( \tilde{p}_1 \), the integration is performed over the Fermi surface, and

\[ f (u) = \int_{-\infty}^{\infty} (\tanh (u - x) + \coth x) \arctan (bx) dx. \]  

(24)

The coefficient \( b = 2\gamma/ (cg) \) in Eq. (24) is of order 1. The function \( \text{Im} \Sigma^R (\varepsilon, p) \) is a smooth function of the position on the Fermi surface and does not depend on the elastic scattering time \( \tau_{el} \). Although Eqs. 19-22  have been derived assuming that the antiferromagnetic fluctuations are two-dimensional, the results are not as sensitive to the form of the Fermi surface that may effectively have both two-dimensional and anisotropic three-dimensional shapes.

Remarkably, the function \( \text{Im} \Sigma^R (\varepsilon, p) \), Eq. (22), factorizes into the energy- and momentum-dependent parts. Therefore, its temperature and energy dependence is the same for all parts of the Fermi surface. One can write

\[ \text{Im} \Sigma^R (\varepsilon, p) \sim -\lambda^2 \max (|\varepsilon|, T) \]  

(25)

in agreement with the findings of Refs. 1, 2.

The electron spectral function has been compared in Ref. 3 with the results of the ARPES measurements of Refs. 4, 5 and a good agreement has been found. Using Eq. (19) one can describe also the other experiments discussed in Refs. 1, 3.

Using Eq. (22) one obtains at \( \varepsilon = 0 \) linear in \( T \) dependence of \( \Sigma^R \) and, hence, of the resistivity \( \rho \). The latter can be written in the form

\[ \rho = \rho_0 + \alpha T \]  

(26)

where

\[ \alpha = \frac{\lambda^2 f (0) A}{32\pi^4 e^2 E_x}, \quad E_x = \int_{FS} \frac{v^2 (\tilde{p})}{|v (\tilde{p})|} \frac{d\tilde{p}}{(2\pi)^d} \]  

(27)

and \( \rho_0 \) is the residual resistivity (dependence of \( A \) on \( p \) is neglected and \( x \) is direction of a bond of CuO planes).

The slope of the \( T \)-dependence does not depend on \( \tau_{el} \) but the residual resistivity \( \rho_0 \) determined by \( \tau_{el} \) does. This agrees with observations of Ref. 10. At the same time, a clear decrease of the slope with the doping has been observed in experiments 3, 4. A more detailed microscopic theory is necessary in order to describe precisely the dependence of \( \alpha \) on the doping \( p \) but a rough estimation can already be done using Eqs. 24-27.

![FIG. 2. (Color online.) Dependence of \( y = \alpha p \) (blue dots) and \( y = 10\alpha p^{2.5} \) (red boxes) on doping \( p \) extracted from Fig.1a of Ref. 4.](image)

We use for comparison between theory and experiment Fig.1a of Ref. 4 displaying the linear temperature dependence of resistivity Bi2Sr2−xLaxCuO6+y for doping \( p = 0.11 - 0.18 \). The slope \( \alpha \) is extracted from the difference \( \Delta \rho = \rho (300K) - \rho (100K) \).

Using the original formulation of MFL, Eq. (3), of Refs. 5, 6 and the fact that the density of states \( \nu \) is independent in 2D on doping \( p \) one comes to the relation \( \alpha \propto v_F^{-2} \propto p^{-1} \), where \( v_F \) is a typical velocity on the Fermi surface. The dependence of \( y = \alpha p \) on \( p \) taken from Fig.1a of Ref. 4 is represented by dots in Fig. 2. Its essential dependence on \( p \) indicates that Eq. 4 should be modified. At the same time, it follows from Eq. 24 that \( A \propto (v_F Q_D)^{-1} \) and \( V_x \propto mv_F^2 \), which leads to \( \alpha \propto v_F^{-3} Q_D^{-1} \propto p^{-5/2} \). The variation of \( y = 10\alpha p^{5/2} \) with \( p \) is represented by boxes in Fig. 2. A weak dependence of \( y = \alpha p^{5/2} \) on the doping \( p \) supports the present theory.

In conclusion, fermions interacting with critical antiferromagnetic fluctuations in two dimensions are considered. Assuming that the CuO planes consist of different domains, such that the coupling constant \( \lambda \) changes the sign when crossing the boarders between them, we have derived the hypothetical mode of the Marginal Fermi Liq-
uid and clarified its dependence on the doping. The slope of the linear temperature dependence of the resistivity calculated here is compared with experimental results and an encouraging agreement is found.

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