Signatures of non-Abelian statistics in non-linear coulomb blockaded transport

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Signatures of the non-Abelian statistics of quasi-particles in the $\nu = 5/2$ quantum Hall state are predicted to be present in the current-voltage characteristics of tunneling through one or two quantum Hall puddles of Landau filling $\nu_a$ embedded in a bulk of filling $\nu_b$ with $(\nu_a, \nu_b) = (2, 5/2)$ and $(\nu_a, \nu_b) = (5/2, 2)$.

Anyons obeying Non-Abelian statistics were predicted to exist in condensed matter systems almost twenty years ago. Although many efforts have been made to suggest and perform experiments to test that hypothesis, there is still no clear-cut proof for the existence of such particles. This work proposes an experimental setup along with a set of measurements designed to unravel signatures of such statistics in the putative non-Abelian quantum Hall state at $\nu = 5/2$. In particular, we show that the current-voltage characteristics of a Coulomb blockaded system in the non-linear regime may detect the neutral Majorana edge mode which is an essential property of the proposed non-Abelian state. Moreover, it allows us to study the spectrum of this mode, and explore the effect of the presence of quasiparticles on that spectrum.

Coulomb blockade effects in the regime of linear response were suggested before as a tool to probe non-Abelian phases, via tunneling of electrons into large dots. Coulomb blockade peaks are observed when the ground state of a dot with $N$ electron is degenerate with that of a dot with $N + 1$ electrons. Thus, their study probes a ground state property as a function of parameters such as the size of the dot and the magnetic field. In contrast, current-voltage characteristics in the non-linear regime of the Coulomb blockade hold information of the many particle excitation spectrum of the system. A peak in the differential conductance $dI/dV$ will appear whenever a proper resonance condition between the source-drain voltage and the excitations of the dot is met.

In this work we analyze $dI/dV$ in a Coulomb blockade measurement as a function of source-drain voltage and magnetic field. We show that the "diamond structure" that characterizes the Coulomb blockade out of the linear response regime could serve to identify the nature of the $\nu = 5/2$ state. We consider a Hall bar or a Corbino disk in which a quantum anti-dot, a puddle of filling fraction $\nu_a$, is embedded in a bulk of filling factor $\nu_b$ (see Fig. 1). The puddle is surrounded by a gapless region. Since this region is compact, its spectrum is quantized. Transport from one edge of the sample to the other is facilitated by tunneling of charge carriers (whose precise nature is discussed below) through the anti-dot, and is characterized by resonances corresponding to the internal states of the gapless region around the antidot. The edges are connected to reservoirs and their spectrum is continuous. We assume that the anti-dot is weakly and symmetrically coupled to the nearby edges, and that tunneling through the anti-dot can be treated in the sequential tunneling approximation.

In the setup we propose, the type of charge-carrier allowed to tunnel across the bulk is determined by the bulk filling fraction $\nu_b$. For an integer $\nu_b$ only electrons are allowed to tunnel, while a fractional $\nu_b$ allows for quasiparticles of fractional charge to tunnel as well. Electron tunneling spectroscopy in the quantum Hall regime has been achieved before for integer filling fraction in Ref. [7]. The experiment we consider probes the spectrum and set of states available to the tunneling excitation in the edge separating the $\nu = 5/2$ phase from the integer $\nu = 2$ phase.

Below, we first consider the case $\nu_b = 5/2, \nu_a = 2$, where our setup should in principle be easier to stabilize with a top-gate due to the robustness of the the integer $\nu = 2$ plateau[21]. We then continue with the combination $\nu_a = 5/2, \nu_b = 2$.

The various descriptions suggested for the $5/2$ phase describe the energy of the edge separating the anti-dot from the bulk as consisting of a sum of several contributions, one arising from the charge mode and the others arising from neutral modes, should those exist. The ground state energy is obtained when that sum is the

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\[ \frac{\nu_a}{\nu_b} = \frac{5}{2} \text{ and } (\nu_a, \nu_b) = (2, 5/2) \text{ and } (\nu_a, \nu_b) = (5/2, 2). \]

FIG. 1: Antidots in a Hall bar geometry. The bulk filling fraction \( \nu_b \) determines the tunneling excitation, and the antidot’s filling fraction is \( \nu_a \). (a) A single antidot. (b) Double antidot geometry, discussed towards the end of the paper.
lowest. The charging energy has the standard form
\[ E_c = \frac{\pi v_e}{\alpha L} (N - N_0 - N_g)^2 \] (1)
where \( v_e \) is the charge velocity, \( L \) is the perimeter of the dot, and \( N \) counts the number of charge-carriers on the edge. Here, \( N_0 \) and \( N_g \) are shifts to the effective number of charge-carriers due to extra flux or the presence of back-gate voltage. If \( N \) counts the number of electrons, \( \alpha = 1/\nu = 2 \), while if \( N \) counts the number of quarter-charged quasiparticles, \( \alpha = 8 \).

The existence and spectrum of the neutral modes vary between different theories of the \( \nu = 5/2 \) state. For concreteness, we first focus on the Moore-Read theory [1] and explore the features it will show in experiment. Then we extend the discussion to include some other known models for the edge, and show that the predictions they lead to are different from those of the Moore-Read state.

For the case of \( \nu = 5/2 \) in a Moore-Read state and \( \nu_a = 2 \), quasi-particles of charge \( e/4 \) will hop onto the edge of the anti-dot, thereby charging it as well as creating neutral excitations on it. The neutral edge mode in this case was extensively studied (See for example Ref. [8]) and shown to be described by a single chiral Majorana fermion theory.

The spectrum of the Majorana edge fermions varies with the number of quasi-particles on the edge. The Hamiltonian is of the form
\[ H = \frac{2\pi v_n}{L} \sum_{n \geq 0} \epsilon_n c_n^{\dagger} c_n + \Delta \] (2)
where \( c_n^{\dagger}, c_n \) are fermionic creation and annihilation operators, while \( v_n \) is the velocity of the excitations. A recent numerical work [9] estimates the neutral velocity \( v_n \) to be between \( 0.75 \times 10^6 \) and \( 1.1 \times 10^6 \) cm/s for GaAs in the thermodynamic limit, and \( v_n \) to be between 6 and 8 times larger. For an even number of quasi-particles, the single fermion states are of energy \( \epsilon_n = n + 1/2 \) and \( \Delta = 0 \), while for an odd number of quasi-particles, the available single fermion states are of energies \( \epsilon_n = n \) and \( \Delta = \frac{1}{4\nu} \). This difference in the single fermion spectrum leads to a pronounced difference in the many fermion spectrum of the Hamiltonian (2). For an odd number of quasi-particles, the spectrum remains composed of non-negative integer multiples of \( 2\pi v_n / L \). In contrast, for an even number of quasi-particles the spectrum is composed of integer (when the number of fermions is even) or half integers (when that number is odd) multiples of \( 2\pi v_n / L \). Remarkably, in the latter case one integer is absent from the spectrum. To get a total energy of \( 2\pi v_n / L \) from half integer single fermion energies two fermions must occupy the state of \( \epsilon_n = 1/2 \), and that is forbidden by the Pauli principle.

These two different many fermion excitation spectra should manifest themselves in the \( I - V \) characteristics of the anti-dot as a function of magnetic field (or the voltage on the gate defining the antidot) and source-drain voltage \( V \), the voltage between the two edges. In the limit when \( e^* V \) is well below twice the charging energy of the anti-dot (with \( e^* \) being the charge of the tunneling charge-carrier), the flow of current takes place through an alternation of the number of quasi-particles on the anti-dot between two values, \( N = n_{qp} \) and \( N = n_{qp} + 1 \). A quasi-particle that tunnels into or out of the anti-dot excites its edge. If the anti-dot is weakly coupled to the two edges, the current through it is small, and in between two consecutive quasi-particles tunneling events the anti-dot relaxes to its ground state. Thus, a tunneling process always starts with the anti-dot in its ground state and ends with it being excited. In view of the symmetry of the coupling between the anti-dot and the two edges, the electrochemical potential of the anti-dot is half-way between those of the edges.

For current to pass through the anti-dot, the minimal voltage \( V_{min} \) should supply enough energy to overcome the difference between ground state energies \( E_{gs} \) of \( n_{qp} \) and \( n_{qp} + 1 \) quasi-particles on the anti-dot,
\[ e^* V_{min}/2 = |E_{gs}(n_{qp} + 1) - E_{gs}(n_{qp})|. \] (3)
For \( V > V_{min} \), resonances in \( dI/dV \) should occur when
\[ E(n_{qp} + 1, N_g) - E_{gs}(n_{qp}, N_g) = e^* V/2 \] (4)
\[ E(n_{qp}, N_g) - E_{gs}(n_{qp} + 1, N_g) = e^* V/2. \] (5)
where \( e^* = e/4 \) is the charge of the tunneling quasi-particle. The first condition (3) is the one allowing tunneling onto the dot. The voltage bias between the edge and the anti-dot has to compensate for the energy difference between the ground state of the dot with \( n_{qp} \) quasiparticles, and an excited state of the anti-dot with \( n_{qp} + 1 \) quasiparticles. Similarly, the second condition (5) allows tunneling from the anti-dot onto the opposite edge.

Since our focus of interest is the neutral mode whose typical energy scale is much smaller than that of the charged mode, we will consider voltages small enough to keep the charged mode always in its ground state. The tunneling quasi-particle may then excite the neutral mode to any excited state to which its energy is sufficient. The excitability of many fermion states of the neutral mode may be understood by considering the description of the Moore-Read state as a \( p \)-wave superconductor of composite fermions, and the neutral Majorana mode as the edge mode that characterizes such a super-conductor [8]. The tunneling quasi-particle is a vortex in this super-conductor. When a \( e/4 \) quasi-particle tunnels through the bulk and onto the anti-dot, it may break some of the pairs in the condensate. Therefore, pairs or individual fermions may be carried with the quasi-particle onto the anti-dot to occupy the single particle energy modes of the Hamiltonian (2).
In equation (1), we find that resonances in \( \frac{dI}{dV} \) for a transition between \( n_{qp} \) and \( n_{qp} + 1 \) quasi-particles on the dot. White region represents a stable state of a dot with \( n_{qp} \) or \( n_{qp} + 1 \) quasiparticles (Coulomb blockaded regions). Green lines are separated by half the distance separating blue lines.

With this picture of the tunneling process, we can now determine the solutions to Eqs. (4) and (5). For \( n_{qp} \) even, \( E(n_{qp}) = E_c(n_{qp}) + 2\pi \nu_n q/L \), and the non-negative \( q \) can obtain integer and half-integer values, with the condition \( q \neq 1 \). For \( n_{qp} \) odd, \( E(n_{qp}) = E_c(n_{qp}) + 2\pi \nu_n q/L + 2\pi \nu_n \Delta/L \), and \( q \) can obtain only integer values. Using equation (4), we find that resonances in \( dI/dV \) as a function of \( V \) and \( N_0 \) appear as lines parallel to the edges of the Coulomb diamonds defined by (8). Plots of the location of resonance for the two possibilities appear in figure (2). As can be seen in the figure, the difference between the even-odd and the odd-even transition from \( n_{qp} \) to \( n_{qp} + 1 \) is manifested in the density of the lines of positive and negative slopes (tunneling-out and tunneling-in resonances). When \( n_{qp} \) is even, lines of positive slope are twice as dense as those of negative slope. When \( n_{qp} \) is odd, the converse is true - the density of lines with a negative slope is twice as large as that of lines with positive slope. This doubling of the number of resonances reflects the difference in the many particle spectrum of the Majorana fermion edge mode - the energy separation between excited states is halved when changing \( n_{qp} \) from odd to even. Note that the second excitation line is missing on the denser set of lines, reflecting the absence of an excitation of energy \( 2\pi \nu_n /L \).

Now, let us consider the case where \( \nu_a = 5/2 \) and \( \nu_b = 2 \). This case is rather similar to the strong backscattering limit of a Fabry-Perot interferometer discussed in Refs. [4, 8]. The charge on the entire \( \nu_a = 5/2 \) anti-dot (bulk+edge) is quantized in units of the electron charge, but the charge on the edge may become fractional, and quantized in units of \( e/4 \). This happens by the creation of a quasi-particle/quasi-hole pair, with one member of the pair located in the bulk of the anti-dot and the other on its edge. The edge spectrum depends on the parity of the number of these pairs, in a similar way to the spectrum we discussed above. The tunneling object is an electron. As the magnetic field is varied the number of electrons that minimizes the energy of the anti-dot varies, and so does also the number of edge quasi-particles. Equations (4) and (5) still describe the conditions for the onset for tunneling, with \( n_{qp} \) replaced by \( N \), the number of electrons on the edge, and \( e^* \) replaced by \( e \). Since the parity of the number of \( e/4 \) quasiparticles on the edge of the anti-dot is not affected by electron tunneling, the energy spectrum of the neutral modes is the same for \( N \) and \( N + 1 \). It may be varied only through changing the magnetic field or the gate voltage. Hence, there should be no difference between the density of resonance lines of positive and negative slope in \( dI/dV \). The density of these lines should be doubled/halved whenever a change in flux or voltage modifies the number of quasiparticles localized inside the anti-dot, and changes its parity.

We now turn to show how the experimental set-up that we discuss may distinguish between different candidate states for \( \nu = 5/2 \), due to their different edge structure. We remind the reader that the structure of lines for the Moore-Read state, seen in Fig. (2), results from a combination of three ingredients: the existence of a neutral slow mode creates the resonances, the dependence of its spectrum on the parity of \( n_{qp} \) leads to the two different densities of peaks, and having just a single Majorana mode leads to the absence of a state of energy \( 2\pi \nu_n /L \) for even \( n_{qp} \).

Although it has been shown numerically that under certain conditions the Pfaffian state faithfully describes the true ground state for \( \nu = 5/2 \), [4] so far only the charge of the tunneling quasi-particle has been measured and verified as \( e/4 \). [10, 12] This is consistent with several other candidates, such as the anti-Pfaffian [13, 15], the 331 Halperin state [16], the \( K = 8 \) state and the \( U(1) \times SU_2 \) state [17, 18]. All these candidate states are made up of condensed pairs of particles with the resulting quasiparticle charge being \( e/4 \). We now examine the \( I - V \) characteristics to be expected for these states.

The anti-Pfaffian: The random edge of the anti Pfaffian supports one charge mode, and three counter propagating neutral Majorana fermion modes [15]. In the infra-red limit, all three modes have the same velocity. There are three different quasi-particle operators defined on that edge, and the presence of a quasi-hole changes the boundary conditions for all three at once. As a consequence of having several flavors of majorana fermions, the analysis we carried out for the Moore-Read state is changed only in one respect: the Pauli principle does not prevent here the formation of an excitation of energy \( 2\pi \nu_n /L \) for an even \( n_{qp} \). We note, however, that a finite quantum anti-dot does not necessarily reach the infra-red limit. If it does not, the velocities of the three Majorana modes will not necessarily be identical, and the spectrum of resonances will be more complicated.

The 331 Halperin state: The edge of the 331 state carries a single chiral complex dirac fermion [8], which is equivalent to two Majorana fermions. The Hamiltonian...
nian of its neutral mode is then similar to Eq. \( \text{[2]} \), but with two flavors of fermions. The results are therefore expected to be similar to those obtained for the Anti-Pfaffian state. We note, however, that the 331 state correspond to a special symmetry point, of zero spin polarization. Any deviation from this point will lead to a different set of resonances.

The \( K=8 \) state: This state does not carry a neutral mode, and hence does not give rise to the resonances we discuss here.

\( SU(2)_1 \times U(1) \): The edge structure of this state is detailed in the appendix of Ref. \([19]\). The edge accommodates a charge density mode, a spin density mode of velocity \( v_s \), and a Majorana mode. Due to the presence of the spin mode, the separation between the resonance lines in the differential conductance may become non-uniform. If \( v_n \ll v_s \), a similar structure to that predicted for the Moore-Read state may emerge, otherwise a more complicated structure is expected.

We now turn to discuss resonant tunneling through a double anti-dot geometry (see Fig. \([1]\)). In this setup, charge-carriers have to tunnel through both anti-dots to get across the Hall bar. We first assume that the two anti-dots are at \( \nu = 2 \), and are Coulomb blockaded in such a way that current flows by tunneling of quasi-particles in the sequence \((m_{qp}, n_{qp}) \rightarrow (m_{qp}+1, n_{qp}) \rightarrow (m_{qp}, n_{qp}+1) \rightarrow (m_{qp}, n_{qp})\), with the two numbers signifying the number of quasi-particles on the first and second anti-dots, respectively. Furthermore, we assume that the coupling between the anti-dots is weak enough such that the first anti-dot relaxes to its ground state before the quasi-particle tunnels on to the second one. Then, tunneling from one edge to the other requires that the ground state of the first anti-dot is aligned with an excited state of the second \([20]\). This allows us to probe the doubling of the many particle density of states of the second anti-dot as a function of the number of quasiparticles its edge accommodates. This time, however, resonance peaks should appear in the current \( I \), rather than in the differential conductance \( dI/dv \). For voltages small enough not to affect \( n_{qp} \), the density of the peaks as a function of gate voltage or source-drain voltage will reflect the many-body spectrum of the neutral mode of the second anti-dot, whose density of states depends on the parity of \( n_{qp} \). The two possible parities will be reflected in two different densities of resonance peaks as a function of gate voltage or source-drain voltage. Similarly, two different densities of peaks should be observed also when the tunneling charge-carrier is an electron.

Before concluding, we comment on an essential ingredient of our analysis, namely the requirement for relaxation of the neutral edge mode(s) of the anti-dot(s) between tunneling events. A mechanism for such relaxation is the coupling of the edge of the anti-dot to quasiparticles localized in the bulk of the \( \nu = 5/2 \) phase \([9]\). This coupling increases with the density of the bulk quasiparticles, and with their proximity to the edge of the anti-dot. Consequently, the rate of relaxation of the anti-dot due to this coupling may be controlled by the magnetic field, which controls the density of bulk quasiparticles. For the experiments we propose here, the width of the excited states of the neutral mode of the anti-dot generated by their relaxation rate to the bulk, should be smaller than their separation, yet larger than the width corresponding to the rate at which charge-carriers pass through the anti-dot. With the relaxation rate being controllable by the magnetic field and the rate at which charge-carriers pass being controllable by the coupling of the anti-dot(s) to the point contact, the desired limit may be obtained. Remarkably, the upper limit on the relaxation rate is far less stringent than that required for the realization of earlier proposals \([6]\).

To summarize, we explored a new geometry for the detection of neutral edge modes in the quantum Hall effect through resonant tunneling experiments. Such experiments at \( \nu = 5/2 \), involving quasi-particle tunneling or electron tunneling, can show unique features for the non-Abelian Moore Read state.

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