Study of acoustic characteristics of the swirling flow generated in ejector

V Vekteris¹, A Styra¹, V Mokshin¹, V Turla², G Viselga¹ and I Tetsman¹

¹Department of Mechanical and Material Engineering, Vilnius Gediminas Technical University, J. Basanavičiaus str. 28, 03224 Vilnius, Lithuania
²Department of Mechatronics, Robotics and Digital Manufacturing, Vilnius Gediminas Technical University, J. Basanavičiaus str. 28, 03224 Vilnius, Lithuania

E-mail: vadim.moksin@vgtu.lt

Abstract. The article deals with changes in the density of pulsating water and air flow generated in pulsating flow ejector. Transformation of the fluid motion equation to Lighthill’s equation describing density change and acoustic radiation of the flow is presented. Calculation results demonstrate suitability of the ejector to cause acoustic agglomeration and effectively remove iron particles from potable water.

1. Introduction

Potable water containing high concentrations of iron compounds causes many problems for consumers. Water soluble iron compounds react with air oxygen and form poorly soluble iron hydroxides, water color changes to brownish and clarity decreases [1–3]. Iron compounds contribute to metallic tasting tap water [1]. They also accumulate on pipe walls, the inside diameter of the pipes decreases as the result of accumulation as well as pipe capacity [4, 5].

Well known iron removal process consists of the oxidation of ferrous iron to less soluble ferric iron and subsequent precipitation of particulate forms of ferric iron [6]. In most cases, atmospheric oxygen is used as an oxidant [7]. Other substances like active chlorine compounds, manganese oxide, potassium permanganate can be used for this purpose. However, they reduce ecological indices of the process and increase water treatment cost. Various hazardous for human health products including carcinogens are produced due to the use of these strong oxidants [8].

To ensure allowable concentrations of iron compounds in potable water according [9], it is necessary to develop advanced, environmentally friendly water purification technologies, for instance, a hydrodynamic non-reagent method. In this method, the efficient mixing of work fluid (water) with ejecting airflow is achieved. It ensures high solubility of oxygen in the liquid and high precipitation rates of iron particles as an outcome.

The use of acoustic field could allow to improve the efficiency of mentioned method [10]. It was proved by scientists that oxidation reactions are significantly accelerated in presence of sound waves as the various physical-chemical effects appear due to mechanical reactions or elastic vibrations [11]. Other reactions like decomposition, polymerization and depolymerization reactions are accelerated as well as molecular rearrangements [12–15].

When high intensity sound waves (>10⁴ W/m²) are generated in liquid, micro explosions occur in continuous medium. Created cavities are filled with dissolved gases and vapor and single-phase liquid
is transformed into a two-phase system. As a rule, degasification processes are accelerated by cavitation in liquids containing large enough amounts of dissolved gases. In most cases, sound-induced chemical reactions are accompanied by particle emission influenced by nature of substances present in liquid medium [16].

High-intensity sound waves enforce the relative motion among particles to produce efficient collisions [17] that lead to the formation of agglomerates (acoustic agglomeration). The most important mechanisms of acoustic agglomeration documented in the literature include the mechanism of hydrodynamic interaction, orthokinetic collision and Brownian agglomeration [18, 19]. These mechanisms promote the formation of particle clusters that settle more easily under the action of gravitational forces. However, acoustic excitation parameters such as sound pressure level and other are very important in the process of acoustic agglomeration [18, 19].

This article theoretically investigates acoustic characteristics of specially designed pulsating flow ejector where intensively mixing in mixing chamber fluid with air is treated at the same time by acoustic radiation generated by swirling flow in the same chamber. The ejector can be used in water treatment systems where water is saturated with oxygen before being purified. It also can be used in other industries, such as chemical and biotechnology.

2. Object of research

The pulsating flow ejector (figure 1) was designed for use in water treatment systems, where water is mixed with oxygen before being treated. It also can be used in other industries, such as chemical and biotechnology, where it is necessary to saturate fluids with gases under variable fluid flow rate condition.

![Diagram of the pulsating flow ejector](image)

**Figure 1.** The pulsating flow ejector [20]: 1 – housing; 2 – diffuser; 3 – mixing chamber; 4 – transition; 5 – liquid chamber; 6 – cylindrical wall; 7 – axis; 8 – bottom wall of mixing chamber; 9 – ejecting medium (air) chamber; 10 – pyramid-shaped bulge; 11 – plane of the bulge; 12, 13 – holes.

Liquid under pressure is supplied to chamber 5 (figure 1) and then tangentially enters the mixing chamber 3 through two holes 13 to generate a swirling flow. Due to the inclination of axes of holes 13, rotating flow is directed towards the bottom wall 8 of chamber 3. When the flow reaches the bottom wall 8, additional turbulence is generated by four bulges 10. As the bulge height is variable and decreases towards the axis of rotation of the flow 7 (or the axis of cylindrical chamber 3), the turbulence intensity remains constant along the edge of bulge. Swirling flow bypassing the bulge 10 creates the negative pressure over four holes 12, which connects chamber 3 with ejecting medium (air) chamber 9. Under the action of negative pressure, air begins to enter chamber 3 through holes 12 and mix with liquid due to the generated turbulence. As the density of mixture is changed, the dynamic pressure of the flow is changed as well as the ratio of dynamic and static pressures. When the static pressure increases, swirling flow moves towards the diffuser 2 thus closing the access of liquid to mixing chamber 3 through holes 13. The static pressure drops as a result of this movement, the dynamic pressure increases proportionally and liquid gains access to mixing chamber 3 again.
3. Analytical and numerical study of pulsating flow

Momentum and mass conservation laws can be expressed as follows [21]:

\[
\rho \left( \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \frac{\partial p_{ij}}{\partial x_j} 
\]

\[
\frac{\partial p}{\partial t} + \frac{\partial}{\partial x_j} (\rho v_j) = 0
\]

(1)

where \( p \) is the pressure, \( \rho \) is the fluid density, \( t \) is the time, \( v \) is the fluid velocity, \( p_{ij} \) is the \((i, j)\)th component of viscous stress tensor:

\[
p_{ij} = p \delta_{ij} + \mu \left[ -\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i} + \frac{2\partial v_k}{\partial x_k} \delta_{ij} \right]
\]

(3)

where \( \mu \) is the dynamic viscosity, \( \delta_{ij} \) is the Kronecker delta: \( \delta_{ij} = 1 \) if \( i = j \) and \( \delta_{ij} = 0 \) if \( i \neq j \).

Multiplying equation (2) by \( v_i \), adding the result to the equation (1), and combining similar terms, the following equation is obtained:

\[
\frac{\partial}{\partial t} (\rho v_i) = -\frac{\partial}{\partial x_j} \left( \rho v_j v_i + \delta_{ij} p - p_{ij} \right)
\]

(4)

After adding and subtracting the term \( \frac{c_0^2}{2} \frac{\partial \rho}{\partial x_i} \), equation (4) becomes:

\[
\frac{\partial}{\partial t} (\rho v_i) + \frac{c_0^2}{2} \frac{\partial \rho}{\partial x_i} = -\frac{\partial T_{ij}}{\partial x_i}
\]

(5)

where \( T_{ij} \) is the Lighthill’s turbulence stress tensor:

\[
T_{ij} = \rho v_i v_j + \delta_{ij} \left[ (p - p_0) - c_0^2 (\rho - \rho_0) \right] - p_{ij}
\]

(6)

where \( c_0 \) is the speed of sound, \( p_0 \) is the atmospheric pressure.

After differentiating equation (2) with respect to \( t \), taking the divergence of equation (5) and subtracting the results, the Lighthill’s equation can be obtained:

\[
\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i \partial x_j} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}
\]

(7)

where \( \rho' \) is the acoustic density fluctuation.

Equation (7) has a waveform as the equation describing the acoustic field generated by the quadrupole source \( \partial^2 T_{ij} / \partial x_i \partial x_j \) in a stationary medium.

Lighthill’s equation (7) can be written in the following form:

\[
\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i \partial x_j} = \frac{\partial m}{\partial t} - \frac{\partial F_i}{\partial x_i} - \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}
\]

(8)

where \( x_1 = x, x_2 = y, x_3 = z, m \) is the sound source.

Since solution of the equation (8) was not obtained, Lighthill proposed to assume known tensor \( T_{ij} \) and examine right side of the equation (8) as a preset. In this way, it was proposed to solve the equation (8) as the wave equation with a known right side. Then solution of the equation (8) consists of three components. The first describes the sound emitted by source \( m \):
\[
\rho^* (\vec{x}, t) = \rho - \rho = \frac{1}{4\pi c_0^2} \int_R \frac{\partial}{\partial t} m \left( \vec{y}, t - \frac{r}{c_0} \right) \frac{dS}{r}
\]

where \(\vec{x}, \vec{y}\) are radii vectors of observation point and the liquid element \(dS\), \(r\) is the distance between points \(\vec{x}\) and \(\vec{y}\) (\(r = |\vec{x} - \vec{y}|\)).

The second component describes sound radiation when force \(F_i\) acts on the flow:

\[
\rho^* (\vec{x}, t) = \frac{1}{4\pi c_0^2} \frac{1}{R} \frac{\partial}{\partial x_i} F_i \left( \vec{y}, t - \frac{r}{c_0} \right) \frac{dS}{r}
\]

The last component describes the turbulent flow induced noise:

\[
\rho^* (\vec{x}, t) = \frac{1}{4\pi c_0^2} \frac{1}{r^3} \int_S \frac{\partial^2 T_{ij}}{\partial t^2} \left( \vec{y}, t - \frac{r}{c_0} \right) dS
\]

When \(m = 0\) and \(F = 0\), equation (8) solution looks like this:

\[
\rho^* (x, t) = \frac{1}{4\pi c_0^2} \frac{1}{\partial x_i \partial x_j} \int_R \frac{\partial^2 T_{ij}}{\partial t^2} \frac{dS}{r} + \frac{1}{4\pi c_0^2} \frac{1}{\partial x_i} \frac{\partial}{\partial x_i} F_i \frac{dS}{r} + \frac{1}{4\pi c_0^2} \frac{1}{\partial t} \left( \rho^* v_n \frac{dS}{r} \right)
\]

where \(f_i\) are the \(x_i\) axis components of the forces with which the ejector’s unit area \(s\) acts on the flow, \(v_n\) is the speed normal to the surface.

\(v_n = 0\) when the surface of the ejector vibrates in its plane and it follows from equation (12) that:

\[
\rho^* (x, t) = \frac{1}{4\pi c_0^2} \frac{1}{\partial x_i \partial x_j} \int_R \frac{\partial^2 T_{ij}}{\partial t^2} \frac{dS}{r} - \frac{1}{4\pi c_0^2} \frac{1}{\partial x_i} \frac{\partial}{\partial x_i} F_i \frac{dS}{r}
\]

If the force \(f_i\) fluctuates:

\[
\rho^* (x, t) \approx \frac{x_i}{4\pi c_0^3} \frac{\partial}{\partial t} F_i \left( t - \frac{r}{c_0} \right)
\]

where \(F_i(t) = \int_S f_i(y, t) ds\) is the total force acting on the flow.

These equations indicate that the sound is generated not only by the ejector, but also due to the turbulization. Therefore, obtained equations are important for calculating the flow induced acoustic radiation. In case of flow turbulization, it is appropriate to consider \(T_{ij}\) as a stationary random function of time.

According to the obtained equations, the numerical simulation of ejected flow was carried out. SolidWorks Flow Simulation 2011 software was used for this aim. Assumptions and conditions used in simulation are presented in work [22]. Results of calculation of volume and mass fractions of air and water in the ejector are shown in figure 2, a and b.

Water flow pressure in the ejector, mixing chamber volume and distance \(L\) (figure 1) were varied during the simulation. From the results obtained the case was chosen when the density variations cause the maximum pressure pulses (figure 2, c).

The examination of Lighthill’s equation (7) or (8) based on dimension theory [23] allows to calculate sound power level of the flow:

\[
L_w = 10 \log \left( k_0 \frac{P_0^2 v^2}{W_0 \rho c^3} \right)
\]

\(k_0, P_0, v, W_0, c, \rho\) are constants.
where $v_0$ is the outflow velocity, $\rho_0$, $\rho_\infty$ are the density of the flow exiting the nozzle and the density of the surrounding medium, $k_0$ is the coefficient, which value varies from $3 \cdot 10^{-5}$ to $(1.5–2.5) \cdot 10^{-5}$, $c_\infty$ is the speed of sound in water, $W_0 = 10^{-12}$ W is the basic sound power.

After inserting into equation (15) numerical simulation results, we obtain the sound power level from 80 to 120 dB which represents sound intensity from $10^{-4}$ W/m$^2$ to 1 W/m$^2$ that exceeds the acoustic aggregation requirements.

Experimental study of iron removal efficiency of the ejector [24] has demonstrated that concentration of ferrous iron in water decreases 2.5 times in presence of acoustic field.

4. Conclusions

Numerical simulation has shown that the flow density varies from 0.66 to 4.64 kg/m$^3$ in the mixing chamber of the pulsating flow ejector during mixing of the air with water. Density fluctuations generate the acoustic field. Analytical calculations showed that the sound power level varies from 80 dB to 120 dB that satisfies the acoustic agglomeration conditions and shows that designed ejector can be used for non-reagent water treatment removing ferrous iron (Fe$^{2+}$).

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