Secure Adaptive Group Testing

Alejandro Cohen * Asaf Cohen * Sidharth Jaggi † Omer Gurewitz*

*Department of Communication Systems Engineering, Ben-Gurion University, Israel, {alejandr, coasaf, gurewitz}@bgu.ac.il
†Department of Information Engineering, Chinese University of Hong Kong, Hong Kong, jaggi@ie.cuhk.edu.hk

Abstract—Group Testing (GT) addresses the problem of identifying a small subset of defective items from a large population, by grouping items into as few test pools as possible. In Adaptive GT (AGT), outcomes of previous tests can influence the makeup of future tests. This scenario has been studied from an information theoretic point of view. Aldridge 2012 showed that in the regime of a few defectives, adaptivity does not help much, as the number of tests required for identification of the set of defectives is essentially the same as for non-adaptive GT.

Secure GT considers a scenario where there is an eavesdropper who may observe a fraction δ of the outcomes, and should not be able to infer the status of the items. In the non-adaptive scenario, the number of tests required is 1/(1−δ) times the number of tests without the secrecy constraint.

In this paper, we consider Secure Adaptive GT. Specifically, when an adaptive algorithm has access to a private feedback link of rate Rf, we prove that the number of tests required for both correct reconstruction at the legitimate user, with high probability, and negligible mutual information at the eavesdropper is 1/min{1, 1−δ + Rf} times the number of tests required with no secrecy constraint. Thus, unlike non-secure GT, where an adaptive algorithm has only a mild impact, under a security constraint it can significantly boost performance. A key insight is that not only the adaptive link should disregard test results and send keys, these keys should be enhanced through a “secret sharing” scheme before usage.

I. INTRODUCTION

Group Testing (GT) was introduced in the seminal study by Dorfman to identify syphilis infected draftees while dramatically reducing the number of required assays [1]. Specifically, the objective of GT is to identify a small subset of K unknown defective items within a much larger set of N items, conducting as few measurements T as possible.

This problem has been analyzed in various scenarios [2], one of which, Non-Adaptive Group Testing (NGT), is when the entire pooling strategy is decided on beforehand. This scenario has also been formulated as a channel coding problem, e.g., [3], where each codeword is associated with the pool tests its associated item participates in.

Secure GT protects the items’ privacy such that an eavesdropper who may observe only a fraction of the pool-tests, will not be able to infer the status of any of the items (negligible mutual information between the captured pool-tests and the status of the items). In [4], the authors addressed Secure Non-adaptive Group Testing (SNGT). To confuse the eavesdropper, instead of each item having a single test vector, determining in which pool-tests it should participate, each item has a random vector, chosen from a known set. [4] proved that when the fraction of tests observed by the eavesdropper (Eve) is 0 ≤ δ < 1, the number of tests required for both correct reconstruction at the legitimate user and negligible mutual information at Eve’s side is 1/(1−δ) times the number of tests required with no secrecy constraint. Thus, the solution in [4] relies on information theoretic security, which offers security at the price of rate, or, in the GT case, trades security and the number of tests.

In this paper, we consider Adaptive Group Testing (AGT), in which the outcomes of previous tests can influence the construction of future pools. In general, the adaptivity may benefit from both a reduced number of tests (T) and efficient decoding techniques [5]. However, it has been shown that in many interesting cases, the decrease in T is negligible. Several studies have analyzed AGT as a channel coding problem with feedback (e.g., [6]–[9]). Utilizing the same techniques as Shannon’s seminal study [10] which proved that feedback does not increase channel capacity, the author in [6] showed that feedback from the lab does not decrease the number of tests required significantly. That is, if T is the minimal number of tests required for reconstruction with negligible error, when N → ∞ the gain due to the adaptivity is marginal. Even with zero-error, the difference in T is only between $O(K^2 \log N / \log K)$ and $O(K \log N)$ [2].

Surprisingly, we show that in secure-adaptive group testing, the adaptive feedback link may decrease the number of tests required significantly, which in some cases coincides with the number of pool tests required with no secrecy constraint at all. While from an information theoretic perspective, previous results indeed show that in communication feedback can increase secrecy capacity [11], [12], herein, previous techniques do not apply directly, as the test transferred are physical entities which cannot be, e.g., one-time-padded, and, as we explain in the sequel, the “encoder” in this case has no knowledge on which $K$ of the $N$ items it needs to protect.

Main Contribution: We propose a new Secure Adaptive Group Testing (SAGT) algorithm. This algorithm significantly reduces the number of tests required, yet is sufficient for the legitimate user to identify the defective items, and to keep an eavesdropper ignorant regarding any of the items.

The model is depicted in Figure 1. In the suggested solution, the set of indices describing the defective items takes the place...
of a confidential message; the testing matrix represents the design of the pools, where each row corresponds to a separate item. Each such matrix row is associated with a codeword where 1 denotes the pool tests that the corresponding item participates. The decoding algorithm is analogous to a channel decoding process, yet now the adaptive link from the lab (who examines the pools) to the mixer (who mixes the samples and creates the pooled tests) takes the place of a feedback link. The eavesdropper observation is analogous to the output of an erasure channel, such that only part of the tests sent from the legitimate source (the mixer) to the legitimate receiver are observed by the eavesdropper. We assume a private adaptive link, which is not observable to the eavesdropper.

We use the feedback link from the lab to the mixer in order to modify the testing matrix, according to the shared information between them, in a way which can be comprehended by both the lab and the mixer, yet confuses the eavesdropper regarding which item participates in each pool test. However, unlike wiretap channels with feedback, we cannot use the data on the link directly, and must use a coding scheme, to increase the number of keys we can generate. Although the keys generated are dependent, any subset which Eve eventually observes is independent and thus protected. Accordingly, this adaptive algorithm decreases the factor $1/(1 - \delta)$ given in SNGT, and one may use a smaller number of tests. In the case that the information rate on the feedback is equal to the rate of the eavesdropper’s observation, this factor is completely rescinded. Thus, we achieve the same sufficiency bound on $T$ as given for the non-secure GT.

II. PROBLEM FORMULATION

In SAGT, a legitimate user wishes to identify a small unknown subset $K$ of defective items form a larger set $N$, while reducing the number of measurements, $T$, as much as possible and keeping an eavesdropper, which is able to observe a subset of the tests results, ignorant regarding the status of the $N$ items. The adaptive link allows the outcomes from previous tests to influence the makeup of future tests in order to further reduce the total number of measurements. $N = |N|$, $K = |K|$ denote the total number of items, and the number of defective items, respectively. The status of the items, defective or not, should be kept secure from the eavesdropper, but detectable by the legitimate user. We assume that in each round the number $K$ of defective items is known a priori. This is a common assumption in the GT literature [13]. Figure 2 gives a graphical representation of the model.

In general, GT is defined by a testing matrix $X = \{X_j(t)\}_{1 \leq j \leq N, 1 \leq t \leq T} \in \{0, 1\}^{N \times T}$, where each row corresponds to a separate item $j \in \{1, \ldots, N\}$, and each column corresponds to a separate pool test $t \in \{1, \ldots, T\}$. For the $j$-th item, entry $X_j(t) = 1$ if item $j$ participates in the $t$-th pool test and $X_j(t) = 0$ if not. Denoting by $A_j \in \{0, 1\}$ an indicator function indicating whether the $j$-th item belongs to the defective set and by superscript $T$ a vector of length $T$, the pool test outcome is

$$Y(t) = \bigvee_{j=1}^N X_j(t)A_j = \bigvee_{d \in K} X_d(t),$$

where $\bigvee$ is used to denote the boolean OR operation.

In AGT, we assume that the makeup of a testing pool can depend on the outcomes of earlier tests, by adaptive feedback link from the lab to the mixer, such that for any test $t > 1$ and any item $j$, $X_j(t) = X_j(t \mid Y(1), \ldots, Y(t-1))$. In the secure model, we assume this link is private, at a limited rate $R_f$. That is, symbols $F_t, t \in \{1, \ldots, T\}$ are sent over an adaptive private feedback link, secretly from the eavesdropper. The feedback alphabets are denoted by $\{F_1, \ldots, F_T\}$. Their cardinalities must satisfy $\frac{1}{T} \sum_j \log(|F_j|) \leq R_f$. At each time instant $t$, $F_{t-1}$ is computed by the lab and revealed to the mixer. The symbol $F_t$ at time instant $t$ may depend on $Y^{t-1}$, the $t - 1$ prior outcomes of the pool tests, the previous feedback symbols, $F^{t-1}$, and some randomness. That is, we assume there exists a distribution $p(F_t | Y^{t-1}, F^{t-1})$. Hence, in the secure AGT $X_j(t) = X_j(t | F^t)$. Note that in the classic adaptive case, where simply the previous outcome is revealed to the mixer, we have $F_t = Y(t - 1)$, hence $|F_t| = 2$ for all $t$ and therefore $R_f = 1$.

In SAGT, we assume an eavesdropper, which observes a noisy vector $Z^T = \{Z(1), \ldots, Z(T)\}$, generated from the outcome vector $Y^T$. We concentrate on the erasure case, where the probability of erasure is $1 - \delta$, i.i.d. for each test. It simplifies the technical aspects and allows us to focus on the key methods. However, it is possible to consider the case where $Z^T$ is generated from $Y^T$ with false positive or false negative errors as given in [3, Section VI], or [14, Section II].

Denote by $W \in \mathcal{W} \triangleq \{1, \ldots, |N|\}$ the index of the subset of defective items. We assume $W$ is uniformly distributed, that is, there is no a priori bias to any specific subset. Denote by $W(Y^T, F^T)$ the index recovered by the legitimate decoder, after observing $Y^T$. We refer to the adaptive procedure of creating the testing matrix, together with the decoder as a SAGT algorithm. We are interested in the asymptotic behavior of a SAGT algorithm.

Definition 1. A sequence of SAGT algorithms with parameters $N, K, T$ and $R_f$ is asymptotically reliable and weakly secure if: (1) At the legitimate receiver, observing $Y^T$, $\lim_{T \to \infty} P(W(Y^T, F^T) \neq W) = 0$. (2) At the eavesdropper, observing $Z^T$, $\lim_{T \to \infty} \frac{1}{T} I(W; Z^T) = 0$.

III. RELATED WORK

Recent works [3], [4], [6]–[9] adopted an information theoretic perspective on GT, presenting it as a channel coding problem. We briefly review the most relevant results. In [3], the authors mapped the NGT model to an equivalent channel model, where the defective set takes the place of the message, the testing matrix rows are codewords, and the test outcomes are the received signal. They let $\hat{S}(X^T, Y^T)$ denote the estimate of the defective subset $S$, which is random due to the randomness in $X$ and $Y$. Furthermore, let $P_e$ denote the average probability of error, averaged over all subsets $S$ of cardinality $K$, variables $X^T$ and outcomes $Y^T$, i.e., $P_e = P_e[\hat{S}(X^T, Y^T) \neq S]$. Then, where $\{S^1, S^2\}$ denote the partition of defective set $S$ into disjoint sets $S^1$
and $S^2$ with cardinalities $i$, the flowing bounds on the total number of tests required in Bernoulli NGT was given by $T \leq T \leq T$, where for some $\varepsilon > 0$ independent of $N$ and $K$, $T = (1 + \varepsilon) \max_{i=1, \ldots, K} \frac{\log(N-K)}{i}$. It is important to note that if $R_f \geq \delta$, as the direct proof will show, the information obtained over the adaptive link between the lab and the mixer is powerful enough to obtain security without increasing $T$. Hence, in this case, the direct bounds of the non-secure and secure adaptive group testing are equal, that is, $T = T^\delta$. Even when $R_f < \delta$, the information obtained over the feedback between the lab and the mixer reduces the upper bound on the number of the tests required in SNGT, thus, $T < T^\delta$. The maximization in Theorem 1 can be solved easily, leading to a simple bound on $T$.

**Corollary 1.** For SAGT with parameters $K \ll N$ and $T$, reliability and secrecy can be maintained with

$$T^\delta_s \geq \frac{1 + \varepsilon}{\min_{1, 1 - \delta + R_f}} K \log(N-K)e.$$

**Remark 1.** In this paper, we consider the asymptotic case in $T$ and negligible error. In this case, without a secrecy constrain, it is well known that feedback does not help [6]. Nonetheless, with a secrecy constraint we show that the link is used only for shared randomness. For finite $T$ and zero error [2], the mixer may use the previous outcomes to reduce the number of tests $T$. Thus, for finite $T$, zero error and a secrecy constraint, is not trivial what should be shared over the link, either pure randomness in order to cope with the secrecy constraint or single previous outcomes in order to try adaptively reduce the number of tests.

**Remark 2.** The result given in Theorem 1 uses ML decoding. To consider a computationally efficient algorithm, and analyze the regime where $K$ is allowed to grow with $N$, we assume the efficient algorithm suggested in [4, Section VII] for the non-adaptive model could be adjusted to the adaptive model given herein.

**Converse (Necessity):** The necessity part is given by the following theorem.

**Theorem 2.** Any reliable and weakly SAGT algorithm with parameters $N, K, R_f$ and $\delta$ satisfies

$$T^\delta_s \geq \frac{1}{\min_{1, 1 - \delta + R_f}} K \log(N-K).$$

The proof is deferred to the extended version [17, Section VII]. Note that, compared to the lower bound without a security constraints, there is an increase by a multiplicative factor of $1/\min_{1, 1 - \delta + R_f}$. When $R_f \geq \delta$, the lower
bounds of the non-secure and secure adaptive group testing are equal, i.e., $\sum = T^{s}$.

V. CODE CONSTRUCTION AND PROOF OF THEOREM 1

The goal, in general, is to design a proper testing matrix, or, specifically, an algorithm to adaptively update it. Remember that each row describes the tests an item participates in. We thus construct this matrix in batches (of $T$ tests each), each time selecting an appropriate row for each item ([17] describes a test-by-test adaptive algorithm based on the one herein). In a batch, for each item we generate a bin, containing several sub-bins, with several rows in each sub-bin (see Figure 3). Internal randomness in the mixer, which is not shared with any other party, is used to select a sub-bin for each item, while data received from the adaptive link is used to select the right row from the sub-bin. While this solution is inspired by codes for wiretap channels with rate limited feedback [12], there are several key differences, which not only change the construction, but also require non-trivial processing of the data received from the feedback. Specifically, first, unlike a wiretap channel, herein there are $N$ items, only $K$ of them, unknown to the mixer, actually participate in the output (“transmit”). Thus, bins and sub-bins sizes should be properly normalized.

More importantly, the mixer, which acts as an encoder, does not know which $K$ messages it should protect. Thus, the mixer should artificially blow-up the data it receives from the private feedback: from bits (used as keys) intended to protect the $K$ defective items, it generates a larger number of keys, sufficient $N$ items, satisfying the property that any $K$ out of the $N$ which will eventually participate, will still be protected. In other words, the keys received from the feedback cannot be used as is, and an interesting secret-sharing-type scheme must be used.

Formally, (a batch-processing) SAGT code consists of an index set $W = \{1, 2, \ldots \binom{N}{K}\}$, its $w$-th item corresponding to the $w$-th subset $\mathcal{S}_{w} \subseteq \{1, \ldots, N\}$; A discrete memoryless source of randomness at the mixer $(\mathcal{R}_{X}, p_{\mathcal{R}_{X}})$; A discrete memoryless source of randomness at the lab $(\mathcal{R}_{Y}, p_{\mathcal{R}_{Y}})$; A feedback at rate $R_{f}$ bits per test, resulting in an index $I \in \{1, \ldots, 2^{T R_{f}}\}$ after $T$ uses; We use a single index due to the batch processing; The mixer, of course, does not know which items are defective, thus it needs to select a row for each item. However, since only the rows of the defective items affect the output $Y^{T}$, it is beneficial to define an "encoder" $\mathcal{G}_{X} : \mathcal{W} \times \mathcal{R}_{X} \times I \rightarrow \mathcal{X}_{S_{w}} \subset \{0, 1\}^{K \times T}$, that is, mapping the message, the randomness and input from the adaptive link to the $K$ codewords which are summed to give the tests output $Y^{T}$. Note that a stochastic encoder and the causally known feedback message $I$ are similar to encoders ensuring information theoretic security, as randomness is required to confuse the eavesdropper about the actual information [18].

A decoder at the legitimate user is a map $\widehat{W} : Y^{T} \times I \rightarrow W$. The probability of error is $P(\hat{W} \neq W)$. The probability that an output test leaks to the eavesdropper is $\delta$. We assume a memoryless model, i.e., each outcome $Y(t)$ depends only on the corresponding input $X_{S_{w}}(t)$, and the eavesdropper observes $Z(t)$, generated from $Y(t)$ according to $p(Y^{T}, Z^{T} | X_{S_{w}}) = \prod_{t=1}^{T} p(Y(t) | X_{S_{w}}(t)) p(Z(t) | Y(t))$. Next we provide the detailed construction and analysis.

1) Codebook Generation: Choose integers $F$ and $M$ such that $\log_{2}(F) = T(R_{f}/K)$ and $\log_{2}(M) = T(\delta - R_{f} - \epsilon)/K$. For each item we generate bin of $M : F$ independent and identically distributed codewords. Each codeword of size $T$ is generated randomly, where each $X_{j}(t)$ is chosen according to $P(x) \sim \text{Bernoulli}(\log(2)/K)$. Consequently, $P(X^{T}) = \prod_{t=1}^{T} P(x_{t})$. Then we split each bin to sub-bins of codewords $x^{T}(m, f), 1 \leq m \leq M$ and $1 \leq f \leq F$ (see Figure 3).

2) Key Generation: We now describe the generation of the shared keys created from the information sent over the adaptive link. This link is of rate $R_{f}$. We do not use it to send information about test results, and simply send random bits. Therefore, in a block of length $T$ we receive $S = T R_{f}$ secret bits. We divide the secret bits to $K$ secret keys, each constituting $T R_{f}/K = S K$ bits.

Our goal is to take these $K$ keys, and use them to create $N$ new keys, with the property that if the original $K$ keys had a random uniform i.i.d. distribution, then any set of $K$ keys out the $N$ new ones will have the same random uniform i.i.d. distribution. This can be done using a generator matrix of an $[N,K]$ MDS code. Such a generator matrix has the property that any $K$ columns are linearly independent. Thus, taking the $K$ original keys, as an $S K \times K$ matrix, and multiplying it by the generator matrix $G_{N \times K}$ creates a matrix of size $S K \times N$, where each column is used as the new key. Since any subset of $K$ columns of $G$ is invertible, each set of $K$ new keys is simply a $1 : 1$ transformation of the $K$ original keys. The importance of this scheme in our context is as follows: for any subset of $K$ new keys (out of $N$), if an eavesdropper has no access to the original $K$ keys, he/she is ignorant regarding the new keys. Moreover, it is important to note that unlike protection using a one-time-pad, these keys cannot be XORed with the rows of the testing matrix, as this operation will change the probability of each item participating in a pool-test, deviating from the optimal distribution of the testing matrix.

3) Testing: The mixer receives the $T R_{f}$ feedback secret bits, divides them to $K$ keys and uses the MDS code to create $N$ new keys. Each new key is of length $T R_{f}/K$. Therefore, at each round and for each item $j$, the mixer selects a sub-bin using its internal randomness, and a message within it...
using the key. The result is a codeword $x^T(n, f)$. Therefore, the SAGT matrix contains $N$ randomly selected codewords of length $T$, one for each item (defective or not).

In the first round of tests, the mixer has no available (feedback) key, hence it operates using a larger number of tests for that round. Amortized over multiple rounds, this loss is negligible. Due to space limitations we omit the details.

4) Decoding at the Legitimate Receiver: The decoder looks for a collection of $K$ codewords $X_{n,K}^T$, one from each sub-bin, for which $Y^T$ is most likely. Namely, $P(Y^T|X_{n,K}^T) > P(Y^T|X_{n,K}^T), \forall w \neq \hat{w}$. Then, the legitimate user declares $\hat{W}(Y^T \times F^T)$ as the set of bins in which the rows reside.

5) Reliability: Let $I(X_{S1}; X_{S2}, Y)$ denote the mutual information between $X_{S1}$ and $(X_{S2}, Y)$, under the i.i.d. distribution with which the codebook was generated and remembering that $Y$ is the output of a Boolean channel. The following lemma is a key step in proving the reliability of the decoding algorithm suggested herein. This Lemma is a direct consequence of [4, Lemma 1], under the enhancement that the index of each sub-bin is set according to the known key sheared between the legitimate receiver and the mixer at each round of the algorithm.

**Lemma 1.** If the number of tests satisfies $T \geq (1 + \varepsilon) \cdot \frac{\log(N - K)}{I(X_{S1}; X_{S2}, Y)}$, then, under the codebook above, as $N \to \infty$ the average error probability approaches zero.

Applying [4, Claim 1], which lower bounds the mutual information between $X_{S1}$ to $(X_{S2}, Y)$ by $i/K$, to the expression in Lemma 1, and substituting $M = 2^{\frac{i - R_f - \varepsilon}{K}}$, a sufficient condition for reliability is

$$T \geq \max_{1 \leq i \leq K} \frac{1 + \varepsilon}{K} \left[ \log \left( \frac{N - K}{i} \right) + \frac{i}{K} T(\delta - R_f) \right]$$

with some $\varepsilon$. Rearranging terms results in

$$T \geq \max_{1 \leq i \leq K} \frac{1 + \varepsilon}{i} \min(1, 1 - (1 + \varepsilon)(\delta - R_f)) \frac{\log(N - K)}{i}.$$

This complete the reliability part. Note that the bound holds for large $K$ and $N$, and $\varepsilon$ independent of $K$ and $N$.

6) Information Leakage at the Eavesdropper: We now prove the security constraint is met. Hence, we wish to show that $I(W; Z^T)/T \to 0$, as $T \to \infty$. Denote by $C_T$ the random codebook and by $X_{S1}^T$ the set of codewords corresponding to the true defective items. We have,

$$\frac{1}{T} I(W; Z^T|C_T) = \frac{1}{T} \left( I(W, R_K R_K^F; Z^T|C_T) - I(R_K R_K^F; Z^T|WC_T) \right),$$

where $R_K$ is the random variable used by the encoder to choose the sub-bins. That is, the encoder has a random variable for each item. $R_K$ denotes the union of $K$ such variables, for the $K$ defective items. $R_K^F$ is again a union, this time of $K$ keys. These are $K$ “shares”, out of the $N$ shares generated by the MDS code. Since $W, R_K, R_K^F$ uniquely define $X_{S1}^T$, continuing from (1), we have:

$$= \frac{1}{T} \left( I(X_{S1}^T; Z_T|C_T) - I(R_K R_K^F; Z^T|WC_T) \right)$$

$$= \frac{1}{T} I(X_{S1}^T; Z_T|C_T) - H(R_K R_K^F|Z^T|WC_T) + H(R_K R_K^F|Z^T, W, C_T)$$

$$\leq \delta + \frac{1}{K} \left( R - R_f + \frac{T}{N} \right) + \frac{1}{T} H(R_K|Z^T, W, C_T) \leq \epsilon_f,$$

where $\epsilon_f \to 0$ as $T \to \infty$. (a) is since both $R_K$ and $R_K^F$ are independent of $W$ and the codebook. (b) is since both keys are uniform, the first includes $K$ variables of $T\left(\frac{1}{N} - \frac{1}{T} \right)$ bits each, and the second $K$ shares of $\frac{T}{N}$ bits each. (c) follows from [4, Section V.B]. In short, given the true $K$ defectors, the codebook and her output $Z^T$, Eve sees a simple MAC channel, at a rate slightly below her capacity. Therefore, she can identify which codeword was selected for each item, hence identify both $R_K$ and $R_K^F$.

**References**

[1] R. Dorfman, “The detection of defective members of large populations,” The Annals of Mathematical Statistics, vol. 14, no. 4, pp. 436-440, 1943.

[2] D. Z. Du and F. K. Hwang, “Combinatorial group testing and its applications (applied mathematics),” 2000.

[3] G. K. Atia and V. Saligrama, “Boolean compressed sensing and noisy group testing,” Information Theory, IEEE Transactions on, vol. 56, no. 3, pp. 1880-1901, 2012. A minor corection appered in vol. 61, no. 3, pp. 1507–1507, 2015.

[4] A. Cohen, A. Cohen, S. Jaggi, and O. Gurewitz, “Secure group testing,” arXiv preprint arXiv:1607.04849, 2016.

[5] J. M. Hughes-Oliver and W. H. Swallow, “A two-stage adaptive group-testing procedure for estimating small proportions,” Journal of the American Statistical Association, vol. 89, no. 427, pp. 982–993, 1994.

[6] M. Aldridge, “Adaptive group testing as channel coding with feedback,” in Information Theory Proceedings (ISIT), 2012 IEEE International Symposium on. IEEE, 2012, pp. 1832–1836.

[7] C. Aksoylar and V. Saligrama, “Information-theoretic bounds for adaptive sparse recovery,” in Information Theory, 2014 IEEE International Symposium on. IEEE, 2014, pp. 1311–1315.

[8] L. Baldassini, O. Johnson, and M. Aldridge, “The capacity of adaptive group testing,” in Information Theory Proceedings (ISIT), 2013 IEEE International Symposium on. IEEE, 2013, pp. 2676-2680.

[9] S. Aaron, V. Saligrama, and M. Zhao, “Information theoretic bounds for compressed sensing,” Information Theory, IEEE Transactions on, vol. 56, no. 10, pp. 5111–5130, 2010.

[10] G. E. Shannon, “The zero error capacity of a noisy channel,” Information Theory, IEEE Transactions on, vol. 2, no. 3, pp. 8–19, 1956.

[11] R. Ahlswede and N. Cai, “Transmission, identification and common randomness capacities for wire-tap channels with secure feedback from the decoder,” in General Theory of Information Transfer and Combinatorics. Springer, 2006, pp. 258–273.

[12] E. Ardestanizadeh, M. Franceschetti, T. Javidi, and Y.-H. Kim, “Wiretap channel with secure rate-limited feedback,” Information Theory, IEEE Transactions on, vol. 55, no. 12, pp. 5353–5361, 2009.

[13] A. J. Macula, “Probabilistic nonadaptive group testing in the presence of errors and DNA library screening,” Annals of Combinatorics, vol. 3, no. 1, pp. 61–69, 1999.

[14] D. Sejdinovic and O. Johnson, “Note on noisy group testing: asymptotic bounds and belief propagation reconstruction,” in Communication, Control, and Computing (Allerton), 2010 48th Annual Allerton Conference on. IEEE, 2010, pp. 998–1003.

[15] A. Cohen, A. Cohen, and O. Gurewitz, “Data aggregation over multiple applications (applied mathematics),” 2000.

[16] D.-Z. Du and F. K. Hwang, “Combinatorial group testing and its applications (applied mathematics),” 2000.

[17] A. J. Macula, “Probabilistic nonadaptive group testing in the presence of errors and DNA library screening,” Annals of Combinatorics, vol. 3, no. 1, pp. 61–69, 1999.

[18] J. Barros and J. Barros, “Secure group testing,” arXiv.org abs/1612.03314, 2017.

[19] M. L. Malloy and R. D. Nowak, “Near-optimal adaptive compressed sensing,” IEEE Transactions on Information Theory, vol. 60, no. 7, pp. 4001–4012, 2014.

[20] M. L. Malloy and R. D. Nowak, “Near-optimal adaptive compressed sensing,” IEEE Transactions on Information Theory, vol. 60, no. 7, pp. 4001–4012, 2014.

[21] A. Cohen, A. Cohen, S. Jaggi, and O. Gurewitz, “Secure adaptive group testing,” arXiv.org abs/1612.03314, 2017.

[22] M. Bloch and J. Barros, Physical-Layer Security: From Information Theory to Security Engineering. Cambridge University Press, 2011.