Adaptive PID Control with Forgetting Factor for Three-phase Asynchronous Motor

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Abstract. In order to solve these problems of large overshoot and poor stability of the speed-control loop for a three-phase asynchronous motor, one adaptive PID controller is applied. The controller combines least square method with forgetting factor to estimate such PID control parameters, which can adjust PID controller’s parameters in real time even when some abrupt disturbance occurs. The simulation results show that the proposed adaptive PID controller has some advantages, such as fast response, good stability, certain adaptability and robustness, compared with traditional one, and can meet the basic requirements for the speed control of asynchronous motor.

1. Introduction
Three-phase alternating current (AC) asynchronous motors are used widely in various industries and human daily life, for providing a variety of mechanical equipment and household appliances with power. However, its speed control is a multivariable, strongly coupled, nonlinear, high-order time-varying system [1]. Traditional double close-loop vector controller of induction motor [2,3] adopts the rotor flux orientation to decompose the three-phase stator current into torque component and excitation component, which control electromagnetic torque and rotor flux respectively. The method can realize the decoupling of stator currents that is equivalent to transform the AC pattern into direct current (DC) motor and obtain good control performances. But the speed control loop of vector control system usually uses classical PID controller [4]. Once these parameters are set, the PID controller is fixed and unchanged during the operating control process. Therefore, the nonlinearity of three-phase asynchronous motor cannot be precisely controlled, which may result in the AC motor control system less robustness and instability [5].

How to obtain these good performances of asynchronous motor on speed? We review many researches, nonlinear PID control [4], adaptive sliding mode control [6], fuzzy adaptive PID controller [7,8,9], neural network [10], genetic algorithm [11] and so on. But these control methods are too complex to realize practically and physically.

In this paper, we propose an improved recursive least squares parameter estimation with forgetting factor [12] that is applied to design one parametric adaptive PID controller [13]. Then we demonstrate the improved adaptive PID controller with Matlab/Simulink by adding some disturbances at different time points compared with traditional PID controller. The results reveal the robust, fast-response and other performances of the improved algorithm.

2. Basic Principles of Vector Control
In vector control systems [1,3,14,15], coordinate transformation is one core and important step. The control algorithm is to turn the stator AC current in static three-phase coordinate system into another
AC current in two-phase one by the rule of the same rotating magnetomotive force, and then to turn the transformed AC current into the equivalent DC current in synchronous rotating coordinate by transforming the two-phase one. Factually, the transformation is to decouple the magnetic flux and torque independently and obtain as the same control effect as the one of DC motor control. That is also to make AC motor control into such equivalent DC motor control and get the approximately good dynamic and steady-state performances from the DC motor control.

2.1. Rule of Equivalent Coordinate Transformation
The principle of the equivalence of motor models in different coordinate systems is that the resultant magnetic emfs, generated by the windings in different coordinate systems, must be are equal. To obtain the DC motor current from the original AC asynchronous current, there are two key transformation matrices [3] as bellowed.

2.2. Transformation Matrices
(a) Transformation matrix 1, also called as 3s/2s transformation, is expressed as Eq. (1), which is used to transform a stationary three-phase coordinate system into a stationary two-phase orthogonal one.

\[
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix}
1 & 1 \\
\sqrt{3} & 0 \\
\sqrt{3} & -\sqrt{3}
\end{bmatrix} \begin{bmatrix}
i_1 \\
i_2 \\
i_3
\end{bmatrix}
\]

(1)

(b) Transformation matrix 2, is also called as 2s/2r transformation, is to turn a stationary two-phase orthogonal coordinate system into a rotating two-phase orthogonal one.

\[
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} = \begin{bmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi
\end{bmatrix} \begin{bmatrix}
i_{\alpha} \\
i_{\beta}
\end{bmatrix}
\]

(2)

Therefore, the transformation matrix of rotating two-phase orthogonal coordinate system to stationary two-phase orthogonal coordinate system can also be obtained as:

\[
\begin{bmatrix}
i_\alpha \\
i_\beta
\end{bmatrix} = \begin{bmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{bmatrix} \begin{bmatrix}
i_d \\
i_q
\end{bmatrix}
\]

(3)

From Eq. (1) and (2) above, 3s/2r transformation and its inverse transformation (2r/3s transformation) can also be deduced:

\[
\begin{bmatrix}
i_\alpha \\
i_\beta \\
i_c
\end{bmatrix} = \begin{bmatrix}
\cos \phi & \cos(\phi - 2\pi/3) & \cos(\phi + 2\pi/3) \\
-\sin \phi & -\sin(\phi - 2\pi/3) & -\sin(\phi + 2\pi/3)
\end{bmatrix} \begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
\]

(4)

\[
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} = \begin{bmatrix}
\sin \phi & -\cos \phi & 1/2 \\
\sin(\phi + 2\pi/3) & -\cos(\phi + 2\pi/3) & 1/2 \\
\sin(\phi - 2\pi/3) & -\cos(\phi - 2\pi/3) & 1/2
\end{bmatrix} \begin{bmatrix}
i_d \\
i_q \\
i_0
\end{bmatrix}
\]

(5)

3. Adaptive PID Controller Design
According to the literature [16], the control system is shown as Fig.1. where is the transfer function of PID controller [13,16], shown by the following.

\[
D(s) = K_p + \frac{K_d}{s} + K_v s
\]

(6)
where $K_p$, $K_i$ and $K_d$ are proportional, integral and differentiate coefficients respectively. $W(s)$ is the transfer function of the control object. In industrial production and manufacturing, most of the controlled objects are non-oscillatory attenuation processes with self-balancing ability, and the transfer function can be approximated by second-order inertia plus hysteresis [16].

$$W(s) = \frac{K e^{-\tau s}}{(T_1 s + 1)(T_2 s + 1)}$$  \hspace{1cm} (7)

where $T_1$ and $T_2$ are time constants, $\tau$ is a time-lag.

Then closed-loop transfer function of the whole system is expressed as follows.

$$G_c(s) = \frac{Y(s)}{R(s)} = \frac{D(s)W(s)}{1 + D(s)W(s)}$$  \hspace{1cm} (8)

When transfer function $W(s)$ is known, the system closed-loop transfer function $G_c(s)$ is obtained according to those system requirements.

$$D(s) = \frac{G_c(s)}{W(s)(1 - G_c(s))}$$  \hspace{1cm} (9)

In order to achieve one optimal output response of the system, choose closed-loop transfer function $D(s)$ [16]. By substituting equation Eq. (7) to (9), we can obtain:

$$D(s) = \frac{(T_1 s + 1)(T_2 s + 1)}{K(1 - e^{-\tau s})}$$  \hspace{1cm} (10)

Take the first-order approximation of $e^{-\tau s}$ as $e^{-\tau s} = 1 - \tau s$, and obtain $D(s)$ as Eq. (11).

$$D(s) = \frac{(T_1 s + 1)(T_2 s + 1)}{K} = \frac{T_1 T_2 s^2 + (T_1 + T_2)s + 1}{K} = \frac{T_1 + T_2}{K} + \frac{1}{K\tau} + \frac{T_1 T_2}{K\tau}$$  \hspace{1cm} (11)

To compare Eq. (11) with (6), we can achieve

$$K_p = \frac{T_1 + T_2}{K}; K_i = \frac{1}{K\tau}; K_d = \frac{T_1 T_2}{K\tau}$$  \hspace{1cm} (12)

According to the above, parameters $K_p, T_1, T_2$ are required first. Now, transfer function $W(s)$ must be discretized, with sampling period $T \ll T_1, T_2$, pure delay $\tau$ being taken as an integer multiple of the sampling period, described as $\tau = LT$, $L$ is an integer. Then, $e^{\tau s} = z^L$. For the backward differential transformation $s = (1 - z^{-1}) / T$ [13], Eq. (7) can be expressed as,

$$W(z) = \frac{\theta_0 z^{-L}}{1 - \theta_0 z^{-1} - \theta_1 z^{-2}}$$  \hspace{1cm} (13)

where,

$$\begin{align*}
\theta_0 &= \frac{K_p T + 2 K_d}{K_p T + K_d + K_i T^2} \\
\theta_1 &= \frac{-K_d}{K_p T + K_d + K_i T^2} \\
\theta_2 &= \frac{K_p T}{(K_p T + K_d + K_i T^2)K_p L}
\end{align*}$$  \hspace{1cm} (14)
The differential equation of the controlled process can be obtained from Eq. (13), as shown (15).

\[
y = \theta_0 y(t-1) + \theta_1 y(t-2) + \theta_2 u(t-L) \tag{15}
\]

The parameter estimation of Eq. (15) is carried out by using the recursive least square method with forgetting factor [16], and then we can obtain these estimated values \( \hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2 \).

\[
K_p = \frac{\hat{\theta}_0 + 2\hat{\theta}_1}{\hat{\theta}_2 L}; \quad K_i = \frac{1 - \hat{\theta}_0 - \hat{\theta}_1}{\hat{\theta}_2 LT}; \quad K_d = \frac{\hat{\theta}_1 T}{\hat{\theta}_2 L} \tag{16}
\]

If it is a time-varying system, the differential equation of the controlled object is shown as Eq. (17).

\[
y = \theta_0 y(t-1) + \theta_1 y(t-2) + \theta_2 u(t-L) + \varepsilon(t) \tag{17}
\]

Let \( \phi^T(t) = [y(t-1) \quad y(t-2) \quad u(t-L)] \), and use least square updating with forgetting factor to get,

\[
\begin{align*}
P(t) &= \frac{(1 - K(t) \phi^T(t)) P(t-1)}{\lambda} \\
K(t) &= \frac{P(t-1) \phi(t)}{\lambda + P(t-1) \phi(t) \phi^T(t)} \\
\varepsilon(t) &= y(t) - \phi^T(t) \hat{\theta}(t-1) \\
\hat{\theta}(t) &= \hat{\theta}(t-1) + K(t) \varepsilon(t)
\end{align*} \tag{18}
\]

For slow time-varying parameters \( \theta \), relatively large ones \( \lambda \) should be selected, like \( \lambda = 0.99 \), and fast time-varying \( \theta \) small \( \lambda \), [16].

The PID parameter values at time \( t \) can be calculated from these estimated values \( \hat{\theta}_0(t), \hat{\theta}_1(t), \hat{\theta}_2(t) \).

\[
K_p = \frac{\hat{\theta}_0(t) + 2\hat{\theta}_1(t)}{\hat{\theta}_2(t)L}; \quad K_i = \frac{1 - \hat{\theta}_0(t) - \hat{\theta}_1(t)}{\hat{\theta}_2(t)L T}; \quad K_d = \frac{\hat{\theta}_1(t) T}{\hat{\theta}_2(t)L} \tag{19}
\]

Based on the above, the adaptive PID control algorithm derived is an implicit self-correcting control method [16], and its structural principle block diagram is shown as Fig. 2.

**Figure 1.** The block diagram of a one-unit feedback control system

**Figure 2.** Schematic diagram of adaptive PID controller
4. Simulated Analysis

4.1. Simulation Model Construction

The simulation model is shown as Fig.3 and 4 by Matlab/Simulink. The specific parameters of the asynchronous motor are selected as follows [3], rated power: \( P_N = 37.28859 \text{ W} \) (50HP), line voltage: \( U_N = 460 \text{ V} \), rated frequency: \( f = 50 \text{ Hz} \), stator internal resistance: \( R_s = 0.09961 \Omega \), stator leakage: \( L_s = 0.867 \text{ mH} \), rotor internal resistance: \( R_r = 0.23 \Omega \), rotor leakage: \( L_r = 0.867 \text{ mH} \), mutual inductance: \( L_m = 30.39 \text{ mH} \), the log of the pole: \( p = 2 \), moment of inertia: \( J = 0.65 \text{ kg} \cdot \text{m}^2 \), the dc voltage of IGBT inverter is set to 780V. In the adaptive PID controller, take the forgetting factor of least square parameter estimation, \( \lambda = 0.96 \), \( L = 6 \), \( T = 1/20000 \) [13].

![Figure 3. Simulation of three-phase AC asynchronous motor](image)

![Figure 4. Vector control module motor control system](image)

4.2. Results

The initial simulation conditions are as follows: given speed is 400rad/s, asynchronous motor load is 5 N\( \cdot \)m, and simulation time lasts 5s. These results are shown as Fig.5 (Blue represents adaptive PID, red represents traditional PID). We find the improved adaptive PID control has a short responding time (traditional PID: 0.20s, adaptive PID: 0.12s), a small overshoot (traditional PID: 12.5\%, adaptive PID: 10.9\%)(The first wave error should be ignored if it is too large), good stability and high accuracy with comparison of traditional PID scheme.
In order to test whether the adaptive PID control of the asynchronous motor has enhanced the adaptive and anti-interference ability compared with the traditional PID control, the load mutation of MATLAB is added at a given time.

The simulation conditions 1 are set as follows: given speed is 400rad/s; The initial motor load 5 N•m operation, and the load is suddenly added 25 N•m at 3s. The simulation results are shown in Fig. 6 (Blue represents adaptive PID, red represents traditional PID).

The simulation conditions 2: given speed is 400rad/s, initial motor load 5 N•m runs, and sudden load mutation increases to 25 N•m at 2s, and then decreases to 10 N•m again at 4s. The final simulation results are shown as Fig. 7 (Blue represents adaptive PID, red represents traditional PID).

These figures demonstrate that the traditional PID control speed fluctuates greatly when the load changes, and the speed has not been stable to 400rad/s, with poor anti-interference ability. However, the speed fluctuation of adaptive PID control is relatively small when the load changes, which is almost equivalent to none. Therefore, it has good adaptability and anti-interference ability.
5. Conclusion

Compared to the traditional PID controller, the adaptive PID controller has the following advantages:

1) From Fig. 5 the response time of adaptive PID controller is faster than that of traditional PID one, and is almost no amount of volatility and overshoot on speed curve. Thus, improved adaptive PID controller is able to better satisfy these basic requirement of AC motor control system.

2) From Fig. 6 and 7, when load changes bring up suddenly, the proposed adaptive PID controller has only a little volatility and quickly restore steady compared with traditional PID one. Therefore, it can be seen that the proposed adaptive PID control scheme has good anti-interference ability and can achieve strong stability in asynchronous motor speed-control.

This article is proposed one adaptive PID controller with forgetting factor recursive least squares parameter estimation, which can real-time correct these control parameters. The results show that the adaptive PID controller in speed control has good stability, anti-interference ability, and the rapid response, for three-phase ac asynchronous motor speed. Moreover, the adaptive PID control method is not only applicable to the control of asynchronous motor, but also applicable to the control of other speed regulation systems. Due to the uncertainty and contingency of the simulation experiment, a large number of experiments are still needed to verify and improve the improved algorithm. All in all, this simulation lays a foundation for further research and has important value in practical application.

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