The Hyperfine Splitting in Charmonium: Lattice Computations using the Wilson and Clover Fermion Actions

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Abstract

We compute the hyperfine splitting \( m_{J/\psi} - m_{\eta_c} \) on the lattice, using both the Wilson and \( O(a) \)-improved (clover) actions for quenched quarks. The computations are performed on a \( 24^3 \times 48 \) lattice at \( \beta = 6.2 \), using the same set of 18 gluon configurations for both fermion actions. We find that the splitting is \( 1.83 \pm 0.13 \) times larger with the clover action than with the Wilson action, demonstrating the sensitivity of the spin-splitting to the magnetic moment term which is present in the clover action. However, even with the clover action the result is less than half of the physical mass-splitting. We also compute the decay constants \( f_{\eta_c} \) and \( f_{J/\psi}^{-1} \), both of which are considerably larger when computed using the clover action than with the Wilson action. For example for the ratio \( f_{J/\psi}^{-1}/f_\rho^{-1} \) we find \( 0.32 \pm \frac{1}{2} \) with the Wilson action and \( 0.48 \pm 3 \) with the clover action (the physical value is 0.44(2)).
The Hyperfine Splitting  

Lattice computations of the vector-pseudoscalar mass-splittings $m_{D^*} - m_D$ and $m_{J/\psi} - m_{\eta_c}$, using the standard Wilson action for the quarks in the quenched approximation, give results which are much too small [1, 2]. At $\beta = 6.2$, for which the inverse lattice spacing $(a^{-1})$ is approximately 2.7 GeV, the discrepancy is about a factor of 2 for $m_{D^*} - m_D$ and a factor of about 4 for $m_{J/\psi} - m_{\eta_c}$. In this letter we compute the hyperfine splitting $m_{J/\psi} - m_{\eta_c}$ using both the Wilson fermion action,

$$S_F^W = a^4 \sum_x \left\{ \bar{q}(x)q(x) + \kappa \sum_{\mu} \left[ \bar{q}(x)(\gamma_{\mu} - r)U_\mu(x)q(x + \hat{\mu}) \right. \
- \left. \bar{q}(x + \hat{\mu})(\gamma_{\mu} + r)U_{\mu}^\dagger(x)q(x) \right] \right\}$$

and the nearest-neighbour $O(a)$–improved (or “clover”) fermion action [3],

$$S_F^C = S_F^W - ir g_0 \kappa a^4 \sum_{x,\mu,\nu} \bar{q}(x) F_{\mu\nu}(x) \sigma_{\mu\nu} q(x)$$

with the same set of 18 gluon configurations in each case. The computations are performed on a $24^3 \times 48$ lattice at $\beta = 6.2$ with $r = 1$. These 18 configurations have been used earlier in our study of light quark spectroscopy and meson decay constants, the results and computational details can be found in ref.[4, 5]. For the quantities studied in [4] the results obtained with the two actions were broadly compatible. However, here we show that the hyperfine splitting in charmonium is almost a factor of 2 larger with the clover action than with the Wilson action (see eq.(3) below), demonstrating the sensitivity of this quantity to the magnetic moment term in eq.(2). We also present the results for the decay constants of the $\eta_c$ and $J/\psi$ mesons.

Qualitatively similar results for the hyperfine splitting were obtained by the Fermilab group [6], who have performed simulations using a fermion action which is similar to that in equation 2 but with a factor of 1.4 multiplying the second term. This factor is their estimate of the effects of higher order perturbative corrections, and was obtained using a mean field theory calculation [7]. These authors also find a larger value of the hyperfine splitting with their action than with the Wilson action. However this comparison is obtained from computations on lattices of different size. Below we will compare our results with those of ref.[6].

For this study we took $\kappa = 0.1350$ for the Wilson action and $\kappa = 0.1290$ for the clover action. These values were chosen so that the pseudoscalar meson masses are almost equal for both actions, and correspond approximately to the physical mass of the $\eta_c$. In table we
Table 1: Masses (in lattice units) of the pseudoscalar and vector heavy-heavy mesons.

|        | $m_{\eta_c}a$ | $m_{J/\psi}a$ | $(m_{J/\psi} - m_{\eta_c})a$ |
|--------|---------------|---------------|-------------------------------|
| Wilson | $1.066^{+6}_{-4}$ | $1.076^{+4}_{-5}$ | $0.0104^{+8}_{-7}$           |
| clover | $1.071^{+6}_{-4}$  | $1.088^{+5}_{-5}$  | $0.0190^{+12}_{-16}$         |

present our results for the masses of the vector and pseudoscalar mesons (in lattice units), and for their difference. Setting the scale from the string tension gives $a^{-1} = 2.73(5)$ GeV \cite{4} and using this value we see that the masses of the mesons are within a few percent of their physical values. The masses were obtained by fitting the correlation functions in the time range $t = 14 - 20$, and the mass differences were obtained by fitting the ratio of the vector and pseudoscalar propagators over the same range. The fits were performed taking the correlations between the values at different time slices into account, and the values of $\chi^2/d.o.f$ were acceptable. The errors presented in this letter were obtained using the bootstrap procedure, described in detail in ref.\cite{4,5}.

The main result of this letter comes from the entries in the third column of table 1, from which it is clear that the hyperfine splittings are very different for the two actions. We stress that the results were obtained using the same gluon configurations and with the same analysis techniques. It is therefore likely that a number of the systematic errors would cancel in the ratio, for which we find:

$$\frac{(m_{J/\psi} - m_{\eta_c})_{\text{clover}}}{(m_{J/\psi} - m_{\eta_c})_{\text{Wilson}}} = 1.83^{+13}_{-15}$$ \hspace{1cm} (3)

In fig. 1 we plot the values of $m_V^2 - m_P^2$ (where $V$ and $P$ represent vector and pseudoscalar respectively) as a function of the square of the pseudoscalar mass. We include not only the values for charmonium obtained from table 1, but also those for mesons composed of light quarks for three different light quark masses \cite{4}. We see that for small masses the two actions give similar results, but as the mass is increased a gap gradually opens, with the clover action giving a larger value for the hyperfine splitting.

It is interesting to note that the suggestion that this quantity might be susceptible to lattice artefacts has been made previously by the APE collaboration in a comparative study \cite{8} of

\footnote{The values of the inverse lattice spacing obtained by comparing the lattice values of the masses of the light hadrons to the physical ones lie within 15\% of the result from the string tension. We take this as an indication of the size of the systematic uncertainty.}
Figure 1: $m_V^2 - m_P^2$ versus $m_P^2$, in lattice units, for the Wilson and clover actions.

Wilson and staggered fermion actions, albeit at stronger coupling and lighter quark masses than the present work.

The result for the hyperfine splitting of charmonium obtained with the clover action is still much smaller than the experimental value. Taking $a^{-1} = 2.73$ GeV, the values in table I correspond to:

$$m_{J/\psi} - m_{\eta_c} = 28 \pm \frac{2}{2} \text{MeV} \quad \text{Wilson Action} \quad (4)$$

$$m_{J/\psi} - m_{\eta_c} = 52 \pm \frac{3}{4} \text{MeV} \quad \text{Clover Action} \quad (5)$$

to be compared to the experimental value of 117(2) MeV. The errors quoted in (4) and (5) are statistical only, and the reader should bear in mind the uncertainty in the value of the lattice spacing. The corresponding values found by El-Khadra et al. [6] are: 51(3) MeV at $\beta = 5.7$, 62(4) MeV at $\beta = 5.9$ and 68(5) MeV at $\beta = 6.1$. Extrapolating linearly in $a^2$ to the continuum limit, these authors quote:

$$m_{J/\psi} - m_{\eta_c} = 73 \pm 10 \text{MeV} \quad (6)$$

where the error includes an estimate of the systematic uncertainty. The lattice spacing in this work was determined from the 1P-1S mass splitting (a quantity which is considerably less sensitive to the form of the action [6]). Thus it appears that there is a difference of about 20 MeV due to the different action used in ref. [6].
**Decay Constants**  The (dimensionless) decay constant of the $J/\psi$-meson is defined by:

$$
\langle 0 | \bar{c}(0) \gamma_\mu c(0) | J/\psi \rangle = \epsilon_\mu \frac{m_{J/\psi}^2}{f_{J/\psi}} 
$$

(7)

where $\epsilon_\mu$ is the polarisation vector of the $J/\psi$. The measured value of the decay constant is $1/f_{J/\psi} = 0.124(5)$. In our computations we take for the lattice vector current, the local operator $Z_V^W \bar{c}(0) \gamma_\mu c(0)$ when using the Wilson action, and the “improved” current

$$
Z_V^C \bar{c}(x)(1 + \frac{ra}{2} \gamma \cdot \vec{D}) \gamma_\mu (1 - \frac{ra}{2} \gamma \cdot \vec{D}) c(x)
$$

(8)

when using the clover action. $Z_V^W$ and $Z_V^C$ are the renormalisation constants (which ensure that the currents are correctly normalised), and can be evaluated in perturbation theory. We obtain the results

$$
\frac{1}{Z_V^W} \frac{1}{f_{J/\psi}} = 0.152 \pm 0.5 \quad \text{Wilson Action} \quad (9)
$$

$$
\frac{1}{Z_V^C} \frac{1}{f_{J/\psi}} = 0.179 \pm 0.7 \quad \text{Clover Action} \quad (10)
$$

The $Z_V$’s have been calculated to one-loop order, $Z_V^W=0.83$ and $Z_V^C=0.90$ if the lattice bare coupling constant $g_0^2$ is used as the expansion parameter of perturbation theory, whereas $Z_V^W=0.71$ and $Z_V^C=0.83$ if an “effective” coupling $g_{eff}^2 = 1.75g_0^2$ is used (following suggestions in ref. [9]). For example using the effective coupling we find $1/f_{J/\psi} = 0.108 \pm 0.4$ using the Wilson action, and $1/f_{J/\psi} = 0.149 \pm 0.7$ with the clover action. Hence the value of $1/f_{J/\psi}$ is about 30–40% larger with the clover action than the Wilson action (although the uncertainty in the values of the renormalisation constants should be borne in mind).

The decay constant of the $\eta_c$ has not been measured, however we present the lattice results in order to compare the values obtained with the two actions. The decay constant is defined by

$$
|\langle 0 | \bar{c}(0) \gamma_\mu \gamma_5 c(0) | \eta_c(p) \rangle| \equiv f_{\eta_c} P_\mu
$$

(11)

(with such a normalisation $f_\pi \simeq 132$ MeV). For the lattice axial current we take the same operators as in the vector case with $\gamma_\mu \rightarrow \gamma_\mu \gamma_5$. The corresponding results are:

$$
\frac{1}{Z_A^W} f_{\eta_c} a = 0.130 \pm 0.6 \quad \text{Wilson Action \quad (12)}
$$

$$
\frac{1}{Z_A^C} f_{\eta_c} a = 0.149 \pm 0.9 \quad \text{Clover Action \quad (13)}
$$
$Z_A^W$ and $Z_A^C$ are equal to 0.87 (0.78) and 0.98 (0.97) when the bare (effective)coupling is used as the expansion parameter. Thus the values of $f_{\eta_c}$ are also about30–40% larger when determined using the clover action than those obtained with the Wilson action.

From eqs.(9)-(13) we note that both the quantities $1/Z_V f_{J/\psi}^{-1}$ and $1/Z_A f_{\eta_c}$ are larger when obtained using the clover action than the Wilson action. This is opposite to the results for thecorresponding quantities for the light mesons $\pi$ and $\rho$. Thus the difference obtained for the decay constants with the two actions is amplified significantly in the ratios $f_{J/\psi}^{-1}$ and $f_{\eta_c}$.

Using the chiral extrapolations for $f_\pi$ and $f_\rho^{-1}$, we find

$$\frac{f_{J/\psi}^{-1}}{f_\rho^{-1}} = 0.32 \pm \frac{1}{2} \quad \text{Wilson Action (14)}$$
$$\frac{f_{J/\psi}^{-1}}{f_\rho^{-1}} = 0.48 \pm \frac{3}{3} \quad \text{Clover Action (15)}$$

where the physical value of this ratio is 0.44(2), and

$$\frac{f_{\eta_c}}{f_\pi} = 2.3 \pm \frac{5}{3} \quad \text{Wilson Action (16)}$$
$$\frac{f_{\eta_c}}{f_\pi} = 4.0 \pm \frac{12}{9} \quad \text{Clover Action (17)}$$

In these ratios the dependence on the renormalisation constants cancels. The differencesbetween the results for the two actions in eqs.(14)-(17) indicate significant errors due to thefiniteness of the lattice spacing (at least, presumably, for the Wilson action) in the decayconstants for the charmonium system.

**Conclusions** In this letter we have shown that the value of the hyperfine splitting incharmonium in lattice simulations is very sensitive to the fermion action which is used, andin particular to the magnetic moment term in the improved action. Using the same gluonconfigurations, we have found that at $\beta = 6.2$, the ratio of the splittings for the clover andWilson actions is about 1.8 (see eq.(3)). Even using the clover action, the value we obtainfor the hyperfine splitting is only about one half of the physical value. The clover action isa “tree-level improved action”, i.e. there are no errors of $O(\alpha_s)$, but the leading remainingerrors due to the finiteness of the lattice spacing are of $O(\alpha_s a)$. Presumably at least someof the discrepancy between the value in eq.(3) and the physical one of 117(2) MeV is due to
these remaining $O(\alpha_s a)$ errors. El-Khadra et al. have tried to reduce these by performing a mean field calculation to estimate the effects of the higher-order perturbative terms on the magnetic moment term in the action $\mathbb{I}-\mathbb{I}$. Their result for the hyperfine splitting, of about 73 MeV, although larger than that in eq.(5), is still considerably smaller than the physical value. In view of the sensitivity of the splitting to the magnetic moment term in the action, it is likely that at least part of the discrepancy is still due to the finiteness of the lattice spacing. Unfortunately it is not possible at present to determine how much of the discrepancy is due to the inadequacy of the mean field calculation, and how much to other systematic errors (such as quenching). We are forced to accept that the hyperfine splitting is currently not amenable to an accurate lattice determination. Nevertheless it is reassuring to find that one of the few quantities for which lattice computations with the Wilson fermion action give a result which disagrees significantly with experiment, is unusually sensitive to known systematic errors.

For the decay constants of charmonium we also found significant differences between the values obtained using the two actions. For the ratio $f_{J/\psi}^{-1}/f_{\rho}^{-1}$ (see eqs.(14) and (15)) we find a result which is about 75% of the physical one with the Wilson action (consistent with the results in ref.[2]), and a result which is consistent with the physical one with the clover action. For the decay constant of the $\eta_c$ (see eqs.(16) and (17)) we find a larger result with the clover than with the Wilson action (note however that the error for the clover action is large, this error is dominated by the uncertainty in the extrapolated value of $f_\pi$).

It will be very interesting to repeat this study for heavy-light mesons. In particular simulations with Wilson fermions indicate a substantial violation of the scaling law $f_P \sqrt{M_P} \simeq$ constant (up to logarithmic corrections) $\mathbb{I}-\mathbb{I}$ for the decay constants of heavy-light pseudoscalar mesons $P$. This would imply a value of $f_B$ of about 200 MeV (one which is consistent with the simulations using the static approximation $\mathbb{I}-\mathbb{I}$), larger than many earlier expectations. It is now important to check whether these results will be stable under the reduction of the errors of $O(a)$ achieved by the use of the clover action.

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