P-modes in rapidly rotating stars – looking for regular patterns in synthetic asymptotic spectra

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The dates of receipt and acceptance should be inserted later

Key words Hydrodynamics - Waves - Chaos - Stars: oscillations - Stars: rotation

According to a recent ray-based asymptotic theory, the high-frequency p-mode spectrum of rapidly rotating stars is a superposition of frequency subsets associated with dynamically independent regions of the ray-dynamics phase space. At high rotation rates corresponding to typical δ Scuti stars, two frequency subsets are expected to be visible: a regular frequency subset described by a Tassoul-like formula and an irregular frequency subset with specific statistical properties. In this paper, we investigate whether the regular patterns can be detected in the resulting spectrum. We compute the autocorrelation function of synthetic spectra where the frequencies follow the asymptotic theory, the relative amplitudes are simply given by the modes’ disk-averaging factors, and the frequency resolution is that of a CoRoT long run. Our first results are that (i) the detection of regular patterns strongly depends on the ratio of regular over irregular modes, (ii) low inclination angle configurations are more favorable than near equator-on configurations (iii) in the absence of differential rotation, the $2\Omega$ rotational splitting between modes $m = 1$ and $m = -1$ modes is an easy feature to detect.

1 Introduction

The launching of the space missions COROT and KEPLER is bringing a new wealth of observational data on the asteroseismology of many different types of stars. For slowly rotating stars, e.g. the sun, the approximate spherical symmetry of the system enables us to classify modes according to well-defined sets of integers. This mode structure allowed us to extract information on the stellar interior from the spectrum of oscillation modes [Christensen-Dalsgaard 2002].

However, for rapidly rotating stars, the deformation of the star breaks the spherical symmetry. It has recently been shown that perturbation theory fails to yield accurate predictions of oscillation spectra even for stars with moderate rotation rates [Lignières et al. 2006, Reese et al. 2006]. In Lignières & Georgeot (2008, 2009), an asymptotic theory of spectra of such stars was built, based on acoustic ray dynamics. It was shown that this dynamics is Hamiltonian, and undergoes a gradual transition from integrability to chaos when the rotation increases. At moderately rapid rotation, the phase space is divided into integrable and chaotic zones, where the dynamics is qualitatively different. It was shown in Lignières & Georgeot (2008, 2009) that this structure modifies the structure of the oscillation spectra. The acoustic mode frequencies cannot be in general associated to well-defined sets of integers. Instead, the spectrum is split into independent sub-spectra, corresponding to different phase space zones for the ray dynamics. The subspectra corresponding to integrable zones give rise to regular sequences of frequencies, whereas a superimposed subspectrum is associated to chaotic dynamics and displays regularity only in a statistical sense.

In order to connect such results to observed spectra, several questions have to be explored. The first one is the validity of the asymptotic theory to the finite range of spectra which can be observed. A first answer was given in Lignières & Georgeot (2008, 2009), where it was shown that the predicted structure can be found in numerically computed relatively low-frequency p-modes of polytropic stellar models. An other important question is to separate these different subspectra in observational data, where the visibility of the modes plays an important role. A first step in this direction is to construct synthetic spectra based on the asymptotic theory where visibility varies from mode to mode and try to extract from them important features of the system, as if they were observed spectra. This is the strategy we follow here. We also note that a first evidence of regular patterns in a p-mode spectrum of a rapidly rotating star has been recently obtained by analyzing the spectra of a δ Scuti CoRoT target [García Hernández et al. 2009].
2 Construction of synthetic spectra in the asymptotic regime

We have built synthetic spectra with the following formulas:

\[ S(\nu, i) = \sum_j L(\nu, \nu_j, A_j(i), \Gamma) \]  

\[ L(\nu, \nu_j, A_j(i), \Gamma) = A_j(i) \exp \left[ - \frac{(\nu - \nu_j)^2}{\Gamma^2} \right] \]

where \( \nu_j \) are the frequencies of the modes and \( A_j(i) \) their amplitudes, which are functions of the inclination angle \( i \).

We have represented modes with Gaussian functions of width \( \Gamma \) to take into account the finite resolution of the spectrum. We have considered in this work a resolution \( \Gamma \) corresponding to 150-day-long runs. Our main assumptions are that the frequencies follow the asymptotic behaviour described in Lignières & Georgeot (2009) and that the relative amplitudes only depend on the relative mode visibilities, the visibility being approximated by the disk-averaging factor computed for high-frequency p-modes in a polytropic stellar model (see formula (37) in Lignières & Georgeot (2009)).

Above a certain rotation rate, the p-modes spectrum is dominated by two families of modes, the island modes and the chaotic modes, as the visibilities of the other modes become negligible. The island modes are associated with island chain structures of the ray dynamics phase space and their spectrum follows a simple formula (Reese et al., 2008).

\[ \omega_{\tilde{n}\tilde{m}} = \Delta\tilde{n} + \Delta\tilde{\ell} + \Delta m|m| - m\Omega + \omega_{\text{ref}} \]

where \( \tilde{n} \) and \( \tilde{\ell} \) are defined from the spatial distribution of the modes in a meridional plane. They correspond respectively to the number of nodes along and across the stable periodic trajectory associated with the island chain. By contrast, chaotic modes are associated with chaotic regions of phase space, and their frequency spectrum is said to be irregular because it is not described by a smooth function of 3 integers. The frequency spectrum has nevertheless specific statistical properties. Indeed, if we consider chaotic modes of the same symmetry class, the distribution of the consecutive frequency spacings \( \sigma_i = \omega_{i+1} - \omega_i \) (scaled by the mean frequency spacing \( \langle \omega_{i+1} - \omega_i \rangle \)) is close to the parameter-free Wigner distribution \( P(\sigma) = \pi\sigma/2 \exp(-\sigma^2/4) \) in accordance with the prediction of Random Matrix Theory. A symmetry class corresponds to a given azimuthal number \( m \) and a given symmetry (+ or −) with respect to the equator. For each symmetry class, a spectrum is thus determined from a realization of the Wigner distribution and the chaotic spectrum is the superposition of all these spectra. Finally in order to construct the total spectrum, we need to know the ratio between chaotic and island modes. This ratio tends to increase with rotation and, for the rotation rate considered in the following \( \Omega = 0.6\Omega_K \) where \( \Omega_K = \sqrt{GMR/\delta} \), it is close to 3.7 (Lignières & Georgeot, 2009).

3 Autocorrelation of the synthetic spectra

The autocorrelation of the synthetic spectra has been investigated, for \( \Omega = 0.6\Omega_K \), by varying three parameters: the frequency range of the spectrum, the inclination angle of the star, and an amplitude threshold that only retains the highest amplitude peaks of the spectrum. The parameter domain that we explored is the following: The frequency range spans \( n_\delta \) large separations \( \delta_n = 2\Delta\delta \), where \( n_\delta \) has been varied from 1 to 6. The amplitude threshold is chosen to keep \( N_\delta \) frequency peaks per large separation interval; \( N_\delta \) has been varied between 10 and 100. Once \( n_\delta \) and \( N_\delta \) are fixed, the total number of frequencies of the spectrum is \( n_\delta \times N_\delta \). The inclination angle has been varied from 0 to 90°.

We find that the inclination angle plays an important role in the search for regular patterns because it affects the relative visibility of the chaotic and island modes. Indeed, the disk-averaging factor of the chaotic modes tends to increase towards equator-on configurations while the disk-averaging factor of island modes tends to decrease.

Let us first consider a high inclination configuration \( i = 63° \). In figures 1 and 2, the autocorrelation of the whole spectrum (top panel), the chaotic spectrum (middle) and the island spectrum (bottom) are displayed for \( N_\delta = 57 \) and for two different frequency ranges \( n_\delta = 3 \) (Fig. 1) and \( n_\delta = 6 \) (Fig. 2). The autocorrelation is defined as in statistics: it is computed after substracting the mean and it is normalized by the variance. The autocorrelation is therefore comprised between -1 and 1. The vertical scale is different for the autocorrelation of the island spectrum because, as expected, the Tassoul-like formula leads to strong peaks associated with the regular spacing and smaller peaks corresponding to linear combinations of \( \Delta\tilde{n}, \Delta\tilde{\ell}, \Delta m \).

No structure is seen in the chaotic mode spectrum except for a peaks at \( 2\Omega \) (corresponding to the \( 2\Omega/\delta_n = 0.836 \) peak of Fig. 1). This lack of structure can be understood from the fact that the chaotic spectrum is a superposition of the subspectra corresponding to the different symmetry classes \( m^\pm \). These subspectra are statistically independent and, as a consequence, the consecutive difference statistics of the chaotic spectrum is generally close to a Poisson statistics thus explaining the featureless autocorrelation of the pole-on configuration is an exception to this rule as only \( 2\Omega \) peaks are visible in this case and the superposition of only two independent subspectra does not lead to Poisson statistics. The \( 2\Omega \) peaks is due to a rotational splitting between \( m = 1 \) and \( m = -1 \) modes. In the p-modes asymptotic regime, the Coriolis force has indeed a negligible effect on the frequency because its characteristic time-scale is much larger than the oscillation time-scale. As shown in (Reese et al., 2006), this asymptotic property is actually already correct at relatively low frequencies. Since the centrifugal force does not distinguish between \( -m \) and \( m \) modes, their frequencies can be considered as degenerate in the rotating frame thus leading to a \( 2m\Omega \) splitting in
Fig. 1  Autocorrelation of a high-frequency p-modes spectrum resulting from the superposition of a chaotic mode spectrum and an island mode spectrum. The top panel displays the autocorrelation function of the whole spectrum, while the middle and bottom panels shows the autocorrelation of the chaotic and island mode spectra, respectively. The x-axis represents the frequency lag in units of the large separation $\delta_n$. The inclination angle is equal to $i = 63^\circ$, the frequency range corresponds to 3 large separations and the 171 highest amplitude frequencies have been retained in the spectrum. The only significative feature of the autocorrelation function is the $2\Omega$ peak also seen in the autocorrelation of the chaotic spectrum.

As shown in the top panel of Fig. 1, signatures of the island mode regular patterns are not apparent in the autocorrelation of the total spectrum. This is due to the fact that the ratio of chaotic modes over island modes is too high in this case. Increasing the frequency range to 6 large separations (Fig. 2) enables us to detect a peak at the large separation $\delta_n$ (the peak seen at a scaled frequency lag of unity on Fig. 1 or Fig. 2). This is expected as a larger frequency range leads to the build up of the peaks associated with the periodic features of the frequency spectrum. For the present high inclination angle configuration, limiting the analysis to the highest amplitude peaks does not help because in average the visibility of the island modes is not higher than the visibility of the chaotic modes.

As illustrated in figures 3 and 4 for $i = 30^\circ$, low inclination configurations are more favorable in order to detect the island mode regular patterns. The autocorrelation functions are shown for a frequency range $n_\delta = 3$ and for two different amplitude threshold $N_\delta = 68$ (Fig. 3) and $N_\delta = 18$ (Fig. 4). In both cases, features associated with the island mode regular patterns are detected in the total autocorrelation function. This is because, as compared to the high inclination case, the proportion of island modes in the total spectrum has increased. The comparison of figures 3 and 4 also shows that, contrary to the high inclination case, the island mode regular patterns are more easily detected if the analysis is limited to the highest amplitude modes. Finally we checked that, as expected, the signature of these patterns is stronger if the frequency range is increased.

4 Discussion and conclusion

Although based on strong simplifying assumptions, the present study should provide some clues to conduct a search for regular patterns in observed p-mode spectra of rapidly
rotating stars. For example, we find that limiting the search for regular patterns to the highest amplitude frequency peaks is not necessarily helpful. This depends on the inclination angle. Generally, low inclination configurations are more favorable because the island modes are relatively more visible than the chaotic modes. A robust feature of the autocorrelation functions computed in the present study is the $2\Omega$ peak. This feature does not rely on the asymptotic regime assumption, it rather requires that the Coriolis force has a negligible effect which is already true at relatively low p-mode frequency. Possible effects of the differential rotation on this peak should nevertheless be tested, for example using the formalism described in [Reese et al. (2009)].

The two most important assumptions in the construction of the synthetic spectra concern the asymptotic regime and the mode amplitudes. We assumed that island mode frequencies strictly follow the asymptotic formula (3) while it is in fact only approximate due to finite wavelength effects (see the discussion in [Lignières & Georgeot (2009)]). The departures from the asymptotic formula (3) have been determined for different stellar models and frequency ranges in [Lignières et al. (2006), Reese et al. (2008) and Reese et al. (2009)]. As a first step, a simple way to model these non-asymptotic effects in the present study would be to decrease the frequency resolution of the synthetic spectra. A second step would be to compute numerically a complete spectrum using the code TOP [Reese et al. (2006)]. Such computations are nevertheless very time consuming. Thus, simple models of the spectra like the one presented in this study will still be useful as they enable us to cover a wider range of parameters. In what concerns the amplitudes of the modes, we lack a consistent theory to describe their excitation and non-linear saturation in rapidly rotating stars. The present model could nevertheless be improved through better determinations of the mode visibilities, including more realistic stellar models as well as non-adiabatic, limb-darkening and gravity darkening effects. The mode inertia could also be taken into account in modelling the amplitudes.

Acknowledgements. We thank D. Reese for fruitful discussions. This work was supported by the Programme National de Physique Stellaire of INSU/CNRS and the SIROCO project of the Agence National de la Recherche.

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Fig. 4  Same as Fig 3. except that only the 54 highest amplitude frequencies have been retained. This helps to detect some of the correlation peaks that were not significant in the previous case.