Hořava gravity: motivation and status

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Abstract. In this short contribution, I will review the status of Hořava gravity as a proposal for quantum gravity. After summarizing the framework and phenomenological constraints, I will present some recent work on the renormalization group flow in 2 + 1 dimensions. The main result is that the theory is asymptotically free and flows to an infrared fixed point connected to a Lorentz-invariant theory.

1. Introduction

The proposal of P. Hořava [1] stems from a simple question: is there a viable, unitary quantum field theory of gravity in 3 + 1 dimensions described by a perturbative fixed point at high energies? The answer to this question seems to be negative in Lorentz-invariant theories. Dropping the different requirements one finds a diversity of proposals for quantum gravity with various levels of complexity and success. As an example, one can consider higher-derivative gravity [2]. This yields a renormalizable theory of quantum gravity, which can even be asymptotically free in 3 + 1 dimensions [3]. However, the theory has problems with perturbative unitarity and stability, both related to the presence of higher time derivatives. Inspired by this proposal, P. Hořava suggested to study gravitational theories with higher spatial derivatives, but only two time derivatives to preserve unitarity. Schematically, one writes for the free part of the graviton field $h$

$$S = \int d^3x dt \left( \dot{h} \dot{h} - h(-\Delta)^z h \right).$$

If this term is dominant at high energies, the proper scaling to understand the relevance of interactions is

$$t \to \lambda^{-z} t, \quad x^i \to \lambda^{-1} x^i, \quad h \to \lambda^{3-z} h.$$

As a consequence, for $z = 3$, $h$ is dimensionless, and the operators with additional factors of $h$, with the number of spatial derivates bounded by $2z$ and only two time derivatives, are all marginal or relevant. If the number of such operators is finite, one deals with a power-counting renormalizable theory.

2. Hořava gravity in a nutshell

The scaling laws [2] and the action [1] break the boost transformations of Lorentz invariance. The implication when building a gravitational theory is that the invariance under diffeomorphisms must be broken to a subgroup compatible with [2]. To do this, one assumes that
space-time is endowed with a preferred time foliation, which selects a preferred time direction \( t \) at each point of space-time. This structure preserves the symmetries

\[ x^i \mapsto \bar{x}^i(x,t), \quad t \mapsto \bar{t}(t), \]

(3)
called foliation-preserving diffeomorphism. A gauge theory invariant under (3) requires the introduction of a metric,

\[ ds^2 = N^2 dt^2 - \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \]

(4)
with the subtlety that the field \( N \) can depend only on time (projectable case) or be generic (non-projectable). The gravitational dynamics of the theory follows from the action built from the metric, invariant under (3) and having a maximum of two time derivatives and six space derivatives (to realize the relevant scaling at high energies). Schematically,

\[ S = \frac{\bar{M}_P}{2} \int d^3x dt \sqrt{\gamma} N \left[ K_{ij} K^{ij} - (1 - \lambda') K^2 - \eta R - \alpha a_i a^i + \cdots + \frac{\Delta^2 R^2}{M_*^2} \right], \]

(5)
where \( K_{ij} \) is the extrinsic curvature of the foliations, and \( a_i \equiv \partial_i \log N \). The constant \( \bar{M}_P \) is a renormalization of the Planck mass \( M_P \) by the parameters in the theory [4]. The symbol \( \Delta \) refers to the three-dimensional Laplacian \( \gamma^{ij} \partial_i \partial_j \) and \( R \) is the three-dimensional Ricci scalar. In fact, here and in the following the quantities referring to intrinsic geometrical properties are those of the three-dimensional metric \( \gamma_{ij} \). For instance, \( R_{ij} \) will be the three-dimensional Ricci tensor. The ellipses represent all the operators with more than two space derivatives. There is a finite number of them and we assume that all of them are suppressed by a common scale \( M_* \).

The terms before the ellipses are enough to describe the theory at energies \( E < M_* \). For the proposal to accomplish its purpose, the new high-energy scaling law should be effective before the theory becomes strongly coupled, \( M_* < \sqrt{c} M_P \), where \( c \) is a combination of the low-energy parameters representing the 'largest' of them [5]. This form of Hořava gravity was first presented in [6].

The projectable case implies \( a_i = 0 \). In this case, the number of spatial operators is relatively small in 3 + 1 dimensions. Denoting by \( \mathcal{V} \) the part of (5) containing only space derivatives, one finds

\[ \mathcal{V} = 2\lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij} R^{ij} + \nu_1 R^3 + \nu_2 R_{ij} R^{ij} + \nu_3 R^i_i R^j_j R^k_k + \nu_4 \nabla_i R \nabla^i R + \nu_5 (\nabla_i R_{jk})^2, \]

(6)
with arbitrary constants. Note that the constants \( \mu_i \) and \( \nu_i \) are dimensionful, and we assume that its order of magnitude is given a common scale \( M_* \). Unfortunately, the Minkowski background is unstable in the projectable case in 3 + 1 dimensions. This can be seen by realizing that the scalar (s) and tensor (TT) perturbations around it obey dispersion relations

\[ \omega_s^2 = \frac{\lambda}{3\lambda - 2} (-\eta k^2 + \mu_1 k^4 + \nu_4 k^6), \quad \omega_{TT}^2 = \eta k^2 + \mu_2 k^4 + \nu_5 k^6. \]

(7)
The different parameters in this formula are combinations of the constants in (6), cf. [7]. Since \( \frac{\lambda}{3\lambda - 2} > 0 \) from the absence of ghosts, we see that there is no choice: either the scalar or the tensor modes are unstable. The non-projectable case does not suffer from this problem, neither from any other known instability. However, since \( a_i \neq 0 \), the number of operators is substantially larger than in the projectable case. Furthermore, for the projectable case, it has been shown that renormalizability happens beyond power counting, which is a non-trivial check for gauge theories [7][8]. The similar proof for the non-projectable case is currently missing due to the difficulty in treating the instantaneous modes present in the theory.
3. Phenomenology of non-projectable Hořava gravity

Since no instability is known for the non-projectable case, it is worth studying further other consequences of the previous proposal for gravitational physics. For the rest of fields in the standard model of particle physics Lorentz invariance has been tested to high accuracy [9, 10, 11], while for dark matter these tests has been proposed in [12, 13].

The gravitational tests of the proposal have been summarized in [4]. They are related to the presence of new massless extra degrees of freedom, modification of the graviton’s properties and other tests of the existence of a preferred frame. One can establish a first classification: tests of the physics at energies $E > M_*$ or at $E < M_*$. In the first case, one explores the core of the proposal: the theory deviates completely from general relativity. For instance, Newton’s law will be modified by an expression of the form

$$\Delta \left(1 + a_1 \frac{\Delta}{M_*^2} + \ldots \right) = \frac{\tau^{00}}{2M_P^2},$$

where by $\tau^{00}$ I mean the time-time component of the energy momentum tensor. The new terms are important to resolve the problem of singularities in this proposal, though the study of the structure of singularities is at its infancy, see e.g. [14]. They also imply that the potential generated by a point-mass $m_{pp}$ will be

$$\phi_{pp} = -\frac{m_{pp}}{8\pi M_P^2} F(rM_*),$$

where $F(rM_*) \rightarrow 1$ at $rM_* \gg 1$. At smaller distances, there is no analytic solution, but it will be less singular than the Newtonian potential, even finite as $r \rightarrow 0$. One can compare this prediction to the standard tests of deviations from Newton’s law, yielding an estimate [15, 16] $M_* \gtrsim \frac{1}{\mu m}$. However, these studies derive the constraints for potentials which are not solutions of (8) and that do not modify the singular behaviour of the Newtonian potential. It would thus be interesting to re-analyze the available data to establish more realistic constraints on the scale $M_*$. At lower energies, the theory is characterized by three dimensionless parameters beyond general relativity: $\lambda', \alpha, \eta$. By using solar system tests [5], observations from pulsars [17] and cosmology [12] these parameters can be constrained to high precision, from $O(10^{-7})$ to $O(10^{-2})$ depending on the combination of parameters. This is remarkable, since one could detect predictions of a quantum gravity proposal at low energies. It is important to state that in the constrained region non-projectable Hořava gravity agrees with all data. The level of precision of the tests has been recently boosted by the strong bound on the speed of propagation of gravitational waves [18, 19]. This means that a particular combination of the infrared parameters is smaller than $O(10^{-15})$. If all the parameters $\lambda', \alpha, \eta$ were so small, the chances to test the proposal with astrophysical observations would be severely reduced. This is not problematic for the proposal, since the only fundamental constraint is $\sqrt{c}M_P > M_*$ (recall our discussion in the previous section).

4. Main open questions for Hořava gravity

Despite the large number of works about Hořava gravity, some of the key questions are still not answered. Remarkably, among them we find the most interesting (and hard) ones:

- Is the theory renormalizable beyond power-counting? As I explained, this has been proven for the projectable case, while for the non-projectable case the question still remains. The difference is the presence of instantaneous modes in the latter, whose effects are harder to track beyond linear theory [7].
• **Is the theory asymptotically free/scale invariant?** A positive answer to the question in the previous point can still hide a Landau pole. In the next section I report on the first answer to this question in 2 + 1 dimensions.

• **How can one explain the high precision at which Lorentz invariance holds for the standard model if it is not a fundamental symmetry?** In a sense, this is the bone in the throat of many people working in Lorentz violation. We have traded a hard problem (quantum gravity in 3 + 1 dimensions) by another hard problem: explaining the emergence of Lorentz invariance. The possibility of Lorentz invariance as an emergent phenomenon has been studied in the past [20]. Recently different ways have been suggested to try to explain the tight tests of Lorentz invariance at low energies [10]:
  – First, it is technically natural to have all the parameters that break Lorentz invariance small. Still, one would like to find ways to make those related to gravity larger (to be able to test the proposal at low energies). Furthermore, these parameters can not be smaller than $\sqrt{c}M_P > M_\star > 1/\mu m$.
  – The hierarchy between gravitational parameters and standard-model parameters can be due to the fact that these sectors are connected by Planck-suppressed operators [21]. This idea has been further developed in [22].
  – By using supersymmetry, Lorentz invariance can be understood as an accidental symmetry [23]. However, it seems hard to extend this idea to Hořava gravity [24].
  – Finally, Lorentz invariance can be emergent in the infrared along renormalization group flows [20][10][25]. This proposal is very interesting, though to explain the small departures from Lorentz invariance at low energies, while allowing a substantial deviation at high-energies, one needs to resort to strong dynamics. The study of the efficiency of this mechanism is still under development. In Hořava gravity, one can hope that the theory is asymptotically free, which would allow for a lattice formulation of the proposal where the problem of emergence through the renormalization group flow can be studied in a fully consistent way.

• Finally, as we discussed before, the nature of black holes in these theories is not clear. Singularities may be solved if the theory is asymptotically free. Also the concept of horizon is modified, since the theory has modes propagating at arbitrary high speeds [14][20][27].

I expect that there will be progress in all the previous points. In particular, the bounds on phenomenology are so tight that it is worth now turning back to more ‘theoretical’ aspects if we want to keep exploring the proposal. In the following I report on a recent result about the ultraviolet behaviour of the theory.

5. **Renormalizability of projectable case in 2 + 1 dimensions**

In the program of studying the renormalizability of the 3 + 1 non-projectable Hořava gravity, there are many obstacles that can be first addressed in simpler settings. In particular, making explicit loop calculations with gravitational degrees of freedom and without Lorentz invariance is technically challenging. This is why, as a warm up, we studied the 2 + 1 projectable case in [8] (see also [28]). This toy model can also be important to understand Hořava gravity as a ultraviolet complete theory of quantum gravity. In particular, the behaviour of singularities, horizons, strong-coupling dynamics (including gravity) or implications to AdS/CFT may be studied first in this context.

Projectable Hořava gravity in 2 + 1 has a very simple action in terms of three couplings,

$$S = \frac{1}{2G} \int dx^2 dt \sqrt{\gamma} \left( K_{ij} K^{ij} - \lambda K^2 - \mu R^2 \right). \quad (10)$$

The term proportional to the 2-dimensional Ricci scalar, $R$, is absent given its topological nature, while we set the cosmological constant to zero. Out of the previous couplings, only two
combinations are physical due to gauge invariance \[8\]: \( \{ G \equiv G/\sqrt{\mu}, \lambda \} \). The technical details on how to perform the one-loop calculation and hence obtain the flow of the parameters in \[10\] are given in \[8\]. The final result is shown in Fig. 1, taken from \[8\].

![Renormalization group flow of the relevant couplings of eq. (10) at one loop.](image)

**Figure 1.** Renormalization group flow of the relevant couplings of eq. (10) at one loop. The arrows point towards the infrared. The region ‘non-unitary’ corresponds to a choice of parameters which has problems with unitarity.

There are several interesting aspects of the flow: first, the theory has two ultraviolet fixed points. The study of the \( \lambda = 1/2 \) fixed point requires a two-loop calculation that is beyond the purpose of \[8\]. More interestingly, the one at \( \lambda = 15/14 \) is asymptotically free, and the one-loop calculation is applicable. Thus, this theory is ultraviolet complete around this gaussian fixed point. Second, this fixed point approaches \( \lambda \to 1 \) in the infrared, which corresponds to the value of a Lorentz-invariant theory at low energies. We do not know if the theory is really close to \( 2 + 1 \) general relativity since it becomes strongly coupled in this limit. This stems from the absence of the \( R \) term. This difficulty is not expected in \( 3 + 1 \), where the new scaling generated by terms of the form \( R \) would stop the flow, and this can happen before the theory becomes strongly coupled.

Thus, one sees that the best possible scenario is satisfied in projectable \( 2 + 1 \) Hořava gravity: asymptotically free theory flowing to a ‘Lorentz invariant’ fixed point in the infrared. Whether this will also be true in the realistic cases in \( 3 + 1 \) dimensions is still to be studied.

6. Summary and outlook
In this contribution I have motivated Hořava gravity as a proposal for perturbative and unitary quantum gravity in \( 3 + 1 \) dimensions. This is done at the expense of breaking of one of the better tested symmetries: boost invariance. Still, the parameters of the theory can be chosen to make the proposal compatible with all phenomenological tests.

There are many aspects of the theory that require further study. In my opinion, the emergence of Lorentz invariance at low energies, aspects of black holes (thermodynamics and singularities) and the computation of quantum properties are the most pressing ones. They are also the most challenging ones. Concerning the last point, I have reported on a fresh result about the one-loop calculation of the projectable case in \( 2 + 1 \) dimensions, where we find a very positive outcome. The extension to the realistic \( 3 + 1 \) non-projectable case or the non-perturbative study of the proposal requires a formidable effort. Still, our findings are encouraging enough to vigorously pursue this endeavour. Finally, it is necessary to include matter fields, to make sure that the nice properties we discussed are still preserved in a realistic scenario.
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