Quantum Zeno and anti-Zeno effects in an asymmetric nonlinear optical coupler

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ABSTRACT
Quantum Zeno and anti-Zeno effects in an asymmetric nonlinear optical coupler are studied. The asymmetric nonlinear optical coupler is composed of a linear waveguide ($\chi^{(1)}$) and a nonlinear waveguide ($\chi^{(2)}$) interacting with each other through the evanescent waves. The nonlinear waveguide has quadratic nonlinearity and it operates under second harmonic generation. A completely quantum mechanical description is used to describe the system. The closed form analytic solutions of Heisenberg’s equations of motion for the different field modes are obtained using Sen-Mandal perturbative approach. In the coupler, the linear waveguide acts as a probe on the system (nonlinear waveguide). The effect of the presence of the probe (linear waveguide) on the photon statistics of the second harmonic mode of the system is considered as quantum Zeno and anti-Zeno effects. Further, it is also shown that in the stimulated case, it is easy to switch between quantum Zeno and anti-Zeno effects just by controlling the phase of the second harmonic mode of the asymmetric coupler.

Keywords: Quantum Zeno effect, quantum anti-Zeno effect, optical coupler, waveguide.

1. INTRODUCTION
Zeno’s paradoxes have been in discussion since fifth century. In 1977, Mishra and Sudarshan\(^{11}\) introduced a quantum analogue of Zeno’s paradox, which is later termed as quantum Zeno effect. Quantum Zeno effect in the original formulation refers to the inhibition of the temporal evolution of a system on continuous measurement\(^{1–3}\) while quantum anti-Zeno or inverse Zeno effect refers to the enhancement of the evolution instead of the inhibition\(^4\) (see\(^5,6\) for reviews on Zeno and anti-Zeno effect). In the last four decades, quantum Zeno effect has been studied in different physical systems, such as two coupled nonlinear optical processes\(^7\), parametric down-conversion\(^8\) and cascaded parametric down-conversion with losses.\(^9\) Similarly, quantum anti-Zeno effect is also reported in various physical systems. Specifically, quantum anti-Zeno effect is observed in parametric down-conversion\(^10\) and in radioactive decay processes, where the measurement causes the system to disintegrate.\(^11\) In Ref.\(^12\) both the effects (quantum Zeno and anti-Zeno) are reported in two-level systems. Further, a geometrical criterion for transition between the Zeno and anti-Zeno effects has also been discussed in Ref.\(^13\) Quantum Zeno effect using environment-induced decoherence theory has also been discussed in the past.\(^14\) Agarwal and Tewari proposed a scheme for an all optical realization of quantum Zeno effect using an arrangement of beam splitters.\(^15\) In addition to the above mentioned theoretical studies, quantum Zeno effect has also been experimentally realized in trapped beryllium ions\(^16\) and with the help of rotators and polarizers.\(^17\)

Recently, the interest on quantum Zeno effect has increased by manifold as it has found its applications in counterfactual quantum computation,\(^18\) where computation is accomplished using the computer in superposition of running and not running states and later to infer the solution from it; counterfactual quantum communication, in which information is sent without sending the information encoded particles through the communication channel\(^19\) and quantum Zeno tomography, which essentially uses interaction free measurement.\(^20\) Interestingly, detection of an absorbing object without any interaction with light, using quantum Zeno effect with higher efficiency has already been demonstrated.\(^21\) These facts motivated the study of Zeno effect in macroscopic systems\(^22\) too. Quantum Zeno and anti-Zeno effects in the nonlinear optical couplers have been studied in the recent past\(^23–25\) by considering that one of the mode in the nonlinear waveguide is coupled with the auxiliary mode in a linear waveguide, and the auxiliary linear mode acts as a probe (continuous observation) on the evolution of the system (nonlinear waveguide) and changes the photon statistics of the other modes of the nonlinear

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waveguide, which are not coupled with the probe. This kind of continuous interaction with an external system is equivalent to the original inhibition or enhancement of evolution of the system on continuous measurement as measurement is strong coupling with a measuring device.\cite{27,29}

An important example of nonlinear optical coupler is an asymmetric nonlinear optical coupler, consisting of a nonlinear waveguide with $\chi^{(2)}$ nonlinearity operating under second harmonic generation coupled with a linear waveguide. This system is studied earlier and the nonclassical properties (such as squeezing, antibunching and entanglement) have been reported in this system for both codirectional and contradirectional propagation of the field in the linear waveguide.\cite{30,33} We will consider here the codirectional propagation of the fields as in contradirectional propagation the solution was only valid at both the ends of the coupler, i.e., not valid for $0 < z < L$, where $L$ is the interaction length of the coupler.\cite{32,33} It has already been established that the nonclassical effects can be observed in the asymmetric nonlinear coupler.\cite{30,33} The presence of quantum Zeno and anti-Zeno effects reported in this work further establishes the existence of nonclassicality in this asymmetric coupler.

To study the quantum Zeno and anti-Zeno effects closed form analytic expressions for different field operators are used here. These expressions were obtained earlier by using Sen-Mandal perturbative approach and a completely quantum mechanical description of the system. The solutions of Heisenberg’s equations of motion used here are better than the conventional short-length solutions as these solutions are not restricted by length.\cite{33} In what follows, we use the solutions reported in Refs.\cite{30,31} to establish the existence of quantum Zeno and anti-Zeno effects in the asymmetric nonlinear optical coupler.

The remaining part of this paper is organized as follows. In Section 2, we briefly describe the momentum operator that provides a completely quantum mechanical description of the system and also note the analytic expressions of the field operators required for the present study. In Section 3, we show the existence of quantum Zeno and anti-Zeno effects in the asymmetric nonlinear optical coupler and show the spatial evolution of the Zeno parameter. The variation of the Zeno parameter has also been studied with the phase of the coherent input nonlinearity. As $\chi^{(2)}$ medium can produce second harmonic generation, we may say that the codirectional asymmetric nonlinear optical coupler studied here for the investigation of possibility of observing quantum Zeno and anti-Zeno effects is operated under second harmonic generation. The momentum operator for this specific coupler in interaction picture is\cite{30}

$$G = -\hbar k a b^{\dagger}_{1} - \hbar \Gamma b_{1}^{\dagger} b_{2}^{\dagger} \exp(i \Delta k z) + \text{H.c.},$$

where H.c. stands for the Hermitian conjugate, and $\Delta k = |2k_{1} - k_{2}|$ is the phase mismatch between the fundamental and second harmonic beams, and $k$ ($\Gamma$) being the linear (nonlinear) coupling constant is proportional to the linear (nonlinear) susceptibility $\chi^{(1)}$ ($\chi^{(2)}$). The value of $\chi^{(2)}$ is considerably smaller than $\chi^{(1)}$ (typically $\chi^{(2)}/\chi^{(1)} \simeq 10^{-6}$) which leads to $\Gamma \ll k$, unless an extremely strong pump is present in the nonlinear waveguide.

In Refs.\cite{30,31} closed form analytic expressions for the evolution of the field operators corresponding to the Hamiltonian were obtained using Sen-Mandal perturbative approach valid up to the linear power of the nonlinear coupling coefficient $\Gamma$. The field operators reported there are:

$$a(z) = f_{1} a(0) + f_{2} b_{1}(0) + f_{3} b_{1}^{\dagger}(0) b_{2}(0) + f_{4} a^{\dagger}(0) b_{2}(0),$$

$$b_{1}(z) = g_{1} a(0) + g_{2} b_{1}(0) + g_{3} b_{1}^{\dagger}(0) b_{2}(0) + g_{4} a^{\dagger}(0) b_{2}(0),$$

$$b_{2}(z) = h_{1} b_{2}(0) + h_{2} b_{1}^{\dagger}(0) + h_{3} b_{1}(0) a(0) + h_{4} a^{\dagger}(0),$$

where

$$\begin{align*}
  f_{1} &= \frac{1}{\Gamma_{1}} \frac{\chi^{(2)}}{\Delta k}, \\
  f_{2} &= \frac{1}{\Gamma_{2}} \frac{\chi^{(2)}}{\Delta k}, \\
  f_{3} &= \frac{1}{\Gamma_{3}} \frac{\chi^{(2)}}{\Delta k}, \\
  f_{4} &= \frac{1}{\Gamma_{4}} \frac{\chi^{(2)}}{\Delta k}, \\
  g_{1} &= \frac{1}{\Gamma_{1}} \frac{\chi^{(2)}}{\Delta k}, \\
  g_{2} &= \frac{1}{\Gamma_{2}} \frac{\chi^{(2)}}{\Delta k}, \\
  g_{3} &= \frac{1}{\Gamma_{3}} \frac{\chi^{(2)}}{\Delta k}, \\
  g_{4} &= \frac{1}{\Gamma_{4}} \frac{\chi^{(2)}}{\Delta k}, \\
  h_{1} &= \frac{1}{\Gamma_{1}} \frac{\chi^{(2)}}{\Delta k}, \\
  h_{2} &= \frac{1}{\Gamma_{2}} \frac{\chi^{(2)}}{\Delta k}, \\
  h_{3} &= \frac{1}{\Gamma_{3}} \frac{\chi^{(2)}}{\Delta k}, \\
  h_{4} &= \frac{1}{\Gamma_{4}} \frac{\chi^{(2)}}{\Delta k}.
\end{align*}$$
Figure 1. (Color online) Schematic diagram of an asymmetric nonlinear optical coupler of interaction length $L$ in codirectional propagation prepared by combining a linear waveguide with a nonlinear (quadratic) waveguide operated by second harmonic generation.

The number of photons in the second harmonic mode for the initial multimode coherent state (5) is given by

$$\langle N_{b_2} (z) \rangle = \langle \alpha \rangle^2 + \langle h_2 b_2^0(0) b_2^2(0) + h_3 b_2^0(0) b_1(0) a(0) + h_4 b_2^0(0) a^2(0) + \text{H.c.} \rangle \right)$$

where $G_\pm = [1 \pm \text{exp}(-i\Delta k z)]$.

The number operator for the second harmonic field mode in the nonlinear waveguide, i.e., $b_2$ mode is given by

$$N_{b_2} (z) = b_2^\dagger (z) b_2 (z) = b_2^\dagger (0) b_2 (0) + \left[ h_2 b_2^0(0) b_2^2(0) + h_3 b_2^0(0) b_1(0) a(0) + h_4 b_2^0(0) a^2(0) + \text{H.c.} \right].$$

The initial state being the multimode coherent state $|\alpha \rangle |\beta \rangle |\gamma \rangle$ with all three modes $|\alpha \rangle$, $|\beta \rangle$ and $|\gamma \rangle$ the eigenkets of annihilation operators $a$, $b_1$ and $b_2$, respectively. For example, after the operation of the field operator $b_2(0)$ on such a multimode coherent state we would obtain

$$b_2(0) |\alpha \rangle |\beta \rangle |\gamma \rangle = |\gamma |\alpha \rangle |\beta \rangle |\gamma \rangle,$$

where $|\alpha |^2$, $|\beta |^2$ and $|\gamma |^2$ are the initial number of photons in the field modes $a$, $b_1$ and $b_2$, respectively. Further, the coupler discussed here can operate under two conditions: spontaneous and stimulated. In the spontaneous case, $|\alpha | \neq 0$, $|\beta | \neq 0$ and $|\gamma | = 0$, whereas in the stimulated case, $|\alpha | \neq 0$, $|\beta | \neq 0$ and $|\gamma | \neq 0$.

### 3. QUANTUM ZENO AND ANTI-ZENO EFFECTS

The number of photons in the second harmonic mode for the initial multimode coherent state (5) is given by

$$\langle N_{b_2} (z) \rangle = |\gamma |^2 + \left[ h_2 b_2^2(0)^* \gamma^* + h_3 a \beta \gamma^* + h_4 a^2 \gamma^* + \text{c.c.} \right].$$

In the absence of the probe mode, i.e., $k = 0$ and $\alpha = 0$, we have

$$\langle N_{b_2} (z) \rangle_{k=0} = |\gamma |^2 + \left[ h_2 b_2^2 \gamma^* + \text{c.c.} \right].$$
where \( h_2' = h_2(k = 0) = \frac{1}{2\pi} \{ 1 - \exp(i\Delta k z) \} \). The effect of the presence of the probe mode can be given as 
\[
\Delta N_Z = \langle N_{b_2}(z) \rangle - \langle N_{b_2}(z) \rangle_{k=0},
\]
where \( \Delta N_Z \) is the Zeno parameter. The non-zero value of the Zeno parameter implies that the presence of the probe affects the evolution of the photon statistics of the system, i.e., the positive (negative) value of the Zeno parameter means the photon generation is increased (decreased) due to the continuous measurement of the probe on the linear mode of the nonlinear waveguide. In the present case, using Eqs. (6) and (7), we may obtain the analytic expression for the Zeno parameter as
\[
\Delta N_Z = \left\{ \left( h_2 - h_2' \right) \beta^2 + h_3\alpha\beta + h_4\alpha^2 \right\} \gamma^* + c.c. \]  
(8)

When the Zeno parameter becomes negative (positive), it implies the existence of quantum Zeno (anti-Zeno) effect. Here, the expression for the Zeno parameter is of the form \( |\gamma| F(h_i, \phi) \), where we have considered \( \gamma \equiv |\gamma| \exp(i\phi) \). So, the Zeno parameter becomes zero in the spontaneous case, and neither the quantum Zeno effect nor the anti-Zeno effect can be observed. In the stimulated case, the functional form of the Zeno parameter suggests that we can control the quantum Zeno and anti-Zeno effects in the asymmetric coupler just by controlling the phase of the input coherent beam of the \( b_2 \) mode, i.e., changing \( \phi \). As the phase change of \( \pi \) in \( \gamma \) changes the quantum Zeno effect into quantum anti-Zeno effect, and vice versa. For example, in Fig. 2 we show the spatial evolution of the Zeno parameter for \( \phi = 0 \) (smooth blue line) and \( \phi = \pi \) (dashed red line). While the choice \( \phi = 0 \) illustrates the existence of quantum Zeno effect, \( \phi = \pi \) is found to illustrate the existence of quantum anti-Zeno effect.

Further, the effect of change of the other parameters on the Zeno parameter can also be observed. Specifically, Fig. 3 shows that the transition between quantum Zeno and anti-Zeno effects can be observed with change in phase mismatch between fundamental and second harmonic modes in the nonlinear waveguide. Similarly, Fig. 4 illustrates the variation of quantum Zeno effect, as the Zeno parameter remains negative, with linear coupling between probe and the system. A similar behaviour can also be observed for quantum anti-Zeno effect. However, the variation of the Zeno parameter with change in nonlinear coupling constant is observed to be linear in nature and decreasing with nonlinear coupling constant (corresponding plot is not included in this paper).

4. CONCLUSION

Existence of quantum Zeno and anti-Zeno effects are reported in an asymmetric nonlinear optical coupler prepared by combing a linear and a nonlinear waveguide of \( \chi^{(2)} \) nonlinearity (cf. Fig. 2). Further, it is also shown that in the stimulated case, it is easy to switch between quantum Zeno and anti-Zeno effects just by controlling the phase of the second harmonic mode in the asymmetric coupler discussed here. This flexibility to switch between the quantum Zeno and anti-Zeno effects was not observed in earlier studies on quantum Zeno and anti-Zeno effects in optical couplers. Further, the effects of change in linear coupling and phase mismatch on
Figure 3. (Color online) The spatial variation of the Zeno parameter ($\Delta N_Z$) in the second harmonic ($b_2$) mode with rescaled length ($\Gamma_z$) and phase mismatch ($\Delta k$) between fundamental and second harmonic mode in the nonlinear waveguide for the initial state $|\alpha\rangle|\beta\rangle|\gamma\rangle$ and $k = 0.1$, $\Gamma = 0.001$, $\alpha = 5$ and $\beta = 2$, and $\gamma = 1$. Transition between quantum Zeno and anti-Zeno effects with change in phase mismatch can be observed.

Figure 4. (Color online) The spatial variation of the Zeno parameter ($\Delta N_Z$) depicts quantum Zeno effect in the second harmonic ($b_2$) mode is shown with rescaled length ($\Gamma_z$) and linear coupling ($k$) between the waveguides for the initial state $|\alpha\rangle|\beta\rangle|\gamma\rangle$ and $\Gamma = 0.001$, $\Delta k = 10^{-4}$, $\alpha = 5$ and $\beta = 2$, and $\gamma = 1$. 
the spatial variation of the Zeno parameter are also illustrated.

The approach adopted here and in Ref.\textsuperscript{33} is quite general and same may be used for the similar studies on other nonlinear optical couplers. Further, the theoretical results reported here seem to be easily realizable in experiment as the coupler used here is commercially available and the photon number statistics required to study quantum Zeno and anti-Zeno effects can be obtained using high efficiency photon number resolving detectors.

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