Studies of $B_s^0 \to \eta^{(')}\eta^{(')}$ decays in the pQCD approach

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Abstract

We calculate the CP averaged branching ratios and CP-violating asymmetries for $B_s^0 \to \eta\eta, \eta\eta'$ and $\eta'\eta'$ decays in the perturbative QCD (pQCD) approach here. The pQCD predictions for the CP-averaged branching ratios are $\text{Br}(B_s^0 \to \eta\eta) = (14.2^{+18.0}_{-7.5}) \times 10^{-6}$, $\text{Br}(B_s^0 \to \eta\eta') = (12.4^{+18.2}_{-7.0}) \times 10^{-6}$, and $\text{Br}(B_s^0 \to \eta'\eta') = (9.2^{+15.3}_{-4.9}) \times 10^{-6}$, which agree well with those obtained by employing the QCD factorization approach and also be consistent with available experimental upper limits. The gluonic contributions are small in size: less than 7% for $B_s \to \eta\eta$ and $\eta\eta'$ decays, and around 18% for $B_s \to \eta'\eta'$ decay. The CP-violating asymmetries for three decays are very small: less than 3% in magnitude.

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Among various $B \to M_1M_2$ decay channels (here $M_i$ refers to the light pseudo-scalar or vector mesons), the decays involving the isosinglet $\eta$ or $\eta'$ mesons in the final state are phenomenologically very interesting and have been studied extensively during the past decade because of the so-called $K\eta'$ puzzle or other special features \cite{1 2 3 4}.

Motivated by the large number of $B_s$ production and decay events expected at the forthcoming LHC experiments, the studies about the $B_s$ meson decays become more attractive than ever before. Very recently, some two-body $B_s \to M_i\eta^{(\prime)}$ decays, such as $B_s \to (\pi, \rho, \omega, \phi)\eta^{(\prime)}$ decays have been studied in Refs. \cite{5 6} in the perturbative QCD (pQCD) factorization approach \cite{7 8 9}. In this paper, we would like to calculate the branching ratios and CP asymmetries for the three $B_s^0 \to \eta\eta', \eta'\eta'$ and $\eta'\eta'$ decays by employing the low energy effective Hamiltonian \cite{10} and the pQCD approach. Besides the usual factorizable contributions, we here are able to evaluate the non-factorizable and the annihilation contributions to these decays.

On the experimental side, only the poor upper limit on $Br(B_s^0 \to \eta\eta)$ is available now \cite{11} (upper limits at 90% C.L.):

$$ Br(B_s^0 \to \eta\eta) < 1.5 \times 10^{-3}, $$

Of course, this situation will be improved rapidly when LHC experiment starts to run at the end of 2007.

This paper is organized as follows. In Sec. I, we calculate analytically the related Feynman diagrams and present the various decay amplitudes for the studied decay modes. In Sec. II, we show the numerical results for the branching ratios and CP asymmetries of $B_s^0 \to \eta^{(\prime)}\eta^{(\prime)}$ decays. A short summary and some discussions are also included in this section.

I. PERTURBATIVE CALCULATIONS

Since the $b$ quark is rather heavy we consider the $B_s$ meson at rest for simplicity. It is convenient to use light-cone coordinate $(p^+, p^-, p_T)$ to describe the meson’s momenta: $p^\pm = (p^0 \pm p^3)/\sqrt{2}$ and $p_T = (p^1, p^2)$. Using the light-cone coordinates the $B_s$ meson and the two final state meson momenta can be written as

$$ P_1 = \frac{M_{B_s}}{\sqrt{2}}(1, 1, 0_T), \quad P_2 = \frac{M_{B_s}}{\sqrt{2}}(1, 0, 0_T), \quad P_3 = \frac{M_{B_s}}{\sqrt{2}}(0, 1, 0_T), $$

respectively, here the light meson masses have been neglected. Putting the light (anti-) quark momenta in $B_s$, $\eta'$ and $\eta$ mesons as $k_1$, $k_2$, and $k_3$, respectively, we can choose

$$ k_1 = (x_1 P^+_1, 0, k_{1T}), \quad k_2 = (x_2 P^+_2, 0, k_{2T}), \quad k_3 = (0, x_3 P^-_3, k_{3T}). $$

Then, after the integration over $k_{1T}, k_{2T}$, and $k_{3T}$, the decay amplitude for $B_s \to \eta\eta'$ decay, for example, can be conceptually written as

$$ \mathcal{A}(B_s \to \eta\eta') \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \cdot \text{Tr} \left[ C(t) \Phi_{B_s}(x_1, b_1) \Phi_{\eta'}(x_2, b_2) \Phi_{\eta}(x_3, b_3) H(x_1, b_1, t) S_t(x_i) e^{-S(t)} \right], \quad (4) $$
where $k_i$ are momenta of light quarks included in each meson, term $\text{Tr}$ denotes the trace over Dirac and color indices, $C(t)$ is the Wilson coefficient evaluated at scale $t$, the function $H(k_1, k_2, k_3, t)$ is the hard part and can be calculated perturbatively, the function $\Phi_M$ is the wave function, the function $S_i(x_i)$ describes the threshold resummation [12] which smears the end-point singularities on $x_i$, and the last term, $e^{-S(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively. We will calculate analytically the function $H(x_i, b_i, t)$ for the considered decays in the first order in $\alpha_s$ expansion and give the convoluted amplitudes in next section.

For the two-body charmless $B_s$ meson decays, the related weak effective Hamiltonian $H_{\text{eff}}$ can be written as [10]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{uq}^* (C_1(\mu)O_1^u(\mu) + C_2(\mu)O_2^u(\mu)) - V_{tb} V_{tq}^* \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right],$$  \hfill (5)

where $C_i(\mu)$ are Wilson coefficients at the renormalization scale $\mu$ and $O_i$ are the four-fermion operators for the case of $b \to q$ ($q = d, s$) transition [5, 10]. For the Wilson coefficients $C_i(\mu)$ ($i = 1, \ldots, 10$), we will use the leading order (LO) expressions, although the next-to-leading order (NLO) results already exist in the literature [10]. This is the consistent way to cancel the explicit $\mu$ dependence in the theoretical formulae. For the renormalization group evolution of the Wilson coefficients from higher scale to lower scale, we use the formulae as given in Ref. [13] directly.

### A. Decay amplitudes

We firstly take $B_s \to \eta\eta'$ decay mode as an example, and then extend our study to $B_s \to \eta\eta$ and $\eta'\eta'$ decays. Similar to the $B_s^0 \to \pi^0\eta(\eta')$ decays in [5], there are 8 type diagrams contributing to the $B_s \to \eta\eta'$ decays, as illustrated in Fig.1. We first calculate the usual factorizable diagrams (a) and (b). Operators $O_{1,2,3,4,9,10}$ are $(V-A)(V-A)$ currents, the sum of their amplitudes is given as

$$F_{\eta\eta} = 8\pi C_F m_B^4 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1, b_1) \times \left\{ (1 + x_3) \phi^A_\eta(x_3, b_3) + (1 - 2x_3) r_\eta (\phi^P_\eta(x_3, b_3) + \phi^T_\eta(x_3, b_3)) \right\} \alpha_s(t_1^e) h_e(x_1, x_3, b_1, b_3) \exp[-S_{ab}(t_1^e)] + 2 r_\eta \phi^P_\eta(x_3, b_3) \alpha_s(t_2^e) h_e(x_3, x_1, b_1, b_3) \exp[-S_{ab}(t_2^e)] \}.$$  \hfill (6)

where $r_\eta = m_\eta^2/m_B^2$; $C_F = 4/3$ is a color factor. The explicit expressions of the function $h_e$, the scales $t^e_i$ and the Sudakov factors $S_{ab}$ can be found Ref. [5]. The form factors of $B_s$ to $\eta$ decay, $F_{0,1}^{B_s \to \eta s}(0)$, can thus be extracted from the expression in Eq. (6).

The operators $O_{5,6,7,8}$ have a structure of $(V-A)(V+A)$. Some of these operators can contribute to the decay amplitude in a factorizable way, but others may contribute after making a Fierz transformation in order to get right flavor and color structure for.
FIG. 1: Typical Feynman diagrams contributing to the $B_s \rightarrow \eta \eta'$ decays, where diagram (a) and (b) contribute to the $B_s \rightarrow \eta$ form factor $F_{B_s \rightarrow \eta}$.

Factorization to work. Such kinds of contributions can be written as

$$F_{e\eta}^{P1} = -F_{e\eta}. \quad (7)$$

$$F_{e\eta}^{P2} = 16\pi C_F m_{B_s}^4 \left( \frac{(f_{\eta'}^s - f_{\eta'}^u)m_{\eta}^2}{2m_s m_{B_s}} \right) \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1, b_1) \times \left\{ \left[ \phi_{e\eta}^A(x_3, b_3) + r_\eta((2 + x_3)\phi_{\eta}^P(x_3, b_3) - x_3\phi_{\eta}^T(x_3, b_3)) \right] \cdot \alpha_s(t_1^1) h_e(x_1, x_3, b_1, b_3) \exp[-S_{ab}(t_1^1)] \\
\quad + [x_1\phi_{\eta}^A(x_3, b_3) - 2(x_1 - 1)r_\eta\phi_{\eta}^P(x_3, b_3)] \cdot \alpha_s(t_2^2) h_e(x_3, x_1, b_3, b_1) \exp[-S_{ab}(t_2^2)] \right\}. \quad (8)$$

For the non-factorizable diagrams 1(c) and 1(d), the corresponding decay amplitudes
can be written as

$$M_{en} = \frac{16\sqrt{6}}{3} \pi C_F m_B^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_\alpha}(x_1, b_1) \phi^{A}_{\eta}(x_2, b_2)$$

\begin{align*}
&\times \left\{ \left[ 2x_3 r_{\eta}^P \phi^P_{\eta}(x_3, b_1) - x_3 \phi_{\eta}(x_3, b_1) \right] \\
&\quad - \alpha_s(t_f) h_f(x_1, x_2, x_3, b_1, b_2) \exp[-S_{cd}(t_f)] \right\},
\end{align*}

(9)

$$M_{en}^{P_1} = 0,$$

(10)

$$M_{en}^{P_2} = -M_{en}.$$

(11)

For the non-factorizable annihilation diagrams 1(e) and 1(f), we find

$$M_{en} = \frac{16\sqrt{6}}{3} \pi C_F m_B^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_\alpha}(x_1, b_1)$$

\begin{align*}
&\times \left\{ \left[ (x_3 - 2) r_{\eta}^P \phi^P_{\eta}(x_3, b_2) (\phi^P_{\eta}(x_3, b_2) + \phi_T^P(x_3, b_2)) - (x_2 - 2) r_{\eta}^P \phi_{\eta}(x_3, b_2) \\
&\quad \phi^P(x_2, b_2) + \phi_T^P(x_2, b_2) \right] \right\} \cdot \alpha_s(t_f^3) h_f^3(x_1, x_2, x_3, b_1, b_2) \exp[-S_{ef}(t_f^3)] \\
&\quad - \left[ x_3 r_{\eta}^A \phi^A_{\eta}(x_3, b_2) (\phi^P_{\eta}(x_3, b_2) + \phi_T^P(x_3, b_2)) \right] \\
&\quad - \alpha_s(t_f^3) h_f^3(x_1, x_2, x_3, b_1, b_2) \exp[-S_{ef}(t_f^3)] \right\},
\end{align*}

(12)

$$M_{en}^{P_1} = \frac{16\sqrt{6}}{3} \pi C_F m_B^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_\alpha}(x_1, b_1)$$

\begin{align*}
&\times \left\{ \left[ (x_3 - 2) r_{\eta}^P \phi^P_{\eta}(x_3, b_2) (\phi^P_{\eta}(x_3, b_2) + \phi_T^P(x_3, b_2)) - (x_2 - 2) r_{\eta}^P \phi_{\eta}(x_3, b_2) \\
&\quad \phi^P(x_2, b_2) + \phi_T^P(x_2, b_2) \right] \right\} \cdot \alpha_s(t_f^3) h_f^3(x_1, x_2, x_3, b_1, b_2) \exp[-S_{ef}(t_f^3)] \\
&\quad - \left[ x_3 r_{\eta}^A \phi^A_{\eta}(x_3, b_2) (\phi^P_{\eta}(x_3, b_2) + \phi_T^P(x_3, b_2)) \right] \\
&\quad - \alpha_s(t_f^3) h_f^3(x_1, x_2, x_3, b_1, b_2) \exp[-S_{ef}(t_f^3)] \right\},
\end{align*}

(13)

$$M_{en}^{P_2} = \frac{16\sqrt{6}}{3} \pi C_F m_B^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_\alpha}(x_1, b_1)$$

\begin{align*}
&\times \left\{ \left[ (x_3 - 2) r_{\eta}^P \phi^P_{\eta}(x_3, b_2) (\phi^P_{\eta}(x_3, b_2) + \phi_T^P(x_3, b_2)) - (x_2 - 2) r_{\eta}^P \phi_{\eta}(x_3, b_2) \\
&\quad \phi^P(x_2, b_2) + \phi_T^P(x_2, b_2) \right] \right\} \cdot \alpha_s(t_f^3) h_f^3(x_1, x_2, x_3, b_1, b_2) \exp[-S_{ef}(t_f^3)] \\
&\quad - \left[ x_3 r_{\eta}^A \phi^A_{\eta}(x_3, b_2) (\phi^P_{\eta}(x_3, b_2) + \phi_T^P(x_3, b_2)) \right] \\
&\quad - \alpha_s(t_f^3) h_f^3(x_1, x_2, x_3, b_1, b_2) \exp[-S_{ef}(t_f^3)] \right\},
\end{align*}

(14)
For the factorizable annihilation diagrams 1(g) and 1(h), we have

\begin{align}
F_{\eta'\eta} &= F_{\eta \eta}^{P1} = -8\pi C_F m_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \left\{ [x_3 \phi_\eta^A(x_3, b_3) \phi_\eta^A(x_2, b_2) \\
&+ 2r_\eta r_\eta' ((x_3 + 1) \phi_\eta^P(x_3, b_3) + (x_3 - 1) \phi_\eta^T(x_3, b_3)) \phi_\eta^P(x_2, b_2)] \\
&\quad \cdot \alpha_s(t_3^3) h_a(x_2, x_3, b_2, b_3) \exp[-S_{gh}(t_3^3)] \\
&- [x_2 \phi_\eta^P(x_3, b_3) \phi_\eta^A(x_2, b_2) \\
&+ 2r_\eta r_\eta' ((x_2 + 1) \phi_\eta^P(x_2, b_2) + (x_2 - 1) \phi_\eta^T(x_2, b_2)) \phi_\eta^P(x_3, b_3)] \\
&\quad \cdot \alpha_s(t_4^4) h_a(x_3, x_2, b_2, b_3) \exp[-S_{gh}(t_4^4)] \right\},
\end{align}

\begin{align}
F_{\eta'\eta}^{P2} &= -16\pi C_F m_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \\
&\quad \times \left\{ [x_3 r_\eta (\phi_\eta^P(x_3, b_3) - \phi_\eta^T(x_3, b_3)) \phi_\eta^A(x_2, b_2) + 2r_\eta r_\eta' \phi_\eta^A(x_3, b_3) \phi_\eta^P(x_2, b_2)] \\
&\quad \cdot \alpha_s(t_3^3) h_a(x_2, x_3, b_2, b_3) \exp[-S_{gh}(t_3^3)] \\
&+ [x_2 r_\eta' (\phi_\eta^P(x_2, b_2) - \phi_\eta^T(x_2, b_2)) \phi_\eta^A(x_3, b_3) + 2r_\eta r_\eta' \phi_\eta^A(x_2, b_2) \phi_\eta^P(x_3, b_3)] \\
&\quad \cdot \alpha_s(t_4^4) h_a(x_3, x_2, b_2, b_3) \exp[-S_{gh}(t_4^4)] \right\}.
\end{align}

For the \(B_s \to \eta \eta'\) decay, besides the Feynman diagrams as shown in Fig. 1 where the upper emitted meson is the \(\eta'\), the Feynman diagrams obtained by exchanging the position of \(\eta\) and \(\eta'\) also contribute to this decay mode. The corresponding expressions of amplitudes for new diagrams will be similar with those as given in Eqs. (6-14), since the \(\eta\) and \(\eta'\) are all light pseudoscalar mesons and have the similar wave functions. The expressions of amplitudes for new diagrams can be obtained by the replacements

\begin{equation}
\phi_\eta^A \longleftrightarrow \phi_\eta', \quad \phi_\eta^P \longleftrightarrow \phi_\eta'^P, \quad \phi_\eta^T \longleftrightarrow \phi_\eta'^T, \quad r_\eta \longleftrightarrow r_\eta'.
\end{equation}

For example, we find that:

\begin{equation}
F_{e\eta'} = F_{\eta \eta}, \quad F_{\eta \eta'} = -F_{\eta \eta}, \quad F_{\eta \eta}^{P1} = -F_{\eta \eta}^{P1}, \quad F_{\eta \eta}^{P2} = F_{\eta \eta}^{P2}.
\end{equation}

Before we write down the complete decay amplitude for the studied decay modes, we firstly give a brief discussion about the \(\eta - \eta'\) mixing and the gluonic component of the \(\eta'\) meson. There exist two popular mixing basis for \(\eta - \eta'\) system, the octet-singlet and the quark flavor basis, in literature. Here we use the \(SU(3)_F\) octet-singlet basis with the two mixing angle \((\theta_1, \theta_8)\) scheme [14] to describe the mixing of \(\eta\) and \(\eta'\) mesons. In the numerical calculations, we will use the following mixing parameters [14]

\begin{equation}
\theta_8 = -21.2^\circ, \quad \theta_1 = -9.2^\circ, \quad f_1 = 1.17 f_\pi, \quad f_8 = 1.26 f_\pi.
\end{equation}

In this paper, we firstly take \(\eta\) and \(\eta'\) as a linear combination of light quark pairs \(u\bar{u}, d\bar{d}\) and \(s\bar{s}\), and then estimate the possible gluonic contributions to \(B_s \to \eta(0)\eta(0)\) decays by using the formulae as presented in Ref. [4]. We found that the possible gluonic contributions are indeed small.
B. Complete decay amplitudes

For $B^0_s \rightarrow \eta \eta'$ decay, by combining the contributions from different diagrams, the total decay amplitude can be written as

$$\mathcal{M}(\eta \eta') = (F_{\text{ef}}F_2f^d_{\eta'} + F_{\text{ef}}F'_2f^d_{\eta'}) \cdot \left[ \xi_u \left( C_1 + \frac{1}{3}C_2 \right) - \xi_t \left( C_3 + \frac{1}{3}C_4 - C_5 - \frac{1}{3}C_6 + \frac{1}{2}C_7 + \frac{1}{6}C_8 - \frac{1}{2}C_9 - \frac{1}{6}C_{10} \right) \right] - (F_{\text{ef}}F_2f^s_{\eta'} + F_{\text{ef'}}F'_2f^s_{\eta'}) \cdot \xi_t \left( \frac{7}{3}C_3 + \frac{5}{3}C_4 - 2C_5 - \frac{2}{3}C_6 - \frac{1}{2}C_7 - \frac{1}{6}C_8 + \frac{1}{3}C_9 - \frac{1}{3}C_{10} \right) - (F_{\text{ef}}F_2 + F_{\text{ef'}}F'_2) \xi_t \left( \frac{1}{3}C_5 + C_6 - \frac{1}{6}C_7 - \frac{1}{2}C_8 \right) + (M_{\text{ef}} + M_{\text{ef'}}) F_2F'_2 \left[ \xi_u C_2 - \xi_t \left( C_3 + 2C_4 - \frac{1}{2}C_9 + \frac{1}{2}C_{10} \right) \right] - (M_{\text{ef}} + M_{\text{ef'}}) F_1F'_2 \xi_t \left( C_4 - \frac{1}{2}C_{10} \right) - (M_{\text{ef}} + M_{\text{ef'}}) F_2F'_2 \xi_t \left( 2C_6 + \frac{1}{2}C_8 \right) - (M_{\text{ef}} F_2 + M_{\text{ef'}} F'_2) \xi_t \left( C_6 - \frac{1}{2}C_8 \right) + (M_{\text{af}} + M_{\text{af'}}) F_1F'_1 \left[ \xi_u C_2 - \xi_t \left( C_3 + 2C_4 - \frac{1}{2}C_9 + \frac{1}{2}C_{10} \right) \right] - (M_{\text{af}} + M_{\text{af'}}) F_2F'_2 \xi_t \left( C_4 - \frac{1}{2}C_{10} \right) - (M_{\text{af}} + M_{\text{af'}}) F_2F'_2 \xi_t \left( C_5 - \frac{1}{2}C_7 \right) - (M_{\text{af}} + M_{\text{af'}}) F_1F'_1 \xi_t \left( 2C_6 + \frac{1}{2}C_8 \right) - (M_{\text{af}} + M_{\text{af'}}) F_2F'_2 \xi_t \left( C_6 - \frac{1}{2}C_8 \right) - f_{B_s} \cdot (F_{\text{af}}F'_2 + F_{\text{af'}}F'_2) F_2F'_2 \xi_t \left( \frac{1}{3}C_5 + C_6 - \frac{1}{6}C_7 - \frac{1}{2}C_8 \right) ,$$

(20)
where \( \xi_u = V_{ub}^* V_{us} \) and \( \xi_t = V_{tb}^* V_{ts} \), and the relevant mixing parameters and decay constants are

\[
F_1 = \sqrt{\frac{1}{6}} \cos \theta_8 - \sqrt{\frac{1}{3}} \sin \theta_1, \quad F_2 = -\sqrt{\frac{2}{3}} \sin \theta_8 + \sqrt{\frac{1}{3}} \cos \theta_1, \tag{21}
\]

\[
F'_1 = \sqrt{\frac{1}{6}} \sin \theta_8 + \sqrt{\frac{1}{3}} \cos \theta_1, \quad F'_2 = -\sqrt{\frac{2}{3}} \sin \theta_8 + \sqrt{\frac{1}{3}} \cos \theta_1, \tag{22}
\]

\[
f_{\eta}^d = \frac{f_8}{\sqrt{6}} \cos \theta_8 - \frac{f_1}{\sqrt{3}} \sin \theta_1, \quad f_{\eta}^s = -\frac{2f_8}{\sqrt{3}} \cos \theta_8 - \frac{f_1}{\sqrt{3}} \sin \theta_1, \tag{23}
\]

\[
f_{\eta'}^d = \frac{f_8}{\sqrt{6}} \sin \theta_8 + \frac{f_1}{\sqrt{3}} \cos \theta_1, \quad f_{\eta'}^s = -\frac{2f_8}{\sqrt{3}} \sin \theta_8 + \frac{f_1}{\sqrt{3}} \cos \theta_1. \tag{24}
\]

Similarly, the decay amplitudes for \( B_s^0 \to \eta \eta \) and \( B_s^0 \to \eta' \eta' \) decay can be obtained easily from Eq. (20) by the following replacements

\[
f_{\eta}^d, f_{\eta}^s \longrightarrow f_{\eta'}^d, f_{\eta'}^s; \quad F_1(\theta_1, \theta_8) \longleftarrow F'_1(\theta_1, \theta_8); \quad F_2(\theta_1, \theta_8) \longleftarrow F'_2(\theta_1, \theta_8). \tag{25}
\]

Note that the contributions from the possible gluonic component of \( \eta' \) meson have not been included here.

### II. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will calculate the CP-averaged branching ratios and CP violating asymmetries for those considered decay modes. The input parameters and the wave functions to be used are given in Appendix A. In numerical calculations, central values of input parameters will be used implicitly unless otherwise stated.

Using the decay amplitudes obtained in last section, it is straightforward to calculate the branching ratios. By employing the two mixing angle scheme of \( \eta - \eta' \) system and using the mixing parameters as given in Eq. (19), one finds the CP-averaged branching ratios for the considered three decays as follows

\[
Br( B_s^0 \to \eta \eta ) = \left[ 14.2_{-4.2}^{+6.0} (\omega_b)_{-6.2}^{+17.7} (m_{s0}^{\text{ps}}) \right] \times 10^{-6}, \tag{26}
\]

\[
Br( B_s^0 \to \eta \eta' ) = \left[ 12.4_{-3.8}^{+5.7} (\omega_b)_{-6.0}^{+17.3} (m_{s0}^{\text{ps}}) \right] \times 10^{-6}, \tag{27}
\]

\[
Br( B_s^0 \to \eta' \eta' ) = \left[ 9.2_{-2.0}^{+3.0} (\omega_b)_{-4.5}^{+15.0} (m_{s0}^{\text{ps}}) \right] \times 10^{-6}, \tag{28}
\]

where the main errors are induced by the uncertainties of \( \omega_b = 0.50 \pm 0.05 \) GeV, and \( m_{s0}^{\text{ps}} = [1.49 - 2.38] \) GeV (corresponding to \( m_s = 130 \pm 30 \) MeV), respectively. The above pQCD predictions agree well with those obtained in the QCD factorization approach [2].

As for the gluonic contributions, we follow the same procedure as being used in Ref. [4] to include the possible gluonic contributions to the \( B_s \to \eta^{(l)} \) transition form factors \( F_{B_s \to \eta^{(l)}}^{B_s} \) and found that the gluonic contributions to the branching ratios are less than 3% for \( B \to \eta \eta \) decay, \( \sim 7 \% \) for \( B \to \eta \eta' \) decay, and around 18% for \( B \to \eta' \eta' \) decay. The central values of the pQCD predictions for \( B_s \to \eta^{(l)} \eta^{(l)} \) decays after the inclusion of possible gluonic contributions are the following

\[
Br( B_s^0 \to \eta \eta ) = \left[ 13.7_{-4.4}^{+6.4} (\omega_b)_{-6.1}^{+16.5} (m_{s0}^{\text{ps}}) \right] \times 10^{-6}, \tag{29}
\]

\[
Br( B_s^0 \to \eta \eta' ) = \left[ 11.6_{-3.4}^{+5.3} (\omega_b)_{-5.7}^{+16.8} (m_{s0}^{\text{ps}}) \right] \times 10^{-6},
\]

\[
Br( B_s^0 \to \eta' \eta' ) = \left[ 10.8_{-3.7}^{+3.7} (\omega_b)_{-5.2}^{+16.2} (m_{s0}^{\text{ps}}) \right] \times 10^{-6}.
\]
Now we turn to the evaluations of the CP-violating asymmetries of $B_s \rightarrow \eta^{(')}\eta^{(')}$ decays in pQCD approach. For $B^0$ meson decays, a non-zero ratio $(\Delta\Gamma/\Gamma)_{B_s}$ is expected in the SM [15, 16]. For $B_s \rightarrow \eta^{(')}\eta^{(')}$ decays, three quantities to describe the CP violation can be defined as follows [16]:

\[
A_{\text{dir}} = \frac{|\lambda_{CP}|^2 - 1}{1 + |\lambda_{CP}|^2}, \quad A_{\text{mix}} = \frac{2Im(\lambda_{CP})}{1 + |\lambda_{CP}|^2}, \quad A_{\Delta\Gamma_s} = \frac{2Re(\lambda_{CP})}{1 + |\lambda_{CP}|^2},
\]

with

\[
\lambda_{CP} = \eta_f \frac{V_{tb}V_{ts}^* \langle f | H_{\text{eff}} | \bar{B}_s^0 \rangle}{V_{tb}V_{ts}^* \langle f | H_{\text{eff}} | B_s^0 \rangle} \approx \frac{\langle f | H_{\text{eff}} | \bar{B}_s^0 \rangle}{\langle f | H_{\text{eff}} | B_s^0 \rangle}
\]

in a very good approximation. Here $A_{\text{CP}}^{\text{dir}}$ and $A_{\text{CP}}^{\text{mix}}$ means the direct and mixing-induced CP violation respectively, while the third term $A_{\Delta\Gamma_s}$ is related to the presence of a non-negligible $\Delta\Gamma_s$. By using the mixing parameters in Eq. (19) and the input parameters as given in Appendix A, one found the pQCD predictions for $A_{\text{CP}}^{\text{dir}}, A_{\text{CP}}^{\text{mix}}$ and $H_f$

\[
A_{\text{CP}}^{\text{dir}}(B^0_s \rightarrow \eta\eta) = \left[-0.2 \pm 0.1(\gamma) \pm 0.1(\omega_b)^{+0.4}_{-0.2}(m_0^{n_s})\right] \times 10^{-2},
A_{\text{CP}}^{\text{dir}}(B^0_s \rightarrow \eta\eta') = \left[+0.6_{-0.2}^{+0.1}(\gamma) \pm 0.1(\omega_b) \pm 0.3(m_0^{n_s})\right] \times 10^{-2},
A_{\text{CP}}^{\text{dir}}(B^0_s \rightarrow \eta'\eta') = \left[-0.8_{-0.2}^{+0.3}(\gamma) \pm 0.1(\omega_b) \pm 0.7(m_0^{n_s})\right] \times 10^{-2},
\]

\[
A_{\text{CP}}^{\text{mix}}(B^0_s \rightarrow \eta\eta) = \left[-0.3 \pm 0.1(\gamma) \pm 0.2(\omega_b) \pm 0.5(m_0^{n_s})\right] \times 10^{-2},
A_{\text{CP}}^{\text{mix}}(B^0_s \rightarrow \eta\eta') = \left[-0.8 \pm 0.2(\gamma) \pm 0.1(\omega_b) \pm 0.2(m_0^{n_s})\right] \times 10^{-2},
A_{\text{CP}}^{\text{mix}}(B^0_s \rightarrow \eta'\eta') = \left[+1.8_{-0.5}^{+0.3}(\gamma) \pm 0.0(\omega_b)^{+0.5}_{-0.3}(m_0^{n_s})\right] \times 10^{-2},
\]

\[
A_{\Delta\Gamma_s}(\eta\eta) \approx A_{\Delta\Gamma_s}(\eta\eta') \approx A_{\Delta\Gamma_s}(\eta'\eta') \approx 1,
\]

where the dominant errors come from the variations of CKM angle $\gamma = 60^\circ \pm 20^\circ$, $\omega_b = 0.50 \pm 0.05$ GeV and $m_0^{n_s} = [1.49 - 2.38]$ GeV (corresponding to $m_s = 130 \pm 30$ MeV), respectively. It is easy to see that both the direct and mixing-induced CP violations of the considered $B_s$ decays are very small in magnitude, and thus almost impossible to measure them even in the LHC experiments. The above pQCD predictions are also consistent with the QCDF predictions [11, 12].

In short, we calculated the branching ratios and CP-violating asymmetries of $B^0_s \rightarrow \eta\eta$, $\eta\eta'$ and $\eta'\eta'$ decays at the leading order by using the pQCD factorization approach. Besides the usual factorizable diagrams, the non-factorizable and annihilation diagrams are also calculated analytically in the pQCD approach. From our calculations and phenomenological analysis, we found the following results:

- Using the two mixing angle scheme, the pQCD predictions for the CP-averaged branching ratios are

\[
Br(B^0_s \rightarrow \eta\eta) = (14.2^{+18.0}_{-7.5}) \times 10^{-6},
Br(B^0_s \rightarrow \eta\eta') = (12.4^{+18.2}_{-7.0}) \times 10^{-6},
Br(B^0_s \rightarrow \eta'\eta') = (9.2^{+15.3}_{-4.9}) \times 10^{-6},
\]

where the various errors as specified previously have been added in quadrature. The pQCD predictions for the three decay channels agree well with those obtained by employing the QCDF approach.
• The gluonic contributions are small in size: less than 7% for $B_s \rightarrow \eta\eta$ and $\eta\eta'$ decays, and around 18% for $B_s \rightarrow \eta'\eta'$ decay.

• The direct and mixing-induced CP violations of the considered three decay modes are very small: less than 3% in magnitude.

Note added: After completion of this paper, the paper in Ref. [18] appeared, and where a systematic study for the $B_s \rightarrow M_1 M_2$ decays in the pQCD factorization approach has been done. Since different mixing-scheme of $\eta - \eta'$ system have been used, the explicit expressions of the decay amplitudes of the relevant decays are different in these two papers, but the numerical predictions for branching ratios and CP violations agree well with each other. The possible gluonic contributions are estimated here.

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APPENDIX A: INPUT PARAMETERS AND WAVE FUNCTIONS

In this Appendix we show the input parameters and the light meson wave functions to be used in the numerical calculations.

The masses, decay constants, QCD scale and $B_s^0$ meson lifetime are

$$\begin{align*}
\Lambda^{(f=4)}_{\overline{MS}} &= 250\text{MeV}, \quad f_\pi = 130\text{MeV}, \quad f_{B_s} = 230\text{MeV}, \\
m_0^{\eta\bar{d}\bar{d}} &= 1.4\text{GeV}, \quad m_s = 130\text{MeV}, \quad f_K = 160\text{MeV}, \\
M_{B_s} &= 5.37\text{GeV}, \quad M_W = 80.41\text{GeV}, \quad \tau_{B_s^0} = 1.46 \times 10^{-12}\text{s} \quad (A1)
\end{align*}$$

For the CKM matrix elements, here we adopt the Wolfenstein parametrization for the CKM matrix, and take $\lambda = 0.2272, A = 0.818, \rho = 0.221$ and $\eta = 0.340$ [11].

For the $B$ meson wave function, we adopt the model

$$\phi_{B_s}(x,b) = N_{B_s}x^2(1-x)^2\exp\left[-\frac{M_{B_s}^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right],$$

where $\omega_b$ is a free parameter and we take $\omega_b = 0.50 \pm 0.05$ GeV in numerical calculations, and $N_{B_s} = 63.67$ is the normalization factor for $\omega_b = 0.50$.

For the distribution amplitudes $\phi_{\eta\bar{d}\bar{d}}^A, \phi_{\eta\bar{d}\bar{d}}^P$ and $\phi_{\eta\bar{d}\bar{d}}^T$, we utilize the result from the light-cone sum rule [17] including twist-3 contribution. For the corresponding Gegenbauer moments and relevant input parameters, we here use $a_2^{\eta\bar{d}\bar{d}} = 0.115, a_4^{\eta\bar{d}\bar{d}} = -0.015, \rho_{\eta\bar{d}\bar{d}} = m_\pi/m_0^{\eta\bar{d}\bar{d}}, \eta_3 = 0.015$ and $\omega_3 = -3.0$. We also assume that the wave function of $u\bar{u}$ is the
same as the wave function of \(d\bar{d}\). For the wave function of the \(s\bar{s}\) components, we also use the same form as \(d\bar{d}\) but with \(m_{0}^{s\bar{s}}\) and \(f_{y}\) instead of \(m_{0}^{d\bar{d}}\) and \(f_{x}\), respectively:

\[
 f_{x} = f_{\pi}, \quad f_{y} = \sqrt{2f_{K}^{2} - f_{\pi}^{2}}. \tag{A3}
\]

These values are translated to the values in the two mixing angle method:

\[
 f_{1} = 152.1 \text{MeV}, \quad f_{8} = 163.8 \text{MeV},
 \theta_{1} = -9.2^{\circ}, \quad \theta_{8} = -21.2^{\circ}. \tag{A4}
\]

The parameters \(m_{0}^{i} (i = \eta_{d\bar{d}(u\bar{u})}, \eta_{s\bar{s}})\) are defined as:

\[
 m_{0}^{\eta_{d\bar{d}(u\bar{u})}} \equiv m_{0}^{\pi} \equiv \frac{m_{\pi}^{2}}{(m_{u} + m_{d})}, \quad m_{0}^{\eta_{s\bar{s}}} \equiv \frac{2M_{K}^{2} - m_{\pi}^{2}}{2m_{s}}. \tag{A5}
\]

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