Quantum-critical superconductivity in underdoped cuprates

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Abstract. – We argue that the pseudogap phase may be an attribute of the non-BCS pairing of quantum-critical, diffusive fermions near the antiferromagnetic quantum critical point. We derive and solve a set of three coupled Eliashberg-type equations for spin-mediated pairing and show that in some $T$ range below the pairing instability, there is no feedback from superconductivity on fermionic excitations, and fermions remain diffusive despite of the pairing. We conjecture that in this regime, fluctuations of the pairing gap destroy the superconducting condensate but preserve the leading edge gap in the fermionic spectral function.

The pseudogap behavior in underdoped cuprates is one of the most unusual and exciting features of condensed matter physics. By all experimental accounts, below optimal doping superconducting-like behavior of fermionic observables sets in at a temperature which increases with underdoping. This temperature correlates with the value of the superconducting gap at $T = 0$, but does not correlate with the transition temperature itself. The latter decreases with underdoping and eventually vanishes [1].

In this paper we argue that the pseudogap behavior may be a part of new, non-Fermi-liquid physics associated with the pairing of fermions in the quantum-critical regime near the antiferromagnetic quantum critical point. The onset of the pairing in this regime has been recently studied by Finkel’stein and two of us (ACF) [2]. ACF demonstrated that the onset temperature $T = T_{\text{ins}}$ is determined by a competition between fermionic incoherence and the absence of a gap for spin excitations which mediate superconductivity [3]. Due to this competition, $T_{\text{ins}}$ tends to a finite value when the spin correlation length, $\xi$, diverges.

An obvious question posed by this result is whether the pairing instability at $T_{\text{ins}}$ leads to true superconductivity, or only gives rise to a formation of spin singlets which still behave incoherently and hence do not superconduct. If this was the case, the phase right below $T_{\text{ins}}$ would be a pseudogap phase, while the actual superconductivity would emerge only at a smaller temperature.

This paper is a first step in addressing this issue. We report the results of our analysis of the system behavior within the Eliashberg-type approach [4]. This approach is mean-field like in the sense that it does not include the feedback effect from the pairing modes (e.g., phase fluctuations) on fermions and spin excitations. However, it does include nontrivial physics associated with non BCS pairing in the quantum-critical regime. We show that pairing of
quantum critical fermions causes a reduction of the superfluid stiffness, $D_s \propto n_s/m^*$. At low $T$, the reduction is predominantly due to an enhancement of the quasiparticle mass, $m^*$. However, in some temperature interval between $T_{ins}$ and $T_0 \leq T_{ins}$ we found an additional reduction of the superfluid stiffness. We argue that the latter effect is an indication of a new physics associated with a quantum-critical, non-BCS pairing near the magnetic instability, and reflects the reduction of the superfluid density $n_s$ due to the fact that fermions remain diffusive at the lowest $\omega$ despite the formation of pairs. We conject that phase fluctuations, acting on top of the Eliashberg solution, likely destroy fermionic coherence at $T \sim T_0$, which then becomes a true phase transition temperature where a continuous $U(1)$ symmetry breaks down. The pairing gap, defined as the scale below which the quasiparticle spectral weight is reduced, however, remains finite at $T_0$ and disappears only at $T_{ins}$.

We next discuss the structure of the Eliashberg equations for spin-mediated superconductors with phonon mediated pairing, a recipe to study the system behavior at strong coupling is the Eliashberg theory. It is justified by Migdal theorem which states that the corrections to the electron-phonon vertex $\delta g/g$ and $\Sigma^{-1} \partial \Sigma(k, \omega)/\partial k$, although increase with the dimensionless coupling $\lambda$, still scale as $\lambda(v_s/v_F)$ where $v_s$ and $v_F \gg v_s$ are the sound velocity and the Fermi velocity. As $v_s/v_F \sim 10^{-4}$, $\delta g/g$ and $\Sigma^{-1} \partial \Sigma(k, \omega)/\partial k$ can be safely neglected for all reasonable $\lambda$. One then has to include only $\Sigma(\omega)$. In our case, Migdal theorem is inapplicable as spin fluctuations are collective modes of fermions and hence the spin velocity (the analog of $v_s$) is of the same order as $v_F$. This implies that the Migdal parameter $\lambda v_s/v_F$ is large in the strong coupling regime. It turns out, however, that in this limit one again can neglect $\delta g/g$ and $\Sigma^{-1} \partial \Sigma(k, \omega)/\partial k$, this time because the dynamics of the collective mode is completely modified by low-energy fermions and becomes diffusive at energies relevant to the pairing problem. [For electron-phonon interaction this happens only for very small frequencies $\omega \ll v_F$.] Specifically, the strong coupling solution of the spin-fermion problem in the normal state yields $d \Sigma/\partial \omega \propto \lambda$ while $\delta g/g$ and $\Sigma^{-1} \partial \Sigma(k, \omega)/\partial k$ scale as $(1/N) \log \lambda \ll \lambda$, where $N(=8)$ is the is the number of hot spots in the Brillouin zone. Furthermore, below $T_{ins}$, we found that $\delta g/g$ even becomes independent on $\lambda$. Physically, the irrelevance of vertex corrections is due to the fact that the theory possesses no SDW precursors, and hence spin excitations are not near Goldstone bosons. Below we formally treat $N$ as a large parameter and perform computations at $N \to \infty$. In this limit, vertex corrections can be totally neglected. Also, following ACF, we neglect the momentum variation of $\Sigma_k(\omega)$ and of the anomalous vertex $\Phi_k(\omega)$ along the Fermi surface, i.e., replace them by $\Sigma_{k_{ins}}(\omega)$ and $\Phi_{k_{ins}}(\omega)$ (respecting the $d_{x^2-y^2}$ symmetry $\Phi_{k_{ins}+Q} = -\Phi_{k_{ins}}$). The last approximation is justified if $\bar{g} \ll v_F k_F$ which we assume to hold. Finally, we verified that both above and below $T_{ins}$, there is no universal thermal contribution to $\xi$ from low-energy fermions, i.e., $\xi(T) \sim O(T = 0)$. At $T > \bar{g}$, the physics is dominated by classical fluctuations, and $\xi^{-1}$ eventually becomes strongly $T$ dependent.

We next discuss the structure of the Eliashberg equations for spin-mediated superconductivity. In conventional superconductors, the Eliashberg theory involves two coupled equations for the fermionic self-energy $\Sigma$, and the pairing vertex $\Phi$. Modifications of the phonon propagator due to fermionic pairing are small and can be neglected. For spin-mediated pair-
ing, the situation is different as the spin dynamics is made out of low-energy fermions and thus is sensitive to the opening of the fermionic gap \( \Delta \). As a result, the Eliashberg theory has to be generalized to include the equation for the dynamical spin susceptibility.

The mutual feedbacks between fermions and spin fluctuations have been earlier studied numerically within the FLEX approximation [1, 2]. Our results agree with these studies, but go beyond them in the understanding of the new physics. The modification of the spin propagator due to fermionic pairing has also been considered in the context of marginal Fermi liquid phenomenology [13], but the feedback effect on fermions has not been studied.

The set of Eliashberg equations for the spin-fermion model is obtained in a standard way, by evaluating diagrammatically the pairing vertex, fermionic self-energy and the spin polarization operator, and integrating over the fermionic energy. The results have been earlier presented in the two limiting cases: for infinitesimally small \( \Phi(\omega) \) and for \( \Phi(\omega) = \text{const} \). A straightforward extension to a general case yields \( \Phi(\omega_m) = \Phi_m \), etc.,

\[
\Phi_m = \frac{\pi T}{2} m \sum_n \frac{\Phi_n}{\sqrt{\Phi_n^2 + \Sigma_n^2}} \left( \frac{\bar{\omega}}{\omega_{sf} + \Pi_{n-m}} \right)^{1/2},
\]

\[
\Sigma_m = \omega_m + \frac{\pi T}{2} \sum_n \frac{\Sigma_n}{\sqrt{\Phi_n^2 + \Sigma_n^2}} \left( \frac{\bar{\omega}}{\omega_{sf} + \Pi_{n-m}} \right)^{1/2},
\]

\[
\Pi_m = \pi T \sum_n \left( 1 - \frac{\Sigma_n \Sigma_{n+m} + \Phi_n \Phi_{n+m}}{\sqrt{\Phi_n^2 + \Sigma_n^2} \sqrt{\Phi_{n+m}^2 + \Sigma_{n+m}^2}} \right).
\]

The dynamical spin susceptibility at antiferromagnetic momentum \( \mathbf{Q} \) and the fermionic Green’s function follow as \( \chi_{\mathbf{Q}}(\omega_n)^{-1} \propto \omega_{sf} - \Pi_m \), and \( G_k(\omega_m) = -(i \Sigma_m + \epsilon_k)/(\Phi_m^2 + \Sigma_m^2 + \epsilon_k^2) \), respectively, where \( \epsilon_k = v_F \cdot (\mathbf{k} - \mathbf{k}_F) \). For further convenience we included the bare \( \omega_n \) term in the fermionic propagator into \( \Sigma_n \) and introduced \( \bar{\omega} = \bar{g}/(2\pi N) \) and \( \omega_{sf} = \bar{\omega}/(4\lambda^2) \propto \xi^{-2} \) instead of \( \lambda \) and \( \bar{g} \) (the 1/\( N \) factor in \( \bar{\omega} \) is eliminated after appropriate rescaling \( \bar{g} \rightarrow \bar{g} N \) and \( v_F \rightarrow v_F N \)).

Analyzing \( \Sigma \) and \( \chi_{\mathbf{Q}} \) in the normal state, we find that \( \omega_{sf} \) separates the Fermi liquid behavior at \( \omega < \omega_{sf} \) from the quantum-critical behavior which holds between \( \omega_{sf} \) and \( \omega \gg \omega_{sf} \). Which of the two energies determines the superconducting properties of the system? The onset of pairing was determined by ACF by linearizing with respect to \( \Phi \). They found that \( \Phi \) emerges at a temperature \( T_{\text{ins}} \) which depends weakly on \( \omega_{sf} \) and saturates at \( T_{\text{ins}} \approx 0.17 \bar{\omega} \) for \( \xi \rightarrow \infty \). Thus, the onset of pairing is produced by quantum-critical fermions with \( \bar{\omega} \geq \omega \gg \omega_{sf} \). We consider the solution of the full set of nonlinear equations (1-3) we analyze whether there are any new crossover scales below \( T_{\text{ins}} \).

At \( T = 0 \), the solution of the Eliashberg set is in many respects similar to the one for \( \Phi(\omega) = \text{const} \). The pairing gap \( \Delta \) (defined in the usual way as \( \Delta(\omega) = \Phi(\omega)\omega/\Sigma(\omega) \) and \( \Delta(\omega = \Delta) = \omega \)) scales with \( T_{\text{ins}} \) up to log \( \lambda \) corrections which, however, remain numerically small even for \( \lambda = 20 \). The presence of the gap eliminates spin-fermion scattering at energies \( \mathcal{O}(\Delta) \) and restores fermionic and bosonic coherence. In particular, spin excitations, which were purely relaxational in the normal state become propagating below \( 2\Delta \) with dispersion \( \Omega_{\mathbf{q}} = (\Delta_{\mathbf{q}}^2 + v_F(\mathbf{q} - \mathbf{Q})^2)^{1/2} \), where \( \Delta_{\mathbf{q}} \sim (\omega_{sf})^{1/2} \propto T_{\text{ins}}/\lambda \). This effect changes the dynamical exponent to \( z = 1 \) and yields the resonance in the inelastic neutron scattering at \( \omega = \Delta_{\mathbf{q}} \). The spin resonance in turn affects fermionic excitations: both, \( \text{Im}\Sigma(\omega) \) and \( \text{Im}\Phi(\omega) \) vanish at low \( \omega \) due to a lack of a phase space for single particle decay and jump to finite values at \( \omega = \Delta + \Delta_{\mathbf{q}} \). This gives rise to a dip in the fermionic spectral function [1, 2].
Fig. 1 – The real part of the pairing vertex $\Phi(\omega)$ and $\text{Im} \Sigma(\omega)$ for $\lambda = 4$ and different temperatures $T < \Delta_s$, $T \sim \Delta_s$, $\Delta_s < T < T_0$ and $T_0 < T < T_{\text{ins}}$. We associate $T \sim \Delta_s$ and $T \sim T_0$ with the onset of impurity-like behavior due to thermal fluctuations, and with the onset of the reduction of the superfluid density, respectively.

These analytical results fully agree with our numerical solution (the lowest $T$ results in Figs. 1 and 2). We found $2\Delta/T_{\text{ins}} \approx 4$ for $\lambda \gg 1$ with downturn deviation at $\lambda \sim 1$. To a reasonable accuracy, $\Delta_s \sim 0.35 \bar{\omega}/\lambda$. The spectral function has a peak-dip-hump structure, and the peak-dip distance exactly equals $\Delta_s$. The numbers which we obtain are also consistent with experiment. We use data for Bi2212 for comparisons. Near optimal doping, which we identify with $\lambda \sim 1$, we obtain $\Delta_s/\bar{\omega} \sim 0.25 - 0.3$ and $\Delta/\bar{\omega} \sim 0.2$. The value of $\bar{\omega}$ can be extracted from the photoemission data as a frequency where nonlinear corrections from $\text{Re} \Sigma(\omega)$ to the quasiparticle dispersion become irrelevant. This yields $\bar{\omega} \sim 150 - 160$ meV. Using this $\bar{\omega}$, we obtain $\Delta_s \sim 38 - 48$ meV and $\Delta \sim 30 - 32$ meV which are in reasonable agreement with the neutron scattering [13], photoemission [14] and tunneling [15] data.

We now turn to $T > 0$. A simple analysis of Eqs. (1, 2) shows that classical thermal spin fluctuations (the ones with zero Matsubara frequency) critically depend on $\omega_s$ and are therefore relevant already at low $T$. These fluctuations account for scattering with zero energy.

Fig. 2 – The dynamical spin susceptibility $\chi''(Q, \omega)$ and the photoemission intensity $I(\omega) = A(\omega) n_F(\omega)$ for $\lambda = 4$ at different $T$. The insets show the the superfluid stiffness $D_s(T)$ and the NMR relaxation rate $1/T_1$ vs $T/T_{\text{ins}}$. The actual behavior of $D_s(T)$ and $1/T_1$ at low $T$ is smoother due to contributions from nodal regions. The arrow indicates the position of the dip in $I(\omega)$. 
transfer and therefore act for spin-mediated d-wave pairing in the same way as nonmagnetic, elastic impurities in s-wave superconductors. We then use the same strategy as for the impurity problem \[^{16}\] and introduce $\Phi_m$ and $\Sigma_m$ via $\Phi_m = \Phi_m n_m$, $\Sigma_m = \Sigma_m n_m$, where

$$\eta_m = 1 + \frac{\pi T \lambda}{\sqrt{(\Phi_m)^2 + (\Sigma_m)^2}} \quad (4)$$

Substituting $\Phi_m$ and $\Sigma_m$ into the Eliashberg set we obtain after some algebra that the quantum contributions to the self energy, $\Phi_m$ and $\Sigma_m$, obey the same Eqs. (1), (2) but without zero frequency ($m = n$) contributions. This implies that the effects of thermal spin fluctuations are completely absorbed into the $\eta_m$ factors. On the other hand, the temperature dependence of $\Phi$ and $\Sigma$ is set by $T_{ins}$ and is weak at $T \sim \Delta_s$. Estimating $\eta$ using $\Sigma^2(\omega) + \Phi^2(\omega) \sim \Delta^2$, we find $\eta - 1 \sim T/\Delta_s$. We see that at low $T \ll T_{ins}$, the system behaves as a dirty superconductor, the role of $\gamma/\Delta$ ratio (where $\gamma$ is the elastic scattering rate due to impurities) is played by $T/\Delta_s$. Using this analogy and the results for dirty superconductors \[^{16, 17}\] we find that the density of states and the two-particle response functions at finite momentum, such as $\Pi(\omega)$, are unaffected by $T/\Delta_s$ ratio as they have the same form in terms of $\Phi_m$ and $\Sigma_m$ as in terms of $\Phi_s$ and $\Sigma_s$. On the other hand, the single particle spectral function and the two particle response functions at zero external momentum (i.e., the Meissner kernel and the superfluid stiffness $D_s$) scale as $\eta^{-1}$ and are substantially reduced above $T \sim \Delta_s$. As in dirty superconductors, this reduction can be absorbed into the renormalization of the quasiparticle mass $m^*/m = d\Sigma/d\omega \sim \eta$ and is not associated with the reduction of the superfluid density.

These features are present in our numerical solution of the Eliashberg equations. In Fig. 1 we show representative results for $\text{Re}\Phi(\omega)$ and $\text{Im}\Sigma(\omega)$ for different $T$. We clearly see that at $T \sim \Delta_s$, sharp structures at $\omega = \Delta + \Delta_s$ transform into broader structures at $\omega = \Delta$. The latter are due to the fact that in real frequencies, $\eta(\omega)$ is peaked at $\omega = \Delta$, and the amplitude of this peak increases with $T$. In Fig. 2 we show the behavior of the dynamical spin susceptibility and the fermionic spectral function (multiplied by $n_F(\omega)$) for different $T$. The insets show the superfluid stiffness, $D_s(T)$, extracted from the computation of the Meissner kernel, and the NMR relaxation rate $1/T_1 \propto \Pi(\omega)|_{\omega \to 0}$. We see that the residue of the peak in the spectral function, and the superfluid stiffness sharply decrease above $\Delta_s$. On the other hand, $1/T_1 T$ and the peak in $\chi_Q^s(\omega)$ are much less sensitive to the ratio $T/\Delta_s$. We explicitly verified by analyzing larger $\lambda = 20$ that $1/T_1 T$ does not change much at $T \sim \Delta_s$.

To this end, we therefore find a crossover in the system behavior at $T \sim \Delta_s$, similar to a crossover at $\gamma \sim \Delta$ in dirty superconductors. The superfluid stiffness is reduced at $T > \Delta_s$ due to mass renormalization. However, just as in dirty superconductors, this reduction of the stiffness does not give rise to a substantial reduction to $T_c$ due to phase fluctuations simply because still $D_s \sim E_F(m/m^*) \gg \Delta, T_c$. This follows from the fact that within Eliashberg theory $\Sigma(\omega \sim \Delta) \sim \eta \Delta \ll E_F$. \[^{14}\]

We now argue that another physics emerges at $T \leq T_{ins}$ and yields an extra reduction of $D_s$ unrelated to a mass renormalization. Indeed, we clearly see in Fig. 3 that the peak in $\text{Re}\Phi(\omega)$ disappears at $T = T_0 < T_{ins}$. Between $T_0$ and $T_{ins}$, $\text{Re}\Phi(\omega) \neq 0$, but it monotonically decreases with frequency. Simultaneously, $\text{Im}\Sigma(\omega)$ becomes roughly the same as in the normal state, the peaks in the dynamical spin susceptibility and in the fermionic spectral function become very broad, $1/T_1 T$ nearly reaches its normal state value, and the superfluid stiffness virtually disappears (see the insets in Fig. 3). This behavior is not associated with the impurity-like scattering of thermal fluctuations as then the peak in $\text{Re}\Phi(\omega)$ at $\omega = \Delta$ would disappear only at $T_{ins}$. Rather it implies that immediately below $T_{ins}$ there is no feedback from pairing on fermionic excitations in the sense that strong $\text{Im}\Sigma(\omega = 0)$ in the normal
The phase diagram which emerges from our consideration is presented in Fig. 3. Without fluctuations, $T = T_{\text{ins}}$ is a transition line, while the other two are the crossover lines. Above $\Delta_s$, superfluid stiffness is reduced due to enhancement of the effective mass, like in dirty superconductors. Above $T_0$, it is further reduced due to a reduction of the superfluid density. The first effect is a quantum one—fluctuations destroy superconductivity at $T \sim T_0 < T_{\text{ins}}$. The second effect is a realization of the scenario suggested by Vojta and Sachdev [18]. Still, however, the leading edge gap in the spectral function remains finite for all $T < T_{\text{ins}}$ and just fills in by fluctuations at $T \sim T_0$.

The phase diagram which emerges from our consideration is presented in Fig. 3. Without fluctuations, there is a true transition at $T = T_{\text{ins}}$, and two crossover lines at $T \sim T_0$ and $T \sim \Delta_s$. For $\lambda \gg 1$, $\Delta_s \ll T_0 \leq T_{\text{ins}}$. We found that $T_0$ and $\Delta_s$ merge at $\lambda \sim 2$, but still both remain smaller than $T_{\text{ins}}$. For smaller $\lambda$, the distance between $T_0$ and $T_{\text{ins}}$ gradually decreases and eventually disappears. With fluctuations, it is likely that coherent superconductivity appears only at $T \sim T_0$. We cannot argue definitely whether this implies that $T_{\text{ins}}$ becomes a crossover temperature below which the system begins creating disordered singlet pairs which condense at $T_0$, or there is an Ising-like transition at $T \sim T_{\text{ins}}$ where singlets are ordered into columnar dimers. The first possibility is, in our opinion, a realization of Anderson’s RVB idea [19]. The second possibility is a realization of the scenario suggested by Vojta and Sachdev [20].

We conclude the paper by summarizing what we obtained. Our key result is the phase diagram in Fig. 3. We found that there are two different fluctuation effects which govern the system behavior below the onset of the pairing instability. First effect is due to thermal spin fluctuations, and its role is equivalent to that of nonmagnetic impurities in $s$–wave superconductors. This effect may account for the reduction of the superfluid stiffness, but cannot destroy superconductivity. The second effect is a quantum one - we found that below $T_{\text{ins}}$, there is no immediate feedback effect on fermionic self-energy, and low-frequency fermionic excitations remain overdamped despite pairing. In this situation, singlet pairs still diffuse rather than propagate, and superconducting condensate is easily destroyed by fluctuations.

The data for cuprates seem to indicate that $T_c$ scales with the superfluid stiffness at $T = 0$ [21] and also with the resonance neutron frequency $\Delta_s$ [13]. To account for these results in our theory, it is necessary that $T_0$ and $\Delta_s$ coincide. This does happen at intermediate $\lambda$ (and our theory does predict that near optimal doping $T_c$ scales with $\Delta_s$) but not at $\lambda \gg 1$. In
other words, the present theory underestimates quantum fluctuations at very strong coupling.

A final remark. Our results bear some similarities but also some discrepancies with the results of the Eliashberg study of phonon superconductors at vanishing Debye frequency [22]. The comparison with phonon case requires a separate study and will be presented elsewhere.

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