Double charge-exchange reactions and the effect of transfer

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\textbf{Abstract.} There is a recently renewed interest on the study of single and double charge exchange reactions with heavy ions. We report here a preliminary theoretical study of double charge exchange reactions in terms of two successive charge exchanges in 2nd order Distorted Wave Born approximation. This allows us to include the effect of the different transfer channels in an approximate way. Evidences show that the charge exchange process is dominant. The following one in importance is the combination of one single charge exchange, one neutron pick-up and one proton stripping. This particular process calls for further investigation.

1. Introduction

Nuclear reactions are one of the most important tools to unravel the nature of the interaction between neutrons and protons inside the atomic nucleus. In principle, we can extract a lot of information by making collide a nucleus of interest with a light ion or with a heavy ion. In the latter case, the theoretical description of the reaction is largely more involved, since we have to deal with the nucleon-nucleon interaction of two different many-body systems. In spite of this extra difficulty, we find in the literature different cases in which we have improve our understanding of certain problems thanks to particular characteristics not available with light nuclei. As an example, we can cite the study of breakup of halo nuclei under extreme electric fields supplied by a heavy target [1] or the search of the Giant Pairing Resonance, where a large Q-value is needed and only reachable with a heavy projectile [2].

For the case of charge exchange reactions, we can study the single charge exchange (SCE) in the nucleus of interest with both light and heavy ions. However, performing double charge exchange (DCE) with light ions presents certain difficulties like dealing with the measurement and reconstruction of the trajectory of three different final projectile particles as we would have with a light ion like \textsuperscript{3}He or \textsuperscript{t}. We can use heavy ions to avoid such problems since both the target-like and projectile-like products of the reactions will be bound in most of the cases. In addition, the final cross section for double charge exchange will depend on the Fermi and Gamow Teller response of both the nucleus of interest (the target) and the heavy ion used as experimental probe in the reaction. By choosing carefully a heavy ion with a large Fermi or Gamow Teller response, we can increase the final cross section, helping to the extraction of the particular response of the nucleus of interest.
Nowadays, there is a large experimental interest on double charge-exchange reactions with heavy ions which has given origin to different campaigns mainly at the radioactive beam facilities of RCNP at Osaka and RIKEN in Japan [3, 4] and at the LNS-INFN of Catania in Italy [5, 6]. The former ones are focused on studying the Gamow-Teller response of nuclei and in the production of exotic nuclei through double charge exchange reactions. The latter is devoted to the study of double charge-exchange reactions in connection with the neutrinoless double-beta decay.

In the present contribution we will focus in one of the reactions performed at the LNS-INFN: 15 MeV/A $^{18}$O projectiles impinging on a $^{40}$Ca target [5]. After the double charge-exchange we will measure $^{18}$Ne and $^{40}$Ar in different states. Here, we do not consider any direct process leading from the initial to the final counterparts. This means that the nucleus-nucleus interaction cannot change two charges acting only once. Such a possibility is under investigation [7]. Therefore, the double charge-exchange is produced by a part of the effective nucleus-nucleus interaction able to change one charge at a time but acting twice, thus being called a second order process.

The final situation can be reached also through different combinations of neutron pick-up, proton stripping, and single charge-exchange processes. Any of these so-called competing processes is of a higher order than the double charge-exchange, so that this latter process should dominate. Nonetheless, at low bombarding energies, the differences can be too small to neglect the contribution of the competing processes. In any case, if we manage to control them we can have access to a complementary structure information of the nucleus considered which can be of great interest when studying double-beta decay candidates.

The study of the competition between charge exchange with heavy ion and the corresponding transfer at different bombarding energies has only been properly done for the case of single charge-exchange [8]. In single charge-exchange, the transfer consists of one neutron (proton) pick-up followed by a one proton (neutron) stripping (or vice versa) what means that can be solved in terms of 2nd order Distorted Wave Born Approximation (DWBA). For double charge-exchange we can have transfer channels involving four particles, which means the need of a 4th order DWBA. Non-orthogonality between prior and post representation in transfer generates a non-orthogonality contribution. This term, together with simultaneous and succesive contributions, contributes coherently to the final total cross section. Full 2nd order DWBA including simultaneous, succesive and non-orthogonality terms has been recently fully implemented, helping us to calculate exact cross sections for these cases, whereas for 4th order DWBA there is no implementation yet for the corresponding non-orthogonalities.

Therefore, our purpose is to preliminary asses the importance of the different contributions in order to study the possibility of extracting the desired response from the experimental data. We will estimate the transfer contribution for the $^{18}$O + $^{40}$Ca reaction at 15 MeV/A and we will compare with the double charge-exchange experimental data from [5].

2. Double charge-exchange and multinucleon transfer cross sections

On one hand, we only consider double charge exchange as a second order charge exchange, so that the form factors needed for the reaction calculation are those for the reactions: $^{18}$O+$^{40}$Ca $\rightarrow$ $^{18}$F+$^{40}$K and $^{18}$F+$^{40}$K $\rightarrow$ $^{18}$Ne+$^{40}$Ar. For the calculation of such form factors we can rely on a double folding procedure of a residual nucleon-nucleon interaction and the corresponding transition densities for target and projectile.

On the other hand, corresponding transfer contributions must be considered. The formalism of multinucleon transfer in DWBA has been known for several decades but only recently we have started to calculate second order DWBA transfer due to its computational demands. For the case of double charge exchange we deal with the transfer of two protons and two neutrons. That would mean the need of a 4th order DWBA which has no implementation yet. Since our purpose is to estimate the order of magnitude and its relative importance with respect to the
charge-exchange, we can approximate the full cross section by the one obtained considering a
second order DWBA two-neutron transfer to the ground states of $^{16}$O+$^{42}$Ca followed by a second
order DWBA two-proton transfer to the final products $^{18}$Ne+$^{40}$Ar.

We have to add here the possibility of having one neutron and one proton transfer followed by
a single charge exchange and vice versa. This process is of third order. Therefore, even though
it can be against our intuition to mix two different responses of nuclei, it could be larger than
the full fourth order transfer.

The structure input needed for the transfer are the different overlaps between the different
target-like products in the different initial, intermediate and final situations considered. The
same reads for the projectile like ones. For the sake of completeness, these overlaps should be
provided by the same structure calculation providing transition densities for the charge exchange
calculation. However, they are usually approximated by the overlap obtained with single particle
wavefunctions multiplied by the corresponding spectroscopic factor. An alternative option is to
look for different spectroscopic factors extracted from experimental transfer reactions on the
considered nuclei.

The last ingredient for calculating the reaction cross section are the optical potential between
the different counterparts of the initial, intermediate, and final situations. All of them can
be obtained again by a double folding procedure with the corresponding central densities of
projectile-like and target-like nuclei.

Finally, the total cross section is the coherent sum of all these contributions. Therefore, we
cannot neglect the processes whose cross sections are small when computed separately. All of
them interfere and, through this interference, their final impact on the total cross section can
be larger than expected.

3. Preliminary Results
In order to tackle the $^{18}$O + $^{40}$Ca reaction at 15 MeV/A we combine two programs HIDEX [9]
and FRESCO [10]. With HIDEX we calculate the single charge exchange form factor by
folding QRPA transition densities of the different nuclei with the Love and Franey nucleon-
nucleon interaction [11]. We approximate the $^{18}$F+$^{40}$K → $^{18}$Ne+$^{40}$Ar part by using the same
form factors as in the first single charge exchange, i.e. $^{18}$O+$^{40}$Ca → $^{18}$F+$^{40}$K. However,
including all the charge exchange strength for all possible multipolarities within FRESCO is very
demanding computationally, so that in this preliminary approach we have constrain ourselves
to the multipolarity leading to the ground states of the intermediate counterparts: $^{18}$F(1$^+$) and
$^{40}$K(4$^-$). Then we increase the total strength of this specific part of the total response of the
nuclei in order to be of the order of the experimental cross section at zero degrees. In this way
the result should be approximate but still representative of the strength of the double charge
exchange process and its comparison with the competing channel remains meaningful.

We use the same interaction to calculate the corresponding optical potentials. We
include the form factors and optical potentials into FRESCO where we can add the different
transfer contributions. For this last part, we rely on experimentally extracted spectroscopic
factors [12, 13, 14, 15] for the different cases. Only ground states are considered for even-even
or odd-odd nuclei. For the odd-even or even-odd ones, we consider all the single particle levels
in the vicinity of the Fermi energy. Therefore, all these levels provide a large basis and we only
lack of completeness in the even-even and odd-odd nuclei.

A final remark has to be done regarding the choice of prior or post interactions for each
steps. We only consider the successive part of the transfer choosing post interactions for all
the steps. In order to have the full transfer cross section, we will need to consider the different
non-orthogonalties. The transfer leading to the same counterparts as DCE is a 4$^{th}$ order DWBA
calculation. The corresponding non-orthogonality terms have not been implemented so far. A
calculation including only succesive terms with post interactions, as the calculation presented
here, should be a good estimation and must reach the exact result if our basis is increased towards completeness [10].

In order to assess the relative importance of pure four particle transfer, the combination of transfer and a single charge exchange and the successive double charge exchange, we have performed three different calculation. In the first one we include all contributions. In the second one, only charge exchange and the combination of charge exchange and transfer. Finally, in a third calculation, only charge exchange is included.

We show the preliminary results in table 1. When only double charge exchange is considered, we get a differential cross section of 18.3 µb/sr at zero degrees. This value is an 8% larger than the calculation that includes all possible competing channels. This last calculation is entitled to be the most correct one and therefore this 8% can be understood as the level of accuracy found when neglecting the effect of transfer. However, if we consider the double charge exchange in combination with one-neutron and one-proton transfer channels, the difference with respect to the full calculation is reduced to less than a 5%.

**Table 1.** Preliminary calculations of the differential cross section at zero degrees and the total cross section for the different processes considered.

| Processes considered                      | \(\frac{d\sigma}{d\Omega}(0)\) (µb/sr) | Integrated \(\sigma\) (µb) |
|------------------------------------------|----------------------------------------|-----------------|
| DCE only                                 | 18.3                                   | 0.060           |
| DCE and (np transfer + SCE)              | 16.2                                   | 0.051           |
| DCE and all transfers considered         | 17.0                                   | 0.054           |
| Experimental [5]                         | 11.1                                   | 0.072           |

This reduced difference is only present in the case of the differential cross section at zero degrees. The angular distribution also changes depending on the processes considered. In fact, the results for the total integrated cross sections show a larger difference as can also be found in table 1.

**4. Conclusions**

We have studied the double charge exchange reaction \(^{18}\text{O} + ^{40}\text{Ca}\rightarrow ^{18}\text{Ne} + ^{40}\text{Ar}\) at 15 MeV/A measured at the LNS-INFN [5] focusing in the relative importance of the different competing channels, i.e. double charge exchange and the different possible transfer combinations leading to the same final nuclei.

We have found that double charge-exchange is the leading process since there is only a difference of an 8% between the differential cross section at zero degrees calculated including all processes considered and the one calculated by considering only the double charge-exchange. This result was as expected according to the fact that this process is the lowest order one considered.

Among the processes involving transfer of particles, the combinations of one single charge exchange, one proton stripping and one neutron pick up is the following in importance. Considering this process and the double charge-exchange we obtain a differential cross section at zero degrees just a 5% different from the one obtained including all processes.

Finally, the 4th order multinucleon transfer is the lowest in importance. This is important since there is not a complete derivation and implementation of the corresponding non-orthogonality terms needed to properly calculate this contribution. This is also consistent with what found for the \(^{20}\text{Ne} + ^{116}\text{Cd}\) reaction [16] where different calculations for the transfer of two neutron and two protons produce strongly suppressed cross sections.
The combinations of one single charge exchange, one proton stripping and one neutron pick up is a third order process. In order to do a full DWBA calculation one has to include non-orthogonality terms for the part that includes transfer. However, these non-orthogonalities are parallel to those of the second order process and its derivation should be much simpler than those for the 4th order transfer.

In any case, these conclusions are based on preliminary, not complete, calculations which should be just representative of the order of magnitude. Further investigations are demanded in order to corroborate these preliminary results. Moreover, a full derivation of the third order process arising from the combination of transfer and charge exchange could be of great interest in order to improve accuracy when comparing our structure models for neutrinoless double-beta decay candidates with the double charge-exchange reaction experimental data. Another issue to investigate is whether the choice of the heavy ion could reduce even more this transfer contribution. As shown in [17], the cross section for transfer of a particle depends of the alignment of its spin with its angular momentum in the initial and final occupied orbits. This feature is not seen in transfer with light ions since the particles transferred are initially in s−orbits. However, the relevance of the transfer can be very different if we use a heavy ion with valence neutrons and protons occupying a spin-aligned or a spin-antialigned orbit.

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