Topologically Massive Abelian Gauge Theory From BFT
Hamiltonian Embedding of A First-order Theory

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Abstract

We start with a new first order gauge non-invariant formulation of massive spin-one theory and map it to a reducible gauge theory viz; abelian $B \wedge F$ theory by the Hamiltonian embedding procedure of Batalin, Fradkin and Tyutin(BFT). This equivalence is shown from the equations of motion of the embedded Hamiltonian. We also demonstrate that the original gauge non-invariant model and the topologically massive gauge theory can both be obtained by suitable choice of gauges, from the phase space partition function of the embedded Hamiltonian, proving their equivalence. Comparison of the first order formulation with the other known massive spin-one theories is also discussed.

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I. Introduction

The construction and the study of massive spin-1 theories which are also gauge invariant has a long history [1], the well-known example being models with Higgs mechanism. Since the existence of Higgs particle has not yet been experimentally verified, it prompts a closer look at other models wherein mass and gauge invariance coexist. One such model which is currently being studied is the topological mechanism for gauge invariant mass for spin-one particle without a residual scalar field, wherein vector and tensor (2-form) fields are coupled in a gauge invariant way by a term known as $B \wedge F$ term [2]. The Lagrangian for this model (for Abelian case) is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2 \cdot 3!} H_{\mu \nu \lambda} H^{\mu \nu \lambda} + \frac{1}{4} \epsilon_{\mu \nu \lambda \sigma} B^{\mu \nu} F^{\lambda \sigma},$$

(1)

where $H_{\mu \nu \lambda} = \partial_\mu B_{\nu \lambda} + \text{cyclic terms}$. This Lagrangian is invariant under

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda,$$

(2)

$$B_{\mu \nu} \rightarrow B_{\mu \nu} + (\partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu).$$

(3)

A similar construction has also been made for non-Abelian theory [3].

The invariance of $B_{\mu \nu}$ when $\lambda_\mu = \partial_\mu \omega$, in (3) necessitates the introduction of ghost for ghost terms in BRST quantization of the $B \wedge F$ theory [4]. It can also be seen in the constraint quantization where this invariance makes the generators of gauge transformation linearly dependent [3]. This theory shows a similar constraint structure as that of massless Kalb-Ramond theory in the existence of first stage reducible constraint [3].

The purpose of this paper is to construct a new first-order formulation of massive spin-one theory involving vector and 2-form fields (see eqn (8) below), which is gauge non-invariant and by following the idea of Hamiltonian embedding due to Batalin, Fradkin and Tyutin (BFT) [7], we show that the resulting theory is equivalent to $B \wedge F$ theory
Thus we have a novel representation of the abelian $B \wedge F$ theory in terms of a gauge non-invariant first order formulation.

The motivation for this study is two fold. One is that the $B \wedge F$ theory, in addition to being a candidate as an alternate for Higgs mechanism also appears in diverse areas like condensed matter physics $[8]$ and black-holes $[9]$. Hence a model which is an equivalent realization of it has potential applications. The other is that the Hamiltonian emebedding procedure, employed here, by itself is of current interest. Several models like abelian and non-abelian self-dual model in 2+1 dimensions, abelian and non-abelian Proca theories have been studied in detail in recent times applying the BFT procedure $[10]$. Here we apply it, along the lines of $[11]$ to demonstrate the equivalence between self-dual model and Maxwell-Chern-Simons theory, to establish the equivalence of this gauge non-invariant theory (8)to the reducible gauge theory (1).

We work with $g_{\mu\nu} = \text{diag} (1 -1 -1 -1)$ and $\epsilon_{0123} = 1$

In order to compare our formulation with the other known formulations of massive spin-1 theories, we first recollect them. The earliest formulation is in the form of a first order relativistic wave equation due to Duffin-Kemmer-Petiau(DKP) $[12]$, which is given by

$$ (i \beta^\mu \partial_\mu + m) \psi = 0, \quad (4) $$

where $\psi$ is a 10 dimensional column vector and $\beta_\mu$ are 10x10 hermitian matrices obeying,

$$ \beta_\mu \beta_\nu \beta_\lambda + \beta_\lambda \beta_\nu \beta_\mu = g_{\mu\nu} \beta_\lambda + g_{\lambda\nu} \beta_\mu. \quad (5) $$

The other well-known theory is that of Proca Lagrangian,

$$ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu \quad (6) $$

There is, another formulation involving 2-form field $[13]$ given by
\[ \mathcal{L} = \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{m^2}{4} B_\mu B^\mu, \]  

(7)

where \( H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \text{cyclic terms} \).

Both these formulations (6, 7) can be made gauge invariant by adding suitable St"{u}ckelberg fields to compensate for the gauge variations of the mass terms, the compensating fields being scalar and vector fields for the Lagrangians (6 and 7) respectively \[14\]. It should be pointed out that the St"{u}ckelberg formulations of both these theories are equivalent by duality transformation to \( B \wedge F \) theory \[15\].

The new first order Lagrangian describing massive spin-1 theory involving a one form and a two form fields, which is proposed and studied in this paper, is given by

\[ \mathcal{L} = -\frac{1}{4} H_{\mu\nu} H^{\mu\nu} + \frac{1}{2} G_\mu G^\mu + \frac{1}{2m} \epsilon_{\mu\nu\lambda\sigma} H^{\mu\nu} \partial^\lambda G^\sigma. \]  

(8)

This Lagrangian obviously has no gauge-invariance. The field equations following from this Lagrangian are

\[ -H_{\mu\nu} + \frac{1}{m} \epsilon_{\mu\nu\lambda\sigma} \partial^\lambda G^\sigma = 0, \]  

(9)

and \[ G_\mu + \frac{1}{2m} \epsilon_{\mu\nu\lambda\sigma} \partial^\nu H^{\lambda\sigma} = 0. \]  

(10)

From these equations, it follows that

\[ \partial_\mu H^{\mu\nu} = 0, \]  

(11)

and \[ \partial_\mu G^\mu = 0. \]  

(12)

The fact that the above Lagrangian describes a massive spin–1 theory can be easily seen by rewriting the coupled equations of motion (using the conditions (11) and (12)) as

\[ (\Box + m^2) H_{\mu\nu} = 0, \]  

(13)

or alternatively
\[(\Box + m^2)G_\mu = 0.\] (14)

Since the equation of motion (14) along with the constraint (12) follows from Proca Lagrangian, we should expect the latter to emerge from the above Lagrangian (8). Indeed by integrating out \(H_{\mu\nu}\) from the Lagrangian (8), Proca Lagrangian (6) is obtained. Similarly, by eliminating \(G_\mu\) from the Lagrangian (8) we arrive at the Lagrangian (7).

It is natural to ask how this Lagrangian (8) is different from the first-order formulation of Lagrangians (6, 7).

The standard first-order form of Proca Lagrangian is given by

\[
\mathcal{L} = \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} B_{\mu\nu} F^{\mu\nu} + m^2 A_\mu A^\mu.\] (15)

Here, by eliminating the linearizing field \(B_{\mu\nu}\), we get back to the Proca Lagrangian (6). But eliminating \(A_\mu\) will not lead to the Lagrangian (7) for 2-form fields. Similarly, the standard first order form corresponding to the Lagrangian (7) is

\[
\mathcal{L} = -\frac{1}{2 \cdot 3!} C_{\mu\nu\lambda} C^{\mu\nu\lambda} - \frac{1}{3!} C_{\mu\nu\lambda} H^{\mu\nu\lambda} + m^2 B_{\mu\nu} B^{\mu\nu}.\] (16)

Here, too, eliminating \(B_{\mu\nu}\) from the above Lagrangian will not lead to the Proca Lagrangian involving 1-form (6).

It should be stressed that (8) is different from the standard first-order formulations (15, 16), by being the first-order formulation for both (6) and (7).

The first order field equations (9) and (10) can be rewritten as

\[
(\beta_\mu \partial^\mu + m) \psi = 0 ,\] (17)

where \(\psi\) is a 10 dimensional column vector whose elements are the independent components of \(G_\mu\) and \(H_{\mu\nu}\). But here the \(\beta_\mu\) matrices are not found to obey the DKP algebra (5).
This paper is organised as follows: In section II, Hamiltonian embedding of the Lagrangian (8) is constructed along the lines of BFT and the embedded Hamiltonian is shown to be equivalent to that of $B\land F$ theory. Section III shows the equivalence in phase space path integral approach using the embedded Hamiltonian. Finally we end up with conclusion.

II. Hamiltonian Embedding

We start with the Lagrangian (8), with the last term expressed in a symmetric form as

$$L = -\frac{1}{4} H_{\mu\nu} H^{\mu\nu} + \frac{1}{2} G_{\mu} G^{\mu} + \frac{1}{4 m} \epsilon_{\mu\nu\lambda\sigma} H^{\mu\nu} \partial^{\lambda} G^{\sigma} - \frac{1}{4 m} \epsilon_{\mu\nu\lambda\sigma} \partial^{\mu} H^{\nu\lambda} G^{\sigma}. \quad (18)$$

This Lagrangian can be rewritten as

$$L = \frac{1}{4 m} \epsilon_{0ijk} H^{ij} \partial^{0} G^{k} - \frac{1}{4 m} \epsilon_{0ijk} \partial^{0} H^{ij} G^{k} - \mathcal{H}_c \quad (19)$$

where $\mathcal{H}_c$, the Hamiltonian density following from the above Lagrangian (19) is

$$\mathcal{H}_c = \frac{1}{4} H_{ij} H^{ij} - \frac{1}{2} G_i G^i + H_{0i} \left( \frac{1}{2} H^{0i} - \frac{1}{m} \epsilon^{0ijk} \partial_j G_k \right) - \frac{1}{2} G_0 \left( G^0 + \frac{1}{m} \epsilon^{0ijk} \partial_i H_{jk} \right), \quad (20)$$

The primary constraints are

$$\Pi_0 \approx 0, \quad (21)$$

$$\Pi_{0i} \approx 0, \quad (22)$$

$$\Omega_i \equiv \left( \Pi_i - \frac{1}{4 m} \epsilon_{0ijk} H^{jk} \right) \approx 0, \quad (23)$$

$$\Lambda_{ij} \equiv \left( \Pi_{ij} + \frac{1}{2 m} \epsilon_{0ijk} G^k \right) \approx 0, \quad (24)$$

The persistence of the primary constraints leads to secondary constraints,

$$\Lambda \equiv \left( G_0 + \frac{1}{2 m} \epsilon_{0ijk} \partial^i H^{jk} \right) \approx 0, \quad (25)$$

$$\Lambda_i \equiv \left( -H_{0i} + \frac{1}{m} \epsilon_{0ijk} \partial^j G^k \right) \approx 0. \quad (26)$$
The non-vanishing Poisson brackets between these linearly independent constraints are

\[
\{ \Pi_0(\vec{x}), \Lambda(\vec{y}) \} = -\delta(\vec{x} - \vec{y}),
\]

(27)

\[
\{ \Pi_{0i}(\vec{x}), \Lambda^j(\vec{y}) \} = \delta^j_i \delta(\vec{x} - \vec{y}),
\]

(28)

\[
\{ \Omega_i(\vec{x}), \Lambda_j(\vec{y}) \} = \frac{1}{m} \epsilon_{oijk} \partial^k \delta(\vec{x} - \vec{y}),
\]

(29)

\[
\{ \Omega_i(\vec{x}), \Lambda_{jk}(\vec{y}) \} = -\frac{1}{m} \epsilon_{oijk} \delta(\vec{x} - \vec{y}),
\]

(30)

\[
\{ \Lambda_{ij}(\vec{x}), \Lambda(\vec{y}) \} = \frac{1}{m} \epsilon_{oijk} \partial^k \delta(\vec{x} - \vec{y}).
\]

(31)

Thus all the constraints are second class as expected of a theory without any gauge invariance. Note that the constraints \(\Omega_i\) and \(\Lambda_{ij}\) are due to the symplectic structure of the Lagrangian (8). Following Fadeev and Jackiw [16], the symplectic conditions, which are not true constraints, are implemented strongly leading to the the modified bracket,

\[
\{ G_i(\vec{x}), H_{jk}(\vec{y}) \} = -m \epsilon_{oijk} \delta(\vec{x} - \vec{y}).
\]

(32)

Consequently \(\Omega_i\) and \(\Lambda_{ij}\) are implemented strongly.

Now we enlarge the phase space by introducing canonically conjugate auxiliary pairs \((\alpha, \Pi_\alpha, p_i, q_i)\) and modify the remaining second class constraints such that they are in strong involution, i.e., have vanishing Poisson brackets. To this end, we define the non-vanishing Poisson brackets among the new phase space variables to be

\[
\{ \alpha(\vec{x}), \Pi_{\alpha}(\vec{y}) \} = \delta(\vec{x} - \vec{y}),
\]

(33)

\[
\{ q^i(\vec{x}), p_i(\vec{y}) \} = \delta^i_i \delta(\vec{x} - \vec{y}).
\]

(34)

The modified constraints which are in strong involution read

\[
\omega = \Pi_0 + \alpha, \quad (35)
\]

\[
\Lambda' = \Lambda + \pi_\alpha, \quad (36)
\]

\[
\theta_i = \Pi_{0i} + q_i, \quad (37)
\]

\[
\Lambda'_i = \Lambda_i - p_i. \quad (38)
\]
Following the general BFT procedure we construct the Hamiltonion which is weakly
gauge invariant and is given by

\[
H_{GI} = \int d^3x \left[ \mathcal{H}_c + \frac{1}{2} \Pi_a^2 + \alpha \partial^i G_i - \frac{1}{2} (\partial^i \alpha)(\partial_i \alpha) - \frac{1}{2} p_i p^i - \frac{1}{2} q_{ij} H^{ij} + \frac{1}{4} q_{ij} q^{ij} \right],
\]

(39)

where \( q_{ij} = (\partial_i q_j - \partial_j q_i) \). The Poisson brackets of modified constraints with \( H_{GI} \) are

\[
\{ \omega, H_{GI} \} = \Lambda',
\]

(40)

\[
\{ \Lambda', H_{GI} \} = 0,
\]

(41)

\[
\{ \theta_i, H_{GI} \} = \Lambda'_i,
\]

(42)

\[
\{ \Lambda'_i, H_{GI} \} = 0.
\]

(43)

Thus all the modified constraints are in involution with the \( H_{GI} \) as one can easily see
from their Poisson brackets. The gauge transformations generated by these first class constraints (35 to 38) are

\[
\left\{ G^0, \int d^3x \omega \bar{\theta} \right\} = \bar{\theta},
\]

\[
\left\{ \Pi_a, \int d^3x \omega \bar{\theta} \right\} = -\bar{\theta},
\]

\[
\left\{ H_{0i}, \int d^3x \theta_i \psi^j \right\} = \psi_i,
\]

\[
\left\{ p_i, \int d^3x \theta_i \psi^j \right\} = -\psi_i,
\]

\[
\left\{ G_i, \int d^3x \Lambda' \bar{\theta} \right\} = -\partial_i \bar{\theta},
\]

\[
\left\{ \alpha, \int d^3x \Lambda' \bar{\theta} \right\} = \bar{\theta},
\]

\[
\left\{ H_{ij}, \int d^3x \Lambda' \psi^i \right\} = -\partial_i \psi_j - \partial_j \psi_i,
\]

\[
\left\{ q_i, \int d^3x \Lambda' \psi^j \right\} = -\psi_i.
\]

(44)

Thus the combinations

\[
\bar{G}_0 = G_0 + \Pi_a,
\]

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\[ G_i = G_i + \partial_i \alpha, \]
\[ \bar{H}_{oi} = H_{oi} + p_i, \]
\[ \bar{H}_{ij} = H_{ij} - q_{ij}. \]  

(45)

are gauge invariant under the transformation generated by the first class constraints (35 to 38). Next we re-express gauge invariant Hamiltonian density \( \mathcal{H}_{GI} \) in terms of these gauge invariant combinations,

\[ \mathcal{H}_{GI} = \frac{1}{4} \bar{H}_{ij} \bar{H}^{ij} - \frac{1}{2} \bar{H}_{oi} \bar{H}^{oi} - \frac{1}{2} \bar{G}_i \bar{G}^i + \frac{1}{2} \bar{G}_0 \bar{G}^0 - G_0 \bar{\Lambda}' - H^0_i \bar{\Lambda}'_i, \]  

(46)

where \( \bar{\Lambda}' \) and \( \bar{\Lambda}'_i \) are the constraints \( \Lambda' \) and \( \Lambda'_i \) expressed in terms of the gauge invariant combinations (45).

The equations of motion following from this Hamiltonian (46) are,

\[ \bar{G}_0 + \frac{1}{2m} \varepsilon_{0ijk} \partial^j \bar{H}^{jk} = 0, \]
\[ \bar{G}_i + \frac{1}{2m} \varepsilon_{i\mu\lambda} \partial^\mu \bar{H}^{\nu\lambda} = 0, \]  

(47)

\[ -\bar{H}_{0i} + \frac{1}{m} \varepsilon_{0ijk} \partial^j \bar{G}^k = 0, \]
\[ -\bar{H}_{ij} + \frac{1}{m} \varepsilon_{ij\mu\nu} \partial^\mu \bar{G}^\nu = 0. \]  

(48)

These equations can be expressed in a covariant way, i.e.,

\[ \bar{G}_\mu + \frac{1}{2m} \varepsilon_{\mu\nu\lambda\sigma} \partial^\nu \bar{H}^{\lambda\sigma} = 0, \]  

(49)

\[ -\bar{H}_{\mu\nu} + \frac{1}{m} \varepsilon_{\mu\nu\lambda\sigma} \partial^\lambda \bar{G}^{\sigma} = 0. \]  

(50)

From these equations it follows that \( \partial^\mu \bar{G}_\mu = 0 \) and \( \partial^\mu \bar{H}_{\mu\nu} = 0 \). These also follow as the Hamiltonian equations of motion for \( \bar{G}_0 \) and \( \bar{H}_{0i} \), respectively. The gauge invariant solution for these equations which also satisfy the divergentless condition for \( \bar{G}_\mu \)and \( \bar{H}_{\mu\nu} \) are

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\[ \tilde{\mathcal{C}}_\mu \equiv \tilde{H}_\mu = \frac{1}{3!} \epsilon_{\mu\nu\lambda\sigma} H^{\nu\lambda\sigma}, \]
\[ \tilde{H}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\lambda\sigma} F^{\lambda\sigma}. \]  
(51)

where \( H^{\mu\nu\lambda} = \partial^\mu B^{\nu\lambda} + \text{cyclic terms} \) and \( F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) \).

Now, by substituting back the solutions for \( \tilde{G}_\mu \) and \( \tilde{H}_{\mu\nu} \) in \( \mathcal{H}_{GI} \) (46), the involutive Hamiltonian density becomes
\[ \mathcal{H}_{\tilde{G}I} = \frac{1}{4} F_{ij} F^{ij} - \frac{1}{2} F_{0i} F^{0i} + \frac{1}{4} H_{0ij} H^{0ij} - \frac{1}{2 \cdot 3!} H_{ijk} H^{ijk} + G_0 \tilde{\Lambda} - H_{0i} \tilde{\Lambda}^i \]  
(52)

where \( \tilde{\Lambda} = (\frac{1}{3!} \epsilon_{0ijk} H^{ijk} - \frac{1}{m} \partial^i F^0_{0i}) \) and \( \tilde{\Lambda}^i = (-\frac{1}{2} \epsilon_{0ijk} F^{jk} + \frac{1}{4} \partial^j H^0_{0ij}) \).

With
\[ \frac{1}{2} \epsilon_{0ijk} F^{jk} = B_i, \quad F_{0i} = - E_i, \]
\[ \frac{1}{3!} \epsilon_{0ijk} H^{ijk} = \bar{B}, \quad \text{and} \quad \frac{1}{2} \epsilon_{0ijk} H^{0jk} = \bar{E}_i, \]  
(53)

\( \mathcal{H}_{\tilde{G}I} \) becomes,
\[ \mathcal{H}_{\tilde{G}I} = \frac{1}{2} \left( E^2 + B^2 \right) + \frac{1}{2} \left( \bar{E}^2 + \bar{B}^2 \right) + G_0 \tilde{\Lambda} - H_{0i} \tilde{\Lambda}^i \]  
(54)

This is the Hamiltonian following from the \( B\wedge F \) Lagrangian (1). Note that \( \tilde{\Lambda} \) and \( \tilde{\Lambda}^i \) are the Gauss law constraints for the \( B\wedge F \) theory. The latter, which was an irreducible constraint in terms of gauge invariant combination becomes a reducible constraint when expressed in terms of the solutions (51), obeying \( \partial^i \tilde{\Lambda}_i = 0 \). By substituting back the solutions for \( \tilde{G}_\mu \) and \( \tilde{H}_{\mu\nu} \) into the equations of motion following from \( \mathcal{H}_{GI} \) (49, 50), they become
\[ - \partial^\nu F_{\mu\nu} + \frac{m}{3!} \epsilon_{\mu\nu\lambda\sigma} H^{\nu\lambda\sigma} = 0, \]  
(55)
\[ \partial^\lambda H_{\mu\nu\lambda} - \frac{m}{2} \epsilon_{\mu\nu\lambda\sigma} F^{\lambda\sigma} = 0, \]  
(56)

which are the same equations as the one following from the \( B\wedge F \) theory. Thus under BFT embedding, the fields appearing in the original Hamiltonian (20) get mapped to
gauge invariant combinations (45) of the embedded Hamiltonian-\(H_{GI}\) (46). The solutions to the equations of motion following from \(H_{GI}\) uniquely map the embedded Hamiltonian to that of \(B\wedge F\) theory. Also the irreducible constraints \(\Lambda_i\) and \(\Lambda\) in the original Hamiltonian (20) get mapped to the reducible constraint \(\tilde{\Lambda}_i\) and \(\tilde{\Lambda}\) respectively.

III. Phase Space Path Integral Approach

The equivalence of the theory described by the Lagrangian (8) to \(B\wedge F\) theory can also be established using phase space path integral method. By suitable choice of gauge fixing conditions, the partition function of the embedded model can become that of original massive spin-1, gauge non-invariant theory or that of \(B\wedge F\) theory, proving their equivalence.

The partition function for the embedded model described by the Hamiltonian (39) is

\[
Z_{emb} = \int D\eta \delta(\omega) \delta(\theta_i) \delta(\Lambda^i) \delta(\psi_i) \exp i \int d^4x \mathcal{L},
\]

where we have omitted the trivial Fadeev-Popov determinant for the gauges chosen below. The measure is,

\[
D\eta = D\Pi_0 D\Pi_{ai} D\Pi_\alpha D\alpha DP_i Dq_i DG_\mu DH_{\mu\nu},
\]

and

\[
\mathcal{L} = \Pi_0 \dot{G}^0 + \Pi_{0i} \dot{H}^{0i} + \Pi_{ai} \dot{\alpha}_i + p_i \dot{q}^i + \frac{1}{4m} \epsilon_{0ijk} \dot{H}^{ij} \dot{G}^k - \frac{1}{4m} \epsilon_{aijk} \dot{G}^i \dot{\alpha}_j H^{jk} - \mathcal{H}_{GI}
\]

where \(\mathcal{H}_{GI}\) is the invariant Hamiltonian given in (39); \(\delta(\psi_i)\) are the gauge fixing conditions corresponding to the first class constraints of the embedded model. Now we have twenty two independent phase space variables (remember \(\Pi_i\), and \(\Pi_{ij}\) are not independent degrees of freedom) along with the eight first class constraints giving the correct degrees of freedom required for a massive spin-one theory.
Choosing the gauge fixing conditions

\[ \delta(\psi_i) = \delta(\Pi_0)\delta(\Pi_{0i})\delta(\Lambda)\delta(\Lambda_i), \]  

(59)

and integrating out the canonical conjugate variables \( \alpha, \Pi_\alpha, q_i \) and \( p_i \) and momenta \( \Pi_0 \) and \( \Pi_{0i} \) from the partition function reduces \( Z_{emb} \) to

\[ Z = \int DG_\mu DH_{\mu\nu}\delta(\Lambda)\delta(\Lambda_i)\exp i \int d^4x \ L, \]  

(60)

where \( L \) is the original first order Lagrangian (8), with the Gauss law constraints imposed through \( \delta(\Lambda) \) and \( \delta(\Lambda_i) \). Note that the original second class constraints are the gauge fixing conditions (59).

Next we choose the gauge fixing conditions as

\[ \delta(\psi_i) = \delta(\partial^i H_{0i})\delta(\partial^i q_i)\delta(\chi_i)\delta(\chi_{ij}), \]  

(61)

where

\[ \chi_i = (G_i - \frac{1}{m} \epsilon_{0ijk} \partial^j H^{0k}) \]  

(62)

\[ \chi_{ij} = (H_{ij} - \frac{1}{m} \epsilon_{0ijk} \partial^k G^0), \]  

(63)

to show the equivalence of the embedded model to the \( B\wedge F \) theory. Owing to the constraints \( \omega \) and \( \theta_i \), the \( D\Pi_0, D\Pi_{0i} \), integrations are trivial. The \( D\Pi_\alpha \), and \( Dp_i \) integrations along with the constraints \( \delta(\Lambda') \) and \( \delta(\Lambda'_i) \) lead to the terms \(-\frac{1}{4m^2}G_{ij}G^{ij}\) and \(\frac{1}{2\cdot 3!}H_{ijk}H^{ijk}\) in the exponent. Using the fact that \( \partial^i G_i = 0 \) and \( \partial^i H_{ij} = 0 \) on the constraint surface of \( \chi_i \) and \( \chi_{ij} \) and the gauge fixing conditions \( \delta(\partial^i q_i) \) and \( \delta(\partial^i H_{0i}) \), we carry out the integrations over \( Dq_i \) and \( D\alpha \), which are just Gaussian. Thus the Lagrangian in the partition function becomes,

\[ L = \frac{1}{2m} \epsilon_{0ijk} \partial^i G^i H^{jk} - \frac{1}{4m^2} G_{ij} G^{ij} + \frac{1}{2 \cdot 3! m^2} H_{ijk} H^{ijk} - \frac{1}{2m^2} G_{0i} G^{0i} + \frac{1}{4m^2} H_{0ij} H^{0ij} \\
- \frac{1}{4} H_{ij} H^{ij} + \frac{1}{2} G_0^2 - \frac{1}{2m^2} H_{0i} \nabla^2 H^{0i} + \frac{1}{2m^2} G_0 \nabla^2 G^0. \]  

(64)
After using the constraints (62, 63) and the conditions \( \nabla^2 H_{0i} = -\frac{m}{2} \epsilon_{0ijk} G^{jk} \) and \( \nabla^2 G_0 = \frac{m}{2} \epsilon_{0ijk} \partial^i H^{jk} \) implied by (62, 63), the partition function becomes,

\[
Z = \int DA_\mu DB_{\mu\nu} \delta(\chi_i) \delta(\chi_{ij}) \exp i \int d^4x \mathcal{L}
\]

where \( \mathcal{L} \), with the identifications

\[
\frac{1}{m} G_\mu = A_\mu, \quad \frac{1}{m} H_{\mu\nu} = B_{\mu\nu}
\]

is the Lagrangian of \( B\wedge F \) theory(18). The constraints \( \chi_i \) and \( \chi_{ij} \), in terms of \( A_\mu \) and \( B_{\mu\nu} \) become,

\[
\chi_i = (mA_i - \epsilon_{0ijk} \partial^j B^{0k}), \quad (68)
\]

and

\[
\chi_{ij} = (mB_{ij} - \epsilon_{0ijk} \partial^k A^0) \quad (69)
\]

\( \chi_i \) and \( \chi_{ij} \) which are the gauge fixing conditions for the linearly independent generators \( \theta_i \) and \( \Lambda_i' \), now play the role of Gauss law constraints of the \( B\wedge F \) theory. Now

\[
\nabla^2 B_{0i} = -\frac{m}{2} \epsilon_{0ijk} F^{jk}, \quad (70)
\]

and

\[
\nabla^2 A_0 = \frac{m}{2} \epsilon_{0ijk} \partial^i B^{jk}, \quad (71)
\]

\( (F_{jk} = \partial_j A_k - \partial_k A_j) \), are the Gauss law constraints in the gauge \( \chi_i = 0 \) and \( \chi_{ij} = 0 \). Since \( \theta_i \) is not a reducible constraint, corresponding gauge fixing condition is also not reducible; but it implies the reducible Gauss law constraint (70) present in the \( B\wedge F \) theory. It is interesting to note the complimentary behavior of Gauss law constraints which comes as the gauge fixing conditions in the partition function for the embedded model.

Conclusion

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In this paper we have started with a new first order formulation of massive spin-one theory which is gauge non-invariant and converted it to a theory with only first class constraints following the line of Hamiltonian embedding of BFT. We showed that the embedded Hamiltonian is equivalent to the Hamiltonian of \( B \wedge F \) theory. This was shown both from the solutions of equations of motion following from the embedded Hamiltonian and from the phase space path integral in a suitable gauge. We also point out how an irreducible constraint of the first order theory gets mapped to the reducible constraint of the \( B \wedge F \) theory. It should be pointed out that the first-order lagrangian (8) and its equivalence to topologically massive gauge theory are both new results.

A similar first order Lagrangian can be formulated for massive spin-zero particle, but now involving a 3-form and a scalar fields as
\[
\mathcal{L} = -\frac{1}{2 \cdot 3!} C_{\mu\nu\lambda} C^{\mu\nu\lambda} - \frac{1}{2} \phi \phi + \frac{1}{4!} \frac{\epsilon_{\mu\nu\lambda\sigma}}{m} C^{\mu\nu\lambda\sigma} \phi, \tag{72}
\]
where \( C_{\mu\nu\lambda\sigma} = \partial_{\mu} C_{\nu\lambda\sigma} + \text{cyclic terms} \). Interestingly here the field content is the same as that of DKP formulation of spin-zero theory.

The equivalence demonstrated here is of the same nature as that between self-dual model \([17]\) and Maxwell-Chern-Simmon theory in 2 + 1 dimensions, shown in \([11]\). The behavior of the fields of the embedded Hamiltonian here is the same as that of 2 + 1 dimension self-dual model; viz, the gauge variant fields of the embedded model can be mapped to the fundamental fields of the \( B \wedge F \) theory or that of the original model. Despite this similarity, model in (8) is different from the self-dual model. The latter describes only half the degrees of freedom compared to that of massive spin-one theory in 2+1 dimensions and consequently is equivalent to the parity violating Maxwell-Chern-Simmon theory. Also the self-duality condition is possible only in \( 4k - 1 \) dimensions. But the former describes all the three states of polarization needed for massive spin-one particle and also this construction is possible in all dimensions and has an even-parity
mass term. Owing to the even-parity mass term described by this model, the $2+1$ dimensional non-abelian generalization of (8) may be related to the recently constructed Jackiw-Pi model [18]. The self-dual model and Maxwell-Chern-Simmon correspondence has proved to be useful in Bosonization in $2+1$ dimensions [19]. It should be interesting to see similarly if the equivalence proved here has a role in studying Bosonization in $3+1$ dimensions. It is also interesting to study the Hamiltonian embedding of the non-abelian version of (8). Work along these lines are in progress.

We have exploited the gauge symmetry arising due to the Hamiltonian embedding procedure to prove the equivalence between two different formulations of massive spin-one theory. There is a different procedure which also has the potential to establish equivalence among different formulations [20]. In this method, the new degrees of freedom are added by hand, which generates abelian gauge algebra and this is used to gauge fix suitably, to arrive at a different formulation of the original theory like for example Bosonisation [21]. It should be interesting to investigate if the quantum equivalence proved here survives the case of coupling with external fields like gravitation and electromagnetism also.

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