I. INTRODUCTION

The invention of the electronic transistor has laid the cornerstone of the information age. With the development of quantum information technology, the research and fabrication of optical transistor have become an important branch. In general, the fabrication of optical transistors is based on optical non-reciprocity [1, 2]. Early, optical non-reciprocity has been achieved in optical waveguides [3–6] or optical nonlinear system [7–9] by breaking the time-reversal symmetry. Recently, several alternative schemes based on different principles are proposed, such as photoacoustic effects [10, 11], indirect interband photon transitions [12, 13], space symmetric fracture structures [14–16], moving systems [10, 17] and parity time-symmetric structures [18–20]. In addition, for the potential applications in photonic quantum information processing, the abilities to be integrated on a chip, non-local control at long-range [21–23], and operated on a single-photon level [24–26] are desirable features for the realization of nonreciprocal all-photon devices in the future. With the rapid development of new micro integrable devices, optomechanical systems have shown enormous potential for application in quantum information processing [27–29]. It has previously been shown that optomechanical systems can be used to induce nonreciprocal effects for light [30–32]. Multi-mode optomechanical systems have been used much attention recently. Numbers of novel and interesting phenomena are noted, such as high-fidelity quantum state transfer [33–35], enhanced quantum nonlinearities [36, 37], phonon lasing [38–40], coherent perfect absorption (CPA) [41–43] and so on. Especially, the CPA could view as an inverse process of laser [44–46], which provides a new mechanism in optomechanical system for controlling optical nonreciprocity. Currently, most of investigations on atom-assisted optomechanical systems coupled with independent cold atoms driven by quantum cavity modes and classical coherent control fields [47–50]. However, Rydberg atoms, coupled by dipole-dipole interactions (DDI), have recently been shown to be efficient nonlinear media in cavities in order to achieve additional control freedom of optomechanical interactions and applications [51–53]. An essential blockade effect based upon DDI prevents the excitation of more than one atom into a Rydberg state within a macroscopic volume of several micrometers in radius [54–57]. Meanwhile, based on dipole blockade, many promising proposals have been put forward for manipulating quantum states of atoms and photons [58–60], simulating many-body quantum systems [61, 62], generating reliable single photons [63–65], and revealing some novel behaviors in EIT [66–72], etc.

In this paper, we study the optical response of a symmetric double-cavity optomechanical system assisted by Rydberg atomic ensemble (Superatom SA) with a movable mirror of perfect reflection and driven by two coupling fields and two probe fields. The transition from ground state to excited state of an atom is coupled by a cavity mode, and the transition from the excited state to Rydberg state is coupled by a classical control field. There are vDW interactions among the Rydberg atoms in the cavity or from outside the system. So the optical response of the hybrid system can be manipulated by controlling the Rydberg excitation of SA.

We focus especially on the transmission and reflection properties of two side weak probe fields, by switching on and off the external control. In case (I), we find a controlled optical diode effect, that photons only pass through the system from one direction. In case (II), we get a controlled photon rectifier, which can change the propagation direction of the photons, instead. In case (III) and (IV), we achieve the function of amplification in two different ways. The first one is based on coherent perfect synthesis (CPS), and the other is owing to FWM effect of atom-assisted opto-mechanical system. We expect that the all-optical controlled transistor (the optical correspondence of classical electrical transistor), which sets controller, rectification, amplification and other functions as a whole, could be explored to build new tunable photonic devices on quantum information networks.
We consider a hybrid optomechanical system with one movable mirror (membrane oscillator) of perfect reflection inserted between two fixed mirrors of partial transmission, which form two mechanically-coupled Fabry–Perot cavities. And in the left cavity there are \( n_{sa} \) atoms with a Rydberg state \( |r\rangle \) [see Fig. [1]]. We describe the two optical modes in the left or right cavity, respectively, by annihilation (creation) operators \( c_i \) (\( c_i^\dagger \)) or \( c_r \) (\( c_r^\dagger \)). And the only mechanical mode is described by \( b \) (\( b^\dagger \)). These annihilation and creation operators \( \hat{O} = \{c_i, c_r, b\} \) are bosons and satisfy the commutation relation \( [\hat{O}_i, \hat{O}_j] = \delta_{i,j} \) (\( i, j = c, c, b \)). Two probe (coupling) fields are used to drive the double-cavity system from either left or right fixed mirrors with their amplitudes denoted by \( \varepsilon_i = \sqrt{2\kappa_{0L}/(\hbar\omega_p)} \) and \( \varepsilon_r = \sqrt{2\kappa_{0R}/(\hbar\omega_r)} \). Here \( \kappa \) is the decay rate of both cavity modes, \( \psi_L, \psi_R, \psi_{cL} \) and \( \psi_{cR} \) are the relevant field powers, \( \omega_p \) (\( \omega_r \)) is the probe (coupling) field frequency. The membrane oscillator has an eigen frequency \( \omega_m \) and a decay rate \( \gamma_m \) and thus exhibits a mechanical quality factor \( Q = \gamma_m/\omega_m \). Two identical optical cavities of lengths \( L \) and frequencies \( \omega_0 \) are got when the membrane oscillator is at its equilibrium position in the absence of external excitation. And in the left Fabry–Perot cavity, cavity field \( c_i \) also drives an ensemble of \( n_{sa} \) cold atoms into the three level ladder configuration together with an external control field with frequency \( \omega_c \). Levels \( |g\rangle, |e\rangle, \) and \( |r\rangle \) correspond, respectively, to \( 5S_{1/2}|F = 1\rangle, 5P_{3/2}|F = 2\rangle \) and \( 60S_{1/2}|F = 1\rangle \) of \(^{87}\text{Rb}\) atoms.

Then the total Hamiltonian of our hybrid system in the rotating-wave frame can be written as

\[
H = H_c + H_m + H_{cp} + H_{mc} + H_a + H_{ac} + H_{vdW},
\]

where the first four terms describe the two free optical cavities, the free movable mirror, and the optomechanical interaction, respectively, with the following expressions:

\[
\begin{align*}
H_c &= \hbar \Delta_c (c_i^\dagger c_i + c_r^\dagger c_r) - \hbar \omega_m b^\dagger b, \\
H_{cp} &= i \hbar \varepsilon_{cL} (c_i^\dagger - c_i) + i \hbar (\varepsilon_i c_i^\dagger e^{-i\delta t} - \varepsilon_i^* c_i e^{i\delta t}) \\
&+ i \hbar \varepsilon_{cR} (c_r^\dagger - c_r) + i \hbar (\varepsilon_{cR} c_r^\dagger e^{-i\delta t} - \varepsilon_{cR}^* c_r e^{i\delta t}), \\
H_{mc} &= \hbar g_0 (c_i^\dagger c_r - c_r^\dagger c_i) (b^\dagger + b),
\end{align*}
\]

where we define \( \Delta_c = \omega_0 - \omega_m \) the detuning between cavity modes and coupling fields, \( \delta = \omega_p - \omega_c \) the detuning between probe fields and coupling fields, \( \theta \) the relative phase between left- and right-side probe fields, \( g_0 = \frac{\alpha^m}{\sqrt{2}} \sqrt{\hbar/(2m\omega)} \) the hybrid coupling constant between mechanical and optical modes (\( m \) quality of the oscillator, \( V \) the volume of the cavity, and \( \varepsilon_0 \) the vacuum dielectric constant. The free atomic ensemble and the atom-light interaction Hamiltonian are \( H_a = \hbar \sum_{i=1}^{n_{sa}} (\omega_{eg} \sigma_{ei}^+ \sigma_{ei}^- + \omega_{eg} \sigma_{ei}^+ \sigma_{ei}^-) \) and \( H_{ac} = -\hbar \sum_{i=1}^{n_{sa}} (\Omega_r \sigma_{ri}^+ e^{-i\omega_r t} + \gamma_{ac} \sigma_{ri}^+ \sigma_{ri}^0 + H.C.) \), respectively, with coupling constant between atomic ensemble and left cavity mode \( g_{ac} = \sqrt{\omega_r/(2\hbar \varepsilon_0)} \) and atomic transition dipole moment \( \gamma_{ac} \). We will focus on the new optomechanical features resulting from DDI as expressed by a vdW potential

\[
H_{vdW} = \hbar \sum_{i<j} C_{0i} \sigma_{ri}^+ \sigma_{rj}^-.
\]

Here \( n_{sa} \) is the atomic number in the microvolume such that only one atom can be excited to the state \( |r\rangle \), with \( C_0 \) denoting the vdW coefficient. And \( R_{ij} \) is the inter-atomic distance between the \( i \)th and \( j \)th atoms in the SA.

Considering dissipation and quantum noise of the system, we can get the following Heisenberg-Langevin equations:
where \( \hat{\sigma}_{ge} = \sum_{i=1}^{n_{sa}} \hat{\sigma}_{ge}^{(i)} = \hat{S} e^{-i(\omega_c + \omega_r)t} \) denotes the collective polarization (spin) operator of atoms, with \( \hat{c}_i = c_i e^{i \omega_c t} \) and \( \hat{P} = \hat{P} e^{-i \omega_r t} \). \( \gamma_m, \gamma_c \) and \( \gamma_r \) correspond to the decay rate of cavities, oscillator, excited state \( |e \rangle \) and that of Rydberg state \( |r \rangle \), respectively. \( \langle V \rangle = \sum_{i=1}^{n_{sa}} c_i \) is the mean value of total vdW potential in a superatom with \( v(i) \) the vdW potential of single atom. \( b_{in} \) being the thermal noise on the movable (zero mean value), and \( c_i^{in} \) is the input quantum vacuum noise from the left (right) cavity with zero mean value. Here we are more interested in the mean optical response of the optomechanical system to probe field in the presence of both strong driving fields. In this regard, we can safely ignore the quantum fluctuations of all relevant operators and use the factorization assumption \( b(c_i) = \langle b(c_i) \rangle \) to generate the mean values in steady state. In order to solve Eqs. (2), we write each operator as the sum of its mean value and its small fluctuation \( \hat{O}_1 = \langle \hat{O}_1 \rangle + \delta \hat{O}_1, \hat{O}_1 = \{ b, c, r, P, S \} \), when both coupling fields are sufficiently strong. Then we get two series of equations about steady-state or fluctuation of operators, respectively,

\[
\begin{aligned}
\dot{b} &= -i \omega_m b - ig_0 |c_i^\dagger c_i - c_i^r c_r\rangle - \frac{\gamma_m}{2} b + \sqrt{\gamma_m} b_{in}, \\
\dot{c}_i &= [- \kappa + i \Delta_c - ig_0 (b_i^\dagger + b_i)] c_i + \varepsilon_{cL} + \varepsilon_r e^{-i \omega_r t} - ig_{ac} \sqrt{n_{sa}} \hat{P} + \sqrt{2 \kappa c_{ir}^\dagger}, \\
\dot{c}_r &= [- \kappa + i \Delta_c + ig_0 (b_i^\dagger + b_i)] c_r + \varepsilon_{cR} + \varepsilon_r e^{i \omega_r t} + \sqrt{2 \kappa c_{ir}^\dagger}, \\
\dot{P} &= [- i \Delta_1 + \gamma_c \delta c_l + i \Omega_r^* \delta S + ig_{ac} \sqrt{n_{sa}} \delta c_l f_1(t), \\
\dot{\delta S} &= [- i \Delta_2 + i \langle V \rangle + \gamma_r \delta S + i \Omega_r \delta \hat{P} + f_2(t),
\end{aligned}
\]

where the effective detuning is \( \Delta_c = \Delta_c - g_c b_s \) and \( c_s = \langle c_i \rangle \) \( (i = l, r) \) is the steady state average of cavities.

It is difficult to solve Eqs. (3) and (4) directly. So we first solve the atomic part of the equations above, taking part of the cavities as a constant. In general, the optical response of the Rydberg atomic ensembles are affected by the vdW interactions. And in our system, the vdW shift reads,

\[
V_s = \left\langle \sum_{j} \frac{C_6}{R_j^{6}} \langle s_j \rangle \right\rangle.
\]

Note that \( V_s \) tends to infinite for one Rydberg ex-
citation \( \langle \hat{\sigma}_{rr} \rangle = 1 \) or vanishing for zero Rydberg excitation \( \langle \hat{\sigma}_{rr} \rangle = 0 \). Then Eqs. (3) cannot be solved straightforwardly owing to the dipole-dipole correlation between different atoms in the rigid blockade regime. Therefore we choose to define the \( n_{sa} \) cold atoms as a single SA whose volume \( V_{sa} \) of radius \( R_{sa} \) is slightly smaller than the blockade volume \( V_{bs} \) of radius \( R_{bs} \). For a very weak cavity field, this SA can be described by three symmetric states containing at most one Rydberg excitation \( |G\rangle = |g\rangle \otimes n_{sa} , \quad |E^{(1)}\rangle = \frac{1}{\sqrt{n_{sa}}} \sum_{i=1}^{n_{sa}} |g_1, g_2, \ldots, c_i, \ldots, g_{n_{sa}} \rangle \), and \( |R^{(1)}\rangle = \frac{1}{\sqrt{n_{sa}}} \sum_{i=1}^{n_{sa}} |g_1, g_2, \ldots, r_i, \ldots, g_{n_{sa}} \rangle \) [21]. According to the Superatom theory model, the Rydberg excitation \( \Sigma_{RR} \) in an SA will determine the effective SA polarization, which is

\[
P_s = (P_2)_{\Sigma_{RR}} + (P_{3L})_{[1 - \Sigma_{RR}]}.
\]

Here \( (P_2) \) is the polarization of a two level atomic system and \( (P_{3L}) \) is that of a three level Ladder type system.

\[
(P_2) = \frac{1}{i \Delta_1 + \gamma_c} \int \gamma_{ac} \langle \hat{\sigma}_{rr} \rangle \hat{P},
\]

\[
(P_{3L}) = \frac{1}{i \Delta_1 + \gamma_c} (i \Delta_2 + i \hat{V} + \gamma_r) \hat{c}_{ls},
\]

And we get

\[
\Sigma_{RR} = \frac{\gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle }{g_{ac} n_{sa} \Omega_1 \Omega_2 \hat{c}_{ls} \hat{c}_{ls} + \langle \gamma_c \rangle \delta_{\gamma} \gamma_2 \gamma_1^2}.
\]

with \( \Sigma_{GG} + \Sigma_{RR} \approx 1 \), neglecting the excitation of the intermediate states of SAs.

We can get the steady-state solutions,

\[
b_s = \frac{i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle }{i \Delta_2 + i \hat{V} + \gamma_r},
\]

\[
\hat{c}_{ls} = \frac{\gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle }{i \Delta_1 + \gamma_c},
\]

\[
c_{rs} = \frac{\gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle }{i \Delta_2 + i \hat{V} + \gamma_r},
\]

(6)

Considering the term of crossing dW interactions \( V_c \), if the Rydberg excitation \( \langle \hat{\sigma}_{rr} \rangle = 1 \), \( (i \Delta_2 + i \hat{V} + \gamma_r) \) ( \( \gg \Omega_1^2 \) ) will tend to be infinite, which causes \( (P_{3L}) \approx \langle P_2 \rangle \) Solving equations, \( \partial_t P_s = -(i \Delta_1 + \gamma_c) P_s + i \Delta_s S_s + i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle \), \( \partial_t S_s = -(i \Delta_2 + i \hat{V} + \gamma_r) S_s + i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle \), we can get

Using the same method, from equation \( \delta \hat{P} = -(i \Delta_1 + \gamma_c) \delta \hat{P} + i \Delta_s \delta S + i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle \), and \( \delta \hat{S} = -(i \Delta_2 + i \hat{V} + \gamma_r) \delta S + i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle \), we can get

\[
\langle \delta \hat{P} \rangle = \langle \delta \hat{P}_{3L} \rangle (1 - \Sigma_{RR}), \quad \langle \delta \hat{P}_{3L} \rangle = \frac{i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle }{i \Delta_1 + \gamma_c}.
\]

(7)

Then keeping only the linear terms of fluctuation operators and moving into an interaction picture by introducing: \( \delta \hat{O} \rightarrow \delta \hat{O} e^{-i \Delta t}, \delta \hat{O}^{in} \rightarrow \delta \hat{O}^{in} e^{-i \Delta t}, \hat{O} = \hat{b}_s, \hat{c}_s, \hat{c}_r, \hat{P}, \hat{S} \), \( \Delta_i = \{ \Delta_1, \Delta_2, V, \Delta_c, \omega_{in} \} \), we obtain the linearized quantum Langevin equations:

\[
\delta \tilde{b} = -i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle - \frac{\gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle }{i \Delta_1 + \gamma_c},
\]

\[
\delta \tilde{c}_s = -i \Delta_s \delta \tilde{c}_s + i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle \delta \hat{S} + i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle \delta \hat{P}_s + \delta \tilde{c}_r = 0,
\]

\[
\delta \tilde{c}_r = -i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle \delta \hat{P}_{3L},
\]

where we set \( G = g_{ac} c_{ls} \) as the effective optomechanical coupling rate and \( n^2 = |c_{rs}/c_{ls}|^2 \) as the photon number ratio of the two cavity modes, with \( \kappa' = i \kappa - \kappa', \gamma_m = i \kappa - \frac{\gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle }{i \Delta_1 + \gamma_c} - i \Delta_2 + i \hat{V} + \gamma_r \), it is easy to attain the following results,

\[
\delta \tilde{b}_+ = \frac{\gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle }{i \Delta_1 + \gamma_c} e^{-i \Delta_1 t} e^{-i \Delta_2 t}
\]

\[
\delta \tilde{c}_s = -i \Delta_s \delta \tilde{c}_s + i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle \delta \hat{S} + i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle \delta \hat{P}_s + \delta \tilde{c}_r = 0,
\]

\[
\delta \tilde{c}_r = -i \gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle \delta \hat{P}_{3L},
\]

(9)

where \( \delta \tilde{b}_+ = \frac{\gamma_{ac} \gamma_{ls} \gamma_{ac} |G\rangle \langle \hat{\sigma}_{rr} | \langle \gamma_c \rangle }{i \Delta_1 + \gamma_c} e^{-i \Delta_1 t} e^{-i \Delta_2 t} \),

(10)
It is possible to determine the output fields \( \varepsilon_{\text{outl}} \) and \( \varepsilon_{\text{outr}} \) leaving from both cavities with the following input-output relation \[44\]

\[
\begin{align*}
\varepsilon_{\text{outl}} + \varepsilon_l e^{-i\omega t} &= 2\kappa (\delta c_l), \\
\varepsilon_{\text{outr}} + \varepsilon_r e^{i\theta} e^{-i\omega t} &= 2\kappa (\delta c_r),
\end{align*}
\]

where the oscillating terms can be removed if we set \( \varepsilon_{\text{outl}} = \varepsilon_{\text{outl}} e^{-i\omega t} + \varepsilon_{\text{outl}} e^{i\omega t} \) and \( \varepsilon_{\text{outr}} = \varepsilon_{\text{outr}} e^{-i\omega t} + \varepsilon_{\text{outr}} e^{i\omega t} \). Note that the output components \( \varepsilon_{\text{outl}}^+ \) and \( \varepsilon_{\text{outr}}^+ \) have the same Stokes frequency \( \omega_p \) as the input probe fields \( \varepsilon_l \) and \( \varepsilon_r \), while the output components \( \varepsilon_{\text{outl}}^- \) and \( \varepsilon_{\text{outr}}^- \) are generated at the anti-Stokes frequency \( 2\omega_c - \omega_p \) in a nonlinear four wave mixing process of optomechanical interaction. Then with Eqs (8) we can obtain

\[
\begin{align*}
\varepsilon_{\text{outl}}^+ &= 2\kappa \delta c_l - \varepsilon_l, \\
\varepsilon_{\text{outr}}^+ &= 2\kappa \delta c_r - \varepsilon_r e^{i\theta},
\end{align*}
\]

oscillating at the Stokes frequency of our special interest. So it is easy to find

\[
\begin{align*}
\varepsilon_{\text{outl}}^+ = 2\kappa n G^2 \varepsilon_l e^{i\theta} + [\gamma_m \delta c_l + 2G^2(2\kappa n^2 - \kappa' - n^2 g'_{AC})] \varepsilon_l, \\
\varepsilon_{\text{outr}}^+ = 2\kappa n G^2 \varepsilon_l - [(ix - 3\kappa)\gamma_m' g'_{AC} + G^2] + n^2 g'_{AC} G^2 \varepsilon_r e^{i\theta}.
\end{align*}
\]

### III. RESULTS AND DISCUSSION

In this section, we numerically simulate the optical response of the hybrid system, which features controlled non-reciprocity and potential use for optical diode and transistor. The transmission coefficient \( T_l = |\varepsilon_{\text{outr}}^+ / \varepsilon_l|^2 \) (\( T_r = |\varepsilon_{\text{outl}}^+ / \varepsilon_r|^2 \)) and reflection coefficient \( R_l = |\varepsilon_{\text{outr}}^- / \varepsilon_l|^2 \) are important. And they are both the functions of the probe frequency \( \varepsilon / \kappa \). In the following discussion, we only consider the two extremes of the system: (a) optical reciprocity and (b) optical non-reciprocity. The case (a) corresponds to decoupling between atomic ensemble and left cavity [see Fig. 2(a)]. Under the condition, we get reflection coefficient of cavities, only in one direction. It is coherent perfect absorption effect (CPA) that is the basis for implementing of this function above. The parameters should be set as \( \gamma = 2\kappa \), \( \theta = n\pi \) and \( \varepsilon_l = \varepsilon_r \neq 0 \). CPA will occur at \( x_{\pm} = \pm\sqrt{(n^2 + 1)G^2 - \kappa'^2} \) \[43, 44\]. However, the coupling between atoms and the left cavity changes the CPA conditions.

In this case, Fig. 3 display the transmission coefficient of left \( T_l \) and right side \( T_r \) of the hybrid system, with different \( G \). It is obvious to find the transmissions of the two sides are absolutely the same \( T_l = T_r \) [see Fig. 3(a1) and h1)], because the atomic ensemble is decoupled with the left cavity, switching off the external control. Photons can flow through the hybrid system in different directions, symmetrically. And then, the optical symmetry of system will be broken \( (T_l \neq T_r) \), when switching on
the control causes the atoms to couple with the cavity system. In particular, \( T_l \approx 1 \) [Fig. 3(a2)] in the range \(-4 < x/\kappa < 4\) because \( g_{AC}^g \gg \kappa \) owing to enough large \( n^2_{\text{na}} \) and \( |\kappa| \approx |3\kappa - ix| \ll g_{AC}^g \),

\[
T_l \approx \left| \frac{(3\kappa - ix)(\gamma_n g_{AC}^g + G^2) + n^2 g_{AC}^g G^2}{\gamma_n g_{AC}^g + n^2 g_{AC}^g G^2 + \kappa G^2} \right|^2 \approx 1 \quad \text{(12)}
\]

but \( T_r \) also can be equal to zero [Fig. 3(b2)]. Furthermore, \( T_l = 1 \) and \( T_r = 0 \) means that we get the photon diode framework which is a controlled unidirectional transmission device [see Fig. 3(e1,2)].

**B. Controlled Photon Rectifier \( (\gamma_m \rightarrow 0, \theta = n\pi, \varepsilon_l \neq 0 \text{ and } \varepsilon_r = 0) \)**

Aiming to achieve optical rectification, we set \( \gamma_m = 0.1\kappa, \theta = n\pi, \varepsilon_l \neq 0 \) and \( \varepsilon_r = 0 \) (without input field of right side). Coherent perfect transmission effect (CPT) is the basis for implementing of this function.

In this case, if there is no coupling between atoms and cavity, we can find \( R_l = 0 \) [Fig. 4(a1)] and \( T_l = 1 \) [Fig. 4(b1)] near the range \(-0.5 < x < 0.5\) with \( G > \kappa \). Photons flow through the hybrid system perfectly without reflection [Fig. 4(c1)]. This perfect transmission is due to quantum coherence of the double-opto-mechanical system, which can be controlled by phase \( \theta \). However, the optical properties of the system have changed, once there are effective coupling between atoms and cavity system. We find

\[
T_l \approx \left| \frac{2\kappa nG^2}{\gamma_n g_{AC}^g} \right|^2 \approx 0, \quad R_l \approx \left| \frac{(ix - 3\kappa)(\gamma_n g_{AC}^g + G^2) + n^2 g_{AC}^g G^2}{\gamma_n g_{AC}^g + n^2 g_{AC}^g G^2 + \kappa G^2} \right|^2 \approx 1, \quad \text{(13)}
\]

[Fig. 4 (a2 and b2)]. Photons are reflected back from the hybrid system around the same range with \( 2\kappa \ll g_{AC}^g \). It will be an framework for photon rectifier, if the manipulation has been achieved to control the direction of photon flow effectively.
C. External control

The purpose of external control is to allow the atomic ensemble in the cavity to be decoupled from the left cavity, controllably. We can achieve it in a variety of ways. Firstly, we consider the control switching on/off the coupling field $\Omega_r$, without the long range Rydberg vdW blockade effect between two Rydberg atomic ensembles. We set $\Delta_1 \gg g_{ac}$ with $\kappa \approx \Delta_2 \approx 0$, which is typical Rydberg-EIT resonance with large single photon detuning condition. Under the case, $\Omega_r \neq 0$ is the necessary condition for effective coupling between cavity system and Rydberg atomic ensembles, because large single photon detuning $\Delta_1$ will result in decoupling without $\Omega_r$. It is a kind of typical coherent control, which is easy to do in the experiments. However, it is obviously not a nonlocal or single photon level quantum control.

Secondly, we consider the control scheme that double-cavity optomechanical system with the long range Rydberg vdW blockade effect between two Rydberg atomic ensembles [see Fig.1(b)]. Similar to self-interactions, external vdW interactions owing to Rydberg excitation of $|r\rangle$ can lead to a large frequency shift of $|r\rangle$. Under the case, $i\Delta_2 + iV + \gamma_r \longrightarrow \infty$, $\langle P_{lL} \rangle \approx \langle P_2 \rangle$ and we dispense with calculating $\sum_{RR}$ . With a large single photon detuning $\Delta_1 \gg g_{ac}$, the atoms are still decoupled from the system. In other words, we can control the excitation of $|r\rangle$ in the external Rydberg atomic ensembles to manipulate the optical response of hybrid systems, which is a long range nonlocal optical manipulation. Moreover, the control of Rydberg excitation of the atomic ensemble outside the system can reach single-photon level.

IV. CONCLUSIONS

In summary, we have studied the optical response of the Rydberg-atom-assisted double-cavity optomechanical system. We switch on and off the Rydberg blockade effect to control whether the atomic ensembles decouple with the left cavity. And whether the optical reciprocity of the original symmetric system will be destroyed is controllable. We consider four special cases depending on the choice of actual parameters: (I) with $\theta = n\pi$, $\varepsilon_l = \varepsilon_r \neq 0$; (II) with $\theta = n\pi$, $\gamma_m \longrightarrow 0$, $\varepsilon_l \neq 0$ and $\varepsilon_r = 0$. In case (I), our numerical calculations show that atomic ensembles coupled with single cavity break the optical reciprocity of the symmetric system. Photons can only flow through the system in one direction, which is the optical diode effect, switching on the control. In case (II), turning off the input field of right side, the system has become an controlled photon rectifier, which controls propagation behavior of the photons. In other words, photons that should have transmitted have been reflected back absolutely. Then, we propose two types of optical amplifier schemes with our hybrid system. Though recent study on optical transistors have achieved great progress, most of them have only achieved optical control. There are no the optical correspondence of classical electric transistors which set controller, rectification, amplification or other functions as a whole. Owing to the blockade effect between Rydberg atoms, it is great promising to make this type of control to the single photon level. We hope our work can provide a new way of thinking and a substantial role in promoting the study of optical transistor and other all-optical devices.

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