Computer Aided Pressure Transient Analysis of a Layered Reservoir System with a Constant Pressure Boundary

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1. Introduction

The petroleum industry has witnessed a massive increase in software applications in recent times, though computers cannot replace human judgement, they have become indispensable to the petroleum engineer in the area of oil and gas prediction, understanding complex processes, planning maintenance and forecasting. Most software are problem or process specific (customised to solve unique challenges) while others are more general and available for public use [1, 2, 3]. The use of computer application in pressure transient analysis has numerous advantages over manual analysis. The benefits include increased productivity and improved quality of test and analysis in the shortest possible time [4, 5, 6 and 7]. Layered reservoirs analysis are complicated, due to the existence of different layers with varying permeability, porosity and formation characteristics, pressure transient test analysis of these systems are difficult. Different mathematical methods employed to obtain solution to the flow equations for the layered models, include the Laplace transform, finite difference, modified Bessel function, linear regression analysis and the source function method [8, 9, 10 and 11]. For these systems with complex configuration, robust computer programs are needed to ease analysis and interpretation. In this work the mathematical solutions to the reservoir models were presented. The models were developed using the Greens and Source function method to model the two layered reservoir subject to a
bottom waterdrive using horizontal wells. An efficient and fast computer algorithm was developed and presented. This algorithm can compute fast and accurate values of pressure and pressure derivative of a two layered reservoir system. In addition a visual display of the solution for quick and easy interpretation of the results was provided. The program created a platform where different configurations of the reservoir and horizontal wells can be envisaged and evaluated. It has the capacity to compute the number of flow periods that will prevail for a particular set of well/reservoir parameters, crossflow coefficient, modification factor and total system permeability. Graphical representations of results were also provided. Typical numerical examples were used to test the performance of the program and highlight its application in well test analysis of layered system with constant pressure boundary.

2. Design of Software Model
The two layered reservoir model with horizontal wells developed in [12] was used in this work. Relevant equations such as equations (9) – (19) were employed in the design of the software using programming technique of Visual Basic.Net [13].

2.1 Dimensionless parameters
For simplicity and easy comparison with other works, parameters were converted to their dimensionless form. Equation (1) was adopted to model dimensionless wellbore radius was modelled using equation (2) [14]

\[ i_D = \frac{2i}{L} \sqrt{\frac{k}{k_i}} \]

\[ R_{bD} = \frac{r_w}{L} \left( \frac{k}{k_z} + \frac{k}{k_i} \right) \]

Dimensionless time and pressure drop were modelled using equation (3), (4) and (5) respectively [15]

\[ t_{DN} = \frac{Kt}{\varphi \mu C_i \left( \frac{L}{2} \right)^2} \]

\[ P_D = \frac{2\pi khDp}{q\mu} \]

The pressure drop caused by production from a continuous source in a well is expressed as

\[ \Delta p(x, y, z, t) = \frac{1}{\varphi C_i} \int q L \cdot S(x, y, z, t) dt \]  

Where \( s(x, y, z, t) \) represents the instantaneous source function (ISF) for the particular reservoir and well configuration.

2.2 Layered system equations
Source and Greens Function were selected to assemble each layer’s dimensionless pressure(P_{Di}) using the Newman’s product rule. Utilising the superposition principle the full P_D for the well and reservoir system was evaluated as (6) and (7). Derivatives of the P_D were computed using equation (8).

\[ P_{Di}(X_{Di}, Y_{Di}, Z_{Di}, t) = 2\pi h_b \int s(X_{Di}, t) s(Y_{Di}, t) s(Z_{Di}, t) dt \]  

\[ P_{Di} = P_a + P_b + \ldots \ldots + P_n \]

Where i represents the layer and \( a, b \) and n the flow periods.

\[ P_D = \frac{\partial P_D}{\partial \ln D} \]

\[ P_D \] for early radial flow period common to both no crossflow and crossflow layered systems was represented by equation (9)

\[ P_D = \alpha \frac{K}{8L_D} \sqrt{\frac{K}{L}} Ei \left( -\frac{r_{Di}^2}{4\tau_{Di}} \right) \]

(9)

2.2.1 No crossflow layered system
P_D for intermediate and late time flow periods of layer one given in equations (10) and (11)
2.2.2 Crossflow layered system

Owing to the permeable interface in the crossflow layered reservoir system, the following additional parameters are required for proper modelling of the system.

Modification factor (Eij)

$E_{ij} = \frac{W_1 + W_2}{2\pi (W_1^2 + W_2^2)}$  \hspace{1cm} (12)

P_D for intermediate and late time flow periods in layer one are determined by equations (14) and (15)

$P_{op} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left[ \left( \frac{K}{X} + Y_{1n} \right) + \left( \frac{K}{X} + Y_{1m} \right) \right] \exp \left( \frac{-2\pi^2}{\mu} \right) \frac{\nu}{\lambda} \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \n...
as numeric values in a data grid and also with an option of graphical representation on Cartesian scale plotting the log of dimensionless time ($\log t_D$) against dimensionless pressure($\log P_D$).

### 3. Program Algorithm

The following is the software’s algorithm.

**Start**

Select type of reservoir interface.

**Option 1:** No-crossflow layered reservoir

**Option 2:** Crossflow layered reservoir

*If* option value $= 1$ *then* if reservoir type is no-crossflow

- If desired parameter $= P_D$ Then Input known parameters $L$, $h$, $k$, $z_m$, $r_w$, $\phi$, $\mu$
- Compute flow periods $t_{DE1}$, $t_{DE2}$, $t_{DE3}$, $t_{D4}$.
- If dimensionless time $t_D < $ first flow period $T_{DE1}$ Then Infinite acting flow period, Exponential integral Equation
- Compute $P_{D1}$ using the Laguerre quadrature

*Else If* $t_{DE3} > $ third flow period $t_{DE3}$ *Then* Compute $P_{DE3}$ using Legendre quadrature

- $P_D = P_D1 + P_D2 + P_D3 + ...$

*End If.*

*If* option’s value’s $= 2$

- If reservoir is crossflow systems

- Input known values of $L$, $r_w$, $h$, $z_m$, $k_y$, $k_x$, $k_z$
- Compute $B_T A$, $H_{DT}$, $K$, $E$
- Compute flow periods $t_{DE1}$, $t_{DE2}$, $t_{DE3}$, $t_{D4}$.
- Compute $P_{D1}$, $P_{D2}$, $P_{D3}$, $P_{D4}$ using same method as no-crossflow reservoir

*End IF* Compute Pressure derivative using differentiation

- Else if $t_D > $ second flow period $T_{DE2}$ Then

- Compute $P_{D2}$ using Lagrange quadrature
- Plot graph

*End*

### 4. Results and Discussion

Numeric examples (Tables 1 and 2) were employed to demonstrate the relevance and robustness of the software. The following well/fluid properties were assumed constant for both (crossflow and no-crossflow) reservoir system. Wellbore radius ($r_w$) = 0.375ft, total compressibility ($C_t$) = 3.0E-6psi$^{-1}$, Porosity($\phi$) = 1%, Viscosity ($\mu$) =1Cp. Using the data from table one, values of pressure and derivative of horizontal wells situated in each layer were generated. The results(increasing pressure and derivative values with time) for layer one indicates that the top layer(layer 2) behaves like a reservoir with no flow boundaries having no communication with the upper layer because the interface is sealed.

![Flowchart for the computer program.](image)

**Table. 1 Showing well/reservoir data for No-Crossflow reservoir system.**

| Parameter | Value |
|-----------|-------|
| $L$, ft   | 1000  |
| $h$, ft   | 200   |
| $Z_{ws}$, ft | 100 |
| $X_{e}$, ft | 6 000 |
| $Y_{e}$, ft | 6 000 |
| Elv, ft   | 20    |
| $K_i$, md | 1:1:1  |
Table 2 Showing well/reservoir data for Crossflow reservoir system.

| Parameter | Layer 1 | Layer 2 |
|-----------|---------|---------|
| L, ft     | 1000    | 1000    |
| h, ft     | 100     | 100     |
| Zx, ft    | 50      | 50      |
| Xx, ft    | 16000   | 16000   |
| Yx, ft    | 10000   | 10000   |
| Elv, ft   | 10      | 10      |
| K, md     | 1:1:1   | 10:10:10 |

reservoir form. These results can be viewed from the immediate window. Results were also displayed in a datagrid shown in figures (6)-(9). Layer one and two display similar trend for pressure and derivative. Similarity in behaviour can be traced to the permeable interface. There is communication (crossflow) between the layers, pressure values increased with time and stabilized at late time while derivative was initially constant but collapsed to zero at latetimes. Both layers behave like reservoirs that have a constant pressure boundary. For proper visual appreciation and interpretation of results cartesian plots of the results were provided. The usefulness and effectiveness of the computer programs can be seen clearly. All aspect of pressure transient test analysis: data preparation, model identification, parameter calculation, and model validation and presentation of results have benefited because of the use of computer [18, 19]

5. Conclusion

This work has presented models for a two layered reservoir with horizontal wells, also formulated and presented a computer program to generate dimensionless pressure and derivative values of the reservoir system with a bottom water drive. User friendly GUI’s were designed for input of data and output of results. From the results shown in figure 2-5 it was observed that the pressure and derivative response of no crossflow reservoir system was higher, layer 1 displayed the behaviour of a bottom water drive reservoir while layer 2 showed the characteristics of a bounded reservoir whose lateral boundaries are infinite. Figure 6-9 showed screenshots of GUI’s for the crossflow reservoir. Similarity in flow behaviour for both layers was observed, at early time pressure values increased and became constant at late times while derivative values became constant and collapsed to zero. This response is typical of that of reservoir with a
constant pressure boundary with the more permeable layer (layer 2) having a lower pressure and derivative response. The graph of the results further highlights the capability of the computer program in model identification and reservoir characterization. Results from this study will improve the quality of test analysis of layered reservoir system with bottom water drive using horizontal wells, especially in the areas of drawdown, build up and interference test. It will also serve as an effective learning tool because it’s easy and fast and results have acceptable level of accuracy.

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Nomenclature

\begin{align*}
C & \quad \text{Compressibility} \quad \text{psi}^{-1} \\
E & \quad \text{Elevation allowance,} \quad \text{ft} \\
h & \quad \text{Pay thickness,} \quad \text{ft} \\
i & \quad \text{Distance in x, y or z direction.} \quad \text{ft} \\
K & \quad \text{Permeability,} \quad \text{md} \\
P & \quad \text{Well length,} \quad \text{ft} \\
q & \quad \text{Pressure,} \quad \text{psi} \\
q & \quad \text{flow rate,} \quad \text{STB/D} \\
r & \quad \text{well radius,} \quad \text{ft} \\
t & \quad \text{time,} \quad \text{hr} \\
X & \quad \text{distance in the x-axis,} \quad \text{ft} \\
Y & \quad \text{distance in the y-axis,} \quad \text{ft} \\
Z & \quad \text{distance in the z-axis,} \quad \text{ft} \\
\Delta t & \quad \text{time increment,} \quad \text{hr} \\
\Phi & \quad \text{Porosity, fraction} \\
\beta & \quad \text{crossflow coefficient} \\
\mu & \quad \text{viscosity} \quad \text{cP} \\
\epsilon & \quad \text{constant} \\
\gamma & \quad \text{integration variable} \\
\text{Dimensionless} & \\
D & \quad \text{External} \\
T & \quad \text{Total} \\
W & \quad \text{Wellbore}
\end{align*}