Visualizing Invisible Dark Matter Annihilation with the CMB and Matter Power Spectrum

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We study the cosmological signatures of Invisibly Annihilating Dark Matter (IAnDM) where DM annihilates into dark radiation particles that are decoupled from the Standard Model (SM). In a large class of dark sector models such invisible annihilation determines the relic abundance of DM via dark thermal freezeout. We demonstrate that IAnDM may reveal itself through observable, novel signatures that are correlated: scale-dependent $\Delta N_{\text{eff}}$ (number of extra effective neutrinos) in the Cosmic Microwave Background (CMB) spectrum due to DM residual annihilation, while the phase of acoustic peaks shift towards the opposite direction relative to that due to SM neutrinos, resembling the effect due to scattering (fluid-like) thermal dark radiation; in addition, IAnDM induces modifications to the matter power spectrum that resemble yet are distinct from that due to warm dark matter. Current data is sensitive to IAnDM with masses up to $\sim$ 200 keV, while future observations will improve the reach, especially if the late-time DM annihilation cross-section is enhanced relative to the standard thermal value, which can be realized in a variety of scenarios.

I. Introduction

Over the past two decades, we have seen overwhelming gravitational evidence for the existence of dark matter (DM) which constitutes 80% of the total matter density in our universe today. Nevertheless, the non-gravitational, particle nature of DM remains mysterious. Among the many theoretical candidates for DM, the Weakly Interacting Massive Particle (WIMP) is a well-motivated scenario that has played a central role in guiding the experimental searches for DM. The key process in WIMP models is the annihilation of DM into Standard Model (SM) particles which determines the relic abundance of DM after its thermal freezeout in the early universe. The residual DM annihilation today is potentially observable through indirect detection experiments. More recently, it has been realized that cosmological observations, in particular, the Cosmic Microwave Background (CMB), may serve as another path for observing DM annihilation that is free of astrophysical uncertainties [1-3]. These CMB studies focus on the effects of visible SM states (e.g., $e^\pm, \gamma$) on the recombination history of the universe.

Recently, driven by the strengthening experimental constraints on WIMP DM, there has been a growing interest in dark sector scenarios, where DM resides in a hidden sector with multiple states and/or self-interactions, in analogy to the complex structure of the SM. In a large class of dark sector models, the merit of predicting DM relic abundance from thermal freezeout is retained, yet the DM predominantly annihilates into stable dark radiation (DR) particles which constitute $\sim 200$ keV, while future observations will improve the reach, especially if the late-time DM annihilation cross-section is enhanced relative to the standard thermal value.

Without additional effective interactions between $\chi$ and the SM or other dark states, $\chi$ freezes out simultaneously as $\chi\chi \rightarrow \psi\psi$ departs from equilibrium. If $\psi$ is massive enough, its relic density today can dominate over that of $\chi$ [16], which is an interesting alternative scenario that we will leave for future investigation. In this work we will assume that $\chi$ is the leading DM and thus the prediction for DM relic density re-
seems that of the standard WIMP DM. This would require ψ to be in form of dark radiation, much lighter than χ and freezes out while it is relativistic. The temperature of a decoupled dark sector (χ, ψ) can be much colder than the SM, depending on the reheating pattern [17] [18] and the number of heavier states that have decoupled in each sector. The temperatures of χ, ψ also redshift differently after their freezeout. For the interest of this work, the relevant effect can be simply parametrized by ξ ≡ T_f/ ˜T_f around the dark thermal freezeout, where ˜T_f is the dark temperature and T_f is the SM one [46].

The relic abundance of χ can be estimated as in [5] [13]. We define freezeout temperature parameters x_f ≡ m_χ/ T_f, ˜x_f ≡ m_χ/ ˜T_f. Assuming ξ < 1 and s-wave annihilation, we have

\[ x_f \simeq \xi \ln \left( \sqrt{\frac{45}{\pi}} \frac{g_s}{\sqrt{g_*}} \hat{\rho}_s \sigma_{v} \langle \sigma v \rangle \xi^2 \right) \sim \xi \hat{x}_f, \]

\[ \Omega_\chi \simeq \sqrt{\frac{45}{\pi}} \frac{s_0}{\rho_c} \frac{g_s}{\sqrt{g_*}} \frac{\hat{x}_f}{\langle \sigma v \rangle}, \]

\[ = 0.32 \left( \frac{\hat{x}_f}{10} \right) \left( \frac{\sqrt{g_*/3}}{\sqrt{3.38 g_*}} \right) \left( \frac{g_s}{g_+} \right) \left( \frac{\langle \sigma v \rangle}{\xi} \right), \]

where M_p is the Planck mass, s_0 and \rho_c are today’s entropy and critical density respectively, g_* is the internal degree of freedom (d.o.f) of χ, g_s, g_+ are the total effective d.o.f.’s dominated by SM states. With Eq. 2 it is important to note that a fixed \langle \sigma v \rangle/\xi ≲ σ_0 ≡ 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} is required to yield correct thermal DM abundance, therefore for ξ < 1, the required \langle \sigma v \rangle is reduced by ξ accordingly. For a light sub-MeV DM, \hat{x}_f \sim 10 (smaller than x_f of a typical WIMP), and around the DM freezeout g_* = 3.38, g_+ = 43/11 (the SM values after neutrino decoupling). In order to make sure that ψ does not over close the Universe, or dominate over Ω_χ, we also check the thermal relic density of ψ. For a massive ψ that freezes out as a hot relic,

\[ \Omega_{\psi, \text{th}} = \frac{\rho_{\psi, \text{th}}}{\rho_c} = 0.038 \left( \frac{m_\psi}{1 \text{ eV}} \right) \hat{g}_\psi \xi^3; \]

in the limit of massless ψ,

\[ \Omega_{\psi, \text{th}} = 2.7 \times 10^{-4} \hat{g}_\psi \xi^4, \]

where the subscript “th” denotes the thermal relic component of ψ, which distinguishes itself from the non-thermal component from χ annihilation that we will focus on (denoted by subscript “nth”). We see that by allowing the general possibility of a colder dark sector, a moderately massive ψ can be cosmologically viable. Note that for ξ ≪ 1, Ω_{ψ, th} is strongly suppressed, easily making ψ a subleading component relative to χ.

III. Cosmological Effects of Invisible DM Annihilation: General Consideration and Analytic Studies

In this section we study potential cosmological effects of IAnDM with analytic approaches, with numerical results following in Section. IV.

The non-thermal free-streaming DR ψ injected from DM χ annihilation contributes an additional radiation-like energy component to the Universe. The accumulated energy density of ψ from χ annihilation, ρ_{ψ, nth}, can be estimated as follows. Assuming m_ψ = 0 for simplicity, a straightforward estimate based on energy conservation gives \[ d \rho_{ψ, nth}(t) = \rho_\chi m_\psi (\langle v \rangle) dt. \]

Upon integration over time with proper redshift factors, the accumulated ψ density by the time t (a) is:

\[ \rho_{ψ, nth}(a) = \frac{\rho_\chi^2 m_\chi \int_a^\infty \frac{1}{a^5} \frac{a^2}{H_0 \sqrt{1 + \frac{a^2}{a_0^2}}} (\frac{a_0}{a})^4}{(3H_0^2\Omega_\chi^2)^2 \langle \sigma v \rangle \frac{\ln(\frac{a_0}{a})}{a_0^4}}, \]

where deep radiation dominated (RD) epoch is assumed to show a neat analytic form, and we have used the convention that current day a_0 = 1. a_t represents the initial time when the net ψ production from χ annihilation becomes effective which is around the freezeout time, a/a_t ∼ T_f/ ˜T_f. Note that in addition to the standard redshift 1/a^4, there is a moderate log-dependence on a which is absent in the energy density of a standard thermal radiation background.

In order to relate to the potential CMB observable, \( \Delta N_{\text{eff}} \), we take the ratio of \( \rho_{ψ, nth} \) over SM neutrino density (one flavor), and find

\[ \Delta N_{\text{eff, nth}}(a) = 0.038 \ln \left( \frac{a}{a_t} \right) \left( \frac{\text{keV}}{m_\chi / \xi} \right) \left( \frac{\langle \sigma v \rangle / \xi}{3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}} \right), \]

where a ≈ 10^{-3} when evaluated around the CMB epoch. Note that \( \Delta N_{\text{eff, nth}} \) inherits the aforementioned ln(a/a_t) dependence. Apparently lighter DM produce more copious ψ and thus more pronounced signals. We can also see that with ξ = 1 and thus \langle \sigma v \rangle taking the standard thermal value σ_0, an O(1) – O(10) keV mass χ could lead to observable \( \Delta N_{\text{eff}} \) at current or upcoming CMB experiments [19] [21], while for ξ < 1 and fixed \langle \sigma v \rangle/ξ ≳ σ_0 a lighter χ is required to yield the same \( \Delta N_{\text{eff}} \). To account for this ξ dependence, in Fig[1] and later figures we parametrize with the rescaled DM mass, m_χ/ξ.

If m_ψ ≲ eV, there would be another irreducible contribution to \( \Delta N_{\text{eff}} \) from the thermal relic of ψ (Eq. 4):

\[ \Delta N_{\text{eff, th}} = 3.046 \frac{\rho_{ψ, th}}{\rho_\nu, 1} = \frac{4}{7} \left( \frac{11}{4} \right)^4 \hat{g}_\nu \xi^4 = 2.2 \hat{g}_\nu \xi^4. \]
Numerical Results: The CMB Signatures

In order to demonstrate the effects of $\psi$ produced from invisible DM annihilation in a visible and representative way, in Fig. 2, we plot the zoomed-in CMB TT spectra around the first and the sixth acoustic peaks, for a set of $m_{\chi}/\xi$ choices. We assume $m_{\psi} = 0$ which represents the general case of $m_{\chi} \gg m_{\psi}$ that we focus on. For comparison we also plot the spectra based on CDM and models with extra neutrinos for a set of $\Delta N_{\text{eff}}$ values. As expected based on our analytic estimate (Eq. 6), the effects from $\psi$ injection at leading order resemble that from standard $\Delta N_{\text{eff}}$, which as shown below are confirmed by numerical studies. However, there are key differences between neutrinos and the $\psi$ produced from DM annihilation. First, unlike neutrinos, the unperturbed partition function $f(\bar{\rho})$ of $\psi$ is not thermal, rather it is given by the annihilation process with a fixed physical momentum (determined by $m_{\chi}$, $m_{\psi}$). The $f(\bar{\rho})$ feeds into the perturbation equations through the factor of $\frac{\partial \ln f(\bar{\rho})}{\partial \ln \bar{\rho}}$, which can be replaced by $-4$ for massless neutrinos. For our non-thermally produced $\psi$, this factor can be calculated in the same way as in [26], which at leading order yields: $\frac{\partial \ln f(\bar{\rho})}{\partial \ln \bar{\rho}} = -\frac{9}{2} + \frac{3g}{\bar{\rho}}$, where $\bar{\rho}$ and $\rho$ are the background pressure and energy density. Deep in RD regime $\bar{\rho} = \frac{3}{2}\rho$, $\frac{\partial \ln f(\bar{\rho})}{\partial \ln \bar{\rho}}$ coincidently gives $-4$. Numerically we also include a small correction to this factor on the order of $O\left(\frac{\Delta N_{\text{eff}}}{H}\right)$. In addition, the annihilation product will slightly change the equation of state, thus changing the Hubble expansion rate, $H$, which we take into account numerically. Beyond these differences, the contribution from annihilation products to the Boltzmann equations are coded in parallel to massless or massive neutrinos. We modify camb [25] to incorporate these new physics considerations, and choose the standard cosmology parameters: $h = 0.678$, $\Omega_{\chi} h^2 \approx \Omega_{\text{DM}} h^2 = 0.1186$, $\Omega_{\gamma} h^2 = 0.02226$, etc. [20]

\[ m_{\text{WDM}} = \left(\frac{2\pi^2 \rho_{\text{WDM}}}{3c^3(3)}\right)^{\frac{1}{4}} \left(\frac{m_{\chi}/\xi}{T_{\gamma} \sqrt{2 \xi}}\right)^{\frac{1}{2}} = 0.08 \left(\frac{m_{\chi}/\xi}{\text{keV}}\right)^{\frac{1}{2}} \text{keV}, \]  

where we took $\rho_{\text{WDM}} = \rho_{\text{DM},0}$. $x_f = 10$.

IV. Numerical Studies

In order to relate to observations in a more precise way, we further perform a numerical analysis by solving the perturbation Boltzmann equations for the evolution of the $\chi, \psi$ system using camb [25]. The general formulation follows [23] with the choice of synchronous gauge. The dominant DM, $\chi$, follows the standard treatment while allowing DM free-streaming effect, and the annihilation term can have a non-negligible impact on its evolution. The initial perturbation of the annihilation product, $\psi$, inherits that of $\chi$ (CDM), while its later evolution follows the same perturbation multipole expansion as the massless or massive neutrinos. However there are key differences between neutrinos and the $\psi$ produced from DM annihilation. First, unlike neutrinos, the unperturbed partition function $f(\bar{\rho})$ of $\psi$ is not thermal, rather it is given by the annihilation process with a fixed physical momentum (determined by $m_{\chi}$, $m_{\psi}$). The $f(\bar{\rho})$ feeds into the perturbation equations through the factor of $\frac{\partial \ln f(\bar{\rho})}{\partial \ln \bar{\rho}}$, which can be replaced by $-4$ for massless neutrinos. For our non-thermally produced $\psi$, this factor can be calculated in the same way as in [26], which at leading order yields: $\frac{\partial \ln f(\bar{\rho})}{\partial \ln \bar{\rho}} = -\frac{9}{2} + \frac{3g}{\bar{\rho}}$, where $\bar{\rho}$ and $\rho$ are the background pressure and energy density. Deep in RD regime $\bar{\rho} = \frac{3}{2}\rho$, $\frac{\partial \ln f(\bar{\rho})}{\partial \ln \bar{\rho}}$ coincidently gives $-4$. Numerically we also include a small correction to this factor on the order of $O\left(\frac{\Delta N_{\text{eff}}}{H}\right)$. In addition, the annihilation product will slightly change the equation of state, thus changing the Hubble expansion rate, $H$, which we take into account numerically. Beyond these differences, the contribution from annihilation products to the Boltzmann equations are coded in parallel to massless or massive neutrinos. We modify camb [25] to incorporate these new physics considerations, and choose the standard cosmology parameters: $h = 0.678$, $\Omega_{\chi} h^2 \approx \Omega_{\text{DM}} h^2 = 0.1186$, $\Omega_{\gamma} h^2 = 0.02226$, etc. [20]

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The most notable effect on the CMB spectrum due to $\Delta N_{\text{eff}}$ or non-thermal $\psi$ in IAnDM is on the heights of the acoustic peaks. In both models, the change to $H$ due to additional radiation energy density leads to the increase in the height of the first peak (i.e., early integrated Sachs-Wolfe (ISW) effect), and the decrease in that of the higher peaks (i.e., enhanced Silk damping). However, the heights of CMB acoustic peaks from IAnDM do not align with those due to a fixed standard $\Delta N_{\text{eff}}$. As expected based on Eq. 6 and Fig. 1, $\Delta N_{\text{eff}}$ varies over time as $\rho_{\text{eff}}$ accumulates while $\chi$ continues to annihilate. This can be seen by comparing the amplitudes of the CMB spectrum shown in Fig. 2 where higher $\ell$ corresponds to an earlier time (when the mode re-enters horizon): consider a fixed $m_\chi/\xi = 1$ keV, around the 6th peak the resulting CMB TT spectrum roughly aligns with that associated with $\Delta N_{\text{eff}} \approx 0.12$; while around the 1st peak, it aligns with a larger $\Delta N_{\text{eff}} \approx 0.35$.

The standard $\Delta N_{\text{eff}}$ also reveals itself via a unique phase shift of the high $\ell$ acoustic peaks due to neutrino-like modes that induce a significant anisotropic stress and propagate faster than the sound speed (free-streaming) [27, 28], which cannot be caused by other standard physics [47]. IAnDM induces a phase shift as well, yet with dramatic difference compared to that caused by standard $\Delta N_{\text{eff}}$. We found that by adjusting $h$, $\Omega_{\text{DM}} h^2$, $Y_\text{He}$, to match the CMB observables of $\theta_\star$, $\theta_\pi$ and $z_{\text{eq}}$ [20], in the IAnDM model the locations of the acoustic peaks shift towards high $\ell$’s, opposite to the direction expected from standard free-streaming $\Delta N_{\text{eff}}$ [27]. (We found that due to the presence of non-thermal DR, the best fit of $h$ shifts to higher values, which helps alleviate the discrepancy in $H_0$ measurements between the CMB observation and local measurements). Such an effect resembles that due to a fluid-like or scattering DR species [13, 28, 29], yet for different reasons. Even though free-streaming, unlike neutrinos, the non-thermal $\psi$ dominantly produced at late times inherits a negligible initial anisotropic stress from $\chi$, and consequently would not induce an additional phase shift along the same direction as neutrinos [48]. However, such $\psi$ does contribute to late-time radiation energy density, and thus reduce the energy fraction of the free-streaming SM neutrinos. As noted in [13] a reduction of free-streaming neutrinos fraction (due to fluid-like DR or non-thermal $\psi$ here) leads to a phase shift along the direction opposite to that due to standard $\Delta N_{\text{eff}}$. Furthermore, just as with peak heights, there is scale dependence in phase shifts, which is a higher order effect that may distinguish IAnDM from fluid-like DR [49].

Numerical Results: The Matter Power Spectra

In Fig. 3, we present the MPS in the scenario of IAnDM, in comparison with other familiar new physics scenario: CDM, and WDM with masses saturating current conservative and aggressive Lyman-α bound [30]. The definition of matter power spectra $P(k)$, $\Delta^2(k)$ are standard, e.g., as given in [31]. Compared to the standard CDM prediction, all these new models cause a suppression of MPS relative to the standard cosmology for $k \gtrsim 1$ h/Mpc. The upper panel $\Delta^2(k)$ shows that the IAnDM spectra well overlaps with WDM spectra with a corresponding mass estimated by Eq. 6. This implies that the $\chi$ free-streaming is the dominant effect determining MPS due to IAnDM. Nevertheless, the zoomed-in bottom panel of Fig. 3 shows slight differences between IAnDM and WDM spectra. We demonstrate such differences with $m_\chi/\xi = 260$ keV, allowing a massive $\psi$ (still require $m_\psi \ll m_\chi$ to be consistent with our starting assumption on cosmology). Here we see that the suppression on small scales is more pronounced for a massive $\psi$. This can be understood as the energetic $\psi$ produced by $\chi$ annihilation eventually clusters as a component of DM and leads to a secondary free-streaming effect.
plunging of the annihilation product’s comoving momentum \[ \frac{\xi}{10} \]
and is beyond the scope of the current Letter. Thus we will not derive the rigorous global constraints by the standard MCMC method, rather will simply recast the current constraints by comparing physical effects analytically.

The results are summarized in Fig. 4 where we label the axes with the scaling factor \( \xi \) to account for the generic colder dark sector possibility. The CMB constraint is estimated by comparing the (projected) experimental sensitivity limit on \( \Delta N_{\text{eff}} \) to our analytical formula Eq. 6. The above numerical studies suggests subtle differences from this estimate based on fixed \( \Delta N_{\text{eff}} \), while as aforementioned a rigorous constraint is beyond the scope of this work. On the other hand, the constraint from MPS can be extracted by comparing the DM free-streaming effect based on our Eq. 6. Assuming the thermal freeze-out annihilation cross section, the current 2σ bound from Planck (combined with BAO data) \( \Delta N_{\text{eff}} \lesssim 0.39 \) [29] (here we used the bound on \( N_{\text{tot}} \) in [29], including both fluid-like and free-streaming dark radiation) can be recast into a bound of \( m_\chi/\xi \gtrsim 0.3 \) keV for \( m_\psi \ll m_\chi \). In contrast, the main effect of IAnDM on MPS only depends on \( m_\chi/\xi \) (Eq. 8), insensitive to \( \langle \sigma v \rangle \). The current Lyman-\( \alpha \) observation constrains IAnDM MPS: the 2σ aggressive bound (\( m \gtrsim 5.3 \) keV for WDM) corresponds to a bound of \( m_\chi/\xi \gtrsim 260 \) keV, and the conservative bound (\( m \gtrsim 3.5 \) keV for WDM) corresponds to a bound of \( m_\chi/\xi \gtrsim 150 \) keV. For a massive \( \psi \) the constraint from Lyman-\( \alpha \) would be stronger due to additional free-streaming suppression as discussed in Sec. IV.

According to Fig. 4 Lyman-\( \alpha \) disfavors IAnDM with \( m_\chi/\xi \lesssim 150 \) keV regardless of \( \langle \sigma v \rangle \), while for larger masses CMB observations have sensitivities if the DM late-time annihilation cross-section is enhanced relative to the standard thermal value. Note that although in this work we considered a particular thermal IAnDM model as a simple example, the inferred novel phenomenology and the sensitivity plot Fig. 4 apply to broader possibilities of IAnDM models allowing enhanced annihilation cross-section relative to the thermal value. Such possibilities have been well considered in the context of the familiar visibly annihilating DM, which generally apply for IAnDM, including: Sommerfeld enhancement [32, 33] (automatic in IAnDM if DR is a light mediator instead of a fermion), enhancement due to non-standard cosmology or non-thermal production of DM [34, 36]. Phenomenological details of these variations require dedicated studies for specific models.

If the sensitivity of the future CMB-S4 experiment can be improved to \( \Delta N_{\text{eff}} \approx 0.02 \) [21, 37–39], then the CMB sensitivity to thermal IAnDM can be improved to \( m_\chi/\xi \gtrsim 10 \) keV. So far Lyman-\( \alpha \) forest observation provides the best probe for MPS relevant for IAnDM, with potential improvement by future 21 cm-line experiments [40]. With the presence of additional interactions beyond our minimal model, an effective DM-DR scattering may induce a more notable effect on large scale structure (\( \langle \sigma v \rangle \) [12] [41] [42].

Astrophysical/cosmological observation is stepping into a high precision era, which enables us to probe well-motivated DM models that are beyond the reach of conventional DM detections. In this Letter we demonstrate a representative example by investigating the novel phenomenology from the generic scenario of Invisibly Annihilating DM (IAnDM). The smoking gun signature of this large class of models includes a correlated combination of scale-dependent, fluid-like \( \Delta N_{\text{eff}} \) in the CMB spectra and a unique pattern of matter power spectrum that resembles WDM. The current data constrains the IAnDM with masses up to \( \sim 200 \) keV, while future experiments can be sensitive to larger masses if the DM annihilation cross-section is enhanced relative to the standard thermal value. These findings motivate new dedicated analyses to optimize the potential for discovering DM residing in a hidden sector.

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[1] N. Padmanabhan and D. P. Finkbeiner, Phys. Rev. D72, 023508 (2005), astro-ph/0503486.
[2] M. S. Madhavacheril, N. Sehgal, and T. R. Slatyer, Phys. Rev. D89, 103508 (2014), 1310.3815.
[3] D. Green, P. D. Meurburg, and J. Meyers (2018), 1804.01055.
[4] M. Pospelov, A. Ritz, and M. B. Voloshin, Phys. Lett. B662, 53 (2008), 0711.4866.
[5] J. L. Feng, H. Tu, and H.-B. Yu, JCAP 0810, 043 (2008), 0808.2318.
[6] C. Cheung, G. Egor, L. J. Hall, and P. Kumar, JHEP 03, 085 (2011), 1010.0024.
[7] J. M. Cline and A. R. Frey, Phys. Lett. B706, 384 (2012), 1109.4639.
[8] G. Belanger and J.-C. Park, JCAP 1203, 038 (2012), 1112.1491.
[9] M. Blennow, E. Fernandez-Martinez, O. Mena, J. Redondo, and P. Serra, JCAP 1207, 022 (2012), 1203.5803.
[10] C. Garcia-Cely, A. Ibarra, and E. Molinaro, JCAP 1402, 0162 (2014), 1405.2246.
[11] K. Agashe, Y. Cui, L. Necib, and J. Thaler, JCAP 1410, 062 (2014), 1410.2246.
[12] Z. Chacko, Y. Cui, S. Hong, and T. Okui, Phys. Rev. D92, 055033 (2015), 1505.04192.
[13] Z. Chacko, Y. Cui, S. Hong, T. Okui, and Y. Tsai, JHEP 12, 108 (2016), 1609.03569.
[14] M. H. Chan, Astropart. Phys. 80, 1 (2016), 1604.01494.
[15] M. Farina, D. Pappadopulo, J. T. Ruderman, and G. Trevisan, JHEP 12, 039 (2016), 1607.03108.
[16] Z. G. Berezhiani, A. D. Dolgov, and R. N. Mohapatra, Phys. Lett. B375, 26 (1996), hep-ph/9511221.
[17] P. Adshhead, Y. Cui, and J. Shelton, JHEP 06, 016 (2016), 1604.02458.
[18] A. Suzuki et al. (POLARBEAR), J. Low. Temp. Phys. 184, 805 (2016), 1604.01494.
[19] F. A. R. Ade et al. (Planck) (2015), 1502.01589.
[20] K. N. Abazajian et al. (CMB-S4) (2016), 1610.02743.
[21] R. Huo, M. Kaplinghat, Z. Pan, and H.-B. Yu (2017), 1709.09717.
[22] C.-P. Ma and E. Bertschinger, Astrophys. J. 455, 7 (1995), astro-ph/9506072.
[23] M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese, and A. Riotto, Phys. Rev. D71, 063534 (2005), astro-ph/0501562.
[24] A. Lewis, A. Challinor, and A. Lasenby, Astrophys. J. 538, 473 (2000), astro-ph/9911177.
[25] R. Huo, Phys. Lett. B701, 530 (2011), 1104.4094.
[26] S. Bashinsky and U. Seljak, Phys. Rev. D69, 083002 (2004), astro-ph/0310198.
[27] D. Baumann, D. Green, J. Meyers, and B. Wallisch, JCAP 1601, 007 (2016), 1508.06342.
[28] C. Brust, Y. Cui, and K. Sigurdson, JCAP 1708, 020 (2017), 1703.10732.
[29] V. Irsic et al., Phys. Rev. D96, 023522 (2017), 1702.01764.
[30] S. Dodelson, Modern Cosmology (Academic Press, Amsterdam, 2003), ISBN 9780122191411.
[31] N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, and N. Weiner, Phys. Rev. D79, 015014 (2009), 0810.0713.
[32] J. L. Feng, M. Kaplinghat, and H.-B. Yu, Phys. Rev. D82, 033525 (2010), 1005.4678.
[33] G. B. Gelmini and P. Gondolo, Phys. Rev. D74, 023510 (2006), hep-ph/0602230.
[34] J. Fan, O. zsoy, and S. Watson, Phys. Rev. D90, 043536 (2014), 1405.7373.
[35] A. L. Erickcek, Phys. Rev. D92, 103505 (2015), 1504.03335.
[36] W. L. K. Wu, J. Errard, C. Dvorkin, C. L. Ku, A. T. Lee, P. McDonald, A. Slosar, and O. Zahn, Astrophys. J. 788, 138 (2014), 1402.4108.
[37] J. Errard, S. M. Feeney, H. V. Peiris, and A. H. Jaffe (2015), 1509.06770.
[38] K. N. Abazajian et al. (Topical Conveners: K.N. Abazajian, J.E. Carlstrom, A.T. Lee), Astropart. Phys. 63, 66 (2015), 1309.5383.
[39] J. R. Pritchard and A. Loeb, Rept. Prog. Phys. 75, 086901 (2012), 1109.6012.
[40] M. A. Buen-Abad, G. Marques-Tavares, and M. Schmaltz, Phys. Rev. D92, 023531 (2015), 1505.03542.
[41] J. Lesgourgues, G. Marques-Tavares, and M. Schmaltz, JCAP 1602, 037 (2016), 1507.04351.
[42] B. Follin, L. Knox, M. Millea, and Z. Pan, Phys. Rev. Lett. 115, 091301 (2015), 1503.07863.
[43] B. Audren, J. Lesgourgues, G. Mangano, P. D. Serpico, and T. Tram, JCAP 1412, 028 (2014), 1407.2418.
[44] V. Foulin, P. D. Serpico, and J. Lesgourgues, JCAP 1608, 036 (2016), 1606.02073.
[45] Such an effect from SM neutrinos was recently detected with Planck data [43].
[46] Qualitatively the novel effects on CMB anisotropy spectrum we discussed here may also originate from the scenario of DM decaying to DR [26, 44, 45]. However, for decaying DM with long enough lifetime to be cosmologically stable and satisfy existing constraints, the amount of DR produced during CMB epoch is tiny and the resultant signal strength is expected to be negligible.
[47] We use \(\xi\) and \(\rho\) to discretized comoving momentum, while camb optimize to 3 to 5 for massive neutrino unless an extra accuracy boost is used.