Application of IAFSA-BP neural network in face orientation recognition

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Abstract In order to improve the recognition accuracy of BP neural network in face orientation recognition, an improved artificial fish swarm algorithm is proposed to optimize the weights and thresholds of BP neural network face orientation recognition model. The improved artificial fish swarm algorithm is based on the standard algorithm, introducing adaptive factors to make the horizon and step size adaptive change, and at the same time learning the solution before the result announcement, so as to improve the accuracy of the final result. Finally, the experimental results show that the effective combination of the improved artificial fish swarm algorithm and the BP neural network algorithm can improve the output accuracy of face orientation recognition of the BP neural network, and the running speed of the improved algorithm is significantly higher than that of the standard artificial fish swarm algorithm.

1. Introduction
Face orientation recognition is a technology that realizes face identification through characteristic analysis of face orientation information in pictures [1]. This technology has good market value and broad application prospect in the fields of identity authentication, financial payment and criminal investigation of public security [2]. At present, some scholars have adopted traditional discriminant analysis methods, such as probabilistic model and geometric model, in the study of face orientation recognition. These methods are only feasible under certain assumptions. When there is a big difference between the assumed conditions and the actual situation, these traditional methods have a very high misjudgment rate [3]. Back Propagation (BP) neural network is the typical representative of artificial neural network. So far, some scholars have studied face orientation recognition using BP neural network [4-8]. However, because BP neural network adopts gradient descent algorithm and selects the initial weights and thresholds randomly, it is easy for BP neural network to fall into the local minimum value during operation and cause the network output shock.

The improved Artificial Fish Swarm Algorithm (IAFSA) and BP neural network Algorithm are proposed to construct the face orientation recognition model. The improved artificial fish swarm algorithm introduces adaptive factors and elite learning into the standard artificial fish swarm algorithm, improves the adaptive change ability of vision and step size, and strengthens the ability of the algorithm to approximate the optimal solution in the later stage. The experimental results show that compared with BP, GA-BP, PSO-BP and AFSA-BP neural network algorithms, IAFSA-BP neural network algorithm not only has high recognition accuracy, but also has better output stability of the model. At the same time, due to the introduction of new optimization strategies, the running time of the improved artificial...
fish swarm algorithm is greatly reduced compared with that of the artificial fish swarm algorithm.

2. Principle of artificial fish swarm algorithm and improvement strategy

2.1. Basic principle of artificial fish swarm algorithm

Artificial fish swarm algorithm is an intelligent optimization algorithm proposed by Li Xiaolei et al. of Zhejiang University in 2002. The design of this algorithm is inspired by the foraging, clustering and rear-ending behaviors of fish swarm in the natural environment [9]. The basic idea of the algorithm is that the artificial fish swarm spontaneously perceives the food concentration in the surrounding area, and then agglomerates to the area with the highest food concentration. Before reaching the area with the highest food concentration, each artificial fish will constantly perceive the change of the food concentration in the surrounding area, and change its own behavior accordingly. In other words, the foraging behavior of fish in groups is determined by the information of surrounding areas, and the foraging, gathering and retracing behavior of each fish also affects the change of information of surrounding areas [10]. When solving the problem, artificial fish simulates the social clustering and rear-ending behaviors of natural fish, selects the best results for subsequent calculation, and takes foraging behavior as the default [11]. The algorithm involves the following three behavior rules:

**Behavior rule 1** Foraging behavior--Set the visual field of artificial fish as \( \text{visual} \), the moving step length as \( \text{step} \), the crowding degree as \( \delta \), the current state as \( X_i \), \( F_c \) as the food concentration function of artificial fish at the position, \( D_y \) as the distance between artificial fish \( i \) and artificial fish \( j \), and select state \( X_j \) randomly within the visual field \( \text{visual} \geq D_y \). If \( F_c(X_j) > F_c(X_i) \), then move one step to \( X_j \), \( X_{i+1} \) updates according to equation (1); otherwise, randomly select \( X_j \), update \( X_{i+1} \) according to equation (2), and re-judge the moving condition. After several temptations, if the moving condition is still not satisfied, update \( X_{i+1} \) according to Equation (3), where \( \text{rand}() \) represents the random number between \([0,1]\).

\[
X_{i+1} = X_i + \text{rand}() \cdot \text{step} \cdot \frac{(X_j - X_i)}{D_y} \quad (1)
\]

\[
X_{i+1} = X_i + \text{rand}() \cdot \text{visual} \quad (2)
\]

\[
X_{i+1} = X_i + \text{rand}() \cdot \text{step} \quad (3)
\]

**Behavior rule 2** Clustering behavior--The artificial fish at position \( X_i \) has \( n_i \) partners in its visual field, the partner at the center is \( X_{\text{center}} \), and \( F_c(X_{\text{center}}) \) represents the food concentration of the partner at the center. When \( F_c(X_{\text{center}})/n_i > \delta F_c(X_i) \), it indicates that the food concentration at the central location of the partner meets the need and moves towards the central location. Then, update \( X_{i+1} \) according to equation (4), or perform foraging behavior.

\[
X_{i+1} = X_i + \text{rand}() \cdot \text{step} \cdot \frac{(X_{\text{center}} - X_i)}{D_{\text{center}}} \quad (4)
\]

**Behavior rule 3** Rear-ending behavior--Among all the partners of the artificial fish located at position \( X_i \) in its visual field, \( F_c \)'s largest partner is \( X_{\text{max}} \). When \( F_c(X_{\text{max}}) > \delta F_c(X_i) \) indicates that the partner at \( X_{\text{max}} \) has the highest food concentration, update \( X_{i+1} \) according to equation (5) and move one step towards \( X_{\text{max}} \). Otherwise perform foraging behavior.

\[
X_{i+1} = X_i + \text{rand}() \cdot \text{step} \cdot \frac{(X_{\text{max}} - X_i)}{D_{\text{max}}} \quad (5)
\]

2.2. Improvement strategy

**Strategy 1** Introduction of adaptive factors--By analyzing three behavioral rules of artificial fish swarm algorithm, the artificial fish optimization movement is mainly adjusted by the field of view \( \text{visual} \) and the step size \( \text{step} \). Usually \( \text{visual} \) and \( \text{step} \) are set to fixed values. The existing research results [12] indicate that the fixed \( \text{visual} \) and \( \text{step} \) may lead to the faster global search speed of the artificial
fish school algorithm in the early stage. And the later local search is slow and easy to produce oscillation problems near the search position. In order to effectively adjust the requirements on visual and step at different running stages of the algorithm, this paper adopts the adaptive factor $w$ shown in Formula (6) to adjust visual and step. At this time, the calculation formulas of visual and step are shown in Equations (7) and (8). Where $w_{\text{max}}$ and $w_{\text{min}}$ are the maximum and minimum weights, $t$ is the current number of iterations, $T_{\text{max}}$ is the maximum number of iterations, visual$_{\text{min}}$ is the minimum visual field value, and step$_{\text{min}}$ is the minimum step size.

$$w = w_{\text{max}} - (w_{\text{max}} - w_{\text{min}}) \cdot \frac{t}{T_{\text{max}}}$$  \hspace{1cm} (6)
$$\text{visual} = \text{visual}_0 \cdot w + \text{visual}_{\text{min}}$$  \hspace{1cm} (7)
$$\text{step} = \text{step}_0 \cdot w + \text{step}_{\text{min}}$$  \hspace{1cm} (8)

**Strategy 2** Elite learning--When the artificial fish swarm algorithm iterates to a certain period, the optimal group solution $X_{\text{ghost}}$ obtained by the algorithm may stop locally because the whole fish swarm lacks the guidance of other excellent individuals. As the iterative process goes on, the whole fish swarm will wander around $X_{\text{ghost}}$. Along with this wandering state, the optimization process of the whole fish will also slow down, presenting a false convergence state. Aiming at this state of false convergence, this paper designs an elite learning mechanism to guide fish swarm to jump out of false convergence. In each iteration to the status announcement, based on the optimal $X_{\text{ghost}}$ and the current optimal $X_{\text{best}}$ of the fish population, an elite individual $X_{\text{new}}$ is introduced by strengthening the learning of $X_{\text{best}}$ to $X_{\text{ghost}}$, and the updating of the fish population is guided by comparing $X_{\text{new}}$ and $X_{\text{ghost}}$. Elite individual $X_{\text{new}}$ is shown in Equation (9).

$$X_{\text{new}} = X_{\text{ghost}} + 2 \cdot \text{rand}() \cdot (X_{\text{best}} - X_{\text{ghost}})$$  \hspace{1cm} (9)

### 3. Construction of IAFSA-BP algorithm

#### 3.1. The core idea of combining IAFSA and BP

The central idea of IAFSA optimization BP neural network is to set the desired optimization variable (the position of artificial fish) of IAFSA as the initial weights and thresholds of the neural network. Each artificial fish represents a neural network, and the food concentration of artificial fish is the fitness function obtained by BP neural network algorithm, the optimization process of IAFSA is to adjust the initial weights and thresholds of BP neural network. IAFSA has good optimization ability, and can be used to adjust the initial weights and thresholds of the BP neural network to the near global optimal values, which makes the BP neural network is no longer a random set of initial weights and thresholds, which can be set as much as possible to avoid random initial weights and thresholds shock problems brought about by the network, makes the model more stable output and higher identification accuracy.

#### 3.2. Implementation steps of IAFSA-BP algorithm

According to the core idea of combining IAFSA with BP, the following steps of IAFSA-BP neural network algorithm are designed:

**Step1** Parameter initialization -- Set BP neural network structure as $m$-$p$-$n$, then the weight value of the input layer to the hidden layer, the hidden layer to the output layer, the threshold value of the hidden layer and the threshold value of the output layer are $\{w\}_i^j$, $\{v\}_s^p$, $\{\theta\}_p$ and $\{r\}_s$, respectively. The essence of the optimization of BP neural network is to adjust the weight threshold parameter combination $\{w, v, \theta, r\}$ of BP network. Then the position of artificial fish can be expressed as $(m+1) \times (p+1) \times n$ vector $X$, as shown in Equation (10):

$$X = \{w_{i1}, w_{i2}, \cdots, w_{in}, v_{j1}, v_{j2}, \cdots, v_{jp}, \theta_{p1}, \theta_{p2}, r_{s1}, r_{s2}, \cdots, r_{sn}\}$$  \hspace{1cm} (10)

**Step2** Design of fitness function--Since the total error of BP neural network $E(w, v, \theta, r)$ is the food
concentration of artificial fish algorithm, the calculation of food concentration $F_c$ of artificial fish at position $X$ is shown in formula (11). Thus, the problem of finding artificial fish food concentration is transformed into the problem of finding BP neural network error. The $F_c$ of each artificial fish in the school is calculated and the size is compared. $X_{\text{gbest}}$, with the maximum $F_c$, enters the bulletin board. Then $X_{\text{gbest}}$ is the optimal parameter combination corresponding to BP neural network in the next iteration.

$$F_c(X) = E(w, v, \theta, r) = \frac{1}{N} \sum_{k=1}^{N} (y_k - y'_k)^2$$

\[ s.t. w \in R^{n \times d}, \ v \in R^{d}, \ \theta \in R^n, \ r \in R^w \tag{11} \]

**Step3** Fish swarm optimization [13]--For any 2 randomly generated artificial fish, set their positions as $X_q$ and $X_t$ respectively, then the distance $D_{qt}$ between the two fish is calculated as shown in equation (12).

$$D_{qt} = \|X_q - X_t\| = \sqrt{\sum_{j=1}^{n} \sum_{j=1}^{d} (w_j(q) - w_j(t))^2 + \sum_{j=1}^{d} (\theta_j(q) - \theta_j(t))^2 + \sum_{j=1}^{n} \sum_{k=1}^{d} (v_{jk}(q) - v_{jk}(t))^2 + \sum_{k=1}^{d} (r_k(q) - r_k(t))^2}$$

\[ s.t. w \in R^{n \times d}, \ v \in R^{d}, \ \theta \in R^n, \ r \in R^w \tag{12} \]

**Step4** Update the extremum--after simulating behavior rule 2 and behavior rule 3 respectively, the fish swarm can generate the optimal cluster individual $X_{\text{swarm}}$ and rear-end individual $X_{\text{follow}}$. Compare $F_c(X_{\text{follow}})$ and $F_c(X_{\text{swarm}})$. If $F_c(X_{\text{swarm}}) < F_c(X_{\text{follow}})$, then $X_{\text{best}} = X_{\text{swarm}}$, otherwise $X_{\text{best}} = X_{\text{follow}}$. Strategy 2 was adopted to generate elite individual $X_{\text{new}}$, if $F_c(X_{\text{new}}) < F_c(X_{\text{best}})$, then $X_{\text{best}} = X_{\text{new}}$, and $X_{\text{new}}$ was put into the fish swarm to replace the current optimal individual, so as to guide the fish swarm to continue iteration. Compare $F_c(X_{\text{best}})$ and $F_c(X_{\text{gbest}})$, if $F_c(X_{\text{best}}) < F_c(X_{\text{gbest}})$, then $X_{\text{gbest}} = X_{\text{best}}$, and update the bulletin board; otherwise, do not update the bulletin board, and continue iteration.

**Step5** Iteration stop judgment--Judge whether the current fitness function value reaches the set accuracy or iterates to the maximum number $T_{\text{max}}$. if the condition is full, the iteration is stopped, and then the value corresponding to $X_{\text{gbest}}$ is the initial optimal weights and thresholds combination of BP neural network, $X_{\text{gbest}}$ was brought into the BP neural network for secondary training until the end of the training to form a face orientation recognition model. Otherwise, go back to Step4 to continue the iteration.

4. **Experimental**

4.1. Sample information generation

In this study, a total of 10 people were collected, each with 5 face-facing pictures and a total of 50 face-facing pictures as research samples. 40 of them were used as BP neural network training samples, and 10 were used as test samples. Each face The images are all 420x420 pixels, and the face orientations are left, front left, front right, front right, and right. Fig.1 shows part of the sample images used in this research.

![Fig.1 Partial face orientation sample.](image)

By observing the images, it can be seen that there are obvious differences in the positions of the five face orientations. In this study, in order to distinguish the five face orientations, the 3-bit binary number was used to define the face orientations as shown in Table 1 as the output of the BP model.
Table 1 Face orientation numeralization

| Face orientation | right | front right | front | front left | left |
|------------------|-------|-------------|-------|------------|------|
| Assignment       | [1 1 0] | [1 0 0] | [0 1 0] | [0 0 1] | [0 1 1] |

4.2. Parameter setting of nodes of hidden layer

In this study, the golden section method proposed by Professor Sharkman was used to determine BP neural network hidden layer node number \( p \). The basic principle is to repeatedly calculate the neural network output error at 0.618 and 0.382 at the value interval \([a, b]\) of the node of the hidden layer initially determined, and to reduce the experimental scope gradually until a satisfactory result is obtained [14]. The calculation of endpoint \( a \) and \( b \) in the initial search interval is shown in equation (13), through calculation, the output error MSE of BP neural network corresponding to nodes of different hidden layers is obtained, as shown in table 2. The number of hidden layer nodes of BP neural network was finally determined to be 16, the structure of BP neural network for face orientation positioning is shown in Fig.2.

\[
a = \frac{(n+m)}{2} \leq p \leq (n+m) + 10 = b
\]

Table 2 Output error comparison of BP network with different hidden layer nodes

| \( p \) | 3 | 8 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|-------|---|---|----|----|----|----|----|----|----|
| MSE   | 0.1520 | 0.0918 | 0.0826 | 0.0718 | 0.0554 | 0.0675 | 0.0527 | 0.0421 | 0.0472 |

Fig. 2 Basic structure of BP neural network for face orientation positioning

After the node of the hidden layer is determined, the next important parameter to be determined for BP neural network is the training algorithm. A large number of studies have shown that levenberg-Marquard (LM) algorithm has a faster convergence speed and higher convergence accuracy than gradient descent algorithm [15]. Therefore, this paper adopts Levenberg-Marquard algorithm as the BP neural network training algorithm.

4.3. Evaluation Description

In order to verify the effectiveness of IAFSA-BP neural network algorithm proposed in this paper in face orientation recognition, the recognition results of IAFSA-BP algorithm are compared with those of BP algorithm, GA-BP algorithm, PSO-BP algorithm and AFSA-BP algorithm. For GA, PSO, AFSA and IAFSA, the initial solution space is all \([-1,1]\), the population size is all 20, and the maximum number of training is all 50. At the same time, in order to compare the optimization efficiency of artificial fish swarm optimization algorithm before and after the improvement, except adding \( w_{max} \), \( w_{min} \), \( visual_{min} \) and \( step_{min} \) to the improved artificial fish swarm algorithm, the other parameters of the improved artificial fish swarm algorithm are exactly the same as the standard artificial fish swarm algorithm. For each algorithm, 10 groups of tests were carried out respectively to calculate the average mean square error Meanmse of the output value for 10 times and the total correct rate of face orientation recognition for 10 test experiments. According to the above provisions on the output value of face orientation, if an absolute error value in the three output direction values of each group exceeds the preset fixed value here, it will be calculated according to the error of orientation recognition, and the preset fixed value here is set as 0.5, 0.4, 0.3, 0.2 and 0.1 respectively.
4.4. Simulation

Face toward the identification results as shown in table 3, as can be seen from the table, the permissible error absolute value or a fixed value is bigger (0.5, 0.4, 0.3), after optimization of BP neural network model toward the recognition accuracy is 100%, and without the optimized BP neural network recognition accuracy is by allowing the fixed value of 0.5 to 0.4 and 0.3, the recognition accuracy from 88% to 86% and 78%. When the fixed value is 0.2, the recognition accuracy of optimized BP neural network is 94%, while the recognition rate of unoptimized BP neural network is 69%. When the fixed value is 0.1, the recognition accuracy rate of the unoptimized BP neural network is only 44%, at which time the network model is basically unavailable, while other optimization algorithms are above 70%, and the optimization effect of IAFSA-BP neural network and AFSA-BP neural network algorithm is the best, reaching 74%. In addition to the recognition accuracy, MSE is also an index that can further measure the performance of the algorithm. The mean square error of IAFSA-BP neural network algorithm decreases to the order of $3 \times 10^{-3}$, and its output accuracy is significantly better than that of other algorithms.

| Table 3 Comparison of face orientation positioning results of several algorithms |
|---------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.5 | 0.4 | 0.3 | 0.2 | 0.1 | MeanMSE |
| BP | 88% | 86% | 78% | 69% | 44% | 0.08882 |
| GA-BP | 100% | 100% | 100% | 89% | 72% | 0.00524 |
| PSO-BP | 100% | 100% | 100% | 92% | 73% | 0.00506 |
| AFSA-BP | 100% | 100% | 100% | 94% | 74% | 0.00431 |
| IAFSA-BP | 100% | 100% | 100% | 94% | 74% | 0.00393 |

In addition to accuracy, operation speed is also an important evaluation index for algorithm improvement effect. Table 4 shows the optimization time comparison of BP neural network optimal initial weights and thresholds before and after artificial fish swarm algorithm improvement. It can be seen from the table that the improved artificial fish swarm algorithm, due to the introduction of adaptive weight and elite learning strategy, can effectively reduce the number of local repeated optimization attempts in the late operation period of the algorithm, and at the same time effectively improve the false convergence problem, greatly increase the solving speed, and shorten the entire algorithm running time.

| Table 4 Operation time comparison of artificial fish swarm algorithm (unit: s) |
|---------------------------------|--------|--------|--------|--------|
| Maxtime | Mintime | Meantime |
| AFSA | 552.079013 | 319.967138 | 454.229355 |
| IAFSA | 337.725560 | 164.403547 | 236.791311 |

5. conclusion

In this paper, the fixed step size and field of view in the standard artificial fish swarm algorithm may lead to the fast global search speed of the artificial fish swarm algorithm in the early stage, while the slow local search in the late stage is likely to produce vibration near the search location point. In order to solve the problem that the step size and field of view are fixed in the standard artificial fish swarm algorithm, the global search speed may be fast in the early stage of the artificial fish swarm algorithm, while the local search in the later stage is slow and easy to produce oscillation near the search location. In this paper, the adaptive weight is used to adjust the adaptive changes of step size and field of view. At the same time, an elite learning mechanism is introduced for the false convergence of the standard artificial fish swarm algorithm. The introduction of elite individuals is used to guide fish swarm to get rid of false convergence and quickly converge to the optimal solution, and it is applied to the optimization of BP neural network in the weights and thresholds of face orientation recognition model. The final experimental results show that the improved strategy of the standard artificial fish swarm algorithm proposed in this paper is successful. The BP neural network face recognition model using the improved artificial fish swarm algorithm not only has small operational error and high recognition accuracy, but also has a great improvement in running speed compared with the standard artificial fish.
swarm optimization algorithm. Therefore, the study in this paper provides a new feasible method for BP neural network weights and thresholds optimization problem, and also provides a new research idea for the study of face recognition technology.

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Reference
[1] Chelali F Z, Djeradi A (2012). Face Recognition System using Discrete Cosine Transform combined with MLP and RBF Neural Networks. International Journal of Mobile Computing and Multimedia Communications, vol.4, iss.4, pp:11-35.
[2] Han-yi G, Fu-wen S, Han-jun G (2018). Face Recognition Method Based on Improved Genetic Algorithm and BP Neural Network. Journal of Wu(Information & Management Engineering), vol.40, iss.5, pp: 22-26.
[3] Miwa S, Kage H, Hirai T, et al (2011). Face Recognition for Access Control Systems Combining Image-Difference Features Based on a Probabilistic Model. IEEJ Transactions on Electronics Information and Systems, vol.131, iss.12, pp: 2165-2171.
[4] Liao B, Wang H F (2015). The Optimization of SIFT Feature Matching Algorithm on Face Recognition Based on BP Neural Network. Applied Mechanics & Materials, iss.743, pp:359-364.
[5] Su-ping L, Zhan-feng W, Jing W (2018). Study of Face Orientation Recognition Based on Neural Network. International Journal of Pattern Recognition and Artificial Intelligence, vol.32, iss.11, pp: 1856015
[6] Tong Z, Wen-Wen L U, Nan-Feng X (2010). Research of Face's Orientation Recognition Based on BP Network. Journal of Chongqing University of Technology (Natural ence), vol.24, iss.4, pp: 61-64
[7] Liu Hao, Fang Wen-yi (2012). New Ideas of Face Orientation Discrimination Based on BP Neural Networks. Computer ence, vol.39, iss. 11, pp: 366-369
[8] Song juan, Zou Shuang, Yin Jian-fang, et al (2017). Facing-orientation Recognition Based on Neural Networks. Industrial Control Computer, vol.30, iss.4, pp:111-113
[9] Xiao-lei L, Zhi-jiang S, Ji-xin Q (2002). An optimizing method based on autonomous animats: Fish-swarm algorithm. Systems Engineering -Theory & Practice, vol.22, iss.11, pp:32-38.
[10] Wen-dai L, Shun-ji J, Hua P (2016). The Global Allocation of Social Capital Under the Control of Government Policy. Journal of Computational and Theoretical Nanoscience, vol.13, iss.12, pp:9539-9542.
[11] Yi-min J, Li-ping S, Xin Y (2019). Transformer fault diagnosis based on wavelet neural network with improved artificial fish-swarm algorithm. Journal of Henan Polytechnic University (Natural Science), vol.38, iss.2, pp:103-109.
[12] Hai-xing B, Zhi-gao Z, Yan-hui Z, et al (2018). An Artificial Fish Swarm Algorithm Based on Variable Visual Field and Step Length. Journal of Hunan University of Technology, vol.32, iss.3, pp: 81-85.
[13] Xue-dian Z, Fu-yan W, Xiao-fei Q (2019). Parameter tuning of fractional order PIλ controller based on artificial fish school algorithm. Application Research of Computers, vol.36, iss.3, pp: 730-735
[14] Ke-wen X, Cheng-biao L, Jun-yi S (2005). An Optimization Algorithm on the Number of Hidden Layer Nodes in Feed-forward Neural Network. Computer Science, vol.32, iss.10, pp:143-145.
[15] Lv C, Xing Y, Zhang J, et al (2018). Levenberg–Marquardt Backpropagation Training of Multilayer Neural Networks for State Estimation of a Safety-Critical Cyber-Physical System. IEEE Transactions on Industrial Informatics, vol.14, iss.8, pp: 3436-3446.