A new mechanism of radiation emission in the polar gap of a pulsar is proposed. It consists in reflected curvature radiation which is emitted by returning positrons moving towards the surface of the neutron star along magnetic field lines and reflects from the surface. Such radiation interferes with transition radiation emitted from the neutron star when positrons hit the surface. It is shown that the proposed mechanism may be applicable for explanation of the mystery of the interpulse shift in the Crab pulsar at high frequencies discovered by Moffett and Hankins [1] twenty years ago (for recent confirmation see [2]). In the proposed model the interpulse phase shift is stipulated by the assumed inclination of the magnetic axis to the pulsar surface normal.

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MEAN PROFILES OF CRAB PULSAR EMISSION AT DIFFERENT FREQUENCIES
(riddle of interpulse shift and appearance of additional High Frequency Components)

The figure from the paper [1] by Moffett & Hankins, 1996 shows the phase dependence of the average registered radiation intensity during one period of the PSR B0531+21 rotation for different radiation frequencies.

At low frequencies two distinct pulses are registered during one period of rotation of the star. Such pulses, the main pulse (MP) and interpulse (IP), are believed to originate from different magnetic poles of the Crab pulsar.

At frequencies around 3 GHz IP disappears. It appears again at higher frequencies having phase shift δ of about 7° comparing to initial IP at lower frequencies. Moreover, two more distinct pulses, known as high-frequency components (HFCs), appear at the same frequencies.

Since the discovery of these peculiar features of radio emission by the Crab pulsar no theoretical explanation of them has been presented. In this work we propose such explanation as reflection from the surface of the neutron star radiation of returning positrons (see also our preprint [3]). The HFCs explanation used the idea of nonlinear reflection from the neutron star surface see in [4].
THE RIDDLE SOLUTION: RADIATION BY RETURNING POSITRONS MOVING IN THE INCLINED MAGNETIC FIELD

As the origin of the shifted high-frequency IP we consider the radiation by positrons moving towards the surface of the pulsar in the polar gap. Such positrons can be returned from the lower layers of the magnetospheric plasma by the same electric field which accelerates the electrons outward the star.

Moving along curved magnetic field lines towards the surface of the star the positrons emit curvature radiation. At sufficiently high frequencies it reflects from the surface and propagates outward the star. Moreover, when the positrons hit the surface the so-called transition radiation is generated in the direction outwards the surface as well.

$$e^+ \text{ radiation} = \text{reflected curvature radiation} + \text{transition radiation}$$

low-frequency interpulse – from $e^-$ radiation
high-frequency (shifted) interpulse – from $e^+$ radiation

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**THE MODEL**

Model of the electric field accelerating the positron:

\[ E(\alpha) = E_1 \alpha \frac{(a_0 - a_1)}{a_0 a_1} \theta(\alpha_1 - \alpha) + \left( E_1 + E_2 \frac{a_1 - a_1}{a_0} \right) \left( 1 - \frac{\alpha}{a_0} \right) \theta(\alpha - a_1) \]

angle corresponding to 'effective' part of positron trajectory:

\[ \alpha_0 \sim \arccos \left( 1 - \frac{R_*}{R} \right) - \frac{R_*}{R} \]

Penetrating into magnetosphere part of the electric field

observed reflected radiation

'limit' of reflected radiation direction

\[ R_* \]

\[ \psi \]

\[ \alpha_0 \]

\[ \alpha \]

\[ \theta \]

\[ E(\alpha) \]

\[ \alpha_0 \]

\[ \alpha \]

\[ E_1 \]

\[ E_2 \]

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In the direction of a certain angle with magnetic field
\[ \psi = \theta + 2\chi \]
(see the picture on the previous page) the radiation is emitted at certain characteristic frequency:
\[ \omega(\psi) \sim c\gamma^3(\alpha) / R \]
depending of gamma

The reflection that get the telescope begins at sufficiently high frequency \( \omega_{\text{min}} \) which is radiated by positrons having the Lorenz-factor:
\[ \gamma_{\text{min}} \sim (\omega_{\text{min}} R / c)^{1/3} \]

Distribution of radiation by positron (reflected curvature radiation + transition radiation) is calculated with the use of method of images (see the previous page) on the basis of the well-known expression of relativistic electrodynamics [5]:

\[ \frac{d^2W}{d\omega\,d\sigma} = \frac{e^2\omega^2}{4\pi^2c^3} \left| \int_{-\infty}^{+\infty} dt \, \vec{n} \times \vec{v}(t) \exp \left\{ i\omega \left( t - \frac{\vec{n}r_0(t)}{c} \right) \right\} \right|^2 \]
ESTIMATION OF THE COHERENT RADIATION FLUX

The observations indicate the necessity of coherent character of radiation emission by charged particles in the pulsar magnetosphere. For the estimation of the positron radiation intensity and derivation of the radiation energy spectrum at first it is enough to accept the very fact of the existence of inhomogeneous (clamped) positron flux in the gap. In this case the main contribution to radiation is made by coherently radiating volumes, which size $V_{coh}$ depends on the wavelength, Lorenz-factor and the curvature of the positrons trajectories. The number of such volumes can be rather roughly estimated through "division" of the total radiating volume by $V_{coh}$.

$$ V_{coh} = r_r^2 \sim \gamma^2 \lambda^3 $$

$ r_r $ and $ r_\perp $ are the linear sizes of the coherently radiating volume in the directions along and perpendicular to the positrons velocity.

$$ J(\omega) \sim \frac{\kappa^2 n_{GJ}^2 e^2 \lambda^{3-1/3}}{2^{5/3} \pi^{14/3} a^2 R_\ast^{4/3}} \sqrt{mc^2 \hbar} \int_0^{R_{PC}} \frac{drr^{5/3}}{2eE_0} \sqrt{1-r^2/R_{PC}^2} \gamma_{\max}^{7/2} \sim $$

we assume linear accelerating field growth on the positron trajectory interval contributing to the radiation flux

$$ \lambda = 2\pi c / \omega \quad \text{radiation wavelength} $$

$$ n_{GJ} \sim \frac{\tilde{G} \tilde{B}}{2\pi c e} \quad \text{the Goldreich-Julian density; } \kappa \cdot n_{GJ} \quad \text{density of positrons; } $$

$$ \overline{\gamma}_{\max} \quad \text{average value of the positron Lorenz-factor at which its radiation ceases to hit the telescope} $$

$$ R_\ast \quad \text{pulsar radius; } \quad R_{PC} \quad \text{polar cap radius; } \quad d \quad \text{distance from the pulsar to the Earth; } $$

$$ E_0 \quad \text{accelerating electric field on the magnetic axis} $$

$$ \overline{h} \quad \text{some average length of the positron trajectory interval which contributes to radiation flux} $$
Here we assume that the initial angular width of the positron radiation diagram \( \gamma_{\text{min}} \), when the particle moves at the beginning of the effective path, exceeds the value of the minimal angle between the magnetic axis and the direction to the telescope. With the increase of the positron Lorenz-factor at lower altitudes the characteristic angle of its radiation diagram becomes less than \( \gamma_{\text{min}} \) (at \( \gamma_{\text{max}} \)) and radiation ceases to be caught by the telescope.

As the expression on the previous page shows, the contribution to the radiation flux in our case grows with the increase of \( \gamma \). Due to this fact the value of the integral with respect to \( \gamma \) is mostly defined by the upper limit \( \gamma_{\text{max}} \) while the exact value of \( \gamma_{\text{min}} \) is not very significant in general position. Let us note that such situation is different from the well known case of synchrotron radiation of the electron component of cosmic rays with the decreasing energy spectrum. Due to such spectrum of the electron energies the integral with respect to \( \gamma \) in this case is merely defined by the lower limit and the contribution to the radiation flux is associated only with \( \gamma_{\text{min}} \). It leads to the well known relation between spectral indices of such cosmic radio emission sources as radio galaxies, quasars, supernova remnants, etc in the case of synchrotron radiation.
THE HIGHEST FREQUENCY FOR THE SHIFTED INTERPULSE

In the framework of our model the minimum value \( \gamma_{\text{min}} \) of the positron Lorenz-factor, at which the radiation begins reflecting from the surface and get the telescope, defines the minimal frequency \( \omega \) of radiation which can reflect from the surface and hit the telescope. The relation between these values is the following: \( \omega \sim c\gamma_{\text{min}}^3 / R \), where \( R = 4R_*^2 / 3r \) is the positron trajectory radius and \( r \) is the distance from the magnetic axis.

Therefore:

\[
\gamma_{\text{min}}(\omega, r) \sim \left( \frac{4R_*^2 \omega}{3cr} \right)^{1/3}
\]

For certain \( \omega \) the reflected radiation mechanism must disappear for \( r = r_0 \) at which: \( \gamma_{\text{min}}(\omega, r_0) = \overline{\gamma}_{\text{max}} \)

The shift of IP has to disappear too.

Thus in the considered model the coherence leads to formation of a hollow cone (in accordance with [6]), restricted from the inner and outer sides by the magnetic field lines situated respectively on distances \( r_0 \) and \( R_{PC} \) from the magnetic axis (in the vicinity of the star surface. The particles moving in the specified region make the main contribution to the radiation flux.

Taking \( r_0 = R_{PC} \) for the maximum frequency at which the reflected radiation from the star surface is still possible we obtain:

\[
\omega_{\text{max}} \sim \frac{3cR_{PC} \overline{\gamma}_{\text{max}}^3}{4R_*^2}
\]

To accurately estimate this frequency it is necessary to use self-consistent models.

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CONCLUSIONS

➢ The explanation of the origin of the shifted high-frequency interpulse in the Crab pulsar as a reflected radiation of the returned positrons in the inclined magnetic field is proposed.

➢ The observed interpulse properties explained by the applied model:
  ❖ appearance of the shifted interpulse at certain high frequency ~ 5 GHz
  ❖ its disappearance at some higher frequency (which is not observed yet)

➢ The total flux of the coherent positron radiation is estimated. The obtained value and energy spectrum quite nicely agrees with observation.

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