Production of dijets with large rapidity separation at colliders

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Abstract. Production of dijets with large rapidity separation at collider energies in modified logarithm approximations is considered. The results for resummations of different types of logarithms: soft-gluon, non-global and Sudakov ones are discussed. Calculations for various dijet observables are confronted with LHC data.

1. Introduction

Search for new physics at hadron colliders is often related to the precise measurement of hadronic jets. Available jet observables are sensitive to various aspects of the new physics. For example, the study of dijets with large rapidity separation between jets would reveal new physics related to new heavy objects beyond the Standard Model (SM) \cite{1, 2, 3} and/or new quantum chromodynamics (QCD) dynamics \cite{4} within the SM. To improve sensitivity to new physics manifestations it can be used different kinematical conditions and cuts, such as veto condition on extra jet production at certain rapidity interval and transverse momentum range \cite{5, 6}. At the moment, one of the most sensitive to possible new QCD dynamics experimental data on large rapidity dijets was obtained at the LHC with 7 TeV \cite{7, 8} using extra jet veto condition. While for inclusive dijets with large rapidity separation between jets \cite{4, 9, 10, 11, 12, 13, 14} it can be used celebrated Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution \cite{15, 16, 17, 18} for resummation of leading small-\(x\) logarithms, whereas for dijet production with imposed jet veto condition there is no established theory technique. The latter circumstance is deferring a clear interpretation of the obtained data.

This work is an attempt to estimate the impact of resummation of various types of logarithms: soft-gluon, non-global and Sudakov ones for dijet observables with extra veto condition. The imposed jet veto condition means that jet activity additional to the observed jets is forbidden above some veto scale \(Q_{\text{veto}}\) which can be of the order of tens GeV at the LHC. However, the multiple soft-gluon radiation below the veto scale leads to energy flow, the transfer of the energy and transverse momentum from the observed jets. When comparing the theoretical predictions to measured data the modern Monte Carlo (MC) event generators based on leading logarithm Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (DGLAP) evolution \cite{19, 20, 21, 22, 23} are able to take some energy flow effects into account. However, there are difficulties in comparison of experimental results with analytical calculations due to complicating phase space configurations. This work tries to improve available analytical methods for the estimation of the energy flow effects.
At high energy asymptotics, the BFKL equation governs the dominant emission of semi-hard gluons (jets) strongly ordered in rapidity and having approximately similar transverse momentum values.

The BFKL evolution predicts the fast rise of dijet cross section with rapidity separation between jets $\Delta y = |y_1 - y_2|$, where $y_1$ and $y_2$ are the rapidities of jets forming the dijet. The veto in dijet observables can be imposed between jets in the dijet (interjet veto) and/or outside dijet. Hence the jet veto in such dijet observables would cover wide rapidity (large polar angles) ranges in which makes them sensitive for testing the energy flow physics.

In the work [24] a partial success was achieved with an application of the Banfi-Marchesini-Smye (BMS) equation [25] to the calculation of dijet observables with interjet veto measured by the ATLAS experiment [8]. The ATLAS collaboration measured the ratio of interjet veto dijet cross section to dijet cross section in proton-proton collisions at $\sqrt{s} = 7$ TeV. The range of measured average transverse momentum of hard jets was $70 < p_T < 270$ GeV whereas the range in rapidity separation was $\Delta y < 6$. The interjet veto was set at transverse momentum $p_T = Q_{\text{veto}} = 20$ GeV.

The BMS equation allows us to sum over possible large-angle soft-gluon emission configurations below $Q_{\text{veto}}$ scale in the region where a veto is imposed (veto region, which we denote $C_{\text{out}}$ – the region away from the observed hard jets). In addition, it sums possible soft-gluon emission in the regions containing hard jets ($C_{\text{in}}$ – the region complementary to $C_{\text{out}}$), as well as virtual corrections, in leading logarithm approximation. In particular, it resumes Sudakov logarithms and non-global ones, where the former are produced by primarily emitted gluons, whereas the latter is originated from secondary gluons in multiple gluon emission. The soft-gluon approximation and large $N_c$ (number of colors) approximation is used in its derivation.

The CMS collaboration in their search for BFKL [7] measured the ratio $R_{\text{incl}}$ of inclusive dijet cross section to veto dijet cross section in proton-proton collisions at $\sqrt{s} = 7$ TeV, for jets with transverse momentum $p_T > 35$ GeV and rapidity $y < 4.7$ ($\Delta y < 9.4$). The veto condition was set in the whole range of rapidity $-4.7 < y < 4.7$ at $p_T = Q_{\text{veto}} = 35$ GeV.

The aim of the present work is to extend the approach of large-angle multiple emission of soft gluons, used in the BMS equation, for calculation of the effects of jet veto which is not restricted to interjet region. In what follows we distinguish interjet veto and jet veto, wherein the former the veto condition is imposed in interjet region and in the later the veto condition is imposed in the whole rapidity acceptance range.

The one difficulty of the application of the BMS equation consists of obtaining its solution, especially when the configuration of the veto region becomes complicated. A far as we concern the only numerical methods, which are requiring a substantial amount of computing resources to get the solution. A simple MC algorithm was proposed in Ref. [26], which is supposed to reproduce the evolution of the BMS equation. In the present work, we implement the algorithm and inspect its performance. Some modifications of the algorithm were implemented incorporating the energy-momentum constraints and recoil effect model.

In section 2 we discuss applications the BMS equation to jet veto calculation. In section 3 we consider the MC algorithm proposed in the work [26], and discuss the modifications of the algorithm in section 4. In section 5 we present the results of application of the discussed methods to jet veto observable measured by the CMS experiment [7].

2. BMS equation and its application to jet veto observables

The BMS equation [25] is proposed for the resummation of Sudakov and non-global logarithms at $Q_{\text{veto}} << Q$, where $Q$ is the hard scale. The Sudakov logarithms are produced by primarily emitted gluons, whereas non-global ones are originated from secondary gluons in multiple gluon emission. The resummation is achieved in the large $N_c$ limit where emission factorised
to incoherent radiation from colour dipoles. The BMS equation reads as the following

$$\partial_\tau \Sigma_{ab}(\tau) = -(\partial_\tau R_{ab}) \Sigma_{ab} + \int_{C_{in}} \frac{d\Omega}{4\pi} w_{ab}(q) [\Sigma_{aq}(\tau) \Sigma_{qb}(\tau) - \Sigma_{ab}(\tau)],$$

where \( \tau = \int_{Q_{veto}}^Q dq \alpha_s(q_t) \) is the evolution variable; \( C_{in} \) is the region adjacent to the veto region \( C_{out} \). The first term in the right part of the equation (1) provides resummation of the Sudakov logarithms, whereas the second term does that for the non-global logarithms.

The solution of the BMS equation \( \Sigma_{ab}(\tau) \) interpreted as the probability for a colour dipole, which is formed by the colour-connected partners \( a \) and \( b \) at some scale \( Q \) moving in the directions \( \Omega_a \) and \( \Omega_b \), not to break the veto condition at the scale \( Q_{veto} \) imposed in the region \( C_{out} \). The first term in the right part of the equation (1) provides resummation of the Sudakov logarithms, whereas the second term does that for the non-global logarithms.

The detailed description of the application of the BMS equation to the calculation of interjet observables is given in [24]. In the work [25] it is noted that the equation (1) can in general be used for more complicated configuration of \( C_{out} \) region, for example when it is not simply connected and consists of several rapidity intervals. However, the use of such \( C_{out} \) region causes significant complications in obtaining the solution.

In the case of jet veto one has the veto condition imposed at the whole rapidity acceptance range. The corresponding interval \( [y_{min}, y_{max}] \) is usually symmetric \( [-y_{max}, y_{max}] \) in the experiments on colliders, where \( y_{min} \) and \( y_{max} \) are the minimal and maximal available rapidities, which are defined by acceptance of the detector. Within this rapidity interval there are the observed hard jets at rapidities \( y_1, y_2, \ldots, y_k \) (let them be ordered in rapidity \( y_1 < y_2 < \ldots < y_k \)). Then it is possible to define \( C_{out} \) region as a composition of the intervals \( [-y_{max}, y_1 - R] \cup [y_1 + R, y_1 - R] \cup \ldots \cup [y_k + R, y_{max}] = C_{out}^{(1)} \cup C_{out}^{(2)} \cup \ldots \cup C_{out}^{(k+1)} \), where \( R \) is jet size.

The rough estimate of the solution of equation (1) for the just discussed \( C_{out} \) region configuration can be obtained by the product

$$\Sigma_{ab}(\tau) = \prod_{i=1}^{k+1} \Sigma_{ab}^{(i)}(\tau),$$

where the \( \Sigma_{ab}^{(i)}(\tau) \) is the solution of equation equation (2) with \( C_{out} = C_{out}^{(i)} \). The cost of such approximation is the loss of possible correlations.

Another simplification one can employ is to calculate just Sudakov logarithms contribution by solving the equation

$$\partial_\tau \Sigma_{ab}(\tau) = -(\partial_\tau R_{ab}) \Sigma_{ab}.$$
break the veto. Arbitrarily complicated $C_{\text{out}}$ region can be used in that case. Colour dipoles can be formed by observed hard jets and the residues of the colliding hadrons. Where the residue is the hadron without kicked off parton, moving toward the initial direction. The formation of colour dipoles corresponds to the colour flow in hard parton subprocess. One needs to employ large $N_c$ approximation to suppress configurations with indefinite colour flow.

To generate an event one first needs to resum the virtual corrections by Sudakov form factor

$$\ln S_{ab}(Q, Q_0) = - \int_{Q_0}^{Q} \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \frac{\pi_s(q_{abl}) w_{ab}(q)}{N_{ab}(\omega_q)} \Theta(q_{abl} - Q_0),$$

(5)

Where the $Q_0$ energy cutoff is introduced, as well as the $q^2_{abl} = \frac{2\omega^2_q}{w_{abl}(q)}$ - the scale transverse to the dipole $ab$.

Starting from $ab$-dipole at scale $Q$ one can use uniform random number $r$ to generate the scale of the next splitting $w_q$ by solving the equation

$$S_{ab}(\omega_q, Q_0) \cdot r = S_{ab}(Q, Q_0)$$

(6)

If $r < S_{ab}(Q, Q_0)$ then dipole does not split, otherwise $\omega_q$ can be found. After the scale of the splitting is found one should generate the direction $\Omega_q$ of the emitted gluon by the distribution

$$\frac{dR_{ab}(\Omega_q)}{d\Omega_q} = \frac{\pi_s(q_{abl}) w_{ab}(q)}{N_{ab}(\omega_q)} \Theta(q_{abl} - Q_0),$$

(7)

where $N_{ab}(\omega_q)$ is the normalising factor. Hence one gets two dipoles $a\bar{q}$ and $b\bar{q}$ at new scale $\omega_q$. One should repeat then the calculations for new dipoles.

However, the introduction of the scale $q_{abl}$ transverse to $ab$-dipole introduces correlation between scale $\omega_q$ and direction $\Omega_q$, which originally was not presented in the BMS equation (1). The harder splitting acquires larger weight, which is calculated as the integral over angles. One needs to introduce the $q_{abl}$ and theta function $\Theta(q_{abl} - Q_0)$ to exclude the singular collinear region. Moreover the introduction of $q_{abl}$ as the argument of the strong coupling $\pi_s(q_{abl})$ is partly an account for the next-to-leading corrections in the soft limit [27, 28].

If one wants to compare the results of MC generation and the BMS equation one needs to use Sudakov form factor corresponding to the equation (2). To exclude the singular region one can introduce some $\theta$ cutoff, much smaller than jet size $R$. The $\theta$ cutoff can be expressed in the maximal allowed value for $\cos \theta_{aq}$ and $\cos \theta_{bq}$, which is denoted as $c_{\max}$. The Sudakov form factor then becomes

$$\ln S_{ab}(Q, Q_0) = - \int_{Q_0}^{Q} \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \frac{\pi_s(q_{abl}) w_{ab}(q)}{N_{ab}(\omega_q)} \Theta(c_{\max} - \cos \theta_{aq}) \Theta(c_{\max} - \cos \theta_{bq}).$$

(8)

In what follows we denote the MC calculation based on equation (5) as $q_{abl}$-on and on equation (8) as $q_{abl}$-off. The results of MC $q_{abl}$-on calculation for different values of cutoff $Q_0 = 2$ and $5$ GeV compared to the measurements of the CMS collaboration [7] are presented in the figure 1 (b), whereas the results of MC $q_{abl}$-off calculation for values of $c_{\max} = 0.9999$ and 0.99999 are in the figure 1 (c).

4. Development of the MC algorithm

The comparison of the results of the application of the BMS equation as well as the soft-gluon MC algorithm to the calculation of jet veto observable of the CMS data [7] showed that the calculation significantly overestimates the data at small rapidity separation $\Delta y$. The small $\Delta y$ region accompanied by large rapidity intervals outside the jets $[y_{\max}, y_1 - R]$ and $[y_2 + R, y_{\max}]$
in the jet veto. If the detector coverage allows to reach large $y_{\text{max}}$, as an usual case of the CMS detector, then the angles between hadron residues and veto region become small. For small angles the algorithm produces emission with rather large energy. Then energy-momentum conservation constrain as well as recoil effect become important and should be taken into account.

The simplified model of the recoil was tested with MC $q_{\text{abt}}$-off calculation. The promising results are obtained. In figure 1 (d) the results of MC $q_{\text{abt}}$-off calculation for value of $c_{\text{max}} = 0.9999$ (green line) together with similar calculation but with the simplified implementation of the energy-momentum conservation constrain (turquoise line) are presented. As one can see, the inclusion of the energy-momentum conservation constrain leads to decrease of the emission excess at small rapidity separation $\Delta y$. More accurate implementation of the energy-momentum conservation constrain and recoil effect can significantly improve the agreement to the measurements.

![Figure 1](image_url)

**Figure 1.** Calculations for ratio of inclusive dijet cross section to dijets with the extra jet veto ones as a function of rapidity separation between jets in comparison with the CMS collaboration measurements [7] in pp-collisions at 7 TeV. The CMS data are represented by black points. The statistical uncertainties of the measurement are represented by black bars, while the systematic uncertainties are represented by yellow band. Colour lines represent the results of the various calculation methods described in the text.

5. Results and Discussion
The various methods of calculation for jet veto observables were studied. The methods based on the large angle soft-gluon emission approximation such as BMS [25] equation and MC algorithm proposed in [26] are tested. The methods applied are meant resumming Sudakov and non-global
logarithms, where former is produced by primarily emitted gluons and latter are originated from secondary gluons in multiple gluon emission. The results are confronted to the CMS measurements [7] and presented in figure 1.

Some of the used methods are able to reproduce large Δy behavior, however at small Δy the experimental data are severely overestimated by all methods. Simplified model of the energy-momentum constrain and recoil effect implemented within the MC method of calculation was tested and showed desirable trend. Further progress could be related with more accurate accounts of the energy-momentum constrain and recoil effect as well as generalised Sudakov resummation [29].

6. Summary
The considered in this work the large angle sof-gluon emission resummation can be a suitable tool for study the extra jet veto conditions in jet production at collider energies. The developed MC algorithm provides a solution faster in computing than straightforward integration methods. Moreover, it is more suitable for the complicated kinematical configurations of the C_out region. However, further improvements are needed for the considered MC algorithm at small jet rapidity separation to take into account the energy-momentum conservation and recoil effects at present collider energies.

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