Uplink Performance of Cell-Free Massive MIMO Over Spatially Correlated Rician Fading Channels

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Abstract—We consider a practical cell-free massive multiple-input-multiple-output (MIMO) system with multi-antenna access points (APs) and spatially correlated Rician fading channels. The significant phase-shift of the line-of-sight component induced by the user equipment movement is modeled randomly. Furthermore, we investigate the uplink spectral efficiency (SE) with maximum ratio (MR)/local minimum mean squared error (L-MMSE) combining and optimal large-scale fading decoding based on the phase-aware MMSE, phase-aware element-wise MMSE and linear MMSE (LMMSE) estimators. Then new closed-form SE expressions with MR combining are derived. Numerical results validate our derived expressions and show that the SE benefits from the spatial correlation. It is important to observe that the performance gap between L-MMSE and MR combining increases with the number of antennas per AP and the SE of the LMMSE estimator is lower than that of other estimators due to the lack of phase-shifts knowledge.

Index Terms—cell-free massive MIMO, spatially correlated Rician fading, phase-shift, spectral efficiency.

I. INTRODUCTION

As one of the most promising technologies for future wireless communication, cell-free massive multiple-input-multiple-output (CF mMIMO) has been widely investigated in [1]–[5]. The key concept is that a large number of access points (APs) are connected to the central processing unit (CPU) via fronthaul connections to jointly serve the user equipments (UEs) on the same time-frequency resource. The number of APs is envisioned to be much larger than the number of UEs, thus distances between the closest AP-UE pairs decrease greatly, which leads to the decrease in path loss and the increase in macro diversity gain. In the uplink (UL), CF mMIMO usually uses maximum ratio (MR) combining for the low complexity but [6] advocates for local minimum mean squared error (L-MMSE) combining for its better performance. Moreover, the large-scale fading decoding (LSFD) method proposed for mMIMO originally has been utilized in CF mMIMO systems to further improve the throughput [6].

The vast majority of scientific papers on CF mMIMO are making the simplifying assumption of Rayleigh fading channels [6]–[9] or Rician fading channels where the line-of-sight (LoS) component has a static phase [10]. Recently, the authors in [11] indicate that the practical channel in CF mMIMO should be composed of a semi-deterministic LoS path component with random phase-shifts and a stochastic non-line-of-sight (NLoS) path component. The phase-shift of the LoS component is modeled as a uniformly distributed random variable due to UE mobility and hardware effects like phase noise. However, [11] is based on the assumption of single-antenna APs, while practical APs are usually equipped with multiple antennas. Furthermore, the authors in [12] consider spatially uncorrelated Rician fading channels with unknown phase shifts and multi-antenna APs. Unfortunately, it did not consider the spatial channel correlation which has a significant impact on CF mMIMO systems [13].

To address these limitations, we consider a CF mMIMO system over spatially correlated Rician fading channels with phase-shifts and multi-antenna APs. The same channel model has been investigated in the cellular mMIMO scenario in [14]. The main contributions of this paper are as follows: (1) We consider three useful channel estimators with different prior information: the phase-aware MMSE with all prior information, the phase-aware element-wise MMSE (EW-MMSE) with phase-shifts and partial large-scale fading knowledge and the linear MMSE (LMMSE) with all large-scale fading but no phase-shift knowledge; (2) Based on these channel estimators and the LSFD method, we derive the UL SE expressions for any combining scheme and compute closed-form SE expressions for MR combining; (3) We analyse the UL SE with MR/L-MMSE combining over correlated/uncorrelated Rician fading channels numerically.

II. SYSTEM MODEL

We consider a CF mMIMO system consisting of $M$ APs with $N$ antennas each and $K$ single-antenna UEs. The channel response is constant in a coherence time-frequency block of length $\tau_c$ channel uses. In the UL, we reserve $\tau_p$ channel uses for the training and $\tau_u = \tau_c - \tau_p$ channel uses for the data transmission. Let $h_{mk} \in \mathbb{C}^N$ denote the channel between AP $m$ and UE $k$. We assume $h_{mk}$ is an independent random variable for every AP $m$-UE $k$ pair and $h_{mk}$ in different coherence blocks are independent and identically distributed (i.i.d.). We consider the spatially correlated Rician fading...
channel which is composed of a semi-deterministic LoS path component and a stochastic NLoS path component as

\[ h_{mk} = \Phi_{mk} \bar{h}_{mk} + g_{mk}, \]

where \( g_{mk} \sim \mathcal{CN}(0, R_{mk}) \) is the NLoS component and \( R_{mk} \in \mathbb{C}^{N \times N} \) is the spatial correlation matrix. \( \bar{h}_{mk} \) is the large-scale fading coefficient for the NLoS propagation. \( \bar{h}_{mk} \in \mathbb{C}^{N} \) represents the deterministic LoS component. Moreover, \( \Phi_{mk} = \mathrm{diag}(e^{j\varphi_{mk1}}, \ldots, e^{j\varphi_{mkN}}) \in \mathbb{C}^{N \times N} \) is the additional phase-shift of the LoS component between the \( n \)-th antenna of AP \( m \) and UE \( k \).

Remark 1. We notice that (1) is a multi-antenna generalization of (11) and an extension of [12] to spatially correlated Rician fading channels.

Remark 2. We can treat each \( N \)-antenna AP as a cluster of \( N \) single-antenna APs only if the channel coefficients to the \( N \) antennas of an AP are independently distributed.

A. Channel Estimation

We use \( \tau_k \) mutually orthogonal pilot sequences for channel estimation. \( \Phi_k \in \mathbb{C}^{\tau \times \tau} \) denotes the pilot sequence of UE \( k \), with \( \| \Phi_k \|^2 = \tau_k \). Notice that \( K > \tau \), so more than one UE use the same pilot sequence. We define \( P_k \) as the index subset of UEs that use the same pilot sequence as UE \( k \) including itself. The received signal \( y_{mk}^p \in \mathbb{C}^{N \times \tau_p} \) at AP \( m \) is given by

\[ y_{mk}^p = \sum_{k=1}^{K} \sqrt{p_k} h_{mk} \Phi_k^T + n_{mk}^p, \]

where \( p_k \) is the pilot transmit power of UE \( k \), \( n_{mk}^p \in \mathbb{C}^{N \times \tau_p} \) is additive noise with independent \( \mathcal{CN}(0, \sigma^2) \) entries, and \( \sigma^2 \) is the noise power. In order to estimate \( h_{mk} \), AP \( m \) multiplies \( y_{mk}^p \) with pilot sequence of UE \( k \) to obtain \( y_{mk}^p = y_{mk}^p \Phi_k^T \) as

\[ y_{mk}^p = \sqrt{p_k} \tau_k h_{mk} + \sum_{l \in P_k \setminus \{k\}} \sqrt{p_l} h_{ml} \Phi_l^T + n_{mk}^p \Phi_k. \]

Based on (3), we can derive three useful channel estimators with different prior information. We will focus on the effects of phase-shifts and spatial correlation matrices in the following.

1) Phase-Aware MMSE Estimator: If \( h_{mk}, R_{mk} \) and \( \varphi_{mk} \) are available for AP \( m \), we can derive the phase-aware MMSE estimate of \( h_{mk} \) as

\[ \hat{h}_{mk}^\text{mmse} = \hat{h}_{mk} e^{j\varphi_{mk}} + \sqrt{p_k} \tau_k \hat{R}_{mk} \Psi_{mk}^{-1} (y_{mk}^p - \tilde{y}_{mk}^p), \]

where \( \tilde{y}_{mk}^p = \sum_{l \in P_k} \sqrt{p_l} \tau_l h_{ml} e^{j\varphi_{ml}} \) and \( \Psi_{mk} = \sum_{l \in P_k} \sqrt{p_l} \tau_l R_{ml} + \sigma^2 I_N \), \( \varphi_{mk} \) and \( \tilde{y}_{mk}^p \) change in every coherence block so that (4) is a single realization. The channel estimate \( \hat{h}_{mk}^\text{mmse} \) and estimation error \( \hat{h}_{mk}^\text{mmse} - h_{mk} \) are independent random variables with

\[ \mathbb{E} \left\{ \hat{h}_{mk}^\text{mmse} | \varphi_{mk} \right\} = \hat{h}_{mk} e^{j\varphi_{mk}}, \quad \mathbb{Cov} \left\{ \hat{h}_{mk}^\text{mmse} | \varphi_{mk} \right\} = \hat{p}_k \tau_k \Omega_{mk}, \]

\[ \mathbb{E} \left\{ \hat{h}_{mk}^\text{mmse} \right\} = 0, \quad \mathbb{Cov} \left\{ \hat{h}_{mk}^\text{mmse} \right\} = C_{mmse}. \]

where \( \Omega_{mk} = R_{mk} \Psi_{mk}^{-1} R_{mk} \) and \( C_{mmse} = R_{mk} - \hat{p}_k \tau_k R_{mk} \Psi_{mk}^{-1} R_{mk} \).

2) Phase-Aware EW-MMSE Estimator: If \( h_{mk}, \varphi_{mk} \) and the diagonals of \( R_{mk} \) are available for AP \( m \), we can obtain the phase-aware EW-MMSE estimation of \( h_{mk} \) as

\[ \hat{h}_{mk}^\text{ew} = \hat{h}_{mk} e^{j\varphi_{mk}} + \sqrt{p_k} \tau_k D_{mk} \Lambda_{mk}^{-1} (y_{mk}^p - \tilde{y}_{mk}^p), \]

where \( D_{mk} \triangleq \mathbb{E} \left\{ | \hat{R}_{mk} |_{nn} : n = 1, \ldots, N \right\} \) and \( \Lambda_{mk} \triangleq \mathbb{E} \left\{ | \hat{\Psi}_{mk} |_{nn} : n = 1, \ldots, N \right\} \). The channel estimate \( \hat{h}_{mk}^\text{ew} \) and estimation error \( \hat{h}_{mk}^\text{ew} - h_{mk} \) are correlated random variables with

\[ \mathbb{E} \left\{ \hat{h}_{mk}^\text{ew} | \varphi_{mk} \right\} = \hat{h}_{mk} e^{j\varphi_{mk}}, \quad \mathbb{Cov} \left\{ \hat{h}_{mk}^\text{ew} | \varphi_{mk} \right\} = \Sigma_{mk}, \]

\[ \mathbb{E} \left\{ \hat{h}_{mk}^\text{ew} \right\} = 0, \quad \mathbb{Cov} \left\{ \hat{h}_{mk}^\text{ew} \right\} = C_{ew}, \]

where \( \Sigma_{mk} \triangleq \hat{p}_k \tau_k D_{mk} \Lambda_{mk}^{-1} \hat{\Psi}_{mk} \Lambda_{mk}^{-1} D_{mk} \) and \( C_{ew} \triangleq \hat{R}_{mk} - \hat{p}_k \tau_k (R_{mk} \Lambda_{mk}^{-1} D_{mk} - D_{mk} \Lambda_{mk}^{-1} R_{mk}) + \Sigma_{mk} \).

3) LMMSE Estimator: If \( h_{mk} \) and \( R_{mk} \) are available and the phase-shift \( \varphi_{mk} \) is unknown at AP \( m \), the LMMSE estimate of \( h_{mk} \) is

\[ \hat{h}_{mk}^\text{lmmse} = \sqrt{\hat{p}_k R_{mk} (\Psi_{mk}^{-1} \Sigma_{mk} + \hat{R}_{mk})^{-1}}, \]

where \( R_{mk} \triangleq R_{mk} + \hat{h}_{mk} \hat{h}_{mk}^H \) and \( \Psi_{mk} \triangleq \sum_{l \in P_k} \hat{p}_l \tau_l R_{ml} + \sigma^2 I_N \). The channel estimate \( \hat{h}_{mk}^\text{lmmse} \) and estimation error \( \hat{h}_{mk}^\text{lmmse} - h_{mk} \) are uncorrelated random variables with

\[ \mathbb{E} \left\{ \hat{h}_{mk}^\text{lmmse} \right\} = 0, \quad \mathbb{Cov} \left\{ \hat{h}_{mk}^\text{lmmse} \right\} = \hat{p}_k \tau_k \hat{\Omega}_{mk}, \]

\[ \mathbb{E} \left\{ \hat{h}_{mk}^\text{lmmse} \right\} = 0, \quad \mathbb{Cov} \left\{ \hat{h}_{mk}^\text{lmmse} \right\} = C_{\text{lmmse}}, \]

where \( \Omega_{mk} = R_{mk} (\Psi_{mk}^{-1} \Sigma_{mk} + \hat{R}_{mk})^{-1} \hat{R}_{mk} \) and \( C_{\text{lmmse}} = \hat{R}_{mk} - \hat{p}_k \tau_k R_{mk} (\Psi_{mk}^{-1} \Sigma_{mk} + \hat{R}_{mk})^{-1} \hat{R}_{mk} \).

B. UL Data Transmission

In the UL, all UEs simultaneously send \( \tau_u \) UL data symbols per coherence block to the APs. The received signal \( y_{mk} \in \mathbb{C}^{N} \) at AP \( m \) is

\[ y_{mk} = \sum_{k=1}^{K} h_{mk} s_k + n_{mk}^u, \]

where \( s_k \sim \mathcal{CN}(0, p_k) \) is the UL signal transmitted by UE \( k \) with power \( p_k = \mathbb{E} \{ |s_k|^2 \} \) and \( n_{mk}^u \sim \mathcal{CN}(0, \sigma^2 I_N) \) is the independent noise. Every AP can detect the UL data locally with a receive combining vector. Let \( \nu_{mk} \) denote the combining vector designed by AP \( m \) for UE \( k \) and the local estimate of \( s_k \) in AP \( m \) is given by

\[ \tilde{s}_{mk} = \nu_{mk}^H h_{mk} s_k + \sum_{l=1, \ell \neq k}^{K} \nu_{ml}^H h_{ml} s_l + \nu_{mk}^H n_{mk}^u. \]
MMSE, EWM-MMSE and LMMSE estimators, respectively, and L-MMSE combining as
\[ \mathbf{v}_{mk} = p_k \left( \sum_{l=1}^{K} p_l \left( \tilde{\mathbf{h}}_{ml}^H \left( \tilde{\mathbf{h}}_{ml}^H + \mathbf{C}_{ml} \right) + \sigma^2 \mathbf{I}_N \right)^{-1} \right) \mathbf{h}_{mk}^H. \]
Note that (9) is optimal for the MMSE and LMMSE estimators since it can minimize MSE
\[ \mathbf{v}_{mk} = E\{ |s_k - \mathbf{v}_{mk}^H \mathbf{v}_m|^2 \} \{ \tilde{\mathbf{h}}_{mk} \}, \]
but suboptimal for the EW-MMSE estimator.

To further mitigate the inter-user interference, the local estimates \( \{ \mathbf{s}_{mk} : m = 1, \ldots, M \} \) are sent to the CPU where they are linearly weighted by the LSFD coefficients to derive \( \hat{s}_k = \sum_{m=1}^{M} a_{mk} s_{mk} \) as
\[ \hat{s}_k = a_k^H \mathbf{b}_{kk} s_k + \sum_{l=1,l \neq k}^{K} a_k^H \mathbf{b}_{kl} s_l + n_k, \]
where \( a_k = [\alpha_{k1}, \ldots, \alpha_{kM}]^T \in \mathbb{C}^M \) is the LSFD coefficient vector, \( \mathbf{b}_{kl} = [v_{h1}^H, \ldots, v_{hM}^H]^T \in \mathbb{C}^M \), and \( n_k = \sum_{m=1}^{M} a_{km}^* v_{hkl}^H \mathbf{n}_{ml} \) respectively.

### III. Spectral Efficiency Analysis

In this section, we study the UL SE of CF mMIMO with different estimators and combining schemes. Based on (10), an achievable SE of UE \( k \) is
\[ \text{SE}_k = \frac{\tau_k}{T_r} \log_2 (1 + \gamma_k) \]
with the effective SINR \( \gamma_k \) given by
\[ \gamma_k = \frac{p_k |a_k|^2 E\{ \mathbf{b}_{kk}^H \mathbf{b}_{kk} \}}{\mathbf{a}_k^H \left( \sum_{l=1}^{K} p_l \mathbf{G}_kl - p_k E\{ \mathbf{b}_{kl}^H \mathbf{b}_{kl} \} \right) \mathbf{a}_k + \sigma^2 \mathbf{Z}_k}, \]
where \( \mathbf{G}_kl = E\{ \mathbf{v}_{ml}^H \mathbf{h}_{ml}^H \mathbf{v}_{ml} \mathbf{h}_{ml} \} : \forall m, m' \in \mathbb{C}^{M \times M} \) and \( \mathbf{Z}_k = \text{diag}(E\{ |\mathbf{v}_{kl}|^2 \}, \ldots, E\{ |\mathbf{v}_{km}|^2 \}) \in \mathbb{R}^{M \times M} \). The expectations are with respect to all sources of randomness [6].

Note that we use the use-and-then-forget (UatF) bound as [11] which serves a lower bound of the UL ergodic channel capacity of UE \( k \) [15]. To maximize the effective SINR in (12), \( a_k \) can be optimized by the CPU as
\[ a_k = \left( \sum_{l=1}^{K} p_l \mathbf{G}_kl - p_k E\{ \mathbf{b}_{kl}^H \mathbf{b}_{kl} \} \right)^{-1} \mathbf{E}\{ \mathbf{b}_{kk}^H \mathbf{b}_{kk} \}, \]
which leads to the maximum SE value
\[ \text{SE}_k = \frac{\tau_k}{T_r} \log_2 (1 + p_k E\{ \mathbf{b}_{kk}^H \mathbf{b}_{kk} \} a_k). \]
The proof of (13) follows from [15, Lemma B.10] since (13) is a generalized Rayleigh quotient with respect to \( a_k \) with a rank-one numerator.

Closed-form SE expressions cannot be obtained when using L-MMSE combining, while Monte Carlo simulations are used to compute the SE with L-MMSE combining. However, we can derive closed-form SE expressions if MR combining adopted. Closed-form SE expressions with different estimators can be similarly formed as SE
\[ \text{SE}_k = \frac{\tau_k}{T_r} \log_2 (1 + \gamma_k) \]
with \( \gamma_k \) shown as [15], where \( b_k = E\{ \mathbf{b}_{kk} \} \in \mathbb{C}^M \) and \( \mathbf{Z}_k = \text{diag}(E\{ |\mathbf{h}_{k1}|^2 \}, \ldots, E\{ |\mathbf{h}_{kM}|^2 \}) \). We define \( \mathbf{G}_kl = \sum_{l=1}^{K} p_l \mathbf{G}_kl^1 + \sum_{l \in \mathcal{P}_k} p_l \mathbf{G}_kl^2 - p_k b_k b_k^H \in \mathbb{C}^{M \times M} \). The SE with maximizing LSFD vector \( a_k = (\mathbf{G}_kl)^{-1} b_k \) is given by
\[ \text{SE}_k = \frac{\tau_k}{T_r} \log_2 \left( 1 + p_k b_k^H (\mathbf{G}_kl)^{-1} b_k \right). \]

### A. SE with the Phase-Aware MMSE Estimator

For MR combining based on the phase-aware MMSE estimator \( \mathbf{v}_{mk} = \hat{\mathbf{h}}_{mk}^\text{mmse} \), we have \( |Z_{mk}^\text{mmse}|_m = \text{tr}(\hat{\mathbf{m}}_{mk}^\text{mmse}) \) and \( b_k^\text{mmse} = \text{diag}(\mathbf{b}_k^\text{mmse}) \). And \( \Gamma_{kl}^\text{mmse}(1) \in \mathbb{C}^{M \times M} \) is a diagonal matrix with the \((m, m)\)-th element given by
\[ \Gamma_{kl}^\text{mmse}(1)_{mm} = \frac{\hat{\mathbf{m}}_{mk}^\text{mmse} \text{tr}(\mathbf{R}_{mk} \mathbf{m}_{mk}^\text{mmse}) + \hat{\mathbf{h}}_{mk}^\text{mmse} \mathbf{R}_{ml} \hat{\mathbf{h}}_{mk}^\text{mmse}}{\hat{\mathbf{m}}_{mk}^\text{mmse} \mathbf{R}_{ml} \hat{\mathbf{m}}_{mk}^\text{mmse} + \hat{\mathbf{h}}_{mk}^\text{mmse} \mathbf{R}_{ml} \hat{\mathbf{m}}_{mk}^\text{mmse}}. \]

The computation of above results follow similar steps as [11] and [13]. Moreover,
\[ \Gamma_{kl}^\text{mmse}(2) = \frac{\hat{\mathbf{m}}_{mk}^\text{mmse} \mathbf{R}_{mk} \mathbf{m}_{mk}^\text{mmse}}{\hat{\mathbf{m}}_{mk}^\text{mmse} \mathbf{R}_{ml} \hat{\mathbf{m}}_{mk}^\text{mmse} + \hat{\mathbf{h}}_{mk}^\text{mmse} \mathbf{R}_{ml} \hat{\mathbf{m}}_{mk}^\text{mmse}}. \]

### B. SE with the Phase-Aware EW-MMSE Estimator

If MR combining based on the phase-aware EW-MMSE estimator \( \mathbf{v}_{mk} = \hat{\mathbf{h}}_{mk}^\text{ew} \) is adopted, \( |Z_{mk}^\text{ew}|_m = \text{tr}(\hat{\mathbf{m}}_{mk}^\text{ew}) \) and \( b_k^\text{ew} = \hat{p}_k \hat{\mathbf{m}}_{mk}^\text{ew} \text{tr}(\mathbf{D}_{mk} \mathbf{A}_{mk}^{-1} \mathbf{D}_{mk}) + |\hat{\mathbf{m}}_{mk}|^2 \). So \( \Gamma_{kl}^\text{ew}(1) \in \mathbb{C}^{M \times M} \) is a diagonal matrix with the \((m, m)\)-th element given by
\[ \Gamma_{kl}^\text{ew}(1)_{mm} = \frac{\text{tr}(\mathbf{D}_{mk} \mathbf{A}_{mk}^{-1} \mathbf{D}_{mk}) + |\hat{\mathbf{m}}_{mk}|^2}{|\hat{\mathbf{m}}_{mk}|^2} + \frac{\text{tr}(\mathbf{R}_{mk} \mathbf{m}_{mk}^\text{ew})}{|\hat{\mathbf{m}}_{mk}|^2} \hat{\mathbf{m}}_{mk}^\text{ew} \mathbf{R}_{ml} \hat{\mathbf{m}}_{mk}^\text{ew}. \]

### C. SE with the LMMSE Estimator

If we use MR combining based on the LMMSE estimator \( \mathbf{v}_{mk} = \hat{\mathbf{h}}_{mk}^\text{lmse} \), we have \( |Z_{mk}^\text{lmse}|_m = \hat{p}_k \mathbf{m}_{mk}^\text{lmse} \). Moreover, \( \Gamma_{kl}^\text{lmse}(1) \in \mathbb{C}^{M \times M} \) is a diagonal matrix with the \((m, m)\)-th element given by
\[ \Gamma_{kl}^\text{lmse}(1)_{mm} = \frac{\hat{p}_k \mathbf{m}_{mk}^\text{lmse} \text{tr}(\mathbf{R}_{mk} \mathbf{m}_{mk}^\text{lmse})}{|\hat{\mathbf{m}}_{mk}|^2}. \]
\[
\gamma_k^i = \frac{p_k a_k^H b_k^i (b_k^i)^H a_k}{\sum_{l=1}^K p_l \gamma_{kl}^{i(1)} + \sum_{l \in P_k} p_l \gamma_{kl}^{i(2)} - p_k b_k^i (b_k^i)^H + \sigma^2 Z_k^i},
\]

\[\left[\mathcal{Y}_{kl}^{(1)}\right]_{mm} = \frac{\tilde{p}_k \tilde{p}_l \tau_p^2}{\text{tr} \left( \left( \mathbb{T}_{ml}^{(1)} \right)^\frac{1}{2} \mathbb{R}_{ml}^2 \right)} + \text{tr} \left( \mathbb{R}_{ml} \mathbb{T}_{ml}^{(1)} \right) + \mathbb{H}_{ml}^H \mathbb{T}_{ml}^{(1)} \mathbb{H}_{ml} + \mathbb{H}_{ml}^H \mathbb{S}_{mk}^H \mathbb{R}_{ml} \mathbb{S}_{mk} \mathbb{H}_{ml} + \left[ \mathbb{H}_{ml}^H \mathbb{S}_{mk} \mathbb{H}_{ml} \right]^2 \]

\[+ 2 \text{Re} \left\{ \text{tr} \left( \left( \mathbb{T}_{ml}^{(1)} \right)^\frac{1}{2} \mathbb{R}_{ml}^2 \right) \mathbb{H}_{ml}^H \mathbb{S}_{mk} \mathbb{H}_{ml} \right\} \]

\[= \frac{\tilde{p}_k \tilde{p}_l \tau_p^2}{\text{tr} \left( \left( \mathbb{T}_{ml}^{(1)} \right)^\frac{1}{2} \mathbb{R}_{ml}^2 \right)} + \text{tr} \left( \mathbb{R}_{ml} \mathbb{T}_{ml}^{(1)} \right) + \mathbb{H}_{ml}^H \mathbb{T}_{ml}^{(1)} \mathbb{H}_{ml} - \left[ \mathbb{H}_{ml}^H \mathbb{S}_{mk}^H \mathbb{H}_{ml} \right]^2.
\]

Besides, we can obtain \(\mathbf{Y}_{kl}^{\text{immsse}}\) as

\[
\mathbf{Y}_{kl}^{\text{immsse},(2)} = \mathbf{Y}_{kl}^{(1)} + d_{kl}^{\text{immsse}} (d_{kl}^{\text{immsse}})^H - \mathbf{Y}_{kl}^{(2)},
\]

where \(\mathbf{Y}_{kl}^{(1)} \in \mathbb{C}^{M \times M}, \mathbf{Y}_{kl}^{(2)} \in \mathbb{C}^{M \times M}\) are diagonal matrices and \(d_{kl}^{\text{immsse}} = \text{diag}\{\mathbf{(Y}_{kl}^{(2)})^2\}\). The \((m, m)\)-th element of \(\mathbf{Y}_{kl}^{(1)}\) is given by (23), where \(\mathbf{S}_{mk} = \mathbf{R}_{mk}^H \mathbf{H}_{mk}^{-1} \mathbf{T}_{mk(l)} \mathbf{S}_{mk}^H\) and \(\mathbf{R}_{mk} = \mathbf{R}_{mk}^H \mathbf{H}_{mk}^{-1} \mathbf{T}_{mk(l)} \mathbf{R}_{mk}^H \mathbf{S}_{mk}^H\), respectively. And the \((m, m)\)-th element of \(\mathbf{Y}_{kl}^{(2)}\) is

\[
\left[\mathbf{Y}_{kl}^{(2)}\right]_{mm} = \frac{\tilde{p}_k \tilde{p}_l \tau_p^2 \text{tr} \left( \mathbb{R}_{ml}^\dagger \mathbb{R}_{mk}^\dagger \left( \mathbf{H}_{mk}^{-1} \right)^2 \right)}{\text{tr} \left( \left( \mathbb{T}_{ml}^{(1)} \right)^\frac{1}{2} \mathbb{R}_{ml}^2 \right)}.
\]

So \(\mathbf{Y}_{kl}^{\text{immsse}} = \sum_{k=1}^K p_k \mathbf{Y}_{kl}^{\text{immsse},(1)} + \sum_{l \in P_k} p_l \mathbf{Y}_{kl}^{\text{immsse},(2)} - p_k b_k (b_k^H)^H + \sigma^2 \mathbf{Z}_{kl}^{(i)}\) and we can obtain the closed-form SE expression based on the LMMSE estimator from (16).

Note that the phase-aware MME estimator achieves better SE than other estimators since it makes use of prior phase knowledge, which will be demonstrated in Section IV.

IV. Numerical Results

We consider APs and UEs are uniformly distributed in a \(1 \times 1\) km\(^2\) area with a wrap-around scheme [15]. All AP-UE pairs have LoS paths and the pathloss is computed by the COST 321 Walfish-Ikegami model as

\[
\beta_{mk} [\text{dB}] = -30.18 - 26 \log_{10} \left( \frac{d_{mk}}{1 \text{m}} \right) + F_{mk},
\]

where \(d_{mk}\) is the distance between AP \(m\) and UE \(k\) taking (11 m) as the reference distance into account. The Rician \(\kappa\)-factor is computed as \(\kappa_{mk} = 10.13 - 0.003d_{mk}\). We model the shadowing fading \(F_{mk}\) as in [11] with \(F_{mk} = \sqrt{\delta_f a_m + 1 - \delta_f b_k}\), where \(a_m \sim N(0, \sigma^2)\) and \(b_k \sim N(0, \sigma^2)\) are independent random variables and \(\delta_f\) is the shadow fading parameter. The covariance functions of \(a_m\) and \(b_k\) are \(E\{a_m a_m\} = 2 \frac{d_{mk}}{d_{mc}}\), \(E\{b_k b_k\} = 2 \frac{d_{mk}}{d_{mc}}\) where \(d_{mm'}\) and \(d_{kk'}\) are the geographical distances between AP \(m\)-AP \(m'\) and UE \(k\)-UE \(k'\), respectively, \(d_{mk}\) is the decorrelation distance depending on the environment. Let \(\delta_f = 0.5, d_{mc} = 100\ m\) and \(\delta_f = 8\) in this paper. The large-scale coefficients of \(h_{mk}\) are given by

\[
\beta_{\text{LoS}} = \frac{\kappa_{mk}}{\kappa_{mk} + 1} \beta_{mk}, \quad \beta_{\text{NLoS}} = \frac{1}{\kappa_{mk} + 1} \beta_{mk}.
\]

Each AP is equipped with a uniform linear array (ULA) with omnidirectional antennas so the \(n\)-th element of the deterministic LoS component \(\tilde{h}_{mk,n} \in \mathbb{C}\) can be written as

\[
\left[\mathcal{H}_{mk}\right]_{n} = \sqrt{\beta_{\text{LoS}} e^{2 \pi d_i (n-1) \sin(\theta_{mk})}}, \quad \text{where } \theta_{mk} \text{ is the angle of arrival to the UE } k \text{ seen from AP } m \text{ and } d_i \text{ denotes the antenna spacing parameter (in fractions of the wavelength).}
\]

\[
\begin{array}{c}
\text{Fig. 1. Average SE against the number of APs } M \text{ with different combining schemes for } K = 40, N = \{1, 2, 4\} \text{ and } \sigma_c = 15^\circ.
\end{array}
\]

The spatial correlation matrix \(\mathbf{R}_{mk}\) is generated based on the Gaussian local scattering model [15]. The \((i, n)\)-th element of \(\mathbf{R}_{mk}\) is given by

\[
\left[\mathbf{R}_{mk}\right]_{in} = \frac{\beta_{\text{LoS}}}{\sqrt{2 \pi \sigma_c}} \int_{-\infty}^{+\infty} e^{2 \pi d_i (1-n) \sin(\theta_{mk} + \delta)} e^{-\frac{(\theta_{mk} + \delta)^2}{2 \pi \sigma_c^2}} d\theta.
\]

where \(\delta \sim N(0, \sigma_c^2)\) is a Gaussian distributed deviation from \(\theta_{mk}\) with angular standard deviation (ASD) \(\sigma_c\). All the UEs transmit with power 200 mW, the bandwidth is 20 MHz, the noise power \(\sigma^2 = -94\ dBm\), and every coherence block contains \(\tau_c = 200\) channel uses where \(\tau_p = 10\) channel uses are reserved for pilot transmission.

Figure 1 shows the UL SE averaged over random UE locations and shadow fading realizations as a function of the number of APs \(M\) for different \(N\) with MR/L-MMSE combining based on the MMSE estimator. The average SE grows with \(N\), e.g., 92.02% improvement with MR combining for \(N = 4, M = 100\) compared with the \(N = 1, M = 100\) scenario. Moreover, L-MMSE combining performs much better than MR combining, e.g., 27.78% SE improvement for L-MMSE combining compared with that of MR combining for \(N = 4, M = 100\). The performance gap between L-MMSE and MR combining becomes larger with the increase of \(N\) since L-MMSE combining can use all antennas on each AP to suppress interference, which means that L-MMSE combining should be advocated in the scenario with multi-antenna APs.

Figure 2 shows the cumulative distribution function (CDF) curves for the SE per UE over spatially correlated/uncorrelated Rician fading channels with MR/L-MMSE combining based on the MMSE estimator. The spatial channel correlation increases as \(\sigma_c\) reduces. Let \(\sigma_c = 5^\circ/30^\circ\) represent strong/moderate spatial correlation, respectively, and \(\mathbf{R}_{mk} = \beta_{\text{LoS}} \mathbf{S}_{N}^H \mathbf{S}_{N}\) is diagonal in the uncorrelated fading scenario. Note that the SE benefits from the spatial correlation since the spa-
For MR/L-MMSE combining, respectively, for $M$ combining. It is important to find that L-MMSE combining performs much better than MR combining in multi-antenna APs scenarios. Moreover, the SE grows with the number of antennas per AP and benefits from the spatial correlation. Finally, the MMSE estimator achieves the best performance and the SE of the LMMSE estimator is lower than the one of other estimators due to the lack of phase-shift knowledge. In the future work, we will consider multi-antenna UEs with uplink precoding, power control, and algorithmic scalability to enable implementation of large CF mMIMO networks.

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