RESEARCH ON ESTABLISHING THE DIFFERENTIAL MODEL OF THE ROBOTIC MECHANISMS

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ABSTRACT
The paper presents the stages of realization of a computer program that allows the establishment of the differential model of the active mechanisms from the component of industrial robots. The computer program was developed using the Maple programming language. Finally, a series of simulation results are presented in the case of an active mechanism of an industrial robot with six degrees of freedom.

Keywords: industrial robot, active mechanism, differential model

INTRODUCTION
Within the realization of various mechanisms, the development of computational algorithms based on the study models and their transposition into computer programs are extremely important stages in their design process [1-16]. In this context, the development of the computer programs for the simulation of the functioning of the industrial robots in order to increase their performances is part of a very topical issue in the field of Robotics [17-20]. The paper presents the stages of realization of a computer program that allows the establishment of the differential model of the active mechanisms from the component of industrial robots. The computer program was developed using the Maple programming language. A series of simulation results are presented in the case of an active mechanism of an industrial robot with six degrees of freedom.

THEORETICAL CONSIDERATIONS AND SIMULATION RESULTS
The differential model of the active mechanisms from the component of industrial robots is expressed by the following equation [17,18]:

\[ dx = J \cdot dq \]  

(1)

where \( x \) represents the vector that contains the operational coordinates:

\[ x = \begin{bmatrix} x_{poz} \\ x_{rot} \end{bmatrix} \]  

(2)

\( x_{poz} \) and \( x_{rot} \) are the vectors that define the position and the orientation of the coordinate system \( (O_T,x_T,y_T,z_T) \) attached between the gripper fingers;
\[ q = \left[ q_1 \ q_2 \ \ldots \ q_n \right]^T \]  

is the vector that contains the coordinates: \( q_1, q_2, \ldots, q_n \) corresponding to the active joints of the robot;

\( J \) represents the Jacobean matrix which achieves the link between the variations of the operational coordinates and those of the coordinates corresponding to the active joints and may be calculated with the following relation [17]:

\[ J = \begin{bmatrix} R_{pos} & 0 & 0 \\ 0 & R_{rot} \end{bmatrix} J_T \]

where:  
\( R_{pos} \) depends on the type of coordinates used to define the position of the component modules. When cartesian coordinates are used: \( R_{pos} = I_3 \), where \( I_3 \) is the unit matrix of rank three; \( R_{rot} \) depends on the type of coordinates used to define the orientation of the coordinate system \((O_Tx_Ty_Tz_T)\);

\[ J_T = \begin{bmatrix} 0 & R_T & 0 \\ 0 & 0 & R_T \end{bmatrix} J_T \]

where:  
\( R_T \) is the rotation matrix corresponding to relative orientation between the coordinate system \((O_Tx_Ty_Tz_T)\) and the fixed system of coordinates \((O_0x_0y_0z_0)\).

\[ 0 = 0^{T}\rightarrow^{T} R_T \ldots^{n-1} R_T = 0^{n} R_T \]

where:  
\( R_{i+1}, i = 0, n-1, \) are the rotation matrices corresponding to relative orientation between the systems of coordinates \((O_{i}x_{i}y_{i}z_{i})\) and \((O_{i+1}x_{i+1}y_{i+1}z_{i+1})\) attached to the component modules \(i\) and \(i+1\);  
\( R_T \) is the rotation matrix corresponding to the relative orientation between the systems of coordinates \((O_nx_ny_nz_n)\) attached to the last module of the robot and \((O_Tx_Ty_Tz_T)\). The column \( k \) of the matrix \( J_T \) of dimensions \((6x n)\) when a type of parameterization of the robot structure is used has the following expression [17]:

\[
\begin{bmatrix}
\sigma_k \mathcal{R}_k^{(k)} k_k + \overline{\sigma}_k \mathcal{R}_k^{(k)} k_k \mathcal{R}_k^{(k)} O_k O_T \\
\overline{\sigma}_k \mathcal{R}_k^{(k)} k_k
\end{bmatrix}
\]

where:  
\( \sigma_k = 0 \) when the module \( k \) is of rotation and \( \sigma_k = 1 \) when the module \( k \) is of translation;  
\( \overline{\sigma}_k = 1 - \sigma_k \);  
\( \mathcal{R}_k = (R_T)^T \) is the rotation matrix corresponding to relative orientation between the systems of coordinates \((O_Tx_Ty_Tz_T)\) and \((O_kx_ky_kz_k)\) attached to the module \( k \) of the robot;

\[ k = k_{i+1} \ldots k_{i+2} \ldots^{n-1} R_T = n R_T \]

\[ (k) k_k = [0 \ 0 \ 1]^T \]
The vector \((k)O_kO_T\) may be extracted from the homogeneous transformation matrix \(\dot{T}_T\), where:

\[
k_T = \begin{bmatrix}
R_k & (k)O_kO_T \\
0 & 1
\end{bmatrix}
k_{k+1} \cdot k_{k+2} \cdot \ldots \cdot k_1 \cdot k = \begin{bmatrix}
0 & -1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

The vector \((k)O_kO_T\) may be extracted from the homogeneous transformation matrix \(\dot{T}_T\), where:

\[
k_T = \begin{bmatrix}
R_k & (k)O_kO_T \\
0 & 1
\end{bmatrix}
k_{k+1} \cdot k_{k+2} \cdot \ldots \cdot k_1 \cdot k = \begin{bmatrix}
0 & -1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

For realization the computer program that allows the establishment of the differential model of the active mechanisms from the component of industrial robots it was used the parameterization presented in [17,18]. In this case the parameters used to establish the relative position and orientation between the coordinates systems attached to the consecutive modules \(i\) and \(i+1\) are presented in Figure 1.

![Figure 1. Parameters used to establish the relative position and orientation between the consecutive modules \(i\) and \(i+1\) [17,18]](image)

When using this parameterization the homogeneous transformation matrix \(\dot{T}_{i+1}\) has the following expression [17,18]:

\[
\dot{T}_{i+1} = \begin{bmatrix}
R_{i+1} & (i)O_iO_{i+1} \\
0 & 1
\end{bmatrix} = \\
\begin{bmatrix}
\cos \theta_{i+1} & -\sin \theta_{i+1} & 0 & d_{i+1} \\
\cos \alpha_{i+1} \cdot \sin \theta_{i+1} & \cos \alpha_{i+1} \cdot \cos \theta_{i+1} & -\sin \alpha_{i+1} & -r_{i+1} \cdot \sin \alpha_{i+1} \\
\sin \alpha_{i+1} \cdot \cos \theta_{i+1} & \sin \alpha_{i+1} \cdot \cos \theta_{i+1} & \cos \alpha_{i+1} & r_{i+1} \cdot \cos \alpha_{i+1} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
A computer program that allows the establishment of the differential model of the active mechanisms from the component of industrial robots was developed using the special capabilities of symbolic calculation of Maple programming language.

A series of simulation results are presented in the case of an active mechanism of an industrial robot with six degrees of freedom (Figure 2).

The values of the parameters that establish the relative position and orientation between the coordinates systems attached to the component modules are given in Table 1.

| $i$ | $\alpha_i$ | $d_i$ | $\theta_i$ | $r_i$  |
|-----|--------------|-------|------------|--------|
| 1   | 0            | 0     | $q_1$      | 0      |
| 2   | 0            | 0     | $q_2$      |        |
| 3   | $-90^\circ$  | 0     | $q_3$      |        |
| 4   | 0            | 0     | $q_4$      | $l_{34}$|
| 5   | $90^\circ$   | 0     | $q_5$      | 0      |
| 6   | $-90^\circ$  | 0     | $q_6$      | 0      |

In the sequel are presented the expressions obtained with the computer program for the elements of all six columns of the matrix $^TJ_T$:
CONCLUSIONS

In the paper a method for achieving the differential model of the active mechanisms from the component of industrial robots has been presented. In this scope, a computer program was developed using the special capabilities of symbolic calculation offered by Maple programming language. A series of simulation results have been presented in the case of an active mechanism of an industrial robot with six degrees of freedom.

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