Theoretical Analysis of a Water Hydraulic Internal Ball Gear Pump

Yousheng Yang1, 2, *, Feihu Yang1, a, Junqian Fan1, b and Richmond Polley Yankey1, c

1College of Engineering, Ocean University of China, Qingdao, China
2Science and Technology on Underwater Vehicle Technology of Harbin Engineering University, Harbin, China

*Corresponding author e-mail: hptc_yang@163.com, yfeihu@126.com, fanjunqian@163.com, Richmondpolley@gmail.com

Abstract. In view of current volumetric water hydraulic pumps, especially water hydraulic internal gear pumps, a scheme of an internal ball gear pump was proposed. Its working principle was expounded and the calculation methods of its performance parameters such as displacement, flow pulsation and maximum speed were studied. Compared and analysed the advantages and disadvantages of traditional displacement water hydraulic pump with ball gear pump, and its key basic technologies. The results show that (1) by engaging the concave/convex ball gears instead of the gears, it can realize the function of the pump; (2) the arrangement of multiple rows of radial staggered not only ensures the continuity and stability of the transmission, but also solves the phenomenon of water trapping; (3) displacement is proportional to the diameter of the ball (modulus) while flow pulsation is inversely proportional to the number of rows of ball gear. Also, the maximum speed is inversely proportional to the diameter of the concave ball gear and the density of the working medium, which is proportional to the inlet pressure of the ball tooth pump, etc... This study could lay a simple theoretical foundation for the development of high-performance water hydraulic internal ball gear pump.

1. Introduction
The hydraulic drive has the advantages of high power density, large output force/torque, convenient speed regulation, stable operation, small impact, fast start/brake/commutation, simple overload protection, etc... It has been widely used in underwater operation equipment and offshore platform [1, 2]. However, due to the incompatibility between the working medium (mineral oil) and the environmental medium (sea water), when working underwater, the traditional hydraulic drive technology has serious disadvantages. The problem of compatibility between the working medium and the environmental medium will be solved, if the underwater operation equipment is driven by water hydraulic, so as to overcome many defects of the oil hydraulic driving and pneumatic driving technologies, and greatly improve the performance of the underwater working equipment [3, 4]. However, the physical and chemical properties of water are quite different from those of mineral oil, which brings a series of technical problems [5, 6].
In view of the fact that water hydraulics is the best driving method for marine electromechanical equipment, the US Navy, Danfoss, Zhejiang University, Huazhong University of Science and Technology, Beijing University of Technology, Ocean University of China and other domestic and foreign research institutions have conducted a series of research and successfully developed products[7-9]. For example, the US Navy developed a series of seawater hydraulic components, underwater working tools and systems in the late 20th century [10]. Huazhong University of Science and Technology developed a seawater hydraulic power system and a variety of underwater operating tools at the beginning of this century [11]. At present, regardless of the type of structure, the following problems exist in the water hydraulic pump:

1. Trapped water phenomenon and the resulting problems of it, such as cavitation, vibration and noise.
2. There are many problems in water hydraulic piston pump, such as complicated structure, high material and processing cost, sensitivity to pollution, large flow pulsation and high noise, etc..
3. It is difficult to form effective lubrication between the friction pairs in the water hydraulic gear pump. In addition, the end face of the water hydraulic gear pump leaks greatly, because of the low viscosity of the water.

These problems restrict the performance of the water hydraulic pump. Therefore, in order to enhance the competitive advantage of water hydraulic technology, a new type of internal ball gear pump is proposed in this paper, and theoretically based on its meshing characteristics, deduce its performance parameter formula.

2. Structure and working principle

2.1. Composition and structure

Figure 1 shows the structure and principle of the water hydraulic internal ball gear pump. For simplicity, Figure 1 is a two-row internal ball gear pump. The pump is mainly composed of concave ball gear, convex ball ring, crescent block, front end cover, rear end cover, shaft, and eccentric shaft hole rotary seal, etc.

![Figure 1](image)

**Figure 1.** Structure and main friction pair of the water hydraulic internal ball gear pump.

2.2. Working principle

The pump is engaged by a concave/convex ball tooth instead of a gear, and the concave ball gear drives the convex ball ring rotation, as shown in Figure 1. The two closed cavities are separated by internal ball gear, crescent block, and eccentric shaft hole rotary seal at both ends. When the pump is working,
the ball teeth on one side of the low pressure chamber are continuously disengaged from meshing, so that the volume of the sealed working chamber is continuously increased to form a partial vacuum. The liquid enters the low pressure chamber under the action of atmospheric pressure and is carried by the continuously rotating ball teeth into the high pressure chamber on the other side. The ball teeth of the high pressure chamber continuously enter meshing, the volume of the sealed working chamber is continuously reduced, and the liquid is squeezed into the system. The liquid is brought from the low pressure chamber to the high pressure chamber to complete the water absorption and water pressure of the pump, through the continuous rotation of the ball gear.

2.3. Characteristics analysis of meshing

Although the water hydraulic internal ball gear pump can be borrowed from the water hydraulic internal gear pump, its meshing principle is different. In this paper, the single-row internal ball gear is taken as an example to analyze its meshing characteristics.

As shown in Figure 2a, the concave ball teeth are slider-on-disc contact with the convex ball teeth, and the slider-on-disc contact becomes line contact in the next moment. From Figure 2a to Figure 2c, the concave ball gear drives the convex ball ring to rotate, and the point C is always the meshing point in the process. From Figure 2c to Figure 2e, the two pitch circle points are the theoretical meshing points, but in fact the friction force is not enough to drive the ball ring gear to rotate, so the point C is the actual meshing point in the process. Until rotating to Figure 2f, point A becomes the meshing point, as shown in Figure 2f, at which point A, C are symmetrical about the longitudinal axis. After that, the cycle is repeated to achieve the cycle of the meshing process.

According to the meshing characteristic analysis, there is an obvious discontinuity in the convex ball ring in the single row internal ball gear pump. However, unlike the internal gear pump, the concave/convex ball teeth mesh with only one linear friction pair during the meshing process, so there is no water trapping phenomenon. When the number of ball gear rows increases to two rows and the ball gear teeth are radially staggered; assuming one of the rows is in Figure 2c, then, it will enter the discontinuous working phase, while the other row will enter the Figure 2e. The ball gear teeth continue to rotate and mesh, and the rear row engages the ball teeth to push the previous row to rotate, which cancels out the discontinuity of the previous row. Based on this feature, the pump is always in continuous operation, and its continuity and smoothness are significantly improved.
The new water hydraulic internal ball gear pump has the following features:

1. The meshing teeth are spheres and ball sockets.
2. The multi-row ball gear teeth are radially staggered, which not only optimizes the water trapping phenomenon of the hydraulic pump, but also ensures the continuity and stability of the hydraulic pump.
3. The eccentric shaft hole rotary seal greatly reduces the end gap leakage, and is expected to improve the volumetric efficiency and working pressure of the water hydraulic pump.
4. As shown in Figure 1, the pump has a variety of line (straight line and arc) contact rolling friction pairs: internal ball gear rolling pair between convex and concave ball gear teeth, crest rolling pair between convex ball gear teeth and crescent block, cylindrical rolling pair between concave and convex ball gear.

3. Performance parameter

3.1. Definition of structure size

As shown in Figure 3, the structure and geometric dimension of the water hydraulic internal ball gear pump are similar to conventional internal gear pumps. The structure naming and geometric dimension relationship are shown in Table 1.

![Figure 3. The structure of water hydraulic internal ball gear pump.](image-url)
Figure 4. The cross-sectional view of the protruding portion of one convex ball.

Table 1. The structure name and geometric dimension relationship of water hydraulic internal ball gear pump.

| Number | Name                                      | Code | Relationship       |
|--------|-------------------------------------------|------|--------------------|
| 2      | The radius of ball gear tooth             | r    | -                  |
| 3      | Number of concave ball gear teeth         | z₁   | -                  |
| 4      | Number of convex ball gear teeth          | z₂   | -                  |
| 5      | Pitch radius of concave ball gear         | r₁   | 0.5mz₁             |
| 6      | Pitch radius of convex ball gear          | r₂   | 0.5mz₂             |
| 7      | Whole depth                               | h    | r·cosα             |
| 8      | Normal center distance                    | e    | r₂ - r₁            |
| 9      | Reference circle radius of concave ball gear | R₁   | r₁ + r - h         |
| 10     | Reference circle radius of convex ball gear | R₂   | R₂ + r - h         |

Note: In order to prevent the ball from falling off, h<r in this design.

3.2. The analysis of performance parameter

3.2.1. Displacement. Based on the working principle, the displacement of the internal ball gear pump is the volume swept by the protruding portion of one convex ball gear teeth when the shaft rotates one revolution. The shaded portion in Figure 4 is a cross-sectional view of the protruding portion of one convex ball.

The displacement $V$ is:

$$ V = N \int_{0}^{2\pi z_2} A(x) R_{OC1} d\alpha = N \frac{z_1}{z_2} 2\pi \left( R_2 - x_c \right) A(x) \quad (1) $$

Where $x_c$ is the centroid to ball centroid distance of the shaded portion of the ball head in Figure 4; $A(x)$ is the area of the shaded area in equation (1). Substituting the geometric relationship into equation (1):
3.2.2. Flow pulsation. According to the relevant theory of conventional internal gear pump. In this paper, the single-row internal ball gear is taken as an example to derive its flow non-uniform coefficient $\delta Q$.

The flow non-uniform coefficient $\delta Q$ is as follows under the condition that the ball gear parameters are determined and the root cut is not considered.

$$\delta Q = \frac{Q_{sh\text{max}} - Q_{sh\text{min}}}{Q} = \frac{Q_{sh\text{max}} - Q_{sh\text{min}}}{nV}$$  \hspace{1cm} (3)

Where $Q_{sh\text{max}}$ is the maximum instantaneous flow rate, $Q_{sh\text{min}}$ is the minimum instantaneous flow rate, $Q$ is the average flow rate of the ball gear pump, $n$ is the rotation speed, and $V$ is the displacement.

Figure 5. The ball gear meshing schematic diagram.

Figure 6. The geometric relationship between the meshing point and the gear center.
Figure 5 shows the ball gear meshing schematic diagram of the water hydraulic internal ball gear pump, which can be obtained from the schematic diagram: $\omega_1 R_1 = \omega_2 R_2$. At some instant $dt$, the concave ball gear rotates through $d\phi_1$ while the convex ball ring rotates through $d\phi_2$, i.e., $d\phi_1 = \omega_1 dt$, $d\phi_2 = \omega_2 dt$. The relationship can be obtained by the above analysis:

$$d\phi_1 = \frac{R_1}{R_2} d\phi_2$$

The volume of liquid discharged by the ball gear pump in $dt$ is as follows:

$$dV = dV_1 + dV_2 = \frac{b}{2} \left[ (r_1^2 - r_n^2) + \frac{R_1}{R_2} (r_n^2 - r^2_3) \right] d\phi_1$$

$$Q_{sh} = \frac{dV}{dt} = \frac{r_1 \omega_1 \left[ R_1 (r_n^2 - r^2_3) + R_2 (r_1^2 - r_n^2) \right] \sin \alpha}{R_2}$$

Where $b$ is the width of the convex ball head. It can be seen from the geometric relationship in Figure 4, $b = 2rsina$. Figure 6 shows the geometric relationship between the meshing point and the gear center. $F$ is the distance between the meshing point $C$ and the node $P$.

$$Q_{sh} = r_1^2 - R_1^2 + R_1 R_2 - \frac{R_1}{R_2} r_3^2 - \left( 1 - \frac{R_1}{R_2} \right) f^2$$

Since the internal ball gear pump is a new type of power component. For $f$, this paper refers to the internal gear pump, i.e., $f = R_1 \phi_1$. Where $\phi_1$ is the corner of the concave ball gear.

$$Q_{sh} = r_1^2 - R_1^2 + R_1 R_2 - \frac{R_1}{R_2} r_3^2 - \left( 1 - \frac{R_1}{R_2} \right) \frac{R_2^2 \phi_1^2}{R_2}$$

It can be seen from equation (7) that $Q_{sh}$ has a maximum value when $f = 0$:

$$Q_{sh\text{max}} = r_1^2 - R_1^2 + R_1 R_2 - \frac{R_1}{R_2} r_3^2$$

$Q_{sh}$ has a minimum value when $f = \pm r [\phi_1 = \pm \pi/z_1]$: 


Substituting equations (9) and (10) into (3). The flow non-uniform coefficient $\delta_Q$ of the single-row water hydraulic internal ball gear pump can be obtained under the theoretical condition.

$$\delta_Q = \frac{Q_{sh\max} - Q_{sh\min}}{nV} = \frac{1}{3(mz_2 + 2r \cos \alpha)(2\pi - \sin 2\alpha) - 8r \sin \alpha} \cdot \frac{6mz_1(z_2 - z_1)}{nN \pi z_1 (mz_2 + 2r \cos \alpha)}$$

Professor Massimo Rundo of the University of Politecnico di Torino pointed out that increasing the number of gears on the shaft of the internal gear pump can not only increase the displacement of the pump but also reduce the flow pulsation. Similarly, increasing the number of rows of ball teeth can also reduce the flow pulsation of the internal ball gear pump.

3.2.3. Maximum speed. The performance of the pump is closely related to its speed. The excessive centrifugal force prevents the liquid from filling the ball gear teeth when the speed is too high, which reduces the flow rate and causes problems such as cavitation, noise and wear.

![Figure 7](image_url)

*Figure 7. The analysis of centrifugal force.*

Figure 7b is the enlarged view of $d\varepsilon$ in Figure 7a, the centrifugal force equation of water contained in the angle is as follows:

$$(p + dp)b(r_4 + dr_4)d\varepsilon = pBr_4d\varepsilon + F_r$$

Where $p$ is the low pressure chamber pressure of the internal ball gear pump, and $F_r$ is the centrifugal force of the water contained in the $d\varepsilon$ and $dr_4$. The equation for $F_r$ is as follows:

$$F_r = br_4dpd\varepsilon + (p + dp)bdr_4d\varepsilon$$
Where \( \rho \) is the density of water. Simultaneous equation (13) (14), \( dp = \rho r_4 \omega^2 dr_4 \).
Integrate both sides of the equal sign and get the following:

\[
p_c = \frac{\rho r_4^2 \omega^2}{2} = \frac{\rho m^2 z_1^2 \omega^2}{8}
\]

(15)

Since \( p_c < p_i \) and the concave ball gear rotates faster than the convex ball gear, the centrifugal force at the edge of the concave ball gear is larger. Therefore, the angular velocity \( \omega = 2\pi n \). Substituting equation (15) the maximum speed \( n_{\text{max}} \) is as follows:

\[
n_{\text{max}} < \frac{1}{\pi mz_1} \sqrt{\frac{p_i}{2\rho}}
\]

(16)

Where \( \rho \) is the density of the hydraulic medium (water); \( p_i \) is the suction pressure of the pump.

4. Conclusion
In this paper, a new type of volumetric water hydraulic internal ball gear pump is designed, which is replaced by concave/convex ball gear instead of gears, and the concave ball gear drives the convex ball gear to rotate, realizing energy transmission and conversion.

The difference, composition and characteristics of the water hydraulic internal meshing ball pump and the traditional internal gear pump are introduced. It is concluded that the water hydraulic internal ball gear pump has better performance.

Based on the conventional internal gear pump, the working principle of the water hydraulic internal ball gear pump is analyzed, and the displacement, flow pulsation and maximum speed equation of the water hydraulic internal ball gear pump are derived. This paper provides a theoretical basis for designing high performance water hydraulic internal ball gear pump.

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