Comparative analysis of two gear-lever differential inter-roller transmission mechanisms

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Abstract: A comparative analysis of the kinematics of two gear-lever differential transmission mechanisms is given in this paper. These mechanisms can be used in engineering and agricultural technological roller machines and can solve important technological and agrotechnical problems. The purpose of comparative analysis of gear-lever differential transmission mechanisms is to determine the values and the patterns of gear ratios of the mechanisms with similar geometrical and kinematic parameters; to estimate the additional angular velocities of the driven gear rings of the considered mechanisms depending on geometrical parameters of the mechanisms and kinematic parameters of the driven gear ring, since this additional angular velocity can lead to geometrical slip between the material being processed and the working rolls. The devices and the operation principle of the considered mechanisms were described in the paper. First, the lever circuits of the mechanisms were considered. In the lever circuits of the mechanisms, the geometrical parameters of the levers, the position of the lever circuits, as well as the rate and direction of the driving links' velocity are known. The kinematic parameters of the lever circuits of the mechanisms were determined analytically. Further, using the results of the lever circuits of these mechanisms, the gear circuits of the mechanisms were also investigated analytically. The centroid method was used in analytical study of the mechanism. The graphs of changes in gear ratios of the mechanisms were plotted depending on changes in the interaxle distance of the driving and driven links. It was proved that in both gear-lever differential mechanisms under consideration the gear ratio changes with a change in the interaxle distance of the driving and driven links. However, the magnitude of the change in the gear ratio in a gear-lever differential transmission mechanism with a six-link double-circuit lever chain is less than in a gear-lever differential transmission mechanism with a four-link lever chain under the same geometrical and kinematic parameters of the mechanisms.

1. Introduction

Engineering and agricultural technological roller machines are widely used in the industrial fields where the rotating rollers present the working bodies. These include all types of technological machines, shredders, conveyors, etc., where a variety of interaxle transmission mechanisms are used [1-6]. However, these transmission mechanisms often do not meet the technological and agrotechnical
requirements. For example, in a picking unit of a cotton harvester and in engineering roller machine to crush stones, the interaxle transmission mechanisms will not provide synchronous rotation of the working shafts at the time of changing their interaxle distance, which worsens the quality of technological process and output product.

In recent years, foreign experts in the field of mechanical engineering and instrument-making have shown an increasing interest in the theory and practice of using gear-lever mechanisms. This is due to the fact, that using gear-lever mechanisms it is possible to implement diverse and complex laws of motion of the links. Numerous combinations of the lever kinematic chains and the kinematic chain composed of gear wheels are diverse and allow one to get a wide range of output link motion. Some remarkable properties of gear-lever mechanisms revealed recently allow us to consider them as one of the most promising for the creation of modern machines and devices [7-9].

In the laboratory "Theory of mechanisms and machines" of the Institute of Mechanics and Seismic Stability of Structures named after M.T. Urazbaev of the Academy of Sciences of the Republic of Uzbekistan, a great work has been done on creation, development and implementation of a group of gear-lever differential inter-roller transmission mechanisms (GLDITM) [10-16] for roll machines with variable interaxle distance of the working shafts [17, 18], which vary both in design and characteristics. As we noted earlier, schematic diagrams of roller machines have various roller modules [19, 20]. These roller modules can be attributed to one of six specific classes of two-roller modules [20].

2. Two Gear-lever Differential Inter-roller Transmission Mechanisms: A comparative analysis

Our studies of existing and newly developed GLDITM showed that the use of similar GLDITM in various designs of roll modules gives different characteristics of roller machines with other identical geometrical parameters, while the use of different designs of GLDITM in the same designs of roller machines gives both similar and different kinematic characteristics of roller machines. For example, we have proved that the use of a GLDITM with a six-link double-circuit lever chain in roller modules of the first type ensures synchronous rotation of the rollers regardless of changes in the interaxle distance [21], and the use of this GLDITM in roller modules of the third type will not provide the effect of synchronous rotation of the rollers [22]. The use of a GLDITM with a four-link single-circuit lever chain will not provide synchronous rotation of rolls in roller modules of the first and third types [23]. Rotation asynchronism in all cases occurs when measuring the interaxle distance of the rolls in the form of an additional angle of rotation of one working roll against another one.

![Diagram](image-url)

**Figure 1.** Scheme of the roller pair, in which the center of the movable working shaft moves along an arc of a circle: 1- rotating working shaft; 2- movable working shaft; 3- lever; 4- the material being processed; $O_1$, $O_3$, $O_5$ - axes of rotation
The roller module of the third type is one of the most widely used modules in roller machines. As noted earlier, GLDITM with a six-link double-circuit lever chain and GLDITM with a four-link single-circuit lever chain in roller machines with a roller module of the third type can lead to an additional angle of rotation of one working roll against another working roll, depending on the change in their interaxle distance. The magnitude of these additional angles of rotation is different in two GLDITM used in roller modules of the third type under other similar geometrical parameters. Therefore, the purpose of the paper is to determine which one of these mechanisms provides the least additional angle of rotation under other similar geometrical and kinematic parameters.

Figure 1 shows a diagram of a roller pair, the center of the movable working shaft of which moves along a circular arc (roller module of the third type). In this roller pair, one of the working shafts (1) rotates on the axis (O1) forming a rotational pair with the frame (0), and the second working shaft (2) rotates on the axis (O2), which is pivotally connected to the frame by means of a lever (3) along the O3 axis. In the roller machines built according to this scheme, at the moment of change in interaxle distance (O1O2), the center of rotation of the movable working shaft (O2) moves in an arc-like manner relative to the center of O3 along an arc of a circle with a radius equal to the lever length (3). Figure 2 shows a diagram of a GLDITM with a four-link lever chain OABSO. The mechanism consists of a frame (0), levers (1, 2, 3) and gears wheels (4, 5, 6, 7). The axis of rotation of the gear wheel 4 is installed on the frame, the axis of rotation of the gear wheel 5 is mounted on the lever 1, and the axes of rotation of the gear wheels 6 and 7 are mounted on the lever units 1, 2, and 3. In the mechanism, all the gear wheels have the ability to rotate around their own axes O1, A, B and C, in addition, the axes of rotation of the gears 5 and 6 have the ability to move circularly around the axis O1, and the axis of rotation of the gear 7 is able to move around the axis O3. Figure 3 shows a diagram of a GLDITM with a six-link double-circuit lever chain. The first lever circuit is a four-link lever OABO – a parallelogram, the second lever circuit is a four-link lever OBCO. The mechanism consists of a frame (0), levers (1, 2, 3, 4, 5) and gear wheels (6, 7, 8, 9). The axis of rotation of the gear 6 is installed on the frame 0, the axes of rotation of the gears 7, 8, 9 are mounted on the lever units 1 - 5. In this mechanism, all gear wheels also have the ability to rotate around their own axes O1, A, B, C and O5, besides, the axes of rotation of the gears 7, 8, 9 have the ability to move circularly around the axes O1, O5, O5, respectively.

Considering that in most roller machines the diameters of the working shafts are equal to each other, consider a special case of the transmission mechanisms for such a roller machine. In roller machines with similar diameters of the working shafts, the gear ratio of the working shafts is a unity, with a
constant interaxle distance of the working shafts; it is provided under the same reference diameter of all gear wheels (gears 4, 5, 6, 7 in Fig. 2 and gears 6, 7, 8, 9 in Fig. 3) or, in pairs, under the same reference diameter of the driving gears (gear 4 in Fig. 2, and gear 6 in Fig. 3) with the driven one (gear 7 in Fig. 2 and gear 9, in Fig. 3) and idle gears (gears 5 and 6 in Fig. 2 and gears 7 and 8 in Fig. 3). Therefore, in a comparative analysis, we take all the corresponding geometrical parameters of the mechanisms to be equal and the positions of the mechanisms to be the same.

That is, \( d_4 = d_5 = d_6 = d_7 = d \) - for the mechanism in Figure 2,
\[ d_6 = d_7 = d_8 = d_9 = d \] - for the mechanism in Figure 3,
where \( d_4, d_5, d_6, d_7, d_8, d_9 \) - are the reference diameters of the corresponding gear wheels.

The initial values of \( \alpha \) and \( \varphi \) in two mechanisms under consideration are the same.
\( \alpha \) is the angle of the mechanism position,
\( \varphi \) is the angle of the position of the levers CO3 and CO5.

\( CO_3 = CO_5 \),

Figure 4 shows the design diagram of the GLDITM lever circuit with a four-link single-circuit lever chain, and Figure 5 shows the design diagram of the GLDITM lever circuit with a six-link double-circuit lever chain. We will determine the lever circuits and the kinematic parameters of the mechanisms depending on the velocity of the centers of rotation of the driven gear wheels (\( V_C \)). The kinematic parameters were determined by the centroid method. Having determined sequentially the instantaneous centers of rotation of all lever links of the mechanisms, we obtain the angular and linear velocities of the characteristic points of the mechanism links.

According to the scheme shown in Figure 4 we can write:
\[ \omega_1 = \frac{V_C}{CP_3}; \] \hspace{1cm} (1)
\[ \omega_2 = \frac{V_C}{CP_2}; \] \hspace{1cm} (2)
\[ V_B = \omega_2 \cdot BP_2; \] \hspace{1cm} (3)
\[ \omega_1 = \frac{BP_2}{BP_1}; \] \hspace{1cm} (4)

**Figure 4.** Design diagram of the GLDITM lever circuit with a four-link single-circuit lever chain: 0 - frame; 1, 2, 3 - levers

**Figure 5.** Design diagram of the GLDITM lever circuit with a six-link double-circuit lever chain: 0 - frame; 1, 2, 3, 4, 5 - levers
\[ \mathbf{V}_A = \omega_1 \cdot \mathbf{A}P_1, \]  
(5)

where \( CP_3, CP_2, BP_2, BP_1 \) are determined by

\[ CP_3 = L_3; \]
(6)

where \( L_3 \) – is the length of the lever \( CO_3 \) (link 3)

\[ \begin{align*} 
CP_2 &= \frac{d \sin \left( \frac{\gamma}{2} - \alpha + \arcsin \left( \frac{BP_1 \sin \left( \frac{\gamma}{2} - \alpha \right)}{d} \right) \right)}{\cos \left( \frac{\gamma}{2} - \alpha + \varphi \right)}; \\
\varphi &= \arccos \left( \frac{d}{L_3} \left( \frac{1}{2 \sin \alpha} + 1 \right) \cos \alpha \right); \\
BP_2 &= \frac{d \cos \left( \arcsin \left( \frac{BP_1 \sin \left( \frac{\gamma}{2} - \alpha \right)}{d} \right) - \varphi \right)}{\cos \left( \frac{\gamma}{2} - \alpha + \varphi \right)}; \\
BP_1 &= d \sqrt{2(1 - \cos \gamma)}. \\
\end{align*} \]
(7)

According to the scheme shown in Fig. 5, we can write:

\[ \omega_3 = \frac{\mathbf{V}_C}{CP_5}; \]
(10)

\[ \omega_2 = \frac{\mathbf{V}_C}{CP_4}; \]
(11)

\[ \mathbf{V}_B = \omega_4 \cdot BP_4; \]
(12)

\[ \omega_1 = \frac{BP_4}{AP_1}; \]
(13)

\[ \mathbf{V}_A = \omega_1 \cdot AP_1, \]
(14)

where - \( CP_4, CP_3, BP_4, AP_1 \) are determined by

\[ CP_3 = L_3; \]
(15)
where \( L_5 \) – is the length of the lever \( \text{CO}_5 \) (link 5)

\[
CP_5 = \frac{d \sin 2\alpha}{\sin(4 - \alpha)}; \\
BP_4 = \frac{d \cdot 2 \sin \alpha \cdot \cos \varphi}{\sin(\varphi - \alpha)} + d; \\
AP_1 = d;
\]

(16)

The gear circuits of the GLDITM are solved after solving the lever circuits of the GLDITM, using the obtained formulas. Fig. 6 shows the design diagram of the gear circuit of the GLDITM with a four-link single-circuit lever chain, and Figure 7 shows the design diagram of the gear circuit of the GLDITM with a six-link double-circuit lever chain. Sequentially determining the instantaneous centers of rotation of gear wheels, we determine the angular velocity of the driven gear wheel 7 with a four-link single-circuit lever chain \(-\omega_7\) and the angular velocity of the gear wheel 9 with a six-link double-circuit lever chain \(-\omega_9\).

According to the scheme shown in Figure 6 we can write:

\[
\omega_7 = \frac{V_C}{P_7 C};
\]

(19)

Where \( P_7 C \) is defined by determining the instantaneous centers of rotation \( P_4, P_5, P_6, P_7 \) of links 4, 5, 6, 7, respectively. The angular velocities of the gear wheels 4, 5, 6, 7 relative to the instantaneous centers of rotation and the linear velocities of the characteristic points of the mechanism \( K, A, E, B, F \) \(- V_{E'}, V_A, V_E, V_B, V_F \), and the angular velocities of links 5, 6, 7 \(-\omega_5, \omega_6, \omega_7\), are determined similarly having previously solved the triangles \( P_5 AE, P_6 BE, P_6 BF, P_7 FC \).

The gear ratio of the mechanism is determined by the formula

\[
U_{47} = \frac{\omega_4}{\omega_7} = \frac{P_5 C}{V_C}.
\]

(20)

According to the scheme shown in Figure 7 we can write:

\[
\omega_9 = \frac{V_C}{P_9 C};
\]

(21)

Where \( P_9 C \) is defined by determining the instantaneous centers of rotation \( P_6, P_7, P_8, P_9 \) of links 6, 7, 8, 9, respectively. The angular velocities of the gear wheels 6, 7, 8, 9 relative to the instantaneous centers of rotation and the linear velocities of the characteristic points of the mechanism \( K, A, E, B, F \) \(- V_{E'}, V_A, V_E, V_B, V_F \), and the angular velocities of links 7, 8, 9 \(-\omega_7, \omega_8, \omega_9\), are determined similarly having previously solved the triangles \( P_7 AE, P_8 EB, P_8 BF, P_9 CF \).

The gear ratio of the mechanism is determined by the formula:

\[
U_{69} = \frac{\omega_6}{\omega_9} = \frac{P_8 C}{V_C}.
\]

(22)
The angular velocities of the driving links of the mechanisms \( \omega_4 \) (the GLDITM with a four-link single-circuit lever chain) and \( \omega_8 \) (the GLDITM with a six-link double-circuit lever chain) are equal to each other, as well as the linear velocities \( V_C \) (the GLDITM with a four-link single-circuit lever chain) and \( V_C \) (the GLDITM with a six-link double-circuit lever chain). So,

\[
U_{47} = \frac{\omega_4 \cdot P_4 C}{V_C}, \quad U_{69} = \frac{\omega_8 \cdot P_8 C}{V_C}
\]  

(23)

Since the changes in the interaxle distance in roller machines during technological processes are directly proportional to the change in the lever 1 (\( \varphi \)) position in both GLDITM, when plotting the graphs of changes in angular velocities of the driving gear wheels (\( \omega_4 \), \( \omega_6 \)), the driven gear wheels (\( \omega_7 \), \( \omega_9 \)) and gear ratios (\( U_{47} \), \( U_{69} \)), the changes in the lever 1 position are taken as the horizontal coordinate axes (see Figure 8 and 9).

**Figure 6.** Design scheme of the gear circuit GLDITM with a four-link single-circuit lever chain: 0 - frame; 1, 2, 3 - levers; 4, 5, 6, 7 - gearwheels

**Figure 7.** Design scheme of the gear circuit GLDITM with a six-link double-circuit lever chain: 0 - frame; 1, 2, 3, 4, 5 - levers; 6, 7, 8, 9 - gearwheels

### Table 1. Initial data for the GLDITM with a four-link single-circuit lever chain

|          | 1     | 2     | 3     | 4     | 5     | 6     |
|----------|-------|-------|-------|-------|-------|-------|
| \( \alpha \), degree | 45    | 49    | 53    | 57    | 61    | 65    |
| \( \varphi \), degree | 15    | 13.36 | 12.12 | 10.48 | 9.24  | 8     |
| \( \omega_4 \), sec\(^{-1} \) | 1.5   | 1.5   | 1.5   | 1.5   | 1.5   | 1.5   |
| \( \omega_7 \), sec\(^{-1} \) | 0.45231 | 0.553137 | 0.712426 | 1.004706 | 1.651966 | 4.975111 |
| \( U_{47} \) | 3.316311 | 2.711805 | 2.10548 | 1.492974 | 0.908009 | 0.301501 |
Figure 8. Graphs of changes in angle $\varphi$, angular velocities $\omega_4$, $\omega_7$ and gear ratio $U_{47}$, depending on the change in the angle position of the GLDITM with a four-link single-circuit lever chain $\alpha$.

Table 2. Initial data for the GLDITM with a six-link double-circuit lever chain

|   | 1   | 2   | 3   | 4   | 5   | 6   |
|---|-----|-----|-----|-----|-----|-----|
| $\alpha$, degree | 45  | 49  | 53  | 57  | 61  | 65  |
| $\varphi$, degree | 15  | 13.36 | 12.12 | 10.48 | 9.24 | 8 |
| $\omega_6$, sec$^{-1}$ | 1.5  | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 |
| $\omega_9$, sec$^{-1}$ | 1.35359 | 1.377591 | 1.394965 | 1.413359 | 1.426575 | 1.438684 |
| $U_{69}$ | 1.108164 | 1.088857 | 1.075296 | 1.061301 | 1.05147 | 1.04262 |

Figure 9. Graphs of changes in angle $\varphi$, angular velocities $\omega_6$, $\omega_9$ and gear ratio $U_{69}$ depending on the change in the angle position of the GLDITM with a six-link double-circuit lever chain $\alpha$. 
3. Conclusions  
1. As seen from the above calculations, the GLDITM with a four-link single-circuit lever chain and the GLDITM with a six-link double-circuit lever chain do not provide a synchronous rotation of the working shafts at the time of changing the interaxle distance of these working shafts;  
2. In the GLDITM with a six-link double-circuit lever chain, the additional angle of rotation of the driven gear wheel is less than in the GLDITM with a four-link single-circuit lever chain, under other similar geometrical and kinematic parameters of the mechanisms.

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