Reliability Evaluation Method of Low-voltage Distribution Network Based on Linear Equation

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Abstract. The research on the reliability of distribution network uses simulation method or analytical method widely. The existing analysis methods for distribution networks usually require network topology analysis for a single node. Based on the topological relationship of low-voltage distribution network, this paper proposes a new analytical model for low-voltage distribution network. Firstly, using the relationship between the upstream and downstream nodes of the line, an algebraic equation of the expected blackout time and the expected blackout frequency of the entire network could be construct. Secondly, the expected power outage time and the expected power outage frequency at each load point can be calculate. Finally, the average availability index and load loss expectations of the entire network are calculate from the expected power outage frequency and expected power outage time at each load point. The model proposed in this article can determine the reliability index of each load point using fewer algebraic equations, and the calculation is fast and accurate, which can provide a reference for the planning and operation of distribution.

1. Introduction
Power system reliability refers to a measure of the ability of a power system to continuously supply power in accordance with acceptable quality standards and the required quantity [1]. In order to improve the reliability of the power supply of the system, investment in construction and management is bound to increase. With the gradual improvement of reliability, the investment will increase exponentially accordingly, so it is necessary to find a balance between reliability demand and investment. The reliability assessment work provides a good reference for this benefit and investment trade-off when planning or expanding the system.

The methods used for Distribution network reliability assessment, can be divided into two categories: simulation methods and analytical methods [2]. The simulation method uses a computer to generate a large number of system states and calculates the probability of system failure state in order to calculate the reliability index [3]. It is suitable for solving the reliability of complex systems, but there is a contradiction between calculation accuracy and calculation events. The analytical method achieves reliability assessment by enumerating system states and calculating state probabilities. As the unavailability of components in the distribution network is usually small, only the first-order failure
event needs to be consider in engineering to meet the accuracy requirements. Therefore, the analytical method is more widely used in the reliability assessment of distribution network.

In the reliability assessment of distribution networks, most research is on medium and high voltage distribution networks. Aiming at the special structure of the medium and high voltage distribution network, scholars have proposed different methods to simplify and improve the calculation speed, such as FMEA, block algorithm [4-5], minimum cut set method [6], and fault diffusion method [7], and the minimal way [8].

The low-voltage power distribution system, as the last link to supply power to the user, is directly connect to the user, and its power supply reliability directly affects the user's power consumption experience [9]. Therefore, the reliability of the low-voltage distribution network should attract sufficient attention. Through the reliability assessment of the low-voltage power distribution network, you can understand the reliability level of the entire network, find weak branch, and guide the management and operation of the system.

At present, there are few researches on the reliability assessment methods for low-voltage distribution networks. In [10], in order to evaluate the reliability of the low-voltage distribution network, an improved Monte Carlo simulation method was proposed, which used the equipment data and operating data of the network and combined with the failure mode analysis to calculate the reliability index. The method is time consuming. References [11] and [12] used machine learning and big data mining methods to solve the reliability index by inputting the component data and statistical data of the system, but lacked in-depth analysis of the network structure. Reference [13] considers the uncertainty of reliability parameters by interval method, and conducts reliability assessment of medium and low voltage distribution networks, focusing on parameter processing, without highlighting the characteristics of low voltage distribution networks. Reference [14] carried out a detailed study of low-voltage structures, but the reliability aspect focused on data processing, and there was no systematic calculation of reliability indicators.

The structure of low-voltage distribution network and medium-high-voltage distribution network are different. Although both are radial, the medium-voltage distribution network has a clear tree structure with trunks. The number of devices, line types, and line structures on the path from the source power supply node to different load are different. In the reliability analysis of the conventional medium-voltage distribution network method, for one node, it need traverse all devices on the path of the source power supply node to the load. Therefore, reliability analysis is cumbersome. The low-voltage distribution network is generally in the form of a transmission, there is no trunk line, and the lines passing from the transformer to the user are almost the same except for the line length, and has a clear hierarchy.

Based on the radial connection relationship of the low-voltage distribution network, this paper analyses the linear relationship between power outages at various load points in detail and establishes relevant algebraic equations to quickly solve the reliability index. Comparing with the Failure Mode and Effect Analysis and the minimum road method, which commonly used method for reliability evaluation and analysis of medium and high voltage distribution networks, the proposed method does not need to perform topology analysis for each node. And the calculation is fast and accurate, which can provide a reference for the scientific management and operation of low voltage distribution networks.

The rest of the paper is organized as follows. The second section proposes a reliability analysis model that considers the structure of the low-voltage power distribution system. The third section uses this model to analyse the reliability of a simple hierarchical network, and the results confirm the effectiveness and convenience of the method.

2. Proposed model
The low-voltage power distribution system generally adopts a radial structure as shown in Fig. 1. From the low-voltage distribution transformer to the user, they pass through the main switch, the low-
voltage power distribution cabinet, the in-building power distribution room, and the indoor power distribution cabinet in order, and finally reach the user node.

It is worth noting that in the low-voltage distribution network, a low-voltage circuit breaker or air switch (protective switch) is configured for each branch line at the distribution node. When a line fails, the protection switch on the line trips to isolate the faulty line and the downstream part of the faulty line.

![Fig.1 Low-voltage distribution network structure](image1)

![Fig.2 Topology of Low-voltage distribution network](image2)

For the convenience of description, the nodes are numbered to obtain the topology structure diagram shown in Fig. 2.

To describe the fault isolation process visually, the distribution node 6 is taken as an example for illustration. There are two users downstream of power distribution node 6, and at the position of node 6, there are two protection switches connected to 6-9 and 6-10 respectively. If the line 6-9 is broken due to lightning strikes, branch growth, insulation aging and other faults, the switch connected to 6-9 at node 6 immediately trips and isolates user 9 from the power distribution network. The remaining nodes are unaffected.

It can be noticed that from the transformer substation to the feeder of user 9, there are four lines: 1-2, 2-3, 3-6, 6-9; there are five switches: node 1, node 2, and node 3 nodes, 6 nodes, 9 nodes (the switch at node 9 is the user's indoor gate). Failure of any one of the lines, or failure of any switch, will cause the load node 9 at the end of the line to lose power.

You can count the upstream line and switch data of a node to evaluate the reliability of the node:

\[
R_i = \sum_{jk \in B^p} \lambda_{jk} L_{jk} + \sum_{k \in B^p} \lambda_k; \forall i \in \Omega^{nodes} \tag{1}
\]

\[
D_i = \sum_{jk \in B^p} t_{jk} \lambda_{jk} L_{jk} + \sum_{k \in B^p} t_k \lambda_k; \forall i \in \Omega^{nodes} \tag{2}
\]

- \(R_i\) Expected outage frequency of the node;
- \(B^p_i\) Set of branches upstream of the node;
- \(\lambda_{jk}\) Fault frequency per unit length of the branch;
- \(L_{jk}\) Length of the branch;
- \(\lambda_k\) Failure frequency of the switch at the node;
- \(\Omega^{nodes}\) Node set other than a substation node;
- \(D_i\) Expected outage time of the node;
\[ \tau_{jk} \quad \text{Outage time caused by a branch fault;} \]
\[ \tau_k \quad \text{Power outage time caused by a switch failure.} \]

To simplify the calculation, this article assumes that the transformer equipment failure rate of the substation node is zero. When the main switch fails, the substation node needs to be powered off in order to repair the fault, that is, the node 1 in Figure 2 is powered off:

\[ R_i = \lambda_i \]
\[ D_i = \tau_i \lambda_i \]

Then, calculate the reliability index of the system through the following types

\[ SAIFI = \sum_{i=\text{Load}}^N \frac{R_i}{N} \]
\[ SAIDI = \sum_{i=\text{Load}}^N \frac{D_i}{N} \]
\[ ASAI = 1 - \frac{SAIDI}{8760} \]
\[ EENS = \sum_{r=1}^{N} \sum_{i=\text{Load}}^N \Delta_r P_i T_r D_i \]

\[ SAIFI \quad \text{Average power failure frequency index of the system;} \]
\[ SAIDI \quad \text{Average system power outage duration index;} \]
\[ ASAI \quad \text{Average system availability index;} \]
\[ EENS \quad \text{Average load loss expectation of the system;} \]
\[ \text{Load} \quad \text{Load node;} \]
\[ \Delta_r \quad \text{Load level as a percentage of the peak load;} \]
\[ T_r \quad \text{Time the load level occupies in a year.} \]

This method is similar to the failure mode consequence analysis method and the minimum road method. Because the network topology analysis needs to be performed once for each load point, the calculation is more complicated when the distribution network structure is complicated.

As the low-voltage distribution network is radial, there is a linear relationship between the power outages of the upstream and downstream nodes [18-19]. Therefore, the length of the branch, the failure rate, the repair time, the faulty circuit of the switch, and the repair time can be used to characterize the corresponding expected power outage frequency and expected power outage time.

At first, taking the upper and lower nodes as an example, the impact of line faults on the upstream and downstream nodes is considered. There is a line between the two nodes. When this line fails, the downstream node loses power. To repair the fault, the switch on the upper end of the line needs to be disconnected, that is, the outlet switch of the distribution box located at the upstream node. At this time, the upstream node is still in the power supply state, which does not affect the upstream distribution node to supply power to the remaining downstream nodes.

Secondly, consider the influence of the switch on two adjacent nodes. There are switches at the upstream and downstream nodes. When the upstream switch fails, the upstream node must be powered off in order to repair the failure. Therefore, the failure of the upstream node switch will not cause a difference in the frequency of power outages between the two points; when the switch at the downstream node fails In order to repair the fault, the switch at the upstream node of the line must be
disconnected, but it does not affect the upstream node to supply power to other branch lines. Therefore, the switch at the downstream node will cause a power outage frequency difference between the upstream and downstream nodes. The above relationship can be expressed by formulas (9) to (10)

\[ R_j = R_i + L_{ij} \lambda_{ij} + \lambda_j; \forall ij \in B \]  \hspace{1cm} (9)

\[ D_j = D_i + \tau_{ij} L_{ij} \lambda_{ij} + \tau_j \lambda_j; \forall ij \in B \]  \hspace{1cm} (10)

Where \( i \) is the upstream node of \( j \).

The upstream and downstream relationships of the nodes are used in the above analysis. There are two problems with this expression. One is to use the breadth search or depth search method to determine the upstream and downstream relationships of all nodes. The network topology optimization model with reliability as a constraint requires the addition of variables representing the upstream and downstream relationships of the nodes, which increases the computational complexity. In order to solve this problem, this paper uses algebraic relations to propose a complete iterative solution model for the reliability evaluation of low-voltage distribution networks as follows:

\[ \sum_{k \in B} \alpha_{ij}^- = 0; \forall i \in S \]  \hspace{1cm} (11)

\[ \sum_{j \in B} \alpha_{ij}^+ + \sum_{j \in B} \alpha_{ij}^- = 1; \forall i \in \Omega \text{nodes} \]  \hspace{1cm} (12)

\[ \alpha_{ji}^+ + \alpha_{ji}^- = 1; \forall ij \in B \]  \hspace{1cm} (13)

\[ R_j = R_i + L_{ij} \lambda_{ij} \alpha_{ij}^+ - L_{ij} \lambda_{ij} \alpha_{ij}^- + \lambda_j \alpha_{ij}^+ - \lambda_j \alpha_{ij}^-; \forall ij \in B \text{ & } i \neq S \]  \hspace{1cm} (14)

\[ D_j = D_i + \tau_{ij} L_{ij} \lambda_{ij} \alpha_{ij}^+ - \tau_{ij} L_{ij} \lambda_{ij} \alpha_{ij}^- + \tau_j \lambda_j \alpha_{ij}^+ - \tau_j \lambda_j \alpha_{ij}^-; \forall ij \in B \text{ & } i \neq S \]  \hspace{1cm} (15)

The distribution transformer node is considered separately:

\[ R_i = L_{ij} \lambda_{ij} + \lambda_i; i = S \]  \hspace{1cm} (16)

\[ D_i = \tau_{ij} L_{ij} \lambda_{ij} + \tau_i \lambda_i; i = S \]  \hspace{1cm} (17)

Equation (5) - (8)

Among them, \( L_{ij}, \lambda_{ij}, \tau_{ij}, \lambda_i, \tau_i \) are input parameters, and, \( \alpha_{ij}^+, \alpha_{ij}^-, D_i \) are variables. \( S \) Represents a substation node.

The binary variables \( \alpha_{ij}^+ \) and \( \alpha_{ij}^- \) can be used to determine the upstream and downstream relationships of all branches. Let \( \alpha_{ij}^+ = 1 \) and \( \alpha_{ij}^- = 0 \) represent the node \( i \) is the upstream node of \( j \), \( \alpha_{ij}^+ = 0 \) and \( \alpha_{ij}^- = 1 \) represent the node \( i \) is the downstream node of \( j \).

Equation (11) indicates that all nodes connected to the substation are all downstream nodes of the substation node; Equation (12) guarantees that all nodes except the substation node receive power flow from a single road. Equation (13) guarantees that \( \alpha_{ij}^+ \) and \( \alpha_{ij}^- \) take different values. Therefore, equations (11)-(13) can determine the upstream and downstream relationships of all nodes in the network topology.

The purpose to be achieved by equation (14) is the same as that of (9), with the difference that the upstream and downstream relationships of the nodes in (9) are known, while (14) has variables representing the upstream and downstream relationships of the nodes, which is convenient for Computer solves. For example, when \( i \) is an upstream node of \( j \), then equation (14) becomes (9), and equations (15) and (10) also have the same meaning.

3. Case study
The following uses the method proposed in this paper to calculate the reliability index and sensitivity analysis for the hypothetical simple model in Figure 1. The parameters in Figure 2 are shown in Table 1.
On this basis, sensitivity analysis is performed. Regarding the description of load characteristics, this paper refers to [19], and divides the load of the entire time of the year into three levels, which account for 70%, 83%, and 100% of the peak load, respectively, and the length of time in a year is 2000 hours, 5760 hours, 1000 hours.

Table 1. Nodes data.

| Load node number | Peak load (kW) |
|------------------|----------------|
| 8                | 1.5            |
| 9                | 1              |
| 10               | 1.4            |
| 11               | 1.8            |

Branch data includes branch length, failure rate per unit length, and fault repair time. The details are list in Table 2.

Table 2. Branch data.

| Branch | Length (km) | Failure rate per unit length (times /year) | Repair time (times / hour) |
|--------|-------------|-------------------------------------------|-----------------------------|
| 1-2    | 0.9         | 0.1                                       | 4                           |
| 2-3    | 0.8         | 0.1                                       | 4                           |
| 2-4    | 0.7         | 0.2                                       | 3                           |
| 3-5    | 0.6         | 0.2                                       | 3                           |
| 3-6    | 0.3         | 0.2                                       | 3                           |
| 4-7    | 0.2         | 0.2                                       | 2                           |
| 5-8    | 0.1         | 1                                         | 2                           |
| 6-9    | 0.07        | 1                                         | 2                           |
| 6-10   | 0.05        | 1                                         | 2                           |
| 7-11   | 0.09        | 1                                         | 2                           |

Table 3. Switch data.

| Switch number | Failure rate (times /year) | Repair time (times / hour) |
|---------------|----------------------------|-----------------------------|
| 1             | 0.1                        | 0.3                         |
| 2             | 0.39                       | 1.06                        |
| 3             | 0.77                       | 1.98                        |
| 4             | 1.53                       | 2.48                        |
| 5             | 2.39                       | 3.84                        |
| 6             | 1.83                       | 3.16                        |
| 7             | 3.07                       | 4.06                        |
| 8             | 4.49                       | 5.04                        |
| 9             | 4.4                        | 4.55                        |
| 10            | 3.88                       | 4.26                        |
| 11            | 5.66                       | 5.49                        |

Table 5. System reliability index

| SAIFI  | SAIDI  | ASAI   | EENS   |
|--------|--------|--------|--------|
| 4.6075 | 4.8350 | 0.999448 | 22.9162 |

By changing the failure rate of each switch separately, the change law of the system EENS can be obtained as shown in Figure 3.
Fig. 3 EENS variation trend with switch failure rate

It can be seen from the figure that the lower the failure rate of switch 7 is, the lower the system EENS is most obviously. By analysing the reliability parameters of the network, it can be seen that the product of the annual failure frequency expectation of switch 7 and the annual failure repair time expectation is higher than that of other switches, indicating that the results are more consistent with the actual situation. This diagram can guide the decision to maintain the system. Under a certain budget, you can first replace the switches at 7 locations with switches with lower failure rates, which can minimize the system's load loss expectations.

4. Conclusions

Aiming at the characteristics of the radiated structure of the low-voltage distribution network, this paper proposes a network-dependent reliability assessment method. First, the binary variables are used to characterize the upstream and downstream relationships of adjacent nodes in the system. Write down the iterative equations of the power outage frequency and power outage time of each node, and obtain the power outage frequency and outage time expectation of each node by solving the linear equation. Finally, calculate the reliability index of the system. Because there is no simulation sampling process and no separate topology network analysis is required for each node, it is more efficient and faster than the traditional simulation method and failure mode consequence analysis method.

However, the method proposed in this paper has more stringent data requirements for low-voltage distribution networks and requires detailed network structure parameters. Due to the problems of unclear network framework, unregulated equipment management, and unclear topology relationships in some areas of low-voltage power distribution system management, adjustments need to be made when using the method proposed in this article. Considering the high similarity of the low-voltage distribution network structure in the same area, it can select the reliability data of typical networks to estimate the reliability of the entire area.

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References

[1] Zhou, J. Xie, K. (2004) Basic terminology of power reliability. China Electric Power Press. Beijing.
[2] Billinton, R. Allan, R. N. (1996) Reliability Evaluation of Power Systems. Plenum, New York, NY, USA.

[3] Liang, H, Chen, L, Liu, S. (2011) Monte Carlo Simulation Based Reliability Evaluation of Distribution System Containing Microgrids. Power System Technology, 35(10):76-81.

[4] Zhou, N, Xie, K. (2005) Reliability evaluation of large scale distribution systems using shortest path algorithm and section technique. Automation of Electric Power Systems, 29(22):39-44.

[5] Liu, B, Xie, K, Ma, C. (2005) Section algorithm of reliability evaluation for complex medium voltage electrical distribution networks. Proceedings of the CSEE, 25(4): 40-45.

[6] Yang, W, Yu, J, Tong, X. (2001) Reliability Assessment Algorithm of Power Distribution System based on Minimal Cut Sets. Journal of Xi'an University of Technology, 17(4): 387-391.

[7] Li. W, Li, Z, Liu, Y. (2003) Evaluation of complex radial distribution system reliability. Proceedings of the CSEE, 23(3): 69-79.

[8] Dai, W, Wu, J. (2002) Fast evaluation for distribution network reliability based on minimal path. Electric Power Automation Equipment, 7: 29-31.

[9] Wei, R. (2012) User Power Supply Reliability Assessment And Prediction In Low-Voltage Distribution System. Shanghai Jiao Tong University, Shanghai.

[10] Zhang, X, Lin, Q, Wu, X. (2017) Sequential Monte Carlo Reliability Prediction Method of low Voltage Distribution Network Based on Failure Effect Analysis. International Conference on Industrial Informatics.

[11] He, S, Wang, W, Zhang, W. (2019) Evaluation Method for Operating State of Low-voltage Distribution Network Based on Data Mining. Guangdong Electric Power, 32(5): 80-86.

[12] Dai, Z, Yang, J, Yu, Q. (2014) Case Study of Distribution Network Reliability Evaluation Considering Parameters Uncertainty [J]. East China Electric Power, 42(6): 1217-1219.

[13] Tan, M. (2016) Closed-Loop Low-voltage Distribution System Design Implementation and Reliability Analysis. North China Electric Power University, Beijing.

[14] Wang, B. (2015) Modelling of LV Distribution Network Based on Metering Data. Power System Technology. 39(11): 3141-3146

[15] Liu, L. (2018) Research on Power Supply Reliability Evaluation of Distribution System Considering User Experience and Power Quality. South China University of Technology, Guangzhou.

[16] Cao, K, Xie, K, Hu, B. (2011) Reliability evaluation algorithm for complex electrical distribution networks using the storage structure of adjacency multilist. Journal of Chongqing University. Chongqing. 34(12):67-71.

[17] Zhang, Z. (2018) Research on the Model and Photovoltaic Allocation Strategy of Low-Voltage Distribution Network. South China University of Technology, Guangzhou.

[18] Munoz-Delgado, Contreras, J, Arroyo, J. M, (2018) Reliability assessment for distribution optimization models: Anon-simulation-based linear programming approach. IEEE Trans. Smart Grid, vol. 9, no. 4, pp. 3048–3059.

[19] Alejandra T, Gregorio M, (2019) An Enhanced Algebraic Approach for the Analytical Reliability Assessment of Distribution Systems, IEEE Trans. Smart Grid, vol. 34, no. 4, pp. 2870–2880.