Surface profile and milling force prediction for milling thin-walled workpiece based on equivalent 3D undeformed chip thickness model

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Abstract
With the development of the aviation manufacturing industry, the application range of thin-walled parts is rapidly increasing. Accurate machining process and surface topography prediction are the keys to ensuring machining quality. This article is based on the relative geometric relationship between the real motion trajectory of the milling cutter and the workpiece; the undeformed chip thickness equation and the two-dimensional surface contour feature algorithm are established. By introducing the parameters of milling cutter critical height, milling cutter helix angle, and lag angle parameters, the three-dimensional surface profile prediction model and the equivalent three-dimensional model considering the chip formation process are derived. And the calculation equations of the instantaneous chip cross-sectional area and the blade-workpiece contact length of the three phases of chip formation are established, respectively. The multiple linear regression method was used to fit the orthogonal test results of titanium alloy Ti-6Al-4 V, so as to complete the dynamic milling force identification, and finally establish the milling force prediction model. The accuracy of the prediction model was verified by the single factor method, and the influence of different parameters on the milling process of thin-walled parts was observed. The results show that the established model has high accuracy in predicting the surface profile and milling force of thin-walled parts. This study provides guidance for efficient prediction of the milling process of thin-walled parts.

Keywords Undeformed chip thickness · Milling force prediction · Surface profile · Thin-walled workpiece

1 Introduction
In recent years, with the development of precision manufacturing fields such as aerospace industry, medical equipment, and electronic industry, the production of assembly parts has been gradually replaced by integrated parts, which means that people have put forward higher demands on the complexity and precision of machined parts. Milling is a manufacturing process that combines the rotary motion of the tool with the feed motion of the workpiece to achieve the purpose of removing the material in the area where the cutting edge and the workpiece surface interfere. It is one of the most widely used processes in metal cutting at present, and it has received extensive attention as the main method for processing complex curved parts.

The existence of the cutter teeth causes the milling process to actually occur intermittently. The cutter teeth are accompanied by the rotary motion and feed movement of the milling cutter, the periodic motion of contact-separation with the workpiece occurs, and the surface material of the workpiece is removed by creating an interference area of variable thickness. However, in precision milling, due to the small processing size and high milling cutter speed, it is extremely difficult to determine the interference area by experimental methods. Therefore, people first proposed to use the undeformed chip thickness to express this relationship. Through a large number of analysis and testing of undeformed chip thickness and establishment of correction models, more influencing factors are considered to make the research on undeformed chip thickness more practical application value.
Wan et al. [1] established a calculation model of instantaneous undeformed chip thickness considering three different tool runout conditions, and realized the expression of milling force coefficient by using the exponential function of undeformed chip thickness. Smaoui et al. [2] used the circular curve to simulate the movement trajectory of the cutter teeth in the process of milling complex surfaces, realized the prediction of the milling force on the entire trajectory, and corrected and compensated the error caused by the tool deflection. Li et al. [3] proposed a method to calculate the real instantaneous undeformed chip thickness using the Taylor series transcendental equation, and the model has higher accuracy and computational efficiency than the traditional method. On this basis, Song et al. [4] used the iterative method to approximate the value of the transcendental equation, and obtained the accurate undeformed chip profile under different cutting widths. Kumanchik and Schmitz [5] established an analytical expression for the undeformed chip thickness considering the trochoidal motion of the cutter teeth, tool runout, and uneven backlash. Liu and Cheng [6] established an undeformed chip thickness model considering the dynamic regeneration effect and the elastic recovery of the workpiece material, which can achieve accurate milling force prediction under the condition of extremely small feed per tooth. Ma et al. [7, 8] established a mathematical prediction model for the surface formed by machining, realized the quantification of the geometric characteristics of the machined surface, and studied the dynamic characteristics of cutting force and material fracture mechanism during the machining process. Tang et al. [9] obtained a three-dimensional undeformed chip thickness calculation method by calculating the penetration curve of the milling cutter edge compared with the workpiece volume. Pu et al. [10] proposed a model for predicting the milling force by using the instantaneous contact length between the milling cutter edge and the workpiece and the instantaneous undeformed chip area. Lu et al. [11] established a calculation model of instantaneous undeformed chip thickness considering the influence of milling cutter radial runout and minimum chip thickness, and used it for milling force prediction. Zhou et al. [12] proposed a generalized algorithm for undeformed chip thickness considering milling cutter radius, material strength, and changing sliding friction coefficient. Qu et al. [13] proposed a combination of theoretical model and numerical simulation model, and conducted in-depth research on the machining performance and material removal mechanism during the machining of difficult-to-machine materials from a microscopic perspective. Oliveira et al. [14] observed the size effect during micro-milling and determined the minimum undeformed chip thickness experimentally, and explained the relationship between the deformation mechanism during chip formation and the variation of cutting force fluctuations. Sun et al. [15] studied the time-varying characteristics of the milling force and the surface formation mechanism for the structural parameters and machining parameters of the EDM milling cutter.

To sum up, in the previous studies on the undeformed chip thickness model, the important factors that have an impact on the milling process have gradually increased, such as tool runout, offset, and material elastic recovery, to ensure that it is more in line with the actual situation of milling processing. However, in previous studies, the two-dimensional undeformed chip thickness was usually used as the basis for the prediction model of milling force, and the influence of axial depth of cut on the change of milling force was ignored to a certain extent. On the other hand, in the current processing of thin-walled parts, high-speed milling technology with less milling force and less temperature transfer is used more frequently. The traditional method usually uses the tool parameters and the relative position relationship between the tool and the workpiece to analytically express the milling force coefficient, but the influence of machining parameters such as milling speed on the change of milling force is ignored. In view of the above problems, this paper proposes an equivalent three-dimensional undeformed chip mathematical model considering the timing process of chip formation. By establishing a parameter equation for the actual motion trajectory of the milling cutter edge, a two-dimensional undeformed chip thickness variation curve is obtained. On this basis, the parameters of the axial lag angle of milling are introduced to generate a three-dimensional surface profile and an equivalent three-dimensional chip thickness model, and the instantaneous chip cross-sectional area and the blade-workpiece contact length are solved by using this model. At the same time, a side milling orthogonal experiment is performed on thin-walled parts to realize dynamic milling force coefficient identification, and finally a milling force prediction model considering axial milling depth, radial milling depth, tool radius, number of teeth, feed per tooth, and milling speed is obtained. The predictive validity of the model was verified by multiple control experiments.

2 Two-dimensional undeformed chip thickness calculation model

2.1 Tool tip trajectory and undeformed chip thickness

Any point on the edge of the milling cutter can be regarded as a radial ray from the center of the tool with a length of $R$. During the milling process, the milling cutter performs a compound motion of rotation around the axial direction ($Z$) and translation along the feed direction ($X$). Therefore, the movement trajectory of the cutter teeth cannot be regarded as a circle. Taking a four-blade milling cutter as an example,
the schematic diagram of four typical instantaneous tool tips positions is shown in Fig. 1.

Let the initial position of the center point of the milling cutter be \(O(0,0)\), and the four tool tips positions are \(A_1(R,0)\), \(A_2(0,R)\), \(A_3(-R,0)\), and \(A_4(0,-R)\), respectively. As the tool tip \(A_1\) continues to rotate forward, it starts to contact the workpiece at point \(C\); the tool tip \(A_1\) enters the processing stage, moves to \(A'_1\) to form the machined surface \(l_1(CA'_1)\), and leaves the workpiece. At the same time, the tool tips \(A_2\), \(A_3\), and \(A_4\) move to \(A'_2, A'_3, \) and \(A'_4\) respectively, and the distance between \(A'_2\) and \(A'_1\) is the feed per tooth. The milling cutter continues to rotate and feed; the tool tip \(A_2\) moves to the point \(D\) to contact the workpiece and enters the processing stage, and moves to the point \(A''_2\) to form the machined surface \(l_2(DA''_2)\). At the same time, the tool tips \(A'_1, A'_3, \) and \(A'_4\) move to \(A''_1, A''_3, \) and \(A''_4\) respectively. So far, the four-blade milling cutter has rotated \(180^\circ\), and the tool tip has made two contact with the workpiece. The area surrounded by the machined surface \(l_1\) and \(l_2\) and the points \(C\) and \(D\) on the upper surface of the workpiece is the theoretical chip formation area \(S\) formed by the tool tip \(A_2\), and the thickness \(d\) pointing to the direction of the current milling cutter center position \(O'\) is the instantaneous undeformed chip thickness. Since the positions of the tool tips on the same milling cutter are fixed relative to the center of the milling cutter, the change in the thickness of the undeformed chips formed in the subsequent machining process is also the same as the above.

It can be seen from Fig. 1 that during the chip formation process, the tool tip \(A_1\) starts to contact the workpiece at point \(C\) and leaves the workpiece at \(A'_1\). In this contact interval, the center point of the milling cutter \(O\) moves to \(O'\) along the positive direction of the \(X\)-axis, and the angle \(\varphi\) at which the tool tip actually performs cutting action is the acute angle between the line segment \(OC\) and the line segment \(O'A'_1\). In the side milling process, the maximum radial milling depth is the cutter radius \(R\), so the process of chip formation occurs in the second quadrant, and the obtuse angle between the line segment \(OC\) and the line segment \(O'A'_1\) is defined as the initial rotation angle \(\theta\) of the tool tip to generate cutting action; it can be expressed as:

\[
\theta = \pi - \arctan\left(\frac{\sqrt{R^2 - (R - a_e)^2}}{R - a_e}\right) \tag{1}
\]

where \(R\) is the radius of the milling cutter (mm); \(a_e\) is the radial depth of cut (mm).

According to the above analysis, the tool tip movement trajectory can be expressed as:

\[
\begin{aligned}
\{ & x_i = R \sin \varphi + f_z \cdot \left(\frac{\varphi}{2\pi/N_t}\right) + f_y \cdot (i - 1) \\
& y_i = R \cos \varphi \\
& i = 1, 2, 3 \ldots 
\end{aligned} \tag{2}
\]

where \(\varphi\) is the milling action angle, \(\varphi=\pi-\theta\); \(f_z\) is the feed per tooth (mm); \(N_t\) is the teeth quantity of the milling cutter; \(i\) is the number of the \(i\)-th tooth.

According to Eq. (2), the motion trajectory curve of adjacent tool tips can be obtained, and the distance between the two points to the center of the circle is the undeformed chip thickness \(d\).

\[
d = \sqrt{x_{i+1}^2 + y_{i+1}^2} - \sqrt{x_i^2 + y_i^2} \tag{3}
\]

From Eqs. (2) and (3), it can be known that by determining the \(R, a_e\) and \(f_z\), the tool tip movement trajectory in

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Fig. 1  Schematic diagram of typical position of tool tip
the undeformed chip thickness variation law can be obtained, as shown in Fig. 2.

In Fig. 2a, it can be seen that since the feed speed is extremely small relative to the tool rotation speed during the milling process, the tool nose movement trajectory is a trochoid that is approximately a circle. As can be seen in Fig. 2b, c, there is an unprocessed area below the intersection of adjacent cutter tooth tracks, which means that there will be a part of the residual material on the machined surface of the workpiece that is not removed, which is a key factor in forming the surface profile. It can be seen in Fig. 2d that the undeformed chip thickness decreases gradually with the increase of the tool rotation angle.

Based on the above model, taking a four-blade milling cutter as an example, the single factor method was used to explore the influence of parameters such as tool radius, feed per tooth, and radial depth of cut on the undeformed chip thickness, as shown in Fig. 3. It can be seen from Fig. 3a that with the increase of the radius of the milling cutter, the starting position participating in the milling action gradually moves to the right, that is, the machining rotation angle of the milling cutter gradually decreases, from \(90^\circ\) at \(R = 1\ mm\) to \(41^\circ\) at \(R = 4\ mm\); this shows that the machining efficiency of the milling cutter per tooth drops from 100 to 45.56%.

At the same time, the maximum value of undeformed chip thickness also decreases with the increase of milling cutter radius. It is also found in Fig. 3a that the undeformed chip thickness curves under different cutter radius parameters are not completely coincident. The closer to the cut-off position, the more obvious the dispersion trend between the curves. The larger the cutter radius, the closer the minimum value of the undeformed chip thickness is to 0, which will help to improve the separation of the chip from the workpiece. It is worth noting that the decrease in machining efficiency is

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**Fig. 2 2D tool tip motion trajectory and undeformed chip thickness.**

(a) The trajectory curve of the tool tip movement on the milling cutter,

\(N_z=4, R = 3\ mm, a_e = 1\ mm, f_z = 0.04\ mm\).

(b) The trajectory curve of

the tool tip movement in the milling area on the second quadrant.

(c) Local magnification of the intersection of adjacent tool tips motion trajectories.

(d) Undeformed chip thickness variation curve.

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not simply caused by the increase in the radius of the milling cutter. In fact, it can be seen from Eq. (1) that the ratio between the radius of the milling cutter and the radial depth of cut jointly determines the initial angle of the cutter teeth, which in turn affects the change of the machining efficiency. When \( R/a_e = 1 \), the machining efficiency is 100\%, and at this time, the max value of undeformed chip thickness is equal to the feed per tooth; the larger the value of \( R/a_e \), the lower the machining efficiency. The machining efficiency mentioned here refers to the ratio between the rotation angle of the cutter teeth participating in the machining process and the inter-tooth angle. In fact, excessive machining efficiency will reduce the rest gap of the cutter teeth, which is not conducive to heat dissipation, resulting in rapid tool wear. Therefore, when machining difficult-to-machine materials with low thermal conductivity, a smaller radial depth of cut is usually selected to suppress the temperature rise.

It can be seen from Fig. 3b that with the increase of the radial depth of cut, the milling cutter’s participating machining range and the initial undeformed chip thickness gradually increase, from 24° and 16.2 \( \mu \text{m} \) when \( a_e = 0.25 \text{ mm} \), respectively, grows to 48° and 29.9 \( \mu \text{m} \) when \( a_e = 1 \text{ mm} \). Different from the effect of the cutter radius on the undeformed chip thickness, the change curves of the undeformed chip thickness in the milling stage are coincident under the conditions of different radial depths of cut, which indicates that radial depth of cut does not affect the minimum undeformed chip thickness.

When \( R/a_e \) is fixed, the range of milling cutter involved in machining will remain unchanged. At this time, the larger the feed per tooth, the larger the initial value of the undeformed chip thickness, as shown in Fig. 3c. It can also be seen from the figure that the lowest point of the undeformed chip thickness does not coincide. The smaller the feed per tooth, the closer the lowest point is to 0, that is, the better the chip separation characteristics.

### 2.2 2D machined surface profile

According to Fig. 2c, the space enclosed by the motion trajectories of adjacent cutter teeth and the theoretical machined surface is not reached by cutter teeth during the entire milling process, and the unmachined material in the space remains to form the machined surface profile. Set the intersection point of adjacent cutter tooth motion trajectories as \( T \). Since the parameter Eq. (2) in MATLAB is discrete, the ratio method is used to determine the point coordinates. The specific calculation method is as follows:

Step 1: Since the expressions of the ordinate \( y \) in the trochoid equation of different teeth are the same, the method of finding the abscissa \( x \) of the trochoid equation is used to find the intersection point, and set \( m = x_i/(x_{i+1} + \text{eps}) \).

Step 2: Use the \textit{find} function to find the position where the ratio \( m \) is infinitely close to 1, that is \( n = \text{find} (m > 1 - 1e^{-4} & m < 1e - 4) \).

Step 3: The \( n \)-th column on the abscissa of the trochoidal curve \( x_i \) and \( x_{i+1} \) is the abscissa of the intersection point \( T \), and the ordinate of the point \( T \) can be determined correspondingly, and then the solution of the height \( H \) of the workpiece surface contour is completed.

Assuming that the lowest points to which the \( i \)-th tooth and the \( i + l \)-th tooth rotate are, respectively, \( U_1 \) and \( U_2 \), since the ordinate of the intersection point \( T \) is extremely small relative to the abscissa distance \( (f_z) \) of \( U_1 \) and \( U_2 \), therefore, the motion trajectories of the \( i \)-th tooth and the \( i + l \)-th tooth can be simplified as line segments \( TU_1 \) and \( TU_2 \), then the profile of the machined surface is an isosceles triangle formed by three points \( T \), \( U_1 \), and \( U_2 \), and the width \( W \) of its base is equal to feed per tooth.

Through the above method, the height \( H \) and width \( W \) of the two-dimensional contour feature of the machined
surface are determined, and the contour line can be expressed by Eq. (4).

\[ y = (-1)^{i+1} \frac{2H}{f_z} \cdot \left\{ x'_i - (i-1) \frac{f_z}{2} \right\} + H \ \{ \right. \begin{array}{l} i = 1, 3, 5, \ldots, H') = 0 \ v
\end{array} \ (i = 2, 4, 6, \ldots, H') = H \}

where \( x'_i \) is the section of the tool nose trajectory from the intersection of adjacent trajectories to the lowest point of the trajectory.

The influence of different parameters on the two-dimensional contour is shown in Fig. 4. According to the previous analysis, the radial depth of cut does not affect the minimum undeformed chip thickness and therefore the machined surface profile. Figure 4a shows that the residual width \( W \) of the material is not affected by the radius of the milling cutter, and the height \( H \) decreases with the increase of the radius of the milling cutter, but the magnitude of the reduction gradually decreases. Figure 4b shows that the material residual width \( W \) is equal to the feed per tooth, and the material residual height \( H \) also increases with the increase of the feed per tooth. It is worth noting that the material residual height \( H \) mentioned in this article is the theoretical value obtained by calculation. In actual processing, it is affected by the phenomenon of tool runout and workpiece yielding, and the residual height \( H \) is generally larger than the theoretical calculation value.

### 3 The prediction model of 3D surface profile and milling force

In the three-dimensional space, the schematic diagram of the relative relationship between the milling cutter and the workpiece is shown in Fig. 5. It can be seen from the figure that with the rotation and feed compound motion of the milling cutter, the cutter teeth gradually cut into the workpiece from bottom to top, and the axial depth of cut will directly determine the length of the cutting edge involved in cutting. Therefore, the axial depth of cut \( a_p \) is also an important parameter that affects the milling process. Due to the existence of the helix angle \( \beta \) of the milling cutter edge in the axial direction, the movement trajectory of the cutter tip at different heights will have different degrees of lag. The lag angle \( \varsigma \) can be expressed by Eq. (5) as:

\[ \varsigma = z \tan \beta / R \]

where \( z \) is the axial height coordinate; \( \beta \) is the helix angle of the milling cutter.

When the lag angle is equal to the inter-tooth angle of the milling cutter, that is, \( \varsigma = 2\pi / N_t \), it indicates that the two-dimensional coordinates of the tool tip in the \( z \)-axis height plane and the reference plane \( (z=0) \) are coincident. Therefore, their movement trajectories of the tool tip are the same, and the variation laws of the undeformed chip thickness are also the same. The \( z \) value here is defined as the critical value \( z_l \), which can be expressed by Eq. (6):

\[ z_l = \frac{2\pi R}{N_t \tan \beta} \]

The range from the reference plane \( (z=0) \) to the critical plane \( (z=z_l) \) is regarded as a milling processing unit. The milling process beyond the critical height is actually a repetition of the processing unit. Therefore, this unit was used as the research object to predict the surface profile and milling force. Based on this, perform linear superposition operations on other axial depth milling processing conditions, which can greatly improve the calculation efficiency.

![Fig. 4 Variation law of 2D machined surface profile. a Milling cutter radius vs profile variation (\( N_t = 4, f_z = 0.5mm, a_p = 1mm, R = 1, 2, 3, 4mm \)). b Feed per tooth vs profile variation (\( N_t = 4, R = 3mm, a_p = 1mm, f_z = 0.25, 0.5, 0.75, 1.00mm \)).](image-url)
3.1 3D Surface profile prediction model

Combined with the previous analysis, it is shown that the two-dimensional contour feature of the workpiece is an isosceles triangle area jointly enclosed by the motion trajectory of the adjacent milling cutter teeth and the theoretical machining surface. With the compound movement of the milling cutter’s rotation and feed, the same isosceles triangle material residual area will be formed between each pair of adjacent cutter teeth, so a set of serrated contour lines arranged along the milling cutter’s feed direction will be obtained. From Eqs. (2) and (5), it can be obtained that the axial depth of cut and lag angle do not affect the characteristic height $H$ and width $W$ of the serrated contour line. Therefore, for three-dimensional machined surfaces, the hysteresis of the milling cutter path at different axial heights will translate the serrated contour lines with the same features. Substituting Eq. (5) into Eq. (4) can get a family of three-dimensional machined surface profile expressions about the axial coordinate $z$:

$$\gamma_z = \left\{ (-1)^{i+1} \frac{2H}{f_z} \cdot \left( \varphi_i - \frac{f_z \cdot N_i}{2\pi} \cdot \zeta_i - (i-1) \frac{f_z}{2} \right) + H^* \right\}$$

$$i = 1, 3, 5, \cdots, H^* = 0 \quad i = 2, 4, 6, \cdots, H^* = H \quad (7)$$

From another point of view, Eq. (7) shows that the three-dimensional machined surface profile is formed by sweeping the two-dimensional serrated contour line on the reference plane along the direction of the milling cutter axis deflection helix angle.

3.2 Milling force prediction model

According to Eqs. (2), (5) and (6), the polar coordinates of the variation of the undeformed chip thickness formed by the same cutting edge at different axial heights with the rotation angle of the milling cutter are calculated, as shown in Fig. 6. It can be seen from the figure that the change curve of undeformed chip thickness generated at different axial height positions on the cutting edge is the same, but the initial contact angle and departure angle are hysteretic to the reference plane. Therefore, in the three-dimensional space, the chips generated by the milling process are geometric bodies formed by the two-dimensional undeformed chips stretched along the axial direction of the milling cutter, as shown in Fig. 7a. However, when predicting the milling force, the timing of chip formation should be considered, so the chip could be regarded as a geometry formed by sweeping along the helix with the tool origin as the center, $R$ as the radius, and the lag angle as the helix angle, as shown in Fig. 7b. This method is used to obtain an equivalent three-dimensional undeformed chip model considering the timing of the chip formation process. Compared with the traditional three-dimensional model, it has the ability to directly calculate the instantaneous chip thickness cross-sectional area and the instantaneous blade-workpiece contact length. According to the chip formation process in Fig. 7b and the change of the length of the cutting edge involved in cutting at different rotation angles of the milling cutter in Fig. 6, the milling process can be divided into three phases, as shown in Fig. 8.

Phase I: In the initial stage of milling, the tool tip on the reference plane first comes into contact with the workpiece. As the milling cutter rotates, the length of the cutting edge involved in milling in the axial direc-
tion increases gradually, and reaches the maximum value when the tool rotation angle is $180^\circ$, as shown in the blue area in Fig. 8. At this phase, both the instantaneous undeformed chip cross-sectional area and the blade-workpiece contact length gradually increase from 0 to the maximum. At the end of this stage, the tool tip on the reference plane completes the milling task, and the bottom surface of the chip is formed.

Phase II: The rotation angle of the milling cutter continues to increase from $180^\circ$, and the bottom of the milling cutter continues to cut out the workpiece. But the height of the cutting edge involved in milling in the axial direction of the milling cutter has not yet reached the critical value $z_l$, so the cutting edge continues to participate in the milling process, as shown in the green area in Fig. 8. At this phase, the length of the milling edge cutting into the workpiece is the same as the length of the workpiece cutting out, that is, the increments of $z_{top}$ and $z_{bottom}$ are the same. The blade-workpiece contact length and instantaneous chip cross-sectional area remain unchanged, the upper end of the chip continues to grow, and the lower end gradually separates from the workpiece.

Phase III: When the height of the cutting edge cutting into the workpiece reaches the critical value $z_l$, the rotation of the milling cutter will not be able to provide new cutting edges to participate in cutting, that is, $z_{top}$ will no longer increase. And the bottom end of the blade continues to cut out the workpiece, that is, $z_{bottom}$ continues to increase, as shown in the yellow area in Fig. 8. At this phase, the instantaneous chip cross-sectional area and the blade-workpiece contact length gradually decrease, and the chips will gradually separate from the workpiece from bottom to top until they are completely separated.

By truncating the equivalent three-dimensional undeformed chip model along the plane parallel to the $z$-axis, the instantaneous chip cross-sectional area and blade-workpiece contact length of the above three phases can be determined, respectively, as shown in Fig. 9.

In phase I, the instantaneous cutting thickness section is approximately a trapezoid, in which the long side of the trapezoid is always the maximum undeformed chip thickness $d_{\text{max}}$.

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**Fig. 6** Polar coordinates of chip thickness variation at different axial depths ($N_t = 4, f_z = 0.04\text{mm/tooth}, a_e = 2\text{mm}, R = 3\text{mm}, z_l = 6.732\text{mm}$)

**Fig. 7** 3D chip morphology.

- **a** Theoretical morphology.
- **b** Equivalent topography

**Fig. 8** Three phases of milling

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and the short side is the instantaneous undeformed chip thickness formed on the reference plane \( d_{z=0} \). The instantaneous milling height \( h_1 \) and the instantaneous chip cross-sectional area \( D_1 \) can be expressed by Eqs. (8) and (9), respectively.

\[
\begin{align*}
    h_1 &= \frac{\theta_1 \cdot R}{\tan \beta} \\
    D_1 &= [d_{\text{max}} + d_{z=0}] \cdot \frac{h_1}{2}
\end{align*}
\]

where \( \theta < \theta_1 < 180^\circ \).

It can be seen from Fig. 8 that the height \( h_1 \) is the sine of the blade-workpiece contact arc length \( L_1 \) according to the helix angle of the tool, and then \( L_1 \) can be expressed as Eq. (10):

\[
L_1 = \frac{h_1}{\sin \beta}
\]

In phase II, the trapezoid bottom of the instantaneous cutting thickness section is constantly cut out by the milling cutter, so the short side of the trapezoid is always the minimum undeformed chip thickness, and the long side of the trapezoid is always the maximum undeformed chip thickness, and the height is \( h_3 = h_{1 \text{max}} \). At this phase, the section of the instantaneous cutting zone moves up gradually, but the area remains unchanged. The phase II ends when the top of the chip moves to the critical height \( z_l \). The tool rotation range at this stage is \( 180^\circ < \theta_2 < 180^\circ + \frac{h_{2 l} - h_{1 \text{max}}}{d_z} \). The instantaneous chip section \( D_2 \) and the blade-workpiece contact arc length \( L_2 \) can be expressed by Eqs. (11) and (12), respectively.

\[
\begin{align*}
    D_2 &= [d_{\text{max}} + d_{\text{min}}] \cdot \frac{h_{1 \text{max}}}{2} \\
    L_2 &= L_{1 \text{max}}
\end{align*}
\]

In phase III, the instantaneous cutting zone section continues to move upward, where the short side of the trapezoid is still the minimum undeformed chip thickness, and the long side is the undeformed chip thickness \( d_z \) at the critical height. Both the height \( h_3 \) and the cross-sectional area of the instantaneous cutting zone \( D_3 \) gradually decrease, and finally drop to zero as the chip is completely separated from the workpiece. They can be represented by Eqs. (13) and (14), respectively.

\[
\begin{align*}
    h_3 &= h_{1 \text{max}} - \frac{\theta_3 \cdot R}{\tan \beta} \\
    D_3 &= [d_{z=l} + d_{\text{min}}] \cdot \frac{h_3}{2}
\end{align*}
\]

Then, \( L_3 \) can be expressed by Eq. (15) as:

\[
L_3 = \frac{h_3}{\sin \beta}
\]

Based on the milling force expressions of Pu et al. [10] and Altintas [16], this paper establishes a new milling force prediction model. It is a function of the instantaneous chip cross-sectional area \( D \), the instantaneous cutting edge-workpiece contact length \( L \), and the coefficient of milling force in each direction, as shown in Eq. (16).
\[
\begin{align*}
F_t &= K_r \cdot \eta \cdot D + K_a \cdot \eta \cdot L \\
F_a &= K_a \cdot \eta \cdot D + K_a \cdot \eta \cdot L \\
F_r &= K_r \cdot \eta \cdot D + K_r \cdot \eta \cdot L
\end{align*}
\]

where \(F_t\) is the tangential milling force; \(K_r\) is the tangential milling force coefficient; \(F_a\) is the radial milling force; \(K_a\) is the radial milling force coefficient; \(F_r\) is the axial milling force; \(K_a\) is the axial milling force coefficient; \(K_r = K \cos \beta\), \(F_a\) is the axial depth of cut correction coefficient, \(\eta = \frac{a_e}{h_{max}}\); \(D\) and \(L\) can be calculated according to Eqs. (9)–(15).

In previous studies [16, 17], the milling force coefficient was generally described in the analytical form of the milling cutter parameters and the relative relationship between the tool and the workpiece. This method ignored the influence of the machining parameters on the milling force coefficient. However, recent studies [18–20] have found that the milling force coefficient varies with the machining parameters. Therefore, this paper uses the method of orthogonal milling experiment \(L_{16}(4^4)\) to perform multiple linear regression fitting on the experiment results to complete the identification of the dynamic milling force coefficient. The experiment is carried out on the SVW80C-3D five-axis machining center produced by Dalian Sanlei Company. The tool is YG8-4B-R3.0 four-blade carbide ball end milling cutter, produced by Dalian Sanlei Company. The tool is YG8-4B-R3.0 four-blade carbide ball end milling cutter, with a rake angle of 6°, a clearance angle of 8°, and a helix angle of 35°. The workpiece material is a titanium alloy Ti-6Al-4 V thin-walled part with a size of 5 mm \(\times\) 50 mm \(\times\) 50 mm. The processing methods are side milling, down milling, and dry milling. The milling force was measured using a three-way piezoelectric force measuring instrument, model Kistler 9257B. The experiment processing site is shown in Fig. 10, and the parameter settings and results of the orthogonal experiment are shown in Table 1.

Since the result of the dynamometer is in the \(X\), \(Y\), and \(Z\) directions, use Eq. (17) to convert it.

\[
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} =
\begin{bmatrix}
\cos \zeta' - \sin \zeta' \\
\sin \zeta' - \cos \zeta'
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
F_t \\
F_r \\
F_a
\end{bmatrix}
\]

where \(\zeta'\) is the lag angle when \(z = h_{\text{max}}\).

Substitute the result of the milling force after coordinate transformation into Eq. (16), and solve the corresponding milling force coefficient of each experiment group. Taking the machining parameters as the independent variable and the milling force coefficient in each direction as the dependent variable, the multiple linear regression analysis was carried out respectively, and Eq. (18) was obtained.

\[
\begin{align*}
K_r &= \begin{bmatrix}
-5.948 - 0.08 \cdot v_c - 5.335 \cdot a_e - 487.203 \cdot f_c + 1.555 \cdot a_p \\
-1.887 + 0.015 \cdot f_c + 17.168 \cdot a_e + 107.656 \cdot f_c - 0.639 \cdot a_p
\end{bmatrix} \\
K_a &= \begin{bmatrix}
0.039 + 0.024 \cdot v_c + 1.486 \cdot a_e + 94.92 \cdot f_c - 0.378 \cdot a_p
\end{bmatrix}
\end{align*}
\]

According to the above method, when the tool parameters and milling parameters are known, the instantaneous chip cross-sectional area of milling, the instantaneous contact length of the blade-workpiece, and the milling force coefficient in each direction can be solved. Substitute them into Eq. (16) to complete the prediction of milling force in all directions.

4 Experiment results and model validity analysis

4.1 3D surface profile

In order to verify the accuracy of the 3D profile prediction model, a set of processing parameters and tool parameters are set in the experiment same as the simulation model. The experiment environment and tool parameters are the same as in the orthogonal experiment. The processing parameters are \(v_c = 120 \text{mm/min}, f_c = 0.1 \text{mm}, a_e = 1 \text{mm},\) and \(a_p = 2 \text{mm}\). The machined surface of the workpiece was observed with a VHX-1000E Ultra-Depth Three-Dimensional Microscope, as shown in Fig. 11a. Figure 11b is the 500\(\times\) magnification of the workpiece surface. It is observed that there are scratches at extremely small intervals in the feed direction of the milling cutter, and a periodic corrugated texture in the direction of the axial deflection of the milling cutter by 35°. Take the texture width of 3 cycles and calculate the average value, which is about 91.98 \(\mu\)m. The 1000\(\times\) magnified and deeply synthesized 3D surface is shown in Fig. 11c, which shows that the texture peak height is about 6.151 \(\mu\)m. The simulation results of the profile prediction model are shown in Fig. 11d. It can be seen from the figure that the angle between the texture and the axis of the milling cutter is equal to the helical angle of the milling cutter, the width is equal to
the feed per tooth 100 μm, and the peak height of the texture is 6.831 μm.

Compared with the experimental results, the simulation prediction values are larger, but the error is less than 10%. The reason for this phenomenon is that in the actual milling process, the radial runout of the tool and the deformation of the workpiece make the center of the milling cutter far away from the workpiece. The actual radial depth of cut $a_e$ is smaller than the preset radial depth of cut $a_e$, causing the theoretical machined surface to move up. Both the width and height of the unprocessed area are thus reduced as shown in Fig. 12. It is also found that the transition between the corrugated crest and the corrugated trough on the surface of the experiment workpiece is smooth, and there is no sharp point such as the predicted profile. This is because the titanium alloy Ti-6Al-4 V is a difficult-to-machine material, and the chips are not easily separated from the workpiece. Part of the residual chips in the second deformation zone generate plastic flow from below the rake face of the milling cutter, and the third deformation zone is formed by the extrusion

| Number | $V_c$ (m/min) | $f_z$ (mm) | $a_e$ (mm) | $a_e$ (mm) | $F_x$ (N) | $F_y$ (N) | $F_z$ (N) |
|--------|--------------|------------|------------|------------|----------|----------|----------|
| 1      | 60           | 0.01       | 0.25       | 2.5        | 19.43    | 1.50     | 1.43     |
| 2      | 60           | 0.02       | 0.5        | 5          | 47.88    | 5.6      | 6.19     |
| 3      | 60           | 0.03       | 0.75       | 7.5        | 112.5    | 23.47    | 12.22    |
| 4      | 60           | 0.04       | 1          | 10         | 153.6    | 39.9     | 23.57    |
| 5      | 80           | 0.01       | 0.75       | 5          | 44.84    | 7.30     | 6.20     |
| 6      | 80           | 0.02       | 1          | 2.5        | 57.21    | 9.678    | 7.99     |
| 7      | 80           | 0.03       | 0.25       | 10         | 105.2    | 25.63    | 13.37    |
| 8      | 80           | 0.04       | 0.5        | 7.5        | 138.6    | 28.19    | 17.06    |
| 9      | 100          | 0.01       | 1          | 7.5        | 83.79    | 8.15     | 7.54     |
| 10     | 100          | 0.02       | 0.75       | 10         | 118.7    | 21.07    | 18.65    |
| 11     | 100          | 0.03       | 0.5        | 2.5        | 64.85    | 23.5     | 16.54    |
| 12     | 100          | 0.04       | 0.25       | 5          | 55.12    | 24.54    | 9.12     |
| 13     | 120          | 0.01       | 0.5        | 10         | 70.15    | 12.03    | 7.00     |
| 14     | 120          | 0.02       | 0.25       | 7.5        | 55.4     | 15.13    | 8.26     |
| 15     | 120          | 0.03       | 1          | 5          | 107.1    | 17.74    | 16.23    |
| 16     | 120          | 0.04       | 0.75       | 2.5        | 64.97    | 19.5     | 10.09    |

Fig. 11 3D profile of workpiece surface and simulation results
of the flank face [21], and finally a transitional smooth corrugated texture is formed.

### 4.2 Milling force

Using the single-factor experiment method with the same experiment environment as the conditions in Sect. 3.2, the effects of milling speed, feed per tooth, radial depth of cut, and axial depth of cut on the change of milling force were studied, respectively, and compared with the simulation prediction results. For the convenience of comparison with the experimental results, the predicted values of milling force $F_x$, $F_y$, and $F_z$ are converted into milling force in the feed direction $F_x$, radial milling force $F_y$, and axial milling force $F_z$, as shown in Fig. 13, where EV is the experimental value and PV is the predicted value. It can be seen from the figure that among the milling forces in all directions, the milling force in the feed direction $F_x$ is much larger than that in the other two directions, and the milling force in the axial direction $F_z$ is the smallest. This is because the tool interacts violently with the workpiece in the feed direction ($X$), so $F_x$ is the largest; In the radial direction of the tool ($Y$), the rigidity of the thin-walled parts is weak, and it is easy to produce large deformation, and the interaction between the tool is significantly reduced, so $F_y$ is smaller than $F_x$; the component of the milling force in the axial direction ($Z$) of the tool is theoretically only provided by the elastic extrusion and friction between the chip and the helical surface of the milling cutter, so $F_z$ is the smallest.

Figure 13a shows the relationship between the milling speed and the milling force in each direction. It can be seen from the figure that as the milling speed increases, $F_x$ first increases and then decreases, and the maximum value appears at the speed of $v_c = 110$ m/min. The milling force in the other two directions is not significantly affected by the milling speed.

Schulz and Moriwaki [22] believes that in the milling process, the increase of the milling speed accelerates the plastic strain rate of the material, resulting in a strain rate hardening effect, which makes the milling force increase. As the milling speed continues to increase, due to the low thermal conductivity of the titanium alloy material, a higher temperature rise occurs. When the thermal softening effect of the material gradually exceeds the strain rate hardening effect and dominates, the hardness and strength of the material decrease, and the milling force will decrease.

Figure 13b shows the relationship between the feed per tooth and the milling force in each direction. It can be seen from the figure that as the feed per tooth increases, the milling force in all directions increases. It can be seen from Fig. 3c that the thickness of the undeformed chips increases due to the increase of the feed per tooth, which will lead to an increase of the instantaneous chip cross-sectional area and an increase of the milling force.

Figure 13c shows the relationship between the radial depth of cut and the milling force in all directions. It can be seen from the figure that with the increase of radial depth of cut $a_e$, both $F_x$ and $F_z$ increase, while $F_y$ decreases. It can be seen from Eq. (16) that the undeformed chip thickness increases with the increase of the radial depth of cut $a_e$, so $F_x$ and $F_z$ increase. However, with the increase of the radial depth of cut, the thickness of the thin-walled workpiece with a thickness of 5 mm decreases gradually, the structural rigidity of the workpiece decreases, and the generated radial reaction force becomes smaller, so the $F_z$ decreases.

Figure 13d shows the relationship between the axial depth of cut and the milling force in all directions. It can be seen from the figure that as the axial depth of cut increases, the milling force increases in all directions. It can be seen from Eq. (16) that the axial depth of cut $a_p$ directly affects the correction coefficient $\eta$, and $\eta$ increases with the increase of $a_p$, indicating that the number of milling units increases, and
the result of the milling force obtained by linear superposition is also greater.

Comparing the predicted results of each group with the experimental results, it can be found that the predicted value is generally slightly smaller than the actual measured value. Among them, the prediction accuracy of the change of milling force under different axial depth of cut conditions is the highest, about 93.52%. The prediction accuracy of the feed per tooth on the change of milling force is 90.53%, and the radial depth of cut is 89.43%. In addition, the lowest prediction accuracy is the influence of milling speed on the milling force, which is about 82.32%. In fact, the influence mechanism of the milling speed on the milling force is relatively complex, involving the strain rate hardening effect caused by the plastic strain rate of the workpiece material and the thermal softening effect caused by the severe temperature rise during the processing of the low thermal conductivity material. All of the above are important reasons for the deviation of predictions. On the whole, the new prediction model of milling surface profile and milling force proposed in this paper has a simple calculation method and has a high prediction accuracy as a whole.

5 Conclusion and discussion

In this paper, the two-dimensional machining surface profile and undeformed chip thickness equations are firstly derived based on the geometrical relationship between the milling cutter and the workpiece. Then, according to the axial hysteresis effect caused by the helix angle of the milling cutter, a three-dimensional machined surface profile model and an equivalent three-dimensional undeformed chip model are
established. According to the three phases of the equivalent three-dimensional undeformed chip formation process, the calculation equations of the instantaneous chip cross-sectional area and the blade-workpiece contact length at each phase are deduced, respectively. Finally, in the $L_1(4^3)$ orthogonal milling experiment, the dynamic milling force coefficient was identified by the multiple linear regression fitting method, and the milling force prediction was realized. Thereby, the conclusion is as follows:

1. The real movement trajectory of the milling cutter is the trochoidal line, and the adjacent cutter teeth will form an unmachined area on the surface of the workpiece. The unmachined area in three-dimensional space is represented as a corrugated texture that deflects the helix angle of the tool along the axial direction of the milling cutter. The corrugation height is affected by the tool radius and the feed per tooth. The smaller the tool radius and the larger the feed per tooth, the larger the texture height; the texture width is approximately equal to the feed per tooth.

2. During the milling process, the milling force in the feed direction $F_x$ is the largest, the radial milling force $F_y$ is the second, and the axial milling force $F_z$ is the smallest. When other parameters are fixed, with the increase of milling speed, $F_x$ first increases and then decreases, and the maximum value appears when $v_c = 110$ m/min. $F_y$ and $F_z$ increase with the increase of milling speed, but the change range is not obvious. With the increase of feed per tooth and axial depth of cut, $F_x$, $F_y$, and $F_z$ all increase. With the increase of radial depth of cut, $F_x$ and $F_z$ gradually increase, and $F_y$ gradually decreases.

3. Both the surface profile prediction model and the milling force prediction model proposed in this paper are in good agreement with the experimental results. The simulation results of the surface profile prediction model can better reflect the actual machined surface profile features, of which the height prediction accuracy is about 81.55%, and the width prediction accuracy is about 91.98%. The simulation results of the milling force prediction model also have high accuracy. The average accuracy of $F_x$ is 93.07%, the average accuracy of $F_y$ is 90.31%, and the average accuracy of $F_z$ is 83.48%.

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