Superluminal propagation and anisotropy on tilted and boosted braneworlds

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Abstract

Braneworlds winding and spinning around an extra compact dimension can manifest surprising phenomena, such as propagation faster than light. We demonstrate that, generically, there are two classes of such worlds, an irreducibly anisotropic one, and one that becomes isotropic in a special frame. In both cases, lightlike fields that can propagate in the bulk will exhibit superluminal propagation on the brane, without violating causality and without tachyons.
1 Introduction

Braneworlds on compactified spacetimes can have some counterintuitive properties due to the possibility of fields propagating over the bulk [1]. Recently, Greene et al. [2] considered the fun problem of light propagation in a flat $M_4$ braneworld embedded in the bulk spacetime $M_4 \times S^1$ [3] and pointed out that, if the braneworld is boosted, light would travel superluminally on the brane. Superluminal transmission is usually associated with acausal effects, and yet there is no violation of causality on the brane.

The essence of what happens is captured by the “scissors effect”: the intersection of the blades of a closing scissors can move faster that the speed of light as the blades become parallel. This is usually dismissed as a non-physical effect, since the intersection point carries no energy or physical information. However, in the case at hand, the propagating wavefront “scissoring” the brane was originally emitted by the brane, thus carrying energy and information. In effect, the compact geometry of the bulk promotes the scissors effect into a (continuous) wormhole effect, resulting in superluminal propagation.

The purpose of this note is to extend this effect to the most general situation in which branes are both boosted and tilted with respect to the compact dimension; that is, braneworlds that wind around and spin around the extra compact dimension (kind of like a barber’s pole). The qualitatively new feature that emerges in this setup is that braneworlds fall generically into two distinct classes: “boostlike” ones, where light propagation is isotropic in an appropriate frame, and “tiltlike” ones with an irreducible light propagation anisotropy. In all these situations causality is preserved, although all light propagation is superluminal, and there are no tachyons. The separator between the two classes is an interesting special case that, in an appropriate lightcone frame, exhibits isotropy and propagation at the speed of light as if it were untilted and unboosted.

2 Tilted and boosted braneworld

We will analyze the propagation of bulk lightlike fields on the braneworld starting with a brane in a frame tilted and boosted with respect to the compact dimension. A special frame that minimizes anisotropy will be studied in the next section.

2.1 Wind and Spin

Consider a flat spacetime $M_4 \times S^1$ and a flat 4-dimensional braneworld in it that is tilted by an angle $\theta$ and boosted by a speed $v$ along the compact dimension. We single out the time coordinate $t$, the compact space coordinate $z$, another space coordinate $x$ along which the braneworld tilts, and the remaining two untilted braneworld coordinates collectively called $y$. The speed of light will be set to 1.
We will generate the tilted and boosted brane by starting from a flat frame \( \bar{x}, \bar{z}, \bar{t} \) in which the periodicity of \( S^1 \) is expressed by the identification

\[
(\bar{x}, y, \bar{z}, \bar{t}) \sim (\bar{x}, y, \bar{z} + R, \bar{t}) \tag{2.1}
\]

with \( R \) the length of the compact dimension. An untilted and unboosted brane would lie on the \( \bar{z} = 0 \) hyperplane in these coordinates. Correspondingly, a tilded and boosted brane would lie on the \( z = 0 \) hyperplane of an appropriately rotated and boosted frame. Performing a Lorentz transformation consisting of a rotation by an angle \( \theta \) in the \( \bar{x} - \bar{z} \) plane and a boost by a speed \( v \) in the \( \bar{z} - \bar{t} \) plane (\( y \) coordinates are not touched), we obtain the brane-fixed coordinates \( x, z, t \) as

\[
\begin{align*}
x &= \cos \theta \bar{x} + \sin \theta \bar{z} \\
z &= \gamma (\cos \theta \bar{z} - \sin \theta \bar{x} - vt) \\
t &= \gamma (\bar{t} - \cos \theta v\bar{z} + \sin \theta v\bar{x})
\end{align*} \tag{2.2}
\]

with \( \gamma = 1/\sqrt{1 - v^2} \) as usual. The position of the brane is at \( z = 0 \), and from (2.2) we see that this implies

\[
\bar{z} = \tan \theta \bar{x} + \frac{v}{\cos \theta} \bar{t} \tag{2.3}
\]

on the brane. This demonstrates that the brane is tilted by an angle \( \theta \) with respect to \( \bar{z} \) and moves normal to itself with a speed \( v \). Note that the speed of the “foot” of the brane on the \( \bar{z} \) axis is \( v/\cos \theta \) which could become bigger than 1, a pure scissors effect. In the brane-fixed coordinates the periodicity of the bulk becomes

\[
(x, y, z, t) \sim (x + \sin \theta R, y, z + \gamma \cos \theta R, t - \gamma v \cos \theta R) \tag{2.4}
\]

### 2.2 Light propagation on the bulk

Consider the emission of a light wave from a point source on the brane, which by assumption propagates as a spherical wave in the bulk. (Emission of light particles in the bulk that return and hit the brane can also be considered, but we will focus on waves.) Assuming that the source was at \( x = y = z = t = 0 \), which is also \( \bar{x} = y = \bar{z} = \bar{t} = 0 \), the periodicity \( \bar{z} \sim \bar{z} + R \) implies that in the covering space of the bulk there is a line of point emitters at \( \bar{z} = Rn, n = 0, \pm 1, \pm 2 \ldots \) emitting spherical waves at the same time \( \bar{t} = 0 \). After time \( \bar{t} = R/2 \) the waves will overlap and, eventually, their interference will result in the cylindrical wavefront caustic

\[
\bar{x}^2 + y^2 - \bar{t}^2 = 0 \tag{2.5}
\]

in a manifestation of the Huygens principle.

An alternative way to see this is to expand the source into Kaluza-Klein modes around the compact dimension:

\[
\sum_{n=-\infty}^{\infty} \delta(\bar{z} - nR) = \frac{1}{R} \sum_{m=-\infty}^{\infty} e^{2\pi i m \bar{z}} \tag{2.6}
\]
(This is essentially the method of images, as done in [4] and repeated in [2].) The zero
mode \( m = 0 \) is massless on the brane and will propagate at the speed of light in the
direction normal to \( \bar{z} \), while higher modes are massive and propagate subluminally, so
they will eventually be left behind. For \( \bar{t} \gg R \) only the constant mode will survive with
a wavefront as in (2.5).

2.3 Light propagation on the brane

The cylindrical wave (2.5) will intersect the brane at \( z = 0 \) and produce a wavefront on
the brane. Its shape can be found by expressing the original coordinates \( \bar{x}, \bar{z}, \bar{t} \) in terms
of \( x, z, t \) by inverting (2.2), setting \( z = 0 \), and substituting in (2.5). We obtain

\[
(cos \theta x - \gamma v \sin \theta t)^2 + y^2 - \gamma^2 t^2 = 0
\]

(2.7)

We see that this wavefront is anisotropic and has a drift. Its spatial sections are oblate
ellipsoids with axes of ratio \( \cos \theta \), and their center drifts in the \( x \) direction with a speed
\( v_d = \gamma v \tan \theta \). The speed of propagation in a direction at an angle \( \phi \) from the axis \( x \) is

\[
c_\phi = \gamma \frac{v \cos \theta \sin \theta \cos \phi + \sqrt{1 - \sin^2 \theta (\cos^2 \phi + v^2 \sin^2 \phi)}}{1 - \sin^2 \theta \cos^2 \phi}
\]

(2.8)

Specifically, the propagation speed in the \( +x, -x \), and \( y \) directions is

\[
c_{\pm x} = \gamma \frac{1 \pm v \sin \theta}{\cos \theta}, \quad c_y = \gamma \sqrt{1 - v^2 \sin^2 \theta}
\]

(2.9)

We remark that all the above speeds are greater than 1. If \( |v| \leq |\sin \theta| \), then \( c_\phi \) will
become equal to the speed of light in the unique azimuthal direction \( \phi_o \)

\[
\cos \phi_o = -\gamma v \cot \theta \quad \Rightarrow \quad c_{\phi_o} = 1
\]

(2.10)

For \( v = \sin \theta \), \( c_{-x} = 1 \), and for \( v = -\sin \theta \), \( c_{+x} = 1 \). For all other angles, and for all
angles when \( |v| > |\sin \theta| \), propagation speeds are superluminal.

3 A driftless frame

We can eliminate the anisotropy and/or the drift in the wave propagation by performing
an additional boost in the \( x - t \) plane, which leaves the brane position and orientation
unchanged at \( z = 0 \). We set

\[
x = g (\hat{x} - u \hat{t}) \quad , \quad g = \frac{1}{\sqrt{1 - u^2}}
\]

\[
t = g (\hat{t} - u \hat{x})
\]

(3.1)

and adjust the boost \( u \) to eliminate the cross-term \( x \cdot t \) in (2.7). We need to consider two
separate cases:
3.1 Tiltlike anisotropic case: $|v| < |\sin \theta|$ 

Performing the transformation (3.1) on (2.7) and setting the coefficient of the term $\hat{x} \cdot \hat{t}$ to zero we obtain

$$u = \gamma v \cot \theta, \quad g = \frac{\sin \theta}{\gamma \sqrt{\sin^2 \theta - v^2}}$$

(3.2)

and (2.7) becomes

$$\gamma^2 \cos^2 \theta \, \hat{x}^2 + y^2 = \hat{t}^2$$

(3.3)

We see that the drift is eliminated, but wave propagation remains anisotropic. The speed of propagation at azimuthal angle $\phi$ with respect to the $\hat{x}$ axis is

$$c_\phi = \frac{1}{\sqrt{\sin^2 \phi + \gamma^2 \cos^2 \phi \cos^2 \theta}}$$

(3.4)

The two extreme propagation speeds are

$$c_{\hat{x}} = \frac{1}{\gamma \cos \theta} > 1, \quad c_y = 1$$

(3.5)

We also note that no additional Lorenz boost can make the wavefront isotropic, even at the price of introducing a drift. The periodicity of bulk space in this frame is

$$(\hat{t}, z, \hat{t}) \sim (\hat{t}, z, \hat{t}) + \left( \gamma \sqrt{\sin^2 \theta - v^2}, \gamma \cos \theta, 0 \right) R$$

(3.6)

3.2 Boostlike isotropic case: $|v| > |\sin \theta|$ 

In this case the elimination of $\hat{x} \cdot \hat{t}$ is achieved with the boost

$$u = \frac{1}{\gamma v} \tan \theta, \quad g = \frac{v \cos \theta}{\sqrt{v^2 - \sin^2 \theta}}$$

(3.7)

and the wavefront becomes

$$\hat{x}^2 + y^2 = \gamma^2 \cos^2 \theta \, \hat{t}^2$$

(3.8)

We note that this is a driftless, isotropic wave with a uniform propagation speed

$$c = \gamma \cos \theta > 1$$

(3.9)

The periodicity of bulk space in this case is

$$(\hat{t}, z, \hat{t}) \sim (\hat{t}, z, \hat{t}) + \left( 0, \gamma \cos \theta, -\gamma \sqrt{\sin^2 \theta - v^2} \right) R$$

(3.10)

The limiting case $|v| = |\sin \theta|$ is interesting. Elimination of cross-terms in this case requires a boost $u = 1$, apparently a singular limit. Nevertheless, the brane spacetime
remains regular in this limit, light propagates at a speed $c = 1$, and the periodicity of bulk space is simply $z \sim z + R$, just as in the untilded unboosted braneworld.

It should be clear that the above two cases exhaust all possibilities for the light propagation dynamics on a braneworld embedded in bulk space $M_4 \times S^1$. Adopting coordinates in the bulk such that the brane is at position $z = 0$, as we can always do, then the compactification of the bulk space can be expressed as

$$ (x, y, z, t) \sim (x, y, z, t) + (a_x, a_y, a_z, a_t) \quad (3.11) $$

The five-vector $\vec{a}$ is necessarily spacelike, otherwise the bulk spacetime would have closed timelike curves guaranteeing the violation of causality. With a rotation within the brane we can set $a_y = 0$. Then a boost along $x$ can eliminate one of the other components of $\vec{a}$: if $(a_x, a_t)$ is spacelike we can eliminate $a_t$, bringing $\vec{a}$ to the form (3.6) of a tiltlike braneworld; if $(a_x, a_t)$ is timelike we can eliminate $a_x$ and bring it to the form (3.6) of a boostlike braneworld. The special intermediate case corresponds to a timelike $(a_x, a_t)$. An infinite boost can drive it to zero, leaving $a_z$ as the only nonzero periodicity variable.

4 Photons and causality

It should be immediately obvious that no causality violation is possible in the previous braneworlds, regardless of superluminal propagation. As also noted in [2], images of the source situated at positions $\vec{X} = n\vec{a}$ in the bulk are spacelike-separated, since $\vec{a}$ is spacelike, and cannot influence each other’s past. Superluminal wave propagation may be disturbing but it does not violate causality.

4.1 Dispersion relations

Causality issues are clarified by considering the dispersion relation of waves propagating at the speed of light in the bulk. A plane wave in the bulk would have the form

$$ \phi = \exp i(k_xx + k_yy + k_zz - k_tt) , \quad k_x^2 + k_y^2 + k_z^2 - k_t^2 = 0 \quad (4.1) $$

This is valid in any Lorentz frame. For a wave emitted from the brane, as was explained in section 2.2, wave propagation will be normal to $\vec{a}$, so the additional condition $\vec{k} \cdot \vec{a} = 0$ holds. Moreover, on the brane $z = 0$. Altogether we have

$$ \phi = \exp i(k_xx + k_yy - k_tt) , \quad \vec{k}^2 = 0 , \quad \vec{k} \cdot \vec{a} = 0 \quad (4.2) $$

The condition $\vec{k} \cdot \vec{a} = 0$ can be solved for $k_z$, and substituting in the light-cone relation $\vec{k}^2 = 0$ yields the dispersion relation for the wave on the brane. For the special drift-free/isotropic frame $\hat{x}, \hat{t}$ the periodicity vector simplifies and we obtain:
a) Tiltlike case:
\[ \frac{k_x^2}{\gamma^2 \cos^2 \theta} + k_y^2 - k_t^2 = 0 , \quad \gamma \cos \theta < 1 \]  \tag{4.3}

b) Boostlike case:
\[ k_x^2 + k_y^2 - \frac{k_t^2}{\gamma^2 \cos^2 \theta} = 0 , \quad \gamma \cos \theta > 1 \]  \tag{4.4}

The group velocities derived from the above dispersion relations reproduce the propagation velocities (3.4, 3.9) obtained before. Upon quantization, they would lead to photons of energy \( E = \hbar k_t \) and momentum \((p_z, p_y) = \hbar (k_x, k_y)\) such that (with \( p^2 = p_x^2 + p_y^2 \))

\[ \begin{align*}
(a) \quad & E^2 - p^2 = \left( \frac{1}{\gamma^2 \cos^2 \theta} - 1 \right) p_x^2 > 0 \\
(b) \quad & E^2 - p^2 = (\gamma^2 \cos^2 \theta - 1) p_y^2 > 0
\end{align*} \]  \tag{4.5}

Therefore, although photons propagate superluminally, they are not tachyonic. What in fact happens is that when a photon is captured by an observer on the braneworld its full energy is released, while only the momentum along the dimensions of the brane is detected. The photon energy contributed by the momentum component normal to the brane results in \( E > p \), without making the photon massive.

## 4.2 Causality

Superluminal speeds can give the impression of causality violation since a particle exchanged between two observers at a speed larger than 1 would change direction of propagation in a boosted frame, therefore appearing as emitted by the receiver and captured by the emitter for an apparent reversal of cause and effect (a phenomenon sometimes called “traveling backwards in time”). Yet this is simply a perception of the boosted observer, and there is no real causality violation. We will demonstrate this by considering a couple of scenaria.

Consider a photon of energy \( E \) in the frame \( \hat{x}, \hat{t} \) emitted by observer A and captured by observer B, transmitting its energy and momentum from A to B. An observer in an appropriately boosted frame could see the photon originating from B and traveling to A, but with a negative energy. This is a tell-tale sign for the observer that this is a frame effect, and the true direction of propagation was from A to B. Observers cannot produce photons of negative energy in their own rest frame.

Another situation is observers A and B relatively at rest exchanging information at maximal speed. This can certainly happen at an overall back-and-forth speed higher than 1. In the \( \hat{x}, \hat{t} \) frame, and for observers on the \( \hat{x} \) axis at positions \( \hat{x}_A = 0 \) and \( \hat{x}_B = L \) (and \( y = 0 \)), the transmission times would be \( L/c \) with \( c > 1 \) the propagation speed, for an
overall back-and-forth communication time $T = 2L/c$. For two boosted observers in the $x$ direction, still relatively at rest, the two speeds would be

$$c_+ = \frac{c - u}{1 - cu}, \quad c_- = \frac{c + u}{1 + cu}$$  \hspace{1cm} (4.6)$$

with $u$ the boost speed. The total communication time would be

$$T = \frac{L}{c_+} + \frac{L}{c_-} = \frac{2Lc(1 - u^2)}{c^2 - u^2}$$  \hspace{1cm} (4.7)$$

We see that this time can never be negative, so communication backwards in time is impossible. An even stronger statement holds: for $c^{-1} < u < 1$, communication between A and B is not even possible: $c_+ < 0$, so propagation in the $+x$ direction is impossible, having instead a birefringent propagation in the $-x$ direction; B can send signals to A, but A cannot reciprocate. The minimal communication time arises for $u = c^{-1}$: A can send signals to B “instantaneously,” but the total communication time is

$$T_{min} = \frac{2Lc}{c^2 + 1}$$  \hspace{1cm} (4.8)$$

certainly less than $2L$ but not arbitrarily small. Note that the limit $c \rightarrow 1$ is discontinuous, giving $T_{min} = L$, half the communication time for $c = 1$.

5 Conclusions

Propagation faster than light seems not just possible but actually generic in braneworlds on compactified bulk spacetimes, all without tachyons or causality violation. If we live in such a braneworld, and unless it happens to be in the pristine and quiescent state of zero tilt and zero boost, we should be able to observe superluminal propagation. In fact, it can be argued that subluminal propagation would be problematic: if it existed, we would be able to “ride” a wave that propagates at the speed of light in the bulk, which is impossible no matter what frame or brane we are stuck on.

Apparent superluminal velocities can arise in curved spacetimes, and are always coordinate frame effects. Co-rotating motion near a Kerr black hole is perhaps the best known example, and it parallels the discussion of section 4.2. In particular, the birefringent domain corresponds to motion inside the Kerr ergosphere, where rotation is possible in only one direction and photons can propagate with two different angular velocities. An example closer to the present flat spacetime situation is the spacetime around a spinning cosmic string \[5\]. The energy per unit length of the string will cause the space around the string to develop a deficit angle, but its angular momentum will cause a time delay/advance for a particle revolving around the string of magnitude

$$\tau = 8\pi GJ$$  \hspace{1cm} (5.1)$$
with $J$ the angular momentum per unit length of the string. Particles winding around
the string in the direction of its angular momentum will complete a full rotation in a time
$t = L/v - \tau$, where $L$ is the proper length traveled by the particle and $v$ its speed. The
effective speed of the particle is

$$c = \frac{v}{1 - \tau v/L}$$

and for $v > L/(L + \tau)$ the effective speed becomes superluminal. Again, no causality
violation occurs as long as $L > \tau$. For $L < \tau$ causality violation would occur, as in this
case there would be closed timelike curves. However, for this to happen the string should
be thinner than $\sim \tau$, so that closed paths of length $L < \tau$ that enclose its full angular
momentum exist. A simple analysis shows that the stress-energy tensor of strings of such
small size would need to be tachyonic in order to produce that much angular momentum.
Therefore, in the absence of tachyonic matter, causality violation is prevented by a kind
of cosmic censorship. By contrast, superluminal propagation on tilted and boosted branes
evades any cosmic censorship, but also does not violate causality.

Experimental observation of superluminal and/or anisotropic light propagation would
be an indication that we may live in a braneworld. How we would detect such effects is
another story. Light, for one, is likely to be produced by, and tied to, physics on the
brane, and therefore unable to propagate in the bulk. “Sterile” neutrinos may be a
candidate for bulk propagation [6], and with a speed close to 1 in the bulk they could
achieve superluminal velocities on a tilted/boosted brane. The only waves guaranteed to
roam the bulk with abandon are gravitational ones, whose mere detection is a challenge,
let alone the measurement of their exact speed. A measurement of their relative speed
compared to photons was nevertheless performed [7] using a binary neutron star event
and found to deviate from 1 by no more than $\sim 10^{-15}$. Assuming that photons propagate
on the brane with speed 1, this puts strict bounds to the tilt and boost of any braneworld
scenario, unless we happen to live in a tiltlike braneworld and the relative position of the
binary star happened to be quite close to the critical direction $\phi_o$, implying a propagation
speed close to 1 (see eq. 2.10). Still, the ingenuity and imagination of experimental
physicists and cosmologists should not be underestimated in devising potential tests of
superluminal and/or anisotropy effects.

At any rate, the theoretical existence of superluminal effects leaves open an exciting
(if wishful) possibility for a glimpse into other dimensions.

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