Experimental diagnostics of quantum repeaters via a collective entanglement witness

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The paper reports on experimental diagnostics of quantum repeaters with an embedded entanglement swapping protocol by means of a collective entanglement witness. We show that this procedure allows to verify functioning of a quantum repeater and the underlying quantum channel with smaller number of measurements than reported previously in the literature. Moreover, our technique allows to identify the type of errors in the entanglement distribution channel which can aid in faster resolution of quality-of-transmission-related problems.

**Introduction.** Quantum communications (QC) is a scientific discipline focussed on the exchange of quantum information between parties connected through a quantum network [1–5]. Due to the unique properties of the quantum world, QC provides, for instance, inherently secure transmission of information [6–9] or improved transmission rate [10–12]. Engineering and diagnostics of multilevel systems further show the possibility for improvement on the robustness and key rate of QC protocols [13, 14]. Quantum teleportation [15–17] appears to be one of the key protocols of QC providing an advantage over classical methods. In fact, early QC networks based on teleportation have already been proposed [17, 20] and realized experimentally [21, 22].

The scaling of probability for a photonic qubit being absorbed, depolarized or dephased grows exponentially with the length of the channel and remains to be the major obstacle to practical long-distance QC [23]. This does not only restrict feasible lengths of quantum channels, but also represents a security threat as the errors could be exploited for a potential attack on the communication protocol [24, 25]. To combat these limitations, quantum repeaters and relays were proposed [23, 26]. Although the working principles of quantum repeaters and relays somewhat differ, they both operate by effectively splitting the communication channel into shorter segments; therefore, lowering the error probability. At their core, quantum repeaters apply the entanglement swapping protocol [27]. This protocol involves teleportation of a quantum state of a particle that shares entanglement with at least one other particle. As a result, entanglement swapping allows to establish entanglement between particles that have never interacted directly. By properly positioning the entanglement sources and measurement device across the communication channel, one can distribute entanglement without the need for physically sending the individual quantum-correlated information carriers through the entire channel length (see Fig 1).

In previous demonstrations of entanglement swapping, quantum repeaters and relays, the authors used various methods to demonstrate successful operation of their schemes. For instance, Li et al. [28] and Yuan et al. [29] used quantum state tomography, Pan et al. [27] and de Riedmatten et al. [30] observed interference visibility and Jennewein et al. [31] and Zhao et al. [32] subjected the resulting state to a Bell type inequality test. In this paper, we present a practical method for diagnostics of quantum repeaters, more specifically of the underlying entanglement swapping protocol, by means of a collective (nonlinear) entanglement witness [33–36]. In particular, we adopt the collectibility witness originally proposed by Rudnicki et al. [37, 38]. Proposed in 2011, the concept of collective witnesses significantly broadened the toolbox of entanglement detection. Our approach...
is preferable to diagnostics by other means because the number of measurement configurations is smaller (especially when compared to complete quantum state tomography [38,41]). We make use of the fact that the geometry of entanglement swapping resembles considerably the layout for measurement of collective entanglement witnesses. This can be observed in Fig. 1. The fact that collective witnesses require simultaneous preparation of multiple copies of the investigated state turns out to be a surprising asset when using them for entanglement swapping diagnostics.

To demonstrate our idea experimentally, we have constructed a memoryless quantum repeater on the platform of linear optics. It consists of two independent EPR-state sources and an entanglement swapping device linking them together. Qubits were encoded into polarization states of individual photons. We used polarizers and wave plates to implement errors occurring in three distinct quantum-information channels, i.e., (a) a depolarizing channel, (b) a phase-damping channel and (c) an amplitude-damping channel. Each channel is characterized by a real-valued parameter determining the probability of disturbing the transmitted quantum state. In this proof-of-principle experiment, both EPR pairs are subjected to the symmetrically distributed state disturbance, i.e., the same value of the channel parameter.

**Experimental implementation.** In the experimental setup depicted in Fig. 2 a frequency-doubled 413 nm femtosecond pulsed laser beam is used to pump spontaneous parametric down-conversion in a BBO crystal cascade [42]. At first, the pump polarization is made diagonal. Next, the beam travels through a polarization dispersion line (PDL) to counter subsequent polarization dispersion of the BBO material. This laser beam impinges on the crystal cascade twice, i.e., after it passes the crystals for the first time, it gets reflected on a mirror and pumps the crystals in the opposite direction. On both occasions, with some probability, a pair of photons in the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|HH\rangle + |VV\rangle)$ is generated, where $H$ and $V$ denote horizontally- and vertically-polarized photons, respectively. For a detailed characterization of this four-photon source see Ref. [39]. While polarization of photons 2 and 4 is projected locally on four states selected using combinations of half- and quarter-wave plates followed by polarizers, the other two photons (1 and 3) are projected onto a singlet state by means of a balanced fiber coupler (FBS) and post-selected onto coincident detection at both its output ports.

The four specific settings of local projections sufficient to estimate collectibility [38] are: $|HH\rangle$, $|HV\rangle$, $|VV\rangle$ and $|+\rangle$, where letters indicate state projections on the two locally-projected photons respectively and $|+\rangle = \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle)$. We denote $p_{XX}$ ($XX \in \{HH, HV, VV, ++\}$) the probability that both locally-projected photons pass the projections conditioned on the other two photons being projected onto a singlet Bell state. However, due to nonremovable jitter between generation of the first and the second pair of photons, probability of two-photon overlap is decreased. These noninteracting photons are seen as noise, which can be estimated and subtracted from the genuine coincidences. In order to estimate the noise level both photon 1 and photon 3 were prepared in the same polarization state ($|H\rangle$) and the achievable Hong-Ou-Mandel bunching effect was measured conditioned on detection of photon 2 and 4 (used as heralds). From the visibility of Hong-Ou-Mandel interference the noise level caused by jitter was estimated (for more details see Ref. [39]). Measuring the probabilities $p_{XX}$, collectibility is calculated using formula

$$W(\hat{\rho}) = \frac{1}{2} \left[ \eta + p_H^2 (1 - 2p_{HH}) + (1 - p_H)^2 (1 - 2p_{VV}) + 2p_H (1 - p_H)(1 - 2p_{HV}) - 1 \right],$$

(1)

where

$$\eta = 8p_H (1 - p_H) \sqrt{p_{HH} p_{VV}} + 2p_{++},$$

(2)

and $p_H$ is the probability of local projection of photon 1 or 3 onto horizontal polarization $|H\rangle$ independently of the singlet Bell state projection. The measurement of all four probabilities $p_{XX}$ was realized in about 60 sequences, each ten minutes long, during which four-photon detections were accumulated. Moreover, for any $p_{XX}$, each sequence was repeated four times introducing the disturbances of the three erroneous channels (depolarizing, phase damping and amplitude damping) as well as observing a perfect channel for comparison. Wave plates used for local polarization projections were used simultaneously to introduce disturbances.
typical for a given type of erroneous quantum channel. Here, we investigate experimentally noisy channels studied theoretically in the context of quantum teleportation in Ref. [44]). Afterwards, the measured sequences for any $p_{XX}$ were summed up randomly alternating the measurements obtained with and without introduced disturbances. The probability to add an either disturbed or undisturbed sequence to the overall value of $p_{XX}$ is determined by the error probability of a channel to be implemented.

**Depolarizing channel.** Qubits transmitted through a depolarizing channel are randomly subjected to three types of transformations causing decoherence. These transformations are bit-flip, phase-flip and combination of bit-flip and phase-flip. It is the randomness and impossibility to predict these transformations that is the effective cause of errors. The action of such channel can be conveniently described using the Kraus operators [45]

\[
\hat{E}_0 = \sqrt{1 - d_D} \hat{I}, \quad \hat{E}_i = \sqrt{\frac{d_D}{3}} \hat{\sigma}_i \quad \text{for } i = \{x, y, z\},
\]

where $d_D$ is the depolarization probability. $\hat{I}$ stands for the identity operator and $\hat{\sigma}_i$ are Pauli matrices. When propagating through such channel, a Bell state is randomly transformed into one of the other three Bell states with equal probability $d_D/3$. To implement a depolarizing channel, we have been randomly switching the half-wave plate between two positions $0^\circ$ and $45^\circ$ to achieve the bit-flip transformation and a quarter-wave plate between $0^\circ$ and $90^\circ$ to achieve the phase-flip transformation. Combined action of both wave plates implements the bit and phase-flip simultaneously. Using the procedure described in the previous paragraph we have been able to measure the collectibility of a Bell state propagating through a depolarizing channel for several values of the depolarization probability $d_D$. The observed collectibility as well as its theoretical prediction are depicted in Fig. 3 a). As expected, collectibility reaches its maximum value for $d_D = 3/4$, $W(\hat{\rho}) = 0.80 \pm 0.09$ (theoretical prediction: $W(\hat{\rho}) = 0.75$). This corresponds to a maximally depolarizing action causing the transmitted state to fully decohere to $\hat{\rho}_D = \hat{I}/4$. Meanwhile, in an ideal channel ($d_D = 0$) the Bell state is propagating undisturbed which coincides with the value of collectibility being $W(\hat{\rho}) = 0.24 \pm 0.06$ (theoretical prediction: $W(\hat{\rho}) = 0.25$).

**Phase-damping channel.** The effect of phase damping causes decoherence between two basis qubit states without, however, causing any bit-flip transformation. Such channel can readily be described by two Kraus operators

\[
\hat{E}_0 = \sqrt{1 - d_P} \hat{I}, \quad \hat{E}_1 = \sqrt{\frac{d_P}{2}} \hat{\sigma}_z,
\]

where $d_P$ is the dephasing probability. Similarly to the previous case, the phase-damping effect was implemented by randomly switching a quarter-wave plate between two positions: $0^\circ$ and $90^\circ$. The resulting collectibility as a function of $d_P$ is presented in Fig. 3 b). Experimental value of collectibility at $d_P = 1$ reaches $W(\hat{\rho}) = 0.32 \pm 0.09$ (theoretical prediction: $W(\hat{\rho}) = 0.25$) as the Bell state propagating through this channel becomes $\hat{\rho}_P = 1/2(|HH\rangle \langle HH| + |VV\rangle \langle VV|)$.

**Amplitude-damping channel.** Typically, amplitude damping causes lossy transmission of qubits through the channel. The overall losses are trivial to detect as they decrease the overall number of coincident detections. Apart from that, white (state-independent) losses do not change the state’s collectibility because the measurement relies solely on successful four-photon detections. It is, therefore, more interesting to analyze state-dependent (polarization sensitive) losses that cause disturbance in superposition of horizontal and vertical polarizations of the state. We describe this channel by an effective

\[
\hat{\rho} \to \hat{E}_A \hat{\rho} \hat{E}_A^\dagger,
\]

where $\hat{E}_A$ is the amplitude damped channel defined by

\[
\hat{E}_A = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1 - d_A} \end{pmatrix}.
\]
Here, in contrast to the above-described channels, the entangled state remains pure but its entanglement decreases. This corresponds to the Bell state being less entangled $\frac{1}{\sqrt{2}}(|HH\rangle + (1 - d_A)|VV\rangle)$ and eventually becoming separable $\hat{\rho}_A = |HH\rangle\langle HH|$, as $d_A \to 1$, where the value of collectibility reaches $W(\hat{\rho}) = -0.05 \pm 0.09$ (theoretical prediction: $W(\hat{\rho}) = 0$). Collectibility allows to capture this transition as shown in Fig. 5(c). Note that the collective witness for pure states can be used as an entanglement measure [37, 38].

**Channel characteristics.** Measurement of collectibility is a powerful tool that allows to detect disturbance occurring in the channel. However, in order to promote our method even further, we have analyzed characteristic effects of the three types of erroneous channels. By more detailed analysis of the individual measurements that take part in the collectibility calculation one can identify which type of errors, i.e. what channel, is inflicted. As explained previously in the text, five probabilities are measured to calculate collectibility: $p_{XX}$ ($XX \in \{HH, HV, VV, ++\}$) and $p_H$. We show experimental and theoretical values of these quantities for comparison between the three tested channels and a reference perfect channel in Fig. 4. The exact results are then summarized in Tab. 1 in a perfect channel both of the Bell states propagate undisturbed. Therefore, the overall state of the system is

$$|\Phi^+\rangle|\Phi^+\rangle = \frac{1}{2}[(|HH\rangle + |VV\rangle)(|HH\rangle + |VV\rangle)],$$

which after projecting the photons 1 and 3 onto a singlet state collapses also to a singlet state

$$|\Phi^+\rangle|\Phi^+\rangle \xrightarrow{|\psi_{24}\rangle \langle \psi_{24}|} |\psi_{24}\rangle.$$

Hence, the only conditioned projection that we observe is the $|HV\rangle$ projection with probability $p_{HV}$ of $1/2$. It follows from the Eq. (6) that the probability $p_H$ of unconditional projection $|H\rangle$ is also $1/2$. In a fully depolarizing channel the state of the system becomes maximally mixed

$$\hat{\rho}_D \otimes \hat{\rho}_D = \frac{1}{16} |\psi_{13}\rangle\langle \psi_{13}| \to \frac{\hat{1}}{24}/4.$$  \hspace{1cm} (8)

Therefore, all of the conditional projections are equally likely with probabilities of $1/4$. The probability $p_H$ of unconditional projection $|H\rangle$ stays at $1/2$. Phase-damping transforms the initial Bell state into a $\rho_P$. The final state of the photons 2 and 4 is then

$$|H\rangle \xrightarrow{\hat{D}} |H\rangle.$$  \hspace{1cm} (9)

The probability of observing a conditional $|HV\rangle$ projection is $1/2$, however, due to the phase-flip transformation we also observe signal in $|++\rangle$ projection with probability $p_{++}$ of $1/4$. The unconditional projection $|H\rangle$ happens with probability $1/2$. In an amplitude-damping channel which is lossy for vertical polarization the state of the photons 2 and 4 becomes

$$|\Phi^+\rangle|\Phi^+\rangle \xrightarrow{|\psi_{24}\rangle \langle \psi_{24}|} |\psi_{24}\rangle.$$  \hspace{1cm} (10)

As $d_A \to 1$ the probability of singlet projection $F(\psi_{24})$ however, tends to $0$ as the state becomes separable. In this limit, conditioned projection $|HV\rangle$ is observed with probability of $1/2$, meanwhile, the probability $p_H$ of unconditional projection $|H\rangle$ raises to $1$.

**Conclusions.** We have reported on experimental diagnostics of quantum repeaters by the method of collective entanglement witness (collectibility). We capitalize on the similarity between the geometry of entanglement swapping protocol and the layout for measurement of collective entanglement witnesses. Our approach allows to detect disturbance in a channel by measuring four probabilities $p_{XX}$ and estimating collectibility. This makes our approach a preferable method as the number of measurement configurations is lower than in other means of diagnostics. We have measured collectibility for three erroneous channels: depolarizing channel, phase-damping and amplitude-damping channel. The uncertainty of conditional probabilities $p_{XX}$ and unconditional probability $p_H$ is $4\%$ and less than $1\%$ respectively. The uncertainty of collectibility measurement is $\pm 0.09$. 

**TABLE I.** Experimental results and theoretical prediction (in parenthesis) of the $p_{XX}$ and $p_H$ probabilities and collectibility obtained for the perfect, fully depolarizing, fully phase-damping and fully amplitude-damping channel. The uncertainty of conditional probabilities $p_{XX}$ and unconditional probability $p_H$ is $4\%$ and less than $1\%$ respectively. The uncertainty of collectibility measurement is $\pm 0.09$. 

| Channel characteristics | Perfect channel | Depolarizing phase-damping | Amplitude-damping |
|-------------------------|-----------------|---------------------------|------------------|
| $p_{HH}$ (%)            | 1 (0)           | 20 (25)                   | 1 (0)            |
| $p_{HV}$ (%)            | 50 (50)         | 21 (25)                   | 50 (50)          |
| $p_{VV}$ (%)            | 3 (0)           | 20 (25)                   | 3 (0)            |
| $p_{++}$ (%)            | 1 (0)           | 32 (25)                   | 0 (0)            |
| $p_{H}$ (%)             | 50 (50)         | 50 (50)                   | 50 (50)          |
| $W(\hat{\rho})$ (%)     | -0.22 (-0.25)   | 0.81 (0.75)               | 0.33 (0.25)      | -0.01 (0.00) |

**FIG. 4.** Characteristic channel signatures allowing to identify type of errors from the individual measurements that constitute collectibility. Gray and blue bars represent theoretical predictions and experimentally obtained values respectively. The uncertainty of the unconditioned probability $p_H$ is negligible, therefore, is not visualized.
channel and amplitude-damping channel. The obtained experimental data are in a good agreement with theoretical predictions. To demonstrate the versatility of our approach we have also analyzed characteristic effects of the three types of erroneous channels. By more detailed analysis of the $p_XX$ probabilities, one can determine which type of error is occurring in the communication channel. We believe that these results can contribute to the field of quantum communications and mainly represent a practical instrument for future deployment of quantum networks or engineering of complex multilevel quantum systems.

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