A developed analytical model for the pressurizer unit in nuclear power plants

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**ABSTRACT**

The pressure control in the pressurized water reactor (PWR) primary loop is key to secure the safe operation. The pressurizer (PZR) unit is responsible for attaining this task. Thus, the PZR unit’s modeling is an important issue for the tracking control purpose and performance analysis concerning the turbine load. This paper develops a mathematical model to accurately predict the PZR unit pressure in the normal and load power changes. The model is a non-equilibrium three-region model developed based on the thermodynamics of mass and energy for the water and steam in the PZR. The variations of the pressure and the temperature due to thermodynamic variables (enthalpy, density) and the mechanical work effect are considered in the developed model. The model also addresses other thermodynamic processes such as bulk flashing, spray condensation, interface condensation, wall condensation, and rainout flow. The inlet and outlet flow rates of the primary circuit (PC) and the average temperature and the hot leg temperature are included in the developed model. Based on data generated from the VVER-1200 simulator, parameter estimation, verification, and validation of the developed model through load power changes are achieved. Besides, a comparison with other models given in literature has been performed. This comparison indicated that the developed model is more sensitive against the minimum variation of PZR dynamics. Finally, a closed-loop PZR pressure control system, including a conventional Proportional Integral Derivative (PID) controller, was designed to test the control purpose’s developed model. The simulation results over typical load change transients have been demonstrated the feasibility, effectiveness, and accuracy of the developed nonlinear model of PZR for dynamic modeling and control purposes.

**1. Introduction**

With developing nuclear fission as a clean-based energy resource, nuclear power plants (NPPs) need to work based on the daily load curve (Bose et al., 2020; Li & Zhao, 2013). The Load following is the possibility that the power plant will adjust its energy output as demand and price fluctuate for electricity throughout the day (Bruens, 1981; Yadav et al., 2020).

One of the common types of NPPs, the pressurized water reactor (PWR), is generally designed to handle heavy load following (Muniglia et al., 2017). In this reactor, the high temperature in the reactor vessel without boiling the water inside the core can be obtained (Liu et al.,). The PZR is one of the most important units in PWR, which is needed to ensure the reactor’s stable operation. The function of this unit is to maintain the pressure of the reactor coolant system (RCS) in steady-state operation; limit the pressure change within allowable range during transient operation, and avoid the overpressure of RCS in accident situations (Baghban et al., 2016; Chung et al., 2019). The PZR is often used to compensate for positive or negative fluctuation caused by load change transient within the PWR (P. Wang et al.,). Hence, the prediction and the control of PZR’s pressure are very important to ensure the stable and safe operation of PWRs.

In literature, the dynamic models of the PZR can be divided into equilibrium and non-equilibrium models. The equilibrium models assume that the water and the steam phases are saturated, and the thermodynamics is equilibrium in the transient state (Kim et al., 2006). These models can simulate the actual working conditions if the transient PZR processes are slow. Hence, the equilibrium models are not suited to simulate the operation of the PZR under rapid transients. For the rapid transient conditions, the non-equilibrium models are developed based on the conservation laws of mass and energy that describe the thermodynamics processes within the PZR. These models are classified into two, three, and multi-region models in terms of the number of control volumes. The PZR space of the two region models is divided into steam and water regions (Fazekas et al., 2007; Gábor et al., 2010; Pini et al., 2018). Fazekas et al. developed a simple dynamic two-region model of the PZR as a part of the PC (Fazekas
et al., 2007). They considered the principles that capture different dynamics of the system in normal operating modes, as they studied the effect of the mass flow rate change of water within the PC. They also considered the effect of the rate change of the temperature inside PZR. This model neglected the effect of mass flow rate and the change in both water and steam temperature inside the PZR. This drawback is avoided in the two-region model developed in (Gábor et al., 2010). However, many transient changes such as bulk flashing (bubble rising) flow rate in the water region of the PZR, the mass flow rates of spray condensation, interface condensation, wall condensation, and rainout (liquid droplet) in the steam region are not included in these models (Fazekas et al., 2007; Gábor et al., 2010). and (Pini et al., 2018) proposed a non-equilibrium two regions triple-volume control-oriented model for the PZR dynamics. (Pini et al., 2018)This model divided the steam and water regions in PZR into a moving-boundary control volume filled with saturated steam and water and two control volumes filled with sub-cooled water. Moreover, most of the developed two region models considered that the insurge flow is thoroughly mixed with the liquid volume.

The non-equilibrium three models were introduced (Kim et al., 2006; Min Baek et al., 1986; G. D. Zhang et al., 2013). These platforms divided the PZR water into main water and surge water (or sometimes called buffer water). The three region models describe most of the important thermal-hydraulics processes inside the water, buffer water, and steam regions. The framework of the three-region model has been employed by fewer researchers for different control volume division approaches. In (You et al., 2001) divided the water region and steam region in the PZR into many equally control volumes. In (W. W. Wang et al.), a three region model was proposed to calculate the pressure and temperature distributions in PZR with satisfactory accuracy. Recently, P. Wang et al. developed a non-equilibrium three-region PZR model based on the basic conservation laws of mass and energy for the steam and water in the PZR (P. Wang et al., 2019).

This model studied the transient of five changes (i.e. the bulk flashing flow rate in the PZR water region, the mass flow rates of spray condensation, interface condensation, wall condensation, and rainout in the steam region) on the PZR pressure change.

The PZR space in the four-region model is divided into regions of continuous vapor, discrete water, discrete vapor, and continuous water (Baghban et al., 2016; Bezrukov et al., 2016; Moghanaki & Rahgoshy, 2014; Zhong et al., 2019). This model considers the effects of vapor bubbles and water droplets’ residence in the continuous water and vapor regions. (Zhong et al., 2019) proposed a multi-region (three layers) PZR model consists of liquid, saturated, and vapor layers. Each layer meshes further into different control volumes. The previous knowledge of the distribution of the surge flow is not required for this model. The effect of vapor bubbles and water droplets’ residence in the continuous water and vapor regions is neglected by the four and multi-region models compared with the two- and three-region models. These models employ different pressures and temperatures for the upper vapor and lower water portions. Also, it employs different pressures and temperatures for the upper vapor and lower water portions (Moghanaki & Rahgoshy, 2014).

Motivated by the above discussion, this study’s main purpose is to develop an efficient model of a non-equilibrium three region PZR unit in PWR by combining many features of the PZR thermodynamics. In summary, the main contributions of this paper are as follows:

1. Derived a mathematical model based on the three volumes’ thermodynamics properties (i.e., water, steam, and buffer water) of the PZR unit in PWR. The physical phonemes that occurred inside the PZR, such as flash out, spray and wall condensation, and rain out in the steam region, are described in this study’s developed model. Moreover, the coolant’s average temperature, the hot leg temperature, and the inlet and outlet mass flow rate for the PC are handled in the developed model. Also, the effect of the load power change on PZR performance is considered.

2. Using the simulator personal computer transient analyzer VVAV –1200 (PCTRAN VVER-1200), the parameter estimation, the validation, and the verifications of the developed model were carried out (Bezrukov et al., 2016; Fyza et al., 2019; Ibrahim et al., 2013; Khan & Islam, 2019; Mollah, 2018).

3. Based on the data generated from PCTRAN VVER-1200 (Khan & Islam, 2019; Mollah, 2018), a comparison between the developed and three region models was carried out.

4. To test the developed model for the control purpose, a conventional PID controller is designed to control the pressure via the spray valve and electric heaters of the PZR unit.

The paper’s reset is organized as follows: Section 2 discusses the preliminaries of the primary circuit of PWRs. Section 3 presents the mathematical model of the developed three region PZR system. Section 4 presents the developed model state-space representation. Section 5 discuss parameter estimation, verification, and validation of the PZR model. Section 6 includes the derivation of applied controllers. Finally, the results of applied controllers for the PZR developed model are compared in the last section.

2. Preliminaries

In PWR reactors, as depicted in Figure 1 (Chung et al., 2019), the PZR unit is located in the primary circuit
consists of a lower liquid and an upper vapor lumps. The lump of liquid is connected through the surge line to the hot leg, and the lump of vapor is connected through the spray line to the cold leg. For heating, the water phase and then increasing the pressure, two heaters (i.e. proportional and backup) are linked with this phase. The heaters increase the pressure in two ways, boiling the water and increasing its internal energy. A spray line connected to the cold leg can decrease the pressure by condensing vapor to water. In case of drastic changes in the pressure, the lumps of water and steam may turn into two phases. Consequently, the PZR may have bulk evaporation and condensation, and hence the pressure change of the PZR will be affected. Accordingly, the PWR performance needs to study the process of modeling the pressure dynamics of the PZR. A typical Schematic diagram of main components of NPP is illustrated at Figure (1).

Among the mathematical models, the non-equilibrium three-phase models divide the PZR control volume according to phase conditions into regions of steam, main water, and buffer water. The considered region buffer water, as shown in Figure 2, acts like a piston, which is connected to the PC heat pipe section through the surge line. Likewise (P. Wang et al., 2019). The Structure of the PZR system in three region division is illustrated at Figure 2. The following assumption is considered in this study:

1-The pressure of water, steam, and buffer regions is equal.
2-Before reaching the main water region, the steam region’s droplets are considered fully heated to become saturated.
3-Neglect the non-condensable gas in the PZR.
4-Neglect the transient time for the transfer of mass between regions of steam and main water.
5-Neglect the transfer of both energy and mass between regions of main and buffer water is.

6-The processes of thermodynamic for the region of buffer water are not considered.

Mass conservation equation for both control volumes can be defined with the flowing equation:

\[
\frac{dM}{dt} = V \frac{\partial p}{\partial t} + \rho \frac{\partial V}{\partial t} + V \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial t} \frac{dh}{dt}
\]

\[
= \sum m_{inlet} - \sum m_{outlet} \quad (1)
\]
Where M, h, ρ, and V are mass (kg), specific enthalpy (MJ/kg), density (kg/m³), and volume (m³) for the region of water or steam, respectively. P denotes the pressure (MPa); \( m_{\text{inlet}} \) and \( m_{\text{outlet}} \) are inlet and outlet mass flow rate of the water or steam, respectively (kg/s). Energy conservation equation for both control volumes can be given by:

\[
M \frac{dh}{dt} - V \frac{dp}{dt} = \sum (mh)_{\text{inlet}} - \sum (mh)_{\text{outlet}} + q - h \left[ \sum m_{\text{inlet}} - \sum m_{\text{outlet}} \right] \tag{2}
\]

Where \( q \) is the net transferred power to the water or steam region (MW). The rate of change of specific enthalpy can be given as in (Ilyas & Aydogan, 2018; Pini et al., 2018; Safaei Arshi et al., 2010) by:

\[
\frac{dh}{dt} = \left( \frac{\partial h}{\partial T} \right) \frac{dT}{dt} + \left( \frac{\partial h}{\partial P} \right) \frac{dP}{dt} \tag{3}
\]

According to the pressure-enthalpy (P-h) and enthalpy-temperature (h-T) diagrams, equation (3) was adapted as:

\[
\frac{dh_i}{dt} = \left( \frac{\partial h_i (T_{\text{ref}})}{\partial T} \right) _{P_i} \frac{dT_i}{dt} + \left( \frac{\partial h_i (P_{\text{pref}})}{\partial P} \right) _{T_i} \frac{dP_i}{dt} + \left( \frac{\partial h_i (P_{\text{pref}})}{\partial P} \right) _{T_i} \frac{dP_i}{dt} \tag{4}
\]

where \( T_{\text{ref}} \) and \( P_{\text{pref}} \) refers to the nominal temperature and pressure of the water and the steam region in PZR, respectively. This equation is used for both saturated steam and hot water by using suffix \( i \), where the suffix \( i \) can take \( G \) for steam and \( F \) for water.

Based on the above thermodynamics equations, the model’s description in this study is given in the following section.

### 3. The developed three region model of the PZR unit

In the developed model, two sets of parameters that affect the performance of the pressure in the PZR are considered. The first one describes the thermodynamic processes at the interface between regions of the water and the steam. These parameters are the flow rate of bulk flashing (bubble rising) in the main water region, the mass flow rates of spray condensation, the interface condensation, the wall condensation, and the rainout (liquid droplet) in the steam region. The second one represented the PC’s variables, including the average temperature of the PC coolant, the temperature of the hot leg, and the inlet and outlet mass flow rates of the PC’s coolant. The following subsections will describe the dynamics equations of the mass, energy, and the temperature of both steam and main water regions and the buffer region dynamic. Consequently, the equation that governs the pressure of the PZR will be given. Finally, the primary circuit variables, such as the water mass dynamics and the surge flow rate, will be described.

#### 3.1. Steam region

For the steam region, \( \sum m_{\text{inlet}} \) and \( \sum m_{\text{outlet}} \) can be given as in (P. Wang et al., 2019):

\[
\sum m_{\text{inlet}} = m_{fl}, \quad \sum m_{\text{outlet}} = (m_{cv} + m_{cw} + m_{sc} + m_{ro}) \tag{5}
\]

where \( m_{fl}, m_{cv}, m_{cw}, m_{sc}, \) and \( m_{ro} \) stand to the bulk flashing flow rate in the main water region (kg/s), the mass flow rate of spray condensation, the wall condensation, the interface condensation, and the liquid droplet mass flow rates (kg/s), respectively. Hence, the mass conservation law defined in (1) can be adopted for the steam region as:

\[
\frac{dM_G}{dt} = V_G \frac{\partial P_G}{\partial P} \frac{dP}{dt} + \rho_G \frac{dV_G}{dt} + V_G \frac{\partial \rho_G}{\partial P} \frac{dh_G}{dt} = m_f - m_{cv} - m_{cw} - m_{sc} - m_{ro} \tag{6}
\]

According to the work of (P. Wang et al., 2019), \( m_{sc}, m_{cv}, m_{fl}, \) and \( m_{ro} \) can be described as in the following. Based on the assumption (2), \( m_{sc} \) is given by:

\[
m_{sc} = \frac{h_{fl} - h_{gw}}{h_{fl} - h_{gw}} \tag{7}
\]

where \( m_{gw} \) denotes to the spray flow rate (kg/s), \( h_{gw} \) (MJ/kg) is the specific enthalpy of spray water, \( h_{sc} \) (MJ/kg) stands to the specific enthalpy of superheated steam, and \( h_{fl} \) (MJ/kg) stands to the specific enthalpy of saturated water. In the case of the main water is subcooled, the flow rate of the condensation (i.e. \( m_{cv} \)) that can occur at the interface between regions of steam and main water is represented by (P. Wang et al., 2019):

\[
m_{cv} = \begin{cases} 0, & h_{fl} \geq h_{fl} \\ \beta m_{gw} \frac{h_{fl} - h_{gw}}{h_{fl} - h_{gw}}, & h_{fl} < h_{fl} \end{cases} \tag{8}
\]

\( \beta \) is a constant, and \( h_{fl} \) (MJ/kg) is the specific enthalpy of subcooled water. During the outsurge transient, the pressure of the PZR begins to decrease. As a result, water immediately flashes out from liquid to steam and reduce pressure increasing rate. The flashed water’s mass flow rate \( m_{fl} \) can be calculated based on the following relation (P. Wang et al., 2019):

\[
m_{fl} = \frac{-\xi_2}{h_{fl} - h_{gw}} \left( M_f \left( \frac{\partial h_{fl}}{\partial P} - V_f \right) \frac{dP}{dt} - \xi_1 m_{gw} (h_{fl} - h_{fl}) - q_{fl} \right) \tag{9}
\]

\( V_f (m^3/kg) \) is the saturated water-specific volume; \( m_{gw} \) (kg/s) is the surge flowrate; \( h_{gw} \) (MJ/kg) is the specific enthalpy of surge flow; \( q_{fl} \) is the power of the electric heater (MW); \( \xi_1 \) is a coefficient that is related to the source of outsurge flow, and it will be defined in the
buffer region subsection; $\xi_2$ is a coefficient that is related to the phase condition of the main water. The bulk flashing $m_{ro}$ in the region of main water has a zero value for the subcooled case, whereas, for the saturated case, we have $h_f = h$, $v_f = v$, and $m_{cv} = 0$

Rainout takes place when steam pressure drops down and leads to a decrease in the pressure and its rate $m_{ro}$ can be computed based on the following relation (P. Wang et al., 2019):

$$m_{ro} = \frac{\xi_2 M_G}{h_g - h_f} \left( \frac{\partial h_g}{\partial P} - v_f \right) \frac{dP}{dt} \cdot \xi_3 \begin{cases} 0, & \text{Superheated steam} \\ 1, & \text{Saturated steam} \end{cases}$$

$v_f$ (m3/kg) refer to the saturated steam specific volume; $\xi_3$ refer to a coefficient that is related to the phase condition of the steam region. When the wall temperature ($T_{wall}$) is lower than dew-point and steam temperature, the steam condenses on the wall and then falling to the water phase in the form of saturated water.

Based on the heat transfer between the wall and the water film, the wall condensation rate can be estimated as follows (Kim et al., 2006; You et al., 2001):

$$m_{cv} = K_{PZR} A_{PZR} \frac{T_{sat} - T_{wall}}{h_g - h_f}$$

$K_{PZR}$ denotes the heat transfer coefficient, $A_{PZR}$ is PZR cross-section area, $T_{sat}$ and $T_{wall}$ are the water and wall temperatures at saturated state, respectively.

To derive the rate of change of the steam temperature $\frac{dT}{dt}$, first the energy balance equation of the PZR steam region according to energy conservation law defined in (2), can be modified as:

$$M_G \frac{d(h_g)}{dt} + h_g \frac{d(M_G)}{dt} = \left( h_g - h_g \right) m_f + \left( h_g - h_f \right) m_{ro} + V_G \frac{dP}{dt} + q$$

where $V_G$ is the volume of the steam region (m3), the term $V_G \frac{dP}{dt}$ represents the mechanical work that arises from water's effect on the steam region during the insurge process. By defining $T_F$ and $T_G$ as the temperature of the water and the steam inside PZR, respectively, and $Q_{G,loss}$ refer to the power losses within the steam region; the energy part $q$ can be defined by:

$$q = K_{PZR}(T_F - T_G) - Q_{G,loss}$$

The term $K_{PZR}(T_F - T_G)$ denotes the part of energy resulting from the difference in temperature between steam and water regions. Second, the change of the steam enthalpy $\frac{d(h_g)}{dt}$ can be adopted based on equation (3) as:

$$\frac{d(h_g)}{dt} = C_{fs} \frac{dT_g}{dt} + C_{pp,s} \frac{dP}{dt}$$

Where,

$$C_{fs} = \frac{\partial h_d(\tau)_{ref}}{\partial T_g} + \frac{\partial^2 h_d(\tau)_{ref}}{\partial T_g^2} \left( T_g - \tau \right) C_{pp,s}$$

$$C_{pp,s} = \frac{\partial h_d(\tau)_{ref}}{\partial P} + \frac{\partial^2 h_d(\tau)_{ref}}{\partial P^2} (P - \tau)$$

From equation (14), then we have:

$$\frac{dT_g}{dt} = \frac{1}{C_{fs}} \left( \frac{dH_d}{dt} - C_{pp,s} \frac{dP}{dt} \right)$$

Finally, based on equations (6) and (12) and some mathematical steps, $\frac{dT}{dt}$ can be calculated as:

$$\frac{dT_g}{dt} = \frac{1}{C_{fs} M_G} \left[ -C_{pp,s} \frac{dP}{dt} \right] M_G$$

$$- h_g (m_f - m_{cv} - m_{cw} - m_{sc} - m_{ro}) + \rho V_G \frac{dP}{dt}$$

$$+ (h_g - h_f) m_f + (h_g - h_f) m_{ro}$$

$$+ K_{PZR}(T_F - T_G) - Q_{G,loss}$$

3.2. Water region

The change rate of water mass inside the PZR system can be described based on the conservation law of the mass balance of equation (1) by:

$$\frac{dM_w}{dt} = V_F \frac{\partial \rho_c}{\partial P} \frac{dP}{dt} + \rho_c \frac{dV_F}{dt} + V_F \frac{\partial \rho_c}{\partial T} \frac{dT}{dt}$$

$$= \xi_1 m_{sw} + m_{cw} + m_{sc} + m_{sp} + m_{ro} - m_f$$

3.2.1. Water regions

The coefficient $\xi_1$ is zero for insurge case and one for out surge case, and the other parameters $m_{cw}, m_{sc}, m_{sp}, m_{ro}$, and $m_f$ are defined before.

Using the same procedures used in the steam region, the dynamic equation of the water temperature inside PZR can be derived. For the water region, the energy balance defined in equation (2) can be modified as:

$$M_f \frac{d(h_f)}{dt} + h_f \frac{d(M_f)}{dt} = \xi_1 m_{uw} (h_{uw} - h_f)$$

$$+ (m_{sp} + m_{cw} + m_{sc} + m_{ro}) (h_f - h_f) - m_{sc} (h_g - h_f) - m_{ro}$$

$$+ K_{PZR}(T_F - T_G) + Q_h - Q_{f,loss}$$

Where, $V_F \frac{dP}{dt}$ is the mechanical work resulting from the effect of steam on the water region during the out surge process, $Q_{f,loss}$ is the power loss in the water region $Q_h$ is the heater power (MW) and $K_{PZR}(T_F - T_G)$ denotes the energy that arises from the temperature difference between water and steam. Like equation (16), the change of the water temperature is represented by:
\[
\frac{dT_F}{dt} = \frac{1}{C_{T,F}} \left( \frac{d\nu_F}{dt} - C_{p,F} \frac{dP}{dt} \right)
\]

(20)

\[
C_{T,F} \text{ and } C_{p,F} \text{ are defined as:}
\]

\[
C_{T,F} = \frac{\partial h_T(\rho_T, T)}{\partial T} + \frac{\partial^2 h_T(\rho_T, T)}{\partial T^2} (T_F - \rho_T, T), \quad C_{p,F} = \frac{\partial h_T(\rho_T, T)}{\partial P} + \frac{\partial^2 h_T(\rho_T, T)}{\partial P^2} (P_F - P)\]

(21)

Thus, using equations (18) and (19), we can obtain \( \frac{dT_F}{dt} \) by:

\[
\frac{dT_F}{dt} = \frac{1}{C_{T,F} \rho_t M_F} \left[ -C_{p,F} \frac{dP}{dt} M_F - h_T(\xi m_w + m_w + m_c + m_p + m_o - m_t) + \xi m_w \rho_t (h_w - h_T) + (m_p + m_c + m_o) \rho_t (h_c - h_T) + m_w (h_o - h_T) - m_o (h_m - h_T) + V_F \frac{dP}{dt} + K_{pZ}(T_F - T_o) - Q_{f,\text{loss}} + \frac{dG}{dt} \right]
\]

(22)

### 3.3. Buffer water region

According to the volume conservation of PZR in normal conditions, the change in the net volume of the buffer region can be obtained by:

\[
\frac{dV_b}{dt} = \left[ \frac{dV_F}{dt} + \frac{dV_G}{dt} \right]
\]

(23)

\( V_F \) (m³) is the water volume of PZR; \( V_G \) (m³) is the steam volume of PZR. In normal operation, the buffer region's liquid is subcooled; there is no mass and heat transfer between the buffer and main water regions. Consequently, only the mass conservation of the buffer region should be considered. In an outburst transient state, a surfue flow comes from the buffer water region before its volume decreases to zero, or an out-surge flow comes from the main water region. Hence, the buffer volume dynamic equation is represented by (P. Wang et al., 2019):

\[
\frac{dV_b}{dt} = \begin{cases} 
(1 - \xi_1)m_w V_b, & \xi_1 \\
0, & V_b > 0 \text{ or } m_w > 0 \\
1, & V_b = 0 \text{ and } m_w < 0
\end{cases}
\]

(24)

where \( V_b \) (m³/kg) refer to the specific volume of surge water.

### 3.4. The pressure of pressurizer system

To derive the equation that governs the estimation of the pressure inside the PZR, recall equations (23) and (24) as (P. Wang et al., 2019):

\[
\frac{dV_F}{dt} + \frac{dV_G}{dt} = -(1 - \xi_1)m_w V_b
\]

(25)

This equation can be rewritten as

\[
\frac{dM_G}{dt} \nu_G + M_G \left( \frac{d\nu_G}{dt} \right) = -(1 - \xi_1)m_w V_b
\]

(26)

Considering \( \frac{d\nu_F}{dt} = \left( \frac{\partial \nu_F}{\partial \rho} \right) \rho_F \frac{d\rho}{dt} \) and \( \frac{d\nu_G}{dt} = \left( \frac{\partial \nu_G}{\partial \rho} \right) \rho_G \frac{d\rho}{dt} \) We have:

\[
\frac{dM_G}{dt} \nu_G + M_G \left( \frac{d\nu_G}{dt} \right) \rho_G \frac{d\rho}{dt} + \left( \frac{\partial\nu_G}{\partial \rho} \right) \rho_G \frac{d\rho}{dt}
\]

(27)

Based on the mass and energy balance equation of the steam region defined in equations (6) and (12), the term \( M_G \frac{d(\rho_G)}{dt} \) can be obtained by:

\[
M_G \frac{d(\rho_G)}{dt} = -h_T(\xi m_w + m_w + m_c + m_p + m_o - m_t) + (h_o - h_T) m_o + V_F \frac{dP}{dt} + Q_{f,\text{loss}} + \frac{dG}{dt}
\]

(28)

Also, for the water region, using equations (18) and (19), the term \( M_F \left( \frac{d\rho_F}{dt} \right) \) is given by:

\[
M_F \left( \frac{d\rho_F}{dt} \right) = \left[ h_T(\xi m_w + m_w + m_c + m_p + m_o - m_t) \right] + \xi m_w \rho_t (h_w - h_T) + (m_p + m_c + m_o) \rho_t (h_c - h_T) - m_t (h_o - h_T) + V_F \frac{dP}{dt} + K_{pZ}(T_F - T_o) - Q_{f,\text{loss}} + \frac{dG}{dt}
\]

(29)

By substituting from equations (29) and (28) into equation (27) and after some mathematical steps, the pressure inside the PZR can be calculated by:

\[
\frac{dP}{dt} = -\left[ D + E \right] \frac{F}{F}
\]

(30)

where

\[
D = \frac{dM_G}{dt} V_G + (1 - \xi_1) m_o \frac{dV_b}{dt} + M_F \left( \frac{d\rho_F}{dt} \right) V_F + \left( \frac{\partial \rho_G}{\partial \rho} \right) \rho_G \frac{d\rho}{dt}
\]

\[
(m_o (h_G - h_T) + m_o (h_o - h_T) + k_{pZ}(T_F - T_o) - Q_{f,\text{loss}})
\]

\[
E = \left( \frac{\partial \rho_G}{\partial \rho} \right) \rho_G \left( \xi m_w (h_w - h_T) + (h_o - h_T) \right)
\]

\[
+ (m_p + m_c + m_o) (h_c - h_T) - m_t (h_o - h_T) + V_F \frac{dP}{dt} + \frac{dG}{dt}
\]

\[
+ Q_{f,\text{loss}} + k_{pZ}(T_F - T_o)
\]

\[
F = \left[ M_G \left( \frac{\partial \rho_G}{\partial \rho} \right) \right] + V_F \left( \frac{\partial \rho_G}{\partial \rho} \right) + M_F \left( \frac{\partial \rho_F}{\partial \rho} \right) + V_F \left( \frac{\partial \rho_F}{\partial \rho} \right)
\]

(31)

### 3.5. Primary circuit variables

In PWRs, changing the load power requires changing the pressure, either by increasing or decreasing. This pressure is related to the heater and sprays operation, and
respectively, it is related to the surge flow of water in/out the PZR. Thus, this subsection aims to derive the equation that describes the surge flow rate (i.e. $m_u$).

The PC internal energy can be defined as:

$$U_{PC} = c_{p,PC} T_{PC} M_{PC}$$

(31)

Then

$$\frac{dU_{PC}}{dt} = c_{p,PC} \left[ M_{PC} \frac{dT_{PC}}{dt} + T_{PC} \frac{dM_{PC}}{dt} \right]$$

(32)

Where, $c_{p,PC}$ is the specific heat at 282°C, $T_{PC}$ is the water temperature in the PC, and $M_{PC}$ is the water mass in PC that its change is given by:

$$\frac{dM_{PC}}{dt} = m_{in} - m_{out}$$

(33)

Based on (Fazekas et al., 2007) $\frac{dU_{PC}}{dt}$ can be computed in terms of the reactor power (i.e. $W_R$), the transferred energy to the secondary circuit through the four steam generators ($4\times W_{SG}$), the energy effect of both the inlet and surge mass flow rates, and the power loss to the environment $Q_{loss,PC}$ as:

$$\frac{dU_{PC}}{dt} = c_{p,PC} m in T_{PC,CL} - c_{p,PC} m out T_{PC,CL} + W_R - 4\times W_{SG} - Q_{loss,PC}$$

(34)

The hot leg, the cold leg temperatures of PC, and $W_{SG}$ are illustrated as:

$$T_{PC,HL} = T_{PC} + 15, \quad T_{PC,CL} = T_{PC} - 15,$$

and,

$$W_{SG} = K_{T,SG}(T_{PC} - T_{SG})$$

(35)

From equations (32–35), the rate of change of temperature of the PC temperature can be reformulated as:

$$\frac{dT_{PC}}{dt} = \frac{1}{c_{p,PC} M_{PC}} \left( c_{p,PC} m in T_{PC,CL} - c_{p,PC} m out (T_{PC,HL} - 30) + W_R - 4\times W_{SG} - Q_{loss,PC} \right)$$

+ $c_{p,PC} T_{PC}(m_{in} - m_{out})$

(36)

Finally, the surge flow rate can be obtained based on the following relation (Fazekas et al., 2007):\n
$$m_u = \frac{\frac{dM_{PC}}{dt}}{V_{PC}} \left[ \frac{\partial P(T_{PC})}{\partial T_{PC}} \right]$$

(37)

$\frac{dM_{PC}}{dt}$ and $\frac{\partial P(T_{PC})}{\partial T_{PC}}$ are given by equations (33) and (36), respectively, and is the density function $P(T_{PC})$ is approximated by a quadratic function as (Gábor et al., 2010):

$$P(T_{PC}) = a_0 + a_1 T_{PC} + a_2 T_{PC}^2$$

(38)

So

$$\frac{\partial P(T_{PC})}{\partial T_{PC}} = a_1 + 2a_2 T_{PC}$$

(39)

4. Developed model state space representation

In this section, the nonlinear equations (6), (17), (18), (22), and (23) of the PZR developed model are arranged in state-space form as defined in equation (40) to investigate their performance operation. The implementation of these state-space equations and the output equation is performed using MATLAB/Simulink (Mathworks, 2018) as illustrated in Figure 3. The implementation of Simulink of PZR system is shown at Figure 3.

\[
\begin{align*}
\dot{x}_1(t) &= f_1(X, U, t), \\
\dot{x}_2(t) &= f_2(X, U, t), \\
\dot{x}_3(t) &= f_3(X, U, t), \\
\dot{x}_4(t) &= f_4(X, U, t), \\
y(t) &= g(X, U, t),
\end{align*}
\]

(40)

Where,

System states: $X(t) = [M_{PC}, M_F, M_G, T_F, T_G]^T$

System Inputs: $U(t) = [m_{in}, m_{out}, T_{PC}, T_{PC,k}]^T$

System Output: $y(t) = [dP/dt]$

\[
\begin{align*}
\dot{x}_1 &= u_1 - u_2 - m_u \\
&= u_1 - u_2 \\
&- [(u_1 - 31 u_2)] \\
x_1 &= V^pca_1(u_1 T_{PC}) \\
&+ V^pca_1(Q_{loss,PC} - W_R + 4\times W_{SG}) \\
&+ 2 V^pca_1(Q_{loss,PC} + 4\times W_{SG}) \\
&+ W_R - 30 u_3 c_{p,PC}^2 \\
&+ u_3 + (2 V^pca_2 u_2^2 + 2 V^pca_2 u_2) X_1
\end{align*}
\]

\[
\begin{align*}
\dot{x}_2 &= \xi_1 \left[ (u_1 T_{PC}) \right] \\
&+ \left[ \phi \left( c_{p,PC} X_1 \right) \right] \\
&+ \left[ \phi \left( u_1 u_2 \right) \right] \\
&+ \left[ \phi \left( u_1 - u_2 \right) \right] \\
&+ u_3 + (2 V^pca_2 u_2^2 + 2 V^pca_2 u_2) X_1
\end{align*}
\]

(41)

\[
\begin{align*}
\dot{x}_3 &= m_{sc} + m_{sp} + m_{ro} - m_{h}
\end{align*}
\]

\[
\begin{align*}
\dot{x}_4 &= m_{ro} - m_{cv} - m_{cw}
\end{align*}
\]

(42)

(43)
5. Results

Through this section, first, the developed model parameters are estimated. Second, the verification of the development is performed. Here, model verification tests the model behaviors in terms of the engineering expectation of the PZR system. Third, the comparison between the developed and three region models is given (P. Wang et al., 2019).

To collect the measured data for the model parameter estimation, model verification, and comparison purpose, a typical PCTRAN VVER-1200 simulator is shown in Figure 4. The PCTRAN VVER-1200 was used in this study. A Personal Transient Computer Analyzer (PCTRAN) is personal computer-based software developed by Micro Simulation Technology Inc., USA, and used for reactor transient and accident analysis simulation for various reactors (Khan & Islam, 2019; Mollah, 2018). The load power of the PZR unit is allowed to increase and decrease between 100% and 70%, and the obtained transient data is used to study a wide operating domain. The PCTRAN VVER-1200 simulator’s recorded values were sampled using a uniform sampler with a 5 sec sampling time. It is noted that the collected data from the VVER-1200 simulator are taken under the closed-loop control operation of the PZR system. Thus, the excitation of the system was provided by the changes in the load power.

5.1. Model parameters estimation

The developed model described in the last section is highly nonlinear. It is also time-dependent that require different parameters to be estimated. The model variables were grouped into input variables, output variables, and the estimated parameters variables. The input variables to the model were the load power change $W_L$ and consequently, the feed water $m_{in}$, the outlet flow rate $m_{out}$, the average temperature of the coolant $T_{PC}$, the temperature of the hot leg $T_{HL}$, the saturation temperature $T_{sat}$, the spray flow rate $m_{spr}$, heater power $q_h$, are illustrated in Figures 5 to 7. The measured pressure of
Figure 4. PCTRAN VVER-1200 simulator for NPP.

(a) (b)

Figure 5. (a) Change of PWR load power, (b) Inlet flow rate $m_{in}$ and the Outlet flow rate $m_{out}$.

Figure 6. Cold-leg $T_{PCCL}$, Hot-leg $T_{PCHL}$, average $T_{PC}$ and saturation $T_{sat}$ Temperatures.
the PZR given from the simulator is considered the desired output for the identified model. Steam tables are used to know the thermo-dynamical properties as the specific heats, temperature-dependent densities, and pressure of saturated vapor and water (Kretzschmar & Wagner, 2019). However, the specific heat of steam is very sensitive to its state, either if it is in a saturated state or not. The Nomenclature is presented at Table 1. The thermo-dynamical and geometrical variables given in Table 2 were the developed model's estimated parameters. The Change of Load power with inlet and outlet mass flow rate is shown at Figure 5.

The aim now is to fit the measured data best, therefore based on the model imperfection and the previously discussed assumptions, the estimation of the mentioned parameters is allowed with a maximum of ± 2–5% change in their values. The water TF and steam TG temperatures and the PC’s coolant mass are initially known according to measured data from the simulator. These initial values are also having the limitation of a 5% change in their estimated values. The other parameters have no limitation in their estimation. The only limitation in their values is that they should have positive values. These parameters are the constants like $K_{PC}$, $Q_{Gloss}$, and $Q_{pc}$, which describe the heat transfer, respectively, through the interface of water and steam and the losses through the steam and water regions.

The estimation of model parameters is an optimization problem used to achieve some objective function, which should have some constraints to have values in a physically meaningful range. The traditional Least Square (LS) algorithm is difficult to be applied for the identification process due to the model non-linearity, the constrained of the objective function, and the embedded controller. Therefore, it is required to select a simple and effective numerical optimization method without estimating the objective function gradient.

**Figure 7.** (a) Heater power $q_{th}$ (b) Spray mass flow rate $m_{sp}$.

**Table 1. Nomenclature .**

| Symbol | Parameter | Value | Unit |
|--------|-----------|-------|------|
| $W_R$ | Reactor power | $1.17 \times 10^9$ | W |
| $m_{PC}$ | Overall mass in the PC | | |
| $T_{pc}$ | Water average temperature | | |
| $m_{th}$ | the flow rate of the inlet mass. | | |
| $m_{out}$ | Outlet mass flow rate | | |
| $T_{pc,HL}$ | Hot leg water temperature | | |
| $T_{pc,CL}$ | Cold leg water temperature | | |
| $T_{pc,i}$ | Inlet temperature to PC | 298 | C |
| $c_{p,pc}$ | Specific heat at 282°C | | |
| $Q_{loss}$ | Heat transfer coefficient | | |
| $V_{pc,0}$ | Heat loss of PC | | |
| $M_{pc,0}$ | Water nominal volume | 242 | m$^3$ |
| $\Delta$ | Water nominal mass | 86,000 | kg |
| $\Delta$ | Differences $T_{pc,HL} - T_{pc} = T_{pc} - T_{pc,CL}$ | 15 | C |
| $T_p$ | Water (Fluid) temperature | 347.9 | C |
| $T_G$ | Steam (Gas) temperature | 347.9 | C |
| $q_{th}$ | Maximum Heating power (back up and variable) | 2520 | kW |
| $I$ | Water level (at rated power) | 11.8 | m |
| $p$ | Pressure | 16.2 | Mpa |
| $c_{p,PC}$ | Specific heat of water at 325°C | 6873.1 | J/kg/K |
| $Q_{loss}$ | Heat loss | | |
| $M_R$ | Water mass | 19,400 | kg |
| $M_W$ | Steam mass | 10,000 | kg |
| $A_{PC}$ | Vessel cross-section area | 33 | cm$^2$ |
| $V_{PC}$ | Vessel volume | 79 | m$^3$ |
| $m_{sp}$ | Maximum spray flowrate | 37 | t/h |
| $V$ | Buffer region volume | 1.5 | m$^3$ |
| $V_{PC}$ | Fully-open pressure of variable heater | 16.04 | Mpa |
| $\alpha_0$ | Opening pressure of backup heater | 16.04 | Mpa |
| $\alpha_1$ | Closing pressure of backup heater | 16.09 | Mpa |
| $\alpha_2$ | Opening pressure of spray valve | 16.25 | Mpa |
| $\alpha_3$ | Fully-open pressure of spray valve | 16.26 | Mpa |
| $\alpha_4$ | Opening pressure of variable heater | 16.19 | Mpa |
| $\alpha_5$ | Density Factors | 581.2 | kg/m$^3$ |
| $\alpha_6$ | $2.98$ | kg/m$^3$ |
| $\alpha_7$ | $-0.00848$ | m$^2$/K |

The Pattern search method (D’Angelo & Palmieri, 2021; Zhang & Zhang, 2020) is the applied...
identification method that is used as an optimization-based parameter estimation method. This method uses the Sum of Square of Error (SSE) as the objective function, which is a factor that uses the 2-norm between the measured and the model computed output signals to measure the data fit. Genetic algorithm (GA) is a meta-heuristic search inspired by the natural evolution process to solve optimization problems. GA utilizes different processes like mutation, selection, crossover, and inheritance. It is characterized by its independence on the starting point of the search functions. Also, it did not require the knowledge of information about the fitness or constraint function. Generally, GA searches for the global optima of a fitness function rather than its local optima (D’Angelo & Palmieri, 2021; D. Zhang & Zhang, 2020). Consequently, the hybrid Pattern search- Genetic (PS-GA) algorithm (Polak & Wetter, 2003; Sayyaadi, 2021) is implemented here to the developed PZR model parameters identification process as shown in Figure 8. The values of estimated parameters are shown in Table 2 throw 1000 iteration with a Mean Squared Error (MSE) of 0.1179.

### 5.2. Model verification

The previous subsystems that were identified separately were integrated into one model. This model is described as shown above by the derived state equations. The state variables trajectories were estimated from the model, and their initial values are determined based on the measured data. Model verification has been performed during a change in the load power from 100% to 75% to evaluate the developed PZR model’s performance. The model’s verification is tested using the identified parameters; the initial states of these parameters are also selected to correspond to the measured data’s steady-state periods. The output pressure behavior of the developed PZR model is analyzed and verified based on the measured data collected from the PCTran VVER-1200 simulator.

These measured data are indicated in Figures 9, 10, and 11 for the load power change $W_{in}$ and consequently the feed water $m_{in}$, the outlet flow rate $m_{out}$, the average temperature of the coolant $T_{PC}$, the temperature of the hot leg $T_{HL}$, the saturation temperature $T_{sat}$, the spray flow rate $m_{spr}$, heater power $q_{he}$ respectively. It is noted that the temperature of the hot leg responds to the decrease of the load power of Figure 9 by decreasing its value as shown in Figure 10(b) through the absorption of more neutrons by the reactor rods, while the cold leg temperature increases as the steam generator use a small portion of its steam for the turbine which increases the returned water temperature of the cold leg. Accordingly, the average temperature of the PC takes the same manner. Also, it is noted that both the heater and spray of Figure 11(a, b) are used to make the PZR pressure around its setpoint value by letting the heater on and spray off when the PZR pressure is less than the adjusted setpoint and vice versa. The selected set point for the PZR pressure of the PCTran VVER-1200 simulator is 162 bar.

Figures 12 to 15 indicate the obtained time variation of the flow rates, states, and output variables for the developed model in response to load power changes. These flow rates are rainout flow rate, spray condensation, bulk flashing flow rate, wall condensation, and interface condensation. The spray and interface condensation showed in Figure 12(a, b),

### Table 2. Estimated parameters for developed model.

| Parameter | Value  | Unit |
|-----------|--------|------|
| $W_{in}$,SG | 1.9$x10^5$ | W    |
| $K_{sg}$     | 9.5296$x10^6$ | W/K  |
| $K_{a2}$     | 811.59  | W/K  |
| $Q_{in}$,PC  | 2.29$x10^5$ | W   |
| $Q_{in}$,loss| 90      | W    |
| $Q_{out}$,PZ | 1.6$x10^2$ | W   |
| $C_{p,PC}$   | 5335    | J/kg/K|
| $C_{p,PC}$   | 4.3439  | J/kg/K|
| $C_{p,s}$    | 2.51225 | J/kg/K|
| $C_{p,he}$   | 9.6988  | J/kg/K|
| $C_{p,spr}$  | 15.72614| J/kg/K|

![Figure 8](image-url) Block diagram of the developed model parameters estimation process.
Figure 9. Change of PWR load power.

Figure 10. (a) Inlet and Outlet flow rates, (b) Cold-leg, Hot-leg, average, and saturation temperatures.

Figure 11. (a) Heater power $P_{\text{heater}}$, (b) Spray mass flow rate $m_{\text{spr}}$. 
Figure 12. (a) Spray condensation flow rate $m_{sc}$ (b) Interface condensation $m_{cv}$.

Figure 13. (a) Wall condensation flow rate $m_{cw}$ (b) Rainout the condensation flow rate $m_{ro}$.

Figure 14. (a) Surge line flow rate $m_{lw}$ (b) Bulk flashing flow rate $m_{fl}$. 
respectively, follow the spray flow rate variation. As shown, they have a zero value when the spray valve is closed, while they increase with the percentage of open of the spray valve. The bulk flashing flow rate of Figure 14 (b) follows the variation of heater power. The surge line flow rate of Figure 14 (a) indicates an increase of its value (i.e. insurge) to increase the PZR pressure when it is below the setpoint and vice versa for the outsurge case. Figure 15 (a, b) indicate the influence of the PZR water and steam temperature on either the heater or the spray, which is affected by how far the PZR pressure from the adjusted set point.

The obtained pressure response of the developed PZR model resulted due to the previously mentioned load power change is shown in Figure 15. It indicates great compatibility between the developed PZR model (Simulation data) with the simulator (Measured data). The expectations are satisfied by the developed model response’s obtained response, as shown in Figures 12 to 15 in each state’s output. The obtained responses have met both the qualitative and the quantitative requirements in both steady-state and approximate time constants. Hence, demonstrating the accuracy and feasibility of the developed model for controller design.

5.3. Model validation

5.3.1. First scenario

The performance of the developed PZR model is validated by analyzing its output pressure behavior for a wide range of the measured input data given in Figures 16 to 18 for the load power change $W_{LH}$ and consequently the feed water $m_{in}$, the outlet flow rate $m_{out}$, the average temperature of the coolant $T_{PC}$, the temperature of the hot leg $T_{HL}$, the saturation temperature $T_{sat}$, the spray flow rate $m_{sp}$, heater power $P_{h}$, respectively. These measured data are collected

![Figure 15](image1.png)

(a) water temperature $T_f$ and steam temperature $T_G$. (b) PZR simulation and measured pressure.

![Figure 16](image2.png)

(a) Change of PWR load power. (b) Inlet flow rate $m_{in}$ and the Outlet flow rate $m_{out}$. 

Figure 15. (a) water temperature $T_f$ and steam temperature $T_G$. (b) PZR simulation and measured pressure.
through an operation time of 900 seconds of the PCTran VVER-1200 simulator. Also, the estimated parameters are used for the validation process. During the validation scenario, the load power is allowed to change from 100% to 75%, and then it increases to 90% and follows by another decrease to 70% to check the performance effectiveness and robustness of the developed model. This comparative simulation aims to investigate the relationship between each parameter of PZR.

The obtained time variation of the flow rates, states, and output variables for the developed model in response to load power changes are indicated in Figures 19 to 22. For example, spray flow rate variation appears on both spray and interface condensation as shown in Figure 19 (a, b), where these condensations have a zero value when the spray valve is closed and increase with the percentage of open of the spray valve. Also, the effect of the change rate of pressure appears obviously on the flow rate of rain out as shown in Figure 20 (b), where the rain outflow rate has a positive or negative value for the case of pressure increase or decrease, respectively. Furthermore, the flow rate of bulk flashing follows the variation of heater power, as indicated in Figure 21 (b). The surge line flow rate of Figure 21 (a) changes its status from insure to outsurge to decrease the PZR pressure when it is above the setpoint and vice versa. Figure 22 (a) indicates the influence of the PZR water and steam temperature on the operation of either the heater or the spray, which in turn are affected by how far the PZR pressure from the adjusted set point.
Figure 19. (a) Spray condensation flow rate \( m_{sc} \) (b) Interface condensation \( m_{cv} \).

Figure 20. (a) Wall condensation flow rate \( m_{cw} \), (b) Rainout the condensation flow rate \( m_{ro} \).

Figure 21. (a) Surge line flow rate \( m_{lw} \) (b) Bulk flashing flow rate \( m_{bl} \).
The developed model’s obtained pressure response according to the previously mentioned load power change, and the considered inputs are illustrated in Figure 22 (b). Firstly, it shows a slight increase due to coolant heat up, and then the pressure decreases during the initial cool down due to the control rod insertion (until about 300 seconds). This pressure is restored to the set point of 162 bar by the actuation of the PZR’s electric heater as the reactor power reaches a steady-state corresponding to the 100% turbine power. The RCS pressure is maintained at a constant value at all operating conditions. The great compatibility between the developed model’s obtained pressures with that of the simulator, as shown in Figure 22 (b). Generally, the different responses of the developed model is shown in Figures 20 to 23 satisfy the state and output variables’ engineering expectations. The developed model response meets both the qualitative and the quantitative requirements in both steady-state and approximate time constants. Hence, demonstrating the accuracy and feasibility of the developed model for controller design.

6. Comparison results

To test, evaluate, and analyze the developed non-equilibrium three-region PZR model, we compare its performance with another old model (Gábor et al., 2010). The performance of the two models is compared with the measured data from the PCTran VVER 1200 simulator. Both models are implemented under MATLAB/Simulink [34], and they are subjected to the same conditions (inputs, disturbances, and load change). The developed model’s implementation is divided into three stages: input, state, and output stages. The block diagram of the PZR system model is illustrated in Figure 23. Finally, the developed model’s performance evaluation is analyzed carefully with other models using data simulation in Table 1 as shown in Figure 24.

6.1. Second scenario

In this part, the malfunction loss of reactor coolant inlet flow is applied (as an impulse signal). The reactor power is reduced by about 30% without
control rod insertion, as shown in Figure 25. This decrease is due to the moderator flow in the core and the negative moderator temperature coefficient’s effect – as the coolant’s temperature increases. After the reactor and turbine trips, a negative flow rate is observed in the reactor coolant pump (RCP), indicating the loop’s flow direction is reversed (Khan & Islam, 2019; Mollah, 2018).

The average PC temperature $T_{avg}$ increases after the RCP is manually tripped due to decreased coolant forced circulation in the loop. Due to the low coolant flow rate, the reactor trip signal is produced within about 10 seconds. The turbine often trips during the reactor ride, and natural circulation is gradually formed in the loop that helps eliminate the residual heat in the heart. The flow rate in the loop decreases when the coolant flow has been changed, and the cold-leg temperature of this loop is close to the temperature of the hot-leg, as shown in Figure 26.

The surge flow rate caused by the swell and shrink of RCS coolant during the transients was calculated using measured data from the PCtran VVER-1200 simulator. The obtained time variation of the flow rates, states, and output variables for the developed model in response to load power changes are indicated in Figures 28 to 30. The obtained pressure response of the developed PZR model resulted due to the previously mentioned load power change is shown in Figure 31 (b). It indicates great compatibility between the developed PZR model (Simulation data) with the simulator (Measured data).

7. Pressure control system for non-linear PZR developed model

PZR is a vital part of the reactor system. It is connected to a loop system and is often used for either positive or negative fluctuation. The transient modification of the load within the NPP is the

Figure 24. PZR Pressure of old model, developed model, and measured data.

Figure 25. (a) Change of PWR load power. (b) Inlet flow rate $m_{in}$, Outlet flow rate $m_{out}$.
Figure 26. Cold-leg $T_{PC,CL}$, Hot-leg $T_{PC,HL}$, average $T_{PC}$ and saturation $T_{sat}$ temperature.

Figure 27. (a) Heater power $q_h$, (b) Spray mass flow rate $m_{sp}$.

Figure 28. (a) Surge flowrate of pressurizer system, (b) Interface condensation.
Figure 29. (a) Spray Condensation, (b) Rainout Condensation.

Figure 30. (a) Bulk flashing, (b) Wall Condensation.

Figure 31. (a) Steam and Water Temperatures, (b) Developed model and PCtran PZR Pressure.
cause of these fluctuations. Regulating the reactor main circuit’s pressure requires maintaining the balance of the PZR water and steam saturation. Its main role is to regulate the pressure of the main loop in the allowable range.

The adjustment of both the flow rate of the spray and the electrical heater is the usual way to control the pressure of PZR. The adjustment is accomplished according to a control signal based on an input error signal. Therefore, the controller is usually called the pressure error controller. Figure 32 illustrates a simplified schematic diagram for a pressure control system of a PZR. The control system aims to restore the pressure’s deviation to a predetermined target value by maintaining the pressure’s overshoot at steady-state conditions no more than 0.2 MPa and below 1 MPa during transient operations. The PZR pressure is controlled by controlling the spray valve operation and electric heaters, including variable and backup ones. The control signal generator receives a signal of pressure error to generate actuation signals to control both the spray valve and electric heaters’ operation. During steady-state conditions, the Backup heater is closed while the variable heater is partly open to compensate for heat loss. The variable heater controls the small variations during transient operations. In case of an actual pressure with a value smaller than the determined set point and a value of absolute error greater than 0.17 MPa, the backup heater will behave in a way that opens completely to increase the pressure until the absolute error of pressure remains lower than a value of 0.117 MPa. When the absolute error is greater than 0.1 MPa, the variable heater will stay at closed status while the spray valve will stay at closed status until increasing the error to a level greater than a 0.17 MPa value. With the increase of the error signal from 0.17 to 0.52, the spray valve responds in a way that opened gradually until opening fully at 0.52 error signal.

The PZR simulation platform is developed using the environment of MATLAB/Simulink. Also, the non-equilibrium three-region model of the PZR and its pressure control system is implemented. The nonlinear model of the PZR is solved by selecting the fixed-step ode45 solver of the Simulink. The proposed block diagram for PZR pressure control is shown in Figure 33.

The PZR heaters are divided into two groups: one bank of variable heaters and several other backup banks on/off heaters. The variable heaters are operated by varied the applied voltage, which determines their heat output over a set pressure range. These heaters maintain the equilibrium heat balance within the PZR throughout steady-state conditions.

The PZR pressure control system’s control strategy using PID controller is illustrated as shown in Figure 34, where a delayed version of the con-

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**Figure 32.** Pressure control signal generating program of the PZR.

**Figure 33.** General block diagram of PZR controller.
trolled output pressure with a time delay value of $\Delta t$ is used as an input to the PZR model. Accordingly, the applied controller will respond to the output pressure variation with another change in the control variables to stabilize the specified set point's output pressure. The plant operation variables are used to provide transients in the system. In our case, the reactor power is used to do transients in the range [70%, 100%]. The implemented model using MATLAB/Simulink with PID controller is illustrated in Figure 35.

The obtained systems response during these transients is indicated in Figures 36 to 38 with a continuous line to indicate the pressure's set point, while the upper and bottom dashed lines indicate the actuation of both heaters and spray, respectively. As indicated in Figure 36, the conventional P controller does not settle the obtained pressure at its target value but retain a steady-state error. The obtained responses conclude the superiority to apply the PI controller for control purposes of the PZR pressure with better pressure response with a similar actuation of spray and heaters than the P conventional controller, as illustrated in Figure 37. The obtained PID response to the PZR system is illustrated in Figure 38. Although the controller's derivative term slows the transient response, this term is highly sensitive to noise in the error term and causes instability in the control process. Table 3 summarizes the best values of controller gains. Table 4 summarizes the response of the three controller types in terms of the ITAE. The ITAE obtained using PID controller showed relative reduction for all controllers compared to P and PI controllers.

8. Conclusion

In this study, a mathematical model of a non-equilibrium three-region PZR system with important thermodynamic processes is developed. Several assumptions are considered during deriving the

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**Figure 34.** Control system inputs/output variables.

**Figure 35.** Implementation of pressurizer system control using MATLAB/Simulink.
Figure 36. Conventional P controller response.

Figure 37. Conventional PI controller response.
The pressure control strategy for the developed non-linear and non-equilibrium PZR model is evaluated by applying the PID controller. In this strategy, the regulation of both the heater’s power and the opening percentage of the spray valve is performed to remain the PZR pressure at a certain predetermined set point. The regulation is performed through a control signal generating program with a PID compensated error signal of the output pressure at its input. Responses of the PZR pressure have shown effective performances of the control. The obtained responses also indicate that the designed controllers have satisfied robustness without challenges for any constraints of the operational or any safety limitation regardless of the adopted PZR simulation model, demonstrating the feasibility and effectiveness of the developed PZR model for dynamic simulation and controller design.

Disclosure statement

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