A LINMAP Method Based on the Bounded Rationality of Evaluators for Property Service Quality Evaluation

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ABSTRACT Property service quality (PSQ) evaluation is the key means of property management. Traditional PSQ evaluation is usually based on the rational hypothesis of the evaluators. The evaluators’ behaviour with bounded rationality is more in line with the characteristics of modern PSQ evaluation practice. At this point, evaluation information is usually incomplete information and heterogeneous data. The existing LINMAP methods have the ability to process incomplete information and heterogeneous information, but cannot adapt to evaluation information processing under bounded rational behaviour. As one of the major achievements of behavioural economics, prospect theory can explain people’s bounded behaviour when making decisions. The purpose of this paper was to develop a linear programming technique for multidimensional analysis of preference (LINMAP) model based on prospect theory for modern PSQ evaluation. First, a data preprocessing method was designed by referring to the TOPSIS method. Then, the value function was used to construct the dominance matrix, and the LINMAP model based on prospect theory was constructed. Finally, all alternatives were ranked in order according to overall performance value, which was solved by the LINMAP model. The feasibility and effectiveness of the proposed method were demonstrated by example analysis. Based on the evaluation results of PSQ of public projects, the important items and excellent property service projects and their reasons were determined, and some suggestions were proposed to improve PSQ.

INDEX TERMS Bounded rationality, linear programming technique for multidimensional analysis of preference, prospect theory, property service quality evaluation.

I. INTRODUCTION

With the popularization of new public service theory, service quality evaluation based on public products has become the basic means of public service management. As the size of the economy continues to grow, the Chinese government began to advocate “high-quality development” in October 2017. In March 2021, the Chinese government promulgated the “National Basic Public Service Standards”, which marked the transformation of public service management from emphasizing quantity to emphasizing quality.

In June 2021, the Chinese government decided to support Zhejiang Province in building a high-quality development demonstration zone with the basic goal of achieving common prosperity. The property service industry needs high-quality development. Research on property service quality (PSQ) of public building has important theoretical and practical significance.

Perceived service quality reflects users’ actual perceived level of service quality. Perceived service quality evaluation is the key link of service quality management. Since Gronroos put forward the concept of perceived service quality [1], a relatively complete method system has been formed for the evaluation of perceived service quality. Evaluation models,
such as SCSB (Sweden, 1989), ACSI (USA, 1994), ECSI (EU, 1999) [2] and CCSI (China, 2002), are constructed and applied in the macro field, while the evaluation method based on the SERVQUAL model [3], [4] is applied in the micro field.

The existing perceived service quality evaluation method is based on the assumption that the evaluator’s behaviour is rational, which is suitable for the evaluation environment of basic information with simple data structure. In the real environment, the subjective behaviour of perceived service quality evaluation is closer to bounded rationality, and the basic data becomes complex. At the same time, the basic evaluation information may come from different types of evaluation subjects and adopt different data types. Taking aim at the problem that the existing evaluation methods cannot effectively measure the perceived service quality in the real environment, this paper introduces classical decision theory and methods, innovates and applies perceived service quality evaluation methods, and develops perceived service quality theory.

The linear programming technique for multidimensional analysis of preference (LINMAP) is a method that can process different types of basic data at the same time [5]–[7]. However, it cannot solve the basic data processing problem based on the evaluator’s bounded rational behaviour. Prospect theory provides a framework for describing people’s bounded rational behaviour in decision making [8], [9]. By selecting appropriate parameters, the value function of prospect theory can be as close as possible to the actual processing of basic evaluation data. Therefore, the research goals of this paper are as follows: (1) It is necessary to propose a normalized data processing method for heterogeneous information that is suitable for the LINMAP model; (2) it is necessary to construct the LINMAP model based on prospect theory under the condition of bounded rationality of decision makers; (3) it is necessary to design a PSQ evaluation method for the characteristics of incomplete, heterogeneous and bounded rationality; and (4) the feasibility and effectiveness of the LINMAP model based on prospect theory need to be verified. Obviously, the LINMAP model based on prospect theory is the core of the proposed method in this paper. It has the performance of processing different types of subject evaluation data, but also has the advantage of accurately describing bounded rational behaviour.

The rest of this paper is arranged as follows: In Section II, the literature related to this research topic is reviewed; in Section III, the value function of prospect theory and the classical LINMAP model are introduced; in Section IV, the principle of the LINMAP method based on bounded rationality of evaluators is described; in Section V, the real example of PSQ evaluation is analysed by the above LINMAP method; in Section VI, the proposed method is compared with the classic method by analysis of a real example, and the advantages of the proposed method are illustrated; and finally in Section VII, we draw a brief conclusion and look ahead to future research.

II. LITERATURE REVIEW
Taking account of the influence of the decision maker’s behaviour on the evaluation method, this paper proposes a new PSQ evaluation method based on the classic decision method. Therefore, the literature review includes the following three aspects: decision behaviour and prospect theory, related decision-making methods and their application, service quality evaluation method and its application.

A. DECISION BEHAVIOUR AND PROSPECT THEORY
Mainstream economics defines subject behaviour as complete rationality or bounded rationality. The rational hypothesis holds that the behaviour subject has ordered preferences, complete knowledge, strong computing power and always makes the best choice [8]. The rational economic people hypothesis is the cornerstone of classical and neoclassical economics, but it is far from the behaviour characteristics of people in real decision-making environments. Simon’s hypothesis of bounded rationality broke through the basic paradigm of neoclassical economics and laid the theoretical foundation of modern management. The bounded rationality hypothesis holds that the behaviour subject in the real economic environment has “human defects”. That is, they pursue rationality rather than maximum pursuit and choose on the basis of the satisfaction standard rather than the optimal standard [10].

Expected utility theory based on the rational hypothesis has a dominant position in traditional multi-attribute decision making [11]. The hypothesis of a rational person in traditional economics holds that people will make decisions by rational calculation and choose the option of utility maximization [12]. Decision-making practice shows that the hypothesis of rational persons not only overestimates people’s cognitive ability, but also ignores the uncertainty of the decision-making environment [13]. The bounded rationality proposed by Simon is more in line with the behavioural characteristics of decision makers in practice, which has been widely recognized [14]. Prospect theory provides a framework for describing bounded rationality in decision making [8]. Based on the hypothesis of decision maker bounded rationality, prospect theory can describe the characteristics of complex psychological behaviours of evaluators in PSQ evaluation practice. Prospect theory holds that the psychological behaviour of decision makers goes through two stages: editing and evaluation [8], [9]. The editing phase includes coding, combination, segregation, cancellation, simplification and detection of dominance. During the editing phase, various heuristic choices are used to simplify the risk situation, and individuals collect and process information by means of the framing effect and reference point. During the evaluation phase, individuals evaluate the edited prospect value and make a choice, which evaluates the information by using the value function and subjective probability weight function [9]. Using prospect theory, some scholars have proposed three-way decisions [11], regret theory [12] and group decision-making [13],
while others have discussed applications in supply chain management [14], multi-attribute reverse auctions [15] and freeway driver route choice [16]. This paper introduces prospect theory to describe the bounded rationality of decision makers.

**B. RELATED DECISION-MAKING METHODS AND THEIR EXTENSIONS**

Some decision methods that could be used to solve the relevant problems of this paper include the linear programming technique for multidimensional analysis of preference (LINMAP) method [5], the technique for order preference by similarity to an ideal solution (TOPSIS) method [17], the interactive and multiple attribute decision making (TODIM) method [18], the multi-index and multi-scale (MAMS) method [7], the ORNESS measure [19], Q-ROFS [20] and MSM Operator [21]. Considering their relevance to the core topic, the LINMAP and TOPSIS methods are mainly reviewed below.

As an objective weight method, the LINMAP method determines the consistency and inconsistency through the relationship between different evaluation values of alternatives and solves the related unknowns by constructing a linear programming model [5]. Scholars have used the LINMAP method to process fuzzy numbers [22], [23], intuitionistic fuzzy numbers [24], [25], heterogeneous data [26], dynamic data [27], [28] and large-scale data [6]. However, the hypothesis of risk neutrality is the basis of the above studies. The existing LINMAP studies do not solve the problem of evaluators’ psychological behaviour based on the hypothesis of bounded rationality.

The TOPSIS method was first proposed by Hwang and Yoon [29] and is a kind of sorting method close to the ideal solution, which only requires that each utility function has a monotonically increasing (or decreasing) property. The Euclidean distance between the evaluated alternative and the two solutions is calculated to determine the merits of the alternative. Obviously, the basic principles of the TOPSIS and LINMAP methods have much in common. The TOPSIS method is usually enhanced in combination with other methods, such as ANN [30], QFD [31], and AHP [32]. The TOPSIS method has been widely used in water resource management [33], emergency rescue [34] and material selection [35].

**C. SERVICE QUALITY EVALUATION METHOD AND ITS APPLICATION**

Service quality includes satisfaction and perceived service quality. In 1965, Cardoso first proposed the concept of customer satisfaction. Subsequently, Anderson et al. carried out a systematic study on the satisfaction measurement theory, in which expectation difference theory and its impact on product performance were mainly discussed [36]. In the early 1980s, Gronroos pioneered the theoretical study of perceived service quality. With the development of the service industry, customer perceived service quality evaluation has become a hot research topic [1]. In the practice of property service quality evaluation in China, the satisfaction evaluation of service projects is usually carried out on-site or online, and the evaluation method is usually a simple average method.

Research on service quality mainly focuses on evaluation dimensions and models, evaluation methods and their applications [37]. The dimension division of service quality evaluation is the basis for constructing an evaluation model. Gronroos divided service quality into technical quality and functional quality [1], and then expanded the elements of service quality into seven dimensions according to the characteristics of employees, customers and service [38].

Letinen et al. believe that service quality is composed of three dimensions: interactivity, tangibility and company quality [39]. Juran [40] proposed the five elements of service quality to include technical quality, psychological quality, time quality, relationship quality and moral quality as the contents. Parasuraman et al. summarized general service quality factors, such as reliability, responsiveness and competence through empirical research [3]. Based on the above research, some scholars have proposed specific views of dimension division in combination with the characteristics of banking, e-government, library and public service industries [41]. The evaluation model is the core of theoretical research on customer perceived service quality, among which the SERVQUAL model [4] proposed by Parasuraman et al. has the greatest impact. The SERVQUAL model is composed of five service quality dimensions, and the measured values of perceived service quality are obtained through a 22-item questionnaire. In order to solve the problem that the empirical test of the SERVQUAL model is insufficient, Cronin and Taylor proposed the SERVPERF evaluation model [42].

The practical application of PSQ evaluation has been very common. Most property service companies take PSQ evaluation as the basic means of enterprise management, but the relevant theoretical research is not enough. Existing theoretical studies use the entropy weight method [43], analytic hierarchy process [44], [45], fuzzy evaluation method [45], [46], quality function deployment method [47], SERVQUAL model [4], [48], etc. The evaluation method based on the SERVQUAL model is the most popular evaluation method in PSQ evaluation. However, the above studies mainly focus on a single type of evaluator, a homogeneous evaluation data structure and psychological behaviour based on rational expectations. Currently, the property service industry is transforming into a modern service industry [49]. The characteristics of the evaluation information in modern PSQ evaluation are incomplete information, heterogeneous data and psychological behaviour based on the bounded rationality of evaluators. Linear programming multidimensional preference analysis (LINMAP) has the basic role of heterogeneous information processing [5]. By extending the LINMAP model, the PSQ evaluation method based on large-scale heterogeneous information [6] and the PSQ evaluation method based on dynamic incomplete information [7] are proposed. However, PSQ evaluation based on the bounded rationality of
evaluators has not been solved. The purpose of this paper is to solve the above problem by extending the LINMAP model.

III. PRELIMINARIES

A. THE VALUE FUNCTION OF PROSPECT THEORY

The value function of prospect theory [8], [50] is the relation between the decision maker’s perceived value and gain (loss). The value function curve can be divided into two parts: gains and losses. The origin of the curve is the reference point of the value function. The gain part is the convex function, and the loss part is the concave function. The whole forms an s-shaped curve. The value function is shown in figure 1.

![Value Function Curve](image)

**FIGURE 1.** The curve of the value function of prospect theory.

Let \( x \) be the deviation degree between the evaluation value and the reference point, \( \Phi(x) \) be the utility value, \( \alpha \) and \( \beta \) be the risk aversion degree of the decision maker, and \( \theta \) be the loss aversion degree of the decision maker. Then, the formula of the value function can be defined as follows:

\[
\Phi(x) = \begin{cases} 
    x^\alpha, & x \geq 0 \\
    -\theta(-x)^\beta, & x < 0 
\end{cases}
\]

where \( 0 < \alpha < 1 \), \( 0 < \beta < 1 \).

The greater the value of \( \alpha \) and \( \beta \), the greater the risk preference of decision making. When \( \theta > 1 \), the decision maker is more sensitive to gains than losses; when \( \theta \leq 1 \), the decision maker is more sensitive to losses than gains. When \( x \geq 0 \), the decision maker obtains positive utility; when \( x < 0 \), the decision maker obtains negative utility. According to the experimental research by Kahneman and Tversky [8], \( \alpha = 0.88, \theta = 2.25 \).

B. BASIC PRINCIPLE OF THE LINMAP METHOD

The construction idea of the classic LINMAP model is as follows: First, the distance between the evaluation value of each alternative and its positive ideal point is calculated. Then, the consistency and inconsistency are determined based on the distances of all alternatives. Finally, the linear programming model is constructed, and the unknowns are solved [5], [23].

Decision makers can be divided into ordinary decision makers or experts. The decision model includes alternative, attribute and its weight. The alternative set can be determined according to the relevant theoretic and practical research results. The decision information of each alternative is come from all attributes of the same set by the ordinary decision makers. That is, all alternatives have the same attribute set. At the same time, experts make preference evaluations between alternatives based on their overall perceptions of all alternatives. The weight of attribute is usually the unknown.

Some mathematical notations to be used are illustrated as follows: Let \( F = \{F_1, F_2, \ldots, F_m\} \) be the set of \( m \) alternatives, \( A = \{A_1, A_2, \ldots, A_n\} \) be the set of \( n \) attributes, and \( \omega = \{\omega_1, \omega_2, \ldots, \omega_n\} \) be the weight set corresponding to the attribute set \( A \). Let \( \Omega \) be the preference set in which the alternative \( F_p \in F \) is not inferior to the alternative \( F_q \in F \) in the opinion of all experts. It can be denoted as follows:

\[
\Omega = \{(p, q) \mid F_p \succeq F_q, (p, q = 1, 2, \ldots, m)\}
\]

The objective function is the total inconsistency index, and the constraint condition is that the total consistency index is equal to or greater than the total inconsistency index. The LINMAP model can be denoted as follows:

\[
\begin{align*}
\min \{B_\Omega\} \\
G_\Omega - B_\Omega & \geq h \\
\sum_{j=1}^{n} \omega_j & = 1 \\
\omega_j & \geq 0
\end{align*}
\]

where \( h \geq 0 \) is an a priori value given by decision makers. \( G_\Omega \) and \( B_\Omega \) denote respectively the total consistency and the total inconsistency, which are determined by the evaluation value of each alternative and its positive ideal point.

Then, the LINMAP model is solved by using different thresholds \( h \). The final index weight and the other unknowns are determined based on existing research, expert experience and sensitivity analysis.

IV. THE LINMAP METHOD BASED ON BOUNDED RATIONALITY OF EVALUATORS

According to the research goals in Section 1, we first propose a method for preprocessing heterogeneous data. Then, the principles of the LINMAP model based on prospect theory are described in detail. Finally, the operation process of the proposed method is summarized.

A. NORMALIZATION OF HETEROGENEOUS EVALUATION INFORMATION

The normalization methods of basic data can be roughly divided into linear and nonlinear, and the linear method is usually used in decision evaluation. Given that the classical LINMAP model determined consistency and inconsistency...
based on the TOPSIS method, the ideas of TOPSIS method is introduced the normalization method. Therefore, this paper combines the linear method and TOPSIS method to construct a data normalization method.

Evaluation information is usually in the form of real numbers, interval numbers, fuzzy numbers and language values, among which fuzzy numbers include triangular fuzzy numbers and hesitant fuzzy numbers. We assume that the evaluation value corresponding to each attribute has the same data type. Referring to the existing methods in references \[51], \[52], this paper proposes a normalization method of heterogeneous data.

Let \( Z = (z_{ij}) (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n) \) be the decision matrix with a normalized value corresponding to the original decision matrix \( U = (u_{ij}) \), where \( z_{ij} \) denotes the normalized evaluation value that corresponds to the alternative \( F_i \in F \) and the attribute \( A_j \in A \). Let \( I = \{I_1, I_2, I_3, I_4, I_5\} \) be the attribute set; \( I_1 = A_1, A_2, \ldots, A_{11} \), \( I_2 = A_{12} + 1, A_{12} + 2, \ldots, A_{22} \), \( I_3 = A_{23} + 1, A_{23} + 2, \ldots, A_{33} \), \( I_4 = A_{34} + 1, A_{34} + 2, \ldots, A_{44} \) and \( I_5 = A_{45} + 1, A_{45} + 2, \ldots, A_5 \) are the subsets of the real numbers, interval numbers, triangular fuzzy numbers, hesitant fuzzy numbers and language values respectively, where \( I_1 \cap I_2 \cap I_3 \cap I_4 \cap I_5 = \emptyset \). The evaluation value \( u_{ij} \) of different data types can be defined as follows:

where \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \).

The formula for calculating the normalized value \( t = 1, 2, \ldots, q \) based on the original evaluation data matrix \( U = (u_{ij}) \) is expressed as follows:

\[
z_{ij} = \frac{d(u_{ij}, u^-_{ij})}{d(u_{ij}, u^-_{ij}) + d(u_{ij}, u^+_{ij})}
\]

where \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \).

The formula for calculating the positive ideal point can be defined as follows:

\[
e^-_j = \left\{ \begin{array}{ll}
1 & \text{if } A_j \in I_1 \\
\{e^-_j, e^-_j\} & \text{if } A_j \in I_2 \\
\{a^-_j, b^-_j, c^-_j\} & \text{if } A_j \in I_3 \\
\{y^-_j, y^-_j, \ldots, y^-_j\} & \text{if } A_j \in I_4 \\
\{s^-_j & \text{if } A_j \in I_5 \\
\end{array}
\right.
\]

where \( r^-_j = \min_{1 \leq i \leq m} \{r_{ij}\}, e^-_j = \max_{1 \leq i \leq m} \{e_{ij}\}, e^-_j = \max_{1 \leq i \leq m} \{e_{ij}\}, a^-_j = \max_{1 \leq i \leq m} \{a_{ij}\}, b^-_j = \max_{1 \leq i \leq m} \{b_{ij}\}, c^-_j = \max_{1 \leq i \leq m} \{c_{ij}\}, y^-_j = \max_{1 \leq i \leq m} \{y_{ij}\}, y^-_j = \max_{1 \leq i \leq m} \{y_{ij}\}, y^-_j = \max_{1 \leq i \leq m} \{y_{ij}\}, s^-_j = \max_{1 \leq i \leq m} \{s_{ij}\}, j = 1, 2, \ldots, n, e^-_j \) denote the attribute of the positive ideal point.

The formula for calculating the negative ideal point can be written as follows:

\[
e^+_j = \left\{ \begin{array}{ll}
1 & \text{if } A_j \in I_1 \\
\{e^+_j, e^+_j\} & \text{if } A_j \in I_2 \\
\{a^+_j, b^+_j, c^+_j\} & \text{if } A_j \in I_3 \\
\{y^+_j, y^+_j, \ldots, y^+_j\} & \text{if } A_j \in I_4 \\
\{s^+_j & \text{if } A_j \in I_5 \\
\end{array}
\right.
\]

where \( r^+_j = \min_{1 \leq i \leq m} \{r_{ij}\}, e^+_j = \min_{1 \leq i \leq m} \{e_{ij}\}, e^+_j = \min_{1 \leq i \leq m} \{e_{ij}\}, a^+_j = \min_{1 \leq i \leq m} \{a_{ij}\}, b^+_j = \min_{1 \leq i \leq m} \{b_{ij}\}, c^+_j = \min_{1 \leq i \leq m} \{c_{ij}\}, y^+_j = \min_{1 \leq i \leq m} \{y_{ij}\}, y^+_j = \min_{1 \leq i \leq m} \{y_{ij}\}, y^+_j = \min_{1 \leq i \leq m} \{y_{ij}\}, s^+_j = \min_{1 \leq i \leq m} \{s_{ij}\}, j = 1, 2, \ldots, n, e^+_j \) denote the attribute of the negative ideal point.

As the attribute indexes are divided into cost type and benefit type, we design the preprocessing method for different types of indexes. Let \( \omega_4 > \omega_1 > \omega_5 > \omega_2 > \omega_3 \) be the benefit type index set corresponding to Eq. (4), where the subsets can be denoted as \( L^b = \{I_1^b, I_2^b, I_3^b, I_4^b, I_5^b\} \). Therefore, the normalized formulas of the benefit type index can be shown in (10), as shown at the bottom of the next page, where \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, l = 1, 2, \ldots, L \).

If the evaluation information is the cost type index set corresponding to Eq. (6), the normalized formula of the cost type index based on the result in Eq. (10) can be calculated by the formula \( z^c = 1 - z \), where \( z^c \) be the normalized value of cost type indexes.

In practice, we can preprocess the basic evaluation information of other numerical types according to the above standardization method.

**B. THE LINMAP MODEL BASED ON PROSPECT THEORY**

In order to adapt to the effective processing of evaluation information of evaluators’ bounded rational behaviour, this paper proposes a LINMAP model based on prospect theory. The construction method of the proposed model is as follows:

First, the value function is used to determine the dominance degree matrix between all alternatives; next, the LINMAP model is constructed based on the dominance degree matrix; and finally, the overall performance value of each alternative is solved, and all alternatives are ranked order according to the overall performance value.

The dominance degree matrix is determined based on the evaluation value between different alternatives. The value
function is used to determine the dominance degree of each alternative over the other alternative. Let \( R_i \) be the performance value. Let \( R = (R_{ik})_{m \times m} (i, k = 1, 2, \ldots, m) \) be the dominance degree matrix, where \( R_{ik} \) is the dominance degree of the alternative \( F_i \) over the alternative \( F_k \). Let \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) be the weight vector of \( n \) attributes, where \( \omega_j \) denotes the weight of the \( j \)-th attribute \( A_j \). The performance value of the alternative \( F_i \) over the alternative \( F_k \) in the attribute \( A_j \) is calculated as follows:

\[
R_{ik} = \sum_{j=1}^{n} \Phi_j(F_i, F_k) \tag{11}
\]

Using Eq. (1), the formula for calculating \( R_{ik} \) can be written as follows:

\[
\Phi_j(F_i, F_k) = \begin{cases} 
(\alpha_j (z_{ij} - z_{kj})^\alpha, & z_{ij} \geq z_{kj} \\
-\theta (\alpha_j (z_{kj} - z_{ij})^\beta, & z_{ij} < z_{kj}
\end{cases} \tag{12}
\]

The positive ideal point vector \( R^* \) of the dominance degree matrix based on the above normalized matrix is calculated using Eqs. (12) and (13). It is denoted as follows:

\[
R^* = (R^*_1, R^*_2, \ldots, R^*_m) \tag{13}
\]

where the positive ideal point \( R^*_k = \max \{R_{ik}\} \) \((k = 1, 2, \ldots, m)\).

**Remark 1:** The positive ideal point in the classical LINMAP model can be obtained from the basic data before solving the model. Since the dominance degree matrix contains weight unknowns, the positive ideal points are taken as unknowns in the following model.

In this paper, the weighted Euclidean distance square value is used to calculate the distance between each alternative and its positive ideal point. The distance between the dominance degree of the alternative \( F_p \in F \) and the reference point \( R^*_k \) is denoted as \( D_p \). It can be defined as follows:

\[
D_p = \sum_{k=1}^{m} (R_{pk} - R^*_k)^2 \tag{14}
\]

Similarly, the distance between the dominance degree of the alternative \( F_q \in F \) and the positive ideal point \( R^*_k \) is denoted as \( D_q \). The calculate formula can be defined as follows:

\[
D_q = \sum_{k=1}^{m} (R_{qk} - R^*_k)^2 \tag{15}
\]

Let the ordered pair \((p, q) \in \Omega\) be the preference of experts between different alternatives, which denotes that the alternative \( F_p \) is not inferior to the alternative \( F_q \). Then, \((D_p - D_q)^-\) and \((D_p - D_q)^+\) denote the inconsistency and consistency between the alternative \( F_p \) and the alternative \( F_q \) respectively.

If \( D_p < D_q \), the value of \((D_p - D_q)^-\) is zero. Therefore, the inconsistency \((D_p - D_q)^-\) can be denoted as follows:

\[
(D_p - D_q)^- = \begin{cases} 
D_q - D_p, & (D_p < D_q) \\
0, & (D_p \geq D_q)
\end{cases} \tag{16}
\]
The inconsistency \((D_p - D_q)^-\) can be rewritten as follows:

\[
(D_p - D_q)^- = \max\{0, D_q - D_p\} \tag{17}
\]

Considering the preference of all experts, the total inconsistency \(B\) can be denoted as follows:

\[
B = \sum_{(p,q)\in\Omega} (D_p - D_q)^- = \sum_{(p,q)\in\Omega} \max\{0, D_q - D_p\} \tag{18}
\]

Similarly, the consistency \((D_p - D_q)^+\) between the alternatives \(F_p\) and \(F_q\) can be denoted as follows:

\[
(D_p - D_q)^+ = \begin{cases} 
  D_p - D_q & (D_p \geq D_q) \\
  0 & (D_p < D_q)
\end{cases} \tag{19}
\]

The consistency \((D_p - D_q)^+\) can be rewritten as follows:

\[
(D_p - D_q)^+ = \max\{0, D_p - D_q\} \tag{20}
\]

Considering the overall preference of all experts, the total consistency \(G\) can be denoted as follows:

\[
G = \sum_{(p,q)\in\Omega} (D_p - D_q)^+ = \sum_{(p,q)\in\Omega} \max\{0, D_p - D_q\} \tag{21}
\]

According to Eqs. (18) and (21), the difference between consistency and inconsistency can be denoted as follows:

\[
G - B = \sum_{(p,q)\in\Omega} (D_p - D_q)^+ - \sum_{(p,q)\in\Omega} (D_p - D_q)^- = \sum_{(p,q)\in\Omega} [(D_p - D_q)^+ - (D_p - D_q)^-] = \sum_{(p,q)\in\Omega} (D_p - D_q) \tag{22}
\]

Let \(\lambda_{pq} = \max\{0, D_p - D_q\}\). For the ordered pair \((p,q)\in\Omega\), we have

\[
\lambda_{pq} \geq D_p - D_q \tag{23}
\]

where \(\lambda_{pq} \geq 0\).

According to whether the fuzzy preference degree of experts is considered, the LINMAP models to be constructed can be divided into the LINMAP model without considering the fuzzy preference degree of experts and the LINMAP model with considering the fuzzy preference degree of experts.

For the case where the fuzzy preference of experts is not considered, we substituted Eqs. (14) \sim (23) into Eq. (3). Then, the LINMAP model without considering the fuzzy preference degree of experts can be written as follows:

\[
\min B = \sum_{(p,q)\in\Omega} \lambda_{pq} \tag{24}
\]

\[
\begin{align*}
\sum_{(p,q)\in\Omega} \sum_{k=1}^m \left[ (R_{pk} - R_{qk}^+)^2 - (R_{qk} - R_{pk}^+)^2 \right] & \geq h \\
\sum_{k=1}^m \left[ (R_{qk} - R_{qk}^+)^2 - (R_{pk} - R_{qk}^+)^2 \right] & \geq \lambda_{pq} > 0
\end{align*}
\]

where \(\Phi_j(F_i, F_k)\) is determined by Eq. (12), \(R_{pk}^+\) is determined by Eq. (13), and the incomplete information on the index weight is represented in the existing way [53], [54]. The unknowns of the above model include index weights, positive ideal points and threshold values.

In contrast to the above model which does not consider the effect of the fuzzy preference degree, the parameters of fuzzy preference are introduced in the following model analysis.

Let \(\hat{C}\) be the fuzzy preference degree set of experts, which can be denoted as follows:

\[
\hat{C} = \{\hat{c}(p,q) | (p,q)\in\Omega\} \tag{25}
\]

where \(\hat{c}(p,q)\) denotes the fuzzy preference degree of the alternative \(F_p\) over \(F_q\) in the opinion of the expert.

Let \(c_{pq}\) be the normalized data after the defuzzification of \(\hat{c}(p,q)\). The process of defuzzification from \(\hat{c}(p,q)\) to \(c_{pq}\) is performed by use of the normalization method mentioned above. Then, the total inconsistency, the total consistency and their difference can be rewritten as follows:

\[
B = \sum_{(p,q)\in\Omega} (D_p - D_q)^- = \sum_{(p,q)\in\Omega} c_{pq} \max\{0, D_q - D_p\} \tag{26}
\]

\[
G = \sum_{(p,q)\in\Omega} (D_p - D_q)^+ = \sum_{(p,q)\in\Omega} c_{pq} \max\{0, D_p - D_q\} \tag{27}
\]

\[
G - B = \sum_{(p,q)\in\Omega} [c_{pq}(D_p - D_q)] \tag{28}
\]

Analogously, Eq. (24) can be rewritten as follows

\[
\lambda_{pq} \geq c_{pq}(D_p - D_q) \tag{29}
\]

and

\[
\lambda_{pq} \geq 0 \tag{30}
\]

The analysis process is the same as Eqs. (14) \sim (23). The new model can be obtained by replacing the parameters in Eq. (24) with Eqs. (26) \sim (30). Therefore, the LINMAP
model considering the fuzzy preference degree of experts can be written as follows:

$$\min B = c_{pq} \sum_{(p,q) \in \Omega} \lambda_{pq}$$

$$\sum_{(p,q) \in \Omega} \sum_{k=1}^{m} c_{pq}[(R_{pq} - R_{pk}^*)^2 - (R_{pq} - R_{kq}^*)^2] \geq h$$

$$\sum_{k=1}^{n} c_{pq}[(R_{pq} - R_{pk}^*)^2 - (R_{pq} - R_{kq}^*)^2] + \lambda_{pq} \geq 0$$

$$R_{ik} = \sum_{j=1}^{n} \Phi_j(F_i, F_k)$$

$$\omega_j \in H, \sum_{j=1}^{n} \omega_j = 1, \omega_j > 0$$

$$\lambda_{pq} > 0$$

where the symbol in Eq. (31) is the same as Eqs. (12) and (24), and the fuzzy parameter $c_{pq}$ is determined in advance by use of the normalization method mentioned above.

Sensitivity analysis is used in the above LINMAP model. The LINMAP model is solved using differing threshold values. Then, the final solution to the above LINMAP model is determined by combining sensitivity analysis, decision makers’ experience and existing research results and decision makers’ experience, and calculate the performance value of each alternative by using Eq. (32).

$$R_{i} = \sum_{k=1}^{m} R_{ik}$$

where $i, k = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.

Lastly, all alternatives are ranked in order from high to low, according to the performance value of each alternative.

**Remark 2:** In the literature [6], the final ranking order of all alternatives is by the comprehensive scores, which are calculated by using the weights and basic data. This paper reveals that the two methods can be replaced by each other, and, if the research focus is on application, method [6] should be used first.

**V. A REAL EXAMPLE OF PSQ EVALUATION**

Wenzhou Sapphires Property Management Co., Ltd. (hereafter called Sapphires) in China is a medium-sized enterprise specializing in property management. In order to better serve the owners, the company has successively implemented the service concepts of “first-question responsibility systems”, “satisfaction projects” and “star services”. Since the existing evaluation method cannot effectively monitor property service performance, Sapphires needs a new method to accurately measure property service quality. The following data from Sapphires are used to verify the proposed method, which can also be used in property service quality evaluation practice.

**A. COLLECTION OF BASIC EVALUATION INFORMATION**

This paper evaluates PSQ with the opinions of visitors, executives, engineers and experts. Visitors are the service object and the direct source of customer experience information. Executives and engineers are familiar with the business of their departments and are suitable for itemized evaluation. Experts are characterized by a wide range of knowledge, which is suitable for overall evaluation. Only by combining the opinions of visitors, executives, engineers and experts, can property service projects be evaluated comprehensively and thoroughly. In order to ensure the reliability of the basic information, we consulted property service experts and property service quality supervision professionals. We determined the basic issues of property service quality evaluation by the collective opinion method. The results are summarized in Table 1.

There are four property service projects: museum (F1), library (F2), science & technology museum (F3) and grand theatre (F4). The evaluators included visitors, executives, engineers and experts. Property service experts were invited from university, government and property service industry association to make preference comparisons according to the overall situation of each property service project. Executives and engineers came from the property service enterprise, and visitors were the people who participated in the property service satisfaction evaluations. Executives, engineers and visitors evaluated property service quality according to safety (A1), service attitude (A2), cleaning (A3), facilities (A4), and greening (A5).

Based on the property service practice from Sapphires, we determine the data type by consulting experts as follows: The safety item evaluation information is determined by three different alternatives, and the LINMAP model based on prospect theory is constructed as shown in Eq. (24) or (31).

**Step 5:** Determine the solution to the above model by combining sensitivity analysis, existing research results and decision makers’ experience, and calculate the performance value of each alternative by using Eq. (32).

**Step 6:** Rank all alternatives according to their overall performance values.
experts in the form of triangular fuzzy number, the service attitude item evaluation information is determined by multiple randomly selected visitors in the form of real number, the cleaning item evaluation information is determined by the indefinite number of supervisors in the form of hesitation number, the facilities item evaluation information is determined by the engineer in the form of linguistic value, the greenery item evaluation information is determined by the negative number of engineering maintenance, and four property service projects, as shown in Table 2.

**TABLE 2. Basic evaluation data of sub-item PSQ.**

| A_1 | A_2 | A_3 | A_4 | A_5 |
|-----|-----|-----|-----|-----|
| F_1 | (4,5,7) | 94.35 | (6,6,7,8) | s4 | [7,9] |
| F_2 | (2,3,4) | 85.21 | (2,3,4) | s4 | [6,7] |
| F_3 | (1,2,3) | 83.52 | (2,3,4,5) | s3 | [5,7] |
| F_4 | (3,4,5) | 90.94 | (3,4,6) | s5 | [8,9] |

The scoring rules for expert preference information were determined in advance by all experts and then scored individually by each expert, according to the same rules. The preference information set of experts is denoted as triangular fuzzy numbers as follows:

\[ \Omega = \{(1, 3), c(1, 3), (1, 4), \hat{c}(1, 4), (2, 3), \hat{c}(2, 3), (4, 2), \hat{c}(4, 2)\} \]

where the fuzzy preference degree can be calculated as follows: \( \hat{c}(1, 3) = (0.3, 0.4, 0.5), \hat{c}(1, 4) = (0.2, 0.5, 0.7), \hat{c}(2, 3) = (0.4, 0.6, 0.8), \hat{c}(4, 2) = (0.3, 0.5, 0.8). \)

In addition, by combining the existing studies [53], [54] and expert opinions, the incomplete information of index weights is denoted as follows:

\[ H = \{(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) | 0.15 \leq \omega_1 \leq 0.45; 0.1 \leq \omega_2 \leq 0.2; 0.1 \leq \omega_3 \leq 0.25; \omega_4 \geq 0.12; \omega_5 \geq 0.15; \omega_1 + \omega_2 + \omega_3 \leq 0.05; \omega_1 - \omega_2 \geq \omega_5 - \omega_4\} \]

**B. THE CALCULATION PROCESS WITHOUT CONSIDERING FUZZY PREFERENCE DEGREE**

The basic evaluation information in Table 1 was processed using Eqs. (4)−(10). The normalized evaluation information is shown in Table 3.

**TABLE 3. Normalized evaluation data of sub-item PSQ.**

| A_1 | A_2 | A_3 | A_4 | A_5 |
|-----|-----|-----|-----|-----|
| F_1 | 1.0000 | 1.0000 | 1.0000 | 0.5000 | 1.0000 |
| F_2 | 0.3000 | 0.1560 | 0.0000 | 0.5000 | 0.2000 |
| F_3 | 0.0000 | 0.0000 | 0.1333 | 0.0000 | 0.0000 |
| F_4 | 0.6000 | 0.6851 | 0.3667 | 1.0000 | 0.8000 |

Based on the data from the normalized matrix, the dominance matrix \((R_k)_{4 \times 4}\) was constructed by using Eqs. (11) and (12). For example, the formula for \(R_{12}\) is:

\[ R_{12} = (0.7000\omega_1)^\alpha + (0.8440\omega_2)^\alpha + (1.0000\omega_3)^\beta + (0.0000\omega_4)^\beta + (0.8000\omega_5)^\beta \]

All the formulas of \(R_{ik}(i, k = 1, 2, 3, 4)\) are summarized in Appendix A.

For the dominance matrix \((R_k)_{4 \times 4}\), the positive ideal point vector can be denoted as \(R^* = (R_1^*, R_2^*, R_3^*, R_4^*)\). When not considering the effect of the fuzzy preference degree, the model in Eq. (24) was selected. The data from Table 3, Eqs. (33) and (34) was substituted into Eq. (24), and the LINMAP model based on prospect theory without considering the fuzzy preference degree is shown in APPENDIX A.

Based on prior research [5], the current paper determined the values of \(\alpha, \beta\) and \(\theta\). Sensitivity analysis was performed based on different threshold values \(h\). When \(h \geq 4.5\), the formula in Appendix A has no feasible solution. And because \(h\) is not less than zero. Therefore, the threshold value \(h\) was determined to be in the range of \([0,4.0]\). Taking 0.5 as the interval of the different values of \(h\), the formula in Appendix A was solved. The calculation results of index weights, performance values and the ranking order of all property service projects are summarized in Table 4.

Based on the results of the above sensitivity analysis, we referred to prior research [5], [6], [23]–[27] and determined \(h = 3.0\). By solving the formula in Appendix A, the overall performance value of each property service project was as follows: \(R_1 = 2.4760, R_2 = 0.2883, R_3 = 0.0501, R_4 = 1.4474\).

Therefore, the ranking order of all property service projects was as follows:

\[ F_1 > F_4 > F_2 > F_3 \]

**C. THE CALCULATION PROCESS CONSIDERING FUZZY PREFERENCE DEGREE**

When fuzzy preference degrees are considered, Eq. (31) was used. Using Eqs. (4)−(10), the defuzzification of fuzzy preference degrees in Eq. (33) were calculated as fol-
According to the above analysis, when the threshold value $h$ changes, the index weights and overall performance values of all property service projects change greatly, but the final ranking order of all property service projects does not change. Furthermore, there is the same ranking order regardless of whether the fuzzy preference degree of experts is considered. In summary, the PSQ evaluation results by the LINMAP model based on prospect theory were stable.

**D. THE EVALUATION RESULTS OF PSQ AND ITS MANAGEMENT SIGNIFICANCE**

The above evaluation results mainly include evaluation index weight, final ranking order of all projects, fuzzy preference degrees and their effect. Based on the calculation results of the above model, we conducted field investigation again and communicated with a number of visitors, executives, engineers and experts. We analysed the reasons in combination with the opinions of field interviews. The evaluation results and the reasons were analysed as follows.

(1) When the fuzzy preference degree is not considered, the index weights are as follows: $\omega_1 = 0.2600, \omega_2 = 0.2000, \omega_3 = 0.2500, \omega_4 = 0.1200$ and $\omega_5 = 0.1700$. When the fuzzy preference degree is considered, the index weights are $\omega_1 = 0.0.2600, \omega_2 = 0.2000, \omega_3 = 0.2500, \omega_4 = 0.1200$ and $\omega_5 = 0.1700$. When the fuzzy preference degree is considered, the index weights are

$$ ...
\begin{array}{cccccccc}
 h & \omega_1 & \omega_2 & \omega_3 & \omega_4 & \omega_5 & R_1 & R_2 & R_3 & R_4 & \text{Ranking order} \\
 0.0 & 0.3050 & 0.2000 & 0.1500 & 0.1200 & 0.2250 & 2.4251 & 0.3186 & 0.0320 & 1.5047 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
 0.1 & 0.1500 & 0.1000 & 0.1000 & 0.1200 & 0.2250 & 2.4251 & 0.3186 & 0.0320 & 1.5047 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
 0.2 & 0.2600 & 0.2000 & 0.1500 & 0.1200 & 0.2250 & 2.4251 & 0.3186 & 0.0320 & 1.5047 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
 0.3 & 0.3050 & 0.2000 & 0.1500 & 0.1200 & 0.2250 & 2.4251 & 0.3186 & 0.0320 & 1.5047 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
 0.4 & 0.2600 & 0.2000 & 0.1500 & 0.1200 & 0.2250 & 2.4251 & 0.3186 & 0.0320 & 1.5047 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
 0.5 & 0.2800 & 0.2000 & 0.2500 & 0.1200 & 0.1500 & 2.4760 & 0.2883 & 0.0501 & 1.4474 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
 0.6 & 0.3800 & 0.1000 & 0.2500 & 0.1200 & 0.1500 & 2.4482 & 0.3034 & 0.0501 & 1.3920 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
 0.7 & 0.4500 & 0.1000 & 0.1800 & 0.1200 & 0.1500 & 2.4082 & 0.3272 & 0.0375 & 1.4082 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
 0.8 & 0.4500 & 0.1000 & 0.1800 & 0.1200 & 0.1500 & 2.4082 & 0.3272 & 0.0375 & 1.4082 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
 0.9 & 0.2600 & 0.2000 & 0.2500 & 0.1200 & 0.1500 & 2.4760 & 0.2883 & 0.0501 & 1.4474 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
 1.0 & 0.2800 & 0.2000 & 0.2500 & 0.1200 & 0.1500 & 2.4755 & 0.2902 & 0.0501 & 1.4357 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
 1.1 & 0.2800 & 0.2000 & 0.2500 & 0.1200 & 0.1500 & 2.4755 & 0.2902 & 0.0501 & 1.4357 & F_1 \succ F_4 \succ F_2 \succ F_3 \\
\end{array}$$

Therefore, the ranking order of all property service projects was as follows:

$$F_1 \succ F_4 \succ F_2 \succ F_3$$

(36)
as follows: \( \omega_1 = 0.3800, \omega_2 = 0.1000, \omega_3 = 0.2500, \) and \( \omega_4 = 0.1500. \) At the same time, when \( h \) is set to the other threshold values, we obtained different evaluation results shown in Table 4 and Table 5. It is the disadvantage of the proposed method, which is explicated in Section VII. All in all, safety and cleaning are the two factors that have the greatest effect on PSQ, whether or not fuzzy preference is considered. Since safety and cleaning are the most basic functions of property service, it also shows that the property service level of these public buildings is not optimal, and there is room for improvement in PSQ.

(2) If the fuzzy preference degree is not considered, the final ranking order of all projects is \( F_4 \succ F_1 \succ F_2 \succ F_3 \) when \( h = 0.0 \) and \( h = 2.0. \) Whether or not the fuzzy preference degree is considered, the final ranking order of all projects is \( F_1 \succ F_4 \succ F_2 \succ F_3 \) in other cases. On the whole, the PSQ of museums (\( F_1 \)) and grand theatre (\( F_4 \)) is better than that of libraries (\( F_2 \)) and science & technology museums (\( F_3 \)). Through our field investigation, we think the reasons are as follows: Visitors to museums (\( F_1 \)) and grand theatre (\( F_4 \)) are usually decent and put less pressure on property management. In contrast, visitors to libraries (\( F_2 \)) and science & technology museums (\( F_3 \)) are mainly teenagers who bring about significant pressure on management. In other words, the characteristics of property service projects and service objects are the main factors that affect the level of PSQ.

Service improvement is the basic issue of service quality management. Good PSQ requires the joint efforts of relevant subjects. Based on the results of the above example analysis, there are three management measures as follows: First, property service companies should pay attention to key property service items, which is critical to improving the service level. The key property service items of the above example are safety and cleaning. Second, public management agencies should formulate targeted measures according to the characteristics of their service objects. For example, museums should strengthen safety management, and libraries should emphasize cleaning. Lastly, visitors are the ultimate service object of a public project, and civilized visitors are conducive to the formation of a good property service environment. Therefore, visitors should abide by the management system of public projects, cooperate with the management measures of property service companies, and create a good environment.

VI. COMPARATIVE ANALYSIS

The rationality hypothesis is the basic characteristic of the existing LINMAP method. This paper especially considers the influence of basic data normalization. The method in the literature [26] is selected as the representative of the existing LINMAP method. Different methods based on LINMAP model are first analysed in the following paragraphs, and then the example from the literature [26] is introduced for data analysis. Finally, the advantages of the proposed method are summarized.

A. THE DIFFERENT METHODS BASED ON LINMAP MODEL

As a new PSQ method, this paper considers the effect of the evaluator’s bounded rational behaviour. It is obvious that the PSQ method proposed in this paper is an innovation compared with the existing methods based on the rationality hypothesis. The LINMAP method based on the bounded rationality of evaluators in this paper has two key components: the normalization method of basic evaluation information based on TOPSIS (hereafter referred to as the “TOPSIS method”) and the LINMAP model based on prospect theory. Among the existing LINMAP method studies, the normalization method corresponding to the TOPSIS method in this paper is “Minimax method”, and the core model corresponding to the proposed method in this paper is the LINMAP model for heterogeneous information.

As shown in Section IV, there are two conditions: without considering the fuzzy preference degree and considering the fuzzy preference degree. At the same time, there are two basic data normalization methods, the TOPSIS method and Minimax method, and there are two core models: the LINMAP model based on prospect theory and the LINMAP model for heterogeneous information. These two conditions, combined with two basic data normalization methods and two core models, constitute eight LINMAP methods, as shown in Table 6. By analysing the ranking results of the PSQ evaluation example using eight LINMAP methods, we can summarize the characteristics of the normalization method and the core model and compare the advantages of different methods.

B. THE PSQ EXAMPLE ANALYSIS BASED ON DIFFERENT LINMAP METHODS

In eight LINMAP methods, the threshold \( h \) in different LINMAP methods is determined by decision makers based on sensitivity analysis combined with existing research, and the other coefficients and incomplete information in different cases are shown in Section V. Corresponding to Table 6, the LINMAP model for heterogeneous information is shown in reference [26], and the LINMAP model based on prospect theory is shown in Eqs. (24) and (31). The normalization method in Section IV can be used to process the expert preference degree data in reference [27]. The expert preference degree considering fuzzy preference after normalized processing is as follows: \( c_{13} = 0.1429, c_{14} = 0.4286, c_{23} = 1.0000, c_{42} = 0.7143. \) Obviously, the expert preference degree without considering fuzzy preference is as follows: \( c_{13} = 1.0000, c_{14} = 1.0000, c_{23} = 1.0000, c_{42} = 1.0000. \) The calculation process is omitted here. The itemized evaluation data after TOPSIS normalization are shown in Table 3. In order to meet the needs of comparative analysis of different evaluation methods, the Minimax method in reference [26] is used to normalize the itemized evaluation data. All heterogeneous evaluation information in Table 2 is benefit type, and the evaluation information in each index has the same data.
TABLE 6. A summary of eight LINMAP methods.

| Cases | Core model | Normalization | Expert preference |
|-------|------------|---------------|-------------------|
| 1     | LINMAP model based on prospect theory | TOPSIS method | Without considering fuzzy preference |
| 2     | LINMAP model based on prospect theory | TOPSIS method | Considering fuzzy preference |
| 3     | LINMAP model based on prospect theory | Minimax method | Without considering fuzzy preference |
| 4     | LINMAP model based on prospect theory | Minimax method | Considering fuzzy preference |
| 5     | LINMAP model for heterogeneous information | Minimax method | Without considering fuzzy preference |
| 6     | LINMAP model for heterogeneous information | Minimax method | Considering fuzzy preference |
| 7     | LINMAP model for heterogeneous information | TOPSIS method | Without considering fuzzy preference |
| 8     | LINMAP model for heterogeneous information | TOPSIS method | Considering fuzzy preference |

TABLE 7. Normalized evaluation data of PSQ by Minimax method.

|    | A1     | A2     | A3          | A4       | A5       |
|----|--------|--------|-------------|----------|----------|
| F1 | (0.57, 0.71, 1.00) | 1.00   | (0.75, 0.75, 0.88, 1.00) | 0.80 | [0.78, 1.00] |
| F2 | (0.29, 0.43, 0.57) | 0.90   | (0.25, 0.38, 0.38, 0.50) | 0.80 | [0.67, 0.78] |
| F3 | (0.14, 0.29, 0.43) | 0.89   | (0.25, 0.38, 0.50, 0.63) | 0.60 | [0.56, 0.78] |
| F4 | (0.43, 0.57, 0.71) | 0.96   | (0.38, 0.50, 0.56, 0.75) | 1.00 | [0.89, 1.00] |

type. Based on data from Table 2, the normalized evaluation information is shown in Table 7.

The PSQ example analysis by the method in Cases 1 and 2 is shown in Section V. The analysis process by the method in Cases 3 and 4 is similar to that in Section V, so they are omitted here. Next, we briefly introduce the basic process of analysing PSQ examples using Cases 5 and 6.

**Case 5:** The PSQ evaluation example is analysed by the LINMAP model for heterogeneous information, the Minimax method and without considering the fuzzy preference degree.

The overall preferences of all experts and the incomplete information of the index weight are the same as in Section V. The above data were substituted into the model in reference [26], and the traditional LINMAP model based on the PSQ example without considering the fuzzy preference degree was obtained. Then, sensitivity analysis was carried out according to the threshold value \( h \). When \( h \geq 0.40 \), the above model has no feasible solution. So the threshold value \( h \) is in the range \([0, 0.20]\). Takes 0.05 as the interval of different values of \( h \) and solves the above model. The results of index weights, performance values and the ranking order of all property service projects considering fuzzy degree are determined. Based on the above sensitivity analysis results, we refer to the existing studies [5], [6], [23]–[27] and determine \( h = 0.20 \). Therefore, the ranking order of four property service projects is as follows:

\[
F_1 \succ F_4 \succ F_2 \succ F_3
\]  

In addition to the data in Case 5, the fuzzy preference degrees of experts are added. The above data were substituted into the model in reference [26], and the traditional LINMAP model based on the PSQ example considering the fuzzy preference degree is obtained. Then, sensitivity analysis is carried out according to the threshold value \( h \). When \( h \geq 0.25 \), the above model has no feasible solution. So the threshold value \( h \) is in the range \([0, 0.20]\). Takes 0.05 as the interval of different values of \( h \) and solves the above model. The results of index weights, the performance values and the ranking order of all property service projects considering fuzzy degree are determined. Based on the above sensitivity analysis results, we refer to the existing studies [5], [6], [23]–[27] and determine \( h = 0.10 \). By solving the above model, the performance values of four property service projects are as follows: \( D_1 = 0.0000, D_2 = 0.0586, D_3 = 0.1140 \) and \( D_4 = 0.0196 \). Therefore, the ranking order of four property service projects is as follows:

\[
F_1 \succ F_4 \succ F_2 \succ F_3
\]  

Similarly, the PSQ example analysis process by the method in Cases 7 and 8 is the same as the above analysis, so they are omitted here.

In conclusion, the final ranking orders by eight LINMAP methods are summarized in Table 8.

According to the research results from Table 8, there are three conclusions: 1) The ranking order \( F_1 \succ F_4 \succ F_2 \succ F_3 \) of PSQ evaluation is the result analysed by most cases, and most of the evaluation results indicate that property service project \( F_1 \) has the best property service quality. 2) The proposed method (including Cases 1 and 2) and the traditional method (including Cases 5 and 6) have stable analysis results regardless of whether the fuzzy preference degree is considered. 3) The ranking order by Case 4 is
quite different from the difference from the ranking order by most cases. This indicates that the Minimax method and LINMAP model based on prospect theory are not a suitable combination. In summary, the proposed method has the same stable performance as the traditional method, and the normalization method in the proposed method should adopt the TOPSIS method instead of the Minimax method.

The ranking order by Case 7 has little difference from the ranking order by most cases. This indicates that the TOPSIS method and LINMAP model for heterogeneous information are reluctant suitable combinations. Therefore, the combination of the LINMAP model based on prospect theory and the Minimax method is an inappropriate method, and the combination of the LINMAP model for heterogeneous information and the TOPSIS method can be an optional method.

Whether incomplete information, heterogeneous data and bounded rational information can be effectively processed is the problem raised in this paper. We compare the performance of the above methods for solving different problems. Except for the inappropriate method in this paper, the performance of three alternative methods is compared and analysed, as shown in Table 9.

Finally, it is comprehensively determined that the traditional method in reference [27] and optional method in this paper are not the optimal methods, and we recommend the proposed method in this paper.

C. THE ADVANTAGES OF THE PROPOSED METHOD

The PSQ method based on the bounded rationality of evaluators has rich theoretical value and application significance. In terms of method innovation, a normalization method based on TOPSIS is designed, and a LINMAP method based on prospect theory is proposed. As a new PSQ evaluation method, it can process evaluation data with complex structure. Combined with the above analysis, the PSQ method based on the bounded rationality of evaluators has the following advantages.

(1) A normalization method based on TOPSIS is designed. Heterogeneous evaluation information is increasingly widely used in modern service quality evaluation, and information normalization is the basis of model design in this paper. This paper uses the Hamming distance to determine the relationship between the basic evaluation value of each alternative and its positive or negative ideal points and uses the TOPSIS method to calculate its normalized value. The information normalization method based on TOPSIS is suitable for the requirements of extending the LINMAP model in this paper. The example analysis shows that the information normalization method based on TOPSIS not only simplifies the calculation process of the proposed method but is also more suitable for the LINMAP model based on prospect theory than the Minimax method.

(2) A LINMAP method based on prospect theory is proposed. The basic idea of the LINMAP method based on prospect theory is as follows: First, the dominance degree matrix is constructed based on the value function. Then, the LINMAP model is constructed and solved. Finally, the performance value of each alternative is calculated, and the ranking order of all alternatives is determined. Obviously, the proposed method contains the idea of prospect theory. Prospect theory shows that people’s risk preference behaviours are inconsistent in the face of gains and losses. They are risk pursuer in the face of “losses”, while they become risk avoider in the face of “gains”. Prospect theory describes the actual behaviour characteristics of bounded rational man. The existing LINMAP methods are based on the evaluator’s rationality, and the LINMAP method based

| Method category                | Incomplete information | Heterogeneous data | Bounded rational information | Whether to recommend the method |
|-------------------------------|------------------------|--------------------|-------------------------------|--------------------------------|
| Proposed method in this paper | ✓                      | ✓                  | ✓                            | ✓                              |
| Traditional method in reference [26] | ✓                      | ✓                  | ×                             | ×                              |
| Optional method in this paper | ✓                      | ✓                  | ×                             | ×                              |
on prospect theory is more in line with the requirements of realistic evaluation scenarios.

(3) A new method for PSQ evaluation is proposed. The characteristics of the new PSQ evaluation method proposed in this paper are as follows. First, it is an effective method that can process heterogeneous information simultaneously. Second, it is a scientific method to fuse information from different types of evaluators. Last, the key point is that the new PSQ evaluation method meets the requirements of bounded rationality of evaluators. On the one hand, with the development of the modern property service industry, the structure of evaluation information becomes complicated, which is mainly reflected in the bounded rational behaviour of evaluators and the heterogeneity of evaluation information. On the other hand, the econometric analysis method is the main traditional PSQ method, which is usually aimed at a single type of evaluator and the same data structure. There is a lack of evaluation methods for multiple types of evaluators and heterogeneous information. This paper proposes a new PSQ evaluation method that can effectively solve the above problems.

VII. CONCLUSION AND FUTURE WORK

According to the requirements of the hypothesis of bounded rationality of evaluators, this paper proposes a LINMAP model based on prospect theory for heterogeneity and
incomplete information processing and designs a new PSQ evaluation method. The evaluation method proposed in this paper can process heterogeneous evaluation information from different types of evaluators simultaneously, but also meet the requirements of information fusion, which is based on the bounded rationality of evaluators. In addition to PSQ evaluation, the proposed method can also be applied in other fields, such as tourism, e-commerce, libraries and public services.

Despite the authors’ efforts, there are still two problems with the method proposed in this paper: 1) Similar to the classical LINMAP model, the parameters $h$ and $\theta$ in the proposed method depend on the sensitivity analysis results and the decision makers’ experience, so the evaluation results have a certain degree of subjectivity. The root cause of this problem lies in the lack of enough empirical analysis. 2) In contrast to the complete rationality hypothesis, this paper is based on the bounded rationality hypothesis, which is closer to the real decision environment. However, there is still room for improvement. There are two ideas corresponding to the above statement in future research. On the one hand, to ensure the objectivity of the calculation results of the LINMAP model, the determination of parameters needs to be based on more empirical analysis. On the other hand, to ensure that the model improvement is reasonable and the evaluation

$$\min \{c_{13} \lambda_{13} + c_{14} \lambda_{14} + c_{23} \lambda_{23} + c_{42} \lambda_{42}\}$$

$${c_{13}((R_{11} - R_{11})^2 + (R_{12} - R_{12})^2 + (R_{13} - R_{13})^2 + (R_{14} - R_{14})^2) - ((R_{31} - R_{11})^2 + (R_{32} - R_{12})^2 + (R_{33} - R_{13})^2 + (R_{34} - R_{14})^2))}$$

$$+ c_{14}((R_{11} - R_{11})^2 + (R_{12} - R_{12})^2 + (R_{13} - R_{13})^2 + (R_{14} - R_{14})^2 - ((R_{41} - R_{11})^2 + (R_{42} - R_{12})^2 + (R_{43} - R_{13})^2 + (R_{44} - R_{14})^2))}$$

$$+ c_{23}((R_{21} - R_{21})^2 + (R_{22} - R_{22})^2 + (R_{23} - R_{23})^2 + (R_{24} - R_{24})^2) - ((R_{31} - R_{21})^2 + (R_{32} - R_{22})^2 + (R_{33} - R_{23})^2 + (R_{34} - R_{24})^2))$$

$$+ c_{42}((R_{41} - R_{41})^2 + (R_{42} - R_{42})^2 + (R_{43} - R_{43})^2 + (R_{44} - R_{44})^2) - ((R_{31} - R_{41})^2 + (R_{32} - R_{42})^2 + (R_{33} - R_{43})^2 + (R_{34} - R_{44})^2))}$$

$$\lambda_{13} - c_{13}((R_{11} - R_{11})^2 + (R_{12} - R_{12})^2 + (R_{13} - R_{13})^2 + (R_{14} - R_{14})^2) \geq h$$

$$\lambda_{14} - c_{14}((R_{11} - R_{11})^2 + (R_{12} - R_{12})^2 + (R_{13} - R_{13})^2 + (R_{14} - R_{14})^2) \geq 0$$

$$\lambda_{23} - c_{23}((R_{21} - R_{21})^2 + (R_{22} - R_{22})^2 + (R_{23} - R_{23})^2 + (R_{24} - R_{24})^2) \geq 0$$

$$\lambda_{42} - c_{42}((R_{41} - R_{41})^2 + (R_{42} - R_{42})^2 + (R_{43} - R_{43})^2 + (R_{44} - R_{44})^2) \geq 0$$

$$R_{11} = 0$$

$$R_{12} = (0.700000)^a + (0.844000)^a + (1.000000)^a + (0.000000)^a + (0.800000)^a$$

$$R_{13} = (1.000000)^a + (1.000000)^a + (0.866700)^a + (0.500000)^a + (1.000000)^a$$

$$R_{14} = (0.400000)^a + (0.314900)^a + (0.633300)^a - \theta(0.500000)^a + (0.200000)^a$$

$$R_{21} = -\theta(0.700000)^a - \theta(0.844000)^a - \theta(1.000000)^a + (0.000000)^a - \theta(0.800000)^a$$

$$R_{22} = 0$$

$$R_{23} = (0.300000)^a + (0.156000)^a - \theta(0.133300)^a + (0.500000)^a + (0.200000)^a$$

$$R_{24} = -\theta(0.300000)^a - \theta(0.529100)^a - \theta(0.366700)^a - \theta(0.500000)^a - \theta(0.600000)^a$$

$$R_{31} = -\theta(1.000000)^a - \theta(1.000000)^a - \theta(0.866700)^a - \theta(0.500000)^a - \theta(1.000000)^a$$

$$R_{32} = -\theta(0.300000)^a - \theta(0.156000)^a + (0.133300)^a - \theta(0.500000)^a - \theta(0.200000)^a$$

$$R_{33} = 0$$

$$R_{34} = -\theta(0.600000)^a - \theta(0.685100)^a - \theta(0.233300)^a - \theta(1.000000)^a - \theta(0.800000)^a$$

$$R_{41} = (0.400000)^a - \theta(0.314900)^a + (0.633300)^a + (0.500000)^a - \theta(0.200000)^a$$

$$R_{42} = (0.300000)^a + (0.529100)^a + (0.366700)^a + (0.500000)^a + (0.600000)^a$$

$$R_{43} = (0.600000)^a + (0.685100)^a + (0.233300)^a + (1.000000)^a + (0.800000)^a$$

$$R_{44} = 0$$

$$R_1 = R_{11} + R_{12} + R_{13} + R_{14}, R_2 = R_{21} + R_{22} + R_{23} + R_{24}, R_3 = R_{31} + R_{32} + R_{33} + R_{34}, R_4 = R_{41} + R_{42} + R_{43} + R_{44}$$

$$0.15 \leq \omega_1 \leq 0.45, 0.1 \leq \omega_2 \leq 0.2, 0.1 \leq \omega_3 \leq 0.25, \omega_4 \geq 0.12, \omega_5 \geq 0.15, \omega_1 \geq 1.3, \omega_2 - \omega_3 \leq 0.05, \omega_1 - \omega_2 \geq \omega_5 - \omega_4$$

$$c_{13} = 0.1429, c_{14} = 0.4286, c_{23} = 1.0000, c_{42} = 0.7143$$
results are accurate, it is necessary to define the evaluator’s behaviour more reasonably in the actual environment.

**APPENDIX A**

In the example analysis of PSQ evaluation from Section V, the LINMAP model based on prospect theory without considering the fuzzy preference degree is shown at the bottom of page 14.

**APPENDIX B**

In the example analysis of PSQ evaluation from Section V, the LINMAP model based on prospect theory considering the fuzzy preference degree is shown at the bottom of page 15.

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[Image of W. Zuo et al.: LINMAP Method Based on Bounded Rationality of Evaluators]