Proper orthogonal decomposition of turbulent swirling flow of a draft tube at part load

S Kumar¹*, S Khullar¹, M J Cervantes², and B K Gandhi¹

¹Indian Institute of Technology, Roorkee, India
Department of Mechanical and Industrial Engineering, Indian Institute of Technology, Roorkee, Uttarakhand 247667
²Lulea University of Technology, Sweden
Division of Fluid and Experimental Mechanics, Department of Engineering Sciences and Mathematics, Norrbotten 97187, Sweden

*Corresponding author: Tel.: +91 9417555636, mechengr03@gmail.com

Abstract: A large scale vortical structure, i.e., rotating vortex rope (RVR), generally forms at part load (PL) operation of a reaction turbine due to the combined effect of swirling flow at the runner outlet and the adverse pressure gradient in the draft tube. Even with the data analysis through advanced measurement techniques like Laser Doppler velocimetry (LDV) and Particle Image Velocimetry (PIV), the information available about the flow instabilities developed due to RVR is still incomplete. This paper presents an application of the proper orthogonal decomposition (POD) method to analyze the flow field inside the draft tube cone at PL operation of a high-head model Francis turbine. The POD analysis is performed on 250 PIV snapshots containing the axial and radial velocities. The results show that the first eight modes contain more than 95% of the total kinetic energy (KE) of the flow field and are associated with the organized motion of the flow. The first mode contains more than 50% of the total energy, and the axial velocity profile reconstructed with the first mode is identical to the mean axial velocity flow field. The maximum dissipation of the turbulent kinetic energy (TKE) occurs through unorganized motion.

1. Introduction
The grid network faces challenges due to energy penetration from intermittent sources and sudden changes in demand. The ability of hydropower plants to quickly respond to load variations and stable operation over a wide range of flow regimes makes them suitable for providing grid stability along with the production of clean energy. This makes the turbine unit operate consistently at off-design conditions, i.e., part load (PL) or high load (HL). At off-design conditions, the flow instabilities and high turbulence are frequently encountered in hydropower plants [1], especially in the draft tube of reaction hydro-turbine. In case of Francis turbines, due to fixed pitched runner blades, off-design operating conditions lead to decelerating swirling flow in the draft tube cone which causes pressure pulsations, power swings, and vibrations [2]. These pressure fluctuations generated due to hydrodynamic flow instability originate in the draft tube and propagate through the entire flow passage, which leads to wear and tear of runner blades and other moving components of the turbine.
The decelerated swirling flow under the strong adverse pressure gradient develops a stagnation and recirculation region along with flow separation, which is presumed to be the origin of the formation of rotating vortex rope (RVR) [4]. The RVR formation in the draft tube is not only unfavorable to turbine life but also reduces its efficiency. It is a low-frequency phenomenon that produces non-periodic pressure pulsations [5]. Qualitative experimental investigations were performed by various researchers to determine the flow pattern and structure of RVR in the draft tube [6-7]. A shear layer forms at the trailing edge of the runner blade by the combination of boundary layers developed on the pressure and suction sides of the runner blade along with wake interaction. The adverse high pressure gradient, together with wake interaction and shear layer separation, generates vortices causing vortex breakdown and formation of RVR [8].

The extraction of information from instantaneous velocity fields in terms of the decomposition of modes provides a better understanding of the vortex rope structure in the draft tube. A large vortical structure can be further decomposed into small organized structures called coherent structures via decomposition through POD. In this paper, the investigation of a vortex rope structure has been attempted via decomposed POD modes. The instantaneous velocity field has been reconstructed using the eigenfunction values and POD coefficients. The different structures of flow field vectors have been explained. This approach gives a better insight to visualize the characteristics of the vortex rope phenomenon like shear layer formation and flow reversal in the stagnation region.

2. Proper Orthogonal Decomposition

Proper orthogonal decomposition (POD) is a tool used for the extraction of coherent structures from the turbulent flows, which are difficult to define and observe. POD is widely used in the field of turbulence and fluid dynamics. This method captures the dominant structures called deterministic functions or POD modes, which give an idea of the organization of flow. This method was first introduced by Lumely [9], and later Sirovich [10] developed the snapshot method. This decomposition of a vector field representing the turbulent fluid motion decouples the spatial variation from the temporal variation [11]. Another advantage of this method is to make an optimal basis that represents most of the data variance, like turbulent kinetic energy, with the help of a few possible functions. Therefore, the most energetic coherent structures, also called modes with high energy, are searched in the turbulent flow field by POD and are likely to be characterized by POD functions. These energetic modes provide a model decomposition of instantaneous velocity fields.

\[
H = \begin{pmatrix}
    u_{1x,y}^1 & u_{1x,y}^2 & \cdots & u_{1x,y}^N \\
    u_{2x,y}^1 & u_{2x,y}^2 & \cdots & u_{2x,y}^N \\
    \vdots & \vdots & \ddots & \vdots \\
    u_{Nx,y}^1 & u_{Nx,y}^2 & \cdots & u_{Nx,y}^N \\
    v_{1x,y}^1 & v_{1x,y}^2 & \cdots & v_{1x,y}^N \\
    v_{2x,y}^1 & v_{2x,y}^2 & \cdots & v_{2x,y}^N \\
    \vdots & \vdots & \ddots & \vdots \\
    v_{Nx,y}^1 & v_{Nx,y}^2 & \cdots & v_{Nx,y}^N
\end{pmatrix}
\]

(1)

\[
G = \frac{1}{N}(HH^T) = \begin{pmatrix}
    G_{11} & \cdots & G_{1N} \\
    \vdots & \ddots & \vdots \\
    G_{N1} & \cdots & G_{NN}
\end{pmatrix}
\]

(2)

\[
GA^i = \lambda^i A^i
\]

(3)
\[ \lambda_1 > \lambda_2 > \ldots \ldots > \lambda_N = 0 \]  

(4)

\[ \alpha_i^m = \overline{v_i} \cdot \xi^m \]  

(5)

In the present work, POD was applied on 250 PIV snapshots (N), available in the literature [12]. These snapshots were taken in a plane in the draft tube cone of a high-head Francis turbine at a steady part-load operation. The instantaneous velocity component data has \( l \) rows and \( c \) columns arranged in a matrix. A matrix \( H \) having \( 2lc \) rows and \( N \) columns, as shown in equation 1, is obtained from the instantaneous data. An auto-covariance tensor matrix \( (G) \) can be derived from equation 1 using equation 2. The eigenvalue problem is then solved for this auto-covariance matrix \( G \) using equation 3 and ranked in decreasing order according to the size of the eigenvalue (equation 4). The snapshot, which represents the instantaneous velocity flow field can be extended to a series of POD modes. When these velocity fields are projected on the \( m \)th eigenfunction value \( \xi^m \), the POD coefficients \( \alpha_i^m \) are obtained, as given in equation 5. Thus, the decomposition of the time-dependent snapshots generates spatial modes and temporal coefficients.

The \( m \)th velocity vector corresponds to \( m \)th decomposition and is given by equation 6. The velocity field represented by a snapshot can be reconstructed using equation 6.

\[
\overline{v_i}(x, y) = \sum_{i=1}^{N} \alpha_i^m \overline{v_i^m}(x, y) = \sum_{i=1}^{N} \alpha_i^m \xi^m(x, y)
\]  

(6)

3. Results and Discussion

The physical behavior of the flow field is determined by the eigenvalues and POD coefficients obtained after decomposition of the instantaneous data. The contribution of the first 10 POD modes in terms of normalized relative energy is shown in Fig.1. Mode 1 has the highest energy, i.e., 50.72% of total kinetic energy, and represents the mean flow field. Mode 2 has almost half of the mode 1 energy and contains about 26% of the total energy, and mode 3 has around half of the energy of mode 2.

Mode 4 and mode 5 have almost the same energy. Mode 6 and mode 7 have energies in the range of 0.03% of the total. The contribution of modes 1 to 5 is more than 95.5% of the total energy. A random instantaneous snapshot of the velocity vector field is reconstructed using all 250 POD modes to get the exact vector fields as represented by the actual snapshot and is presented in Fig. 2 (a-b). The x-coordinate has been normalized with respect to the draft tube inlet radius. The blue and red colors regions are shown in Fig. 2(a-b) represent the region of local maximum and minimum velocity magnitudes, respectively. The bulk fluid flow is towards the negative y-direction, as represented by blue-colored regions in Fig. 2(a-b). Thus, the velocity vectors in the direction of flow through the draft tube are blue, while the red color region shows the velocity vectors in the reverse direction. The streamline of the first eight POD modes is shown in Fig. 3 (a-h). The streamlines of mode 1 in Fig. 3 (a) shows the mean flow field. The inclination of streamlines in the direction of runner rotation can be attributed to the tangential component of the velocity. Modes 2 to 8 reveal the organized motion of coherent structures. These coherent structures are formed due to instabilities like Kelvin-Helmholtz instability of shear layer. Mode 2 in Fig. 3(b) has flow in direction opposite to runner rotation. Here the kinetic energy is half of mode 1, and the effect of the axial velocity component may be suppressed by the dominant tangential velocity component in that region. Two large vortical structures can be seen in mode 3 (Fig. 3 c). The orientation of these coherent structures is similar and appears like a large vortex being dissociated into two small structures. Mode 4 in Fig. 3 (d) shows two vortices that are mirror images of each other. Mode 5 in Fig. 3 (e) is a flipped image of mode 4 and has almost same energy as mode 4. This shows that the incoherent turbulence and pairing interaction of coherent structure leads to the formation of new structures with lesser energy, which indicates that the coherent structure formed in mode 4 further dissociate into smaller structures. Each coherent structure has its own boundary and has its own independent domain so that they do not spatially overlap each other. Modes 3, 4, and 5 can be associated with the bursting phenomenon [13], which refers to a release of
the burst of turbulent kinetic energy (TKE), and rapid diffusion of vorticity. Mode 6 is similar to the two vortical structures present in Mode 3, which results in reduced core vorticity and cause the incoherent turbulence. The energy of mode 6 is 0.34 % and has 4 small vortical structures. Mode 7 also has a very small KE of around 0.30 %. It is formed by dissociation of the large vortical structures into further smaller vortical structures. Mode 8 (Fig. 3h) shows the flow reversal in the middle of the flow field plane, and it looks like a vortex core with the reverse flow, which feeds the vortical structures present in the turbulent flow field.

**Figure 1.** Modes versus energy plot

**Figure 2.** (a) Quiver plot of a random instantaneous snapshot; (b) Reconstruction of snapshot from 250 modes.
Figure 3. Streamline plot of first eight POD modes; (a) POD mode 1; (b) POD mode 2; (c) POD mode 3; (d) POD mode 4; (e) POD mode 5; (f) POD mode 6; (g) POD mode 7; (h) POD mode 8.
3.1. Kinetic Energy of POD Modes

The energy carried by the coherent structures in the turbulent flow is represented by the eigenvalues of the velocity field covariance matrix. Therefore, the eigenvalues correspond to the kinetic energy carried by each mode, which is normalized by the sum of the eigenvalues. Thus, kinetic energy carried by the fluid can be given by equation 7 as:

$$K(x, y) = \frac{1}{2} \sum_{i=1}^{N} \lambda_i \bar{\xi}_i(x, y) \cdot \bar{\xi}_i(x, y)$$

Figure 4 shows the mean kinetic energy of the flow or kinetic energy associated with mode 1, which has 50.72% of total kinetic energy. Moreover, the left-hand region from the center of the draft tube has lower energy as compared to the right-hand side as the flow is directed toward the right side due to runner rotation.

Figure 5 shows the contribution of kinetic energy by the coherent structures of mode 2 to mode 8, which have a contribution of 45.93% to the total kinetic energy of the flow field, as shown in Fig. 5 (a).
remaining contribution of 3.43% can be attributed to the incoherence or turbulence in the flow which is shown in Fig. 5 (b).

4. Conclusion
The decomposition of all the snapshots of instantaneous velocity fields by POD provides an insight into the structure of the vortex rope. The mean flow (mode 1), organized vortical structures (mode 2 to 7), flow reversal (mode 8), and turbulence (mode 9 to 250) are all visible distinctively. The KE in mode 1 is maximum (50.72%) as the flow rate is higher near the wall due to the formation of the stagnation region in the central core. For modes 2-8, the central region has maximum energy because of the energy exchanges between the eddies formed in these coherent structures. The magnitude of energy contribution of these coherent structures is less than that of mode 1. These incoherent structures can be held responsible for viscous losses in the fluid flow.

References
[1] Trivedi C, Cervantes M J, Gandhi B K, Ole D G 2014 Journal of Hydrodynamics 26(2) 277-90.
[2] Dörfler P, Sick M, Coutu A, 2012, Flow-induced pulsation and vibration in hydroelectric machinery: engineer’s guidebook for planning, design and troubleshooting Springer Science & Business Media.
[3] Frunzăverde D, Muntean S, Mârginean G, Campian V, Mașăvina L, Terzi R and Şerban V 2010 IOP conference series: earth and environmental science 12(1) 012115.
[4] Goyal R and Gandhi B K 2018 Renewable energy 1(116) 697-709.
[5] Sotoudeh N, Maddahian R and Cervantes M J 2019, Renewable Energy 1(151)238-54.
[6] Iliescu MS, Ciocan GD and Avellan F 2002 In ASME Joint US-European fluids engineering division conference 311-316.
[7] Müller A, Dreyer M, Andreini N and Avellan F 2013 Experiments in fluids 54(4) 1514.
[8] Zhang R K, Mao F, Wu JZ, Chen S Y, Wu Y L and Liu S H 2009 Journal of Fluids Engineering 131(2) 021101.
[9] Lumley J L 1967 Atmospheric Turbulence and Wave Propagation 166-78.
[10] Sirovich L 1987 Quarterly of applied mathematics 45(3) 561-71.
[11] Andrianne T, Razak N A and Dimitriadiis G 2011 Wind Tunnels 87-104.
[12] Kumar S, Khullar S, Goyal R, Cervantes M J and Gandhi B K 2019 Proceedings of the 25th National and 3rd International ISHMT-ASTFE Heat and Mass Transfer Conference (IHMTC 2019), IIT Roorkee, India.
[13] Hussain A F 1983 The Physics of fluids 26(10) 2816-50.