Three-Dimensional Adaptive Mesh Refinement
Simulations of Point-Symmetric Nebulae

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Abstract. Previous analytical and numerical work shows that the generalized interacting stellar winds model can explain the observed bipolar shapes of planetary nebulae very well. However, many circumstellar nebulae have a multipolar or point-symmetric shape. With two-dimensional calculations, Icke showed that these seemingly enigmatic forms can be easily reproduced by a two-wind model in which the confining disk is warped, as is expected to occur in irradiated disks. In this contribution we present the extension to fully three-dimensional adaptive mesh refinement simulations of such an interaction.

1. Introduction

The interaction between a slow disk-shaped inner AGB nebula and a fast stellar wind as a mechanism to produce bipolar bubbles is by now well established. In this generalized interacting stellar winds model (see Balick & Frank 2002 for a review) the shape of the dense gas around the star is assumed to be a disk or a toroid. Analytical (Icke 1988; Icke et al. 1989) and numerical (e.g. Soker & Livio 1989; Icke et al. 1992; Mellema et al. 1991) work shows that this mechanism works very well.

However, many circumstellar nebulae have a multipolar or point-symmetric shape (Schwarz 1993; Sahai & Trauger 1998). Icke (2003, this volume) proposed that these nebulae are formed in a wind-disk interaction where the disk confining the fast wind is warped. One can produce such a warp around a single star, from the combined effects of irradiation and cooling (e.g. Pringle 1996; Maloney et al. 1996).

Whereas Icke’s computations were restricted to a two-dimensional proof-of-principle, we now present a first series of fully three-dimensional hydrodynamic computations of such a wind-disk interaction.

2. Radiation driven warping

When an accretion disk is subject to external torques, it may become unstable to warping and when irradiated by a sufficiently luminous central star even an initially flat disk will warp (e.g. Pringle 1996; Maloney et al. 1996). For this mechanism to work, it is essential that the disk is optically thick for the stellar radiation and for its own cooling flux. This restricts the disks in our model to a specific subclass with relatively high density and low temperature.
For a PN the luminosity of the central star alone is sufficiently high to induce a radiation driven warp. Following Pringle (1997), an expression for the radius $R_{\text{crit}}$ beyond which the disk is unstable to radiation driven warping is found:

$$R_{\text{crit}} = (2\pi/A)^2,$$

with the constant $A$ defined by $A^2 \equiv 1/4 c^{-2} G^{-1} M_*^{-1} L_*^2 \eta^{-2} \dot{M}_{\text{acc}}^{-2}$. Here $c$ is the speed of light, $G$ the gravitational constant, $\eta \equiv \nu_2/\nu_1$ is the ratio of the azimuthal to the radial viscosity, $M_*$ is the mass and $L_*$ the luminosity of the central star and $\dot{M}_{\text{acc}}$ is the disk’s accretion rate. We assumed a surface density $\Sigma_d = \dot{M}_{\text{acc}}/(3\pi \nu_1)$ (e.g. Pringle 1981).

In a cartesian coordinate system, the warped disk surface is given by

$$\mathbf{x}(R, \phi) = R \begin{pmatrix} \cos \phi \sin \gamma + \sin \phi \cos \gamma \cos \beta \\ - \cos \phi \cos \gamma + \sin \phi \sin \gamma \cos \beta \\ - \sin \phi \sin \beta \end{pmatrix},$$

with local disk tilt angle $\beta(R, \phi)$, and orientation angle of the line of nodes $\gamma(R, \phi)$. Here, $R$ and $\phi$ are the non-orthogonal radial and azimuthal coordinates, respectively, pointing to the surface of the disk (Pringle 1996). In our model calculations we adopt the case of a steady precessing disk with no growth and zero torque at the origin. This gives $\gamma = A \sqrt{R}$ and $\beta = \sin \gamma/\gamma$ (Maloney et al. 1996) in the precessing frame.

### 3. 3D AMR Simulations

We used the three-dimensional hydrocode *Flash* (Fryxell et al. 2000) to model the interaction between a spherical wind and a warped disk. This parallelized
code implements block-structured adaptive mesh refinement (AMR) initially developed by Berger & Oliger (1984) and a PPM type hydrosolver (Woodward & Colella 1984).

To construct the warped disk, Eq. (1) was combined with a constant ‘wedge angle’ $\theta_d$ and a proper value for $A$, i.e. $R_d$ was taken to be a few times $R_{\text{crit}} \simeq 1 \text{ AU}$, see Eq. (2). This disk was given a constant density which, through the density contrast $\chi$, resulted in a value for the environment density $n_e$. The spherical wind was implemented as an inner boundary condition and given a $1/r^2$ density profile and a constant wind velocity $v_w$. The pressure was calculated from an equation of state with a constant Poisson index $\gamma = 1.1$, resulting in a highly compressible, momentum-driven flow.

To test our code we ran a number of two-dimensional simulations, an example of which is shown in Fig. 1. After this we ran wind-disk simulations in three dimensions on a cartesian grid with an effective resolution of up to $512^3$ cells using five levels of refinement. We used the following parameters: $n_e = 10^9 \text{ cm}^{-3}$, $\chi = 100$, $\theta_d = 5^\circ$, and $v_w = 200 \text{ km s}^{-1}$. In Fig. 2 we present a visualization of the three-dimensional shape of the swept up shell through isosurfaces at different viewing angles. We also derived the corresponding synthesized $H\alpha$ images by projecting the three-dimensional data cube onto the plane of the sky. For this, we simply integrated the density squared along the line of sight and used this as a rough estimate for the emission. The images show that the interaction of a spherical stellar wind with a warped disk results in a wide va-
riety of point-symmetric shapes. Movies of these simulations can be found at http://www.strw.leidenuniv.nl/AstroHydro3D/

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