Non-universal corrections to the tension ratios in softly broken $\mathcal{N} = 2$ $SU(N)$ gauge theory

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Abstract. Calculation by Douglas and Shenker of the tension ratios for vortices of different $N$-alities in the softly broken $\mathcal{N} = 2$ supersymmetric $SU(N)$ Yang–Mills theory is carried to the second order in the adjoint multiplet mass $m$. Corrections to the ratios violating the well known sine formula are found, showing that it is not a universal quantity.

Recently the tension ratios among the confining vortices corresponding to sources of different $N$-alities in $SU(N)$ gauge theories have been the subject of some attention, as a quantity characterizing quantitatively the confining phase of these systems. After an interesting suggestion from MQCD that such ratios might have universal values [1],

$$\frac{T_k}{T_1} = \frac{\sin \frac{\pi k}{N}}{\sin \frac{\pi}{N}}, \quad (1)$$

a more recent study in string theory based on supergravity duals [2], gave model-dependent results for two $\mathcal{N} = 1$ SQCD-like theories. The result of direct measurement in the lattice (non-supersymmetric) $SU(N)$ gauge theories is consistent with equation (1) [3, 4].

Derivation of formulae such as equation (1) in the standard, continuous $SU(N)$ gauge theories still defies us. The first field-theoretic result on this issue was obtained by Douglas and Shenker [5], in the $\mathcal{N} = 2$ supersymmetric $SU(N)$ pure Yang–Mills theory, with supersymmetry softly broken to $\mathcal{N} = 1$ by a small adjoint scalar multiplet mass $m$. They found equation (1) for the ratios of the tensions of Abelian (Abrikosov–Nielsen–Olesen (ANO)) vortices corresponding to different $U(1)$ factors of the low-energy effective (magnetic) $U(1)^{N-1}$ theory.
The \( n \)th colour component of the quark has charges
\[
\delta_{n,k} - \delta_{n,k+1} \quad (k = 1, 2, \ldots, N - 1; n = 1, 2, \ldots, N)
\]
with respect to the various electric \( U_k(1) \) gauge groups. The source of the \( k \)th ANO string thus corresponds to the \( N \)-ality \( k \) multiquark state, \( |k\rangle = |q_1 q_2, \ldots, q_N\rangle \), allowing a reinterpretation of equation (1) as referring to the ratio of the tension for different \( N \)-ality confining strings [6].

However, physics of the softly broken \( \mathcal{N} = 2 \) \( SU(N) \) pure Yang–Mills theory is quite different from what is expected in QCD. Dynamical \( SU(N) \to U(1)^{N-1} \) breaking introduces multiple meson Regge trajectories with different slopes at low masses [6, 7], a feature which is neither seen in nature nor expected in QCD†. For instance, another \( N \)-ality \( k \) state \( |k\rangle' = |q_2 q_3, \ldots, q_k+1\rangle \) acts as the source of the \( U_{k+1}(1) \) vortex and as the sink of the \( U_2(1) \) vortex, which together bind \( |k\rangle' \) and anti-\( |k\rangle' \) states with a tension different from \( T_k \). The Douglas–Shenker prediction is, so to speak, a good prediction for a wrong theory! Only in the limit of \( \mathcal{N} = 1 \) does one expect to find only one stable vortex for each \( N \)-ality, corresponding to the conserved \( Z_N \) charges [6].

Within the softly broken \( \mathcal{N} = 2 \) \( SU(N) \) theory, the two regimes can be in principle smoothly interpolated by varying the adjoint mass \( m \) from zero to infinity, adjusting \( \Lambda \) appropriately. At small \( m \) one has a good local description of the low-energy effective dual, magnetic \( U(1)^{N-1} \) theory. The transition towards a large \( m \) regime involves both perturbative and non-perturbative effects. Perturbatively, there are higher corrections due to the \( \mathcal{N} = 1 \) perturbation, \( m \langle \text{Tr} \Phi^2 \rangle \). Non-perturbatively—in the dual theory—there are productions of massive gauge bosons of the broken \( SU(N)/U(1)^{N-1} \) generators, which mix different \( U(1)^{N-1} \) vortices and eventually lead to the unique stable vortex with a given \( \mathcal{N} \)-ality. There seem to be no general reasons to believe that the tension ratios found in the small \( m \) limit are not renormalized in such processes.

Below we report the result on the first type of effects: perturbative corrections to the tension ratios equation (1) due to the next-to-lowest contributions in \( m \). We shall find a small non-universal correction to the sine formula equation (1). Our point is, of course, not that such a result is of interest in itself as a physical prediction but that it gives a strong indication for the non-universality of this formula, even though it could be an approximately good one.

The problem of the next-to-lowest contributions in \( m \) has already been studied in \( SU(2) \) theory, by Vainshtein and Yung [7] and by Hou [12], although in that case there is only one \( U(1) \) factor so that the author’s interest was different. When only up to the order \( A_D \) term in the expansion
\[
m \langle \text{Tr} \Phi^2 \rangle = m U(A_D) = m \Lambda^2 \left(1 - \frac{2i A_D}{\Lambda} - \frac{A_D^2}{4 \Lambda^2} + \cdots\right)
\]
is kept, the effective low-energy theory turns out to be an \( \mathcal{N} = 2 \) SQED, \( A_D \) being an \( \mathcal{N} = 2 \) analogue of the Fayet–Iliopoulos term. As a result, the vortex remains BPS-saturated, and its tension is proportional to the monopole charge. When the \( A_D^2 \) term is taken into account, the vortex ceases to be BPS-saturated: the correction to the vortex tension can be calculated perturbatively, giving rise to the result that the vacuum behaves as a type I superconductor.

† In fact, the same problem is expected in any confining vacuum in which such a dynamical breaking takes place. t’ Hooft’s original suggestion for QCD ground state [8] is of this type.
Our aim here is to generalize the analysis of Vainshtein, Yung and Hou [7, 12] to $SU(N)$ theory. In fact, the Douglas–Shenker result (equation (1)) in $SU(N)$ theory was obtained in the BPS approximation, by keeping only the linear terms in $a_{Dk}$ in the expansion

$$U(a_{Di}) = U_0 + U_{0k} a_{Dk} + \frac{U_{0mn}}{2} a_{Dm} a_{Dn} + \cdots, \quad U_{0k} = \frac{-4i\Lambda}{N} \sin \frac{\pi k}{N}. \quad (4)$$

The coefficients $U_{0k}$ were computed by Douglas and Shenker [5]. Our first task is then to compute the coefficients of the second term $U_{0mn}$. In principle it is a straightforward matter, as one must simply invert the Seiberg–Witten formula:

$$a_{Dm} = f_{a_m} \lambda, \quad a_m = f_{a_m} \lambda, \quad \lambda = \frac{1}{2\pi i} \int y \frac{\partial P(x)}{\partial x} \, dx, \quad (5)$$

which is explicitly known to second order. The only trouble is that $a_{Dm}$ and $a_m$ ($m = 1, 2, \ldots, N - 1$) are given simply in terms of $N$-dependent vacuum parameters $\phi_i$, $\sum_{i=1}^{N} \phi_i = 0$.

By denoting the formal derivatives with respect to $\phi_i$ as $\frac{\delta}{\delta \phi_i}$, one finds

$$\sum_{i=1}^{N} \frac{\delta a_{Dm}}{\delta \phi_i} \frac{\partial \phi_i}{\partial a_{Dm}} = \delta_{mn}, \quad \sum_{m=1}^{N-1} \frac{\delta a_{Dm}}{\delta \phi_i} \frac{\delta a_{Dm}}{\delta \phi_j} = \delta_{ij} - \frac{1}{N}, \quad (6)$$

which follow easily by using the constraint, $\sum_{i=1}^{N} \phi_i = 0$. In terms of $B_{mi} \equiv -i \frac{\delta a_{Dm}}{\delta \phi_i}$, which are explicitly given at the $N$ confining vacua in [5], one then finds

$$\frac{\partial \phi_i}{\partial a_{Dm}} = -i B_{mi}, \quad \sum_{m=1}^{N} B_{mi} B_{ni} = \delta_{mn}, \quad \sum_{m=1}^{N-1} B_{mi} B_{mj} = \delta_{ij} - \frac{1}{N}. \quad (7)$$

The explicit values of $B_{mi}$ are (see [5]):

$$B_{mi} = \frac{1}{N} \frac{\sin[\hat{\theta}_m]}{\cos[\hat{\theta}_i] - \cos[\hat{\theta}_m]}, \quad \hat{\theta}_n = \frac{\pi n}{N}, \quad \theta_n = \frac{\pi(n - 1/2)}{N}. \quad (8)$$

The definition of $u(a_{Di})$ is the following:

$$u(a_{Di}) = \sum_i \phi_i^2. \quad (9)$$

Then the desired coefficients can be found by the following expression, computed at $a_{Di} = 0$:

$$U_{0mn} = \frac{\partial^2 u}{\partial a_{Dm} \partial a_{Dn}} = 2 \sum_k \frac{\partial \phi_k}{\partial a_{Dm}} \frac{\partial \phi_k}{\partial a_{Dn}} + 2\phi_k \frac{\partial^2 \phi_k}{\partial a_{Dm} \partial a_{Dn}}. \quad (10)$$

The first part of equation (10) becomes:

$$2 \sum_k \frac{\partial \phi_k}{\partial a_{Dm}} \frac{\partial \phi_k}{\partial a_{Dn}} = -2 \sum_k B_{km} B_{kn} = -2 \sum_{k,s} \frac{2}{N} \sin \left[ \frac{\pi m s}{N} \right] \sin \left[ \frac{\pi n k}{N} \right] \delta_{ks} = -2 \delta_{mn}. \quad (11)$$

The evaluation of the second term in equation (10) is reported in the appendix. The result is the following:

$$2 \sum_k \phi_k \frac{\partial^2 \phi_k}{\partial a_{Dm} \partial a_{Dn}} = \left( 2 - \frac{1}{N} \right) \delta_{mn}. \quad (12)$$

† We follow the notation of [5], with $y^2 = P(x)^2 - \Lambda^2; P(x) = \frac{1}{2} \prod_{i=1}^{N} (x - \phi_i)$. 

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thus
\[ U_{0mn} = (-\frac{1}{N})\delta_{mn}. \tag{13} \]

We now use this result to calculate the corrections to the tension ratios (1) found in the lowest order. The effective Lagrangian near one of the \( N \) confining \( \mathcal{N} = 1 \) vacua is
\[
\mathcal{L} = \sum_{i=1}^{N-1} \text{Im} \left[ \frac{i}{e_{D_1}} \left( \int d^4 \theta A_{D_1} A_{D_1}^\dagger + \int d^2 \theta (W_{D_1})^2 \right) \right] + \text{Re} \left[ \int d^4 \theta \left( M_i^+ e^{V_{D_1} M_i} + \tilde{M}_i^+ e^{-V_{D_1} \tilde{M}_i} \right) \right] + 2\text{Re} \left[ \sqrt{2} \int d^2 \theta A_{D_1} M_i \tilde{M}_i + mU[A_{D_1}] \right]. \tag{14} \]

The coupling constant \( e_{D_1}^2 \) is formally vanishing, as
\[
\frac{4\pi}{e_{D_k}^2} \simeq \frac{1}{2\pi} \ln \frac{\Lambda \sin(\bar{\theta}_k)}{a_{D_k} N} \]
where \( \bar{\theta}_m \equiv \frac{\pi m}{N} \) and \( a_{D_k} = 0 \) at the minimum. Physically, the monopole loop integrals are in fact cut off by masses caused by the \( \mathcal{N} = 1 \) perturbation. The monopole becomes massive when \( m \neq 0 \), and \( \sqrt{2}a_{D_k} \) should be replaced by the physical monopole mass \( (m\Lambda \sin(\bar{\theta}_k))^{1/2} \) which acts as the infrared cutoff for the coupling constant evolution. This is equivalent to the prescription of taking \( a_{D_n} = \langle M \tilde{M} \rangle_n^{1/2} \), which is used in [5]. One finds thus
\[
\frac{e_{D_m}^2}{2\pi} \simeq \frac{16\pi^2}{\ln \frac{\Lambda \sin(\bar{\theta}_m)}{mN^2}}. \tag{15} \]

As \( U_{0mn} \) is found to be diagonal, the description of the ANO vortices [9, 10] in terms of effective magnetic Abelian theory description continues to be valid for each \( U(1) \) factor. In the linear approximation \( U(A_D) = mA^2 + \mu A_D \), where \( \mu \equiv |4m\Lambda \sin(\bar{\theta}_k)| \) for the \( k \)th \( U(1) \) theory, the theory can be (for the static configurations) effectively reduced to an \( \mathcal{N} = 4 \) theory in 2+1 dimensions. In this way, Bogomolny’s equations for the BPS vortex can be easily found from the condition that the vacuum is supersymmetric:
\[
\begin{align*}
F_{12} &= \sqrt{2}(\sqrt{2}M^+ \tilde{M}^+ - \mu), \quad (D_1 + iD_2)M = 0, \quad (16) \\
M &= \tilde{M}^+, \quad A_D = 0. \quad (17)
\end{align*}
\]
The solutions of these equations are similar to the one considered by Nielsen and Olesen:
\[
M = \left( \frac{\mu}{\sqrt{2}} \right)^{1/2} e^{in\phi} f[re^{\sqrt{\mu}}], \quad A_\phi = -2n \frac{g(re^{\sqrt{\mu}})}{r} \tag{18}
\]
where
\[
f' = \frac{f}{r}(1 - 2g)n, \quad g' = \frac{1}{2n}r(1 - f^2) \tag{19}
\]
with boundary conditions \( f(0) = g(0) = 0, f(r \to \infty) = 1, g(r \to \infty) = +1/2 \). The tension turns out to be independent of the coupling constant: for the minimum vortex:
\[
T = \sqrt{2}\pi \mu = 4\sqrt{2}\pi |m\Lambda| \sin \frac{\pi k}{N}. \tag{20}
\]
\[\dagger\] The fact that the absolute value of \( m \) appears in equation (20), as it should, may not be obvious. This can be shown by an appropriate redefinition of the field variables, used in [11], which makes all equations real. The correction term in (23) is thus negative independently of the phase of \( m \).
When the second-order term in \( U(A_D) = \mu A_D + \frac{1}{2} \eta A_D^2 \), \( \eta = U_{kk} \), is taken into account, the vortex ceases to be BPS-saturated. The corrections to the vortex tension due to \( \eta \) can be taken into account by perturbation theory, following [12]. To first order, the equation for \( A_{Dk} = A_D \) is
\[
\nabla^2 A_D = -2e^4 \eta (\mu - \sqrt{2} \hat{M} \hat{M}) + 2e^2 A_D (MM^+ + \hat{M} \hat{M}^+) \tag{21}
\]
where unperturbed expressions from equation (18) can be used for \( M, \hat{M} \). The vortex tension becomes simply
\[
T = \int d^2 x \left[-\sqrt{2} \mu F_{12} - 2e^2 \eta A_D (\mu - \sqrt{2} M^+ \hat{M}^+)\right] \tag{22}
\]
where the second term represents the correction. By restoring the \( k \) dependence, we finally get for the tension of the \( k \)th vortex,
\[
T_k = 4\sqrt{2}\pi |m| \Lambda \sin \left(\frac{\pi k}{N}\right) - C \frac{16\pi^2 |m|^2}{N^2 \Lambda \sin(k\pi/N)} \tag{23}
\]
where \( C = 2\sqrt{2}\pi(0.68) = 6.04 \). The correction term has a negative sign, independently of the phase of the adjoint mass. Note that the relation \( T_k = T_{N-k} \) continues to hold. Equation (23) is valid for \( m \ll \Lambda \).

We end with a few remarks. In the above consideration, we have taken into account exactly the \( m^2 \) corrections in the \( F \) term of the effective low-energy action. On the other hand, the corrections to the \( D \) terms are subtler. Indeed, based on the physical consideration, \( a_D \) in the argument of the logarithm in the effective low-energy coupling constant was replaced by the monopole mass of \( O(\sqrt{m}\Lambda) \). This amounts to \( m \) insertion to all orders in the loops. Such a re-summation is necessitated by the infrared divergences, just as in the case of chiral perturbation theory. This explains the non-analytic dependence on \( m \) as well as on \( 1/N \) [13].

Also, there are corrections due to non-diagonal elements in the coupling constant matrix \( \tau_{ij} \), which mix the different \( U(1) \) factors [14], neglected in equation (14). These non-diagonal elements are suppressed by \( O\left(\frac{1}{\log \Lambda/m}\right) \) relatively to the diagonal ones, apparently of the same order of suppression as the correction calculated above. However, these non-diagonal elements give rise to terms of the form of the effective potential [14]
\[
\Delta V = \text{(Im} \tau)_{ij}^{-1} \left(\sqrt{2} M_i \hat{M}_i - m \frac{\partial U[A_D]}{\partial A_{Di}}\right)^* \left(\sqrt{2} M_j \hat{M}_j - m \frac{\partial U[A_D]}{\partial A_{Dj}}\right) + \frac{(\text{Im} \tau)_{ij}^{-1}}{2} (|M_i|^2 - |\hat{M}_i|^2)(|M_j|^2 - |\hat{M}_j|^2). \tag{24}
\]
When this is used in the equations of motion, one finds that the correction to the tension due to the non-diagonal \( \text{(Im} \tau)_{ij}^{-1} \) is actually one order higher, \( O\left(\frac{1}{\log^2 \Lambda/m}\right) \), hence is negligible to the order considered.

We thus find a non-universal correction to the Douglas–Shenker formula, equation (1). In the process of transition towards fully non-Abelian superconductivity at large \( m \), non-perturbative effects such as \( W \) boson production are probably essential. Nevertheless, the presence of a calculable deviation from the sine formula is qualitatively significant and shows that such a ratio is not a universal quantity.
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Appendix. Computation of equation (12)

We use the following identity (found by partial derivation of the identity $\frac{\partial \rho_{ik}}{\partial a_{ik}} = \delta_{ik}$—the first part of equation (6)—with respect to $a_{Dl}$):

$$\sum_{i,j} \frac{\partial^2 \phi_i}{\partial a_{Dl} \partial a_{Dm}} \delta^2 a_{Dk} - \sum_{t} \frac{\delta a_D}{\delta a_{Dm} \partial a_{Dl}} \frac{\partial^2 \phi_t}{\partial a_{Dm} \partial a_{Dl}} = - \sum_{i,j} \frac{\partial \phi_i}{\partial a_{Dm} \partial a_{Dl}} \frac{\partial^2 \phi_k}{\partial a_{Dm} \partial a_{Dl}}.$$

The expression (12) now becomes:

$$\sum_{i,j} \phi_i \frac{\partial^2 \phi_j}{\partial a_{Dm} \partial a_{Dl}} = - \sum_{i,j,k,t} \frac{\phi_i}{\partial a_{Dk} \partial a_{Dl}} \frac{\partial \phi_k}{\partial a_{Dm} \partial a_{Dl}} \frac{\partial^2 \phi_j}{\partial a_{Dm} \partial a_{Dl}}.$$

(A.1)

$\frac{\delta^2 a_{Dm}}{\delta \phi_i \delta \phi_j}_{aD=0}$ can be found by first considering $\frac{d^2 \lambda}{d \phi_i d \phi_j}$ and integrating this along the circles $\alpha_m$ (see [5] for the conventions; the variable $\theta$ is defined by $x = 2 \cos[\theta]$):

$$\frac{d^2 \lambda}{d \phi_i d \phi_j} = \frac{1}{2\pi} \left[ \frac{P(1 - \delta_{ij})}{y(x - \phi_i)(x - \phi_j)} \, dx + \frac{P^3}{y^3(x - \phi_i)(x - \phi_j)} \right].$$

(A.2)

We perform the integrations taking the residues at the poles: $\hat{\theta}_m = \frac{\pi m}{N}$:

$$A = \frac{\cot[N\theta] \sin[\theta](1 - \delta_{ij})}{(\cos[\theta] - \cos[\theta_i])(\cos[\theta] - \cos[\theta_j])},$$

(A.3)

$$B = \frac{\cot^3[N\theta] \sin[\theta]}{(\cos[\theta] - \cos[\theta_i])(\cos[\theta] - \cos[\theta_j])}.$$

(A.4)

At the end of the integration, one has:

$$\frac{2}{\delta \phi_i \delta \phi_j} \frac{\delta^2 a_{Dl}}{\delta \phi_i \delta \phi_j} = \frac{- \sin(\hat{\theta}_i)}{(\cos[\theta_i] - \cos[\theta_i])(\cos[\theta_i] - \cos[\theta_j]) \left( \frac{1}{2N^3} + \frac{\delta_{ij}}{N} \right)}$$

$$- \frac{1}{4N^3} \frac{\sin(\hat{\theta}_i)(\cos[2\hat{\theta}_i] + 2 \cos[\theta_i] \cos[\theta_j] - 3)}{(\cos[\theta_i] - \cos[\theta_i])^3(\cos[\theta_i] - \cos[\theta_j])}$$

$$- \frac{1}{4N^3} \frac{\sin(\hat{\theta}_i)(\cos[2\hat{\theta}_i] + 2 \cos[\theta_j] \cos[\theta_i] - 3)}{(\cos[\theta_i] - \cos[\theta_i])(\cos[\theta_i] - \cos[\theta_j])^3}$$

$$+ \frac{1}{2N^3} \frac{2 \sin^3(\hat{\theta}_i) + \sin(2\hat{\theta}_i)(2 \cos[\theta_i] - (\cos[\theta_i] + \cos[\theta_j]))}{(\cos[\theta_i] - \cos[\theta_i])^2(\cos[\theta_i] - \cos[\theta_j])^2}.$$ 

(A.5)
Substituting this in (A.1) one finds

\[
2 \sum_k \phi_k \frac{\partial^2 \phi_k}{\partial a_{Dn} \partial a_{Dm}} = 4i \sum_{t,r,s} \sin \left( \frac{\pi t}{N} \right) \frac{\partial \phi_s}{\partial a_{Dn}} \frac{\partial \phi_r}{\partial a_{Dm}} \frac{\delta^2 a_{DL}}{\delta \phi_r \delta \phi_s}
\]

\[
= -4i \sum_{t,r,s} \sin \left( \frac{\pi t}{N} \right) B_{ns} B_{mr} \frac{\delta^2 a_{DL}}{\delta \phi_r \delta \phi_s} = \left( 2 - \frac{1}{N} \right) \delta_{mn}. \tag{A.7}
\]

The last equality involves rather cumbersome trigonometric expressions: we found equation (A.7) by using Mathematica up to \( N = 50 \).

References

[1] Hanany A, Strassler M and Zaffaroni A 1998 Nucl. Phys. B 513 87 (Preprint hep-th/9707244)
[2] Herzog C P and Klebanov I R 2002 Phys. Lett. B 526 388 (Preprint hep-th/0111078)
[3] Lucini B and Teper M 2001 Phys. Lett. B 501 128 (Preprint hep-lat/0012025)
    Lucini B and Teper M 2001 Phys. Rev. D 64 105019 (Preprint hep-lat/0107007)
[4] Del Debbio L, Panagopoulos H, Rossi P and Vicari E 2002 Phys. Rev. D 65 021501 (Preprint hep-th/0106185)
    Del Debbio L, Panagopoulos H, Rossi P and Vicari E 2002 J. High Energy Phys. JHEP02(2001)009 (Preprint hep-th/0111090)
[5] Douglas M R and Shenker S H 1995 Nucl. Phys. B 447 271 (Preprint hep-th/9503163)
[6] Strassler M 1998 Prog. Theor. Phys. Suppl. 131 439 (Preprint hep-lat/9803009)
[7] Yung A 2000 Preprint hep-th/0005088 3rd Moscow School of Physics and 28th ITEP Winter School of Physics, Moscow
    Vainshtein A and Yung A 2001 Nucl. Phys. B 614 3 (Preprint hep-th/0012250)
[8] ’t Hooft G 1981 Nucl. Phys. B 190 455
    Mandelstam S 1975 Phys. Lett. B 53 476
    Mandelstam S 1976 Phys. Rep. C 23 245
[9] Abrikosov A A 1957 JETP 5 1174
[10] Nielsen H and Olesen P 1973 Nucl. Phys. B 61 45
[11] Di Pierro M and Konishi K 1996 Phys. Lett. B 388 90 (Preprint hep-th/9605178)
[12] Hou Xinrui 2001 Phys. Rev. D 63 045015 (Preprint hep-th/0005119)
[13] Ferrari F 2001 Nucl. Phys. B 612 151 (Preprint hep-th/0106192)
[14] Edelstein J D, Fuertes W G, Mas J and Guilarte J M 2000 Phys. Rev. D 62 065008 (Preprint hep-th/0001184)