Improved predictions for magnetic moments and M1 decay widths of heavy hadrons

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In the framework of an extended bag model the magnetic moments, M1 transition moments, and decay widths of all ground-state heavy hadrons are calculated. For the heavy baryons containing three quarks of different flavors the effect of hyperfine mixing of the states is taken into account. The additional care is taken to get more accurate theoretical estimates for the mass splittings of heavy hadrons. The use of such improved values enables one to provide more accurate predictions for the decay widths. These values of the hyperfine splittings between baryons may be also useful for the further experimental searches of new heavy hadrons. For instance, we predict $M(\Xi_{cc}^{++}) = 3695 \pm 5$ MeV. The agreement of our results for the M1 decay rates with available experimental data is good. We also present a wide comparison of the predictions obtained in our work with the results obtained using various other approaches.

I. INTRODUCTION

The heavy hadrons are the permanent subject of interest from both experimental and theoretical sides \cite{1–18}. A typical example could be recently discovered by the LHCb Collaboration the doubly charmed baryon $\Xi_{cc}^{++}$ with the mass near 3621 MeV. From the experimental side, the long-standing puzzle of the $\Xi_{cc}$ was at last solved. On the other side, the utility of various theoretical approaches has been proven. The $\Xi_{cc}^{++}$ was found almost exactly where it was predicted to reside. The observed value agrees well with the quark model prediction 3.61 GeV \cite{19} and more recent estimate by Karliner and Rosner $M(\Xi_{cc}) = 3627 \pm 12$ MeV \cite{20}. The calculations in the framework of the nonrelativistic model with different potentials \cite{8, 9} provide values in the range from 3607 to 3631 MeV. The estimate obtained using the relativistic approach in the quark-diquark approximation \cite{21} is 3620 MeV. Similar results were also obtained in the recent lattice QCD calculations: 3603 ± 31 MeV \cite{22}, 3610 ± 45 MeV \cite{23}, and 3606 ± 19 MeV \cite{24}.

Many heavy hadrons are still waiting to be discovered. Moreover, the understanding of the properties of yet undiscovered hadrons is not complete. Therefore the study of various properties of hadrons is important task. Recently we have studied the magnetic properties (magnetic moments, M1 decay rates) of ground-state mesons using the modified version of the bag model \cite{25}. Now we extend our investigation to include the magnetic properties of heavy baryons. It could be one more step towards the comprehensive analysis of the magnetic properties of all hadrons. We expect our approach could be suitable for this purpose because the bag model treats the mesons and baryons on equally the same footing. There are many other attempts devoted to the unified treatment of mesons and baryons using various approaches \cite{15, 21}. Lattice QCD and QCD sum rules can also serve as appropriate framework.

In the baryon sector several additional difficulties emerge, such as the state mixing problem of the baryons $(\Xi_{c}, \Xi_{c}', (\Xi_{b}, \Xi_{b}'), (\Xi_{cb}, \Xi_{cb}')$, and $(\Omega_{cb}, \Omega_{cb}')$. Another problem is possibly insufficient accuracy of the available theoretical predictions for the hyperfine splittings of doubly and triply heavy baryons. To deal with these problems is the object of the present work, too.

There is a vast number of publications devoted to the theoretical treatment of the magnetic properties of various hadrons. The magnetic moments of the mesons were studied in Refs. \cite{26–28, 30–32}, the M1 decays of the mesons in Refs. \cite{33–35}, the magnetic moments of heavy baryons in Refs. \cite{36–38, 42, 166, 167, 186, 212, 238}, and the M1 decay properties of heavy baryons in Refs. \cite{68, 69, 166, 167, 186, 212, 238}. References are representative, not necessarily exhaustive. It is interesting to perform a detailed comparison between results obtained using various approaches. Such comparison yields some information which can help us to get a comprehensive picture of the contemporary capabilities to predict the magnetic properties of the hadrons.

The plan of the paper is the following one. In the next section the method used for our investigation is presented, and the expressions needed for the calculation of the magnetic observables of the hadrons are given. The details of the derivations are skipped. In Sec. \textbf{III} the problem of the state mixing is discussed. In Sec. \textbf{IV} the method to improve theoretical predictions for hyperfine splittings of doubly (and triply) heavy baryons is proposed, and the new estimates are obtained. Subsequently these values are used in the calculations of the decay widths of heavy baryons. In Sec. \textbf{V} the model parameter setting procedure is discussed, and the magnetic properties of the mesons are analyzed. Sec. \textbf{VI} is devoted to the study of the magnetic properties of heavy baryons. Magnetic moments, magnetic dipole transition moments, and M1 decay widths of these baryons are calculated. Our predictions are compared with the results obtained using various other approaches and with available experimental data. Finally, Sec. \textbf{VII} contains the concluding discussion.

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II. FORMALISM: BAG MODEL AND MAGNETIC PROPERTIES OF THE HADRONS

The model used for the present investigation is the same as in our previous work [14] except for the specific values of the scale factors used for the parameterization of the quark magnetic moments. It is a version of the MIT bag model designed to reconcile the initial model with the heavy quark spectroscopy [23]. In the bag model [24, 25, 26], the hadrons are considered as extended objects localized in space. The internal structure of a particle is associated with quark and gluon fields. The model possesses many desirable features inspired by QCD and relativity. For practical calculations it is convenient to use the static spherical cavity approximation, in which the bag surface is frozen. The motion of the quarks inside the cavity of radius $R$ is described by the free Dirac equation. The wave function of the quark in the $s$-mode is written in the form

$$\Psi^{1/2}_m(r) = \frac{1}{\sqrt{4\pi}} \left( \begin{array}{c} G(r) \\ -i(\sigma \cdot \hat{r}) F(r) \end{array} \right) \Phi^{1/2}_m, \quad (1)$$

where $\sigma$ are Pauli matrices, $\hat{r}$ is the unit radius-vector, $\Phi^{1/2}_m$ is usual two-component spinor. $G(r)$ and $F(r)$ are radial parts of the upper and lower components respectively. The eigenenergy of the quark is determined by the boundary condition $G(R) = -F(R)$.

The wave functions of the ground-state mesons are characterized by the total spin $J$. In the baryon sector the situation is more complex. The baryon wave function can be constructed by coupling the spins of the first two quarks to the intermediate spin $S$ and then adding the spin of the third quark to form the total spin $J$.

$$|B_{(q_1 q_2) q_3}\rangle = (q_1 q_2)^{S=0} q_3, J = \frac{1}{2} \rangle, \quad (2)$$

$$|B_{(q_1 q_2) q_3}\rangle = (q_1 q_2)^{S=1} q_3, J = \frac{1}{2} \rangle, \quad (3)$$

$$|B^*\rangle = (q_1 q_2)^{S=1} q_3, J = \frac{3}{2} \rangle. \quad (4)$$

The energy associated with a specific hadron is

$$E = \frac{4\pi}{3} BR^3 + \sum_i \varepsilon_i + E_{\text{int}}. \quad (5)$$

The bag radius $R_B$ of the particular hadron is determined from the equilibrium condition $\frac{\partial E}{\partial R} = 0$. The three terms on the right-hand side are the so called volume energy (parameter $B$ is the bag constant), the sum of single-particle eigenenergies, and the quark interaction energy due to one-gluon-exchange $E_{\text{int}} = E^e + E^m$. The $E^e$ and $E^m$ are the color-electric and color-magnetic pieces of the interaction energy.

### Table I. Parameters which specify the color-magnetic interaction energy of baryons.

| $(S_1, S_2)$ | $J$ | $a_{12}$ | $a_{13}$ | $a_{23}$ |
|--------------|-----|----------|----------|----------|
| (0, 0)       | $\frac{1}{2}$ | $-3$ | 0 | 0 |
| (0, 1)       | $\frac{1}{2}$ | 0 | $-\sqrt{3}$ | $\sqrt{3}$ |
| (1, 1)       | $\frac{1}{2}$ | 1 | $-2$ | $-2$ |
| (1, 1)       | $\frac{3}{2}$ | 1 | 1 | 1 |

$$E^e = \lambda \alpha_c(R) \sum_{J>i} I_{ij}(R), \quad (6)$$

$$E^m = \alpha_c(R) \sum_{J>i} a_{ij} K_{ij}(R), \quad (7)$$

where

$$\lambda = \begin{cases} -1 & \text{for baryons,} \\
-2 & \text{for mesons.} \end{cases} \quad (8)$$

Parameters $a_{ij}$ that specify the color-magnetic energy for mesons with the total spin $J$ are

$$a_{ij} = \begin{cases} -6 & (J = 0), \\
2 & (J = 1). \end{cases} \quad (9)$$

The corresponding parameters for the color-magnetic energy of baryons are presented in Table I. Note that in the case of $J = 1/2$ baryons containing three differently flavored quarks the off-diagonal (with respect to intermediate spin $S$) matrix elements are present. Therefore, in general case, the physical states $|B\rangle$ and $|B^*\rangle$ are the linear combinations of the states $|B(q_1 q_2) q_3\rangle$, $|B(q_1 q_2) q_3\rangle$ and are obtained by the diagonalization of the interaction energy matrix.

Functions $I_{ij}(R)$ and $K_{ij}(R)$ can be written as

$$I_{ij}(R) = \frac{2}{3} \int_0^R r^2 dr \rho'_i(r) V_j(r, R), \quad (10)$$

$$K_{ij}(R) = \frac{4}{3} \int_0^R r^2 dr \mu'_i(r) A_j(r, R), \quad (11)$$

where

$$\rho'_i(r) = G_i^2(r) + F_i^2(r) \quad (12)$$

is the charge density, and

$$\mu'_i(r) = -\frac{2r}{3} G_i(r) F_i(r) \quad (13)$$
is the scalar magnetization density of an $i$-th quark. $V_i(r, R)$ and $A_i(r, R)$ are semiclassical scalar and vector potentials generated by the $i$-th quark.

\begin{equation}
V_i(r, R) = \rho_i(r) \left( \frac{1}{r} - \frac{1}{R} \right) + \int_0^R x^2 dx \frac{\rho'_i(x)}{x},
\end{equation}

...(14)

\begin{equation}
A_i(r, R) = \frac{\mu_i(r)}{r^3} + \frac{\mu_i(R)}{2R^3} + M_i(r, R),
\end{equation}

...(15)

where

\begin{equation}
\rho_i(r) = \int_0^r x^2 dx \rho'_i(x),
\end{equation}

...(16)

\begin{equation}
\mu_i(r) = \int_0^r x^2 dx \mu'_i(x),
\end{equation}

...(17)

\begin{equation}
M_i(r, R) = \int_R^r x^2 dx \frac{\mu'_i(x)}{x^3}.
\end{equation}

...(18)

The interaction energy depends on the effective running coupling constant $\alpha_c(R)$. We allow this constant to vary with the bag radius in a manner consistent with asymptotic freedom and adopt the parameterization proposed in Ref. [244].

\begin{equation}
\alpha_c(R) = \frac{2\pi}{9 \ln(A + R_0/R)},
\end{equation}

...(19)

where $R_0$ is the scale parameter analogous to the QCD constant $\Lambda^{-1}$, and the parameter $A$ is introduced in order to avoid divergences when $R_0 \to R$. We also introduce effective $R$-dependent (running) quark mass

\begin{equation}
m_f(R) = \tilde{m}_f + \alpha_c(R) \cdot \delta_f,
\end{equation}

...(20)

where parameters $\tilde{m}_f$ and $\delta_f$ define the mass functions $m_f(R)$ for each quark flavor.

The mass of the hadron is defined as the corresponding bag energy corrected for the center-of-mass motion (c.m.m.) according to the prescription

\begin{equation}
M^2 = E^2 - \gamma \sum_i p_i^2,
\end{equation}

...(21)

where $p_i = \sqrt{\epsilon_i^2 - m_i^2}$ is the momentum of the $i$-th quark, $m_i$ is the effective quark mass given by Eq. (20), and $\gamma$ is the free parameter.

The model parameters $B, \gamma, A, R_0, \tilde{m}_s, \delta_s, \tilde{m}_c, \delta_c, \tilde{m}_b, \delta_b$ were taken from the Ref. [14] ($B = 7.301 \times 10^{-4}$ GeV$^4$, $\gamma = 1.785$, $A = 0.7719$, $R_0 = 3.876$ GeV$^{-1}$, $\tilde{m}_s = 0.2173$ GeV, $\delta_s = 0.1088$ GeV, $\tilde{m}_c = 1.456$ GeV, $\delta_c = 0.1003$ GeV, $\tilde{m}_b = 4.746$ GeV, and $\delta_b = 0.0880$ GeV). The lightest (up and down) quarks are assumed to be massless.

The magnetic moment of a hadron $\mu(H)$ and the static transition moment $\mu(H \to H_2)$ can be calculated by the matrix element of the operator $\mu$

\begin{equation}
\mu(H) = \langle H \mid [\mu] H \rangle,
\end{equation}

...(22)

\begin{equation}
\mu(H \to H_2) = \langle H \mid [\mu] H_2 \rangle,
\end{equation}

...(23)

\begin{equation}
\mu = \frac{1}{2} \int d^3 x [\sigma \times j_{em}],
\end{equation}

...(24)

where $j_{em}$ is the Dirac electromagnetic current, and $\langle H \rangle$ is the hadron state of definite polarization. After some algebra these expressions can be rewritten as

\begin{equation}
\mu = \sum \tilde{\mu}_i \langle H_1 \uparrow \mid e_i \sigma_i \mid H_2 \uparrow \rangle,
\end{equation}

...(25)

where $e_i$ is the charge of the corresponding quark, and $\tilde{\mu}_i$ is the reduced (charge-independent) quark magnetic moment

\begin{equation}
\tilde{\mu}_i = \int r^2 dr' \frac{2r}{3} G_i(r) F_i(r),
\end{equation}

...(26)

\begin{equation}
\mu_i = e_i \tilde{\mu}_i.
\end{equation}

...(27)

where $e_q$ ($\tau_q$) is the charge of the corresponding quark (antiquark).

The static transition moment connecting vector and pseudoscalar mesons is given by

\begin{equation}
\mu(V \to PS) = \mu_1 - \mu_2 = e_1 \tilde{\mu}_1 - e_2 \tilde{\mu}_2.
\end{equation}

...(28)

For the baryons one has (see Ref. [176]):

\begin{equation}
\mu(B_{[q_1q_2]q_3}) = \mu_3,
\end{equation}

...(29)

\begin{equation}
\mu(B_{[q_1q_2]q_3}) = \frac{1}{3} (2\mu_1 + 2\mu_2 - \mu_3),
\end{equation}

...(30)

\begin{equation}
\mu(B^*) = \mu_1 + \mu_2 + \mu_3,
\end{equation}

...(31)

\begin{equation}
\mu(B_{[q_1q_2]q_3} \to B_{[q_1q_2]q_3}) = \frac{1}{\sqrt{3}} (\mu_2 - \mu_1),
\end{equation}

...(32)

\begin{equation}
\mu(B^* \to B_{[q_1q_2]q_3}) = \sqrt{\frac{2}{3}} (\mu_1 - \mu_2),
\end{equation}

...(33)

\begin{equation}
\mu(B^* \to B_{[q_1q_2]q_3}) = \sqrt{\frac{2}{3}} (\mu_1 + \mu_2 - 2\mu_3).
\end{equation}

...(34)
For the M1 decay widths we use the following expressions

\[ \Gamma(H_1 \rightarrow H_2) = \frac{\alpha k^3}{M_p^2} \frac{1}{2J+1} \mu^2(H_1 \rightarrow H_2) \quad (35) \]

for mesons, and

\[ \Gamma(H_1 \rightarrow H_2) = \frac{\alpha k^3}{M_p^2} \frac{2}{2J+1} \mu^2(H_1 \rightarrow H_2) \quad (36) \]

for baryons. In these expressions \( \alpha \approx \frac{1}{137} \) is the fine structure constant, \( M_p \) is the proton mass,

\[ k = (M_1^2 - M_2^2)/(2M_1) \quad (37) \]

is the photon momentum in the rest frame of a decaying particle, \( J \) is the spin of decaying hadron, and \( \mu(H_1 \rightarrow H_2) \) is the corresponding M1 transition moment expressed in nuclear magnetons \( \mu_N = 1/(2M_p) \). The transition moment depends on the momentum of the emitted photon and can be obtained from Eqs. (32–34) by replacing static moments with the corresponding \( k \)-dependent M1 transition moments

\[ \bar{\mu}_1^{BR}(k) = \int r^2 dr j_1(kr) [G_{1i}(r) F_{2i}(r) + G_{2i}(r) F_{1i}(r)]. \quad (38) \]

Indices 1 and 2 stand for initial and final particles, \( j_1(kr) \) is the Bessel function. When the radii of the bags differ we choose the smaller one as the upper limit of the integral. It can be checked, that Eqs. (35), (36) and the expression (17) for the M1 decay width obtained in Ref. (68) are equivalent.

It is well-known that magnetic observables calculated in the bag model should be corrected for the center-of-mass motion (c.m.m.), recoil, and possibly other effects. We use simple prescription adopted in Ref. (14)

\[ \mu_L = C_L \mu_L^0, \quad \mu_H = C_H \mu_H^0. \quad (39) \]

where \( C_L, C_H \) are the model parameters and \( \mu_L^0, \mu_H^0 \) are the original, uncorrected bag model quantities (magnetic or M1 transition moments) for the light (\( u, d, \) or \( s \)) and heavy (\( c \) or \( b \)) quarks, respectively.

### III. STATE MIXING

As was mentioned earlier the color-magnetic interaction mixes the states with different intermediate spins \( |B(q_1 q_2)q_3) \) and \( |B(q_1 q_3)q_2) \). The physical states \( |B \) and \( |B' \) are the linear combinations of these states

\[ |B \rangle = C_1 |B(q_1 q_2)q_3) + C_2 |B(q_1 q_3)q_2), \]
\[ |B' \rangle = C_1 |B(q_1 q_2)q_3) - C_2 |B(q_1 q_3)q_2). \quad (40) \]

The magnetic observables for such baryons are given by

\[ \mu(B) = C_1^2 \mu(B(q_1 q_2)q_3) + C_2^2 \mu(B(q_1 q_3)q_2) + 2C_1 C_2 \mu(B(q_1 q_2)q_3 \rightarrow B(q_1 q_3)q_2), \quad (41) \]
\[ \mu(B') = C_1^2 \mu(B(q_1 q_2)q_3) + C_2^2 \mu(B(q_1 q_3)q_2) - 2C_1 C_2 \mu(B(q_1 q_2)q_3 \rightarrow B(q_1 q_3)q_2), \quad (42) \]
\[ \mu(B' \rightarrow B) = (C_1^2 - C_2^2) \mu(B(q_1 q_2)q_3 \rightarrow B(q_1 q_3)q_2) + C_1 C_2 [\mu(B(q_1 q_2)q_3) - \mu(B(q_1 q_3)q_2)], \quad (43) \]
\[ \mu(B' \rightarrow B') = C_1 \mu(B' \rightarrow B(q_1 q_2)q_3) - C_2 \mu(B' \rightarrow B(q_1 q_3)q_2). \quad (45) \]

In Eqs. (41) and (42) the static values of the transition moments must be used. On the other hand, in Eqs. (43)–(45) all the entries are \( k \)-dependent moments.

We have performed the detailed analysis of the state mixing effect using all possible quark ordering schemes. In Table III we present the expansion coefficients obtained for all possible quark orderings. A symbol \( q \) is used for the lightest (\( u \) or \( d \)) quarks. Of course, if the state mixing is taken into account, the final results for the calculated physical quantities do not depend on the basis. If the state mixing is ignored, the results, in general, become dependent on the quark ordering scheme.

As pointed out in Ref. (176), there is a favored ordering scheme (the optimal basis), which to some extent minimizes the state mixing effect. This is the basis with the specific quark ordering scheme, in which the heaviest quark \( q_3 \) is chosen as the third one in the wave function \( |(q_1 q_2)^c q_3) \). In this basis the effect of the hyperfine mixing on the baryon masses is generally negligible. This is because in the case of weak mixing the mass shifts are second order in the mixing angles. Thus, if this optimal basis is chosen for the calculation of baryon masses, one could not bother very much about the state mixing problem. On the other hand, the shifts due the hyperfine state mixing for the magnetic moments are first order in the mixing angle, and in this case the effect can be important. We postpone the detailed analysis of this effect on the magnetic observables of heavy baryons to subsequent sections.

In Table III we also compare our results for the expansion coefficients with the results obtained using some other approaches. These are the nonrelativistic quark
model (NR), two different nonrelativistic potential models (PM), and the approach based on the QCD sum rules (QCDSR). We see that our results are very close to the values obtained in the simple nonrelativistic model (NR). Presumably the reason is the same algebraic structure of the hyperfine interaction in both models. The results obtained in other, slightly different version of the bag model [248] are also practically indistinguishable from the present ones.

The results obtained using potential models and QCDSR method demonstrate some model dependence, however, the main pattern remains similar in all cases. For instance, in the case of \( \Xi_c \) type baryons all methods give very similar predictions. In the case of \( \Xi_b \) type baryons the potential model [245] as well as QCDSR method predict stronger mixing than our model. For the baryons of \( \Xi_{cb} \) and \( \Omega_{cb} \) type our results are similar to the results given by both potential models, but QCDSR method predicts weaker mixing. In general, our predictions seem to be quite reasonable, with possible exception of the \( (\Xi_b, \Xi_b') \) baryons, where the real state mixing presumably should be somewhat stronger.

### IV. HYPERFINE SPLITTINGS

If one seeks reliable predictions for the decay widths, it is important to have accurate values of the photon momentum \( k \), because the decay widths are third order in \( k \). For this reason, we prefer to use in the calculations of decay widths the experimental values of the hadron masses. Unfortunately, not all masses of heavy ground-state hadrons are measured. Such are the \( B^*_c \) meson, singly heavy baryon \( \Omega'_{bc} \), and all (now, except for the \( \Xi_c \)) doubly and triply heavy baryons. In these cases we need as accurate as possible theoretical estimate. For heavy hadrons the momentum of the emitted photon is approximately equal to the corresponding mass difference. In what follows we will use the semiphenomenological approach to estimate these mass splittings. The mass spectra of doubly heavy baryons were studied using various approaches, such as lattice QCD [11–13, 249–250], QCD sum rules [251, 262], nonrelativistic phenomenological quark model [14, 15], semiempirical sum rules [253, 264], NRQCD based effective theory [253, 254], potential models [9, 245, 256, 266], covariant quark model [267], model based on Bethe-Salpeter equations [274], relativistic quark model with the hyperspherical expansion [275], relativistic quark-diquark model [10], Salpeter model with AdS/QCD inspired potential [276]. The spectra of triply heavy baryons were calculated using lattice QCD [12], QCD sum rules [277, 278], potential models [245, 279], relativistic quark model [275], and the model with AdS/QCD inspired potential [276].

In Tables [III] and [IV] we have collected some predictions for the hyperfine splittings of heavy baryons obtained using lattice QCD, three variants of the potential model (labeled as AL1, RP, VGV), and our current variant of the bag model, accompanied with available experimental data (Exp). We see from Table [IV] that all these approaches give many or less similar estimates. Seemingly, one can use any of them, but it is not quite clear to what extent these results are trustworthy. To test the quality of the predictions it could be useful to compare the estimates given by these approaches with the available experimental data (see Table [III]). The results of a comparison are a little bit disappointing. We see that the predictions given by various approaches differ considerably. The best agreement with experiment provides the VGV potential model with quark-quark interaction mediated via one-gluon plus scalar and pseudoscalar boson exchanges [271]. However, it is not very helpful for us, because in this case there are no predictions for \( \Xi_{cb}, \Omega_{cb}, \) and triply heavy baryons. The lattice QCD predictions, as a rule, exceed experimental data (sometimes by \( \sim 50\% \)), but remain compatible with them within errors.

Our strategy to obtain the improved baryon mass splittings is based on the assumption that these mass

| Table II. Expansion coefficients \( C_1, C_2 \) in the expression \( |B\rangle = C_1 |B_{(c,q_2,q_2)}\rangle + C_2 |B_{(c,q_1,q_2)}\rangle \). |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Baryons Quark ordering | Our | NR [176] | PM [222] | PM [245] | QCDSR [246, 247] |
| \( \Xi_c, \Xi_b \) | \((qs)c\) | 0.9975, 0.0703 | 0.9978, 0.0660 | \cdots | 0.9919, 0.0438 | 0.996, 0.086 |
| \( \Xi_c, \Xi_b \) | \((qc)s\) | -0.5597, 0.8287 | \cdots | \cdots | \cdots | \cdots |
| \( \Xi_c, \Xi_b \) | \((sc)q\) | 0.4379, 0.8990 | \cdots | \cdots | \cdots | \cdots |
| \( \Xi_{cb}, \Xi_{cb} \) | \((qs)b\) | 0.9998, 0.0175 | 0.9999, 0.0170 | \cdots | 0.9913, 0.0330 | 0.995, 0.100 |
| \( \Xi_{cb}, \Xi_{cb} \) | \((qc)b\) | 0.9918, 0.1278 | 0.9916, 0.1296 | \cdots | \cdots | \cdots |
| \( \Xi_{cb}, \Xi_{cb} \) | \((qb)c\) | -0.6066, 0.7950 | \cdots | \cdots | \cdots | \cdots |
| \( \Xi_{cb}, \Xi_{cb} \) | \((cb)q\) | -0.3852, -0.9228 | \cdots | 0.431, 0.902 | 0.3839, 0.8976 | 0.249, 0.969 |
| \( \Omega_{cb}, \Omega_{cb} \) | \((sc)b\) | 0.9937, 0.1120 | 0.9928, 0.1197 | \cdots | \cdots | \cdots |
| \( \Omega_{cb}, \Omega_{cb} \) | \((sb)c\) | -0.5939, 0.8046 | \cdots | \cdots | \cdots | \cdots |
| \( \Omega_{cb}, \Omega_{cb} \) | \((cb)s\) | -0.3998, -0.9166 | \cdots | 0.437, 0.899 | 0.4149, 0.8947 | 0.279, 0.960 |
The first uncertainty is statistical, the second one is systematic.

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The first uncertainty is statistical, the second one is systematic.
\[ M(\Xi_{cb}^*) - M(\Xi_{cb}) = 6K_{qb}, \]
\[ M(\Omega_{cb}^*) - M(\Omega_{cb}) = 6K_{sb}, \]

etc. The expressions for doubly heavy baryons of \( \Xi_{cb} \) and \( \Omega_{cb} \) type are

\[ M(\Xi_{cb}^*) - M(\Xi_{cb}) = 3K_{qb} + 3K_{cb}, \]  
\[ M(\Xi_{cb}^*) - M(\Xi_{cb}) = 4K_{qc} + K_{qb} + K_{cb}, \]  
\[ M(\Xi_{cb}^*) - M(\Xi_{cb}) = 4K_{qc} - 2K_{qb} - 2K_{cb}, \]  
\[ M(\Omega_{cb}^*) - M(\Omega_{cb}) = 3K_{sb} + 3K_{cb}, \]  
\[ M(\Omega_{cb}^*) - M(\Omega_{cb}) = 4K_{sc} + K_{sb} + K_{cb}, \]  
\[ M(\Omega_{cb}^*) - M(\Omega_{cb}) = 4K_{sc} - 2K_{sb} - 2K_{cb}. \]

For the triply heavy baryons one has

\[ M(\Omega_{cb}^*) - M(\Omega_{cb}) = 6K_{cb}, \]  
\[ M(\Omega_{cb}^*) - M(\Omega_{cb}) = 6K_{cb}. \]

In the expressions above the index \( q \) denotes the lightest \((u \text{ or } d)\) quarks. In general, parameters \( K_{ij} \) depend on the hadron. To proceed with we make an assumption that in baryons with the same heavy quark content the interaction strengths between quarks do not differ substantially, i.e. \( K_{ij}(qqQ) \approx K_{ij}(qsQ) \), \( K_{ij}(qsQ) \approx K_{ij}(ssQ) \), and \( K_{ij}(qQ_1Q_2) \approx K_{ij}(sQ_1Q_2) \). Now we can invert the equations above in order to estimate the parameters of the quark-quark interaction. For singly charmed baryons we obtain

\[ K_{qq} = \frac{1}{12} \{ 2 [M(\Sigma_{cc}^*)] - M(\Lambda_c) \} + [M(\Sigma_c) - M(\Lambda_c)], \]  
\[ K_{qs} = \frac{1}{12} \{ 2 [M(\Xi_{cc}^*)] - M(\Xi_{cc}) \} + [M(\Xi_c^*)] - M(\Xi_c)], \]  
\[ K_{qc} = \frac{1}{6} \{ 2 [M(\Xi_{cc}^*)] - M(\Xi_{cc}) \} - [M(\Omega_{cc}^*)] - M(\Omega_{cc})], \]  
\[ K_{qc} = \frac{1}{6} \{ 2 [M(\Xi_c^*)] - M(\Xi_c) \} - [M(\Lambda_c^*)] - M(\Lambda_c)], \]  
\[ K_{qc} = \frac{1}{6} [M(\Sigma_{cc})] - M(\Sigma_c), \]  
\[ K_{sc} = \frac{1}{6} [M(\Omega_{cc})] - M(\Omega_c). \]

In analogy with the above relations for the bottom baryons we have

\[ K_{qq} = \frac{1}{12} \{ 2 [M(\Sigma_{bb}^*)] - M(\Lambda_b) \} + [M(\Sigma_b) - M(\Lambda_b)], \]  
\[ K_{qs} = \frac{1}{12} \{ 2 [M(\Xi_{bb}^*)] - M(\Xi_{bb}) \} + [M(\Xi_b^*)] - M(\Xi_b)], \]  
\[ K_{qc} = \frac{1}{6} \{ 2 [M(\Xi_{bb}^*)] - M(\Xi_{bb}) \} - [M(\Omega_{bb}^*)] - M(\Omega_{bb})], \]  
\[ K_{qb} = \frac{1}{6} \{ 2 [M(\Xi_{bb}^*)] - M(\Xi_{bb}) \} - [M(\Omega_{bb})] - M(\Omega_{bb})], \]  
\[ K_{sb} = \frac{1}{6} [M(\Omega_{bb})] - M(\Omega_b). \]

For the doubly heavy baryons \( \Xi_{QQ} \) and \( \Omega_{QQ} \) the corresponding expressions are

\[ K_{qc} = \frac{1}{6} [M(\Xi_{cc})] - M(\Xi_{cc}), \]  
\[ K_{sc} = \frac{1}{6} [M(\Omega_{cc})] - M(\Omega_{cc}), \]  
\[ K_{qb} = \frac{1}{6} [M(\Xi_{bb})] - M(\Xi_{bb}), \]  
\[ K_{sb} = \frac{1}{6} [M(\Omega_{bb})] - M(\Omega_{bb}). \]

and similarly for the triply heavy baryons

\[ K_{cc} = \frac{1}{6} [M(\Omega_{cc})] - M(\Omega_{cc}), \]  
\[ K_{cb} = \frac{1}{6} [M(\Omega_{cb})] - M(\Omega_{cb}). \]

The case of the \( B_{cb} \) type baryons is more complicated. We can write down

\[ K_{qc} = \frac{1}{6} \{ 2 [M(\Xi_{cb}^*)] - M(\Xi_{cb}) \} + [M(\Xi_{cb}^*)] - M(\Xi_{cb})], \]  
\[ K_{sc} = \frac{1}{6} \{ 2 [M(\Omega_{cb}^*)] - M(\Omega_{cb}) \} + [M(\Omega_{cb}^*)] - M(\Omega_{cb})], \]  
\[ K_{qb} + K_{cb} = \frac{1}{3} [M(\Xi_{cb})] - M(\Xi_{cb}), \]  
\[ K_{sb} + K_{cb} = \frac{1}{3} [M(\Omega_{cb})] - M(\Omega_{cb}). \]

but the problem is that the information we can get from the mass splittings is insufficient to disentangle the three interaction parameters \( K_{qb}, K_{sb}, \) and \( K_{cb} \). We will use some interpolation to relate parameters \( K_{qb} \) and \( K_{sb} \) to
the corresponding parameters obtained from the Eqs. 79 and 80.

In Table V, we present interaction parameters $K_{ij}$ obtained using as an input the mass splittings given in Table III. In the calculations of the experimental interaction parameters the value $M(\Sigma^+_b) - M(\Sigma_b) = 20$ MeV was used instead of 21 MeV. With the original value one would have $6K_{qb} = 21$ MeV and $6K_{sb} = 19$ MeV. For singly charmed baryons $K_{qc} < K_{sc}$, and for bottom baryons we expect $K_{qb} < K_{cb}$. The new choice, while staying within the error bars, is consistent with this expectation. We have not included the results of Roberts and Pervin [245] into our analysis because this model, due to its complexity, is not completely compatible with the simple picture we have used to describe the quark-quark interactions.

From Table V we see that some regularities exist. For instance, the approximate relations $K_{ij}(qqc) \approx K_{ij}(qqb)$ and $K_{ij}(qsc) \approx K_{ij}(qsb)$ are valid. This is the manifestation of the heavy quark symmetry (HQS) [283–288], which states that the light degrees of freedom are to some extent independent of heavy flavors. As we see, it works perfectly in the systems with one heavy quark. Indeed, the coincidences for the experimental values 314 \approx 311, and 230 \approx 229 are really impressive. On the other hand, for the heavy baryons ($\Xi_{cb}, \Xi'_{cb}$) and ($\Omega_{cb}, \Omega'_{cb}$) the utility of HQS is limited. For example, the HQS suggestion that the spin of heavy diquark is conserved for these baryons could be reliable only if the quark-quark hyperfine interaction was ignored.

Next, we see that our assumptions $K_{ij}(qqQ) \approx K_{ij}(qsQ)$ and $K_{ij}(qsQ) \approx K_{ij}(ssQ)$ work very well. In fact, for the experimental values we have $K_{qc}(qqc) = K_{qc}(qsc)$ and $K_{sc}(qsc) = K_{sc}(ssc)$. Therefore, we expect $K_{qb}(qsb) = K_{sb}(ssb)$, and, as a consequence,

$$M(\Omega^+_b) - M(\Omega_b) = 6K_{sb}(ssb) = 20 \text{ MeV}. \quad (87)$$

This result is similar to the prediction $M(\Omega^+_b) - M(\Omega_b) = 23$ MeV, obtained using mass sum rules [264], and the prediction 19.8 \pm 3.1 \text{ MeV}, obtained using $1/N_c$ expansion [284]. The predictions obtained using various potential models are: 21 MeV [245], 23 MeV [271], and 20.4 MeV [290]. The relativistic quark model [291] gives $M(\Omega^+_b) - M(\Omega_b) = 24$ MeV.

Another symmetry that is worthwhile to discuss is the approximate independence of the hyperfine interaction on light quark flavor ($u$, $d$, or $s$) observed in the meson spectra

$$M(D^*) - M(D) \approx M(D_{s}^*) - M(D_s), \quad (88)$$

$$M(B^*) - M(B) \approx M(B_{s}^*) - M(B_s), \quad (89)$$

which looks like some reminiscence of the flavor SU(3) symmetry. The interpretation of this phenomenon is not clear: is it a kind of an accidental symmetry [292], or a consequence of relativistic kinematics [293]. In any case, it could be useful to check if such symmetry also survives in the heavy baryon sector. For the singly charmed baryons this symmetry implies $K_{qc}(qqc) \approx K_{sc}(qsc)$. However, from Table V, we see that $K_{sc}(qsc)$ obtained using as an input experimental data is \approx 10% larger than $K_{qc}(qqc)$. Neither AL1 type potential model, nor the bag model can reproduce such behavior. Moreover, the lattice QCD predictions [12] seem also to suffer from this drawback. On the other hand, the potential model used in Ref. [271] has succeeded, but at the expense of the additional chiral interaction. For the singly bottom baryons this symmetry seems to be restored $M(\Sigma^+_b) - M(\Sigma_b) \approx M(\Xi^*_b) - M(\Xi_b)$, and consequently $K_{qb}(qsb) \approx K_{sb}(qsb)$. We expect similar behavior in the case of doubly heavy baryons, too.

In order to obtain the improved predictions for the hyperfine splittings of doubly and triply heavy baryons we have chosen as an input the predictions of AL1 potential model. Alternative choice could be the bag model predictions. In the case of doubly heavy baryons both approaches give similar results, however, for singly heavy baryons AL1 predictions are evidently more reliable. This is a serious indication that internal quark dynamics is better described in this approach. Using Eqs. 77–83 we immediately obtain

$$6K_{qc}(qqc) = 94 \text{ MeV}, \quad (90)$$

$$6K_{sc}(qcc) = 83 \text{ MeV}, \quad (91)$$

$$6K_{gb}(qsb) = 39 \text{ MeV}, \quad (92)$$

$$6K_{sb}(ssb) = 38 \text{ MeV}, \quad (93)$$

$$6K_{cb}(ccb) = 28 \text{ MeV}, \quad (94)$$

$$6K_{cb}(cbb) = 31 \text{ MeV}, \quad (95)$$

$$6K_{qc}(qcb) = 101 \text{ MeV}, \quad (96)$$

$$6K_{as}(scb) = 90.5 \text{ MeV}. \quad (97)$$

In order to get an estimation for the interaction parameters $K_{qb}(qcb)$ and $K_{sb}(scb)$ some additional assumption is necessary. After the inspection of the listed above parameters we note that the substitution of the bottom
quark instead the charmed one increases the strengths of
the interaction by \( \approx 10\% \). Assuming

\[
\frac{K_{gb}(qcb)}{K_{gb}(qbb)} = \frac{K_{sb}(scb)}{K_{sb}(sbb)} = \frac{K_{cb}(ccb)}{K_{cb}(cbb)},
\]

we obtain

\[
6K_{gb}(qcb) = 35.2 \text{ MeV}, \quad 6K_{sb}(scb) = 34.3 \text{ MeV}, \quad 6K_{cb}(ccb) = 23.7 \text{ MeV},
\]

Now, using Eqs. (85) and (86), we get

\[
\begin{align*}
6K_{gb}(qcb) & = 22.8 \text{ MeV}, \\
6K_{sb}(scb) & = 29 \text{ MeV}.
\end{align*}
\]

The next step is to rescale the interaction parameters obtained above. We define

\[
K_{ij}^\prime(q_0 Q_1 Q_2) = K_{ij}(q_0 Q_1 Q_2) \frac{K_{ij}(q_0 q_0 Q)}{K_{ij}(q_0 q_0 Q)^\text{AL1}},
\]

where \((i, j) = (q, c), (s, c), (q, b),\) and \((s, b)\). Parameters \(K_{ij}^\text{Exp}\) and \(K_{ij}^\text{AL1}\) belong to the singly heavy baryon sector. In order to rescale the interaction parameters \(K_{cb}\) we need some reliable theoretical estimate, because the experimental data are absent. In this case we turn to the lattice QCD result \(M(\Omega_{bb}^\prime) - M(\Omega_{cb}) = 33.5(0.6)(4.1)\) MeV [12]. From Table VI we see that for the singly heavy baryons lattice QCD predictions always overestimate the experimental data. We expect similar tendencies to hold also in the sectors of doubly and triply heavy baryons. Having this in mind we take the smallest still compatible with lattice QCD value and define \(K_{cb}^\prime(ccb) = 29 \text{ MeV}\). Now we can rescale the remaining parameters using relations

\[
K_{cb}^\prime(\cdots) = K_{cb}(\cdots) \frac{K_{cb}^\prime(ccb)}{K_{cb}(ccb)}.
\]

The new (improved) parameters are

\[
\begin{align*}
6K_{gb}^\prime(qcc) & = 77 \text{ MeV}, \\
6K_{gb}^\prime(scc) & = 79 \text{ MeV}, \\
6K_{gb}^\prime(qcb) & = 83.1 \text{ MeV}, \\
6K_{gb}^\prime(qbc) & = 22.7 \text{ MeV}, \\
6K_{cb}^\prime(qcb) & = 21.3 \text{ MeV}, \\
6K_{cb}^\prime(scb) & = 86.9 \text{ MeV}, \\
6K_{cb}^\prime(sbc) & = 22.9 \text{ MeV}, \\
6K_{cb}^\prime(sbb) & = 22 \text{ MeV}, \\
6K_{cb}^\prime(qbb) & = 25 \text{ MeV}, \\
6K_{cb}^\prime(sbb) & = 25 \text{ MeV}, \\
6K_{cb}^\prime(ccb) & = 26 \text{ MeV}, \\
6K_{cb}^\prime(cbb) & = 29 \text{ MeV}.
\end{align*}
\]

TABLE VI. Improved predictions for the hyperfine splittings of doubly and triply heavy baryons (in MeV).

| Baryons          | PM(AL1) | PM(VGV) | PM(YHHOS) | Bag   |
|------------------|---------|---------|------------|-------|
| \(\Xi_{cc}^-\)   | 77      | 75      | 71         | 67    |
| \(\Omega_{cc}^-\)| 79      | 75      | 75         | 73    |
| \(\Xi_{cb}^+\)   | 63      | \cdots | \cdots     | 52    |
| \(\Omega_{cb}^+\)| 22      | \cdots | \cdots     | 24    |
| \(\Omega_{cb}^+\)| 41      | \cdots | \cdots     | 28    |
| \(\Omega_{cb}^+\)| 65      | \cdots | \cdots     | 55    |
| \(\Omega_{cb}^+\)| 22      | \cdots | \cdots     | 23    |
| \(\Omega_{cb}^+\)| 43      | \cdots | \cdots     | 32    |
| \(\Xi_{bb}^+\)   | 25      | 26      | 23         | 23    |
| \(\Omega_{bb}^+\)| 25      | 24      | 22         | 24    |
| \(\Omega_{bb}^+\)| 26      | \cdots | \cdots     | 25    |
| \(\Omega_{bb}^+\)| 29      | \cdots | \cdots     | 29    |

We see that the interaction parameter \(K_{gb}^\prime(qcc)\) now became larger than \(K_{gb}^\prime(qcb)\) in agreement with the expectations from the singly heavy baryon sector. Moreover, the approximate independence of quark-quark interaction strength on light quark flavor holds in the bottom sector, i.e. \(K_{gb}^\prime(qbb) \approx K_{gb}^\prime(sbb)\), \(K_{gb}^\prime(qcb) \approx K_{gb}^\prime(scb)\), and is slightly broken in the case of doubly charmed baryons.

In addition one new interesting feature emerges. We see that for the baryons \(\Xi_{cb}\) and \(\Omega_{cb}\) the interaction strength between light and bottom quarks is very similar to the interaction strength between charmed and bottom quarks. This could be an indication that in the systems containing one bottom quark the charmed quark behaves in some ways more like the light quark than the heavy one. Such tendency was also observed in the full lattice QCD calculations [294].

Putting new (primed) interaction parameters into Eqs. (83) and (84) we obtain the new (improved) predictions for the hyperfine splittings of doubly and triply heavy baryons. The results are presented in Table VII. We also performed analogous analysis taking as input the bag model results and the predictions given by two potential models. These are the model used in Ref. [271] (labeled as VGV) and the model used in Ref. [272] (labeled as YHHOS). For the sake of comparison we have included in Table VII these results, too. We see that new hyperfine splittings are almost always smaller than the original ones. At this point we can make one (more or less reasonable) prediction. Taking the average of the potential model based predictions given in Table VII we can write down \(M(\Xi_{cc}^-) - M(\Xi_{bc}^-) = 74 \pm 4 \text{ MeV}\). Next, adding this value to the experimental mass of \(\Xi_{cc}^+\) we obtain an estimate \(M(\Xi_{cc}^+) = 3695 \pm 5 \text{ MeV}\).

We now have at our disposal the values for the hyperfine splittings (and corresponding photon momenta) of all ground-state heavy baryons. However, in the meson sector one mass splitting needed for the further analysis is
still missing. This is the mass difference \( M(B^+_c) - M(B_c) \).

There exist a number of theoretical predictions for this quantity obtained using various approaches, such as nonrelativistic QCD \(^{[295]}\), QCD sum rules \(^{[296]}\), lattice QCD \(^{[297,299]}\), semiempirical mass formulae \(^{[300]}\), nonrelativistic potential models \(^{[301,304]}\), various variants of relativistic or semirelativistic models \(^{[305,310]}\), Bethe-Salpeter model \(^{[311,314]}\), and so on. More references can be found in Ref. \(^{[314]}\). The latest predictions for this hyperfine splitting \(^{[297]}\) are more or less similar. Let us take the typical nonrelativistic potential model prediction \( M(B^+_c) - M(B_c) = 68 \pm 8 \text{ MeV} \) \(^{[7]}\) and lattice QCD result \( 53 \pm 7 \text{ MeV} \) \(^{[297]}\). The only estimate compatible with these two predictions is \( M(B^+_c) - M(B_c) = 60 \text{ MeV} \). It is this value we will use in the further calculations. Some other approaches also provide results compatible with our choice. For instance, another potential model based estimate can be obtained using the semiempirical relationship \(^{[302]}\):

\[
\Delta B_c = (0.7)[M(J/\psi) - M(\eta_c)]^{0.65}[M(\Upsilon) - M(\eta_b)]^{0.35},
\]

which is expected to hold at about 10\% level. Using the experimental masses of \( J/\psi, \eta_c, \Upsilon, \) and \( \eta_b \) mesons one gets \( \Delta B_c = 64 \pm 6 \text{ MeV} \), the result similar to the one obtained in Ref. \(^{[7]}\). The full lattice QCD including the charm quarks in the see \(^{[298]}\) gives very accurate prediction \( \Delta B_c/\Delta B_s = 1.166(79) \). Taking PDG average value \( \Delta B_c = 48.6(1.8) \text{ MeV} \) one obtains \( \Delta B_c = 56.7(5.8) \text{ MeV} \) in good agreement with our choice. Their original prediction is somewhat lower \( (54 \pm 3 \text{ MeV}) \) because of the different choice of \( \Delta B_s \). The recent free-form smearing lattice QCD \(^{[299]}\) prediction \( \Delta B_c = 57.5(3) \text{ MeV} \) is slightly lower than our choice. On the other hand, the relativistic quark model \(^{[310]}\), which gives good fits to the meson spectrum, predicts \( \Delta B_c = 61 \text{ MeV} \). Note that these two predictions are respectively \( \approx 0.5 \text{ MeV} \) and \( \approx 1 \text{ MeV} \) lower than the estimates for the \( \Upsilon - \eta_b \) hyperfine splittings obtained using these approaches. If one defines this splitting as the difference between averaged PDG masses \( M(\Upsilon) = 9460.30(26) \text{ MeV} \) and \( M(\eta_b) = 9399.0(2.3) \text{ MeV} \), one obtains \( M(\Upsilon) - M(\eta_b) = 61(2) \text{ MeV} \). Subtracting 1 MeV from this value one again obtains \( M(B^+_c) - M(B_c) \approx 60 \text{ MeV} \).

Now we have all ingredients to estimate the momenta of the outgoing photons necessary for the calculation of the M1 decay widths. The results for the mesons and singly heavy baryons are presented in Tables \(^{[VII, VIII]}\) Where available, the experimental data of the hadron masses were used. When the experimental data for the masses of the isospin multiplet members are absent, the isospin symmetry is assumed. For the mass splittings \( \Delta Q_b \) and \( \Delta B_c \), the estimates obtained above are given. In the case of doubly and triply heavy baryons the momenta of the emitted photons and the hyperfine splittings practically coincide. The corresponding results are given in Table \(^{[VIV]}\) (labeled as AL1).

### V. Parameter Setting and the Magnetic Properties of Mesons

In the previous paper \(^{[14]}\) we have proposed the improved version of bag model suitable for the description of the magnetic properties of light and heavy mesons. Now we want to extend our analysis to the heavy baryon sector. However, before switching to this new field, it could be useful in this context to discuss the magnetic properties of the mesons once more. The utility of the meson sector for our investigation is twofold. First, the existing experimental data may be used to determine the free parameters of the model. Next, the comparison of the model predictions with other data and with some most reliable estimates obtained using other approaches may serve as a preliminary test of the capability of the model to describe the magnetic properties of other hadrons.

#### Table VII. Mass splittings and momenta of emitted photons (in MeV) for the ground-state mesons.

| Transition | \( \Delta E \) (MeV) | \( k \) |
|------------|----------------------|-------|
| \( \rho^0 \to \pi^0 \) | 640 | 376 |
| \( \rho^+ \to \pi^+ \) | 635 | 375 |
| \( \omega^0 \to \pi^0 \) | 647 | 379 |
| \( K^{*0} \to K^0 \) | 398 | 310 |
| \( K^{*+} \to K^+ \) | 399 | 310 |
| \( D^{*0} \to D^0 \) | 142 | 137 |
| \( D^{*+} \to D^+ \) | 140 | 135 |
| \( D_{s1}^* \to D_{s1}^0 \) | 144 | 139 |
| \( J/\psi \to \eta_c \) | 114 | 112 |
| \( B^{*0} \to B^0 \) | 46 | 46 |
| \( B^{*+} \to B^+ \) | 46 | 46 |
| \( B_c^{*0} \to B_c^0 \) | 48 | 48 |
| \( B_c^{*+} \to B_c^+ \) | 60 | 60 |
| \( \Upsilon \to \eta_b \) | 61 | 61 |

#### Table VIII. Mass splittings and momenta of emitted photons (in MeV) for the singly heavy baryons.

| Transition | \( \Delta E \) (MeV) | \( k \) |
|------------|----------------------|-------|
| \( \Sigma_c^{*0} \to \Sigma_c^0 \) | 65 | 64 |
| \( \Sigma_c^{*+} \to \Sigma_c^+ \) | 65 | 64 |
| \( \Sigma_b^{*0} \to \Sigma_b^0 \) | 60 | 60 |
| \( \Sigma_b^{*+} \to \Sigma_b^+ \) | 64 | 64 |
| \( \Xi_c^{*0} \to \Xi_c^0 \) | 175 | 169 |
| \( \Xi_c^{*+} \to \Xi_c^+ \) | 178 | 172 |
| \( \Xi_b^{*0} \to \Xi_b^0 \) | 160 | 158 |
| \( \Xi_b^{*+} \to \Xi_b^+ \) | 107 | 105 |
| \( \Omega_c^{*0} \to \Omega_c^0 \) | 71 | 70 |
| \( \Omega_c^{*+} \to \Omega_c^+ \) | 70 | 69 |
| \( \Omega_b^{*0} \to \Omega_b^0 \) | 137 | 135 |
| \( \Omega_b^{*+} \to \Omega_b^+ \) | 108 | 106 |
We think it could be useful to change slightly the procedure used to fit the model parameters $C_L$ and $C_H$ necessary to set the scale of the magnetic observables of the light ($u,d$ or $s$) and heavy ($c$ or $b$) quarks. We expect this new choice to be more appropriate for the description of the magnetic properties of heavy hadrons. In Ref. [14] parameter $C_L$ was adjusted to reproduce exactly the value of the proton magnetic moment. Our new choice is to set the scale factor $C_L$ to reproduce the decay widths of the light mesons $\rho \to \pi\gamma$ and $\omega \to \pi\gamma$ simultaneously. From the 2017 PDG update of Review of Particle Physics [281], we have

$$\Gamma(\rho^+ \to \pi^+\gamma) = 68 \pm 7 \text{ keV},$$
$$\Gamma(\rho^0 \to \pi^0\gamma) = 70 \pm 9 \text{ keV},$$
$$\Gamma(\omega^0 \to \pi^0\gamma) = 713 \pm 26 \text{ keV}.$$  

Using relation inverse to Eq. (35) the experimental values of the corresponding $M1$ transition moments can be deduced:

$$\mu(\rho^+ \to \pi^+\gamma) = 0.68 \pm 0.04 \mu_N,$$
$$\mu(\rho^0 \to \pi^0\gamma) = 0.69 \pm 0.05 \mu_N,$$
$$\mu(\omega^0 \to \pi^0\gamma) = 2.18 \pm 0.04 \mu_N.$$  

Setting $C_L = 1.52$ we obtain $\mu(\rho \to \pi\gamma) = 0.72 \mu_N$, and $\mu(\omega \to \pi\gamma) = 2.15 \mu_N$, the predictions within error bars compatible with the data. The scale factor $C_H$ for the heavy quarks is now adjusted to reproduce the PDG average of the partial decay width $\Gamma(J/\psi \to \eta_c) = 1.59 \pm 0.42 \text{ keV}$. The experimental $M1$ transition moment in this case is $\mu(J/\psi \to \eta_c\gamma) = 0.64 \pm 0.09 \mu_N$. We have set $C_H = 0.94$. This choice leads to $\mu(J/\psi \to \eta_c\gamma) = 0.634 \mu_N$.

Now, the calculation of the magnetic observables is rather simple task. We only need to use the expressions presented in Sec. [1] Let us begin with the magnetic moments of the mesons. The results are presented in Tables [X][XI][XII] where they are compared with the estimates obtained using various other approaches, such as the model based on relativistic Hamiltonian (RH), various models based on the Dyson-Schwinger equation (DSE), the effective field theory (EFT), standard and light cone QCD sum rules (QCDSR), the relativistic quark model (RQM), various light front quark models (LFM), lattice QCD simulations (Latt), two variants of the Nambu-Jona-Lasinio model (NJL), and the formalism based on the Blankenbecler-Sugar equation (BSLT). The label NR stands for the simple nonrelativistic quark model. The input values for the quark magnetic moments in this model $\mu_u = 1.86 \mu_N$, $\mu_d = -0.93 \mu_N$, $\mu_s = -0.61 \mu_N$, $\mu_c = 0.39 \mu_N$, and $\mu_b = -0.06 \mu_N$ were taken from Ref. [176]. Note that in this approach, as in almost all nonrelativistic quark models [315], the values of the magnetic moments of the light quarks are adjusted to reproduce the magnetic moments of light baryons.

The adopted in Ref. [176] values $\mu_c$ and $\mu_b$ are about 10% smaller than the corresponding values deduced in Ref. [316] using nonrelativistic framework from the hyperfine splittings of $\Sigma_c$ and $\Sigma_b$ baryons.

The experimental data for the magnetic moments of the mesons are so far absent (the possibilities to measure the magnetic moments of another short living particles, i.e. heavy baryons, are discussed in the Refs. [317, 318]). Nevertheless, there exist indirect, to some extent model dependent, estimate of the $\rho^+$ magnetic moment based on the analysis of preliminary experimental data from the BaBar Collaboration. We have included this result in Tables [X][XI][XII] under the label Exp.

In this paper we prefer to express magnetic moments in the units of nuclear magneton ($\mu_N$). Nevertheless, it is sometimes convenient to express them in so-called natural magnetons $1/(2M_i)$, where $M_i$ is the mass of the particle under consideration. For instance, it is a frequent practice in the analysis of the magnetic moments of light mesons. For the discussion about the utility of natural magnetons in the lattice QCD calculations see Ref. [319].

We present, the predictions for the magnetic moments of light mesons expressed in nuclear magnetons in Tables [X][XI][XII] and expressed in natural magnetons in Tables [XIII][XIV][XV]. The differences between the two approaches is visible, for example, in the comparison of our results for the $\rho^+$ meson with the corresponding prediction obtained using lattice QCD with three degenerate flavors of dynamical quark tuned to approximately the physical strange quark mass [42]. We see that for the quantities expressed in natural magnetons the coincidence is excellent, but this is not the case when the nuclear magnetons are used. The similar behavior we observe for the predictions obtained in Ref. [34] using Dyson-Schwinger formalism. On the other hand, the comparison of our predictions for the $\rho^+$ meson with the results obtained using full lattice QCD near physical masses [41] shows good agreement, no matter in which units the results are expressed. With a few exceptions the agreement with the results obtained using other approaches is also good.

Note the excellent agreement between our results and the predictions obtained using instant-form relativistic quark models [48, 49]. The agreement with the predictions obtained in Ref. [29] in the framework of the light-front quark model is also very good. Our estimates for $K^{*+}$ meson are lower than quenched lattice QCD predictions [39], larger than the predictions obtained using approach based on a relativistic Hamiltonian [36], and approximately 5% larger than the results obtained using Dyson-Schwinger [33] and NJL [46] frameworks. In the case of $K^{*0}$ meson almost all approaches predict small and negative ($\sim -0.2\mu_N$) magnetic moment value. In the heavy meson sector there are only few other predictions to compare our results with. The overall agreement is good, and this could be an indication that all these approaches provide more or less reasonable predictions.

The inclusion of the results obtained in the framework of simple nonrelativistic quark model (NR) needs some
comment. In all cases these estimates are larger in magnitude than ours. However, we see that these naive predictions in all likelihood are not so bad as would be expected. More specifically, the whole pattern provided by NR estimates could serve as a useful guide if one 

wants to gain some preliminary insight how the things 

take effect come into the game the NR approach plausibly 

also becomes useless. But sometimes for heavy hadrons 

there is little benefit of it. In the cases when various sub-

dictions (SIE), and lattice QCD (Latt). The experimen-

tation theory (BSE), Blankenbecler-Sugar equation (BSLT), 

heavy quark effective theory (HQET), chiral perturba-

tion methods based on the spectral integral equa-

ions (SIE), and lattice QCD (Latt). The experimental 

data are from the 1917 update of Review of Particle 

Physics [281], or are derived from. 

From Table XVI we see that our results for the light 

mesons our predictions 

are somewhat larger than experimental values, and the predictions obtained in the extended quark model also 
suffer from this drawback. More detailed comparison 

shows that in the light meson sector the results obtained 
in our model are of similar quality as the predictions ob-
tained using other approaches. The lattice QCD for the 
time being likely has some trouble in the describing the 
magnetic transition moments of light mesons. The full 
lattice QCD prediction for the $\rho \rightarrow \pi \gamma$ transition 
moment obtained near physical value of $m_{\pi} = 156$ MeV 

is lower by about $33\%$ than the experimental es-
timate. This is a serious contradiction between experi-
mental data and the theoretical prediction obtained using 
method expected to provide sufficiently accurate results. 

The situation looks somewhat strange because similar 
calculation of $\rho^+$ magnetic moment [41] seems to provide 
quite reasonable prediction consistent with other recent 
estimates, including ours. 

The comparison of our results for the transition mo-
mments of heavy mesons (Table XVII) with the predictions 

obtained using semirelativistic and relativistic potential 
models shows a good agreement in almost all cases. The 
exception is the decay $B^{*+}_{s} \rightarrow B^{+}_{s}\gamma$. In this case other 

predictions are about $25\%$ larger than ours. 

Somewhat more complicated picture emerges when we 

compare our predictions for the decay widths. The agree-
ment with the results obtained within the framework of 
RPM [121, 122, 129, 130] remains good. But the predic-
tions obtained using other approaches are varied. The 
lattice QCD predictions for the decay widths of heavy-
light mesons (see Table XVII) suffer from large uncertain-
ities. Moreover, the prediction for the decay $D^{*+}_{s} \rightarrow D^{+}_{s}\gamma$ is order of magnitude smaller than other estimates. 

Often the reason of differences in the calculated decay 
widths is the differences in photon energies used in 
calculations. The model dependence of the transition 
moments sometimes also plays his role. In order to il-
\n
\begin{table}[h]
\centering
\caption{Magnetic moments of light mesons (in nuclear magnetons $\mu_N$).}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
Particle & Our & NR & RH & DSE & NJL & NJL & QCDSR & Latt & Latt & Latt \\
\hline
$\rho^+$ & 2.65 & 2.79 & 2.37 & 2.54 & 3.14 & 2.54 & 2.9(5) & 3.25(3) & 2.61(10) & 2.00(9) \\
$K^{*0}$ & 0.229 & 0.329 & 0.326 & 0.26 & $\cdots$ & $\cdots$ & 0.29(4) & $\cdots$ & $\cdots$ & $\cdots$ \\
$K^{*+}$ & 2.35 & 2.47 & 2.19 & 2.23 & $\cdots$ & 2.26 & 2.1(4) & 2.81(1) & $\cdots$ & $\cdots$ \\
\hline
\end{tabular}
\end{table}
TABLE XI. Magnetic moments of light mesons in natural magnetons (1/(2M_{s})).

| Particle | Our | NR | BSLT | NJL | EFT | QCDSR | Latt | Latt | Latt | Latt |
|----------|-----|-----|------|-----|-----|-------|------|------|------|------|
| \( \rho^{+} \rightarrow \pi^{0} \) | 0.69(5) | 0.720 | 0.72 | 0.69 | 0.70 | 0.68 | 0.93 |
| \( \rho^{+} \rightarrow \pi^{+} \) | 0.68(4) | 0.720 | 0.72 | 0.69 | 0.70 | 0.68 | 0.93 |
| \( \omega^{0} \rightarrow \pi^{0} \) | 2.18(4) | 2.15 | 2.14 | 2.07 | 2.07 | 2.02 | 2.79 |
| \( K^{*0} \rightarrow K^{0} \) | 1.19(5) | 1.27 | 1.25 | 1.20 | 1.35 | 1.17 | 1.54 |
| \( K^{*+} \rightarrow K^{+} \) | 0.78(4) | 0.905 | 0.90 | 0.91 | 0.79 | 0.84 | 1.25 |

TABLE XII. Magnetic moment of \( \rho^{+} \) meson in natural magnetons (1/(2M_{s})).

| Our | Exp | EFT | QCDSR | DSE | RQM | LMF | LFM | LFM | LFM | LFM |
|-----|-----|-----|-------|-----|-----|-----|-----|-----|-----|-----|
| 2.17 | 2.1(5) | 2.24 | 2.0(3) | 2.11 | 2.20 | 2.16 | 2.35 | 1.83 | 1.92 | 2.21 | 2.06 |

TABLE XV. Transition moments of heavy mesons (in nuclear magnetons \( \mu_{N} \)).

| Transition | Exp | Our | EQM | SRPM | RPM | RPM | NR |
|------------|-----|-----|-----|------|-----|-----|----|
| \( \rho^{0} \rightarrow \pi^{0} \) | 0.69(5) | 0.720 | 0.72 | 0.69 | 0.70 | 0.68 | 0.93 |
| \( \rho^{+} \rightarrow \pi^{+} \) | 0.68(4) | 0.720 | 0.72 | 0.69 | 0.70 | 0.68 | 0.93 |
| \( \omega^{0} \rightarrow \pi^{0} \) | 2.18(4) | 2.15 | 2.14 | 2.07 | 2.07 | 2.02 | 2.79 |
| \( K^{*0} \rightarrow K^{0} \) | 1.19(5) | 1.27 | 1.25 | 1.20 | 1.35 | 1.17 | 1.54 |
| \( K^{*+} \rightarrow K^{+} \) | 0.78(4) | 0.905 | 0.90 | 0.91 | 0.79 | 0.84 | 1.25 |

TABLE XVI. Transition moments of light vector mesons (in magnetic moments \( T \)).

| Transition | Exp | Our | EFT | QCDSR | DSE | RQM | LMF | LFM | LFM | LFM | LFM |
|------------|-----|-----|-----|-------|-----|-----|-----|-----|-----|-----|-----|
| \( \rho^{0} \rightarrow \pi^{0} \) | 0.69(5) | 0.720 | 0.72 | 0.69 | 0.70 | 0.68 | 0.93 |
| \( \rho^{+} \rightarrow \pi^{+} \) | 0.68(4) | 0.720 | 0.72 | 0.69 | 0.70 | 0.68 | 0.93 |
| \( \omega^{0} \rightarrow \pi^{0} \) | 2.18(4) | 2.15 | 2.14 | 2.07 | 2.07 | 2.02 | 2.79 |
| \( K^{*0} \rightarrow K^{0} \) | 1.19(5) | 1.27 | 1.25 | 1.20 | 1.35 | 1.17 | 1.54 |
| \( K^{*+} \rightarrow K^{+} \) | 0.78(4) | 0.905 | 0.90 | 0.91 | 0.79 | 0.84 | 1.25 |

the \( K^{*+} \rightarrow K^{+} \) transition being an exception. For heavy mesons the tendency becomes opposite, and these predictions are, as a rule, larger than ours. On the other hand, the predictions obtained using relativistic quark model [100, 101] are substantially lower than our results. BSLT approach [139, 140] also gives lower predictions, however, in the case of doubly heavy mesons the estimates given by this approach become close to our results. Our prediction for the transition \( \Upsilon \rightarrow \eta_{b} \gamma \) practically coincide with the typical nonrelativistic result \( \Gamma(\Upsilon \rightarrow \eta_{b} \gamma) = 0.00895 \text{ keV} [141] \). This result seems to differ from the pNRQCD prediction \( \Gamma(\Upsilon \rightarrow \eta_{b} \gamma) = 0.0152 \text{ keV} \). However, the latter estimate was obtained using old experimental data \( \Delta M = 70 \text{ MeV} \) for the \( \Upsilon \rightarrow \eta_{b} \) hyperfine splitting. After simple rescaling one obtains \( \Gamma(\Upsilon \rightarrow \eta_{b} \gamma) = 0.0101 \text{ keV} \), much closer to the usual nonrelativistic result. The comparison of the rescaled pNRQCD prediction with our result enables us to estimate the accuracy of the decay width obtained in our model. We conclude that our model predicts bottomium decay widths with the accuracy \( \approx 16\% \), and, consequently, the possible uncertainty for the magnetic moment of the bottom quark is approximately \( 8\% \).

The pNRQCD result [142] for the bottomium decay width is not the only one that requires rescaling. For example, such are also predictions obtained in Refs. [62, 51, 108, 140]. For illustration, we have listed the corresponding rescaled values in Table XXII. From this table we see that new, rescaled predictions are essentially closer to our result. In addition, our prediction for the \( \Upsilon \rightarrow \eta_{b} \gamma \) decay is close to the estimates obtained in Refs. [93, 95] using potential models, semirelativistic potential model (Ref. [105]), and the approach based on the spectral integral equations (Ref. [157]).

Our result for the decay width \( \Gamma(J/\psi \rightarrow \eta_{c} \gamma) \) is not the partial decay width \( \Gamma(J/\psi \rightarrow \eta_{c} \gamma) = 1.59 \pm 0.42 \text{ keV} \). Some approaches (such as the quark model used in Ref. [54], relativistic quark models [100, 117], and the model based on BSLT framework [140]) give lower predictions, other (light front quark model [62], potential models [51, 84], and one variant of semirelativistic potential model [108])
### TABLE XVI. M1 decay widths of light vector mesons (in keV).

| Transition   | Exp   | Our   | QM   | PM(AL1) | PM(AP1) | RPM   | RPM   | LFM   | LFM   |
|--------------|-------|-------|------|---------|---------|-------|-------|-------|-------|
| $\rho^0 \rightarrow \pi^0$ | 70(9) | 76    | 74.6 | 48.7    | 60.6    | 69.0  | 65.4  | 81.3  | 69    |
| $\rho^+ \rightarrow \pi^+$ | 68(7) | 76    | 74.6 | 48.5    | 60.4    | 68.3  | 64.8  | 81.3  | 69    |
| $\omega^0 \rightarrow \pi^0$ | 713(26) | 694 | 716  | 450    | 572    | 645   | 613   | 667   |
| $K^{*0} \rightarrow K^0$ | 116(10) | 134  | 114  | 98.3   | 116   | 150   | 112   | 124   | 117   |
| $K^{*+} \rightarrow K^+$ | 50(5) | 68    | 82.3 | 79.1    | 104    | 51.1  | 58.1  | 54.4  | 71.4  |

### TABLE XVII. M1 decay widths of heavy vector mesons (in keV).

| Transition   | Exp   | Our   | QM   | PM(AL1) | PM(AP1) | RPM   | RPM   | HQET  | $\chi$PT | Latt | Latt |
|--------------|-------|-------|------|---------|---------|-------|-------|-------|----------|------|------|
| $D^{*0} \rightarrow D^0$ | 22.9  | 21.7  | 33.6 | 44.7    | 33.5   | 32    | 22.7  | 2(2.2) | 26(6)    | 27(14) | 163  |
| $D^{*+} \rightarrow D^+$ | 1.34(36) | 1.19 | 1.42 | 2.48    | 3.58   | 1.63  | 1.83  | 0.9(3) | 1.54(35) | 0.8(7) | 163  |
| $D_s^{*+} \rightarrow D_s^+$ | 0.430  | 0.21  | 0.23 | 0.31    | 0.43   | 0.2   | 1.04  | 0.12   | 0.18     | 1.06  | 163  |
| $J/\psi \rightarrow \eta_c$ | 1.59(42) | 1.56 | 1.27 | 1.85    | 1.87   | 0.28  | 0.36  | 0.24   | 0.14(20) | 0.13  | 163  |
| $B_s^{*0} \rightarrow B_s^0$ | 0.115  | 0.39  | 0.097 | 1.20   | 0.78   | 0.468 | 0.75  | 0.15   | 0.069    | 0.15  | 163  |

### TABLE XVIII. M1 decay widths of heavy vector mesons (in keV).

| Transition   | Exp   | Our   | QM   | pNRQCD | PM   | PM   | PM   | RQM   | RQM   | RQM   | RQM   | RQM   | RQM   | BSLT   | QCDSR | LFM   | LFM   |
|--------------|-------|-------|------|--------|------|------|------|-------|-------|-------|-------|-------|-------|--------|-------|-------|-------|
| $D^{*0} \rightarrow D^0$ | 22.9  | 28.4  | 19.5 | 26.5   | 23.6 | 11.5 | 1.25 | 14.4  | 21.7  | 20.0  |       |       |       |        |        |       |       |
| $D^{*+} \rightarrow D^+$ | 1.34(36) | 1.19 | 1.08 | 1.63   | 0.932 | 0.950 | 1.04 | 1.10  | 1.56  | 0.90  |       |       |       |        |        |       |       |
| $D_s^{*+} \rightarrow D_s^+$ | 0.430  | 0.38  | 0.44 | 0.213  | 0.19  | 0.337 | 1.05 | 1.25  | 1.69  |       |       |       |       |        |        |       |       |
| $J/\psi \rightarrow \eta_c$ | 1.59(42) | 1.56 | 0.21 | 0.14   | 0.181 | 0.131 | 0.070 | 0.0096 | 0.16  | 0.142 | 0.13  |       |       |        |        |       |       |
| $B_s^{*0} \rightarrow B_s^0$ | 0.115  | 0.13  | 0.06 | 0.119  | 0.054 | 0.148 | 0.054 | 0.068 |       |       |       |       |       |        |        |       |       |
| $B_s^{*+} \rightarrow B_s^+$ | 0.039  | 0.020 | 0.030 | 0.023  | 0.033 | 0.034 | 0.024 |       |       |       |       |       |       |        |        |       |       |

### TABLE XIX. $J/\psi \rightarrow \eta_c \gamma$ and $\Upsilon \rightarrow \eta_b \gamma$ decay widths (in keV).

| Transition   | Exp   | Our   | QM   | pNRQCD | PM   | PM   | PM   | PM   | SRPM  | SRPM  | RPM   | RPM   | SIE   |
|--------------|-------|-------|------|--------|------|------|------|------|-------|-------|-------|-------|------|
| $J/\psi \rightarrow \eta_c$ | 1.59(42) | 1.56 | 1.96 | 2.12(4) | 1.8  | 3.28 | 2.44 | 1.8  |       |       |       |
| $\Upsilon \rightarrow \eta_b$ | 0.0087 | 0.0095 | 0.0152(5) | 0.004 | 0.0093 | 0.0154 | 0.01 | 0.001 | 0.01  | 0.0031 | 0.010 |

### TABLE XX. $J/\psi \rightarrow \eta_c \gamma$ decay widths (in keV).

| Exp   | Our   | PM   | PM   | SRPM  | RQM   | QCDSR | QCDSR | QCDSR | QCDSR | QCDSR | Latt | Latt | Latt | Latt |
|-------|-------|------|------|-------|-------|-------|-------|-------|-------|-------|------|------|------|------|
| 1.59(42) | 1.56 | 2.85 | 2.2  | 2.4  | 1.05 | 2.1(4) | 2.7(5) | 2.6(5) | 2.9  | 2.51(8) | 2.84(6) | 2.49(19) | 2.64(14) |
give similar or slightly larger results. On the other hand, many variants of potential model [78, 90, 92, 94], semirelativistic potential model [104], relativistic potential model [129], and lattice QCD [158, 159, 161, 162] predict substantially larger decay widths. In this context our choice looks like some kind of compromise.

In summary, we have chosen the free parameter $C_L$ to reproduce the experimental values of the decay widths $\rho \rightarrow \pi \gamma$ and $\omega \rightarrow \pi \gamma$ simultaneously. Another parameter $C_H$ was chosen to reproduce the PDG average of the decay width $\Gamma(J/\psi \rightarrow \eta_c \gamma)$. Predictions for the static magnetic moments seem to be more or less reliable and are similar to almost all other predictions obtained using various approaches including recent lattice QCD results. Since the transition $J/\psi \rightarrow \eta_c \gamma$ was chosen for the fit of the model parameter $C_H$, the only experimental value we can use to test our approach in the heavy meson sector remains the transition moment of the $D^{*+} \rightarrow D^+ \gamma$ decay. We see from Tables XV–XVII that the agreement is quite good. Thus the predictions of our model for $M_1$ transitions of the ground-state heavy mesons seem to be consistent with available data. The problem is that only two decay widths are measured, and the uncertainties in data are large (15–25%). One more field in which the quality of the predictions can be tested is the nonrelativistic limit. For this purpose the decay $\Upsilon \rightarrow \eta_b \gamma$ can be used. Strictly speaking, this is the exclusive quark-antiquark system in which the nonrelativistic approximation should be valid. Our prediction for the transition moment $\mu(\Upsilon \rightarrow \eta_b)$ is very close to the NR result (0.12 $\mu_N$). Prediction for the decay width $\Gamma(\Upsilon \rightarrow \eta_b \gamma)$ approximately coincides with the typical nonrelativistic estimate, indicating the proper behavior of the model in the nonrelativistic limit.

VI. MAGNETIC PROPERTIES OF HEAVY BARYONS

A. Magnetic moments

We begin our analysis of the magnetic properties of heavy baryons with the calculation of magnetic moments. The results are presented in Tables XXIII–XXXIII. The mixing of the baryons $(\Xi_c, \Xi^+_c, \Xi_0, \Xi^0_0, \Xi^{*}_{cb}, \Xi^{'*}_{cb}, \Omega_{cb}, \Omega^{*}_{cb})$ is taken into account. For the transition moments the static ($k = 0$) values are given.

We compare our predictions with the results obtained using other approaches, such as simple nonrelativistic quark model (NR), various potential models (PM), hyperlocal approach (Hyp), effective mass and charge scheme (EM&C), chiral quark model ($\chi$QM), relativistic quark models (RQM), relativistic potential models (RPM), QCD sum rules (QCDSR), lattice QCD (Latt), and chiral perturbation theory ($\chi$PT). From Ref. [3] we have taken predictions corresponding to two potentials (BD and AL1) in order to illustrate the sensitivity of the results to the form of the potential. For the baryons $(\Xi_c, \Xi^+_c, \Xi^0_c, \Xi^{*}_{cb}, \Xi'^*_{cb}, \Omega_{cb}, \Omega'^*_{cb})$ we have included our results with and without state mixing. This is done for two reasons. First, the comparison of the mixed predictions with the corresponding unmixed results allows us to estimate the effect of this mixing on the values of the magnetic (transition) moments. Second, it is not very meaningful to compare the mixed estimates obtained in one framework with the unmixed results obtained using other approaches, such as simple nonrelativistic quark models (RQM), relativistic potential models (RPM), QCD sum rules (QCDSR), lattice QCD (Latt), and chiral perturbation theory ($\chi$PT). From Ref. [3] we have taken predictions corresponding to two potentials (BD and AL1) in order to illustrate the sensitivity of the results to the form of the potential. For the baryons $(\Xi_c, \Xi^+_c, \Xi^0_c, \Xi^{*}_{cb}, \Xi'^*_{cb}, \Omega_{cb}, \Omega'^*_{cb})$ we have included our results with and without state mixing. This is done for two reasons. First, the comparison of the mixed predictions with the corresponding unmixed results allows us to estimate the effect of this mixing on the values of the magnetic (transition) moments. Second, it is not very meaningful to compare the mixed estimates obtained in one framework with the unmixed results obtained using other approaches. For some baryons $(\Xi_c, \Xi^+_c, \Xi_0, \Xi^0_0, \Xi^{*}_{cb}, \Xi'^*_{cb})$ there exist the predictions of the magnetic moments with the state mixing taken into account. These calculations (Refs. [8, 9]) are based on the solution of the Fadeev equations in the framework of nonrelativistic potential model, and the state mixing effect is present in this method by construction. In all other cases it is more reasonable to compare presented estimates with our unmixed predictions. We include also the mixed NR results in order to have one more prediction to compare our results with. From Tables XXIII–XXXIII we see that our predictions agree well with the results obtained using Fadeev formalism, and the best agreement is achieved in the case of BD potential. For the $\Lambda_Q, \Sigma_Q, \Xi_Q$ and $\Xi'_Q$ baryons the agreement is almost excellent, however, for the baryons $\Omega_Q$ PM(BD) predictions are 15–20% smaller than ours. For doubly heavy baryons the agreement is good, but not so impressive, and for triply heavy baryons the agreement is sufficiently good again. Note the excellent agreement between our result, the PM (BD and AL1) predictions, and the NR estimate in the obviously nonrelativistic case of $\Omega^0_{cb}$ baryon. This is an indication that in the baryon sector our model also has the proper nonrelativistic limit, the feature desirable for any more or less reliable phenomenological model. Next, we see that almost all predictions of various potential models are similar. Moreover, these predictions in almost all cases are close to the predictions
The predictions obtained using various other approaches are closer to the predictions obtained in RQM than to the results obtained in PM or NR models. The predictions obtained using various other approaches are lower than NR results, while the predictions obtained using potential models remain, as a rule, similar to the NR estimates. For example, for the baryons $\Xi_c^-$ and $\Xi_b^-$ our results are closer to the predictions obtained in RQM [192] than to the results obtained in PM or NR models. The predictions obtained using various other approaches differ. For instance, relativistic potential model [193] give predictions similar to the ones obtained using potential models or NR approach. On the other hand, the predictions obtained in the relativistic quark model (Ref. [192]) are lower by $\approx 30\%$. The predictions obtained using light cone QCD sum rules (QCDSR) are similar to ours. Lattice QCD results for $\Sigma_c^0$ and $\Sigma_c^{++}$ baryons within the error bars agree with other predictions, but for $\Omega_c^0$ and $\Omega_c^{+\prime}$ are by $\approx 30\%$ lower. In the case of $\Xi_c^-$, $\Xi_c^{+\prime}$, doubly heavy ($\Xi_c^{+\prime\prime}$, $\Omega_c^{+\prime\prime}$), and triply heavy ($\Omega_c^{++\prime\prime}$) baryons the lattice QCD results are again substantially lower, and the disagreement between lattice QCD results and the predictions obtained using other approaches is evident.

In order to estimate the effect of the state mixing one can compare the results for the mixed and unmixed cases presented in Tables XXXVI, XXXVII, and XXXVIII. We see that for the magnetic moments of $\Sigma_c^0$, $\Xi_c^0$, $\Xi_c^{+\prime}$, $\Xi_c^{+\prime\prime}$ baryons the effect is not large. The transition moment $\mu(\Xi_c^0 \rightarrow \Xi_c^{+\prime})$ in the physical basis is smaller by $\approx 4$ times. For the ($\Xi_b^0$, $\Xi_b^{+\prime}$) baryons the transition moment is lower by $\approx 10\%$. For ($\Xi_c^+$, $\Xi_c^{+\prime}$) and ($\Xi_b^0$, $\Xi_b^{+\prime}$) baryons the tendency is opposite, i.e. the state mixing has practically no effect on the transition moments, but the effect on the values of the corresponding magnetic moments is significant. The magnetic moment of $\Xi_c^{+\prime}$ baryon becomes larger by $\approx 25\%$, and magnetic moment of $\Xi_c^{+\prime\prime}$ smaller by $\approx 2.4$ times. For ($\Xi_c^0$, $\Xi_c^{+\prime}$) baryons the effect is slightly weaker. The magnetic moment of $\Xi_b^0$ baryon is larger by $\approx 10\%$, and magnetic moment of $\Xi_b^{+\prime}$ becomes larger twice. Note that we compare the predictions obtained in the physical (mixed) basis with the results obtained in the optimal ($q_1 q_2)Q$ basis. No one uses the spin coupling scheme such as $(q_1 Q)q_2$. It can be readily checked that the effect of the state mixing in this basis would be very large. In the case of doubly heavy baryons ($\Sigma_c^0$, $\Xi_c^0$), and ($\Omega_c^0$, $\Omega_c^{+\prime\prime}$) the mixing effect is always important, no matter which basis is used. The effect is not so severe for the optimal $(q_1 Q)b$ basis, however, traditionally another basis ($cb)q$ is hooked up. We have compared our unmixed predictions obtained in this basis with the corresponding results obtained using various other approaches in Table XXX. As expected, when calculated in the same basis, the results obtained using various approaches are similar.

As we have seen, the state mixing effect for the singly heavy ($\Xi_c^-$, $\Xi_c^{+\prime}$), ($\Xi_b^0$, $\Xi_b^{+\prime}$), as well as for doubly heavy ($\Xi_c^0$, $\Xi_c^{+\prime\prime}$), ($\Omega_c^0$, $\Omega_c^{+\prime\prime}$) baryons is important and in the realistic calculations must be taken into account.

### B. M1 transition moments and decay widths

In this section we present our results for the M1 decay characteristics of heavy baryons. In Tables XXXIV–XXXIX the magnetic dipole transitions moments are given. Alongside we present the static moments in order to demonstrate the dependence of the transition moment on the photon momentum $k$. Static values also may be useful for the comparison of our predictions with the estimates obtained using other approaches. We see from Tables XXXIV–XXXIX that the static moments are slightly ($\lesssim 10\%$) larger. So, the effect of setting $k = 0$ is not large, but it is not negligible.

We see that for singly heavy baryons our predictions are similar to the corresponding results obtained using other approaches. The exception is the transition $\Sigma_c^{+\prime\prime} \rightarrow \Sigma_c^{+\prime\prime}$, and $\Omega_c^{+\prime\prime} \rightarrow \Omega_c^{+\prime\prime}$. In this case the transition moment is very sensitive to the choice of the model, and the accuracy of the predicted values is not quite clear. Our prediction for this transition practically coincides with the result obtained using hypercentral approach [221], while other models predict lower values. For the doubly charmed baryons our predictions are similar to the effective mass and charge scheme (EM&C) results, and lower than $\chi$QM or $\chi$PT estimates. For the doubly bottom and triply heavy baryons we have not found other predictions. The comparison with NR results shows that our predictions are somewhat lower. This is usual, expected behavior. Analyzing results presented in Tables XXXVI, XXXVII, we see that the state mixing effect is especially large for the transitions $\Xi_c^0 \rightarrow \Xi_c^0$ and $\Xi_c^{+\prime} \rightarrow \Xi_c^{+\prime}$. In these cases the values of the mixed transition moments are approximately four times smaller than the unmixed ones. The mixed transition moment $\mu(\Xi_c^0 \rightarrow \Xi_c^0)$ is $\approx 20\%$ larger, the transition moment $\mu(\Xi_c^{+\prime} \rightarrow \Xi_c^{+\prime})$ is $\approx 20\%$ smaller, and the remaining moments do not undergo large changes. In order to estimate the state mixing effect on the transition moments of $\Xi_c$ and $\Omega_c$ baryons in Table XXXVIII the unmixed moments in the two spin coupling schemes $(qc)b$ and $(cb)q$ are given. The unmixed results calculated in the $(cb)q$ basis are also compared with recent predictions obtained in the framework of heavy hadron chiral perturbation theory [218]. We see that the effect of the mixing is appreciable, especially for the transitions $\Xi_c^0 \rightarrow \Xi_c^0$, $\Xi_c^{+\prime} \rightarrow \Xi_c^{+\prime}$, and $\Omega_c^0 \rightarrow \Omega_c^0$.

As the last step, we have used the obtained above transition moments to calculate the corresponding decay widths. In the calculations the values of the photon momentum given in Sec. IV were used. The results are collected in Tables XXXII, XXXV. The predictions are compared with other available results given by various approaches. In the case of $\Sigma_Q$ baryons our predictions are more or less compatible with the potential model (PM, Hyp) results, but the agreement is not very good. The
predictions obtained using other approaches differ from each other. For the \( \Xi_b \) baryons again all predictions are different. Note that the decay widths \( \Gamma(\Xi_b^0 \rightarrow \Xi_b^0 \gamma) \) and \( \Gamma(\Xi_b^{*+} \rightarrow \Xi_b^{*+} \gamma) \) in the physical basis are of order of magnitude smaller than corresponding unmixed values. Available lattice QCD estimates \( \Gamma(\Omega_c^{*+} \rightarrow \Omega_c^0 \gamma) \), \( \Gamma(\Xi_c^* \rightarrow \Xi_c^0 \gamma) \), and \( \Gamma(\Xi_c^{*+} \rightarrow \Xi_c^{*+} \gamma) \) are considerably lower than other predictions. For the decay \( \Xi_c^0 \rightarrow \Xi_c^0 \gamma \) they practically even did not find any signal.

The case of doubly heavy \( \Xi_{cb} \) and \( \Omega_{cb} \) baryons is the nice exception for us, because we can compare our predictions with the mixed estimates obtained using other approaches. These are the predictions obtained in nonrelativistic potential model \([222]\) and in relativistic three quark model \([224]\). Models are rather different and provide to some extent different results. Our predictions for the transitions \( \Xi_{cb}^{0} \rightarrow \Xi_{cb}^{0} \), \( \Xi_{cb}^{0} \rightarrow \Xi_{cb}^{0} \), \( \Xi_{cb}^{+} \rightarrow \Xi_{cb}^{+} \), \( \Omega_{cb}^{0} \rightarrow \Omega_{cb}^{0} \), and \( \Omega_{cb}^{0} \rightarrow \Omega_{cb}^{0} \) are somewhere between their results, for the transitions \( \Xi_{cb}^{*} \rightarrow \Xi_{cb}^{*} \), \( \Xi_{cb}^{*} \rightarrow \Xi_{cb}^{*} \), \( \Xi_{cb}^{*} \rightarrow \Xi_{cb}^{*} \), and \( \Omega_{cb}^{0} \rightarrow \Omega_{cb}^{0} \) are larger than both, and for the transition \( \Xi_{cb}^{0} \rightarrow \Xi_{cb}^{0} \) our result is close to the PM prediction. For the sake of comparison we also included in Table XLIV the unmixed results obtained in \((c\bar{b})q\) basis using potential model and heavy hadron chiral perturbation theory. We see that \( \chi PT \) predictions are approximately order of magnitude larger than PM results. We have no our own predictions for the decay widths in this case because we have no good estimates for the hyperfine mass splittings of unmixed states in \((c\bar{b})q\) basis.

For the transitions of doubly heavy baryons \( \Xi_{QQ}^{0} \rightarrow \Xi_{QQ}^{0} \) and \( \Omega_{QQ}^{0} \rightarrow \Omega_{QQ}^{0} \) our results are smaller than corresponding results obtained using other approaches. The models are different, and we can expect different predictions. But in one case (i.e., the RQM \([224]\)) the main reason of the discrepancies is clear. In this model the hyperfine splittings \( \Delta\Omega_{cc} \) and \( \Delta\Omega_{bb} \) are \( \approx 1.2 \) times, \( \Delta\Omega_{bb} \approx 1.4 \) times, and \( \Delta\Omega_{cc} \) even \( \approx 2.7 \) times larger than ours. Simple rescaling brings RQM results close to ours. For comparison, these rescaled values are also included in Table XLIV (labeled as Resc).

The decay widths for the transitions \( \Xi_{cc}^{*+} \rightarrow \Xi_{cc}^{*+} \) and \( \Xi_{cc}^{*+} \rightarrow \Xi_{cc}^{*+} \) were also analyzed in the framework of chiral quark model in Ref. [213]. Using their results one can obtain the following estimates

\[
\Gamma(\Xi_{cc}^{*+} \rightarrow \Xi_{cc}^{*+} \gamma) \approx (2.6 - 3.5) \text{ keV}, \quad (123)
\]
\[
\Gamma(\Xi_{cc}^{*+} \rightarrow \Xi_{cc}^{*+} \gamma) \approx (2.3 - 3.4) \text{ keV}. \quad (124)
\]

These estimates are compatible with our predictions.

VII. DISCUSSION AND SUMMARY

The main purpose of this work was to obtain better estimates for the magnetic moments and magnetic dipole decay widths of heavy ground-state hadrons. For this end several improvements were implemented. One such improvement is the new prescription proposed in our previous work \([14]\) to deal with c.m.m. corrections for the magnetic observables calculated in the bag model framework. Usually c.m.m. corrections are applied to the whole observable, e.g., magnetic moment of the hadron. On the other hand, the expression \([39]\) is defined at the quark level. This choice allows us to overcome the difficulty present in the case of heavy-light hadrons, where the usual approach can not ensure reliable predictions. The point is that in the heavy-light systems the size of the c.m.m. correction is governed mainly by heavy quarks, but it is applied to the whole observable (all quarks) in contradiction with the expectations from the HQS. Eq. \([39]\) solves this problem because the magnetic observables of heavy and light quarks are scaled independently. In addition, in the present paper the new procedure, seemingly more suitable for the description of the magnetic properties of heavy hadrons, is used to determine the model parameters \((C_L \text{ and } C_H)\) responsible for the scaling of quark magnetic observables.

In order to reduce the uncertainties in the calculation of M1 decay widths the experimental values of the hadron masses (if available) were used. When the experimental data are absent we resort to the theoretical estimates. Some efforts were undertaken to make these estimates more reliable. For this purpose we have employed the semiempirical approach based on the quark model relations using as input the experimental data, lattice QCD predictions, and theoretical predictions obtained in the AL1 potential model. So, strongly speaking, the decay widths obtained in our work are not pure bag model predictions. We have no intent to test the bag model facilities. The advantages and drawbacks of this model are well known. Instead, our strategy was to combine various methods in order to obtain as good predictions as possible. As a byproduct we also obtain an estimate for the mass of the \( \Xi_{cc} \) baryon \( M(\Xi_{cc}^{*}) = 3695 \pm 5 \text{ MeV} \).

The essential ingredient in our analysis of the baryons containing three differently flavored quarks is the proper treatment of the state mixing problem. As we have seen above, if one seeks to get the reliable predictions for the magnetic moments and M1 decay widths of these baryons the effect of the hyperfine mixing must be taken into account (see also Refs. \([176, 222, 224]\)).

It is a difficult task to obtain the meaningful error estimates in any model, especially when a model is extended to a new region. In the heavy meson sector one has the experimental data (not very accurate) only for two M1 decays \( J/\psi \rightarrow \eta_c \gamma \) and \( D^{*+} \rightarrow D^{+} \gamma \). We have set the parameter \( C_H \) to reproduce the PDG average of \( \Gamma(J/\psi \rightarrow \eta_c \gamma) \). Therefore, the only one experimental result remains for the comparison. The agreement is very good, but this is not enough to trust all other predictions. Also one can check if the predictions in the cases when nonrelativistic approximation is expected to hold (M1 transition moment \( \mu(T \rightarrow \eta_c \gamma) \), magnetic moment of triply heavy baryon \( \Omega_{bb}^{0} \)) agree with the estimates obtained using NR approach. We see that the agreement is
quarks is magnetic moments of the up, down, strange, and bottom quarks is \(\approx 8\%\), for the charmed quark \(\approx 15\%\). Because the magnetic moment of the bottom quark is small the uncertainty in its value does not affect other predictions very much. The main source of possible errors is the uncertainty in the value of the magnetic moment of charmed quark, but the substantial improvement in this field at the present time seems to be hardly possible. Though these estimates come from the meson sector, we optimistically expect similar uncertainties to hold in the heavy baryon sector, too. To estimate the accuracy of the predictions for the magnetic properties of particular hadrons in each case more detailed analysis is necessary.

Having in mind that the accuracy of the model predictions can not be higher than the accuracy of the data used in the fitting of the model parameters we can obtain a crude estimate of possible errors. Such estimate for the magnetic moments of the up, down, strange, and bottom quarks is again good.

To conclude, we have performed a comprehensive analysis of the magnetic properties (magnetic moments, magnetic dipole decay widths) of the ground-state heavy hadrons. The agreement of the predictions with the available experimental data and with some (but not all) theoretical predictions is good. To our knowledge, some of our predictions, such as magnetic moments of neutral heavy mesons \((D^{*0}, B^{*0}, B_s^{*0})\), and the decay widths of triply heavy baryons \(\Gamma(\Omega_{cc} \rightarrow \Omega_{cc} \gamma)\), \(\Gamma(\Omega_{cc} \rightarrow \Omega_{cc} \gamma)\) still are the only available theoretical estimates.

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TABLE XXIII. Magnetic moments of $\Sigma_0$ and $\Omega_c$ baryons (in nuclear magnetons $\mu_N$).

|          | Our | NR  | PM(BD) | PM(AL1) | PM | Hyp | EM&c | $\chi$ | QM | RPM | RQM | QCDSR | Latt |
|----------|-----|-----|--------|---------|----|-----|------|--------|-----|-----|-----|-------|------|
| $\Sigma^0_0$ | $-1.31$ | $-1.37$ | $-1.35$ | $-1.44$ | $-1.16$ | $-1.01$ | $-1.17$ | $-1.60$ | $-1.39$ | $-1.04$ | $-1.50$ | $-1.12$ |
| $\Sigma^+_0$ | $0.487$ | $0.49$ | $0.507$ | $0.548$ | $0.392$ | $0.50$ | $0.63$ | $0.30$ | $0.525$ | $0.36$ | $0.50$ | $0.15$ |
| $\Lambda^+_0$ | $0.335$ | $0.39$ | $0.335$ | $0.341$ | $0.408$ | $0.384$ | $0.37$ | $0.392$ | $0.341$ | $0.42$ | $0.40$ |
| $\Sigma^+_0 \rightarrow \Lambda^+_0$ | $-1.56$ | $-1.61$ | $\ldots$ | $\ldots$ | $\ldots$ | $-1.51$ | $1.56$ | $\ldots$ | $-1.54$ |
| $\Omega_c^0$ | $2.28$ | $2.35$ | $2.36$ | $2.53$ | $1.95$ | $2.27$ | $2.18$ | $2.20$ | $2.44$ | $1.76$ | $2.4(5)$ | $2.03(39)$ |
| $\Omega_c^+ - 0.950$ | $-0.94$ | $-0.806$ | $-0.835$ | $-0.950$ | $-0.958$ | $-0.92$ | $-0.90$ | $-0.85$ | $-0.85$ | $-0.9(2)$ | $-0.668(31)$ |

TABLE XXIV. Magnetic moments of $\Sigma_b$ and $\Omega_b$ baryons (in nuclear magnetons $\mu_N$).

|          | Our | NR  | PM(BD) | PM(AL1) | PM | PM | Hyp | EM&c | $\chi$ | QM | RPM | RQM | QCDSR | Latt |
|----------|-----|-----|--------|---------|----|----|-----|------|--------|-----|-----|-----|-------|------|
| $\Sigma^0_b$ | $-1.15$ | $-1.22$ | $-1.22$ | $-1.31$ | $-1.16$ | $-1.03$ | $-1.05$ | $-1.11$ | $-1.26$ | $-1.01$ | $-1.3(3)$ |
| $\Sigma^+_b$ | $0.603$ | $0.64$ | $0.639$ | $0.682$ | $0.609$ | $0.547$ | $0.591$ | $0.53$ | $0.659$ | $0.53$ | $0.6(2)$ |
| $\Lambda^+_b$ | $-0.060$ | $-0.06$ | $-0.059$ | $0.062$ | $0.609$ | $0.547$ | $0.591$ | $0.53$ | $0.659$ | $0.53$ | $0.6(2)$ |
| $\Sigma^0_b \rightarrow \Lambda^+_b$ | $-1.53$ | $-1.61$ | $\ldots$ | $\ldots$ | $-1.55$ | $\ldots$ | $\ldots$ | $-1.54$ | $\ldots$ | $-1.6(4)$ |
| $\Sigma^+_b$ | $2.25$ | $2.50$ | $2.50$ | $2.67$ | $2.38$ | $2.12$ | $2.23$ | $2.17$ | $2.58$ | $2.07$ | $2.4(5)$ |
| $\Omega'_b$ | $-0.806$ | $-0.79$ | $-0.676$ | $-0.703$ | $\ldots$ | $-0.805$ | $-0.958$ | $-0.863$ | $-0.714$ | $-0.82$ | $-0.8(2)$ |

TABLE XXV. Magnetic moments of $\Xi_c$ and $\Xi'_c$ baryons (in nuclear magnetons $\mu_N$).

|          | Mixed | Unmix | Our | NR  | PM(BD) | PM(AL1) | Our | PM | EM&c | $\chi$ | QM | RPM | RQM | QCDSR | Latt |
|----------|-------|-------|-----|-----|--------|---------|-----|----|------|--------|-----|-----|-----|-------|------|
| $\Xi^0_c$ | $-1.13$ | $-1.18$ | $\ldots$ | $\ldots$ | $-1.12$ | $-0.987$ | $-0.93$ | $-1.32$ | $-1.12$ | $-0.95$ | $-1.2(3)$ | $-0.599(71)$ |
| $\Xi^+_c$ | $0.346$ | $0.41$ | $0.357$ | $0.360$ | $0.334$ | $\ldots$ | $0.366$ | $0.28$ | $0.341$ | $0.39$ | $0.35(5)$ | $0.192(17)$ |
| $\Xi^0'_c \rightarrow \Xi^0_c$ | $0.035$ | $0.08$ | $\ldots$ | $\ldots$ | $0.138$ | $\ldots$ | $0.13$ | $-0.31$ | $\ldots$ | $\ldots$ | $0.18(2)$ | $0.009(13)$ |
| $\Xi^+_c$ | $0.825$ | $0.89$ | $\ldots$ | $\ldots$ | $0.633$ | $0.509$ | $0.76$ | $0.76$ | $0.796$ | $0.47$ | $0.8(2)$ | $0.315(144)$ |
| $\Xi^+_c$ | $0.142$ | $0.20$ | $0.166$ | $0.211$ | $0.334$ | $\ldots$ | $0.37$ | $0.40$ | $0.341$ | $0.37$ | $0.50(5)$ | $0.235(25)$ |
| $\Xi^0'_c \rightarrow \Xi^+_c$ | $-1.35$ | $-1.40$ | $\ldots$ | $\ldots$ | $-1.38$ | $\ldots$ | $-1.39$ | $1.30$ | $\ldots$ | $\ldots$ | $1.3(1)$ | $0.729(103)$ |

TABLE XXVI. Magnetic moments of $\Xi_b$ and $\Xi'_b$ baryons (in nuclear magnetons $\mu_N$).

|          | Mixed | Mixed | Our | NR  | PM(BD) | PM(AL1) | Our | PM | EM&c | $\chi$ | QM | RPM | RQM | QCDSR | Latt |
|----------|-------|-------|-----|-----|--------|---------|-----|----|------|--------|-----|-----|-----|-------|------|
| $\Xi^-_b$ | $-0.968$ | $-1.02$ | $\ldots$ | $\ldots$ | $-0.966$ | $-0.941$ | $-0.902$ | $-0.996$ | $-0.985$ | $-0.91$ | $-1.2(3)$ |
| $\Xi^-_b$ | $-0.0555$ | $-0.05$ | $-0.052$ | $-0.055$ | $-0.0596$ | $\ldots$ | $\ldots$ | $-0.066$ | $\ldots$ | $-0.06$ | $-0.08(2)$ |
| $\Xi^-_b \rightarrow \Xi^-_b$ | $0.113$ | $0.16$ | $\ldots$ | $\ldots$ | $0.128$ | $\ldots$ | $\ldots$ | $0.142$ | $\ldots$ | $\ldots$ | $0.21(1)$ |
| $\Xi^0_0$ | $0.782$ | $0.90$ | $\ldots$ | $\ldots$ | $0.737$ | $0.658$ | $0.766$ | $0.676$ | $0.893$ | $0.66$ | $0.7(2)$ |
| $\Xi^0_0$ | $-0.106$ | $-0.11$ | $-0.106$ | $-0.086$ | $-0.0596$ | $\ldots$ | $\ldots$ | $-0.060$ | $\ldots$ | $-0.06$ | $-0.045(5)$ |
| $\Xi^0_0 \rightarrow \Xi^0_0$ | $-1.33$ | $-1.41$ | $\ldots$ | $\ldots$ | $-1.34$ | $\ldots$ | $1.35$ | $\ldots$ | $\ldots$ | $1.4(1)$ |
TABLE XXVII. Magnetic moments of $J = \frac{3}{2}$ singly charmed baryons (in nuclear magnetons $\mu_N$).

| Baryon | Our | NR | PM | Hyp | Hyp | Hyp | EM&C | $\chi$QM | QCDSR | Latt |
|--------|-----|----|----|-----|-----|-----|-------|----------|-------|------|
|        | [182] | [183] | [185] | [186] | [180] | [197] | [206] | [210]      |
| $\Sigma^+_c$ | $-1.49$ | $-1.47$ | $-1.15$ | $-0.848$ | $-1.44$ | $-1.02$ | $-1.18$ | $-1.99$ | $-0.81(20)$ | ... |
| $\Sigma^{++}_c$ | $1.25$ | $1.32$ | $1.13$ | $1.25$ | $1.32$ | ... | $1.18$ | $0.97$ | $2.00(46)$ | ... |
| $\Sigma^{++}_c$ | $3.98$ | $4.11$ | $3.41$ | $3.84$ | $4.10$ | ... | $3.63$ | $3.92$ | $4.81(122)$ | ... |
| $\Xi^+_c$ | $-1.20$ | $-1.15$ | $-0.987$ | $-0.888$ | $-1.18$ | $-0.825$ | $-1.02$ | $-1.43$ | $-0.68(18)$ | ... |
| $\Xi^{++}_c$ | $1.47$ | $1.64$ | $1.26$ | $1.51$ | $1.04$ | ... | $1.39$ | $1.59$ | $1.68(42)$ | ... |
| $\Omega^+_c$ | $-0.936$ | $-0.83$ | $-0.834$ | $-0.865$ | $-0.92$ | $-0.625$ | $-0.84$ | $-0.86$ | $-0.62(18)$ | $-0.730(23)$ |

TABLE XXVIII. Magnetic moments of $J = \frac{1}{2}$ singly bottom baryons (in nuclear magnetons $\mu_N$).

| Baryon | Our | NR | PM | Hyp | Hyp | EM&C | $\chi$QM | QCDSR | Latt |
|--------|-----|----|----|-----|-----|-------|----------|-------|------|
|        | [189] | [183] | [185] | [181] | [197] | [206] |         |       |      |
| $\Sigma^{-}_b$ | $-1.82$ | $-1.92$ | $-1.82$ | $-1.66$ | $-1.91$ | $-1.75$ | $-1.63$ | $-1.50(36)$ |
| $\Sigma^{0}_b$ | $0.820$ | $0.87$ | $0.819$ | $0.791$ | $0.89$ | $0.705$ | $0.724$ | $0.50(15)$ |
| $\Sigma^{+}_b$ | $3.46$ | $3.56$ | $3.46$ | $3.23$ | $3.69$ | $3.10$ | $3.08$ | $2.52(50)$ |
| $\Xi^{-}_b$ | $-1.55$ | $-1.60$ | $-1.10$ | $-1.65$ | $-1.59$ | $-1.48$ | $-1.42(35)$ |
| $\Xi^{0}_b$ | $1.03$ | $1.19$ | $1.04$ | $1.16$ | $0.915$ | $0.875$ | $0.50(15)$ |
| $\Omega^{-}_b$ | $-1.31$ | $-1.28$ | $-1.20$ | $-1.38$ | $-1.39$ | $-1.29$ | $-1.40(35)$ |

TABLE XXIX. Magnetic moments of $B_{cb}$ and $B'_{cb}$ baryons (in nuclear magnetons $\mu_N$) with and without state mixing.

|        | Our | NR | PM(BD) | PM(AL1) | Mixed | Mixed | [s] | [s] | Unmix |
|--------|-----|----|--------|---------|-------|-------|-----|-----|-------|
| $\Xi^{0}_{cb} \rightarrow \Xi^{0}_{cb}$ | $-0.452$ | $-0.53$ | ... | ... | ... | ... | $-0.291$ |        |
| $\Xi^{0}_{cb} \rightarrow \Xi^{0}_{cb}$ | $0.102$ | $0.13$ | $0.117$ | $0.058$ | $\Xi^{0}_{[dc]b}$ | $0.0596$ | | | |
| $\Xi^{0}_{cb} \rightarrow \Xi^{0}_{cb}$ | $0.600$ | $0.70$ | ... | ... | ... | ... | $0.651$ | $\Xi^{0}_{[dc]b}$ |
| $\Xi^{+}_{cb}$ | $1.46$ | $1.71$ | ... | ... | ... | ... | $1.30$ | $\Xi^{+}_{[uc]b}$ |
| $\Xi^{+}_{cb}$ | $-0.222$ | $-0.25$ | $-0.254$ | $-0.198$ | $\Xi^{+}_{[uc]b}$ | $-0.0596$ | | | |
| $\Xi^{0}_{cb} \rightarrow \Xi^{+}_{cb}$ | $-0.532$ | $-0.62$ | ... | ... | ... | ... | $-0.729$ | $\Xi^{+}_{[uc]b}$ |
| $\Omega^{0}_{cb}$ | $-0.275$ | $-0.27$ | ... | ... | ... | ... | $-0.157$ | $\Omega^{0}_{[uc]b}$ |
| $\Omega^{0}_{cb}$ | $0.058$ | $0.08$ | $0.047$ | $0.009$ | $\Omega^{0}_{[uc]b}$ | $-0.0596$ | | | |
| $\Omega^{0}_{cb} \rightarrow \Omega^{0}_{cb}$ | $0.510$ | $0.56$ | ... | ... | ... | ... | $0.534$ | $\Omega^{0}_{[uc]b}$ |

TABLE XXX. Magnetic moments of $B_{cb}$ and $B'_{cb}$ baryons (in nuclear magnetons $\mu_N$) without state mixing.

|        | Our | PM(AL1) | Hyp | Hyp | EM&C | RQM | RQM | $\chi$PT |
|--------|-----|----------|-----|-----|-------|-----|-----|----------|
|        | Unmix | [178] | [183] | [187, 188] | [181] | [192] | [193] | [211] |
| $\Xi^{0}_{[cb]d}$ | $-0.796$ | $-0.993$ | ... | ... | $-0.814$ | $-0.76$ | ... | $-0.59$ |
| $\Xi^{0}_{[cb]d}$ | $0.446$ | $0.518$ | $0.477$ | $0.354$ | $0.480$ | $0.42$ | $0.63$ | $0.56$ |
| $\Xi^{0}_{[cb]d} \rightarrow \Xi^{0}_{[cb]d}$ | $-0.225$ | ... | ... | ... | $0.242$ | ... | ... | ... |
| $\Xi^{0}_{[cb]u}$ | $1.59$ | $1.99$ | ... | ... | $1.72$ | $1.52$ | ... | $0.69$ |
| $\Xi^{0}_{[cb]u}$ | $-0.350$ | $-0.475$ | $-0.400$ | $-0.204$ | $-0.369$ | $-0.12$ | $-0.52$ | $-0.54$ |
| $\Xi^{0}_{[cb]u} \rightarrow \Xi^{0}_{[cb]u}$ | $-0.225$ | ... | ... | ... | $0.250$ | ... | ... | ... |
| $\Omega^{0}_{[cb]s}$ | $-0.595$ | $-0.542$ | ... | ... | $-0.624$ | $-0.61$ | ... | $0.24$ |
| $\Omega^{0}_{[cb]s}$ | $0.378$ | $0.368$ | $0.395$ | $0.439$ | $0.407$ | $0.45$ | $0.49$ | $0.49$ |
| $\Omega^{0}_{[cb]s} \rightarrow \Omega^{0}_{[cb]s}$ | $-0.225$ | ... | ... | ... | $0.243$ | ... | ... | ... |
### TABLE XXXI. Magnetic moments of $J = 1/2$ doubly heavy baryons (in nuclear magnetons $\mu_N$).

| Baryon | Our | NR | PM (BD) | PM (AL1) | Hyp | Hyp | EM & C | RPM | RQM | Latt | $\chi$PT | Latt |
|--------|-----|----|---------|----------|-----|-----|--------|------|-----|-------|----------|-------|
| $\Xi_{cc}^+$ | 0.719 | 0.83 | 0.775 | 0.784 | 0.859 | 0.784 | 0.77 | 0.774 | 0.72 | 0.853 | 0.85 | 0.43(3) |
| $\Xi_{cc}^{-}$ | 0.110 | -0.10 | -0.172 | -0.206 | -0.137 | -0.031 | -0.11 | -0.184 | 0.13 | -0.169 | -0.25 | ... |
| $\Omega_{cc}^+$ | 0.645 | 0.72 | 0.620 | 0.635 | 0.783 | 0.692 | 0.70 | 0.639 | 0.67 | 0.74 | 0.78 | 0.407(7) |
| $\Xi_{bb}^-$ | 0.171 | 0.23 | 0.230 | 0.251 | 0.190 | 0.196 | 0.218 | 0.236 | 0.18 | 0.32 | 0.26 | ... |
| $\Xi_{bb}^0$ | -0.581 | -0.70 | -0.698 | -0.742 | -0.656 | -0.663 | -0.630 | -0.722 | -0.53 | -0.89 | -0.84 | ... |
| $\Omega_{bb}^-$ | 0.112 | 0.12 | 0.095 | 0.101 | 0.109 | 0.108 | 0.139 | 0.10 | 0.04 | 0.16 | 0.19 | ... |

### TABLE XXXII. Magnetic moments of $J = 3/2$ doubly heavy baryons (in nuclear magnetons $\mu_N$).

| Baryon | Our | NR | PM (AL1) | Hyp | Hyp | EM & C | RPM | RQM | Latt | $\chi$PT | Latt |
|--------|-----|----|---------|-----|-----|--------|------|-----|-------|----------|-------|
| $\Xi_{cc}^{++}$ | -0.178 | -0.15 | -0.311 | -0.168 | 0.068 | 0.035 | -0.23 | -0.18 | 0.095 | 0.139 | 0.139 | 0.121 | 0.285 | 0.21 | 0.16 | 0.17 |
| $\Xi_{cc}^{++}$ | 2.35 | 2.64 | 2.67 | 2.75 | 2.22 | 2.52 | 2.72 | 2.61 | 0.140 | 1.74 | 1.87 | 1.58 | 1.61 | 1.51 | 2.30 | 2.83 |
| $\Xi_{bb}^{--}$ | -0.880 | -1.05 | -1.11 | -0.951 | -1.74 | -1.052 | -1.32 | -1.33 | -0.697 | -0.73 | -0.662 | -0.711 | -1.24 | -0.805 | -0.86 | -1.54 |
| $\Xi_{bb}^{0}$ | -0.534 | -0.60 | -0.712 | -0.567 | -0.372 | -0.508 | -0.76 | -0.84 | -0.010 | -0.280 | -0.261 | -0.316 | -0.181 | -0.309 | -0.32 | -1.09 |
| $\Omega_{bb}^{++}$ | 1.88 | 2.19 | 2.27 | 2.05 | 1.56 | 2.02 | 2.68 | 3.72 | 0.329 | -0.280 | -0.261 | -0.316 | -0.181 | -0.309 | -0.32 | -1.09 |

### TABLE XXXIII. Magnetic moments of triply heavy baryons (in nuclear magnetons $\mu_N$).

| Baryon | Our | NR | PM (BD) | PM (AL1) | PM | Hyp | EM & C | RPM | RQM | Latt | $\chi$PT | Latt |
|--------|-----|----|---------|----------|-----|-----|--------|------|-----|-------|----------|-------|
| $\Omega_{ccc}^{++}$ | 0.989 | 1.17 | 1.00 | 1.02 | 1.18 | 1.19 | 1.16 | ... | ... | 0.67(6) |
| $\Omega_{ccc}^{++}$ | 0.455 | 0.54 | 0.466 | 0.475 | 0.565 | 0.502 | 0.522 | 0.476 | 0.53 | ... |
| $\Omega_{cbb}^{++}$ | 0.594 | 0.72 | ... | ... | 0.751 | 0.651 | 0.703 | ... | ... |
| $\Omega_{bcb}^{++}$ | -0.187 | -0.21 | -0.191 | -0.193 | -0.223 | -0.203 | -0.200 | -0.197 | -0.20 | ... |
| $\Omega_{bbb}^{++}$ | 0.204 | 0.27 | ... | ... | 0.285 | 0.216 | 0.225 | ... | ... |
| $\Omega_{ccc}^{++}$ | -0.178 | -0.18 | -0.178 | -0.180 | -0.196 | -0.195 | -0.198 | ... | ... | ... | ... |

### TABLE XXXIV. Transition moments of $\Sigma_c$ and $\Omega_c$ baryons (in nuclear magnetons $\mu_N$).

| Transition | Our | Our | NR | PM | Hyp | Hyp | EM & C | $\chi$QM |
|------------|-----|-----|----|-----|-----|-----|--------|----------|
| $\Sigma_{cc}^{0} \rightarrow \Sigma_{cc}^{0}$ | -1.14 | -1.17 | -1.24 | -1.06 | -1.12 | -1.04 | 1.07 | 1.48 |
| $\Sigma_{cc}^{+} \rightarrow \Sigma_{cc}^{+}$ | 0.102 | 0.111 | 0.07 | 0.008 | 0.100 | ... | 0.08 | -0.003 |
| $\Sigma_{cc}^{0} \rightarrow \Lambda_{c}^{+}$ | 2.07 | 2.23 | 2.2 | 1.86 | 2.12 | 1.84 | 2.15 | 2.40 |
| $\Sigma_{c}^{+} \rightarrow \Lambda_{c}^{+}$ | -1.48 | -1.56 | -1.61 | -1.35 | -1.54 | ... | -1.54 | 1.56 |
| $\Sigma_{cc}^{++} \rightarrow \Sigma_{cc}^{++}$ | 1.34 | 1.39 | 1.39 | 1.08 | 1.32 | ... | 1.23 | -1.37 |
| $\Omega_{c}^{0} \rightarrow \Omega_{c}^{0}$ | -0.892 | -0.911 | -0.94 | -0.908 | -0.916 | -0.876 | 0.90 | 1.24 |
### TABLE XXXV. Transition moments of $\Sigma_b$ and $\Omega_b$ baryons (in nuclear magnetons $\mu_N$).

| Transition | Our | Our | NR | PM | PM | Hyp |
|------------|-----|-----|----|----|----|-----|
|            |Static | [189] | [221] | [221] |
| $\Sigma_b^+ \to \Sigma_b^-$ | -0.760 | -0.768 | -0.82 | -0.774 | -0.683 | -0.692 |
| $\Sigma_b^0 \to \Sigma_b^0$ | 0.464 | 0.468 | 0.49 | 0.473 | 0.429 | 0.459 |
| $\Sigma_b^0 \to \Lambda_b^0$ | 2.02 | 2.16 | 2.28 | 2.19 | 1.93 | 2.00 |
| $\Sigma_b^0 \to \Lambda_b^0$ | -1.43 | -1.53 | -1.61 | -1.55 | -1.37 | -1.42 |
| $\Sigma_b^+ \to \Sigma_b^+$ | 1.69 | 1.70 | 1.81 | 1.72 | 2.22 | 2.30 |
| $\Omega_b^- \to \Omega_b^-$ | -0.523 | -0.528 | -0.52 | $\cdots$ | -0.523 | -0.476 |

### TABLE XXXVI. Transition moments of $\Xi_b$ baryons (in nuclear magnetons $\mu_N$).

| Transition | Our | Our | NR | PM | Hyp | EM&C | $\chi$QM |
|------------|-----|-----|----|----|-----|-------|---------|
|            | Mixed | Static | Mixed | Unmix | [221] | [221] | [179] | [180] | [197] |
| $\Xi_b^0 \to \Xi_b^0$ | -0.994 | -1.01 | -1.07 | -1.03 | $\cdots$ | $\cdots$ | 0.99 | 1.24 |
| $\Xi_b^0 \to \Xi_b^-$ | -0.249 | -0.268 | -0.33 | -0.193 | -0.120 | -0.208 | 0.18 | -0.50 |
| $\Xi_b^0 \to \Xi_b^0$ | 0.0339 | 0.035 | 0.08 | 0.139 | $\cdots$ | $\cdots$ | 0.13 | -0.31 |
| $\Xi_b^+ \to \Xi_b^+$ | 0.0664 | 0.0738 | 0.09 | 0.216 | $\cdots$ | $\cdots$ | 0.17 | -0.23 |
| $\Xi_b^+ \to \Xi_b^+$ | 1.86 | 1.96 | 2.03 | 1.97 | 0.991 | 1.11 | 1.94 | 2.08 |
| $\Xi_b^0 \to \Xi_b^0$ | -1.33 | -1.35 | -1.40 | -1.38 | $\cdots$ | $\cdots$ | -1.39 | 1.30 |

### TABLE XXXVII. Transition moments of $\Xi_b$ baryons (in nuclear magnetons $\mu_N$).

| Transition | Our | Our | NR | Our | PM | Hyp |
|------------|-----|-----|----|-----|----|-----|
|            | Mixed | Static | Mixed | Unmix | [221] | [221] |
| $\Xi_b^+ \to \Xi_b^-$ | -0.6229 | -0.636 | -0.66 | -0.641 | $\cdots$ | $\cdots$ |
| $\Xi_b^- \to \Xi_b^-$ | -0.182 | -0.193 | -0.26 | -0.181 | -0.124 | -0.196 |
| $\Xi_b^- \to \Xi_b^-$ | 0.109 | 0.113 | 0.16 | 0.128 | $\cdots$ | $\cdots$ |
| $\Xi_b^0 \to \Xi_b^0$ | 0.521 | 0.529 | 0.61 | 0.563 | $\cdots$ | $\cdots$ |
| $\Xi_b^0 \to \Xi_b^0$ | 1.83 | 1.91 | 2.03 | 1.89 | 1.04 | 1.05 |
| $\Xi_b^0 \to \Xi_b^0$ | -1.30 | -1.39 | -1.41 | -1.34 | $\cdots$ | $\cdots$ |

### TABLE XXXVIII. Transition moments of $\Xi_b$ and $\Omega_b$ baryons (in nuclear magnetons $\mu_N$).

| Transition | Our | Our | NR | Transition | Our | Transition | Our | $\chi$PT |
|------------|-----|-----|----|------------|-----|------------|-----|---------|
|            | Mixed | Static | Mixed | Unmix | Unmix | [216] |
| $\Xi_b^0 \to \Xi_b^0$ | -0.0416 | -0.0445 | -0.06 | $\Xi_b^0 \to \Xi_b^0$ | -0.162 | $\Xi_b^0 \to \Xi_b^0$ | 0.319 | -0.36 |
| $\Xi_b^0 \to \Xi_b^0$ | -0.919 | -0.934 | -1.09 | $\Xi_b^0 \to \Xi_b^0$ | -0.899 | $\Xi_b^0 \to \Xi_b^0$ | 0.879 | 1.34 |
| $\Xi_b^0 \to \Xi_b^0$ | 0.508 | 0.600 | 0.70 | $\Xi_b^0 \to \Xi_b^0$ | 0.649 | $\Xi_b^0 \to \Xi_b^0$ | -0.225 | $\cdots$ |
| $\Xi_b^0 \to \Xi_b^0$ | 0.814 | 0.823 | 0.95 | $\Xi_b^0 \to \Xi_b^0$ | 0.958 | $\Xi_b^0 \to \Xi_b^0$ | 0.319 | -0.36 |
| $\Xi_b^0 \to \Xi_b^0$ | 1.12 | 1.15 | 1.33 | $\Xi_b^0 \to \Xi_b^0$ | 0.998 | $\Xi_b^0 \to \Xi_b^0$ | -1.37 | -2.56 |
| $\Xi_b^0 \to \Xi_b^0$ | -0.531 | -0.532 | -0.62 | $\Xi_b^0 \to \Xi_b^0$ | -0.727 | $\Xi_b^0 \to \Xi_b^0$ | -0.225 | $\cdots$ |
| $\Omega_b^0 \to \Omega_b^0$ | 0.0173 | 0.0161 | 0.05 | $\Omega_b^0 \to \Omega_b^0$ | -0.0684 | $\Omega_b^0 \to \Omega_b^0$ | 0.318 | -0.36 |
| $\Omega_b^0 \to \Omega_b^0$ | -0.748 | -0.758 | -0.82 | $\Omega_b^0 \to \Omega_b^0$ | -0.741 | $\Omega_b^0 \to \Omega_b^0$ | 0.688 | 1.33 |
| $\Omega_b^0 \to \Omega_b^0$ | 0.508 | 0.510 | 0.56 | $\Omega_b^0 \to \Omega_b^0$ | 0.532 | $\Omega_b^0 \to \Omega_b^0$ | -0.225 | $\cdots$ |
### TABLE XXXIX. Transition moments of doubly and triply heavy baryons (in nuclear magnetons $\mu_N$).

| Transition | Our Static | Our [180] | NR [107] | EM&C [216] | $\chi_{QM}$ | $\chi_{PT}$ |
|------------|------------|-----------|---------|-----------|-------------|-------------|
| $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{++}$ | 1.13 | 2.28 | 1.13 | 2.31 | 0.028(3) | 0.0274 |
| $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{++}$ | -1.24 | -2.46 | -1.23 | -2.43 | -0.04(4) | -0.043 |
| $\Omega_c^{++} \rightarrow \Omega_c^{++}$ | 0.94 | 1.93 | 0.93 | 1.91 | 0.029(3) | 0.0278 |
| $\Sigma_{cc}^{++} \rightarrow \Sigma_{cc}^{++}$ | 0.93 | 1.93 | 0.93 | 1.92 | 0.029(3) | 0.0278 |
| $\Xi_{cc}^{0} \rightarrow \Xi_{cc}^{0}$ | 1.13 | 2.26 | 1.13 | 2.23 | 0.028(3) | 0.0274 |
| $\Xi_{cc}^{0} \rightarrow \Xi_{cc}^{0}$ | -1.24 | -2.46 | -1.23 | -2.43 | -0.04(4) | -0.043 |
| $\Omega_c^{0} \rightarrow \Omega_c^{0}$ | 0.94 | 1.93 | 0.93 | 1.91 | 0.029(3) | 0.0278 |
| $\Sigma_{cc}^{0} \rightarrow \Sigma_{cc}^{0}$ | 0.93 | 1.93 | 0.93 | 1.92 | 0.029(3) | 0.0278 |

### TABLE XL. M1 decay widths (in keV) of $\Sigma_c$ and $\Omega_c$ baryons.

| Transition | Our | PM | PM | Hyp | QM | RQM | $\chi_{PT}$ | QCDSR | Latt |
|------------|-----|----|----|-----|----|-----|-------------|-------|------|
| $\Sigma_c^0 \rightarrow \Sigma_c^0$ | 1.13 | 1.2 | 1.12 | 1.44 | 3.43 | 1.2 | 2.52 | 0.08(3) | 0.083 |
| $\Sigma_c^+ \rightarrow \Sigma_c^+$ | 0.001 | 0.001 | 0.00 | 0.004 | 0.14 | 0.002 | 0.4 | 0.4 | 0.4 |
| $\Sigma_c^+ \rightarrow \Lambda_c^+$ | 7.41 | 100 | 60.6 | 98.0 | 80.0 | 80.6 | 60.7 | 88 | 164 | 50.17 |
| $\Xi_c^{0+} \rightarrow \Xi_c^{0+}$ | 1.96 | 1.60 | 1.15 | 1.98 | 3.94 | 1.4 | 11.6 | 0.265(1)8 |
| $\Omega_c^{0} \rightarrow \Omega_c^{0}$ | 0.85 | 0.69 | 0.20 | 0.82 | 0.89 | 4.82 | 0.932 | 0.074(8) |

### TABLE XLI. M1 decay widths (in keV) of $\Sigma_b$ and $\Omega_b$ baryons.

| Transition | Our | PM | PM | Hyp | QM | $\chi_{PT}$ | QCDSR |
|------------|-----|----|----|-----|----|-------------|-------|
| $\Sigma_b^0 \rightarrow \Sigma_b^0$ | 0.0192 | 0.023 | 0.00 | 0.004 | 0.14 | 0.002 | 0.4 | 0.4 | 0.4 |
| $\Sigma_b^0 \rightarrow \Sigma_b^0$ | 0.0083 | 0.0086 | 0.0 | 0.006 | 0.14 | 0.002 | 0.4 | 0.4 | 0.4 |
| $\Sigma_b^0 \rightarrow \Lambda_b^0$ | 158 | 114 | 129 | 142 | 335 | 435 | 114(45) |
| $\Sigma_b^0 \rightarrow \Lambda_b^0$ | 116 | 78 | 94.8 | 100 | 130 | 288 | 152(60) |
| $\Omega_b^+ \rightarrow \Omega_b^+$ | 0.110 | 0.13 | 0.08 | 0.11 | 0.25 | 0.60 | 0.46(22) |
| $\Omega_b^{0} \rightarrow \Omega_b^{0}$ | 0.0091 | 0.03 | 0.20 | 0.10 | 0.92 |

### TABLE XLII. M1 decay widths (in keV) of $\Xi_b$ baryons.

| Transition | Our | Mixed | Unmix | PM | PM | Hyp | QM | RQM | $\chi_{PT}$ | QCDSR | Latt |
|------------|-----|-------|-------|----|----|-----|----|-----|-------------|-------|------|
| $\Xi_b^{0} \rightarrow \Xi_b^{0}$ | 1.23 | 1.33 | 1.1 | 3.03 | 3.83 | 2.14 |
| $\Xi_b^{0} \rightarrow \Xi_b^{0}$ | 1.24 | 0.745 | 1.22 | 0.30 | 1.15 | 0.0 | 0.68 | 0.36 | 0.66(32) |
| $\Xi_b^{0} \rightarrow \Xi_b^{0}$ | 0.011 | 0.185 | 0.23 | 0.0 | 0.17 | 0.02 | 0.27(6) | 0.0024(4) |
| $\Xi_b^{0} \rightarrow \Xi_b^{0}$ | 0.006 | 0.063 | 0.03 | 0.004 | 1.10 | 0.274 |
| $\Xi_b^{0} \rightarrow \Xi_b^{0}$ | 72.7 | 81.6 | 16 | 63.3 | 99.9 | 139 | 54 | 502 | 52(25) |
| $\Xi_b^{0} \rightarrow \Xi_b^{0}$ | 17.3 | 18.6 | 5.7 | 42.3 | 12.7 | 54.3 | 8.5(2.5) | 5.5(1.5) |
### TABLE XLIII. M1 decay widths (in keV) of $\Xi_b$ baryons.

| Transition          | Our                  | Our                  | PM [221] | Hyp [224] | QM [215] | $\chi$PT [216] | QCDSSR [227, 228, 230] |
|---------------------|----------------------|----------------------|-----------|-----------|-----------|-----------------|-------------------------|
| $\Xi_b^{-} \to \Xi_b^{-}$ | 0.0131               | 0.0116               | $\ldots$ | $\ldots$ | 15.0      | $\ldots$       | 0.303                   |
| $\Xi_b^{-} \to \Xi_b^{0}$ | 0.542               | 0.536               | 0.69      | 0.03      | 1.87      | 1.50(75)        |                         |
| $\Xi_b^{-} \to \Xi_b^{+}$ | 0.259               | 0.357               | $\ldots$ | $\ldots$ | 0.00      | $\ldots$       | 3.3(13)                 |
| $\Xi_b^{0} \to \Xi_b^{0}$ | 0.009               | 0.0105              | $\ldots$ | $\ldots$ | 5.19      | $\ldots$       | 0.131                   |
| $\Xi_b^{0} \to \Xi_b^{+}$ | 51.8                | 55.3                | 18.8      | 3.60      | 104       | 136             | 135(65)                 |
| $\Xi_b^{0} \to \Xi_b^{0}$ | 34.3                | 36.4                | $\ldots$ | $\ldots$ | 8.46      | $\ldots$       | 47(21)                  |

### TABLE XLIV. M1 decay widths (in keV) of $\Xi_{cb}$ and $\Omega_{cb}$ baryons.

| Transition          | Our [222]           | PM [224]           | QM [224]           | Transition          | PM [222] | $\chi$PT [216] |
|---------------------|----------------------|---------------------|---------------------|---------------------|-----------|-----------------|
| $\Xi_{cb}^{-} \to \Xi_{cb}^{-}$ | 7.6 - 10^{-5}       | 2 - 10^{-6}        | $\Xi_{cb}^{-} \to \Xi_{cb}^{0}$ | 0.0404               | 0.52            |
| $\Xi_{cb}^{0} \to \Xi_{cb}^{0}$ | 0.876               | 1.03               | $\Xi_{cb}^{0} \to \Xi_{cb}^{0}$ | 0.505               | 7.19            |
| $\Xi_{cb}^{+} \to \Xi_{cb}^{+}$ | 0.204               | 0.209              | $\Xi_{cb}^{+} \to \Xi_{cb}^{+}$ | 0.00992              | $\ldots$       |
| $\Xi_{cb}^{0} \to \Xi_{cb}^{0}$ | 0.0923              | 0.0605             | $\Xi_{cb}^{0} \to \Xi_{cb}^{0}$ | 0.0015              | $\ldots$       |
| $\Xi_{cb}^{0} \to \Xi_{cb}^{0}$ | 1.31                | 0.739              | $\Xi_{cb}^{0} \to \Xi_{cb}^{0}$ | 1.05                | 26.2           |
| $\Xi_{cb}^{+} \to \Xi_{cb}^{+}$ | 0.161               | 0.124              | $\Xi_{cb}^{+} \to \Xi_{cb}^{+}$ | 0.14                | $\ldots$       |
| $\Omega_{cb}^{0} \to \Omega_{cb}^{0}$ | 1.3 - 10^{-5}       | 0.0031             | $\Omega_{cb}^{0} \to \Omega_{cb}^{0}$ | 0.0369              | 0.52           |
| $\Omega_{cb}^{0} \to \Omega_{cb}^{0}$ | 0.637               | 0.502              | $\Omega_{cb}^{0} \to \Omega_{cb}^{0}$ | 0.209               | 7.08           |
| $\Omega_{cb}^{0} \to \Omega_{cb}^{0}$ | 0.170               | 0.0852             | $\Omega_{cb}^{0} \to \Omega_{cb}^{0}$ | 0.00568             | $\ldots$       |

### TABLE XLIV. M1 decay widths (in keV) of doubly and triply heavy baryons.

| Decay             | Our | QM [218] | SRPM [225] | RQM [224] | RQM [224] | $\chi$PT [216] | Resc [216] |
|-------------------|-----|----------|------------|-----------|-----------|----------------|-------------|
| $\Xi_c^{-} \to \Xi_c^{-}$ | 2.17   | 14.6      | 3.90       | 28.8      | 1.46      | 9.57           |             |
| $\Xi_c^{++} \to \Xi_c^{++}$ | 2.79   | 16.7      | 7.21       | 23.5      | 1.19      | 22.0           |             |
| $\Omega_c^{+} \to \Omega_c^{+}$ | 1.60   | 6.93      | 0.82       | 2.11      | 1.22      | 9.45           |             |
| $\Xi_{bb}^{-} \to \Xi_{bb}^{-}$ | 0.0268 | 0.24      | 0.21       | 0.059     | 0.0215    | 5.17           |             |
| $\Xi_{bb}^{0} \to \Xi_{bb}^{0}$ | 0.137  | 1.19      | 0.98       | 0.31      | 0.113     | 31.1           |             |
| $\Omega_{bb}^{-} \to \Omega_{bb}^{-}$ | 0.0148 | 0.08      | 0.04       | 0.0226    | 0.0131    | 5.08           |             |
| $\Omega_{bb}^{+} \to \Omega_{bb}^{+}$ | 0.0096 | $\ldots$ | $\ldots$  | $\ldots$ | $\ldots$ | $\ldots$          |             |
| $\Omega_{bb}^{0} \to \Omega_{bb}^{0}$ | 0.0130 | $\ldots$ | $\ldots$  | $\ldots$ | $\ldots$ | $\ldots$          |             |