Could the variation in quasar luminosity, due to extra dimension 3-brane in RS model, be measurable?

R. da Rocha
IFGW, Universidade Estadual de Campinas,
CP 6165, 13083-970 Campinas, SP, Brazil.

C. H. Coimbra-Araújo
Departamento de Astronomia, Universidade de São Paulo,
05508-900 São Paulo, SP, Brazil.

We propose an alternative theoretical approach showing how the existence of an extra dimension in RS model can estimate the correction in the Schwarzschild radius of black holes, and consequently its measurability in terms of the variation of quasar luminosity, which can be caused by a imprint of an extra dimension endowing the geometry of a brane-world scenario in an AdS

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bulk. This paper is intended to investigate the variation of luminosity due to accretion of gas in black holes (BHs) in the center of quasars, besides also investigating the variation of luminosity in supermassive BHs by brane-world effects, using RS model.

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I. INTRODUCTION

The possibility concerning the existence of extra dimensions is one of the most astonishing aspects on string theory and the formalism of \( p \)-branes. In spite of this possibility, extra dimensions still remain up to now unaccessible and obliterated to experiments. An alternative approach to the compactification of extra dimensions, provided by, e.g., Kaluza-Klein (KK) and string theories \[1,2,3,4\], involves an extra dimension which is not compactified, as pointed by, e.g., RS model \[5,6\]. This extra dimension implies deviations on Newton’s law of gravity at scales below about 0.1 mm, where objects may be indeed gravitating in more dimensions. The electromagnetic, weak and strong forces, as well as all the matter in the universe, would be trapped on a brane with three spatial dimensions, and only gravitons would be allowed to leave the surface and move into the full bulk, constituted by an AdS

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spacetime, as prescribed by, e.g., in RS model \[5,6\].

At low energies, gravity is localized on the brane and general relativity is recovered, but at high energies, significant changes are introduced in gravitational dynamics, forcing general relativity to break down to be overcome by a quantum gravity theory \[5,6\]. A plausible reason for the gravitational force appear to be so weak in relation to other forces can be its dilution in possibly existing extra dimensions related to a bulk, where \( p \)-branes \[1,2,3,4,5\] are embedded. \( p \)-branes are good candidates for brane-worlds \[6\] because they possess gauge symmetries \[2,3,4\] and automatically incorporate a quantum theory of gravity. The gauge symmetry arises from open strings, which can collide to form a closed string that can leak into the higher-dimensional bulk. The simplest excitation modes of these closed strings correspond precisely to gravitons. An alternative scenario can be achieved by Randall-Sundrum model (RS) \[5,6\], which induces a volcano barrier-shaped effective potential for gravitons around the brane \[10\]. The corresponding spectrum of gravitational perturbations has a massless bound state on the brane, and a continuum of bulk modes with suppressed couplings to brane fields. These bulk modes introduce small corrections at short distances, and the introduction of more compact dimensions does not affect the localization of matter fields. However, true localization takes place only for massless fields \[11\], and in the massive case the bound state becomes metastable, being able to leak into the extra space. This is shown to be exactly the case for astrophysical massive objects, where highly energetic stars and the process of gravitational collapse, which can originate black holes, leads to deviations from the 4D general relativity problem. There are other interesting and astonishing features concerning RS models, such as the AdS/CFT correspondence of a RS infinite AdS

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brane-world, without matter fields on the brane, and 4-dimensional general relativity coupled to conformal fields \[5,6,12\].

We precisely investigate the consequences of the deviation of a Schwarzschild-like term in a 5D spacetime metric, predicted by RS1 model in the correction of the Schwarzschild radius of a BH. We show that, for fixed effective extra dimension size, supermassive BHs (SMBHs) give the upper limit of variation in luminosity of quasars, and although the method used holds for any other kind of BH, such as mini-BHs and stellar-mass ones, we shall...
use SMBHs parameters, where the effects are seen to be more notorious. It is also analyzed how the quasar luminosity variation behaves as a function of the AdS$_5$ bulk radius $\ell$, for various values of BH masses, from $10$ to $10^6$ solar masses.

The search for observational evidence of higher-dimensional gravity is an important way to test the ideas that have been coming from string theory. This evidence could be observed in particle accelerators or gravitational wave detectors. The wave-form of gravitational waves produced by black holes, for example, could carry an observational signature of extra dimensions, because brane-world models introduce small corrections to the field equations at high energies. But the observation of gravitational waves faces severe limitations in the technological precision required for detection. This is a undeniable fact. Possibly, an easier manner of testing extra dimensions can be via the observation of signatures in the luminous spectrum of quasars and microquasars. This is the goal of this paper, which is the first of a series of papers we shall present. Here we show the possibility of detecting brane-world corrections for big quasars by their luminosity observation. In the next article we shall discuss the relationship between the electric part of Weyl tensor and KK modes in RS1 model, the deviation in Schwarzschild form and its corrections in 4-dimensional gravity wave modes. These nonlocal corrections cannot be determined purely from data on the brane. Indeed, mini-BHs are to be shown to be much more sensitive to brane-world effects. In the last article of this series we also present an alternative possibility to detect electromagnetic KK modes due to perturbations in black strings.

This article is organized as follows: in Section 2 after presenting Einstein equations in AdS$_5$ bulk and discussing the relationship between the electric part of Weyl tensor and KK modes in RS1 model, the deviation in Newton’s 4D gravitational potential is introduced in order to predict the deviation in Schwarzschild form and its consequences on the variation in quasar luminosity. For a static spherical metric on the brane the propagating effect of 5D gravity is shown to arise only in the fourth order expansion in terms of the Talor’s of the normal co-ordinate out of the brane. In Section 3 the variation in quasar luminosity is carefully investigated, by finding the correction in the Schwarzschild radius caused by brane-world effects. All results are illustrated by graphics and figures.

II. BLACK HOLES ON THE BRANE

In a brane-world scenario given by a 3-brane embedded in an AdS$_5$ bulk the Einstein field equations read

$$G_{\mu\nu} = -\frac{1}{2} \Lambda_5 g_{\mu\nu} + \frac{1}{4} \kappa^4 \left[ T T_{\mu\nu} - T_{\mu}^{\alpha} T_{\nu}^{\alpha} + \frac{1}{2} g_{\mu\nu} (T^2 - T_{\alpha\beta}^{\alpha\beta}) \right] - E_{\mu\nu},$$

where $T = T_{\mu}^{\alpha} T_{\nu}^{\alpha}$ denotes the trace of the momentum-energy tensor $T_{\mu\nu}$, $\Lambda_5$ denotes the 5-dimensional cosmological AdS$_5$ bulk constant, and $E_{\mu\nu}$ denotes the ‘electric’ components of the Weyl tensor, that can be expressed by means of the extrinsic curvature components $K_{\mu\nu} = -\frac{1}{2} E_{\alpha\beta} g_{\mu\nu}$

$$E_{\mu\nu} = E_{\alpha\gamma} K_{\mu\nu} + K_{\mu}^{\alpha} K_{\nu}^{\alpha} - \frac{1}{\ell^2} g_{\mu\nu}$$

(1)

where $\ell$ denotes the AdS$_5$ bulk radius. It corresponds equivalently to the effective size of the extra dimension probed by a 5D graviton.

The constant $\kappa_5 = 8\pi G_5$, where $G_5$ denotes the 5-dimensional Newton gravitational constant, that can be related to the 4-dimensional gravitational constant $G$ by $G_5 = G \ell_{\text{Planck}}$, where $\ell_{\text{Planck}} = \sqrt{\hbar c G}$ is the Planck length.

As indicated in [5, 12], “table-top tests of Newton’s law currently find no deviations down to the order of 0.1 mm”, so that $\ell \lesssim 0.1$ mm. Emparan et al [15] provides a more accurate magnitude limit improvement on the AdS$_5$ curvature $\ell$, by analyzing the existence of stellar-mass BHs on long time scales and of BH X-ray binaries. In this paper we relax the stringency $\ell \lesssim 0.01$ mm to the former table-top limit $\ell \lesssim 0.1$ mm.

The Weyl ‘electric’ term $E_{\mu\nu}$ carries an imprint of high-energy effects sourcing KK modes. It means that highly energetic stars and the process of gravitational collapse, and naturally BHs, lead to deviations from the 4-dimensional general relativity problem. This occurs basically because the gravitational collapse unavoidably produces energies high enough to make these corrections significant. From the brane-observer viewpoint, the KK corrections in $E_{\mu\nu}$ are nonlocal, since they incorporate 5-dimensional gravity wave modes. These nonlocal corrections cannot be determined purely from data on the brane. The component $E_{\mu\nu}$ also carries information about the collapse process of BHs. In the perturbative analysis of Randall-Sundrum (RS) positive tension 3-brane, KK modes consist of a continuous spectrum without any gap. It generates a correction in the gravitational potential $V(\rho) = \frac{G c}{c^2 r}$ to 4D gravity at low energies from extra-dimensional effects [12], which is given by

$$V(\rho) = \frac{Gc}{c^2 r} \left[ 1 + \frac{2k^2}{3r^2} + O \left( \frac{\ell}{r} \right)^4 \right].$$

(2)

The KK modes that generate this correction are responsible for a nonzero $E_{\mu\nu}$. This term carries the modification to the weak-field field equations, as we have already seen. The Gaussian coordinate $y$ denotes here the direction normal out of the brane into the AdS$_5$ bulk, in each point of the 3-brane.

The RS metric is in general expressed as

$$(5) \, ds^2 = e^{-2k|y|} g_{\mu\nu} dx^\mu dx^\nu + dy^2,$$

(3)

where $k^2 = 3/(2c^2)$, and the term $e^{-2k|y|}$ is called the warp factor [5, 6, 12], which reflects the confinement role.
AdS5 BULK

3-BRANE

FIG. 1: Schematic diagram of a slice of a 3-brane embedded in an AdS5 bulk. The Gaussian coordinate $y$ is normal to the brane and $x$ denotes spacetime coordinates in the brane.

of the bulk cosmological constant $\Lambda_5$, preventing gravity from leaking into the extra dimension at low energies \[6, 12\]. The term $|y|$ clearly provides the $\mathbb{Z}_2$ symmetry of the 3-brane at $y = 0$.

Concerning the anti-de Sitter (AdS$_5$) bulk, the cosmological constant can be written as $\Lambda_5 = -6/\ell^2$ and the brane is localized at $y = 0$, where the metric recovers the usual aspect. The contribution of the bulk on the brane can be shown only to be due to the Einstein tensor, and can be expressed as $\nabla \nu (E^\nu - S^\nu) = 0$, which implies that $\nabla \nu (E^\nu - S^\nu) = 0 \ [14]$, where

$$S_{\mu\nu} := \frac{1}{4} \kappa_5^4 \left[ TT_{\mu\nu} - T^\alpha_{\mu\nu} T_{\alpha\beta} + \frac{1}{2} g_{\mu\nu} (T^2 - T_{\alpha\beta} T^{\alpha\beta}) \right]$$

A vacuum on the brane, where $T_{\mu\nu} = 0$ outside a BH, implies that

$$\nabla \nu E^\nu = 0.$$  \hspace{1cm} (5)

Eqs. (5) are referred to the nonlocal conservation equations. Other useful equations for the BH case are

$$G_{\mu\nu} = \frac{1}{2} \Lambda_5 g_{\mu\nu} - E_{\mu\nu}, \quad R_{\mu\nu} = 0 = E^\alpha_{\mu\nu}.$$ \hspace{1cm} (6)

Therefore, a particular manner to express the vacuum field equations in the brane given by eq. (3) is $E_{\mu\nu} = -R_{\mu\nu}$, where the bulk cosmological constant is incorporated to the warp factor in the metric. One can use a Taylor expansion in order to probe properties of a static BH on the brane \[17\], and for a vacuum brane metric, we have, up to terms of order $O(y^5)$ on, the following:

$$g_{\mu\nu}(x, y) = g_{\mu\nu}(x, 0) - E_{\mu\nu}(x, 0)y^2 - \frac{2}{\ell} E_{\mu\nu}(x, 0) ||y||^3 + \frac{1}{12} \left[ \left( \frac{32}{\ell^2} \right) E_{\mu\nu} + 2R_{\mu\nu\alpha\beta} E^{\alpha\beta} + 6E^\alpha_{\mu} E^\alpha_{\nu} \right] y^4 \quad \text{for} \quad y \to 0,$$

where $\Box$ denotes the usual d’Alembertian. It shows in particular that the propagating effect of 5D gravity arises only at the fourth order of the expansion. For a static spherical metric on the brane given by

$$g_{\mu\nu}dx^\mu dx^\nu = -F(r)dt^2 + \frac{dr^2}{H(r)} + r^2 d\Omega^2,$$ \hspace{1cm} (7)

where $d\Omega^2$ denotes the spherical 3-volume element related to the geometry of the 3-brane, the projected electric component Weyl term on the brane is given by the expressions

$$E_{\theta\theta} = -1 + H \frac{r}{2} \left( \frac{F'}{F} + \frac{H'}{H} \right).$$ \hspace{1cm} (8)

Note that in eq. (4) the metric is led to the Schwarzschild one, if $F(r)$ equals $H(r)$. The exact determination of these radial functions remains an open problem in BH theory on the brane \[12, 18, 19, 20, 21, 22\].

These components allow one to evaluate the metric coefficients in eq. (4). The area of the 5D horizon is determined by $g_{\theta\theta}$. Defining $\psi(r)$ as the deviation from a Schwarzschild form for $H(r)$ \[12, 18, 19, 20, 21, 22\],

$$H(r) = 1 - \frac{2GM}{c^2 r} + \psi(r),$$ \hspace{1cm} (9)

where $M$ is constant, yields

$$g_{\theta\theta}(r, y) = r^2 - \psi' \left( 1 + \frac{2}{\ell} ||y|| \right) y^2 + \left[ \psi' + \frac{2}{\ell} (1 + \psi') (r \psi' - \psi') \right] \frac{y^4}{6r^2} + \cdots.$$ \hspace{1cm} (10)

It can be shown $\psi$ and its derivatives determine the change in the area of the horizon along the extra dimension \[12\]. For a large BH, with horizon scale $r \gg \ell$, it follows from eq. (2) that

$$\psi(r) \approx -\frac{4GM\ell^2}{3c^2 r^3}.$$ \hspace{1cm} (11)

III. VARIATION IN THE LUMINOSITY OF QUASARS AND ADS CURVATURE RADIUS

The observation of quasars (QSOs) in X-ray band can constrain the measure of the AdS$_5$ bulk curvature radius $\ell$, and indicate how the bulk is curled, from its geometrical and topological features. QSOs are astrophysical objects that can be found at large astronomical distances (redshifts $z > 1$). For a gedanken experiment involving a static BH being accreted, in a simple model, the accretion efficiency $\eta$ is given by

$$\eta = \frac{GM}{6c^2 R_{\text{brane}}},$$ \hspace{1cm} (12)
where $R_{\text{Sbrane}}$ is the Schwarzschild radius corrected for the case of brane-world effects. The luminosity $L$ due to accretion in a BH, that generates a quasar, is given by
\[ L(\ell) = \eta(\ell) \dot{M} c^2, \tag{13} \]
where $\dot{M}$ denotes the accretion rate and depends on some specific model of accretion.

In order to estimate $R_{\text{Sbrane}}$, fix $H(r) = 0$ in eq. (8), resulting in
\[ 1 - \frac{2GM}{c^2 R_{\text{Sbrane}}} - \frac{4GM\ell^2}{3c^2 R_{\text{Sbrane}}} = 0. \tag{14} \]
This equation can be rewritten as
\[ R_{\text{Sbrane}}^3 - \frac{2GM}{c^2} R_{\text{Sbrane}}^2 - \frac{4GM\ell^2}{3c^2} = 0. \tag{15} \]
Using Cardano’s formulae \[27\], it follows that
\[ R_{\text{Sbrane}} = (a + \sqrt{b})^{1/3} + (a - \sqrt{b})^{1/3} + \frac{2GM}{3c^2}, \tag{16} \]
where
\[ a = \frac{2GM}{3c^2} \left( \ell^2 + \frac{4G^2M^2}{9c^4} \right), \tag{17} \]
\[ b = \frac{4G^2M^2\ell^2}{9c^4} \left( \ell^2 + \frac{8G^2M^2}{9c^4} \right). \tag{18} \]

Writing $a$ and $b$ explicitly in terms of the Schwarzschild radius $R_S$ it follows from eqs. \[18\] that
\[ a = \frac{R_S}{3} \left( \ell^2 + \frac{R_S^2}{9} \right), \tag{19} \]
\[ b = \frac{R_S^2 \ell^2}{9} \left( \ell^2 + \frac{2R_S^2}{9} \right). \tag{20} \]

Now, substituting the values of $G$ and $c$ in the SI, and adopting $\ell \sim 0.1\text{ mm}$ and $M \sim 10^9M_\odot$ (where $M_\odot \approx 2\times10^{33}\text{ g}$) denotes solar mass, corresponding to the mass of a SMBH, it follows from eq. \[10\] that the correction in the Schwarzschild radius of a SMBH by brane-world effects is given by
\[ R_{\text{Sbrane}} - R_S \sim 100\text{ m}, \tag{21} \]
and since the Schwarzschild radius $R_S$ is defined as $\frac{2GM}{c^2} = 2.964444 \times 10^{12}\text{ m}$, the relative error concerning the brane-world corrections in the Schwarzschild radius of a SMBH is given by
\[ 1 - \frac{R_S}{R_{\text{Sbrane}}} \sim 10^{-10} \tag{22} \]

These calculations shows that there exists a correction in the Schwarzschild radius of a SMBH caused by brane-world effects, although it is negligible. This tiny correction can be explained by the fact the event horizon of the SMBH is $10^{15}$ times bigger than the AdS$_3$ bulk curvature radius $\ell$. As shall be seen in a sequel paper these corrections are shown to be outstandingly wide in the case of mini-BHs, wherein the event horizon can be a lot of magnitude orders smaller than $\ell$. As proved in \[28\], the solution above for $R_{\text{Sbrane}}$ can be also found in terms of the curvature radius $\ell$. It is then possible to find an expression for the luminosity $L$ in terms of the radius of curvature, regarding formula \[13\].

Here we shall adopt the model of the accretion rate given by a disc accretion, given by \[29\]. Having observational values for the luminosity $L$, it is possible to estimate a value for $\ell$, given a BH accretion model. For a typical supermassive BH of $10^9M_\odot$ in a massive quasar the accretion rate is given by
\[ \dot{M} \approx 2.1 \times 10^{16}\text{ kg s}^{-1}. \tag{23} \]

Supposing the quasar radiates in Eddington limit, given by (see, e.g., \[30\])
\[ L(\ell) = L_{\text{Edd}} = 1.263 \times 10^{45} \left( \frac{M}{10^9M_\odot} \right) \text{ erg s}^{-1}. \tag{24} \]
for a quasar with a supermassive BH of $10^9M_\odot$, the luminosity is given by $L \sim 10^{47}\text{ erg s}^{-1}$. From eqs. \[23\] and \[24\], the variation in quasar luminosity of a SMBH is given by
\[ \Delta L = \frac{GM}{6c^2} (R_{\text{Sbrane}}^{-1} - R_S^{-1}) \dot{M} c^2 = \frac{1}{12} \left( \frac{R_S}{R_{\text{Sbrane}}} - 1 \right) \dot{M} c^2. \tag{25} \]
For a typical SMBH eq. \[26\] reads
\[ \Delta L \sim 10^{28}\text{ erg s}^{-1}. \tag{26} \]

In terms of solar luminosity units $L_\odot = 3.9 \times 10^{33}\text{ erg s}^{-1}$ it follows that the variation of luminosity of a SMBH quasar due to the correction of the Schwarzschild radius in a brane-world scenario is given by
\[ \Delta L \sim 10^{-5} L_\odot. \tag{27} \]

Naturally, this small correction in the Schwarzschild radius of SMBHs given by eqs. \[24\] implies in a consequent correction in quasar luminosity via accretion mechanism. This correction has shown to be a hundred thousand weaker than the solar luminosity. In spite of the huge distance between quasars and us, it is probable these corrections can be never observed, although they indeed exist in a brane-world scenario. This correction is clearly regarded in the luminosity integrated in all wavelength. We look forward the detection of these corrections in particular selected wavelengths, since quasars also use to emit radiation in the soft/hard X-ray band.

In the graphics below we illustrate the variation of luminosity $\Delta L$ of quasars as a function of the SMBH mass and $\ell$, and also for a given BH mass, $\Delta L$ as is depicted as a function of $\ell$. 


FIG. 2: 3D graphic of $\frac{\Delta L}{Mc^2} \times \ell \times M$ where the SMBH mass $M$ varies from $10$ to $10^6 M_\odot$ and the radius $\ell$ of the AdS$_5$ bulk varies from $10^{-7}$ to $10^{-1}$ mm.

IV. CONCLUDING REMARKS AND OUTLOOKS

In the present model the variation of quasar luminosity is regarded as an extra dimension brane effect, and can be immediately estimated by eq. (25), involving the Schwarzschild radius calculated in a brane-world scenario, and the standard Schwarzschild radius of a BH in 3-brane. It is desirable also to calculate the variation of quasar luminosity in a Kerr and in a Reissner-Nordstrøm (RN) geometry, where while the latter is caused by an electrically neutral, rotating BH, the former is generated by a charged, static black-hole. It shall be done in a sequel paper, using a formula equivalent to eq. (9) but now concerning RN metric [17, 19, 22].

The 2-brane model by contrast, for suitable choice of the extra dimension length and of $\ell$ does predict tracks and/or signatures in LHC [12, 31]. Black holes shall be produced in particle collisions at energies possibly below the Planck scale. ADD brane-worlds [32, 33, 34] also provides a possibility to observe black hole production signatures in the next-generation colliders and cosmic ray detectors [35]. In the sequel article we will show that, since mini-BHs possess a Reissner-Nordstrøm-like effective
behavior under gravitational potential, they feel a 5D gravity and are more sensitive to extra dimension brane effects.

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In general the vector field cannot be globally defined on the brane, and it is only possible if the 3-brane is considered to be parallelizable.