FIELD THEORY DUALS FROM (NON)-CRITICAL TYPE 0 STRINGS

Dario Martelli†

SISSA/ISAS, Via Beirut 2-4 34014 Trieste and
INFN Sezione di Trieste, Italy
dmartell@sissa.it

Abstract We review some aspects of Polyakov’s proposal for constructing non-supersymmetric field theories from non-critical Type 0 string theory.

Getting some insight into nonsupersymmetric field theory via AdS/CFT approach is a difficult task to achieve. There are different proposals to address this issue, dealing with possible mechanisms to break supersymmetry. Polyakov in [1] has proposed considering a nonsupersymmetric string theory with non-chiral GSO projection, known as Type 0 [2, 3]. This proposal was elaborated further in a series of papers starting from [4]. In this short note we present the results obtained in [5, 6].

Type 0 string theories are purely bosonic strings with modular invariant partition function and four sectors whose low laying fields are here summarized

\[
\begin{array}{c|cccc}
  & (NS+, NS+)& (NS-, NS-) & (R+, R\mp) & (R-, R\pm) \\
\hline
0A & \Phi, B_{\mu\nu}, g_{\mu\nu} & T & A_\mu^{(1)} A_{\mu\rho}^{(1)} & A_\mu^{(2)} A_{\mu\rho}^{(2)} \\
0B & \Phi, B_{\mu\nu}, g_{\mu\nu} & T & A^{(1)} A_{\mu\nu}^{(1)} A_{\mu\rho\sigma}^{(+)} & A^{(2)} A_{\mu\nu}^{(2)} A_{\mu\rho\sigma}^{(-)} \\
\end{array}
\]

There is a corresponding doubled set of D-branes coupling to RR-fields.

†Preprint SISSA 110/99/EP.
†Talk presented at the NATO-ASI and TMR Summer School ‘Progress in String Theory and M-Theory’ Cargese, May 24 - June 5, 1999.
1The two entries in parenthesis refer respectively to left and right movers and the signs correspond to the choice of GSO projection. The upper (lower) signs in the RR sectors define Type 0A (0B) theory.
As pointed out in [1] there are no open string tachyons on the world-volume of these branes, while there is a perturbative closed string tachyon that renders the Minkowski vacuum unstable. Nevertheless one should regard this as an indication that ten dimensional flat background is not stable, while there should exist other vacua in which the theory makes sense. AdS space seems in this respect a good candidate in that it allows for tachyonic modes.

\( \mathcal{N} = (1, 1) \) supersymmetry on the world-sheet makes these theories similar in some respect to supersymmetric Type II. For example all tree level correlators of vertex operators of \((\text{NS}+, \text{NS}+)\) and \((\text{R}+, \text{R}+)\) fields are the same as Type II. Using these and other properties it is possible to derive an expression for the effective gravity action [4] that, split in the NSNS and RR contributions, reads:

\[
S_{\text{NSNS}} = \int d^{10}x \sqrt{-g} \left\{ e^{-2\Phi} \left( R - \frac{1}{12} |dB|^2 + 4|d\Phi|^2 - \frac{1}{2} |dT|^2 - V(T) \right) \right\}
\]

\[
S_{\text{RR}} = \int d^{10}x \sqrt{-g} \left\{ f(T) |F_{\mu
u}|^2 + \cdots + (\text{CS terms}) \right\}.
\]

The main novelties are coming from the tachyon couplings. In particular, there is a potential \( V(T) \) which is an even function of the tachyon field, as well as functions \( f(T) \) multiplying RR terms, that can be worked out perturbatively.

The original proposal in [1] was to consider a string theory in dimension \( d < 10 \). Even if a microscopic description of string theory out of criticality is still far, there are at least indications for a possible extension of Type 0 theories in non-critical dimensions:

- a diagonal partition function, whose modular invariance doesn’t rely on \( d = 10 \) (as opposed to Type II theories)
- the tachyon should condense, providing an effective central charge \( c_{\text{eff}} \sim V(\langle T \rangle) \). It doesn’t seem unnatural to shift \( c_{\text{eff}} \) by the central charge deficit \((10 - d)\)

\( c_{\text{eff}} \) provides a tree level cosmological constant in the low energy theory, and this agrees with the expectation that the inconsistencies possibly arise only in flat background.

We work at the level of the effective gravity action. In \( d < 10 \) one should guess the field content of the theory and write down the relative action. In [5] we assume the NSNS sector (gravity + tachyon) is always present and the RR sector is worked out on group theory grounds, considering tensor products of \( SO(d - 2) \) spinors. In five dimensions for
instance, $2 \times 2 = 1 + 3$, and one is led to include a scalar potential $A$ and a vector $A_\mu$. We further assume the existence of a 4-form potential to accommodate the would be D3-brane: This really amounts to consider massive gravity.

With these assumptions, the equations of motion following from the relative action have interesting solutions, whose interpretation may give some insight into their field theory duals, and eventually provide hints in favor of the consistency of either critical or even non-critical Type 0 string theory.

The relevant piece of the $d$-dimensional action, for the ansatz that we consider is

$$
S = \int d^d x \sqrt{-g} \left\{ R - \frac{1}{2} (\partial_M \Phi)^2 - \frac{1}{2} (\partial_M T)^2 - V(T) \sqrt{\frac{2}{d-2}} e^{\Phi} \right. \\
- \frac{1}{2 (p+2)!} f(T) e^{\frac{1}{2} \sqrt{\frac{2}{d-2}}(d-2p-4)\Phi} \left( F_{M_1 \cdots M_{p+2}} \right)^2 \left\},
$$

where $V(T) = -10 + d - \frac{d-2}{8} T^2 + \cdots$ is the tachyon potential, including central charge deficit.

Nonzero fields are the metric, constant dilaton ($\Phi_0$) and tachyon ($T_0$), and a RR ($p+2$)-form field strength:

$$
R_{\mu \nu \rho \lambda} = -\frac{1}{R_0^2} \left( g_{\mu \rho} g_{\nu \lambda} - g_{\mu \lambda} g_{\nu \rho} \right), \quad R_{ijkl} = +\frac{1}{L_0^2} \left( g_{ik} g_{jl} - g_{il} g_{jk} \right),
$$

$$
F_{\mu_1 \cdots \mu_{p+2}} = F_0 \sqrt{-g_{(p+2)}} \epsilon_{\mu_1 \cdots \mu_{p+2}}.
$$

With such an ansatz the equations of motion become a set of algebraic equations. The tachyon VEV is determined implicitly by the following equation

$$
\frac{f''(T_0)}{f(T_0)} = \frac{1}{2} (d - 2p - 4) \frac{V''(T_0)}{V(T_0)}.
$$

Now, without a precise knowledge of the functions $f(T)$ and $V(T)$ one cannot infer whether or not it admits solutions. One should really assume it has, and extract some information.

The remaining equations fix the value of the radii of the two maximally symmetric spaces and that of the dilaton. It turns out that such solution is a $AdS_{p+2} \times S^{d-p-2}$ space, with tachyon VEV, and fixed 't Hooft coupling $\lambda = e^{\Phi_0} N$.³

²Uppercase indices run from 1 to $d$, while Greek and Latin indices run from 1 to $p+2$ and from $p+3$ to $d$ respectively.

³$N$ is the number of branes, which has to be evaluated in the string frame.
Notice that string loop corrections are suppressed in the large N limit, while $\alpha'$-corrections are important because the curvature is $O(1)$.

By the AdS/CFT correspondence the field theory dual of this solution should be at a conformal point. However it is difficult to make contact with perturbative field theory because $\lambda \sim O(1)$. Nevertheless one can still get additional information applying the correspondence. It is in fact possible to count the number of degrees of freedom that should live in the theory dual to this background [6].

Consider a thermal deformation of the solution. Identifying the Hawking temperature with the finite temperature of the field theory one can compute the entropy. By either computing the free energy from the Euclidean action, or computing the area of the horizon, it turns out that it has the following behavior

$$S \sim N^2 V^p_T T_H^p$$

for any value of $p$. This is an indication that the dual field theory should have $N^2$ degrees of freedom, and could be YM theory in some non Gaussian limit. Notice the different scaling power in the analogous relation one gets from evaluating the entropies of black M2 and M5 branes (3/2 and 3 respectively).

We conclude pointing out that this kind of approach is potentially predictive. Consider in fact the formula for scaling dimensions of dual operators:\footnote{These results are obtained in the case $d=p+2$.}

$$\Delta = \frac{(d - 1) + \sqrt{(d - 1) + 4m^2R_0^2}}{2}$$

$$m^2R_0^2 = d(d - 1) \left( 1 + \frac{\tau}{2} \pm \frac{1}{2} \sqrt{\tau^2 + (2d - 4)\frac{V''(T_0)^2}{V(T_0)^2}} \right)$$

with

$$\tau = d \frac{V'(T_0)^2}{V(T_0)^2} - \frac{2 f''(T_0)}{d f'(T_0)} - \frac{V''(T_0)}{V(T_0)} - 1.$$
be able to predict an infinite tower of dimensions from KK analysis. Moreover, some control on the theory at this conformal point may shed light on Type 0 string theory and/or on non-critical string theory via AdS/CFT correspondence.

Further details concerning this short note can be found in [5, 6].

Acknowledgments

I wish to thank G. Ferretti for reading the manuscript and J. Kalkkinen for useful discussions. I also thank the organizers of the school for providing a nice environment in which to learn and discuss. This work was supported in part by European Union TMR program CT960045.

References

[1] A.M. Polyakov, “The wall of the cave,” Int. J. Mod. Phys. A14 (1999) 645 hep-th/9809057.
[2] L.J. Dixon and J.A. Harvey, “String Theories In Ten-Dimensions Without Space-Time Supersymmetry,” Nucl. Phys. B274, 93 (1986).
[3] N. Seiberg and E. Witten, “Spin Structures In String Theory,” Nucl. Phys. B276, 272 (1986).
[4] I.R. Klebanov and A.A. Tseytlin, “D-branes and dual gauge theories in type 0 strings,” Nucl. Phys. B546 (1999) 155 hep-th/9811035.
[5] G. Ferretti, J. Kalkkinen and D. Martelli, “Non-critical type 0 string theories and their field theory duals,” Nucl. Phys. B, to appear hep-th/9904013.
[6] G. Ferretti and D. Martelli, “On the construction of gauge theories from non critical type 0 strings,” Adv. Theor. Math. Phys. 3 (1999) 119 hep-th/9811208.