Correlation between Rail Vibration and Sound Radiation Using a Hammer Impact

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Abstract. When the track structure is subject to certain excitation, the vibration characteristics and sound radiation characteristics will show certain correlation with each other. However, the study on finding the direct relationship between rail vibration and radiated sound is very limited. In this paper, the correlation between the rail vibration and radiated sound pressure under a hammer impact was investigated with a full-scale track slab. The rail acceleration, rail surface sound pressure and radiated sound pressure nearby at the rail web and rail foot were measured and analyzed in frequency domain. The transfer function, coherence function were obtained to reveal the relevance between each pair of input and output signals. Finally, by choosing rail vibration acceleration and surface sound pressure as input excitation and choosing the surface sound pressure and the sound pressure in near field as output response successively, the transfer function and the transmission loss for each pair of input and output were analyzed and compared to reveal the signal transmission in different frequency ranges.

1. Introduction

As the railway traffic continuously grows in highly populated areas, the environmental vibration and noise induced by moving vehicles has aroused more and more focus [1, 2]. Usually the mitigation track structure [3, 4] and different vibration isolation mats [5, 6] were adopted to reduce the vehicle induced environmental vibration at the vibration source and the transmission path. Rafał Burdzik et al. [7] have conducted experimental test aiming at identifying the vibration characteristics of the railway infrastructure. Antonio D’Andrea et al.[8] have performed a series of vibration measurements at a test section of a railway interconnection in Turin-Milan HS Railway and a quantity of piezoelectric accelerometers were adopted to determine the influence of new hot mix asphalt layer on the vibration transmission. David Thompson et al. [9] have investigated the influence of stiffening the subgrade beneath the railway track on reducing the vibration transmission. Mingjing Fang et al.[10] had presented a model of a Portland cement concrete slab track bed, and a sub- track asphalt roadbed to analyze their responses to ground vibration attenuation under high-speed train loads by considering different thick sub-track asphalt layer replaced on the top of the upper subgrade. However, when the railway mitigation tracks were adopted more and more frequently, certain problem has emerged. It has
been discovered that after adopting the ballast mat, although the environmental vibration transmitted to the infrastructure beneath has been reduced, but the radiated sound near the wheel rail contacting zone has been increased significantly, which on the contrary will increase the overall noise radiation [11].

In the past, researchers have conducted theoretical and experimental studies in order to determine the sound radiation by the moving vehicles. Xiaodong Song et al. [12, 13] had studied the sound radiation of the single-span and multi-span bridge by combining the inverse boundary element method and in-site measured data. The model was further verified against the field measurements of a U-shaped girder bridge. Xun Zhang et al.[14, 15] had carried out a field measurement on a simply-supported box girder bridge to measure the accelerations of the slabs and the associated sound pressures induced by running trains and developed a simplified vehicle–track–bridge coupling vibration model to calculate the wheel–rail interaction force in a frequency range of 20–200Hz. From the study above, it can be seen that most of the research focused on the wheel-rail interaction radiated noise, and most regarding the rail to be simulated as an infinite beam and later worked as a line sound source in the simulation. However, the railway track structure are consisted of different solid components with complex profiles, thus each small area on the solid surface will give different contributions to the overall sound at certain environment point according to the infinitesimal element method [16–21]. Thus it is more reasonable to study the rail radiated sound through a solid element model. [16–19, 22–24] From the literature above, it can be seen that most researchers are focusing on verifying the numerical model with the near field sound pressure, and little research have been conducted on finding the direct relationship between the rail vibration and sound radiation.

In this paper, the hammer impact load was applied as the system excitation mainly due to the fact that the hammer impact load will provide a wide frequency band of excitation spectra and thus the system response can be regarded as a representation of the track structure characteristics of its own [25, 26]. A laboratory test was carried out for a full-scale track, and the acceleration, the surface sound pressure and the radiated sound pressure near the hammer location have been chosen as evaluation indexes. The responses for each index were analyzed in the frequency domain. Finally, by choosing rail vibration acceleration and surface sound pressure as input excitation and choosing the surface sound pressure and the sound pressure in near field as output response successively, the transfer function and the transmission loss for each pair of input and output were analyzed and compared to reveal the signal transmission in different frequency ranges.

2. Methodology

2.1 General information of the full-scale slab track

The full scale slab track is a typical CRTS II ballastless slab track [27, 28], which is consisted of the rail, the slab, the cement asphalt(CA) mortar and the hydraulically bonded layer(HBL). The sketch of the track slab and full scale model in the lab are shown in Figure 1.
Figure 1. Sketch of the track slab and the full-scale model in the lab.

2.2 Equipment and sensor locations

The data collector is type INV3062C1(L) manufactured by the China Orient Institute of Noise and Vibration. 8 channels of signal can be collected at the same time with the maximum sampling rate of 216kHz and 24 bit precision per channel. Three types of transducers were chosen for the aim of the research, namely are the accelerometers for the rails, surface microphone and 1/2 inch microphones. The technical specification for each type of the transducer is listed in Table. 1. The sampling rate is set to 10.24kHz for each channel.

| Transducer specification | Type       | Sensitivity | Technical parameter                   |
|--------------------------|------------|-------------|---------------------------------------|
| Accelerometer            | ICP        | 4.72~4.88mV/g | Frequency range: 1 to 15KHz            |
|                          |            |             | Dynamic range: 0~1000g                |
| 1/2 inch microphone      | ICP        | 44.4~49.7mV/Pa | Frequency range: 20~20kHz             |
|                          |            |             | Dynamic range: 20~146dB               |
| Surface microphone       | ICP        | 11.2 mV/Pa  | Frequency range: 5 to 50000Hz         |
|                          |            |             | Dynamic range: 55~160dB               |

In this test, all sensors are installed at the mid-span position between two adjacent sleepers along the lateral direction. In together, two accelerometers are installed on the rail web and rail foot. One surface microphone is installed first near the accelerometer at the rail web with 5 cases of hammer test and then installed near the accelerometer at the rail foot mainly because currently there is only one surface microphone ready to work with. Three 1/2 sound microphones are installed near the vibration position, among which two of them are fixed near the accelerometer at the rail web and rail foot, and one in the near field which keeps a distance of 0.35m from the rail web in the lateral position and in the same height as the rail neutral axis. The location of all the sensors are shown in Figure 2. The details of the fixed transducers are shown in Figure 3.

Figure 2. Sketch of the location of the transducer.
The hammer is directly forced on the rail head along the center line of the rail at the mid-span position between two sleepers. The hammer is able to excite 80kN to the maximum. In order to excite the high frequency response of the rail, the steel hammer head was selected in the test. The sensitivity of the hammer impact force sensor is 0.0639mV/N. All together 5 set of tests were performed, and in each test, the top of the rail was hit three times with a nearly even force with a time lag of 1~2 seconds in order to get a smooth transfer function. By taking the case when the surface microphone is installed at rail foot, the typical hammer forces in five test scenarios and its frequency distribution was shown in Figure 4. It can be seen that with the steel head, the hammer can generate a force with a frequency band from 10~4000Hz with an flat excitation force spectrum, with a nearly 10dB decrease at 4000Hz. Thus it is reasonable to only analyze the results in the following chapters focusing only on the frequency range from 10Hz~4000Hz. However, one thing should be mentioned that this doesn't account for that there are not frequency peaks over 4000Hz. On the contrary, although the excitation force is small comparing with the force below 4000Hz, there are also obvious peaks over 4000Hz, which means the impact load can easily excite the vibration mode with resonance frequency over 4000Hz.

![Figure 4. Transducers fixation in the test for both scenarios.](image)

**Figure 4.** Transducers fixation in the test for both scenarios.

![Figure 4. Hammer force in time and frequency domain.](image)

**Figure 4.** Hammer force in time and frequency domain.
In order to measure the real vibro-acoustics data, it is important to know the background noise before carrying out formal test. According to ISO 11204-2010 and ISO 1680-2013, when the background noise level is at least 10dB lower than the acoustic event, it can be regarded that the background noise can be neglected when the microphone is located at the same position for two measurements[29]. Thus before the formal measurement starts, a test measurement was carried out to obtain the sound level in the presence and absence of the hammer impact to see the difference. Figure 5 shows the comparison between the impact noise and the background noise and it can be seen for all the sound sensors the difference between the impact noise and background noise lies in the range from 34.13 to 62.46dB(A). Thus it can be concluded that the background noise can be neglected in the whole test.

![Comparison of sound level between impact noise and background noise.](image)

3. Results and Discussion
The data collector is type INV3062C1(L) manufactured by the China Orient Institute of Noise and Vibration. 8 channels of signal can be collected at the same time with the maximum sampling rate of 216kHz and 24 bit precision per channel. Three types of transducers were chosen for the aim of the research, namely are the accelerometers for the rails, surface microphone and 1/2 inch microphones. The technical specification for each type of the transducer is listed in Table. 1. The sampling rate is set to 10.24kHz for each channel.

The measured accelerations, surface sound pressure and sound pressure in the near field is first analyzed in the frequency domain through Fast Fourier Transformation(FFT) and 1/3 octave analysis according to ISO266-1997. Then the transfer function and coherence function between each pair of signals are obtained and analyzed further. The transfer function is mainly contributed to characterize the system energy transfer capability[30–32] and is defined by equation 1:

\[
H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{Y(\omega) \cdot X^*(\omega)}{X(\omega) \cdot X^*(\omega)}
\]

where \(X(\omega)\) and \(Y(\omega)\) are the FFT of the input and output signal respectively, \(X^*(\omega)\) is the conjugation of \(X(\omega)\).

The coherence function[33–35] is usually adopted to evaluate the extent of correlation between each pair of input and output signal. The coherence function is obtained by using equation 2:

\[
\gamma^2(\omega) = \frac{G_{yy}(\omega)}{G_{xx}(\omega) \cdot G_{yy}(\omega)}
\]

where \(G_{yy}(\omega)\) is the cross power spectral density function between the input and output signal, \(G_{xx}(\omega)\) and \(G_{yy}(\omega)\) are the self power spectral density function respectively. When the values of coherence function are equal to 1, it means that the relationship between the input and the output are totally correlated to each other without any noise contained. Because the noise in the entire system cannot be avoid, the value of coherence function is always smaller than 1. Usually a coherence
function with a value larger than 0.8 means the output signal has strong correlation with the input signal.

In each set of test, the measured data get good convergence in time domain and frequency domain, thus in the following chapter all the curves are combined into one curve using the linear average results at each discrete frequency to give a better illustration.

3.1 Rail acceleration and radiated sound in frequency domain

Figure 6 shows the rail acceleration in both frequency domain and 1/3 octave band. From the results it can be seen that the rail foot vertical acceleration and the rail web lateral acceleration have a different energy distribution in the frequency domain. For the vertical acceleration at rail foot, the main frequency components are at around 487Hz, 895.3Hz, 1535Hz, 2113~2356Hz, 2606Hz, and 2841Hz. Over 3000Hz, the energy get damped and peaks are comparatively smaller. For the lateral acceleration at rail web, the main frequency components are at around 1535Hz, 2574Hz and 3867Hz. According to [20], when the vibration propagation waves in the rail is higher than 3000 Hz (for example 5000, 50,000, and 80,000 Hz in the literature), the rail profile's local deformation becomes the major vibration mode, thus it can be regarded that the hammer impact have excited the local deformation of the rail web and thus there are certain frequency components over 3000Hz. However, for rail foot, the local deformation vibration mode is not excited by the hammer impact.

From the 1/3 octave analysis, it can be seen that at low frequency range from 10~125Hz, the rail web shows a predominant vibration amplitude larger than that at the rail foot, and the same trend can be observed at around 2000~3150Hz. In other frequency band, both curves show nearly the same frequency component distribution. Another thing can be discovered is that although the excitation is only applied on the rail head in the vertical direction, the lateral vibration of the rail web can also be excited.

Figure 6. Rail acceleration in frequency domain and 1/3 octave band.

Figure 7 shows the surface sound pressure in both frequency domain and 1/3 octave band. From the results it can be seen that the surface sound pressure at rail foot shows a predominant frequency distribution at 388Hz, 488Hz, 608Hz, 892Hz and 1556Hz. The energy in the range from 1556Hz to 2000Hz is comparatively smaller and has no obvious distribution above 3000Hz. The surface sound pressure at rail web shows a predominant frequency distribution at 1535Hz, 2045Hz and 2578Hz, and the amplitude is smaller than the predominant frequency component of that at the rail foot. According to [36, 37], the track has several resonant frequencies which correspond to the entire structural vibration mode below 3000Hz, such as the pinned-pinned frequency, so it can be concluded that the predominant frequency components of the surface sound pressure generated by the rail foot vibration mainly distributes in the frequency range from 388~1500Hz. On the other hand, the predominant frequency components of the surface sound pressure generated by the rail web vibration mainly distributes in the frequency range from 1535~2578Hz.

From the 1/3 octave analysis, it can be seen that at low frequency range from 10~1250Hz, the surface sound pressure generated by the rail foot vibration shows a predominant sound level with
4.9–20.7dB larger than that at the rail web, and at the frequency range over 1600Hz the surface sound pressure at rail web is larger than that at rail foot.

**Figure 7.** Surface sound pressure in frequency domain and 1/3 octave band.

Figure 8 shows the sound pressure near the accelerometer in both frequency domain and 1/3 octave band. It can be seen the sound pressure near the rail web shows a larger amplitude in almost all frequency range. The predominant frequency range of the sound pressure at rail web is 491Hz, 893Hz, 1623Hz, 2115Hz and above 3000Hz. For the sound pressure near rail foot, the predominant frequency range only distributes at around 488Hz and 891Hz. From the 1/3 octave analysis, it can be seen that the sound level near the rail web is 3–14dB larger at each central frequency than that near the rail foot. It can also be observed that the frequency domain curves for both measuring points shows a similar distribution. This means the influence of the surface sound pressures are becoming less predominant after the vibration wave propagated and transmitted into the air volume. In other words, the sound pressure near the solid surface can be regarded as an overall sum up of all the contributions from the infinitesimal sound source at the solid surface and get damped with nearly the same decay rate at each frequency band. In this way the sound pressure at both measuring points show a similar distribution in the frequency domain.

**Figure 8.** Sound pressure near accelerometer in frequency domain and 1/3 octave band.

Figure 9 shows the sound pressure at near field in both frequency domain and 1/3 octave band which is 35cm away from the rail web surface. From the results it can be discovered that there are several peaks in the frequency domain curve, which namely are at around 365Hz, 487Hz, 1539Hz, 2089Hz and 2288Hz. From the 1/3 octave analysis, it shows several peaks at the central frequency of 160Hz, 400Hz and 1600Hz. The maximum sound level over all central frequencies is 71.5dB.
3.2 Transfer function and coherence function analysis

Figure 10 shows the transfer function and coherence function between the rail acceleration and surface sound pressure in both frequency domain. From the results it can be seen that for the energy transmission between the rail acceleration and surface sound pressure, in the frequency range from 20–1447Hz the transfer function at rail foot shows a larger amplitude than that at rail web. In the frequency above, the rail web begins to show a better energy transmission than the rail foot. This means with the same acceleration level, the rail vertical vibration can induce the surface sound in low and mid-frequency more easily, and the rail web can induce the surface sound in higher frequency range more easily.

From the coherence function, it can be discovered below 77Hz, the rail acceleration and surface sound generation has no direct relevance with each other. It can also be seen that there are some decreasing points at around 50Hz, 100Hz, 300Hz and 440Hz, which means that there exists little relevance between each pair of input and output signal at around these frequencies.

Figure 11 shows the transfer function and coherence function between the surface sound pressure and sound pressure near the accelerometer in both frequency domain. It has clearly shown from the transfer function results that the rail web has a better energy transmission for converting the surface sound to radiated sound into the air nearby, and thus caused a higher amplitude in nearly all the frequency range in the transfer function curve.

From the coherence function, it can be discovered below 101Hz, the rail surface sound and the sound pressure caused in the air nearby has weak relevance with each other, especially for the rail foot. It can also be seen that there are some decreasing points at around 117Hz, 221Hz, 259Hz, 290Hz and 457Hz, which means that there exists little relevance between each pair of input and output signal at around these frequencies.
Figure 11. Transfer function and coherence function between sound pressure near accelerometer and at surface.

Figure 12 shows the transfer function and coherence function between the surface sound pressure and sound pressure at near field in both frequency domain. Because the output signal in this analysis is the same for both inputs, the transfer function here can also be regarded as an evaluation of the contribution from each measuring location. It can be seen from the transfer function results that the rail web generally have a larger amplitude in most frequency range, which represents that the rail web basically contributes more to the sound pressure than the rail foot to the point in the near field which has a certain distance from the rail surface.

From the coherence function, it can be discovered below 98Hz, the rail surface sound and the sound pressure in the nearby field has weak relevance with each other, especially for the rail foot. It can also be seen that there are some decreasing points at around 118Hz, 218Hz and 457Hz, which means that there exists little relevance between each pair of input and output signal at around these frequencies.

Figure 12. Transfer function and coherence function between sound pressure at near field and at surface.

3.3 Vibration-sound transmission loss analysis

The wave propagation and attenuation from the input excitation to the output response in each transmission path is identified by the transmission loss at each 1/3 octave band central frequency which is defined using equation 3:

\[
\text{Transmission Loss} = 20 \times \log \left( \frac{P_{\text{incident}}}{P_{\text{transmitted}}} \right) = 20 \times \log \left( P_{\text{incident}} \right) - 20 \times \log \left( P_{\text{transmitted}} \right)
\]

where the \( P_{\text{incident}} \) refers to the input excitation which namely are the rail vibration acceleration and the surface sound pressure successively; the \( P_{\text{transmitted}} \) refers to the output response which namely are the surface sound pressure and the sound pressure in near field successively. The transmission loss for
each pair of input and output were analyzed and compared to reveal the signal transmission in different frequency ranges.

Figure 13 shows the 1/3 octave band for each measuring point both at the rail foot and rail web. Here, in order to see more clearly the vibration transmission loss, the vibration accelerations have been converted with the same reference which equals 0.00002m/s² as the sound pressure. From the comparison, it can be found that for the rail foot scenario, the rail acceleration and surface sound pressure are nearly the same below 160Hz. Then the rail surface sound pressure shows a magnitude 8dB~11dB larger than rail acceleration, and above 1600Hz, the surface sound pressure starts to decrease. For both the sound pressure measured at near the accelerometer and in the near field, the 1/3 octave band curve are nearly the same, with a slight difference less than 8dB. For the rail web scenario, the rail acceleration only shows a similar distribution in the frequency range from 400~2000Hz, and in other frequency range the rail web acceleration is larger than the surface sound pressure. In the frequency range from 10~400Hz, the surface sound pressure is nearly the same as the sound pressure in near field measuring points, but the surface sound pressure shows larger amplitudes than the field measuring points, which indicates that the high frequency noise are getting damped through transmission in the air.

Figure 14 shows the transmission loss between each pair of input-output signals at the rail web and at rail foot respectively. By comparing that with the transmission loss between each pair of input and output signals, it can be discovered for the rail foot, the vibration energy loss occurs during the procedure when the surface sound pressure is propagating to the air, and the transmission loss is nearly 14~38dB at different central frequencies. As for the rail web, the vibration energy loss occurs during the procedure when the rail web acceleration induced the surface sound pressure, and the transmission loss is from 5.6~29.8dB at different central frequencies below 400Hz.
4. Conclusions and Future Developments

In this paper, a hammer test was conducted for a full-scale track to reveal vibration propagation and transmission from the rail to the air. The acceleration, surface sound pressure and sound pressure nearby have been measured and analyzed in the frequency domain. By choosing rail vibration acceleration and surface sound pressure as input excitation and choosing the surface sound pressure and the sound pressure in near field as output response successively, the transfer function and the transmission loss for each pair of signals were analyzed and compared to reveal the signal transmission in different frequency ranges. The following conclusions can be drawn from this study.

When the excitation is applied on the rail head in the vertical direction, the lateral vibration of the rail web can also be excited and have a predominant energy distribution above 3000Hz. The rail foot only shows a predominant energy distribution below 3000Hz. The predominant frequency of the surface sound pressure generated by the rail foot vibration mainly distributes in 500–1500Hz. The predominant frequency of the surface sound pressure generated by the rail web vibration mainly distributes in 1500–2500Hz.

The influence of the surface sound pressures becomes less predominant after the vibration wave propagated and transmitted into the air. With the same acceleration level at the rail foot and rail web, the rail vertical vibration can induce the surface sound in low and mid-frequency below 1400Hz more easily, and the rail web can induce the surface sound in frequency higher than 1400Hz more easily. The rail web has a better transmission efficiency for converting the surface sound to radiated sound into the air nearby than the rail foot, and contributes more to the sound pressure than the rail foot to the field point.

For the rail foot, the vibration transmission loss occurs during the procedure when the surface sound pressure is propagating to the air, for the rail web, the vibration transmission loss occurs during the procedure when the rail web acceleration induced the surface sound pressure.

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Acknowledgments
The authors acknowledge the Ministry of Science and Technology of the People’s Republic of China [grant number 2017YFB1201104]; the Fundamental Research Funds for the Central University [grant number 2018JBM042]; the National Natural Science Foundation of China [grant numbers 51708021] and China Railway Corporation [grant number 2016G009-B].