SOME ASPECTS OF
DEEPLY VIRTUAL COMPTON SCATTERING

by

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We consider different aspects of the virtual Compton amplitude in QCD on two examples: small-x physics accessible in the Regge regime and twist-3 approximation in the description of DVCS through the general parton distributions. Using this model, we give an estimate for the cross section of deeply virtual Compton scattering for the kinematics of CEBAF at Jefferson Lab.
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CHAPTER I
INTRODUCTION

One of the inexhaustible studies in physics is the study of hadron structure. It is believed that hadrons and nuclei are built from quarks and gluons, but it is still under investigation how those building blocks reveal themselves in nuclear reactions and how we could get information about those blocks from modern experiments.

Theory offers Quantum Chromodynamics (QCD) as the gauge theory of strong interactions describing the dynamics between colored quarks and gluons. According to QCD, the interaction between the quarks becomes weak at very short distances. This phenomenon, known as asymptotic freedom, allows us to use perturbation theory to describe high energy strong interaction processes. At low energy, quarks and gluons are confined into colorless mesons and baryons which are the real world particles. The QCD coupling constant grows at these energies plus the chiral symmetry of QCD is spontaneously broken by almost massless quarks. It brings us to the necessity of using non-perturbative methods at this regime.

In search of ways describing the complexity of hadrons and hadronic systems one should perform precise low-energy experiments to gain information about the dynamics inside the hadron. However, one should also look at the specific high energy kinematical regimes where the factorization theorems have been proven. Those experiments could reveal some new non-perturbative hadron structure information with the help of an accurate perturbative QCD description of the reaction dynamics [6–12].

The experiments performed with electromagnetic probes (Compton scattering) are the best ones. In Compton scattering, a real or virtual photon, emitted by a lepton, interacts with the nucleon with initial momentum $p$ and as a result a real photon is emitted. Due to the new generation of electron accelerators and high precision, large acceptance detectors, very high precision Compton scattering experiments have become a reality. Despite the small cross sections of the process, it serves as a clean probe of hadron structure.

It is convenient to examine this process in a parton model [13]. In this model, the nucleon consists of (a) three valence quarks, (b) quark-antiquark sea, and (c)
gluons, the carriers of quark interactions. The sea and gluons is created and dissipated through time and is expected to have almost no influence on the process’ outcome. The leading role in the nucleon belongs to the valence quarks. The typical time between interactions should be $1/\Lambda$ because $\Lambda \approx 200\text{MeV}$ is the only genuine scale in light quark QCD. The virtual photon, emitted by a lepton and carrying the momentum $q$ ($q^2 = -Q^2$), has a lifetime $1/Q$, which varies inversely with the momentum transferred by the photon. The momentum $Q$ defines the virtuality of the photon. The higher virtuality, the shorter is a photon’s life. At some point one may view the photon as being absorbed instantaneously by one of the quarks in the hadron.

Suppose the quark that absorbs the photon has longitudinal momentum $r$. Upon absorbing the virtual photon, the struck quark becomes highly virtual with a lifetime $r/Q^2$. Since this time is much shorter than the normal interaction time between quarks in the proton, the struck quark must reemit the photon before any interactions with the other quarks and gluons in the proton take place.

Finally, since the transverse momentum of the absorbed photon is $|q| = Q$ the photon must be absorbed and re-emitted over a transverse coordinate region having $|\Delta x| \approx 1/Q$. The quark which absorbs the virtual photon is point-like (bare) down to a transverse size $|\Delta x| \approx 1/Q$.

Thus, the scattering by the virtual photon takes place essentially instantaneously and over a very small, almost point-like, spatial region. Since the photon interacts only with a single quark and the transverse momenta $r_\perp$ of partons inside, the moving proton is assumed to be independent of photon virtuality $Q^2$. It is expected that the amplitude of the process should be given in terms of the number density of quarks in the proton times the scattering amplitude of an individual quark. Charge and energy conservation brings us to the following sum rules for the parton distributions $n_i(x)$ inside the proton

$$1 = \sum_i e_i \int_0^1 dx \left( n_i(x) - n^\perp_i(x) \right), \quad 1 = \sum_i \int_0^1 dx \, x \, n_i(x), \quad (1)$$

while the cross-section $\sigma_L$ for the longitudinally polarized virtual photon is zero

$$\sigma_L = 0, \quad (2)$$

and the cross-section $\sigma_T$ for the transversely polarized photon is expressed in the impulse approximation as a sum of photon-quark cross-sections averaged over the
distributions of quarks $n_i(x)$ and anti-quarks $n_i^c(x)$ in the proton:

$$\sigma_T = \sum_i e_i^2 \frac{4\pi^2\alpha}{Q^2} x (n_i(x) + n_i^c(x)), \quad (3)$$

where $e_i$ is the $i$-quark charge measured in the units of the electron charge $e$ and $\alpha = e^2/4\pi$.

FIG. 1: The diagram for deep inelastic scattering as an imaginary part of the virtual Compton amplitude.

Most of the internal structure of the nucleon has been revealed during the last few decades through Deep Inelastic Scattering (DIS). DIS is a type of inclusive scattering of high energy leptons on the nucleon in the Bjorken regime, which means that the photon virtuality $Q^2$ is very large, $x_B = Q^2/2p\cdot q$ is finite, and the total center-mass energy of the photon-nucleon system $s = (p + q)^2$ (one of the Mandelstam variables) is above the resonance region.

The range of $x_B$

$$0 \leq x_B \leq 1$$

is given by the fact that the invariant mass of the unobserved final hadronic state is larger than the nucleon mass $M$

$$(p + q)^2 = q^2 + 2p\cdot q + M^2 \geq M^2$$

and for the elastic scattering $x_B = 1$.

In the process of deep inelastic scattering, the photon emitted by the lepton and absorbed by the proton creates an infinite number of possible final states. Summing up all those possible outcomes gives the intermediate state of elastic lepton-hadron scattering. This means that deep inelastic scattering could be described using the optical theorem by the imaginary part of forward Compton scattering.
Elastic scattering has the same initial and final momentum for both of the participating particles. The amplitude of this process would be dominated by short distance interactions in the Bjorken regime defined above.

The leading contribution comes from so-called handbag diagram. The factorization theorem states that in the large-$Q^2$ limit the perturbatively calculable hard quark propagators completely factorize from parton distribution functions $n_i(x)$ ($i = u, d, s, \ldots$), which describe non-perturbative long-distance information about hadronic structure.

It was this particular process to which the perturbative quantum chromodynamics (QCD) was traditionally and successfully applied and where the approximate scaling behavior of the structure functions (independence of hadronic structure functions from the virtuality of the photon) was discovered. Unpolarized DIS experiments have mapped out the quark and gluon distributions in the nucleon, while polarized DIS experiments have shown that quarks carry only a small fraction of the nucleon spin. As a result, new investigations to understand the nucleon spin, both experimental and theoretical, became necessary.

Yet, even more complex and intriguing information could be extracted from Deeply Virtual Compton Scattering (DVCS) [1, 3]. As well as in DIS, the virtual photon carries large negative momentum $q^2 = -Q^2$, but now the final momentum of the particles differs from its initial values. One should make sure that the momentum transfer $t$ is as small as possible [1,2]. Again, the hard short-distance part factorizes from the non-perturbative long-distance part, general parton distributions (GPD), which now would be much more complex.

\[ \gamma(\gamma) \rightarrow e^+ e^- \]

\[ k, k' \]

\[ q, q' \]

\[ p, p' \]

\[ q = \gamma^\mu \]

\[ \gamma = \gamma^\nu \]

\[ FIG. 2: \ The \ diagram \ for \ virtual \ Compton \ scattering. \]

In addition to usual variable $x$, the fraction of the nucleon momentum carried by
the photon absorbing quark, which runs from 0 to 1 for the quark and from -1 to 0 for antiquark, depends on two other variables. One of them is the momentum transfer \( t = (q - q')^2 = (p' - p)^2 \) which is an independent Mandelstam variable, that should be small and negative. The other one is the so-called skewedness \( \xi \), defined as a fraction of the average nucleon momentum carried by the overall momentum transfer in the process. These two extra degrees of freedom, \( t \) and \( \xi \), make the dynamics of deeply virtual Compton scattering rich and diverse \([14-16]\).

\[ \begin{align*}
q + \frac{\Delta}{2} & \\
\Delta & \\
q - \frac{\Delta}{2} & \\
(\xi + x)p & \\
(\xi - x)p & \\
P - \frac{\Delta}{2} & \\
P + \frac{\Delta}{2} & \\
\end{align*} \]

FIG. 3: Parton picture for DVCS. Here \( P = (p + p')/2 \) is an average nucleon momenta, \( \Delta = (p' - p) \) is the overall momentum transfer.

Let us look at the handbag diagram of deeply virtual Compton scattering. From this figure it is easy to see that, depending on values of \( x \) and \( \xi \), the non-forward parton distribution functions can represent either the correlation between two quarks (when \( x > \xi \)), two antiquarks (\( x < -\xi \)), or between a quark and antiquark (\( -\xi < x < \xi \)). In the last case, GPDs behave like a meson distribution and uncover completely new information about nucleon structure which is inaccessible in DIS (which corresponds to the limit \( \xi \to 0 \)). That is why DVCS is so compelling and intriguing for physicists nowadays.

There is only one disadvantage of measuring the virtual Compton scattering amplitude, namely that the final photon can be emitted not only by the proton, the process we are most interested in, but also by an electron, which is referred to as the Bethe-Heitler (BH) process. This process is well known and it can be calculated exactly in quantum electrodynamics. The only thing which must be known is the elastic form factors of the nucleon. Unfortunately, light particles such as electrons radiate much more than the heavy proton. Therefore the BH process generally dominates the DVCS amplitude, especially at small \( t \).
FIG. 4: a) the DVCS part of the amplitude; b), c) the Bethe-Heitler part.

One way to minimize this problem is to find the kinematical regions where the BH process is suppressed (or at least is comparable with the DVCS amplitude). One of those cases will be discussed in Chapter 2.

Another way is to exploit the interference between the DVCS and the calculable Bethe-Heitler process, by independently measuring both the real and imaginary parts of the amplitude. This approach requires the theoretical models which will allow one to calculate the interference term with high precision and better understanding. In Chapter 3, the leading (twist-2) approximation for the DVCS process will be discussed as well as the influence of the next to leading (twist-3) approximation on the final result.

There are other promising non-forward hard exclusive processes such as longitudinal electroproduction of vector and pseudoscalar mesons at large $Q^2$, but they are not discussed here.
CHAPTER II

DVCS AT SMALL-X

II.1 INTRODUCTION

In this chapter, the specific kinematical regime for the DVCS is considered when the energy of an incoming virtual photon \( E_e \) is very large \( p \cdot q \to \infty \) in comparison to its virtuality \( Q^2 \to \infty \), while the Bjorken variable \( x_B = Q^2/2ME_e \) is finite and small. It is the so called small-\( x \) region. To be specific, we calculate the DVCS amplitude in the region

\[
s \gg Q^2 \gg -t \gg M^2,
\]

where \( s = (p+q)^2 \approx 2ME_e \) is the Mandelstam variable corresponding to the center-of-mass energy squared of the photon-hadron system, \( M \) is the nucleon mass, \( t = (q - q')^2 \) is another independent Mandelstam variable equal to the momentum transfer. The mass of the lepton is neglected.

The first study of the small-\( x \) DVCS was undertaken in Ref. [5]. The DVCS in this region is a semihard process in which quark ladders are dominated by gluon ladders well-known as the Balitsky-Fadin-Kuraev-Lipatov (BFKL) pomeron. It turns out that at large momentum transfer the coupling of the BFKL pomeron to the nucleon is essentially equal to the Dirac form factor of the nucleon \( F_1(t) \). Thus, the DVCS amplitude in this region (4) can be calculated without any model assumptions. Since there are only model predictions for the small-\( x \) DVCS in the current literature [19], even the approximate calculations of the cross section in QCD are very timely. The results obtained in this region can be used for the estimates of the amplitude at experimentally accessible energies where one or more conditions in Eq. (4) are relaxed.

To proceed further, at high energies it is convenient to use the Sudakov variables. Let us define

\[
q' = p_1, \\
p' = p_2 + \frac{M^2}{s}p_1,
\]

where \((p_1, p_2)\) is the light-cone basis of the final particles momenta plane

\[
p_1^2 = p_2^2 = 0, \quad 2p_1p_2 = s \to \infty.
\]

One advantage of these coordinates is their simple scaling properties when one takes the high energy limit.
All the other momenta are introduced in this basis as \( k = \alpha k_p + \beta k_p + k_\perp \) with \( p_1 k_\perp = p_2 k_\perp = 0 \) by definition. For example:

\[
r = \alpha_r p_1 + \beta_r p_2 + r_\perp.
\]

From \( p^2 = (p' - r)^2 = M^2 \) and \( q^2 = (q' + r)^2 = -Q^2 \) one can estimate \( \alpha_r \) and \( \beta_r \) to be:

\[
\begin{align*}
\alpha_r &\simeq \frac{r_\perp^2}{s}, \\
\beta_r &\simeq -\frac{(Q^2 + r_\perp^2)}{s}.
\end{align*}
\]

II.2 AMPLITUDE FACTORIZATION

The amplitude of deeply virtual Compton scattering is determined by contracting the nucleon matrix element [20] of the T-product of two electromagnetic currents with the photon polarization vectors

\[
H = -i\epsilon_\mu (q)\epsilon_\mu^*(q') \int d^4z e^{-iqz} \langle p'| T\{j_{e.m.}^\mu (z) j_{e.m.}^\nu(0)\} | p\rangle.
\]

It is known (see for example the review [22]) that in the leading order in perturbation theory the amplitude at high energy is purely imaginary up to the \( \frac{Q^2}{s} \) corrections. At high orders in perturbation theory the amplitude will be purely imaginary in the leading logarithmic approximation (LLA) and one will restore the real part using the dispersion relations.

Hence, we will first calculate the imaginary part \( \Im \) of the amplitude \( H \)

\[
V = \frac{1}{\pi} \Im H.
\]

The typical diagram for the DVCS amplitude in the lowest order in perturbation theory is shown in Fig. 5 (recall that the diagrams with gluon exchanges dominate at high energies). We are primarily looking for the imaginary part of the amplitude. At high energy the amplitude for the colorless particle scattering is described by the Feynman diagrams containing only two intermediate gluons with momenta \( k \) and \( r + k \) in the \( t \)-channel (Fig. 6). Simple estimations show that with good accuracy we can neglect the longitudinal momenta in their propagators:

\[
k^2 \simeq k_\perp^2, \quad (r + k)^2 \simeq (r + k)_\perp^2.
\]

Let us consider the integral over gluon momentum \( d^4k = d^2k_\perp \frac{d\alpha_k d\beta_k}{2s} \):

\[
V = \frac{2}{\pi} \int \frac{d^4k}{16\pi^4} \frac{1}{k^2 (r + k)^2} \Im \Phi_{ab}^{\mu\nu}(k + r, -k) \Im \Phi^{\mu\nu}_{Nab}(-k - r, k),
\]

\[
\Phi_{ab}^{\mu\nu}(k + r, -k) = \frac{1}{N} \int d^4z e^{-i(k + r)z} \langle p'| T\{j_{e.m.}^\mu (z) j_{e.m.}^\nu(0)\} | p\rangle.
\]

\[
\Phi^{\mu\nu}_{Nab}(-k - r, k) = \frac{1}{N} \int d^4z e^{-i(-k - r)z} \langle p'| T\{j_{e.m.}^\mu (z) j_{e.m.}^\nu(0)\} | p\rangle.
\]

\[
\Phi^{\mu\nu}_{Nab}(-k - r, k) = \frac{1}{N} \int d^4z e^{-i(-k - r)z} \langle p'| T\{j_{e.m.}^\mu (z) j_{e.m.}^\nu(0)\} | p\rangle.
\]
where $\Phi^{ab}_{\mu\nu}(k+r,-k)$ and $(\Phi_N)^{ab}_{\mu\nu}(-k-r,k)$ are the upper and the lower blocks of the diagram with two gluon exchanges in Fig. 6 (where $a, b$ and $\mu, \nu$ are the color and Lorentz indices, respectively) corresponding to the impact-factor representation.

Due to our definition of the light-cone basis, all of the $\alpha$-s in the upper block could be neglected as well as all of the $\beta$-s in the lower block. At high energies, the metric tensor in the numerator of the Feynman-gauge gluon propagator reduces to $g^{\mu\nu} \to \frac{2}{5}p'^2k'^2$ so the integral (11) for the imaginary part factorizes into a product of two "impact factors" integrated with two-dimensional propagators

$$V = \frac{2s}{4\pi}g^4 \left( \sum e_q^2 \right) \int \frac{d^2k_{\perp}}{4\pi^2} \frac{1}{k_{\perp}^2} \frac{1}{(r+k)_{\perp}^2} I(k_{\perp},r_{\perp})I_N(k_{\perp},r_{\perp}),$$  \quad (12)
where
\[
I(k_{\perp}, r_{\perp}) = \frac{1}{2s} p_2^\mu p_2^\nu \int \frac{d\beta}{2\pi} \frac{d^4 p}{i(2\pi)^4} \Phi_{\mu\nu}^{aa}(k + r, -k) \bigg|_{\alpha_k = 0},
\]
\[
I_N(k_{\perp}, r_{\perp}) = \frac{1}{2s} p_1^\mu p_1^\nu \int \frac{d\alpha}{2\pi} \frac{d^4 p}{i(2\pi)^4} \Phi_{\mu\nu}^{aa}(-k - r, k) \bigg|_{\beta_k = 0},
\]
and \(\sum c_i^2\) is the sum of squared charges of active flavors (u, d, s, and possibly c).

II.3 PHOTON IMPACT FACTOR

The photon impact factor is given by the two one-loop diagrams shown in Fig. 7.

The first diagram yields:
\[
- \frac{2}{s} \int \frac{d\beta_k}{2\pi i} \int \frac{d^4 p}{i(2\pi)^4} \frac{Tr[\hat{\epsilon}\hat{p}_2 \hat{p}_2 (\hat{p} + \hat{k}) \hat{p}_2 (\hat{p} - \hat{r}) \hat{\epsilon}' (\hat{p} - \hat{q})]}{(p^2 + i\varepsilon)((p - r)^2 + i\varepsilon)} \times
2\pi i\delta((q - p)^2)\theta(q_0 - p_0)2\pi i\delta((p + k)^2)\theta(p_0 + k_0). \quad (15)
\]

The second diagram differs only by trace and quark propagator:
\[
- \frac{2}{s} \int \frac{d\beta_k}{2\pi i} \int \frac{d^4 p}{i(2\pi)^4} \frac{Tr[\hat{\epsilon}\hat{p}_2 \hat{p}_2 (\hat{p} + \hat{k}) \hat{p}_2 (\hat{p} + \hat{k} + \hat{r} - \hat{q}) \hat{p}_2 (\hat{p} - \hat{q})]}{(p^2 + i\varepsilon)((p + k + r - q)^2 + i\varepsilon)} \times
2\pi i\delta((q - p)^2)\theta(q_0 - p_0)2\pi i\delta((p + k)^2)\theta(p_0 + k_0). \quad (16)
\]

In these formulae, the following notation was used: \(\hat{p} = \gamma^\mu p_\mu\), \(\delta((q - p)^2)\) is a Dirac delta-function and \(\theta(q_0 - p_0)\) is a Heaviside step function. The singularities in the physical region were replaced using the Cutkosky rule [23].

In a physical region the singularity is related to external 4-momenta. It is true for the normal thresholds. Anomalous thresholds do not show up as main singularities.
in the physical region if the external particles are stable, which means that the 
4-momenta squared for each vertex is less than the smallest value of the normal 
threshold in the given channel.

S. Coleman and R.E. Norton [24] found the following simple interpretation. In the 
physical region the main singularity of the diagram occurs only if all the vertices could 
be considered as some space-time points while the internal lines could be regarded 
as the trajectories of the real relativistic particles on mass shell.

Due to the δ-function all the particles are on the mass shell

$$\beta \to \frac{q^2}{s} + \frac{p_\perp^2}{\bar{\alpha}s}, \quad \beta_k \to -\frac{q^2}{s} - \frac{p_\perp^2 + \bar{\alpha}(k_\perp^2 + 2p_\perp k_\perp)}{\alpha \bar{\alpha}s},$$

and their energy is positive ($\theta(q - p) \to \theta(q - p, p_2) \to \theta(\bar{\alpha}\frac{s}{2})$ and $\theta(p + k) \to \theta(p + k, p_2) \to \theta(\alpha\frac{s}{2})$). Here we introduced the notation $\bar{\alpha} \equiv 1 - \alpha$.

The trace in the integral (16) contains $k$-dependence:

$$\frac{-1}{s^2} Tr[\hat{\epsilon} \hat{p}_2(\hat{p} + \hat{k}) \hat{\epsilon}'(\hat{p} + \hat{k} + \hat{r} - \hat{q}) \hat{p}_2(\hat{p} - \hat{q})] =$$

$$-\left(\epsilon, \epsilon'\right) 2(p_\perp^2 + R_\perp p_\perp)$$
$$-\left(\epsilon, r_\perp\right)(\epsilon', \frac{p_\perp^2}{s}) 4p_\perp^2$$
$$+(\epsilon, r_\perp)(\epsilon', p_\perp) 2$$
$$+(\epsilon, \frac{p_\perp^2}{s})(\epsilon', p_\perp) 4((1 - 2\alpha\bar{\alpha})p_\perp^2 + R_\perp p_\perp)$$
$$-(\epsilon, \frac{p_\perp^2}{s})(\epsilon', \frac{p_\perp^2}{s}) 8p_\perp^2(p_\perp^2 - r_\perp p_\perp)$$
$$-(\epsilon, \frac{p_\perp^2}{s})(\epsilon', p_\perp) 4p_\perp^2(1 - 2\alpha)$$
$$-(\epsilon, \frac{p_\perp^2}{s})(\epsilon', R_\perp) 4p_\perp^2$$
$$-(\epsilon, p_\perp)(\epsilon', p_\perp) 4\alpha\bar{\alpha}(1 - 2\alpha)$$
$$-(\epsilon, p_\perp)(\epsilon', \frac{p_\perp^2}{s}) 4((1 - 2\alpha)p_\perp^2 + 2R_\perp p_\perp)$$
$$+(\epsilon, p_\perp)(\epsilon', p_\perp) 8\alpha\bar{\alpha}$$
$$-(\epsilon, p_\perp)(\epsilon', R_\perp) 2(1 - 2\alpha)$$
$$-(\epsilon, p_\perp)(\epsilon', p_\perp) 8\alpha^2\bar{\alpha}^2$$
$$+(\epsilon, p_\perp)(\epsilon', \frac{p_\perp^2}{s}) 4((1 - 2\alpha\bar{\alpha})p_\perp^2 + (1 - 2\alpha)R_\perp p_\perp)$$
$$-(\epsilon, p_\perp)(\epsilon', p_\perp) 4\alpha\bar{\alpha}(1 - 2\alpha)$$
$$-(\epsilon, p_\perp)(\epsilon', R_\perp) 4\alpha\bar{\alpha}$$
$$+(\epsilon, R_\perp + \bar{\alpha}r_\perp)(\epsilon', \frac{p_\perp^2}{s}) 4R_\perp p_\perp$$
\[ + (\epsilon, p_{\perp}) (\epsilon', R_{\perp} + \bar{\alpha} r_{\perp}) 4\alpha (1 - 2\alpha) \]
\[ + (\epsilon, p_1) (\epsilon', R_{\perp} + \bar{\alpha} r_{\perp}) 8\alpha^2 \bar{\alpha} \]
\[ + (\epsilon, \frac{p_2}{s}) (\epsilon', R_{\perp} + \bar{\alpha} r_{\perp}) 8\alpha p_1 \]
\[ - (\epsilon, r_{\perp}) (\epsilon', \frac{p_2}{s}) 4(R_{\perp} r_{\perp} + \bar{\alpha} r_{\perp}^2) \]
\[ - (\epsilon, p_1) (\epsilon', \frac{p_2}{s}) 4(R_{\perp} + \bar{\alpha} r_{\perp} ((1 - 2\alpha) R_{\perp} - \alpha (4 p_{\perp} + (1 - 2\alpha) r_{\perp})) \]
\[ - (\epsilon, p_1) (\epsilon', \frac{p_2}{s}) 8\alpha (R_{\perp} + \bar{\alpha} r_{\perp}) ((1 - 2\alpha) p_{\perp} + \bar{\alpha} R_{\perp} - \alpha \bar{\alpha} r_{\perp}) \]
\[ - (\epsilon, \frac{p_2}{s}) (\epsilon', \frac{p_2}{s}) 8(R_{\perp} + \bar{\alpha} r_{\perp}) (2p_{\perp} + R_{\perp} - \alpha r_{\perp}) p_{\perp}^2, \] (17)

hidden in the definition of \( R_{\perp} \): \( R_{\perp} \equiv k_{\perp} + \alpha r_{\perp} \).

This formula is rather long, but its beauty is in its generality. Here one has no restrictions on photon polarizations or virtualities. It simplifies a lot for every specific case. For example, when both photons are transverse \((\epsilon, p_1) = (\epsilon, p_2) = (\epsilon', p_1) = (\epsilon', p_2) = 0\), this integral reduces to:

\[
\int_0^1 \frac{d\alpha}{2\pi} \int \frac{d^2 p_{\perp}}{(2\pi)^2} \frac{1}{\alpha \bar{\alpha} q^2 + p_{\perp}^2 \alpha \bar{\alpha} q^2 + (p_{\perp} + R_{\perp})^2} \times (-2) \left[ (\epsilon, \epsilon') (p_{\perp}^2 + R_{\perp} p_{\perp}) \right.
\]
\[ - (\epsilon, p_{\perp}) (\epsilon', r_{\perp}) 2\alpha \bar{\alpha} (1 - 2\alpha) \]
\[ - (\epsilon, p_{\perp} + R_{\perp}) (\epsilon', p_{\perp}) \]
\[ + (\epsilon, p_{\perp}) (\epsilon', R_{\perp}) (1 - 2\alpha)^2 \right]. \] (18)

The integral for the first diagram differs only in overall sign and the definition of \( R_{\perp} \equiv -\bar{\alpha} r_{\perp} \). By introducing the Feynman parameter \( \alpha' \) (with simultaneous shifting to \( p_{\perp} \leftarrow p_{\perp} + \alpha' R_{\perp} \)) and using formulae of Feynman parametrization, one is able to get the photon impact factor for the case of arbitrary photon polarization as the following:

\[
\int_0^1 \frac{d\alpha}{2\pi} \int_0^1 \frac{d\alpha'}{2\pi} \left\{ \alpha' \bar{\alpha}' (R_{\perp}^2 + \Omega^2) \right\}^{-1} \times \]
\[ \left\{ (\epsilon, \epsilon') (\alpha \bar{\alpha} (1 - 2\alpha \bar{\alpha})(1 - 2\alpha') (q^2 - q'^2) + (1 - 2\alpha \bar{\alpha} - 2\alpha' \bar{\alpha'} + 8\alpha \bar{\alpha} \alpha' \bar{\alpha'}) R_{\perp}^2 \right\} \]
\[ - (\epsilon, p_1) (\epsilon', p_1) 8\alpha^2 \bar{\alpha}^2 \]
\[ - (\epsilon, \frac{p_2}{s}) (\epsilon', p_1) 2 \left( \alpha \bar{\alpha} (1 - 2\alpha \bar{\alpha})(1 - 2\alpha') (q^2 - q'^2) + \right. \]
\[ \left. \right. \]
\begin{align*}
&(1-2a\bar{a}-2a'\bar{a}'+4a\bar{a}a'(2-\alpha'))R_\perp^2) \\
+&(\epsilon, R_\perp)(\epsilon', p_1)4a\bar{a}(1-2\alpha)a' \\
-&(\epsilon, p_1)(\epsilon', p_2^0)2\left(a\bar{a}(1-2a\bar{a})(1-2a')\left(q^2 - q^2\right) - 4a^2\bar{a}^2r_\perp^2 + \\
&\quad 4a\bar{a}(1-2\alpha)\bar{a}'(R_\perp, r_\perp) + (1 + 2a\bar{a} - 2a'\bar{a}' - 4a\bar{a}a^2)R_\perp^2) \\
-&(\epsilon, p_2^0)(\epsilon', p_2^0)\left(4a\bar{a}(q^2 + r_\perp^2)(\alpha\bar{a}(1-2a')(q^2 - q^2) + (1 - 4a' + 2a^2)R_\perp^2) - \\
&\quad (1 - 2a)(R_\perp, r_\perp)(\alpha\bar{a}(1-4a'\bar{a}')(q^2 - q^2) + (1 - 6a' + 10a^2 - 6a^3)R_\perp^2) \\
-&(\epsilon, R_\perp)(\epsilon', \frac{p_2^0}{s})4a\bar{a}a'((1-2\alpha)(q^2 + r_\perp^2) + 4a'(R_\perp, r_\perp)) \\
+&(\epsilon, r_\perp)(\epsilon', \frac{p_2^0}{s})2\left(a\bar{a}(1-2a\bar{a})(1-2a')(q^2 - q^2) + \\
&\quad (1 - 2a\bar{a} - 2a'\bar{a}' + 8a\bar{a}a'\bar{a})R_\perp^2) \\
-&(\epsilon, p_1)(\epsilon', R_\perp)4a\bar{a}(1-2\alpha)a' \\
+&(\epsilon, p_1)(\epsilon', r_\perp)8a^2\alpha^2 \\
+&(\epsilon, p_2^0)(\epsilon', R_\perp)2(1-2a\bar{a})(\alpha\bar{a}(1-2a')^2(q^2 - q^2) + (1 - 6a' + 10a^2 - 6a^3)R_\perp^2) \\
-&(\epsilon, p_2^0)(\epsilon', r_\perp)4a\bar{a}(\alpha\bar{a}(1-2a')(q^2 - q^2) + (1 - 4a' + 2a^2)R_\perp^2) \\
-&(\epsilon, R_\perp)(\epsilon', R_\perp)8a\bar{a}a'\bar{a}' \\
-&(\epsilon, R_\perp)(\epsilon', r_\perp)4a\bar{a}(1-2a\alpha') \\
-\left[R_\perp = k_\perp + a\epsilon_\perp \rightarrow R'_\perp = -\bar{a}\epsilon_\perp, \\
\Omega^2 \equiv \frac{\alpha\bar{a}}{\alpha'\bar{a}'}(\alpha'q^2 + \bar{a}'q^2)\right]
\end{align*}

One can see from Eq. (19), that even in the most general case, the difference between the two diagrams of Fig. 7 is in the presence of $k_\perp$ in one and the absence ($k_\perp \to 0$) in the other. It means one can write [25]:

\begin{align*}
I(k_\perp, r_\perp) = \tilde{I}(k_\perp, r_\perp) - \bar{I}(0, r_\perp).
\end{align*}

For the particular case of two transverse photons one gets (cf. [26]):

\begin{align*}
\tilde{I}^{TT}(k_\perp, r_\perp) &= \frac{1}{2} \int_0^1 \frac{da}{2\pi} \int_0^1 \frac{da'}{2\pi} \left\{\alpha'\bar{a}'(R_\perp^2 + \Omega^2)\right\}^{-1} \times \\
&\quad \left\{(1 - 2a\bar{a})R_\perp^2(\epsilon, \epsilon') \right. \\
&\quad + 4a\bar{a}a'[R_\perp^2(\epsilon, \epsilon') - 2(\epsilon, R_\perp)(\epsilon', R_\perp)] \\
&\quad - 4a\bar{a}(1 - 2a)(r, \epsilon)_\perp(R, \epsilon')_\perp \right\},
\end{align*}

(22)
and for the longitudinal polarization

$$\epsilon^3(q) = \frac{1}{Q}(p_1 + xp_2)$$  \hspace{1cm} (23)

of the incoming photon the formula is also simple:

$$I^{LT}(k_\perp, r_\perp) = \frac{1}{2Q} \int_0^1 \frac{d\alpha}{2\pi} \int_0^1 \frac{d\alpha'}{2\pi} \left\{ (1 - 2\alpha\bar{\alpha})R_\perp^2 (r, \epsilon')_\perp + 4\alpha\bar{\alpha}\bar{\epsilon}' \left[ R_\perp^2 (r, \epsilon')_\perp - 2(r, R)_\perp (\epsilon', R)_\perp \right] ight\}^{-1} \times \left\{ (1 - 2\alpha\bar{\alpha})R_\perp^2 (r, \epsilon')_\perp ight\}.$$

(24)

Here $(a, b)_\perp$ denotes the (positive) scalar product of transverse components of vectors $a$ and $b$. At large transverse momenta $k_\perp^2 \gg r_\perp^2$, the impact factor (21) reduces to:

$$I(k_\perp, r_\perp) \rightarrow \frac{(\epsilon, \epsilon')_\perp k_\perp^2}{4\pi^2 Q^2} \ln \frac{Q^2}{r_\perp^2}.$$ \hspace{1cm} (25)

The impact factor for the proton, which describes the pomeron-nucleon coupling, cannot be calculated in perturbation theory. However, in the next section it is demonstrated that at high momenta $k_\perp^2 \gg M^2$ this impact factor reduces to

$$I_N(k_\perp, r_\perp) \overset{k_\perp^2 \gg M^2}{=} F_1^{p+n}(t),$$ \hspace{1cm} (26)

where $F_1^{p+n}(t)$ is the sum of the proton and neutron Dirac form factors.

II.4 NUCLEON IMPACT FACTOR

In the lowest order in perturbation theory there is no difference between the diagrams for the nucleon impact factor shown in Fig. 8 and similar diagrams with two gluons replaced by two photons. One has to add the trivial numerical factor $c_F = \frac{4}{3}$ and to replace $e \leftrightarrow g$. In this case the lower part of the diagram can be formally written as follows:

$$\Phi_N(-k - r, k) \overset{\text{def}}{=} \frac{2}{3} \frac{p_1^\mu p_1'^\nu}{s} \int dz e^{ikz} \langle p' | T^* \{ J_\mu(z)J_\nu(0) \} | p \rangle,$$ \hspace{1cm} (27)

where $J_\mu = \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d$. The $T^*$ means that we take into account only the $T$-product of the diagrams with pure gluon exchanges in the $t$-channel excluded. By definition, such diagrams contribute to subsequent ranks of the BFKL ladder rather than to the impact factor. This is also the reason why we have not included strange quarks in
the definition of electromagnetic current $J$. Since $k^2$ in our case is large and negative ($-k^2 = k_\perp^2 \gg M^2$) we can expand the T-product of two currents near the light cone (see e.g. [27])

$$\Phi_N(-k-r,k) = \frac{2}{3s} \int dz e^{ikz} \frac{zp_1}{\pi^2 z^4} (p'[-\bar{\psi}(z)[z,0]p_1\psi(0) + \bar{\psi}(0)[0,z]p_1\psi(z)][p]^*_{z=0}. \quad (28)$$

where again $\langle \ldots \rangle^*$ stands for the matrix element with pure gluon exchanges excluded. Here $[x,y] = P \text{exp} \left( ig \int_0^1 du (x-y)^\mu A_\mu (ux + (1-u)y) \right)$. The matrix element (28) can be parametrized in terms of skewed parton distributions [28] as follows:

$$\langle p', \lambda' | q(0)[0,z]p_1q(z)|p, \lambda \rangle^*_{z=0} = \bar{u}(p', \lambda') \hat{p}_1 u(p, \lambda) \int_0^1 dX e^{i(X-x)pz} \mathcal{V}_x^q(X,t) + \frac{1}{2M} \bar{u}(p', \lambda') \hat{p}_1 \hat{r}_\perp u(p, \lambda) \int_0^1 dX e^{i(X-x)pz} \mathcal{W}_x^q(X,t), \quad (29)$$

$$\langle p', \lambda' | q(0)[z,0]p_1q(z)|p, \lambda \rangle^*_{z=0} = \bar{u}(p', \lambda') \hat{p}_1 u(p, \lambda) \int_0^1 dX e^{-iXpz} \mathcal{V}_x^q(X,t) + \frac{1}{2M} \bar{u}(p', \lambda') \hat{p}_1 \hat{r}_\perp u(p, \lambda) \int_0^1 dX e^{-iXpz} \mathcal{W}_x^q(X,t), \quad (29)$$

where $\mathcal{V}_x^q(X,t)$ and $\mathcal{W}_x^q(X,t)$ are the non-flip and spin-flip skewed parton distributions for the valence $u$ quark (recall that one must take into account only valence quarks since diagrams with pure gluon exchanges are forbidden). Similarly, $\mathcal{V}_x^d(X,t)$ and $\mathcal{W}_x^d(X,t)$ refer to the valence $d$-quark distributions. At large energies $\bar{u}(p', \lambda') \hat{p}_1 u(p, \lambda) = s \delta_{\lambda' \lambda}$, so

$$\langle p', \lambda' | q(0)[z,0]p_1q(z) - \bar{q}(z)[z,0]p_1q(0)|p, \lambda \rangle^*_{z=0} = \int_0^1 dX \left( e^{-iXpz} - e^{i(X-x)pz} \right) \left[ s \delta_{\lambda' \lambda} \mathcal{V}_x^q(X,t) + \bar{u}(p', \lambda') \frac{\hat{p}_1 \hat{r}_\perp}{2M} u(p, \lambda) \mathcal{W}_x^q(X,t) \right]. \quad (30)$$
After integration over $z$ the lower block (27) reduces to

$$\Phi_N(-k - r, k) = \frac{2}{3s} \int_0^1 dx \left[ \frac{(X - x)s + 2p_1 \cdot k}{-k^2 - 2p \cdot k(X - x) - i\varepsilon} - \frac{-Xs + 2p_1 \cdot k}{-k^2 + 2p \cdot kX - i\varepsilon} \right]$$  \hspace{1cm} (31)

$$\left( \delta_{\lambda\lambda'}(\mathcal{V}_x^u(X, t) + \mathcal{V}_x^d(X, t)) + \bar{u}(p', \lambda') \frac{\hat{p}\hat{p}_\perp}{2Ms} u(p, \lambda)(\mathcal{W}_x^u(X, t) + \mathcal{W}_x^d(X, t)) \right).$$

The nucleon impact factor (14) is the integral of the imaginary part of right hand side of Eq. (31) over energy:

$$I_N(k_\perp, r_\perp) = \int_0^1 d\alpha_k \frac{3\Phi_N(-\alpha_k - \frac{r^2}{s})p_1 - k_\perp - r_\perp, \alpha_k p_1 + k_\perp}{2\pi}$$  \hspace{1cm} (32)

$$= \frac{1}{3} \int_0^1 d\alpha_k \int_x^1 dX \left[ s(X - x)\delta(k_\perp^2 - \alpha_k s(X - x)) - sX\delta(k_\perp^2 + \alpha_k sX) \right]$$

$$\left( \delta_{\lambda\lambda'}(\mathcal{V}_x^u(X, t) + \mathcal{V}_x^d(X, t)) + \bar{u}(p', \lambda') \frac{\hat{p}\hat{p}_\perp}{2Ms} u(p, \lambda)(\mathcal{W}_x^u(X, t) + \mathcal{W}_x^d(X, t)) \right).$$

And finally,

$$I_N(k_\perp, r_\perp) = \frac{1}{3} \int_x^1 dX \left( \delta_{\lambda\lambda'}(\mathcal{V}_x^u(X, t) + \mathcal{V}_x^d(X, t)) \right.$$

$$\left. + \frac{1}{2Ms} \bar{u}(p', \lambda') \hat{p}\hat{p}_\perp u(p, \lambda)(\mathcal{W}_x^u(X, t) + \mathcal{W}_x^d(X, t)) \right).$$

(33)

Since valence quark distributions decrease at $x \to 0$ one can extend the lower limit of integration in the r.h.s. of Eq. (32) to 0 and obtain:

$$I_N(k_\perp, r_\perp) \hspace{1cm} \frac{k_\perp^2 \gg M^2}{3} \int_0^1 dX \left( \delta_{\lambda\lambda'}(\mathcal{V}_x^u(X, t) + \mathcal{V}_x^d(X, t)) \right.$$

$$\left. + \frac{1}{2Ms} \bar{u}(p', \lambda') \hat{p}\hat{p}_\perp u(p, \lambda)(\mathcal{W}_x^u(X, t) + \mathcal{W}_x^d(X, t)) \right).$$

(34)

Let us recall the sum rules [1,28]

$$\int_0^1 dX (\mathcal{F}_x^q(X, t) - \mathcal{F}_x^q(X, t)) = F_1^q(t),$$

$$\int_0^1 dX (\mathcal{K}_x^q(X, t) - \mathcal{K}_x^q(X, t)) = F_2^q(t),$$

(35)

where $\mathcal{F}_x^q(X, t)$ and $\mathcal{K}_x^q(X, t)$ are the total (valence + sea) non-flip and spin-flip skewed quark distributions while $\mathcal{F}_x^q(X, t)$ and $\mathcal{K}_x^q(X, t)$ are the antiquark ones. In these equations, $F_1^q(t)$ and $F_2^q(t)$ stand for the $q$-quark components of the Dirac and Pauli form factors of the proton. Since the contribution of sea quarks drops from
the difference $F^q - F^\bar{q}$ one can rewrite Eqs. \(35\) as the sum rules for valence quark distributions:

\[
\int_0^1 dX V^q_\perp (X, t) = F^q_1(t), \quad \int_0^1 dX W^q_\perp (X, t) = F^q_2(t).
\]  

Substituting this estimate to Eq. \(34\) and using the isospin invariance, one can get the final result for the nucleon impact factor at large transverse momenta

\[
I_N(k_\perp, r_\perp) \sim \frac{k_\perp^2 \gg M^2}{\delta_{\lambda\lambda'} F_1^{p+n}(t) + \frac{1}{2Ms} \bar{u}(p', \lambda') \hat{p}_\perp \cdot \bar{u}(p, \lambda) F_2^{p+n}(t)},
\]  

where $F_1^{p+n}(t) \equiv F_1^p(t) + F_1^n(t)$ and $F_2^{p+n}(t) \equiv F_2^p(t) + F_2^n(t)$. As usual, $F_1^{p(n)}$ and $F_2^{p(n)}$ are the Dirac and Pauli form factors of the proton (neutron), respectively. With our accuracy they can be approximated by the dipole formulas:

\[
\begin{align*}
F_1^p + F_2^p t/4M^2 &= G_E^p = 1/(1 + |t|/\alpha_t)^2, \\
F_1^p + F_2^p &= G_M^p = 2.79/(1 + |t|/\alpha_t)^2,
\end{align*}
\]

\[
\begin{align*}
F_1^n + F_2^n t/4M^2 &= G_E^n = 0, \\
F_1^n + F_2^n &= G_M^n = -1.91/(1 + |t|/\alpha_t)^2,
\end{align*}
\]  

which leads to

\[
\begin{align*}
F_1^{p+n}(t) &= \frac{1}{(1 + |t|/\alpha_t)^2} \frac{1+0.88|t|/4M^2}{1+|t|/4M^2} \to \frac{1}{(1 + |t|/\alpha_t)^2}, \\
F_2^{p+n}(t) &= \frac{0.12}{(1 + |t|/\alpha_t)^2} \to 0.
\end{align*}
\]  

Here the notation $\alpha_t = 0.71$ GeV$^2$ is introduced. In further calculations we will use the estimate \(39\). It is obvious that the spin-flip term turned out to be negligible for our values of $t$. Besides, it vanishes at $t = 0$, which suggests that it is numerically small at all $t$.

Our final estimate of the nucleon impact factor is:

\[
I_N(k_\perp, r_\perp) \sim \delta_{\lambda\lambda'} F_1^{p+n}(t),
\]  

where $F_1^{p+n}$ is given by the dipole formula \(39\).

The dipole formula for the neutron form factor does not seem to work as well as the dipole formula for the proton form factor. As a measure of the uncertainty one can compare the results obtained from Eq. \(39\) to those obtained using the model
from Ref. [29] (which was fit only to the proton form factor):

\[
F_1^{p+n}(t) = \frac{1}{3} \int_0^1 dX \left( \mathcal{V}_x^u(X, t) + \mathcal{V}_x^d(X, t) \right),
\]

\[
\mathcal{V}_x^u(X, t) = 1.89X^{-0.4}X^{3.5}(1 + 6X) \exp \left( -\frac{X}{X \, 2.8\text{GeV}^2} \right),
\]

\[
\mathcal{V}_x^d(X, t) = 0.54X^{-0.6}X^{4.2}(1 + 8X) \exp \left( -\frac{X}{X \, 2.8\text{GeV}^2} \right). \tag{41}
\]

The results for the DVCS cross section in this model are about 1.5 times bigger than the results obtained from the dipole formula (39).

In what follows the factor \( \delta_{\lambda'} \) is omitted (as it was done in Eq. (26)) since all the amplitudes are diagonal in the proton’s spin.

### II.5 THE BFKL LADDER

As we shall see below, the characteristic transverse momenta in our gluon loop are large, so the estimate (26) is sufficient for our purposes. Substituting the nucleon impact factor (26) into Eq. (12) one obtains:

\[
V = \frac{2s}{\pi} g^4 \left( \sum e_q^2 \right) F_1^{p+n}(t) \int \frac{d^2 k_\perp}{4\pi^2} \frac{I(k_\perp, r_\perp)}{k_\perp^2(r + k_\perp)^2}. \tag{42}
\]

The final integration over \( k_\perp \) reveals the logarithmic dependence of the photon impact factor \( \ln r_\perp^2/\Omega^2 \to \ln |t|/Q^2 \):

\[
I_0 \equiv \int \frac{d^2 k_\perp}{k_\perp^2(k_\perp + r_\perp)^2} \left( \frac{1}{(k_\perp + \alpha r_\perp)^2 + \Omega^2} - \frac{1}{\alpha^2 r_\perp^2 + \Omega^2} \right) \]

\[= \frac{\pi}{\Omega^2 - \alpha^2 r_\perp^2} \left\{ -\alpha \left( \frac{\Omega^2 + \alpha^2 r_\perp^2}{\Omega^2} \right) \left( 2 \ln \frac{\Omega^2 + \alpha^2 r_\perp^2}{\Omega^2} + \eta \right) + \frac{-\alpha}{\Omega^2 + \alpha^2 r_\perp^2} \left( 2 \ln \frac{r_\perp^2}{\Omega^2} + 2\eta \right) \right\}, \tag{43}
\]

\[
I_1 \equiv \int \frac{d^2 k_\perp}{k_\perp^2(k_\perp + r_\perp)^2} \left( \frac{(k_\perp + \alpha r_\perp)^\mu}{(k_\perp + \alpha r_\perp)^2 + \Omega^2} - \frac{(-r_\perp + \alpha r_\perp)^\mu}{\alpha^2 r_\perp^2 + \Omega^2} \right) \]

\[= \frac{\pi}{\Omega^2 - \alpha^2 r_\perp^2} \left( \frac{r_\perp^\mu}{\Omega^2 + \alpha^2 r_\perp^2} \ln \frac{\Omega^2 + \alpha^2 r_\perp^2}{\Omega^2} - \frac{\Omega^2}{\Omega^2 + \alpha^2 r_\perp^2} \ln \frac{\Omega^2 + \alpha^2 r_\perp^2}{\Omega^2} \right) \]

\[+ \frac{\Omega^2 - \alpha^2 r_\perp^2}{\Omega^2 + \alpha^2 r_\perp^2} \ln \frac{r_\perp^2}{\Omega^2} + (1 - \frac{\alpha^2 r_\perp^2}{\Omega^2 + \alpha^2 r_\perp^2}) - \frac{\alpha^2 r_\perp^2}{\Omega^2 + \alpha^2 r_\perp^2} \eta \}, \tag{44}
\]

\[
\int \frac{d^2 k_\perp}{k_\perp^2(k_\perp + r_\perp)^2} \left( \frac{(k_\perp + \alpha r_\perp)^2}{(k_\perp + \alpha r_\perp)^2 + \Omega^2} - \frac{\alpha r_\perp^2}{\alpha^2 r_\perp^2 + \Omega^2} \right) = -\Omega^2 * I_0, \tag{45}
\]
\[ \int \frac{d^2 k_{\perp}}{k_{\perp}^2 (k_{\perp} + r_{\perp})^2} \left( \frac{(k_{\perp} + \alpha r_{\perp})^2 (k_{\perp} + \alpha r_{\perp})^\mu}{(k_{\perp} + \alpha r_{\perp})^2 + \Omega^2} - \frac{\bar{\alpha}^2 r_{\perp}^2 (-r_{\perp} + \alpha r_{\perp})^\mu}{\bar{\alpha}^2 r_{\perp}^2 + \Omega^2} \right) = -\Omega^2 * I_1 + \frac{\pi r_{\perp}^\mu}{r_{\perp}^2} (\ln \frac{r_{\perp}^2}{\Omega^2} + \eta), \] (46)

where

\[ \eta \equiv \frac{1}{\varepsilon} + \gamma_E + \ln \pi + \ln \Omega^2, \] (47)

\[ R_\mu R_\nu \leftrightarrow \frac{1}{2} g_{\mu\nu} R^2, \] (48)

and \( \Omega^2 \) is defined in Eq. (20).

For the case of transverse photon polarizations one gets

\[ V^{TT} = 2 \left( \frac{\alpha_s}{\pi} \right)^2 \left( \sum_q e_q^2 \right) F_1^{p+n}(t) \left( (\epsilon, \epsilon')_\perp \left( \frac{1}{2} \ln^2 \frac{Q^2}{|t|} + 2 \right) - (\epsilon, \epsilon')_\perp \frac{2}{r_{\perp}^2} (\epsilon, r, r')_\perp + O(\frac{t}{Q^2}) \right), \] (49)

and when the incoming photon has longitudinal polarization we have

\[ V^{LT} = -\frac{2}{x} \left( \frac{\alpha_s}{\pi} \right)^2 \left( \sum_q e_q^2 \right) F_1^{p+n}(t) \frac{(r, \epsilon')_\perp}{Q} \left( \frac{1}{2} \ln^2 \frac{Q^2}{|t|} - 5 \ln \frac{Q^2}{|t|} + \frac{15}{2} - \frac{\pi^2}{3} + O(\frac{t}{Q^2}) \right). \] (50)

The longitudinal amplitude (50) is twist-suppressed as \( \sqrt{|t|/Q^2} \) in comparison to the transverse amplitude (50) (as it should, due to the fact that \( t \to 0 \) corresponds to a real incoming photon).

Since the integral over \( k_{\perp} \) (42) converges at \( k_{\perp} \sim Q \), the region \( k_{\perp} \sim M \), where we do not know the nucleon impact factor, contributes to the terms \( \sim O(|t|/Q^2) \) which we neglect.

In the next-to-leading order (NLO) in perturbation theory the most important diagrams are those of the type shown in Fig. 9. Actually, this diagram gives the total contribution in LLA if one replaces the three-gluon vertex in Fig. 9 by the effective Lipatov’s vertex [22].

Calculation of this diagrams in the leading log approximation yields:

\[ V = \frac{2 s g^4}{\pi} \left( \sum_q e_q^2 \right) \left( 6 \alpha_s \ln \frac{1}{x} \right) \int \frac{d^2 k_{\perp}}{4\pi^2} \frac{d^2 k'_{\perp}}{4\pi^2} \frac{I(k_{\perp}, r_{\perp})}{k_{\perp}^2 (r + k_{\perp})^2} K(k_{\perp}, k'_{\perp}, r_{\perp}) \frac{I_N(k'_{\perp}, r_{\perp})}{(k'_{\perp})^2 (r + k'_{\perp})^2}, \] (51)
FIG. 9: Typical diagram in the next-to-leading order in perturbation theory.

where $K(k, k', r)$ is the BFKL kernel [30]

\[
K(k', k, r) = 2 \left[ \frac{k' \cdot (k' + r)}{k'^2 (k' + r)^2} - \frac{k' \cdot (k' - k)}{k'^2 (k' - k)^2} - \frac{(k' - k) \cdot (k' + r)}{(k' - k)^2 (k' + r)^2} \right. \\
\left. + \frac{k' \cdot (k' - k)}{(k' - k)^2 [k'^2 + (k' - k)^2]} + \frac{(k' - k) \cdot (k' + r)}{(k' - k)^2 [(k' - k)^2 + (k' + r)^2]} \right].
\] (52)

As we shall see below, the integral over $k'$ converges at $|k'| \gg M$ so we can again use the approximation (26) for the nucleon impact factor. One obtains

\[
\int d^2 k' K(k, k', r) \frac{I_N(k', r)}{(k')^2 (r + k')^2} = \pi F_1^{p+n}(t) \left( \ln \frac{k^2}{r^2} + \ln \frac{(k + r)^2}{r^2} \right),
\] (53)

and therefore the amplitude (51) takes the form

\[
V = \frac{g^4 s}{\pi} F_1^{p+n}(t) \left( \frac{3 \alpha_s}{\pi} \ln \frac{1}{x} \right) \int d^2 k \frac{I(k, r)}{4 \pi^2} \frac{1}{k^2 (r + k)^2} \left( \ln \frac{k^2}{r^2} + \ln \frac{(k + r)^2}{r^2} \right).
\] (54)

Finally, the integration over $k$ yields:

\[
V = \frac{2}{x} \left( \frac{\alpha_s}{\pi} \right)^2 \left( \sum_q e_q^2 \right) F_1^{p+n}(t) \left( \frac{3 \alpha_s}{\pi} \ln \frac{1}{x} \right) \\
\left( (e, e') \left( \frac{1}{6} \ln^3 \frac{Q^2}{|t|} + 2 \ln \frac{Q^2}{|t|} - 2 + \zeta(3) \right) + \left( \frac{2}{r^2} (e, r)(e', r) - (e, e') \right) \right).
\] (55)
where the accuracy is \( O(1/\ln x) \) and \( \zeta(3) \) is a Riemann zeta function.

In the next order in the BFKL approximation (see Fig. 10) it is still possible to obtain the DVCS amplitude \( (9) \) in the explicit form. It is possible to write down the

\[
\begin{align*}
V &= \frac{g^4 s}{\pi} \left( \sum q^2 \right) \left( 6 \alpha_s \ln \frac{1}{x} \right)^2 \int \frac{d^2 k_\perp}{4\pi^2} \frac{d^2 k'_\perp}{4\pi^2} \frac{d^2 k''_\perp}{4\pi^2} \frac{I(k_\perp, r_\perp)}{k_\perp^2 (r+k_\perp)^2} \frac{K(k_\perp, k'_\perp, r_\perp)}{(k'_\perp)^2 (r+k'_\perp)^2} \frac{I_N(k''_\perp, r_\perp)}{(k''_\perp)^2 (r+k''_\perp)^2} \end{align*}
\]

(56)

Once again, if we use the fact that the integral over \( k'_\perp \) converges at \( |k'_\perp| \gg M \) we can approximate the nucleon impact factor by Eq. (40), and obtain:

\[
\begin{align*}
\int \frac{d^2 k'_\perp}{4\pi^2} \int \frac{d^2 k''_\perp}{4\pi^2} K(k_\perp, k'_\perp, r_\perp) \frac{1}{(k'_\perp)^2 (r+k'_\perp)^2} K(k'_\perp, k''_\perp, r_\perp) \frac{I_N(k''_\perp, r_\perp)}{(k''_\perp)^2 (r+k''_\perp)^2} &= \frac{1}{4\pi} F_1^{p+n}(t) \int \frac{d^2 k'_\perp}{4\pi^2} K(k_\perp, k'_\perp, r_\perp) \left( \ln \frac{(k'_\perp)^2}{r_\perp^2} + \ln \frac{(k'_\perp + r)^2}{r_\perp^2} \right) \end{align*}
\]
\[
V = \frac{1}{16\pi^2} F_1^{p+n}(t) \left( \ln^2 \frac{k_1^2}{r_\perp^2} + \ln^2 \frac{(k + r)^2_\perp}{r_\perp^2} \right). \tag{57}
\]

The resulting integration over \( k_\perp \) yields:

\[
V = \frac{9}{x} \left( \frac{\alpha_s}{\pi} \right)^4 \left( \sum_q e_q^2 \right) F_1^{p+n}(t) \ln^2 x \left[ (\epsilon, \epsilon')_\perp \left( \frac{1}{24} \ln^4 \frac{Q^2}{|t|} + \ln^2 \frac{2Q^2}{|t|} - 2 \ln \frac{Q^2}{|t|} + 2(\zeta(3) - 1) + 1.46 \right) + \left( \frac{2}{r_\perp^2} (\epsilon, r)_\perp (\epsilon', r)_\perp - (\epsilon, \epsilon')_\perp \right) \right]. \tag{58}
\]

As it was mentioned, we are unable yet to obtain the explicit expressions for the amplitude in higher orders in perturbation theory. It turns out, however, that for HERA energies the achieved accuracy is reasonably good. The estimation of the next term gives \( \sim 30\% \) of the answer at not too low \( x \) (see the discussion in next section).

In the leading logarithmic approximation it is impossible to distinguish between \( \alpha_s(Q) \) and \( \alpha_s(\sqrt{|t|}) – to this end one needs to use the NLO BFKL approximation [31] (see also [32]) which is beyond the scope of this paper.

The final result for the DVCS amplitude with transversely polarized photons is:

\[
V = \frac{2}{x} \left( \frac{\alpha_s(Q)}{\pi} \right)^2 \left( \sum_q e_q^2 \right) F_1^{p+n}(t) \left[ (\epsilon, \epsilon')_\perp v + \left( \frac{2}{r_\perp^2} (\epsilon, r)_\perp (\epsilon', r)_\perp - (\epsilon, \epsilon')_\perp \right) v' \right]. \tag{59}
\]

where

\[
v(x, Q^2/t) = \left( \frac{1}{2} \ln^2 \frac{Q^2}{|t|} + 2 \right) + \frac{3\alpha_s(Q)}{\pi} \ln \frac{1}{x} \left( \frac{1}{6} \ln^3 \frac{Q^2}{|t|} + 2 \ln \frac{Q^2}{|t|} - 2 + \zeta(3) \right) + \frac{1}{2} \left( \frac{3\alpha_s(Q)}{\pi} \ln \frac{1}{x} \right)^2 \left( \frac{1}{24} \ln^4 \frac{Q^2}{|t|} + \ln^2 \frac{2Q^2}{|t|} + 2(\zeta(3) - 1) \ln \frac{Q^2}{|t|} + 1.46 \right), \tag{60}
\]

\[
v'(x, Q^2/t) = 1 + \frac{3\alpha_s(Q)}{\pi} \ln \frac{1}{x} + \frac{1}{2} \left( \frac{3\alpha_s(Q)}{\pi} \ln \frac{1}{x} \right)^2. \tag{61}
\]

Note that the spin-dependent part \( \sim v' \) does not contain any \( \ln Q^2/|t| \) and is, hence, much smaller than the spin-independent part \( \sim v \). For the longitudinal polarization [23] the amplitude is twist-suppressed as \( \sim \sqrt{|t|/Q^2} \) so we have not calculated any terms beyond Eq. (50). In the numerical analysis carried out in the next sections only the spin-independent part of the amplitude is kept:

\[
V_\perp \equiv \frac{1}{4} \sum \epsilon_\perp \epsilon'_\perp V = \frac{2}{x} \left( \frac{\alpha_s(Q)}{\pi} \right)^2 \left( \sum_q e_q^2 \right) F_1^{p+n}(t) v(x, Q^2, t). \tag{62}
\]
The expressions above give us the imaginary part of the DVCS amplitude. For the calculation of the DVCS cross section one needs to know also the real part $\Re H$ of this amplitude, which can be estimated via the dispersion relation. For the positive-signature amplitude $H_\perp (\equiv \frac{1}{4} \sum \epsilon_\perp \epsilon'_\perp H)$ we get \cite{33} (see also \cite{19})

$$\Re H_\perp(s) = \frac{\pi}{2} \tan \left( s \frac{d}{ds} \right) \Im H_\perp(s),$$

which amounts to the substitution

$$\ln s \rightarrow \frac{1}{2} (\ln(-s - i\varepsilon) + \ln s)$$

in our amplitude \cite{62}. Thus, the real part is:

$$R \equiv \frac{1}{\pi} \Re H_\perp = \frac{2}{x} \left( \frac{\alpha_s}{\pi} \right)^2 \left( \sum_q \epsilon_q^2 \right) F_1^{q+n}(t)r(x, Q^2, t),$$

$$r(x, Q^2, t) = \frac{\pi}{2} \left[ \frac{3\alpha_s}{\pi} \left( \frac{1}{6} \ln^3 \frac{Q^2}{|t|} + 2 \ln \frac{Q^2}{|t|} - 2 + \zeta(3) \right) + \left( \frac{3\alpha_s}{\pi} \right)^2 \ln \frac{1}{x} \left( \frac{1}{24} \ln^4 \frac{Q^2}{|t|} + \ln^2 \frac{Q^2}{|t|} + 2(\zeta(3) - 1) \ln \frac{Q^2}{|t|} + 1.46 \right) \right].$$

II.6 DVCS CROSS SECTION

It is instructive to compare the DVCS amplitude $V$ given by Eq. \cite{9} with the corresponding amplitude for the forward $\gamma^*$ scattering

$$H = -i\epsilon_\nu \epsilon'_\mu \int dze^{-iq\cdot z} \langle p|T\{j^\mu(z)j^\nu(0)\}|p\rangle.$$ 

The imaginary part of this amplitude is the total cross section for deep inelastic scattering (DIS)

$$\frac{1}{\pi} \Im H = W =$$

$$\epsilon_\nu \epsilon'_\mu \left[ \left( \frac{q_\nu q_\mu}{q^2} - g_{\mu\nu} \right) F_1(x, Q^2) + \frac{1}{pq} \left( p_\mu - q_{\mu q} \right) \left( p_\nu - q_{\nu q} \right) F_2(x, Q^2) \right].$$

For example, $W$ averaged over the transverse polarizations of the photons is:

$$W_\perp \overset{\text{def}}{=} \frac{1}{4} \sum \epsilon_\perp \epsilon'_\perp W = F_1(x, Q^2) = \frac{1}{2x} F_2(x, Q^2),$$

(at the leading twist level we have the Callan-Gross relation $F_2 = 2xF_1$). We will compare the imaginary part of the DVCS amplitude $V_\perp$ given by Eq. \cite{62} to the result
for $W_\perp$ calculated with the same accuracy. (The notation $W_\perp(x)$ is used rather than $F_1(x)$ in order to avoid confusion with $F_1(t)$).

Similarly to the DVCS case, the DIS amplitude has the form (cf. Eqs. (12), (51), and (56)):

\[
W_\perp = \frac{2g^2s}{\pi} \left( \sum e_q^2 \right) \int \frac{d^2k_\perp}{4\pi^2} \frac{k_\perp}{k_\perp^4} I_\perp(k_\perp,0) \left[ 1 + \frac{3g^2}{8\pi^3} \ln \frac{1}{x} \int d^2k'_\perp K(k_\perp,k'_\perp,0) \left( \frac{1}{k'_\perp^2} I_N(k'_\perp,0) + \frac{9g^4}{128\pi^6} \ln \frac{1}{x} \int d^2k''_\perp K(k'_\perp,k''_\perp,0) \left( \frac{1}{k''_\perp^2} I_N(k''_\perp,0) \right) \right] \right],
\]

where $I_\perp(k_\perp,0)$ is the virtual photon impact factor averaged over the transverse polarizations [34]:

\[
I_\perp(k_\perp,0) = \frac{1}{2} \int_0^1 \frac{d\alpha}{2\pi} \int_0^1 \frac{d\alpha'}{2\pi} \frac{k_\perp^2 (1 - 2\alpha\bar{\alpha})(1 - 2\alpha'\bar{\alpha}')}{\alpha\alpha'(k_\perp^2 + \Omega^2)}.
\]

The nucleon impact factor $I_N(k'_\perp,0)$ cannot be calculated in perturbation theory since it is determined by the large-scale nucleon dynamics. However, we know the asymptotics at large $k_\perp \gg M$

\[
I_N(k_\perp,0) \quad k_\perp \gg M^2 \quad F_1^{p+}(0) = 1.
\]

Also, at $I_N(k_\perp,0) \to 0$ at $k \to 0$ due to the gauge invariance. It seems reasonable to model this impact factor by the simple formula:

\[
I_N(k_\perp,0) = \frac{k_\perp^2}{k_\perp^2 + M^2},
\]

which has the correct behavior both at large and small $k_\perp$. With this model, the DIS amplitude (70) takes the form:

\[
W_\perp = \frac{F_2}{2x} = \frac{4}{3x} \left( \frac{\alpha_s(Q)}{\pi} \right)^2 \left( \sum e_q^2 \right) \left[ \frac{1}{2} \ln^2 \frac{Q^2}{M^2} + \frac{7}{6} \ln \frac{Q^2}{M^2} + \frac{77}{18} \right] + \frac{3\alpha_s}{\pi} \ln \frac{1}{x} \left[ \frac{1}{6} \ln^3 \frac{Q^2}{M^2} + \frac{7}{12} \ln^2 \frac{Q^2}{M^2} + \frac{77}{18} \ln \frac{Q^2}{M^2} + \frac{131}{27} + 2\zeta(3) \right] + \frac{1}{2} \left( \frac{3\alpha_s}{\pi} \ln \frac{1}{x} \right)^2 \left[ \frac{1}{24} \ln^4 \frac{Q^2}{M^2} + \frac{7}{36} \ln^3 \frac{Q^2}{M^2} + \frac{77}{36} \ln^2 \frac{Q^2}{M^2} + \frac{131}{27} + 4\zeta(3) \right] \ln \frac{Q^2}{M^2} + \frac{1396}{81} - \frac{\pi^4}{15} + \frac{14}{3} \zeta(3) \right].
\]
Note that the coefficients in front of leading logs of $Q^2$, determined by the anomalous dimensions of twist-2 operators, coincide up to the factor $2/3$. The graph of the model (74) versus the experimental data is presented in Fig. 11 for $Q^2 = 10\text{GeV}^2$ and $Q^2 = 35\text{GeV}^2$ (we take $\sum e_q^2 = \frac{10}{9}$).

For DIS it is possible to write down the total BFKL sum as a Mellin integral and unlike DVCS, the integrals of impact factors with the BFKL eigenfunctions $(k_{\perp}^2)^{-\frac{1}{2}+i\nu}$ can be calculated explicitly. Eqs. (74) and (75) correspond to the expansion of this explicit expression in powers of $\alpha_s \ln x$.

For example, the next term in BFKL series (74) has the form:

$$\frac{4}{3x} \left( \frac{\alpha_s(Q)}{\pi} \right)^2 \left( \sum_{\text{flavors}} e_q^2 \right) \frac{1}{6} \left( \frac{3\alpha_s}{\pi} \ln \frac{1}{x} \right)^3 \left( \frac{1}{120} \ln^5 \frac{Q^2}{M^2} + \frac{7}{144} \ln^4 \frac{Q^2}{M^2} + \frac{77}{108} \ln^3 \frac{Q^2}{M^2} + \left( \frac{131}{54} + 3\zeta(3) \right) \ln^2 \frac{Q^2}{M^2} + \right. $$

$$\left. + \left( \frac{1396}{81} - \frac{\pi^4}{15} + 7\zeta(3) \right) \ln \frac{Q^2}{M^2} + \frac{4736}{243} - \frac{7\pi^4}{90} + \frac{77}{3} \zeta(3) + 6\zeta(5) \right).$$

The ratio of this $(\alpha_s \ln x)^3$ term to the sum of the first three ones (74) is presented in Fig. 12 for $Q^2 = 10\text{GeV}^2$ and $Q^2 = 35\text{GeV}^2$. From these graphs we see that the sum of the first three terms gives the reliable estimate of the DIS amplitude at not too low $x$. It is expected that the same will also be true for the DVCS amplitude.

At very small $x \sim 10^{-3} - 10^{-5}$ the full BFKL result for $F_2$ in our model is growing more rapidly than Fig. 11. On the other hand if one takes into account the NLO BFKL corrections [31, 32] the result for $F_2$ at very small $x$ goes well under the experimental points. This indicates, that at such $x$ we need to unitarize the
BFKL pomeron, which is currently an unsolved problem. (The best hope is to find the effective action for the BFKL pomeron (see e.g. [35, 36])). On the contrary, at “intermediate” $x \sim 0.1 - 0.001$, we see from Fig. 11 that, since the corrections almost cancel each other, it makes sense to take into account only a few first terms in BFKL series.

It is instructive to compare the $t$-dependence of our DVCS amplitude (60) with the model used in the paper [19]

$$V_1(x, t, Q^2) = \frac{1}{R} F_1(x, Q^2) e^{bt/2},$$

(76)

$$V_2(x, t, Q^2) = \frac{1}{R} F_1(x, Q^2) \frac{1}{(1 + |t|/\alpha_t)^2},$$

(77)

where $R \simeq 0.5$ for our energies. (Literally, the model used in ref. [19] corresponds to $V_1$ but it is more natural to approximate the $t$-dependence by the dipole formula [37]).

The comparison is shown in Fig. 13 for $Q^2 = 10$GeV$^2$, $Q^2 = 35$GeV$^2$ and $x=0.01$, $x=0.001$.

In order to estimate the cross section for DVCS at HERA kinematics ($Q^2 > 6$GeV$^2$ and $x < 10^{-2}$) we will use formulae from Ref. [19] (see also Ref. [38]) with the trivial substitution $\frac{1}{2x} F_2(x) R^{-1} e^{bt/2} \to V_\perp(x, Q^2, t)$. The expressions for the DVCS cross section, the quantum electrodynamics (QED) Compton (Bethe-Heitler) cross section, and the interference term have the form ($\bar{y} \equiv 1 - y$):

$$\frac{d\sigma_{\text{DVCS}}}{dxdydt} = \pi \alpha^3 x \frac{1 + \bar{y}^2}{Q^4 y} (V_\perp^2(x, Q^2, t) + R^2(x, Q^2, t)), \quad (78)$$
\[
\frac{d\sigma^{QEDC}}{dxdydt d\phi_r} = \frac{\alpha^3 y(1 + \bar{y}^2)}{\pi x |t| Q^2 \bar{y}} \left( (F_1^p(t))^2 + \frac{|t|}{4M^2} (F_2^p(t))^2 \right),
\]
(79)

\[
\frac{d\sigma^{INT}}{dxdydt d\phi_r} = \mp 2\alpha^3 (1 + \bar{y}^2) \frac{R_{\perp}(x, Q^2, t) F_1^p(t) \cos \phi_r}{Q^2 \sqrt{|t|}},
\]
(80)

The expression for the interference term from ref. [19] is corrected by factor 2 [37].

FIG. 13: The ratio \( V_1/V_\perp \) (lower curve) and \( V_2/V_\perp \) (upper curve).

Here \( y = 1 - \frac{E'}{E} \) (\( E \) and \( E' \) are the incident and scattered electron energies, respectively, as defined in the proton rest frame) and \( \phi_r = \phi_e + \phi_N \) where \( \phi_N \) is the azimuthal angle between the plane defined by \( \gamma^* \) and the final state proton and the \( x-z \) plane and \( \phi_e \) is the azimuthal angle between the plane defined by the initial and final state electron and \( x-z \) plane (see Ref. [19]). As mentioned above, we approximate the Dirac and Pauli form factors of the proton by the dipole formulas [39].

At first let us discuss the relative weight of the above cross sections. We start with the asymmetry defined in ref. [40]

\[
A = \frac{\int_{-\pi/2}^{\pi/2} d\phi_r d\sigma^{DQI} - \int_{-\pi/2}^{3\pi/2} d\phi_r d\sigma^{DQI}}{\int_0^{2\pi} d\phi_r d\sigma^{DQI}},
\]
(81)
where
\[ \frac{d\sigma^{DQI}}{d\sigma} \equiv d\sigma^{DVCS} + d\sigma^{QEDC} + d\sigma^{INT}. \] (82)

The asymmetry shows the relative importance of the interference term, which is proportional to the real part of the DVCS amplitude. In our approximation the asymmetry is
\[ A(y, t) = \frac{4y\sqrt{Q^2/|t|} (\sum e_q^2) \left( \frac{\alpha_s}{\pi} \right)^2 \left( 1 + 2.8 \frac{|t|}{4M^2} \right) r}{\frac{4\pi^2(\sum e_q^2)^2(v^2 + r^2) \left( \frac{\alpha_s}{\pi} \right)^4 \left( 1 + \frac{|t|}{4M^2} \right) + \frac{v^2\Delta^2}{g|t|} \left( 1 + 7.84 \frac{|t|}{4M^2} \right)}}. \] (83)

The plots of asymmetry versus \( y \) and \(|t|\) are given by Fig. 14.

Second, we define the ratio of the DVCS and Bethe-Heitler cross sections [19]
\[ D(y, t) \equiv \frac{d\sigma_{DVCS}}{d\sigma_{QEDC}} = \frac{4\pi^2(\sum e_q^2)^2(v^2 + r^2) \left( \frac{\alpha_s}{\pi} \right)^4 \left( 1 + \frac{|t|}{4M^2} \right)}{\frac{v^2\Delta^2}{g|t|} \left( 1 + 7.84 \frac{|t|}{4M^2} \right)}. \] (84)

This ratio is presented on Fig. 15.
We see that there is a sharp dependence on $y$; at $y > 0.2$ the DVCS part is negligible in comparison to Bethe-Heitler background whereas at $y < 0.05$ the QEDC background is small in comparison to DVCS.

Finally let us estimate the relative weight of the DVCS signal (starting from $|t| = 1$ GeV$^2$) as compared to the DIS background. We define (cf. ref. [19])

$$R_\gamma = \frac{\sigma(\gamma^* + p \rightarrow \gamma + p)}{\sigma(\gamma^* + p \rightarrow \gamma^* + p)} \sim \frac{4\pi \alpha}{Q^2 F_2(x, Q^2)} \left( \frac{\alpha s}{\pi} \right)^2 \sum e_i^2 \int_1^{Q^2} dt \left( F_1^{p+n}(t) \right)^2 \left( v^2(x, Q^2/t) + r^2(x, Q^2/t) \right) .$$ (85)

At $Q^2 = 10$ GeV$^2$ we find $R_\gamma = 1.56 \times 10^{-5}$ for $x = 0.01$ and $R_\gamma = 2.36 \times 10^{-5}$ for $x = 0.001$, while for $Q^2 = 35$ GeV$^2$ we find $R_\gamma = 6.2 \times 10^{-5}$ for $x = 0.01$ and $R_\gamma = 7.1 \times 10^{-5}$ for $x = 0.001$.

The expressions (78)-(80) are correct if $Q^2 \gg |t|$ up to $O(\frac{|t|}{Q})$ accuracy with the notable exception of the correction $O(\sqrt{\frac{|t|}{Q}})$ coming from the expansion of electron propagator in the u-channel of the Bethe-Heitler amplitude. As suggested in ref. [39], at intermediate $t$ one can keep the propagator in unexpanded form (and expand the
rest of the amplitude, as we have done above). This amounts to the replacement
\[
\bar{y} \rightarrow \bar{y} \left[ (1 + \frac{|t|}{Q^2 \bar{y}}) (1 + \frac{|t|}{Q^2 \bar{y}}) - 2 \frac{2 - y}{\sqrt{y}} \sqrt{\frac{|t|}{Q^2}} \cos \phi_R + 4 \frac{|t|}{Q^2} \cos^2 \phi_R \right]
\]  
(86)
in the numerator in Eqs. (79) and (80) (see ref. [38]).

The resulting asymmetry (81) is presented in Fig. 16. We see that the correction factor (86) crucially changes the behavior of the asymmetry due to the fact that it restores the azimuthal dependence of the QEDC amplitude which was not taken into account in Eqs. (78-80). In order to find asymmetry at these \(Q^2\) and \(t\) with greater accuracy one should take into account other twist-4 contributions as well. On the contrary, the ratio \(D(x, Q^2/t)\) does not change much (see Fig. 17) so we hope that our leading-twist results for the ratio presented in Fig. 15 are reliable.

II.7 CONCLUSION

The DVCS in the kinematical region (4) is probably the best place to test the momentum transfer dependence of the BFKL pomeron. Without this dependence, the
model (77) would be exact, hence the upper curves in Fig. 13 indicate how important is the $t$-dynamics of the pomeron. We see that the $t$-dependence of the BFKL pomeron changes the cross section at $t > 2$GeV$^2$ by orders of magnitude and therefore it should be possible to detect it [41].

The pQCD calculation of the DVCS amplitude in the region (4) is in a sense even more reliable than the calculation of usual DIS amplitudes since it does not rely on the models for nucleon parton distribution. Indeed, all the non-perturbative nucleon input is contained in the Dirac form factor of the nucleon, which is known to a reasonable accuracy. There are, of course, the non-perturbative corrections to the BFKL pomeron itself. It is not clear how to take them into account at this moment. Of course, any reasonable models of nucleon parton distributions such as (32) should reproduce the form factor after integration over $X$.

Finally, let us discuss uncertainties in our approximation and possible ways to improve it. One obvious improvement would be to calculate (at least numerically) the next $\sim (\alpha_s \ln x)^3$ term in the BFKL series for the DVCS amplitude. Hopefully,
it will be as small as the corresponding calculation of the DIS amplitude suggests. Besides, there are non-perturbative corrections to the BFKL pomeron which are mentioned above. These non-perturbative corrections correspond to the situation like the “aligned jet model” when one of the two gluons in Fig. 5 is soft and all the momentum transfers through the other gluon. It is not clear how to take these corrections into account, but one should expect them to be smaller than the corresponding corrections to $F_2(x)$ which come from two non-perturbative gluons in Fig. 5 (in other words, from the “soft pomeron” contribution to $F_2(x)$).

The biggest uncertainty in our calculation is the argument of coupling constant $\alpha_s$ which we take to be $Q^2$. As mentioned above, it is not possible to fix the argument of $\alpha_s$ in the LLA, so one could have used $\alpha_s(|t|)$ instead. We hope to overcome this difficulty by using the results of NLO BFKL in our future work.
CHAPTER III

GENERALIZED PARTON DISTRIBUTIONS

III.1 INTRODUCTION

Let me start with the historical overview of the problem. The first information about quark distributions inside the nucleon was received from the experiments on deep inelastic scattering:

\[ e(k) + N(p) \rightarrow e(k') + X(p_n), \]

(87)

where \( X(p_n) \) are possible final hadronic states with the 4-momentum \( p_n \).

The scattering amplitude for the \( n \)-channel is given by:

\[ T_n = e^2 \bar{u}(k', \lambda') \gamma_\mu u(k, \lambda) \frac{1}{q^2} \langle n \mid J_{e.m}^\mu(0) \mid p, \sigma \rangle, \]

(88)

where \( e \) is electron’s charge, \( k(k') \) is the initial(final) momentum of the electron, \( \lambda(\lambda') \) is electron’s initial(final) polarization, \( q \) is the momentum of the virtual photon, that hits the nucleon \( (q^2 = (k - k')^2 = -4EE'\sin^2 \frac{\theta}{2} \leq 0, Q^2 = -q^2) \), \( \theta \) is electron’s scattering angle and \( J_{e.m}^\mu \) is hadronic electromagnetic current.

The differential cross section for this scattering for unpolarized particles has the form:

\[ d\sigma_n = \frac{1}{v} \frac{1}{2M 2E (2\pi)^3 2k_0} \prod_{i=1}^n \left[ \frac{d^3 p_i}{(2\pi)^3 2p_{i0}} \right] \times \frac{1}{4} \sum_{\sigma, \lambda, \lambda'} | T_n |^2 (2\pi)^4 \delta^4(p + k - k' - p_n), \]

(89)

where \( p_n = \sum_{i=1}^n p_i \).

If we sum over all possible final hadronic states, we will get the inclusive cross section

\[ \frac{d^2 \sigma}{d\Omega dE'} = \frac{\alpha^2}{q^4} \left( \frac{E'}{E} \right) l_{\mu\nu} W^{\mu\nu}, \]

(90)

where \( \alpha = e^2/4\pi \) - fine structure constant. Lepton tensor \( l_{\mu\nu} \) has the following form:

\[ l_{\mu\nu} = \frac{1}{2} Tr[\hat{k}'\gamma_\mu \hat{k}\gamma_\nu] = 2 \left( k_\mu k'_\nu + k'_\mu k_\nu + \frac{q^2}{2} g_{\mu\nu} \right), \]

(91)

and the hadronic one \( W^{\mu\nu} \) has the form:

\[ W^{\mu\nu} = \frac{1}{4M} \sum_\sigma \int \frac{d^4x}{2\pi} e^{iq\cdot x} \langle p, \sigma \mid [J_{e.m}^\mu(x) J_{e.m}^\nu(0)] \mid p, \sigma \rangle. \]

(92)
From the current conservation $\partial_\mu J^\mu_{\text{e.m.}} = 0$ one gets the following equations:

$$q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0,$$

which brings us to the requirements that hadronic tensor $W^{\mu\nu}$ should satisfy: it should be a Lorentz invariant tensor of rank two and it should depend on momenta $p_\mu$ and $q_\mu$ (the only momenta we have in this case), which brings us to the following representation of the hadronic tensor:

$$W_{\mu\nu}(p, q) = \left[-W_1 \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + W_2 \frac{1}{M^2} \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right)\right],$$

where $W_{1,2}$ are the first Lorentz invariant structure functions of the nucleon target that had ever been introduced. Those structure functions depend on two variables $q^2$ and $\nu = E - E'$.

In the case, when the final hadronic state $X(p_n)$ is also a nucleon (the case of the elastic electron-nucleon scattering), the matrix element for the electromagnetic current has the form:

$$\langle \tilde{N}(p') | J^\mu_{\text{e.m.}}(0) | \tilde{N}(p) \rangle = \bar{u}(p') \left[\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu} q^\nu \frac{1}{2M} F_2(q^2)\right] u(p),$$

with $q = p - p'$. For the proton $F_1^p(0) = 1$ is equal to the electric charge and $F_2^p(0) = 1.79$ is equal to the anomalous magnetic moment. For the neutron those number are $F_1^n(0) = 0$ and $F_2^n(0) = -1.91$, respectively.

Therefore, the differential cross section measurements for the elastic electron-nucleon scattering give us information about electrical and magnetic form factors. If the nucleon had no internal structure, those form factors were independent on $q^2$. The experimental fact that they have momentum transfer dependence opened a new era in nucleon structure investigations.

The new parton model was created by Feynman, which described a nucleon as a particle consisting of three partons (quarks) interacting by gluon exchange. Lepton-nucleon scattering has been transformed to the scattering of virtual photon off one of the partons. Accordingly, the structure functions $W_{1,2}(p, q)$ describing the response of the nucleon with momentum $p$ to the momentum transfer $q$ were replaced by structure functions $F_{1,2}(x)$:

$$MW_1(p, q) \rightarrow F_1(x) = \frac{1}{2} n(x),$$

$$\nu W_2(p, q) \rightarrow F_2(x) = x n(x),$$
where $n(x)$ represents the probability to find the parton (quark) inside the nucleon carrying the fraction $x$ of initial longitudinal nucleon momenta $xp$ within the range from $x$ to $x + dx$. The corresponding function for the antiquark is denoted as $\bar{n}(x)$.

Thus, through the experiments on deep inelastic lepton-hadron scattering one can extract information on forward quark distribution functions (Fig. 18).

In the following, we introduce the kinematical variables entering the DVCS process. We then discuss the DVCS amplitude in the leading power in $Q^2$ (twist-2 accuracy), and show its dependence upon the general parton distribution (GPD) functions.

**III.2 DVCS IN TWIST-2 APPROXIMATION**

As was already mentioned in the first chapter, the deeply virtual Compton scattering process

$$\gamma^*(q) + N(p) \rightarrow \gamma(q') + N(p'),$$

in the limit of vanishing momentum transfer $t = (p - p')^2$ and large virtuality $Q^2 \rightarrow \infty$ offers us much more complex insight on parton dynamics in the nucleon. In the lowest approximation (twist-2) the structure functions $F(x,y)$, called double distributions [1, 2], carry information on the quark distributions with different fraction $x$ of the initial nucleon momentum and different fraction $y$ of the momentum transfer $r = p' - p$ carried by the quark. This nucleon structure information can be parametrized...
in terms of four structure functions. Those functions look like usual distribution functions with respect to \( x \) and like distribution amplitudes with respect to \( y \).

### III.2.1 Kinematical variables and distribution functions

Since \( q'^2 = 0 \) (\( q' \) - is the final real photon four-momenta), it is natural to use it as one of the basic Sudakov light-cone 4-vectors. Another basic light-cone vector we use is \( p \), which is also understandable, since \( p^2 = 0 \) can be neglected compared to the virtuality \( Q^2 = -q^2 \) of the initial photon and the energy invariant \( p \cdot q = M\nu \).

In this limit (\( p^2 = 0 \) and \( r^2 = 0 \)), the simple equation \( p'^2 = (p + r)^2 = p^2 \) requires \( p \cdot r = 0 \) which can be satisfied in the light-cone basis only if they are proportional to each other \( r = \zeta p \).

Despite the proportionality between \( p \) and \( r \), they specify the momentum flow in two different channels, \( s \) and \( t \) respectively. The coefficient of their proportionality \( \zeta \) is the parameter characterizing the “asymmetry” or the “skewedness” of the matrix elements.

The total momentum fraction of the quark can either be positive or negative. Since positive momentum fraction corresponds to quark and the negative one - to antiquark, there are three major regions that could be found. Two of them, namely when we have two active quarks or antiquarks, are the regions accessible in inclusive processes like DIS and giving us the usual forward parton distributions. The third region represents the one-quark-one-antiquark case, which cannot be measured through DIS, since it vanishes in the forward limit \( \Delta \to 0 \). In the limit \( r = 0 \) we
come up with the following reduction formulas for the double distributions $F(x, y)$:

$$
\int_0^{1-x} F(x, y; t = 0)|_{x>0}dy = n(x); \quad \int_{-x}^1 F(x, y; t = 0)|_{x<0}dy = -\bar{n}(-x). \quad (99)
$$

$$
\int_0^{1-x} \tilde{F}(x, y; t = 0)|_{x>0}dy = n(x); \quad \int_{-x}^1 \tilde{F}(x, y; t = 0)|_{x<0}dy = \bar{n}(-x). \quad (100)
$$

Positive-$x$ and negative-$x$ components of the double distributions can be treated as non-forward generalizations of quark and antiquark densities, respectively: $F(x, y; t)|_{x>0} (-F(-x, 1 - y; t)|_{x<0})$.

The reduction formulas and interpretation of the $x$-variable suggests, that the profile of $F(x, y)$ in $x$-direction is basically determined by the shape of $n(x)$. The

![Diagram](https://example.com/diagram.png)

FIG. 20: "Handbag" diagrams for DVCS.

profile in the $y$-direction at the same time characterizes the spread of the parton momentum induced by the momentum transfer $r$. This brings us to the following relation:

$$
F(x, y) = h(x, y)n(x), \quad (101)
$$

where $h(x, y)$ is an even function of $y$ and

$$
\int_{-1+|x|}^{1-|x|} h(x, y)dy = 1. \quad (102)
$$

Integrating each particular double distribution over $y$ gives the non-forward parton distributions:

$$
\mathcal{F}_\zeta(X) = \int_0^1 dx \int_0^{1-x} \delta(x + \zeta y - X)F(x, y)dy = \\
\theta(X \geq \zeta) \int_0^{X/\zeta} F(X - \zeta y, y)dy + \theta(X \leq \zeta) \int_0^{X/\zeta} F(X - \zeta y, y)dy, \quad (103)
$$
where $X \equiv x + \zeta y$ is the total momentum fraction and the notation $\zeta \equiv 1 - \zeta$ is used.

### III.2.2 Twist-2 DVCS amplitude

The leading \cite{1,3,42} handbag contribution (Fig. 20) to the DVCS amplitude can be represented as \cite{14}

\[
T^{\mu\nu}(p,q,q') = \frac{1}{2(pq')} \left\{ \sum_a e_a^2 \left( \frac{1}{2(pq')} (p^\mu q'^\nu + p'^\nu q'^\mu) - g^{\mu\nu} \right) \right.
\]

\[
\times \left[ \bar{u}(p') q' u(p) T^a_F(\zeta) + \frac{1}{2M} \bar{u}(p') (q' \gamma_5 - \gamma_5 q') u(p) T^a_R(\zeta) \right] + i\epsilon^{\mu\nu\alpha\beta} p_\alpha q'_\beta
\]

\[
\times \left[ \bar{u}(p') q'^5 u(p) T^a_G(\zeta) + \frac{(q'^5 r)}{2M} \bar{u}(p') \gamma_5 u(p) T^a_P(\zeta) \right] \} .
\] (104)

The invariant amplitudes $T^a(\zeta)$ can be calculated using model \cite{43} for the non-forward quark parton distributions with $t = 0$:

\[
T^a_F(\zeta) = - \int_0^1 \left[ \frac{1}{X - \zeta + i\epsilon} + \frac{1}{X - i\epsilon} \right] \left( \mathcal{F}^a_\zeta(X) + \mathcal{F}^a_{\bar{\zeta}}(X) \right) dX .
\] (105)

Note, that because the non-forward distributions are real, the imaginary part of $T^a(\zeta)$ comes only from the singularities of the expression in the square brackets. Since all non-forward distributions vanish at $X = 0$, only the first term in the square brackets generates the imaginary part:

\[
\Im T^a_F(\zeta) = \pi \left( \mathcal{F}^a_\zeta(\zeta) + \mathcal{F}^a_{\bar{\zeta}}(\zeta) \right) .
\] (106)

The real part of the invariant amplitudes $T^a(\zeta)$ (Fig. 21) is given by the “principle value integration” in Eq.\cite{103}

\[
\Re T^a_F(\zeta) = - P \int_0^1 \left( \mathcal{F}^a_\zeta(X) + \mathcal{F}^a_{\bar{\zeta}}(X) \right) \frac{dX}{X - \zeta} .
\] (107)

The non-forward distributions $\mathcal{F}^a_\zeta(X)$ are expressed through the double distributions by Eq.\cite{103}.

In our calculations we use the model \cite{43} for the double distribution given by the factorized ansatz

\[
\mathcal{F}_a(x,y) = f_a(x) h(x,y), \quad h(x) = \int_0^1 h(x,y) dy,
\] (108)

In our calculations we use the model \cite{43} for the double distribution given by the factorized ansatz

\[
\mathcal{F}_a(x,y) = f_a(x) h(x,y), \quad h(x) = \int_0^1 h(x,y) dy,
\] (108)
where $f_a(x)$ is the forward quark–parton distribution and $h(x,y)$ is the profile function. The realistic profile for the double quark distributions, both valence and sea, is close to the “asymptotic” form

$$h_{as}(x,y) = 6y(1 - x - y).$$

The resulting curves for the real and imaginary parts of the invariant amplitude $T(\zeta) = \sum_a c_a^2 T^a(\zeta)$ are shown in Fig. 22.

**III.2.3 Twist-2 DVCS cross section**

The DVCS amplitude, as was mentioned before, may be observed in the exclusive lepton-nucleon scattering. In the $\gamma^*(q) + N(p) \rightarrow \gamma(q') + N(p')$ reaction, the final photon can be emitted either by the proton or by the lepton. The three relevant diagrams are shown in Fig. 23. The blob with two photon legs in diagram Fig. 23.
stands for the DVCS nucleon amplitude (i.e. the amplitude $T_{DVCS}$ in Eq. (110)) of scattering of the virtual photon on the nucleon, with the real photon in the final state, whereas the one-photon blob in diagrams Fig. 23\textit{b, c} represents the nucleon electromagnetic form factor.

We will refer to diagram Fig. 23\textit{a} as the DVCS part of the amplitude of the electron-nucleon scattering. Figs. 23\textit{b, c} give together the Bethe-Heitler part.

The invariant cross section of the $\gamma^*(q) + N(p) \to \gamma(q') + N(p')$ reaction differential with respect to $Q^2$, $x_B$, $t$, and out-of-plane angle $\phi$ ($\phi = 0^\circ$ corresponds to the situation where the real photon is emitted in the same half plane as the leptons) is given by:

$$
\frac{d\sigma}{dQ^2 \, dx_B \, dt \, d\Phi} = \frac{1}{(2\pi)^4 \, 32 \, Q^4} \, x_B \, y^2 \, \left(1 + \frac{4M^2 x_B^2}{Q^2}\right)^{-1/2} \, \left|T_{BH} + T_{DVCS}\right|^2, \quad (110)
$$
where $M$ is the nucleon mass, $\frac{y}{\gamma} \equiv \frac{(p \cdot q)}{(p \cdot k)}$, and $k$ is the initial lepton four-momentum.

The process where the photon is emitted from the initial or final lepton is referred to as the Bethe-Heitler (BH) process (amplitude $T_{BH}$ in Eq. (IIU)), and can be calculated exactly:

$$
|T_{BH}|^2 = -\frac{e^6}{M^2 x_B y^2 \tau^2 \beta (1 + 2 \beta + \tau)} \left( x_B y^2 \mu \tau (1 + 8 \beta^2 + \tau^2 + 4 \beta (1 + \tau)) ((2 F_1(t) + F_2(t))^2 + 2(2 \mu / \tau - 1) F_1^2(t)) + \tau (1 + \bar{y}^2 - x_B y (1 + 2 \beta (1 + \bar{y}) + \bar{y} \tau)) (4 \mu F_1^2(t) - \tau F_2^2(t)) \right),
$$

where we introduced the dimensionless variables $\beta = \frac{(k \cdot q')}{Q^2}$, $\tau = \frac{t}{Q^2}$, and $\mu = \frac{M^2}{Q^2}$. The $\phi$-dependence is hidden in $\beta$ and it alone determines the $\phi$-dependence of the Bethe-Heitler amplitude $T_{BH}$.

Light particles such as electrons radiate much more than the heavy proton. Therefore the BH process generally dominates the DVCS amplitude, especially at small $t$. The best way to extract the information on general parton distribution functions and still be in an accessible region for the experimentalists is to measure the interference term. We could benefit from the situation if we consider the difference in cross section for electrons with opposite helicities. This way, in the difference of cross sections $\sigma_{e^+} - \sigma_{e^-}$, the BH (whose amplitude is purely real) drops out. The DVCS contribution is strongly suppressed and the term dominating the cross section would be the BH-DVCS interference [44] term

$$
\sigma_{e^+} - \sigma_{e^-} \sim \Im \left[ T_{BH}^* T_{DVCS} \right],
$$

FIG. 23: a) the DVCS part of the amplitude; b), c) the Bethe-Heitler part.
where the Bethe-Heitler process will project out the imaginary part of DVCS amplitude magnifying it with its own full magnitude.

The imaginary part of the DVCS amplitudes is directly proportional to the GPDs, evaluated along the line $x = \xi$, and measures in this way the ‘envelope functions’ $H(\xi, \xi, t)$, $E(\xi, \xi, t)$, $\tilde{H}(\xi, \xi, t)$, and $\tilde{E}(\xi, \xi, t)$ (Eq. (106)).

Figures 24 through 32 show the results of our calculations for the energies and kinematics accessible at Jefferson Lab. The values of transferred momentum squared run from $t = -1$ to $t = -0.5$ limited by

$$t_{\text{min}} = -4M^2\xi^2/(1 - \xi^2) \quad (113)$$

for each of kinematical sets.

One can see, that the interference term is larger at lower momentum transfer $t$ even though it is responsible for a smaller percentage of the total cross section. As the initial energy of the electron $E_e$ goes up, the interference cross section value drops down by one order. The ratio of the interference term to the total cross section, nevertheless, stays the same: about 40 to 60 percents.

III.3 TWIST-3 CORRECTIONS FOR THE DVCS PROCESS

The twist-2 DVCS amplitude (104) is independent of $Q$. This means it has a scaling behavior. However, this amplitude is not complete beyond the leading order in $Q$. Although it is exactly gauge invariant $q_\mu T^{\mu\nu} = 0$ with respect to the virtual photon, the electromagnetic gauge invariance is violated by the real photon except in the forward direction $t = 0$. This violation of gauge invariance is a higher twist (twist-3) effect compared to the leading order term $T^{\mu\nu}$. Since the product of final photon 4-momentum and the DVCS amplitude at twist-2 accuracy is proportional to the transverse component of the transferred momentum $\Delta_\perp$, an improved DVCS amplitude has been used [45, 46] to restore the gauge invariance in the nonforward direction:

$$T^{\mu\nu}_{\text{DVCS}} = T^{\mu\nu}_{\text{DVCS-2}} + \frac{P^\nu}{(P \cdot q')}(\Delta_\perp)_{\lambda} T^{\mu\lambda}_{\text{DVCS-2}}. \quad (114)$$

It creates a correction term of higher order in $Q$ to the twist-2 DVCS amplitude $T^{\mu\nu}_{\text{DVCS-2}}$.

It is necessary to have an estimate of the effect of power suppressed (higher twist) contributions to those observables in order to be able to extract the twist-2 GPDs from DVCS observables at accessible values of the hard scale $Q$. 
FIG. 24: Cross section calculated at twist-2 level for $E_e = 4.25\text{GeV}$, $x_B = 0.2$, and $Q^2 = 1.3\text{GeV}^2$ for the different values of momentum transfer: $t = -0.1$ – solid curve, $t = -0.2$ – dotted curve, $t = -0.3$ – dashed curve, and $t = -0.5$ – dash-dotted curve.
FIG. 25: The same for $E_e = 4.25\text{GeV}$, $x_B = .2$, and $Q^2 = 1.5\text{GeV}^2$. 
FIG. 26: The same for $E_e = 6\text{GeV}$, $x_B = .15$, and $Q^2 = 1.5\text{GeV}^2$. 
FIG. 27: The same for $E_e = 6\text{GeV}$, $x_B = .2$, and $Q^2 = 1.5\text{GeV}^2$. 
FIG. 28: The same for $E_e = 6\text{GeV}$, $x_B = .2$, and $Q^2 = 2\text{GeV}^2$. 
FIG. 29: The same for $E_e = 6\text{GeV}$, $x_B = .3$, and $Q^2 = 1.5\text{GeV}^2$. 
FIG. 30: The same for $E_e = 6\text{GeV}$, $x_B = .3$, and $Q^2 = 2\text{GeV}^2$. 
FIG. 31: The same for $E_e = 6\text{GeV}$, $x_B = .3$, and $Q^2 = 2.5\text{GeV}^2$. 
FIG. 32: The same for $E_e = 11\text{GeV}$, $x_B = .3$, and $Q^2 = 2.5\text{GeV}^2$. 
The first power correction to the DVCS amplitude of order $O(1/Q)$ is called twist-3.

The twist-3 corrections to the DVCS amplitude, which have been derived and calculated recently by several groups [47–56] using different approaches. In our study we used the notations and definitions from [57]. The following section is a short overview of these definitions.

### III.3.1 Kinematical variables of the DVCS process

Let us start with the comparison of the kinematical variables used for the description of twist-3 amplitude with the set of variables we used in the previous section. There is only a slight difference, but it is worth mentioning.

There are four generalized structure functions used for the description of DVCS amplitude. They are denoted by $H$, $E$, $\tilde{H}$ and $\tilde{E}$ and depend on three variables. The light-cone momentum fraction $x$ is defined by $k^+ = xP^+$, where $k$ is the quark loop momentum and $P$ is the average nucleon momentum ($P = (p + p')/2$, where $p$ ($p'$) are the initial (final) nucleon four-momenta respectively). In the previous chapter we have used the initial momentum of the hadron $p$ as one of the basic light-cone momenta and up to twist-3 accuracy those momentum coincide as will be shown later. The skewedness variable $\xi$ is defined by $\Delta^+ = -2\xi P^+$, where $\Delta = p' - p$ is the same as $r$ in the previous chapter, the overall momentum transfer in the process, and where $2\xi \to x_B/(1 - x_B/2)$ in the Bjorken limit. The third variable entering the GPDs is given by the Mandelstam invariant $t = \Delta^2$, exactly as it was before.

Using momenta $P$ and $\Delta$ instead of $p = P - \Delta/2$ and $p' = P + \Delta/2$ we can define the active quark momentum as $k^+ - \Delta^+/2 = (x + \xi)P^+$ before the virtual photon impact and $k^+ + \Delta^+/2 = (x - \xi)P^+$ afterwards. The light-cone basis is chosen along the positive and negative $z$-direction (both $q^\mu$ and $P^\mu$ are collinear along the $z$-axis and have opposite direction): $\bar{p}^\mu = P^+/\sqrt{2}(1,0,0,1)$ and $n^\mu = 1/P^+ \cdot 1/\sqrt{2}(1,0,0,-1)$, satisfying $\bar{p} \cdot \bar{p} = 0$, $n \cdot n = 0$, $\bar{p} \cdot n = 1$ and using the notation $P^+ = P \cdot n$.

In this frame, the physical momenta entering the DVCS process have the following decomposition [45]:

$$P^\mu = \frac{1}{2} (p^\mu + p'^\mu) = \bar{p}^\mu + \frac{m^2}{2} n^\mu, \tag{115}$$

$$q^\mu = -\left(2\xi'\right) \bar{p}^\mu + \left(\frac{Q^2}{4\xi'}\right) n^\mu, \tag{116}$$
\[ \Delta^\mu \equiv p'^\mu - p^\mu = -(2\xi) \tilde{p}^\mu + \left(\xi \tilde{m}^2\right) n^\mu + \Delta^\mu_\perp, \]  
\[ q'^\mu \equiv q^\mu - \Delta^\mu = -2 \left(\xi' - \xi\right) \tilde{p}^\mu + \left(\frac{Q^2}{4\xi'} - \xi \tilde{m}^2\right) n^\mu - \Delta^\mu_\perp, \]

where \( \Delta_\perp \) is the perpendicular component of the momentum transfer \( \Delta \) (i.e. \( \tilde{p} \cdot \Delta_\perp = n \cdot \Delta_\perp = 0 \)), and where the variables \( \tilde{m}^2, \xi' \) and \( \xi \) are given by

\[ \tilde{m}^2 = M^2 - \frac{\Delta^2}{4}, \]
\[ 2\xi' = -q^+ = \frac{P \cdot q}{m^2} \left[ -1 + \sqrt{1 + \frac{Q^2 \tilde{m}^2}{(P \cdot q)^2}} \right] \xrightarrow{Bj} \frac{x_B}{1 - \frac{2\mu}{2}}, \]
\[ 2\xi = -\Delta^+ = 2\xi' \frac{Q^2 - \Delta^2}{Q^2 + m^2(2\xi')^2} \xrightarrow{Bj} \frac{x_B}{1 - \frac{2\mu}{2}}. \]

To twist-3 accuracy, Eqs. (115-118) reduce to

\[ P = \tilde{p}, \]
\[ \Delta = -2\xi P + \Delta_\perp, \]  
\[ q = -2\xi P + \frac{Q^2}{4\xi} n, \]
\[ q' = \frac{Q^2}{4\xi} n - \Delta_\perp. \]

Thus, the final (real) photon 4-momentum squared \( q'^2 - \Delta^2_\perp \) suggests the twist-3 accuracy \( t = \Delta^2 \rightarrow \Delta^2_\perp \rightarrow 0 \).

### III.3.2 Twist-3 DVCS amplitude

As a starting point we use the DVCS amplitude on the nucleon to the order \( \mathcal{O}(1/Q) \) derived explicitly in a parton model approach [48] and in a light-cone expansion framework [49]:

\[ H^{\mu\nu} = \frac{1}{2} \int_{-1}^{1} dx \left\{ \left[ (g^{\mu\nu})_\perp - \frac{P^\mu P^\nu}{(P \cdot q')} \right] n^\beta \mathcal{F}_\beta(x, \xi) C^+(x, \xi) \right\} \]

\[ - \left[ (g^{\rho\kappa})_\perp - \frac{P^\rho P^\kappa}{(P \cdot q')} \right] i(\epsilon_{\mu\rho\kappa})_\perp n^\beta \tilde{\mathcal{F}}_\beta(x, \xi) C^-(x, \xi) \]

\[ - \left[ (g^{\rho\kappa})_\perp - \frac{P^\rho P^\kappa}{(P \cdot q')} \right] \left[ (g^{\mu\nu})_\perp - \frac{P^\mu P^\nu}{(P \cdot q')} \right] \left\{ \mathcal{F}_k(x, \xi) + C^+(x, \xi) \right\} \]

\[ - i(\epsilon_{\mu\rho\kappa})_\perp \tilde{\mathcal{F}}^{\rho}(x, \xi) C^-(x, \xi) \} \]
where the functions $F_{\mu}$ and $\tilde{F}_{\mu}$ are given by:

\[
F_{\mu}(x, \xi) = \frac{\Delta_{\mu}}{2M\xi} \bar{N}(p') N(p) E(x, \xi) - \frac{\Delta_{\mu}}{2\xi} \bar{N}(p') \hat{n} N(p) (H + E)(x, \xi)
+ \int_{-1}^{1} du \ G_{\mu}(u, \xi) W_{+}(x, u, \xi) + i\epsilon_{\perp \mu k} \int_{-1}^{1} du \ \tilde{G}^{k}(u, \xi) W_{-}(x, u, \xi),
\]

\[(124)\]

\[
\tilde{F}_{\mu}(x, \xi) = \frac{\Delta_{\mu}}{2M} \bar{N}(p') \gamma_{5} N(p) \tilde{E}(x, \xi) - \frac{\Delta_{\mu}}{2\xi} \bar{N}(p') \hat{n} \gamma_{5} N(p) \tilde{H}(x, \xi)
+ \int_{-1}^{1} du \ \tilde{G}_{\mu}(u, \xi) W_{+}(x, u, \xi) + i\epsilon_{\perp \mu k} \int_{-1}^{1} du \ G^{k}(u, \xi) W_{-}(x, u, \xi).
\]

\[(125)\]

The following notations are used:

\[
G_{\mu}(u, \xi) = \bar{N}(p') \gamma_{\mu} N(p) (H + E)(u, \xi)
+ \frac{\Delta_{\mu}}{2\xi M} \bar{N}(p') N(p) \left[ u \frac{\partial}{\partial u} + \xi \frac{\partial}{\partial \xi} \right] E(u, \xi)
- \frac{\Delta_{\mu}}{2\xi} \bar{N}(p') \hat{n} N(p) \left[ u \frac{\partial}{\partial u} + \xi \frac{\partial}{\partial \xi} \right] (H + E)(u, \xi),
\]

\[(126)\]

\[
\tilde{G}_{\mu}(u, \xi) = \bar{N}(p') \gamma_{\mu} \gamma_{5} N(p) \tilde{H}(u, \xi)
+ \frac{\Delta_{\mu}}{2M} \bar{N}(p') \gamma_{5} N(p) \left[ 1 + u \frac{\partial}{\partial u} + \xi \frac{\partial}{\partial \xi} \right] \tilde{E}(u, \xi)
- \frac{\Delta_{\mu}}{2\xi} \bar{N}(p') \hat{n} \gamma_{5} N(p) \left[ u \frac{\partial}{\partial u} + \xi \frac{\partial}{\partial \xi} \right] \tilde{H}(u, \xi).
\]

\[(127)\]

The functions $W_{\pm}(x, u, \xi)$ are called [58] Wandzura-Wilczek kernels. They were introduced in Ref. [49, 51] and are defined as:

\[
W_{\pm}(x, u, \xi) = \frac{1}{2} \left\{ \theta(x > \xi) \frac{\theta(u > x)}{u - \xi} - \theta(x < \xi) \frac{\theta(u < x)}{u - \xi} \right\}
\pm \frac{1}{2} \left\{ \theta(x > -\xi) \frac{\theta(u > x)}{u + \xi} - \theta(x < -\xi) \frac{\theta(u < x)}{u + \xi} \right\}.
\]

\[(128)\]

We also introduce the metric and totally antisymmetric tensors in the two dimensional transverse plane ($\epsilon_{0123} = +1$):

\[
(-g^{\mu\nu})_{\perp} = -g^{\mu\nu} + n^{\mu} \tilde{p}^{\nu} + n^{\nu} \tilde{p}^{\mu}, \quad \epsilon_{\perp \mu\nu} = \epsilon_{\mu\nu\alpha\beta} n^{\alpha} \tilde{p}^{\beta}.
\]

\[(129)\]

and the coefficient functions $C_{\pm}(x, \xi)$ are defined as:

\[
C_{\pm}(x, \xi) = \frac{1}{x - \xi + i\epsilon} \pm \frac{1}{x + \xi - i\epsilon}.
\]

\[(130)\]
In the expression Eq. (123) for the DVCS amplitude to the twist-3 accuracy, the first two terms correspond to the scattering of transversely polarized virtual photons. Applying all the definitions to the first two terms of the expression Eq. (123) brings us to the twist-2 DVCS amplitude like we had in the previous chapter. The vector part

$$T_1 = \frac{1}{2} \left( -g_\perp^{\mu\nu} - \frac{P_\mu \Delta^\nu}{(P \cdot q')} \right) \left( -\frac{1}{M} \bar{u}(p') u(p) C_{11} + \bar{u}(p') \gamma_5 u(p) C_{12} \right),$$

(131)

with

$$C_{11}(\xi) = \int_{-1}^{1} E(x, \xi) C^+(x, \xi) dx,$$

(132)

$$C_{12}(\xi) = \int_{-1}^{1} (E(x, \xi) + H(x, \xi)) C^+(x, \xi) dx,$$

(133)

and the axial-vector part

$$T_2 = -\frac{i e_k k^{\mu}}{2} \left( -g_\perp^{\mu k} - \frac{P_\mu \Delta^\mu}{(P \cdot q')} \right) \left( -\frac{\xi}{M} \bar{u}(p') \gamma_5 u(p) C_{21} + \bar{u}(p') \gamma_5 u(p) C_{22} \right),$$

(134)

with

$$C_{21}(\xi) = \int_{-1}^{1} \tilde{E}(x, \xi) C^-(x, \xi) dx,$$

(135)

$$C_{22}(\xi) = \int_{-1}^{1} \tilde{H}(x, \xi) C^-(x, \xi) dx.$$

(136)

This part of the amplitude, containing \(n_\beta F^\beta\) and \(n_\beta \tilde{F}^\beta\), depends only on the twist-2 GPDs \(H, E\) and \(\tilde{H}, \tilde{E}\) and was elaborated in Refs. [45,46]. Comparison with structure blocks of Eq. (104) gives us the idea of how the structure functions \(T^a\) are related to the structure functions parametrizing the tensor \(E(x, \xi)\), vector \(H(x, \xi)\), pseudoscalar \(\tilde{E}(x, \xi)\), and axial-vector \(\tilde{H}(x, \xi)\) transitions:

$$C_{11}(\xi) \leftrightarrow \frac{2}{1 + \xi} T^a_R(\xi),$$

(137)

$$C_{12}(\xi) - C_{11}(\xi) \leftrightarrow \frac{1}{(1 + \xi)} T^a_F(\xi),$$

(138)

$$C_{21}(\xi) \leftrightarrow \frac{1}{2(1 + \xi)} T^a_P(\xi),$$

(139)

$$C_{12}(\xi) \leftrightarrow -\frac{1}{2(1 + \xi)} T^a_G(\xi).$$

(140)

The generalized parton distributions coincide with the quark distributions at vanishing momentum transfer and the first moments of the GPDs are related to the elastic
form factors of the nucleon through model independent sum rules [1,2]:

\[\int_{-\frac{1}{2}}^{+\frac{1}{2}} dx H^q(x, \xi, t) = F_1^q(t), \quad (141)\]
\[\int_{-\frac{1}{2}}^{+\frac{1}{2}} dx E^q(x, \xi, t) = F_2^q(t), \quad (142)\]
\[\int_{-\frac{1}{2}}^{+\frac{1}{2}} dx \bar{H}^q(x, \xi, t) = g_A^q(t), \quad (143)\]
\[\int_{-\frac{1}{2}}^{+\frac{1}{2}} dx \bar{E}^q(x, \xi, t) = h_A^q(t), \quad (144)\]

while the second Mellin moment is closely related to the quark orbital momentum contribution to the proton spin [48].

The third term in Eq. (123) corresponds to the contribution of the longitudinal polarization of the virtual photon. Defining the polarization vector of the virtual photon as

\[\varepsilon_L^\mu(q) = \frac{1}{Q} \left(2\xi P^\mu + \frac{Q^2}{4\xi} n^\mu \right), \quad (145)\]

we can easily calculate the DVCS amplitude for longitudinal polarization of the virtual photon \((L \rightarrow T\) transition), which is purely of twist-3:

\[\langle \varepsilon_L \rangle_H^{\mu\nu} = \frac{2\xi}{Q} \int_{-1}^{1} dx \left(\mathcal{F}_1^\nu C^+(x, \xi) - i\varepsilon_\perp^{\nu\kappa} \mathcal{F}_\perp\perp^{\kappa} C^-(x, \xi) \right), \quad (146)\]

It is therefore seen that this term depends only on new ‘transverse’ GPDs \(\mathcal{F}_\perp^\mu\) and \(\mathcal{F}_\perp^{\mu\kappa}\), which can be related to the twist-2 GPDs \(H, E, \bar{H}\) and \(\bar{E}\) with help of Wandzura-Wilczek relations given by Eqs. (124-127). Twist-3 part of the amplitude could be divided into three separate terms. Two of them

\[T_3 = -\frac{1}{2} \frac{\xi^2}{(P \cdot q) q^\mu} (g_{\perp\kappa} - P^\nu \Delta^k) \left(\bar{u}(p') u(p) \Delta_{\perp k} C_{31} + \bar{u}(p') \gamma_{\perp k} u(p) \Delta_{\perp k} C_{32}\right), \quad (147)\]

are similar to \(T_1\) and \(T_2\) and give no output to the cross section correction at twist-3 accuracy. The third one

\[T_5 = -\frac{1}{2} \frac{\xi^2}{(P \cdot q) q^\mu} (g_{\perp\kappa} - P^\nu \Delta^k) \left(\bar{u}(p') \gamma_{\kappa} u(p) C_{33} + i \gamma_{\perp l} \bar{u}(p') \gamma_{l} \gamma_{\perp 5} u(p) C_{41}\right), \quad (149)\]
with
\[
C_{33}(\xi) = \int_{-1}^{1} du \left\{ (E(u,\xi) + H(u,\xi)) \right. \\
\left. \int_{-1}^{1} dx \left( C^+(x,\xi)W^+(x,u,\xi) - C^-(x,\xi)W^-(x,u,\xi) \right) \right\},
\]
(150)
\[
C_{41}(\xi) = \int_{-1}^{1} du \left\{ \tilde{H}(u,\xi) \right. \\
\left. \int_{-1}^{1} dx \left( C^+(x,\xi)W^-(x,u,\xi) - C^-(x,\xi)W^+(x,u,\xi) \right) \right\},
\]
(151)
is the only one that needs to be calculated for the estimate of twist-3 corrections.

### III.3.3 Twist-3 DVCS cross section

In the leading twist-2 approximation, the amplitude squared falls off as \(1/Q^4\):
\[
|T_{\text{twist2}}|^2 = \frac{2e^6}{Q^4\xi^2(1-\xi^2)} \left( (\beta^2 + \xi^2)Q^2 - 4\beta\xi(k_\perp \cdot \Delta_\perp) \right) \\
\left( |C_{11} - (1 - \xi^2)C_{12}|^2 + |\xi^2C_{21} - (1 - \xi^2)C_{22}|^2 \right).
\]
(152)
Next to the leading, twist-3, approximation is suppressed by \(1/Q^2\):
\[
|T_{\text{twist3}}|^2 = \frac{8e^6(Q^2 - 2t)M^2}{Q^4(Q^2 + t)^2(1-\xi^2)} \left( (\beta^2 + 3\xi^2)Q^2 - 4\beta\xi(k_\perp \cdot \Delta_\perp) \right) \\
\left( - 12\xi^2|C_{33}|^2 + (4\xi C_{33} + C_{41}|^2 \right).
\]
(153)
Here we used the notation:
\[
\beta \equiv \beta_k + \xi = \xi - 4\xi^2 k_\perp \cdot q' / Q^2.
\]
(154)

Coefficient functions \(C_{33}\) and \(C_{41}\) defined by Eqs. (150) and (151) are affected by the Wandzura-Wilczek transformation. To better understand the properties of the WW transformations let us consider two limiting cases of the WW transformation: the forward limit \(\xi \to 0\) and the ‘meson’ limit \(\xi \to 1\). In the forward limit we easily obtain:
\[
\lim_{\xi \to 0} \int_{-1}^{1} du W_+(x,u,\xi) f(u,\xi) = \theta(x \geq 0) \int_{x}^{1} \frac{f(u,0)}{u} du \\
- \theta(x \leq 0) \int_{-1}^{x} \frac{f(u,0)}{u} du,
\]
(155)
\[
\lim_{\xi \to 0} \int_{-1}^{1} du W_-(x,u,\xi) f(u,\xi) = 0.
\]
(156)
From the second expression it follows that the $W$ kernel is a ‘genuine non-forward’ object as it disappears in the forward limit.

In the limit $\xi \to 1$ the generalized parton distributions have properties of meson distribution amplitudes [64,65]. In this limit the WW transformations have the form:

$$\lim_{\xi \to 1} \int_{-1}^{1} du W_\pm(x,u,\xi) f(u,\xi) = \frac{1}{2} \left\{ \int_{-1}^{x} \frac{du}{1-u} f(u,1) \pm \int_{x}^{1} \frac{du}{1+u} f(u,1) \right\}.$$

In Refs. [50,53,54] it was demonstrated that the twist-3 skewed parton distributions in the WW approximation exhibit discontinuities at the points $x = \pm \xi$. Using general properties of the WW transformation (158) and Eqs. (124,125) one obtains:

$$\lim_{\delta \to 0} \int_{-1}^{1} du (W_\pm(\xi + \delta, u, \xi) - W_\pm(\xi - \delta, u, \xi)) f(u,\xi) = \frac{1}{2} \mathcal{P} \int_{-1}^{1} \frac{f(u,\xi)}{u-\xi} du,$$

$$\lim_{\delta \to 0} \int_{-1}^{1} du (W_\pm(-\xi + \delta, u, \xi) - W_\pm(-\xi - \delta, u, \xi)) f(u,\xi) = \pm \frac{1}{2} \mathcal{P} \int_{-1}^{1} \frac{f(u,\xi)}{u+\xi} du. \tag{157}$$

Here $\mathcal{P}$ means an integral in the sense of principal value. We see that for a very wide class of functions $f(u,\xi)$, the discontinuity of the corresponding WW transforms is nonzero. This feature of the WW transformation may lead to the violation of the factorization for the twist-3 DVCS amplitude.

Fortunately, some combinations of the distributions $F_\mu$ and $\tilde{F}_\mu$ are free of discontinuities due to a certain symmetry of the Eqs. (124) and (125). For example:

$$F_\mu(x,\xi) - i\varepsilon_{\mu\rho} \tilde{F}_\rho(x,\xi), \tag{158}$$

is free of the discontinuity at $x = \xi$ while its ‘dual’ combination:

$$F_\mu(x,\xi) + i\varepsilon_{\mu\rho} \tilde{F}_\rho(x,\xi), \tag{159}$$

has no discontinuity at $x = -\xi$. The cancellation of discontinuities in these particular combinations of the GPDs ensures the factorization of the twist-3 DVCS amplitude on the nucleon.

One of the non-trivial properties of the generalized parton distributions is the polynomiality of their Mellin moments which follows from the Lorentz invariance of nucleon matrix elements [1,2,59]:

$$\int_{-1}^{1} dx \ x^N H^q(x,\xi) = h_0^q(N) + h_2^q(N) \xi^2 + \ldots + h_{N+1}^q(N) \xi^{N+1}, \tag{160}$$

$$\int_{-1}^{1} dx \ x^N E^q(x,\xi) = e_0^q(N) + e_2^q(N) \xi^2 + \ldots + e_{N+1}^q(N) \xi^{N+1}.$$
The time reversal invariance requirement [60, 61] leaves only even powers of the skewedness parameter $\xi$. This fact implies that the highest power of $\xi$ is $N + 1$ for odd $N$ (singlet GPDs) and $N$ for even $N$ (nonsinglet GPDs). Furthermore, there is a relation between the highest power of $\xi$ for the singlet functions $H^q$ and $E^q$ due to the fact that the nucleon has spin $1/2$ [1, 2, 61]:

$$e^{q(N)}_{N+1} = -h^{q(N)}_{N+1}.$$  \hspace{1cm} (161)

The polynomiality conditions (160) strongly restrict the class of functions of two variables $H^q(x, \xi)$ and $E^q(x, \xi)$. In the previous section the polynomiality conditions was implemented by using the double distributions [3,14,42]. In this case the generalized distributions are obtained as a one-dimensional section (Eq. 103) of the two-variable double distributions $F(x, y)$ (Eq. 101) [62].

It is easy to check that the GPDs obtained by reduction from the double distributions satisfy the polynomiality conditions (160) but always lead to $h^{q(N)}_{N+1} = e^{q(N)}_{N+1} = 0$, i.e. the highest power of $\xi$ for the singlet GPDs is absent. In other words the parametrization of the singlet GPDs in terms of double distributions is not complete. It can be completed by adding the so-called D-term to Eq. (103) [63]:

$$F_\xi(X) = \int_0^1 dx \int_0^{1-x} \delta(x + \zeta y - X)F(x, y)dy \pm \theta \left[1 - \frac{X^2}{\zeta^2}\right] D\left(\frac{X}{\zeta}\right).$$  \hspace{1cm} (162)

Note that for both tensor $E^q(x, \xi)$ and vector $H^q(x, \xi)$ GPDs the absolute value of the D-term is the same and has an opposite sign.

For the quark helicity dependent GPDs $\tilde{H}(x, \xi)$ and $\tilde{E}(x, \xi)$ the D-term is absent.

III.4 CONCLUSION

In this chapter, we give the predictions for the DVCS cross section for the kinematics reachable by CEBAF at Jefferson Lab. The DVCS amplitude was estimated using the model for the non-forward parton distributions from [43]. The results for the DVCS part of the total cross section are close to those obtained in [57].

As was pointed out, the contribution of the interference term to the differential cross section is small compared to that of the Bethe-Heitler part alone. This gives the idea of how difficult it could be to get the information on real and imaginary parts of the DVCS amplitude from the experiment. However, it is a well known fact that
the interference term for the cross section of DVCS with the electrons of opposite helicity will give us an estimate of the imaginary part of the DVCS amplitude, while the asymmetry of electron-positron cross sections will reveal the real part of the amplitude.

Moreover, at sufficiently large $\Theta_{CM}^{\gamma\gamma}$ angles the DVCS part of the total cross section dominates, but those angles correspond to the large values of $|t|$. That is why it is also necessary to create a “non-zero-$t$” model of the non-forward distributions to better estimate the DVCS input from the generalized parton distributions.
CHAPTER IV

CONCLUSION

In this dissertation two aspects of deeply virtual Compton scattering were studied.

One aspect considers the small-$x$ DVCS where the energy of the incoming virtual photon is very large in comparison to its virtuality. The DVCS in this region can be described by the BFKL pomeron. At the same time these kinematics \(x \sim 10^{-2} - 10^{-4}, Q^2 \geq 6\text{GeV}^2\), and \(-t \sim 1 - 5\text{GeV}^2\) could be accessed at HERA [18].

It is shown that at large momentum transfer the coupling of the BFKL pomeron to the nucleon is equal to the Dirac form factor of the nucleon. It allows us to calculate the DVCS cross section and to investigate \(t\)-dependence of the amplitude in a model-independent way. These approximate calculations of the DVCS cross section in the Regge regime in QCD are very timely because all the other predictions for the small-$x$ DVCS rely on some model assumptions.

The algorithm for the calculation of the photon impact factor was elaborated. It allows also to estimate the meson production processes amplitude.

The study showed that the \(t\)-dependence of the BFKL pomeron is very important and it should be possible to detect at \(t < 2\text{GeV}^2\).

Unfortunately, this approximation contains some uncertainties. There is no evidence that the next \(\sim (\alpha_s \ln x)^3\) term in the BFKL series is as small as we assume. Also, we are unable to fix the \(\alpha_s\) in the leading logarithmic approximation.

Thus, the first part of this study is devoted to the exploration of the deeply virtual Compton scattering process in the Regge regime. It gives an estimation for the cross section in soon to be accessible kinematics without any model assumptions. On the other hand, it does not reveal any information about the structure of the targeted nucleon.

To study the internal structure of the nucleon, we used the concept of Generalized Parton Distributions within the quantum chromodynamics framework. Those distribution functions are accessible through deeply virtual exclusive reactions and mainly through the deeply virtual Compton scattering at the kinematical region of high virtuality \(Q^2\) and low momentum transfer \(t\).

In our study we analyzed the leading twist-2 approximation of DVCS amplitude and prepared the algorithm for the estimation of the next-to-leading order, twist-3 term of the amplitude. In our analysis we neglected the \(\Delta^2_\perp\) (twist-3 approximation)
FIG. 33: $\phi$ dependence of the beam spin asymmetry $A$ [66]. $Q^2 = 1.25(\text{GeV/c})^2$, $x_B = 0.19$, and $-t = 0.19 \ (\text{GeV/c})^2$.

which is natural for the parton model.

DVCS is regarded as the cleanest tool to access the underlying GPDs. Unfortunately, virtual Compton scattering is always in competition with the dominating BH process.

In our study we created a computer program for precise calculation of the Bethe-Heitler process in the required kinematics.

To exclude the unnecessary input from BH process, the study of asymmetry with two electrons of opposite helicities was invoked. It was shown that the main input in this case comes from the interference DVCS-BH term, which simultaneously filters DVCS observables and magnifies them by the Bethe-Heitler magnitude.

Numerical algorithms for calculating the DVCS amplitude in twist-2 and twist-3 approximations were developed. The results for the DVCS cross section at different kinematical regions accessible on CEBAF at Jefferson Lab in twist-2 approximation were presented. The first measurements of the beam spin asymmetry in the DVCS regime were performed at CEBAF at Jefferson Lab [66, 71]. As was expected, the interference of the DVCS and Bethe-Heitler processes provide a clear asymmetry (Eq. (112)). The results of our study based on models from [43] agree in sign, and in magnitude to predictions made before [46, 57] and to the experimental results represented on Figs. 33 and 34.

It proves that DVCS could be a good tool to access GPDs at relatively low energies and momentum transfers.
FIG. 34: The new data sample at 5.75 GeV [71] averaged over the entire kinematic range of the data.

With the upcoming CEBAF upgrade to 12 GeV [70] at Jefferson Lab further measurements at higher beam energy are expected. Construction of new detectors, increase in beam energy, and an improvement of polarization, luminosity, acceptance, and precision will broaden the range of the $Q^2$ and $x_B$. This will make possible a better understanding of nucleon structure.

From the theoretical side of the study, higher order perturbative QCD corrections as well as new models of $t$-dependence of GPDs are expected.
BIBLIOGRAPHY

[1] Ji X., Phys. Rev. Lett. 78, 610 (1997).
[2] Ji X., Phys. Rev. D 55, 7114 (1997).
[3] A.V. Radyushkin, Phys. Lett. B 380, 417 (1996).
[4] K. Watanabe, Prog. Theor. Phys. 67, 1834 (1982).
[5] J. Bartels and M. Loewe, Z. Phys. C 12, 263 (1982).
[6] A. Airapetian, et al. (HERMES Coll.), Phys. Rev. Lett. 87, 182001 (2001).
[7] P.R.B. Saull (ZEUS Coll.), Prompt photon production and observation of deeply virtual Compton scattering, hep-ex/0003030.
[8] C. Adloff et al. (H1 Coll.), Phys. Lett. B 517, 47 (2001).
[9] S. Stepanyan et al. (CLAS Coll.), Phys. Rev. Lett. 87, 182002 (2001).
[10] V. Burkert, D. Crabb, R. Minehart (CLAS Coll.), E-91-023; V. Burkert, L. Elouadrhiri, M. Garçon, S. Stepanyan (CLAS Coll.), Deeply virtual Compton scattering with CLAS at 6 GeV, E-01-113.
[11] P. Bertin, C.E. Hyde-Wright, F. Sabatié et al. (Hall A Jefferson Lab), Deeply virtual Compton scattering at 6 GeV, PCCF-RI-0013.
[12] V.A. Korotkov, W.D. Nowak, Eur. Phys. J. C 23, 455 (2002); W.D. Nowak, European perspectives for electron-nucleon scattering at the luminosity frontier, Nucl. Phys. Proc. Suppl. 105, 171 (2002).
[13] R. P. Feynman, The Photon-Hadron Interaction (Benjamin, Reading 1972).
[14] A. V. Radyushkin, Phys. Rev. D 56, 5524 (1997).
[15] X. Ji and J. Osborne, Phys. Rev. D 58, 094018 (1998).
[16] J. C. Collins and A. Freund, Phys. Rev. D 59, 074009 (1999).
[17] A.V. Radyushkin, Phys. Lett. B 385, 333 (1996).
[18] [ZEUS Collaboration], Observation of DVCS in $e^+e^-$ Interactions at HERA, paper submitted to EPS HEP99 Conference, Tampere, 1999 (see also http://www-zeus.desy.de).

[19] L.L. Frankfurt, A. Freund, and M.I. Strikman, Phys. Rev. D 58, 114001 (1998), Erratum- Phys. Rev. D 59, 119901 (1999).

[20] P.A.M. Guichon, M. Vanderhaeghen, Prog. Part. Nucl. Phys. 41, 125 (1998).

[21] J.C. Collins, L.L. Frankfurt, and M.I. Strikman, Phys. Rev. D 56, 2982 (1997).

[22] L.N. Lipatov, Phys. Reports 286, 131 (1997).

[23] R.E. Cutkosky, J. Math. Phys. 1, 429 (1960).

[24] S. Coleman, R.E. Norton, Nuovo Cimento 38, 438 (1965).

[25] H. Cheng and T.T. Wu, Phys. Rev. 182, 1852 (1969), Phys. Rev. D 1, 2775 (1970); V.N. Gribov et al., Sov. J. Nucl. Phys. 12, 543 (1971).

[26] I. Balitsky, Nucl. Phys. B 463, 99 (1996).

[27] I. Balitsky, Phys. Lett. B 124, 230 (1983).

[28] A.V. Radyushkin, Phys. Rev. D 56, 5524 (1997).

[29] A.V. Radyushkin, Phys. Rev. D 58, 114008 (1998).

[30] V.S. Fadin, E.A. Kuraev, and L.N. Lipatov, Phys. Lett. B 60, 50 (1975); I.I. Balitsky and L.N. Lipatov, Sov. Journ. Nucl. Phys. 28, 822 (1978).

[31] L.N. Lipatov ans V.S. Fadin, Phys. Lett. B 429, 127 (1998), Nucl. Phys. B 477, 767 (1997), Nucl. Phys. B 406, 259 (1993).

[32] G. Carnici and M. Ciafaloni, Phys. Lett. B 412, 396 (1997), Phys. Lett. B 430, 349 (1998).

[33] J.B. Bronzan, G.L. Kane, and U.P. Sukhatme, Phys. Lett. B 49, 272 (1974).

[34] I.I. Balitsky and L.N. Lipatov, JETP Letters 30, 355 (1979).

[35] L.N. Lipatov, Nucl. Phys. B 452, 369 (1996).
[36] I. Balitsky, Phys. Rev. D 60, 014020 (1999).

[37] M.I. Strikman, private communication.

[38] M. Diehl, T. Gousset, B. Pire, J.P. Ralston, Phys. Lett. B 411, 193 (1997).

[39] A.V. Belitsky, D. Mueller, L. Niedermeier, and A. Shaefer, Nucl. Phys. B 593, 289 (2001).

[40] L.L. Frankfurt, A. Freund, and M.I. Strikman, Phys. Lett. B 460, 417 (1999).

[41] I. Balitsky, E. Kuchina, Phys. Rev. D 62, 074004 (2000).

[42] D. Müller, D. Robaschik, B. Geyer B., F.-M. Dittes, and J. Horejsi, Fortschr. Phys. 42, 101 (1994).

[43] I.V. Musatov, A.V. Radyushkin, Phys. Rev. D 61, 074027 (2000).

[44] S.J. Brodsky, F. E. Close and J. F. Gunion, Phys. Rev. D 6, 177 (1972).

[45] P.A.M. Guichon, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 41, 125 (1998).

[46] M. Vanderhaeghen, P.A.M. Guichon, and M. Guidal, Phys. Rev. D 60, 094017 (1999).

[47] I.V. Anikin, B. Pire, and O.V. Teryaev, Phys. Rev. D 62, 071501 (2000).

[48] M. Penttinen, M.V. Polyakov, A.G. Shuvaev, and M. Strikman, Phys. Lett. B 491, 96 (2000).

[49] A.V. Belitsky, and D. Müller, Nucl. Phys. B 589, 611 (2000).

[50] A.V. Radyushkin, and C. Weiss, Phys. Lett. B 493, 332 (2000).

[51] N. Kivel, and M.V. Polyakov, Nucl. Phys. B 600, 334 (2001).

[52] N. Kivel, M.V. Polyakov, and M. Vanderhaeghen, Phys. Rev. D 63, 114014 (2001).

[53] N. Kivel, M.V. Polyakov, A. Schäfer and O.V. Teryaev, Phys. Lett. B 497, 73 (2001).
[54] A.V. Radyushkin, and C. Weiss, Phys. Rev. D 63, 114012 (2001).

[55] A.V. Belitsky, D. Müller, A. Kirchner and A. Schäfer, Phys. Rev. D 64, 116002 (2001).

[56] A.V. Belitsky, and D. Müller, Phys. Lett. B 507, 173 (2001).

[57] K. Goeke, M. V. Polyakov, M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47, 401 (2001).

[58] S. Wandzura and F. Wilczek, Phys. Lett. B 72, 195 (1977).

[59] X. Ji, and R. F. Lebed, Phys. Rev. D 63, 076005 (2001).

[60] L. Mankiewicz, G. Piller, and T. Weigl, Eur. Phys. J. C 5, 119 (1998).

[61] X. Ji, J. Phys. G 24, 1181 (1998).

[62] A.V. Radyushkin, hep-ph/0101225.

[63] M.V. Polyakov, and C. Weiss, Phys. Rev. D 60, 114017 (1999).

[64] P. Ball, and V.M. Braun, Phys. Rev. D 54, 2182 (1996).

[65] P. Ball, V.M. Braun, Y. Koike and K. Tanaka, Nucl. Phys. B 529, 323 (1998).

[66] S. Stepanyan et al., Phys. Rev. Lett. 87, 182002 (2001).

[67] L. Elouadrhiri, Nucl. Phys. A 711 (2002) 154.

[68] V. Burkert, L. Elouadrhiri, M. Garçon, S. Stepanyan et al., Jefferson Lab Experiment E01-113.

[69] P. Bertin, C. Hyde-Wright, F. Sabatié et al., Jefferson Lab Experiment E00-110.

[70] “The Hall B 12 GeV Upgrade Preconceptual Design Report,” December, 2002, and references therein.

[71] E. S. Smith for the CLAS Collaboration, AIP Conf. Proc. 698, 129 (2004).
APPENDIX A

BETHE-HEITLER CROSS-SECTION IN DVCS REGIME

This program was created on the request of experimentalists at Jefferson Lab. It calculates the cross-section of the Beth-Heitler process at DVCS kinematics. The cross-section is calculated in nb/GeV$^4$ with respect to the initial and final electron energy and azimuthal and out-of-the-plane angles of the final electron and photon.

This program uses the Dirac $F_1^p(t)$ and Pauli $F_2^p(t)$ form factors of the proton as a nucleon form factors.

Subroutine BH(E0,Ee,ae,ag,fe,fg,cs) calculates the cross section of the $e+p\rightarrow e'+p'+\gamma$ reaction, which is differential with respect to $Q^2$, Bjorken $x$, $t$ and out-of-lepton-plane angle $f$ for the process where the photon is emitted from the initial or final lepton (Bethe-Heitler process).

Input parameters include:

- E0 - initial electron energy, Gev
- Ee - final electron energy, Gev
- ae - azimuthal angle of final electron, rad
- ag - azimuthal angle of emitted photon, rad
- fe - out-of-plane electron angle, rad
- fg - out-of-plane photon angle, rad

Outcoming parameter is

- cs - Bethe-Heitler cross section, nb/GeV**4

Program BetheHeitler

Pi=3.141592663
E0=5.75
Ee=3.75
ag=20.*Pi/180.
fe=Pi
fg=0.
do 1 i=-24,24
Subroutine BH(E0,Ee,ae,ag,fe,fg,cs)

Real M,k1q2
Pi=3.141592663
M=.938
x=E0*Ee*(1.-Cos(ae))/((E0-Ee)*M)
y=(E0-Ee)/E0
Eg=-(-E0*(Ee*(1-Cos(ae))-M)-Ee*M)/(-E0*(1-Cos(ag))-M+
+Ee*(1-Cos(ae)*Cos(ag)-Cos(fe-fg)*Sin(ae)*Sin(ag))))
if(Eg.lt.0) return
Q2=2*E0*Ee*(1-Cos(ae))
t=2*(-E0*(Ee*(1-Cos(ae))+Eg*(1-Cos(ag)))+
+Ee*Eg*(1-Cos(ae)*Cos(ag)-Cos(fe-fg)*Sin(ae)*Sin(ag)))
k1q2=Eg*(1-Cos(ag))/(2*Ee*(1-Cos(ae)))
elch=2.*Sqrt(Pi/137.)
vph=(x*y**2)/(512.*Pi**4*Q2**2*Sqrt(1+(4*M**2*x**2)/Q2))
bhsq=-(2*(2*M**2*(1+8*k1q2**2+t**2/Q2)**2+2)+
+k1q2*(4+4*t/Q2))**x**2*y**2+t*(4-2*(2+x+4*k1q2*x+t*x/Q2)*y+
+((t**2*x**2+2*Q2*t*x*(1+2*k1q2*x)+Q2**2*(2+x**2+8*
+k1q2**2*x**2+4*k1q2*x*(1+x)))*y**2)/Q2**2)**2)
*F1(t)**2)/(x**2*y**2)+4*t*(1+8*k1q2**2+t**2/Q2**2+k1q2*
*(4+4*t/Q2))*F1(t)*F2(t)+(t*(M**2*(1+8*k1q2**2+t**2)/
/Q2**2+k1q2*(4+4*t/Q2))**x**2*y**2+t*(-2+(2+x+4*k1q2*x+
t*x/Q2)+y-(1+2*k1q2*x+t*x/Q2)*y**2))*F2(t)**2)/
/(M**2*x**2+y**2)))/(k1q2*t**2+(1+2*k1q2+t/Q2))
cs=elch**6*bhsq*vph/(2.56942E-6)
return
end
Function F1(t)
\[
\begin{align*}
f_1 &= \frac{1}{(1-t/0.706)^2} \times \frac{1-2.79t/(4 \times 0.938^2)}{1-t/(4 \times 0.938^2)} \\
\text{Function } F_2(t) \\
f_2 &= \frac{1}{(1-t/0.706)^2} \times \frac{1.79}{1-t/(4 \times 0.938^2)} \\
\end{align*}
\]
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