Top quark electric and chromo electric dipole moments in the general two Higgs Doublet model.

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Abstract

We study the electric and chromo electric dipole moment of top quark in the general two Higgs Doublet model (model III). We analyse the dependency of this quantity to the new phases coming from the complex Yukawa couplings and masses of charged and neutral Higgs bosons. We observe that the electric and chromo electric dipole moments of top quark are at the order of $10^{-21} \, e \, cm$ and $10^{-20} \, g_s \, cm$, which are extremely large values compared to the ones calculated in the SM and also the two Higgs Doublet model with real Yukawa couplings.

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1 Introduction

Among all fermions, included in the standard model (SM), the top quark reaches great interest, since it breaks the $SU(2) \times U(1)$ symmetry maximally due to its large mass. The investigation of top quark properties, such as mass, decay width, decay products and some fundamental quantities like electric dipole moment (EDM), chromo electric dipole moment (CEDM),..., etc., becomes important.

EDM (CEDM) of a fermion is induced by the CP violating interaction and it provides comprehensive informations in the determination of the free parameters of the various theoretical models. There are large number of experimental studies in the literature about EDMs of fermions. Neutron EDM has a special interest and the experimental upper bound has been found as $d_N < 1.1 \times 10^{-25} e \text{cm}$ \cite{1}. The electron, muon and tau EDMs have been measured experimentally as $d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27} e \text{cm}$ \cite{2}, $d_\mu = (3.7 \pm 3.4) \times 10^{-19} e \text{cm}$ \cite{3} and $d_\tau = (3.1) \times 10^{-16} e \text{cm}$ \cite{4} respectively. With these measurements, it would be possible to get powerful clues about the internal structure of the particles, if it exists.

The complex Cabbibo-Kobayashi-Maskawa (CKM) matrix elements are the sources of CP violation in the SM and therefore nonzero EDM or CEDM. However, EDM (CEDM) of quarks in the SM exists at the three loop level \cite{5} and estimated as $\sim 10^{-30} (e (g_s) \text{cm})$ even for top quark case. Notice that quark EDM vanishes at one loop order in the SM, since moduli of matrix element is involved in the relevant expression. Furthermore, it also vanishes at two loop order after sum over internal flavours \cite{6, 7}. When QCD corrections are taken into account, nonzero EDM exists \cite{8}. Therefore, one has an open window to go beyond the SM and test a new physics. There are many sources of CP violation in the models beyond the SM, such as multi Higgs doublet models (MHDM), supersymmetric model (SUSY), extra dimensions \cite{9},..., etc.

The quark EDM was calculated in the multi Higgs doublet models in the literature \cite{10, 11, 12, 13}. EDM, induced by the neutral Higgs boson effects, was studied in the two Higgs doublet model (2HDM) \cite{11} and in \cite{13}, the necessity of more scalar fields than just two Higgs doublets was emphasised for non-zero EDM when only the charged Higgs boson effects were taken into account in \cite{12}. The EDM and weak EDM were calculated in the 2HDM and models with three and more Higgs doublets. Weak EDM of b-quark was predicted in the range $10^{-21} - 10^{-20} e \text{cm}$, following the scenario where CP violation may only come from the neutral Higgs sector. Furthermore, b-quark EDM was obtained in the range $10^{-23} - 10^{-22} e \text{cm}$ when the CP violating effect comes from the charged sector. In \cite{14}, quark EDM was calculated in
the 2HDM if the CP violating effects are due to the CKM matrix elements and it was thought that $H^\pm$ particles also mediate CP violation besides $W^\pm$ bosons, however, at the two loop order these new contributions vanish. In [13], the electric and weak electric dipole form factors of heavy fermions in a general two Higgs doublet model were studied and it was concluded that the enhancement of three orders of magnitude in the electric dipole form factor of the $b$ quark with respect to the prediction of 2HDM I and II was possible. The EDM of $b$-quark in the general 2HDM (model III) [16] and the general 3HDM with $O(2)$ symmetry in the Higgs sector ($3HDM(O_2)$) has been studied in [17] and it was observed that a large EDM, at the order of $10^{-20} \text{e cm}$, could be obtained, using the complex Yukawa couplings. In [18], the leading contribution to the EDM and CEDM of the top quark was calculated in Higgs-boson-exchange models of CP nonconservation. The dipole moments were estimated of the order $10^{-20} (e(g_s) \text{cm})$. In this work, EDM (CEDM) was assumed to arise at one loop order through neutral Higgs boson exchange.

In our work, we study EDM and CEDM of top quark in the model III, including charged Higgs contribution. In this case the sources of CP violation are the complex Yukawa couplings $\xi^U_{N,tt}$ and $\xi^D_{N,bb}$, which bring two independent CP violating parameters, $\sin \theta_{tt}$ and $\sin \theta_{tb}$ (see section two for their definitions). These parameters play the important role for the CP violating interactions, which are responsible for the EDM (CEDM) of top quark. It is interesting to study the sensitivity of top quark EDM (CEDM) to these parameters, since it gives a comprehensive information about the new physics beyond the SM and also the sign of the Wilson coefficient $C^\text{eff}_7$ (see Discussion). Furthermore, the dependencies of EDM (CEDM) to the new Higgs boson masses, namely $m_{H^\pm}$, $m_{h^0}$ and $m_{A^0}$, are informative in the determination of model parameters. This work is devoted to above analysis and presents the upper and lower limits of EDM (CEDM), which are obtained by using the present experimental results. The numerical values of EDM (CEDM) of top quark can be estimated at the order of the magnitude of $10^{-21} (e\text{ cm}) (10^{-21} - 10^{-20} (g_s \text{ cm})).$

The paper is organized as follows: In Section 2, we present EDM and CEDM of top quark in the framework of model III. Section 3 is devoted to discussion and our conclusions.

2 Electric and chromo electric dipole moments of top quark in the general two Higgs Doublet model

Non-zero EDMs (CEDMs) of quarks are the sign of CP violation and they are extremely small in the framework of the SM. This forces one to study the CP violating interactions in the new
physics beyond the SM. In this section, we calculate top quark EDM and CEDM in the model III and take complex Yukawa couplings which are the possible sources of CP violation. The starting point is the general Yukawa interaction

\[ L_Y = \eta^U_{ij} \bar{Q}_{iL} \tilde{\phi}_1 U_{jR} + \eta^D_{ij} \bar{Q}_{iL} \tilde{\phi}_1 D_{jR} + \xi^U_{ij} \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi^D_{ij} \bar{Q}_{iL} \tilde{\phi}_2 D_{jR} + \text{h.c.} \, , \]  

(1)

where \( L \) and \( R \) denote chiral projections \( L(R) = 1/2 (1 \mp \gamma_5) \), \( \phi_i \) for \( i = 1, 2 \), are the two scalar doublets, \( \bar{Q}_{iL} \) are left handed quark doublets, \( U_{jR} (D_{jR}) \) are right handed up (down) quark singlets, with family indices \( i,j \). The Yukawa matrices \( \eta^U,D \) and \( \xi^U,D \) have in general complex entries. It is possible to collect SM particles in the first doublet and new particles in the second one by choosing the parametrization for \( \phi_1 \) and \( \phi_2 \) as \[ \phi_1 = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2} \chi^+ \\ i \chi^0 \end{pmatrix} \right] ; \phi_2 = \frac{1}{\sqrt{2}} \left( \begin{pmatrix} \sqrt{2} H^+ \\ H_1 + iH_2 \end{pmatrix} \right) . \]  

(2)

with the vacuum expectation values,

\[ < \phi_1 > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} ; < \phi_2 > = 0 . \]  

(3)

Here, \( H_1 \) and \( H_2 \) are the mass eigenstates \( h^0 \) and \( A^0 \) respectively since no mixing occurs between two CP-even neutral bosons \( H^0 \) and \( h^0 \) at tree level, for our choice.

The part of the Yukawa lagrangian which is responsible for the Flavor Changing (FC) interaction is

\[ L_{Y,FC} = \xi^U_{ij} \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi^D_{ij} \bar{Q}_{iL} \phi_2 D_{jR} + \text{h.c.} \, , \]  

(4)

where the couplings \( \xi^U,D \) for the FC charged interactions are

\[ \xi^U_{ch} = \xi^U_N V_{CKM} , \]

\[ \xi^D_{ch} = V_{CKM} \xi^D_N , \]  

(5)

and \( \xi^U,D_N \) is defined by the expression

\[ \xi^U(D)_N = (V_{R(L)}^{-1} \xi^U(D) V_{L(R)}). \]  

(6)

Notice that the index "N" in \( \xi^U,D_N \) denotes the word "neutral".

The effective EDM and CEDM interactions for \( t \)-quark are

\[ L_{EDM} = i e d_t \bar{t} \gamma_5 \phi^{\mu \nu} t F_{\mu \nu} \, , \]  

(7)

\[ L_{CEDM} = i g_s d_t \bar{t} \gamma_5 \phi^{\mu \nu} \frac{\lambda^a}{2} t G_{\mu \nu}^a \, , \]  

(8)
where $F_{\mu\nu}$ and $G^a_{\mu\nu}$ are the electromagnetic and chromodynamic field tensors, $\lambda^a$ are Gell Mann matrices with color indices $a$, and ”$d_\gamma$ ($d_g$)“ is EDM (CEDM) of top quark and it is a real number by hermiticity. In the model III with complex Yukawa couplings, the charged $H^\pm$ and neutral Higgs bosons $h^0, A^0$ can induce CP violating interactions at loop level and this is the source of non-zero EDM and CEDM. We present the 1-loop diagrams due to charged and neutral Higgs particles in Figs. [1] and [2]. Notice that only vertex diagrams (c, d in Fig. [1] and c in Fig. [2]) contribute to the CP-violating interactions.

The most general vertex operator for on-shell top quark and off-shell photon (gluon) can be written as

$$\Gamma^{(\alpha)}_\mu = F_1(q^2) \gamma_\mu (\frac{\lambda^a}{2}) + F_2(q^2) \sigma_{\mu\nu} (\frac{\lambda^a}{2}) q^\nu + F_3(q^2) \sigma_{\mu\nu} \gamma_5 (\frac{\lambda^a}{2}) q^\nu$$

(9)

where $q_\nu$ is photon (gluon) 4-vector and $q^2$ dependent form factors $F_1(q^2)$ and $F_2(q^2)$ are proportional to the charge and anomalous (chromo) magnetic moment of top quark respectively. Existence of the form factor $F_3(q^2)$ is the reason for the CP violating interactions and therefore for nonvanishing EDM (CEDM) of top quark. The EDM and CEDM of top quark are obtained as a sum of contributions coming from charged and neutral Higgs bosons,

$$d_\gamma = d_{\gamma H^\pm} + d_{\gamma h^0} + d_{\gamma A^0}, \quad (10)$$

and

$$d_g = d_{g H^\pm} + d_{g h^0} + d_{g A^0}. \quad (11)$$

Using the equations

$$d_{H^\pm} = \frac{4 G_F}{\sqrt{2}} \frac{1}{16\pi^2} \frac{m_b}{m_t} I m(\xi_{N,bb}^{D*} \xi_{N,tt}^U) |V_{tb}|^2 \int_0^1 dx \frac{(-1 + x) (Q_b (-1 + x) + x \kappa) y_t}{r_b y_t + x^2 y_t - x (-1 + y_t + r_b y_t)};$$

$$d_{h^0} = \frac{4 G_F}{\sqrt{2}} \frac{1}{16\pi^2} \frac{1}{m_t} I m(\xi_{N,tt}^U) R e(\xi_{N,tt}^U) Q_t \left\{ 1 - \frac{r_1}{2} \ln r_1 \right\} + \frac{r_1 (r_1 - 2)}{\sqrt{r_1 (r_1 - 4)}} \left( \text{Arcatan} \left( \frac{r_1}{\sqrt{r_1 (r_1 - 4)}} \right) - \text{Arcatan} \left( \frac{r_1 - 2}{\sqrt{r_1 (r_1 - 4)}} \right) \right), \text{ for } r_1 < 4,$$

$$d_{h^0} = -\frac{4 G_F}{\sqrt{2}} \frac{1}{16\pi^2} \frac{1}{m_t} I m(\xi_{N,tt}^U) R e(\xi_{N,tt}^U) Q_t \left\{ 1 - \frac{r_1 (r_1 - 2)}{\sqrt{r_1 (r_1 - 4)}} \ln \frac{\sqrt{r_1} - \sqrt{r_1 - 4}}{2} \right\} - \frac{1}{2} r_1 \ln r_1 \right\}, \text{ for } r_1 > 4,$$

$$d_{A^0} = -d_{h^0} (r_1 \to r_2), \quad (12)$$
we get \(d_{\gamma}^{H^\pm}, d_{\gamma}^{h^0}, \) and \(d_{\gamma}^{A^0}\) as

\[
\begin{align*}
  d_{\gamma}^{H^\pm} &= d_{\gamma}^{H^\pm}(\kappa = 1), \\
  d_{\gamma}^{h^0(A^0)} &= d_{\gamma}^{h^0(A^0)},
\end{align*}
\]

and

\[
\begin{align*}
  d_{g}^{H^\pm} &= d_{g}^{H^\pm}(\kappa = 0, Q_b \rightarrow 1), \\
  d_{g}^{h^0(A^0)} &= d_{g}^{h^0(A^0)}(Q_t \rightarrow 1).
\end{align*}
\] (13)

Here \(r_b = m_b^2/m_t^2, r_1 = m_{b_0}^2/m_t^2, r_2 = m_{A_0}^2/m_t^2, y_t = m_t^2/m_{tt^\pm}\), \(Q_b\) and \(Q_t\) are charges of \(b\) and \(t\) quarks respectively and \(\xi_{N,ij}^{U(D)} = \sqrt{\frac{4G_F}{\sqrt{2}}} \xi_{N,ij}^{U(D)}\).

In eqs. (12), we take only internal \(b\) (\(t\))-quark contribution for charged (neutral) Higgs interactions. Here, we assume that the Yukawa couplings \(\bar{\xi}_{N,ij}^{U,tt}, i = u, c,\) and \(\bar{\xi}_{N,bj}^{D} j = d, s\) are negligible compared to \(\bar{\xi}_{N,tt}^{U}\) and \(\bar{\xi}_{N,bb}^{D}\) (see [21]).

The Yukawa couplings \(\bar{\xi}_{N,tt}^{U}\) and \(\bar{\xi}_{N,bb}^{D}\) are complex in general and we take,

\[
\begin{align*}
  \bar{\xi}_{N,tt}^{U} &= |\bar{\xi}_{N,tt}^{U}| \exp(i\theta_{tt}) , \\
  \bar{\xi}_{N,bb}^{D} &= |\bar{\xi}_{N,bb}^{D}| \exp(i\theta_{bb}) .
\end{align*}
\] (15)

With this parametrization, EDM and CEDM of top quark can be obtained as

\[
d_{\gamma} = \frac{4G_F}{\sqrt{2}} \frac{1}{16\pi^2} \frac{1}{m_t} |\bar{\xi}_{N,bb}^{D}| |\bar{\xi}_{N,tt}^{U}| \left\{ |V_{tb}|^2 \sqrt{r_b \sin(\theta_{tt} - \theta_{tb})} \int_0^1 dx \frac{(-1 + x)(Q_b(-1 + x) + x\kappa)y_t}{r_b y_t + x^2 y_t - x(-1 + y_t + r_b y_t)} \right. \\
+ \frac{1}{2} \sin 2\theta_{tt} Q_t \left( \frac{r_2}{2} \ln r_2 - \frac{r_1}{2} \ln r_1 \right) \\
+ \frac{r_1(r_1 - 2)}{r_1(r_1 - 4)} \left( \arctan \left( \frac{r_1}{r_1(r_1 - 4)} \right) - \arctan \left( \frac{r_1 - 2}{r_1(r_1 - 4)} \right) \right) \\
- \frac{r_2(r_2 - 2)}{r_2(r_2 - 4)} \left( \arctan \left( \frac{r_2}{r_2(r_2 - 4)} \right) - \arctan \left( \frac{r_2 - 2}{r_2(r_2 - 4)} \right) \right) \right\} ,
\] (16)

and

\[
d_{g} = d_{\gamma}(\kappa = 0, Q_b \rightarrow 1, Q_t \rightarrow 1) ,
\] (17)

for \(r_1, r_2 < 4\). In the case of other possibilities for \(r_1\) and \(r_2\) the expression for \(d_{\gamma}\) and \(d_{g}\) can be obtained by using the eqs. (12), (13) and (14).

Now, for the completeness, we present the \(q^2\) dependencies of the form factors \(d_{\gamma}(q^2)\) and \(d_{g}(q^2)\). Here \(q^2\) is the virtuality of the outgoing photon and gluon, respectively. Using the
functions
\[ d^{H^\pm}(q^2) = -\frac{4G_F}{\sqrt{2}} \frac{1}{16\pi^2} |V_{tb}|^2 \left\{ m_b \text{Im}(\bar{\xi}_{N,bb}^D \xi_{N,tt}^U) f_1 - i \frac{m_t}{2} (|\xi_{N,bb}^D|^2 - |\xi_{N,tt}^U|^2) f_2 \right\}, \]
\[ d^{h^0}(q^2) = \frac{4G_F}{\sqrt{2}} \frac{1}{4\pi^2 m_t} \text{Im}(\xi_{N,tt}^U) \text{Re}(\xi_{N,tt}^U) Q_t f_3(r_1), \]
\[ d^{A^0}(q^2) = -d^{h^0}(q^2, r_1 \rightarrow r_2), \] (18)

and the equation
\[ d_\gamma(q^2) = d^{H^\pm}(q^2, \kappa \rightarrow 1) + d^{h^0}(q^2) + d^{A^0}(q^2), \] (19)

we get
\[ d_\gamma(q^2) = -\frac{4G_F}{\sqrt{2}} \frac{1}{16\pi^2} \left\{ |V_{tb}|^2 \left( m_b |\bar{\xi}_{N,bb}^D| |\xi_{N,tt}^U| \sin(\theta_{tt} - \theta_{tb}) f_1(\kappa \rightarrow 1) \right. \right. \]
\[ \left. \left. - i \frac{m_t}{2} (|\xi_{N,bb}^D|^2 - |\xi_{N,tt}^U|^2) f_2(\kappa \rightarrow 1) \right) \right. \]
\[ \left. \left. - \frac{2}{m_t} \sin 2\theta_{tt} Q_t (f_3(r_1) - f_3(r_2)) \right\}, \] (20)

and
\[ d_g(q^2) = d_\gamma(q^2, \kappa \rightarrow 0, Q_t \rightarrow 1, Q_b \rightarrow 1). \] (21)

Here the functions \( f_1, f_2 \) and \( f_3(z) \) are
\[ f_1 = \int_0^1 \int_0^{1-x} dx \, dy \left( \frac{Q_b (-1 + x)}{\Delta} + \frac{\kappa x}{\Delta'}, \right), \]
\[ f_2 = \int_0^1 \int_0^{1-x} dx \, dy \left( -1 + x + 2y \right) \left( \frac{Q_b}{\Delta} + \frac{\kappa}{\Delta'}, \right), \]
\[ f_3(z) = \int_0^1 dx \left( \frac{1 + x}{\sqrt{r_q}} \frac{\text{Arctan} \left( \sqrt{-r_q (-1 + x)^2 + 4(1 + (-2 + z)x + x^2)} \right)}{-r_q (-1 + x)^2 + 4(1 + (-2 + z)x + x^2)} \right), \] (22)

with
\[ \Delta = \frac{m_t^2}{y_t} \left( x^2 y_t + q_y y_t (-1 + y) + x(1 + y_t (r_q y + r_b - 1)) \right), \]
\[ \Delta' = \frac{m_t^2}{y_t} \left( 1 + x^2 y_t + q_y y_t (-1 + y) + x(-1 + y_t (r_q y + r_b - 1)) \right), \]
\[ r_q = \frac{q^2}{m_t^2}. \] (23)
3 Discussion

This section is devoted to the analysis of dependencies of EDM and CEDM of top quark on the CP violating parameters $\sin \theta_{tb}$, $\sin \theta_{tt}$, the masses of charged and neutral Higgs bosons. Notice that EDM (CEDM) is obtained in the limit $q^2 \to 0$ (see eqs. (24) and (21)). Since there are large number of free parameters in the model III, such as Yukawa couplings, $\zeta_{N,j}^{D}$, the masses of new Higgs bosons, $H^\pm$, $h^0$ and $A^0$, there is a need to restrict them using the experimental measurements. To find a constraint region for these free parameters we restrict the Wilson coefficient $C_7^{eff}$, which is the effective coefficient of the operator $O_7 = \frac{e}{16\pi^2} \delta_\alpha \sigma_{\mu\nu}(m_b R + m_s L)b_\alpha F^{\mu\nu}$ (see [22] and references therein), in the region $0.257 \leq |C_7^{eff}| \leq 0.439$. Here upper and lower limits were calculated using the CLEO measurement [23]

$$Br(B \to X_s\gamma) = (3.15 \pm 0.35 \pm 0.32) \times 10^{-4}.$$  

(24)

and all possible uncertainties in the calculation of $C_7^{eff}$ [22]. Using this restriction we get a constraint region for the couplings $\xi_{N,tt}^U$, $\xi_{N,bb}^D$ and the CP violating parameters $\sin \theta_{tt}$ and $\sin \theta_{tb}$. In our calculations we neglect all the Yukawa couplings except $\xi_{N,tt}^U$ and $\xi_{N,bb}^D$ since they are negligible due to their light flavor contents [24]. Furthermore, we also respect the constraint for the angle $\theta_{tt}$ and $\theta_{bb}$, due to the experimental upper limit of neutron electric dipole moment, $d_n < 10^{-25}$e·cm (or the more recent results $|d_n| < 6.3 \times 10^{-26}$e·cm (90 % C.L.) [25]), which leads to $\frac{1}{m_\nu m_b} Im(\xi_{N,tt}^U \xi_{N,bb}^D) < 1.0$ for $M_{H^\pm} \approx 200$ GeV [26]. Notice that we take $h^0$ as the lightest Higgs boson and assume that the coupling $\xi_{N,tt}^U$ has a small imaginary part.

In Fig. 3, we plot EDM "d" with respect to $\sin \theta_{tb}$ for $m_{H^\pm} = 400$ GeV, $\xi_{N,bb}^D = 40 m_b$ and $|r_{tb}| = |\frac{\xi_{N,tt}^U}{\xi_{N,bb}^D}| < 1$, when the coupling $\xi_{N,tt}^U$ is real. In this case the neutral Higgs bosons $h^0$ and $A^0$ do not have any contribution to the EDM of top quark. EDM is restricted in the region between solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). This physical quantity is at the order of the magnitude of $10^{-21}$e·cm for the large values of $\sin \theta_{tb}$, especially for $C_7^{eff} > 0$. It can get both signs and even vanish for $C_7^{eff} < 0$. With the increasing values of $\sin \theta_{tb}$, "d" becomes large as expected and the restricted region becomes wider, for both $C_7^{eff} > 0$ and $C_7^{eff} < 0$.

Fig. 4 is devoted to the EDM "d" with respect to $\sin \theta_{tb}$ for $m_{H^\pm} = 400$ GeV, $\xi_{N,bb}^D = 40 m_b$ and $|r_{tb}| < 1$, when the coupling $\xi_{N,tt}^U$ is complex. Here we take a small imaginary part for $\xi_{N,tt}^U$, namely $\sin \theta_{tt} = 0.1$. This is the case where the neutral Higgs bosons $h^0$ and $A^0$ have also contributions to the EDM of top quark. The magnitude of EDM slightly decreases for both $C_7^{eff} > 0$ and $C_7^{eff} < 0$ compared to the case where $\xi_{N,tt}^U$ is real.

In Fig. 5 we present $R_{neutr} = \frac{m_{h^0}}{m_{A^0}}$ dependence of EDM for $\sin \theta_{tb} = 0.5$, $\sin \theta_{tt} = 0.1$, 

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\[ m_{A^0} = 90 \text{ GeV}, \ m_{H^\pm} = 400 \text{ GeV}; \ \bar{\xi}_{N,bb}^D = 40 \ m_b \ \text{and} \ |r_{tb}| < 1. \] Here ”d” lies in the region bounded by solid lines for \( C_{7}^{\text{eff}} < 0 \) and by dotted lines for \( C_{7}^{\text{eff}} > 0 \), when the neutral Higgs contributions are not taken into account. With the addition of the neutral Higgs boson contributions, ”d” can reach to \( 3 \times 10^{-21} (e \text{ cm}) \) for \( C_{7}^{\text{eff}} > 0 \) and the small values of the ratio \( R_{\text{neutr}} \). In the case of degenerate masses of \( h^0 \) and \( A^0 \), the neutral Higgs contributions vanish.

For completeness we plot ”d” with respect to the charged Higgs mass \( m_{H^\pm} \) for \( \sin \theta_{tb} = 0.5, \ \sin \theta_{tt} = 0.1, \ m_{h^0} = 80 \ \text{GeV}, \ m_{A^0} = 90 \ \text{GeV}; \ \bar{\xi}_{N,bb}^D = 40 \ m_b, \ C_{7}^{\text{eff}} > 0 \) and a special choice of \( |r_{tb}| < 1, \ r_{tb} = 0.001 \). Here dashed (solid) line represents the charged (total) contribution. This figure shows that ”d” is sensitive to the charged Higgs mass \( m_{H^\pm} \).

CEDM is the EDM due to the external gluon instead of photon and it is also a clue about the existence of CP violating interactions in the theory.

In Fig. 4, we plot CEDM ”d” with respect to \( \sin \theta_{tb} \) for \( m_{H^\pm} = 400 \ \text{GeV}, \ \bar{\xi}_{N,bb}^D = 40 \ m_b, \ |r_{tb}| < 1 \) and the real coupling \( \bar{\xi}_{N,tt}^U \). CEDM is restricted in the region between solid (dashed) lines for \( C_{7}^{\text{eff}} > 0 \) (\( C_{7}^{\text{eff}} < 0 \)). It can reach to the order of magnitude of \( \sim 10^{-20} g_s \text{ cm} \) for large values of \( \sin \theta_{tb} \), for \( C_{7}^{\text{eff}} > 0 \). Similar to EDM, it can get both signs and even vanish for \( C_{7}^{\text{eff}} < 0 \). However, the sign of CEDM is opposite to that of EDM. With the increasing values of \( \sin \theta_{tb} \), ”d” becomes large as expected and the restricted region becomes wider, for both \( C_{7}^{\text{eff}} > 0 \) and \( C_{7}^{\text{eff}} < 0 \), similar to EDM.

Fig. 8 is devoted to the CEDM ”d” with respect to \( \sin \theta_{tb} \) for \( m_{H^\pm} = 400 \ \text{GeV}, \ \bar{\xi}_{N,bb}^D = 40 \ m_b \) and \( |r_{tb}| < 1 \), when the coupling \( \bar{\xi}_{N,tt}^U \) is complex, with a small imaginary part, \( \sin \theta_{tt} = 0.1 \). In this case, the neutral Higgs bosons \( h^0 \) and \( A^0 \) have also contributions to the CEDM of top quark and they decrease the magnitude of CEDM slightly for both \( C_{7}^{\text{eff}} > 0 \) and \( C_{7}^{\text{eff}} < 0 \) compared to the case where \( \bar{\xi}_{N,tt}^U \) is real.

In Fig. 9, we represent \( R_{\text{neutr}} = \frac{m_{h^0}}{m_{A^0}} \) dependence of CEDM for \( \sin \theta_{tb} = 0.5, \ \sin \theta_{tt} = 0.1, \ m_{A^0} = 90 \ \text{GeV}, \ m_{H^\pm} = 400 \ \text{GeV}; \ \bar{\xi}_{N,bb}^D = 40 \ m_b \ \text{and} \ |r_{tb}| < 1 \). Here ”d” lies in the region bounded by solid lines for \( C_{7}^{\text{eff}} < 0 \) and by dotted lines for \( C_{7}^{\text{eff}} > 0 \), when neutral Higgs contributions are not taken into account. A weak enhancement in the magnitude of CEDM appears with the addition of neutral Higgs boson contributions, for \( C_{7}^{\text{eff}} > 0 \) and small values of the ratio \( R_{\text{neutr}} \).

Finally, we plot CEDM with respect to the charged Higgs mass \( m_{H^\pm} \) for \( \sin \theta_{tb} = 0.5, \ \sin \theta_{tt} = 0.1, \ m_{h^0} = 80 \ \text{GeV}, \ m_{A^0} = 90 \ \text{GeV}; \ \bar{\xi}_{N,bb}^D = 40 \ m_b, \ C_{7}^{\text{eff}} > 0 \) and \( r_{tb} = 0.001 \). Here dashed (solid) line represents the charged (total) contribution. CEDM is sensitive to the charged Higgs mass \( m_{H^\pm} \), similar to the EDM case.
Now we would like to summarize our results:

• EDM (CEDM) is generated by the one loop diagrams with the choice of complex Yukawa couplings in the model III. In the case of real $\xi_{N,tt}^U$, only the charged Higgs sector contributes. The additional imaginary part of $\xi_{N,tt}^U$ ensures that the neutral Higgs part can also have contribution. In general, EDM and CEDM depends on the CP violating parameter $\sin(\theta_{tt} - \theta_{tb})$ in the charged part and on $\sin 2\theta_{tt}$ in the neutral part.

• Top quark EDM (CEDM) is sensitive to the ratio $R_{\text{neutr}}$, especially for $C_{C}^{\text{eff}} > 0$. The sensitivity to the charged Higgs mass $m_{H^\pm}$ is stronger.

• If EDM (CEDM) is negative (positive), $C_{C}^{\text{eff}}$ can have both signs. However, if it is negative, $C_{C}^{\text{eff}}$ must be positive (negative). This is observation is useful in the determination of the sign of $C_{C}^{\text{eff}}$.

• EDM (CEDM) of top quark is at the order of the magnitude of $\sim 10^{-21} e\text{ cm} \ (\sim 10^{-20} g_s \text{ cm})$.

Therefore, the experimental investigations of the top quark EDM and CEDM give powerful informations about the physics beyond the SM.

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Figure 1: One loop diagrams contribute to EDM (CEDM) of top quark due to $H^\pm$ in the 2HDM. Wavy lines represent the electromagnetic (chromomagnetic) field and dashed lines the $H^\pm$ field.
Figure 2: The same as Fig. 1 but for neutral Higgs bosons $h^0$ and $A^0$. 
Figure 3: Top quark EDM $"d"$ as a function of $\sin \theta_{tb}$ for $m_{H^\pm} = 400 \text{GeV}$, $\sin \theta_{tt} = 0$ and $|r_{tb}| < 1$, in the model III. Here $d$ is restricted in the region bounded by solid lines for $C_T^{\text{eff}} > 0$ and by dashed lines for $C_T^{\text{eff}} < 0$. 
Figure 4: The same as Fig. 3 but for $\sin \theta_{tt} = 0.1$, $m_{h^0} = 80\, GeV$ and $m_{A^0} = 90\, GeV$.

Figure 5: Top quark EDM $"d"$ as a function of the ratio $R_{neutr} = \frac{m_{h^0}}{m_{A^0}}$, for $m_{H^\pm} = 400\, GeV$, $\sin \theta_{tb} = 0.5$, $\sin \theta_{tt} = 0.1$, $\tilde{\xi}^{D}_{N,bb} = 40\, m_b$ and $|r_{tb}| < 1$, in the model III. Here $"d"$ lies in the region bounded by dashed (solid) lines for $C_T^{eff} < 0$ and by small dashed (dotted) lines for $C_T^{eff} > 0$, when neutral Higgs contributions are (not) taken into account.
Figure 6: Top quark EDM "d" with respect to the charged Higgs mass $m_{H^\pm}$ for $\sin \theta_{tb} = 0.5$, $\sin \theta_{tt} = 0.1$, $m_{h^0} = 80\, GeV$, $m_{A^0} = 90\, GeV$, $\xi_{N,bb}^D = 40\, m_b$, $C_{eff}^7 > 0$ and $r_{tb} = 0.001$. Here dashed (solid) line represents EDM (not) including neutral Higgs contributions.

Figure 7: The same as Fig. 6, but in for top quark CEDM.
Figure 8: The same as Fig. 4, but in for top quark CEDM.

Figure 9: The same as Fig. 5, but in for top quark CEDM.
Figure 10: The same as Fig. 6 but for top quark CEDM.