Extra equation of gravity
induced by spontaneous local Lorentz symmetry breakdown

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The broken gauge theory of a Weyl doublet allows two types of spontaneous local Lorentz symmetry breakdown consistent with Lorentz invariance of the emergent theory. In these cases, the Nambu-Goldstone boson induces the new interaction, which is described by an asymmetric equation for the vierbein appearing in association with the Einstein equation of gravity. The extra interaction has the combined properties of gravity and electromagnetism, and can be extremely stronger than the Newton-Einstein gravity, enhanced by the ratio of the Planck mass squared to the breaking scale squared. The detection of strong inertial force appearing in association with the electromagnetic phenomena will be therefore an evidence for discerning whether the Lorentz invariance confirmed by observations and experiments is the primary symmetry, or the secondary one emergent after spontaneous local spacetime symmetry breakdown.

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I. INTRODUCTION

Broken symmetry is well known to play an essential role in understanding various physical phenomena. This concept, which was originated from superconductivity theory [1], was applied in fundamental theories for aiming at the unified model of elementary particles [2], and then fructified as the electro-weak theory [3], owing to the discovery of Higgs mechanism [4–6]. This paper considers to extend the concept to spacetime symmetry for proceeding further the approach of unification.

Continuous symmetries considered in physics consist of gauge symmetries and spacetime symmetries. Although quantum mechanics can break every symmetry, the broken symmetries hitherto considered in elementary particle physics are almost solely gauge symmetries, while spacetime symmetries, in particular, Lorentz symmetry or general relativity, are rarely considered broken except possibly in Planck-scale physics. The reason seems clear, since we have had no positive evidence to reveal the breakdown of spacetime symmetries, at least, in energy scales accessible by experiments nor by observations.

However, it is not evident that spontaneous spacetime symmetry breakdown immediately contradicts the Lorentz invariance confirmed by experiments and observations. This paper shows that there exist in fact possibilities to emerge exact Lorentz invariant effective theories even after the violation of local spacetime symmetry.

The spacetime symmetry which may not directly come into conflict with observational relativity is local Lorentz symmetry. It occupies an exceptional position among spacetime symmetries, since it is not directly connected with general relativity nor global Lorentz invariance. Accordingly, there will be a room for expecting that a Lorentz invariant effective theory emerges.

Local Lorentz symmetry appears when fermions are introduced in curved spacetime, since the description of spinor fields in curved spacetime requires the transformation matrix called vierbein, which relates a local Lorentz frame at each spacetime point to the general coordinate system. Local Lorentz symmetry is necessary for guaranteeing that physics should not be affected by any choice of a local Lorentz frame at any spacetime point.

It has been shown in the previous papers [7, 8] that the SU(2) gauge theory of a Weyl doublet in the presence of a Higgs doublet can break Lorentz symmetry spontaneously by the gauge field developing the vacuum expectation value, when the gauge bosons become massive by the Higgs mechanism. Accordingly, the same system considered in the general coordinate system will break local Lorentz symmetry spontaneously, from which we may expect the emergence of relativistic effective theory.

The remaining concern for the relativistic effective theory is the relativity of the emergent fermions or quasi fermions. Since in perturbation theory the vacuum expectation value of the gauge field modifies the equation of motion for a Weyl doublet, requiring relativity of the emergent fermions restricts in turn the form the vacuum expectation value. As we will see, the gauge field actually allows two types of vacuum expectation values which can lead to the relativistic
quasi fermions. The quasi fermion doublet corresponding to a leptonic doublet, which was reported in the previous papers [7, 8], is one of the two possibilities.

The merit of embedding the system in curved spacetime is that it provides us with the field representation of the Nambu-Goldstone boson [9, 10], which serves to examine by explicit calculations the relativistic properties of the emergent theory. As we will see, the vierbein plays the role of the field representation of Nambu-Goldstone boson as the result of spontaneous local Lorentz symmetry breakdown. In this sense, the Nambu-Goldstone boson is essentially the graviton, though obeying a modified equation of gravity.

The modified equation of gravity can be properly decomposed into the symmetric Einstein equation of gravity for the metric tensor and the extra asymmetric equation for the vierbein, from which physical insights are obtained. Then we examine the properties of the Nambu-Goldstone boson, as well as its interaction with the emergent fermions.

We will see that the extra equation of gravity does not involve the Newton constant $G$. Instead, the spacetime symmetry breaking scale $m$ appears in the place of the Planck mass $M_P = G^{-1/2}$. Therefore, the Nambu-Goldstone graviton will attract two objects roughly $(M_P/m)^2$ times stronger than the Einstein graviton. The breaking scale $m$ appears also as the mass of a quasi fermion interpretable as a charged lepton. Assuming that $m$ equals the electron mass, we have $(M_P/m_e)^2 \approx 10^{44}$.

In the separation of the equation for the Nambu-Goldstone graviton from the modified equation of gravity, the energy-momentum tensor of a quasi fermion doublet is also decomposed into the ordinary kinetic energy-momentum tensor and the residual part, which will be called the tensor current, and the latter can be regarded as four vector currents numbered by the local Lorentz index. We find that some linear combination of these currents constitutes a conserved $U(1)$ current of the quasi fermions, which is interpretable as the electromagnetic current. Then, the quantum of vierbein couples both to the energy-momentum tensor and to the $U(1)$ current of the quasi fermions. In this sense, the quantum of vierbein is not only the graviton, but also the photon.

The two types of spontaneous local Lorentz symmetry breakdown investigated in this paper show the possibilities of the relativistic effective theories emergent even after spontaneous violation of spacetime symmetry. This observation suggests that relativity confirmed by observations and experiments may be an effective symmetry emergent after spontaneous spacetime symmetry breakdown. The answer of the question of how to discern the original Lorentz symmetry from the emergent one will be provided by detecting strong gravitational interactions appearing in conjunction with the electromagnetic phenomena.

II. MODEL OF SPONTANEOUS LOCAL LORENTZ SYMMETRY BREAKDOWN

We consider the local Lorentz symmetry breakdown in the system consisting of a left-handed Weyl doublet $\varphi$, an SU(2) Yang-Mills gauge field $Y_\mu$, a Higgs doublet $\Phi$, and the vierbein $e_\mu^\alpha$. The gauge theory of a Weyl doublet is apt to be considered inconsistent due to a global SU(2) anomaly [11]. However, the reported inconsistency will be irrelevant for our system, the explanation of which is given in Appendix A. In the following, indices $\mu, \nu, \cdots$ refer to the general coordinate system, while $\alpha, \beta, \cdots$ refer to the local Lorentz frame vectors. Each of both indices runs from 0 to 3. The vierbein satisfies the relations $\eta_{\alpha\beta} e^\mu_\alpha e^\beta_\nu = g_{\mu\nu}$ and $g^{\mu\nu} e^\alpha_\mu e^\beta_\nu = \eta^{\alpha\beta}$, where $g_{\mu\nu}$ is the metric tensor in the general coordinate system, while $\eta_{\alpha\beta}$ is the Lorentz metric defined by $\eta_{00} = -\eta_{11} = -\eta_{22} = -\eta_{33} = 1$, with the other components being 0. Raising or lowering the index of the vierbein is performed by contracting with the metric tensors $g_{\mu\nu}$ and $\eta_{\alpha\beta}$. The vierbein guarantees the local Lorentz invariance as well as the general coordinate invariance of the system. The action of the system is given by

$$S = \int d^4x e \mathcal{L}, \quad \mathcal{L} = \mathcal{L}_G + \mathcal{L}_Y + \mathcal{L}_\Phi + \mathcal{L}_\varphi,$$

where $e = \text{det} e_\mu^\alpha$. We have included the Einstein action for the dynamics of the metric tensor field, which is given by

$$\mathcal{L}_G = -\frac{R}{16\pi G},$$

where $R = g^{\mu\nu} R_{\mu\nu}$ is the scalar curvature, while the Ricci tensor is defined by $R_{\mu\nu} = R^{\sigma}_{\mu\rho\sigma\nu}$. The Riemann curvature $R^{\rho}_{\sigma\mu\nu}$ is given through the relation $[\nabla_\mu, \nabla_\nu] V^\rho = R^{\rho}_{\sigma\mu\nu} V^\sigma$ for an arbitrary vector field $V^\rho$. Since the vierbein does not have its own action, it contains auxiliary degrees of freedom except for the part indirectly governed by the Einstein action [12]. The Lagrangian density for the gauge field in a local Lorentz frame $Y_\alpha = e_\mu^\alpha Y_\mu$ is given by

$$\mathcal{L}_Y = -\frac{1}{4} Y^{\alpha\beta} \cdot Y_{\alpha\beta}, \quad Y_{\alpha\beta} = \nabla_\alpha Y_\beta - \nabla_\beta Y_\alpha - g_{\alpha\beta} \times Y, \quad \nabla_\alpha Y_\beta = \epsilon^\mu_\alpha \partial_\mu Y_\beta + \omega_{\mu\beta} Y_\gamma,$$

where
where $g$ is the coupling constant. The local Lorentz connection $\omega_{\mu\beta\gamma}$ is defined by requiring that the covariant derivative of the vierbein $e_\mu^\alpha$ should vanish:

$$\nabla_\mu e_\nu^\alpha = \partial_\mu e_\nu^\alpha - \Gamma^\alpha_\mu_\nu e_\rho^\alpha + \omega^\alpha_\mu_\beta e_\nu^\beta = 0, \quad (4)$$

where the Christoffel symbol $\Gamma^\rho_\mu_\nu = \frac{1}{2}g^{\rho\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})$ is symmetric under the exchange of $\mu$ and $\nu$, while the local Lorentz connection $\omega_{\mu\beta\gamma}$ is antisymmetric under the exchange of $\beta$ and $\gamma$, as a consequence of the requirement $\nabla_\alpha \eta_{\beta\gamma} = 0$.

The Lagrangian density for the Higgs field is given by

$$L_\Phi = D^\alpha \Phi^\dagger D_\alpha \Phi - \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 - \frac{\lambda \eta^2}{2} \Phi^\dagger \Phi - \Lambda_0, \quad D_\alpha \Phi = e^\mu_\alpha (\partial_\mu + igY_\mu) \Phi, \quad Y_\mu = \frac{\rho}{2} \cdot Y_\mu, \quad (5)$$

where $\rho = (\rho^1, \rho^2, \rho^3)$, and $\rho^a/2$ with $a = 1, 2, 3$ are the SU(2) generators. $\Lambda_0$ is the bare cosmological constant.

The Lagrangian density for a Weyl doublet is defined by the real part of

$$\mathcal{L}_\psi = \varphi^\dagger \tilde{\sigma}^a i D_\alpha \varphi, \quad D_\alpha \varphi = e^\mu_\alpha (\nabla_\mu + igY_\mu) \varphi, \quad \nabla_\mu \varphi = (\partial_\mu + \frac{1}{8} \omega_{\mu\beta\gamma} \tilde{\sigma}^{\beta\gamma}) \varphi, \quad \varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}, \quad (6)$$

where the constant spin matrices $\sigma^a$, $\tilde{\sigma}^a$, $\sigma^{a\beta}$, and $\sigma^{a\beta}$ are given in Appendix 1. The difference between the Hermitian density $\epsilon L_\varphi = \frac{1}{2} (L_\varphi + L_\varphi^\dagger)$ and $\epsilon L_\varphi$ is merely a total divergence. Then, we have explicitly

$$\tilde{L}_\varphi = \varphi^\dagger \tilde{\sigma}^a i D_\alpha \varphi, \quad D_\alpha \varphi = e^\mu_\alpha (\nabla_\mu + igY_\mu) \varphi, \quad \nabla_\mu \varphi = (\partial_\mu + \frac{1}{8} \omega_{\mu\beta\gamma} \tilde{\sigma}^{\beta\gamma}) \varphi, \quad \varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}, \quad (7)$$

where the relation: $\tilde{\sigma}^\alpha \sigma^{\beta\gamma} - (\tilde{\sigma}^\alpha \sigma^{\beta\gamma})^\dagger = -4i \epsilon^{a\beta\gamma\delta} \tilde{\sigma}_\delta$ has been used, which follows from the identity:

$$\tilde{\sigma}^\alpha \sigma^{\beta\gamma} = \eta^{\alpha\beta} \sigma^{\gamma} - \eta^{\alpha\gamma} \sigma^{\beta} + \eta^{\beta\gamma} \sigma^{\alpha} - i \epsilon^{a\beta\gamma\delta} \tilde{\sigma}_\delta. \quad (8)$$

The constant antisymmetric tensor $\epsilon^{a\beta\gamma\delta}$ is defined with convention $\epsilon^{0123} = 1$.

We next explain how local Lorentz symmetry breaks spontaneously in this system. In order to make the argument as simple as possible, we consider only the case that local Lorentz symmetry breaks after gauge symmetry breakdown. Though the both may break simultaneously, this restriction serves to avoid complexities inessential for our argument. We first simplify the Lagrangian by replacing the Higgs boson field with its vacuum expectation value. Since the both may break simultaneously, this restriction serves to avoid complexities inessential for our argument. We henceforth consider the ground state of the system in a local inertial frame where the gravitational fields $\Gamma_{\mu\nu}^\rho$ and $\omega_{\mu\nu}^\rho$ vanish locally.

The classical ground state for the gauge field $Y^\alpha$ is determined by the minimum of the potential density $\mathcal{V}(Y^\alpha)$, which is obtained by regarding all the boson fields in the the total Lagrangian (10) as constant. Concerning the spinor field, we simply disregard it at first as the zeroth order approximation, and introduce the potential term $-J^\alpha_0 \cdot Y_\alpha$ in $\mathcal{L}$, by replacing the fermion current $j^\alpha$ in the interaction term $-j^\alpha \cdot Y_\alpha$ with an external constant current $J^\alpha_0$. This prescription will be unavoidable for taking the contribution from a Weyl doublet properly into account, since fermions are essentially quantum mechanical, and can not be regarded as constant. The vacuum expectation value $\langle j^\alpha \rangle$ as the constant current $J^\alpha_0$ is determined only after the establishment of the vacuum configuration. It can not be taken into account beforehand for determining the ground state itself.

Then we can give $\mathcal{V}(Y^\alpha)$ in the form independent of the general coordinate system:

$$\mathcal{V}(Y^\alpha) = \frac{g^2}{4} (Y^\alpha \times Y^\beta) \cdot (Y_\alpha \times Y_\beta) - \frac{1}{2} m_Y^2 Y^\alpha \cdot Y_\alpha + J^\alpha \cdot Y_\alpha + \Lambda_0 - \frac{\lambda \eta^4}{4}. \quad (12)$$

Under the variation with respect to $Y^\alpha$, it takes the extremum value at $Y^\alpha = Y^\alpha_0$ satisfying the condition

$$m_Y^2 Y^\alpha_0 + g^2 (Y^\alpha_0 \times Y^\beta_0) \times Y_{0\beta} = J^\alpha_0. \quad (13)$$
from which we see that the gauge field has a non-zero vacuum expectation value \( \langle Y^\alpha \rangle = Y_0^\alpha \neq 0 \) for \( J_0^\alpha \neq 0 \).

In this case, the gauge current \( j^\alpha \) will also develop the vacuum expectation value \( \langle j^\alpha \rangle_0 \), since \( Y_0^\alpha \) breaks both the gauge invariance and the local Lorentz invariance of the free equation of motion for a Weyl doublet.

We may proceed to obtain a more precise ground state configuration by replacing \( J_0^\alpha \) with \( J_1^\alpha = J_0^\alpha + \langle j^\alpha \rangle_0 \) in (12) to have a more precise vacuum expectation value \( Y_1^\alpha \), and recalculate \( \langle j^\alpha \rangle_1 \) in the presence of \( Y_1^\alpha \). The same procedure can be repeated further ad infinitum to obtain the exact result. If we achieve in this way the convergent value \( \langle Y^\alpha \rangle \), we can regard it as corresponding to the true ground state of the system. This procedure obeys the same principle employed in the Hartree-Fock approximation for the ground state of the many electron system in the atom.

Non-zero vacuum expectation values \( \langle Y^\alpha \rangle \) and \( \langle j^\alpha \rangle \) can be obtained even for \( J_0^\alpha = 0 \), if the equation

\[
\langle j^\alpha \rangle = m_Y^2 \langle Y^\alpha \rangle + g^2 \langle Y^\alpha \times (Y^\beta) \rangle \times \langle Y_\beta \rangle,
\]

has a non-trivial solution, where we may calculate \( \langle j^\alpha \rangle \) from the free equation of motion of a Weyl doublet in a local inertial frame, by assuming that the quantization volume is small enough compared with the scale of the background curved spacetime:

\[
(\bar{\sigma}^\alpha \partial_\alpha - \bar{M}) \varphi = 0, \quad \bar{M} = \frac{g}{2} \bar{\sigma}^\alpha \rho \cdot \langle Y_0 \rangle.
\]

From the consideration on symmetry property, \( \langle j^\alpha \rangle \) will have the form

\[
\langle j^\alpha \rangle = \Gamma_Y \langle Y^\alpha \rangle,
\]

where the coefficient \( \Gamma_Y \) should be invariant under local Lorentz transformations as well as global SU(2) transformations, though it may have dependence on \( \langle Y^\alpha \rangle \). The extremum of \( \mathcal{V}(Y^\alpha) \) in this case becomes

\[
\mathcal{V}_{\text{ext}} = \frac{1}{2} m_Y^2 Y^\alpha \cdot Y_\alpha - \frac{3g^2}{4} (Y^\alpha \times Y^\beta) \cdot (Y_\alpha \times Y_\beta) + \Lambda_0 - \frac{\lambda_0^4}{4}.
\]

If \( \mathcal{V}_{\text{ext}} < \mathcal{V}(0) \) for the non-zero solution \( \langle Y^\alpha \rangle \) of the consistency equation (14), we expect that local Lorentz symmetry breaks spontaneously. The estimation of \( \Gamma_Y \) will be given in the next section after narrowing down the candidates of \( \langle Y^\alpha \rangle \), in view of the consistency with relativity for quasi particles, and then the occurrence of spontaneous local Lorentz symmetry breakdown will be confirmed.

Incidentally, we see from (14) that what become constant by spontaneous local Lorentz symmetry breakdown are the local Lorentz vectors: \( \langle Y^\alpha \rangle \) and \( \langle j^\alpha \rangle \). These are frame-dependent quantities, though invariant under general coordinate transformations. This observation reflects in turn how we are to formulate the broken spacetime symmetry in curved spacetime.

We may consider the same local Lorentz symmetry breakdown in another local frame at another spacetime point. Whereas we will have the same form of equation (14), the vacuum expectation values can be different for local frames at different spacetime points. We formulate spontaneous local Lorentz symmetry breakdown as that the vacuum expectation value of a local Lorentz vector is the same for all the local frames at different spacetime points. The requirement can be fulfilled owing to the local Lorentz symmetry in the original system, whereas the orientation of local Lorentz frames at different spacetime points are in turn mutually fixed. Then the constant local Lorentz vector in a general coordinate system acquires spacetime dependence to become a vector field. In this formulation, we will not detect a specific spacetime direction even after local Lorentz symmetry breaks spontaneously, unless gravitational fields are completely ignored. We will come back to this viewpoint later again.

### III. RELATIVISTIC QUASI PARTICLES

The previous section showed how the massive gauge field \( Y^\alpha \) develops the vacuum expectation value in the presence of the gauge current of fermions, and breaks local Lorentz symmetry spontaneously. We further argued that the vacuum expectation value of a local Lorentz vector will not be detectable as a specific direction in spacetime by an observer in the general coordinate system.

Our main concern here is relativity of the quasi fermions emergent after spontaneous local Lorentz symmetry breakdown.

In perturbation theory, the vacuum expectation value of the gauge field modifies the free equation of motion of a Weyl doublet (15), which may break relativistic properties of an emergent quasi fermion doublet, though the violation of local Lorentz symmetry seemed to have no influence on the general relativity of the emergent theory. Fortunately, however, it has been already known that there exists a class of Lorentz-violating terms in the Lagrangian, which does not in fact contradict the Lorentz invariance of the system (13,14).
In relativistic quantum mechanics where the coordinate operators $\hat{x}^\mu$ are represented as classical numbers, the momentum operators defined by $\hat{p}^\mu = i\partial^\mu - \dot{\hat{x}}^\mu F(x)$ satisfy the ordinary canonical commutation relations $[\hat{x}^\mu, \hat{p}_\nu] = i\eta^\mu_\nu$, and $[\hat{p}^\mu, \hat{p}_\nu] = 0$, for an arbitrary scalar function $F(x)$ [15]. In particular, for $F(x) = \delta \cdot x$ with a constant 4-vector $\delta^\mu$, we have $\hat{p}^\mu = i\partial^\mu - \delta^\mu$. In this representation, there appear superficially Lorentz-violating terms in the Schrödinger equation, though these terms are removable by a local phase transformation for the wave function. Inversely, if the Lorentz-violating terms emergent in the equation of motion for the quasi fermions belong to this class, we may still expect Lorentz invariance of the emergent theory. This requirement in turn severely restricts the form of the vacuum expectation value $\langle Y^\alpha \rangle$.

In this section and the next, we treat gravity as perturbation. In the zeroth order approximation: $\epsilon_\mu^\alpha = \eta_\mu^\alpha$, it is not necessary to distinguish the general coordinate indices and the local Lorentz indices.

We consider as an extension of the relativistic dispersion relation for a quasi particle with mass $m$:

$$ (p - \delta) \cdot (p - \delta) = m^2, \tag{18} $$

where $p^\mu$ is the canonical 4-momentum, and $\delta^\mu = (\delta^0, \delta)$ is a constant 4-vector. A fermion obeying the dispersion relation of this form will be equivalent to an ordinary relativistic fermion, as $\delta^\mu$ will be removable by a local linear phase transformation of the spinor field. We may call the dispersion relation (18) “quasi-relativistic”, and a particle which satisfies it a quasi-relativistic particle.

In special relativity, the kinetic 4-momentum $k^\mu$ of a relativistic particle with mass $m$ satisfies the relation $k \cdot k = m^2$. The kinetic 4-momentum of a quasi particle satisfying the dispersion relation $(p - \delta) \cdot (p - \delta) = m^2$ is therefore $k^\mu = p^\mu - \delta^\mu$, which implies that the canonical 4-momentum $p^\mu$ of a quasi particle is the sum of the kinetic 4-momentum $k^\mu$ and the constant 4-potential $\delta^\mu$. In the representation of 4-momentum operator $\hat{p}^\mu = i\partial^\mu$, the kinetic 4-momentum operator is expressed by $k^\mu = i\partial^\mu - \delta^\mu$. The constant 4-potential $\delta^\mu$ will be understood in fact as one of the characteristic features of the relativistic effective theory emergent from spontaneous spacetime symmetry breakdown.

The derivation of the dispersion relation for a relativistic quasi fermion doublet is as follows. Treating gravity and gauge force as perturbation, and putting $\langle \epsilon_\mu^\alpha \rangle = \eta_\mu^\alpha$, and $\frac{1}{2} \langle Y_{\mu \alpha} \rangle = m_{\mu \alpha}$, we express the free equation of motion for a quasi fermion doublet as

$$ (\hat{\sigma}^\mu i\partial_\mu - \hat{\mathcal{M}})\varphi = 0, \quad \hat{\mathcal{M}} = \delta^\mu \rho^\mu m_{\mu \alpha}. \tag{19} $$

The requirement for the dispersion relation $|\hat{\sigma} \cdot p - \hat{\mathcal{M}}| = 0$ to be quasi relativistic is expressed by the following identity

$$ |\hat{\sigma} \cdot p - \hat{\mathcal{M}}| = |(p - \delta_1) \cdot (p - \delta_1) - m_1^2| \times |(p - \delta_2) \cdot (p - \delta_2) - m_2^2|, \tag{20} $$

with respect to 4-momentum $p^\mu$, where $\delta_1^\mu$ and $\delta_2^\mu$ are two constant 4-vectors, while $m_1$ and $m_2$ are the masses of quasi fermions. Expanding the left-hand side of (20), we have

$$ |\hat{\sigma} \cdot p - \hat{\mathcal{M}}| = p_0^2 - 2p_0^2 \left( \frac{p^2 + m_0^2 + \sum m_a^2}{\sqrt{m_0^2 + \sum m_a^2}} \right) $$

$$ + m_0 \left( 2 m_2 \cdot m_3 + \sum m_0 m_a \cdot m_a \right) $$

$$ + (p^2)^2 $$

$$ - 2p_0^2 \left( \sum m_0^2 - m_a^2 \right) - 4 \sum m_0 m_a \cdot m_a $$

$$ + m_0^2 + \sum m_a^2 $$

$$ - 4 \sum_{\mu \nu} \epsilon_{\mu \nu} m_0 m_0 \cdot m_b \cdot m_a $$

$$ + \left( m_0^2 + \sum m_a^2 \right)^2 $$

$$ - 4 \sum_{\mu \nu} \epsilon_{\mu \nu} m_0 m_a \cdot m_b - 2 \sum \left( m_0 m_a \times m_b \right)^2, \tag{21} $$

where $m_0 = (m_0^1, m_0^2, m_0^3)$, and $m_a = (m_a^1, m_a^2, m_a^3)$. Comparing the coefficients of $p_0^3$ and $p_0^2$ in the both sides of (20), we find that $\delta_1^\mu + \delta_2^\mu = 0$. We therefore introduce $\delta^\mu$ by $\delta^\mu = \delta_1^\mu = -\delta_2^\mu$. All the other requirements are as follows:

$$ m_0^2 + \sum m_a^2 = (\delta^0)^2 + \delta^2 + \frac{1}{2} (m_1^2 + m_2^2), \tag{22} $$

$$ m_1 \cdot (m_2 \times m_3) = \frac{\delta^0}{4} (m_2^2 - m_1^2), \tag{24} $$

$$ m_0 \cdot m_a = \delta_0 \delta, \tag{23} $$
\[ \delta^i \delta^j - \sum_a m_a ^i m_a ^j = \frac{\delta^{ij}}{2} \left[ m_0 ^2 - \sum_a m_a ^2 - \delta \cdot \delta + \frac{1}{2} (m_1 ^2 + m_2 ^2) \right], \quad (25) \]

\[ \sum_{abc} \epsilon_{abc} m_{a0} (m_b \times m_c) = \frac{\delta}{2} (m_2 ^2 - m_1 ^2), \quad (26) \]

\[ \left( m_0 ^2 + \sum_a m_a ^2 \right) ^2 - 4 \left( \sum_a m_{0a} m_a \right) ^2 - 2 \sum_{ab} (m_a \times m_b) ^2 = (m_1 ^2 - \delta \cdot \delta) (m_2 ^2 - \delta \cdot \delta). \quad (27) \]

Introducing an auxiliary parameter \( m^2 \), which is not necessarily positive, we can decompose the condition (24) into two relations:

\[ \delta^i \delta^j - \sum_a m_a ^i m_a ^j = - \frac{m^2}{4} \delta^{ij}, \quad (28) \]

\[ m_0 ^2 - \sum_a m_a ^2 = \delta \cdot \delta - \frac{1}{2} (m_1 ^2 + m_2 ^2 + m^2). \quad (29) \]

Then, we have from (28)

\[ \sum_a m_a ^2 = \delta^2 + \frac{3}{4} m^2. \quad (30) \]

Applying this relation to (22) and (29), we obtain

\[ m^2 = m_1 ^2 + m_2 ^2, \quad (31) \]

\[ (\delta^0) ^2 - m_0 ^2 = \frac{m^2}{4}. \quad (32) \]

The relation (31) shows that \( m^2 \) should be positive or zero. If we extend matrix \( m_{\mu a} \) to \( m_{\mu \alpha} \) by introducing the iso-scalar component \( m_{\mu 0} = \delta_{\mu} \), then (24) combined with (23) and (32) can be rewritten as

\[ \eta^{\alpha \beta} m_{\alpha \mu} m_{\beta \nu} = \frac{m^2}{4} \eta^{\mu \nu}, \quad (33) \]

where \( \eta^{\mu \nu} \) and \( \eta^{\alpha \beta} \) are the same Minkowski metric with signature \((1, -1, -1, -1)\). The indices appearing twice are understood to be summed.

**A. Type-I \( (m = 0) \)**

If \( m = 0 \), we find from (33), (31) and (32) that

\[ m_1 = m_2 = 0, \quad m_0 ^2 = (\delta^0) ^2, \quad \sum_a m_a ^2 = \delta^2. \quad (34) \]

Then, from (23) and (24), we obtain for arbitrary \( a \) and \( b \)

\[ m_a \times m_b = 0, \quad (35) \]

which implies that three vectors \( m_a \) lie on the same line. Furthermore, from (23), every \( m_a \) should be proportional to \( \delta \). Then we have

\[ m_a = \epsilon_a \delta, \quad (36) \]

\[ \sum_a \epsilon_a ^2 = 1, \quad (37) \]
by taking the third condition of \( (34) \) into account. If \( \delta \neq 0 \), we see from \( (23) \) and the second equation in \( (34) \) that
\[
\sum_a m_{0a} \epsilon_a = \delta_0, \quad \sum_a (m_{0a} - \delta_0 \epsilon_a)^2 = 0,
\]
which show that \( m_{0a} - \delta_0 \epsilon_a = 0 \), and therefore we have
\[
m^\mu_a = \delta^\mu \epsilon_a.
\]
If \( \delta = 0 \), on the other hand, we find \( m_a = 0 \) from \( (30) \), and \( m_{0a} = \epsilon_a \delta_0 \) from \( (32) \), which imply that \( (39) \) holds also in this case. The result is expressible in the form:
\[
|\vec{\sigma} \cdot \vec{p} - \vec{M}_1| = (p - \delta) \cdot (p - \delta)(p + \delta) \cdot (p + \delta),
\]
with
\[
2\epsilon(Y^\mu_a) = \delta^\mu \epsilon_a, \quad \vec{M}_1 = \vec{p} \cdot \vec{\epsilon} \cdot \vec{\sigma} \cdot \delta,
\]
for an arbitrary \( \delta^\mu \). The condition in the previous section
\[
\nu_{\text{ext}}(1) = \frac{2 m^2_\lambda}{g^2} \delta \cdot \delta < 0,
\]
shows that local Lorentz symmetry breaks for a space-like \( \delta^\mu \).

We could prepare in advance two spinors in a quasi fermion doublet as the eigenstates of \( \vec{p} \cdot \vec{\epsilon} \). Then, \( \vec{M}_1 \) becomes simply \( \vec{M}_1 = \rho^2 \vec{\sigma} \cdot \delta \). In this case, the quasi fermion with up-isospin \( \varphi_1 \), and that with down-isospin \( \varphi_2 \) satisfy the equations of motion
\[
\vec{\sigma}^\mu (i \partial_\mu - \delta_\mu) \varphi_1 = 0, \quad \vec{\sigma}^\mu (i \partial_\mu + \delta_\mu) \varphi_2 = 0,
\]
respectively. The energy eigenvalues and corresponding eigenstates are given by
\[
\begin{align*}
p_0 &= |p - \delta| + \delta^0, \quad \varphi_{q1 p} = L_{p - \delta}, \\
p_0 &= -|p - \delta| + \delta^0, \quad \varphi_{q1 p} = R_{p - \delta}, \\
p_0 &= |p + \delta| - \delta^0, \quad \varphi_{q2 p} = L_{p + \delta}, \\
p_0 &= -|p + \delta| - \delta^0, \quad \varphi_{q2 p} = R_{p + \delta}.
\end{align*}
\]
The explicit representation of \( L_k \) and \( R_k \) with \( k = p \pm \delta \) are given in Appendix B. The anti-particle interpretation applies for the solutions with the negative kinetic energy. Then \( (44) \) shows the emergence of a quasi fermion \( q_1 \) with energy \( E_{1p} = |p - \delta| + \delta^0 \), and its anti-particle \( \bar{q}_1 \) with energy \( E_{1\bar{p}} = |p + \delta| - \delta^0 \), in addition to a quasi fermion \( q_2 \) with energy \( E_{2p} = |p + \delta| - \delta^0 \), and its anti-particle \( \bar{q}_2 \) with energy \( E_{2\bar{p}} = |p - \delta| + \delta^0 \), since the absence of a particle with momentum \( -k^\mu \) implies the presence of an anti particle with momentum \( k^\mu \) in the hole theory.

We can construct by canonical quantization the Schrödinger operator \( \varphi(x) \) in terms of the creation and annihilation operators for \( q_1, \bar{q}_1, q_2, \) and \( \bar{q}_2 \) as
\[
\varphi_1(x) = \sum_p \left( q_{1p} L_{p - \delta} + \bar{q}_{1\bar{p}} R_{p - \delta} \right) \frac{e^{ip \cdot x}}{\sqrt{V}},
\]
\[
\varphi_2(x) = \sum_p \left( q_{2p} L_{p + \delta} + \bar{q}_{2\bar{p}} R_{p + \delta} \right) \frac{e^{ip \cdot x}}{\sqrt{V}},
\]
with the quantization volume \( V \), from which we calculate \( \langle j^\alpha \rangle \) to obtain \( (10) \) with
\[
\Gamma_Y = g^2 \left( \frac{2}{3} k_1 - \frac{\delta \cdot \delta}{60 \pi^2} \right), \quad k_1 = \int \frac{d^3 p}{(2\pi)^3} \frac{i}{p^2 + \epsilon},
\]
where the divergent integral is estimated as
\[
\int \frac{d^3 p}{(2\pi)^3} \frac{p - \delta}{|p - \delta|} = -\delta \left( \frac{4}{3} k_1 - \frac{\delta \cdot \delta}{30 \pi^2} \right).
\]
In this calculation, the \( \delta^0 \)-dependence of \( \Gamma_Y \) drops out. From the self consistency equation \( (14) \), spontaneous local Lorentz symmetry breakdown of type-I occurs for
\[
\delta \cdot \delta = \frac{60 \pi^2}{g^2} \left( \frac{2}{7} g^2 k_1 - m^2 \right) > 0.
\]
B. Type-II (m ≠ 0)

If m ≠ 0, the matrix \( m^\mu_\alpha \) becomes invertible, and we have the reciprocal relation:

\[
\eta_{\mu\nu}m^\mu_\alpha m^\nu_\beta = \frac{m^2}{4}\eta_{\alpha\beta},
\]

or, equivalently

\[
\delta \cdot \delta = m^2/4,
\]

\[
\delta^0 m^0_a = \delta \cdot m_a,
\]

\[
m_a \cdot m_b = m_{a0}m_{b0} + \frac{1}{4}m^2\delta_{ab}.
\]

The evaluation of the left-hand side of (24) with (54) gives

\[
m^0_a = v_a \cdot \delta, \quad m^i_a = \frac{m}{2}v^i_a + \frac{v_a \cdot \delta}{\delta_0 + m/2}\delta^i.
\]

The relation (51) implies that \( \delta^\mu \) should be a time-like 4-vector. Then, there is a Lorentz frame in which \( \delta^\mu = (0, 0) \). In this frame, we find that \( m^0_a = 0 \) from (52), and the relation of (53) shows that \( m_a \) can be written as \( m_a = m^0_a/2 \) with the help of some orthonormal 3-vectors \( v_a \) satisfying \( v_a \cdot v_b = \delta_{ab} \). We can also assume in addition that \( v_a \times v_b = \epsilon_{abc}v_c \) holds. Pulling back to the original frame, we obtain \( m^\mu_a \) as the solution of (52) and (53).

\[
m^0_a = v_a \cdot \delta, \quad m^i_a = \frac{m}{2}v^i_a + \frac{v_a \cdot \delta}{\delta_0 + m/2}\delta^i.
\]

The vacuum expectation value \( \langle Y^\mu_a \rangle \) becomes simplest in a Lorentz frame in which \( \delta^\mu = \frac{m}{2}\eta^\mu_0 \) and \( v^i_a = \delta^i_a \). Then we have

\[
\frac{\ddot{\varphi}}{2}(Y^\mu_a) = \frac{m}{2}\eta^\mu_0, \quad \ddot{M}_2 = \frac{m}{2}\rho \cdot \sigma.
\]

In this case, the condition for the vacuum energy density

\[
\nu_{\text{ext}}(II) = -\frac{3m^2}{2g^2}(m^V_\gamma + 3m^2) < 0,
\]

implies that the spacetime symmetry breaking is unavoidable if \( m \) exists.

The energy eigenvalues and the corresponding eigenfunctions for (19) are in this case given by

\[
p_0 = p + \frac{m}{2}, \quad \varphi_{vp} = L_p L_p,
\]

\[
p_0 = -p + \frac{m}{2}, \quad \varphi_{vp} = R_p R_p,
\]

\[
p_0 = \omega - \frac{m}{2}, \quad \varphi_{vp} = \lambda_+ R_p L_p + \lambda_- L_p R_p,
\]

\[
p_0 = -\omega - \frac{m}{2}, \quad \varphi_{vp} = -\lambda_- R_p L_p + \lambda_+ L_p R_p,
\]

where \( p = |p| \) and \( \omega = \sqrt{p^2 + m^2} \). They show the emergence of a “quasi neutrino” with energy \( E_{vp} = p + \frac{m}{2} \), and a “quasi anti-neutrino” with energy \( E_{\bar{v}} = p - \frac{m}{2} \), in addition to a “quasi electron” with energy \( E_{vp} = \omega - \frac{m}{2} \), and a “quasi positron” with energy \( E_{\bar{v}} = \omega + \frac{m}{2} \), according to the hole theory applied to the negative kinetic energy.

The wave functions have been concisely represented in terms of the direct product \( IS \), where the first entry \( I = R_p \), or \( L_p \) is a “iso-helicity” eigenstate, and the second entry \( S = R_p \), or \( L_p \) is an ordinary helicity eigenstate of a fermion with 3-momentum \( p \). The explicit representation of the coefficients \( \lambda_\pm \) are given in Appendix (B).
As done in the type-I case, we can construct the Schrödinger operator \( \varphi(x) \) in terms of the creation and annihilation operators for the quasi neutrino and the quasi electron:

\[
\varphi(x) = \varphi_\nu(x) + \varphi_e(x),
\]

\[
\varphi_\nu(x) = \sum_p \left( \nu_p \varphi_{\nu p} + \bar{\nu}_{-p} \varphi_{\bar{\nu} p} \right) e^{ip \cdot x} \sqrt{V},
\]

\[
\varphi_e(x) = \sum_p \left( e_p \varphi_{e p} + e_{-p} \varphi_{\bar{e} p} \right) e^{ip \cdot x} \sqrt{V},
\]

from which we calculate \( \langle j^n \rangle \) to obtain \( \langle 10 \rangle \)

\[
\Upsilon = g^2 \int \frac{d^3p}{(2\pi)^3 3\omega} \frac{1}{\sqrt{V}} \frac{2}{3} g^2 \left[ k_1 - m^2 \left( -\frac{1}{2} + \ln \frac{2p_{\max}}{m} \right) \right],
\]

where \( p_{\max} \) is the 3-momentum cut off. The right-hand side of the self consistency equation \( \langle 14 \rangle \) becomes in this case \((m_Y^2 + 2m^2)Y^\nu\), and therefore the type-II spontaneous local Lorentz symmetry breakdown occurs for

\[
m^2 = \frac{2g^2k_1 - m_Y^2}{2 + \frac{g^2}{12\pi^2}(-\frac{1}{2} + \ln \frac{2p_{\max}}{m})} > 0.
\]

### IV. RIGHT-HANDED QUASI ELECTRON

Though our model contains only a left-handed Weyl doublet, we have seen in the previous section that a massive quasi electron emerges in the case of type-II breakdown, which implies that a right-handed quasi electron has been created from a left-handed Weyl doublet, since the mass term of an electron requires both a left-handed chiral electron and a right-handed chiral electron. This section explains how to obtain the field representation of a right-handed chiral electron.

We first introduce the operator \( T \) by

\[
T = \frac{1}{2}(1 + \rho \cdot \sigma),
\]

which is unitary, and exchanges the spin and the isospin:

\[
T^\dagger = T^{-1} = T, \quad T \rho T^{-1} = \sigma, \quad T \sigma T^{-1} = \rho.
\]

The formulae given in Appendix B show that \( \varphi_{\nu p} \) and \( \varphi_{\bar{\nu} p} \) are invariant under the operation of \( T \):

\[
T \varphi_{\nu p} = \varphi_{\nu p}, \quad T \varphi_{\bar{\nu} p} = \varphi_{\bar{\nu} p},
\]

while \( \varphi_{e p} \) and \( \varphi_{\bar{e} p} \) are transformed into the different wave functions \( \chi_{e p} \) and \( \chi_{\bar{e} p} \) given by

\[
\chi_{e p} = T \varphi_{e p} = \lambda_+ L_p R_p + \lambda_- R_p L_p, \quad \chi_{\bar{e} p} = T \varphi_{\bar{e} p} = -\lambda_+ L_p R_p + \lambda_- R_p L_p,
\]

respectively. If the arbitrary solution of \( \langle 19 \rangle \) for the type-II case is expressed by \( \varphi = \varphi_\nu + \varphi_e \), in the same way as we obtained the representations \( \langle 64 \rangle, \langle 65 \rangle, \) and \( \langle 66 \rangle \), we find that

\[
T \varphi_\nu = \varphi_\nu, \quad T \varphi_e = \chi_e.
\]

By operating \( T \) on \( \langle 19 \rangle \), we also find that the spinor field \( \chi_e \) satisfies the equation of motion

\[
(\not{\rho} i \partial_\mu - \not{M})\chi_e = 0,
\]

which can be rewritten in the form

\[
(\not{\sigma} i \partial_\mu - \not{\bar{M}})\chi_e = 0,
\]

due to \( \rho \cdot \nabla \chi_e = -\sigma \cdot \nabla \chi_e \). The equation \( \langle 74 \rangle \) shows that \( \chi_e \) is a right-handed Weyl doublet. By noticing the relation \( \bar{M} = mT - \frac{\bar{\mu} \rho}{T} \), we can represent the free equations of motion for \( \varphi_\nu, \varphi_e, \) and \( \chi_e \) as

\[
\bar{\sigma}^\mu (i \partial_\mu - \delta_\mu) \varphi_\nu = 0,
\]

\[
\bar{\sigma}^\mu (i \partial_\mu + \delta_\mu) \varphi_e = m \chi_e,
\]

\[
\bar{\sigma}^\mu (i \partial_\mu + \delta_\mu) \chi_e = m \varphi_e,
\]
where $\delta_\mu = \frac{\partial}{\partial \eta_\mu}$. Since the above three equations have no more isospin matrices, it is redundant to represent $\varphi_\nu$, $\varphi_e$, and $\chi_e$ as doublets. We may represent them in terms of the two component spinors $\psi_L$, $\psi_eL$, and $\psi_eR$ as

$$\varphi_\nu = \phi_\nu \psi_eL, \quad \varphi_e = \phi_e \psi_eL, \quad \chi_e = \phi_e \psi_eR.$$  \hspace{1cm} (78)

where $\phi_\nu$ and $\phi_e$ are constant scalar iso-doublets with normalization $\phi_\nu^\dagger \phi_\nu = \phi_e^\dagger \phi_e = 1$. We may assume further that $\phi_\nu^\dagger \phi_e = 0$. In this representation, the equations for $\psi_L$, $\psi_eL$, and $\psi_eR$ are expressible, by using the Dirac matrices $\gamma^\mu$ of the chiral representation $^{15}$, in the form

$$\gamma^\mu (i \partial_\mu - \delta_\mu) \psi_\nu = 0, \quad \psi_\nu = \begin{pmatrix} 0 \\ \psi_eL \end{pmatrix},$$

$$[\gamma^\mu (i \partial_\mu + \delta_\mu) - m] \psi_e = 0, \quad \psi_e = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_eR \\ \psi_eL \end{pmatrix},$$

which are identical with the Dirac equations for the neutrino and the electron, respectively, except for the appearance of the kinetic 4-momentum operators $i \partial_\mu \pm \delta_\mu$, instead of the canonical 4-momentum operator $i \partial_\mu$.

V. KINETIC ENERGY-MOMENTUM TENSOR FOR A QUASI FERMION DOUBLET

One more preparation is necessary before arguing the equation of gravity modified by spontaneous local Lorentz symmetry breakdown. It is on the energy-momentum tensor for the emergent quasi fermion doublet $X^{\mu\nu}$, which is obtained from the variation of $\langle \mathcal{L}_\varphi \rangle$ with respect to the vierbein:

$$\delta \int d^4x e \langle \mathcal{L}_\varphi \rangle = \int d^4x e [ - \delta e_{\mu\alpha} e_\nu^{\alpha} X^{\mu\nu} ],$$

where the vacuum expectation value is taken only for the gauge field $Y_\mu$. The explicit form of $X^{\mu\nu}$ is expressed as

$$X^{\mu\nu} = K^{(\mu\nu)} + J^{\mu\nu} - g^{\mu\nu} \langle \mathcal{L}_\varphi \rangle,$$

$$K^{(\mu\nu)} = \frac{1}{2} (K^{\mu\nu} + K^{\nu\mu}), \quad K_{\mu\nu} = \frac{1}{2} (K^{\mu\nu} + K^{\nu\mu}^\dagger),$$

$$J^{\mu\nu} = \varphi^\dagger \delta^\mu (i \nabla^\nu - g(Y^\nu)) \varphi,$$

$$J^{\mu\nu} = j^{\mu} \cdot (Y^\nu).$$

The derivation is not simple regardless of its appearance. The last term in the right-hand side of (84) is removable owing to the equation of motion. The tensor $X^{\mu\nu}$ is asymmetric due only to the existence of the tensor current $J^{\mu\nu}$, which could not be incorporated into the Einstein equation of gravity as the source term without any modifications.

The reason for the emergence of $J^{\mu\nu}$ is the following. Originally, the gauge field $Y_\mu$ and the vierbein field are independent, and $\delta Y_\mu = 0$ under the variation with respect to the vierbein. After the breakdown, $Y_\mu$ develops the vacuum expectation value in a local Lorentz frame, and acquires the dependence on the vierbein: $\langle Y_\mu \rangle = e_{\mu\alpha} \langle \gamma^\alpha \rangle$ in the general coordinate system. The extra term $J^{\mu\nu}$ comes from the variation of $\langle Y_\mu \rangle$ with respect to the vierbein. It may be worth remarking in advance that the emergence of an extra kinetic term for the vierbein obtained in the next section will remedy the superficial inconsistency on this symmetry property.

We have seen that the canonical 4-momentum $p^\mu$ of a quasi fermion involves a potential term $\delta^\mu$ in addition to the kinetic 4-momentum $k^\mu$. The energy-momentum tensor of an emergent doublet $X^{\mu\nu}$ involves of course the contribution from $\delta^\mu$ through the vacuum expectation value $\langle Y^\alpha \rangle$, which will be also the source of gravity.

If $K^{(\mu\nu)}$ in $X^{\mu\nu}$ represents the kinetic energy-momentum tensor for the quasi fermion doublet, the resultant effect on gravity coming from broken spacetime symmetry is only the tensor current $J^{\mu\nu}$. We check it in the following for the quasi fermions individually. Otherwise, we will re-decompose $X^{\mu\nu}$ in order to obtain better physical insights.

A. $X^{\mu\nu}$ for the type-I doublet

For the type-I case, $K^{\mu\nu}$ defined in (85) becomes

$$K^{\mu\nu} = \varphi^\dagger \delta^\mu (i \nabla^\nu - \rho^3 \delta^\nu) \varphi.$$  \hspace{1cm} (88)
As we see from (13), the kinetic energy-momentum tensor is \( k^\mu = p^\mu - \delta^\mu, \) while that of the isospin-down quasi fermion is \( k^\mu = p^\mu + \delta^\mu. \) Then we find that \( \bar{K}^{(\mu \nu)} \) in this case has the property appropriate for calling it the kinetic energy-momentum tensor. We obtain further in this case the representation

\[
X^{\mu \nu} = \bar{K}^{(\mu \nu)} + j^{\mu \nu}A^\nu, \tag{89}
\]

\[
j^{\mu \nu} = \frac{g}{2} \bar{\sigma}^\nu \rho^\mu \varphi, \tag{90}
\]

\[
A^\mu = \frac{2}{g} \delta^\mu = \frac{2}{g} e^\mu \delta^\alpha. \tag{91}
\]

**B. \( X^{\mu \nu} \) for the type-II doublet**

For the type-II case, we have from (86)

\[
K_{\nu}^{\mu \nu} = \varphi^\dagger \bar{\sigma}^\mu (i \nabla^\nu - \frac{m}{2} \delta^\nu) \varphi \tag{92}
\]

\[
J_{\nu}^{\mu \nu} = \varphi^\dagger \bar{\sigma}^\mu (i \nabla^\nu - \delta^\nu + \frac{m}{2} \delta^\nu) \varphi \tag{93}
\]

\[
\bar{\delta}^\mu = e^{\nu \alpha} \delta^\alpha = \frac{m}{2} \rho^\mu, \tag{94}
\]

where \( \bar{\rho}^\alpha = (0, \rho), \) \( \rho^\alpha = (1, \rho), \) while \( \bar{\rho}^\mu, \) \( \rho^\mu, \) and \( \bar{\rho}^\mu \) are obtained by contracting \( \bar{\rho}^\alpha, \rho^\alpha, \) and \( \bar{\rho}^\alpha \) with the vierbein \( e^\mu \alpha, \) respectively. As we see from (75), (76), and (77), on the other hand, the kinetic energy-momentum tensor for the quasi neutrino and that for the quasi electron should be given by

\[
K_{\nu}^{\mu \nu} = \varphi^\dagger \bar{\sigma}^\mu (i \nabla^\nu - \delta^\nu) \varphi \nu, \tag{96}
\]

\[
J_{\nu}^{\mu \nu} = \varphi^\dagger \bar{\sigma}^\mu (i \nabla^\nu + \delta^\nu) \varphi e, \tag{97}
\]

\[
\delta^\mu = e^{\nu \alpha} \delta^\alpha = \frac{m}{2} \rho^\mu, \tag{98}
\]

where the lower index \( \nu \) in (96) implying the quasi neutrino should not be confused with the upper index \( \nu \) representing the general coordinate index. Then, it is more appropriate to decompose \( X^{\mu \nu} \) for the quasi neutrino \( \varphi = \varphi_\nu, \) and that for the quasi electron \( \varphi = \varphi_e \) as

\[
X_{\nu}^{\mu \nu} = \bar{K}_{\nu}^{(\mu \nu)} + J_{\nu}^{\mu \nu}, \quad J_{\nu}^{\mu \nu} = \frac{m}{4} \varphi^\dagger \bar{\sigma}^\nu (\bar{\rho}^\mu \rho^\nu + \bar{\rho}^\nu \rho^\mu) \varphi \nu, \tag{99}
\]

\[
X_e^{\mu \nu} = \bar{K}_e^{(\mu \nu)} + J_e^{\mu \nu}, \quad J_e^{\mu \nu} = -\frac{m}{4} \varphi^\dagger \bar{\sigma}^\nu (\bar{\rho}^\mu \rho^\nu + \bar{\rho}^\nu \rho^\mu) \varphi e. \tag{100}
\]

It is worth remarking, moreover, that owing to the relations (73) and (72), we have another expressions

\[
J_{\nu}^{\mu \alpha} = \frac{m}{4} \varphi^\dagger \bar{\sigma}^\nu (\bar{\rho}^\mu \rho^\alpha + \bar{\rho}^\alpha \rho^\mu) \varphi \nu = \bar{\psi}_\nu \gamma^\mu \psi_\nu \delta^\alpha, \tag{101}
\]

\[
J_e^{\mu \alpha} = -\frac{m}{4} (\varphi_e \bar{\sigma}^\nu \rho^\alpha \varphi_e + \gamma_e \sigma^\alpha \rho^\mu \chi_e) = -\frac{m}{2} \bar{\psi}_e \gamma^\mu \psi_e \delta^\alpha, \tag{102}
\]

The last equality in (101), and that in (102) show the representations in terms of the effective Dirac fields \( \psi_\nu \) and \( \psi_e \) given by (81) and (82). It is found, in particular, that \( J^{\mu \alpha}_e \) is not zero only for the iso-scalar component \( J^{0 \alpha}_e, \) while the iso-scalar component of \( J^{\mu \alpha}_e \) is expressible as \( -\bar{\psi}_e \gamma^\mu \psi_e \delta^\alpha. \)

Each of those iso-scalar components is individually eliminable by a suitable phase transformations for a quasi fermion doublet. Since our model does not have a U(1) gauge symmetry, a local phase transformation leads to a different expression for the equation of motion as well as the energy-momentum tensor. For example, the phase transformation \( \varphi \to e^{i \theta} \varphi \) applied to the Lagrangian \( \bar{L}_\varphi \) defined by (71) gives rise to the changes

\[
e \bar{L}_\varphi \to e \bar{L}_\varphi - e j^{\mu 0} \partial_\mu \theta, \quad j^{\mu 0} = \varphi^\dagger \bar{\sigma}^\mu \varphi, \tag{103}
\]

where the iso-scalar current \( j^{\mu 0} \) conserves: \( \nabla_\mu j^{\mu 0} = 0. \) Since the extra term in (103) represents a total divergence, it does not affect the dynamics of a Weyl doublet. Similarly, if \( J^{\mu \nu} \) is an arbitrary conserved current, adding \(-J^{\mu \nu} \partial_\mu \theta\) to the Lagrangian will not affect the dynamics of the system. We assume here that the conserved current in a local
Lorentz frame is a bi-linear form of an effective Dirac field: \( J^\alpha = e^\mu_\alpha J^\mu = \bar{\psi} \gamma^\alpha \psi \), which is definable independent of the vierbein. The result of adding this total divergence term is easier to see, if we consider the vierbein as a small fluctuation from the identity: \( e_{\mu\alpha} = \eta_{\mu\alpha} + \omega_{\mu\alpha} \) with \( \omega_{\mu\alpha} \) small. Up to the first order of \( \omega_{\mu\alpha} \), the action of the additional term is expressible as
\[
\Delta S = \int dx^4 e^{-i e_\mu e_\nu J^\mu \partial_\mu \theta} = \int dx^4 \left[ - (1 + \omega^\rho_\mu) \bar{\psi} \gamma^\mu \psi \partial_\mu \theta + \omega_{\mu\nu} \bar{\psi} \gamma^\mu \psi \partial^\nu \theta + O(\omega^2) \right],
\] (104)
while the free action for a quasi fermion doublet may be rewritten in terms of an effective Dirac field as
\[
S_\psi = \int dx^4 [(1 + \omega^\rho_\mu) \bar{\psi} \gamma^\mu \psi (i \partial_\mu - a_\mu) \psi - \omega_{\mu\nu} (\bar{K}^{\mu\nu} + J^{\mu\nu}) + O(\omega^2)],
\] (105)
in which \( a^\mu = \delta^\mu \) for \( \psi = \psi_\nu \), while \( a^\mu = -\delta^\nu \) for \( \psi = \psi_\nu \). Adding the two expressions (104) and (105), we find that the phase transformation changes the constant potential \( a^\mu \) and the tensor current \( J^{\mu\nu} \) as
\[
\begin{align*}
a^\mu &\to a^\mu + \partial^\mu \theta, \\
J^{\mu\nu} &\to J^{\mu\nu} - \bar{\psi} \gamma^\mu \psi \partial^\nu \theta,
\end{align*}
\] (106)
provided that \( \bar{K}^{\mu\nu} \) is regarded as left unchanged. Accordingly, \( J^{\mu\nu}_\rho \) vanishes for \( \partial^\nu \theta = \delta^\nu \), while the iso-scalar component of \( J^{\mu\nu}_\rho \) vanishes for \( \partial^\nu \theta = -\delta^\nu \). In either case, however, \( \bar{K}^{\mu\nu} \) becomes no more the kinetic energy-momentum tensor, since the constant potential for the quasi neutrino changes from \( \delta^\mu \) to \( 2\delta^\mu \) in the first case, while that for the quasi electron changes from \( -\delta^\mu \) to \( -2\delta^\mu \) in the second case. In these gauges, the gravitational field generated by \( \bar{K}^{\mu\nu} \) will involve contributions from the 4-potential \( \delta^\mu \).

The constant 4-potentials for the quasi neutrino and the quasi electron can not be removed simultaneously by a single phase transformation for a quasi fermion doublet, from which physical effects will result. The presence of \( \delta^\mu \) will affect the thermal equilibrium states in the argument on the matter dominance in cosmology, as already mentioned in the paper.

VI. MODIFIED EQUATION OF GRAVITY BASED ON THE VIERBEIN

The section showed that the gauge field has two types of vacuum expectation values consistent with the relativistic emergent theory. This section shows that the vacuum expectation value of the gauge field generates also the kinetic term for the vierbein.

In the case of spontaneous gauge symmetry breakdown, the Nambu-Goldstone bosons are field theoretically represented by broken gauge parameters. We will see that the same parallelism applies to spontaneous local Lorentz symmetry breakdown, where the vierbein becomes the field representation of the Nambu-Goldstone bosons. Therefore, the Nambu-Goldstone bosons in this case are essentially gravitons. We are here naturally invited into the realm of quantum theory of gravity without waiting for the Planck scale energy. We derive for two types of relativity preserving local Lorentz symmetry breakdown the equation of gravity based on the vierbein as the equation of motion for the Nambu-Goldstone bosons.

Before entering into the derivation, we consider again the implication of the constancy of a local Lorentz vector. The vacuum expectation values of the gauge potential and the gauge current developed by spontaneous local Lorentz symmetry breakdown are local Lorentz vectors, the direction of which depend on the orientation of the Lorentz frame taken at each spacetime point. Then the direction of a constant local Lorentz vector can differ for each spacetime point. As we have already mentioned in sec. we deal with this situation by formulating that the vacuum expectation value of a local Lorentz vector is the same for every local frames at different spacetime points. The freedom of the vierbein under local Lorentz transformations serves to satisfy this requirement, and the orientations of local Lorentz frames at different spacetime points are in turn mutually fixed.

According to this understanding, what is constant is a triplet of local Lorentz vectors \( \{Y_\alpha\} \), while that with respect to the general coordinate system \( \{Y_\mu\} = e^\mu_\alpha \{Y_\alpha\} \) becomes the triplet of vector fields. We can separate in a local Lorentz frame the vacuum expectation value from the gauge field by \( Y_\alpha = \{Y_\alpha\} + A_\alpha \) with \( \langle A_\alpha \rangle = 0 \). Expressing in the general coordinate system, we have
\[
Y_\mu = e^\mu_\alpha \{Y_\alpha\} + A_\mu,
\] (107)
where \( A_\mu \) represents the ordinary SU(2) gauge field without the vacuum expectation value. Since \( \{Y_\mu\} = e^\mu_\alpha \{Y_\alpha\} \) after the breakdown of local spacetime symmetry, the Lagrangian \( \mathcal{L}_Y \) provides also the kinetic term for the vierbein, and \( e^\mu_\alpha \) acquires its own dynamics.
As mentioned in Sec V, the gauge field $Y_\mu$ is originally independent of the vierbein, which is clear from the definition of the covariant derivative for gauge transformations. Only after spacetime symmetry breakdown, it partly acquires the vierbein dependence through the vacuum expectation value. It is due to this formulation that the Nambu-Goldstone boson can acquire its field representation.

The Lagrangian for the vierbein $L_V$ is therefore expressible as

$$L_V = \langle L_Y + L_\Phi \rangle,$$

where the vacuum expectation value is taken for both the Higgs field $\Phi$ and the gauge field $Y_\mu$. The vacuum expectation value of the Higgs Lagrangian becomes in this case constant, providing the cosmological constant $\Lambda$:

$$\langle L_\Phi \rangle = \frac{m^2}{2} \langle Y^\alpha \cdot Y_\alpha \rangle + \frac{\lambda \eta}{4} - \Lambda_0 = -\Lambda.$$

The variation of the action for the vierbein consists of the part coming from the variation $\delta e_{\mu\alpha}$ for $\langle Y_\mu \rangle$, and the part coming from the variation of the metric tensor $\delta g_{\mu\nu}$:

$$\delta \int d^4x e^{L_V} = \int d^4x \delta e_{\mu\alpha} e^{\alpha} \left[ \langle D_\rho Y_\rho^{\mu} \cdot Y_\nu \rangle - \Omega^{\mu\nu} - g^{\mu\nu} \Lambda \right],$$

where

$$\Omega^{\mu\nu} = \langle Y_\rho^{\mu} \cdot Y_\nu \rangle - \frac{1}{4} g^{\mu\nu} F_\sigma F^{\rho\sigma},$$

$$F_\mu^{\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

where the first term in the right-hand side of equation (110) gives dynamics to the vierbein even in the absence of the Einstein action of gravity, while the remaining terms are regarded as the energy-momentum tensor for the Nambu-Goldstone boson and the cosmological constant. Taking the contribution from the Einstein action of gravity and that of a quasi fermion doublet into account, we obtain the equation of gravity modified by spontaneous local Lorentz symmetry breakdown in the form:

$$\frac{1}{8\pi G} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + N_{\mu\nu} = \bar{K}_{(\mu\nu)} + \Omega_{\mu\nu} + g_{\mu\nu} \Lambda,$$

$$N_{\mu\nu} = \langle D_\rho Y_\rho^{\mu} \cdot Y_\nu \rangle - \bar{J}_{\mu\nu},$$

where we have expressed the energy-momentum tensor of a quasi fermion doublet $X_{\mu\nu}$ in the form (84). In this expression, asymmetry exists only in the term $N_{\mu\nu}$, and the antisymmetric part of it should vanish:

$$N_{[\mu\nu]} = 0.$$
The extra equation (115) is satisfied, in particular, when
\[ \nabla^\alpha F_{\mu\nu} = j_3^\mu, \]  
(119)
We have in this case \( N_{\mu\nu} = 0 \), and the modified equation of gravity (118) returns to the ordinary form of Einstein equation:
\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G \left[ \tilde{K}_{(\mu\nu)} + \Omega_{\mu\nu} + g_{\mu\nu}\Lambda \right], \]  
(120)
The explicit dependence on \( \delta^\mu \) disappears, and we obtain two covariant equations (119) and (120). Accordingly, spontaneous local Lorentz symmetry breakdown will not show the violation of relativity nor the anisotropy in physical phenomena.

The extra equation of gravity (119) is completely identical in form with that for the electromagnetic potential \( A_\mu \), where \( j_3^\mu \) is the electromagnetic current of a quasi fermion doublet. As the result, the Nambu-Goldstone graviton is interpretable as the photon. Incidentally, the emergent U(1) gauge invariance under \( \delta A_\mu = \partial_\mu \theta \) is found to have the same origin as the general coordinate invariance in the Einstein gravity, since the infinitesimal general coordinate transformation of the vierbein is expressed by \( \delta e_{\mu a} = \partial_\mu \xi_a \) with an arbitrary infinitesimal 4-vector field \( \xi_a \). The current \( j_3^\mu \) defined by (90) shows that the electric charge of a quasi fermion doublet is \( q = \frac{2}{3} \rho^3 \).

It should be noticed, however, that the metric tensor \( g_{\mu\nu} \) and the electromagnetic potential \( A_\mu \) are not independent, but are connected by (111), which suggests that the electromagnetic force can generate the gravitational force directly, without mediation of the energy-momentum tensor. The strength of electromagnetic excitation of gravity depends on the scale of symmetry breaking parameter \( \delta^\mu \). This is one of the predictions derived in this paper, if the origin of electromagnetic force is the strong mode of gravity induced by spontaneous spacetime symmetry breakdown.

VIII. EXTRA EQUATION OF GRAVITY: TYPE-II

In the type-II case: \( \langle Y^\alpha_a \rangle = \frac{m}{g^2} \eta^\alpha_a \), we have
\[ N_{\mu\nu} = N'_{\mu\nu} + 2 \frac{m^4}{g^2} e_{\mu a}e_{\nu a}, \]  
(121)
\[ \Delta_{\mu a} = \nabla^\nu e_{\mu a} + m e_{abc} [e_{\mu bc} + \nabla^\nu (e_{\mu b}e_{\nu c})], \quad e_{\mu a} := \partial_\mu e_{\nu a} - \partial_\nu e_{\mu a}. \]  
(122)
The antisymmetric tensor \( \epsilon_{\mu a} \) is defined with convention \( \epsilon_{123} = 1 \). The covariant derivative with a check symbol implies that it lacks the local Lorentz connection; it is covariant only under the general coordinate transformations, but not under the local Lorentz transformations. For example, \( \nabla^\mu e_{\nu a} = \partial_\mu e_{\nu a} - \Gamma^\mu_{\nu\rho} e_{\rho a} \neq 0 \), in contrast to \( \nabla_\nu e_{\nu a} = 0 \). Furthermore, we have
\[ \Omega_{\mu\nu} = \Omega'_{\mu\nu} + \frac{m^4}{g^2} \left[ \frac{3}{2} g_{\mu\nu} + 2 e_{\mu a}e_{\nu a} \right], \]  
(123)
\[ \Omega'_{\mu\nu} = \frac{m^2}{g^2} \left[ e_{\mu a}e_{\nu a} + \frac{1}{4} g_{\mu\nu}e_{\alpha a}e_{\rho a}a + m e_{abc} (e_{\mu a}e_{\nu bc} + e_{\nu a}e_{\mu bc} - \frac{1}{2} g_{\mu\nu}e_{abc}) \right], \]  
(124)
Then, the modified equation of gravity has the form
\[ \frac{1}{8\pi G} (R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) + N'_{\mu\nu} = \tilde{K}_{(\mu\nu)} + \Omega'_{\mu\nu} + g_{\mu\nu}\Lambda', \]  
(125)
\[ \Lambda' = \Lambda + \frac{3m^4}{2g^2}, \]  
(126)
from which the extra equation of gravity \( N'_{\mu\nu} = 0 \) will be obtained. However, the local U(1) phase transformation discussed in sec.6 allows us to have \( N'_{\mu\nu} = 0 \) as the extra equation of gravity, since in the gauge where the iso-scalar component of the tensor current vanishes, the iso-scalar component of the equation \( N'_{\mu a} = \frac{m^2}{g^2} \Delta_{\mu a} \eta_{aa} - J_{\mu a} = 0 \), which would be inconsistent in other U(1) gauges, becomes simply the identity \( 0 = 0 \). In this case, the modified equation of gravity (125) is decomposed into two equations
\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G (\tilde{K}_{(\mu\nu)} + \Omega'_{\mu\nu} + g_{\mu\nu}\Lambda'), \]  
(127)
\[ \Delta_{\mu a} = - \frac{g^2}{m^2} J_{\mu a}. \]  
(128)
The factor $\frac{\omega^2}{m^2}$ appearing in the extra equation of gravity (128) at the place corresponding to the gravitational constant $G$ in the Einstein equation (127) implies that the Nambu-Goldstone graviton is much effectively excited than the graviton obeying the Einstein equation, enhanced roughly by the ratio $(\frac{\omega}{m})^2$. Accordingly, we may expect classical and quantum gravity phenomena to be observed in energy scales available even at laboratories, since we have assumed that spacetime symmetry breaks at an energy scale lower than the electroweak scale.

Expanding $\Delta_{\mu \alpha}$ with respect to the small fluctuation of the vierbein $\delta e_{\mu \alpha} = \omega_{\mu \alpha}$, we have

$$
\Delta_{\mu \alpha} = \Box \omega_{\mu \alpha} - \partial_\mu \partial_\nu \omega_{\nu \alpha} + Z_{\mu \alpha} + O(\omega^2),
$$

(129)

$$
Z_{\mu \alpha} = m e_{\mu \nu \delta} \delta \partial_\mu \omega_{\nu \delta} + \partial_\delta (\omega_{\mu \nu} - \omega_{\nu \mu}) + \eta_{\mu \delta} (\partial_\nu \omega^\rho \partial_\nu \omega_{\rho \delta}).
$$

(130)

Since the source term $J_{\mu \alpha}$ for the quasi neutrino does not have the iso-vector components, only the quasi electron contributes to $J_{\mu \alpha}$. If $\omega_{\mu \alpha}$ satisfies the constraint

$$
Z_{\mu \alpha} = 0,
$$

(131)

the linear approximation of equation (128) is expressible as

$$
\Box A_\mu - \partial_\mu \partial_\nu A_{\nu} = \frac{g}{2} \bar{\psi}_e \gamma_\mu \psi_e, \quad (132)
$$

$$
A_\mu = \frac{m}{g} \omega_{\mu \alpha} \phi^\alpha \phi_e, \quad (133)
$$

by contracting the equation (128) with a constant iso-vector $\frac{m}{g} \phi^\alpha \phi_e$. The constraint $Z_{\mu \alpha} = 0$ is solved in the next section. We find that (132) is identical with the electromagnetic equation for the gauge potential $A_\mu$ with the electron current as the source, where the “electric charge” of the quasi electron is $\frac{g}{2}$. Also in this case, the Nambu-Goldstone graviton can be regarded as the photon. Even though regarded as the photon, it differs from the ordinary photon in the sense that it can also couple to the energy-momentum tensor as the graviton, since the both are simply different realizations of the common quantum of vierbein.

We have considered up to now the quasi electron and the quasi neutrino separately. For the case of quantum transitions from a quasi electron to a quasi neutrino, or vice versa, the separation of $X_\mu$, into the gravitational part and the electromagnetic part becomes ambiguous, since we can not define the kinetic energy-momentum tensor properly in this case. Furthermore, the consideration on the charged currents in weak interactions are out of scope for our “electro-gravity theory”, which would require another paper of the “electroweak-gravity theory” taking the SU(2) gauge field properly into account.

**IX. GRAVITATIONAL WAVES IN TYPE-II GRAVITY**

As clearly seen from (130), if $\omega_{\mu \nu}$ is symmetric and traceless, the constraint (131) reduces to

$$
\partial_\mu \omega_{\mu \alpha} = 0, \quad (134)
$$

which is the same as the Lorentz gauge condition on the electromagnetic potential: $\partial_\mu A^\mu = 0$ in (133). As we can impose on the vierbein the gauge condition $\partial^\mu \omega_{\mu \alpha} = 0$, due to the general coordinate transformation invariance, the reduced constraint (134) is satisfied automatically. As the result, if the source term is absent, the linearized extra equation of gravity (128) has the plane wave solution

$$
\omega_{\mu \nu}(x) = \epsilon_{\mu \nu} e^{-ik \cdot x}, \quad k \cdot k = 0,
$$

(135)

with a constant tensor $\epsilon_{\mu \nu}$, which is symmetric, traceless, and satisfying $k^\mu \epsilon_{\mu \nu} = 0$. We can further simplify the constant tensor $\epsilon_{\mu \nu}$ by assuming that the wave propagates in the direction of the third axis of the spacial coordinates: $k^\mu = (k, 0, 0, k)$, to have $\epsilon_{0 \nu} + \epsilon_{3 \nu} = 0$. Then, the residual gauge transformation, $\epsilon'_{\mu \nu} = \epsilon_{\mu \nu} - k^\mu \epsilon_{\nu}$, which preserves the gauge condition $k^\mu \epsilon_{\mu \nu} = 0$, can make $\epsilon_{0 \nu}$ and $\epsilon_{3 \nu}$ vanish simultaneously, and the remaining non-zero components are only $\epsilon_{11}$, $\epsilon_{12}$, $\epsilon_{21}$, and $\epsilon_{22}$. Accordingly, the physical modes of the plane wave solutions are represented explicitly by

$$
\begin{pmatrix}
\omega_{11} \\
\omega_{12} \\
\omega_{21} \\
\omega_{22}
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix} e^{-ik \cdot x}, \quad \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix} e^{-ik \cdot x},
$$

(136)

with other components being 0. These plane wave solutions are identical with the gravitational waves obtained from the ordinary Einstein equation of gravity, since $\delta g_{\mu \nu} = \omega_{\mu \nu} + \omega_{\nu \mu}$. The result shows that the two equations of gravity (127) and (128) have consistently the common gravitational wave solutions.
If we remove the condition on $\omega_{\mu\nu}$ to be symmetric and traceless, two more solutions are found:

$$\omega_{10}, \omega_{13} = (1, -1) e^{-ik \cdot x}, \quad \omega_{20}, \omega_{23} = (1, -1) e^{-ik \cdot x},$$

with other components being 0, which were gauge modes under the additional condition.

**X. SUMMARY**

In phenomenology of elementary particle physics, the gravitational interaction is ordinarily ignored due to its weakness. However, this paper showed that the inclusion of gravity is inevitable for the systematic investigation of spontaneous spacetime symmetry breakdown.

Furthermore, in contrast to gauge symmetry, Lorentz symmetry is apt to be considered unbroken owing to the perfect experimental confirmation of relativity. Nevertheless, the emergence of relativistic effective theory showed that the possibility of spontaneous local Lorentz symmetry breakdown can not be excluded from phenomenology.

Since in curved spacetime the bi-spinor gauge current needs primarily a local Lorentz frame to be defined, what become constant in local Lorentz symmetry breakdown by fermions are local Lorentz vectors, while the vacuum expectation values in the general coordinate system still remain as vector fields. Therefore, we will not detect a specific spacetime direction nor violation of relativity unless gravity is completely ignored.

The remaining concern for considering spacetime symmetry breaking in phenomenology is the relativistic properties of the emergent quasi particles. We have shown that there are in fact two types of broken local Lorentz symmetry in which emergent quasi particles are still relativistic.

According to the understanding in this paper, the vierbein plays two roles; one is as the constituent of the metric tensor representing classical gravity, and the other is as the Nambu-Goldstone graviton. The dynamics of the vierbein is governed by the modified equation of gravity, which differs from that of Einstein by the extra kinetic term and the tensor current of the emergent quasi fermions.

We have also seen the congruence between the lost local Lorentz symmetry and the broken gauge symmetry. After the local Lorentz symmetry breakdown, the local Lorentz index becomes indistinguishable from the isospin index. Even after local Lorentz symmetry is lost, Lorentz invariance of the emergent theory is guaranteed as the general coordinate invariance of the original theory in the limit of flat Minkowski spacetime.

The decomposition of the modified equation of gravity into the conventional form of Einstein equation for the metric tensor and the equation for the vierbein shows that the scale of broken spacetime symmetry appears in the latter equation instead of the Planck mass, which implies that the quantum of vierbein can be much sensitive to the source than the graviton in the Einstein gravity. We showed that the asymmetric part separated from the modified gravitational equation involves the electromagnetic equation for the quasi fermions in either case of relativity preserving spacetime symmetry breakdown.

The emergent electromagnetism differs from that in the standard theory in the sense that the electromagnetic potential is directly connected through the constant vector to the vierbein, and therefore to the gravitational field. Accordingly, the electromagnetic interaction in the emergent theory will be associated with gravitational interactions, possibly much stronger than the Einstein gravity.

Furthermore, from the viewpoint obtained in this paper, quantum electrodynamics is interpretable in a sense as a realization of quantum gravity. Then, quantum gravity phenomena may be observed even in the energy scales available at laboratories, since local spacetime symmetry breaks at an energy scale after the gauge bosons become massive.

From the phenomenological point of view, this observation provides us with the means to know whether the vacuum of our universe is in the broken phase of spacetime symmetry or not. The answer will be obtained, not by detecting the specific spacetime direction nor the violation of Lorentz invariance, but by detecting classical or quantum mechanical strong inertial force acting on the electrically neutral objects associated with the electromagnetic interactions.

This paper partly revises the previous papers [7, 8] which express the expectation that the Nambu-Goldstone bosons emergent from the spontaneous breakdown of Lorentz symmetry will be the photon in the standard theory. The systematic investigation based on the conceptions presented in this paper reveals that the derived electrodynamics differs from that in the standard theory in the point already mentioned.

Eguchi argued in his renormalization theory [16] that gauge symmetry emerges from global symmetry, and that the massless condition of emergent gauge bosons is derivable from the equation revealing the breakdown of Lorentz invariance, which is similar to [14]. According to his result, it might be expected that the massive gauge bosons obtained by the Higgs mechanism would return to massless again by spontaneous Lorentz symmetry breakdown, which we called in the previous paper [8] “the inverse Higgs mechanism”. As we have already argued in this paper, the Nambu-Goldstone bosons are not the SU(2) gauge bosons, and therefore the inverse Higgs mechanism does not realize.
Acknowledgments

The author would like to thank H. Kanno for the correspondence concerning a global SU(2) anomaly.

Appendix A: Irrelevance of a global SU(2) anomaly in Minkowski spacetime

Though the paper reporting a global SU(2) anomaly \cite{11} seems to be known as the proof of the inconsistency of the SU(2) gauge theory of a Weyl doublet, what is proved is the inconsistency of that theory defined in a Euclidean space, the topology of which is a four dimensional sphere. Intrinsically, however, a Weyl doublet in four dimensional Euclidean space is undefinable, rather inconsistent, in quantum theory. There will be no room for a global SU(2) anomaly of that theory in four dimensional Minkowski spacetime.

A Weyl doublet in Euclidean four dimension can only be defined through the Lagrangian

\[
\mathcal{L}_E = -i \chi \gamma^a \sigma^a D_a \varphi - i \varphi \gamma^a \sigma^a D_a \chi = \psi \psi \psi \psi, \quad \sigma^a = (\sigma, i), \quad \bar{\sigma}^a = (\sigma, -i), \quad \psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix},
\]

where the coordinate index \(a\) is of a four dimensional Euclidean space, which is not related to the isospin index in the main text, while \(H\) is the Hamiltonian of a Dirac doublet \(\psi\) defined in the five dimensional Minkowski spacetime.

It is inevitable to double a Weyl doublet in quantum theory, since the canonical conjugate variable of a “left-handed” Weyl doublet \(\varphi\) is a “right-handed” Weyl doublet \(\chi\), though the equation of motion for each is expressible separately. For obtaining a Euclidean Weyl doublet forcibly, we have to take a square root of the Hamiltonian squared \(\sqrt{H^2}\), from which we may have symbolically the Lagrangian for a Weyl doublet in Euclidean four dimension

\[
\mathcal{L}_W = \varphi \sqrt{-\sigma^a D_a \bar{\sigma}^b D_b \varphi}.
\]

What Witten showed is that taking a specific branch of the square root is impossible from the consideration on the spectral flow \cite{17,19} of the Hamiltonian \(H\) of the five dimensional Dirac doublet in the presence of an instanton. Though the paper \cite{11} insists that the inconsistency exists also in four dimensional Minkowski spacetime, the substantial argument is absent.

Appendix B: notations and formulas

This section summarizes the notations and formulas used in this paper. It is often convenient to use the frame in which the direction of the third axis coincides with that of the given 3-momentum \(p\). The orthonormal frame vectors \(v^i\) are definable by

\[
v^1 = \frac{\partial v^3}{\partial \theta} = \begin{pmatrix} \cos \theta \cos \phi \\ \cos \theta \sin \phi \\ -\sin \theta \end{pmatrix}, \quad v^2 = \frac{1}{\sin \theta} \frac{\partial v^3}{\partial \phi} = \begin{pmatrix} -\sin \phi \\ \cos \phi \\ 0 \end{pmatrix}, \quad v^3 = \frac{p}{|p|} = \begin{pmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{pmatrix},
\]

where \(\theta\) and \(\phi\) are the colatitude and the azimuthal angle of momentum \(p\), respectively. The frame vectors \(v^i\) satisfy the right-handed orientation \(v^1 \times v^2 = v^3\). The helicity eigenstates are given by

\[
R_p = \begin{bmatrix} e^{-i \frac{\phi}{2} \sin \frac{\theta}{2}} \\ e^{i \frac{\phi}{2} \sin \frac{\theta}{2}} \end{bmatrix}, \quad L_p = \begin{bmatrix} -e^{-i \frac{\phi}{2} \sin \frac{\theta}{2}} \\ e^{i \frac{\phi}{2} \sin \frac{\theta}{2}} \end{bmatrix},
\]

which satisfy

\[
v^3 \cdot \sigma R_p = R_p, \quad v^3 \cdot \sigma L_p = -L_p,
\]

where \(\sigma = (\sigma^1, \sigma^2, \sigma^3)\) are the Pauli matrices. In these notations, we obtain the following relations

\[
\sigma R_p = v^3 R_p + v^+ L_p, \quad \sigma L_p = -v^3 L_p + v^- R_p, \quad (v^\pm = v^1 \pm iv^2).
\]

The Dirac matrices \(\gamma^\alpha\) and \(\gamma_5\) in the chiral representation are defined by

\[
\gamma^\alpha = \begin{pmatrix} 0 & \bar{\sigma}^\alpha \\ \sigma^\mu & 0 \end{pmatrix}, \quad \sigma^\alpha = (1, \sigma), \quad \bar{\sigma}^\alpha = (1, -\sigma), \quad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

\[
(A1)
\]

\[
(A2)
\]

\[
(B2)
\]

\[
(B3)
\]

\[
(B4)
\]

\[
(B5)
\]
The free equation of motion for the Dirac fermion with mass $m$ in the general coordinate system is given by

$$[\gamma^\mu i \nabla_\mu - m] \psi = 0, \quad \gamma^\mu = e^\mu_\alpha \gamma^\alpha, \quad \nabla_\mu \psi = \left( \partial_\mu + \frac{1}{8} \omega_{\mu \alpha \beta} \Sigma^{\alpha \beta} \right) \psi, \quad \Sigma^{\alpha \beta} = [\gamma^\alpha, \gamma^\beta] = \begin{pmatrix} \sigma^{\alpha \beta} & 0 \\ 0 & \overline{\sigma}^{\alpha \beta} \end{pmatrix}, \quad \sigma^{\alpha \beta} = \sigma^\alpha \sigma^\beta - \sigma^\beta \sigma^\alpha, \quad \overline{\sigma}^{\alpha \beta} = \sigma^\alpha \overline{\sigma}^\beta - \sigma^\beta \sigma^\alpha. \quad \text{(B6)}$$

where

$$[\gamma^\alpha, \gamma^\beta] = \left( \begin{array}{cc} \sigma^{\alpha \beta} & 0 \\ 0 & \overline{\sigma}^{\alpha \beta} \end{array} \right), \quad \sigma^{\alpha \beta} = \sigma^\alpha \sigma^\beta - \sigma^\beta \sigma^\alpha, \quad \overline{\sigma}^{\alpha \beta} = \sigma^\alpha \overline{\sigma}^\beta - \sigma^\beta \sigma^\alpha. \quad \text{(B7)}$$

In the flat Minkowski spacetime: $e^\mu_\alpha = \eta^\mu_\alpha$, the explicit representations of the 4-momentum eigenstates, $u_{p_R}$ for $p^\mu = (\omega, p)$, and $v_{-p_S}$ for $p^\mu = (-\omega, p)$ are given by

$$u_{p_R} = \begin{pmatrix} \lambda_+ \\ \lambda_- \end{pmatrix} R_k, \quad u_{p_L} = \begin{pmatrix} \lambda_- \\ \lambda_+ \end{pmatrix} L_p, \quad v_{-p_R} = \begin{pmatrix} -\lambda_- \\ \lambda_+ \end{pmatrix} R_p, \quad v_{-p_L} = \begin{pmatrix} \lambda_+ \\ -\lambda_- \end{pmatrix} L_p, \quad \text{(B8)}$$

where

$$\lambda_\pm = \frac{1}{2} \left[ 1 + \frac{m}{\omega} \pm \sqrt{1 - \frac{m^2}{\omega^2}} \right], \quad \omega = \sqrt{p^2 + m^2}. \quad \text{(B9)}$$

Incidentally, the following relations hold:

$$\lambda_+^2 + \lambda_-^2 = 1, \quad \lambda_+^2 - \lambda_-^2 = \frac{k}{\omega}, \quad \lambda_- = \frac{\omega - p}{m}, \quad \lambda_+ = \frac{\omega + p}{m}. \quad \text{(B10)}$$