The uniqueness of visual levels in resolving geometry of shape and space content based on van hieles’s theory

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Abstract. The process of learning mathematics in school has several problems related to the characteristics of mathematics, concepts, principle of mathematics and abstract of mathematical objects. Students also have difficulties in solving geometry of shape and space content. This can be seen from the research carried out by the researchers at one of schools in Thailand. Some students who were at the visualization level still could not do optimally in the level of visualization. However, there was a unique level of students’ visualization who could complete several levels above it. The basis that the author had was to examine these students more deeply. The type of this research was qualitative descriptive conducted at one of secondary schools by giving Van Hiele geometry test to get the groups concerning the level of students, then it was proceed by PISA test of shape and space content. SV00 was defined as a code of students’ visualization at level 0. At the visualization level, SV00 was able to reach 50% of indicators at the visualization stage and 33.3% at the analysis level.

1. Introduction
One of the objectives in students’ learning is having the ability to think critically, creatively and problem solving. This capability is considered very necessary especially in facing the development of science and technology which is increasing rapidly and dominated by global atmosphere that is always changing, uncertain and competitive. Learning should be based on constructivism, student-centered and learning with understanding also should be through learning by doing and referring to four UNESCO pillars, namely: learning to know, learning to do, learning to be, and learning to live together [1].

The ability to think critically and creatively plays an important role in modern society since it is capable to make people become more mentally flexible, open and easily adapt to various situations and problems. The community’s critical and creative thinking can develop themselves in making decisions, judgements, and problem solving [2].

Critical and creative thinking allow the students to study problems systematically, face above millions of challenges in an organized way, formulate innovative questions, and design problems which are seen as relatively new [3]. On the contrary, the students just imitate and record how to solve the problems done by the teacher. If they are given different questions with practice questions, they become confused as they do not know where to start and where they work [4]. This kind of learning makes the students tend to learn to remember or memorize and without understanding of what is being taught by the teacher [4].

In fact, Mathematics is difficult both on teaching and learning or mathematics is a lesson that is difficult to teach and learn [5]. This difficulty occurs since mathematics is a vertical structured lesson in which there is a sequence to study mathematical material [6]. The students only tend to memorize a
number of formulas, calculations and steps in solving the problems done by the teacher or provided on
the textbook. Thus, the students’ critical and creative thinking skills do not develop optimally [7].
Therefore, the students while learning mathematics in school should be trained to have critical and
creative thinking skills in acquiring, choosing, and processing information in order to survive in
changing and competitive conditions [1].

Thinking is a mental activity experienced by someone if they face a problem or situation that must
be solved [8]. Thinking as a mental activity helps to formulate or solve a problem, make a decision, or
fulfill curiosity desire [9]. This opinion shows that when someone formulates and solves a problem, or
wants to understand something, then he does a thinking activity. Thinking as a person’s mental ability
can be divided into several types, covering logical, analytical, systematic, critical, and creative
thinking [2,10]. Logical thinking can be interpreted as the student’s ability to think and draw
legitimate conclusions according to the rules of logic and is able prove that the conclusion is true
(valid) in accordance with the previous known knowledge. Analytical thinking is the student’s ability
to think about describing, elaborating, and analyzing the information used to understand a knowledge
by using logical reasoning, and it is not based on feelings or guesses [11]. Systematic thinking is the
student’s ability to think in doing or completing a task related to the sequences, stages, steps, or
planning that is appropriate, effective, and efficient. Those three types of thinking are interrelated.
Someone can be said to think systematically, then he needs to think analytically to understand the
information used. Then, to be able to think analytically, it requires the ability to think logically in
drawing conclusion from a situation[4]. Critical thinking and creative thinking are manifestations of
higher order thinking. Critical thinking can be seen as the student’s ability to think in comparing two
or more information, for instance, the information received from outside and the information which is
already owned. If there are differences or similarities, then he will ask questions or comments in order
to get the explanation. Critical thinking is often associated with creative thinking [7].

Geometry is an important branch of mathematics and it has been identified as a basic
mathematical skill [12,13]. According to Sherard [14] and Hong [15], geometry is important for
students as it is also applied in other branches of mathematics. For instance, geometry is applied in
other subjects such as engineering drawing, geometry drawing and so on. Basically, there are two
objectives of geometry learning, which are to develop logical thinking skill and spatial intuitions
which refer to how one views space and area in real world [12,16].

According to 2015 PISA Survey, Indonesian students were still weak in completing Space and
Shape content, most students were only able to reach level 3 and very few students who could reach
level 4 and 5, even no students was able to reach level 6[17]. Based on this statement, this research
focused on knowing the students’ thinking skills on Shape and Space content with the aim of
increasing the students’ ability levels, especially on Shape and Space content.

This research discussed the development level of students’ thinking in geometry based on van
Hiele’s level from the result of his research, this research revealed that as many as 576 third grade
students of junior high school in Jember, there were 13.89%, 44.62%, 34.55%, 6.77% and 0.17%
students who were at the level of previsualization, visualization, analysis, informal deduction, and
deduction [18].

2. The Method
The method used in this study was descriptive qualitative by selecting and formulating problems that
required conception and usefulness of the problem in which it could be investigated with existing
sources to determine the objectives of the research. The purpose of this research was consistent with
the formulation and definition of the problem, provided a limitation of the area or scope as well as the
extent to which the descriptive research would be carried out. These included the geographical area in
which the research would be carried out, chronological boundaries, the size of the shallow and how to
keep intact the research area that would be reached.

The type of research used in this study was descriptive research with qualitative approach. Thus,
this study would describe the uniqueness of mathematics students in solving PISA problem in Space
and Shape content. This research would be analyzed based on the achievement of indicators of each
Van Hiele levels and the percentage of each Van Hiele levels would be obtained as the final results. The used indicators had been developed by the researchers. In this study, the chosen subject was Thai students of Mathayom 4/2 Hatyaiwitayalaisomboonkulanya School. The subject of the research consisted of 4 students based on the Van Hiele test on level 0, 1, 2 and 3 only. The subject was obtained by giving Van Hiele question test in which the results would be categorized in the level of visualization, analysis, informal deduction and deduction, the researchers could not find any students with rigor level. The level was determined by using van Hiele level so that it did not need to be revalidated because its validity had been proven. After categorizing the results of the Van Hiele test, one student was selected from each category. One subject from each category with high skill in English language was taken due to the need for interviews at the time of research for each levels. This was done to find out whether or not there was difference in the results of written or oral tests from the interview of each level.

3. Results and Discussion

Before determining the research subject, it was necessary to determine the Van Hiele level of Mathayom 4/2 students. Determining Van Hiele’s level was done by giving Van Hiele test quoted in Van Hiele. Van Hiele test was done by the students within 60 minutes. After obtaining the results of student level based on Van Hiele’s level, the students were grouped in several levels and four students would be chosen as the research subjects with one student represented each level. After getting the research subjects, the next stage was conducting an interview. The students were interviewed interchangeably and analyzed their thinking process in solving the standard PISA problem in shape geometry and space content.

3.1 The Analysis of Students’ Thinking Profiles in PISA Standard Test Questions

The analysis of students’ thinking profiles was based on the students’ achievement of each indicators. The results of the tests and interviews would strengthen the results of analysis of students’ thinking profiles. The following were the results of the analysis of students’ geometry thinking profiles. SV00 as the research subjects was the representatives of level 0 group, namely visualization. The following was the results of SV00’s test and interview as well as its analysis.

3.1.1 The Question of Number One

The first indicator that became a reference to look at the profile of geometry thinking for level 0 was that the student could identify the structure based on the form they had seen in its entirety. In Figure 1, it could be seen that SV00 used of written language did not accompany by an explanation of the image, it only explained that in the graph, there were 3 curves up and down and had 3 curved sides, when writing the answer, SV00 seemed to have lack of understanding about the function of the graph and the questions in number 1. SV00 also did not answer the problem by concluding the chosen answers, however, through interview and after being given understanding by the author, SV00’s could explain and choose the answer.

Figure 1. The answer of SV00 for question number 1
The indicator that could be achieved by SV00 was identifying the structure based on the form seen in its entirety. Students with SV00 code could answer the problem of number 1 correctly, this was conveyed when SV00 was given a question by the author to get clarity related to the answers given during the written test. It could be seen in the interview excerpt, SV00 explained what had been written down in the written test, SV00 explained that the graph in question number 1 gone up and down 3 times according to the pictures A, B, C, D but D was too curved, and C also still curved, so the answer was B. From SV00 explanation, it could be said that picture A had 3 angles, but from what could be seen in image A, it had more than 3 angles, this indicated that SV00 still could not identify the structure parts in the stage of visualization.

During the second interview related to the structure formed from images A, B, C, D and E, SV00 explained that B, C, D was a triangle, images A and E were not triangles because they had many angles. This made SV00 could reach indicator 1 from visualization stage. The next indicator from visualization stage determined the example and that was not the example of geometric image. SV00 could not fulfill this indicator when he was given a question related to images A and E in question number 1, SV00 explained that A and E looked like squares, whereas before he answered that A and E had many angles, and square had only 4 angles. SV00 still did not know which was the examples and not the examples of geometry builds.

### Figure 2. The excerpt of interview with SV00 on question number 1

During the second interview related to the structure formed from images A, B, C, D and E, SV00 explained that B, C, D was a triangle, images A and E were not triangles because they had many angles. This made SV00 could reach indicator 1 from visualization stage. The next indicator from visualization stage determined the example and that was not the example of geometric image. SV00 could not fulfill this indicator when he was given a question related to images A and E in question number 1, SV00 explained that A and E looked like squares, whereas before he answered that A and E had many angles, and square had only 4 angles. SV00 still did not know which was the examples and not the examples of geometry builds.

#### 3.1.2 The Question Of Number Two

In the problem of number two, the question was to find out the achievement level of analysis, SV00 was not able to meet the indicators comparing to the builds based on their characteristics. It can be seen in the Figure 4.5 showed that SV00 only looked at the picture, it explained that the APQ triangle was congruent with the PBR, QRC and PRQ triangles. SV00 did not explain that triangle could be said to be congruent, SV00 in the written test explained that PBR and QRC triangles were parallel to this, it can be seen in the question of number 2 that R was the midpoint of the BC line, so it was true that PBR and QRC triangles were parallel. SV00 also explained that the PQR triangle was a reflection of the RQC. In this case, SV00 was correct if it explained that PQR as a reflection of RQC because reflection is a transformation that moves each point on a field by using a mirror image properties of
the points being moved. Mirror image properties is the similar distance between the original object and the mirror with the distance of the point of the shadow to the mirror, as well as the size and shape are similar. The answer written on the answer sheet by SV00 did the problem solving that involved recognizable wake-up traits.

Figure 3. SV00’s answer for question of number 2

SV00 in the figure 3 re-illustrated the triangle in the question of number 2 and was separated and given symbols and information such as reflection and parallel, from figure 3 it can be analyzed that SV00 was able to determine the properties of a wake by making observations, experiments, and drawing, but it was not able to see the relationship between several geometric shapes.

3.1.3 The Question Of Number 3

In the problem of number three, SV00 answered questions correctly. SV00’s answer explained that ABCD square had a side of 12 meters, it gave a conclusion to know the area of a square, it was necessary to multiply side by side, and got 12 x 12 equal to 144. The answer from SV00 was square meters, this means that SV00 understood the function of using mathematical symbols. In the visualization level, SV00 verbally described the wake with its full appearance, this showed that the build was a square and SV00 was able to solve the problem by operating the wake without using the properties applied in general, according to Van Hiele descriptors.

Figure 4. SV00’s answer for the question of number 3
SV00 in the Figure 5 on the problem of number 3, the second problem described the shape of the prism to get the answer of the question. SV00 explained that the AT line had a length of 12 meters and had a midpoint, so the midpoint divided the line into 6 meters, and it was connected, it also had a length of 6 meters. This was clearly explained in the interview picture 6.

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PF0025: why you say 144?
SV0025: because length of AB is 12 m and length of BC is 12 m too
PF0026: so?
SV0026: we can multiply together
PF0027: okay, now mention shape and space that you find in question number three?
SV0027: many pyramid, big pyramid, cubic and prism.
PF0028: now what can you define from question number three?
SV0028: angle side angle, side angle side, and side side side.
PF0029: what that for? For what?
SV0029: for congruent triangle
PF0030: can you mention the theorem?
SV0030: can not
PF0031: okay thank you, bye bye
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**Figure 5.** Quote of SV00 interview in the question of number 3

SV00 in the interview quote explained the square length of AB was 12 meters and BC was also 12 meters, so it multiplied the two sides of the square and got the result 144 m². SV00 fulfilled the level 1 of visualization indicator, which is identifying the building based on the form that is seen in its entirety by mentioning the build in the picture, which SV00 saw only the visible images, such as prisms and cubes, which were mentioned in three-dimensional images. This indicated that SV00 did not pay attention to other shapes such as squares and triangles which were two-dimensional constructs. SV00 also could not explain the theorem that could be explained in the picture about problem number 3.

Based on the results of the analysis, conclusions that could be drawn that the students in the visualization category were able to meet 1 indicator at the visualization level, which was students could identify the building based on the form that is seen in full and 1 indicator at the analysis level that was solving problems that had been identified. Students only recognized the geometric shapes of visual characteristics and their appearance but had not been able to understand and determine the geometry characteristics and the build shown. This is in line with the opinion of Yudianto, et all[19,20] The cause of students' mistakes in determining answer choices are that students assume that the test given is not a mathematical test because it is in the form of writings not numbers, whereas to answer questions in the form of build up images, students need concrete objects to help in answering questions. Students do not yet know the properties possessed by geometry and the use of language in test questions that are less familiar to them.

4. Conclusions

Students’ visualization in this research can solve the problems above the level of analysis. Based on van Hiele's theory, said that above level is guaranteed to work on all below levels, the students’ level of analysis is guaranteed to be able to do visualization level, if the below level ever been able to work on the above level. In this case, level 0 (visualization) will never be able to do level 1 (analysis). So, for further research, we will look for information related to the uniqueness that occurred in this
research which was level 0 capable of doing level 1. This allows van Hiele levels to still be modified according to the region or country of each student.

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