Stability of event horizons against neutrino flux

Koray Düztaş

1 Introduction

A space-time possesses a singularity if it fails to satisfy causal geodesic completeness which requires that every time-like and null geodesic can be extended to arbitrarily large affine parameter value both into the future and into the past. In essence, causal geodesic completeness means that photons or freely moving particles can not appear or disappear off the edge of the universe.

According to the singularity theorems of Hawking and Penrose [1], a space-time $M$ can not satisfy causal geodesic completeness if –in addition to some generic conditions– Einstein’s equations are satisfied and $M$ contains a trapped surface. Trapped surfaces arise when the gravitational collapse of a localized body (e.g. a star) to within its Schwarzschild radius takes place, provided that the deviation of the model from spherical symmetry is negligible. Thus, a singularity ensues as a result of gravitational collapse in classical general relativity.

Keywords Black hole · neutrino field · Cosmic censorship · Dirac equation

Received: date / Accepted: date

Abstract We construct a thought experiment in which an extremal Kerr black hole interacts with a test massless Dirac field, i.e. a “neutrino field”. Evaluating the flux integrals imposed by the energy momentum tensor for fermionic fields and the Killing vectors of the space-time, we prove that this interaction can indeed destroy the event horizon of the black hole and convert it to a naked singularity. The range of frequencies of the test neutrino field that can be used to destroy the black hole turns out to be the superradiant range for bosonic fields. We comment on back reaction and quantum effects.
If a space-time contains a well defined external infinity $I^+$ the black hole region of the space-time is defined by (see e.g. [2])

$$\mathcal{B} = M - J^-(I^+)$$

(1)

with an event horizon that constitutes the boundary of $\mathcal{B}$ in $M$.

$$\mathcal{H} = \partial \mathcal{B} = J^-(I^+) \cap M$$

(2)

where $J^-$ denotes the causal past of a set. In an asymptotically predictable space-time $M$, a trapped surface $\mathcal{T}$ is entirely contained within the black hole region $\mathcal{B}$; i.e. $\mathcal{T} \subset \mathcal{B}$. Therefore if the singularity has formed in the way prescribed by Hawking and Penrose it is hidden behind an event horizon, so it is invisible to the rest of the space time. In the opposite case where the space-time contains a singularity that lies to the past of $I^+$, so time-like curves may be drawn into the past that terminate on the singularity, the singularity is said to be naked. Whether a singularity is naked or clothed is crucial to preserve causal structure. In the presence of naked singularities, initial conditions on a Cauchy surface become undefined since the surface necessarily intersects the singularity, thus asymptotic predictability is disabled. Causal behaviour is characterized by prohibiting the existence of closed time-like or causal curves. It turns out that for asymptotically flat space-times, closed time-like curves which violate causality can evolve in the regions which contain naked singularities [3]. It becomes impossible to predict the behaviour of space-time in the causal future of a singularity.

It is an open question whether physically realistic collapse situations may arise without trapped surfaces, resulting in singularities that are not necessarily hidden behind an event horizon. For example the singularity theorem does not apply in the presence of a positive cosmological constant so it is unknown how the collapse will develop in that case. This led Penrose to propose the Cosmic Censorship Conjecture (CCC) in 1969 [4], to avoid the physical and philosophical pathologies that could arise in a space-time containing a naked singularity. CCC asserts that gravitational collapse of a body always ends up in a black hole rather than a naked singularity, so all singularities are hidden within the event horizons of black holes; i.e. they are invisible to distant observers (see [5] for a review). Distant observers neither encounter any singularities nor any effects propagating out of singularities. Conjecturing singularities to be hidden behind event horizons without any access to distant observers allows us to preserve causal structure despite the fact that the formation of singularities is inevitable in classical general relativity.

As a concrete proof of CCC has been elusive, it has become customary to attack the closely related –though not identical– problem of the stability of event horizons in the interactions of the black hole with test particles or fields. In these problems the initial state is a stationary Kerr-Newman space-time uniquely defined by three parameters (Mass $M$, charge $Q$, and angular momentum per unit mass $a$), satisfying

$$M^2 \geq Q^2 + a^2.$$ 

(3)
which defines a black hole surrounded by an event horizon as opposed to a naked singularity. To test the stability of the event horizon one lets the black hole absorb some particles or fields coming from infinity. At the end of the interaction the space-time is expected to settle to another stationary configuration with new values of $M$, $Q$, and $a$. Then one may check if it is possible to increase charge or angular momentum of the black hole beyond the extremal limit saturating (3) so that the final configuration of the parameters violates (3) and defines a naked singularity.

The first thought experiment to test the stability of black hole horizons was constructed by Wald [6]. He showed that particles having enough charge or angular momentum to exceed extremality are not captured by the black hole. This work motivated many authors to construct similar thought experiments. We shall not attempt to review all the related work here, rather we will consider the specific case of a fermionic field interacting with an extremal Kerr black hole. We will follow the recipes developed in [7] and [8], which evaluate the bosonic cases of a fermionic field interacting with an extremal Kerr black hole. We shall not attempt to review all the related work here, rather we will consider the specific case of a fermionic field interacting with an extremal Kerr black hole. We will follow the recipes developed in [7] and [8], which evaluate the bosonic cases of spin-0 (scalar field) and spin-1 (electromagnetic field) respectively. (For a review of related work see the introduction of [8]).

In the present work we consider the scattering of neutrino fields from an extremal Kerr black hole saturating (3). Neutrino field refers to a test massless Dirac field in Kerr background. This Kerr background, uniquely parametrized by $M$ and $a$, is the initial stationary state of the problem. The impact of the test field on the background geometry is negligible. The field coming in from infinity, is partially transmitted through the horizon and partially reflected back to infinity. At the end of the interaction the field decays away and the space-time settles down to another stationary state described by the Kerr metric, with new values of the black hole parameters $M$ and $a$. To calculate the changes in the black hole parameters, we construct expressions for the fluxes of energy and angular momentum carried by the neutrino field and evaluate them at the horizon of the black hole. We treat the free neutrino test field using Newman-Penrose (NP) [9] formalism. We use the separation of Dirac equations for the relevant NP variables in Kerr space-time by Chandrasekhar [10], and asymptotic solutions at the horizon and at infinity for the massless case by Teukolsky [11]. In this context we check if neutrino fields can be used to over-spin an extremal Kerr black hole.

2 Neutrino fields in Kerr geometry

This section consists of a brief review of spinor formalism and previous results involving neutrino fields in Kerr space-time especially by Chandrasekhar and Teukolsky. We start with Dirac equation which couples two fermion fields via

$$\nabla_{AA'} P^A + i \mu_f Q_{A'} = 0$$
$$\nabla_{AA'} Q^{A'} + i \mu_f P_A = 0$$

where $\nabla_{AA'}$ is the spinor covariant derivative defined axiomatically as a map $\nabla_x = \nabla_{XX'}: \theta \rightarrow \theta_{XX'}$ [12] and $\sqrt{2} \mu_f$ is the mass of the fermion field.
Every spin basis induce a tetrad of null vectors.

\[ l^a = o^A \bar{o}^{A'} \quad n^a = i^{A'} \bar{i}^A \quad m^a = o^A i^{A'} \quad \bar{m}^a = i^{A'} \bar{o}^A \]  

(5)

\( l \) and \( n \) are real while \( m \) and \( \bar{m} \) are complex conjugates. Null vectors satisfy orthogonality relations

\[ l_a n^a = n_a l^a = -m_a \bar{m}^a = -\bar{m}_a m^a = 1 \]

\[ l_a m^a = l_a \bar{m}^a = n_a m^a = n_a \bar{m}^a = 0 \]  

(6)

In 1962, Newman and Penrose [9] have developed the idea of adapting tetrads of null vectors to spinors. The directional derivatives along the null directions are denoted by conventional symbols

\[ D = l^a \nabla_a, \quad \Delta = n^a \nabla_a, \quad \delta = m^a \nabla_a, \quad \bar{\delta} = \bar{m}^a \nabla_a \]  

(7)

\( \nabla_a \) can be expressed as a linear combination of these operators.

\[ \nabla_a = g_a^b \nabla_b \]

\[ = (n_a l^b + l_a n^b - m_a \bar{m}^b - m_a \bar{m}^b) \nabla_b \]

\[ = n_a D + l_a \Delta - \bar{m}_a \delta - m_a \bar{\delta} \]  

(8)

Dirac’s equations [1] can explicitly be written in the form:

\[ (D + \epsilon - \mu) P^0 + (\bar{\delta} + \pi - \alpha) P^1 = i \mu f Q^1 \]  

(9)

\[ (\Delta + \mu - \gamma) P^1 + (\bar{\delta} - \tau + \beta) P^0 = -i \mu f \bar{Q}^\delta \]  

(10)

\[ (D + \bar{\epsilon} - \bar{\mu}) \bar{Q}^\delta + (\bar{\delta} + \bar{\pi} - \bar{\alpha}) \bar{Q}^1 = -i \mu f P^1 \]  

(11)

\[ (\Delta + \bar{\mu} - \bar{\gamma}) \bar{Q}^1 + (\bar{\delta} + \bar{\bar{\beta}} - \bar{\tau}) \bar{Q}^\delta = i \mu f P^0 \]  

(12)

where \( P^0 \) and \( P^1 \) are components of \( P^A \) along the spinor dyad basis \( o^A \) and \( i^A \) respectively. Chandrasekhar [10] showed that equations (4) can be solved by a separation of variables.

\[ P^0 = (r - ia \cos \theta)^{-1}(-1/2 R(r))(-1/2 S(\theta)) e^{-i\omega t} e^{im\phi} \]

\[ P^1 = (1/2 R(r))(1/2 S(\theta)) e^{-i\omega t} e^{im\phi} \]

\[ \bar{Q}^\delta = -(r + ia \cos \theta)^{-1}(-1/2 R(r))(1/2 S(\theta)) e^{-i\omega t} e^{im\phi} \]

\[ \bar{Q}^1 = (1/2 R(r))(-1/2 S(\theta)) e^{-i\omega t} e^{im\phi} \]  

(13)

This separation leads to a pair of equations for both \( (\pm 1/2 R(r)) \) and \( (\pm 1/2 S(\theta)) \) which can be expressed as a single equation with \( s = \pm 1/2 \) as a parameter.

As we let \( \mu_f = 0 \) for neutrino fields the radial equation takes the form

\[ \Delta^{-s} \frac{\partial}{\partial r} \left( A^{s+1} \frac{\partial (sR)}{\partial r} \right) + \left( \frac{K^2 - 2is(r - M)K}{\Delta} + 4i\omega r - \lambda \right) (sR) = 0 \]  

(14)
where \( K \equiv (r^2 + a^2)\omega - am \) and \( \lambda \equiv A + a^2\omega^2 - 2am\omega \). The angular equation is given by

\[
\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial S}{\partial \theta} \right) + \left( a^2\omega^2 \cos^2 \theta - \frac{m^2}{\sin^2 \theta} - 2a\omega s \cos \theta - \frac{2ms \cos \theta}{\sin^2 \theta} - s^2 \cot^2 \theta + s + A \right) S = 0
\]

(15)

These are exactly Teukolsky’s equations for massless fields. The asymptotic solutions at infinity for the radial functions are \[11\]

\[
Y_{in} e^{-i\omega r^*}, \quad Y_{out} e^{i\omega r^*} \quad s = 1/2
\]

\[
Z_{in} e^{-i\omega r^*}, \quad Z_{out} e^{i\omega r^*} \quad s = -1/2
\]

(16)

where we have adopted the notation of Teukolsky and Press \[13,14\]; \( Y_{in}, Y_{out}, Z_{in}, Z_{out} \) are the normalizations of the ingoing and outgoing waves at infinity for the cases \( s = 1/2 \) and \( s = -1/2 \) respectively, and \( r^* \) is the tortoise coordinate defined by \( dr^*/dr = (r^2 + a^2)/\Delta \), so that \( r^* \to -\infty \) as the horizon is approached.

Only the ingoing solutions are physical at the horizon \[14\]. Therefore the asymptotic solutions of the radial equation (14) near the horizon are given by

\[
Y_{\text{hole}}(\Delta^{-1/2}) e^{-ikr^*} \quad s = 1/2
\]

\[
Z_{\text{hole}}(\Delta^{1/2}) e^{-ikr^*} \quad s = -1/2
\]

(17)

where \( k = \omega - m\Omega, \Omega = a/2Mr_+ \) is the rotational frequency of the black hole.

The angular equation \[15\] constitutes a Sturm-Liouville eigenvalue problem for the separation constant \( A = sA_m^m(a\omega) \), together with boundary conditions of regularity at \( \theta = 0 \) and \( \pi \). From Sturm-Liouville theory the eigenfunctions \( sS_m^m \) are complete and orthogonal on \( 0 \leq \theta \leq \pi \) for each \( s \) and \( a\omega \). One can also define \( sZ_m^m = sS_m^m e^{im\phi} \) with orthonormalization:

\[
\int_0^{2\pi} d\phi \int_0^\pi d\theta (sZ_{lm}(\theta, \phi, a\omega))(sZ_{lm}^*(\theta, \phi, a\omega)) \sin \theta = \delta_{ll'} \delta_{mm'}
\]

(18)

The functions \( sZ_{lm}(\theta, \phi, a\omega) \) reduce to spin weighted spherical harmonics \( sY_{lm}(\theta, \phi) \) when \( a\omega = 0 \).

3 The stability of the event horizon

In this section we evaluate the changes in the mass and angular momentum of the black hole due to its interaction with the test neutrino field. For that
purpose we are going to use the current conservation equations imposed by
the Killing vectors of the space-time and the local conservation of energy
and momentum in general relativity. If $K$ is a Killing vector, by definition
it satisfies $\mathcal{L}_K g = 0$, where $\mathcal{L}$ is the Lie derivative. This can be re-arranged
to give the Killing equation $\nabla_a (T^a_{\mu} K_{\mu}) = 0$. The current conservation equation
$\nabla_a (T^a_{\mu} K_{\mu}) = 0$ is derived by combining the expression of the space-time sym-
metry in terms of Lie derivatives or equivalently the Killing equation with the
local conservation of energy-momentum $\nabla_c T^a_{ac} = 0$, where $T^{ac}_{ac}$ is the energy
momentum tensor. This allows us to express the rates of change in the corre-
sponding black hole parameters as fluxes into the black hole. Kerr space-
time is stationary and axi-symmetric with corresponding Killing vectors
$\partial/\partial t$ and $\partial/\partial \phi$. Therefore the net radial flux of energy and the net radial flux of angular
momentum across any sphere centered at the black hole are given by surface
integrals of $-T^r_t$ and $T^r_\phi$ respectively.

$$\left(\frac{dM}{dt}\right)_{\text{b.h.}} = - \int_S \sqrt{-g} T^1_0 d\theta d\phi$$

(19)

Since $dM = dE$ for the black hole, and

$$\left(\frac{dL}{dt}\right)_{\text{b.h.}} = \int_S \sqrt{-g} T^1_3 d\theta d\phi$$

(20)

where the label b.h. stands for black hole..

To test the stability of the horizon of a Kerr black hole, we define an
indicator

$$C = M^2 - a^2$$

(21)

Then, using $a = L/M$

$$\delta C = \int \frac{dC}{dt} dt = \int \frac{2}{M} \left\{ (M^2 + a^2) \frac{dM}{dt} - a \frac{dL}{dt} \right\} dt$$

(22)

implying

$$\frac{dC}{dt} = \int_S \sqrt{-g}[M^2 + a^2](T^1_0) - a T^1_3 d\theta d\phi$$

(23)

For an extremal black hole saturating the main criterion $\delta C$ should al-
ways remain positive to preserve the event horizon. If the initial state is an
extremal black hole and $\delta C$ has a negative value, the final state describes a
naked singularity.

Kerr space-time can be represented by an NP tetrad of the form:

$$l^\mu = [(r^2 + a^2)/\Delta, 1, 0, a/\Delta],$$

$$n^\mu = [(r^2 + a^2), -\Delta, 0, a]/(2\Sigma)$$

$$m^\mu = [ia \sin \theta, 0, 1, i/\sin \theta]/[\sqrt{2}(r + ia \cos \theta)]$$

(24)

where $\Sigma = r^2 + a^2 \cos^2 \theta$ and $\Delta = r^2 - 2Mr + a^2$, which should not be confused
with the NP derivative operator $\Delta$. Note that $\Delta \rightarrow r^2 - 2Mr + a^2 + Q^2$ gives
the Kerr-Newman space-time, and $a \to 0$ gives the Schwarzschild space-time. Using tetrad (24) one derives

$$T_{ab}^{\rho b} = -\frac{\Delta^2}{4} - \Sigma T_{ab}^{\sigma b} = -(r^2 + a^2)T^1_0 - T^1_3$$  \hspace{1cm} (25)$$

We are going to evaluate the fluxes (19) and (20) at the horizons ($r = r_+$) of extremal Kerr black holes ($r_+ = M$). In that case we recognize that the right-hand-side of (25) is the integrand of (23).

For fermionic fields, the energy momentum tensor in terms of the corresponding NP scalars is given by (see [15], [16])

$$T_{AA'B'B'} = -\frac{1}{2} \{ P_A \nabla_{BB'} P_{A'} - P_A \nabla_{BB'} P_A + P_B \nabla_{AA'} P_{B'} - P_B \nabla_{AA'} P_B - Q_A \nabla_{BB'} Q_{A'} + Q_A \nabla_{BB'} P_{QA} - Q_B \nabla_{AA'} Q_{B'} + Q_B \nabla_{AA'} Q_B \}$$

implying

$$T_{ab}^{\rho l b} = T_{AA'B'B'} = \phi^{A'} \phi^B \phi^{B'}$$

$$= -i \{ P^1 D \bar{P}_1 - \bar{P}_1 D P^1 + \bar{Q}^1 D Q^1 - Q^1 D \bar{Q}^1 \}$$  \hspace{1cm} (27)$$

and

$$T_{ab}^{\rho l b} = T_{AA'B'B'} = \phi^{A'} \phi^B \phi^{B'}$$

$$= -i \{ P^0 \Delta \bar{P}_0 - \bar{P}_0 \Delta P^0 + \bar{Q}^0 \Delta Q^0 - Q^0 \Delta \bar{Q}^0 \}$$  \hspace{1cm} (28)$$

where $D$ and $\Delta$ are NP derivative operators. Now we may plug in the solutions near the horizon for $P^0$ and $P^1$ following (13) and (17) to evaluate the left-hand-side of (25). We first write the general solution in terms of separated modes:

$$P^1 = \int d\omega e^{-i\omega t} \sum_{l,m} e^{im\phi} Y_{lm}(\theta, \omega) Y_{hole}(\Delta^{-1/2}) e^{-ikr}$$

$$\bar{Q}^1 = \int d\omega e^{-i\omega t} \sum_{l,m} e^{im\phi} Y_{lm}(\theta, \omega) Y_{hole}(\Delta^{-1/2}) e^{-ikr}$$

$$P^0 = \int d\omega e^{-i\omega t} \sum_{l,m} e^{im\phi} Y_{lm}(\theta, \omega) \rho Y_{hole}(\Delta^{1/2}) e^{-ikr}$$

$$\bar{Q}^0 = \int d\omega e^{-i\omega t} \sum_{l,m} e^{im\phi} Y_{lm}(\theta, \omega) \rho^* Y_{hole}(\Delta^{1/2}) e^{-ikr}$$  \hspace{1cm} (29)$$

where $\rho = -(r - ia \cos \theta)^{-1}$. Then

$$P^1 D \bar{P}_1 = \Delta^{-2} \int d\omega d\omega' \sum_{l,m} |Y|^2 e^{-i(k-k')r} (S_{k'} S_{l'}) e^{i(m-m')\phi} e^{-i(\omega-\omega')t}$$

$$\times [i(r^2 + a^2)(w' + k') - (r - M) - im']$$

$$\Delta^{-2} \int d\omega d\omega' \sum_{l,m} |Y|^2 e^{-i(k-k')r} (S_{k'} S_{l'}) e^{i(m-m')\phi} e^{-i(\omega-\omega')t}$$

$$\times [i(r^2 + a^2)(w' + k') - (r - M) - im']$$  \hspace{1cm} (30)$$
\[ P^1 D^1 P^1 = \Delta^{-2} \int d\omega d\omega' \sum |Y|^2 e^{-i(k-k')r^*} (S_\pm S'_\pm) e^{i(m-m')\phi} e^{-i(\omega'-\omega)t} \times [-i(r^2+a^2)(w+k) - (r-M) + ima] \] (31)

\[ Q^1 D^1 Q^1 = \Delta^{-2} \int d\omega d\omega' \sum |Y|^2 e^{-i(k-k')r^*} (S_- S'_-) e^{i(m-m')\phi} e^{-i(\omega-\omega')t} \times [i(r^2+a^2)(w' + k') - (r-M) - ima] \] (32)

\[ \tilde{Q}^1 D^1 \tilde{Q}^1 = \Delta^{-2} \int d\omega d\omega' \sum |Y|^2 e^{-i(k-k')r^*} (S_- S'_-) e^{i(m-m')\phi} e^{-i(\omega-\omega')t} \times [i(r^2+a^2)(w' + k') - (r-M) - ima] \] (33)

where the summation is over \( l, m, l', m' \). \(|Y|^2 = Y_{\text{hole}} Y_{\text{hole}}^* \) and \( S_{l\pm} = [\pm 1/2 S_{l\pm} (\theta, a\omega)] \).

Note that all of the four terms (30-33) that construct \( T_{\alpha\beta} \) in (27) have a common factor of \( \Delta^{-2} \) which will cancel with the \( \Delta^2 \) term in the first term of the left-hand-side of (25). We proceed to evaluate the terms in (28).

\[ P^0 D^0 \tilde{P}^0 = (\Delta/2\Sigma) \int d\omega d\omega' \sum |\rho|^2 |Z|^2 e^{-i(k-k')r^*} (S_- S'_-) e^{i(m-m')\phi} e^{-i(\omega-\omega')t} \times [i(r^2+a^2)(w' - k') - \Delta \rho^* - (r-M) - ima] \] (34)

\( P^0 D^0 \tilde{P}^0 \) and the remaining three terms in \( T_{\alpha\beta} n^\alpha n^\beta \) terms turn out to be at least first order in \( \Delta \) so the second term on the left-hand-side of (25) will not contribute to the flux at a surface at the horizon where \( \Delta \to 0 \).

Now we can evaluate (28) and (22) for a surface at the horizon \( r = r_+ \), \( \Delta \to 0 \Rightarrow M^2 + a^2 = 2Mr_+ \) of an extremal Kerr black hole \( (r_+ = M) \). Note that \( \sqrt{-g} = \Sigma \sin \theta \) at the horizon. We first take the time integral in (22) which gives a delta function in \( \omega \) and \( \omega' \) allowing us to evaluate the integral over \( \omega' \). Now the angular functions are all functions of \( \omega \) so we can use the orthonormality relation (18) and evaluate \( \theta \) and \( \phi \) integrals. This gives the kronecker delta \( \delta_{ll'} \delta_{mm'} \), so we evaluate the sums over \( l' \) and \( m' \). Having \( \omega = \omega' \) (from the integration of the delta function in \( \omega \) and \( \omega' \) over \( d\omega' \)) and \( m = m' \) (as a result of \( \theta \) and \( \phi \) integrals using the orthonormality relation) leads to \( k = k' \). The expressions in the square brackets in (30,33) reduce to \( \pm i2Mr_+ (k + \omega - am/(2Mr_+)) = \pm i2Mr_+ (2k) \). Finally \( \delta(C) \) takes the form

\[ \delta C = \int d\omega \sum_{lm} |Y|^2 \delta_{ll'} \delta_{mm'} \] (35)
4 Conclusions

The expression (35) becomes negative in the region $0 < \omega < am/(2Mr_+)$ where $k$ is negative. This is exactly the superradiant region for bosonic fields \[17,18\]. If the frequency of the incoming neutrino field is in this region, the interaction of the field with the black hole leads to the destruction of the horizon, exposing the singularity of the black hole to outside observers. However in almost identical analysis involving bosonic fields and extremal black holes (see \[7\] and \[8\] for scalar and electromagnetic cases), cosmic censorship conjecture has been shown to remain valid (the horizons of extremal black holes can not be destroyed using integer spin test fields). This is in accord with the fact fermionic fields do not exhibit superradiant behaviour like integer spin fields \[19\]. Between the initial and final states there can be a net absorption of the superradiant modes which carry more angular momentum than energy.

We have neglected back reaction effects in this analysis, however neutrino fields lead to a generic destruction of black holes which can not be fixed by back reaction effects. A hope to avoid the naked singularity lies in the quantum treatment of the problem. Unruh studied neutrino fields in Kerr background by using quantum field theory in curved space-times and found that black holes spontaneously emit neutrino fields in superradiant modes and spin themselves down \[20\]. (This pre-dates the well-known Hawking effect \[21\]) In that work he refers to a possibility suggested by Feynman: if we send in neutrino fields with frequency $\omega < am/(2Mr_+)$ towards the black hole, part of this flux is suppressed because of exclusion principle; black hole only absorbs the exact modes which it has emitted. This represents a quantum form of superradiance. To the extent that we accept that the Unruh radiation fills the phase space, one can at best suppress the constant spinning down of the black hole. In that case the modes with $0 < \omega < am/(2Mr_+)$ are harmless because they are either not absorbed or a field in the same mode has been emitted so they do not contribute to the flux at the horizon. We have shown that these are the only modes that make $\delta C$ negative, so if we exclude these modes from our analysis it will not be possible to over-spin the black hole. In this way, quantum approach provides a possibility to avoid the naked singularity.

References

1. S.W. Hawking and R. Penrose, *The Singularities of Gravitational Collapse and Cosmology*, Proc. R. Soc. London **314** (1970) 529.
2. R.M. Wald, *General Relativity*, The University of Chicago Press, London U.K., (1984).
3. F.J. Tipler,, *Causality Violation in Asymptotically Flat Space-Times*, Phys. Rev. Lett. **37** (1976) 879.
4. R. Penrose, *Gravitational Collapse : The Role of General Relativity*, Riv. Nuovo Cimento **1** (1969) 252.
5. R.M. Wald, *Gravitational Collapse and Cosmic Censorship*, arXiv:gr-qc/9710068 (1997).
6. R.M. Wald, *Gedanken Experiments to Destroy a Black Hole*, Annals Phys. **82** (1974) 548.
7. I. Semiz, *Dyonic Kerr-Newman black holes, complex scalar field and Cosmic Censorship*, *Gen. Relativ. Gravit.* **43** (2010) 833.

8. K. Düztaş, *Electromagnetic field and cosmic censorship*, *Gen. Relativ. Gravit.* **46** (2014) 1709.

9. E. Newman and R. Penrose, *An Approach to Gravitational Radiation by a Method of Spin Coefficients*, *J. Math. Phys.* **3** (1962) 566.

10. S. Chandrasekhar, *The solution of Dirac’s equation in Kerr geometry*, *Proc. R. Soc. Lond. A* **349** (1976) 571.

11. S.A. Teukolsky, *Perturbations of a Rotating Black Hole. I. Fundamental Equations for Gravitational, Electromagnetic, and Neutrino-Field Perturbations*, *Astrophys. J.* **185** (1973) 635.

12. J. Stewart, *Advanced General Relativity*, Cambridge University Press, Cambridge U.K., (1991).

13. S.A. Teukolsky and W.H. Press, *Perturbations of a Rotating Black Hole. II. Dynamical Stability of the Kerr metric*, *Astrophys. J.* **185** (1973) 649.

14. S.A. Teukolsky and W.H. Press, *Perturbations of a Rotating Black Hole. III. Interaction of the Hole with Gravitational and Electromagnetic Radiation*, *Astrophys. J.* **193** (1974) 443.

15. R. Güven, *Wave Mechanics of Electrons in Kerr Geometry*, *Phys. Rev. D.* **16** (1977) 1706.

16. J.A.H. Futterman, F.A. Handler and R.A. Matzner, *Scattering from black holes*, Cambridge University Press, Cambridge U.K., (2009).

17. C.W. Misner, *Interpretation of Gravitational-Wave Observations*, *Phys. Rev. Lett.* **28** (1972) 994.

18. W. H. Press and S.A. Teukolsky, *Floating Orbits, Superradiant Scattering and the Black-hole Bomb*, *Nature* **238** (1972) 211.

19. S. Chandrasekhar, *The Mathematical Theory of Black Holes*, Oxford University Press, New York U.S.A. (1983).

20. W.G. Unruh, *Second quantization in the Kerr metric*, *Phys. Rev. D.* **10** (1974) 3194.

21. S.W. Hawking, *Particle creation by black holes*, *Commun. Math. Phys.* **43** (1975) 199.