ELECTROWEAK SYMMETRY BREAKING WITH
NON-UNIVERSAL SCALAR
SOFT TERMS AND LARGE $\tan \beta$ SOLUTIONS

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ABSTRACT

We discuss radiative electroweak symmetry breaking with non-universal scalar masses at the GUT scale. Large $\tan \beta$ solutions are investigated in detail and it is shown that qualitatively new (as compared to the universal case) solutions exist, with much less correlation between soft terms. We identify two classes of non-universalities which give solutions with $A_o \approx B_o \approx M_{1/2} \approx O(M_Z)$, $\mu \approx O(0.5m_o)$, $m_o \gg M_Z$ and $A_o \approx B_o \approx M_{1/2} \approx \mu \approx O(M_Z)$, $m_o \geq M_Z$, respectively. In each case, after imposing gauge and Yukawa coupling unification, we discuss the predictions for $m_t, m_b$ and the spectrum of supersymmetric particles. One striking consequence is the possibility of light charginos and neutralinos which in the second option can be higgsino-like. Cosmological constraint on the relic abundance of the lightest neutralino is also included.

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The mechanism of radiative electroweak symmetry breaking in supersymmetric models [1] has been extensively studied in the literature. The $SU(2) \times U(1)$ gauge symmetry is broken by quantum corrections to the tree level potential which is postulated at the GUT scale. The quantum corrections are included by means of the renormalization group evolution of the parameters of the lagrangian from the GUT scale to the electroweak scale. Most of the work [2] has so far been done under the assumption that the soft supersymmetry breaking parameters: gaugino masses, scalar masses and trilinear couplings have at the GUT scale some universal values $M_{1/2}$, $m_o$ and $A_o$, respectively. Then, with two additional parameters: $B_o$ (the soft Higgs mixing term) and $\mu_o$ (the Higgs mixing in the superpotential), the model depends on five free parameters. However, exact universality of the soft mass terms at the GUT scale is neither phenomenologically necessary nor theoretically sound.

From the phenomenological point of view, the usual argument based on the smallness of FCNC does not constrain mass terms which are diagonal in the family space and even for the off–diagonal terms the constraint is weaker than usually believed [3].

On the theoretical side, one may argue that exact universality is realized at the string scale but no longer at the GUT scale, where one can envision several types of non–universal corrections to the soft terms.

Radiative electroweak breaking is expected to be sensitive to the non–universal corrections to the soft terms in those regions of the parameter space where the breaking with universal terms requires a large degree of correlation (fine tuning) among them. This is the case for the large $\tan \beta$ solutions [4,5] and the purpose of this letter is to study such solutions in the presence of non–universal scalar soft mass terms. Some aspects of non-universal breaking terms have already been addressed in ref.[6].

The issue of the ”large $\tan \beta$” vacuum has been discussed for some time [4,7,8] as the possible explanation to the breaking of the $SU(2)_V$ symmetry of the quark masses:

$$\tan \beta \equiv \frac{v_2}{v_1} \simeq \frac{mt}{mb}$$

with

$$h_t \simeq h_b$$

where $v_1$, $v_2$ are the vacuum expectation values of the two Higgs fields which couple to the down and up quarks, respectively, and $h_t$ and $h_b$ are Yukawa couplings. Two interesting questions emerge in this context. One is this: can large breaking of the $SU(2)_V$ by the vacuum be obtained by amplification (via the mechanism of radiative breaking) of weak effects such as e.g. different hypercharge assignment of the up and down quarks and squarks or weak $SU(2)_V$ breaking by the soft scalar mass terms at the GUT scale (or small deviation from $h_t = h_b$ etc.)? Another question is whether with large $\tan \beta$ one can also explain in a natural way the gradual restoration of the $SU(2)_V$ symmetry for the lighter generations, but it will not be addressed here.

Special attention to the large $\tan \beta$ scenario has been given in the framework of the models with underlying $SO(10)$ symmetry [8], which predict the equality of the
top and bottom Yukawa couplings at the GUT scale.

In a recent paper [5] a detailed study has been performed of radiative electroweak breaking with complete gauge and Yukawa unification and universal soft mass terms at the GUT scale. It has been found that for large values of $h_t$ (as required for the heavy top quark) solutions with large tan $\beta$ are obtained only for strongly correlated values of the GUT scale parameters.

This can be easily understood in a qualitative way by studying the tree level scalar potential and the 1-loop RG evolution of its parameters from the $M_Z$ scale to the GUT scale. For easy reference we recall the main points here. The Higgs potential

$$V = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_3^2 \left( H_1^\dagger i\tau_2 H_2 + h.c. \right) + \text{quartic terms}$$

(1)

has, for large tan $\beta$ values, two characteristic features. It follows from the minimization conditions that

a) $m_2^2 \simeq -\frac{M_Z^2}{2}$

and

b) $m_3^2 \simeq \frac{M_A^2}{\tan \beta} \simeq 0$

(3)

with

$$M_A^2 \simeq m_1^2 + m_2^2 > 0.$$  

(4)

Equations (2) and (4) combined together give a useful constraint on the low energy parameters

$$m_1^2 - m_2^2 > M_Z^2.$$  

(5)

Equations (2) and (3) are the two main constraints on the parameters of the scalar potential, which are characteristic for large tan $\beta$ solutions.

The next information we need is the running of the parameters from the GUT scale to the $M_Z$ scale. For universal soft supersymmetry breaking parameters at the GUT scale and for $Y_t = Y_b = Y$ ($Y_{t,b} = \frac{h_t,b}{4\pi}$) and large (such that $Y(M_Z)$ is close to its infrared quasi–fixed point value $Y_f$, as required by the heavy top quark), the approximate solutions to the RG equations read[5]:

$$m_{H_1}^2 \simeq m_{H_2}^2 = m_o^2 + 0.5 M_{1/2}^2 - \frac{3}{7} \Delta m^2$$

(6)

where $m_i^2 = \mu^2 + m_{H_i}^2$,

$$\Delta m^2 \simeq 3 m_o^2 \frac{Y}{Y_f} - 4.6 A_o M_{1/2} \frac{Y}{Y_f} \left( 1 - \frac{Y}{Y_f} \right)$$

$$+ A_o^2 \frac{Y}{Y_f} \left( 1 - \frac{Y}{Y_f} \right) + M_{1/2}^2 \left[ 14 \frac{Y}{Y_f} - 6 \left( \frac{Y}{Y_f} \right)^2 \right]$$

(7)
and
\[ \mu^2 = 2\mu_o^2 \left(1 - \frac{Y}{Y_f}\right)^{6/7}. \]  
\hfill (8)

We observe that for large values of \( Y \) the masses \( m_{H_1}^2 \) and \( m_{H_2}^2 \) tend to become large and negative for increasing values of \( m_o \) and/or \( M_{1/2} \). For the running of the soft supersymmetry breaking bilinear and trilinear couplings we have:
\[ A_t \simeq A_o \left(1 - \frac{Y}{Y_f}\right) - M_{1/2} \left(4.2 - 2.1 \frac{Y}{Y_f}\right), \]  
\hfill (9)
\[ B \simeq \delta(Y) + M_{1/2} \left(2 \frac{Y}{Y_f} - 0.6\right) \]  
\hfill (10)
with
\[ \delta(Y) = B_o - \frac{6}{7} \frac{Y}{Y_f} A_o. \]  
\hfill (11)

Finally, the squark masses read
\[ m_U^2 \simeq m_D^2 = m_o^2 + 6.7 M_{1/2}^2 - \frac{2}{7} \Delta m^2, \]  
\hfill (12)
\[ m_Q^2 \simeq m_o^2 + 7.2 M_{1/2}^2 - \frac{2}{7} \Delta m^2. \]  
\hfill (13)

It is clear that in the approximation (6) the condition (5), necessary for the proper symmetry breaking, is not satisfied. However, eq. (6) neglects small differences in the running of the two Higgs masses which follow from the different hypercharges of the right top and bottom squarks, from the difference in the running of the bottom and top Yukawa couplings (equal at the GUT scale) and from the effects due to the \( \tau \) lepton Yukawa. As can be inferred from the RG equations, the inclusion of those effects leads to the following result:
\[ m_1^2 - m_2^2 = \alpha M_{1/2}^2 + \beta m_o^2 \]  
\hfill (14)
where \( \beta < 0 \) due to the \( Y_\tau \) effects, \( \alpha > 0 \) due to the other effects mentioned above and both are \( O(0.1) \) in absolute values. With the condition (5) we see now that large \( \tan \beta \) solutions must be driven by large values of \( M_{1/2} \):
\[ M_{1/2} > \frac{M_Z}{\sqrt{\alpha}}, \quad M_{1/2} > \sqrt{\frac{\beta}{\alpha}} m_o \]  
\hfill (15)

Also, it follows from eqs. (2), (6) and (7) that the low energy superpotential parameter \( \mu \) (or its GUT value \( \mu_o \); see eq. (8)) is strongly correlated with \( M_{1/2} \). For large \( M_{1/2} \), large values of \( \mu^2 \) are needed to cancel large negative values of \( m_{H_2}^2 \) and to satisfy the condition (2). Indeed, in the limit \( Y_t \rightarrow Y_f \) we get
\[ \mu^2 \approx 3 M_{1/2}^2. \]
Finally, in the limit $Y \to Y_f$, eq. (3) and (10) give (remember $m_3^2 \equiv B\mu$):

$$\delta(Y) \equiv B_o \ - \ \frac{6}{7}A_o \ \approx \ -1.4 \ M_{1/2}. \quad (16)$$

In summary, in the model with universal soft supersymmetry breaking terms large $\tan\beta$ solutions to radiative electroweak symmetry breaking are characterized by large ($\gg M_Z$) and linearly correlated values of the parameters $M_{1/2}$, $\mu$ and $\delta(Y)$, with the constraint $M_{1/2} > m_o$. Numerical calculations [5] based on the 2–loop RGE for the gauge and Yukawa couplings, and one–loop equations for the Higgs and supersymmetric mass parameters with all the leading supersymmetric threshold corrections included (bottom–up approach of ref.[9]) confirm the above qualitative considerations.

Another question one must address in the context of the large $\tan\beta$ scenario, with $Y_t = Y_b = Y_\tau$ at the GUT scale, is about the supersymmetric threshold corrections to the bottom mass. We refer the reader to ref.[10,5] for a detailed discussion and here repeat only the main points. Fig.1 (taken from ref.[5]) shows the predictions for the top quark (pole) mass as a function of the strong coupling constant $\alpha_s(M_Z)$ under the assumption of the gauge and Yukawa coupling unification for several values of the bottom mass, when supersymmetric corrections to $m_b$ are ignored. The shaded region shows the $(m_t, \alpha_s)$ values consistent with those assumptions and with $m_b$ in the experimental range (4.9±0.3) GeV. Let us suppose experiment will confirm the values of $m_t$ and $\alpha_s$ in that range. Then the question emerges: is the parameter space obtained from radiative breaking consistent with the assumed small supersymmetric corrections to the $b$ mass? Another option is also open: experimental data for $m_t$ and $\alpha_s$ will place us outside the shaded region of Fig.1, i.e. in the region where the corresponding bottom mass (with $Y_t = Y_b = Y_\tau$ and no SUSY corrections) is larger than the experimental value (4.9±0.3) GeV. Then large SUSY correction to $m_b$ will be necessary to reconcile the measured values of $m_b$, $m_t$, $m_\tau$ and $\alpha_s$ with full unification of the gauge and Yukawa couplings. One of the main results of ref. [5] is that in the model with universal soft terms the supersymmetric loop correction to the bottom mass (mainly gluino and higgsino exchange):

$$\frac{\delta m_b}{m_b} \sim \left( \frac{\alpha_3}{3\pi} \ \frac{M_3 \mu}{m_\tilde{q}^2} \ + \ \frac{Y_t}{8\pi} \ \frac{A_t \mu}{m_\tilde{q}^2} \right) \tan\beta \quad (17)$$

are large, 0(20–50%). This follows from the discussed above pattern of radiative electroweak breaking and selects the second option as the only one consistent with this mechanism. Another important physical feature of those solutions is heavy superpartner spectrum, with only the Higgs pseudoscalar remaining light [5].

Strong correlations between parameters signal a high degree of fine tuning which is needed for large $\tan\beta$ solution with universal soft terms. Of course, it is not excluded that such, very regular correlations may follow from the future theory of the soft supersymmetry breaking terms. However, at the present purely phenomenological level they are triggered by the smallness and the signs of the coefficients $\alpha$ and
\(\beta\) in eq. (14), which have their origin in the effects mentioned after eq. (13). It is important to realize that solutions consistent with eq. (14) and (5) are very unstable with respect to small perturbations, if they can reverse the sign of \(\beta\): for \(\beta > 0\), qualitatively new solutions become possible, with \(M_{1/2} \simeq 0\) and

\[
m_o > \frac{M_Z}{\sqrt{\beta}}
\]  

From inspection of the RGE, it is easy to see that there are (at least) three ways to reverse the sign of \(\beta\) by perturbing slightly the universal boundary conditions at the GUT scale:

\[
\begin{align*}
\text{a)} & \quad Y_t > Y_b \\
\text{and/or} & \quad \text{b)} & \quad (m_U^2) = (1 + \delta_U) m_o^2 > (1 + \delta_D) m_o^2 \equiv (m_D^2)^2 \\
\text{and/or} & \quad \text{c)} & \quad (m_{H_1}^2) = (1 + \delta_1) m_o^2 > (1 + \delta_2) m_o^2 \equiv (m_{H_2}^2)^2
\end{align*}
\]

Already O(20-30\%) perturbation is sufficient to reverse the sign of \(\beta\). The possibility of small deviation from the equality \(Y_t = Y_b\) has already been discussed in ref. [5]. Here we focus on the non-universal scalar masses.

In the presence of non-universal boundary values for the scalar masses the coefficients of the \(m_o^2\) terms in eqs. (6) and (7) are appropriately modified. For a qualitative insight one can use the following approximate formula (in the limit \(Y \to Y_f\)):

\[
\Delta \beta = \left[ \frac{2}{5} + \frac{3}{5} \left( 1 - \frac{Y}{Y_f} \right)^{5/7} \right] (\delta_1 - \delta_2) + \frac{3}{5} \left[ 1 - \left( 1 - \frac{Y}{Y_f} \right)^{5/7} \right] (\delta_U - \delta_D). \tag{19}
\]

It takes values O(0.1) for \((m_{H_1}^2)^2 - (m_{H_2}^2)^2\) or \((m_U^2)^2 - (m_D^2)^2\) of order O(20\%)\(m_o^2\) and O(0.5) for \(O(m_o^2)\), respectively.

It should be stressed that the pattern (18) of symmetry breaking is now determined by the positive sign of the coefficient \(\beta\) and essentially does not depend on the origin of this sign. However, as we shall see, there are important properties of the solutions which do depend on the character of the non-universality at the GUT scale.

With \(\beta > 0\), it follows from eq. (18) that there exists a new class of solutions with \(M_{1/2} \simeq 0\) and \(m_o \gg M_{1/2}\). The allowed region in the parameter space \((M_{1/2}, m_o)\) is shown in Fig.2. In this and the other figures we present the results of our numerical calculations (based on the method developed in ref.[9] and ref.[5]) for a) \(m_t = 170\) GeV, \(\tan \beta = 49\) and with the GUT scale values \(m_{H_1}^2 = 1.2 m_o^2, m_{D}^2 = 0.8 m_o^2\), b) \(m_t = 180\) GeV, \(\tan \beta = 53, m_{H_2}^2 = 0.7 m_o^2\), c) \(m_t = 180\) GeV, \(\tan \beta = 53, m_{H_1}^2 = 2.0 m_o^2, m_{H_2}^2 = 1.5 m_o^2\), where \(m_o\) is the universal mass of the other scalars.

Here we refer to the overall regions in Fig.2 where the solutions to radiative breaking exist: the only constraints which are taken into account are the present experimental limits on the sparticle masses and the requirement that the lightest neutralino is the lightest supersymmetric particle. The other regions in Fig.2 will be discussed shortly.
Further properties of the solutions and certain classification of non-universalities follow from the equation for $\mu^2$ modified by the non-universal scalar terms. In the limit $Y \to Y_f$ it reads:

\[ \mu^2 = -\frac{M_Z^2}{2} + \Delta_M m_o^2 + \Delta_M M_{1/2}^2 + \frac{3}{t} \left( 1 - \frac{Y}{Y_f} \right) A_o \left( A_o - 4.6 M_{1/2} \right) \] (20)

where

\[ \Delta_M \simeq 6 \frac{Y}{Y_f} - \frac{18}{7} \left( \frac{Y}{Y_f} \right)^2 - 0.5, \quad \Delta_m \simeq \frac{9 Y}{t Y_f} - 1 + \Delta_s \]

with

\[ \Delta_s = -\delta_2 + \frac{3Y}{14Y_f} (\delta_U + \delta_D + \delta_2 + \delta_1) + \frac{3}{10} \left[ 1 - \left( 1 - \frac{Y}{Y_f} \right)^5 \right] (\delta_U - \delta_D + \delta_2 - \delta_1) \]

Several observations follow from eq. (20): The strong $(M_{1/2}, \mu)$ correlation triggered by eq. (20) in the limit of universal scalar terms disappears, due to the importance of both $m_o$ and $M_{1/2}$ contributions, depending on the particular solution. Nevertheless, the values of $\mu^2$ remain correlated with $m_o$ or $M_{1/2}$ in the limit $m_o \gg M_{1/2}$ and $M_{1/2} \gg m_o$, respectively. In addition, in the limit $m_o \gg M_{1/2}$ which is of interest for us the values of $\mu^2$ depend, contrary to the sign of $\beta$, on the pattern of the deviation from universality in the scalar masses. In the limit $Y \simeq Y_f$ we obtain the following classification:

A) $\mu^2 > \Delta_M M_{1/2}^2$ for $\Delta_s > -2/7$

B) $\mu^2 < \Delta_M M_{1/2}^2$ for $\Delta_s < -2/7$

It is clear that with $\beta > 0$, as in eq. (19), the first option can be realized eg. for $(m_o^{H_2})^2 < m_o^2$ or $(m_U^2)^2 > m_o^2$, with the masses of the other scalars at $m_o^2$. Generically the values of $\mu$ remain then large $\mu \gg M_Z$. This case is illustrated by the examples (a) and (b) of our numerical calculations.

The possibility (B) (with $\beta > 0$) can be realized when $m_o^D < m_U^2 < m_o$ or/and $m_o < m_h^D < m_h^{H_2}$, for large enough deviations from universality. The parameter $\mu$ can then be arbitrarily small. Thus, radiative breaking can be driven by $m_o \ge M_{1/2} \simeq O(M_Z)$, with $\mu \simeq O(M_Z)$ and uncorrelated with $m_o$.

In our numerical calculations example (c) illustrates this case. We stress that the above approximate considerations are only meant as a qualitative guideline to the complete numerical calculations.

We can now understand all the details of the full solution regions in Fig.2 as well as the solution regions in the planes $(\mu, M_{1/2})$ and $(\mu, m_o)$ shown in Fig.3 and 4, respectively. In case (A) the $(m_o, M_{1/2})$ region is bounded from below by the constraints (15) and (18). The values of $(\alpha, \beta)$ are generically $(O(0.2), O(0.02))$ and

\footnote{With the same mechanism one can obtain $\mu \simeq 0$ also when $\beta < 0$, i.e. when radiative breaking is driven by large $M_{1/2}$. However, one needs then $\delta_2 \gg 1$ and, in addition, large $M_{1/2}$ brings us back to the strong ($\delta(Y), M_{1/2}$) correlation present in the universal case.}
(O(0.2), O(0.01)) for case (a) and (b), respectively. For large values of \( M_{1/2} \) we have a lower bound on \( m_o \) from the requirement \( m_t > m_{\chi^0_1} \) (the lightest neutralino). The lower bound on \( M_{1/2} \) is the experimental one obtained from the CDF limit on the gluino mass. In case (B) additional constraints follow from the condition \( \mu^2 > (50\,\text{GeV})^2 \) (experimental limit) which for a given \( m_o \) gives a lower bound on \( M_{1/2} \). The absolute lower bound on \( M_{1/2} \) is therefore higher than the experimental bound of \( \sim 50 \,\text{GeV} \). The plots in Fig.3 and Fig.4 are also nicely consistent with our qualitative discussion. The lower limit on \( M_{1/2} \) as a function of \( \mu \), in case (c), follows from eq. (20). The upper bound for \( M_{1/2} \) as a function of \( \mu \) in case (a) and (b) corresponds to the linear correlation present in the universal case. Comparing cases (a) and (b), we see the effects of approaching the limit \( Y = Y_f \).

The correlation between \( \delta(Y) \) and \( M_{1/2} \) remains as given by eq. (16). However, for the solutions with \( M_{1/2} \approx O(M_Z) \) eq. (16) can be satisfied with \( B_0 \sim A_0 \sim M_{1/2} \sim O(M_Z) \) (21).

We conclude that with non-universal scalar masses we have the interesting new range (21) of the parameter space which gives large \( \tan \beta \) solutions with \( \mu \geq O(0.5m_o), m_o \gg M_Z \) in case (A)

\[
\mu \sim O(M_Z), m_o \geq M_Z \text{ in case (B).}
\]

The main source of the potential fine–tuning are the constraints \( m_2^2 \equiv -\frac{1}{2}M_Z^2 \) and \( m_3^2 \equiv 0 \) which have to be satisfied for varying values of the parameters at the GUT scale. The degree of fine–tuning can be measured by the derivatives

\[
\frac{m_o^2}{m_o^2} \frac{\partial m_o^2}{\partial \mu_o^2} \approx O\left(\frac{m_o^2}{M_Z^2}\right)
\]

(22)

\[
\frac{M_{1/2}^2}{m_o^2} \frac{\partial m_o^2}{\partial M_{1/2}^2} \approx O\left(\frac{M_{1/2}^2}{M_Z^2}\right)
\]

(23)

\[
\frac{Y_t}{m_o^2} \frac{\partial m_o^2}{\partial Y_t} \approx O\left(\frac{m_o^2}{M_Z^2}\right)
\]

(24)

\[
\frac{M_{1/2} \tan \beta}{\tan \beta} \frac{\partial \tan \beta}{\partial M_{1/2}} \approx \frac{\delta(Y) \partial \tan \beta}{\tan \beta} \approx \frac{\delta(Y)}{B} \approx \frac{M_{1/2}}{B}
\]

(25)

\[
\frac{Y_t}{\tan \beta} \frac{\partial \tan \beta}{\partial Y_t} \approx \frac{A_o}{B} \approx \frac{\mu A_o}{m_o^2}
\]

(26)

where eqs. (6–11) and (15) have been used in the calculation. (We have also conservatively assumed that the derivative of \( Y_t(M_Z) \) over its values at \( M_{GUT} \) is 1 and neglected in eqs. (22–26) several small terms.) The derivatives in eqs. (22),(23), and (24) are of the order \( \geq O(\frac{1}{\alpha}) \) or \( \geq O(\frac{1}{\beta}) \) and reflect some fine tuning needed to satisfy eq. (2) for solutions with universal and non-universal soft terms, respectively.

The derivative in eq. (25) (and similarly for (26)) is \( O(M_{1/2} \tan \beta/\beta m_o) \) and \( O(\tan \beta \)
\(\alpha\) for non-universal and universal case, respectively. In the latter case its value is \(O(500)\) whereas in the former – depending on the values of \(\beta\) and \(m_o\) – it can be even as low as \(O(10)\). At this point our conclusion is that, with non-universal scalar mass terms, radiative electroweak breaking has qualitatively new solutions for large \(\tan \beta\), with moderate fine tuning.

There are, however, two additional important constraints which have to be considered: we have not addressed yet the question of consistency of radiative breaking with full unification of gauge and Yukawa couplings and, secondly, the relic abundance of the stable neutralino should not overclose the Universe. As we know from ref.[5], the first requirement is a strong constraint for the model with universal soft terms due to the potentially large supersymmetric corrections to the bottom mass. We recall again that for \((m_t, \alpha_s)\) values in the shaded region of Fig.1 the \(\delta m_b/m_b\) correction given in eq. (17) has to be small whereas for \((m_t, \alpha_s)\) values outside that region the correction has to be non-negligible and in the right range, in order to be consistent with coupling unification. For given values of \((m_t, \alpha_s)\) (or equivalently \(m_t\) and \(\tan \beta\); see Fig. 1) it is, therefore, necessary to study the parameter space obtained after radiative breaking and constrained by the requirement of the right magnitude for the correction \(\delta m_b\). In particular, it is interesting to understand if there are now consistent solutions for the shaded region in Fig.1 i.e. with \(\delta m_b/m_b \leq 10\%\), say. For a qualitative discussion, we can consider the first term in eq. (17) (this is a conservative estimate as the second term partially cancels the first one; see eq. (9)). The condition

\[
\frac{\delta m_b}{m_b} \approx 0.5 \frac{M_{\tilde{g}} \mu}{m_{\tilde{q}}} < 1/10 \quad (27)
\]

can be fulfilled in case (A) (which gives \(\mu \sim O(m_{\tilde{q}})\)) for \(m_{\tilde{q}} > 5M_{\tilde{q}}\). Taking into account the experimental limit \(M_{\tilde{g}} > 150\) GeV we need \(m_{\tilde{q}} > O(1 - 2)\) TeV and correspondingly \(m_o \sim O(2 - 4)\) TeV (since \(M_{\tilde{g}} \ll m_{\tilde{q}}\), the squark masses must be almost entirely determined by \(m_o\)). The regions constrained by the condition (27) are marked in Fig.2, 3 and 4 and are characteristic by upper bounds on \(M_{1/2}\) for given \(m_o\) (i.e. \(m_{\tilde{q}}\)) or given \(\mu\). Again, comparing (a) and (b) we see the effect of approaching the limit \(Y \rightarrow Y_f\). The regions in \((m_o, \mu)\) plane reflect the correlation following from eq. (20) in the limit \(m_o \gg M_{1/2}\).

In case (B), due to the lower limit on \(M_{1/2}\) as a function of \(m_o\) shown in Fig.2 (which follows from the condition \(\mu^2 > (50\text{GeV})^2\)), the squark masses \(m_{\tilde{q}}^2 \sim O(0.2)m_o^2 + 4M_{1/2}^4\) are determined by the gluino contribution and \(M_{\tilde{g}}/m_{\tilde{q}} \sim O(1)\). The condition (27) is satisfied for \(m_{\tilde{q}} > O(5\mu)\) which is achieved for \(m_{\tilde{q}} > O(300\text{GeV})\), as \(\mu\) can take small values. Clearly, eq. (27) gives a lower bound on \(M_{1/2}\) as a function of \(\mu\) which is seen in Fig.3. But at the same time \(\mu\) is given by eq. (20) so this bound must be consistent with eq. (20). We obtain then a lower bound on \(m_o\) as a function of \(M_{1/2}\), whereas the upper bound is given by the condition \(\mu > (50\text{GeV})^2\). With \(\Delta m_o < 0\), the allowed region is never empty. This explains the pattern seen in Fig.2. Similarly can be understood Fig.4. One should stress the importance of the
non-zero gaugino mass in satisfying simultaneously all the constraints in case (B). In particular it assures the positivity of the squark masses squared and of the \( \mu^2 \) for a given deviation from universality in the scalar masses.

It is obvious from Figs. 2, 3 and 4 that the constraint (27), by imposing certain cuts on the otherwise large parameter space with large \( \tan \beta \) solutions, requires somewhat more fine tuning. In option (A) it follows mainly from large values of \( m_o \) (eq. (22) and (24)) and in case (B) it is the \( M_{1/2} > O(M_Z) \) which is important for eq. (25) and (26). Generically, after the cut (27), the derivatives (22)–(26) are \( O(100) \).

The superpartner mass spectra obtained with non-universal scalar masses show several new and interesting features as compared to the universal case. Some of them are presented in Figs. 5 and 6. The most important aspects are (we discuss the spectra with eq. (27) taken into account):

1. Light charginos and neutralinos; they reflect very small values of \( M_{1/2} \) (case (A)) or \( \mu \) (case (B)). This latter case is particularly interesting as it revives our interest in light higgsino–like neutralinos.

2. Clear differences between sparticle spectra in case (A) and (B): they reflect the available parameter regions in both cases. We get, respectively
   a) very heavy, \( \geq O(1 \text{ TeV}) \), or light, \( \geq O(100 \text{ GeV}) \), sleptons.
   b) very heavy, \( \geq O(1 \text{ TeV}) \), or moderately light, \( \geq O(300 \text{ GeV}) \), squarks.
   c) very light, \( \geq O(150 \text{ GeV}) \), or somewhat heavier, \( \geq O(300 \text{ GeV}) \), gluinos.

3. The Higgs sector is characterized by generically light pseudoscalar \( A \). This follows from the equation \( M_A^2 = \alpha M_{1/2}^2 + \beta m_o^2 - M_Z^2 \), with small coefficients \( \alpha \) and \( \beta \). In the allowed parameter range, both in cases (A) and (B), the mass \( M_A \) can reach the present experimental limit of about 50 GeV and increases with increasing \( m_o \) and/or \( M_{1/2} \). The mass of the light scalar \( h \) is strongly correlated with \( M_A \) and changes from \( M_h \sim 60 \text{ GeV} \) for \( M_A \sim 50 \text{ GeV} \) to \( M_h \sim (120 - 140) \text{ GeV} \) for \( M_A > 100 \text{ GeV} \).

Finally, we discuss the condition \( \Omega h^2 < 1 \) for the stable neutralino. In case (A) it is strongly gaugino–like (\( \mu > M_{1/2} \)) and annihilation proceeds mainly through the Higgs pseudoscalar exchange (all sfermions are heavy). Explicit calculation gives \( M_A \leq O(200 \text{ GeV}) \) for \( \Omega h^2 < 1^{[11]} \). Therefore we get the constraint

\[
m_o \leq \frac{M_Z \sqrt{6 \beta}}{\sqrt{\beta}}
\]

which has to be satisfied together with the lower bound \( m_o \geq O(2 \text{ TeV}) \). This constraints \( \beta \) to small values \( \beta \sim O(0.01) \). The non–universal boundary conditions chosen for the numerical calculations give \( \beta \) in this range.

In case (B) the stable neutralino is often dominantly higgsino–like or at least a mixture of gaugino and higgsino. Its annihilation is more effective and uncorrelated
with $M_A$. The neutralino relic abundance is generically small, $\Omega h^2 \sim O(0.01-0.1)$, and the requirement $\Omega h^2 < 1$ does not constrain the parameter space.

The discussed solutions to radiative breaking are consistent with some of the approximate symmetries suggested in ref. [10] in order to stabilize large $\tan \beta$ solutions and to protect $m_b$ from large supersymmetric corrections. In case (A) these are

$$\frac{A}{m_{\tilde{q}}} \sim \frac{M_{\tilde{g}}}{m_{\tilde{q}}} \sim \frac{B}{m_{\tilde{q}}} \sim 0$$

(29)

and in case (B)

$$\frac{B}{m_{\tilde{q}}} \sim \frac{\mu}{m_{\tilde{q}}} \sim 0.$$  

(30)

However, it is not possible to satisfy simultaneously all the symmetries of ref.[10]. For instance, in case (B), the result $B/m_{\tilde{q}} \approx 0$ is obtained by some moderate cancellation between $A_0 \sim B_0 \sim M_{1/2} \sim O(M_Z)$, with $M_{\tilde{g}}/m_{\tilde{q}} \sim A/m_{\tilde{q}} \sim O(1)$. It is due to the fact that, for large $\tan \beta$, $B \ll M_Z$ and only solutions with $M_{1/2} \approx 0$ would be free of this type of fine tuning. On the other hand, in the first case $\mu/m_{\tilde{q}} \sim O(1)$ and there is moderate fine tuning in $m_2^2$ between $\mu_o^2$ and $m_{\tilde{q}}^2$.

With non–universal scalar terms top quark mass up to the IR quasi fixed point values, $m_t \sim (190–200)$ GeV, can be accommodated in the model. In case (A) this can be achieved at the expense of heavy squarks but in case (B) is as easy as for $m_t \sim (170-180)$ GeV.

In summary, non–universal scalar terms at the GUT scale open qualitatively new possibilities for radiative electroweak breaking with natural large $\tan \beta$ solutions. Two different patterns of non–universality can be identified, with distinct predictions for the particle spectra. Solutions with $A_o \sim B_o \sim M_{1/2} \sim \mu \sim O(M_Z)$, $m_o \geq M_Z$ look particularly interesting. Top quark masses up to the IR fixed point values (190–200) GeV can be easily accommodated in this scheme.

The main results of this paper have been presented in a talk by SP at the SUSY’94 workshop in Ann Arbor, May 14–17. Related results were presented at SUSY’94 by H.P. Nilles [12] and A. Pomarol [6]. We would like to thank these people for discussions.
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FIGURE CAPTIONS

Fig. 1. Top quark mass predictions as a function of the strong gauge coupling, within the framework of exact unification of the three Yukawa couplings of the third generation. The solid lines represent constant values of the pole bottom mass $m_b$ (without supersymmetric corrections), equal to A) 4.6 GeV, B) 4.9 GeV, C) 5.2 GeV, D) 5.5 GeV and E) 5.8 GeV. The dashed lines represent constant values of $\tan \beta$, equal to a) 40, b) 45, c) 50, d) 55 and e) 60. The region to the right of the dotted line is consistent with the unification of gauge couplings and the experimental correlation between $m_t$ and $\sin^2 \theta_W(M_Z)$. The long-dashed line represents the infrared quasi-fixed-point values for the top quark mass, for which $Y_t(0) \simeq 1$.

Fig. 2. The regions in $(M_{1/2}, m_o)$ parameter space which give radiative electroweak symmetry breaking and consistent with all experimental limits on sparticle masses.
- Short dashed curves: $m_t = 170$ GeV, $\tan \beta = 49$, $m^2_{Q}(0) = 1.2m_o^2$, $m^2_{D}(0) = 0.8m_o^2$.
- Long dashed curves: $m_t = 180$ GeV, $\tan \beta = 53$, $m^2_{H_2}(0) = 0.7m_o^2$.
- Solid curves: $m_t = 180$ GeV, $\tan \beta = 53$, $m^2_{H_1}(0) = 2.0m_o^2$, $m^2_{H_2}(0) = 1.5m_o^2$.
- The marked areas correspond to $\delta m_b/m_b < 10\%$, in each case.

Fig. 3. The same as Fig. 2 for $(M_{1/2}, \mu)$.

Fig. 4. The same as Fig. 2 for $(m_o, \mu)$.

Fig. 5. The same as Fig. 2 for the predictions for chargino and slepton masses.

Fig. 6. The same as Fig. 2 for the predictions for stop and gluino masses.
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