Multi-domain feature selection aimed at the damage detection of historical bridges

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Abstract. The aim of this study is the analysis and comparison of several sets of features selected from different domains for the detection of damage induced by scour on historical bridges. A scaled experimental model of a masonry arch bridge was subjected to differential settlements of the pier to simulate the occurrence of scour events. Damage states of increasing extent were introduced to verify the sensitivity of each feature and the accuracy of the damage detection method. The vibration measurements were acquired after each damage step using sensors located in different positions on the structure and different sources of excitation. The Kernel Density Estimation (KDE) was employed to characterise the correlation between the vibration signatures acquired in the time domain. The identified natural frequencies and the sampled range of the transmissibility spectrum were used in the Outlier Analysis (OA) to identify the novelties coming from the damage steps. All the selected features proved to be sensitive to damage, while showing pros and cons depending on the feature domain.

1. Introduction

Vibration measurements are commonly used within the Structural Health Monitoring (SHM) of major bridges to assess the structural integrity from the changes in their dynamics. Data-driven methods represent a valuable alternative to the numerical simulations whose reliability could be affected by the construction uncertainty and complexity. For this approach the damage identification problem is treated as a pattern recognition one, where data (patterns) are classified according to either a priori knowledge or a statistical information extracted from the patterns. These techniques are particularly appealing for the assessment of historical structures since the estimation’s efficacy does not rely on an accurate physical model rather on a statistical model established on data-extracted features. The main issue in the application of data-driven methods concerns the selection of those features which can provide the most discriminating information about the states of the structure. The sensitivity and the ability to distinguish among different damage states must be coupled with the requirement of physical meaningfulness to ensure the reliability and interpretability of the assessment. In the following sections the problem of the features selection is addressed with particular emphasis to the damage identification of historical bridges. The model of a masonry arch bridge provided the experimental case study to investigate the advantages and the drawbacks of damage features extracted form different domains. For each the adopted methodology is described and the results are presented.

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2. Damage assessment using vibration measurements
The identification of damage using vibration measurements is a well-established practice and its application to civil engineering structures dates back to early Eighties. Its theoretical foundation derives from the principle that the dynamic response of the structural system is affected by the alteration of the stiffness, mass or energy dissipation properties when damage occurs. The most widely accepted interpretation of the damage identification problem is that of statistical pattern recognition. In this approach the system is represented by a statistical model whose parameters are directly derived from the data. Each data (here referred as pattern) is condensed and expressed in terms of selected damage-sensitive features. The features extraction is generally recognised as the most crucial step in the diagnostic procedure. Its role is essential and it can highly bias the damage recognition stage. Indeed, most of the damage identification methods are unable to deal with the raw data records because their massive dimension does not fit with the inputs limitation requirement common to every pattern recognition algorithm. Moreover, the selected features shall maximise the capability of the diagnostic method to discriminate among the different structural states. The most sensitive information are condensed in low-dimension arrays while discarding further parts which could be source of noise and affect the accuracy of the damage assessment.

Several damage symptoms have been identified to detect the anomalous behaviour of the system due to damage. For an extensive documentation about the parameters that can be employed within the vibration-based SHM the reader may refer to [1]. The features extracted from vibration measurements can be subdivided into three main domains: the time, frequency (or spectral) and modal domains. This classification is not only philosophically but entails substantial distinctions in the implementation of the damage identification algorithm depending on the selected features. In the time domain, the coefficients estimated by means of the Auto-Regressive (AR) or the Auto-Regressive Moving Average (ARMA) models are commonly used to fit the time histories acquired from the undamaged state of the structure. A base line is derived from these features and damage is detected when the coefficients estimated from new acquisitions are seen to deviate from it. In [2] Sohn and Farrar applied a control chart based on the coefficients extracted from the vibration test data acquired from a concrete column progressively damaged.

The features selection in the frequency domain rely on the application of the Fourier Frequency Transform (FFT) which allows to reduce drastically the volume of data and compensate the little loss of information averaging the effects of random noise. The selected features are commonly the shift in the resonance and anti-resonances or changes affecting the amplitudes. Alternative approaches consider limited portions of the frequency spectrum sampled around the resonance peaks. When the input force is unknown like in the output-only measurements, the computation of the Frequency Response Function (FRF) is substituted by the Power Spectral Density (PSD) or the Transmissibility Functions (TF). In [3] the transmissibility spectra computed by Fourier transforming the signals acquired from couples of piezoelectric accelerometers are sampled to detect, localise and assess damage simulated on an aircraft wing by the removal of pre-installed panels. A pattern recognition approach is pursued and an artificial neural network is trained to classify the patterns into damage classes corresponding to the removed panel.

The modal domain provides a large set of damage sensitive features. The first are the natural frequencies of the system whose shifts are commonly employed to compute damage detection indices. Methods based on modal displacements can be used to objectively measure the similarity between two mode shapes. The Modal Assurance Criterion (MAC) compares sets referred to the undamaged and damaged states of the structure. Low values of the index represent a dissimilarity between the modes and can be interpreted as an indication of damage. Information about the localisation of structural changes due to damage can be obtained also by means of mode shape curvatures or strain mode shapes. Since they are derivatives of mode shapes their changes are highly localised to the region of damage and their sensitiveness is more pronounced compared to modal displacements. Modal curvatures are also employed in the determination of the modal strain energy which was used by Stubbs et al. in [4] to define an index for damage localisation in off-shore platforms.
Several other features can be used for damage identification but, up to now, none of them has proved to work satisfactorily for every type of structure and for every type of damage. Unfortunately, the best features for damage detection are typically application specific. This issue finds an actual confirmation in the particular case of the damage assessment of historical structures. Indeed, ancient masonry constructions are complex and uncertain systems, single exemplars with no repetitive characteristics and thus hardly identifiable in a common and well-known prototype. A large variety of possible damage scenarios may occur and the selection of the best features depends on the extraction of the most explanatory information related to the most likely damage events. This crucial stage of the diagnostic procedure requires a priori knowledge about the structure, the expected damage scenario and their interaction, i.e. the response of the structure to that specific type of damage. This amount of required information varies depending on the selected features but does not automatically imply an handicap for the damage assessment. The evidence of an expected structural behaviour represents a precious proof of the method’s reliability and should not be considered secondary. From a different point of view, we can say that the physical meaningfulness is a further condition which should be satisfied in order to prevent erroneous misclassifications due to the scarce interpretability of the results. Finally, one can claim that the optimality in the features selection resides in a trade-off between their damage sensitiveness and the concrete possibility to ensure their consistency with the expected structural response to damage.

3. The case study

3.1. The bridge model description
The consideration presented so far are justified by means of the simulation of a scour event at the piers foundation of historical arch bridges. A scaled model of a masonry arch bridge was built in the laboratory and subjected to differential settlements of the central pier to reproduce the effect of scour in a controlled manner. Figure 1(a) shows the twin-arch bridge model whose dimensions are 5.90m of length, 1.60m of width and 1.75m of height. The reader is referred to [5] for a more detailed description of the model geometry and the employed materials. The main structural elements which compose the bridge model are the two arch barrels, supported by the abutments at the edges and by the pier at the mid-section. The longitudinal and transversal spandrels walls over the arch barrels retain the backfill material, i.e. a mixture of sand, gravel and debris of different shapes and sizes, which was used in the historical bridges to load and stabilise the arch barrels. The most peculiar characteristic of the model is represented by the mechanical device shown in figure 1(b) which was placed under the pier at an hypothetical middle-height section and used to reproduce the effects of scour.

3.2. Damage steps
Several differential settlements steps were introduced by lowering the plate at the top of the mechanical device according to the results of specific hydraulic flume tests carried out on a further
scaled model of the pier. Two settlements application campaigns were arranged into different lapses of time. In the former, four steps of settlement were carried out acting only on the two screws at the front-side of the model, this leading to increasing differential settlements coherent with the first stages of the streambed erosion localised at the upstream section of the bridge’s pier. The displacements realized at the front-side were equal to 0.5, 1.5, 2.5 and 4.0mm, respectively. Slight cracks opening were observed at the edge between the arch barrels and the longitudinal spandrel walls after the third step application. At the conclusion of the forth step a partial detachment between the bottom of the pier and the top of the plate was noticed, meaning that the pier was suspended and further settlements application would have resulted ineffective.

The settlements campaign was resumed at the ending of the relaxation phenomena which restored the contact between the pier and the plate. Since the evolution of scour involves larger and larger portions of foundation soil underneath the pier, in the final stages the undermining effect produces a compensation of the previous differential settlements which lead to a quasi-uniform configuration. Therefore, the steps of the second campaign were applied acting also on the back-side screws of the settlement application system. The resulting evolution of the prescribed displacements throughout the whole destructive tests campaign is presented in the plot of figure 2. In the last five steps of the settlements application the opening of new cracks in the arch barrels and the propagation of the former ones were observed. The application of the prescribed displacements was coupled with the removal of larger and larger portions of the polystyrene ring surrounding the pier base in order to simulate the concomitant erosion of the streambed around the foundation.

3.3. Vibration measurements

Some vibration tests were carried out on the undamaged state of the bridge model and were repeated after each settlement step. A set of 18 capacitive accelerometers was deployed on the bridge model according to two different configurations in order to investigate all the positions characterised by the largest modal deformations evaluated through a finite element analysis. The free dynamic response of the model was acquired, using the random environmental noise present in the laboratory as source of excitation. The free decay response of the model was obtained by means of the impacts of a sledge hammer applied in several positions and along different directions in order to properly excite the largest number of modes estimated by the numerical modal analysis. The sampling frequency was set to a value of 400 Hz for both the ambient noise and the impact hammer excitations. The free response acquisitions lasted for 180 seconds. All the impacts collected throughout the 60 seconds lapse of time were processed in the following modal identification.

![Figure 2. The settlements evolution throughout the tests.](image-url)
3.4. Signal processing

The signals acquired throughout the free decay vibration tests were used to perform a preliminary modal identification. The Eigenvalue Realisation Algorithm (ERA) [6] was the technique selected to process the data by virtue of the great spectral resolution it offers and its modal uncoupling capability. The results of the experimental modal analysis were compared with those obtained from the numerical analysis carried out on a finite element model of the bridge. The correspondence between the numerical and the experimental results allowed to identify the first six modes of vibration and provided at the same time a reliable proof of the accuracy of the experimental campaign. Although the dynamic behaviour of the structure was exhaustively characterised at the low frequencies, slight differences between the numerical and the experimental natural frequencies were observed. This result proves the significant level of uncertainty which characterises the masonry bridge model and limits the adoption of a model-based method for the diagnosis of such a complex structure.

The modal identification has been here described as preliminary signal processing operation. Indeed, the results derived from this step represented the prerequisites for the implementation of the following feature selection in the time domain. The experimental modal analysis provided the a priori knowledge about the response of the structure to the simulated damage scenario, as explained in section 2. Moreover, the repetition of the dynamic identification for all the datasets acquired after each settlement application supplied the features to perform the damage detection in the modal domain. Further details about the importance of signal processing for the feature selection in the different domains are given in section 4.

4. Feature selection and damage detection

4.1. Time domain

The damage assessment employing features derived from the time domain was aimed at the detection of the asymmetric effects introduced by the scour simulation in the bridge model. The comparison between the identified modal displacements for the sound and the damaged configurations highlighted the transition from a symmetric to an asymmetric mode shape for the first vertical bending mode [7]. The observed behaviour, clearly ascribable to the change in the boundary conditions, suggested the possibility to investigate the same effect in the response of the sensors placed in the middle portion of the bridge and measuring accelerations along the vertical direction.

In order to detect the occurrence of a lack of correlation in the response of the upstream and downstream side of the bridge model, a “symmetric” source of excitation was required. The free vibrations acquired under the random environmental noise present in the laboratory were thus selected. A couple of sensors was considered for the acquisition of the reference state and after each settlement step. A band-pass signals filtering was then applied about the natural frequencies of the first vertical bending mode, with a confidence interval of +/- 3Hz to take in account its variation throughout the settlements application. This signal filtering was introduced to enhance the capability of the approach to detect the corruption of the symmetry of the first bending mode produced by the scour effect on the pier. The signals where also normalized by their r.m.s. value in order to prevent misclassifications due to the variable extent of the random excitation among the acquired sets.

The signals acquired from the couple of selected sensors were treated as two independent random variables and their cross-correlation was used to represent a measure of their similarity, exploiting a well-known property of this type of random variables. Indeed, the cross-correlation between two independent random variables can be expressed by means of the Probability Density Function (PDF) of their difference. The PDF of each dataset was estimated using the Kernel Density Estimation (KDE) method which assumes that data are normally distributed [8]. The outcome of the KDE is a Gaussian bell function which represents the probability distribution of the residual between the two signals. Sharps bells are referred to highly correlated couples of signals, while the loss of correlation belongs to smooth bells characterized by most likely higher values for the residuals. The variation in the estimated peak amplitude, standard deviation, skewness and kurtosis throughout the stages
experienced by system can be employed to objectively characterize the symptomatic change of the structural state.

4.2. Spectral domain

The Transmissibility Functions (TF) are the features selected for the damage assessment in the spectral domain. Differently from the Frequency Response Functions (FRF), these features employ only the measurement of the structural responses and do not require knowledge of the excitation input. This operational advantage is coupled with a remarkable damage sensitivity as proved in the work by Manson and Worden [9]. The transmissibility function \( TF_{i,j,k} \) between two measuring points \( i \) and \( j \), applying the impact at the position \( k \) are defined by:

\[
TF_{i,j,k} = \frac{A_{i,k}(\omega)}{A_{j,k}(\omega)}
\]

where \( A_{i,k}(\omega) \) and \( A_{j,k}(\omega) \) are the Fourier Transforms of the acceleration signals acquired at the two points, applying the impact in the position \( k \).

The number of possible combinations for the definition of the transmissibility functions obviously depends on the number of available sensors and impacts positions. Moreover, the selection of features from the transmissibility spectrum is constrained to a limited set of spectral lines as required by the damage detection algorithm to correctly discriminate the patterns. The possibility to select the features from different regions of the transmissibility spectrum and according to different criteria introduces a further variability and complexity in the feature extraction. The investigation of all the possible combinations of sensors and impacts locations, coupled with the search for the spectrum sample capable to distinguish unambiguously between sound and damaged patterns is not feasible in the present application. A more automatic and objective approach based on genetic optimisation is here pursued to identify the best features among a numerous set of possible candidates. The optimised parameters were the sensors and impact positions selected to compute the TFs, the number of sampled spectral lines and the first sampled spectral line. Each set of parameters allowed to define a specific sample of the transmissibility spectrum, arranged in vector format. The selected features set was then submitted to the damage assessment stage, whose outcome was used to drive the optimisation process.

The following damage detection was accomplished by means of a discordancy test capable to deal with the multivariate nature of the problem to solve. In the statistical pattern recognition terminology the problem here addressed is a novelty detection problem, since the purpose is the indication of the damage occurrence based only on data from the undamaged state of the structure. No prior information about the damaged state of the system is required, a statistical model is derived from the sound measurements and damage is inferred as soon as a new observation fails to fit with the modelled normal condition. The Outlier Analysis was the employed damage detection algorithm and its choice among the several available methods is mainly related to its limited computational effort and the accuracy in the obtained results proved in the literature. The Mahalanobis squared-distance is used to assess the inconsistency of an observation with the rest of normal data. This statistical measure is given by:

\[
D_\xi = \left( \{ \bar{x}_\xi \} - \{ \bar{x} \} \right)^T [S]^{-1} \left( \{ \bar{x}_\xi \} - \{ \bar{x} \} \right)
\]

where \( \bar{x}_\xi \) is the candidate observation, \( \{ \bar{x} \} \) is the mean vector of the data set and \([S]\) is the data set covariance matrix. A threshold is computed from the “undamaged” system data using a Monte Carlo approach based on extreme value statistics and the candidate observations are discriminate as outliers and inliers whether they exceed or not the threshold.
4.3. Modal domain
The repetition of the experimental modal analysis for all the datasets acquired after each settlement step allowed to employ the identified natural frequencies as features to perform the damage detection in the modal domain. In this case the feature extraction process is immediate compared to the other domains. We can also say that there is no concrete distinction between the signals pre-processing and the following features selection. However, as in the time domain, the preliminary experimental modal analysis represents an essential pre-requisite. Indeed, the results of the pre-processing allowed to identify the most reliable and recurrent modes of the structure which represented the targets for the following identifications. Each acquisition carried out for the different setups and settlement application under the sledge hammer excitation was processed, automatically selecting the free decay responses at each impact and discarding the solutions not matching with preliminary results. The check on the identified results was performed in terms of natural frequencies, damping ratios and MAC values, setting admissible ranges according to the variation of the modal parameters within the preliminary analysis. The most important result we obtained was the automatization of the features extraction, which is fundamental for the execution of the damage detection in an on-line manner. The genetic optimisation integrated in the damage detection framework in the spectral domain was here replaced by a sensitivity analysis carried out a posteriori in order to identify the best set of features varying the number of considered modes. The damage detection algorithm implemented in the modal domain is the same multivariate Outlier Analysis used in the spectral domain.

5. Results and discussion
The sensitivity of the features extracted from the time domain is depicted in the plots of figure 3. The suspected decrease of signal correlation along with the application of settlements of increasing extent is clearly recognisable from the reduction of the sharpness of the PDFs shown in figure 3(a). The couple of sensing positions here selected involves the mid-span sections of the left-side arch barrel symmetric about the longitudinal axis of the bridge model. None remarkable distinctions characterise the PDFs of the reference and the following step, here labelled as step 0 since it entails only the partial removal of the polystyrene ring surrounding the bottom of the pier without applying any settlement. The smooth bell functions related to the second and third settlement steps highlight the lack of correlation between the response of the upstream and downstream sides of the bridge. A similar result is obtained for the sensors placed on the two opposite spandrel walls at the pier section. In figure 3(b) a couple of sensors both placed at the downstream side is selected. The choice of transversally symmetric positions leads to high signals correlation invariant with the damage states and provides a clear suggestion about the nature of the structural response to the damage source. The investigation of the second settlements campaign confirms the coherence of the change in the PDFs of the signals residuals with the displacements applied at the pier base. The PDF of the set assumed as reference in the second campaign is depicted by a dashed line whose peak is slightly lower than the peak of the reference set curve shown in figure 3(a). The restoration of the original contact between the pier base and the supporting plate was not completely achieved because of the cracks openings occurred in the last steps of the first settlements campaign. The application of the first settlement steps leads to a marked discordancy between the signals acquired at the mid-span sections of the right-side arch barrel, as shown by the smooth bell functions depicted in figure 3(c). From the 7th step the pier settlement is applied also at the downstream side and the asymmetric response of the first bending mode is progressively reduced. This effect is accurately reproduced by the trend of the PDF of the last steps, whose higher peak values and sharpness are clear symptoms of the regained correlation between the signals, now due to the lack of contact with the support for both the sides. The four sensors positions considered in the PDFs estimation are shown in figure 3(d). The couples A-B and B-D are used for the first settlements campaign, while the second involves the couple C-D.
The features extracted from the spectral domain undertook an optimisation process to select the most sensitive set. The optimality criterion that was assumed relies on the outcome of the novelty detection stage which was distinguished between the two settlements campaigns. The maximisation of the discordancy between the reference set and the 4 steps of the first campaign was chosen to deal with the monotonic increase of the applied damage state. The downstream mid-span section of the right arch barrel and the mid-length section of the upstream spandrel wall provided the optimal positions for the sensors couple. The impact given at the one-quarter section from the abutment of the left arch barrel at the downstream side was selected to compute the TFs. The transmissibility spectra obtained for the reference set and the damaged steps adopting the optimal parameters are depicted in figure 4(a). The sample selected by the genetic optimisation effectively provide the spectral range characterised by the highest deviation among the sets. The result of the Outlier Analysis shown in figure 4(b) highlights the ability to label all the observations acquired for the four damage steps as outliers. Moreover, the increase of the Mahalanobis squared distance with the damage steps follows the actual trend of the damage extent applied to the structure.

The data set acquired after the completion of the relaxation phenomena and immediately before the fifth damage step was assumed as reference for the second settlement campaign. This provided a confirmation of the robustness of the damage detection methodology employing spectral features. Although the optimisation procedure selects sensors and impact locations different from the previous analysis, the deviation from the reference set is detected and reproduces the same trend observed in the time domain. The optimal spectral features selected for the second settlement campaign and the corresponding damage detection results are shown in figure 4(c) and figure 4(d), respectively.
Figure 4(a). Transmissibility spectra for the 1st settlement campaign. Features selected between spectra lines 370 and 390, i.e. 145 and 153 Hz.

Figure 4(b). Outlier Analysis results: Mahalanobis squared distance of the damage steps of the 1st settlements campaign.

Figure 4(c). Transmissibility spectra for the 2nd settlement campaign. Features selected between spectral lines 283 and 303, i.e. 110 and 118 Hz.

Figure 4(d). Outlier Analysis results: Mahalanobis squared distance of the damage steps of the 2nd settlements campaign.

For the sake of brevity, only the natural frequencies are here presented as examples of features extracted from the modal domain. Further results obtained employing damping ratios and mode shapes are collected in [10]. The same discordancy maximisation criterion used for the optimisation of the spectral features was adopted also for the sensitivity analysis carried out a posteriori on the frequency of the first six identified modes. The number of available modes was varied in order to identify the best set of features. Table 1 collects the best solutions of the two test campaigns. The fitness value, i.e. the measure of the deviation from normality computed for all the damaged data sets, and the indexes of the selected modes are presented for each feature dimension. The fitness increases with the number of modes and the whole set of available modes provides the best feature in both the cases. The results of the OA carried out with the whole set of natural frequencies for the two tests campaigns are plotted in figure 5(a) and figure 5(b). In the first, all the damage steps are correctly labelled as outliers, apart from step 0. A slightly increasing trend of the damage index with the data set is observed for the two tests. In the second, this result disagrees with the trends obtained in the spectral and time domains.

Table 1. Sensitivity analysis of the natural features used as damage features.

|                   | Number of features | 1    | 2    | 3    | 4    | 5    | 6    |
|-------------------|--------------------|------|------|------|------|------|------|
| **1st settlement campaign** | Fitness           | 179.2| 208.6| 222.2| 247.0| 273.9| 286.7|
|                   | Modes              | 1    | 15   | 15   | 25   | 35   | 35   |
| **2nd settlement campaign** | Fitness           | 630.2| 1106.5| 1478.4| 1747.6| 1926.6| 2010.5|
|                   | Modes              | 1    | 15   | 15   | 15   | 35   | 35   |
6. Conclusions
The selection of damage sensitive features was carried out in different domains to assess the structural integrity of an historical bridge model subject to pier settlement. The features extracted from the time, frequency and spectral domain proved to suitably deal with the requirement of damage sensitivity. However, a different amount of prior information about the response of the structure to the expected damage scenario distinguishes the implementation and the assessment of each technique. The use of the spectral features requires a limited knowledge of the system and the investigated problem, while the extraction of the modal features relies on their physical meaningfulness to assure the reliability. The KDE method is driven to detect the asymmetric response to the expected damage effect and provides the best interpretability of the results. The integration of features selected from different domains within a single diagnostic framework should be encouraged to exploit their specific benefits.

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