A fast Gaussian filtering algorithm for three-dimensional surface roughness measurements

Y B Yuan 1, W Y Piao and J B Xu

1 Harbin Institute of Technology (HIT), Harbin, China, 150001

E-mail address: pwyingpwy@yahoo.com.cn

Abstract. The two-dimensional (2-D) Gaussian filter can be separated into two one-dimensional (1-D) Gaussian filters. The 1-D Gaussian filter can be implemented approximately by the cascaded Butterworth filters. The approximation accuracy will be improved with the increase of the number of the cascaded filters. A recursive algorithm for Gaussian filtering requires a relatively small number of simple mathematical operations such as addition, subtraction, multiplication, or division, so that it has considerable computational efficiency and it is very useful for three-dimensional (3-D) surface roughness measurements. The zero-phase-filtering technique is used in this algorithm, so there is no phase distortion in the Gaussian filtered mean surface. High-order approximation Gaussian filters are proposed for practical use to assure high accuracy of Gaussian filtering of 3-D surface roughness measurements.

1. Introduction

Three-dimensional surface roughness measurement and analysis technology is used in an increasing number of industrial applications. For 3-D surface texture assessment, a reference surface, known as the mean surface, must be established, so the determination of the mean surface is an important procedure for describing and assessing 3-D surface characteristics. The Gaussian filtered mean line is established as the reference line in 2-D surface metrology by the ISO 11562-1996 standard [1] and ASME B46-1995 standard [2]. Many approaches for implementing Gaussian filtering of surface profiles have been studied for high-efficiency and high-accuracy applications [3-10]. Similarly, the Gaussian filtered surface should also be an option for the mean surface for 3-D surface assessment. The computational efficiency is relatively more important for 3D surface roughness measurement than for 2-D surface profiling measurement because of a large number of measured data points to be processed. Early in 1993, Luo et al. developed such a Gaussian filtering algorithm using a Fast Fourier Transform (FFT) technique. To simplify the Gaussian filtering approach and to improve the filtering efficiency in 3-D surface roughness measurements, we present a new recursive approximation algorithm for calculating the Gaussian filtered mean surface, which is an extension of our research on Gaussian filtering for surface profiles [8-10]. The approximation algorithm can ensure both amplitude accuracy and zero phase characteristic, and it can also realize a high computational efficiency.

2. Gaussian filter and its approximated implementation

2.1. One-dimensional Gaussian filter
In surface metrology, the impulse response, or the weighting function, of a 1-D Gaussian filter is given as

\[ h(t) = \frac{1}{\alpha \lambda_c} e^{-\pi \left( \frac{t}{\lambda_c} \right)^2} \]  

(1)

where \( t \) is the independent variable in the spatial domain, \( \lambda_c \) is the cut-off wavelength (in the units of \( t \)), and \( \alpha \) is a constant. This is a low-pass filter, whose amplitude transmission characteristic in the spatial domain may be written as

\[ H(\lambda / \lambda) = e^{-\pi \left( \frac{\alpha \lambda}{\lambda} \right)^2} \]  

(2)

If the constant \( \alpha = (\ln 2/\pi)^{1/2} \approx 0.4697 \), then when \( \lambda = \lambda_c \), the transmission ratio of the Gaussian filter is 50%.

2.2. Two-dimensional Gaussian filter

Similarly, the impulse response of the 2-D Gaussian filter for 3-D surface topography is defined as

\[ h(x, y) = \frac{1}{\beta \lambda_x \lambda_y} \cdot \exp \left\{ -\frac{\pi}{\beta} \left[ \left( \frac{x}{\lambda_x} \right)^2 + \left( \frac{y}{\lambda_y} \right)^2 \right] \right\} \]  

(3)

and the amplitude transmission characteristic is

\[ H(\lambda_x, \lambda_y) = \exp \left\{ -\pi \beta \left[ \left( \frac{\lambda_x}{\lambda_x} \right)^2 + \left( \frac{\lambda_y}{\lambda_y} \right)^2 \right] \right\} \]  

(4)

where \( \beta = \ln 2/\pi \) and \( \lambda_x, \lambda_y \) are the cut-off wavelengths in the \( x \) and \( y \) directions respectively. The filter has an attenuation ratio of 50% at the cut-off wavelength, for example at \( \lambda_x = \lambda_{xc} \) with \( \lambda_y = \infty \) or at \( \lambda_y = \lambda_{yc} \) with \( \lambda_x = \infty \).

From Eq. (4), we can see

\[ H(\lambda_x, \lambda_y) = \exp \left\{ -\pi \beta \left( \frac{\lambda_x}{\lambda_x} \right)^2 \right\} \cdot \exp \left\{ -\pi \beta \left( \frac{\lambda_y}{\lambda_y} \right)^2 \right\} = H(\lambda_x) \cdot H(\lambda_y) \]  

(5)

Given this separable property of the 2-D Gaussian filter, a 2-D Gaussian filter can be implemented by using two independent 1-D Gaussian filters, one in the \( x \) direction, and the other in the \( y \) direction.

2.3. Implementation of the 1-D Gaussian filter

According to the Central limit theorem [7], the random variable \( X \), which is the sum of statistically independent and identically distributed random variables \( X_i \), approaches to a Gaussian random variable on the condition that the number of \( X_i \) is large enough.

\[ X = \sum_{i=1}^{n} X_i \]  

(6)

It means that the convolution of density function \( f_{X_i} \) that corresponds to \( X_i \) approaches to the Gaussian density function \( f_X \).

\[ f_X(x) = \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{(x-m)^2}{2\delta^2}} \approx f_{X_1} * f_{X_2} * \ldots * f_{X_n} \]  

(7)
where "**" denotes convolution, \( m \) denotes the mean value of variable \( X \), and \( \delta^2 \) denotes the variance of variable \( X \). The transform of self-convolution in the time domain is equal to a self-multiplication in the frequency domain. If the function \( 1/(1+u^2) \) is used as approximation of \( H(\lambda_c/\lambda) \) in Eq. (2), then, we can define the function:

\[
H_{\beta_h}(\lambda_c/\lambda) = \frac{1}{1 + \beta_h(\lambda_c/\lambda)^2}
\]

A series of Gaussian approximation filters can be constructed by self-multiplication of the function Eq. (8) in the frequency domain [8]:

\[
H_{\beta_n}(\lambda_c/\lambda) = \left[ \frac{1}{1 + \beta_n(\lambda_c/\lambda)^2} \right]^n
\]

if the constant \( \beta_n=2^{l/n} - 1 \), then when \( \lambda=\lambda_c \), the transmission ratio of the Gaussian filter is 50%.

From Eq. (9) we obtain the digital implementation of the Gaussian filter

\[
H(z,n) = \left( \frac{a}{1 - e^{-a} \cdot z^{-1}} \right)^n \cdot \left( \frac{a}{1 - e^{-a} \cdot z} \right)^n
\]

where \( H(z,n) \) is the z-transform expression of the \( n \)th-order Gaussian filter and is calculated by taking the impulse response invariant transformation of Eq. (9), and where \( a = 2\pi(\beta_n^{1/2} \times N_c)^{-1} \), \( N_c \) is the number of sampled points within a cut-off length \( \lambda_c \). When \( N_c > 200 \), \( e^{-\alpha} \approx 1 - \alpha \), Eq. (10) is a zero-phase-shift infinite impulse response (IIR) digital filter. The filter is implemented in the spatial domain by a set of difference equations obtained by taking the inverse z-transform of Eq. (10). The difference equations use as input the measured surface profile and produce as output the filtered mean line. This approximation of the Gaussian filter procedure has been shown to have very high computational efficiency, high accuracy, and no phase distortion.

2.4. Implementation of the 2-D Gaussian filter

The 1-D approximation for the Gaussian filter can be extended to the 2-D case, that is,

\[
H_n(\lambda_x, \lambda_y) = \left( \frac{1}{1 + \beta_{nx}(\lambda_x / \lambda_c)^2} \right)^n \cdot \left( \frac{1}{1 + \beta_{ny}(\lambda_y / \lambda_c)^2} \right)^n
\]

so the 2-D approximation filter can be expressed as

\[
H(z_x, z_y, n) = \left( \frac{a}{1 - e^{-a} \cdot z_x^{-1}} \right)^n \cdot \left( \frac{a}{1 - e^{-a} \cdot z_x} \right)^n \cdot \left( \frac{a}{1 - e^{-a} \cdot z_y^{-1}} \right)^n \cdot \left( \frac{a}{1 - e^{-a} \cdot z_y} \right)^n
\]

Where \( H(z_x, z_y, n) \) is the z-transform of the \( n \)th-order 2-D approximation Gaussian filter;

2.5. Transmission characteristic deviation of the \( H(\lambda_x, \lambda_y, n) \) from the 2-D Gaussian filter

The amplitude transmission characteristic function of the 2-D Gaussian filter is shown in Fig. 1. For the approximation filter, we take \( H(\lambda_x, \lambda_y, 8) \) for example, which is shown in Fig. 2. The deviation between the Gaussian filter and the approximation function is shown in Fig. 3. From Fig. 3, we can see that the transmission characteristic deviation of the 2-D approximation filter \( H(\lambda_x, \lambda_y, 8) \) from the 2-D Gaussian filter is less than 2.5%. Higher-order 2-D approximation filters result in smaller transmission characteristic deviations, or higher accuracy. The transmission characteristic deviation of the 2-D approximation filter \( H(\lambda_x, \lambda_y, 16) \) from the 2-D Gaussian filter is less than 1.25%.
3. Simulation experiments

Figure 4 represents an unfiltered 3-D surface topography over an area of 0.56 mm × 0.56 mm. The sampling interval is 1 μm in both directions and the total number of the sampled data points is 560 × 560 = 313600. The simulated 2D-Gaussian filtering results obtained from $H(\lambda_x, \lambda_y, \delta)$ for $\lambda_x = \lambda_y = 0.25$ mm are shown in Fig. 5 and Fig. 6. Figure 5 shows the Gaussian filtered mean surface and Fig. 6 shows the roughness topography obtained by subtracting the Gaussian filtered mean surface from the original surface topography of Fig. 4. The Gaussian filtering algorithms above have been programmed in Visual Basic NET for our purposes and less than one second is required on the computer with CPU Celeron D 2.13G.
4. Conclusion
The 2-D Gaussian filter for 3-D surface topography can be separated into two 1-D Gaussian filters. This important property makes it possible that two independent 1-D Gaussian filters be used for implementing a 2-D Gaussian filter. The 2-D Gaussian filter, like a 1-D Gaussian filter, can be implemented by using a recursive filtering algorithm without phase distortion. The recursive algorithm for 2-D Gaussian filtering requires a relatively small number of simple mathematical operations such as addition, subtraction, multiplication, or division, so that it has considerable computational efficiency. The accuracy of the amplitude transmission characteristic of the 2-D Gaussian filter for 3-D surface topography is determined by the order of the approximation function. Higher order results in a smaller approximation error and higher amplitude transmission characteristic accuracy, but lower data processing efficiency, and vice versa.

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