Impedance Optimization for Uncertain Contact Interactions Through Risk Sensitive Optimal Control

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Abstract—This paper addresses the problem of computing optimal impedance schedules for legged locomotion tasks involving complex contact interactions. We formulate the problem of impedance regulation as a trade-off between disturbance rejection and measurement uncertainty. We extend a stochastic optimal control algorithm known as Risk Sensitive Control to take into account measurement uncertainty and propose a formal way to include such uncertainty for unknown contact locations. The approach can efficiently generate optimal state and control trajectories along with local feedback control gains, i.e. impedance schedules. Extensive simulations demonstrate the capabilities of the approach in generating meaningful stiffness and damping modulation patterns before and after contact interaction. For example, contact forces are reduced during early contacts, damping increases to anticipate a high impact event and tracking is automatically traded-off for increased stability. In particular, we show a significant improvement in performance during jumping and trotting tasks with a simulated quadruped robot.

I. INTRODUCTION

State of the art locomotion controllers include a model predictive control scheme that computes trajectories of some reduced model. This model predictive scheme is then realized through a pre-designed impedance controller or a QP based inverse dynamics solver. These strategies have proven to be successful in completely structured and controlled environments. However, current robot control strategies still lack the ability to reason about uncertainty in the environment. High stiffness feedback controllers are usually used to track precise trajectories. This approach is usually limiting for a robot in multi-contact scenarios where the robot depends on intermittent contact interactions to move itself or some object around. Contact interactions increase the complexity of the control design problem. A stiff controller will counteract an unpredicted contact by increasing the control input to ensure tracking, which might generate high contact forces that destabilize the system. On the other hand, excessive compliance could lead to large deviations from the desired task. Studies from the field of neuroscience suggest that human beings modulate their impedance during contact interactions [1], [2]. Other studies suggest that sensorimotor commands are the result of an optimal feedback control mechanism that controls a trade-off between accuracy and stability [3]. More recent studies [4], [5], [6] suggest that this impedance "sweet spot" is a result of reasoning not only about the desired task, but also the uncertainties present during contact interactions.

Impedance control for robotics was introduced in [7] and a controller was derived in [8]. Later, [9] demonstrated experimentally that this controller is capable of stabilizing contact interactions if proper impedance parameters are chosen. Following the results presented by Hogan, Park [10] used a similar approach to design a bipedal walking controller. The results show a biped capable to walk on uneven terrain. A similar approach was used for a quadruped trotting gait in [11]. Another application of impedance control was presented in [12] for the MIT Cheetah, where an impedance controller is shown to be stable at various speeds of a periodic trotting gait. In [13] the authors provided a thorough study on the impedance controller of the HyQ quadruped robot using hydraulic actuators. The results achieved through impedance control have proven to be superior for control strategies involving contact interactions. However, all these approaches design the impedance schedule through an exhaustive trial and error process. It remains an open question how to systematically optimize impedance profiles for robotic tasks involving contact interactions.

Optimal feedback control theory has many promising aspects that could help approach this problem. An optimal control formulation results in motions from an abstract task formulation, namely the cost function along with feedback gains to stabilize the motion, hence impedance profiles. This aligns well with the suggested concepts in [3], [4]. An algorithm that computes local quadratic approximations of both the dynamics and the cost functions and then iteratively solves the nonlinear optimal control problem was introduced by Mayne [14]. This algorithm is commonly known as the Differential Dynamic Programming Algorithm (DDP). It has the advantage of providing an optimal control trajectory together with local feedback controller. Many variations of DDP appeared later in the literature Li [15], Sideris [16], Tassa [17] and [18]. In [19] the issue of control bandwidth in optimal control is addressed. The work in [20] extended the previous results to optimize a reduced model of the quadruped robot ANYmal. These algorithms are deterministic in nature and favor tracking over stability, making them prone to failure in situations where tracking cannot be perfectly achieved, and in attempting to do so, the controller can destabilize the system, uncertain contact interactions being a clear example.

Todorov [21] added multiplicative process noise to the optimal control problem violating the certainty equivalence principles. This led to control policies that are dependent
on the process noise. Li [22] derived similar results for partially observable systems with control constraints resulting in control policies that are a function of both process and measurement uncertainties. Another method to break the certainty equivalence principle is achieved through an exponential transformation of the cost function, this was first introduced by Jacobson in [23] for linear systems and later extended for nonlinear optimal control using a DDP-like algorithm by Farshidian in [24]. This formally synthesizes a controller that could obtain risk neutral, risk sensitive or risk seeking behaviors depending on the parameterization of the role of the uncertainty in the cost function.

Medina [25] used the exponential cost transformation with process noise to perform manipulation tasks through a model predictive control scheme. The exponential transformation was extended to accommodate for measurement uncertainties in [26] obtaining a risk sensitive optimal control algorithm that accounts for higher order statistics in both process and measurement models making it a suitable framework for designing feedback controls that reason about the trade-off between disturbance rejection and measurement uncertainty. However, the approach was only tested on toy problems and never analyzed to achieve more complex robotic tasks.

In this paper, we build on the ideas of [26] to propose a systematic manner to compute impedance schedules for legged robots. We extend the algorithm to work with hard contact transitions and introduce a way of incorporating contact measurement uncertainty into the whole-body optimal control formulation. This results in systematically optimized impedance profiles that exhibit desirable stiffness and damping patterns to handle uncertain, high impact, contact transitions. Extensive numerical simulations demonstrate the properties of the approach when compared to usual DDP algorithms and other measurement noise models. In particular, we show a significant increase in performance for hard impacts during jumping and for trotting over uneven terrains.

II. BACKGROUND

In this section, we provide background on the robot and contact models, risk-sensitive stochastic optimal control and its extension to include measurement uncertainty.

A. Multi-Contact Robot Dynamics

The dynamics of a legged robot in contact with its environment can be described by the follow equation

$$M(q) \ddot{q} + h(q, v) = S^T \tau + J(q)^T \lambda_{ext}$$

(1)

where $q \in \mathbb{R}^n_q$ is the vector of generalized coordinates, $v \in \mathbb{R}^n_v$ is the vector of generalized velocities, $M(q)$ is the inertial matrix, $h(q, v)$ is the vector combining the nonlinear terms such as Coriolis acceleration and gravity, $S^T$ is the selection matrix mapping the controls to the actuated degrees of freedom, $\lambda_{ext}$ is the vector of contact forces and $J(q)$ is the contact Jacobian. The notation indicating the dependence on $q$ and $v$ will be omitted for the remainder of the text. The robot dynamics in (1) can then be expressed as a first order differential equation

$$\frac{d}{dt} \begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} M^{-1}(\tau - h + J^T \lambda_{ext}) \\ f(x, u) \end{bmatrix}$$

(2)

This defines the state vector as $x^T = [q^T, v^T] \in \mathcal{X}$, the set of all possible states. Then the change in the state vector during a time interval $\delta t$ can be written as

$$dx_t = \begin{bmatrix} dq_t \\ dv_t \end{bmatrix} = \delta t \begin{bmatrix} v \\ \dot{v} \end{bmatrix} = \delta t.f(x, u)$$

(3)

which leads to the discretized state dynamics

$$x_{t+1} = f_t(x_t, u_t) = x_t \oplus dx_t$$

(4)

where $\oplus$ handles the Lie group composition operation for the base orientation.

B. Rigid Contact Model

While different contact models can be chosen to compute the contact forces $\lambda_{ext}$ [27], we will use a rigid contact model for the optimal control computation. Let $p$, $\dot{p}$ and $\ddot{p}$ denote any contact point position, velocity and acceleration respectively. During an active contact phase, the rigid contact assumption can be stated as

$$\dot{p} = Jv = 0$$

(5)

$$\ddot{p} = J\dot{v} + J\ddot{v} = 0$$

(6)

In order to resolve the contact forces for all active contacts, the robot dynamics (1) is projected to the contact space using $(JM^{-1}J^T)^{-1}JM^{-1} = \Lambda JM^{-1}$ to result in

$$\lambda_{ext} = -\Lambda \dot{J} v + \Lambda J M^{-1} h - \Lambda J M^{-1} S^T \tau$$

(7)

Once $\lambda_{ext}$ is obtained, the motion vector $\dot{x}$ can be constructed and the state vector $x_{t+1}$ can be obtained from (4).

C. Risk Sensitive Optimal Control

We are interested in stochastic optimal control approaches that can explicitly handle uncertainty. In particular, we use a nonlinear iterative risk sensitive optimal control formulation [28], [24], [26] which enables to explicitly take into account the distribution of uncertainty while being numerically efficient. The algorithm is similar to what can be obtained using DDP [18], [29] while reasoning about the higher order statistics of the problem. Consider the following dynamics expressed as a nonlinear stochastic difference equation

$$x_{t+1} = f(x_t, u_t) + F(x_t, u_t) \omega_t$$

(8)

where $\omega_t \sim \mathcal{N}(0, \Omega_t)$ is the process noise (i.e. it accounts for unmodeled disturbances) and $F(x_t, u_t)$ maps the noise to the full dynamics. A typical optimal control problem will optimize an objective function of the form

$$\mathcal{L}(x, \pi(x)) = l_T(x_T) + \sum_{0}^{T-1} l_t(x_t, \pi(x_t))$$

(9)
where $l_T(x_T)$ is the cost at the terminal time $T$, and $l_t(x_t, \pi(x_t))$ is the cost at time $t$ along the horizon. In order to include the higher order statistics into the problem, risk sensitive optimal control minimizes the following transformation of the objective

$$J^* = \min_{\pi(x_t)} \mathbb{E}[\exp (\sigma \mathcal{L}(x, \pi))]$$

(10)

where $J^*$ is the optimal risk sensitive cost and $\sigma$ is the risk sensitive parameter. Farshidian [24] proved that the cumulant generating function of $J$ can be expressed as

$$\frac{1}{\sigma} \log J = \mathbb{E}[\mathcal{L}] + \frac{\sigma}{2} \mu_2[\mathcal{L}] + \frac{\sigma^2}{6} \mu_3[\mathcal{L}] + \ldots$$

(11)

where $\mu_i[\mathcal{L}]$ is the $i$'th moment of the random variable $\mathcal{L}$. The risk sensitivity parameter $\sigma$ then provides a tool to control the contribution of the higher order moments on the cost. The exponential transformation is such that the optimal risk sensitive cost will minimize a weighted sum of the expectation of the objective function and its higher moments. In particular, $\sigma > 0$ (risk-averse mode) will increase the cost of having a high variance of the objective. The solution to the optimal control problem then proceeds from that point in a similar fashion to what is done in DDP by constructing a local quadratic approximation of the cost function and a linear approximation of the system dynamics along nominal state and control trajectories, then finding improvements to the deviations (12) along the state and control trajectories iteratively.

$$\delta x_t = x_t - x^n_t, \quad \delta u_t = u_t - u^n_t$$

(12)

Risk-sensitive backward Riccati equations [23], [24] are used to iteratively improve the solution. Notably, this results in a control law of the form

$$\delta u_t = K_t \delta x_t + k_t$$

(13)

where $K_t$ are the error feedback gains and $k_t$ the feedforward commands. Notably, the control law explicitly incorporates the covariance of the noise distribution in $K_t$ and $k_t$ [24].

D. Including Measurement Uncertainty

The formulation presented above constructs the optimal control solutions for a problem with process noise. Speyer [30] incorporated measurement noise into the risk sensitive formulation by introducing a state vector that grows at each time step to include the entire history of the states. Recently, Ponton [26] suggested that this could be avoided by extending the linearized system dynamics with that of an Extended Kalman Filter (EKF). To do so, consider the nonlinear measurement model

$$y_{t+1} = h(x_t, u_t) + H(x_t, u_t) \gamma_t$$

(14)

where $\gamma_t \sim \mathcal{N}(0, \Gamma_t)$ is the measurement noise (i.e. uncertainty from sensing). Assuming an observable system, an EKF can be used to compute the estimate deviations $\delta \hat{x}_t$ along the nominal trajectory at each iteration. The extended system dynamics then become

$$\begin{bmatrix} \delta x_{t+1} \\ \delta \hat{x}_{t+1} \end{bmatrix} = \begin{bmatrix} A_t \delta x_t + B_t \delta u_t \\ A_t \delta \hat{x} + B_t \delta u_t + G_t F_t (\delta x_t - \delta \hat{x}_t) \end{bmatrix}$$

$$+ \begin{bmatrix} C_t \\ 0 \end{bmatrix} G_t D_t \begin{bmatrix} \omega_t \\ \gamma_t \end{bmatrix}$$

(15)

where $A_t$, $B_t$, $C_t$, $F_t$ and $D_t$ are the linear local approximations of $f(x_t, u_t)$, $F(x_t, u_t)$, $h(x_t, u_t)$ and $H(x_t, u_t)$ from (8) and (14) respectively. At each iteration, the Kalman gains $G_t$ are computed along the nominal trajectory, then the optimal control law is optimized by iterating over modified backward Riccati equations. The optimal feedback gains $K_t$ explicitly include the covariance of both process and measurement noises, i.e. the control law depends on a trade-off between external disturbances and measurement uncertainty.

III. Multi-Contact Risk Sensitive Control

Now we detail how we use the risk-sensitive optimal control algorithm including measurement uncertainties to optimize motions and impendence schedules for legged robots.

A. Including Multiple Contact Switching

Since we assume a rigid contact model, we need to define the switching dynamics to handle discontinuous contact transitions. We use a predefined contact sequence and timing for contact switching and compute an initial guess for the open-loop optimal trajectory using an existing kino-dynamic optimizer [31].

We also align the contact transitions with the collocation points such that we have the objective function and dynamics of the problem in the form

$$\min_{\delta x, \delta u} l_T(\delta x_T) + \sum_{n=0}^{N} \sum_{t=0}^{T} l_t(\delta x_t, \delta u_t)$$

(16)

$$\text{s.t.} \quad \delta x_{t+1} = f_n(\delta x_t, \delta u_t)$$

(17)

where $N$ is the number of contact switches along the trajectory and $T$ is the horizon of each phase. This procedure avoids non-smooth switches in the contacts by solving the optimal control problem on multiple smooth intervals. We enforce consistency at the transition between the switching intervals through additional cost terms (i.e. soft constraints).

In particular, we enforce linearized friction cone constraints and smooth contact force transitions. We compute analytical derivatives for all the quantities, including the contacts, using the Pinocchio library [32].

B. Error State Kalman Filter on Smooth Manifolds

To synthesize the controller, we assume that the measurement consists in the full robot state. We then design an estimator capable of estimating floating base configurations including orientation. The estimation error $\delta \hat{x}$ is defined on the tangent space $T \mathcal{S}$ so its covariance $[33]$. The error in the state and its covariance is represented as

$$\delta \hat{x} = x \otimes \hat{x}$$

$$\Sigma = \mathbb{E}[\delta \hat{x} \delta \hat{x}^T]$$

(18)

(19)
where $\oplus$ represents the difference between the two states $\hat{x}$ and $x$ with the proper Lie group operations [33]. The propagation of the dynamics can be handled similarly using the $\oplus$ operation [29]. The gaussian noise $\epsilon \sim N(0, \Sigma)$ is then defined on the same space as the estimation error, and the covariance can be propagated through the adjoint transformation while the state prediction follows the relative group composition operation described earlier in (4).

$$\Sigma_{t+1} = Ad_{d_x}\Sigma_t Ad_{d_x}^T + \Omega_{t+1}$$

A full treatment of EKFs on lie groups can be found in [33], [34].

C. Uncertainty in Contact Interactions

Consider a robot during a dynamic multi-contact scenario such as the quadruped in Fig. 1. A robot control algorithm computes control torques based on the perception of the robot’s own states along with its surrounding environment. Both the perception of the robot environment and its states are susceptible to sensor noise and perception errors, introducing uncertainty in the measurements. We propose to model the contact uncertainty as an uncertainty at the tip of the swing foot $\Gamma_c$ as shown in the ellipsoid in Fig. 1. This has the advantage that we do not need to keep track of the environment model nor the next contact location in our state vector. We then map this uncertainty back to the space where the full state covariance matrix $\Gamma_f s$ is defined. Adding both covariance matrices then results in the total uncertainty in the contact interaction.

![Fig. 1: Uncertainty in Contact Interactions](image)

A deviation in the swinging end-effector of the robot can be linearly mapped to a deviation in its state vector through

$$\begin{bmatrix} \delta p \\ \delta \dot{p} \end{bmatrix} = A_c \begin{bmatrix} \delta q \\ \delta \dot{q} \end{bmatrix}$$

(21)

Then the minimum norm change in the state vector corresponding to a change in the end-effector is given by

$$\begin{bmatrix} \delta q \\ \delta \dot{q} \end{bmatrix} = A^\dagger_c \begin{bmatrix} \delta p \\ \delta \dot{p} \end{bmatrix}$$

(22)

where $A_c^\dagger = A^T_c (A_c A^T_c)^{-1}$ is the Moore-Penrose inverse of $A_c$. Different norms could be chosen using a weighted inverse if desired. Now that the deviations of the end-effector are in the same vector space as the full state errors, it is possible to add the noise resulting from the robot states such as joint encoders along with the contact uncertainty. However, a deviation in the swing foot might induce a deviation in the feet that are actively in contact and break our rigid constraint assumption in (5) and (9). To avoid inconsistency, the deviations of the swing foot must be projected to the null space of the feet actively in contact

$$P_c = I - A_c^\dagger A_c$$

(23)

where $A_c$ is similar in structure to $A_s$ however containing the Jacobians of the active contacts described in (5) and (6) along with their time derivatives. The final form of the deviations transformation can then be written as

$$\begin{bmatrix} \delta q \\ \delta \dot{q} \end{bmatrix} = \left( (I - A_c^\dagger A_c) A_s \right) \begin{bmatrix} \delta p \\ \delta \dot{p} \end{bmatrix}$$

(24)

We use this transformation to map the mean of the deviations in a certain end-effector frame to that of the full state vector of the robot. Since the noise was assumed to be Gaussian, the covariance matrix can also be transformed using the affine property of Gaussians. The total covariance of the state vector becomes

$$\Gamma = \Gamma_f s + P_c A^\dagger_c \Gamma_c A^T_c P^T_c$$

(25)

In summary, we define Gaussian noise models both in end-effector and state space, with covariance $\Gamma_f s$ and $\Gamma_c$ respectively and combine them using (25). It will enable us to explicitly change the uncertainty associated to the swinging feet before contact.

IV. Simulations Results

To demonstrate the capabilities of the proposed method for controlling multi-contact interactions under contact uncertainty, we present three different simulation experiments using an accurate model of our open-source quadruped robot Solo [35]. In the first experiment, we study the effect of different measurement noise models on the computed impedance profiles and resulting impact forces when encountering an unexpected contact. With the second experiment, we explore how the trade-off between stiffness and damping changes when using our risk-sensitive control approach when compared to standard DDP methods. Finally, in the third experiment, we systematically quantify how the stability of the system is favored relative to the accuracy of the controller in the risk sensitive case during locomotion tasks.

The DDP controller used as a baseline in the presented experiments is from [29]. The kino-dynamic optimizer described in [31] is used to generate reference trajectories around which both iterative controllers DDP and Risk Sensitive are initialized. A linear spring damper contact model is used for the simulations with an explicit Euler integration scheme [36] at a time step $\delta t_{sim} = 1.0 - 4s$ with a spring stiffness parameter of $k = 1.0 + 5N/m$ and a spring damping parameter of $b = 3.0 + 2Ns/m$. The coefficient of static friction used in simulation is $\mu = 0.7$. The simulated feedback control frequency runs at 1kHz and the discretization step for the optimal control problems is set to $\delta t_{opt} = 1.0 - 2s$.

For all the experiments, the same cost function, weights and reference trajectories are used for both DDP and Risk
Sensitive Control. All the planning and control is designed for perfectly flat floor in all three experiments. This results in the same whole body trajectories $x_t$ and the same feedforward torque control profiles $\tau$ for both DDP and Risk Sensitive control. The sensitivity parameter is set to $\sigma = 10$ for the Risk Sensitive solver, leaving the uncertainty models as the only variable in the experiments. The only differences in the optimized plans are the impedance profiles (i.e. the feedback gains $K_f$).

### A. Effect of Noise Models on Impedance Regulation

In this experiment, the task is to swing a single leg forward with a maximum height of $10\, \text{cm}$ and a step length of $8\, \text{cm}$ similar to what is shown in Fig. [1](#). A $3\, \text{cm}$ high block is added at the next contact location to simulate an unpredicted contact of $9.2\%$ of the total leg length. The contact with the block occurs at $t = 0.43\, \text{s}$ whereas the contact with flat ground was planned for $t = 0.55\, \text{s}$.

![Fig. 2: Tracking error and feedback norms for uncertainty models and DDP. The grey zone corresponds to the time between the unexpected contact and the planned one.](image)

The first uncertainty model, **Risk-Uniform**, is simply a diagonal matrix with equal variance on all of its entries. In the second uncertainty model, **Risk-SwingJoints**, the variance terms on the joints of the swinging foot are increased. In the third model, **Risk-Unconstrained**, we add a contact noise term similar to (22) without using the nullspace projection due to the active contacts. The last model, **Risk-Contact**, includes the projection of the swing contact uncertainty into the null space of the active contacts (24).

Figure 2 compares the tracking behavior of each control scheme. The state error is divided into the error in the configuration $\delta q$ (Fig. 2a) and the error in the velocity tracking $\delta v$ (Fig. 2b). Similarly the optimized feedback gains are divided into the feedback gains associated to the configuration $K_p$ and the feedback gains from the velocity $K_d$. Their norms are shown in Fig. 2c and Fig. 2d respectively. The vector norm $\|v\| = \sqrt{v^T \cdot v}$ is used to compute the state error norms whereas the Frobenius Norm $\|M\|_F = \sqrt{\text{Trace}(M^T M)}$ is used to compute the feedback norms.

We notice in Fig. 2 that the risk sensitive control with Risk-Uniform and Risk-SwingJoint noise models diverge after the unexpected impact. DDP also diverges but accumulates less error than these two risk sensitive schemes. All three controllers have high gain norms at the time of impact $|K_p| \approx 400$ and a stiffness to damping ratio at $|K_p|/|K_d| \approx 20$. It is important to note that such gains are too high and would not be usable on the real robot [35]. The maximum reduction in the overall stiffness and damping is obtained by using the Contact uncertainty model where $|KpDDP|/|KpContact| \approx 10$ and $|KdDDP|/|KdContact| \approx 4$. With this reduction in the impedance magnitudes, a less accurate tracking is observed for both Unconstrained and Contact earlier in time $t = 0.3\, \text{s}$. However, thanks to the
less aggressive feedback gains, the robot can handle the unpredicted impact. Importantly, these reduced gains fit well within ranges acceptable for execution on the real robot.

The effect of the controllers is clearer when looking at the normal contact forces (Fig. 3) where the impact force on the swinging leg \( FL \) after \( t = 0.43 \text{ s} \) is lowest for Risk-Contact. Additionally, the impact force propagates its effect to the front right leg for DDP, Risk-Uniform and Risk-SwingJoints. Whereas for the case where the uncertainty about the contact is included (Risk-Unconstrained and Risk-Contact), we see that the robot absorbs the impact force and we do not notice any propagation of the disturbance to the other feet.

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Inspecting the structure of the feedback matrices computed at the time of impact can shed light on these differences in behavior. The blocks of the feedback matrix that map the base states to the control are depicted in Fig. 4. Risk Sensitive control changes the structure of the gain matrices, where the base error is modulated mainly through the feet on the ground namely FR, HL and HR, which is expected. The advantage of Risk-Contact is observed in the portion of the feedback matrix that maps the joint errors to the control commands (Fig. 5). DDP has aggressive \( K_p \) gain to track the motion of the swinging foot \( FL \), that is the first three diagonal elements of \( K_p \)-DDP relative to the remaining diagonal elements corresponding to the remaining joints. Unlike \( K_p- \) DDP and \( K_p\)-Unconstrained, \( K_p\)-Contact has lower gains on the joints of \( FL \) relative to the joints of the support feet FR and HL. This allows the swing foot to behave as a soft spring relative to the support feet which explains the significantly lower impact force observed and the good tracking of the contact forces on the other feet. These results underline the importance of choosing appropriate noise models and supports the choice of the Risk-Contact noise model.

### B. Stiffness vs. Damping & Impact Forces

We now only consider the Risk-Contact noise model. This experiment discusses how the introduced model results in an improvement of the performance of an extremely dynamic motion, a jump of a total height of 0.5 m. Halfway through the flight phase, a block of 3 cm height is placed on the floor. We model this uncertainty by increasing the contact-specific measurement noise.

![Stiffness vs. Damping & Impact Forces](image)

Both \( DDP \) and Risk-Contact feedback policies achieve the desired jump height with a slightly better performance for the DDP controller. However, inspecting the tracking and impedance profiles reveals a more natural behavior emerging in the Risk Sensitive case. During the flight phase between \( t = 1 \text{ s} \) and \( t = 1.5 \text{ s} \) both controllers exhibit a significant decrease in both the base and joint stiffness as shown in Fig. 6. For the damping portion of the feedback matrix, the base damping also shows a decrease during the flight phase. Remarkably, we notice a stark difference in the damping modulation of the joints. DDP significantly decreases damping on the joints while Risk Sensitive significantly increases it. This substantial increase in the joint damping allows to more quickly absorb the unexpected impact, i.e. an abrupt change in the velocity of the feet. As a result, we notice 20% lower impact forces (Fig. 7), which in turn avoids the bouncing behavior observed with the DDP controller (Fig. 9). This comes however at the cost of larger deviations for the joint positions. When measuring the stiffness to damping ratio of both the base and the joints, we notice that during active contact phases the base stiffness to damping ratio is
relatively the same for DDP and risk sensitive. However, for the joints, during the support phase, the stiffness to damping ratio of risk sensitive is 1.5 times higher than that of DDP, explaining the good tracking with lower overall gain profiles.

C. Trade-off between Stability & Accuracy

In this experiment, we study the capabilities of our controller when trotting on unknown terrain. Both DDP and risk sensitive policies are optimized to track a trotting gait including a total of 14 contact switches. During simulations, we introduce unexpected blocks at contact locations (Fig. 10) in order to study the policies robustness. An execution is successful if the robot manages to reach the desired terminal base configuration and velocity without falling.

We conduct 1000 simulations, divided into two batches.

| Method        | # of Simulations | Maximum # of blocks | Maximum block height | % of leg length | % of Successful Sim. |
|---------------|------------------|---------------------|----------------------|-----------------|-----------------------|
| Experiment 1  | 500              | 14                  | 45 mm                | 14 %            | 47.4%                 |
| Experiment 2  | 500              | 14                  | 34 mm                | 11 %            | 65.6%                 |

TABLE I: Parameters and results of the trotting experiments

In the first batch, the contact disturbances are sampled to represent up to 14% of the total leg length, which is quite aggressive. In the second batch, the maximum contact variation is 11% of the leg length. The results are summarized in Table I. In both cases, we observe a significant increase in trotting performance when using the risk sensitive controller. Moreover, the magnitude of the gains of the DDP controller are outside of the range of gains admissible on the real robot while the risk sensitive controller gains are not. This result demonstrates the improved robustness to contact uncertainties provided by the risk-sensitive controller.

For successful executions, we show the distribution of base position and velocity tracking errors in Fig. 11. When successful, the risk sensitive control policy finishes the trotting gait with larger deviations in the base configuration while damping large velocity disturbances to stabilize the robot, underlying the trade-off between accurate tracking and robustness to contact uncertainty.

V. CONCLUSION

In this paper, we extended the idea of risk sensitive optimal control with measurement uncertainty to legged locomotion problems. We showed the importance of the choice of the noise models to generate meaningful impedance modulation patterns. Through extensive simulations, we demonstrated that our approach can generate stiffness and damping profiles that lead to better responses in face of hard impacts and contact uncertainty when compared to typical DDP algorithms. Moreover, the computed gains are significantly smaller than the ones computed through DDP, and within ranges that are realistically executable on the real robot. Our approach provides a systematic approach to automatically compute optimal impedance modulations, at the same computational cost as modern DDP algorithms. Future work will include experiments on real robots and efforts to extend the approach to receding horizon control.
Fig. 10: Snapshots of the trotting gait and the contact disturbances.

Fig. 11: Trotting Base Tracking Error Norms Statistics

(c) Linear Velocity error norm
(d) Angular Velocity error norm

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