Investigation of the Effect of Material on Undamped Free Vibration of Cantilever Beams with Uniform Single Surface Crack

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Abstract. Crack detection in structures is a critical area of research where the developments have been made out since decades. Various techniques are available for early identification and quantification of cracks to predict and prevent the unexpected sudden failure of structure and ensure uninterrupted service. Use of vibration analysis for detecting crack is one of the widely used techniques which offer lots of advantages over other like it is easier and less costly method and can be used for inaccessible components. The present work attempts to use modal analysis through FEA to investigate the effect of crack on natural frequency of vibration in beams of different materials, for three different crack location. From the result, it has been inferred that among Structural Steel (SS), Aluminium alloy (Al) and Gray Cast Iron (CI), the natural frequency is highest for Al beam and lowest for CI beam. Introduction of crack reduces the natural frequency of vibration, however, the effect of crack location on frequency is not uniform for different modes. Also, the trend is similar in beams of all the materials.

Index Terms— Finite Element Analysis, Natural frequency, Mode shapes, Vibration Modes, Crack location

1. Introduction
Beams are common structures used to carry and transfer high lateral loads in machines and civil structures. High level of stress concentration due to heavy and consistent loads may often generate cracks, which may result in sudden or sometimes, catastrophic failure of structures. Therefore, machines and structural components should be continuously monitored for the early detection of cracks for structural safety as well as for uninterrupted service. The detection of crack-like defects in mechanical systems and civil engineering structures has been considerably attempted by many researchers in the past few decades. Several non-destructive techniques have been developed for early prediction of cracks to prevent unexpected failure.
Rizos et al. [1] proposed a method for crack identification of uniform beams, based on the flexural vibration where the crack is modelled as rotational massless springs to represent the local flexibility because of the introduction of crack. This technique requires a knowledge of vibration amplitude of beam, vibrating at one of the natural frequencies. The method was further simplified by Liang et al. [2] which only requires measurement of two or three transverse natural frequencies of the beam. This procedure of detection of crack and determination of its location in beams was further extended to short beams by Lele and Maiti [3] based vibration signatures.
Pandey et al. [4] made use of curvature mode shapes for locating damage in structures. Unlike displacement mode shapes, curvature mode shapes are localized in damage region, and therefore, is a good indicator for damage identification.

Kisa and Gurel have developed a method based on combined finite element and component mode synthesis wherein the beam is detached into parts from the crack region. These substructures are joined by using the flexibility matrices taking into account the interaction forces derived by virtue of fracture mechanics [5].

Wavelet theory is another field that has also been exploited as a common tool for vibration analysis where application of wavelet theory in the spatial domain is made for crack identification in structures [6] [7].

Since the crack results in reduction in stiffness and consequent reduction in natural frequencies and increase in modal damping, research studies have been utilized to non-destructively detect the crack location and magnitude through changes in modal characteristics. Such methods offer several advantages, like measurement of vibration parameters is less time consuming, less costly and can be easily measured from any single location of a structure. This is particularly useful in the situations where surface cracks are not fully accessible, due to external insulations and hence, where visual inspection techniques may not be useful. In these cases, the modal data provides useful information for the determination of structural defects such as cracks. Various attempts have been made to relate the changes in modal characteristics to the changes in beam geometrical properties for identifying crack location and magnitude [8] [9]. The technique involves two stages: Forward analysis, wherein the modal parameters are determined using beam geometrical details and crack parameters and; Inverse analysis in which the presence of crack and its location for a given beam geometry is identified for a particular value of natural frequencies or mode shapes [3]. A detailed review of using vibration characteristics for locating damages in a structure can be found in [10] [11].

Vibration analysis of a cantilever beam with single open transverse crack for different crack depth and crack locations is done experimentally by Shinde and Katekar for both no load and with transverse load conditions [12].

Many researches have exploited the use of Finite Element Analysis (FEA) and software based on it for damage detection in structures. Khan and Parhi [13] analysed the effects of crack depth on natural frequency and mode shape of cantilever and fixed- fixed beam with varying crack depth and location using FEA. Kumar et al. [14] studied the effect of crack on mode shapes and natural frequencies in cantilever, simply supported and fixed beam using theoretical and experimental analysis and simulation in FEA. Chaudhary and Patil [15] performed static and modal analysis of beam to find deflection and natural frequencies before and after development of crack. Ghodke et al. [16] have investigated the effect of crack on continuous I- section simply supported beam on modal frequencies and mode shapes in Structural steel and Aluminium using FEA. In [17] the effect of single edged notch has been analysed for various crack depth using Modal analysis with FEA.

In the present work, modal analysis of a cantilever beam with a single open transverse crack has been performed using FEA to determine the effect of crack on the natural frequencies and mode shapes in cantilever beams of different material for different crack locations. For study, only the first five modes of vibration have been considered.

2. Finite Element Modeling and Analysis

2.1. Introduction

When a crack is introduced in a structure, life of the system is reduced as a result of high stress concentration at the edges of the cracks, promoting its further growth. This may result into serious system failure and therefore its presence needs to be identified as soon as it comes into existence. The generation of crack results in reduction in stiffness, consequently reducing the natural frequency and manipulating the vibration signatures of the system. This change may be experimentally determined and correlated to the size and location of crack, for early detection of cracks. Since, different materials
behave in different manner in the presence of damage, the study of the influence of materials is important. In this work, the change in modal frequency due to the change in material property has been investigated using FEA in beams before and after damage. Firstly, the solid model of the beam without crack has been created and its modal characteristics determined. Next, the crack is introduced in the solid model and its modal analyses have been carried out. The crack location has been changed to investigate its effect on the modal parameters for different vibration modes.

Table 1 summarizes the geometry and material properties of different beams considered in this study.

| Beam Geometry | Material Properties |
|---------------|---------------------|
| Length (mm)   | 200                 |
| X-section (width x depth) (mm) | 12 x 18 Structural steel |
| Cross-sectional Area (mm²) | 5832 aluminium alloy |
| Area Moment of Inertia (mm⁴) | 216 Gray Cast Iron |
| Young’s Modulus (GPa) | 200 |
| Poisson ratio | 0.3 |
| Density (Kg/mm³) | 7845 |
| Structural steel  | 0.3 7845 |
| Aluminium alloy   | 0.33 2775 |
| Gray Cast Iron    | 0.28 7210 |

2.2. Beams without crack
The modal analyses of the three beams (of materials as Structural Steel (SS), Aluminium alloy (Al) and Gray Cast Iron (CI)), have been carried out in FEA. The first five natural frequencies of vibration for the three beams before the introduction of crack, have been listed in Table 2.

| Vibration Modes | Frequency (Hz) |
|-----------------|----------------|
| Mode 1          | Structural steel (SS) 244.91 |
|                 | Aluminium alloy (Al) 245.86 |
|                 | Gray Cast Iron (CI) 189.56 |
| Mode 2          | 365.77 |
|                 | 367.11 |
|                 | 283.14 |
| Mode 3          | 1509.7 |
|                 | 1515.2 |
|                 | 1168.7 |
| Mode 4          | 2211.3 |
|                 | 2218.0 |
|                 | 1712.4 |
| Mode 5          | 3343.1 |
|                 | 3315.5 |
|                 | 2608.8 |

2.3. Beams with crack
Now, the finite element analysis has been done for beams with a transverse surface crack of width (w) = 0.1mm and depth (d) = 4mm, spreading uniformly along the width of beams. The location (l) of crack from the fixed end has been varied at 60mm, 100mm and 160mm. Table 3, Table 4 and Table 5 show the results of the analyses for different beams at different crack locations.
### Table 3 Natural frequencies of cracked SS beam

| Vibration Modes | Frequency (Hz) | Without Crack | l= 60mm | l= 100mm | l= 140mm |
|-----------------|----------------|---------------|---------|---------|---------|
| Mode 1          | 244.91         | 242.75        | 244.19  | 244.81  |
| Mode 2          | 365.77         | 354.82        | 362.04  | 365.19  |
| Mode 3          | 1509.7         | 1505.7        | 1491.3  | 1499.3  |
| Mode 4          | 2211.3         | 2192.6        | 2122.0  | 2159.9  |
| Mode 5          | 3343.1         | 3317.5        | 3327.0  | 3336.8  |

### Table 4 Natural frequencies of cracked Al beam

| Vibration Modes | Frequency (Hz) | Without Crack | l= 60mm | l= 100mm | l= 140mm |
|-----------------|----------------|---------------|---------|---------|---------|
| Mode 1          | 245.86         | 243.68        | 245.13  | 245.75  |
| Mode 2          | 367.11         | 356.19        | 363.39  | 366.53  |
| Mode 3          | 1515.2         | 1511.1        | 1496.7  | 1504.7  |
| Mode 4          | 2218.0         | 2199.5        | 2129.3  | 2167.0  |
| Mode 5          | 3315.5         | 3290.4        | 3299.7  | 3309.4  |

### Table 5 Natural frequencies of cracked CI beam

| Vibration Modes | Frequency (Hz) | Without Crack | l= 60mm | l= 100mm | l= 140mm |
|-----------------|----------------|---------------|---------|---------|---------|
| Mode 1          | 189.56         | 187.89        | 189.0   | 189.48  |
| Mode 2          | 283.14         | 274.63        | 280.24  | 282.69  |
| Mode 3          | 1168.7         | 1165.6        | 1154.5  | 1160.7  |
| Mode 4          | 1712.4         | 1697.8        | 1642.9  | 1672.4  |
| Mode 5          | 2608.8         | 2588.7        | 2596.1  | 2603.8  |

### 3. Results and Discussion

The results of analyses clearly show that the natural frequency of Al beam is largest for the first four vibration modes. For the fifth mode, the SS structure possesses the largest value. Natural frequency of CI beam is lowest for all the five modes. This difference indicates that the selection of material is critical while making decision regarding material for machines or structures. The final design should take into consideration the vibration behaviour for the range of frequencies that may be encountered in service conditions, so as to prevent resonance and its subsequent undesirable after effects.

The present analysis also shows that the introduction of crack reduces the natural frequency, though by very small value. Also, the variations in natural frequencies with crack location is not following a regular trend. Fig. 1 shows the percentage change in natural frequency of the three beams with crack location. It can be observed, that all the materials follow the similar though irregular trend. The change in crack location does not have similar effect on the modal frequencies. For instance, the natural frequencies increase with increase in crack location for mode 1, mode 2 and mode 5 but the frequencies corresponding to mode 3 and mode 4 (Fig. 1), are first decreasing with the crack location and then increasing thereafter. This non-uniform behaviour may be attributed to the combined
influence of location of crack (i) relative to the nodes of particular vibration mode and (ii) from the fixed support.

4. Conclusion

In the present work, the influence of material property on the modal behaviour of cantilever beams having a single surface crack, spreading uniformly along the width has been investigated using Finite Element Analysis (FEA), for the first five modes of vibration. The following conclusions can be drawn from the results of analysis:

1. Al Beam has the highest fundamental frequency, followed by Structural Steel for the first four modes. Gray Cast Iron beam has the lowest frequency in all the modes.
2. In all the beams, introduction of crack reduces the natural frequency of vibration, though by very small value.
3. There is no regularity in the trend of variation in natural frequencies due to change in crack location, however, the trend is similar in all the three beams.

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