The Drell-Yan process and Deep Inelastic Scattering from the lattice

M. Göckeler\(^{a,b,c}\), R. Horsley\(^d\), E.-M. Ilgenfritz\(^d\), H. Oelrich\(^e\), H. Perl\(^f\), P. E. L. Rakow\(^{a,e}\), G. Schierholz\(^{a,e,g}\), A. Schiller\(^f\) and P. Stephenson\(^e\)

\(^{a}\)Hochleistungsrechenzentrum HLRZ, c/o Forschungszentrum Jülich, D-52425 Jülich, Germany
\(^{b}\)Institut für Theoretische Physik, RWTH Aachen, D-52056 Aachen, Germany
\(^{c}\)Institut für Theoretische Physik, J. W. Goethe Universität, D-60054 Frankfurt, Germany
\(^{d}\)Institut für Physik, Humboldt-Universität zu Berlin, Invalidenstraße 110, D-10115 Berlin, Germany
\(^{e}\)DESY-IfH Zeuthen, D-15735 Zeuthen, Germany
\(^{f}\)Fak. f. Physik und Geowiss., Universität Leipzig, Augustusplatz 10–11, D-04109 Leipzig, Germany
\(^{g}\)Deutsches Elektronen-Synchrotron DESY, Notkestraße 85, D-22603 Hamburg, Germany

We report on measurements of the \(h_1\) structure function, relevant to calculating cross-sections for the Drell-Yan process. This is a quantity which can not be measured in Deep Inelastic Scattering, it gives additional information on the spin carried by the valence quarks, as well as insights on how relativistic the quarks are.

1. INTRODUCTION

Deep Inelastic Scattering (DIS) gave us the first evidence that quarks are true physical objects, not just “book-keeping” devices for a flavour symmetry group. The structure functions measured in DIS give us an important insight into the internal workings of a hadron, allowing us to measure how the energy and spin is shared out between the different constituents.

Perturbative QCD can explain the evolution of these structure functions as we change the scale at which we probe the hadron, but is unable to give us a starting value for this evolution. Lattice QCD has been reasonably successful for masses, the next step is to use it to calculate structure function moments, form factors and matrix elements. At present this is our only hope of finding these numbers from first principles.

2. THE INTERPRETATION OF \(h_1\)

Deep Inelastic Scattering (DIS) is not the only useful probe of the hadrons’ parton distributions. In hadron-hadron colliders the Drell-Yan process can be observed, in which a quark in one hadron and an anti-quark in the other annihilate to form an extremely virtual time-like photon, which then decays to a lepton-antilepton pair. Measurements of the lepton pair’s total momentum give enough information to extract \(x\) and \(y\), the fraction each parton carries of its hadron’s momentum.

If we look at the unpolarised Drell-Yan process we find the same structure functions occurring as in DIS. For example the total cross-section is proportional to \(\sum a e_a^2 f_a^q (x) f_{\bar{a}}^{\bar{q}} (y)\) where \(a\) runs over all flavours. The asymmetry in cross-sections when two longitudinally polarised nucleons collide can again be expressed in terms of a quantity known from polarised DIS, namely the structure function \(g_1\). However if we consider the cross-section when two transversely polarised nucleons with spins \(S_A\) and \(S_B\) collide, and try to find the asymmetry on flipping one spin,

\[
A_T \equiv \frac{\sigma (S_A, S_B) - \sigma (S_A, -S_B)}{\sigma (S_A, S_B) + \sigma (S_A, -S_B)} \propto \frac{\sum a e_a^2 h_{a}^{q} (x) h_{\bar{a}}^{\bar{q}} (y)}{\sum a e_a^2 f_a^q (x) f_{\bar{a}}^{\bar{q}} (y)},
\]

(1)
we find that it cannot be expressed in terms of the familiar structure functions, a new structure function, \( h_1 \), is needed \([1]\).

Moments of the structure function \( h_1 \) can be related, through the operator product expansion, to the matrix elements of the operators

\[
O^{\sigma, \mu_1, \cdots, \mu_n} = \bar{\psi} \sigma^{\mu_1} \gamma_5 D^{\mu_2} \cdots i D^{\mu_n} \psi - \text{tr.} \tag{2}
\]

This operator is very similar in structure to the operators that give the moments of \( f_1 \) and \( g_1 \). An important difference is in the Dirac structure of the operator. In \( f_1 \) and \( g_1 \) this is proportional to \( \gamma_\mu \) or \( \gamma_\mu \gamma_5 \) respectively, in both cases these matrices anti-commute with \( \gamma_5 \). However the operator in Eq. (2) is proportional to a \( \sigma \) matrix, which commutes with \( \gamma_5 \), implying that \( h_1 \) has the opposite chiral properties to \( f_1 \) and \( g_1 \). It is this difference that explains why \( h_1 \) is observable in the Drell-Yan process, but not in DIS. With massless quarks chirality is conserved along a quark line. In the Feynman diagram for DIS (Fig. 1) there is only a single quark line passing through the hard part of the process, so only the chirally-even structure functions \( f_1 \) and \( g_1 \) can be observed. On the other hand the Drell-Yan process involves two separate fermion lines, which can have the same or opposite chirality. This means that, even at leading twist, both chirally-even and chirally-odd structure functions are observable.

![Figure 1. The Feynman diagram for deep inelastic scattering. The hard scattering process conserves chirality.](image1)

What does the structure function \( h_1 \) tell us about the quarks in the proton? If we consider a stationary proton (momentum in the 0-direction) the operator in Eq. (2) differs from the operator for \( g_1 \) by a factor \( \gamma_0 \). In the non-relativistic limit

fermions are in eigenstates of \( \gamma_0 \) with eigenvalue 1, and so \( h_1 \) and \( g_1 \) are identical. By comparing \( h_1 \) and \( g_1 \) for a real proton we can gain an insight into how relativistic the constituents are.

The structure functions \( h_1 \) and \( g_1 \) have the opposite behaviour under charge conjugation, so it is expected that the contributions of the quarks and anti-quarks in the sea will largely cancel in \( h_1 \), making it a quantity given mostly by the valence quarks. This means that we might hope that a quenched calculation of \( h_1 \) is likely to give an answer close to the true value.

Beyond the non-relativistic approximation, what does \( h_1 \) measure? By considering the effect of operating with the operator from Eq. (2) on a quark state which is an eigenstate of the operator \( \not\! \gamma_5 \) it can be seen that what is actually being measured by \( h_1 \) is the distribution of this quantity, which is given the name “transversity”.

3. RESULTS

The operator in Eq. (2) can be discretised and its expectation value measured on the lattice using the methods used earlier \([2]\) for the more familiar \( f_1 \) and \( g_1 \) structure functions.

We have undertaken measurements of the quenched QCD structure functions. The measurements I report here were made with the quenched Sheikholeslami-Wohlert action on a \( 16^3 \times 32 \) lattice with \( \beta = 6.0 \) and \( \epsilon_{SW} = 1.769 \) (the value recommended in \([3]\), chosen to eliminate \( O(a) \) discretisation effects). To completely
remove all $O(a)$ effects it is not enough to simply use the optimal fermion action, the lattice operators must also be improved. At present we are using tree-level operator improvement. To produce a final answer we need to know the $Z$ factors of renormalisation theory. We are using the 1-loop perturbative calculation of [4]. Ideally we would like to determine both the operator improvement and the $Z$ factors non-perturbatively, we are currently working on this calculation.

Figure 3. The proton tensor charge $\delta q \equiv \int_0^1 dx (h_1(x) - \bar{h}_1(x))$ for the valence $u$ (upper two lines) and $d$ (lower two lines) quarks. The open points are the data, the solid points the chiral extrapolation.

In Fig. 3 we show our results for the proton $\delta q \equiv \int_0^1 dx (h_1(x) - \bar{h}_1(x))$ for the $u$ quarks (upper two lines) and $d$ quarks (lower lines). This lowest moment of $h_1$ is found from the $n = 1$ case of Eq. (3). We have made two determinations of $\delta q$, once from the expectation value of the operator $O^{1,2}$ calculated with a stationary nucleon (squares), and once from the operator $O^{1,2}$ measured for a nucleon with one unit of momentum in the 1-direction (circles). (In both cases our nucleon spin is polarised in the 2-direction.) If Lorentz symmetry has been restored on the lattice, both determinations would agree, which they do within the errors, though for $O^{1,2}$ the errors are large. In the heavy quark limit the $u$ and $d$ quark contributions to $\delta q$ are $+4/3$ and $-1/3$ respectively. We see that the $u$ contribution has dropped below this value for our quark masses.

It is interesting to compare the lattice results with the results of a recent QCD sum rule calculation [5], which finds $\delta u = 1.33 \pm 0.53$ and $\delta d = 0.04 \pm 0.02$ at the scale $1 \text{GeV}^2$. The small value of $\delta d$ differs from the results of lattice gauge theory, (see Fig. 3 or [7]).

As mentioned in the previous section, the comparison between $h_1$ and $q_1$ gives us an impression of how relativistic the quarks in a nucleon are. Comparing $\delta q$ and the valence quark contribution to $\Delta q \equiv \int_0^1 dx q_1(x)$, as reported in [6], we see that they are rather similar, (a conclusion also reached in [7,8]) showing that for the quark masses used, which are approximately in the strange quark range, a non-relativistic description of the spin structure is reasonable.

ACKNOWLEDGMENTS

We wish to thank DESY-Zeuthen, where the numerical calculations were performed on an APE (Quadrics QH2).

REFERENCES

1. R. L. Jaffe and X. Ji, Phys. Rev. Lett. 67 (1991) 552; Nucl. Phys. B375 (1992) 527.
2. M. Göckeler, R. Horsley, E.-M. Ilgenfritz, H. Perlt, P. Rakow, G. Schierholz and A. Schiller, Phys. Rev. D53 (1996) 2317, hep-lat/9508004.
3. ALPHA collaboration, Phys. Lett. B372 (1996) 275; R. Sommer, Lattice 96, hep-lat/9608049.
4. H. Perlt, Lattice 96, hep-lat/9608033.
5. H. He and X. Ji, hep-ph/9607408.
6. P. Stephenson, Lattice 96, hep-lat/9608081.
7. S. Aoki, M. Doui and T. Hatsuda, hep-lat/9606006.
8. A. Pochinsky, Lattice 96, hep-lat/9608069.