NEW MECHANISM FOR REPEATED POSTED PRICE AUCTION WITHOUT DISCOUNTING

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Abstract. On ad exchange platforms the place for advertisement is sold through different kinds of auctions. However, it is not uncommon the situation where the seller repeatedly encounters only one buyer, thus the posted price auction degenerates into a monopoly-monopsony game with asymmetric information and nearly an infinite number of rounds; on each round the seller proposes the price and the buyer accepts or rejects it.

I learned this problem from a discussion with members of Yandex research team and my main motivation was to find an incentive-compatible seller’s strategy. In this short paper such a strategy is proposed and a corresponding distortion at the top type lower bound (Spence-Mirrlees property, actually) for the surplus of the buyer is established; this shows that the proposed strategy is the best possible.

The key ingredients are the following. The main leash that the buyer has is the frequency of accepted deals. Once this frequency (as a function on the buyer’s type) is fixed, the strategy randomly chooses between the

1. Setup

1.1. The game. The repeated posted price auction, also known as the fishmonger problem, is an archetypical repeated monopoly-monopsony game with asymmetric information. This is a game with two players: the seller (fisher) and the buyer (cook). In each round $i = 1, 2, \ldots$ the fisher proposes to the cook a unit of the good (think of fish or a place for advertisement) with the price $Q_i$. Let the cook’s valuation of this good be $q \in \mathbb{R}_{>0}$, the type of the cook. In each round the cook has two options: accept the deal (and then we set $a_i = 1$) or reject it (we set $a_i = 0$). In this round, the cook’s surplus is $a_i(q - Q_i)$ and the fisher’s revenue is $a_i Q_i$. This is an example of one-sided ignorance: the seller does not know the value of his good for the buyer.

The repeated posted price auction appears in ad exchange conducted by the leading Internet companies, see [2, 8, 11, 10, 15, 17, 15] and the references therein. It is common to assume that $q$ is drawn from a known distribution. In line with [6], recently there have appeared a bunch of papers (e.g. [7, 5, 12, 22, 4]) which weaken this assumption since it is practically unrealistic if the buyer is unique of their kind. We will study the case when the fisher has no information at all about $q$ (it is even worse than the worst case in terminology of [15] where $q$ was at least in the interval $[0, 1]$). Consequently, to make sense of the problem we must further assume that the game has an infinite number of rounds.

It is also common to discount the surplus of the cook or introduce the stopping time, thus making the problem more accessible: the cook prefers less loss now to the bigger gain in the future. However, it is hard to justify any choice of discount factor in practice because nowadays many auctions are run by robots and are performed many times per second. So we unleash the cook by imposing no discounting on his surplus.
1.2. Contribution of this paper. A distortion at the top upper bound for the surplus of the fisher is established. Namely, if the fisher commits to a certain strategy in advance, then the higher the type $q$ of the cook, the bigger share of the total welfare $q$ distributed in each round goes to the cook.

Then, we propose a strategy such that if the fisher commits to it, then the optimal in expectation response of the cook will be to play naively, i.e. accept the deal if and only if $Q_i \leq q$, thus this strategy is incentive-compatible. Furthermore, the revenue of the fisher in this strategy attains the upper bound discussed above, so this bound is tight. The proofs are technically simple and consist merely of changing the point of view.

The proposed mechanism is credible (cf. [1]), the fisher exercises “the power to commit”: the cook can verify that the fisher is using exactly this strategy, so there will be no mistrust.

1.3. The objectives of the players. Fix a strategy $S_{fisher}$ of the fisher and a strategy $S_{cook}$ of the cook. In the spirit of Wald’s maxmin model, we denote

$$Fisher(S_{fisher}, S_{cook}, q) = \liminf_{k \to \infty} \frac{1}{k} \sum_{i=1}^{k} a_i Q_i,$$

$$Cook(S_{fisher}, S_{cook}, q) = \liminf_{k \to \infty} \frac{1}{k} \sum_{i=1}^{k} a_i (q - Q_i),$$

and say that the fisher’s objective is to maximize $Fisher(S_{fisher}, S_{cook}, q)$ and the cook’s objective is to maximize $Cook(S_{fisher}, S_{cook}, q)$. Roughly speaking, they maximize the minimal average revenue and the minimal average surplus respectively. Since the total welfare is $q = (q - Q_i) + Q_i$ one may think than in each round the fisher and the cook divide $q$ among themselves, if the deal is accepted.

A strategy (we allow mixed strategies too) is any rule which maps the previous history of the game to a distribution from where the next price (or a decision to accept or reject) is drawn.

2. Observables and Spence-Mirrlees property

Fix any strategy $S_{fisher}$ of the fisher. Suppose that the cook is of the type $q'$ and the cook knows $S_{fisher}$ in advance and chooses a strategy $S_{cook}^0$. Let them play and write the history of all the moves, i.e. $a_i, Q_i, i = 1, \ldots$. Taking the physical point of view we may try to extract observables, i.e. some quantitative data from this history. The first observable coming in mind is the empirical proportion of accepted deals. Namely, define

$$p(q') = p(q', S_{fisher}, S_{cook}^0) = \liminf_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} a_i.$$

Again, we would like to take the limit of the average frequency of accepted deals. Since it may not exist, we consider $\liminf$ instead.

**Lemma 1.** Suppose that $p(q') > 0$. Then, for each $\varepsilon > 0$, for $q$ big enough we have

$$\min Fisher(S_{fisher}, S, q) \leq q \cdot (\varepsilon + 1 - p(q')),$$

where we take the minimum by all strategies $S$ of the cook. In other words, the fisher can not guarantee himself more than the share $1 - p(q')$ of the total welfare for all sufficiently large $q \in \mathbb{R}_{>0}$.

**Proof.** For $q > q'$ by playing $S_{cook}^0$ the cook of type $q$ has

$$Cook(S_{fisher}, S_{cook}^0, q) \geq Cook(S_{fisher}, S_{cook}^0, q') + (q - q') p(q').$$

This gives

$$\min Fisher(S_{fisher}, S, q) \leq Fisher(S_{fisher}, S_{cook}^0, q) \leq q - Cook(S_{fisher}, S_{cook}^0, q') - (q - q') p(q') =$$

$$= q(1 - p(q')) + [q' p(q') - Cook(S_{fisher}, S_{cook}^0, q')] < q \cdot (1 - p(q') + \varepsilon)$$

\]
for \( q \) big enough, which finishes the proof.

This proof amounts to the fact that by pretending of being of the type \( q' \) the cook takes roughly at least \( 1 - p(q') \) share of the total welfare in each round.

Let the cook of type \( q \) play a strategy \( S_{cook}(q) \) against \( S_{fisher} \). It is natural to assume that the revenue of the cook of type \( q \) using strategy \( S_{cook}(q) \) is at least that of playing the strategy \( S_{cook}(q') \) with \( q' < q \), because otherwise we may set \( S(q) := S(q') \).

Using the history of playing \( S_{cook}(q) \) against \( S_{fisher} \) we define \( p(q) \) for all \( q > 0 \) as

\[
p(q) = p(q, S_{fisher}, S_{cook}(q)) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} a_i.
\]

**Lemma 2.** Under the above assumptions, for each \( q, x \geq 0 \) we have

\[
Cook(S_{fisher}, S_{cook}(q + x), q + x) \geq Cook(S_{fisher}, S_{cook}(q), q) + x \cdot p(q).
\]

**Proof.** If the cook’s type is \( q + x \) and he plays the strategy \( S_{cook}(q) \), his average surplus is at least

\[
Cook(S_{fisher}, S_{cook}(q), q) + x \cdot p(q)
\]

which must be at most \( Cook(S_{fisher}, S_{cook}(q + x), q + x) \). \( \square \)

Morally, this estimate tells us that the derivative of \( R(q) = Cook(S_{fisher}, S_{cook}(q), q) \) (if it exists) is at least \( p(q) \). Also, this is the old good Spence-Mirrlees property.

It is not difficult to show that if \( p(q) \) locally decreases with \( q \) then is not possible that both fisher and cook are better when \( q \) increases. Therefore it is natural to assume that \( p(q) \) grows with \( q \). Additionally, one is likely to expect that the higher the type \( q \) of the buyer, the more frequently the deals are made. Also it is natural to assume that \( \lim_{q \to \infty} p(q) = 1 \).

### 2.1. Distortion at the top.

It directly follows from Lemma 2 that the following property holds.

**Corollary 1.** If \( p(q) \) is non-decreasing function on \( q \) with \( \lim_{q \to \infty} p(q) = 1 \) then

\[
\lim_{q \to \infty} \frac{Fisher(S_{fisher}, S_{cook}(q), q)}{Cook(S_{fisher}, S_{cook}(q), q)} = 0,
\]

i.e the fraction that the fisher gets out of the total welfare \( q \) tends to zero.

Note that using an arbitrary strategy \( S_{fisher} \) of the fisher and the responses \( S_{cook}(q) \) of the cook we constructed a function \( p(q) \), the observable which measures the proportion of accepted deals. Now we switch our attention to this monotone function \( p(q) : [0, \infty) \to [0, 1] \) and show how to use it in order to construct an incentive-compatible strategy \( S_{fisher} \) of the fisher, such that the best response strategy \( S_{cook}(q) \) of the cook will amount to exactly this given proportion \( p(q) \) of accepted deals.

### 3. A new mechanism

Our mechanism is introduced after the following informal motivation.

#### 3.1. Commitment.

Let \( S_{cook}^{naive}(q) \) be the following naive strategy for the cook: accept the price \( Q_i \) if \( Q_i \leq q \), refuse otherwise. For the cook of type \( q \), playing \( S_{cook}^{naive}(q) \) is not always optimal: the fisher quickly determines \( q \) and the cook’s surplus \( q - Q_i \) will tend to zero.

On the other hand, imagine that the cook makes a commitment: he tells to the fisher that he is going to play \( S_{cook}^{naive}(1) \) regardless the actions of the fisher. If the cook sticks to this strategy (fortunately for the fisher in many cases the cook has no such a commitment power), then the best response for the fisher is to always propose \( Q_i = 1 \), other strategies bring strictly less revenue. In any case, it is in the interest of the fisher to incentivise the cook not to play this strategy if the cook’s type is much bigger than \( q = 1 \). The only way to do this is to assure the cook that the higher prices he accepts, higher will be his average surplus.
If the fisher has a strategy, it makes sense to reveal it. Indeed, otherwise a strategic cook tricks the fisher and is afraid to buy even for a small \( Q \), because this gives the fisher an information about the cook's type, and the fisher will use this information in an unknown way. It is proven that the absence of a fisher's commitment may crash the market.

3.2. Strategy background ideas, fisher's guess of the cook's type. A strategy of the fisher (choosing the price \( Q_{n+1} \) after the round \( n \)) can depend on the history of proposed prices \( Q_i \) and the responses \( a_i \) of the cook in all rounds \( i \) preceding a given round. It seems reasonable for the fisher to squash this bunch of information to a number: we suppose that before playing the round \( i \) the fisher has a guess \( q_i \) of the cook's type, and his next proposal depends only on \( q_i \).

A good strategy \( S_{fisher} \) of the fisher has the following qualities:

- **type adaptation.** \( S_{fisher} \) should depend on \( q_i \) and adapt to it (one may imagine that the true type \( q \) of the cook slowly changes over time an it would be good if the fisher's strategy adapts to this change);
- **rewarding.** to incentivize the cook, the surplus of the cook should be monotone with respect to \( q_i \), and even more: since the cook can pretend that his type is lower than it is, the growth of the cook's surplus when the fisher's estimate \( q_i \) increases must include what the cook can obtain by pretending that he is of a lower type;
- **type confirmation.** the fisher should not incentivise the cook to pretend that his type is higher than it is, because stimulating to lie is not good by itself and because this again can crash the market and impose additional revenue loss for the fisher.

To disentangle all these features we take each of them to the extreme and propose each of them with some probability (because mixed strategies frequently work better in the context of learning or truthful mechanism design, cf. \[9\]). Start with **rewarding**: the fisher will sometimes propose \( Q_i = 0 \), i.e. will give the object to the cook for free. To implement **type adaptation** it is enough to propose a price higher than the current estimate \( q_i \), for example, \( Q_i \) is drawn from a uniform distribution on \([q_i, 1 + q_i]\) (or any other distribution with the support on \([q_i, +\infty)\)). To ensure that the cook is indeed of the type \( q_i \), one might propose the price \( q_i \) sometimes, and if the cook refuses, then set \( q_{i+1} < q_i \), which should cause a revenue loss for the cook.

3.3. The mechanism, formally. The fisher knows that \( q \in [0, \infty) \) (the mechanism can be easily adapted to the case when the fisher knows for sure that \( q \) belongs to a fixed interval). The fisher fixes any increasing function \( p : [0, \infty) \to [0, 1], p(0) = 0, \lim_{q \to \infty} p(q) = 1 \). Let \( R \) be the solution of the following differential equation: \( R(0) = 0, R'(q) = p(q) \). The fisher commits to use the following strategy, all the ingredients (\( p \) and the following algorithm) of which are announced to the cook in advance. We will show that the optimal response \( S_{cook}(q) \) of the cook of type \( q \) gives him \( Cook(S_{fisher}, S_{cook}(q), q) = R(q) \).

In each round the fisher has an estimate \( q_n \) of \( q \) \((q_0 = 0)\). The fisher plays the following mixed strategy \( S_{fisher}(q_n) \) randomly choosing between three types of prices in this round:

- (with probability \( 1 - p(q_n) \)) the fisher plays **type adaptation**: the price \( Q_n \) is uniformly drawn from \([q_n, q_n + 1]\), accepting this price the cook achieves that \( q_{n+1} > q_n \). If \( Q_n \) is accepted, then the fisher sets \( q_{n+1} = Q_n \). If \( Q_n \) is refused, then \( q_{n+1} = q_n \).
- (with probability \( \frac{R(q_n)}{q_n} \)) the fisher plays **rewarding**: the price \( Q_n = 0 \), which is profitable for the cook, and the probability of this price appearing grows with \( q_n \) thereby stimulating the cook to reveal his type.
- (with probability \( p(q_n) - \frac{R(q_n)}{q_n} \)) the fisher plays **type confirmation**: the price \( Q_n = q_n \), which is assumed to give no surplus to the cook, but refusing it incurs lowering the estimate of the type of the cook. If \( Q_n \) is accepted, then \( q_{n+1} = q_n \). If \( Q_n \) is refused, then we should set \( q_{n+1} < q_n \). This can be done in many ways, e.g. let \( k = \sum_{i=1}^{n} a_i \). Let us reorder the set \( \{Q_i\}_{i=1}^{n} \),

\[
Q'_1 \geq Q'_2 \geq Q'_3 \geq \ldots \geq Q'_n.
\]

The fisher sets \( q_{n+1} = \frac{Q'_{n+1} + Q'_{n+2}}{2} \). The intuition for this formula is as follows: it is natural to define \( q_n \) (the empirical guess of \( q \)) as the average between the maximal accepted price and the minimal rejected price. But, depending on the strategy of the cook, the latter can be less than the former. So we reorder all proposed prices and choose \( q_n \) such that the proportion of proposed prices higher than \( q_{n+1} \) is equal to the proportion of refused prices.
While the fisher plays \( S_{\text{fisher}}(q_n) \), the average surplus of the cook of type \( q \neq q_n \), playing \( S_{\text{cook}}^{\text{naive}}(q) \) is
\[
(q - q_n) \left( p(q_n) - \frac{R(q_n)}{q_n} \right) + q \left( \frac{R(q_n)}{q_n} \right) = (q - q_n)p(q_n) + R(q_n) < R(q),
\]
where the last inequality follows from the definition of \( R(q) \) if \( q_n < q \), and from the convexity of \( R(q) \) when \( q_n > q \). This kind of inequality is common in Revenue Equivalence type theorems.

From this we derive the following theorem.

Theorem 1. For any strategy \( S_{\text{cook}}(q) \) of the cook we have
\[
\mathbb{E}(\text{Cook}(S_{\text{fisher}}, S_{\text{cook}}^{\text{naive}}(q), q)) \geq \mathbb{E}(\text{Cook}(S_{\text{fisher}}, S_{\text{cook}}(q), q)),
\]
where \( \mathbb{E}(\cdot) \) stands for the expectation (recall that the strategy \( S_{\text{fisher}} \) is not pure).

Proof. The cook can manipulate \( q_n \). The incentives are as follows: 1) If \( q_n < q \) then for the cook it is profitable because the cook gains additionally on accepting type adaptation price; 2) If \( q_n > q \), then for the cook it is profitable because the reward price \( Q_i = 0 \) appears more frequently.

By the mechanism, to sustain \( q_n < q \) the cook has to always reject prices \( Q_i > q_n \), i.e. this strategy can not be more profitable than \( S_{\text{cook}}^{\text{naive}}(q_n) \). In order to make \( q_n \) bigger than \( q \), the cook should accept type adaptation prices (which are rare, so the process will take a long time) and type confirmation prices, both have negative surplus for the cook, so in the expectation it is not profitable. Then the cook may enjoy reward prices for some time, but refusing a status confirmation price results in setting \( q_n \) lower.

To summarise, due to the strategy of the fisher, in order to make the fisher believe that \( q \) is \( q' \) the cook should, in fact, play a strategy \( S_{\text{cook}}^{\text{naive}}(q') \) for a substantial time, which is not profitable on average by construction.

Surely, a risk loving cook can play \( S_{\text{cook}}^{\text{naive}}(x) \) with \( x > q \) and this can be more profitable than playing \( S_{\text{cook}}^{\text{naive}}(q) \) on short sequences of rounds.

4. Discussion

The main idea of this paper is to put the situation upside down: for a third party observing the game the most salient number which can be extracted from the game is the proportion \( p \) of accepted deals. It is a function \( p(q) \) on the type \( q \) of the cook. Then to construct the mechanism we use this function to produce the strategy \( S_{\text{fisher}} = S_{\text{fisher}}(p(\cdot)) \) of the fisher, and if the cook uses his best response to \( S_{\text{fisher}} \), then the proportion of accepted deals is exactly \( p(q) \). Any good strategy must have the following features: it should reward the cook for the revelation of his true type (and here we see a common phenomenon: higher the type, bigger should be the reward), it should adapt to the cook’s type, and it should not incentivise the cook to pretend that his type is higher than it is, because in this case the fisher should be giving the cook a bigger reward. Our mechanism satisfies these three requirements.

As far as we know the proposed mechanism is novel (and very simple). Another distribution for mixed strategies may be considered in the fisher’s strategy, but they all should have an atom at \( q \): indeed, if the surplus is growing with \( q \), the cook has an incentive to pretend that his type is higher than it is. To prevent it we must always check (by offering the price \( q \) that the cook is indeed of type \( q \) or lower. In the contract theory paradigm (cf. recent [16]) one can give the following metaphor: 1) an employee should have opportunities to show that she is more capable that her status suggests, 2) higher the status, more free benefits (ratchet effect), 3) there should be always work to do, to confirm her high status.

The idea that in order to reveal the true type of the cook, the fisher must give him the substantial part of the total welfare (almost 100% of welfare when the cook’s type is huge) is not new and can be traced back to [19].

Note another advantage of the proposed mechanism: if the valuation \( q \) of the cook changes, the algorithm can be easily adapted – for example, we may set the rule for \( q_n \), taking into account only the last 100 deals. Similar statements can be found in [2] (Section 6), but under a different abstract disguise, in another context, and without a concrete mechanism. The mechanism is credible: by playing enough time in a stable position (meaning that \( q_i \) does not change) the cook can calculate the frequency of reward, adaptation, and confirmation prices, and compare them with the probabilities which can be derived from the function \( p(\cdot) \) which was announced in advance.
4.1. How the fisher chooses $p(q)$? Indeed, instead of saying that the fisher knows the distribution of $q$ we need to choose $p(q)$, which, again, means to make a guess about distribution. A subtle difference is that it is hard to guess the distribution if we suspect that (surely, with small probability, but what is the order of magnitude?) $q$ can be really huge. Also, our mechanism can be used to find $q$ of a given customer, and the customers are reluctant to reveal that information if they know that the game is playing for many rounds in the future.

4.2. Further questions. How do real people behave playing this game? (cf. [20]) Let us say, two participants play 20 rounds, and it is known that $q$ belongs to a given interval, but the distribution of $q$ is not known to the fisher role player. The players should be paid: e.g. a fixed amount of money is equal to $20q$, and then the players get the corresponding proportion of that money according to their results. In such a way both are interested in making the deal (any rejection is a waste of $q$ money that they should “divide” among them). I guess that the fairness will not be an issue here because actual $q$ may be small or may be big, the fisher-player has no estimate of it and subsequently can not perceive a deal as fair or not until the end of game when $q$ is revealed.

A multi-person game may be considered in the spirit of [3]: let the fisher face several cooks (of a priori different types), the game be infinite, and there be no discounting. It seems that for any mechanism there will be a distortion at the top estimate, but only for the cook of the highest type, whose presence coerces all other cooks to reveal their types. Finally, it would be interesting to find a formal setup with only one buyer where there is a force similar to the presence of a higher type buyer but weaker than that (an instance of such a force is a common trick when a seller tells you that there is another buyer who agreed to pay way more than you, but tomorrow).

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