Neutrinoless double beta decay and chiral $SU(3)$

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Abstract

TeV-scale lepton number violation can affect neutrinoless double beta decay through dimension-9 $\Delta L = \Delta I = 2$ operators involving two electrons and four quarks. Since the dominant effects within a nucleus are expected to arise from pion exchange, the $\pi^- \to \pi^+ ee$ matrix elements of the dimension-9 operators are a key hadronic input. In this letter we provide estimates for the $\pi^- \to \pi^+ ee$ matrix elements of all Lorentz scalar $\Delta I = 2$ four-quark operators relevant to the study of TeV-scale lepton number violation. The analysis is based on chiral $SU(3)$ symmetry, which relates the $\pi^- \to \pi^+ ee$ matrix elements of the $\Delta I = 2$ operators to the $K^0 \to \bar{K}^0$ and $K \to \pi\pi$ matrix elements of their $\Delta S = 2$ and $\Delta S = 1$ chiral partners, for which lattice QCD input is available. The inclusion of next-to-leading order chiral loop corrections to all symmetry relations used in the analysis makes our results robust at the 30% level or better, depending on the operator.
**Introduction** – Neutrinoless double beta decay ($0\nu\beta\beta$) is a rare nuclear process in which two neutrons inside a nucleus convert into two protons with emission of two electrons and no neutrinos, thus changing the number of leptons by two units. Since lepton number is conserved in the Standard Model (SM) at the classical level, observation of $0\nu\beta\beta$ would be direct evidence of new physics, with far reaching implications: it would demonstrate that neutrinos are Majorana fermions \[1\], shed light on the mechanism of neutrino mass generation, and probe lepton number violation (LNV), a key ingredient needed to generate the matter-antimatter asymmetry in the universe via “leptogenesis” \[2\]. The current experimental limits on the half-lives are already impressive \[3–9\], at the level of $T_{1/2} > 2.1 \times 10^{25} \text{ y}$ for $^{76}\text{Ge}$ \[6\] and $T_{1/2} > 1.07 \times 10^{26} \text{ y}$ for $^{136}\text{Xe}$ \[3\], with next generation ton-scale experiments aiming at a sensitivity of $T_{1/2} \sim 10^{27–28} \text{ y}$.

By itself, the observation of $0\nu\beta\beta$ would not immediately point to the underlying physical origin of LNV. While $0\nu\beta\beta$ searches are commonly interpreted in terms of the exchange of a light Majorana neutrino, other new physics mechanisms deserve careful evaluation. In an effective theory approach to new physics, the light Majorana neutrino exchange dominates whenever the scale of lepton number violation, $\Lambda_{\text{LNV}}$, is very high compared to the electroweak scale: as long as $\Lambda_{\text{LNV}} \gg \text{TeV}$, the only low-energy manifestation of this new physics is a Majorana mass for light neutrinos, encoded in a single gauge-invariant dimension-5 operator \[10\]. However, as $\Lambda_{\text{LNV}}$ is lowered, new contributions to $0\nu\beta\beta$ are possible, which typically involve the exchange of new TeV-mass Majorana fermions, for example R-handed neutrinos in left-right symmetric models or neutralinos in certain supersymmetric models (for recent reviews see Refs. \[11–14\]). At low energy, the effects of this TeV scale LNV dynamics can be encoded in a set of local dimension-9 operators (involving two leptons and four quarks) that change lepton number by two units. The operators have been classified both according to $SU(3)_C \times U(1)_{\text{EM}}$ gauge invariance \[15, 16\], directly relevant at low-energy, and $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge invariance \[17\], important to connect $0\nu\beta\beta$ to possible LNV signals at the Large Hadron Collider.

To interpret positive or null $0\nu\beta\beta$ results in the context of TeV-scale LNV dynamics, it is essential to quantify the hadronic and nuclear matrix elements involving the $\Delta L = 2$ dimension-9 operators. This is conveniently tackled by first matching the dimension-9 quark level operators onto appropriate operators at the pion-nucleon level, and subsequently computing the nuclear matrix elements. As illustrated in Fig. 1 the dimension-9 operators induce a variety of effective vertices at the pion-nucleon level. The use of chiral power counting has led to the identification of the two-pion exchange in Fig. 1 as the dominant contribution \[16, 18\]. It is therefore very important to estimate as accurately as possible the $\pi^-\pi^- \rightarrow ee$ matrix elements of the dimension-9 operators. Current knowledge of these matrix elements is based on vacuum saturation or naive dimensional analysis, with the exception of two operators for which chiral symmetry was used to relate the two-pion matrix elements to the $K^+ \rightarrow \pi^+\pi^0$ amplitude \[19\].

In this paper we generalize the chiral symmetry analysis to all Lorentz scalar $\Delta I = 2$ four-quark operators, $O_{1,...,5}$ defined in Eq. \[2\] below. We provide estimates for the $\pi^- \rightarrow \pi^+$ matrix elements of $O_{2,...,5}$ by relating them to the $K^0 \rightarrow \bar{K}^0$ matrix elements of their $\Delta S = 2$ chiral partners, which have been computed by several lattice QCD groups \[20–24\]. By including the leading chiral loop corrections, we are able to estimate the uncertainty on the symmetry relations, finding that it does not exceed 30%.

**Operator basis and chiral transformation properties** – At the hadronic scale, short-distance contributions to $0\nu\beta\beta$ can be parameterized by a number of dimension-9 operators \[16\]
SU(2)\langle \pi \rangle + SU(3)_R \rangle \) group we chose to work in the following basis $^3$

\[
\begin{align*}
O_1 &= q_L^\alpha \gamma^\mu \tau^+ q_L^\beta \gamma^\mu \tau^+ q_L^\gamma q_L \\
O_2 &= q_R^\alpha \gamma^\mu \tau^+ q_R^\beta \gamma^\mu \tau^+ q_R^\gamma q_R \\
O_3 &= q_R^\alpha \gamma^\mu \tau^+ q_L^\beta \gamma^\mu \tau^+ q_L^\gamma q_R \\
O_4 &= q_L^\alpha \gamma^\mu \tau^+ q_R^\beta \gamma^\mu \tau^+ q_R^\gamma q_L \\
O_5 &= q_L^\alpha \gamma^\mu \tau^+ q_L^\beta \gamma^\mu \tau^+ q_L^\gamma q_L
\end{align*}
\]

where $q^T = (u, d, s)$, $q_{L,R} = (1/2)(1 \mp \gamma_5)q$, $\alpha, \beta$ denote color indices, and $\tau^+ = T^1 + iT^2$ in terms of the $SU(3)$ generators $T^a$. Three additional operators $O_{1,2,3}'$ are obtained from $O_{1,2,3}$ by the interchange $L \leftrightarrow R$ everywhere. Parity invariance of QCD implies \langle \pi^+|O_{1,2,3}'\pi^-\rangle = \langle \pi^+|O_{1,2,3}\pi^-\rangle.

The operators $O_i$ belong to irreducible representations of the chiral symmetry group $SU(3)_L \times SU(3)_R$ ($q_{L,R} \rightarrow U_{L,R} q_{L,R}$ with $U_{L,R} \in SU(3)_{L,R}$). $O_1$ transforms as $27_L \times 1_R$, $O_{2,3}$ transform as $6_L \times \bar{6}_R$, and finally $O_{4,5}$ transform as $8_L \times 8_R$. The transformation properties of $O_1$ were exploited in Ref. 19 to relate the matrix element of $\langle \pi^+|O_1|\pi^-\rangle$ to the $\Delta I = 3/2$ $K^+ \rightarrow \pi^-\pi^0$ amplitude. Here we exploit the transformation properties of $O_{2,3,4,5}$ to relate their two-pion matrix elements to the matrix elements of their chiral partners between a $K^0$ and a $\bar{K}^0$ meson, which have been computed with lattice QCD by several groups $^20$ $^24$. Strictly speaking the symmetry relation is valid only to leading order in the chiral expansion, and is expected to receive $O(30\%)$ corrections. To make our analysis more robust, we also estimate the size of next-to-leading order (NLO) quark-mass corrections by computing the leading chiral loops.

$^2$ This is consistent with the bases used in Refs. 26 and 27 for the $\Delta S = 2$ effective Hamiltonian beyond the Standard Model. Compared to the basis presented in Ref. 17, we are able to eliminate the operator involving tensor densities $\sigma_{\mu\nu} \otimes \sigma^{\mu\nu}$.
Determination of $\langle \pi^+|O_{2,3,4,5}\pi^-\rangle$ – The argument proceeds as follows. $O_{2,3}$ and $O_{4,5}$ can be written as linear combinations of operators transforming according to the $6_L \times 6_R$ and $8_L \times 8_R$ representations of the chiral group, respectively. These operators in turn admit a unique hadronic realization to leading order in the chiral expansion

$$O_{6,8}^{a,b} = \bar{q}_RT^a q_L \bar{q}_RT^b q_L \rightarrow g_{6,8} \frac{F_0^4}{4} \text{Tr} \left( T^a U T^b U \right)$$

where the trace is over flavor and $U$ is the usual matrix of pseudo-Nambu-Goldstone boson fields transforming as $U \rightarrow U_L U_R^\dagger$ under $SU(3)_L \times SU(3)_R$.

$$U = \exp \left( \frac{\sqrt{2\pi i}}{F_0} \right), \quad \pi = \begin{pmatrix} \frac{\pi_+ + \pi_0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi_+ + \pi_0}{\sqrt{6}} & K^0 \\ K^- & K^0 & -\frac{2}{\sqrt{6}}\pi_8 \end{pmatrix}$$

and $F_0$ is the pseudoscalar decay constant in the chiral limit (in our normalization $F_\pi \simeq 92.4$ MeV). The non-perturbative dynamics is encoded in the low-energy constants $g_{6,8}$, and for each representation there are two independent constants, corresponding to different color contractions (e.g. $O_2$ and $O_3$).

The operators in Eqs. (2) are obtained by setting $T^a = T^b \rightarrow T^1 + i T^2$ in Eq. (3). The same representations, however, contain $\Delta S = 2$ operators that contribute to $K^0,\bar{K}^0$ mixing in extensions of the Standard Model ($T^a = T^b \rightarrow T^0 - i T^7$ in Eq. (3)). The relevant $K^0,\bar{K}^0$ matrix elements have been computed in lattice QCD, thus providing the couplings $g_{6,8}$ and $g_{8,8}$ to leading order in the chiral $SU(3)$ expansion. Note that the $g_{8,8}$ couplings can be independently extracted through their contributions to $K^0 \rightarrow (\pi\pi)_{I=2}$ amplitudes via the $\Delta S = 1$ electroweak penguin operators, that transform as $8_L \times 8_R$.

We first focus on the relation to $K^0,\bar{K}^0$ mixing. A straightforward calculation based on the leading chiral realization of Eq. (3) leads to:

$$\mathcal{M}_{6,6}^{K\bar{K}} \equiv \langle K^0|O_{6,6}^{6+7,6-7}|K^0\rangle \equiv \mathcal{M}_{6,6}^{K\bar{K}} \quad \text{(5a)}$$

$$\mathcal{M}_{8,8}^{K\bar{K}} \equiv \langle K^0|O_{8,8}^{6+7,6-7}|K^0\rangle \equiv \mathcal{M}_{8,8}^{K\bar{K}} \quad \text{(5b)}$$
NLO chiral corrections arising from the one-loop diagrams of Fig. 2 and $O(p^2)$ counterterms could alter the above relation (see for example Refs. [28] and [29] for the analogous discussion of $K^0-ar{K}^0$ mixing and $K^\pm \to \pi^\pm \pi^0$ amplitudes in the Standard Model). For the relations of interest here, we find to NLO at zero momentum transfer (defining $L_{\pi,K,\eta} \equiv \log \mu_\chi^2/m_{\pi,K,\eta}^2$)

$$\mathcal{M}_{8\times 8}^{KK} = g_{8\times 8} F_{\pi}^2 \left\{ 1 + \frac{1}{(4\pi F_0)^2} \left( m_K^2 (-1 + 2L_K) - \frac{m_{\eta}^2}{4} L_{\pi} - \frac{3}{4} m_{\eta}^2 L_{\eta} + \delta_{8\times 8}^{KK} \right) \right\} \quad (6)$$

$$\mathcal{M}_{8\times 8}^{\pi\pi} = g_{8\times 8} F_{\pi}^2 \left\{ 1 + \frac{1}{(4\pi F_0)^2} \left( m_{\pi}^2 (-1 + L_{\pi}) + \delta_{8\times 8}^{\pi\pi} \right) \right\} \quad (7)$$

To identify the finite parts of the loops we have followed the modified $\overline{\text{MS}}$ scheme commonly used chiral perturbation theory [30][31]. Moreover, $\delta_{8\times 8}^{KK}$ and $\delta_{8\times 8}^{\pi\pi}$ denote linear combinations of $O(p^2)$ counterterms, that reabsorb the $\mu_\chi$ dependence of $L_{\pi,K,\eta}$ and contain additional finite corrections. Using the NLO effective Lagrangian [32], we find

$$\delta_{8\times 8}^{KK} = a_{8\times 8} m_K^2 + b_{8\times 8} \left( m_K^2 + \frac{1}{2} m_{\pi}^2 \right) \quad \delta_{8\times 8}^{\pi\pi} = a_{8\times 8} m_{\pi}^2 + b_{8\times 8} \left( m_K^2 + \frac{1}{2} m_{\pi}^2 \right) \quad (8)$$

with $a_{8\times 8}$ and $b_{8\times 8}$ dimensionless constants. Similarly, for the $6_L \times \bar{6}_R$ representation we find

$$\mathcal{M}_{6\times 6}^{KK} = -g_{6\times 6} F_{\pi}^2 \left\{ 1 + \frac{1}{(4\pi F_0)^2} \left( m_K^2 (-1 + 2L_K) + \frac{m_{\eta}^2}{4} L_{\pi} - \frac{7}{12} m_{\eta}^2 L_{\eta} + \delta_{6\times 6}^{KK} \right) \right\} \quad (9)$$

$$\mathcal{M}_{6\times 6}^{\pi\pi} = -g_{6\times 6} F_{\pi}^2 \left\{ 1 + \frac{1}{(4\pi F_0)^2} \left( m_{\pi}^2 (-1 + L_{\pi}) + \frac{2}{3} m_{\eta}^2 L_{\eta} + \delta_{6\times 6}^{\pi\pi} \right) \right\} \quad (10)$$

with counterterm contributions analogous to the ones in Eq. (8). The loop corrections to $\mathcal{M}_{8\times 8,6\times 6}^{KK}$ have been calculated in Ref. [33] and we agree with them. Eqs. (6)-(10) lead to the central result of our work, namely a relation between $\mathcal{M}_{8\times 8}^{\pi\pi}$ and $\mathcal{M}_{8\times 8}^{KK}$ valid to NLO in the chiral expansion

$$\mathcal{M}_{8\times 8}^{\pi\pi} = \mathcal{M}_{8\times 8}^{KK} \times \frac{F_{\pi}^2}{F_K^2} \times (1 + \Delta_{8\times 8}) = \mathcal{M}_{8\times 8}^{KK} \times R_{8\times 8} \quad (11a)$$

$$\mathcal{M}_{6\times 6}^{\pi\pi} = \mathcal{M}_{6\times 6}^{KK} \times \frac{F_{\pi}^2}{F_K^2} \times (1 + \Delta_{6\times 6}) = \mathcal{M}_{6\times 6}^{KK} \times R_{6\times 6} \quad (11b)$$

with

$$\Delta_{8\times 8} = \frac{1}{(4\pi F_0)^2} \left[ \frac{m_{\pi}^2}{4} (-4 + 5L_{\pi}) - m_K^2 (-1 + 2L_K) + \frac{3}{4} m_{\eta}^2 L_{\eta} - a_{8\times 8} \left( m_K^2 - m_{\pi}^2 \right) \right] \quad (12a)$$

$$\Delta_{6\times 6} = \frac{1}{(4\pi F_0)^2} \left[ \frac{-m_{\pi}^2}{4} (4 - 3L_{\pi}) - m_K^2 (-1 + 2L_K) + \frac{5}{4} m_{\eta}^2 L_{\eta} - a_{6\times 6} \left( m_K^2 - m_{\pi}^2 \right) \right] \quad (12b)$$

Eq. (8) implies that the low-energy constants $a_{8\times 8,6\times 6}$ could be extracted from lattice QCD calculations of $K^0-\bar{K}^0$ mixing at different values of both $m_u$ and $m_d$. Moreover, at NLO one can derive counterterm-free relations that connect $\mathcal{M}_{8\times 8}^{\pi\pi}$, $\mathcal{M}_{6\times 6}^{KK}$ and $K \to \pi\pi$ matrix elements ($\mathcal{M}_{8\times 8}^{K\to\pi\pi}$). For the $6_L \times \bar{6}_R$ case, using the NLO Lagrangian of Ref. [32], we obtain

$$(2m_{\pi}^2 - m_K^2) F_{\pi}^2 \mathcal{M}_{8\times 8}^{\pi\pi} = m_K^2 F_K^2 \mathcal{M}_{8\times 8}^{KK}$$
where $\Delta^{\text{loop}}$ is a calculable loop correction. In practice, at the moment neither of these two approaches is feasible, due to missing or not sufficiently precise lattice input. So in our estimates we adopt the following strategy: to obtain central values for $\Delta_{8\times8,6\times6}$, we evaluate the chiral loops at the scale $\mu = m_{\rho}$ and set the counterterms to zero. We then assess the counterterm uncertainty in two ways: first, assuming naive dimensional analysis (NDA), namely $|a_{8\times8,6\times6}| \sim O(1)$, we find $\Delta_{8\times8} = 0.02(20)$ and $\Delta_{6\times6} = 0.07(20)$. Second, requiring that the counterterms be of comparable size to their beta-functions, namely $\Delta_n^{(ct)} = \pm |d\Delta_n^{(\text{loops})}/d(\log \mu)|$ ($n = 8 \times 8$ or $6 \times 6$), we find $\Delta_{8\times8} = 0.02(36)$ and $\Delta_{6\times6} = 0.07(16)$. To account for the strong scale dependence of loops in $\Delta_{8\times8}$, we enlarge the NDA estimate to $\Delta_{8\times8}^{(ct)} = \pm 0.3$ and use in the subsequent analysis $\Delta_{8\times8} = 0.02(30)$ and $\Delta_{6\times6} = 0.07(20)$. The above results point to the fact that the dominant $SU(3)_L \times SU(3)_R$ correction to the relations in Eq. (5) is captured by the ratio $(F_{\pi}/F_K)^2$ in Eq. (11). Putting together the effect of chiral loops and $F_K/F_\pi = 1.19$ [34], the total chiral corrections in Eq. (11) amount to $R_{8\times8} = 0.72(21)$ ($\sim 30\%$ uncertainty) and $R_{6\times6} = 0.76(14)$ ($\sim 20\%$ uncertainty). Given that the chiral expansion is well behaved for these quantities, we expect residual higher order corrections not to exceed $10\%$, well within the assigned ranges.

Using Eqs. (11) and the matrix elements of the $\Delta S = 2$ operators calculated in Refs. [20,24], we find for the two-pion matrix elements renormalized in the $\overline{\text{MS}}$ scheme at the scale $\mu = 3$ GeV

$$
\langle \pi^+|O_2|\pi^-\rangle = \frac{5}{12} B_2 K \times R_{6\times6} \quad K = \frac{2 F_K^2 m_K^4}{(m_d + m_s)^2} \quad (14a)
$$

$$
\langle \pi^+|O_3|\pi^-\rangle = \frac{1}{12} B_3 K \times R_{6\times6} \quad (14b)
$$

$$
\langle \pi^+|O_4|\pi^-\rangle = -\frac{1}{3} B_5 K \times R_{8\times8} \quad (14c)
$$

$$
\langle \pi^+|O_5|\pi^-\rangle = -B_4 K \times R_{8\times8} \quad , \quad (14d)
$$

where the dimensionless scale- and scheme-dependent $B_{2,3,4,5}$ are reviewed in [34]. To obtain the central value estimates for the matrix elements we use $F_K = 110$ MeV, $m_d(3\text{GeV}) = 4.3$ MeV, $m_s(3\text{GeV}) = 87.5$ MeV [35]. For the $B_i$ we take the midpoint of a conservative range encompassing the maximum and minimum values of the $N_f = 2+1$ [22], and $N_f = 2+1+1$ [20] results summarized in Ref. [34]. The uncertainty associated with this treatment of the $B_i$ is at the level of $10\%$ for $B_{2,3}$, $20\%$ for $B_4$, and $30\%$ for $B_5$. For the matrix elements of $O_{2,3}$ the uncertainty due to the quark masses is non negligible, but subdominant compared to the effect of NLO chiral corrections. We summarize our current best estimates for the matrix elements and their uncertainties in Table 1. The fractional uncertainty is at the $20\%$ level for $\langle \pi^+|O_{2,3}|\pi^-\rangle$ (dominated by chiral corrections), at the $35\%$ for $\langle \pi^+|O_4|\pi^-\rangle$ (dominated by chiral corrections), and at the $40\%$ level for $\langle \pi^+|O_5|\pi^-\rangle$ (equally shared by chiral correction and lattice QCD input).

The effective coupling $g_{8\times8}$ can also be extracted from the electroweak penguin matrix elements $\langle (\pi\pi)_{f=2} Q_{7\times8} K^0 \rangle$ [36,37]. This extraction was recently updated in Ref. [38] to LO in the chiral expansion (in [38] the notation $g_{8\times8}^{(i)} \to -A_{iLR}$ was used). Using the value of $g_{8\times8}$ from Ref. [38] in Eq. (7) and neglecting chiral corrections leads to $\langle \pi^+|O_4|\pi^-\rangle = -1.9 \times 10^{-2}$ GeV$^4$.

$^2$Our operators $O_i$ are related to the $Q_i$ of Ref. [54] as follows: $O_{1,2,3} = (1/4)Q_{1,2,3}$, $O_{4,5} = -(1/2)Q_{5,4}$. 

5
Table 1: Pionic matrix elements of the operators in Eqs. (2) in the $\overline{\text{MS}}$ scheme at the scale $\mu = 3$ GeV. The first uncertainty refers to the lattice QCD input on kaon matrix elements. The second uncertainty is associated to the size of partially known NLO chiral corrections (only loops are taken into account) and possible higher order effects. See text for discussion.

and $\langle \pi^+ | O_5 | \pi^- \rangle = -8.5 \times 10^{-2}$ GeV$^4$, in reasonable agreement with the estimate of these matrix elements based on $K^0 \bar{K}^0$ mixing given in Eq. (14) and Table 1. NLO chiral effects in $K^0 \rightarrow (\pi \pi)_{l=2}$ change the extracted low-energy constant as follows, $g_{8 \times 8} \rightarrow g_{8 \times 8}(1 + \Delta_2)$, with $\Delta_2 = -0.30 \pm 0.20$ GeV$^2$, where the central value stems from chiral loop and known counterterms, while the error encompasses an estimate of the unknown counterterms. Taking this into account and keeping the chiral logs in Eq. (7) leads to $\langle \pi^+ | O_4 | \pi^- \rangle = -2.7 \times 10^{-2}$ GeV$^4$ and $\langle \pi^+ | O_5 | \pi^- \rangle = -12.7 \times 10^{-2}$ GeV$^4$, in excellent agreement with the results of Table 1.

**Determination of $\langle \pi^+ | O_1 | \pi^- \rangle$** — For completeness, we also update the analysis of Ref. [19]. First, note that $O_1$ belongs to the $27_L \times 1_R$ representation of $SU(3)_L \times SU(3)_R$, along with $O_{\Delta S=2} = \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{s} \gamma_\mu (1 - \gamma_5) d$ and the component $Q_2^{(27 \times 1)}$ of the $\Delta S = 1$ operator $Q_2 = \bar{s} \gamma_\mu (1 - \gamma_5) u \bar{u} \gamma_\mu (1 - \gamma_5) d = Q_2^{(27 \times 1)} + Q_2^{(8 \times 1)}$. Using the normalization conventions of Ref. [31], the leading order chiral realization of these operators is

$$Q_2^{(27 \times 1)} \rightarrow g_{27 \times 1} F_0^4 \left( L_{\mu 32} L_{11}^\mu + \frac{2}{3} L_{\mu 31} L_{12}^\mu \right) \quad (15a)$$

$$O_{\Delta S=2} \rightarrow \frac{5}{3} g_{27 \times 1} F_0^4 L_{\mu 32} L_{12}^\mu \quad (15b)$$

$$4 O_1 \rightarrow \frac{5}{3} g_{27 \times 1} F_0^4 L_{\mu 12} L_{12}^\mu \quad (15c)$$

with $L_{ij}^\mu = i(U^\dagger \partial^\mu U)_{ij}$. The factor of 4 multiplying $O_1$ in (15) accounts for the different normalization of $O_1$ compared to $Q_2$ and $O_{\Delta S=2}$. In principle, Eqs. (15) allow one to relate $\langle \pi^+ | O_1 | \pi^- \rangle$ to both $K^0 - \bar{K}^0$ mixing (as done for $\langle \pi^+ | O_{2,\ldots,5} | \pi^- \rangle$) and $\langle \pi^+ | \pi^- | Q_2 | K^+ \rangle$. However, it turns out that the determination in terms of $K^0 - \bar{K}^0$ mixing suffers from potentially large chiral corrections. The NLO analysis leads to

$$\langle \pi^+ | O_1 | \pi^- \rangle = \frac{1}{4} m_\pi^2 F_\pi^2 \left( \frac{1}{m_K^2 F_K^2} \langle \bar{K}^0 | O_{\Delta S=2} | K^0 \rangle (1 + \Delta_{27 \times 1}) \right) \quad (16)$$

with a strongly scale-dependent one-loop correction given by $\Delta_{27 \times 1}^{(\text{loops})} = \{ +0.24, -0.11, -0.39 \}$ at $\mu = \{ m_\pi, m_\mu, 1 \text{ GeV} \}$, respectively. This is not unexpected, as large chiral corrections to

Explicitly the projection reads $Q_2^{(27 \times 1)} = 2/5 \left[ \bar{s} \gamma_\mu (1 - \gamma_5) u \bar{u} \gamma_\mu (1 - \gamma_5) d + \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{u} \gamma_\mu (1 - \gamma_5) u \right] - 1/5 \left[ \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{d} \gamma_\mu (1 - \gamma_5) d + \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{d} \gamma_\mu (1 - \gamma_5) s \right].$
\[ \langle \bar{K} | O_{\Delta S=2} | K^0 \rangle \] were already found in Refs. \cite{29} and \cite{33}. Taking for \( B_K \) the midpoint of a range that includes the \( N_f = 2 + 1 \) and \( N_f = 2 + 1 + 1 \) lattice results \cite{34}, namely \( B_K (3 \text{ GeV}) = 0.52(3) \), from \cite{16} at \( \mu = m_\pi \) we obtain \( \langle \pi^+ | O_1 | \pi^- \rangle = 0.97 \times 10^{-4} \text{ GeV}^4 \). The uncertainty from chiral corrections is at least 40-50\%, given the strong scale dependence of the one-loop effects. In light of this, we focus next on the relation of \( \langle \pi^+ | O_1 | \pi^- \rangle \) to \( K^+ \to \pi^+ \pi^0 \) \cite{19}.

At one loop in chiral perturbation theory we find

\[ \langle \pi^+ | O_1 | \pi^- \rangle = \frac{5}{3} g_{27 \times 1} m_\pi^2 F_\pi^2 \left\{ 1 + \frac{m_\pi^2}{(4\pi F_0)^2} (1 - 3L_\pi) + \frac{\delta^{\pi\pi}_{27 \times 1}}{4} \right\}, \tag{17} \]

\[ \langle \pi^+ \pi^0 | iQ_2 | K^+ \rangle = \frac{5}{3} g_{27 \times 1} F_\pi \left( m_K^2 - m_\pi^2 \right) \left\{ 1 + \Delta_{27}^{K^+ \pi^+ \pi^0} \right\}. \tag{18} \]

where \( \delta^{\pi\pi}_{27 \times 1} \) denotes a linear combination of counterterms. \( \Delta_{27}^{K^+ \pi^+ \pi^0} \) receives both loop and counterterms contributions: the loops have been computed in Ref \cite{31} and they are small while displaying a very mild scale dependence. The counterterms have been estimated in the large-\( N_C \) approximation and are negligible \cite{31}. Overall, one finds \( 1 + \Delta_{27}^{K^+ \pi^+ \pi^0} = 0.98 \pm 0.05 \) \cite{31}.

Comparing Eq. (18) to the lattice QCD results for \( \langle \pi^+ \pi^0 | iQ_2 | K^+ \rangle \) \cite{36, 37} (in the \( \overline{\text{MS}} \) scheme at \( \mu = 3 \text{ GeV} \)) we obtain \( g_{27 \times 1} = 0.34(3)_{\text{LQCD}}(2)_{\chi} \), where the first error is from the lattice input and the second from the chiral corrections in Eq. (18). Using this value in Eq. (17), and assigning a conservative 20\% error due to the unknown counterterms in \( \delta^{\pi\pi}_{27 \times 1} \), we obtain the result reported in Table I.

Finally, note that the determination of \( \langle \pi^+ | O_1 | \pi^- \rangle \) from \( K^0 \to K^0 \) mixing, though plagued by larger uncertainty, is quite consistent with the result of Table I.

**Discussion and conclusion** – In this letter we have provided estimates for the \( \pi^\pm \to \pi^\pm \) matrix elements of all Lorentz scalar \( \Delta I = 2 \) four-quark operators relevant to the study of TeV-scale lepton number violation. The analysis is based on (i) chiral \( SU(3) \) symmetry, which relates the \( \pi^- \to \pi^+ \) matrix elements of \( O_{1,..,5} \) defined in Eq. (2) to the \( K^0 \to \bar{K}^0 \) and \( K \to \pi \pi \) matrix elements of their \( \Delta S = 2 \) and \( \Delta S = 1 \) chiral partners; (ii) lattice QCD input for the relevant kaon matrix elements. Our main results are summarized in Eqs. (14) and Table I.

A preliminary lattice QCD calculation of the matrix elements considered in this letter has appeared in Ref. \cite{40}. A complete comparison is not yet possible because Ref. \cite{40} presents results for bare matrix elements. Nonetheless, already at this level, we find the hierarchy of bare matrix elements in \cite{40} to be in qualitative agreement with our results.

For all the symmetry relations used here, we have included the NLO chiral loop corrections, showing that the chiral expansion is well behaved and the relations are robust at the 20-30\% level, depending on the operator under consideration. The remaining uncertainty can be further reduced as the precision on \( K^0 \to \bar{K}^0 \) matrix elements improves. Our results provide a first controlled estimate of the hadronic matrix elements needed to assess the sensitivity of \( 0\nu\beta\beta \) to TeV-scale sources of lepton number violation, and can be used as input in nuclear structure calculations of the leading pion-exchange operators \cite{14}.

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\footnote{This result is in good agreement with \( g_{27} \approx 0.29 \) found by a fit to the \( K \to \pi \pi \) decay rates \cite{31}. The identification \( g_{27} = g_{27 \times 1} \), however, neglects mixing of \( Q_2 \) with other operators and its scale dependence. Our result is also in good agreement with Ref \cite{19}, once we take into account that the coupling \( g^{(27)} \) of Ref. \cite{19} is related to our \( g_{27 \times 1} \) by \( g^{(27)} = (5/12)g_{27 \times 1} \).}

\footnote{There is a slight difference between the operators used here \( (O_i) \) used here and the ones used in Ref. \cite{40} \( (O_i^{(3)}) \). Using parity-invariance of QCD, for the \( \pi^\pm \to \pi^\pm \) matrix elements we have the following relations: \( \langle O_i^{(3)} \rangle = \langle O_i \rangle \), \( \langle O_i^{(3)} \rangle = \langle O_i \rangle \), \( \langle O_i^{(3)} \rangle = \langle O_i \rangle \).}
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