Modifying the optical path in a nonlinear double-slit experiment

Vassilis Paltoglou\textsuperscript{1} and Nikolaos K. Efremidis\textsuperscript{1}\textsuperscript{*}

\textsuperscript{1}Department of Mathematics and Applied Mathematics, University of Crete, 70013 Heraklion, Crete, Greece

In this letter, we study a nonlinear interferometric setup based on diffraction rather than beam combining. It consists of a nonlinear analogue of Young’s double-slit experiment where a nonlinear material is placed exactly after one of the slits. The presence of nonlinearity breaks the transverse spatial symmetry of the system and thus modifies the optical path. For moderate nonlinearities this leads to a self-induced shift of the intensity pattern in the transverse plane. A simple theoretical model is developed which is surprisingly accurate in predicting the intensity profile of the main lobes for a wide range of parameters. We discuss about possible applications of our model in nonlinear interferometry, for example in measuring the nonlinearities of optical materials.

We expect that our results might be useful in nonlinear interferometry, for example, in measuring the nonlinear properties of optical materials \cite{12}.

Let us consider the double-slit configuration shown in Fig. 1. A coherent monochromatic laser light source with intensity \( I_0 \) is normally incident at the aperture plane \((z = 0)\). The two rectangular slits have dimensions \( w_x \times w_y \) and their centers are separated by a distance \( w \) along the \( x \)-direction. A nonlinear slab having length \( L_D \) in the \( z \)-direction is placed in front of the left slit and completely covers it. The nonlinear dependence of the refractive index of the material is assumed to be of the Kerr type and is given by \( n(I) = n_0 + \gamma I \), where \( I \) is the beam intensity, \( n_0 \) is the linear refractive index of the material and \( \gamma \) is the Kerr coefficient. After the beams have propagated through the air and the nonlinear material, the intensity pattern is recorded at the observation plane \( z = z_f \).

In the paraxial approximation the beam dynamics inside the nonlinear material is given by

\[
iv_\psi + \frac{1}{2k} \nabla_{x,y}^2 \psi + \gamma k_0 |\psi|^2 \psi = 0
\]

(1)

where \( \psi \) is the field amplitude, \( \nabla_{x,y}^2 = \partial_x^2 + \partial_y^2 \), \( k = n_0 \omega/c = n_0 k_0 \) is the wavenumber, \( \omega \) is the optical frequency, and \( c \) is the speed of light. When the beam propagates through the air \( k = k_0 \) and the nonlinearity is ignored in Eq. (1). At this point it is important to identify the relevant length scales of the problem. To this end we introduce normalized coordinates: We scale the field to the square root of the input intensity \( \psi = r_0^{1/2} \Psi \). Furthermore, we measure distances in the transverse plane according to the aperture size along the \( x \)-direction \((x_0 = w_x)\), \( X = x/x_0, Y = y/x_0 \) and in the propagation direction according to the diffraction (Rayleigh) length \( Z = z/z_0 \), where \( z_0 = L_D/k x_0^2 \).

Therefore Eq. (1) can be written as

\[
iv_\Psi + \frac{1}{2} \nabla_{X,Y}^2 \Psi + \text{sgn}(\gamma) \frac{L_D}{L_{NL}} |\Psi|^2 \Psi = 0
\]

(2)

where the nonlinear length is defined as \( L_{NL} = 1/(\gamma I_0 k_0) \). The length scales can be written in dimensionless form in terms of the diffraction length as

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\* Corresponding author: nefrem@uoc.gr
The relations between the relevant length scales of the nonlinear material play an important role in the dynamical behavior of the system. For weak beam intensities the nonlinear length is large. However, as the intensity increases the nonlinear length can take values that are much smaller than the diffraction length \( L_{NL} \ll L_D \). We select a slab length and a slit aperture such that in the dynamical limit the effect of diffraction is not dominant in the nonlinear length and the slab length become of the same order of magnitude \( L_{NL} \sim L_S \). Although a beam that passes from a square aperture diffracts quite fast due to the presence of discontinuities, we assume that the semiclassical, or ray optics, limit can be applied. This might look like a crude approximation however, as we are going to see, it captures surprisingly well the fundamental features of the diffracted pattern.

Assuming that the apertures are equally and evenly illuminated leads to the following initial condition

\[
\psi(z = 0) = \sqrt{I_0 \left[ \text{rect} \left( \frac{x + w/2}{w_x} \right) + \text{rect} \left( \frac{x - w/2}{w_x} \right) \right] \text{rect} \frac{y}{w_y}}
\]

In the semiclassical limit, we can find a simplified expression for the field that passes through the slab: The amplitude remains invariant while the phase accumulates the nonlinear contribution

\[
\phi_{NL} = \text{sgn}(\gamma) \frac{L_S}{L_{NL}} = \text{sgn}(\gamma) \frac{l_S}{l_{NL}}.
\]

In addition inside the slab the beam obtains a constant linear phase \( \phi_L = n_0 k_0 L_S \). Since \( \phi_L \) does not affect the dynamics as a function of the intensity through the rest of the paper for simplicity we assume that \( n_0 = 1 \). Note that through the rest of the paper we are going to use \( \phi_{NL} \) not only as an estimate of the accumulated phase but, more importantly, as a measure of the relative scales between the nonlinear length \( L_{NL} \) and the length of the slab \( L_S \). Thus \( \phi_{NL} \) increases by decreasing the normalized nonlinear length (i.e. increasing \( \gamma \) or \( I_0 \)) or by increasing the normalized length of the slab. Because of this duality we prefer to present \( \phi_{NL} \) in the figures instead of \( L_{NL} \) and \( L_S \). Asymptotically, as we described above, we can take into account the presence of the slab by modifying the initial condition to

\[
\psi = \sqrt{I_0} \left[ e^{i\phi_{NL}} \text{rect} \left( \frac{x + w/2}{w_x} \right) + \text{rect} \left( \frac{x - w/2}{w_x} \right) \right] \text{rect} \frac{y}{w_y}
\]

which then effectively propagates through the air for \( z = z_f \). In the Fresnel limit the diffraction dynamics lead to

\[
\psi = \sqrt{I_0} \left[ e^{i\phi_{NL}} I(x, w/2, w_x) + I(x, -w/2, w_x) \right] I(y, 0, w_y)
\]

where

\[
I(x, \xi_c, w_x) = \frac{1}{\sqrt{2\pi}} \left[ F(W_+(x, \xi_c, w_x)) - F(W_-(x, \xi_c, w_x)) \right]
\]

the function \( F(t) \) is the following sum of the Fresnel integrals

\[
F(t) = C(t) + iS(t) = \int_0^t e^{i\pi \xi^2/2} d\xi,
\]

and \( W_\pm(x, \xi_c, w_x) = \sqrt{k/(\pi z)} (x + \xi \pm w_x/2) \). The limiting case of 1D diffraction is obtained from the above equations by taking the limit \( w_y \to \infty \) or \( I(y, 0, w_y) \to 1 \).

In Fig. 2 we see typical one-dimensional numerical results for the diffraction dynamics in the case of a slab of
constant length with self-focusing nonlinearity in comparison with the asymptotic expression of Eq. (4). Our calculations are surprising accurate in the case where the nonlinear phase is \( \phi_{NL} = \pi/4 \) and \( \phi_{NL} = \pi/2 \) taking into account the simplicity of our assumptions as well as the relative large amount of diffraction inside the slab. Not only can we predict the location of the intensity maxima but also the amplitude profile of the main lobes. By further increasing the light intensity to \( \phi_{NL} = \pi \) the numerical results start to slightly deviate from the predicted values. The main reason is that the beam inside the slab, due to its higher intensity profile, starts to experience self-focusing. This behavior can be qualitatively understood in comparison with Fig. 2(a), where the location of the two first intensity maxima as a function of \( \phi_{NL} \) for constant slab length is shown along with the theoretical predictions of Eq. (4). The beam passing from the left slab experiences self-focusing and its optical path length (OPL = \( \int n(s) \, ds \)) increases with the intensity. As a result, the interference between the two slits is shifted almost linearly towards the negative \( x \) direction. For \( \phi_{NL} \lesssim \pi \) we see that the numerical data are slightly shifted in the negative \( x \)-direction as compared to the theoretical curve. This is the outcome of the weak self-focusing that further increases the accumulated phase (and thus the optical path) as compared to the predicted values. As the nonlinearity increases, and for \( \phi_{NL} \approx 0.74\pi \), the absolute maximum of the output intensity pattern is shifted to the adjacent maximum curve located to the right. The theoretical value for this jump is \( \phi_{NL} = \pi \). We attribute this difference to the nonlinear spectral broadening of the beam passing through the slab that leads to increased diffraction, which eventually overcomes the amplitude increase due to self-focusing. Specifically, in Fig. 2(d), where \( \phi = \pi \) we see that the left main lobe has significantly reduced intensity as compared to the theory, whereas the right main lobe compares well to the theoretical curve. By further increasing \( \phi_{NL} \) up to \( 2\pi \) the same behavior is repeated: the intensity maximum is shifted to the next adjacent branch that is located to the right for \( \phi_{NL} \approx 1.66\pi \), whereas the Fresnel theory predicts this value to be \( \phi_{NL} = 2\pi \). An even further increase of the nonlinearity \( \phi_{NL} \) results to strong self-focusing inside the slab. The diffraction dynamics lead to more complicated intensity patterns at the observation plane that can not be predicted in terms of our theory.

The case of a nonlinear self-defocusing slab is shown in
lobes is very good even in the case where \( \varphi_{NL} = \pi/4 \). The agreement between theory and numerics for the two main intensity maxima, occurring due to constructive interference, is overall in even better agreement for larger values of the nonlinearity as compared to the thin slab. Of particular interest it to study the quantum limit of such a nonlinear double slit configuration.

Fig. 3. As one can see, in this regime the theoretical and numerical results are overall in even better agreement for larger values of the nonlinearity as compared to the thin slab. The defocusing nonlinearity reduces the effective index and thus the optical path length inside the nonlinear slab. As a result we expect that the intensity maxima, occurring due to constructive interference, are going to be shifted in the right direction. The agreement between theory and numerics for the two main lobes is very good even in the case where \( \varphi_{NL} = -2\pi \). In all cases, \( L_S/L_D = 1/40 \), each aperture has dimensions \( w_x = (1,5) \), and aperture separation is \( w = 5/3 \).

In Fig. 4 we analyze the effect of the normalized slab length \( L_S = L_S/L_D \) on the location of the intensity maxima for constant values of the total accumulated phase \( \varphi_{NL} = \text{sgn}(\gamma)S/L_{NL} \). Thus by increasing the slab length \( L_S \) we also need to proportionally increase the nonlinear length \( L_{NL} \) to keep a constant value of \( \varphi_{NL} \). According to our model a constant phase accumulation \( \varphi_{NL} \) in slabs of different lengths is going to have the same effect. However, this is not in general true. In the limit where the slab length goes to zero, the effect of diffraction becomes negligible inside the slab and the Kerr effect results to purely self-phase modulation. Our model should be in perfect agreement with this limit. However, as the width of the slab increases, diffraction starts to become more and more important. The Kerr effect in combination with diffraction lead to the self-focusing or to the self-defocusing of the beam.

In Fig. 5 we see that for \( \varphi_{NL} = \pi/4, \pi/2 \) and for slab lengths up to 1/5 of the diffraction length we can accurately predict the location of the two first maxima for both signs of the nonlinearity. Small deviations occur in the case of self-focusing nonlinearity especially in the prediction of the location of the second intensity maximum.

Finally, we have carried out numerical simulations of the proposed system in the case of two transverse directions. Typical results are shown in Fig. 4 for \( \varphi_{NL} = \pi/4 \). As one can see the agreement between the theory and the simulations is excellent.

It might be interesting to present an example in physical units of the nonlinear double-slit configuration presented in this work. Specifically, for aperture size \( x_0 = 100 \mu m \), wavelength \( \lambda = 1 \mu m \), index \( n_0 = 2 \) and slab size \( L_S = L_S/L_D = 1/40 \) we obtain \( L_D = 63 \) mm and \( L_S = 1.57 \) mm. A maximum phase accumulation \( \varphi_{NL} = \pi \) is obtained for index contrast \( \Delta n_{NL} = \gamma L_S = 1.59 \times 10^{-4} \). Let us point out that an analytical expression can be obtained for the total power \( P = 2\varphi_{NL}/(n_0\gamma\kappa_0^2S) \) which depends on the material and light properties as well as on \( L_S \). For \( \gamma = 10^{-13} cm^2/W \) this results to a maximum double-slit laser power \( P = 318 kW \).

In conclusion, we have studied a nonlinear interferometric setup that is based on diffractive interference of waves rather than beam combining. We expect that such a double-slit setup might be useful in nonlinear interferometry, for example in measuring the optical nonlinearities of materials. Of particular interest can be the possibility to explore the dynamics beyond the thin slab approximation where stronger nonlinearities can lead to more complicated dynamics. For example, strong self-focusing nonlinearities can lead to soliton generation. On the other hand, strong self-defocusing field exhibit increased nonlinear diffraction and amplitude decrease inside the slab. Of particular interest it to study the quantum limit of such a nonlinear double slit configuration.
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