Digital Root-mean-square Signal Meter

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\textbf{PAPER INFO}

\textbf{A B S T R A C T}

This paper introduces a meter of the root-mean-square value of deterministic and stochastic signals of an arbitrary shape that are generated over the set time interval. Such a meter involves only the minimum number of simple arithmetic operations to obtain results, and it ensures a high degree of measurement accuracy. For this purpose, the direct calculation of the signal root-mean-square value is applied while the measurement of the half-period average straightened signal value is carried out by means of the traditional measurement devices. Implementation of this meter requires neither the knowledge of what the signal period is, nor the synchronization with the processed sampling. Simulation is then carried out demonstrating the high efficiency of the proposed measurement algorithm. We analyze the characteristics of the meter operating within a wide frequency range of the measurable signals. The recommendations concerning the hardware implementation of such a meter by means of the field programmable gate arrays are considered. The meter can be used when designing digital high-frequency AC voltmeters and ammeters and it can provide the readings that do not depend upon the signal waveform.

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\textbf{1. INTRODUCTION}

Measuring the root-mean-square (RMS) values of alternating signals (currents and voltages) is a common task in various areas of electronic engineering [1-4]. There are various ways to perform such measurement [5, 6] that can be divided into several groups.

In the first group, the measuring principle is that the AC voltage is converted into the constant one using a rectifier, then the result is measured. On this basis, various high-precision measuring chips (RMS-to-DC converters) have been designed, for example, the ones provided by Analog Devices [7] and some other companies. In these devices, the half-period average straightened value of a signal is measured and its scaling into the RMS value is carried out. A drawback of such devices is that their measurement results depend upon the waveform. Thus, for the common non-harmonic signals, it is necessary to recalculate the readings using a pre-calculated coefficient (Crest factor) specified for a particular waveform. If the waveform is unknown or if it changes during the operation, then the results of such measurements are invalid.

The second group of meters includes the devices with the direct reaction to the signal RMS value. First of all, these are thermal meters [5, 6] operable only when the input signal is powerful enough. Their drawback is their low accuracy.

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The third group belongs to the analog RMS signal meters (True RMS-to-DC Converters) [7, 8]. They operate via generating the analog voltage RMS value by multiplying, integrating and extracting the square root from the result. They allow for a relatively simple technical implementation, but their accuracy is not very high, because performance of mathematical operations for signal processing is only approximately correct as it is influenced by the changes in environmental parameters.

A separate group is also formed by the digital meters of signal RMS values that use a digital signal processor (DSP) to process a particular sample of binary sampling codes by carrying out digital multiplication, integration (summation, accumulation) and square root operations [6]. Digital meters make it possible to apply the method of determining the signal RMS value by calculating its amplitude spectrum [7], but it presupposes significant computational costs for spectral analysis.

If the signal period is known or can be measured, a relatively small sample size can be chosen within this interval. Such an approach is implemented, for example, in digital oscilloscopes 2. At the same time, under the unknown signal period, a big sample size is required and that leads to a larger number of summations, which results in increasing the computational costs and decreasing the measurement operating speed. In addition, in common devices [6], the signal sampling frequency should be significantly greater than the signal bandwidth.

To implement the integration operation, a digital first-order low-pass infinite impulse response filter (IIR filter) is applied. However, in this case, the measurement accuracy depends on the measured signal frequency properties. This can only be acceptable in special cases, for example measuring either the voltage or the current in a power network.

The RMS value of a signal can be obtained more accurately by directly processing the recorded samples of the input signal as provided by the standard calculation algorithm [9]. However, in this case, the problem of real-time measurement implementation takes place. In this paper, in order to overcome this difficulty, it is proposed to use the fast processing algorithm in the procedures described in [10, 11]. The mathematical substantiation of the measuring procedure and the block diagram of the meter suitable for its software and hardware implementation are presented. The results of measuring deterministic signals and noise are analyzed by means of simulation. The problems of choosing the signal sampling frequency and the hardware implementation of the meter are also considered. It is shown that application of the proposed approach allows us to design high-precision devices that can measure the signal RMS value that are invariant to the shape of the input signal.

The structure of the paper includes the following parts. In Section II, we present algorithms for calculating the signal RMS value for the cases when signal period is either known or unknown. We analyze the maximum errors in estimation of both the harmonic RMS values and the rectangular pulse sequences with different pulse ratios. It is noted that the brute force computation of the arbitrary signal RMS value requires a large number of addition operations. Section III presents the fast algorithm for measuring the arbitrary signal RMS value together with the block diagram of the corresponding meter. Analysis for the accuracy of the harmonic RMS value measurement is carried out by means of simulation. The artificial network voltage measurement accuracy is also tested using the same method. In Section IV, an example for measuring the band Gaussian process RMS value is provided, and it supports the operability and the efficiency of the introduced algorithm that are demonstrated during processing random signals. In Section V, a technique is described for choosing the sampling frequency for the analog-to-digital converter (ADC) of the measurement. It is shown that the proposed meter ensures high accuracy measurements within a wide range of signal frequencies. Finally, in Section VI, the hardware components required for the measurement implementation are specified.

2. CALCULATING THE ROOT-MEAN-SQUARE SIGNAL

The RMS value of a periodic signal with an arbitrary shape (current or voltage) \( s(t) \) is determined by the expression [9, 12]:

\[
S_{\text{rms}} = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} s^2(t) \, dt,
\]

where \( T_0 \) is the signal period, and \( t_0 \) is any arbitrary reference time of integration upon which the value of the integral (1) does not depend on. In order to calculate the value (1), it is necessary to know the signal period, but that is not always implementable, especially, when the signal frequency changes during measurements.

For an arbitrary integration interval \( T \), firstly, one defines the value

\[
S_{\text{rms}} = \frac{1}{T} \int_{t_0}^{t_0+T} s^2(t) \, dt.
\]

It coincides with (1) when \( T \) is a multiple of \( T_0 \) and, in the general case, depends upon \( T \) and \( t_0 \). The value

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\( \tilde{S}_{\text{RMS}} \) can be considered as an estimation of the signal RMS value and its calculation does not require the knowledge of \( T_0 \).

If the harmonic signal

\[ s(t) = S \cos(\omega t + \varphi) \]

is processed, then, from (2), one gets

\[ \tilde{S}_{\text{RMS}} = S_{\text{RMS}} \sqrt{1 + \frac{1}{\omega^2 T^2} \cos(2\omega t_0 + 2\varphi + \omega T) \sin(\omega T)} , \]

where \( S_{\text{RMS}} = S/\sqrt{2} \) is the exact RMS value of the periodic signal [5]. In (3), the notations are: \( S \) is the amplitude, \( \omega = 2\pi/T_0 \) is the frequency, \( \varphi \) is the initial phase of the signal. As the product of trigonometric functions in (4) is not greater than unity in absolute value, the relative error of estimating (4) from the integral (2) is determined by the inequality

\[ \delta = \frac{\tilde{S}_{\text{RMS}} - S_{\text{RMS}}}{S_{\text{RMS}}} \leq \delta_{\text{max}} = \frac{1}{4\pi K} , \]

where \( K = \text{int}\{T/T_0\} \) is the number of signal periods within the integration interval, \( \text{int}\{\} \) is an integer part.

Figure 1 shows the dependence of \( \delta_{\text{max}} \) (5) upon the normalized integration time \( T/T_0 \). As it can be seen, this error is less than 0.8% under \( K = 10 \).

One of the most common models of inharmonic signals is the rectangular pulse. If, under the period \( T_0 \) and the duration \( \tau \) (pulse ratio \( Q = T_0/\tau \)), the RMS value of positive rectangular pulses with the amplitude \( S \) is measured, then the maximum relative measurement error can be represented as [5], [6]

\[ \delta_{\text{max}} = \sqrt{\frac{Q(K+1)}{KQ+1} - 1} . \]

The values of \( \delta_{\text{max}} \) (6) with \( Q = 2 \) and \( Q = 4 \), are shown in Figure 2a and Figure 2b, respectively. It follows that, in these cases, the measurement error is high, but it becomes acceptable under \( T/T_0 > 10 \ldots 20 \). Thus, the RMS estimation using (2) does not require knowledge of the signal period and provides a sufficiently high accuracy when \( K > 10 \ldots 20 \).

If \( N \) samples are available from signal \( s_i \), then the integral in (2) can be calculated by the method of rectangles as follows [13]:

\[ \tilde{S}_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{k=0}^{N-1} s^2_{i,k}} . \]
The input signal $s(i)$ arrives at the ADC input that generates the sequential samples $s_i$, where $i$ is the number of the current sample, with the sampling frequency $f_S$. The samples $s_i$ are passed to the square-law converter that can be implemented, for example, by means of the digital multiplier (U). In the first summator $SUM_1$, the square of the sample $s_i^2$ produced at the output of the square-law converter is added to the preceding value $s_{i-1}^2$ that has been previously stored in the multi-bit one cell shifter MR_1. Thus, at the output of the summator $SUM_1$ the value $s_i^2 + s_{i-1}^2$ appears. In the summator $SUM_2$, this result is added to the value $s_{i-2}^2 + s_{i-3}^2$ that has been stored in the multi-bit two cell shifter MR_2. After that, at the output of the summator $SUM_2$, the sum $s_i^2 + s_{i-1}^2 + s_{i-2}^2 + s_{i-3}^2$ is formed. Further, similar calculations are carried out and at the output of the last summator $SUM_n$ one gets:

$$G_i = \sum_{k=1}^{n} s_k^2,$$

where $n = \log_2 N$ is the number of summation stages and $N = 2^n$ is the sample size by which the signal RMS value is determined. It should be noted that the summation algorithm presented in Figure 3 requires a minimum number of operations. Therefore, the minimum hardware resources are used for its implementation by means of the field programmable gate arrays, for example.

The values $G_i$ are moved to the non-linear converter (NC) that generates the value $\sqrt{G_i}$ at its output. The easiest way to calculate the square root is to use the storage device (SD), that is a hardware implementation. In this case, at the SD address input, the $G_i$ binary code is received while the $\sqrt{G_i}$ binary code has been pre-recorded in the specified SD memory cell. For example, if the bus-widths of address and data are the same--$D = 16$, then the SD capacity is 1 Mbit, and even when $D = 24$, the SD capacity of only 384 Mbit is required and that is technically feasible. Software implementation, on the other hand, requires that the square root should be calculated using the standard algorithms, i.e., the Heron formula [13] or the power series, for example.

The analysis of the accuracy of the harmonic RMS value measurement using Equation (7) is carried out by means of simulation. In Figure 4a, the dependence is plotted for the normalized RMS value $S_{RMS}/S$ upon the current normalized time $i = t/\Delta$ (where $\Delta = 1/f_S$ is the sampling interval). It is assumed that the signal frequency is $f_0 = 0/2\pi = 10$ kHz (the signal period is $T_0 = 1/f_0 = 100 \text{ µs}$), the sampling frequency is $f_S = 1$ MHz ($\Delta = 1$ µs), the sample size is $N = 4096$, the number of samples within the period is $K_0 = 100$, and the number of periods within the averaging interval is $K \approx 41$. At the initial stage, the shifters are filled during 4.096 ms, and then the current measurements are performed, and they are, as one can see, fairly accurate. The right normalized result is equal to $1/\sqrt{S}$, and it is drawn by dashed line.

Figure 4b shows the precision errors of the measurements (hundredths of a percent). Their fluctuations are caused by sample shifting during the realization of the harmonic signal.

Of particular interest is the measurement of an artificial network with the voltage of 220 V and the frequency of 50 Hz. In Figure 5, the normalized response of the RMS value meter is presented for the case when the sample size is $N = 2^{13} = 8192$ and the sampling frequency is $f_S = 10$ kHz ($\Delta = 10^{-4}$ s).
However, the shape of the artificial network voltage and current may often differ from the sinusoidal one. Figures 6 and 7 show examples of distorted normalized signal and the result of its measurement by the introduced device. In the process, 40 periods are averaged approximately and 100 samples are generated within each period.

If the signal presented in Figure 6 is processed by a device that produces the half-period average voltage at its output while its scale is calibrated by the harmonic RMS values (such a measurement procedure is typical), then, as one can see from Figure 7, the measurement result is $S_{RMS}/S = 0.793$ (dashed line) while the theoretical RMS value is $S_{RMS}/S = 0.894$ (solid line after 0.82 s).

The analysis of the effect of $m$ that is ADC bit-width indicates that the precision error in measuring the signal RMS value decreases significantly as $m$ increases from 3 to 6. However, if ADC bit-width increases further ($m \geq 8$), its precision error decreases only slightly. A significantly greater effect on decreasing the measurement precision error is produced by the increasing the sample size.

4. MEASURING THE NOISE RMS VALUE

The device presented in Figure 3 allows us to measure the RMS value of a random signal (noise).

Figure 8 shows the realization of the samples $s_i$ of the band Gaussian random process with zero mathematical expectation and dispersion (mean power) $S^2$.

Figure 9a draw dependence of the measured normalized value $S_{RMS}/S$ upon the number $i$ of the processed sample, and in Figure 9b one can see the same dependence but for $i > N$. Under $N = 4096$ and ADC bit-width $m = 12$, the RMS relative measurement

![Figure 5](image)

Figure 5. The results of measuring the normalized RMS value of an artificial network voltage

![Figure 6](image)

Figure 6. A distorted artificial network voltage

![Figure 7](image)

Figure 7. The results of measuring the normalized RMS value of a distorted artificial network voltage

![Figure 8](image)

Figure 8. The realization of the centered band Gaussian random process

![Figure 9](image)

(a)

Figure 9. The results of measuring the normalized RMS value of the band Gaussian random process
error is equal to 1.4%. And if $N = 1024$, then it increases up to 2.4%, while if $N = 65536$, then it decreases down to 0.44%.

It can be noted that in order to determine the mean value and the dispersion of a random process [1], the fast-operation meter introduced in [14] can be used.

5. CHOOSING THE SAMPLING FREQUENCY

One should choose the sampling frequency $f_s$ (sampling interval $\Delta$) from the range $f_s = (50 \ldots 200)f_0$ depending on the signal frequency $f_0$. If the signal frequency varies within a wide limit, then it can be chosen so that, at the minimum frequency $f_0$, a sufficiently large number of samples $N$ occupies several periods $T_{0\text{max}} = 1/f_0\text{min}$. This corresponds to the condition $T/T_{0\text{max}} = T/f_{0\text{min}} = (3 \ldots 5)$. As $T = N\Delta = N/f_s$, for the sampling frequency one gets:

$$f_s = N/T = Nf_{0\text{min}}/(3 \ldots 5). \tag{9}$$

For example, if $f_{0\text{min}} = 50$ Hz and $N = 2^{10} = 1024$, then, by applying (9), one can get $f_s = 10$ kHz while with $N$ increasing to $2^{16} = 65536$ one gets $f_s = 655$ kHz.

If, according to the rules of numerical integration [19], it is assumed that at least 20 samples are required to be generated within the signal period, then, for the maximum signal frequency, one would get the relation $f_{0\text{max}} = f_s/20. \tag{10}$

For example, if $f_s = 655$ kHz and $N = 2^{16}$, then one gets $f_{0\text{min}} = 50$ Hz and $f_{0\text{max}} = 32$ kHz. When implementing the meter, estimation of the signal frequency can be automatically generated and, in accordance with it, the desired sampling frequency can be chosen.

It should be noted that the measurement accuracy of the algorithm presented in Figure 3 significantly decreases, if the ratio $f_s/f_0$ lies in the $\pm (0.2 \ldots 1)$ % vicinity of the values $n/2, n = 1, 2, \ldots$. Figure 10 shows the results of simulating the meter operation when measuring the RMS value of the harmonic signals with the amplitude of 5 V and the different frequencies $f_{0\text{max}}$. It is assumed that $N = 4096, f_s = 10$ kHz and the ADC is applied with the spread of $\pm 5$ V and the bit-width $m = 8$, while the sampling frequency differs for different signal frequencies. If the signal frequency is $f_0 = f_s/2 = 5$ kHz and the initial phase $\varphi$ in (3) is equal to $\pi/2$ (the samples at the ADC output correspond to zero signal values), then the measured value of $S_{RMS}$ is equal to zero. From Figures 10b, 10c, it follows that the precision error of measurement dramatically increases in $f_0 = (f_s/2) \pm 10$ Hz vicinity.

In order to eliminate this effect, 1% random fluctuations can be introduced into the sampling frequency, for example.

In Figures 11, one can see the results of simulating the meter operation under the signal frequency $f_0 = f_s/2 = 5$ kHz and when the random Gaussian oscillations of the sampling frequency of $f_s = 10$ kHz are introduced with a standard deviation of 1 Hz.

From Figure 11, it follows that the measurement results become acceptable, and the precision error of measurement can be reduced as $N$ increases.

The measurements of the signal RMS value can be carried out with a high accuracy at the signal frequencies that are much higher than the sampling frequency: $f_0 > f_s$. Thus, one sample takes place during several signal periods. Figure 12 shows the dependences similar to those drawn in Figure 11, if $f_s = 10$ kHz and $f_0 = 53$ kHz (Figure 12a) or

![Image](image-url)
for example, the integrated chips AD9211 or ADC1175 can be used. If the sample sizes $N = 2^{10} \ldots 2^{14}$ are required to be processed, then relatively simple FPGA Cyclone III (produced by Altera) or Spartan-6 families (i.e., XC6SL25 produced by Xilinx, for example) can be applied.

7. CONCLUSION

The digital meter of the harmonic RMS value has been considered. It is shown that it provides the minimum number of arithmetic operations together with the high accuracy of direct measurement of the RMS value of both deterministic (harmonic and non-harmonic) signals and random processes. The precision error of measurement decreases significantly as the processed sample size increases. Based on the introduced meter, the digital high-frequency AC voltmeters and ammeters can be designed providing the readings that do not depend on the waveform and that can be implemented by means of field programmable gate arrays while utilizing minimal FPGA resources.

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**Persian Abstract**

در این مقاله یک مترینگ از مقدار میانگین مربع ریشه‌سیگنال‌های قطعی و تصادفی یک شکل دلخواه که در زمان‌های تعیین شده تولید می‌شوند، معرفی می‌شود. برای دستیابی به نتایجی صحیح، فقط شامل حداقل تعداد عملیات محاسباتی است و در جریان الگوریتم به‌طور مشابه یک تکرار و یک تظیم می‌گردد. برای این منظور، مجوز استفاده از مقدار میانگین مربع ریشه‌سیگنال یک عامل مهم در تولید می‌شود. در حالت که این الگوریتم می‌تواند در این الگوریتم، مقدار میانگین مربع ریشه‌سیگنال صاف شده نیمه دوره به‌طور سنتی اندازه‌گیری می‌شود. اجرای این کوچک با بهره‌مندی در نمایشگر سیگنال و تنظیم‌های دهگان کاری جهت استفاده از گیرنده‌های پردازش ناحیه محیط ایجاد می‌گردد. نتایج شبیه‌سازی نشان داد که کارکرد الگوریتم مطرح شده در این کارکرد با استفاده از آرایه‌های دوکان در بهبود پردازش ناحیه و کاهش اینکارکرد قابل استفاده و همچنین، کارکرد مثبتی در بهبود پردازش ناحیه در استفاده با دیتاسیگنال‌های میانگین، و در نهایت به بهبود قابل حساب در دستگاه‌های اندازه‌گیری نیمه دوره سیگنال خواهد کرد. این الگوریتم ممکن است به‌طور کلی به کار در دستگاه‌های اندازه‌گیری AC و DC بکار برده و روش مثبتی برای کاهش از اینکارکرد قابل استفاده در شرایط مختلف دارد.