A VIM3-compliant measurement model and some related issues

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Abstract. The paper aims at introducing a model of measurement allowing to interpret some of the several significant novelties of the VIM3 in a consistent conceptual framework.

1. Introduction

Consistently with its nature of vocabulary, the *International Vocabulary of Metrology – Basic and General Concepts and Associated Terms* (VIM3) [1] does not assume any specific measurement model as underlying its lexical and conceptual choices. Rather, its Introduction acknowledges that, despite of the “different current philosophies and descriptions of measurement”, that “sometimes lead to difficulties in developing definitions that could be used across the different descriptions”, “no preference is given in this third edition to any of the particular approaches”. This *super partes* position is explicitly maintained, in particular, in reference to “the treatment of measurement uncertainty from an Error Approach (sometimes called Traditional Approach or True Value Approach) to an Uncertainty Approach”.

On the other hand, the several significant novelties of the VIM3 can be surely better highlighted and understood, and the VIM3 itself can become better usable by its many target categories of readers (“scientists and engineers [...] as well as for both teachers and practitioners involved in planning or performing measurements [...], governmental and intergovernmental bodies, trade associations, accreditation bodies, regulators, and professional societies” [Scope]), if the concepts are interpreted in a framework allowing to relate them with each other¹. This holds in particular for the definitions

¹ The VIM3 itself includes, in its introductory section [0.1], some hints towards such a framework, and a presentation of the “VIM3 rationale” can be found in [2]. The reader should be warned of a further reason of complexity of the VIM3, due to the fact that it supports the so-called “substitution principle”, i.e., “it is possible in any definition to replace a term referring to a concept defined elsewhere in the VIM by the definition corresponding to that term, without introducing contradiction or circularity.” While this is surely an elegant strategy, the price to be paid is not trivial, as the *complete* characterization of some concepts requires browsing the notes of the same or other definitions (the substitution principle does not apply to the notes, and this allows introducing such “backward links” in them). See for example the comment below about the role of measuring systems and measurement procedures in measurement.

It should also mentioned that the term adopted in this paper to designate this framework, “measurement model”, is not compliant with the VIM3, that for ‘measurement model’ gives the definition “mathematical relation among all quantities known to be involved in a measurement” [2.48], a very specific meaning as it assumes that “model” stands for “mathematical model”. Accordingly, the contents presented here can be thought of as a *measurement meta-model*.
around the measurement process, that the VIM3 aptly presents according to a functional, i.e., black box, model: as some of the mentioned new concepts, or newly defined ones, relate to the inputs to and the outputs from such black box (e.g., the definition of ‘measurand’ and ‘measured quantity value’ respectively), some analysis on this matter can be useful.

This analysis will be developed in black box terms in its turn: those that are deemed to be the fundamental elements for a VIM3-compliant measurement model will be introduced at first, and only subsequently and where specifically relevant some further details will be added, thus with a top-down refinement process. Despite of some occasional criticisms expressed towards the VIM3, my claim is that the framework presented in this paper is at the same time faithful to the VIM3 itself and Approach-agnostic.

(The following analysis is structured as a list of numbered sentences, that synthetically introduce the model, complemented with some comments; in the numbered sentences underlined terms denote concepts defined in the VIM3. The relevant VIM3 definitions and notes are quoted in footnotes.)

2. Minimal model of measurement

S1. A measuring system experimentally interacts with a system under measurement relatively to a quantity being measured and according to a measurement procedure.

S2. The result of this interaction is an indication.

S3. The indication is associated with a measurement result by means of the information acquired in the calibration of the measuring system.

S4. The measurement result is interpreted as related to a measurand.

2.1. Comments

Despite its simplicity, this basic version of the model already presents some non trivial issues.

- The definition of ‘measurement’ \footnote{Measurement: “process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity”} would not require in principle that measurement is performed by means of a measuring system \footnote{[2.1 Note 3]: “Measurement presupposes […] a measurement procedure and a calibrated measuring system”}, and only a note \footnote{[2.1 Note 3]} makes this requirement explicit. On the other hand, S1 is critical to distinguish measurement from other types of symbolic assignments such as guess and subjective evaluation by experience. Exactly the same issue arises about measurement procedures \footnote{Measurement procedure: “detailed description of a measurement according to one or more measurement principles and to a given measurement method, based on a measurement model and including any calculation to obtain a measurement result”}, and an analogous conclusion holds: any measurement is required to be performed according to a specified measurement procedure, thus highlighting that measurement is a designed-to-target (and not a “spontaneous”) process.

- Neither the entity under measurement nor the quantity being measured are explicitly defined by the VIM3, that uses different terms for them (e.g., “phenomenon, body, or substance” \footnote{[1.1] but also “object” \footnote{[2.15] and “system undergoing measurement” \footnote{[4.13 Note 2] for the former).}

- An indication \footnote{Indication: “quantity value provided by a measuring instrument or a measuring system”} is a quantity value \footnote{[1.19], i.e., a “number and reference together expressing magnitude of a quantity”. Although not explicitly stated, the VIM3 seems to suppose that such number, called “numerical quantity value” \footnote{[1.20]}, is a real number. Indeed, the same quantity value is claimed \footnote{[1.20 Note 2 Example]} to be numerically expressed, e.g., as 5.7, if the reference is in kg, and as 5 700, if the reference is in g. In other terms, it does not seem that for quantity values, and then indications in particular, any issue arises of the number of significant digits and the like. This also highlights that indications are assumed here as unproblematic (as “pure data”, according to philosophy of science), thus justifying why they are possibly dealt with as the evidence in inferential processes}.

\footnote{[2.1] Measurement: “process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity”.

[2.1 Note 3]: “Measurement presupposes […] a measurement procedure and a calibrated measuring system”.

[2.6] Measurement procedure: “detailed description of a measurement according to one or more measurement principles and to a given measurement method, based on a measurement model and including any calculation to obtain a measurement result”.

[4.1] Indication: “quantity value provided by a measuring instrument or a measuring system”.

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such as Bayesian estimation when aimed at computing measurement results as probability distributions.

- The distinction between indications and measurement results [2.9] cannot be neglected, at least because (i) the latter is defined as a “set of quantity values”, instead of a single quantity value, and (ii) “an indication and a corresponding value of the quantity being measured are not necessarily values of quantities of the same kind” [4.1 Note 2], a situation that happens whenever the measuring system operates as a transducer from a quantity (in a general sense) being measured X to a quantity, whose values are indications, Y, X ≠ Y. The VIM3 explicitly aims at removing some confusion on this matter.6

- As measurement is required to produce measurement results, i.e., indications are not enough, and the transformation from indications to measurement results derives from calibration, the very concept of uncalibrated measuring system is contradictory.

- Finally, the set of quantity values that constitutes a measurement result is not necessarily attributed to the quantity being measured, but to a measurand [2.3], defined as the “quantity intended to be measured”. This highlights that the quantity actually measured could be different from the one which the measurer is interested to, and “intentions” could not be enough to guarantee their identity.

As interpreted in this model, measurement has therefore a symmetric structure:

![Symmetric measurement structure diagram]

### 3. Refined model of measurement (focusing on outputs of the experimental interaction)

The previous sentence S3 is so important that is worth of some further specification, as follows.

S3.1. As the indication is a scalar entity and instead the measurement result includes information on measurement uncertainty, such information is obtained by the calibration of the measuring system.

S3.2. Depending on its expected usage and the measurement uncertainty, the measurement result can be expressed in different forms, in particular as a probability distribution, a measured quantity value and a measurement uncertainty, or even possibly a measured quantity value.

### 3.1. Comments

- The fact that calibration [2.39] is defined as a two-step operation (why “operation” instead of “process” is not clear, but a hypothesis will be suggested below) stresses the functional distinction of such steps, but does not affect the requirement that both of them must be performed. The first step is aimed at generating a relation between the quantity values with measurement uncertainties provided by measurement standards and the corresponding indications with associated measurement uncertainties: under the hypothesis that the (general) quantity whose definition is (partially) realized by such measurement standards [5.1] is (the general quantity including as an individual quantity) the

### Footnotes

6 [2.9] Measurement result: “set of quantity values being attributed to a measurand together with any other available relevant information”.

7 [2.9 Note 3] “In the traditional literature and in the previous edition of the VIM, measurement result was defined as a value attributed to a measurand and explained to mean an indication, or an uncorrected result, or a corrected result, according to the context”.

8 [2.39] Calibration: “operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a measurement result from an indication”.

9 [5.1] Measurement standard: “realization of the definition of a given quantity, with stated quantity value and associated measurement uncertainty, used as a reference”.
measurand, this relation is formally the inverse of a calibration diagram. The second step “uses this information to establish a relation for obtaining a measurement result from an indication”, as it can be expressed by a calibration diagram itself. That the first relation is not simply the inverted version of the second one could depend on the fact that the information required to obtain “a measurement result from an indication” generally takes into account also the effects of the influence quantities, in the specific measurement situation, on the measurement result.

- The VIM3 itself acknowledges that “often, the first step alone [as presented in the definition of ‘calibration’] is perceived as being calibration” [2.39 Note 3]. Indeed, several Authors would designate this second step as “measurand reconstruction”, and consider it a component of the measurement process instead of calibration, actually the second step of measurement, the first one being experimental transduction, that produces an indication. This possibly throws some light on the issue of the relation between measurement and calibration: the condition that measurement requires calibrated measuring systems can be interpreted here either in the strong sense that a component of calibration is a part of measurement, or in the weak sense that calibration establishes the general relation between possible indications and “potential” measurement results, while measurement, once an indication has been obtained, simply instances such relation relatively to the given indication and produces an “actual” measurement result accordingly.

- The position of the VIM3 on the relations between the concepts of measurement result and measured quantity value is subtle: as measurement results are sets of quantity values together with relevant information (and therefore a probability distribution can be a measurement result, provided that its support is interpreted as the set and the sequence of probability values as the relevant information), a measured quantity value, with or without a measurement uncertainty, cannot be a measurement result. On the other hand, a measurement result can be represented, or expressed, by a measured quantity value, as in the definition of ‘measured quantity value’ and in the critical [2.9 Note 2]: “A measurement result is generally expressed as a single measured quantity value and a measurement uncertainty. If the measurement uncertainty is considered to be negligible for some purpose, the measurement result may be expressed as a single measured quantity value. In many fields, this is the common way of expressing a measurement result”. Introducing the non-trivial distinction between what a measurement result is and how a measurement result is expressed seems to be required to maintain the consistency of this note to the definition of “measurement result”.

- The option that a measurement result can be expressed without any measurement uncertainty is refused in the interpretation of the Uncertainty Approach given by the VIM3: “even the most refined measurement cannot reduce the interval [of values that can reasonably be attributed to the measurand] to a single value because of the finite amount of detail in the definition of a measurand” [0.1]. This position is understandable under the assumption that the values attributable to the measurand are real numbers, but it is very questionable as far as it is acknowledged that in some cases – paradigmatically: counting (a case explicitly allowed by the VIM3) – such values are indeed chosen in a discrete set.

- Finally, a further note, [2.10 Note 1], contributes to make things even more elusive: “For a measurement involving replicate indications, each indication can be used to provide a corresponding measured quantity value. This set of individual measured quantity values can be used to calculate a resulting measured quantity value, such as an average or median, usually with a decreased associated measurement uncertainty”. The compatibility of this note with the definition of ‘measured quantity value’ is not clear, as it is not clear which measurement results are represented by which measured quantity values. In particular, the measured quantity values “corresponding to each indication” seem to
be the same ones involved in a calibration curve [4.31]\(^{13}\), that, properly, “does not supply a measurement result as it bears no information about the measurement uncertainty” [4.31 Note]. Hence such measured quantity values could “represent” a measurement result but “do not supply” it at the same time. A more consistent position would simply drop the qualifier “measured” whenever not referred to an entity obtained after both the calibration steps, thus, e.g., rephrasing the definition of ‘calibration curve’ as “expression of the relation between indication and corresponding quantity value for the quantity being measured”.

As interpreted in this model, measurement outputs have therefore the following functional structure:

\[
\text{indication} \xrightarrow{\text{calibration}} \text{(calibrated) quantity value} \xrightarrow{\text{calibration}} \text{measurement result} \xrightarrow{\text{representation}} \text{measured quantity value}
\]

that can be optionally detailed as:

\[
\text{indication} \xrightarrow{\text{(step 1)}} \text{(calibrated) \text{quantity value}} \xrightarrow{\text{(step 2)}} \text{measurement result} \xrightarrow{\text{representation}} \text{measured quantity value}
\]

4. Conclusions

One of the most important contributions of the VIM3 is its acknowledgment that models play an important role in measurement. Indeed, “measurement presupposes […] a measurement procedure” [2.1 Note 3], i.e., a “detailed description […] based on a measurement model”, i.e., a “mathematical relation among all quantities known to be involved in a measurement” [2.48], that, in particular, in the simple case of explicit functional form for a single measurand can be written as \( Y = f(X_1, \ldots, X_n) \), where \( Y \) is the measurand and \( X_1, \ldots, X_n \) are the input quantities in the measurement model [2.49 Note 1]. On the other hand, this meaning for “measurement model” seems to be quite narrow, as a measurement explicitly or implicitly involves more than one model. While still focusing on the role of the measurement function \( Y = f(X_1, \ldots, X_n) \), the following diagram suggests a structure for the models involved in a measurement and their relations, with a pragmatic emphasis on the uncertainty contributions coming from each model (some consequences of this structure have been explored in [3] from the mathematical point of view, and in [4] by means of a case study).

This diagram is not fully VIM3-compliant, as it supposes that:

\(^{13}\)[4.31] Calibration curve: “expression of the relation between indication and corresponding measured quantity value”.

\[\text{definition}\]

\[\text{acquisition}\]

\[\text{processing}\]

\[\text{expression}\]

\[\text{input quantities } X_1, \ldots, X_n\]

\[\text{definitional uncertainty of } X_1, \ldots, X_n\]

\[\text{input quantities } X_1, \ldots, X_n\]

\[\text{measurement uncertainty for } X_1, \ldots, X_n\]

\[\text{instrumental measurement uncertainty}\]

\[\text{processing uncertainty}\]

\[\text{processing uncertainty of } Y\]

\[\text{measurand}\]

\[\text{expressed}\]

\[\text{model}\]

\[\text{measuring system model}\]

\[\text{measurement model}\]

\[\text{input quantities model}\]

\[\text{propagated uncertainty of } Y\]
- the experimental (sub)process of acquisition of the input quantities \( X_i \), if actually performed for one or more of them, is a critical component of measurement, and therefore its specification should be included in the measurement model; this holds under the (non radically operational) assumption that the same quantity can be evaluated by means of different processes, so that such specification is not part of the quantity model;
- each quantity involved in the measurement, both the input quantities \( X_i \) and the measurand \( Y \), has an associated model; the VIM3 acknowledges the importance of these models, but it is unfortunately not so clear as to their required features (e.g.: “Measurement presupposes a description of the quantity commensurate with the intended use of a measurement result” [2.1 Note 3]; “The specification of a measurand requires knowledge of the kind of quantity, description of the state of the phenomenon, body, or substance carrying the quantity, including any relevant component, and the chemical entities involved” [2.3 Note 1]). The most critical example of the relevance of quantity models comes from the very definition of ‘true value’, “quantity value consistent with the definition of a quantity” [2.11].

Given the importance of these concepts, it is plausible that some future developments of measurement science will be devoted to better understand the role of models in measurement.

References
[1] JGCM 200:2008, International Vocabulary of Metrology – Basic and General Concepts and Associated Terms (VIM), Sèvres, France, 2008; available online: http://www.bipm.org/en/publications/guides/vim.html
[2] Ehrlich C, Dybkaer R, Wöger W 2007 J. Accreditation and Quality Assurance 12, 3-4 201-218
[3] Macii D, Mari L, Petri D 2010 IEEE Trans. Instr. Meas. 59, 1 238-246
[4] Mari L, Ugazio E 2010, in these proceedings