RESEARCH ARTICLE

Linguistic neutrosophic power Muirhead mean operators for safety evaluation of mines

Suizhi Luo¹, Weizhang Liang²*, Guoyan Zhao²

¹ College of Systems Engineering, National University of Defense Technology, Changsha, Hunan, China, ² School of Resources and Safety Engineering, Central South University, Changsha, Hunan, China

* wzlian@csu.edu.cn

Abstract

Safety is the fundamental guarantee for the sustainable development of mining enterprises. As the safety evaluation of mines is a complex system engineering project, consistent and inconsistent, even hesitant evaluation information may be contained simultaneously. Linguistic neutrosophic numbers (LNNs), as the extensions of linguistic terms, are effective means to entirely and qualitatively convey such evaluation information with three independent linguistic membership functions. The aim of our work is to investigate several mean operators so that the safety evaluation issues of mines are addressed under linguistic neutrosophic environment. During the safety evaluation process of mines, many influence factors should be considered, and some of them may interact with each other. To this end, the Muirhead mean (MM) operators are adopted as they are powerful tools to deal with such situation. On the other hand, to diminish the impacts of irrational data provided by evaluators, the power average (PA) operators are under consideration. Thus, with the combination of MM and PA, the power MM operators and weighted power MM operators are proposed to aggregate linguistic neutrosophic information. Meanwhile, some key points and special cases are studied. The advantages of these operators are that not only the interrelations among any number of inputs can be reflected, but also the effects of unreasonable information can be reduced. Thereafter, a new linguistic neutrosophic ranking technique based on these operators is developed to evaluate the mine safety. Moreover, in-depth discussions are made to show the robust and flexible abilities of our method. Results manifest that the proposed method is successful in dealing with mine safety evaluation issues within linguistic neutrosophic circumstances.

Introduction

Mineral resources are important raw materials for other downstream industries, which play a fundamental role in economic development [1,2]. Safety production is a prerequisite for the exploitation of mineral resources, and is an important guarantee for the sustainable development of mining enterprises. Nevertheless, mining is full of high risks due to the industry
particularity. Although safety issues are getting more and more attentions, many dangers still exist in the mining process. Every year, plenty of casualties are caused by mining accidents around the world, especially in developing countries [3,4]. Only take coal mines in China as an example, the number of deaths has reached at least 275 in 2018 [5]. To guarantee the safety of miners and take effective control measures in advance, it is significant to adopt effective approach to conduct mine safety evaluation.

Mine safety evaluation can be regarded as an decision making process with the purpose of picking out the safest mine or ranking mines based on their safety performance. Due to the complexity of mine system, the mine safety is affected by many criteria, such as the individual, environmental and managerial factors. Accordingly, many multi-criteria decision making (MCDM) methods have been used to solve mine safety evaluation problems, such as the analytic hierarchy process (AHP) [3], ordered weighted aggregation operator [6], and Frank Heronian mean (HM) operator [7]. In addition, as numerous uncertainties are contained in the working environment of mines, decision makers (DMs) often have a vague understanding when conducting safety assessment. In this case, they are accustomed to using linguistic phrases (i.e., “very good”, “good” and “bad”) instead of numerical values [8].

Zadeh [9] firstly put forward the notion of linguistic variables to represent linguistic evaluation information. Thereafter, plenty of decision making methods based on linguistic variables have been developed [10,11,12]. On the other hand, considering the vagueness of human cognitions, linguistic terms have been combined with various fuzzy sets to express more uncertain information [13,14,15]. For example, Chen et al. [16] defined the linguistic intuitionistic fuzzy numbers (LIFNs) based on linguistic terms and intuitionistic fuzzy numbers (IFNs). In IFNs, two crisp numbers are respectively adopted to describe the membership and non-membership functions [17,18]. In LIFNs, both the membership and non-membership degrees are linguistic values, rather than crisp numbers. As a result, the incomplete evaluation information can be qualitatively described. However, for both IFNs and LIFNs, there is still an obvious limitation, that is: (1) In a group decision making process, inconsistent results are likely to be produced among several DMs. Another situation is that people may be hesitant about their evaluations when facing with complex objectives. Nevertheless, IFNs and LIFNs cannot address such situations because they don’t contain indeterminate or inconsistent linguistic data.

To conquer the limitation (1) of IFNs, Smarandache [19] first proposed the notion of neutrosophic sets (NSs). When there is only an element in NSs, it is reduced as a single-valued neutrosophic number (SVNN) [20]. Three membership functions (namely, the truth, hesitance and falsity membership degrees) are within a SVNN [21,22]. As a result, all the consistent, hesitant and inconsistent information of DMs can be contained in a SVNN. From then on, various decision making models related to NSs have been presented [23,24,25,26]. Besides, considering the advantages of NSs, they have been extended with some other fuzzy numbers to deal with complex real problems [27,28,29]. For example, Ji et al. [30] proposed a combined neutrosophic linguistic approach to pick out ideal providers; Liu et al. [31] put forward the Dombi power HM operators under 2-tuple linguistic neutrosophic environment; Abdel-Basset et al. [32] used type-2 neutrosophic number to describe linguistic phrases in the decision process; Wang et al. [33] extended the Muirhead mean operators with neutrosophic 2-tuple linguistic information; Dat et al. [34] discussed the interval complex neutrosophic sets within linguistic decision circumstances.

Borrowed the idea of single-valued neutrosophic numbers (SVNNs), the concept of linguistic neutrosophic numbers (LNNs) [35] was raised to overcome this drawback (1) of LIFNs. They extended linguistic terms with SVNNs. This combination can make full use of the advantages of linguistic variables and neutrosophic sets. Three autonomous linguistic membership degrees exist in LNNs so as to comprehensively describe qualitative evaluation information
Consequently, many researchers show great interest in solving decision making problems under linguistic neutrosophic environments [37,38,39]. For example, Shi and Ye [40] defined the cosine measure of LNNs to settle MCDM issues; Liang et al. [41] evaluated the investment risk of metallic mines by using an improved technique for order performance by similarity to ideal solution (TOPSIS) approach within linguistic neutrosophic circumstances; Pamučar et al. [42] selected the best power-generation technique with an extended combinative distance-based assessment (CODAS) model based on LNNs; Liang et al. [43] chose a satisfactory mining method with a linguistic neutrosophic multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA) method. Moreover, some extensions of LNNs have been studied in existent literature [44,45]. Recently, Liu and You [46] defined a novel distance measure and the bidirectional projection measure of LNNs; Li et al. [47] extended the evaluation based on distance from average solution (EDAS) technique to LNNs for selecting the optimal property management company; Wang et al. [48] combined linguistic neutrosophic information with the visekriterijumska optimizacija i kom-promisno resenje (VIKOR) method to pick out a suitable fault handling point.

Besides, information aggregation operators are another basic and powerful decision making techniques [49,50]. Consequently, many aggregation operators related to LNNs are also presented. For instance, Fang and Ye [35] described the basic mean operators of LNNs; Garg and Nancy [51] considered the priority relations of linguistic neutrosophic information. However, there are two noticeable shortcomings of these existing operators: (2) There is a hypothesis: All criteria have no relevance with each other. Clearly, it is uncommon in real life; (3) the correlations among inputs are not taken into considerations at all.

To conquer the weaknesses of (2) and (3), some researchers have explored new linguistic neutrosophic aggregation operators. For example, considering that Bonferroni mean operators [52,53] contain the relationships of inputs, Fan et al. [54] proposed the linguistic neutrosophic weighted Bonferroni mean (LNWBM) and linguistic neutrosophic weighted geometric Bonferroni mean (LNWGBM) operators; Wang and Liu [55] generalized the partitioned Bonferroni mean operators within linguistic neutrosophic conditions; Liang et al. [43] proposed several HM operators to do with LNNs. They abandoned the inputs independence assumption and took the roles of arguments into account. Yet, there is still a deficiency in these operators: (4) Even though they regard the inputs are dependent, the interrelationships can only be reflected between two arguments. That is, these operators are useless in the situation where there are more than two inputs are interrelated.

Subsequently, for overcoming this flaw (4), scholars tried to investigate other useful aggregation operators to capture more correlations. For instance, Liu and You [56] combined the weighted Hamy mean operators with linguistic neutrosophic information; Liang et al. [57] extended the Hamacher aggregation operators with LNNs to obtain aggregated results. Except them, another renowned aggregation operators are the family of Muirhead mean (MM) operators [58]. They are powerful and flexible in dealing with correlations among any number of inputs. The largest highlight of these operators is that they can perform diverse functions by allocating different values to the parameter vector according to different conditions. In this sense, some of the mentioned-above operators, such as the basic mean operators and Bonferroni mean operators, can be regarded as the special cases of MM operators. Because the classical MM operators can only treat with crisp numbers, they have been modified with various fuzzy sets to resolve fuzzy decision making methods [59,60,61]. Yet, the imperfections are: (5) As far as we know, the MM operators have not been integrated with LNNs until now; (6) the bad effects of unreasonable inputs on the final aggregated values are ignored.

To surmount the disadvantage of (6), the idea of power average (PA) operators was put forward by Yager [62]. The PA operators have a great performance in eliminating the influence of
awkward information provided by DMs [63]. Different with prioritized operators, PA operators allow inputs to support each other in the process of aggregation by defining support degrees (instead of prioritization relationships) [64]. Since then, the PA operators have been either improved with fuzzy extensions to dispose decision making issues under dissimilar settings, or combined with other operators to achieve more goals. For instance, Liu et al. [65] integrated the PA operators with HM operators to aggregate linguistic neutrosophic information. Particularly, the PA operators have been combined with MM operators under many fuzzy environments [66,67]. However, a defect is that: (7) Their integration has not been studied in the linguistic neutrosophic situation. To overcome the limitations of (5) and (7), this study takes this idea for reference and aims to recommend several power Muirhead mean (PMM) operators for LNNs to better resolve complex decision making problems.

In summary, the main motivations of this study are three-fold. First, in the safety evaluation process of mines, consistent, hesitant and inconsistent information may be included in a decision making group at the same time. LNNs are suitable for describing such information with three independent linguistic membership degrees. Second, due to the complexity of objectives or the limitation of DMs’ knowledge, unreasonable data may be provided by DMs. In this case, the PA operators can be adopted to reduce the impacts of these irrational values. Third, as some mine safety evaluation criteria has interactions with each other, proper techniques should be used to capture these relationships. Thus, the objective of our work is to assess the safety of mines through integrating PA with MM operators under linguistic neutrosophic environment.

The key novelties and contributions are:

First, the fuzzy assessment information of the mine safety is expressed with LNNs, so that the disadvantage of (1) is overcome and the preferences or opinions of DMs can be fully conveyed with three independent linguistic membership degrees.

Second, the linguistic neutrosophic power Muirhead mean (LNPMM) and weighted linguistic neutrosophic power Muirhead mean (WLNPMM) operators are suggested to aggregate evaluation information under linguistic neutrosophic environment. Besides, some important properties are certified and special cases are discussed. As a result, the limitations of (2)-(7) can be all surmounted.

Third, a new framework on the basis of these operators is established to solve multi-criteria evaluation problems within linguistic neutrosophic circumstances. An example of assessing safety status of gold mines is provided to explain the utilization of the new method. In addition, its flexibility and superiority are certified after thorough discussions.

The rest of this study is: Section 2 briefly introduces related knowledge of LNNs and PMM operators. In Section 3, the LNPMM and WLNPMM operators, are recommended to aggregate LNNs. After that, a novel approach with these two operators is proposed in Section 4. Next, an illustration instance of safety evaluation of mines is provided to display the application of the presented method in Section 5. At the same time, sensitivity analyses and comparison analyses are conducted in Section 6 to show the features and highlights of this method. Some main conclusions are provided in the end.

**Basic knowledge**

Some preliminaries are provided in this section to advance the following studies.

**Linguistic neutrosophic numbers**

**Definition 1.** [68] Let \( \tilde{a}_i (i = 0, 1, \ldots, 2b) \) be a linguistic phrase, then a collection of \( \tilde{a}_i \) is regarded as a disconnected linguistic term set \( \tilde{A} = \{ \tilde{a}_i | i = 0, 1, \ldots, 2b \} \). If \( A = \{ a_i | i \in [0, 2c] \} \), it is a continuous linguistic term set.
For two arbitrary linguistic phrases \( a_i (i \in [0, 2]) \) and \( a_j (j \in [0, 2]) \) in \( A \), the basic operations contain: \( a_i \oplus a_j = a_{i+j} \) and \( \partial a_i = a_{i_0} (\partial > 0) \).

Besides, the preference relations between two linguistic phrases are: (1) \( a_i > a_j \) if \( i > j \); (2) \( a_i = a_j \) if \( i = j \); (3) \( a_i < a_j \) if \( i < j \).

**Definition 2.** [35] Suppose \( A = [a_i]_{i \in [0, 2]} \) is a linguistic term set, then three linguistic membership functions (namely, the linguistic true membership degree \( a_{T \in A} \), the linguistic indeterminate membership degree \( a_{I \in A} \), and the linguistic false membership degree \( a_{F \in A} \)) are composed of a linguistic neutrosophic number (LNN), denoted as \( a = (a_{T}, a_{I}, a_{F}) \).

**Definition 3.** [35] Given two LNNs \( a_1 = (a_{T_1}, a_{I_1}, a_{F_1}) \) and \( a_2 = (a_{T_2}, a_{I_2}, a_{F_2}) \), a linguistic term set \( A = [a_i]_{i \in [0, 2]} \), and \( \partial > 0 \), their operational rules are

1. \( a_1 \oplus a_2 = (a_{T_1}, a_{I_1}, a_{F_1}) \oplus (a_{T_2}, a_{I_2}, a_{F_2}) = (a_{T_1 + T_2}, a_{I_1 + I_2}, a_{F_1 + F_2}) \)

2. \( a_1 \otimes a_2 = (a_{T_1}, a_{I_1}, a_{F_1}) \odot (a_{T_2}, a_{I_2}, a_{F_2}) = (a_{T_1 \cdot T_2}, a_{I_1 \cdot I_2}, a_{F_1 \cdot F_2}) \)

3. \( \partial a_i = \partial(a_{T_i}, a_{I_i}, a_{F_i}) = (a_{2^{-1} - 2(1 - \partial)}, a_{2^{-1} - (1 - \partial)}, a_{2^{-1} - (1 - \partial)}) \)

4. \( a_i^0 = (a_{T_i}, a_{I_i}, a_{F_i})^0 = (a_{2^{-1} - (1 - \partial)}, a_{2^{-1} - (1 - \partial)}, a_{2^{-1} - (1 - \partial)}) \)

**Definition 4.** [43] Let \( a = (a_{T}, a_{I}, a_{F}) \) be an LNN, \( T, I \) and \( F \) are respectively the subscripts of \( a_{T \in A}, a_{I \in A} \) and \( a_{F \in A} \), then its score function \( B(a) \) and accuracy function \( C(a) \) are

\[
B(a) = \frac{4c + T - I - F}{6c},
\]

\[
C(a) = \frac{T - F}{2c}.
\]

**Definition 5.** [43] For two arbitrary LNNs \( a_1 = (a_{T_1}, a_{I_1}, a_{F_1}) \) and \( a_2 = (a_{T_2}, a_{I_2}, a_{F_2}) \), their preference relations are

1. if \( B(a_1) > B(a_2) \), then \( a_1 > a_2 \);
2. if \( B(a_1) = B(a_2) \) and \( C(a_1) > C(a_2) \), then \( a_1 > a_2 \);
3. if \( B(a_1) = B(a_2) \) and \( C(a_1) = C(a_2) \), then \( a_1 = a_2 \).

**Definition 6.** [41] If \( a_1 = (a_{T_1}, a_{I_1}, a_{F_1}) \) and \( a_2 = (a_{T_2}, a_{I_2}, a_{F_2}) \) are two LNNs, then their distance can be defined as

\[
L(a_1, a_2) = \left( \frac{1}{3} \cdot \frac{1}{2} \left( |T_1 - T_2| + |I_1 - I_2| + |F_1 - F_2| \right)^\lambda \right)^\frac{1}{\lambda} (\lambda > 0).
\]

When \( \lambda = 1 \), the equation is reduced to the Hamming distance \( L_H(a_1, a_2) = \frac{1}{3} \cdot \frac{1}{2} \left( |T_1 - T_2| + |I_1 - I_2| + |F_1 - F_2| \right) \); when \( \lambda = 2 \), the equation is reduced to the Euclidean distance \( L_E(a_1, a_2) = \left( \frac{1}{3} \cdot \frac{1}{2} \left( |T_1 - T_2|^2 + |I_1 - I_2|^2 + |F_1 - F_2|^2 \right) \right)^\frac{1}{2} \).

**Power Muirhead mean operators**

**Definition 7.** [58] Assume \( \beta_i (i = 1, 2, \ldots, n) \) is a group of real numbers, \( D = (d_1, d_2, \ldots, d_n) \in R^n \) is a vector of parameters, \( \sigma(j) (j = 1, 2, \ldots, n) \) is an arrangement of \( i (i = 1, 2, \ldots, n) \), and \( E_n \) is a set of
all possible arrangements, then the MM operators are

$$\text{MM}^D(\beta_1, \beta_2, \ldots, \beta_n) = \left( \frac{1}{n!} \sum_{\sigma \in \mathcal{E}_n} \prod_{j=1}^{n} \beta_{\sigma(j)}^{d_j} \right) \sum_{i=1}^{n} d_i.$$  (4)

**Definition 8.** [62] Assume $\beta_i (i = 1, 2, \ldots, n)$ is a set of crisp numbers, $F(\beta_i) = \sum_{j \neq i}^n G(\beta_i, \beta_j)$, and $0 \leq G(\beta_i, \beta_j) \leq 1$ is the support of $\beta_i$ to $\beta_j$, then the PA operators are defined as

$$PA(\beta_1, \beta_2, \ldots, \beta_n) = \sum_{i=1}^{n} \left( \frac{1 + F(\beta_i)}{\sum_{j=1}^{n} (1 + F(\beta_j))} \right)^{d_i}.$$  (5)

Note that: the support $G(\beta_i, \beta_j) = G(\beta_j, \beta_i)$; and if $H(\beta_i, \beta_j) < H(\beta_j, \beta_i)$, then $G(\beta_i, \beta_j) < G(\beta_j, \beta_i)$, where $H(\beta_i, \beta_j)$ is the distance between $\beta_i$ and $\beta_j$.

**Definition 9.** [66] If $\beta_i (i = 1, 2, \ldots, n)$ is a set of crisp numbers, $\sigma(i) (i = 1, 2, \ldots, n)$ is any permutation of $i = 1, 2, \ldots, n$, $D = (d_1, d_2, \ldots, d_n) \in \mathbb{R}^n$ is a vector of parameters, $E_n$ is a set of all possible permutations, $F(\beta_i) = \sum_{j \neq i}^n G(\beta_i, \beta_j)$ and $G(\beta_i, \beta_j) \in [0, 1]$ is the support of $\beta_i$ to $\beta_j$, then the PMM operators are defined as

$$\text{PMM}^D(\beta_1, \beta_2, \ldots, \beta_n) = \left( \frac{1}{n!} \sum_{\sigma \in \mathcal{E}_n} \prod_{j=1}^{n} \left( \frac{n(1 + F(\beta_{\sigma(i)}))}{\sum_{j=1}^{n} (1 + F(\beta_j))} \right)^{d_j} \right) \sum_{i=1}^{n} d_i.$$  (6)

### Some linguistic neutrosophic power Muirhead mean operators

In this section, the PMM operators are extended under linguistic neutrosophic environment. As a result, the LNPM and WLNPM operators are put forward to aggregate linguistic neutrosophic information. The largest advantage of these operators is: They could capture the relationships among any number of inputs, at the same time, the influence of unreasonable information can be diminished.

**Linguistic neutrosophic power Muirhead mean operator**

**Definition 10.** If $\alpha_i (i = 1, 2, \ldots, m)$ is a group of LNNs, $K = (k_1, k_2, \ldots, k_m) \in \mathbb{R}^m$ is a vector of parameters, $\sigma(i)$ is any permutation of $i = 1, 2, \ldots, m$, $E_m$ is a set of all possible permutations, $F(\alpha_i) = \sum_{j \neq i}^m G(\alpha_i, \alpha_j)$ and $G(\alpha_i, \alpha_j) = 1 - L(\alpha_i, \alpha_j) \in [0, 1]$ is the support for $\alpha_i$ and $\alpha_j$, $0 \leq w_i = \sum_{j \neq i}^m (1 + F(\alpha_j)) \leq 1$, and $w_1 + w_2 + \cdots + w_m = 1$, then the LNPM operators is

$$\text{LNPM}^D(\alpha_1, \alpha_2, \ldots, \alpha_m) = \left( \frac{1}{m!} \sum_{\sigma \in \mathcal{E}_m} \prod_{j=1}^{m} (m w_{\alpha_{\sigma(j)}})^{k_j} \right) \sum_{i=1}^{m} k_j.$$  (7)

**Theorem 1.** Suppose $\alpha_i = (\alpha_{i_1}, \alpha_{i_2}, \alpha_{i_3}) (i = 1, 2, \ldots, m)$ is a group of LNNs, then the result based on Eq (7) is still an LNN, and

$$\text{LNPM}^D(\alpha_1, \alpha_2, \ldots, \alpha_m) = \left( \prod_{k=1}^{m} \left( 1 - \prod_{l=1}^{m} \left( 1 - \left( \frac{1}{2} \alpha_{i_l} \alpha_{i_l} \right) \right) \right) \right)^{\frac{1}{m!}} \sum_{k=1}^{m} k_j \sum_{i=1}^{m} k_j \sum_{i=1}^{m} k_j \sum_{i=1}^{m} k_j \sum_{i=1}^{m} k_j \sum_{i=1}^{m} k_j.$$  (8)

**Proof.**

Based on Definition 4, $mw_{\alpha_{\sigma(j)}} = (a_{\sigma(j)_1}, a_{\sigma(j)_2}, a_{\sigma(j)_3})$ and
The expression is:

\[
\prod_{i=1}^{m} (mw_{\mathbf{x}(i)})^k = \left( a_{2e-1} \left( 1 - T_{\mathbf{x}(i)}(\frac{mw_z}{2}) \right) \right) \left( a_{2e-2} \left( \frac{T_{\mathbf{x}(i)}(\frac{mw_z}{2})}{\frac{mw_z}{2}} \right) \right) \left( a_{2e-3} \left( \frac{T_{\mathbf{x}(i)}(\frac{mw_z}{2})}{\frac{mw_z}{2}} \right) \right) \ldots \left( a_1 \right)
\]

where 
\[
w_m = \frac{\left( \prod_{i=1}^{m} \left( 1 - \left( 1 - \frac{T_{\mathbf{x}(i)}(\frac{mw_z}{2})}{\frac{mw_z}{2}} \right) \right)^{\frac{1}{2}} \right)}{\sum_{i=1}^{m} \frac{1}{2}}
\]

The aggregated value is 
\[
w_m = \frac{\left( \prod_{i=1}^{m} \left( 1 - \left( 1 - \frac{T_{\mathbf{x}(i)}(\frac{mw_z}{2})}{\frac{mw_z}{2}} \right) \right) \right)}{\sum_{i=1}^{m} \frac{1}{2}}
\]

Example 1. Assume \( \alpha_1 = (a_{0,1,3,1}), \alpha_2 = (a_{4,2,5,2}), \alpha_3 = (a_{5,4,1,2}) \) and \( \alpha_4 = (a_{3,1,2,2}) \) are four LNNs, and \( K = \{1,1,1,1\} \), on the basis of Eq (8), their aggregated value is 
\[
g_{\mathbf{x}1} = (a_{4,3,5,2,6,8,1,8,4})
\]

Property 1. (Idempotency) Assume \( \mathbf{x}_i = (a_{i1,1,2,\ldots,m}) \) is a collection of LNNs, and \( \alpha_j = a_j = \alpha \) is true, then \( LNPMM^K(\alpha, \alpha, \ldots, \alpha) = \alpha \).

Proof. As \( \alpha_j = a_j = \alpha \) is true, then based on Eq (3), \( L(\alpha_j, \alpha_j) = 0 \Rightarrow G(\alpha_j, \alpha_j) = 1-L(\alpha_j, \alpha_j) = 1 \).

Hence, \( F(\mathbf{x}_j) = \sum_{i=1}^{m} w_i \Rightarrow \sum_{i=1}^{m} \frac{G(\alpha_j, \alpha_j)}{1 + F(\mathbf{x}_j)} = \frac{1}{\sum_{i=1}^{m} \frac{G(\alpha_j, \alpha_j)}{1 + F(\mathbf{x}_j)}} \)

In addition, by using Eq (8), \( LNPMM^K(\alpha, \alpha, \ldots, \alpha) = LNPMM^K(\alpha, \alpha, \ldots, \alpha) \).
According to Eq (8),

\begin{align*}
\text{Proof.} & \\
& \text{As } (\alpha_{1}, \alpha_{2}, \ldots, \alpha_{m}) = (a_{T}, a_{1}, a_{F}), \text{ then }
\end{align*}

\begin{align*}
& \text{Property 2. (Boundedness) If } \tau_{i} = (a_{T}, a_{i}, a_{F}) (i = 1, 2, \ldots, m) \text{ is a set of LNNs, } \\
& \alpha^{+} = \max \{\alpha_{1}, \alpha_{2}, \ldots, \alpha_{m}\} = (a_{T}, a_{1}, a_{F}), \text{ and } \alpha^{-} = \min \{\alpha_{1}, \alpha_{2}, \ldots, \alpha_{m}\} = (a_{T}, a_{1}, a_{F}), \text{ then }
\end{align*}

\begin{align*}
& \text{LNPMM}^{K}(x^{+}, x^{-}, \ldots, x^{-}) \leq \text{LNPMM}^{K}(x_{1}, x_{2}, \ldots, x_{m}) \leq \text{LNPMM}^{K}(x^{+}, x^{-}, \ldots, x^{-})
\end{align*}
When $1 = 1$, the LNPMM operator is degenerated into the linguistic neutrosophic power geometric operator, denoted as:

$$LNPMM_{1,1}\ldots1(x_1, x_2, \ldots, x_m) = \prod_{i=1}^{m} (x_i)^{1} = \prod_{i=1}^{m} (1 + F(x_i)).$$

(10)

Special case 4:
When $K = (1, 1, 1, 1, 0, 0, \ldots, 0)$, the LNPMM operator is degenerated into the linguistic neutrosophic power MacLaurin symmetric mean operator, denoted as:

$$LNPMM_{1,1,1,1,0,0,\ldots,0}(x_1, x_2, \ldots, x_m) =$$

$$= \left( \prod_{i,j=1}^{m} (1 - \left(1 - \left(1 - \left(1 - \frac{1}{m} \right) \right) \right)) \right)^{1/m - 1}$$

$$= \left( \prod_{i,j=1}^{m} (1 - \left(1 - \left(1 - \left(1 - \frac{1}{m} \right) \right) \right)) \right)^{1/m - 1}$$

$$= \left( \prod_{i,j=1}^{m} (1 - \left(1 - \left(1 - \left(1 - \frac{1}{m} \right) \right) \right)) \right)^{1/m - 1}$$

(11)
When $K = (1, 1, \ldots, 1)$, the LNPMM operator is degraded into the linguistic neutrosophic power geometric average operator, denoted as:

$$\text{LNPMM}^{(1, 1, \ldots, 1)}(z_1, z_2, \ldots, z_m) = \prod_{i=1}^{m} (w_i z_i)^{\frac{1}{m}} = \prod_{i=1}^{m} \left( \frac{1 + F(z_i)}{\sum_{j=1}^{m} (1 + F(z_j))} z_i \right)^{\frac{1}{m}}. \quad (13)$$

Unlike the linguistic neutrosophic power geometric average operator in Special case 1, the linguistic neutrosophic power geometric average operator emphasizes the equilibrium of arguments and the coordination (instead of complementarity) among individuals.

### Weighted linguistic neutrosophic power Muirhead mean operator

Clearly, the weights of LNNs are not under considerations in the LNPMM operators. Thus, the WLNPMM operators are proposed in this subsection, so that the weights of LNNs can be contained. In other words, if the weights of criteria need to be considered, DMs should choose the WLNPMM operators, otherwise the LNPMM operators can be selected.

**Definition 11.** Given several LNNs $z_i (i = 1, 2, \ldots, m)$, $K = (k_1, k_2, \ldots, k_m) \in \mathbb{R}^m$ is a vector of parameters, $k(i)$ is any permutation of $(i, 1, \ldots, m)$, $E_m$ is a set of all possible permutations, $F(z_i) = \sum_{j=1}^{m} G(z_i, z_j)$, and $0 \leq G(z_i, z_j) = 1 - L(a_i, a_j) \leq 1$ is the support for $a_i$ and $a_j$,

$$w_i = \sum_{j=1}^{m} \left(1 + F(z_j)\right) \in [0, 1], \quad \sum_{i=1}^{m} w_i = 1, \ 0 \leq \sum w_i \leq 1$$

is the weight value, and $w_1 + w_2 + \ldots + w_m = 1$, then WLNPMM operator is

$$\text{WLNPMM}^W(z_1, z_2, \ldots, z_m) = \left( \frac{1}{m!} \sum_{\epsilon \in E_m} \prod_{i=1}^{m} \left( \frac{mw_i \bar{G}(a_{\epsilon(i)}, z_i)}{\sum_{j=1}^{m} w_j \bar{G}(a_{\epsilon(j)}, z_j)} \right)^{\frac{1}{m}} \right)^{\frac{1}{m}} \sum_{i=1}^{m} k_i. \quad (14)$$

**Theorem 2.** If there are several LNNs $z_i = (a_{i1}, a_{i2}, a_{i3})$ $(i = 1, 2, \ldots, m)$, then the aggregated value based on Eq (14) is a LNN, where

$$\text{WLNPMM}^W(z_1, z_2, \ldots, z_m) = \left( \frac{1}{m!} \sum_{\epsilon \in E_m} \prod_{i=1}^{m} \left( \frac{1}{1 - \frac{1}{m} \sum_{j=1}^{m} w_j \bar{G}(a_{\epsilon(j)}, z_j)} \right) \right)^{\frac{1}{m}} \sum_{i=1}^{m} k_i. \quad (15)$$

**Example 2.** Suppose $a_1 = (a_{11}, a_{12}, a_{13})$, $a_2 = (a_{21}, a_{22}, a_{23})$, $a_3 = (a_{31}, a_{32}, a_{33})$ and $a_4 = (a_{41}, a_{42}, a_{43})$ are four LNNs, $K = (1, 1, 1, 1)$ and $\omega_1 = \omega_2 = \omega_3 = \omega_4 = 1/4$, based on Eq (15), their aggregated value is $ga_2 = (a_{43}, a_{21}, a_{13}, a_{43})$.

### New linguistic neutrosophic evaluation approach

In this section, a new approach is presented with the WLNPMM operator to address mine safety evaluation problems within linguistic neutrosophic circumstaces.

#### Problem description

Given that there are $p$ mines $\{R_1, R_2, \ldots, R_p\}$, and DMs are required to assess the safety of these mines under $q$ criteria $\{S_1, S_2, \ldots, S_q\}$, so that the safest mine can be selected. $\omega_j$ is the corresponding weight value of criterion $S_j (j = 1, 2, \ldots, q)$, where $\omega_j \in [0, 1]$ and $\sum_{j=1}^{q} \omega_j = 1$. Besides, experts decide to express their preference by means of LNNs. Hence, a linguistic neutrosophic
evaluation matrix is constructed, denoted as $U = (a_{ij})_{p \times q}$, where $a_{ij} = (a_{T_{ij}}, a_{I_{ij}}, a_{F_{ij}})$ is the linguistic neutrosophic evaluation information of mine $R_i$ ($i = 1, 2, \cdots, p$) against criterion $S_j$ ($j = 1, 2, \cdots, q$).

### Decision making process

The decision making procedures for coping with mine safety evaluation problems are:

1. **Step 1: Normalize the original assessment matrix.**
   - Generally, when both benefit and cost criteria exist in the matrix simultaneously, the cost criteria need to be converted to the benefit one for convenience. The transformation rule is
     \[
     a_{N_{ij}} = \begin{cases} 
     (a_{T_{ij}}, a_{I_{ij}}, a_{F_{ij}}) & \text{for benefit criteria } S_j \\
     (a_{F_{ij}}, a_{I_{ij}}, a_{T_{ij}}) & \text{for cost criteria } S_j
     \end{cases}
     \] (16)
   - Consequently, the normalized decision making matrix is $U^N = (a_{N_{ij}})_{p \times q}$.

2. **Step 2: Obtain the power weight values.**
   - The power weight value $w_{ij}$ of the corresponding LNN $x^N_{ij}$ can be computed with
     \[
     \begin{align*}
     G(x^N_{ij}, x^N_{ir}) &= 1 - L(x^N_{ij}, x^N_{ir})(j, r = 1, 2, \ldots, q; r \neq j), \\
     F(x^N_{ij}) &= \sum_{r=1, r \neq j}^{q} G(x^N_{ij}, x^N_{ir}), \\
     w_{ij} &= \frac{1 + F(x^N_{ij})}{\sum_{r=1}^{q} (1 + F(x^N_{ir}))}
     \end{align*}
     \] (17, 18, 19)
   - Step 3: Acquire the comprehensive assessment values.
     - Based on the WLNPM operator defined in subsection 3.2, the overall assessment value is
       \[
       V_i = WLNPM^{K}(x^N_{i1}, x^N_{i2}, \ldots, x^N_{iq}),
       \] (20)

3. **Step 4: Compute the score function or accuracy function.**
   - According to Eq (1), the score function $B(V_i)$ of each mine is computed. When two score function values are the same, the accuracy function values of them should be computed based on Eq (2).

4. **Step 5: Determine the safest mine.**
   - In accordance with Eqs (1) and (2), the safest mine $R^*$ can be obtained.

### Case study

In this section, an case of safety assessment for gold mines is illustrated to justify the practicability of our approach.

### Project profile

Laizhou city is located in Shandong Province of China. It is an important gold production base, where distributes numerous gold mines. Nevertheless, near-to surface mineral resources in most of these mines are gradually becoming depleted with the accelerating rate of mining in recent decades. Exploiting deep mineral resources has become unavoidable. However, the situation of safety production is becoming more and more serious because of the higher ground
stress, temperature, and water pressure in deep mining areas. To protect the lives and property of workers effectively, it is essential to conduct a safety evaluation for these mines firstly.

To understand the mine safety status, the local mine safety supervision bureau intends to evaluate the safety conditions of four typical gold mines (denoted as $R_1$, $R_2$, $R_3$ and $R_4$) in this area recently.

**Evaluation criteria**

Identifying the criteria is the first step for the mine safety evaluation. Based on the concrete characteristics of mines and some literature [3,6,7], four criteria are selected after thorough investigations: the human factor, environmental conditions, technological equipment, and management quality (denoted as $S_1$, $S_2$, $S_3$ and $S_4$). The details of these criteria are described in Table 1.

**Determining the safest mine**

Suppose the importance degrees of these four criteria are equal, that is, $\omega_1 = \omega_2 = \omega_3 = \omega_4 = 1/4$. Considering the fuzziness of human cognitions, LNNs are suggested to describe these four qualitative evaluation indexes for reserving initial evaluation information as much as possible.

A decision making group, which contain ten experts, is planned to make evaluations. The used linguistic term set is

$$A = \{ a_0 = \text{very low}, \ a_1 = \text{low}, \ a_2 = \text{a little low}, \ a_3 = \text{medium}, \ a_4 = \text{a little high}, \ a_5 = \text{high}, \ a_6 = \text{very high}\}.$$  

After mutual discussions, the initial evaluation information is obtained with LNNs in Table 2.

Then, the new methodology is used to pick out a gold mine with best safety conditions. The detailed procedures are described in the following.

Step 1: Normalize the original evaluation matrix.

As all criteria are benefit, they don’t need to be transformed, then the normalized matrix is still $U^N = U$.

Step 2: Obtain the power weight values.

### Table 1. Details of evaluation criteria for mine safety.

| Evaluation criteria       | Benefit/ Cost | Descriptions                                                                 |
|---------------------------|---------------|------------------------------------------------------------------------------|
| Human factor $S_1$        | Benefit       | It refers to the personal protection, emergency training, violation, and total mining experience. |
| Environmental conditions $S_2$ | Benefit | It refers to the geological feature, dust content, temperature, and humidity. |
| Technological equipment $S_3$ | Benefit | It refers to the mining mechanization, ventilation, dust-proof, fire-fighting, drainage, and transport equipment. |
| Management quality $S_4$  | Benefit       | It refers to the monitoring, defective design, safety culture, rules and regulations. |

### Table 2. Original assessment matrix $U$.

|       | $S_1$                     | $S_2$                     | $S_3$                     | $S_4$                     |
|-------|---------------------------|---------------------------|---------------------------|---------------------------|
| $R_1$ | $(a_5,a_2,a_1)$           | $(a_6,a_3,a_2)$           | $(a_6,a_1,a_3)$           | $(a_6,a_2,a_3)$           |
| $R_2$ | $(a_5,a_2,a_1)$           | $(a_6,a_3,a_2)$           | $(a_6,a_1,a_3)$           | $(a_6,a_2,a_3)$           |
| $R_3$ | $(a_6,a_3,a_1)$           | $(a_6,a_2,a_3)$           | $(a_6,a_1,a_3)$           | $(a_6,a_2,a_3)$           |
| $R_4$ | $(a_5,a_1,a_2)$           | $(a_6,a_2,a_3)$           | $(a_6,a_1,a_3)$           | $(a_6,a_2,a_3)$           |
By using Eq (17), the supports for two LNNs are calculated (let $\lambda = 1$) (See the second to fourth columns in Tables 3–6).

Then, based on Eq (18), the values of $F(x_i^w)$ are computed (See the sixth columns in Tables 3–6).

Thereafter, the power weight values are calculated on the basis of Eq (19) (See the last columns in Tables 3–6).

Step 3: Acquire the comprehensive evaluation results.

Based on the WLNPMMD defined in subsection 3.2, the comprehensive evaluation results are (Without loss of generality, we assume $K = (1, 1, 1, 1)$): $V_1 = WLNPMMD(x_{11}^w, x_{12}^w, x_{13}^w, x_{14}^w) = (a_{3.40}, a_{2.30}, a_{1.20}, a_{1.30})$, $V_2 = (a_{3.67}, a_{2.25}, a_{1.88})$, $V_3 = (a_{4.35}, a_{2.68}, a_{1.84})$ and $V_4 = (a_{4.12}, a_{2.46}, a_{2.53})$.

Step 4: Compute the score or accuracy values.

By using Eq (1), the score function of each mine is computed as: $B(V_1) \approx 0.664$, $B(V_2) \approx 0.641$, $B(V_3) \approx 0.657$ and $B(V_4) \approx 0.618$.

Step 5: Determine the optimal alternative.

### Table 3. Values of $G(x_i^w, x_j^w)$, $F(x_i^w)$ and $w_j$ ($i = 1$).

| $G(x_i^w, x_j^w)$ | $x_{i1}^w$ | $x_{i2}^w$ | $x_{i3}^w$ | $x_{i4}^w$ | $F(x_i^w)$ | $w_j$ |
|------------------|-----------|-----------|-----------|-----------|------------|-------|
| $x_{11}^w$       |           |           |           |           | 2.388      | 0.252 |
| $x_{12}^w$       | 0.833     |           |           |           | 2.728      | 0.244 |
| $x_{13}^w$       |           | 0.667     |           |           | 0.778      |       |
| $x_{14}^w$       |           |           | 0.889     |           | 2.278      | 0.244 |

https://doi.org/10.1371/journal.pone.0224090.t003

### Table 4. Values of $G(x_i^w, x_j^w)$, $F(x_i^w)$ and $w_j$ ($i = 2$).

| $G(x_i^w, x_j^w)$ | $x_{i1}^w$ | $x_{i2}^w$ | $x_{i3}^w$ | $x_{i4}^w$ | $F(x_i^w)$ | $w_j$ |
|------------------|-----------|-----------|-----------|-----------|------------|-------|
| $x_{21}^w$       |           | 0.722     |           |           | 2.222      | 0.246 |
| $x_{22}^w$       | 0.722     |           |           |           | 2.222      | 0.246 |
| $x_{23}^w$       |           |           | 0.833     |           | 2.222      | 0.246 |
| $x_{24}^w$       |           | 0.833     | 0.833     |           | 2.444      | 0.262 |

https://doi.org/10.1371/journal.pone.0224090.t004

### Table 5. Values of $G(x_i^w, x_j^w)$, $F(x_i^w)$ and $w_j$ ($i = 3$).

| $G(x_i^w, x_j^w)$ | $x_{i1}^w$ | $x_{i2}^w$ | $x_{i3}^w$ | $x_{i4}^w$ | $F(x_i^w)$ | $w_j$ |
|------------------|-----------|-----------|-----------|-----------|------------|-------|
| $x_{31}^w$       |           | 0.722     | 0.889     | 0.667     | 2.278      | 0.252 |
| $x_{32}^w$       | 0.722     |           | 0.722     | 0.833     | 2.277      | 0.252 |
| $x_{33}^w$       | 0.889     | 0.722     |           | 0.667     | 2.278      | 0.252 |
| $x_{34}^w$       | 0.667     | 0.833     | 0.667     |           | 2.167      | 0.244 |

https://doi.org/10.1371/journal.pone.0224090.t005

### Table 6. Values of $G(x_i^w, x_j^w)$, $F(x_i^w)$ and $w_j$ ($i = 4$).

| $G(x_i^w, x_j^w)$ | $x_{i1}^w$ | $x_{i2}^w$ | $x_{i3}^w$ | $x_{i4}^w$ | $F(x_i^w)$ | $w_j$ |
|------------------|-----------|-----------|-----------|-----------|------------|-------|
| $x_{41}^w$       |           | 0.778     | 0.889     | 0.778     | 2.445      | 0.254 |
| $x_{42}^w$       | 0.778     |           | 0.778     | 0.667     | 2.223      | 0.238 |
| $x_{43}^w$       | 0.889     | 0.778     |           | 0.889     | 2.556      | 0.262 |
| $x_{44}^w$       | 0.778     | 0.667     | 0.889     |           | 2.334      | 0.246 |

https://doi.org/10.1371/journal.pone.0224090.t006
As \( B(V_1) > B(V_3) > B(V_2) > B(V_4) \), then the ranking order of all mines is \( R_1 > R_3 > R_2 > R_4 \). 

**Discussions**

The impact of parameters is discussed, and the strengths of the proposed aggregation operators are justified in this section.

**Sensitivity analyses**

In this subsection, the impacts of the parameter vector \( K = (k_1,k_2,k_3,k_4) \) in the LNPMM operator are analyzed. Ranking results are obtained when dissimilar values are assigned to \( K \) (See Table 7).

From Table 7, it is clear that dissimilar rankings are derived with different \( K \) values. When the criteria are independent with each other, i.e., \( K = (1,0,0,0) \), the score values of alternatives are greatest, and the best one is \( R_2 \). When the relations between two LNNs are captured, the best alternative is changed as \( R_3 \). However, the ranking result is stable at \( R_1 > R_3 > R_2 > R_4 \) when more interrelations among linguistic neutroshophic criteria values are reflected. In other words, the best alternative is \( R_1 \) in most cases (i.e., \( K = (1,1,1,0) \), \( K = (1,1,1,1) \) and \( K = (1/4,1/4,1/4,1/4) \)). Therefore, the proposed method is robust in some extent. Meanwhile, the choice of parameter can reflect DMs’ risk preference and increase the flexibility of this method. That is, when the DM is optimistic, she/he can choose a smaller \( K (K = (1,0,0,0) \) or \( K = (1,1,0,0) \)) to obtain more flexibility; On the contrary, when the DM is pessimistic, he/she may choose a larger \( K \) to retain more stability.

**Validation of the proposed approach**

In this subsection, an example from literature [57] is used to verify the feasibility of our method firstly. Then, comparison analyses with several literature \([35,40,41,43]\) are made to show the strengths of our approach.

Part 1: Validation with a same example

In this part, our method is adopted to solve the problem in literature [57]. The dataset can be seen in [57] and the detailed process is:

Step 1: Normalize the original evaluation matrix.

The normalized original evaluation matrix is the same with that in [57].

Step 2: Obtain the power weight values.

Based on Eq (16)–(18), the power weight values are calculated as: \( w_{1j} = [0.254,0.254,0.246,0.246] \), \( w_{2j} = [0.246,0.254,0.254,0.246] \), \( w_{3j} = [0.256,0.256,0.232,0.256] \) and \( w_{4j} = [0.254,0.254,0.238,0.254] \).

Step 3: Acquire the comprehensive evaluation results.

| Parameter vector \( K \) | Score function value | Ranking order | Best alternative \( R^* \) |
|--------------------------|----------------------|---------------|---------------------------|
| \( (1,0,0,0) \)          | \( B(V_1) = 0.782, B(V_2) = 0.792, B(V_3) = 0.754, B(V_4) = 0.790 \) | \( R_1 > R_3 > R_2 > R_4 \) | \( R_2 \) |
| \( (1,1,0,0) \)          | \( B(V_1) = 0.526, B(V_2) = 0.512, B(V_3) = 0.527, B(V_4) = 0.486 \) | \( R_1 > R_3 > R_2 > R_4 \) | \( R_1 \) |
| \( (1,1,1,0) \)          | \( B(V_1) = 0.673, B(V_2) = 0.655, B(V_3) = 0.669, B(V_4) = 0.629 \) | \( R_1 > R_3 > R_2 > R_4 \) | \( R_1 \) |
| \( (1,1,1,1) \)          | \( B(V_1) = 0.664, B(V_2) = 0.641, B(V_3) = 0.657, B(V_4) = 0.618 \) | \( R_1 > R_3 > R_2 > R_4 \) | \( R_4 \) |
| \( (1/4,1/4,1/4,1/4) \) | \( B(V_1) = 0.664, B(V_2) = 0.642, B(V_3) = 0.657, B(V_4) = 0.621 \) | \( R_1 > R_3 > R_2 > R_4 \) | \( R_4 \) |

https://doi.org/10.1371/journal.pone.0224090.t007
Based on the WLNPMM defined in subsection 3.2, the comprehensive evaluation results are \( K = (1,1,1,1) \): \( e_1 = (a_{1,37},a_{2,33},a_{1,32}) \), \( e_2 = (a_{1,36},a_{2,34},a_{1,32}) \), \( e_3 = (a_{4,35},a_{2,33},a_{1,33}) \) and \( e_4 = (a_{4,36},a_{2,33},a_{1,32}) \).

Step 4: Compute the score or accuracy values.
By using Eq (1), the score function of each mine is computed as: \( U(e_1) \approx 0.7067 \), \( U(e_2) \approx 0.7056 \), \( U(e_3) \approx 0.7050 \) and \( U(e_4) \approx 0.7061 \).

Step 5: Determine the optimal alternative.
As \( U(e_1) > U(e_4) > U(e_2) > U(e_3) \), then the ranking order is \( a_1 > a_4 > a_2 > a_3 \).
As the ranking result is the same with that in [57], it demonstrates the feasibility of our method to some extent.

Part 2: Comparison of ranking results
At first, the ranking orders of alternatives in Section 5 with different approaches are obtained, as listed in Table 8.

As dissimilar rankings exist in Table 8, the best ranking among them needs to be determined to certify the effectiveness of the proposed approach. For this purpose, the technique in literature [69] is suggested.

Step 1: Compute the number of times for alternatives under various ranks in Table 9. For example, it can be seen that \( R_1 \) ranks No.1 once and No.3 five times.
Step 2: Smooth the alternatives in terms of ranking distribution, as shown in Table 10.
Step 3: Establish a programming model with several constraints as follows:

\[
\begin{align*}
\text{Max } & \Psi = \sum_{i=1}^{4} \sum_{s=1}^{4} \left( \Pi_u \cdot \frac{4!}{s} \cdot \Phi_u \right) \\
\text{s.t. } & \sum_{i=1}^{4} \Phi_u = 1, \quad s = 1, 2, 3, 4 \\
& \sum_{i=1}^{4} \Phi_u = 1, \quad i = 1, 2, 3, 4 \\
& \Phi_u = 0 \text{ or } \Phi_u = 1, \quad i, s = 1, 2, 3, 4 
\end{align*}
\]

After addressing this model, the optimum ranking is \( R_1 \succ R_3 \succ R_2 \succ R_4 \).
The optimal ranking and other rankings in Table 8 are portrayed, which can be seen in Fig 1. Obviously, same rankings are derived with Model (21) and our method. It demonstrates

| Approach                | Ranking basis                                      | Value          | Ranking order   |
|-------------------------|----------------------------------------------------|----------------|-----------------|
| LNWAM [35]              | Score function value                               | \( B(V_1) \approx 0.782 \), \( B(V_2) \approx 0.792 \), \( B(V_3) \approx 0.790 \), \( B(V_4) \approx 0.753 \) | \( R_2 \succ R_3 \succ R_1 \succ R_4 \) |
| LNWGM [35]              | Score function value                               | \( B(V_1) \approx 0.664 \), \( B(V_2) \approx 0.641 \), \( B(V_3) \approx 0.657 \), \( B(V_4) \approx 0.619 \) | \( R_1 \succ R_2 \succ R_3 \succ R_4 \) |
| Cosine-based [40]       | Cosine measure                                     | \( cm(V_1) \approx 0.958 \), \( cm(V_2) \approx 0.946 \), \( cm(V_3) \approx 0.959 \), \( cm(V_4) \approx 0.933 \) | \( R_2 \succ R_1 \succ R_3 \succ R_4 \) |
| TOPSIS [41]             | Distance measure                                   | \( dm(V_1) \approx 0.375 \), \( dm(V_2) \approx 0.346 \), \( dm(V_3) \approx 0.375 \), \( dm(V_4) \approx 0.337 \) | \( R_1 \succ R_3 \succ R_2 \succ R_4 \) |
| MULTIMOORA [43]         | Ratio system, reference point and multiplicative model | Rank 1: \( R_1 \succ R_2 \succ R_3 \succ R_4 \) Rank 2: \( R_1 \succ R_4 \succ R_3 \succ R_2 \) Rank 3: \( R_1 \succ R_3 \succ R_4 \succ R_2 \) | \( R_1 \succ R_2 \succ R_3 \succ R_4 \) |
| The proposed approach   | Score function value                               | \( B(V_1) \approx 0.664 \), \( B(V_2) \approx 0.641 \), \( B(V_3) \approx 0.657 \), \( B(V_4) \approx 0.618 \) | \( R_1 \succ R_2 \succ R_3 \succ R_4 \) |

https://doi.org/10.1371/journal.pone.0224090.0008
that the proposed method is preponderant in disposing such issues where extreme values exist or the interrelations among criteria should be captured.

Part 3: Further comparisons

As indicated in Table 8, various techniques have been adopted to resolve evaluation issues under linguistic neutrosophic environment. They are based on dissimilar ranking bases and have distinctive characteristics.

(1) Compared with existent methods based on other aggregation operators

The LNWAM and LNWGM operators in literature [35] are two basic linguistic neutrosophic mean operators. However, both of them don’t take the mutual relationships of LNNs into account. Instead, our approach can reflect the interrelations among several inputs. Even though the approach in literature [43,54,55,56,57] also considers the relationship of arguments, it neglects the influences of unreasonable information. On the other hand, just the correlations between two inputs can be captured in [43,54,55], while the relations among more than two arguments are reflected in [56,57]. Compared with them, the proposed method can describe the relationships among any number of inputs through the adjustment of parameter vector. Furthermore, the idea of power weighting is borrowed in our method, so that the impacts of some irrational values can be diminished. In this sense, the advised approach is more influential and supple.
(2) Compared with other existent decision making approaches

The approach proposed by Shi and Ye in literature [40] is based on cosine measure. That is, the cosine measures of pairwise LNNs need to be calculated before ranking alternatives. Similarly, the distance measures of pairwise LNNs are required to be compute when the extended TOPSIS in literature [41] is adopted. Clearly, a lot of additional calculations are produced in these methods. More seriously, the approach in [43] is based on three modes, which makes it complicated. In addition, a satisfactory ranking may be not derived in some cases, especially when three induced ranks are contradictory. In contrast, the proposed method in this study is on the basis of aggregation of LNNs under each alternative. In other words, the score functions of aggregated values can directly obtain the final rankings. Compared with them [40,41,43], our method is simple and has less computing work.

Summary of advantages

In summary, the highlights of our approach are:

1. Due to the vagueness of DMs, the fuzzy evaluation data are expressed by LNNs. In LNNs, three independent linguistic membership functions are included. In this case, not only the consistent and inconsistent information, but also the hesitant degrees of DMs can be conveniently and fully depicted.

2. The constructed framework is based on the WLNPMM, so that the superiority of PA and MM operators can be exploited. That is, the proposed method can capture the relationships among any number of inputs with the MM operators. At the same time, our method can avoid negative influence of bad data with the PA operators.

3. Many existent aggregation operators (e.g., the arithmetic/geometric/Bonferroni mean operators) can be regarded as the special cases of MM operators. Thus, the proposed method, which combined MM operators with PA operators and LNNs, is more general. Besides, the alterable parameter vector makes our method more flexible as it can alter with the change of the number of inputs whose relations can be reflected.

Conclusions

In this study, the LNPMM operator and WLNPMM operator were explored to aggregate linguistic neutrosophic information. The highlight of these operators is that they can exert the advantages of LNNs, PA and MM operators. Some main properties and special cases of them were revealed as well. Then, the new decision making methodology with these operators was adopted to evaluate the safety of mines under linguistic neutrosophic environment. The strengths include: The correlations among criteria can be reflected and the negative impacts of anomalous values on ranking orders can be diminished in the evaluation process. The sensitivity analysis certified that our method is flexible because it contains a changeable parameter vector. Meanwhile, the comparison analyses with other methods showed that our method is robust and efficient when solving complex decision making problems under linguistic neutrosophic conditions.

The limitation of this study is that the way of determining criteria weight values is not discussed. Because plenty of objective and subjective weight determination models have been developed in existent literature, maybe they can be directly used or duly modified according to the characteristics of issues. Second, the proposed method in this study may be adopted to settle linguistic neutrosophic decision making issues in other fields. Third, maybe our method can combined with other extensions of neutrosophic sets (such as the plithogenic set [70,71].
and neutrosophic linguistic overset/underset/offset [72] according to the real conditions. In addition, the proposed method can be applied into other fields, and new applications (especially applications in industry) with knowledge of neutrosophic and plithogenic sets and logic are worthy to be researched in the future.

Acknowledgments

This work was supported by National Key Research and Development Program of China (2018YFC0604606), and National Natural Science Foundation of China (51774321)

Author Contributions

Conceptualization: Weizhang Liang.

Formal analysis: Suizhi Luo.

Funding acquisition: Guoyan Zhao.

Methodology: Suizhi Luo, Weizhang Liang.

Validation: Weizhang Liang, Guoyan Zhao.

Writing – original draft: Suizhi Luo.

Writing – review & editing: Suizhi Luo, Weizhang Liang, Guoyan Zhao.

References

1. Badri A, Nadeau S, Gbodossou A. A new practical approach to risk management for underground mining project in Quebec. Journal of Loss Prevention in the Process Industries 2013; 26: 1145–1158
2. Liang WZ, Luo SZ, Zhao GY. Evaluation of cleaner production for gold mines employing a hybrid multi-criteria decision making approach. Sustainability 2019; 11(1): 146. https://doi.org/10.3390/su11010146
3. Wang QX, Wang H, Qi ZQ. An application of nonlinear fuzzy analytic hierarchy process in safety evaluation of coal mine. Safety Science 2016; 86: 78–87.
4. Liang WZ, Zhao GY, Wu H, Chen Y. Assessing the risk degree of goafs by employing hybrid TOWIM method under uncertainty. Bulletin of Engineering Geology & the Environment 2018; 78(5): 3767–3782. https://doi.org/10.1016/j.bulge.2015.11.014
5. Liu H, Zeng LH. Statistical analysis of national coal mine safety accidents in 2018. Inner Mongolia Coal Economy 2019; 6: 92–93.
6. Wei CF, Pei Z, Li HM. An induced OWA operator in coal mine safety evaluation. Journal of Computer and System Sciences 2012; 78(4): 997–1005.
7. Peng HG, Wang JQ, Cheng PF. A linguistic intuitionistic multi-criteria decision-making method based on the frank heronian mean operator and its application in evaluating coal mine safety. International Journal of Machine Learning and Cybernetics 2017; 9: 1053–1068.
8. Liang WZ, Zhao GY, Wang X, Zhao J, Ma CD. Assessing the rockburst risk for deep shafts via distance-based multi-criteria decision making approaches with hesitant fuzzy information. Engineering Geology 2019; https://doi.org/10.1016/j.enggeo.2019.105211
9. Zadeh LA. The concept of a linguistic variable and its application to approximate reasoning—I. Information Sciences 1975; 8(3): 199–249.
10. Liu PD. Two-dimensional uncertain linguistic generalized normalized weighted geometric Bonferroni mean and its application to multi-criteria decision making. Scientia Iranica. Transaction E, Industrial Engineering 2018; 25(1): 450–465.
11. Mendoza-Sanchez J, Silva F, Rangel L, Jaramillo L, Mendoza L, Garzon J, Quiroga A. Benefit, risk and cost of new oral anticoagulants and warfarin in atrial fibrillation: A multicriteria decision analysis. PLOS One 2016; 13(5): e0196361. https://doi.org/10.1371/journal.pone.0196361 PMID: 27923207
12. Liang WZ, Zhao GY, Wu H, Dai B. Risk assessment of rockburst via an extended MABAC method under fuzzy environment. Tunnelling and Underground Space Technology 2019; 83: 533–544.
13. Chang KH, Chang YC, Chain K, Chung HY. Integrating soft set theory and fuzzy linguistic model to evaluate the performance of training simulation systems. PLOS One 2016; 11(9): e0162092. https://doi.org/10.1371/journal.pone.0162092 PMID: 27598390

14. Liu PD, Zhang XH. Approach to multi-attributes decision making with intuitionistic linguistic information based on Dempster-shafer evidence theory. IEEE Access 2018; 6: 52969–52981.

15. Luo SZ, Zhang HY, Wang JQ, Li L. Group decision-making approach for evaluating the sustainability of constructed wetlands with probabilistic linguistic preference relations. Journal of the Operational Research Society 2019; 1–17. https://doi.org/10.1080/01605682.2018.1510806

16. Chen ZC, Liu PH, Pei Z. An approach to multi-attribute group decision making based on linguistic intuitionistic fuzzy numbers. International Journal of Computational Intelligence Systems 2015; 8(4): 747–760.

17. Liu PD, Tang GL. Some intuitionistic fuzzy prioritized interactive Einstein choquet operators and their application in decision making. IEEE Access 2018; 6: 72357–72371.

18. Liu PD, Li DF. Some Muirhead mean operators for intuitionistic fuzzy numbers and their applications to group decision making. PLOS One 2017; 12(1): e0168767. https://doi.org/10.1371/journal.pone.0168767 PMID: 28103244

19. Smarandache F. Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis. 1st ed. Rehoboth: American Research Press; 1998.

20. Biswas P, Pramanik S, Giri BC. TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. Neural Computing and Applications 2016; 27(3): 727–737.

21. Garg H, Nancy. New logarithmic operational laws and their applications to multiattribute decision making for single-valued neutrosophic numbers. Cognitive Systems Research 2018; 52: 931–946.

22. Rashno A, Nazari B, Kooszekanani DD, Drayna PM, Sadri S, Rabbani H, Parhi KK. Fully-automated segmentation of fluid regions in exudative age-related macular degeneration subjects: Kernel graph cut in neutrosophic domain. PLOS One 2017; 12(10): e0186949. https://doi.org/10.1371/journal.pone.0186949 PMID: 29059257

23. Pramanik S, Biswas P, Giri BC. Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. Neural Computing and Applications 2017; 28 (5): 1163–1176.

24. Garg H, Nancy. Some hybrid weighted aggregation operators under neutrosophic set environment and their applications to multicriteria decision-making. Applied Intelligence 2018; 46(12): 4871–4888.

25. Li DP, Cheng SJ, Cheng PF, Wang JQ, Zhang HY. A novel financial risk assessment model for companies based on heterogeneous information and aggregated historical data. PLOS One 2018; 13(12): e0208166. https://doi.org/10.1371/journal.pone.0208166 PMID: 30586437

26. Nancy, Garg H. A novel divergence measure and its based TOPSIS method for multi criteria decision-making under single-valued neutrosophic environment. Journal of Intelligent & Fuzzy Systems 2019; 36(1): 101–115.

27. Dey PP, Pramanik S, Giri BC. An extended grey relational analysis based multiple attribute decision making in interval neutrosophic uncertain linguistic setting. Neutrosophic Sets and Systems 2016; 11: 21–30.

28. Garg H, Nancy. Non-linear programming method for multi-criteria decision making problems under interval neutrosophic set environment. Applied Intelligence 2018; 48(8): 2199–2213.

29. Abdel-Basset M, Saleh M, Gamal A, Smarandache F. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing 2019; 77: 438–452.

30. Ji P, Zhang HY, Wang JQ. Selecting an outsourcing provider based on the combined MABAC–ELEC-TRE method using single-valued neutrosophic linguistic sets. Computers & Industrial Engineering 2018; 120: 429–441.

31. Liu PD, Khan Q, Mahmood T, Smarandache F, Li Y. Multiple attribute group decision making based on 2-tuple linguistic neutrosophic Dombi power Heronian mean operators. IEEE Access 2019; 1–1. https://doi.org/10.1109/ACCESS.2019.2925344

32. Abdel-Basset M, Saleh M, Gamal A, Smarandache F. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing 2019; 77: 438–452.

33. Wang J, Lu JP, Wei GW, Lin R, Wei C. Models for MADM with single-valued neutrosophic 2-tuple linguistic Muirhead mean operators. Mathematics 2019; 7(5): 442. https://doi.org/10.3390/math7050442

34. Dat LQ, Thong NT, Ali M, Smarandache F, Abdel-Basset M, Long HV. Linguistic approaches to interval complex neutrosophic sets in decision making. IEEE Access 2019; 7: 38902–38917.
35. Fang ZB, Ye J. Multiple attribute group decision-making method based on linguistic neutrosophic numbers. Symmetry 2017; 9(7): 111. https://doi.org/10.3390/sym9070111
36. Luo SZ, Liang WZ, Xing LN. Selection of mine development scheme based on similarity measure under fuzzy environment. Neural Computing and Applications 2019; 1–12. https://doi.org/10.1007/s00521-019-04026-x
37. Liang WZ, Zhao GY, Hong CS. Performance assessment of circular economy for phosphorus chemical firms based on VIKOR-QUALIFLEX method. Journal of Cleaner Production 2018; 196: 1365–1378.
38. Mondal K, Pramanik S, Giri BC. Multi-criteria group decision making based on linguistic refined neutrosophic strategy. In: Smarandache F, Pramanik S, editors. New Trends in Neutrosophic Theory and Applications. Brussels: Pons Editions; 2018. pp. 125–139.
39. Luo SZ, Liang WZ. Optimization of roadway support schemes with likelihood–based MABAC method. Applied Soft Computing 2019; 80: 80–92.
40. Shi LL, Ye J. Cosine measures of linguistic neutrosophic numbers and their application in multiple attribute group decision-making. Information 2017; 8(4): 117. https://doi.org/10.3390/info8040117
41. Liang WZ, Zhao GY, Wu H. Evaluating investment risks of metallic mines using an extended TOPSIS method with linguistic neutrosophic numbers. Symmetry 2017; 9(8): 149. https://doi.org/10.3390/sym9080149
42. Pamučar D, Badi I, Sanja K, Obradović R. A novel approach for the selection of power-generation technology using a linguistic neutrosophic CODAS method: A case study in Libya. Energies 2018; 11(9): 2489. https://doi.org/10.3390/en11092489
43. Liang WZ, Zhao GY, Hong CS. Selecting the optimal mining method with extended multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA) approach. Neural Computing and Applications 2018; https://doi.org/10.1007/s00521-018-3405-5
44. Garg H, Nancy. Algorithms for possibility linguistic single-valued neutrosophic decision-making based on COPRAS and aggregation operators with new information measures. Measurement 2019; 138: 278–290.
45. Wang J, Gao H, Wei GW. Some 2-tuple linguistic neutrosophic number Muirhead mean operators and their applications to multiple attribute decision making. Journal of Experimental & Theoretical Artificial Intelligence 2019; 31(3): 409–439.
46. Liu PD, You XL. Bidirectional projection measure of linguistic neutrosophic numbers and their application to multi-criteria group decision making. Computers & Industrial Engineering 2019; 128: 447–457.
47. Li YY, Wang JQ, Wang TL. A linguistic neutrosophic multi-criteria group decision-making approach with EDAS method. Arabian Journal for Science and Engineering 2019; 44(3): 2737–2749.
48. Wang XG, Geng YS, Yao PP, Yang MJ. Multiple attribute group decision making approach based on extended VIKOR and linguistic neutrosophic set. Journal of Intelligent & Fuzzy Systems 2019; 36(1): 149–160.
49. Liu PD, Wang P. Some q-Rung Orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making. International Journal of Intelligent Systems 2018; 33(2): 259–280.
50. Liu S, Yu W, Liu L, Hu YA. Variable weights theory and its application to multi-attribute group decision making with intuitionistic fuzzy numbers on determining decision maker’s weights. PIOS One 2019; 14(3): e0212636. https://doi.org/10.1371/journal.pone.0212636 PMID: 30840647
51. Garg H, Nancy. Linguistic single-valued neutrosophic prioritized aggregation operators and their applications to multiple-attribute group decision-making. Journal of Ambient Intelligence and Humanized Computing 2018; 9(6): 1975–1997.
52. Liu PD, Liu JL. Some q-Rung Orthopair fuzzy Bonferroni mean operators and their application to multi-attribute group decision making. International Journal of Intelligent Systems 2018; 33(2): 315–347.
53. Liu PD, Wang P. Multiple-attribute decision making based on Archimedean Bonferroni operators of q-Rung Orthopair fuzzy numbers. IEEE Transactions on Fuzzy Systems 2018; https://doi.org/10.1109/TFUZZ.2018.2826452
54. Fan CX, Ye J, Hu KL, Fan E. Bonferroni mean operators of linguistic neutrosophic numbers and their multiple attribute group decision-making methods. Information 2017; 8(3): 107. https://doi.org/10.3390/info8030107
55. Wang YM, Liu P. Linguistic neutrosophic generalized partitioned Bonferroni mean operators and their application to multi-attribute group decision making. Symmetry 2018; 10(5), 160. https://doi.org/10.3390/sym10050160
56. Liu PD, You XL. Some linguistic neutrosophic Hamy mean operators and their application to multi-attribute group decision making. PIOS One 2018; 13(3): e0193027. https://doi.org/10.1371/journal.pone.0193027 PMID: 29513697
57. Liang WZ, Zhao GY, Luo SZ. Linguistic neutrosophic Hamacher aggregation operators and the application in evaluating land reclamation schemes for mines. PloS one 2018; 13(11): e0206178. https://doi.org/10.1371/journal.pone.0206178 PMID: 30399171

58. Muirhead RF. Some methods applicable to identities and inequalities of symmetric algebraic functions of n letters. Proceedings of the Edinburgh Mathematical Society 1902; 21: 144–162.

59. Wang J, Wei GW, Wei C, Wei Y. Dual hesitant q-Rung Orthopair fuzzy Muirhead mean operators in multiple attribute decision making. IEEE Access 2019; 7(1): 67139–67166.

60. Tang XY, Wei GW, Gao H. Models for multiple attribute decision making with interval-valued Pythagorean fuzzy Muirhead mean operators and their application to green suppliers selection. Informatica 2019; 30(1): 153–186.

61. Wang R, Wang J, Gao H, Wei GW. Methods for MADM with picture fuzzy Muirhead mean operators and their application for evaluating the financial investment risk. Symmetry 2019; 11(1): 6. https://doi.org/10.3390/sym11010006

62. Yager RR. The power average operator. IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans 2001; 31(6): 724–731.

63. Liu PD, Chen SM, Wang P. Multiple-attribute group decision-making based on q-Rung Orthopair fuzzy power Maclaurin symmetric mean operators. IEEE Transactions on Systems, Man, and Cybernetics-Systems 2018; (99): 1–16. https://doi.org/10.1109/TSMC.2018.2852948

64. Garg H, Nancy. (2018). Multi-criteria decision-making method based on prioritized muirhead mean aggregation operator under neutrosophic set environment. Symmetry, 10(7), 280. https://doi.org/10.3390/sym10070280

65. Liu PD, Mahmood T, Khan Q. Group decision making based on power Heronian aggregation operators under linguistic neutrosophic environment. International Journal of Fuzzy Systems 2018; 20(3): 970–985.

66. Li L, Zhang RT, Wang J, Zhu XM, Xing YP. Pythagorean fuzzy power Muirhead mean operators with their application to multi-attribute decision making. Journal of Intelligent & Fuzzy Systems 2018; 35(2): 2035–2050.

67. Khan Q, Hassan N, Mahmood T. Neutrosophic cubic power Muirhead mean operators with uncertain data for multi-attribute decision making. Symmetry 2018; 10(10): 444. https://doi.org/10.3390/sym10100444

68. Luo SZ, Cheng PF, Wang JQ, Huang YJ. Selecting project delivery systems based on simplified neutrosophic linguistic preference relations, Symmetry 2017; 9(8): 151. https://doi.org/10.3390/sym9080151

69. Jahan A, Ismail MY, Shuib S, Norfazidah D, Edwards KL. An aggregation technique for optimal decision-making in materials selection. Materials & Design 2011; 32(10): 4918–4924.

70. Smarandache F. Plithogenic Set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-revisited. Neutrosophic Sets and Systems 2018; 21: 153–166.

71. Smarandache F. Plithogenic, plithogenic set, logic, probability, and statistics. 1st ed. Brussels: Pons Publishing House; 2017.

72. Smarandache F. Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset, Similarly for Neutrosophic Over-/Under-/Off- Logic, Probability, and Statistics. 1st ed. Brussels: Pons Edition; 2016.