RG flows on the phase spaces and the $\tau$ functions for the generic Hamiltonian systems

A. S. Gorsky

Institute of Theoretical and Experimental Physics, Moscow 117259, B. Cheryomushkinskaya 25

Abstract

We discuss the generic definition of the $\tau$ function for the arbitrary Hamiltonian system. The different approaches concerning the deformations of the curves and surfaces are compared. It is shown that the Baker-Akhiezer function for the secondary integrable system of the Toda lattice type can be identified with the coherent wave function of the initial dynamical system. The $\tau$ function appears to be related to the filling of the interior of the classical trajectory by coherent states. Transition from dispersionless to dispersionful Toda lattice corresponds to the quantization of the initial dynamical system.

1. Recently it was recognized that it is possible to attribute the so called $\tau$ function to any analytic curve in two dimensions [1, 2]. This can be done as follows. In the simplest case one considers the evolution of the curve when the area inside grows linearly while the moments of the curve being fixed. In more general situation all moments become new variables ("times") too. It appears that in all cases the relevant object which encodes the information on
the properties of the deformation of the curve is the quasiclassical $\tau$ function of the Toda lattice with the additional constraint which selects the solution to the hierarchy.

On the other hand it was shown in [3] that it is possible to attribute the prepotential which is the quasiclassical version of the $\tau$ function to any holomorphic dynamical system. It plays the role of the generating function for the analogue of the S-duality transformation in the context of Hamiltonian dynamics. Moreover it was argued that generically there exists a pair of dynamical systems such that their coordinates and actions get interchanged. Actually one can say that along this viewpoint the quasiclassical $\tau$ function can be attributed to the Riemann surface which is the solution to the equation of motion of the holomorphic system.

The third general setting involving prepotential is the theory of the effective actions. It appears that generically effective actions manifest a lot of universality property which is governed by the hidden integrability. The examples are the identification of the effective action of the N=2 SYM theory with $\tau$ function of the quasiclassical hierarchy [4] and similar integrable structures behind d=2 theories which have been uncovered in [3, 6, 7]. Along this way times in the hierarchy are identified with the coupling constants in the corresponding theories. In more general terms integrability amounts from the deformations of the topological theories or topological sectors in nontopological theories by the proper observables [9].

The goal of this note is to attempt to glue these ideas altogether and formulate the place of $\tau$ function for the arbitrary Hamiltonian system without any appealing to the integrability.

2. Let us remind the key points from [1]. One can consider complex coordinates $\bar{z}, z$ and the curve determined by the equation

$$\bar{z} = S(z)$$

(1)

The Schwarz function $S(z)$ is analytic in a domain including the curve. One more ingredient to be defined is the map of the exterior of the curve to the exterior of the unit disk

$$\omega(z) = \frac{z}{r} + \sum_j p_j z^{-j}$$

(2)

where $\omega$ is defined on the unit circle. It is useful to introduce the moments of the curve

$$t_n = \frac{1}{2\pi in} \oint z^{-n} S(z) dz$$

(3)

$$v_n = \frac{1}{2\pi i} \oint z^n S(z) dz$$

(4)

$$v_0 = \oint \log|z| dz$$

(5)

which yield the following expansion for the Schwarz function

$$S(z) = \sum k t_k z^{k-1} + t_0 z^{-1} + \sum k v_k z^{-k-1}$$

(6)
Let us define the generating function

$$S(z) = \partial_z \Omega(z)$$  \hspace{1cm} (7)

with the following expansion

$$\Omega(z) = \sum_{k=1} t_k z^k + t_0 \log z - \sum_{k=1} v_k z^{-k} - 1/2 v_0$$  \hspace{1cm} (8)

One can easily prove the following relations

$$\partial_{t_0} \Omega(z) = \log \omega(z)$$  \hspace{1cm} (9)

$$\partial_{t_n} \Omega(z) = (z^n(\omega))_+ + 1/2(z^n(\omega))_0$$  \hspace{1cm} (10)

$$\partial_{\bar{t}_n} \Omega(z) = (S^n(\omega))_+ + 1/2(S^n(\omega))_0$$  \hspace{1cm} (11)

The symbol \((S(\omega))_+\) means the truncated Laurent series with only positive powers of \(\omega\) kept and the \((S(\omega))_0\) is the constant term in the series. The differential \(d\Omega\)

$$d\Omega = Sdz + \log \omega dt_0 + \sum (H_k dt_k - \bar{H}_k d\bar{t}_k)$$  \hspace{1cm} (12)

provides the Hamiltonians and \(\Omega\) itself can be immediately identified with the generating function for the canonical transformation from the pair \((z, \bar{z})\) to the canonical pair \((t_0, \log \omega)\).

The dynamical equations read

$$\partial_{t_n} S(z) = \partial_z H_n(z)$$  \hspace{1cm} (13)

$$\partial_{\bar{t}_n} S(z) = \partial_{\bar{z}} \bar{H}_n(z)$$  \hspace{1cm} (14)

and the consistency of (13) (14) yields the zero-curvature condition which amounts to the equations of the dispersionless Toda lattice hierarchy. The first equation of the hierarchy reads as follows

$$\partial^2_{t_1 \bar{t}_1} \phi = \partial_{t_0} e^{\partial_{t_0} \phi}$$  \hspace{1cm} (15)

where \(\partial_{t_0} \phi = 2\log r\). The Lax operator \(L\) coincides with \(z(\omega)\) and its eigenfunction - Baker-Akhiezer function looks as follows \(\Psi = e^{i\Omega}\). Hamiltonians are expressed in terms of the Lax operator

$$H_k = (L^k)_+ + 1/2(L^k)_0$$  \hspace{1cm} (16)

3. In this section we shall interpret the evolution above in terms of dynamics on the phase space assuming that \(\bar{z}, z\) pair yields the phase space of some dynamical system. The curve itself corresponds to the energy level of this dynamical system. With this setup it is clear that Poisson bracket between \(\bar{z}\) and \(z\) is fixed by the standard symplectic form . The next step is the identification of \(\log \omega\) as angle variable hence the identification of the area inside the curve \(t_0\) as the action variable is evident.
The holomorphic variables on the phase space are usually considered as the operators in the Fock space so we identify $z = b; \bar{z} = b^+$. Typically the creation-annihilation operators are expressed in terms of the $(p,q)$ variables however for our needs we assume the action-angle representation. The variable $z = b$ can be expressed in terms of action-angle variables in the particular dynamical system and just this expression provides the Toda lattice Lax operator

$$z(\log w, t_0) = L(\log w, t_0)$$

(17)

Baker-Akhiezer (BA) function is solution to the equation

$$L\Psi(z, t_0) = z\Psi$$

(18)

and the complex $z$ plane is identified with the surface where the spectral parameter of the Toda system lives on.

Now we turn to clarification of the meaning of the BA function in the generic dynamical system. The answer follows from the equation above; BA function is nothing but the coherent wave function in the action representation. Indeed the coherent wave function is the eigenfunction of the creation operator

$$\hat{b}\Psi = b\Psi$$

(19)

The main properties of the coherent states can be summarized as follows [11]. The coherent state can be represented in the form

$$|\alpha> = D(\alpha)|0>$$

(20)

where $|0>$ is the normalized vacuum vector annihilated by the operator $b$ and the $D(\alpha) = \exp(\alpha b^+ - \bar{\alpha} b)$ yields the action of the Heisenberg-Weyl group. The operators $D(\alpha)$ obey the following relation

$$D(\alpha)D(\beta) = e^{2i\text{Im}(\alpha\bar{\beta})}D(\beta)D(\alpha)$$

(21)

The coherent states $|\alpha>$ and $|\beta>$ are not orthogonal moreover the system of the coherent states $|\alpha>$ is overcompleted generically. To get the complete system one can proceed as follows. The lattice on the complex plane $\alpha_{nm} = n\omega_1 + m\omega_2$ can be introduced with the area of the elementary cell equals to the elementary Planck cell $\pi$. In this case the system of the coherent states $|\alpha_{nm}>$ can be expressed in terms of the standard theta-functions on the torus and turns out to be complete.

Note that the system of the coherent states can be defined for the generic group as follows

$$|\psi_g> = T(g)|\psi_0>$$

(22)

If $H$ is the stabilizer of the vector $|\psi_0>$ the coherent state is defined by the point $x=x(g)$ from the homogeneous space $G/H$ corresponding to the element $g$. Moreover the state corresponding to $|x>$ can be identified with the one dimensional projector $P_x = |x><x|$. Now we are ready to recognize the meaning of the generating function $\Omega$. From the equations above it becomes clear that it is the generating
function for the canonical transformations from the $b, b^+$ representation to the angle-action variables. Let us also remark that the role of the Orlov-Shulman operator becomes transparent. This operator looks like $b^+b$ and its eigenvalue counts the "number of particles".

Having identified the BA function for the generic system let us consider the role of the $\tau$ function in the generic case. To this aim it is convenient to use the following formulae for the $\tau$ function

$$\tau(t, W) = <t, \bar{t}|W>$$

where the bra vector depends on times while the ket vector is fixed by the so called point of Grassmanian. One more useful representation is provided by the fermionic language

$$\tau(t, W) = \frac{<N|\Psi(z_1)\cdots\Psi(z_N)|W>}{\Delta(z)}$$

where $\Delta(z)$ is Vandermonde determinant.

Let us briefly comment on the definition of the point of the Grassmanian W. Generally speaking the Grassmanian itself can be considered as a collection of all fermionic Bogolyubov transforms of the vacuum state. Hence we can say that W belongs to the Grassmanian if it is annihilated by some linear combination of the fermionic creation and annihilation operators. Equivalently one could consider the following definition

$$|W> = S|0>, S = \exp \sum_{nm} A_{nm} \bar{\psi}_{-n-1/2} \psi_{-m-1/2}$$

that is S can be considered as the element of $GL(\infty, C)$.

The consideration above suggests the following picture behind the definition of the $\tau$ function. The peculiar classical trajectory of the dynamical system yields the curve on the phase space. Then the domain inside the trajectory is filled by the coherent states for this particular system. Since the coherent state occupies the minimal cell of the phase space the number of the coherent states packed inside the domain is finite and equals N. Since there is only one coherent state per cell for the complete set it actually behaves like a fermion implying a kind of the fermionic representation.

Therefore we can develop the second dynamical system of the Toda lattice type based on the generic dynamical system. The number of the independent time variables in the Toda system amounts from the independent parameters in the potential in the initial system plus additional time attributed to the action variable. The situation is similar to the consideration of the pair of the dual dynamical systems in \cite{3}. The subtle point in that paper is the choice of the Hamiltonians for the dual systems. It seems that consideration in this note provides the unified viewpoint on this issue. Indeed formulae above suggest the set of commuting Hamiltonians for the dual system. Let us emphasize that the choice of the particular initial dynamical system amounts to the choice of the particular solution to the Toda lattice hierarchy.
It is important that the energy level of the dynamical system doesn’t develop the discontinuity in some range of the energies. This happens only if the system approaches separatrices generically existing on the phase space.

Consider the example of the oscillator. Corresponding phase trajectories

\[ E = p^2 + q^2 \]  

(26)

are circles on the phase space (ellipses in the generic case) and the natural complex variables are

\[ z = p + iq \]  

(27)

If we calculate the expression for \( v_0 \) the following prepotential for the oscillator emerges

\[ F = \frac{1}{2} I^2 \log I - \frac{3}{4} I^2 \]  

(28)

where I is the action variable. To get the first dispersionless Toda equation one has to consider the phase trajectories with the shifted center. The position of the center is parameterized by times \( t_1, \bar{t}_1 \). If the Toda equation with three nonvanishing times is solved the prepotential above can be reproduced. Let us also note that the dual Hamiltonian in the sense of [3] is

\[ H = b = \sqrt{t_0} e^{i\theta_0} \]  

(29)

The wave function of the dual system coincides with the coherent wave function in the action representation.

The prepotential for the complex system is defined by its spectral curve

\[ a_D = \frac{\partial F}{\partial a} \]  

(30)

where \( a_D \) and \( a \) are the integrals of the meromorphic differential over A and B cycles. The variable \( a \) is just the action variable. The S duality transformation \( a \to a_D \) actually maps the region of small energies to the region of the large energies. Prepotential obeys the Matone type [3] relation which has the meaning of the Ward identity

\[ \frac{\partial F}{\log \Lambda} = \text{const} H \]  

(31)

where \( H \) is the Hamiltonian of the complex system and \( \Lambda \) is the scale factor. The simple calculation shows that such relation doesn’t hold in the real case.

Let us remind how the prepotential provides the S-duality transformation for the complex dynamical system [3]. The action variables in dynamical system are the integrals of meromorphic differential \( \lambda \) over the A-cycles on the spectral curve. The reason for the B-cycles to be discarded is simply the fact that the B-periods of \( \lambda \) are not independent of the A-periods. On the other hand, one can choose as the independent periods the integrals of \( \lambda \) over any lagrangian subspace in \( H_1(T_b; \mathbb{Z}) \).

This leads to the following structure of the action variables in the holomorphic setting. Locally over a disc in B one chooses a basis in \( H_1 \) of the
fiber together with the set of $A$-cycles. This choice may differ over another
disc. Over the intersection of these discs one has a $Sp(2m, Z)$ transformation
relating the bases which has a natural interpretation as a S-duality trans-
formation. Altogether they form an $Sp(2m, Z)$ bundle. It can be easily shown
the two form:

$$dI_i \wedge dI_i^D$$

vanishes. Therefore one can always locally find a function $F$, such that:

$$I_i^D = \frac{\partial F}{\partial I_i}$$

The angle variables are uniquely reconstructed once the action variables are
known. Since the real system to some extend can be considered as a real
section of the dynamical system one could expect the analogue of the S-
duality for the real system too.

An important outcome from our identification is the natural recipe for
the transition from the dispersionless to dispersionful Toda lattices. Indeed
at the first step of quantization the system one imposes the Bohr-Sommerfeld
quantization condition on the action variable. This is in agreement with the
discretization of $t_0$ variable in Toda lattice. The Planck constant in the initial
system plays the transparent role in the Toda lattice.

4. Let us argue now that there is clear correspondence between the inte-
grable dynamics behind the deformation of the curves which are the phase
space trajectories for the real dynamical systems and the deformation of the
surfaces which represent the trajectories of the complex dynamical systems.
Remind that to formulate the integrable dynamics behind the deformation
of the curves one introduces the following main objects; generating function
$\Omega$, $\tau$ function depending on time variables $t_k$ which are the moments of the
exterior of the classical trajectory as well as the dual variables $v_k$ representing
the moments of interior.

It is not a difficult task to recognize their counterparts in the case of the
surfaces. First let us represent the time variables in the following form

$$v_k = \oint z^k d\Omega(z)$$

$$t_k = \oint z^{-k} d\Omega(z)$$

and compare it with the definition of times in the case of surfaces

$$T_n = res_{\xi=0} \xi^n dS_{SW}$$

$$\frac{\partial F}{\partial T_n} = \frac{1}{2\pi i n} res_{\xi} \xi^{-n} dS_{SW}$$

where $S_{SW}$ in the context of the N=2 SYM theories is the so-called Seiberg-
Witten meromorphic differential whose derivatives with respect of the moduli
of the curve are the holomorphic differentials. We see that the variables in
cases of curves and surfaces are differed just by the Legendre transform. The variable $t_0$ is nothing but the variable

$$a = \oint_A dS$$

(38)

where the integration is over the A cycle on the spectral surface is the complex dynamical system with only one degree of freedom is considered. The definition of conjugated variables $v_n$ is more delicate issue since naively there are no the direct analogue of the integrals over the B cycles

$$a_D = \oint_B dS$$

(39)

in the real case. However it can be seen that the analogous variable for the one degree of freedom can be identified with variable $v_0$ from [1, 2]. If there are several degrees of freedom the corresponding variables $a_i$ are defined as the integrals over the complete set of A cycles.

It is interesting to recognize the real analogue of the modulus of the spectral curve in the complex case

$$\tau = \frac{da_D}{da}$$

(40)

which plays the role of the effective coupling constant in the field theory context. In the problem under consideration the corresponding variable is

$$\log r = \frac{dv_0}{dt_0}$$

(41)

and also exhibits the behaviour similar to the perturbative regime in the field theory. In what follows we shall see more indications that dynamics of the curve has a perturbative regime as a field theory counterpart.

In the context of the Seiberg-Witten solution to the N=2 SUSY YM theory the Whitham dynamics has a transparent renormalization group meaning [10]. Indeed the derivative of the prepotential with respect to the first time in SUSY YM context looks as follows

$$\frac{\partial F}{\partial T_1} = a \frac{\partial F}{\partial a} - 2F = 2H(a)$$

(42)

where the term at the r.h.s. coincides with the Hamiltonian for the dynamical system providing the corresponding spectral curve. The dependence $H(a)$ on the action variable is the intrinsic characteristics of the dynamical system, moreover this term has an anomalous nature from the field theory viewpoint. The analogous equation has been also found for the real case [1] where the observed quadratic dependence on the action in the anomaly term corresponds to the perturbative limit in the field theory framework.

Given such RG interpretation in the complex case one could look for the reasonable RG structure behind the real dynamical system. The following starting point can be chosen. Assume that the RG scale is fixed by the energy corresponding to the classical trajectory or by the variable $t_0 = \mu$. Then the
phase space is naturally divided into the ”IR theory” inside the trajectory and the ”UV theory” outside. Therefore a kind of the RG problem can be formulated. To this aim introduce the classical expectation value of the observable in the theory

$$<O> = \int dp dq O(p, q) \rho(p, q)$$

(43)

where $\rho(p, q)$ is the phase space density (Wigner function). The vacuum expectation value at the scale $\mu$ is naturally introduced if the integration over the phase space up to the energy $\mu$ is performed.

Now we can formulate the following observation - correlators in UV theory in the outer region amount to the coupling constants in the inner region and vice versa

$$\tau(t_0, t_k) = <\exp \sum t_k O_k>$$

(44)

where the matrix element is taken in the IR theory. The key point is that usually effective action $S(\mu)$ identified with the prepotential is defined via integrating out all modes up to the scale $\mu$. From [1] it becomes clear that is has to be done in a clever way; one has to integrate out the part of the phase space outside of the phase trajectory corresponding to the energy $E$. Then the Riemann-Hilbert problem is defined on the RG scale $\mu$ - one has to divide the whole theory into UV and IR parts consistently with the RG flows. Remark that interpretation of the area inside the curve as the RG parameter agrees with the interpretation of the size of the matrix in the matrix model as RG scale [12]. Let us emphasize that the picture emerged is formally identical to the consideration in [10] where $\tau$ function approach amounts to the explicit calculation of the RG dependence of the correlators. Along this viewpoint one could expect that the similar correlators which now carry the information about the spectrum of the system could be found. Therefore instead of the determination of the RG invariant characteristics of the field theories we expect the RG like evaluation of the spectral invariants which would distinguish different behaviour of the Hamiltonian systems.

Let us emphasize that the Toda lattice equations are written for the variable $b(\omega)$ which yields the dependence of the creation operator on the angle variable. If only zero time is involved then the evolution doesn’t change the system however with all times switched on the dynamics proceeds in the space of the dynamical systems or can be interpreted as the RG flows. The integrals of motion in the Toda system are the ”classical correlators” in the IR theory.

5. It was shown in [1] that the $\tau$ function of the curve allows two additional complementary interpretations. First, it has the interpretation of the energy of the fermions with the unit charge inside the curve with the Coulomb interaction taken into account. Secondly it can be considered as the partition function of the normal matrix model with the potential yielding the set of times in the Toda lattice.

Let us speculate on the appearance of the matrix model in a given context. To this aim consider the phase space as a two dimensional noncommutative
plane. Evidently the Planck constant $h$ fixes the scale of noncommutativity due to the canonical commutation relation

$$[x_1, x_2] = i h$$  \hspace{1cm} (45)

The Moyal multiplication law for the functions on the phase space reads

$$f(x) * g(x) = \exp(ih\partial_x \partial_y)_{x=y} g(y) f(x)$$ \hspace{1cm} (46)

Now let us remind the notion of the Morita equivalence. Qualitatively it means that for the rational noncommutativity ($h = p/q$) the noncommutative theory is equivalent to the commutative theory on the rescaled noncommutative manifold. The functions on the commutative manifold become matrix valued with the size of the matrix fixed by the noncommutativity.

Let us apply these circle of ideas to the generic dynamical system. Suppose that the Planck constant is a rational number $h = \frac{1}{N}$. If $N$ is large we are dealing with the quasiclassical approximation. Applying the Morita equivalence, all functions become $N \times N$ matrix valued and the system becomes effectively classical. Therefore Morita transform maps the quantum system into the classical matrix theory. The ”classical” partition function of the matrix model is the prepotential for the dynamical system.

One more indication of the effective classicality of the matrix model follows from the matrix relation $[M^+, M] = 0$ which corresponds to the ”classical” commutation relation. On the other hand the corresponding Planck constant in the Morita equivalent system is $\tilde{h} = N$ which indicates the regime of the strong noncommutativity on the phase plane. Along this viewpoint one could expect the appearance of the peculiar objects relevant for the such regime, namely the noncommutative solitons which in the simplest case correspond to the projector operators $|n><n|$. One the other hand we have already remarked that the coherent states also have the description in terms of the projector operators on the homogeneous spaces. This implies the intriguing conjecture to consider the matrix model as the system of the interacting noncommutative solitons.

Let us make one more remark concerning the different interpretations of the prepotential. It has a interpretation of the energy in the matrix model, action for the c=1 string and somewhat involved interpretation in the context of particle dynamics. This could presumable have the following picture behind; it is energy for the membrane and therefore the action for string. With this picture in mind the matrix model corresponds to the M(atrix) model representation for the membrane. This is in a rough agreement with the chain of T duality transformations of the phase space. Let us also note that in the context of c=1 string ”times” correspond to the vacuum expectation values of tachyons with the different momenta which could be related to the known phenomena of an appearance of the noncommutative solitons(coherent states) as the result of the tachyon condensation.

6. To conclude we have argued that the Toda lattice governs both the deformations of the complex surfaces and analytic curves. These deformations can be considered as the motion in the space of the complex and real dynamical systems respectively or as a kind of RG flows in a peculiar system. The
dynamics with respect to the time $t_0$ can be considered as the RG evolution with $t_0$ playing the role of the scale factor. One could also expect that the discrete symmetries could provide an important tool for investigation of the RG flows like in [15].

The Baker-Akhiezer function was definitely identified as the coherent wave function in the action-angle representation and the generating functional has the interpretation of the generating function of the canonical transformation from holomorphic to action-angle representation. The role of the quantization of the initial dynamical system in the integrability framework is clarified.

The author is grateful to V. Roubtsov, P. Wiegmann and A. Zabrodin for the interesting discussions and University of Angers where the paper has been completed for the hospitality. The work was partially supported by grants INTAS-99-1705 and CRDF-RP1-2108 and by grant for senior scientist fellowship of CNRS 2000.

References

[1] M. Mineev-Weinstein, P. Wiegmann and A. Zabrodin, Phys. Rev. Lett. 84 (2000), 5106

[2] P. Wiegmann and A. Zabrodin, hepth/9909149
I. Kostov, I. Krichever, M. Mineev-Weinstein, P. Wiegmann and A. Zabrodin, hepth/0005259

[3] V. Fock, A. Gorsky, N. Nekrasov and V. Rubtsov, JHEP, 0007, 028, (2000)

[4] A. Gorsky, I. Krichever, A. Marshakov, A. Mironov and A. Morozov, Phys. Lett. B355 (1995) 466

[5] M. Fukuma, H. Kawai and R. Nakayama, IJMP A6 (1991) 1385

[6] S. Cecotti and C. Vafa, Comm. Math. Phys. 150 (1993) 569

[7] R. Dijgraaf, G. Moore and R. Plesser, Nucl. Phys. B394 (1993) 356

[8] M. Matone, Phys. Lett. 357 (1995) 342
E. D’Hoker and D. Phong, B513 (1998) 405

[9] I. Krichever, Fun. Anal Appl. 22 (1989) 200;
I. Krichever, Comm. Math. Phys. 47 (1992) 437
B. Dubrovin, Nucl. Phys. B379 (1992) 627
K. Takasaki and T. Takebe, Rev. Math. Phys. 7 (1995) 743

[10] A. Gorsky, A. Marshakov, A. Mironov and A. Morozov, Nucl. Phys. B527 (1998) 690; hepth/9802007
J. Edelstein, M. Mariño and J. Mas, Nucl. Phys. B541 (1999) 671
K. Takasaki, hepth/9905224
[11] A. Perelomov, "Generalized coherent states and their applications", Springer Verlag, (1986)

[12] E. Brezin and J. Zinn-Justin, Phys. Lett. B228 (1992) 54

[13] R. Gopakumar, S. Minwalla and A. Strominger, JHEP 9909, 032 (1999)

[14] A. Sen, JHEP, 9808, 012 (1998)
    J. Harvey, P Kraus, F. Larsen and E. Martinec, JHEP 0007, 042, (2000)

[15] A. Ritz, Phys. Lett. B434, 54 (1998)
    B. P. Dolan, Phys. Lett. B312, 97 (1993).
    J. I. Latorre and C. A. Lutken, Phys. Lett. B421, 217 (1998)