Scalar meson production in proton-proton and proton-antiproton collisions

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Taking into account the exchange forces between protons of scalar, pseudoscalar, vector and axial vector type the cross sections of neutral and charged scalar mesons $a_0(980)$, $a_+(980)$, $f_0(980)$, $\sigma(600)$ production are calculated. The estimation for the facilities of moderately high energies such as PANDA and NICA are presented. Similar analysis is given for processes of charged and neutral Higgs boson production at high energy proton-proton colliders such as Tevatron, RHIC and LHC. The possible signal of neutral Higgs boson decay into two oppositely charged leptons of different kinds is discussed. Numerical estimations for parameters of modern colliders are given.

I. INTRODUCTION

Proton-proton as well as proton-antiproton colliders of moderately high energies with sufficiently high luminosities provide the source of light scalar mesons. We investigate here the simplest mechanism of meson creation which is the emission from the nucleon while the exchange forces between the nucleons assumed to be mediated by neutral scalar, pseudoscalar, vector and axial-vector mesons.

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We introduce the nucleon-meson interaction vertices as a product of coupling constants estimated in the Nambu-Jona-Lasinio model [1] and the relevant form factors.

For the case of large transfer momentum we chose the form of form factors predicted by quark counting behavior [2].

In such a facilities as NICA (pp collisions with $s = 20 \text{GeV}^2$) and PANDA ($p\bar{p}$ collisions, $s \sim 30 \text{GeV}^2$) the cross-sections of creation of scalar mesons ($a_0(980)$, $a_+(980)$, $f_0(980)$, $\sigma(600)$) for large c.m. angles kinematics of pp-scattering is about $10^3 \text{nb}$. These estimations was done in Sections II and III.

Neutral and charged Higgs bosons production in $pp$ collisions is investigated in the same approach in Section IV. We assume the absence of formfactor suppression considering the scattered proton to produce jet. In that case we estimate the average cross section of the Higgs-boson (charged and neutral) production on the level of several pb at the LHC energy range.

The possible signature of neutral Higgs boson production through it’s decay into two different lepton pairs was investigated in Section V.

The results are illustrated by the relevant spectral distributions.

II. DIFFERENTIAL CROSS SECTIONS OF SCALAR MESON PRODUCTION.

COLLINEAR APPROXIMATION

Keeping in mind the smallness of meson masses compared with the center of mass (c.m.) total energy of the initial nucleons ($M_S \sim M_P \sim M_V \sim M_A \sim M_p \ll \sqrt{s}$) we can apply the chiral amplitudes method. Besides we will consider the cross sections of production of scalar mesons with their emission in the collinear kinematics.
Let us consider the following processes of neutral scalar meson production

\[ p(p_1, \lambda_1) + p(p_2, \lambda_2) \rightarrow p(p_1', \lambda_1') + p(p_2', \lambda_2') + S(k), \quad (2.1) \]

\[ p(p_1, \lambda_1) + \bar{p}(p_2, \lambda_2) \rightarrow p(p_1', \lambda_1') + \bar{p}(p_2', \lambda_2') + S(k), \quad (2.2) \]

where \( S = \sigma(600), f_0(980), a_0(980) \) is the neutral scalar meson. In case of charged scalar meson \( a_+ (980) \) production one of proton in the final state must be replaced by the neutron.

The sufficiently high energies of colliders allows to neglect masses of all particles and use the chiral amplitudes method [4].

The collinear kinematics of scalar meson emission dominates. Using the "quasi-real electron" method [3] we obtain for process (2.1) for scalar meson emission from initial nucleons:

\[ M_{\lambda_1' \lambda_2' \lambda_1 \lambda_2} = \sum_{r=\pm 1} M_{B_{\lambda_1' \lambda_2', \lambda_1 \lambda_2}}^r (p_1 - k, p_2; p_1', p_2') \frac{\bar{u}^r (p_1 - k) u^{\lambda_1} (p_1)}{2 (p_1 k)} g_s + \]

\[ + \sum_{r=\pm 1} M_{B_{\lambda_1' \lambda_2, \lambda_1 \lambda_2}}^r (p_1, p_2 - k; p_1', p_2') \frac{\bar{u}^r (p_2 - k) u^{\lambda_2} (p_2)}{2 (p_2 k)} g_s, \quad (2.3) \]

for the case of emission from final nucleons we obtain

\[ M_{\lambda_1' \lambda_2' \lambda_1 \lambda_2} = \sum_{r=\pm 1} M_{B_{\lambda_1' \lambda_2', \lambda_1 \lambda_2}}^r (p_1, p_2; p_1' + k, p_2') \frac{\bar{u}^{\lambda_1} (p_1') u^r (p_1' + k)}{2 (p_1' k)} g_s + \]

\[ + \sum_{r=\pm 1} M_{B_{\lambda_1' \lambda_2, \lambda_1 \lambda_2}}^r (p_1, p_2; p_1', p_2' + k) \frac{\bar{u}^{\lambda_2} (p_2') u^r (p_2' + k)}{2 (p_2' k)} g_s, \quad (2.4) \]

where \( M_{B_{\lambda_1' \lambda_2, \lambda_1 \lambda_2}}^r \) is the matrix element of \( pp \rightarrow pp \) subprocess in Born approximation.

Chiral states of nucleon and antinucleon are defined as [4]

\[ u^\lambda (p) = \omega_\lambda u(p), \quad v^\lambda (p) = \omega_{-\lambda} v(p), \quad (2.5) \]

\[ \omega_\lambda = \frac{1}{2} (1 + \lambda \gamma_5), \quad \lambda = \pm. \]
The relevant cross sections then have a form (center of mass of initial particles implied):

\[
\frac{d\sigma^{\lambda_1\lambda'_1\lambda_2\lambda'_2}}{dCdx} = \frac{g_5^2 (1-x) L(x)}{64\pi^2 s (2-x(1-C))^2} \left| M_B^{-\lambda_1\lambda'_1\lambda_2\lambda'_2} \right|^2, \quad \text{when } \vec{k}||\vec{p}_1, \quad (2.6)
\]

\[
\frac{d\sigma^{\lambda_1\lambda'_1\lambda_2\lambda'_2}}{dCdx} = \frac{g_5^2 (1-x) L(x)}{64\pi^2 s (2-x(1+C))^2} \left| M_B^{\lambda_1\lambda'_1\lambda_2\lambda'_2} \right|^2, \quad \text{when } \vec{k}||\vec{p}_2, \quad (2.7)
\]

\[
\frac{d\sigma^{\lambda_1\lambda'_1\lambda_2\lambda'_2}}{dCdx} = \frac{g_5^2 (1-x) L(x)}{256\pi^3 s} \left| M_B^{\lambda_1\lambda'_1\lambda_2\lambda'_2} \right|^2, \quad \text{when } \vec{k}||\vec{p}'_1, \quad (2.8)
\]

\[
\frac{d\sigma^{\lambda_1\lambda'_1\lambda_2\lambda'_2}}{dCdx} = \frac{g_5^2 (1-x) L(x)}{256\pi^3 s} \left| M_B^{\lambda_1\lambda'_1\lambda_2\lambda'_2} \right|^2, \quad \text{when } \vec{k}||\vec{p}'_2, \quad (2.9)
\]

where \( x = k_0/E \), where \( k_0 \) and \( E \) are the energies of created scalar meson and initial proton correspondingly. \( C = \cos(\theta) \) and \( \theta \) is the angle between direction of initial proton beam direction \( \vec{p}_1 \) and scattered proton \( \vec{p}'_1 \). The factor \( L(x) \) is the large logarithm which enhances the probability of meson production in collinear kinematics:

\[
L(x) = \ln \left( \frac{4E^2x^2}{M_p^2x^2 + M_S^2} \right), \quad (2.10)
\]

where \( M_p \) and \( M_S \) are the masses of proton and the created scalar meson correspondingly.

The kinematical invariants we define in the following manner

\[
s = 2 (p_1p_2) = 4E^2, \quad t = -2 (p_1p'_1) = -\frac{s}{2} (1 - C),
\]

\[
u = -\frac{s}{2} (1 + C), \quad s + t + u = 0.
\]

In case of colliders of moderate energies the collinear approximation allows to evaluate the cross section with the accuracy of order 10%.

In the next section we will evaluate the Born amplitudes \( M_B^{\lambda_1\lambda'_1\lambda_2\lambda'_2} \) from (2.6), (2.7), (2.8), (2.9).
III. CHIRAL AMPLITUDES OF $pp \rightarrow pp$ SUBPROCESS IN BORN APPROXIMATION

Matrix element of the subprocess

$$p (p_1, \lambda_1) + p (p_2, \lambda_2) \rightarrow p (p'_1, \lambda'_1) + p (p'_2, \lambda'_2) \quad (3.1)$$

have a form:

$$M_{B}^{\lambda_1 \lambda'_1 \lambda_2 \lambda'_2} = \sum_{i=S,P,V,A} \frac{F^2_i (t)}{t} \left[ \bar{u}^{\lambda'_1} (p'_1) \Gamma_i \omega_{\lambda_1} u (p_1) \right] \left[ \bar{u}^{\lambda'_2} (p'_2) \Gamma_i \omega_{\lambda_2} u (p_2) \right] - \sum_{i=S,P,V,A} \frac{F^2_i (u)}{u} \left[ \bar{u}^{\lambda'_2} (p'_2) \Gamma_i \omega_{\lambda_1} u (p_1) \right] \left[ \bar{u}^{\lambda'_1} (p'_1) \Gamma_i \omega_{\lambda_2} u (p_2) \right], \quad (3.2)$$

where $\Gamma_i$ are the vertexes of interaction of the scalar, pseudoscalar, vector and axial-vector mesons with the protons:

$$\Gamma_S = 1, \quad \Gamma_P = \gamma_5, \quad \Gamma_V = \gamma_\mu, \quad \Gamma_A = \gamma_\mu \gamma_5. \quad (3.3)$$

Meson-proton form factors are model-dependent ones. At zero value of momentum transferred they are determined by the relevant current-algebra values. The behavior at large momentum is determined by quark-counting rule:

$$F_i (t) = \frac{g_i}{\left(1 + \frac{-t}{M_0^2}\right)^2}, \quad i = S, P, V, A,$$

$$F_i (u) = \frac{g_i}{\left(1 + \frac{-u}{M_0^2}\right)^2}, \quad Re [F_i (s)] = \frac{g_i}{\left(1 + \frac{s}{M_0^2}\right)^2},$$

where $M_0 \sim 1 - 2 \text{ GeV}$ is the mass scale characterizing the region the quark-counting approximation is valid. The vertex constants $g_i$ we take following the prescription of Nambu-Jona-Lazinio model [1]:

$$g_S = 10, \quad g_P = 12, \quad g_V = g_A = 6. \quad (3.4)$$

Inserting the projection operators

$$P_1 = \frac{N_1}{N_1} = 1, \quad N_1 = [\bar{u} (p_1) \hat{p}'_1 \omega_+ u (p_1)] [\bar{u} (p_2) \hat{p}'_2 \omega_+ u (p'_2)], \quad (3.5)$$

$$P_2 = \frac{N_2}{N_2} = 1, \quad N_2 = [\bar{u} (p_1) \hat{p}'_2 \omega_+ u (p'_1)] [\bar{u} (p_2) \hat{p}'_1 \omega_+ u (p'_2)], \quad (3.6)$$
into (3.2) and using the completeness condition of type \( u^\lambda \rho (p) \bar{u}^\lambda \rho (p) = \omega \rho \), we obtain for \( M_B^{+++} \):

\[
M_B^{+++} = \frac{S_1}{N_1} \frac{1}{t} - \frac{S_2}{N_2} \frac{1}{u},
\]

(3.7)

\[
S_1 = 2ts^2 \left( F_V^2 (t) + F_A^2 (t) \right), \quad S_2 = 2us^2 \left( F_V^2 (u) + F_A^2 (u) \right).
\]

Using further the explicit values of \( N_{1,2} \):

\[
|N_1|^2 = (st)^2, \quad |N_2|^2 = (su)^2, \quad N_1 N_2^* = -s^2 tu,
\]

(3.8)

we obtain for \( |M_{pp}^{+++}|^2 \):

\[
|M_{pp}^{+++}|^2 = 4s^2 \left| \frac{F_V^2 (t) + F_A^2 (t)}{t} + \frac{F_V^2 (u) + F_A^2 (u)}{u} \right|^2.
\]

(3.9)

Performing the similar algebraic manipulations we calculate the remaining non-vanishing chiral amplitudes \( |M_{pp}^{\lambda_1 \lambda_1' \lambda_2 \lambda_2'}|^2 \):

\[
|M_{pp}^{+++--}|^2 = \left| \frac{2u}{t} \left( F_V^2 (t) - F_A^2 (t) \right) - \left( F_S^2 (u) - F_P^2 (u) \right) \right|^2,
\]

(3.10)

\[
|M_{pp}^{-++-}|^2 = \left( F_S^2 (t) - F_P^2 (t) \right) - \frac{2t}{u} \left( F_V^2 (u) - F_A^2 (u) \right)^2,
\]

\[
|M_{pp}^{-+-+}|^2 = F_S^2 (t) + F_P^2 (t) + F_S^2 (u) + F_P^2 (u)^2.
\]

Similar calculation can be done for case of proton-antiproton collision \( |M_{pp}^{\lambda_1 \lambda_1' \lambda_2 \lambda_2'}|^2 \):

\[
|M_{pp}^{---++}|^2 = \left| \frac{2s}{t} \left( F_V^2 (t) - F_A^2 (t) \right) - \left( F_S^2 (s) - F_P^2 (s) \right) \right|^2,
\]

(3.11)

\[
|M_{pp}^{+++--}|^2 = 4u^2 \left| \frac{F_V^2 (t) + F_A^2 (t)}{t} + \frac{F_V^2 (s) + F_A^2 (s)}{s} \right|^2,
\]

\[
|M_{pp}^{-++-}|^2 = \left( F_S^2 (t) - F_P^2 (t) + F_S^2 (s) - F_P^2 (s) \right)^2,
\]

\[
|M_{pp}^{-+-+}|^2 = \left( F_S^2 (t) + F_P^2 (t) \right) - \frac{2t}{s} \left( F_V^2 (s) + F_A^2 (s) \right)^2.
\]
In case of proton-neutron and antiproton-neutron scattering we obtain:

\[
|M_{pn}^{++-}|^2 = \frac{4s^2}{t^2} |F_V^2(t) + F_A^2(t)|^2,
\]

\[
|M_{pn}^{++-}|^2 = \frac{4u^2}{t^2} |F_V^2(t) - F_A^2(t)|^2,
\]

\[
|M_{pn}^{+-+-}|^2 = |F_S^2(t) - F_P^2(t)|^2,
\]

\[
|M_{pn}^{++-}|^2 = |F_S^2(t) + F_P^2(t)|^2,
\]

\[
|M_{pn}^{++-}|^2 = \frac{4s^2}{t^2} |F_V^2(t) - F_A^2(t)|^2,
\]

\[
|M_{pn}^{++-}|^2 = \frac{4u^2}{t^2} |F_V^2(t) + F_A^2(t)|^2,
\]

\[
|M_{pn}^{++-}|^2 = |F_S^2(t) - F_P^2(t)|^2,
\]

\[
|M_{pn}^{++-}|^2 = |F_S^2(t) + F_P^2(t)|^2.
\] (3.13)

Due to parity conservation we have \( |M^\lambda|^2 = |M^{-\lambda}|^2 \), so we can choose \( \lambda_1 = + \).

The spectral distributions

\[
\frac{1}{\sigma_0} \frac{d\sigma}{dcdx} = \Phi(x, c), \quad \sigma_0 = \frac{g_s^2}{64\pi^3s} = \frac{20 \text{ \mu b}}{s(\text{GeV}^2)},
\] (3.14)

for several fixed values of \( C = \cos(\theta) \) are presented in Fig. 1, 2, 3, 4.

IV. NEUTRAL AND CHARGED HIGGS BOSON PRODUCTION IN \( pp \) COLLISIONS

Consider the simplest channel of neutral Higgs boson production at proton-proton collision:

\[
p(p_1) + p(p_2) \rightarrow p(p'_1) + p(p'_2) + H(k),
\] (4.1)

\[
s = (p_1 + p_2)^2 \gg k^2 = M^2 \gg M_p^2.
\]

We suppose Higgs boson interact coherently with quarks of a proton, resulting in the vertex \( g_H \) of Higgs boson-proton-proton-proton interaction of a form [6]:

\[
g_H = \frac{M_p}{v}, \quad v = 256 \text{ GeV}.
\] (4.2)
FIG. 1: The value $\Phi(x)$ for production of $f_0(980)$ meson in case when angle between the momenta of initial and scattered protons is $\theta = 60^\circ$ (where $C = \cos(\theta)$. $x = 2E_{f_0}/\sqrt{s}$ is the energy fraction of $f_0$ meson. The solid line is the case then $f_0$ meson is produced along $\vec{p}_1$ (i.e. $d\sigma^{+++(\vec{k}||\vec{p}_1})$, dashed line is the case then $f_0$ meson is produced along $\vec{p}'_1$ (i.e. $d\sigma^{+++(\vec{k}||\vec{p}'_1})$).

FIG. 2: The notations are the same as in Fig. 1. The case when $\theta = 90^\circ$. 
FIG. 3: The notations are the same as in Fig. 1. The case when $\theta = 60^\circ$ and the spiral amplitude spin state is $\{\lambda\} = \{- + - -\}$.

FIG. 4: The notations are the same as in Fig. 1. The case when $\theta = 90^\circ$ and the spiral amplitude spin state is $\{\lambda\} = \{- + - -\}$.
Taking into account the identity of protons in the final state only 8 tree type Feynman diagrams must be considered. The relevant contributions schematically can be written as

$$g_H \mathcal{M} = g_H [M_1 + M_2 + M_3 + M_4],$$

$$M_1 = \sum_{i=S,P,V,A} F_i^2(t_1) \frac{\bar{u}(p_2') \Gamma_i u(p_2)}{t_1} \bar{u}(p_1') O_i^i u(p_1), \quad O_1^i = -\frac{\Gamma_i \hat{k}}{\chi_1} + \frac{\hat{k} \Gamma_i}{\chi_1'};$$

$$M_2 = \sum_{i=S,P,V,A} F_i^2(t) \frac{\bar{u}(p_1') \Gamma_i u(p_1)}{t} \bar{u}(p_2') O_2^i u(p_2), \quad O_2^i = -\frac{\Gamma_i \hat{k}}{\chi_2} + \frac{\hat{k} \Gamma_i}{\chi_2'};$$

$$M_3 = -\sum_{i=S,P,V,A} F_i^2(u_1) \frac{\bar{u}(p_1') \Gamma_i u(p_1)}{u_1} \bar{u}(p_2') O_3^i u(p_1), \quad O_3^i = -\frac{\Gamma_i \hat{k}}{\chi_1} + \frac{\hat{k} \Gamma_i}{\chi_1'};$$

$$M_4 = -\sum_{i=S,P,V,A} F_i^2(u) \frac{\bar{u}(p_1') \Gamma_i u(p_1)}{u} \bar{u}(p_2') O_4^i u(p_2), \quad O_4^i = -\frac{\Gamma_i \hat{k}}{\chi_2} + \frac{\hat{k} \Gamma_i}{\chi_2'},$$

with the kinematical invariants now defined as

$$s = 2p_1 p_2, \quad t = -2p_1 p_1', \quad u = -2p_1 p_2',$$

$$s_1 = 2p_1 p_2', \quad t_1 = -2p_2 p_2', \quad u_1 = -2p_2 p_1',$$

$$\chi_i = 2p_i k, \quad \chi' = 2p_i' k, \quad p_i^2 = p_i'^2 = 0, \quad i = 1, 2; \quad k^2 = M_H^2,$$

$$s + s_1 + t + t_1 + u + u_1 = M_H^2,$$

(4.3)

indexes $S, P, V, A$ denote the type of exchange forces between the nucleons. With the choice $\lambda_1 = +1$ only 4 chiral amplitudes entering in the matrix element squared given above are nonzero:

$$\mathcal{M}^{+++}, \quad \mathcal{M}^{++-}, \quad \mathcal{M}^{+-+}, \quad \mathcal{M}^{++-}.$$ 

In terms of $M_i$ they are:

$$\mathcal{M}^{+++} = M_1^{s,p} + M_2^{v,a} + M_3^{v,a} + M_4^{s,p};$$

$$\mathcal{M}^{++-} = M_1^{v,a} + M_2^{s,p} + M_3^{v,a} + M_4^{s,p};$$

$$\mathcal{M}^{+-+} = M_1^{v,a} + M_2^{s,p} + M_3^{s,p} + M_4^{v,a};$$

$$\mathcal{M}^{++-} = M_1^{s,p} + M_2^{v,a} + M_3^{s,p} + M_4^{v,a}.$$
Now we use again the projection operators

\[ P_{12} = \frac{N_{12}}{N_{12}}, \quad P_{34} = \frac{N_{34}}{N_{34}}. \]  

(4.4)

We can choose the quantities \( N_{12}, N_{34} \) for each chiral state to provide their equality

\[ |N_{12}|_\{\lambda\}^2 = |N_{34}|_\{\lambda\}^2 = N_{12}^{\{\lambda\}} N_{34}^{\{\lambda\}} \equiv N^{\{\lambda\}}, \]  

(4.5)

where \( \{\lambda\} = \{\lambda_1 \lambda_1' \lambda_2 \lambda_2'\} \). Using the completeness condition for the chiral states of protons \( u_\lambda(p) \bar{u}_\lambda(p) = \omega_\lambda \hat{p} \) chiral amplitudes can be written in form

\[ \mathcal{M}^{\{\lambda\}} = \left[ \frac{1}{N_{12}} \left( \frac{S_1}{t_1} + \frac{S_2}{t} \right) - \frac{1}{N_{34}} \left( \frac{S_3}{u_1} + \frac{S_4}{u} \right) \right]_{\{\lambda\}}. \]  

(4.6)

And the matrix element square then reads as

\[ |\mathcal{M}^{\{\lambda\}}|^2 = \frac{1}{N^{\{\lambda\}}} \left| \frac{S_1}{t_1} + \frac{S_2}{t} - \frac{S_3}{u_1} - \frac{S_4}{u} \right|_{\{\lambda\}}^2. \]  

(4.7)

Below we present the explicit form of the values from (4.7) for definite choice of \( \{\lambda\} = \{\lambda_1 \lambda_1' \lambda_2 \lambda_2'\} \). In case then \( \{\lambda_1 \lambda_1' \lambda_2 \lambda_2'\} = \{+++-\} \) we have:

\[ N_{12} = [\bar{u}_2 \hat{p}'_2 \omega_+ u'_1'] [\bar{u}_1 \omega_- u'_2]; \quad N^{+++-} = s_1 t_1 u; \]

\[ N_{34} = [\bar{u}_2 \omega_- u'_1'] [\bar{u}_1 \hat{p}'_2 \omega_+ u'_1]; \]

\[ S_1 = F^2_S(t_1) Tr \left[ \hat{p}'_2 \Gamma S \hat{p}'_2 \hat{p}'_1 O_1^S \hat{p}_1 \omega_- \right] + F^2_P(t_1) Tr \left[ \hat{p}'_2 \Gamma P \hat{p}'_2 \hat{p}'_1 O_1^P \hat{p}_1 \omega_- \right]; \]

\[ S_2 = F^2_V(t) Tr \left[ \hat{p}'_1 \Gamma V \hat{p}_1 \hat{p}'_2 O_2^V \hat{p}_2 \omega_+ \right] + F^2_A(t) Tr \left[ \hat{p}'_1 \Gamma A \hat{p}_1 \hat{p}'_2 O_2^A \hat{p}_2 \omega_+ \right]; \]

\[ S_3 = F^2_V(u_1) Tr \left[ \hat{p}'_1 \Gamma V \hat{p}_1 \hat{p}'_2 O_3^V \hat{p}_2 \omega_+ \right] + F^2_A(u_1) Tr \left[ \hat{p}'_1 \Gamma A \hat{p}_1 \hat{p}'_2 O_3^A \hat{p}_2 \omega_+ \right]; \]

\[ S_4 = F^2_S(u) Tr \left[ \hat{p}'_2 \Gamma S \hat{p}_2 \hat{p}'_1 O_4^S \hat{p}_1 \omega_- \right] + F^2_P(u) Tr \left[ \hat{p}'_2 \Gamma P \hat{p}_2 \hat{p}'_1 O_4^P \hat{p}_1 \omega_- \right]. \]

For chiral state \( \{+-+-\} \) we have

\[ N_{12} = [\bar{u}_2 \hat{p}_1 \omega_- u'_1'] [\bar{u}_1 \omega_- u'_2]; \quad N^{+-+-} = s t u; \]

\[ N_{34} = [\bar{u}_2 \hat{p}_1 \omega_- u'_2] [\bar{u}_1 \omega_- u'_1]; \]

\[ S_1 = F^2_V(t_1) Tr \left[ \hat{p}'_2 \Gamma V \hat{p}_2 \hat{p}_1 \hat{p}'_1 O_1^V \hat{p}_1 \omega_- \right] + F^2_A(t_1) Tr \left[ \hat{p}'_2 \Gamma A \hat{p}_2 \hat{p}_1 \hat{p}'_1 O_1^A \hat{p}_1 \omega_- \right]; \]
\[ S_2 = F_S^2(t) \, T_r \left[ \hat{p}_1' \Gamma_S \hat{p}_1 \hat{p}_2 O_S^2 \hat{p}_2 \hat{p}_1 \omega_- \right] + F_P^2(t) \, T_r \left[ \hat{p}_1' \Gamma_P \hat{p}_1 \hat{p}_2 O_P^2 \hat{p}_2 \hat{p}_1 \omega_- \right]; \]

\[ S_3 = F_V^2(u_1) \, T_r \left[ \hat{p}_1' \Gamma_V \hat{p}_2 \hat{p}_1 \hat{p}_2 O_V^2 \hat{p}_2 \hat{p}_1 \omega_- \right] + F_A^2(u_1) \, T_r \left[ \hat{p}_1' \Gamma_A \hat{p}_2 \hat{p}_1 \hat{p}_2 O_A^2 \hat{p}_2 \hat{p}_1 \omega_- \right]; \]

\[ S_4 = F_S^2(u) \, T_r \left[ \hat{p}_1' \Gamma_S \hat{p}_1 \hat{p}_1' O_S^2 \hat{p}_2 \hat{p}_1 \omega_- \right] + F_P^2(u) \, T_r \left[ \hat{p}_2' \Gamma_P \hat{p}_1 \hat{p}_1' O_P^2 \hat{p}_2 \hat{p}_1 \omega_- \right]. \]

For chiral state \( \{++--\} \) we have

\[ N_{12} = [\bar{u}_2 \omega_+ u_1'] [\bar{u}_1 \hat{p}_2 \omega_+ u_2'] ; \quad N^{++--} = s t_1 u_1; \]

\[ N_{34} = [\bar{u}_2 \omega_+ u_2'] [\bar{u}_1 \hat{p}_2 \omega_+ u_1'] ; \]

\[ S_1 = F_S^2(t_1) \, T_r \left[ \hat{p}_2' \Gamma_S \hat{p}_2 \hat{p}_1' O_1^S \hat{p}_1 \hat{p}_2 \omega_+ \right] + F_P^2(t_1) \, T_r \left[ \hat{p}_2' \Gamma_P \hat{p}_2 \hat{p}_1' O_1^P \hat{p}_1 \hat{p}_2 \omega_+ \right]; \]

\[ S_2 = F_V^2(t) \, T_r \left[ \hat{p}_1' \Gamma_V \hat{p}_2 \hat{p}_1 \hat{p}_2 \hat{p}_2' O_V^2 \hat{p}_2 \hat{p}_1 \omega_+ \right] + F_A^2(t) \, T_r \left[ \hat{p}_1' \Gamma_A \hat{p}_2 \hat{p}_1 \hat{p}_2 \hat{p}_2' O_A^2 \hat{p}_2 \hat{p}_1 \omega_+ \right]; \]

\[ S_3 = F_S^2(u_1) \, T_r \left[ \hat{p}_1' \Gamma_S \hat{p}_2 \hat{p}_1' O_3^S \hat{p}_2 \hat{p}_1 \omega_+ \right] + F_P^2(u_1) \, T_r \left[ \hat{p}_1' \Gamma_P \hat{p}_2 \hat{p}_1' O_3^P \hat{p}_2 \hat{p}_1 \omega_+ \right]; \]

\[ S_4 = F_V^2(u) \, T_r \left[ \hat{p}_2' \Gamma_V \hat{p}_1 \hat{p}_2 \hat{p}_1' O_4^V \hat{p}_2 \hat{p}_1 \omega_+ \right] + F_A^2(u) \, T_r \left[ \hat{p}_2' \Gamma_A \hat{p}_1 \hat{p}_2 \hat{p}_1' O_4^A \hat{p}_2 \hat{p}_1 \omega_+ \right]. \]

For chiral state \( \{-++-\} \) we have

\[ N_{12} = [\bar{u}_2 \omega_- u_1'] [\bar{u}_1 \hat{p}_1' \omega_- u_2'] ; \quad N^{-+++} = s t_1 u_1; \]

\[ N_{34} = [\bar{u}_2 \hat{p}_1' \omega_- u_2'] [\bar{u}_1 \omega_- u_1'] ; \]

\[ S_1 = F_V^2(t_1) \, T_r \left[ \hat{p}_2' \Gamma_V \hat{p}_2 \hat{p}_1' O_1^V \hat{p}_1 \hat{p}_2 \omega_+ \right] + F_A^2(t_1) \, T_r \left[ \hat{p}_2' \Gamma_A \hat{p}_2 \hat{p}_1' O_1^A \hat{p}_1 \hat{p}_2 \omega_+ \right]; \]

\[ S_2 = F_S^2(t) \, T_r \left[ \hat{p}_1' \Gamma_S \hat{p}_1 \hat{p}_1' O_2^S \hat{p}_2 \hat{p}_1 \omega_- \right] + F_P^2(t) \, T_r \left[ \hat{p}_1' \Gamma_P \hat{p}_1 \hat{p}_1' O_2^P \hat{p}_2 \hat{p}_1 \omega_- \right]; \]

\[ S_3 = F_S^2(u_1) \, T_r \left[ \hat{p}_1' \Gamma_S \hat{p}_2 \hat{p}_1' O_3^S \hat{p}_2 \hat{p}_1 \omega_- \right] + F_P^2(u_1) \, T_r \left[ \hat{p}_1' \Gamma_P \hat{p}_2 \hat{p}_1' O_3^P \hat{p}_2 \hat{p}_1 \omega_- \right]; \]

\[ S_4 = F_V^2(u) \, T_r \left[ \hat{p}_2' \Gamma_V \hat{p}_1 \hat{p}_2 \hat{p}_1' O_4^V \hat{p}_2 \hat{p}_1 \omega_- \right] + F_A^2(u) \, T_r \left[ \hat{p}_2' \Gamma_A \hat{p}_1 \hat{p}_2 \hat{p}_1' O_4^A \hat{p}_2 \hat{p}_1 \omega_- \right]. \]

For the case of the charged Higgs boson production we must put \( S_3 = S_4 = 0 \) in (4.7). The cross section of Higgs meson production have a form

\[
\frac{1}{\sigma_H} \frac{d\sigma_H^{(\lambda)}}{dC d\omega} = \frac{dx'dC_0}{\sqrt{D(C_0, C, C')}} |M^{(\lambda)}|^2 s, \quad (4.8)
\]

\[
\sigma_H = \frac{g_H^2}{512 \pi^4 s} = \frac{0.13}{s (\text{GeV}^2)}. \]
where \( x = \omega / E, \ x' = E' / E, \ \sigma = M^2_H / s, \ C_0 = \cos (\vec{p}_1, \vec{p}_1'), \ C = \cos (\vec{k}, \vec{p}_1), \) and

\[
C' = 1 + \frac{2}{x x'} (1 - x - x' + \sigma),
\]

\[
D (C_0, C, C') = 1 - C^2 - C'^2 - C'^2_0 + 2 C C' C_0, \quad D > 0.
\]

The relevant parametrization of kinematical invariants using the conservation laws can be written as:

\[
\chi_1 = \frac{s x}{2} (1 - C), \ \chi_2 = \frac{s x}{2} (1 + C), \ \chi'_1 = s (1 - x_2 - \sigma)
\]

\[
t = -\frac{s x'}{2} (1 - C_0); \ u_1 = -\frac{s x'}{2} (1 + C_0); \ \chi'_2 = s (1 - x' - \sigma);
\]

\[
t_1 = t + \chi'_1 - \chi_1 + M^2_H; \ u = u_1 + \chi'_1 - \chi_2, \ x = \frac{\omega}{E},
\]

\[
x' = \frac{E'_1}{E}, \ x_2 = \frac{E'_2}{E}, \ s = 4 E^2. \quad (4.9)
\]

We will argue below that estimating the Higgs boson production we can replace \( F_i(t) = g_i \) (see Discussion). To make some numerical illustration we present the spectral distributions over produced Higgs boson energy fraction with fixed angles of produces Higgs boson momenta

\[
\frac{1}{\sigma_H} \frac{d \sigma}{d C d x} = \Phi_H (C, x) = \int_{-1}^{1} d C_0 \int_{x_0}^{1} d x' \cdot \frac{s \cdot |M^\lambda|^2}{\sqrt{D}}, \quad D > 0. \quad (4.10)
\]

Keeping in mind the condition of possibility to measure the final particles (we avoid collinear and planar kinematics) we put additional constrains: \( D > 0.01. \)

The mass of Higgs boson we took as \( M_{H_0} = M_{H_+} = 140 \) GeV. The spectral distributions \( \Phi_H (C, x) \) for neutral Higgs boson production within these assumptions for several values of Higgs boson emission angle \( C = \cos(\theta), \ \theta = 50^o - 80^o \) are presented in Fig.5. The obtained results for neutral Higgs boson production are in the satisfactory agreement with estimations given in [6].
FIG. 5: The cross section for production of Higgs boson in $pp$ collisions at $\sqrt{s} = 14$ TeV as a function of Higgs energy fraction $x = 2\omega/\sqrt{s}$. Different curves correspond to different angle of between the initial beam direction and the direction of produces Higgs boson momenta $C = \cos(\theta)$. Non-complanarity condition is $D > 0.01$.

V. SIGNATURE OF NEUTRAL HIGGS BOSON

One of the clear signal of Higgs boson existence can be seen in its decay $H \rightarrow W^+W^- \rightarrow \mu^+\nu_\mu + e^-\bar{\nu}_e$. Really in final state we will have two different leptons of opposite sign of electric charge. In the experiment the energy fractions of leptons and the angle between the directions of their emission in the system of rest of Higgs boson can be measured. We show below that this quantities obey non-trivial kinematical inequality. To obtain this inequality we consider the phase volume of reaction:

$$H \rightarrow W^+(q_+) + W^-(q_-) \rightarrow (\mu^+(p_+) + \nu_\mu(p_{\nu 1})) + (e^-(p_-) + \bar{\nu}_e(p_{\nu 2})), \quad (5.1)$$

which has the form

$$\frac{d^3p_{\nu 1}}{2E_{\nu 1}} \frac{d^3p_{\nu 2}}{2E_{\nu 2}} \frac{d^3p_+}{2E_+} \frac{d^3p_-}{2E_-} \delta^4(p_H - p_{\nu 1} - p_{\nu 2} - p_e - p_{e}) = d^4p_{\nu 1}\delta \left(p_{\nu 1}^2\right) d^4p_{\nu 2}\delta \left(p_{\nu 2}^2\right) \times$$
\[
\frac{d^3p_+ d^3p_-}{2E_+ 2E_-} \delta^4(p_H - p_{\nu 1} - p_{\nu 2} - p_{e 1} - p_{e 2}).
\] (5.2)

Let’s make some transformations in this formula:

\[
d^4p_1 d^4p_2 \to d^4q_+ d^4q_- , \quad \text{(using momenta conservation laws)},
\]

\[
\frac{d^3p_+ d^3p_-}{2E_+ 2E_-} \to \frac{1}{4} E_+ dE_+ E_- dE_- d\Omega_+ d\Omega_-,
\] (5.3)

where \(d\Omega_+\), \(d\Omega_-\) are the angular dependence of final leptons momenta \(p_+\) and \(p_-\). Let us now select the pivot direction, say \(\vec{q}_-\), which will be the initial direction to measure all the angles from. Then:

\[
d\Omega_+ d\Omega_- = 2\pi dC_- dC_+ d\phi,
\] (5.4)

where \(C_\pm = \cos \theta_\pm\) and \(\theta_\pm\) are the angles between \(\vec{p}_\pm\) and \(\vec{q}_-\) and \(\phi\) is the angle between the plane \((\vec{q}_-, \vec{p}_+\)) and the plane \((\vec{q}_-, \vec{p}_-\)). So, now we introduce angle \(\theta\) (the angle between \(p_+\) and \(p_-\)) and rewrite the part of phase volume as:

\[
dC_- dC_+ d\phi = dC_- dC_+ d\phi dC \delta (C - C_+ C_- + S_+ S_- \cos \phi) = \frac{2dC_- dC_+ dC}{|S_+ S_- \sin \phi|}(5.5)
\]

where we introduced new integration over \(dC\) \((C = \cos \theta)\) with \(\theta\) is the angle between the momenta of the charged leptons (rest frame of Higgs boson is implied). The additional \(\delta\)-function and then we cut off the integration over \(\phi\) using this \(\delta\)-function. The denominator \((S_+ S_- \sin \phi)\) can be transformed as follows:

\[
|S_+ S_- \sin \phi|^2 = S_+^2 S_-^2 - S_+^2 S_-^2 C^2 = 1 - C_-^2 - C_+^2 - C^2 + 2C_+ C_- C = D_H,
\] (5.6)

where we used the \(\delta\)-function from (5.5). After that we use mass-shell \(\delta\)-functions of neutrinos \(\delta (p_{\nu 1,2}^2)\) to integrate over \(dC_-\) and \(dC_+\) in (5.5):

\[
\delta (p_{\nu 1,2}^2) = \delta ((q_+ - p_\mp)^2) = \delta (M_W^2 + m_{\mp}^2 - 2E E_+ \mp 2\beta E_{C_{\mp}}) =
\]
\[
\frac{1}{2\beta E_{\mp}} \delta (C_{\mp} - a_{\mp}), \quad (5.7)
\]
where \( \beta = |\vec{q}_{\pm}| / |E_{\pm}| = \sqrt{1 - \frac{4M_W^2}{M_H^2}} m_{\pm} \) is the mass of corresponding final lepton and

\[
a = \frac{-M_W^2 + 2EE_{-}}{2\beta EE_{-}} = -\frac{1 - \beta^2 - x}{x\beta}, \quad x = \frac{4E_-}{M_H}; \quad (5.8)
\]
\[
b = \frac{M_W^2 - 2EE_{+}}{2\beta EE_{+}} = \frac{1 - \beta^2 - y}{y\beta}, \quad y = \frac{4E_+}{M_H}. \quad (5.9)
\]

After that the phase volume (5.5) reads as

\[
\frac{2dC_\mp dC_+ dC}{\sqrt{D}}, \quad (5.10)
\]
which leads to constrain:

\[
D = 1 - \left( -\frac{1 - \beta^2 - x}{x\beta} \right)^2 - \left( \frac{1 - \beta^2 - y}{y\beta} \right)^2 - C^2 + 2 \left( -\frac{1 - \beta^2 - x}{x\beta} \right) \left( \frac{1 - \beta^2 - y}{y\beta} \right) C > 0. \quad (5.11)
\]
So now the size and the shape of this kinematically allowed region depend on the Higgs-mass (see (5.8), (5.9)). And putting the actual events to 3D-Dalitz plot over three variables \((x, y \text{ and } C)\) and fitting the 3D-body which this events points fill we can find out the mass of Higgs, i.e. the smallest mass of Higgs which embraces tightly all the measured points by the surface \(D = 0\).

\section{VI. DISCUSSION}

We have presented a calculated cross section for Higgs boson production at the LHC, in the decay \(H \to WW \to l\nu l\nu\). The calculation takes into account all the experimental cuts designed to isolate the Higgs boson signal \([7, 8]\). In the case of the decay mode \(H \to WW \to l\nu l\nu\), we confirm previous findings that the effect of radiative corrections is strongly reduced by the selection cuts.
We present the spectral distribution on Higgs energy fraction at some its emission angles. Our calculations for Higgs boson production was performed in suggestion of independence of all vertex functions on the momenta of the particles entering them. We argue now that this approach is rather realistic. Really the main mechanisms consists in creation of the jets. The elastic formfactors contributions are suppressed by Sudakov type mechanism, whereas the inelastic formfactors or the creation of a jets are effectively constant of order of ones used above.

We do not consider the two-gamma mechanism of Higgs boson production as well as it is irrelevant for the large angles kinematics considered here. The ”total” cross section obtained by the phase volume integration with the reasonable experimental cuts imposed (the invariant mass of jets of order of proton mass, the angles between the beam axis and emitted particles as well as between the final particles supposed to exceed some value $\theta_i > 30^0$ is the quantity of order $1pb$. From the detailed analysis of the background situation the Higgs boson decay channel through intermediate two Z- boson state with the subsequent decay each of them to the pair of leptons can be identified in data analysis on the level of five standard deviations (see [5]). Further tagging of a pair of different leptons and the angle between their direction of motion is described in the previous section. As for the channel $H \rightarrow W^+W^- \rightarrow l_1\bar{\nu}_1l_2\nu_2$, the formidable background appears from the processes with creation of $b\bar{b}$ (see [5]).

The accuracy of cross section of light scalar meson production is estimated on the level of $15 - 20\%$. It is caused by the frames of validity of chiral amplitudes method (omission of terms of order $O(M^2_p/s)$), collinear kinematics approximation and some uncertainty in the choice of the nucleon formfactors.
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