Designing Fee Tables for Retail Delivery Services by Third-party Logistics Providers

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Abstract

Manufacturers are increasingly relying on third-party logistics service providers to distribute their products to retail stores. Fee tables, specifying how much to pay for each delivery based on weight and distance, are commonly used as the basis for compensating distributors for their delivery services. This paper proposes and applies an optimization model and methodology to help a large building-products manufacturer design an appropriate fee table for payments to its distributors for delivering products from regional distribution centers to retail stores. The model selects the weight and distance ranges and sets the fees for each combination of ranges, taking into account the distances and distribution of shipment weights to each store, so as to ensure adequate total compensation for each distributor while satisfying fee monotonicity and other requirements. Since our mixed integer programming model is difficult to solve using commercial solvers, we develop a tailored approach to obtain near-optimal solutions quickly. Specifically, we identify valid inequalities based on the problem’s special structure to strengthen the model formulation, and develop an optimization-based procedure, that solves related network flow problems, to generate a heuristic solution. When applied to actual data from the building products manufacturer, our composite solution method, combining cutting planes and heuristic, proved to be effective (yielding heuristic solutions that are within 1% of optimality) and generated substantial savings (of nearly 10%) over the current fee table.
1. Introduction

As retail supply chains have grown and evolved, manufacturers are increasingly relying on third-party logistics providers (3PLs) to deliver product replenishments to the numerous retail outlets. This trend is evident in the dramatic growth of the 3PL industry in the U.S. from $56 billion in 2000 to an estimated $142.2 billion in 2012. In 2010, more than 82% of Fortune 500 companies used 3PLs (Armstrong & Associates 2012). Such partnerships permit manufacturing firms to focus on their core competencies while assuring high service levels to retail stores. These collaborative arrangements raise the important issue of how much the manufacturer should pay the 3PL provider for their delivery services. This paper stems from a project to address this issue for a leading building products manufacturer.

The firm sells its products through national hardware and home improvement retail chains that operate thousands of stores nationwide. The retailers want the manufacturer to periodically (once or twice a week) deliver product replenishments directly to each store. Stores can order any needed quantities of the manufacturer’s products up to 24 hours before the scheduled delivery time; for most stores, the total quantity to be delivered is less than truckload. To meet these delivery requirements, the manufacturer established 13 regional distribution centers (RDCs) nationwide, located so that each store is within 300 miles or so (to meet the short delivery lead time requirement) from its assigned RDC. Each RDC receives bulk shipments of products from the manufacturer’s factories, and is responsible for fulfilling the replenishment orders from its assigned stores. To deliver the products from RDCs to stores, the manufacturer decided to partner with independent regional logistics providers, one for each RDC. These service providers, who we call distributors, are small firms that own and operate the delivery trucks and have limited geographical coverage (i.e., they only operate within the region, rather than nationally); moreover, the manufacturer is typically their largest, if not only, client and commits to using their services for several years. In this setting, the manufacturer wished to develop a systematic approach to compensate the distributors.

Following industry practice, the manufacturer adopted a fee-based payment scheme that compensates distributors for each store delivery they make. The payment depends on the store’s distance from the RDC and the actual weight of the shipment to the store, and is based on a mutually agreed fee table that divides the possible delivery distances and weights into a discrete set of ranges and specifies the delivery fee to be paid in each distance and weight range. Published commercial less-than-truckload (LTL) rates provide some guidance on the desired characteristics of delivery fees (e.g., increasing range widths) and their magnitude (e.g., upper and lower bounds). However, neither the specific ranges of these tariff tables nor their published rates were considered appropriate for the manufacturer’s delivery operations due to
the distinctive characteristics of these operations. For instance, published LTL rates are typically applicable to one-off and/or long-haul shipments (from hundreds to thousands of miles) by nationwide carriers, do not reflect any discounts for overall shipment volume (across stores) or long-term commitments, and do not necessarily apply to scheduled deliveries with tight time windows. On the other hand, for deliveries to its retail customers, the manufacturer relies on small independent and regional distributors whose cost structure and capacity sharing opportunities differ from those of long-haul carriers. Delivery distances are relatively short, ranging from tens to a few hundred miles. The manufacturer commits to using a distributor for a year or more, and exclusively assigns over a hundred stores, essentially establishing a long-term partnership with the distributors rather than simply viewing them as contract carriers for ad hoc shipments. Finally, since all store shipments originate at the RDC and are less-than-truckload, the distributor can deploy a single truck to deliver to multiple stores on each trip (versus point-to-point movements). For these reasons, the manufacturer wanted to develop a “customized” fee table that takes into account the nature and scope of its delivery needs, while also ensuring that each distributor is able to cover costs and make profits. This paper proposes an optimization model to address this problem, which we call the Fee Table Design problem, develops an effective solution methodology, and applies the model to actual data from the manufacturer’s distribution network.

The academic literature related to compensation for 3PL delivery services is relatively sparse, and largely focuses on conceptual or qualitative discussions of contracting and best practices, but does not provide methods to determine the actual fees to be paid for these services. Recognizing the importance of contracts to ensure successful 3PL partnerships, some researchers have attempted to identify key factors that drive the viability and sustainability of these alliances. For instance, Lambert et al. (1999) and Logan (2000) examined 3PL partnerships using various strategic theories, listed some important criteria for the partners to consider, and emphasized the need to design outcome-based rather than behavior-based contracts. van Hoek (2000) observed the prevalence of two types of contract relationships—“detailed and fixed” and “open-ended and broad”—in 3PL partnerships, and delineated the factors that lead to the use of each type of contract. Using a case study, Halldorsson and Skjoett-Larson (2006) highlighted the importance of using contracts that are dynamic (i.e., can be revisited periodically) for the success of long-term partnerships. We note that the fee table scheme that the manufacturer used to compensate distributors incorporates many of the desirable contract attributes discussed in these papers; this scheme is outcome-based (fees per delivery made), detailed (specifying an unambiguous pricing mechanism), and dynamically adaptable (permitting changes to fees to reflect changes to operations and costs over time). Industry experts (Cram 1996, Carter 1998) have supplemented academic theory on 3PL compensation by
offering practical tips and guidance on commonly-used pricing mechanisms. These papers and many others in the 3PL literature have focused on applying empirical models and methods to identify desirable attributes of 3PL contracts, but do not offer specific decision support models or methodologies (Lukassen and Wallenburg 2010, Marasco 2008, Selviardis and Spring 2007, Maloni and Carter 2006).

Another stream of literature describes model-based approaches for pricing shipments and determining tariffs. Brotcorne et al. (2000) proposed and solved a bi-level optimization model for tariff-setting by a commercial carrier for truckload transport on various lanes that it operates, taking into account the response of shippers to these lane tariffs. King and Topaloglu (2007) developed a model for dynamic fleet management and lane pricing, considering revenues from price-dependent lane demands and costs for re-positioning vehicles on the lane network. Using a bi-level optimization framework, Yano and Newman (2007) developed a model and method for determining day-of-the-week and speed-of-service pricing for an express package delivery service provider to better utilizes its resources while incorporating customers’ shipping patterns as a function of the prices. These models do not directly apply to the problem of setting fees for periodic delivery services of varying loads to multiple stores from a distribution center.

For the problem of developing a fee table, Balakrishnan, Natarajan, and Pangburn (2000) formulated a linear program to decide fee values for a given fee table structure, i.e., when the distance and weight ranges are specified. We consider the higher level problem of determining both the structure of the table and the fee values to use for each combination of ranges in this table. For this new Fee Table Design problem, we propose a novel mixed-integer programming model, and develop a tailored methodology to effectively solve the problem. Among the methodological contributions, we identify several new classes of valid inequalities (and a variable elimination method) to strengthen the problem formulation, develop a cutting plane approach to iteratively add these inequalities, and propose an optimization-based heuristic algorithm to generate good quality feasible solutions. These enhancements significantly improve computational performance. We test this approach using actual data from the manufacturer’s distribution network, and demonstrate the effectiveness of both the model and the method by comparing the solutions with current practice.

The rest of the paper is organized as follows. Section 2 elaborates on the Fee Table Design problem, and formulates it as a mixed-integer program. Section 3 develops valid inequalities to strengthen the problem formulation. Section 4 outlines our cutting plane procedure to incorporate these inequalities, and an optimization-based method that uses the solution to the strong model formulation to generate good heuristic solutions (that also serve as the starting upper bounds for a branch-and-bound procedure).
Section 5 reports on the application of our model and methodology to actual data for the manufacturer’s retail delivery operations, and validates the economic benefits of using our approach. Section 6 offers concluding remarks.

2. Problem context and model formulation

2.1 Problem description

During the past decade, two important trends have characterized the transformation of the retail industry – the emergence and growth of large nationwide retail chains, and increasing reliance on third-party logistics (3PL) providers for transportation in retail supply chains. For instance, in the home improvement sector of retailing, large retail chains such as Home Depot and Lowes have become dominant by offering a wide assortment of products at competitive prices. According to its website (www.homedepot.com), Home Depot operates over 2,000 retail stores in the U.S. and Canada, each of which carries over 40,000 products. These retailers leverage their market share and aggregate volume by negotiating low prices with suppliers, and specifying stringent delivery requirements, in the form of frequent deliveries in small lot sizes to individual stores, so as to ensure high product availability while controlling store inventories. These demanding requirements, combined with strategic choices by manufacturers to focus on their core competence, have led firms to develop partnerships with 3PL providers to deliver their products to retail stores.

The building products manufacturer we worked with relied on large retail chains for the bulk of its sales of residential products. Faced with the challenges of providing responsive and cost-effective service to each of these customers’ stores, the manufacturer set up a nationwide distribution network with 13 regional distribution centers, and selected independent regional trucking companies, one for each RDC, to deliver products from the RDC to stores. Each RDC serves an exclusive geographical region, and is responsible for fulfilling the replenishment orders from all the retail stores in this region. On average, RDCs were assigned 150 stores each, located from tens of miles to a few hundred miles from the distribution center. Figure 1 shows the histogram of delivery distances for stores served by a particular RDC. The average delivery distance is about two hundred miles; most stores are located within 350 miles from the RDC, with some relatively close to the RDC.
Each retail store places and receives orders from its assigned RDC, and requires one or more deliveries per week at scheduled dates and times. Stores can order products in small quantities (e.g., cartons rather than pallet loads), and are permitted to place their orders up to 24 hours before the scheduled delivery time to the store. Thus, the total quantity ordered by each store varies from week to week, but is often significantly less than a truckload. Figure 2 shows a histogram of the mean weekly shipment weight across all stores assigned to the RDC. As the figure shows, over 30% of the stores have mean shipment weight of less than 2,500 pounds per week, and the average shipment weight across stores does not exceed 20,000 pounds whereas a truck can carry up to 43,000 pounds.  

Distributors are responsible for picking up the items ordered by the stores (in its coverage area) from the RDC and delivering it to the store locations at the scheduled times. The distributors own and operate their own trucks, and plan their delivery routes. The manufacturer and distributors agreed that payments for deliveries will be based on a fee table that specifies the fee payable for weights and distances that lie within specified ranges. Table 1 shows a sample fee table with nine distance ranges, covering the possible distances from 0 to 500 miles, and seven weight ranges spanning 0 to 20,000 pounds. We refer to the specification of the ranges (their respective starting values and widths) along the two dimensions, distance and weight, as the fee table structure, and to each pair of distance range and weight range as a block. The table specifies the fee value for each block of the table; this value represents the payment to be made to the distributor for a delivery whose distance from the RDC to the store and weight falls within that block. For instance, if a distributor makes a delivery of 2,200 pounds to a store that is 140 miles

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Figure 1. Histogram of delivery distances for a RDC

Figure 2. Histogram of mean weight per shipment for a RDC

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1 Since transported items can vary in density, the freight industry uses the so-called dimensional weight which is the pound equivalent adjusted for density. Throughout the paper, “pounds” refers to the dimensional weight applicable to the manufacturer’s products (which fall under the category of dry goods).
from the RDC, this delivery falls within the block defined by the fifth distance range and the third weight range in the fee table shown in Table 1; so, the distributor receives a fee of $508.28 for this delivery. Observe that, the distance of a store from its RDC is fixed, but the shipment weights can vary from week to week. Hence, the payments that the manufacturer makes to the distributor for delivering to the same store can also vary from week to week if the shipment weights span more than one block.

| Delivery distance (miles) | 0 – 24 | 25 – 49 | 50 – 79 | 80 – 119 | 120 – 169 | 170 – 219 | 220 – 274 | 275 – 349 | 350 – 500 |
|--------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0 – 499                  | 100.40 | 146.84 | 156.37 | 166.16 | 175.70 | 194.53 | 213.35 | 232.18 | 251.00 |
| 500 – 999                | 142.82 | 199.80 | 219.63 | 229.16 | 242.22 | 272.59 | 302.96 | 333.58 | 363.95 |
| 1000 – 2499             | 205.57 | 215.36 | 268.07 | 333.58 | 375.25 | 428.96 | 482.42 | 536.14 | 589.85 |
| 2500 – 4999             | 221.13 | 277.61 | 354.41 | 431.47 | 508.28 | 585.08 | 661.89 | 738.94 | 815.75 |
| 5000 – 7499             | 240.96 | 341.11 | 441.26 | 541.16 | 641.31 | 741.45 | 841.35 | 941.50 | 1041.65 |
| 7500 – 12499            | 301.20 | 436.24 | 571.03 | 706.06 | 840.85 | 975.89 | 1110.68 | 1245.71 | 1380.50 |
| 12500 – 20000           | 657.62 | 889.04 | 1120.72 | 1352.14 | 1583.81 | 1815.23 | 2046.91 | 2278.33 | 2510.00 |

Table 1. Sample fee table

The fee table mimics the tariff structure used by commercial carriers (e.g., Middlewest Motor Freight Bureau, SMC). It divides each dimension into a discrete set of distance ranges and weight ranges, and specifies the rate for each block. For the building products delivery context, this type of payment scheme was preferred because it is well-accepted in the industry, is unambiguous and easy to administer, and is outcome-based (i.e., payments are based on actual deliveries).

As we noted earlier, published LTL freight rate tables – their structure or values of fees – were not considered appropriate for the building products retail delivery setting because of the distinctive characteristics of this setting. Instead, the firm wanted to develop a customized fee table that accounts for the distribution of shipment weights and delivery distances to stores within each region and the distributors’ costs, but broadly conforms to commercial rates. We next discuss these requirements for the fee table structure and fee values.

First, as illustrated in the fee table shown in Table 1, the widths of the ranges need not be equal. However, in practice, the widths are non-decreasing in distance and weight. So, if we index the ranges from smaller to higher distances (or weights), higher ranges must not have smaller widths than lower ranges. Second, although the fee values in the table may change from year to year (e.g., due to changing commercial rates vary by class of product, where each class is determined by freight characteristics such as density, stowability, handling, and liability (National Motor Freight Traffic Association, Inc., www.nmfta.org). All of the building product manufacturer’s products belong to the same class, and so one fee table suffices.

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2 Commercial rates vary by class of product, where each class is determined by freight characteristics such as density, stowability, handling, and liability (National Motor Freight Traffic Association, Inc., www.nmfta.org). All of the building product manufacturer’s products belong to the same class, and so one fee table suffices.
geographical coverage of RDCs, increase in costs), the firm preferred to keep the fee table structure, i.e., the specification of distance and weight ranges, fixed for three years or so. We refer to each year (or the time span over which the fee values are fixed) as a period, and the longer span of the fee table structure as the planning horizon. Third, fee values must be non-decreasing with distance, weight, and time. Fourth, the fee values for different distance-weight combinations must lie within certain specified upper and lower bounds that may be agreed upon or derived from benchmarks rates. Finally, the fee table should not have too many or too few ranges, both to ensure that using the table is not operationally cumbersome and to provide flexibility in fee setting. We, therefore, assume that the user specifies (and can parametrically vary) the number of distance and weight ranges that the table should contain. These fee structure and value requirements collectively govern the shape of the fee value function.

For the fee table to be acceptable to distributors, it must also be adequate to cover their costs and yield some profits. The costs incurred by the distributors are not solely determined by distances and weights. For instance, distributors have to invest in trucks, hire and pay drivers, incur recurring costs for maintenance, depreciation, and insurance, and so on. Moreover, their actual distance- and weight-dependent costs (e.g., for fuel) depend on how they combine store deliveries into single trips originating at the RDC. Rather than match the fee paid for each delivery to the cost associated with that delivery distance and weight (which is anyway difficult to determine due to allocations of various fixed costs, and so on), the firm and distributors wanted to ensure that distributors costs are covered in “aggregate,” i.e., the total anticipated payments made to each distributor over all deliveries over, say, a month must at least cover the distributor’s monthly cost plus a profit margin. Based on information from the distributors on underlying costs and through benchmarking, the manufacturer was able to develop estimates of the total cost for each distributor in every year of the planning horizon to meet the anticipated store demands in their respective coverage areas. We refer to the desired total payment (equal to the estimated cost plus profit margin) to a distributor in each period as the compensation target for that distributor.

In summary, the problem of designing a fee table, applicable to all distributors in the retail delivery system, entails selecting a fee table structure for the planning horizon and deciding the fee values for each block of this table for every year of the planning horizon so as to satisfy the shape requirements and ensure that the total expected payment to each distributor, given the distribution of shipment weights for each store assigned to that distributor, equals or exceeds the annual compensation target for that distributor. To ensure equity and avoid adverse comparisons, the manufacturer wanted to construct a fee table that minimizes the excess payments to distributors over their respective compensation targets. This
objective also serves to minimize the total payments that the firm makes to cover all of its retail delivery needs during the planning horizon. We refer to this problem as the Fee Table Design (FTD) problem.

Manually deciding the structure of the fee table and the fee values for each period during the planning horizon is difficult both because of the vast number of choices and the various requirements. We next discuss our approach for modeling this problem, and formulate it as a mixed-integer program that incorporates all of the fee table design requirements.

2.2 Modeling approach

The fee paid for a particular shipment depends on the distance and weight ranges in which the delivery falls, and the fee value for that block, both of which must be decided. Moreover, shipment weights are random (with known empirical distributions or parametric distributions that are not necessarily analytically tractable), and we need to capture the expected payments in terms of the decision variables. With these characteristics, it is not obvious how to model the FTD problem without nonlinearities; further, none of the known classes of optimization models are appropriate for this problem. We propose a novel modeling approach that discretizes each dimension (delivery distance and shipment weight) into small intervals, permitting us to represent the FTD problem as a (linear) mixed integer program. Specifically, we divide the distance axis into $N$ intervals that span all delivery distances from zero to the maximum delivery distance. Similarly, we consider $M$ intervals that cover all shipment weights from zero to the maximum shipment weight over all stores, and aggregate these intervals to form the weight ranges for the table. Together, the distance and weight intervals create a $M \times N$ interval grid. Given this grid, the range selection decisions translate to deciding which intervals, on each axis, to combine to form the desired number of ranges. Accordingly, we define binary variables to represent the choice of contiguous intervals that are aggregated into each range; these variables determine the “structure” of the recommended fee table. Using additional (continuous) decision variables to represent fee values in each cell on the interval grid, we ensure through forcing constraints that cells belonging to the same ranges have the same fee value. We also incorporate the practical requirements on the choice of ranges and fee values, i.e., that the range widths must be increasing, and fees for each range should lie within specified bounds, and monotonically increase with distance, weight, and time.

2.3 Notation and problem formulation

We consider the fee design problem over a planning horizon of $T$ periods, indexed as $t = 1, 2, \ldots, T$. The structure of the table, i.e., the specification of the distance and weight ranges (to be decided), remains the same for the entire horizon, but the fee values in the table may change (increase) from period to period to
accommodate changes in delivery requirements and costs. Let $S$ denote the set of all retail locations to which the manufacturer’s products need to be delivered, and let $R$ be the set of regional distributors that the firm uses to deliver products. Each regional distributor $r \in R$ has an exclusive geographic coverage area, and is responsible for deliveries to the stores within that coverage area. Due to new store openings and closures in each period, the set of stores assigned to a distributor can change from one period to the next. Let $\Omega(r,t)$ denote the set of stores that are assigned to distributor $r$ in period $t$. For every store $s \in \Omega(r,t)$, let $q_s$ be the store’s delivery distance from its assigned distribution center to the store location, $f_s$ the number of deliveries per period, and $\Phi_{st}(\mathcal{g})$ the probability distribution of shipment weights to the store in period $t$. Since store demands change over time, we permit the delivery frequency and shipment weight distribution to vary by period $t$. Table 2 summarizes our notation.

| Indices and Sets | Parameters | Decision variables |
|------------------|------------|-------------------|
| $h, H$ Weight range index, $h = 1, 2, \ldots, H$ | $a_{ijrt}$ Expected number of deliveries by distributor $r$ in period $t$ that fall in weight interval $i$ and distance interval $j$ | $P_{ijt}$ Fee value for cell $<i, j>$ in period $t$ |
| $k, K$ Distance range index, $k = 1, 2, \ldots, K$ | $C_{rt}$ Compensation target for distributor $r$ in period $t$ | $X_{h,j_1,j_2}^t$ Range-indexed distance range selection; 1 if the $h^{th}$ distance range starts at $j_1$ and $j_2$, and 0 otherwise |
| $i, I$ Weight (row) interval index, $i \in I = \{1, 2, \ldots, M\}$ | $d_{ij}^t$ Upper limit of distance interval $j$ | $\gamma_{h,i_1,i_2}^t$ Range-indexed weight range selection; 1 if the $h^{th}$ weight range starts at $i_1$ and $i_2$, and 0 otherwise |
| $j, J$ Distance (column) interval index, $j \in J = \{1, 2, \ldots, N\}$ | $w_i$ Upper limit of weight interval $i$ | $\Omega(r,t)$ Set of stores covered by distributor $r$ in period $t$ |
| $<i, j>$ Interval grid cell corresponding weight interval $i$ and distance interval $j$ | $f_{st}$ Frequency of deliveries to store $s$ in period $t$ | |
| $r, R$ Distributor index, $r \in R$ | $q_s$ Delivery distance for store $s$ from its assigned distributor | |
| $s, S$ Store index, $s \in S$ | $l_{ijt}$ Minimum permissible fee value in cell $<i, j>$ in period $t$ | |
| $t, T$ Time period index, $t = 1, 2, \ldots, T$ | $u_{ijt}$ Maximum permissible fee value in cell $<i, j>$ in period $t$ | |
| $\Phi_{st}(\mathcal{g})$ Distribution of shipment weight to store $s$ in period $t$ | |

Table 2. Summary of notation

The FTD problem entails selecting weight and distance ranges for the planning horizon, and fee values for each period of the horizon, while ensuring adequate compensation for each of the distributors.
Let $C_r$ be the compensation target (minimum expected total payment needed) for distributor $r$ in period $t$.

As discussed earlier, our modeling approach first divides the distance axis into $N$ fine-grained intervals, indexed by $j = 1, 2, \ldots, N$, as shown in Figure 3. Distance interval $j$ corresponds to delivery distances from $d_{j-1}$ to $d_j$, where $0 = d_0 < d_1 < \ldots < d_N \geq \max_{s \in S} \{q_s\}$. Similarly, we divide the weight axis into $M$ intervals (see Figure 3), with each interval $i$ corresponding to shipment weights from $w_{i-1}$ to $w_i$. Every pair of weight and distance intervals forms a cell $<i, j>$ in the interval grid. Let $K (< M)$ and $H (< N)$ respectively denote the desired number of distance and weight ranges for the fee table, spanning all possible values of shipment distances and weights.

With this framework, the decisions on the table structure separate into two sets of range selection choices, one each for weight and distance. For $1 \leq j_1 \leq j_2 \leq N$ and $k = 1, 2, \ldots, K$, we define a distance range selection binary variable $X_{j_1, j_2}^k$ that takes the value 1 if the $k^{th}$ distance range begins at interval $j_1$ and ends at interval $j_2$ (inclusive), and is 0 otherwise. Similarly, for $1 \leq i_1 \leq i_2 \leq M$ and $h = 1, 2, \ldots, H$, we define a weight range selection binary variable $Y_{i_1, i_2}^h$ that is 1 when the $h^{th}$ weight range spans the intervals from $i_1$ to $i_2$ (including both $i_1$ and $i_2$), and is 0 otherwise. The curved arrows in Figure 3 illustrate these range selection decisions for the weight and distance axes. To model the fee value decisions, we define

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3 The intervals need not be uniform, i.e., their widths ($d_j - d_{j-1}$) can vary with $j$. 

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continuous variables \( P_{ijt} \) denoting the delivery fee in each cell \(<i,j>\) for period \( t \).

We next discuss how to express the expected fees that a distributor \( r \) receives in every period \( t \) in terms of our decision variables. For each store \( s \), let \( f(s) \) denote the index of the distance interval corresponding to the RDC-to-store delivery distance \( q_s \) to this store, i.e., \( f(s) \) is the interval index such that \( d_{f(s)-1} \leq q_s < d_{f(s)} \). With \( \Phi_s(g) \) denoting the cumulative distribution of shipment weight to store \( s \) in period \( t \), the probability that the weight of a shipment to this store lies in the \( i^{th} \) weight interval, i.e., between \( w_{i-1} \) and \( w_i \), is \( \left( \Phi_s(w_i) - \Phi_s(w_{i-1}) \right) \). Hence, the expected number of deliveries in period \( t \) to store \( s \) that fall in the \( i^{th} \) weight interval is \( \left( \Phi_s(w_i) - \Phi_s(w_{i-1}) \right) \times f_{s,t} \). Consequently, the expected total number of deliveries by a distributor \( r \) to all of its assigned stores that fall in cell \(<i,j>\) in period \( t \) is:

\[
a_{ijt} = \sum_{s \in I(r)} \left( \Phi_s(w_i) - \Phi_s(w_{i-1}) \right) \times f_{s,t}.
\]

Based on the given data on delivery weight distributions and frequencies, we can compute this parameter \( a_{ijt} \) for all cells, distributors, and periods. The total expected payment to distributor \( r \) in period \( t \) is then \( \sum_j \sum_{ij} a_{ijt} P_{ijt} \). We require this payment to exceed distributor \( r \)'s target (minimum) total compensation \( C_{rt} \) for the period. The minimum and maximum permissible fee values translate to lower and upper bounds \( l_{ijt} \) and \( u_{ijt} \) on the fee value decision variable \( P_{ijt} \) for each cell \(<i,j>\) in period \( t \). Given the fee monotonicity requirements, these lower and upper bounds must be non-decreasing with shipment weight, delivery distance, and time.

Using these decision variables and parameters, we can formulate the Fee Table Design problem as the following mixed-integer program, which we denote as [FTD]:

\[
\text{Minimize} \quad \sum_I \sum_J \sum_{ij} \sum_{jt} a_{ijt} P_{ijt}
\]

subject to:

\[
\sum_{j \in I} X_{1,j}^I = 1 \quad \text{and} \quad \sum_{j \in J} X_{j,N}^F = 1
\]

\[
\sum_{j \in I} X_{k,j}^I = \sum_{j \in J} X_{k+1,j}^I \quad \text{for all } j \in J \setminus \{N\}, k = 1, 2, \ldots, K - 1,
\]

\[
P_{i,j+1,t} - P_{ijt} \leq (u_{ij+1,t} - l_{ijt}) \sum_{j \in J} X_{k,j}^I \quad \text{for all } i \in I, j \in J \setminus \{N\}, t = 1, 2, \ldots, T,
\]

\[
\sum_{j \in J} Y_{1,j}^H = 1 \quad \text{and} \quad \sum_{j \in J} Y_{j,H}^M = 1
\]

\[
\sum_{j \in J} Y_{h,j}^H = \sum_{j \in J} Y_{h+1,j}^H \quad \text{for all } i \in I \setminus \{M\}, h = 1, 2, \ldots, H - 1,
\]

\[
P_{i+1,j,t} - P_{ijt} \leq (u_{i+1,j,t} - l_{ijt}) \sum_{j \in J} Y_{h,j}^H \quad \text{for all } i \in I \setminus \{M\}, j \in J, t = 1, 2, \ldots, T,
\]

\[
\sum_I \sum_{jt} a_{ijt} P_{ijt} \geq C_{rt} \quad \text{for all } r = 1, 2, \ldots, R, t = 1, 2, \ldots, T,
\]

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\[ l_{ijt} \leq P_{ijt} \leq u_{ijt} \quad \text{for all } i \in I, j \in J, t = 1, 2, \ldots, T, \]

\[ P_{i,j+1,t} - P_{ijt} \geq 0 \quad \text{for all } i \in I, j \in J \setminus \{N\}, t = 1, 2, \ldots, T, \]

\[ P_{i+1,j,t} - P_{ijt} \geq 0 \quad \text{for all } i \in I \setminus \{1\}, j \in J, t = 1, 2, \ldots, T, \]

\[ P_{i,j,t+1} - P_{ijt} \geq 0 \quad \text{for all } i \in I, j \in J, t = 1, 2, \ldots, T - 1, \]

\[ \sum_{j \in \{1\}} \sum_{i \in \{1\}} (d_{j2} - d_{j1})X_{j1j2}^k \leq \sum_{j \in \{1\}} \sum_{i \in \{1\}} (d_{j2} - d_{j1})X_{j1j2}^{k+1} \quad \text{for all } k = 1, 2, \ldots, K - 1, \]

\[ \sum_{j \in \{1\}} \sum_{i \in \{1\}} (w_{j2} - w_{j1})Y_{j1j2}^h \leq \sum_{j \in \{1\}} \sum_{i \in \{1\}} (w_{j2} - w_{j1})Y_{j1j2}^{h+1} \quad \text{for all } h = 1, 2, \ldots, H - 1, \]

\[ X_{j1j2}^k = 0 \text{ or } 1 \quad \text{for } j_1 = 1, 2, \ldots, N, j_2 \geq j_1, k = 1, 2, \ldots, K, \text{ and } \]

\[ Y_{j1j2}^h = 0 \text{ or } 1 \quad \text{for } i_1 = 1, 2, \ldots, M, i_2 \geq i_1, h = 1, 2, \ldots, H. \]

The firm’s goal is to minimize the excess payments it makes to distributors (over their respective compensation targets) over the planning horizon using the chosen fee table. Since the compensation targets are constants, the model achieves this goal by minimizing the total expected payments (1) to all distributors over all T periods, which is \( \sum_{i,j} d_{ij} P_{ij} \). Constraints (2) and (3) model the distance range selection decisions. Constraints (2) specify that the first and last ranges must, respectively, include intervals 1 and N. Constraints (3) ensure range contiguity by requiring that, if the \( k^{th} \) distance range ends at interval \( j \), then the next range \( (k+1) \) must begin at interval \( (j+1) \). Constraints (4) link the range selection decision variables to the fee value variables \( P_{ij} \). Specifically, if two contiguous distance intervals \( j \) and \( (j+1) \) belong to the same distance range (i.e., if \( \sum_i \sum_j X_{j1j2}^k = 0 \)), then the constraint requires the fee values in cells \( (i, j) \) and \( (i, j+1) \) to be the same. Constraints (5) and (6), model the interval coverage and range contiguity requirements, analogous to the distance range selection constraints (2) and (3), for the weight ranges. And, like constraint (4), constraint (7) specifies that, if two contiguous weight intervals \( i \) and \( (i+1) \) belong to the same weight range, then the fee values in the cells \( (i, j) \) and \( (i+1, j) \) must be the same. Thus, constraints (4) and (7) together require that all the cells within a particular block of delivery distances and shipment sizes have the same fee values. Constraints (8) are the fee adequacy requirements, specifying that, for every distributor \( r \), the expected payment in each period \( t \) must be greater than or equal to the distributor’s target compensation. Constraints (9) apply the lower and upper bounds on the fee value of each cell \( <i, j> \) in each period \( t \). Next, constraints (10) – (14) incorporate the desired “shape” restrictions on the fee values (monotonicity) and range choices (increasing widths).

Constraints (10), (11), and (12) ensure that the fee values are non-decreasing with distance, weight, and time, respectively. Constraint (13) requires the width of the \( (k+1)^{th} \) range to be greater than or equal to
the width of range \( k \). The expressions on the left-hand side and right-hand side of this constraint represent the range widths of the \( k \)th and \((k+1)\)th ranges in terms of the range selection variables. The validity of these expressions stem from the property (implied by constraints (2) and (3)) that \( \sum_{h} \sum_{j_{2}>j_{1}} X^{k}_{h,j_{2}} = 1 \) for every range index \( k \). Constraints (14) similarly impose the increasing width requirement for the weight ranges. Finally, constraints (15) and (16) are the integrality requirements for the range selection variables.

As with most integer programs, there are alternative ways to formulate the FTD problem. For instance, in formulation \([FTD]\) we have chosen to define separate range selection variables, \( X^{k}_{h,j_{2}} \) and \( Y^{h}_{h,j_{2}} \), for each range index \((k \text{ or } h)\). Instead of using these range-indexed variables to model the range choices, we can alternatively use variables that omit the range indices \( k \) and \( h \) and just indicate which contiguous (distance and weight) intervals to combine into a single range. For illustrative purposes, we discuss this alternate representation for just the distance range selection decisions. For every pair of distance intervals \( j_{1} \) and \( j_{2} \), with \( 1 \leq j_{1} \leq j_{2} \leq N \), suppose we define the binary variable \( V^{k}_{h,j_{2}} \) to represent the distance range selection decisions; this variable that takes the value one if a distance range starts at interval \( j_{1} \) and ends at \( j_{2} \), and zero otherwise. The new \( V \)-variables are aggregated versions of our previous \( X \)-variables, i.e., \( V^{k}_{h,j_{2}} = \sum_{h} X^{k}_{h,j_{2}} \) for all \( 1 \leq j_{1} \leq j_{2} \leq N \). We can modify the constraints of the FTD problem formulation in terms of these variables, instead of the \( X \)-variables, while retaining linearity. Although using the \( V \)-variables (and analogous variables for weight range selection) reduces the number of variables in the model, it also weakens the formulation, i.e., the model’s linear programming (LP) relaxation is not as tight as that of model \([FTD]\). Further, the new model does not permit the modeling enhancements, such as problem reduction and strengthening using valid inequalities, that we discuss in Section 3. Since having a strong LP relaxation is a key requirement for the success of our solution strategy, we work with (and enhance) formulation \([FTD]\) containing the range-indexed variables.

Observe that, in practice, the desired number of ranges, \( H \) and \( K \), are relatively small (e.g., less than ten), and so the range-indexed model remains well within capabilities (in terms of model size limits) of common solvers.

In preliminary computations, commercial solvers with standard branch-and-bound algorithms were not able to solve formulation \([FTD]\) effectively due to poor linear programming (LP) lower bounds and initial upper bounds (from heuristic solutions generated by the solver) that were far from optimal. We, therefore, pursued a two-pronged strategy of (i) strengthening the model formulation by eliminating variables and developing several classes of valid inequalities (Section 3), and (ii) improving the initial upper bound using an optimization-based heuristic procedure (Section 4).
3. Strengthening the model formulation

Strong formulations with tight LP relaxation values are valuable for two main reasons. First, they provide superior lower bounds on the optimal values, thereby accelerating exact procedures such as branch-and-bound and providing more accurate assessment of the quality of heuristic solutions. Second, the solution to the LP relaxation of the strong formulation often provides a better starting point for LP-rounding methods to generate integer feasible solutions.

Identifying ways to strengthen the model formulation requires first understanding the structure of the problem’s LP solutions and the drivers of the integrality gap between the LP value and optimal value of the integer program. For the [FTD] model, the LP relaxation achieves a lower objective value by choosing fractional values for the range selection variables \( X_{j,l}^k \) and \( Y_{l}^{h} \) (effectively selecting multiple fractional starting and ending intervals for each range, but satisfying the range selection requirements) so that the solution can set different fee values for different cells even if these cells belong to the same block. To limit the fractional values of range selection variables and raise the LP value, we tighten the range width constraints of formulation [FTD], eliminate some variables in this model (without eliminating feasible solutions), and develop several families of additional valid inequalities. These strategies exploit formulation [FTD]’s use of range-indexed variables for range selection, and also utilize the following two properties of the FTD problem: (a) the increasing range width property, i.e., higher-indexed ranges must not have smaller widths than lower-indexed ranges, and (b) the fee value in a block must satisfy the fee upper and lower bounds of every cell in the block. The following subsections elaborate on these model strengthening strategies.

3.1 Disaggregated increasing range width constraints

To satisfy the distance range width constraints (13) in formulation [FTD], the LP solution only needs to ensure that the “average” width of the \((k+1)^{th}\) range is greater than or equal to the “average” width of the \(k^{th}\) range, where the averages are taken over all the cell pairs \(\{j’, j’’\}\) spanned by the respective ranges. We can strengthen these constraints by disaggregating them, i.e., by imposing such constraints for every possible ending interval \(j\) of the \(k^{th}\) range, as follows:

\[
\sum_{j \in J} (d_{j} - d_{j-1}) X_{j,l}^k \leq \sum_{j \in J} (d_{j+1} - d_{j}) X_{j+1,l}^{k+1} \quad \text{for all } j \in J \setminus \{N\}, k = 1, 2, \ldots, K - 1. \tag{17}
\]

The validity of these disaggregated constraints stems from (and exploits) the additional contiguity requirement on the range choices, namely, if range \(k\) ends in interval \(j\), then range \((k+1)\) must begin in interval \((j+1)\). We can show that any fractional solution that satisfies constraints (17) must also satisfy
(13) (since the latter constraints are aggregate versions of the former), but the reverse need not hold; hence, constraints (17) are tighter. We can similarly replace the increasing range width constraints (14) for the weight ranges with the following stronger disaggregated version:

$$\sum_{i \in \mathbb{W}} (w_i - w_{i-1})Y_{i,j}^h \leq \sum_{i \in \mathbb{W}} (w_{i+1} - w_i)Y_{i+1,j}^h$$

for all $i \in \mathbb{W} \setminus \{M\}, h = 1, 2, \ldots, H - 1$. (18)

Note that, since we have one disaggregated constraint for each interval ($i$ or $j$), the number of increasing range width constraints for distance and weight respectively increase by factors of $N$ and $M$.

Nevertheless, for practical problems, the benefit of the revised model’s tighter LP lower bounds outweigh the increase in formulation size. Henceforth, we designate the version of the model with constraints (17) and (18), instead of constraints (13) and (14), as formulation [FTD].

### 3.2 Cumulative range width forcing inequalities

Even with the disaggregate versions (17) and (18) of the increasing range width constraints, the LP solution of [FTD] (which includes the disaggregate constraints) can assign positive values for two adjacent distance range selection variables $X_{h,j}^k$ and $X_{j+1,j+2}^{k+1}$ even if $(d_j - d_{j-1}) > (d_{j+1} - d_j)$ (this choice of adjacent ranges violates the increasing range width requirement and so is prohibited for integer solutions). Despite this infeasible choice of $X$-values, the LP solution satisfies the increasing range width constraints (17) by choosing suitable fractional values for other range selection variables $X_{j+1,j}^t$ and $X_{j+1,j+1}^{t+1}$ for one or more distance intervals $j_3 > j_1$ and/or $j_4 > j_2$. The following class of inequalities prevents such fractional assignments:

$$\sum_{j_3 \in \mathbb{D}, d_{j_3} = d_j} X_{j_3,j}^{k+1} \geq \sum_{j_4 \in \mathbb{D}, d_{j_4} \leq d_j} X_{j_4,j}^k$$

for all $j \in \mathbb{J} \setminus \{N\}, j_1 \leq j, k = 1, 2, \ldots, K - 1$. (19)

The constraint states that, because of the increasing range width requirement, if the solution includes in range $k$ all the cells from interval $j_1$ to $j$ (and possibly other intervals before $j_1$), then it must include in range $(k+1)$ all the cells from interval $(j+1)$ to the lowest-indexed interval $j_3$ (and possibly other intervals after $j_3$) for which $(d_{j_3} - d_j) \geq (d_j - d_{j-1})$. The tightness of constraints (19) stems in part from the unit coefficients for all the variables in these constraints (whereas the variables have coefficients greater than one in the distance range width constraints (17) of model [FTD]). Thus, they can strengthen formulation [FTD]; however, they do not dominate constraints (17), i.e., both sets of constraints can eliminate LP solutions that are feasible for the other. We can also develop constraints analogous to (19) for the weight ranges using the range selection variables $Y_{i,j}^h$ as follows:
\[
\sum_{i \in I, 1 \leq i \leq H} Y^i_{i+1} \geq \sum_{i \in I} Y^h_{i,j} \quad \text{for all } i \in I \setminus \{M\}, \; i_i \leq i, \; h = 1, 2, \ldots, H - 1. \quad (20)
\]

We refer to constraints (19) and (20) as cumulative range width forcing constraints; the following proposition formally establishes their validity.

**PROPOSITION 1.** The cumulative range width constraints (19) and (20) are valid for the FTD problem.

**Proof.** See Appendix 1A.

We note parenthetically that we can formulate another set of inequalities, complementary to (19) and (20), that ensure consistency in range widths between one range and its previous range.

### 3.3 Eliminating some range selection variables

In formulation \([FTD]\), we define range selection variables \(X^k_{j_1, j_2}\) for all \(j_2 \geq j_1\) and \(Y^h_{i,j}\) for all \(i \geq i_1\). We can exploit the increasing range width requirement to omit many of these variables from the formulation.

For the distance range selection variables, the following proposition narrows the feasible interval pairs \((j_1, j_2)\) that the \(k^{th}\) distance range can span, for \(k = 1, 2, \ldots, K\).

**PROPOSITION 2.** In any feasible integer solution to the FTD problem, the range selection variable \(X^k_{j_1, j_2}\) can be non-zero only if the indices \(k, j_1, j_2\) satisfy one of the following three conditions:

1. **First range**: if \(k = 1\), then (a) \(j_1 = 1\), (b) \(j_2 \leq N - (K - 1)\), and (c) \(d_{j_2} \leq (d_N - d_{j_1})/(K - 1)\);
2. **Middle range**: if \(1 < k < K\), then (a) \(j_1 \geq k\), (b) \(j_2 \leq N - (K - k)\), (c) \(d_{j_2} - d_{j_1 - 1} \geq d_{j_1 - 1}/(k - 1)\), and (d) \(d_{j_2} - d_{j_1 - 1} \leq (d_N - d_{j_1})/(K - k)\); or,
3. **Last range**: if \(k = K\), then (a) \(j_1 \geq K\), (b) \(j_2 = N\), and (c) \(d_N - d_{j_1 - 1} \geq d_{j_1 - 1}/(K - 1)\).

**Proof.** See Appendix 1B.

To understand this result, let us consider condition (1) regarding the feasible spans of the first range (with \(k = 1\)). Condition 1(a) states that the first range must necessarily include, and hence start from, the first interval \(j_1 = 1\). Condition 1(b) exploits the fact that each of the ranges following the first range must include at least one interval. Condition 1(c) uses the increasing range width condition in the following way: since the fee table contains \(K\) distance ranges and the widths of the ranges are non-decreasing with the range index, the width of the first range must not exceed the average range width \((d_N - d_{j_1})/(K - 1)\) of the remaining \((K - 1)\) ranges. This observation yields an upper limit on the index \(j_2\) of the highest-indexed cell that the first range can span. The other results (for middle and last ranges) of the proposition similarly use the increasing range width requirement and range definitions to restrict the range spans.

Proposition 2 permits us to reduce the size of the problem formulation significantly by eliminating (omitting from the formulation) the \(X^k_{j_1, j_2}\) variables corresponding to all of the combinations of indices \(k, j_1, j_2\) that do not satisfy any of the conditions of the proposition. Moreover, eliminating these
variables (which is equivalent to adding constraints setting them to zero) also strengthens the model by tightening its LP relaxation (otherwise, the LP relaxation may choose positive values for the eliminated variables even though they are not feasible in the integer solution). After this reduction, for each distance range \( k \), let \( \Pi(k) \subseteq J \times J \) denote the set of interval pairs \( \{i_1, j_2\} \) satisfying the appropriate conditions of Proposition 1 for which the variable \( X^k_{j_1,j_2} \) remains in the model formulation.

We can develop a result analogous to Proposition 2 to eliminate many \( Y^h_{i,j} \) variables a priori by restricting the spans of the weight ranges based on the increasing range width requirement and other range selection constraints. For each weight range \( h \), let \( \Psi(h) \subseteq I \times I \) denote the set of interval pairs \( \{i_1, j_2\} \) for which the variable \( Y^h_{i,j} \) remains in the model formulation.

### 3.4 Variable lower and upper bounds on fee values

#### 3.4.1 Variable lower bounds

Constraints (9) of formulation [FTD] impose the given minimum and maximum fee values in each cell as simple lower and upper bounds on the corresponding fee value variable in constraint (9). In the model’s solution, however, cells are combined into blocks of the proposed fee table, and the fee values for all the cells in a block must be the same. Hence, the fee value for a cell \( <i,j> \) must also satisfy the lower and upper bounds for all the other cells that the solution includes within the same block as cell \( <i,j> \). For instance, if the solution includes intervals \( j \) and \( j_2 (> j) \) in the same distance range, then the fee value \( P_{ijt} \) of cell \( <i,j> \) in period \( t \) must exceed not only its own lower bound \( l_{ijt} \) but also the lower bound \( l_{i_2,j} \) of cell \( <i,j_2> \). Although all integer solutions of [FTD] satisfy this condition, the solutions to the LP relaxation with fractional range selection values may violate this requirement. To limit such violations, we can impose the following variable lower bounding constraints for fee values.

\[
P_{ijt} \geq \sum_{1 \leq i_1 < i_2 \leq |I|} \sum_{j_1 < j_2 \leq |J|} l_{i_1,j_2} X^k_{j_1,j_2} \quad \text{for all } i \in I, j \in J, t = 1, 2, \ldots, T. \tag{21}
\]

Constraint (21) dominates constraint (9) because the lower bound values are non-decreasing in distance and because each interval \( j \) is covered by exactly one distance range. Constraint (21) incorporates only the lower bounds implied by the distance range selection variables; we can further strengthen it by also incorporating the weight range selection variables as follows. Suppose the solution includes cell \( i \) in a weight range \( h \) by setting \( Y^h_{i,j} \) equal to one, for some \( <i_1,i_2> \subseteq \Psi(h) \) with \( i_1 < i < i_2 \). In this case, the fee value \( P_{ijt} \) must also satisfy the lower bound \( l_{i_2,j} \) which can exceed \( l_{ijt} \) due to monotonicity of bounds. To add the weight range selection decisions to the variable lower bound in constraint (21), we consider the smallest necessary increment in cell \( <i,j> \)’s lower bound for various weight range choices. For this purpose, let \( \lambda \) be the highest-indexed distance interval that a distance range can span while including

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interval $j$, i.e., $\lambda_j = \max \{ j_2 : k, < j_1, j_2 > \in \Pi(k) \text{ for } j_1 \leq j \leq j_2 \}$. Then, $l_{i,\lambda_j}$ is the highest coefficient in the right-hand side of constraint (21). Hence, if the solution includes weight interval $i$ in the $h^{th}$ weight range by setting $Y^h_{i,j_t}$ equal to one, we can increase the lower bound that the fee value $P_{ijt}$ must satisfy by at least $[l_{i,j_t} - l_{i,\lambda_j}]$. The following constraint incorporates this strengthening by considering the weight range selection variables in addition to the previous distance range variables:

$$P_{ijt} \geq \sum_{k, <j_1,j_2 > \in \Pi(k)} l_{i,j_t} X^k_{h,j_t} + \sum_{h, <i_1,i_2 > \in \Psi(h)} [l_{i,j_t} - l_{i,\lambda_j}] Y^h_{i,j_t} \text{ for all } i \in I, j \in J, t = 1, 2, \ldots, T. \quad (22)$$

We can similarly develop another inequality that first accounts for lower bounds implied by the weight range choices, and then adds increments to the lower bound based on the distance range choices. To formulate this version of the variable lower bounding constraint, let $\mu_i$ be the highest-indexed interval that can be part of a weight range that includes interval $i$, i.e., $\mu_i = \max \{ i_2 : h, <i_1,i_2 > \in \Psi(h) \text{ for } i_1 \leq i \leq i_2 \}$. Then, the following constraint is the complementary (to constraints (22)) version of the variable lower bounding constraints:

$$P_{ijt} \geq \sum_{h, <i_1,i_2 > \in \Psi(h)} l_{i,j_t} Y^h_{i,j_t} + \sum_{k, <j_1,j_2 > \in \Pi(k)} [l_{i,j_t} - l_{i,\lambda_j}] X^k_{h,j_t} \text{ for all } i \in I, j \in J, t = 1, 2, \ldots, T. \quad (23)$$

We refer to constraints (22) and (23) as the *block-based fee value variable lower bound constraints*.

**Proposition 3.** The block-based fee value variable lower bound constraints (22) and (23) are valid for FTD problem.

**Proof.** See Appendix 1C.

### 3.4.2 Variable upper bounds

As with the lower bounds, we can also strengthen the upper bounds on the fee values (constraint (9)) by making them contingent on the range choices. The fee value in a particular distance and weight range must not exceed the given upper bounds for all of the cells included in that range. Since the upper bounds are non-decreasing in distance and weight, the fee upper bound corresponding to the starting interval of a range provides the tightest upper bound among all the cells in the range. Building on this observation and using the distance range selection variables, the following constraints, analogous to constraint (21), establish a variable upper bound on fee values:

$$P_{ijt} \leq \sum_{k} \sum_{<i_1,i_2 > \in \Pi(k)} u_{i,j_t} X^k_{h,j_t} \text{ for all } i \in I, j \in J, t = 1, 2, \ldots, T. \quad (24)$$

Since $u_{ijt} \geq u_{ij't}$ for $j' \leq j$, constraint (24) is at least as strong as the simple upper bound constraint (9). The upper bound values are also non-decreasing with respect to shipment weight, and so constraint (24) can be further strengthened by incorporating the weight range selection variables. Let $\rho_j$ denote the lowest-
indexed starting interval for a distance range that includes interval \( j \). Then, \( u_{i,s_{i,j}} \) is the smallest among the upper bound values in (24). Using this definition, we can add the weight range selection variables to right-hand side of constraint (24) (as we did for constraint (21)) to account for weight ranges to impose a tighter variable upper bound on the fee value \( P_{ijt} \).

\[
P_{ijt} \leq \sum_{k < s_{i,j} \in \Pi(h), j_s \in SF_{j2}} u_{i,s_{i,j}} X_{h,t}^k - \sum_{k < s_{i,j} \in \Pi(h), j_s \in SF_{j2}} [u_{i,s_{i,j}} - u_{i,s_{i,j}+1}] Y_{h,t}^k \quad \text{for all } i \in I, j \in J, t = 1, 2, \ldots, T.
\] (25)

Again, analogous to constraints (23), we can derive an alternative version of constraints (25) by first incorporating the weight range selection variables and then accounting for the incremental impact of distance range selection variables. Let \( \sigma_i \) denote the lowest-indexed starting interval for a weight range that includes weight interval \( i \). Then, the following constraints can tighten the fee value upper bound constraints (9) in [FTD].

\[
P_{ijt} \leq \sum_{h < s_{i,j} \in \Pi(h), j_s \in SF_{j2}} u_{i,s_{i,j}} Y_{h,t}^k - \sum_{h < s_{i,j} \in \Pi(h), j_s \in SF_{j2}} [u_{i,s_{i,j}} - u_{i,s_{i,j}+1}] X_{h,t}^k \quad \text{for all } i \in I, j \in J, t = 1, 2, \ldots, T.
\] (26)

**Proposition 4.** The block-based fee value variable upper bound constraints (25) and (26) are valid for the FTD problem.

**Proof.** See Appendix 1D.

### 3.5 Fee difference forcing constraints

As another approach (in addition to the block-based constraints ((22), (23), (25), and (26)) to constraining the fee values using information on both the distance and weight range selection variables, we next develop bounds on the maximum permissible fee difference between adjacent cells by simultaneously considering distance and weight range choices. For instance, consider a weight range that begins at interval \( i_1 \leq i \), spans weight interval \( i \), and ends at interval \( i_2 \geq i \). In this case, the lower bound \( l_{i_2,j} \) is a valid and tighter bound than \( l_{ij} \) on the fee value \( P_{ij} \) (since \( i_2 \geq i \) and lower bounds are monotonic in weight), and \( u_{i,j+1} \) is a valid and tighter bound than \( u_{i,j+1} \) on \( P_{ij+1} \) (since \( i_1 \leq i_2 \) and upper bound values are monotonic in weight). Using this observation, we can write the following constraints that are stronger than the original range-to-fee linkage constraint (4).

\[
P_{i,j+1} - P_{ij} \leq (u_{i,s_{i,j+1}} - l_{i,j+1}) \sum_{h < s_{i,j+1} \in \Pi(h)} X_{h,t}^k + \sum_{h < s_{i,j+1} \in \Pi(h), s_{i,j+1} \leq \sigma_i} \left( u_{i,s_{i,j+1}} - u_{i,s_{i,j+1}} + (l_{i,j+1} - l_{i,j+1}) \right) Y_{h,t}^k
\]

for all \( i \in I, j \in J \setminus \{N\}, t = 1, 2, \ldots, T. \]

(27)

Analogously, to ensure that fees within a weight range are the same, we can employ the following tighter version of the range-to-fee linkage constraints (7).
\[ P_{i+1,j,t} - P_{i,j,t} \leq (u_{i+1,j,t} - l_{i,j,t}) \sum_{h \in \mathbb{H}(h)} Y_{i,h}^k + \sum_{k < j_j,j \in \Pi(h), j_j \neq j_1} \left( (u_{i+1,j,t} - u_{i+1,j,t}) + (l_{i,j,t} - l_{i,j,t}) \right) X_{j,j,t}^k \]

for all \( i \in I \setminus \{ M \}, j \in J, t = 1, 2, \ldots, T \). (28)

The fee difference forcing constraints (27) and (28) are valid and stronger than the linkage constraints (4) and (7) in [FTD].

**Proposition 5.** The fee difference forcing constraints (27) and (28) are valid for FTD problem.

**Proof.** See Appendix 1E.

### 3.6 Block selection forcing constraints

Since formulation [FTD] models the weight and distance range choices separately, the LP relaxation solution can fractionally select blocks whose fee values are not feasible in the integer solution. For instance, suppose cell \(<i, j>\) is part of a weight range that begins at \(i_1\) and ends at \(i_2\) (with \(i_1 \leq i \leq i_2\)) and a distance range that spans intervals \(j_1\) through \(j_2\) (\(j_1 \leq j \leq j_2\)). Although these two range choices may permit feasible fee values (within bounds) independently (i.e., \(l_{i,j,t} \leq u_{i,j,t}\) and \(l_{j,t} \leq u_{j,t}\) for all \(t\)), selecting both ranges together may not be feasible due to the bounds on the fee values. That is, if \(l_{i,j,t} \geq u_{i,j,t}\) for any \(t\), the FTD solution must not simultaneously select these two ranges. The following constraints capture this restriction.

\[ \sum_{h < j_j,j \in \Pi(h), j_j \neq j_1} Y_{h,j}^k \geq \sum_{k < j_j,j \in \Pi(h), j_j \neq j_2} X_{j,j,j}^k \quad \text{for all } i \in I, j_1 \in J, j_2 \geq j_1. \] (29)

Analogous constraints, contingent on weight range choices, are as follows.

\[ \sum_{k < j_j,j \in \Pi(h), j_j \neq j_2} X_{j,j,j}^k \geq \sum_{h < j_j,j \in \Pi(h), j_j \neq j_1} Y_{h,j}^k \quad \text{for all } i \in I, i_2 \geq i_1, j \in J. \] (30)

**Proposition 6.** The block selection forcing constraints (29) and (30) are valid for the FTD problem.

**Proof.** See Appendix 1F.

### 3.7 Summary

In this section, taking advantage of the characteristics and requirements of the FTD problem, namely, increasing range width and bounds on fee values, we have developed a problem reduction approach and five new classes of valid inequalities that do not exclude any feasible integer FTD solutions but eliminate fractional LP solutions. Since the FTD problem is new, these inequalities have not been previously proposed in the literature. Our valid inequalities tighten different aspects of the formulation, and are complementary. The block-based fee value variable lower bound ((22) and (23)), block-based fee value variable upper bound ((25) and (26)), and the fee difference forcing ((27) and (28)) inequalities specify mixed-integer cuts that ensure a tighter linkage between the binary range selection and the continuous fee.
setting decision variables. The cumulative range width forcing ((19) – (20)) and block selection forcing ((29) and (30)) inequality classes develop binary cuts that further streamline (beyond the base formulation) the range selection choices to ensure increasing range width and consistency with interval lower and upper bounds. As our computational tests (Section 5) confirm, these constraints are effective in strengthening the LP relaxation of the FTD problem and accelerating the solution procedure.

4. Solution methodology

We implemented a LP-based solution method for the FTD problem that incorporates the valid inequalities (Section 3) to obtain a strong LP relaxation whose solution not only yields a tight lower bound on the optimal value, but also provides a good starting point for LP-based heuristics. Since the number of possible valid inequalities is large, we do not add them all a priori. Instead, we developed a cutting plane procedure (Section 4.1) to iteratively add violated constraints from the five classes of valid inequalities discussed in Section 3 to obtain the strong LP relaxation. If the solution to this relaxation is fractional, our method: (i) generates an LP-based heuristic solution (Section 4.2) by solving a maximum flow problem on a related layered network, (ii) refines this initial solution using local improvement rules (Section 4.3), and (iii) initiates a branch-and-bound procedure. We refer to the overall algorithm, with problem reduction, model strengthening, and heuristic, as the composite solution procedure. The following sections discuss each component of the algorithm.

4.1 Cutting plane method

If the LP relaxation solution of the base formulation [FTD] is fractional, our approach adds select valid inequalities from the five different classes, consisting of constraints (19), (20), (22), (23), and (25) – (30), using a cutting plane procedure. At each iteration, after solving the LP relaxation of the current model, the procedure identifies constraints in each inequality class that the LP solution violates, adds these constraints to the current model, and re-solves the LP. The cutting plane procedure terminates when the current LP solution does not violate any of the valid inequalities. We refer to the augmented formulation at the end of this procedure as the strong model. Since the five inequality classes contain only a polynomial number of constraints, at each iteration, we can identify the violated constraints by evaluating the inequalities for all relevant weight and distance intervals and ranges, and time periods. At each iteration, we can either add all the violated constraints or just the most violated subset. The former strategy rapidly increases the model size and hence computational effort at each iteration, whereas the latter strategy may require a large number of iterations. We adopted the following intermediate approach to determine the subset of violated constraints to add. Given an LP relaxation solution $\mathbf{x}^{LP} = (\mathbf{P}^{LP}, \mathbf{X}^{LP})$,
to the incumbent model, we say that a constraint $ax \leq b$ is $\varepsilon$-violated by this solution if $ax^{\text{LP}} \geq b + \varepsilon$.

Using this definition, at each iteration, we only add those constraints that are $\varepsilon$-violated. In our computations, we initially set $\varepsilon = 0.1$; if no constraints are $\varepsilon$-violated at a particular iteration, we reduce $\varepsilon$ by half and repeat the iteration, continuing until the value of $\varepsilon$ becomes less than 0.005.

4.2 LP-rounding heuristic method

When the strong model’s LP relaxation solution is fractional, we first apply a rounding method that solves two related network flow problems to determine the weight and distance ranges. Using this range choice, we then solve an LP to determine the corresponding optimal fee values.

We first decompose the problem of determining the table structure into two separate problems, one each for the weight and distance ranges. We focus here on the procedure to determine distance ranges from the LP solution; a similar procedure determines the weight ranges. We begin by framing the problem of selecting distance ranges based on the LP solution values as a network flow problem. For this purpose, we construct a layered network with $K$ layers, one for each distance range. For each distance range selection variable $X^k_{j_1,j_2}$, we include a corresponding node $\{j_1,j_2\}$ in layer $k$ of the network. We then connect nodes in a layer to appropriate nodes in the next layer that are consistent with range contiguity and increasing range width requirements. That is, a node $\{j_1,j_2\}$ in layer $k$ is connected to a node $\{j_3,j_4\}$ in layer $(k+1)$ only if $j_2 + 1 = j_3$ and $d_{j_1} - d_{j_{k-1}} \leq d_{j_4} - d_{j_2}$. Figure 4 shows an illustrative network for a problem instance that requires selecting three distance ranges (layers) from ten equal width intervals. For this example, after eliminating infeasible range selection combinations (based on the problem reduction method discussed in Section 3.3), layers 1, 2, and 3 have three, eight, and five nodes respectively. The arcs in the network connect only those pairs of nodes in successive layers that satisfy range contiguity and increasing range width requirements. The origin node (O) is connected to all nodes in layer 1 and the destination node (D) is connected to nodes in the last layer. With this construction, any path in the network from the origin to destination defines a distance range structure that satisfies the range selection constraints.

We wish to use this network to decide which among the range selection values $(X^k_{j_1,j_2})^{\text{LP}}$ in the LP solution to round up to get a feasible set of distance range choices, i.e., a set of range choices that satisfies range contiguity and increasing range width requirements. In particular, we favor rounding up $X^{\text{LP}}$-values that are higher since such values indicate good range choices from the perspective of minimizing the FTD model’s objective function value. For this purpose, we assign the LP value $(X^k_{j_1,j_2})^{\text{LP}}$ for each distance range index $k$ and all $\{j_1,j_2\} \in \Pi(k)$ as the length of every incoming arc entering node $\{j_1,j_2\}$ in layer $k$. 22
Aris connecting to the destination node have a zero cost. In Figure 4, for instance, the arc from the origin to node (1, 2) in Layer 1 has a length of \( \left( X_{1,2}^1 \right)^{LP} \); similarly, the arc connecting node (1, 4) in Layer 1 to node (4, 7) in Layer 2 has a length of \( \left( X_{4,7}^2 \right)^{LP} \), and so on. With these arc lengths, our desired origin-to-destination path is the one with the longest total length. Since our network is acyclic, solving a longest path problem from origin to destination requires only modest computational effort (polynomial time). This path then yields the set of distance ranges to use in our heuristic solution. Applying the same procedure to the weight intervals and using the fractional \( Y^{LP} \) values, we solve another longest path problem to obtain the weight ranges. We denote the distance and weight range solution obtained using this procedure as \( \hat{X} \) and \( \hat{Y} \).

Given these distance and weight ranges, we can solve the following fee-setting linear program (Balakrishnan, et al. 2000), which we denote as \([FSP]\), to determine the optimal fees for every period for the chosen fee ranges.

\[
\begin{align*}
\text{Minimize} & \quad \sum_i \sum_r \sum_s \sum_j d_{ijr} P_{ijr} \\
\text{subject to:} & \\
P_{i,j+1,r} - P_{ijr} &\leq (u_{i,j+1,r} - l_{ijr}) \sum_{k \in \{N\}} \hat{X}_{k,i,j} & \text{for all } i \in I, j \in J \setminus \{N\}, t = 1, 2, \ldots, T, \\
P_{i+1,j,r} - P_{ijr} &\leq (u_{i+1,j,r} - l_{ijr}) \sum_{k \in \{M\}} \hat{Y}_{k,i,j} & \text{for all } i \in I \setminus \{M\}, j \in J, t = 1, 2, \ldots, T, \text{ and constraints (8) - (12).}
\end{align*}
\]

The solution \( \hat{P} \) of the \([FSP]\) together with \( \hat{X}, \hat{Y} \) is a heuristic solution to the FTD problem. Because our procedure considers the range selection decisions independently and determines fee setting decisions in a hierarchical manner, we cannot guarantee the optimality of the solution. Nevertheless, as the computations in Section 5 illustrate, this heuristic approach yields near optimal solutions for practical instances.
4.3 Local improvement procedure

Beginning with the range choices and the objective function value of the heuristic solution, the local improvement step attempts to refine the range choices in the incumbent solution to reduce the total fees. The procedure first generates a candidate range structure by modifying a subset of the range choices in the incumbent solution. Next, the algorithm solves the $[FSP]$ problem corresponding to these modified ranges, and updates the incumbent to this new solution if it improves upon the current objective function value. To systematically identify candidate range structures in the neighborhood of an incumbent solution, the procedures adopt the following two rules. For each range $k < K$, the first rule re-assigns the ending interval to include the starting interval of the following range. This move increases the width of the first range, reduces the width of the last range, and possibly alters the widths of other intermediate ranges. The second rule re-assigns the starting interval of every range $k > 1$ to include the ending interval of the previous range. This rule decreases the width of the first range and increases that of the last range. Since we can apply either rule to the weight and/or distance ranges, we can generate eight neighbors of the incumbent fee structure. For each neighbor that satisfies the increasing range width requirement, we solve the corresponding $[FSP]$ problem and select as the new incumbent the neighbor that provides the highest cost reduction. The procedure terminates when none of the neighbors reduces the total fees.
These neighborhood generating rules are analogous to rules used to obtain the so-called cyclic transfer neighborhood structure that researchers (e.g., Ahuja, Orlin, and Sharma 2000) have applied to other problems (e.g., partitioning).

4.4 Summary

To meet our goal of generating verifiably near-optimal solutions quickly, our composite algorithm first strengthens the model by iteratively adding the new classes of inequalities we developed (Section 3). Using an $\varepsilon$-violation threshold, we add only a small number of the numerous possible violated constraints at each iteration to obtain most of the LP improvement benefit. The resulting strong LP value provides a tight lower bound on the optimal objective value. We then apply a rounding heuristic, based on the strong LP solution, to generate a starting feasible solution. The heuristic method constructs a layered network satisfying the range width requirements, with LP fractional values as arc costs, and solves a longest path problem on the network to identify the best weight and distance ranges. Finally, starting with the LP-based heuristic solution, the solution method begins a local improvement procedure that evaluates the incumbent solution’s neighbors, which are generated through cyclic exchange rules, to identify cost reduction opportunities. If the gap between the strong LP value and the locally optimal solution is greater than a pre-determined threshold (for our computations, we set this value as 1%), our method initiates branch-and-bound, allowing two hours for this step. We refer to this integrated solution procedure as the composite method. The next section discusses the successful use of this method to generate a practical and effective fee table based on actual data.

5. Application: Developing a fee table for distributing building products

We applied our model and method to data from the nationwide distribution network, with 13 RDCs and over 2,000 stores, for the building products manufacturer. We describe the application context, data, and the results next.

5.1 Application context and data summary

The manufacturer and its distributors were using a fee table based on delivery distance and shipment weight to determine delivery fees. This table, chosen manually, had five weight and four distance ranges. We refer to this table as the original 5 X 4 table. With its limited number of weight and distance ranges that were not optimized, the payments based on this original fee table led to significant excess payments to several distributors. Therefore, the firm wanted to explore the development of a new fee table with more weight and distance ranges. The structure of the table, i.e., the weight and distance ranges, would be fixed for three years, but the fee values can change each year to reflect operational changes.
As part of the fee planning process, the firm first assessed demand at the store level. Based on inputs from the firm’s retail customers, the company generated the list of stores that would be operational (taking into account anticipated store closings and new store openings) in each time period of the planning horizon, and the delivery frequency for each store. Each store was assigned to an appropriate RDC in the manufacturer’s distribution system. Next, the firm determined the distribution of shipment weights (in pounds) to each store, based on historical data (for existing stores) as well as inputs from planners and the retail chains. Based on information regarding the anticipated shipments and delivery distances to the stores in a distributor’s coverage area in each period, together with information from the distributors, the firm estimated each distributor’s costs and developed distributor compensation targets for each period.

To apply our model to the firm’s data, we analyzed the delivery distances and shipment weights to determine an appropriate fine-grained grid of weight and distance intervals. Our analysis of store delivery distances (for all distributors) revealed a relatively high number of stores within 200 miles from the RDC, with store density decreasing at delivery distances beyond 200 miles. Consistent with these observations, our interval grid starts with narrow five mile intervals for the first 50 miles, followed by progressively wider intervals of 10, 25, 50, and 100 miles; altogether, the grid has 43 distance intervals (i.e., \( N = 43 \)). Similarly, our analysis of store shipment weights showed that more stores had mean shipment weights that were less than 5,000 pounds, with a decreasing number of stores having higher mean shipment weights. Accordingly, for the weight axis, we chose narrow initial intervals with a width of 100 pounds and increased the interval width progressively to 250, 500, 1000, and 2,500 pounds. In total, the grid includes 48 weight intervals (\( M = 48 \)) from 0 to 43,000 lbs. For the cells in this interval grid, we chose minimum and maximum fee values based on commercial LTL rates.

We implemented the FTD model and our composite solution method using the CPLEX 9.0 callable optimization library running on a Windows server with a 3.2 GHz processor, and applied it several scenarios base on the above data. First, we determined the best possible fee table with five weight and four distance ranges by setting \( H = 5 \) and \( K = 4 \) in the FTD model. We refer to this table as the optimized 5 X 4 table; this table has optimal widths for the five weight and four distance ranges whereas the original 5 X 4 table had pre-specified (manually chosen) widths for the weight and distance ranges. Comparing the objective values of the FTD model for these two tables (using optimized fee values for the original table) helps us assess the value of optimizing the range widths for a given number of desired distance and weight ranges. Next, to assist the decision to increase the number of ranges, we conducted two sets of computational runs. To establish a baseline for these runs, we set the desired number of weight and
distance ranges at $H = 7$ and $K = 7$ respectively. Then, to understand the impact of increasing the number of distance ranges, we held the desired number of weight ranges $H$ constant at seven, and varied the number of distance ranges from five to nine. Next, keeping the desired number of weight ranges $K$ at seven, we applied the model to different settings for the number of weight ranges to gauge the effect of fewer or more weight ranges.

For the baseline case with $H = 7$ and $K = 7$, even the base model [$FTD$] (without our valid inequalities) has more than 10,000 variables and nearly 30,000 constraints.

5.2 Results

We first discuss our composite algorithm’s effectiveness to generate near-optimal solutions quickly, and then address the improvement that the model’s solution provides compared to the firm’s original fee table. To understand the method’s effectiveness, we assess the performance of its two main components, namely, the strong formulation incorporating our valid inequalities and the optimization-based heuristic method. For this purpose, we examine: (i) the improvement in the LP relaxation value when using the strong formulation, and (ii) the ability of the heuristic approach to generate solutions that are close to optimality. For this assessment, we compare the performance of the composite method with the results using a branch-and-bound procedure (CPLEX), without a heuristic warm start, for the base [$FTD$] formulation, allowing two hours of computational time for each approach. We refer to this latter approach as Base B&B. Table 4 summarizes the results for the baseline scenario (with seven weight and distance ranges) and the eight other scenarios obtained by varying the desired number of ranges on each dimension.

Performance of the composite method. In Table 4, to compare the composite method’s ability to generate near-optimal solutions with that of the Base B&B, we measure the IP Gap for each method as follows. Using the best integer solution and best lower bound (LB) values from each method at the end of two hours of computational time, we compute the IP gap for each method as $(\text{Best integer value} - \text{Best LB value})/\text{Best LB value}$. Columns 2 and 3 show these IP gaps for both the composite method and the Base B&B. The composite method outperforms the Base B&B approach in all of the instances, consistently yielding much smaller integrality gaps. The superior performance of the composite method (overall average gap of 1.5%) is due both to the strong valid inequalities and the LP-rounding heuristic solution. We measure the improvement in the LP relaxation value due to the strong model by expressing the difference between the strong and base LP values as a percentage of the maximum possible improvement, which is the difference between the best LB and the base LP value. As column 4 shows,
the valid inequalities that we developed were very effective in tightening the LP relaxation of the base model, improving the LP value by nearly 50%. Moreover, the cutting plane procedure achieves this improvement efficiently, taking only about fifteen minutes (column 5) on average. We measure the performance of the LP-rounding heuristic procedure by expressing the difference between the heuristic value and the best upper bound value as a percentage of the best upper bound value. Column 6 shows that the solution from the LP-based heuristic procedure yields a solution that is remarkably close to the best upper bound value (within 0.8% on average), with only modest computational effort (column 7). Together, for different settings of the desired number of ranges, the strong LP and the heuristic procedure yield near-optimal solutions quickly even before initiating the branch-and-bound procedure.

**FTD Savings.** To estimate the savings from applying the FTD approach, we first determined the expected fee range compensation using the *original* 5 X 4 table for the projected store demands and distances. For each fee range in this table, we imposed the range choices of the *original* 5 X 4 table as constraints on the Y and Z variables, and solved the corresponding fee-setting [FSP] linear program (Sec. 4.2) to minimize the total excess payments. This value, which we denote as $F_{\text{orig}}$ and refer to as *original fees*, serves as the benchmark to compare with the (near-) optimal best upper bound value of the [FTD] model, denoted as $F_{\text{FTD}}$, that we obtain using the composite solution procedure. We express the savings due to the FTD model over the original fees as $(F_{\text{orig}} - F_{\text{FTD}})/F_{\text{FTD}}$. Column 8 in Table 3 shows this savings % for each scenario. First, by optimizing the range widths of a table with five weight ranges and four distance ranges, the composite method gives an *optimized* 5 X 4 table that reduces the total excess payments compared to the original table by 6.2%. Next, the results in Table 4 show that increasing the number of ranges beyond five ranges provides substantial additional savings (around 4%). But, as we might expect, the benefit of having additional ranges (distance or weight) yields diminishing returns in terms of reducing excess payments. Overall, the FTD model reduces total fees by nearly 10%.

Beyond the FTD Savings, the FTD model offers other important benefits. The FTD model’s constraints ensure fair, adequate, and industry-benchmarked compensation for the distributors. Hence, the model and its results promote and are consistent with the goals of the long-term partnership between the manufacturer and the distributors. In turn, the optimized fee table reassured the distributors that their considerations and costs were taken into account in determining delivery fees, allowing the firm to reach agreements with the distributors more easily. The results from parametrically varying the desired number of distance and weight ranges and reapplying the model helped to quantify the value of increasing the number of ranges, permitting the firm to make appropriate tradeoffs between operational and economic considerations in dimensioning the fee table.
### Table 3. Performance of the composite method and FTD savings.

| Desired # of ranges | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------------------|---|---|---|---|---|---|---|---|
|                     |   | IP Gaps @ 2 Hrs* (%) | LP improvement gap* (%) | Strong LP time (min.) | Heur. Gap~ (%) | Heur. Time (min.) | FTD Savings^ (%) |
|                     |   | Base B&B | Composite | | | | |
| $H = 5, K = 4$      | 6.8% | 2.8% | 43.6% | 11.2 | 1.9% | 7.9 |
| Increasing num. of distance ranges | | | | | | | |
| $H = 7, K = 5$      | 4.2% | 1.6% | 47.8% | 17.1 | 0.6% | 17.9 |
| $H = 7, K = 6$      | 4.9% | 1.5% | 46.6% | 10.4 | 0.6% | 13.8 |
| $H = 7, K = 7$      | 4.0% | 1.4% | 46.6% | 11.7 | 3.2% | 10.3 |
| $H = 7, K = 8$      | 2.9% | 1.0% | 53.3% | 17.0 | 0.0% | 8.9 |
| $H = 7, K = 9$      | 3.1% | 0.9% | 56.6% | 14.2 | 0.0% | 5.4 |
| Average             | 3.8% | 1.3% | 50.2% | 14.1 | 0.9% | 11.3 |
| Increasing num. of weight ranges | | | | | | | |
| $H = 5, K = 7$      | 4.2% | 1.6% | 55.1% | 16.7 | 0.5% | 10.6 |
| $H = 6, K = 7$      | 3.8% | 1.5% | 50.8% | 12.7 | 0.5% | 8.0 |
| $H = 7, K = 7$      | 4.0% | 1.4% | 46.6% | 11.7 | 3.2% | 10.3 |
| $H = 8, K = 7$      | 2.5% | 1.3% | 45.3% | 18.7 | 0.5% | 11.3 |
| $H = 9, K = 7$      | 2.7% | 1.3% | 41.8% | 13.6 | 0.3% | 7.6 |
| Average             | 3.4% | 1.4% | 47.9% | 14.7 | 1.0% | 9.5 |
| Grand average       | 3.9% | 1.5% | 48.7% | 14.3 | 0.8% | 10.2 |

* - IP Gap @ 2 hours = (Best UB – Best LB)/Best LB (%)

~ - LP Improvement Gap = (Strong LP value - Base LP value)/(Best LB - Base LP value) (%)

~ - Heuristic Gap = (Heuristic value – Best UB)/Best UB (%)

^ - FTD Savings = (F_{ORIG} – F_{FTD})/F_{FTD} (%)

6. Conclusions

As logistics alliances between manufacturers and third-party distributors grow, new decision requirements are emerging. The design of fee tables based on weight and distance emerged as an important decision problem for a large building-products manufacturer that outsourced its retail deliveries to independent regional distributors. Motivated by this problem, we describe a novel optimization model to determine weight and distance ranges, along with fee values for each fee block implied by the ranges. By developing a model that can determine LTL delivery fees while incorporating important practical considerations, we have addressed a gap in the 3PL pricing and contracts literature. Application of this model to real data has demonstrated the significant savings that optimization can yield for delivery services.
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Appendix 1: Proofs of validity of inequalities

Part A: Proof of Proposition 1 (cumulative range width constraints)

Constraint (19) is valid: If the [FTD] solution does not select a range $k$ that spans intervals $j_1$ through $j$ and ends at interval $j$, then $\sum_{j_{s_h}^j} X_{i,j}^k = 0$, and constraint (19) follows from the non-negativity of the range selection variables. If $\sum_{j_{s_h}^j} X_{i,j}^k = 1$, then the width of range $k$ is at least $(d_j - d_{j-1})$. In this case, the [FTD] must choose a range $k + 1$ beginning with interval $j + 1$ (to satisfy range contiguity constraints (3)) and have a width of at least $(d_j - d_{j-1})$ (from the increasing range width constraints (13)); consequently, the solution must select an ending interval $j_k$ for range $k + 1$ such that $(d_{j_k} - d_{j_{k-1}}) \geq (d_j - d_{j-1})$, implying (19).

The proof of the validity of constraints (20) follows a similar logic.

Part B: Proof of variable elimination rules

First range. The first range must start with the first interval, therefore $j_1 = 1$. Next, the width of the first range must be the least of the range widths and as a result must be less than or equal to the average range width $\left(\frac{d_{ij}}{K}\right)$ of the subsequent $K - 1$ ranges that span a distance of $(d_N - d_{j_1})$. Finally, because each of the remaining $K - 1$ ranges must occupy at least one interval, $j_2 + (K - 1)$ must be less than or equal to $N$.

Middle range. Because $k - 1$ ranges must have been completed with each occupying at least one interval, we know that $j_1 \geq k$. Analogously, the $K - k$ ranges that follow range $k$ must each cover at least one interval. Therefore, $j_2 + (K - k)$ must be less than or equal to $N$ and part (b) follows. To see part (c), consider a distance range selection $X_{i,j_2}^k$ that implies a width of $(d_{j_2} - d_{j_{k-1}})$ for range $k$. In this case, the previous ranges, each with a width no greater than that of range $k$, can have an average width of at most $\frac{d_{ij}}{K-1}$, which range $k$’s width must exceed and (c) follows. Next, each following range must have a width of at least $d_{j_2} - d_{j_{k-1}}$. Therefore, the average width $\frac{d_{ij}}{K-1}$ of the $K - k$ ranges that follow $k$ must be greater than or equal to range $k$’s width and (d) follows.

Last range. The last range must include the last interval $N$, therefore $j_2 = N$. Next, the previous $K - 1$ ranges must include at least one interval each; consequently, $j_k \geq K$. Finally, the width $(d_{j_1} - d_{j_k})$ of the last range must be at least as much as the average width $\frac{d_{ij}}{K-1}$ of the previous $K - 1$ ranges and (c) follows.

Part C: Proof of Proposition 3 (block-based fee value variable lower bound constraints)

Constraint (22) is valid: Suppose the [FTD] solution selects some ending interval $j' \geq j$ for the distance range that includes interval $j$ (from (2) and (3), we know that interval $j$ must belong to exactly one of the $K$ distance ranges). Similarly, suppose weight interval $i' \geq i$ is the ending interval for a range that spans weight interval $i$. In this case, the fee lower bound constraints (9) and the range-to-fee linkage constraints (4) and (7) ensure that $P_{ij}^k$ must be at least as much as the highest lower bound $(l_{ij})$ in the block containing cell $<i, j>$, and constraint (22) follows. A similar argument establishes constraint (23).

Part D: Proof of Proposition 4 (block-based fee value variable upper bound constraints)

Constraint (25) is valid: Suppose the [FTD] solution selects some starting interval $j' \leq j$ for the distance range that includes interval $j$. Similarly, suppose weight interval $i' \leq i$ is the starting interval for a range that spans weight interval $i$. In this case, the fee upper bound constraints (9) and the range-to-fee linkage constraints (4) and (7) ensure that $P_{ij}^k$ is no more than the lowest upper bound $(u_{ij})$ of the block containing cell $<i, j>$ and constraint (25) follows. A similar argument establishes constraint (26).
**Part E: Proof of Proposition 5 (fee difference forcing constraints)**

*Constraint (27) is valid:* Consider an [FTD] solution that includes both distance interval \(j\) and \(j + 1\) in the same distance range. In this case, \(\sum_{k,j \notin j} x_{k,j}^k = 0\) and constraints (27) follow from the range-to-fee linkage constraints in [FTD]. If not and if the solution chooses weight range \(Z_{i1, i2}\) to span weight interval \(i\) (i.e., \(i_1 \leq i \leq i_2\)), then the RHS evaluates to \((u_{i1,j+1,t} - l_{i1,j,d})\) which is valid because cells \((i_1, j+1), (i_2, j), (i, j),\) and \((i, j+1)\) belong to the same fee block. Similarly, starting from the weight ranges and then integrating the distance range selections, we can show the validity of constraint (28). ■

**Part F: Proof of Proposition 5 (block selection forcing constraints)**

*Constraint (29) is valid:* If the intervals \(j_1\) and \(j_2\) belong to different distance ranges, the RHS of constraint (29) is zero and the constraint follows from the non-negativity of the weight range selection variables. If \(j_1\) and \(j_2\) belong to the same range, then the RHS of the constraint is one. Consider all of the eligible weight ranges that span weight interval \(i\). The weight range selection constraints (5) and (6) ensure that the sum of all the weight ranges that span any interval \(i\) is one. If the solution selects \(i_1 \leq i \leq i_2\) with \(u_{i1,i2,t} < l_{i1,i2,d}\) for any \(t\), then the fee value for the block containing cell \(<i,j>\) cannot simultaneously satisfy its lower and upper bounds. Therefore, the LHS of constraint (29) must be one in this case. We can establish the validity of constraint (30) following a similar argument. ■