GRAVITATIONAL WAVES FROM INSPIRALING COMPACT BINARIES
WITH MAGNETIC DIPOLE MOMENTS
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ABSTRACT

We investigate the effects of magnetic dipole-dipole coupling and electromagnetic radiation on the frequency evolution of gravitational waves from inspiraling binary neutron stars with magnetic dipole moments. This study is motivated by the discovery of superstrongly magnetized neutron stars, i.e., magnetars. We derive the contributions of the magnetic fields to the accumulated cycles in gravitational waves as \( N_{\text{mag}} \sim 6 \times 10^{-3}(H/10^{16} \text{ G})^2 \), where \( H \) denotes the strength of the polar magnetic fields of each neutron star in the binary system. It is found that the effects of the magnetic fields will be negligible for the detection and the parameter estimation of gravitational waves if the upper limit for the magnetic fields of neutron stars is less than \( \sim 10^{16} \text{ G} \), which is the maximum magnetic field observed in soft gamma repeaters and anomalous X-ray pulsars to date. We also discuss the implications of electromagnetic radiation from inspiraling binary neutron stars for the precursory X-ray emission prior to the gamma-ray burst observed by the Ginga satellite.

Subject headings: binaries: close — gamma rays: bursts — gravitation — stars: magnetic fields — stars: neutron — waves

1. INTRODUCTION

Direct detection of gravitational waves (GWs) is one of the most exciting challenges in the history of science. Long-baseline interferometers for detection of GWs such as TAMA300 (Kuroda et al. 1997), GEO600 (Hough 1992), VIRGO (Bradaschia et al. 1990), and LIGO (Abramovici et al. 1992) will be in operation within five years. One of the most promising sources of GWs for such detectors is inspiraling binary neutron stars (BNSs) since we may expect several coalescing events per year within 200 Mpc (Phinney 1991; Narayan, Piran, & Shemi 1991; van den Heuvel & Lorimer 1996).

As the orbital radii of BNSs decay because of gravitational radiation reactions, the frequency of GWs sweeps upward in the detector’s sensitive bandwidth from \( \sim 10 \) to \( \sim 1000 \text{ Hz} \). In the early inspiraling phase of a BNS, each neutron star (NS) can be treated as a point particle and the post-Newtonian (PN) expansion will converge (Cutler et al. 1991; Narayan, Piran, & Shemi 1991; van den Heuvel & Lorimer 1996).

BNSs are needed in order to extract physical information about BNSs from GWs since any effect that causes only 1 cycle ambiguity over 16,000 accumulated cycles in the theoretical templates will reduce the signal-to-noise ratio (S/N). For the inspiraling BNSs in the sensitive bandwidth, with \( v^2 \sim m/r \) (hereafter \( G = c = 1 \)) typically around \( 10^{-2} \), the correction of \( 1/16,000 \sim 10^{-4} \) corresponds to second-PN (2PN) order, \( (m/r)^2 \sim 10^{-4} \) (Blanchet et al. 1995). Many efforts are devoted to calculating higher order PN corrections to theoretical templates (e.g., Blanchet 1996; Jaranowski & Schäfer 1998a, 1998b; Damour, Jaranowski, & Schäfer 1999; Tagoshi & Nakamura 1994; Tagoshi & Sasaki 1994; Poisson 1995). However, these studies pay attention only to the gravitational effects on the theoretical templates, and how large corrections to the theoretical templates are caused by electromagnetic effects, i.e., the magnetic fields of NSs, has not been studied as far as we know.

NSs observed as radio pulsars are believed to have strong magnetic fields, typically \( \sim 10^{12} \text{ G} \), assuming that the spin-down of pulsars is due to magnetic dipole radiation (e.g., Taylor, Manchester, & Lyne 1993). In addition, it has begun to be recognized recently that NSs with superstrong magnetic fields \( \gtrsim 10^{14} \text{ G} \) really exist (see below). We can crudely estimate the correction to the waveform due to magnetic dipole fields of NSs by comparing the gravitational force \( F_G \sim m_1 m_2/r^2 \) and the magnetic force \( F_M \sim 3\mu_1 \mu_2 r^4 \) between BNSs of masses \( m_1 \) and \( m_2 \) with magnetic dipole moments \( \mu_1 \) and \( \mu_2 \), as

\[
F_M/F_G = 1 \times 10^{-4} \left( \frac{m_1 + m_2}{m_1 m_2} \right)^{-2} \left( \frac{H_1}{2 \times 10^{15} \text{ G}} \right)^2 \times \left( \frac{H_2}{2 \times 10^{16} \text{ G}} \right) \left( \frac{R_1}{10^6 \text{ cm}} \right)^3 \left( \frac{R_2}{10^6 \text{ cm}} \right)^3 \times \left[ \frac{1.4 M_\odot}{m_1} \right] \left[ \frac{1.4 M_\odot}{m_2} \right] \left( \frac{2.8 M_\odot}{m_1 + m_2} \right)^2,
\]

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where $H_p = 2\mu_p/R_p^3$ ($p = 1, 2$) are the magnetic fields at the pole of the star and $R_p$ are radii of NSs. This corresponds to the 2PN-order correction. Therefore, the magnetic fields of order $\sim 10^{16}$ G might cause about 1 rotation error. Note that the $r$-dependence of the magnetic correction is the same as that of 2PN order, $v^2 \propto r^{-2}$, so that this argument is independent of the value of the separation $r$.

Theoretically, in a newborn NS, such superstrong magnetic fields $\sim 10^{16}$ G can be generated if the initial spin is in the millisecond range since the conditions for helical dynamo action are met during the first few seconds after gravitational collapse (Duncan & Thompson 1992; Thompson & Duncan 1993). Observationally, such superstrongly magnetized NSs, or “magnetars,” may be found as soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs); Duncan & Thompson 1992; Thompson & Duncan 1996). The dipolar magnetic fields of SGRs are estimated as $\sim 10^{15}$ G by using the measured spin periods with spin-down rates for SGR 1900+14 (Kouveliotou et al. 1999; Woods et al. 1999a) and SGR 1806–20 (Kouveliotou et al. 1999) and with the peak luminosity for SGR 1627–41 (Woods et al. 1999b). AXPs that have measured spin-down rates (Israel et al. 1999; Mereghetti, Israel, & Stella 1998; Baykal et al. 1998; Vasisht & Gotthelf 1997; Haberl et al. 1997) can be considered to have magnetic fields of $10^{14}$–$10^{15}$ G. Although there are some other models for SGRs and AXPs, these objects are best understood within the framework of magnetars (see, e.g., Thompson & Duncan 1995; Kouveliotou et al. 1998; Vasisht & Gotthelf 1997). Therefore, from both theoretical and observational results, it may be possible that a NS has superstrong magnetic fields $\sim 10^{16}$ G.

In this paper we investigate the effects of the magnetic dipole fields of NSs on the frequency evolution of GWs from the inspiraling BNSs since magnetic fields of order $\sim 10^{16}$ G might cause about 1 rotation error. Throughout the paper, we use the relation $H = 2\mu/R^3$ to connect the magnetic moment $\mu$ to the magnetic field at the magnetic pole $H$ and $R = 10^6$ cm as the radius of a NS. For later convenience, note that $\mu = 1.4 \times 10^{20}(H/10^{16}$ G) cm$^5$ in units of $G = c = 1$.

2. EQUATIONS OF MOTION

We consider a binary system of two compact bodies of masses $m_1$ and $m_2$ with magnetic dipole moments $\mu_1$ and $\mu_2$, respectively. Since we pay particular attention to the effects of magnetic fields, we treat the orbital motion of BNSs in Newtonian gravity. Although a spherical symmetry of the stellar configuration is, in general, incompatible with the presence of magnetic fields ( Chandrasekhar 1981, p. 577), we ignore quadruple effects, which are caused by magnetic fields for a moment (see § 5 and Appendix B). We also neglect tidal effects, which are expected to be small until the premerging phase of BNSs ( Bildsten & Cutler 1992).

By eliminating the motion of the center of mass of a BNS and setting the origin of the coordinate frame at the center of mass of a BNS, the effective one-body equations of motion can be derived from a Lagrangian,

$$\mathcal{L} = \frac{1}{2} \eta m v^2 + \frac{\eta m^2}{r} + \mathcal{L}_{DD},$$

where

$$\mathcal{L}_{DD} = \mu_1 \cdot H_2 = \frac{1}{r^3} [3(\hat{n} \cdot \mu_1)(\hat{n} \cdot \mu_2) - \mu_1 \cdot \mu_2].$$

Here $m = m_1 + m_2$, $\eta = m_1 m_2 / m^2$, $r = |x|$, $x = x_1 - x_2$, $\hat{n} = x/r$, $\phi = \dot{x}$, and $H_2 = [3(\hat{n} \cdot \mu_2)\hat{n} - \mu_2]/r^3$ is the magnetic field at $x_1$ produced by the magnetic moment $\mu_2$. By using the Euler-Lagrange equations, we obtain the equations of motion as

$$a = - \frac{m}{r^2} \dot{\phi} + a_{DD},$$

where $a = \ddot{x}$ and

$$a_{DD} = \frac{3}{\eta m^2} \left[ (\mu_1 \cdot \mu_2) \hat{n} + (\hat{n} \cdot \mu_2) \mu_1 \right]$$

$$+ (\hat{n} \cdot \mu_1) \mu_2 - 5(\hat{n} \cdot \mu_1)(\hat{n} \cdot \mu_2) \hat{n}. $$

From equation (2), the energy of this system is given by

$$E = \frac{1}{2} \eta m v^2 - \frac{\eta m^2}{r} + E_{DD},$$

where

$$E_{DD} = - \frac{1}{r^3} [3(\hat{n} \cdot \mu_1)(\hat{n} \cdot \mu_2) - \mu_1 \cdot \mu_2].$$

The total angular momentum can be defined as $L = L_N + S$, where $L_N = \eta m(x \times \phi)$ is the Newtonian orbital angular momentum and $S = S_1 + S_2$ is the total spin angular momentum. We can show explicitly $\dot{E} = 0$ and $\dot{L} = 0$ with the equations of motion (4) and the evolution equations of the spins,

$$\dot{S}_p = \mu_p \times H_q = \frac{1}{r^3} [3(\hat{n} \cdot \mu_p)(\mu_p \times \hat{n}) - \mu_p \times \mu_q]$$

$$(p, q = 1, 2).$$

Assuming NSs as spherical compact bodies, the spin angular velocities $\Omega_p$ are related to the spins $S_p$ as $S_p = I_p \Omega_p$, where $I_p$ is the principle moment of inertia of the bodies. Since the magnetic moments evolve as $\mu_p = \Omega_p \times \mu_p = S_p \times \mu_p$, the angular velocities of the magnetic moments $\Omega_p$ will be of order $\Omega_p \sim (\mu_1 \mu_2 / m R^2 r^3)^{3/2}$ from dimensional analysis with equation (8). Note also that the orbital angular velocity $\omega$ is of order $\omega \sim (m r^3/2 m)^{3/2}$ and the orbital inspiral rate $w_{ins} = (dE/dt)_{GW}/E$ is of order $w_{ins} \sim (m r^3/m)$.2

3. GRAVITATIONAL WAVES AND ELECTROMAGNETIC WAVES

As a first step, we use the quadrupole formula to derive the rate of energy loss from a binary system due to GWs (e.g., Thorne 1980). The symmetric, trace-free parts of the quadrupole moments of this system are given by $I_{ij} = \eta m (x_i x_j - \frac{1}{3} r^2 \delta_{ij})$. Taking third time derivatives of these quadrupole moments, we obtain the energy loss rate from

\[ \text{Note that there are no spin-orbit and spin-spin interactions since we consider Newtonian gravity. The spins are generated by the torque due to the magnetic dipole-dipole interaction.} \]
the quadrupole formula as
\[
\left(\frac{dE}{dt}\right)_{GW} = -\frac{1}{5} \langle \mathbf{I}_1 \cdot \mathbf{I}_2 \rangle \\
= -\frac{8}{15} \left( 12v^2 - 11\dot{v}^2 \right) \\
+ \frac{1}{\eta m^2 r^4} \left( 6(-12v^2 + 13\dot{v}^2)(\mu_1 \cdot \mu_2) \right) \\
+ 12(21v^2 - 34\dot{v}^2)[(\mathbf{n} \cdot \mathbf{v})(\mathbf{n} \cdot \mathbf{v})] \\
- 36(\mathbf{v} \cdot \mu_1)(\mathbf{v} \cdot \mu_2) \\
+ 87\mathbf{r}[(\mathbf{n} \cdot \mu_1)(\mathbf{v} \cdot \mu_2) + (\mathbf{v} \cdot \mu_1)(\mathbf{n} \cdot \mu_2)] \right),
\] (9)

where we have used the equations of motion (4) and assumed \( \mu_1 \mu_2/m^2 r^2 \ll 1 \).

On the other hand, electromagnetic (EM) waves are also emitted from this binary system since the magnetic moments are moving (see Appendix A). Using the linearity in the EM fields, the radiation fields \( B^{rad}_{\mu} \) in equation (A4) for this binary system are given by
\[
B^{rad}_{\mu} = \frac{1}{D} (\mathbf{a} \cdot \mathbf{\mu}_{\text{eff}}) [\mathbf{\dot{a}} (\mathbf{\dot{a}} - \mathbf{\hat{a}})],
\] (10)

where
\[
\mu_{\text{eff}} = \frac{1}{m} (m_2 \mu_1 - m_1 \mu_2) .
\] (11)

Therefore, since the radiated power is given by equation (A6), the rate of energy loss due to the EM radiation is calculated as
\[
\left(\frac{dE}{dt}\right)_{EM} = -\frac{2}{15} m^2 \left( \frac{2}{r^5} \left( 2\mu_{\text{eff}}^2 (v^2 - 6\dot{v}(\mathbf{n} \cdot \mathbf{v}) + 9\dot{v}^2) \right) \\
- 3\left[ \mu_{\text{eff}} \cdot (v - 3\dot{v}\mathbf{n}) \right]^2 \right),
\] (12)

where we have substituted \( \mu_{\text{eff}} \) into \( \mu \) in equation (A6).

Note that the assumption of the constant magnetic moments in equations (9), (10), and (A4) is valid since the angular velocities of the magnetic moments \( \Omega_\mu \approx (\mu_1 \mu_2/mR^2 r^3)^{1/2} \) are much smaller than the orbital angular velocities \( \omega \approx (m/r)^{3/2}/m \) when \( (H_1 \cdot H_2)^{1/2} \ll 10^{18} \, \text{G} \).

4. INSPIRAL OF CIRCULAR ORBITS

For calculational simplicity we assume that the orbital motion of a BNS has decayed to become circular apart from the adiabatic inspiral (Peters & Mathews 1963; Peters 1964). In general, circular orbit solutions of equation (4) do not exist unless the magnetic moments are aligned perpendicular to the orbital plane. However, since the angular velocities of the magnetic moments \( \Omega_\mu \approx (\mu_1 \mu_2/mR^2 r^3)^{1/2} \) are much smaller than the orbital angular velocity \( \omega \approx (m/r)^{3/2}/m \) when \( (H_1 \cdot H_2)^{1/2} \ll 10^{18} \, \text{G} \), we can regard the magnetic moment vectors and \( \mathbf{L} \) as time-independent ones over an orbit where \( \mathbf{L} \) is a unit vector orthogonal to the orbital plane. Then, after taking an average of the magnetic term in the acceleration (5), we can obtain orbits of constant separation, \( \ddot{r} = \dot{\mathbf{n}} \cdot \mathbf{v} = 0 \), \( w = \dot{v}/r \), and \( L_N = \eta m^2 \omega \mathbf{L} \).

For similar discussions on spin precessions see Kidder et al. 1993; Kidder 1995. From the equations of motion for circular orbits, \( \mathbf{n} \cdot \mathbf{a} = \ddot{r} - rw^2 \), we can calculate the orbital angular velocity as
\[
w^2 = \frac{m}{r^3} \left( 1 + \frac{3}{2}\eta m^2 \left( \frac{m}{r} \right)^2 [\mu_1 \cdot \mu_2 - 3(\mathbf{L} \cdot \mu_1)(\mathbf{L} \cdot \mu_2)] \right),
\] (13)

where we have used the orbit-averaged relation,
\[
(\mathbf{n} \cdot \mu_1)(\mathbf{n} \cdot \mu_2) = \frac{1}{2} [\mu_1 \cdot \mu_2 - (\mathbf{L} \cdot \mu_1)(\mathbf{L} \cdot \mu_2)] .
\] (14)

The total energy and the energy loss rate for a circular orbit, averaged over an orbit, can be obtained as
\[
-\dot{E} = \eta m^2 \left( 1 + \frac{1}{2}\eta m^2 \left( \frac{m}{r} \right)^2 \right) \times [\mu_1 \cdot \mu_2 - 3(\mathbf{L} \cdot \mu_1)(\mathbf{L} \cdot \mu_2)] ,
\] (15)

\[
\frac{dE}{dt} = \left( \frac{dE}{dt} \right)_{GW} + \left( \frac{dE}{dt} \right)_{EM}
\]
\[
= -\frac{32}{5} \eta^2 \left( \frac{m}{r} \right)^5 \left( 1 + \frac{9}{2}\eta m^2 \left( \frac{m}{r} \right)^2 \right) \times [\mu_1 \cdot \mu_2 - 3(\mathbf{L} \cdot \mu_1)(\mathbf{L} \cdot \mu_2)] + \frac{1}{96\eta^2 m^2}
\]
\[
\left( \frac{m}{r} \right)^2 \left[ 3\mu_{\text{eff}}^2 + (\mu_{\text{eff}} \cdot \mathbf{L})^2 \right] ,
\] (16)

by using equations (6), (9), (12), and (13).

Combining equations (13), (15), and (16), we can express the change rate of the orbital angular velocity \( \dot{w} \) as a function of \( w \),
\[
\dot{w} = \frac{96}{5} \eta m^5 w^{4/3} \left[ 1 + \sigma_{\text{mag}} (m\omega)^{4/3} \right] ,
\] (17)

where
\[
\sigma_{\text{mag}} = \frac{5}{\eta m^2} [\mu_1 \cdot \mu_2 - 3(\mathbf{L} \cdot \mu_1)(\mathbf{L} \cdot \mu_2)] + \frac{1}{96\eta^2 m^2} [3\mu_{\text{eff}}^2 + (\mu_{\text{eff}} \cdot \mathbf{L})^2] .
\] (18)

By using equation (17), we calculate the accumulated number of GW cycles
\[
N = N_{\text{grav}} + N_{\text{mag}} ,
\] (19)

where \( N_{\text{grav}} \) and \( N_{\text{mag}} \) denote the contributions from the Newtonian gravity term and the magnetic term, respectively, and are expressed as
\[
N_{\text{grav}} = -\frac{1}{32\eta} (\pi m \omega)^{-5/3} f_{\text{max}}^{1/3} f_{\text{min}} ,
\] (20)
\[
N_{\text{mag}} = \frac{5}{32\eta} \sigma_{\text{mag}} (\pi m \omega)^{-1/3} f_{\text{max}}^{1/3} f_{\text{min}} .
\] (21)
Here \( f_{\text{max}} \) is the exit frequency and \( f_{\text{min}} \) is the entering frequency of the detector’s bandwidth.

5. DISCUSSION

Using 10 Hz as the entering frequency and 1000 Hz as the exit one, we obtain the contribution to the total number of GW cycles from the magnetic term as

\[
N_{\text{mag}} = -5.9 \times 10^{-3} \left( \frac{H_1}{10^{16} \, G} \right) \left( \frac{H_2}{10^{16} \, G} \right) \times \left( \frac{\mu}{2 \times 1.4 \, M_{\odot}} \right)^{-13/3},
\]

where we assume \( \mu_1 \cdot \mu_2 < 0, \mu_1 \parallel \vec{L}, \mu_2 \parallel \vec{L} \), and \( m_1 = m_2 \). Note that the contribution of the EM radiation reaction (the second term in eq. [18]) is much less than that of the dipole-dipole interaction (the first term in eq. [18]). The maximum magnetic field allowed by the scalar virial theorem is \( \sim 10^{18} \, G \) for NSs (Chandrasekhar 1981; see also Bocquet et al. 1995). If NSs in the inspiraling BNSs have such magnetic fields as \( 10^{17} \, G \lesssim H \lesssim 10^{18} \, G \), the effects of the magnetic fields can change more than one cycle in the accumulated cycles. However, if we consider that the maximum value of the observed fields \( \sim 10^{16} \, G \) is the upper limit for the magnetic fields of NSs, the magnetic term will make negligible contributions to the accumulated phase, contrary to the crude estimate in § 1 and equation (1). Consequently, the magnetic fields of NSs will not present difficulties for the detection of GWs from BNSs, if the upper limit for the magnetic fields of NSs is less than \( \sim 10^{16} \, G \).

The magnetic term in equation (17) has the same dependence on the angular velocity \( \omega \) as 2PN terms. We can see this dependence from the 2PN expression for the frequency sweep (Blanchet et al. 1995),

\[
\dot{\omega} = \frac{96}{5} \eta m^{5/3} w^{11/3} \left[ 1 - \left( \frac{743}{336} + \frac{11}{4} \eta \right) (\text{mw})^{2/3} \right. \\
+ (4\pi - \beta)(\text{mw}) \\
+ \left. \frac{34,103}{18,144} + \frac{13,661}{2016} \eta + \frac{59}{18} \eta^2 + \sigma \right] (\text{mw})^{4/3},
\]

where the spin-orbit parameter is \( \beta = \frac{1}{2} \Sigma \chi \left( 113 m^2 + 75 \mu L \cdot \chi \right) \), the spin-spin parameter is \( \sigma = \frac{\eta}{48} \left( 247 \chi_3 + 721 \chi \right) \), and \( \chi_3 = S_{\mu}/m^3 \). As we can see from equations (17) and (23), we cannot distinguish the magnetic term \( \sigma_{\text{mag}} \) from the spin-spin term \( \sigma \) at 2PN order. Only the sum of the magnetic term and the spin-spin term can be deduced from the frequency evolution of 2PN order. This degeneracy might be broken by examining the waveform modulations caused by the spin-induced precession of the orbit (Apostolatos et al. 1994; Kidder 1995). However, the analysis of Poisson & Will (1995) shows that the measurement error on the spin-spin term is \( \Delta \sigma \sim 17.3 \), assuming that the S/N is 10, \( \beta = 0, \sigma = 0 \), and there is no prior information.\(^6\) On the other hand, the contribution of the magnetic term is estimated as

\[
\sigma_{\text{mag}} = 2.9 \times 10^{-3} \left( \frac{H_1}{10^{16} \, G} \right) \left( \frac{H_2}{10^{16} \, G} \right) \\
\times \left( \frac{m}{2 \times 1.4 \, M_{\odot}} \right)^{-4}, \tag{24}
\]

where we assume \( \mu_1 \cdot \mu_2 < 0, \mu_1 \parallel \vec{L}, \mu_2 \parallel \vec{L} \), and \( m_1 = m_2 \). We can see from this equation that the contribution of the magnetic fields of NSs is much smaller than the measurement error on the spin-spin term. Therefore, the effects of the magnetic fields of NSs are also negligible for parameter estimation with moderate S/N if we consider that the maximum value of the observed fields \( \sim 10^{16} \, G \) is the upper limit for the magnetic fields of NSs.

Conservatively, there are some other reasons to consider that the magnetic fields of inspiraling NSs are smaller than \( \sim 10^{16} \, G \). There are three observed BNSs in our Galaxy and nearby globular cluster that will merge in less than a Hubble time: PSR B1913+16 (Taylor & Weisberg 1989), PSR B1534+12 (Wolszczan 1991; Stairs et al. 1998), and PSR B2127+11C (Prince et al. 1991). From the spin-down rate, the magnetic field strength is estimated as order \( 10^{10} \, G \) for all pulsars in these binary systems. If such a value is typical of the magnetic field strength,\(^6\) the magnetic terms will be negligible. Moreover, if the decay time of the magnetic fields is shorter than the coalescence time (Heyl & Kulkarni 1998; Thompson & Duncan 1996; Goldreich & Reisenegger 1992; Shalybkov & Urpin 1995), of course, the magnetic fields will not be concerned, although the magnetic field evolution of isolated NSs is an unresolved issue. It may also be difficult for BNSs including magnetars to be formed because of large recoil velocities (Duncan & Thompson 1992).

In this paper, we regard NSs as spherical compact bodies. However, when we consider NSs as extended bodies, we have to take into account the quadrupole effects induced by magnetic fields. The magnetic fields are a source of nonhydrostatic stress in the interiors of NSs. A magnetic dipole moment \( \mu \) would give rise to moment differences of order

\[
\epsilon = \frac{I_c - I_a}{I_a} \sim \frac{R^4 H^2}{m^2}, \tag{25}
\]

where \( I_c \) and \( I_a \) refer to the moments of inertia about the dipole axis and about an axis in the magnetic equator. Then, the gravitational potential \( \eta m^2/r \) between NSs is modified by an amount of order \( m_1 \epsilon \gamma^2 \sim \mu^2/m^2 \). This is the same order as the EM interaction term in equation (2) when \( \mu_1 \sim \mu_2 \). Therefore, an accurate ellipticity \( \epsilon \) in equation (25) is needed to determine the magnetic effects within a factor (Bocquet et al. 1995; Bonazzola & Gourgoulhon 1996; Konno, Obata, & Kojima 1999). Note that there are such quadrupole effects even if only one companion has magnetic moment.\(^7\)

\(^6\) This set of configurations, \( \mu_1 \cdot \mu_2 < 0, \mu_1 \parallel \vec{L}, \mu_2 \parallel \vec{L} \), is the most stable one; i.e., the EM interaction energy in equation (7) becomes the minimum value. However, note that the alignment rate of the magnetic moments \( w_{\text{mag}} \) is smaller than the orbital inspiral rate \( w_{\text{ins}} \sim (m/r)^4 m \) when \( H \lesssim 10^{14} (r/10^{11} \, \text{cm})^{-1} \, G \) since the alignment rate \( w_{\text{mag}} \) can be estimated as \( w_{\text{mag}} \sim (\mu^2 m^2/|\mu|)^{1/2} \sim \mu^2 m^2 R^4 \) when \( \mu_1 \sim \mu_2 \sim \mu \).

\(^7\) Because of some simplifying assumptions in Poisson & Will (1995), the true measurement error is still uncertain. However, this will not affect the discussion since the true measurement error will be within a factor of the order of 2 (Balasubramanian, Sathyaprakash, & Dhurandhar 1996; Balasubramanian & Dhurandhar 1998; Nicholson & Vecchio 1998).
Although the effects of the magnetic fields of NSs will be negligible for observations of GWs, they might be concerned with gamma-ray bursts (GRBs). The BNS merger is one of several models of GRB sources. (see, e.g., Piran 1999 for a review). If an NS in the binary system has strong magnetic fields, the total energy emitted by EM waves until coalescence can be estimated from equations (13), (16), and (17) as

$$\frac{\Delta E_{\text{EM}}}{\Delta t} \sim \pi^{2} \mu_{\text{eff}}^{2} (f_{\text{max}}^{2} - f_{\text{mid}}^{2}) / 144 \eta m \sim 10^{44} (H / 10^{16} \text{ G})^{2} (f_{\text{max}} / 10^{3} \text{ Hz})^{2} \text{ ergs.}$$

This energy will be radiated at very low frequency \(\sim 10^{3} \text{ Hz, which is difficult to observe with the present radio telescope. Furthermore, such radiation cannot propagate a plasma if an electron density is larger than } \sim 0.01 \text{ cm}^{-3} \text{ since the plasma frequency is larger than the radiation frequency (e.g., Spitzer 1962). However, this energy may be converted to the thermal energy of the surrounding plasma efficiently if the electron density } n_e \text{ is sufficiently high since the electron-electron relaxation time is about } t_{\text{rel}} \sim (n_e / 10^{11} \text{ cm}^{-3})^{-1} (kT_e / 2 \text{ keV})^{3/2} \text{ s, where } T_e \text{ is the electron temperature (e.g., Spitzer 1962). Then, this thermal radiation might explain the precursory X-ray emission } \sim 10^4 \text{ s before the onset of the GRB observed by the } \text{Ginga} \text{ satellite, in which the total energy of the X-ray precursor emission is estimated to be about } \sim 10^{46} (d / 100 \text{ Mpc})^2 \text{ ergs (Murakami et al. 1991). Even though the strong magnetic fields are not relevant to the X-ray precursor in GRBs, the EM radiation could be the EM signature of the coalescing BNSs. Therefore, it is an interesting future problem to investigate the conversion of the low-frequency EM radiation to the higher frequency one.}

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APPENDIX A

ELECTROMAGNETIC RADIATION FROM A MOVING MAGNETIC MOMENT

In this section we review the radiation from a moving magnetic dipole moment. We consider a particle with only a magnetic dipole moment \(\mu\) in its rest frame \(K\). A moving magnetic dipole moment with velocity \(v = \dot{x}\) relative to an observer frame \(K\) also has an associated electric dipole moment. The apparent electric dipole moment is

$$\mathbf{p} = \mathbf{v} \times \mathbf{\mu}, \quad (A1)$$

where \(\mathbf{\mu} = \mathbf{\mu}' - \gamma (\gamma + 1) (v \cdot \mathbf{\mu}') \mathbf{v}\) is the magnetic moment observed in \(K\) and \(\gamma = (1 - v^2)^{-1/2}\) (Jackson 1998). Therefore, in the observer frame \(K\), the magnetization density \(\mathbf{M}\) and electric polarization density \(\mathbf{P}\) are given by

$$\mathbf{M}(t, z) = \mu(t) \delta[z - x(t)], \quad \mathbf{P}(t, z) = p(t) \delta[z - x(t)], \quad (A2)$$

where \(p\) is given by equation (A1). Recalling that the moving magnetic moment is equivalent to a current \(\mathbf{J} = \nabla \times \mathbf{M} + \dot{\mathbf{P}}\) (Jackson 1998), we can calculate the electric and magnetic fields. For our purpose, it is sufficient to obtain the radiative parts that fall off as the inverse of the distance \(D^{-1}\). The radiation field \(\mathbf{B}^{\text{rad}}\) of this moving magnetic moment is given by (e.g., Heras 1994)

$$\mathbf{B}^{\text{rad}}(t, z) = \frac{3 \dot{\mathbf{a}} \times (\dot{\mathbf{a}} \times \mathbf{\mu} - p)(\dot{\mathbf{a}} \cdot \mathbf{a})}{D W^3} + \frac{3 \dot{\mathbf{a}} \times (\dot{\mathbf{a}} \times \mathbf{\mu} - p)(\dot{\mathbf{a}} \cdot \mathbf{a})}{D W^4}$$

$$\quad + \frac{\dot{\mathbf{a}} \times (\dot{\mathbf{a}} \times \mathbf{\mu} - p)(\dot{\mathbf{a}} \cdot \mathbf{a})}{D W^5} + \frac{\dot{\mathbf{a}} \times (\dot{\mathbf{a}} \times \mathbf{\mu} - p)(\dot{\mathbf{a}} \cdot \mathbf{a})}{D W^6}, \quad (A3)$$

where \(D(t) = z - x(t), \dot{\mathbf{a}} = D(t) / |D(t)| = D(t) / D, \ W = 1 - v \cdot \dot{\mathbf{a}}, a = \dot{\mathbf{v}},\) and the right-hand side of this equation is evaluated at the retarded time \(t'\); i.e., \(t' + D(t) = t\). When we assume that the magnetic dipole moment vector \(\mathbf{\mu}(t)\) is a constant one, the above equation (A3) can be calculated as

$$\mathbf{B}^{\text{rad}}_0 = \frac{1}{D} [\dot{\mathbf{a}} \times (\dot{\mathbf{a}} \times \mathbf{\mu})(\dot{\mathbf{a}} \cdot \mathbf{a}) - \dot{\mathbf{a}} \times \mathbf{p}] = \frac{1}{D} (\dot{\mathbf{a}} \cdot \mathbf{\mu})[(\dot{\mathbf{a}} \cdot \mathbf{a}) \dot{\mathbf{a}} - \dot{\mathbf{a}}], \quad (A4)$$

up to the leading order term of \(v\) and \(|x| / D\). Note that the radiation field \(\mathbf{B}^{\text{rad}}_0\) becomes the same equation as (A4) up to leading order even if we use \(\mathbf{\mu}'\) instead of \(\mathbf{\mu}\). The power radiated per unit solid angle is given by (e.g., Landau & Lifshitz 1975)

$$\frac{dP_0}{d\Omega} = \frac{1}{4\pi} (B^{\text{rad}}_0)^2 D^2 = \frac{1}{4\pi} (\dot{\mathbf{a}} \cdot \mathbf{\mu})^2 [\dot{\mathbf{a}}^2 - (\dot{\mathbf{a}} \cdot \mathbf{a})^2]. \quad (A5)$$

The total instantaneous power is obtained by integrating equation (A5) over all solid angles as

$$P_0 = \frac{2}{15} [2 \mu^2 |\dot{\mathbf{a}}|^2 - (\mathbf{\mu} \cdot \dot{\mathbf{a}})^2]. \quad (A6)$$
APPENDIX B

THE CONTRIBUTION OF ELECTROMAGNETIC FIELDS TO THE MASS

Thus far we have ignored the contribution of the EM fields to the mass of a compact body in a binary system. In this section, we estimate the correction to the mass by the EM fields. We have implicitly defined the mass $m$ as that of the isolated spherical body; i.e., the orbit separation of the binary system is infinity. Therefore, the mass $m$ includes the self-energy of the EM fields. The self-energy $m_{\text{mag}}$ of the magnetic dipole moment $\mu$ is obtained by

$$\begin{align*}
m_{\text{mag}} &= \int \frac{H^2}{8\pi} d^3\mathbf{x}' = \frac{1}{8\pi} \int \frac{3\mu^2 \cos^2 \theta' + \mu^2}{r^6} d^3\mathbf{x}' = \mu^2 \left[ -\frac{1}{3r^3} \right]_R^\infty = \frac{\mu^2}{3R^3},
\end{align*}$$

(B1)

where $R$ is the radius of the compact body. (The effects of the EM fields inside the compact body are discussed in § 5.)

Next, we consider the gravitational interaction between the two compact bodies with magnetic moments. Here we note that at a finite separation the gravitational field of a mass with magnetic dipole moment $\mu$ is different from that of a point mass without magnetic dipole moment even if the masses are the same value since EM fields have an extent. Therefore, the mass $m$ in the gravitational potential term $\eta m^2/r$ in equation (2) suffers a small correction. This correction can be evaluated by considering the gravitational potential produced by the energy of the EM fields of the magnetic dipole moment $\mu$. The gravitational potential $\phi$ at $x$ is obtained by

$$\phi(x) = -\frac{1}{r} \int \frac{H^2}{8\pi|x-x'|} d^3\mathbf{x}' = -\frac{\mu^2}{8\pi} \int \frac{3 \cos^2 \theta' + 1}{r^6|\mathbf{x-x}'|} d^3\mathbf{x}' .$$

(B2)

Here we expand $1/|\mathbf{x-x}'|$ by the spherical harmonics $Y_{lm}(\theta, \phi)$,

$$\frac{1}{|\mathbf{x-x}'|} = 4\pi \sum_{l=0}^\infty \sum_{m=-l}^l \frac{1}{2l+1} \frac{r}{r_3} Y_{lm}^\dagger(\theta', \phi)Y_{lm}(\theta, \phi),$$

(B3)

where $r_3(r_\infty)$ is the smaller (larger) of $|\mathbf{x}|$ and $|\mathbf{x}'|$. Noting the orthonormality of the spherical harmonics, $\int Y_{lm}^\dagger(\theta, \phi)Y_{lm}(\theta, \phi) d\Omega = \delta_{l1}\delta_{m0}$, and a relation, $3\cos^2 \theta' + 1 = 2(4\pi/5)^{1/2}Y_{20}(\theta', \phi) + 2(4\pi)^{-1/2}Y_{00}(\theta', \phi')$, equation (B2) is calculated as

$$\phi(x) = -\frac{1}{r} \left[ \frac{\mu^2}{3R^3} + \frac{\mu^2(3 \cos \theta - 1)}{10R^2} - \frac{\mu^2 \cos^2 \theta}{4r^3} \right].$$

(B4)

The first term in the bracket on the right-hand side of the above equation comes from the total self-energy of the EM fields in equation (B1), and the last two terms are corrections due to the extent of the EM fields. The order of the correction is estimated as $\delta m/m \sim (\mu^2/R^2)m \sim \mu^2/mR^2$. On the other hand, the correction due to the dipole-dipole interaction is of order $\mu_1 \mu_2/mR^2$ from equation (17). Therefore, the correction due to the extent of the EM fields is smaller than that due to the dipole-dipole interaction when $\mu_1 \sim \mu_2$. We have also confirmed that the contribution of the EM fields to the quadrupole moments $I_{ij}$ is of order $\mu^2/mR^2$.

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