Interacting Ricci Dark Energy and its Statefinder Description

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In this paper we have considered an interacting Ricci dark energy in flat FRW universe. We have reconstructed the Hubble’s parameter under this interaction. Also, we have investigated the statefinder diagnostics. It has been revealed that the equation of state parameter behaves like quintessence in this interaction and from the statefinder diagnostics it has been concluded that the interacting Ricci dark energy interpolates between dust and ΛCDM stages of the universe.

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A. Introduction

The “dark energy” that is responsible for the present accelerated expansion of the universe occupies about 70% of today’s universe. Reviews on DE include [1], [2], [3] and [4]. The basic characteristic of DE is that its equation of state (EOS) parameter $w = p/\rho$, where $\rho$ is the energy density and $p$ is the pressure that has a negative value. The simplest candidate of dark energy is a tiny positive cosmological constant [2] corresponds to a fluid with a constant equation of state $w = -1$. However, as is well known, it is plagued by the so-called “cosmological constant problem” and “coincidence problem” [2]. Other dark energy models include quintessence [5], phantom [6], quintom [1], Chaplygin gas [7], tachyon [8], hessence [9] etc. All DE models can be classified by the behaviors of equations of state as following [1]:

- Cosmological constant: its EoS is exactly equal to $-1$, that is $w_{DE} = -1$.
- Quintessence: its EoS remains above the cosmological constant boundary, that is $w_{DE} \geq -1$.
- Phantom: its EoS lies below the cosmological constant boundary, that is $w_{DE} \leq -1$.
- Quintom: its EoS is able to evolve across the cosmological constant boundary.
In recent times, an interesting attempt for probing the nature of dark energy within the framework of quantum gravity is the so-called holographic dark energy (HDE) proposal [10]. The holographic principle is an important result of the recent research for exploring the quantum gravity [11]. Inspired by the holographic principle, Gao et al. [12] took the Ricci scalar as the IR cut-off and named it the Ricci dark energy (RDE), in which they take the Ricci scalar $R$ as the IR cutoff. With proper choice of parameters the equation of state crosses $-1$, so it is a ‘quintom’ [13]. The Ricci scalar of FRW universe is given by $R = -6\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right)$, where $H$ is the Hubble parameter, $a$ is the scale factor and $k$ is the curvature. It has been found that this model does not only avoid the causality problem and is phenomenologically viable, but also naturally solves the coincidence problem [14]. The energy density of RDE is given by $\rho_{RDE} = 3c^2\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right)$. In flat FRW universe, $k = 0$ and hence $\rho_{RDE} = 3c^2\left(\dot{H} + 2H^2\right)$.

Interacting DE models have gained immense interest in recent times. Works in this direction include [15], [16], [17]. Interacting RDE was considered in [14], where the observational constraints on interacting RDE were investigated. In this work, we have considered an interacting RDE. We have reconstructed the Hubble’s parameter $H$ under this interaction and subsequently calculated the equation of state parameter $w$, deceleration parameter $q$ and statefinder parameters $\{r, s\}$ in terms of redshift $z$. This study deviates from [14] in the choice of the interaction term. Moreover, contrary to [14], we have not considered radiation while considering interaction and expressed $H$ in terms of redshift $z$. In [13], statefinder diagnostics of RDE was investigated without any interaction. Our study deviated from [13] in this regard.

### B. Interacting RDE

The metric of a spatially flat homogeneous and isotropic universe in FRW model is given by

$$ds^2 = dt^2 - a^2(t) \left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

(1)

where $a(t)$ is the scale factor. The Einstein field equations are given by

$$H^2 = \frac{1}{3}\rho$$

(2)
\[ \dot{H} = -\frac{1}{2}(\rho + p) \]  

(3)

where \( \rho \) and \( p \) are energy density and isotropic pressure respectively (choosing \( 8\pi G = c = 1 \)).

The conservation equation is given by

\[ \dot{\rho} + 3H(\rho + p) = 0 \]  

(4)

As we are considering interaction between RDE and dark matter, the conservation equation will take the following form

\[ \dot{\rho}_{\text{total}} + 3H(\rho_{\text{total}} + p_{\text{total}}) = 0 \]  

(5)

where, \( \rho_{\text{total}} = \rho_{\text{RDE}} + \rho_{\text{m}} \) and \( p_{\text{total}} = p_{\text{RDE}} \) (as we are considering pressureless dark matter, \( p_{\text{m}} = 0 \)). As in the case of interaction the components do not satisfy the conservation equation separately, we need to reconstruct the conservation equation by introducing an interaction term \( Q \). It is important to note that the conservation equations imply that the interaction term should be a function of a quantity with units of inverse of time (a first and natural choice can be the Hubble factor \( H \)) multiplied with the energy density. Therefore, the interaction term could be in any of the forms \[ \text{[18]}: \ Q \propto H\rho_{\text{RDE}}, \ Q \propto H\rho_{\text{m}} \text{ and } Q \propto H\rho_{\text{total}}. \]

Considering the interaction term \( Q \) as \( Q = 3H\delta\rho_{\text{m}} \), where \( \delta \) is the interaction parameter, the conservation equation (13) takes the form

\[ \dot{\rho}_{\text{RDE}} + 3H(\rho_{\text{RDE}} + p_{\text{RDE}}) = Q \]  

(6)

and

\[ \dot{\rho}_{\text{m}} + 3H\rho_{\text{m}} = -Q \]  

(7)

It should be noted that we are considering pressureless dark matter. It may be noted that similar choice of the interaction term has been made in \[ \text{[18]} \]. If \( Q > 0 \), there is a flow of energy from dark matter to dark energy \[ \text{[19]} \].
Fig. 1 shows the variations of equation of state parameter $w_{RDE}$ against $z$ for $\delta = 0.05$ and $\rho_{m0} = 0.23$.

The red, green and blue lines correspond to $c^2 = 0.7, 0.75$ and 0.8 respectively.

From equations (3) and (7) we express $p_{RDE}$ under interaction in a flat FRW universe as

$$p_{RDE} = - \left[ (2 + 3c^2)\dot{H} + 6c^2H^2 + \rho_{m0}a^{-3(1+\delta)} \right]$$  \hspace{1cm} (8)

Using the energy density and pressure of RDE in (6) we express $H(z)$ as

$$H(z)^2 = \frac{c^2}{-1 + 2c^2} + B_0(1+z)^{-\frac{2}{c}+4c} + \frac{2c(1+z)^{3(1+\delta)}\rho_{m0}}{6 + 3c(3 - 4c + 3\delta)}$$ \hspace{1cm} (9)

Using the above form of Hubble's parameter in $p_{RDE}$ and $\rho_{RDE}$ we get the pressure and energy density of RDE under interaction. Subsequently we calculate the equation of state parameter $w_{RDE} = \frac{p_{RDE}}{\rho_{RDE}}$ and plot in figure 1 against $z$ for various values of $c^2$.

C. Statefinder diagnostics

The problem of discriminating different dark energy models is now emergent. In order to solve this problem, a sensitive and robust diagnostic for dark energy is a must. The statefinder parameter pair $r, s$ introduced by [20] is proven to be useful tools for this purpose. The statefinder pair is
a ‘geometrical’ diagnostic in the sense that it is constructed from a space-time metric directly, and it is more universal than ‘physical’ variables which depends upon properties of physical fields describing dark energy, because physical variables are, of course, model-dependent [13]. First, we consider deceleration parameter $q$

$$q = -\frac{\ddot{a}}{a^2} = -1 - \frac{\dot{H}}{H^2} = -1 - \frac{a}{2H^2} \frac{d\dot{H}^2}{da} = -1 + \frac{(1 + z) \ d\dot{H}^2}{H^2 \ dz} \quad (10)$$

and subsequently we consider the statefinder parameters

$$r = 1 + 3 \frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \quad (11)$$

and

$$s = -\frac{3H \dot{H} + \ddot{H}}{3H(2\dot{H} + 3H^2)} \quad (12)$$

Or, equivalently [21]

$$r = 1 + \frac{2a}{H^2} \frac{d\dot{H}^2}{da} + \frac{a^2}{2H^2} \frac{d^2\dot{H}^2}{da^2} = 1 + \frac{(1 + z) \ d\dot{H}^2}{H^2 \ dz} + \frac{(1 + z)^2 \ d^2\dot{H}^2}{2H^2 \ dz^2} \quad (13)$$

and

$$s = -\frac{4a \frac{d\dot{H}^2}{da} + a^2 \frac{d^2\dot{H}^2}{da^2}}{3(3H^2 + a \frac{dH^2}{da})} = -\frac{2(1 + z) \frac{d\dot{H}^2}{dz} - (1 + z)^2 \frac{d^2\dot{H}^2}{dz^2}}{3 \left(3\dot{H}^2 - (1 + z) \frac{d\dot{H}^2}{dz}\right)} \quad (14)$$

Using $\ddot{H} = H(z)$ we get under the present interaction the deceleration parameter as

$$q(z) = -1 + \frac{2(-1 + 2c^2)B_0(1 + z)^{-\frac{2}{c} + 4c}}{c} + \frac{6c(1 + z)^{3(1 + \delta)}(1 + \delta)\rho_{mo}}{6 + 3c(-4c + 3(1 + \delta))} \quad (15)$$

The statefinder pair $\{r, s\}$ are obtained as

$$r = 1 + \frac{\zeta_1}{\zeta_2} \quad (16)$$

where

$$\zeta_1 = 3(-1 + 2c^2) \times$$

$$\left\{(-1 + 2c^2)(-2 + c(-3 + 4c))B_0(1 + z)^{-\frac{2}{c} + 3\delta}(-2 + 4c^2 - 3c(1 + \delta)) - 3c^3(1 + z)^{3 - 4c\delta}(1 + \delta)\rho_{mo}\right\} \quad (17)$$
Fig. 2 plots the deceleration parameter \( q \) against \( z \) for \( \delta = 0.05 \) and \( \rho_{mo} = 0.23 \). The red, green and blue lines correspond to \( c^2 = 0.7, 0.75 \) and 0.8 respectively.

\[
\zeta_2 = c^2 \left\{ 3(1 + z)^{-3\delta}(-1 + 2c^2)B_0(1 + z)^{-\frac{2}{3} + c^2(1 + z)^{-4c}} \right\} \\
\times (-2 + 4c^2 - 3c(1 + \delta)) - 2c(-1 + 2c^2)(1 + z)^{3-4c}\rho_{mo}
\]
\[
(18)
\]

\[
s = \frac{\xi_1}{\xi_2}
\]
\[
(19)
\]

where

\[
\xi_1 = 2(-1 + 2c^2) \times \\
\left\{ (-1 + 2c^2)(-2 + c(-3 + 4c))B_0(1 + z)^{-\left(\frac{2}{3} + 3\delta\right)}(-2 + 4c^2 - 3c(1 + \delta)) - 3c^3(1 + z)^{3-4c}\delta(1 + \delta)\rho_{mo} \right\}
\]
\[
(20)
\]

and

\[
\xi_2 = 3c(-(-1 + 2c^2))(-2 + c(-3 + 4c))B_0(1 + z)^{-\left(\frac{2}{3} + 3\delta\right)}(-2 + 4c^2 - 3c(1 + \delta)) + \\
c^2(1 + z)^{-4c} \left\{ 3c(1 + z)^{-3\delta}(-2 + 4c^2 - 3c(1 + \delta)) + 2(-1 + 2c^2)(1 + z)^{3\delta}\rho_{mo} \right\}
\]
\[
(21)
\]

The deceleration parameter is plotted against \( z \) in figure 2 and the \( \{s - r\} \) trajectory is plotted in figure 3 for different values of \( c^2 \).
Fig. 3 plots the \( s - r \) trajectory for \( \delta = 0.05 \) and \( \rho_{mo} = 0.23 \). The red, green and blue lines correspond to \( c^2 = 0.7, 0.75 \) and \( 0.8 \) respectively.

D. Concluding remarks

Considering the interaction between RDE and pressureless dark matter in a flat Friedman-Robertson-Walker universe we observe the following:

From figure 1 we observe that the equation of state parameter \( w_{RDE} \) is gradually decaying towards \(-1\) with the evolution of the universe. Throughout the evolution of the universe it is staying above \(-1\) and at lower redshifts it is tending to \(-1\). However, it never crosses the \(-1\) boundary. This indicates ‘quintessence’ behavior \[1\]. This observation is consistent with the observation of
where for $c^2 > 1/2$ the equation of state parameter for RDE behaved like ‘quintessence’ in presence of dark matter without interaction.

From figure 2 we find that when $c^2 = 0.7$, the deceleration parameter $q$ is negative throughout the evolution of the universe. However, for $c^2 = 0.75$ and 0.8, the deceleration parameter is transiting from positive to negative side with the evolution of the universe. This leads us to conclude that for $c^2 = 0.7$ the interacting RDE is giving an ever-accelerating universe. However, for the other values of $c^2$ the interacting RDE is producing the transition from decelerated to the current accelerated universe. Moreover, we observe that $q$ is increasing in the negative side with decrease in the redshift. This indicates an increase in the acceleration of the universe.

A study of statefinder diagnostics of Ricci dark energy was done by [13], where it was found that the $\{s-r\}$ trajectory is a vertical segment, i.e. $s$ is a constant during the evolution of the universe for a particular choice of $c^2$. Figure 3, suggests a different behaviour of the $\{s-r\}$ trajectory. The trajectory is mostly confined in the second quadrant of the $s-r$ plane. The spatially flat $\Lambda$CDM (cosmological constant $\Lambda$ with cold dark matter) scenario corresponds to a fixed point in the diagram $\{s,r\}|_{\Lambda CD M} = \{0, 1\}$. We find that the $\{s-r\}$ can not cross $\{r = 1, s = 0\}$. This means that it can not go beyond $\Lambda$CDM. We also find from figure 3 that for finite $r$, $s \to -\infty$. This corresponds to the dust stage of the universe. Thus, the interacting RDE interpolates between dust and $\Lambda$CDM stage of the universe.

The study reported in this paper has investigated the behaviors of the deceleration parameter, equation of state parameters $\{r, s\}$ in presence of interacting Ricci dark energy. As a future study we propose to investigate current observational constraints on this model from SNIa, CMB and BAO observations. In particular, it may be examined whether the present interaction can affect the CMB constraint. We expect that the future high precision observation data may enlighten Ricci dark energy further and may reveal some significant features of the underlying theory of dark energy.

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