Parity breaking and phase transition induced by a magnetic field in high \( T_c \) superconductors

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Abstract

We suggest that the 2+1 dimensional Gross–Neveu model can give an effective field theory description of low-energy quasiparticles in high temperature superconductors. The magnetic catalysis of dynamical symmetry breaking is examined. The model shows that a magnetic field can induce a phase transition associated with parity breaking. In particular, it is intended to give an explanation of a second phase transition observed in a recent experiment by Krishana \textit{et al.}

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1 Introduction

The catalysis of chiral symmetry breaking by a magnetic field was studied in Nambu–Jona-Lasinio models in $2 + 1$ and $3 + 1$ dimensions. It was also extended to the case of QED, external non-Abelian chromomagnetic fields and finite temperatures, curved space, supersymmetry, etc. confirming the universality of the phenomenon. By far it is already well known that a magnetic field is a strong catalyst of dynamical symmetry breaking, leading to generating a fermion dynamical mass even at the weakest attractive interactions between fermions.

In this paper we promote the hypothesis that this magnetic catalysis of dynamical symmetry breaking can be used to explain the results of a recent experiment on Bi$_2$Sr$_2$CaCu$_2$O$_8$ (BSCCO) by Krishana et al. At temperature well below the superconducting critical temperature ($T_c$), they discovered a kink-like feature of thermal conductivity at a transition magnetic field, which depends on temperature approximately in a quadratic form. They suggested that the field induced a second phase transition in the superconducting state: a large gap appeared abruptly and time reversal symmetry was broken. By now, several possible mechanisms of this phenomenon have been discussed. Laughlin argued that a small parity-violating $d_{xy}$ superconducting order parameter was developed, and his hypothesis led to a model free-energy functional which could display a phase transition induced by a magnetic field. But he did not tell how the $d_{xy}$ order parameter was developed.

In the following, we shall argue that an extended version of the Gross–Neveu model in $2 + 1$ dimensions is suitable for studying the low energy excitations of high temperature superconductors. The theory shows that an external magnetic field $B$ can
induce a continuous phase transition at temperatures below the superconducting critical
temperature: an additional gap is developed and parity is broken. The temperature de-
pendence of the critical magnetic field shall be calculated and fitted into the experiment
of Krishana et al. \[6\] Unlike those parity-conserving gauge models studied in Refs. \[8, 10\],
parity is broken in our model. Since parity violation is becoming popular in studying high
$T_c$ superconductors (see e.g. \[14, 7, 15, 16\]), our model is thus of great interest.

2 The model

We limit our discussion to clean two spatial dimensional superconductors in respect of the
layered structure of cuprates. Following Lee and Wen \[17\], we assume that the elementary
excitations in the superconducting state are well defined fermionic quasiparticles with dis-

erption $E(p) = \sqrt{(\epsilon(p) - \mu)^2 + \Delta^2(p)}$, where $p = (p_1, p_2)$, $\epsilon(p) = 2t_f[\cos(p_1a) + \cos(p_2a)]$
and $\Delta(p) = \frac{1}{2}\Delta_0[\cos(p_1a) - \cos(p_2a)]$ with $a$ the square lattice constant. Obviously, $\Delta(p)$
is a $d_{x^2-y^2}$-wave gap. The Fermi surface consists of four isolated nodes at $p = (\pm \frac{\pi}{2a}, \pm \frac{\pi}{2a})$
where the gap is vanishing. In the following, we shall consider the case of chemical potential
$\mu \approx 0$ \[17, 18\].

In the BCS formulation of the high $T_c$ superconductor, the above quasiparticle spectrum
can be obtained from the Bogoliubov–Nambu Hamiltonian \[19\]

$$H = \sum_p \begin{pmatrix} c_{\uparrow}^\dagger(p), c_{\downarrow}(-p) \end{pmatrix} \begin{pmatrix} \epsilon(p) & -i\tilde{\Delta}(p) \\ i\tilde{\Delta}^*(p) & -\epsilon(p) \end{pmatrix} \begin{pmatrix} c_{\uparrow}(p) \\ c_{\downarrow}^\dagger(-p) \end{pmatrix}, \quad (1)$$

where $c_{\sigma}(p)$ and $c_{\sigma}^\dagger(p)$ are the quasi-electron annihilation and creation operators with spin
indices $\sigma = \uparrow, \downarrow$. $\tilde{\Delta}(p)$ is related to $\Delta(p)$ by $\tilde{\Delta} = \exp(i\phi)\Delta$ where $\phi$ can be interpreted as
the phase of the complex superconducting order parameter. (By definition, \( \Delta(p) \equiv |\tilde{\Delta}(p)| \) is real.) Since our aim is to obtain an effective description at low energies and long length-scales, it is sufficient to focus on the gapless modes near the \( d \)-wave nodes (known as nodons [20]), integrating out those electrons far away in the Brillouin zone. Hence, it is legitimate to linearize the quasiparticle Hamiltonian (1) and perform a rotation in momentum space. In the vicinity of each node, say \( (\frac{\pi}{2a}, \frac{\pi}{2a}) \), we have

\[
\epsilon(p) \rightarrow v_F p_2', \quad \Delta(p) \rightarrow v_\Delta p_1',
\]

where \( p_1' = (1/\sqrt{2})(-p_1 + p_2) \), \( p_2' = (1/\sqrt{2})(-p_1 - p_2 + \pi/a) \), \( v_F = 2\sqrt{2}t_f a \) and \( v_\Delta = \Delta_0 a/\sqrt{2} \).

In the coordinate space, the resulting continuum theory reads

\[
H = \sum_r \int d^2x \begin{pmatrix} c_\uparrow(x) \\ c_\downarrow(x) \end{pmatrix}_r \begin{pmatrix} v_F \hat{p}_2 & -i\tilde{\Delta}(\hat{p}) \\ i\tilde{\Delta}^*(\hat{p}) & -v_F \hat{p}_2 \end{pmatrix} \begin{pmatrix} c_\uparrow(x) \\ c_\downarrow(x) \end{pmatrix}_r,
\]

where the index \( r \) runs over four nodal points, \( \hat{p}_i \equiv -i\partial_i \) with index \( i = 1, 2 \), and \( \tilde{\Delta}(\hat{p}) = e^{i\phi}v_\Delta \hat{p}_1 \).

The presence of an external constant magnetic \( B \) field perpendicular to the superconducting plane can be incorporated into our effective Hamiltonian (3) by introducing an external (classical) gauge field \( A \) and replacing \( \hat{p} \rightarrow \hat{p} \pm eA \) with \( \pm \) depending on whether the quasiparticle is electron- or hole-like [21, 22]. Here is a subtle point. In order to maintain the Hamiltonian both gauge invariant and hermitian, we should not introduce gauge fields through the complex superconducting order parameter (or gap) function. That is, we replace \( \epsilon(\hat{p}) \rightarrow \epsilon(\hat{p} \pm eA) \) whereas keeping \( \tilde{\Delta}(\hat{p}) \) unchanged [23]. Alternatively, one can understand this by an intuitive argument: a gap function \( \tilde{\Delta} \) is by definition expressed in terms of the
paring fields and the interaction potential and therefore should be naturally interpreted as a function of coordinate variable $x$ rather than momentum operator $\hat{p}$, while $\epsilon (\hat{p})$ can be thought as a kinematic term. Thus,

$$H = \sum_r \int d^2 x \left( c_\uparrow^\dagger(x), c_\downarrow(x) \right)_r \left( \begin{array}{cc} v_F(\hat{p}_\perp + eA_2) & -i\tilde{\Delta}(\hat{p}) \\ i\tilde{\Delta}^*(\hat{p}) & -v_F(\hat{p}_\perp - eA_2) \end{array} \right) \left( \begin{array}{c} c_\uparrow(x) \\ c_\downarrow^\dagger(x) \end{array} \right)_r. \quad (4)$$

(In our notation, the electron has charge $-e$.) Accordingly, let us choose a gauge:

$$A = (A_1, A_2) = (0, Bx_1). \quad (5)$$

The above Hamiltonian is manifestly invariant under a U(1) gauge transformation $\Lambda(x)$:

$$A_i(x) \rightarrow A_i(x) + \partial_i \Lambda(x),$$

$$c_\sigma(x) \rightarrow e^{-ie\Lambda(x)} c_\sigma(x), \quad (6)$$

$$\tilde{\Delta} \rightarrow e^{-i2e\Lambda(x)} \tilde{\Delta}.$$  

If $\tilde{\Delta}(\hat{p})$ had been treated in the same way as $\epsilon (\hat{p})$, we would have got into trouble of either violating the U(1) gauge invariance or having a non-hermitian Hamiltonian [23].

Following Anderson [22], we transform away the $\phi$ dependence of the Hamiltonian (4) by a unitary transformation $H' = UHU^{-1}$ with

$$U = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}. \quad (7)$$

We then have

$$H' = \sum_r \int d^2 x \chi_r^\dagger [\sigma_3 v_F(\hat{p}_\perp - eA_2) + \sigma_2 v_\Delta \hat{p}_1] \chi_r \quad (8)$$

where $\sigma_a$’s ($a = 1, 2, 3$) are the usual Pauli matrices and the fermionic fields

$$\chi_r = U \left( \begin{array}{c} c_\uparrow^\dagger \\ c_\downarrow^\dagger \end{array} \right)_r.$$
To obtain Eq. (8), we have used the fact that
\[ \nabla \phi = -2eA + m\nu_s, \]  
(9)
and assumed that, deep inside a superconductor, the supercurrent velocity \( \nu_s \approx 0 \) (i.e., the contribution of vortex lattice has been ignored). Anderson suggested that Eq. (8) is the effective Hamiltonian appropriate for hole-like excitations in high \( T_c \) superconductors.

There exists another unitary matrix \( U \) that would lead to an effective Hamiltonian for electron-like excitations, identical with (8) except for \( e \to -e \) (see Ref. [22] for detail). The Hamiltonian (8) can be equivalently recast into a Dirac (“relativistic”) form. To achieve this, let us rewrite \( H' \) in terms of new fermion fields, related to the spinors \( \chi_r \) via a unitarity transformation,
\[ \psi_r = \frac{1}{\sqrt{2}}(\sigma_1 + \sigma_3) \chi_r = \frac{1}{\sqrt{2}} \left( e^{i\phi} c^r_\uparrow + c^r_\downarrow \right), \]  
(10)
and rescale our coordinate space such that \( x_1 \to (v_\Delta/v_D)x_1, x_2 \to (v_F/v_D)x_2 \), where \( v_D \equiv \sqrt{v_F v_\Delta} \) takes place of the velocity of light in a true relativistic quantum theory. Also, we are free to replace \( p_1 \to (p_1 - eA_1) \) for convenience since we are working in the gauge in which \( A_1 = 0 \). Thus, the effective Hamiltonian for nodal particles (quasiholes, for instance) reads
\[ H' = v_D \sum_r \int d^2x \psi^\dagger_r \gamma_3(\gamma_i D_i) \psi_r, \]  
(11)
where we have introduced the covariant derivative \( D_i = \partial_i - ieA_i \) (with \( i = 1, 2 \)) and the 2 \( \times \) 2 Dirac matrices \( \gamma_\mu = (\sigma_1, \sigma_2, \sigma_3) \) which satisfy the Clifford algebra \( \{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \) \( (\mu, \nu = 1, 2, 3) \).

We are now in a position to propose a low energy effective field theory for quasiparticles in the high \( T_c \) superconducting state. The partition functional and Lagrangian read (in
Euclidean space with \( \hbar \equiv v_D \equiv 1 \) and \( \beta = 1/k_B T \) respectively

\[
Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \ e^{-\int_0^\beta d\tau \int d^2x \mathcal{L}}
\]  

(12)

and

\[
\mathcal{L} = \bar{\psi}_r (\gamma_3 \partial_\tau + \gamma_i D_i) \psi_r - \frac{g}{2N_f} (\bar{\psi}_r \psi_r)^2, \quad (i = 1, 2),
\]  

(13)

where \( \bar{\psi}_r \equiv \psi^*_r \gamma_3 \). The 2-component spinor \( \psi_r \) represents the nodal particle (e.g., quasi-hole) field with a “flavor” index \( r = 1, 2, \cdots, N_f \) \( (N_f = 4 \) for the physical case of four nodes). (Hereafter, “flavor” indices \( r \) shall be suppressed in our notation.) In this theory, we have introduced a four-fermion interaction \( V_{\text{int}} = -\frac{g}{2N_f} (\bar{\psi}\psi)^2 \) with \( g \) the coupling constant of dimensionality of \([\text{mass}]^{-1}\) to mimic the quasiparticle interactions in superconducting states. The theory (13) is known to particle physicists as one version of the \( U(N_f) \) Gross–Neveu model in \( d = 2 + 1 \) in a constant magnetic field. A mass term is excluded by parity (space-reflection invariance),

\[
\psi(x_1, x_2, \tau) \rightarrow \gamma_1 \psi(-x_1, x_2, \tau).
\]  

(14)

As we shall see, it is this discrete symmetry that suffers breakdown.

Similar models [8, 10] were proposed to study the magnetic catalysis phenomenon in high \( T_c \) superconducting states. However, our model is different. To see this, one can rewrite the Lagrangian (13) as

\[
\mathcal{L} = \mathcal{V}_l (\tilde{\gamma}_3 \partial_\tau + \tilde{\gamma}_i D_i) \Psi_l - \frac{g}{2N_f} (\mathcal{V}_l \tau \Psi_l)^2 \quad (\text{where } l = 1, 2)
\]

by combining two 2-component fermions in a new four-component spinor \( \Psi_l \) and introducing \( 4 \times 4 \) matrices \( \tilde{\gamma}_\mu = \text{diag}(\sigma_\mu, -\sigma_\mu) \) and \( \tau = \text{diag}(1, -1) \). Comparing our theory with those of Refs. [8] and [10] shows that we have chosen a different four-fermion interaction. Our choice, which shall lead to a mass term that breaks parity, is motivated by the popular studies of parity-breaking in high \( T_c \) superconductors [14, 4, 13, 16].
3 Free energy and gap equation

The quartic term in Eq. (13) can be canceled by adding an expression that is quadratic in an auxiliary field $\sigma(x, \tau)$, and that vanishes when $\sigma$ is integrated out. This results in the replacement of Eq. (13) with the equivalent Lagrangian

$$L = \overline{\psi} \left( \gamma_3 \partial_\tau + \gamma_i D_i \right) \psi + \sigma \overline{\psi} \psi + \frac{N_f}{2g} \sigma^2. \quad (15)$$

Apparently, $L$ has a stationary point at $\sigma = -\frac{g}{N_f} \overline{\psi} \psi$. Now, the partition functional is written as

$$Z = \int D\overline{\psi} D\psi D\sigma \ e^{-\int_0^\beta d\tau \int d^2 x L}. \quad (16)$$

To leading order in $1/N_f \ [26]$, the effective action $\Gamma[\sigma]$ is defined by

$$e^{-\Gamma[\sigma]} = \int D\overline{\psi} D\psi e^{-\int_0^\beta d\tau \int d^2 x L}. \quad (17)$$

The Lagrangian (15) is quadratic in fermion fields and we can formally integrate out them. Thus,

$$\Gamma[\sigma] = \beta \mathcal{V}_2 \frac{N_f}{2g} \sigma^2 - \ln(\det K), \quad (18)$$

where $\mathcal{V}_2 = \int d^2 x$ is the area of the space and the “matrix”

$$K_{rr', \tau' \tau} = -\left[ \gamma_i D_i + \gamma_3 \partial_\tau + \sigma \right] \delta^2(x - x') \delta(\tau - \tau') \delta_{rr'}. \quad (19)$$

We only have even powers of $\sigma$ because the trace of an odd number of Dirac matrices vanishes (alternatively, because $\sigma$ changes sign under parity transformation (14)). At first glance, it would seem that $\langle \sigma \rangle = 0$ is automatically a stationary point since $\Gamma$ is even in $\sigma$. However, if we are interested in the breakdown of parity, the issue is precisely whether there
are stationary points other than $\langle \sigma \rangle = 0$. Fortunately, to settle this issue, we only need to compute $\Gamma$ for constant $\sigma$ \[27\].

Since $K$ is invariant under imaginary time translations, it is convenient to perform a Fourier transformation on $\tau$:

$$K_{r\mathbf{x}n,r'\mathbf{x}'n'} = \int_0^\beta \frac{d\mathbf{\tau}}{\sqrt{\beta}} e^{i\omega_n\mathbf{\tau}} \int_0^\beta \frac{d\mathbf{\tau}'}{\sqrt{\beta}} e^{-i\omega_n\mathbf{\tau}'} K_{r\mathbf{x}r'\mathbf{x}'n'n'} = -[\gamma_i D_i - i\omega_n\gamma_3 + \sigma] \delta^2(\mathbf{x} - \mathbf{x}') \delta_{nn'} \delta_{rr'},$$

(20)

where $\omega_n = (2n + 1)\pi/\beta$ is the usual Matsubara frequency for fermions. Then, we have

$$\ln(\det K) = N_f \int d^2 x \sum_{n=-\infty}^{+\infty} \text{tr} \left\{ (\mathbf{x} | \ln(\gamma_i D_i - i\omega_n\gamma_3 + \sigma) | \mathbf{x} ) \right\}$$

$$= \frac{1}{2} N_f \int d^2 x \sum_{n=-\infty}^{+\infty} \text{tr} \left\{ (\mathbf{x} | \ln \left( (i\gamma_i D_i)^2 + \omega_n^2 + \sigma^2 \right) | \mathbf{x} ) \right\}$$

(21)

in virtue of the fact that

$$\sum_n \text{tr} \ln (\gamma_i D_i - i\omega_n\gamma_3 + \sigma) = \sum_n \text{tr} \ln (\gamma_3 [\gamma_i D_i - i\omega_n\gamma_3 + \sigma] \gamma_3)$$

$$= \sum_n \text{tr} \ln (-\gamma_i D_i + i\omega_{-(n+1)}\gamma_3 + \sigma) = \sum_n \text{tr} \ln (-[\gamma_i D_i - i\omega_n\gamma_3] + \sigma).$$

(22)

Therefore, $\ln(\det K)$ can be expressed in terms of an integral over the proper time $s$,

$$\ln(\det K) = -\frac{N_f}{2} \sum_n \int d^2 x \int_0^\infty ds \frac{ds}{s} \text{tr} (\mathbf{x} | e^{-is[(i\gamma_i D_i)^2 + \omega_n^2 + \sigma^2]} | \mathbf{x} ),$$

(23)

where $(i\gamma_i D_i)^2 = -D_i D_i + ieB\gamma_1\gamma_2$. Following Schwinger’s proper time approach \[28\], we find the matrix element

$$\langle \mathbf{x}' | e^{-is[(i\gamma_i D_i)^2]} | \mathbf{x}'' \rangle = \frac{-i}{4\pi} R(\mathbf{x}', \mathbf{x}''; s) eB [\cot(eBs) + i\gamma_3],$$

(24)

where

$$R(\mathbf{x}', \mathbf{x}''; s) = \exp \left[ \frac{i}{4} (\mathbf{x}' - \mathbf{x}'')^2 eB \cot(eBs) + ie \int_{\mathbf{x}''}^{\mathbf{x}'} dx_i A_i(\mathbf{x}) \right],$$
in which the integration path is a straight line. Substituting Eq. (24) into Eq. (23), we get
\[
\ln(\det K) = \frac{i N_f e B}{4 \pi} \sum_n \int d^2 x \int_{0}^{\infty} \frac{ds}{s} e^{-is(\omega_n^2 + \sigma^2)} \cot(eBs). \tag{25}
\]
Inserting Eq. (25) in Eq. (18) gives us the effective action. Let \( \Gamma[\sigma] \equiv \beta \mathcal{V}_2 F(\sigma) \) where \( F(\sigma) \) is called the free energy. After a Wick rotation \( s \to -is \),
\[
F(\sigma) = \frac{N_f \sigma^2}{2 g} + \frac{N_f e B}{4 \pi \beta} \sum_n \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} e^{-s(\omega_n^2 + \sigma^2)} \coth(eBs), \tag{26}
\]
where we have explicitly introduced the ultraviolet cutoff (Debye frequency) \( \Lambda \). An integral very analogous to that of Eq. (26) was evaluated in Appendix B of Ref. [1]. We thus skip all intermediate analyses and directly go the final result of the free energy,
\[
F(\sigma) = \frac{N_f}{2\pi} \left[ \frac{M_0}{2} \sigma^2 - \sqrt{2} \frac{l^2}{l^3} \zeta\left( -\frac{1}{2}, \frac{\sigma^2 l^2}{2} + 1 \right) - \frac{\sigma}{2l^2} \right]
- \frac{N_f}{2\pi l^2} \left[ \ln(1 + e^{-\beta|\sigma|}) + 2 \sum_{k=1}^{\infty} \ln(1 + e^{-\beta \sqrt{\sigma^2 + 2k/l^2}}) \right] + O\left( \frac{1}{\Lambda} \right), \tag{27}
\]
where \( l = |eB|^{-1/2} \) is the magnetic length, \( M_0 = \frac{\pi}{g} - \frac{\Lambda}{\sqrt{\pi}} \) is of the dimension of mass, and \( \zeta(z, q) \) is the generalized Riemann Zeta function [29]. This result is exact for arbitrary strength of \( B \).

The gap equation, \( \partial F/\partial \sigma = 0 \), reads
\[
0 = \sigma \left[ M_0 - \frac{1}{\sqrt{2l}} \zeta\left( 1, \frac{\sigma^2 l^2}{2} + 1 \right) - \frac{1}{2l^2 \sigma} \tanh \frac{\beta|\sigma|}{2} + \frac{2}{l^2} \sum_{k=1}^{\infty} \frac{(\sigma^2 + 2k/l^2)^{-1/2}}{(e^{\beta \sqrt{\sigma^2 + 2k/l^2}} + 1)} \right]. \tag{28}
\]
The nontrivial solution to Eq. (28) yields the quantum thermal average \( \langle \sigma \rangle \). As Eq. (13) implies, a non-vanishing \( \langle \sigma \rangle \) means that fermions gain dynamical mass, \( m = \langle \sigma \rangle \). In other words, a gap opens at the nodes of the quasiparticle spectrum. Meanwhile, parity is broken by the mass term \( \langle \sigma \rangle (\bar{\psi} \psi) \) to appear in the effective Lagrangian. As \( B \to 0 \), Eq. (28) can
be explicitly solved, yielding the known result \[12\]

\[
\langle \sigma \rangle = - M_0 + 2\beta^{-1} \ln\left(\frac{1}{2} \left[ 1 + \sqrt{1 - 4 \exp(\beta M_0)} \right] \right).
\]

Therefore, for some range of parameter space \((T, M_0)\) there is spontaneous parity breakdown. Obviously, as \(T \to 0\), Eq. (29) has a nontrivial solution only if \(M_0 < 0\) (i.e., \(g > \pi^{3/2}/\Lambda\)).

For finite \(B\), the magnetic field however changes the situation dramatically. Firstly, an external magnetic field itself explicitly breaks the parity invariance of the Lagrangian (13) since it requires that the Lagrangian be invariant under transformation (14) together with transformations \((A_1, A_2) \to (-A_1, A_2)\) and \(B \to -B\). Secondly, as we learned from Gusynin et al. \[1\], the magnetic field can catalyze parity breaking. This shall be discussed further in the following section.

4 Comparison with experiment

Next, we shall investigate the free energy (27) and the gap equation (28) numerically for the experiment of Krishana et al. \[6\]. The theory has two free parameters, \(g\) and \(v_D\). The latter determines the typical energy scale of the theory. (In this paragraph only, we shall explicitly restore \(\hbar\) and \(v_D\) in order to discuss experiments.) The lattice constant for the cuprate BSCCO is \(a \sim 5.41\AA\) (see, for example, \[31\]). The point-contact tunneling experiment on BSCCO \[32\] shows that the ratio of \(2\Delta_0/k_B T_c\) is roughly \(11.6 \sim 12.4\). \((T_c\) denotes the superconducting critical temperature, and \(\Delta_0\) is the gap associated with the occurrence of superconductivity and needs not to be confused with the gap given by Eq. (28).) If we take \(12.4\) as the ratio and \(T_c = 92K \[6\], then \(\Delta_0 \approx 49.15\)meV. This gives \(\hbar v_2 = \Delta_0 a/\sqrt{2} \approx \).
0.188eV·Å. On the other hand, the superconducting coherence length for BSCCO can be taken typically as \( \xi_0 = 20\text{Å} \) [10]. By definition, \( h v_F = \pi \Delta_0 \xi_0 \approx 3.09\text{eV} \cdot \text{Å} \). Therefore, we find \( h v_D = h \sqrt{v_F v_2} \approx 0.762\text{eV} \cdot \text{Å} \). The Debye cutoff \( \Lambda \) is of the order of \( h v_D / a \sim 0.14\text{eV} \), and should be regarded as the highest energy scale in our effective field theory.

The free energy (27) is plotted in Fig. 1. It exhibits that at low temperatures a magnetic field induces a (continuous) phase transition: \( F(\sigma) \) can have a stationary point \( \langle \sigma \rangle \neq 0 \), which infers that parity is spontaneously broken. As shown in the insert of Fig. 1, a slight increase of the field above the critical point \( (\delta B = 0.2T) \) results in a gap \( 2\langle \sigma \rangle \approx 16\text{K} \) at \( T = 10\text{K} \). It follows that the quasiparticle density is substantially suppressed by a factor \( \exp(-2\beta\langle \sigma \rangle) \approx 20\% \). Fig. 2 shows the critical line of \( B-T \) determined by Eq. (28) in comparison with the experimental data from Krishana et al. [6]. We find that the experimental data can be well fitted by a very small \( M_0 \) [33]. According to our theory, the critical line means that parity is restored above some temperature that depends on \( B \). We thus conclude that the basic feature of the experiment [3] can be well explained by the present theory.

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$$B \ll H_{c2}$$
where vortex cores comprise a very small relative volume $\propto \frac{B}{H_{c2}}$.

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[30] The Hamiltonian (11) is obviously of a “relativistic” form, which is known to have a spectrum of eigenvalues $E_n = \hbar v_D \sqrt{eBn/\hbar}$ with $n = 0, 1, 2, \cdots$. By contrast, a non-relativistic 2D free electron gas in the presence of a constant magnetic field has Landau levels $E_n = \hbar (eB/m)(n + \frac{1}{2})$.

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[33] $M_0 = 0$ (equivalently $g = \pi^{3/2}/\Lambda$) corresponds to an ultraviolet renormalization group fixed point when $B = 0$ and $T = 0$ [12]. It is also interesting to notice that, as Eq. (13) implies, $g > 0$ represents an attractive quartic interaction.
Figure 1: The free energy $F' \equiv F(\sigma) - F(0)$ as a function of $\sigma$ at $T = 10\text{K}$, $M_0 = 0$, and $\hbar v_D = 0.762\text{eV} \cdot \text{Å}$. Insert: (a) $B = 1.44\text{T}$ is just below the critical point; and (b) $B = 1.64\text{T}$ slightly above the critical point.
Figure 2: The critical line of $B-T$ with parameters $M_0 = 0$ and $\hbar v_D = 0.762\text{eV}\cdot\text{Å}$. The experimental data is the courtesy of Krishana et al.