Can Inhomogeneities Accelerate the Cosmic Volume Expansion?

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If expanding and contracting regions coexist in the universe, the speed of the cosmic volume expansion can be accelerated. We construct simple inhomogeneous dust-filled universe models in which the speed of the cosmic volume expansion is accelerated for finite periods. These models are constructed by removing spherical domains from the Einstein-de Sitter universe and filling each domain with a Lemaître-Tolman-Bondi dust sphere possessing the same gravitational mass as the removed region. This represents an exact solution of the Einstein equations. We find that acceleration of the cosmic volume expansion is realized in some cases when the size of the contracting region is comparable to the horizon radius of the Einstein-de Sitter universe though this model is very different from the universe observed today. This result implies that non-linear general relativistic effects of inhomogeneities are very important to realize the acceleration of the cosmic volume expansion.

§1. Introduction

The accelerating expansion of the universe indicated by observational data for the luminosity distance of Type Ia supernovae and the cosmic microwave background radiation is a great mystery in modern cosmology. The acceleration of a homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe implies the existence of a dark energy component of matter fields which violates the energy conditions and whose nature is unknown. Conversely, if the universe is not homogeneous and isotropic, the observational data do not necessarily indicate an accelerating expansion of the universe, or even if the cosmic expansion is accelerating, it does not necessarily imply the existence of dark energy. Thus, to account for the observational data without introducing dark energy, arguments concerning the effects of inhomogeneities have been made. Roughly speaking, there are two such arguments. One is that the apparent acceleration of our universe can be regarded as a result of an almost spherically symmetric but inhomogeneous peculiar velocity field, assuming that we are located in the vicinity of its symmetry center. With this argument, the acceleration of the cosmic volume expansion is not necessary. The other argument is that the apparent acceleration of our universe results from backreaction effects due to inhomogeneities in the background FRW universe. In spite of its viability, the former argument is simple and clear. By contrast, at present it is unclear whether the backreaction effects of inhomogeneities can actually accelerate the cosmic volume expansion and, further, account for the observed distance-redshift relation.
Perturbative analysis of backreaction effects on the background FRW universe shows that spatial inhomogeneities behave as a positive spatial curvature, and their presence reduces the expansion rate of the FRW universe.\textsuperscript{10–12} Further, recently Kasai et al. conjectured that a non-linear backreaction cannot accelerate the cosmic volume expansion even if the inhomogeneities are very large.\textsuperscript{13} By contrast, one of the present authors, Nambu, and a collaborator, Tanimoto, pointed out the possibility that the non-perturbative features of inhomogeneities are necessary to realize an accelerating expansion of the universe through backreaction effects.\textsuperscript{14} They proposed a model of the universe containing both expanding and contracting regions and showed that a spatially averaged scale factor defined in terms of the volume of a spatial domain can exhibit accelerated expansion if the size of each region is properly chosen.

At present, it is still unclear whether the acceleration of the cosmic volume expansion could be realized through the effects of inhomogeneities in the universe without introducing dark energy. Thus this point must be investigated before we attempt to explain the observed distance-redshift relation with this approach. As Nambu and Tanimoto have not presented a correct example of an accelerating universe, it is necessary to determine whether their mechanism actually works. In this paper, in order to do this, we study the comoving volume of a highly inhomogeneous dust-filled universe.

This paper is organized as follows. In §2, we give a qualitative description of a scenario in which inhomogeneities lead to the acceleration of the cosmic volume expansion. In §3, in order to obtain quantitative information about the acceleration of the cosmic volume expansion, we study the stuffed Swiss-cheese model. Section 4 is devoted to a summary and discussion. In this paper, we employ units in which $c = G = 1$ and basically follow the conventions for the Riemann and metric tensors and the notation used in the textbook of Hawking and Ellis.\textsuperscript{15}

\section*{§2. One scenario, a qualitative argument}

We consider an inhomogeneous dust-filled universe which is initially expanding everywhere. We assume that at some stage, the dust begins contracting in some domains and continues expanding elsewhere (see Fig. 1). Such a situation is consistent with our conventional picture of the real universe: Structures (stars, galaxies, etc.) form within contracting domains, or there might be large-scale bulk velocity fields due to large mass concentrations, in an overall expanding universe. Further, we assume that the universe is almost periodic and thus homogeneous in an average sense. We adopt a dust-comoving Gaussian normal coordinate system in which the line element is given by

\begin{equation}
\begin{aligned}
ds^2 &= -dt^2 + \gamma_{ij}(t, x^k)dx^idx^j, \\
\end{aligned}
\end{equation}

where $i, j, k = 1, 2, 3$ represent the spatial components, and $\gamma_{ij}(t, x^k)$ is the induced metric on the spacelike hypersurface $t = \text{[constant]}$, which is orthogonal to the tra-
Can Inhomogeneities Accelerate the Cosmic Volume Expansion?

The stress-energy tensor of dust is given by

\[ T^{\mu \nu} = \rho u^\mu u^\nu, \]  

(2.2)

where \( \rho \) is the rest mass density of dust and \( u^\mu = \delta^\mu_0 \) is the 4-velocity of dust particles.

We consider a compact spatial domain \( D \) in each \( t = \text{[constant]} \) hypersurface by assuming that \( D \) is dust-comoving. By definition, \( D \) contains a fixed rest mass of dust. We assume that the domain \( D \) is so large that the spatial periodicity is recognizable in this domain. The volume \( V_D \) of \( D \) is defined by

\[ V_D = \int_D \sqrt{\gamma} \, d^3x, \]  

(2.3)

where \( \gamma \) is the determinant of the spatial metric \( \gamma_{ij} \). Following Refs. 13) and 16), we define the spatially averaged scale factor \( a_D(t) \) of the domain by \( 3\dot{a}_D/a_D = \dot{V}_D/V_D \), where the dot represents the derivative with respect to \( t \). This definition is equivalent to the relation

\[ a_D(t) \propto V_D^{1/3}(t). \]  

(2.4)

Because we assume that the expanding and contracting domains coexist within the domain \( D \), we rewrite the volume \( V_D \) in the form

\[ V_D = V_e + V_c, \]  

(2.5)

where \( V_e \) and \( V_c \) are the comoving volumes of the union of the expanding domains and that of the contracting domains, respectively. The time derivative of the volume is given by

\[ \dot{V}_D = \dot{V}_e + \dot{V}_c = |\dot{V}_e| - |\dot{V}_c|. \]  

(2.6)

We see from the above equation that even though \( \dot{V}_D \) is initially positive, once \( \dot{V}_c \) becomes dominant, \( \dot{V}_D \) can be small or even negative. In other words, due to the appearance of contracting domains in \( D \), the cosmic volume expansion of \( D \) can greatly
slow down, or the volume $V_D$ of $D$ can decrease. However, the contracting domains will collapse, and then their contributions to the volume $V_D$ will necessarily become negligible; in realistic situations, almost stationary structures, like stars, black holes and galaxies, are formed. This means that $\dot{V}_D$ will be dominant in $\dot{V}_D$, and thus the volume $V_D$ begins to increase again. Here it should be noted that acceleration of the volume expansion is realized near the end of the slowdown or contraction stage. During this period, the second-order time derivative of the effective scale factor $a_D(t)$ becomes positive. This is the mechanism of the acceleration of the cosmic volume expansion proposed by Nambu and Tanimoto.\textsuperscript{14}

§3. Stuffed Swiss-Cheese Model

In order to quantitatively study the effects of inhomogeneities in the universe, we construct a specific model. First, we consider the Einstein-de Sitter universe (EdS) and remove disconnected spherical domains from it, so as to guarantee spatial periodicity. Next, each removed region is stuffed with spherically symmetric but inhomogeneous dust whose Misner-Sharp mass\textsuperscript{17} is the same as that of the original EdS regions. The dynamics of the inhomogeneous dust ball are described by the so-called Lemaitre-Tolman-Bondi (LTB) solution. We call this model the stuffed Swiss-cheese (SSC) model. We assume that the LTB regions are initially expanding but eventually begin to contract and form singularities and black holes.

The line element of the LTB region is given by

$$ds^2 = -dt^2 + \frac{Y''(t, \chi)}{1-k(\chi)\chi^2}d\chi^2 + Y^2(t, \chi)(d\theta^2 + \sin^2 \theta d\varphi^2),$$ \hspace{1cm} (3.1)

where the prime denotes differentiation with respect to the radial coordinate $\chi$, which is assumed to be non-negative. This coordinate system is a dust-comoving Gaussian normal coordinate system, as in the previous section. The Einstein equations lead to the equations for the areal radius $Y(t, \chi)$ and the rest mass density $\rho(t, \chi)$ of the dust,

$$\dot{Y}^2 = -k(\chi)\chi^2 + \frac{2M(\chi)}{Y},$$ \hspace{1cm} (3.2)

$$\rho = \frac{M'(\chi)}{4\pi Y^3 Y'^2},$$ \hspace{1cm} (3.3)

where $k(\chi)$ and $M(\chi)$ are arbitrary functions of the radial coordinate. We assume that $\rho$ is non-negative and $Y$ is monotonic with respect to $\chi$, i.e., $Y'>0$ in the regular region. This assumption leads to $M' \geq 0$, and thus we can set

$$M(\chi) = \frac{4\pi \rho_0}{3} \chi^3,$$ \hspace{1cm} (3.4)

where $\rho_0$ is a non-negative arbitrary constant. Our treatment does not lose generality with the above choice of $M(\chi)$. Equations (3.1)-(3.3) are invariant under the rescaling of the radial coordinate $\chi$,

$$\chi \rightarrow \tilde{\chi} = \tilde{\chi}(\chi).$$ \hspace{1cm} (3.5)
By virtue of this property, the above form of \( M(\chi) \) is entirely general.

The solution of Eq. (3.6) is given by

\[
Y = \frac{4\pi \rho_0}{3k(\chi)} \left( 1 - \cos \left( \sqrt{k(\chi)} \eta \right) \right) \chi, \tag{3.6}
\]

\[
t - t_i(\chi) = \frac{4\pi \rho_0}{3k(\chi)} \left( \eta - \frac{1}{\sqrt{k(\chi)}} \sin \left( \sqrt{k(\chi)} \eta \right) \right), \tag{3.7}
\]

where \( t_i(\chi) \) is an arbitrary function. We can use this form of the solution for any sign of \( k(\chi) \). Note that \( t_i(\chi) \) is the time at which a shell focusing singularity appears, where 'shell focusing singularity' means \( Y = 0 \) for \( \chi > 0 \) and \( Y' = 0 \) at \( \chi = 0 \). In this paper, we consider the region satisfying \( t > t_i \), and hence the time \( t = t_i \) corresponds to the Big Bang singularity. Hereafter, we assume a simultaneous Big Bang, i.e., \( t_i = 0 \). Using the terminology of cosmological perturbation theory, there are only growing modes near the Big Bang singularity in this scenario.

We denote the boundary between the LTB region and the EdS region by \( \chi = \chi_b \). Then, we divide the LTB region into four regions, \([0, \chi_1)\), \([\chi_1, \chi_2)\), \([\chi_2, \chi_3)\) and \([\chi_3, \chi_b)\). We use the following spatial profile of the curvature function:

\[
k(\chi) = \begin{cases} 
  k_0 & \text{for } 0 \leq \chi < \chi_1, \\
  \frac{k_0}{2\chi^2} \left( \frac{\chi^2 - \chi_3^2}{\chi_1^2 - \chi^2} + \chi_1^2 + \chi_2^2 \right) & \text{for } \chi_1 \leq \chi < \chi_2, \\
  \frac{k_0}{2\chi^2} (\chi_1^2 + \chi_2^2) & \text{for } \chi_2 \leq \chi < \chi_3, \\
  \frac{k_0}{2\chi^2} (\chi_1^2 + \chi_2^2) \left\{ \frac{\chi^2 - \chi_3^2}{\chi_2^2 - \chi_3^2} - 1 \right\}^2 & \text{for } \chi_3 \leq \chi < \chi_b,
\end{cases} \tag{3.8}
\]

where \( k_0 \) is constant. In order to guarantee the relation \( 1 - k\chi^2 > 0 \), the following inequality should hold:

\[
\kappa := \frac{k_0}{2} (\chi_1^2 + \chi_2^2) < 1. \tag{3.9}
\]

Since we are interested in the case that the LTB regions contract, we assume \( k_0 > 0 \).

The parameter \( \kappa \) is closely related to the value of the comoving volume in the LTB region \( V_{LTB} \), which is bounded below as

\[
V_{LTB} := 4\pi \int_{0}^{\chi_b} \frac{Y'^2 Y''}{\sqrt{1-k\chi^2}} d\chi = 4\pi \left( \int_{\chi_0}^{\chi_2} + \int_{\chi_2}^{\chi_3} + \int_{\chi_3}^{\chi_b} \right) \frac{Y'^2 Y''}{\sqrt{1-k\chi^2}} d\chi
\]

\[
= \frac{4\pi}{3\sqrt{1-\kappa}} (Y_3^3 - Y_2^3) + 4\pi \left( \int_{\chi_0}^{\chi_2} + \int_{\chi_2}^{\chi_3} \right) \frac{Y'^2 Y''}{\sqrt{1-k\chi^2}} d\chi
\]

\[
> \frac{4\pi}{3\sqrt{1-\kappa}} (Y_3^3 - Y_2^3) + 4\pi \left( \int_{\chi_0}^{\chi_2} + \int_{\chi_3}^{\chi_b} \right) Y'^2 Y'' d\chi
\]

\[
= \frac{4\pi(1-\sqrt{1-\kappa})}{3\sqrt{1-\kappa}} (Y_3^3 - Y_2^3) + \frac{4}{3} \pi Y_b^3, \tag{3.10}
\]
where, for notational simplicity, we denote $Y_{|\chi=\chi_i}$ by $Y_i$. Note that the last term in the above equation, $4\pi Y_i^3/3$, is equal to the volume of the original EdS region, since the areal radius $Y$ is continuous across the boundary between the LTB and EdS regions. Also, the first term in the last inequality gives a lower bound on the volume increase $\delta V$ due to the inhomogeneous geometry:

$$\delta V > \frac{4\pi(1 - \sqrt{1 - \kappa})}{3\sqrt{1 - \kappa}} (Y_3^3 - Y_2^3).$$  \hfill (3.11)

In the limit $\kappa \to 1$, the volume increase becomes infinite. Thus if $\kappa$ is almost equal to unity, the volume of the LTB region can be dominant in the SSC universe, even if $Y_3$ becomes small. Further, note that this result is not related to how the LTB region is divided into the four regions $[0, \chi_1)$, $[\chi_1, \chi_2)$, $[\chi_2, \chi_3)$ and $[\chi_3, \chi_b)$, if $\kappa$ is sufficiently close to unity.

The physical meaning of the volume increase is the decrease of the binding energy in the LTB region. The binding energy $E_{\text{bind}}(\chi)$ within the spherical domain of the comoving radius $\chi$ is defined by the difference between the gravitational mass $M(\chi)$ and the rest mass of the dust, i.e., we have

$$E_{\text{bind}}(\chi) := M(\chi) - 4\pi \int_0^\chi \frac{\rho Y^2 Y'}{\sqrt{1 - k\chi^2}} d\chi,$$ \hfill (3.12)

where the second term on the right-hand side corresponds to the rest mass of the dust included within the spherical domain of the comoving radius $\chi$. In the case of vanishing $k_0$, the LTB region is identical to the original EdS region, and $E_{\text{bind}}(\chi)$ vanishes. By using Eq. (3.3) and the same manipulation as in Eq. (3.10), we obtain an upper bound on the binding energy $E_{\text{bind}}(\chi_b)$ of the LTB region as

$$E_{\text{bind}}(\chi_b) < -\frac{1}{\sqrt{1 - \kappa}} [M(\chi_3) - M(\chi_2)].$$ \hfill (3.13)

The limit $\kappa \to 1$ leads to a negatively infinite binding energy. In other words, if $\kappa$ is positive, the rest mass of dust in the LTB region is larger than that included in the original EdS region. An arbitrarily large rest mass can be put into the removed domain if $\kappa$ is set sufficiently close to unity. In this sense, the LTB region with $\kappa \simeq 1$ is a very high density region compared with the original EdS region.

The singularity formed at the origin, $\chi = 0$, can be a null naked singularity, whereas that formed in the domain $\chi > 0$ is spacelike. In the case of the present model, since the innermost region $[0, \chi_1)$ is a Friedmann universe with a positive spatial curvature, the singularity formed at $\chi = 0$ is spacelike and thus not naked. We see from Eq. (3.7) that the singularity formation time is $t = 8\pi^2 \rho_0/(3k^{3/2})$. Because $k$ is monotonically decreasing with respect to $\chi$, the singularity formation time is monotonically increasing with respect to $\chi$, and thus no shell crossing singularity is realized in the present model. Therefore, singularities formed through gravitational collapse in the LTB regions are necessarily spacelike in the present SSC model. There is no causal influence of spacelike singularities on the regular domains. This property allows us to ignore the contribution of singularities in the calculation of the volume.
Can Inhomogeneities Accelerate the Cosmic Volume Expansion?

$V_D$ of the domain $D$, and thus we can follow the evolution of the cosmic volume expansion even after singularity formation. Further, it should be noted that the present choice of $k(\chi)$ guarantees that the LTB region smoothly connects to the EdS universe. The line element of the EdS region is given by the form

$$ds^2 = -dt^2 + a_e^2(t) \left[ d\chi^2 + \chi^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right], \quad (3.14)$$

where

$$a_e(t) = \left( \frac{6\pi \rho_0 t^2}{3} \right)^{1/3}. \quad (3.15)$$

We assume that the universe has the periodicity of the comoving interval, $2l$, in the original Einstein-de Sitter universe. With this assumption, a cubic region of comoving edge length $2l$ corresponds to the compact domain $D$ considered in the previous section. We define the scale factor $a(t)$ of this inhomogeneous universe as

$$a_D(t) \equiv \frac{V_D^{1/3}}{2l}. \quad (3.16)$$

3.1. One-scale model

We first consider a model with one LTB region with $\chi_b = l$ in the domain $D$ (see Fig. 2).

![Fig. 2. The compact domain $D$ of the one-scale SSC model.](image)

The volume $V_D$ of the domain $D$ is given by

$$V_D = \left( 8 - \frac{4\pi}{3} \right) a_e^3(t) l^3 + 4\pi \int_0^{a_e(t)} \frac{Y^2 dY}{\sqrt{1 - k(\chi)\chi^2}}. \quad (3.17)$$

The integral on the right-hand side of the above equation corresponds to the comoving volume of the LTB region. It should be noted that $\chi$ in this integral is a function of the areal radius $Y$ at time $t$ through Eqs. (3.6) and (3.7). Thus, this integration
Fig. 3. The left panel plots the evolution of the averaged scale factor $a_D$ of one scale SSC model for various values of $\kappa$. The right panel plots the acceleration of the averaged scale factor $a_D$.

covers the only region which has not yet collapsed to the singularity, i.e., the domain satisfying $Y > 0$ at time $t$.

In Fig. 3, we plot several examples of the evolution of the scale factor $a_D(t)$. We set $H_0l = \sqrt{6}/2, H_0\chi_1 = \sqrt{6}/8, H_0\chi_2 = \sqrt{6}/4$ and $H_0\chi_3 = 3\sqrt{6}/8$ with $H_0 = \sqrt{8\pi\rho_0/3}$. We chose these values so that the areal radius of the boundary of the LTB region is equal to the horizon radius of the EdS universe at $t = l$. These curves represent the scale factors as functions of $t$ for various values of $\kappa$ defined by Eq. (3.9).

As mentioned in the preceding section, the slowdown of the volume expansion or the contraction of the domain $D$ actually does occur, due to the contraction of the LTB region. After the LTB region collapses to a singularity covered by a horizon and forms a black hole, the volume expansion of the domain $D$, which has been slowed by the contracting LTB region, accelerates until the speed of the volume expansion becomes the same as that of the EdS universe. Thus, at a time near the end of the slowdown or contraction stage, an acceleration period appears. In the cases depicted in Fig. 3, acceleration of the volume expansion occurs in four cases ($\kappa \geq 0.999453$), and in these four cases, volume contraction before the acceleration period occurs in the two cases with $\kappa \geq 0.999922$, whereas the other two cases with smaller $\kappa$ do not have a contraction period, although acceleration does occur. In the case of the smallest $\kappa (= 0.984375)$ depicted in Fig. 3, no acceleration period appears.

The necessary condition of the appearance of an acceleration period is that the volume of the LTB region becomes dominant in $V_D$. As mentioned above, if the parameter $\kappa$ is almost equal to unity, the contribution of $\chi$ in the range $\chi_2 \leq \chi < \chi_3$ is dominant in the volume of the LTB region and, further, makes the volume of the LTB region dominant in $V_D$ for a long time before the domain $\chi \leq \chi_3$ completely collapses. Thus, it is conjectured that acceleration of the cosmic volume expansion is realized if and only if $\kappa$ is almost equal to unity, and we can see from Fig. 3 that this is in fact the case in the present model.

The size of the LTB region relative to the cosmological horizon is very important. If there are many contracting LTB regions within the cosmological horizon of the EdS
universe in the acceleration period, the SSC model might mimic our real universe. By contrast, if the size of the LTB region is comparable to the cosmological horizon in the acceleration period, the behavior of this SSC model might be very different from that of our universe. To clarify the situation, we investigate which of these two cases corresponds to this SSC model. It should again be noted that when the volume of the domain $\chi < \chi_3$ is dominant in the volume $V_D$ and is contracting, the volume expansion of the domain $D$ greatly slows down, or the domain $D$ contracts. The acceleration of the volume expansion is realized when the volume of the domain $\chi < \chi_3$ becomes so small that its contribution to $V_D$ is negligible. In other words, the acceleration period appears around the time at which the dust in the domain $\chi < \chi_3$ collapses to a singularity, and this time is given by

$$t_{ac} := \frac{8\pi^2\rho_0\chi_3^3}{3\kappa^{3/2}}. \quad (3.18)$$

At $t = t_{ac}$, the ratio of the areal radius of the boundary of a LTB region $a_e(t)l$ to the Hubble horizon radius of the EdS region, $H^{-1}(t) := a_e/\dot{a}_e = 3t/2$, becomes

$$\frac{a_e(t_{ac})l}{H^{-1}(t_{ac})} = \left(\frac{2}{3\pi}\right)^{1/3} \kappa^{1/2} \frac{l}{\chi_3} \simeq 0.60 \kappa^{1/2} \left(\frac{l}{\chi_3}\right). \quad (3.19)$$

Here it should be noted that $\kappa$ has to be almost equal to unity so that the acceleration of the cosmic volume expansion is realized. Therefore, the size of the LTB region is comparable to the horizon radius of the EdS region in the acceleration period. This means that non-linear general relativistic effects are important for the realization of the acceleration period.

3.2. Two-scale model

As the second model, we consider the situation that there are two kinds of LTB regions, one with radius $\chi_b = l$ and the other with radius $\chi_b = (\sqrt{3}-1)l$. We assume that the large LTB region is put in the cubic domain $D$ in the same manner as in the first model, whereas an eighth of the small LTB region is put on each vertex of $D$ (see Fig. 4).
In this model, the volume of the compact domain $D$ is given by

$$V_D = \left[ 8 - \frac{4\pi}{3} \left\{ 1 + \left( \sqrt{3} - 1 \right)^3 \right\} \right] a_D(t) l^3$$

$$+ 4\pi \int_0^{a_D(t)} \frac{Y^2 dY}{\sqrt{1 - k(\chi_L)\chi_L^2}} + 4\pi \int_0^{(\sqrt{3} - 1)a_D(t)} \frac{Y^2 dY}{\sqrt{1 - k(\chi_S)\chi_S^2}},$$

(3.20)

where $\chi_L = \chi_L(Y)$ and $\chi_S = \chi_S(Y)$ are the radial coordinates as functions of the areal radius $Y$ at time $t$ in the large and small LTB regions, through Eqs. (3.6) and (3.7), respectively. We present one example of the evolution of the scale factor $a_D(t)$ in Fig. 5. As expected, there are two acceleration periods. The first is realized when the small LTB regions collapse to the singularities, and the second appears when the large LTB region collapses. Thus, the coexistence of various scales of LTB regions can lead to more than one acceleration period.
Can Inhomogeneities Accelerate the Cosmic Volume Expansion?

§4. Summary and discussion

Using the stuffed Swiss-cheese model, we investigated how the cosmic volume expansion is accelerated by inhomogeneities in a dust-filled universe. We found that the existence of contracting regions is a necessary condition to realize the acceleration of the cosmic volume expansion. Near the time at which the central contracting region collapses to form a singularity, the acceleration of the averaged scale factor becomes positive, and the acceleration phase of the spatially averaged universe appears. Such a situation might be the same as that in the real universe, with structures (e.g., galaxies, clusters of galaxies) forming within the contracting regions, or there might be large-scale bulk velocity fields due to large mass concentrations, in our expanding universe. However, acceleration of the cosmic volume expansion is realized only when the size of each inhomogeneity is comparable to the cosmological horizon scale in the case of the stuffed Swiss-cheese universe model. We see from this result that this model is very different from the universe observed today in the period of accelerating cosmic volume expansion. However, for the model with various scales of inhomogeneities, the temporal variation of the cosmic volume expansion is nontrivial from a theoretical point of view, and a complete analysis is left as a future work. The analysis of the backreaction effect for the dust-filled universe based on a second-order cosmological perturbation reveals no signs of acceleration of the volume expansion.\(^\text{10),11,13)\) Thus, we conclude that the appearance of the acceleration phase in our model is due to a highly non-linear effect expected to appear beyond third order in the perturbation analysis.

Here we note that the issue of cosmic acceleration first arose from observational data of the distance-redshift relation and that we have not directly observed the acceleration of the ‘volume’ expansion. If the universe is homogeneous and isotropic, then we can conclude from the observational data that the cosmic volume expansion is indeed accelerating. By contrast, in the inhomogeneous universe model considered...
in this paper, the cosmic expansion of the comoving volume does accelerate, but it is not yet clear whether the distance-redshift relation is similar to that of the homogeneous and isotropic universe with accelerating volume expansion. There are several studies of an inhomogeneous but isotropic dust-filled universe whose distance-redshift relation is consistent with the observational data.\textsuperscript{1–4} In these models, there is no contracting region, and therefore the cosmic volume expansion does not accelerate. Conversely there is a possibility that the acceleration of the cosmic volume expansion due to inhomogeneities does not lead a distance-redshift relation similar to that obtained in the observation of Type Ia supernovae. Investigating this point is also a future work.

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References

1) M. Celerier, Astron. Astrophys. 353 (2000), 63.
2) K. Tomita, Mon. Not. R. Astron. Soc. 326 (2001), 287.
3) H. Iguchi, T. Nakamura and K. Nakao, Prog. Theor. Phys. 108 (2002), 809.
4) R. A. Vanderveld, É. É. Flanagan and I. Wasserman, astro-ph/0602476
5) E. W. Kolb, S. Matarrese, A. Notari and A. Riotto, hep-th/0503117
6) E. W. Kolb, S. Matarrese, and A. Riotto, astro-ph/0506594
7) É. É. Flanagan, Phys. Rev. D 71 (2005), 103521.
8) C. M. Hirata and U. Seljak, Phys. Rev. D 72 (2005), 083501.
9) A. Ishibashi and R. M. Wald, Class. Quantum Grav. 23 (2006), 235.
10) H. Russ, M. H. Soffel, M. Kasai and G. Börner, Phys. Rev. D 56 (1997), 2044.
11) Y. Nambu, Phys. Rev. D 62 (2000), 104010.
12) H. Kozuki and K. Nakao, Phys. Rev. D 66 (2002), 104008.
13) M. Kasai, H. Asada and T. Futamase, Prog. Theor. Phys. 115 (2006), 827.
14) Y. Nambu and M. Tanimoto, gr-qc/0507057
15) S. W. Hawking and G. F. R. Ellis, The large scale structure of spacetime (Cambridge University Press, 1973).
16) M. Kasai, Phys. Rev. Lett. 69 (1992), 2330.
17) C. W. Misner and D. H. Sharp, Phys. Rev. 136 (1964), B571.
18) D. Christodoulou, Commun. Math. Phys. 93 (1984), 171.
19) R. A. C. Newman, Class. Quantum Grav. 3 (1986), 527.
20) P. S. Joshi and I. H. Dwivedi, Phys. Rev. D 47 (1993), 5357.