Formation of First Galaxies inside Density Peaks and Voids under the Influence of Dark Matter–Baryon Streaming Velocity. I. Initial Condition and Simulation Scheme

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Abstract

We present a systematic study of the cosmic variance that existed in the formation of the first stars and galaxies. We focus on the cosmic variance induced by the large-scale density and velocity environment engraved at the epoch of recombination. The density environment is predominantly determined by the dark-matter overdensity, and the velocity environment by the dark matter–baryon streaming velocity. Toward this end, we introduce a new cosmological initial condition generator, BCCOMICS, which solves the quasi-linear evolution of small-scale perturbations in the large-scale density and streaming-velocity environment and generates the initial condition for dark matter and baryons, as either particles or grid data at a specific redshift. We also describe a scheme to simulate the formation of the first galaxies inside density peaks and voids, where a local environment is treated as a separate universe. The resulting cosmic variance in the number density of minihalos and the amount of cooling mass are presented as an application. Density peaks become a site for enhanced formation of the first galaxies, which compete with the negative effect from the dark matter–baryon streaming velocity on structure formation.

Key words: cosmology: theory – dark ages, reionization, first stars – Galaxy: formation

1. Introduction

The first stars and first galaxies formed out of the primordial environment, which was devoid of any metal, and they are categorized as Population III objects. For such objects to form, at least high-density, efficient gas-phase cooling and shielding from the coolant-dissociating radiation field are necessary. Such conditions can first be met inside minihalos, which are the first nonlinear objects in the universe. Pioneering numerical simulations found that the first stars were very massive \( M_\star \geq 100-1000 M_\odot \) and formed in isolation inside minihalos (Abel et al. 2002; Bromm et al. 2002; Yoshida et al. 2006), but later, higher-resolution simulations found that multiple formation of intermediate-mass \( (M_\star \lesssim 30 M_\odot) \) stars was likely as well (Turk et al. 2009; Stacy et al. 2010; Greif et al. 2011a). Semi-analytical studies also find that intermediate- or even small-mass Population III stars may have formed out of the primordial environment (Omukai 2001; Nagakura & Omukai 2005). Different physical conditions inside minihalos likely lead to a wide spectrum of the mass of Population III stars (Hirano et al. 2014, 2015). Direct observation of truly metal-free stars is yet to be made in future surveys. Current observations of ultra-iron-poor stars in the Milky Way (e.g., Caffau et al. 2011; Howes et al. 2015; Jacobson et al. 2015) cannot be taken as convincing evidence of small-mass Population III stars because of the existence of other heavy elements.

Because the first stars and first galaxies formed first inside minihalos, varying physical conditions of minihalos (forming through cooling of hydrogen molecules) would lead to a variation in the outcome of the first-star formation. In addition to the minihalo-to-minihalo variance (Hirano et al. 2014, 2015), a large-scale variance in the streaming flow of baryons against dark matter particles was found to impact the formation and evolution of minihalos (Tseliakhovich & Hirata 2010, TH hereafter). This effect can be parameterized by the large-scale streaming velocity \( V_{bc} \equiv V_b - V_c \), where \( V_b \) and \( V_c \) are the bulk velocities of baryons and cold dark matter (CDM), respectively. In terms of the comoving wavenumber, \( k \sim [10–1000] h \text{Mpc}^{-1} \) is the range strongly affected, and it results in the overall suppression of the matter density fluctuation at this scale (TH). The coherence length of \( V_{bc} \), below which \( V_{bc} \) hardly varies, is set by the process of baryon acoustic oscillation (BAO) during the epoch of recombination and is of the order of a few comoving megaparsecs.

Subsequent studies have focused on both cosmological and astrophysical implications of the impact of \( V_{bc} \), and indeed have found quantitative changes and new physical phenomena. Both the amplitude and the peak location of the BAO feature, when observed through galaxy surveys, will be affected mostly through the impact of \( V_{bc} \) on the galaxy formation (Dalal et al. 2010; Yoo et al. 2011; Lewandowski et al. 2015; Selipan & Eisenstein 2015; Blazek et al. 2016; Schmidt 2016). A new type of modulation on the 21 cm intensity mapping due to \( V_{bc} \) and the corresponding boost of the signal may allow easier high-redshift cosmology (McQuinn & O’Leary 2012). The minimum mass of halos that can host the first stars will be increased from that without \( V_{bc} \), or first-star formation will be delayed, due to the advection of gas across the dark matter potential well, as found by numerical simulations (Greif et al. 2011b; Maio et al. 2011; Stacy et al. 2011; O’Leary & McQuinn 2012). The mismatch of CDM and baryon over-densities may induce baryon-dominated objects such as globular clusters (Naoz & Narayan 2014) or yield the cosmic variance in the gas content of halos (Fialkov et al. 2012). The increase in the effective Jeans mass may induce the formation of large-mass, direct-collapse black holes (Hirano et al. 2017; Schauer et al. 2017; Regan & Downes 2018).

Unfortunately, numerical simulations mentioned above (Greif et al. 2011b; Maio et al. 2011; Stacy et al. 2011; O’Leary & McQuinn 2012; Hirano et al. 2017; Schauer et al. 2017), except for O’Leary & McQuinn (2012), are likely to underestimate the negative effect of \( V_{bc} \) on star formation and other similar effects. This is because these works generated the
initial condition based on a typical Boltzmann code (such as CAMB) and imposed a sudden $V_{bc}$ at some initial time $t_i$. This procedure would then completely underestimate the negative effect that had existed before $t_i$. Instead, one should calculate the cumulative effect of $V_{bc}$ from the recombination epoch to $t_i$ and then generate the initial condition, which would then contain the negative effect that had existed before $t_i$. O’Leary & McQuinn (2012) followed this track by first calculating the evolution of perturbations (given in terms of equations by TH) under the influence of $V_{bc}$ and then generating initial conditions, which were used for numerical simulations of their nonlinear growth. As a result, they provided a new initial condition generator, Cosmological Initial Conditions for AMR and SPH Simulations (CICsASS) that can be used to simulate the nonlinear evolution of small-scale structures in a given $V_{bc}$ environment.

In the meantime, improvements to the original formulation of TH were made by considering long-range modes that had been neglected in TH but found to be of greater significance than $V_{bc}$. It was found that the large-scale overdensity, or equivalently the velocity divergence, would impact the small-scale density fluctuation more efficiently than $V_{bc}$ (Ahn 2016, Ahn16 hereafter; Blazek et al. 2016; Schmidt 2016). Ahn16 adopted the original peak-background split scheme of TH but with additional large-scale mode contributions, approximating long-range (small wavenumber) modes as a uniform local patch with given overdensities ($\Delta_c$ and $\Delta_b$), velocity divergences ($\Theta_c$ and $\Theta_b$), $V_{bc}$, and the temperature fluctuation $\Delta_T$. Ahn16 found that small-scale perturbations would evolve in a biased way: the higher $\Delta_c$ was, the faster $\delta$ would grow. Schmidt (2016) showed that the galaxy bias and subsequently the clustering of galaxies are more strongly affected by the large-scale differential overdensity ($\Delta_{bc} \equiv \Delta_c - \Delta_b$), the difference between the CDM overdensity $\Delta_c$ and the baryon overdensity $\Delta_b$ and the differential velocity divergence ($\Theta_{bc} \equiv \Theta_c - \Theta_b$, the difference between the CDM velocity divergence $\Theta_c$ and the baryon velocity divergence $\Theta_b$) than $V_{bc}$. Blazek et al. (2016) invoked a similar perturbative approach to Schmidt (2016), including all one-loop contributions, and found that the BAO features (both the amplitude and the peak location) should be affected.

In order to study the growth of small-scale structures in the given large-scale environment, characterized mainly by $\Delta$, $\Theta$, and $V_{bc}$, the most accurate way would be to (1) generate an initial condition at a redshift high enough to make perturbative calculations valid and then (2) numerically simulate the evolution of perturbations to the nonlinear regime. For (1), even CICsASS falls short of the requirement: initial conditions are adequately generated only for mean-density ($\Delta = 0$) regions in CICsASS because the original formulation of TH is used. Ahn16 showed clearly that $\delta$ would grow faster in a larger $\Delta$ environment, and thus the full set of equations treating large-scale impact, given by Ahn16, should be integrated in order to generate an initial condition. This requirement becomes even stronger if one is interested in a very high-density or a low-density environment, because then the error from just using equations of TH instead of Ahn16 would get larger. Here we introduce our initial condition generator, the Baryon–CDM Cosmological Initial Condition Generator for Small Scales3 (BCCOMICS), which fulfills this requirement.

For (2), because we want to cover any overdensity environment, the usual set-up with zero overdensity and a periodic boundary condition would not work. This is allowed by using instead the well-known trick of treating the environment as a separate universe with local cosmological parameters, which are different from those of the global universe (e.g., Goldberg & Vogeley 2004).

This paper is organized as follows. We describe the framework for BCCOMICS in Section 2, including the spatial variance of the large-scale environment, how this enters the evolution equation of small-scale perturbations, and the schematics for generating initial conditions from the transfer functions that become anisotropic. In Section 3 we describe a strategy to identify an overdense (underdense) patch as a separate universe by defining local parameters. In Section 4, as an application of the schemes in Sections 2 and 3, we describe a suite of numerical simulations of high-redshift, small-scale structure formation inside varying large-scale overdensity and $V_{bc}$ environments. In Section 5, we conclude the paper with a summary. Unless noted otherwise, length scales are expressed in comoving length units.

2. Initial Condition: BCCOMICS

In order to generate initial conditions for this study, we need an initial condition generator that implements environmental effects from both streaming velocity and density. This requires first solving the perturbation equations for the evolution of dark-matter and baryon components at the least. In the early universe, a perturbative description of their evolution is justified when the scales of interest have not yet entered the nonlinear regime.

The perturbation theory for the evolution of small-scale structures under the influence of only the large-scale streaming velocity environment was formulated by TH, and there exists an initial condition generator that implements this theory (CICsASS, O’Leary & McQuinn 2012). In terms of overdensity ($\delta \equiv (\rho - \bar{\rho})/\bar{\rho}$, defined with the local density $\rho$ and the average density $\bar{\rho}$), peculiar velocity ($v$, with its divergence $\theta = (1/a) \nabla \cdot v$ where $a$ is the scale factor and $\nabla$ is the gradient in the comoving frame), and temperature fluctuation ($\delta_T \equiv (T - \bar{T})/\bar{T}$, with the local baryon temperature $T$ and the mean baryon temperature $\bar{T}$), they solve the following linearized perturbation equation in Fourier space:

$$\frac{\partial \delta_c}{\partial t} = -\theta_c,$$
$$\frac{\partial \theta_c}{\partial t} = -\frac{3}{2}H^2\Omega_m(f_c\delta_c + f_b\delta_b) - 2H\theta_c,$$
$$\frac{\partial \delta_b}{\partial t} = -ia^{-1}V_{bc} \cdot k\delta_b - \theta_b,$$
$$\frac{\partial \theta_b}{\partial t} = -ia^{-1}V_{bc} \cdot k\theta_b - \frac{3}{2}H^2\Omega_m(f_c\delta_c + f_b\delta_b) - 2H\theta_b + a^{-2}\frac{k_B T}{\mu m_H} k^2 (\delta_T + \delta_b),$$
$$\frac{\partial \delta_T}{\partial t} = \frac{2}{3} \frac{\partial \delta_b}{\partial t} + \frac{x_s(t)}{t_\gamma} a^{-2}\frac{T_c^2}{T} \delta_T,$$

(1)

where the subscripts $c$ and $b$ denote the cold dark matter and baryons, respectively. $V_{bc} = (V_c - V_b)$ is the large-scale streaming velocity of baryons against dark matter, $k$ is a given

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3 The code is available at https://www.github.com/KJ-Ahn/BCCOMICS.
wavenumber, $H$ is the Hubble parameter, $\overline{T}$, is the average radiation temperature, $k_b$ is the Boltzmann constant, $\mu$ is the mean molecular weight, $m_H$ is the hydrogen mass, $t_r = 1.17 \times 10^{12}$ yr, and $x_e(t)$ is the global ionized fraction.\footnote{The original work by TH, for simplicity, ignores $\delta_T$ and assumes a spatially constant sound speed, and works in the baryon rest frame. Equation (1) instead uses the CDM rest frame.} As long as one limits the density environment to that of the mean density, CICsASS correctly solves Equation (1) and provides initial conditions that are accurate to the linear order.

However, in order to include the effect of the density environment in addition to that of the streaming-velocity environment, Equation (1) is no longer valid and a more accurate description is required. In addition to $V_{bc}$, a given large-scale patch will have in general non-zero overdensity ($\Delta$), velocity divergence ($\Theta$), and temperature fluctuation ($\Delta_T$).

This induces coupling of large-scale and small-scale fluctuations (denoted “mode–mode coupling” hereafter), and was formulated by Ahn16 into the following perturbation equation:

$$\frac{\partial \delta_c}{\partial t} = -(1 + \Delta_c) \theta_c - \Theta_c \delta_c,$$

$$\frac{\partial \theta_c}{\partial t} = -\frac{3}{2} H^2 \Omega_m (f_c \delta_c + f_b \delta_b) - 2 \theta_c,$$

$$\frac{\partial \delta_b}{\partial t} = -a^{-3} V_{bc} \cdot k \delta_b - (1 + \Delta_b) \theta_b - \Theta_b \delta_b,$$

$$\frac{\partial \theta_b}{\partial t} = -a^{-3} V_{bc} \cdot k \theta_b - \frac{3}{2} H^2 \Omega_m (f_c \delta_c + f_b \delta_b) - 2 \theta_b,$$

$$+ a^{-2} \frac{k_T}{\mu m_H} k^2 \{ (1 + \Delta_b) \theta_T + (1 + \Delta_T) \delta_T \},$$

$$\frac{\partial \delta_T}{\partial t} = \frac{2}{3} \left\{ \frac{\partial \Delta_b}{\partial t} + \frac{\partial \Delta_b}{\partial t} + \frac{\partial \theta_b}{\partial t} (\Delta_T - \Delta_b) \right\}$$

$$- \frac{x_e(t)}{t_r} a^{-4} \frac{T_c}{T} \delta_T. \tag{2}$$

Equation (2) is the governing equation for the evolution of small-scale perturbations ($\delta_c, \delta_b, \theta_c, \theta_b, \delta_T$) under the influence of a local large-scale environment ($\Delta_c, \Delta_b, \Theta_c, \Theta_b, \Delta_T$), accurate to the leading order. Equation (2) is developed in the CDM rest frame, such that an observer is moving at the bulk velocity of CDM particles, $V_{bc}$, inside the patch. This equation is based on the assumption that the two scales are well separated, and thus this formalism is a peak–background split scheme. We consider the variation of the large-scale fluctuations at a scale of 4 comoving Mpc, which is a natural choice because the streaming velocity at recombination is coherent at a few comoving Mpc (TH).

We introduce a newly developed, cosmological initial condition generator, BCCOMICS, that incorporates the full impact of the large-scale environment on small-scale fluctuations. Obviously, we developed BCCOMICS because there has not been any initial condition generator that fully incorporates this effect. The large-scale environments are realized as a set of three-dimensional fields and their evolution is calculated properly (Section 2.1). BCCOMICS then solves the perturbation equation (Equation (2)) and generates three-dimensional fields of small-scale quantities (Section 2.2). Incorporating a non-zero overdensity environment requires a careful reassignment of cosmological parameters and scaling laws, which will be described in Section 3 in detail.

2.1. Evolution of Large-scale Variance

We need to understand how large-scale ($\gtrsim$ a few Mpc) fluctuations vary in space, and how they evolve in time until a dedicated epoch for the initial condition is reached. This is because the impact of the environment is continuous in time as seen in Equation (2). For any given patch chosen out of many large-scale patches with varying physical properties, once we are able to track its evolution, we obtain the full understanding of a given environment, and this can be fed into Equation (2). This evolution, if in the linear regime, can be easily obtained by Boltzmann solvers such as CAMB, which solves Eulerian linear evolution equations of all components in the standard $\Lambda$CDM cosmology. Nevertheless, it would be somewhat more efficient computationally (we will justify this below) and also intuitive to have an approximation based on a simplified evolution equation.

Indeed, the evolution of large-scale variables after recombination can be well approximated by solving for the growth of four independent modes subject to a very simplified evolution equation (Ahn16) given by

$$\frac{\partial \Delta_c}{\partial t} = -\Theta_c,$$

$$\frac{\partial \theta_c}{\partial t} = -\frac{3}{2} H^2 \Omega_m \Delta_c - 2 \Theta_c,$$

$$\frac{\partial \Delta_b}{\partial t} = -\Theta_b,$$

$$\frac{\partial \delta_T}{\partial t} = -2 H \Theta_b. \tag{3}$$

where $\Delta_+ \equiv f_c \Delta_c + f_b \Delta_b$, $\Theta_+ \equiv f_c \Theta_c + f_b \Theta_b$, $\Delta_\pm \equiv \Delta_c - \Delta_b$, and $\Theta_\pm \equiv \Theta_c - \Theta_b$ with $f_c \equiv \rho_c / (\rho_c + \rho_\Lambda)$ and $f_b \equiv 1 - f_c$. These are growing, decaying, compensated, and streaming modes, which are solutions to these linear equations (Equation (3)). The growing and decaying modes compose the adiabatic perturbation, and the compensated and streaming modes compose the isocurvature perturbation. Note that the isocurvature perturbation can be sourced primordially by the cosmic inflation in some inflation models, and secularly from BAO during the recombination epoch (Tsaiakovich & Hirata 2010; Barkana & Loeb 2011). We only consider the latter possibility, which is self-consistently calculated by CAMB.

Equation (3) is in this simple form because (1) fluctuation in the radiation component quickly decays in time after recombination, (2) the large-scale ($\gtrsim$ a few Mpc) variance is likely to be free from the mode–mode coupling, and (3) the pressure term $a^{-2} \frac{k_T}{\mu m_H} k^2 \{ \Delta_T + \Delta_\pm \}$ is negligible due to the smallness of $k$. Then the temporal evolution of these variables for a given patch is given by a linear combination of these modes. Finally, we evolve $\Delta_T$ passively, after solving for the evolution of the modes $\{ \Delta_c, \Delta_b, \Theta_c, \Theta_b \}$ beforehand, by integrating the following equation (identical to Equation (8) of Naoz & Barkana 2005):

$$\frac{\partial \Delta_T}{\partial t} = \frac{2}{3} \frac{\partial \Delta_b}{\partial t} + \frac{x_e(t)}{t_r} a^{-4} \left\{ \Delta \left( \frac{5 T_c}{4 T} - 1 \right) - \Delta_T \frac{T_c}{T} \right\}. \tag{4}$$
where we use the output transfer function from CAMB for the radiation fluctuation $\Delta_\gamma(z)$, and the initial temperature fluctuation $\Delta_T(z)$ is fixed by the scheme of Naoz & Barkana (2005):

$$
\Delta_T(z) = \Delta_T \left( 5 - \frac{4T}{T_r} \right) + \frac{1}{r} \left( \frac{2}{3} \frac{\partial \Delta_b}{\partial r} - \frac{\partial \Delta_T}{\partial r} \right),
$$

where all the terms on the right are evaluated at $z_r$. Note that Equations (3)–(5) can all be solved in $k$-space, and an $r$-space map can be obtained from a $k$-space quantity by randomization (Equation 12 of Ahn16) and Fourier transformation.

We note that it is important to consider all the four modes in evolving Equation (3), because the motion of baryons is different from that of CDM particles. This difference in motion among the two fluid components results in non-zero values of the compensated and streaming modes. These modes even dominate over the growing and decaying modes around the recombination epoch (see Figure 9 of Ahn16). Therefore, it is a bad practice to use only the matter component (growing and decaying modes), or even only the growing mode as is done in some simulations of structure formation. At the same time, one should consider the non-negligible radiation component ($\Omega_r \equiv \rho_r / \rho_{\text{crit}}$) at least in the global evolution of $\Omega_m$, which makes $\Omega_m = 1$ for quite long after recombination. Cosmological $N$-body+hydrodynamics codes such as Enzo and Gadget used not to include the radiation component. For Enzo at least, therefore, we have now implemented non-zero $\Omega_r$, and Enzo correctly calculates the corresponding $\Omega_m(z)$ and the Hubble parameter $H(z)$. Otherwise, a spurious effect in simulation of structure formation will occur if one applies the transfer function from CAMB, which of course considers the radiation component, to $N$-body+hydro codes, which do not have the radiation component.

In practice, the following steps are performed for calculating the evolution of a patch. We refer readers to Section 2.2 and the Appendix of Ahn16 for details. First, at $z_{re}$, we use the transfer function from the widely used Boltzmann solver CAMB. By convolving the transfer function with a Gaussian random seed, 3D maps of $\{\Delta_c, \delta_\gamma, \Theta_0, \Theta_\gamma, V_c, V_\gamma\}$ or $\{\Delta_c, \Delta_\gamma, \Theta_0, \Theta_\gamma, V_c, V_\gamma\}$ at $z_{re}$ can be realized. Second, we numerically solve for the temporal evolutions of $\{\Delta_c, \Delta_\gamma, \Theta_0, \Theta_\gamma, V_c, V_\gamma\}$. Evolution of each of these mode variables is fully described by a single corresponding numerical solution (let’s say $F(z)$), because Equation (3) is linear. Third, at any $z$, one simply multiplies $F(z)$ by the fluctuation value at $z_{re}$ and obtains the evolved value at $z$. For example, if $F_\Delta(z)$ is the solution of $\Delta_\gamma$ with normalization convention $F_\Delta(z_{re}) = 1$, then $\Delta_\gamma(z) = \Delta_\gamma(z_{re})F_\Delta(z)$. Fourth, as for $\Delta_\gamma$, one can use either the actual evolution (obtained by integrating Equation (4)) or a fitting formula given by Equation (30) of Ahn16: thanks to the quick coupling of $\Delta_\gamma$ to $\Delta_\delta$ after $z \approx 500$, the fitting formula is given in terms of $\Delta_b$, and the smallness of $\Delta_\delta \approx 10^{-5}$ before $z \approx 500$ allows us to simply set $\Delta_T = 0$ when $z \gtrsim 500$. Then these solutions are fed into Equation (2) when solving for the evolution of small-scale fluctuations for any given patch.

An important physical intuition can be gained by comparing spatial maps and correlations of these variables at $z \approx z_{re}$ and a target redshift, e.g., $z_t = 200$. Figures 1 and 2 show maps and histograms at $z = 1000$ and $z = 200$, respectively. As seen in Figure 1(a), $V_\delta \equiv V_c - V_\gamma$ (arrows) is dominated by $V_c$ and thus shows convergence into peaks and divergence from voids. Comparing Figures 1(a) and (b), we find that $\Delta_\gamma$ dominates over $\Delta_b$ in amplitude at $z = 1000$, and these two are very weakly correlated, as also seen in Figure 1(e). This is due to the tight coupling of baryons to photons during recombination, which can also be seen in the map of $\Delta_T$ (Figure 1(c)) showing the characteristic feature of the BAO. CDM moves mainly through its self-gravity, producing a very tight correlation between $\Delta_\gamma$ and $\Theta_\gamma$ (Figure 1(d)). $V_\gamma$ and $\Delta_\gamma$ are not correlated at all (Figure 1(f)).

The main difference between the two epochs lies in the behavior of baryon fluctuations. At $z = 200$, aside from the amplitude, maps of $\Delta_\gamma$, $\Delta_b$, and $\Delta_T$ are almost indistinguishable (Figures 2(a)–(c)). This is because the baryon fluctuation is now governed predominantly by gravity rather than by photon fluctuation. The correlation between $\Delta_\gamma$ and $\Theta_\gamma$ is even tighter (Figure 2(d)) than it is at $z = 1000$, and now $\Delta_\gamma$ is strongly correlated with $\Delta_\delta$ (Figure 2(e)). The latter fact is in stark contrast with the loose correlation seen at $z = 1000$ (Figure 1(e)). Because of this, one may assume that baryons are “locked” into CDM such that $\rho_b/\rho_c = \Omega_b/\Omega_c$ in all patches, and consider $\Delta_c$ as the only important density environment after $z = 200$. This is indeed an approximation appropriate for the study of first-galaxy formation, and allows one to use a periodic boundary condition without worrying the net inflow of baryons to the simulation box. We therefore take this approximation and isolate the simulation box with baryon and CDM contents fixed in this paper. However, we note that the full degree of such a “locking” has not happened yet, because $\Delta_b/\Delta_c \approx 1$ but $\Delta_b/\Delta_\delta \approx 0.5$. This makes $\rho_b/\rho_c$ slightly different from the cosmic abundance $\Omega_b/\Omega_c$, even though the smallness of $\Delta_b$ and $\Delta_\delta$ at $z = 200$ makes this approximation acceptable. Nevertheless, in time, the value of $\Delta_b/\Delta_c$ gradually approaches 1, but still with some variance over the large scale ($k \lesssim 0.1 \text{ Mpc}^{-1}$) (Figure 3). This is indeed a result of BAO. For a precise cosmology with galaxies or the intergalactic medium, for example, one should therefore consider other large-scale variables as well as $\Delta_c$. $V_\gamma$ and $\Delta_\gamma$ still remain uncorrelated (Figure 2(f)).

2.2. Anisotropic Transfer Function and Real-space Fluctuations

All the transfer functions (let us denote such a function by $T_f(k, z; V_{bc}, \{\Delta_c\}$) for small-scale component $s$ under the influence of large-scale component $L$; $\{\Delta_L\}$ is the abbreviation for all large-scale fluctuations) of small-scale perturbations $\{(\delta_s) = (\delta_c, \delta_b, \delta_\gamma, \delta_L)\}$ are anisotropic due to the impact of $V_{bc}$, as expected from Equation (2). This makes it necessary to calculate $T_f(k, z; V_{bc}, \{\Delta_c\})$ at every wavenumber $k$. Or, we can use an implied dependence $T_f(k, z; V_{bc}, \{\Delta_c\}) = T_f(k, \mu, z; V_{bc}, \{\Delta_c\})$, where $\mu \equiv k \cdot V_{bc}/(k V_{bc})$. In practice, we first integrate Equation (2) and generate $T_f(k, \mu, z; V_{bc}, \{\Delta_c\})$ with dozens of logarithmic samples $k$ and uniformly gridded samples of $\mu$, for a given patch with a

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\*The up-to-date development version of Enzo, which is downloadable from http://enzo-project.org, reflects this implementation.
specific set of large-scale fluctuations \{\Delta_L\} and \(V_{bc}\). (1) This forms a 2D table of \(T_\ell\) in terms of \(\{k, \mu\}\) for a given patch, where subscript \(m\) is the integer index for the discretized values of \(\mu\) or \(\mu_{\text{orig}}\). (2) Then, for a given \(k\) and \(V_{bc}\), we perform a 2D interpolation at the actual set of points \(\{k, \mu\}\) and obtain \(T_\ell(k, z; V_{bc}, \{\Delta_L\})\). Once we obtain \(T_\ell(k, z; V_{bc}, \{\Delta_L\})\), with proper normalization (to fluctuations at \(z = 1000\)), the power spectrum of quantity \(l\) becomes \(P_l(k, z; V_{bc}, \{\Delta_L\}) = P_l(k, z = 1000)T_l^2(k, z; V_{bc}, \{\Delta_L\})\). (3) We apply two Gaussian random seeds \(G_1\) and \(G_2\) to obtain real and imaginary parts of the \(k\)-space field, where both \(G_1\) and \(G_2\) have mean 0 and variance 1:

\[
\begin{align*}
\text{Re}(\delta_q(k)) &= G_1N^3 \left( \frac{P(k)}{2V_{\text{box}}} \right)^{1/2} \text{sign}[T(q,k)], \\
\text{Im}(\delta_q(k)) &= G_2N^3 \left( \frac{P(k)}{2V_{\text{box}}} \right)^{1/2} \text{sign}[T(q,k)].
\end{align*}
\]

(4) Of course, real-space variables are all in real numbers, thus we use the condition \(\delta_q^*(k) = \delta_q(-k)\) after filling only half of the allowed \(k\)-space (\(^*\) denotes the complex conjugate). In addition, all monopole terms \((k = 0)\) are assumed to be zero. Removing monopole terms allows one to use the usual periodic boundary condition, and we will explain in Section 3 how this becomes possible even in the presence of non-zero \(\Delta_c\) and \(\Theta_c\).

(5) Finally, we take the Fourier transform of \(\delta_q(k)\) and obtain \(\delta_q(x)\). Vectorization of the baryon velocity field is performed through the relation \(v(k) = -(ik/k^2)\delta(k)\) assuming a curl-free velocity field.

The whole process (1)–(5) is straightforward for generating uniform-grid quantities. For particle quantities (of CDM particles; of baryon particles if smoothed particle hydrodynamics is used), we follow the usual Lagrangian perturbation theory, relating the displacement vector to the Eulerian overdensity (Bouchet et al. 1992). Because we calculate the Eulerian overdensity based on the linear-order Boltzmann solver CAMB, we restrict our calculation to the first-order Lagrangian perturbation theory. A particle is displaced from its Lagrangian point \(q\) to its Eulerian point \(x\) by the displacement vector \(\Psi\), as \(x = q + \Psi\). To linear order,

\[
\delta(q) = -\nabla \cdot \Psi(q),
\]

which can be again cast into the \(k\)-space quantities

\[
\Psi = (ik/k^2)\delta
\]

and allows \(T_\ell(k)\) to be used for \(\delta(k)\). It is easy to generate the real-space displacement field through the Fourier transformation of \(\Psi(k)\) if displacing particles from cubically and uniformly spaced positions; in the case of a glass, an interpolation of this uniform-space \(\Psi(q)\) field onto glassy positions is further required. For the particle velocity field, consequently, its generation first takes the steps for the
uniform-grid data, and this uniform-grid velocity field is interpolated onto the displaced positions. In practice, however, such interpolation becomes unnecessary if $Y$ is very small compared to the length scale $Y \sim Y_{\sim} |\cdot|$.  

3. How to Simulate Density Peaks and Voids

Because BCComics solves Equation (2), which includes possible couplings between large-scale modes (in terms of $\{\Delta_l\}$) and small-scale modes (in terms of $\{\Delta_s\}$), it provides so far the most accurate (and only) initial condition regarding the large-scale environmental effect on small-scale fluctuations. To take advantage of this fact and simulate structure formation in overdense and underdense regions, however, we need to take further steps than those required for a mean-density environment. In this section, we lay out a strategy for achieving this goal.

3.1. Local Hubble Parameter and the Friedmann Equation

We will take an overdense (underdense) patch as a separate universe. There can be other ways of simulating an overdense patch, such as taking a much larger volume as a simulation box and zooming into a patch of interest with, e.g., nested grids (e.g., O’Shea et al. 2015). Nevertheless, this scheme has the merit of allowing a periodic boundary condition, because an overdense (underdense) patch in our universe is now treated as a mean-density patch in a universe with different cosmological parameters.

Because the patch will detach from the global Hubble flow, almost all relevant cosmological parameters should be redefined. Anything redefined in such a separate universe will be called “local” and the relevant symbol will be capped by $\tilde{\cdot}$. The mass conservation of such a patch first defines the local Hubble parameter (Goldberg & Vogeley 2004):

$$\tilde{H} = H - \frac{\Delta}{3(1 + \Delta)}.$$  

where $\Delta$ is the matter overdensity of a given patch, $\Delta = \frac{d\Delta}{dt}$ with cosmic time $t$, and we assume that the net influxes of CDM and baryons are both zero on this patch from the local viewpoint, or $\tilde{\Theta}_c = \tilde{\Theta}_b = 0$, in order to incorporate the usual periodic box condition for simulation. Nevertheless, because we take a patch in a CDM rest frame, there will be a bulk flow of baryons with velocity $\tilde{V}_b = V_{bc}$. Note also that $\Delta_c = \Delta_b = 0$ by definition, because we take a patch as a separate mean-density universe.

Figure 2. Same as Figure 1, except that all panels are now based on quantities at $z = 200$.

Figure 3. Overdensity ratio $\Delta_b/\Delta_c$ calculated by the linear Boltzmann solver at different wavenumbers $k$ and redshifts $z$.  

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The local Friedmann equation can be written in various forms:
\[
1 = \tilde{\Omega}_m + \tilde{\Omega}_r + \tilde{\Omega}_\Lambda + \tilde{\Omega}_K, \tag{10}
\]

\[
\dot{H} = \dot{H}_0 [\tilde{\Omega}_m, a^{-3} + \tilde{\Omega}_r, a^{-4} + \tilde{\Omega}_\Lambda, 0 + \tilde{\Omega}_K, a^{-2}t^2 / 2], \tag{11}
\]

\[
\dot{H} = \dot{H}_i \left[ \frac{a}{a_i} \right]^{-3} + \tilde{\Omega}_r, \left( \frac{a}{a_i} \right)^{-4} + \tilde{\Omega}_\Lambda, i + \tilde{\Omega}_K, \left( \frac{a}{a_i} \right)^{-2}t^2 / 2, \tag{12}
\]

where the convention \(a_0 = 1\) is used in Equation (11), and the subscript \(i\) refers to an initial time with the initial scale factor \(a_i\). Equation (9) gives

\[
\frac{\dot{H}_i}{H_i} = 1 - \frac{(\dot{\Delta})_i}{3H_i(1 + \Delta_i)}. \tag{13}
\]

The initial local cosmological parameters become

\[
\tilde{\Omega}_{m,i} = \frac{\tilde{\rho}_{m,i}}{\rho_{\text{crit},i}} = \left(1 + \Delta_i\right) \Omega_{m,i} \left(\frac{\dot{H}_i}{H_i}\right)^{-2}, \tag{14}
\]

\[
\tilde{\Omega}_{r,i} = \frac{\tilde{\rho}_{r,i}}{\rho_{\text{crit},i}} = \Omega_{r,i} \left(\frac{\dot{H}_i}{H_i}\right)^{-2}, \tag{15}
\]

\[
\tilde{\Omega}_{\Lambda,i} = \frac{\tilde{\rho}_{\Lambda,i}}{\rho_{\text{crit},i}} = \Omega_{\Lambda,i} \left(\frac{\dot{H}_i}{H_i}\right)^{-2}, \tag{16}
\]

\[
\tilde{\Omega}_{K,i} = 1 - (\tilde{\Omega}_{m,i} + \tilde{\Omega}_{r,i} + \tilde{\Omega}_{\Lambda,i}), \tag{17}
\]

where the global \(\Omega_i\) values (in a flat LCDM universe) for each component are of course given by, e.g.,

\[
\Omega_{m,i} = \frac{\rho_{m,i}}{\rho_{\text{crit},i}} = \frac{\Omega_{m,0} a_i^{-3}}{\Omega_{m,0} a_i^{-3} + \Omega_{r,0} a_i^{-4} + \Omega_{\Lambda,0}}. \tag{18}
\]

### 3.2. Local Cosmological Parameters and Redshift Mapping

We take Enzo as our model simulation code, and show how cosmological parameters are assigned when an overdense (underdense) patch is simulated. Application to other codes will be similar. Because the Enzo parameter file requires parameters at the “present,” such as CosmologyOmegaMatterNow, we need to get the local values of these. However, we can arbitrarily define the “present,” only if the local scale factor \(a\) at that time, \(a_0\), is normalized to 1. This convention is used in Enzo. We note that the following assignment procedure is provided as a separate function code in BBCCOMICS.

Let us take some global redshift \(z'\) and scale factor \(a' = 1/(1 + z')\) to be those of the local present. We then need to connect this information to the actual evolution of \(a(t)\).

We need to integrate \(d\tilde{a}/dt\) (Equation (12)):

\[
\frac{d\tilde{a}}{d(tH_i)} = \tilde{a} \frac{\dot{H}_i}{H_i} \left[ \frac{\dot{\tilde{\Omega}}_{m,i} }{\tilde{\rho}_{m,i} \tilde{a}_i} - \frac{\dot{\tilde{\Omega}}_{r,i} }{\tilde{\rho}_{r,i} \tilde{a}_i} \right]^{-1} + \left( \frac{\tilde{\Omega}_{r,i} }{\tilde{a}_i} \right)^{-2} + \left( \frac{\tilde{\Omega}_{\Lambda,i} }{\tilde{a}_i} \right)^{-2} + \left( \frac{\tilde{\Omega}_{K,i} }{\tilde{a}_i} \right)^{-2}, \tag{19}
\]

and then form a table of \(tH_i, \tilde{a}(t)\). When performing numerical integration in practice, we start from some time \(t_i\) deep inside the radiation-dominated epoch, with \(t_i H_i \ll 1\), and then use the analytical expression during this epoch

\[
\tilde{a}_i = \tilde{a}_i \sqrt{2t_i H_i \frac{\dot{H}_i}{H_i} \frac{\dot{\tilde{\Omega}}_{r,i} }{\tilde{\rho}_{r,i} \tilde{a}_i}}, \tag{20}
\]

where we set \(\tilde{a}_i = a_i\) “temporarily.” We find that \(t_i H_i = 10^{-5}\) provides an excellent accuracy for the post-recombination epoch, and we use ODE45 (Runge–Kutta fourth-order equivalent) of Matlab and gnu octave, or simply Simpson’s rule with appropriate time-binning. Either way, we can easily obtain an accuracy of less than \(10^{-4}\) at all times.

We then sample specific global time \(tH_i\)’s from the starting (global) scale factor \(a_0\) to the final (global) scale factor \(a'\), in terms of the global time variable \(tH_i = \{ t_1 H_i, t_2 H_i, \ldots, t_N H_i \}\). This becomes the set of \(N\) epochs for data output. The corresponding local scale factor of the patch will be \(a = \{ a_1, a_2, \ldots, a_N \}\), which can be obtained from the table of \(\{ tH_i, \tilde{a}(t) \}\). \(\tilde{a}_N = a'\) in general. For convenience, we match \(\tilde{a}_N\) to the local present. We thus rescale \(a\) to \(\tilde{a} = \{ \tilde{a}_1, \tilde{a}_2, \ldots, 1 \}\), and the corresponding redshifts become \(\tilde{z} = \{ z_1, z_2, \ldots, 0 \}\) where \(\tilde{z} = 1 - \tilde{a}_N^{-1} \tilde{a}_N^{-1} - 1, \ldots, 1 - 1\). This list is assigned to CosmologyOutputRedshift in Enzo. When necessary, one can always map \(\tilde{z}\) to \(z\) through the table of \(\{ tH_i, \tilde{a}(t)\}\).

We also need to change several other cosmological parameters. To obtain CosmologyOmegaMatterNow, for example, we calculate it from

\[
\tilde{\Omega}_{m,0} = \tilde{\Omega}_{m,i} \left( \frac{\tilde{a}_N}{\tilde{a}_i} \right)^{-3} + \tilde{\Omega}_{r,i} \left( \frac{\tilde{a}_N}{\tilde{a}_i} \right)^{-3} + \tilde{\Omega}_{\Lambda,i} \left( \frac{\tilde{a}_N}{\tilde{a}_i} \right)^{-2} + \tilde{\Omega}_{K,i} \left( \frac{\tilde{a}_N}{\tilde{a}_i} \right)^{-2}, \tag{21}
\]

and follow similar steps with the correct power law of \(\tilde{a}_N / \tilde{a}_i\) in the numerator for all other CosmologyOmegaNow’s, where \(^*\) = \{ CDM, Baryon, Matter, Lambda, Radiation \}. A non-zero curvature term for non-zero \(\Delta_m\) should be taken carefully, and in the case of (recent-version) Enzo, this term is calculated internally instead of being accepted as an input parameter. CosmologyHubbleConstantNow, which is the present Hubble constant in units of \(100 \text{ km s}^{-1} \text{ Mpc}^{-1}\),
becomes

$$
\bar{h} = \frac{\bar{R}_i}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}} \left[ \frac{\bar{\rho}_m}{\bar{\rho}_i} \left( \frac{\bar{\delta}_N}{\bar{a}_i} \right)^{-3} + \bar{\Omega}_{\lambda,i} \right] + \bar{\Omega}_{r,i} \left( \frac{\bar{\delta}_N}{\bar{a}_i} \right)^{-4} + \bar{\Omega}_{K,i} \left( \frac{\bar{\delta}_N}{\bar{a}_i} \right)^{-2} \right]^{1/2} \tag{22}
$$

In addition, because the comoving box size is the proper length at "present," this needs to be reassigned too. First, let us denote the comoving length of the mean-density box by $L$. Then the proper length of the box at $\bar{a}_i$ is $L\bar{a}_i$. The proper length of the box at $\bar{a}_N$, which is the comoving length of the box, is then $L\bar{a}_i(\bar{a}_N/\bar{a}_i) = L\bar{a}_N$. This is then assigned to CosmologyComovingBoxSize.

### 3.3. Scaling Laws and Halo Identification

Even when peaks and voids are treated as a separate universe, an observer there can measure quantities based on the global properties of the universe. For example, an observer who estimated the local matter content $\bar{\Omega}_m$ through an observation inside a small volume (e.g., only inside a 4 Mpc patch) will realize that the global value $\bar{\Omega}_m$ has been underestimated only after enlarging the survey volume. Therefore, scaling laws for time and length are required.

The scaling laws are given trivially. If the local length and timescales are $\bar{L}$ and $\bar{t}$, the global length scales $L$ and $t$ are given by

$$L = \bar{L} \frac{\bar{a}}{a} \tag{23}$$

and

$$T = \bar{T} \frac{\bar{a}}{a} \tag{24}$$

respectively, where $t$ and $\bar{t}$ are ages of the actual universe and the local universe (patch), respectively.

A few obvious but important applications are imminent. If one were to restrict the volume of a galaxy survey to, e.g., $200^3 \text{ Mpc}^3$ centered at the observer, there would occur a danger of wrongfully measuring the BAO scale (true value of $\sim 150 \text{ Mpc}$) as $150(\bar{a}/a) \text{ Mpc}$ where $\bar{a}$ is the local scale factor of the $200^3 \text{ Mpc}^3$ volume. Similarly, the cosmological wavenumber of fluctuations should be scaled properly. If one conducted an autocorrelation analysis on samples of galaxies from a limited-volume survey, the local wavenumber $k$ and the local correlation length $\bar{l}$ should be mapped to the global values as

$$k = k \frac{\bar{a}}{a} \tag{25}$$

and

$$l = l \frac{\bar{a}}{a} \tag{26}$$

Equation (25) should be applied to the scaling law suggested for the $k$-space fluctuation and the power spectrum by Goldberg & Vogele 2004, Equations (21) and (26)), where they forgot to scale the wavenumber.

Halo identification schemes should also be approached carefully, which is of great interest to us. Let us restrict the discussion to one specific halo identification scheme: the friends-of-friends (FoF) algorithm. A halo is identified if a collection of particles are connected in lengths that are smaller than the FoF linking length $b$, in units of the mean particle separation. A given linking length $b$ is an implicit indicator of the mean density of resulting halos $\langle \rho_m \rangle$: $\langle \rho_m \rangle \sim 180(b/0.2)^{-3} \bar{\rho}_m$ or $\delta_{\text{lin}} > \delta_{\text{lin}} \sim 1.67$, as in Lacey & Cole (1994). Because the relation

$$\frac{\langle \rho_m \rangle}{\bar{\rho}_m} \sim 180 \left( \frac{b}{0.2} \right)^{-3} \tag{27}$$

roughly holds regardless of cosmology for any simulation box (Lacey & Cole 1994), this can be interpreted in our case as

$$\frac{\langle \rho_m \rangle}{\bar{\rho}_m} \sim 180 \left( \frac{\bar{b}}{0.2} \right)^{-3} \tag{28}$$

for any local patch, where halos with a local linking length $\bar{b}$ in units of the local mean particle separation will have the local overdensity $180(\bar{b}/0.2)^{-3}$. Now, any halo inside our universe is defined in terms of the global mean density $\bar{\rho}_m$ to make $\langle \rho_m \rangle/\bar{\rho}_m = 180 (b/0.2)^{-3}$, and thus

$$\bar{b} = b \left( \frac{a}{\bar{a}} \right). \tag{29}$$

Equation (29) is indeed consistent with the simple fact that a single proper linking length, $b$ ($\times$ global mean particle separation), should be applied universally if one imagines a very large, mean-density simulation box that encloses many overdense and underdense patches.

Once a local patch is treated as a separate universe, then FoF halos there should be generated using $\bar{b}$ given by Equation (29), which requires scaling $\bar{b}$ in a time-dependent way if a constant $b$, e.g., $b = 0.2$, is used. The minimum number of $N$-body CDM particles for halo identification need not change, because a universal criterion should be used. For example, let us imagine a certain overdense patch with $\bar{a} = 0.05$ but $a = 0.1$ at the same cosmic time $tH_0 = 10$. If the “usual” linking length is $0.2$ of the mean particle separation in the mean-density patch, then the linking length of the overdense patch should be $0.2 \times (0.1/0.05) = 0.4$ of the “local” mean particle separation.

Finally, if some sort of overdensity threshold is used, e.g., $\delta_{\text{th}} = 1$ for triggering a certain astrophysical process, this needs to be scaled too. To have a universal density threshold, $\rho_{\text{th}} = (1 + \delta_{\text{th}}) \bar{\rho}_b = (1 + \delta_{\text{th}}) \rho_b$, we need $\delta_{\text{lin}} = (\bar{a}/a)^3 (1 + \delta_{\text{th}}) = 1$. If a threshold is given instead in terms of $\rho_b$ or $\rho_{\text{th}}$ because a physical process of interest is dependent on the proper density, then there is no need for scaling.

### 4. Application

As an application, we use BCCOMICS for generating initial conditions and perform a suite of cosmological simulations of structure formation using Enzo (Bryan et al. 2014), sampling a few patches of varying $\Delta_c$ and $V_{\text{bc}}$. One can consider other large-scale variables ($\Theta_c$, $\Delta_c$, $\Theta_b$, and $\Delta_b$) as well. However, strong correlation between $\Theta_c$ and $\Delta_c$ alleviates the need to consider $\Theta_c$, and $\{\Delta_b, \Theta_b, \Delta_T\}$ are of less importance than $\Delta_c$ and $V_{\text{bc}}$. The rather quick transition of a very loose correlation between $\Delta_c$ and $\Delta_b$ at $\bar{z} = 1000$ to a very strong correlation at
Table 1
Sampling Parameters

| Parameter         | D0V0 | D0V2 | D2V2 | D2V3 |
|-------------------|------|------|------|------|
| $\Delta_c$        | 0    | 0    | $2\sigma_{\Delta_c}$ | $2\sigma_{\Delta_c}$ |
| $V_{bc}$ (km s$^{-1}$) | 0    | 26.5 | 26.5 | 38   |

Note. All cases use initial conditions with a universal random seed, a grid resolution and the CDM particle number of 512$^3$ each, and the initial box size of 1 Mpc. High-density cases D2V2 and D2V3 gradually detach from the Hubble expansion and thus the comoving size of these boxes shrinks in time, in contrast to the cases D0V0 and D0V2.

We show three types of field maps at $z = 200$ in Figure 4, which are CDM overdensity, baryon overdensity, and the ratio of kinetic-to-thermal energies of baryons, in varying overdensity and streaming-velocity environments. In this figure, maps of CDM overdensity look indistinguishable from one another, because the dynamics of CDM is dominated by its self-gravity, while maps of baryon overdensity show a distinguishable smear that gets stronger as $V_{bc}$ increases. The ratio of kinetic-to-thermal energies of baryons increases overall as $V_{bc}$ increases. If we imagine the practice of a sudden imposition of $V_{bc}$, the maps of baryon overdensity would not show any mutual difference. The relative importance of $V_{bc}$ in terms of energetics, also gets stronger as $V_{bc}$ increases, as is seen in figures of the energy ratio.

After simulating structure formation based on initial conditions generated by BCCOMICS, we identified halos using the FoF scheme. As described in Section 3.3, a universal linking length $b = 0.2$ was used for all cases. For overdense patches (D2V2, D2V3), this translates to the local linking length $b = 0.2(a/\bar{a})$. We used the $\gamma L$ analysis tool for halo identification, and because this tool is sensitive only to local values including the local mean particle separation, we fed this time-varying $\tilde{b}(z)$ into $\gamma L$ for any redshift $z$. Figure 5 is the halo mass function $dn/dM$ for the net DM mass $M$ of FoF halos.

A few features are notable. First, overdense patches contain more halos across the full mass range than mean-density patches, as expected. Second, when the overdensity environment is the same, higher $V_{bc}$ yields a stronger suppression. Third, the impact of $V_{bc}$ weakens over time (see bottom panels of Figure 5), which is expected from the fact that $V_{bc} \propto 1/a$. These features are consistent with the positive effect of overdensity and the negative effect of $V_{bc}$ on the clumping of baryons inside DM halos and even DM clumping itself, as seen in the $k$-space variance of matter (CDM+baryon), $\Delta_c^2 \equiv P_b(k)k^3/(2\pi^2)$, and that of CDM, $\Delta_m^2 \equiv P_m(k)k^3/(2\pi^2)$. Figure 6 shows that $\Delta_c$ boosts both $P_m(k)$ and $P_b(k)$ at all values of $k$, while $V_{bc}$ suppresses both $P_m(k)$ and $P_b(k)$ in a bound region of $k$. Aside from very rare and massive halos ($M \gtrsim 10^7 M_\odot$) we could identify until $z \approx 20$, the number density of halos is $\sim[2-4]$ times as high as that in mean-density patches. Because we allowed gas cooling, it is probable that the negative effect of $V_{bc}$ is somewhat reduced compared to the case without cooling (e.g., O’Leary & McQuinn 2012). One subtle feature is that suppression of halo formation is not biased toward the low-mass end. TH predicts that at $z \sim 40$, halos with mass $M \sim 10^6 M_\odot$ will be more strongly suppressed than those with, e.g., $M \sim 10^8 M_\odot$. In Figure 5, this tendency can be barely observed. For the high-mass end, however, such a comparison is not meaningful, due to poor statistics. We plan to obtain better statistics in Paper II by enlarging the size of the simulation box (including larger-scale modes of fluctuation, equivalently) and thus allowing more frequent formation of massive halos.

The amount of gas that is first gravitationally bound in halos or filaments and then undergoes cooling is closely related to the process of star formation. Because we did not allow adaptive mesh refinement (AMR), we defer our analysis of star formation to Paper II but instead analyze the total amount of cooling gas. We define the cooling gas by the criterion $t_{cool} < t_{dyn}$, where $t_{cool}$ and $t_{dyn}$ are the local cooling time and
the local dynamical time, respectively. Figure 7 shows the evolution of the total amount of cooling gas ($M_{\text{cool}}$) in each simulation box. We can take this quantity as a rough indicator of star formation activity. As nonlinear structures grow in time, $M_{\text{cool}}$ increases in all cases. The positive effect of overdensity and the negative effect of $V_{bc}$ on gas clumping (or cooling) are also clearly observed. In the early phase, at $z \sim 30$, the D0V0 case has the largest $M_{\text{cool}}$ because the density environments of D2V2 and D2V3 have not deviated too much from those of D0V0 and D0V2, and therefore the hierarchy roughly follows that of $V_{bc}$ in descending order: $M_{\text{cool}}(D0V0) > M_{\text{cool}}(D0V2) \approx M_{\text{cool}}(D2V2) > M_{\text{cool}}(D2V3)$. After that, however, the positive effect of overdensity takes over as density peaks are more detached from the global Hubble flow, and the D2V2 case becomes the site for the most efficient cooling. At $z \lesssim 20$, the positive effect from density dominates over the early-phase negative effect from $V_{bc}$, but it still retains the memory of the negative effect from $V_{bc}$, such that $M_{\text{cool}}(D2V2) > M_{\text{cool}}(D2V3) > M_{\text{cool}}(D0V0) > M_{\text{cool}}(D0V2)$. This is indeed an epoch where minihalos provide a significant contribution to cosmic reionization (e.g., Ahn et al. 2012), and thus our study is expected to improve upon the existing scenarios of reionization.

We will extend this application with a more self-consistent treatment in Paper II, especially with the AMR capability on. The quantitative result of this section, therefore, is likely to be changed. Nevertheless, the qualitative result is expected to remain the same.
Figure 5. Halo mass functions in different \( \{\Delta_c, V_{bc}\} \) environments (D0V0: black solid; D0V2: red dotted; D2V2: blue dotted-dashed; D2V3: orange dashed), obtained from N-body-hydro simulations based on initial conditions at \( z = 200 \) depicted in Figure 4. Plotted are halo mass functions (upper) and the ratios with respect to the D0V0 case (lower) at \( z = 30 \) (left) and \( z = 20 \) (right).

Figure 6. (a) Matter power spectrum at \( z = 200 \), expressed in terms of the \( k \)-space variance \( \Delta_m^2 \), for different patches (D0V0: black solid; D0V2: red dotted; D2V2: blue dotted-dashed; D2V3: orange dashed). Except for the D0V0 patch, cases with constant \( \mu = \cos(k, V_{bc}) = 0.5 \) are selected for other patches for a fair comparison of the negative impact of \( V_{bc} \). The ratio in the bottom is with respect to the D0V0 case. (b) Same as (a), but for the CDM power spectrum.
and the ratio to the D0V0 case (bottom). The line-type convention is the same as in Figure 5. Note that this figure should be considered only for a qualitative comparison, because the numerical resolution of these simulations, a uniform grid of $512^3$ on a simulation box of volume $(1\, h^{-1}\text{Mpc})^3$, is currently not adequate for accurately predicting this quantity.

5. Summary

The first stars and first galaxies were created inside minihalos, which are the first nonlinear structure in the history of the universe. The evolution of small-scale fluctuations of both CDM and baryons is found to be strongly affected by the large-scale environment, leading to cosmic variance in the formation and evolution of these first objects. This effect is caused predominantly by the large-scale density and the baryon–CDM streaming velocity. Because this effect has not been fully incorporated in existing initial condition generators, we have developed an initial condition generator, BCCOMICS, that fully incorporates this effect for the first time. BCCOMICS first calculates the evolution equations, which were given by TH for only the mean-density environment and by Ahn16 for a fully generic, non-zero overdensity environment, and then generates three-dimensional fields of grid and particle quantities.

Study of this cosmic variance requires realizing the local environment in simulations. This can be done as zoom-in patches in one big simulation box, or as an individual patch with a periodic box condition. For the latter, we have developed a systematic scheme to simulate the growth of small-scale structure, inside density peaks and voids, by treating the environment as a separate universe with local cosmological quantities. The affected quantities are the local Hubble parameter, the local cosmic abundance of various contents (CDM, baryons, radiation, cosmological constant), and the local scale factor. Analysis of simulation data requires a scaling law that maps the local quantity to the global quantity. For example, a correlation length of galaxy clustering or the spatial BAO peak of the correlation function, which are found through a local galaxy survey inside an overdense environment, will differ from the global values found through an unlocalized galaxy survey. We have provided a trivial but important scaling law that allows one to easily deduce the corresponding global quantities.

As a pilot study, we generated initial conditions by BCCOMICS and performed a suite of $N$-body+hydro simulations of small-scale structure formation in varying large-scale environments. As expected, the overdensity environment yields positive feedback effects and the streaming-velocity environment negative feedback effects. More halos are generated than in the mean-density environment, and gas cooling becomes more efficient in the overdense environment. The higher the overdensity is, the stronger this positive feedback will become. In contrast, the streaming velocity tends to suppress halo formation and also the gas cooling.

We need to improve upon the quantitative prediction of this pilot study, and will conduct a more self-consistent study by allowing AMR and possibly radiation transfer as well. At the same time, a wider parameter space of environmental variation will be considered, and the result will be described in Paper II (in preparation).

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References

Abel, T., Bryan, G. L., & Norman, M. L. 2002, Sci, 295, 93
Ahn, K. 2016, ApJ, 830, 68
Ahn, K., Iliev, I. T., Shapiro, P. R., et al. 2012, ApJL, 756, L16
Barkana, R., & Loeb, A. 2011, MNRAS, 415, 3113
Blazek, J. A., McEwen, J. E., & Hirata, C. M. 2016, PhRvL, 116, 121303
Bouchet, F. R., Juszkiewicz, R., Colombi, S., & Pellat, R. 1992, ApJL, 394, L5
Bromm, V., Coppi, P. S., & Larson, R. B. 2002, ApJL, 564, 23
Bryan, G. L., Norman, M. L., O’Shea, B. W., et al. 2014, ApJS, 211, 19
Caffau, E., Bonifacio, P., François, P., et al. 2011, Natur, 477, 67
Dalal, N., Pen, U.-L., & Seljak, U. 2010, JCAP, 11, 007
Fialkov, A., Barkana, R., Tseliakhovich, D., & Hirata, C. M. 2012, MNRAS, 424, 1335
Goldberg, D. M., & Vogeley, M. S. 2004, ApJ, 605, 1
Greif, T. H., Springel, V., White, S. D. M., et al. 2011a, ApJ, 737, 75
Greif, T. H., White, S. D. M., Klessen, R. S., & Springel, V. 2011b, ApJL, 736, 147
Hirano, S., Hosokawa, T., Yoshida, N., et al. 2014, ApJL, 781, 60
Hirano, S., Hosokawa, T., Yoshida, N., & Kuiper, R. 2017, Sci, 357, 1375
Hirano, S., Hosokawa, T., Yoshida, N., Omukai, K., & Yorke, H. W. 2015, MNRAS, 448, 568
Howes, L. M., Casey, A. R., Asplund, M., et al. 2015, Natur, 527, 484
Jacobson, H. R., Keller, S., Frebel, A., et al. 2015, ApJ, 807, 171
Lacey, C., & Cole, S. 1994, MNRAS, 271, 676
Lewandowski, M., Perko, A., & Senatore, L. 2015, JCAP, 5, 019
Maio, U., Koopmans, L. V. E., & Ciardi, B. 2011, MNRAS, 412, L40
McQuinn, M., & O’Leary, R. M. 2012, ApJL, 760, 3
Nagakura, T., & Omukai, K. 2005, MNRAS, 364, 1378
Naoz, S., & Barkana, R. 2005, MNRAS, 362, 1047
Naoz, S., & Narayan, R. 2014, ApJL, 791, L8
O’Leary, R. M., & McQuinn, M. 2012, ApJL, 760, 4
Omukai, K. 2001, ApJL, 546, 635
O’Shea, B. W., Wise, J. H., Xu, H., & Norman, M. L. 2015, ApJL, 807, L12
Regan, J. A., & Downes, T. P. 2018, MNRAS, 478, 5037
Schauer, A. T. P., Regan, J., Glover, S. C. O., & Klessen, R. S. 2017, MNRAS, 471, 4878
Schmidt, F. 2016, PhRvD, 94, 063508
Slepian, Z., & Eisenstein, D. J. 2015, MNRAS, 448, 9
Stacy, A., Bromm, V., & Loeb, A. 2011, ApJL, 730, L1

Stacy, A., Greif, T. H., & Bromm, V. 2010, MNRAS, 403, 45
Tseliakhovich, D., & Hirata, C. 2010, PhRvD, 82, 083520
Turk, M. J., Abel, T., & O’Shea, B. 2009, Sci, 325, 601
Yoo, J., Dalal, N., & Seljak, U. 2011, JCAP, 7, 018
Yoshida, N., Omukai, K., Hernquist, L., & Abel, T. 2006, ApJ, 652, 6