Electrons acceleration in a $TE_{113}$ cylindrical cavity affected by a static inhomogeneous magnetic field

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Abstract. The relativistic dynamics of an electron accelerated in a cylindrical cavity mode $TE_{113}$ in the presence of a static inhomogeneous magnetic field is studied. This type of acceleration is known as Spatial AutoResonance Acceleration (SARA). The magnetic field profile is such that it keeps the phase difference between the electric microwave field and the electron velocity vector within the acceleration phase band. We study the dynamic of the electron through simulations of the relativistic Newton-Lorentz equation. Numerical experiments with $TE_{113}$ cylindrical microwave fields of a frequency of $2.45\,GHz$ and an amplitude of the order of $7\,kV/cm$ show that it is possible accelerate the electrons up to energies of the order of $300\,keV$. This energy is about of $30\%$ higher than those obtained in previous studies by using the $TE_{112}$ mode.

1. Introduction

The study of cyclotron resonance has been extensively studied and has led to several mechanisms to accelerate charged particles [1–13]. The spatial autoresonance acceleration SARA [7–10,12,13] has emerged as an excellent option to accelerate electric charged particles by using cylindrical $TE_{11p}$ or rectangular $TE_{10p}$ modes ($p = 1, 2, ..$). For example, particles that reach energies of the order of $230\,keV$ by using cylindrical $TE_{112}$ mode or rectangular $TE_{102}$ mode have already been reported [7, 12, 13]. An X ray source based on in the SARA concept has been ceritficated recently [14].

In the SARA mechanisms the axial motion is restricted by the diamagnetic force limiting the energy which can be achieved. Such force arise from the longitudinal inhomogeneity of the magnetostatic field. One way to reach higher value energies is to use higher order $TE_{11p}$ or $TE_{10p}$ modes. In this work we show that it is possible to accelerate an electrons beam up to energies of the order of $300\,keV$ by using a cylindrical $TE_{113}$ mode of a frequency of $2.45\,GHz$ and an amplitude of the order of $7\,kV/cm$.

The manuscript is organized as follows. Section 2 represents our theoretical approach. Section 3 is about the results for the conditions selected for the autoresonant system. Our findings are summarized in section 4.

2. Theoretical formalism

The physical scheme of the autoresonant accelerator is shown in Figure 1. A magnetron emits $2.45\,GHz$ microwaves into the cylindrical cavity (2) through a waveguide (4). The magnetostatic
The electric and magnetic field components of a cylindrical $TE_{113}$ mode circular polarized excited in the cavity are described by the following expressions:

$$E_r^{hf} = -2E_0\frac{J_1(K_\perp r)}{K_\perp r}\text{sen}(K_\parallel z)\cos(\theta - \omega t),$$  \hspace{1cm} (1)

$$E_\theta^{hf} = 2E_0 J'_1(K_\perp r)\text{sen}(K_\parallel z)\cos(\theta - \omega t),$$  \hspace{1cm} (2)

$$B_r^{hf} = 2E_0\frac{K_\parallel}{\omega} J'_1(K_\perp r)\cos(K_\parallel z)\cos(\theta - \omega t),$$  \hspace{1cm} (3)

$$B_\theta^{hf} = -2E_0\frac{K_\parallel}{\omega} J_1(K_\perp r)\cos(K_\parallel z)\text{sen}(\theta - \omega t),$$  \hspace{1cm} (4)

$$B_z^{hf} = 2E_0\frac{K_\parallel}{\omega} J_1(K_\perp r)\text{sen}(K_\parallel z)\cos(\theta - \omega t),$$  \hspace{1cm} (5)

where $K_\perp = S_{11}/a$, $S_{11} = 1.8411$, $K_\parallel = 3\pi/d$. $d$ and $a$ are the length and radius of the cavity, respectively. $E_0$ is the electric field strength and

$$\omega = c(K_\parallel^2 + K_\perp^2)^{1/2}$$  \hspace{1cm} (6)

is the angular frequency of the microwave field.

For our simulations we consider a cylindrical cavity whose length and diameter are 30cm and 9.1cm, respectively. An appropriate magnetostatic field profile is obtained using the parameters showed in Table 1. where $R_i$, $R_e$, $L_b$, and $z$ are the internal radius, the external radius, the width of each coil, and the positions of the coils, respectively, and $J$ is the coil current density.

| Bobina | $R_i$(cm) | $R_e$(cm) | $L_b$(cm) | $J$(A/mm$^2$) | $z$(cm) |
|--------|-----------|-----------|-----------|--------------|--------|
| 1      | 6.0       | 20.0      | 6.0       | 1.39         | -5.75  |
| 2      | 6.0       | 20.0      | 7.5       | 1.08         | 8.25   |
| 3      | 6.0       | 20.0      | 6.9       | 1.18         | 19.50  |
| 4      | 6.0       | 20.0      | 6.1       | 2.07         | 32.00  |
The two-dimensional profile of the magnetic field in units of \( B_0 = m_e \omega / e = 0.0875T \) is shown in the Figure 2.

Figure 2. The profile of the magnetostatic field in the \( y = 0 \) plane.

The single particle dynamics is described with the following motion equations:

\[
\frac{d\vec{r}}{dt} = \vec{v}, \quad \frac{d(\gamma m_e \vec{v})}{dt} = -e(\vec{E} + \vec{v} \times \vec{B})
\]

(7)

Where \( \gamma = \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} \) is the relativistic factor, \(-e\) and \(m_e\) are the charge and mass of an electron, \( \vec{E} = \vec{E}_{rf} \) and \( \vec{B} = \vec{B}_{hf} + \vec{B}_{coils} \) are the total electric and magnetic fields at the electron position, respectively. The equation (7) is solved numerically in a finite difference scheme by using the Boris method, which let us calculate the electron position, particle energy and longitudinal velocity at any time step.

In the simulations all the quantities are taken in dimensionless form: \( \vec{E}^* = \vec{E}/(-B_0c) \), \( \vec{B}^* = \vec{B}/(-B_0) \), \( r^* = \frac{r}{r_{l0}} \) and \( t^* = \frac{\omega t}{c} \); where: \( B_0 = m_e \omega / e \) is the magnetic field corresponding to the classic cyclotron resonance for the electrons and \( r_{l0} = c / \omega \) is the relativistic Larmor radius.

3. Results and discussion

The numerical simulations is run under the above-mentioned physical parameters. The Figure 3(a) shows the spatial evolution of the energy of a single electron injected with an energy of 20\( keV \) into the cavity for three different electric field strength of the \( TE_{113} \) mode: 7.2\( kV/cm \) (red line), 7.4\( kV/cm \) (green line) and 7.6\( kV/cm \) (blue line). It is shown that the electron can be accelerated up to energies of the order of 300\( keV \) for the 3 cases.

The Figure 3(b) shows the evolution of the phase-shift between the electric field and the transversal velocity of the electron when the electron advances along the z-direction. In the injection position \( z = 0 \), where the electric field has a node, the phase-shift \( \varphi \) acquire the value \( \pi/2 \) due to the deviation produced by the magnetic field component of the microwave field. Then the phasing is produced by the electric field up to the exact resonance \( \varphi = \pi \) in the position \( z = 6cm \). Despite of the electron acceleration is not produced in exact resonance, the phase-shift is maintained in the range \( \pi/2 < \varphi < 3\pi/2 \). In such range the microwave electric field can transfer power effectively to the electron, in this sense this interval is known as acceleration band [7].

There is some intervals in the Figure 3(b) where the phase-shift is out of the acceleration band, e.g. for the blue line case these intervals are 10 – 12.5\( cm \), 19 – 20\( cm \) and 25 – 29\( cm \). It is
easy to check that the electron energy decrease in these intervals (See Figure 3(a)). We can note that the phase-shift changes a $\pi$ value ($\varphi \rightarrow \varphi + \pi$) in the positions $z = 10cm$ and $z = 20cm$ because the electric field of $TE_{113}$ has nodes in such points.

Figure 3. (a) Spatial evolution of the energy of a single electron for different electric field strength of the $TE_{113}$ mode. (b) The corresponding phase-shift between the electric field and the transversal velocity of the electron.

Figure 4. (a) Spatial evolution of the energy of a single electron for different injection energies. (b) The corresponding phase shift between the electric field and the transversal velocity of the electron.

The Figure 4(a) shows the spatial evolution of the energy of a single electron for different injection energies while Figure 4(b) the corresponding phase shift between the electric field and the transversal velocity of the electron by using a electric field strength $E_0 = 7.6kV/cm$ for all cases. The relation between these graphics can be understood using a similar analysis as was previously described for the Figure 3. We can see that the electron injected with an energy of 20keV advances a greater longitudinal distance, thereby prolonging the resonant interaction with the microwave field and increasing its energy. To understand this behaviour let us check
the Figure 5, which shows the longitudinal velocity for the case considered in the Figure 4. For all energies injections appears a diamagnetic force which acts in the direction opposite to the magnetic field gradient and impedes the advance of the electrons into a higher magnetic field. However for the green line case the electron accelerates longitudinally in the range 18 – 22cm (see Figure 5). This effect is attributed to the magnetic component of the microwave field whic depend of the phase-shift $\varphi$ and $z$ coordinate of each electron.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{Transversal ($\beta_T = \frac{v_T}{c}$) and longitudinal ($\beta_Z = \frac{v_Z}{c}$) velocities for the case presented in Figure 4.}
\end{figure}

4. Conclusions

The realized numerical experiment shows that electrons can be accelerated up to energies of the order of 300keV in spatial autoresonance acceleration conditions by using a $TE_{113}$ mode. This results could be usefull to design a hard X-ray device based on de SARA concept which have many applications, e.g. in airport security.

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References

[1] A Kolomenskii and A Levedev 1962 Dokl Akad Nauk USSR 145 1251
[2] V Davydovskii 1962 Zh Eksp. Teor. Fiz. 43 886
[3] V Milantiev 2013 Physics Uspekhi 56 823
[4] K S Golovanivsky 1983 IEEE Trans. Plasma Sci. 11 28
[5] Gal 1989 IEEE Trans. Plasma Sci. 17 622
[6] R Geller and K S Golovanivsky 1999 Nucl. Instrum. Methods Phys. Res. Sect. B 68 479
[7] V Dugar-Zhabon, E A Orozco and A Umnov 2008 Phys. Rev. Spec. Top. Acceler. and Beams 11 041302 1-5
[8] V Dugar-Zhabon and E A Orozco 2009 Phys. Rev. Spec. Top. Acceler. and Beams 12 041301 1-8
[9] V Dugar-Zhabon and E A Orozco 2010 IEEE Transaction on Plasma Science 38(10) 2980
[10] V Dugar-Zhabon and E A Orozco 2009 Journal of Plasma Fusion Research Series 8 473-476
[11] H Gutierrez, E A Orozco and V Dugar-Zhabon 2011 Rev. Colombiana de Física 44(3) 220-224
[12] V Dugar-Zhabon, E A Orozco and A M Herrera 2016 Journal of Physics: Conference Series 687 012076
[13] V Dugar-Zhabon, J D González and E A Orozco 2016 Journal of Physics: Conference Series 687 012077
[14] V D Dugar-Zhabon and E A Orozco Ospino 2015 Compact self-resonant X-ray source (USA: Patent office)