FINAL-STATE RADIATION AND LINE-SHAPE DISTORTION
IN RESONANCE PAIR PRODUCTION

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ABSTRACT

In this letter it is shown how final-state QED corrections to the production of a pair of resonances can distort the line shape of such a resonance in a sizeable way. This effect depends on the definition of the line shape and can reach up to 30%, depending on the final state. The mechanism is first displayed for a particular case of $ZZ$ production, for which an exact and approximate treatment can be given. The approximate method is then applied to $W$-pair production. In addition some simple rules of thumb are given for accurately estimating the characteristic distortion effects, like the mass shift and peak reduction.
1 Introduction

As is well known [1], the $Z$ line shape as measured in $e^+e^- \rightarrow Z \rightarrow 2f$ is distorted due to initial-state radiation (ISR). Without ISR the total cross-section $\sigma(s)$ as a function of the square of the centre-of-mass energy $s$ gives the line shape. With ISR the centre-of-mass energy available to produce a $Z$ boson changes and, as a consequence, so does the shape of the total cross-section $\sigma(s)$. Experimentally the latter is measured. If one would measure the square of the modified centre-of-mass energy $s'$, one would determine $\sigma(s')$ and thereby the pure line-shape. It should be noted that final-state radiation (FSR) only marginally corrects the overall size of $\sigma(s)$, but not its shape. Therefore FSR is less relevant for the usual $Z$ line-shape measurement.

When one produces two resonances, or one resonance and a stable particle, the line shape of such a resonance will be measured from the invariant-mass distribution of its decay products. Examples are pair production of $W$ bosons, $Z$ bosons, $tt$ or $HZ$. Depending on how one measures the invariant-mass distribution of the decay products of the particular resonance, one finds the pure line shape or a distorted one. This time also FSR can cause the distortion.

It is the main purpose of this letter to point out that such a FSR-induced distortion can arise. For exhibiting the effect we take an example for which we can perform both exact and approximate calculations. An ideal example is the double-resonance process

$$\nu_\mu \bar{\nu}_\mu \rightarrow ZZ \rightarrow e^+e^-\nu_\tau \bar{\nu}_\tau. \quad (1)$$

Here QED corrections apply only to the decay $Z \rightarrow e^+e^-$ and not to the other $Z$ decay or the initial state. When the $Z$ line shape is obtained from measuring the invariant-mass distribution of the $e^+e^-$ pair, FSR will distort it in a way reminiscent of the usual ISR distortion in single $Z$ production. The virtue of the example is threefold. In the first place, process (1) is free of the gauge-invariance problems that are inherent in the production of unstable particles. This holds in spite of the fact that we have left out all non-double-resonant mechanisms for producing the $e^+e^-\nu_\tau \bar{\nu}_\tau$ final state. Secondly, the QED radiative corrections only lead to FSR. So, the effect of FSR on the line shape can be studied without the additional presence of ISR phenomena. Thirdly, the effect can be calculated exactly. In more realistic examples, involving for instance $e^+e^-$ initial states, $Z$-pair production with both $Z$ bosons decaying into charged particles, or $W$-pair production, additional classes of QED radiative corrections emerge, like ISR [2] or non-factorizable interference corrections [3]. Moreover, in order to avoid gauge-invariance problems, the QED corrections often have to be calculated in an approximation, which for instance restricts the calculation to the leading logarithmic corrections and/or the leading terms in a pole-scheme expansion around the resonances [2, 4]. Nevertheless the FSR distortion of the line shape will still be one of the main features. In these more complicated cases Monte Carlo studies including radiative corrections would be needed. Here we focus exclusively on the line-shape deformation and its impact on the determination of the resonance mass.

Although we start with reaction (1), we shall also comment on the more realistic case of the $W$ line shape at LEP2.
2 The \( Z \)-pair example: exact calculation

For process (1) we first consider the Born approximation, to which two double-resonant diagrams contribute. After integration over the \( Z \) production angle and the fermion decay angles, one obtains

\[
\frac{d\sigma_0(M_1^2, M_2^2)}{dM_1^2 dM_2^2} = \Pi(M_1^2, M_2^2) \frac{\Delta_1(M_1^2)}{|D_1(M_1^2)|^2} \frac{\Delta_2(M_2^2)}{|D_2(M_2^2)|^2} = F(M_1^2, M_2^2),
\]

where \( M_1^2 \) and \( M_2^2 \) denote the invariant masses of the \( e^+e^- \) and \( \nu_\tau\bar{\nu}_\tau \) pairs, respectively. The spin-averaged production cross-section takes the form

\[
\Pi(M_1^2, M_2^2) = \frac{G_F^2 M_Z^4}{4\pi s} (2g_\nu)^4 \sqrt{\lambda} \left[ -2 + \frac{s^2 + (M_2^2 + M_2^2)^2}{(s - M_1^2 - M_2^2)\sqrt{\lambda}} \ln\left( \frac{s - M_1^2 - M_2^2 + \sqrt{\lambda}}{s - M_1^2 - M_2^2 - \sqrt{\lambda}} \right) \right],
\]

with \( \lambda \) the Kallen function

\[
\lambda = s^2 + M_1^4 + M_2^4 - 2(sM_1^2 + sM_2^2 + M_1^2 M_2^2),
\]

which is in agreement with the literature [4]. The decay parts are given by

\[
\Delta_1(M_1^2) = \frac{G_F M_Z^2}{6\pi \sqrt{2}} (g^2 + g^2_{A\ell}) \frac{1}{\pi} M_1^2 = \frac{1}{\pi} M_1^2 \frac{\Gamma_{Z \rightarrow e^+e^-}}{M_Z},
\]

\[
\Delta_2(M_2^2) = \frac{G_F M_Z^2}{6\pi \sqrt{2}} 2g_\nu \frac{1}{\pi} M_2^2 = \frac{1}{\pi} M_2^2 \frac{\Gamma_{Z \rightarrow \nu_\ell \nu_\ell}}{M_Z},
\]

whereas the resonance shapes are dominated by

\[
D_{1,2}(M_{1,2}^2) = M_{1,2}^2 - M_Z^2 + iM_{1,2} \frac{\Gamma_Z}{M_Z}.
\]

Note that we have used the standard LEP1 representation in the above formulae, involving \( G_F \) and the effective couplings of the \( Z \) boson to leptons (\( g_{v\ell}, g_{A\ell} \)) and neutrinos (\( g_\nu \)).

Applying virtual and soft photonic corrections to (2) yields

\[
\frac{d\sigma_{vs}(M_1^2, M_2^2)}{dM_1^2 dM_2^2} = \frac{d\sigma_0(M_1^2, M_2^2)}{dM_1^2 dM_2^2} \left[ 1 + \frac{2\alpha}{\pi} (L - 1) \ln \epsilon + \frac{\alpha}{\pi} \left( \frac{3}{2} L + \frac{\pi^2}{3} - 2 \right) \right],
\]

with

\[
L = \ln \left( \frac{M_1^2}{m_\gamma^2} \right).
\]

Here we have defined the soft photons in the rest frame of the \( Z \): \( E_\gamma < \epsilon M_1/2 \ll \Gamma_Z \). Photon bremsstrahlung involving more energetic photons introduces an explicit dependence on the photon energy \( E_\gamma \), resulting in a distribution in the invariant masses of both the \( e^+e^- \) pair (\( M_1^2 \)) and the \( e^+e^-\gamma \) system (\( M_2^2 = \text{virtuality of the } Z \) boson):

\[
\frac{d\sigma_{brem}(M_1^2, M_1^2, M_2^2)}{dM_1^2 dM_1^2 dM_2^2} = \frac{d\sigma_0(M_1^2, M_2^2)}{dM_1^2 dM_2^2} \frac{\alpha}{\pi} (L' - 1) \frac{1 + z^2}{1 - z} \frac{1}{M_1^2},
\]

where \( z = M_1^2 / M_Z^2 \).
where
\[ z = \frac{\hat{M}_1^2}{M_1^2} = \frac{1}{\zeta}, \quad L' = \ln \left( \frac{\hat{M}_1^2}{m_e^2} \right) = L + \ln z. \]  
(10)

When correction (7) is combined with (9) and an integration over \( \hat{M}_1^2 \) is performed, the correction to \( d\sigma_0/(dM_1^2 dM_2^2) \) takes on the form of the usual FSR factor \( 1 + 3\alpha/(4\pi) \). This is in agreement with the KLN theorem, which implies that the large logarithmic contributions \( (\propto L, L') \) vanish upon summation (integration) over all degenerate final states. So, the resonance shape is not deformed when one measures the \( M_1^2 \) distribution, i.e. the invariant-mass distribution of the \( e^+e^-\gamma \) system.

In our special example this choice of distribution is, of course, the natural one. However, in more realistic processes it is in general unclear whether the photon is radiated from the initial state, the unstable particles, or the final state. This introduces the freedom to either choose \( M_1^2 \) or \( \hat{M}_1^2 \) for the definition of the invariant-mass distribution of the unstable particle.

If one measures the \( \hat{M}_1^2 \) distribution one will find a distorted line shape. The reason is that (9) now has to be integrated over \( M_1^2 \) values ranging from \( \hat{M}_1^2 \) to \( (\sqrt{s} - M_2)^2 \). This causes the \( M_1^2 \) line shape to receive contributions from effectively higher \( Z \)-boson virtualities. This is to be compared with the single-\( Z \)-production case where the ISR-corrected line shape receives contributions from effectively lower \( Z \)-boson virtualities. Due to the fact that roughly speaking the resonance shape is symmetric around the resonance mass, one expects now a distortion of the resonance shape that is approximately the LEP1 distortion reflected with respect to the resonance mass.

The bremsstrahlung contribution to the line shape \( d\sigma/(d\hat{M}_1^2 dM_2^2) \) arising from (9) reads
\[ \frac{d\sigma_{\text{brem}}(\hat{M}_1^2, M_2^2)}{d\hat{M}_1^2 dM_2^2} = \frac{\alpha}{\pi} (L' - 1) \int_1^{\zeta_{\text{max}}} d\zeta F(\zeta \hat{M}_1^2, M_2^2) \left[ 2 - \frac{1 + \zeta}{\zeta^2} \right], \]
(11)
where the function \( F \) is defined in (2) and \( \zeta_{\text{max}} = (\sqrt{s} - M_2)^2/\hat{M}_1^2 \). Combining (7) and (11) gives the \( \mathcal{O}(\alpha) \) correction to the line shape. Just like at LEP1 it will be necessary to resum the soft corrections, as will become clear from the discussion in the following section. Based on LEP1 experience (9) a suitable expression for this resummation is given by
\[ \frac{d\sigma_{\text{res}}(\hat{M}_1^2, M_2^2)}{d\hat{M}_1^2 dM_2^2} = \int_1^{\zeta_{\text{max}}} d\zeta G(\zeta) F(\zeta \hat{M}_1^2, M_2^2), \]
(12)
with
\[ G(\zeta) = \beta (\zeta - 1) \beta^{-1} (1 + \delta_{\text{res}}^{\text{vs}}) - \frac{\beta}{2} \frac{1 + \zeta}{\zeta^2} \]
\[ \beta = \frac{2\alpha}{\pi} (L' - 1) \]
\[ \delta_{\text{res}}^{\text{vs}} = \frac{\alpha}{\pi} \left( \frac{3}{2} L' + \frac{\pi^2}{3} - 2 \right). \]
(13)
\[ \frac{d\sigma}{dM^2_1 dM^2_2} \left[ \text{pb}/\text{GeV}^4 \right] \]

\[ \tilde{M}_1 \text{ [GeV]} \]

Figure 1: The FSR-induced distortion of the line shape \( \frac{d\sigma}{(d\tilde{M}^2_1 dM^2_2)} \) corresponding to process (1) for \( M_2 = M_Z \). Centre-of-mass energy: \( \sqrt{s} = 200 \text{ GeV} \).

In Fig. 1 we display the FSR-induced distortion effects on the line shape \( \frac{d\sigma}{(d\tilde{M}^2_1 dM^2_2)} \) for a centre-of-mass energy of \( \sqrt{s} = 200 \text{ GeV} \) and a fixed invariant mass \( M_2 = M_Z \). However, the actual distortion phenomena do not depend on the precise value of \( M_2 \). The parameter input used in the numerical evaluation is:

\begin{align*}
M_Z &= 91.1867 \text{ GeV}, \quad \Gamma_Z = 2.4948 \text{ GeV}, \quad G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}, \quad m_e = 0.51099906 \text{ MeV}, \\
\alpha^{-1} &= 137.0359895, \quad g_\nu = 0.50125, \quad g_{V\ell} = -0.03681, \quad g_{A\ell} = -0.50112.
\end{align*}

The sizeable distortion effects are clearly visible, just as the importance of the soft-photon resummation. Compared with the Born line shape, the \( \mathcal{O}(\alpha) \) (resummed) QED corrections induce a shift in the peak position of \(-199 \text{ MeV} \) \((-112 \text{ MeV})\) and a reduction of the peak height by 29\% \((26\%)\). The size of these effects are a direct result of the non-cancellation of the leading logarithmic corrections, which can be understood from the observation that a fixed value for \( \tilde{M}_1 \) makes it impossible to sum over all degenerate final states. Another noteworthy observation is the close similarity of the curves in Fig. 1 to the ones for the Z line shape at LEP1 [1]. As predicted, the two sets of curves are approximately related by reflection with respect to the Born peak position.

3 The Z-pair example: approximations

As mentioned before, in order to calculate QED corrections to more realistic processes like \( W \)-pair production one in general has to resort to approximations. First of all, the fact that we are dealing with unstable (charged) particles introduces the problem of a gauge-invariant treatment of the finite-width effects [2]. An appropriate way of handling this problem is by applying the pole scheme, i.e. by performing an expansion around the resonances. When it comes to \( \mathcal{O}(\alpha) \)
corrections, it is sufficient to consider only the leading (double-pole) term in this expansion, leaving out terms that are formally suppressed by at least $\alpha L \Gamma/(\pi M)$. From now on we will refer to this procedure as the double-pole approximation and indicate quantities that are calculated in this approximation by a bar. Note that the approximation only makes sense near the resonance of the unstable particle and sufficiently far above the production threshold of the underlying on-shell production process. The latter is caused by the direct relation between the double-pole residues and the on-shell production and decay processes. Secondly, as we have seen in the previous section, the leading logarithmic corrections ($\propto L'$) constitute the bulk of the FSR distortion effects. Moreover, these leading-log effects are universal and gauge invariant, being directly related to the collinear limit of photon radiation off light particles (like $e^\pm$). In particular the universality property is appealing, since it implies that the description of the leading-log corrections does not depend on the specific features of the unstable particles and their photonic interactions. Therefore, it is worthwhile to further restrict the double-pole calculations to the leading logarithms. This additional approximation is referred to as the leading-log approximation (LLA). Before making any comments on processes like $W$-pair production, we first concentrate on our $Z$-pair example and check the validity of the indicated approximations.

We start off with the definition of the double-pole approximation. At Born level it amounts to

$$\frac{d\sigma_0(M^2_{Z1}, M^2_{Z2})}{dM^2_{Z1} dM^2_{Z2}} = \Pi(M^2_{Z1}, M^2_{Z2}) \frac{\Delta_1(M^2_{Z2})}{|D_1(M^2_{Z2})|^2} \frac{\Delta_2(M^2_{Z1})}{|D_2(M^2_{Z1})|^2}. \quad (14)$$

with

$$\tilde{D}_{1,2}(M^2_{1,2}) = M^2_{1,2} - M^2_Z + i M_Z \Gamma_Z. \quad (15)$$

As mentioned before, the QED radiative corrections introduce an ambiguity in defining the invariant-mass distributions of the unstable particles. The most transparent way of illustrating this is by considering the case that the photon is radiated from such an unstable particle. For the description of the resonance before (after) radiation the natural choice of invariant mass involves the fermion pair with (without) the photon. The pole expansion can now in principle be performed around either resonance. In practice one has to choose one particular invariant mass for the distributions. For the purpose of studying FSR-induced distortion effects, we shall choose the $e^+e^-$ invariant mass $\tilde{M}_1$ in the following, although the $e^+e^-\gamma$ invariant mass $M_1$ would have been more natural for our special example (1). The corresponding double-pole approximation forces us to replace $M^2_1 = \zeta M^2_Z$ by $\zeta M^2_Z$, introducing an explicit dependence on the photon energy in the double-pole residues. This would even affect the (neutral) resonance-pair-production stage of the process. However, as can be verified explicitly, only semi-soft photons with energy $E_\gamma = \mathcal{O}(\Gamma_Z) \ll M_Z$ contribute to the $\mathcal{O}(\alpha)$ corrected double-pole residues. As a result, $\zeta$ can be effectively replaced by unity whenever possible, re-establishing the usual form of the double-pole approximation in terms of off-shell Breit–Wigner distributions and on-shell production/decay processes. The effects from hard photons ($E_\gamma \gg \Gamma_Z$) are suppressed by at least $\Gamma_Z/M_Z$ and are therefore neglected in the double-pole approximation. The picture underlying this phenomenon is that hard photons move the $Z$-boson virtuality ($M^2_1$) far off resonance for near-resonance $M^2_1$ values, resulting in a suppressed contribution to the $M^2_1$ line.
shape. In fact, only the (soft) \( 1/(\zeta - 1) \) term in (11) contributes to the \( \tilde{M}_1^2 \) line shape in the double-pole approximation. It should be noted that this very suppression of hard-photon effects serves as \textit{a posteriori} justification of the soft-photon resummation proposed in (12).

With this observation in mind, the bremsstrahlung contribution (11) takes the following form in double-pole approximation:

\[
\frac{d\sigma_{\text{brem}}(\tilde{M}_1^2, M_2^2)}{dM_1^2 dM_2^2} = \frac{d\sigma_0(\tilde{M}_1^2, M_2^2)}{dM_1^2 dM_2^2} \int_{1+\epsilon}^{\infty} d\zeta \frac{\bar{\beta}}{\zeta - 1} \frac{|D_1(\tilde{M}_1^2)|^2}{|D_1(\zeta M_1^2)|^2}.
\]

(16)

Here \( \bar{\beta} \) can be derived from \( \beta \) by setting \( \bar{M}_1^2 = M_2^2 \). Note that the upper integration boundary \( \zeta_{\text{max}} \) has been extended to infinity, which is motivated by the fact that hard-photon effects are sufficiently suppressed. The remaining integral can be performed analytically. Combining with the virtual and soft corrections, which can be readily derived from (7), we obtain in the double-pole LLA

\[
\frac{d\sigma(M_1^2, M_2^2)}{dM_1^2 dM_2^2} = \frac{d\sigma_0(M_1^2, M_2^2)}{dM_1^2 dM_2^2} \left\{ 1 + \frac{3}{4} \frac{\bar{\beta} + \bar{\beta} \Re \left[ \frac{iD_1^*(\tilde{M}_1^2)}{M_Z \Gamma_Z} \ln \left( \frac{D_1(\tilde{M}_1^2)}{M_2^2} \right) \right]}{\sin(\pi \bar{\beta})} \right\}.
\]

(17)

In a way similar to the previous section the soft-photon corrections can be resummed, but this time the integral can be carried out explicitly in the double-pole LLA:

\[
\frac{d\sigma_{\text{res}}(\tilde{M}_1^2, M_2^2)}{dM_1^2 dM_2^2} = \frac{d\sigma_0(\tilde{M}_1^2, M_2^2)}{dM_1^2 dM_2^2} \left( 1 + \frac{3}{4} \frac{\bar{\beta}}{\sin(\pi \bar{\beta})} \right) \frac{\pi \bar{\beta}}{\Gamma_Z} \Re \left[ \frac{iD_1^*(\tilde{M}_1^2)}{M_Z \Gamma_Z} \left( \frac{D_1(\tilde{M}_1^2)}{M_2^2} \right)^{\bar{\beta}} \right] \left( 1 + \frac{3}{4} \bar{\beta} \right).
\]

(18)

Having calculated the same quantities as in the previous section, we are now in the position to check the validity of the double-pole LLA. It turns out that the approximated results exhibit the same FSR distortions as the exact ones. Upon closer investigation, we observe for the \( \mathcal{O}(\alpha) \) (resummed) QED corrections a shift in the peak position of \(-193 \text{ MeV} \) (\(-113 \text{ MeV} \)) and a reduction of the peak height by 29% (26%). This is in excellent agreement with the distortion parameters of the exact calculation, proving the viability of the adopted approximations.

Based on the results in the double-pole LLA, it is possible to derive simple and sufficiently accurate rules of thumb for the distortion parameters:

- \( \mathcal{O}(\alpha) \) corrections: the shift in the peak position \( \Delta \tilde{M}_1^{\text{peak}} \) and the corresponding peak reduction factor \( \kappa^{\text{peak}} \) with respect to the Born line shape can be approximated by

\[
\Delta \tilde{M}_1^{\text{peak}} \approx - \frac{\pi \bar{\beta} \Gamma_Z/8}{\kappa^{\text{peak}} - 3\bar{\beta}/2} = -196 \text{ MeV},
\]

\[
\kappa^{\text{peak}} \approx 1 + \bar{\beta} \ln \left( \frac{\Gamma_Z}{M_Z} \right) + \frac{3}{4} \bar{\beta} + \frac{\pi^2}{16} \bar{\beta}^2 = 0.70; \quad (19)
\]

\footnote{For completeness we note that all curves in the double-pole LLA are displaced by a small amount with respect to the exact ones. This is caused by the fact that the Born results differ by subleading terms in the pole expansion.}
resummed corrections: now the distortion parameters read

$$\Delta \tilde{M}_1^{\text{peak}} \approx -\frac{\pi}{8} \bar{\beta} \Gamma_Z (1 + \frac{\tilde{\beta}}{2}) = -111 \text{ MeV},$$

$$\kappa^{\text{peak}} \approx \left( \frac{\Gamma_Z}{M_Z} \right)^{\beta} (1 + \frac{3}{4} \bar{\beta}) (1 + \frac{5\pi^2}{48} \beta^2 - \frac{\pi^2}{32} \beta^3) = 0.74. \quad (20)$$

This is in perfect agreement with the observed exact and double-pole distortion parameters. The analogy with the rules of thumb derived for the Z line shape at LEP1 \[\text{(3)}\] confirms the relation between the FSR-induced distortion effects in double Z-resonance production and the ISR-induced distortion effects in single Z-resonance production at LEP1.

4 Some comments on the W line shape at LEP2

As has been shown in the previous section, the double-pole LLA constitutes a reliable framework for a gauge-invariant and universal description of FSR-induced distortion phenomena in double-resonance production. The essence of these phenomena is fully contained in the correction factor presented in \( \text{(18)} \), which applies to each individual distorted Breit–Wigner distribution. For two distorted distributions the effect is hence multiplicative. Consequently, the reduction factor for a double-invariant-mass distribution is given by the product of the reduction factors for the individual single-invariant-mass distributions. However, the shift in the peak position does not change in the presence of more than one resonance; it only depends on the decay products of the unstable particle that is investigated. The only differences between process \( \text{(1)} \) and the more realistic process of \( \text{W} \)-pair production at LEP2 are the resonance parameters \( (M_\text{W} = 80.22 \text{ GeV} \text{ and } \Gamma_\text{W} = 2.08 \text{ GeV}) \) and the fact that \( \bar{\beta} \) depends on the decay products of the decaying particle. For instance, the leptonic \( \text{W} \) decays involve only \textit{one} charged lepton instead of two. As a result, we should use \( \bar{\beta} \rightarrow \frac{2}{\pi} \left[ \ln(M_\text{W}^2/m_\ell^2) - 1 \right] \) for \( \ell = e, \mu, \tau \), which is scaled down by at least a factor of two compared with the Z-pair example. For \( \text{W} \) bosons decaying into an electron or positron, the resummed FSR distortion effects amount to a shift in the peak position of \(-45 \text{ MeV}\) and a peak reduction factor of 0.86 per distorted resonance (i.e. 0.74 for a double-invariant-mass distribution), as can also be read off from \( \text{(20)} \).

From the previous discussions it should be clear that FSR-induced distortion effects can be sizeable and should be taken into account properly in the Monte Carlo programs that are used for the \( \text{W} \)-mass determination at LEP2.

References

\[ ^2 \text{In more realistic event-selection procedures also a minimum opening angle } (\theta_0) \text{ between the lepton and photon might be required for a proper identification of both particles. The effect of this can be represented by using } \ln(4/\theta_0^2) \text{ instead of } [\ln(M_\text{W}^2/m_\ell^2) - 1] \text{ in the definition of } \bar{\beta}. \]
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