Polemic Notes On IR Perturbative Quantum Gravity

Ilya L. Shapiro

Departamento de Física, Universidade Federal de Juiz de Fora, Juiz de Fora, CEP: 36036-330, MG, Brazil

Abstract. Quantum gravity is an important and to great extent unsolved problem. There are many different approaches to the quantization of the metric field, both perturbative and non-perturbative. The current situation in the perturbative quantum gravity is characterized by a number of different models, some of them well elaborated but no one perfect nor mathematically neither phenomenologically, mainly because there are no theoretically derived observables which can be experimentally measured. A very interesting one is an effective approach which separates the low-energy quantum effects from the UV sector. In this way one can calculate quantities which are potentially relevant for establishing certain universal features of quantum gravity. In this presentation we give a polemic consideration of the effective approach to the infrared quantum gravity. We question the validity of the recent results in this area and also discuss how one can check the alleged universality of the effective approach.

Keywords: Quantum Gravity; Effective Quantum Field Theory; Quantum Corrections.

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1 Introduction

General Relativity and Quantum Field Theory are very successful theories of the most fundamental phenomena from the scale of elementary particles up to the scale of the whole Universe. The problem of creating the quantum theory of gravity, that means a quantization of the metric field, is in the agenda of theoretical physicists for more than 70 years [1] and nowadays we may be proud of many important achievements and ideas. However, there is no real solution of the problem yet, in part due to the theoretical difficulties but also because there are no experimental data which could help to distinguish more successful theories and models from the other ones. In this situation the advent of an idea of an effective approach to the infrared quantum gravity, [2] (we abbreviate it as IRQG in what follows) is very welcome, for it paves

E-mail address: shapiro@fisica.ufjf.br

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the way for the use of the conventional Feynman diagram technique to derive some concrete observables, such as quantum corrections to the Newton potential.

Despite the idea of such calculations was not really new, the proposal of Ref. 2 has a few very attractive features, namely:

1) Universality of the IRQG. This means that one does not need to know what is the fundamental theory of quantum gravity at high energy scale in order to calculate at least some of the low-energy observables. The higher derivative terms and corrections which are inevitable in the framework of semi-classical approach or in string theory, do not concern the IR sector, which is completely related to the Einstein-Hilbert term in the gravitational action.

2) The massive modes (degrees of freedom) of the gravitational field which may be present in an unknown fundamental theory, do decouple at low energies and we arrive at the quantum corrections which are due to the diagram with only graviton and (massive) matter field internal lines. The same is true for the possible high derivative interactions of the gravitational degrees of freedom, which are Planck suppressed and therefore irrelevant at low energy scale.

After the papers 2 were published, people readily noticed that, technically, the calculations presented there were not perfect. In particular, some diagrams were calculated incorrectly 4 and some relevant ones were missed. The last point was explained using the analog model based on scalar QED 5. Starting from the set of diagrams presented in the original publication 2, one ends up with the quantum correction to the Newton potential which depends on an arbitrary choice of the gauge-fixing condition, making all calculations senseless. The gauge-fixing invariance of the amplitudes is restored when all relevant diagrams, in the given loop order, are taken into account. This property has been also verified for IRQG, but unfortunately the corresponding work was never published 6.

Several calculations of the IRQG corrections to the Newton potential have been presented in the meantime 7,4,8,9. It is stated that there is a correct result for the IRQG improved Newton potential 9 (there is, however, a divergence with the alternative calculation in Ref. 8). This result for the quantum correction includes r^2 and r^3 (proportional terms which come from two different sets of diagrams: the ones which have only massless (graviton) internal lines give L-type contributions, which lead to the r^3 terms in the potential 2, while more complicated Q-type contributions come from the diagrams with two types of internal lines: both massless gravitons and massive (e.g. scalar) propagators are there. This kind of diagrams lead to both r^2 and r^3 type corrections. Let us mention that in one of the cited above calculations 7 the subject of quantization was only a metric field, but not the matter sources, which have the form of point particles. In this case the result for improved Newton potential is quite different, because only the L-type contributions are present. It is important that such "reduced" quantum corrections are gauge-fixing independent by themselves, without taking the Q-type contributions into account.

The purpose of the present communication is to critically discuss the main points of the standard IRQG approach. In particular, we present some arguments questioning the universality of the IRQG. In our opinion this issue is not completely clear and requires an explicit verification. Furthermore, we argue that only the easy to calculate L-type contributions have reasonable physical interpretation while the complicated Q-type ones should be perhaps disre-
garded. We present only qualitative arguments and postpone the corresponding calculations for the next occasion.

The paper is organized as follows. In the next section we discuss the choice of the diagrams and in section 3 consider the implications for the universality of the IRQG and how it can be, in principle, checked. In section 4 we draw our conclusions.

2 Which Diagrams Must Be Taken And Which Not?

In the rst paper on IRQG [2] there was a following important result: the sum of the corresponding terms from the \(Q\)-type diagrams reproduce the post-Newtonian limit of classical GR. It was correctly stressed that any other output would make the whole scheme of IRQG being rather suspicious, because the mentioned post-Newtonian correction is one of very successful tests of GR. However, the consequent analysis of quantum corrections has shown that the successful output of the original calculations was just a result of an unintentional wrong use of Feynman diagrams [4]. In particular, the most complicated diagrams were disregarded in [2]. As we have already mentioned above, without the full set of diagrams the quantum corrections are gauge xing dependent and the whole calculation has no sense [5]. Finally, in the mathematically correct result [3] there is no correspondence to the conventional post-Newtonian limit and also there is some intrinsic arbitrariness of the quantum corrections. One can think that something may be wrong either in the de nition of the quantum theory or in the interpretation of the results. The immediate conclusion can be, for instance, that gravity should not be quantized at all. Fortunately, as we shall see in a moment, we do not need to go so far.

Let us look at the situation from another viewpoint. We can remember, once again, that the \(Q\)-type diagrams consist of the loops with mixed internal lines content, that means there are, at the same time, massless graviton and massive matter lines. As a model for the matter it is usually taken a massive scalar eld. This model works well for the one-graviton exchange between the two masses \(m_1\) and \(m_2\), e.g., it is perfectly producing a Newton law in the non-relativistic regime. So, it looks natural to go beyond the tree-level approximation and try to evaluate loop corrections [3,2].

In the path integral interpretation of Quantum Field Theory the presence of massive matter lines in the internal part of Feynman diagrams means that the matter eld is a subject of functional integration, and it means that this eld must be quantized. In order to see this one can compare the two possible forms of the generating functionals of the Green functions

\[
Z_1[J;J] = \int Dg \ e^{ig \mathcal{S} + \frac{1}{2}g J + iJ} \quad (1)
\]

and

\[
Z_2[J;J] = \int Dg \ e^{ig \mathcal{S} + \frac{1}{2}g J + iJ} \quad (2)
\]

In both expressions we meet a functional integration over the metric, but in the rst case there is an additional integral over the matter eld. Only this integration enables one to have
propagators of this \( \mathbf{e}_I \) in the internal part of the loops. At the same time, the presence of the source \( J \) for the \( \mathbf{e}_I \) lets us to have the diagram with external lines of \( \mathbf{e}_I \) in both cases. From the other side, the functional integration over the \( \mathbf{e}_I \) is possible only if this \( \mathbf{e}_I \) is an elementary quantum object \([11]\) and not a classical source or a composite \( \mathbf{e}_I \), because the last should be treated in a different way.

The first option \([1]\) looks more general, the calculations of Ref. \([2]\) and most of the previous and consequent calculations (except Ref. \([3]\)) of the quantum corrections to the Newton potential were based on \([1]\), with being a scalar \( \mathbf{e}_I \). The bad news for this approach is that, after all, the macroscopic bodies (e.g. planets or stars, satellites etc) which take part in the phenomenon relevant gravitational interactions are not made from a scalar \( \mathbf{e}_I \). Much on the contrary, they do consist from a baryonic matter, that means interacting protons, neutrons and electrons. These particles are not elementary (except electron) and none of them may be properly described by a scalar \( \mathbf{e}_I \). Of course, nucleons consist from quarks and gluons, so one may think to replace the scalar \( \mathbf{e}_I \) by the spinor one and try to obtain the quantum gravity corrections taking, e.g., mixed graviton-quark diagrams. However, this would not be a right step, because quarks are not free particles. One of the manifestations of this fact is that, e.g., the total mass of the \( u, u \) and \( d \) quarks inside the proton is essentially smaller than the mass of the whole proton. Therefore, if we calculate such (even tree-level) diagrams with quarks we have no chance to arrive at the correct result. The same is true for the protons and neutrons, which are not free but instead interact within the nucleus. After all, the \( Q \)-type diagrams imply the quantization of macroscopic bodies, e.g., of the Earth, Moon or Mercury. It seems to us that such quantization is something odd with respect to the principles of quantum theory and therefore it should be better avoided. Finally, the most correct approach is to treat massive sources correctly, that means to regard them as massive macroscopic bodies which should not be quantized.

Finally, we arrive at the conclusion that the "correct" set of diagrams includes only the \( L \)-type ones. Indeed this point of view was already presented in Ref. \([7]\), where the corresponding quantum corrections were calculated through the functional method for the massive source which consists of the point particles. The analysis of these calculations shows that one can chose any other form of external source (e.g., scalar or fermion \( \mathbf{e}_I \)) without changing the result. Also, the same quantum corrections can be probably obtained through the \( L \)-type Feynman diagrams. These diagrams, with only graviton internal lines are in fact easy to evaluate. In fact, the quantum contributions coming from the massless diagrams are always easy to evaluate, because the IR non-local terms are dual to the UV divergences, which can be obtained, e.g., through the Schwinger-Dyson technique. The resulting quantum corrections will be only of the \( r^3 \) type and hence they are not very relevant from the phenomenological viewpoint \([2]\). However, the very fact we can separate the IRQG effects from the UV sector remains very interesting by itself.
3 Is The IRQG Really Universal?

The type quantum corrections to the Newton potential are very small, but the possibility to evaluate them in a unique and consistent way is exciting, especially taking into account the existing variety of the Quantum Gravity models. However, let us inspect whether the universality of the IRQG is a certain thing or it is only a hypothesis which has to be verified.

One can distinguish the possible corrections to the Einstein-Hilbert action in the following two kinds of theories: (super)string theory and the quantum theory of matter fields on curved gravitational background. In the first case the action of gravity is the low energy effective action which emerges after we quantize the fundamental object—a (super)string. Within a standard Polyakov approach [12] this effective action has a form of an expansion in the parameter $\lambda$. At the first order one meets the Einstein-Hilbert action with additional dilaton field and at the next orders there are higher derivative corrections to this action. After one obtains the low energy string effective action, the two additional operations are executed. From one side, one needs to compactify extra dimensions. Furthermore, it is custom ary to perform the Zweibach reparametrization of the metric [13] in order to make the higher orders in $\lambda$ terms free of the high derivative ghosts. Indeed, this operation is very ambiguous [14] but one can hope this feature disappears when using an exact result which is nonperturbative in $\lambda$. From the IRQG viewpoint, the Zweibach transformation means the graviton propagator can be derived from the Einstein-Hilbert action alone, without taking the higher derivative (that means higher order in $\lambda$) corrections into account. These corrections show up only in the vertices and are suppressed by the Planck scale. Therefore, in this case the scheme of IRQG works perfectly well and we have the desired universality of the quantum corrections.

One has to keep in mind that the known low energy quantum effects are described not by the string theory but by the quantum field theories, such as the Standard Model of particle physics or its generalizations. Following this pattern, in the presence of a gravitational field one has to use the formalism of quantum field theory (QFT) in curved space (see, e.g., Refs. [15,16] for the introduction and Ref. [17] for a recent review). The bad news for IRQG is that the standard formalism of QFT in curved space implies the formulation of a classical action of external metric and that this classical action includes dynamical higher derivative terms, like the square of the Weyl tensor $C^2 = C \cdot C$ and of the scalar curvature,

$$S_t = \frac{1}{16} \int d^4 x \sqrt{g} \left( \frac{1}{G} R + 2 \frac{1}{2} C^2 + \frac{1}{3} R^2 \right). \tag{3}$$

The fourth derivative terms are necessary for renormalizability of the theory and do not lead to the problem with unphysical massive spin-2 ghosts because in this approach we do not need to consider the S-matrix for the gravitational excitations. Therefore, despite the procedure of metric reparametrization similar to the one used in string theory [13] is possible, there is no reason to apply it, especially in view of serious ambiguities which follow from this procedure.

In the model of IRQG, however, we have to quantize the metric and therefore meet the usual problem of unphysical ghosts. In this situation one can apply the same logic as people use in the string theory: first calculate quantum corrections to any interesting physical observable starting from the higher derivative action (3) or from its superrenormalizable generalizations [18] and
then perform the transition to the "observable" metric via the Zweibach reparametrization. It is obvious that the higher derivative quantum gravity becomes a perfectly consistent theory within this approach. However, by the end of the day we meet the mentioned ambiguity related to the transformation [13].

Now, let us come back to the IRQG and see what are the implications of the general quantum gravity situation in this case. The low-energy quantum corrections depend on the following three elements: gravitational propagator, vertices of gravitational self-interaction and vertices of gravitational interaction with matter. The last does not depend on the presence of the higher derivative terms in the gravitational action. Next, looking at the action (3) it is clear that contributions of the higher derivative terms to the vertices of gravitational self-interaction are suppressed, in the IR, by the ratio of typical energy of the process to the Planck mass. Therefore the only potentially doubtful element of the IRQG technique is the propagator of the gravitational field.

Here we meet the following dilemma: since the graviton propagator has to be derived from the total action, how can we know that the low-energy sector of the propagator comes from the Einstein term and not from the fourth derivative term? The standard argument in favor of universality of GR as IRQG is covariance which means $P^{-g}^{R}$ term is the unique second derivative covariant term. However, the whole action (3) is covariant and this does not mean that the relevant at low energies part of the propagator has tensor structure of the Einstein term and does not depend on the fourth derivative ($P^{-g} c^2$ and $P^{-g} R^2$) terms.

It is well known that the propagator of the gravitational field depends on the gauge fixing conditions and, in case of the higher derivative quantum gravity, there are more degrees of freedom in the propagator than in the theory based on General Relativity. The theory (3) can be seen as describing the interaction of two different particles: massless graviton and massive (including spin-2) ghosts (we do not discuss the problem of unitarity here). Therefore, the relevant Feynman diagrams include the loops of massive components of the metric and also the mixed loops with both massless and massive internal lines. Therefore the problem of the quantum calculations in this theory is technically similar to the one addressed in Refs. [2,3,5,4,9,10,8]. One can consider the propagator of higher derivative gravity as an algebraic sum of the graviton propagator and the propagator of a massive unphysical ghost [19,20]. If we consider the diagrams which do contribute to, for example, quantum corrections to the Newton potential, these extra degrees of freedom will show up in the mixed loops, with both graviton and ghost internal lines. Using the method of [8,9], we can reduce these diagrams to the ones without ghost internal lines, but they remain relevant and therefore can contribute to the corrections to the Newton potential. It looks like the cancelation of the corresponding quantum contributions is required to provide the irrelevance of the higher derivative terms.

In other words, we have to check that the contributions of the higher derivative ghosts really decouple at low energies [21].

Finally, we can see that simple qualitative considerations do not ensure the universality of the IRQG and one needs an explicit verification. Such verification cannot be performed in the framework of the quantum GR and requires the analysis of some general theory, e.g., the one based on the action (3) or some of its alternatives [18]. This means one has to start from one
of the more general actions and calculate its low-energy prediction. If the main hypothesis of Ref. [2] is right, the effect of all terms except the ones of $\rho R$-term will be negligible in the IR. Practical realization of such calculations is possible, at least in the framework of the fourth derivative model [3], where we have an extensive experience of loop calculations [20,22,23,21]. However, what is actually requested for the IRQG purpose is much more complicated, because one has to go beyond the conventional minimal subtraction renormalization scheme, because one need to extract some relevant information about the finite part of the effective action.

4 Conclusions

We have considered some qualitative arguments concerning the model of IRQG. There are serious arguments in favor of restricting the set of relevant diagrams such that the massive matter source is not quantized. Also, if we quantize only the metric and look for the corresponding logarithmic corrections to the amplitudes, it is not obvious that the low-energy limit of the theory possesses the alleged universality. In order to ensure that this nice property really takes place one has to start from the theory more general than the GR, derive some observable in this framework and compare it to the one obtained within the quantum GR. The most important example of a more general gravitational theory is the model based on the action [3], because this form of the action is dictated by the renormalizability of the semi classical theory.

On the other hand, there is a possibility to define both semi classical theory and perturbative quantum gravity with an additional procedure of metric reparametrization, which must be performed after the quantum corrections are calculated, in the same way as it is done in the string theory [13]. In this way one can construct a theory of quantum metric which would be both renormalizable and free of high derivative ghosts. However, there is a serious price to pay. The disadvantages of this scheme are the noncanonical use of the quantum field theory procedure and, also, a vast ambiguity in the quantum corrections.

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