Photonic interface for long-distance entanglement of logical-qubits

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A scalable fault-tolerant quantum-computer hardware with current noisy intermediate-scale quantum (NISQ) devices requires the juxtaposition of different types of quantum systems. In this sense, long-distance entanglement of stationary error-corrected logical qubits requires a photonic bus facilitating inter/intra-connection among the cores of quantum processors, the units of quantum memories, and the worldwide quantum internet. This article proposes a photonic interface for 4 and 6-qubit encoding of surface-code logical-qubits in an atomic-lattice platform. Accommodating the lattice inside a cavity, the gate emits photonic-qubits that are entangled by the logical-qubits. The entangling mechanism is provided by the Fermi scattering of a Rydberg electron from the plaquette atoms trapped in a qubit-dependent lattice. Therefore, different arrangements of logical-qubits derive the central atom over distinguished eigen-states, featuring photon emission at the early or late times distinguished by quantum interference. Finally, entanglement swapping of two emitted photons would make the far separated plaquettes entangled in the logical basis.

I. INTRODUCTION

The development of quantum computing architectures with current NISQ devices requires clustering the error-corrected quantum processors. The correction capabilities come with encoding the logical qubits in multiple physical qubits and are protected by error-correction codes [1, 2]. Clustering requires the ability to interconvert the logical and the flying qubits, which has remained a challenge. This photonic interface is valuable for clustering the intermediate-size fault-tolerant processors [3], connecting them with the quantum memories [4, 5], and with the worldwide quantum internet [6].

The most promising approach towards quantum error correction is provided by topological codes, such as the surface code [7, 8] that only requires nearest neighbor interactions in two-dimensional (2D) architectures [9, 11]. This article proposes a photonic interface for the 4-qubit encoding surface-code [12] and discusses the possible extension to the 6-qubit [13] version. The distance-2 surface code, uses four physical qubits to encode logical computational basis as

\[
\begin{align*}
|0\rangle_L &= \left(|0000\rangle + |1111\rangle\right)/\sqrt{2}, \\
|1\rangle_L &= \left(|0101\rangle + |1010\rangle\right)/\sqrt{2}.
\end{align*}
\]

(1)

A logical qubit is a highly entangled two-dimensional subspace in the larger Hilbert space of multiple physical qubits. Hence, the logic gates are performed by an overload of operations at the level of physical qubits and require costly techniques such as complex optimization [14, 15], magic state distillation [16], transversal gates [1] and lattice surgery [17, 19]. Using system-specific properties significantly reduces the number of operations on the physical qubits and hence the errors incurred during execution. Laser excited Rydberg atoms are an ideal example where the long-range interactions provide the possibility of simultaneous operations on multiple qubits [20, 26], with the bonus opportunities in quantum optics [27, 29]. However, simultaneous multi-qubit operation requires Rydberg population in all interacting qubits. This would affect the Rydberg stabilizer operations [23] due to unwanted cross-talk between physical qubits with demolishing effects on the logical encoding.

This article proposes a photonic interface for logical qubits. The scheme is based on cavity QED photon emission from an auxiliary centred atom, conditioned on the logical qubit encoded on the surrounding plaquette atoms. The single-step logical operation is performed by Fermi scattering of central atom’s Rydberg electron from the plaquette atoms [30, 32] in a spin-dependent lattice [19, 28]. Therefore, the logical-qubit determines which eigenstate the system would follow, either containing or excluding the atom-cavity coupling. As a result, the logical-qubit would get entangled with the time-bin photonic qubit emitted by the central atom, generating the entangled state

\[
(|\text{early}\rangle|0_L\rangle + |\text{late}\rangle|1_L\rangle)/\sqrt{2}.
\]

(2)

Subsequent projective Bell state measurement on the emitted photons from two far separated logical qubits in time-bin basis [7, 8, 44, 46–49], makes the logical qubits entangled. The time-bin qubit is an ideal choice due to intrinsic robustness against phase fluctuations [8], with new advances in high-fidelity operations [9]. In terms of the multi-qubit operation, unlike the dipolar scheme, the current proposal operates via a sole auxiliary Rydberg atom, closing the unwanted cross-talk of physical qubits over the logical operation. Furthermore, the Rydberg-Fermi interaction provides a molecular type potential that eliminates the frozen gas regime requirement of the Rydberg dipolar schemes and also operates at much shorter interatomic distances, addressing the scalability.
FIG. 1. Long-distance entanglement in logical basis. (a) The auxiliary atom at the centre of plaquette is emitting a photon at the early or late time, depending on the number of plaquette atoms in the Rydberg wave-function. Subsequent projective measurement of photons in the Bell state basis entangles far-separated plaquette atoms in logical basis. (b) The level scheme of the auxiliary atom. The lambda configuration consists of atom-laser and atom-cavity couplings and is responsible for photon emission. The interaction-induced level-shift of the Rydberg state depends on the plaquette’s spin-state $S_p$. The $|0_L⟩$ logical state is associated with $S_p = 0$ and 4, making $Ω_r$ laser out of resonance with the Rydberg level. Hence, the Raman transition from $|s⟩$ to $|e⟩$ would generate a single photon at the [early] time. On the other hand, $|1_L⟩$ logical state accommodates two plaquette atoms in Rydberg wave-function $S_p = 2$, making the $Ω_r$ laser in-resonance with the Rydberg level. Consequently, destructive interference inhibits the transition to $|e⟩$ and therefore blocks the early photon emission. The following exclusive $Ω_s$ pulse shown in (c) leads to [late] photon emission in case of $|1_L⟩$ state.

II. SCHEME

The setup consists of cesium $^{133}$Cs atoms trapped in a 2D atomic lattice accommodated in a cavity, see Fig. 1h. The system atoms are placed on square plaquettes consisting of physical qubit states $|0⟩ = |6S, F = 3⟩$ and $|1⟩ = |6S, F = 4⟩$, while auxiliary atom responsible for conditional photon emission is at the centre of the plaquettes with the electronic level scheme presented in Fig. 1j. The lower lambda configuration is responsible for on-demand single-photon emission, similar to $^{51,52}$. The $Λ$ transition would be (allowed) prohibited by the (un)resonant blue laser controlled by quantum interference $^{53}$. This laser tuning is determined by the Fermi scattering of the Rydberg electron from the plaquette atoms, designed to make the photon emission conditioned on the logical qubit. In the lower lambda system the transition between $|s⟩ = |6S_{1/2}, F = 3⟩$ and $|p⟩ = |7P_{3/2}⟩$ states is derived by $Ω_s$ laser while the $|p⟩$ to $|e⟩ = \{6S_{1/2}, F = 4⟩$ or $5D_{3/2}⟩\}$ is governed by the Jaynes-Cummings interaction with the coupling constant $g$ and the cavity mode with frequency $ω_c \approx \{450, 1300\} \text{nm} \,^{54}$. Both transitions are detuned from the intermediate state by $Δ$. The state basis $|i, n⟩$ in Fig. 1b includes the central atom’s electronic state $|i⟩$ and cavity number state $|n⟩$. Over a Raman $π$ pulse, the transition from $|s, 0⟩$ to $|e, 1⟩$ would generate a photon in the cavity. The decay of the cavity mode causes single-photon emission and the system settles itself in the state $|e, 0⟩$.

Fermi scattering of central atom’s Rydberg electron from plaquette atoms in a spin-dependent lattice results in an effective level-shift quantified by the plaquette spin $S_p = \sum_{j∈p}σ_{00}^{(j)}$, with $σ_{00} = |0⟩⟨0|$ being the projective operator and $j$ goes over the plaquette atoms, see Fig. 1f and Sec. III. Having logical qubit $|0⟩_L = (|0000⟩ + |1111⟩)\sqrt{2}$, two plaquette atoms would be in Rydberg electron wave-function. Consequently, the blue laser would get in resonance with the Rydberg level, blocking the photon emission over the first $Ω_s$ pulse. The remained $|s, 0⟩$ population would then get transferred over the second $Ω_s$ pulse (not accompanied by the Rydberg lasers $Ω_r$) and hence emits a late photon [late], leading to the desired state of Eq. 2. The time ordering of the pulses is shown in Fig. 1c, and the physics behind the blocking and transmission are discussed in Sec. IV. In an entanglement swapping station, the emitted photons from distinct sources would undergo a C-NOT gate followed by projective Bell state measurement. This would make the two far separated plaquettes entangled in the logical basis $^{55}$.

III. RYDBERG-FERMI INTERACTION

To apply a qubit-dependent Rydberg-fermi interaction, the lattice undergoes spin-dependent shift along the $z$ direction. Hence, the Rydberg electron of the central atom would only scatter from the plaquette atoms in qubit state $|0⟩$. This would cause a level-shift that depends on the plaquette spin $S_p = \sum_{j∈p}σ_{00}^{(j)}$.

The spin-dependent lattice shift along $z$ direction is formed by counter-propagating linearly polarized lights as depicted in Fig. 2. Having a relative shift between the fields’ polarizations $2θ$, the total electric field could be written in terms of the sum of right and left circularly polarized lights $E = E_0 \exp(-ivt)(π^+ \sin(kz + θ) + π^- \sin(kz − θ))$. To make a spin-dependent lattice-shift, the spin polarizabilities should be linked to different circular polarization components of lights $^{12}$. To cancel the polarizabilities with unwanted light elements shown by dashed lines in Fig. 2, the trapping laser must be tuned between $P_{3/2}$ and $P_{1/2}$ states so that the ac-Stark shifts of these two levels cancel each other. As a re-
(b) \( \theta \) \nolimits
\( x \ yz \)
(a)
(c)
6S_{1/2}
6P_{1/2} \pi^+ \nolimits
6P_{3/2} \pi^- \nolimits
(d) (e)
|0\rangle \nolimits
|1\rangle \nolimits
Z \text{ (nm)} \nolimits
Y \text{ (nm)} \nolimits
X \text{ (nm)} \nolimits
X \text{ (nm)}

FIG. 2. Rydberg-Fermi interaction in spin-dependent lattice. (a) In a 2D lattice with a single atom per site, applying a qubit-dependent lattice-shift along \( z \) makes each atom in a spatial superposition of being in red and green sites where the components are controlled by the internal electronic qubit-states \( |0\rangle \) and \( |1\rangle \). (b) Counter propagating linearly polarized lights along \( z \) direction with relative polarization shift of \( \theta \) forms two standing-waves of \( \pi^- \) and \( \pi^+ \) circular polarizations. (c) Tuning the trapping laser between \( 6P_{1/2} \) and \( 6P_{3/2} \), the polarizability of qubit states \( |0\rangle \) and \( |1\rangle \) are given by the distinctively circularly polarized lights \( \pi^- \) and \( \pi^+ \) respectively, resulting to spin-dependent trapping. The cross-sections of \( \{45D_{5/2,5/2}\} \) Rydberg wave-function along (d) \( xy \) and (e) \( xz \) are plotted. The position of the qubit states \( |0\rangle \) and \( |1\rangle \) are marked by the red and green circles. (e) Note to the spin-dependent overlap of plaquette atoms with the Rydberg electron wave-function.

Thus the \( m_j \) = \pm 1/2 levels of the ground state would be trapped by \( V_z = \alpha_1|E_0|^2\sin(kz \pm \theta) \) respectively. The hyperfine qubit states \( |0\rangle \) \( |F = 3, m_F = 3\rangle \) and \( |1\rangle \) \( |F = 4, m_F = 4\rangle \) would experience \( V_0 = (V_+ + 3V_-)/4 \), \( V_1 = (V_+ - V_-)/2 \).

The Rydberg laser \( \Omega_r \) is the auxiliary atom at the center of the desired plaquette, see Fig. 2. Applying a spin-dependent lattice shift perpendicular to the 2D lattice would result in dual spin/spatial encoding of the plaquette qubits. Thus, exciting the central atom to Rydberg level, the electron wave-function would exclusively overlap with plaquette qubits in spin-state \( |0\rangle \), see Fig. 2. The resulting Rydberg-Fermi interaction caused by scattering of Rydberg electron from a single neutral atom would be quantified by \[ V_{RF} = \frac{2\pi \tan(\delta_\sigma)}{k(R)} - 6\pi \frac{\tan(\delta_\rho)}{k^3(R)} \delta(r - R) \] with \( r \) and \( R \) being the positions of the Rydberg electron and a ground state atom with respect to the ionic core, and \( \delta(\xi, \rho) \) are the triplet s- and p-wave scattering phase shift of Rydberg electron from a neighboring ground state atom \( S_p \). Since \( S_p \) presents the number of plaquette atoms in the Rydberg wave-function of central atom, the effective level-shift caused by the Fermi scattering would be \( S_p V_{RF} \), see Fig. 1b.

IV. ENTANGLING GATE

This section discusses the main physics behind the conditional photon emission that entangles the stationary logical and flying qubits. In the case of \( |1_L\rangle \), the physical qubits’ arrangement in the qubit-dependent lattice would accommodate two plaquette atoms in the Rydberg wave-function of the central atom, making the \( \Omega_r \) laser in-resonance with the shifted Rydberg level \( \delta_{RF}(|1_L\rangle) = 0 \). Having \( |0_L\rangle \), the plaquette’s arrangement contains either \( S_p = 0 \) or \( 4 \) atoms in the Rydberg wave-function, resulting in the effective detuning of \( \delta_{RF}(|0_L\rangle) = 2V_{RF} \). The Hamiltonian of the central atom in the electronic basis is given by

\[ H = \Omega_s/2(\sigma_\rho + \text{h.c.}) + \Omega_R/2(\sigma_\rho + \text{h.c.}) + g/2(\sigma_\rho + \text{h.c.}) + \Delta \sigma_\rho + \delta_{RF} \sigma_\rho \]

where \( \delta_{\alpha\beta} = |\alpha\rangle\langle\beta| \). To simplify the analytic discussion here the large detuning regime \( \Delta \gg \Omega_s, g \) and equal couplings \( g = \Omega_s \) are considered. After adiabatic elimination of the intermediate state \( |p\rangle \), the Hamiltonian is represented in new basis \( |\pm\rangle = (|s\rangle \pm |e\rangle)/\sqrt{2} \) and \( |r\rangle \) as

\[ H/\epsilon = \lambda^2|\rangle\rangle + |1 - \delta||r\rangle\rangle + \lambda(|+\rangle(r) + \text{h.c.}) \]

where \( \lambda = \Omega_s/\Omega_r \) is the dimensionless Rabi frequency and \( \delta = \delta_{RF}/\epsilon \) is the Rydberg-Fermi interaction scaled by \( \epsilon = \Omega_s^2/4\Delta \). In the regime of \( \lambda, \delta \ll 1 \) the lower bright \( |b\rangle = |1 - \delta||+\rangle - \lambda|r\rangle\rangle/\sqrt{1 - \delta^2 + \lambda^2} \) state, with the energy \( 2\delta\lambda^2 \) would interfere with the dark state \( |d\rangle = |1\rangle \) having \( |\rangle\rangle \). Having \( |1_L\rangle \), the Rydberg laser would be in resonance \( \delta = 0 \). Hence, the system would follow the new dark state \( |D\rangle = (|d\rangle + |b\rangle)/\sqrt{2} = (|s\rangle - \lambda|r\rangle)/\sqrt{1 + \lambda^2} \) featuring destructive interference that blocks the transition to \( |e\rangle \) state. A Ryd-Fermi induced level-shift of \( \delta = \delta_{RF}/(\Omega_s^2/4\Delta) \) \( \gtrsim 1 \) lifts the \( |d\rangle \) and \( |b\rangle \) states degeneracy, the system would exclusively follow the dark state \( |d\rangle \) and emits an early photon, see Fig. 3b.

While a large cavity decay rate \( \Gamma_c \) enhances the emission rate and narrows the emission probability profile, it reduces the coherence. In the regime of \( \lambda = \Omega_s/\Omega_r \ll 1 \) the steady-state of the master equation could be obtained perturbatively as \( \rho = \rho_0 + \lambda \rho_1 + \lambda^2 \rho_2 \). The desired transition coherence is given by

\[ \rho_{se} = \frac{\Omega_s g}{4\Delta} \frac{2i\delta_{RF}}{\Gamma_c(\Omega_s^2/4\Delta - \delta_{RF})} \]

Cavity decay rates \( \Gamma_c \) larger than effective transition Rabi frequency \( \Omega_{eff} = \Omega_s g/4\Delta \) reduces the coherence between \( \rho \) and \( |e\rangle \). Since an incoherent superposition of these states does not exclusively project into the dark state \( |d\rangle \), some of the population would transfer to bright states.
VI. EXTENSION TO SIX-QUBIT ENCODING

The current proposal could be extended to the six-qubit code [13], with the logical basis being the odd and even parity states of the $Z = IZIZIZ$ logical operator [62]. Exciting Rydberg superposition state $R_{\text{opt}}(t)(Y_2^Z(\theta, \phi) + Y_2^{-Z}(\theta, \phi))/\sqrt{2}$ could realize the desired parity-photon entangling gate in a triangular lattice, see Fig. 4c. Fermi scattering of central atom’s Rydberg electron from every other site in a hexagonal plaquette structure is conditioned on the physical qubits to be in $|0\rangle$ state. This would cause an effective level-shift quantified by three atoms’ spin-number, see Fig. 4d. The two-color $\Omega_r$ in Fig. 4d are tuned in-resonance with odd-parity of the $Z$ stabilizer. The two-color blue transitions could be obtained by a single laser in a setup of beamsplitters and acusto-optical modulators. With even parity of $Z$, the blue laser would be out of resonance with the Rydberg level. Hence, the first $\Omega_s$ pulse in Fig. 1c derives an early photon emission (early). Having an odd number of spin-downs in the three targeted atoms, makes one of the blue lasers in resonance with the Rydberg level, while the other laser causes a level-shift to $|p\rangle$ state resulting in a small correction in the detuning value $\Delta \equiv \Delta \pm \Omega_s^2/4\Omega_{\text{RF}}$. The resonant laser would block the transition over the first $\Omega_s$ pulse as discussed in Sec. [14]. The remained $|s, 0\rangle$ population would then transfer over the second $\Omega_s$ pulse (not accompanied with $\Omega_r$) and hence emits a late photon $|\text{late}\rangle$, providing the desired state of Eq. 2 in the 6-qubit encoding.

VII. CONCLUSION AND OUTLOOK

This article proposes a hardware architecture and protocol for long-distance entanglement generation between logical qubits encoded on atomic-lattice within cavities. The Fermi-scattering of the Rydberg electron from the
FIG. 4. Photonics interface for 6-qubit surface code. The physical qubits are dual encoded in the spin/spatial basis of a spin-dependent lattice placed over the hexagonal structure. (a) The logical operator \( Z = \sigma_z \) is implemented by exciting the auxiliary atom at the centre of hexagonal plaquette to the Rydberg superposition state with angular part \( Y_\theta^z(\theta, \phi) + Y_\phi^{-1}(\theta, \phi)/\sqrt{2} \). (b) The energy splitting of the Rydberg state depends on the number of atoms in the Rydberg orbitals \( S_p = \sum_{i=2,4,6}^\infty a_i \). The two-colored Rydberg lasers \( \Omega_s \) get in resonance with the odd parity of \( Z \), associated with \( |1\rangle \), blocking the early photon emission.

\[ |p,0\rangle \Omega_s \Omega_r |s,0\rangle |r,0\rangle |e,1\rangle \gamma \ |e,0\rangle \Gamma - \Delta \]

\( \Delta = 0 = 1 = 2 S_p = 3 \)

\[ xyz \]

\[ \text{plaque}t\text{t atoms in a spin-dependent lattice forms a level-shift that is quantified by the logical qubit. This level-shift would control the central atoms coupling with the cavity mode and hence the ordering of photon emission would be entangled by the logical qubit. The two characteristic of the Rydberg-fermi interaction in working at small interatomic constant and in making bound state with restoring force allows long-time operation in ultra-dense atomic processors, making a significant advantage over the Rydberg dipolar counterparts. The proposed photonic quantum bus solution, promises the scalability of fault-tolerant processors at the current NISQ-era devices. This would resemble the integrated circuits (IC) technology of silicon-based processors characterized by Moore’s law [83]. From a fundamental perspective, the presented technique of entangling multiple physical qubits in the stabilizer basis facilitates the investigation of phenomena, properties, and protocols that arise in quantum information over a wide dimension of state space, while operating on the basis that is growing polynomially.} \]

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The logical basis states for the six-qubit code \textsuperscript{13} are:

\[ |0\rangle_L = |000000\rangle - |100111\rangle + |001111\rangle - |101000\rangle - |011011\rangle + |111100\rangle, \]
\[ |1\rangle_L = |001100\rangle + |101011\rangle + |000011\rangle + |100100\rangle - |011000\rangle - |111111\rangle + |010111\rangle + |110000\rangle, \]

which are the odd and even parity of logical operator \( \bar{Z} = IZIZIZ \).

\[ \text{SUPPLEMENTAL DOCUMENT: PHOTONIC INTERFACE FOR LONG-DISTANCE ENTANGLEMENT OF LOGICAL-QUBITS} \]

This section discusses the steps in calculating the potential energy curves (PEC) under the Rydberg-Fermi interaction \[1\textsuperscript{3},2\textsuperscript{3} \]

\[ V_{RF} = (2\pi \frac{\tan(\delta_s)}{k(R)} - 6\pi \frac{\tan(\delta_p)}{k^3(R)} \nabla_r \cdot \nabla_r \delta(r - R)). \]  

The coupling of the Rydberg state \(|45D_{5/2}, 5/2\rangle\) with the neighbouring states \(|43H + 45D_{3/2} + 46P_{1/2, 3/2} + 47S_{1/2}\rangle\) is considered at the position of neighbouring lattice sites \(R\). Here \(|nH\rangle = \sum_{l,m} |n,l,m\rangle\) represents the Hydrogen state encountering semi degenerate orbital angular momentum numbers \(2 < l < n\). The matrix elements in the manifold of coupled states are given by

\[ H_{nlm,n'lm'}(R) = \langle \psi_{nlm}(R)|V_{RF}|-\psi_{n'l'm'}(R) \rangle \]
\[ H_{nlm,nlm} = -\frac{Ry}{n^*} \]

where \(Ry\) is the Rydberg constant of \(Cs\) atoms and \(n^*\) is the effective Rydberg principal number. Diagonalizing 8000 coupled states, the energy potential is plotted in Fig. 5. In the UV optical-lattice with \(\lambda = 350\text{nm}\), the Fermi scattering of Rydberg electron from the neighbouring lattice site would result in 400MHz level-shift of the Rydberg level ideal for fast quantum operations.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{PEC_Cs.png}
\caption{PEC with S- and P-wave scattering in Cs atoms. Here the coupling of the Rydberg state \(|45D_{5/2}, 5/2\rangle\) with the neighbouring states \(|43H + 45D_{3/2} + 46P_{1/2, 3/2} + 47S_{1/2}\rangle\) is considered under Eq. 8. Interaction strength is plotted across radial direction with \(\theta = \pi/2\).}
\end{figure}

For the p-wave scattering of Rydberg electron from the neighbouring ground state atom, the gradient of the Rydberg wave-function \(\psi = R_{nl}(r)Y_{l}^{m}(\theta, \phi)\) at the position of the neighbouring lattice site is required which is

\[ \nabla \psi(r, \theta, \phi) = \begin{bmatrix}
\frac{\partial R_{nl}}{\partial r} Y_{l}^{m} \\
\frac{1}{r} \frac{\partial R_{nl}}{\partial \theta} Y_{l}^{m} \\
\frac{1}{r \sin \theta} \frac{\partial R_{nl}}{\partial \phi} Y_{l}^{m}
\end{bmatrix} = \]

\[ \text{[62]} \quad \text{The logical basis states for the six-qubit code \textsuperscript{13} are:} \]

\[ |0\rangle_L = |000000\rangle - |100111\rangle + |001111\rangle - |101000\rangle - |011011\rangle + |111100\rangle, \]
\[ |1\rangle_L = |001100\rangle + |101011\rangle + |000011\rangle + |100100\rangle - |011000\rangle - |111111\rangle + |010111\rangle + |110000\rangle, \]

\[ \text{which are the odd and even parity of logical operator \( \bar{Z} = IZIZIZ \).} \]

\[ \text{[63]} \quad \text{G. E. Moore, Electronics 38, 114 (1965).} \]
where the rotated Bell states are obtained by applying a Hadamard on the second element i.e. |\psi\rangle = \frac{1}{\sqrt{2}} \left[ \sqrt{m+1}(\theta, \phi)e^{-i\phi} - (l + m + 1)Y_l^{m+1}(\theta, \phi)e^{i\phi} \right]

in the spherical coordinate. The radial wave-function and its derivative are calculated numerically using Numerov technique [4].

S2. ENTANGLEMENT SWAPPING

Entangling far separated atomic qubits in the logical basis requires Bell state projective measurement of the cavity emitted photons. The proposed Rydberg-Fermi cQED gate in this article would provide atom-photon entanglement in each cavity setup, see top line of Eq. [11]. The state could then rearranged in a separate logical and photonic qubit pairs in the following line

\[ |\psi\rangle = \frac{(|1_L\rangle|late\rangle + |0_L\rangle|early\rangle)}{\sqrt{2}} \]

where the rotated Bell states are obtained by applying a Hadamard on the second element i.e. |\hat{\phi}^\pm\rangle = |1\rangle_1|+\rangle_2 \pm |0\rangle_1|−\rangle_2 and |\tilde{\psi}^\pm\rangle = |0\rangle_1|+\rangle_2 \pm |1\rangle_1|−\rangle_2 where |\pm\rangle = (|1\rangle \pm |0\rangle)/\sqrt{2}. The photonic states follow the same presentation format in the early and late basis. Applying a CZ gate on the photonic states [5] written in the four rotated Bell states would result in

\[ CZ_p|\psi\rangle = (|++\rangle_p|\tilde{\psi}^+\rangle_L - |−−\rangle_p|\tilde{\psi}^-\rangle_L - |−+\rangle_p|\tilde{\psi}^+\rangle_L - |+-\rangle_p|\tilde{\psi}^-\rangle_L)/2. \] (12)

Subsequent projective measurement of the photonic pair in the Bell basis [6] guarantees the entanglement of logical qubits. The time-bin qubit has been widely used for entanglement swapping [7] due to intrinsic robustness against phase fluctuations [8], with new advances in high-fidelity operations [9].

APPENDIX D: PRESERVING THE GROUND MOTIONAL STATE

Over the gate operation, the Rydberg-Fermi potential modifies the optical trapping experienced by the plaquette atoms in |l\rangle Wannier states. To preserve the ground motional state, it is important to apply the \(\Omega_s\) adiabatically. Over the operation, plaquette atoms in |l\rangle Wannier state would experience trap evolution \(U_{trap} = U_{op} + \frac{P_r}{(\hbar)^2}Y_{RF}\) where \(U_{op}\) is the optical trap, \(P_r = (\hbar)^2\) is the Rydberg population of the central atom. As long as the dynamic is adiabatic, i.e. \(\dot{\omega}_{trap} < \omega_{trap}^2\) [10], the Wannier states of the |l\rangle can adapt continuously and stays close to the instantaneous ground motional state. For the setup discussed in the main text, the \(\Omega_s\) pulses as short as 0.5\(\mu s\) could be designed to preserve the adiabaticity. This is significantly shorter than the Gaussian pulse with \(\sigma = 3.5\mu s\) considered in the main text. Also, note that applying the dark-state optical-lattice with better site confinement [11], allows faster adiabatic operations.

S3: CLOSING THE RYDBERG-MOLECULE DECOHERENCE CHANNEL

In the Bose-Einstein condensate (BEC), the Fermi scattering of Rydberg electron from a free ground state atom could enhance the decoherence rate [12]. This decoherence is due to attractive Rydberg-Fermi potential close to the ionic core, which moves the two interacting atoms to a very small separation of about 2 nm, where the binding energy of the molecules can ionize the Rydberg electron and form a Cs\(^+\) molecule [13]. Confining atoms in the lattice, the interatomic separation in this scheme is tuned to be at the last lobe of Ryd-Fermi interaction, preserving the interatomic distance. Without the mass transport, step-wise decay or ionization of the Rydberg atom is ruled out. This is because the Rydberg-Fermi binding energy at 170nm lattice constant used in this paper is orders of magnitude smaller than the closest Rydberg levels for the principal numbers applied here. The ion-pair formation is also highly unlikely in this system [13].
VIII. A. 3: ADDRESSING INDIVIDUAL SITES

Considering the small lattice constant, single site addressing would be challenging. There are two approaches to avoid laser cross talk. In the first approach, the sub-wavelength localized population rotation via semi interferometer techniques [14][15] could be used to improve single site addressing.

In the second approach, a dual species optical lattice of $^{133}\text{Cs}$ and $^{87}\text{Rb}$ is considered [10][18]. The trapping in the lattice plane is formed by 820nm linearly polarized light that is blue and red detuned for $^{133}\text{Cs}$ and $^{87}\text{Rb}$ and hence they would be stored on the nodes and antinodes of the standing wave respectively, see Fig. 6c,d. The qubit-dependent trapping perpendicular to the lattice plane is formed by a 870nm standing waves of left and right circularly polarized lights as depicted in Fig. 2b of the main text. The $|0\rangle$ qubit state of Rb (Cs) would be trapped on the antinodes of $\pi^{-}$ ($\pi^{+}$) standing wave and $|1\rangle$ gets trapped on the opposite polarization lattice, see Fig. 6a,b,e. The physical qubits are encoded in the Cs atoms placed on the plaquette while the central auxiliary atoms are Rb. This arrangement optimizes the interaction since the electron scattering from Cs atoms features resonance at lower kinetic energies close to last lobe [29]. The corresponding Rydberg-Fermi interaction over a single Cs site would be $|0\rangle V_{RF}|0\rangle/2 = 50\text{MHz}$. Considering the adjusted $\Omega_{s} = 428\text{nm}$ laser for the new Rb central atom, focusing the laser beam to a single site with NA=0.8 microscope [30], the probability of photon emission from the neighboring Rb atoms after an $\Omega_{s}$ pulse would be 1%. The Rydberg-Fermi interaction PEC of $|48D_{5/2}, 5/2\rangle$ Rydberg state targeting the new inter-atomic separation of 205nm is plotted in Fig. 6f.

Finally it should be noted that the spin-dependent lattice has been widely studied [19][28] and the coherence of the system has been tested via the atomic interferometer techniques [21][26][27].

FIG. 6. Dual-species spin-dependent lattice. (a) The 870nm trapping laser perpendicular to the lattice plane is aligned between $6P_{3/2}$ and $6P_{1/2}$ for Cs, hence the polarizability of qubit states $|0\rangle$ and $|1\rangle$ are given by distinguished circularly polarized lights $\pi^{-}$ and $\pi^{+}$ respectively. (b) the same polarization elements traps opposite qubit states in Rb. (c,d) For in-plane trapping the 2D standing wave is formed by 820nm linearly polarized light trapping both Cs and Rb at the nodes and anti-nodes of the standing wave via blue and red detuned transitions. The resulting qubit-dependent arrangement is depicted in (e) where the green and red ovals present the position of $|0\rangle$ and $|1\rangle$ qubit states respectively. (f) PEC with S- and P-wave scattering of the Rb atom’s Rydberg electron from a ground state Cs atom. Here the coupling of the Rydberg state $|48D_{5/2}, 5/2\rangle$ with the neighbouring states $|48H + 48D_{3/2}/2, 49P_{1/2,3/2} + 50S_{1/2}\rangle$ is considered under Eq. 8 Interaction strength is plotted across radial direction with $\theta = \pi/2$ being perpendicular to the lattice plane.

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