Robust Model Predictive Control for Takagi-Sugeno Model with Bounded Disturbances – Pólya Approach

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ABSTRACT This paper proposes a general robust model predictive control (MPC) approach for the constrained Takagi-Sugeno (T-S) fuzzy model with additive bounded disturbances. We adopt the homogeneous polynomially parameter-dependent (HPP) Lyapunov matrix with the arbitrary complexity degree and the corresponding HPP control law for the controller design. By applying the Pólya’s theorem and the extended nonquadratic boundedness property, a systematic approach to construct a set of sufficient conditions for assessing robust stability described by parameter-dependent linear matrix inequalities (LMIs) is established. The proposed approach is an improvement over existing approaches in terms of control performance and stabilizable model range. Numerical examples are provided to show the effectiveness of the proposed robust MPC approach.

INDEX TERMS Robust model predictive control, T-S fuzzy model, bounded disturbances, extended nonquadratic boundedness

I. INTRODUCTION

Takagi-Sugeno (T-S) fuzzy model has been widely used to approximate or even exactly represent nonlinear systems, whose basic idea is transforming the original system into a family of linear submodels [1]–[3]. For stability analysis of T-S model, many efforts are made based on the Lyapunov function, such as the common quadratic Lyapunov function in [4], the parameter-dependent quadratic Lyapunov function in [5], the piecewise-quadratic Lyapunov function in [6], the nonquadratic Lyapunov function in [7], and barrier Lyapunov functions in [8], [9]. To further improve the performance and reduce the conservatism, a general nonquadratic stabilization conditions are presented by the multiple-parameterization approach in [10]. In [11], [12], other general forms of relaxed stabilization conditions are derived by means of affine parameter-dependent Lyapunov functions. More details are available in [2], [3].

Model predictive control (MPC), as a widespread control technique being implemented in a receding horizon fashion, has advantages in constraints handling for multi-variable plants, such as distributed MPC [14], industrial hierarchical MPC [15], and stochastic MPC [16]. Usually, at each sampling interval, MPC solves an optimal control problem, with the performance index being associated with the system evolutions over a prediction horizon, subject to physical constraints, where a sequences of control moves are treated as decision variables, but only the first control move among this sequence is implemented on the actuator. These actions are repeated in a receding horizon fashion. Since the future predictions for input/state/output are needed, which are obtained based on the system dynamic model, the accuracy of this model is crucial for the future prediction, which as a result, can influence the control performance [13].

The stability analysis and control synthesis for T-S fuzzy model by MPC approaches have been studied with variety. In [17], an interval type-2 fuzzy MPC approach is proposed for nonlinear networked systems. The model and controller are not required to share the same lower and upper membership functions. In [18], to improve the performance, a local stability approach is applied and an estimation of the domain of attraction is provided. The work of [19] investigates the robust fuzzy MPC, which uses the nonlinear local models. More relaxed results are achieved, and on-line computational cost is significantly reduced. The authors in [20] proposes a
fuzzy generalized predictive control for T-S systems based on Kernel Ridge Regression strategy which learns the T-S fuzzy parameters from the input and output data. [21] utilizes the zonotopic set and interval matrices to bound the membership function errors. The controller parameters are stored in an off-line table for searching and robust tubes can be time-varying. In [22], the nonlinear multivariable predictive control is proposed for vehicle systems. For maintaining the robustness and stability, the controller design is based on LMI convex optimization. The cooperative fuzzy MPC is represented in [23] where the overall nonlinear plant consists of a group of parallel input-coupled T-S fuzzy models. For this cooperation, convergence and stability are guaranteed.

In order to deal with unknown disturbances, a series of paradigms have been elaborated (see e.g., [26]–[31], [33]). In [26], the homothetic tube-based approach is proposed, which maintains the state predictions of the linear model in the presence of disturbance within an on-line scaling tube centered at the disturbance-free model trajectory. The work of [27] proposes a tube-based MPC for nonlinear continuous-time model, and the feedback control law is optimized off-line. The authors in [28] utilized an integral non-squared stage cost and a non-squared terminal cost, so that the robustness of the resultant MPC is ensured without additional stability constraints. In [29], the input-to-state stability property is utilized for the quasi-min-max MPC design, and the first control move from the control sequence can be optimized directly. In [30], the notion of quadratic boundedness (QB) is utilized, based on which, the system is guaranteed to be quadratically bounded in the presence of disturbance. In [31], the notion of quadratic boundedness (QB) is utilized, based on which, the system is guaranteed to be quadratically bounded in the presence of disturbance. [31] proposes the full and partial dynamic output feedback MPC applying full Lyapunov matrix. The elliptical estimation error set is refreshed on-line based on optimized information of the last sampling instant. [32] aims at the norm-bounded model parametric uncertainty. The estimated state feedback gain and state estimator matrix are optimized on-line while the state estimator gain is designed off-line. In [33], sufficient conditions for computing the positively invariant set for T-S fuzzy systems are derived, and the terminal constraint set for 0-step and N-step control strategies are obtained.

This paper characterizes MPC synthesis, based on improving the Lyapunov function, for constrained T-S fuzzy model with the bounded disturbance. Some general results for the positiveness of polynomials with matrix-valued coefficients (based on Pólya’s theorem) is given in [35], where some complete characterization of the solution of parameter-dependent LMIs, usually arising in the robust stability analysis, is proposed. However, when the model parameters are time-varying uncertain, the results in [35] cannot be directly invoked. We deal with this issue in this paper, and contributions are summarized as follows.

1) The potentiality of Pólya’s theorem is exploited. The general homogeneous polynomially parameter-dependent (HPP) Lyapunov matrix whose complexity degree is tunable, and corresponding HPP control law, are applied.

2) By a generalization of the methods based-on Pólya’s theorem and parameter-dependent LMIs, a series of finite-dimensional LMI relaxations, as sufficient stability conditions, are developed to robustly stabilize the resultant closed-loop system.

In [34], a general robust MPC approach for linear parameter varying (LPV) systems in the absence of bounded disturbance has been proposed, which can include many existing approaches with common quadratic Lyapunov matrices and state feedback laws (e.g., [24], [25]) as special cases. As compared with [12], [34], this paper handles the unknown but bounded disturbance. Since the bounded disturbance is incorporated in the model, the characterization of the closed-loop stability is different from that in [34]. There are two other differences as compared with [34].

- While [34] introduces a free control move (i.e., the control move is the immediate decision variable), this paper does not.
- While [34] utilizes the dilution parameter $G$, this paper does not.

The rationale of approximation is the same as in [34]: the complexity degree of HPP solutions is tunable for the proposed approach and, when it increases, the conservatism of the results reduces; the HPP Lyapunov matrix and HPP feedback gain matrix can asymptotically approximate any Lyapunov and feedback gain matrices which are continuous on the combining coefficient functions.

**Notation:** The symbol ⋆ induces a symmetric structure in the matrix inequalities. A variable with superscript ⋆ means the optimal solution to the optimization problem. $\mathbb{R}^{m \times n}$ denotes the $m \times n$-dimensional real matrix set. $\mathbb{N}_+$ is the set of nonnegative integers. For the vector $x$ and positive-definite matrix $P > 0$, $\|x\|_P^2 = x^T P x$. $M!$ denotes factorial of $M$. $x([i])$ is the value of $x$ at the future interval $k + i$, predicted at interval $k$. $I$ is the identity matrix with appropriate dimension. For the column vectors $x$ and $y$, $[x; y] = [x^T, y^T]^T$. $\mathcal{E}_P = \{\xi \in \mathbb{R}^n | \xi^T P \xi \leq 1\}$ denotes the ellipsoid that is associated with the symmetric positive-definite matrix $P$. The time-dependence of the MPC decision variables is often omitted for brevity.

**II. PROBLEM STATEMENT**

Consider a class of T-S fuzzy systems, with its $j$th rule represented by

$$\text{Rule } j : \text{IF } \theta_1(k) \text{ is } H_{1j}, \ldots, \text{and } \theta_d(k) \text{ is } H_{dj}, \text{ THEN } x(k + 1) = A_j x(k) + B_j u(k) + D_j w(k),$$

(1)

where $j \in \{1, \ldots, r\}$ with $r$ rules. Let $\theta(k) = [\theta_1(k); \theta_2(k); \ldots; \theta_d(k)]$ be the measurable premise variable. $H_{1j}, H_{2j}, \ldots, H_{dj}$ are the fuzzy sets. $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $w(k)$ are measurable state, input, and bounded disturbance vectors, respectively. The disturbance is persis-
tent and satisfies \( w(k) \in \varepsilon_{P_w} \). Then the acquisition of T-S model can be described as
\[
x(k+1) = A(\theta(k))x(k) + B(\theta(k))u(k) + D(\theta(k))w(k),
\]
\[
A(\theta(k)) = \sum_{j=1}^{r} h_j(\theta(k))A_j, \quad \Lambda \in \{A, B, D\}. \tag{2}
\]
where \( H_j^{(\theta)}(\theta_j(k)) \) denotes the grade of membership of \( \theta_j(k) \) in \( H_j^{(\theta)} \), and \( h_j(\theta(k)) \) is the normalized membership function (abbreviated as the membership function), with \( h_j(\theta(k)) \geq 0 \), \( \sum_{j=1}^{r} h_j(\theta(k)) = 1 \). For the sake of simplicity, \( h_j \) is short for \( h_j(\theta(k)) \) and \( h_{j+} \) is short for \( h_j(\theta(k) + 1) \) for the rest of the paper.

The physical constraints to be considered is as follows,
\[
-u \leq u(i|k) \leq \bar{u}, \quad i \geq 0, \tag{4}
\]
\[
-\psi \leq \Psi x(i+1|k) \leq \bar{\psi}, \quad i \geq 0, \tag{5}
\]
where \( u = [u_1; u_2; \cdots; u_m] \), \( \bar{u} = [\bar{u}_1; \bar{u}_2; \cdots; \bar{u}_m] \), \( \bar{\psi} = [\bar{\psi}_1; \bar{\psi}_2; \cdots; \bar{\psi}_r] \), \( i \geq 0 \), \( s = 1, 2, \cdots, m \), and \( \bar{\psi}_s > 0, \psi_s > 0, s = 1, 2, \cdots, r \).

III. MODEL PREDICTIVE CONTROL SYNTHESIS

In this section, we propose the main result. Firstly, the standard MPC synthesis approach with the nonquadratic Lyapunov function is proposed with guaranteed recursive feasibility and closed-loop stability. Then, we apply the Pólya’s theorem to extend the result such that the LMI conditions in the MPC optimization problem is relaxed.

A. FORMULATION OF OPTIMIZATION PROBLEM

This paper utilizes a \( g \)-degree HPP control law in the form of
\[
u(k) = F_g x(k), \tag{8}
\]
where \( F_g = Y_g S_g^{-1} \) is the parameter-dependent feedback gain, which is define as follows.

Firstly, introduce some definitions in order to be consistent with [35]. Let the HPP matrix with tunable complexity degree \( g \) be
\[
G_g(\eta) = \sum_{p \in \mathcal{K}(g)} \eta_{1}^{p} \eta_{2}^{p} \cdots \eta_{r}^{p} G_p, \quad p = p_1 p_2 \cdots p_r. \tag{9}
\]
where \( \eta \in \Omega_r, \quad p_i \in \mathbb{N}_+, \quad i = 1, 2, \cdots, r \), each \( \eta_{1}^{p_1} \eta_{2}^{p_2} \cdots \eta_{r}^{p_r} \) is a monomial. For all \( p \in \mathcal{K}(g) \), \( G_p \in \mathbb{R}^{n \times n} \) are parameter-based matrices. \( \mathcal{K}(g) \) is the family of \( r \)-tuples, which is comprised of all the terms \( p_1 p_2 \cdots p_r \), \( p_i \geq 0, \quad i = 1, 2, \cdots, r \), with \( p_1 + p_2 + \cdots + p_r = g \). \( \mathcal{K}(g) \) includes \( J(g) \) number of elements with
\[
J(g) = \frac{(r + g - 1)!}{g!(r - 1)!}.
\]

For example, for the case with \( g = 2 \) and \( r = 2 \), \( \mathcal{K}(g) = \{02, 11, 20\} \) and \( J(2) = 3 \), subject to the form \( G_2(\eta) = \eta_{12} \bar{G}_{20} + \eta_{2} \bar{G}_{11} + \eta_{2} \bar{G}_{20} \).

Hence, in light of (9), we have parameter-dependent HPP Lyapunov matrices \( Y_g = \sum_{p \in \mathcal{K}(g)} \eta_{12}^{p} \eta_{11}^{p} \cdots \eta_{20}^{p} Y_p, \quad S_g = \sum_{p \in \mathcal{K}(g)} \eta_{12}^{p} \eta_{11}^{p} \cdots \eta_{20}^{p} S_p \), with \( p = p_1 p_2 \cdots p_r \).

Lemma 1: If there exist a parameter-dependant LMI satisfies
\[
\begin{align*}
\lambda(\mu, \eta) &\triangleq P_0(\eta) + \mu P_1(\eta) + \cdots + \mu M P_M(\eta) > 0,
\end{align*}
\]
where \( P_0(\cdot), P_1(\cdot), \ldots, P_r(\cdot) \) are continuous functions with respect to parameters \( \eta = [\eta_{1}, \eta_{2}, \ldots, \eta_{r}]^T \in \Omega_r \) and unknown \( \mu \in \mathbb{R}^M \). If there exists \( \mu(\eta) \) ensuring \( P(\mu(\eta), \eta) > 0 \) for all \( \eta \), then a homogenous polynomial function \( \tilde{\mu}(\eta) \) exists such that \( P(\tilde{\mu}(\eta), \eta) > 0 \) holds.

Proof. See [35].

Remark 1: Lemma 1 implies that a homogeneous polynomial form of solutions, whose parameters lie in the unit simplex, is a very general form, i.e., can be readily transformed into any other continuous solutions of parameter-dependent LMIs. By considering the famous Weiherstrass approximation theorem and applying Lemma 1, it infers that \( P_g \) and corresponding \( F_g \) can represent any Lyapunov matrix and feedback gain matrix that are parameterized by \( h_j, \quad j \in \{1, 2, \ldots r\} \) as \( g \) increases.

By applying (8) on (2), the closed-loop system is obtained as
\[
x(k+1) = (A(\theta(k)) + B(\theta(k))Y_g S_g^{-1}) x(k) + D(k)u(k), \tag{10}
\]
In order to guarantee the stability of closed-loop system (10), the technique of quadratic boundedness (QB) ([37]), which is primarily utilized for the state estimation problem and can be particularly useful for handling the system with bounded disturbance, is utilized to ensure that the state will stay in a quadratically bounded set. The definition and theorem of QB according to [37] are reviewed as follows.

Definition 1: For all allowable \( w(k) \in \varepsilon_{P_w}, \quad k \geq 0 \), the autonomous linear system \( x(k+1) = Ax(k) + Dw(k) \) is quadratically bounded with a common Lyapunov matrix \( P \),
if \( x(k)^T P x(k) \geq 1 \) implies that \( x(k + 1)^T P x(k + 1) \leq x(k)^T P x(k) \).

The QB condition is applied to many MPC problems with common quadratic Lyapunov functions. However, the resulting control performance can be conservative due to the utilization of the form of Lyapunov function. Since it is known that the extended nonquadratic Lyapunov method outperforms the common quadratic one, the extended nonquadratic boundedness is applied to characterize the closed-loop property in this paper.

Let \( P_g \) be the nonquadratic Lyapunov matrix. At interval \( k + 1 \), define \( P_{g+} = \sum_{p \in K(p)} h_1(k+1)^p h_2(k+1)^p \cdots h_r(k+1)^p P_g = \sum_{q \in K(q)} h^1 q h^2 q \cdots h^r q P_g \), with \( q = q_1 q_2 \cdots q_r \), satisfying \( q_1 + q_2 + \cdots + q_r = q \), \( \sum_{j=1}^r h_{j+} = 1 \).

**Definition 2:** System (10) is strictly nonquadratically bounded with an extended nonquadratic Lyapunov matrix \( P_g \) if \( x(k)^T P_g x(k) \geq 1 \) implies that \( x(k + 1)^T P_g x(k + 1) \leq x(k)^T P_g x(k) \) for any \( w(k) \in \varepsilon_{P_g} \), \( k \geq 0 \).

By inheriting the results in [37] and extending to the case with the extended nonquadratic boundedness, the following conclusion can be obtained.

**Lemma 2:** For all allowable \( w(k + i), i \geq 0 \), the following statements are equivalent.

a) System (2) is nonquadratically bounded with an extended nonquadratic Lyapunov matrix \( P_g \).

b) The ellipsoid \( \varepsilon_{P_g} \) is a positively invariant set for (2).

c) \( x(i)^T P_g x(i) \geq 1 \) implies that \( x(i + 1)^T P_g x(i + 1) \leq x(i)^T P_g x(i) \).

From Lemma 2, we can obtain that the system (2) is nonquadratically bounded if at each sampling interval \( k \), the following condition is satisfied:

\[
x(i)^T P_g x(i) \geq 1 \Rightarrow \\
\|x(i + 1)|^2_{P_g} + \|x(i)|^2_{P_g} + x(i)^T R x(i), i \geq 0.
\]

Since \( w(k + i) \in \varepsilon_{P_g} \), \( i \geq 0 \), \( x(i)^T P_g x(i) \geq 1 \) is equivalent to \( w(k + i)^T P_g w(k + i) \leq x(i + 1)^T P_g x(i) \). Hence, the condition (11) is equivalent to

\[
w(k + i)^T P_g w(k + i) \leq x(i + 1)^T P_g x(i) \Rightarrow \\
\|x(i + 1)|^2_{P_g} + \|x(i)|^2_{P_g} + x(i)^T R x(i), i \geq 0.
\]

According to (10), (12) can be expressed in quadratic form as

\[
\begin{bmatrix}
x(i) \\
w(k + i)
\end{bmatrix}^T 
\begin{bmatrix}
P_g & 0 \\
0 & -P_w
\end{bmatrix} 
\begin{bmatrix}
x(i) \\
w(k + i)
\end{bmatrix} \geq 0 \Rightarrow \\
\begin{bmatrix}
x(i) \\
w(k + i)
\end{bmatrix}^T 
\begin{bmatrix}
\Delta \\
0
\end{bmatrix} 
\begin{bmatrix}
-D(k + i)^T P_g + A & -D(k + i)^T P_g + A
\end{bmatrix} \times \\
\begin{bmatrix}
x(i) \\
w(k + i)
\end{bmatrix} \leq 0.
\]

where \( \Delta := P_g - A^T P_g A - Q - S_g^{-1} Y_g Y_g S_g^{-1} \), and \( A = A(\theta(k + i) + B(\theta(k + i) + Y_g Y_g S_g^{-1}) \).

By eliminating the variables \( [x(i)^T w(k + i)]^T \) and invoking the S-procedure, it is shown that (12) is satisfied if and only if there exists a scalar \( \alpha > 0 \) such that (see [37])

\[
\alpha \begin{bmatrix}
P_g & 0 \\
0 & -P_w
\end{bmatrix} \geq 0.
\]

By substitute \( P_g = \gamma S_g^{-1}, P_g^+ = \gamma S_g^{-1}, \) pre- and post-multiplying (14) by \( diag\{S_g, I\} \) (which leaves the inequality unaffected), and applying the Schur complement, it is shown that (14) is guaranteed by

\[
\begin{bmatrix}
(1 - \alpha) S_g & * & * & * \\
0 & \alpha P_w & * & * \\
A_j S_g + B_j Y_g & D_j & S_{g+} & * & * \\
Q_{1/2} S_g & 0 & 0 & \gamma I & *
\end{bmatrix} \geq 0,
\]

\[
\begin{bmatrix}
1/2 Y_g & 0 & 0 & \gamma I
\end{bmatrix} 
\begin{bmatrix}
j \in \{1, 2, \ldots, r\}.
\end{bmatrix}
\]

**Proposition 1:** Reference model (6) is quadratically stable with the Lyapunov matrix \( P_g \).

**Proof:** According to Lemma 2, if (2) is quadratically bounded with the Lyapunov matrix \( P_g \), then condition (15) is satisfied. By neglecting the disturbance, (15) is reduced to \( (1 - \alpha) \gamma S_g^{-1} - A^T \gamma S_g^{-1} A \geq Q + S_g^{-1} Y_g Y_g S_g^{-1} \) which guarantees that \( \gamma S_g^{-1} - A^T \gamma S_g^{-1} A \geq Q + S_g^{-1} Y_g Y_g S_g^{-1} \). Thus, by defining the HPP quadratic function \( V(\tilde{x}(i)) = \tilde{x}(i)^T P_g \tilde{x}(i) \), and substituting \( V(\tilde{x}(i + 1)) = \tilde{x}(i + 1)^T P_g \tilde{x}(i + 1) \), we have

\[
V(\tilde{x}(i + 1)) - V(\tilde{x}(i)) \leq \\
- \tilde{x}(i)^T Q \tilde{x}(t|k) - \tilde{u}(i|k)^T R \tilde{u}(i|k), i \geq 0.
\]

Thus, the conclusion holds.

**Remark 2:** The HPP quadratic function \( V(\cdot) \) is in a very general form, which implies that it covers many existing Lyapunov functions. For example, by taking \( g = 0 \), the Lyapunov function in [24] is recovered; by taking \( g = 1, [25] \) is recovered.

Based on Proposition 1, the state and input prediction of reference model (6) will converge to the origin, i.e., \( \lim_{j \to \infty} \tilde{x}(i|k) = 0 \) and \( \lim_{i \to \infty} \tilde{x}_0(i) = 0 \). Hence, summing (16) from \( i = 0 \) to \( \infty \) yields

\[
J_{\infty} \leq V(\tilde{x}(0|k)) = \tilde{x}(0|k)^T P_g \tilde{x}(0|k).
\]

Since \( P_g = \gamma S_g^{-1} \) and \( \tilde{x}(0|k) = x(k) \), let \( \gamma \) be the upper bound of (17), i.e., \( V(\tilde{x}(0|k)) \leq \gamma \), then the following holds:

\[
\begin{bmatrix}
1 \\
x(k)
\end{bmatrix} S_g \geq 0.
\]
Similar to the procedure in [24], the input and state constraints in (4) and (5) are guaranteed by
\[
\begin{bmatrix}
S_g \\
Y_g \\
Z
\end{bmatrix} \succcurlyeq 0, \quad Z_{ss} \preceq z_{s,inf}^2, \quad s \in \{1, 2, \ldots, m\},
\]
\[(19)
\]
\[
\begin{bmatrix}
S_g \\
\Psi(A_j S_g + B_j Y_g) \quad \Gamma
\end{bmatrix} \succcurlyeq 0, \quad j \in \{1, 2, \ldots, r\},
\]
\[(20)
\]
where \(z_{s,inf} = \min\{a_s, b_s\}\), \(a_s, b_s \in \mathbb{R}\) and \(Z_{ss}\) is the \(s\)th diagonal element of \(Z(\Gamma)\).

As the usual practice in robust MPC, the optimization problem is formulated as the following min-max form:
\[
\min_{S_g, S_{pp}, Y_g, r, Z, \Gamma} \gamma \text{ s.t. } (15), (18), (19), (20).
\]

Remark 3: For simplifying the presentation, we only consider the case when switching horizon \(N = 0\) which is consistent with the benchmark work [24], when \(N > 0\), free control moves are added before the control law (8), this can be achieved easily by generalization, which is omitted here for brevity.

B. OPTIMIZATION PROBLEM VIA HPP SOLUTIONS

Theorem 2: Suppose \(\Omega_{t}\) is the simplex set satisfies \(\Omega_{t} = \{\eta \in \mathbb{R}^r | \sum_{i=1}^{r} \eta_i = 1, \eta_i \geq 0\}\). If \(f \in \mathbb{R}^r\) is homogeneous and positive on \(\Omega_{t}\), then for a sufficiently large scalar \(d\), all the coefficients of \((\eta_1 + \eta_2 + \cdots + \eta_r)^d f(\eta_1, \eta_2, \ldots, \eta_r)\) are positive.

In this paper, we consider the homogeneous polynomial matrix \(X\) with degree \(g \times q\) in the following form
\[
X = \sum_{g \in K(g)} \eta_{1,g}^q \eta_{2,g}^q \cdots \eta_{r,g}^q \sum_{p \in K(g)} \eta_{1,p}^p \eta_{2,p}^p \cdots \eta_{r,p}^p X_{p,q},
\]
\[(23)
\]
where each \(X_{p,q}\) is a parameter-based matrix. Also, we have \(\sum_{i=1}^{\rho} \eta_i = 1, \sum_{i=1}^{g} \eta_i = 1, \sum_{i=1}^{j} \eta_i = 1, \sum_{i=1}^{d} \eta_i = 1\).

Proposition 2: For the condition when (23) is positive, it is guaranteed that a set of sufficiently large \(d\) and \(d_+\) exist which ensures the positiveness of all the coefficients of \((\eta_1 + \cdots + \eta_r)^d (\eta_1 + \cdots + \eta_r + d_+)^d X\).

Proof. This is referred to Theorem 2. Let us consider the following matrix-valued function:
\[
f(\zeta_1, \zeta_2, \ldots, \zeta_{2r}) = \sum_{\rho \in K(2g)} \zeta_{1,p}^q \zeta_{2,p}^q \cdots \zeta_{2r,p}^q X_{p},
\]
\[(24)
\]
where \(\rho = \rho_1 \rho_2 \cdots \rho_{2r}\) such that \(\rho_1 + \rho_2 + \cdots + \rho_{2r} = 2p\). Note that each \(\zeta_{1,p}^q \zeta_{2,p}^q \cdots \zeta_{2r,p}^q\) is a monomial and \(K(2g)\) is the set of \(2r\)-tuples. If we rewrite
\[
\zeta_j = \frac{\eta_j}{2}, \quad \zeta_{j+r} = \frac{\eta_j}{2}, \quad j \in \{1, 2, \ldots, r\}
\]
\[
X_{p} = 4^p X_{p,q}, \quad p = \rho_1 \rho_2 \cdots \rho_{2r}, q = \rho_{r+1} \rho_{r+2} \cdots \rho_{2r-1}
\]
then
\[
f(\zeta_1, \zeta_2, \cdots, \zeta_{2r}), \sum_{j=1}^{r} \rho_j = g, \sum_{j=1}^{r} \rho_{j+r} = g
\]
By the extension of scalar-valued function case in [36, Th1], we can obtain that a set of sufficiently large \(d\) and \(d_+\) exist which ensures the positiveness of all the coefficients of \((\eta_1 + \cdots + \eta_r + \eta_{1+} + \cdots + \eta_{r+})^d (\eta_{1+} + \cdots + \eta_{r+})^d X\). Since \((\eta_1 + \cdots + \eta_r)^d (\eta_{1+} + \cdots + \eta_{r+})^d X\) is a composition of part of terms in \((\eta_1 + \cdots + \eta_r + \eta_{1+} + \cdots + \eta_{r+})^d (\eta_{1+} + \cdots + \eta_{r+})^d X\), all the coefficient of \((\eta_1 + \cdots + \eta_r)^d (\eta_{1+} + \cdots + \eta_{r+})^d X\) are required to be positive.

We utilize the procedure in [35] based-on Pólya’s theorem to further handle (15), (18), (19), and (20) in order to obtain a non-conservativeness result.

Remark 3: For simplifying the presentation, we only consider the case when switching horizon \(N = 0\) which is consistent with the benchmark work [24], when \(N > 0\), free control moves are added before the control law (8), this can be achieved easily by generalization, which is omitted here for brevity.
satisfying inequalities (25)-(28), then constraints (15), (18), (19), (20) are guaranteed.

\[
\frac{(g + d_+)!}{\pi(q)} \sum_{p' \in \mathcal{K}(d), \ j \in \{1, \ldots, r\}, \ p_j > p_j'} \frac{d!}{\pi(p')} \begin{bmatrix}
(1 - \alpha) S_{p-p'-e_j} & \ast & \ast & \ast \\
0 & 0 & \ast & \ast \\
A_j S_{p-p'-e_j} + B_j Y_{p-p'-e_j} & D_j & 0 & \ast \\
Q^{1/2} S_{p-p'-e_j} & 0 & 0 & 0 \\
R^{1/2} Y_{p-p'-e_j} & 0 & 0 & 0
\end{bmatrix} + \\
\frac{(g + d_+)!}{\pi(p)} \begin{bmatrix}
0 & 0 & \ast & \ast \\
0 & 0 & \ast & \ast \\
0 \ast & \ast & \ast & \ast \\
0 & 0 & \frac{(g + d_+)!}{\pi(q)} \gamma I & \frac{(g + d_+)!}{\pi(q)} \gamma I
\end{bmatrix} \geq 0,
\]

\[
\forall p \in \mathcal{K}(g + d + 1), \ \forall q \in \mathcal{K}(g + d_+),
\left[\frac{(g + d_+)!}{\pi(p)} x(k) \sum_{p' \in \mathcal{K}(d), p \geq p'} \frac{(d)!}{\pi(p')} S_{p-p'} \ast \right] \geq 0, \ \forall p \in \mathcal{K}(g + d),
\]

\[
\left[\sum_{p' \in \mathcal{K}(d), p \geq p'} \frac{(d)!}{\pi(p')} S_{p-p'} \ast \sum_{p' \in \mathcal{K}(d), p \geq p'} \frac{(d)!}{\pi(p')} S_{p-p'} \ast \sum_{p' \in \mathcal{K}(d), p \geq p'} \frac{(d)!}{\pi(p')} S_{p-p'} \ast \right] \geq 0, \ Z_{ss} \leq \gamma_{s,inf}^2, \ s \in \{1, 2, \ldots, m\}, \ \forall p \in \mathcal{K}(g + d),
\]

\[
\Gamma_{ss} \leq \psi_{s,inf}^2, \ s \in \{1, 2, \ldots, r\}, \ p \in \mathcal{K}(g + d + 1).
\]

**Proof.** Consider constraint (26) first. Let \( \Theta = \begin{bmatrix} 1 & \ast \\ x(k) & S_g \end{bmatrix} \). Since \( \sum_{j=1}^r h_j = 1 \) and \( \Theta = (h_1 + h_2 + \cdots + h_r)^d \) hold, by applying the equality

\[
\sum_{p \in \mathcal{K}(g + d)} h_1^{p_1} h_2^{p_2} \cdots h_r^{p_r} \frac{(g + d)!}{\pi(p)} = 1, \ p = p_1 p_2 \cdots p_r,
\]

and utilizing the procedure in [35], \( \Theta \) is rewritten as

\[
\Theta = \sum_{p \in \mathcal{K}(g + d)} h_1^{p_1} h_2^{p_2} \cdots h_r^{p_r} \begin{bmatrix}
\frac{(g + d)!}{\pi(p)} x(k) \sum_{p' \in \mathcal{K}(d), p \geq p'} \frac{(d)!}{\pi(p')} S_{p-p'} \\
\frac{(g + d)!}{\pi(p)} x(k) \sum_{p' \in \mathcal{K}(d), p \geq p'} \frac{(d)!}{\pi(p')} S_{p-p'} \\
\end{bmatrix},
\]

\( \forall p \in \mathcal{K}(g + d). \) \hspace{1cm} (29)

It is clearly shown that \( \Theta \geq 0 \) is guaranteed by (26).

According to Proposition 2, the proof of (25) contains two steps.

1) handle \( S_g, Y_g. \) Let

\[
\Upsilon = \begin{bmatrix}
(1 - \alpha) S_g & \ast & \ast & \ast \\
0 & \alpha P_w & \ast & \ast \\
A(k + i) S_g + B(k + i) Y_g & D(k + i) & S_{g+} & \ast \\
Q^{1/2} S_g & 0 & 0 & \gamma I \\
R^{1/2} Y_g & 0 & 0 & \gamma I
\end{bmatrix}.
\]

Since \( \sum_{j=1}^r h_j = 1, \ \Upsilon = (h_1 + h_2 + \cdots + h_r)^d \) \( \Upsilon. \) By the fact that

\[
\sum_{p \in \mathcal{K}(g + d + 1)} h_1^{p_1} h_2^{p_2} \cdots h_r^{p_r} \frac{(g + d + 1)!}{\pi(p)} = 1, \ p = p_1 p_2 \cdots p_r.
\]
Then we can conclude that $\Upsilon \geq 0$, i.e., constraint (18) is guaranteed by $X_p \geq 0$.  
2) handle $S_{g+}$. Since $\sum_{j=1}^r h_{j+} = 1$, by the fact

$$\sum_{q \in \mathcal{K}(g+d_+)} h_{q1}^q h_{q2}^q \ldots h_{qr}^q \frac{(g + d_+)}{\pi(q)} = 1, \quad q = q_1 q_2 \ldots q_r,$$

we can obtain that

$$(h_{1+} + h_{2+} + \ldots + h_{r+}) \Phi_1 + X_p = \sum_{q \in \mathcal{K}(g+d_+)} h_{q1}^q h_{q2}^q \ldots h_{qr}^q X_{p,q},$$

$$X_{p,q} = \frac{(g + d_+)}{\pi(q)} \sum_{p' \in \mathcal{K}(d_+)} \sum_{j \in \{1, \ldots, r\}} \frac{d!}{\pi(p')} \Phi_1 \frac{(g + d_+)}{\pi(q)} \Phi_2,$$

where

$$\Phi_1 = \begin{bmatrix}
(1 - \alpha)S_{p-p'}e_j & * & * & * \\
0 & 0 & * & * \\
A_j S_{p-p'}e_j + B_j Y_{p-p'}e_j & D_j & 0 & * \\
Q_{1/2} S_{p-p'}e_j & 0 & 0 & * \\
R_{1/2} Y_{p-p'}e_j & 0 & 0 & 0
\end{bmatrix},$$

$$\Phi_2 = \begin{bmatrix}
0 & * & * & * \\
\frac{(g + d_+)}{\pi(q)} \alpha P_{w} & * & * & * \\
0 & 0 & \sum_{q' \in \mathcal{K}(d_+), q' \leq q} \frac{(d_+)^q}{\pi(q')} (S_{q-q'}) & * & * \\
0 & 0 & 0 & \frac{(g + d_+)}{\pi(q)} \gamma I \\
0 & 0 & 0 & 0 & \frac{(g + d_+)}{\pi(q)} \gamma I
\end{bmatrix}, \quad \forall p \in \mathcal{K}(g + d_+), \forall q \in \mathcal{K}(g + d_+).$$

We can find that $X_{p,q} \geq 0$ ensures $X_p \geq 0$. Analogously, it is easy to prove that (19), (20) are satisfied if (27), (28) hold, respectively.

Thus, the proof is complete.

In summary, optimization problem (21) is reexpressed as

$$\min_{S_{p}, Y_{p}, \beta, \Phi, \gamma} \gamma \text{ s.t. } (25) - (28).$$

(30)

Similar to (21), problem (30) can be solved via convex optimization if $\alpha$ is pre-specified. However, note that the computational burden of (30) can be much heavier but the result is non-conservative.

**Corollary 3.1:** If optimization problem (30) has feasible solution for a particular set of $\{g_0, d_0, d_0+, \}$, then it also holds for $g > g_0$, $d > d_0$, $d_+ > d_0+$.

**Proof.** see [35].

**Remark 4:** An alternative methodology to calculate the HPP Lyapunov solution to (30) is by only increasing $g$ and choosing $d = 0, d_d = 0$. However, more decision variables are emerged by increasing $g$ while the increase in $\{d, d_d\}$ brings the larger number of LMIs. If the computational efficiency is a crucial factor, one can simply reduce $g$, $d$ and $d_d$ to a satisfactory level.

**IV. ILLUSTRATIVE EXAMPLE**

**Example 1.**

Consider the following discrete-time nonlinear system:

$$x_1(k + 1) = x_1(k) - x_1(k)x_2(k) + (5 + x_1(k))u(k) + 0.5x_1(k)w(k),$$

$$x_2(k + 1) = -x_1(k) - 0.5x_2(k) + 2x_1(k)u(k),$$

(31)

where $x_1(k) \in [-\beta, \beta]$ with $\beta > 0$ and disturbance $|w(k)| \leq 0.5$. Let membership function $h_1 = (\beta + x_1(k))/(2\beta)$ and $h_2 = (\beta - x_1(k))/(2\beta)$ be the combination coefficients.

Then, nonlinear system (31) can be represented by the T-S
The T-S model can be different by simply changing $\beta$. Larger $\beta$ implies the model in a larger region, which is more difficult to control.

Choose $Q = I, R = I, P_w = 2, \alpha = 0.3, w(k) = 0.5\sin(k)$. In order to show the effectiveness of the proposed approach, (30) is solved for a variety of pairs $\{g, d, d_+\}$.

For each different $\{g, d, d_+\}$, there exists a crucial $\beta_0$ that if $\beta \leq \beta_0$, then (30) becomes feasible, i.e., the T-S model can be stabilizable.

TABLE 1: Feasible values of $\beta_0$ ((30) is feasible when $\beta \leq \beta_0$)

| $g$ | $d$ | $d_+$ | $\beta_0$ |
|-----|-----|-------|---------|
| 0   | 0   | 0     | 0.73    |
| 1   | 0   | 0     | 1.28    |
| 1   | 1   | 0     | 1.47    |
| 2   | 0   | 1     | 1.52    |
| 2   | 2   | 0     | 1.52    |
| 2   | 3   | 0     | 1.55    |
| 2   | 3   | 1     | 1.57    |

Different pairs of $\{g, d, d_+\}$ and $\beta_0$ are chosen for the simulation and they are listed in Table 1. It can be inferred that as values of $\{g, d, d_+\}$ increases, the stabilizable range of T-S model is enlarged by applying the approach in this paper. Moreover, in order to get the exact same $\beta_0$, one can increase $g$ while maintaining $d$ and $d_+$ to be zero, or increase the set $\{g, d, d_+\}$ as a whole.

To illustrate the effectiveness of different $g$, three special cases with $\{d = 0, d_+ = 0\}$ for the same T-S model are considered, i.e., the case $g = 0$ that is applied in benchmark work [24], the case $g = 1$ that is applied in another work [25], and case $g = 2$ that is tuned in this paper. Choose $\beta_0 = 0.73, x(0) = [0.73; 1]$. The resultant trajectories of states and inputs are shown in Figure 1 and Figure 2, respectively. Define the cumulated cost $J_{\text{sum}} = \sum_{k=0}^{k_1} \left[ \|x(k_1)\|^2_Q + \|u(k_1)\|^2_R \right]$ to assess the performance. Calculate $J_{\text{sum}}$ for $g = 0, g = 0.5$.
where $C_A$ is the concentration of irreversible exothermic reaction A in the reactor, $T$ is the measurable reactor temperature, $T_c$ is the temperature of the coolant stream. The disturbance $|w| \leq 1$. The objective is to regulate $C_A$ and $T$ by manipulating $T_c$. Corresponding parameters used are summarized in Table 2.

The constraints are $328K \leq T_c \leq 348K$, $340K \leq T \leq 360K$, $0 \leq C_A \leq 1mol/l$. Denote the non-zero equilibrium as $\{C_{eq}^A, T_{eq}, T_{eq}^c\}$ where $C_{eq}^A = 0.5mol/l$, $T_{eq} = 350K$, $T_{eq}^c = 338K$. Define the state vector $x = [C_A - C_{eq}^A, T - T_{eq}]^T$ and the input $u = T_c - T_{eq}^c$. Denote the constraint on $x$ as $x_2$, $x_3 \leq x_2 \leq \bar{x}_2$. Moreover, define $\varphi_1(x_2) = k_0 \exp(-E/R)/(x_2 + T_{eq})$, $\varphi_2(x_2) = k_0 \exp(-E/R)/(x_2 + T_{eq}) - \exp(-(E/R)/T_{eq})C_{eq}^A/\bar{x}_2$, $H_1 = \frac{1}{2}(\varphi_1(x_2) - \varphi_1(\bar{x}_2))/(\varphi_1(\bar{x}_2) - \varphi_1(x_2))$, $H_2 = \frac{1}{2}(\varphi_2(x_2) - \varphi_2(\bar{x}_2))/(\varphi_2(\bar{x}_2) - \varphi_2(x_2))$, $H_3 = \frac{1}{2}(\varphi_2(x_2) - \varphi_2(\bar{x}_2))/(\varphi_2(\bar{x}_2) - \varphi_2(x_2))$. By discretizing the continuous system with sampling period $\Delta t_s = 0.2$minutes, the nonlinear system (32) can be approximated by following four rules of T-S fuzzy model ($h_1 = H_1$, $h_2 = H_2$, $h_3 = H_3$, and $h_4 = H_4$):

**Rule 1:** IF $x_2(k)$ is $H_1$, THEN

$$x(k+1) = \begin{bmatrix} 0.5347 & -0.0073 \\ 27.7559 & 0.7640 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0.4184 \end{bmatrix}$$

$$u(k) + \begin{bmatrix} 0.01 \\ 0 \end{bmatrix} w(k)$$

**Rule 2:** IF $x_2(k)$ is $H_2$, THEN

$$x(k+1) = \begin{bmatrix} 0.8271 & -0.0073 \\ -2.8341 & 0.7640 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0.4184 \end{bmatrix}$$

$$u(k) + \begin{bmatrix} 0.01 \\ 0 \end{bmatrix} w(k)$$

**Rule 3:** IF $x_2(k)$ is $H_3$, THEN

$$x(k+1) = \begin{bmatrix} 0.6809 & -0.0060 \\ 12.4609 & 1.0059 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0.4184 \end{bmatrix}$$

$$u(k) + \begin{bmatrix} 0.01 \\ 0 \end{bmatrix} w(k)$$

### Table 2: System parameters

| Parameter | Value | Units |
|-----------|-------|-------|
| $q$       | 100   | l/min |
| $V$       | 100   | l     |
| $C_{eq}$  | 0.9   | mol/l |
| $k_0$     | $3.456 \times 10^{10}$ | min$^{-1}$ |
| $E/R$     | 8750  | K     |
| $T_f$     | 350   | K     |
| $\Delta H$ | $2.5 \times 10^4$ | J/mol |
| $\rho$    | 1000  | g/l   |
| $C_p$     | 0.239 | J/gK  |
| $UA$      | $5 \times 10^4$ | J/minK |
| $D$       | 0.01  | -     |

**Example 2.**

Consider another benchmark example, i.e., a continuous stirred tank reactor (CSTR) whose continuous dynamics is

$$C_A = \frac{q(C_{AF} - C_A)}{V} - k_0 \exp\left(-\frac{E}{RT}\right) C_A + D w$$

$$\dot{T} = \frac{q}{V} (T_f - T) + \frac{1}{\rho C_p} \left[-\Delta H \kappa \exp\left(-\frac{E}{RT}\right) C_A + \frac{UA}{\rho C_p} (T_c - T)\right]$$

(32)
TABLE 3: Feasible values of maximal $\sigma$

| $g$ | 0 | 1 | 2 | 2 | 2 |
|-----|---|---|---|---|---|
| $d$ | 0 | 0 | 1 | 0 | 1 |
| $d_+$ | 0 | 0 | 0 | 0 | 1 |
| $\sigma_{\text{max}}$ | 1.28 | 1.45 | 1.5 | 1.73 | 1.75 |

FIGURE 7: Responses of states via different $g$

Rule 4: IF $x_2(k)$ is $H_4$, THEN

$$x(k+1) = \begin{bmatrix} 0.6809 & -0.0013 \\ 12.4609 & 0.5220 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0.4184 \end{bmatrix}$$

$$u(k) + \begin{bmatrix} 0.01 \\ 0 \end{bmatrix} w(k)$$

Then choose $Q = I$, $R = I$, $w(k) = \sin(k)$.

We use another way to modify the model through multiplying submodels by $\sigma$ simultaneously. The maximal values of $\sigma$ for the stabilizable T-S models are listed in Table 3. From the Table 3, it is shown that with values of $\{g, d, d_+\}$ increasing, $\sigma$ becomes larger. Thus, the stabilizable model range is enlarged.

To illustrate the effectiveness via different complexity degrees $g$, we consider three cases for the same T-S model, i.e., $\{g = 0, d = 0, d_+ = 0\}$, $\{g = 1, d = 0, d_+ = 0\}$, $\{g = 2, d = 0, d_+ = 0\}$. Choose $\sigma = 1$, $x(0) = [0.2, 4]^T$. The trajectories of states and input are depicted in Figures 7–8. Figures 7–8 show that the state and input evolve to the neighborhood of the origin without the constraints violation over the whole simulation horizon, so the closed-loop system is nonquadratically bounded. Choose $J_g = \sum_{k=0}^{30} \left( \|x(k)\|_Q^2 + \|u(k)\|_R^2 \right)$ as the performance criterion. Calculate $J_g$ via $g = 0, g = 1, g = 2$, respectively, and we obtain that $J_{(g=0)} = 154.3521$, $J_{(g=1)} = 133.0989$, $J_{(g=2)} = 110.1759$ (see Figure 9). As can be shown $J_g$ reduces as the value of $g$ increases. It implies that performance is improved with a larger value of $g$.

V. CONCLUSIONS

In this paper, a less conservative MPC approach for T-S fuzzy model with bounded disturbance is proposed. A general form of HPP Lyapunov function and the corresponding HPP control law are adopted to extend the previous approaches which are taken as special cases in this paper. The complexity degree is allowed to be tuned in order to balance the control performance and the computational efficiency. The proposed technique brings less conservatism as well as enlarging the stabilizable model range. Controlled systems subject to measurement noises widely exist in practice, our future attention is therefore paid on extending the proposed method to other systems with this issue, such as Markov system [38], [39], linear parameter-varying system [40], and networked control systems [41].

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