Constraints on $R$-parity violating interactions in supersymmetric standard model from leptonic decays of $D_s$ and $B^+$ mesons

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Abstract

We study leptonic decays $D_s \to \tau \nu$ and $B^+ \to \tau \nu$ in $R$-parity violating (RPV) supersymmetric standard model. The interference between the $s$-channel slepton exchange and the $t$-channel squark exchange diagrams could be destructive in a certain model parameter region so that sizable RPV couplings are allowed. Contributions from the RPV interactions to the final states with the lepton flavor violation (LFV), $\tau \nu_\mu (\tau \nu_\tau)$, are also examined taking account of constraints on the RPV couplings from the other LFV processes.
1 Introduction

Although the standard model (SM) of particle physics has shown a good agreement with almost all the experimental results (for example, see [1]), the gauge hierarchy problem motivates us to explore physics beyond the SM. Supersymmetry (SUSY) has been expected as a promising idea to solve the gauge hierarchy problem [2], and looking for direct or indirect signatures of supersymmetric extension of the SM is one of the most important tasks of collider experiments at energy frontier such as LHC.

It is well known that, in the supersymmetric SM, the baryon ($B$) and lepton ($L$) numbers are not conserved in general. The $R$-parity is, therefore, introduced to protect the proton longevity or to suppress unobserved $B$- or $L$-number violating processes. An important consequence of $R$-parity is that the lightest supersymmetric particle (LSP) is stable and it could be a candidate of the cold dark matter. On the other hand, if some of the $R$-parity violating (RPV) interactions are allowed, phenomenology of supersymmetric SM is drastically changed. For example, tiny neutrino mass can be explained through the RPV interactions without introducing a heavy Majorana neutrino [3]. Even without $R$-parity, the light gravitino could play a role of cold dark matter [4].

In ref. [5], we studied the leptonic decays $D_s \rightarrow \tau \nu$ and $B^+ \rightarrow \tau \nu$ in the RPV supersymmetric SM. It was pointed out that the experimental data of $D_s \rightarrow \tau \nu$ [6] shows the deviation from the Lattice QCD calculations about $2.4\sigma$ [7], while the discrepancy between the experimental measurement of $B^+ \rightarrow \tau \nu$ and the SM expectation is about $2.5\sigma$ [10]. In the supersymmetric SM with RPV interactions, the leptonic decays occur at tree level through (i) charged slepton exchange in the $s$-channel diagram, and (ii) down-squark exchange in the $t$-channel diagram. We found that the supersymmetric contributions could either constructively or destructively interfere between the $s$- and $t$-channel diagrams so that the $R$-parity couplings could be sizable when the contributions are cancelled each other which has not been pointed out in earlier works [11, 12, 13, 14, 15, 16]. Although the RPV interactions lead to the final states where the charged lepton and neutrino are not only flavor diagonal ($\tau \nu_\tau$) but also off-diagonal ($\tau \nu_\mu$ or $\tau \nu_e$), the latter was neglected in the previous works.

Recently, the Lattice QCD calculation of the decay constant $f_{D_s}$ has been updated [17] and the world average of the experimental data of $D_s \rightarrow \tau \nu$ is smaller than the previous one [18]. As a result, the deviation between the SM expectation and the experimental data of $D_s \rightarrow \tau \nu$ decreased from $2.4\sigma$ to $1.6\sigma$ and so called the “$f_{D_s}$-puzzle”

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1 The flavor violating final states in the leptonic decay of $D_s$ meson have been discussed in the framework of leptoquark models in ref. [32].
may disappear.

In this paper, we study constraints on the RPV interactions from $D_s \rightarrow \tau \nu$ and $B^+ \rightarrow \tau \nu$ taking account of the recently reported result on $f_{D_s}$. Although the deviation in $D_s \rightarrow \tau \nu (f_{D_s})$ is now $1.6 \sigma$, we show that there are still sizable allowed parameter space of the RPV couplings where the supersymmetric contributions are cancelled between the $s$- and $t$-channel diagrams. We also study in this paper the contributions of the RPV interactions to the leptonic decays with the lepton flavor violation (LFV) ($D_s, B^+ \rightarrow \tau \nu, i = e, \mu$). The RPV interactions which lead to the LFV in the final state of the leptonic decays also contribute to the other LFV process, such as $\tau \rightarrow \mu \gamma$ and $\tau \rightarrow \mu \eta$ for $D_s \rightarrow \tau \nu$, and $B^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $B^0 \rightarrow \ell_+^\pm \ell_j^\mp$ for $B^+ \rightarrow \tau \nu$. We show that contributions of RPV interactions to $D_s \rightarrow \nu \nu$ are suppressed by a few order of magnitude compared to the flavor diagonal case ($\tau \nu_{\tau}$) owing to the constraints from the other LFV processes, while those for $B^+$ can be as large as $\tau \nu_{\tau}$.

This paper is organized as follows. In the next section, we briefly review the relevant RPV interactions for the leptonic decays of $D_s$ and $B^+$ mesons. The numerical results are presented in Sec. 3. Sec. 4 is devoted to summary and discussions.

2 Set up

The trilinear interactions with $R$-parity violations are described by the following superpotential

$$W_R = \frac{1}{2} \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k + \frac{1}{2} \lambda''_{ijk} U_i D_j D_k,$$

where $Q$ and $L$ are SU(2)$_L$ doublet quark and lepton superfields, respectively. The up- and down-type singlet quark superfields are represented by $U$ and $D$, while the lepton singlet superfield is $E$. The generation indices are labeled by $i, j$ and $k$. The SU(2)$_L$ and SU(3)$_C$ gauge indices are suppressed. The dimensionless coefficient $\lambda_{ijk}$ is anti-symmetric for $i$ and $j$, while $\lambda''_{ijk}$ is anti-symmetric for $j$ and $k$. For a comprehensive review of the $R$-parity violating supersymmetric SM, see ref. [19]. Constraints on the RPV couplings $\lambda_{ijk}, \lambda'_{ijk}$ and $\lambda''_{ijk}$ from various processes have been studied in the literature [20, 21, 22, 23, 13]. Since the baryon number violating coupling $\lambda''_{ijk}$ induces too fast proton decay, we take $\lambda''_{ijk} = 0$ in the following. Then, the leptonic decays of $D_s$ and $B^+$ mesons occur through the slepton exchange in the $s$-channel diagram with a product of $\lambda$ and $\lambda'$, and the squark exchange in the $t$-channel diagram with a product of two $\lambda'$ couplings.

Let us briefly summarize the leptonic decay of a pseudo scalar meson $P$ which consists
of the up (and anti-) down-type quarks $u_a$ and $\bar{d}_b$, where $a, b$ are generation indices of quarks. The decay width of $P \to \ell_i \nu_j$ is given as

$$\Gamma(P \to \ell_i \nu_j) = \frac{1}{8\pi} r_P^2 G_F^2 |V^{*}_{ua db}|^2 f_P^2 m_P^2 m_P \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2$$

(2)

where $G_F, V_{ua db}, m_\ell, m_P$ are the Fermi constant, the Cabibbo-Kobayashi-Maskawa matrix element, the mass of a charged lepton $\ell_i$ and the mass of a pseudo scalar meson $P$, respectively. The flavor indices of charged leptons and neutrinos are expressed by $i$ and $j$, respectively. The decay constant is denoted by $f_P$. A parameter $r_P$ is defined as,

$$r_P^2 \equiv \left|\frac{G_F V^{*}_{ua db} + A_{ii}^P}{G_F^2 |V^{*}_{ua db}|^2}\right|^2 + \sum_{j(\neq i)} \left|\frac{A_{ij}^P}{G_F^2 |V^{*}_{ua db}|^2}\right|^2,$$

(3)

where $A_{ij}^P$ represents new physics contribution. Note that, in the second term of r.h.s. in eq. (3), one should take a sum only for $j$ (neutrinos), because that the neutrino flavor cannot be detected experimentally. If there is no new physics contribution, $r_P = 1$.

The interaction Lagrangian of the $t$-channel contribution to the decay width (2) can be obtained from the superpotential (1);

$$\mathcal{L} = \lambda_{ijk} \left\{ -(\ell_R^*)_i (u_L)_j (\bar{d}_R)_k \right\} + \lambda_{ijk} \left\{ (\nu^*_L)_i (d_L)_j (\bar{d}_R)_k \right\} + \text{h.c..}$$

(4)

Using the Fierz transformation, the effective Lagrangian which describes the $t$-channel squark exchange is given as

$$\mathcal{L}_{\text{eff}}^t = \frac{1}{8} \sum_{k=1}^{3} \frac{\lambda'_{iak} \lambda'_{jbk}}{m^2_{d_Rk}} \bar{d}_j \gamma^\mu (1 - \gamma_5) \ell_i \tilde{d}_b \gamma_\mu (1 - \gamma_5) u_a.$$  

(5)

For comparison, we show the effective Lagrangian for the $W$-boson exchange

$$\mathcal{L}_{\text{eff}}^{SM} = \frac{G_F}{\sqrt{2}} V^{*}_{ua db} \bar{d}_i \gamma^\mu (1 - \gamma_5) \ell_i \tilde{d}_b \gamma_\mu (1 - \gamma_5) u_a.$$  

(6)

Using the decay constant $f_P$ which is defined by

$$\langle 0 | \bar{d}_b \gamma^\mu \gamma_5 u_a | P(q) \rangle = i f_P q^\mu,$$

(7)

we find the $t$-channel squark contribution to the decay $P(u_a \bar{d}_b) \to \ell_i \nu_j$ as

$$(A_{ij}^P)_{ij} = \frac{1}{4\sqrt{2}} \sum_{k=1}^{3} \frac{\lambda'_{iak} \lambda'_{jbk}}{m^2_{d_Rk}}.$$  

(8)

The $s$-channel contribution can be calculated from the interaction Lagrangian

$$\mathcal{L} = \lambda_{ijk} \left\{ -(\ell_R^*)_k (u_L)_j (\bar{d}_L)_i \right\} + \lambda_{ijk} \left\{ -(\nu^*_L)_k (d_L)_j (\bar{d}_L)_i \right\} + \text{h.c..}$$  

(9)
The effective Lagrangian is given by

\[ \mathcal{L}_{\text{eff}}^s = -\frac{1}{4} \sum_{k=1}^{3} \frac{\lambda_{kji}^* \lambda_{kab}'}{m_{LLk}^2} \bar{\nu}_j (1 + \gamma_5) \ell_i \bar{d}_b (1 - \gamma_5) u_a. \] (10)

From eq. (7) and equations of motion for \( u \)- and \( d \)-quarks, we find

\[ \langle 0 | \bar{d} \gamma_5 u | P(q) \rangle = -i \frac{m_{\bar{d}}}{m_{u} + m_{\bar{d}}} f_P. \] (11)

Using eq. (11), we obtain the \( s \)-channel contribution as

\[ (A_P^s)_{ij} = -\frac{1}{2\sqrt{2}m_{\ell_i}} \frac{m_{P}^2}{m_{u} + m_{\bar{d}}} \sum_{k=1}^{3} \frac{\lambda_{kji}^* \lambda_{kab}'}{m_{LLk}^2}. \] (12)

The charged Higgs contribution can be calculated from the interaction Lagrangian,

\[ \mathcal{L} = V_{uad}^* \left\{ \frac{gm_{\bar{d}}}{\sqrt{2}m_W} \tan \beta \bar{d}_b P_L u_a H^- + \frac{gm_{ua}}{\sqrt{2}m_W} \cot \beta \bar{d}_b P_R u_a H^- \right\} \]
\[ + \frac{gm_{\ell_i}}{\sqrt{2}m_W} \tan \beta \bar{\nu}_i P_R \ell_i H^+ + \text{h.c.}, \] (13)

where \( g \) denotes the SU(2)\(_L\) gauge coupling constant, and \( \tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle \) is a ratio of the vacuum expectation values of two Higgs doublets \( H_u \) (the weak hypercharge \( Y = 1/2 \)) and \( H_d \) (\( Y = -1/2 \)). We obtain the charged Higgs contribution \( A_H^P \) from eq. (13) as

\[ A_H^P = -G_F V_{uad}^* \frac{m_{\bar{d}}}{m_{u} + m_{\bar{d}}} \frac{m_{P}^2}{m_H^-} \left( \tan^2 \beta - \frac{m_{ua}}{m_{\bar{d}}} \right). \] (14)

Since leptons in the final state due to the charged Higgs exchange are flavor diagonal, the indices \( i, j \) are suppressed in l.h.s. of eq. (14). Note that the overall sign of the r.h.s. of eq. (14) is opposite compared to the \( W \)-boson exchange (6), the charged Higgs boson contribution destructively interferes with the \( W \)-boson contribution when \( \tan^2 \beta > m_{ua}/m_{\bar{d}} \).

Next we summarize the experimental data and constraints on the parameter \( r_P \) for \( P = D_s \) and \( B^+ \). Comparison of the experimental results of leptonic decay of \( D_s \) meson is often presented in terms of the decay constant \( f_{D_s} \). The world average of the experimental data of \( D_s \to \tau \nu \) has been given by HFAG as [18]

\[ f_{D_s}^{\text{exp}} = 257.3 \pm 5.3 \ [\text{MeV}], \] (15)

while the HPQCD collaboration has updated their evaluation of the decay constant \( f_{D_s} \) as [17]

\[ f_{D_s}^{\text{SM}} = 248 \pm 2.5 \ [\text{MeV}]. \] (16)
The discrepancy between eqs. (15) and (16) is 1.6σ, and we find that the constraint on \( r_{D_s} \)-parameter as

\[
r_{D_s} = 1.04 \pm 0.03. \tag{17}
\]

For the leptonic decay \( B^+ \to \tau \nu \), the experimental data of the branching ratio have been given by Belle and BABAR \cite{8, 9}. The average of the data and the SM prediction given by the UTfit collaboration \cite{10} is

\[
\text{Br}(B^+ \to \tau \nu)_{\text{exp}} = (1.73 \pm 0.34) \times 10^{-4}, \tag{18}
\]

\[
\text{Br}(B^+ \to \tau \nu_{\tau})_{\text{SM}} = (0.84 \pm 0.11) \times 10^{-4}. \tag{19}
\]

The difference between (18) and (19) is 2.5σ and we find

\[
r_{B^+} = 1.44 \pm 0.23. \tag{20}
\]

3 Numerical Study

As is shown in eq. (3), the new physics contributions to the leptonic decay of a pseudo scalar meson \( P \) consist of the lepton flavor diagonal part \( A^P_{ii} \) and off-diagonal part \( A^P_{ij} (i \neq j) \). We study the contributions of \( R \)-parity violating interactions to the diagonal part \( A^P_{ii} \) \cite{5} in sec. 3.1, and the off-diagonal part \( A^P_{ij} (i \neq j) \) taking account of the other decay processes in sec. 3.2. In our analysis, we assume that the RPV couplings are real. We consider exchanges of the left-handed smuon and the right-handed sbottom in the \( s \)- and \( t \)-channel diagrams, respectively. Recall that the operator \( L_i L_j E_k \) in eq. (11) is anti-symmetric for \( i \) and \( j \). This is why we consider the smuon instead of the stau in the \( s \)-channel diagram. Throughout in our numerical analysis, the sfermion masses are fixed at 100 GeV, and we adopt the central values of the following parameters \cite{24}

\[
|V_{cs}| = 1.023 \pm 0.036, \quad |V_{ub}| = (3.89 \pm 0.44) \times 10^{-3}, \quad |V_{cb}| = (40.6 \pm 1.3) \times 10^{-3},
\]

\[
m_{D_s} = 1968.47 \pm 0.33 \text{ MeV}, \quad m_{B^+} = 5279.17 \pm 0.4 \text{ MeV}, \quad m_{B^0} = 5279.5 \pm 0.5 \text{ MeV}. \tag{21}
\]

3.1 Constraints on the RPV couplings from the flavor diagonal final state

In this section, we focus on the final state where the lepton pair is flavor diagonal \( (\tau \nu_{\tau}) \), i.e., we set \( A_{ij} = 0 \) in eq. (3). The experimental data of both \( D_s \to \tau \nu \) and \( B^+ \to \tau \nu \) favor the positive interference between the SM \( W \)-boson exchange and the
new physics contribution as shown in eqs. (17) and (20). Due to the sign of r.h.s. in the $s$-channel slepton exchange (12) and the $t$-channel squark exchange (8), those contributions could be cancelled each other when the products of RPV couplings in both $s$- and $t$-channel diagrams have the same sign. In Fig. 1, we show the 1-$\sigma$ allowed region of the $r_{D_s}$-parameter (left) and the $r_{B^+}$-parameter (right) for $m_{\tilde{\mu}_L} = m_{\tilde{\tau}_R} = 100$GeV.

The charged Higgs boson contributions is negligibly small for $D_s \to \tau \nu$ but sizable for $B^+ \to \tau \nu$ when $\tan \beta$ is large. The dependence of the charged Higgs boson contributions to the $r_{B^+}$-parameter is shown in the right side of Fig. 1 for $\tan \beta = 50$. The bands with solid line are obtained without the charged Higgs boson contribution while those with dashed line are obtained for $m_{H^-} = 300$ GeV, respectively.

In each bands, the inner lines denote the 1-$\sigma$ lower bounds while the outers are the upper bounds. We find from the figures that the allowed region of $s$-channel couplings shows positive correlations with the $t$-channel couplings. This is because the interference between the $s$- and $t$-channel contributions is destructive. For $D_s \to \tau \nu$, since the $t$-channel coupling is always positive ($|\lambda'_{323}|^2 \geq 0$), not only the magnitude but also the sign of $s$-channel coupling $\lambda_{222}^s \lambda_{233}^s$ is strongly constrained. For $\lambda_{222}^s \lambda_{233}^s \leq 0$, the $t$-channel coupling $|\lambda'_{323}|^2$ should be smaller than 0.07, and $-0.08 \leq \lambda'_{222} \lambda_{233}^s \leq 0$ is experimentally allowed in the 1-$\sigma$ level. For $B^+ \to \tau \nu$, the $s$- and $t$-channel couplings

Figure 1: Constraints on the RPV couplings from $D_s \to \tau \nu$ (left) and $B^+ \to \tau \nu$ (right). The horizontal axis represents the RPV couplings for the $t$-channel while the vertical axis denotes the couplings for the $s$-channel diagram. The bands correspond to the 1-$\sigma$ allowed range for the $r_{D_s}$ (left) and $r_{B^+}$ (right) parameters, respectively.

\[
\lambda'_{323}^s \lambda_{233}^s \
\lambda_{222}^s \lambda_{233}^s \
|\lambda'_{323}|^2 \
\lambda'_{222} \lambda_{233}^s
\]
with opposite signs are strongly constrained. As can be seen in Fig. 1 when the $t$-channel coupling is positive ($\lambda'_3 \lambda_3^\ast \geq 0$), the negative $s$-channel coupling is constrained to be $-0.0004 \lesssim \lambda'_{23} \lambda_{23}^\ast \leq 0$ when there is no charged Higgs contribution. Although the leptonic decays of $D_s$ and $B^+$ mesons are useful to constrain the sign of the relevant RPV couplings, the size of the couplings cannot be restricted because of the cancellation between the $s$- and $t$-channel diagrams. However, since several RPV couplings could be large simultaneously, it may lead to observation of productions or decay processes of SUSY particles due to the RPV interactions at LHC.

3.2 Constraints on the RPV couplings from the final state with lepton flavor violation

We have so far neglected the contributions of RPV interactions to the flavor off-diagonal part, $A_{ij}^P$ in eq. (3). In this subsection we examine $A_{ij}^P$ taking account of experimental bounds on the RPV couplings from the other lepton flavor violating processes. Since the lepton flavor off-diagonal terms in eq. (3) are always positive, one may expect that the deviation from the SM prediction could be explained by these terms when the flavor diagonal contribution $A_{ii}^P$ is negligible due to the cancellation between the $s$- and $t$-channel diagrams. We set, therefore, $A_{ii}^P = 0$ in the following analysis.

3.2.1 $D_s \to \tau \nu$

First we examine $D_s \to \tau \nu$. As shown in Fig. 2 replacing $(s_L, \nu_\mu)$ in the $t$-channel diagram by $(c_L, \mu_L)$, we obtain the Feynman diagram for the lepton flavor violating process $\tau \to \mu \gamma$. For $\tau \to \mu \gamma$, the relevant RPV couplings are $\lambda'_{32i} \lambda_{22i}$ where $i$ denotes the generation index of the right-handed down squark. Constraints on $\lambda'_{32i} \lambda_{22i}$ from $\tau \to \mu \gamma$ have been studied in ref. [16] using the experimental data given by Belle [25] as,

$$\text{Br}(\tau \to \mu \gamma) < 4.5 \times 10^{-8}. \quad (22)$$

Figure 2: The Feynman diagram of the $t$-channel squark exchange in $D_s \to \tau \nu$ (left) and the LFV process $\tau \to \mu \gamma$ (right).
They found the constraint on the RPV couplings $\lambda'_{32i} \lambda'_{22i} < 9 \times 10^{-2}$ for $m_{\tilde{d}_R} = 300$ GeV. Since we adopt $m_{\tilde{d}_R} = 100$ GeV as a reference value in our numerical study, the constraint on the RPV coupling can be read as

$$\lambda'_{32i} \lambda'_{22i} < 1 \times 10^{-2}.$$  \hfill (23)

The bound (23) will be as small as $\lambda'_{32i} \lambda'_{22i} < 4 \times 10^{-3}$ for $\text{Br}(\tau \to \mu \gamma) < 1 \times 10^{-8}$ which is expected to be achieved at future super B-factory with the integrated luminosity 5ab$^{-1}$. \footnote{26}

We introduce a parameter $\Delta r_P(P = D_s, B^+)$ to estimate the contributions of the RPV interactions as

$$\Delta r_P \equiv r_P - (r_P)_{SM}, \quad (P = D_s, B^+),$$  \hfill (24)

where $(r_P)_{SM} = 1$. In Fig. 3, the parameter $\Delta r_{D_s}$ is shown as a function of the RPV coupling $\lambda'_{32i} \lambda'_{22i}$. The vertical line denotes the experimental bound on $\lambda'_{32i} \lambda'_{22i}$ given in eq. (23). When $\lambda'_{32i} \lambda'_{22i} = 1 \times 10^{-2}$, the magnitude of $\Delta r_{D_s}$-parameter is $\sim 10^{-4}$. Since the experimental bound on the $r_{D_s}$-parameter is $r_{D_s} = 1.04 \pm 0.03$ \footnote{17}, $\Delta r_{D_s} \lesssim 10^{-4}$ is small enough so that the contributions from the RPV coupling $\lambda'_{32i} \lambda'_{22i}$ is negligible.

A product of RPV couplings $\lambda_{i23} \lambda'_{i22}$ which appears in the s-channel diagram of $D_s \to \tau \nu_\mu$ also lead to the lepton flavor violating process $\tau \to \mu \eta$ as shown in Fig. 4. The constraint on $\lambda_{i23} \lambda'_{i22}$ has been studied by Li et al. \footnote{27} as $\lambda_{i23} \lambda'_{i22} < 8.03 \times 10^{-4}$.
Figure 4: The Feynman diagram of the s-channel slepton exchange in $D_s \to \tau \nu_\mu$ (left) and the LFV process $\tau \to \mu \eta$ (right).

For $m_{\tilde{\ell}_L} = 100\text{GeV}$ from the experimental limit $\text{Br}(\tau \to \mu \eta) < 1.5 \times 10^{-7}$ given by Belle in 2005 [29]. We update the bound given in ref. [27] using the new data in 2010 by Belle [28]

$$\text{Br}(\tau \to \mu \eta) < 2.3 \times 10^{-8},$$

and find that the bound on the RPV couplings is stronger than a previous one as

$$\lambda_{i23} \lambda'_{22} < 3.1 \times 10^{-4}.$$  \hspace{1cm} (26)

We show $\Delta r_{D_s}$ as a function of the RPV couplings $\lambda_{i23} \lambda'_{22}$ in Fig. 3 (right). The vertical line denotes the experimental limit on $\lambda_{i23} \lambda'_{22}$ from $\text{Br}(\tau \to \mu \eta)$ shown in eq. (26). It is easy to see that, taking account of the constraint on RPV coupling (26), the magnitude of $\Delta r_{D_s}$-parameter is at most $\sim 10^{-5}$, which is negligibly small as compared to the experimental uncertainty (20).

We conclude that, in $D_s \to \tau \nu$, the contributions of the RPV interactions to the lepton flavor off-diagonal part $A_{ij}$ in eq. (3) is highly suppressed from the constraints on the other LFV processes, $\tau \to \mu \gamma$ and $\tau \to \mu \eta$. The deviation from the SM contribution in the $r_{D_s}$-parameter is predicted to be smaller than $10^{-4}$ ($t$-channel) or $10^{-5}$ ($s$-channel) so that we can safely neglect these contributions.

### 3.2.2 $B^+ \to \tau \nu$

For $B^+ \to \tau \nu$, the relevant RPV couplings in $A_{3j}^{B^+}$ (3) are $\lambda'_{3i} \lambda'_{j3i}$ and $\lambda_{ij3} \lambda_{13j}$ for the $t$- and the $s$-channel diagrams, respectively. Note that the indices $i$ and $j$ denote the flavor of slepton $\tilde{\ell}_{Li}$ and neutrino $\nu_j$. Hereafter we consider $j = 2$ in our analysis for simplicity.

The couplings in the $t$-channel diagram, $\lambda'_{3i} \lambda'_{23i}$, also appear in the semileptonic decays of $b$-quark such as $b \to u \ell \nu$ or $b \to d \nu \nu$. Since the latter is a typical process
Figure 5: The $r_{B^+}$-parameter as a function of the RPV couplings in the $t$-channel diagram, $\lambda'_{31i}\lambda'_{23i}$ for the slepton mass $m_{\tilde{\ell}_L} = 100$ GeV. The contributions from the flavor diagonal part in the final leptons are set to be zero, i.e., $A_{ii}^{B^+} = 0$ in eq. (3).

due to the flavor changing neutral current, the SM contribution is given by the 1-loop diagram while the RPV interactions lead to the same process at the tree level. Therefore strong constraints can be expected from $b \to d \nu\bar{\nu}$.

Constraints on the RPV coupling $\lambda'_{31i}\lambda'_{23i}$ from $B^+ \to \pi^+ \nu\bar{\nu}$ have been studied in ref. [30]. The authors in ref. [30] found the constraint on the RPV coupling $\lambda'_{31i}\lambda'_{23i} < 2.5 \times 10^{-2}$ for $m_{\tau} = 500$GeV. It can be read, for $m_{\tau} = 100$GeV, as

$$\lambda'_{31i}\lambda'_{23i} < 1.0 \times 10^{-3}.$$  \hfill (27)

We depict the $r_{B^+}$-parameter as a function of the RPV couplings $\lambda'_{31i}\lambda'_{23i}$ for the squark mass $m_{\tilde{d}_R} = 100$GeV in Fig. 5. In the figure, we set the contributions to the $r_{B^+}$-parameter due to the RPV interactions in the diagonal part of the lepton flavor to be zero, i.e., $A_{ii}^{B^+} = 0$ in eq. (3). It is easy to see that, taking account of the bound from $B^+ \to \pi^+ \nu\bar{\nu}$, the contributions from the couplings $\lambda'_{31i}\lambda'_{23i}$ are not enough to explain the data of $r_{B^+}$-parameter in the 1-$\sigma$ level, $r_{B^+} = 1.44 \pm 0.23$ in eq. (20), though the contributions slightly increase the $r_{B^+}$-parameter from unity.

Next we study contributions of the RPV interactions in the $s$-channel diagram. The combination of the RPV couplings $\lambda_{ij3}\lambda'_{1i3}$ in the $s$-channel diagram of $B^+ \to \tau \nu_j$ ($j = 1, 2$ or $e, \mu$) can also contribute to the lepton flavor violating processes $B^0 \to \tau^\pm e^\mp$ or
$B^0 \rightarrow \tau^\pm \mu^\mp$. The decay rate of $B^0 \rightarrow \ell^+_i \ell^-_j$ is given by

$$\Gamma(B^0 \rightarrow \ell^+_i \ell^-_j) = \frac{1}{128\pi} \frac{|\lambda_{kij}X'_{1k13}|^2}{m_{\tau_k}^4} f_B \frac{m_B^5}{m_B^4} \left(1 - \frac{m_{\ell_i}^2}{m_B^2}\right)^2,$$

$$\approx 2.2 \times 10^{-10} \frac{|\lambda_{kij}X'_{1k13}|^2}{m_{\ell_i}^4} \left(\frac{100\text{GeV}}{m_{\ell_i}}\right)^4,$$  \hspace{1cm} (28)

where $m_{\ell_i} \gg m_{\ell_j}$ is assumed. The branching ratio of $B^0 \rightarrow \ell^+_i \ell^-_j$ is obtained by

$$\text{Br}(B^0 \rightarrow \ell^+_i \ell^-_j) = \frac{\Gamma(B^0 \rightarrow \ell^+_i \ell^-_j)}{\Gamma(b \rightarrow c\bar{\ell} \nu)} \text{Br}(b \rightarrow c\bar{\ell} \nu).$$ \hspace{1cm} (29)

The experimental bounds on $\text{Br}(B^0 \rightarrow \tau^\pm e^\mp)$ and $\text{Br}(B^0 \rightarrow \tau^\pm \mu^\mp)$ are summarized by HFAG as $^{31}$

$$\text{Br}(B^0 \rightarrow \tau^\pm e^\mp) < 28 \times 10^{-6},$$ \hspace{1cm} (30a)

$$\text{Br}(B^0 \rightarrow \tau^\pm \mu^\mp) < 22 \times 10^{-6}.$$ \hspace{1cm} (30b)

We show the branching ratio of $B^0 \rightarrow \ell^+_i \ell^-_j$ as a function of the RPV couplings for the $s$-channel diagram of $B^+ \rightarrow \tau \nu$ in Fig. 6 (left). The sneutrino mass is fixed at 100GeV. In the figure, two horizontal lines denote the experimental upper bound of $B^0 \rightarrow \tau^\pm e^\mp$ (dashed) and $B^0 \rightarrow \tau^\pm \mu^\mp$ (dotted), respectively. Due to the experimental
bounds on $B^0 \rightarrow \tau^\pm e^\mp$ and $B^0 \rightarrow \tau^\pm \mu^\mp$ in eqs. (30a) and (30b), the upper limits of the RPV couplings slightly differ between $\lambda_{i23}\lambda'_{i13}$ and $\lambda_{i13}\lambda'_{i13}$. From the figure, we find constraints on the RPV couplings as

$$\lambda_{i23}\lambda'_{i13} \lesssim 1.7 \times 10^{-4}, \quad (31a)$$

$$\lambda_{i13}\lambda'_{i13} \lesssim 1.9 \times 10^{-4}. \quad (31b)$$

Now we are ready to study contribution to the $r_{B^+}$-parameter through the RPV interactions with couplings $\lambda_{i23}\lambda'_{i13}$ and $\lambda_{i13}\lambda'_{i13}$ taking account of the experimental constraints from $B^0 \rightarrow \ell_i^+ \ell_j^-$, (31a) and (31b). We show the $r_{B^+}$-parameter as a function of the RPV couplings $\lambda_{i23}\lambda'_{i13}$ in Fig. 6 (right). The 1- and 2-$\sigma$ bounds of the $r_{B^+}$-parameter are shown by the horizontal dashed lines as indicated in the figure. The vertical line denotes the constraints on the RPV couplings from the branching ratio of $B^0 \rightarrow \tau^\pm \mu^\mp$. We find that, taking account of the experimental data of $\text{Br}(B^0 \rightarrow \tau^\pm \mu^\mp)$, the constraints on $r_{B^+}$-parameter given by the lepton flavor off-diagonal coupling $\lambda_{i23}\lambda'_{i13}$ is $r_{B^+} \lesssim 1.15$, which is smaller than the 1-$\sigma$ lower bound, $r_{B^+} = 1.21$. However, $r_{B^+}$ may be enhanced when we add contributions from $\lambda_{i23}\lambda'_{i13}$ and $\lambda_{i13}\lambda'_{i13}$. Under the constraints given in eqs. (31a) and (31b), we find the $r_{B^+}$-parameter could be at max 1.22 which is slightly larger than the 1-$\sigma$ lower bound, $r_{B^+} = 1.21$. The deviation in the $r_{B^+}$-parameter from the SM expectation can be, therefore, explained by the RPV couplings $\lambda_{i23}\lambda'_{i13}$ and $\lambda_{i13}\lambda'_{i13}$ which lead to the lepton flavor off-diagonal final state in $B^+ \rightarrow \tau \nu$ without conflicting the other LFV processes such as $B^0 \rightarrow \tau^\pm \mu^\mp, \tau^\pm e^\mp$.

4 Summary

We have studied the contributions from $R$-parity violating interactions to $D_s \rightarrow \tau \nu$ and $B^+ \rightarrow \tau \nu$. Owing to the latest Lattice QCD calculation of the decay constant $f_{D_s}$, the difference between the data and the SM expectation is only 1.6$\sigma$ in $D_s \rightarrow \tau \nu$ while there is still 2.5$\sigma$ deviation in $B^+ \rightarrow \tau \nu$.

The $R$-parity violating interactions could contribute to the leptonic decays through the slepton exchange in the $s$-channel diagram and the down-squark exchange in the $t$-channel diagram. In addition, the RPV interactions predict the final states not only the flavor diagonal lepton pair ($\tau \nu_i$) but also flavor off-diagonal pair ($\tau \nu_i, i = e, \mu$). In the flavor diagonal ($\tau \nu_i$) case, the supersymmetric contributions could interfere with the SM $W$-boson contribution either constructively or destructively, see eq. (3). We showed that, since the supersymmetric contributions consist of the $s$-channel and $t$-channel diagrams, the interference between them is also either constructive or destructive, owing to a choice
of relative sign of the RPV couplings between two diagrams. It was also shown that the experimentally allowed region of the RPV couplings are found to be strongly restricted when the relative sign of RPV couplings between two diagrams is opposite. On the other hand, when the RPV couplings in $s$- and $t$-channel diagrams have the same sign, these couplings are allowed to be large simultaneously.

In the flavor off-diagonal case, as shown in eq. (3), the new physics contributions $A_{ij}$ to the leptonic decay always interfere with the SM contribution constructively. The RPV interactions in $A_{ij}^{s}$ also contribute to the LFV processes $\tau \rightarrow \mu \gamma$ and $\tau \rightarrow \mu \eta$. Taking account of experimental bounds on the RPV couplings from these decay processes, we found that $A_{ij}^{s}$ is highly suppressed and the contribution is smaller than the experimental uncertainty by two or more orders of magnitude. The RPV couplings in the $t$-channel diagram of $B^{+} \rightarrow \tau \nu_{\mu}$ are constrained from $B^{+} \rightarrow \pi^{+} \nu \bar{\nu}$ so that the $t$-channel contribution to $A_{ij}^{B^{+}}$ cannot explain the 2.5$\sigma$ discrepancy between the data and the SM prediction in the leptonic decay of the $B^{+}$ meson. The RPV couplings in the $s$-channel diagram also contribute to the LFV processes $B^{0} \rightarrow \tau^{\pm} \mu^{\mp}, \tau^{\pm} e^{\mp}$. We found that, taking account of the experimental bound on the RPV couplings $\lambda_{i23}\lambda_{i13}'$ from $B^{0} \rightarrow \tau^{\pm} \mu^{\mp}$, the $r_{B^{+}}$-parameter is given as $r_{B^{+}} \lesssim 1.15$ which is smaller than the 1-$\sigma$ lower bound ($r_{B^{+}} = 1.44 \pm 0.23$). However, taking a sum of contributions from $\lambda_{i23}\lambda_{i13}'$ in $B^{+} \rightarrow \tau \nu_{\mu}$ and $\lambda_{i13}'\lambda_{i13}'$ in $B^{+} \rightarrow \tau \nu_{e}$, the $r_{B^{+}}$-parameter could be at max 1.22 which is consistent with the 1-$\sigma$ lower bound.

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