We consider self-magnetization of charged and neutral vector bosons bearing a magnetic moment in a gas and in vacuum. For charged vector bosons (W bosons) a divergence of the magnetization in both the medium and the electroweak vacuum occurs for the critical field \( B = B_{wc} = m_w^2/e \). For \( B > B_{wc} \) the system is unstable. This behavior suggests the occurrence of a phase transition at \( B = B_c \), where the field is self-consistently maintained. This mechanism actually prevents \( B \) from reaching the critical value \( B_c \). For virtual neutral vector bosons bearing an anomalous magnetic moment, the ground state has a similar behavior for \( B = B_{nb} = m_{nb}^2/q \). The magnetization in the medium is associated to a Bose-Einstein condensate and we conjecture a similar condensate occurs also in the case of vacuum. The model is applied to virtual electron-positron pairs bosonization in a magnetic field \( B \sim B_{pc} \lesssim 2m_e^2/e \), where \( m_e \) is the electron mass. This would lead also to vacuum self-magnetization in QED, where in both cases the symmetry breaking is due to a condensate of quasi-massless particles.

I. INTRODUCTION

Macroscopic bodies become unstable when its rest energy is of the same order than the interaction energy with some field. For instance, a classical instability is produced when the gravitational and rest energies of a body of mass \( M \) and radius \( R \) are of the same order \( Mc^2 \sim GM^2/R \), leading to a gravitational collapse. Quantum magnetic collapse for macroscopic magnetized objects is also claimed to occur for high magnetic fields \[1, 2, 3\] when the magnetic energy density tends to be equal to the internal energy density.

Of especial interest is the instability of the energy ground state resulting from the solution of the Dirac equation for an electron in a Coulomb field large enough. The eigenvalues \[4\]
are $E = mc^2/\sqrt{1 + (Z^2\alpha^2)/(s+n')^2}$, where $s = \pm (k^2 - Z^2\alpha^2)^{1/2}$, and $k = \pm 1, \pm 2, ...$, $n' = 0, 1, 2...$ It is well known that for $n' = 0$, $k = 1$ the denominator diverges if $Z = 1/\alpha$. That is, there is a critical electric field $E_c = Ze/\lambda^2 = m^2c^3/\epsilon\hbar$ for which electrons and positrons can be created spontaneously from the decay of vacuum: vacuum boils. Thus, atoms with atomic number $Z \geq 1/\alpha \approx 137$ are not stable, due to the QED vacuum instability for electric fields $E > E_c$.

The usual electroweak vacuum in an external magnetic field $B$ has also an instability for fields greater than some critical value $B_{wc} = m^2w/e \sim 10^{24}G$, due to the presence of charged vector bosons $W$ ($m_w$ is the W boson mass). This problem was studied by Ambjorn and Olesen [5, 6] who found solutions $B > B_{wc}$ for classical equations of motion.

Here we analyze the problem of quantum stability of degenerate $W$ boson gas and vacuum in a magnetic field, starting from the quantum statistical point of view and methods. The present authors interpret the mentioned instability of bosonic vacuum as indicating a phase transition to a self-magnetized state. We want to remark that for much lower fields, $B \sim 10^{20}G$ the similar instability might appear for instance, for $\rho, \omega$ vector mesons or paired fermions in states of spin unity. A similar behavior has been found in the case of neutral vector bosons with an anomalous magnetic moment which suggests the applicability of this model to describe the positronium behavior in a strong magnetic field and to discuss the possibility of QED vacuum self-magnetization.

II. CHARGED VECTOR BOSONS

For $W$ bosons in an external magnetic field $B_j = B\delta_{j3}$ ($j = 3$) the energy spectrum is

$$E_{wn} = \sqrt{p_3^2 + m_w^2 + 2eB(n + \frac{1}{2})}, \quad E_{wg} = \sqrt{p_3^2 + m_w^2 - eB},$$

where $n = 0, 1, 2...$ are the Landau quantum numbers and $p_3$ is the momentum component along the field direction. The ground state $E_{wg}(p_3 = 0)$ vanishes for $B = B_{wc}$, and becomes imaginary (unstable) for $B > B_{wc}$.

We started from the expression for the thermodynamic potential at the tree level approximation

$$\Omega_w = \Omega_{sw} + \Omega_{0w}. $$
The first term in (2) is the statistical contribution\[11\]
\[
\Omega_{sw} = \frac{eB}{4\pi^2}\beta \int_{-\infty}^{\infty} d\nu_3 \ln[(1 - e^{-(E_{wg}-\mu_w)\beta})(1 - e^{-(E_{wg}+\mu_w)\beta})] + \\
+ \frac{eB}{4\pi^2\beta} \sum_{n=0}^{\infty} b_n \int_{-\infty}^{\infty} d\nu_3 \ln[(1 - e^{-(E_{wn}-\mu_w)\beta})(1 - e^{-(E_{wn}+\mu_w)\beta})].
\] (3)

Here \(b_n = 3 - \delta_{n0}\), \(\beta = 1/T\) is the inverse of temperature and \(\mu_w\) is the chemical potential.

The second one
\[
\Omega_{0w} = \frac{eB}{4\pi^2} \int_{-\infty}^{\infty} d\nu_3 (\sum_{n=0}^{\infty} E_{wg} + b_n E_{wn}),
\] (4)
corresponds to the zero point energy density of vacuum, obtained as the zero temperature and zero density limit of \(\Omega_w\). After regularization, we get an Euler-Heisenberg\[12\] like term
\[
\Omega_{0w} = -\frac{e^2B^2}{16\pi^2} \int_0^{\infty} e^{-B_w x/B} \left( \frac{1 + 2\cosh 2x}{\sinh x} - \frac{3x}{2} \right) \frac{dx}{x^2} < 0.
\] (5)

The \(W\) boson density \(N_w = -\frac{\partial\Omega_{sw}}{\partial \mu_w}\) looks like
\[
N_w = \frac{eB}{4\pi^2} \left[ \int_{-\infty}^{\infty} d\nu_3 (n_{ow-} - n_{ow+}) + \sum_{n=0}^{\infty} \beta_n \int_{-\infty}^{\infty} d\nu_3 (n_{nw-} - n_{nw+}) \right],
\] (6)
\[
n_{ow\pm} = \left[ e^{(E_{wg}+\mu_w)\beta} - 1 \right]^{-1}, \quad n_{nw\pm} = \left[ e^{(E_{wn}+\mu_w)\beta} - 1 \right]^{-1}.
\] (7)

The magnetization \(M_w = -\partial\Omega_w/\partial B\) contains the contributions of both real and virtual \(W\) bosons
\[
M_w = M_{sw} + M_{0w},
\] (8)
being
\[
M_{sw} = -\frac{\Omega_w}{B} + \frac{e^2B}{4\pi^2} \int_{-\infty}^{\infty} d\nu_3 \frac{(n_{ow-} + n_{ow+})}{\varepsilon_{ow}} - \\
- \frac{e^2B}{4\pi^2} \sum_{n=0}^{\infty} \beta_n \left( n + \frac{1}{2} \right) \int_{-\infty}^{\infty} d\nu_3 \frac{(n_{nw-} + n_{nw+})}{\varepsilon_{nw}}.
\] (9)

It can be observed that the expression (9) contains positive (ferromagnetic) and negative (diamagnetic) contributions, coming from the ground and excited Landau states, respectively. Vacuum shows a paramagnetic behavior, described by
\[
M_{0w} = -\frac{2\Omega_{0w}}{B} + \frac{em_w^2}{16\pi^2} \int_0^{\infty} e^{-B_w x/B} \left( \frac{1 + 2\cosh 2x}{\sinh x} - \frac{3x}{2} \right) \frac{dx}{x} > 0.
\] (10)
A. Degenerate limit

For $eB \gg T^2$, the average W boson population in excited Landau states is negligible small, and most of the W density is in the Landau ground state, which near the zero momentum along $B$ behaves as

$$\Omega_{sw} \sim \frac{eB}{4\pi^2 \beta} \int_0^{p_0} dp_3 \ln[(\sqrt{p_3^2 + m_w^2} - eB) - \mu_w)\beta].$$

(11)

If $\mu_w \to (m_w^2 - eB)$, we would have an infrared divergence at $p_3 = 0$. Actually, the population in the Landau ground state increases as the parameter $T/N_{w}^{1/3}$ decreases: there is a Bose condensation but no critical temperature. For such conditions if $N_{0w}$ is the density in the ground state, the magnetization is

$$\mathcal{M}_{sw} = eN_{0w}/2\sqrt{m_w^2 - eB},$$

(12)

and the condition of self magnetization for $\frac{B}{B_{wc}} \equiv \eta$ can be written as

$$\eta = \frac{a}{\sqrt{1 - \eta}}, \quad a = \frac{2\pi e^2 N_w}{m_w^3}.$$

(13)

We see that for $eB \to m_w^2$, the field can be maintained self-consistently for $N_w \leq \frac{m_w^3 a_{max}}{2\pi e^2} \sim 10^{47}$ ($a_{max} = 0.3849$), the instability of the thermodynamic potential at $\mu_w = (m_w^2 - eB) = 0$ due to the arising of effectively massless vector charged particles, can be avoided and the field intensity is kept always as $B < B_{wc} \sim m_w^2/e$. The condensate behaves as ferromagnetic.

From the general expression for the energy momentum tensor, we get anisotropic pressures $P_{w3}$ and $P_{w\perp}$ in the directions parallel and perpendicular to the magnetic field, respectively,

$$P_{w3} = -\Omega_w$$

(14)

$$P_{w\perp} = -\Omega_w - BM_w,$$

(15)

We see that the contribution of observable particles, given by the statistical term in the expression for the total thermodynamic potential, vanishes in the zero temperature and zero chemical potential limit. The remaining term leads to the zero point energy of vacuum. For vacuum, the average particle density vanishes, but other quantities like energy density, magnetization and pressures are non-zero.
Experimental results in condensed matter have shown \[15\] that a fermion gas bosonize for temperatures close to zero (in usual CGS units, the adimensional parameter \( T/\hbar cN^{1/3} \) small enough), leading either to BCS pairing or to Bose-Einstein condensation. The last case shows ferromagnetic properties \[16\]. Thus, if the thermal disorder decreases enough, it leads through a phase transition to a lower energy highly ordered state. As the increasing magnetic field produces also an increasing order of the fermion system, and if \( eB \gg T^2 \), the mechanism of bosonization might lead to lower energy states through Bose-Einstein condensation. Thus, the occurrence of such mechanism is interesting in connection to the origin of large magnetic fields in some white dwarfs (e.g. \( 10^{10} \) G), if vector pairing electrons occur and condense \[17\]. (Also, if charged vector di-quarks are formed in neutron stars, a version of our model might be of interest in explaining the arising of larger fields (\( \geq 10^{13} \) G) in neutron stars).

B. \( W \) boson vacuum

In electroweak vacuum the magnetization \((10)\) diverges for \( B \geq B_{wc} \). One can pick up the logarithmical infrared divergence by considering again a neighborhood of zero momentum for the Landau ground state. One gets the term

\[
M_{0w} \sim -\frac{e^2 B_{wc}}{8\pi^2} \ln \left( \frac{B_{wc}}{B} - 1 \right) > 0.
\]  

This divergence (Fig. 1) is indicating a phase transition to a ferromagnetic state for \( B \approx B_{wc} \), which we may understand as due to a sort of Bose-Einstein condensation of electrically positive and negative virtual quanta whose effective mass is non-zero but arbitrary small.

By equating \( B = 4\pi M_{vac} \), one can obtain an electroweak vacuum self-magnetization satisfying

\[
\eta = \frac{1}{1 + e^{-\frac{B}{m_w}}}.
\]  

Thus, the self-magnetization avoids the divergence of both \( \Omega_{0w} \) and \( M_{0w} \).

Using the previous expressions \((14), (15)\) we find a positive pressure in the direction parallel to the field \( P_{0w3} = -\Omega_{0w} \), and a negative perpendicular pressure

\[
P_{0w\perp} = \Omega_{0w} - \frac{eBm_w^2}{16\pi^2} \int_0^{\infty} e^{-B_{wc}/B} \left( \frac{1 + 2 \cosh 2x}{\sinh x} - \frac{3}{x} - \frac{7x}{2} \right) \frac{dx}{x} < 0.
\]  


FIG. 1: \(W\) vacuum magnetization vs. magnetic field in a logarithmic scale. Note that \(M_{0w}\) diverges for \(B \to B_{wc}\). A similar behavior is expected for the neutral boson case.

This leads to magnetostrictive effects for any value of the magnetic field since \(W^+ - W^-\) vacuum is compressed perpendicularly to \(B\), due to the negative pressures, and as the pressure \(P_{0w3}\) is positive, it is stretched in along \(B\).

III. NEUTRAL VECTOR BOSONS

It is believed that neutron stars magnetic fields could be produced due to ferromagnetic spin coupling of neutrons. The boson state resulting from such pairing is more favorable energetically, since its Gibbs free energy is smaller than that of the original neutron system [18]. For neutral vector bosons with an anomalous magnetic moment we use (from Ref. [18]) the following spectrum

\[
E_{nb}(\eta) = \sqrt{p_3^2 + p_\perp^2 + m_{nb}^2 + \eta q B \sqrt{p_\perp^2 + m_{nb}^2}}.
\]  

(19)

\(\eta = -1, 0, 1\), leading to states with magnetic moment

\[
\mu(\eta) = -\frac{\partial E_{nb}}{\partial B} \bigg|_{p=0} = \frac{\eta q}{2 \sqrt{m_{nb} + \eta q B}}.
\]  

(20)
The ground state contain again effectively massless particles,
\[ E_{nb}(\eta = -1, p = 0) = \sqrt{m_{nb}^2 - qBm_{nb}} \] (21)
vanishes for \( B = B_{nbc} = \frac{m_{nb}}{q} \), and becomes imaginary for \( B > B_{nbc} \), in analogy to the charged case, leading to the instability.

The statistical part of the thermodynamic potential is
\[ \Omega_{snb} = -\frac{1}{4\pi^2 \beta} \sum_{\eta} \int_0^\infty p_\perp dp_\perp \int_{-\infty}^\infty dp_3 \ln[(1 - e^{-(E_{nb} - \mu_{nb})\beta})(1 - e^{-(E_{nb} + \mu_{nb})\beta})]. \] (22)

The zero point energy density is
\[ \Omega_{0nb} = -\frac{1}{4\pi^2} \int_{-\infty}^\infty dp_3 \int_0^\infty p_\perp dp_\perp \sum_{\eta} E_{nb}. \] (23)

By summing and integrating over all degrees of freedom and after regularization it leads to the Euler-Heisenberg like expression
\[ \Omega_{0nb} = -\frac{(qm_{nb}B)^{2}}{8\pi^2} (I_0^{(2)} + I_1^{(3)} + I_2^{(2)}) \] (24)
where \(-\frac{(qm_{nb}B)^{2}}{8\pi^2} I_0^{2}\) is the contribution of the states \( E_{nb}(p_\perp = 0) \), and
\[ I_0^{(k)} = \int_0^\infty e^{-\frac{p_\perp^2 x}{4}} (\cosh x - 1) \frac{dx}{x^k}, \] (25)
\[ I_1^{(k)} = \int_0^\infty e^{-\frac{p_\perp^2 x}{4}} (\cosh x - 1 - \frac{x^2}{2}) \frac{dx}{x^k}, \] (26)
\[ I_2^{(k)} = \int_0^\infty \int_0^\infty e^{-\frac{p_\perp^2 (u+1)^2}{4}} (\sinh x(u+1) - x(u+1) - \frac{x^3(u+1)^3}{6}) \frac{du}{u} \frac{dx}{x^k}. \] (27)

The neutral boson vacuum magnetization is
\[ M_{0nb} = -2\frac{\Omega_{0nb}}{B} + \frac{qm_{nb}^3}{8\pi^2} (I_0^{(1)} + I_1^{(2)} + I_2^{(1)}) > 0 \] (28)
The magnetization (28) is a positive quantity and diverges for \( B \to B_{nbc} = \frac{m_{nb}}{q} \), due to the behavior of the states \( E_{nb}(p_\perp = 0) \). But this means that neutral boson vacuum also can self-consistently maintain the field, keeping \( B < B_{nbc} \).

Again, vacuum paramagnetic properties conduce to the achievements of anisotropic pressures \( P_{0nb\perp} = -\Omega_{0nb} > 0 \) and
\[ P_{0nb\perp} = \Omega_{0nb} - \frac{qBm_{nb}^3}{8\pi^2} (I_0^{(1)} + I_1^{(2)} + I_2^{(1)}) < 0. \] (29)

For a gas of density \( N_{nb} \) in the condensate \( M_{nb} = qm_{nb}N_{nb}/(2\sqrt{m_{nb}^2 - m_{nb}qB}) \), and according to the value of \( N_{nb} \), self consistent fields \( B = 4\pi M_{nb} \) may occur up to \( B \sim 10^{17} \text{G} \). This might be another mechanism for production of extremely large fields in neutron stars [18]. For \( B \sim m_{nb}/q \sim 10^{20} \text{G} \), \( M_{nb} \) diverges.
A. QED vacuum

For an electron (positron) in an external magnetic field

\[ E_n = \sqrt{p_n^2 + m_e^2c^4 + 2eBn}, \quad n = 0, 1, 2, \ldots \]  \hspace{1cm} (30)

and the zero point energy density (Euler-Heisenberg term) in the tree level approximation is \[ \Omega_{0e} = \frac{e^2 B^2}{8 \pi^2} \int_0^{\infty} e^{-B_{ec}x/B} \left( \frac{\coth x}{x} - \frac{1}{x^2} - \frac{1}{3} \right) \frac{dx}{x} < 0 \]  \hspace{1cm} (31)

\( B_{ec} = \frac{m_e^2}{e} = 4.41 \cdot 10^{13} \text{G} \) is the critical Schwinger field.

\[ \mathcal{M}_{0e} = -2 \frac{\Omega_{0e}}{B} - \frac{e^2 B_{ec}}{8 \pi^2} \int_0^{\infty} e^{-B_{ec}x/B} F(x)_{HE} dx > 0. \]  \hspace{1cm} (32)

The electron-positron vacuum shows a paramagnetic behavior, but \( \mathcal{M}_{0e} \ll B \).

But for \( B \sim B_{ec} = m_e^2/e = 4.41 \times 10^{13} \text{G} \), the QED vacuum polarization effects, like the creation of electron-positron pairs by a photon, become important. Photons coexist with mutually independent virtual \( e^+ - e^- \) pairs and with bound \( e^+ - e^- \) virtual states (positronium), which is related to the singular behavior of the polarization operator \( \Pi_{\mu\nu} \) near the thresholds for these processes \[21\]. Such singularity contributes with an absorptive term: vacuum becomes unstable and decays in observable \( e^+ - e^- \) pairs or positronium. The first threshold for free pair creation occurs for the minimal photon energy \( \omega = 2m_e \). For a smaller energy, \( \omega = m_p = \lambda \cdot 2m_e \) vacuum decay in \( e^+ - e^- \) bound states, i.e., positronium, where \( \lambda = 1 - \Delta\varepsilon/2m_e \) and \( \Delta\varepsilon \) is the positronium binding energy. We want to address the problem of the positronium vacuum in a magnetic field.

The positronium mass in the Landau ground state of the electron and positron is \( m_p = 2m_e - \Delta\varepsilon \), where \( \Delta\varepsilon \) is the binding energy which is due to the Coulomb interaction between the electron and the positron in presence of a high magnetic field \( (m_p(B) \lesssim 2m_e) \). For the Coulomb ground state, \( \Delta\varepsilon \) reaches high values when the distance between the Larmor orbits of the electron and the positron tends to zero \[22\]. We introduce as fundamental assumption that the relativistic expression for the energy of such one-dimensional positronium state bearing an anomalous magnetic moment is a particular case of \[19\] \[ E_p = \sqrt{p_p^2 + m_p^2 - qBm_p}, \]

with \( q = 2\mu_B \), and a degeneracy factor \( eB \).

This is equivalent to assume the bosonization of the pair resulting from the parallel and antiparallel spin coupling of virtual electrons and positrons, leading to a neutral boson
with a magnetic moment \( q = 2\mu_B \) and confined to move parallel to the field \( B \). These virtual positronium states lead to a logarithmic divergence of the neutral boson vacuum magnetization for \( B \to B_{pc} \lesssim 2m_e^2/e \).

The energy density is
\[
\Omega_{0p} = -\left(\frac{eB}{2\pi^2}\right)^2 \int_0^\infty \frac{e^{-\nu_{pc}B}x}{x^2} \frac{dx}{x^2} \quad (33)
\]

One can obtain easily the magnetization
\[
\mathcal{M}_{0p} = -\frac{\partial \Omega_{0p}}{\partial B} \quad (34)
\]

It is easily seen that \( \mathcal{M}_{0p} \) diverges logarithmically. This divergence is indicating a phase transition to a ferromagnetic state for \( B \approx B_{pc} \). By equating \( B = 4\pi M_{0p} \) and calling \( \eta' = B/B_{pc}' \), one obtain a vacuum self-magnetization satisfying
\[
\eta' = \frac{1}{1 + e^{-\eta' / \eta}}. \quad (35)
\]

This suggests the arising of a ferromagnetic phase transition for QED vacuum, which we may understand as due to a sort of Bose-Einstein condensation of positronium in its ground state whose effective mass is arbitrary small.

It means that near \( B_{pc} \) all the previous considerations about neutral boson vacuum behavior for \( B \to B_{pc} \) are applicable to the positronium vacuum; in particular, vacuum self-magnetization might be possible in QED.

**IV. CONCLUSIONS**

For electroweak vacuum, the contribution from \( W^\pm \) vector bosons to the ground state energy shows an instability for \( B > B_{wc} = m_{w^2}/e \). The magnetization \( \mathcal{M}_w \) diverges in both the dense medium and the vacuum cases for \( B \to B_{wc} \). By equating \( B = 4\pi \mathcal{M}_w \), the field can be self-consistently maintained, i.e. becomes a ferromagnet. This mechanism actually prevents \( B \) from reaching \( B_{wc} \). For neutral vector bosons with an anomalous magnetic moment \( q \) the ground state also shows an instability for \( B > B_{nbc} = m_{n^2}/q \) in a medium and in vacuum. \( \mathcal{M}_{0nb} \) also diverges for \( B \to B_{nbc} \) and, as a consequence, can be self-consistently maintained, keeping \( B < B_{nbc} \). We conjecture this mechanism might be applied to magnetized QED vacuum by assuming virtual positronium as the neutral vector particle.
with anomalous magnetic moment. Such phase transition would mean the arising of an “order parameter”, or symmetry breaking of vacuum. It is due to the condensate of quasi-massless particles, bearing some analogy with the Goldstone case.

[1] A. Pérez Martínez, H. Pérez Rojas and H. J. Mosquera Cuesta, *Eur.Phys.J.*, **C29**, 111-123 (2003)

[2] R. Gonzalez Felipe, H. J. Mosquera Cuesta, A. Pérez Martínez, H. Pérez Rojas, *Chin.J.Astron.Astrophys.*, **5**, 399 (2005)

[3] M. Chaichian, S. Masood, C. Montonen, A. Pérez Martínez, H. Pérez Rojas, *Phys. Rev. Lett.*, **84**, 5261 (2000).

[4] L. I. Schiff, *Quantum mechanics*, New York, McGraw-Hill Book Co.(1955).

[5] J. Ambjorn and P. Olesen, *Nucl. Phys. B* **315** 606-614 (1989).

[6] J. Ambjorn and P. Olesen, *Nucl. Phys. B* **330** 193-204 (1990).

[7] C. Bernard, *Phys. Rev. D* **9** (1974) 3312.

[8] J.I. Kapusta, *Phys. Rev. D* **24** (1981) 486.

[9] H. Perez Rojas, O.K. Kalashnikov, *Nucl. Phys. B* **293** (1987) 241.

[10] E. S. Fradkin, *Quantum Field Theory and Hydrodynamics*, Proc. of the P.N. Lebedev Inst. No. **29**, Consultants Bureau (1967).

[11] H. Perez Rojas, *Acta Phys. Pol.*, **B17**, 861 (1986).

[12] W. Heisenberg and H. Euler, *Z. Phys.*, **98**, 714 (1936)

[13] E. Rodriguez Querts, A. Martin Cruz and H. Perez Rojas, *Int. J. Mod. Phys. A*, **17**, 561-573 (2002)

[14] E. Rodriguez Querts, H. Perez Rojas and Perez Martinez, *Int. J. Mod. Phys. D*, **13**, 1261-1265(2004).

[15] M. W. Zwerlein, J. R. Abo-Shaeer, A. Schirotzkek, C. H. Schunck and W. Ketterle, 3 [cond-mat/0505635](http://arxiv.org/abs/cond-mat/0505635) 3.

[16] N. N. Klausen, J. L. Bohn Chris and H. Greene, *Phys. Rev. A* **64**, 053602-1 (2001).

[17] A. Perez Martinez, H. Perez Rojas, and H. Mosquera Cuesta, *Int. J. Mod. Phys. D*, **13** 1207-1211 (2004).

[18] H. Perez Rojas, A. Perez Martinez and H. Mosquera Cuesta,*Int. J. Mod. Phys. D*, **14** 1855-
[19] H. Perez Rojas, E. Rodriguez Querts, hep-ph/0406284.
[20] H. Perez Rojas, E. Rodriguez Querts, hep-ph/0402213.
[21] A. E. Shabad, *Annals of Phys.*, 90, 166(1975).
[22] A. E. Shabad, U. U. Usov, *Astrophys. and Space Sci.*, 117, 309-325(1985).