Twisted speckle entities inside wavefront reversal mirrors.

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The previously unknown property of the optical speckle pattern reported. The interference of a speckle with the counter-propagating phase-conjugated speckle wave produces a randomly distributed ensemble of a twisted entities (ropes) surrounding optical vortex lines. These entities appear in a wide range of a randomly chosen speckle parameters inside the phase-conjugating mirrors regardless to an internal physical mechanism of the wavefront reversal. These numerically generated interference patterns are relevant to the Brillouin PC-mirrors and to a four-wave mixing PC-mirrors based upon laser trapped ultracold atomic cloud.

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Phase singularities of the optical beams attracted a substantial interest in a recent decades from the point of view of optical information processing [1]. Helical wavefronts had been shown to affect a processes of the second harmonic generation [1], image processing with a photorefractive mirrors [2], phase-conjugated reflection via nondegenerate four-wave mixing in a cold atoms cloud [3]. The photons with helical wavefunction had been shown to possess quantized angular momentum ħ per photon [4]. The optical nonlinearities are capable to transfer the angular momentum from photons to an ensemble of ultracold atoms [5]. This effect is considered as a possible tool for the light storage in addition to a photorefractive mirrors [2], phase-conjugated reflections from a Brillouin PC - mirror [9]. The goal of the current communication is to study the spiral interference patterns with the period of "l/2 around the nodes of optical speckle pattern. These coarse interference patterns turn in rotation the Brillouin medium producing acoustical vortices carrying OAM [11].

The traditional experimental and numerical technique for the visualization of the optical phase singularities is the standard model of the phase conjugation via stimulated Brillouin scattering (SBS) is the Bragg reflection of the phase-conjugated LG photons from 10^7 cesium atoms cooled down to T ≈ 10^-4K and reversal of the orbital angular momentum (OAM) had been interpreted as a consequence of internal macroscopic rotations inside atomic cloud [8]. Afterwards these experimental results were analyzed from a point of view of the angular momentum conservation for the incident and reflected photons from a Brillouin PC - mirror [9]. The goal of the current communication is to study the spiral interference patterns with the period of λ/2 around the nodes of optical speckle pattern. These coarse interference patterns turn in rotation the Brillouin medium producing acoustical vortices carrying OAM [11].

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The previous finding [12, 13] is that the ideal (or close to an ideal) PC mirror visualizes in the speckle patterns a peculiar optical entities, which might be called the ropes. Typically each such a rope is composed of a 2-3 optical vortex lines (fig.2). In contrast to the well studied straight optical vortex lines e.g. LG [1, 4], the OV in a speckle field are self-twisted as it seen from fig.2. In the real PC-mirrors the contrast of such gratings may be reduced due to the interference with a non-phase-conjugated component of the radiation.

The novel feature compared to the previous findings [12, 13] is that the ideal (or close to an ideal) PC mirror visualizes in the speckle patterns a peculiar optical entities, which might be called the ropes. Typically each such a rope is composed of a 2-3 optical vortex lines (fig.2). In contrast to the well studied straight optical vortex lines e.g. LG [1, 4], the OV in a speckle field are self-twisted as it seen from fig.2. Having in mind the well known fact [12], that OV appear and annihilate as a pairs of whirls with opposite circulations [9, 10] it is easy to conclude that at least two adjacent OV with opposite topological charges are needed to produce a rope (fig.3). The mean length of each OV in Z direction is the Rayleigh range of a speckle pattern LR ≈ D^2/λ, where D is an average transverse size (in the plane X, Y) of the OV core, the wavelength λ = 2π/k_p. The fig.4 shows the numerically generated fragment of a speckle pattern which contains three ropes each composed of a set of a vortex lines. Let us describe the numerical procedure for generating the interference pattern of the two counter-propagating speckle fields.

The standard model of the phase conjugation via stimulated Brillouin scattering (SBS) is the Bragg reflection from the sound grating with period λ/2 moving with
The evolution in space of the two counter-propagating paraxial laser beams inside Brillouin active medium obeys to Maxwell equations with the cubic nonlinear polarization induced by electrostrictive effect \[13\]. The linearly polarized pump field \(E_p\) moves in a positive direction of \(Z - \text{axis}\), the reflected Stockes field \(E_s\) with the same polarization propagates in opposite direction(fig.1). The acoustic field \(Q\) is excited via electrostriction \[13\]. The three-wave equations for the interaction of \(E_p, E_s\) with adiabatic elimination of the \(Q \approx E_p E_s^*\) are:

\[
\frac{\partial E_p(z, x, y, t)}{\partial z} + \frac{i}{2k_p} \Delta_\perp E_p = -\frac{\gamma^2 \omega_p k_a^2}{32 \rho_0 n_c \omega_{ac}} |E_s|^2 E_p, \\
\frac{\partial E_s(z, x, y, t)}{\partial z} - \frac{i}{2k_s} \Delta_\perp E_s = \frac{\gamma^2 \omega_s k_s^2}{32 \rho_0 n_c \omega_{ac}} |E_p|^2 E_s,
\]

(1)

where \(\gamma = \rho (\partial \epsilon / \partial \rho)_{S}\) is the electrostrictive coupling constant, \(\rho_0\) is the unperturbed density of medium \[13\]. Consider the phase conjugation with a random phase plate which produces the chaotic transverse modulation of the phase of the incident \(E_p\) with a characteristic size of the transverse inhomogeneity of the order of \(5 - 50 \mu m\) \[13\]. In such a case the complex envelope amplitude of the inhomogeneous speckle field \(E_p\) at the entrance to PC-mirror is given as a multimode random field \[16\]:

\[
E_p(\vec{r}, z = 0) \approx E_p^0 \sum_{0 < j_x, j_y < N_g} A_{j_x,j_y} \times \exp \left\{ i2\pi \left\{ \frac{x j_x}{p_x} \kappa_{j_x} + \frac{y j_y}{p_y} \kappa_{j_y} + i \theta_{j_x,j_y} \right\} \right\},
\]

(3)

where random phases \(\theta_{j_x,j_y}\) are the random numbers from interval \([0, \pi]\), \(A_{j_x,j_y}\) are the real amplitudes of the spatial harmonics, \(p_x, p_y\) are maximal transverse sizes (in \(X, Y\)-plane), \(\vec{r} = (x, y) = (r, \phi)\), \(j_x, j_y\) are integers corresponding to \(N_g = (16, 32, 64)\) plane waves with random phases \(\theta_{j_x,j_y}\). The random numbers \(\kappa_{j_x} = p_x / p_{j_x}\) and \(\kappa_{j_y} = p_y / p_{j_y}\) having the uniform distribution in the small vicinity of \(1\) are responsible for a random tilt of a plane waves constituting the spatial Fourier spectrum of the light transmitted through a random phase plate. For the paraxial beam propagation the amplitude and phase structure of the complex field \(E_p\) have the following form \[17\] in an arbitrary \(Z\)-plane:

\[
E_p(\vec{r}, z > 0) \approx E_0 \exp \left\{ i \kappa_{j_x} z \right\} \sum_{0 < j_x, j_y < N_g} \exp \left\{ i \theta_{j_x,j_y} \right\} A_{j_x,j_y} \exp \left\{ i2\pi \left\{ \frac{x j_x}{p_x} \kappa_{j_x} + \frac{y j_y}{p_y} \kappa_{j_y} + \frac{z}{2} \left( \frac{j_x^2}{p_{j_x}^2} + \frac{j_y^2}{p_{j_y}^2} \right) \right\} \right\},
\]

(4)

The interference pattern \(I(\vec{r}, z)\) is responsible for the sound grating in a Brillouin PC-mirror. For the grating produced by a linear superposition of the random plane waves we have:

\[
\text{FIG. 1: Gray scale intensity plot for a small volume of a speckle pattern inside PC-mirror in } (X, Z) \text{-plane } (Y = 23 \mu m) \text{ The mean transverse size of a speckle is } D \sim 8 \mu m. \text{ The mean longitudinal length of a speckle entity is of the order of the Rayleigh range } L_R \approx D^2 / \lambda \text{ for } \lambda \sim 1 \mu m. \text{ The size of the volume is } 32 \mu m \text{ in transverse } (X, Y) \text{ directions and 128 } \mu m \text{ in longitudinal } (Z) \text{ direction. The step of longitudinal modulation } \cos((k_p + k_s)z + 2\phi) \text{ induced by the interference of counter-propagating pump and Stockes waves is enlarged here 8 times for visualization purposes. The characteristic } \pi \text{-phase-jump is clearly visible in between adjacent Bragg’s cos-modulated roll patterns.}
\]

the speed of sound \(v_{ac}\) \[13\]. Due to the conservation of momentum \(\vec{p} = h k_p = h k_s + h k_{ac}\) the length of the wavevector of sound \(k_{ac}\) is equal to doubled length of wavevector of pump light \(k_p\) with an accuracy about \(10^{-5}\) \[14\]. The Doppler effect defines the small (of the order of \(10^{-5}\)) frequency shift of the reflected (Stockes) wave \(\omega_{ac} = 2\omega_p n v_{ac} / c\), where \(n\) is refractive index of medium, \(c\) is the speed of light in vacuum, \(\omega_p, \omega_s\) is the frequency of the pump and Stockes light respectively.
FIG. 2: The plot of the light intensity maxima at a given moment $t$ in a small volume inside the PC-mirror. The four pairs of spiral interference patterns with opposite handedness are located randomly in space. Their diameters and directions are changed smoothly due to diffraction. The step of longitudinal modulation $\cos((k_p + k_s)z \pm 2)\phi$ is enlarged here 16 times.

$$I(\vec{r}, z > 0) \approx |E(p)(\vec{r}, z) + E_s(\vec{r}, z)|^2,$$

where the phase-conjugated Stockes wave $E_s$ is given by:

$$E_s(\vec{r}, z > 0) \approx \displaystyle E_p^* \exp(-ik_s z) \sum_{0<j_x,j_y<N_y} \exp[-i \theta_{j_x,j_y}] A_{j_x,j_y} \exp[-i2\pi \left( \frac{x j_x}{p_{j_x}} + \frac{y j_y}{p_{j_y}} + \frac{z}{2k_s} \left( \frac{j_x^2}{p_{j_x}^2} + \frac{j_y^2}{p_{j_y}^2} \right) \right)]. \quad (6)$$

provided that $E_s \approx E_p^*$ (noiseless PC-mirror). Such an approach presumes the ideally perfect phase-conjugation with ultimate reflection $R = 1$ or interference of the non-interacting fields $E_s \sim E_p^*$ when the right hand sizes of Eq. (12) are equal to zero.

The intensity snapshots given by [4] were obtained numerically by summation of the series (1) and (6) on standard Intel platform with dual-core 1.86 Ghz processor and 1 Gb memory using standard educational software. The numerical simulations show that optical vortex lines intertwine and form ropes. The average length of a rope is of the order of Rayleigh range $D^2/\lambda$. This may be interpreted as a consequence of diffractive divergence within the angle $\lambda/D$ (fig.1). The longitudinal modulation by the Bragg factor $\cos((k_p + k_s)z \pm 2)\phi$ is accompanied by an additional bending and twisting at characteristic wavelength $\lambda K$. The latter is of the order of a several tenth of optical $\lambda$, resembling Kelvin modes of the vortex lines [13][14]. The handedness of helical ropes changes randomly from one entity to another: the clockwise and counterclockwise ropes appear with the equal probability. Despite the statistical nature of a speckle pattern (fig.1) the ropes are structurally stable: they appear at different locations and have a variable sizes but their morphology is reproduced from a given statistical realization to another one.

The physical interpretation of these numerically observed patterns is given in the fig.3 using the (5) with the electrical fields $E_p = E_s^*$ in the form of a two overlapping elementary $LG_0^1$ optical vortices:

$$E_p \approx \exp (ik_p z - (x_1^2 + y_1^2) + i\ell \phi_1) \cdot \sqrt{(x_1^2 + y_1^2)} + \exp (ik_p z - (x_2^2 + y_2^2) \pm i\ell \phi_2) \cdot \sqrt{(x_2^2 + y_2^2)}, \quad (7)$$

where the location of the vortex cores $(x_1 D = x + M \cdot \cos(2\pi z/\lambda K), (y_1 D = y + M \cdot \sin(2\pi z/\lambda K))$ and $(x_2 D = x + M \cdot \cos(2\pi z/\lambda K + \pi/2)), (y_2 D = y + M \cdot \sin(2\pi z/\lambda K + \pi/2))$ changes sinusoidally with period $\lambda K$ along $Z$-axis, $\phi_1$ and $\phi_2$ are the local azimuthal angles around vortex cores. Such vortex-vortex (fig.3a) and vortex-antivortex (fig.3b) pair interference pattern is used here as a variational anzatz in eq. (5). The fig.3 reproduces qualitatively some features of the numerically generated speckle patterns (fig.1).

In summary the nontrivial topology of the mul-

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tiaply connected optical speckle patterns was demonstrated numerically. Using the interference with the counter-propagating phase-conjugated speckle field we have shown the hidden twisted geometry of the multimode wave field composed of the randomly tilted plane waves \[16\]. Noteworthy the ropes exist without phase-conjugated counterpart: the reflection from PC-mirror makes these twisted entities visible due to the characteristic $\pi$-phase-jump in between the adjacent Bragg’s sinusoidally-modulated roll patterns (fig.1). Thus it seems probable that a rope entities exist in a wide class of a superpositional physical fields like an electromagnetic or acoustical ones \[11\]. The other most evident examples are the blackbody radiation field in a cavity or a monochromatic field in a cavity with a rough mirrors. Taking into account the phenomena of a so-called nonlinear superposition which take place for collisions of the optical solitons, vortices \[20\] and formation of the optical vortex lattices \[21\] one might expect the finding of a stable ropes in a nonlinear fields. The relevant examples are photorefractive PC-mirrors \[2\], ultracold matter, e.g. phase-conjugating PC-mirrors based on nondegenerate four-wave mixing in a trapped gases \[3, 8\] and the situations of the slow \[7, 8, 22\] and the fast light \[23, 24\] propagation in a resonant medium.

The reported result extends the set of the possible forms of the photon’s wavefunction. In addition to the well known photon wavefunctions in the form of the elementary optical vortex lines, e.g. LG beams which possess a helical wavefronts \[1, 4\], the twisted entities each composed of the several vortex lines offer a new form of the photon’s wavefunction \[25, 26\].

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