Adapting CBPP Platforms for Instructional Use

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February 1, 2008

Abstract

Commons based peer-production (CBPP) is the de-centralized, net-based approach to the creation and dissemination of information resources. Underlying every CBPP system is a virtual community brought together by an internet tool (such as a web site) and structured by a specific collaboration protocol. In this talk we will argue that the value of such platforms can be leveraged by adapting them for pedagogical purposes.

We report on one such recent adaptation. The Noösphere system is a web-based collaboration environment that underlies the popular Planetmath website, a collaboratively written encyclopedia of mathematics licensed under the GNU Free Documentation License (FDL). Recently, the system was used to host a graduate-level mathematics course at Dalhousie University, in Halifax, Canada. The course consisted of regular lectures and assignment problems. The students in the course collaborated on a set of course notes, encapsulating the lecture content and giving solutions of assigned problems. The successful outcome of this experiment demonstrated that a dedicated Noösphere system is well suited for classroom applications. We argue that this “proof of concept” experience also strongly suggests that every successful CBPP platform possesses latent pedagogical value.

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1 Introduction

1.1 Background and motivation

The capacity of communications networks to create value is well recognized (Metcalf, 1995). There is a theoretical argument that internet value creation is an even more dramatic process, because it is dominated by exponential rather than polynomial scaling effects (Reed, 1999). To put it another way, the internet engenders powerful emergent phenomena, because every potential group with a shared interest can interact, collaborate, and create intellectual value through internet (and especially WWW) software applications.

Thus, with the advent of powerful search and indexing technologies, the world wide web is evolving into a ubiquitous reference resource (Berners-Lee et al., 2001). The network transforms the disconnected efforts of millions of web page authors into something of practical value. Another noteworthy project is Wikipedia (Wales and Sanger, 2001), a knowledge-oriented virtual community that successfully employs the wiki collaboration protocol (Leuf and Cunningham, 2001) to unite the efforts of thousands of volunteers around the scholarly goal of a public domain encyclopedia (Kantor, 2004).

In both of the above examples, the underlying process lacks explicit organization and is non-hierarchical. In both cases the value is governed by an emergent phenomenon: the value of the whole is significantly greater than the sum of the individual parts. A recent economics-based theory attempts to explain such emergent value phenomena as instances of commons-based peer production (CBPP), an idealized mode of production that is complementary to firms and markets, and one that manifests naturally on the internet (Benkler, 2002). However, economic theory is insufficient to fully understand and exploit the complex, emergent phenomena that underly internet value creation (Iannacci and Mitleton-kelly, 2005). The study of the internet is inherently cross-disciplinary; no one discipline, or even a blend of two will suffice.

In the present article we report on and discuss a recent adaptation of Noösphere (Krowne, 2003a), a web platform for mathematics collaboration, for the purpose of teaching a graduate course in mathematics. A convenient categorizing label for our project is computer supported collaborative learning (CSCL), a field that brings together perspectives from cognitive science, computer and information science, education, and philosophy (Stahl, 2006).
Our thesis is inherently cross-disciplinary. We argue that CBPP, the phenomenon of internet value creation, crosses over naturally into the world of CSCL. We argue that the infrastructure of collaborative, knowledge-related projects, like Wikipedia and Noösphere, can be leveraged to yield concrete educational assets.

This value stems in large part from the inherent unity and collaborative nature of the scholarly enterprise. A context that fosters the formation of communities which acquire, organize, generate, synthesize, and transmit knowledge will also be a context where learning and pedagogy are of central importance. These qualities naturally lead us to the concept of a digital library. Traditionally, libraries have been the cornerstone of scholarship, providing a space for both research and learning, and other, more intangible benefits. It would therefore be surprising if emergent collaboration phenomena and educational scenarios did not play a role in the evolution of the digital library (Robertson and Reese, 1999).

1.2 Re-conceptualizing the digital library

The concept of a digital library is a natural outgrowth of the development of modern, network-oriented information technology. Information, once digitally encoded, can be stored electronically and distributed over the internet. Physical and geographical barriers disappear. There are no limits to the size of the library. Its contents are potentially available to everyone, everywhere, all the time.

The word library carries with it connotations of a nearly static archive, one where the primary information-related activity are storage, classification and retrieval. The shift of information content from the physical to the digital realm undermines this traditional conceptualization (Levy and Marshall, 1995). Various recent internet-focused developments—powerful and ubiquitous search engines, virtual communities and the free culture movement, to name just a few—challenge us to move beyond the simple notion of an “electronic traditional library,” and to embrace benefits beyond the elimination of space and scarcity concerns.

Older information technologies, such as paper, foster a dichotomy between information and knowledge. The latter is the more dynamic concept; knowledge implies research, dissemination, debate, synthesis, activation, history and evolution. As well, knowledge cannot be conceived as something separate from people; knowledge implies a community of scholars, teachers,
learners, and practitioners (Ehrlich and Cash, 1994).

Therefore, the digital library concept needs to evolve to more fully realize the potential of the underlying network technology and software technology. New library tools and modalities that address collaboration, superimposed information, knowledge creation, and education will have to be developed (Delcambre et al., 2001; Frumkin, 2005; Krowne, 2003b; McRobbie, 2003).

1.3 CBPP

In this regard, commons based peer production (CBPP) shapes up to become a key phenomenon in the digitally mediated transition from information to knowledge. Internet-based CBPP has its origins in the open-source software movement, a collaborative, extra-commercial process of software creation\(^1\). The existence of numerous successful internet projects, Wikipedia and Project Gutenberg/Distributed Proofreaders (Lebert, 2004), to cite just two examples, indicate that the phenomenon of collaborative internet value creation has pertinence well beyond generating software programs.

With peer production on the Internet, distributed ensembles of people share open production of complex products and services—generally for no financial compensation. While the idea of non-market, non-corporate production is not new (science has traditionally worked this way), large-scale, decentralized, sustained, open production by diverse groups of peers is a new phenomenon: a development that has been enabled and encouraged by the confluence of computers, networking and the information economy. This form of non-market, internet-based peer production has been applied to create a wide variety of significant knowledge assets (Galiel, 2004).

The impact of a knowledge-centric community like Wikipedia on the digital library landscape cannot be ignored. Neither should the enormous productive leverage of a project like Distributed Proofreaders. Therefore, it makes good sense (for both practical and idealistic reasons) to expand the “digital library” concept to incorporate an internet-based CBPP aspect.

PlanetMath (Krowne and Egge, 2001) is another CBPP project, of special connection to our study. Planetmath is a collaboratively written encyclopedia of mathematics licensed under the GNU Free Documentation License.

\(^1\)This is not imply that open source software is without commercial value. Rather, the process of creation is governed by something other than a simple exchange of money for software end products.
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(FDL), and implemented using the Noösphere system. The PlanetMath project is an instance of CBPP; the aim is to create a community-oriented, web-based repository for mathematical knowledge. The project attracts a diverse and international body of participants. These people are students and members of the wider public with an interest in mathematics, graduate students pursuing advanced mathematics degrees, professional mathematicians who make their living by practicing or teaching mathematics classes and by conducting mathematics research. Planetmath and Noösphere also have an extended role as a testbed for research and development in semantic extraction, digital information exchange, and collaborative authority models (Krowne and Bazaz, 2004).

1.4 Academia, instruction, and engagement

Academic communities are concerned with knowledge in all its manifestations; both the information and community-related aspects are important. Certainly, instruction and the teacher-learner relationship are central academic concerns.

Instruction can be conceptualized as a structured interaction between senior and junior members of a knowledge community. The instructor is more than just a particular medium for the storage and transmission of information. Rather, for the student, the lecture hall is a portal to the community of knowledge (Clancey, 1995). Let us use the term engagement to describe the process of active student participation and scholarly development (Stahl, 2005).

In addition to the immediate goals of any particular course of academic instruction, there is, in the teacher-student relationship, an implicit invitation to “do as we do”; to join the community, and to become involved in knowledge-related activities. Pedagogical structures: exercises, discussions, individual and group projects, examinations and other assessment modalities, are the devices of guided scholarship. Engagement, rather than skill-set and information “download” is the deeper goal of academic instruction. The ultimate measure of success is the metamorphosis of the student, an individual at the outset capable and interested only in passive, assisted knowledge activities, into the scholar, an individual engaged in independent knowledge activities.

It is worth briefly examining the critical elements of scholarship. Of paramount importance is that for scholar, no “oracle” exists to provide the
answer to a research question. Peers can provide critique but not guaranteed answers. The scholar also lacks a roadmap towards a solution, and must prioritize his/her efforts, evaluate the intellectual contributions of others, and act upon their own judgments. This is the universal situation of the scholar, and it is utterly different from the environment of the formal student. While attempts are made to deliberately teach students many of the tools upon which scholars rely, the aims and trajectory of classroom activities are by definition preset. Thus, the characteristics of the true scholarly environment induce a sharp division of students who have meaningfully become scholars from those who have merely learned to regurgitate information with relative success.

CBPP projects like Wikipedia and Noösphere possess a remarkable capacity for fostering engagement in scholarly activity. We suggest that it is reasonable to tap such free-culture phenomena for the purposes of academic instruction. Indeed, nothing could be more natural, because of the inherent compatibility between academic and free-culture goals and values. Let us make a sketch of how such an evolution can take place.

A re-conceptualized, more dynamic and community-oriented digital library is a natural context for both public domain knowledge activity and for pedagogical efforts that involve students in online knowledge activities. Such activities should include not just information retrieval, but collaborative knowledge creation and organization (Brown, 1999). The physical community of the classroom can be extended to the network. The same community and collaboration tools and technologies that enable CBPP projects can be used to create a virtual space in which the participating students can carry out knowledge-related activities, albeit in an assisted and structured fashion.

We hypothesize that such an approach can lead to a heightened level engagement, because of the subtle but important shift of emphasis from “I will teach, you will learn” to “let us collaborate on a knowledge project”. The change of attitude is natural and desirable from an academic point of view, but is difficult to implement using traditional classroom methods and technologies.

Our hypothesis is that adoption of CBPP technologies into an instructional setting will facilitate just such a shift of emphasis. The student goal-set and motivations will be enriched by incorporating a network-based, collaborative aspect into the classroom experience. At one level, the instruction

\[2\] The open access movement illustrates this nicely (Suber, 2004).
process can proceed in the traditional manner: the teacher guides the students through a fixed syllabus, assigns tasks, and performs evaluation. However, since the setting is now a “research library” as well as the classroom, since the medium of interaction includes a virtual collaboration environment, and since the goal-set includes the incorporation of individual efforts into a digitally encoded body of knowledge, the end result will manifest as a collaboration between all involved. Such a process should lead to heightened levels of student engagement.

2 A Trial of Noösphere as a platform for collaborative instruction

2.1 Test scenario and goals

In the Winter of 2003, the Noösphere system was used to host Math 5190: Ordinary Differential Equations, a graduate mathematics course at Dalhousie University, in Halifax, Canada. One of the current authors served as course instructor. A “tabula rasa” Noösphere system was set up on a dedicated server. The primary course goal was the collaborative creation of a set of course notes, including a number of worked-out exercises to illustrate the key concepts. Assessment criteria included the quantity and quality of the online participation, as well as a more conventional final project.

The course attracted 3 graduate students and an auditor, who in the coming semester created and organized an online body of knowledge on the topic of differential equations. The end result was a 70 page document containing definitions, theorems, proofs, and examples. When taken together, these constitute a mini-treatise on certain aspects of the theory of ordinary differential equations.

The trial addressed the following research goals:

1. Our main hypothesis was that CBPP platforms are suitable for advanced mathematics instruction, and that a course structured around collaborative principles and online tools can serve and advance conventional academic goals.

2. We evaluated the feasibility of deploying Noösphere as a CSCL environment. Experiences with CoWeb (Guzdial et al., 2001), show that
CSCL-type mathematics courses present special challenges related to specialized notation and division of labor issues. Noösphere’s \LaTeX-based design incorporates the full range of advanced mathematical notation. As well, Noösphere possesses a unique authority model and groupware capabilities. The trial examined the capacity of these designs to address the above challenges. In particular, we wanted to compare the patterns of student activity in a collaborative, online environment with those in a traditional mathematics courses, and to consider the impact on student engagement. Our secondary hypothesis is that student engagement benefits from the introduction of CBPP elements.

3. We also considered the impact of a collaborative, online course environment on the students’ scholarly development.

2.2 Methodology

Math 5190 is a one-semester course at Dalhousie University on the theory and methods of ordinary differential equations. Such courses, typically aimed at beginning graduate students and advanced undergraduates, are offered, with certain variations, by most mathematics departments in North American universities.

In the Winter of 2003 this course served as a proof-of-concept study of the Noösphere system in an educational setting. The course included a number of conventional instructional components: 3 hours/week of lectures, a reading list, regular meetings with of the instructor with individual students, a final project, and student presentations. The core component, however, was a dedicated website set up as a “tabula rasa” Noösphere environment.

The basic unit of content in Noösphere is the entry, which any registered user can create. The entries comprise the main section of the system, which is called the “encyclopedia”. This reflects the general orientation and pedagogical style of the system.

Noösphere entries consist of title, content, and various metadata. The entries are interlinked, which means that the text of each entry contains hyperlinks pointing to other entries where appropriate. The general intent of this is to provide definitions for each concept utilized, in an easily navigable fashion. Entries are written in \LaTeX (Lamport, 1986), which serves as the basis for Noösphere’s mathematics support in addition to allowing for the expression of general document formatting. Displayed in rendered form, the
mathematical portions of each entry “look right” with a standard browser (with no plug-ins), a considerable improvement over most other attempts to publish mathematics to the web to date. This mathematics support makes Noösphere a good candidate for use in all of the mathematical sciences.

A key feature of Noösphere is the corrections system. If any registered user determines there is a problem with an entry, he or she can voice concern by filing a correction to that entry. Until addressed, this correction is displayed when the entry is shown, ensuring that the critique is “out in the open”.

Finally, each entry in Noösphere has an owner, who is initially the person who created the entry. An owner has the option of orphaning an entry, or transferring ownership to another user. Orphaned entries are flagged by the system and may be adopted by any interested user.

Noösphere has a number of other services that provide direct community support.

1. The requests service, which functions as a global “to-do” list of content addition for the Noösphere site. Users can fulfill particular requests, rendering them inactive, by creating an appropriate entry.

2. The discussion service provides threaded, asynchronous messaging. A discussion can be attached to most of the core objects of Noösphere. This includes encyclopedia entries, corrections, and requests.

3. Noösphere’s notification system keeps members of the community aware of activity relevant to them through e-mail and a Noösphere system “inbox”. Corrections to an entry result in a notice to the entry’s owner. A resolved correction results in a notice to the filer, indicating what action was taken and why. Similarly, replies to a message posted result in a notice that makes the initial poster aware of the reply. An important part of the notification system is the ability to create configurable watches. Watches placed on any object by any user result in (e-mail or web) notices about events to that object being sent to the user.

At the outset, the students were informed that the main course objective was the collaborative creation of a set of lecture notes using the online environment. The instructor’s role was to facilitate and to structure this effort. As such, the instructor mirrored lecture topics and contents with Noösphere request objects that enumerated the key definitions, theorems, proofs, and
techniques covered in the lectures. The students were responsible for filling these requests by creating the requisite entries and subsequently evolving and improving them based on corrections received from the instructor and fellow classmates. The students had to cooperate to decide how to divide the requests and to share the corresponding workload.

It is well recognized that mathematics instruction is greatly facilitated by supplementary problems and exercises. In place of the conventional system of regular assignments with specific deadlines, course exercises were presented to the students as illustrative examples to be included in the collaborative notes. The instructor, on a regular basis, created and orphaned exercise-type entries. The students were then responsible for adopting the entries and furnishing solutions. Again, students were given the opportunity to evolve and improve their solutions through interactions with instructor and classmates. As such, an incorrect solution did not necessarily result in a poorer evaluation, but rather served as an additional learning opportunity in the context of Noösphere’s system of corrections. Students had the opportunity to continuously improve their entries up to the course termination deadline.

The collaborative, online aspect of student progress was assessed according to the number of owned entries, and according to the extent the entries were developed. At the termination of the course, a score of 1, 2, or 3 was assigned to each student entry according to the following criteria:

- Degree of participation was measured by the number of filled requests, and adopted exercises. An adopted entry with even a minimal amount of content was assigned a score of 1.

- A reasonably well developed entry with unresolved corrections was assigned a score of 2.

- A correct, well written entry with no outstanding corrections was assigned a score of 3.

The instructor issued corrections in response to student errors, and to suggest improvements to the mathematical content and presentation format.

Course assessment did not include an examination component. Rather, an assessment of scholarly development was based on a final project, which was implemented conventionally, and involved both an oral presentation and a written report. With input from the instructor, students selected a relevant
topic\(^3\), delivered a classroom presentation, and submitted a written report. The project component played a particularly important role in the trial, providing a measure of student progress independent of the online activity.

### 2.3 Results data

By the end of the course, the 3 registered participants, all first year MSc students, had created a total of 122 entries. The entry totals and the corresponding scores (see above) are displayed in Table 1. A score of 0 indicates an entry with non-existent or negligible content. At the conclusion of the course there were a total 12 unfilled requests and unadopted exercise problems. A total of 78 corrections were issued\(^4\).

Subsequently, the website contents were converted into document form and redistributed to the students. The resulting document spans 74 typeset pages. The table of contents of the resulting document is displayed in Appendix A. Some representative entries are shown in Appendix B.

The Noösphere collaboration protocol proved to be very suitable for student-instructor interactions. The entry ownership system and email updates allowed the instructor to easily follow student progress, and to issue timely feedback in the form of corrections. With minor adjustments, the Noösphere scoring system proved valuable as a highly visible indicator of individual participation levels.

| Student | Entry score | Total |
|---------|-------------|-------|
|         | 0 | 1 | 2 | 3 |     |
| 1       | 0 | 10 | 26 |    | 37  |
| 2       | 1 | 2 | 10 | 27 | 39  |
| 3       | 3 | 6 | 10 | 16 | 32  |

Table 1: Student entries and assessment scores.

Student-instructor interactions stabilized around the following cyclical pattern. The instructor delivered lectures and suggested deadlines for the fulfillment of requests and the adoption of exercise entries. This was followed by posted corrections and occasional email “nags” and feedback. As

\(^3\)The 3 registered students chose the following topics: convergence of iterative integral solutions, predator-prey models, differential equation modeling of guerrilla vs. conventional warfare.

\(^4\)All but one of these corrections originated with the instructor.
is often the case in conventional courses, the students functioned as largely passive knowledge agents. There was no evidence of direct online collaboration among the students. Students did not give each other corrections, nor did they use the online forums to discuss mathematical content. Rather, students reported collaborating in more conventional ways. They held study group meetings to discuss course material, and to decide on the division of labor for their online tasks.

Student behavior and outlook in the trial was typical for courses at the beginning graduate level. Students at this level still require explicit goal structure and assessment criteria, and are often passive in their approach to the material. Students in the trial displayed typical procrastination behaviors, and regarded their participation as “necessary duty” to be balanced against time requirements from other courses and from outside jobs. As such, their online efforts tended to occur in bursts of concentrated activity. An example of this behavior pattern is visible in Figure 1, which shows the temporal distribution of student responses to corrections.
The conventional educational objectives of the course were fulfilled. The content of the final projects and the website entries, especially the exercises, provided clear and substantial evidence of progress toward mastery of the subject matter, and progress in scholarly development. Relative to these metrics (exercise solutions and final projects), progress of the students in the trial was directly comparable to the progress of students in the same course taught by the same instructor conventionally in other years.

2.4 Findings

Given the limited enrollments and the advanced nature of the material characteristic of graduate courses, and keeping in mind the natural variation of student backgrounds and abilities, it is not feasible to render a judgment on the relative merit of conventional pedagogy versus collaborative, online learning. However, our observations allow us to make the following points.

1. Our experience with Math 5190 and Noösphere provides strong support for the hypothesis that conventional educational objectives can be met by a course based on online learning and CBPP principles. Importantly, we found no evidence that the inclusion of a CBPP component diminished or disrupted traditional classroom learning. Our outcomes should be reproducible by groups of advanced students at other institutions, and with other courses in the mathematics curriculum. To make sense of this claim, however, one must incorporate assessment components that can provide an objective measure of student progress.

2. The students in the trial readily accepted the mechanics of Noösphere and expressed appreciation at being able to do their work in an online setting. Nowadays knowledge of \( \text{LaTeX} \) is a near-universal prerequisite for the scholarly development of mathematics students. The \( \text{LaTeX} \) component of Noösphere provided our students with a useful opportunity to develop their typesetting skills.

Based on the instructor’s observations and communication with the students, Noösphere’s protocol of entry adoption and ownership allowed the students to exercise control over their participation, and thereby facilitated engagement. The fulfillment of requests and the adoption of exercises manifested as an act of commitment on the part of a student. Thus, the authority model allowed the students to pursue a division
of labor, but in a transparent and principled fashion that is usually lacking in conventional courses.

A potential weakness of this approach is the possibility that an overly selective focus on the part of some students may lead to a spotty coverage of essential topics. The instructor has an important role to play here, and must encourage students to contribute to a variety of course topics. Such difficulties did not visibly manifest in the trial under discussion. However, without a comprehensive final examination it is difficult to discount the possibility that some of the students received inadequate exposure to some of the topics.

3. The collaboratively compiled course notes are a valuable asset that is not readily available in the context of conventional instruction. From the point of view of the students, the document is far more than a transcription of the instructor’s lectures. In a very real sense, the students are the authors of the document. As such, the notes concretely encapsulate their learning experience.

There are a number of benefits to producing such a document. The notes can serve as a source of reference for future work in the subject. Perhaps, more importantly, the very existence of the notes embodies a latent, but powerful message about the students’ capacity for scholarship, and about the nature of the academic enterprise. In an important sense, the creation of the notes transforms the asymmetrical relationship between instructor and the students into something more closely resembling scholarly collaboration.

There is also the intriguing possibility that collaboratively produced course notes can serve as contributions to public domain knowledge repositories.\footnote{The students in the trial were encouraged to convert their course contributions into PlanetMath entries—though none of them chose to pursue such activity.}

The primary responsibility of the course instructor centers around the student learning experience. As such, it would not be appropriate to make full scholarly use of the course notes without addressing issues of consent and attribution. Still, it is important to provide students with opportunities for independent scholarly activity. If nothing else, the format of the trial made the students aware of ongoing CBPP efforts, and served as an invitation to contribute to them.
3 Discussion

The Noösphere/Math5190 trial constitutes a limited, proof-of-concept experiment regarding the application of CBPP tools in an educational, academic setting. Though our experiment was a success, the small scale of the trial limits the inferences we can draw in support for our hypothesis regarding CBPP and education. It will be necessary to subject the hypothesis to further testing: one needs to organize more CBPP-based courses, involve more students and instructors, employ control and experimental groups, and to consider diverse academic subject material.

One also has to come to grips with the limitations revealed by our experience. Collaborative learning methods are not a panacea for improving student engagement (Guzdial et al., 2002). Indeed, it would be useful to undertake a systematic examination of the effects of CBPP on academic engagement. Methodologically, the undergraduate curriculum, with its larger enrollments, may be a more appropriate setting for such studies.

Wiki-based courses in the humanities and the social sciences are the subject of ongoing research and discussion (Boyd and Lohnes, 2005). Wiki software is widely available, notational demands are lower, and the wiki interface is easier to learn than the \LaTeX-based Noösphere. The ostensible aim of such a course should be a well-developed body of “wikified” content that encapsulates a subject of interest, and that provides a concrete record of individual students’ participation. An initial study on this topic (Scharff, 2002) supports the conclusions of our own trial. It would also be interesting to study the effect of such an experience on scholarly evolution. To what extent does student exposure to wikis as an instructional medium encourage contributions to sites like Wikipedia, or the pursuit of more conventional scholarly publications?

4 Conclusion

The joining together of the themes of collaborative education, the internet, and digital libraries is not a new idea (Roschelle and Pea, 1999). Rather, the relatively recent emergence of successful CBPP knowledge projects should be viewed as a timely and complementary development (Tomek, 2003). Much of the infrastructure, interface, and design issues are the same for both contexts. There is strong common focus on extraction of semantics, collaboration in-
terfaces, and educational applications. We believe the potential for mutual benefit and a convergence of interests is evident.

In the context of a symposium on Digital Libraries and Free Culture, it is also appropriate to note the relevance of our hypotheses to the continuing debate about intellectual property and the public domain. Pragmatism and utility are strong arguments for commons-based knowledge activity. The impact of the open source and the free software movements on development of information technology is, at this point, beyond question. Likewise, projects like Wikipedia, PlanetMath, and Distributed Proofreaders are beginning to make a significant contribution to the intellectual commons.

As is the case with emergent internet value phenomena, the potential value of such projects is unconstrained and will manifest in unforeseen ways. But, this is just one instantiation of the general argument in support of public domain knowledge and culture (Lessig, 2004). Synergy and flexibility is the point here, and a libre free project like PlanetMath is good example. This project began as a mathematics encyclopedia, then evolved into a groupware platform and test-bed for digital library research (Noösphere), and is now being used as an educational delivery system.

Academic involvement in CBPP projects allows researchers, librarians, and educators to exploit the kind of internet value that IT companies enjoy when they employ open-source software. Conversely, free-culture projects benefit from academic attention and investment. Successful adaptation of CBPP technologies for academic instruction is a powerful argument in support of free culture. However, much work remains to be done in the cross-disciplinary exploration of CBPP, CSCL, and digital libraries.

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A  Math 5190 course notes: table of contents

Topics in Ordinary Differential Equations
Collaborative course notes compiled by the students of Math 5190
Dalhousie University
April 18, 2003

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B Math 5190 collaborative course notes: representative entries

58 Proof of the Banach fixed point theorem

Theorem Let $T: X \to X$ be a contraction transformation of a complete metric space. Then there exists a unique fixed point $\hat{x} \in X$ i.e. $T(\hat{x}) = \hat{x}$.

Proof Choose a point in $X$ and call that point $x_1$. Set

$$x_2 = T x_1, x_3 = T x_2, \ldots, x_{n+1} = T x_n.$$ 

Set

$$k = d(x_1, x_2).$$ 

By hypothesis,

$$d(x_2, x_3) \leq qk, 0 < q < 1.$$ 

Continuously, we get

$$d(x_n, x_{n+1}) \leq kq^{n-1}.$$ 

Thus,

$$d(x_1, x_{n+1}) \leq d(x_1, x_2) + d(x_2, x_3) + \cdots + d(x_n, x_{n+1})$$

$$\leq k + qk + \cdots + q^{n-1}$$

$$= k \frac{1 - q^n}{1 - q}.$$ 

More generally,

$$d(x_m, x_{n+m}) \leq kq^{m-1} \frac{1 - q^n}{1 - q}$$

$$\leq \frac{dq^{m-1}}{1 - q}.$$
So as \( m \to \infty \), \( d(x_m, x_{n+m}) \to 0 \). Since \( X \) is assumed to be complete, there exists a limit of the sequence \( x_n \) call that the limit \( \hat{x} \). Note that for all \( n = 1, 2, \ldots \),
\[
d(\hat{x}, T\hat{x}) \leq d(\hat{x}, x_n) + d(x_n, x_{n+1}) + d(x_{n+1}, T\hat{x}) \leq (1 + q)d(\hat{x}, x_n) + kq^{n-1}.
\]
As \( n \to \infty \), the right hand side \( \to 0 \). Thus, \( d(\hat{x}, T\hat{x}) = 0 \). Therefore, \( T\hat{x} = \hat{x} \). If \( \hat{x}, \hat{y} \) are such that \( T\hat{x} = \hat{x} \) and \( T\hat{y} = \hat{y} \), then \( d(\hat{x}, \hat{y}) = d(T\hat{x}, T\hat{y}) \leq qd(\hat{x}, \hat{y}), q < 1 \) Thus, \( d(\hat{x}, \hat{y}) = 0 \). Therefore, \( \hat{x} = \hat{y} \).

**74 Symmetry**

A symmetry of a scalar ODE \( \frac{dy}{dx} = \omega(x, y) \) is a transformation
\[
\begin{pmatrix}
\hat{x} \\
\hat{y}
\end{pmatrix} = \begin{pmatrix} f(x, y) \\ g(x, y) \end{pmatrix}
\]
such that
\[
\omega(\hat{x}, \hat{y}) = \frac{\hat{y}_x + \omega(x, y)\hat{y}_y}{\hat{x}_x + \omega(x, y)\hat{x}_y}.
\]

**Example 1:**
Consider the ODE
\[
\frac{dy}{dx} = -\frac{x}{y}.
\]
We claim that
\[
\begin{pmatrix}
\hat{x} \\
\hat{y}
\end{pmatrix} = \begin{pmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}
\]
is a symmetry of the ODE
\[
\begin{align*}
\frac{d\hat{y}}{d\hat{x}} &= \frac{\sin(t) + (-x/y)\cos(t)}{\cos(t) + (-x/y)(-\sin(t))} \\
&= \frac{-x \cos(t) + y \sin(t)}{x \sin(t) + y \cos(t)} \\
&= -\frac{\hat{x}}{\hat{y}} \\
&= \omega(x, y).
\end{align*}
\]
Therefore the transformation is a symmetry of the ODE.

**Example 2:**
Consider the ODE
\[ \frac{dy}{dx} = x + y. \]

We claim that the transformation

\[ \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} x \\ y + ke^x \end{pmatrix}, \]

where \( k \) is a constant, is a symmetry of the ODE. So we check:

\[ \frac{d\hat{y}}{d\hat{x}} = \frac{ke^x + (x + y)}{1} = x + y + ke^x = \hat{x} + \hat{y} = \omega(\hat{x}, \hat{y}). \]

Therefore the transformation is a symmetry of the ODE.

**81.9 Flows problem 9a**

What is the flow corresponding to the following differential equation:

\[ \frac{dy}{dx} = -\frac{x}{y}. \]

Letting \( \Phi_t(x, y) = \Phi(x, y, t) \) denote the flow mapping, verify the following:

\[ \Phi_0(x, y) = (x, y); \]
\[ \Phi_r \circ \Phi_t = \Phi_{r+t}; \]
\[ \frac{\partial \Phi}{\partial t} = \dot{\Phi} \circ \Phi; \]
\[ \frac{\partial \Phi}{\partial t} = J[\Phi] \dot{\Phi}. \]

We first autonomize the ODE:

\[ \frac{dx}{dt} = 1 \]
\[ \frac{dy}{dt} = -\frac{x}{y}. \]
If we solve the original ODE we see that the implicit solution is $x^2 + y^2 = c$, where $c$ is a constant (the implicit solution can be found by separation of variables). To find the flow for the above ODE, we begin by letting $\hat{x} = x + t$. Then we note that

$$\dot{x}^2 + \dot{y}^2 = c.$$  

Solving for $\dot{y}$ gives us

$$\dot{y} = \sqrt{c - \dot{x}^2} = \sqrt{(x^2 + y^2) - (x + t)^2} = \sqrt{y^2 - 2xt - t^2}.$$  

Then

$$\Phi(t, x, y) = \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} x + t \\ \sqrt{y^2 - 2xt - t^2} \end{pmatrix}.$$  

The first condition is satisfied easily:

$$\Phi_0(x, y) = \begin{pmatrix} x \\ y \end{pmatrix}.$$  

Next we verify the condition $\Phi_t \circ \Phi_\tau = \Phi_{t+\tau}$. Let

$$\Phi_t = \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} x + t \\ \sqrt{y^2 - 2xt - t^2} \end{pmatrix},$$  

and let

$$\Phi_\tau = \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} \hat{x} + \tau \\ \sqrt{\hat{y}^2 - 2\hat{x}\tau - \tau^2} \end{pmatrix}.$$  

Then

$$\Phi_\tau \circ \Phi_t = \begin{pmatrix} x + t \\ \sqrt{y^2 - 2xt - t^2 - 2(x + t)\tau - \tau^2} \end{pmatrix} = \begin{pmatrix} x + (\tau + t) \\ \sqrt{y^2 - 2x(\tau + t) - (\tau + t)^2} \end{pmatrix} = \Phi_{t+\tau}.$$  

Now consider

$$\frac{\partial \Phi}{\partial t} = \begin{pmatrix} 1 \\ \frac{x + t}{\sqrt{y^2 - 2xt - t^2}} \end{pmatrix}.$$
We test the two remaining conditions. First we check to see if \( \frac{\partial \Phi}{\partial t} = \dot{\Phi} \circ \Phi \).

Note that

\[
\dot{\Phi} = \left( -\frac{1}{x + t} - \frac{y^2 - 2xt - t^2}{y^2 - 2xt - t^2} \right)_{t=0} = \left( 1 \right) \cdot \left( 1 \right).
\]

Then

\[
\dot{\Phi} \circ \Phi = \dot{\Phi}(\hat{x}, \hat{y})
\]

\[
= \left( 1 \right) \cdot \left( \frac{1}{\hat{x}} \right)
\]

\[
= \left( \frac{1}{x + t} \right) \cdot \left( \frac{1}{\sqrt{y^2 - 2xt - t^2}} \right).
\]

Therefore \( \frac{\partial \Phi}{\partial t} = \dot{\Phi} \circ \Phi \). Finally we test the condition \( \frac{\partial \Phi}{\partial t} = J[\Phi] \dot{\Phi} \).

\[
J[\Phi] \dot{\Phi} = \begin{pmatrix}
-\frac{1}{\sqrt{y^2 - 2xt - t^2}} & 0 \\
\frac{2y}{\sqrt{y^2 - 2xt - t^2}} & -\frac{1}{y}
\end{pmatrix}
\begin{pmatrix}
1 \\
-\frac{x}{y}
\end{pmatrix}
\]

\[
= \left( -\frac{1}{\sqrt{y^2 - 2xt - t^2}} \right) \cdot \frac{\partial \Phi}{\partial t}.
\]

Therefore our four conditions stated in the problem above are satisfied by the flow we had determined, \( \Phi \).