Effects of time and rate on the stress-strain-strength behavior of soils

Jian-Hua Yin

i) Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Hong Kong, China

ABSTRACT

This paper presents main data from both oedometer tests and triaxial tests on Hong Kong Marine Clay (HKMC) and a mixture of bentonite-silica. The oedometer tests include (i) multi-staged loading tests with unloading/reloading and enough time for creep and swelling in 1D straining and (ii) step-changed constant rate of strain compression tests with unloading/reloading in 1D straining as well. The triaxial tests include K₀-consolidated undrained compression or extension tests with constant (or stepped changed) strain rates. This paper will discuss the time effects and rate effects with a special attention to the non-linear and elastic visco-plastic behavior. Based on the test data presented above, this paper presents a brief review of the works of elastic visco-plastic (EVP) modelling of the time-dependent stress-strain behaviour of soils in one-dimensional straining (1D) and in 3D stress state. A few important concepts and their physical meanings are explained. The 1D EVP model is briefly reviewed with a comparison with the classic Maxwell’s rheological model. It is found that Yin and Graham’s 1D EVP model is an extension of Maxwell’s rheological model for considering the nonlinear behaviour of soils. The recent extension of the EVP modelling framework to consider the swelling of a saturated soil is introduced. A 3D EVP model is also introduced and discussed. A nonlinear creep function proposed by the author is presented. This function has been used in refined 1D and 3D EVP models. It is concluded that the time and rate effects of clayey soils shall be considered in a suitable constitutive model.

Keywords: clay, soil, creep, time effects, rate effects, elastic, visco-plastic, stress-strain

1. INTRODUCTION

The stress-strain behaviour of many soils is time-dependent or rate-dependent. Creep and swelling are two special aspects of the time-dependence. Some clayey soils exhibit more or less both creep and swelling. In this paper, “creep” means viscous compression under a constant load in an oedometer condition; while, “swelling” means viscous expansion under a constant load in an oedometer condition and is a reverse behaviour to “creep”, all for saturated soils. Clayey soils containing clay mineral montmorillonite show strong creep and swelling characteristics. Both creep and swelling characteristics have a significant influence on the deformation and failure of geotechnical structures. Stress relaxation is another time-dependent phenomenon in which the effective stress is decreasing with time when strain is keep constant. The rate effects include the strain rate and stress-rate effects in which the effective stress increases with the rate.

This paper presents main data from both oedometer tests and triaxial tests on Hong Kong Marine Clay (HKMC) and a mixture of bentonite-silica, discuss the effects of time and rate, and elastic visco-plastic modelling.

2. OBSERVATIONS FROM LABORATORY TESTS

2.1 Oedometer tests on HKMC

Fig. 1 shows the curves of log(time) and vertical strain from a multi-stage loading (MSL) oedometer test on undisturbed Hong Kong Marine Clay (HKMC) (Yin and Cheng 2006). It is seen from the figure that the HKMC exhibits creep strain after the End of the Primary Consolidation (EOP). Fig. 2 shows curves of effective vertical stress and vertical strain a multi-stage CRS (Constant Rate of Strain) oedometer test and MSL test on the same HKMC (Yin and Cheng 2006). The CRS rates are 0.018%/hours, 0.18%/hours, and 1.8%/hours. The strain rate effects are seen clearly. The larger the strain rate, the higher the effective vertical stress.
2.2 Triaxial tests on HKMC

Fig. 3 shows results from four multi-stage CRS $K_o$-consolidated compression triaxial tests ($q$ vs axial strain in top and porewater pressure $u$ vs axial strain in the bottom) (Yin and Cheng 2006). The CRS rates are 0.2%/hours, 2%/hours, and 20%/hours. It is seen clearly that the strain rates affect curves of stress-strain and the porewater pressure and strain. The larger the strain, the higher deviator stress and larger the excess porewater pressure.

Fig. 4 shows results from multi-stage CRS $K_o$-consolidated extension triaxial tests ($q$ vs axial strain in top and porewater pressure $u$ vs axial strain in the bottom) (Yin and Cheng 2006). The extension test was done that the confining stress was kept constant while the vertical stress was reduced at a step-changed strain rate. The strain rate effects are seen clearly.

2.3 Oedometer tests on a mixture of bentonite-silica

Fig. 5 shows curves of long(time) and vertical strains from a multi-stage loading and unloading/reloading oedometer test on a mixture of bentonite and 70% of silica (Yin and Tong 2011). It is seen that the mixture shows both creep and swelling.

Fig. 1. Results from a multi-stage loading oedometer test

Fig. 2. Results from a multi-stage CRS oedometer test

Fig. 3. Results from multi-stage CRS $K_o$-consolidated compression triaxial tests ($q$ vs axial strain in top and porewater pressure $u$ vs axial strain in the bottom)

Fig. 4. Results from multi-stage CRS $K_o$-consolidated extension triaxial tests ($q$ vs axial strain in top and porewater pressure $u$ vs axial strain in the bottom)
Fig. 5 shows the curve of effective vertical stress in logarithmic scale and vertical strain from data in Fig. 5. Large unloading and reloading loops are observed. This is caused by significant swelling of the bentonite.

3. ELASTIC VISCO-PLASTIC MODELLING

Constitutive modelling of the viscous stress-strain behavior of soils is very important and practically needed, not matter simple and comprehensive models. This is because (a) analysis, design and construction of most geotechnical structures need to consider the time and strain effects and non-linear and plastic deformations, and (ii) a constitutive relation is a mathematic expression of the physical stress-strain behavior including the time and strain effects, non-linearity, plastic strains etc. with in-depth understandings and logical expression of these physical phenomena. The item (ii) is not an easy task.

3.1 An elastic visco-plastic model and extension of the classic Maxwell’s linear rheological model

Based on the concepts (“equivalent time”, “reference time” line, etc.) and understandings of the time-dependent behavior, Yin and Graham (1989, 1994) derived rigorously a constitutive equation of a one-dimensional Elastic Visco-Plastic (1D EVP) model:

\[ \dot{e}_z = \frac{\kappa}{V} \frac{\sigma'_z}{\sigma_z} + \frac{\psi}{Vt_0} \exp\left[-\left(e_z - e_{zr}\right) \frac{V}{\psi} \frac{\sigma'_z}{\sigma_{zr}} \right] \frac{\sigma'_z}{\sigma_z}^{\psi/V} \]  

(1)

where \( \dot{e}_z \) and \( e_z \) are vertical strain rate and strain; \( \sigma'_z \) and \( \sigma_z \) are vertical effective stress rate and stress; \( \kappa/V \) is a constant related to elastic compression (see Fig.6); \( \lambda/V \) is a constant related to the “reference time line”. The meaning of \( \sigma_{zr} \) and \( e_{zr} \) are the define a point where the “reference time line” passes. As explained before \( \psi/V \) and \( t_0 \) (in units of time) are two constants related to creep of the soil. More details can be found in Yin and Graham (1989, 1994).

In (1), the elastic strain rate is \( \dot{e}_z = \kappa \frac{\sigma'_z}{V \sigma_z} \); the visco-plastic strain rate is:

\[ \dot{e}_z^{vp} = \frac{\psi}{Vt_0} \exp\left[-\left(e_z - e_{zr}\right) \frac{V}{\psi} \frac{\sigma'_z}{\sigma_{zr}} \right] \frac{\sigma'_z}{\sigma_z}^{\psi/V} \]  

(2)

It is seen from Eqn.(1) that the visco-plastic strain rate (the creep strain rate) is a function of the stress-strain state \( (\sigma'_z, e_z) \) only, nothing to do with how to reach this state.

It is well known that the classic Maxwell’s rheological model can be expressed as:
\[ \dot{e}_z = \frac{\sigma'_z}{E} + \sigma'_{ij} \frac{1}{\eta} \]  

(3)

where \( E \) is the elastic modulus and \( \eta \) is a viscous constant. Maxwell’s rheological model is derived from a series connection of a linear spring and a linear dashpot. In this approach, the total strain rate is divided into elastic strain rate and a visco-plastic strain rate. It is noted that Maxwell’s rheological model is not a visco-elastic model since the strain due to the dashpot is not recoverable. Maxwell’s rheological model is, in fact, a linear elastic and linear visco-plastic model. Comparing Eqn.(3) with Eqn. (1), it is found that Yin and Graham’s 1D EVP (Yin and Graham 1989, 1994) is a nonlinear elastic and nonlinear visco-plastic model, that is, a nonlinear rheological model. Yin and Graham’s 1D EVP can be considered to be an extension of the classic Maxwell’s linear rheological model (Yin 2012).

### 3.2 An elastic visco-plastic model considering both creep and swelling

The swelling here is the time-dependent expansion of the skeleton of a soil, in opposite to creep (compression). For saturated soil with both creep and swelling, Yin and Tong (2011) proposed the following constitutive model, based on Yin and Graham (1989, 1994):

\[ \dot{e}_z = \frac{\sigma'_z}{V} + \sigma'_{ij} \frac{1}{V} \left[ \exp \left( -\frac{e_z - e_{z0}}{\psi} \right) \frac{\sigma'_{ij}}{\sigma'_{ij0}} \right] \frac{1}{\psi'} \]  

\[ -\frac{\psi'}{V} \frac{1}{V} \left[ \exp \left( -\frac{e_z - e_{z0}}{\psi} \right) \frac{\sigma'_{ij}}{\sigma'_{ij0}} \right] \frac{1}{\psi'} \]  

(4)

Eqn. (4) is a general constitutive model for the time-dependent stress-strain behaviour of soils exhibiting both creep and swelling in 1D straining. This model is valid for all loading conditions such as constant rate of strain (CRS) loading, relaxation, unloading, reloading etc. This new model may be called one-dimensional Elastic Visco-Plastic model considering swelling, namely 1D EVPS model. The explanation of all parameters in (4) can be found in Yin and Tong (2011).

### 3.3 3D Elastic visco-plastic models

Using the concept of ‘equivalent time’, the Modified-Cam Clay model, and the approach in 1D EVP model (Yin and Graham 1989, 1994), Yin and Graham (1999) derived a three-dimensional Elastic Visco-Plastic (3D EVP) model:

\[ \dot{e}_z = \dot{e}_z^e + \dot{e}_z^p = \left( \frac{1}{2G} \dot{\epsilon}_y + \frac{\kappa}{V} \dot{\epsilon}_y \right) + \]  

\[ \psi \exp \left[ \left( e_{m0} + \frac{\lambda}{V} \ln \frac{p_m}{p_{m0}} - e_m \right) \frac{1}{V} \right] \frac{1}{\psi} \frac{\psi'}{p_m - p_{m0}} \frac{\partial F}{\partial \sigma_{ij}} \]  

(5)

where \( \dot{e}_z \) is the total strain rate \((i = 1,2,3; j = 1,2,3)\); \( \dot{e}_z^e \) and \( \dot{e}_z^p \) are the elastic and visco-plastic strain rate; \( \sigma_{ij} \) is effective stress; the mean effective stress \( p' \) is defined as \( p' = \sigma_{kk} / 3 \); \( \delta \) is the deviator stress rate; the deviator stress \( s \) is defined as \( s = \sigma_{ij} - \delta_{ij} \sigma_{kk} / 3 \), where \( \delta_{ij} = 0 \) if \( i \neq j \); \( \delta_{ij} = 1 \) if \( i = j \); \( G \) is the elastic shear modulus; \( \kappa / V \) \((V \) is specific volume), \( \psi / V \), \( t \), \( \lambda / V \), \( p_m \) and \( e_m \) are model parameters.

The \( F \) in Eqn.(5) is a function describing the visco-plastic flow surface (Yin and Graham 1999):

\[ F = p^2 + \frac{q^2}{M^2} - p p_m = 0 \]  

(6)

where \( M \) is the slope of the Critical State strength envelope in the \( q-p' \) plane; and \( q \) is the generalized deviator stress \( q / \sqrt{2} s_{ij} \). In Eqn.(5) and Eqn.(6), \( p_m \) is the mean effective stress at which the flow surface in Eqn.(6) intercepts the \( p' \)-axis in the \( q-p' \) plane. The sub-index ‘\( m \)’ represents the mean stress or volume strain under isotropic stressing conditions, that is, \( p' = p_m \) with \( q = 0 \). For example, \( e_m \) in Eqn.(5) is the total volumetric strain under isotropic stressing. Yin and Graham (1999) show that the rate of \( e_m \) can be expressed:

\[ \dot{e}_m = \frac{\psi}{V} \frac{\dot{p}_m}{p_m} + \frac{\psi}{V} \exp \left[ \left( e_{m0} + \frac{\lambda}{V} \ln \frac{p_m}{p_{m0}} - e_m \right) \frac{1}{V} \right] \frac{1}{\psi} \frac{\psi'}{p_m - p_{m0}} \frac{\partial F}{\partial \sigma_{ij}} \]  

(7)

Eqns.(5), (6) and (7) are differential equations of a 3D EVP model for describing time-dependent stress-strain behaviour of clays. The proposed model has been verified using data from a number of soils.

It is noted that the 3D EVP model in Eqn.(5) uses the natural logarithmic functions for creep, instant time line, and the reference time line without limit. To overcome the limitations, Yin et al. (2002) extended the 3D EVP model to describe the time-dependent behaviour of normally and overconsolidated clays using nonlinear functions (see Section 4 below) for creep. The validation of the model in Eqn.(5) can be found in Yin and Graham (1999).

### 4. A NON-LINEAR FUNCTION FOR CREEP OF SOILS IN 1D STRAINING

A semi-logarithmic function using \( e = e - C_0 \log t \)

where \( C_0 \) is the so-called the coefficient of “secondary consolidation” has a problem: when the time \( t \) is very big or infinite, the void ratio \( e \) becomes
negative. A semi-logarithmic function using 
\( \varepsilon_z = \varepsilon_{z0} + \left[ C_c / (1 + e_0) \right] \log \left( \sigma'_{z0} / \sigma'_{z0} \right) \) or 
\( e = e_0 - C_c \log \left( \sigma'_{z0} / \sigma'_{z0} \right) \) has the same problem (\( C_c \) is the compression index; \( e_0 \) is the void ratio corresponding stress). Yin (1999, 2011) suggested a new function for fitting non-linear creep behaviour of soils. In order to help readers to understand, we use the common log-function and parameters to express this non-linear function:

\[
\varepsilon_z^p = \frac{C_{z0} \log \left( \frac{t + t_o}{t_o} \right)}{1 + \frac{C_{z0}}{V} \log \left( \frac{t + t_o}{t_o} \right)}
\]  

(8)

where \( \varepsilon_z^p \) is creep strain (not including the initial strain before creep) and \( t \) is the duration of loading and \( \varepsilon_z^p \) is the creep limit. In Eqn.(8) \( C_{z0} \) and \( t_o \) stand for parameters at time \( t = 0 \) and can be determined by fitting Eqn.(8) to creep test data. It is noted that Eqn.(8) is valid for time \( t = 0 \). When the time \( t \) is infinite, \( \log \left( \frac{t + t_o}{t_o} \right) \to \infty \), \( \varepsilon_z^p = \varepsilon_z^0 \). It is noted that if we treat \( \log \left( \frac{t + t_o}{t_o} \right) \) as a new variable “x”, Eqn.(8) becomes 

\[
\varepsilon_z^p = \frac{x}{V / C_{z0} + (1 / \varepsilon_z^0)x} = \frac{x}{a + bx}
\]

which is the well-known hyperbolic function (\( a = V / C_{z0} \) and \( b = 1 / \varepsilon_z^0 \)). It is noted that exponential function (\( \varepsilon = \varepsilon_{z0} + c_i e^{-b t} \)) and the hyperbolic function in terms of time \( t \) (\( \varepsilon = \frac{t}{a + bt} \)) are not suitable for the nonlinear creep in 1D straining.

We can use the same type of non-linear function to fit the relation between the vertical effective stress and vertical strain (Yin, Zhu and Graham 2002). For example for the reference time line (similar to the normal consolidation line):

\[
\varepsilon_z = \frac{C_{z0} \log \left( \sigma'_{z} / \sigma'_{z0} \right)}{1 + \frac{C_{z0}}{V} \log \left( \sigma'_{z} / \sigma'_{z0} \right)}
\]  

(9)

If \( \sigma'_{z} = \sigma'_{z0} \), we have \( \varepsilon_z = 0 \). When the stress \( \sigma'_{z} \to \infty \), \( \varepsilon_z = \varepsilon_{zlf} \) which is the limit.

We can have the same function type in Eqn.(9) for the limit time line in Figure 5:

\[
\varepsilon_{zlf} = \frac{C_{zlf} \log \left( \sigma'_{z} / \sigma'_{z0} \right)}{1 + \frac{C_{zlf}}{V} \log \left( \sigma'_{z} / \sigma'_{z0} \right)}
\]

(10)

If \( \sigma'_{z} = \sigma'_{z0} \), we have \( \varepsilon_z = 0 \). When the stress \( \sigma'_{z} \to \infty \), \( \varepsilon_z = \varepsilon_{zlf} \) which is the limit.

5. CONCLUSIONS

From the previous presentation and discussion, the following conclusions are made:

(a) Most soils, such as Hong Kong Marine Clay (HKMC) and a mixture of bentonite and sand, exhibit various viscous phenomena, such as creep, swelling, rate effects, nonlinearity, and plastic strains.

(b) All these time effects, rate effects, nonlinearity, and plastic strains shall be considered a constitutive model.

(c) The statement of “the magnitude of a creep strain rate at a stress-strain state point is unique, independent of the loading path to reach this point” can be seen in the 1D EVP model (Yin and Graham 1989, 1994).

(d) The one-dimensional Elastic Visco-Plastic (1D EVP) (Yin and Graham 1989, 1994) is rigorously derived. This 1D EVP model is an extension of Maxwell’s linear rheological model for considering the nonlinear behaviour of soils.

(e) The 1D EVP model has been extended to consider both creep and swelling of a mixture of bentonite and sand.

(f) The 3D EVP model is rigorously derived based on the 1D EVP model approach and the modified Cam-Clay model for the time-dependent stress-strain behaviour of clayey soils. Further improvements of this model have been done and are still needed.

(g) The nonlinear functions proposed by the author are good for fitting the creep compression (with time) and the compression under high stress of most soft soils in 1D straining.

ACKNOWLEDGEMENTS

The work in this paper is supported by a research grant (project No. 51278442) from National Natural Science Foundation of China (NSFC), a grant (PolyU 152196/14E) from Research Grants Council (RGC) of Hong Kong Special Administrative Region Government of China, PolyU Shenzhen Research Institute in Mainland, China, and The Hong Kong
REFERENCES

1) Yin, J-H. 1999, ‘Non-linear creep of soils in oedometer tests’, Geotechnique, 49, No.5, pp.699-707.
2) Yin, J-H (2011). “From constitutive modeling to development of laboratory testing and optical fiber sensor monitoring technologies”, Chinese J of Geotechnical Engineering, 33(1), 1-15. (14th “Huang Wen-Xi Lecture” in China).
3) Yin, J-H. (2012). Review of elastic visco-plastic modeling of the time-dependent stress-strain behaviour of soils and its extensions and applications. “Constitutive Modelling of Geomaterials – Advances and New Applications”, Springer Series in Geomechanics and Geoengineering, Yang, Q., Zhang, JM, Zheng, H. and Yao YP (eds.), 149-158.
4) Yin, J-H and Cheng, CM, (2006). Comparison of Strain-rate Dependent Stress-Strain Behaviour from K0-consolidated Compression and Extension Tests on Natural Hong Kong Marine Deposits. Marine Georesources and Geotechnology, Vol.24, No.2, pp119-147.
5) Yin, J. H. and Graham, J. 1989. Viscous elastic plastic modelling of one dimensional time dependent behavior of clays. Canadian Geotechnical Journal 26, pp.199-209.
6) Yin, J. H., and Graham, J. 1994. Equivalent times and one dimensional elastic visco-plastic modelling of time dependent stress strain behaviour of clays., Canadian Geotech. Journal 31, 42 52.
7) Yin, J-H. and Graham, J., 1999, ‘Elastic visco-plastic modelling of the time-dependent stress-strain behaviour of soils,’ Canadian Geotech. Journal, 36, 736-745.
8) Yin, JH and Tong, F. (2011). Constitutive Modelling of the Time-dependent Stress-strain Behaviour of Saturated Soils Exhibiting both Creep and Swelling. Canadian Geot J. 48, 1870-1885.
9) Yin J-H., Zhu J-G. and Graham, J (2002): A new elastic visco-plastic model for time-dependent behaviour of normally and overconsolidated clays: theory and verification. Canadian Geotechnical Journal 39, 157-173.