Comment on “Intensity correlations and mesoscopic fluctuations of diffusing photons in cold atoms”

In a recent Letter \[1\], O. Assaf and E. Akkermans claim that the angular correlations of the light intensity scattered by a cloud of cold atoms with internal degeneracy (Zeeman sublevels) of the ground state overcome the usual Rayleigh law. More precisely, they found that they become exponentially large with the size of the sample. In what follows, we will explain why their results are wrong and, in contrary, why the internal degeneracy leads to lower intensity correlations.

Following the Authors’ proposed experimental scheme and notations (their Eq.(3)), the correlation of the transmission coefficient reads:

\[
T_{ab}T_{a'b'} = \sum_{ijkl} A_i^{(R,m)} A_j^{(R,m')} A_k^{(R,m')} A_l^{(R,m')} \tag{1}
\]

As explained by the Authors, the configuration average of the preceding equation leads to two possible pairing among the photon paths, either \(i = j, k = l\) which corresponds to \(T_{ab}T_{a'b'}\), or \(i = l, j = k\). In the latter case, the correlation of the transmission coefficient reads:

\[
\delta T_{ab}\delta T_{a'b'} = \sum_{m,m'} \sum_i A_i^{(m)} A_i^{(m')} \sum_j A_j^{(m)} A_j^{(m')} \tag{2}
\]

The preceding equation corresponds to Eq.(4) of the Letter, with the only difference that the average over the internal states is explicitly written (the configuration probabilities \(p(m), p(m')\) are left implicit). Since the sum over \(i\) is just the complex conjugate of the sum over \(j\), Eq. (2) reads

\[
\delta T_{ab}\delta T_{a'b'} = \sum_{m,m'} \sum_i |A_i^{(m)} A_i^{(m')}|^2 \tag{3}
\]

Now, for fixed \(\{m\}, \{m'\}\), applying the Cauchy-Schwartz inequality (i.e. \(\sum_i x_i y_i^* \leq \left( \sum_i |x_i|^2 \sum_j |y_j|^2 \right)^{1/2}\)) to the sum over \(i\) one obtains:

\[
\delta T_{ab}\delta T_{a'b'} \leq \sum_{m,m'} \sum_i |A_i^{(m)}|^2 \sum_j |A_j^{(m')}|^2 = T_{ab}T_{a'b'} \tag{4}
\]

This inequality proves that the condition \(C_{aba'b'} < 1\) is always fulfilled, even in the case of internal degeneracy.

To pinpoint the error in [1], please note that in our Eq. (3), the sum over the internal states is outside the modulus square in contradiction with the Eq.(5) of the Letter, where this sum appears inside the modulus square. This results in a completely different mesoscopic situation. From Eq. (3), the intensity correlation arises as an incoherent sum of the square of different correlation diffusons (one for each \(\{m\}, \{m'\}\) pair), whereas in Eq.(7) of the Letter, the authors are calculating the square of a correlation diffuson in which all \(\{m\}, \{m'\}\) contributions add coherently. Obviously, transforming a sum of intensities into a square of a sum of amplitudes can lead to a very different physical behavior, like an exponential grow with the size of the system.

The preceding points, alone, are enough to prove that the large intensity fluctuations calculated by the Authors are unphysical. Still, we would like to emphasize an additional crucial point: as explained by the Authors, the dominant contribution in Eq. (2) is obtained only when the two paths \(i\) and \(j\) are not sharing any scatterers. In the case of internal degeneracy, this condition actually implies that, along these two paths, the atoms can only undergo Rayleigh transitions (i.e. for which the initial and final internal states are the same), in full agreement with the experimental results of Ref. [2]. Indeed, the fact that both photon paths \(i\) and \(j\) correspond to the same global configuration of internal states (see Eq.(2)), (i.e. \(\{m\}\) during the first pulse and \(\{m'\}\) during the second pulse), means that, if a Raman transition occurs for a given atom along the photon path \(i\), then the same atom must undergo the same Raman transition along the path \(j: i\) and \(j\) must share, at least, this particular atom. One must mention that this point is also crucial for the coherent backscattering effect: the latter arises from interference between photon paths not only visiting the same atoms in the reverse order, but also along which each atom performs the same internal transition [3], resulting in a lower enhancement factor, in perfect agreement with the experimental observations [4]. In the present case, this means that only photon paths along which all the atoms have undergone Rayleigh transitions contribute to the disorder averaged intensity correlations. Since the number of those paths is smaller than the total possible paths, we expect \(T_{ab}T_{a'b'} < T_{aba'b'}\), i.e. \(C_{aba'b'} < 1\). Only when the ground state is not degenerate (i.e. Rayleigh scatterers), all paths contributing to the average intensity also contribute to the correlation function, leading to \(x_i = y_i\) and the Cauchy-Schwartz inequality becomes an equality, implying \(C_{aba'b'} = 1\).

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