Modeling the closed circulation of thermal water

V I Pen’kovskii and N K Korsakova
Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk, 630090, Russia
E-mail: penkov@hydro.nsc.ru, kors@hydro.nsc.ru

Abstract. Two schemes of closed circulation of thermal waters are considered. The first scheme is filtration in a single stratum. In the second scheme, the flow in a formation with a weakly permeable bottom is considered. The effective for designers formulas are obtained. The thermal formation model is used in the form of a heterogeneous structure consisting of two embedding continuums.

1. Introduction
Use of coal, oil, and gas as general energy carriers leads to aggravating the situation with the depletion of their non-renewable deposit reserves. So, the use of thermal water will be more extensive in future. There is a great historical experience of using thermal waters for growing vegetables and fruits on the territory of the Solovetsky Archipelago, on the banks of the Anadyr River and otherwhere in Russia [1-3]. Thermal waters in Siberia are located on a depth of 600-900 meters. Sometimes the water temperature reaches 90 degrees Celsius. The existing geothermal gradient is on average 300°C per 1 km depth of the rocks. A number of Kuznetsk Basin sites fall into the list of territories with the highest thermal conditions, as well as Kamchatka, Kurile Islands, Baikal, Pamir, Tien Shan. Interest in the problem of modeling the movement of thermal waters and their practical use has been expressed in many countries of the world [4]. There are specialized institutes; regular international congresses are held, and packages for implementation of computing programs are created [5].

The main problem faced by the designers of plants for the application of underground thermal waters is the creation of a closed water circulation regime. Thermal waters often contain a large amount of salts of various toxic chemical compounds. The discharge of the polluting elements into natural water systems located on the surface of the earth is extremely undesirable.

In this paper, model problems of the process of heat energy transfer by thermal waters in the closed circulation regime are considered based on the representation of the aquifer as a heterogeneous structure consisting of two continuums nested into each other [6].

2. Closed circulation of thermal water in single formation
Let us consider a simple model problem of circulation in the case of one-dimensional flow of thermal waters in a finite pressure formation. We denote the length and thickness of the formation by symbols $L, M$, respectively. Let $Q = \nu M$ be the magnitude of extraction or injection of water, calculated per unit of stratum width, $T_c$ and $T_0$ be the temperatures of the injected and pumped water, respectively, $T_a$ be geothermal initial temperature at a given stratum depth, $\nu$ be flow velocity. If there are no additional sources or sinks in the formation, then one can set the values of pressure
\( h(x) = \Delta h / 2 - x\Delta h / L \) instead of discharges. At the input \( x = 0 \) and output \( x = L \) sections we obtain \( h = \Delta h / 2 \) and \( h = -\Delta h / 2 \), respectively. Here \( \Delta h = LQ / (2k_fM) \), \( k_f \) is seepage coefficient. Scheme of flow is presented in figure 1.

Let us write down the equations of heat transfer in a heterogeneous continuous medium consisting of two interpenetrating continua: a pore space of a reservoir saturated with thermal water and an immovable solid matrix of porous medium. Taking into account the kinetics of internal heat exchange between the continuums and the influx of heat through the top and subface of the stratum from the surrounding rocks, the system of heat transfer equations has the form:

\[
\begin{align*}
-mD \frac{\partial^2 T}{\partial x^2} + c\rho \frac{\partial(T_v)}{\partial x} + mc\rho \frac{\partial T}{\partial t} + (T - T_s) / \alpha_s & = mq_a; \\
-(1 - m)D \frac{\partial^2 T}{\partial x^2} + (1 - m)c_s\rho_s \frac{\partial T_s}{\partial t} - (T - T_s) / \alpha_s & = (1 - m)q_a.
\end{align*}
\]

Here \( D, T, m, \rho, c, v, x, t \) are thermal-conductivity coefficient, temperature, porosity, density, heat capacity, flow velocity, coordinate, and time, respectively; values with index \( s \) refer to solid matrix. We assume that the sources of heat coming from the surrounding rocks through the top and subface of the stratum are proportional to the difference between the temperature of surrounding rocks and the current temperature of water in formation \( q_a = \alpha_a(T_a - T) \).

The first terms in system (1) reflect external diffusive process. The convective transfer process is determined by the second term (taking into account \( v_s = 0 \)). The third terms of the equations are the time variation of the internal thermal energy of the continuums. The intensity of interpore heat exchange between the continuums is proportional to the difference in local temperatures. Since heat exchange occurs at distances comparable to the pore size and substantially depends on pore specific surface, then \( \alpha_s \) can be considered as small value in system (1), and, therefore, \( T_s \approx T \).

Under these assumptions, the system of equations (1) is transformed by addition to the equation

\[
-(1 - m)D \frac{\partial^2 T}{\partial x^2} + c\rho \frac{\partial(T_v)}{\partial x} + [mc\rho + (1 - m)c_s\rho_s] \frac{\partial T}{\partial t} = \alpha_s(T_a - T)
\]

of general convective-diffusive heat transfer over the entire section of the formation.
Equation (2) must be supplemented by the equation of the kinetics of rapid heat exchange between the rock skeleton and water moving in it (there is a complete analogy with the process of salt transfer in rocks with double porosity [8])

\[(1 - m)c_s \rho_s \frac{\partial T_s}{\partial t} = \frac{1}{\alpha_s}(T_s - T_c).\]  

This equation is obtained from the second equation of system (1) by its formal expansion in a small parameter with the conservation of two terms. The physical meaning of equation (3) lies in describing the kinetics of the heat pore diffusion process with diffusion coefficient \(D_s = r_s/\alpha_s\), where \(r_s\) is the average pore radius. On the other hand, the right-hand side of equation (3) is the heat transfer between the continua such as the solid matrix and formation water.

Let the initial temperature of the stratum be \(T_a\). Water is injected at a given temperature \(T_c < T_a\) through section \(x = 0\). In section \(x = L\) it is pumped out with the same flow rate \(Q\). The temperature of the pumped water will be determined in the process of establishing a closed circulation. Neglecting the first term in equation (2), which reflects the total process of heat diffusion in the continua, and assuming that the filtration rate is constant, we write the relations on the characteristics of the transformed equation:

\[\frac{dx}{v \rho c} = \frac{dt}{m c \rho + (1 - m)c_s \rho_s} = \frac{dT}{\alpha_s(T_a - T)}.\]  

The cooled injected water front is determined by the characteristic \(x = x_s(t)\), coming out of the point \(x = 0\), \(x_s(t) = \frac{vt}{m + (1 - m)\delta}\), where \(\delta = \frac{c_s \rho_s}{c \rho}\). This front lags behind the front of the marked water particles \(x_m(t) = vt / m\). The width of the expanding in time zone of heating the injected water is determined by the difference of the fronts

\[x_s(t) - x_m(t) = \frac{vt}{m} - \frac{(1 - m)\delta}{m + (1 - m)\delta}.\]

Using the relations (4) and integrating subject to the boundary condition, we obtain temperature distribution in \(0 \leq x < x_s(t)\): \(T = T_a - (T_a - T_c) \exp(-\alpha_s^f x)\), where \(\alpha^f_s = \alpha_s L(c \rho h / \Delta h)^{-1}\). In the interval \(x_s(t) \leq x \leq L\), the temperature remains the initial \(T = T_a\).

The steady state circulation is achieved with the following ratio between the parameters of the heat transfer process

\[\frac{T_a - T_0}{T_a - T_c} = \exp(-\alpha_s^f L).\]

Figure 2 presents the plots of the temperature distribution in the stratum and the locations of the water cooling front and the front of marked water particles at time points of 10, 15, 20, 30 and 50 days after the start of injection.
**Figure 2.** The distribution of temperature and location of water fronts.

Figure 3 shows the dependence of the coefficient $\alpha^f_u$ on the geometrical parameters of the formation for the closed circulation mode with values $T_1 = 20$, $T_0 = 80$, $T_u = 90$ degrees Celsius.

**Figure 3.** Relation between the parameters of the problem.

The calculations were made at the following parameters: $L = 100$ m, $M = 5$ m, $Q = 5$ m$^3$/day per unit stratum width, $v = 1$ m/day, $\rho_s = 2400$ kg/m$^3$, $\rho = 1000$ kg/m$^3$, $c_s = 0.8$ kJ/kgK, $c = 4.2$ kJ/kgK, $m = 0.2$.

3. **The thermal water circulation in the stratum with a weakly permeable bottom**

Consider a flow that differs from the previous scheme by the presence of a weakly permeable bottom with seepage coefficient $k_1$ and thickness $M_1$, separating the exploited formation from the underlying formation, in which the pressure $h_1 = 0$ and the initial temperature of the water are retained. In this case, we will follow the well-known Myatiev-Girinsky flow scheme in layered-uniform formations [7].
According to this theory, the movement of fluid in the main aquifers is considered horizontal, and in weakly permeable interlayers, the flow is considered vertical, occurring at a velocity proportional to the difference of aquifers pressures.

Let us suppose for simplicity that the dependence of the seepage coefficients on temperature can be neglected. In this case, the horizontal velocity of thermal water flow \( v(x) = -k_j \frac{\partial h}{\partial x} \) in the main formation depends only on the coordinate \( x \), and the pressure, as some value averaged over the vertical coordinate \( z \), in the absence of additional volume sources or sinks must satisfy the filtration equation, in which the right-hand side determines the amount of vertical cross-flows with velocity \( w(x) = k_i \frac{h}{M_1} \) between two layers. The water flow equation in the main formation has the form [7]

\[
\frac{\partial^2 h}{\partial x^2} = \alpha^2 h; \quad (\alpha^2 = k_i (k_j, M_1, M)^{-1}).
\]

We subject its general solution \( h = C_1 h(x) + C_2 c(x) \) to the boundary conditions \( h(0) = \Delta h / 2; \quad h(L) = -\Delta h / 2 \). As result, we obtain

\[
C_1 = -\frac{\Delta h}{2} \frac{1 + h(c(L))}{sh(c(L))}, \quad C_2 = \frac{\Delta h}{2}.
\]

Taking into account that \( \frac{\partial v}{\partial x} = -\alpha^2 k_j h(x) \) and convective heat transfer in the vertical direction through a weakly permeable bottom is estimated by the value \( w(x) c_\rho \frac{\partial T}{\partial z} \approx -k_h c c(T_a - T) / M_1^2 \), the equation (2) (without a diffusion term) after some transformations takes the form

\[
- \frac{\partial h}{\partial x} \frac{\partial T}{\partial x} + [m + (1 - m)\delta] \frac{\partial T}{\partial \tau} = \alpha_1^2 (T_a - T) + \alpha^2 hM / M_1 (T_a - T) + \alpha^2 hT.
\]

Here we denote \( \tau = k_j t \). Satisfying the condition \( \tau = 0, x = x_c = 0 \), a characteristic of the written equation is determined implicitly by formula \( \tau = \int_0^\infty \frac{dx}{-\partial h/\partial x} \) in which the integral can be calculated explicitly.

In the steady state circulation regime, the temperature distribution in the formation is described by an ordinary differential equation:

\[
\frac{dT}{dx} = -[\alpha_1^2 (T_a - T) + \alpha^2 hM (T_a - T) / M_1 + \alpha^2 hT] / (dh/\partial x) \quad \text{with boundary condition} \quad x = 0: T = T_c.
\]

Figure 4 presents the curves of temperature distribution in a geothermal formation with a weakly permeable layer for different values of the parameter \( \alpha = 0, 0.01, 0.011, 0.014, 1.5, 1.75 \), marked with numbers 1-6.
As it can be seen from the numerical results, the regime of closed circulation in the formation with weakly permeable bottom depends essentially on the above parameter. In this case, the distance between the wells can be reduced in comparison with the circulation scheme in one isolated formation.

**Conclusions**

The proposed approach makes it possible to identify the reactive zone in the process of the closed circulation of thermal waters, in which the formation water is actively cooled or heated. A big role in choosing the “base” (the distance between the injection and pumping wells) is played by the ratio between the geological and geometrical parameters of the projected closed circulation system.

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