Bearing capacity of strip footings on jointed rock mass

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Abstract
Rock masses are non-homogenous, discontinuous media composed of rock material and naturally occurring discontinuities such as joints, fractures and bedding planes. Due to the presence of the geological discontinuities such as joints, faults and bedding planes, the compressive strength and modulus of elasticity of jointed rock mass are significantly reduced and the measurement of the strength behaviour of these jointed rock masses below the foundation becomes a challenging task. Previous researches have dealt with the bearing capacity of strip footings on the jointed rock mass for concentric, eccentric, inclined loading, separately. But, very limited work has been carried out for determining the bearing capacity of footings on jointed rock mass under eccentric-inclined loading together. In this study, the behaviour of rock masses under the pressure of strip footing has been investigated. To make the problem, more realistic, eccentric-inclined load was applied on the strip footing resting on horizontal jointed rock mass. A parametric study has also been carried out to develop some non-dimensional correlation between different parameters including GSI, e/B ratio, inclination, bearing capacity, etc. Three-dimensional analysis has been carried out by the finite element method using PLAXIS 3D software. Modified Hoek–Brown criteria was used to simulate the behaviour of rock mass and elastic behaviour of foundation was taken into the consideration for analysis. From the results, it can be concluded that the bearing capacity values drop as the eccentricity of the load increases. This indicates that as the eccentricity of the load increases, the bearing capacity of jointed rock mass diminishes. The bearing capacity value decreases with increasing loading inclination with respect to vertical. In the current study, non-dimensional correlations have been developed using data from non-linear elasto-plastic FEA to forecast footing’s bearing capacity, settlement and tilt of shallow foundation. These connections rely on the inclination of the load as well as the eccentricity to breadth ratio. The results obtained from the non-dimensional correlations holds goods on comparing the results obtained from the FEM analysis.

Keywords Eccentric inclined · Strip footing · Jointed rock mass · Finite element analysis

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Introduction

It is evident that little investigation is needed for the intact rocks because of very high strength characteristics and their resistance to displacement even under very high load application. But, for jointed rocks, the behaviour is very uncertain. As the behaviour of heavily jointed rock mass is uncertain, therefore, it requires intensive investigation before any construction work is carried out over such rock mass. A jointed rock mass may fail by several mechanisms. Various failure mechanism may include cracking, crushing, wedging, punching and shearing. Moreover, adequate attention has not been given to the load-settlement characteristics of shallow footings over jointed rock mass. These can be seen as the grey areas to conduct further studies in these areas. A geotechnical engineer many times faces problems of foundations subjected to eccentric-inclined loads. Eccentric-inclined load arises in case of foundations of various structures such as transmission towers, bridge abutments and retaining walls. Also, many times foundations are needed to be laid down on jointed rock mass, whose behaviour is hard to predict due to the presence of joints, fissure, cracks, etc. Taking all these into account, a geotechnical engineer needs to design foundations which involves determination of ultimate bearing capacity, tilt and settlement of the footing. Figure 1 depicts the settlement of a footing under the influence of an eccentric-inclined load. In Fig. 1, B is the width of the footing, e is the eccentricity of the applied load, i is the inclination of the applied load with respect to the vertical, t is the tilt of the footing, $S_e$ is the settlement of the footing at the eccentric-inclined load application point and $S_m$ is the maximum settlement of the footing under eccentric-inclined load.

Many studies have been conducted to assess the bearing capacity of soils. And, a wide range of theoretical and/or numerical models have been developed taking into account the strength parameters of soil for determining the bearing capacity of the foundations built on a variety of soils. However, due to the highly altering nature and huge effect of geometric and physical characteristics of the joints, it becomes very difficult to develop an explicit solution for obtaining the bearing capacity of foundations laid on rock/rock mass. In this study, an attempt has been made to develop non-dimensional correlations to assess the settlement, tilt and ultimate bearing capacity of rigid surface strip footings resting on weak or highly fragmented Hoek–Brown rock mass subjected to eccentric-inclined loads. The correlations have been developed through an extensive parametric study which was carried out to predict the pressure versus settlement characteristics using elasto-plastic finite element analysis and considering rigid interface characteristics at the footing-rock mass interface.

Review of literature

The well-established methodologies, used for determination of ultimate bearing capacity of shallow foundation resting on jointed rock mass, can be classified into following four groups: (1) analytical methods which includes limit equilibrium analysis, limit analysis and characteristic line method; (2) numerical methods; (3) experimental methods and (4) empirical methods.

The limit equilibrium method is a widely known technique for obtaining approximate solution for stability problems which include bearing capacity of footing, lateral earth pressure of retaining walls and stability of slopes. This method has simple formulation but it did not consider the stress–strain response of rock mass. As this method considers the equilibrium conditions only, so the solutions obtained are mostly approximate. Many researchers namely Terzaghi (1943), Meyerhof (1951), Zhu et al. (2001), Silvestri (2003), Bishnoi (1968) and Kulhawy and Goodman (2005) have developed bearing capacity solutions using this methodology.

In comparison to limit equilibrium method, the limit analysis method considers the stress–strain response of rock mass in an idealized approach. This idealization, known as normality principle or plastic flow rule, establishes two limit theorems which forms the foundation of limit analysis. The plastic limit theorems of Drucker et al. (1952) are conveniently utilized to obtain the lower and upper bounds of the collapse load. Kulhawy and Carter (1992) proposed...
a simple lower bound solution for the bearing capacity of a weightless rock mass obeying a non-linear Hoek–Brown yield criterion. Bindlish (2007) and Bindlish et al. (2012) carried out an experimental programme to investigate the ultimate bearing capacity of anisotropic rock mass specimens. Plaster of Paris specimens were used as a model material to simulate intact rock material behaviour. Experimental findings were validated through UDEC (Universal Distinct Element Code) numerical modelling. Prakoso and Kulhawy (2004) and Singh and Rao (2005) developed lower bound bearing capacity models for the determination of ultimate bearing capacity of shallow strip footings resting on anisotropic jointed rock mass. Yang and Yin (2005) proposed an upper bound method based on plastic theorems proposed by Drucker et al. (1952) for calculating ultimate bearing capacity using the generalized tangential technique. Merifield et al. (2006) carried out numerical limit analysis for obtaining ultimate bearing capacity. The finite element method was used to obtain rigorous upper and lower bound solutions. Bearing capacity factors were presented for different rock mass conditions.

Serrano and Olalla (1994, 1996) and Serrano et al. (2000) developed a more comprehensive approach for determining the bearing capacity of strip footing on jointed rock mass obeying Hoek–Brown failure criterion. They made use of the characteristic line method as was developed by Sokolovski (1960) and Sokolovskii (1965).

These methods have been proposed for the solution of this problem and almost all the methods come with a set of conventions such as infinite strip footing, plane strain condition, no inertial force, weightless rock mass and undisturbed rock mass makes the situation impractical or unrealistic. This limitation can be overcome by using the numerical method. Sutcliffe et al. (2004) have carried out rigorous lower bound limit analysis of surface strip footings to evaluate the bearing capacity of the jointed rock mass. In this analysis, linearized Mohr–Coulomb yield criterion has been used. The problem has been formulated assuming plane strain conditions. The jointed rock mass has been treated as homogeneous, anisotropic and perfect plastic material. Only small deformations were considered in this analysis under limit load. From the analysis, it was found that the presence of one or two joint sets in rock mass can reduce the bearing capacity by up to 60% or 87%, respectively. In comparison to the results for a rock mass with two joint sets, the addition of a third joint that is vertically oriented results in a further loss in ultimate bearing capacity of up to 40%. The bearing capacity solutions for strip footings on jointed rock masses taking into account one and two sets of discontinuities were presented by Prakoso and Kulhawy (2004). It was shown that the bearing capacity of rock mass is significantly influenced by strength and geometrical factors. The main drawback of this approach is that bearing capacity solutions did not take into account the effects of the rock mass weight, embedment and joint set spacing. Using finite element analysis, Clausen (2013) looked into the issue of the bearing capacity of a circular surface footing resting on a horizontal rock mass. The issue has been handled as an axis-symmetric problem. The generalized Hoek–Brown failure criterion has been used to characterize how rock masses behave. It has been discovered that for rocks of low grade (GSI 30), which lead to displacement failure, the weight of the rock has a considerable effect on the bearing capacity. For higher quality rocks, the self-weight has essentially no impact. Utilizing a unique element-based software called UDEC, Salari-Rad et al. (2013) looked into the issue of the bearing capacity of shallow foundations supported by anisotropic discontinuous rock mass. A 5-m-wide strip footing has been modelled on a rock mass with a single joint set. Sensitivity analysis was used to choose the model’s dimension (width 70 m and depth 30 m). The Hoek–Brown and Mohr–Coulomb failure criterion for joints and rock material, respectively, were used in the numerical analysis. The results of this investigation showed that a rock mass with one joint set has an ultimate bearing capacity that ranges between 27 and 86% of intact rock. Additionally, it was shown that as the shear strength of the plane of weakness reduces, the bearing capacity of the rock mass does as well. Utilizing the unique element-based software UDEC, Bindlish et al. (2013) simulated the issue of ultimate bearing capacity of a strip footing on jointed rock mass subjected to central vertical load and compared the analysis results with the experimental results of Bindlish et al. (2012). Numerous numerical tests were run under conditions of planar strain. The elastic model, Mohr–Coulomb model and Coulomb–Slip model, in that order, were used to describe how the foundation, blocks and joints of rock masses behaved. The results of the numerical simulations looked to be extremely close to the experimental data. Additionally, it was believed that splitting and shearing were the predominant failure modes. The bearing capacity of two interfering strip footings made of Hoek–Brown materials was examined by Javid et al. (2015). In this study, interference between two neighbouring rough rigid strip footings was investigated using the finite-difference code UDEC. The findings of numerical modelling supported the idea that interference between two strip footings resulted in bearing capabilities that are significantly higher than those of the same footings that are isolated. It was discovered that calculating a rock mass’s maximum bearing capacity using equivalent Mohr–Coulomb parameters significantly overestimated the bearing capacity. Mansouri et al. (2019) used three-dimensional finite element studies to examine the ultimate bearing capacity of Hoek–Brown rock masses under square and rectangle footings. Analyses were done for various rock mass attributes as well as various footing dimensions. The outcomes demonstrate that as rock mass Hoek–Brown parameters are increased, the bearing
capacity also increases. Additionally, it is stated that three-dimensional bearing capacity assessments should be done for square and rectangular footings because the current two-dimensional methodologies result in an overestimation of bearing capacity. Deb et al. (2022) has performed structural analysis of piled raft foundation in soft soil by experimental simulation and also conducted parametric study by numerical method. With $2 \times 2$ and $3 \times 3$ pile groups, respectively, two distinct types of small-scaled combination piled raft foundation CPRF models were created, and the pile spacing as well as the clayey soil thickness were also changed. A numerical model was also created based on the findings of the experimental tests, and a parametric analysis was then carried out numerically while taking into account a 10-story residential building. The impact of various pile lengths on CPRF was also examined, and parametric research using a variety of pile layouts was carried out. Chavda and Dodagoudar (2018) have evaluated the ultimate capacity of strip footing using FEM. According to the findings of the FE study, the strip footing’s ultimate load is influenced by the strength parameters, footing width, soil unit weight and surcharge at the footing’s base level. The ultimate capacity is unaffected by the deformation parameters and will essentially remain the same for materials with models MC, HS, HS small and SS.

**Constitutive model for rock mass**

In the present study, Hoek et al. (2002) edition of the Hoek–Brown model has been adopted in the PLAXIS-3D finite element package to simulate the isotropic behaviour of rock materials. The adoption of the model along with the factorization of strength part is based on Benz et al. (2008). The generalized form of Hoek–Brown failure criterion can be devised basically as a non-linear relationship between the major and minor effective principal stresses:

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left( \frac{\sigma'_c}{\sigma_{ci}} + s \right)^a$$  

where $\sigma_{ci}$ represents reduced value of the intact rock parameter $m_i$ and is also dependent on geological strength index (GSI) and the disturbance factor. $s$ and $a$ are auxiliary material constants corresponding to the rock mass. $\sigma_{ci}$ is the ucs value of the intact rock material.

**Finite element modelling**

A plane strain bearing capacity problem is being considered in Fig. 2. A rigid surface strip footing with rough base is lying on weightless jointed Hoek–Brown rock mass with uniaxial compressive strength of intact rock $\sigma_c$, geological strength index GSI and intact rock yield parameter $m_i$.

The non-linear behaviour of rock mass was modelled using the 2002 edition of Hoek–Brown model utilizing three-dimensional finite element models and conducting parametric studies by PLAXIS-3D software, Brinkreve et al. (2013a, b), the effect of eccentric-inclined loading on the ultimate bearing capacity and settlement of footing was studied. In this study, strip rigid footings of width 2 m were considered. The eccentric-inclined load was applied incrementally in order to draw the load-settlement curve of the rock mass, which was used for obtaining the bearing capacity. The software allows the automatic generation of 10-noded tetrahedral elements for rock mass discretization and 6-noded beam elements for the footing to simulate the behaviour of rigid footing. Meshing is one of the most important parts of FEM modelling because it controls the average element size and number of generated triangular elements. To check the effect of mesh conversion, load-settlement curves have been plotted for each type of automatic generation of mesh (very coarse, coarse, medium, fine, very fine). On comparing these graphs, it can be clearly concluded that the influence of mesh conversion is minor in this scenario. Hence, for optimization, medium meshing has been chosen for further exploration. Mesh is refined near the footing area and coarseness of meshing increases as we move away from it. The dimension of the soil profile is $100 \times 30 \times 25$ m and strip footing is placed at the centre of it of width 2 m., as shown in Fig. 3. Width and height of model are taken as 15 times and 12.5 times of width of footing, respectively. These extent of boundary conditions are decided such that effect of boundary condition on the structure is minimized. The length of the strip footing is taken as 100 m. A line load is also applied on the rigid footing with varying eccentricity and inclination. The properties of the
jointed rock mass and rigid footing that are being used in the numerical modelling are tabulated below in Table 1 and Table 2, respectively. The boundary conditions were applied such that the nodes along XZ-plane of model were restrained in Y direction and were free to move in X and Z directions, whereas nodes along YZ-plane of model were restrained in X direction and were free to move in Y and Z directions. The bottom boundary was restrained in all the directions and top surface was kept free in all directions, as is clear in Fig. 3.

Evaluation of ultimate bearing capacity

The pressure-settlement characteristic curves are the most widely used tools in determining the ultimate bearing capacity of the ground below the foundation. At present, there are four published methods for estimating the ultimate capacity using the pressure-settlement curves. Lutenegger and Michael T. Adams (1998) explained these methods, which includes the tangent intersection, the 0.1B, the hyperbolic and the log–log methods. Among these methods, the tangent intersection method works better when there is a significant change in the settlement of foundation. Such a pressure-settlement characteristic is generally observed when the shear surfaces present in the material below the foundation reaches the ground surface. In case if these shear surfaces fail to reach the ground surface, there is no significant settlement change of the footing. In such a case as per Frank et al. (2004), the strength of the soil/rock body is quite large, and thus, the criterion for the determination of ultimate bearing capacity is the excessive settlement. Such pressure-settlement curves do not possess a discrete peak value; the ultimate pressure on the footing is governed by the settlement of the footing. In those cases, the 0.1B method will work efficiently. Tangent intersection and 0.1B methods are shown in Fig. 4. The ultimate bearing capacity can be written as,

\[ q_u = \sigma_{ci}N_{et0} \]  

where

\[ q_u \]  

ultimate bearing capacity

Table 1 Properties of rock mass

| S. No | Properties                          | Value    | Unit  |
|-------|-------------------------------------|----------|-------|
| 1     | Young’s modulus                     | E        | 27.50E+6 | kN/m² |
| 2     | Poisson’s ratio                      | \( \mu \) | 0.2    |       |
| 3     | Uniaxial compressive strength of intact rock | \( \sigma_{ci} \) | 50.00E+3 | kN/m² |
| 4     | Intact rock parameter                | \( m_i \) | 1      |       |
| 5     | Disturbance factor                   | D        | 0      |       |
| 6     | Dilatancy angle                      | \( \psi_{\max} \) | 0      | deg   |
| 7     | Geological strength index            | GSI      | 10, 20, 30, 40, 50 |       |

Table 2 Properties of rigid footing

| S. No | Properties                          | Value    | Unit  |
|-------|-------------------------------------|----------|-------|
| 1     | Equivalent plate thickness          | d        | 0.5   | m     |
| 2     | Young’s modulus in first axial direction | \( E_1 \) | 2.1E+12 | kN/m² |
| 3     | Young’s modulus in second axial direction | \( E_2 \) | 2.1E+12 | kN/m² |
| 4     | Poisson’s ratio                      | \( \mu_{12} \) | 0.25 |       |
| 5     | In plane shear modulus              | \( G_{12} \) | 840E+9 | kN/m² |
| 6     | Out of plane shear modulus related to shear deformation over first direction | \( G_{13} \) | 840E+9 | kN/m² |
| 7     | Out of plane shear modulus related to shear deformation over second direction | \( G_{23} \) | 840E+9 | kN/m² |
\( \sigma_{ci} \) uniaxial compressive strength of intact rock

\( N_{e0} \) bearing capacity factor for weightless rock mass

**Validation of analysis**

For the validation of analysed finite element model for a rigid surface strip footing resting on weightless Hoek–Brown rock mass, bearing capacity factors \( N_{e0} \) as obtained in the present elasto-plastic FEA study have been compared with those present in the previous literature. Bearing capacity factors \( N_{e0} \) as obtained by Kulhawy and Carter (1992) and Serrano et al. (2000) have been tabulated along with those obtained as per the present FEA study in Table 3 and corresponding plot is shown in Fig. 5. From Table 3 and Fig. 5, it can be concluded that result of present study well matched with Serrano et al. (2000).

**Parameters considered in analysis**

An extensive parametric study was carried out for different eccentricity to width ratio, load inclination and the geological strength index of the rock mass. Values of these parameters considered in studying the non-linear elasto-plastic behaviour of rock mass-footing system and their range have been stated in Table 4.

**Procedure for developing non-dimensional correlations**

In the finite element analysis of rock mass-footing system, the pressure was applied incrementally and for each and every load increment; attempt was made to satisfy the equilibrium condition in order to achieve convergence. The process was
continued until the collapse of the rock body occurred. A through and comprehensive parametric study was carried out to predict the characteristics such as pressure-settlement and pressure-tilt using non-linear elasto-plastic analysis. The variation of pressure-settlement characteristics with respect to eccentricity and load inclination is shown in Figs. 6 and 7, respectively. Similar procedure was adopted by Viladkar et al. (2013) for obtaining non-dimensional correlations for the case of strip footing on cohesionless soils. From Fig. 6, it can be seen that collapse load decreases with increase in $e/b$ ratio. When a footing is eccentrically loaded, the foundation tilts in that direction, and the contact pressure rises on the side of the tilt and falls on the opposite side. Therefore, settling of the footing will be connected to the base tilting toward the eccentric side. Additionally, as shown in Fig. 7, the ultimate collapse load of the rock mass falls slightly as the load’s inclination increases. These findings are connected to the fact that inclined loading generates additional moments on the footing, making

Fig. 6 Variation of pressure-settlement characteristics with respect to load eccentricity

![Graph showing the variation of pressure-settlement characteristics with respect to load eccentricity.](image)

Fig. 7 Variation of pressure-settlement characteristics with respect to load inclination

![Graph showing the variation of pressure-settlement characteristics with respect to load inclination.](image)
it weak and prone to collapse. Based on these characteristics, non-dimensional correlations were developed to estimate settlement, tilt and ultimate bearing capacity of surface strip footings subjected to eccentric-inclined loading. This involved in all 100 runs of non-linear elasto-plastic finite element analysis for the five GSI values (i.e. 10, 20, 30, 40 and 50).

Non-dimensional correlations

Non-dimensional correlations have been developed for the following parameters:

\[ \text{GSI} = 10, 20, 30, 40, 50 \]

\[ \text{Inclination, } i = 0, 5, 10, 15, 20 \]

\[ \text{Eccentricity to with ratio, } e/B = 0, 0.1, 0.2, 0.3 \]

Ultimate bearing capacity

Variation of bearing capacity factor for the case of rock mass with GSI value equal to 40 is presented in Table 5. This data has also been plotted in a non-dimensional form between bearing capacity factor \( (N_{so}) \) versus \( (e/B) \) in Fig. 8 for different values of load inclination, i. The relationship, \( (N_{so}) \) versus \( (e/B) \) in Fig. 8, can be expressed in the form of the following equation for different values of load inclination and has been obtained by regression analysis (curve fitting) using the method of least squares.

\[ N_{so} \sigma_{ci} = A_0 \left( \frac{e}{B} \right)^2 + A_1 \left( \frac{e}{B} \right) + A_2 \] (3)

where \( A_0, A_1 \) and \( A_2 \) are constants that depend on the value of load inclination \( (i) \). The individual equation for the variation of \( (N_{so}) \) versus \( (e/B) \) is presented in Fig. 8 for different load inclination \( (i) \) values, from which values of constants \( A_0, A_1 \) and \( A_2 \) are obtained, which are presented in Table 6.

| S No | \( e/B \) | \( i \) | \( N_{so} \) |
|------|----------|------|----------|
| 1    | 0        | 0    | 0.1073   |
| 2    | 0        | 5    | 0.0981   |
| 3    | 0        | 10   | 0.0952   |
| 4    | 0        | 15   | 0.0804   |
| 5    | 0        | 20   | 0.0658   |
| 6    | 0.1     | 0    | 0.0934   |
| 7    | 0.1     | 5    | 0.0927   |
| 8    | 0.1     | 10   | 0.0898   |
| 9    | 0.1     | 15   | 0.0766   |
| 10   | 0.1     | 20   | 0.063    |
| 11   | 0.2     | 0    | 0.0738   |
| 12   | 0.2     | 5    | 0.0737   |
| 13   | 0.2     | 10   | 0.0711   |
| 14   | 0.2     | 15   | 0.0602   |
| 15   | 0.2     | 20   | 0.0534   |
| 16   | 0.3     | 0    | 0.0536   |
| 17   | 0.3     | 5    | 0.0528   |
| 18   | 0.3     | 10   | 0.0484   |
| 19   | 0.3     | 15   | 0.046    |
| 20   | 0.3     | 20   | 0.0425   |

\[ A_0 = \frac{0.002i^2 - 0.044i - 0.181}{2} \] (4)

\[ A_1 = 0.013i - 0.12 \] (5)

\[ A_2 = -(7E - 0.05)^2 + 0.105 \] (6)

Substituting these expressions for \( A_0, A_1 \) and \( A_2 \), in Eq. (3), the equation can be rewritten as,

\[ \left( \frac{q_u}{\sigma_{ci}} \right) = \frac{(0.002i^2 - 0.044i - 0.181)(\varepsilon B)^2 + (0.013i - 0.12)(\varepsilon B)^2}{0.105} \] (7)

(for GSI = 40, \( e = 0 \) to 0.2 and \( i = 0 \) to 20)

The above developed equation can be used to estimate ultimate bearing capacity \( (q_u) \) of a rigid surface strip footing lying on rock mass having GSI value equal to 40 and uniaxial compressive strength equal to \( \sigma_{ci} \). Similarly, the correlation has been developed to estimate ultimate bearing capacity \( (q_u) \) of a rigid surface strip footing lying on rock mass having GSI value equal to10, 20, 30 and 50 and presented in Appendix.

Evaluation of footing settlement

Using the data obtained from non-linear elasto-plastic analysis of surface strip footing lying on weightless rock mass, non-dimensional correlations have been obtained for \( S/S_0 \) and \( S/S_{so} \), where \( S \) is the settlement of footing at the point

\[ S/S_0 = \frac{1}{1 + \frac{e}{B} + \frac{e}{B}^2} \] (8)

\[ S/S_{so} = \frac{1}{1 + \frac{e}{B} + \frac{e}{B}^2} \] (9)

where \( S/S_0 \) and \( S/S_{so} \) are the non-dimensional correlation factors for the non-dimensional settlement of surface strip footing lying on weightless rock mass having GSI value equal to 10, 20, 30 and 50 and eccentricity to weightless ratio, \( e/B \).
of application of eccentric-inclined load; \( S_m \), the maximum settlement of the footing under an eccentric-inclined load; \( B \), width of the footing and \( S_o \), the settlement of same footing when subjected to central vertical load. Variation of bearing capacity factor for the case of rock mass with GSI value equal to 40 is presented in Table 7.

This data has also been plotted in a non-dimensional form between \((S_e/S_o)\) versus \((e/B)\) and \((S_m/S_o)\) versus \((e/B)\) in Figs. 9 and 10, respectively for different values of loading inclination, \(i\).

The relationship \((S_e/S_o)\) versus \((e/B)\) in Fig. 9 and \((S_m/S_o)\) versus \((e/B)\) in Fig. 10 can be expressed in terms of the following equation for different values of load inclination and has been obtained by curve fitting through the method of least squares.

\[
\left( \frac{S_e}{S_o} \right) = B_0 \left( \frac{e}{B} \right)^2 + B_1 \left( \frac{e}{B} \right) + B_2
\]  

(8)

### Table 6 Values of constants \(A_0\), \(A_1\) and \(A_2\) for different values of load inclination, \(i\)

| \(i\) | \(A_0\) | \(A_1\) | \(A_2\) |
|------|--------|--------|--------|
| 0    | -0.157 | -0.133 | 0.107  |
| 5    | -0.387 | -0.038 | 0.098  |
| 10   | -0.43  | -0.03  | 0.095  |
| 15   | -0.26  | -0.041 | 0.081  |
| 20   | -0.202 | -0.018 | 0.066  |
where \((B_0, B_1, B_2)\) and \((C_0, C_1, C_2)\) are constants that depend on the value of load inclination, \(i\). From Fig. 9 and Fig. 10, it is possible to obtain values of constants \((B_0, B_1, B_2)\) and \((C_0, C_1, C_2)\), which are presented in Table 8.

The equations for constants \((B_0, B_1, B_2)\) and \((C_0, C_1, C_2)\) have been obtained by plotting them against the values of load inclination, \(i\), and using the method of least squares. These have been expressed as,

\[
\left(\frac{S_m}{S_o}\right) = C_0 \left(\frac{e}{B}\right)^2 + C_1 \left(\frac{e}{B}\right) + C_2
\]  

\[
B_0 = 0.027i^2 - 0.282i - 5.408
\]  

\[
B_1 = -0.011i^2 + 0.291i - 1.163
\]  

\[
C_0 = 0.005i^2 + 0.963i - 21.81
\]  

\[
C_1 = -0.001i^2 - 0.122i + 3.605
\]
Substituting these expressions for \((B_0, B_1, B_2)\) and \((C_0, C_1, C_2)\), in Eqs. (8) and (9), the equation can be rewritten as,

\[
B_2 = -0.061i^2 + 1.044
\]

\[
C_2 = -0.045i + 1.108
\]

Substituting these expressions for \((B_0, B_1, B_2)\) and \((C_0, C_1, C_2)\), in Eqs. (8) and (9), the equation can be rewritten as,

\[
\left(\frac{S_m}{S_o}\right) = (0.005i^2 + 0.963i - 21.81)\left(\frac{i}{2}\right)^3 + (-0.01i^2 - 0.122i + 3.605)\left(\frac{i}{2}\right) + (-0.061i + 1.044)
\]

\[
\left(\frac{S_m}{S_o}\right) = -22.90x^2 + 4.213x + 1.011
\]

\(R^2 = 0.995\)

\[
\left(\frac{S_m}{S_o}\right) = -13.56x^2 + 1.369x + 1.079
\]

\(R^2 = 0.947\)

\[
\left(\frac{S_m}{S_o}\right) = -15.12x^2 + 3.299x + 0.572
\]

\(R^2 = 0.971\)

\[
\left(\frac{S_m}{S_o}\right) = -0.554x^2 + 0.203x + 0.125
\]

\(R^2 = 0.848\)

Therefore, by knowing the settlement, \(S_o\) of footing under a central vertical load though conventional methods available in the literature, settlement of the footing, \(S_e\) and \(S_m\) of a rigid footing subjected to an eccentric-inclined load can be obtained for any given eccentricity ratio, \(e/B\) and inclination of load, \(i\). Similarly, the correlation have been developed to estimate \(S_e\) and \(S_m\) of a rigid surface strip footing lying on
rock mass having GSI value equal to 10, 20, 30 and 50 and presented in Appendix.

Tilt

After obtaining values of $S_e$ and $S_m$ from the non-dimensional correlations, one can estimate tilt ($t$) of the footing. The tilt ($t$) of the footing can be calculated from the following equation (Fig. 1),

$$ t = \sin^{-1} \left( \frac{S_m - S_e}{B - e} \right) \quad (18) $$

### Illustrative example

An example has been solved here to illustrate the application of the procedure to analyse a strip footing using non-linear elasto-plastic analysis.

| Table 7 | Variation of vertical settlement with eccentric inclined |
|---------|-------------------------------------------------------|
| S No   | e/B | i  | $S_e$ | $S_m$ | $S_e/S_o$ | $S_m/S_o$ |
| 1      | 0   | 0  | 4.327 | 4.327 | 1          | 1          |
| 2      | 0   | 5  | 3.953 | 4.823 | 0.91356598 | 1.11462907 |
| 3      | 0   | 10 | 2.01  | 2.41  | 0.46452508 | 0.55696788 |
| 4      | 0   | 15 | 0.7537| 0.7581| 0.17418535 | 0.17520222 |
| 5      | 0   | 20 | 0.5218| 0.5481| 0.12059163 | 0.12666975 |
| 6      | 0.1 | 0  | 3.478 | 5.357 | 0.80379015 | 1.23804021 |
| 7      | 0.1 | 5  | 2.771 | 4.222 | 0.6403975  | 0.97537376 |
| 8      | 0.1 | 10 | 2.323 | 3.456 | 0.53686157 | 0.7987058  |
| 9      | 0.1 | 15 | 0.991 | 1.313 | 0.22902704 | 0.30344349 |
| 10     | 0.1 | 20 | 0.5272| 0.5894| 0.12183961 | 0.13621447 |
| 11     | 0.2 | 0  | 2.692 | 3.91  | 0.62214005 | 0.90362838 |
| 12     | 0.2 | 5  | 2.751 | 3.963 | 0.63577536 | 0.9158705  |
| 13     | 0.2 | 10 | 1.787 | 2.511 | 0.41298821 | 0.58030968 |
| 14     | 0.2 | 15 | 0.8803| 1.136 | 0.20344349 | 0.26253755 |
| 15     | 0.2 | 20 | 0.5384| 0.6391| 0.12442801 | 0.14700499 |
| 16     | 0.3 | 0  | 0.8064| 0.9758| 0.18636469 | 0.22551421 |
| 17     | 0.3 | 5  | 0.834 | 1.014 | 0.19274324 | 0.2343425  |
| 18     | 0.3 | 10 | 0.7724| 0.9392| 0.17850705 | 0.2170557  |
| 19     | 0.3 | 15 | 0.6921| 0.8405| 0.15994916 | 0.19424544 |
| 20     | 0.3 | 20 | 0.5022| 0.5844| 0.11606194 | 0.13505893 |

| Table 8 | Values of constants ($B_0$, $B_1$, $B_2$) and ($C_0$, $C_1$, $C_2$) for different values of load inclination, ($i$) |
|---------|-------------------------------------------------------|
| $i$    | $S_e/S_o$ | $S_m/S_o$ | $B_0$ | $B_1$ | $B_2$ | $C_0$ | $C_1$ | $C_2$ |
| 0      | $-5.989$ | $-0.825$ | 0.986 | $-22.9$ | 4.213 | 1.011 |
| 5      | $-4.246$ | $-0.893$ | 0.878 | $-13.56$ | 1.369 | 1.079 |
| 10     | $-7.67$  | 1.319   | 0.468 | $-15.12$ | 3.299 | 0.572 |
| 15     | $-2.458$ | 0.669   | 0.177 | $-4.913$ | 1.49  | 0.182 |
| 20     | $-0.24$  | 0.061   | 0.12  | $-0.554$ | 0.203 | 0.125 |

### Problem statement

A rigid surface strip footing of length 100 m and width 2 m is subjected to an eccentric-inclined line load equal to 8800 kN/m, at an inclination of 7° with vertical and an eccentricity of 0.3 m from the centre of the footing. Determine ultimate bearing capacity of the footing, settlement below the load and maximum settlement of the footing. Properties of the rock mass beneath the footing are as follows,

Phyllite: $E_i = 27,500$ MPa, $E_j = 4390.44$ MPa, $\mu_i = 0.2$, $\mu_j = 0.35$, $\sigma_{ci} = 50$ MPa, GSI = 40, $m_i = 1$.

### Solution

Analysis is being carried out for 1-m length of the footing.

Uniform load intensity, $p = (\text{line load/area of the footing}) = (8800/2*1) = 4400$ kPa
The FEM analysis of the model in problem yielded a pressure-settlement curve (Fig. 11), which can be used to obtain ultimate bearing capacity, settlement below point of application of load and maximum settlement of footing.

As obtained from the pressure-settlement curve, Ultimate bearing capacity, \( q_u = 4050 \) kPa Settlement below the point of application of load, \( S_e = 2.52 \) mm Maximum settlement, \( S_m = 3.72 \) mm

Tilt \((t)\) of the footing can be obtained using the following relation,

\[
t = \sin^{-1} \left( \frac{S_m - S_e}{\frac{B}{2} - e} \right) = \sin^{-1} \left( \frac{3.72 - 2.52}{\frac{2000}{2} - 300} \right) = 0.14 \text{deg}
\]

**Non-dimensional correlations**

Settlement of footing under central vertical load; \( S_o \) is to be obtained using conventional methods present in the literature. Here relation provided by Ramamurthy (2010) is to be used,

\[
S_o = \frac{s_f \rho B \left( 1 - \mu_j^2 \right)}{E_j}
\]

\( s_f \) = shape factor, \( \rho \) = uniform load intensity, \( B \) = width of footing, \( \mu_j \) = Poisson’s ratio of rock mass, \( E_j \) = deformation modulus of rock mass.

Using chart prepared by Winterkorn and Fang (1991), \( s_f = 2.894 \) for \( L/B = (100/2) = 50 \).

Therefore, \( S_o = \frac{2.894 + 4400 \times 2000 \times (1 - 0.35^2)}{4390440} = 5.09 \text{mm} \)

For GSI 40,

\[
\left( \frac{q_u}{\sigma_{ci}} \right) = \left( 0.002 \right)^2 - \left( 0.044 \right) \left( \frac{s_f}{B} \right)^2 + \left( 0.013 \right) \left( 0.005^2 + 0.963 - 21.81 \right) \left( \frac{s_f}{B} \right)^2 + \left( 0.011^2 + 0.291 \right) \left( \frac{s_f}{B} \right) + \left( -0.061 \right) \left( 1.044 \right) \left( \frac{s_f}{B} \right) + \left( -0.045 \right) \left( 3.605 \right) \left( \frac{s_f}{B} \right)
\]

Assigning a value of \( e/B = 0.15 \) and \( i = 7^\circ \) in the above equations, we get:

\( q_u = 0.09136; \sigma_{ci} = 0.09136 \times 50 = 4.568 \text{ MPa} = 4568 \text{ kPa} \)

\( S_e = 0.53, S_o = 0.53 \times 5.09 = 2.69 \text{ mm} \)

\( S_m = 0.86, S_o = 0.86 \times 5.09 = 4.37 \text{ mm} \)

Tilt,

\[
t = \sin^{-1} \left( \frac{S_m - S_e}{\frac{B}{2} - e} \right) = \sin^{-1} \left( \frac{4.37 - 2.69}{\frac{2000}{2} - 300} \right) = 0.14 \text{deg}
\]

Values obtained from FEM analysis and using non-dimensional correlations have been tabulated in Table 9.
From Table 9, it can be noticed that values obtained from FEM analysis and using non-dimensional correlations are quite closer.

**Concluding remarks**

A series of numerical analysis has been carried out to predict pressure-settlement characteristics of strip footing resting on jointed rock mass under eccentric-inclined loading. The non-dimensional correlations have also been developed in the present study on the parametric study. The parameters for the non-dimensional correlations include GSI, e/B ratio, inclination, bearing capacity, etc. Hoek–Brown criterion has been utilized to conduct a rock mass analysis using Finite Element software PLAXIS 3D. The study primarily aimed at determining the bearing capacity, settlement and tilt characteristics of such footings. On the basis of current study, the following conclusions are drawn:

a. Using non-linear elastoplastic analysis, pressure-settlement characteristics of a rigid surface strip footing can be predicted for eccentric-inclined loading condition resting on weightless Hoek–Brown rock mass. With regard to the ultimate bearing capacity of shallow strip footings, it is shown that results of present study obtained using the method of tangent intersection agree well with those of F. Kulhawy and Carter (1992) and Serrano et al. (2000).

b. The bearing capacity values drop as the eccentricity of the load increases. This indicates that as the eccentricity of the load increases, the bearing capacity of jointed rock mass diminishes. The bearing capacity values decrease with increasing loading inclination with respect to vertical.

c. Non-dimensional correlations have been developed in the present study to predict the ultimate bearing capacity, settlement and tilt of the footing using the data obtained from non-linear elastoplastic FEA. These correlations have been presented in Appendix. These correlations are dependent on (i) eccentricity to width ratio and (ii) inclination of the load. The results obtained from the non-dimensional correlations hold goods on comparing the results obtained from the FEM analysis.

The non-dimensional correlations are limited to the surface footing condition. The study of the behaviour of single and two isolated interfering square footing resting on jointed rock mass can further be studied. Also, in the present study, horizontal jointed rock mass was considered; therefore, behaviour of strip/square footings placed on sloping ground mainly consisting of jointed rock mass can be topic of future research. Effect of earthquake on footings on jointed rock mass is another important topic for future research.

**Appendix**

**Summary of non-dimensional correlations**

**GSI-10** (applicable for $e = 0$ to 0.2 and $i = 0$ to 20)

\[
\frac{q_u}{\sigma_{ci}} = (5E - 5)^2 - 0.001i - 0.028 \left( \frac{e}{B} \right)^2 + (-0.003) \left( \frac{e}{B} \right) + (-0.008i - 0.069i - 0.008) \quad (A.1)
\]

\[
\frac{S_n}{S_m} = (0.013i^2 + 0.259i - 1.33) \left( \frac{e}{B} \right)^2 + (-0.008i^2 + 0.157i + 0.46) \left( \frac{e}{B} \right) + (0.001i^2 - 0.078i + 0.033) \quad (A.2)
\]

\[
\frac{S_m}{S_n} = (-0.025i^2 + 0.005i - 29.93) \left( \frac{e}{B} \right)^2 + (0.005i^2 - 0.404i + 6.257) \left( \frac{e}{B} \right) + (-0.067i^2 + 1.097) \quad (A.3)
\]

**GSI-20** (applicable for $e = 0$ to 0.2 and $i = 0$ to 20)

\[
\frac{q_u}{\sigma_{ci}} = (-4E - 5)^2 - 0.096 \left( \frac{e}{B} \right)^2 + (17E - 5)^2 - 0.01 \left( \frac{e}{B} \right)^2 + (-2E - 5)^2 + (2E - 5)i + 0.023 \left( \frac{e}{B} \right) = 0.026i^2 - 0.39i - 3.572 \left( \frac{e}{B} \right) + (-0.01i^2 + 0.31i - 1.646) \left( \frac{e}{B} \right) + (0.001i^2 - 0.068i + 1.028); \quad (A.4)
\]

\[
\frac{S_n}{S_m} = (0.855i^2 - 18.66) \left( \frac{e}{B} \right)^2 + (-0.113i^2 + 2.131i) \left( \frac{e}{B} \right) + (-4E - 5)^2 = -0.052i + 1.082 \quad (A.5)
\]

**GSI-30** (applicable for $e = 0$ to 0.2 and $i = 0$ to 20)

\[
\frac{q_u}{\sigma_{ci}} = (0.001i^2 - 0.03i - 0.084) \left( \frac{e}{B} \right)^2 + (0.01i - 0.066) \left( \frac{e}{B} \right) + (-2E - 5)^2 + 0.055 \left( \frac{e}{B} \right) = 0.116i^2 - 1.909i - 6.14 \left( \frac{e}{B} \right)^2 + (-0.043i^2 + 0.87i - 0.659) \left( \frac{e}{B} \right) + (0.002i^2 - 0.88i + 0.951) \left( \frac{S_m}{S_n} \right) \quad (A.6)
\]

\[
= (0.136i^2 - 1.5i - 21.8) \left( \frac{e}{B} \right)^2 + (-0.049i^2 + 0.752i + 3.909) \left( \frac{S_m}{S_n} \right) + (0.002i^2 - 0.085i + 0.969) \quad (A.7)
\]

**Table 9** Comparison of results

| Method   | $q_u$ (kPa) | $S_n$ (mm) | $S_m$ (mm) | Tilt, $\theta$ (deg) |
|----------|-------------|------------|------------|---------------------|
| FEM      | 4050        | 2.52       | 3.72       | 0.1                 |
| Correlations | 4568        | 2.69       | 4.37       | 0.14                |

From Table 9, it can be noticed that values obtained from FEM analysis and using non-dimensional correlations are quite closer.
GSI-40 (applicable for $e = 0$ to 0.2 and $i = 0$ to 20)
\[
\left( \frac{q_u}{s_{uc}} \right) = \left( 0.002 i^2 - 0.044i - 0.181 \right) \left( \frac{e}{E} \right)^2 + \left( 0.013i - 0.12 \right) \left( \frac{e}{E} \right) + (-7E - 05)i^2 + 0.105 \left( \frac{v_E}{E} \right) + (-0.027i^2 - 0.282i - 5.408) \left( \frac{e}{E} \right)^2 \\
+ (-0.011i^2 + 0.291i - 1.613) \left( \frac{v_E}{E} \right) + (-0.061i + 1.044) \left( \frac{v_E}{E} \right) + (0.005i^2 + 0.963i - 21.81) \left( \frac{e}{E} \right)^2 + (-0.001i^2 - 0.122i + 3.605) \left( \frac{e}{E} \right) + (-0.045i + 1.108) 
\]

GSI-50 (applicable for $e = 0$ to 0.2 and $i = 0$ to 20)
\[
\left( \frac{q_u}{s_{uc}} \right) = \left( 0.003i^2 - 0.042i - 0.94 \right) \left( \frac{e}{E} \right)^2 + (0.006i + 0.018) \left( \frac{e}{E} \right) + (0.0166) \left( \frac{e}{E} \right) = \left( 0.050i^2 - 0.614i - 6.728 \right) \left( \frac{e}{E} \right)^2 + (-0.020i^2 + 0.438i - 0.753) \left( \frac{e}{E} \right) + (0.001i^2 - 0.075i + 1.060) \left( \frac{e}{E} \right) + (0.042i^2 + 0.446i - 24.3) \left( \frac{e}{E} \right)^2 + (-0.016i^2 + 0.103i + 4.351) \left( \frac{e}{E} \right) + (-0.067i + 1.130) 
\]

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Declarations

Conflict of interest The authors declare no competing interests.

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