Reply

Reply to ‘Comment “On the fusion triple product and fusion power gain of tokamak pilot plants and reactors”’

A.E. Costley, P.F. Buxton and J. Hugill

Tokamak Energy Ltd, Culham Innovation Centre, D5, Culham Science Centre, Abingdon, OX14 3DB, UK

E-mail: Alan.Costley@physics.org

Received 15 July 2016, revised 3 October 2016
Accepted for publication 21 November 2016
Published 23 December 2016

Abstract

In reply to the Comment by Biel et al (2016 Nucl. Fusion 57 038001) on our recent papers Costley et al (2015 Nucl Fusion 55 033001) and Costley (2016 Nucl. Fusion 56 066003), we point out that the fusion triple product, $nT\tau_E$, and fusion power gain, $Q_{\text{fus}}$, cannot be expressed solely in terms of independent engineering design variables such as major radius, $R$, and toroidal field, $B$; output performance variables such as normalised beta, $\beta_N$, safety factor, $q$, and fusion power $P_{\text{fus}}$ have to be invoked. Further, we show that the density limit has the effect of largely cancelling the size dependence in $nT\tau_E$ and $Q_{\text{fus}}$, which would otherwise be present, when these parameters are expressed in terms of $P_{\text{fus}}$. Considerations of engineering aspects are also briefly discussed.

Keywords: tokamaks, pilot plants, fusion reactors, steady state operation

Biel et al [1] have criticised our claim in recent papers that for steady state tokamaks operating at fixed fractions of the density and beta limits the fusion gain, $Q_{\text{fus}}$, and the triple product, $nT\tau_E$, depend only weakly on device major radius, $R$, when expressed in terms of the fusion power, $P_{\text{fus}}$, and energy confinement time enhancement factor, $H$ [2, 3]. Judging from the comments of Biel et al it seems that this criticism stems from a misunderstanding of what we did and what we found, and so we are grateful to have this opportunity to respond. We believe that some of our findings have important implications for the design of tokamak pilot plants and reactors, and so this discussion is important.

First, we note that Biel et al have not reported any errors in our papers: they do not criticise the underlying physics model, our analytical derivations or our calculations. Rather, they criticise some of the choices we have made for variables used in our investigations, and some of the conclusions we have drawn. On the other hand, we believe that it is our choice of variables and our methodology of carrying out our parameter scans that has led to our findings. Biel et al are also concerned about some of the engineering and technological aspects of our suggestions for possible future devices; such considerations were largely not in the scope of our investigations, which concentrated mainly on the physics aspects, but we did make some calculations in those areas.

Biel et al raise an issue relating to input/output variables, where they define input variables as independent engineering design variables, such as $R$, aspect ratio, $A$, and toroidal field, $B$, and output plasma performance variables such as normalised beta, $\beta_N$, and safety factor, $q$, they also consider as input variables.

The Biel et al expression for $nT\tau_E$ is derived in terms of input variables according to their definition, equation (2) in the Comment [1]. For a fusion plasma, there is a link between current, field and power due to the beta limit, captured in $P_{\text{fus}} \propto (\beta_N B^4 R^3/(q^2 A^4))$, and so it is straightforward to eliminate...
but not exactly at this level because an increase of is shown in the paper by Zohm, figure 1 in [4]. The simultaneous selection criteria on the output calculations. They can be applied on the value of the input variables, or as course, the performance limits have to be properly respected. Any expression can be used but, of course, the performance limits have to be properly respected. They can be applied on the value of the input variables, or as selection criteria on the output calculations.

The Biel et al expression for \( nT^2 \), equation (2) in the Comment, shows a strong dependence on \( R \) when expressed in terms of \( H, R, B, A \) and \( q \), and demonstrates that \( nT^2 \) will increase with \( R \) when the values of the other variables are held constant. Because \( P_{\text{fus}} \propto \beta_N^2 \beta_B^2 R^3 / (q^8 A^3) \), a scan in \( R \) with the value of the other parameters held constant leads to a simultaneous increase in \( P_{\text{fus}} \) and \( Q_{\text{fus}} \). A good example of such a scan is shown in the paper by Zohm, figure 1 in [4]. The simultaneous increase of \( nT^2 \) and \( Q_{\text{fus}} \) with \( P_{\text{fus}} \) is inevitable from the conditions of the scan and illustrates the close relationship between these three quantities.

In our investigations, we carry out scans in a different way. Our goal is to find the minimum feasible device size for a set device performance; that is to achieve specified values of \( Q_{\text{fus}} \) (or \( P_{\text{fus}} \)) for a given \( H \) factor. Hence we specify \( Q_{\text{fus}} \) (or \( P_{\text{fus}} \)) and \( H \) and keep them constant during our scans. Since we want the highest possible performance at any size, we set both the density and the normalised beta to be at fixed (high) fractions of their limits at all points in the scan. This means that our scans represent a set of devices and each is operating at the highest possible performance from a physics perspective, but remains within physics limits. The feasibility of the devices depends, of course, on engineering and technological aspects. These were not included in our initial investigations but we are now extending our system code to include them (below).

In contrast to our scans, in the scan carried out by Zohm the density tends to be low at small \( R \) and so the devices at this size are relatively under performing.

To illustrate this comparison we repeat the scan made by Zohm with the same fixed parameters that he used and also for the same device using our method; in both cases we use our system code. The results are presented in figure 1. The variation of \( P_{\text{fus}} \) and \( Q_{\text{fus}} \) with \( R \) is very different for the two types of scans. All operation points are valid at least from a physics perspective in both cases. The relative under performance of the devices at low values of \( R \) in the Zohm scan is clearly visible.

It is notable that in our scan \( nT^2 \) remains approximately constant with size, and so, from a physics performance perspective, there is no benefit in increasing the device size in terms of this parameter. Biel et al are correct that such a scan implies a reduction in field with \( R \) approximately according to \( B \sim R^{-3/4} A q^{1/2} \beta_N^{-1/2} \) but not exactly at this level because the bootstrap current contributes to the current in the safety factor; a calculation of the bootstrap current is included in our code and typically is quite high (~50%).

**Figure 1.** The blue and red lines indicate the two approaches to size scaling: blue is the Zohm’s scaling where: \( B_1, \beta_0 \) and \( q_0 \) are held constant as \( R \) changes. The red lines are the Costley, Hugill and Buxton (CHB) size scaling where the Greenwald fraction (\( f_{\text{GW}} \)), \( \beta_N \), and \( P_{\text{fus}} \) are held constant. The point where the blue and red lines cross approximately corresponds to the ITER steady-state scenario. In both plots the following parameters are held constant: \( H(\text{IPB98y2}) = 1.5, A = 3.43, \kappa = 1.8, \delta = 0.5, f_{\text{inj}} = 0.02, f_{\text{fus}} = 0.01 \), and the current drive normalised efficiency \( = 0.5 \). The field at \( R \leq 3 \) for the CHB size scaling is unrealistically high. For comparison we also show results for a low aspect ratio case \( (A = 1.8, \beta_N = 4.5, H(\text{IPB98y2}) = 1.9) \) (in green). In this case the field is much lower, due to operation at higher \( \beta_N \), but is still a technical challenge.
independent of plasma size in our scans given that $\tau_\text{E}$ scales as $\sim R^2?$. One would have expected $nT_\text{E}$ to go as $R^2$ too. We submit that the reason is due to the impact of the density limit. Effectively the positive size scaling in the confinement time, which would otherwise lead to an increase of $nT_\text{E}$ and $Q_\text{fus}$ with size, is significantly diminished due to this limit.

At the simplest level one can understand this by recognising that the density limit scales as $\sim1/R^2$. Hence, for operation at fixed (high) fractions of the density limit, which is where the highest plasma performance is achieved, $nT_\text{E}$ will not change significantly with size. Effectively the inverse scaling of the density limit negates the positive size scaling in $\tau_\text{E}$. The situation is actually more complicated because the density limit goes as $I_p/R^2$ and $I_p$ has to increase with size in order to keep $q > 2$. This is handled in the analysis in our paper [3].

It is possible to gain additional insight by repeating our analysis but using a generalised form for the size scaling in the density limit; $n_{\lim} \propto I_p(A/R)^y$. One finds:

$$nT \propto \frac{H^2 B^4 R^{4-y}}{A^{3-y} q^3} \propto \frac{H^2 P_{\text{fus}}^{3/4} A^{y-2} R^{4-y}}{\gamma N^2 q^{3/2}}. \tag{1}$$

Of course, when the empirical value of $y = 2$ is inserted the former result is recovered. This shows the significant impact of the density limit: when $nT_\text{E}$ is expressed in terms of $P_\text{fus}$, the residual size dependence becomes very weak. We note also that it effectively eliminates any dependence on $A$. Similarly, $Q_\text{fus}$ becomes only weakly dependent on size and has no dependence on $A$ when expressed in terms of $P_\text{fus}$. Hence the density limit has the effect of eliminating any significant size impact in terms of the global performance parameters, $P_\text{fus}$ and $Q_\text{fus}$, which are the principal performance parameters of a tokamak fusion reactor. The size scalings in $\tau_\text{E}$ and the density limit are not obviously related so this is almost certainly a coincidence but it does have significant consequences for the design of tokamak pilot plants and reactors, as we discuss in our papers.

Turning to the engineering and technological aspects, which Biel et al highlight and which are certainly important. Our specific goal and unusual way of carrying out our performance scans, has led us to develop a novel approach for investigating the engineering aspects. Our approach is to bring into our system code performance parameterisations of the main critical elements, and then for a fixed aspect ratio and set plasma performance parameters ($Q_\text{fus}$, $P_\text{fus}$, $H$ factor), search for the smallest device that satisfies all engineering requirements by adjusting the radial build for the optimum engineering solution. Since writing our papers [2, 3], parameterisations of the engineering aspects of key elements of the central core, which is the most critical area for relatively small size, low $A$, devices, have been included in the code; particularly, the attenuation of the neutron flux and associated heat deposition in the central core due to an inboard shield (based on MCNP calculations of the effectiveness of candidate shield materials), estimates of the power requirement of the associated cryoplant, and estimates of the peak crushing stress based on a simplified model. Estimates of divertor loads are also made but not yet fully integrated into the code. For the magnet, we are specifically considering high temperature superconductors (HTS), which can provide relatively high current density and withstand relatively high fields. Key performance parameters of the HTS tape, for example the maximum field on the conductor, are included as boundary conditions. With these additions we can begin to optimise the design of the central core.

The recent developments in the code and some example optimisation results are described in a paper presented at the recent IAEA Fusion Energy Conference [5]. Further code developments are planned and are described in our paper. Thus far encouraging results have been obtained for a device with $R_0 \sim 1.5$ m and $A = 1.8$. We believe that these justify investigation with a more advanced code and this is something that we seek to do in the near future.

References

[1] Biel W., Lackner K., Sauter O., Wenninger R. and Zohm H. 2016 Nucl. Fusion 57 038001
[2] Costley A.E., Hugill J. and Buxton P.F. 2015 Nucl. Fusion 55 033001
[3] Costley A.E. 2016 Nucl. Fusion 56 066003
[4] Zohm H. 2010 Fusion Sci. Technol. 58 613–24
[5] Sykes A. et al 2016 Compact fusion energy based on the spherical tokamak, paper EXP/P3-26th IAEA Fusion Energy Conf.—IAEA CN-234 (Kyoto, October 2016)