TOPOLOGY OF $\text{H} \, \text{I}$ GAS DISTRIBUTION IN THE LARGE MAGELLANIC CLOUD

Sungeun Kim$^1$ and Changbom Park$^2$

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ABSTRACT

We have analyzed the $\text{H} \, \text{I}$ aperture synthesis image of the Large Magellanic Cloud (LMC), using an objective and quantitative measure of topology to understand the $\text{H} \, \text{I}$ distribution, which hosts a number of holes and clumps of various sizes, in the interstellar medium. The $\text{H} \, \text{I}$ distribution shows different topologies at four different chosen scales. At the smallest scales explored (19–29 pc), the $\text{H} \, \text{I}$ mass is distributed in such a way that numerous clumps are embedded on top of a low-density background. At larger scales, from 73 to 194 pc, it shows a generic hole topology. These holes might have been formed mainly by stellar winds from hot stars. At scales from 240 to 340 pc, slightly above the disk scale height of the gaseous disk, major clumps in the $\text{H} \, \text{I}$ map change the distribution to have a slight clump topology. These clumps include the giant cloud associations in the spiral arms and the thick filaments surrounding superholes. At the largest scales studied (390–485 pc), the hole topology is present again. Responsible for the hole topology at this scale are a few superholes that seem to be mainly associated with supernova explosions in the outer disk. The gaps between the bar and the spiral arms have a minor effect on the topology at this scale.

Subject headings: galaxies: individual (Large Magellanic Cloud) — galaxies: ISM — Magellanic Clouds

1. INTRODUCTION

The topology of the interstellar medium (ISM) is critical to understanding the nature of the underlying physical structure that gives us observed spatial and velocity structures. The global structure of the ISM is manifested by the spatial and velocity structures of the neutral hydrogen gas. Atomic hydrogen is known to be an important component of the ISM (Burton et al. 1992). Relatively high resolution $\text{H} \, \text{I}$ images of nearby galaxies obtained with radio aperture synthesis interferometers have shown overall clumpy $\text{H} \, \text{I}$ distributions and various structures such as holes, shells, loops, filaments, and bubbles (Brinks & Bajaja 1986; Deul & den Hartog 1990; Kamphuis et al. 1991; Puche et al. 1992; Staveley-Smith et al. 1997; Kim et al. 1998; Stanimirovic et al. 1999; Thilker et al. 2000; Walter & Brinks 2001).

One of the most striking results of these surveys is that $\text{H} \, \text{I}$ supergiant shells (SGSs) occupy a large volume of the ISM, such as the $\text{H} \, \text{I}$ filaments seen in the Galaxy (Heiles 1984). About 20 SGSs with diameters larger than 750 pc are identified in the Large Magellanic Cloud (LMC), of which 1/3 are associated with optical counterparts detected in $\text{H}$ $\alpha$ emission (Kim et al. 1999). Many of these $\text{H} \, \text{I}$ shells are found in regions of very active star formation in the LMC. Massive stars interact with the ambient ISM through fast stellar winds and supernova ejecta to form interstellar shell structures with sizes ranging from 10 pc to greater than 1000 pc. However, the correlation between the $\text{H} \, \text{I}$ shells and the 122 OB stellar associations in the LMC (Lucke & Hodge 1970) is not very tight. No star clusters were found at the center of the $\text{H} \, \text{I}$ holes in Holmberg II (Rhode et al. 1999) and the Galaxy (Heiles 1984). Deul & den Hartog (1990) have reported that $\text{H} \, \text{I}$ holes larger than 500 pc are in general located at the interarm cavities and are not likely to be produced by $\text{H} \, \text{I}$ regions and OB associations. The energy required to produce such an $\text{H} \, \text{I}$ hole is at least $10^{53}$ ergs (Kamphuis et al. 1991). These observations have raised an interesting question about the origin of these structures and whether these $\text{H} \, \text{I}$ holes were formed by the interaction between massive stars and the ISM or not. In fact, high-resolution two-dimensional hydrodynamic simulations indicate that the large cavities of the ISM may be formed by nonlinear development of the combined thermal and gravitational instabilities, without a need for stellar energy injection in a galaxy modeling of the LMC (Wada et al. 2000). In that study, dense clumps and filamentary structures are formed as a natural consequence of the nonlinear evolution of the multiphase ISM.

In this paper, we present a method to disentangle the character of the structures seen in the $\text{H} \, \text{I}$ distribution in the LMC as a function of scale using the genus statistic. We find that the genus curve can give information on the shape and topological properties of $\text{H} \, \text{I}$ structure at different scales. We also investigate the relationship between the amplitude of the genus curve and the slope of the power spectrum in the current paper.

2. OBSERVATIONAL DATA

We use an $\text{H} \, \text{I}$ aperture synthesis image from the high-resolution $\text{H} \, \text{I}$ survey of the LMC that was performed with the Australia Telescope Compact Array (ATCA) at 1421 MHz with velocity coverage from $-33$ to 627 km s$^{-1}$ (Kim et al. 1998). The $\text{H} \, \text{I}$ aperture synthesis mosaic of the LMC was made by combining data from 1344 separate pointing centers, and it corresponds to 11.1$^\circ$ × 12.4$^\circ$ on the sky. We have produced an $\text{H} \, \text{I}$ brightness temperature map of the LMC from the brightest $\text{H} \, \text{I}$ component at each position. The spatial resolution of the map is 50"–55", corresponding to 12–14 pc at the LMC’s distance of 50.1 kpc (Alves 2004). At this distance, the LMC can be mapped with a high spatial resolution, and both the interstellar structures and their underlying stars can be resolved and examined in great detail. Since it has low inclination angles, the LMC can be studied with little confusion along the line of sight. In addition, since it has small foreground and internal extinctions, the LMC can be easily observed at UV and X-ray wavelengths. The pixel size of the synthesis map is 20". Figure 1 shows the resulting $\text{H} \, \text{I}$ map of the LMC. The lines are circles with radii of 700, 600, and 500 pixels (from outermost to innermost). We mainly choose to analyze the region within the circle of radius 600 pixels (200" or 3.0 kpc). This

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$^1$ Astronomy and Space Science Department, Sejong University, 98 Kwangjin-gu, Seoul 143-747, Korea; sek@sejong.ac.kr.

$^2$ Korea Institute for Advanced Study, Dongdaemun-gu, Seoul 130-722, Korea.
region contains most of the interesting structures and is not much affected by the outer edge of the disk.

3. THE GENUS STATISTIC

We use the genus statistic to quantify the geometric shape of the projected \( \text{H} \text{i} \) distribution of the LMC. The intrinsic topology of isotemperature or isodensity contours can be measured by the genus. The genus of an object indicates the largest number of cuts that can be made through it without dividing it into two pieces. Following Coles (1988), Melott et al. (1989), and Gott et al. (1990), we adopt, for repeatedly or infinite connected contours in the plane, a modified definition of the two-dimensional genus:

\[
G(\nu) = N_{\text{high}} - N_{\text{low}},
\]

where \( N_{\text{high}} \) is the number of isolated high-density regions and \( N_{\text{low}} \) is the number of low-density holes. This genus is equal to 1 minus the mathematically defined genus. For example, the genus of a ring will be 1 because it has one connected high-density region and one hole. On the other hand, the genus of a disk is +1 because it has one isolated high-density region and no hole. In the case of the \( \text{H} \text{i} \) map of the LMC, we first smooth the map over a scale we are interested in and construct the isodensity contours at a set of threshold levels. The genus is then calculated by integrating the local curvature along the contours. With a threshold level of \( \nu \), the mathematical form of the genus is

\[
G(\nu) = \frac{1}{2\pi} \sum C_i \int_{C_i} \kappa \, ds,
\]

where \( \kappa \) is the local (signed) curvature on an isodensity contour \( C_i \). We define the sign of \( \kappa \) so that it is positive when a contour encloses a high-density region counterclockwise (Gott et al. 1990; Park et al. 1992, 2001). A genus curve is obtained by measuring the genus at a set of threshold levels. A good feature of the genus statistic is that the analytic formula of the genus curve is known for a Gaussian random field. The genus for a Gaussian random phase field is

\[
g(\nu) = A \text{e}^{-\nu^2/2},
\]

where the amplitude \( A \) depends only on the shape of the power spectrum of the field and not on its amplitude (see eqs. (8) and (9)). Any deviation of a measured genus curve from this formula is evidence for non-Gaussianity and reveals the nature of the geometrical shape of the fluctuation. The threshold levels are chosen so that they represent area fractions. A threshold level with a label of \( \nu \) corresponds to the area fraction

\[
f(\nu) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \text{e}^{-x^2/2} \, dx.
\]

Contours with \( \nu = 1.0, 0.0, \) and \( -1.0 \) enclose 16%, 50%, and 84% of the total area, respectively.

To get an idea of what the genus curve indicates about the shape of structures in the disk of a galaxy, we show in Figure 2 the genus curves of three toy models. The solid line shows the genus curve of a disk filled with randomly fluctuating but statistically uniform matter smoothed over a Gaussian filter with width \( \delta_{\text{FWHM}} = 2' \). This Poisson matter fluctuation yields a Gaussian random field, and the genus curve has the form of equation (3). The dashed line with filled circles shows the genus curve for a uniform disk with additional 857 randomly distributed clumps with a diameter of 4'. In this case of clump topology, the genus curve is shifted to the left and becomes asymmetric. The dotted line with open circles shows the genus curve for a uniform disk with 857 randomly distributed empty holes with a diameter of 4'. The genus curve is now shifted to the right and again becomes asymmetric, indicating that the shape of the low-density regions is altered by
the holes. Since the area fraction is fixed at a given \( A \), the low amplitude of the genus curve indicates fewer structures with larger sizes compared to the expectation for a Gaussian field. Therefore, the genus curve tells not only the deviation of a field from Gaussianity, but also the statistical nature of the field. Even though the true nature of the \( H_i \) distribution in the LMC is three-dimensional, the projected view of the distribution shown in Figure 1 reveals a wealth of interesting structures that deserve a two-dimensional topological study. The two-dimensional genus statistic has been applied to the distribution of galaxies when the data are the projected distribution of galaxies on the sky or the distribution of galaxies in a thin slice (Melott et al. 1989; Park et al. 1992) and plasma turbulence (Lazarian 1999).

4. RESULTS

To measure the genus, we smooth the \( H_i \) image of the LMC and cut the circular area of the disk within 200' from the center. The genus curves of the \( H_i \) distribution of the LMC for 12 selected smoothing scales are shown in Figure 3. In each panel, three genus curves are plotted, and their Gaussian smoothing lengths are denoted in units of arcminutes. The solid line in each panel represents the Gaussian curve (eq. [3]) that provides the best fit to the filled circles, which correspond to the median smoothing length. The open circles represent the shortest smoothing length. The genus curve for the \( \lambda_{FWHM} = 1.7' \) (bottom right, open circles) smoothing shows no shift, but the negative side has an amplitude higher than that of the positive side. This means that the low-density regions are broken into numerous pieces, while the high-density regions are relatively more connected into fewer pieces at the same volume fraction. Therefore, the genus curve indicates that there are coherent high-density clumps distributed on a noisy (low fluctuation amplitude) background at this shortest scale. On the other hand, the genus curves corresponding to \( \lambda_{FWHM} = 10' \) and 13' (bottom left; filled circles and open squares, respectively) show a slight shift to the right, and the low-density part has a lower amplitude. This is the generic shape of a hole topology, as shown in Figure 2. At larger smoothing lengths, the genus curves become noisier, and the topological structure of the \( H_i \) disk is not obvious from Figure 3.

To make more quantitative measurements of the shift and asymmetry of the genus curve, we develop pseudostatistics derived from the genus curve. We first find the best-fit amplitude of the Gaussian genus curve for each measured genus curve over the interval \(-2 \leq \nu \leq +2\). We quantify the shift by

\[
\Delta \nu = \frac{\int_{-1}^{1} \nu G(\nu) \, d\nu}{\int_{-1}^{1} G(\nu) \, d\nu},
\]

Fig. 3.—Genus curves of the \( H_i \) map of the LMC at various smoothing scales. The smoothing lengths are in units of pixels, and the radius of the circular region under study is set to 200'. In each panel, open circles are for the shortest smoothing length, and squares are for the longest smoothing length. The solid line is the Gaussian curve that best fits the filled circles, which correspond to the median smoothing length.
where $G_G$ is the best-fit Gaussian genus curve (Park et al. 2001). The asymmetry of the genus curve at high density levels is quantified by

$$A_C = \frac{\int_{0.7}^{2} G(\nu) \, d\nu}{\int_{0.7}^{2} G_G(\nu) \, d\nu}. \quad (6)$$

If the measured genus is lower than the best-fit Gaussian curve at high threshold levels, $A_C$ will be less than 1, indicating that the high-density regions are more connected into fewer and larger pieces compared to the Gaussian case. The asymmetry parameter $A_H$ at low density levels is similarly defined, with an integration interval from $-0.7$ to $-2.0$:

$$A_H = \frac{\int_{-2}^{-0.7} G(\nu) \, d\nu}{\int_{-2}^{-0.7} G_G(\nu) \, d\nu}. \quad (7)$$

If the low-density regions are well connected to one another or are dominated by a few large holes, the genus becomes lower than the Gaussian expectation and $A_H$ becomes less than 1.

Figure 4 shows interesting variations of the shift and asymmetry parameters as functions of the smoothing scale. The circles and triangles in the top panel are the shift parameters measured from the circular area within radii of 200$'$ (600 pixels) and 167$'$, respectively. In the bottom panel the asymmetry parameters $A_C$ (filled circles) and $A_H$ (open circles) are plotted. These parameters reveal an interesting change of topology as the scale changes. At the smallest scales studied (1.3$'$–2$'$ or 19–29 pc), the shift parameter is nearly zero, while $A_C < 1$ and $A_H > 1$, as we saw in the bottom right panel of Figure 3. This corresponds to the situation in which clumps are embedded in a background H\textsc{i} distribution that has low-amplitude random fluctuations. At high density thresholds, the clumps are detected and their number density falls below that of a Gaussian random field. However, at low thresholds the isolated low-density holes are found in the fluctuating background, and their number density exceeds that of the best-fit Gaussian model.

At the next largest scales (5$'$–13$'$ or 73–194 pc) the H\textsc{i} map shows a generic hole topology. The shift parameter is positive, and the genus curve is asymmetric in the sense that the low-density holes are fewer (or larger in size) than the Gaussian expectation. This was also shown in the bottom left panel of Figure 3. The transition from clump to hole topology occurs at about 3$'$ or 48 pc. At the still larger scales of 17$'$–23$'$ or 240–340 pc, the asymmetry of the genus curve remains the same, but the shift parameter suddenly drops to slightly negative values. The clump topology is not statistically significant when the uncertainty at these scales is taken into account, but the change in topology is clear. It seems that there are large dense clumps, as well as large holes of similar sizes, at these scales. The large clumps are large enough to show up in the shift parameter, but they are not prominent and numerous enough to change the whole topology of the disk. The scale is about 1/10 the size of the disk, and the corresponding clumps are mainly distributed along the bar and the spiral arms. Some of them are filaments around the biggest holes.

At the largest scales explored (27$'$–33$'$ or 390–485 pc), the genus curves are very noisy because of the small number of resolution elements at these large smoothing scales. However, the genus-related parameters suggest a hole topology at these scales. The shift parameter, although uncertain, is positive, and the genus curve is asymmetric, again with low amplitude at low density thresholds, as in the second case above. There are a few big holes causing this behavior of the genus curve. They are mainly the roughly circular giant holes in the upper left part of the disk, rather than the gaps between the spiral arms (see Fig. 1). The lower right part, where there are several connected holes of smaller sizes located between the spiral arms, also contributes to the hole shift.

In addition to the topology of the H\textsc{i} distribution, we also study its power spectrum, because the characteristic scales showing topological changes could appear in the shape of the power spectrum as well. The amplitude of the genus curve can be used to explore the slope of the power spectrum near the smoothing scale. Consider a two-dimensional Gaussian random field, whose power spectrum is $P(k)$, that is smoothed by a Gaussian kernel $F(k) = \exp(-k^2R_G^2/2)$ over a smoothing length $R_G$. Then the genus curve per unit area in equation (2) is given by

$$A = \frac{1}{(2\pi)^{3/2}} \left< \frac{k^2}{2} \right>, \quad (8)$$

$$\left< k^2 \right> = \frac{\int k^2 P(k) F^2(k) \, dk}{\int P(k) F^2(k) \, dk}. \quad (9)$$

When the power spectrum is a power-law one, $P(k) \propto k^n$, the moment of the power spectrum is given by

$$\left< k^2 \right> = (n_{\text{eff}} + 2)/2R_G^2. \quad (10)$$

Note that the Gaussian smoothing length is related to the FWHM by $\lambda_{\text{FWHM}} = 2(2 \ln 2)^{1/2}R_G$. For Poisson noise of $n = 0$, the total genus at $\nu = 1$ is expected to be 3350 for an area with a radius of
200' and a smoothing scale of $\lambda_{\text{FWHM}} = 2'$. This is exactly what is seen for the random mass distribution in Figure 2 (solid line).

The top panel of Figure 5 shows the amplitude of the genus curve per square degree as a function of the smoothing scale. For a Gaussian matter fluctuation with a perfect power-law power spectrum, it will scale as $i^{-2}$. However, it drops faster than that at larger scales. The bottom panel shows the effective power index of the power spectrum at the smoothing scale calculated from equation (10), if we assume that the H i distribution is a Gaussian random field. Even though the H i distribution is not exactly Gaussian, the deviation is not great, as can be seen in Figures 3 and 4, and the index of the power spectrum measured by equation (10) can serve as an estimate of the true index. Figure 5 shows that the power index continues to decrease as the smallest scale studied is $i_{\text{FWHM}} = 20'$, where it stays more or less constant at $-1$. The 20' scale is where the clump and hole topologies are competing with each other. The power index of $-1.8$ at the smallest scale means that the H i mass fluctuation is nearly scale-free. On the other hand, the value of $n \gg -2$ at larger scales means that the mass fluctuation is higher at small scales and lower at large scales. The constancy of the power index at scales larger than 20' might mean that there is some mechanism generating additional matter fluctuations at those scales and above.

5. DISCUSSION

We performed a topological analysis of the H i map, which hosts a number of holes and clumps of various sizes. The H i distribution shows different topologies at four different scales. At the smallest scale explored (19–29 pc), the H i mass is distributed in such a way that numerous clumps are embedded on top of a low-density background. At the next largest scales, from 73 to 194 pc, it shows a generic hole topology. These holes might have been mainly formed by stellar winds from massive stars and supernovae. Therefore, the structure of the neutral atomic interstellar gas is dominated by numerous small clumps and relatively larger holes at scales less than the scale height of the gaseous disk, which has been estimated to be approximately 180 pc (Kim et al. 1999).

At the scales from 240 to 340 pc, major clumps in the H i map change the distribution to a slight clump topology. These H i clumps include the giant cloud associations in the spiral arms and the thick filaments surrounding large holes. At the largest scales studied (390–485 pc), a hole topology is again detected. Responsible for the hole topology are a few large holes that seem to be mainly associated with supernova explosions at the outer disk. The gaps between the bar and the spiral arms have minor effects.

We have also measured the effective power index of the power spectrum of the H i map. The H i distribution has a nearly scale-free power spectrum at the smallest scales explored. However, the power index continuously increases at larger scales. This means that the H i mass fluctuation at these scales is dominated by the power at smaller scales. At scales larger than 290 pc, the power index stays at a roughly constant value of $-1.05 \pm 0.05$. This transition scale is comparable to the H i scale height estimated from the average vertical velocity dispersion and the average surface density of both the H i and the stellar components of the LMC disk (Kim et al. 1999). However, a wavelength of 290 pc is still shorter than the warm H i diffuse layer, which has a thickness of 350 pc, as estimated on the basis of the supershell model for LMC 2 (Wang & Helfand 1991).

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