Long-distance contribution to the forward-backward asymmetry in decays $K^+ \rightarrow \pi^+ \ell^+ \ell^-$

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Abstract

The long-distance contribution via the two-photon intermediate state to the forward-backward asymmetries in decays $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ ($\ell = e$ and $\mu$) has been studied within the standard model. In order to evaluate the dispersive part of the $K^+ \rightarrow \pi^+ \gamma^* \gamma^* \rightarrow \pi^+ \ell^+ \ell^-$ amplitude, we employ a phenomenological form factor to soften the ultraviolet behavior of the transition. It is found that, this long-distance transition, although subject to some theoretical uncertainties, can lead to significant contributions to the forward-backward asymmetries, which could be tested in the future high-precision experiments.

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Rare kaon decays provide interesting information on the structure of the weak interactions at low energies \([1 \ 2 \ 3]\). Among them, the flavor-changing neutral-current processes \(K^\pm \to \pi^\pm \ell^+\ell^-\ (\ell = e \text{ and } \mu)\), induced at the one-loop level in the standard model (SM), are well suited both to explore the quantum structure of the SM and to search for new physics beyond it \([4]\). The total decay rates for these transitions are dominated by the long-distance contribution via one-photon exchange, which have been successfully described within the framework of chiral perturbation theory \([5]\) up to \(O(p^0)\) in terms of a vector form factor \([6]\) fixed by experiments \([7]\).

Many useful observables in these transitions, such as P- and T-violating muon polarization effects in \(K^+ \to \pi^+\mu^+\mu^-\ \([8 \ 9 \ 10 \ 11 \ 12 \ 13]\), as well as the CP-violating charge asymmetries in \(K^\pm \to \pi^\pm \ell^+\ell^-\ \([14 \ 15]\), were investigated in the past literatures. Recently another interesting observable, the forward-backward asymmetry \(A_{FB}\) in \(K^+ \to \pi^+\ell^+\ell^-\), has been studied \([16]\). As pointed out by Chen, Geng, and Ho \([16]\), present experimental constraints allow large values of \(A_{FB}\)'s; thus it is expected that the measurement of these asymmetries in future experiments could be very interesting both to test the SM and to probe new physics scenarios. The purpose of this paper is devoted to the analysis of this forward-backward asymmetry in the SM, which is induced by the long-distance transition via the two-photon intermediate state, \(K^+ \to \pi^+\gamma^*\gamma^* \to \pi^+\ell^+\ell^-\). Since at present the dispersive part of the \(K^+ \to \pi^+\gamma^*\gamma^* \to \pi^+\ell^+\ell^-\) amplitude, which contains the logarithmic divergences, cannot be evaluated in a model-independent way, we will employ a phenomenological form factor proposed in Refs. \([17 \ 18]\) to soften the ultraviolet behavior of this transition. As we shall see, this long-distance transition will lead to a scalar form factor and an extra antisymmetric vector form factor under the exchange of the lepton momenta \((p+ \leftrightarrow p_-)\), and both of them can induce contributions to the forward-backward asymmetries in \(K^+ \to \pi^+\ell^+\ell^-\) decays.

The general invariant amplitude for \(K^+(p) \to \pi^+(p_+)\ell^+(p_+)\ell^-(p_-)\) can be parameterized as \([9 \ 16]\)

\[
\mathcal{M} = F_S \bar{\ell} \ell + iF_P \bar{\ell} \gamma_5 \ell + F_V p^\mu \bar{\ell} \gamma_\mu \ell + F_A p^\mu \bar{\ell} \gamma_\mu \gamma_5 \ell, 
\]

where \(p, p_+, p_-\) are the four-momenta of \(K^+, \pi^+, \) and \(\ell^\pm\), and \(F_S, F_P, F_V, \) and \(F_A\) are scalar, pseudoscalar, vector, and axial-vector form factors, respectively. The differential decay rate takes the form

\[
\frac{d\Gamma}{dzd\cos \theta} = \frac{m_K^5 \beta_\ell \lambda^{1/2}(1, z, r_\pi^2)}{2^8 \pi^3} \left\{ \left| \frac{F_S}{m_K} \right|^2 z \beta_\ell^2 \left[ z + \left| F_P \right|^2 \frac{1}{4} \lambda(1, z, r_\pi^2)(1 - \beta_\ell^2 \cos^2 \theta) \right. \right.
\]

\[
\left. + \left| F_A \right|^2 \left[ \frac{1}{4} \lambda(1, z, r_\pi^2)(1 - \beta_\ell^2 \cos^2 \theta) + 4 r_\ell^2 \right] \right. \left. + \frac{\text{Re}(F_S F_P^*)}{m_K} 2 r_\ell \beta_\ell \lambda^{1/2}(1, z, r_\pi^2) \cos \theta + \frac{\text{Im}(F_P F_A^*)}{m_K} 2 r_\ell (r_\pi^2 - 1 - z) \right\},
\]

where \(\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc), r_\ell = m_\ell / m_K, r_\pi = m_\pi / m_K, z = (p_+ + p_-)^2 / m_K^2, \beta_\ell = \sqrt{1 - 4 r_\ell^2 / z}, \) and \(\theta\) is the angle between the three momentum of the kaon and the three momentum of \(\ell^-\) in the dilepton rest frame. The phase space in terms of \(z\) and \(\cos \theta\) is given by

\[
4 r_\ell^2 \leq z \leq (1 - r_\pi^2)^2, \quad -1 \leq \cos \theta \leq 1.
\]
Thus the forward-backward asymmetry is defined as

$$A_{\text{FB}}(z) = \frac{\int_0^1 \left( \frac{d\Gamma}{dz d\cos \theta} \right) d\cos \theta - \int_{-1}^0 \left( \frac{d\Gamma}{dz d\cos \theta} \right) d\cos \theta}{\int_0^1 \left( \frac{d\Gamma}{dz d\cos \theta} \right) d\cos \theta + \int_{-1}^0 \left( \frac{d\Gamma}{dz d\cos \theta} \right) d\cos \theta}.$$  (4)

As seen from Eqs. (2) and (4), in general only two form factors $F_S$ and $F_V$ will play relevant roles in obtaining the significant $A_{\text{FB}}$. In the SM, one-photon exchange transition $K^+ \to \pi^+ \gamma^* \to \pi^+ \ell^+ \ell^-$ dominates the form factor $F_V$; while two-photon exchange transition $K^+ \to \pi^+ \gamma^* \gamma^* \to \pi^+ \ell^+ \ell^-$ can contribute to both $F_S$ and $F_V$. It has been shown in (6), one-photon exchange contribution to $F_V$ can be written as

$$F_V^\gamma = -\frac{\alpha G_F}{2\pi} (a_+ + b_+) z - \frac{\alpha}{2\pi m_K^2} W^{\pi\pi}_+, \quad (5)$$

where the real parameters $a_+$ and $b_+$ encode local contributions starting from $O(p^4)$ to $O(p^6)$ in chiral perturbation theory, and the experimental measurement of $K^+ \to \pi^+ e^+ e^-$ by BNL E865 (7) has determined them to be

$$a_+ = -0.587 \pm 0.010, \quad b_+ = -0.655 \pm 0.044.$$  (6)

The non-analytic term $W^{\pi\pi}_+$ denotes pion-loop contribution, which is estimated to $O(p^6)$ using the physical $K^+ \to \pi^+ \pi^+ \pi^-$ data, and its full expression can be found in Ref. (6).

The general invariant amplitude for $K^+ \to \pi^+ \gamma \gamma$ is given by

$$A[K^+(p) \to \pi^+(p_\pi)(\gamma(q_1, \epsilon_1)\gamma(q_2, \epsilon_2))] = \epsilon_{1\mu}(q_1)\epsilon_{2\nu}(q_2)\mathcal{M}^{\mu\nu}(p, q_1, q_2),$$  (7)

and $\mathcal{M}^{\mu\nu}$ can be decomposed as

$$\mathcal{M}^{\mu\nu} = \frac{A(y, z)}{m_K^2} (q_1^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu}) + \frac{2B(y, z)}{m_K^4} (p \cdot q_1 q_2^\mu p^\nu + p \cdot q_2 q_2^\mu q_1^\nu - p \cdot q_1 q_2 g^{\mu\nu})
- q_1 \cdot q_2 p^{\mu} p^\nu + \frac{C(y, z)}{m_K^2} \varepsilon^{\mu\nu\alpha\beta} q_1\alpha q_2\beta + \frac{D(y, z)}{m_K^4} \varepsilon^{\mu\nu\alpha\beta} (p \cdot q_2 q_1\alpha + p \cdot q_1 q_2\alpha)p_\beta
+(p^\mu \varepsilon^{\nu\alpha\beta\sigma} + p^\nu \varepsilon^{\mu\alpha\beta\sigma})p_\alpha q_1\beta q_2\sigma),$$  (8)

where $y = p \cdot (q_1 - q_2)/m_K^2$ and $z = (q_1 + q_2)^2/m_K^2$. For our purposes, now we are concerned about $A(y, z)$ and $B(y, z)$ amplitudes since $C(y, z)$ and $D(y, z)$ are irrelevant to the present discussion. Within the framework of chiral perturbation theory, the amplitude $A(y, z)$ will receive non-vanishing contribution at $O(p^4)$, which has been computed in Ref. (19); while the leading order contribution to $B(y, z)$ starts from $O(p^3)$, and only the unitarity corrections and vector resonance contributions to it were evaluated (20 21). It is easy to find that, via the transition $K^+ \to \pi^+ \gamma^* \gamma^* \to \pi^+ \ell^+ \ell^-$, the leading order $A$ amplitude only contributes to $F_S$, and the $B$ amplitude contributes to both $F_S$ and $F_V$, which is similar to the case of $K_L \to \pi^0 \gamma^* \gamma^* \to \pi^0 \ell^+ \ell^-$ (19 22 23 24). Therefore one can expect that the contribution
of the scalar form factor $F_S$ in $K^+ \to \pi^+ \ell^+ \ell^-$ due to the two-photon intermediate state is
dominantly given by the $O(p^4)$ $A$ amplitude, and in the following we will neglect high order
contributions to $F_S$ from the $B$ and $O(p^6)$ $A$ amplitudes.

The leading $\Delta I = 1/2$ $O(p^4)$ $A(y, z)$ amplitude for $K^+ \to \pi^+ \gamma \gamma$ can be expressed as \cite{20}

$$A(y, z) = \frac{G_S m_K^2 \alpha}{2\pi z} \left[(r_\pi^2 - 1 - z)F\left(\frac{z}{r_\pi^2}\right) + (1 - r_\pi^2 - z)F(z) + \hat{c}z\right],$$  \hspace{1cm} (9)

where $|G_S| = 9.2 \times 10^{-6}$ GeV$^{-2}$. $F(z/r_\pi^2)$ and $F(z)$ are generated from $\pi$ and $K$ loop
diagrams respectively, which could be defined as

$$F(x) = \begin{cases} 
1 - \frac{4}{x} \arcsin^2 \left(\frac{\sqrt{x}}{2}\right) & x \leq 4, \\
1 + \frac{1}{x} \left(\ln \frac{1 - \sqrt{1 - 4/x}}{1 + \sqrt{1 - 4/x}} + i\pi\right)^2 & x \geq 4.
\end{cases}$$  \hspace{1cm} (10)

$\hat{c}$ in eq. (9) is from $O(p^4)$ non-anomalous local counter-terms \cite{25}, which is a quantity $O(1)$. The first
observation of the decay $K^+ \to \pi^+ \gamma \gamma$ was reported in \cite{26}, and a maximum likelihood fit of $\hat{c}$ to the
decay spectrum using the absolutely normalized rate has been performed to fix the value of $\hat{c}$: without the unitarity corrections, $\hat{c} = 1.6 \pm 0.6$ with $\chi^2$/DOF = 6.3/7; with the unitarity corrections, $\hat{c} = 1.8 \pm 0.6$ with $\chi^2$/DOF = 4.6/7.

In Ref. \cite{20}, the $O(p^6)$ contribution to $K^+ \to \pi^+ \gamma \gamma$ including unitarity corrections from $K^+ \to \pi^+ \pi^+ \pi^-$ and local terms generated by vector resonance exchange has been evaluated. As pointed out by D’Ambrosio and Portolés \cite{20}, the unitarity corrections are relevant while the vector meson contributions are likely to be negligible. Thus the corresponding $B$
amplitude can be written as \cite{20}

$$B(y, z) = \frac{\alpha}{\pi} \left\{ \frac{1}{3r_\pi^4}(4\xi_1 + \xi_1) \left[-\frac{1}{6} \left(1 + 2 \ln \frac{m_\pi^2}{\mu^2}\right) + \frac{z}{18r_\pi^2} - \frac{2r_\pi^2}{z} F\left(\frac{z}{r_\pi^2}\right) \right] \\
+ \frac{1}{3} \left(\frac{z}{r_\pi^2} - 10\right) R\left(\frac{z}{r_\pi^2}\right) \right\},$$  \hspace{1cm} (11)

where parameters $\xi_1$ and $\xi_1$ have been determined from the phenomenology of $K^+ \to \pi^+ \pi^+ \pi^-$ \cite{27}, the mass scale $\mu$ is generally taken as $m_\rho$ in the numerical calculation, and

$$R(x) = \begin{cases} 
-\frac{1}{6} + \frac{2}{x} - \frac{2}{x}\sqrt{4/x - 1} \arcsin \left(\frac{\sqrt{x}}{2}\right) & x \leq 4, \\
-\frac{1}{6} + \frac{2}{x} + \sqrt{1 - 4/x} \left(\ln \frac{1 - \sqrt{1 - 4/x}}{1 + \sqrt{1 - 4/x}} + i\pi\right) & x \geq 4.
\end{cases}$$  \hspace{1cm} (12)

Now following the similar procedure as in the case of the neutral channels $K_L \to \pi^0 \gamma^* \gamma^* \to \pi^0 \ell^+ \ell^-$ \cite{24,18}, one can get the amplitudes from the $A$ and $B$ terms via the
two-photon intermediate state for the charged channels, which read

\[
\mathcal{M}(K^+ \rightarrow \pi^+\gamma^*\gamma^* \rightarrow \pi^+\ell^+\ell^-)^A = \frac{i e^2}{m_K^2} \int \frac{d^4q}{(2\pi)^4} \frac{A(y, z)}{q^2(Q - q)^2[(p_+ - q)^2 - m_\ell^2]} \times \bar{u}(p_-)[3q^2 - 2(Q + p_-) \cdot q + s]\gamma_{\mu}q^\mu - 2m_\ell Q \cdot q + 2m_\ell q^2]v(p_+),
\]

and

\[
\mathcal{M}(K^+ \rightarrow \pi^+\gamma^*\gamma^* \rightarrow \pi^+\ell^+\ell^-)^B = \frac{2ie^2}{m_K^2} \int \frac{d^4q}{(2\pi)^4} \frac{B(y, z)\bar{u}(p_-)\gamma_{\mu}v(p_+)}{q^2(Q - q)^2[(p_+ - q)^2 - m_\ell^2]} \times \{p^\mu[2p \cdot q_{\mu} - Q - 2p \cdot q\cdot Q - p \cdot (p_+ - p_-)q^2]
+ q^\mu[2p \cdot q\cdot (p_+ - p_-) - 2p \cdot Q p \cdot q + 2p \cdot q_{\mu}q^\mu]\},
\]

where \(Q = p_+ + p_-\), \(s = Q^2\), and \(q\) is the loop momentum for the internal photon. Since the integrals in Eqs. (13) and (14) are logarithmically divergent, only their absorptive part contributions can be calculated unambiguously. Actually for the off-shell photons, the \(A(y, z)\) and \(B(y, z)\) amplitudes corresponding to the on-shell photons, should be replaced by \(A[y, z, q^2, (Q - q)^2]\) and \(B[y, z, q^2, (Q - q)^2]\), respectively. At present, there is no model-independent way to obtain these off-shell form factors. Analogous to the analysis of \(K_L \rightarrow \pi^0\gamma^*\gamma^* \rightarrow \pi^0\ell^+\ell^-\) presented in Ref. [18], we employ the following ansatz to regularize the above integrals

\[
A[y, z, q^2, (Q - q)^2] = A(y, z) \times f[q^2, (Q - q)^2],
\]

\[
B[y, z, q^2, (Q - q)^2] = B(y, z) \times f[q^2, (Q - q)^2]
\]

with the form factor

\[
f[q^2, (Q - q)^2] = 1 + a\left[q^2 + \frac{(Q - q)^2}{(Q - q)^2 - m_V^2}\right] + b\frac{q^2(Q - q)^2}{(Q - q)^2 - m_V^2}
\]

is defined in analogy with the analysis of the \(K_L \rightarrow \gamma^*\gamma^* \rightarrow \mu^+\mu^-\) in Ref. [17], and the parameters \(a\) and \(b\) are expected to be \(O(1)\) by naive dimensional chiral power counting. This structure is dictated by the assumption that vector meson dominance (VMD) plays a crucial role in the matching between short and long distance physics (in the numerical calculation \(m_V\) is conventionally chosen to be \(m_V \simeq 770\) MeV). As shown in [18], in order to obtain the ultraviolet convergent integrals, we need to impose the condition\(^1\)

\[
1 + 2a + b = 0.
\]

In a special case for \(a = -b = -1\), the form factor (16) will be identical to the one adopted in Ref. [24] for \(K_L \rightarrow \pi^0\gamma^*\gamma^*\). It is then straightforward to perform the integrals in Eqs. (13)\(^1\)

\(^1\)when one includes the \(O(p^6)\) contribution to the \(A\) amplitude, this condition will be not enough to guarantee the convergent integral in Eq. (13); however, as discussed above and as a good approximation, we neglect this high order contribution.
and \(\text{Eq. (15)}\) after including the form factors in Eq. \(\text{Eq. (15)}\). Neglecting terms which are suppressed by powers of \(1/m_K^2\) and eliminating \(b\) by means of Eq. \(\text{Eq. (17)}\), we get

\[
\mathcal{M}(K^+ \rightarrow \pi^+ \gamma^* \gamma^* \rightarrow \pi^+ \ell^+ \ell^-)^A = F_S^{\gamma\gamma} \bar{u}(p_-)v(p_+),
\]  

and

\[
\mathcal{M}(K^+ \rightarrow \pi^+ \gamma^* \gamma^* \rightarrow \pi^+ \ell^+ \ell^-)^B = F_V^{\gamma\gamma} p^\mu \bar{u}(p_-)\gamma_\mu v(p_+),
\]  

where

\[
F_S^{\gamma\gamma} = \frac{\alpha m_e A}{4\pi m_K^2} \left\{ \frac{7}{2} - 6a - 3 \ln \frac{r_V^2}{z} - \int_0^1 dx \int_0^x dy \left( \frac{(1 + \frac{2r^2}{z})(1 - x)^2}{r^2 z(1 - x)^2 - y(x - y)} \right) - (12 - 9x) \ln \left[ \frac{r^2}{z}(1 - x)^2 - y(x - y) \right] \right\}
\]  

is the scalar form factor,

\[
F_V^{\gamma\gamma} = \frac{\alpha B}{2\pi m_K^4} p \cdot (p_+ - p_-) \left\{ \frac{2}{3} \ln \frac{r_V^2}{z} - \frac{8}{9} + \frac{4}{3}a - \int_0^1 dx \int_0^x dy (2 - x) \ln \left[ \frac{r^2}{z}(1 - x)^2 - y(x - y) \right] \right\}
\]  

is the extra vector form factor [relative to the one-photon exchange contribution to \(F_V\) in Eq. \(\text{Eq. (5)}\) via two-photon intermediate state, and \(r_V = m_V/m_K\). The absorptive parts of \(F_S^{\gamma\gamma}\) and \(F_V^{\gamma\gamma}\) for on-shell two photons can be extracted directly from Eqs. \(\text{Eq. (20)}\) and \(\text{Eq. (21)}\) as

\[
F_S^{\gamma\gamma}\text{absorptive} = \frac{i\alpha m_e A}{4\pi m_K^2} \beta_\ell \ln \left[ \frac{1 - \beta_\ell}{1 + \beta_\ell} \right],
\]  

and

\[
F_V^{\gamma\gamma}\text{absorptive} = \frac{i\alpha B}{8m_K^4} p \cdot (p_+ - p_-) \left[ \frac{2}{3} \beta_\ell^2 + \frac{2}{3} \beta_\ell^2 - \left( \frac{1}{\beta_\ell^2} - \beta_\ell^2 \right) \right] \frac{1}{\beta_\ell} \ln \left[ \frac{1 + \beta_\ell}{1 - \beta_\ell} \right].
\]  

Actually these results [Eqs. \(\text{Eq. (22)}\) and \(\text{Eq. (23)}\)] are model independent, which are consistent with ones using the methods presented in Refs. \(\text{Ref. [19, 22, 23]}\).

Contributions to the forward-backward asymmetry \(A_{FB}\) due to the interference between \(F_S^{\gamma\gamma}\) and \(F_V^{\gamma\gamma}\) can be easily derived from Eqs. \(\text{Eq. (2)}\) and \(\text{Eq. (4)}\) as

\[
A_{FB}^a(z) = \frac{m_K^4 r_\ell \beta_\ell^2 \lambda(1, z, r_\ell^2)}{2^7 \pi^3} \text{Re}(F_S^{\gamma\gamma} F_V^{\gamma\gamma})/(d\Gamma/dz),
\]  

where \(F_S^a\) denotes \((F_S^{\gamma\gamma})^*\), and \(d\Gamma/dz\) is the differential decay rate after integrating the angle \(\theta\) in Eq. \(\text{Eq. (2)}\) (neglecting the contributions from \(F_P\) and \(F_A\)). Meanwhile, since \(F_V^{\gamma\gamma}\) is proportional to \(p \cdot (p_+ - p_-)\), the significant asymmetry \(A_{FB}\) can also be generated from the interference between \(F_V^{\gamma\gamma}\) and \(F_V^{\gamma}\). Using the relation

\[
p \cdot (p_+ - p_-) = -\frac{m_K^2}{2} \beta_\ell \lambda^{1/2}(1, z, r_\ell^2) \cos \theta,
\]
one can get the asymmetry

\[ A_{FB}^b(z) = \frac{m_K^7 \beta^2 \lambda^2 (1, z, r_2^2)(1 - \beta^2_t/2))}{2^{10} \pi^3} \text{Re}(\tilde{f}_V F_V^\gamma) / (d\Gamma/dz), \]

where \( \tilde{f}_V = -F_V^\gamma/p \cdot (p_+ - p_-) \).

One can find that there is a free parameter \( a \) in the expressions of \( F_S^{\gamma\gamma} \) and \( F_V^{\gamma\gamma} \) [Eqs. (20) and (21)], which should be \( O(1) \) from the naive dimensional chiral power counting, however, cannot be fixed from both the theoretical and phenomenological analysis at present. It is expected that the future experimental study of \( K^+ \to \pi^+ \gamma \gamma^* \to \pi^+ \gamma^\ell^+ \ell^- \) could provide some interesting information on it \[28\]. In the following, we will take \( a = -1, 0, +1 \), respectively, to illustrate the numerical results for \( A_{FB}^a \) and \( A_{FB}^b \). Numerical calculations show that contributions from both \( F_S^{\gamma\gamma} \) and \( F_V^{\gamma\gamma} \) to the decay rate of \( K^+ \to \pi^+ \gamma \ell^+ \ell^- \) are negligible, consistent with one-photon exchange dominant mechanism in this decay, and the values of \( F_S^{\gamma\gamma} \) from Eq. (20) are smaller than the experimental bound on the scalar form factor, \( |F_S/(G_F m_K)| \lesssim 6.6 \times 10^{-5} \), for \( K^+ \to \pi^+ e^+ e^- \) given in [7].

Since \( A_{FB}^a \) in Eq. (24) is proportional to \( m_\ell^2 \), the scalar contribution to the forward-backward asymmetry in \( K^+ \to \pi^+ e^+ e^- \) is strongly suppressed. The numerical analysis gives that it is at most \( O(10^{-4}) \). However, \( A_{FB}^b \) in \( K^+ \to \pi^+ \mu^+ \mu^- \) can be \( O(10^{-2}) \), which has been plotted in Fig. 1. Interestingly in the region of large \( z \), the sign of \( A_{FB}^b \) for the muon mode is sensitive to the value of \( a \). This is not surprising because, in Eq. (24), \( \text{Re}(F_S^a F_V^\gamma) \) in this region can change sign for the value of \( a \) varying from 1 to \(-1 \), as shown in Fig. 2, while the differential decay rate \( d\Gamma/dz \) in this region, dominated by \( F_V^\gamma \), almost remains the same for different \( a \), as shown in Fig. 3. Thus the measurement of \( A_{FB}^a \) in \( K^+ \to \pi^+ \mu^+ \mu^- \) in the region of large \( z \) might impose some interesting constraints on the value of \( a \).

The forward-backward asymmetries \( A_{FB}^b \)'s in \( K^+ \to \pi^+ e^+ e^- \) and \( K^+ \to \pi^+ \mu^+ \mu^- \) have been plotted in Fig. 4 and Fig. 5, respectively. Now there is no similar \( m_\ell^2 \) suppression in \( A_{FB}^b \) as that in \( A_{FB}^a \). It is seen that \( A_{FB}^b \) can be \( O(10^{-3}) \) for the electron mode, and about from \( 10^{-4} \) to \( 10^{-3} \) for the muon mode. On the other hand, if we only consider the absorptive parts of \( F_S^{\gamma\gamma} \) [Eq. (22)] and \( F_V^{\gamma\gamma} \) [Eq. (23)] in Eqs. (24) and (20), \( A_{FB}^b \) for the muon mode can also be \( O(10^{-2}) \) in the region of large \( z \) but with the positive sign; \( A_{FB}^a \)'s for both the electron and muon modes will be very small, which are only \( O(10^{-5}) \).

In summary, we have studied the long-distance contribution via the two-photon intermediate state to the forward-backward asymmetries in decays \( K^+ \to \pi^+ \ell^+ \ell^- \). In order to estimate the dispersive part of the \( K^+ \to \pi^+ \gamma \gamma^* \to \pi^+ \ell^+ \ell^- \) amplitude, a phenomenological parameterization of the \( K^+ \to \pi^+ \gamma^* \gamma^* \) form factor has been used. Our analysis shows that these asymmetries \( A_{FB}^a \) and \( A_{FB}^b \) could be accessible to future experiments such as the CKM experiment at Fermilab, where on the order of \( 10^8 \) events for these decays can be produced [29]. It is found that, however, at present the theoretical uncertainty from the parameter \( a \) may obscure the standard model prediction to these quantities. Therefore further study on the general parameterization of the \( K^+ \to \pi^+ \gamma^* \gamma^* \) form factor both experimentally and theoretically is needed to improve our understanding of the forward-backward asymmetries in \( K^+ \to \pi^+ \ell^+ \ell^- \) decays.
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Figure 1: Forward-backward asymmetry $A_{FB}^a$ in decay $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ as a function of $z$. The full line is for $a = -1$, the dashed line for $a = 0$, and the dotted line for $a = 1$. 
Figure 2: $\text{Re}(F_s^*F_v^\gamma)$ in decay $K^+ \rightarrow \pi^+\mu^+\mu^-$ as a function of $z$ with $0.35 \leq z \leq (1 - r_\pi)^2$. The full line is for $a = -1$, the dashed line for $a = 0$, and the dotted line for $a = 1$. 
Figure 3: Differential decay rate for $K^+ \to \pi^+\mu^+\mu^-$ as a function of $z$ with $0.35 \leq z \leq (1 - r_\pi)^2$. 
Figure 4: Forward-backward asymmetry $A_{\text{FB}}^{b}$ in decay $K^+ \rightarrow \pi^+ e^+ e^-$ as a function of $z$. The full line is for $a = -1$, the dashed line for $a = 0$, and the dotted line for $a = 1$. 
Figure 5: Forward-backward asymmetry $A_{FB}^{b}$ in decay $K^+ \to \pi^+\mu^+\mu^-$ as a function of $z$. The full line is for $a = -1$, the dashed line for $a = 0$, and the dotted line for $a = 1$. 