Dark vertical conductance of cavity-embedded semiconductor heterostructures

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Abstract
We present a linear-response nonlocal theory of the electronic conductance along the vertical (growth) direction of a semiconductor heterostructure embedded in a single-mode electromagnetic resonator in the absence of illumination. Our method readily applies to the general class of n-doped semiconductors with parabolic dispersion. The conductance depends on the ground-state properties and virtual collective polaritonic excitations that have been determined via a bosonic treatment in the electromagnetic vacuum. Our method readily applies to the general class of n-doped semiconductors with parabolic dispersion. The conductance depends on the ground-state properties and virtual collective polaritonic excitations that have been determined via a bosonic treatment. We show that, depending on the system parameters, the cavity vacuum effects can enhance or reduce significantly the dark vertical conductance with respect to the bare heterostructure.

1. Introduction

Semiconductor heterostructures such as quantum wells (QW) and superlattices are the building blocks of important optoelectronic devices working in the mid and far infrared, such as photodetectors [1, 2] and quantum cascade lasers [3, 4]. In the case of infrared photodetectors, the photo-induced vertical electrical current along the growth direction of the heterostructure is the key detected quantity. The sensitivity of a photodetector is limited by the so-called dark current, i.e. the current without illumination [2]. Several experimental and theoretical studies have investigated such systems [5–7], pointing out that the main origin of the dark current at low temperatures is the electron tunneling through potential barriers [8]. Recently, an improvement of the performances of quantum-well infrared photodetectors has been demonstrated for arrays of heterostructures embedded in photonic resonators [9].

Recent transport experiments in cavity-embedded organic semiconductors [10, 11] and in-plane magnetotransport measurements in cavity-embedded high-mobility two-dimensional electron systems [12] have shown that the linear-regime electronic conductance of a material can be significantly affected by a cavity electromagnetic resonator even in the absence of illumination. Early theoretical works exploring the role of a cavity on transport have focused on cavity-modified excitonic linear transport [13, 14] and charge nonlinear conduction [15] for chains of two-level systems in the nonlinear regime where electrons are injected in the excited levels. A recent theoretical work [16] on in-plane magnetotransport in the presence of Landau electronic levels has shown how virtual polariton excitations can control the charge transport in the linear regime. Indeed, light–matter interaction can play a pivotal role in determining the electronic transport properties without illumination especially if the strong [17] or ultra-strong coupling regime [18–21] is achieved.

In this paper, we report a theoretical study of the electronic linear-regime conductance of a cavity-embedded generic n-doped semiconductor heterostructure with parabolic dispersion, such as n-doped GaAs. Our theory considers the electromagnetic-vacuum effects on the conductance: no real photons are injected or created in the resonator. Based on a bosonized Hamiltonian, we calculate the vertical conductance in the growth direction within a nonlocal linear-response Kubo approach [22]. In order to describe the collective electromagnetic effects, we consider the system Hamiltonian in the dipole gauge [23–26] where two interaction terms emerge, describing respectively the so-called depolarization shift and the light–matter coupling. The present general framework is applied to QW heterostructures. Numerical results showing how the dark conductance is controlled by the cavity without illumination are presented.
The effective mass and
\[ j \]
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determine collective light–matter excitations. Numerical results for speci
c\nprovide the general expression for the dark vertical conductance for n-doped heterostructures depending on the
presented and discussed in section 4. Finally, we draw our conclusions and perspectives in section 5. The most
technical details about the theoretical model are reported in appendices A and B.

2. Hamiltonian framework and collective light–matter states

The system considered here is an arbitrary n-doped semiconductor heterostructure with negligible non-
parabolicity epitaxially grown along the z direction and embedded in a single-mode electromagnetic cavity. Single-mode electromagnetic resonators with tunable frequency can be obtained, for example, via LC-like structures [27–30] (see figure 1). We assume the electrons to be free in the transverse direction where
\[ S = L_x \times L_y \]
\[ S \]
is the transverse area. The single-particle electronic eigenfunctions can be written as

\[ \varphi_{j, q}(r) = \frac{e^{iq \cdot r}}{\sqrt{S}} \phi_j(z), \]  

(1)

where \( r = \{ x, y \} \) is the in-plane electron position and \( q = \{ k_x, k_y \} \) the in-plane wavevector. The eigenfunctions \( \phi_j \) and their energies \( E_j \) are found by solving the one-dimensional Schrödinger equation for the motion along \( z \). Within the effective mass approximation [31], the latter reads:

\[ \left(-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} + V(z)\right)\phi_j(z) = E_j \phi_j(z) \]  

(2)

with \( m^* \) the effective mass and \( V(z) \) the potential describing an heterostructure of arbitrary shape. The in-plane energy dispersion of the state \( \varphi_{j, q} \) is \( E_{j, q} = E_j + \hbar^2 q_j^2/(2m^*), \) as sketched in figure 2. In the second quantization framework, it is convenient to introduce the fermionic operators \( \hat{c}_{j, q} (\hat{c}_{j, q}^\dagger) \) which annihilate (create) an electron in the \( j \)th band with wave-vector \( q \). In the absence of interactions, the many-body ground state is a Fermi sea with only the first \( j_F \) subbands populated (see figure 2), namely:

\[ |FS\rangle = \prod_{j \leq j_F} \prod_{k < k_F} \hat{c}_{j, k}^\dagger |\text{vacuum}\rangle, \]  

(3)

where \( k = |k| \), \( k_F \) is the Fermi wave vector associated with the \( j \)th band, while \( |\text{vacuum}\rangle \) is the electronic and photonic vacuum. To describe the light–matter interaction for the heterostructure it is convenient to introduce collective electronic excitations [18, 24, 26] described by the operators

\[ \hat{b}_{j,l}^\dagger = \frac{1}{\sqrt{N_{l,j}}} \sum_q \hat{c}_{j,q} \hat{c}_{l,q}^\dagger, \]  

(4)

for \( j \leq j_F \) and \( j < l \). The number \( N_{l,j} = N_j - N_l > 0 \) is the difference between the occupation numbers in the two conduction subbands and ensures the normalization of the state \( \hat{b}_{j,l}^\dagger |FS\rangle \). To simplify the notation, we

1 For simplicity, we neglect here the variation of effective mass along the heterostructure.
compact the double index \( l, j \) into \( \nu = \{ l, j \} \), assuming that any sum on such index runs over all the index pairs satisfying \( l > j \) and \( j \leq j_0 \), differently specified. Note that in the limit where \( N_\nu \gg 1 \) and the number of excitations is small, the \( \hat{b}_\nu \) operators satisfy bosonic commutation relations, that is \( [b_\nu, b_\nu^\dagger] = \delta_{\nu \nu'} \) \(^2\).

The total Hamiltonian of the considered system can be arranged as the sum of four terms:

\[
\hat{H}_\text{tot} = \hat{H}_e + \hat{H}_c + \hat{H}_\text{LM} + \hat{H}_\text{dep},
\]

where we will neglect all constant contributions.

The noninteracting electronic contribution \( \hat{H}_e \) can be written in terms of the operators introduced in equation (4) and of the transition energies \( \hbar \omega_\nu = E_l - E_j \):

\[
\hat{H}_e = \hbar \sum_\nu \omega_\nu \hat{b}_\nu \hat{b}_\nu^\dagger.
\]

In the following, we wish to consider the Hamiltonian for an arbitrary semiconductor heterostructure embedded in a single-mode photonic cavity. Calling \( \omega_c \) the frequency of the photon mode, the bare cavity contribution to the system Hamiltonian is given by

\[
\hat{H}_c = \hbar \omega_c \hat{a} \hat{a}^\dagger,
\]

where \( \hat{a} (\hat{a}^\dagger) \) is the bosonic annihilation (creation) operator for a cavity photon.

To describe electromagnetic interactions, we will work here in the dipolar gauge \(^2\![23, 24]\), giving rise to two interaction terms. The first describe the coupling between the cavity photon mode and the collective intersubband excitations:

\[
\hat{H}_\text{LM} = i\hbar (\hat{a}^\dagger - \hat{a}) \sum_\nu \Omega_\nu (\hat{b}_\nu^\dagger + \hat{b}_\nu),
\]

where the collective vacuum Rabi frequency \( \Omega_\nu \) reads \(^2\![24]\)

\[
\Omega_\nu = \sqrt{\frac{\hbar N_\nu \epsilon_\nu^2}{8\epsilon_0 c m_e^2 \xi L_c}} \sqrt{\frac{\omega_c}{\omega_\nu}} \int_0^L \xi_\nu (z) dz,
\]

with \( L \) the cavity length, \( \epsilon_0 \epsilon_r \) the dielectric constant of the material filling the cavity, and

\[
\xi_\nu (z) = [\partial_\nu \phi_\nu (z)] \phi_\nu (z) - \phi_\nu (z) [\partial_\nu \phi_\nu (z)].
\]

A relevant quantity indicating the strength of the light–matter coupling of a given transition is the resonant Rabi coupling \( \Omega_\nu^{\text{res}} = \Omega_\nu \omega_c / \omega_\nu \). Note that the light–matter interaction Hamiltonian (8) includes both resonant and anti-resonant (counter-rotating wave) terms. The second interaction term is called depolarization shift Hamiltonian, which describes the self-interaction of the electronic polarization. In the considered system, it

\[
^2\text{More specifically, } [\hat{b}_\nu, \hat{b}_\nu^\dagger] = \frac{1}{2} \{N_j - N_\nu\} \text{ with } N_\nu = \sum \hat{b}_\nu^\dagger \hat{b}_\nu \text{ the population operator in subband } j \text{ and } N_j = N_i - N_j \gg 1. \text{ As long as one considers states with a total number of excitations much smaller than } N_\nu, \text{ one has } [32] \frac{1}{N_\nu} \{N_j - N_\nu\} \ll 1. \text{ This implies that within a few-excitation subspace one can consider } [\hat{b}_\nu, \hat{b}_\nu^\dagger] = 1.
\]

\[
^3\text{This can be done by projecting the full electronic Hamiltonian over the manybody excited states } \hat{b}_\nu^\dagger |\text{FS}\rangle \text{ [24, 48].}
\]
reads \[ \mathcal{H}_{\text{dep}} = \hbar \sum_{\nu} \sum_{\nu'} \Xi_{\nu'} (\hat{b}_{\nu} + \hat{b}_{\nu}^\dagger) (\hat{b}_{\nu'} + \hat{b}_{\nu'}^\dagger), \] (11)

where

\[ \Xi_{\nu'} = \frac{\hbar e^2}{8\epsilon_0 m_e^2 S} \sqrt{N_{\nu} N_{\nu'}} \int dz \, \xi_{\nu'}(z) \xi_{\nu'}(z). \] (12)

The depolarization Hamiltonian \( \mathcal{H}_{\text{dep}} \) is due to the collective part of the Coulomb interaction [33, 34]. The other Coulomb terms are responsible for non-collective electron–electron scattering which will be neglected here.

In this work, we will focus on the case where all the subbands \( j \leq j_0 \) are macroscopically occupied. In this limit, the collective excitations behave as bosons and the Hamiltonian (5) is quadratic in the bosonic operators \( \hat{a} \) and \( \hat{a}^\dagger \). Hence, the eigenstates of the system can be determined by performing a Hopfield–Bogoliubov transformation [18, 34–36]. The Hamiltonian can be diagonalized by introducing the bosonic hybrid light–matter polaritonic annihilation operators

\[ \hat{p}_r = w_r \hat{a} + \chi_r \hat{a}^\dagger + \sum_{i} (x_{r,i} \hat{b}_i + z_{r,i} \hat{b}_i^\dagger). \] (13)

To ensure the bosonic commutation relations \( \{ \hat{p}_r, \hat{p}_r^\dagger \} = \delta_{r,r'} \), the coefficients must satisfy the hyperbolic normalization

\[ |w_r|^2 - |\chi_r|^2 + \sum_{\nu} (|x_{r,\nu}|^2 - |z_{r,\nu}|^2) = 1. \] (14)

In terms of such polariton operators, the total Hamiltonian (5) reads

\[ \hat{h}_b = \hbar \sum_{r} \omega_r \hat{p}_r \hat{p}_r^\dagger. \] (15)

The Hopfield–Bogoliubov coefficients and the polariton frequencies \( \omega_r \) are determined by solving the eigenvalue equation \( \mathbf{M} \hat{v}_r = \omega_r \hat{v}_r \), where the vector \( \hat{v}_r = (w_r, x_{r,1}, x_{r,2}, \ldots, x_{r,J}, z_{r,1}, z_{r,2}, \ldots) \) and the matrix \( \mathbf{M} \) is given explicitly in appendix A, equation (A.1).

### 3. Dark vertical conductance

Our goal in this section is to derive an expression for the dark vertical conductance of the cavity-embedded heterostructure, i.e. the linear-regime electronic transport along the growth direction without illumination. In order to do so, we first determine the nonlocal conductivity via a linear-response Kubo approach [22, 37]. Specifically, we are interested in the linear response when a small bias voltage \( \delta U \) is applied along the vertical (growth) direction \( z \) between the two leads at the edges of the sample separated by the cavity spacer length \( L_c \) (see figure 1).

Assuming that the heterostructure is translationally invariant in the \( xy \)-plane, but not along the growth direction \( z \), the electric field within the sample depends on \( z \) but not on \( r_x \). This allows us to simplify the general Kubo expression by integrating out the transverse directions, as detailed in appendix B. The field in the \( z \) direction and the voltage are related by \( E_z(z) = -\partial_z U(z) \) with

\[ \delta U = -\int_{-L_z/2}^{L_z/2} dz E_z(z) = U(L_z / 2) - U(-L_z / 2). \] (16)

The variation of current density in the low-temperature limit is determined through the nonlocal response function \( \chi(z, z') \) via

\[ \delta \langle j_z \rangle(z) = \int_{-L_z/2}^{L_z/2} dz' \chi(z, z') E_z(z'), \] (17)

with

\[ \chi(z, z') = 2x S \sum_{b \geq 1} \frac{\eta_b}{\epsilon_b - \epsilon_z - \epsilon_0} \frac{(\Psi_{b\downarrow}^\dagger \tilde{S}(z) \Psi_{b\downarrow} (z')) (\Psi_{b\downarrow}^\dagger \Psi_{b\downarrow})}{(\epsilon_b - \epsilon_z - \epsilon_0)^2 + \eta_b^2}, \] (18)

where \( \{ \Psi_{b\downarrow} \}_{b=1,2,\ldots} \) are the many-body ground \((b = 1)\) and excited \((b > 1)\) states of the system considered, being \( \epsilon_b \) the corresponding energies. The quantity \( \eta_b = \hbar / \tau_b \) depends on the phenomenological scattering time \( \tau_b \) [22, 38]. For purely electronic excitations, \( \tau_b \) corresponds to the Drude transport scattering time \( \tau_0 \). Finally, the operator \( \tilde{S}(z) \) is obtained from the \( z \)-component of the paramagnetic current density operator, upon integration on the transverse plane:
Note that the inhomogeneity of the semiconductor heterostructure implies that the induced current \( \delta \langle J_z \rangle \) actually depends on \( z \) (see equation (17)). As a result, applying an infinitesimal bias \( \delta U \) between the leads at \( z = \pm L_c/2 \), implies a \( z \)-dependent change \( \delta \langle \hat{\rho}_z \rangle (z) \) of the electronic charge density where \( \int_{-L_c/2}^{L_c/2} \delta \langle \hat{\rho}_z \rangle (z) \, dz = 0 \). In other words, while the total charge is conserved in the semiconductor heterostructure, there are local variations of the charge density. This explains why \( \delta \langle \hat{J}_z \rangle \) is not constant along the semiconductor heterostructure. As an example, it can be analytically proven from the general linear response Kubo expression applied to \( \delta \langle \hat{\rho}_z \rangle \) and \( \delta \langle \hat{J}_z \rangle \) for the case of a noninteracting system, that the following continuity expression holds

\[
\frac{\partial \delta \langle \hat{J}_z \rangle}{\partial z} (z) = - \frac{1}{\tau_0} \delta \langle \hat{\rho}_z \rangle (z),
\]

where \( \tau_0 = 1/\hbar \) is the inelastic scattering time (the same for all subbands). Indeed, this expression shows the link between the induced current and the dissipation through charge accumulations. In the case of an interacting system, an analogous continuity equation holds with a more complex form.

The derivation of the conductance from the two-point response function \( \chi \) in non-translationally-invariant systems is a subtle task which has been addressed in detail for elastic scattering in disordered systems [39, 40]. The application of a bias \( \delta U \) across the sample induces a current \( \delta I \) coming out of the lead at \( z = L_c/2 \) (figure 1). The latter is related to the current density \( \langle \psi(r) \rangle \) by \( \delta I = \delta \langle \hat{J}_z \rangle \langle L_c/2, z \rangle \).

Assuming a homogeneous flow in the metallic contacts, such a \( \delta I \) must be also equal to the current entering through the lead at \( z = -L_c/2 \), that is, \( \delta I = \delta \langle \hat{J}_z \rangle \langle -L_c/2, z \rangle \). To ensure this, we impose periodic boundary conditions to the functions \( \phi_j \) when solving the Schrödinger equation (2). With this choice, definitions (10), (18), and (19) imply \( \langle J_z \rangle \langle -L_c/2, z \rangle = \langle J_z \rangle \langle L_c/2, z \rangle \), which in turns ensure the periodicity of the current at the leads.

Qualitatively, we can assume the variation of the electric field to be negligible, so that the current reads:

\[
\delta I = S E_z \int_{-L_c/2}^{L_c/2} \text{d}z' \chi (L_c/2, z')
\]

from which the general expression for the two-probe conductance is

\[
\frac{\delta I}{\delta U} = - \frac{S}{L_c} \int_{-L_c/2}^{L_c/2} \text{d}z' \chi (L_c/2, z').
\]

### 3.1. Conductance for noninteracting electrons

Let us first consider the behavior of a noninteracting system (no light–matter interaction, no depolarization shift, i.e. \( \hat{H}_{LM} = \delta \hat{H}_{dep} = 0 \)). In this case \( |\psi_0\rangle = |\text{FS}\rangle \) and the only excited states connected to the ground state via \( \hat{J}_x \) (z) are \( \hat{b}_r^\dagger |\text{FS}\rangle \), which are states containing a collective electron–hole excitation above the Fermi sea. This simplifies equation (18) to the form

\[
\chi_{NI} (z, z') = \frac{\hbar e^2}{2m^*} \sum_{\nu} \frac{\tau_0}{\omega_\nu} \frac{\tilde{\xi}_\nu (z) \tilde{\xi}_\nu (z')}{{1 + (\tau_0 \omega_\nu)^2}},
\]

where \( \tilde{\xi}_\nu (z) = \sqrt{N_\nu/N} \xi_\nu (z) \). The noninteracting conductance reads

\[
G_{NI} = \frac{S}{L_c} \frac{\hbar e^2}{2m^*} \sum_{\nu} \frac{\tau_0}{\omega_\nu} \frac{\tilde{\xi}_\nu (L_c/2) \int_{-L_c/2}^{L_c/2} \tilde{\xi}_\nu (z') \text{d}z'}{{1 + (\tau_0 \omega_\nu)^2}}.
\]

### 3.2. The cavity case: conductance in the absence of illumination

In presence of interaction with the cavity quantum field, the manybody ground state of the system differs from the noninteracting Fermi sea, that is, \( |\text{GS}\rangle \neq |\text{FS}\rangle \). The cavity-dressed polaritonic excited states are defined by \( \hat{p}_r^\dagger |\text{GS}\rangle \). The current operator (19) can be suitably rewritten exploiting the inverse of equation (13):

\[
\hat{b}_r = \sum_j (\hat{x}_{r,j}^\dagger \hat{p}_r - \hat{z}_r \hat{a}_r^\dagger \hat{p}_r). \]

Thus, equation (18) can be easily applied to the new set of dressed manybody states to get

\[
\chi (z, z') = \frac{\hbar e^2}{2m^*} \sum_{\nu} \frac{\tau_0}{\omega_\nu} \frac{\tilde{\xi}_\nu (z) \tilde{\xi}_\nu (z')}{{1 + (\tau_0 \omega_\nu)^2}}.
\]
with
\[ \tilde{\xi}_{t}^{\text{eff}}(z) = \sum_{\nu} (x_{r,\nu} + z_{r,\nu}) \tilde{\xi}_{\nu}(z). \]  

(26)

This finally gives the dark vertical conductance
\[ G = -\frac{\hbar m_{e} c^{2}}{L_{c} 2m_{e}^{2}} \sum_{t} \frac{\tau_{t}}{\omega_{t}^{2}} \tilde{\xi}_{t}^{\text{eff}+}(I_{t}/2) \int_{-I_{t}/2}^{I_{t}/2} \tilde{\xi}_{t}^{\text{eff}}(z') dz'. \]  

(27)

Note that several effects contribute in determining how the cavity dark conductance (27) differs from the noninteracting one (24). First, the electronic states are dressed by photonic ones due to the interactions, which can strongly modify the spatial localization of the manybody wavefunctions. This determines the spatial shape of \( \tilde{\xi}_{t}^{\text{eff}} \) and is responsible for what we call the orbital renormalization of the conductance. Second, the interactions change the energetic cost associated with a polaritonic transition, thus altering the conductance. The third effect is the modification of the scattering times \( \tau_{e} \), which are associated with excited states having a mixed light–matter nature [16]. These hybrid scattering times are a weighted combination of the Drude electronic scattering time \( \tau_{0} \) and of a photonic transport scattering time \( \tau_{p} \):
\[ \frac{1}{\tau_{e}} = \frac{W_{e,r}}{\tau_{0}} + 1 - \frac{W_{e,r}}{\tau_{p}}, \]  

(28)

where the electronic weight of the polariton created by \( p_{r}^{\dagger} \) is
\[ W_{e,r} = \sum_{\nu} |x_{r,\nu}|^{2} - \sum_{\nu} |z_{r,\nu}|^{2}. \]  

(29)

Note that \( \tau_{e} \) is not the photon lifetime inside the resonator, but a transport scattering time expected to be much longer [16]. In what follows, we will consider \( \tau_{e} \gg \tau_{0} \), implying \( \tau_{e} = \tau_{0}/W_{e,r} \).

### 3.3. Two-subband approximation

In general, there is a subtle interplay between orbital, energetic and scattering-time effects determining the conductance of a cavity-embedded heterostructure. One can get some analytical insight in the special case in which one transition gives a dominant contribution to the conductance. This is the case, for instance, when one transition frequency is significantly smaller than the others, since high-frequency terms are quickly suppressed in the sums of equations (24) and (27). In this context, the four-by-four Hopfield–Bogoliubov matrix (A.2) can be analytically diagonalized.

Let \( D \) be the two-component index associated with the dominant transition, the noninteracting conductance reads
\[ G_{NLD} = -\frac{\hbar m_{e} c^{2}}{L_{c} 2m_{e}^{2}} \frac{\tau_{p}}{\omega_{D}} \tilde{\xi}_{p}(I_{c}/2) \int_{-I_{c}/2}^{I_{c}/2} \tilde{\xi}_{p}(z') dz'. \]  

(30)

Making use of the exact relation holding for Hopfield coefficients [16]
\[ |x_{r,D} + z_{r,D}|^{2} = W_{e,r} \frac{\omega_{r}^{2}}{\omega_{D}}, \]  

(31)

the cavity dark conductance can be recast as
\[ G_{D} = G_{NLD} \sum_{r=LP,UP} \frac{1 + (\tau_{0}\omega_{p})^{2}}{1 + (\tau_{e}\omega_{r})^{2}}. \]  

(32)

Note that in this one-transition approximation there are only two polaritonic branches: the lower (\( r = \text{LP} \)) and upper (\( r = \text{UP} \)) one.

Two relevant limits can be addressed. First, let us consider the limit \( \omega_{c} \rightarrow 0 \), where only the depolarization shift effect matters. In this case, we get
The second limit is that of a high-frequency resonator: \( \omega \to \infty \). The asymptotic value of the conductance reads

\[
G_{\omega \to 0, \pi} = G_{NL, \pi} \frac{1 + (\gamma \omega)^2}{1 + (\gamma \omega)^2 + (1 + 4\gamma^2 \omega^2)^{1/2}} \quad \text{and} \quad G_{\omega \to \infty, \pi} = G_{NL, \pi} \frac{\gamma \omega^2 \pi \delta \pm 1}{1 + 4 \gamma^2 \omega^2 + (2\gamma \omega^2)^2}. \tag{33}
\]

The second limit is that of a high-frequency resonator: \( \omega \to \infty \). The asymptotic value of the conductance reads

\[
G_{\omega \to 0, \pi} = G_{NL, \pi} \frac{1 + (\gamma \omega)^2}{1 + (\gamma \omega)^2 + (1 + 4\gamma^2 \omega^2)^{1/2}} \quad \text{and} \quad G_{\omega \to \infty, \pi} = G_{NL, \pi} \frac{\gamma \omega^2 \pi \delta \pm 1}{1 + 4 \gamma^2 \omega^2 + (2\gamma \omega^2)^2}. \tag{34}
\]

Note that one always has \( \frac{\omega^2}{\omega_0^2} \geq \left( \frac{\omega_{ph}}{\omega_0} \right)^2 \) due to the Cauchy–Schwartz inequality

\[
\int_{-L/2}^{L/2} dz \xi_\rho^2(z) \geq \frac{1}{L} \left[ \int_{-L/2}^{L/2} dz \xi_\rho(z) \right]^2. \tag{35}
\]

In the considered limits, since there is no scattering-time hybridization effect, the conductance modification is solely due to orbital and energetic effects. We see that the interactions always reduce the conductance with respect to the noninteracting case in the considered limits. It is also interesting to note that

\[
G_{\omega \to 0, \rho} \leq G_{\omega \to \infty, \rho} \leq G_{NL, \rho}. \tag{36}
\]

4. Numerical results for QW structures

In order to understand the impact of the cavity on the physical properties of the embedded heterostructure, we consider some paradigmatic examples. We study both the case of a single QW and of multiple QWs' heterostructure. For all the numerical results presented in this paper, we carefully verified that convergence was reached by increasing the number of intersubband transitions up to a large enough value.

4.1. Single QW

Let us start by studying a single QW whose potential barrier is \( V_0 \) and whose spatial width is \( L_{QW} \), located at the middle of the semiconductor cavity spacer of length \( L_c \) (figure 3(a)). We set \( V_0 \) in order to have a single quantum confined state in the QW, followed by a quasi-continuum of states which are delocalized over the whole semiconductor region.
In figure 3(b) we present the polariton spectrum as a function of the cavity frequency $\omega_c$. In order to maximize the light–matter coupling, we set $E_2 = E_3$ (see panel (a)). We can see that some ‘dark’ states are completely uncoupled to the photonic mode, while the others show the typical anti-crossing behavior of polaritonic excitations [41–43]. The region in which the polaritonic excitations manifest a hybrid light–matter nature is depicted in figure 3(b) via the color of the dispersion curve. The width of this region is proportional to the Rabi frequency $\Omega_p^0$.

For the same configuration of figures 3, 4(b) and (c) show the cavity-embedded conductance $G$ normalized to $G_{\omega_c=0}$ as a function of the resonator frequency $\omega_c$ in two different configurations for the Fermi energy (see panel (a)). We first consider an electronic density $n_e$ in such a way that the Fermi energy level is $E_{13}^{\text{FI}} = E_2$. In this configuration, the confined subband is maximally-populated without filling the delocalized subband. The blue curve of figure 4(c) represents the conductance, exhibiting sharp resonances while changing the cavity mode frequency. These features match the polaritonic resonances observed in figure 3(b) (coupled 1 $\rightarrow$ 3 and 1 $\rightarrow$ 5 transitions). At the main resonance, the conductance reduction is approximately 50%. This effect is due to the scattering–time mixing (equation (28)). Note that the difference between $G_{\omega_c=0}$ and the noninteracting conductance $G_{NI}$ (the dotted red line) is about a 20% reduction coming from the competition of orbital and energetic effects. In this configuration, the energy subbands are sufficiently spaced for the two–subband approximation of section 3.3 to hold. The dashed black curve of figure 4(c) has been obtained considering the 1 $\rightarrow$ 3 transition. As we can see, the two-subband approximation gives a good qualitative insight about the behavior of the conductance even if we miss the peak corresponding to the 1 $\rightarrow$ 5 transition. The dotted–dashed pink line represents the limit of equation (34) and shows how the light–matter coupling, even for a far-detuned cavity, renormalizes the conductance compared to the cavityless case. We note that the inequality (36) holds and that the cavity tends to reduce the effects of the depolarization shift with respect to the noninteracting case.

In figure 4(b), the Fermi Energy $E_F^{\text{FI}}$ was fixed just below $E_n$, in the continuum. Thus further transitions are allowed, giving significant contributions to the conductance. Here the depolarization shift reduces the conductance of almost one order of magnitude with respect to the noninteracting case. This large effect is smoothly weakened by the light–matter coupling when the cavity frequency is increased. We still can observe residual effects of 2 $\rightarrow$ $n$ resonances, but in this configuration the scattering–time mixing is much less relevant than the orbital and energetic effects. Remarkably, by increasing the cavity frequency the dark conductance increases by nearly an order of magnitude for the considered range of parameters. In panel (b), we also perform the two–subband approximation by restricting to the 2 $\rightarrow$ 3 transition. Such approximation shows a good agreement, since the dashed black line matches well the blue curve.
A critical parameter in determining the sign of the cavity-induced effects is the value of the Drude transport scattering time $\tau_0$. To elucidate its role, we take the configuration of figure 3 and show in figure 5 the ratio $G G_{c=0}$ for different values of $\tau_0$. When we scan over the cavity frequency, the scattering times $\tau_r$ get a mixed light–matter nature around $\omega_c$; $\omega_\nu$ and, for $\tau_p \gg \tau_0$, increases compared to the noninteracting case (see equation (28)). According to equation (27), if $\tau_0 \omega_\nu \gg 1$ the subband $\nu$ gives a contribution to the conductance $\propto 1/\tau_\nu$, so that the conductance due to this transition decreases with respect to the cavity-less case. On the contrary when $\tau_0 \omega_\nu \ll 1$, the conductance is proportional to $\tau_\nu$, which leads to an enhancement of the contribution of the $\nu$ transition.

### 4.2. Multiple QWs

In view of experimental realizations, it can be more practical to consider larger cavity spacers. Increasing the cavity length decreases the light–matter interaction (see equation (9)). However, this can be compensated by

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**Figure 5.** Cavity conductance $G$ (on log scale and normalized to $G_{c=0}$) as a function of the cavity frequency $\omega_c$ (in units of the first transition frequency $\omega_{(2,1)}$). Same parameters as in figure 4 when $E_F = E_F^{(1)}$. The different curves correspond to different values of the scattering time $\tau_0$ (see legend). Convergence for these results have been obtained including 40 subbands.

**Figure 6.** Panel (a): single-electron energy levels and wavefunctions (square modulus, arbitrary units, offset vertically for clarity) for a cavity-embedded heterostructure with 5 identical QWs. System parameters: $V_0 = 100$ meV, $L_{QW} = 5$ nm, $L_c = 100$ nm, $m_e = 0.067 m$, with $m_e$ the bare electron mass. Panel (b): normalized cavity conductance as a function of $\omega_c/\omega_{(N_{QW}+1,1)}$. The curves correspond to $N_{QW} = 2, 3, 4, 5$ (see legend), where $N_{QW}$ is the number of equally-spaced QWs of width $L_{QW} = 5$ nm giving a total cavity length $L_c = N_{QW} \times 20$ nm. The other system parameters are unchanged with respect to the case described in panel (a). The Fermi energy is always fixed just below the energy of the first delocalized subband in order to maximize the interactions. The Drude scattering time is $\tau_0 = 1$ ps and we obtained these results including 40 subbands (necessary to have accurate convergence).
considering multiple QWs. For example, in figure 6(a) we represent the single-particle energy levels of a multiple QW heterostructure with a number \( N_{qw} = 5 \) of QWs. In figure 6(b) we consider different values of \( N_{qw} \): for each case, we consider a cavity length \( L_c = N_{qw} \times 20 \) nm and the Fermi energy is adjusted just below the quasi-continuum. All the confined subbands are approximately equally populated since the tunneling coupling between the different QWs is negligible. The results in figure 6(b) show the normalized conductance as a function of the cavity frequency for the different values of \( N_{qw} \), that we have considered. It is apparent that we observe a similar behavior while changing \( N_{qw} \). Indeed, the collective coupling due to a succession of QWs approximately compensates the decrease of coupling due to the increase of the cavity length [44–47].

5. Conclusions

In this work we have introduced a linear-response manybody formalism for the vertical conductance of an arbitrary n-doped parabolic semiconductor heterostructure. Within our formalism, it is possible to account for the effect of the light–matter coupling when such an heterostructure is embedded in a single-mode electromagnetic resonator in absence of illumination. We diagonalized the many-body Hamiltonian in the dipole gauge, accounting for the collective Coulomb interaction (depolarization shift) and the light–matter coupling in a bosonized formalism. As summarized by equations (17) and (18), the conductance is controlled by virtual transitions from the manybody ground state to the excited states via the collective current operator \( \mathbf{J}_c^{\dagger} \).

The conductance depends on the matrix elements of the current operator between ground and polariton excited states, on the energy of such polariton excitations, and on the scattering times.

After presenting our general bosonized theory, we focused on the paradigmatic example of cavity-embedded QW heterostructures. We have shown that the cavity dark vertical conductance (no illumination) can be largely modified. This effect can be measured by observing the dependence of the conductance on the cavity mode frequency for a fixed cavity length \( L_c \). The hybridization of the scattering times leads to the appearance of resonances in the conductance as a function of the cavity frequency. These peaks can correspond to enhancement or suppression of the conductance depending on the value of the Drude electronic transport scattering time. Then we focused on the case of multiple QWs, showing that a similar phenomenology can be observed by using larger cavity lengths. Using parameters for GaAs-based semiconductor heterostructures, we have shown both qualitative effects (appearance of a multi-peak/dip structure) and quantitative effects (large enhancement or suppression). Note that for a given single QW structure, the largest vacuum effects are obtained when the Fermi energy lies above the QW barrier. In this configuration we can see variations of one order of magnitude because we reached a very strong coupling. For a configuration where only one subband is populated the effects are smaller. As possible future developments of the present theory, we mention the case of dispersive multimode cavities and the investigation of the regime where the number of electrons is not large enough to allow for a bosonic treatment of the elementary excitations.

Our findings show the role of electromagnetic-vacuum effects on the vertical electronic transport of cavity-embedded semiconductor heterostructure. The flexibility of the formalism allows to generalize our theory on any kind of heterostructure, which is all the more significant since several devices are based on this technology. Among the possible generalizations, we mention the extension to systems with non-parabolic dispersions and band wave-mixing.

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Appendix A. Hopfield–Bogoliubov matrix for the considered system

The Hopfield–Bogoliubov matrix \( \mathbf{M} \) associated with the Hamiltonian (5) can be written in the following block form:

\[
\mathbf{M} = \begin{pmatrix}
\omega_{fi} & -O & 0 & O \\
-O^{\dagger} & W + D & -O^{\dagger} & -D \\
0 & O & -\omega_{fi} & -O \\
-O^{\dagger} & D & -O^{\dagger} & W + D \\
\end{pmatrix}
\]

(A.1)

with \( O = (\Omega_{\nu_1}, \Omega_{\nu_2}, \ldots) \), \( W = \begin{pmatrix}
\omega_{\nu_1} \\
\omega_{\nu_2} \\
\vdots
\end{pmatrix} \) and \( D = \begin{pmatrix}
2\pi v_{\nu_1} & 2\pi v_{\nu_1} & \cdots \\
2\pi v_{\nu_2} & 2\pi v_{\nu_2} & \cdots \\
\vdots & \vdots & \ddots
\end{pmatrix} \).
When restricting to a single main transition of index \( \mathcal{D} \), \( \mathbf{M} \) is a four by four matrix:

\[
\mathbf{M} = \begin{pmatrix}
\omega_c & -i\Omega_p & 0 & i\Omega_p \\
if\Omega_p & \omega_D + 2\Xi_p & if\Omega_p & -2\Xi_p \\
0 & if\Omega_p & -\omega_c & -if\Omega_p \\
if\Omega_p & 2\Xi_p & if\Omega_p & -\omega_D - 2\Xi_p
\end{pmatrix}
\]  \quad (A.2)

### Appendix B. Details about the linear–response formalism for the vertical nonlocal conductivity

In this appendix we wish to show some intermediate steps of the derivation of equations (17)–(19) from a general linear–response function formalism. In the absence of external magnetic field and for an applied electric field \( E_z \) applied along the growth direction \( z \), the current density variation in the sample is given by [22]

\[
\delta \langle J_z (r) \rangle = \int d^3r' \chi (r, r') E_z (r'), \quad \text{(B.1)}
\]

The general nonlocal current–current response function \( \chi (r, r') \) reads

\[
\chi (r, r') = 2\hbar \sum_{\text{exc}} \frac{\eta_{\text{exc}}}{E_{\text{exc}} - E_{\text{GS}}} \frac{|\langle \text{GS} | \hat{J}_z^\text{V} (r) | \text{exc} \rangle |^2}{(E_{\text{exc}} - E_{\text{GS}})^2 + \eta_{\text{exc}}^2},
\]  \quad (B.2)

where \( \hat{J}_z^\text{V} (r) \) is the \( z \)-component of the paramagnetic current operator, namely

\[
\hat{J}_z^\text{V} (r) = -i\frac{e\hbar}{2m^*_s} [\hat{\partial}_z \hat{\psi}^+(r) \hat{\psi} (r) - \hat{\psi}^+(r) (\hat{\partial}_z \hat{\psi} (r))].
\]  \quad (B.3)

In the formalism of second quantization on the basis of noninteracting one-electron wave functions (1), the current density operator reads

\[
\hat{J}_z^\text{V} (r) = -i\frac{e\hbar}{2m^*_s} \sum_{i,j,k,q} \xi_{ij} (z) \frac{e^{iqz}}{S} \hat{c}_{ik} \hat{c}_{j-k-q}.
\]  \quad (B.4)

Since the heterostructure is translationally invariant in the \( xy \)-plane, the application of a bias voltage along \( z \) can only create a field \( E_z (r) = E_z (z) \). Similarly, the current variation \( \delta \langle \hat{J}_z (r) \rangle \) can not depend on \( r_z \). We can thus integrate over the transverse surface \( S \) both sides of equation (B.1) and get

\[
\delta \langle \hat{J}_z (z) \rangle = \int d^2r' \left[ \frac{1}{S} \int d^3r \int d^3r' \chi (r, r') \right] E_z (z').
\]  \quad (B.5)

The only dependence on \( r \) and \( r' \) of equation (B.2) is in the current density operators. Hence, by introducing

\[
\hat{J}_z^\text{V} (z) \equiv \frac{1}{S} \int_S d^2r \hat{J}_z^\text{V} (r),
\]  \quad (B.6)

the term in square brackets in equation (B.5) reads as \( \chi (z, z') \) of equations (18) and (B.5) becomes (17). Upon insertion of equation (B.4) into (B.6) one finally gets equation (19).

### References

[1] Gendron L, Carras M, Huynh A, Ortiz V, Koeniguer C and Berger V 2004 Appl. Phys. Lett. 85 2824
[2] Jagadish C, Gunapala S and Rhiager D 2011 Advances in Infrared Photodetectors vol 84 (Amsterdam: Elsevier)
[3] Faist J, Capasso F, Sicco D L, Sirtori C, Hutchinson A L and Cho A Y 1994 Science 264 553
[4] Köhler R, Tredicucci A, Bellarm F, Beere H E, Linfield E H, Davies A G, Ritchie D A, Iotti R C and Rossi F 2002 Nature 417 156
[5] Levine B F, Bethera G, Hassanain G, Shen V O, Pelve E and Abbott R R 1990 Appl. Phys. Lett. 56 851
[6] Zussman A, Levine B F, Kuo J M and de Jong J 1991 J. Appl. Phys. 70 5101
[7] Liu H C, Steele A G, Buchanan M and Wasilewski Z R 1993 J. Appl. Phys. 73 2029
[8] Levine B F 1993 J. Appl. Phys. 74 R1
[9] Palafiori D et al 2018 Nature 556 85 EP
[10] Orgiul E et al 2015 Nat. Mater. 14 1123
[11] Nagarajan K et al 2018 (https://doi.org/10.26434/chemrxiv.7498622.v1)
[12] Paravicini-Bagiani G L et al 2019 Nat. Phys. 15 186
[13] Schachenmayer J, Genes C, Tignon E and Pupillo G 2015 Phys. Rev. Lett. 114 196403
[14] Feist J and Garcia-Vidal F J 2015 Phys. Rev. Lett. 114 196402
[15] Hagenmüller D, Schachenmayer J, Schön S, Genes C and Pupillo G 2017 Phys. Rev. Lett. 119 223601
[16] Bartolo N and Ciuti C 2018 Phys. Rev. B 98 205301
[17] Dini D, Köhler R, Tredicucci A, Biasiol G and Sorba L 2003 Phys. Rev. Lett. 90 116401
[18] Ciuti C, Bastard G and Carusotto I 2005 Phys. Rev. B 72 115303
[19] Anappara A A, De Liberato S, Tredicucci A, Ciuti C, Biasiol G, Sorba L and Beltram F 2009 Phys. Rev. B 79 201303
[20] Günter G et al 2009 Nature 458 178
[21] Todorov Y, Andrews A M, Colombelli R, De Liberato S, Ciuti C, Klang P, Strasser G and Sirtori C 2010 Phys. Rev. Lett. 105 196402
[22] Bruus H and Flensberg K 2004 Many-Body Quantum Theory in Condensed Matter Physics (Oxford: Oxford University Press)
[23] Babiker M and Loudon R 1983 Proc. R. Soc. A 385 439
[24] Todorov Y and Sirtori C 2012 Phys. Rev. B 85 045304
[25] De Bernardis D, Pilar P, Jaako T, De Liberato S and Rabl P 2018 Phys. Rev. A 98 053819
[26] Cortese E, Carusotto I, Colombelli R and Liberato S D 2019 Optica 6 354
[27] Lee B, Lee I-M, Kim S, Oh D-H and Hesselink L 2010 J. Mod. Opt. 57 1479
[28] Todorov Y and Sirtori C 2014 Phys. Rev. X 4 041031
[29] Paulillo B, Manceau J M, Degiron A, Zerounian N, Beaudoin G, Sagnes I and Colombelli R 2014 Opt. Express 22 21302
[30] Paulillo B et al 2017 Optica 4 1451
[31] Klinglshin C 2007 Semiconductor Optics, Advanced Texts in Physics (Berlin: Springer)
[32] De Liberato S and Ciuti C 2009 Phys. Rev. Lett. 102 136403
[33] Lee S-C and Galbraith I 1999 Phys. Rev. B 59 15796
[34] De Liberato S and Ciuti C 2012 Phys. Rev. B 85 125302
[35] Hopfield J J 1958 Phys. Rev. 112 1555
[36] Natal P and Ciuti C 2010 Nat. Commun. 1 72
[37] Girvin S M and Yang K 2019 Modern Condensed Matter Physics (Cambridge: Cambridge University Press)
[38] Allen P B 2006 Conceptual Foundations of Materials: A Standard Model for Ground- and Excited-State Properties (Contemporary Concepts of Condensed Matter Science) ed S Louie and M Cohen (Amsterdam: Elsevier)
[39] Baranger H U and Stone A D 1989 Phys. Rev. B 40 8169
[40] Kane C L, Serota R A and Lee P A 1988 Phys. Rev. B 37 6701
[41] Weisbuch C, Nishioka M, Ishikawa A and Arakawa Y 1992 Phys. Rev. Lett. 69 3314
[42] Liu A 1997 Phys. Rev. B 55 7101
[43] Dini D, Kohler R, Tredicucci A, Biasiol G and Sorba L 2003 Phys. Rev. Lett. 90 116401
[44] Dicke R H 1954 Phys. Rev. 93 99
[45] Bonifacio R and Preparata G 1970 Phys. Rev. A 2 336
[46] Bonifacio R, Schwendimann P and Haake F 1971 Phys. Rev. A 4 302
[47] Laurent T, Todorov Y, Vasaneli A, Delteil A, Sirtori C, Sagnes I and Beaudoin G 2015 Phys. Rev. Lett. 115 187402
[48] De Liberato S 2009 Cavity quantum electrodynamics and intersubband polaritonics of a two dimensional electron gas PhD Thesis UniversitéParis-Diderot—Paris VII