**Coupling constants of the $S_{11}$ resonances to pseudoscalar mesons and octet baryons**

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**Abstract.**
Coupling constants of $S_{11}(1535)$ and $S_{11}(1650)$ resonances to meson-baryon ($\pi N, \eta N, \eta'N, KY$) are investigated in a chiral quark model, complemented by inclusion of five-quark components in those resonances. The known strong decay partial widths of both resonances are well reproduced, provided sizeable strangeness components in the relevant wave functions.

**Keywords:** $S_{11}$-meson-baryon Coupling constants  
**PACS:** 12.39.-x, 13.30.Eg, 14.20.Gk

**INTRODUCTION**
Understanding the structure of the $S_{11}(1535)$ and $S_{11}(1650)$ resonances and their dynamical properties are among important issues in baryon spectroscopy. At the hadronic level, $S_{11}(1535)$ is advocated to be a $K\Lambda - K\Sigma$ quasi-bound state [1, 2], and in a recent paper [3], $S_{11}(1650)$ is also reported to be a $K\Sigma$ bound state. In the quark models, both resonances are usually treated as the first orbitally excited states of the nucleon [4]. It is well-known that the structure of resonances can be investigated via photon- and hadron-induced production of mesons. So, the $S_{11}$-meson-baryon coupling constants are crucial issues in hadronic physics.

Recently, an Extended Chiral Constituent Quark Model (E$\chi$CQM) was developed to include higher Fock components in the resonances wave functions. Within that approach, the electromagnetic and strong decays properties of the $P_{31}(1232)$ [5], $P_{11}(1440)$ [6], $S_{11}(1535)$ [7] and $S_{01}(1405)$ [8, 9] resonances are well described.

Here, we present our recent calculations on the couplings of $S_{11}(1535)$ and $S_{11}(1650)$ to pseudoscalar mesons and octet baryons in the E$\chi$CQM. In our model [10], both resonances are considered as admixtures of three-quark and strangeness five-quark components, consistent with the proposed $KY$ bound-state structures at hadronic level.

**THEORETICAL FRAMEWORK**
In this work, we employ the traditional wave functions [4] for the three-quark components of $S_{11}(1535)$ and $S_{11}(1650)$. For the five-quark components, explicit wave functions can be found in Ref. [7]. Here we will follow the formalism for strong decays in Ref. [8], where the coupling of a resonance to meson-baryon channel is obtained by
calculating the matrix element of the following operators:

\[
H_{M}^{NR(3)} = \sum_{j} g_{j}^{A} \frac{\omega_{M}}{2 f_{M}^{2} E_{f} + M_{f}} \mathbf{P}_{f} \cdot \mathbf{P}_{f} + \omega_{M} \frac{\omega_{M}}{E_{f} + M_{i}} \mathbf{P}_{f} \cdot \mathbf{P}_{f} + \frac{\omega_{M}}{2 \mu} \mathbf{P}_{f} \cdot \mathbf{P}_{f} X_{j} \exp\{-i \mathbf{k}_{M} \cdot \mathbf{r}_{j}\},
\]

\[
H_{M}^{NR(5)} = \sum_{j} g_{j}^{A} C_{XFS}^{j}(m_{i} + m_{f}) \mathbf{k}_{j} \left( \begin{array}{c} 1 \\ 0 \\ 0 \\ 1 \end{array} \right) \mathbf{X}_{M} \exp\{-i \mathbf{k}_{M} \cdot \mathbf{r}_{j}\},
\]

with the meson emission operators being

\[
X_{\pi}^{j} = \lambda_{3}^{j}, \quad X_{\pi}^{±} = \pm \frac{1}{\sqrt{2}}(\lambda_{1}^{j} \mp \lambda_{2}^{j}),
\]

\[
X_{K}^{±} = \pm \frac{1}{\sqrt{2}}(\lambda_{4}^{j} \pm \lambda_{5}^{j}), \quad X_{k^{0}}^{j} = \pm \frac{1}{\sqrt{2}}(\lambda_{6}^{j} \pm \lambda_{7}^{j}),
\]

\[
X_{\eta}^{j} = \cos \theta \lambda_{8}^{j} - \sin \theta \sqrt{\frac{2}{3}} \mathcal{J}, \quad X_{\eta}^{j} = \sin \theta \lambda_{8}^{j} + \cos \theta \sqrt{\frac{2}{3}} \mathcal{J},
\]

where \(\lambda_{i}^{j}\) are the \(SU(3)\) Gell-Mann matrices, and \(\mathcal{J}\) the unit operator in the \(SU(3)\) flavor space. \(\theta\) denotes the mixing angle between \(\eta_{1}\) and \(\eta_{8}\), which leads to the physical states for \(\eta\) and \(\eta'\).

**NUMERICAL RESULTS**

The input parameters in our model are: constituent quark masses (\(m_{c}\) and \(m_{s}\)), oscillator parameters (\(\omega_{1}\) and \(\omega_{2}\)), mixing angle \(\theta_{S}\) between \(N_{Q}^{2}P_{M}\) and \(N_{Q}^{3}P_{M}\) states, probabilities for five-quark components in \(S_{11}(1535)\) (\(P_{3q}\)) and \(S_{11}(1650)\) (\(P_{3q}^{'}\)). In line with Refs. [7, 8], we take \(m = 290\) MeV, \(m_{s} = 430\) MeV, \(\omega_{1} = 340\) MeV and \(\omega_{2} = 600\) MeV.

In order to calculate partial decay widths, for the remaining three adjustable parameters (\(\theta_{S}, P_{3q}\) and \(P_{3q}^{'}\)) the whole phase space was mapped out in the following ranges: \(0^\circ \leq \theta_{S} \leq 90^\circ, 0\% \leq P_{3q} \leq 100\%\) and \(0\% \leq P_{3q}^{'} \leq 100\%\).

Then, ranges were determined allowing to reproduce the known [11] partial decay widths for \(S_{11}(1535) \rightarrow \pi N, \eta N, \) and \(S_{11}(1650) \rightarrow \pi N, \eta N, K\Lambda, \) and turned out to be \(26.9^\circ \leq \theta_{S} \leq 29.8^\circ, 21\% \leq P_{3q} \leq 31\%\) and \(11\% \leq P_{3q}^{'} \leq 18\%\).

In Fig. 1 the relevant ranges for the five-quark probabilities at various mixing angles are depicted, showing smooth but significant angle dependence. Within that subspace, extreme values for the partial widths have been extracted (Table 1), and compared to the data, resulting in from good to excellent agreements.

**TABLE 1.** Strong decay widths for \(S_{11}(1535)\) and \(S_{11}(1650)\).

| \(N^{\circ}\)     | \(\pi N\)   | \(\eta N\) | \(K\Lambda\) | Ref.        |
|-------------------|-------------|-------------|--------------|-------------|
| \(S_{11}(1535)\)  | 68 ± 15     | 79 ± 11     |              | PDG [11]    |
|                   | 58 ± 5      | 79 ± 11     |              | Present work|
| \(S_{11}(1650)\)  | 128 ± 29    | 3.8 ± 3.6   | 4.8 ± 0.7    | PDG [11]    |
|                   | 143 ± 5     | 4.5 ± 2.8   | 4.5 ± 0.5    | Present work|

The obtained model is used to put forward predictions for \(S_{11}\)-meson-baryon coupling constants, Fig. 2, for the following meson-baryon sets: \(\pi^{0}p, \pi^{+}n, \eta p, K^{+}\Lambda, K^{0}\Sigma^{+},\)
FIGURE 1. Ranges for five-quark components probabilities in $S_{11}$ as a function of mixing angle.

$K^+\Sigma^0$, $\eta'p$. In Fig. 2, the couplings values at null five-quark probabilities correspond to pure qqqq configuration within broken $SU(6) \otimes O(3)$ symmetry.

FIGURE 2. Coupling constants $g_{S_{11}MB}$ for $S_{11}(1535)$ (left panel) and $S_{11}(1650)$ (right panel) as a function of five-quark components probabilities at $\theta_S = 28^\circ$, for meson-baryon sets: $\pi^0p$ (full), $\pi^+n$ (dashed), $\eta p$ (dotted), $K^+\Lambda$ (dash-dotted), $K^0\Sigma^+$ (dash-dot-dotted), $K^+\Sigma^0$ (short dashed), $\eta'p$ (short dotted). The vertical bands correspond to the probability ranges determined in this work.
The coupling $g_{S_{11}(1535)\eta N}$ turns out to be the most sensitive one to the five-quark components. Significant effects are also anticipated for $g_{S_{11}(1535)K^0\Sigma^+}$ and $g_{S_{11}(1650)K^0\Sigma^+}$.

With respect to the magnitude of couplings, we find the following orderings:
- for $S_{11} \equiv S_{11}(1535)$:
  \[ |g_{S_{11}\pi^0 p}| < |g_{S_{11}\pi^+ n}| < |g_{S_{11}K^+\Sigma^0}| < |g_{S_{11}K^+\Lambda}| < |g_{S_{11}K^0\Sigma^+}| < |g_{S_{11}\eta p}| < |g_{S_{11}\eta' p}|, \]
- for $S_{11} \equiv S_{11}(1650)$:
  \[ |g_{S_{11}\eta p}| < |g_{S_{11}K^+\Lambda}| < |g_{S_{11}\pi^0 p}| < |g_{S_{11}\pi^+ n}| < |g_{S_{11}K^+\Sigma^0}| < |g_{S_{11}\eta' p}| < |g_{S_{11}K^0\Sigma^+}|. \]

**CONCLUSION**

The partial decay widths for the low-lying $S_{11}$ resonances are known with about 14% accuracy for $\Gamma_{S_{11}(1535)\rightarrow \eta N}$ and $\Gamma_{S_{11}(1650)\rightarrow K^+\Lambda}$, 22% for $\Gamma_{S_{11}\rightarrow \pi N}$ for both resonances, and 95% for $\Gamma_{S_{11}(1650)\rightarrow \eta N}$. Previous studies \[2,12\] have allowed to reproduce a significant number of those data. Chiral constituent quark model without or with $SU(6) \otimes O(3)$ breaking effects lead \[10\] to significant discrepancies with the data.

In the present work we extended the $\chi$CQM approach by complementing it with five-quark components in the $S_{11}$ resonances wave functions. The phase space formed by the mixing angle and the probabilities of those components, leads *simultaneously* to satisfactory results with respect to all five measured widths, within the following ranges: $26.9^0 \leq \theta_S \leq 29.8^0$, $21% \leq P_5 \leq 31\%$ and $11% \leq P_5' \leq 18\%$. Here we have reported predictions for the coupling constants of $\pi^0 p$, $\pi^+ n$, $\eta p$, $K^+\Lambda$, $K^0\Sigma^+$, $K^+\Sigma^0$, $\eta' p$ pseudoscalar meson - octet baryon sets to both $S_{11}$ resonances.

A worthy to be noticed outcome of the present work is that the $S_{11}(1535)\eta N$ system turns out to be very appealing for two reasons: i) the partial decay width $\Gamma_{S_{11}(1535)\rightarrow \eta N}$ is sizeable, ii) the coupling constant $g_{S_{11}(1535)\eta N}$ is rather large and sensitive enough to the higher Fock space components in the resonance. Accordingly, possible role of the baryon five-quark components in the $\eta$ meson production processes is under investigation.

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