Study on Control Strategy for Tilt-rotor Aircraft Conversion Procedure

Xufei Yan¹, Renliang Chen², Bin Lou¹, Ye Xie¹, Anhuan Xie¹ and Dan Zhang¹,*

¹ Zhejiang Lab, Hangzhou, Zhejiang, 310000, China
² Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu, 210016, China

Corresponding Author: *dzhang@zhejianglab.com

Abstract. This paper studies the control strategy in tilt-rotor aircraft dynamic conversion procedure. A nonlinear flight dynamics model with full flight modes is established. On this basis, a nonlinear optimal control model of dynamic conversion is built by constructing the Bolza form of nonlinear optimal control problem. It contains the limitations and effects of conversion corridor, pilot control, flight attitude, engine rated power, wing stall, and the cooperation between lift and thrust on the procedure of dynamic conversion. An efficient numerical solution method with good convergence is designed to obtain the trajectory and control strategy of dynamic conversion between the modes of helicopter and fixed-wing aircraft. Through the weight parameter analysis, it can be seen that when the subitem of pilot workload is included in the cost function, the displacements of collective stick input as well as the longitudinal stick input are significantly reduced, and the height and pitch attitude change more gently, but the time has been extended. In addition, the pilot workload weight coefficient should not be dominant, otherwise the overly smooth manipulations of collective and longitudinal stick will cause a decrease of height.

1. Introduction

The tilt-rotor aircraft has a good performance of vertical flight and high speed cruise. It consists of helicopter mode, conversion mode and fixed-wing aircraft mode. The conversion process between helicopter and fixed-wing aircraft mode is completed by tilting the nacelle of the engine [1]. The control process during conversion is complicated because of the cooperation between the lift and thrust, complex unsteady aerodynamic effects, body motion and inertial coupling as well as the switch of control between helicopter mode and fixed-wing mode [2-4]. A number of works have studied the conversion procedure by designing control law on a linear model [5-7]. However, in the dynamic tilting process, the faster the rotor tilting rate, the higher level of nonlinearity of the model, and the more obvious the error between the linear model and the nonlinear model. Therefore, the nonlinear optimal control theory based on nonlinear model can more accurately reflect the characteristics of the dynamic conversion procedure [8-12].

This paper studies the control strategy in the conversion procedure of tilt-rotor aircraft based on nonlinear optimal control theory. A nonlinear flight dynamics model of XV-15 tilt-rotor aircraft with full flight modes is established. On this basis, a nonlinear optimal control model of dynamic conversion is built in the Bolza form of optimal control problem constructed by cost function, optimal variables, differential equation set, boundary conditions and path constraints. It contains the...
limitations and effects of conversion corridor, pilot control, flight attitude, engine rated power, wing stall, and the cooperation between lift and thrust on the procedure of dynamic conversion. An efficient numerical solution method with good convergence is designed to obtain the trajectory and control strategy of dynamic conversion between the modes of helicopter and fixed-wing aircraft. Finally, the pilot control strategy and flight state calculated by applying the performance index with different weight coefficient of pilot workload are compared and discussed.

2. Flight dynamics model

2.1. Modelling
The flight dynamics model of XV-15 tilt-rotor aircraft [1] consists of two rotor models, a wing-pylon model, a fuselage model, a horizontal stabilizer model, a vertical stabilizer model and a mixed control system. The description of each component is as follow,

1. Blade aerodynamic force and moment is formulated according to the blade element theory. For each blade element, the coefficients of lift, drag and moment are obtained based on table look-up data to account for flow separation and transonic compressibility effects.

2. The Pitt-Peters dynamic inflow model with rotor wake distortion effect and side-by-side rotor effect is applied to simulate the dynamic characteristics of rotor inflow during tilting.

3. The aerodynamic effect of rotor wake on the wing is made separately from the aerodynamic force and moment caused by freestream flow. The effect between rotor and wing is formulated as a function of contracted wake radius, angle of tilting, angle of wing attack, angle of sideslip and dynamic pressure of the wing immersed in the wake (see details in GTRS model Ref [1]).

4. The fuselage, wing-pylon assembly, horizontal tail and vertical fins are modelled separately to describe the influence of rotor wake on the airframe aerodynamics, see details in GTRS model Ref [1].

Finally, according to the influence of nacelle tilting motion on rotor aerodynamics, the interference between aerodynamic components and the inertial coupling during dynamic conversion as well as the characteristics of control switch between the modes of helicopter and fixed-wing, the nonlinear flight dynamics model with full flight modes is established below

\[ \dot{x} = f(x,u,\tau) \]

where \( x = [x_B;x_F;x_I] \) represents the state that contains the fuselage motion state \( x_B \), the rotor flapping state \( x_F \) (Left and Right), and rotor inflow state \( x_I \) (Left and Right). \( u = [\delta_{col};\delta_{lat};\delta_{ped};i_n] \) is the control vector, where \( \delta_{col} \) is the pilot collective input, \( \delta_{lat} \) is the pilot lateral input, \( \delta_{ped} \) is the pilot longitudinal input, \( i_n \) is the pilot collective input, the nacelle tilting angle \( i_n \) is applied to study the engine nacelle tilting law which can be realized by automatic tilting system, \( \tau \) is the time.

\[
\begin{align*}
    x_B &= [u,v,w,p,q,r,\phi,\theta,\psi,x,y,h]^T \\
    x_F &= [\dot{a}_0,R,\dot{a}_1,R,\dot{b}_R,\dot{a}_0,L,\dot{a}_1,L,\dot{b}_L,\dot{a}_0,L,\dot{a}_1,L,\dot{b}_L]^T \\
    x_I &= [v_{0,R},v_{1s,R},v_{1c,R},v_{0,L},v_{1s,L},v_{1c,L}]^T
\end{align*}
\]

2.2. Validation
The XV-15 tilt-rotor aircraft flight test data is utilized for steady-state analysis in three modes of flight. Figure 1 and 2 below show the predicted steady-state results and the flight test data [13].
Figure 1. Conversion Corridor of XV-15

Figure 2. Trim vs. Flight data
3. Construction of nonlinear optimal control problem

3.1. Formulation of nonlinear optimal control problem

Based on the flight dynamics model and the characteristics of conversion between the modes of helicopter and fixed-wing, the conversion procedure is formulated into a nonlinear optimal control problem (NOCP). A Bolza form of the NOCP includes differential equation set, optimal variables, cost function, boundary conditions and path constraints.

1) Differential equation set: which is basically an augmented flight dynamics model. To reflect the control rate limits of pilot control as well as the nacelle tilting angle, this paper applies the derivatives of $\delta_{\text{col}}, \delta_{\text{lon}}, \delta_{\text{lat}}, \delta_{\text{ped}}, i_n$ as control variables (denoted by $u_c, u_{\text{lon}}, u_{\text{lat}}, u_{\text{ped}}, u_n$), thus $\delta_{\text{col}}, \delta_{\text{lon}}, \delta_{\text{lat}}, \delta_{\text{ped}}, i_n$ are set as state variables. The developed differential equation set is expressed as

$$\dot{x}_a = f_a(x_a, u_a, t)$$

where

$$x_a = \begin{bmatrix} u, v, w, p, q, r, \phi, \theta, \psi, x, y, h, \\
\dot{h}, \dot{\theta}, \dot{\psi}, \dot{x}_{\text{lon}}, \dot{x}_{\text{lat}}, \dot{x}_{\text{ped}}, \dot{i}_n, \dot{\delta}_{\text{col}}, \dot{\delta}_{\text{lon}}, \dot{\delta}_{\text{lat}}, \dot{\delta}_{\text{ped}}, \dot{i}_n, \dot{\phi}, \dot{\theta}, \dot{\psi}
\end{bmatrix}^T$$

$$u_a = \begin{bmatrix} u_c, u_{\text{lon}}, u_{\text{lat}}, u_{\text{ped}}, u_n \end{bmatrix}^T$$

The tilt-rotor aircraft has a longitudinally symmetrical configuration, thus the conversion procedure is in the longitudinal plane under no crosswind conditions. To improve the calculation efficiency in solving the nonlinear optimal control problem, this paper assumes that the state variables and control variables related to lateral and heading motion remain in the initial state, and do not participate in the numerical calculation of dynamic tilting.

2) Optimal variables: differential state variables $x_a$, control variables $u_a$ of the differential equation set Eq. (3) and free time $t_f$ at terminal point.

3) Cost function: the performance index of whole conversion process. The determination of cost function should consider the integration of multiple factors of the time of dynamic tilting, flight attitude, feasibility of pilot control and pilot workload etc. The basic formulation is expressed as

$$\min_u J = \phi(x_a(t_0), t_0, x_a(t_f), t_f) + \int_{t_0}^{t_f} L(x_a(t), u_a(t), t) dt$$

where $\phi$ is the function for state performance index at initial $(t_0)$ and terminal point $(t_f)$, the integration of function $L$ is the performance index for state and control in the conversion procedure. Specific expression of $J$ will be given in the case study. The optimal control strategy of dynamic conversion can be successfully solved under the following boundary conditions, path constraints and terminal constraints.

4) Constraints: the initial boundary conditions are the current flight state of the aircraft. The terminal constraints are set as the target tilting angle and forward flying speed

$$\begin{cases}
\dot{i}_n(t_f) = \dot{i}_n \\
u_n(t_f) = 0\text{ m/s}
\end{cases}$$

$$\begin{cases}
\dot{x}_{\text{min}} \leq \dot{x}(t_f) \leq \dot{x}_{\text{max}} \\
\dot{h}_{\text{min}} \leq \dot{h}(t_f) \leq \dot{h}_{\text{max}}
\end{cases}$$
where $i_{nt}$ is the target engine nacelle tilting angle, $\dot{x}$ is the target forward speed, $\dot{h}$ is the target ascent rate, the specific values and additional items can be determined according to the requirements of the conversion flight mission.

The path constraints should be determined by the boundary of conversion corridor (as shown in Figure 1). In low-speed conversion, the aerodynamic lift provided by wing is restricted by the critical stall attack angle. Therefore, at the lower conversion envelope, the attack angle of wing equals the critical value. The corresponding path constraint is

$$\alpha_{W,\min} \leq \alpha_W(t) \leq \alpha_{W,\max}$$

(7)

where the critical attack angle $\alpha_{W,\min}$ and $\alpha_{W,\max}$ can be obtained from wind tunnel data [1]. The maximum forward speed in the conversion procedure is restricted by the stall effect of retreating rotor blade, the compressibility of advancing rotor blade, the available power and dynamic stability of the rotor, among which the available power of the rotor is dominant. Therefore, the path constraint determined by the upper conversion envelope is

$$0 \leq P_n(t) \leq P_n$$

(8)

where $P_n$ is the rated power output of the engine (two). To further guarantee the flight safety during conversion process, the speed corresponding to the engine nacelle tilting angle of 45° on the upper conversion envelope is taken as the stop speed, and the flight speed during the tilting process shall not exceed the stop speed $V_{\text{stop}}$

$$V_{\text{max}} \leq V_{\text{stop}}$$

(9)

In addition, the constraints of altitude, pitch attitude as well as the pilot control inputs should also be considered in the path constraints. Notice that in order to study the pilot control strategy on height change during dynamic tilting, the height constraint is appropriately relaxed. The constraints of control rates are selected according to physical limits of actuator in XV-15

$$\Delta h_{\text{min}} \leq \Delta h(t) \leq \Delta h_{\text{max}}$$

$$-10^\circ \leq \theta(t) \leq 10^\circ$$

$$-5^\circ / s \leq q(t) \leq 5^\circ / s$$

(10)

$$0 \leq \delta_{\text{col}}(t), \delta_{\text{on}}(t) \leq 1$$

$$0^\circ \leq \delta_h(t) \leq 90^\circ$$

$$-0.3 / s \leq u_{\text{col}}(t), u_{\text{on}}(t) \leq 0.3 / s$$

$$-15^\circ / s \leq u_h(t) \leq 15^\circ / s$$

(11)

3.2. Numerical solution methods

The NOCP of conversion procedure are complex, which can only be solved numerically. First of all, the optimal variables $x, u, t$ are normalized and scaled to improve the convergence speed in numerical optimization

$$\left(\bar{u}, \bar{v}, \bar{w} \right) = \frac{k_1}{\Omega_0 R} (u, v, w), \quad \left(\bar{p}, \bar{q}, \bar{r} \right) = \frac{k_2}{\Omega_0} (p, q, r)$$

$$\left(\bar{a}_{0,LR, \bar{a}_{1,LR, \bar{b}}_{1,LR}} \right) = \frac{k_3}{\Omega_0 \tau} (a_{0,LR, a_{1,LR, b_{1,LR}}}), \quad \left(\bar{x}, \bar{y}, \bar{h} \right) = \frac{k_4}{R} (x, y, h)$$

(12)

where $\Omega_0$ is the standard main rotor rotational speed, $k_1 \sim k_4$ are the constant scaling factors. Take $k_1 = k_2 = 100, k_3 = 1, k_4 = 0.01$, the corresponding normalized-scaled flight dynamics model is
\[ \frac{d\bar{x}_k}{d\tau} = f(\bar{x}_k, \bar{u}_k, \tau) \]  \hspace{1cm} (13)

By far the most feasible numerical solving method for NOCP is to transform it to nonlinear programming problem (NLP) by node collocation. This paper adopts direct multiple shooting collocation approach [14-15] to break the continuous NOCP into multi-nodes NLP with shorter time segments, which is an effective numerical solution technique for the problem with large degrees of freedom and high complexity [15]. As shown in Figure 3.

\[ \hat{x}_{k+1} - \hat{x}_{k+1} = 0, \quad k = 1, \ldots, N - 1 \]  \hspace{1cm} (14)

where

\[ \hat{x}_{k+1} = \bar{x}_k + \int_{\tau_k}^{\tau_{k+1}} f(\bar{x}, \bar{u}, \tau) d\tau \]  \hspace{1cm} (15)

In the same way, the original cost function is discretized to the performance index of NLP as follow

\[
\min J = \phi(\bar{x}_a(\tau_i), \tau_i, \bar{x}_a(\tau_N), \tau_N) + \sum_{k=1}^{N-1} \int_{\tau_k}^{\tau_{k+1}} L(\bar{x}_a(\tau), \bar{u}_a(\tau), \tau) d\tau
\]  \hspace{1cm} (16)

Path constraints are enforced at each node (except \( \tau_0 \) and \( \tau_i \)), the initial boundary condition is imposed at \( \tau_0 \), and the terminal constraints is imposed at \( \tau_i \). Then the optimal solution of NLP is solved by sequential quadratic programming (SQP) method [16]. Finally, the continuous control process \( u(t) \) is approximated by piecewise linear interpolation of the numerical solution, and the continuous states \( x(t) \) are approximated by integrating differential equation set from \( \tau_0 \) to \( \tau_i \) with control process \( u(t) \).

4. Tilt-rotor aircraft conversion and reconversion procedure

4.1. Conversion procedure

In this paper, the conversion procedure (helicopter mode to fixed-wing mode) of XV-15 tilt-rotor aircraft is taken for case study. The initial conditions are steady-state in helicopter mode \( (i_o=90^\circ) \) with...
weight 5897 kg, flap setting 40/25, altitude 100 m, forward speed 35 m/s, no sideslip, track angle 0°, 2 sec pilot delay.

In fact, if tilt-rotor aircraft encounters danger in conversion mode, it is unable to quickly make maneuver to avoid it. Besides, it cannot quickly enter into autorotation when the engine fails in conversion mode. Therefore, it is one of the most important index to complete the conversion as soon as possible within the safety range. In addition, the pilot workload also needs to be kept as low as possible. Since the limitations of altitude and attitude in the conversion process can be expressed by path constraints according to different missions, the performance index in this paper is set as follows

$$
\min J_C = w_l \left( t_f - t_0 \right) + \frac{w_p}{\tau_f - \tau_0} \int_{\tau_0}^{\tau_f} \left[ w_{col} \cdot u_{col}^2 (r) + w_{lon} \cdot u_{lon}^2 (r) + w_n \cdot u_n^2 (r) \right] dr
$$

(17)

In actual flight, the pilot focuses on the collective (thrust) and longitudinal control in the process of conversion, so the weight coefficients $w_{col}$ and $w_{lon}$ are set higher than $w_n$. The specific values are selected as $w_{lon}=0.5$, $w_{col}=0.35$, $w_n=0.15$. The distribution of the weight coefficient $w_p$ of pilot workload and the weight coefficient $w_n$ of time has great influence on the quality of conversion procedure. Figure 4 shows the pilot control strategy and flight state under different $w_p/w_n$.

As can be seen from Figure 4, the displacements of controls are significantly reduced when $J_C$ includes the weight coefficient of pilot workload ($w_p$), and the height as well as pitch attitude change more gently, but the time has been extended. In addition, $w_p$ should not be dominant, otherwise the overly smooth manipulation will lead to a decrease of height. When the ratio of $w_p/w_t$ is 3/7 for conversion procedure, the pilot manipulation is smooth, the height change is small, and the tilting process of the engine nacelle can be basically realized by the constant angular rate automatic tilting system (stabilized at 7.5°/s).
4.2. Reconversion procedure

This paper takes the XV-15 tilt-rotor aircraft reconversion procedure (fixed-wing mode to helicopter mode) as case study. The initial conditions are steady-state in fixed-wing mode ($i_0=0^\circ$) with weight 5897 kg, flap setting 40/25, altitude 100 m, forward speed 65 m/s, no sideslip, track angle $0^\circ$, 2 sec pilot delay. The analysis process of reconversion procedure is similar to that of the conversion procedure as mentioned above, the performance index is set as follows:

$$
\min J_R = w_1 \left( t_f - t_0 \right) + \frac{w_p}{t_f - t_0} \int_{t_0}^{t_f} \left[ w_{\text{col}} \cdot u_{\text{col}}^2(r) + w_{\text{lon}} \cdot u_{\text{lon}}^2(r) + w_n \cdot u_n^2(r) \right] \mathrm{d}r
$$

(18)

The specific values of $w_{\text{col}}, w_{\text{lon}}$, and $w_n$ are selected as $w_{\text{lon}}=0.5$, $w_{\text{col}}=0.35$, $w_n=0.15$. Figure 5 shows the pilot control strategy and flight state under different $w_p/w_1$. It can be concluded that when the ratio of $w_p/w_1$ is $1/1$ for reconversion procedure, the pilot manipulation is smooth, the height change is small, and the tilting process of the engine nacelle can be basically realized by the constant angular rate automatic tilting system (stabilized at $7.5^\circ$/s).
5. Conclusion
This paper studies the control strategy of tilt-rotor aircraft dynamic conversion procedure. A nonlinear flight dynamics model with full flight modes is established. A nonlinear optimal control model of dynamic conversion is constructed, and an efficient numerical solution method with good convergence is designed to obtain the trajectory and control strategy of dynamic conversion between helicopter and fixed-wing mode. The results yield the following conclusions:

1. The optimal control model and the numerical solution method applied are feasible for the study of tilt-rotor aircraft dynamic conversion procedure.

2. As can be seen from Figure 4 and Figure 5, when the subitem of pilot workload is included in the cost function, the displacements of collective stick input as well as the longitudinal stick input are significantly reduced, and the height and pitch attitude change more gently, but the time has been extended. In addition, the pilot workload weight coefficient $w_{p}$ should not be too dominant, otherwise the overly smooth collective stick and longitudinal stick manipulation will lead to a decrease of height.

3. Based on the analysis above, it can be concluded that when the ratio of $w_{p}/w_{t}$ is 3/7 for conversion procedure, and 1/1 for reconversion procedure, the pilot manipulation is smooth, the height change is small, and the tilting process of the engine nacelle can be basically realized by the constant angular rate automatic tilting system (stabilized at 7.5°/s).

Acknowledgment
This work was supported by the program of Zhejiang Lab No. 2018DF0ZX01, program of The Leading Innovation and Entrepreneurship Team in Zhejiang Province, No. 2018R01006, and the National Natural Science Foundation of China (No. 11672128).

References
[1] Ferguson S W. 1988. A mathematical model for real time flight simulation of a generic tilt-rotor aircraft. NASA Report CR-166536; pp. 1–20.
[2] Muscarello V, Colombo F, Quaranta G, et al. 2018. Aeroelastic Rotorcraft–Pilot Couplings in Tiltrotor Aircraft. Journal of Guidance, Control, and Dynamics, Vol. 1, (1); pp. 1-14.
[3] Felker F F, Maisel M D, Betzina M D. 1986. Full-Scale Tilt-Rotor Hover Performance. Journal of the American Helicopter Society, Vol. 31, (2); pp. 10-8.
[4] Johnson W, Lau B H, Bowles J V. 1987. Calculated performances, stability, and maneuverability of high speed tilting proprotor. Vertica, Vol. 11, (112); pp. 317-39.
[5] Righetti A, Muscarello V, Quaranta G. 2017. Linear Parameter Varying Models for the Optimization of Tiltrotor Conversion Maneuver. In: Proceedings of American Helicopter Society 73rd Annual Forum. Fort Worth, Texas, USA: AHS, May 9–11; pp. 280-7.
[6] Klein G D. 1996. Linear modeling of tiltrotor aircraft (in helicopter and airplane modes) for stability analysis and preliminary design. Dissertation. USA: Naval Postgraduate School; pp.23-50.
[7] Miller M, Narkiewicz J. 2006. Tiltrotor modelling for simulation in various flight conditions. Journal of theoretical and applied mechanics, Vol. 44, (4); pp. 881-906.
[8] Carlson E B, Zhao Y J. 2004. Optimal City-center Takeoff Operation of Tiltrotor Aircraft in
One Engine Failure. *Journal of Aerospace Engineering*, Vol. 17, (1); pp. 26-39.

[9] Carlson E B, Zhao Y J. 2002. Optimal Short Takeoff of Tiltrotor Aircraft in One Engine Failure. *Journal of Aircraft*, Vol. 39, (2); pp. 280-9.

[10] Bottasso C L, Maisano G, Scorcelletti F. 2010. Trajectory optimization procedures for rotorcraft vehicles, their software implementation, and applicability to models of increasing complexity. *Journal of the American Helicopter Society*, Vol.55, (3); p. 32010.

[11] Bibik P, Narkiewicz J. 2012. Helicopter optimal control after power failure using comprehensive dynamics model. *Journal of Guidance, Control, and Dynamics*, Vol. 35, (2); pp. 1354-62.

[12] Meng W, Chen R L. 2013. Study of helicopter autorotation landing following engine failure based on a six-degree-of-freedom rigid-body dynamics model. *Chinese Journal of Aeronautics*, Vol. 26, (6); pp. 1380-8.

[13] Ferguson S W. 1989. Development and validation of a simulation for a generic tilt-proprotor aircraft. *NASA report CR-166537*; pp A1-A185.

[14] Kim C J, Sung S, Park S H, et al. 2014. Nonlinear optimal control analyses. *Journal of Guidance, Control, and Dynamics*, Vol. 37, (2); pp. 658-73.

[15] Bottasso C L, Maisano G, Scorcelletti F. 2009. Trajectory optimization procedures for rotorcraft vehicles including pilot models, with applications to ADS-33 MTES, CAT-A and engine off landings. In: *Proceedings of the 65th American Helicopter Society*, Grapevine, TX, May; pp. 1-9.

[16] Gill P E, Murray W, Saunders M A. 2007. *User’s guide for SNOPT version 7: Software for large-scale nonlinear programming*. San Diego: University of California; pp. 4-29.