Light vector meson and heavy baryon strong interaction

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We calculate the coupling constants between the light vector mesons and heavy baryons within the framework of the light-cone QCD sum rule in the leading order of heavy quark effective theory. Most resulting sum rules are stable with the variations of the Borel parameter and the continuum threshold. The extracted couplings will be useful in the study of the possible heavy baryon molecular states.

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I. INTRODUCTION

Heavy quark effective theory (HQET) is a systematic approach to study the spectra and transition amplitudes of heavy hadrons. In HQET, the expansion is performed in terms of \(1/m_Q\), where \(m_Q\) is the mass of the heavy quark involved. In the leading order of HQET, namely \(m_Q \to \infty\), heavy hadrons form a series of degenerate doublets due to the heavy quark symmetry. The two states in a doublet share the same quantum number \(J_l\), where \(J_l\) is the angular momentum of the light components.

Light-cone QCD sum rules (LCQSR) has been used as a useful nonperturbative approach to extract various hadronic transition form factors. In this framework, one considers the T-product of the two interpolating currents sandwiched between the vacuum and an hadronic state, instead of two vacuum states as in the conventional QCD sum rules. Now the operator product expansion is performed near the light-cone rather than at a small distance as in the conventional sum rules. The double Borel transformation is always invoked to suppress the excited state and the continuum contribution.

The \(\rho\) and \(\omega\) couplings between the heavy meson doublets \(H, S,\) and \(T\) were systematically studied with LCQSR in the leading order of HQET. The \(\pi\) coupling constants between \(\Sigma^*, \Sigma,\) and \(\Lambda\) were calculated within the same framework in Ref. [5].

In this work we employ LCQSR to calculate the vector meson coupling constants between low-lying heavy baryons in the leading order of HQET. These baryons can be grouped into two flavor multiplets \(6_F\) and \(\bar{3}_F\), according to the flavor of their two light quarks. On the other hand, the members in the multiplet \(6_F\) are degenerate doublets in the limit \(m_Q \to \infty\). Because of the heavy quark symmetry, the vector couplings with the same \((l, j_h)\) between two doublets are not independent, where \(l\) and \(j_h\) denote the orbital and total angular momentum of the final vector meson respectively. Therefore the following calculations on the channels involving the \(J^P = \frac{3}{2}^+\) members in \(6_F\) and the members in \(\bar{3}_F\) include all independent coupling constants under consideration.

II. SUM RULES FOR THE \(\rho\) COUPLING CONSTANTS

According to the flavor of their two light quarks, heavy baryons can be decomposed into two flavor multiplets \(6_F\) and \(\bar{3}_F\). As far as the ground states are concerned, the total spin of the two light quarks must be 1 for \(6_F\) and 0 for \(\bar{3}_F\) due to the symmetric property of their colors and flavors. This leads to \(J^P = \frac{1}{2}^+ / \frac{3}{2}^+\) for \(6_F\) and \(J^P = \frac{1}{2}^+\) for \(\bar{3}_F\). We use * as a superscript differentiating the \(J^P = \frac{3}{2}^+\) heavy baryons from the \(J^P = \frac{1}{2}^+\) ones in \(6_F\).

We introduce the same interpolating currents as Ref. [5] in our calculation for the heavy baryons with \(J^P = \frac{1}{2}^+\) in \(\bar{3}_F\), \(J^P = \frac{1}{2}^+\) in \(6_F\) and \(J^P = \frac{3}{2}^+\) in \(6_F\):

\[
\eta_B(x) = \epsilon_{abc} \left[ q_1^{aT}(x) C \gamma_5 q_2^b(x) \right] h_c^v(x), \tag{1}
\]
our approximation, we need the 2- and 3-particle wave functions. \( \gamma_{l,j} \) is the orbital and total angular momentum.(iS) reads as where 

\[ \eta_{BP}(x) = \epsilon_{abc} \left[ g_{1}^{T}(x)C\gamma_{\mu}q_{2}^{b}(x) \right] \gamma_{\mu}^{\nu}h_{\nu}^{c}(x), \]  

(2) \[ \eta_{B*}(x) = \epsilon_{abc} \left[ g_{1}^{T}(x)C\gamma_{\mu}q_{2}^{b}(x) \right] \left[-g_{\mu}^{\nu} + \frac{1}{3} \gamma_{\mu}^{\nu} \right] h_{\nu}^{c}(x). \]  

Here, \( a, b, \) and \( c \) are color indices, the subscript of \( g_{1}(x) \) denotes the flavor of the quark field: \( u, d \) or \( s \). \( h_{\nu}^{c} \) is the heavy quark with velocity \( v \). \( T \) is the transpose matrix and \( C \) is the charge conjugate matrix. \( g_{\mu}^{\nu} \equiv g_{\mu}^{\nu} - \hat{v} \mu \nu, \) \( \gamma_{\mu}^{\nu} \equiv \gamma_{\mu}^{\nu} - \hat{v} \mu \nu. \)

We denote the \( \rho \) coupling constant between \( \Sigma^{*} \) and \( \Lambda \) as \( g^{1}_{\Sigma^{*}\Lambda_{p}} \), where the superscript \( p \) and the number \( 1 \) indicate the orbital and total angular momentum \( (l,j_{h}) \) of the final \( \rho \) meson respectively. To obtain the sum rules for \( g^{1}_{\Sigma^{*}\Lambda_{p}} \), we consider the correlation functions defined as

\[ \int d^{4}xe^{-ik\cdot x} \langle \rho(q)||T \{ \eta_{\Sigma}^{c}(0)\bar{\eta}_{\Lambda}(x) \} ||0 \rangle = \left[ g_{1}^{c} \frac{1}{3} \gamma_{\mu}^{\nu} \gamma_{\mu}^{\nu} \right] \frac{1 + \hat{v} \cdot \hat{t}}{2} \int dt \int dxe^{-ik\cdot x} \delta(-x - vt) \times 4Tr(\rho(q)||u(0)d(x)||0)\gamma_{5}C\Sigma^{*}(-x)C\gamma_{5}, \]  

(4) where \( q \) is the momentum of the final \( \rho \) meson, \( q^{2} = m_{p}^{2}. \) In the leading order of HQET, the heavy quark propagator reads as

\[ \langle 0||T \{ h_{\nu}(0)\bar{h}_{\nu}(x) \} ||0 \rangle = \frac{1}{2} \int dt\delta^{4}(-x - vt). \]  

(5) \( iS(-x) \) is the full light quark propagator

\[ iS(x) \equiv \langle 0||T \{ q(x)\bar{q}(0) \} ||0 \rangle = \frac{i\vec{\sigma}}{2\pi^{2}x^{4}} - \frac{m_{q}}{4\pi^{2}x^{2}} - \frac{\langle \bar{q}q \rangle}{12} + \frac{im_{q}\langle \bar{q}q \rangle}{48} - \frac{m_{q}^{2}\langle \bar{q}q \rangle}{192}(1 - \frac{im_{q}\hat{x}}{6}) - \frac{ig_{s}}{16\pi^{2}} \int_{0}^{1} du \left\{ \frac{x}{2}G(ux) - 4\text{Tr}G(ux) \right\} + \ldots. \]  

(6)

At the quark level, the correlation function can be calculated using the \( \rho \) meson light-cone wave functions [8]. To our approximation, we need the 2- and 3-particle wave functions.

At the hadron level, the correlation function can be written as

\[ \int d^{4}xe^{-ik\cdot x} \langle \rho(q)||T \{ \eta_{\Sigma}^{c}(0)\bar{\eta}_{\Lambda}(x) \} ||0 \rangle = \left[ -g_{1}^{c} \frac{1}{3} \gamma_{\mu}^{\nu} \gamma_{\mu}^{\nu} \right] \frac{1 + \hat{v} \cdot \hat{t}}{2} \epsilon_{\mu\nu\rho} \nu_{\mu}q_{\nu}v_{\rho}G^{1}_{\Sigma^{*}\Lambda_{p}}(\omega, \omega'), \]  

(7) where \( \epsilon_{\mu}^{*} \equiv \epsilon_{\mu} - (\epsilon^{*} \cdot \nu)v_{\mu}, q_{\mu} \equiv \mu_{\nu} - (q \cdot \nu)v_{\mu}, \omega \equiv 2v \cdot k, \omega' \equiv 2v \cdot (k - q). \) \( G^{1}_{\Sigma^{*}\Lambda_{p}}(\omega, \omega') \) has the following pole terms

\[ \frac{4g^{1}_{\Sigma^{*}\Lambda_{p}}}{f_{\rho}} \left( \frac{f_{\Sigma^{*}}f_{\Lambda}}{(2\Lambda_{\Sigma^{*}} - \omega')(2\Lambda_{\Lambda} - \omega)} + \frac{c}{2\Lambda_{\Sigma^{*}} - \omega'} + \frac{c'}{2\Lambda_{\Lambda} - \omega} \right), \]  

(8)
where \( \Lambda_{\Sigma^*} \equiv m_{\Sigma^*} - m_Q \), \( \overline{\Lambda}_\Lambda \equiv m_\Lambda - m_Q \). \( f_{\Sigma^*} \) etc. are the overlapping amplitudes of the interpolating currents with their corresponding states \([3]\):

\[
\langle 0 | J_B | B \rangle = f_B u_B, \\
\langle 0 | J_{B'} | B' \rangle = f_{B'} u_{B'}, \\
\langle 0 | J_{B''} | B'' \rangle = f_{B''} u_{B''}.
\]

(9) (10) (11)

where \( u_{B''} \) is the Rarita-Schwinger spinor in HQET, and we have \( f_{B'} = \sqrt{3} f_B \) in the leading order of HQET \([7]\).

We define the coupling constant \( g_{\Sigma^* A \rho}^{p1} \) by the following decay amplitude

\[
M(\Sigma^* \rightarrow A \rho) = \frac{g_{\Sigma^* A \rho}^{p1}}{f_\rho} \bar{u}_{\Lambda} u_{\Sigma^*} \delta \epsilon^{\mu \nu \rho} e_{\mu} e^*_\nu v_\rho.
\]

(12)

\( G_{\Sigma^* A \rho}^{p1}(\omega, \omega') \) can be expressed by the \( \rho \) meson light-cone wave functions as

\[
G_{\Sigma^* A \rho}^{p1}(\omega, \omega') = \frac{1}{192 \pi^2} \int_0^1 dt \int_0^1 du e^{i(1-u)\frac{\omega}{T}} e^{iu\frac{\omega'}{T}} \\
\left[ -24 f_T^T m_\rho^2 A_T(u) \frac{1}{t} + \pi^2 f_\rho m_\rho m_0^2 \langle \bar{q}q \rangle g_\perp^{(1)}(u) t^3 \\
+16 \pi^2 f_m m_\rho \langle \bar{q}q \rangle g_\perp^{(1)}(u) t - 384 f_T^T \varphi_\perp(u) \frac{1}{t^3} \right] \\
- \frac{f_T^T m_\rho^2}{4 \pi^2} \int_0^\infty dt \int_0^1 du e^{i(1-\alpha - \omega \alpha)} \beta_t \epsilon^{(\alpha + \omega \alpha)} \delta_t \\
(T_1 - T_2 - T_3 + T_4 - S - \bar{S} = 2 w T_3 - 2 w T_4 + 2 w \bar{S}).
\]

(13)

Here \( \varphi_\perp, g_\perp^{(1)} \), and \( A_T \) are twist-2, 3, and 4 2-particle \( \rho \) meson light-cone distribution amplitudes respectively. They are normalized to satisfy \( f_T^T du \phi(u) = 1 \). \( T_1, T_2, T_3, T_4, S, \) and \( \bar{S} \) are twist-4 3-particle \( \rho \) meson distribution amplitudes \([6]\).

After the Wick rotation and the double Borel transformation with \( \omega \) and \( \omega' \), the single-pole terms in \([8]\) are eliminated and we arrive at

\[
g_{\Sigma^* A \rho}^{p1} f_{\Sigma^*} f_\Lambda = \frac{f_R}{4} \frac{\Lambda_{\Sigma^* + \Lambda_\Lambda}}{f_\rho} \left( \frac{f_T^T}{2 \pi^2} \varphi_\perp(u_0) T^4 f_3 \frac{\omega_c}{T} - \frac{f_T^T m_\rho^2}{8 \pi^2} A_T(u_0) T^2 f_1 \frac{\omega_c}{T} \\
- \frac{f_T^T m_\rho}{3} \langle \bar{q}q \rangle g_\perp^{(1)}(u_0) + \frac{f_T^T m_\rho}{12 T^2} m_0^2 \langle \bar{q}q \rangle g_\perp^{(1)}(u_0) \\
- \frac{f_T^T m_\rho^2}{4 \pi^2} T^2 f_1 \frac{\omega_c}{T} \left[ (T_1 - T_2 - T_3 + T_4 - S - \bar{S}) - \omega_0 \right] (u_0) \\
+ 2(T_3 - T_4 + \bar{S})^{[0,1]}(u_0) \right),
\]

(14)

where \( f_n(x) \equiv 1 - e^{-x} \sum_{k=0}^{n-1} \frac{x^k}{k!} \) is the continuum subtraction factor, \( \omega_c \) is the continuum threshold, \( u_0 = \frac{T_1}{T_1 + T_2} \), \( T \equiv \frac{T_1 T_2}{T_1 + T_2} \). \( T_1 \) and \( T_2 \) are the two Borel parameters. In the above equation we have used the following definitions

\[
F^{[n]}(u_0) = \int_0^u \cdots \int_0^{x_3} \int_0^{x_2} \int_0^{x_1} F(x_1) dx_1 dx_2 \cdots dx_n,
\]

(15)

\[
F^{[0,0]}(u_0) = \int_0^u \int_0^{1-\alpha_1} F(\alpha_1, 1-\alpha_1 - \alpha_3, \alpha_3) d\alpha_3 d\alpha_1,
\]

(16)

\[
F^{[0,1]}(u_0) = \int_0^u \int_0^{1-\alpha_1} (u_0 - \alpha_1) F(\alpha_1, 1-\alpha_1 - \alpha_3, \alpha_3) d\alpha_3 d\alpha_1,
\]

(17)

\[
F^{[1,0]}(u_0) = \int_0^u \int_0^{1-\alpha_1} \frac{F(\alpha_1, 1-\alpha_0, \alpha_0 - \alpha_1)}{\alpha_3} d\alpha_3 d\alpha_1,
\]

(18)

\[
F^{[1,1]}(u_0) = \int_0^u \int_0^{1-\alpha_1} \frac{F(\alpha_1, 1-\alpha_0, \alpha_0 - \alpha_1)}{\alpha_3} d\alpha_3 d\alpha_1 - \int_0^{1-u_0} \frac{F(u_0, 1-u_0 - \alpha_3, \alpha_3)}{\alpha_3} d\alpha_3,
\]

(19)
\begin{align}
\mathcal{F}[-1,0](u_0) &= \int_0^{u_0} \int_0^{u_0-\alpha_1} \mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \, d\alpha_3 \, d\alpha_1 \\
&\quad + \int_0^{u_0} \int_0^{1-\alpha_1} \left. \frac{(u_0 - \alpha_1) \mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3)}{\alpha_3} \right|_{\alpha_3 = 0} \, d\alpha_3 \, d\alpha_1, \\
(20) \\
\mathcal{F}[-1,1](u_0) &= \frac{1}{2} \left\{ \int_0^{u_0} \int_0^{u_0-\alpha_1} \mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \, d\alpha_3 \, d\alpha_1 \\
&\quad + \int_0^{u_0} \int_0^{1-\alpha_1} \left. \frac{(u_0 - \alpha_1)^2 \mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3)}{\alpha_3^2} \right|_{\alpha_3 = 0} \, d\alpha_3 \, d\alpha_1 \right\}, \\
(21) \\
\mathcal{F}[-3,0](u_0) &= \int_0^{u_0} \int_0^{u_0-\alpha_1} \int_0^{\alpha_3} \int_0^{x_2} \mathcal{F}(\alpha_1, 1 - \alpha_1 - x_1, x_1) \, dx_1 \, dx_2 \, d\alpha_3 \, d\alpha_1 \\
&\quad + \frac{1}{2} \int_0^{u_0} \int_0^{u_0-\alpha_1} \int_0^{\alpha_3} x \mathcal{F}(\alpha_1, 1 - \alpha_1 - x, x) \, dx \, d\alpha_3 \, d\alpha_1 \\
&\quad + \frac{1}{6} \int_0^{u_0} \int_0^{u_0-\alpha_1} \int_0^{\alpha_3} \alpha_3 \mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \, d\alpha_3 \, d\alpha_1 \\
&\quad + \frac{1}{6} \int_0^{u_0} \int_0^{u_0-\alpha_1} \int_0^{\alpha_3} \left( \frac{(u_0 - \alpha_1)^3}{\alpha_3} \right) \mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \, d\alpha_3 \, d\alpha_1, \\
(22) \\
\mathcal{F}[-3,1](u_0) &= \frac{1}{2} \left\{ \int_0^{u_0} \int_0^{u_0-\alpha_1} \int_0^{\alpha_3} \int_0^{x_2} \mathcal{F}(\alpha_1, 1 - \alpha_1 - x_1, x_1) \, dx_1 \, dx_2 \, d\alpha_3 \, d\alpha_1 \\
&\quad + \frac{1}{6} \int_0^{u_0} \int_0^{u_0-\alpha_1} \int_0^{\alpha_3} x \mathcal{F}(\alpha_1, 1 - \alpha_1 - x, x) \, dx \, d\alpha_3 \, d\alpha_1 \\
&\quad + \frac{1}{24} \int_0^{u_0} \int_0^{u_0-\alpha_1} \int_0^{\alpha_3} \alpha_3 \mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \, d\alpha_3 \, d\alpha_1 \\
&\quad + \frac{1}{24} \int_0^{u_0} \int_0^{u_0-\alpha_1} \int_0^{\alpha_3} \left( \frac{(u_0 - \alpha_1)^4}{\alpha_3^2} \right) \mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \, d\alpha_3 \, d\alpha_1 \right\}, \\
(23)
\end{align}

We have used the Borel transformation \( \hat{B}^T \), \( e^{\alpha \omega} = \delta(\alpha - \frac{1}{T}) \) to obtain \[40\].

The other \( \rho \) coupling constants between \( B \) and \( B^* \) can be calculated in the same way. The definitions of these coupling constants are

\begin{align}
\mathcal{M}(\Sigma^* \rightarrow \Sigma \rho) &= \frac{g_{\Sigma^* \Sigma \rho}^\rho}{f_\rho} \, i u_{\Sigma^*}^\rho \, u_{\Sigma}^\rho \, g_{\eta \rho} (e^* \cdot q_t) \\
&\quad + \frac{g_{\Sigma^* \Sigma \rho}^1}{f_\rho} \, i u_{\Sigma^*}^\rho \, u_{\Sigma}^\rho \left( q_{\Sigma^*} e_t^\rho - q_{\Sigma} e_t^{\rho^*} \right) \\
&\quad + \frac{g_{\Sigma^* \Sigma \rho}^2}{f_\rho} \, i u_{\Sigma^*}^\rho \, u_{\Sigma}^\rho \left[ q_{\Sigma^*} e_t^\rho + q_{\Sigma} e_t^{\rho^*} - \frac{2}{3} q_{\Sigma^*} q_{\Sigma} (e^* \cdot q_t) \right] \\
&\quad + \frac{g_{\Sigma^* \Sigma \rho}^3}{f_\rho} \, i u_{\Sigma^*}^\rho \, u_{\Sigma}^\rho (e^* \cdot q_t) \left[ q_{\Sigma^*} q_{\Sigma} - \frac{1}{3} q_{\Sigma^*} q_{\Sigma}^2 \right], \\
(24) \\
\mathcal{M}(\Xi \rightarrow \Xi \rho) &= \frac{g_{\Xi \Xi \rho}^\rho}{f_\rho} \, i u_{\Xi} \, u_{\Xi}^\rho (e^* \cdot q_t). \\
(25)
\end{align}

The definitions of \( g_{\Xi^* \Xi \rho} \) and \( g_{\Xi \Xi \rho} \) are similar to the above definitions. To derive their sum rules, we consider the following correlators

\begin{align}
\int d^4x e^{-i k \cdot x} \langle \rho(q) | T \{ \eta_{\Xi^*}^\rho(0) \eta_{\Xi}^\rho(x) \} | 0 \rangle &= \left[ - \frac{g_t^4}{3} + \frac{g_t^2}{3} \eta_\rho \right] \int dt \int dxe^{-i k \cdot x} \delta(-x - vt) \\
&\quad \times 4 \text{Tr} \left[ (\rho(q) | u(0) \bar{d}(x) | 0 ) \gamma_\rho C_i S_T^\rho(-x) C_{\gamma_5} \right], \\
(26) \\
\int d^4x e^{-i k \cdot x} \langle \rho(q) | T \{ \eta_{\Xi}^\rho(0) \eta_{\Xi}^\rho(x) \} | 0 \rangle &= - \frac{1}{2} \int dt \int dxe^{-i k \cdot x} \delta(-x - vt) \\
&\quad \times 2 \text{Tr} \left[ (\rho(q) | u(0) \bar{d}(x) | 0 ) \gamma_\rho C_i S_T^\rho(-x) C_{\gamma_5} \right]. \\
(27)
\end{align}
Here \( \eta_{\Sigma^*}^{0}(x) \) is the interpolating current for \( I = 0 \) \( \Sigma^* \) baryon.
The hadron level expression for the above two correlators reads:

\[
\int d^4x e^{-ik \cdot x} \langle \rho(q) | T(\eta^0_{\Sigma^*},(0) \eta^0_{\Sigma^*}(x)) | 0 \rangle = \left[ -g_t^{\alpha \delta} + \frac{1}{3} g_t^{\alpha \delta} \right] \frac{1}{2} \left[ 1 + \hat{\nu} \right] \left[ -g_t^{\xi \eta} + \frac{1}{3} g_t^{\xi \eta} \right] \left\{ g_{\eta \eta}(e^* \cdot q_t) G_{\Sigma^* \Sigma^*, \rho}(\omega, \omega') \\
+ (q_{\eta} e^*_\eta - q_{\eta} e^*_\eta) G_{\Sigma^* \Sigma^*, \rho}(\omega, \omega') \\
+ [q_{\eta} e^*_\eta + q_{\eta} e^*_\eta - \frac{2}{3} g_{\eta \eta}(e^* \cdot q_t)] G_{\Sigma^* \Sigma^*, \rho}(\omega, \omega') \\
+ (e^* \cdot q_t) \left[ q_{\eta} q_{\eta} - \frac{1}{3} g_{\eta \eta} q^2_t \right] G_{\Sigma^* \Sigma^*, \rho}(\omega, \omega') \} \right.
\]

\[
\int d^4x e^{-ik \cdot x} \langle \rho(q) | T(\eta_{\Sigma^*}(0) \eta_{\Sigma^*}(x)) | 0 \rangle = \frac{1}{2} \left( e^* \cdot q_t \right) G_{\Sigma^* \rho}(\omega, \omega').
\]
\[ +72f_ρm_ρ^4\left[(\Psi - \Phi - \frac{\omega}{2} - \tilde{\Psi} + \tilde{\Phi} + \frac{A}{2})^{[-3.0]}(u_0) - 2(\Psi - \Phi - \frac{\omega}{2})^{[-3.1]}(u_0)\right] \frac{1}{T}, \] (34)

The sum rules for \(g_{\Xi^* \bar{\Xi}^* \rho}^\rho\) and \(\Xi^* \bar{\Xi}^* \rho\) can be derived in a similar way and we list them below:

\[ g_{\Xi^* \bar{\Xi}^* \rho}^{\rho 0} f_{\Xi^* \bar{\Xi}^*}^2 = \frac{1}{96\pi^2} e^{\frac{2\lambda_\rho}{3}} f_\rho m_\rho \left\{ 36 f_\rho m_\rho m_\rho \left( 36 f_\rho m_\rho \right)^{1/3} (u_0) T^3 f_2\left(\frac{\omega_c}{T}\right)^2 \right\} - 9f_\rho m_\rho^2 \left( 36 f_\rho \right)^{2/3} (u_0) T f_2\left(\frac{\omega_c}{T}\right)^2 + 36 f_\rho \left[ m_\rho m_\rho m_\rho \right]^{1/3} (u_0) T f_2\left(\frac{\omega_c}{T}\right)^2 + 9f_\rho m_\rho^2 \left( 36 f_\rho \right)^{2/3} (u_0) T f_2\left(\frac{\omega_c}{T}\right)^2 \] (35)

\[ g_{\Xi^* \bar{\Xi}^* \rho}^{\rho 0} f_{\Xi^* \bar{\Xi}^*}^2 = \frac{1}{192\pi^2} e^{\frac{2\lambda_\rho}{3}} f_\rho m_\rho \left\{ 36 f_\rho m_\rho m_\rho \left( 36 f_\rho m_\rho \right)^{1/3} (u_0) T^3 f_2\left(\frac{\omega_c}{T}\right)^2 \right\} + 18f_\rho m_\rho \left( 36 f_\rho \right)^{2/3} (u_0) T f_2\left(\frac{\omega_c}{T}\right)^2 - 48f_\rho \left[ m_\rho m_\rho m_\rho \right]^{1/3} (u_0) T f_2\left(\frac{\omega_c}{T}\right)^2 + 12f_\rho m_\rho \left( 36 f_\rho \right)^{2/3} (u_0) T f_2\left(\frac{\omega_c}{T}\right)^2 \] (36)

\[ g_{\Xi^* \bar{\Xi}^* \rho}^{\rho 0} f_{\Xi^* \bar{\Xi}^*}^2 = -\frac{3}{64\pi^2} e^{\frac{2\lambda_\rho}{3}} f_\rho m_\rho \left\{ f_\rho \left[ 36 f_\rho m_\rho m_\rho \left( 36 f_\rho m_\rho \right)^{1/3} (u_0) T^3 f_2\left(\frac{\omega_c}{T}\right)^2 \right] \right\} + 2f_\rho m_\rho \left( 36 f_\rho \right)^{2/3} (u_0) T f_2\left(\frac{\omega_c}{T}\right)^2 - 4f_\rho m_\rho \left( 36 f_\rho \right)^{2/3} (u_0) T f_2\left(\frac{\omega_c}{T}\right)^2 \] (37)

\[ g_{\Xi^* \bar{\Xi}^* \rho}^{\rho 0} f_{\Xi^* \bar{\Xi}^*}^2 = \frac{3}{64\pi^2} e^{\frac{2\lambda_\rho}{3}} f_\rho m_\rho \left\{ f_\rho \left[ 36 f_\rho m_\rho m_\rho \left( 36 f_\rho m_\rho \right)^{1/3} (u_0) T^3 f_2\left(\frac{\omega_c}{T}\right)^2 \right] \right\} - 2f_\rho m_\rho \left( 36 f_\rho \right)^{2/3} (u_0) T f_2\left(\frac{\omega_c}{T}\right)^2 \] (38)

\[ g_{\Xi^* \bar{\Xi}^* \rho}^{\rho 0} f_{\Xi^* \bar{\Xi}^*}^2 = \frac{i}{576\pi^2} e^{\frac{2\lambda_\rho}{3}} f_\rho m_\rho \left\{ f_\rho \left[ 36 f_\rho m_\rho m_\rho \left( 36 f_\rho m_\rho \right)^{1/3} (u_0) T^3 f_2\left(\frac{\omega_c}{T}\right)^2 \right] \right\} + 4f_\rho m_\rho \left( 36 f_\rho \right)^{2/3} (u_0) T f_2\left(\frac{\omega_c}{T}\right)^2 + 2f_\rho m_\rho \left( 36 f_\rho \right)^{2/3} (u_0) T f_2\left(\frac{\omega_c}{T}\right)^2 \] (39)

### III. NUMERICAL ANALYSIS

We need the mass parameters \(\hat{\Lambda}\)'s and the overlapping amplitudes of these interpolating currents \(f_\rho\)'s in our numerical analysis. The values of these parameters can be extracted from the mass sum rules derived in Ref. [8]. We adopt the following values in our calculation, noticing that the \(f_\rho\)'s defined in Ref. [8] is \(\sqrt{3}\) times of those used in our work:

| \(\Sigma^*\) | \(\Xi^*\) | \(\Omega^*\) | \(\Lambda\) | \(\Xi\) |
|---|---|---|---|---|
| 1.0 | 1.12 | 1.25 | 0.8 | 1.0 |

| \(f[GeV^3]\) | 0.023 | 0.028 | 0.037 | 0.018 | 0.024 |
We take the distribution amplitudes of the $\rho$ meson (and the $\omega$, $\phi$, and $K^*$ mesons involved in the subsequent sections) from Ref. [3], there the SU(3)-breaking contributions to high-order conformal partial waves were considered in the framework of renormalon model. Also, the parameters that appear in the distribution amplitudes of the various mesons take the values from Ref. [6]. We use the values at the scale $\mu = 1$ GeV in our calculation under the consideration that the heavy quark behaves almost as a spectator of the decay processes in our discussion in the leading order of HQET.

We will work at the symmetry point, i.e., $T_1 = T_2 = 2T$, $u_0 = 1/2$. This comes from the observation that the mass difference between the baryons involved are less than 0.45 GeV in the leading order of HQET. They are much smaller than the Borel parameter $T_1$, $T_2$ used below. On the other hand, every reliable sum rule has a working window of the Borel parameter $T$ within which the sum rule is insensitive to the variation of $T$. So it is reasonable to choose a common point $T_1 = T_2$ at the overlapping region of $T_1$ and $T_2$. Furthermore, choosing $T_1 = T_2$ will enable us to subtract the continuum contribution cleanly while the asymmetric choice will lead to the very difficult continuum subtraction [4].

From the requirement that the pole contribution is larger than 30%, we get the upper bound $T_{\text{max}}$ of the Borel parameter. In principle, every decay channel has its own $T_{\text{max}}$. The convergence requirement of the operator product expansion leads to the lower bound of the Borel parameter $T_{\text{min}} = 1.0$ GeV, starting from which the stability of the sum rule develops. We find that several sum rules have no working interval, namely $T_{\text{min}} > T_{\text{max}}$. This may arise from the high dimension of the baryon interpolating currents. Two typical resulting sum rules are plotted in Fig. 2 and 3 in the working interval 1.0 GeV $< T < 1.3$ GeV and 1.0 GeV $< T < 6.5$ GeV, with $\omega_c = 2.4, 2.5, 2.6$ GeV.

![FIG. 2: The sum rule for $g_{\rho, \Sigma^* \rho}^\rho f_{\Sigma^* \rho}^2$ with $\omega_c = 2.4, 2.5, 2.6$ GeV](image)

Now we can extract the numerical value of these coupling constants using the following input parameters [6]: $f_\rho = 0.216$ GeV, $f_\rho^T = 0.165$ GeV, and $m_\rho = 0.77$ GeV. The masses of $u$, $d$, and $s$ quark and their condensations are taken as: $m_u = m_d = 0$, $m_s = 0.133$ GeV, and $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -0.243$ GeV$^3$, $m_0^2 = 0.8$ GeV$^2$.

| $g_{\Sigma^* \Lambda_\rho}$ | $g_{\Sigma^* \Sigma^* \rho}$ | $g_{\Xi^* \Xi^* \rho}$ | $g_{\Xi^* \Xi^*_\rho}$ | $g_{\Xi \Xi^*_\rho}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $p_0$           | 2.65 ± 0.20     | 1.15 ± 0.13     |                 | -1.22 ± 0.17    |
| $p_1$           |                 | 2.27 ± 0.20     | 0.90 ± 0.13     |                 |
| $p_2$           | -0.02 ± 0.004   | -0.005 ± 0.001  |                 |                 |
| $f_2$           | 0.10 ± 0.004 GeV$^{-2}$ | 0.03 ± 0.001 GeV$^{-2}$ |                 |                 |

where "×" indicates the non-existence of a working interval of the corresponding sum rule. The errors come from the variations of $T$ and $\omega_c$ in the working region and the central value corresponds to $(T_{\text{min}} + T_{\text{max}})/2$ and $\omega_c = 2.5$ GeV.
Here we take the following values for the parameters 

\[ \begin{align*} 
\Sigma & = \omega \phi, m_\Sigma \to m_q, \langle \bar{s}s \rangle \to \langle \bar{q}q \rangle, \\
\rho & = \omega, m_\rho \to m_q, \langle \bar{s}s \rangle \to \langle \bar{q}q \rangle, \\
\omega & = \Sigma \phi, m_\omega \to m_q, \langle \bar{s}s \rangle \to \langle \bar{q}q \rangle, \\
\end{align*} \]

IV. SUM RULES FOR THE \( \omega, K^*, \) AND \( \phi \) COUPLING CONSTANTS

Replacing the \( \rho \) meson parameters by those for the \( \omega \) meson, one obtains the sum rules for the \( \omega \) meson couplings with the heavy mesons:

\[ \begin{align*} 
g_{\Sigma^* \omega} & = \sqrt{2} g_{\Sigma^* \rho} f_{\Sigma^*}^2 (\Sigma^* \to \Sigma, \rho \to \omega, m_s \to m_q, \langle \bar{s}s \rangle \to \langle \bar{q}q \rangle), \\
g_{\Lambda \omega}^0 & = \sqrt{2} g_{\Xi^* \rho} f_{\Xi^*}^2 (\Xi \to \Lambda, \rho \to \omega, m_s \to m_q, \langle \bar{s}s \rangle \to \langle \bar{q}q \rangle), \\
g_{\Xi^* \omega} & = \sqrt{2} g_{\Xi^* \rho} f_{\Xi^*}^2 (\rho \to \omega), \\
g_{\Xi \omega}^1 & = \sqrt{2} g_{\Xi \Xi}^1 f_{\Xi} (\omega \to \rho), \\
g_{\Xi \omega}^2 & = \sqrt{2} g_{\Xi \Xi}^2 f_{\Xi} (\rho \to \omega), \\
g_{\Xi \omega}^3 & = \sqrt{2} g_{\Xi \Xi}^3 f_{\Xi} (\rho \to \omega), \\
\end{align*} \]

where \( g_{\Sigma^* \omega} = g_{\Sigma^* \rho}^0, g_{\Xi^* \omega}^1, g_{\Xi^* \omega}^2, g_{\Xi \omega}^3, g_{\Xi \omega}^4, \) etc. It is straightforward to get the numerical values for various \( \omega \) coupling constants from these sum rules.

\[ \begin{array}{cccccc}
g_{\Lambda \omega}^0 & g_{\Sigma \omega}^0 & g_{\Xi^* \omega}^0 & g_{\Xi \omega}^0 & g_{\Xi \omega}^0 \\
\hline
\phantom{0}p0 & -1.85 \pm 0.06 & 1.51 \pm 0.18 & 0.64 \pm 0.13 & -0.87 \pm 0.15 \\
\phantom{0}p1 & 1.32 \pm 0.18 & 0.51 \pm 0.13 & \times \\
\phantom{0}p2 & -0.09 \pm 0.002 & -0.005 \pm 0.001 \\
\phantom{0}f2 & 0.04 \pm 0.002 \text{ GeV}^{-2} & 0.03 \pm 0.001 \text{ GeV}^{-2} \\
\end{array} \]

Here we take the following values for the parameters \( f_\omega, f_\rho^T \) \(^{10}\), and \( m_\omega \): \( f_\omega = 0.195 \) GeV, \( f_\rho^T = 0.145 \) GeV, and \( m_\omega = 0.78 \) GeV.

The sum rules for the \( \phi \) and \( K^* \) coupling constants can be derived in a similar way:

\[ \begin{align*} 
g_{\Xi \phi} & = g_{\Xi \rho} f_{\Xi}^2 (\rho \to \phi, m_s \to m_q = 0, \langle \bar{s}s \rangle \to \langle \bar{q}q \rangle), \\
g_{\Xi \Xi \phi}^1 & = -g_{\Xi \Xi \rho} f_{\Xi} (\rho \to \phi, m_s \to m_q = 0, \langle \bar{s}s \rangle \to \langle \bar{q}q \rangle), \\
g_{\Xi \Xi \phi}^2 & = g_{\Xi \Xi \rho} f_{\Xi}^2 (\rho \to \phi, m_s \to m_q = 0, \langle \bar{s}s \rangle \to \langle \bar{q}q \rangle), \\
\end{align*} \]
\[
\begin{align*}
g_{\xi^0 \varphi}^{\rho^0} f_{\Omega^0}^2 &= 2 g_{\xi^0 \rho}^{\rho^0} f_{\Omega^0}^2 (\Xi^* \rightarrow \Omega^*, \rho \rightarrow \varphi), \\
g_{\xi^0 \gamma^* K^*} f_{\Omega^0} f_{\Xi^*} &= g_{\xi^0 \rho}^{\rho^0} f_{\Omega^0} f_{\Xi^*} (\Xi^* \rightarrow \Sigma^*, \rho \rightarrow K^*, m_s \rightarrow m_q = 0, (\bar{s}s) \rightarrow (\bar{q}q)), \\
g_{\xi^0 \gamma^* K^*} f_{\Omega^0} f_{\Xi^*} &= 2 g_{\xi^0 \rho}^{\rho^0} f_{\Omega^0} f_{\Xi^*} (\Xi^* \rightarrow \Omega^*, \rho \rightarrow K^*), \\
g_{\xi^0 \Lambda K^*} f_{\Omega^0} f_{\Xi^*} &= g_{\xi^0 \rho}^{\rho^0} f_{\Omega^0} f_{\Xi^*} (\Xi^* \rightarrow \Lambda, \rho \rightarrow K^*, m_s \rightarrow m_q = 0, (\bar{s}s) \rightarrow (\bar{q}q)), \\
g_{\xi^0 \Lambda K^*} f_{\Omega^0} f_{\Xi^*} &= g_{\xi^0 \rho}^{\rho^0} f_{\Omega^0} f_{\Xi^*} (\Xi^* \rightarrow \Lambda, \rho \rightarrow K^*, m_s \rightarrow m_q = 0, (\bar{s}s) \rightarrow (\bar{q}q)), \\hline
\end{align*}
\]

The input parameters of their numerical analysis are taken as \[\epsilon: f_\rho = 0.215 \text{ GeV, } f_\phi = 0.186 \text{ GeV, } m_\rho = 1.020 \text{ GeV, and } f_{K^*} = 0.220 \text{ GeV, } f_{K^*} \equiv 0.185 \text{ GeV, } m_{K^*} = 0.89 \text{ GeV.}

Also, we list the numerical results below:

\[
\begin{array}{cccc}
g_{\xi^0 \varphi} & g_{\xi^0 \rho} & g_{\xi^0 \phi} & g_{\Omega^0 \varphi} \\
p0 & 1.15 \pm 0.13 & -1.39 \pm 0.17 & 1.61 \pm 0.15 \\
p1 & 0.89 \pm 0.13 & \times & 1.31 \pm 0.15 \\
p2 & -0.008 \pm 0.001 & -0.015 \pm 0.0015 & \\
f2 & 0.025 \pm 0.001 \text{ GeV}^{-2} & 0.037 \pm 0.0015 \text{ GeV}^{-2} & \\
\end{array}
\]

\[
\begin{array}{cccc}
g_{\xi^0 \gamma^* K^*} & g_{\Omega^0 \gamma^* K^*} & g_{\xi^0 \Lambda K^*} & g_{\Omega^0 \Lambda K^*} & g_{\xi^0 \Xi K^*} & g_{\Omega^0 \Xi K^*} & g_{\Xi^0 \Xi K^*} \\
p0 & 3.11 \pm 0.16 & 4.83 \pm 0.19 & -4.63 \pm 0.70 & \\
p1 & 1.09 \pm 0.16 & 1.93 \pm 0.19 & \times & \times & \times \\
p2 & 0.015 \pm 0.003 & 0.03 \pm 0.002 & \\
f2 & 0.03 \pm 0.003 \text{ GeV}^{-2} & 0.019 \pm 0.001 \text{ GeV}^{-2} & \\
\end{array}
\]

V. CONCLUSION

We have calculated the light vector meson couplings with heavy baryons multiplets \(6_F\) and \(3_F\) in the leading order of HQET, using the LCQSR approach. Most sum rules for these coupling constants are stable with the variations of the Borel parameter and the continuum threshold. Some possible sources of the errors in our calculation come from the inherent inaccuracy of LCQSR: the omission of the higher order terms in operator product expansion, the choice of \(\omega_c\), the variation of the coupling constant with the Borel parameter \(T\) in the working region and the approximation in the light-cone distribution amplitudes of the vector meson. The uncertainty in \(f_s\)’s and \(\bar{A}_s\)’s also leads to errors. The 3-particle light-cone distribution amplitudes of the vector meson are not well-known as the 2-particle ones. This may lead to additional systematical errors in our calculation.

Recently, Belle observed a significant near-threshold enhancement called the X(4630) in the e^+e^- \rightarrow \Lambda_c \bar{\Lambda}_c exclusive cross section with initial-state radiation \[11\]. There have been extensive discussion of the X(4630), the interpretation of which includes a conventional charmonium state, or a baryon-antibaryon threshold effect, etc. The extracted coupling constants may play an important role in the study of the interaction between \(\Lambda_c\) and \(\bar{\Lambda}_c\), and therefore be helpful to clarify the nature of the X(4630). Besides, they may be useful in the study of the formation of possible molecular candidates composed of two heavy baryons, such as \(B_Q \bar{B}_Q\) or \(B_Q B_Q\).
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