Evidence for a dynamic origin of charge

W. A. Hofer

Dept. of Physics and Astronomy, University College London
Gower Street, London WC1E 6BT, E-mail: w.hofer@ucl.ac.uk

The fundamental equations of particle motion lead to a modified Poisson equation including dynamic charge. This charge derives from density oscillations of a particle; it is not discrete, but continuous. Within the dynamic model of hydrogen it accounts for all features of electron proton interactions, its origin are density oscillations of the proton. We propose a new system of electromagnetic units, based on meter, kilogram, and second, bearing on these findings. The system has none of the disadvantages of traditional three-unit systems. On the basis of our theoretical model we can genuinely derive the scaling factor between electromagnetic and mechanic variables, which is equal, within a few percent, to Planck’s constant \( h \). The implications of the results in view of unifying gravity and quantum theory are discussed. It seems that the hypothetical solar gravity waves, in the low frequency range of the electromagnetic spectrum, are open to experimental detection.

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I. THE NATURE OF CHARGE

Since the discovery of the electron by J. J. Thomson \[1\] the concept of electric charge has remained nearly unchanged. Apart from Lorentz’ extended electron \[2\], or Abraham’s electromagnetic electron \[3\], the charge of an electron remained a point like entity, in one way or another related to electron mass \[4,5\]. In atomic nuclei we think of charge as a smeared out region of space, which is structured by the elementary constituents of nuclear particles, the quarks \[6\].

The first major modification in this picture occurred only in the last decades, when experiments on the quantum hall effect \[7,8\] suggested the existence of “fractional charge” of electrons. Although this effect has later been explained on the basis of standard theory \[9\], its implications are worth a more thorough analysis. Because it cannot be excluded that the same feature, fractional or even continuous charge, will show up in other experiments, especially since experimental practice more and more focuses on the properties of single particles. And in this case the conventional picture, which is based on discrete and unchangeable charge of particles, may soon prove too narrow a frame of reference. It seems therefore justified, at this point, to analyze the very nature of charge itself. A nature, which would reveal itself as an answer to the question: What is charge?

It must be noted, in this respect, that the picture of continuous charge, in classical theories, is due to the omission of the atomic structure of matter. In any modern sense, continuous charge can only be recovered by considering dynamic processes within the very particles themselves.

With this problem in mind, we reanalyze the fundamental equations of intrinsic particle properties \[10\]. The consequences of this analysis are developed in two directions. First, we determine the interface between mechanic and electromagnetic properties of matter, where we find that only one fundamental constant describes it: Planck’s constant \( h \). And second, we compute the fields of interaction within a hydrogen atom, where we detect oscillations of the proton density of mass as their source. Finally, the implications of our results in view of unifying gravity and quantum theory are discussed and a new model of gravity waves derived, which is open to experimental tests.

II. THE ORIGIN OF DYNAMIC CHARGE

The intrinsic vector field \( \mathbf{E}(r, t) \), the momentum density \( \rho(r, t) \), and the scalar field \( \phi(r, t) \) of a particle are described by (see \[10\], Eq. (18)): \n
\[
\mathbf{E}(r, t) = -\nabla \phi(r, t) + \frac{1}{\sigma} \frac{\partial}{\partial t} \rho(r, t)
\]

Here \( \sigma \) is a dimensional constant introduced for reasons of consistency. Rewriting the equation with the help of the definitions:

\[
\beta := \frac{1}{\sigma} \quad \beta \phi(r, t) := \phi(r, t)
\]

we obtain the classical equation for the electric field, where in place of a vector potential \( \mathbf{A}(r, t) \) we have the momentum density \( \rho(r, t) \). This similarity, as already noticed, bears on the Lorentz gauge as an expression of the energy principle (\[10\] Eqs. (26) - (28)).

\[
\mathbf{E}(r, t) = -\nabla \phi(r, t) + \beta \frac{\partial}{\partial t} \rho(r, t)
\]

Note that \( \beta \) describes the interface between dynamic and electromagnetic properties of the particle. Taking the gradient of (3) and using the continuity equation for \( \rho(r, t) \):
\[ \nabla \mathbf{p}(r, t) + \frac{\partial}{\partial t} \rho(r, t) = 0 \quad (4) \]

where \( \rho(r, t) \) is the density of mass, we get the Poisson equation with an additional term. And if we include the source equation for the electric field \( \mathbf{E}(r, t) \):

\[ \nabla \mathbf{E}(r, t) = \sigma(r, t), \quad (5) \]

\( \sigma(r, t) \) being the density of charge, \( \epsilon \) set to 1 for convenience, we end up with the modified Poisson equation:

\[ \Delta \phi(r, t) = - \underbrace{\sigma(r, t)}_{\text{static charge}} - \underbrace{\beta \frac{\partial^2}{\partial t^2} \rho(r, t)}_{\text{dynamic charge}} \quad (6) \]

The first term in \( \Delta \) is the classical term in electrostatics. The second term does not have a classical analogue, its novelty is due to the fact, that no dynamic interpretation of the vector potential \( \mathbf{A}(r, t) \) exists, whereas, in the current framework, \( \mathbf{p}(r, t) \) has a dynamic meaning: that of momentum density.

To appreciate the importance of the new term, think of an aggregation of mass in a state of oscillation. In this case the second derivative of \( \rho \) is a periodic function, which is, by virtue of Eq. (5), equal to periodic charge. Then this dynamic charge gives rise to a periodic scalar field \( \phi \). This field appears as a field of charge in periodic oscillations: hence its name, dynamic charge. It should be noted that dynamic charge is essentially different from a classical dipole: in that case the field can appear zero (cancellation of opposing effects), whereas in case of dynamic charge it is zero. Even, as shall be seen presently, for monopole oscillations.

### III. OSCILLATIONS OF A PROTON

We demonstrate the implications of Eq. (5) on an easy example: the radial oscillations of a proton. The treatment is confined to monopole oscillations, although the results can easily be generalized to any multipole. Let a proton’s radius be a function of time, so that \( r_p = r_p(t) \) will be:

\[ r_p(t) = R_p + d \cdot \sin \omega_H t \quad (7) \]

Here \( R_p \) is the original radius, \( d \) the oscillation amplitude, and \( \omega_H \) its frequency. Then the volume of the proton \( V_p \) and, consequently, its density of mass \( \rho_p \) depend on time. In first order approximation we get:

\[ \rho_p(t) = \frac{3M_p}{4\pi} \left( R_p + d \sin \omega_H t \right)^{-3} \]

\[ \rho_p(t) \sim \rho_0 \left( 1 - x \sin \omega_H t \right) \quad x := \frac{3d}{R_p} \quad (8) \]

The Poisson equation for the dynamic contribution to proton charge then reads:

\[ \Delta \phi(r, t) = -\beta x \rho_0 \omega_H^2 \sin \omega_H t \quad (9) \]

Integrating over the volume of the proton we find for the dynamic charge of the oscillating proton the expression:

\[ q_D(t) = \int_{V_p} d^3r x \rho_0 \omega_H^2 \sin \omega_H t = \beta x M_p \omega_H^2 \sin \omega_H t \quad (10) \]

This charge gives rise to a periodic field within the hydrogen atom, as already analyzed in some detail and in a slightly different context [11]. We shall turn to the calculation of a hydrogen’s fields of interaction in the following sections. But in order to fully appreciate the meaning of the dynamic aspect it is necessary to digress at this point and to turn to the discussion of electromagnetic units.

### IV. NATURAL ELECTROMAGNETIC UNITS

By virtue of the Poisson equation (6) dynamic charge must be dimensionally equal to static charge, which for a proton is \( + e \). But since it is, in the current framework, to the geometry of the problem the interaction energy depends on time. In first order approximation we get:

\[ \Delta \phi(r, t) = -\beta x \rho_0 \omega_H^2 \sin \omega_H t \quad (9) \]

From \( \rho_0 \) we get, dimensionally:

\[ [\epsilon] = \beta [M_p \omega_H^2] \quad \Rightarrow \quad [\beta] = \left[ \frac{\epsilon}{M_p \omega_H^2} \right] \quad (11) \]

The unit of \( \beta \) is therefore, in SI units:

\[ [\beta] = C \cdot \frac{s^2}{kg} = C \cdot \frac{m^2}{J} \quad [SI] \quad (12) \]

We define now the natural system of electromagnetic units by setting \( \beta \) equal to 1. Thus:

\[ [\beta] := 1 \quad \Rightarrow \quad [C] = \frac{J}{m^2} \quad (13) \]

The unit of charge \( C \) is then energy per unit area of a surface. Why, it could be asked, should this definition make sense? Because, would be the answer, it is the only suitable definition, if electrostatic interactions are accomplished by photons.

Suppose a \( \delta^3(r - r') \) like region around \( r' \) is the origin of photons interacting with another \( \delta^3(r - r'') \) like region around \( r'' \). Then \( r' \) is the location of charge. Due to the geometry of the problem the interaction energy will decrease with the square of \( |r' - r''| \). What remains constant, and thus characterizes the charge at \( r' \), is only the interaction energy per surface unit. Thus the definition, which applies to all \( r^{-2} \) like interactions, also, in principle, to gravity.

Returning to the question of natural units, we find that all the other electromagnetic units follow straightforward from the fundamental equations [11]. They are displayed in Table I.
If we analyze the units in Lorentz’ force equation, we observe, at first glance, an inconsistency.

\[ \mathbf{F}_L = q \left( \mathbf{E} + \mathbf{u} \times \mathbf{B} \right) \] (14)

The unit on the left, Newton, is not equal to the unit on the right. As a first step to solve the problem we include the dielectric constant \( \epsilon^{-1} \) in the equation, since this is the conventional definition of the electric field \( \mathbf{E} \).

Then we have:

\[ [\mathbf{F}_L] = \frac{Nm}{m^2} \left( \frac{m^4 N}{N m^3} + \frac{m N s}{s m^4} \right) = N + N \cdot \frac{N}{m^4} \] (15)

Interestingly, now the second term, which describes the magnetic forces, is wrong in the same manner, the first term was before we included the dielectric units. It seems thus, that the dimensional problem can be solved by a constant \( \eta \), which is dimensionally equal to \( \epsilon \), and by rewriting the force equation (14) in the following manner:

\[ \mathbf{F}_L = \frac{q}{\eta} \left( \mathbf{E} + \mathbf{u} \times \mathbf{B} \right) \] (16)

\[ [\eta] = Nm^{-4} = Cm^{-3} = \left[ \sigma \right] \] (17)

The modification of (14) has an implicit meaning, which is worth being emphasized. It is common knowledge in special relativity, that electric and magnetic fields are only different aspects of a situation. They are part of a common field tensor \( F_{\mu\nu} \) and transform into each other by Lorentz transformations. From this point of view the treatment of electric and magnetic fields in the SI, where we end up with two different constants (\( \epsilon, \mu \)), seems to go against the requirement of simplicity. On the other hand, the approach in quantum field theory, where one employs in general only a dimensionless constant at the interface to electrodynamics, the fine structure constant \( \alpha \), is over the mark. Because the information, whether we deal with the electromagnetic or the mechanic aspect of a situation, is lost. The natural system, although not completely free of difficulties, as seen further down, seems a suitable compromise. Different aspects of the intrinsic properties, and which are generally electromagnetic, are not distinguished, no scaling is necessary between \( \mathbf{p}, \mathbf{E} \) and \( \mathbf{B} \). The only constant necessary is at the interface to mechanic properties, which is \( \eta \). This also holds for the fields of radiation, which we can describe by:

\[ \phi_{\text{Rad}}(r, t) = \frac{1}{8\pi \eta} \left( \mathbf{E}^2 + c^2 \mathbf{B}^2 \right) \] (18)

Note that in the natural system the usage, or the omission, of \( \eta \) ultimately determines, whether a variable is to be interpreted as an electromagnetic or a mechanic property. Forces and energies are mechanic, whereas momentum density is not. The numerical value of \( \eta \) has to be determined by explicit calculations. This will be done in the next sections. We conclude this section by comparing the natural system of electromagnetic units to existing systems.

From an analysis of the Maxwell equations one finds three dimensional constants \( k_1, k_2, k_3 \), and a dimensionless one, \( \alpha \), which acquire different values in different systems (see Table 4).

Judging by the number of dimensional constants it seems that the natural system is most similar to the electrostatic system of units. However, since we have defined a separate interface to mechanic properties, it is free of the usual nuisance of fractional exponents without a clear physical meaning. The other difference is that \( c \), in the esu, is a constant, whereas it only signifies the velocity of a particle in the natural system. For photons this velocity equals \( c \), but for electrons it is generally much smaller. We note in passing that all the fundamental relations for the intrinsic fields remain valid. Also the conventional relations for the forces of interaction and the radiation energy remain functionally the same. Only the numerical values will be different.

Comparing with existing systems we note three distinct advantages: (i) The system reflects the dynamic origin of fields, and it is based on only three fundamental units: \( m, \text{kg}, \text{s} \). A separate definition of the current is therefore obsolete. (ii) There is a clear cut interface between mechanics (forces, energies), and electrodynamics (fields of motion). (iii) The system provides a common framework for macroscopic and microscopic processes.

V. INTERACTIONS IN HYDROGEN

Returning to proton oscillations let us first restate the main differences between a free electron and an electron in a hydrogen atom [11]: (i) The frequency of the hydrogen system is constant \( \omega_p \), as is the frequency of the electron wave. It is thought to arise from the oscillation properties of a proton. (ii) Due to this feature the wave equation of momentum density \( \mathbf{p}(r, t) \) is not homogeneous, but inhomogeneous:

\[ \Delta \mathbf{p}(r, t) - \frac{1}{u^2} \frac{\partial^2}{\partial t^2} \mathbf{p}(r, t) = f(t) \delta^3(\mathbf{r}) \] (19)

for a proton at \( r = 0 \) of the coordinate system. The source term is related to nuclear oscillations. We do not solve (19) directly, but use the energy principle to simplify the problem. From a free electron it is known that the total intrinsic energy density, the sum of a kinetic component \( \phi_K \) and a field component \( \phi_{EM} \) is a constant of motion [10]:

\[ \phi_K(\mathbf{r}) + \phi_{EM}(\mathbf{r}) = \rho_0 u^2 \] (20)

where \( u \) is the velocity of the electron and \( \rho_0 \) its density amplitude. We adopt this notion of energy conservation also for the hydrogen electron, we only modify it to account for the spherical setup:
\[
\phi_K(r) + \phi_{EM}(r) = \frac{\rho_0}{r^2} u^2
\]  

(21)

The radial velocity of the electron has discrete levels. Due to the boundary values problem at the atomic radius, it depends on the principal quantum number \( n \). From the treatment of hydrogen we recall for \( u_n \) and \( \rho_0 \) the results:

\[
u_n = \frac{\omega_H R_H}{2\pi n} \quad \rho_0 = \frac{M_e}{2\pi R_H}
\]

(22)

where \( R_H \) is the radius of the hydrogen atom and \( M_e \) the mass of an electron. Since \( \rho_0 \) includes the kinetic as well as the field components of electron "mass", e.g. in Eq. (21), we can define a momentum density as well as the field components of electron "mass", e.g. the momentum density

\[
u_n(t) = u_n \cos \omega_H t \mathbf{e}^r
\]

(23)

the momentum density \( \mathbf{p}_0(r, t) \) is given by:

\[
\mathbf{p}_0(r, t) = \frac{\rho_0 u_n}{r^2} \cos \omega_H t \mathbf{e}^r
\]

(24)

The combination of kinetic and field components in the variables has a physical background: it bears on the result that photons change both components of an electron wave [10]. With these definitions we can use the relation between the electric field and the change of momentum, although now this equation refers to both components:

\[
\mathbf{E}_0(r, t) = \frac{\partial}{\partial t} \mathbf{p}_0(r, t) = -\frac{\rho_0 u_n}{r^2} \omega_H \sin \omega_H t \mathbf{e}^r
\]

(25)

Note that charge, by definition, is included in the electric field itself. Integrating the dynamic charge of a proton from Eq. (21) and accounting for flow conservation in our spherical setup, the field of a proton will be:

\[
\mathbf{E}_0(r, t) = \frac{q D}{r^2} = \frac{M_p \omega_H^2}{r^2} x \sin \omega_H t \mathbf{e}^r
\]

(26)

Atm from a phase factor the two expressions must be equal. Recalling the values of \( u_n \) and \( \rho_0 \) from (22), the amplitude \( x \) of proton oscillation can be computed. We obtain:

\[
x = \frac{3d}{R_p} = \frac{M_e}{(2\pi)^2 M_p} \frac{1}{n}
\]

(27)

In the highest state of excitation, which for the dynamic model is \( n = 1 \), the amplitude is less than \( 10^{-5} \) times the proton radius: Oscillations are therefore comparatively small. This result indicates that the scale of energies within the proton is much higher than within the electron, say. The result is therefore well in keeping with existing nuclear models. For higher \( n \), and thus lower excitation energy, the amplitude becomes smaller and vanishes for \( n \rightarrow \infty \).

It is helpful to consider the different energy components within the hydrogen atom at a single state, say \( n = 1 \), to understand, how the electron is actually bound to the proton. The energy of the electron consists of two components:

\[
\phi_K(r, t) = \frac{\rho_0 u_1^2}{r^2} \sin^2 k_1 r \cos^2 \omega_H t
\]

(28)

is the kinetic component of electron energy (\( k_1 \) is now the wavevector of the wave). As in the free case, the kinetic component is accompanied by an intrinsic field, which accounts for the energy principle (i.e. the requirement, that total energy density at a given point is a constant of motion). Thus:

\[
\phi_{EM}(r, t) = \frac{\rho_0 u_1^2}{r^2} \cos^2 k_1 r \cos^2 \omega_H t
\]

(29)

is the field component. The two components together make up for the energy of the electron. Integrating over the volume of the atom and a single period \( \tau \) of the oscillation, we obtain:

\[
W_{el} = \frac{1}{\tau} \int_0^\tau dt \int_{V_H} d^3r (\phi_K(r, t) + \phi_{EM}(r, t))
\]

\[
= \frac{1}{2} M_e u_1^2
\]

(30)

This is the energy of the electron in the hydrogen atom. \( W_{el} \) is equal to 13.6 eV. The binding energy of the electron is the energy difference between a free electron of velocity \( u_1 \) and an electron in a hydrogen atom at the same velocity. Since the energy of the free electron \( W_{free} \) is:

\[
W_{free} = \hbar \omega_H = M_e u_1^2
\]

(31)

the energy difference \( \Delta W \) or the binding energy comes to:

\[
\Delta W = W_{free} - W_{el} = \frac{1}{2} M_e u_1^2
\]

(32)

This value is also equal to 13.6 eV. It is, furthermore, the energy contained in the photon field \( \phi_{Rad}(r, t) \) of the proton’s radiation

\[
W_{Rad} = \Delta W = \frac{1}{\tau} \int_0^\tau dt \int_{V_H} d^3r \phi_{Rad}(r, t)
\]

\[
= \frac{1}{2} M_e u_1^2
\]

(33)

This energy has to be gained by the electron in order to be freed from its bond, it is the ionization energy of hydrogen. However, in the dynamic picture the electron is not thought to move as a point particle in the static field of a central proton charge, the electron is, in this model, a dynamic and oscillating structure, which emits and absorbs energy constantly via the photon field of the central proton. In a very limited sense, the picture is still a statistical one, since the computation of energies involves the average over a full period.
VI. THE MEANING OF \( \eta \)

The last problem, we have to solve, is the determination of \( \eta \), the coupling constant between electromagnetic and mechanic variables. To this end we compute the energy of the radiation field \( W_{\text{Rad}} \), using Eqs. (18), (26), and (27). From (18) and (26) we obtain:

\[
\phi_{\text{Rad}}(r,t) = \frac{1}{8\pi\eta}E^2 = \frac{1}{8\pi\eta} \frac{M_p\omega_H^4}{r^4} x^2 \sin^2 \omega_H t \quad (34)
\]

Integrating over one period and the volume of the atom this gives:

\[
W_{\text{Rad}} = \frac{1}{\tau} \int_0^\tau dt \int_{R_p}^{R_H} 4\pi r^2 dr \phi_{\text{Rad}}(r,t)
\approx -\frac{1}{4\eta} \cdot \frac{M_p\omega_H^4 x^2}{R_p} \quad (35)
\]

provided \( R_p \), the radius of the proton is much smaller than the radius of the atom. With the help of (27), and remembering that \( W_{\text{Rad}} \) for \( n = 1 \) equals half the electron’s free energy \( \hbar \omega_H \), this finally leads to:

\[
W_{\text{Rad}} = \frac{1}{4\eta} \cdot \frac{M_p^2\omega_H^4 x^2}{R_p} = \frac{1}{2} \hbar \omega_H \quad (36)
\]

\[
\eta = \frac{M_p^2\nu_H^3}{2\hbar R_p} = \frac{1.78 \times 10^{20}}{R_p} \quad (37)
\]

since the frequency \( \nu_H \) of the hydrogen atom equals \( 6.57 \times 10^{15} \) Hz. Then \( \eta \) can be calculated in terms of the proton radius \( R_p \). This radius has to be inferred from experimental data, the currently most likely parametrization being [13]:

\[
\frac{\rho_p(r)}{\rho_{p,0}} = \frac{1}{1 + e^{(r-1.07)/0.55}} \quad (38)
\]

radii in fm. If the radius of a proton is defined as the radius, where the density \( \rho_{p,0} \) has decreased to \( \rho_{p,0}/e \), with e the Euler number, then the value is between 1.3 and 1.4 fm. Computing \( 4\pi \) the inverse of \( \eta \), we get, numerically:

\[
\frac{4\pi}{\eta} = 0.92 \times 10^{-34} \quad (R_p = 1.3 fm)
= 0.99 \times 10^{-34} \quad (R_p = 1.4 fm) \quad (39)
= 1.06 \times 10^{-34} \quad (R_p = 1.5 fm)
\]

Numerically, this value is equal to the numerical value of Planck’s constant \( \hbar \) [2]:

\[
\hbar_{\text{UIP}} = 1.0546 \times 10^{-34} \quad (40)
\]

Given the conceptual difference in computing the radius the agreement seems remarkable. Note that this is a genuine derivation of \( \hbar \), because nuclear forces and radii fall completely outside the scope of the theory in its present form. If measurements of \( R_p \) were any different, then we would be faced, at this point, with a meaningless numerical value. Reversing the argument it can be said, that the correct value - or rather the meaningful value - is a strong argument for the correctness of our theoretical assumptions. Since these assumptions involve to a greater or lesser extent the whole theory of matter waves developed so far, we devote the rest of this section mainly to a critical analysis of this result and shall show the most striking physical implications only at the end.

Starting with the approximations involved, we note (i) a first order approximation in \( d \), and (ii) an approximation in the integration. Since \( d \approx 10^{-5} R_p \), and \( R_p \approx 10^{-5} R_H \), both errors are negligible. In view of the standard experimental error margins, also the deviation of a few percent, depending on how we define the proton radius, seems acceptable. On the positive side, there are two plausibility arguments, indicating that we deal not only with a numerical coincidence: (i) The Planck constant describes the interface between frequency and energy in all fundamental experiments. Since we started with a frequency (= proton oscillations), and calculated an energy, it must have, at some point, entered the calculation. The only unknown quantity in the calculation was \( \eta \); therefore it should contain \( \hbar \). (ii) What we have in fact developed with this model of hydrogen, is in spirit very close to the harmonic oscillator in quantum theory: the rest energy term is related to the energy of our photon field. In order to be compatible with quantum theory, the energy must contain \( \hbar \). Again, the only variable, which could contain it, is \( \eta \).

It can also be asked, why electromagnetic variables are multiplied by \( \hbar \) to give the energy of radiation. Especially, since the finestructure constant contains a division by \( \hbar \):

\[
\alpha = \frac{e^2}{\hbar c} \quad (41)
\]

To answer this question, consider a variable in electrodynamics \( A_{\text{ED}} \) and its correlating variable \( A_M \) in mechanics. Then the transition from \( A_{\text{ED}} \) to \( A_M \) is described by a transformation \( T \), so that:

\[
A_M = T_{M,ED}A_{ED} \quad (42)
\]

Since the inverse transformation must exist and the variables are assumed to be unique, the transformation is unitary:

\[
T_{M,ED}T_{ED,M} = T_{M,ED}^{-1} = 1 \quad (43)
\]

In our case the primary variables are the electromagnetic ones: \( p, E, B \). And the transformation involves a multiplication by \( \hbar \):

\[
A_M = \hbar A_{ED} \quad (44)
\]
The fundamental units m, kg, s are, in this system, the natural system, tied to the electromagnetic variables. In quantum theory, on the other hand, the fundamental variables are Newtonian. Then the transformation between electromagnetic and mechanic variables involves the inverse transformation

\[ A_M(QM) = \hbar^{-1} A_{ED}(QM) \]  

(45)

If we consider, in addition, that charge has been included in the definition of \( E \), the transformation, in conventional units and in quantum theory should read:

\[ A_M(QM) = \frac{e^2}{\hbar} A_{ED}(QM) \]  

(46)

which is the finesture constant multiplied by \( c \). And \( c \) is, generally, only a matter of convention. Therefore we think, the conclusion, that \( 4\pi/\eta \) really is \( \hbar \), is a reasonable and safe one. But in this case Planck’s constant has not much bearing on a different scale of measurement, as is often invoked, when there is talk about a “basic scale”. This scale is determined by the inverse transformation.

\[ \Phi = \frac{\hbar}{2} \left( \frac{E}{4\pi} \right)^2 + \frac{c^2}{4\pi} \left( \frac{B}{4\pi} \right)^2 \]  

(50)

Every calculation of mechanic properties involves a multiplication by \( \hbar \). Since \( \hbar \) is a scaling constant, the term “quantization”, commonly used in this context, is misleading. Furthermore, it is completely irrelevant, whether we compute an integral property (the force in \( (48) \)), or a density \( (\Phi_{Rad} \text{ in } (50)) \), a force density can also be obtained by replacing charge \( q \) by a density value. From the interaction fields within a hydrogen atom, e.g. Eq. (52), it is clear that the field varies locally and temporarily and can reach any value between zero and its maximum. Although it is described by:

\[ \Phi_{Rad}(r,t) = \frac{\hbar M^2 \omega^4}{r^4} x^2 \sin^2 \omega t \]  

(51)

it is not "quantized". Neither would be the forces based on the field \( E_0 \), or the angular momenta. Although, in both cases, they are proportional to \( \hbar \). What is, in a sense, discontinuous, is the mass contained in the shell of the atom. But this mass depends, as does the amplitude of \( \Phi_{Rad}(r,t) \), on the mass of the atomic nucleus. So the only discontinuity left on the fundamental level, is the mass of atomic nuclei. That the energy spectrum of atoms is discrete, is a trivial observation in view of boundary conditions and finite radii. To sum up the argument: There are no quantum jumps.

All our calculations so far focus on single atoms. To get the values of mechanic variables in SI units used in macrophysics, we have to include the scaling between the atomic domain and the domain of everyday measurements. Without proof, we assume this value to be \( N_A \), Avogadro’s number. The scale can be made plausible from solid state physics, where statistics on the properties of single electrons generally involve a number of \( N_A \) particles in a volume of unit dimensions \([1\, \text{cm}^3]\). And a dimensionless constant does not show up in any dimensional analysis.

VII. SOLAR GRAVITY FIELDS

We conclude this paper, which seems to open a new perspective on a number of very fundamental problems, by a brief discussion. The first issue concerns the nature of gravity. From the given treatment it is possible to conclude, that there is maybe no fundamental difference between electrostatic and gravitational interaction. Both seem to be transmitted with the velocity of light, both obey a \( r^{-2} \) law, both are related to the existence of mass, whether its static or its dynamic features. So the conjecture, that also gravity is transmitted by a "photon", has it least some basis. But here the similarities end. Because of the vast differences in the coupling \( \varepsilon_G \approx 10^{-22} \) one must assume a very different frequency scale. From Eq. (26):

\[ |E| = \frac{qD}{r^2} \approx \frac{M \omega^2}{r^2} \]  

(52)

it can be inferred that the frequency scale for gravity and electrostatics would differ by about \( 10^{-11} \).

\[ \omega_G \approx 10^{-11} \omega_E \]  

(53)

Here \( \omega_E \) is the characteristic electromagnetic frequency, \( \omega_G \) its gravitational counterpart. The hypothesis can in principle be tested. If we assume that \( \nu_G \), the frequency of gravity radiation, is about \( 10^{-11} \) times the frequency of proton oscillation, we get:

\[ \nu_G \approx 1 - 100 \quad kHz \]  

(54)

If therefore electromagnetic fields of this frequency range exist in space, we would attribute these fields to
solar gravity. To estimate the intensity of these, hypothetical, waves, we use Eq. (55):

$$G_S(r, t) = \frac{\partial}{\partial t} P_E(r, t)$$

Here $G_S$ is the solar gravity field. The momentum density and its derivative can be inferred from centrifugal acceleration.

$$\rho_E = \frac{3M_E}{4\pi R_E^3} \quad a_C = \omega_E^2 R_O$$

where $R_O$ is the earth’s orbital radius and where we have assumed isotropic distribution of terrestrial mass. Then Eq. (54) leads to:

$$\phi_C(r = R_O) = \frac{\hbar}{2} \left( \frac{G_S}{4\pi} \right)^2 = \frac{\hbar}{2} \left( \frac{3M_E R_O}{4R_E^2} \right)^2$$

Note the occurrence of Planck’s constant also in this equation, although all masses and distances are astronomical. The intensity of the field, if calculated from (57), is very small. To give it in common measures, we compute the flow of gravitational energy through a surface element at the earth’s position. In SI units we get:

$$J_G(R_O) = \phi_G(R_O) \cdot N_A \cdot c \approx \frac{70mW}{m^2}$$

Compared to radiation in the near visible range - the solar radiation amounts to over 300 Watt/m^2 [10] - the value seems rather small. But considering, that also radiation in the visible range could have an impact on terrestrial motion, the intensity of the gravity waves could be, in fact, much higher.

**VIII. IS THERE STATIC CHARGE?**

In the conventional models a particle’s charge is not only discrete, but has also a defined sign. Although anti-particles are thought to exist, the charge of protons is positive, the charge of electrons negative. Dynamic charge is neither discrete, nor does it possess a defined sign. Depending on the exact moment, the charge of a proton:

$$q_p = M_p \omega_H^2 x \sin \omega_H t$$

either has a positive or a negative value, which determines the direction of the energy flow within the hydrogen atom. The difference between electrons and protons in this model is mainly due to their density of mass.

Related to this feature is a shift of focus within the dynamic model of atoms. Although the states of the atom are described by quantum numbers (n for the principal state, l and m if multipoles are included), these numbers refer primarily to nuclear states of oscillation. States of the atom’s electron are merely a reaction to them. Therefore the properties of an atom, in the dynamic model, refer to properties of the atomic nucleus. How this model bears on chemical properties, remains to be seen.

The last issue is a consequence of our treatment of the hydrogen atom. In this case the main features, the energy spectrum as well as ionization energy and the energy of emitted photons can be explained from dynamic charge alone. There is, in contrast to the conventional treatment, no necessity to invoke static potentials. It will also have been noted that in natural units and based on dynamic processes interactions are generally free of any notion of ”charge” in its proper sense. So does that mean, it could be asked, that there is no charge? Based on the current evidence and considering the situation in high energy physics, this is definitely too bold a statement. Considering, though, that the notion of a fixed ”elementary charge” lies at the heart of all current accounts of these experiments, the degree of theoretical freedom in the dynamic picture is incomparably higher. So that still, after a few years of development, we might end up with a tentative answer: maybe not. And in this case, the question about the true nature of charge will have been answered. It is dynamic in nature, we would then say.

**IX. CONCLUSIONS**

In this paper we presented evidence for the existence of a dynamic component of charge. It derives, as shown, from the variation of a particle’s density of mass. A new system of electromagnetic units, the natural system, has been developed, which bears on these dynamic sources. We have given a fully deterministic treatment of hydrogen, where we used our theoretical model to determine the fundamental scaling constant between electromagnetic and mechanic variables. The constant, we found, is $\hbar$, Planck’s constant. The constant thus has no bearing on any length scale, as frequently thought. And finally we have discussed these results in view of unifying gravity and quantum theory. The intensity of the postulated solar gravity waves seems sufficiently high, so that these waves, in the low frequency range of the electromagnetic spectrum, can in principle be detected.

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[1] J. J. Thomson, Phil. Mag. 44 (1897) 293
[2] H. A. Lorentz, Proc. R. Acad. Sci. Amsterdam 6 (1904) 809
[3] M. Abraham, Ann. Physik 10 (1903) 105
[4] D. Bender et al., Phys. Rev D 30 (1984) 515
[5] The Theory of the Electron, edited by J. Keller and Z. Oziewicz, UNAM Mexico (1997)
[6] L. Montanet et al., Phys. Rev. D 50 (1994) 1173
[7] D. C. Tsui, H. L. Störmer, and A. C. Grossard, Phys. Rev. Lett. 48 (1982) 1559
[8] The Quantum Hall effect, edited by R. E. Prange and S. H. Girvin, Springer-Verlag New York (1990)
[9] R. B. Laughlin, Phys. Rev. Lett. 50 (1983) 1395
[10] W. A. Hofer, Physica A 256 (1998) 178
[11] W. A. Hofer, A dynamic model of atoms: structure, internal interactions and photon emissions of hydrogen, preprint at quant-ph/9801044
[12] J. D. Jackson, Classical Electrodynamics, 3rd edition, Wiley & Sons, New York (1999) 779
[13] R. Eisberg and R. Resnick, Quantum Physics, 2nd edition, Wiley & Sons, New York (1985) 517
[14] UIPAP document UIP 20. Physica A 93 (1978) 1
[15] N. W. Ashcroft and N. D. Mermin, Solid State Physics, Holt-Saunders, Philadeslphia (1974) 4
[16] D. Morrison and T. Owen, The Planetary System, 2nd edition, Addison Wesley, New York (1996) 243

| Quantity         | Symbol | Units             |
|------------------|--------|-------------------|
| Charge           | $C_{SI}$ | $Jm^{-2}$        |
| Ampere           | $A_{SI}$ | $Jm^{-2}s^{-1}$  |
| Current density  | $J$    | $Jm^{-2}s^{-1}m^{-2}$ |
| Electric field   | $E$    | $Nm^{-3}$         |
| Magnetic field   | $B$    | $Nsm^{-1}m^{-3}$  |
| Scalar potential | $\phi$ | $Jm^{-3}$         |
| Dielectric constant | $\epsilon$ | $Nm^{-4} = Cm^{-3}$ |
| Magnetic permeability | $\mu$ | $(Cm^{-3})^{-1}m^2s^{-2}$ |

TABLE I. Electromagnetic quantities in natural units

| System of units | $k_1$  | $k_2$  | $k_3$  | $\alpha$ |
|-----------------|--------|--------|--------|----------|
| Electrostatic (esu) | 1      | $c^{-2}$ | 1      | 1        |
| Electromagnetic  | $c^2$  | 1      | 1      | 1        |
| Gaussian         | 1      | $c^{-2}$ | $c$   | $c^{-1}$ |
| Heavyside-Lorentz| $1/4\pi$ | $1/4\pi c^2$ | $c$   | $c^{-1}$ |
| SI               | $1/4\pi\epsilon$ | $\mu/4\pi$ | 1      | 1        |
| Natural system   | $1/4\pi$ | $c^{-2}$ | 1      | $1/4\pi$ |

TABLE II. Magnitude and dimension of the electromagnetic constants. Note that we have taken the constants as they appear in the Maxwell equations (Eq. (A8) of [12]).