Cavitation from bulk viscosity in neutron stars and quark stars

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Abstract
The bulk viscosity in quark matter is sufficiently high to reduce the effective pressure below the corresponding vapor pressure during density perturbations in neutron stars and strange stars. This leads to mechanical instability where the quark matter breaks apart into fragments comparable to cavitation scenarios discussed for ultra-relativistic heavy-ion collisions. Similar phenomena may take place in kaon-condensed stellar cores. Possible applications to compact star phenomenology include a new mechanism for damping oscillations and instabilities, triggering of phase transitions, changes in gravitational wave signatures of binary star inspiral, and astrophysical formation of stranelets. At a more fundamental level it points to the possible inadequacy of a hydrodynamical treatment of these processes in compact stars.

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It is well-known that the bulk viscosity in dense nuclear matter containing hyperons as well as in quark matter can be very high and therefore may play an important role in the damping of compact star oscillations and instabilities [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. But so far it has not been realized that bulk viscosity may in fact be so high that a hydrodynamical treatment of the oscillations and instabilities becomes unphysical because the effective pressure can become lower than the corresponding vapor pressure and cause the fluid to break up into fragments. This may introduce interesting new phenomenology and at least call for a re-evaluation of various processes in compact stars. The effect is the high chemical potential, low temperature analogy of the recent demonstration at low temperature [17, 18, 19], the bulk viscosity calculated from lattice QCD at the high temperatures and low chemical potentials relevant for heavy-ion collisions is at the borderline of being large enough to drive the effective pressure negative on the short (strong interaction) time-scales involved [17, 18, 19, 20] (for 1-dimensional boost expansion $\Theta = 1/t$, where $t$ is the proper time). The same lattice calculations are relevant for the cosmological quark-hadron transition. Here $\Theta = 3H$ where $H$ is the Hubble-parameter. For $H = 1/2t$ and $t \approx 10^{-5}$ s the effects of the bulk viscosity in the cosmological quark-hadron transition are negligible.

But there is another astrophysical setting with a much higher bulk viscosity where mechanical instability could be relevant, namely neutron stars or quark stars. For example $P_{\text{eff}} < 0$ occurs for a pressure of $10^{35}$ dyn/cm$^2$, a bulk viscosity of $10^{31}$ g/cm s, and an expansion scalar $\Theta > 10^4$ s$^{-1}$, numbers which are realistic in a compact star setting. In the following the conditions necessary for cavitation are derived, the relevant adiabatic indices for quark matter in compact stars are calculated, and the parameter ranges where $P_{\text{eff}} < P_{\text{vap}}$ are presented. Mechanical instability turns out not to occur for nuclear matter, but it can play an important role in compact stars containing kaon condensation or quark matter. This may have interesting consequences for compact star phenomenology.

The conditions necessary for mechanical instability in dense matter are best described in terms of adiabatic indices, defined as $\Gamma \equiv d\ln P/d\ln \rho$, where $P$ is the pressure and $\rho$ is the baryon density. As demonstrated in [7] the bulk viscosity in a dense, relativistic fluid can be ex-
pressed as
\[ \zeta = \frac{P(\Gamma_{Fr} - \Gamma_{Eq})\tau}{1 + (\omega\tau)^2}, \]  
where \( \omega \) is the frequency of an oscillation, \( \tau \) is the equilibration time-scale for the relevant microscopic processes, and \( \Gamma_{Fr} \) and \( \Gamma_{Eq} \) are the adiabatic indices for frozen chemical composition and equilibrium composition respectively. The bulk viscosity measures the impact of deviation from equilibrium via the competition between the time-scale characterizing the microscopic response, \( \tau \), and the variation in fluid density with frequency \( \omega \) and it enters the relativistic hydrodynamical equations in the combination
\[ P_{\text{eff}} = P - \zeta \Theta = P \left( 1 - \frac{(\Gamma_{Fr} - \Gamma_{Eq})\omega\tau \Theta}{(1 + (\omega\tau)^2)} \right). \]  
This equation demonstrates that the effect of bulk viscosity is largest for \( \omega\tau \approx 1 \), and it shows that the condition for cavitation, \( P_{\text{eff}} < P_{\text{vap}} \), is
\[ \frac{(\Gamma_{Fr} - \Gamma_{Eq})\omega\tau \Theta}{(1 + (\omega\tau)^2)} > 1 - \frac{P_{\text{vap}}}{P}. \]  
In particular a necessary condition for cavitation is
\[ \Delta \equiv (\Gamma_{Fr} - \Gamma_{Eq}) > \frac{2\omega}{\Theta} \left( 1 - \frac{P_{\text{vap}}}{P} \right) \equiv \Delta_C, \]  
for which \( P_{\text{eff}} < P_{\text{vap}} \) in the following range,
\[ \frac{\Delta - \sqrt{\Delta^2 - \Delta_C^2}}{\Delta_C} < \omega\tau < \frac{\Delta + \sqrt{\Delta^2 - \Delta_C^2}}{\Delta_C}. \]  
For a sinusoidal volume oscillation with amplitude \( A \) \((V = V_0(1 + A \sin(\omega t)))\) and non-relativistic flow velocities, \( \Theta = A \omega \cos(\omega t)/(1 + A \sin(\omega t)) \), so for not too large amplitude \( \Delta_C \approx 2(1 - P_{\text{vap}}/P)/A \cos(\omega t) \). This shows that cavitation is most likely near a phase boundary (\( P \rightarrow P_{\text{vap}} \)), that it is facilitated by a large amplitude, and that the effect of bulk viscosity is most important when \( \cos(\omega t) \rightarrow 1 \), i.e. when the expansion rate is maximal, whereas the impact on phase-transformation from the change in thermodynamic pressure is most important for \( \sin(\omega t) \rightarrow -1 \).

The bulk viscosity of nuclear matter in neutron stars with and without hyperons has been presented for various equations of state in the literature [4, 8, 11, 13]. Published numbers are often close to the interesting regime for mechanical instability, but the necessary condition Eq. 5 may be hard to fulfill unless the amplitude is very large. For example the values of \( \Gamma_{Fr} \) and \( \Gamma_{Eq} \) published in Ref. [22] differ at most by 0.8. The highest published value for \( \Delta \) (for neutron stars with kaon condensates [11]) is around 5. For equations of state that involve phase transitions, such as neutron stars with a kaon condensed core [22], the relevant vapor pressure for the core is the pressure of the low-density phase at the core boundary, so \( \Delta_C \rightarrow 0 \) near the boundary, which means that there will be an outer layer in the core where cavitation triggered by bulk viscosity can take place for a range of \( \omega\tau \) in addition to phase transformation caused by the oscillation in the thermodynamic pressure.

Bulk viscosities in dense quark matter are known to be even higher than in nuclear matter for some ranges of parameters [11, 12, 13, 14, 16], and therefore the most likely setting for cavitation is quark matter in strange stars if quark matter is absolutely stable or in the inner parts of neutron stars if quark matter becomes stable only at high pressure. As demonstrated below, the necessary condition for mechanical instability is indeed fulfilled in the outer tens of meters of strange stars, and for other parameters where strange matter is not absolutely stable cavitation can occur in quark matter cores deeper inside neutron stars.

Quark matter will be treated within the MIT bag model where the pressure is given by
\[ P = -B + P_u(\mu_u) + P_d(\mu_d) + P_s(\mu_s) + P_e(\mu_e), \]  
where \( B \) is the bag constant to be thought of as the energy density of the confining vacuum, and \( P_i(\mu_i) \) are the Fermi-gas contributions from the constituent particles, which are up, down, and strange quarks as well as electrons [23, 24, 25, 26]. The star is assumed transparent to neutrinos. The temperature is set to zero for calculations of adiabatic indices (but not for calculations of \( \tau \)), which is an excellent approximation after the first few seconds in the life of a proto-neutron star. The pressure contributions are (for massless up and down quarks and electrons, and strange quarks with mass \( m \) and Fermi momentum \( k_s \equiv (\mu_s^2 - m^2)^{1/2} \))
\[ P_u(\mu_u) = \frac{\mu_u^4}{4\pi^2}, \quad P_d(\mu_d) = \frac{\mu_d^4}{4\pi^2}, \quad P_e(\mu_e) = \frac{\mu_e^4}{12\pi^2}, \]
\[ P_s(\mu_s) = \frac{1}{4\pi^2} \left[ k_s^2 \left( \mu_s^2 - \frac{5}{2}m^2 \right) + \frac{3}{2}m^4 \ln \left( \frac{\mu_s + k_s}{m} \right) \right], \]  
and the corresponding number densities are \( n_i(\mu_i) = \partial P/\partial \mu_i \).

The total baryon density \( n_B = [n_u(\mu_u) + n_d(\mu_d) + n_s(\mu_s)]/3 \), and local charge neutrality (which is obeyed under most conditions except in a mixed quark-hadron phase) gives the constraint
\[ \frac{2}{3}n_u(\mu_u) - \frac{1}{3}n_d(\mu_d) - \frac{1}{3}n_s(\mu_s) - n_e(\mu_e) = 0. \]  
If reactions are very slow compared to the time-scale for density change, the composition is effectively frozen, and the highest possible value of the adiabatic index is obtained. To calculate \( \Gamma_{Fr} = d \ln P / d \ln n_B |_{\text{Freeze}} = \)
(n_B/P)dP/dn_B|_{\text{freeze}} one notes that
\[
\frac{dP}{dn_B|_{\text{constraints}}} = \sum_{i=u,d,s,e} \frac{\partial P}{\partial n_i} \frac{\partial n_i}{\partial n_B|_{\text{constraints}}} \equiv \sum_{i=u,d,s,e} n_i \frac{\partial P}{\partial n_i} \frac{\partial n_i}{\partial n_B|_{\text{constraints}}}. \tag{9}
\]

Fixed composition means that the individual densities vary in proportion to the baryon density, leaving \(x_i \equiv n_i/n_B\) constant, so for example \(\partial \mu_u/\partial n_B = \partial (\pi^2 x_u n_B)^{1/3}/\partial n_B = \mu_u/3n_B\), and \(\partial \mu_s/\partial n_B = k_s^2/3\mu_s n_B\). Collecting terms this gives
\[
\Gamma_{\text{Fr}} = 4\left[P + B + (n_u k_u^2/4\mu_u - P_u)\right]/3P, \tag{11}
\]

reproducing the well-known limit for massless quarks, \(\Gamma = 4(P + B)/3P\), which approaches the equally well-known value \(\Gamma = 4/3\) for a gas of extremely relativistic particles in the limit where \(P \gg B\).

If all particle reactions are very fast compared to the dynamical time-scale for density change, the composition has time to equilibrate, and the adiabatic index will take its minimum value, \(\Gamma_{\text{Eq}}\). The values of \(x_i\) will no longer be constant. Instead chemical equilibrium conditions are determined by the possible processes, which are
\[
s + u \leftrightarrow u + d \tag{12}
\]
\[
d \leftrightarrow u + e^- + \nu_e
\]
\[
s \leftrightarrow u + e^- + \bar{\nu}_e
\]
leading to constraints on the chemical potentials
\[
\mu_s = \mu_d = \mu_u + \mu_e, \tag{13}
\]
which together with the definition of baryon density and the condition for charge neutrality gives 4 constraints on the 5 variables taken to be the four chemical potentials plus \(n_B\) or the four particle densities and \(n_B\), which again allows the calculation of all necessary derivatives in Eq. (9) as a function of \(n_B\). The calculation involves implicit differentiation of a fairly long expression for the relation between \(n_u\) and \(n_B\). Therefore only the numerical results are given here. Again, in the limit of massless quarks, the expression for \(\Gamma_{\text{Eq}}\) converges to \(4(P + B)/3P\), which is a useful consistency check on the results, and also confirms that \(\Gamma_{\text{Eq}} = \Gamma_{\text{Fr}}\) for massless quarks.

To appreciate the numerical results for \(\Delta \equiv (\Gamma_{\text{Fr}} - \Gamma_{\text{Eq}})\) shown in Figure 1 as a function of baryon density, \(n_B\), it is important to note that the surface of a stable, self-bound strange star (possible for 145 MeV < \(B^{1/4}\) < 165 MeV, depending on the value of \(m\) \(\approx 23, 24, 25, 26\)) is characterized by \(P = 0\), and that the density increases from zero to well above nuclear matter density when crossing the surface. For massless quarks the surface density is exactly \(4B\) and contrary to an ordinary neutron star (where the density, not just the pressure, goes to zero at the stellar surface), this density increases only by a factor of a few from stellar surface to stellar center \(\approx 23, 24, 25, 26\). This unusual behavior of self-bound matter near the stellar surface is reflected in a very special behavior of the adiabatic indices. Again for massless quarks one sees that \(\Gamma = 4(P + B)/3P\) diverges at the surface, since \(P \to 0\), so non-interacting quark matter has the special property that the equation of state is extremely stiff (high \(\Gamma\)) at the lowest (albeit still very high) densities, whereas the equation of state is soft \((\Gamma \to 4/3)\) at the highest densities. And for \(m > 0\) it leaves room for a significant numerical difference between the adiabatic index calculated for frozen composition and equilibrium composition respectively as illustrated in Figure 1.

Figure 1 illustrates that the necessary condition for mechanical instability, Eq. (5), is obeyed near the surface of a strange star, where cavitation involves \(P_{\text{vap}} = 0\) and therefore \(\Delta C \approx 2/\cos(\omega t)\) for sinusoidal perturbations. For a typical compact star mass of 1.4 solar masses \(\Delta > 10\) in a surface layer stretching to a depth of 100 meters and \(\Delta > 100\) in the outer few meters. If quark matter is not absolutely stable but found only as a quark matter core inside a neutron (or so-called hybrid) star, the outer meters of this core will obey Eq. (5) for high \(A\). In this case the relevant vapor pressure is the pressure of the nuclear matter phase at the core boundary, \(P_{\text{vap}} = P_{\text{ph}}\), so \(\Delta C \to 0\) near the quark core boundary where \(P \to P_{\text{ph}}\). If quark
matter occurs in a mixed quark-hadron phase the results above cannot be directly applied, because quark matter in such a mixed phase does not obey local charge neutrality. However there is reason to believe that a similar effect will occur there.

When the necessary condition for cavitation is fulfilled, the actual behavior is determined by $\omega \tau$. The microscopic relaxation time, $\tau$, has been calculated for the relevant processes, and can take essentially all values from $10^{-10} - 10$ s depending on temperature and strange quark mass in particular $[1, 2, 3, 4, 5, 9, 10, 12, 13, 14, 16]$. Cavitation is most likely for $\omega \tau \approx 1$ (Eq. (6)), and this can very well occur, since many dynamical processes in compact stars reach such time-scales, including stellar radial oscillations, $r$-mode instabilities $[27]$, and tidal deformation during the final stages of binary compact star inspiral $[28]$. Therefore the consequences can be important for detailed studies of these phenomena and related observables such as the gravitational wave emission signatures, pulsar glitches and possible triggering of phase transitions that could be relevant for gamma-ray bursts and other energetic sources. Also, the detailed break-up process in binary inspiral of strange stars will decide the mass distribution of fragments in form of strangelets sought for in cosmic rays $[28, 29, 30]$. Quark matter has here been treated within the simplest version of the MIT bag model without inclusion of gluon-exchange corrections. Such corrections have been shown (as far as the equation of state is concerned for small strong coupling) to be equivalent to changing the value of $B$ $[24]$, so the qualitative scenario should remain unchanged. Going beyond the bag model will clearly change the numerical estimates, but the possibility of having large and even divergent values of the adiabatic indices, and therefore possibly also large values of $\Delta$, appears linked to the special properties of self-bound matter rather than to the specific equation of state. Quark pairing as is expected to occur with color superconductivity and color-flavor locking in the infinite density limit $[31]$ may have only minor effect on the equation of state and adiabatic indices, but it significantly (often exponentially) decreases the bulk viscosity via increasing the characteristic microscopic time-scale, $\tau$ $[3, 9, 10, 12, 13, 14, 16]$. This would significantly change the mechanical instability window, at least for full color-flavor locking whereas the change in bulk viscosity is smaller for phases with partial pairing, such as 2SC. However, pairing is expected to be most important at the highest densities, and it could well be that quark matter is unpaired at the lowest densities where the mechanical instability occurs.

The bulk viscosity in dense quark matter has been shown to make the effective pressure smaller than the vapor pressure under conditions relevant for neutron stars or strange stars. This indicates the onset of cavitation where the quark matter breaks apart into fragments which may survive as strangelets (if quark matter is stable) or hadronize in a manner comparable to recent scenarios for ultra-relativistic heavy-ion collisions. At the very least it points to the need for a more detailed investigation of instabilities and oscillations in compact stars containing quark matter. In particular, the cavitation process represents a new mechanism for damping oscillations, $r$-modes, and other perturbations in compact stars. Similar phenomena can occur in the outer parts of kaon condensed cores in neutron stars (and for other systems with a phase transition separating a core from the outer parts). Other types of dense nuclear matter do not seem to fulfill the necessary condition for cavitation, but the numbers are sufficiently close that also this may deserve further study. The effect of bulk viscosity is complementary to phase transformations caused by the perturbation in the thermodynamic pressure itself. The interplay between these contributions and questions related to the spectrum and growth rates of cavities $[18]$ remain to be studied.

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