Threshold Resummation and Determinations of Parton Distribution Functions

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The precise knowledge of parton distribution functions (PDFs) is indispensable to the accurate calculation of hadron-initiated QCD hard scattering observables. Much of our information on PDFs is extracted by comparing deep inelastic scattering (DIS) and lepton pair production (LPP) data to convolutions of the PDFs with the partonic cross sections of these processes. It is known that partonic cross sections receive large corrections in regions of phase space near partonic thresholds that can be resummed using threshold resummation techniques. The effect of threshold resummation on DIS and LPP differs because partonic thresholds for the two processes occur in different kinematic regions. Recent global fits for PDFs have included DIS data from the large Bjorken $x$ and moderate $Q^2$ region where threshold effects have been shown to be large. The present project explores the effects of simultaneously incorporating threshold resummation in both DIS and LPP and to evaluate the effects of such additions on global fits.

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1 Introduction: Inclusive Cross Section

A differential inclusive cross section for a process $H1 + H2 \rightarrow A + X$ with observed final state $A$ takes the generic form of a convolution of hard scattering cross sections and parton distribution functions,

$$d\sigma(x, Q^2) = \sum_{a,b} dx_1 dx_2 dz f_{a/H_1}(x_1, Q)f_{b/H_2}(x_2, Q)d\hat{\sigma}_{ab}(z, Q^2)\delta(x - x_1 x_2 z),$$  \hspace{1cm} (1)

where $\hat{\sigma}_{ab}$ is the hard scattering cross section describing the interactions of two partons $a$ and $b$, and $f_{a/H}$ is a parton distribution function (PDF). The PDFs can be interpreted as the probability of obtaining a parton $a$ from a hadron $H$ at an energy scale $Q$ and momentum fraction of the parent hadron between $x$ and $x + dx$. Unlike $\hat{\sigma}$, which is calculated via a perturbative expansion in $\alpha_s$, PDFs cannot be calculated perturbatively and are determined by comparing data from hadron-initiated processes to the theoretical predictions of the data. PDFs have thus far been successfully inferred from data with fixed-order techniques [1–6] using global sets of data that include deep inelastic scattering (DIS) and lepton pair production (LPP). The goal of this study is to qualitatively demonstrate that the inclusion of threshold resummation, an all-orders technique, can impact nucleon PDFs.

2 Threshold Resummation

Logarithmic corrections appear in the perturbative expansion of $\hat{\sigma}$ that can become large in threshold regions, when the leading order process takes all available energy and final state gluons are soft, potentially spoiling the perturbative series. In this situation, an all-orders resummation of these corrections is needed to preserve the perturbative series [7,8]. These threshold logarithms appear at all orders of perturbation theory beyond LO in a predictable manner, but are convoluted with the PDFs and cannot be easily resummed. Threshold resummation is therefore naturally performed in a Mellin conjugate space with Mellin moments $N$, where the convolution integrals are reduced to simple products. In this space, the threshold regions manifest as $N \rightarrow \infty$ and the threshold logarithms as powers of $\ln(N)$. It has been shown that when the threshold logarithms are resummed, one obtains an exponent [7–9]:

$$C_N^{res}(\alpha_s) = g_0(\alpha_s) \exp(G_N(\alpha_s)), \hspace{1cm} (2)$$

where $g_0$ is a series in $\alpha_s$ that is finite as $N \rightarrow \infty$ and $G_N$ contains the resummed threshold logarithms. Resummation corrections are known exactly for many processes up to next-to-next-to-leading logarithmic order (NNLL) [10,11], and $N^3LL$ [12] corrections are partially known for some processes.

One must invert the resummation corrections from Mellin space in order to compare them with data, a task that requires a method for avoiding the Landau pole.
divergence. One of the more common methods, and the one used in this study, is the minimal prescription [13], which simply chooses an inversion integral path to the left of the Landau pole, but to the right of all other poles. By subtracting the expansion of the resummation exponent to an appropriate order in $\alpha_s$, one can also match the resummed corrections to fixed-order calculations. For the purposes of this study, it is sufficient to consider an NLO fixed-order calculation with matched NLL corrections, meaning that logarithmic terms of the order $\alpha_s^m \ln^{m+1}(N)$ and $\alpha_s^m \ln^m(N)$ are fully resummed.

3 Deep Inelastic Scattering and Lepton Pair Production

DIS and LPP data are often used in global fits of PDFs to constrain the lighter quark and antiquark distributions. DIS data is a primary source of information on the valence quarks, particularly the up quark, since $F_2$, the familiar DIS structure function, behaves as $F_2 \sim (4u + d)$ at high $x$. The comparison of proton-proton and proton-neutron LPP data provides constraints to the $\bar{u} - \bar{d}$ distribution and the $d/u$ ratio.

DIS is a single hadron process initiated by vector boson exchange between a lepton and a parton inside the hadron (schematically $l + H \rightarrow l' + X$). The hadronic final state $X$ has an invariant mass squared of $W^2 = M^2 + Q^2(1/x - 1)$, where $M^2$ is the squared nucleon mass, $Q^2$ is the negative of the invariant mass squared of the exchanged boson in the process, and $x$ is the Bjorken scaling variable. Threshold for DIS corresponds to $W^2 = M^2$, when all final state gluons are soft, and therefore occurs at $x = 1$. In the left side of Figure 1 it is demonstrated that the threshold resummation corrections are indeed greater at high $x$. The right side of Figure 1 shows the $Q^2$ dependence of the resummation corrections. It can be seen that this high $x$ behavior is consistent across this range of $Q^2$. Therefore, the inclusion of data sets in these kinematic regions [14,15] will impact global fits of PDFs.

LPP is a hadron-hadron scattering event with center of mass energy squared $S$, where a parton from one hadron interacts with a parton from the other hadron to produce a massive vector boson that decays into a lepton pair of invariant mass squared $Q^2$. In LPP, the threshold region corresponds to $Q^2$ taking all the available energy of the partonic system, $\hat{\tau} = x_1 x_2 S$ with $x_1$ and $x_2$ being the parton’s momentum fraction, so that any final state gluons are soft. Therefore, the threshold region occurs when $\hat{\tau} = Q^2/\hat{S} = 1$. In some LPP observables, one integrates across all available momentum fractions $x_1$ and $x_2$, implying that threshold conditions can be reached for any value of $Q^2/S$.

LPP data is often a doubly-differential cross section in both $Q^2$ and $x_F = \frac{2p_L}{\sqrt{S}}$ (or $Y$, the lepton pair’s rapidity), where $p_L$ is the longitudinal momentum of the
lepton pair. In Figure 2, the resummation corrections to $q^2 \sigma \frac{d^2}{dq^2 dx_F}$ are shown. Here, $Q, \sqrt{S}$, and the range of $x_F$ were chosen to match E866/NuSea [17, 18] conditions to demonstrate that resummation corrections are sizable in kinematic regions with LPP data. Notice that the resummation effects grow with increasing $x_F$; this is because at higher values of $x_F$, one of either $x_1$ or $x_2$ becomes large, and the steeply falling PDFs at higher $x$ push the fixed-order calculation toward threshold. Therefore, the resummation corrections become the most dominant contribution to the observable.

4 Conclusions

Recent global fits of PDFs [3, 5] have included data from threshold regions, where threshold resummation corrections to DIS and LPP have been shown to be sizable. In addition, these corrections occur in different kinematic regions between DIS and LPP, and their simultaneous inclusion in global fits will affect different PDFs. The valence PDFs at large $x$ and moderate $Q^2$ will be particularly affected by the DIS resummation corrections, and we should expect to see more constraints in this region. This implies that it may be necessary to include threshold resummation in global fits of PDFs in order to gain a more accurate understanding of the PDFs. To this end, a PDF fitting program has been modified to include threshold resummation corrections at NLO+NLL for DIS and LPP, and preliminary global fits have been performed. The results of this study are expected to be published at a later date.
Figure 2: $\frac{d^2\sigma}{dQ^2dx_F}$ at NLL+NLO normalized to NLO with $Q = 8$ GeV, $\sqrt{S} = 38.76$ GeV (using the same PDFs and scale choices as in Figure 1).

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