A study on the nonlinear elastic behavior of ballistic gelatin using a rotational rheometer

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Abstract. The present work was focused on the study of large elastic behavior and constitutive modeling of ballistic gelatin. The simple shear tests were conducted using a rotational rheometer. It is observed that the mechanical parameters of the material should be analyzed directly by the data of rotational angle and torque measured by the rheometer but not the apparent shear stress and strain data provided by the rheometer directly. A hyperelastic constitutive model, a 2-term reduced polynomial strain energy potential, was adopted to describe the nonlinear stress-strain relationship. The material parameters were identified and the dependency of the parameters on the temperature was analyzed. It’s found that the modulus at small deformation decreases with increasing temperature following an approximately linear relationship; while, the modulus at large deformation showed a process of first increasing and then decreasing with increasing temperature.

1. Introduction
Ballistic Gelatin (BG) is a tissue simulant whose properties can be easily modified to match various human organ tissue properties by simply varying water to powder ratio. Due to its super mimic ability, gelatin has become a popular model material or surrogate in many bioengineering or biomedical studies [1-3]. There are several reports on the uniaxial compression or elongation experiments of gelatin [4-7]; and found that gelatin can bear very large elastic deformation. When the elastic limit is exceeded, the material fails due to fracture. To evaluate the mechanical stiffness of BG, the elastic or hyperelastic constitutive models have been developed to describe the stress-strain relationship, in which the stiffness parameters were calibrated for individual strain rates. Cronin and Falzon [5] used the Neo-Hookean constitutive model to describe the compressional stress-strain relationship for strain rates from 0.01/s to 1/s. Amborn et al. [8] adopted three forms of hyperelastic models (Marlow form, Neo-Hookean form and Ogden form), and obtained the values of the material parameters using the compression data at a specific strain rate of 300/s, which was considered to be consistent with the application in ballistic penetration. Ravikumar et al. [9] employed an Ogden type strain energy density function for the nonlinear elastic response and a single Prony exponential term for the rate-dependent response of BG for multiple strain rates. The calibration of the material parameters was based on uniaxial compression tests in which the gelatin samples were compressed to the maximum engineering strain of 0.25 and the nominal compression strain rates in the range of 0.001/s~0.1/s. In our previous work [10], the Mooney-Rivlin model was adopted to describe the hyper-elastic behavior of BG and the material parameters
were obtained as functions of the strain rate from the quasi-static compression tests (~0.208/s) and SHPB tests of Salisbury and Cronin (2009) (~1550/s). The drawbacks of this method are that the fitting errors in calibrating the material parameters are quite large.

The shearing test has great advantage in studying the mechanical properties of soft materials[11]. The volume of the samples can be very small (in the present study, the diameter of the samples is 25 mm and thickness 2 mm), making the temperature control not as difficult as that in the compression or elongation test. The deformation rate can be easily controlled to be a constant value and the experimental results are much more repetitive. In the present study, a rotational rheometer with parallel plates (see Figure 1a) was used to study the non-linear elastic behaviors of gelatin. The parallel plates geometry was sketched in Figure 1b. With one plate stationary and the other rotating at angular velocity of \( \omega \), neglecting inertial forces, the shearing strain rate and shear strain in the sample must be,

\[
\dot{\gamma}(r) = \frac{\omega}{h}, \quad \gamma(r) = \frac{r \theta}{h}
\]

In which \( \theta \) is the rotating angle of the upper plate which is measured by the rheometer to calculate the shear strain or strain rate. Therefore, the shear strain and stress in the sample is nonhomogeneous and depends on position. From a torque balance, the load torque of the rotating shaft is,

\[
M = 2\pi \int_{0}^{h} \tau(r) r dr
\]

In which \( \tau(r) \) is the shear stress. The value of \( M \) can be measured by the rheometer to calculate the shear stress at the edge of the sample, approximately,

\[
\tau(R) = \frac{2M}{\pi R^3}
\]

This approximation is valid for a wide range of liquids[11]. The rheometer provides the representative stress and strain at the point \( r/R=3/4 \) to study the viscosity of the sample,

\[
\gamma_s = \frac{3R}{4h} \theta, \quad \tau_s = \frac{3M}{2\pi R^3}
\]

However, for BG under 29°C, the sample is a viscoelastic solid and the approximation in Eq. (3) is not necessarily valid and the shear stress can not be calculated directly. In the present study, it’s supposed that BG is incompressible and its large elastic behavior can be described by the following hyperelastic constitutive model,

\[
U = C_{10}(I_1 - 3) + C_{20}(I_1 - 3)^2
\]

In which, \( U \) is the strain energy per unit of reference volume; \( C_{10} \) and \( C_{20} \) are material parameters; \( I_1 \) is the first deviatoric strain invariant. Then, the shear stress and strain can be related by,

\[
\tau = c_1\gamma + c_2\gamma^2 \quad (c_1 = 2C_{10}, c_2 = 4C_{20})
\]

Then the torque of the rotating shaft can be calculated by substituting Equation (6) to equation (2),

\[
M = 2\pi \int_{0}^{h} \left[ c_1 \frac{\theta}{h} r^2 + c_2 \left( \frac{\theta}{h} \right)^3 r^3 \right] dr
\]

In the present work, the elastic modulii of the gelatin material, \( c_1 \) and \( c_2 \), were estimated both by the apparent shear stress and strain using equation (6) (method 1) and by the torque vs. apparent strain using equation (7) (method 2) and the two methods were compared to study the error of calculating the shear stress with equation (3). Further, the mechanical parameters of BG and their dependency on sample concentration was analyzed.
Figure 1. Rheometer used in the present study (a) and the sketched configuration of the shear test (b).

2. Materials and methods

2.1. Gelatin gel preparation
The Type-A gelatin powder with a Bloom number of 250, provided by QingHai Gelatin Factory (China), was used in the present study. Preparation of the gelatin solution was described in a previous publication [12]. The solution was left in a incubator chamber at 40°C for hydration for 24 hours. After that, the solution was poured gently onto the stationary plate of the rheometer. After the rotational plate descending to contact with the solution and the gap between the two plates achieving the predetermined value (2mm), the superfluous solution was scraped off and the water-cooling system connected to the stationary plate was turned on to keep the sample at 4-19°C. After 24 hours’ aging, the gelatin gel was ready to be tested.

2.2. Shearing test
The selected shearing rates are 0.0005/s, 0.01/s, and 0.1/s. The angle of rotation and torque of the rotational shaft were recorded, and the shearing stress and strain were provided using Eq. (4). Each test was replicated at least three times. It’s observed from the test that BG can bear very large elastic deformation before fracture. Figure 2a and figure 2b show a typical set of test results displayed with $\tau_a$ vs. $\gamma_a$ and $M$ vs. $\gamma_a$, respectively.

The results showed that the material behaves nonlinearly at large deformation and the stiffness of the material is dependent on strain rate. The elastic modulii, $c_1$ and $c_2$, can be estimated both by the apparent shear stress and strain in figure 2a using equation (6) (method 1) and by the torque vs apparent strain in figure 2b using equation (7) (method 2). The comparison of the results obtained by the two methods was presented in table 1.

The errors of the parameters obtained by method 1 were close to 20% compared that with method 2. This observation confirmed that it is not proper to use the apparent shear stress and strain directly provided by the rheometer with parallel plates to study the mechanical parameters of BG. Otherwise, the torque of the rotating shaft should be used for analysis indirectly, e.g. equation (7). It should be pointed out the specific relationship between the $M$ vs. $\gamma_a$ is dependent on the constitutive model chosen as in equation (5).
Figure 2. Typical sets of shear test results: (a) apparent shear stress vs. shear strain and (b) torque vs. apparent shear strain.

Table 1. Material parameters, $c_1$ and $c_2$, obtained by two methods.

| Strain rate | Method 1 | Method 2 |
|-------------|----------|----------|
|             | $c_1$    | $c_2$    | $c_1$    | $c_2$    |
| 0.0005/s    | 19300    | 4500     | 16300    | 3800     |
| 0.01/s      | 22000    | 4600     | 18500    | 3900     |
| 0.1/s       | 24800    | 8300     | 21300    | 7000     |

2.3. Temperature dependence of the mechanical parameters

The shearing test under different temperatures were carried out with a selected shearing rate, 0.01/s, to study the temperature dependency of the mechanical parameters of BG. The torque vs. apparent strain were used to estimate the elastic modulii, $c_1$ and $c_2$, using equation (7). The curve fitting results were shown in figure 3a and the elastic modulii variation with temperature in figure 3b.

Figure 3b indicated that the modulus $c_1$ changes monotonically with the temperature. As anticipated, decreasing the temperature increased the material stiffness. Nevertheless, the value of $c_2$ seemed to reach a maximal value between the temperature of 4℃ and 14℃. This phenomenon needs further study.

3. Discussions and conclusions

The mechanical behaviors of ballistic gelatin can be obtained using a rheometer with parallel plates. However, for ballistic gelatin, direct using the apparent shear stress and strain reported by the rheometer to estimate the mechanical parameters produces relatively large errors. Instead, the shear stress-strain relationship can be pre-assumed, e.g., equation (6), and the torque of the rotating plate can be calculated, equation (7) and used to fit the test results. It is observed that the errors of the elastic modulii of 10% ballistic gelatin at 4℃ obtained by the traditional method (method 1) were close to 20% compared that with the new method (method 2). And the large errors exist for different loading rates in the range of 0.0005~0.1/s.

The temperature dependence of the mechanical parameters of ballistic gelatin was also studied by using the shearing test under different temperatures with a selected shearing rate, 0.01/s. The elastic
modulus, \( c_1 \), which is the shear modulus (under small deformation) of the material, decreases monotonically with the increasing temperature. But the variation of \( c_2 \) with the temperature seems more complicated, which reaches a maximal value approximately at 9°C and further tests are needed to verify this conclusion.

(a)  
(b)  

**Figure 3.** Shear test results under different temperature: (a) test data of \( M \) vs. \( \gamma_a \) and fitted curves, and (b) elastic modulii variations with temperature.

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