Measurement of the Branching Fraction, Polarization, and CP Asymmetries in $B^0 \to \rho^0 \rho^0$ Decay, and Implications for the CKM Angle $\alpha$

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We study $B^0 \to \rho^0 \rho^0$ decays in a sample of $465 \times 10^6 \ Upsilon(4S) \to B \bar{B}$ events collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ collider located at the Stanford Linear Accelerator Center (SLAC). We measure the branching fraction $B = (0.92 \pm 0.32 \pm 0.14) \times 10^{-6}$ and longitudinal polarization fraction $f_L = 0.75^{+0.11}_{-0.14} \pm 0.04$, where the first uncertainty is statistical and the second is systematic. The evidence for the $B^0 \to \rho^0 \rho^0$ signal has a significance of 3.1 standard deviations, including systematic uncertainties. We investigate the proper-time dependence of the longitudinal component in the decay and measure the CP-violating coefficients $S_L = (0.3 \pm 0.7 \pm 0.2)$. 

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and \( C_{L}^{00} = (0.2 \pm 0.8 \pm 0.3) \). We study the implication of these results for the unitarity triangle angle \( \alpha \).

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Measurements of \( CP \)-violating asymmetries test the flavor structure of the standard model by over-constraining the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix \([1]\). The time-dependent \( CP \) asymmetry in the decays of \( B^0 \) or \( \bar{B}^0 \) mesons to a \( CP \) eigenstate dominated by the tree-level amplitude \( b \rightarrow u \bar{d}d \) measures \( \sin 2 \alpha_{\text{eff}} \), where \( \alpha_{\text{eff}} \) differs from the CKM unitarity triangle angle \( \alpha \equiv \arg [-V_{td}V_{tb}^*/V_{ud}V_{ub}^*] \) by a quantity \( \Delta \alpha \) accounting for the contributions from loop (penguin) amplitudes. The value of \( \Delta \alpha \) can be extracted from an analysis of the full set of isospin-related channels \([2]\).

Since the tree contribution to the \( B^0 \rightarrow \rho^0 \rho^0 \) decay is color-suppressed, the decay rate is much smaller than those of the \( B^0 \rightarrow \rho^+ \rho^- \) and \( B^+ \rightarrow \rho^+ \rho^0 \) channels \([4, 5, 6, 7, 8]\), and is sensitive to the penguin amplitude. Therefore a stringent limit on \( \Delta \alpha \) can be set \([2, 7, 9]\). This makes the \( \rho \rho \) system particularly effective for measuring \( \alpha \).

In \( B \rightarrow \rho \rho \) decays the final state is a superposition of \( CP \)-odd and \( CP \)-even states. An isospin-triangle relation \([2]\) holds for each of the three helicity amplitudes \( A_{\lambda=1,0,+1} \), which can be separated through an angular analysis. The longitudinal polarization fraction \( f_L = |A_0|^2/\sum |A_{\lambda}|^2 \) can be determined through the measurement of the distribution of the helicity angles \( \theta_1 \) and \( \theta_2 \), defined as the angles between the direction of the \( \pi^+ \) and the direction of the \( B \) meson in the rest system of each of the \( \rho^0 \rightarrow \pi^+ \pi^- \) candidates.

In this paper, we update our previous measurement \([4]\) of the branching fraction \( B \) and longitudinal polarization fraction \( f_L \) in \( B^0 \rightarrow \rho^0 \rho^0 \) decays, along with \( B \) for \( B^0 \rightarrow \rho^0 f_0 \), \( f_0 f_0 \), and \( \rho^0 \rho^+ \rho^- \), \( \pi^+ \pi^- \pi^+ \pi^- \). In addition, we present the first study of the time-dependent \( CP \) asymmetry \( A_{CP} \) in this mode. We determine the coefficients \( C_{L}^{00} \) and \( S_{L}^{00} \) of \( A_{CP} \) for the longitudinal component, expressed as a function of \( \Delta t \), the proper time difference between the two \( B \) decays in \( \Upsilon(4S) \rightarrow B^0 \bar{B}^0 \):

\[
A_{CP}(\Delta t) = -C_{L}^{00} \cos \Delta m \Delta t + S_{L}^{00} \sin \Delta m \Delta t .
\]

where \( \Delta m = (0.507 \pm 0.005) \) ps\(^{-1} \) is the mass difference between two \( B^0 \) mass eigenstates \([10]\). When combined with measurements of \( B^0 \rightarrow \rho^+ \rho^- \) and \( B^+ \rightarrow \rho^+ \rho^0 \), \( A_{CP} \), \( B \), and \( f_L \) in \( B^0 \rightarrow \rho^0 \rho^0 \) allow a complete isospin analysis and improve the constraints on the penguin contribution to \( B \rightarrow \rho \rho \) decays. Changes with respect to our previous analysis \([4]\) include a larger data sample, improved track-selection techniques, and inclusion of the \( B \)-decay time information.

We use a sample of \( (465 \pm 5) \times 10^6 \) \( \Upsilon(4S) \rightarrow B \bar{B} \) decays into \( B \bar{B} \) pairs collected with the \( \text{BABAR} \) detector \([11]\) at the PEP-II asymmetric-energy \( e^+ e^- \) collider \([12]\). A detailed description of the \( \text{BABAR} \) detector is available elsewhere \([4, 11]\).

We select \( B \rightarrow M_1 M_2 \rightarrow (\pi^+ \pi^-)(\pi^+ \pi^-) \) candidates, where \( M_{1,2} \) stands for a \( \rho^0 \) or \( f_0(980) \) candidate, from neutral combinations of four charged tracks that are consistent with originating from a single vertex near the \( e^+ e^- \) interaction point. We veto tracks that are positively identified as kaons or electrons. The identification of signal \( B \) candidates is based on several kinematic variables. The beam-energy-substituted mass \( m_{\text{ES}} = [(s/2 + p_{\rho B}^2)/E_i^2 - p_B^2]^{1/2} \), where the initial \( e^+ e^- \) four-momentum \( (E_i, p_i) \) and the \( B \) momentum \( p_B \) are defined in the laboratory frame, is centered near the \( B \) mass with a resolution of 2.6 MeV/c\(^2\) for signal candidates. The difference \( \Delta E = E_{\text{ES}} - \sqrt{s}/2 \) between the reconstructed \( B \) energy in the center of mass (c.m.) frame and its known value \( \sqrt{s}/2 \) has a maximum near zero with a resolution of 20 MeV for signal events. Four other kinematic variables describe two possible \( \pi^+ \pi^- \) pairs: invariant masses \( m_1, m_2 \) and helicity angles \( \theta_1, \theta_2 \).

We use the kinematic selection of signal candidates described in \([4]\). We require 5.245 \( < m_{\text{ES}} < 5.290 \) GeV/c\(^2\), \( |\Delta E| < 85 \) MeV, 0.55 \( < m_{1,2} < 1.050 \) GeV/c\(^2\), and \( |\cos \theta_{1,2}| < 0.98 \). The extended di-pion invariant mass range allows us to study the non-resonant decays \( B^0 \rightarrow \rho^0 \pi^+ \pi^- \) and \( B^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^- \), as well as \( B^0 \rightarrow \rho^0 f_0 \) and \( B^0 \rightarrow f_0 f_0 \). The contributions from the higher mass resonances in this range are relatively small. We suppress the dominant \( e^+ e^- \rightarrow q \bar{q} \) \((q = u, d, s, c)\) continuum background using a neural network-based discriminant \( \mathcal{E} \), which combines eight topological variables \([4]\).

We use multivariate \( B \)-flavor tagging algorithms trained to identify primary leptons, kaons, soft pions, and high-momentum charged particles from the other \( B \), called \( B_{\text{tag}} \) \([13]\). The effective tagging efficiency is \((31.1 \pm 0.3)\% \). Additional background discrimination power arises from the difference between the tagging efficiencies for signal and background in seven tagging categories \( (c_{\text{tag}}) \). We determine \( \Delta t \) and its error \( \sigma_{\Delta t} \) from the spatial separation between the decay vertices of the signal \( B \) and \( B_{\text{tag}} \) and require \( |\Delta t| < 15 \) ps and \( \sigma_{\Delta t} < 2.5 \) ps.

After application of all selection criteria, 72154 events are retained. On average, each selected event has 1.05 signal candidates, while in Monte Carlo (MC) \([14, 15]\) samples of longitudinally (transversely) polarized \( B^0 \rightarrow \rho^0 \rho^0 \) decays we find 1.15 (1.03) candidates. When more than one candidate is present in the same event, the candidate that yields the smallest \( \chi^2 \) for the four-pion vertex is selected. Simulation shows that 18\% of longitudinally and 4\% of transversely polarized \( B^0 \rightarrow \rho^0 \rho^0 \) events are misreconstructed with one or more tracks that do not
originates from the $B^0 \rightarrow \rho^0 \rho^0$ decay. These are mostly low-momentum tracks from the other $B$ meson in the event. Such partially reconstructed candidates are included in the definition of the signal probability density functions (PDFs).

We use an unbinned extended maximum likelihood fit to extract the $B^0 \rightarrow \rho^0 \rho^0$ event yield, $f_s$, $c^{60}_{L}$, and $S^{60}_{L}$. We also fit for the event yields of $B^0 \rightarrow f_0f_0$, $B^0 \rightarrow f_0\rho^0$, $B^0 \rightarrow \rho^0\pi^+\pi^-$, and $B^0 \rightarrow \pi^+\pi^-\pi^-\pi^-$ decays. The likelihood function includes the background components from non-signal $B$ decays and continuum. The PDFs for each component depend on ten discriminating variables: $m_{ES}, \Delta E, \mathcal{E}, m_1, m_2, \cos \theta_1, \cos \theta_2, \epsilon_{\text{ang}}, \Delta t,$ and $\sigma_{\Delta t}$.

Since the statistical correlations among the variables are found to be negligibly small, we take the total PDF as the product of the PDFs for the separate variables. Exceptions are the kinematic correlation between the two helicity angles in signal, and mass-helicity correlations in several $B$-decay classes and misreconstructed signal.

We use double-Gaussian functions to parameterize the $m_{ES}$ and $\Delta E$ PDFs for signal, and relativistic Breit-Wigner functions for the resonance line-shapes of $\rho^0$ and $f_0$(980), with the $f_0$(980) mass and width taken from [16]. The angular distribution for $B$ decays [13] (expressed as a function of $f_L$ for $B^0 \rightarrow \rho^0\rho^0$) is multiplied by a detector acceptance function determined from simulations. We assume that the $\rho^0$ in $B^0 \rightarrow \rho^0\pi^+\pi^-$ is longitudinally-polarized (i.e., $\pi^+\pi^-$ are produced in the S-wave), and we use the phase-space distributions for $B^0 \rightarrow \pi^+\pi^-\pi^-\pi^+$. The $(\pi\pi)$ invariant mass and helicity distributions of misreconstructed signal events are parameterized with empirical shapes in a way similar to that used for $B$ background discussed below. The neural network discriminant $\mathcal{E}$ is described by the sum of three asymmetric Gaussian functions with different parameters for signal and background distributions.

The PDFs for non-signal $B$-decay modes are generally modeled with empirical analytical distributions. Several variables have distributions identical to those for signal, such as $m_{ES}$ when all four tracks come from the same $B$, or $m_{1,2}$ when both tracks come from a $\rho^0$ meson. Also for some of the modes the two $\pi^+\pi^-$ pairs can have different mass and helicity distributions, e.g., when only one of the two combinations comes from a genuine $\rho^0$ or $f_0$ meson, or when one of the two pairs contains a high-momentum pion (as in $B \rightarrow a_1\pi$). In such cases, we use a four-variable correlated mass-helicity PDF. The proper-time distribution for signal and background $B$ decays is convolved with a resolution function [13], while the time distribution of continuum background is assumed to have zero lifetime.

The signal and $B$-background PDF parameters are extracted from simulation. The MC parameters for the $m_{ES}$, $\Delta E$, and $\mathcal{E}$ PDFs are adjusted by comparing data and simulation in control channels with similar kinematics and topology, such as $B^0 \rightarrow D^-\pi^+$ with $D^- \rightarrow K^+\pi^-\pi^-$. The continuum background PDF shapes are extracted from on-resonance sideband data ($m_{ES} < 5.27\text{GeV}/c^2$), with parameters of most PDFs (for $m_{ES}$, $\Delta E$, $\mathcal{E}$, $\theta_1$, $\theta_2$, and $\Delta t$) left free in the final fit. The tagging efficiencies, mistag fractions, and the parameters of the proper-time distributions for signal modes are obtained in dedicated fits to events with identified exclusive $B$ decays [13]. For inclusive $B$ backgrounds these parameters are determined by MC and their systematic uncertainties are evaluated in data.

We study the contributions of the dominant backgrounds by using high-statistics exclusive MC samples. We single out the $B^0 \rightarrow a_1^\pm\pi^\mp$, $B^0 \rightarrow \rho^0K^{*0}$ and $B^0 \rightarrow f_0K^{*0}$ modes, which have kinematic distributions similar to those of the signal events, and parameterize their PDFs individually. The event yield for $B^0 \rightarrow a_1^\pm\pi^\mp$ is allowed to vary in the fit, while the yields for $B^0 \rightarrow \rho^0K^{*0}$, $B^0 \rightarrow f_0K^{*0}$ are fixed to the expected values [17].

In addition, we construct a charmless event category consisting of $B^0 \rightarrow \rho^+\rho^-$, $B^0 \rightarrow \rho^+\pi^-$, $B^+ \rightarrow \rho^+\rho^-$, $B^+ \rightarrow a_1^\pm\pi^+$, $B^+ \rightarrow a_1^\pm f_0$, $B \rightarrow \eta'K$, and $B^+ \rightarrow \rho^0\pi^+$ backgrounds. Kinematic distributions in these events, especially events in which at least one charged particle is not correctly associated to the $B$ candidate, are similar to each other, and also to those of other, poorly measured, charmless decays. We allow the overall event yield for this category of events to vary in the fit, after fixing the relative weight of each mode to the expected value [10].

The remaining events, which mostly originate from open $b \rightarrow c$ transitions, are parameterized as a separate background component, with its yield left free in the fit.

Table 1 summarizes the results of the fit. The $B^0 \rightarrow \rho^0\rho^0$ decay is observed with a significance of 3.1 standard deviations ($\sigma$), as determined by the quantity $S = \sqrt{-2\ln(\mathcal{L}_0/\mathcal{L}_{\text{max}})}$, where $\mathcal{L}_{\text{max}}$ is the maximum likelihood value, and $\mathcal{L}_0$ is the likelihood for a fit with the sig-
nal contribution set to zero. Both likelihoods include systematic uncertainties, which are assumed to be Gaussian-distributed and are discussed below. This significance level corresponds to a probability of $1.0 \times 10^{-3}$ that the observed signal yield is consistent with a background fluctuation. We do not observe significant event yields for $B^0 \rightarrow \rho^0 f_0$ or $B^0 \rightarrow f_0 f_0$ decays, nor for the non-resonant decays $B^0 \rightarrow \rho^0 \pi^+ \pi^-$ and $B^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$. We find $280 \pm 53, 670 \pm 96, 2329 \pm 147,$ and $6870 \pm 281$ events for the $B^0 \rightarrow a_1^\pm \pi^\mp$, charmless, $b \rightarrow c$, and continuum backgrounds, respectively, consistent with expectations. In Fig.1 we show the projections of the fit results onto $m_{ES}$ and $m_{\pi\pi}$.

From the fit to the proper-time distribution of the data sample, we determine the CP-violating parameters
\[
S_{L}^{00} = 0.3 \pm 0.7 \text{ (stat.)} \pm 0.2 \text{ (syst.)}
\]
\[
C_{L}^{00} = 0.2 \pm 0.8 \text{ (stat.)} \pm 0.3 \text{ (syst.)}
\]
for the longitudinal component of the $B^0 \rightarrow \rho^0 \rho^0$ sample. The statistical correlations between fit parameters for $B^0 \rightarrow \rho^0 \rho^0$ are given in Table II.

### Table II: Correlation matrix for $B^0 \rightarrow \rho^0 \rho^0$ parameters.

| Parameter | Yield | $f_L$ | $S_{L}^{00}$ | $C_{L}^{00}$ |
|-----------|-------|-------|-------------|-------------|
| Yield     | 1.000 | 0.086 | -0.136      | -0.273      |
| $f_L$     |       | 1.000 | -0.006      | -0.174      |
| $S_{L}^{00}$ |     |       | 1.000       | -0.035      |
| $C_{L}^{00}$ |     |       |             | 1.000       |

Dominant systematic uncertainties in the fit originate from statistical errors in the PDF parameterizations due to the limited number of events in the control samples, variations in the $B$ background branching ratios fixed in the fit, and from potential fit bias. The PDF parameters are varied by their respective uncertainties to derive the corresponding systematic errors. The fit bias is studied in a large number ($\sim 1000$) of MC experiments, in which signal and dominant charmless $B$ background events are fully simulated, while other backgrounds are sampled from their respective PDFs. We correct for the bias of $7.9 \pm 2.0$ events for $B^0 \rightarrow \rho^0 \rho^0$, $0.03 \pm 0.02$ for $C_{L}^{00}$ and $0.07 \pm 0.03$ for $S_{L}^{00}$. The uncertainties associated with the $B$ background model are $4$ events for the signal yield, $0.01$ for $f_L$, $0.01$ for $C_{L}^{00}$ and $0.11$ for $S_{L}^{00}$. The systematic uncertainties due to the charmless background composition, arising from the uncertainties in the individual branching ratios and the CP content of the $B$ background [18, 19], are $5$ events for the signal yield, $0.01$ for $f_L$, $0.18$ for $C_{L}^{00}$ and $0.14$ for $S_{L}^{00}$. The above systematic uncertainties do not scale with event yield and are included in the calculation of the significance of the result.

We estimate the systematic uncertainty due to the interference between the $B^0 \rightarrow \rho^0 \rho^0$ and $a_1^\pm \pi^\mp$ decays using simulated samples in which the decay amplitudes for $B^0 \rightarrow \rho^0 \rho^0$ are generated according to this measurement and those for $B^0 \rightarrow a_1^\pm \pi^\mp$ correspond to a branching fraction of $(33.2 \pm 4.8) \times 10^{-6}$ [20]. The strong phases and CP content of the interfering $a_1^\pm \pi^\mp$ state are varied between zero and a maximum value using uniform distributions. We take the RMS variation of the fitted values (13 events for the $\rho^0 \rho^0$ yield, $0.02$ for $f_L$, and $0.04$ for $S_{L}^{00}$ and $C_{L}^{00}$) as a systematic uncertainty.

Uncertainties in the reconstruction efficiency arise from track finding and particle identification, and are determined by dedicated studies on control data samples. Uncertainties due to other selection requirements, such as vertex probability, track multiplicity, and thrust angle, amount to $2.4\%$ for the event yields, and are negligible for the polarization and CP observables.

We perform an isospin analysis of $B \rightarrow \rho \rho$ decays, by minimizing a $\chi^2$ that includes the measured quantities expressed as the lengths of the sides of the isospin triangles [2]. We use the measured branching fraction and fraction of longitudinal polarization of $B^+ \rightarrow \rho^+ \rho^0$ decays [4], the measured branching fraction, polarization, and CP parameters $S_{L}^{+0}$ and $C_{L}^{+0}$ determined from the time evolution of the longitudinally polarized $B^0 \rightarrow \rho^+ \rho^-$ decays [7], and the branching fraction, polarization, and CP parameters $S_{L}^{00}$ and $C_{L}^{00}$ of $B^0 \rightarrow \rho^0 \rho^0$ reported here. We assume uncertainties to be Gaussian distributed and neglect $I = 1$ isospin contributions, electroweak loop amplitudes, non-resonant, and isospin-breaking effects.
We obtain a 68% (90%) CL limit \( |\alpha - \alpha_{\text{eff}}| < 15.7^\circ \) (<17.6\(^\circ\)) where \( \alpha_{\text{eff}} \) is defined by \( \sin(2\alpha_{\text{eff}}) = S_L^\pi/(1-C_L^\pi/2)^{1/2} \). Fig. 2 shows the confidence level with and without use of \( S_L^\pi \) and \( C_L^\pi \) in the isospin analysis fit. We observe four solutions near zero, as in the \( B \rightarrow \pi \pi \) isospin analysis [21]. The additional constraint from \( S_L^\pi \) provides some discrimination among the four solutions.

In summary, we confirm our earlier evidence [4] for \( B^0 \rightarrow \rho^0 \rho^0 \) decays with a significance of 3.1\( \sigma \) and measure the branching fraction, longitudinal polarization fraction, and \( CP \) asymmetries in these decays. These measurements combined with those for \( B^+ \rightarrow \rho^+ \rho^- \) and \( B^0 \rightarrow \rho^\pm \rho^- \) decays provide a constraint on the penguin uncertainty in the determination of the CKM unitarity angle \( \alpha \). We find no significant evidence for the decays \( B^0 \rightarrow \rho^0 f_0 \), \( B^0 \rightarrow f_0 f_0 \), \( B^0 \rightarrow \rho^0 \pi^+ \pi^- \) or \( B^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^- \).

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\[ \text{FIG. 2: Confidence level (CL) on } \alpha - \alpha_{\text{eff}} \text{ determined from the isospin analysis. The long-dashed curve is obtained without the two } CP \text{ parameters } S_L^\pi \text{ and } C_L^\pi. \text{ The dotted curve corresponds to the isospin analysis without } S_L^\pi, \text{ and the solid curve } \text{CL} \text{ includes both } C_L^\pi \text{ and } S_L^\pi \text{ in the fit. The horizontal dotted lines correspond to the 68\% (top) and 90\% (bottom) CL intervals.} \]

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