The Braginskii model of the Rayleigh-Taylor instability. I. Effects of self-generated magnetic fields and thermal conduction in two dimensions

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Abstract

There exists a substantial disagreement between computer simulation results and high-energy density laboratory experiments of the Rayleigh-Taylor instability \cite{1}. Motivated by the observed discrepancies in morphology and growth rates, we attempt to bring simulations and experiments into better agreement by extending the classic purely hydrodynamic model to include self-generation of magnetic fields and anisotropic thermal conduction.

We adopt the Braginskii formulation for transport in hot, dense plasma, implement and verify the additional physics modules, and conduct a computational study of a single-mode RTI in two dimensions with various combinations of the newly implemented modules. We analyze physics effects on the RTI mixing and flow morphology, the effects of mutual physics interactions, and the evolution of magnetic fields.

We find that magnetic fields reach levels on the order of 11 MG (plasma $\beta \approx 9.1 \times 10^{-2}$) in the absence of thermal conduction. These fields do not affect the growth of the mixed layer but substantially modify its internal structure on smaller scales. In particular, we observe denting of the RT spike tip and generation of additional higher order modes as a result of these fields. Contrary to interpretation presented in earlier work \cite{2}, the additional mode is not generated due to modified anisotropic heat transport effects but due to dynamical effect of self-generated magnetic fields. The overall flow morphology in self-magnetized, non-conducting models is qualitatively different from models with a pre-existing uniform field oriented perpendicular to the interface. This puts the usefulness of simple MHD models for interpreting the evolution of self-magnetizing HED systems with zero-field initial conditions into doubt.

The main effects of thermal conduction are a reduction of the RT instability growth rate (by about 20% for conditions considered here) and inhibited mixing on small scales. In this case, the maximum self-generated magnetic fields are weaker (approximately 1.7 MG; plasma $\beta \approx 49$). This is due to reduction of temperature and density gradients due to conduction. These self-generated magnetic fields are of very similar strength compared to magnetic fields observed recently in HED laboratory experiments \cite{3}.

We find that thermal conduction plays the dominant role in the evolution of the model RTI system considered. It smears out small-scale structure and reduces the RTI growth rate. This may account for the relatively featureless RT spikes seen in experiments, but does not explain mass extensions observed in experiments.

Resistivity and related heat source terms were not included in the present work, but we estimate
their impact on RTI as modest and not affecting our main conclusions. Resistive effects will be discussed in detail in the next paper in the series.

Keywords: magnetohydrodynamics, hydrodynamic instabilities, thermal conduction, Braginskii equations, laboratory astrophysics

1. Introduction

The Rayleigh-Taylor instability (RTI) occurs when a dense fluid is accelerated by a light fluid. At early times, RTI produces characteristic morphology with heavy material penetrating into a light fluid in forms of fingers or spikes which are well-separated by bubbles filled with the light fluid [4,5]. The resulting material mixing plays an important and sometimes critical role in many problems including industrial applications [6,7], inertial confinement fusion [8,9], and evolution of stellar objects such as supernovae and their remnants (see [10,11,12] and references therein). In the case of core-collapse supernovae (ccSNe), RTI occurs when a supernova shock expands with a varying speed through the progenitor envelope (see, e.g., [13] and references therein).

Conditions relevant to mixing of the chemical elements in ccSNe were successfully reproduced in a series of high-energy density (HED) laboratory experiments [14,15,1]. In the HED experiments, the RT instability is created when a laser-driven blast wave moves through either planar or spherical target from a high to low-density material. The target is designed to mimic a structure characteristic of compositional interfaces in a ccSN progenitor envelope. Experiments are routinely aided by extensive computational studies at both the design and analysis stages.

In this paper, we focus on unusual morphological features of RTI observed in the Kuranz et al. [1] experiment. The reported RTI morphology was significantly different from the results of hydrodynamic simulations, showing strongly suppressed growth of small scale structure and mass extensions of RT spikes. In their analysis, Kuranz et al. [1] focused on the discussion of RT spike mass extensions. They concluded that pure hydrodynamic models cannot account for the observed features and theorized that magnetic fields might be responsible for extended structures emanating from tips of RT spikes.

Large-scale magnetic fields are known to affect the RTI dynamics and its growth rate [16]. Simulations performed by Jun et al. [17] demonstrated that magnetic fields reduced RTI growth for single-mode perturbations, in good agreement with linear theory. However, in the case of multi-mode perturbations they observed faster growth of RTI when the field was tangential to the interface. They also found stronger amplification of magnetic fields in 3-D than in 2-D. More recently, Stone and Gardiner [18] found that in 3-D, strong magnetic fields parallel to the interface increase RTI growth rates by inhibiting shear at the interface and suppressing material mixing. Given that neither of the above studies indicated presence of RT spike mass extensions, Kuranz et al. [1] proposed that the target is self-magnetized. The relevant physical process is known as the Biermann battery effect [19,20]. In it, a thermoelectric current is spontaneously created whenever the electron temperature and electron density gradients are misaligned and results in (self-)generation of magnetic field. This process does not require any seed magnetic
field to operate. The in situ generated field can subsequently be modified due to fluid flow and additional interactions between ions and electrons in the plasma.

Thermal conduction is also known to have varying effects on plasma dynamics. Numerous studies have shown that heat transfer can decrease RTI growth rates [21, 22]. The opposite effect was found by Ryutov [23] who demonstrated that it is possible to create conditions under which thermal conduction actually has a destabilizing effect and induces RTI. When plasma is magnetized, it is expected that heat transfer by electrons across magnetic field lines becomes suppressed if the electron gyroradius gets smaller than its mean free path [24]. Lecoanet et al. [25] studied systems stable and unstable against RTI. They showed that anisotropic heat transport affects only compressible modes, and these modes can actually grow faster.

In the context of inertial confinement fusion (ICF), Nishiguchi [2] found the magnetic fields generated by RTI strong enough to make the electron thermal conduction anisotropic. In particular, Nishiguchi found thermal conduction to be most efficient near the center of a RT spike. Nishiguchi also noted that an additional mode was growing at the interface and claimed conduction responsible for this new flow feature. Generated magnetic fields in megagauss range were obtained in simulations by several groups Nishiguchi [2], Mima et al. [26]; comparable field level was observed recently in experiments Manuel et al. [3], Gao et al. [27].

In this work, motivated by the recent results of laser-driven high-energy density experiments of Kuranz et al. [1], we study the effects of self-generated magnetic fields and thermal conduction on the Rayleigh-Taylor instability in a basic plasma physics setting in two spatial dimensions. Our physics model is a simplified version of the Braginskii model of a single fluid MHD. We implement and verify suitable additional physics modules in our hydrocode and perform a series of simulations to assess numerical model convergence. In application to RTI, we consider conditions relevant to the Kuranz et al. experiment, and focus on individual physics effects and their interactions. We discuss the results and propose directions for the future research in this area.

2. Computational methods

2.1. Extended MHD model with anisotropic conduction

An electrically conducting fluid can be described by one-fluid and two-fluid plasma models. In a one-fluid plasma model, fluid flow equations are written in terms of the average fluid velocity only, and are coupled with Maxwell’s equations of electromagnetism via Ohm’s law. This description is adequate when the plasma can be considered locally electrically neutral. In a two-fluid plasma model, the dynamics of electrons and ions are described individually, for example, by the Braginskii equations [24]. This is a more accurate description of the plasma than a one-fluid model, but is also more expensive requiring to solve twice as many fluid flow equations.

In this paper we numerically solve the magnetohydrodynamic (MHD) equations in conservation form using the Proteus code. Proteus is based on the FLASH code [28], and includes additional physics and algorithms developed to model anisotropic thermal conduction and magnetic field generation. This extended physics model is based on a single-fluid formulation by Braginskii [24], and described by the following set of equations,

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} &= 0, \\
\nabla \cdot \vec{B} &= 0, \\
\frac{\partial \rho \vec{V}}{\partial t} + \nabla \cdot \left( \rho \vec{V} \cdot \vec{V} - \frac{\vec{B} \cdot \vec{B}}{4\pi} \right) + \nabla P &= \rho \vec{g},
\end{align*}
\]
\[
\frac{\partial \rho E}{\partial t} + \nabla \cdot \left( (\rho E + P) \vec{v} - \frac{\vec{B} \vec{B}}{4\pi} \right) = \rho \vec{v} \cdot \vec{g} + \vec{g} \cdot (\vec{q}_T^e + \vec{q}_T^i),
\]
\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B}) + \frac{c}{\epsilon} \left[ \nabla \times \frac{\nabla P_e}{n_e} - \nabla \times \left( \frac{\nabla \times \vec{B}}{4\pi n_e} \right) \right] - \nabla \times \vec{R}_T.
\]

In these equations, \(\rho\) is the plasma density, \(\vec{v}\) is the plasma velocity, \(E\) is the total plasma specific energy, \(P\) is the plasma pressure, and \(\vec{B}\) is the magnetic field. We also include effects due to constant gravitational acceleration, \(\vec{g}\). In the last equation above, \(n_e\) is the electron number density, \(P_e\) is the electron pressure, and other symbols have their usual meaning. The last term inside the square brackets above is simplified compared to the Braginskii model, and only includes a contribution due to the thermal force,

\[
\vec{R}_T = -\vec{g} \left[ \begin{array}{c} q_e^e \nabla T_e \vec{e}_e \parallel \vec{B} \nabla \times \vec{B} \parallel \vec{B} \nabla \times [\vec{h} \times \nabla T_e] \end{array} \right].
\]

Here \(T_e\) is the electron temperature, \(\nabla \parallel\) and \(\nabla \perp\) are mean components of the gradient operator (here acting on the electron temperature) parallel and perpendicular to the magnetic field, and \(\vec{h}\) is a unit vector parallel to the magnetic field.

The last term on the right hand side of the energy equation describes thermal conduction, where \(\vec{q}_T^e\) and \(\vec{q}_T^i\) are the electron and ion heat flux vectors, respectively:

\[
\vec{q}_T^e = -\vec{\kappa}_T^e (\nabla T_e - \vec{\kappa}_T^e \nabla \parallel T_e - \vec{\kappa}_T^e \nabla \perp T_e - \vec{\kappa}_T^e [\vec{h} \times \nabla T_e]),
\]
\[
\vec{q}_T^i = -\vec{\kappa}_T^i (\nabla T_i - \vec{\kappa}_T^i \nabla \parallel T_i + \vec{\kappa}_T^i [\vec{h} \times \nabla T_i]),
\]

where \(T_i\) is the temperature of ions. In agreement with the Braginskii model, we assume both species are in thermal equilibrium, \(T_e = T_i\). The electron heat conduction coefficients are

\[
\begin{align*}
\vec{\kappa}_T^e & = \frac{c \tau_e T_e}{m_e} \bar{\gamma} \bar{\gamma} \bar{\gamma}, \\
\vec{\kappa}_T^e & = \frac{c \tau_e T_e}{m_e} \bar{\gamma} \bar{\gamma} \bar{\gamma}, \\
\vec{\kappa}_T^e & = \frac{c \tau_e T_e}{m_e} \bar{\gamma} \bar{\gamma} \bar{\gamma}, \quad (1)
\end{align*}
\]

and the ion conduction coefficients are

\[
\begin{align*}
\vec{\kappa}_T^i & = \frac{3.906 a_i T_i}{m_i}, \\
\vec{\kappa}_T^i & = \frac{a_i T_i (2 \bar{\gamma} + 2.645)}{m_i}, \\
\vec{\kappa}_T^i & = \frac{a_i T_i (2 \bar{\gamma} + 2.645)}{m_i}. \quad (2)
\end{align*}
\]

Here \(\Delta_e = \chi_e^4 + \delta_1 \chi_e^2 + \delta_0\) and \(\Delta_i = \chi_i^4 + 2.70 \chi_i^2 + 0.677\), with \(\chi_e = \omega_e \tau_e\) and \(\chi_i = \omega_i \tau_i\). \(\omega_e\) and \(\omega_i\) are the cyclotron frequency of electrons and ions, respectively,

\[
\omega_e = \frac{eB}{m_e c}, \quad \omega_i = \frac{ZeB}{m_i c}.
\]
and $\tau_e$ and $\tau_i$ are the ion-electron and ion-ion collision times, respectively,

$$\tau_e = \frac{3 \sqrt{m_e T_e} \hat{e}^3}{4 \sqrt{2} \pi \lambda e^4 \bar{Z}^2 n_i},$$

$$\tau_i = \frac{3 \sqrt{m_i T_i} \hat{e}^3}{4 \sqrt{2} \pi \lambda e^4 \bar{Z}^4 n_i}.$$ 

$\lambda$ is the Coulomb logarithm; all other symbols are defined in [24].

Compared to the original Braginskii formulation, our model does not take into account electric resistivity effects. This is primarily due to severe time step restriction imposed by corresponding friction force term in the induction equations, which would make computations infeasible. However, our results and estimates demonstrate resistivity will not affect the dynamics of the system considered here or change our main conclusions. Also, Budde et al. [29] found that viscosity plays only a minor role in the evolution of RTI targets in the HED experiments. Consequently, we decided not to included viscous effects in this work. Omission of the above two physics processes is the only difference between our model and the Braginskii formulation.

We used the ideal equation of state with $\gamma = 5/3$, and assumed a plasma composed of a single species with atomic mass $A$, and atomic charge $Z$. In this case, the ion number density is $n_i = \rho N_A / A$, $N_A$ being the Avogadro constant. In order to calculate the electron number density $n_e$, we first compute the (average) plasma charge, $\bar{Z}$, using the Thomas-Fermi equation of state [30]. The corresponding electron number density is $n_e = \bar{Z} n_i$.

### 2.2. Implementation of the model

The additional source terms for self-generation of magnetic fields and anisotropic thermal conduction were implemented in PROTEUS and integrated with the directionally unsplit MHD FLASH solver, USM [31]. The required modifications to the USM algorithm are rather modest. Assuming 2-D planar geometry, we proceed as follows:

- At the beginning of the time step, $n$, compute a stable time step, $\Delta t$, based on thermal conductivity coefficients and magnetic field growth rates.

- Apply source terms to magnetic fields at cell centers:

$$B_{i,j}^{n+1} = B_{i,j}^n + \frac{\Delta t}{2} \frac{\partial B_{i,j}^n}{\partial t}.$$ 

- Compute fluxes at cell interfaces at half time step using the USM solver.

- Compute heat fluxes due to thermal conduction at cell interfaces and add them to the total energy fluxes.

- Execute the remaining part of the USM solver to advance the solution to the next time level, $t^{n+1} = t^n$.

- Using the updated state, compute the magnetic field source terms at cell centers and correct the magnetic fields:
\[
\vec{B}_{i,j}^{n+1} \rightarrow \vec{B}_{i,j}^{n+1} + \frac{\Delta t}{2} \frac{\partial \vec{B}_{i,j}^{n+1}}{\partial t}.
\]

- End of step; advance the time step counter, \( n \rightarrow n + 1 \).

The extension of the above algorithm to 3-D is straightforward. The algorithm is explicit and first-order accurate in time. A higher-order method would be much desired, but considerably more complex to implement and its development is beyond the scope of the current paper.

2.3. Solution verification tests

We performed several tests in order to verify that our extended code correctly generates magnetic fields and accounts for the effects of anisotropic heat conduction. To this end, we designed a simple test to verify that differential operators produce correct results and used problems with known exact solutions to assess performance of the Biermann battery source term and anisotropic thermal conduction solver. In the following sections we present the results of the above solution verification tests. In these tests we used dimensionless units.

2.3.1. Gradient orientation test

Assuming \( P_e = n_e T_e \), the Biermann battery term can be written as

\[
\frac{\partial \vec{B}}{\partial t} = \frac{c}{e} [\nabla T_e \times \nabla \ln(n_e)].
\]

Therefore, provided \( \nabla T_e \) and \( \nabla \ln(n_e) \) are constant, one expects a steady and uniform production of magnetic field in direction perpendicular to the plane defined by the above two gradients. Starting with the zero field initial conditions, no field should occur when the two gradients are aligned and the field growth should reach a maximum when the gradients are perpendicular.

In Fig. 1 we show the magnetic field component \( B_z \), due to the Biermann battery source term for various orientations of \( \nabla \ln(n_e) \) and \( \nabla T_e \). The 2-D Cartesian domain covers a square region, \((x, y) \in [0, 360] \times [0, 360]\). The orientation of \( \nabla \ln(n_e) \) (not shown) varies from 0 to 360 degrees with the \( x \) axis, while the orientation of \( \nabla T_e \) (not shown) changes from 0 to 360 degrees along the \( y \) axis, where 0 degrees angle corresponds to a gradient in the direction of the \( x \) axis. The magnitude of the temperature gradient increases with \( z \). The results shown in Fig. 1 demonstrate that the generated magnetic field reaches a maximum when the two gradients are orthogonal, and is zero when they are aligned, as expected.

2.3.2. Single-step Biermann battery test

One can compute the rate of magnetic field generation due to the Biermann battery,

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times \frac{\nabla P_e}{n_e},
\]

by defining the initial distribution of the electron pressure and the electron number density. If in addition, one assumes the medium is initially static and there is no magnetic field, the problem simplifies still further and one can verify the Biermann battery source term implementation using essentially unmodified code.
Figure 1: The value of magnetic field component, $B_z$, as a function of orientation of electron number density gradient and temperature gradient. The values of both gradients are constant. The generated magnetic field is strongest when the two gradients are orthogonal and no field is generated when they are parallel, as expected. See Sect. 2.3.1 for details.

We follow Tóth et al. [32], and assume the following initial conditions,

\[
\begin{align*}
n_e &= n_0 + n_1 \cos(k_x x), \\
P_e &= P_0 + P_1 \cos(k_y y), \\
k_x &= k_y = \frac{\pi}{10},
\end{align*}
\]

with $n_0 = p_0 = 1$ and $n_1 = p_1 = 0.1$. For the above conditions, the exact solution for the rate of magnetic field generation is,

\[
\frac{\partial B_z}{\partial t} = -\frac{k_x k_y n_1 p_1 \sin(k_x x) \sin(k_y y)}{[n_0 + n_1 \cos(k_x x)]^2}.
\]

Figure 2 shows the numerical and exact solutions, as well as the numerical error. By performing

Figure 2: Solution verification of the Proteus code Biermann battery magnetic field source term. The results are shown at $t = 0.05$ on the mesh 160×160 zones. (left panel) Computed solution; (center panel) exact solution; (right panel) absolute value of the difference between the exact and computed solution. Note the scale change between panels. See Sect. 2.3.2 for details.
this test using meshes with different resolutions, we have confirmed that our implementation is second order accurate.

2.3.3. Anisotropic conduction test

Figure 3 shows the results of a test of anisotropic thermal conduction in a circular magnetic field, as originally proposed by Parrish and Stone [33]. In this test, the computational domain is a two-dimensional square region, \((x, y) \in [-1, 1] \times [-1, 1]\). The initial conditions include a small region of higher temperature,

\[
T(r, \theta) = \begin{cases} 
12 & \text{if } (0.5 \leq r \leq 0.7) \text{ and } (11 \leq \theta \leq 12), \\
10 & \text{otherwise}.
\end{cases}
\]

In the ideal case and assuming heat cannot be transported across the field lines, as the time proceeds, all heat should remain confined to the annulus with the inner and outer radii of the small hot region. In reality, however, one may expect that due to numerical errors some heat will be transported across magnetic field lines and diffuse outside the ring.
In our test calculations, we set the parallel diffusion coefficient \( D_\parallel = 10 \) and \( D_\perp = D_\wedge = 0 \), where,

\[
D = \frac{\kappa}{\rho C_v},
\]

and \( C_v \) is the specific heat at constant volume. For the above choice of diffusion coefficients, and as we mentioned earlier, the exact solution predicts that heat flows only within an annulus defined by the magnetic field lines initially bounding the hot region. Also, the temperature should be nearly evenly distributed within the annulus by \( t = 200 \).

The residuals between simulation results and the exact solution for this test problem are shown in Fig. 4 along with the results obtained by Parrish and Stone [33] for several different mesh resolutions. The temperature converges as the mesh resolution increases, albeit slowly.

Parrish and Stone [33] also studied a rate at which heat diffuses across the magnetic field lines and estimated the corresponding effective numerical perpendicular conduction coefficient, \( \kappa_\perp \). The smaller the value of this coefficient, the less diffuse is the anisotropic thermal conduction solver.

The value of the numerical conduction coefficient characteristic of the Proteus solver is shown in Fig. 5 for different mesh resolutions. The results demonstrate the numerical heat diffusion decreases with the resolution, as desired. We also note that the performance of the Proteus heat conduction module matches that of the Parrish and Stone [33] implementation.

### 3. Application to Rayleigh-Taylor problem

In this section, we report the results of simulations of the single-mode RTI in a basic plasma physics setting and study the effects of thermal conduction and self-generated magnetic fields.

#### 3.1. Initial model

We considered a two-layer plasma system in two-dimensional Cartesian geometry, \((x, y)\). In our initial model, the top plasma layer consisted of high density carbon \((\rho = 2 \text{ g cm}^{-3})\) while the
bottom layer was made of a lower density carbon ($\rho = 1 \text{ g cm}^{-3}$). The system was initially at rest and placed in a uniform gravitational field, $\vec{g} = (0, -2 \times 10^{14}) \text{ cm s}^{-2}$, i.e. directed from the high density fluid to the low density fluid. This configuration is RT-unstable.

We imposed a sinusoidal single-mode density perturbation at the interface between the two layers, with peak-to-peak amplitude $A = 5 \mu m$ and wavelength $\lambda = 71 \mu m$, as in Kuranz et al. [1]. We defined the initial pressure profile by integrating the equation of hydrostatic equilibrium with the minimum ambient pressure of $5 \times 10^{11} \text{ dyn cm}^{-2}$. The resulting temperature varied from about $3.8 \times 10^{4} \text{ K}$ at the top of the domain to approximately $1.3 \times 10^{6} \text{ K}$ at the bottom.

In order to prevent the growth of spurious modes on small scales (due to a finite mesh resolution), we used a diffuse interface with a hyperbolic tangent density profile with 98% of amplitude variation over a distance of $5 \mu m$. This procedure largely regularizes the initial conditions and allows for more meaningful comparison of simulation results obtained at different mesh resolutions. In the case of diffuse interface, the RTI growth rate remains unchanged provided the interface thickness is small compared to the perturbation wavelength [34]. It is worth noting that in the HED laser-driven RTI experiments some smearing of material interfaces is expected to occur due to laser preheat [35].

In application simulations, the mesh was uniformly resolved and covered a rectangular region, $(x, y) \in [0, 71] \times [0, 284] \mu m$. To assess convergence of the numerical solution, we varied the number of mesh zones per the perturbation wavelength from 40 (L40 models) to 160 (L160 models). We used periodic boundary conditions in the horizontal direction and enforced hydrostatic equilibrium at the bottom and at the top of the computational domain.

### 3.2. Simulation results

#### 3.2.1. Pure hydrodynamics

In this model, the initial perturbation grows and retains its general sinusoidal profile on large scale until $t \approx 15$ ns. Soon after that time, the fluid on the spike shoulders begins to roll up resulting in the familiar mushroom-like RTI morphology (Fig. [5]). At $t \approx 25$ ns, the initial vortex is shed and a new one is formed at the leading section of the spike. The vorticity associated with
Figure 6: Rayleigh-Taylor instability: L160 model with various physics effects. Shown is the density distribution at $t = 20$ ns. (a) Hydrodynamics only. (b) Hydrodynamics with self-generation of magnetic fields. (c) Hydrodynamics with thermal conduction. (d) Hydrodynamics with self-generation of magnetic fields and thermal conduction.
those large scale flow features eventually induces a complex mixed flow structure on small scales
dominating the mid-section of the spike at the final time (see Fig. 7a).

Figure 8a shows lateral (in the direction perpendicular to gravity) averages of density in a
model with hydrodynamics only at the final time for various mesh resolutions. In this case,
positions of spike and bubble tips converge already at L40. The lateral density averages also
appear to converge to a single distribution inside the mixed region (e.g. low density region around
55 µm and a broad low density area around 150 µm). These results are qualitatively consistent
with the trends reported by Budde et al. [29].

3.2.2. Hydrodynamics with self-generated magnetic field

Compared to the pure hydro model, the influence of self-generated magnetic fields on RTI
appears negligible during the linear growth phase (Fig. 9a). However, around the time when
the width of mixed layer reaches about 50% of the perturbation wavelength, two small bulging
Figure 8: Lateral averages of density as a function of vertical position for L20 (squares), L40 (circles), L80 (triangles), and L160 (asterisks) models at the final time. (a) Hydrodynamics only. (b) Hydrodynamics with self-generation of magnetic fields. (c) Hydrodynamics with thermal conduction. (d) Hydrodynamics with self-generation of magnetic fields and thermal conduction.
structures become clearly visible at the interface located roughly half way between the spike tip and the bubble tip (Fig. 9b). We identify this new flow feature as an additional RTI mode. This mode is observed only in models with self-generated magnetic fields and without conduction (see Sect. 3.2.4 below for a description of the corresponding model with conduction).

During that time, the magnetic field rapidly builds up inside the interface with the average field reaching 1 MG around \( t = 10 \) ns (dotted line in Fig. 10). At that time, the maximum field strength is \( B_z \approx 3.7 \) MG (plasma \( \beta \approx 11 \)) along leading sides of the sinking spike. This additional pressure modifies the flow dynamics in the interface region with the most profound effect occurring near the spike tip. Figure 11 allows for comparison of the flow structure in the current model (right panel) to that in the pure hydro model (left panel). The important difference between the two solutions is a contribution of the self-generated magnetic field to the plasma pressure. The ratio between magnetic pressure and the gas pressure (inverse of plasma \( \beta \)) is shown as a gray contour map. The maximum pressure ratio is reached very near the middle of the interface (second from the top dashed contour line corresponds to mass fraction of the heavy fluid of 50%) and closer to the spike tip. The contribution due to magnetic pressure then gradually decreases away from that point and reaches zero at the very spike tip (no field is generated in that area). The fluid elements close to the spike center line are no longer smoothly accelerated (spacing between the velocity contours gradually increases until the mid-center of the interface profile in the left panel in Fig. 11, but are deflected toward the center line by pressure gradients due to generated magnetic field. In the process, the flow “chokes” with fluid elements located closest to the spike’s center line being decelerated most. This process of slowing down the flow near the spike’s center becomes increasingly more efficient as the magnetic field grows. Figure 9c (and also Fig. 6b) shows the central section of the spike surface bent inward by about 4 \( \mu \)m (\( \approx 5\% \) of the mixing...
Figure 10: Time evolution of the maximum horizontally averaged magnetic field strength in L160 RTI models with self-magnetization. (dotted) without conduction; (solid) with conduction.

Figure 11: Structure of the flow near the tip of RT spike at $t = 10$ in non-conducting models (a) without and (b) with self-generation of magnetic field. The resolution of both models is L160. Boundaries of the interface region between light and heavy fluid are marked with contours of mass fraction of the heavy fluid of 0.01, 0.5, and 0.99 (dashed lines). The magnitude of fluid velocity is shown with 20 equispaced contour lines between $1 \times 10^5$ cm/s and $2.5 \times 10^5$ cm/s. The darker the contour line the higher the velocity. See Sect. 3.2.2 for discussion.
length width) over the course of 10 ns. The rapid initial growth of the field saturates around $t = 20$ ns with the average maximum field strength reaching approximately 2.5 MG (dotted line in Fig. 10). At those intermediate times, $\beta$ is typically < 1 inside the interface and the strongest fields are observed at the outer edges of the RT mushroom cap. The rapid increase of the maximum field strength that starts around $t \approx 18$ ns also occurs in those regions and is due to fast bending of the magnetic field lines when the bottom sections of the cap roll inward. Strong fluctuations of the magnetic field observed at later times occur on small scales (see below) and their amplitude will likely be smaller in 3-D models and in realistic situations when the symmetry of the flow is broken by perturbations.

3.2.3. Hydrodynamics with thermal conductivity

The large temperature gradients present in the initial conditions make thermal conductivity a potentially important physical effect in our RTI system. Indeed, the most striking and common feature of all thermally conducting models considered here is strong mass diffusion across the interface, the resulting dramatically reduced amount of density structure, and much smoother morphology of the flow. In particular, the RT mushroom cap seen in the pure hydro model is much smoother when conduction is accounted for, and we do not observe any significant density roll-up (see Fig. 6c).

3.2.4. Hydrodynamics with thermal conductivity and self-generated magnetic field

The addition of self-generation of magnetic fields does not qualitatively change the evolution of thermally conducting model discussed in the previous section. The interface suffers strong diffusion at early times (Fig. 6d) and the RT mushroom cap remains underdeveloped (Fig. 7d). The initial magnetic field growth rate is by a factor of about 5 smaller than in the non-conducting case (solid line in Fig. 10), and the field strength reaches approximately 190 kG at $t = 10$ ns. Moreover, we do not see enhancement of small scale structure found in the non-conducting self-magnetized model (Fig. 7b). This is expected given the strong diffusive effects due to thermal conductivity. In consequence, rapid fluctuations of the magnetic field are absent in this case.

4. Discussion

4.1. Comparison of physics effects on RTI mixing

4.1.1. Mixing on large scales

To quantify the effects of mixing on large scales, we introduced a scalar passively advected with the plasma in the simulations. At the initial time, the value of the passive scalar was set to one inside the dense layer and to zero inside the light material. As the simulation progressed, the tracer was advected and tracked distribution of the dense carbon.

Figure 12 shows the evolution of positions of tips of RT spikes (where the lateral average of the tracer reaches 0.01) and bubbles (where the lateral average of the tracer decreases to 0.99) in L160 models with different physics. By comparing models without (top row of panels in Fig. 12) and with conduction (bottom row of panels in Fig. 12), one can clearly see that the mixed layer growth is chiefly controlled by thermal conduction. For example, the mixed layer width reaches about 185 \(\mu\)m in a pure hydrodynamic model and is reduced about 20% (mixed layer width $\approx 146\ \mu$m) when conduction is allowed for (Fig. 13). The observed reduction of the RTI growth is qualitatively consistent with theoretical predictions (see [34] and references therein). The effect is largely due to reduced speed of the RT spike; positions of bubble tips
Figure 12: Dependence of positions of tips of RT spikes and bubbles on model physics. Absolute distances normalized to the perturbation wavelength, $L$, for the RT spikes are shown with solid and dashed lines, respectively, as a function of normalized time, $Agr^2/L$. Here $A$ is the Atwood number corresponding to the fluid densities outside the smeared interface profile. The mesh resolution is $L_{160}$. (a) Hydrodynamics only. (b) Hydrodynamics with self-generation of magnetic fields. (c) Hydrodynamics with thermal conduction. (d) Hydrodynamics with self-generation of magnetic fields and thermal conduction. See Sect. 4.1.1 for details.

Figure 13: Dependence of laterally averaged structure of the RT mixed region on model physics. Lateral averages of density (left panel) and heavy fluid tracer mass fraction (right panel) are shown as a function of vertical distance at the final time for models with hydrodynamics only (squares), hydrodynamics with self-generation of magnetic fields (asterisks), hydrodynamics with thermal conduction (circles), and hydrodynamics with self-generation of magnetic fields and thermal conduction (triangles). See Sect. 4.1.1 for details.
remain essentially unchanged. In the case of non-conducting models, the mixed layer widths are very similar and self-magnetization leads to only slight increase of the mixed layer width (to about 186 µm).

The relative importance of thermal conduction compared to the contribution of self-generated magnetic fields is confirmed by close similarity of lateral averages of the density in thermally conducting models without and with self-generation of magnetic fields (see Figs. 8c and 8d, respectively). When thermal conduction is inefficient, the role of self-generated magnetic fields on the overall RTI growth appears negligible with the mixing layer width essentially identical to that obtained in the pure hydro case (cf. panels (a) and (b) in Fig. 12).

In passing we would like to note that these conclusions remain unchanged when analyzing relatively coarse L40 models. We believe that our best resolved simulations truthfully capture physics effects and are largely unaffected by numerical diffusion.

4.1.2. Mixing on small scales

We found that in the case of self-magnetized non-conducting flows much richer flow structure is created compared to pure hydro situations (see Figs. 7b and 7a, respectively), presumably due to presence and subsequent decay of additional higher-order modes. However, this additional small scale structure is no longer present when conduction is allowed for (Fig. 7d). We conclude that thermal conduction dominates the evolution of RTI on all scales and that self-generated magnetic fields play a relatively unimportant role in the evolution.

4.1.3. Characteristics of the self-generated magnetic field

In simulations that incorporate self-generation of magnetic field without thermal conduction, the magnetic field at \( t = 35 \) ns is about 7.5 MG. This corresponds to a maximum magnetic pressure on the order of 1.4 times the thermal pressure (plasma \( \beta \approx 0.72 \)). Furthermore, at intermediate times following saturation of the field, we found magnetic fields locally reaching 11 MG (\( \beta \approx 9.1 \times 10^{-2} \)). This can be compared to magnetic field strength of approximately 1.7 MG (\( \beta \approx 49 \)) found when the thermal conduction is allowed for. The magnetic field is lower in this case because magnetic source terms depend on density and temperature gradients, which are quickly decreased due to efficient thermal transport. Also, the field strengthens gradually and evolves smoothly both in time and space.

Using similar initial conditions but with the pre-existing magnetic field perpendicular to the interface (aligned with the direction of gravity), Fryxell et al. [36] has demonstrated that even relatively weak fields can significantly modify the RTI morphology on large scales. In particular, they found that the RT mushroom cap does not form in the case with a pre-existing magnetic field for \( \beta = 2500 \). However, the resulting morphology of their model (see Fig. 4 in [36]) does not resemble complex structure of our non-conducting model with self-generated field (cf. Fig. 7b). Instead, it is similar to morphology of featureless RT spikes seen in our thermally conducting models at early times (cf. Fig. 6c). We conclude that pre-existing smoothly ordered fields produce qualitatively different effects than self-generated magnetic fields and the results of such studies should not be in general used to interpret the evolution of self-magnetizing systems with zero-field initial conditions.

4.1.4. Anisotropy of heat fluxes

The degree of anisotropy of thermal transport due to magnetic field can be estimated based on the value of parameter \( \chi \) (cf. Eqs. 1 and 2, which depends on the magnetic field strength. The heat transport is essentially isotropic in situations with \( \chi \ll 1 \), it becomes progressively
more anisotropic as the field strengthens, and one may expect new effects due to anisotropy as $\chi$ reaches values of order $1 \times 10^{-2}$. In our subset of models without conduction (that produces the strongest fields), we find $\chi \approx 0.001$, which indicates that heat fluxes are essentially isotropic and effects due to anisotropic thermal conduction are negligible.

4.1.5. Modified RTI growth

As we mentioned in Sect. 3.2.2, the important effect of the magnetic field generated in the absence of thermal conduction is the creation of an additional RTI mode. Presence of this mode is clearly visible by comparing the results of a purely hydrodynamic model at $t = 20$ ns (Fig. 6a) to the corresponding non-conducting model with self-generated magnetic field (Fig. 6b). Nishiguchi [2] also found an additional mode in his simulations of RTI in the ICF setting. However, contrary to Nishiguchi’s interpretation, thermal conduction is unnecessary for creation of additional modes in our model. Nishiguchi’s mechanism requires strong magnetic fields in order to make the conduction anisotropic and its perpendicular components suppressed.

The essence of Nishiguchi’s mechanism is that it is the suppressed conductivity that causes the laser heat flux decrease at the corresponding section of the interface. In consequence, the ablation rate is locally decreased and hence the acceleration that drives RTI. In effect, the RTI growth near the spots with maximal $B$ is decreased, and so, similar to our model, an additional mode, not present in the pure hydro RTI, is generated. As we pointed in Sect. 3.2.2 the self-generated magnetic field is most likely directly responsible for creation of additional perturbation modes in our setup. In this interpretation, thermal conduction may only enhance the effect; our dynamical mechanism must be present in the Nishiguchi’s model as well. Unfortunately, Nishiguchi does not discuss the effects of self-generated magnetic fields in the absence of thermal conduction, and the data presented in his paper is insufficient to judge which of the two mechanisms dominates in his model. The problem of generation of additional RTI modes in the ICF regime deserves a careful study of individual physics effects, similar to presented in this work.

The condition for either of the two mechanisms of the additional mode generation to be effective is that $\beta$ should become $\lesssim 1$. Suitable physical conditions for this are high enough temperatures (a few hundred eV) so that resistive diffusion is small enough to allow the magnetic field to grow to MG-level values while localized close to the interface region (we estimate the resistive diffusion below, see Sect. 4.3); and substantial acceleration to keep the interface between the light and the dense plasmas thin. Both of these conditions appear satisfied in Nishiguchi’s setup.

4.2. Technical aspects

4.2.1. Numerical mesh convergence

We found that in the case of pure hydrodynamics the large scale flow features are well-resolved already in a relatively coarse L40 (40 zones per the perturbation wavelength) model. As we increase the resolution, the shear flow near the interface between the spike material and the ambient medium becomes better resolved allowing for more vigorous development of structure on small scales.

The increase of the amount of structure with the resolution and the corresponding lack of convergence on small scales is a well-known characteristic of RTI models. This is chiefly due to a lack of physics (such as viscosity) that would control flow development on small scales but above the mesh resolution limit. In this situation, one has to resort to a convergence metric insensitive to variations on small scales such as lateral averages.
The lateral averages of density for simulations of pure hydro and hydro with self-generated magnetic field simulations in Figs. 8a and 8b show that the spike position has converged by the L80 resolution. Oscillatory changes of the lateral averages seen near the spike in the L160 model with self-generated magnetic fields indicate magnetic fields are responsible for substantially different density structure in this model compared to pure hydro. The convergence in this model also appears the slowest. By contrast, the lateral averages for the two simulations that incorporate thermal conduction are nearly identical at all resolutions and appear to have converged already in the L40 model.

The flow structure on small scales is significantly richer in the case magnetic fields are self-generated and thermal conduction is absent (see Fig. 7b). This leads to visibly slower convergence of lateral averages of density with mesh resolution (Fig. 8b). The enhanced small scale structure is likely due to perturbations induced by the additional RTI mode, and it is conceivable that similar phenomena develop on smaller scales leading to perturbations at higher frequencies as the interface becomes more convoluted. We defer analysis of this interesting possibility to the future work.

Finally, the overall symmetry of the flow is well-preserved on large scales at all times, although the morphology appears asymmetric on small scales (Fig. 7b). Such asymmetries are typically observed in RTI simulations in the nonlinear phase when otherwise negligible numerical discretization errors become sufficiently large to perturb the flow.

4.2.2. Heat flux limiting

When electron heat flux according to Braginskii expressions becomes extreme, reaching a few percent of the free-streaming value, Braginskii’s value is no longer justified and flux-limiters are used:

\[
|\vec{q}_e| = \min(\kappa_e \cdot \nabla T_e, f n_e T_e (T_e/m_e)^{3/2}),
\]

with \( f \) usually chosen \( f \in [0.03; 0.1] \). At still larger gradients (of the temperature in part), when thermodynamical quantities change significantly over distances of order of electron mean free path \( l_e = \tau_e (T_e/m_e)^{1/2} \), MHD description becomes invalid as well. In our simulations the temperature along the interface varied in \([4 \times 10^{5}; 9 \times 10^{5}] K\) range (about the largest value we had in the simulations) and \( \kappa_e = 2.8 \times 10^9 \) g cm s \(^{-1} \) K \(^{-1} \) (at \( T_e = 9 \times 10^5 \) K) is at most \( 1.3 \times 10^{-4} \). Consequently there was no need to use flux limiters in our simulations.

4.2.3. Computational limitations due to thermal transport

We have also found that conducting models become prohibitively expensive at resolutions higher than L160 due to a time step limited by the thermal conduction timescale,

\[
\Delta t \leq \frac{1}{2} \frac{\Delta x^2}{D},
\]

where \( D \) is the temperature diffusion coefficient (cf. Eq. 3). For the current application, the thermal diffusion is dominated by free electrons and shows strong dependence on temperature,
$D \propto T^{5/2}$. This imposes a practical limit on maximum temperatures attainable in our computational models using explicit time integration to $1 \times 10^8$ K. Modeling situations at higher temperatures will require using implicit solvers for thermal conduction for the simulations to remain computationally feasible and numerically accurate.

4.2.4. Magnetic field growth limiter

Under high-energy density conditions considered here, magnetic fields are generated very quickly. In the absence of thermal conduction and with only minimal numerical diffusion due to zero initial velocities, temperature and density gradients remain large and close to their initial values for prolonged amounts of time. At the same time, the generated field is neither distributed (a numerical Z-pinch-like effect occurs due to assumed 2-D geometry) nor advected. Combination of those two effects occasionally results in production of magnetic fields so strong that the resulting magnetic pressure leads to a complete evacuation of computational zones and unphysical solutions. In order to avoid such situations, we limit the magnetic field growth to a small (typically a few percent) fraction of the existing field, provided the field is sufficiently strong,

$$\Delta B = \begin{cases} \min \left[ C_{\text{Bmax}} B, \Delta t \left( \frac{\partial B}{\partial t} \right) \right] & \text{if } (P_{\text{mag}} \geq C_\beta P_{\text{therm}}), \\ \Delta t \left( \frac{\partial B}{\partial t} \right) & \text{otherwise.} \end{cases}$$

where we have set $C_\beta = 1 \times 10^{-3}$. We experimented using various values of the field growth limiter, $C_{\text{Bmax}}$, and found that $C_{\text{Bmax}} = 3 \times 10^{-3}$ was sufficiently small to produce stable solutions in all cases with exception of L160 non-conducting model, which required $C_{\text{Bmax}} = 1 \times 10^{-3}$.

We believe the above field growth limiting strategy is justified as the procedure is only applied locally and does not affect large scale features of the solution. Also, we anticipate the above problem will not be present in 3-D due to qualitatively different geometry of the generated field.

4.3. Resistive effects

One can provide a rough estimate of magnetic field resistive diffusion by considering a one-dimensional diffusion problem with magnetic field source term of the form $f(t)g(x)$, located in a vicinity of the interface at $x = 0$. We take $g(x) = (\pi b^2)^{-1/2} \exp(-x^2/b^2)$, where $b$ is the characteristic thickness of magnetic field generating region ($b \approx 3 \mu m$ in our L160 model with no heat conduction, and $b \approx 15 - 20 \mu m$ in the models with Braginskii heat conduction.)

The solution of the diffusion problem with constant isotropic magnetic diffusivity $D_m$ and the above source term

$$\partial B(x,t)/\partial t = \nabla (D_m \nabla B) + f(t)g(x),$$

and with $B(x,0) = 0$ is

$$B(x,t) = \int_0^t d\tau f(t-\tau) \frac{\exp(-x^2/(b^2 + 4D_m \tau))}{\pi(b^2 + 4D_m \tau)^{1/2}}.$$  

When $D_m \to 0$, $B(x,t) \propto \int_0^t f(\tau) d\tau$, is similar at all $x$. We thus take $f(t) = f_0 = \text{const}$ to reproduce approximately linear in time growth of $B(x,t)$ at the interface at early times. At late times, $t \gg b/D_m$, the distribution of magnetic field has characteristic thickness of $(b^2 + 4D_m \tau)^{1/2}$, whereas $B(0,t)/B(0,0)|_{D_m=0} \approx b/\sqrt{D_m \tau}$.

Magnetic diffusivity coefficient $D_m$ can be expressed in terms of Braginskii’s $\alpha$ coefficients as $D_m = \frac{\alpha}{\beta} \left( \frac{\alpha}{\beta} \right)^2$. As with heat conductivity, resistivity (and hence magnetic field diffusion)
is essentially isotropic in our problem since $c = \omega_0 \tau_r \ll 1$. In our simulations, $D_m$ varies approximately between $1.7 \times 10^4$ and $0.6 \times 10^5$ cm$^2$ s$^{-1}$ along the interface. The corresponding temperatures are approximately $4 \times 10^3$ K and $9 \times 10^3$ K, respectively, with the electron number density, $n_e \approx 4.5 \times 10^{23}$ cm$^{-3}$. For the estimated average value of the diffusivity coefficient, $D_m = 1 \times 10^4$ cm$^2$ s$^{-1}$ (or $D_m = 1 \times 10^5$ µm$^2$ ns$^{-1}$), it takes $10^{-3}$ ns for the field to diffuse across the distance of 10 µm. Consequently, for the present application, we can expect resistivity starting to affect the magnetic field evolution for time intervals longer than about $10^{-3}$ ns.

At $t = 20$ ns and assuming characteristic thickness of the interface in our simulations with Braginskii heat conduction, $b = 15$ µm, we estimate the field will be reduced by a factor of $\approx \sqrt{D_m t/b} \approx 100$ compared to the non-resistive problem we consider in this work. One should note that at higher temperatures which are achieved in experiments [11], the role of resistivity will be smaller.

Another effect related to resistivity that we omitted in the present work is the thermoelectric heat flux, $\bar{q}^e_{\text{ff}} = -\frac{1}{2} \frac{\tau_e}{\tau_r} \mathbf{b} \cdot \mathbf{rot} \mathbf{B}$ [24]. Adopting the values for the maximal magnetic field strength and the characteristic length scale of the region where this field is localized at $t = 20$ ns, and depending on which physics is included, we estimate $|\bar{q}^e_{\text{ff}}| \in [10^{13}; 10^{17}]$ erg cm$^{-2}$ s$^{-1}$. For instance, if both resistivity and heat conduction are included, characteristic $B$ is about $5 \times 10^3$ G, spread over 3 mm, leading to $|\mathbf{rot} \mathbf{B}| \approx 2 \times 10^4$ G cm$^{-1}$ and the lower bound for $|\bar{q}^e_{\text{ff}}|$ in the estimate above; whereas not taking resistivity into account would yield $|\mathbf{rot} \mathbf{B}| \approx 2 \times 10^5$ G cm$^{-1}$ and the upper bound for the $|\bar{q}^e_{\text{ff}}|$ given. On the other hand, using our estimates presented in Sect. 4.2.2 above yields thermal heat flux $|\bar{q}^e_{\text{ff}}| \approx 7 \times 10^{17}$ erg cm$^{-2}$ s$^{-1}$.

We conclude that taking the thermoelectric heat flux into account should not change our simulation results significantly. Also, since magnetic fields generated in the thermally conducting model are already relatively weak, the resistive effects should not change RTI evolution in any qualitatively new way. These expectations will be tested against RTI models with resistivity taken into account in the next paper in the series.

4.4. Comparison with previously published experimental and theoretical work

Experiments. In a series of HED experiments on the OMEGA laser, Manuel et al. [3] studied the generation of magnetic fields induced by the Rayleigh-Taylor instability. Using the proton radiography technique and path integrated measurements they were able to estimate the magnetic field strength. Their estimates of the maximum field strength were obtained by assuming the field was localized in a region with a thickness on the order of the perturbation amplitude. The perturbation amplitude grew exponentially in time at a rate about 2.2 ns$^{-1}$, and the magnetic field reached its maximum strength of 0.15 MG at $t = 1.5$ (the end of the linear RTI stage). The authors did not provide estimates for the field strength at later times.

Megagauss-level magnetic fields were observed for the first time in HED experiments by Gao et al. [27]. The target used was a 15 µm thick polystyrene foil, compared to 21 µm thick foil used in Manuel et al. [3]. Gao et al. [27] were able to measure the fields well into nonlinear RTI regime, up to 2.56 ns after the laser beam arrival at the target. At that time the foil was broken apart by the instability. Maximal field strengths up to 2 MG were inferred from path-integrated measurements. Gao et al. [27] also considered a 25 µm thick foil and found sub-MG fields; the foil stayed unbroken, suggesting that the fastest field growth was attained in the late, highly nonlinear stage of RTI.

The relevant model for comparison to these experiments is our conducting model with self-generated magnetic field (see Sect. 3.2.4). In this model, the maximum magnetic field strength
initially rapidly increases to 83 kG at \( t = 1 \text{ ns} \), after which the growth slows and fields on the order of 103 kG are obtained at \( t = 2 \text{ ns} \). Between \( t = 2 \text{ ns} \) and \( t = 10 \text{ ns} \) the average rate of field growth is 27 kG ns\(^{-1}\). We find these results to be in good agreement with the experimental works, considering the differences between the initial conditions in our model (motivated by the Kuranz et al. experiment) and the actual conditions of the experiments described above. The field growth rate is higher in these experiments due to a target made of a thin foil. This is because for the same laser drive parameters, one obtains greater accelerations and so also greater RTI growth rates for lighter targets. (We estimate the characteristic RTI growth rate in our conducting model with self-magnetization to be about 0.14 ns\(^{-1}\) compared to 2.2 ns\(^{-1}\) obtained in the Manuel et al. experiment.) Indeed, Gao et al. \cite{27} reported that the foil remnants had a velocity of \((3 \pm 1) \times 10^7\) cm s\(^{-1}\) at \( t = 2.56 \text{ ns} \), implying average acceleration of \( \approx 10^{16} \text{ cm s}^{-2} \). This acceleration is nearly 2 orders of magnitude larger than the acceleration adopted in our study \(( g = 2 \times 10^{14} \text{ cm s}^{-2})\), which is representative of the Kuranz et al. experiment. In what follows, we discuss additional differences between our setup and the experimental designs.

No seed perturbations were machined on the foil used in the Gao et al. experiment. The authors reported \( \approx 82 \mu m \) wavelength perturbations dominating at \( t = 2.11 \text{ ns} \), and \( \approx 115 \mu m \) perturbations at \( t = 2.56 \text{ ns} \). These are comparable to \( \lambda = 71 \mu m \) perturbations used in Kuranz et al. \cite{1} experiment and in this study. Similar perturbation wavelengths were used in \cite{3}, with \( \lambda = 120 \mu m \) (and with an amplitude of \( a_0 = 0.27 \mu m \)). The differences in wavelengths between various setups are relatively small and unlikely to cause significant deviations in magnetic field growth rates. Reported densities of the target material during the experiments/simulations are also very close. The main factors that we link to the difference in \( B(t) \) are the interface acceleration, the temperatures near the interface (these strongly influence transport properties, and enter the Biermann battery term directly as the \( \nabla T_e \) factor), differences in the flow structure in ablative RTI setup compared to our classical RTI configuration.

**Theoretical and computational studies.** We begin a comparison of our simulation results to those reported by other groups with one of the early numerical studies of magnetic field generation by Mima et al. \cite{26}. The hydrodynamical process studied by that group was not RTI in its classical meaning, as no gravity or sufficiently smooth (on interface thickness scale) global acceleration was present. Instead, the authors studied equilibration in two setups. The first configuration included discontinuities in both pressure and temperature (“implosion setup”). In the second configuration the plasma was initially in pressure equilibrium but with a temperature discontinuity. The process of hydrodynamical equilibration was accompanied by the perturbations growing on initially flat interface separating two sections of the computational domain that contained plasmas at constant but different pressures and temperatures. The temperature varied between 50 eV and 200 eV, with the maximum temperature higher by a factor roughly 2.5 than the maximum temperature at the interface in our model \(( \approx 80 \text{ eV})\). The magnetic field growth was computed only using the Biermann battery, and the maximal fields obtained in the above two configurations were 3 MG and 0.3 MG, respectively. The magnetic field saturated after one half the sound crossing time of the domain, and thermodynamic equilibrium was achieved by about that time in both setups. Differences in the flow morphology obtained by \cite{26} and reported here are significant and do not allow for meaningful comparison of the results of the two studies.

A setup similar to ours was studied more recently by Nishiguchi \cite{2}. The simulation code used by Nishiguchi solved the full set of Braginskii’s equations in 2-D, and modeled laser energy deposition in the 50 \( \mu m \) thick target of CH\(_2\) composition. The target surface was perturbed using a single mode sinusoidal perturbation with \( \lambda = 100 \mu m \). Results of two types of simu-
lations were reported. In the first case, transport coefficients were fixed at their $B = 0$ values; in the second case, these coefficients depended on the magnetic field according to Braginskii’s formulation. Maximal B-field values reported for the above two cases at $t = 3.4$ ns were 2 MG and 8 MG, respectively. Such a substantial difference in field strengths was due to the increase of the temperature gradient in the case where thermal conductivity depended on the magnetic field (i.e., heat flux was reduced in the direction orthogonal to the magnetic field). This makes the Nishiguchi’s system substantially different from ours, because in our model the magnetic field was relatively weak and thermal conduction remained essentially isotropic.

The discrepancy between our models and Nishiguchi’s simulations stems from differences in temperature at the interface region. Although Nishiguchi does not provide the temperature near the interface in his model, he reports the maximal temperature in his simulations as $2.6 \times 10^7$ K, which is nearly 20 times higher than the maximum temperature recorded in our models. This results in $\chi = \omega_e \tau_e$ in Nishiguchi’s simulations being a factor of 200 greater than ours for the same magnetic field strength ($\tau_e$ scales approximately as $T_e^{3/2}/\overline{Z}$, with $\chi \approx 0.5$ for $B = 2$ MG). The thermal conductivity is thus reduced as $\kappa_\perp/\kappa_\parallel \approx 0.55$. In consequence, the temperature diffusion across the field lines is suppressed resulting in thinner interface and faster magnetic field growth. As the field progressively strengthens, thermal transport is reduced even more, increasing the field growth rate still further.

The higher temperatures found in the Nishiguchi’s study appear to be caused by both target design and driving energy. The target proposed by Nishiguchi is thinner and is continuously driven by a much stronger laser source. The laser intensity used by Nishiguchi was a constant $2 \times 10^{15}$ W cm$^{-2}$, while the laser drives used in experiments were weaker and lasted only for a short amount of time ($9 \times 10^{14}$ W cm$^{-2}$ for 1 ns in [1]; $4 \times 10^{14}$ for 2 ns in [3, 27]). Although [2] does not provide the effective acceleration value, assuming a 2 MG magnetic field and a few micron thick interface, we can use the results shown in his Fig. 1 to estimate $g \approx 10^{16}$ cm s$^{-2}$. This estimate is consistent with the results presented by [27], and again much higher than in our model.

Apart from the computer simulations discussed above, Nishiguchi [2] and Manuel et al. [3] independently provided estimates for the resistive diffusion. In particular, Manuel et al. [3] estimated the resistive diffusion time on the order of 1 ns. This implied a reduction of the magnetic field by a factor 2.5 compared to the non-resistive result. We note that we used a different approach to estimate resistive diffusion (see Sect. 4.3). Specifically, we did not substitute the spatial differential operator with a wavenumber as Nishiguchi and Manuel et al. did. This difference in the analysis method is especially important in case of thin interfaces and at early times. This is because, and as we have shown, during the initial stages magnetic field mostly diffuses away from the interface, not along it. This means that the magnetic field is distributed in space with characteristic thickness different than that of the interface or the interface perturbation wavelength, invalidating the approach adopted by Manuel et al. . In consequence, Manuel et al. overestimated the diffusion timescale and underestimated the field reduction factor.

We conclude that the magnetic field strength and its growth rate obtained in our numerical simulations are consistent with those obtained in simulations by other groups and in the recent HED experiments provided differences in the problem setting are taken into account.

5. Conclusions

Motivated by discrepancies between the results of the Rayleigh-Taylor instability (RTI) high-energy density laser experiments and computer simulations, we investigated the effects of physics
previously not included in computer models on the instability growth and morphology of the flow.

Our physics model is based on the Braginskii formulation \cite{24}, but we do not account for resistive effects. We implemented anisotropic thermal conduction and source terms for self-generation of magnetic fields in the FLASH-based Proteus code. The new computational modules were subjected to a verification process and produced correct results for several test problems.

Using the verified code, we performed a series of simulations of a single mode RTI in 2-D planar geometry for conditions relevant to the experiments. Models differed in mesh resolution and physics. We found that in models with thermal conduction the mixed layer width converged in simulations with 40 mesh cells per perturbation wavelength; twice higher resolution was required to obtain convergence when the thermal conduction was absent.

For relatively moderate HED plasma conditions considered in the present work, we found that thermal conduction inhibits development of small scale structure and adversely affects the RTI growth due to strong mass diffusion stemming from heat transport and the related density gradient reduction. Our result provides a plausible explanation for the relatively featureless RTI spikes characteristic of the laboratory experiments. However, we do not observe the mass extensions seen in those experiments, even in simulations with no thermal conduction that produce relatively strongest RTI growth.

We found that magnetic fields grow up to approximately 11 MG (plasma $\beta \approx 9.1 \times 10^{-2}$) in the absence of thermal conduction, with the average fields on the order of 2.5 MG. These fields do not affect the integral RTI growth rate but do change RTI morphology on scales comparable to the perturbation wavelength. Specifically, we found evidence of additional higher order mode development due to dynamical effect of these fields. Generation of such a mode was reported by Nishiguchi \cite{2} who interpreted the additional mode as being due to the (perpendicular) thermal conduction components getting reduced by the field. Importantly, in the Nishiguchi’s model, this field is maximal in the same region of the interface as in our simulations.

We also found that self-magnetization enhances RTI mixing on small scales, possibly due to contribution and decay of additional higher-order modes. Furthermore, we found that magnetic fields are responsible for deceleration of the flow near and along the spike’s center line resulting in significant “denting” of the spike’s surface.

We identified generation of additional RTI mode(s) and changes of the spike morphology as unique features of our non-conducting self-magnetized models. This finding suggests that the pre-existing, smoothly ordered fields produce qualitatively different effects than those in which magnetic fields are self-generated. We conclude that the results of studies using such pre-existing fields in general are not helpful in understanding the evolution of self-magnetizing systems with zero-field initial conditions.

In our fully integrated model with self-generation of magnetic fields and thermal conduction, temperature gradient and density gradient were quickly reduced resulting in weaker magnetic fields, $B \approx 1.7$ MG (plasma $\beta \approx 49$). These magnetic fields were too weak to make heat transport strongly anisotropic or produce additional RTI modes. More importantly, we found these fields of comparable strength to self-generated magnetic fields observed recently in HED laboratory experiments by Manuel et al. \cite{3}.

The present study offers several directions for future research. For example, 3-D simulations are necessary to describe the dynamics involved in RTI more truthfully, especially with regard to the process of competition between hydrodynamic drag and buoyancy \cite{38, 39}. The topology of self-generated magnetic fields in 3-D will also be different and may result in new phenomena, and their morphology might be modified on small scales due to resistive effects we did not include
in this work. Finally, future studies should be conducted in the regime more closely matching conditions of the past and planned high-energy density RTI laboratory experiments.

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