Energy dependence of the $\bar{K}N$ interaction and the two-pole structure of the $\Lambda(1405)$ – are they real?

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Abstract. It is shown, that the energy-dependence of the chiral based $\bar{K}N$ potentials, responsible for the occurrence of two poles in the $I=0$ sector is the consequence of applying the on-shell factorization approximation [1]. When the dynamical equation is solved without this approximation, the $T$-matrix has only one pole in the region of the $\Lambda(1405)$ resonance.

Introduction

The $\Lambda(1405)$ is one of the basic objects of strangeness nuclear physics (SNP). Experimentally it is a well pronounced bump in the $\pi\Sigma$ missing mass spectrum in various reactions somewhat below the $K^-p$ threshold with PDG resonance parameters $E - i\Gamma/2 = (1405 - 25i)\text{MeV}$. Theoretically it is an $I = 0$ quasi-bound state in the $\bar{K}N - \pi\Sigma$ system, which decays into the $\pi\Sigma$ channel.

Constructing any multichannel $\bar{K}N$ interaction, the starting point of any SNP study, one of the first questions is: "What kind of $\Lambda(1405)$ it produces?" At present, it is believed, that theoretically substantiated $\bar{K}N$ interactions can be derived from the chiral perturbation expansion of the $SU(3)$ meson-baryon Lagrangian. For these interactions the widely accepted answer to the above question is, that the observed $\Lambda(1405)$ is the result of the interplay of two $T$-matrix poles. Our aim is to challenge this opinion.

The full and on-shell factorized WT potentials $\hat{V}$ and $\hat{U}$.

Our starting point is the lowest order Weinberg-Tomozawa (WT) term of the chiral Lagrangian (eq.(7) from the basic paper [1]):

$$\langle q_i|v_{ij}|q_j\rangle \sim -\frac{c_{ij}}{4f_\pi^2}(q_i^0 + q_j^0),$$

where $i$ and $j$ denote the different meson-baryon channels ($i,j = 1, 2, 3, 4, 5 = [\bar{K}N]^{I=0}, [\bar{K}N]^{I=1}, [\pi\Sigma]^{I=0}, [\pi\Sigma]^{I=1}, [\pi\Lambda]^{I=1}$), $q_i$ and $q_i^0 = \sqrt{m_i^2 + q_i^2}$ denote the meson c.m. momentum and energy, $c_{ij}$ are $SU(3)$ Clebsch-Gordan coefficients and $f_\pi$ is the pion decay constant, $m_i(M_i)$ are the meson (baryon) masses.
Physical quantities can be derived from this expression via a certain dynamical framework, relativistic (BS equation, relativistic kinematics) or non-relativistic (LS equation, non-relativistic kinematics). We shall use the second option, having in mind applications for \( n > 2 \) systems. According to our choice and the usual practice, eq. (1) has to be supplemented: adding appropriate cut-off factors, applying a relativistic correction to meson energies and introducing two meson decay constants instead of \( f_x \):

\[
\langle q_i|v_{ij}|q_j \rangle = -\frac{c_{ij}}{64\pi^3 F_i F_j \sqrt{m_i m_j}} (q_i^{0'} + q_j^{0'}),
\]

where

\[
q_i^{0'} = q_i^0 + \frac{q_i^0 - m_i}{2M_i} = q_i^0 + \frac{q_i^2}{2M_i} \approx m_i + \frac{q_i^2}{2\mu_i}
\]

with the reduced mass \( \mu_i = m_i M_i/(m_i + M_i) \) and \( F_i, i = K, \pi \) are the new meson decay constants. In order to use the potential (2) in LS equation a regularization procedure has to be applied to ensure convergence of the occurring integrals. We use the separable potential representation of the interaction, which amounts to multiplying the potential (2) by suitable cut-off factors \( u_i(q_i) \) and \( u_j(q_j) \). Finally, the potential \( V_{ij} \) entering the LS equation for total energy \( W \)

\[
\langle q_i | T_{ij}(W) | q_j \rangle = \langle q_i | V_{ij} | q_j \rangle + \sum_s \int \langle q_i | V_{is} | q_s \rangle G_s(q_s; W) \langle q_s | T_{sj}(W) | q_j \rangle d q_s
\]

has the form

\[
\langle q_i | V_{ij} | q_j \rangle = u_i(q_i) \langle q_i | v_{ij} | q_j \rangle u_j(q_j) = \lambda_{ij} (g_i A(q_i) g_j B(q_j) + g_i B(q_i) g_j A(q_j))
\]

which is a two-term multichannel separable potential with form factors

\[
g_i A(q_i) = u_i(q_i); \quad g_i B(q_i) = g_i A(q_i) \gamma_i(q_i); \quad \gamma_i(q_i) = (m_i + q_i^2)/(2\mu_i),
\]

and coupling matrix

\[
\lambda_{ij} = -\frac{c_{ij}}{64\pi^3 F_i F_j \sqrt{m_i m_j}}.
\]

The non-relativistic propagator \( G_s(q_s; W) \) has the form

\[
G_s(q_s; W) = (W - m_s - M_s - q_s^2/(2\mu_s) + i\epsilon)^{-1} = \frac{2\mu_s}{k_s^2 - q_s^2 + i\epsilon},
\]

where \( k_s = \sqrt{2\mu_s(W - m_s - M_s)} \) is the on-shell c.m. momentum in channel \( s \).

A commonly used procedure before solving the integral equation (4) is to remove the inherent \( q \)-dependence of the potential by replacing \( q_i \) in \( \gamma_i(q_i) \) by its on-shell value \( k_i; \quad \gamma_i(q_i) \rightarrow \gamma_i(k_i) = W - M_i \). This is the so-called on-shell factorization approximation, introduced in [1] and never checked afterwards. The
separable potential representation of the interaction allows an exact solution of the LS equation \( \text{(4)} \) for both versions of the potential: the "full" WT potential

\[
\hat{V} = \sum_{i,j} |g_{iA}\rangle \lambda_{ij} \langle g_{jB}| + |g_{iB}\rangle \lambda_{ij} \langle g_{jA}|
\] (7)

and its on-shell factorized energy-dependent counterpart

\[
\hat{U}(W) = \sum_{i,j} |g_{iA}\rangle \lambda_{ij} (2W - M_i - M_j) \langle g_{jA}|
\] (8)

providing thus a check of the effects of this approximations. Moreover, the separable potential approach offers an insight into the nature of the on-shell factorization approximation. When calculating the Green’s function matrix elements containing \( g_{iB} \)-type form-factors, e.g.

\[
\langle g_{iA}|G_i(W)|g_{iB}\rangle = 2\mu_i \int \frac{u_i(q)^2 \gamma_i(q)}{k_i^2 - q^2 + i\varepsilon} \, dq,
\]

the on-shell factorization replaces \( \gamma_i(q) \) under the integration sign by \( \gamma_i(k_i) = W - M_i \) and puts it outside the integral. It can be seen, that for real, positive \( k_i \), when the integral is singular this might have some justification, however, for complex (or imaginary) \( k_i \), which is the case when bound states or complex pole positions are sought, the approximation seems to be meaningless.

**Numerical results**

Practical solution of eq. (4) starts with an appropriate choice of the form- or cut-off factors \( u_i(q) \), which ensures the convergence of all occuring integrals. In our case it was the dipole Yamaguchi form with adjustable cut-off (or range) parameters \( \beta_i \):

\[
u_i(q) = \left( \frac{\beta_i^2}{q^2 + \beta_i^2} \right)^2
\]

The details of the formalism for the actual calculations can be found in [2]. Both potentials \( \hat{V} \) and \( \hat{U} \) depend on the same set of 7 parameters \( F_K, F_\pi, \beta_1, \beta_2, \beta_3, \beta_4 \) and \( \beta_5 \) which have to be fitted to the available experimental data, which are the 6 low-energy elastic and inelastic \( K^- N \) cross sections, the 3 threshold branching ratios \( \gamma, R_n, R_e \) and the 1s level shift \( \Delta E \) in kaonic hydrogen. The results of the fit for the two potentials are shown in Table 1 and Fig 1. Table 2 shows the obtained parameter values.

More or less equal quality fits can be obtained for both potentials but for very different parameter values. This means, that \( \hat{U} \) can not be considered as an approximation to \( \hat{V} \) – they are basically different interactions. Their most significant difference is, that, while the full WT potential \( \hat{V} \) produces a single pole in the region of \( \Lambda(1405) \), the on-shell factorized potential \( \hat{U} \) for any reasonable

\[1 \text{ For their definition see [2]}\]
Fig. 1: Elastic and inelastic $K^- p$ cross sections for the potentials $U$ and $V$.

combination of parameters produces the familiar two poles. The pole positions for the two potentials are

$$z_1 = (1425 - 21i)\text{MeV} \quad \text{for} \quad \hat{V} \quad \text{and}$$

$$z_1 = (1428 - 35i)\text{MeV} \quad \text{and} \quad z_2 = (1384 - 62i)\text{MeV} \quad \text{for} \quad \hat{U}.$$ 

The position of the single $\hat{V}$ pole does not confirm the strong $K^- p$ binding, which is the main feature of the phenomenological potentials adjusted to the PDG pole position.

|   | $V$  | $U$  | Exp               |
|---|------|------|-------------------|
| $\gamma$ | 2.32 | 2.35 | 2.36 ± 0.04       |
| $R_c$   | 0.671| 0.664| 0.664 ± 0.011    |
| $R_n$   | 0.202| 0.194| 0.189 ± 0.015    |
| $\Delta E$ (eV) | 350 − 279 $i$ | 302 − 294 $i$ | (283 ± 36) − (271 ± 46) $i$ |

Table 1: Calculated and experimental values of the discrete data for potentials $\hat{U}$ and $\hat{V}$.

Most recent and accurate information on the $\Lambda(1405)$ resonance comes from the CLAS photoproduction experiment $\gamma + p \rightarrow K^+ + \pi^0 + \Sigma^0$ [3] in which the $M(\pi^0 \Sigma^0)$ missing mass spectra give the $\Lambda(1405)$ line shape. For the analysis of these spectra at present we have the semi-phenomenological final state
interaction formula of Roca and Oset [4], which contains some adjustable parameters $c_i$ and the $T$-matrix elements of the $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential. Using energy-dependent (two-pole) $\bar{K}N$ potentials, with simultaneous variation of $c_i$ and the potential parameters, acceptable fits to the CLAS data were obtained. This fact then was considered as the ultimate proof of the two-pole structure of the $\Lambda(1405)$.

Using the same formula (corrected for the repeated use of on-shell factorization), we have made a preliminary fit to the few lowest $\gamma$-energy bin CLAS data varying only the $c_i$-s with our unchanged single-pole potential $\hat{V}$. The results are shown in Fig. 2.

It does not seem, that another pole is necessarily needed to improve these fits. A complete analysis of the CLAS data, including the charged channels is the subject of a forthcoming work.

|   | $F_e$ | $F_K$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ |
|---|---|---|---|---|---|---|---|
| $\hat{V}$ | 80.8 | 132 | 1094 | 960 | 516 | 537 | 629 |
| $\hat{U}$ | 107 | 109 | 1247 | 1622 | 919 | 959 | 443 |

Table 2: Parameters of the potentials $\hat{V}$ and $\hat{U}$ (all values are given in MeV).

Conclusions

- It was shown, that the energy-dependence of the WT term of the $\bar{K}N$ interaction, derived from the chiral $SU(3)$ Lagrangian and responsible for the appearance of a second pole in the $\Lambda(1405)$ region, follows from the unjustified application of the on-shell factorization approximation.
- Without this approximation a new, chiral based, energy-independent $\bar{K}N$ interaction was derived, which supports only one pole in the region of the $\Lambda(1405)$ resonance.
- The widely accepted ”two-pole structure” of the $\Lambda(1405)$ state thus becomes questionable.
• In coordinate space calculations for \( n > 2 \) systems the use of the new potential avoids the not easily (and not uniquely) surmountable difficulties arising from the energy dependence of the two-body interaction.

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References

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