Density dependence of quark masses and stability of color-flavor locked phases
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Considering the density dependence of quark masses, we investigate the color-flavor-locked matter and its stability relative to the (unpaired) strange quark matter. We find that, when the current mass of strange quark $m_s$ is small, the strange quark matter remains stable for the moderate baryon densities. When $m_s$ is large, the gapless phase of the color-flavor-locked matter is found to be difficult to be stable. A schematic phase diagram of three-flavor quark matter is presented, in which the color-flavor-locked phase region is suppressed in comparison with the previous results.

PACS number(s): 25.75.Nq, 12.39.Ba, 12.38.-t

I. INTRODUCTION

The quark matter with three flavors ($u$, $d$ and $s$) has been intensively studied for two decades. When the down quark chemical potential is larger than the strange quark masses, the strange quark matter (SQM) might be energetically favored with respect to two-flavor quark matter and even nuclear matter so that it should be the ground state of strongly interacting matter [1]. Within the framework of the bag model, Farhi and Jaffe pointed out that SQM with the strange quark mass $m_s < 140$MeV (and with appropriate bag constant) becomes the stable ground state for low baryon densities [2]. Based on this consideration, it is further speculated that some compact stars are made up not of neutrons but SQM, which are termed as strange quark stars [3]. On the other hand, the study of dense quark matter draws much attention due to the recent progress in understanding of color superconductivity. At high densities, the original color and flavor symmetries of three-flavor QCD, namely $SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R$, is suggested to be broken down to a diagonal subgroup $SU(3)_{\text{color}+L+R}$ via the Bardeen-Cooper-Schrieffer (BCS) pairing [4]. The three-flavor quark matter with the particular symmetry is called the color-flavor locked (CFL) matter and it is different from SQM which is the matter without the BCS pairing. As another state of strongly
interacting matter, the CFL matter is widely believed to become "absolutely" stable for sufficiently high densities [5].

Thus there are two candidates for the ground state of three-flavor quark matter, CFL and SQM, which are stable for high and low densities respectively. The question is, in the moderate density region, which one of them is the ground state. In the other words, one concerns how SQM undergoes a phase transition to CFL with increase of density. Investigation on these issues is important for exploring the physics of strange quark stars and/or the interior structure of compact stars. Ignoring the $u$ and $d$ quark masses, the CFL free energy takes the form

$$\Omega_{CFL} = - \frac{3\mu^4}{4\pi^2} + \frac{3m_s^2\mu^2}{4\pi^2} - \frac{m_s^4 - 12m_s^4\ln\frac{m_s}{2\mu}}{32\pi^2} - \frac{3\Delta^2\mu^2}{\pi^2},$$

(1)

to the fourth order in $m_s$, where $\mu$ is the quark chemical potential and $\Delta$ denotes the color superconducting gap. By comparing Eq.(1) with the free energy of the neutral unpaired quark matter at the same order, Alford et. al. concluded that CFL is more stable than SQM as long as [6]

$$\mu \geq \frac{m_s^2}{4\Delta}.$$  

(2)

As the necessary condition for the CFL presence, Eq.(2) is valid only for high densities [7]. This inequality can not fully answer the question raised above because it does not address the phase structure for moderate densities.

To illustrate this point more clearly, we draw the phase diagram based on Eq.(2) in the $(m_s, \mu)$ plane (Fig. 1)\(^1\). When the strange quark mass is small as $m_s < 175$MeV, Fig. 1 shows that SQM is excluded completely from the moderate density region $\mu = 0.3 - 1$GeV and CFL (including its gapless phase, see the following) dominates all over. However, for small $m_s$, SQM has been predicted to be the stable ground state [2] so that it should be favorable at least for low densities such as $\mu \sim 0.3$GeV. If assuming that CFL emerges in strange quark stars, this contradiction becomes more

\(^1\)Until now the actual value of $\Delta$ and its dependence on $\mu$ and $m_s$ are not been well known, which are closely linked to the gap equation. In the literature, $\Delta$ was estimated to be of order tens to $100$MeV. In Fig. 1 it is simply treated as a given parameter $\Delta \sim 25$MeV for moderate $\mu$, say, $\mu \sim 0.5$GeV [8].
obvious. Starting at very low density and increasing the matter density by increasing \( \mu \), the CFL formation must be preceded by the appearance of the stable SQM state. From this point of view, SQM remains stable for relatively low densities; otherwise, the self-bound quark stars could not exist and then the CFL formation would be impossible. Therefore, the phase diagram Fig. 1 is problematic especially for relatively low densities.

On the other hand, the so-called gapless CFL phase (gCFL) is shown in Fig. 1, where gCFL separates CFL (namely the conventional CFL phase) and SQM from each other. As suggested by the authors of Ref.[8], the nonzero value of \( m_s \) plays a key role in triggering the gCFL phase. When \( m_s \) is relatively large, thus, it seems reasonable that gCFL replaces CFL to be more stable in the moderate density region. However, Casalbuoni et al. have pointed out recently that gCFL is actually unstable due to the pure imaginary masses of the gluons in this phase [9]. It implies that the gCFL phase region shown in Fig. 1 might be incorrect. In addition, the SQM-gCFL transition curve in Fig. 1 was simply determined by the critical relation of Eq.(2). Since the validity of Eq.(2) is worthy of doubt for moderate densities, the gCFL phase region shown in Fig. 1 is problematic also.

In fact, the implicit assumption for Fig. 1 is that only the current mass of strange quark \( m_s \) was considered in the descriptions for SQM, CFL and gCFL. According to the low-density QCD, the strange quark mass not merely originates from the explicit breaking of chiral symmetry. For low densities where SQM exists as the stable ground state, there is no reason to neglect the dynamical mass induced by the spontaneous chiral breaking. Once the dynamical mass is taken into account, it has been found that the SQM stability window, e.g. the allowed region of the current mass \( m_s \), is widened [10, 11]. This motivates us to reexamine the phase diagram involving SQM and CFL (gCFL) in the framework where the dynamical mass is incorporated. In the present paper, we will introduce the density dependence of quark mass to investigate the phenomenological effects of the dynamical mass on the moderate-density phase.
diagram. This approach should be closer to reality and obviously helpful to clarify the
problem of Fig. 1. In Sec.II, we briefly review the mass-density-dependent model [10]
and consider the free energies of CFL and gCFL when the density-dependent quark
mass is incorporated. In Sec.III, we investigate the phase transitions from SQM to
CFL and/or gCFL and then present a new phase diagram which is very different from
Fig. 1.

II. THE MODEL

Following Ref.[10], the density-dependent quark mass is given by

\[ m_D = C/(3\rho), \]  

where \( \rho \) denotes the matter density and \( C \) is a model parameter which is constrained
by the SQM stability conditions. If ignoring the current masses of \( u \) and \( d \) quarks, the
masses for the light and strange quarks in this model are

\[ M_u = M_d = m_D; \quad M_s = m_s + m_D, \]  

respectively.

The SQM free energy contributed by the Fermi gas reads [10]

\[
\Omega(\mu_i, M_i, p^i_F) = \sum_{i = u, d, s} \int_0^{p^i_F} \frac{3}{\pi^2} p^2 (\sqrt{p^2 + M_i^2} - \mu_i) dp
\]

\[
= - \sum_{i = u, d, s} \frac{1}{4\pi^2} [\mu_i p^i_F (\mu_i^2 - \frac{5}{2} M_i^2) + \frac{3}{2} M_i^4 \ln(\frac{\mu_i + p^i_F}{M_i})].
\]  

For each flavor the Fermi momentum \( p^i_F \) is defined by \( p^i_F = \sqrt{\mu_i^2 - M_i^2} \) where \( \mu_u = \mu - 2\mu_e/3 \) and \( \mu_d = \mu_s = \mu + \mu_e/3 \) if the electron chemical potential \( \mu_e \neq 0 \). On
the SQM side, the Fermi momenta of \( u, d \) and \( s \) quarks are different and are related
to the corresponding densities via \( \rho_i = (p^i_F)^3 / \pi^2 \). Therefore, for SQM, the electrical
neutrality is realized by

\[
\frac{2}{3} \rho_u - \frac{1}{3} (\rho_d + \rho_s) = \rho_e = \frac{\mu_e^2}{3\pi^2},
\]  

and the baryon density is
\[ \rho = \frac{1}{3}(\rho_u + \rho_d + \rho_s). \] (7)

When the contribution from electrons is included, the total free energy for SQM becomes
\[ \Omega_{SQM} = \Omega(\mu_i, M_i, p_F^i) - \frac{\mu_e^4}{12\pi^2}. \] (8)

With respect to nuclear matter, SQM becomes energetically stable for low densities as long as its energy per baryon satisfies
\[ \left(\frac{\mathcal{E}}{\rho}\right)_{SQM} \leq 930\text{MeV}, \] (9)
at zero pressure, where 930MeV corresponds to a typical value of the energy per baryon in nuclei. For our purpose, Eq.(9) needs to be considered seriously to guarantee that not nuclear matter but SQM undergoes a phase transition to CFL (if without this constraint the nuclear-CFL transition [6] would be very likely). In this work, we fix the parameter \( C \) by the critical condition of Eq.(9) for certainty. For instance, the value of \( C \) is adopted to be 110 and 70MeV/fm\(^3\) as \( m_s = 0 \) and 180MeV respectively (see Ref.[10] for details).

On the other hand, the energy per baryon for two-flavor quark matter (2QM) is required to satisfy the inequality
\[ \left(\frac{\mathcal{E}}{\rho}\right)_{2QM} > 930\text{MeV}, \] (10)
at zero pressure [2]. By using Eqs.(9) and (10), the stability window can be obtained in which SQM corresponds to the stable ground state at low densities. But Eq.(10) does not apply to the moderate-density case. Due to the appearance of strange flavor, SQM is favored over the regular two-flavor matter as long as
\[ \mu_d = \mu + \mu_e/3 \geq M_s. \] (11)

Instead of Eq.(10), thus, Eq.(11) needs to be taken into account in the following calculation.
Different from the unpaired one, the CFL quark matter is an insulator in which no
electrons are required for the electrical neutrality [12]. The Fermi momenta have the
common value [6, 12]
\[ \nu = 2\mu - \sqrt{\mu^2 + m_s^2/3}, \] (12)
for all three flavors and \( \mu_e \) does not influence the CFL free energy directly \(^2\). Thus the
CFL free energy contributed by the Fermi gas is obtained by replacing the variables
\( \mu_i \) and \( p_F^i \) in Eq.(5) by \( \mu \) and \( \nu \) respectively. Together with the contribution from the
CFL pairing ( the last term in the RHS of Eq.(1) ), the total free energy for CFL takes
the form
\[ \Omega_{CFL} = \Omega(\mu, M_i, \nu) - \frac{3\Delta^2 \mu^2}{\pi^2}, \] (13)
when the density dependence of quark mass is considered. At high density, the value
of \( m_D \) is close to zero so that the difference between Eqs.(1) and (13) becomes neg-
ligible. For the concerned density region, Eq.(13) means that not only \( m_s \) but also
\( m_D \) contribute to the CFL free energy. As a consequence, the previous results of the
SQM-CFL transition e.g. Eq.(2) are no longer valid at low/moderate densities ( see
Sec.III for details ).

Then we turn to consider the gCFL phase in the model where the density depen-
dence of the quark mass is included. At sufficiently high densities, it is well known
that \( m_s^2/(2\mu) \) is regarded as the chemical potential associated with strangeness, i.e.
\( \mu_S = m_s^2/(2\mu) \) [14]. For the Cooper pairs between the blue-down ( \( bd \) ) and green-
strange ( \( gs \) ) quasi quarks, the effective chemical potential for the \( gs \) modes is influ-
enced by \( \mu_S \) while that for the \( bd \) modes is independent of \( \mu_S \). In this case, the effective

\(^2\)Assuming that the SQM-CFL transition is of first order, there is an interface between the electron-
rich SQM and the electron-free CFL. In this case, the effective value of electron chemical potential
is zero on the CFL side because of the electrostatic potential at the metal-insulator boundary ( see
Refs.[6, 13] for details ). In the present work, a possibility of the mixed state consisting of the
electrical-opposite SQM and CFL will be ignored. The reason is that the nonzero \( m_D \) provides the
additional instanton interaction so that the electrical-negative phase such as CFL\( K^- \) is very difficult
to emerge.
chemical potentials $\mu_{gs}^{\text{eff}}$ and $\mu_{bd}^{\text{eff}}$ become different and the relative chemical potential of the paired $bd$ and $gs$ modes becomes \[ \delta \mu = \frac{\mu_{bd}^{\text{eff}} - \mu_{gs}^{\text{eff}}}{2} = \frac{m_s^2}{2\mu}, \]

where the contribution from the chemical potential associated with the color charge has been included \[7\]. When the variation $\delta \mu$ is larger than the color superconducting gap, i.e.

\[ \frac{m_s^2}{2\mu} \geq \Delta, \]

gCFL is more stable than CFL. As a consequence, the free energy contributed from the gapless phenomenon depends on the comparison between the variation $\delta \mu$ and the color superconducting gap $\Delta$ mainly \[15, 16\]. A natural question is whether the variation $\delta \mu$ and then the gapless phenomenon are influenced by the nonzero $m_D$ also. In view of the fact that $m_D$ is the dynamical mass for all three flavors, it cannot enter the strangeness chemical potential $\mu_S$ via a simple replacement $m_s \rightarrow M_s = m_s + m_D$. Therefore, an extrapolation like $\delta \mu \rightarrow \delta \mu' = \frac{M^2_s}{2\mu} = \frac{(m_s + m_D)^2}{2\mu}$ is not feasible in principle. Thus it is reasonable to assume that $\delta \mu$ is independent of $m_D$ and Eq.\((14)\) holds unchanged when the density-dependent quark mass is considered. Based on the simple ansatz, Eq.\((15)\) is still valid as the condition for the gCFL formation although the SQM-gCFL transition needs to be reexamined seriously.

**III. NUMERICAL RESULTS AND DISCUSSIONS**

As a strong-coupling effect, the nonzero $m_D$ is expected to affect the phase diagram of three-flavor quark matter for not-very-high densities. Before going to be specific, let us firstly discuss *whether or not* the phase transition between SQM and CFL/gCFL occur in the moderate density region. The answer is not always positive and it is actually linked to the value of $m_s$, as argued in the following. Now both SQM and CFL/gCFL are the deconfined phases, therefore the physics of confinement does not play a role in determining the SQM-CFL/gCFL transitions. So the negative values of
the free energies obtained in Sec. II are related to the corresponding pressure directly
and then the Gibbs condition for the pressure equilibrium reads

\[ P_{CFL/gCFL} - P_{SQM} = \Omega_{SQM} - \Omega_{CFL/gCFL} = 0. \]

(16)

For \( m_s = 10, 50, 100 \) and \( 150 \text{MeV} \), we show \( \delta P = P_{CFL} - P_{SQM} \) as a function of
\( 1/\mu \) in Fig. 2. It is found that \( \delta P \) does no longer approach to zero monotonously with
increasing \( 1/\mu \), i.e. decreasing \( \mu \). As shown in Fig. 2, there exists a rising tendency
of \( \delta P \) in the vicinity of \( \mu \simeq 0.3 \text{GeV} \). This leads to the fact that no any pressure
equilibrium appears in the moderate density region so that a first-order SQM-CFL
phase transition does not occur. Although the CFL pressure is relatively large, the
absence of the phase transition means that the CFL matter is impossible to exist at
least in our concerned density region. Therefore, SQM with small \( m_s \) still remains as
a stable state for moderate densities while CFL with small \( m_s \) is prohibited unless the
density is very large.\(^3\)

The above physical picture holds valid until \( m_s \) is large. For larger \( m_s \), more
pressure is paid to maintain the common Fermi momentum so that the pressure of CFL
decreases. As long as \( m_s \) is large enough, the SQM-CFL transition in moderate density
region becomes possible to occur. Our numerical calculation shows that, as \( m_s \) is about
150MeV, the pressure equilibrium comes to appear in the vicinity of \( \mu \simeq 0.3 \text{GeV} \) ( see
Fig. 2 also ). Noticing that such pressure equilibrium behaves like ”crossover” of
CFL and SQM, \( m_s \simeq 150 \text{MeV} \) is regarded as the minimal value allowed for the CFL
existence at moderate density. Once \( m_s \) is larger than 150MeV, the transition from
SQM to CFL occurs in moderate density region. As a typical example, the result of
\( \delta P = P_{CFL} - P_{SQM} \) for \( m_s = 200 \text{MeV} \) is given by the solid line in Fig. 3. As shown in
Fig. 3, the critical chemical potential \( \mu_c \) for the SQM-CFL transition is about 0.4\text{GeV}.
This means that, for \( m_s = 200 \text{MeV} \), SQM remains stable in the region of \( \mu < \mu_c \) and

\(^3\)At a very high density, the pressures for SQM and CFL can become close to each other as long
as the pairing gap \( \Delta \) is small enough compared with the value of \( \mu \). In the asymptotic sense, the
SQM-CFL transition always occur regardless of whether \( m_s \) is small. But this is not the case being
concerned in the present work.
CFL emerges as $\mu \geq \mu_c$.

Then, we turn to consider the possibility of the SQM-gCFL phase transition. When $m_s$ is small such as $m_s < 150\text{MeV}$, the CFL matter is not stable for moderate densities so that its gapless phase does not exist at all. Even if $m_s$ is large, we emphasize that the presence of gCFL is still difficult because of the absence of the SQM-gCFL transition. For illustrative purpose, let us examine the gCFL phase in the case of $m_s = 200\text{MeV}$. In Fig. 3, the critical chemical potential $\mu_g$ for the gCFL formation (i.e. the gCFL-CFL transition) is shown to be about 0.67GeV, which is obviously larger than the critical value $\mu_c$ for the SQM-CFL transition. Since the gapless phenomenon is determined by $m_s^2/(2\mu)$ mainly, the difference between the gCFL and CFL pressures rises with increasing $1/\mu$. Therefore, the value of $P_{gCFL} - P_{SQM}$ does not approach to zero even if $\mu$ is close to $\mu_c$, as shown by the dashed line in Fig. 3. As a result, there is not the pressure equilibrium between gCFL and SQM so that the phase transition to gCFL actually does not occur in the quark star environment. Although gCFL is more energetically favorable than CFL, thus, it is impossible to emerge as the stable state for $m_s = 200\text{MeV}$.

The above argument is similar as that involving the CFL unstability in the case of $m_s$ is small (Fig. 2) and it holds valid for the most values of $m_s$. For example, if $m_s$ is larger than $200\text{MeV}$, more pressure is gained from the gapless phenomenon and the raising tendency of $P_{gCFL} - P_{SQM}$ becomes more obvious as $\mu$ is small. So the SQM-gCFL transition and then the presence of gCFL are prohibited. The exception happens only if the critical points $\mu_g$ and $\mu_c$ are close to each other. In that case, the difference between the gCFL and CFL pressures becomes small, so that the SQM-gCFL transition occurs also and the corresponding chemical potential approaches to $\mu_c$. This implies that gCFL emerges as the stable (exactly, metastable) state for a very narrow region of quark chemical potential. Our numerical calculation shows that such gCFL phase is impossible unless $(m_s, \mu)$ is limit in the vicinity of $(183\text{MeV}, 0.37\text{GeV})$, which corresponds to the intersection of the SQM-CFL and CFL-gCFL transitions (see Fig.
Based on the above arguments, a schematic phase diagram of the CFL quark matter is given for the moderate density region in Fig. 4. There are three kinds of different structures in the phase diagram according to the value of $m_s$:

(i) As $m_s$ is small such as $m_s < 150\text{MeV}$, the effect of $m_D$ prohibits the CFL formation for not-very-high densities. In this case, it is not CFL but SQM to be the stable state in the whole moderate density region, as shown in Fig. 4. This conclusion is very different from that obtained by Fig. 1, but agrees with the original prediction that CFL with zero (or small) $m_s$ becomes possible only when the density is high enough [4].

(ii) When $m_s$ is larger than 150MeV, a first-order transition from SQM to CFL takes place. Due to the density-dependent quark mass $m_D$, the SQM-CFL transition curve shown in Fig. 4 is deviated from the relation Eq.(2) obviously. With respect to the result of Fig. 1, we find that the CFL phase region is strongly suppressed for moderate densities. More importantly, the effect of $m_D$ prohibits the presence of the so-called gCFL phase also. For the most values of $m_s$, we find that SQM does not undergo a first order phase transition to gCFL and thus gCFL can not emerge as the stable state in the strange quark-star environment. This point is qualitatively different from the result of Fig. 1, in which gCFL was shown to be dominant as long as $m_s$ is relatively large. With respect to the result of Fig. 1, we find that the presence of the stable gCFL phase is reduced to almost nil.

(iii) As $m_s$ is very large, the existence of three-flavor quark matter including SQM becomes difficult. By using Eq.(11) we give the boundary curve of SQM (namely the 2QM-SQM transition curve) in Fig. 4, which shows that 2QM seems irrelevant to the presence of CFL. Once the color superconductivity is incorporated into 2QM, however, the possibilities of the 2QM-CFL/gCFL transitions could not be ruled out simply [17]. In that case, the dashed line shown in Fig. 4 needs to be modified, which is beyond the scope of the present work.
In summary, we extend the description of color-flavor-locked matter from high-density case to the moderate density region where the density dependence of quark masses can not be ignored. Starting at low density and raising the matter density, the physical picture that SQM remains as the stable state at first and then undergoes a first-order phase transition to CFL is reexamined in details. We predict a very different phase diagram of three-flavor quark matter, in which both the CFL and gCFL phase regions are suppressed for moderate densities. The present phase diagram is helpful to better understand the ground states of strongly interacting matter especially in the environment of strange quark stars. Of course, there are some uncertainties of the color superconducting gap used in this work. When the value of the gap is chosen in other ways, we can give the similar phase diagram as Fig. 4. For instance, if the gap is large such as $\Delta \sim 80\text{MeV}$ [18] we find that the minimum value of $m_s$ allowed for the CFL existence increases so that the CFL phase region for moderate densities might be further suppressed. Even if the density dependence of the gap is included, the change of $\Delta$ in the finite region of $\mu = 0.3 - 1\text{GeV}$ is not too drastic and the conclusion obtained from Fig. 4 is still qualitatively correct. In the further work one should construct the dynamical quark mass within a more realistic framework such as that beyond the bag model as well as take the two-flavor color superconducting phases into account. Some of the problems are being investigated.

Acknowledgements

This work was supported by National Natural Science Foundation of China (NSFC) under Contract No.10405012.

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Figure 1: Schematic phase diagram of the CFL quark matter in the \((m_s,\mu)\) plane, where the solid line is obtained from Eq.(2) and the dashed line corresponds to the phase transition from the conventional CFL to gCFL phases (see Eq.(15) in the following).

Figure 2: The CFL pressure vs. the SQM pressure. The solid lines from top to bottom are the relative pressures for \(m_s = 10, 50, 100\) and \(150\text{MeV}\) respectively.

Figure 3: The CFL/gCFL pressures vs. the SQM pressure for \(m_s = 200\text{MeV}\), where the solid and dashed lines are the relative pressures for CFL and gCFL respectively.

Figure 4: Similar as Fig. 1 but the SQM-CFL transition is considered in the case of including effects of \(m_D\). The solid line is the boundary of color-flavor-locked matter while the dashed line is the boundary of three-flavor quark matter. The square marks the possible region for the gCFL existence as the metastable phase.
\begin{figure}
\centering
\includegraphics[width=\textwidth]{plot.png}
\caption{Graph showing the relation between \(m_\nu\) [MeV] and \(\mu\) [GeV].}
\end{figure}

- SQM
- gCFL
- CFL
