Wave propagation in layered poroelastic cylinder in contact with fluid

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Abstract. The present paper investigates the propagation of time harmonic waves in a two layered hollow poroelastic cylinder in contact with inviscid fluids. The hollow poroelastic two layered cylinder is assumed to be infinite in axial direction. The outer and inner surfaces of the two layered cylinder are in contact with fluids and at the interface stresses and displacements are continuous. The frequency equation for wave propagation for a pervious is obtained. Non-dimensional phase velocity is computed as a function of wave number and presented graphically for two types of two layered cylinder for arbitrary parameters of the fluid in contact. Numerical results show that, in general, phase velocity is more in absence of outer fluid compared to the case of absence of inner fluid. Several other particular cases are discussed.

1. Introduction
Mechanical behavior of multilayered cylindrical structures in contact with dense fluid has been the interest of research in the recent years because these models are widely used as one of the components in different engineering branches. The vibration analyses of multilayered structures in various problems contribute to noise and structural vibrations. When the natural frequency of an acting force on a multilayered cylinder coincides with the natural frequency of the structure, the phenomenon of resonance occur causing failure in the complete system. Hence it is important to predict the natural frequencies that help in the design of the multilayered structures in contact with fluid. In the considered problem, we study the wave propagation in a two layered poroelastic cylinder in contact with fluid on the free surface of the cylinder using Biot’s theory. Frequency equation for a permeable surface is obtained. When the fluid at the outer surface of the two layered cylinder vanish, we obtained a two layered cylinder filled with fluid and the frequency equations are obtained. Gazis [5] studied the propagation of free harmonic waves in elastic hollow circular cylinder. McNiven et al. [7] discussed propagation of axially symmetric waves in composite elastic rods. Ahmed shah [1] discussed axially symmetric vibrations in a fluid-loaded poroelastic hollow cylinder. Malla Reddy and Tajuddin [6] studied axially symmetric vibrations of composite poroelastic cylinders. Flexural wave
propagation in coated poroelastic cylinders is presented by Ahmed shah [2]. Shanker et al [10] analyzed the vibrations in fluid-filled poroelastic shell surrounded by a fluid. The variation of frequency in fluid-loaded poroelastic spherical shell surrounded by fluid is more than that of poroelastic spherical shell surrounded by fluid for both pervious and impervious surfaces in case of radial and rotatory vibrations. A detailed study of Plane-strain vibrations in a poroelastic hollow cylinder immersed in an inviscid fluid is presented by Shanker et al [11]. The frequency equations for axially symmetric, flexural and anti-symmetric vibrations are obtained each for pervious and impervious surface. Axially symmetric shear vibrations are independent of the nature of the surface and the frequency is steady and increasing for a moderately thick poroelastic cylinder in the case of flexural vibrations. Ramesh et al. [8] studied Radial vibrations in poroelastic cylinder immersed in acoustic medium. It is observed that the vibrations are highly dispersive when the thickness of cylinder is very small, also there is a decrease in dispersion with an increase in thickness. Sandhyarani et al [9] has made a study on Radial vibrations in a fluid-filled transversely isotropic composite hollow poroelastic cylinder. A comparative study has been made on the composite thick-walled hollow cylinder and the composite cylinder filled with fluid. It is observed that the anisotropic ratio affects frequency in all the cases.

The present paper investigates the propagation of time harmonic waves in a two layered hollow poroelastic cylinder in filled and surrounded by inviscid fluids. The outer and inner surfaces of the two layered cylinder are in contact with fluids and at the interface stresses and displacements are continuous. Non-dimensional phase velocity is computed as a function of wave number and presented graphically for two types of two layered cylinder for arbitrary parameters of the fluid in contact. Several other particular cases viz composite hollow cylinder containing fluid, composite hollow cylinder surrounded by fluid, composite solid cylinder immersed in fluid and composite bore containing fluid are discussed.

2. Governing Equations
The equations of motion of a homogeneous, isotropic poroelastic solid (Biot, [3]) in presence of dissipation $b$ are

$$
NV^2 \ddot{u} + (A + N) \nabla e + Q \nabla \varepsilon = \frac{\partial^2}{\partial t^2} (\rho_1 \ddot{u} + \rho_1 \ddot{U}) + b \frac{\partial}{\partial t} (\ddot{u} - \ddot{U}),
$$

$$
Q \nabla e + R \nabla \varepsilon = \frac{\partial^2}{\partial t^2} (\rho_{12} \ddot{u} + \rho_{22} \ddot{U}) - b \frac{\partial}{\partial t} (\ddot{u} - \ddot{U}).
$$

(1)

where $V^2$ is the Laplacian, $\mathbf{u}=(u, v, w)$ and $\mathbf{U}=(U, V, W)$ are displacements of solid and liquid respectively, $e$ and $\varepsilon$ are the dilatations of solid and liquid; $A$, $N$, $Q$, $R$ are all poroelastic constants and $\rho_{ij}$ (i,j=1,2) are the mass coefficients following Biot [3]. The poroelastic constants $A$, $N$ correspond to familiar Lame constants in purely elastic solid. The coefficient $N$ represents the shear modulus of the solid. The coefficient $R$ is a measure of the pressure required on the liquid to force a certain amount of the liquid into the aggregate while total volume remains constant. The coefficient $Q$ represents the coupling between the volume change of the solid to that of liquid.

For axially symmetric vibrations, the displacements of solid $\ddot{u}(u, 0, w)$ and liquid $\ddot{U}(U, 0, W)$ are

$$
u = \frac{\partial \Phi_1}{\partial r} - \frac{\partial \Psi_1}{\partial z}, \quad w = \frac{\partial \Phi_1}{\partial z} + \frac{\partial \Psi_1}{\partial r} + \frac{\Psi_1}{r},$$
where $\Phi_1, \Phi_2, \Psi_1, \Psi_2$ are functions of $r, z$ and time $t$.

Substitution of equation (2) into equation (1) yield

$$P \nabla^2 \Phi_1 + Q \nabla^2 \Phi_2 = \frac{\partial^2}{\partial t^2} \left( \rho_{11} \Phi_1 + \rho_{12} \Phi_2 \right) + b \frac{\partial}{\partial t} \left( \Phi_1 - \Phi_2 \right),$$

$$Q \nabla^2 \Phi_1 + R \nabla^2 \Phi_2 = \frac{\partial^2}{\partial t^2} \left( \rho_{12} \Phi_1 + \rho_{22} \Phi_2 \right) - b \frac{\partial}{\partial t} \left( \Phi_1 - \Phi_2 \right),$$

$$N \nabla^2 \Psi_1 = \frac{\partial^2}{\partial t^2} \left( \rho_{11} \Psi_1 + \rho_{12} \Psi_2 \right) + b \frac{\partial}{\partial t} \left( \Psi_1 - \Psi_2 \right),$$

$$0 = \frac{\partial^2}{\partial t^2} \left( \rho_{12} \Psi_1 + \rho_{22} \Psi_2 \right) - b \frac{\partial}{\partial t} \left( \Psi_1 - \Psi_2 \right),$$

where $P = A + 2N$.

The stresses $\sigma_{ij}$ and the liquid pressure $s$ of the poroelastic solid are

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Q \varepsilon)\delta_{ij}, \quad (i, j = r, \theta, z)$$

$$s = Qe + R \varepsilon,$$

where $e_{ij}$ are strain components of poroelastic solid and $\delta_{ij}$ is the well-known Kronecker delta function.

3. Solution of the problem

Let $(r, \theta, z)$ be the cylindrical polar coordinates. Consider a two layered poroelastic hollow cylinder of infinite extent where each layer is homogeneous and isotropic and two layers are made up of different poroelastic materials. Inner surface of the inner poroelastic shell and outer surface of the outer poroelastic shell is in contact with inviscid elastic fluid. Let the inner and outer radii of the two layered cylinder be $r = r_1$ and $r = r_2$ respectively and the interface is at $r = a$. The axis of the poroelastic shell is in the direction of $z$-axis. Two layered poroelastic shell consists of circular inner shell of one poroelastic material bounded by and bonded to circular outer shell of different poroelastic material. The quantities related to inner shell are denoted by $^*$ as a superscript. For example poroelastic constants of outer shell are $A, N, Q, R$ and the poroelastic constants of inner shell are denoted by $A^*, N^*, Q^*, R^*$. 

![Fig. 1 Geometry of the problem](image)
Then for axially symmetric vibrations, the displacement of solid \(u = (u, 0, w)\) and stresses can be readily be evaluated from equations (2) and (3) is

**For Coating (outer poroelastic cylindrical shell):**

\[
\sigma_r + s = [C_1a_{11}(r) + C_2a_{12}(r) + C_3a_{13}(r) + C_4a_{14}(r) + A_1a_{15}(r) + B_1a_{16}(r)]e^{ikz + \omega t},
\]
\[
\sigma_\theta = [C_1a_{21}(r) + C_2a_{22}(r) + C_3a_{23}(r) + C_4a_{24}(r) + A_2a_{25}(r) + B_2a_{26}(r)]e^{ikz + \omega t},
\]
\[
u = [C_1a_{31}(r) + C_2a_{32}(r) + C_3a_{33}(r) + C_4a_{34}(r) + A_4a_{35}(r) + B_4a_{36}(r)]e^{ikz + \omega t},
\]
\[
w = [C_1a_{41}(r) + C_2a_{42}(r) + C_3a_{43}(r) + C_4a_{44}(r) + A_4a_{45}(r) + B_4a_{46}(r)]e^{ikz + \omega t},
\]
\[
s = [C_1a_{51}(r) + C_2a_{52}(r) + C_3a_{53}(r) + C_4a_{54}(r)]e^{ikz + \omega t},
\]

where the elements \(a_{ij}(r)\) and \(d_{ij}(r)\) are

\[
a_{11} = [(Q + R)\delta_{r}^2 - (A + Q)k^2][(Q + R)\delta_{r}^2 - (P + Q)\alpha_n^2]J_0(\alpha_n r) + \frac{2N\alpha_1}{r}J_1(\alpha_n r),
\]
\[
a_{12} = [(Q + R)\delta_{r}^2 - (A + Q)k^2][(Q + R)\delta_{r}^2 - (P + Q)\alpha_n^2]Y_0(\alpha_n r) + \frac{2N\alpha_1}{r}Y_1(\alpha_n r),
\]
\[
a_{13} = [(Q + R)\delta_{r}^2 - (A + Q)k^2][(Q + R)\delta_{r}^2 - (P + Q)\alpha_n^2]J_0(\alpha_n r) + \frac{2N\alpha_2}{r}J_1(\alpha_n r),
\]
\[
a_{14} = [(Q + R)\delta_{r}^2 - (A + Q)k^2][(Q + R)\delta_{r}^2 - (P + Q)\alpha_n^2]Y_0(\alpha_n r) + \frac{2N\alpha_2}{r}Y_1(\alpha_n r),
\]
\[
a_{15} = -2ikN\alpha_3 J_0(\alpha_3 r) + \frac{2ikN}{r}J_1(\alpha_3 r), \quad a_{16} = -2ikN\alpha_3 Y_0(\alpha_3 r) + \frac{2ikN}{r}Y_1(\alpha_3 r),
\]
\[
a_{21} = -2ikN\alpha_1 J_1(\alpha_1 r), \quad a_{22} = -2ikN\alpha_1 Y_1(\alpha_1 r), \quad a_{23} = -2ikN\alpha_1 J_1(\alpha_2 r),
\]
\[
a_{24} = -2ikN\alpha_2 J_1(\alpha_2 r), \quad a_{25} = -N(k^2 - \alpha_n^2)J_1(\alpha_n r), \quad a_{26} = -N(k^2 - \alpha_n^2)Y_1(\alpha_n r),
\]
\[
a_{31} = (R\delta_{r}^2 - Q)(\alpha_n^2 + k^2)J_0(\alpha_n r), \quad a_{32} = (R\delta_{r}^2 - Q)(\alpha_n^2 + k^2)Y_0(\alpha_n r),
\]
\[
a_{33} = (R\delta_{r}^2 - Q)(\alpha_n^2 + k^2)J_0(\alpha_n r), \quad a_{34} = (R\delta_{r}^2 - Q)(\alpha_n^2 + k^2)Y_0(\alpha_n r),
\]
\[
a_{35} = 0, \quad a_{36} = 0, \quad a_{41} = -\alpha_1 J_1(\alpha_1 r), \quad a_{42} = -\alpha_1 Y_1(\alpha_1 r), \quad a_{43} = \alpha_1 J_1(\alpha_2 r),
\]
\[
a_{44} = -\alpha_2 Y_1(\alpha_2 r), \quad a_{45} = -\alpha_2 J_1(\alpha_3 r), \quad a_{46} = -\alpha_2 Y_1(\alpha_3 r), \quad d_{41} = -\alpha_2 Y_1(\alpha_3 r),
\]
\[
a_{51} = ikJ_0(\alpha_n r), \quad a_{52} = ikY_0(\alpha_n r), \quad a_{53} = ikJ_0(\alpha_2 r),
\]
\[
a_{54} = ikY_0(\alpha_2 r), \quad a_{55} = \alpha_2 J_0(\alpha_2 r), \quad a_{56} = \alpha_2 Y_0(\alpha_2 r),
\]
\[
d_{31} = -(R\delta_{r}^2 - Q)(\alpha_n^3 + k^2\alpha_n)J_1(\alpha_n r), \quad d_{32} = -(R\delta_{r}^2 - Q)(\alpha_n^3 + k^2\alpha_n)Y_1(\alpha_n r),
\]
\[
d_{33} = -(R\delta_{r}^2 - Q)(\alpha_n^3 + k^2\alpha_n)J_1(\alpha_n r), \quad d_{34} = -(R\delta_{r}^2 - Q)(\alpha_n^3 + k^2\alpha_n)Y_1(\alpha_n r),
\]
\[
d_{35} = 0, \quad d_{36} = 0.
\]

where \(C_1, C_2, C_3, C_4, A_1\) and \(B_1\) are constants, \(\omega\) is frequency of wave, \(k\) is wave number, \(J_n\) and \(Y_n\) are Bessel functions of first and second kind, respectively, each of order \(n\),
\[ \alpha_i^2 = \frac{\omega^2}{V_i^2} - k^2, \text{ for } i = 1, 2, 3, \]

\[ \delta_k^2 = \frac{(PR - Q^2) - V_k^2 (RM_{11} - QM_{12})}{V_k^2 (RM_{12} - QM_{12})} \text{ for } k = 1, 2 \]

\[ M_{11} = \rho_{11} - ib \omega, M_{12} = \rho_{12} + \frac{ib}{\omega} \omega, M_{22} = \rho_{22} - \frac{ib}{\omega}, \]

For Core (inner poroelastic shell):

Similarly, the displacement and stress components of inner poroelastic shell can be obtained as

\[ (\sigma_{rr} + s)^* = [D_1 b_{11}(r) + D_2 b_{12}(r) + D_3 b_{13}(r) + D_4 b_{14}(r) + A_2 b_{15}(r) + B_2 b_{16}(r)]e^{i(kz + \omega t)}, \]

\[ (\sigma_{rr})^* = [D_1 b_{21}(r) + D_2 b_{22}(r) + D_3 b_{23}(r) + D_4 b_{24}(r) + A_2 b_{25}(r) + B_2 b_{26}(r)]e^{i(kz + \omega t)}, \]

\[ s^* = [D_1 b_{31}(r) + D_2 b_{32}(r) + D_3 b_{33}(r) + D_4 b_{34}(r) + A_2 b_{35}(r) + B_2 b_{36}(r)]e^{i(kz + \omega t)}, \]

\[ u^* = [D_1 b_{41}(r) + D_2 b_{42}(r) + D_3 b_{43}(r) + D_4 b_{44}(r) + A_2 b_{45}(r) + B_2 b_{46}(r)]e^{i(kz + \omega t)}, \]

\[ w^* = [D_1 b_{51}(r) + D_2 b_{52}(r) + D_3 b_{53}(r) + D_4 b_{54}(r) + A_2 b_{55}(r) + B_2 b_{56}(r)]e^{i(kz + \omega t)}, \]

where \( D_1, D_2, D_3, D_4, A_2 \) and \( B_2 \) are constants and elements \( b_{ij}(r) \) and \( e_{ij}(r) \) are

\[ b_{11} = \{(Q^* + R^*) \delta_{11}^{(\omega)} - (A^* + Q^*) \delta_{12}^{(\omega)} - (P^* + Q^*)|\alpha_i^2|J_0(\alpha_i^2 r) + \frac{2N^* \alpha_i^2}{\omega} J_1(\alpha_i^2 r), \]

\[ b_{12} = \{(Q^* + R^*) \delta_{11}^{(\omega)} - (A^* + Q^*) \delta_{12}^{(\omega)} - (P^* + Q^*)|\alpha_i^2|Y_0(\alpha_i^2 r) + \frac{2N^* \alpha_i^2}{\omega} Y_1(\alpha_i^2 r), \]

\[ b_{13} = \{(Q^* + R^*) \delta_{11}^{(\omega)} - (A^* + Q^*) \delta_{12}^{(\omega)} - (P^* + Q^*)|\alpha_i^2|J_0(\alpha_i^2 r) + \frac{2N^* \alpha_i^2}{\omega} J_1(\alpha_i^2 r), \]

\[ b_{14} = \{(Q^* + R^*) \delta_{11}^{(\omega)} - (A^* + Q^*) \delta_{12}^{(\omega)} - (P^* + Q^*)|\alpha_i^2|Y_0(\alpha_i^2 r) + \frac{2N^* \alpha_i^2}{\omega} Y_1(\alpha_i^2 r), \]

\[ b_{15} = -2ikN^* \alpha_i^2 J_0(\alpha_i^2 r) + \frac{2ikN^*}{\omega} J_1(\alpha_i^2 r), \]

\[ b_{16} = -2ikN^* \alpha_i Y_0(\alpha_i^2 r), \]

\[ b_{21} = -2ikN^* \alpha_i J_1(\alpha_i^2 r), \]

\[ b_{22} = -2ikN^* \alpha_i Y_1(\alpha_i^2 r), \]

\[ b_{23} = -2ikN^* \alpha_i J_2(\alpha_i^2 r), \]

\[ b_{24} = -2ikN^* \alpha_i Y_2(\alpha_i^2 r), \]

\[ b_{25} = N^* (k^2 - \alpha_i^2) J_1(\alpha_i^2 r), \]

\[ b_{26} = N^* (k^2 - \alpha_i^2) Y_1(\alpha_i^2 r), \]

\[ b_{31} = (R^* \delta_{21}^{(\omega)} - Q^*)(\alpha_i^{(\omega)} + k^2) J_0(\alpha_i^2 r), \]

\[ a_{32} = (R^* \delta_{21}^{(\omega)} - Q^*)(\alpha_i^{(\omega)} + k^2) Y_0(\alpha_i^2 r), \]

\[ b_{33} = (R^* \delta_{21}^{(\omega)} - Q^*)(\alpha_i^{(\omega)} + k^2) J_0(\alpha_i^2 r), \]

\[ a_{34} = (R^* \delta_{21}^{(\omega)} - Q^*)(\alpha_i^{(\omega)} + k^2) Y_0(\alpha_i^2 r), \]

\[ b_{35} = 0, \]

\[ b_{36} = 0, \]

\[ b_{41} = -\alpha_i^2 J_1(\alpha_i^2 r), \]

\[ b_{42} = -\alpha_i Y_1(\alpha_i^2 r), \]

\[ b_{43} = -\alpha_i^2 J_1(\alpha_i^2 r), \]

\[ b_{44} = -\alpha_i Y_1(\alpha_i^2 r), \]

\[ b_{45} = -ik J_1(\alpha_i^2 r), \]

\[ a_{46} = -ik Y_1(\alpha_i^2 r). \]
\[
\begin{align*}
b_{51} &= ik J_0(\alpha_1 r), \quad b_{52} = ik J_0(\alpha_2 r), \quad b_{53} = ik J_0(\alpha_3 r), \\
b_{54} &= ik Y_0(\alpha_1 r), \quad b_{55} = \alpha_1 J_0(\alpha_1 r), \quad b_{56} = \alpha_1 Y_0(\alpha_1 r), \\
m_{31} &= -(R^* \delta_j^{\alpha_1^2} - Q^*)(\alpha_1^3 + k^2 \alpha_1^*) J_1(\alpha_1^* r), \quad m_{32} = -(R^* \delta_j^{\alpha_2^2} - Q^*)(\alpha_2^3 + k^2 \alpha_2^*) Y_1(\alpha_2^* r), \\
m_{33} &= -(R^* \delta_j^{\alpha_3^2} - Q^*)(\alpha_3^3 + k^2 \alpha_3^*) J_1(\alpha_3^* r), \quad m_{34} = -(R^* \delta_j^{\alpha_4^2} - Q^*)(\alpha_4^3 + k^2 \alpha_4^*) Y_1(\alpha_4^* r), \\
m_{35} &= 0, \quad m_{36} = 0.
\end{align*}
\]

In equation (10), \(a_1^{\alpha_1^2}, a_2^{\alpha_2^2}, a_3^{\alpha_3^2}, a_4^{\alpha_4^2}\) are similar to equation (8), with its material parameters having * as a superscript.

**Inviscid elastic fluid**

The equation of motion for a homogeneous, isotropic, inviscid elastic fluid is

\[
\nabla^2 \Phi^{(j)} = \frac{1}{V_f^{(j)}} \frac{\partial^2 \Phi^{(j)}}{\partial t^2}, \quad (j = 1, 2)
\]

where \(\Phi^{(j)}\) is the displacement potential function and \(V_f^{(j)}\) is the velocity of sound in fluid.

The parameters with super fix \(j = 1\) represents the parameters of outer fluid, whereas the parameters of inner fluid are represented by super fix \(j = 2\).

The fluid pressure \(P_f^{(j)}\) is given by

\[
P_f^{(j)} = -\rho_f^{(j)} \frac{\partial^2 \Phi^{(j)}}{\partial t^2}, \quad (j = 1, 2)
\]

where \(\rho_f^{(j)}\) is the density of the fluid. The superscripts \(j = 1, 2\) represent outer and inner fluids, respectively.

Using displacement potential function, the displacement \(u_f^{(1)}(u_f^{(1)}, 0, w_f^{(1)})\) and fluid pressure \(P_f^{(1)}\) of outer fluid are

\[
\begin{align*}
  u_f^{(1)} &= A_f^{(1)} \xi_f (H_0^{(1)}) \xi_f (r) e^{i(kj+\omega t)}, \\
  w_f^{(1)} &= -A_f^{(1)} \frac{1}{r} H_0^{(1)} \xi_f (r) e^{i(kj+\omega t)}, \\
  p_f^{(1)} &= A_f^{(1)} \rho_f^{(1)} \omega \xi_f (r) e^{i(kj+\omega t)}.
\end{align*}
\]

The displacement \(u_f^{(2)}(u_f^{(2)}, 0, w_f^{(2)})\) and fluid pressure \(P_f^{(2)}\) of inner fluid are obtained as

\[
\begin{align*}
  u_f^{(2)} &= A_f^{(2)} \xi_f (J_0^{(2)}) \xi_f (r) e^{i(kj+\omega t)}, \\
  w_f^{(2)} &= -A_f^{(2)} \frac{1}{r} J_0^{(2)} \xi_f (r) e^{i(kj+\omega t)}, \\
  p_f^{(2)} &= A_f^{(2)} \rho_f^{(2)} \omega J_0^{(2)} \xi_f (r) e^{i(kj+\omega t)}.
\end{align*}
\]
where $\zeta^ {(i)} = \frac{\omega}{V^ {(i)}}, A^ {(i)}$ is a constant, $J_0$ is Bessel function of first kind and of order 0 and $H_0^{(i)}$ is Hankel function of first kind and of order zero.

4. Frequency equation

The outer and inner surfaces of the poroelastic layered cylinder are in contact with outer and inner fluids respectively where continuity of radial displacements of solid and fluid are assumed. At the interface of two solid layers the displacements and stresses are continuous. Thus the boundary conditions for a poroelastic layered cylinder in contact with fluid in case of a permeable surface are

$$r = r_1, \quad (\sigma_{rr} + s) = - p^ {(i)}_r, \quad (\sigma_{\theta\theta}) = 0, s = p^ {(i)}_s, u = u^ {(i)}_s$$

$$r = a, \quad (\sigma_{rr} + s) = (\sigma_{\theta\theta})^*, \quad (\sigma_{\phi\phi}) = (\sigma_{\phi\phi})^*, s = 0, s^* = 0, u = u^*, w = w^*$$

$$r = r_2, \quad (\sigma_{rr} + s)^* = - p^ {(2)}_r, \quad (\sigma_{\theta\theta})^* = 0, s^* = p^ {(2)}_s, u^* = u^*$$

Eqs. (14) results in a system of fourteen homogeneous equations in constants $C_i, C_2, C_3, C_4, A_1, A_2, A_3, D_1, D_2, D_3, D_4, A_5, A_6,$ and $A_7$ such a homogeneous system has non-trivial solution only if the determinant of the coefficients of the unknowns vanishes identically. Thus by eliminating the constants, the frequency equation of vibrations for poroelastic composite hollow cylinder for a surface is obtained as

$$|A_{ij}| = 0 \quad \text{for } i, j = 1, 2, ..., 14$$

where the elements $A_{ij}$ in the above equation are given by

$$A_{ij} = a_{ij} (r_2), j = 1, 2, ..., 6; A_{ij} = \rho^ {(2)}_j \omega^2 J_0 (\zeta^ {(2)}_j r); A_{ij} = 0, j = 8, ..., 14$$

$$A_{ij} = a_{ij} (r_2), i = 2, 3 & j = 1, 2, ..., 6; A_{ij} = 0, i = 2, 3 & j = 7, 8, ..., 14$$

$$A_{ij} = a_{ij} (r_2), j = 1, 2, ..., 6; A_{ij} = \zeta^ {(2)}_j J_0^' (\zeta^ {(2)}_j r); A_{ij} = 0, j = 8, 9, ..., 14$$

$$A_{ij} = a_{ij} (a), i = 5, 6 & j = 1, 2, ..., 6; A_{ij} = b_{ij} (a), i = 5, 6 & j = 7, 8, ..., 14$$

$$A_{ij} = a_{ij} (a), j = 1, 2, ..., 6; A_{ij} = 0, j = 7, 8, ..., 14$$

$$A_{ij} = 0, j = 1, 2, ..., 6; A_{ij} = b_{ij} (a), j = 7, 8, ..., 14$$

$$A_{ij} = a_{ij} (a), j = 1, 2, ..., 6; A_{ij} = b_{ij} (a), j = 8, 9, ..., 14$$

$$A_{ij} = a_{ij} (a), j = 1, 2, ..., 6; A_{ij} = b_{ij} (a), j = 8, 9, ..., 14$$

$$A_{ij} = 0, j = 1, 2, ..., 6; A_{ij} = b_{ij} (r_1), j = 8, ..., 13; A_{ij} = \rho^ {(1)}_j \omega^2 H_0^{(1)} (\zeta^ {(2)}_j r)$$

$$A_{ij} = 0, i = 12, 13 & j = 1, 2, ..., 6; A_{ij} = b_{ij} (r_1), i = 12, 13 & j = 7, 8, ..., 14$$

$$A_{ij} = 0, j = 1, 2, ..., 6; A_{ij} = b_{ij} (r_1), j = 8, 9, ..., 14; A_{ij} = \zeta^ {(1)}_j (H_0^{(1)})^' (\zeta^ {(1)}_j r)$$
Eq. (16) represents frequency equation of axially symmetric vibrations in a composite poroelastic cylinder immersed in a fluid in case of pervious surface.

Some of cases are discussed as particular cases:

I. Composite poroelastic hollow cylinder containing fluid

When density of outer fluid is zero i.e. \( \rho_f^{(1)} = 0 \), the frequency equation (16) reduces to frequency equation of composite poroelastic hollow cylinder containing fluid.

II. Composite poroelastic hollow cylinder surrounded by fluid

When density of inner fluid is zero i.e. \( \rho_f^{(2)} = 0 \), the frequency equation (16) reduces to frequency equation of composite poroelastic hollow cylinder containing fluid.

5. Non-dimensionalization of frequency equation

When the dissipation coefficient i.e. \( b \) is zero, the natural frequency is real. Hence to discuss the results numerically the dissipation coefficient \( 'b' \) is assumed as zero. The above formed frequency equations of vibrations of poroelastic composite cylinders immersed in fluid can be discussed numerically by introducing the following non-dimensional parameters:

\[
\begin{align*}
    a_1 &= \frac{2}{H}, \quad a_2 = \frac{3}{H}, \quad a_3 = \frac{2}{H}, \quad a_4 = \frac{3}{H}, \\
    d_1 &= \frac{2}{\rho_1}, \quad d_2 = \frac{2}{\rho_2}, \quad d_3 = \frac{2}{\rho_2}, \\
    b_1 &= \frac{1}{P}, \quad b_2 = \frac{1}{Q}, \quad b_3 = \frac{1}{R}, \\
    b_4 &= \frac{1}{N}, \quad g_1 = \frac{1}{\rho_1}, \quad g_2 = \frac{1}{\rho_2}, \quad g_3 = \frac{1}{\rho_2}, \\
    c &= \frac{\omega}{k}, \\
    x_1 &= \left( \frac{V_0}{V_1} \right)^2, \quad y_1 = \left( \frac{V_0}{V_2} \right)^2, \quad z_1 = \left( \frac{V_0}{V_3} \right)^2, \\
    x_2 &= \left( \frac{V_0}{V_4} \right)^2, \quad y_2 = \left( \frac{V_0}{V_5} \right)^2, \quad z_2 = \left( \frac{V_0}{V_6} \right)^2, \\
    \end{align*}
\]

where \( C \) is non-dimensional phase velocity and

\[
\begin{align*}
    H &= 1 + 2 \cdot 1 + 1, \quad \rho = 1 \cdot \rho_1 + 2 \cdot \rho_2 + 1 \cdot \rho_2, \\
    \rho_0^2 &= \frac{1}{\rho_1}, \quad \rho_0^2 = \frac{1}{\rho_2}. \\
\end{align*}
\]

Non-dimensional work is calculated for two types of composite cylinders, namely composite cylinder-I and composite cylinder-II for each pervious and impervious surface. Composite cylinder-I consists of core made up of sandstone saturated with water (Yew and Jogi, [12]) and casing is made up of sandstone saturated with kerosene (Fatt, [4]), where as in composite cylinder-II, the core is sandstone saturated with kerosene and casing is sandstone saturated with water. The physical parameters of these poroelastic composite materials following equation (18) are given in Table 1.
Table. 1

| Material Parameters | a₁ | a₂ | a₃ | a₄ | d₁ | d₂ | d₃ | x₂ | y₂ | z₂ |
|---------------------|----|----|----|----|----|----|----|----|----|----|
| Composite Cylinder-I | 0.445 | 0.034 | 0.015 | 0.123 | 0.887 | -0.001 | 0.099 | 1.863 | 8.884 | 7.183 |
| Composite Cylinder-II | 1.819 | 0.011 | 0.054 | 0.780 | 0.891 | 0 | 0.125 | 0.489 | 2.330 | 1.142 |

| b₁ | b₂ | b₃ | b₄ | g₁ | g₂ | g₃ | x₁ | y₁ | z₁ |
|----|----|----|----|----|----|----|----|----|----|
| 0.960 | 0.006 | 0.028 | 0.412 | 0.877 | 0 | 0.123 | 0.913 | 4.347 | 2.129 |
| 0.843 | 0.065 | 0.028 | 0.234 | 0.901 | -0.001 | 0.101 | 0.999 | 4.763 | 3.851 |

6. Results and Discussion

For a given poroelastic parameters, frequency equations when non-dimensionalized using equation (17), constitute a relation between non-dimensional phase velocities and wave number.

In Fig. 2, phase velocity for thin inner cylinder and thick outer cylinder for Fluid-loaded poroelastic hollow cylinder surrounded by a fluid is plotted. Here, more variations are observed in composite cylinder I than in cylinder II. From Fig. 3 a similar observation is noted for thick inner cylinder and thin outer cylinder. Fig. 4 depicts all the possible three cases on composite poroelastic cylinder I. The variations in phase velocity are less when inner and outer cylinders are moderately thin (or thick) and equal case compared to other two cases. A similar comparison is depicted in Fig. 5 for composite poroelastic cylinder II. Here phase velocity for moderately thin and equal case is more compared to other cases. The effect of presence of inner and outer fluids on composite poroelastic cylinders I and II are presented in Fig. 6 and Fig. 7 respectively. The phase velocity is more in the presence of only inner
7. Conclusion:

The analysis of vibrations of fluid-loaded poroelastic composite hollow cylinder surrounded by fluid has lead to the following conclusions:

- Poroelastic Composite cylinder I has more variations in phase velocity compared to cylinder II.
- Variations in phase velocity is more in the cases of thin core–thick casing and thin casing-thick core compared to the case where they have equal ratios for composite cylinder I.
- Phase velocity is nearly same in the cases of thin core–thick casing and thin casing-thick core and less compared to the case where they have equal ratios for composite cylinder II.
- Phase velocity is more in absence of outer fluid compared to the case of absence of inner fluid for composite cylinder I.
Variations in Phase velocity are more for small values of wave numbers and nearly same for higher values in the cases of absence of inner fluid, absence of outer fluid and presence of both for composite cylinder II

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