On the radial acceleration of disk galaxies

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ABSTRACT
The physical processes defining the dynamics of disk galaxies are still poorly understood. Hundreds of articles have appeared in the literature over the last decades without arriving at an understanding within a consistent gravitational theory. Dark matter (DM) scenarios or a modification of Newtonian dynamics (MOND) are employed to model the non-Keplerian rotation curves in most of the studies, but the nature of DM and its interaction with baryonic matter remains an open question and MOND formulates a mathematical concept without a physical process. We have continued our attempts to use the impact theory of gravitation for a description of the peculiar acceleration and velocity curves and have considered five more galaxies. Using published data of the galaxies NGC 3198, NGC 2403, NGC 1090, UGC 3205 and NGC 1705, it has been possible to find good fits without DM for the observed disk velocities and, as example, also for the extraplanar matter of NGC 3198.

Key words: galaxies: spiral – galaxies: kinematics and dynamics – gravitation – astroparticle physics

1 INTRODUCTION
The main topic of this study will be the nearly flat outer rotation curves (RCs) of disk galaxies which cannot be understood by the visible baryonic mass and Keplerian dynamics.

Since Oort (1932) and Zwicky (1933) introduced the concept of dark matter (DM; dunkle Materie) in order to resolve problems discovered in the velocity distributions of our Galaxy and in the Coma galaxy cluster, the DM concept has been applied to explain the flat RCs of disk galaxies (cf. e.g. Rubin 1983, 1986). Even before, Babcock (1939) found for the Andromeda Nebula (M 31) a rotation of the outer portions of the disk with constant angular velocity indicating that the mass-luminosity coefficient was too small. He suggested that absorption could be the cause or that new dynamical considerations might be required. In the outer parts of the nebula NGC 3115, Oort (1940) also found that the ratio of mass-density to light-density is very high.

In a detailed summary of the observations of galaxies and their interpretations, Rubin (2000) concludes that galaxy dark halos exist, but, at the same time, states that details on the amount of DM as well as its composition and distribution still have to be discovered. Even a modification of the gravitational potential rather than DM might provide an explanation for the observations.

Burkert (1995); Treu & Koopmans (2004); McGaugh (2008); Boylan-Kolchin, Bullock & Kaplinghat (2011);

Weinberg et al. (2015); Battaglieri et al. (2017) and Salucci (2018) also conclude that the nature of DM and its exact role in galaxy formation is still unknown, although it apparently dominates the dynamics of large-scale cosmic structures. Freeman (2008) and Kaplinghat, Ren & Yu (2019) state that the cold dark matter (CDM) concept is successful in the Universe, but that there are challenges on galactic scales. Star formation by bosonic self-interacting dark matter (SIDM) is considered as possibility (Eby et al. 2016). In an investigation of luminous and DM distributions with the help of extended RCs of 12 disk galaxies Bottema & Pestana (2015) conclude that DM halos have central cores and that the mass ratios between the dark and baryonic components are between \( \approx 9 \) to \( \approx 5 \) for small and large galaxies, respectively. Giraud (2000) shows that the DM profiles have a main component (the “coupled halo”) and one with a gaslike distribution. Kregel, van der Kruit & de Grijs (2002) find a trend that disks flatten with increased mass, but conclude that many questions remain open. Many more DM candidates are mentioned by Schneider et al. (2017).

It is thus not surprising that statements about DM include “mysterious material” (Binney, May & Ostriker 1987), “DM around galaxies … as the direct manifestation of one of its most extraordinary mysteries” (Donato et al. 2009; Gentile et al. 2009; Karukes & Salucci 2017), mysteries of DM and dark energy (Bekenstein 2010). “Our results may point to the need for a revision of the current DM paradigm” (Lelli et al. 2017). It is “… arguably deepest mystery of...
modern physics” (Ghari et al. 2019); de Blok (2018) asks whether there is a universal alternative and at the beginning of their abstract Rodrigues et al. (2018) write: “Dark matter is currently one of the main mysteries of the Universe.” Salucci (2019) concludes his paper with reference to the new observational opportunities: “… if one believes or not that this will lead to a solution of the old mystery of dark matter.”

A modification of the Newtonian dynamics (MOND) has been proposed by Milgrom (1983, 1994, 2015, 2016, 2020), assuming that for small accelerations characterized by \( a_0 \approx 1.2 \times 10^{-8} \text{ cm s}^{-2} \), the \( 1/r^2 \) dependence in Newton’s law of gravity effectively changes to \( 1/r \). Ghari, Haghi & Zonoozi (2019) find support that \( a_0 \) could be a universal constant. MOND can describe the dynamics of many galaxies without DM (cf. e.g. Kroupa, Pawłowski & Milgrom 2012; Samurović 2016). It has to be noted, though, that it is a mathematical assumption for which a physical process is still in need (cf. e.g. McGaugh 2012). MOND formulates a fundamental modification of gravity or inertia in low-acceleration regime – “MOND is the key to new gravitational physics” (Sanders & Noordermeer 2007).

We have considered the rotation dynamics of three disk galaxies in a paper entitled “A physical process of the radial acceleration of disc galaxies” with a proposal which explains the observations without the need to introduce any DM (Wilhelm & Dwivedi 2018). It is our aim to follow up this proposal by applying the process to five more galaxies with flat or slightly declining RCs.

2 NEWTON’S LAW

Soffe & Rubin (2001) hope that the research about galaxies will confirm the Newtonian gravitational theory or will lead to its successor. Newton’s inverse square law of gravitation is, of course, also a mathematical abstraction and Newton himself was not convinced that “action at a distance” was an appropriate physical process. Since he could not discover such a process, he formulated his famous “Hypotheses non fingo”. Nicolas Fatio de Duillier influenced by Isaac Newton presented in 1690 his impact theory of gravitation to the Royal Society London. He had to incorporate a shadow effect to produce an attraction (Fatio de Duillier 1690); (cf. Nijhoff 1901; Bopp 1929; Zehe 1983).

Discrepancies between Newton’s law and observations, namely, the perihelion precession of Mercury (Le Verrier 1859) and the deflection of light twice as strong as predicted (Dyson et al. 1920; Soldner 1804), could be resolved in a formal way by the General Theory of Relativity (GTR, Einstein 1916). Later von Laue (1959) differentiated between the physical world and its mathematical formulation: A four-dimensional ‘world’ is only a valuable mathematical trick; deeper insight, which some want to see behind it, is not involved.

A physical process based on Nicolas Fatio’s idea consistent with GRT – as far as the perihelion precession and the light deflection are concerned - and in line with strict momentum and energy conservation has been proposed by Wilhelm, Wilhelm & Dwivedi (2013) and is discussed in a wider context in Wilhelm & Dwivedi (2020). The solution is based on gravitons which lose energy and momentum when interacting with baryonic matter. We have invoked the basic idea of impacting gravitons – originally called quadrupoles – with no mass and a speed of light \( c \). They are absorbed by massive particles and re-emitted with reduced energy \( T_G \) according to \( T_G = T_c (1 - Y) \), where \( T_c \) is the energy (very small) of a graviton in the background flux and \( Y \) with \( 0 < Y \ll 1 \) is defined as the reduction parameter. The energy difference \( Y T_G \) leads to a mass increase of the interacting particle. The corresponding momentum equation is \( p_G = -p_c (1 - Y) \), where \( |p_c| = T_c / c \). This implies that the diminished graviton is re-emitted in the anti-parallel direction relative to the incoming one. This geometry had to be assumed in a study defining the mass equivalent of the potential energy in a gravitationally bound two-body system. It is to be noted here that the omni-directional emission, as originally postulated, led to the conflicts with energy and momentum conservation (Wilhelm & Dwivedi 2015a).

For small particles and bodies the gravitational interaction obeys the inverse square law, but in large mass conglomerations multiple interactions lead to deviations that could explain the perihelion anomalies (Wilhelm & Dwivedi 2014), anomalous Earth flybys (Wilhelm & Dwivedi 2015b) and the non-Keplerian RCs of disk galaxies (Wilhelm & Dwivedi 2018). Multiple interactions in disk galaxies can only occur within their planes and are responsible for the flat RCs. The last topic is also the subject of this paper and will be detailed in later sections.

The main assumption in this paper is that the flat RCs of spiral galaxies can be understood without DM. The baryonic mass of galaxies we denote with \( M_{\text{bar}} \) and the corresponding velocity and acceleration quantities with \( V_{\text{bar}} \) and \( g_{\text{bar}} \). We follow Karukes, Salucci & Gentile (2015) and others in identifying the “luminous” components of spiral galaxies with the stellar and gaseous disks and a central bulge. The corresponding velocity, acceleration and mass quantities are written as \( V_{\text{star}}, V_{\text{gas}}, V_{\text{bulge}}; g_{\text{star}}, g_{\text{gas}}, g_{\text{bulge}}; M_{\text{star}}, M_{\text{gas}} \) and \( M_{\text{bulge}} \), respectively. Not all components are present in all spiral galaxies. The stellar and gaseous parts of the disk are combined to \( V_{\text{disk}}, g_{\text{disk}} \) and \( M_{\text{disk}} \). The centripetal acceleration \( g(R) \) exerted by a certain component at radius \( R \) must cancel the centrifugal force and is thus related to the rotational velocity \( V \) by

\[
g(R) = -\frac{V^2}{R} \tag{1}
\]

(cf. e.g. Eq. (5) Babcock 1939). We use the definitions of Freeman (1970) and Persic, Salucci & Stel (1996):

\[
\Sigma_d(R) = \frac{M_{\text{disk}}}{2\pi R D} e^{-R/R_D} ,
\]

where \( \Sigma_d(R) \) is the stellar surface density and \( R_D \) the exponential disk scale length. The radius \( R_{\text{gal}} = 3.2 R_D \) then encloses 83 % of the total light of a galaxy (cf. Salucci 2018).

Before we present our results, it is important to attempt a summary of the recent literature on disk galaxies. It is meant to show the enormous progress that has been achieved in many aspects, but, at the same time, that fundamental questions are not resolved – most of them related to DM.
3 SPIRAL GALAXIES

MOND provides reasonable predictions both for galaxies with low masses and rotation velocities (Milgrom & Sanders 2007) as well as for massive baryonic systems, whereas galactic bulges pose a complication (Sanders & Noordermeer 2007). Modified gravity thus an alternative for DM (Bekenstein 2010; Sanders 2019). Li, Tang & Lin (2017) compare DM models, but cannot prefer one model and exclude others. Since DM particle have not been found, MOND is the best fit for RCs. It breaks down, however, for very low masses of $M_{\text{bar}} < 2 \times 10^{16}$ kg (i.e. $\approx 10^6 \, M_\odot$) (Dutton et al. 2019).

There is general agreement that a close relationship exists between baryonic mass and the observed radial acceleration $g_{\text{obs}}$ (McGaugh 2005a). It can be described by the radial acceleration relation (RAR)

$$g_{\text{obs}} = \frac{g_{\text{bar}}}{1 - e^{-\sqrt{g_{\text{obs}}/g}}}$$

(3)

according to McGaugh, Lelli & Schombert (2016) and McGaugh et al. (2019), where $g_{\text{bar}} = 1.2 \times 10^{-10}$ m s$^{-2}$. Lelli et al. (2017) point out that there is no room for a substantial variation of $g_{\text{bar}}$. Assuming the DM concept, the DM halo contribution is then fully specified by that of the baryons (see e.g. Tenneti et al. 2018; Gharı et al. 2019). The authors add that black holes are not important in this context and Richards et al. (2018) find it remarkable that the tight relationship between DM and baryonic mass is not affected by the different distributions of the baryons in galaxies. Many more studies demonstrate that DM halos and stellar disks are indeed strongly correlated (e.g. Sancisi 2004; Fraternali, Sancisi & Kamphuis 2011; Salucci & Turini 2017; Wechsler & Tinker 2018; Li et al. 2019a,b). This may imply new physics for DM or a modification of gravity (McGaugh 2005a). Recent observations, related to improved observation and evaluation techniques (cf. e.g. Behroozi et al. 2019), have revealed very low halo masses relative to the disk mass adding more constraints on the galaxy–DM halo connection (cf. Sofue 2016; Posti et al. 2019).

The mass discrepancy-acceleration relation (MDAR)

$$\frac{V_{\text{obs}}^2(R)}{V_{\text{bar}}^2(R)} = D$$

(4)

describes a force law in disk galaxies at radius $R$ (McGaugh 2014; Lelli et al. 2017; Navarro et al. 2017; Desmond 2017). McGaugh (2008) discusses this relation and MOND and concludes that the physics behind these empirical relations is unclear. In the inner regions of dwarf galaxies the baryonic mass distribution cannot be responsible for the RCs and thus MDAR is not valid there according to Santos-Santos et al. (2019).

The Tully-Fisher relation (TFR) between luminosity and flat rotation velocity (Tully & Fisher 1977) as well as the corresponding baryonic Tully-Fisher relation (BTFR) (McGaugh 2005b) depend on the fourth power of the flat rotation velocity $V_f$ at large $R$:

$$M_{\text{bar}} = A_f V_f^4,$$

(5)

where $A_f = 9.94 \times 10^{19}$ kg s$^4$ m$^{-4}$ is the best fit for the coefficient $A_f$ (cf. McGaugh et al. 2019). It is valid over many decades in mass (McGaugh 2008) and describes a tight relation between rotation speed and the luminosity or the baryonic mass of disk galaxies (cf. e.g. Navarro 1998; Bekken 2010; Lelli et al. 2017), although its relation to MOND is unclear (McGaugh 2012).

Low surface brightness (LSB) galaxies have larger mass discrepancies between visible and Newtonian dynamical mass than high surface brightness (HSB) ones (Sanders & Noordermeer 2007; McGaugh 2014; McGaugh et al. 2019). We feel that these observations provide important support for our interaction model as will be discussed in Section 4.

The RC shapes also depend on the surface brightness: LSB galaxies have slowly rising RCs, whereas in HSB systems the speed rises sharply and stays flat or even declines beyond the optical radius (Navarro 1998).

In the abstract Noordermeer et al. (2007) write: “At intermediate radii, many RCs decline, with the asymptotic rotation velocity typically 10 to 20 percent cent lower than the maximum. The strength of the decline is correlated with the total luminosity of the galaxies, more luminous galaxies having on average more strongly declining RCs. At large radii, however, all declining RCs flatten out, indicating that substantial amounts of dark matter must be present in these galaxies.” These findings might be directly relevant for our study, cf. Sections 4.1, 4.3 and 4.4.

4 MULTIPLE INTERACTIONS OF GRAVITONS IN GALACTIC DISKS

For isolated masses Newton’s law is valid for the impact model, however, in large mass conglomerations multiple interactions of the proposed gravitons happen and lead to their multiple energy decreases. In a spherical configuration, this does not significantly change the $1/r^2$ dependence of the acceleration field, but in a flat geometry, such as that of disk galaxies, the multiply affected gravitons spread out predominantly in the plane of the disk and thus display a $1/r$ dependence consistent with the flat RCs. It must be noted, however, that a disk galaxy has a certain thickness, presumably related to its surface brightness. LSB galaxies will thus, in general, comply with the $1/r$ rule better than HBS galaxies that are expected to have a $1/r^3$ shape with $\beta > 1$.

We assumed in table 2 of Wilhelm & Dwivedi (2018) that the mean free path length of gravitons $\lambda_0$ is equal to $R_{\text{gal}}$ and found that an amplification factor $F$ of 1.75 to 2 could be achieved by 5 to 6 interactions. With $\lambda_0 = R_{\text{gal}}/4$ the calculation has been extended for this study and gives a factor of about 5 for 10 iterations.

Details on the disk galaxies NGC 3198, NGC 2403, NGC 1090, UGC 3205 and NGC 1705 will be presented in the following subsections.

4.1 NGC 3198

The spiral disk galaxy NGC 3198 is shown in Fig. 1. Begeman (1987) observed a large discrepancy between the actual RC and that predicted from the light distribution in this galaxy. Karukes, Salucci & Gentile (2015) state that it
provides spectacular evidence of a dark force in action as baryons are unable to account for the kinematics. NGC 3198 has been studied by many groups (e.g. van Albada et al. 1985; Taga & Iye 1994; Blais-Ouellette, Amram & Carignan 2001; Kassim, de Jong & Weiner 2006; Kostov 2006; Lovas & Kielkopf 2014; Karukes & Salucci 2017; de Almeida, Amendola & Niro 2017; Daod & Zeki 2019) and we compiled some of the results in Figs. 1 to 3 as well as in Table 1. We did, however, not refer to the many statements related to DM as we feel that our multiple interaction model can explain the flat RCs without a DM component.

The RCs published by Karukes, Salucci & Gentile (2015) are plotted in the upper panel of Fig. 2 and converted to accelerations in the lower panel with equation (1). Using equation (3), i.e. Eq. (4) of McGaugh, Lelli & Schombert (2016), the curve $g_{\text{obs}}$ was approximated by the shaded area enclosing the data points. The required $g_{\text{fit}}$ as a modification of $g_i$ is given in Table 2 and the uncertainties are indicated on the right-hand scale of the diagram. For large radii three-dimensional fits gave mass estimates of $M_{\text{baryon}}^\text{3D}$ and $M_{\text{gas}}^\text{3D}$.

Under the assumption that most of the baryonic mass is inside a certain radius $R = \lambda_0$, we can write

$$\lambda_0 \approx \sqrt{\frac{G \cdot M_{\text{gas}}}{g_{\text{fit}}}},$$

where $G = 6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ is the Newtonian constant of gravitation (Source: 2018 CODATA) and $g_{\text{fit}}$ is the acceleration near $\lambda_0$.

On the abscissa of Fig. 2, we have plotted $R_D$, $\lambda_0$ and $R_{\text{gal}}$ and notice that at $R_{\text{fit}}$ (near $R_{\text{gal}}$) the factor between $g_{\text{fit}}$ and $g_{\text{obs}}$ is close to 1.75, the value expected, if the mean free path $\lambda_0$ of the gravitons is comparable to $R_{\text{gal}}$. A near perfect fit to the observations at larger radii is then obtained by

$$g_{\text{mult2D}}(R) = g_{\text{mult2D}}(R_{\text{fit}}) \left(\frac{R_{\text{fit}}}{R}\right)^\beta$$

with an exponent $\beta = 1.05$. The flat and slightly declining RC in the upper panel, i.e. $V_{\text{mult2D}}(R)$, then follows with equation (1). The physical interpretation of an exponent $\beta > 1$ obviously is that the disk of the galaxy has a certain thickness and the graviton interactions do not exactly occur in a two-dimensional plane. This conclusion is also supported by the slower rotation velocity of the extraplanar gas observed by Gentile et al. (2013) as summarized in their conclusions: “We revealed for the first time in this
galaxy the presence of extraplanar gas over a thickness of a few ($\sim 3$) kpc. Its amount is approximately 15% of the total mass, and one of its main properties is that it appears to be rotating more slowly than the gas close to midplane, with a (rather uncertain) rotation velocity gradient in the vertical direction (lag) of (7 to 15) km s$^{-1}$ kpc$^{-1}$. Other observations of extraplanar gas showing a decrease in velocity above the galactic plane are mentioned in the next subsection.

In Section 6 we will discuss this configuration in more detail taking into account energy and angular momentum conservation.

The acceleration $g_{\text{mult2D}}(R_{\text{fit}})$ also provides a starting point for approximations at smaller radii. The multiple interaction scenario implies that inside the disk gravitons travel back and forth after multiple energy reductions. The amplification factor should, therefore, be close to zero at $R = 0$. A linear interpolation between $R_{\text{fit}}$ and $R = 0$ gives a very good fit. The values of four of the factors are plotted in the diagram.

The irregular behaviour of $g_{\text{local}}$ near the centre of NGC 3198 indicates the presence of a bulge. In Fig. 3 this component is explicitly included with data of de Blok et al. (2008). The treatment of the data is very similar to that of Fig. 2. As a result only $\lambda_0$ and $R_{\text{fit}}$ are shifted to somewhat larger values and $g_{\text{fit}}$ decreased, but still is close to the acceleration, where the amplification factor $F$ and the exponent $\beta$ are not affected.

4.2 NGC 2403

The spiral galaxy NGC 2403 is shown in Fig. 5. This LSB galaxy is also the subject of many publications (e.g. Jarrett et al. 2003; Lovas & Kielkopf 2014; McGaugh 2014).

It is discussed by Jalocha et al. (2011) in the context of a “global thin-disc model of spiral galaxies” as a system with vertical gradients of the azimuthal velocity.
Figure 5. Spiral galaxy NGC 2403. Type: SAB(s)cd, apparent size $21.9' \times 12.3'$, which corresponds to about $(10.6 \times 5.9) \times 10^{20}$ m (i.e. 34.3 kpc \times 19.1 kpc) at a distance of 2.69 Mpc (NASA/IPAC Extragalactic Database). Credit and Copyright Adam Block/Mount Lemmon SkyCenter/University of Arizona - www.caelumobservatory.com/gallery/n2403.shtml.

Figure 6. Spiral Galaxy NGC 2403 (cf. Fig. 5). The $V_{\text{obs}}$, $V_{\text{bar}}$, and $V_{\text{gas}}$ velocity data are taken from figure 1 of Frigerio Martins & Salucci (2007). Characteristic scales are: $R_D = 6.42 \times 10^{19}$ m (i.e. 2.08 kpc) (McGaugh 2005b), $R_{\text{b, half}} = 1.21 \times 10^{20}$ m (i.e. 3.92 kpc) (Santos-Santos et al. 2019), $\lambda_0 = 1.64 \times 10^{23}$ m (i.e. 5.31 kpc), $R_{\text{gal}} = 2.05 \times 10^{20}$ m (i.e. 6.64 kpc) and $R_{\text{eff}} = 2.08 \times 10^{20}$ m (i.e. 6.74 kpc). The flat RC is not declining, i.e. $\beta \approx 1$.

Figure 7. Spiral galaxy NGC 1090. Type: SB(rs)bc, apparent size $3.9' \times 1.8'$, which corresponds to about $(2.6 \times 1.2) \times 10^{21}$ m (i.e. 84.3 kpc \times 38.9 kpc) at a distance of 37.5 Mpc (Bratton 2011). Image: NGC1090-SDSS-DR14.jpg. Credit: Sloan Digital Sky Survey (SDSS), DR14.

Based on data from Fraternali et al. (2002), they find a near linear decrease at heights above the plane between (0.6 and 3.0) kpc of $(−10 \pm 4)$ km s$^{-1}$ kpc$^{-1}$. This decrease is consistent with other observations of peculiar kinematics (Schaap, Sancisi & Swaters 2000; Fraternali et al. 2001; Fraternali, Oosterloo & Sancisi 2004; Fraternali & Binney 2008; Fraternali et al. 2010), but Fraternali & Binney (2006) point out that their model does not predict the observed inflow.

In Fig. 6 the RCs taken from figure 1 of Frigerio Martins & Salucci (2007) display a perfectly flat $V_{\text{obs}}$ that could best be fitted with $\beta \approx 1$. For a thin, LSB galaxy such a result could be expected as most of the graviton interactions have to occur in the plane of the disk. Outside this plane, the extraplanar gas will experience normal gravitational attraction by $M_{\text{bar}}$ with a Keplerian rotation velocity, i.e. slower than the $V_{\text{disk}}$.

4.3 NGC 1090

In Fig. 8 we have modelled the observed RC of spiral galaxy NGC 1090 shown in Fig. 7 quite successfully as $V_{\text{mult2D}}(R)$ under the assumption of an amplification factor $F = 1.5$ and an exponent $\beta = 1.15$. In this case, the high exponent might be caused by the small amplification factor $F$ and a relatively great $\lambda_0$ indicating fewer multiple interactions of gravitons.

Lovas & Kielkopf (2014) write that there is a clear need for CDM for explaining the RCs of NGC 1090 (and other galaxies), however, its distribution is unclear. Kassin, de Jong & Weiner (2006) define a radius $R_X$, where the relative contribution of DM to the ob-
the bulge component is distributed (Frigerio Martins & Salucci 2007). The characteristic scales are: \( R_D = 1.05 \times 10^{20} \) m (i.e. 3.40 kpc) (Frigerio Martins & Salucci 2007), \( R_{\text{bul}} = 3.36 \times 10^{20} \) m (i.e. 10.9 kpc), \( \lambda_0 = 3.72 \times 10^{20} \) m (i.e. 12.1 kpc), \( R_{\text{fit}} = 3.86 \times 10^{20} \) m (i.e. 12.5 kpc) and \( R_X = 23.3 \) kpc with 0.1 dex uncertainties (see table 2 in arXiv:astro-ph/0602271v1, Kassin, de Jong & Weiner 2006). The amplification factor at \( R_{\text{fit}} \) is \( F = 1.50 \), whereas a value of 1.75 was found for NGC 3198 and NGC 2403. The acceleration \( g_{\text{mult2D}}(R) \) is declining with \( (R_{\text{fit}}/R)^{1.15} \) for \( R \geq R_{\text{fit}} \). The dashed line in the upper panel indicates a constant flat RC. \( R_2 \) will be discussed in the text.

For NGC 1090 they find \( R_X = 23.3 \) kpc and emphasize the findings of e.g. Persic & Salucci (1990) and Persic, Salucci & Stel (1996) that for LSB galaxies, such as NGC 1090 as extreme case, cf. (Gentile et al. 2004) and also http://www.kopernik.org/images/archive/n1090.htm, the RCs deviate from the baryonic contribution at smaller radii than for HSB ones. The radius \( R_2 \), where the value of \( g_{\text{mult2D}} \) is twice that of \( g_{\text{bar}} \), can be compared to \( R_X \) indicating that the multiple interaction process can explain the apparent mass discrepancy.

4.4 UGC 3205

In Figs. 9 to 11, we plot RCs of the early-type (HSB) disk galaxy UGC 3205 with data taken from Sanders & Noordermeer (2007), which are slightly modified in the next two figures. In Fig. 10 the bulge component is not included and in Fig. 11 the bulge component is distributed over the disk.

The observed RC is declining at large radii. In this context, it may be of interest that many galaxies with declining RCs are found at \( z > 1 \), interpreted as possibly indicating smaller DM contributions (e.g. Genzel et al. 2017). Drew et al. (2018), however, published a counterexample. Sanders & Noordermeer (2007) present a fit of UGC-3205 with MOND and find deviations from the observed velocities below 10 kpc, tentatively attributed to the weak bar of this galaxy, but quite good agreement at large radii.

We apply our multiple interaction model in Fig. 9 and obtain a good fit even at small radii. This can be seen, in particular, in the acceleration diagram in the lower panel. The similarity between the distribution of the bulge component of this galaxy with that of NGC 3198 in Fig. 9 motivated us to apply the same procedure of eliminating the bulge contribution in Fig. 10. The result again is that the presumably spheroidal bulge contribution does not affect the RC at large radii. The claim of McGaugh (2014) that the BTFR is insensitive to the distribution of the baryonic mass...
Figure 10. These diagrams are based on the same data as in Fig. 9 (Sanders & Noordermeer 2007, figure 1), but the bulge component is not included. An important result of the \( V_{\text{mult}2D} \) and \( g_{\text{mult}2D} \) calculations is that the elimination of the bulge baryons does not effect the RC at large radii. The characteristic scales \( \lambda_0 = 3.20 \times 10^{20} \) m (i.e. 10.4 kpc) and \( R_{\text{fit}} = 3.63 \times 10^{20} \) m (i.e. 11.8 kpc) changed only very little, but the amplification factor \( F \) and \( \beta \) remain the same.

Figure 11. These diagrams are again based on the same data as in Fig. 9 taken from figure 1 of Sanders & Noordermeer (2007), but the bulge component is now distributed over the disk by assuming a modified \( g_{\text{disk}} = g_{\text{disk}} \times M_{\text{mod}2D}/M_{\text{disk}} \) (cf. Table 1). This has a major impact on the extended RC with \( V_{\text{mod}2D} \) higher than \( V_{\text{mult}2D} \) by more than 10 km s\(^{-1}\). The characteristic scales are \( \lambda_0 = 3.34 \times 10^{20} \) m (i.e. 10.8 kpc) and \( R_{\text{fit}} = 3.62 \times 10^{20} \) m (i.e. 11.7 kpc).

in a galaxy, has been tested in Fig. 11, where the bulge mass has been distributed over the disk. This has a major impact on the extended RC. The relative increase of the resulting \( V_{\text{mod}2D} \) is greater than 5 % and thus would give a fractional gain in baryonic mass of more than 20 % in equation (5). Santos-Santos et al. (2019) also found that the RCs are affected by the distribution of the baryonic mass.

4.5 NGC 1705

The four galaxies discussed in the preceding sections are all characterized by regular disk shapes. The irregular dwarf galaxy NGC 1705 we want to include in our sample in order to test the applicability of our procedure on a broader scale. This compact galaxy is much smaller and contains in its nucleus a luminous super star cluster with a mass of \( \approx 2 \times 10^{35} \) kg (i.e. \( 10^5 \) M\(_{\odot}\)) and a significant cold dust component (Tosi et al. 2001; Annibali et al. 2003; O’Halloran et al. 2010).

The RCs in Fig. 13 demonstrate that there is a very large discrepancy between \( V_{\text{bar}} \) and \( V_{\text{obs}} \). A conclusion of Santos-Santos et al. (2019) on the shapes of dwarf galaxy RCs is – stated explicitly for NGC 1705 – that they are difficult to reproduce with DM scenarios, if the central baryon component is small.

If we use the same procedure for this galaxy as for the other ones, we find a very large \( g_{\text{fit}} \) and a small \( \lambda_0 \) relative to \( 2r_{b,\text{half}} \). As mentioned in Sect. 4 this configuration allows many multiple interactions and produces an amplification factor of 5 for 10 iterations. This factor gives a good fit for \( R > R_{\text{fit}} \approx 2r_{b,\text{half}} \). Inside \( R_{\text{fit}} \) the amplification factor has to remain high down to \( \lambda_0 \). The data do, however, not allow any conclusion for \( R < \lambda_0 \).

5 SUMMARY

As a summary of the evaluations of the five galaxies, we compile in Table 1 all mass values found in the literature and those obtained through the various fits. Uncertainty ranges are not included, but the large variations both of the published and the deduced values indicate significant uncertainties. In Table 2 the observed and derived velocities of the RCs are listed together with the fit parameter \( g_{\text{fit}} \) and the exponent \( \beta \).
Table 1. Baryonic mass values of the five disk galaxies from the literature (without uncertainty margins) and from fits in the figures.

| Galaxies | Mass(*) | $M_{\text{bulge}}$/ 10^{40} kg | $M_{\text{gas}}$/ 10^{40} kg | $M_{\text{star}}$/ 10^{40} kg | $M_{\text{disk}}$/ 10^{40} kg | References |
|----------|---------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|------------|
| NGC 3198 | 6.76    | 0.994                         | 12.9                         |                               |                                | Begeman (1989) |
|          |         |                               |                               |                               |                                | Kostov (2006) |
|          |         |                               |                               |                               |                                | Stark, McGaugh & Swaters (2009) |
|          | Fig. 2  | (6.48) (b)                    | (3.59)                        | 8.75                          | (10.1)                        | Karukes, Salucci & Gentile (2015) |
|          | Fig. 3  | 0.573                         | 5.60                          |                               | (11.0)                        | de Blok et al. (2008), figure 37 |
|          | Fig. 4  | { - } (c)                     | (6.18)                        |                               |                                |                        |
| NGC 2403 | Fig. 6  | 2.19                          | 0.934                         | 2.41                          | [3.12] (d)                    | Frigerio Martins & Salucci (2007) |
|          |         |                               |                               |                               | [3.12]                        | McGaugh (2005b)               |
|          |         |                               |                               |                               | 1.84                          | Santos-Santos et al. (2019)     |
|          |         |                               |                               |                               |                               | de Blok et al. (2014)           |
|          |         |                               |                               |                               |                               | Fraternali et al. (2002)        |
|          |         |                               |                               |                               |                               | Stark, McGaugh & Swaters (2009) |
| NGC 1090 | Fig. 8  | [1.68] (e)                    | 1.69                          | 9.34                          | [11.3]                        | Frigerio Martins & Salucci (2007) |
|          |         |                               |                               |                               | (13.1)                        | Gentile et al. (2004)           |
|          |         |                               |                               |                               |                               | Kassin, de Jong & Weiner (2006)  |
|          |         |                               |                               |                               |                               |                        |
|          | Fig. 9  | (1.66)                        | (15.1)                        | (3.32)                        | (18.4)                        | Santos-Santos et al. (2019)     |
|          | Fig. 10 | { - } (f)                     |                               |                               |                               |                        |
|          | Fig. 11 | { - } (f)                     |                               |                               |                               |                        |
| NGC 1705 | Fig. 13 | ≥ 0.0557                      | ≈ 0.0239                      |                               |                               | Annibali et al (2003)           |
|          |         |                               |                               |                               |                               | Santos-Santos et al. (2019)     |

(*) : Mass of Sun (1.98847 ± 0.00007) × 10^{30} kg = 1 M_{\odot} (International Astronomical Union);
(b) : Results of three-dimensional fits in figures are given in parentheses; (c) : $M_{\text{bulge}}$ neglected;
(d) : Values in brackets with modifications explained in text; (e) : 18 % of $M_{\text{star}}$ (Frigerio Martins & Salucci 2007);
(f) : $M_{\text{bulge}}$ distributed over the disk.

Table 2. Observed rotation velocities of the five disk galaxies together with derived mass and acceleration values.

| Galaxy   | Figures | $V_{\text{obs}}$/ km s^{-1} | $V_{\text{fit}}$/ km s^{-1} | $A_{\text{fit}}$/ kg m s^{-1} | $M_{\text{fit}}$/ 10^{40} kg | $V_{\text{min}}$/ km s^{-1} | $g_{\text{multid}}$/ m s^{-2} | $g_{\text{fit}}$/ m s^{-2} | $\beta$ (b) |
|----------|---------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------|
| NGC 3198 | Fig. 2  | 161.0                         | 154.2                         | 1.76 × 10^{20}                | [9.98, 5.63]                  | 148.7                         | 1.49 × 10^{-11}               | 8.50 × 10^{-11}               | 1.05        |
|          | Fig. 3  | 158.7                         | 151.0                         | 3.00 × 10^{20}                | [15.6, 5.17]                  | 147.3                         | 1.85 × 10^{-11}               | 5.00 × 10^{-11}               | 1.05        |
|          | Fig. 4  | 158.7                         | 152.8                         | 2.72 × 10^{20}                | [14.9, 5.42]                  | 148.5                         | 1.88 × 10^{-11}               | 5.50 × 10^{-11}               | 1.05        |
| NGC 2403 | Fig. 6  | 136.2                         | 133.3                         | 2.50 × 10^{20}                | [7.88, 3.13]                  | 133.3                         | 3.01 × 10^{-11}               | 6.00 × 10^{-11}               | ≈ 1.00      |
| NGC 1090 | Fig. 8  | 177.2                         | 172.7                         | 3.33 × 10^{20}                | [29.6, 8.84]                  | 161.7                         | 2.83 × 10^{-11}               | 4.50 × 10^{-11}               | 1.15        |
| UGC 3205 | Fig. 9  | 242.9                         | 221.2                         | 1.25 × 10^{20}                | [29.9, 23.8]                  | 215.7                         | 3.86 × 10^{-11}               | 1.20 × 10^{-10}               | 1.05        |
|          | Fig. 10 | 242.9                         | 233.3                         | 1.25 × 10^{20}                | [31.0, 24.7]                  | 216.7                         | 3.89 × 10^{-11}               | 1.20 × 10^{-10}               | 1.05        |
|          | Fig. 11 | 242.9                         | 233.2                         | (e)                           | (e)                           | 226.2                         | 4.24 × 10^{-11}               | 1.20 × 10^{-10}               | 1.05        |
| NGC 1705 | Fig. 13 | 73.44                         | 73.31                         | 3.33 × 10^{19}                | [0.0062, 0.287]               | 73.31                         | 2.89 × 10^{-11}               | 4.50 × 10^{-10}               | ≈ 1.00      |

(*) : Coefficient $A_{\text{fit}} = 9.94 × 10^{19}$ kg s^{-4} m^{-4} in equation (5) (McGaugh 2008, 2012); (b) : Exponent of declining RCs; (e) : Unrealistic test.

6 DISCUSSION AND CONCLUSIONS

With the exception of the irregular galaxy NGC 1705, the other acceleration diagrams in the lower panels indicate that for a good fit an amplification factor $F$ between 1.5 and 1.8 was required to raise the $g_{\text{bar}}$ values to $g_{\text{obs}}$. According to equation (6) the corresponding galactic radius has to be $R_{\text{fit}} \approx \lambda_0$ provided most of the baryonic mass is within this limit. Equations (1), (6) and (7) then give for the flat RCs with $V_1$ an estimate of $A_{\text{fit}} \approx 1/(g_{\text{fit}} G_N)$ for the coefficients in equation (5) and the corresponding baryonic masses $M_{\text{fit}}$ in Table 2. A comparison with mass values in Table 1 shows that the masses are in reasonable agreement for NGC 3198, NGC 2404, UGC 3205, and even for NGC 1705, but not for NGC 1090.

The observed non-Keplerian behaviour of RCs could be modelled for five disk galaxies without difficulties under the assumption of multiple interactions of gravitons in the planes of the disks. This can be considered as a substantial support for the graviton impact theory—a modification of the old impact theory proposed by Nicolas Fatio de Duillier in 1990.

As mentioned in Subsection 4.1, the physics of the extraplanar gas observations needs to be studied. This can
only be done in a very crude approximation at this stage. It will be helpful to visualize the gravitational potential at $R > \lambda_0 \approx R_{\text{fit}}$ both for the case of Newton’s law outside the galactic plane (a) and near that plane (b). The situation will be considered explicitly at four radii $R = R_{\text{fit}} \times [1, 2, 3, 4]$. Case (a) gives the usual gravitational potential of

$$U(R) = -\frac{G N M_{\text{bar}}}{R}$$

with $U_0 = 0$ at $\infty$. In case (b), we cannot assume a perfectly flat case, corresponding to $g(R) \propto 1/R$, because this would lead to $U_{\text{plane}}(R) = \ln(R)$, which is not zero at infinity. However, $g_{\text{mod}}(R) \propto 1/R^\beta$ with $1 < \beta \ll 2$ is a good approximation of the acceleration in the plane and can be integrated to give the potential

$$U_{\text{mod}}(R) = -F \frac{G N M_{\text{bar}}}{(\beta - 1) R_{\text{fit}}} \left(\frac{R}{R_{\text{fit}}}\right)^{(\beta - 1)}$$

when we require an amplification factor $F$ at $R_{\text{fit}}$. The shape of the potential near the galactic plane can be compared to a canyon, if $\beta$ is increased to 2 just above the plane.

We will apply these results to NGC 3198 in Fig. 2 and get with a baryonic mass of $M_{\text{bar}} = 6.91 \times 10^{10}$ kg (i.e. $3.46 \times 10^{10} M_\odot$) the accelerations indicated by the triangle marks and with equation (1) the velocities marked in the upper panel, where the slow speeds correspond to the Keplerian case outside the plane. The ratio of the potentials is $U_{\text{mod}}(R_{\text{fit}})/U(R_{\text{fit}}) = 35$, i.e. the “depth” of the canyon at $R_{\text{fit}}$. The extraplanar material cannot directly “fall” into the canyon, as this would violate the angular momentum conservation (assuming a closed system). Energy and angular momentum can, however, be conserved, if the falling material moves inwards at the same time, because the radius decreases and the speed is nearly constant.

Our model thus provides consistent pictures both for the in plane and the extraplanar material. Nevertheless, many questions remain, in particular, on the physical properties of the gravitons. Since it was possible to explain the anomalous RCs without assuming any DM that is also a complete mystery, there is hope that further studies will clarify the situation. Considering that MOND models are good fits for many galaxies, the multiple interaction might even point to a physical process behind the mathematical modification of the gravitational acceleration at small values.

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