The choice of polynomials for feed drives CNC machines synthesis for machining complex surfaces.

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Abstract. The article analyzes the problem of choosing the reference polynomials for the feed drives of machines intended for machining curved surfaces. The evaluation of accuracy of a rolling organ movement on a circular trajectory was performed under and without load. The authors proposed a polynomial that provides high accuracy of the coordinated operation of the drives, and having great prospects for use.

The workpiece contour is produced by coordinated motion of the tool and workpiece. The drives in the mechatronic shaping systems of CNC machine tools coordinate the motion of the tool and workpiece. The corresponding specifications are transmitted through the information network of the control device. By switching from mechanical to informational links, the flexibility of the equipment may be significantly increased, and the variety of surfaces that may be produced in the machine tool is expanded. In addition, the machining precision is increased [1-3].

The main problem in the design of the CNC machine drives is to find the optimality criterion for the drive design, since, the dynamics of control signal is not known in advance. The proximity criterion of the output and specified contours cannot be used because the machining program is introduced in the machine tool immediately before manufacture of the part. Therefore, at the design phase, the designer can only determine the transfer function of the drive. However, the time lag between the output and input signals in the feed drives is not so important as in the classical servo drive. An important condition is the lack of signal perturbation thru the unit values of the transfer function modules and the equality of the time delays of the drives of the machine shaping system. Thus, the drive characteristics of the machine shaping system must match the characteristics of the ideal low-frequency filter. Since it is impossible to create an ideal frequency in practice, the transfer function for each drive must be chosen for the class of filters whose frequency characteristics are the best approximation to the ideal characteristic within the frequency range of the useful component of the input signal.

Parametric synthesis of a two-mass supply drive [4] is considered, in the case where one mass is the moment of inertia of the drive’s rotating part \( J_1 \), while the other is the reduced moment of inertia of the drive’s linearly moving part \( J_2 \) (Figure 1). The elastic coupling between the masses \( c_{1,2} \) determines the axial rigidity of the drive. The features of modeling the mechanical part, engine and drive control system are described in [5]. In synthesis, the parameters of the electromechanical system are determined by setting the coefficients of the characteristic polynomial equal to the coefficients of the polynomial approximating an ideal low-frequency filter [6]. The drive’s transfer functions, obtained using the polynomials, have different frequency and time characteristics, have different theoretical and practical realization [7, 8].

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In the machining of complex contour surfaces, the machining error depends on the cutting forces change and the dynamic characteristics of two or more drives and their consistency with each other. The circular form trajectory was used to verify the accuracy of the joint operation of the two drives [9]. The standard ISO 230-4: 2005 contains a similar test to verify the accuracy of the machine.

The trajectory of the two drives system around a circle with a diameter of 32 mm and feed of 4 m/min is shown in figure 2. The dynamic characteristics of both drives correspond to Butterworth or Bessel filter, the first of which minimizes the amplitude, and the second - the phase error [10]. The table 1 contains more detailed research results (for different feed speeds and circle diameters). In the column «D/v_s», «D» is the diameter of the contour, and «v_s» is the feed rate. The task signals for the drives have the form of functions

\[ x(t) = D \sin(2\pi ft) \]  
\[ y(t) = D \cos(2\pi ft) \]

for obtaining a circle trajectory, where \( D \) is the contour diameter (mm), \( f = \frac{1000v_s}{60\pi D} \) – signal frequency (Hz), \( t \) - time (s). The table data shows, that the error in approximation a circle trajectory is directly proportional to its diameter for all the study polynomials at the same frequency of the input signal. The error occurs due to a deviation of the drive’s amplitude-frequency characteristics from the unit at the operating frequency \( f \).

The research results confirmed the need for identical dynamic characteristics of the drives. So, if all feed drives have the same dynamic characteristic, then the maximum trajectory approximation is ensured with their different design features. The error of approximation of the circular trajectory becomes less than 1 \( \mu \)m when the speed of circular feed is less than 4 m / min and the drive was tuned to a single reference polynomial. This is true for all represented polynomials. Therefore, for non-cutting or low-cutting machining methods (laser, EDM, etc.), it is more important to provide the same frequency response of the feed drives than to achieve a higher dynamic quality of the frequency response of each drive (in terms of its proximity to the frequency response of an ideal low pass).
Table 1. Maximum approximation error of a circular trajectory for different reference polynomials (micron).

| D, mm / vLinear phase filter 0.05 | Filter | Butterworth Integral weighted estimate | Linear phase filter 0.5 | Bessel | Binomial filter | Gauss | Filter Gauss 6 dB | Filter Gauss 12 dB | 40 / 10 | 16 / 4 | 16 / 1 | 80 / 20 | 16 / 10 | 40 / 10 | 80 / 20 |
|-----------------------------------|--------|---------------------------------------|------------------------|--------|----------------|-------|----------------|----------------|--------|--------|--------|--------|--------|--------|--------|
| 0.05                              |        |                                       |                        |        |                |       |                |                |        | 0.03   | 0.02   | 0.8    | 0.05   | 0.03   | 0.8    |
| 0.1                               |        |                                       |                        |        |                |       |                |                |        | 0.04   | 0.04   | 1.4    | 1.6    | 1.6    | 1.4    |
| 0.2                               |        |                                       |                        |        |                |       |                |                |        | 0.3    | 0.3    | 2.5    | 4.8    | 4.8    | 2.5    |
| 0.3                               |        |                                       |                        |        |                |       |                |                |        | 0.1    | 0.1    | 0.4    | 0.2    | 0.2    | 0.4    |
| 0.5                               |        |                                       |                        |        |                |       |                |                |        | 0.01   | 0.01   | 0.2    | 0.4    | 0.4    | 0.2    |

Figure 3 shows the simulation results of the approximation of circle and polygonal contours by a system of drives tuned to the Butterworth and Bessel filters under the action of a sinusoidal external load with an amplitude of 1 Nm with frequencies of 50 and 130 Hz. It can be seen that the external harmonic load is of direct importance for the appearance of contour error, and the error of the coordinated operation of the drives is practically absent. If the frequency of exposure is 50 Hz, tuning to the Butterworth filter provides the best accuracy of the drive system, and at 130 Hz - the Bessel filter.

![Figure 3](image1.png)

**Figure 3.** Errors in the simulation of a circle by a drive system tuned to the Butterworth (red) and Bessel (green) filters under load with a frequency of 50 Hz (a) and 130 Hz (b). (a).

Figure 4 shows the graphs of errors caused by a harmonic load of 1 Nm on one of two drives. A diameter of approximated circle is equal 16 mm, a feed speed is equal 4 m/minute, and a frequency is changes from 10 to 500 Hz.

![Figure 4](image2.png)
Figure 4. The relationship between the error and the frequency of the harmonic force.

The graphs show that the maximum error is formed when the frequency of an external force reaches the upper limit of the transmission band (bandwidths). For all considered polynomials, the effect of external load will be the smaller, the larger the drive’s bandwidths. The Butterworth Filter is an exception. Although its bandwidth is wider than the Integral weighted estimate filter drive bandwidths, the Butterworth filter error is greater. The choice of cutting conditions can reduce the error, if the bandwidth is known. Conversely, if we know the frequency of the external force that will be exerted on the drive, it is possible to construct a feed drive using a polynomial with the smallest error.

The study of circular accuracy of the drives (Fig. 2) showed that the drive with Bessel transmission function has amplitude less than the specified task and the drive with Butterworth transmission function has amplitude greater than the specified task. The method of obtaining filters with transient frequency characteristics is described in [11]. These filters minimize the total drive error. The method consists in placing the poles on the complex plane between the corresponding poles of the original polynomials.

The scheme for obtaining the Bessel – Butterworth transition polynomial uses the formulas $r = r_{Be}^m$ and $\varphi = \varphi_{Bu} - m(\varphi_{Bu} - \varphi_{Be})$, where:

- $r_{Be}$ – the radius of the Bessel filter pole;
- $\varphi_{Bu}$ and $\varphi_{Be}$ – the angular coordinates of the Butterworth filter pole and Bessel filter pole respectively;
- $m$ – the degree of transition. Figure 5 shows the transition scheme from the Bessel filter ($P_{Be}$) to the Butterworth filter ($P_{Bu}$) for one of the poles (P). Reducing the value of $m$ from 1 to 0 corresponds to the conversion of the Bessel filter ($m = 1$) to the Butterworth filter ($m = 0$).

Figure 5. The circuit creates a pole of a transition polynomial.

Figure 6 contains graphics of logarithmic phase-frequency characteristics (LPFC) and group delay of transient Bessel – Butterworth filters with varying transition degree $m$ in increments of 0.1. The graphs show that this transition is characterized by a decrease in bandwidths and overshoot, as well as a change in group delay.
Figure 6. LPFC (a) and group delay (b) of transition polynomials by varying the degree of transition \( m \) with a step of 0.1. (red color - Butterworth filter, green - Bessel).

Figure 7 shows the relationship between the degree of transition \( m \) and the frequency \( f_{0.01} \). The deviation of LPFC from the zero value at the frequency \( f_{0.01} \) is less than ± 0.01 dB. It is seen that at \( m \approx 0.4 \) the value of this frequency reaches a maximum.

Figure 8. The relationship between the error and the frequency of a harmonic force when for the transition polynomial \( m = 0.427 \), Bessel and Butterworth filters

Table 2 contains the results of numerical experiments implemented to find the transition polynomial with the smallest error of circular trajectory approximation. The experiments were carried out for a circle with a radius of 16 mm and node movement speeds of 4 and 10 m/min (numerator and denominator in the first column). The smallest error was obtained with \( m = 0.427 \).

Table 2. Error of a circle approximation (\( \mu m \)) by the Bessel – Butterworth transition polynomial with different values \( m \).

| \( m \) | 0   | 0.1 | 0.2 | 0.3 | 0.4 | 0.427 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1   |
|-------|-----|-----|-----|-----|-----|-------|-----|-----|-----|-----|-----|-----|
| 16/1  | 1.9 | 1.55| 1.12| 0.66| 0.16| 0.017 | 0.3 | 0.95| 1.55| 2.19| 2.8 | 3.5 |
|       | 0.3 | 0.25| 0.18| 0.11| 0.03| 0.004 | 0.0 | 0.15| 0.25| 0.35| 0.4 | 0.5 |
|       | 1   | 0.25| 0.18| 0.11| 0.03| 0.004 | 0.0 | 0.15| 0.25| 0.35| 0.4 | 0.5 |
The form of reference polynomial with \( m = 0.427 \):

\[
G_p(s_p) = s_p^7 + 4.785s_p^6 + 11.274s_p^5 + 16.789s_p^4 + 16.949s_p^3 + 11.548s_p^2 + 4.903s_p + 1.
\]

Figure 8 shows the change of maximum error from harmonic load (1 Nm) acting to one of two drives. The diameter of the approximate circle is 16 mm, the feed rate is 4 m/min, the frequency range of the load is from 10 to 500 Hz. The drives are tuned to the Bessel and Butterworth filters and the transition filter \((m = 0.427)\). The maximum error of the transition filter was approximately 20 \(\mu\)m at a frequency of 111 Hz.

Analysis of the results showed that the proposed transition polynomial has a high accuracy of circle approximation. It is comparable with the best results provided by complex polynomials (if there isn’t external force). In this case, the drive, with its corresponding transfer function, is easier implemented on modern components. This polynomial criterion is comparable to the Gauss filter, and is superior to the Bessel and Butterworth polynomials.

Conclusions

1. The error of approximation of a circle path does not exceed 1 \(\mu\)m if the machine drives are tuned to a single reference polynomial and there isn’t external load. This is true for all polynomials discussed above at moderate feed (less than 1000 mm / min). The error significantly depends on the correspondence of the AFC (Amplitude Frequency Characteristic) transfer function to a single value at the frequency of the drive. Therefore, for non-cutting or low-cutting machining methods (laser, EDM, etc.), it is more important to provide the same frequency response of the feed drives than to achieve a higher dynamic quality of the frequency response of each drive.

2. The approximation error of the circle trajectory depends on the drive bandwidths if an external harmonic load acts on the drive. Expansion of the bandwidth reduces the error in almost any reference polynomial. In this case, the maximum error occurs at the frequency of external force is equal to the maximum frequency of the bandwidth. For a known drive bandwidth, it is possible to reduce the magnitude of the error by selecting the cutting mode.

3. The use of different filters gives a minimum of different errors: the Butterworth filter for the amplitude error; Bessel filter for the phase error; the transient filter "Bessel - Butterworth \( m = 0.427\)" for the integral error. Using the last filter for a circle path (without external load) reduced the integral error by 100 and 13 times compared to Butterworth and Bessel filters, respectively. Using the drive with the Bessel-Butterworth transfer function \( m = 0.427 \) for a circle path gives an error decrease of more than 9% and 17% compared with Butterworth and Bessel filters, respectively, under the action of a harmonic load.

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