Modeling of Electron Transmittance and Tunneling Current through a Trapezoidal Potential Barrier by Considering the Spin Polarization Effect

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Abstract. A modeling of the transmittance and tunneling current through a trapezoidal potential barrier was developed by taking into account the spin polarization of the electrons. The transmittance was calculated by using an Airy wavefunction approach and employing a zinc-blende material for the barrier in the heterostructure, which depends on the spin states indicated as ‘up’ and ‘down’. The obtained transmittance was then used to calculate the tunneling current by using the Gauss-Laguerre quadrature method. It was shown that the transmittance is not symmetric with the incident angle of the electron. It was also shown that the tunneling current increases with increasing the bias voltage and tends to be constant in a high voltage regime.

1. Introduction
It is well known that spin control of electronics or magnetism is the base of spintronic devices [1]. One of the important study subjects in spintronics is the technique of spin injection into semiconductors [2]. Initially, spin-polarized carriers from magnetic materials were injected to achieve the spin orientation [2]. However, a common impediment of electrical injection of ferromagnetic materials into semiconductors is the conductivity mismatch of the metal-semiconductor structure, as reported by Schmidt et al. [2-3]. On the other hand, Voskoboynikov, et al. reported that a spin filter could be obtained from nonmagnetic semiconductor materials because of the Rashba spin-orbit coupling due to the Dresselhaus effect occurring in zinc-blende [2, 4-5].

Several authors have studied the tunneling process in heterostructures with a zinc-blende semiconductor under spin polarization [6-9]. However, they did not consider the effect of the bias voltage on the heterostructure, where the rectangular barrier becomes a trapezoidal one. In this paper, we studied the electron transmittance and tunneling current through the trapezoidal potential barrier under spin consideration. The transmittance was derived by applying the Dresselhaus effect and solving the Schrödinger equation. The effects of electron incident angle, energy, and bias voltage are reported.

2. Theoretical Model
Figure 1 illustrates the potential profile of the heterostructure by applying a bias voltage to the barrier with a zinc-blende semiconductor. Here, $\Phi_0$ is the barrier height, $L$ is the barrier width, $e$ is the electronic charge, $V_b$ is the bias voltage, and $E$ is the energy. We considered an electron tunneling through GaSb as the zinc-blende semiconductor from Region 1 to Region 3. The potential profile can therefore be expressed as:
**Figure1.** Potential profile of heterostructures when applying a bias voltage to the barrier.

The model starts from the Hamiltonian equation describing the electron behavior in a material as given by:

\[ H \Psi = E \Psi \]  
(2)

where \( E \) is the total electron energy, \( \Psi \) is the electron wavefunction and \( H \) is the Hamiltonian. Noting that the Hamiltonian, \( H \), consists of \( H_0 \) and \( H_D \) for the heterostructure without and with spin polarization (Dresselhaus term), respectively, the Hamiltonian, \( H \), is then written as \( H = H_0 + H_D \). Here, \( H_D \) is expressed by [10]:

\[ H_D = \gamma \left( \alpha_x k_x - \alpha_y k_y \right) \left( \partial^2 / \partial z^2 \right) \]  
(3)

where \( \alpha_i \) are Pauli matrices \( (i \in \{x,y\}) \) and \( \gamma \) is a Dresselhaus constant.

By solving the Schrödinger equation and applying the boundary conditions at each interface [11], the transmittance can easily be found by using an Airy wavefunction approach following Ref. [12].

\[ T_{\pm} = t_{\pm}^* t_{\pm} \]  
(4)

Where \( t_{\pm} \) is the transmission coefficient, \( t_{\pm}^* \) is the conjugate of \( t_{\pm} \), and the signs '+' and '-' denote spin up and spin down respectively. The \( t_{\pm} \) is written as:

\[ t_{\pm} = -2i \frac{k_{1\pm} \delta_{1\pm}}{m_1} \exp(-ik_{3\pm} L) \times \left\{ \frac{2eV_b}{m_2^2 \hbar^2 L} \right\}^{\frac{1}{3}} \delta_{2\pm} + i \left( \frac{k_{1\pm} \delta_{3\pm} k_{3\pm} \delta_{4\pm}}{m_1 m_3} \right) \left( \frac{2eV_b}{m_2^2 \hbar^2 L} \right)^{\frac{1}{3}} \delta_{5\pm} \]  
(5)
with

$$k_{1±} = \left( \frac{2m_1E_{z±}}{\hbar^2} \right)^{1/2} \left( 1 \mp \frac{2m_1k}{\hbar^2} \right)^{1/2},$$  \hspace{2cm} (6)

$$k_{3±} = \left( \frac{2m_1(E_z + eV_b)}{\hbar^2} \right)^{1/2},$$  \hspace{2cm} (7)

$$\eta_{±}(z) = \left( \frac{2m_2eV_b(E_z + eV_b)}{\hbar^2L} \right)^{1/3} \left\{ \Phi_0 - E_z + \frac{L}{eV_b} - z \right\} \left( \Phi_0 - E_z - \frac{L}{eV_b} - z \right),$$  \hspace{2cm} (8)

$$\delta_{1±} = A\left( \eta(L) \right) - A\left( \eta(L) \right),$$  \hspace{2cm} (9)

$$\delta_{2±} = A\left( \eta(0) \right) - A\left( \eta(0) \right),$$  \hspace{2cm} (10)

$$\delta_{3±} = A\left( \eta(0) \right) - A\left( \eta(0) \right),$$  \hspace{2cm} (11)

$$\delta_{4±} = A\left( \eta(L) \right) - A\left( \eta(L) \right),$$  \hspace{2cm} (12)

$$\delta_{5±} = A\left( \eta(L) \right) - A\left( \eta(L) \right).$$  \hspace{2cm} (13)

Subsequently, the obtained transmittance is employed to calculate the electron tunneling current as given in Ref. [13].

$$J_z = \frac{em}{2\pi^2h^3} \int_0^\infty T(E_z) \ln \left\{ \frac{1 + \exp[(E_F - E_z)/kT]}{1 + \exp[(E_F - E_z - eV_b)/kT]} \right\} dE_z,$$  \hspace{2cm} (14)

where \( k \) is the Boltzmann constant, \( T \) is the temperature, \( E_F \) is the Fermi energy of metal, and \( T(E_z) = T_+ (E_z) - T_- (E_z) \) is the total transmittance. The electron tunneling current in Equation (14) can be easily evaluated by using the Gauss-Laguerre quadrature method [14].

3. Calculated Results and Discussion

To study the transmittance and tunneling current in a metal/GaSb/metal structure, we used the following parameters: \( \gamma_1 = 0, \gamma_2 = 187 \text{ eV/Å}^3, \delta = 0.2 \text{ eV}, d = 5 \text{ nm}, V_b = 0.1 \text{ V}. \) Since the Dresselhauss constant, \( \gamma \), in Region 1 is the same as that in Region 3, the wave vectors are also the same. The electron transmittances as a function of the incident electron energy for an electron energy of 0.1 eV and 0.2 eV are depicted in Figure 2. It can be seen that at an electron energy of 0.1 eV, the transmittance has the highest values at an incident angle of 0° (perpendicular to the interface) for spin up and down polarization. However, the transmittances are the highest at incident angles of -30° and +30°, or spin up and down polarization, respectively. It can also be seen that the transmittance is quasi symmetric with the incident angle of the electron for each state. This appears because of the effect of the bulk inversion asymmetry properties of the zinc-blende.
Figure 2. The transmittance of the electron depends on its incident angle.

Figure 3. Dependence of the electron tunneling current on the bias voltage.

Figures 3 shows the tunneling current versus the oxide voltage for an electron energy of 0.1 eV. It can be seen that the tunneling current increases with the bias voltage. This happens because the potential barrier is lowered by applying a bias voltage to the barrier. Consequently, the electron more easily tunnels through the barrier so that the tunnelling current is enhanced.
4. Conclusions
We have studied the transmittance and tunneling current through a trapezoidal potential barrier under spin polarization consideration and by adding the Dresselhause term. A zinc-blende material was employed for the barrier in the heterostructure. The transmittance depends on the spin states denoted as ‘up’ and ‘down’ states. It was shown that the transmittances are quasi symmetric with the incident angle due to the anisotropy properties of the zinc-blende material. It was also shown that the tunneling current became enhanced as the bias voltage was increased because of the decrement of the potential barrier.

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