Group targets Doppler unmixing based on Viterbi difference algorithm

CHEN Shuai1, *, WANG Yijin1, FENG Cunqian2

1Graduate college of Air Force Engineering University, Xian China
2Air and missile defense College of Air Force Engineering University, Xian China
acsto1995@163.com

ABSTRACT — There are still translational velocity components after coarse compensation of midcourse ballistic target, which will cause the aliasing of time-frequency images when the radar pulse repetition frequency is low, so it is impossible to estimate translational parameters by extracting time-frequency curve. Firstly, Viterbi algorithm is used to extract a time-frequency curve of the strongest signal component, and then the number of Doppler aliasing is determined by difference method. Then, the time-frequency image is expanded in frequency domain, and the unmixing curve is extracted by digital image processing. The simulation results show that the method is effective and robust.

1. Introduction

Ballistic targets will release many conical warheads and decoys in ballistic midcourse. It is difficult for narrow-band radar to separate targets from range. Until the concept of micro motion was proposed, a lot of research on the micro-Doppler characteristics of warheads made it possible for narrow-band radar to distinguish mid-range ballistic group targets.

Narrow-band radar echo signal of micro motion target is a typical non-stationary signal. Because the target is micro motion accompanied by high-speed translation, when the echo signal is analyzed directly in time-frequency domain, there will be aliasing in the time-frequency curve, which is not conducive to the characteristic analysis of the time-frequency curve. In order to solve the above problems, Reference [1] first extracts the time-frequency ridge of the target echo, and then differentiates the ridge, then the number of peak points of the difference function is searched to determine the number of target Doppler aliasing. On this basis, time-frequency expansion is carried out, and the correct time-frequency curve is finally extracted. Reference [2] directly constructs the restoration matrix, which is based on the time-frequency image, expands several images in the positive and negative direction of the frequency dimension, and finally extracts the correct time-frequency curve by the nearest neighbor processing of the restoration matrix. Considering the different intensities of multiple signal components on time-frequency image, there will be a sudden change when the maximum curve is extracted directly, and different degrees of pseudo peaks will be produced in differential operation.

2. Midcourse Target Motion Model

In addition to its micro motion, the midcourse target will also move along the established orbit. Because of the large translation speed and the relatively small phase change caused by micro motion, the echo signal can be pre-compensated by the velocity $V_0$ measured by a pulse. The pre-compensated translational phase can be expressed as
\[ \phi_p' = \frac{4\pi f_c}{c} \left( r_0(t) - V_0 t \right) \quad (1) \]

According to reference [7], the translational phase can be further expressed by a second order polynomial.

\[ \phi_p'' = \frac{4\pi f_c}{c} (a_1 t + a_2 t^2) \quad (2) \]

Among them, \( a_1 \) is the error value of group target velocity after initial compensation, and \( a_2 \) is the acceleration of group target.

The signal echo of multi-target can be equivalent to the sum of several strong scattering points in optical region. Therefore, the multi-target rotational fretting model can be simplified to several strong scattering centers with different spin periods. The single target is modeled and the micro-motion model as shown in Figure 1 is established.

![Fig. 1 Rotating micro-motion warhead model](image)

The reference coordinate system is established by taking the target spin axis as \( z \) axis and the plane intersecting the LOS and \( z \) axis as \( xoz \) plane. It is assumed that the initial position \((x_p^0, y_p^0, z_p^0)^T\) of point P of the rotating scattering center is located on the \( xoz \) plane, and the LOS is expressed as \( n = (\sin \alpha, 0, \cos \alpha)^T \). Point P rotates around oz axis at angular velocity \( \omega \), and the real-time position is represented by coordinate \((x_p(t), y_p(t), z_p(t))^T\).

\[
\begin{align*}
x_p(t) &= \sqrt{x_p^2 + y_p^2} \cos(\omega t) \\
y_p(t) &= \sqrt{x_p^2 + y_p^2} \sin(\omega t) \\
z_p(t) &= z_p
\end{align*}
\]

With the radar line of sight as a reference, the spin micro motion distance of point P can be obtained from the projection position of point P on LOS. It can be seen from reference [6] that the rotational micro motion distance of scattering point P can be expressed as

\[ r_p(t) = z_p \cos \alpha + \sqrt{x_p^2 + y_p^2} \sin \alpha \cos(\omega t) \quad (3) \]

According to formulas (2), (3), the phase change caused by the mid-range target motion can be expressed as

\[ \phi_p = -\frac{4\pi f_c}{c} (a_1 t + a_2 t^2 + r_p(t)) \quad (4) \]

Micro-Doppler frequency of rotating scattering points can be obtained by phase derivation

\[ f_p = \frac{d\phi_p}{dt} \quad (5) \]

Assuming that the frequency of radar transmitting signal is a single frequency signal of \( f_c \), the radar echo of ballistic group targets can be expressed as follows


\[ s(t) = \sum_{i} \sigma_i \exp(-j \frac{4\pi f_c}{c}(a_i t + a_i t^2 + r_{pi})) \]  \hspace{1cm} (6)

Among them, \( i \) is the target number and \( r_{pi} \) is the rotational micro motion distance of \( i \).

3. Viterbi time-frequency curve difference method

Based on the energy rule, the frequency curve can be obtained by processing time-frequency image. The most commonly used method is to find the frequency point with the maximum energy at each time, without considering the amplitude correlation of two adjacent moments on the time-frequency curve, which is not effective in processing multi-component signals. Therefore, the continuity of time-frequency curve and the energy of time-frequency distribution should be taken into account when extracting multi-component time-frequency curve.

3.1 Extraction of time-frequency curve based on Viterbi

To construct the objective optimization function (7), the task of separating the curve is equivalent to finding the path with the maximum sum of energy and the minimum change rate of any two points in the time-frequency image.

\[ f(n) = \arg \min_{k(n):k(n) \neq 0} \left\{ \sum_{n=0}^{n_{-}} C_1(RS_y(n,f(n))) + \sum_{n=0}^{n_{+}} C_2(f(n),f(n+1)) \right\} \]  \hspace{1cm} (7)

In the formula, \( C_1(u) \) and \( C_2(x,y) \) are penalty functions, and \( RS_y(n,k(n)) \) are the magnitude of corresponding points on the selected path. As shown in Figure 2, \( f_1(n) \) and \( f_2(n) \) are two hypothetical curves on the time-frequency graph. \( C_1(u) \) is a subtractive function and \( C_2(x,y) \) is an incremental function. When the difference between \( x \) and \( y \) is less than the threshold value, \( C_2 \) is the smallest, otherwise \( C_2 \) grows linearly.

Viterbi algorithm is used to solve the objective function, and the optimal path corresponding to the time-frequency curve of each scattering point is found sequentially on the time-frequency distribution image. Viterbi algorithm is a commonly used hidden state dynamic programming algorithm for estimating signals. It can find the current optimal state at every moment and reduce the computational complexity effectively.

![Fig.2 Separation diagram of time-frequency curve](image)

3.2 Time-frequency curve difference

The bandwidth of translational Doppler can reach thousands of hertz, and even after coarse compensation, the bandwidth of micro-Doppler can reach hundreds of hertz. When the pulse repetition frequency is less than the micro-Doppler bandwidth, the position of the center of gravity frequency cannot be found when estimating the velocity. Therefore, it is necessary to choose a larger pulse repetition frequency. In fact, under the pulse repetition frequency of long-range early warning radar, there will still be Doppler curve aliasing in observation time, assuming that the true frequency value is \( f \), and the observed frequency value \( f_o \) can be expressed as
\[ f_s = \begin{cases} f_z & |f_z| \leq \text{prf}/2 \\ f_z \pm N \times \text{prf} & |f_z| \leq \text{prf}/2 \end{cases} \quad (8) \]

Formula (8) shows that the pulse repetition frequency differs by N times between the real frequency and the observed frequency, so determining the repetition number N is the key to de-mixing.

Because the time-frequency curves before and after aliasing time are discontinuous on the time axis, there will be obvious peaks when the time-frequency curves are differentiated. Firstly, the time-frequency curve of the target is extracted by viterbi algorithm, and the trajectory of the time-frequency curve is obtained. Then, the difference equation is constructed as follows.

\[ CF(t) = f(t + \Delta t) - f(t) \quad (9) \]

In the formula, \( \Delta t \) is the time interval, and the pulse repetition period is taken as \( \Delta t \). By searching the number of peaks of difference curve \( CF(t) \), the number of aliasing times in time-frequency domain can be determined.

4. Time-frequency curve extraction
In order to get the real time-frequency image, the original time-frequency image \( S_b(t, f) \) is extended in frequency domain, and The obtained extended frequency domain matrix \( KS_b(t, f) \) can be expressed as

\[ KS_b(t, f) = [S_b(t, f); S_b(t, f + \text{prf}); \ldots; S_b(t, f + N \times \text{prf})] \quad (10) \]

In formula (10), N is solved by 2.2-section difference method. It can be seen that \( KS_b(t, f) \) contains not only real Doppler time-frequency curves, but also some aliasing time-frequency curves have been expanded. Direct extraction will cause target velocity ambiguity and can not complete the disambiguation.

The method of digital image processing is introduced to extract the connected region of \( KS_b(t, f) \).

Firstly, the extended time-frequency image is binarized, and the threshold is set to a. The result of binarization can be expressed as

\[ KS_{b,a}(t, f) = \begin{cases} 0 & KS_b(t, f) \leq a \\ 1 & KS_b(t, f) > a \end{cases} \quad (11) \]

Assuming that the matrix \( F_{\beta} \) represents the connected region of the real Doppler time-frequency curve, \( B \) is a structural element of appropriate size. \( F_{0} = f_{0}, \) when \( F_{\beta(k)} = F_{\beta(k-1)} \), the iteration is stopped and \( F_{\beta(k)} \) is equal to \( F_{\beta} \) which represents the connecting area of the true curve. The real time-frequency image of the target is obtained by extracting the connected region from the highest point of the amplitude in the extended frequency domain image. The real time-frequency image of the target is obtained by extracting the connected region from \( f_{0} \) which is the highest point of the amplitude in the extended frequency domain image.

5. Simulation experiment
The simulation parameters are as follows. Radar transmits narrowband single-frequency pulse with pulse repetition frequency of 2000Hz and working frequency of 8GHz. Suppose there are two spin targets in space, spin angular velocities are \( 2\pi \text{ rad/s} \) and \( 4\pi \text{ rad/s} \), respectively.

The true frequency of the spin target is shown in Figure 3. It can be seen that when \( \text{prf} = 2000\text{Hz} \), the target Doppler frequency is aliasing at 0.6s. The aliasing time-frequency image is shown in Fig. 4. The time-frequency analysis method used in simulation is Gabor distribution. It can be seen that there are three time-frequency curves on the whole time-frequency plane. The aliasing occurred at 0.6 s and 1.8 seconds, respectively.
The Viterbi method of Section 2.1 is used to dynamically plan the path of the objective function, and a time-frequency curve in the time-frequency graph is extracted, as shown in Fig. 5.

The extracted aliasing time-frequency curves are processed differently by formula (9). The results are shown in Fig. 6. It can be seen that there are peaks at 0.6 s and 1.8 seconds of aliasing time. Search the number of peaks and extend the initial time-frequency image, as shown in Figure 7. It can be seen that in addition to the complete real time-frequency signal, the aliasing time-frequency curve has also been extended.
6. Summary
It is difficult to extract micro-motion information of micro-motion target because of the existence of translation. The existence of residual velocity and first-order acceleration after coarse translation compensation will lead to the aliasing of time-frequency curve in signal time-frequency image, which is more disadvantageous to the estimation of fretting information.

The simulation results show that the algorithm is effective and can accurately judge the aliasing times of Doppler curves under the condition of low signal-to-noise ratio, which has strong anti-noise performance.
Reference

[1] V.C.Chen. Micro-Doppler effect in radar [M]. [S. l]: Artech House, 2011.
[2] V.C.Chen. Advances in applications of radar micro-Doppler signatures[C]//Proceedings of IEEE Antenna Measurements & Application France:2014.1-4.
[3] Chen V C, Li F Y, Ho S S, et al. Analysis of micro-Doppler signatures[J]. IEE Proceedings on Radar, Sonar and Navigation, 2003, 150(4): 271-276.
[4] DAVID L R. Ballistic missile defense[J]. Journal of Electronic Defence, 2006, 29(1): 46-52.
[5] S. Victoria. American Missile Defense [M]. California: Praeger Security International, 2010: 44-78.
[6] Cohort 311–121O/Team LCS, Missile Defense in the 21st Century Acquisition Environment: Exploring a BMD-Capable LCS Mission Package [A]. Naval Postgraduate School, Monterey, CA, 2013: 1-8.
[7] S. B. Johnson. Technical and institutional factors in the emergence of project management [J]. International Journal of Project Management, 2013, 31(5): 670-681.
[8] S. Victoria. American Missile Defense [M]. California: Praeger Security International, 2010: 44-78.