On the Non-linear Motion of IGS Station

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Abstract With the daily SINEX files of the IGS, the time series of IGS stations are obtained using an independently developed software under generalized network adjustment models with coordinate patterns. From the time series, non-linear motions are found. With spectral analysis method, the variation frequency (annual period and semi-annual period) of the site velocity is found. Moreover, the empirical model of the velocity variation of the station has been established by regression analysis method based on the weekly solution coordinate series of the station. With respect to the velocity of the IGS tracking station, it was better to model the variation periodically or to give a velocity periodically using a piece-wise linear function rather than a linear variable to estimate its bias.

Keywords IGS station; non-linear; spectral analysis; empirical model

1 Problems on the estimation of the station velocity parameter

Nowadays, in the realization of ITRF and the application of GPS, station velocities are supposed to be linear constant velocity motions in station velocity parameter estimation models based on baseline patterns or on coordinate patterns. So, two problems, as follows, are inevitable.

1) Is it reasonable to consider the station velocity as the constant velocity motion? Do periodic variations exist?

2) The significance determination problem of station velocity parameters: does it cause overparameterization while introducing the station velocity parameters directly without the significance analysis of the velocity parameters? If the station velocity parameter is not notable when introducing it, it will have an effect on the intensity of the result and may cause the singular solution of other parameter estimation like the station coordinate parameter.

2 Time series of IGS stations

In order to test whether it is reasonable to suppose the coordinate variation of all stations as the constant velocity motion, the daily SINEX files for 3 a from 2000 to 2002(sioigis10430.snx-sioigis11996.snx) were used, in which the coordinates and velocities of 30 IGS core tracking stations under the ITRF2000 frame and at the epoch 2000.0 were constrained. The generalized network adjustment software based on coordinate patterns is used to estimate weekly station coordinate parameters, and the three-dimensional coordinates accu-
racy of the weekly solutions is about 2-3 mm.

Now, some weekly solutions of IGS continuous tracking stations like LHAS, IRKT are used to explain.

From Fig.1 and Fig.2, we can see that from the weekly solutions, the coordinate variation is not completely linear and the periodic differences may exist, which is notable especially in the U direction.

Although at present there are more than 200 IGS continuous tracking stations around the world, and some countries also set their own continuous operational systems according to a certain density, the density is not enough for establishing the global velocity field model. Multi-period GPS observations are used to estimate velocities for the purpose of denseness. In a simple and convenient way, the method for velocity estimation is:

\[ v = \frac{X_2 - X_1}{t_2 - t_1} \]  

where \( v \) is the station velocity; \( X_2 \) is the station coordinate at epoch \( t_2 \); and \( X_1 \) is the station coordinate at epoch \( t_1 \). The formula above is based on the following assumptions.

1) The accuracy of the observations for each period is almost at the same level.
2) The station coordinate variation is linear.
3) Each station has at least two periods of observations.

Generally speaking, the longer the observation time interval of two periods is, the more exact the velocity estimated by Eq.(1) is. When the observation time interval is longer, the station’s movement is more obvious, but when the three preconditions above cannot be satisfied, the station velocity cannot be estimated correctly.

Recently, in the practical process of using GPS for deformation measurement, these problems often exist. It cannot explain why the separate accuracy of n-period observed results are high, whereas the deformation trend obtained using the n-period observed results according to Eq.(1) is not consistent. It is worth making further researches into whether the effect of station periodic variation exists.

Also, the other problem in using Eq.(1) to get the velocity is that for a station’s n-period observations, it needs to observe the exact same point for the different periods. For a continuous tracking station, there should not be a relative displacement for the station antenna during the process of continuous observation. Now, we will use the weekly result of the WUHN IGS continuous tracking station to explain the preponderance of this problem.

From Fig.3, we can see that for WUHN station, there is an obvious jump between GPS week 1 150 and GPS week 1 151 in the U direction, which is caused obviously by the antenna motion. By referring to the station log of IGS (wuhn_20021027[1].log), we found that the antenna of WUHN station has moved on January 26, 2002, which is just the sixth day of the GPS week 1 150. The quantity of movement is \(-0.0094\) m in N direction, \(-0.0022\) m in E direction and \(0.0410\) m in the U direction.
3 Periodic analysis of station velocity parameter

IGS analysis center provides not only the station coordinate under a certain frame and at a certain referenced epoch, but also the station velocity of this frame for the IGS tracking stations. When providing the velocity, all the IGS tracking stations are supposed to be in constant motion. Because of the effect of outer environment variations, such as the solid earth environment variations, it is reasonable for us to doubt that all the IGS tracking stations are supposed to be in constant motion, which will have a great effect on the research methods and the research conclusion on crustal deformation.

From the coordinate series of IGS tracking station weekly solutions, we can see that the variation of the station coordinates is not absolutely linear in each direction but may have periodic differences. Various data indicates that the variation of the station coordinates is the overlay of a periodic trend and a linear trend.

3.1 Frequency analysis of station velocity variation

The data of station coordinate time series can be processed and analyzed in time domain, which is because station coordinate variation is a function of time, and the estimation of station velocity parameter is dependent on station coordinate variation. Therefore, we can say that the estimation of station velocity is based on time. According to Fourier transform, the signal in time domain can be transformed into one in the frequency domain. The transforming formula is as follows:

$$ F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} \, dt $$

Generally speaking, power is proportional to the square of amplitude, and the corresponding spectrum is called the power spectrum. In the time domain, the average power spectrum of function \( f(t) \) can be defined as follows:

$$ s(t) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 \, dt $$

We can carry on the frequency analysis of the time series data of IGS tracking station so as to get the power spectrum in the N, E and the U direction. Through a mass of spectrum data analysis, we can identify that there is a one-year period variation in most IGS station coordinates or a certain component.

3.2 Empirical model of station velocity estimation

As to the IGS station including one-year period variation, we can use the following empirical model to regress each component of the station velocity:

$$ \nu(\Delta t_i) = a + b \cdot \sin(2 \pi \cdot \Delta t_i) + c \cdot \cos(2 \pi \cdot \Delta t_i) + \epsilon_i $$

where \( \Delta t_i \) is the time variable relative to the referenced epoch, and its unit is a; \( \nu(\Delta t_i) \) is the velocity component at epoch \( \Delta t_i \) in the N, E or the U direction or the X, Y or Z direction; \( a \) is the linear trend term; \( b \) and \( c \) are the coefficients of one-year period term; and \( \epsilon_i \) is the residual term.

According to the time series of LHAS, IRKT and ALIC IGS tracking stations from 2000 to 2002, the empirical model as Eq.(4) can be created. In the process of creating the model, \( t^2 \) test method is used to carry on preliminary analysis of the significance for regressive coefficients. The results are shown in Table 1.

From Table 1 we can see following.

1) From the station time series data, using the empirical model as Eq.(4) to estimate the station velocity, the periodic term parameter is notable in some directions of some stations. This shows that the station coordinate variation is not completely linear but has a one-year period.

2) The linear trend term \( a \) of the velocity in each direction estimated by the empirical model as Eq.(4) is coincident with the linear velocity provided by the
IGS analysis center, as shown in Table 2. This can explain two problems. One is to prove that the adjustment program developed independently for the processing of SINEX files is correct. The other is to explain that in the adjustment process, it is feasible to estimate the station velocity through the time series data instead of considering the station velocity as parameter, which not only simplify the computation, but also has more flexibility in choosing the velocity model.

### 3.3 Primary explanation and countermeasure of the station velocity periodic variation

From the analysis mentioned above, we can see that the periodic variation may exist in station velocity. When analyzing the deformation in the past, it usually based on the assumption that the station coordinate variation is linear, but it cannot give the reasonable explanation for some nonlinear variation results. This condition is usually due to the observation error.

In fact, the time series data processing mentioned above also has a serious defect. We used the time series of station coordinate variation to analyze the periodicity of the station coordinate variation. However, while using the SINEX files to obtain the station coordinate series, the 30 IGS core tracking stations, which were used as datum, were all supposed to have constant motion. This has certain contradictions. Because the datum (the 30 IGS core tracking stations) may have periodic motion, the estimated station coordinate may not be exact when supposing the core tracking stations to have constant motion. On the reduction of absurdity, however, if all the station varia-

### Table 1 Empirical model of stations velocity estimation

| Station | Velocity direction | Empirical velocity model | IGS velocity model |
|---------|--------------------|--------------------------|-------------------|
|         |                    | $a/\text{mm} \cdot \text{a}^{-1}$ | $t = \frac{\hat{a}}{\sigma_a}$ | $b/\text{mm} \cdot \text{a}^{-1}$ | $t = \frac{\hat{b}}{\sigma_b}$ | $c/\text{mm} \cdot \text{a}^{-1}$ | $t = \frac{\hat{c}}{\sigma_c}$ |
| $v_n$   | 13.4               | 116.44                   | 0.3               | **1.18**                      | 0.7                            | 2.58                      | 13.8                     |
| LHAS    | $v_e$              | 41.1                     | 188.20            | 0.5                           | **0.90**                       | -1.7                      | 46.3                     |
|         | $v_n$              | -5.6                     | -23.25            | 2.7                           | 4.67                          | 4.6                       | 8.05                     | 0.2                     |
|         | $v_n$              | -12.4                    | -53.03            | 0.3                           | **0.53**                       | -0.9                      | **-1.59**                | -8.7                    |
| IRKT    | $v_e$              | 20.7                     | 91.71             | -1.2                          | -2.18                         | -1.5                      | -2.63                    | 24.7                    |
|         | $v_n$              | -1.0                     | -3.28             | -8.3                          | -10.94                        | -1.3                      | **-1.65**                | -0.1                    |
|         | $v_n$              | 57.3                     | 687.21            | 0.7                           | 3.69                          | -0.2                      | **-0.81**                | 57.8                    |
| ALIC    | $v_e$              | 33.6                     | 280.55            | -1.0                          | -3.56                         | -0.1                      | **-0.24**                | 30.9                    |
|         | $v_n$              | 4.8                      | 19.26             | 1.0                           | **1.61**                      | -0.6                      | **-1.07**                | 9.1                     |

**Note:** In the form, the bold terms indicate $| t | < t_{0.05} (\alpha = 0.05)$, the corresponding parameters are not apparent.

### Table 2 Difference of linear item in empirical velocity model and IGS velocity model

| Station | Velocity direction | Linear trend term of empirical model / mm • a⁻¹ | IGS velocity model / mm • a⁻¹ | Difference / mm • a⁻¹ |
|---------|--------------------|-----------------------------------------------|-----------------------------|------------------------|
| $v_n$   | 13.4               | 13.8                                         | -0.4                        |                        |
| LHAS    | $v_e$              | 41.1                                         | 46.3                        | -5.2                   |
|         | $v_n$              | -5.6                                         | 0.2                         | -5.8                   |
|         | $v_n$              | -12.4                                        | -8.7                        | -3.7                   |
| IRKT    | $v_e$              | 20.7                                         | 24.7                        | -4                     |
|         | $v_n$              | -1.0                                         | -0.1                        | -0.9                   |
|         | $v_n$              | 57.3                                         | 57.8                        | -0.5                   |
| ALIC    | $v_e$              | 33.6                                         | 30.9                        | 2.7                    |
|         | $v_n$              | 4.8                                          | 9.1                         | -4.3                   |
tions is linear, the obtained other station coordinate variation is also linear when supposing the 30 IGS core tracking stations to have constant motion. Therefore, to some extent, we can say that the nonlinear assumption of station coordinate variation is reasonable.

As for the research on the periodic variation of station velocity, it is still at the initial stage. Usually, the possible reasons for coordinate nonlinear variations are the variations of solid earth environment, the incomplete elimination of observation error in GPS baseline disposal, and the unreasonable data disposal model. It is worth our attention that as this study has revealed, there are regional features in the periodicity of the station coordinate variation. These problems need further research.

Thus, station velocity estimation needs further study. It seems that the method of using the time series data to estimate velocity is more reasonable than the one of obtaining the velocity by adding independent parameters in adjustment process with the parameter transformation method. The reasons are as follows.

1) When considering the station velocity as independent parameter in adjustment process, the numbers of parameters increase obviously, which undoubtedly reduces the stability of the solution.

2) When considering the station velocity as independent parameter in adjustment process, it is difficult to process the jumpiness of the station coordinate caused by the antenna motion and so on.

3) When considering the station velocity as an independent parameter in the adjustment process, the precondition is the linear variation of the station coordinate, but this precondition is suspicious.

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