Research Article

New Traveling Wave Solutions for the (2 + 1)-Dimensional Heisenberg Ferromagnetic Spin Chain Equation

Tianyong Han, Jiajin Wen, Zhao Li, and Jun Yuan

1 College of Computer Science, Chengdu University, Chengdu 610106, China
2 Key Laboratory of Pattern Recognition and Intelligent Information Processing, Institutions of Higher Education of Sichuan Province, Chengdu University, Chengdu 610106, China
3 School of Information Engineering, Nanjing Xiaozhuang University, Nanjing, Jiangsu 211171, China

Correspondence should be addressed to Jun Yuan; yuanjun_math@126.com

Received 9 March 2022; Revised 5 April 2022; Accepted 9 April 2022; Published 6 May 2022

1. Introduction

In this paper, we are devoted to studying the traveling wave solutions for (2 + 1)-dimensional nonlinear equation describing nonlinear wave propagation in Heisenberg ferromagnetic spin chain equation (HFSC) which was derived by Latha and Christal Vasanthi [1],

\[ \dot{u} + \alpha_1 u_{xx} + \alpha_2 u_{yy} + \alpha_3 u_{xy} - \alpha_4 |u|^2 u = 0, \]  

where \( \alpha_1 = \gamma^4 (J + J_1) \), \( \alpha_3 = 2 \gamma^4 J_2 \), and \( \alpha_4 = 2 \gamma^4 \Omega \). In (1), \( u(x, y, t) \) is the complex function of the normalized spatial variables \( x \) and \( y \) and temporal variable \( t \) and represents the appropriate continuum approximation of the coherent magnetism amplitude to the bosonic operators at spin-lattice sites; \( \gamma \) is lattice parameter; \( J \) and \( J_1 \) correspond to the coefficients of bilinear exchange interactions along the \( x \) and \( y \) directions, respectively; \( J_2 \) refer to the neighboring interaction along the diagonal; and \( \Omega \) is the uniaxial crystal field anisotropy parameter.

Traveling wave solution is an important research content of nonlinear partial differential equations. It is difficult to find the traveling wave solution of nonlinear partial differential equations (PDE), but in recent years, many methods of spherical traveling wave solution have been studied. A large number of reports show that researchers have obtained traveling wave solutions of partial differential equations involving many fields, such as nuclear physics, chemical reaction, signal processing, optical fiber, hydrodynamics, plasma, nonlinear optics, and ecology. Traveling wave solutions play an important role in revealing the properties of these nonlinear partial differential equations and predicting the trend of these phenomena [2–21].

Heisenberg ferromagnetic spin chain equation is used to describe the nonlinear wave propagation in a ferromagnetic spin chain system. It is also a generalization of (2 + 1)-dimensional nonlinear Schrödinger equation. With the development of technology, the transistor size is reduced to the nanometer scale. How to develop electronic components...
with higher density and faster storage speed is the practical reason why the HFSC equation is valued [22–25].

Since the traveling wave solution is helpful to explain many physical properties of magnetic materials, many scholars are attracted to the study of the traveling wave solution of the HFSC equation. In Reference [1], equation (1) was derived by using the coherent state ansatz connected to a Holstein–Primakoff bosonic representation of spin operators. Then, the multisoliton solution is constructed by the Darboux transform, and the modulation instability is discussed. Three kinds of traveling wave solutions of equation (1) are derived by using traveling wave transformation [26]. The bilinear form and dark soliton solution of equation (1) are derived by the auxiliary function method, and the soliton interaction is studied. There are two types of elastic and inelastic collisions between these solitons [27]. In references [28, 29], dark multiple solitons are derived, and then, the propagation, interaction, and linear stability analysis of solitons are discussed, respectively. Some complex solutions were obtained by the auxiliary ordinary differential equation method in [30]. In [31], a series of new solutions are constructed by the improved F-expansion method combined with the Jacobian elliptic method. They are exported to understand the constraints that exist. These solutions include periodic wave solutions, double periodic wave solutions, dark soliton solutions, and bright soliton solutions. Lax pair and generalized Darboux transform are used in equation (1) to construct a class of n-order strange waves [32]. In reference [33], the bifurcation of the solution of equation (1) and some traveling wave solutions are obtained by using the dynamic system method. In [34], the authors find the soliton solutions for equation (1) by considering the Bäcklund transformation. By using the Hirota bilinear method, the one-order rogue waves solutions are obtained in [35], and the interaction behaviors between breather and rogue wave are studied in [36]. By applying the bilinear method, the lump wave solution is constructed in [37]. Recently, Li studied a (2 + 1)-dimensional nonlinear ferromagnetic spin chain system with variable coefficients in [38] and obtained the breather and rogue wave by using the algebraic iteration method.

The scholars mentioned above have obtained different traveling wave solutions by different transformations, so they have obtained the properties of HFSC from different directions. Although many traveling wave solutions of the HFSC equation have been obtained in the above literature, there are few studies on the classification of all possible traveling wave solutions. Driven by the above reasons, we will use the complete discrimination system method to find more traveling wave solutions of HFSC to enrich the research results of HFSC.

The rest of this continuing article is methodized as follows: in Section 2, we propound the formation of the polynomial complete discrimination system. In Section 3, we implement this technique to find traveling wave solutions to the HFSC equation. In Section 4, with the help of Maple, the graphical illustration of the modulus of the traveling wave solutions is described by using 2-dimensional and 3-dimensional plots. Finally, some conclusions are given in Section 5.

2. Overview of the Complete Discrimination System

To show the basic idea of our method, consider the following nonlinear differential equation:

\[ P(u, u_x, u_y, u_t, u_{xx}, u_{xt}, u_{yt}, \cdots) = 0, \]

where \( u \) is an unknown function and \( P \) is a polynomial of \( u \) and its partial fractional derivatives. Using the traveling wave transformation,

\[ u(x, y, t) = U(\xi), \]

\[ \xi = \eta_1 x + \eta_2 y - ct, \]

where \( c \) is a nonzero velocity of the traveling wave in (3). We get an ordinary differential equation of the polynomial form

\[ Q(u, u', u'', \cdots) = 0, \]

where \( Q \) is a polynomial in \( u \) and its derivatives and notation (\( u' \)) is the derivative with respect to \( \xi \). Equation (4) can be written as

\[ (u' (\xi))^2 = G(u, \theta_1, \theta_2, \ldots, \theta_m), \]

where \( \theta_1, \theta_2, \ldots, \theta_m \) are parameters. Then, integrating the above formula once, we have

\[ \pm (\xi - \xi_0) = \int \frac{1}{G(u, \theta_1, \theta_2, \ldots, \theta_m)} \, du, \]

where \( G(u) \) is a polynomial function.

According to the complete discrimination system for \( G(u) \), the roots of \( G(u) \) can be classified, and the detailed classification will be given in Section 3.

3. Traveling Wave Solution of the (2 + 1)-Dimensional HFSC

To find the traveling wave solutions of equation (1), we assume that

\[ u(x, y, t) = \varphi(\xi) e^{\rho(x, y, t)}, \]

where

\[ \xi = \eta_1 x + \eta_2 y - ct, \]

\[ \rho(x, y, t) = k_1 x + k_2 y - \mu t, \]

where \( \xi \) is the traveling coordinate, \( \varphi(\xi) \) is the real amplitude function to be determined, and \( \rho(x, y, t) \) is the phase of the envelope. The parameters \( \eta_1 \) and \( \eta_2 \) represent the wave-numbers in the \( x \) and \( y \) directions, respectively, \( c \) is the wave velocity, and \( \mu \) is the frequency of the pulse.
Using the wave transformation (8) into equation (1), separating the real part and the imaginary part, respectively, we obtain that
\[
(\mu - \alpha_i k_i^2 - \alpha_j k_j^2 - \alpha_i \alpha_j k_i k_j) \phi + (\alpha_i \phi_i^4 + \alpha_k \eta_i \eta_j) \phi_{i\xi} - \alpha_i \phi^3 = 0,
\]
and
\[
[2\alpha_i k_i \eta_1 + 2\alpha_j k_j \eta_2 + \alpha_3 (k_1 \eta_2 + k_2 \eta_1) - c] \phi_{\xi} = 0.
\]
(9)

Letting
\[
A = \mu - \alpha_i k_i^2 - \alpha_j k_j^2 - \alpha_i \alpha_j k_i k_j,
B = \alpha_i \phi_i^4 + \alpha_k \eta_i \eta_j,
C = 2\alpha_i k_i \eta_1 + 2\alpha_j k_j \eta_2 + \alpha_3 (k_1 \eta_2 + k_2 \eta_1) - c.
\]
(11)
It follows
\[
C \phi_{\xi} = 0, \quad (12)
\]
\[
A \phi + B \phi_{\xi \xi} - \alpha_i \phi^3 = 0. \quad (13)
\]

We can see that systems (12) and (13) will be compatible in the case \( C = 0 \). We always assume that \( B \neq 0 \), otherwise equation (13) is not a differential equation. Multiply both sides of equation (13) by \( \phi^4 (\xi) \) and integrate once, we get
\[
\phi_{\xi}^2 = \frac{A}{2B} \phi^4 - \frac{A}{B} \phi^2 + C_0, \quad (14)
\]
where \( C_0 \) is the integration constant.

Let \( a_1 = \alpha_i / 2B, \ a_2 = -A / B, \ a_0 = C_0 \). Equation (14) can be written as
\[
\phi_{\xi}^2 = a_1 \phi^4 - a_2 \phi^2 + a_0. \quad (15)
\]

Taking
\[
\begin{align*}
\phi (\xi) &= \pm \sqrt{(4a_1)^{-1} / 2 w (\xi_1)} , \\
b_1 &= 4a_2 (4a_1)^{-2 / 3} , \\
b_0 &= 4a_0 (4a_1)^{-1 / 3} , \\
\xi_1 &= (4a_1)^{1 / 3} \xi.
\end{align*}
\]
(16)
Equation (15) can be written as follows:
\[
w^2 = w (w^2 + b_1 w + b_0).
\]
(17)
Equation (17) becomes the following integration form:
\[
\pm (\xi_1 - \xi_0) = \int \frac{dw}{\sqrt{wF(w)}}, \quad (18)
\]
where \( F(w) = w^2 + b_1 w + b_0 \), and noting \( \Delta = b_1^2 - 4b_0 \).

**Case 1.** \( \Delta = 0 \). Since \( w > -b_1 \), equation (19) can be written as
\[
\pm (\xi_1 - \xi_0) = \int \frac{dw}{(w + b_1/2) \sqrt{w}}.
\]
(19)
If \( b_1 < 0 \), the solution of equation (19) is
\[
\pm (\xi_1 - \xi_0) = \frac{2}{b_1} \ln \left| \frac{\sqrt{2w}}{\sqrt{2w} + \sqrt{-b_1}} \right| .
\]
(20)
The traveling wave solutions of equation (17) are
\[
w_1 (\xi_1) = -\frac{b_1}{2} \tanh^{1 / 2} \left[ \frac{1}{2} \sqrt{\frac{b_1}{2} (\xi_1 - \xi_0)} \right],
\]
(21)
w_2 (\xi_1) = -\frac{b_1}{2} \coth^{1 / 2} \left[ \frac{1}{2} \sqrt{\frac{b_1}{2} (\xi_1 - \xi_0)} \right] .
\]
(22)
If \( b_1 > 0 \), the traveling wave solution of (17) is
\[
w_3 (\xi_1) = \frac{b_1}{2} \tan^{1 / 2} \left[ \frac{1}{2} \sqrt{\frac{b_1}{2} (\xi_1 - \xi_0)} \right].
\]
(23)
If \( b_1 = 0 \), the traveling wave solution of equation (17) is
\[
w_4 (\xi_1) = \frac{1}{(\xi_1 - \xi_0)^2}.
\]
(24)
That is to say, when \( \Delta = 0 \), we get the following solutions of equation (17): solution (21) is the solitary wave solution, solution (22) is the hyperbolic function solution, solution (23) is the trigonometric function solution, and solution (24) is the rational function solution of the equation.

**Case 2.** \( \Delta > 0, \ b_0 = 0 \). Since \( w > -b_1 \), equation (19) can be written as
\[
\pm (\xi_1 - \xi_0) = \int \frac{dw}{w \sqrt{w + b_1}}.
\]
(25)
If \( b_1 > 0 \), then equation (25) is
\[
\pm (\xi_1 - \xi_0) = \frac{2}{b_1} \ln \left| \frac{\sqrt{2(w + b_1)} - \sqrt{\Delta}}{\sqrt{2(w + b_1)} + \sqrt{\Delta}} \right| .
\]
(26)
The solutions of equation (17) are
\[
w_5 (\xi_1) = \frac{b_1}{2} \tanh^{1 / 2} \left[ \frac{1}{2} \sqrt{\frac{b_1}{2} (\xi_1 - \xi_0)} \right] - b_1 ,
\]
(27)
w_6 (\xi_1) = \frac{b_1}{2} \coth^{1 / 2} \left[ \frac{1}{2} \sqrt{\frac{b_1}{2} (\xi_1 - \xi_0)} \right] - b_1 .
\]
(28)
If \( b_1 < 0 \), then, the solution of equation (25) is
\[
\pm (\xi_1 - \xi_0) = -2 \sqrt{-\frac{2}{b_1}} \arctan \left( \sqrt{\frac{2(w + b_1)}{-b_1}} \right).
\]
(29)
So, the solution of (17) is
\[
w_7 (\xi_1) = \frac{b_1}{2} \tan^{1 / 2} \left[ \frac{1}{2} \sqrt{\frac{b_1}{2} (\xi_1 - \xi_0)} \right] - b_1 .
\]
(30)

**Case 3.** \( \Delta > 0, \ b_0 \neq 0 \). Suppose \( a_1 < a_3 \), and one of them is zero, and the other two are the roots of \( F(w) \).
If \( \alpha_1 < w < \alpha_2 \), take \( w = \alpha_1 (\alpha_2 - \alpha_1) \sin^2 \theta \), then, equation (18) can be rewritten as
\[
\pm (\xi_1 - \xi_0) = \frac{2}{\sqrt{\alpha_3 - \alpha_1}} \int_{\xi_1}^{\xi_0} \frac{d\theta}{\sqrt{1 - m^2 \sin^2 \theta}} \tag{31}
\]

Here, \( m^2 = \alpha_2 - \alpha_1/\alpha_3 - \alpha_1 \). According to the definition of Jacobian elliptic function \( sn \), we obtain the solution of equation (17) in the following form:
\[
w_8(\xi_1) = \alpha_1 + (\alpha_2 - \alpha_1) \sin^2 \left( \frac{1}{2} \sqrt{\alpha_3 - \alpha_1} (\xi_1 - \xi_0), m \right). \tag{32}
\]

If \( w > \alpha_3 \), take \( w = \alpha_3 - \alpha_2 (\sin^2 \theta / \cos^2 \theta) \), the solution of (17) can be constructed as follows:
\[
w_9(\xi_1) = \frac{-\alpha_2 \sin(1/2 \sqrt{\alpha_3 - \alpha_1} (\xi_1 - \xi_0), m) + \alpha_3}{\cos(1/2 \sqrt{\alpha_3 - \alpha_1} (\xi_1 - \xi_0), m)}. \tag{33}
\]

Because \( sn \) and \( cn \) are periodic functions, we get two periodic solutions. Note that \( \lim_{k \to 1} \sin(x, k) = \tanh(x) \), \( \lim_{\alpha_2 \to \alpha_3} \cos(x, k) = \sech(x) \). When \( \alpha_2 \to \alpha_3 \), \( m \to 1 \). Then,
\[
w_8(\xi_1) = \alpha_1 + (\alpha_2 - \alpha_1) \tanh^2 \left( \frac{1}{2} \sqrt{\alpha_3 - \alpha_1} (\xi_1 - \xi_0) \right),
\]
\[
w_9(\xi_1) = \frac{-\alpha_2 \tanh(1/2 \sqrt{\alpha_3 - \alpha_1} (\xi_1 - \xi_0)) + \alpha_3}{\sech(1/2 \sqrt{\alpha_3 - \alpha_1} (\xi_1 - \xi_0))}. \tag{34}
\]

Case 4. \( \Delta < 0 \). Since \( w > 0 \), take the transformation
\[
w = \sqrt{b_0 \tan^2 \frac{\theta}{2}} \tag{35}
\]

By (18) and (32),
\[
\pm (\xi_1 - \xi_0) = b_0^{-1/4} \int_{\theta}^{\xi_1} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \tag{36}
\]

where \( k^2 = 1/2(1 - b_1/2\sqrt{b_0}) \). From the definition of Jacobian elliptic function \( cn \), we obtain
\[
\cos \left( b_0^{-1/4} (\xi_1 - \xi_0), k \right) = \cos \theta. \tag{37}
\]

From equation (34), we get
\[
\cos \theta = \frac{2\sqrt{b_0}}{w + \sqrt{b_0}} - 1. \tag{38}
\]

By using equations (36) and (37), the traveling wave solution of equation (17) can be derived in the following form:
\[
w_{10}(\xi_1) = \frac{2\sqrt{b_0}}{1 + \cos(b_0^{-1/4} (\xi_1 - \xi_0), k) - \sqrt{b_0}}. \tag{39}
\]

Now, we get another periodic solution of (17).

With the help of (7) and (16), we get the classification of all single traveling wave solutions of (1) as follows:

\[
u_1(x, y, t) = \pm \left( \frac{A}{\sqrt{a_4}} \right) \tanh \left( \sqrt{2^{-5/3} A \left( 2^{1/3} (a_4/B) \right)^{1/3} (\eta_1 x + \eta_2 y - ct) - \xi_0} \right)
\times \exp(i(k_1 x + k_2 y - \mu t)),
\]

\[
u_2(x, y, t) = \pm \left( \frac{A}{\sqrt{a_4}} \right) \coth \left( \sqrt{2^{-5/3} A \left( 2^{1/3} (a_4/B) \right)^{1/3} (\eta_1 x + \eta_2 y - ct) - \xi_0} \right)
\times \exp(i(k_1 x + k_2 y - \mu t)),
\]

\[
u_3(x, y, t) = \pm \left( \frac{A^3}{2^{5/3} B a_4^2} \right) \tan \left( \sqrt{\sqrt{2a/B} \left( \sqrt{2a/B} \right)^{1/3} (\eta_1 x + \eta_2 y - ct) - \xi_0} \right)
\times \exp(i(k_1 x + k_2 y - \mu t)),
\]
\[ u_4(x, y, t) = \pm \left( \frac{B}{2a_4} \right)^{1/6} \exp \left( i(k_1x + k_2y - \mu t) \right) \]
\[ u_5(x, y, t) = \pm \left( \frac{A}{a_4} \right) \sqrt{2 + \tan^2 \left( \frac{2^{-5/6} A^{1/2}}{B \left( \frac{a_4}{b} \right)^{1/3}} \left( ct - \eta_1x - \eta_2y + \xi_0 \right) \right) - \exp \left( i(k_1x + k_2y - \mu t) \right) \]
\[ u_6(x, y, t) = \pm \left( \frac{a_4}{2} \right)^{1/6} \sqrt{a_1 + (a_2 - a_1) \tan^2 \left( \frac{a_3 - a_1}{2} \right) \left( \eta_1x + \eta_2y - ct + \xi_0 \sqrt{a_2 - a_1/a_3 - a_1} \right) - \exp \left( i(k_1x + k_2y - \mu t) \right) \]
\[ u_7(x, y, t) = \pm \left( \frac{a_4}{2} \right)^{1/6} \sqrt{1 - \tan^2 \left( \frac{a_3 - a_1}{2} \right) \left( \eta_1x + \eta_2y - ct + \xi_0 \sqrt{a_2 - a_1/a_3 - a_1} \right) + \exp \left( i(k_1x + k_2y - \mu t) \right) \]
\[ u_8(x, y, t) = \pm \left( \frac{a_4}{2} \right)^{1/6} \sqrt{1 - \tan^2 \left( \frac{a_3 - a_1}{2} \right) \left( \eta_1x + \eta_2y - ct + \xi_0 \sqrt{a_2 - a_1/a_3 - a_1} \right) + \exp \left( i(k_1x + k_2y - \mu t) \right) \]
\[ u_9(x, y, t) = \pm \left( \frac{a_4}{2} \right)^{1/6} \sqrt{1 - \tan^2 \left( \frac{a_3 - a_1}{2} \right) \left( \eta_1x + \eta_2y - ct + \xi_0 \sqrt{a_2 - a_1/a_3 - a_1} \right) + \exp \left( i(k_1x + k_2y - \mu t) \right) \]
\[ u_{10}(x, y, t) = \left( \frac{32BC^2}{a_4} \right)^{1/12} \left( 1 - \cosh^2 \left( \frac{a_3 - a_1}{2} \right) \left( \eta_1x + \eta_2y - ct + \xi_0 \sqrt{a_2 - a_1/a_3 - a_1} \right) + \exp \left( i(k_1x + k_2y - \mu t) \right) \right) \]

\( (40) \)

### 4. Numerical Simulation

This section contains the 2-dimensional and 3-dimensional solution graph of some of the obtained traveling wave solutions of the HFSC equation. Here, the numerical simulation has been performed in Figures 1–5 for showing the nature of the obtained solution. The plots of the modulus of \( u_1(x, y, t), u_2(x, y, t), u_4(x, y, t), u_6(x, y, t), \) and \( u_{10}(x, y, t) \), by choosing suitable parameters are shown in Figures 1–4 and Figure 5.
Figure 1: The plots of the modulus of $u_1(x, y, t)$ with $A = 2^{5/6}$, $B = -2$, $\alpha_4 = 1/2$, $C_0 = -1$, $\eta_1 = 1$, $\eta_2 = 2$, $c = 1$, and $\xi_0 = 0$. (a) The modulus of $u_1$ when $t = 1$. (b) The modulus of $u_1$ at $y = 0$.

Figure 2: The plots of the modulus of $u_3(x, y, t)$ by choosing suitable parameters: $A = 2^{5/6}$, $B = -2$, $\alpha_4 = 1/2$, $C_0 = 1$, $\eta_1 = 1$, $\eta_2 = 2$, $c = 1$, and $\xi_0 = -3$. (a) The modulus of $u_3$ when $t = 3$. (b) The modulus of $u_3$ at $y = t = 0$. 
Figure 3: The plots of the modulus of $u_4(x, y, t)$ with $A = 0$, $B = 2$, $\alpha_4 = 1/2$, $C_0 = 0$, $\eta_1 = 1$, $\eta_2 = 2$, $c = 1$, and $\xi_0 = 0$. (a) The modulus of $u_4$ when $t = 3$. (b) The modulus of $u_4$ at $y = t = 0$.

Figure 4: The plots of the modulus of $u_6(x, y, t)$ with $A = -3$, $B = 1$, $\alpha_4 = \sqrt{2}$, $C_0 = 1$, $\eta_1 = 1$, $\eta_2 = 2$, $c = 1$, and $\xi_0 = 0$. (a) The modulus of $u_6$ when $t = 3$. (b) The modulus of $u_6$ at $y = 0$, $t = 2$. 
5. Concluding Remarks

In this paper, the $(2+1)$-dimensional Heisenberg ferromagnetic spin chain equation has been investigated via the complete discriminant system method. A range of new traveling wave solutions is obtained, such as periodic solutions, rational wave solutions, Jacobi elliptic solutions, triangular functions solutions, and hyperbolic function solutions. By selecting appropriate parameters, some representative solutions are drawn. These solutions may help us to explore new phenomena which appear in equation (1). This paper gives a new idea to study the dispersive traveling wave solutions of the Heisenberg ferromagnetic spin chain equation. In addition, these results are also helpful to understand the dynamics of nonlinear waves in optics, hydrodynamics and magnetic materials. This method can solve the single wave solutions of more types of PDE, such as fractional or random terms.

Data Availability

No data are required for this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Institutions of Higher Education of Sichuan Province under grant no. MSSB-2021-13.

References

[1] M. M. Latha and C. Christal Vasanthi, “An integrable model of $(2+1)$-dimensional Heisenberg ferromagnetic spin chain and soliton excitations,” *Physica Scripta*, vol. 89, Article ID 065204, 2014.

[2] J. G. Liu, M. S. Osman, W. H. Zhu, L. Zhou, and G. P. Ai, “Different complex wave structures described by the Hirota equation with variable coefficients in inhomogeneous optical fibers,” *Applied Physics B*, vol. 125, no. 175, pp. 1–9, 2019.

[3] K. U. Tariq, M. Younis, H. Rezazadeh, S. T. R. Rizvi, and M. S. Osman, “Optical solitons with quadratic-cubic nonlinearity and fractional temporal evolution,” *Modern Physics Letters B*, vol. 32, no. 26, Article ID 1850317, 2018.

[4] M. S. Osman, D. Lu, M. M. A. Khater, R. A. M. Attia et al., “Complex wave structures for abundant solutions related to the complex Ginzburg-Landau model,” *Optik*, vol. 192, Article ID 162927, 2019.

[5] X. F. Yang, Z. C. Deng, and Y. Wei, “A Riccati-Bernoulli subODE method for nonlinear partial differential equations and its application,” *Advances in Difference Equations*, vol. 117, no. 1, 2015.

[6] A. M. Wazwaz, “A sine-cosine method for handling nonlinear wave equations,” *Mathematical & Computer Modelling*, vol. 40, no. 5–6, pp. 499–508, 2004.

[7] T. Han, Z. Li, and X. Zhang, “Bifurcation and new exact traveling wave solutions to time-space coupled fractional nonlinear Schrödinger equation,” *Physics Letters A*, vol. 395, no. 1, Article ID 127217, 2021.

[8] Z. Li and T. Y. Han, “Bifurcation and exact solutions for the $(2+1)$-dimensional conformable time-fractional Zoomeron equation,” *Advances in Difference Equations*, vol. 656, no. 1, pp. 1–13, 2020.

[9] T. Y. Han, J. J. Wen, and Z. Li, “Bifurcation analysis and single traveling wave solutions of the variable coefficient davey-stewartson system,” *Discrete Dynamics in Nature and Society*, vol. 2022, Article ID 9230723, 2022.

[10] C. Wu and W. Rui, “Method of separation variables combined with homogenous balanced principle for searching exact solutions of nonlinear time-fractional biological population model,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 63, pp. 88–100, 2018.

[11] L. Tang and S. Chen, “Traveling wave solutions for the diffusive Lotka-Volterra equations with boundary problems,” *Applied Mathematics and Computation*, vol. 413, Article ID 126599, 2022.
[12] L. Tang and S. Chen, “The classification of single traveling wave solutions for the fractional coupled nonlinear Schrödinger equation,” *Optical and Quantum Electronics*, vol. 54, no. 2, pp. 1–17, 2022.

[13] L. Tang, “Exact solutions to conformable time-fractional Klein-Gordon equation with high-order nonlinearities,” *Results in Physics*, vol. 18, Article ID 103289, 2020.

[14] L. F. Li, Y. Y. Xie, and S. H. Zhu, “New exact solutions for a generalized KdV equation,” *Nonlinear Dyn*, vol. 92, pp. 215–219, 2018.

[15] J. Zhou, R. Zhou, and S. Zhu, “Peakon, rational function and periodic solutions for Tzitzéica-Dodd-Bullough type equations,” *Chaos, Solitons & Fractals*, vol. 141, Article ID 110419, 2020.

[16] L. X. Du, Y. H. Sun, and D. S. Wu, “Bifurcations and solutions for the generalized nonlinear Schrödinger equation,” *Physics Letters A*, vol. 383, Article ID 126028, 2019.

[17] B. Zhang, Y. H. Xia, and W. J. Zhu, “Explicit exact traveling wave solutions and bifurcations of the generalized combined double sinh-cosh-Gordon equation,” *Applied Mathematics and Computation*, vol. 363, pp. 1–26, 2019.

[18] M. Eslami, “Exact traveling wave solutions to the fractional coupled nonlinear Schrödinger equations,” *Applied Mathematics and Computation*, vol. 285, pp. 141–148, 2016.

[19] B. Sturdevant, “Topological 1-soliton solution of the Biswas-Milovic equation with power law nonlinearity,” *Nonlinear Analysis Real World Applications*, vol. 11, pp. 2871–2874, 2016.

[20] N. Taghizadeh, M. Mirzazadeh, and F. Farahrooz, “Exact solutions of the nonlinear Schrödinger equation by the integral method,” *Journal of Mathematical Analysis & Applications*, vol. 374, pp. 549–553, 2011.

[21] L. Tang, “Dynamical behavior and traveling wave solutions in optical fibers with Schrödinger-Hirota equation,” *Optik*, vol. 245, Article ID 167750, 2021.

[22] P. Martins, D. Silva, M. P. Silva, and S. Lanceros-Mendez, “Improved magnetodielectric coefficient on polymer based composites through enhanced indirect magnetodielectric coupling,” *Applied Physics Letters*, vol. 109, Article ID 112905, 2016.

[23] E. Lizundia, A. Maceiras, J. L. Vilas, P. Martins, and S. Lanceros-Mendez, “Magnetic cellulose nanocrystal nanocomposites for the development of green functional materials,” *Carbohydrate Polymers*, vol. 175, pp. 425–432, 2017.

[24] R. Nakane, G. Tanaka, and A. Hirose, “Reservoir computing with spin waves excited in a garnet film,” *IEEE Access*, vol. 6, pp. 4462–4469, 2018.

[25] B. Q. Li and Y. L. Ma, “Rich soliton structures for the Kraenkel-Manna-Merle (KMM) system in ferromagnetic materials,” *Journal of Superconductivity & Novel Magnetism*, vol. 31, pp. 1773–1778, 2018.

[26] H. Triki and A. M. Warwaz, “New solitons and periodic wave solutions for the (2+1)-dimensional Heisenberg ferromagnetic spin chain equation,” *Journal of Electromagnetic Waves and Applications*, vol. 30, pp. 788–794, 2016.

[27] Q.-M. Wang, Y.-T. Gao, C.-Q. Su, B.-Q. Mao, Z. Gao, and J.-W. Yang, “Dark solitonic interaction and conservation laws for a higher-order (2+1)-dimensional nonlinear Schrödinger-type equation in a Heisenberg ferromagnetic spin chain with bilinear and biquadratic interaction,” *Annals of Physics*, vol. 363, pp. 440–456, 2015.

[28] D. Y. Liu, B. Tian, Y. Jiang, X. Y. Xie, and X. Y. Wu, “Analytic study on a (2+1)-dimensional nonlinear Schrödinger equation in the Heisenberg ferromagnetism,” *Computers & Mathematics with Applications*, vol. 71, pp. 2001–2007, 2016.

[29] X.-H. Zhao, B. Tian, D.-Y. Liu, X.-Y. Wu, J. Chai, and Y.-J. Guo, “Dark solitons interaction for a (2+1)-dimensional nonlinear Schrödinger equation in the Heisenberg ferromagnetic spin chain,” *Superlattices and Microstructures*, vol. 100, pp. 587–595, 2016.

[30] G. S. Tang, S. H. Wang, and G. W. Wang, “Solitons and complexions solutions of an integrable model of (2+1)-dimensional Heisenberg ferromagnetic spin chain,” *Nonlinear Dynamics*, vol. 88, pp. 2319–2327, 2017.

[31] Y. L. Ma, B. Q. Li, and Y. Y. Fu, “A series of the solutions for the Heisenberg ferromagnetic spin chain equation,” *Mathematical Methods in the Applied Sciences*, no. 41, pp. 3316–3322, 2018.

[32] B.-Q. Li and Y.-L. Ma, “Lax pair, Darboux transformation and Nth-order rogue wave solutions for a (2+1)-dimensional Heisenberg ferromagnetic spin chain equation,” *Mathematics with Applications*, vol. 77, no. 2, pp. 514–524, 2019.

[33] D. Yang, “Traveling waves and bifurcations for the (2+1)-dimensional Heisenberg ferromagnetic spin chain equation,” *Optik*, vol. 248, Article ID 168058, 2021.

[34] D. Guo, S.-F. Tian, and T.-T. Zhang, “Integrability, soliton solutions and modulation instability analysis of a(2+1)-dimensional nonlinear Heisenberg ferromagnetic spin chain equation,” *Computers & Mathematics with Applications*, vol. 77, no. 3, pp. 770–778, 2019.

[35] B.-Q. Li and Y.-L. Ma, “Characteristics of rogue waves for a (2 + 1)-dimensional Heisenberg ferromagnetic spin chain system,” *International Journal of Computer Mathematics*, vol. 99, pp. 506–519, 2022.