Naturally Small $x_s$?

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Abstract:

Within the Standard Model, $x_s$ (the mixing parameter in the $B_s - \bar{B}_s$ system) is constrained to the range $7 \leq x_s \leq 40$. We point out that if New Physics contributes significantly to $x_d$ (the mixing parameter in the $B_d - \bar{B}_d$ system), then $2 \leq x_s \leq 7$ is possible without any fine-tuned cancellations between the Standard Model and the New Physics contributions.

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1. $x_s$ in the Standard Model

A measurement of $x_s$, the mixing parameter in the $B_s - \bar{B}_s$ system, would be of much interest \[1\]. Within the Standard Model it will determine the ratio $|V_{td}/V_{ts}|$ with relatively small hadronic uncertainties. Furthermore, it will constrain or may even discover New Physics.

Within the Standard Model, mixing in the $B_s - \bar{B}_s$ system is dominated by box diagrams with intermediate top quarks. This gives

$$x_s^{SM} = \frac{G_F^2 m_W^2}{6\pi^2} \eta_{QCD} (y_t f_2(y_t)) (\tau_{B_s} m_{B_s}) (B_{B_s} f_{B_s}^2) |V_{ts} V_{tb}|^2$$

(1.1)

where $y_t = m_t^2 / m_W^2$ and

$$f_2(y) = 1 - \frac{3y(1+y)}{4(1-y)^2} \left[ 1 + \frac{2y}{1-y} \ln(y) \right].$$

(1.2)

One way to calculate the Standard Model constraints on $x_s$ is to directly use (1.1). The significant sources of uncertainty are $m_t$, $f_{B_s}$, and $\tau_{B_s}|V_{ts}|^2 \approx \tau_b |V_{cb}|^2$:

$m_t = 165 \pm 35 \text{ GeV},$

$$\sqrt{B_{B_s} f_{B_s}} = 0.22 \pm 0.06 \text{ GeV},$$

$$\frac{\tau_b}{1.49 \text{ ps}} |V_{cb}| = 0.037 \pm 0.007.$$ (1.3)

(We use $B_B = 1.16 \pm 0.07$. Note that this corresponds to the renormalization group invariant definition of $B_B$. Accordingly, we use for $\eta_{QCD}$ the value of $\eta_{QCD}(m_t = 150 \text{ GeV}) = 0.5 \ [8]$.) Allowing these parameters to vary independently within their $1\sigma$ ranges, we get

$$3 \leq x_s \leq 40.$$ (1.4)

Another option is to use the theoretical expression for the ratio $R \equiv x_s/x_d$,

$$R^{SM} = \left( \frac{m_{B_s}}{m_{B_d}} \frac{\tau_{B_s}}{\tau_{B_d}} \right) \left( \frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2} \right) \left| \frac{V_{ts}}{V_{td}} \right|^2,$$

(1.5)

together with the experimental value of $x_d$ to find $x_s$. The significant sources of uncertainty here are $x_d$, $f_{B_s}/f_{B_d}$, and $|V_{ts}/V_{td}|$:

$$x_d = 0.69 \pm 0.07,$$

$$\frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2} = 1.35 \pm 0.15,$$

$$|V_{ts}/V_{td}| = 5 \pm 2.$$ (1.6)
The upper bound on $|V_{td}|$ arises from CKM unitarity (we used the recent CLEO range $|V_{ub}/V_{cb}| = 0.08 \pm 0.03$), and the lower bound from the $x_d$ constraint. This leads to $11 \leq R_{SM} \leq 75$ and consequently

$$7 \leq x_s \leq 60.$$  \hspace{1cm} (1.7)

Combining the two methods (1.4) and (1.7), we finally get the Standard Model prediction

$$7 \leq x_s \leq 40.$$  \hspace{1cm} (1.8)

Experiments will soon be able to explore the region near the lower bound in eq. (1.8). The question addressed in this work is whether a violation of this bound is likely in the presence of New Physics.

2. $x_s$ beyond the Standard Model

There are several possible ways in which New Physics could lead to violation of the bounds in (1.8):

(a) The ratio $|V_{ts}/V_{td}|$ is outside the bounds (1.3).

(b) There are significant new contributions to $x_s$.

(c) There are significant new contributions to $x_d$.

1. We would first like to argue that the first effect (a) is not really of much significance. The lower bound in (1.8) corresponds to the upper bound on $|V_{td}|$. This, as mentioned above, is a result of CKM unitarity; therefore it can only be violated in models where the quark sector is extended beyond the three sequential generations of the Standard Model. It was shown, however, in ref. [12] that if CKM unitarity were even moderately violated, then New Physics contributions – $t'$-mediated box-diagrams in models of a fourth quark generation and $Z$-mediated tree-diagrams in models of non-sequential quarks – would dominate the mixing of neutral $B$-mesons. Consequently, either or both of effects (b) and (c) are guaranteed to be much more significant.

The upper bound on $|V_{td}|$ comes from the assumption that the Standard Model contribution saturates $x_d$. Therefore, its violation means that effect (c) is important. We
conclude that even if $|V_{ts}/V_{td}|$ is outside of its Standard Model range, it would not be the dominant source of violation for either bound in (1.8).

2. In many extensions of the Standard Model, $R = R^{SM}$ independently of whether there are significant new contributions to neutral $B$ mixing. The most obvious example is the Minimal Supersymmetric Standard Model (MSSM) [13]: $R^{MSSM} = R^{SM}$ is a result of the fact that the mixing matrix for the gluino couplings to down quarks and squarks is equal to the CKM matrix. A second example is multi-scalar doublet models with Natural Flavor Conservation (NFC): $R^{NFC} = R^{SM}$ is a result of the fact that the relevant charged scalar couplings are proportional, in most of the parameter space, to $m_t V_{ij}$ where $V_{ij}$ is the appropriate CKM element.

As all the considerations that lead to (1.7) remain valid in this class of models, the lower bound in (1.8) remains valid, independent of whether the new contributions to $x_s$ are significant.

On the other hand, the upper bound in (1.8) does not necessarily hold. If there are significant new contributions (in this case to both $x_d$ and $x_s$), the upper bound is relaxed to at least that of eq. (1.7). Actually, with significant new contributions to $x_d$, the lower bound on $|V_{td}|$ is relaxed to the CKM unitarity bound: $|V_{ts}/V_{td}| \leq 9$, leading to $x_s \leq 90$.

In some models, $R \geq R^{SM}$. An example is a multi-scalar doublet model with NFC where $|X| \geq \mathcal{O}(m_{H^\pm}/\sqrt{m_b m_s})$ [14]. $X$ is a parameter that arises from mixing of charged scalars and determines the size of the lightest charged scalar Yukawa couplings that are proportional to down-type masses. ($|X|$ can be large enough only in models with more than two scalar doublets.) In such models, again, the lower bound in (1.8) holds, but the upper bound could be significantly violated [14].

In various other models, $R \approx R^{SM}$ is a good order of magnitude estimate. For example, in multi-scalar models with no NFC but with horizontal symmetries [15] one typically estimates $R^{Hor} \sim \frac{m_s}{m_d}$ which is well within the range of $R^{SM}$. Another example is that of Extended Technicolor (ETC) interactions that generate the top quark mass [16]. In these models we do not expect a strong violation of the lower bound in (1.8), though it is not rigorously excluded.

Finally, there are models where the New Physics contribution is much smaller than
the Standard Model one. For example, in Left-Right Symmetric models, $W_R$-mediated box-diagrams are constrained to contribute less than about 15% of the Standard Model diagrams. Moreover, the new contribution obeys $R^{\text{LRS}} = R^{\text{SM}}$: this is a result of the fact that the mixing matrix for $W_R$ couplings is similar to the CKM matrix. (The situation could be different in models of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry without the discrete LRS and with fine-tuned mixing angles [17].) In supersymmetric models with quark–squark alignment (QSA) [18] the Supersymmetric diagrams contribute negligibly to $x_s$ and modify $x_d$ by no more than 15%. In this type of models, the Standard Model constraints on $x_s$ (1.8) remain essentially unchanged.

3. A third observation is that if $x_d$ is dominated by the Standard Model contribution, then a violation of the lower bound in (1.8) is unlikely. The reason for that is simple: if $x_d$ is accounted for by the $t$-mediated box diagrams, then (1.7) gives the correct bounds on the Standard Model contribution to $x_s$. Therefore, in order that the lower bound in (1.8) is violated, the New Physics has to interfere destructively with the Standard Model. This requires that the two contributions are of the same order of magnitude and of opposite signs. In the large parameter space of New Physics models, such a possibility usually requires fine-tuning.

4. The most interesting models, as far as near-future measurements of $x_s$ are concerned, are those where large contributions from New Physics to $x_d$ are possible and where $R \neq R^{\text{SM}}$. Is this case, the scaling from the experimental value $x_d^{\text{exp}}$ is misleading: the Standard Model contributes $x_s^{\text{SM}} = R^{\text{SM}} x_d^{\text{SM}}$ which could be smaller than the lower bound in (1.7) (though not significantly smaller than the lower bound in (1.4)). This makes the search for $x_s$ in the range $2 \leq x_s \leq 7$ very interesting: if $x_s$ is found to lie in this range it will most likely imply that there are significant new contributions to $x_d$! We next describe two examples of such models.

3. Models that Allow Small $x_s$

1. Our first example is a model with extra mirror down quarks, $D(3, 1)_{-1/3}$ and $\bar{D}(3, 1)_{+1/3}$. Such particles are predicted by $E_6$ GUTs and in “string inspired” frameworks.
If the masses of these vector quarks are not much larger than the electroweak breaking scale, the Z-boson is likely to have non-negligible flavor changing couplings to quarks $U_{ij}$. Z-mediated tree diagrams will contribute to $x_s$:

$$x_s^Z = \frac{\sqrt{2} G_F}{6} \eta_{QCD}(\tau_B, m_B)(B_{B_s} f^2_{B_s}) |U_{sb}|^2.$$  \hspace{1cm} (3.1)

The ratio between the new contributions to $x_s$ and to $x_d$,

$$R^Z = \left( \frac{m_{B_s}}{m_{B_d}} \right) \left( \frac{B_{B_d} f^2_{B_d}}{B_{B_s} f^2_{B_s}} \right) \left| \frac{U_{sb}}{U_{db}} \right|^2,$$  \hspace{1cm} (3.2)

could be very different from $R^{SM}$. Moreover, for $0.01 \leq |U_{db}/V_{cb}| \leq 0.04$, the Z-contribution to $x_d$ is significant [19]. On the other hand, the experimental bound on $BR(B \to X_s \mu^+ \mu^-)$ gives $|U_{sb}/V_{cb}| \leq 0.04$, implying that the Z contribution to $x_s$ is, at most, 25% of the Standard Model contribution [20]. We conclude that in models with Z-mediated FCNCs, $x_s$ is dominated by the Standard Model contribution and the constraints replacing (1.8) are

$$2 \leq x_s \leq 50$$  \hspace{1cm} (3.3)

(where we have taken into account a possible 25% effect due to the Z contribution).

2. The second example is a model with a fourth quark generation. (Of course, four quark generation models are not a very likely possibility in view of the LEP and SLC bounds on the number of light left-handed neutrinos.) Diagrams with one or two $t'$ propagators replacing the the Standard Model $t$ propagators contribute

$$x_s^{4G} = \frac{G_F^2 m_W^2}{6 \pi^2} \eta_{QCD}(\tau_B, m_B)(B_{B_s} f^2_{B_s}) \times \left| 2 y_t y_{t'} g_3(y_t, y_{t'})(V^*_{ts} V_{tb} V^*_{t's} V_{t'b}) + y_{t'} f_2(y_{t'}) (V^*_{t's} V_{t'b})^2 \right|,$$  \hspace{1cm} (3.4)

where

$$g_3(y_i, y_j) = \left[ \frac{1}{4} - \frac{3}{2(y_j - 1)} - \frac{3}{4(y_j - 1)^2} \right] \ln \frac{y_j}{y_i} + (y_i \leftrightarrow y_j) - \frac{3}{4(y_i - 1)(y_j - 1)}.$$  \hspace{1cm} (3.5)

The bounds from $BR(B \to X_s \mu^+ \mu^-)$ (see [21] for the experimental bound and [22] for the theoretical expression) and from $BR(B \to X_s \gamma)$ (see [23] for the experimental bound and [24] for the theoretical expression) are rather mild and allow the new contributions to
dominate $x_s$. As $R^{4G} \neq R^{SM}$ and as $x_d$ may be dominated by $t'$ contributions, the lower bound on $x_s$ is relaxed.

To be more precise, we distinguish three cases:

(a) $\left| \frac{V^*_{ts} V'_{tb}}{V_{ts} V_{tb}} \right|^2 \ll \frac{y_t}{y_{t'}}$: the top contribution dominates and the fourth generation induces small corrections only. We have $3 \leq x_s \leq 40$.

(b) $\left| \frac{V^*_{ts} V'_{tb}}{V_{ts} V_{tb}} \right|^2 \gg \frac{y_t}{y_{t'}}$: the $t'$ contribution dominates and $x_s$ could be significantly enhanced over its Standard Model value. We have $3 \leq x_s$ while the upper bound could be $\mathcal{O}(10)$ weaker than in the Standard Model.

(c) $\left| \frac{V^*_{ts} V'_{tb}}{V_{ts} V_{tb}} \right|^2 \sim \frac{y_t}{y_{t'}}$: the $t$ and $t'$ contributions are of the same order of magnitude. If the relative phase between the two CKM combinations is real, $x_s$ is enhanced. Only if $\arg \left( \frac{V^*_{ts} V'_{tb}}{V_{ts} V_{tb}} \right) \sim \pi/2$ a significant destructive interference becomes possible. Thus, to suppress $x_s$ below, say, 1 would require fine-tuning of both the magnitude and the phase of the mixing matrix. This confirms the results of ref. [25] that finds that a small $x_s$ arises in only a tiny region of the four generation model parameter space. (For previous studies of $x_s$ in four generation models, see [26].)

We conclude that in four generation models, if no fine-tuned cancellations take place,

$$2 \leq x_s^{4G}$$

(3.6)

(where we allowed a reasonable destructive interference) while the upper bound is high above the Standard Model bound.

4. Conclusions

Values of $x_s \leq 7$ do not require fine-tuned cancellations between Standard Model and New Physics contributions. Instead, $2 \leq x_s \leq 7$ is possible under two conditions: (a) There are significant new contributions to $x_d$; and (b) The ratio of these contributions to $x_s$ and to $x_d$ is not proportional to $(V_{ts}/V_{td})^2$. The types of New Physics most likely to fulfill these conditions are extensions of the quark sector by either sequential or non-sequential quarks.

The “window” that we find for naturally small $x_s$ depends on the lower bounds on $m_t$ and $f_{B_s}$. For example, if experiments find $m_t \geq 160 \text{ GeV}$, the lower bound on $x_s$ in eq.
will change from 3 to 4; if lattice calculations imply \( f_{B_s} \geq 0.19 \, \text{GeV} \), the bound will change to 5. (The window will be closed if \( f_{B_s} \geq 0.22 \, \text{GeV} \) is established.)

We present the \( x_s \) bounds in various extensions of the Standard Model in Table 1. The numbers presented in this Table are often a result of a more detailed calculation than presented above. For example, in the MSSM, we take into account that supersymmetric diagrams may enhance the Standard Model result by about 20%, while in quark–squark alignment models \([18]\) supersymmetric diagrams may modify \( x_d \) in either direction by about 15% and do not affect \( x_s \).

### Table 1

**Bounds on \( x_s \)**

| Model      | SM | MSSM | QSA \([13]\) | NFC | Hor \([13]\) | ETC \([16]\) | LRS | Z-FCNC | 4 Gen |
|------------|----|------|-------------|-----|-------------|-----------|-----|--------|-------|
| \( x_s \geq \) | 7  | 7    | 6           | 7   | \~ 7        | \~ 7       | 7   | 2      | 2     |
| \( x_s \leq \) | 40 | 50   | 40          | Large | \~ 90      | \~ 90      | 45  | 50     | Large |

If experiments find \( x_s < 7 \), it would have interesting implications for CP asymmetries in neutral \( B \) decays. As the likely explanation of small \( x_s \) is a large New Physics component in \( x_d \), then CP asymmetries in \( B_d \) decays may differ significantly from the Standard Model predictions. The combination of \( x_s \) and CP asymmetry measurements would be useful in closing in on the source of deviations from the Standard Model.

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