The photo-balls and static solutions in NCQED with time attended

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We drive the potential of photon interaction from Feynman diagrams amplitudes, and we show that the photo-balls, can be produced in noncommutative electrodynamics with time attended but for the static and localized fields, the static solutions (the lumps) can not be exited.

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I. INTRODUCTION

Ordinary noncommutative theories are based on an antisymmetric quantity of rank two, so it does not exhibit the Lorentz symmetry and it does not make much sense to look for noncommutative theories invariant by general coordinates [1]. But, many people believe that this problem can be solved by Hopf algebra. This hope has led to further study of noncommutative dynamics. These studies revealed some peculiar features of noncommutative quantum models. Much attention has been paid also to quantum field theories on noncommutative space time, in particular noncommutative Yang-Mills theory as well as noncommutative QED.

The aim of this paper is to study another aspect of the noncommutivity framework adapted to the source-free static solutions of noncommutative Maxwell equations and extracting photo-balls. As well known, the Maxwell’s four laws describe the evolution in time and space of the electric and magnetic fields and the photo-balls are the bound states of photon interactions. In this paper we will study of values of $E$ and $B$ and we show that there are no static solutions for source-free noncommutative $U(1)$ in the case of $\theta^{\mu} \neq 0$ only. In Ref.[2] S. Deser presents the static solutions in source-free Yang-Mills theory are forbidden. His work is on the nonabelian electrodynamics in the commutative space time and we think this idea can not be generalized to noncommutative electrodynamics, comprehensively.

II. THE PRESENTATION OF PHOTO-BALLS WITH TIME ATTENDED

Interestingly one finds the situation very reminiscent to that of non-Abelian gauge theories, and then the question is whether there are some kinds of bound states in analogy with glue-balls of QCD, here might be called photo-balls. In previous work Ref.[3], in ncqed with space noncommutativity case, we have shown the photo-balls can be excited. Now, we show that in contrast to QED on ordinary space time, noncommutative QED with time attended $[x^\mu, x'^\nu] = i\theta^{\nu\mu}$ is involved by direct interactions between photons. This work is based on the potential model which this model is a Furrier transformation of Feynman diagrams amplitudes.

We consider the possibility that photons of noncommutative QED can make bound states on potential model. The basic ingredient of potential model is that the self-interacting massless gauge particles may get mass by inclusion non-perturbative effects.

There are two related issues when we are considering the effective gauge theory of constituent photons as massive vector particles. First, it is known that the gauge symmetry is lost via the mass term, and the second, massive gauge theories are known to be perturbatively non-renormalizable. Here we remember of given comments on these issues in Ref.[4]. The non-renormalizability of massive gauge theories for QCD and NCQED is same and it is under this assumption that the mass in the theory appears as a fixed parameter, surviving at large momentum. In fact the insufficient decrease of propagator of a massive vector particle at large momentum, due to simple power counting, suggests that the theory can not be renormalizable. But the situation might be different in a theory with constituent mass. At very large momentum, where coupling constant is small due to asymptotic freedom, the perturbation is valid and gluons or photons appear as massless particles. So the mass of constituent gluon or photon, which are generated dynamically, depends on momentum and vanish at large momentum. In a theory for gluons or photons, it is argued that if one can keep the dependence of constituent mass on momentum, which of course is possible only by including the non-perturbative effects, the theory may appear to be non-perturbatively renormalizable. Although
the argument above is for a model involved by dynamically generated mass, due to lack of a systematic treatment of non-perturbative effects, much can be learned via a kinematical description of gluon or photon mass, it is to assume mass as a fix parameter, though the problem still remains with local gauge symmetry.

Following the procedure developed for NCQED case, we insert a mass term to noncommutative QED. As described this is done by introducing an extra scalar field, so the extra scalars do not appear as external legs of diagrams, but the situation is even simpler as far as one considers just the tree diagrams, in which one can ignore the scalars. There are 3 and 4 photon vertices.

In the each vertex energy momentum conservation should be understood and we work in the non-relativistic limit, namely \( p^\mu = ( m + \frac{p^2}{2m}, p ) \), and for polarization vector \( e^\mu = ( \frac{p}{m} e, \frac{p}{m} e ) \), where \( e \) is a 3-vector satisfying \( e^* \cdot e = 1 \) and from Lorentz gauge-fixing condition, we have \( p \cdot e = p^\mu e_\mu = 0 \). Although, there are four diagrams at tree level, those coming from s-, t-, u- and seagull channels. When extracting the potential, by the properly symmetrized wave function for identical particle systems, the “exchange” or “symmetry” diagrams are automatically taken care of, that only one of t- and u-channels’ contributions should be added to others’ contributions. We define the vector \( \theta \) based on \( p_0 \theta^0 \) due to \( q_0 = 0 \) and \( p_0 = m + \frac{p^2}{2m} \).

By this vector, for \( b = ( 0, b ) \), we can write the \( \wedge \)-product as
\[
a \wedge b = \theta_{\alpha \beta} a^\alpha b^\beta = \theta \cdot b \tag{1}
\]

Where \( \theta^i = p_0 \theta^0, \theta^i \). There are two diagrams in seagull channel, one gives the contribution \( \mathcal{M}_{s,g}(1) \propto 1 + 3 \rightarrow 2 + 4 \) and the other, \( \mathcal{M}_{s,g}(2) \), is obtained with replacements \( 3 \rightarrow 4 \). We continue in the center-of-mass frame. By referring to Ref.[4], and for the small noncommutativity parameter, we obtain
\[
\mathcal{M}_{s,g}(1) = 8 e^2 \sin(\frac{P_1 \wedge P_3}{2}) \sin(\frac{P_2 \wedge P_4}{2}) (\cdots) \propto \theta_0^2 p^2 \tag{2}
\]

and
\[
\mathcal{M}_{s,g}(2) = -4 e^2 \sin^2(\frac{p \wedge q}{2q^2 + m^2}) [4m^2 + 3q^2 - 2S^2 q^2 + 2(S \cdot q)^2 + 6iS \cdot (q \times p)] + O(p^2) \tag{3}
\]

Which the seagull channel is in order of \( p^2 \) and we can ignore it. By replacing the non-relativistic limit of \( \epsilon \)‘s, we see that, even without considering coefficient involving \( \sin(\cdots) \), the leading order contribution of s-channel is order of \( |p|^2 \ll m^2 \), that we can ignore it in comparison with the zeroth orders. This observation is exactly as the same as that happens in the QCD case. And a similar one for \( \mathcal{M}_{s,g}(2) \) by replacements \( 3 \rightarrow 4 \) occurred. This observation is different from that for QCD glue-balls, for them the contribution of seagull channel is in zeroth order of momentum and thus should be kept.

**The effective potential between photons**

For the t-channel contribution with help of above equation we have
\[
\mathcal{M}_{f_i} = -4 e^2 \sin^2(\frac{q \cdot \theta}{q^2 + m^2}) \mathcal{Y}(q), \tag{4}
\]

where
\[
\mathcal{Y}(q) = 4m^2 + 3q^2 - 2S^2 q^2 + 2(S \cdot q)^2 + 6iS \cdot (q \times p), \tag{5}
\]

By using the total amplitude, the potential can be written
\[
V_{2\gamma}(r) = \int \frac{d^3q}{8\pi^3} \frac{e^{i\mathbf{q} \cdot \mathbf{r}}}{4\sqrt{E_1 E_2 E_3 E_4}} \mathcal{M}_{f_i} \tag{6}
\]

Now, we define \( U(R) = \int \frac{d^3q}{8\pi^3} \frac{e^{i\mathbf{q} \cdot \mathbf{r}}}{q^2 + m^2} \) and by replacing \( q \rightarrow -i \nabla \) also we keep the \( V_{2\gamma}(r) \) up to first power of \( \theta^2 \) and we ignore from higher power of \( (\theta^4) \) or we have
\[
V_{2\gamma}(r) = \frac{e^2}{4m^2} \mathcal{Y}(-i \nabla) \left[ 2U(r) - U(r_+) - U(r_-) \right], \tag{7}
\]
with \( \mathbf{r}_\perp = \mathbf{r} \pm \theta \). We mention that, only for \( \theta = 0 \) the potential vanishes and this is different from space noncommutativity QED where in the space noncommutativity qed this happens when \( \theta = 0, \quad \mathbf{p} = 0 \) and \( \mathbf{p} \parallel \theta \). It is reasonable to see the behavior of potential for small noncommutativity parameter, the first surviving terms are given by

\[
V_{2\gamma}(r) = -\frac{e^2}{4m^2} \mathcal{Y}(-i\nabla)\left(\theta \cdot \nabla\right)^2 U(r) + O(\theta^4)
\]

Recalling that for a function \( f(r) \), \( \partial_\gamma f(r) = x_\gamma \nabla_r f \), with \( \nabla_r = r^{-1} \partial_r \), and using \( (\mathbf{p} \times \mathbf{S}) \cdot \mathbf{r} = (\mathbf{r} \times \mathbf{p}) \cdot \mathbf{S} = \mathbf{L} \cdot \mathbf{S} \), \( \nabla^2 U(r) = m^2 U(r) - \delta(r) \) with \( \mathbf{L} \) as the total angular momentum, we get the expression for potential

\[
V_{2\gamma}(r) = -\frac{e^2}{4m^2} \left\{ m^2 (1 + 2S^2) \left[ (\theta \cdot \theta)^2 \nabla_r + (\theta \cdot r)^2 \nabla_r \right] - 2 \left[ (\theta \cdot \theta)^2 + 2(\theta \cdot \mathbf{S})^2 \right] \nabla_r \nabla_r + (\theta \cdot r)^2 \right\} U(r)
\]

\[
+ \text{D.D.} + O(\theta^4).
\]

Where \( S \equiv |\mathbf{S}| \), and D.D. is for the distributional derivatives of the function \( U(r) \), containing \( \delta \)-function and its derivatives. We make comments on the potential given by Eq. (9). First we mention that due to \( \mathbf{r}_\perp \)'s in the inner products, the effective lowest power is \( r^{-5} \). Second, the strength of the potential, through the definition of \( \theta \), depends on mass and momentum. Third, the spin-independent part of the potential, \( S = 0 \), we have

\[
V^{S=0}_{2\gamma}(r) = -\frac{e^2}{4m^2} \frac{e^{-mr}}{4\pi} \left[ - (\theta \cdot \theta)^2 \frac{m^2 r^2 + 3mr + 3}{r^3} \right],
\]

this potential is not included the photons momentums and this is in contrast of the space noncommutativity potential with \( S = 0 \). The following modes can be considered

\[
\theta = p_0 \theta^{0\perp} \hat{z}
\]

so we have \( \theta \cdot \hat{r} = p_0 \theta^{0\perp} \cos \theta \) so \( V^{S=0}_{2\gamma}(r, \theta) \) is not a central force! but the \( p_\perp \) is still constant of motion so the motion stays at plane, the meaning of this statement is for a specified amount of \( \theta \), according to the behavior of potential, the bound state(s) can be still exist. For the general \( \theta \) the potential is \( V^{S=0}_{2\gamma}(r, \theta, \phi) \) which there are not constant of motion so this case is not interesting.

### III. Static Solutions and Photo-Balls

We now turn to the case of noncommutative \( U(1) \) theory on the \( D \)-dimensional Minkowski space time with noncommutative coordinates. The \( U(1) \) action is

\[
S = -\frac{1}{4} \int d^4x \left( F_{\mu\nu} \star F_{\alpha\beta} \right) \eta^{\mu\alpha} \eta^{\nu\beta}
\]

where \( F_{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha - \imath e [A^\alpha, A^\beta]_\star \) denotes the strength of the noncommutative \( U(1) \) gauge fields. In space time noncommutativity with condition of \( [\mathbf{x}_1, \mathbf{x}_2]_\star = i\theta^{ij} \) In the standard \( U(1) \) theory the canonical energy momentum tensor is

\[
4T_{\mu\nu} = -2\{F_{\mu\alpha}, F_{\alpha\nu}\}_\star - \eta^{\mu\nu}(F_{\alpha\beta} \star F_{\alpha\beta}),
\]

where

\[
D_\mu \star T_{\mu\nu} = \partial_\mu T_{\mu\nu} - \imath e [A_\mu, T_{\mu\nu}]_\star = 0
\]

Here, we show that for the case \( d = 4 \) and \( \theta^{0i} \neq 0 \) there are no static solutions for electric fields \( (F_{0i} = E_i) \) to self interacting models of \( U(1) \) type.
Lemma: In the noncommutativity QED, the static solutions of electric fields \(F_{0i}\) are absence.

Proof: The time independent solutions of the gauge fields are chosen so the electric field reduces to \(F_{0i} = \partial_0A_i - \partial_iA_0 - ie[A_0, A_i]\). For static solutions \(\partial^\mu J_{\mu} = 0\), it follows that \(D^i \times F_{0i} = 0\) and consequently that for any values of \(d\) we have \(F_{0i} = 0\) so its result is that absence of the electric fields. In this case, there are no sentences about the magnetic fields.

Lemma: In the time noncommutativity QED, the static solutions are absence, expect for \(d = 5\).

Proof: In this case of noncommutativity, \([\dot{x}^\mu, \dot{x}^\nu] = i\theta^{0\mu}\theta_{0\nu}\). The all details in Ref. [2] are correct or in this case the previous lemma is still valid and the stress tensor for a \(U_c(1)\) field has following components

\[
\int d^{d-1}x \, T_{\mu\nu} = \int d^{d-1}x \, \frac{1}{4}(4 - d)F_{\alpha\beta}F_{\alpha\beta},
\]

then \(\int d^{d-1}x \, T_{0}^0 = \int d^{d-1}x \, \frac{1}{2}(F_{0i}^2 + \frac{1}{2}F_{ij}^2)\) where \(F^2 = F_{\mu\nu}F_{\mu\nu}\) now, compactness of gauge group indeed to \(F_{0i}^2\) and \(F_{ij}^2\) be positive. We assume that the fields will be vanished on space boundary because all of \(F_{\mu\nu}\) to fall of faster than \(|\vec{r}|^{-\frac{1}{2}(d-1)}\), so

\[
\int d^{d-1}y \, T_{j}^{j}(y) = 0
\]

The vanishing of the integration implies that

\[
\int d^{d-1}y \, T_{i}^{i} = \int d^{d-1}y \, \frac{1}{2}((d - 3)F_{0i}^2 + \frac{1}{2}(5 - d)F_{ij}^2) = 0
\]

For \(d = 4\), \(F_{0i}\) and \(F_{ij}\) must all vanish. For \(d > 5\) we learn nothing further above from Eq. (17).

For earlier case \([\dot{x}^\mu, \dot{x}^\nu] = i\theta^{0\mu}\theta_{0\nu}\) the photon self interaction will be removed because the field strength tensor becomes \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = f_{\mu\nu}\) and there is no self interaction, so we can write

Lemma: In the noncommutativity with time attended, the photo-balls can not be exited.

IV. DISCUSSION

In this work by using the electrodynamics in noncommutative space time, we drive the potential of \(\gamma - \gamma\) interaction based on Furrier transformation of the Feynman diagrams amplitudes. For special cases we show that the photo-balls can be excited but it can not be produced in noncommutativity with time attended. Also we show that the vanishing of self-stress for static systems excludes finite energy time-independent solutions of source-free \(U_c(1)\) theory in \((3+1)\) dimensions. This implies that static solutions in the case of \(\theta^{ij} \neq 0\), for non-commutative electromagnetic fields are forbidden. For the case of spatial noncommutativity, we show that just the electric fields is absence and the magnetic fields is not.

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