Simulation of satellite attitude control using Single Gimbal Control Moment Gyro (SGCMG) system

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Abstract. Control Moment Gyro (CMG) is a type Attitude Control System (ACS), a system used to change the attitude of satellite along its orbit. CMG generates required gyroscopic torque by spinning a number of rotors, and directs the torque by changing the orientation of those rotors using gimbals. CMG is capable of generating a large amount of torque with relatively low power which makes it suitable for high agility mission requiring 1-10 deg/s slew rate. A major problem encountered with the use of CMG in practice is the presence of singularity, a state in which, the CMG is unable to generate torque along certain direction. Various steering algorithms have been developed to overcome the singularity problems. In this paper, the attitude control performance of Single Gimbal Control Moment Gyro (SGCMG) system is investigated. Steering algorithm also implemented to overcome singularity problem that might be encountered. The results show that SGCMG capable of generating torque required for agile attitude maneuver in the presence of singularity.

1. Introduction
Attitude control is the process of stabilizing and changing the attitude of satellite during mission. Current mission for imaging satellite requires a capability to rapidly change the attitude of the satellite with slew rate requirement of 1-10 deg/s [1]. To fulfill the slew rate requirement, a powerful attitude control system capable of generating large amount of torque will be needed. Control Moment Gyro (CMG) is a type of Attitude Control System (ACS) that generates torque by changing the angular momentum vector of spinning rotor with respect to spacecraft frame of reference. CMG is capable of generating a large amount of torque with relatively low power which makes it suitable for high agility mission requiring 1-10 deg/s slew rate. Despite the capability of CMG to produce large amount of torque, the use of CMG in practice is hindered by inherent geometric singularities.
In general, there are three types of CMG, the earliest CMG developed is called Double Gimbal Control Moment Gyro (DGCMG). It utilizes two gimbals to change the orientation of the rotor and providing additional degree of freedom for spinning rotor. This type of CMG was already used in several satellites like MIR, Skylab, and International Space Station (ISS). However using DGCMG requires a lot of space and the hardware is complicated to build which makes it unsuitable for smaller satellite. The second type is called Single Gimbal Control Moment Gyro where only one gimbal is used to change the orientation of spinning rotor. This type of CMG has less degree of freedom but capable of generating higher torque than any other type of CMG. In addition the hardware of SGCMG is the simplest to build and does not require a lot of space which makes it suitable for smaller satellite [3]. The third type is called Variable Speed Control Moment Gyro (VSCMG) which basically the same as SGCMG because it utilizes only one gimbal. However this type of CMG is able to change the speed of the spinning rotor, essentially adding extra degree of freedom compared to SGCMG [3].

Singularity for CMG can be defined as a set of gimbal angles where no torque can be generated along certain direction [2][4]. Generally there are two types of singularity of CMG. The first type is when the maximum momentum available for CMG is achieved or in other words when all CMG point at the same direction. This type is called external or saturation singularities, because it occurs at the surface of CMG momentum envelope. This singularity also represent hardware limit of CMG where no more momentum can be stored along that direction. The second type of singularity occurs at internal surface of CMG momentum envelope which the total angular momentum of CMG is smaller than the maximum available. This type is called internal singularities [5] which also the most troublesome singularity that might be encountered during reorientation maneuver using CMG. This type of singularities are discussed comprehensively in reference [6] while the visualization of singularities is discussed in reference [7].

During attitude maneuver this singularity problem might be encountered and many methods have been reported in literatures to overcome the CMG singularities. This method is also known as steering algorithm and has many variations. The most common type of steering algorithm is based on the minimum two-norm solution in the form of pseudo inverse of gimbal steering equation. There are also method that utilize the null motion to avoid any kind of singularities.

In this paper, the utilization of SGCMG for attitude control of satellite is investigated using a simulator developed in MATLAB and Simulink environment. Steering algorithm is also implemented in the simulation to escape or avoid any kind of singularity that might be encountered during maneuver.

2. Mathematical modeling
The equation of motion to describe rigid spacecraft dynamics with a cluster of CMGs is given by :

$$\dot{H}_s + \omega \times H_s = M_{ext}$$  \hspace{2cm} (1)
where \( \dot{H}_s \) represent spacecraft angular momentum and \( M_{ext} \) is the external torque acting on spacecraft, including external disturbances. Assuming the CMG system is statically and dynamically balanced, the total angular momentum of spacecraft with a cluster of CMG is given by:

\[
H_s = I\omega + h_{cmg}
\]  

(2)

where \( I \) is the inertia matrix of spacecraft, \( \omega \) is the angular velocity of spacecraft and \( h_{cmg} \) is the total angular momentum of CMG. Substituting the equation (2) to equation (1) we get

\[
I\dot{\omega} + \dot{h}_{cmg} + \omega \times (I\omega + h_{cmg}) = M_{ext}
\]  

(3)

To further simplify the equation, the term internal control torque generated by CMG, \( u \) will be introduced, thus the equation becomes

\[
\dot{h}_{cmg} + \omega \times h_{cmg} = -u
\]  

(4)

\[
I\dot{\omega} + \omega \times h_{cmg} = u + M_{ext}
\]  

(5)

The angular momentum of CMG is a function of gimbal angles \( \delta_i \) (\( i = 1,2,3,\ldots n \) and \( n \) is the number of CMG) given by:

\[
h_{cmg} = \sum_{i=1}^{n} h_i(\delta_i)
\]  

(6)

The output torque of CMG, \( \dot{h}_{cmg} \) can be obtained by taking the time derivation of angular momentum given by

\[
\dot{h}_{cmg} = \frac{dh_{cmg}}{dt} = \frac{dh_{cmg}}{d\delta} \cdot \frac{d\delta}{dt}
\]  

(7)

where \( \frac{dh_{cmg}}{d\delta} \) is \( 3 \times n \) Jacobian matrix (J). Thus the torque generated by CMG can written as

\[
\dot{h}_{cmg} = J(\delta)\dot{\delta}
\]  

(8)

As mentioned before, the major problem of using CMG as ACS is its inherent geometric singularities. It represents a set of gimbal angles where no torque can be generated in certain direction. To overcome this singularity problems, various steering algorithms have been developed in literatures. These steering algorithms determine the optimal gimbal angle trajectories (in the form of gimbal rate) necessary to generate commanded torque while avoiding or escaping singularities encountered during operation.

3. Methodology

3.1. Simulation scheme

In this paper, Single Gimbal Control Moment Gyro (SGCMG) will be used in attitude control simulation. To ensure 3 axis control and higher momentum capability, a cluster of 4 SGCMG in pyramid configuration as shown in figure 1 will be implemented.
The angular momentum of 4 SGCMG in pyramid configuration is given by
\[
\vec{H} = h_0 \begin{bmatrix}
-\cos \beta \sin \delta_1 \\
\cos \delta_1 \\
\sin \beta \sin \delta_1 \\
-\cos \beta \sin \delta_2 \\
-\cos \beta \sin \delta_2 \\
\cos \beta \sin \delta_2 \\
\sin \beta \sin \delta_3 \\
\cos \beta \sin \delta_3 \\
\sin \beta \sin \delta_3 \\
\cos \delta_4 \\
\sin \beta \sin \delta_4 \\
\cos \delta_4
\end{bmatrix} + \begin{bmatrix}
\cos \beta \sin \delta_3 \\
-\cos \beta \sin \delta_3 \\
\sin \beta \sin \delta_3 \\
\cos \beta \sin \delta_4 \\
\sin \beta \sin \delta_4 \\
\cos \beta \sin \delta_4
\end{bmatrix} (9)
\]
and the Jacobian matrix is given by
\[
J = \begin{bmatrix}
-\cos \beta \cos \delta_1 & \sin \delta_2 & \cos \beta \cos \delta_3 & -\sin \delta_4 \\
-\sin \delta_1 & -\cos \beta \cos \delta_2 & \sin \delta_3 & \cos \beta \cos \delta_4 \\
\sin \beta \cos \delta_1 & \sin \beta \cos \delta_2 & \sin \beta \cos \delta_3 & \sin \beta \sin \delta_4
\end{bmatrix} (10)
\]

The attitude control strategy using CMG requires two steps, first, the torque necessary to change the attitude of the satellite must be determined and secondly the gimbal rate of CMG required to generate the torque. The required torque will be computed using simple controller, while the gimbal rate will be computed using steering algorithm. Steering algorithm will also be used to overcome singularity problem that might be encountered during attitude control. Figure 2 represents the closed loop system of attitude control simulation using a cluster of 4 SGCMG.

Figure 2. SGCMG in pyramid configuration relative to spacecraft frame [4][8].

3.2 Controller
To compute the necessary torque for attitude maneuver, a quaternion feedback controller will be used. Using the kinematics equation from previous section, the attitude of spacecraft in the form of
quaternion will be obtained. The quaternion along with the angular velocity of the satellite will then be fed to a quaternion feedback controller [4] which results to the control torque $\tau$. For real-time implementation, the following linear state feedback controller will be used [4]:

$$u = -Kq_e - C\omega$$  \hspace{1cm} (11)

where $q_e = (q_{1e}, q_{2e}, q_{3e})$ is attitude error in quaternion and $K$ and $C$ are gain matrices to be properly selected. Adding the gyroscopic term in spacecraft rotational dynamics, the linear state feedback controller becomes

$$u = -Kq_e - C\omega + \omega \times I\omega$$  \hspace{1cm} (12)

To guarantee global and asymptote stability of the controller, the gain matrices $K$ and $C$ were selected as [4]:

$$K = kl_s$$

$$C = cl_s$$

where $l_s$ is the inertia matrix of spacecraft and $k$ and $c$ are positive scalar constant to be selected properly. Furthermore, to limit the control torque computed by controller a variable limiter will be utilized. Thus, the linear state feedback controller with variable limiter will have the following form:

$$\tau = -l_s\left(2k sat_{L_s}(q_e) + c\omega\right)$$  \hspace{1cm} (13)

$$u = sat_{U}(\tau) + \omega \times l_s\omega$$  \hspace{1cm} (14)

$$L_i = \pm \frac{c}{2k} \min \left(\sqrt{4a_i|e_i|}, \omega_{\text{max}}\right)$$  \hspace{1cm} (15)

$$U = \pm h_0\delta_{\text{max}}$$  \hspace{1cm} (16)

where $a_i$ is control acceleration from $U/l_s$ [4] and $U$ is maximum torque generated by 1 CMG. The controller also utilized slew rate limit in equation 16 To accommodate sensor limit.

3.3 Steering Algorithm

As mentioned previously, steering algorithm for a cluster of $N$ CMG will compute gimbal rate necessary to produce commanded torque while avoiding or escaping singularities encountered during maneuver. In this paper, steering algorithm based on off-diagonal singularity robust developed by Wie [8] will be used. This algorithm is a modified version of SR steering algorithm with the utilization of weighting matrix with non-zero off diagonal. This steering algorithm is derived from the following minimization problem:

$$\min_{\delta} \left((J\dot{\delta} - \dot{h})^T P (J\dot{\delta} - \dot{h}) + \delta^T Q\delta\right)$$  \hspace{1cm} (17)

The solution of equation yields the following form

$$\dot{\delta} = J^\# \dot{h}$$  \hspace{1cm} (18)

where

$$J^\# = (A^T PA + Q)^{-1}A^T P$$  \hspace{1cm} (19)

$$\equiv Q^{-1}A^T(AQ^{-1}A^T + P^{-1})^{-1}$$

$$\equiv W A^T(AW A^T + V)^{-1}$$
P and Q are weighting matrices where

\[
Q^{-1} \equiv W = \begin{bmatrix}
    W_1 & \lambda & \lambda & \lambda \\
    \lambda & W_2 & \lambda & \lambda \\
    \lambda & \lambda & W_3 & \lambda \\
    \lambda & \lambda & \lambda & W_4
\end{bmatrix} > 0
\] (20)

\[
P^{-1} \equiv V = \lambda \begin{bmatrix}
    1 & \epsilon_3 & \epsilon_2 \\
    \epsilon_3 & 1 & \epsilon_1 \\
    \epsilon_2 & \epsilon_1 & 1
\end{bmatrix} > 0
\] (21)

The singularity parameter \(\lambda\) and \(\epsilon_i\) have the following form

\[
\lambda = \alpha_0 e^{-\mu m} \\
m = \det(JJ^T) \\
\epsilon_i = \epsilon_0 \sin(\omega t + \phi_i)
\]

where \(\alpha_0, \mu, \epsilon_0, \omega\) and \(\phi_i\) are singularity parameter to be properly selected to obtain tolerable error and gimbal rates to escape all kind of singularities [4, 8]. It is demonstrated in references [8, 9] that this algorithm is capable of escaping any kind of singularities and has better performance than other algorithm.

### 3.4 Simulation parameters

For the simulation, it is assumed that the satellite has the following inertia matrix

\[
l = \begin{bmatrix}
    3.73 & -0.000246 & 0.03 \\
    -0.000246 & 2.0596 & -0.0167 \\
    0.03 & -0.0167 & 3.4121
\end{bmatrix} \text{ kg.m}^2
\]

Where the pyramid configuration of 4 SGCMG is skewed at 54.73° which is considered to be the most optimum arrangement in term of uniformity of momentum envelope [3]. This configuration gives almost spherical angular momentum and could provide equal control authority in any axis. The CMG specifications were based on the work of Lappas [3] with angular momentum of 0.2288 Nms for each CMG. Moreover the gimbal rate also explicitly limited to 10 deg/s to represent hardware limit. Thus, the CMG is capable of generating maximum about 0.03 Nm of torque.

The simulation started with initial condition of 25 deg roll, 20 deg pitch and -30 deg yaw and gimbal angles of [-90 0 90 0] which correspond to well known elliptic singularity for pyramid configuration. Then the satellite is commanded to do inertial pointing by changing and holding its attitude at 60 deg roll, -75 deg pitch and 90 deg yaw with slew rate limit of 6 deg/s. To summarize, the simulation parameters in this paper are listed in the table below.
### Table 1. Simulation parameters.

| Parameter | Value       |
|-----------|-------------|
| $h_0$     | 0.2288 Nm.s |
| $\beta$  | 54.73 deg   |
| $\delta_{\text{max}}$ | 10 deg/s |
| $R Y_{\text{init}}$ | [25 20 -30] |
| $\delta_{\text{init}}$ | [-90 0 90 0] |
| $R Y_{\text{ref}}$ | [60 -75 90] |
| $\omega_{\text{max}}$ | 6 deg/s |

### 4. Results and discussion

The results presented in this paper were obtained from simulator created using MATLAB and Simulink. The simulation results show below were performed using microsatellite orbiting in Low Earth Orbit (LEO) with assumption no external disturbances present during attitude maneuver. The singularity parameters in this simulation were carefully chosen through tuning to obtain tolerable torque error and gimbal rate to escape/avoid singularity. The following singularity parameters were used

#### Table 2. Singularity parameter for steering algorithm.

| Singularity Parameter | Value |
|-----------------------|-------|
| $\alpha$              | 0.01  |
| $\mu$                 | 10    |
| $\epsilon_0$          | 0.1   |
| $\omega$              | 1 rad/s |
| $\phi$                | 90 deg |
| $W$                   | $\text{diag}[1 \ 3 \ 5 \ 7]$ |

Figure 4, 5 and 6 represent the results of three axis attitude maneuver of satellite using SGCMG, it can be seen that the attitude maneuver is successfully completed within 35 seconds without large overshoot even though the CMG cluster initially trapped in elliptic singularity. However in figure 4 on the right column, it can be seen that the spacecraft angular rate during attitude maneuver slightly exceeds the slew rate limit of 6 deg/s. This happens due to the initial condition of gimbal angle is already trapped in elliptic singularity and to escape this condition, inevitable torque error will be generated as shown in figure 6.
Figure 4. Spacecraft attitude (left) and angular rate (right) during maneuver.

Figure 5. SGCMG singularity measure (left) and angular momentum during maneuver.

Figure 6. SGCMG gimbal angle (left) and gimbal rate (right) during maneuver.
Figure 5 on the left column shows the performance of steering algorithm and the chosen parameters to escape singularity condition during simulation. As mentioned previously, the system start off at singularity condition which can be seen in figure 5 where the Jacobian singular value is zero. However, it can further be seen that the system able to escape the singularity after about 5 seconds where the Jacobian singular value is started to rise and have non zero value even though the gimbal rates were explicitly limited to 10 deg/s as shown in figure 6. This verifies that the off-diagonal singularity robust algorithm with chosen parameters and physical limitation is capable of rapidly escaping the singularity while staying away from singularities during the rest of simulation.

Finally, figure 7 show the comparison of requested torque by controller and generated torque by CMG cluster. Generally the torque generated by CMG is very close to the torque requested by controller. However it can be seen that initially the CMG cluster is unable to generate torque along x axis which corresponds to singularity problem previously discussed where no torque can be generated along certain direction or in this case singular direction.

![CMG Torque](image)

![Torque (N.m)](image)

Figure 7. SGCMG and controller torque during maneuver.

5. Conclusion and future works
The simulations performed in this paper confirm the ability of SGCMG system to provide agile attitude maneuver with slew rate of 1-10 deg/s. The cluster of 4 SGCMG in pyramid configuration is shown to be able to perform three axis attitude maneuver with maximum slew rate of 6 deg/s. Furthermore, the steering algorithm is capable of generating gimbal rate to produce torque requested by controller in the presence of singularities.

In conclusion, SGCMG system can be alternative attitude control system for satellites that require agility in term of high slew rate maneuver and the singularity problem that might be encountered during maneuver can be handled properly using various steering algorithm.

For future works, more robust controller can applied to utilize the maximum capability of SGCMG system. Furthermore, sensitivity study should be conducted to determine the effect of singularity parameters of steering algorithm on attitude control behavior. In addition, sensor dynamics and external disturbance should be implemented for more realistic environment.
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