Next to new minimal standard model

Naoyuki Haba\textsuperscript{1,2}, Kunio Kaneta\textsuperscript{2,3,4} and Ryo Takahashi\textsuperscript{2}

\textsuperscript{1}Graduate School of Science and Engineering, Shimane University, Matsue, Shimane 690-8504, Japan
\textsuperscript{2}Department of Physics, Faculty of Science, Hokkaido University, Sapporo, Hokkaido 060-0810, Japan
\textsuperscript{3}Kavli IPMU (WPI), The University of Tokyo, Kashiwa, Chiba 277-8568, Japan
\textsuperscript{4}Department of Physics, Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan

Abstract

We suggest a minimal extension of the standard model, which can explain current experimental data of the dark matter, small neutrino masses and baryon asymmetry of the universe, inflation, and dark energy, and achieve gauge coupling unification. The gauge coupling unification can explain the charge quantization, and be realized by introducing six new fields. We investigate the vacuum stability, coupling perturbativity, and correct dark matter abundance in this model by use of current experimental data.
1 Introduction

The standard model (SM) in particle physics has achieved great success in the last few decades. In particular, a recent discovery of the Higgs particle with the mass of 126 GeV at the CERN Large Hadron Collider (LHC) experiment\cite{1} filled the last piece of the SM. So far the results from the LHC experiment are almost consistent with the SM, and no signatures of new physics such as the supersymmetry (SUSY) or extra-dimension(s) are discovered. However, there are some unsolved problems in the SM, for example, there is no candidate of dark matter (DM) in the SM, which are expected to be solved by the new physics beyond the SM.

The SUSY is an excellent candidate for the physics beyond the SM since it solves the gauge hierarchy problem and realizes the gauge coupling unification (GCU) as well as contains the DM candidate. But, the recent discovery of the Higgs with the 126 GeV mass and no signature of the SUSY may disfavor the SUSY at low energy. Actually, the magnitude of the fine-tuning in the gauge hierarchy problem is much less than that of the cosmological constant problem. So it should be meaningful to reconsider the minimum extension of the SM by forgetting about the gauge hierarchy problem. A model suggested in Ref.\cite{2} was a minimal extension of the SM, which is called new minimal SM (NMSM). In addition to the SM fields, the NMSM contains a gauge singlet scalar, two right-handed neutrinos, an inflaton, and the small cosmological constant, which can explain the DM, small neutrino masses and baryon asymmetry of the universe (BAU), inflation, and dark energy (DE), respectively. Although a favored parameter space in the NMSM for the vacuum stability, triviality bounds, and the correct DM abundance was shown in Ref.\cite{2}, experimental data was old. For example, the allowed region for the scalar singlet DM is also updated \cite{4} by utilizing the results of the LHC searches for invisible Higgs decays, the thermal relic density of the DM, and DM searches via indirect and direct detections, recently. The parameter search must be investigated again with the current experimental data. This is one motivation of this paper.

It is worth noting that the GCU can not be achieved in the NMSM. The charge quantization is one of the biggest problems in the SM, which should be solved in a grand unified theory (GUT). The GCU can be a sufficient condition of the GUT, and the great merit of the SUSY SM is just the realization of the GCU. Thus, here we suggest next to new minimal SM (NNMSM) in order to achieve the GCU by extending the NMSM. Our model includes six new fields, two adjoint fermions and four vector-like $SU(2)_L$ doublet fermions, in addition to the particle contents of the NMSM. We also revisit the stability and triviality bounds with the 126 GeV Higgs mass, the recent updated limits on the DM particle, and the latest experimental value of the top pole mass as 173.5 GeV. The vacuum stability and triviality bounds are quit sensitive to the Higgs and top masses. We will point out that there are parameter regions in which the stability and triviality bounds, the correct abundance of DM, and the Higgs and top masses can be realized at the same time.

\*See also \cite{k} and references therein for related works.
2 Next to new minimal standard model

We suggest next to new minimal standard model (NNMSM) by extending the NMSM, which has the gauge singlet real scalar boson $S$, two right-handed neutrinos $N$, the inflaton $\phi$, and the small cosmological constant $\Lambda$ in addition to the SM. Our model introduces six new fields such as two adjoint fermions $\lambda_{a}$ ($a = 2, 3$) and four vector-like $SU(2)_L$-doublet fermions, $L'_i$ and $\overline{L'}_i$ ($i = 1, 2$), in addition to the particle contents of the NMSM. The quantum numbers of these particles are given in Table. 1, where the quantum number of $L'_i$ and $\overline{L'}_i$ is the same as that of the SM lepton doublet.

The gauge singlet scalar and two adjoint fermions have odd-parity under an additional $Z_2$-symmetry while other additional particles have even-parity. We will show the singlet scalar becomes DM as in the NMSM. Runnings of gauge couplings are changed from the SM due to new particles with the charges. The realization of the GCU is one of important results of this work as we will show later.

We consider the NNMSM as a renormalizable theory, and thus, the relevant Lagrangian of the NNMSM is given by

$$L_{\text{NNMSM}} = L_{\text{SM}} + L_{S} + L_{N} + L_{\phi} + L_{\Lambda} + L', \quad (1)$$

$$L_{\text{SM}} \supset -\lambda \left( |H|^2 - \frac{v^2}{2} \right)^2, \quad (2)$$

$$L_{S} = -m_S^2 S^2 - \frac{k}{2} |H|^2 S^2 - \frac{\lambda_S}{4!} S^4 + \text{(kinetic term)}, \quad (3)$$

$$L_{N} = - \left( \frac{M}{2} N_i^c N_i + h_{\nu_i}^\alpha \overline{N}_i L_\alpha \tilde{H} + \text{c.c.} \right) + \text{(kinetic term)}, \quad (4)$$

$$L_{\phi} = -B \phi^4 \left[ \ln \left( \frac{\phi^2}{\sigma^2} \right) - \frac{1}{2} \right] - \frac{B_1^2}{2} - \mu_1 \phi |H|^2 - \mu_2 \phi S^2 - \kappa_H \phi^2 |H|^2 - \kappa_S \phi^2 S^2$$

$$- (y_N^{ij} \phi \overline{N}_i N_j + y_3 \phi \overline{\lambda}_3 \lambda_3 + y_2 \phi \overline{\lambda}_2 \lambda_2 + y_{\nu_i}^{ij} \phi \overline{L'}_i L'_{j} + \text{c.c.}) + \text{(kinetic term)}, \quad (5)$$

$$L_{\Lambda} = (2.3 \times 10^{-3} \text{ eV})^4, \quad (6)$$

$$L' = \left[ (y_L^{\alpha} L'_i \tilde{H} + y_L^{\alpha} \overline{L'}_i H^\dagger) E_\alpha + M_3 \overline{\lambda}_3 \lambda_3 + M_2 \overline{\lambda}_2 \lambda_2 + M_E L'_i \overline{L'}_i + \text{h.c.} \right] + \text{(kinetic terms),} \quad (7)$$

with $\alpha = e, \mu, \tau$ and $\tilde{H} = i \sigma_2 H^*$ where $L_{\text{SM}}$ is the Lagrangian of the SM, which includes the Higgs potential. $v$ is the vacuum expectation value (VEV) of the Higgs as $v = 246 \text{ GeV}$. $L_{S,N,\phi,\Lambda}$ are Lagrangians for the dark matter, right-handed neutrinos, inflaton, and the cosmological constant, respectively. $L_{\text{SM}} + L_{S,N,\Lambda}$ are the same as those of the NMSM. $L'$ is new Lagrangian in the NNMSM, where $E$ is right-handed charged lepton in the SM. Mass matrix, $M_L$, is assumed to be diagonal, for simplicity.

$\dagger$Other possibilities for particle contents are studied in Ref. 5

$\ddagger$For the present cosmic acceleration, we simply assume that the origin of DE is the tiny cosmological constant, which is given in $L_{\Lambda}$ of Eq. (6), so that the NNMSM predicts the equation of state parameter as $\omega = -1$, like the NMSM. We will not focus on the DE in this work anymore.

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Table 1: Quantum numbers of additional particles ($i = 1, 2$).

There are several mass scales of new particles, i.e., masses of DM, right-handed neutrinos, adjoint fermions, and inflaton. For the minimal setup, we introduce two mass scales in addition to the EW (TeV) scale. One is the mass of the new particles, $M_{NP}$, and all new fermions have the mass scale as $M_3 \simeq M_2 \simeq M_{L_i} \simeq M_{NP}$. The other is the scalar DM with the mass

$$m_S = \sqrt{\bar{m}_S^2 + kv^2/8}, \quad (8)$$

which is constrained by experiments and a realization of the correct abundance of the DM. Actually, there are other options for the setup of building the model, which will be shown later.

2.1 Gauge coupling unification

At first, we investigate the runnings of the gauge couplings in the NNMSM. Since we introduce two adjoint fermions $\lambda_3$ and $\lambda_2$, and four vector-like $SU(2)_L$-doublet fermions, $L_i'$ and $\bar{L}_i$ ($i = 1, 2$), listed in Tab. 1 the beta functions of the RGEs for the gauge couplings become

$$2\pi \frac{d \alpha_j^{-1}}{dt} = b_{j}^{SM} + b_{j}'^\prime, \quad (9)$$

where $(b_1^{SM}, b_2^{SM}, b_3^{SM}) = (41/10, -19/6, -7)$ for the SM, and $(b_1', b_2', b_3') = (4/5, 8/3, 2)$ for new contributions in the NNMSM. $t \equiv \ln(\mu/1$ GeV$)$ and $\mu$ is the renormalization scale, and $\alpha_j \equiv g_j^2 / (4\pi)$ ($j = 1, 2, 3$) with $g_1 \equiv \sqrt{5/3}g'$. Since all masses of new particles are around the same scale, $\Lambda_{EW} < M_{NP} \simeq M_3 \simeq M_2 \simeq M_{L_i}'$, where $\Lambda_{EW}$ is the EW scale, we should utilize the RGEs of Eq. (9) at high energy scale ($M_{NP} \leq \mu$) while the right-handed side of Eq. (9) must be $b_j^{SM}$ at low energy scale (\$ $\Lambda_{EW} \leq \mu < M_{NP}$).

According to the numerical analyses, taking a free parameter $M_{NP}$ as $1.40 \times 10^3$ TeV can realize the GCU with a good precision at 1-loop level as shown in Fig. 1. We show the threshold of new particles with $1.40 \times 10^3$ TeV mass by a black solid line. The NNMSM suggests the GCU at

$$\Lambda_{GCU} \simeq 2.45 \times 10^{15} \text{ GeV} \quad (10)$$

In this analysis, we take the following values as $[6]$, $\sin^2 \theta_W(M_Z) = 0.231, \alpha_{em}^{-1}(M_Z) = 128, \alpha_s(M_Z) = 0.118$, for the parameters in the EW theory, where $\theta_W$ is the Weinberg angle, $\alpha_{em}$ is the fine structure constant, and $\alpha_s$ is the strong coupling, respectively.
Figure 1: The runnings of the gauge couplings in the NNMSM. The horizontal axis is the renormalization scale and the vertical axis is the values of $\alpha^{-1}_i$. The runnings of $\alpha^{-1}_1$, $\alpha^{-2}_2$, and $\alpha^{-1}_3$ are described by black, blue, and red solid curves, respectively. We take $M_{NP} = 1.40 \times 10^3$ TeV, and the coupling unification is realized at $\mu = \Lambda_{GUT} \simeq 2.45 \times 10^{15}$ GeV with $\alpha^{-1}_{GCU} \simeq 36.1$. Dotted (dashed) contour shows the experimental limit of $p \to \pi^0 e^+ \gamma > 8.2 \times 10^{33}$ yrs, by use of $\alpha_H = -0.0146 \ (0.0078 \text{ GeV}^3)$.

with the unified coupling as

$$\alpha^{-1}_{GCU} \simeq 36.1.$$ (11)

Suppose the minimal $SU(5)$ GUT at $\Lambda_{GCU}$, the protons decay of $p \to \pi^0 e^+$ occurs by exchanging heavy gauge bosons of the GUT gauge group, and here we estimate a limit from the proton life time. A constraint from the proton decay experiments is $\tau(p \to \pi^0 e^+) > 8.2 \times 10^{33}$ years [6], and the partial decay width of proton for $p \to \pi^0 e^+$ is given by

$$\Gamma(p \to \pi^0 e^+) = \alpha^2_H \frac{m_p}{64\pi f^2_\pi} (1 + D + F)^2 \left(\frac{4\pi \alpha_{GCU}}{\Lambda_{GCU}} A_R\right)^2 \left(1 + (1 + |V_{ud}|^2)^2\right),$$ (12)

where $\alpha^2_H$ is the hadronic matrix element, $m_p$ is the proton mass, $f_\pi$ is the pion decay constant, $D$ and $F$ are the chiral Lagrangian parameters, $A_R$ is the renormalization factor, and $V_{ud}$ is an element of the CKM matrix (e.g., see [7, 8]). In our analysis, we take these parameters as $m_p = 0.94$ GeV, $f_\pi = 0.13$ GeV, $A_R \simeq 0.93$, $D = 0.80$ and $F = 0.47$. A theoretical uncertainty on the proton life time comes mainly from the hadronic matrix element as $\alpha_H = -0.0112 \pm 0.0034 \text{ GeV}^3$ [9]. When $\alpha_H = -0.0146$ GeV$^3$, which is the lowest value, the proton life time is evaluated as $\tau \simeq 5.7 \times 10^{33}$ years. On the other hand, when $\alpha_H = -0.0078$ GeV$^3$, which is the largest value, the proton life time is $2.0 \times 10^{34}$ years. As for the center value, $\alpha_H = -0.0112$ GeV$^3$, the proton life time is $\tau \simeq 9.7 \times 10^{33}$ years. Thus, the NNMSM can be consistent with the proton decay experiment, although the conservative limit can not.
shows the experimental limit, \( \tau(p \rightarrow \pi^0 e^+) = 8.2 \times 10^{33} \), by use of Eqs. (10) and (11) with \( \alpha_H = -0.0146 \) \( (-0.0078) \) GeV\(^3\). Since the future Hyper-Kamiokande experiment is expected to exceed the life time \( \mathcal{O}(10^{35}) \) years \( [10] \), which corresponds to \( \Lambda_{GCU} \simeq 4.39^{+0.62}_{-0.72} \times 10^{15} \) GeV for \( \alpha_H = -0.0112 \pm 0.0034 \) GeV\(^3\), the proton decay is observed if the NNMSM is true.

Here let us examine other numbers of \( L'_i \) and \( \overline{L}_i \). When only one pair of \( L'_i \) and \( \overline{L}_i \) is introduced, the GCU is never realized even with taking \( M_{NP} \) as any other scales. If we introduce more pairs of \( L'_i \) and \( \overline{L}_i \) than two, a heavier mass scale of new particles \( (M_{NP} \gg 10^3 \text{ TeV}) \) can also realize the GCU. For examples, three (four) pairs of \( L'_i \) and \( \overline{L}_i \) with \( M_{NP} \simeq 4.26 \times 10^8 \) GeV \( (7.67 \times 10^9 \text{ GeV}) \) realize the GCU at \( \Lambda_{GCU} = 6.87 \times 10^{14} \) GeV \( (3.62 \times 10^{14} \text{ GeV}) \). However, these cases cannot satisfy the constraint on the proton stability, and introduction of more pairs of \( L'_i \) and \( \overline{L}_i \) leads to smaller \( \Lambda_{GCU} \). Thus, we conclude two pairs of \( L'_i \) and \( \overline{L}_i \) is consistent with the phenomenology.

We have taken the initial setup that there are two adjoint fermions and all new fermions have the same scale masses. Under this condition, the above field content is the minimal as the the NNMSM. However, there are other initial setups for building the NNMSM. One is introducing different mass scales for the new fermions. For examples, the GCU can be achieved by different mass scales between \( M_3 \) and \( M_2 \) \([7]\). In this case, we do not need \( L'_i \) and \( \overline{L}_i \). It has less degrees of freedom of the fields but contains different mass scales. Another is introducing several generations of adjoint fermions with \( M_{NP} \sim 10^8 \) GeV, where the GCU can also be realized. This initial setup can induce tiny neutrino mass without two right-handed neutrinos like the NMSM and NNMSM, through the type-III seesaw mechanism. Taking these initial setups is alternative way of constructing “another” NNMSM, which will be investigated in a separate publication \([11]\).

### 2.2 Abundance and stability of new fermions

Next, we discuss an abundance and stability of new fermions, \( \lambda_3, \lambda_2, \) and \( L'_i, \overline{L}_i \). \( \lambda_3 \) and \( \lambda_2 \) are expected to be long lived since they cannot decay into the SM sector due to the \( Z_2 \)-symmetry. A stable colored particle is severely constrained by experiments with heavy hydrogen isotopes, since it bounds in nuclei and appears as anomalously heavy isotopes (e.g., see \([12]\)). The number of the stable colored particles per nucleon should be smaller than \( 10^{-28} \) \( (10^{-20}) \) for its mass up to \( 1 \) \( (10) \) TeV \([13][14]\). But the calculation of the relic abundance of the stable colored particle is uncertain because of the dependence on the mechanism of hadronization and nuclear binding\([15]\).

In this paper, we apply a simple scenario in order to avoid the problem of the presence of the stable colored particle. It is to consider few production scenario for the stable particle, i.e., the stable particles were rarely produced in the thermal history of the universe and clear the constraints of the colored particles. In fact, a particle with mass of \( M \) is very rarely produced thermally if the reheating temperature after the inflation is lower than \( M/(35 \sim 40) \)\(^4\) Therefore,

\(^4\) We thank S. Matsumoto for pointing out it in a private discussion.
we consider a relatively low reheating temperature as

\[ T_{RH} \lesssim \frac{M_{NP}}{40} = 25 \text{ TeV}, \]  

(13)

since \( M_{NP} = 10^3 \text{ TeV}. \) \( \lambda_2 \) is also rarely produced in the thermal history of the universe. Therefore, the presence of two new adjoint fermions in the NNMSM for the GCU is not problematic. The vector-like fermions \( L_i \) and \( \overline{L_i} \) are also rarely produced (if they are produced, they decay into the SM particles through the Yukawa interactions in Eq. (7) before the Big Bang nucleosynthesis (BBN)). Therefore Yukawa couplings of \( L_i' \) in Eq. (7) are not constrained.

If we introduce an additional gauge singlet fermion \( N' \) with odd parity, the decay of \( \lambda_3 \) can be induced through dimension-6 operator, \( \frac{\lambda_3}{\Lambda_2} Q_3 N'. \) However, in order for \( \lambda_3 \) to decay before the BBN, \( \Lambda < \mathcal{O}(10^{13}) \text{ GeV} \) is required. We consider the NNMSM as the renormalizable theory, and we do not want to introduce this new scale which could induce various higher dimensional operators. Thus, we do not introduce the above operator in the NNMSM.

### 2.3 Inflation

Next, we discuss the inflation, and the relevant Lagrangian is given by \( \mathcal{L}_\phi \) in Eq. (5). The WMAP [16, 17] and the Planck [18] measurements of the cosmic microwave background (CMB) constrain the cosmological parameters related with the inflation in the early universe. In particular, the first results based on the Planck measurement with a WMAP polarization low-multipole likelihood at \( \ell \leq 23 \) (WP) [16, 17] and high-resolution (highL) CMB data gives

\[
\begin{align*}
  n_s & = 0.959 \pm 0.007 \, (68\%; \text{ Planck+WP+highL}), \\
  r_{0.002} & < \begin{cases} 0.11 & (95\%; \text{ no running, Planck+WP+highL}) \\
                       0.26 & (95\%; \text{ including running, Planck+WP+highL}) \end{cases}, \\
  \frac{dn_s}{d\ln k} & = -0.015 \pm 0.017 \, (95\%; \text{ Planck+WP+highL}),
\end{align*}
\]  

(14) \hspace{1cm} (15) \hspace{1cm} (16)

for the scalar spectrum power-law index, the ratio of tensor primordial power to curvature power, the running of the spectral index, respectively, in the context of the ΛCDM model. Regarding \( r_{0.002} \), the constraints are given for both no running and including running cases of the spectral indices.

In the NMSM, the relevant Lagrangian for the inflation is given by

\[
\mathcal{L}_\phi = -\frac{m^2}{2} \varphi^2 - \frac{\mu}{3!} \varphi^3 - \frac{\kappa}{4!} \varphi^4.
\]  

(17)

If the inflaton starts with a trans-Planckian amplitude, the model corresponds to the chaotic inflation model [19]. The benchmark point discussed in [2] was \( m \simeq 1.8 \times 10^{13} \text{ GeV}, \mu \lesssim 10^6 \text{ GeV}, \text{ and } \kappa \lesssim 10^{-14}. \) Since the terms \( \frac{\mu}{3!} \varphi^3 \) and \( \frac{\kappa}{4!} \varphi^4 \) are dominated by the quadratic term of \( \frac{m^2}{2} \varphi^2 \) at this point, this inflation model is similar to the simplest inflation model with a quadratic potential. This type of the inflationary model can be on the absolute edge of the constraint from
the Planck (95%) when the $e$-folds is $N \approx 60$ [18]. The values of coupling of inflaton with the Higgs, DM, and right-handed neutrinos, are appropriately chosen by the reheating temperature for the thermal leptogenesis [20] and keeping the flatness of the inflaton potential. However, we must require the relatively low reheating temperature as $T_{RH} \lesssim 25$ TeV for the few production scenario of additional fermions. Such a low reheating temperature leads to smaller $e$-folds as $N < 60$ in the chaotic inflation, which lies outside the joint 95% CL for Planck+WP+highL data. In order to realize the $e$-folds as $60 \lesssim N$ in this chaotic inflation model, $4 \times 10^{16}$ GeV$ \lesssim T_{RH}$ should be taken.

Therefore, we adopt a different inflation model for the NNMSM, which is given by $\mathcal{L}_p$ in Eq.(5). The inflaton potential is the Coleman-Weinberg (CW) type [21, 22], which is generated by radiative corrections. In this potential Eq.(5), the VEV of $\varphi$ becomes $\sigma$. When we take $(\phi, \sigma, B) \simeq (6.60 \times 10^{19}$ GeV, $9.57 \times 10^{19}$ GeV, $10^{-15})$, the model can lead to $n_s = 0.96$, $r = 0.1$, $dn_s/d\ln k \simeq 8.19 \times 10^{-4}$, and $(\delta \rho/\rho) \sim \mathcal{O}(10^{-5})$, which are consistent with the cosmological data. The values of couplings of inflaton with the Higgs, DM, right-handed neutrinos, and new fermions are also constrained because there is an upper bound on the reheating temperature after the inflation as $T_{RH} \lesssim 25$ TeV. This upper bound leads to $\mu_{1,2} \lesssim 9.23 \times 10^3$ GeV and $(y_N^{ij}, y_3, y_2, y_{\varphi L}) \lesssim 2.41 \times 10^{-10}$. Since $\kappa_{H,S}$ should be almost vanishing at the low energy for the realizations of the EW symmetry breaking and the DM mass, we take the values of $\kappa_{H,S}$ as very tiny at the epoch of inflation. The smallness of $\kappa_{H,S}$ does not also spoil the stability and triviality bounds, which will be discussed in the next section. As for the lower bound of the reheating temperature, it depends on the baryogenesis mechanism. When the baryogenesis works through the sphaleron process, the reheating temperature must be at least higher than $\mathcal{O}(10^2)$ GeV.

There are a large number of inflation models even in the context of single-field inflationary models (e.g., see [23] for the Planck constraints on single-field inflation), so it is interesting to investigate whether other inflation models can be embedded into the NNMSM. Where we should consider or construct an inflation model satisfying non-trivial constraints in the NNMSM in addition to the cosmological data. These are that the inflationary model must (i) realize low reheating temperature for the tiny abundance of the adjoint fermions (upper bound), and (ii) take coupling constant(s) to the scalar sector of the NNMSM as small enough not to spoil the stability and triviality conditions, EW symmetry breaking, and the DM mass. As mentioned above, the upper bound on the reheating temperature in the inflation model depends on the mass scale of the new particles for the GCU. If one can realize the GCU with different particle contents and the corresponding mass scale, there might be other possible inflation and suitable baryogenesis models.
3 Stability, triviality, dark matter, neutrino, and baryogenesis

In this section, we investigate parameter region where not only stability and triviality bounds but also correct abundance of the DM are achieved. Realizations of the suitable tiny active neutrino mass and baryogenesis are also discussed.

3.1 Stability, triviality, and dark matter

The ingredients of Higgs and DM sector in the NNMSM is the same as the NMSM\cite{2}, which are given by $\mathcal{L}_{SM}$ and $\mathcal{L}_{S}$ in Eqs.(2) and (3). The singlet scalar $S$ becomes the DM. In Ref.\cite{2}, the Higgs boson mass was predicted to be in the range of $130 \text{ GeV} \lesssim m_h \lesssim 180 \text{ GeV}$ for values of $\lambda_S(M_Z) = 0, 1, 1.2$ with top Yukawa coupling $y(M_Z) = 1$ (corresponding to the top MS mass $m_t(M_Z) \simeq 174 \text{ GeV}$). However, the stability and triviality bounds are very sensitive to the top mass, and then, it is important to reanalyze the stability and triviality bounds with the 126 GeV Higgs mass and the latest experimental value of the top pole mass\cite{6, 24},

$$M_t = 173.5 \pm 1.4 \text{ GeV}.$$  

We should also use the present limits for the singlet DM model. The RGEs for three quartic couplings of the scalars are given by [2],

$$\frac{(4\pi)^2 d\lambda}{dt} = 24\lambda^2 + 12\lambda y^2 - 6y^4 - 3\lambda(g'^2 + 3g^2) + \frac{3}{8} \left[ 2g^4 + (g'^2 + g^2)^2 \right] + \frac{k^2}{2},$$  

$$\frac{(4\pi)^2 dk}{dt} = k \left[ 4k + 12\lambda + \lambda_S + 6y^2 - \frac{3}{2} (g'^2 + 3g^2) \right],$$  

$$\frac{(4\pi)^2 d\lambda_S}{dt} = 3\lambda_S^2 + 12k^2.$$  

We comment on Eq.(20) that right-hand side of the equation is proportional to $k$ itself. Thus, if we take a small value of $k(M_Z)$, evolution of $k$ tends to be slow and remained in a small value, and the running of $\lambda$ closes to that of SM. In our analysis, boundary conditions of the Higgs self-coupling and top Yukawa coupling are given by

$$\lambda(M_Z) = \frac{m_h^2}{2v^2} = 0.131, \quad y(M_t) = \frac{\sqrt{2}m_t(M_t)}{v}$$  

for the RGEs, where the vacuum expectation value of the Higgs field is $v = 246 \text{ GeV}$. 

Let us solve the RGEs, Eqs.(19)~(21), and obtain the stable solutions, i.e., the scalar quartic couplings are within the range of $0 < (\lambda, k, \lambda_S) < 4\pi$ up to the Planck scale $M_{pl} = 10^{18} \text{ GeV}$. Figure 2 shows the case of $M_t = 172.1 \text{ GeV}$ (corresponding to $m_t(M_t) = 156 \text{ GeV}$), which is the smallest value of the top pole mass in Eq.(18). The solutions of the RGEs are described by gray plots in Fig. 2 where the horizontal and vertical axes are $\log_{10}(m_S/1 \text{ GeV})$ and $\log_{10}k$ at the
$M_Z$ scale. The smaller top pole mass becomes, the smaller top Yukawa contribution in Eq. (19) becomes. Thus the stability bound tends to be relaxed by comparing larger top mass cases, and actually this case does not suffer from stability condition. We also show the contour satisfying $\Omega_S/\Omega_{DM} = 1$ with $\Omega_{DM} = 0.115$, where $\Omega_S$ and $\Omega_{DM}$ are density parameter of the singlet DM and observed value of the parameter [16], respectively. The contour is calculated by micronEOMAs [23]. Since there is no other DM candidate except for the $S$ to compensate $\Omega_S/\Omega_{DM} < 1$, which is above the contour, we focus only on the contour. The relic density depends on $k$ and $m_S$ but not $\lambda_S$, meanwhile $\lambda_S$ affects the stability and triviality bounds. In the figure, $\lambda_S(M_Z)$ is randomly varied from 0 to 4$\pi$, where $\lambda_S$-dependence of the stability and triviality bounds is not stringent, and most of $\lambda_S(M_Z) \in [0, 1]$ as the boundary condition can satisfy the bounds. A direct DM search experiment, XENON100 (2012), gives an exclusion limit [4], which is described by the (red) dashed line in Fig. 4. There are two regions, $R_{1,2}$, which satisfy both the correct DM abundance and the triviality bound simultaneously,

$$R_{1}^{(M_t=172.1)} = \begin{cases} 
63.5 \text{ GeV} \lesssim m_S \lesssim 64.0 \text{ GeV} \\
2.40 \times 10^{-2} \lesssim k(M_Z) \lesssim 2.63 \times 10^{-2} \\
1.803 \lesssim \log_{10}(m_S/1 \text{ GeV}) \lesssim 1.806 \\
-1.64 \lesssim \log_{10}k(M_Z) \lesssim -1.58
\end{cases}$$

(23)

$$R_{2}^{(M_t=172.1)} = \begin{cases} 
81.3 \text{ GeV} \lesssim m_S \lesssim 2040 \text{ GeV} \\
3.16 \times 10^{-2} \lesssim k(M_Z) \lesssim 6.31 \times 10^{-1} \\
1.91 \lesssim \log_{10}(m_S/1 \text{ GeV}) \lesssim 3.31 \\
-1.50 \lesssim \log_{10}k(M_Z) \lesssim -0.20
\end{cases}$$

(24)

The future XENON100 experiment with 20 times sensitivity, which is described by the (blue) dotted lines in Fig. 2 will be able to rule out the lighter $m_S$ region, $R_1$, completely. On the other hand, the heavier $m_S$ region, $R_2$, can be currently allowed by all experiments searching for DM. It is seen that the future XENON100×20 can check up to $m_S \lesssim 1000$ GeV ($\log_{10}(m_S/1 \text{ GeV}) \lesssim 3$). The future XENON1T experiment and combined data from indirect detections of Fermi+CTA+Planck at 1$\sigma$ CL may be able to reach up to $m_S \simeq 5$ TeV [4].

Next, let us show the heaviest top pole mass, $M_t = 174.9$ GeV, in Eq. (18), where the allowed regions become narrow as

$$R_{1}^{(M_t=174.9)} = R_{1}^{(M_t=172.1)},$$

(25)

$$R_{2}^{(M_t=174.9)} = \begin{cases} 
1862 \text{ GeV} \lesssim m_S \lesssim 2040 \text{ GeV} \\
5.74 \times 10^{-1} \lesssim k(M_Z) \lesssim 6.31 \times 10^{-1} \\
3.27 \lesssim \log_{10}(m_S/1 \text{ GeV}) \lesssim 3.31 \\
-0.24 \lesssim \log_{10}k(M_Z) \lesssim -0.20
\end{cases}$$

(26)

This is because the larger top Yukawa coupling gives stringent bound on the vacuum stability. On the other hand, the small $m_S$ region, $R_1$, does not change from the case of $M_t = 172.1$ GeV. The reason is as follows. In the RGE analyses, $k$ in the R.H.S. of Eq. (19) is effective above the energy scale of $m_S$. Then, the triviality bound of $\lambda$ becomes severe as the $m_S$ becomes small, and the left-edge of gray dots shows this bound. This does not depend on the top Yukawa coupling, so that the region $R_1$ is independent of the top pole mass.

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II The NMSM [2] predicted the larger Higgs mass region as 130 GeV $\lesssim m_h \lesssim$ 180 GeV. It is because the top mass was taken as $m_t(M_Z) = 174$ GeV, and such the large top Yukawa coupling induces vacuum instability.
Figure 2: A contour of fixed relic density $\Omega_S/\Omega_{DM} = 1$ and a region, which is described by gray plots, satisfying the stability and triviality bounds with $M_t = 172.1$ GeV ($m_t(M_t) = 156$ GeV). The upper boundary of the gray region is determined by the triviality condition, where “triviality” in the both figures represents the corresponding condition. The (red) dashed and (blue) dotted lines are experimental limits from XENON100 (2012) and 20 times sensitivity of XENON100, respectively. (a) The mass region is $63$ GeV $\leq m_S \leq 100$ GeV ($1.8 \leq \log (m_S/1 \text{ GeV}) \leq 2.0$). (b) The mass region is $10$ GeV $\leq m_S \leq 5000$ GeV ($1.0 \leq \log (m_S/1 \text{ GeV}) \leq 3.7$).
Figure 3: The same plots as Fig. 2 with $M_t = 174.9$ ($m_t(M_t) = 165$ GeV). The lower and upper boundaries of the gray region are determined by the stability and triviality conditions, respectively, where “stability” and “triviality” in the both figures represent the corresponding conditions. (a) The mass region is $63$ GeV $\leq m_S \leq 100$ GeV ($1.8 \leq \log(m_S/1 \text{ GeV}) \leq 2.0$). (b) The mass region is $10$ GeV $\leq m_S \leq 5000$ GeV ($1.0 \leq \log(m_S/1 \text{ GeV}) \leq 3.7$).
Finally, let us show the case of the center value of the top pole mass, $M_t = 173.5$ GeV, in Fig. 4. In the figure, we can find the regions satisfying the correct DM abundance and the stability and triviality bounds as,

$$R_1^{(M_t=173.5)} = R_1^{(M_t=172.1)},$$

$$R_2^{(M_t=173.5)} = \begin{cases} 
955 \text{ GeV} \lesssim m_S \lesssim 2040 \text{ GeV} \\
2.94 \times 10^{-1} \lesssim k(M_Z) \lesssim 6.31 \times 10^{-1}
\end{cases} \begin{cases} 
(2.98 \lesssim \log_{10}(m_S/1 \text{ GeV}) \lesssim 3.31) \\
(-0.53 \lesssim \log_{10} k(M_Z) \lesssim -0.20)
\end{cases}. \quad (27)$$

We can show that $R_1$ region is the same as other top pole mass cases. As for the region $R_2$, it is the middle of above two figures, and we notice again that the top Yukawa dependence is quite large.

Here let us show a typical example of the RGE running of scalar quartic couplings, Eqs. (19)-(21), with $M_t = 173.5$. In Fig. 5 the horizontal axis is the renormalization scale and the vertical axis is the value of the scalar quartic couplings. The black, red, and blue solid curves indicate the runnings of $\lambda$, $k$, and $\lambda_S$, respectively, and we take values of the couplings as

$$k(M_Z) = 0.496, \quad \lambda_S(M_Z) = 0.1, \quad (29)$$

with $m_S = 1600$ GeV, which realize the correct relic density and stability and triviality to the Planck scale.
3.2 Neutrinos and baryogenesis

The neutrino sector is shown in Eq. (4), where tiny active neutrino mass is obtained through the type-I seesaw mechanism [26]. Since there are two right-handed neutrinos, one of active neutrinos is predicted to be massless $m_1 = 0$ ($m_3 = 0$) for the normal (inverted) mass hierarchy. Reminding the low reheating temperature in the NNMSM, masses of the right-handed neutrino must be lighter than 25 TeV. What mechanism can induce the suitable baryon asymmetry in such a low reheating temperature? One possibility is the resonant leptogenesis [27]** in which the right-handed neutrinos can be light such as 1 TeV. Thus, the reheating temperature, $1 \text{ TeV} \lesssim T_{RH} \lesssim 25 \text{ TeV}$, can realize the resonant leptogenesis, which means the couplings of inflaton as $369 \text{ GeV} \lesssim \mu_{1,2} \lesssim 9.23 \times 10^8 \text{ GeV}$ and $9.63 \times 10^{-12} \lesssim y_i^{ij} \lesssim 2.41 \times 10^{-10}$ in Eq.(5). For the suitable light active neutrino mass, neutrino Yukawa couplings should be small. If one allows fine-tunings among the neutrino Yukawa couplings, larger neutrino Yukawa couplings can also reproduce experimental values in the neutrino sector in the context of the low scale seesaw mechanism.††

4 Summary

The SM has achieved great success in the last few decades, however, there are some unsolved problems such as explanations for DM, gauge hierarchy problem, tiny neutrino mass scales,**

** It is known that the singlet DM model can induce a strong EW phase transition for the EW baryogenesis in some parameter regions[28]. However, in the parameter regions searched in the previous subsection, the singlet DM model cannot explain total energy density of DM, and requires other candidates of the DM. Thus, in the NNMSM, we the resonant leptogenesis is preferable than the EW-baryogenesis.

†† In the case, the searches of the lepton flavor violating (LFV) processes such as $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, and $\mu^- - e^-$ conversion may constrain and/or check the sizes of the neutrino Yukawa couplings (e.g., see [29, 30, 31]).
baryogenesis, inflation, and the DE. The minimally extended SM without the SUSY, so-called NMSM, could explain the above problems except for the gauge hierarchy problem and GCU by adding two gauge singlet real scalars and two right-handed neutrinos, small cosmological constant. In this paper, we suggested the NNMSM for the realization of the GCU by extending the NMSM. We take a setup that all new fermions have the same mass scale of new physics. Under the condition, the GCU with the proton stability determines the field contents of the NNMSM, i.e., six new fields such as two adjoint fermions under $SU(3)_C$ and $SU(2)_L$, and four vector-like $SU(2)_L$ doublet fermions are added to the particle contents of the NMSM. The GCU can occur at $\Lambda_{\text{GCU}} \simeq 2.45 \times 10^{15}$ GeV with the mass scale of the new particles as $10^3$ TeV. We consider low reheating temperature, $T_{RH} \lesssim 25$ TeV, in order not to produce the stable adjoint fermions in the early universe. This low reheating temperature requires the following issues. The masses of right-handed neutrino should be lighter than 25 TeV, so that tiny neutrino mass is realized through the Type-I seesaw with relatively small neutrino Yukawa couplings. The BAU should be achieved through, for example, the resonant leptogenesis. For the inflation model, it should (i) realize low reheating temperature, and (ii) take coupling constants to the scalar sector of the NNMSM as small enough not to spoil the stability and triviality conditions, EW symmetry breaking, and the DM mass.

We have also analyzed the stability and triviality conditions by use of recent experimental data of Higgs and top masses. We found the parameter regions in which the correct abundance of dark matter can be also realized at the same time. One is the lighter $m_S$ region as $63.5 \text{ GeV} \lesssim m_S \lesssim 64.0 \text{ GeV}$, and the other is heavier ones as $708 \text{ GeV} \lesssim m_S \lesssim 2040 \text{ GeV}$ with the center value of top pole mass. We have shown the top mass dependence is quite large even within the experimental error of top pole mass. The future XENON100 experiment with 20 times sensitivity will completely check out the lighter mass region. On the other hand, the heavier mass region will also be completely checked by the future direct experiments of XENON100×20, XENON1T and/or combined data from indirect detections of Fermi+CTA+Planck at 1σ CL.

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