Isotope effects in high-$T_c$ cuprate superconductors as support for the bipolaron theory of superconductivity

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Abstract. We provide a unified parameter-free explanation of the observed oxygen-isotope effects on the critical temperature, the magnetic-field penetration depth and on the normal-state pseudogap for underdoped cuprate superconductors within the framework of the multi-(bi)polaron theory with strong Coulomb and Fröhlich interactions. We also quantitatively explain the measured critical temperature and the magnitude of the magnetic-field penetration depth. This paper thus represents an important support for the bipolaron theory of high-temperature superconductivity, compatible with many other independent observations.

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On the long journey towards a microscopic understanding of superconductivity, the observation of an isotope effect on the critical temperature, $T_c$, in 1950 [1, 2] provided an important clue to the microscopic mechanism of superconductivity. The presence of an isotope effect thus implies that superconductivity is not of purely electronic origin. In the same year, Fröhlich [3] pointed out that the electron–phonon interaction gave rise to an attractive interaction between electrons, which might be responsible for superconductivity. Fröhlich’s theory played a decisive role in establishing the correct mechanism. Finally, in 1957, Bardeen, Cooper and Schrieffer (BCS) [4] developed the BCS theory that was the first successful microscopic theory of superconductivity. The BCS theory implies an isotope-mass dependence of $T_c$, with an isotope-effect exponent $\alpha = -\frac{d \ln T_c}{d \ln M} = \frac{1}{2}$, in excellent agreement with the reported isotope exponents in simple metallic superconductors such as Hg, Sn and Pb.

The doping-dependent oxygen-isotope effect (OIE) on the critical temperature $T_c$, $\alpha^O = -\frac{d \ln T_c}{d \ln M_O}$ (where $M_O$ is the oxygen-isotope mass) [5] and the substantial OIE on the in-plane supercarrier mass $m^{**}_{ab}$, $\alpha^O_{m} = \frac{d m^{**}_{ab}}{d \ln M_O}$ [6–11], provide direct evidence for a significant electron–phonon interaction (EPI) also in high-temperature cuprate superconductors. High-resolution angle-resolved photoemission spectroscopy (ARPES) [12] provides further evidence for strong EPI with c-axis-polarized optical phonons [13]. These results, along with optical [14], neutron scattering [15, 16] and tunneling data [17–19], unambiguously show that lattice vibrations play a significant but unconventional role in high-temperature superconductivity. The interpretation of the optical spectra of high-$T_c$ materials as the polaron absorption [20, 21] strengthens the view [22] that the Fröhlich EPI is important in those structures. Operating together with a shorter-range deformation potential and molecular-type (e.g. Jahn–Teller [23]) EPIs, the Fröhlich EPI can readily overcome the Coulomb repulsion at a short distance of about the lattice constant for electrons to form real-space inter-site bipolarons [24].

Despite all these remarkable and well-performed experiments that lead to the consistent conclusion about the important role of EPI in high-temperature superconductors, there is no consensus on the microscopic origin of the observed unconventional isotope effects on the in-plane magnetic-field penetration depth and the normal-state pseudogap. The doping
dependent $\alpha^O$ has been explained as being due to the doping-independent OIE on the in-plane carrier concentration $n$, that is, $\alpha^O_n = -d \ln n / d \ln M_0 = 0.146$ [25]. This interpretation contradicts other independent experiments [8, 9, 11] that consistently show that the carrier concentrations of the two oxygen-isotope samples are the same within 0.0004 per Cu. Also the low value of $\alpha^O_n = 0.146$ in [25] is in sharp contrast to the observed very large OIE on the low-temperature magnetic-field penetration depth, $\lambda_{ab} \propto (m^*_{ab}/n)^{1/2}$, in both La$_{1.94}$Sr$_{0.06}$CuO$_4$ (see [9]) and Y$_{0.55}$Pr$_{0.45}$Ba$_2$Cu$_3$O$_{7-y}$ (see [26]), which would lead to $\alpha^O_n \sim 2$ if one assumed that the supercarrier mass is independent of the oxygen mass. Another model based on the pair-breaking effects due to impurities, disorder and/or pseudogap can also explain the observed OIEs on the penetration depth and critical temperature in deeply underdoped samples [27]. But this model cannot consistently explain the negligibly small $\alpha^O$ but large OIE on the penetration depth in optimally doped samples [10].

Alternatively, the bipolaron theory of superconductivity [24] can naturally account for the substantial $\alpha^O_m$ and large $\alpha^O$ in deeply underdoped cuprates [28]. There is a qualitative difference between ordinary metals and polaronic conductors. The renormalized effective mass of electrons is independent of the ion mass $M$ in ordinary metals (where the Migdal adiabatic approximation is believed to be valid), because the EPI coupling constant $\lambda$ does not depend on the isotope mass. However, when electrons form polarons (new quasiparticles dressed by lattice distortions), their effective mass $m^*$ depends on $M$ [28].

Although the bipolaron theory can qualitatively explain both $\alpha^O_m$ and $\alpha^O$ in deeply underdoped samples, some important issues have not been well addressed by the theory. The first issue is why $\alpha^O_m$ is not equal to $\alpha^O$ even in deeply underdoped cuprates. The second issue is why $\alpha^O$ is much smaller than $\alpha^O_m$ for slightly underdoped samples. The third issue is why there is a giant OIE on the pseudogap formation temperature $T^*$ in HoBa$_2$Cu$_4$O$_8$ (see [29]).

Here we provide parameter-free explanations of the observed OIEs on the critical temperature, on the in-plane supercarrier mass and on the normal-state pseudogap in HoBa$_2$Cu$_4$O$_8$ within the framework of the bipolaron theory. This paper thus represents an important support for the bipolaron theory of high-temperature superconductivity.

2. Low-energy excitations in doped polar insulators

In highly polarizable ionic lattices such as cuprate superconductors both the Coulomb repulsion and the Fröhlich electron–phonon interaction (EPI) are quite strong (of the order of 1 eV) compared with the low Fermi energy of doped carriers because of a poor screening by non- or near-adiabatic carriers [30]. In those conditions the BCS–Eliashberg theory [31] breaks down because of the polaronic collapse of the electron bandwidth [32] so that one has to apply a non-adiabatic small polaron theory [24].

Here we adopt the bipolaronic low-energy excitation structure of cuprates (figure 1) recently derived by one of us from the microscopic Hamiltonian with the strong unscreened (bare) Coulomb and Fröhlich interactions [33]. It has been shown that, in highly polarizable ionic lattices, the bare long-range Coulomb and EPIs almost negate each other, giving rise to a novel physics described by the polaronic $t$–$J_p$ model with a short-range polaronic spin-exchange $J_p$ of phononic origin,

$$
\mathcal{H} = - \sum_{i,j} (t_{ij} \delta_{\sigma\sigma'} + \mu \delta_{ij}) c_i^{\dagger} c_j + 2 \sum_{m\neq n} J_p(m-n) \left( S_m \cdot S_n + \frac{1}{4} m^a n^a \right),
$$

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Figure 1. Low-energy excitation bands for two oxygen isotopes in underdoped cuprate superconductors.

where \( \mathbf{S}_m = (1/2) \sum_{\sigma, \sigma'} \epsilon_{m\sigma}^* \tau_{\sigma\sigma'} c_{m\sigma'} \) is the spin 1/2 operator (\( \tau \) are the Pauli matrices), \( i = (m, \sigma) \) and \( j = (n, \sigma') \) include both site \((m, n)\) and spin \((\sigma, \sigma')\) indices; \( t_{ij} \) and \( \mu \) are the polaron hopping integral and chemical potential, respectively, while \( J_p(m - n) > t \) represents the exchange interaction between polarons on different sites from a residual polaron–multiphonon interaction. It has been proposed that the \( t - J_p \) Hamiltonian, equation (1), has a high-\( T_c \) superconducting ground state protected from clustering [33].

Different from any model proposed so far, all quantities in the polaronic \( t - J_p \) Hamiltonian equation (1) are defined through the material parameters, in particular \( t_{ij} = T(m - n) \exp[-g^2(m - n)] \) with

\[
g^2(m) = \frac{2\pi e^2}{\kappa h \omega_0 V} \sum_q \frac{1 - \cos(q \cdot m)}{q^2} \tag{2}
\]

and

\[
J_p(m) = \frac{T^2(m)}{2g^2(m) h \omega_0}, \tag{3}
\]

where \( \kappa = \epsilon_\infty \epsilon_0 / (\epsilon_0 - \epsilon_\infty) \) and \( V \) is the normalization volume. Here the high-frequency, \( \epsilon_\infty \), and the static, \( \epsilon_0 \), dielectric constants, as well as the optical phonon frequency, \( \omega_0 \), and the bare hopping integrals in a rigid lattice, \( T(m) \), are measured and/or found using first-principles density functional theory [34] in a parent polar insulator.

Neglecting the first term in \( \mathcal{H} \), which is the polaron kinetic energy, one can readily diagonalize the remaining spin-exchange part of the Hamiltonian [33]. Its ground state is an ensemble of inter-site singlet bipolarons with the binding energy \( \Delta = J_p \) localized on nearest-neighbor sites. Such small bipolarons repel each other and single polarons via a short-range repulsion of about \( J_p \). The kinetic energy operator in equation (1) connects singlet configurations in the first and higher orders with respect to the polaronic hopping integrals \( t \ll J_p \). Taking into account only the lowest-energy degenerate singlet configurations and discarding all other configurations, one can project the \( t - J_p \) Hamiltonian onto the inter-site bipolaronic Hamiltonian using the bipolaron annihilation operators [22, 33]. Such inter-site bipolarons are perfectly mobile since they tunnel via single-polaron transitions [22, 35].
At finite temperatures single polarons, thermally excited above the pseudogap, coexist with these bipolarons, as shown in figure 1.

Small bipolarons are hard-core bosons with the short-range repulsion and a huge anisotropy of their effective mass since their inter-plane hopping is possible only in the second order of the polaron hopping integral [36]. The occurrence of superconductivity in bipolaronic systems is not controlled by the pairing strength, but by the phase coherence among the electron pairs.

3. Different isotope effects on the critical temperature and the London penetration depth

The critical temperature of quasi-two-dimensional (2D) bipolarons, which are hard-core bosons, depends on their density \( n_b(T_c) \) at the critical temperature and the in-plane bipolaron mass \( m_{ab}^{**} \) as

\[
T_c \propto \frac{n_b(T_c)}{m_{ab}^{**}},
\]

which is consistent with the universal linear relation between \( T_c \) and the inverse magnetic-field penetration depth squared found in cuprate superconductors [37]. The bipolaron density slightly depends on temperature due to bipolaron depletion into unbound single polarons, \( n_b(T) = [x - n_p(T)]/2 \), with the density \( n_p \) given by

\[
n_p(T) = \frac{k_B T}{W} \ln(1 + e^{-\Delta/2k_B T}),
\]

where \( W \) is the polaron half-bandwidth, and \( x \) is the in-plane doping level (figure 1). This expression is obtained by integrating the Fermi–Dirac distribution function with a constant (2D) density of states in the polaron band, \( N(E) = 1/2W \), and assuming that the polaron half-bandwidth is large enough, \( W \gg k_B T_c \).

\[
n_p(T) = \int_0^{2W} dE \frac{2N(E)}{1 + \exp(\frac{\Delta}{2} + E)/k_B T}.
\]

Here we take into account that the chemical potential is zero in the superconducting state at and below \( T_c \), if all energies are taken with respect to the bipolaron ground state. The polaron in-plane effective mass, \( m_{ab}^{**} \propto \exp(A\sqrt{M}) \) (\( A \) is a constant), the polaron inverse bandwidth, \( 1/W \), and the inter-site bipolaron mass \( m_{ab}^{**} \) have the same isotope exponent \( \frac{d\ln m_{ab}^{**}}{d\ln M} = \alpha_{m^*} = (1/2) \ln (m_{ab}^{**}/m) \) [28], where \( m \) is the band mass in a rigid lattice. The isotope effect on the pseudogap is given by

\[
\delta\Delta = -\frac{3}{2} \delta W = \frac{3W \delta m_{ab}^{**}}{2 m_{ab}^{**}}
\]

and

\[
\frac{d\ln \Delta}{d\ln M} = \alpha_{m^*} \frac{3W}{2\Delta}.
\]

The above expressions are obtained by taking into account that the pseudogap in figure 1 is given by \( \Delta/2 = J_p/2 - (W - t/2) = J_p/2 - 3W/4 \), where \( J_p \) is the phonon-induced inter-site attraction, which is independent of the ionic mass equation (3), and \( t \approx W/2 \) in the intermediate
coupling regime [35]. Then using equations (4) and (5) and neglecting the terms of the order of $k_B T_c / W \ll 1$, one readily obtains the ratio

$$\frac{\alpha}{\alpha_{m^*}} = 1 - \frac{1}{[x - n_p(T_c)][1 + \exp(\Delta / 2k_B T_c)]}. \quad (9)$$

Equation (9) can naturally explain why $\alpha_{m^*}^O$ is always larger than $\alpha^O$ [8, 9].

4. Quantitative explanation of isotope effects, $T_c$ and the magnetic-field penetration depth

It is worth noting that equation (9) is valid only if d ln $\Delta$ / d ln $M$ is small. When d ln $\Delta$ / d ln $M$ is large, we need to use equations (4) and (5) to directly calculate the $T_c$ and $n_p(T_c)$ changes upon the isotope exchange, that is,

$$\frac{\delta T_c}{T_c} = \frac{\delta m_{ab}^{**}}{m_{ab}^{**}} - \frac{\delta n_p(T_c)}{x - n_p(T_c)}. \quad (10)$$

Using the above equations, we can quantitatively explain the OIEs on the pseudogap and the critical temperature in slightly underdoped HoBa$_2$Cu$_4$O$_8$ (see [29]). The OIE on the relaxation rate of crystal-field excitations in this compound was investigated by means of inelastic neutron scattering [29]. The relaxation rate, which is related to the free-carrier spin density, clearly shows a large OIE (see figure 2). For the $^{16}$O sample there is evidence for the opening of an electronic gap in the normal state at $T^* \simeq 170$ K, while for the $^{18}$O sample $T^*$ is shifted to about 220 K. In contrast, the $T_c$ is shifted from 79.0 to 78.5 K upon replacing $^{16}$O with $^{18}$O (the $^{18}$O concentration is about 75%) [29].

In a normal state with no pseudogap, the relaxation rate $\Gamma_1(T)$ is proportional to $[J_{ex} N(E_F)]^2 T$, where $J_{ex}$ is the exchange integral between the $4f$ electrons of the Ho$^{3+}$ ions and charge carriers and $N(E_F)$ is the electronic density of states at the Fermi energy [29]. Within the polaron/bipolaron framework, $N(E_F) = 1/2W \propto m_{ab}^{**} \propto m_{ab}^{**}$, so the OIE on $N(E_F)$ is the same as the OIE on $m_{ab}^{**}$. For a slightly underdoped YBa$_2$Cu$_3$O$_{6-y}$ film, $\delta m_{ab}^{**}/m_{ab}^{**}$ was found to be 5.5% upon replacing $^{16}$O with $^{18}$O (the $^{18}$O concentration is about 95%) [11]. If we assume that the OIE on $m_{ab}^{**}$ for HoBa$_2$Cu$_4$O$_8$ is similar to that for the slightly underdoped YBa$_2$Cu$_3$O$_{6-y}$ film, we expect that $\delta m_{ab}^{**}/m_{ab}^{**} = 4.3\%$ in HoBa$_2$Cu$_4$O$_8$.

In the superconducting state or below $T^*$, the relaxation rate is suppressed due to the opening of the gap. Then the relaxation rate is given by [29]

$$\Gamma \propto T \exp(-\Delta / 2k_B T). \quad (11)$$

The solid lines in figure 2 represent the best fits of equation (11) to the data below $T^*$. The best fits yield $\Delta / k_B = 94.6 \pm 8.8$ K for the $^{16}$O sample and $\Delta / k_B = 286 \pm 19$ K for the $^{18}$O sample. Therefore, there is a giant OIE on the pseudogap, in agreement with equation (7). Substituting $\delta \Delta = 16.5$ meV and $\delta m_{ab}^{**}/m_{ab}^{**} = 4.3\%$ into equation (7), we find $W = 0.256$ eV and $t = 0.128$ eV. Using $m_{ab}^{**} = 2h^2 / ta^2$, we calculate $m_{ab}^{**} = 8.1 m_e$, which is very close to that (8.3 $m_e$) inferred for the slightly underdoped YBa$_2$Cu$_3$O$_{6.88}$ with $T_c = 87.9$ K (see [38]).

From the $\Delta$, $T_c$ and $W$ values of the $^{16}$O and $^{18}$O samples, we can directly calculate $n_p(T_c)$ to be 0.0117 and 0.0038 for the $^{16}$O and $^{18}$O samples, respectively. Substituting $\delta T_c / T_c = -0.63\%$ and $\delta m_{ab}^{**}/m_{ab}^{**} = 4.3\%$ into equation (10), we obtain $\delta n_p(T_c) / [x - n_p(T_c)] = -3.67\%$. With the
Figure 2. The relaxation rate $\Gamma$ of crystal-field excitations for the $^{16}\text{O}$ and $^{18}\text{O}$ samples of slightly underdoped HoBa$_2$Cu$_4$O$_8$. The data are taken from [29]. The solid red lines represent the best fits of equation (11) to the data below $T^*$. The best fits yield $\Delta/k_B = 94.6 \pm 8.8$ K for the $^{16}\text{O}$ sample and $\Delta/k_B = 286 \pm 19$ K for the $^{18}\text{O}$ sample.

For $n_p(T_c)$ values for the $^{16}\text{O}$ and $^{18}\text{O}$ samples, we find that $x = 0.227$. Since the in-plane doping level $x$ in the optimally doped YBa$_2$Cu$_3$O$_{6.95}$ was found to be 0.264 (see [39]), our inferred doping level of 0.227 for HoBa$_2$Cu$_4$O$_8$ is consistent with the fact that this compound is slightly underdoped. With $m_{ab}^{**} = 8.1 m_e$ and $x = 0.227$, we calculate $\lambda_{ab}(0) = 2248$ Å, which is close to $\lambda_a(0) = 2000$ Å for YBa$_2$Cu$_4$O$_8$ (see [40]).

The exponent $\alpha_{m}^{O}$ of the OIE on $m_{ab}^{**}$ is calculated to be 0.46 for the slightly underdoped YBa$_2$Cu$_3$O$_{7-y}$ and HoBa$_2$Cu$_4$O$_8$. Since the exponent $\alpha_{Cu}^{Cu}$ of the copper-isotope effect on $T_c$ is similar to $\alpha^{O}$ in the underdoped regime [41, 42], we expect that $\alpha_{m}^{O} \simeq \alpha_{m}^{Cu}$ and $\alpha_{m}^{O} \simeq 2\alpha_{m}^{O}$. 

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The mass enhancement factor is then equal to \( \exp(4a_0^O/16) = 6.3 \). The bare bandwidth \( D = 2W \exp(4a_0^O) = 3.2\) eV, in quantitative agreement with the band-structure calculations [43].

In the absence of charge localization, as in the case of the stoichiometric HoBa\(_2\)Cu\(_4\)O\(_8\), the Bose–Einstein condensation (BEC) temperature \( T_c \) is given by [44]

\[
k_B T_c = \frac{2t n_b(T_c)}{1 + \ln(k_B T_c/2t_c)}, \tag{12}
\]

where \( t \) is the bipolaron half-bandwidth and \( t_c \) is related to the out-of-plane bipolaron mass \( m_c^{\ast\ast} \) as \( t_c = \hbar^2/2m_c^{\ast\ast}d^2 \) (\( d \) is the inter-plane distance). The above equation is applied to a quasi-2D bipolaron energy spectrum when \( t_c \ll k_B T_c \). It can be readily generalized for any anisotropy of the boson energy spectrum (see equation (3) and figure 1 in [38]). The out-of-plane bipolaron mass \( m_c^{\ast\ast} \) is deduced to be 518 \( m_c \) using \( m_c^{\ast\ast}/m_c^{\ast\ast} = 64 \) at \( T_c \) ([45]) and \( m_c^{\ast\ast} = 8.1m_c \). Then we calculate \( t_c = 0.160 \) meV. Substituting \( t = 0.128 \) eV, \( t_c = 0.160 \) meV and \( n_b(T_c) = 0.1077 \) into the above equation, we calculate \( T_c = 78.9 \) K, in quantitative agreement with the measured \( T_c \) (79.0 K). We further show (see appendix) that equation (12) can also qualitatively explain the underdoped La\(_{1.9}\)Sr\(_{0.1}\)CuO\(_4\) with \( T_c = 29 \) K but overestimates the \( T_c \) of some optimally doped samples. This implies that optimally doped cuprates are in the crossover regime from bipolaronic real-space pairing to the Cooper pairing of polarons [24].

5. Summary

Apart from the striking isotope effects explained quantitatively here, there is abundant independent evidence in favor of bipolarons and the BEC in underdoped cuprate superconductors [46]. In particular, the parameter-free estimates of the Fermi energy using the magnetic-field penetration depth [47] and the magnetic quantum oscillations [48] pointed to a very low value (below 50 meV), supporting the real-space pairing in underdoped cuprate superconductors. Magneto-transport and thermal magneto-transport data strongly support preformed bosons in cuprates. Many high-magnetic-field studies revealed the non-BCS upward curvature of the upper critical field \( H_{c2}(T) \) as a function of temperature [49] as predicted for the BEC of charged bosons in the magnetic field [50]. The Lorenz number differs significantly from the conventional Sommerfeld value of the standard Fermi-liquid theory because the carriers are double charged bosons [51]. Direct measurements of the Lorenz number using the thermal Hall effect just above \( T_c \) [52] produce its value, which is about the same as that predicted by the bipolaron model. The unusual normal-state diamagnetism uncovered by torque magnetometery has been convincingly explained as the normal state (Landau) diamagnetism of charged bosons [53].

Single polarons, localized within an impurity band-tail, coexist with bipolarons in the charge-transfer-doped Mott–Hubbard insulator, where the chemical potential is pinned within the charge-transfer gap due to bipolaron formation. This band-tail model accounts for two energy scales in ARPES and in the extrinsic and intrinsic tunnelling, their temperature and doping dependence, and for the asymmetry and inhomogeneity of extrinsic tunnelling spectra of cuprates [54].

\[ L \equiv \ln(k_BT_c/2t_c) \] in the denominator of expression (12) slightly affects equation (9) since both \( T_c \) and \( t_c \propto t^2 \) [36] depend on the isotope mass. However, the corrections are small as soon as \( L \gg 1 \).
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Appendix. Bose–Einstein condensation temperature of some cuprate superconductors

Equation (12) can be written in terms of measurable parameters such as the inplane penetration depth and the supercarrier mass anisotropy constant \( \gamma^2 = m_{c}^{*}/m_{ab}^{*}, \)

\[
k_{B} T_{c} = \frac{d h_{c}^{2} c^{2}}{16 \pi^{3} e^{2} \lambda_{ab}^{2}(0)} \left[ 1 + \ln \left( \frac{32 \pi^{3} x e^{2} k_{B} T_{c} \lambda_{ab}^{2}(0) \gamma^{2} d}{a^{2} h_{c}^{2} c^{2}} \right) \right]^{-1}. \tag{A.1}
\]

It is worth noting that the \( T_{c} \) value calculated from equation (A.1) should be somewhat overestimated due to the fact that \( n_{b}(T_{c}) \) is slightly lower than \( n_{b}(0) = x/2 \). For \( \text{La}_{1.90}\text{Sr}_{0.10}\text{CuO}_{4+y} \), \( \lambda_{ab}(0) = 291 \text{ nm} \) (see [37]), \( \gamma = 43 \) (see [55]) and \( x = 0.1 \). These parameters lead to \( T_{c} = 31.8 \text{ K} \), in quantitative agreement with the measured value of 29 K. For the optimally doped \( \text{YBa}_{2}\text{Cu}_{3}\text{O}_{6.95} \) with \( T_{c} = 93 \text{ K} \), \( \lambda_{ab}(0) = 1600 \text{ Å} \) ([40]), \( x = 0.264 \) (see [39]) and \( \gamma = 8 \) (see [56]), \( T_{c} \) is calculated to be 162 K, which is higher than the measured value of 93 K, pointing to the BEC–BCS crossover [32].

References

[1] Maxwell E 1950 Phys. Rev. 78 477
[2] Reynolds C A, Serin B, Wright W H and Nesbitt L B 1950 Phys. Rev. 78 487
[3] Fröhlich H 1950 Phys. Rev. 79 845
[4] Bardeen J, Cooper L N and Schrieffer J R 1957 Phys. Rev. 108 1175
[5] Zhao G M 2007 Polaron in Advanced Materials ed A S Alexandrov (Dordrecht: Springer) pp 569–97
[6] Bussmann-Holder A and Keller H 2007 Polaron in Advanced Materials ed A S Alexandrov (Dordrecht: Springer) pp 599–621
[7] Zhao G M and Morris D E 1995 Phys. Rev. B 51 R16487
[8] Zhao G M, Singh K K, Sinha A P B and Morris D E 1995 Phys. Rev. B 52 6840
[9] Zhao G M, Hunt M B , Keller H and Müller K A 1997 Nature 385 236
[10] Zhao G M, Konder K, Keller H and Müller K A 1998 J. Phys.: Condens. Matter 10 9055
[11] Zhao G M, Kirtikar V and Morris D E 2001 Phys. Rev. B 63 R220506
[12] Khasanov R et al 2004 Phys. Rev. Lett. 92 057602
[13] Lanzara A et al 2001 Nature 412 510
[14] Meevasana W et al 2006 Phys. Rev. Lett. 96 157003
[15] Mihailovic D, Foster C M , Voss K and Heeger A J 1990 Phys. Rev. B 42 7989
[16] Sendyka T R, Dmowski W, Egami T, Seiji N, Yamauchi H and Tanaka S 1995 Phys. Rev. B 51 6747
[17] Reznik D, Pintschovius L, Ito M, Ikubo S, Sato M, Goka H, Fujita M, Yamada K, Gu G D and Tranquada J M 2006 Nature 440 1170
[18] Shim H, Chaudhari P, Logvenov G and Bozovic I 2008 Phys. Rev. Lett. 101 247004
[19] Zhao G M 2009 Phys. Rev. Lett. 103 236403
[20] Tempea J and Devreese J T 2001 Phys. Rev. B 64 104504
[21] Alexandrov A S and Devreese J T 2009 Advances in Polaron Physics (Heidelberg: Springer)

New Journal of Physics 14 (2012) 013046 (http://www.njp.org/)
[22] Alexandrov A S 1996 Phys. Rev. B 53 2863
[23] Müller K A 2007 J. Phys.: Condens. Matter 19 251002
[24] Alexandrov A S 2003 Theory of Superconductivity: From Weak to Strong Coupling (Bristol: Institute of Physics Publishing)
[25] Weyeneth S and Müller K A 2011 J. Supercond. Nov. Magn. 24 1235
[26] Khasanov R, Strassle S, Conder K, Pomjakushina E, Bussmann-Holder A and Keller H 2008 Phys. Rev. B 77 104530
[27] Tallon J L, Islam R S, Storey J, Williams G V M and Cooper J R 2005 Phys. Rev. Lett. 94 237002
[28] Alexandrov A S 1992 Phys. Rev. B 46 14932
[29] Weyeneth S and Müller K A 2011 J. Supercond. Nov. Magn. 24 1235
[30] Eilenberger G M 1960 Zh. Eksp. Teor. Fiz. 39 1437
Eilenberger G M 1960 Sov. Phys.—JETP 12 1000 (Engl. Transl.)
[31] Alexandrov A S 1983 Zh. Fiz. Khim. 57 273
Alexandrov A S 1983 Russ. J. Phys. Chem. 57 167 (Engl. Transl.)
[32] Alexandrov A S 2011 Eur. Phys. Lett. 95 27004
[33] Bauer T and Falter C 2009 Phys. Rev. B 80 094525
[34] Hague J P, Kornilovitch P E, Samson J H and Alexandrov A S 2007 Phys. Rev. Lett. 98 037002
[35] Alexandrov A S, Kabanov V V and Mott N F 1996 Phys. Rev. Lett. 77 4796
[36] Uemura Y et al 1989 Phys. Rev. Lett. 62 2317
[37] Alexandrov A S and Kabanov V V 1999 Phys. Rev. B 59 13628
[38] Chmaissem O, Eckstein Y and Kuper C G 2001 Phys. Rev. B 63 174510
[39] Basov D N, Liang R, Bonn D A, Hardy W N, Dabrowski B, Quijada M, Tanner D B, Rice J P, Ginsberg D M and Timusk T 1995 Phys. Rev. Lett. 74 598
[40] Franck J P, Harker S and Brewer J H 1993 Phys. Rev. Lett. 71 283
[41] Zhao G M, Kirtikar V, Singh K K, Sinha A P B, Morris D E and Inyushkin A V 1996 Phys. Rev. B 54 14956
[42] Pickett W E 1989 Rev. Mod. Phys. 61 433
[43] Alexandrov A S and Mott N F 1995 Polaronics and Bipolarons (Singapore: World Scientific) p 144
[44] Hussey N E, Nozawa K, Takagi H, Adachi S and Tanabe K 1997 Phys. Rev. B 56 R11423
[45] Alexandrov A S 2007 J. Phys.: Condens. Matter 19 125216
[46] Alexandrov A S 2001 Physica C 363 231
[47] Doiron-Leyraud N, Proust C, LeBoeuf D, Levallois J, Bonnemaison J, Liang R, Bonn D A, Hardy W N and Taillefer L 2007 Nature 447 565
[48] Zavaritsky V N, Kabanov V V and Alexandrov A S 1998 Europhys. Lett. 60 127
[49] Alexandrov A S 1993 Phys. Rev. B 48 10571
[50] Alexandrov A S and Mott N F 1995 Phys. Rev. Lett. 71 1075
[51] Zhang Y, Ong N P, Xu Z A, Krishana K, Gagnon R and Taillefer L 2000 Phys. Rev. Lett. 84 2219
[52] Alexandrov A S 2006 Phys. Rev. Lett. 96 147003
[53] Alexandrov A S and Beanland J 2010 Phys. Rev. Lett. 104 026401
[54] Willemin M, Rossel C, Hofer J, Keller H and Revcolevschi A 1999 Phys. Rev. B 59 R717
[55] Tajima S, Schutzmann J, Miyamoto S, Terasaki I, Sato Y and Hauff R 1997 Phys. Rev. B 55 6051