Evidence for Cold Plasma in Planetary Nebulae From Radio Observations With the LOw Frequency ARray (LOFAR)

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Abstract

We present observations of planetary nebulae with the LOw Frequency ARray (LOFAR) between 120 and 168 MHz. The images show thermal free–free emission from the nebular shells. We have determined the electron temperatures for spatially resolved, optically thick nebulae. These temperatures are 20%–60% lower than those estimated from collisionally excited optical emission lines. This strongly supports the existence of a cold plasma component, which co-exists with hot plasma in planetary nebulae. This cold plasma does not contribute to the collisionally excited lines, but does contribute to recombination lines and radio flux. Neither of the plasma components are spatially resolved in our images, although we infer that the cold plasma extends to the outer radii of planetary nebulae. However, more cold plasma appears to exist at smaller radii. The presence of cold plasma should be taken into account in modeling of radio emission of planetary nebulae. Modelling of radio emission usually uses electron temperatures calculated from collisionally excited optical and/or infrared lines. This may lead to an underestimate of the ionized mass and an overestimate of the extinction correction from planetary nebulae when derived from the radio flux alone. The correction improves the consistency of extinction derived from the radio fluxes when compared to estimates from the Balmer decrement flux ratios.

Unified Astronomy Thesaurus concepts: Planetary nebulae (1249); Radio continuum emission (1340); Post-asymptotic giant branch stars (2121); Interstellar dust extinction (837)

1. Introduction

Planetary nebulae (PNe) are detectable at a broad range of wavelengths, from X-rays up to radio frequencies. Continuum radio emission originates from thermal free–free emission of ionized elements. It traces all of the ionized ejecta in PNe. Radio emission is not affected by interstellar or circumstellar extinction caused by dust.

Radio observations constrain the physical parameters of astrophysical plasma. In particular, optically thick free–free emission allows the electron temperature to be determined from the Rayleigh–Jeans law. Optically thick free–free emission decreases very quickly as frequency squared, making it difficult to detect. However, the LOw Frequency ARray (LOFAR, van Haarlem et al. 2013) provides enough sensitivity and spatial resolution to image optically thick radio emission of PNe.

The electron temperature is one of the most important parameters in studying PNe. It governs the energy balance and is a very important parameter when assessing the chemical composition of PNe (Stasińska 2002). Electron temperatures can be measured from the flux ratios of collisionally excited lines (CEls; Kaler 1986). These depend on the electron density and rely on the accuracy of the determination of transition probabilities and collision strengths. CEls are suppressed by collisional de-excitation when the critical density is exceeded in plasma. They are also weak at low electron temperatures. Thus, electron temperatures measured from CEls are weighted toward hot regions that do not exceed the critical density and are less sensitive to dense and cool plasma.

Electron temperatures can be alternatively derived from recombination lines (RLs) or from free–bound emission (e.g., Balmer jumps). RLs are in general much fainter than CEls and therefore more difficult to measure. However, electron temperatures derived from RLs are systematically lower than from CEls. Moreover, the abundances derived from RLs are higher than the abundances obtained from CEls (Peimbert 1971; Stasińska & Szczepanek 2001; Zhang et al. 2004; Wesson et al. 2005). The difference between these two determinations is referred to as an abundance discrepancy factor. Peimbert (1971) attributed this discrepancy to temperature fluctuations in PNe. If large-temperature fluctuations occur, then the temperature measured from CEls is overestimated and RLs appear stronger. However, photoionization models failed to reproduce temperature fluctuations sufficiently large enough to account for the temperature and abundance discrepancy (Kingdon & Ferland 1995).

Liu et al. (2000) showed that the inclusion of high-density hydrogen-deficient plasma can explain the RL and CE temperature and abundance discrepancies. This has been subsequently confirmed by Tsiakis et al. (2004), Zhang et al. (2005), and Wesson et al. (2005). A further insight came from...
Corradi et al. (2015), who linked the abundance discrepancy with binarity of their central stars. García-Rojas et al. (2019) present the most recent review of the topic.

Optically thick free–free radio emission provides an alternative method to assess electron temperatures in PNe. Brightness temperature is simply equal to electron temperature for optically thick thermal radiation. Unlike other methods, this determination does not depend on electron density. The only assumption is a Maxwellian distribution of electrons in nebular plasma, which is most likely fulfilled (Draine & Kreisch 2018).

In this paper, we report electron temperatures for a sample of PNe using LOFAR observations of optically thick free–free emission.

2. Radio Emission from Planetary Nebulae

In the case of an ionized nebula with constant electron density and temperature $T_e$ (hereinafter referred to as a homogeneous nebula or a homogeneous model) which covers a solid angle of $\Omega$, the free–free flux density is given by

$$ S_{ff} = \frac{2\nu^2kT_e}{c^2}(1 - e^{-\tau_\nu})\Omega \tag{1} $$

in the Rayleigh–Jeans approximation. The optical depth is given as $\tau_\nu = 0.0544 \times T_e^{-1.5} \nu^{-2}g_\nu(T_e)EM$ (Olhön 1975). $EM$ denotes the emission measure, $k$ the Boltzmann constant, $\nu$ frequency, and $c$ the speed of light. van Hoof et al. (2014) computed non-relativistic Gaunt factors $g_\nu$ for a wide range of frequencies.

The radio spectra of PNe appear nearly flat ($S_{ff} \propto \nu^{-0.1}$) in the optically thin part at $\tau_\nu \rightarrow 0$. The spectrum steeply declines with decreasing frequency squared ($S_{ff} \propto \nu^2$) when $\tau_\nu \gg 1$. The emission peaks at the turnover frequency close to $\tau_\nu \approx 1$.

The brightness temperature depends on the surface brightness of the object,

$$ T_B(\nu) = \frac{c^2S_{ff}}{2k\nu^2\Omega} \tag{2} $$

The brightness temperature approaches electron temperature in an optically thick case. It is often assumed that PNe with $T_B$ determined from Equation (2) lower than some arbitrary value (e.g., 1 kK in Ruffle et al. (2004), 3 kK in Stasieńska et al. (1992)) are optically thin, which is well justified in the case of homogeneous nebula for a typical electron temperature of 10 kK. However, a study of radio spectral indices ($SI$) $F(5\text{ GHz})/F(1.4\text{ GHz})$ and brightness temperatures by Phillips (2007) revealed that the majority of PNe show an excess of $F(5\text{ GHz})/F(1.4\text{ GHz})$ ratios over the value predicted by a homogeneous model. Phillips (2007) has attributed this excess to the existence of strong radial density gradients in the nebulae. In such cases, nebulae become partially optically thick over a wide range of frequencies (Wright & Barlow 1975).

Siódmiak & Tylenda (2001) attempted to explain the $F(5\text{ GHz})/F(1.4\text{ GHz})$ index excess using an alternative approach. They used two components instead of one in Equation (1). One of the components covers only a fraction of the solid angle $\xi\Omega$ and has an optical thickness of $\tau_\nu$. The other component covers the rest of the solid angle $(1 - \xi\Omega)$ and has an optical thickness of $\eta\tau_\nu$:

$$ S_{ff} = \frac{2\nu^2kT_e}{c^2}[(1 - e^{-\tau_\nu})\xi\Omega + (1 - e^{-\eta\tau_\nu})(1 - \xi\Omega)] \tag{3} $$

Siódmiak & Tylenda (2001) achieved the best fit for Equation (3) using $\xi = 0.27$ and $\eta = 0.19$.

Hajduk et al. (2018) studied radio spectra of PNe and excluded the presence of strong density gradients. They showed that a prolate ellipsoid shell model (Masson 1990; Aaquist & Kwok 1996) has a higher $F(5\text{ GHz})/F(1.4\text{ GHz})$ index compared to the homogeneous model. Other studies have shown that the prolate ellipsoid shell model provides a better fit to the observed surface brightness distribution of PNe than the homogeneous model. However, ellipsoidal shells would have to be enormously elongated to account for the high excesses observed in some PNe.

Equation (1) gives a satisfactory fit to most of the PNe using electron temperatures derived from CELs given that $\Omega$ is smaller than the observed size of the nebula, i.e., when the bulk of the emission comes from a fraction of the solid angle. This is equivalent to Equation (3) for $\eta = 0$ and $0 < \xi < 1$. It is impossible to find a single value of $EM$ which would allow fitting the optically thin and optically thick parts of the spectrum simultaneously for $\xi = 1$ in most cases. With higher $EM$ Hajduk et al. (2018) were able to reproduce the turnover frequency, but overestimated the optically thin flux. Lower values of $EM$ allowed us to fit the optically thin part of the spectrum but shifted the turnover to lower frequencies than observed.

3. Observations and Data Analysis

LOFAR is a radio interferometer which consists of 52 stations distributed in Europe. The Netherlands host 24 core and 14 remote stations operating at the shortest baselines. The remaining 14 stations are located in other countries and provide the longest baselines. Each single station consists of a set of low-band and high-band antennas observing in the 30–80 and 110–240 frequency ranges, respectively (van Haarlem et al. 2013).

We used the radio continuum 120–168 MHz images (central frequency of 144 MHz) of PNe collected by the LOFAR Two-Metre Survey (LoTSS; Shimwell et al. 2019). The survey uses only the data from core and remote stations. The collected visibilities are processed with direction-dependent calibration (van Weeren et al. 2016). The clean algorithm is replaced with a spectral-dependent deconvolution algorithm, which improves the dynamic range of the obtained images (Tasse et al. 2018). de Gasperin et al. (2019) present the calibration strategy and examples.

The survey provides low- and high-resolution images with the full width at half maximum of the restoring beam being 20 and 6″, respectively. We assumed an absolute flux density scale accuracy of 10% (Shimwell et al. 2019). The median positional accuracy of the high-resolution images is 0″2, though it may range from 0″1 to 4″8 for individual fields. LoTSS fields reach a flux accuracy of 100–500 µJy beam$^{-1}$.

Good sampling of the $uv$ plane by short baselines provides LOFAR with an excellent sensitivity to extended emission. Some examples are presented in Shimwell et al. (2019). An upgraded pipeline improved the reduction of extended emission and removed artifacts which were present in the preliminary LoTSS release (Shimwell et al. 2017). With the shortest
baseline of about 80 m the largest angular scale of LoTSS reaches 40’ (Savini et al. 2018).

The LoTSS observations and data processing are still ongoing. We included observations which were processed before 2021 April. This largely overlapped with the upcoming LoTSS-DR2 (T. Shimwell et al., 2021, in preparation.). LoTSS-DR2 includes overlapping fields that are mosaiced to produce the final survey images. We also included additional pointings that have not been yet mosaiced. Their quality will improve in the future after LoTSS completes observations and produces final mosaics.

We selected 165 PNe in the observed part of sky using the SIMBAD database (Wenger et al. 2000) and the catalog by Parker et al. (2016). Out of them, 30 were detected. Table 1 presents the nebular sizes and flux densities of these PNe at 144 MHz. The fluxes and diameters of compact PNe Θ₁ were measured with Gaussian deconvolution using CASA (McMullin et al. 2007). We multiplied the deconvolved diameters by correction factors to account for more realistic surface brightness distribution than a simple Gaussian (van Hoof 2000). We applied the correction factors computed for disk geometry for optically thick PNe. This choice is justified by a flat surface brightness profile of a spherically symmetric nebular model at optically thick 20 cm (equivalent to frequency of 500 MHz) computed by van Hoof (2000). The flat surface brightness profile represents a circular, constant surface brightness disk.

The correction factors were not applied for the large PNe and for unresolved PNe. For well resolved PNe we fitted an ellipse to the emission that exceeded the background by 3σ, which is marked with a thick line in Figures 2, 3, 4, and 5. To measure the flux density, we integrated the emission within this area.

4. Results

4.1. Spectral Fitting

We combined our new 144 MHz flux densities with flux densities collected at different frequencies in other surveys, which are listed in Hajduk et al. (2018). We fitted the spectra with Equation (3) for η = 0 using the derived sizes (Figures 2–5). Only two of the three unknown parameters, ξ, EM, and T_e, could be fitted independently. EM is parameterized in the optical depth term. We used electron temperatures derived from CELs by Kaler (1986), leaving ξ and EM as free parameters. Using an electron temperature derived from CELs or fixing it to an arbitrary value (e.g., 10^4 K) is a common practice in fitting the radio spectral energy distribution (SED) of PNe (Pazderska et al. 2009; Hajduk et al. 2018; Bojić et al. 2021). This reduces the number of unknown variables in the fit to two. However—as we will show later—a cool plasma component may also contribute to radio emission and bias the results. Optical depth does not strongly depend on the assumed

Note. The deconvolved and corrected diameters are not given for unresolved PNe. Large PNe were not fitted with a Gaussian and their diameters Θ₁ refer to the size, which exceeded 3σ (see text).

| Name     | F(144 MHz) (mJy) | Θ₁ (arcsec × arcsec) | Θ (arcsec) | Θcor (arcsec) |
|----------|------------------|----------------------|------------|---------------|
| BV 5-1   | 6.1 ± 1.0        | 12.4 × 5.1           | 14.4 ± 7.8 | 42 × 10       |
| BV 5-2   | 4.0 ± 1.1        | 19.5 ± 14.3          | 24 ± 11    | 23.8 ± 23     |
| H 3-29   | 21.6 ± 2.8       | 18.1 ± 15.0          | 22.8 ± 1.6 | 2.7 ± 2.7     |
| H 4-1    | 0.95 ± 0.20      | 13.7 ± 8.0           | 14.7 ± 3.4 | 12.5 ± 8.0    |
| IC 2149  | 20.4 ± 4.3       | 12.5 ± 11.9          | 17.24 ± 0.57 | 17.8 ± 17.8  |
| IC 3568  | 16.7 ± 2.0       | 13.2 ± 9.4           | 15.6 ± 1.1 | 15.3 ± 14.7   |
| IC 4593  | 10.1 ± 1.7       | 7.0 ± 3.9            | 8.9 ± 1.9  | 9.4 ± 6.3     |
| J 320    | 52 ± 12          | 31.9 ± 14.5          | 29.6 ± 3.8 | 18.6 ± 11.9   |
| K 3-17   | 6.7 ± 1.2        | 5.7 ± 5.2            | 8.0 ± 1.5  | 6 ± 6         |
| M 1-1    | 7.6 ± 1.4        | 8.1 ± 3.6            | 8.2 ± 1.9  | 7 ± 6         |
| M 2-51   | 24.4 ± 3.8       | 36.9 ± 27.4          | 45.8 ± 5.6 | 64 ± 48       |
| M 2-52   | 13.8 ± 2.2       | 12.2 ± 10.1          | 15.9 ± 1.9 | 16 ± 13       |
| NGC 1514 | 191 ± 69         | 108.0 ± 69.2         | 188 ± 182  | 18     |
| NGC 2242 | 8.3 ± 1.4        | 15.8 ± 14.2          | 20.1 ± 2.1 | 20 ± 20       |
| NGC 2371 | 33.6 ± 8.1       | 56.3 ± 36.6          | 48.9 ± 30.6 | 202 ± 30   |
| NGC 3587 | 105 ± 21         | 185.9 ± 182.8        | 208 ± 34   | 56 ± 34       |
| NGC 40   | 115 ± 30         | 47.2 ± 42.4          | 36 ± 28    | 14 ± 14       |
| NGC 6058 | 5.8 ± 1.0        | 16.1 ± 14.1          | 24.4 ± 4.5 | 36 ± 28       |
| NGC 6210 | 26.7 ± 3.6       | 19.2 ± 13.9          | 24.5 ± 3.4 | 14 ± 14       |
| NGC 650/651 | 86 ± 15         | 126 ± 68             | 168 ± 111  | 15 ± 13       |
| NGC 6543 | 56.1 ± 6.1       | 23.0 ± 18.2          | 28.2 ± 1.1 | 26.5 ± 23.5   |
| NGC 6572 | 11.5 ± 2.1       | 84.6 ± 59.1          | 89 ± 66    | 27 ± 24       |
| NGC 6720 | 155 ± 51         | 22.3 ± 18.7          | 28.1 ± 1.0 | 15.6 ± 12.0   |
| NGC 6826 | 77.6 ± 8.4       | 12.7 ± 5.6           | 12.2 ± 1.5 | 6 ± 4         |
| PM 1-305 | 7.7 ± 2.5        | 21.7 ± 5.0           | 17.7 ± 6.6 | 6 ± 4         |

Table 1 The Flux Densities, Deconvolved Diameters Θ₁, Corrected Diameters Θ, and Optical Diameters Taken from Frew et al. (2016) of PNe Detected in the LoTSS Survey

* The present coverage of LoTSS is shown in https://lofar-surveys.org/lotss-tier1.html
Inclusion of cold plasma with... 

Comparison of Mean Electron Temperatures Derived from Radio Observations of Optically Thick PNe at 144 MHz, Balmer Jump, Helium Lines, [O III] and [N II] CELs, and RL of O II in Kelvin

| Name       | 144 MHz | BI | He I λ7281/λ6678 | He I λ7281/λ5876 | (O III) e | (N II) e | O I f |
|------------|---------|----|-----------------|-----------------|----------|----------|-------|
| IC 2149    | 7700 ± 3000 |    |                  |                  |          |          |       |
| IC 3568    | 5760 ± 680 | 9500 ± 900 | 8100 ± 1000 | 7800 ± 1450 | 8900 | 11400 | 630  |
| IC 4593    | 5680 ± 850 | 6000 ± 2000 | 12000 ± 500 | 10360 ± 1100 | 12400 | (13700) | 450  |
| K 3-17     | 5100 ± 1500 | (1900) |                  |                  |          |          |       |
| NGC 40     | 4700 ± 1300 | 7000 ± 700 | 10240 ± 1900 | 10580 ± 4200 | 11000 | 7900 | 400  |
| NGC 6543   | 6010 ± 730 | 6800 ± 1400 | 6010 ± 1400 | 5450 ± 1400 | 8100 | 9000 | 500  |
| NGC 6826   | 8360 ± 980 | 8700 ± 700 | 8290 ± 1500 | 8520 ± 2000 | 11200 | 12200 | 800  |
| NGC 7027   | 9600 ± 2000 | 12000 ± 400 | 10360 ± 1100 | 9030 ± 2200 | 12400 | (13700) | 450  |

Notes.  
*a* This work  
*b* Zhang et al. (2004)  
*c* Zhang et al. (2005)  
*d* Kaler (1986)  
*e* Kaler et al. (1996)  
*f* McNabb et al. (2013)

The inclusion of cold plasma with a uniform electron temperature of 10 kK randomly distributed, filling 10% of the radius (red solid line). Bottom: the same as above, with an electron temperature of the uniform model of 5 kK.

Figure 1. Top: Comparison of a computed radio spectrum of a PN with a uniform electron temperature of 10 kK (blue dashed line) and a PN with an inclusion of cold plasma with $T_e = 2$ kK randomly distributed, filling 10% of the radius (red solid line). Bottom: the same as above, with an electron temperature of the uniform model of 5 kK.

4.2. Electron Temperatures

The nebular images and spectra are shown in Figures 2 through 5. We converted the intensity scale in the images from $F_{144 \text{ MHz}}$/beam to $T_B$/beam. The converted images map the electron temperature for resolved and optically thick PNe. However, for PNe more compact than the instrument beam, the peak flux and brightness temperature are diluted by the squared ratio of the source size to the beam size $(\theta / \theta_{\text{beam}})^2$.

We determined average electron temperatures for well resolved PNe which are optically thick at 144 MHz. For this purpose, we substituted the measured size and the integrated the 144 MHz flux to Equation (2) with $T_B = T_e$. The flux and diameter uncertainties propagate to the calculated $T_e$ error. The derived temperatures are presented in Table 2 along with electron temperatures from the literature obtained using...
alternative methods. The electron temperatures derived from the 144 MHz images do not exceed 9.6 kK. Table 2 shows that our derived temperatures are 20% to 60% lower than the temperatures derived from CELs of [O III] and [N II]. They are also lower from the temperatures derived from the Balmer jump, although they agree within 1 $\sigma$ in two cases (NGC 6543 and NGC 6826). The mean temperature determined from the 144 MHz images is about 7.0 kK, which is about 35% lower than the 10.7 kK mean temperature derived for [O III]. The low $T_e$ derived from 144 MHz optically thick emission results from the presence of the cold plasma component, which is observed in RLs (Liu et al. 2000). Radio flux is strongly affected by the coldest and most dense regions, even if they contain only a small fraction of the total ionized mass in PNe. The optical

Figure 2. Left: Images of PNe at 144 MHz. We converted the flux intensity scale to brightness temperature, represented by the color bar. The contours levels are spaced by 3$\sigma$. The white circle marks the size of the beam. Right: observed and fitted radio spectra of PNe. The lower panel shows the difference between the fit and the observed fluxes.
thickness of plasma at radio wavelengths is approximately proportional to $T_e^{-1.35}$. Thus, low electron temperature plasma has much higher opacity from hot plasma and can become a strong opacity source for low-frequency radio emission.

We modeled a radio spectrum from an ionized nebula filled with the plasma with $T_e$ of 10 kK, a typical value for PNe, with 10% of the volume filled with a randomly distributed cool plasma component with $T_e$ of 2 kK. The electron temperature averaged over volume is thus 9.2 kK. Analysis of RLs confirms that the cold component can indeed have an electron temperature as low as $T_e \approx 1$ kK (Corradi et al. 2015). The resulting free–free radio continuum spectrum is compared to the spectrum of the homogeneous model with a temperature of 10 kK (Equation (1)) in Figure 1, upper panel.

An inclusion of cold plasma increases flux emitted at optically thin high frequencies with respect to the homogeneous model.

Figure 3. Images and spectra of PNe—continued.
(Figure 1). This is because optically thin cold plasma emits much more flux than hot plasma. The turnover is shifted to higher frequency. The inclusion of low-temperature plasma reduces the brightness temperature in the optically thick part of the spectrum compared to that expected from an homogeneous model by as much as 50\% at 144 MHz. Hence, in this scenario, the electron temperature determined from an optically thick radio image would be around 50\% lower than the temperature of the hot component. This is consistent with our observations, with $T_e(144$ MHz) lower by 20\%–60\% than $T_e([\text{O III}])$, which represents the hot component.

The model with two plasma components is compared with the homogeneous model with a lower electron temperature of $T_e = 5$ kK (Figure 1, lower panel). The spectra appear quite similar. The turnover is approximately at the same frequency. Both models emit similar amounts of optically thick and

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**Figure 4.** Images and spectra of PNe—continued.
optically thin flux. However, the optical depth effects become visible before the turnover frequency in the two-component model. This would produce an excess of the 5–1.4 GHz flux ratio with respect to a homogeneous model.

The distribution, temperature, chemical composition, and density of cold plasma may vary from one PN to another. In every case, cold plasma adds more flux to the optically thin part of the spectrum compared to the homogeneous model, assuming \( T_e \) derived from CELs, and suppresses optically thick flux if it extends to the outer boundary of the PN. \( T_e \) should be treated as a free parameter in modeling radio spectra of PNe, but unfortunately it is not independent of \( \xi \) and \( EM \).

### 4.3. Comparison of Radio and Optical Emission

We compared radio continuum images of PNe with the Hubble Space Telescope (HST) and the Izaak Newton Telescope (INT), collected in most cases by the INT Photometric H-Alpha Survey (Drew et al. 2005, IPHAS) H\( \alpha \) images in Figures 6, 7, and 8. We convolved H\( \alpha \) images with a 6" or 20" Gaussian in order to match the resolution of radio and optical images. H\( \alpha \) emission is optically thin in PNe. The brightness distribution in H\( \alpha \) should be similar to optically thin at 144 MHz emission, as both of them depend primarily on the emission measure. However, cold and hydrogen-deficient regions should stand out in the 144 MHz images. On the other hand, the 144 MHz surface brightness distribution for optically thick PNe is not expected to correlate with the H\( \alpha \) image since it depends on the local electron temperature close to the outer radius of a PN.

PNe K 3-17, IC 4593, NGC 6543, NGC 6826, and NGC 7027 are optically thick in radio. IC 4593 (Figure 6) is not very well resolved and does not allow for a detailed comparison of optical and radio surface brightness distribution. NGC 7027 (Figure 6) is slightly more resolved. It shows a maximum of the radio emission in the northwestern part of the nebula. NGC 6826, NGC 6543 (Figure 6), and NGC 40 (Figure 8) have similar sizes in radio and H\( \alpha \) (Table 1). Low brightness temperatures of these three PNe indicate that cold plasma exists near the outer radius of the shell. Otherwise, their brightness temperature would reflect the temperature of the hot plasma component, close to the [O III] or [N II] temperatures (Table 2). We were able to measure temperature fluctuations (Peimbert 1967) since these three PNe are well...
resolved and optically thick. The fluctuations clearly exceed the 3σ noise in the 144 MHz images (Figure 6). The point-to-point temperature fluctuations measured in the plane of sky in the central part of the disk are $\tilde{r} = 0.006 \pm 0.001$ in NGC 6826, $\tilde{r} = 0.004 \pm 0.001$ for NGC 6543, and $\tilde{r} = 0.013 \pm 0.006$ for NGC 40. $\tilde{r}$ is defined as the standard deviation of the temperature distribution computed close to the nebular center, so that it would not be affected by a drop of the flux at the edges. We subtracted the background root mean square error (rms). In order to compute the background rms we first measured the rms in the whole image. Then we flagged all the regions exceeding 3 rms and we computed a new value of the rms in the unflagged image. We repeated the process if there were still regions exceeding 3 rms in the image. The temperature fluctuations in NGC 6543 agree with the 0.004 estimate by Wesson & Liu (2004) from optical imaging. Temperature fluctuations which could explain the abundance discrepancy would need to be one order of magnitude bigger. However, the temperature fluctuations which could be responsible for the dichotomy of abundance determination (Peimbert 1971) may exist on lower spatial scales.

NGC 40 shows patchy structure in the 144 MHz image. The optical and radio images are not correlated. NGC 40 a born-again candidate (Toalá et al. 2019). In such a case, new, hydrogen-free ejecta can be mixed with the previously ejected hydrogen-rich envelope and cause significant inhomogeneities of the chemical composition and electron temperature within the nebula.

K 3-17 shows a weak trace of the bipolar structure in radio (Figure 7). The waist of the hourglass nebula is optically thick in radio, whereas the bipolar structure is optically thin.

The radio images of NGC 6720 (Figure 6) and NGC 2371 (Figure 6) resemble the Hα images. However, their radio spectra indicate considerable optical thickness at 144 MHz. The 144 MHz flux is dominated by the brightest regions in these two nebulae. This is confirmed by non-uniform brightness distributions observed in the 144 MHz images and small values of $\xi$ derived in the fit of the radio spectra. The small value of $\xi$ indicates that most of the emission comes from a small fraction of the nebulae, while the remaining part remains optically thin. Both nebulae have significantly smaller $\tau_{144\text{ MHz}}$ than PNe which are fully optically thick.

NGC 6720 is very well resolved in the LOFAR image. Both 144 MHz and Hα images show an oval ring. The brightest part, reaching a temperature of 5000 K at maximum, is optically thick. The center of the nebula and the part of the ring close to the long axis, which contribute less to the radio flux, remain optically thin. O’Dell et al. (2013) modeled the optical image of NGC 6720 with a triaxial ellipsoid seen nearly pole-on. The projected ring is brighter on its shorter axis. The maximum of the optical emission along the shorter axis is close to the outer edge of the ring. The maximum of radio emission is shifted toward the center of the nebula with respect to Hα emission. The reason for this could be that more cold plasma exists closer to the central star. As a result, the 144 MHz opacity increases toward the central star, so the inner part of the ring is brighter.
Figure 9 compares the 70 μm and 144 MHz continuum images of NGC 6720. The maximum of the 144 MHz emission is closer to the central star than the 70 μm emission. It appears that cold plasma is not associated with dust emitting at 70 μm.

NGC 2371 is a bipolar nebula. The brightest, barrel-like structure contains a collection of knots. Two pair of brightest knots in the Hα image lie at the position angle of 60 degrees in the NW and SE direction from the central star, although they are not perfectly aligned with the central star (Figure 6). The 144 MHz image of NGC 2371 does not trace the Hα emission in detail. In particular, the maxima of 144 MHz emission are not centered on the brightest knots in the Hα image, but located on fainter knots closer to the central star. This is more clearly seen when comparing a full resolution optical image with the radio emission (Figure 10).

Gómez-González et al. (2020) used spatially resolved spectroscopy to study the electron temperature in NGC 2371. The brightest clump in the nebula, located in the NE direction from the central star (designated as A7, Figure 10) has a temperature of 13.8 kK. It is classified as a low-ionization knot (Gonçalves et al. 2001). For comparison, the neighboring region A6, which is the brightest region in the 144 MHz map, has a significantly higher temperature of 18 kK. The A7 clump should stand out at 144 MHz since it is brighter in the optical and cooler. However, it is at least two times fainter in radio than the A6 clump. This suggests that the low-ionization knot...
A7 does not contain cold plasma or contains less cold plasma than the A6 clump.

5. Discussion and Conclusions

We observed 144 MHz free–free radio emission in a sample of PNe using LOFAR. Optically thick emission allows for a relatively straightforward measurement of the local electron temperature. The observations confirm the presence of a significant amount of cold plasma, which was first proposed from the study of RLs and CELs. Cold and hot components remain spatially unresolved in the 144 MHz images. However, cold plasma has much higher opacity compared to hot plasma. In the result, the determined electron temperatures are significantly weighted toward a cold plasma component. Thus, the previous approaches, which assumed a homogeneous model of PNe with the temperature derived from CEL or an arbitrary value of 10 kK, are incorrect.

Different studies assumed homogeneous models with the temperature corresponding to the hot plasma component. In particular, ionized mass determination relies on the assumption that PNe are optically thin at 5 GHz and their plasma temperature is similar to $T_e$ derived from CELs (Buckley & Schneider 1995). This approximation remains valid as soon as the electron temperatures used in the computation are reduced by 40% on average. Taking this into account it would scale down the ionized masses of PNe derived from optically thin radio emission. Lower $T_e$ would also result in lower diameters derived from the radio SED fit compared to observed diameters (Hajduk et al. 2018; Bojičić et al. 2021).

Stasińska et al. (1992) used a homogeneous model for extinction determination from the ratio of the optically thin radio to hydrogen flux. They used electron temperatures from Kaler (1986) or derived them using his formulae. The radio flux was used to determine the dereddened $H\beta$ flux. This dereddened flux was compared with the observed $H\beta$ flux to derive the extinction $C_{\text{rad}}$. Another extinction determination $C_{\text{opt}}$ comes from the observed $H\alpha$ to $H\beta$ ratio. Stasińska et al. (1992) showed that $C_{\text{opt}}$ is systematically larger by a factor of about 1.2 than $C_{\text{rad}}$ for the PNe in the direction to the Galactic center. Ruffle et al. (2004) postulated a steeper extinction law toward the Galactic center to explain the difference between radio and optical extinction, which was later confirmed by Hajduk & Žijlstra (2012). Finally, Pottasch & Bernard-Salas (2013) suggested that the 5 GHz emission in PNe is not optically thin, which, however, contradicted most of the other studies.

The ratio of the optically thin radio to $H\beta$ flux depends on electron temperature $\sim T_e^{0.53}$ (Pottasch 1984). If PNe were
modeled with the temperature lower by 40%, the dereddened H2 fluxes would be higher by a factor of 1.3. This would decrease $C_{\text{rad}}$ by 0.12 and improve the consistency of both extinction determinations though not yet fully explain it.

The 144 MHz images allowed us to constrain the spatial distribution of cold plasma. It extends to the outer radii of the nebulae. However, the partially optically thick image of NGC 6720 shows that cool plasma is more abundant toward the center of the nebula. It is noteworthy that the abundance discrepancy factor also increases toward the center of the PN (Garnett & Dinerstein 2001). The 70 $\mu$m image also has a different brightness distribution from radio emission, which suggests that dusty regions observed at 70 $\mu$m do not harbour cold plasma. Another example, in which radio emission is more concentrated in the inner ring of the nebula than dust emission is the Helix nebula (Planck Collaboration et al. 2015).

Low-ionization structures are often present in PNe-hosting binary central stars (Miszalski et al. 2009), and they are one of the candidates to explain the abundance discrepancy (Corradi et al. 2015). If low-ionization structures in NGC 2371 contained cold plasma, they would stand out in the 144 MHz radio image. Instead, they are fainter. This could be explained if they did not contain cold plasma or contained significantly less cold plasma than other regions of the nebula. Observations of a larger sample when LoTSS is completed will allow for more deep and statistically important study of the cold plasma component in PNe. Multi-frequency analysis of PNe radio-continuum images may allow us to better constrain the spatial distribution of cold plasma in PNe.

We will continue to study low-frequency radio emission of PNe using more complete data from the LoTSS survey. The number of observed PNe will increase rapidly when the survey improves completeness at low Galactic latitudes. The low-frequency survey LoLSS will observe at 42–66 MHz, but with a reduced spatial resolution of 15$''$ compared to LoTSS. Further advances will be made with the Square Kilometer Array (SKA; Umana et al. 2015), which will carry out an extremely sensitive radio continuum survey at 1.4 GHz. Multi-frequency images will allow us to obtain accurate spectral index maps of PNe and possibly model the spatial distribution of cold plasma in PNe.

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### Appendix

The appendix contains supplementary data for flux density analysis of known PNe which were undetected in the studied LoTSS fields. Table 3 lists upper flux density limits for these PNe corresponding to $3 \times \text{rms}$. Table 4 lists a subset of PNe detected in the NVSS survey at 1.4 GHz. It combines NVSS flux densities with the LoTSS 144 MHz upper limit to estimate lower limits for the spectral index for PNe.

#### Table 3

| Name          | 3×rms (mJy beam$^{-1}$) | $T_B$ K |
|---------------|-------------------------|---------|
| A 16          | 0.791                   | 1853    |
| A 28          | 0.722                   | 1692    |
| A 30          | 0.570                   | 1335    |
| A 39          | 3.330                   | 7806    |
| A 43          | 3.180                   | 7452    |
| A 43          | 0.762                   | 1786    |
| A 46          | 0.669                   | 1569    |
| A 53          | 4.980                   | 11660   |
| A 59          | 5.090                   | 11932   |
| A 61          | 0.927                   | 2172    |
| A 63          | 3.890                   | 9109    |
| A 73          | 3.810                   | 8935    |
| A 74          | 0.726                   | 1700    |
| A 79          | 5.810                   | 13606   |
| A 84          | 5.340                   | 12520   |
| BI 2-1        | 0.639                   | 1496    |
| CN 3-1        | 5.790                   | 13563   |
| DDDM 1        | 0.391                   | 916     |
| EGB 1         | 0.411                   | 962     |
| ETHOS 4       | 1.060                   | 2486    |
| FBP 8         | 1.560                   | 3644    |
| FSMV 1        | 1.380                   | 3226    |
| GLMP 879      | 10.200                  | 23976   |
| HaTr 12       | 3.280                   | 7674    |
| HaTr 14       | 7.210                   | 16897   |
| Hen 1-1       | 1.730                   | 4061    |
| Hen 1-2       | 1.350                   | 3173    |
| Hen 2-447     | 5.360                   | 12561   |
| IC 2003       | 0.955                   | 2237    |
| IC 351        | 2.150                   | 5040    |
| IPHAS J185321.76+055641.9 | 1.850 | 4333 |
| IPHAS J185322.1+083018   | 1.750 | 4090 |
| IPHAS J185744.4+105053   | 1.690 | 3949 |
| IPHAS J185815.8+073753   | 1.820 | 4271 |
| IPHAS J185957.0+073544   | 1.940 | 4537 |
| IPHAS J190718.1+044056   | 7.120 | 16681 |
| IPHAS J192553.5+165331.4 | 5.060 | 11854 |
| IPHAS J193517.8+223120   | 8.250 | 19320 |
| IPHAS J193652.96+171940.7 | 2.350 | 5495 |
| IPHAS J193718.6+202102   | 1.150 | 2701 |
| IPHAS J221118.0+552841   | 5.810 | 13606 |
| IPHAS J185225.0+080843   | 1.950 | 4565 |
| IPHAS J185309.4+075241   | 1.740 | 4069 |
| IPHAS J190340.7+094639   | 4.280 | 10017 |
| IPHAS J190417.9+084916   | 3.220 | 7555 |
| IPHAS J190432.9+091656   | 4.400 | 10305 |
### Table 3 (Continued)

| Name                | $3 \times$ rms (mJy beam$^{-1}$) | $T_b$ K |
|---------------------|----------------------------------|---------|
| IPHASX J190454.0+101801 | 6.500                            | 15224   |
| IPHASX J192146.7+172055  | 8.250                            | 19320   |
| IPHASX J193009.3+192129  | 1.040                            | 2437    |
| IPHASX J194301.3+215424  | 3.610                            | 8456    |
| IPHASX J194648.2+193608  | 5.630                            | 13187   |
| IRAS 19086+0603        | 1.940                            | 4537    |
| IRAS 19297+1954        | 0.929                            | 2176    |
| Jn 1                 | 0.289                            | 678     |
| JnEr 1               | 2.060                            | 4837    |
| K 1-15               | 1.240                            | 2906    |
| K 1-16               | 0.422                            | 988     |
| K 1-20               | 4.070                            | 9527    |
| K 3-14               | 9.270                            | 21720   |
| K 3-15               | 3.240                            | 7603    |
| K 3-31               | 4.880                            | 11441   |
| K 3-32               | 5.320                            | 12471   |
| K 3-35               | 1.280                            | 3004    |
| K 3-38               | 2.380                            | 5566    |
| K 3-40               | 5.260                            | 12325   |
| K 3-42               | 1.400                            | 3284    |
| K 3-43               | 1.720                            | 4039    |
| K 3-58               | 2.330                            | 5448    |
| K 3-73               | 2.860                            | 6702    |
| K 3-76               | 6.470                            | 15150   |
| K4 -30               | 2.080                            | 4881    |
| KLSS 1-1             | 2.850                            | 6672    |
| KLSS 1-2             | 1.170                            | 2737    |
| KLSS 2-1             | 2.990                            | 7008    |
| KLSS 2-6             | 2.130                            | 4990    |
| Kn 132               | 0.865                            | 2026    |
| Kn 20                | 2.600                            | 6087    |
| Kn 21                | 2.360                            | 5524    |
| Kn 23                | 0.675                            | 1580    |
| Kn 43                | 1.700                            | 3988    |
| Kn 49                | 0.708                            | 1659    |
| Kn 58                | 1.700                            | 3979    |
| Kn 59                | 1.230                            | 2873    |
| Kn 68                | 0.694                            | 1626    |
| Kn 7                 | 2.760                            | 6461    |
| Kn 9                 | 8.750                            | 20495   |
| KnFe 1               | 5.270                            | 12347   |
| LoTr 5               | 0.401                            | 940     |
| M 1-64               | 2.340                            | 5486    |
| M 1-71               | 1.040                            | 2429    |
| M 1-72               | 2.630                            | 6172    |
| M 2-53               | 2.580                            | 6044    |
| M 3-35               | 6.070                            | 14215   |
| MSX 6c               | 2.580                            | 6048    |
| NGC 6742             | 5.190                            | 12166   |
| NGC 6765             | 3.670                            | 8589    |
| NGC 6833             | 0.902                            | 2114    |
| Ou 2                 | 1.090                            | 2558    |
| Ou 3                 | 6.870                            | 16095   |
| Ou 5                 | 1.560                            | 3665    |
| Pa 157               | 1.540                            | 3619    |
| Pa 18                | 1.440                            | 3373    |
| Pa 27                | 2.390                            | 5595    |
| Pa 4                 | 5.500                            | 12883   |
| Pa 5                 | 6.340                            | 14853   |
| PK 020-02 1          | 1.680                            | 3930    |
| PM 1-262             | 1.460                            | 3424    |
| PM 1-264             | 1.770                            | 4146    |
| PM 1-267             | 1.780                            | 4163    |

### Table 4

Lower Limits for 144 MHz–1.4 GHz Spectral Indices

| Name            | $3 \times$ rms (mJy beam$^{-1}$) | $F_{1.4\,\text{GHz}}$ (mJy) | $S_{1.4\ldots0.144\,\text{GHz}}$ |
|-----------------|----------------------------------|------------------------------|----------------------------------|
| A66 53          | 4.98                             | 33.6 ± 1.1                  | 0.84                             |
| A66 63          | 3.89                             | 4.5 ± 0.5                   | 0.06                             |
| A66 73          | 3.81                             | 11.1 ± 1.3                  | 0.47                             |
| A66 79          | 5.81                             | 16.6 ± 2.1                  | 0.46                             |
| Cn 3-1          | 5.79                             | 59.5 ± 1.8                  | 1.02                             |
| EGB 1           | 0.41                             | 8.6 ± 1.7                   | 1.34                             |
| Hen 1-1         | 1.73                             | 16.0 ± 0.7                  | 0.98                             |
| Hen 1-2         | 1.35                             | 14.6 ± 0.6                  | 1.05                             |
| Hen 2-447       | 5.36                             | 22.9 ± 1.9                  | 0.64                             |
| IC 2003         | 0.96                             | 54.8 ± 1.7                  | 1.78                             |
| IC 351          | 2.15                             | 31.9 ± 1.3                  | 1.19                             |
| IPHASX J185309.4| 1.74                             | 9.0 ± 0.5                   | 0.72                             |
| IPHASX J185815.8| 1.82                             | 16.6 ± 1.1                  | 0.97                             |
| IPHASX J190340.7| 4.28                             | 6.1 ± 0.6                   | 0.16                             |
| IPHASX J194329.2| 4.40                             | 17.5 ± 0.7                  | 0.61                             |
| IPHASX J192533.5| 5.06                             | 44.6 ± 1.8                  | 0.96                             |
| IPHASX J193718.6| 1.15                             | 6.6 ± 0.5                   | 0.77                             |
| IPHASX J221118.0| 5.81                             | 3.1 ± 0.5                   | 0.29                             |
| IRAS 19086+0603 | 1.94                             | 8.5 ± 0.7                   | 0.65                             |
| IRAS 19297+1954 | 0.93                             | 2.1 ± 0.5                   | 0.36                             |
| K 3-31          | 4.88                             | 16.8 ± 0.7                  | 0.54                             |
| K 3-35          | 1.28                             | 14.5 ± 0.6                  | 1.07                             |
| K 3-38          | 2.38                             | 28.7 ± 1.1                  | 1.09                             |
Table 4
(Continued)

| Name   | 3 × rms (mJy beam$^{-1}$) | $F_{1.4 \text{ GHz}}$ (mJy) | $S_{1.4-0.144 \text{ GHz}}$ |
|--------|--------------------------|-----------------------------|-----------------------------|
| K 3-40 | 5.26                     | 17.1 ± 0.7                  | 0.52                        |
| K 3-42 | 1.40                     | 11.2 ± 1                    | 0.91                        |
| K 3-43 | 1.72                     | 4.8 ± 0.5                   | 0.45                        |
| K 4-30 | 2.08                     | 20 ± 1                      | 1.00                        |
| NGC 6742 | 5.19                  | 3.7 ± 0.5                   | −0.15                       |
| NGC 6765 | 3.67                  | 10.1 ± 0.5                  | 0.45                        |
| NGC 6833 | 0.90                  | 4.2 ± 0.5                   | 0.68                        |
| Sh 1-118 | 1.81                  | 39.8 ± 3.1                  | 1.36                        |
| Tk 1    | 0.27                     | 14 ± 0.6                    | 1.73                        |

References

Aaquist, O. B., & Kwok, S. 1996, ApJ, 462, 813
Bojičić, I. S., Filipović, M. D., Urošević, D., Parker, Q. A., & Galvin, T. J. 2021, MNRAS, 503, 2887
Buckley, D., & Schneider, S. E. 1995, ApJ, 446, 279
Condon, J. J., & Kaplan, D. L. 1998, ApJS, 117, 361
Corradi, R. L. M., García-Rojas, J., Jones, D., & Rodríguez-Gil, P. 2015, ApJ, 803, 99
de Gasperin, F., Dijkstra, T. J., Drabent, A., et al. 2019, A&A, 622, A5
de Gasperin, F., Williams, W. L., Best, P., et al. 2021, A&A, 648, A104
Draine, B. T., & Kreisch, C. D. 2018, ApJ, 862, 30
Drew, J. E., Greimel, R., Irwin, M. J., et al. 2005, MNRAS, 362, 753
Frew, D. J., Parker, Q. A., & Bojičić, I. S. 2016, MNRAS, 455, 1459
García-Rojas, J., Wesson, R., Boffin, H. M. J., et al. 2019, arXiv:1904.06763
Garnett, D. R., & Dinerstein, H. L. 2001, ApJ, 558, 145
Gómez-González, V. M. A., Toalá, J. A., Guerrero, M. A., et al. 2020, MNRAS, 496, 959
Gonçalves, D. R., Corradi, R. L. M., & Mampaso, A. 2001, ApJ, 547, 302
Hajduk, M., van Hoof, P. A. M., Śniadkowski, K., et al. 2018, MNRAS, 479, 5657
Hajduk, M., & Zijlstra, A. A. 2012, in IAU Symp. 283, Planetary Nebulae: An Eye to the Future (Cambridge: Cambridge Univ. Press), 380

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Kaler, J. B. 1986, ApJ, 308, 322
Kaler, J. B., Kwitter, K. B., Shaw, R. A., & Browning, L. 1996, PASP, 108, 980
Kingdon, J. B., & Ferland, G. J. 1995, ApJ, 450, 691
Liu, X. W., Storey, P. J., Barlow, M. J., et al. 2000, MNRAS, 312, 585
Masson, C. R. 1990, ApJ, 348, 580
McMullin, J. P., Waters, B., Schiebel, D., Young, W., & Golap, K. 2007, in ASP Conf. Ser. 376, Astronomical Data Analysis Software and Systems XVI, ed. R. A. Shaw, F. Hill, & D. J. Bell (San Francisco, CA: ASP), 127
McNabb, I. A., Fang, X., Liu, X. W., Bastin, R. J., & Storey, P. J. 2013, MNRAS, 426, 3443
Miszalski, B., Acker, A., Parker, Q. A., & Moffat, A. F. J. 2009, A&A, 505, 249
O’Dell, C. R., Ferland, G. J., Henney, W. J., & Peimbert, M. 2013, AJ, 145, 92
Olson, F. M. 1975, A&A, 39, 217
Parker, Q. A., Bojičić, I. S., & Frew, D. J. 2016, JPhCS, 728, 032008
Pazderska, B. M., Gawroński, M. P., Feiler, R., et al. 2009, A&A, 498, 463
Peimbert, M. 1967, ApJ, 150, 825
Peimbert, M. 1971, BOTT, 6, 29
Phillips, J. P. 2007, MNRAS, 378, 231
Planck Collaboration, Arnaud, M., Atrio-Barandela, F., et al. 2015, A&A, 573, A6
Pottasch, S. R. 1984, Planetary Nebulae. A study of Late Stages of Stellar Evolution, Vol. 107 (Dordrecht: Reidel)
Pottasch, S. R., & Bernard-Salas, J. 2013, A&A, 550, A35
Ruffle, P. M. E., Zijlstra, A. A., Walsh, J. R., et al. 2004, MNRAS, 353, 796
Savini, F., Bonafede, A., Brüggen, M., et al. 2018, MNRAS, 474, 5023
Shimwell, T. W., Röttgering, H. J. A., Best, P. N., et al. 2017, A&A, 598, A104
Shimwell, T. W., Tasse, C., Hardcastle, M. J., et al. 2019, A&A, 622, A1
Siódmiak, N., & Tylenda, R. 2001, A&A, 373, 1032
Stasińska, G. 2002, in Revista Mexicana de Astronomía y Astrofísica Conf. Ser. 12, ed. W. J. Henney, J. Franco, & M. Martos (Cambridge: Cambridge Univ. Press), 62
Stasińska, G., & Szczepański, R. 2001, A&A, 379, 1024
Stasińska, G., Tylenda, R., Acker, A., & Stenholm, B. 1992, A&A, 266, 486
Tasse, C., Hugo, B., Mirmont, M., et al. 2018, A&A, 611, A87
Toalá, J. A., Ramos-Larios, G., Guerrero, M. A., & Todt, H. 2019, MNRAS, 485, 3360
Tsamis, Y. G., Barlow, M. J., Liu, X. W., Storey, P. J., & Danziger, I. J. 2004, MNRAS, 353, 953
Umana, G., Trigilio, C., Cerrigone, L., et al. 2015, in Proc. of Advancing Astrophysics with the Square Kilometre Array (AASKA14) (Trieste: PoS), 118
van Haarlem, M. P., Wise, M. W., Gunat, A. W., et al. 2013, A&A, 556, A2
van Hoof, P. A. M. 2000, MNRAS, 314, 99
van Hoof, P. A. M., van de Steene, G. C., Barlow, M. J., et al. 2010, A&A, 518, L137
van Hoof, P. A. M., Williams, R. J. R., Volk, K., et al. 2014, MNRAS, 444, 420
van Weeren, R. J., Williams, W. L., Hardcastle, M. J., et al. 2016, ApJS, 223, 2
Wenger, M., Ochsendorf, F., Egeit, D., et al. 2000, A&AS, 143, 9
Wesson, R., & Liu, X. W. 2004, MNRAS, 351, 1026
Wesson, R., Liu, X. W., & Barlow, M. J. 2005, MNRAS, 362, 424
Wright, A. E., & Barlow, M. J. 1975, MNRAS, 170, 41
Zhang, Y., Liu, X. W., Liu, Y., & Rubin, R. H. 2005, MNRAS, 358, 457
Zhang, Y., Liu, X. W., Wesson, R., et al. 2004, MNRAS, 351, 935