Effect of wavelength of sinusoidal wavy wall surface on drag and heat transfer at turbulent thermal boundary layer flow

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Abstract
Direct numerical simulations of thermal turbulent boundary layer flows over a wavy wall surface are performed to investigate the effect of a wavelength on drag coefficients and heat transfer performance. The Reynolds number based on an inlet boundary layer thickness is set to be 2820 and the Prandtl number is set to be Pr = 0.71 and 2.0. The wavy wall surface is homogeneous in the spanwise direction and the wave amplitude is fixed at 2a = 20. The six wavelength cases of λ/2a = 7.5 ~ 45 are examined. As the wavelength decreases, the skin-friction drag decreases and the pressure drag and heat transfer increase. The total drag peaks at λ/2a = 12.5 and the flow separation occurs at λ/2a < 15. In the separation region, the backward flow transfers the heat and results in a negative correlation coefficient between the velocity and temperature of R(u′t′) at the bottom of the wavy wall. Spindle-shaped spots of the Nusselt number are also observed on the upslope of the wavy wall.

Keywords : Direct numerical simulation, Turbulent boundary layer flow, Heat transfer enhancement, Wavy wall surface, Wavelength

1. Introduction

The heat transfer enhancement in wall turbulence is significant from the viewpoint of engineering fields, and a simple method is to use wall undulation. The flow over the sinusoidal wavy wall surface, therefore, has been extensively studied. Here, the sinusoidal wavy wall indicates that the bottom wall shape varies sinusoidally in the streamwise direction and is homogeneous in the spanwise direction.

Previous studies have employed linear analysis to investigate the flow over the wavy wall; however, the analysis could be only applied to wavy wall surfaces with a small amplitude (e.g., Thorsness et al., 1978). In contrast, for the large wave amplitude cases, investigations have been conducted through experiments and numerical simulations. For example, Hudson et al. (1996) conducted laser Doppler velocimetry (LDV) measurement to show the turbulence production in the flow over a wavy wall surface. Nakagawa and Hanratty (2001) performed particle image velocimetry (PIV) measurements to show two-point correlation of the velocity fluctuations, and revealed that the turbulent flow structure over the wavy wall is similar to that on the flat surface, while mechanisms of maintaining the turbulence are different. Nakagawa and
Hanratty (2003) and Nakagawa et al. (2003) performed LDV measurements for a flow field with the highly rough wavy wall surfaces, the intermediate roughened surfaces, and the hydraulically smooth surfaces. Günther and von Rohr (2003) conducted the PIV measurement and discussed the role of streamwise-oriented large-scale structures over the wavy wall surface. Direct numerical simulations (DNS) and large eddy simulations have also been conducted, and have revealed significant effects of the wavy boundary on flow statistics, flow structure, pressure, and skin-friction drag coefficient (e.g., de Angelis et al. (1997), Henn and Sykes (1999), Yoon et al. (2009)).

As discussed above, there are numerous studies of turbulent channel flow with wavy wall surface and the effect of wave amplitude. In contrast, Gong et al. (1996) investigated the turbulent boundary layer flow over the wavy wall surface numerically and experimentally, and found that the mean flow separated over each wave was approximately two-dimensional for small amplitude cases. For a relatively smooth surface, however, the flow is attached after the first wave and a three-dimensional secondary flow is observed after the first few waves. In general, numerical simulations of the turbulent boundary layer flows require larger computational cost as compared to the channel flow simulation. Consequently, few studies have been conducted on the turbulent boundary layer flows over the wavy wall. In the present study, therefore, we focus on the turbulent thermal boundary layer flow over the wavy wall surface. The expected contribution is twofold.

First is the drag prediction. For the prediction of the drag over the roughness, Moody’s diagram (Moody, 1944) is well-known and summarizes the pressure drop of the pipe flow as a function of Reynolds number and the relative roughness. The relative roughness is defined as the ratio of roughness height to pipe diameter, and the diagram is very useful for predicting the drag if the relative roughness is known. However, if the relative roughness is not known, the Moody diagram cannot be used to predict the pressure drop straightforwardly; rather, unknown relative roughness is determined from a fitting of roughness height to match the measured pressure drop. In this study, turbulent flows over the wavy wall with a different wavelength and constant amplitude are investigated using DNS. According to the definition of relative roughness, it is unique and the diagram predicts the same drag. However, the resultant drag is a function of the wavelength, and we will discuss the reason of the variation.

Second is to estimate the heat transferred by the wavy wall in the turbulent thermal boundary layer flow. Owing to the inherent similarity between momentum and heat transfer, the heat transfer enhancement is expected where turbulence is promoted. Previously, e.g., Choi and Suzuki (2005) performed the large eddy simulation of a channel flow equipped with a heated wavy wall surface and investigated the response for different wave amplitudes; as for the experiments, Günther and von Rohr (2002) used liquid crystal thermometry to quantify the temperature field and assessed the large-scale structure; Kruse and von Rohr (2006) performed a particle thermometry technique to determine the turbulent heat flux between a heated wavy wall surface and investigated the response for different wave amplitudes; as for the experiments, de Angelis et al. (1997) investigated the turbulent boundary layer flow over the wavy wall surface of different wave amplitudes; as for the experiments, Hanratty (2003) and Nakagawa et al. (2003) performed LDV measurements for a flow field with the highly rough wavy wall surfaces, the intermediate roughened surfaces, and the hydraulically smooth surfaces. Günther and von Rohr (2003) conducted the PIV measurement and discussed the role of streamwise-oriented large-scale structures over the wavy wall surface. Direct numerical simulations (DNS) and large eddy simulations have also been conducted, and have revealed significant effects of the wavy boundary on flow statistics, flow structure, pressure, and skin-friction drag coefficient (e.g., de Angelis et al. (1997), Henn and Sykes (1999), Yoon et al. (2009)).

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This study aims to investigate the wavelength effect on the thermal turbulent boundary layer flow by using DNS. The thermal turbulent boundary layer flow is chosen for threefold reasons: first, it is more practical than the channel flow in the real situation; second, the wavelength of the wavy wall can be easily changed in the turbulent boundary layer flow if a large computational domain is provided; third, the similarity between momentum and heat transport is held under the no-slip and isothermal boundary conditions. The rest of this paper is organized as follows: in section 2, detail of the computational condition is summarized; in section 3, we show DNS results and discuss the relation between the velocity and temperature fields; and in section 4, we draw our conclusion.

2. DNS

Figure 1(a) shows schematics of computational domains. Both main and driver parts are thermal boundary layer flows. The Cartesian coordinate system is employed, where \(x\), \(y\), and \(z\) denote the streamwise, wall-normal, and spanwise coordinates and the corresponding velocities are \(u\), \(v\), and \(w\), respectively. For notation convenience, \(x_i\) and \(u_i\) (for \(i = 1...3\)) are interchangeably used. The computational domain is \(L_x \times L_y \times L_z\) and the corresponding number of grids is \(N_x \times N_y \times N_z\). The periodic condition is applied in the spanwise direction. The governing equations are an incompressible continuity, Navier-Stokes, and energy equations, i.e.,

\[
\frac{\partial u_i}{\partial x_i} = 0, \tag{1}
\]

\[
\frac{\partial u_i}{\partial t} = \frac{\partial u_i u_j}{\partial x_j} - \frac{\partial p}{\partial x_i} + \frac{1}{Re_{99,1a}} \frac{\partial^2 u_i}{\partial x_i \partial x_j}. \tag{2}
\]
The asterisk indicates dimensional variables. Nondimensional variables are defined as stream velocity $u$, where an outlet convective velocity $U_w$ is constant (i.e., is not a function of $y$) and is determined such that the mass conservation is satisfied in the computational domain. The Neumann condition is imposed for the pressure. At the upper boundary, velocity and temperature are:

$$u = U_w = 1.0, \quad v = \frac{\partial \delta_1}{\partial x}, \quad \frac{\partial u}{\partial y} = 0, \quad T = T_w = 1.0,$$

where $\delta_1$ is the displacement thickness. The no-slip velocity condition is imposed on the smooth wall in the driver and main parts and the periodic condition is imposed in the spanwise direction. In this paper, the drag coefficients in the range of $0 < x < 50$ of the main part are mainly discussed since an influence of the outlet condition appears at $x > 50$.
The present study focuses on velocity and temperature fields over the wavy wall surface, as shown in Fig. 1(b). The wavy wall is installed on the lower wall in the main part. The height of the wall varies sinusoidally in the streamwise direction. The wavy wall is characterized by the amplitude \( a \) and wavelength \( \lambda \). We examine six cases of \( \lambda/2a = 7.5, 10, 12.5, 15, 22.5, \) and 45, where the amplitude of the wave is fixed at \( 2a^* = 20 \) and the wavelength is varied. Here, the subscript of the plus sign denotes a nondimensionalized variable by the friction velocity at the inlet of the main part and the kinematic viscosity. The wavy wall starts at \( x = 3 \) from the inlet of the main part. The velocity and temperature on the surface of the wavy wall are set to be zero. The wall shape is expressed by an immersed boundary method (IBM)(Kim et al., 2001). The computational code of the IBM used in this study has been verified in Sasamori et al. (2017): the IBM was used to express the sinusoidal riblet surface in the channel flow and the grid number dependency was small.

Figure 2 shows the streamwise variation in the local skin-friction drag coefficient \( c_f \) and the Stanton number \( St \) in the main part for the flat surface case, together with those computed by empirical formulas (Schoenherr, 1932; Kays and Crawford, 1993; Kong et al., 2000), as

\[
c_f = 0.31 \left( \ln(2\mathrm{Re}_f) \right)^2 - 2 \ln \mathrm{Re}_f^{-1},
\]

\[
St = C ((1 n)/C y^{(n-1)} \mathrm{Re}_f^{n/(n-1)}, \quad \text{where} \quad n = 0.1879 \mathrm{Pr}^{-0.18} \text{ and } C = 0.02426 \mathrm{Pr}^{-0.895},
\]

where \( \mathrm{Re}_f \) and \( \mathrm{Re}_s \) are the Reynolds numbers based on the momentum thickness \( \delta_f \) and enthalpy thickness \( \Delta_2 \), respectively. The momentum thickness \( \delta_f \) and enthalpy thickness \( \Delta_2 \) are defined as, respectively,

\[
\delta_f = \int_0^\infty u(U_\infty - u)dy, \quad \Delta_2 = \int_0^\infty u(T_\infty - T)dy.
\]

There is a good agreement between the present DNS and the empirical formula for both \( c_f \) and \( St \).

Figures 3 show the mean and rms values of the velocities on the flat surface to compare the existing DNS and experimental data. As shown in Fig. 3(a), the mean velocity in the present study shows good agreement with the linear law in the viscous sublayer and its slope is higher than the log law in the logarithmic layer. The profile also shows good agreement with the experimental data. As shown in Fig. 3(a), the mean velocity in the present study shows good agreement with the linear law in the viscous sublayer and its slope is higher than the log law in the logarithmic layer. The profile also shows good agreement with the experimental data. As shown in Fig. 3(b), the mean temperature in the present study shows good agreement with the empirical formula suggested by Kader (1981) as

\[
T^+ = \frac{T_{\text{rms}}}{\Gamma} \exp \left(-\Gamma \right) + \left( 2.12 \ln \left( 1 + y^+ \right) + 2.5 \left( \frac{x + y/\delta_{99}}{4 \left( 1 + y/\delta_{99} \right)} \right) + \beta(\mathrm{Pr}) \right) \exp \left(-1/\Gamma \right),
\]

\[
\Gamma = \frac{10^{-2} (\Pr_{y^+})^4}{1 + 5\Pr_{y^+}^4 y^+},
\]

\[
\beta(\Pr) = \left( 3.85(\Pr_{y^+}^{1/3} - 1.3) \right)^2 + 2.12 \ln \Pr,
\]

where \( \delta_{99} \) is the local 99 \% thickness. The mean temperature shows good agreement with the DNS data and Kader’s formula, while \( T_{\text{rms}}^+ \) is overestimated. The overestimation in \( u'_{\text{rms}}^+ \) and \( T_{\text{rms}}^+ \) is due to the grid resolution in the spanwise direction.
3. Result and discussion

3.1. Effect of wavelength

First, the drag coefficient and heat transfer performance of the wall undulation are discussed. The local skin-friction and pressure drag coefficients in the streamwise component, \( c_{fx} \) and \( c_{px} \), are defined as

\[
\begin{align*}
c_{fx} &= \frac{\tau_{w,x}}{\frac{1}{2} \rho U_{inj}^2} = 2 \tau_{w,x}, \\
c_{px} &= \frac{p_{w,x}}{\frac{1}{2} \rho U_{inj}^2} = 2 p_{w,x}.
\end{align*}
\]

The subscript of \( x \) indicates the streamwise component. Here, the wall shear stress \( \tau_{w,x} \) and the wall pressure \( p_{w,x} \) are

\[
\begin{align*}
\tau_{w,x} &= \frac{1}{L_x} \int_0^{L_x} \left( \frac{1}{\text{Re}_{\text{inj}}} \frac{\partial(\bar{u} \cos \theta - \bar{v} \sin \theta)}{\partial n} \cos \theta \right) dz, \\
p_{w,x} &= \frac{1}{L_x} \int_0^{L_x} (\bar{p}_w - \bar{p}_{\text{ref}}) \cos \theta dz,
\end{align*}
\]

where \( \theta \) is the local angle between the wall slope and the horizontal plane and is defined in \( 0 < \theta < \pi \), and \( \bar{p}_{\text{ref}} \) is the reference pressure. In this study, the reference pressure is the wall pressure on the top of the wavy wall (i.e., \( \bar{p}_{\text{ref}} = \bar{p}_w \) at \( x/L = 0 \)), and \( n \) is a unit normal direction to the wavy wall surface. The bar denotes the averaged value in the time and the spanwise direction. For reader’s convenience, we remove the subscript \( x \) in the following. We note that the total coefficient is denoted by \( c_T = c_f + c_p \). The local Nusselt number is defined as

\[
\text{Nu} = \left. \frac{\partial T}{\partial n} \right|_{\text{wall}}.
\]

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Since the surface area of the wavy wall increases with decreasing wavelength, we integrated the wall values of $c_f$, $c_p$, and $\text{Nu}$ over the wavy wall surface and averaged by projected area on “the bottom wall” of the main part ($aL_c$), i.e.,

$$
\overline{c_f} = \frac{1}{\lambda} \int_{x-\lambda/2}^{x+1/2} c_f dx, \quad \overline{c_p} = \frac{1}{\lambda} \int_{x-\lambda/2}^{x+1/2} c_p dx, \quad \overline{\text{Nu}} = \frac{1}{\lambda} \int_{x-\lambda/2}^{x+1/2} \text{Nu} dx.
$$

The streamwise variation in $\overline{c_f}$, $\overline{c_p}$, and $\overline{\text{Nu}}$ for different wavelengths are shown in Figs. 5-6. The single marker is plotted at the center of one wave, which corresponds to the top of each wave. As shown in Fig. 5(a), $\overline{c_f}$ in the case of $\lambda/2a = 7.5$ is maximized at the first wave, minimized at the second wave, recovers, and then gradually decreases in the downstream direction. The maximum and minimum $\overline{c_f}$ are due to the flow impingement to the first wave and the flow separation behind the first wave. The skin-friction coefficient $\overline{c_f}$ is lower than that of the flat case, except the first wave. This trend is similar in other cases, except the case of $\lambda/2a = 45$. In the case of $\lambda/2a = 45$, there is no large skin-friction increase at the first wave and $\overline{c_f}$ almost agrees with that of the flat case. The variation in the pressure drag coefficient is shown in Fig. 5(b), which is similar to that in the skin-friction drag coefficient: $\overline{c_p}$ is maximized, minimized, recovers, and gradually decreases in the streamwise direction. In the case of $\lambda/2a = 45$, $\overline{c_p}$ almost agrees with that of the flat case.

Figure 6(a) shows the Nusselt number at $\text{Pr} = 0.71$. The Nusselt number increases in the range of $5 < x/\delta_{99,\text{in}} < 10$ and decreases gradually in the streamwise direction. With decreasing wavelength, the Nusselt number is found to increase. A similar trend is observed in the case of $\text{Pr} = 2.0$, as shown in Fig. 6(b).

To clarify the wavelength effect, the Nusselt number and drag coefficients are averaged in the range of $30 < x < 50$, and are shown in Fig. 7. For longer wavelength case, the skin-friction coefficient and Nusselt number decrease to those of the flat-surface case and the pressure drag decreases to zero. With decreasing wavelength, the skin-friction coefficient decreases and the pressure drag increases due to the flow separation. As discussed later, the flow separation occurs in the cases of $\lambda/2a < 15$. For shorter-wavelength cases of $\lambda/2a = 10$ and 7.5, the pressure drag is almost unchanged, whereas the friction drag decreases. The total drag, therefore, peaks at $\lambda/2a = 12.5$. On the other hand, the Nusselt number gradually increases with decreasing wavelength.

The decrease in wavelength with a constant wave amplitude shows a similar effect with an increase in wave amplitude with constant wavelength, while some points are different. Yoon et al. (2009) developed the DNS of the turbulent channel flow to clarify the effect of the wave amplitude with constant wavelength at the bulk Reynolds number of $\text{Re}_b = 6760$. With increasing wave amplitude, the total drag and pressure drag increase, and the skin-friction drag increases, peaks at $a/\lambda = 0.03$ (i.e., $\lambda/2a \approx 16.7$), and then decreases. The peak of the skin-friction drag coefficient is slightly larger than that of the flat surface, where $(a/\lambda = 0.03)$ induces the flow separation. In the present study, the skin-friction drag sustains

Fig. 5  Streamwise variation in (a) skin-friction and (b) pressure drag coefficients for different wavelength cases:

\( - \circ - \), $\lambda/2a = 7.5$; \( - \times - \), $\lambda/2a = 10$; \( - \square - \), $\lambda/2a = 12.5$; \( - \circ - \), $\lambda/2a = 15$; \( - \times - \), $\lambda/2a = 22.5$; \( - \square - \), $\lambda/2a = 45$; \( - \circ - \), flat surface case. The first wave starts at $x = 3.0$.

Fig. 6  Streamwise variation in the Nusselt number for different wavelength cases. The legends are the same as those for Fig. 5.

\( \overline{c_f} \), \( \overline{c_p} \), and \( \overline{\text{Nu}} \) are shown in Fig. 7.
Fig. 7 Skin-friction drag coefficient and Nusselt number averaged at $30 < x < 50$ as a function of wavelength.

Fig. 8 The $j/f$ factor as a function of wavelength: solid line, $Pr = 2.0$; broken line, $Pr = 0.71$.

below that of the flat surface regardless of the wavelength and no peak appears.

Figure 8 shows the $j/f$ factor, which is an indicator of the dissimilarity between the momentum and heat transfers: the heat transfer is larger than the momentum transfer if $j/f$ is larger than 1. Here, two different $j/f$ factors are defined as

$$j/f_t = \frac{Nu Pr^{-1/4}}{4c_T} , \quad j/f = \frac{Nu Pr^{-1/4}}{4c_f} . \quad (17)$$

The former and latter are for the total drag coefficient and skin-friction drag coefficient, respectively. These are normalized by that of the flat surface ($j/f)_0$ in Fig. 8. In the longer wavelength case, the $j/f$ factor for the skin-friction drag coefficient increases because of the increase in $Nu$ and the decrease in $c_f$. On the other hand, the $j/f$ factor for the total drag coefficient is smaller than that of the flat surface.

Figure 9 shows the streamwise component of the drag coefficient and the Nusselt number over one wavy wall. This profile is taken at $x = 40$. The horizontal axis is normalized by the wavelength and the black patch represents the shape of the wavy wall (not to be scaled). The obtained profile is similar to other numerical and experimental studies e.g., Buckles et al. (1984), de Angelis et al. (1997), Yoon et al. (2009).

Figure 9(a) shows the profiles of $c_f$. In the longest wavelength case, $c_f$ varies sinusoidally. In the cases of $\lambda/2a < 15$, we find a very small or negative $c_f$ due to the reversed flow in the downslope. In the upslope, $c_f$ increases due to the flow reattachment and the peak moves in the downstream direction with an increase in $\lambda$. Because the separation region covers the valley of the wavy wall, the negative $c_f$ region extends toward the upslope for small wavelength cases. Thorsness et al. (1978) analyzed a linear momentum equation for the flow over the wavy wall and categorized the flow pattern into three regimes. For long wavelength with small amplitude, wall shear stress distributes sinusoidally in the downstream direction, which is named as the “linear response regime.” With an increase in the amplitude or wavelength, the wall shear stress does not distribute sinusoidally and the regime is named as the “non-linear response regime.” The rest of the
regime is named as the “reversed flow regime,” where a reversed flow occurs. According to them, $\lambda/2a = 45$, and in the present simulation, is categorized into the linear regime; $\lambda/2a = 22.5$ is in the non-linear regime; and the other cases are in the reversed flow regime.

The local pressure coefficient $c_p$ is shown in Fig. 9(b). For the longest wavelength case, the positive and negative pressure drags are almost canceled out and the positive drag remains slightly. However, with decreasing wavelength, the positive $c_p$ is enhanced drastically due to the flow reattachment at the upslope. Therefore, the positive $c_p$ exceeds the negative $c_p$, which results in a large pressure drag. Mirzaei et al. (2013) pointed out that not only the mean pressure but also the pressure fluctuation ($p_{rms}$) is maximized around the reattachment point due to the impingement behavior of the separated flow. With decreasing wavelength, the impingement behavior is activated and results in an increase in the pressure drag on the upslope.

As displayed in Fig. 9(c), for longer wavelength cases, the Nusselt number is smaller and larger than that of the flat case in the downslope and upslope regions, respectively. With decreasing wavelength, the smaller Nusselt number region expands to the beginning of the upslope. On the other hand, the peak of the Nusselt number increases on the upslope where the flow reattachment occurs. For the case of $Pr = 2.0$, as shown in Fig. 9(d), the Nusselt number is larger than that of $Pr = 0.71$, while the profiles are similar. In the case of $\lambda/2a = 7.5$, the decrement in the Nusselt number at the minimum point from the flat level is about 70% for both the Prandtl cases, whereas the increment at the maximum location is about 150% and 190% in $Pr = 0.71$ and $Pr = 2.0$, respectively. As reported by Mirzaei et al. (2013), the proportion of the reduced Nusselt number around a separation point to the flat level is almost unchanged for different Prandtl number cases, whereas that of the increment around a reattachment point is enhanced for larger Prandtl number cases.

In summary, with decreasing wavelength, $C_f$ decreases, $C_p$ increases, and $Nu$ increases. The decrease in $C_f$ is due to the enhancement of the backward flow and the increase in $C_p$ is due to the impingement behavior around the reattachment point. The resultant total drag peaks at $\lambda/2a = 12.5$, which is interesting because the peak of the total drag is not observed if the amplitude of the wave is increased. For example, Yoon et al. (2009) performed a DNS of the turbulent flows in wavy wall channels. With an increase in the amplitude with a constant wavelength, the pressure drag increased, whereas the skin-friction drag peaked at $a/\lambda = 0.03$; however, the total drag increased until their maximum amplitude of $a/\lambda = 0.05$. On the other hand, Choi and Suzuki (2005) investigated the heat transfer in a wavy wall channel flow through large eddy
3.2. Flow field at $\lambda/2a = 12.5$

Figure 10 shows distributions of statistics and correlation functions in the region close to the wavy wall surface at $x = 40$, together with the mean velocity vector. The bar indicates the averaged value in the time and the spanwise direction. Figure 10(a-c) shows the mean velocities and temperature. Since the flow separation appears, negative $u$ and small positive $v$ are found in the region close to the valley of the wavy wall. The thermal boundary layer is thickened at the region of the flow separation and thinned after the flow reattachment.

The distributions of the Reynolds shear stress (RSS) and turbulent heat flux (THF) are displayed in Fig. 10(d-f). The maximum $-u'v'$ and $-u'T'$ are observed away from the wavy wall surface. These distributions are similar to those in e.g., Hudson et al. (1996) and Yoon et al. (2009). On the other hand, $-u'v'$ and $-u'T'$ are negative over the upslope due to a Cartesian coordinate system pointed out by Hudson et al. (1996). Accordingly, we project the velocity fluctuations on the local mean velocity vector as follows:

$$ u_t = \mathbf{u} \cdot \mathbf{a}, \quad u_n = \mathbf{u} \cdot \mathbf{b}. \tag{18} $$

Here, $\mathbf{u}$ is the vector of the local mean velocity, $\mathbf{a}$ is the unit vector of the local mean velocity vector, and $\mathbf{b}$ is the unit vector normal to $\mathbf{a}$. The subscripts of $t$ and $n$ show the velocity component of tangential and normal to the local mean velocity vector, respectively.

Figure 10(g) shows distribution of $-u'_t u'_n$. The distribution is similar to that of $-u'v'$ in the outer region, whereas negative $-u'_t u'_n$ is found in the separation region and close to the reattachment point. A similar trend is observed for $-u'_t T'$, as shown in Fig. 10(h). In addition, $u'_t T'$, as shown in Fig. 10(i), remains almost unchanged from $u' T'$.

To clarify the correlation between velocity and temperature fluctuations, the correlation coefficients are defined as

$$ R(-u'_t u'_n) = \frac{-u'_t u'_n}{u'_{rms} u'_{rms}}, \quad R(-u'_t T') = \frac{-u'_t T'}{u'_{rms} T'_{rms}}, \quad R(u'_t T') = \frac{u'_t T'}{u'_{rms} T'_{rms}}, \tag{19} $$

where the subscript of “rms” means the root-mean-square value. These correlation coefficients are displayed in Fig. 10 (j)-(l). In the separation region, the correlation coefficients of $R(-u'_t u'_n)$ and $R(-u'_t T')$ are very small, whereas the correlation

...
of $R(u'T')$ is negative at the bottom of the wavy wall.

The instantaneous wall-shear stress and Nusselt number distributions on the wavy wall surface are visualized as shown in Fig. 11. The vortical structures are identified by the isosurface of the second invariant of the velocity deformation tensor. In the separation region, the local wall shear stress is negative from the middle of the downslope to the beginning of the upslope. The Nusselt number is small, but larger than one. Because the reversed flow transports the heat, the correlation coefficient between streamwise velocity fluctuation and temperature is negative, as shown in Fig. 10(l). From the beginning of the upslope, the Nusselt number increases and “spindle-shaped” spots are found. As indicated by Choi and Suzuki (2005), the spindle-shaped spots are created by the vortical structure generated downstream the reattachment point.

4. Conclusion

Direct numerical simulations of thermal turbulent boundary layer flows over the wavy wall surface are performed. The wavy wall surface is homogeneous in the spanwise direction and the amplitude of the wave is fixed at $2\alpha = 20$. We examined six wavelength cases of $\lambda/2\alpha = 7.5 \sim 45$ in order to investigate the heat transfer and drag coefficient. The Reynolds number is based on the inlet boundary layer thickness at the inlet, the uniform velocity is set to 2820, and the Prandtl number is set to $Pr_e = 0.71$ and 2.0.

With decreasing wavelength, the skin-friction drag decreases and the pressure drag increases. The total drag peaks at $\lambda/2\alpha = 12.5$. The flow separation occurs at $\lambda/2\alpha < 15$ and the backward flow dominates in the separation region. On the other hand, the heat transfer is enhanced with decreasing wavelength.

The flow field in the largest drag case of $\lambda/2\alpha = 12.5$ is discussed. To avoid the negative RSS and THF over the upslope due to the Cartesian coordinate system, these statistics are projected onto the local mean velocity vector. In the separation region and the bottom of the wavy wall, the correlations of $R(u'u'_n)$ and $R(\bar{u}'T')$ are very small, while $R(u'T')$ is negative. The visualization of the wall-shear stress and Nusselt number on the wavy wall surface indicates that the reversed flow transports heat. Additionally, the spindle-shaped spots of the Nusselt number are also observed on the upslope of the wavy wall.

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