Calculation of the Momentum Dependence of Hadronic Current Correlation Functions at Finite Temperature

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Abstract

We have calculated spectral functions associated with hadronic current correlation functions for vector currents at finite temperature. We made use of a model with chiral symmetry, temperature-dependent coupling constants and temperature-dependent momentum cutoff parameters. Our model has two parameters which are used to fix the magnitude and position of the large peak seen in the spectral functions. In our earlier work, good fits were obtained for the spectral functions that were extracted from lattice data by means of the maximum entropy method (MEM). In the present work we extend our calculations and provide values for the three-momentum dependence of the vector correlation function at $T = 1.5 T_c$. These results are used to obtain the correlation function in coordinate space, which is usually parametrized in terms of a screening mass. Our results for the three-momentum dependence of the spectral functions are similar to those found in a recent lattice QCD calculation for charmonium [S. Datta, F. Karsch, P. Petreczky and I. Wetzorke, hep-lat/0312037]. For a limited range we find the exponential behavior in coordinate space that is usually obtained for the spectral function for $T > T_c$ and which allows for the definition of a screening mass.

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I. INTRODUCTION

In a number of recent works [1-3] we have calculated various hadronic correlation functions and compared our results to results obtained in lattice simulations of QCD [4-6]. The lattice results for the correlators, $G(\tau, T)$, may be used to obtain the corresponding spectral functions, $\sigma(\omega, T)$, by making use of the relation

$$G(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) K(\tau, \omega, T),$$

(1.1)

where

$$K(\tau, \omega, T) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}.$$

(1.2)

The procedure to obtain $\sigma(\omega, T)$ from the knowledge of $G(\tau, T)$ makes use of the maximum entropy method (MEM) [7-9], since $G(\tau, T)$ is only known at a limited number of points.

In our studies of meson spectra at $T = 0$ and at $T < T_c$ we have made use of the Nambu–Jona-Lasinio (NJL) model. The Lagrangian of the generalized NJL model we have used in our studies is

$$L = \bar{q}(i\gamma^\mu - m^0_q)q + \frac{G_S}{2} \sum_{i=0}^{8} [\sum_{i}^{8} \left( \bar{q} \lambda^i q \right)^2 + \left( \bar{q} \gamma_5 \lambda^i q \right)^2] - \frac{G_V}{2} \sum_{i=0}^{8} \left( \bar{q} \gamma_5 \gamma_\mu q \right)^2 + \frac{G_D}{2} \left\{ \det[\bar{q}(1 + \lambda_5)q] + \det[\bar{q}(1 - \lambda_5)q] \right\} + L_{conf}.$$

(1.3)

Here, $m^0_q$ is a current quark mass matrix, $m^0_q=\text{diag}(m^0_u, m^0_d, m^0_s)$. The $\lambda_i$ are the Gell-Mann (flavor) matrices and $\lambda^0 = \sqrt{2/3} \mathbf{1}$, with $\mathbf{1}$ being the unit matrix. The fourth term is the ’t Hooft interaction and $L_{conf}$ represents the model of confinement used in our studies of meson properties.

In the study of hadronic current correlators it is important to use a model which respects chiral symmetry, when $m^0_q = 0$. Therefore, we make use of the Lagrangian of Eq. (1.3), while neglecting the ’t Hooft interaction and $L_{conf}$. Thus, there are essentially three parameters to consider, $G_S$, $G_V$ and a cutoff parameter $k_{max}$, which restricts the momentum integrals so that $k < k_{max}$. When we use the NJL model to study matter at finite temperature, we
introduce the temperature-dependent parameters $G_s(T)$ and $G_V(T)$. (We have also used a Gaussian cutoff for the momentum integrals in our earlier work.) These parameters have been adjusted to obtain fits to the spectral functions $\sigma(\omega, T)$ for $T/T_c = 1.5$ and $T/T_c = 3.0$, which are the values of $T/T_c$ studied in the lattice simulations of QCD [10].

The temperature-dependent coupling constants and cutoff parameters of our work are analogous to the corresponding density-dependent parameters introduced in Ref. [11] and [12]. Further study of models with temperature-dependent and density-dependent parameters are of interest and a general theoretical formalism for the introduction of such dependencies should be considered.

In this work we limit our study to the data for the vector correlator at $T = 1.5 T_c$ [10, 13] and therefore only need to specify $G_V(T)$ and the momentum cutoff. (We remark that the results for the scalar, vector, pseudoscalar and axialvector correlators are quite similar in the deconfined region.) In Figs. 1 and 2 we show the data obtained by the MEM method at $T/T_c = 1.5$ and $T/T_c = 3.0$ for both pseudoscalar and vector correlators [10, 13]. The second peaks in these correlators are known to be a lattice artifacts [13].

The organization of our work is as follows. In Appendix A we present the formalism for calculation of pseudoscalar and vector correlation functions. In Appendix A we discuss the calculation of the correlator in the case the quark and antiquark carry zero total momentum. In Appendix B we show how the formalism is modified for the correlator calculated at finite momentum $\vec{P}$. In Section II we present the results of our calculation of the imaginary part of the correlator $\sigma(\omega, \vec{P})$. (Since we place $\vec{P}$ along the $z$-axis this quantity may be written as $\sigma(\omega, 0, 0, P_z)$ in accord with the notation of Ref. [10].) Our results for $\sigma(\omega, \vec{P})$ will be presented for a series of values of $|\vec{P}|$ in Section II. In Section III we present our result for the coordinate-dependent correlator $C(z)$ which is proportional to the correlator defined in Eq. (1) of Ref. [10],

$$C(z) = \frac{1}{2} \int_{-\infty}^{\infty} dP_z e^{iP_z z} \int_0^\infty d\omega \frac{\sigma(\omega, 0, 0, P_z)}{\omega}. \quad (1.4)$$

We may also use the form

$$C(z) = \frac{1}{4} \int_{-\infty}^{\infty} dP_z e^{iP_z z} \int_0^\infty dP^2 \frac{\sigma(P^2, 0, 0, P_z)}{P^2}. \quad (1.5)$$

Finally, in Section IV we present our conclusion and further discussion.
II. TEMPERATURE DEPENDENT HADRONIC CURRENT CORRELATORS AT FINITE MOMENTUM

We make use of the formalism presented in Appendices A and B to obtain values of the vector correlator at \( T = 1.5 T_c \). Here we take \( T_c = 270 \) MeV, since we have usually made comparison to lattice calculations made in the quenched approximation.

In Fig. 3 we present \( \sigma(\omega)/\omega^2 \), for various values of \( |\vec{P}| \), as function of \( \omega^2 \). Comparison may be made to the lattice data shown in Fig. 2 [10, 13]. (We again note that our calculation does not reproduce the second peak in the lattice data which is known to be a lattice artifact [13].) The curves shown in Fig. 3 are given for values of \( |\vec{P}| \) ranging from 0.10 GeV to 2.10 GeV in steps of 0.20 GeV. For these calculations we have used \( k_{\text{max}} = 1.22 \) GeV. Our results for the various values of \( |\vec{P}| \) given in Fig. 3 may be compared to Fig. 20 of Ref. [14]. We see that the results calculated by completely different methods are similar.
FIG. 2: The spectral functions $\sigma/\omega^2$ for vector states obtained by MEM are shown [10, 13]. See the caption of Fig. 1. The second peak is a lattice artifact [13].

FIG. 3: The imaginary part of the correlator $\sigma(\omega)/\omega^2$ is shown for various values of $|\vec{P}|$ as a function of $\omega^2$. Starting with the topmost curve the values of $|\vec{P}|$ in GeV units are 0.10, 0.30, 0.50, 0.70, 0.90, 1.10, 1.30, 1.50, 1.70, 1.90 and 2.10. Here we have used $G_S = 1.2$ GeV$^{-2}$ and $k_{max} = 1.22$ GeV.
FIG. 4: The correlation function $C(z)$ defined in Eq. (1.5) is shown. The dotted line represents a fit using an exponential function.

III. THE COORDINATE-SPACE CORRELATION FUNCTION

We have used the results of our calculations which were presented in Fig. 3 to calculate $C(z)$ of Eq. (1.5). The result of that calculation is shown in Fig. 4. We note that the simple assumption for the behavior of this correlator that is usually made, $C(z) \sim \exp[-m_{sc} z]$, is born out in our calculation for $1 \text{GeV}^{-1} < z < 3 \text{GeV}^{-1}$. For the study of charmonium on the lattice, values found for the screening mass are given in Ref. [14].

For the result shown in Fig. 4 we obtain a screening mass of 1.02 GeV which may be compared to the value of $\pi T=1.27 \text{ GeV}$. The dotted line represents an exponential fit to our result.

IV. DISCUSSION

Recent theoretical work concerning the quark-gluon plasma has been discussed by Shuryak [16]. He notes that the physics of excited matter produced in heavy-ion collisions in the region $T_c < T < 4T_c$ is different from that of a weakly coupled quark-gluon plasma. That is due to the strong coupling generated by the bound states of the quasipar-
articles. These bound states appear as resonant structures when the imaginary parts of the hadronic correlators are extracted from lattice data using the maximum entropy method (MEM) [4-9]. These resonances lead to very strong interactions between the quasiparticles and are, in part, responsible for very small mean free paths and collective flow in heavy-ion collisions. That flow may be described by hydrodynamics. Indeed, Shuryak suggests that “... if the system is macroscopically large, then its description via thermodynamics of its bulk properties (like matter composition) and hydrodynamics for space-time evolution should work” [10].

In our present work we have presented a simple chiral model that is able to reproduce the resonances extracted from the lattice data via the MEM procedure. We have calculated the imaginary parts of the vector correlator for various values of the total external momentum $\vec{P}$. We have also calculated the correlation function, $C(z)$, and find the simple exponential behavior $C(z) \sim \exp[-m_{sc}z]$ for a limited range of $z$. We have shown in an earlier work that exponential behavior with the appropriate screening mass may be obtained for the full range of $z$ values if quite small values of $k_{\text{max}}$ (of the order of 0.4 GeV) are used [20]. Finally, we note that an extensive discussion of screening masses appears in Ref. [21].

**APPENDIX A**

For ease of reference, we present a discussion of our calculation of hadronic current correlators taken from Ref. [3]. The procedure we adopt is based upon the real-time finite-temperature formalism, in which the imaginary part of the polarization function may be calculated. Then, the real part of the function is obtained using a dispersion relation. The result we need for this work has been already given in the work of Kobes and Semenoff [17]. (In Ref. [17] the quark momentum is $k$ and the antiquark momentum is $k - P$. We will adopt that notation in this section for ease of reference to the results presented in Ref. [17].) With reference to Eq. (5.4) of Ref. [17], we write the imaginary part of the scalar
polarization function as
\[
\text{Im } J_S(P^2, T) = \frac{1}{2} N_c \beta_S \epsilon(P^0) \int_{\kappa_{\text{max}}}^{k_{\text{max}}} \frac{d^3k}{(2\pi)^3} \left( \frac{2\pi}{2E_1(k)2E_2(k)} \right) \times \{ [1 - n_1(k) - n_2(k)] \delta(P^0 - E_1(k) - E_2(k)) \\
- [n_1(k) - n_2(k)] \delta(P^0 + E_1(k) - E_2(k)) \\
- [n_2(k) - n_1(k)] \delta(P^0 - E_1(k) + E_2(k)) \\
- [1 - n_1(k) - n_2(k)] \delta(P^0 + E_1(k) + E_2(k)) \}.
\]  

Here, \( E_1(k) = [\vec{k}^2 + m_1^2(T)]^{1/2} \). Relative to Eq. (5.4) of Ref. [17], we have changed the sign, removed a factor of \( g^2 \) and have included a statistical factor of \( N_c \). In addition, we have used a sharp cutoff, \( k_{\text{max}} \), for the momentum integral. We also note that
\[
n_1(k) = \frac{1}{e^{\beta E_1(k)} + 1}, \tag{A2}
\]
and
\[
n_2(k) = \frac{1}{e^{\beta E_2(k)} + 1}. \tag{A3}
\]
For the calculation of the imaginary part of the polarization function, we may put \( k^2 = m_1^2(T) \) and \( (k - \vec{P})^2 = m_2^2(T) \), since in that calculation the quark and antiquark are on-mass-shell. In Eq. (A1) the factor \( \beta_S \) arises from a trace involving Dirac matrices, such that
\[
\beta_S = -\text{Tr}[\bar{k} + m_1)(\bar{k} - \vec{P} + m_2)] \tag{A4}
\]
\[
= 2P^2 - 2(m_1 + m_2)^2, \tag{A5}
\]
where \( m_1 \) and \( m_2 \) depend upon temperature. In the frame where \( \vec{P} = 0 \), and in the case \( m_1 = m_2 \), we have \( \beta_S = 2P_0^2(1 - 4m^2/P_0^2) \). For the scalar case, with \( m_1 = m_2 \), we find
\[
\text{Im } J_S(P^2, T) = \frac{N_c P_0^2}{8\pi} \left( 1 - \frac{4m^2(T)}{P_0^2} \right)^{3/2} [1 - 2n_1(k)], \tag{A6}
\]
where
\[
\bar{k}^2 = \frac{P_0^2}{4} - m^2(T), \tag{A7}
\]
with \( k < k_{\text{max}} \).
For pseudoscalar mesons, we replace $\beta_S$ by

$$\beta_P = -\text{Tr}[i\gamma_5(\slashed{k} + m_1)i\gamma_5(\slashed{k} - \slashed{P} + m_2)] \quad (A8)$$

$$= 2P^2 - 2(m_1 - m_2)^2, \quad (A9)$$

which for $m_1 = m_2$ is $\beta_P = 2P_0^2$ in the frame where $\vec{P} = 0$. We find, for the $\pi$ mesons,

$$\text{Im} J_P(P^2, T) = \frac{N_c P_0^2}{8\pi} \left( 1 - \frac{4m^2(T)}{P_0^2} \right)^{1/2} \left[ 1 - 2n_1(k) \right], \quad (A10)$$

where $\vec{k}^2 = P_0^2/4 - m^2_u(T)$, as above, with $k < k_{max}$. Thus, we see that the phase space factor has an exponent of 1/2 corresponding to a $s$-wave amplitude. For the scalars, the exponent of the phase-space factor is 3/2, as seen in Eq. (A6).

For a study of vector mesons we consider

$$\beta_{V\mu\nu} = \text{Tr}[\gamma_\mu(\slashed{k} + m_1)\gamma_\nu(\slashed{k} - \slashed{P} + m_2)], \quad (A11)$$

and calculate

$$g^\mu\nu\beta_{V\mu\nu} = 4[2P^2 - m_1^2 - m_2^2 + 4m_1 m_2], \quad (A12)$$

which, in the equal-mass case, is equal to $4P_0^2 + 8m^2(T)$, when $m_1 = m_2$ and $\vec{P} = 0$. This result will be needed when we calculate the correlator of vector currents. Note that, for the elevated temperatures considered in this work, $m_u(T) = m_d(T)$ is quite small, so that $4P_0^2 + 8m_u^2(T)$ can be approximated by $4P_0^2$, when we consider the vector current correlation functions. In that case, we have

$$\text{Im} J_V(P^2, T) \simeq \frac{2}{3}\text{Im} J_P(P^2, T). \quad (A13)$$

At this point it is useful to define functions that are not for $k > k_{max}$:

$$\text{Im} \tilde{J}_P(P^2, T) = \frac{N_c P_0^2}{8\pi} \left( 1 - \frac{4m^2(T)}{P_0^2} \right)^{1/2} \left[ 1 - 2n_1(k) \right], \quad (A14)$$

and

$$\text{Im} \tilde{J}_V(P^2, T) = \frac{2N_c P_0^2}{3} \frac{8\pi}{8\pi} \left( 1 - \frac{4m^2(T)}{P_0^2} \right)^{1/2} \left[ 1 - 2n_1(k) \right], \quad (A15)$$

For the functions defined in Eq. (A14) and (A15) we need to use a twice-subtracted dispersion relation to obtain $\text{Re} \tilde{J}_P(P^2, T)$, or $\text{Re} \tilde{J}_V(P^2, T)$. For example,

$$\text{Re} \tilde{J}_P(P^2, T) = \text{Re} \tilde{J}_P(0, T) + \frac{P_0^2}{P_0^2} \left[ \text{Re} \tilde{J}_P(P_0^2, T) - \text{Re} \tilde{J}_P(0, T) \right]$$

$$+ \frac{P^2(P^2 - P_0^2)}{\pi} \int_4^{\Lambda^2} ds \frac{\text{Im} \tilde{J}_P(s, T)}{s(P^2 - s)(P_0^2 - s)}, \quad (A16)$$

where $\Lambda$ is the scale for the phase space factor. Both $\text{Re} \tilde{J}_P$ and $\text{Re} \tilde{J}_V$ are renormalized and the functions $\text{Im} \tilde{J}_P$ and $\text{Im} \tilde{J}_V$ are defined to be zero for $s > 4m^2(T)$.
where \( \tilde{\Lambda}^2 \) can be quite large, since the integral over the imaginary part of the polarization function is now convergent. We may introduce \( \tilde{J}_P(P^2, T) \) and \( \tilde{J}_V(P^2, T) \) as complex functions, since we now have both the real and imaginary parts of these functions. We note that the construction of either \( \text{Re} \, J_P(P^2, T) \), or \( \text{Re} \, J_V(P^2, T) \), by means of a dispersion relation does not require a subtraction. We use these functions to define the complex functions \( J_P(P^2, T) \) and \( J_V(P^2, T) \).

In order to make use of Eq. (A16), we need to specify \( \tilde{J}_P(0) \) and \( \tilde{J}_P(P^2_0) \). We found it useful to take \( P^2_0 = -1.0 \ \text{GeV}^2 \) and to put \( \tilde{J}_P(0) = J_P(0) \) and \( \tilde{J}_P(P^2_0) = J_P(P^2_0) \). The quantities \( \tilde{J}_V(0) \) and \( \tilde{J}_V(P^2_0) \) are determined in an analogous function. This procedure in which we fix the behavior of a function such as \( \text{Re} \, \tilde{J}_V(P^2) \) or \( \text{Re} \, \tilde{J}_V(P^2) \) is quite analogous to the procedure used in Ref. [18]. In that work we made use of dispersion relations to construct a continuous vector-isovector current correlation function which had the correct perturbative behavior for large \( P^2 \rightarrow -\infty \) and also described the low-energy resonance present in the correlator due to the excitation of the \( \rho \) meson. In Ref. [18] the NJL model was shown to provide a quite satisfactory description of the low-energy resonant behavior of the vector-isovector correlation function.

We now consider the calculation of temperature-dependent hadronic current correlation functions. The general form of the correlator is a transform of a time-ordered product of currents,

\[
iC(P^2, T) = \int d^4xe^{iP \cdot x} \ll T(j(x)j(0)) \gg,
\]

where the double bracket is a reminder that we are considering the finite temperature case.

For the study of pseudoscalar states, we may consider currents of the form \( j_{P,i}(x) = \tilde{q}(x)i\gamma_5\lambda^i q(x) \), where, in the case of the \( \pi \) mesons, \( i = 1, 2 \) and \( 3 \). For the study of scalar-isoscalar mesons, we introduce \( j_{S,i}(x) = \tilde{q}(x)\lambda^i q(x) \), where \( i = 0 \) for the flavor-singlet current and \( i = 8 \) for the flavor-octet current [19].

In the case of the pseudoscalar-isovector mesons, the correlator may be expressed in terms of the basic vacuum polarization function of the NJL model, \( J_P(P^2, T) \). Thus,

\[
C_P(P^2, T) = J_P(P^2, T) \frac{1}{1 - G_P(T)J_P(P^2, T)},
\]

where \( G_P(T) \) is the coupling constant appropriate for our study of \( \pi \) mesons. We have found \( G_P(T) = 13.49 \ \text{GeV}^{-2} \) by fitting the pion mass in a calculation made at \( T = 0 \), with
\( m_u = m_d = 0.364 \text{ GeV} \). The result given in Eq. (A18) is only expected to be useful for small \( P^2 \), since the Gaussian regulator strongly modifies the large \( P^2 \) behavior. Therefore, we suggest that the following form is useful, if we are to consider the larger values of \( P^2 \).

\[
\frac{C_P(P^2, T)}{P^2} = \left[ \frac{\tilde{J}_P(P^2, T)}{P^2} \right] \frac{1}{1 - G_P(T)J_P(P^2, T)}. \tag{A19}
\]

(As usual, we put \( \vec{P} = 0 \).) This form has two important features. At large \( P^2 \), \( \text{Im} C_P(P_0, T)/P_0^2 \) is a constant, since \( \text{Im} \tilde{J}_P(P_0^2, T) \) is proportional to \( P_0^2 \). Further, the denominator of Eq. (A19) goes to 1 for large \( P_0^2 \). On the other hand, at small \( P_0^2 \), the denominator is capable of describing resonant enhancement of the correlation function. As we have seen, the results obtained when Eq. (A19) is used appear quite satisfactory. (We may again refer to Ref. \[18\], in which a similar approximation is described.)

For a study of the vector-isovector correlators, we introduce conserved vector currents \( j_{\mu,i}(x) = \bar{q}(x)\gamma_\mu \lambda_i q(x) \) with \( i=1, 2 \) and 3. In this case we define

\[
J_{\mu\nu}^V(P^2, T) = \left( g_{\mu\nu} - \frac{P\mu P\nu}{P^2} \right) J_V(P^2, T) \tag{A20}
\]

and

\[
C_{\mu\nu}^V(P^2, T) = \left( g_{\mu\nu} - \frac{P\mu P\nu}{P^2} \right) C_V(P^2, T), \tag{A21}
\]

taking into account the fact that the current \( j_{\mu,i}(x) \) is conserved. We may then use the fact that

\[
J_V(P^2, T) = \frac{1}{3} g_{\mu\nu} J_{\mu\nu}^V(P^2, T) \tag{A22}
\]

and

\[
\text{Im} J_V(P^2, T) = \frac{2}{3} \left[ \frac{P_0^2 + 2m_u^2(T)}{8\pi} \right] \left( 1 - \frac{4m_u^2(T)}{P_0^2} \right)^{1/2} [1 - 2n_1(k)] \tag{A23}
\]

\[
\approx \frac{2}{3} \text{Im} J_P(P^2, T). \tag{A24}
\]

(See Eq. (A7) for the specification of \( k = \vert \vec{k} \vert \).) We then have

\[
C_V(P^2, T) = \tilde{J}_V(P^2, T) \frac{1}{1 - G_V(T)J_V(P^2, T)}, \tag{A25}
\]

where we have introduced

\[
\text{Im} \tilde{J}_V(P^2, T) = \frac{2}{3} \left[ \frac{P_0^2 + 2m_u^2(T)}{8\pi} \right] \left( 1 - \frac{4m_u^2(T)}{P_0^2} \right)^{1/2} [1 - 2n_1(k)] \tag{A26}
\]

\[
\approx \frac{2}{3} \text{Im} J_P(P^2, T). \tag{A27}
\]
In the literature, $\omega$ is used instead of $P_0$ [4-6]. We may define the spectral functions

\[
\sigma_V(\omega, T) = \frac{1}{\pi} \text{Im} C_V(\omega, T), \tag{A28}
\]

and

\[
\sigma_P(\omega, T) = \frac{1}{\pi} \text{Im} C_P(\omega, T), \tag{A29}
\]

Since different conventions are used in the literature [4-6], we may use the notation $\sigma_P(\omega, T)$ and $\sigma_V(\omega, T)$ for the spectral functions given there. We have the following relations:

\[
\sigma_P(\omega, T) = \sigma_P(\omega, T), \tag{A30}
\]

and

\[
\frac{\bar{\sigma}_V(\omega, T)}{2} = \frac{3}{4} \sigma_V(\omega, T), \tag{A31}
\]

where the factor $3/4$ arises because, in Refs. [4-6], there is a division by 4, while we have divided by 3, as in Eq. (A22).

**APPENDIX B**

Here we extend the work of Appendix A to consider case of finite three-momentum, $\vec{P}$. We consider the calculation of $\text{Im} J_P(P^0, \vec{P}, T)$. The momenta $P^0$ and $\vec{P}$ are the values external to the loop diagram. Internal to the diagram, we have a quark of momentum $k + P/2$ leaving the left-hand vertex and an antiquark of momentum $k - P/2$ entering the left-hand vertex. It is useful to define

\[
E_1(k) = \left| \vec{k} + \vec{P}/2 \right| \tag{B1}
\]

\[
= \left( k^2 + \frac{P^2}{4} + kP \cos \theta \right)^{1/2} \tag{B2}
\]

and

\[
E_2(k) = \left| \vec{k} - \vec{P}/2 \right| \tag{B3}
\]

\[
= \left( k^2 + \frac{P^2}{4} - kP \cos \theta \right)^{1/2}. \tag{B4}
\]

Here $k = |\vec{k}|$ and $P = |\vec{P}|$. 

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We have
\[
\text{Im } J_V(P_0, \vec{P}, T) = \frac{1}{2} N_c \beta V \epsilon(P_0) \int_{k_{\text{max}}}^{k_{\text{max}}} \frac{d^3k}{(2\pi)^3} \left( \frac{2\pi}{2E_1(k)2E_2(k)} \right) (B5)
\]
\[
\times \{[1 - n_1(k) - n_2(k)]\delta(P_0 - E_1(k) - E_2(k))
- [n_1(k) - n_2(k)]\delta(P_0 + E_1(k) - E_2(k))
- [n_2(k) - n_1(k)]\delta(P_0 - E_1(k) + E_2(k))
- [1 - n_1(k) - n_2(k)]\delta(P_0 + E_1(k) + E_2(k))\}.
\]

Here,
\[
n_1(k) = \frac{1}{e^{\beta E_1(k)} + 1}, \quad (B6)
\]
and
\[
n_2(k) = \frac{1}{e^{\beta E_2(k)} + 1}. \quad (B7)
\]

In Eq. (B5), the second and third terms cancel and the fourth term does not contribute. It is useful to rewrite \(\delta(P_0 - E_1(k) - E_2(k))\) using
\[
\delta[f(\cos \theta)] = \frac{2}{|\partial f / \partial \cos \theta|_x} \delta(\cos \theta - x), \quad (B8)
\]
where
\[
x^2 = \cos^2 \theta
= \frac{4P_0^2(k^2 + P^2/4) - P_0^4}{4k^2P^2}. \quad (B9)
\]

We find
\[
\left| \frac{\partial f}{\partial \cos \theta} \right| = \frac{1}{2} kP \left| \frac{E_1(k) - E_2(k)}{E_1(k)E_2(k)} \right|, \quad (B10)
\]
and obtain
\[
\text{Im } J_P(P_0, \vec{P}, T) = \frac{1}{2} N_c \beta P \epsilon(P_0)(2\pi)^2 \int_{k_{\text{max}}}^{k_{\text{max}}} \frac{k^2dk}{(2\pi)^3} (B11)
\]
\[
\int \frac{1}{2E_1(k)E_2(k)}[1 - n_1(k) - n_2(k)] \left| \frac{\partial f(\cos \theta)}{\partial \cos \theta} \right|
\times \delta(\cos \theta - x)d(\cos \theta).
\]

14
We note there is a singularity when $E_1(k) = E_2(k)$. That occurs when $\cos \theta = 0$ or $\theta = \pi/2$. For our calculations we eliminate the point with $\theta = \pi/2$ when evaluating the angular integral over $d(\cos \theta)\delta(\cos \theta - x)$ in the last expression. We obtain

$$\text{Im} J_P(P^0, \vec{P}, T) = N_c \beta_P \epsilon(P^0) \frac{4\pi^2}{(2\pi)^3} \int_{k_{\text{max}}}^{k_{\text{max}}} k^2 dk \times \left| \frac{1 - n_1(k) - n_2(k)}{k P |E_1(k) - E_2(k)|} \right|_x,$$

where $x$ is obtained from Eq. (B9),

$$x = \frac{P^0}{k P} \left[ k^2 + \frac{P^2}{4} - \frac{P^2_0}{4} \right]^{1/2}$$

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