Trajectory Tracking Error Using PID Control Law for Two-Link Robot Manipulator via Adaptive Neural Networks

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Abstract

This paper presents the application of adaptive neural networks to robot manipulator control. The main methodologies, on which the approach is based, are recurrent neural networks and Lyapunov functions methodology and Proportional-Integral-Derivative (PID) control for nonlinear systems. The proposed controller structure is composed of a neural identifier and a control law defined by using the PID approach. The proposed new control scheme is applied via simulations to control a robot manipulator two-link model. Experimental results in two degrees of freedom of the robot arm shown the usefulness of the proposed approach. To verify the analytical results, an example of dynamical network is simulated and a theorem is proposed to ensure the tracking of the nonlinear system.

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1. Introduction

Artificial neural networks, computational models of the brain, are widely used on engineering applications due to their ability to estimate the relation between inputs and outputs from a learning process. Motivated by the seminal paper [1], there exists a continuously increasing interest in applying neural networks to identification and control of nonlinear systems. Most of these applications use feedforward structures [2], [3].

Recently, recurrent neural networks are being developed; as an extension of the static neural networks capability to approximate nonlinear functions, recurrent neural networks can approximate nonlinear systems. They allow more efficient modeling of the underlying dynamical systems [4]. Three representative books [5], [6] and [7] have reviewed the application of recurrent neural networks for nonlinear systems identification and control. In particular,
[5] uses off-line learning, while [6] analyzes adaptive identification and control by mean of on-line learning, where stability of the closed-loop system is established based on the Lyapunov function method. In [6], the trajectory tracking problem is reduced to a linear model following problem, with application to DC electric motors. In [7], analysis of Recurrent Neural Networks for identification, estimation and control are developed, with applications on chaos control, robotics and chemical processes. Control methods that are applicable to general nonlinear systems have been intensely developed since the early 1980’s. Main approaches include, for example, the use of differential geometry theory [8]. Recently, the passivity approach has generated increasing interest for synthesizing control laws [9]. An important problem for these approaches is how to achieve robust nonlinear control in the presence of unmodelled dynamics and external disturbances. In this direction, there exists the so-called $H_{\infty}$ nonlinear control approach [10]. One major difficulty with this approach, alongside it’s possible system structural instability, seems to be the requirement of solving some resulting partial differential equations. In order to alleviate this computational problem, the so-called inverse optimal control technique was recently developed, based on the input-to-state stability concept [11]. On the basis of the inverse optimal control approach, a control law for generating chaos in a recurrent neural network was designed in [12]. In [13] and [14], this methodology was modified for stabilization and trajectory tracking of an unknown chaotic dynamical system. The proposed adaptive control scheme is composed of a recurrent neural identifier and a controller, see fig. (1).

Where the former is used to build an on-line model for the unknown plant and the latter, to ensure that the unknown plant tracks the reference trajectory. In this paper, we further improve the design by adequately it to systems with less inputs than states. The approach is based on the methodology developed in [13] and [14], in which the control law is optimal with respect to a well-defined Lyapunov function.

Robot manipulators present a practical challenge for control purposes due to the nonlinear and multivariable nature of their dynamical behavior. Motion control in joint space is the most fundamental task in robot control; it has motivated extensive research work in synthesizing different control methods such as fuzzy computed torque control [15], PI+PD fuzzy control [16] and static neural network control [17].

2. Modelling of the plant

The nonlinear plant unknown, it is given as:

$$\dot{x}_p = F_p(x_p, u) \triangleq f_p(x_p) + g_p(x_p)u$$  \hspace{1cm} (Eq. 1)

Where $x_p, f_p \in \mathbb{R}^n, u \in \mathbb{R}^m$ where $g_p \in \mathbb{R}^{n \times m}$. Both $f_p$ and $g_p$ are unknown, and we propose to model (1) by the neural network state space representation, \( x' = A(x) + W^*\Gamma(x) + \Omega u \) plus one more term modelling error.
We define the modelling error between the neural network and the plant by:

\[ w_{per} = x - x_p \]  
(Eq. 2)

Which assumes fulfills the following hypotheses.
Hypotheses 1. (Objective of Modelling): Modelling error is exponentially stable, this is:

\[ \dot{w}_{per} = -k w_{per} \]  
(Eq. 3)

In this work we consider \( k = 1 \), and now, from (2) we have \( w_{per} = \hat{x} - \hat{x}_p \) where: \( \hat{x}_p = \hat{x} + w_{per} \)
The unknown plant, it can be modelling as:

\[ \dot{x}_p = \hat{x} + w_{per} = A(x) + W^* \Gamma_z(x) + w_{per} + \Omega u \]  
(Eq. 4)

\( W^* \) are the fixed weights but unknown from the neural network, which minimize the modelling error.

3. Trajectory Tracking

We proceed now to analyze the modelling error between the unknown plant, modelling by (4) and the reference signal defined by

\[ \dot{x}_r = f_r(x_r, u_r), \text{ with } u_r \text{ and } x_r \in \mathbb{R}^n \]  
(Eq. 5)

where \( x_r \) are the states, \( u_r \) the input and \( f_r \) is a nonlinear function.

For this proposal, we define the modelling error between the plant and the reference signal by:

\[ e = x_p - x_r \]  
(Eq. 6)

Whose derivative in the time is

\[ \dot{e} = \dot{x}_p - \dot{x}_r = A(x) + W^* \Gamma_z(x) + w_{per} + \Omega u - f_r(x_r, u_r) \]  
(Eq. 7)

Adding and subtracting, to the right hand side from (7) the terms \( \hat{W} \Gamma_z(x_r), \alpha_r(t, \hat{W}), A\epsilon \), and taking into account that \( w_{per} = x - x_r \), we have

\[ \begin{align*}
\dot{e} &= A(x) + W^* \Gamma_z(x) + x - x_p + \Omega u - f_r(x_r, u_r) + \hat{W} \Gamma_z(x_r) - \hat{W} \Gamma_z(x_r) + \hat{\Omega}_r(t, \hat{W}) - \hat{\Omega}_r(t, \hat{W}) + A\epsilon - A\epsilon \\
\dot{e} &= A\epsilon + W^* \Gamma_z(x) + \Omega u - f_r(x_r, u_r) + \hat{W} \Gamma_z(x_r) + \hat{\Omega}_r(t, \hat{W}) - \hat{W} \Gamma_z(x_r) - \hat{\Omega}_r(t, \hat{W}) \\
&- Ax_r - x_r + x + A(x)
\end{align*} \]  
(Eq. 8)

In this part we consider the next similar supposition to the proposal. The neural network will follow the reference signal, and yet with the presence of disturbances, if:

\[ Ax_r + \hat{W} \Gamma_z(x_r) + x_r - x_p + \Omega \alpha_r(t, \hat{W}) = f_r(x_r, u_r), \text{ where} \]

\[ \Omega \alpha_r(t, \hat{W}) = f_r(x_r, u_r) - Ax_r - \hat{W} \Gamma_z(x_r) - x_r + x_p \]  
(Eq. 9)
\[
\dot{e} = Ae + W^* \Gamma_z(x) - \hat{W} \Gamma_z(x_r) - Ae + (A + I)(x - x_r) + \Omega(u - \alpha_r(t, \hat{W}))
\]  
(Eq. 10)

With \(\hat{W}\) is the first approach from \(W^*\). Now, adding and substracting in (10) the term \(\hat{W} \Gamma_z(x)\) and defining \(\Gamma_z(x) = \Gamma(z(x) - z(x_r))\)

\[
\dot{e} = Ae + (W^* - \hat{W}) \Gamma_z(x) + \hat{W} \Gamma(z(x) - z(x_r)) + (A + I)(x - x_r) - Ae + \Omega(u - \alpha_r(t, \hat{W}))
\]  
(Eq. 11)

We define

\[
\hat{W} = W^* - \hat{W} \quad \text{and} \quad \tilde{u} = u - \alpha_r(t, \hat{W})
\]  
(Eq. 12)

and we replace (12) in (11), we obtain:

\[
\dot{e} = Ae + \hat{W} \Gamma_z(x) + \hat{W} \Gamma(z(x) - z(x_r)) + (A + I)(x - x_r) - Ae + \Omega \tilde{u}
\]  
(Eq. 13)

Now:

\[
\tilde{u} = u_1 + u_2
\]  
(Eq. 14)

So, we define:

\[
\Omega u_1 = - \hat{W} \Gamma(z(x) - z(x_p)) - (A + I)(x - x_p)
\]  
(Eq. 15)

and (13) it is reduced to:

\[
\dot{e} = Ae + \hat{W} \Gamma_z(x) + \hat{W} \Gamma(z(x_p) - z(x_r)) + (A + I)(x_p - x_r) - Ae + \Omega u_2
\]  

Considering that \(e = x_p - x_r\), the last equation, it is possible to be written as:

\[
\dot{e} = (A + I)e + \hat{W} \Gamma_z(x) + \hat{W} \Gamma(z(x) + x_r) - z(x_r)) + \Omega u_2 = (A + I)e + \hat{W} \sigma(x) + \hat{W} (\sigma(e) + x_r) - \sigma(x_r) + \Omega u_2
\]

If \(\phi(e) = \sigma(e + x_r) - \sigma(x_r)\), and we have

\[
\dot{e} = (A + I)e + \hat{W} \sigma(x) + \hat{W} \phi(e) + \Omega u_2
\]  
(Eq. 16)

Now, the problem is to find the control law \(\Omega u_2\), which it stabilizes to the system (16). The control law, we will obtain using the Lyapunov methodology.

4. Stability of the tracking Error

Once (16) is obtained, we consider it is stabilization in feedforward. We note \((e, \hat{W}) = 0\), is an asymptotically stable equilibrium point of the undisturbed autonomous system \(\{A = -\lambda I \text{ and } \lambda > 0\}\). For its stability, we propose the next PID control law:

\[
\Omega u_2 = K_p e + K_i \dot{e} + K_i \int_0^t e(\tau)d\tau - \Upsilon(\frac{1}{2} + \frac{1}{2} || \hat{W} ||^2 L_2^2) e
\]  
(Eq. 17)

The parameters \(K_p, K_i\), and \(K_i\) will be determined later, and \(L_2^2\) is the Lipschitz constant of \(\phi_z\), with \(\Upsilon > 0\), this control law (17) is similar to [18]. We will show, the feedback system is asymptotically stable. Replacing (17) in (16) we have

\[
\dot{e} = (A + I)e + \hat{W} \sigma(x) + \hat{W} \phi(e) + K_p e + K_i \int_0^t e(\tau)d\tau - \Upsilon(\frac{1}{2} + \frac{1}{2} || \hat{W} ||^2 L_2^2) e
\]

then

\[
(1 - K_v)\dot{e} = (A + I)e + \hat{W} \sigma(x) + \hat{W} \phi(e) + K_p e + K_i \int_0^t e(\tau)d\tau - \Upsilon(\frac{1}{2} + \frac{1}{2} || \hat{W} ||^2 L_2^2) e. \text{If } a = (1 - K_v),
\]

then:

\[
\dot{e} = \frac{1}{a} (A + I)e + \frac{1}{a} \hat{W} \sigma(x) + \frac{1}{a} \hat{W} \phi(e) + \frac{1}{a} K_p e + \frac{1}{a} K_i \int_0^t e(\tau) d\tau - \Upsilon(\frac{1}{2} + \frac{1}{2} || \hat{W} ||^2 L_2^2) e
\]  
(Eq. 18)
\[ \dot{e} = -\frac{1}{a} (\lambda - 1 - K_p) e + \frac{1}{a} \tilde{W} \sigma(x) + \frac{1}{a} \tilde{W} \phi(e) + \frac{1}{a} K_i \int_0^t e(\tau) d\tau - \frac{\gamma}{a} \left( \frac{1}{2} + \frac{1}{2} \| \tilde{W} \|^2 L^2_0 e \right) \]  \tag{Eq. 19}

and if \( w = \frac{1}{a} K_i \int_0^t e(\tau) d\tau \), then \( \dot{w} = \frac{1}{a} K_i e(\tau) \), then (19) we rewrite as:

\[ \dot{e} = -\frac{1}{a} (\lambda - 1 - K_p) e + \frac{1}{a} \tilde{W} \sigma(x) + \frac{1}{a} \tilde{W} \phi(e) + w - \frac{\gamma}{a} \left( \frac{1}{2} + \frac{1}{2} \| \tilde{W} \|^2 L^2_0 e \right) \]  \tag{Eq. 20}

We will show, the new state \((e, w)^T\) is asymptotically stable, and the equilibrium point is \((e, w)^T = (0, 0)^T\), when \( \tilde{W} \sigma(x_r) = 0 \), as an external disturbance.

Let \( V \) be, the next candidate Lyapunov function as:

\[ V = \frac{1}{2} (e^T, w^T)(e, w)^T + \frac{1}{2a} tr \left\{ \tilde{W} \tilde{W} \right\} \]  \tag{Eq. 21}

The time derivative of (21) along the trajectories of (20) is:

\[ \dot{V} = (e^T, w^T)(\dot{e}, \dot{w})^T + \frac{1}{a} tr \left\{ \tilde{W} \dot{\tilde{W}} \right\} = e^T \dot{e} + w^T \dot{w} + \frac{1}{a} tr \left\{ \tilde{W} \dot{\tilde{W}} \right\} \]  \tag{Eq. 22}

\[ \dot{V} = e^T (\frac{-1}{a} (\lambda - 1 - K_p) e + \frac{1}{a} \tilde{W} \sigma(x) + \frac{1}{a} \tilde{W} \phi(e) + w - \frac{\gamma}{a} \left( \frac{1}{2} + \frac{1}{2} \| \tilde{W} \|^2 L^2_0 e \right)) + \frac{1}{a} w^T K_i e + \frac{1}{a} tr \left\{ \tilde{W} \dot{\tilde{W}} \right\} \]  \tag{Eq. 23}

In this part, we select the next learning law from the neural network weights as in [6] and [19]:

\[ tr \left\{ \dot{\tilde{W}} \tilde{W} \right\} = -e^T \tilde{W} \sigma(x) \]  \tag{Eq. 24}

Then (23) is reduced to

\[ \dot{V} = \frac{-1}{a} (\lambda - 1 - K_p) e^T e + e^T \phi(e) + (1 + \frac{K_i}{a}) e^T w - \frac{\gamma}{a} \left( \frac{1}{2} + \frac{1}{2} \| \tilde{W} \|^2 L^2_0 e \right) e^T e \]  \tag{Eq. 25}

We apply the next inequality to the second term right hand side from (25) then:

\[ x^T y \leq \frac{1}{2} x^T x + \frac{1}{2} y^T y \]  \tag{Eq. 26}

\[ \dot{V} \leq \frac{-1}{a} (\lambda - 1 - K_p) e^T e + \frac{1}{a} (\frac{e^T e}{2} + \frac{1}{2} \| \tilde{W} \|^2 L^2_0) e^T e + (1 + \frac{K_i}{a}) e^T w - \frac{\gamma}{a} \left( \frac{1}{2} + \frac{1}{2} \| \tilde{W} \|^2 L^2_0 e \right) e^T e \]  \tag{Eq. 27}

Here, we select \((1 + \frac{K_i}{a}) = 0\) and \( K_V = K_i + 1 \), with \( K_V \geq 0 \) then \( K_i > -1 \), with this selection of the parameters from (27) is reduced to:

\[ \dot{V} \leq \frac{-1}{a} (\lambda - 1 - K_p) e^T e - \frac{1}{a} (\gamma - 1) \left( \frac{1}{2} + \frac{1}{2} \| \tilde{W} \|^2 L^2_0 e \right) e^T e \]  \tag{Eq. 28}

In this part, if \( \lambda - 1 - K_p > 0 \), \( a > 0 \) and \( \gamma - 1 > 0 \), then \( \dot{V} < 0 \), \( \forall e, w, \tilde{W} \neq 0 \), the error tracking is asymptotically stable and it converges to cero for everything \( e \neq 0 \), this mean, the plant follow asymptotically to the reference. Finally, the control law, which affects the plant and the neural network, is given by:
This control law gives asymptotic stability of error dynamics and thus ensures the tracking to the reference signal. To this end, the results obtained can summarized as follows.

**Theorem** For the unknown nonlinear system modeled by (4), the on-line learning law (24) and the control law (29) together ensure the tracking to the nonlinear reference model (7)

**Remark 2** From (28) we have
\[
\dot{V} \leq -\frac{1}{a} (\lambda - 1 - K_p) e^T e - \frac{1}{a} (C - 1) (\frac{1}{2} + \frac{1}{2} \| \hat{W} \|^2 L_{\alpha}^2) e^T e < 0 \quad \forall e \neq 0, \forall \hat{W}
\]
where \( V \) is decreasing and bounded from below by \( V(0) \), and then: \( V = \frac{1}{2} (e^T, w^T)(e, w)^T + \frac{1}{2a} tr \{ \hat{W}^T \hat{W} \} \), then we conclude that \( e, \hat{W} \in L_1 \); this means that the weights remain bounded.

## 5. Simulation

The manipulator used for simulation is a two revolute joined robot (planar elbow manipulator), as show in Fig. 2. The meaning of the symbols and numerical values have been taken from [20].

The entries of the dynamics of this two degree-of-freedom robotic manipulator are given by the elements \( M_{ij}(q)(i, j = 1, 2) \) of the inertia matrix \( M(q) \), as in [21].

\[
M_{11}(q) = m_1 l_1^2 + m_2 (l_1^2 + l_2^2 + 2l_1 l_2 \cos(q_2)) + I_1 + I_2; \quad M_{12}(q) = m_2 (l_1 l_2 \cos(q_2)) + I_1
\]

\[
M_{21}(q) = m_2 (l_1^2 + l_1 l_2 \cos(q_2)) + I_2; \quad M_{22}(q) = m_2 l_2^2 + I_2
\]

The elements \( C_{ij}(q, \dot{q})(i, j = 1, 2) \) of the centrifugal and coriolis matrix \( C(q, \dot{q}) \):

\[
C_{11}(q, \dot{q}) = -m_2 l_1 l_2 \sin(q_2) \dot{q}_2; \quad C_{21}(q, \dot{q}) = -m_2 l_1 l_2 \sin(q_2)(\dot{q}_1 + \dot{q}_2)
\]

And the elements of gravitational torque vector \( g(q) \):

\[
g_1(q) = (m_1 l_1 + m_2 l_1) g \sin(q_1) + m_2 l_2 g \sin(q_1 + q_2); \quad g_2(q) = m_2 l_2 g \sin(q_1 + q_2)
\]

With the end of supporting the effectiveness of the proposed controller we have used a Duffing equation:

![Fig. 2 Diagram of the prototype planar robot with 2 degrees of freedom](image)
(24), $\sigma(x) = (\tanh(x_1), \tanh(x_2), \ldots, \tanh(x_n))^T$, $\Omega = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ and the $n$ is calculated using (29). The plant is stated in [18], [22], and it is given by:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau$$  \hspace{1cm} (Eq. 30)

The time evolution for the position angles and applied torque are shown in Figs. 4-7. As can be seen in Figs. 4 and 5, the trajectory tracking is successfully obtained where plant and reference signals are the same.

We try to force this manipulator to track a reference signal given by undamped Duffing equation:

$$\ddot{x} - x + x^3 = 0.114 \cos(1.1t); x(0) = 1, \dot{x}(0) = 0.114$$  \hspace{1cm} (Eq. 31)

and its phase space trajectory, it is given in Fig. 3.

We can see that the Recurrent Neural Controller ensures rapid convergence of the system outputs to the reference trajectory. The controller is robust in presence of disturbances applied to the system. another important issue of this approach related to other neural controllers, is that most neural controllers are based on indirect control, first the neural network identifies the unknown system and when the identification error is small enough, the control is applied. In our approach, direct control is considered, the learning laws for the neural networks depend explicitly of the tracking error instead of the identification error. This approach results in faster response of the system.

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6. Conclusions

We have extended the adaptive recurrent neural control previously developed in [13], [14] and [23] for trajectory tracking control problem in order to consider less inputs than states. Stability of the tracking error is analyzed via Lyapunov control functions and the control law is obtained based on the PID approach. A robot model with friction terms and unknown external disturbances is used to verify the design for trajectory tracking, with satisfactory performance. Research along this line will continue to implement the control algorithm in real time and to further test it in a laboratory environment as well as consider the two-link manipulator with an unknown load see example 4.9-pp 151-154, Two-Link Manipulator[24].

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