Analysis and Visualization of the Instantaneous Spatial Energy Density and Poynting Vector of the Wireless Power Transfer System

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Abstract: This study analyzes the instantaneous spatial energy density and Poynting vector in the WPT system and presents time-varying distributions and animations of this energy density and Poynting vector. First, the energy density is decoupled by two self-energy densities of each coil and the mutual energy density of the two coils. Result reveals how the energy is stored in the WPT system. Second, the Poynting vector is analyzed, and it is found that the power is transferred only in the last half period of the Poynting vector, not at every moment of the whole period. This instantaneous Poynting vector also possesses a characteristic that shows no power flow on the condition that the current phase difference equals zero. This finding is different from the energy density and indicates that the instantaneous Poynting vector can perfectly interpret how power is transferred in the WPT system. Finally, a simulation and an experiment were conducted to verify the correctness of the analysis. This study contributes to a deeper and better understanding of the intrinsic characteristics of energy storage and power flow in the WPT system, and can be referred to for WPT system design and optimization when one considers the EMC or human electromagnetic field exposure problem.

Keywords: wireless power transfer; electromagnetic energy density; Poynting vector; animation

1. Introduction

A wireless power transfer (WPT) system is a prospective power transfer technology in electrical areas [1–4]. The basic model of a typical two-coil WPT system is a mutual inductance circuit. The key factor to determine whether the power flows from one coil to another is the current phase difference of the two currents carried in the transmit coil and receive coil [5]. If the current phase difference is not equal to zero, then an energy flow occurs. Although the circuit theory has perfectly interpreted the principle of WPT and provides an efficient approach to design or optimize a WPT system, some problems still need to be solved or to be explained more deeply.

One problem is to explain how power is transferred from the transmit coil to the receive coil and how the energy is stored in the coil system at every moment and at every point in space. For the far-field WPT system, if one obtains the power flow characteristic, the power-efficient or transfer path of the power flow can be designed and optimized to decrease the exposure hazard of the far-field WPT. Refs. [6,7] presents a deeper understanding of the power flow phenomena and presents power-efficient algorithms. Hence, for the near-field WPT system, which is analyzed in this paper, the analysis on the energy store and power flow presents a deeper understanding of the near-field WPT system. Ref. [8] calculated the Poynting vector and found that power distributes as a donut shape in a three-dimensional (3D) space. Additionally, a better understanding of the Poynting vector also may have helped to understand the parasitic capacitive effects affecting the power flow from the transmitting coil to the receiving one [9]. Analysis on the energy store and power flow...
can also be used to design or optimize the WPT system when one considers the EMC or exposure hazard \[10,11\].

To solve and explain this question in the near field, the WPT system should be analyzed from the electromagnetic views. Many studies [12–15] have calculated the magnetic field of a WPT system. The magnetic field of a WPT system is important when one analyzes the magnetic field or designs of the coil system. However, only analyzing the magnetic field to analyze the energy flow is insufficient. This is because the energy transferred in space not only contains magnetic field energy but also electric field energy. Some studies [8,16] have analyzed the electric field of the WPT system to estimate the influence of the WPT on humans. These results show that the function of the electric field in the WPT system cannot be neglected when analyzing the energy distribution, although the WPT system is a magnetic coupling system. Ref. [17] initially analyzed WPT by using the Poynting vector, and then presented a physics approach based on the Maxwell equations. Refs. [8,10] used the complex Poynting vector and showed that the active power possesses a characteristic that the direction of the active Poynting vector is transferred from the transmit coil to the receive coil. This result interprets the principle of how the power flows from one coil to another in a frequency domain. However, the characteristic of the energy or power flow in a time domain has not been analyzed. In research related to the time domain, [18] built a time domain simulator to calculate the electromagnetic field of the WPT system. Ref. [19] presented a time domain magnetic field result to verify the designed coil system. However, analysis of the instantaneous spatial energy and power of the WPT system is still unavailable.

The energy store and power flow in the time domain are important to interpret and understand the problem of how the energy stores and the power flows in a WPT system. The absence of analysis on the instantaneous spatial energy and power of the WPT system may have three reasons. First, instantaneous analysis is more complicated in the time domain than in the frequency domain. The numbers of the distribution figures of the complex Poynting vector only have two, which are as follows: one is the distribution of the active Poynting vector, and another is the reactive Poynting vector. However, the numbers of instantaneous distributions of the Poynting vector are much more than two and will be determined by the time step used in the analysis. Second, the instantaneous distribution is a 3D matrix, resulting in a difficult visualization. Third, the relationship of the stored energy and transfer power is coupling in a time domain situation, and decoupling into parts according to some regulations is difficult.

Therefore, to solve the problem of how the energy stores and the power flows in the WPT system, this study presents an analysis on the instantaneous spatial energy and power in a WPT system and discovers the energy storing and power flowing characteristics. The results are presented in a figure and an animation to obtain a better visualization. The decoupling methods and results are also analyzed.

The instantaneous spatial energy density of the WPT system is initially calculated and analyzed. The results show that the energy density distribution can be decoupled into three parts. Under this decoupling, the mutual energy is introduced and can be considered the stored energy of the mutual inductance. The energy results show the characteristic of the stored energy but not of the transferred energy.

The instantaneous spatial Poynting vector is analyzed to discover the characteristic of the transferred power. Decoupling methods are initially discussed, and the results show that the Poynting vector cannot be decoupled similar to that in energy decoupling. The instantaneous spatial Poynting vector can only be decoupled using different orthogonal vector components. We found that the Poynting vector is a period function, the period of which is half the WPT system period, and the power transferring appears only in the last half period of the Poynting vector period.

The remainder of this paper is arranged as follows: In Section 2, we deduce the magnetic field intensity and the electric field intensity based on a two-coil WPT system. In Section 3, the instantaneous energy density distributions are analyzed, and one decoupling
The remainder of this paper is arranged as follows: In Section 2, we deduce the decoupling method is presented. Section 4 presents studies on the instantaneous Poynting vector, and the decoupling methods are discussed. In Section 5, the theoretical analysis is proven using the Ansys Electromagnetic Suites. The last Section 6 presents the conclusion.

The Supplementary Materials animations can be downloaded at the journal’s website or at https://www.mathworks.com/matlabcentral/fileexchange/115840-animations-of-the-paper-titled-analysis-and-visualization, accessed on 4 August 2022.

2. Theoretical Analysis

2.1. Magnetic Field Intensity of the WPT System

A two-coil eight-turn WPT system is simplified and shown in Figure 1. The fields on the xoz plane will be calculated and analyzed due to the axial symmetry of the system. Coils 1 and 2 represent the transmit coil and the receive coil, respectively. The radius of these coils is 0.1 m, and the distance between the two coils is 0.2 m.

The currents in the two coils are obtained as follows:

\[ i_1(t) = I_1 \cos(\omega t) \]  
\[ i_2(t) = I_2 \cos(\omega t + \Delta \phi) \]

where \( \omega \) is the angular frequency, and the frequency of the WPT system is set to 1 MHz. Subscripts 1 and 2 represent coils 1 and 2, respectively. \( I_1 \) and \( I_2 \) are the magnitudes of the two currents. For simplicity, the magnitude of the two currents is set to 1A. \( \Delta \phi \) is the current phase difference. According to the theory of the WPT system, the power transferring condition is \( \Delta \phi \neq 0 \).

![Figure 1. A two-coil WPT system. (a) the structure of the system. (b) the x-z view of the system.](image)

The magnetic field intensities generated by coils 1 and 2 are \( H_1(x,z,t) \) and \( H_2(x,z,t) \), respectively. \( H_1(x,z,t) \) is superposed by the x-component \( H_{x1}(x,z,t) \) and the z-component \( H_{z1}(x,z,t) \). That is,

\[ H_1(x,z,t) = H_{x1}(x,z,t)e_x + H_{z1}(x,z,t)e_z \]  

where \( e_x \) and \( e_z \) are the unit vectors of the x and the z directions, respectively, and

\[ H_{x1}(x,z,t) = I_1 \cos(\omega t)T_{x1}(x,z) \]  
\[ H_{z1}(x,z,t) = I_1 \cos(\omega t)T_{z1}(x,z) \]

where \( T_{x1}(x,z) \) and \( T_{z1}(x,z) \) are the geometrical functions, which are defined in [20] as follows:

\[ T_{x1}(x,z) = \frac{1}{4\pi} \int_0^{2\pi} \frac{\cos \Phi_1 \cdot R(z-z_1) d\Phi_1}{\left[ x^2 + R^2 - 2xR \cos \Phi_1 + (z - z_1)^2 \right]^{3/2}} \]
$$T_{x_1}(x, z) = \frac{1}{4\pi} \int_0^{2\pi} \frac{(R^2 - \cos \Phi_1 R x) d\Phi_1}{\left[ x^2 + R^2 - 2R \cos \Phi_1 + (z - z_1)^2 \right]^{3/2}}$$  \hspace{1cm} (7)

The magnetic field intensity generated by coil 2 is as follows:

$$H_2(x, z, t) = H_{x_2}(x, z, t) e_x + H_{z_2}(x, z, t) e_z$$  \hspace{1cm} (8)

and

$$H_{x_2}(x, z, t) = I_2 \cos(\omega t + \Delta \phi) T_{x_2}(x, z)$$  \hspace{1cm} (9)

$$H_{z_2}(x, z, t) = I_2 \cos(\omega t + \Delta \phi) T_{z_2}(x, z)$$  \hspace{1cm} (10)

where $T_{x_2}(x, z)$ and $T_{z_2}(x, z)$ are the geometrical functions of coil 2 [20]. This method is also called the filament method, which is a kind of integral method. Additionally, another method is the partial element equivalent circuit method [21], which can also be used to calculate the magnetic field of the WPT system.

Considering the linear superposability of the magnetic field, the superposed magnetic field of coils 1 and 2 is obtained as follows:

$$H(x, z, t) = H_1(x, z, t) + H_2(x, z, t) = H_x(x, z, t) e_x + H_z(x, z, t) e_z$$  \hspace{1cm} (11)

where $H_x(x, z, t)$ and $H_z(x, z, t)$ are the $x$ and $z$ components of the superposed magnetic field, respectively, and can also be expressed as follows:

$$H_x(x, z, t) = H_{x_1}(x, z, t) + H_{x_2}(x, z, t)$$  \hspace{1cm} (12)

$$H_z(x, z, t) = H_{z_1}(x, z, t) + H_{z_2}(x, z, t)$$  \hspace{1cm} (13)

2.2. Inductance Electric Field Intensity of the WPT System

The electric field on the xoz plane only contains the y-direction component. The electric field generated by coil 1 can be expressed as follows [10]:

$$E_1(x, z, t) = -\mu_0 \omega I_1 \sin(\omega t) T_{y_1}(x, z) e_y$$  \hspace{1cm} (14)

where $\mu_0$ is the magnetic permeability of free space. $T_{y_1}(x, z)$ is the geometrical function of the electric field generated by coil 1, and is obtained as follows:

$$T_{y_1}(x, z) = \frac{1}{4\pi} \int_0^{2\pi} \frac{\cos \Phi_1 R d\Phi_1}{\left[ x^2 + R^2 - 2R \cos \Phi_1 + (z - z_1)^2 \right]^{1/2}}$$  \hspace{1cm} (15)

The electric field of coil 2 is obtained as follows:

$$E_2(x, z, t) = -\mu_0 \omega I_2 \sin(\omega t + \Delta \phi) T_{y_2}(x, z) e_y$$  \hspace{1cm} (16)

Consequently, similar to the total magnetic field and its components, the total electric field can be expressed as follows:

$$E(x, z, t) = E_1(x, z, t) + E_2(x, z, t)$$

$$= -\mu_0 \omega I_1 \sin(\omega t) T_{y_1}(x, z) + I_2 \sin(\omega t + \Delta \phi) T_{y_2}(x, z) e_y$$  \hspace{1cm} (17)

For convenience in a further discussion, the y-component of the electromagnetic field is expressed as follows:

$$E_y(x, z, t) = E_{y_1}(x, z, t) + E_{y_2}(x, z, t)$$  \hspace{1cm} (18)
The equations calculating the magnetic field and the electric field will be coded to obtain the field result for further use. This method is the filament method, which can be used to calculate the electromagnetic field of the WPT system in the near field [20].

3. Instantaneous Energy Density

3.1. Decoupling of the Instantaneous Energy Density

The energy density in the WPT system is the electromagnetic field energy density, which is composed of the magnetic field energy density \( w_m(x, z, t) \) and the electric field energy density \( w_e(x, z, t) \) [22]. In the following discussion, the parameters \( x, z, \) and \( t \) are omitted for convenience.

Hence, the electromagnetic field energy density is obtained as follows:

\[
    w_{em} = w_m + w_e
\]  

(19)

The magnetic field energy density is calculated as follows:

\[
    w_m = \frac{1}{2} \mu_0 H^2 = \frac{1}{2} \mu_0 (H_x^2 + H_z^2)
\]  

(20)

According to Equations (12) and (13), \( w_m \) can be calculated as follows:

\[
    w_m = \frac{1}{2} \mu_0 (H_{x1}^2 + 2 |H_{x1} H_{x2}| + H_{x2}^2 + H_{z1}^2 + 2 |H_{z1} H_{z2}| + H_{z2}^2)
\]  

(21)

Therefore, \( w_m \) is naturally decoupled by the x-components and the z-components of coils 1 and 2, respectively. For coil 1, the magnetic field energy contains two energy components, which are as follows:

\[
    w_{mx1} = \frac{1}{2} \mu_0 H_{x1}^2
\]  

(22)

\[
    w_{mx2} = \frac{1}{2} \mu_0 H_{x2}^2
\]  

(23)

For coil 2, the magnetic field energy also contains two energy components, which are as follows:

\[
    w_{mx1} = \frac{1}{2} \mu_0 H_{x1}^2
\]  

(24)

\[
    w_{mx2} = \frac{1}{2} \mu_0 H_{x2}^2
\]  

(25)

In Equation (21), two multiplied items exist, and they can be regarded as the mutual magnetic field energy density of coils 1 and 2. According to the physical meaning, no negative energy exists; hence, absolute values are used.

\[
    w_{mx12} = \mu_0 |H_{x1} H_{x2}|
\]  

(26)

\[
    w_{mx21} = \mu_0 |H_{x1} H_{x2}|
\]  

(27)

According to Equation (18), the electric field energy density is obtained as follows:

\[
    w_e = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} \varepsilon_0 \left( E_y^2 + 2 |E_{y1} E_{y2}| + E_{y2}^2 \right)
\]  

(28)

and it can also be decoupled into three parts, as follows:

\[
    w_{ey1} = \frac{1}{2} \varepsilon_0 E_{y1}^2
\]  

(29)

\[
    w_{ey2} = \frac{1}{2} \varepsilon_0 E_{y2}^2
\]  

(30)
\[ \text{where } w_{ey1} \text{ and } w_{ey2} \text{ are the self-electric field energy density of coils } 1 \text{ and } 2, \text{ respectively. } \\
\]

\[ w_{ey12} \text{ is the mutual electric field energy density of coils } 1 \text{ and } 2. \]

Equations (22) to (27) and (29) to (31) are used to obtain nine energy densities theoretically. However, determining the relations and phenomena among the nine results is difficult. Therefore, we need to superpose and classify the nine energy densities to provide a better result to discover the characteristic of the decoupled energy density of the WPT system.

We are interested in the self-electromagnetic field energy of coils 1 and 2, as well as the mutual energy. The self-electromagnetic energy of coils 1 and 2 is familiar to us by this classification, indicating that the energy is proportional to the square of the current. Although the mutual energy has been less studied previously, the mutual energy stands for the coupling energy of the mutual inductance. By analyzing the mutual energy, the proposed problem of how the energy is stored in the WPT may be answered.

Therefore, the self-electromagnetic field energy densities of coils 1 and 2 are obtained as follows:

\[ w_{em1} = w_{mx1} + w_{mz1} + w_{ey1} \quad (32) \]
\[ w_{em2} = w_{mx2} + w_{mz2} + w_{ey2} \quad (33) \]

and the mutual electromagnetic field energy density of the coils is obtained as follows:

\[ w_{em12} = w_{mx12} + w_{mz12} + w_{ey12} \quad (34) \]

The total electromagnetic field of the WPT system is computed as follows:

\[ w_{em} = w_{em1} + w_{em2} + w_{em12} \quad (35) \]

### 3.2. Distributions of Energy Densities

The instantaneous distributions of the self-electromagnetic field energy density of coil 1 \( w_{em1} \) are shown in Figure 2. The distribution of the mutual energy density \( w_{em12} \) is shown in Figure 3. The total electromagnetic field energy density \( w_{em} \) is shown in Figure 4. The blank distributions at 90° or 0° in Figures 2 and 3 are the state where the electromagnetic field energy densities are equal to zero. The animations are presented on the journal’s website.

The time parameter \( \omega t \) is depicted on top of each figure. The time parameters vary from 0° to 180° because the period of the energy density is half of that of the currents or the period of the WPT system. The period of the current is 0° to 360°.

These figures show the different energy densities varying with time. In Figure 2, the energy of coil 1 varies with time and distributes around coil 1, indicating that the energy of coil 1 has no relationship with coil 2. This electromagnetic energy can be regarded as the self-energy density of coil 1. When \( \omega t = 90^\circ \), the energy density is equal to 0, since the current of coil 1 is equal to 0 at this moment. Similarly, the electromagnetic field energy of coil 2 will vary with time but possess a time delay \( \Delta \phi \) and distribute around coil 2.

Figure 3 shows the mutual electromagnetic field energy density distributions varying with time. The period of this energy density is also half of the WPT system. The distributions of the mutual energy density form a bridge-like shape between coils 1 and 2, indicating the mutual energy density of coils 1 and 2. When \( \omega t = 0^\circ \) and \( \omega t = 90^\circ \), the mutual energy densities are equal to zero, since one of the currents is equal to zero at these moments.

Figure 4 shows the total electromagnetic field energy density distributions of the WPT system. These distributions are the linear superpositions of the self-energy densities of coils 1 and 2 and the mutual distributions at each moment. Hence, the period of the total electromagnetic field energy density is also half of the WPT system period.

The mutual energy density shows a coupled energy between coils 1 and 2. However, this energy cannot be regarded as the transferred energy between coils 1 and 2. When
the current phase difference $\Delta \phi = 0$, no power is transferred. However, when $\Delta \phi = 0$, the mutual energy density is not equal to 0. Thus, the mutual energy density cannot be considered the transfer energy between coils. This energy density can only be regarded as the stored energy of the mutual inductance. This mutual energy density is similar to the mutual inductance; the mutual inductance always exists whether or not power is transferred.

Figure 4 shows the total electromagnetic field energy density distributions of the WPT system. These distributions are the linear superpositions of the self-energy densities of coils 1 and 2 and the mutual distributions at each moment. Hence, the period of the total electromagnetic field energy density is also half of the WPT system period. The mutual energy density shows a coupled energy between coils 1 and 2. However, this energy cannot be regarded as the transferred energy between coils 1 and 2. When the current phase difference $\Delta \phi = 0$, no power is transferred. However, when $\Delta \phi = 0$, the mutual energy density is not equal to 0. Thus, the mutual energy density cannot be considered the transfer energy between coils. This energy density can only be regarded as the stored energy of the mutual inductance. This mutual energy density is similar to the mutual inductance; the mutual inductance always exists whether or not power is transferred.

Figure 2. Instantaneous distributions of the self-electromagnetic field energy density of coil 1.

Figure 3. Instantaneous distributions of the mutual electromagnetic field energy density of coils 1 and 2.
4. Instantaneous Poynting Vector

4.1. Poynting Vector and Power Flow Phenomenon

The instantaneous distributions of the different energy densities present a time-varying characteristic of the stored energy of the WPT system. To analyze the power flow and discover the problem of how power is transferred in the WPT system, the Poynting vector is analyzed. The instantaneous Poynting vector on the xoz plane can be calculated as follows:

\[ S = E \times H = E_y H_z e_x - E_y H_x e_z \]  

where the bold character represents the vector. The x-component and z-component of the Poynting vector can be defined as follows:

\[ S_x = E_y H_z \]  

\[ S_z = -E_y H_x \]  

The magnitude of the Poynting vector is obtained as follows:

\[ S = \sqrt{S_x^2 + S_z^2} \]  

The electromagnetic field components \( H_x, H_z, \) and \( E_y \) are initially calculated by Equations (12), (13), and (18), and then the field components are integrated into Equations (37) to (39) to obtain the x- and z-direction components and magnitude of the Poynting vector. This Poynting vector is a function of the time and the position. The overall characteristic of the power flow that varies with time and space can be observed using this instantaneous Poynting vector, which contains Equations (37)–(39). The result is shown in Figure 5, and the animation of the Poynting vector is presented on the journal’s website.

The time variable \( \omega t \) is added on top of each subfigure. These subfigures in Figure 5 shows that the period of the Poynting vector is from 0° to 180°, which is a half of the WPT system’s period. The colors on the xoz plane stand for the magnitude of the Poynting vector, which is calculated by Equation (39). The arrows stand for the unit magnitude directions of the Poynting vector on different positions. The unit magnitude x-direction arrow is the ratio of Equations (37)–(39). The unit magnitude z-direction arrow is the ratio of Equations (38) and (39).
In Figure 5, from 0° to 90° (excluding 0° and 90°), the Poynting vector is surrounded by the four cross-sections of the two coils. In the region around coil 1, the arrows of the Poynting vector are from a far area to coil 1. In the region around coil 2, the arrows of the Poynting vector are emitted from the cross-sections of coil 2 to the far area. A dark blue Poynting vector is from a far area to coil 1. In the region around coil 2, the arrows of the Poynting vector are emitted from the cross-sections of the two coils. In the region around coil 1, the arrows of the Poynting vector are from coils 1 to 2 and in the middle region between the two coils. Some vertical arrows exist, indicating that the power flows from coil 1 to coil 2. This finding is in accordance with the direction of the power transfer in the WPT system.

The phenomena indicate a significant result, in which the power transferred can be found in the instantaneous distributions of the Poynting vector. In addition, power is transferred in the last half period of the Poynting vector. This interpretation indicates how power is transferred in the WPT system. Additionally, the power distribution forms a circular column in the 3D space in the power-transferred period. The distribution of the power or the maximum value can be used in the evaluation of the EMC or human electromagnetic field exposure problems.

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4.2. Discussion on a Decoupling Method of the Poynting Vector

The overall Poynting vector exhibits the characteristics of the power flow, and this flowing power contains the stored energy and the transferred energy. A natural idea is to decouple the overall Poynting vector to find which component stores the energy and which component transfers the energy.

To decouple the Poynting vector, four component Poynting vectors in the frequency domain were defined in [8]. In the time domain, these components can also be defined as follows:

\[
S_x = (E_{y1} + E_{y2})(H_{z1} + H_{z2}) \\
= E_{y1}H_{z1} + E_{y1}H_{z2} + E_{y2}H_{z1} + E_{y2}H_{z2} \\
= S_{x11} + S_{x12} + S_{x21} + S_{x22} \\
\]

\[
S_z = -(E_{y1} + E_{y2})(H_{x1} + H_{x2}) \\
= -E_{y1}H_{x1} - E_{y1}H_{x2} - E_{y2}H_{x1} - E_{y2}H_{x2} \\
= S_{z11} + S_{z12} + S_{z21} + S_{z22} \\
\]  

(40)  

(41)
The magnitudes of the four-component Poynting vectors are computed as follows:

\[ S_{11} = \sqrt{S_{111}^2 + S_{112}^2} \]  
(42)

\[ S_{12} = \sqrt{S_{121}^2 + S_{122}^2} \]  
(43)

\[ S_{21} = \sqrt{S_{211}^2 + S_{212}^2} \]  
(44)

\[ S_{22} = \sqrt{S_{221}^2 + S_{222}^2} \]  
(45)

Although this decoupling method is used in the frequency domain of the complex Poynting vector [8], this decoupling is invalid in the time domain because of two reasons. The first reason is that the following inequality that does not support the decoupling method can be easily proven.

\[ S \neq S_{11} + S_{12} + S_{21} + S_{22} \]  
(46)

This result is different from Equation (35), in which the total energy density is equal to the sum of self-energy densities and the mutual energy density. The reason is that the Poynting vector is a vector field, the magnitude of the field is equal to the square root of the sum of self-energy densities and the mutual energy density. The reason is that the Poynting vector is a vector field, the magnitude of the field is equal to the square root of two direction components, and the square root does not possess linear superposability.

The second reason is that the directions of \( S_{11}, S_{12}, S_{21}, \) and \( S_{22} \) are not functions of time. This finding indicates that if we apply this decoupling method, then we decouple an instantaneous quantity \( S(x, z, t) \), which is a function of time into \( S_{11}(x, z) \), \( S_{12}(x, z) \), \( S_{21}(x, z) \), and \( S_{22}(x, z) \), whose directions are not functions of time. Hence, this decoupling method is not suited in the time domain situation.

Therefore, we cannot decouple the Poynting vector into several self parts and mutual parts similar to the results in the decoupling of the energy density.

However, a method for decoupling the overall Poynting vector still exists because the Poynting vector is a vector physical quantity. The only method for decoupling the overall Poynting vector is to use different orthogonal vector components, which will be discussed in the subsequent section.

4.3. \( x \)- and \( z \)-Direction Components of the Poynting Vector

The only approach to decouple the overall Poynting vector is to directly use the orthogonal direction components. This finding is because a vector field can always be superposed by two orthogonal components. Thus, the overall Poynting vector on the \( xoz \) plane is decoupled into \( x \)- and \( z \)-components, which can be calculated as follows:

\[ S_x = (E_{y1} + E_{y2})(H_{z1} + H_{z2}) \]
\[ = \mu_0 \omega (I_1 \sin \omega T_{y1} + I_2 \sin(\omega t + \Delta \phi)T_{y2}) \cdot (I_1 \cos \omega T_{z1} + I_2 \cos(\omega t + \Delta \phi)T_{z2}) \]  
(47)

\[ S_z = (E_{y1} + E_{y2})(H_{z1} + H_{z2}) \]
\[ = \mu_0 \omega (I_1 \sin \omega T_{y1} + I_2 \sin(\omega t + \Delta \phi)T_{y2}) \cdot (I_1 \cos \omega T_{z1} + I_2 \cos(\omega t + \Delta \phi)T_{z2}) \]  
(48)

The instantaneous distributions of \( S_x \) and \( S_z \) are shown in Figures 6 and 7. Figures 6 and 7 show that the stored power and the transferred power can be easily classified and observed. The function of \( S_x \) is main to store energy because the arrows of \( S_x \) move back and forth only on the horizontal direction, and the power is not transferred from coil 1 to coil 2. The function of \( S_z \) is that it stores energy and transfers energy. In the period from \( 0^\circ \) to \( 90^\circ \), \( S_z \) stores the power, and in the period from \( 90^\circ \) to \( 180^\circ \), it transfers the power. This finding is because, from \( 0^\circ \) to \( 90^\circ \), a gap exists preventing the power transfers from coil 1 to coil 2, but from \( 90^\circ \) to \( 180^\circ \), the arrows are directly from coil 1 to coil 2, indicating the power flow.

Therefore, the overall Poynting vector is decoupled, and the decoupled components can be used to illustrate the characteristics of energy storage and power transfer.
transferred from coil 1 to coil 2. The function of $z_S$ is that it stores energy and transfers energy. In the period from $0^\circ$ to $90^\circ$, $z_S$ stores the power, and in the period from $90^\circ$ to $180^\circ$, it transfers the power. This finding is because, from $0^\circ$ to $90^\circ$, a gap exists preventing the power transfers from coil 1 to coil 2, but from $90^\circ$ to $180^\circ$, the arrows are directly from coil 1 to coil 2, indicating the power flow. Therefore, the overall Poynting vector is decoupled, and the decoupled components can be used to illustrate the characteristics of energy storage and power transfer.

4.4. Instantaneous Poynting Vector on Different Points

To present a more direct relation between the overall Poynting vector and its components, the Poynting vector and its component curves that vary with time on two different points on the xoz plane are plotted in Figures 8 and 9. In Figures 8 and 9, the blue triangle line stands for the overall Poynting vector, and the red triangle line and the yellow square line represent the x- and z-components of the Poynting vector, respectively. The curves of the x- and z-components are similar to the sine wave, but not the sine wave. The power transferring time is the time when the z-component value is larger than zero. The relations between the Poynting vector and its components can be clearly observed by those curves.

4.5. Instantaneous Poynting Vector on $0^\phi_\Delta=$

As mentioned in Section 1, on the condition that $0^\phi_\Delta=$, the WPT system does not transfer the energy. In Section 3, we analyzed the instantaneous energy density distribution on the condition that $0^\phi_\Delta=$, and the result shows that the energy density exists, implying that the instantaneous distributions of the energy density do not explain the flowing power in the WPT system. However, the instantaneous Poynting vector can perfectly interpret the power flow, even on the condition $0^\phi_\Delta=$. The reason for that can be found in the instantaneous distributions of the Poynting vector on the condition $0^\phi_\Delta=$, whose animation is presented on the journal's website. There are no vertical arrows of the Poynting vector on the condition $0^\phi_\Delta=$, which indicates that the power does not transfer from coil 1 to coil 2.

Figure 6. Instantaneous distributions of $x_S$.

Figure 7. Instantaneous distributions of $z_S$.

4.4. Instantaneous Poynting Vector on Different Points

To present a more direct relation between the overall Poynting vector and its components, the Poynting vector and its component curves that vary with time on two different points on the xoz plane are plotted in Figures 8 and 9.

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5. Verification and Extended Analysis

5.1. Verification by Simulation

A simulation has been conducted to verify the correctness of the analysis. The simulation uses the Ansys Electromagnetic Suites. Initially, a WPT model with two coils was built. This model is the same as the model in the analysis. The instantaneous Poynting vector is simulated from 0° to 360°. We only select two moments, namely, 45° and 135°, to illustrate the energy flow in the WPT system.

4.5. Instantaneous Poynting Vector on $\Delta \phi = 0$

As mentioned in Section 1, on the condition that $\Delta \phi = 0$, the WPT system does not transfer the energy. In Section 3, we analyzed the instantaneous energy density distribution on the condition that $\Delta \phi = 0$, and the result shows that the energy density exists, implying that the instantaneous distributions of the energy density do not explain the flowing power in the WPT system. However, the instantaneous Poynting vector can perfectly interpret the power flow, even on the condition $\Delta \phi = 0$. The reason for that can be found in the instantaneous distributions of the Poynting vector on the condition $\Delta \phi = 0$, whose animation is presented on the journal’s website. There are no vertical arrows of the Poynting vector on the condition $\Delta \phi = 0$, which indicates that the power does not transfer from coil 1 to coil 2.
the correctness of the analysis. The directions and magnitudes of the Poynting vector on 45° and 135° are shown in Figure 10. A gap on the 45° condition is shown in Figure 10c, and the vertical arrows are shown in Figure 10b. These phenomena are the same in the analysis, proving the correctness of the analysis.

To present more details on the verification, the Poynting vector on two points of the xoz plane is plotted in Figure 11. The coordinates of points 1 and 2 are (0.1, 0.074) and (0.1, 0.131), respectively. In Figure 11, the blue line and the red broken line represent the analyzed result of the normalized magnitudes of the Poynting vector on points 1 and 2, respectively. The yellow triangle line and the purple square line represent the simulated results of the normalized magnitudes of the Poynting vector on points 1 and 2.

Normalization is used to eliminate the position error between the simulation and the analysis. Figure 11 shows that on each point, the analyzed curve and simulated curve have the same tendency, confirming the correctness of the analysis.

**Figure 10.** Poynting vector results by simulation. Poynting vector directions on (a) 45° and on (b) 135° and the magnitude on (c) 45° and on (d) 135°.

**Figure 11.** Curves of the Poynting vectors on different points by analysis and simulation.
5.2. Verification by Experiment

A WPT system is built to verify the theoretical analysis. The system contains two 8-turn coils. The distance between two coils is 0.2 m. The system is shown in Figure 12. The operation frequency of the system is tuned to 1.412 MHz. The voltages and currents of coils 1 and 2 are measured and shown in Figure 13. \( U_1 = 77.95 \text{ V} \) and \( U_2 = 191.86 \text{ V} \) stand for the voltages of coils 1 and 2, respectively. \( I_1 = 0.36 \text{ A} \) and \( I_2 = 0.88 \text{ A} \) are the currents in coils 1 and 2, respectively. The phase difference of \( I_1 \) and \( I_2 \) is \(-97^\circ\). The magnitudes of the current and the phase difference are close to that used in theoretical analysis. The Poynting distributions on this experimental condition are shown in Figure 14, which are similar to that in the theoretical analysis result shown in Figure 5. This result proves the correctness of the analysis again.

![Experimental WPT system](image1)

**Figure 12.** Experimental WPT system.

![Currents and voltage waves](image2)

**Figure 13.** Currents and voltage waves of the WPT system.
Figure 14. Instantaneous distributions of the overall Poynting vector in the WPT system under the experimental condition.

5.3. Extended Analysis

The parameters and coil conditions of the above analysis are chosen for theoretical analysis. An extended analysis is also presented to meet the real parameters and conditions, which also shows that the result and the analysis method used in this paper are suited for the real situations. The frequency of the system is chosen at 6.78 MHz, which is the operation frequency according to CISPR11 for ISM devices, and the coils have a 0.05 m misalignment on the x-axis. The Poynting vector distribution with this frequency and on this misalignment condition is shown in Figure 15.

Figure 15. Instantaneous distributions of the overall Poynting vector in the WPT system under 6.78 MHz with 0.05 m misalignment.

The distribution of the Poynting vector shown in Figure 15 is under 6.78 MHz with 0.05 m misalignment. The 6.78 MHz frequency is higher than 1 MHz, which is used in the theoretical analysis in Figure 5. According to the relationship of the frequency to the
electromagnetic field, which can be found in Equations (9), (10) and (14), the magnitude of the Poynting vector increases when the frequency increases. Hence, the value in Figure 15 is higher than that in Figure 5.

When the coil is misaligned, the overall characteristic of the distributions and the directions of the Poynting vector is similar to the condition with no misalignment. The transfer period shown in Figure 15 is the same as that in Figure 5.

The above analysis also proves that the analysis method and results can be extended into the real condition of the WPT system analysis.

6. Conclusions

This study analyzes the instantaneous characteristics of the energy density and the Poynting vector in the space of the WPT system, and provides a better understanding of how the energy is stored and how power is transferred in the WPT system. A deeper understanding of the stored energy and power flow can provide a reference for the further design and optimization of the WPT system in the near field when one considers EMC or human electromagnetic exposure problems.

The electromagnetic field energy density of the WPT system is initially analyzed. The energy density is decoupled into three components, including the self-energy density of each coil and the mutual energy density of two coils. The varying distribution figures of all the energy densities are presented, and their animations are also presented. This study is the first to discuss the time-varying mutual energy density. The self-energy densities of each coil are surrounded by the coils. The distributions of the mutual energy density form a bridgelike shape between coils. It is also found that one cannot interpret that the energy is transferred from one coil to another by using the energy densities. The distributions of the energy densities only exhibit the characteristics of the energy storage but not the energy transfer.

The instantaneous Poynting vector is subsequently analyzed. The overall time-varying Poynting vector is presented in figures and animations. We found that the period of the Poynting vector is half of that of the WPT system. In the first half period of the Poynting vector, a gap between two coils exists, preventing the power transfer from coil 1 to coil 2. In the last half period of the Poynting vector, several vertical arrows exist, indicating that power is transferred from coil 1 to coil 2. The power transfer does not occur every moment of the whole Poynting vector period, but only in the last half period of the Poynting vector. This finding interprets the problem of how power is transferred in the WPT system more deeply.

Moreover, the decoupling method used in the analysis of the energy density is discussed. The result shows that this method is invalid in the decoupling of the instantaneous Poynting vector. Another decoupling method is also discussed, and the Poynting vector is decoupled into two orthogonal direction components. The x-direction component mainly stands for the energy storage, and the z-direction component mainly represents the transferred power. The no-power transfer condition of the WPT system is also considered. The instantaneous Poynting vector has no vertical arrows under that condition. This result shows that the instantaneous Poynting vector can perfectly interpret and exhibit the power flow in the WPT system.

Finally, a simulation and an experiment are conducted, confirming the correctness of the analysis. This paper presents a comprehensive analysis on the instantaneous energy density and the Poynting vector of the WPT system. It also presents an interpretation on the problem of how energy is stored and how power is transferred in the WPT system by the instantaneous and spatial energy and power view. The analysis and the results provide better understanding on the mechanism of the WPT system. In the future, the WPT design or optimization can be performed by referring to the energy or power values in the distributions presented in this paper. The maximum values of the distributions can be used as the target value, and the design or optimization method also can refer the method presented in this paper.
Supplementary Materials: The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/en15165764/s1, Video S1: The whole EM energy density.avi, S2: The mutual EM energy density of the coils.avi, S3: The whole Poynting vector on the power transfer condition.avi, S4: The whole Poynting vector not on the power transfer condition.avi.

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