Inclusive $B \to sg$ decay in QCD

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The inclusive nonleptonic transition rate of $B$ meson in gluonic penguin channel is calculated in QCD using the heavy quark mass expansion. We found the branching ratio to be approximately 0.25%.

12.38.Lg, 13.25.Hw

I. INTRODUCTION

Recently, there has been a significant interest in the physics of beauty hadrons. From the theoretical viewpoint, its attractiveness is due to the fact that both perturbative and nonperturbative corrections are under control on a scale of the heavy $b$-quark mass. On the other hand, it is the $B$-physics where the Standard Model can be tested with a high precision and a possible New Physics is expected to be eventually caught. If a discrepancy between the predictions of the Standard Model and the data is established, it would be necessary to invoke the New Physics for its resolution.

The aim of the present paper is to calculate a contribution to the nonleptonic inclusive decay rate of the $B$-meson due to the gluon penguin channel $(b \to sg)$. This study is motivated by the existence of the semileptonic BR problem $^1$. Despite many theoretical efforts, this problem so far remains unsolved within the Standard Model with the 20 \% gap between the data and theoretical predictions. In attempts to its resolution, it is essential to take into account all relevant contributions which could enhance the BR$_{sl}$. On the other hand, the gluon penguin contribution to the nonleptonic inclusive width has not been yet calculated. It is our purpose to fill this gap in the literature and calculate the corresponding partial width beyond the perturbation theory. We find that no considerable enhancement of BR$_{sl}$ occurs due to this decay channel, in agreement with the common belief. Thus, if one is expecting some significant enhancement of the BR$_{sl}$ in the gluon penguin then an appearance of a New Physics is anticipated $^2$.

To calculate the inclusive transition rate in the $b \to sg$ channel nonperturbatively we use a powerful method of pre-asymptotic expansion $^3$ $^4$. It is based upon an incorporation of elements of the heavy quark effective theory (HQET) $^5$ into the Wilson operator product expansion (OPE). In this method, one uses the optical theorem to connect the decay rate $\Gamma$ with the transition operator $T$ defined by

$$T(Q \to f \to Q) = i \int d^4x T[H_{eff}^+(x)H_{eff}(0)], \quad (1.1)$$

where $H_{eff}$ denotes an effective weak Hamiltonian relevant for the $Q \to f$ transition. The nonlocal operator $T$ is expanded in a series of local operators with the heavy quark mass serving as a large parameter of the expansion. The long distance physics is parametrised by a set of matrix elements of the local operators over the hadronic states. The transition rate is calculated in a systematic way through an expansion in inverse powers of the heavy flavour quark mass. Thus, inclusive decay rates are determined in terms of this universal set of matrix elements without further phenomenological input.

The structure of the paper is as follows. In Sec. II we present in short the method of pre-asymptotic heavy quark mass expansion. The inclusive width in the $b \to sg$ channel is calculated in Sec. III. The final Section IV provides a summary of our results.

II. PRE-ASYMPTOTIC EXPANSION FOR INCLUSIVE B-DECAYS

The starting point of our analysis is the expression $^{[1]}$ for the transition operator $T$. In the case $b \to sg$ decay the effective weak Hamiltonian $^6$ is

$$H_{eff}(x) = \frac{G_F}{\sqrt{2}} \frac{g}{4\pi^2} C_W V_{bt} V_{st} s_L(x) \sigma_{\mu\nu} G_{\mu\nu}(x) b_R(x). \quad (2.1)$$

Here, $G_F$ is the Fermi constant, $V_{bt}$ and $V_{st}$ stand for the CKM matrix elements. The effective three point interaction couples the right-handed $b$ quark with the left-handed $s$ quark and the gluon. The Wilson coefficient $C_W$ is a result of integrating out hard gluons which generate the effective vertex $^{[2]}$. For the effective Hamiltonian $^{[2]}$, $C_W(m_b) \approx -0.150$ $^{[1]}$. The calculation of the decay rate $\Gamma$ is based on the optical theorem, which relates the inclusive width to the imaginary part of the forward scattering amplitude:

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\[
\Gamma(B \rightarrow f) = \frac{1}{M_B} \text{Im}(B|\hat{T}|B) \tag{2.2}
\]

The heavy meson mass \(M_B\) in Eq. (2.2) accounts for the proper normalisation. As was pointed out in Ref. [3–5], the expression (2.2) suggests a way for calculation of \(\Gamma\) through a systematic expansion in inverse powers of the heavy quark mass. The nonlocal operator \(\hat{T}\) is expanded in a series of local operators, the operators of lowest dimensions being \(O_1 = bb\) and \(O_2 = \bar{b}i\sigma Gb\). Thus, \(\Gamma\) can be written in the form

\[
\Gamma = C_1 \langle B|O_1|B\rangle + C_2 \langle B|O_2|B\rangle + \ldots \tag{2.3}
\]

where dots denote contributions of higher order operators. Since these terms are suppressed by powers of the heavy quark mass their contributions will be omitted in the following calculations.

In the limit of heavy quark mass, the expectation value of operator \(O_1\) over a meson state is

\[
\langle B|O_1|B\rangle = \frac{1}{4m_b^2} \langle B|\bar{b}\gamma_0 b|B\rangle - \frac{1}{2m_b^2} \langle B|\bar{b}\gamma^2 b|B\rangle + O(\frac{1}{m_b^3}). \tag{2.4}
\]

The first term in the expression (2.4) is equal, up to normalisation, to the density of \(b\) quark in \(B\) meson. Taking into account the normalisation of the \(B\) meson state \(\langle B|B\rangle = 2M_B\), we get

\[
\frac{\langle B|\bar{b}\gamma_0 b|B\rangle}{2M_B} = 1. \tag{2.5}
\]

The second and third terms in (2.4) are suppressed by inverse \(b\) quark mass squared. Note, that the second operator is just the \(O_2\) operator which appears in the expansion for the \(O_1\) matrix element and plays the role of chromomagnetic interaction [8]. The operator \(O_2\) arises in the expression (2.3) for \(\Gamma\) also in a direct way through the expansion of the transition operator \(\hat{T}\). This happens when the interaction of \(s\) quark and gluon with an external gluonic field is taken into account. The relevant propagators are

\[
S(x) = \frac{i\tilde{x}}{2\pi^2x^4} + \frac{xk\gamma^5G_{ks}(0)}{8\pi^2x^2}, \tag{2.6}
\]

\[
D_{\alpha\beta}(x - y) = \frac{g_{\alpha\beta}}{4\pi^2(x - y)^2} + \frac{1}{8\pi^2} G_{\alpha\beta}(0) \ln[-(x - y)^2] - \frac{1}{8\pi^2} G_{\mu\nu}(0)x_\mu y_\nu \frac{g_{\alpha\beta}}{(x - y)^2}. \tag{2.7}
\]

Here \(\tilde{G}\) denotes the dual of the field strength. The matrix element of the chromomagnetic operator is proportional to the splitting between squared masses of \(B^*\) and \(B\) mesons:

\[
\frac{\langle B|\bar{b}\sigma Gb|B\rangle}{2M_B} = \frac{3}{2}(M_{B^*}^2 - M_B^2) \simeq 0.7 \text{ GeV}^2 \tag{2.8}
\]

The third operator on the right-hand side of equation (2.4) describes the kinetic energy of the \(b\) quark inside the hadron. Its matrix element can be estimated [9] to be

\[
\frac{\langle B|\bar{b}p^2 b|B\rangle}{2M_B} \simeq 0.5 \text{ GeV}^2 \tag{2.9}
\]

### III. GLUON PENGUIN CHANNEL IN INCLUSIVE DECAY

In order to calculate the inclusive transition rate of \(B\) meson in the gluon penguin channel, we use Eq. (1.1) with the effective weak Hamiltonian defined in (2.1):

\[
\hat{T} = A_i \int dx T\{\bar{b}(x) \frac{1}{2} \sigma_{\alpha\beta} G_{\alpha\beta}(x)s(x) \times \bar{s}(0) \sigma_{ij} \frac{1}{2} G_{ij}(0)b(0)\} \tag{3.1}
\]

In the limit of the heavy quark mass, the \(b\) quark wave function is rescaled and the mechanical part is extracted [10]:

\[
\bar{b}(x) = \bar{b}(0)e^{ipx} \tag{3.2}
\]

Here \(p\) denotes the four-momentum of the \(b\) quark inside the hadron. The lowest order contributions to the expression (3.1) for the transition operator \(\hat{T}\) are given by the three diagrams (Figs. 1, 2).

![Fig. 1. Zero order contribution to the transition operator \(\hat{T}\).](image-url)
The explicit calculations for the coefficients $C$ we finally determine the transition rate:

for the matrix elements and using all numerical values in (2.3) yield the following answers:

The appropriate branching ratio is $2.5 \times 10^{-3}$.

These diagrams are obtained when the s quark and gluon fields are contracted in Eq. (3.1). Expressions (2.6), (2.7) are used for the propagators in the external gluonic field.

The first diagram (Fig. 1) gives rise to the operator $O_1$ while the other two (Fig. 2) produce the operator $O_2$. The explicit calculations for the coefficients $C_1$ and $C_2$ in (2.3) yield the following answers:

$$C_1 = A \left( \frac{1}{8\pi^2} \right) C_F m_b^3 \ln(-m_b^2)$$

$$C_2 = A \left( \frac{1}{8\pi^2} \right) \frac{N_c}{2} m_b \ln(-m_b^2)$$

Here, $N_c$ stands for the number of colours and $C_F = (N_c^2 - 1)/2N_c$. Using the fact that $\text{Im} \ln(-m_b^2) = \pi$ we arrive at the final expression for the inclusive decay rate

$$\Gamma = \frac{G_F^2}{2} \frac{g^2}{16\pi^4} V_{st}^2 V_{ub}^2 C_W^2 \frac{1}{M_B} C_F \frac{m_b}{8\pi^2} (B[\bar{b}b]B)$$

$$+ \frac{m_b}{16\pi^2} \left( \frac{1}{2N_c} - N_c \right) \langle B[\bar{b}aG_b|B] \rangle \quad (3.3)$$

Substituting expressions (2.4), (2.5), (2.6) and (2.7) for the matrix elements and using all numerical values we finally determine the transition rate:

$$\Gamma \simeq 1.1 \times 10^{-15} \text{ GeV}.$$  

The appropriate branching ratio is $2.5 \times 10^{-3}$.

IV. CONCLUSIONS

In the present paper, we calculated nonperturbatively the inclusive nonleptonic transition rate of the B meson in the gluonic penguin channel $b \to s g$ by means of the pre-asymptotic heavy quark mass expansion. The obtained BR is about 0.25% while the nonperturbative contribution to the transition rate is about 4%. Our result provides the Standard Model predictions beyond the perturbation theory and is to be checked experimentally. The number obtained means that a significant contribution to the $b \to s g$ decay mode is only possible within the New Physics scenarios.

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