Data Fusion Technique for Multibeam Echosoundings

HUANG Motao ZHAI Guojun OUYANG Yongzhong LIU Yanchun

1 Introduction

In recent years multibeam echosounding systems, especially shallow-water mapping systems, have been developing rapidly for various applications. The main motivation for going from single beam echosounding to multibeam echosounding is the capability of producing maps of much higher quality and with much more details. Unlike conventional survey echosounders, the multibeam systems are designed for mapping 100% of the seafloor. The multibeam sonars transmit hundreds of beams from each side of the transmitter and receive the returned signal within its beamwidth. So multibeam echosounders can achieve swath coverage of the seafloor along the survey line with higher density and better resolution. The quality of the maps are higher because of the dense sounding pattern. The dense soundings reveal all the significant underwater features, and eliminate the large errors generated by having to guess (or interpolate) what the seafloor is like between the survey lines.

At present, the most important problem which remains to be resolved as soon as possible is how so high a volume of data from a multibeam system, particularly in case of very shallow water, can be managed and processed effectively. In China, the first set of homemade multibeam system named H/HCS-017 has been put into use in 1999. Hence there still exist many problems to be studied, and the data processing is more significant to the H/HCS-017 system. To evaluate or check the accuracy of a multibeam echosounder, in general, we often apply multibeam system and single beam echosounder to work simultaneously in a same vessel.
2 Analysis of error sources in multibeam echosounding

The observations in marine survey are affected not only by atmosphere, but also by the movement and physical property of ocean water. There exist, therefore, more noise sources in marine survey than in terrestrial survey. Taking the conventional shipboard depth sounding for example, in addition to the influences of well-known white noises from the sensor and positioning errors, the transducer arrays draught and the tidal level have to be taken into account in data processing due to the movement of ocean water. And the sound velocity profiles will become necessary to correct the observations due to the physical property change of ocean water. Moreover, the pulse signals emitted from echosounder could be reflected by some zooplanktons (e.g., fishes) and phytoplankton during their propagation. These false echoes and additional round trip echoes may result in a big discrepancy between the observed value and the true depth. As mentioned above, multibeam echosounding systems consist of multi-sensors. Therefore, they have not only several sources of error in common with single beam echosounders but also additional sources of the measurement error and errors due to measurements made by other sensors. That is to say, the accuracy of soundings in multibeam systems is dependent not only on their own sensors but also on the assistant sensors. With the multibeam operating principle, the additional error sources include the following aspects.

1) Signal detection errors. Multibeam echosounder, opposed to the conventional single beam echosounder, transmits its pulse of acoustic energy vertically, and in a wide fan. It means that the beam incident angles in multibeam systems change from the center beams to the outer beams. As we know, the amplitude detection method will lose its capability for detecting the return pulse from outer beams, the phase detection method has been introduced into the new generation equipment, multibeam systems. The amplitude detection is more suitable to map complex underwater terrain in which echo signals from more than one direction may arrive at transducer simultaneously, whereas the phase detection is a good method to map simple underwater terrain (more or less flat seabed). So the combination of the two methods can be complementary to each other. The two methods are characterized by different errors. The amplitude detection is related to the measurement of range, and the detection errors get bigger with the increase of beam incident angles. The phase detection errors affect the accuracy of beam incident angles.

2) Calibration residuals. After fitting the transducer head of multibeam system to the vessel, extensive calibrations must be performed to obtain the precise mounting offset angles and time delay between the transducer head and the other sensors of the system. However, because of some invisible disturbances, it is impossible to get a perfect result using the general calibration methods. There surely exist some calibration residuals in the installations of transducer head and gyrocompass. In this case, the transducer will not be oriented exactly the same as the vertical reference unit (VRU) during the survey. The roll and pitch transducer misalignment angles affect not only the echosoundings themselves (i.e., the range and beam angle) directly, but also the positions of the soundings on the seafloor. The heading misalignment has no effect on depth measurement, but shifts the positions of observed points. The pitch transducer misalignment angle makes the soundings shift in the direction of vessel travel, whereas the heading misalignment lets soundings rotate around the footprints of the center beams. Therefore, the influences of calibration residuals on the measured ranges and beam angles are complicated.

3) Sound speed errors. There will be errors introduced into the final depth and position as a result from imperfectly known sound speed profiles. As we know, the mean sound velocity is a common contribution for both single beam and multibeam
systems. The additional sound velocity errors are mainly related to the ray-bending caused by variations of the sound velocity with depth. Due to the different densities of the water column through which the sound is travelling, the sonar signal will bend (effect of Snell's law) either toward or away by the center beam, causing a distortion of seafloor topography. This bending effect is stronger for beams that travel under an incident angle with the water layers (usually the outer beams). Although this effect can be compensated to a great extent based on the sound speed profiles, there will normally remain some residual error, due to the uncertainty remained in measuring these profiles, as well as small variations with time and also over the survey area.

In addition, the measured roll and pitch angles, as determined by the VRU, also contain measurement noise and may be affected by a time delay from the VRU. The errors mentioned above may affect the echosoundings themselves directly, because roll and beam angle are always additive. A more detailed account of the errors in multibeam system has been given in Reference[1].

According to the above analysis of error sources, it can be seen that the influences of disturbing factors on the measurement in multibeam system become more apparent from center beams to outer beams. That is to say, in general, the accuracy of center beams is higher than that of outer beams. It means that multibeam system belongs to a non-uniform accuracy echosounding. However, the non-uniform accuracy is characterized by certain regularity. On the basis of this fact, an error compensation model can be reasonably built in the following section.

3 Mathematical model for data fusion

3.1 Mathematical model for data fusion within multibeam echosoundings

As mentioned above, multibeam echosounder is a bathymetric swath survey system. In order to arrive at 100% coverage of the seafloor, the survey procedures should be changed slightly from earlier practice. One significant change is to plan the survey lines to be overlapping, in general, as much as 10%. This survey practice will remove all uncertainty related to interpolation, and is close to an absolute guarantee that all underwater obstacles and features are detected. In addition, it will make the quality of the collected sounding data easier to assess than before, because comparison between results from neighboring swaths easily reveals inconsistencies in the data. Apparently, due to the disturbance of different internal and external factors, the multibeam soundings during their acquisitions are unavoidably affected by different kinds of error sources. As a result, there surely exists a discrepancy between results from neighboring swaths. To obtain continuous and smooth sounding data covering the full survey area, it is necessary to carry out a data fusion in the data overlapping area. It can be considered as an important part of data post-processing in multibeam survey.

It is clear from the preceding error analysis that the performance of the errors from different stages of the multibeam survey appears mainly systematic influence on the measurements. The combined effect of the errors will vary in a very complicated way. It may consist of linear, periodic, and irregular trends. In this paper, we will regard the systematic error effect as a special kind of signal and express it with a trend surface function as follows:

\[ \delta = F_1(x, y) = a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2 \]  

or

\[ \delta = F_2(x, y) = F_1(x, y) + \sum_{i=1}^{2}(c_i \cos \alpha_i x + d_i \sin \alpha_i x) + \sum_{i=1}^{2}(e_i \cos \omega_i y + f_i \sin \omega_i y) \]  

where \( \delta = F(x, y) \) represents the systematic error; \((x, y)\) is the coordinate of the observed point; \( \omega \) indicates the angular frequency corresponding to a period covering a survey line, i.e., \( \omega_x = 2\pi/(x_e - x_b) \) and \( \omega_y = 2\pi/(y_e - y_b) \), where \( x_e \) and \( x_b \) are the end and beginning coordinates of a survey line in X axis direction, respectively, and \( y_e \) and \( y_b \) are the corresponding values in Y axis direction; \( a_i, c_i, d_i, e_i, \) and \( f_i \) are the unknown
model parameters to be evaluated. According to Eq. (1) or Eq. (2), a measurement of sounding at an arbitrary observed point can be divided into three parts as follows:

\[ z = z_0 + F(x, y) + \Delta \]  

where \( z \) represents the measurement of sounding; \( z_0 \) is the true value of \( z \); \( \Delta \) indicates the random error. Letting \( v \) be the correction of the random error, and then introducing the difference of two measurements at common points in the overlapping area of neighboring swaths such as survey swath \( i \), and \( j \) as a new kind of measurement, we can define an error equation as

\[ v_{ij}^{(j+1)} - v_{ij} = -F_{ij}^{(j+1)}(x, y) + F_{ij}(x, y) + \left( z_{ij}^{(j+1)} - z_{ij} \right) \]  

where the subscript \( i \) represents the order number of common points in the overlapping area of neighboring swaths; the superscript \( j \) is the order number of survey swaths. As far as the neighboring swath \( j \) and \( (j + 1) \) are concerned, the error equations can be expressed in matrix notation as follows:

\[ \mathbf{V}(j, j+1) = \mathbf{A}(j, j+1) \mathbf{X}(j, j+1) - \mathbf{L}(j, j+1) \]  

where \( \mathbf{V} \) represents the correction vector of random error; \( \mathbf{X} \) is the unknown parameter vector of error model; \( \mathbf{A} \) is the coefficient matrix; \( \mathbf{L} \) is the discrepancy vector; and the superscript \((j, j+1)\) indicates the concerned values corresponding to the neighboring swath \( j \) and \((j + 1)\). In every overlapping area of neighboring swaths, an equation with the same form as Eq. (5) can be built through the measurements. Suppose the total number of swaths in overall survey area to be \( m \), then the number of equations such as Eq. (5) is \((m - 1)\). Since the neighboring equations share half of the unknown parameters in one equation, there exists a function relation between the neighboring equations. Therefore, we have to solve all the equations in an integrated way.

In order to simplify the procedure of solving the equations, the sequential adjustment method \(^{(2)}\) can be used to achieve their rigorous solutions. However, it should be pointed out here that only relative measurements (differences) are in Eq. (5), only relative parameters can be determined uniquely (completely similar to the crossover adjustment problem in processing satellite altimeter data). It is because the differences of measurements will be invariant with respect to a certain transformation of the two error models for neighboring swaths, for example, a constant is subtracted from or added to the two error models. It means that the adjustment problem here is singular. If the parameters in absolute sense are desired, some additional constraints have to be fixed. In theory, we can apply the same method as Huang et al. (1999) used in processing the marine gravity data. That is to say, by introducing pseudo-observations into the error equations and regarding the unknowns as a special kind of signals with a priori statistical characteristic, the hybrid system is solved with a proper weighting between real observations and pseudo-observations. And finally, the rank deficiency of the adjustment problem can be overcome successfully. In practice, however, due to the high volume of data in multibeam survey, it is difficult to realize the procedure of solving the equations, if we apply indiscriminately the rigorous method used in Huang et al. (1999). It is very complicated and time-consuming.

We propose an approximate approach to deal with the systematic error compensation in multibeam echosoundings. The key of the method is to divide the data fusion of neighboring swaths into two steps. Firstly, a conventional overlapping point adjustment is carried out by using the condition adjustment. The adjustment model is:

\[ B \mathbf{V} - L = 0 \]  

where \( \mathbf{V} \) represents the correction vector including systematic and random errors; \( B \) is the coefficient matrix which consists of \( 1 \) and \(-1\); \( L \) denotes the discrepancy vector. The least square solution of Eq. (6) is:

\[ \mathbf{V} = P^{-1}B^T(BP^{-1}P^T)^{-1}L \]  

The cofactor matrix is:

\[ Q\mathbf{V} = P^{-1}B^T(BP^{-1}P^T)^{-1}BP^{-1} \]  

According to the modern adjustment theory, after the correction vector is calculated from Eq. (7), it can be further considered as a new kind of observations and also can be filtered by using the same error model as Eq. (1) or Eq. (2) shows. The difference between the previous rigorous method and
the approximate approach here is that each swath in the latter can be processed separately, i.e., regarding it as independent of other swaths, by neglecting the statistical correlation of errors between neighboring swaths. Suppose that the “observation” (i.e., the \( \mathbf{V} \) values) vector of the \( j \)-th survey swath is \( \mathbf{V}_j = \mathbf{V}_j^l \), and the corresponding unknown vector of error model is \( \mathbf{X}_j \). Then, similar to Eq. (5), we can build a group of error equations at this stage as follows:

\[
\mathbf{V}_j = \mathbf{A}_j \mathbf{X}_j - \mathbf{L}_j 
\]

Its least square solution is:

\[
\mathbf{X}_j = \left( \mathbf{A}_j^T \mathbf{P}_j \mathbf{A}_j \right)^{-1} \mathbf{A}_j^T \mathbf{P}_j \mathbf{V}_j 
\]

where \( \mathbf{A}_j \) and \( \mathbf{P}_j \) represent the coefficient matrix of error equation and the weight matrix of “observation” vector \( \mathbf{V}_j \), respectively.

Compared with the rigorous method using sequential adjustment applied in Huang (1994), it should be admitted that, in theory, there exist some approximations in the two-step approach suggested here, due to neglecting the statistical correlation of errors between neighboring swaths. In practice, however, the approximate approach has great advantage over the rigorous method, which is shown particularly in both computation procedure and compensation effect. Apparently, it will greatly simplify the computation procedure for the data fusion of multibeam echosoundings by using the two-step approach, because the neighboring swaths can be processed separately. In this case, the rank deficiency will not appear again in the error equation. Therefore, the adjustment results of the latter should be more stable and reliable.

3.2 Mathematical model for data fusion between multibeam and single beam echosoundings

In the previous section, it has proposed that the discrepancies of depth in the overlapping area of neighboring swaths were used as observations to build error equations. And then a two-step approach was suggested to solve the adjustment problem, in spite of the fact that we can achieve a reasonable mosaic seafloor surface between the neighboring swaths by using the method mentioned above, it should not be thought to be a perfect approach yet. The reason is that the overlapping area is always located on the edges of neighboring swaths. And it is difficult to characterize completely the systematic errors of overall swaths by using such local information to build error equations.

In order to remedy the defect in the previous method, a new way to improve the compensation effect of systematic error in multibeam survey is proposed. The key idea of the new method is to merge the single beam soundings into the data fusion with the multibeam measurements. Having been developed for several decades, the single beam echosounders are very adequate to the hydrographic survey. In addition, there exists no ray-bending problem in the center beams of multibeam systems. So we can regard reasonably the single beam soundings and the center multibeam data as of equivalent accuracy, and merge the difference information between them into the discrepancy “observations” of neighboring swaths to determine the compensation model of systematic errors. Since we still maintain to use the two-step adjustment, apparently, the mathematical models used in the new approach are completely the same as in the previous method. The only difference is that, on different solving stages, some new equations built from the discrepancies between single beam soundings and multibeam measurements are added to the original condition equations and the original error equations for every swath. Thanks to the addition of comparison “observations” in center beams as control information, the obtained error compensation model will show an overall feature and an expected reasonability. This will be further illustrated in the case study of this paper.

3.3 The significance test of compensation efficiency

Once the form of error model is determined, we can eventually calculate a set of unknown parameters corresponding to the error model through the two-step adjustment method. In practice, however, according to the theory of mathematical statistics, only when there exists statistically a significant correlation between the discrepancy “observations” and the model parameters, the error compensation equations obtained from the adjustment
is of practical value. Otherwise, the adjustment procedure will treat mistakenly a part of white noise as signals (i.e., systematic errors). In this case, the residual discrepancy of the soundings between neighboring swaths can become small superficially. But in fact, the adjustment will result in the deformation of observations and conversely reduced the survey accuracy. Therefore, we still need to do the significance test of compensation efficiency after the adjustment. This problem is now discussed as follows:

Let $L_i (i = 1, 2, \ldots, m_1)$ be the discrepancy of soundings at overlapping points, and $L_i$ be the corresponding compensation values of systematic errors. With $Q$ being the total sum of square deviation, i.e.,

$$Q = \sum_i (L_i - \bar{L})^2$$

and further decomposing it, we have:

$$Q = \sum_i (L_i - \bar{L})^2 = Q_1 + Q_2$$

where

$$\bar{L} = \frac{\sum L_i}{m_1}$$

$Q_1$ is called the sum of square deviation in compensation and $Q_2$ the sum of residual square deviation. It can be seen from Eq. (13) that the compensation efficiency of systematic errors depends on the ratio of $Q_1$ to $Q_2$. If $Q_1$ is greatly larger than $Q_2$, it is shown that the compensation effect is major in the total sum of square deviation, i.e., the compensation efficiency is of significance. On the contrary, if $Q_2$ is major in $Q$, the compensation efficiency is of no significance. It can be proved that the well-known $F$-test for parameter significance with the null hypothesis $H_0$: can be used to see whether the compensation efficiency is significant or not\(^{[3]}\). The statistic variable is:

$$F = \frac{Q_1/n_1}{Q_2/(m_1 - n_1 - 1)}$$

where $n_1$ is the number of the unknown parameters to be evaluated. If the $F$-test statistic variable $F$ is larger than the critical value $F_a$ at a given significance level $\alpha$, the hypothesis does not hold and the compensation efficiency is significant. Conversely, the hypothesis holds and the compensation efficiency is not significant.

### 4 A case study

In order to evaluate the new methods proposed in this paper, a practical network surveyed with the homemade multibeam system named H/HCS-017 is used as a case study to make some necessary tests and comparisons. The survey network consists of eight main swaths in north-south direction and three cross swaths in east-west direction. The length of the main swaths is about 6 km. The positioning in survey is performed by a difference-GPS system, of which the standard deviation is less than $5$ m. To save space, only the processed results of swaths No. 5 to No. 8 will be given here. The statistics of the observed depths themselves for the four swaths above are shown in Table 1. The discrepancies of soundings between the neighboring swaths are given in Table 2.

| Order number of swaths | Number of points | Number of overlapping points | Max.      | Min.      | Mean   | RMS   | STD  |
|------------------------|------------------|------------------------------|-----------|-----------|--------|-------|------|
| 5                      | 17 056           | 875                          | 106.7     | 101.2     | 104.1  | 104.1 | 1.24 |
| 6                      | 14 896           | 828                          | 106.5     | 100.5     | 103.6  | 103.7 | 1.27 |
| 7                      | 17 280           | 1 016                        | 107.5     | 100.8     | 104.3  | 104.3 | 1.43 |
| 8                      | 16 400           |                               |           |           |        |       |      |

Comparing Table 1 with Table 2, it can be seen that the variation amplitude of sounding discrepancies is even greater than that of soundings themselves. This fact shows that there exist, apparently, some uncertain factors affecting the data set of soundings corresponding to that cruise. We take Eqs. (1) and (2), respectively, as the error model for data fusion within multibeam echosoundings. And the systematic errors are compensated by using the
two-step adjustment mentioned above. The comparison of results between neighboring swaths after data fusion are listed in Table 3.

| Error model | Order number of swaths | Max. | Min. | Mean | RMS | F test values |
|-------------|------------------------|------|------|------|-----|--------------|
| Eq. (1)     | 5                      | 2.35 | -1.41| 0.01 | 0.48| 1 070        |
|             | 6                      | 1.76 | -1.80| -0.01| 0.45| 593          |
|             | 7                      | 1.93 | -1.92| -0.02| 0.59| 1 250        |
|             | 8                      | 1.36 | -1.09| -0.01| 0.39| 1 592        |
| Eq. (2)     | 5                      | 1.69 | -1.55| -0.00| 0.42| 1 291        |
|             | 6                      | 1.47 | -1.35| 0.00 | 0.44| 1 494        |

It can be seen from Table 3 that the mosaic effect of neighboring swaths has been improved greatly after data fusion. When the significance level \( \alpha \) is taken to be 0.05, \( F_{\alpha} \) is less than 5.0, thus all the \( F \) test values in Table 3 are larger than \( \alpha \). It shows that the procedure of compensating systematic errors above is effective. Comparatively, the compensation efficiency of taking Eq. (2) as error model is better than that corresponding to Eq. (1).

By taking into account the fact that a dual-frequency single beam echosounder is used to sound together with H/HCS-017 multibeam system, simultaneously, it is possible to merge the two sets of information into the adjustment procedure by using the data fusion model mentioned above. The discrepancies between the single beam soundings and the center beams of multibeam soundings before data fusion are shown in Table 4.

| Order number of swaths | Number of compared points | Max. | Min. | Mean | RMS | STD |
|------------------------|----------------------------|------|------|------|-----|-----|
| 5                      | 351                        | -0.70| -1.60| -1.27| 1.28| 0.14|
| 6                      | 307                        | -1.11| -2.48| -1.31| 1.32| 0.10|
| 7                      | 366                        | 0.24 | -2.60| -1.20| 1.25| 0.33|
| 8                      | 368                        | -0.62| -1.73| -1.29| 1.29| 0.13|

It can be seen from Table 4 that there exist, apparently, some systematic differences between the single beam and multibeam soundings. Eq. (1) and Eq. (2) are used as error models, respectively, to carry out the data fusion among the mixed soundings through the two-step adjustment. The comparison results after data processing are listed in Table 5 and Table 6, respectively.

| Error model | Order number of swaths | Max. | Min. | Mean | RMS | STD |
|-------------|------------------------|------|------|------|-----|-----|
| Eq. (1)     | 5                      | 0.22 | -0.52| -0.17| 0.20| 0.11|
|             | 6                      | 0.28 | -0.47| -0.10| 0.15| 0.11|
|             | 7                      | 0.43 | -0.38| -0.18| 0.19| 0.07|
|             | 8                      | 0.40 | -0.97| -0.32| 0.36| 0.16|
| Eq. (2)     | 5                      | 0.53 | -0.41| -0.03| 0.12| 0.12|
|             | 6                      | 0.19 | -0.37| -0.01| 0.09| 0.09|
|             | 7                      | 0.52 | -0.28| -0.11| 0.12| 0.07|
|             | 8                      | 0.41 | -0.92| -0.26| 0.31| 0.17|

Comparing Table 5 with Table 4, and Table 6 with Table 2, we can see that after having added the differences between the single beam and center multibeam soundings to the procedure of data fu-
sion as control information, the original systematic errors have been removed successfully. And the mosaic effect of neighboring swaths also has been improved greatly. This fact shows again that the data fusion methods of echosoundings proposed in this paper are really feasible and effective.

| Error model | Order number of swaths | Max. | Min. | Mean | RMS | F-test values |
|-------------|------------------------|------|------|------|-----|---------------|
| Eq. (1)     | 5                      | 1.55 | −1.73| −0.03| 0.54| 918           |
|             | 6                      | 1.70 | −1.64| 0.00 | 0.46| 474           |
|             | 7                      | 2.10 | −1.89| −0.06| 0.62| 735           |
|             | 8                      | 1.30 | −1.47| 0.00 | 0.41| 952           |
| Eq. (2)     | 5                      | 1.65 | −1.49| 0.00 | 0.44| 1003          |
|             | 6                      | 2.37 | −1.53| −0.07| 0.49| 1068          |
|             | 7                      |      |      |      |     |               |

5 Conclusion

The following conclusions can be drawn from the preceding discussions and the case study:

1) The data fusion technique through building error model and dealing with an adjustment system can effectively compensate the systematic errors in multibeam survey and improve the mosaic smoothness of neighboring swaths.

2) Using single beam soundings as control information is valuable to the improvement of overall swath accuracy.

References

1. Hare R (1995) Depth and position error budgets for multibeam echosounding. *International Hydrographic Review*, LXXII(2): 37-69
2. Huang M T (1994) Configuration optimization of the disturbing point masses model and its sequential solution, *Acta Geodaetica et Cartographica Sinica*, 23(2): 81-89 (in Chinese)
3. Li Q H, Tao B Z (1982) The application of probability and mathematical statistics in survey adjustment. Beijing: Publishing House of Surveying and Mapping. (in Chinese)
4. Huang M T, Zhai G J, Guan Z, et al. (1999) On the compensation of systematic errors in marine gravity measurements. *Marine Geodesy*, 22: 183-194
5. Basu A, Saxena N K (1999) A review of shallow water mapping systems. *Marine Geodesy*, 22(3): 249-257
6. Li J B, Wang X B, Zhang Z H, et al. (1999) Multibeam sounding principles, survey technologies and data processing methods. Beijing: Oceanographic Publishing House. (in Chinese)
7. Chen F F (1999) The development of researches on multibeam swath bathymeter. *Ocean Technology*, 18(2): 26-32. (in Chinese)
8. Mocaffrey E K (1981) A review of the bathymetric swath survey system. *International Hydrographic Review*, LVI(1): 19-27
9. Tyce R C (1986) Deep seafloor mapping systems—a review. *Journal of Marine Technology Society*, 20(4): 4-16