Influence of losses on the output voltage of ferrite transformer in case of strong magnetic field in the core

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Taking into account the nonlinear properties of ferrites the output voltage of toroidal transformer with magnetically soft ferrite core is calculated. Calculations are based on methods analogous to those used in nonlinear optics. Initially demagnetized ferrite is assumed to be isotropic. At this condition apart from the harmonic on fundamental frequency the output voltage of transformer contains also higher harmonics of odd order. Nonlinear susceptibilities of third and fifth order are used in calculations Magnetic losses are taken into account considering linear and nonlinear susceptibilities as being complex. These losses affect amplitudes of harmonics in the output voltage of transformer and create phase shifts between components of magnetization on different frequencies in the core and the intensity of exciting harmonic magnetic field. Results obtained in calculations are confirmed by experimental measurement of output voltage of toroidal transformer with MnZn ferrite core. In order to calculate amplitudes of harmonics in the output voltage of transformer numerical values of susceptibilities in question should be known.

1. Introduction
In last decades due to their magnetic and other properties ferrites have found applications in different areas of electronics. In general the magnetization of ferrites is a nonlinear function of the strength of the magnetic field. If the magnetic field applied is sufficiently weak ferrites can be considered as being linear substances. However, there are applications where nonlinear properties of ferrites play an important role, e.g., in [1] an application is considered where intermodulation distortions of ferrites should be taken into account. In [2], we considered the influence of nonlinear properties of ferrite on the voltage of toroidal transformer with a soft ferrite (narrow hysteresis loop) core. The output voltage of transformer was calculated in a simplified case when losses in the core were not taken into account. The goal of this paper is to generalise the phenomenological description developed in [2] by taking into account also losses in the core.

From the experimental point of view it was very well known that the sufficiently strong magnetic field in the core leads to the distortion of the signal shape and the appearance of harmonics. Results obtained in [2] are based on the following statements. Magnetization $\mathbf{M}$ of ferrite can be expanded in power series of the intensity of the magnetic field $\mathbf{H}$ [3,4]. In symbolic form one can write

$$\mathbf{M} = \chi^{(1)} \mathbf{H} + \chi^{(2)} \mathbf{H}\mathbf{H} + \chi^{(3)} \mathbf{H}\mathbf{H}\mathbf{H} + \chi^{(4)} \mathbf{H}\mathbf{H}\mathbf{H}\mathbf{H} + \chi^{(5)} \mathbf{H}\mathbf{H}\mathbf{H}\mathbf{H}\mathbf{H} + \cdots,$$

(1)

where $\chi^{(1)}$ is the linear but $\chi^{(2)} - \chi^{(5)}$ are nonlinear magnetic susceptibilities. In fact susceptibilities in (1) are tensors of corresponding rank.

Initially demagnetised ferrite core as polycrystalline substance is isotropic due to the arbitrary orientation of crystallites. This means that properties of ferrite do not depend from the direction in space, respectively, should stay invariant under the transform $\mathbf{H} \rightarrow -\mathbf{H}$. The consequence of this is that susceptibilities of even order in (1) are zero. Indeed, the product of even number of $\mathbf{H}$ doesn’t
change the sign at transform $\overline{H} \rightarrow -\overline{H}$ while the magnetization $\overline{M}$ does. From this it follows that at transform $\overline{H} \rightarrow -\overline{H}$ $\chi^{(2k)} = -\chi^{(2k)}$ ($k$ is a natural number). This is possible only if $\chi^{(2k)} = 0$. In contrast, susceptibilities of odd number don’t change the sign at transform $\overline{H} \rightarrow -\overline{H}$ and, therefore, in general are nonzero. In this paper we will confine ourselves to the consideration of susceptibilities up to the fifth order including.

2. Magnetization of the core

For toroidal cores the cylindrical system of reference is appropriate. In what follows it is supposed that only the component $H_\varphi$ of the magnetic field is nonzero. Due to isotropy the magnetization has the same direction. Besides it is also supposed that the harmonic magnetic field acts on the core and the wavelength of the field is much greater than the linear dimensions of the core. Then

$$H_\varphi(\omega) = H_\varphi \cos \omega t = \left[ H_\varphi(\omega)\exp(-i\omega t) + H_\varphi(-\omega)\exp(i\omega t) \right] H_\varphi(-\omega) = \frac{1}{2} H_\varphi . \tag{2}$$

In (2) $i$ is the imaginary unit and $*$ means the complex conjugated expression.

Let us write the magnetization of the core as a sum of three terms

$$M_\varphi = M_\varphi^{(1)} + M_\varphi^{(3)} + M_\varphi^{(5)} , \tag{3}$$

where the first one is determined by the linear susceptibility, the second one by the cubic susceptibility and the last one by the susceptibility of the fifth order. Let us first consider $\overline{M}_\varphi^{(3)} = \overline{M}_\varphi^{(3)} + \overline{M}_\varphi^{(3)*}$. Due to analogy between the electrical polarisation and the magnetization methods used in nonlinear optics [3,4] can be applied here. For magnetization one can write

$$M_\varphi^{(3)} = \sum_{k\ell m} \chi_{4\varphi}^{(3)}(\omega_k, \omega_\ell, \omega_m) H_\varphi(\omega_k) H_\varphi(\omega_\ell) H_\varphi(\omega_m) \exp[-i(\omega_k + \omega_\ell + \omega_m)] . \tag{4}$$

In (4) $\chi_{4\varphi}^{(3)}$ means $\chi_{\varphi\varphi\varphi}^{(3)}$ and $\omega_k + \omega_\ell + \omega_m$. At perturbation of the core with a harmonic field only two frequencies $\omega$ and $-\omega$ are present. Then $\overline{M}_\varphi^{(3)}$ has two time dependent terms, one with the fundamental frequency $\omega$ and another with the triple frequency $3\omega$. The first one is

$$\overline{M}_\varphi^{(3)}(\omega) = \left[ \chi_{4\varphi}^{(3)}(\omega, \omega, -\omega) + \chi_{4\varphi}^{(3)}(\omega, -\omega, \omega) + \chi_{4\varphi}^{(3)}(\omega, -\omega, \omega) \right] \times \left| H_\varphi(\omega) \right|^2 H_\varphi(\omega) \exp[-i\omega t] \tag{5}$$

From the symmetry properties of nonlinear susceptibilities [3,4] it follows that all susceptibilities in (5) are equal and, therefore

$$\overline{M}_\varphi^{(3)}(\omega) = 3\chi_{4\varphi}^{(3)}(\omega, \omega, -\omega) \left| H_\varphi(\omega) \right|^2 H_\varphi(\omega) \exp[-i\omega t] . \tag{6}$$

The second one is the following

$$\overline{M}_\varphi^{(3)}(3\omega) = \chi_{4\varphi}^{(3)}(3\omega, \omega, \omega) H_\varphi^3(\omega) \exp[-i3\omega t] . \tag{7}$$

Completely analogously one can consider terms caused by susceptibilities of the first and the fifth order. Magnetization caused by the susceptibility of the first order is

$$M_\varphi^{(1)}(\omega) = \chi_{2\varphi}^{(1)}(\omega) H_\varphi(\omega) \exp[-i\omega t] . \tag{8}$$
Correspondingly, susceptibility of the fifth order determine 3 terms each on different frequencies. They are as follows

\[ \tilde{M}^{(5)}_{\varphi}(\omega) = 10 \chi_{6p}^{(5)}(\omega, \omega, \omega, \omega, -\omega) \left[ H_{\varphi}(\omega) \right]^4 H_{\varphi}(\omega) \exp[-i\omega t], \] (9)

\[ \tilde{M}^{(5)}_{\varphi}(3\omega) = 5 \chi_{6p}^{(5)}(3\omega, \omega, \omega, \omega, -\omega) \left[ H_{\varphi}(\omega) \right]^3 H_{\varphi}(\omega) \exp[-i3\omega t]. \] (10)

\[ \tilde{M}^{(5)}_{\varphi}(5\omega) = \chi_{6p}^{(5)}(5\omega, \omega, \omega, \omega, \omega) H_{\varphi}^5(\omega) \exp[-i5\omega t]. \] (11)

In (9) and (10) symmetry properties of the susceptibility of the fifth order analogous to those used in (5) are taken into account.

3. Magnetic flux and the output voltage of transformer

The magnetic flux in the core is given by \( \Phi = B_\varphi S \) where the induction of the magnetic field \( B_\varphi \) on the basis of (6) to (11) can be expressed as a sum of three terms

\[ B_\varphi = B_\varphi(\omega) + B_\varphi(3\omega) + B_\varphi(5\omega), \] (12)

where

\[ B_\varphi(\omega) = \mu_0 \left( 1 + \chi^{(1)}_{2p}(\omega) + \frac{3}{4} \chi^{(1)}_{4p}(\omega) H_{\varphi}^2 + \frac{5}{8} \chi^{(5)}_{6p}(\omega) H_{\varphi}^4 \right) \frac{H_{\varphi}}{2} \exp(-i\omega t) + c.c., \] (13)

\[ B_\varphi(3\omega) = \mu_0 \left( \chi^{(3)}_{4p}(3\omega) + \frac{5}{4} \chi^{(5)}_{6p}(3\omega) H_{\varphi}^2 \right) \frac{H_{\varphi}^3}{8} \exp(-i3\omega t) + c.c., \] (14)

\[ B_\varphi(5\omega) = \mu_0 \chi^{(5)}_{6p}(5\omega) \frac{H_{\varphi}^5}{32} \exp(-i5\omega t) + c.c. \] (15)

In (13) to (15) c.c. stands for the complex conjugated expression and for the sake of simplicity the following designations are introduced

\[ \chi^{(1)}_{4p}(\omega) = \chi^{(1)}_{4p}(\omega, \omega, \omega, -\omega), \quad \chi^{(5)}_{6p}(\omega, \omega, \omega, \omega, -\omega), \quad \chi^{(1)}_{4p}(3\omega) = \chi^{(1)}_{4p}(3\omega, \omega, \omega, \omega). \]

\[ \chi^{(5)}_{6p}(3\omega) = \chi^{(5)}_{6p}(3\omega, \omega, \omega, \omega, -\omega) \quad \text{and} \quad \chi^{(5)}_{6p}(5\omega) = \chi^{(5)}_{6p}(5\omega, \omega, \omega, \omega, \omega). \]

Due to magnetic losses in the core all susceptibilities in (13) to (15) become complex, respectively have the form

\[ \chi^{(2k+1)} = \text{Re} \chi^{(2k+1)} + \text{Im} \chi^{(2k+1)}, \quad k=0,1,2. \] (16)

Inserting (16) into (13) to (15) one obtains

\[ B_\varphi = A \cos(\omega t - \delta_1) + C \cos(3\omega t - \delta_3) + D \cos(5\omega t - \delta_5), \] (17)

where
\[ A = \mu_0 \left[ 1 + \text{Re} \chi_{2p}^{(1)}(\omega) + \frac{3}{4} \text{Re} \chi_{4p}^{(3)}(\omega)H_{\varphi}^2 + \frac{5}{8} \text{Re} \chi_{6p}^{(5)}(\omega)H_{\varphi}^4 \right] \left[ \text{Im} \chi_{2p}^{(1)}(\omega) + \frac{3}{4} \text{Im} \chi_{4p}^{(3)}(\omega)H_{\varphi}^2 + \frac{5}{8} \text{Im} \chi_{6p}^{(5)}(\omega)H_{\varphi}^4 \right]^{\frac{1}{2}} H_{\varphi}, \]  
\[ (18) \]

\[ C = \frac{\mu_0}{4} \left[ \text{Re} \chi_{4p}^{(3)}(3\omega) + \frac{5}{4} \text{Re} \chi_{6p}^{(5)}(3\omega)H_{\varphi}^2 \right]^{\frac{1}{2}} H_{\varphi}^3 \]
\[ + \left[ \text{Im} \chi_{4p}^{(3)}(3\omega) + \frac{5}{4} \text{Im} \chi_{6p}^{(5)}(3\omega)H_{\varphi}^2 \right]^{\frac{1}{2}} H_{\varphi}^3 \]
\[ (19) \]

\[ D = \frac{\mu_0}{16} \left[ \text{Re} \chi_{6p}^{(5)}(5\omega) \right]^2 + \left[ \text{Im} \chi_{6p}^{(5)}(5\omega) \right]^2 H_{\varphi}^5 \]
\[ (20) \]

and

\[ \delta_1 = \arctan \frac{\text{Im} \chi_{2p}^{(1)}(\omega) + \frac{3}{4} \text{Im} \chi_{4p}^{(3)}(\omega)H_{\varphi}^2 + \frac{5}{8} \text{Im} \chi_{6p}^{(5)}(\omega)H_{\varphi}^4}{1 + \text{Re} \chi_{2p}^{(1)}(\omega) + \frac{3}{4} \text{Re} \chi_{4p}^{(3)}(\omega)H_{\varphi}^2 + \frac{5}{8} \text{Re} \chi_{6p}^{(5)}(\omega)H_{\varphi}^4}, \]
\[ (21) \]

\[ \delta_3 = \arctan \frac{\text{Im} \chi_{4p}^{(3)}(3\omega) + \frac{5}{4} \text{Im} \chi_{6p}^{(5)}(3\omega)H_{\varphi}^2}{\text{Re} \chi_{4p}^{(3)}(3\omega) + \frac{5}{4} \text{Re} \chi_{6p}^{(5)}(3\omega)H_{\varphi}^2}, \]
\[ (22) \]

\[ \delta_5 = \arctan \frac{\text{Im} \chi_{6p}^{(5)}(5\omega)}{\text{Re} \chi_{6p}^{(5)}(5\omega)}. \]
\[ (23) \]

E.m.f. per turn \( E \) of toroidal transformer with ferrite core equals to the voltage per turn and is given by formula

\[ E = -\frac{d\Phi}{dt} = A\omega \sin(\omega t - \delta_1) + 3C\omega \sin(3\omega t - \delta_3) + 5D\omega \sin(5\omega t - \delta_5). \]
\[ (24) \]

In order to obtain the output voltage of transformer \( E \) given by (24) should be multiplied by the number of turns \( N \) in the output winding. If losses are absent, respectively, imaginary parts of all susceptibilities are zero then from (17) to (24) follow corresponding formulae obtained in [2]. Besides, the phase shifts between all magnetization components on different frequencies and the exciting magnetic field disappear.

For illustrative purposes we present here experimental measurements of the output spectrum of toroidal transformer with MnZn ferrite core (Fig. 1). It was checked that the spectrum of generator used in the input of transformer contained only the harmonic of fundamental frequency. Measurements
were performed at the saturation of the magnetic induction in the core (see Fig. 2). From Fig. 1 the presence of third and fifth harmonics in the output voltage of transformer is clearly seen. Measured output quantities at fundamental frequencies 1 and 10 kHz are similar to those shown on Fig. 1 and 2.

4. Conclusions
Methods used in nonlinear optics successfully can be applied to description of nonlinear properties of soft ferrites. An initially demagnetized ferrite can be considered as being isotropic. Due to this apart from fundamental harmonics of odd order appear in the output of transformer with soft ferrite core. Magnetic losses present in the core create the phase shifts between components of magnetization on different frequencies and the intensity of exciting magnetic field. They affect also numerical values of amplitudes of harmonics. In order to calculate these amplitudes numerical values of susceptibilities in question should be known.

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