Flavour alignment of New Physics in light of the $(g - 2)_\mu$ anomaly

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ABSTRACT: We investigate the flavour alignment conditions that New Physics (NP) models need to satisfy in order to address the $(g-2)_\mu$ anomaly and, at the same time, be consistent with the tight bounds from $\mu \to e\gamma$ and $\tau \to \mu\gamma$. We analyse the problem in general terms within the SMEFT, considering the renormalisation group evolution of all the operators involved. We show that semileptonic four-fermion operators, which are likely to generate a sizeable contribution to the $(g-2)_\mu$ anomaly, need to be tightly aligned to the lepton Yukawa couplings and the dipole operators in flavour space. While this tuning can be achieved in specific NP constructions, employing particular dynamical assumptions and/or flavour symmetry hypotheses, it is problematic in a wide class of models with broken flavour symmetries, such as those proposed to address both charged- and neutral-current $B$ anomalies. We quantify this tension both in general terms, and in the context of explicit NP constructions.

KEYWORDS: Beyond Standard Model, Heavy Quark Physics, Quark Masses and SM Parameters

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1 Introduction

The anomalous magnetic moment of the muon, $a_\mu = (g_\mu - 2)/2$, is a very powerful probe of possible physics beyond the Standard Model (SM). The recent experimental measurement of $a_\mu$ by the E989 experiment at FNAL [1], combined with the previous BNL result [2], has strengthened the discrepancy with the SM prediction reported in ref. [3] (see also [4–13]). While there is still some debate on the precise value of the SM prediction $a_\mu^{\text{SM}}$ (see in particular ref. [14]), the recent experimental result has stimulated a renewed interest in possible beyond-the-SM (BSM) contributions to this observable. A general analysis of $a_\mu$ within the SM Effective Field Theory (SMEFT), i.e. under the hypothesis of new degrees of freedom above the electroweak scale, has been presented in refs. [15–18].

The interest in BSM contributions to $a_\mu$ is further reinforced by the deviations from lepton flavour universality observed in neutral-current [19–22] and charged-current [23–28] semileptonic $B$ decays, often referred to as the $B$ physics anomalies. Indeed, also these observables indicate deviations from SM predictions in processes involving charged leptons and in particular muons (in the case of the neutral-current anomalies). Extensions of the SM aiming at combined explanations of $a_\mu$ and (some of) the $B$ physics anomalies have been presented in refs. [29–40]. The $a_\mu$ anomaly involves only leptons of the second generation and, taken alone, does not indicate any violation of flavour quantum numbers. It also does not provide a clear indication on the energy scale of the associated new dynamics [16, 17].
On the other hand, the $B$ physics anomalies involve fermions of different generations (if combined), necessarily implicate flavour changing dynamics (at least on the quark side), and point to New Physics above the electroweak scale. The goal of this paper is to analyse the general implications of the putative $a_\mu$ anomaly on the lepton flavour structure of the underlying NP model, assuming the latter is well described by the SMEFT. A key question to address in general terms is the compatibility of the $a_\mu$ anomaly with other non-standard phenomena and, moreover, to investigate the class of NP models favoured by this anomaly.

Within the SM, the three separate lepton flavour quantum numbers are conserved. However, this is an accidental property of the $d=4$ operators of the SMEFT, which might well be violated in the ultraviolet (UV) completion of the theory. Indeed already at $d=5$, in the neutrino mass matrix, we observe large lepton flavour mixing effects. As we shall show using general EFT arguments, if the $a_\mu$ anomaly is confirmed as clear evidence of NP, we are forced to assume that the conservation of lepton flavour plays an important role also above the electroweak scale. More precisely, the interplay between the possible evidence for NP associated to the $a_\mu$ anomaly and the tight bounds from $\mu \to e\gamma$ and $\tau \to \mu\gamma$, imply that lepton flavour (and in particular the electron flavour) must be conserved to a very good accuracy in a whole sector of the $d=6$ SMEFT operators. As we shall discuss, this symmetry property of the SMEFT is unlikely to hold accidentally.

The paper is organised as follows: in section 2 we briefly summarise the size of the muon dipole operator at the electroweak scale implied by the $a_\mu$ anomaly, and the bounds on the flavour violating dipole operators following from the non-observation of $\mu \to e\gamma$ and $\tau \to \mu\gamma$. In section 3 we show how, via Renormalisation Group (RG) evolution, this information translates into constraints on other SMEFT operators at higher scales, implying a non-trivial series of alignment conditions in flavour space. In section 4 we discuss how to fulfil these alignment conditions in general terms, either using dynamical hypotheses or via flavour symmetries. An illustrative implementation of these general mechanisms into a simple NP model based on scalar mediators is presented in section 5. The results are summarised in the Conclusions.

## 2 Experimental constraints on the leptonic dipole operators

We work under the assumption that NP is heavy and can be well described by the SMEFT Lagrangian, including effective operators up to $d=6$. The focus of our analysis are the leptonic dipole operators

$$\mathcal{O}_{e\gamma} = \frac{v}{\sqrt{2}} \bar{e}_L^r \sigma^{\mu\nu} e_R^s F_{\mu\nu}$$

written here below the scale of electroweak symmetry breaking, where $r$ and $s$ are generic flavour indices and $F_{\mu\nu}$ is the electromagnetic field strength tensor. Depending on the flavour structure, these operators can describe both NP effects in $a_\mu$, as well as non-vanishing rates for $\mu \to e\gamma$ and $\tau \to \mu\gamma$. The key point we want to investigate is the interplay between the evidence of a non-vanishing value for (some of) the Wilson coefficients of these operators, following from the $a_\mu$ anomaly, and the tight constraints derived by the non-observation of $\mu \to e\gamma$ and $\tau \to \mu\gamma$. 

The combined result from the E989 experiment at FNAL [1] and the E821 experiment at BNL [2] on $a_\mu$, together with the SM prediction in [3], imply

$$\Delta a_\mu = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}. \quad (2.2)$$

The tree-level expression for $\Delta a_\mu$ in terms of the Wilson coefficient of the dipole operator is

$$\Delta a_\mu = \frac{4m_\mu}{e} \frac{v}{\sqrt{2}} \text{Re} C'_{\gamma 22}, \quad (2.3)$$

where $v = (\sqrt{2}G_F)^{-1/2} \approx 246 \text{ GeV}$. Here, the Wilson coefficient is understood to be evaluated at the weak scale (we neglect the small effect of running below the weak scale) and the prime indicates the flavour basis corresponding to the mass-eigenstate basis of charged leptons.¹ Saturating the experimental results leads to

$$\text{Re} C'_{\gamma 22} \approx 1.0 \times 10^{-5} \text{ TeV}^{-2}. \quad (2.4)$$

The tree-level expression of a generic radiative LFV rate in terms of the $C_{e\gamma}$ coefficients is

$$\mathcal{B}(\ell_i \rightarrow \ell_j \gamma) = \frac{m_{\ell_i}^2 v^2}{8\pi \Gamma_{\ell_i}} \left( |C'_{\gamma ij}|^2 + |C'_{\gamma ji}|^2 \right). \quad (2.5)$$

Using this expression, the experimental bound $\mathcal{B}(\mu^+ \rightarrow e^+ \gamma) < 4.2 \times 10^{-13}$ (90% C.L.) obtained by the MEG experiment [42] can be translated into the upper bound

$$|C'_{e\gamma 12(21)}| < 2.1 \times 10^{-10} \text{ TeV}^{-2}. \quad (2.6)$$

Taking into account eq. (2.4), the requirement of fitting the $a_\mu$ anomaly and, at the same time, being consistent with the $\mathcal{B}(\mu^+ \rightarrow e^+ \gamma)$ bound, leads to the following tight constraints on off-diagonal over diagonal entries in the $2 \times 2$ light lepton sector:

$$|\epsilon_{12}^L|, |\epsilon_{12}^R| < 2 \times 10^{-5}, \quad (2.7)$$

where we have defined

$$\epsilon_{12}^{L(R)} \equiv \frac{C'_{\gamma 12(21)}}{C'_{\gamma 22}}, \quad \epsilon_{23}^{L(R)} \equiv \frac{C'_{\gamma 23(32)}}{C'_{\gamma 33}}. \quad (2.8)$$

The parameters $\epsilon_{23}^{L(R)}$ can be constrained by the bounds on radiative LFV decays of the $\tau$ lepton. In particular, $\mathcal{B}(\tau^\pm \rightarrow \mu^\pm \gamma) < 4.4 \times 10^{-8}$ (90% CL) as measured by the BaBar experiment [43] implies

$$|C'_{e\gamma 23(32)}| < 2.7 \times 10^{-6} \text{ TeV}^{-2} \quad (2.9)$$

¹The one-loop relation can be found in [41].
that, in turn, leads to
\[ |\epsilon_{23}^L| , |\epsilon_{23}^R| < 1.6 \times 10^{-2} \times \left| \frac{y_\tau C_{e\gamma}'}{\mu C_{e\gamma}''} \right|. \]

(2.10)

In absence of a direct experimental constraint on the anomalous magnetic moment of the \( \tau \) lepton, the normalisation of the bounds in eq. (2.10) has been chosen following the natural expectation
\[ |C_{e\gamma}''/y_\tau| \sim |C_{e\gamma}'/y_\mu|. \]

(2.11)

3 RG evolution of the leptonic dipoles in the SMEFT

In this section we analyse how the low-energy constraints derived before translate into high-scale constraints. To this purpose, we consider all possible \( d = 6 \) operators with the same leptonic flavour structure, i.e. operators of the type
\[ \ell_r \Gamma(A, H, \psi) e_s, \quad \ell_r \equiv (\nu_L^r e_L^r), \quad e_s \equiv e_{R_s}. \]

(3.1)

Those operators undergo a non-trivial mixing together with the dipole operators and/or the Yukawa couplings. On the other hand, we can safely ignore operators with a different flavour structure since either they do not mix with dipole (or Yukawa) operators or they provide only a trivial multiplicative renormalisation.

Adopting the SMEFT Warsaw basis [44] for the \( d = 6 \) effective operators, the list of relevant terms can be decomposed as
\[ \Delta L_{\text{unbroken}} = \Delta L_H + \Delta L_{4f} + \text{h.c.}, \]

(3.2)

where
\[ \Delta L_H = -[Y_e]_{pr}^\top (\bar{\ell}_p e_r) H + C_{eH} (\bar{\ell}_p e_r) H (H^\dagger H) \]
\[ + C_{eB} (\bar{\ell}_p \sigma^{\mu\nu} e_r) B_{\mu\nu} + C_{eW} (\bar{\ell}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I, \]

(3.3)

\[ \Delta L_{4f} = C_{\mu\nu}^{(3)} (\bar{\ell}_p \sigma^{\mu\nu} e_r) e_{jk} (\bar{d}_s^k \sigma^{\mu\nu} u_t) + C_{\mu\nu}^{(1)} (\bar{\ell}_p e_r) e_{jk} (\bar{d}_s^k u_t) \]
\[ + C_{\ell\text{eq}} (\bar{\ell}_p e_r) (\bar{d}_s q_t). \]

(3.4)

In order to identify the mass-eigenstate basis for the leptons, and the dipole operators defined in (2.1), we need to work in the broken phase of the SMEFT. To this purpose, we can rewrite \( \Delta L_H \) in the broken phase as
\[ \Delta L_H^{\text{broken}} = -\frac{\nu}{\sqrt{2}} (\bar{e}_L p e_R) - \frac{h}{\sqrt{2}} (\bar{e}_L p e_R) + C_{e\gamma} \frac{\nu}{\sqrt{2}} (\bar{e}_L p \sigma^{\mu\nu} e_R) F_{\mu\nu} \]
\[ + C_{eZ} \frac{\nu}{\sqrt{2}} (\bar{e}_L p \sigma^{\mu\nu} e_R) Z_{\mu\nu} + \mathcal{O}(h^2, h F_{\mu\nu}, h Z_{\mu\nu}), \]

(3.5)
where $Z_{\mu\nu}$ is the field strength tensor for the $Z$ boson and $h$ is the physical Higgs boson. The relations between terms in the broken and unbroken phase are

\[
\begin{pmatrix}
C_{e\gamma} \\
C_{eZ}
\end{pmatrix}_{rs} = \begin{pmatrix}
c_9 & -s_9 \\
-s_9 & c_9
\end{pmatrix}
\begin{pmatrix}
C_{eB} \\
C_{eW}
\end{pmatrix}_{rs},
\]  

(3.6)

\[
\begin{pmatrix}
[Y_{c}]_{rs} \\
[Y_{he}]_{rs}
\end{pmatrix} = \begin{pmatrix}
1 - \frac{1}{2} \\
1 - \frac{3}{2}
\end{pmatrix}
\begin{pmatrix}
[Y_{e}]_{rs} \\
v^2 C_{eH}
\end{pmatrix},
\]  

(3.7)

where

\[
c_9 = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} = \frac{e}{g_1}, \quad s_9 = \frac{g_1}{\sqrt{g_1^2 + g_2^2}} = \frac{e}{g_2}
\]  

(3.8)

with $g_2$, $g_1$, and $e$, the coupling constants of SU(2)$_L$, U(1)$_Y$, and U(1)$_{em}$ respectively.

### 3.1 Renormalization group equations

The RG equations for the SMEFT Wilson coefficients can be written in the form

\[
\mu \frac{d}{d\mu} C_i = \frac{d}{d \log \mu} C_i = \frac{1}{16\pi^2} \gamma_{ij} C_j = \frac{1}{16\pi^2} \beta_i,
\]  

(3.9)

where the $\gamma_{ij}$ (and $\beta_i$) in the Warsaw basis have been derived in [45–47] and are summarised in ref. [48]. We follow the Higgs potential normalisation conventions and the hypercharge assignments of the latter reference. In the limit in which we set all Yukawa $[Y_{u(d)}]_{33} = y_{t(b)}$, the $\beta_i$ relevant to our analysis are

\[
\beta_{Y_{rs}} = 2m^2 \left[ 3C_{eH} + N_c y_b C_{lequ} - N_c y_t C_{lequ}^{(1)} \right]_{rs33},
\]  

(3.10)

\[
\beta_{eH} = \left[ 12\lambda + 3N_c \left( \left| y_t \right|^2 + \left| y_b \right|^2 \right) - 3 \left( 3y_e^2 + 3y_e^2 - 4y_t y_e \right) g_1^2 + \frac{27}{4} g_2^2 \right] C_{eH}^{rs33}
\]  

\[
-6 \left[ 4g_1^2 y_b^2 (y_e + y_t) + g_2^2 y_t y_b \right] C_{eB}^{rs33}
\]  

\[
-3 \left[ 4g_1^2 g_2^2 y_b (y_e + y_t) + 3g_2^3 \right] C_{eW}^{rs33}
\]  

\[
+2N_c y_b \left( \lambda - 2 \left| y_b \right|^2 \right) C_{lequ}^{rs33} - 2N_c y_t \left( \lambda - 2 \left| y_t \right|^2 \right) C_{lequ}^{(1)}_{rs33},
\]  

(3.11)

\[
\beta_{eW} = N_c \left[ \left( \left| y_t \right|^2 + \left| y_b \right|^2 \right) C_{eW}^{rs} - 2g_2 z_c y_t^3 C_{lequ}^{(3)}_{rs33} \right]
\]  

\[
+ \left[ \left( 3c_{F,2} - b_{0,2} \right) g_2^2 + \left( -3y_e^2 + 8y_e y_t - 3y_t^2 \right) g_1 \right] C_{eW}^{rs} + g_1 g_2 \left( 3y_t - y_e \right) C_{eB}^{rs33},
\]  

(3.12)

\[
\beta_{eB} = N_c \left[ \left( \left| y_t \right|^2 + \left| y_b \right|^2 \right) C_{eB}^{rs} + 4g_1 (y_u + y_q) y_t y_b^3 C_{lequ}^{(3)}_{rs33} \right]
\]  

\[
+ \left[ -3c_{F,2} g_2^2 + \left( 3y_e^2 + 4y_e y_t + 3y_t^2 - b_{0,1} \right) g_1 \right] C_{eB}^{rs} + 4c_{F,2} g_1 g_2 \left( 3y_t - y_e \right) C_{eW}^{rs33},
\]  

(3.13)

where $c_{F,2} = 3/4$, $b_{0,1} = -1/6 - 20n_g/9$, and $b_{0,2} = 43/6 - 4n_g/3$ with the number of generations $n_g$. 


In terms of these expressions, working at one-loop accuracy, the solutions to the RG equations of the electromagnetic dipole operators and the mass Yukawa in the broken phase assume the form

\[
C_{e\gamma}(\mu_L) = C_{e\gamma}(\mu_H) + \frac{1}{16\pi^2} \log \left( \frac{\mu_L}{\mu_H} \right) \left( c_0 \beta_{eB} - s_0 \beta_{eW} \right), \quad \text{(3.14)}
\]

\[
[Y_e]_{rs} (\mu_L) = [Y_e]_{rs} (\mu_H) + \frac{1}{16\pi^2} \log \left( \frac{\mu_L}{\mu_H} \right) \left( \beta_{Y} - \frac{v^2}{2} \beta_{eH} \right), \quad \text{(3.15)}
\]

where we evolve the coefficients from a low scale \( \mu_L \) to the high scale \( \mu_H \). Further neglecting the terms proportional to the quartic Higgs coupling or to at least two powers of the gauge couplings, assuming \( y_t \) to be real, and defining \( \tilde{L} = \frac{1}{16\pi^2} \log \left( \frac{\mu_H}{\mu_L} \right) \), leads to

\[
C_{e\gamma}(\mu_L) = \left[ 1 - 3\tilde{L} \left( y_t^2 + y_b^2 \right) \right] C_{e\gamma}(\mu_H) - \left[ 16\tilde{L} y_t^2 \right] C_{lequ}^{(3)}(\mu_H), \quad \text{(3.16)}
\]

\[
[Y_e]_{ij} (\mu_L) = [Y_e]_{ij} (\mu_H) - \frac{v^2}{2} C_{eH}(\mu_H) + 6v^2 \tilde{L} \left[ y_t^3 C_{lequ}^{(1)} - y_t^3 C_{ledq}^{(1)} + \frac{3}{4} \left( y_t^2 + y_b^2 \right) C_{eH} \right] \quad \text{(3.17)}
\]

### 3.2 Rotation to the mass basis

Focusing on the 2 \times 2 sector of light lepton indices, we define the following ratios of Wilson coefficients at the high scale:

\[
\theta_{L}^Y = \frac{[Y_e]_{12}}{[Y_e]_{22}} \mu_H, \quad \theta_{L}^{e\gamma} = \frac{C_{e\gamma}}{C_{e\gamma}} \mu_H, \quad \theta_{L}^{eH} = \frac{C_{eH}_{12}}{C_{eH}_{22}} \mu_H, \quad \theta_{L}^{Y} = \frac{C^{(1)}_{lequ}}{C^{(1)}_{ledq}} \mu_H, \quad \theta_{L}^{d} = \frac{C_{ledq}}{C_{ledq}} \mu_H.
\]

We denote these parameters the left-handed flavour phases of the operators. In a similar fashion we define the right-handed flavour phases \( \theta_{R}^{e\gamma} \) from the ratios of 21 over 22 entries on a given Wilson coefficient. These flavour phases define the alignment in flavour space of these five operators. We assume \( |\theta_{L(R)}^{Y}| \ll 1 \), and ever smaller values for the ratios of 11 over 22 entries (for each Wilson coefficient).

We are interested in estimating the size of the (flavour) off-diagonal entries of the dipole operator in the flavour basis where the mass Yukawa at the low scale is diagonal. In the limit of small off-diagonal entries, the rotation to the mass basis in the 12 sector is determined by

\[
\Theta_{L(R)}^{Y} = - \frac{[Y_e]_{12(21)}}{[Y_e]_{22}} \mu_L \quad \text{(3.19)}
\]

and, in general, we expect \( \Theta_{L(R)}^{Y} \neq \theta_{L(R)}^{Y} \). In the mass basis the off-diagonal elements of the dipole operator are given by

\[
C'_{e\gamma} (\mu_L) = C_{e\gamma} (\mu_L) + \Theta_{L(R)}^{Y} C_{e\gamma} (\mu_L) \quad \text{(3.20)}
\]

while the diagonal element receives negligible corrections i.e. \( C'_{e\gamma} (\mu_L) \approx C_{e\gamma} (\mu_L) \).
Focusing on the 12 sector, i.e. on left-handed flavour rotations, and expressing the $C_{\epsilon \gamma}$ and $\mathcal{Y}_e$ coefficients in terms of their low-scale values, we obtain

$$C_{\epsilon_{12}}(\mu_L) = (\theta_{L}^{\epsilon \gamma} - \theta_{L}^{Y})(\mu_L) + (\theta_{L}^{\epsilon \gamma} - \theta_{L}^{u_3})(16\hat{L}e\gamma)(\mu_L)$$

$$+ \left[ (\theta_{L}^{Y} - \theta_{L}^{u_1})(6y_t^3)C_{\text{lequ}}^{(1)}(\mu_H) + (\theta_{L}^{d} - \theta_{L}^{Y})(6y_b^3)C_{\text{ledq}}(\mu_H) \right] \frac{1}{[\mathcal{Y}_e]_{22}(\mu_L)} \tilde{L}^2C_{\epsilon \gamma}(\mu_L)$$

$$+ (\theta_{L}^{H} - \theta_{L}^{Y}) \frac{1}{2}(y_t^2 + y_b^2)\tilde{L}^{2}C_{\epsilon \gamma}(\mu_H) \frac{1}{[\mathcal{Y}_e]_{22}(\mu_L)}v^2C_{\epsilon \gamma}(\mu_L).$$

The above equation allows us to derive a few important considerations:

- Even if $C_{\epsilon \gamma}^{12} = 0$ at the high scale (i.e. if $\theta_{L}^{\epsilon \gamma} = 0$), a non-vanishing $C_{\epsilon \gamma}(\mu_L)$ is naturally generated at the low scale if any of the other operators in (3.21) has a non-vanishing 22 entry (i.e. if any of the $\theta_{L}^{X}$ in eq. (3.18) is non-zero).

- $C_{\epsilon \gamma}^{12}(\mu_L) = 0$ is obtained only aligning all the flavour phases in eq. (3.18).

- All terms but one in eq. (3.21) are proportional to the 22 entry of the dipole operator, which is required to be non-vanishing and sizeable by the $a_\mu$ anomaly (see section 2).

Of the terms in eq. (3.21), the one proportional to $C_{\text{ledq}}$ has a numerically small coefficient which does not imply a sizeable tuning on the difference $(\theta_{L}^{d} - \theta_{L}^{Y})$. The combination $(\theta_{L}^{H} - \theta_{L}^{Y})C_{\epsilon \gamma}$ controls the $e-\mu$ flavour violating coupling of the physical Higgs boson, which is tightly constrained by other observables [49, 50] and can be safely ignored in the present analysis. The non-trivial contributions to $\epsilon_{12}^{L}$ defined in eq. (2.8) can be thus simplified to

$$\epsilon_{12}^{L} = (\theta_{L}^{\epsilon \gamma} - \theta_{L}^{Y}) + (\theta_{L}^{u_3} - \theta_{L}^{\epsilon \gamma})\Delta_3 + (\theta_{L}^{d} - \theta_{L}^{Y})\Delta_1$$

with

$$\Delta_3 = \frac{-16\hat{L}e\gamma C_{\text{lequ}}^{(3)}(\mu_H)}{C_{\epsilon \gamma}^{22}(\mu_L)} = \frac{\Delta a_\mu|_{\epsilon_{12}^{\text{lequ}}}}{\Delta a_\mu^{\exp}}$$

$$\Delta_1 = \frac{-6y_t^3\tilde{L}^2}{[\mathcal{Y}_e]_{22}(\mu_L)}C_{\text{equ}}^{(-)}(\mu_H) \approx 1 \times 10^{-3} \left[ \frac{C_{\text{equ}}^{(1)}(\mu_H)}{C_{\text{equ}}^{(2233)}(\mu_H)} \right] \times \Delta_3. \ (3.24)$$

By construction, the coefficient $\Delta_3$ is of $\mathcal{O}(1)$ if $\Delta a_\mu^{\exp}$ is saturated by the (radiatively induced) contribution of the triplet semileptonic operator. In this limit, assuming further triplet and singlet semileptonic operators of similar size, we expect $\Delta_1 = \mathcal{O}(10^{-3})$. In the case where the muon Yukawa is saturated by the (radiatively induced) contribution of the singlet semileptonic operator we expect $\Delta_1$ of $\mathcal{O}(1)$.

Given the $10^{-5}$ bound on $|\epsilon_{12}^{L}|$ in eq. (2.7), the result in eq. (3.22) implies a tight alignment in flavour space of four a priori independent SMEFT operators. Note that the
bound is expressed in terms of the left-handed flavour phases \( (\theta_L^X) \), which characterise the orientation of the operators in flavour space, and not in terms of the size of the Wilson coefficients. In the case of \( C_{\text{lequ}}^{(1,3)} \), the impact of the size is encoded by the coefficients \( \Delta_{1,3} \) which, as shown in eqs. (3.23)–(3.24), are expected to be sizeable.

A completely analogous formula holds for the 23 sector; however, in that case we can neglect the term proportional to \( \Delta_1 \), whose size is small enough (in case of similar size triplet and singlet semileptonic operators) to satisfy the constraint on \(|\epsilon_{23}|\) even in presence of \( \mathcal{O}(1) \) misalignments in flavour space.

### 3.3 Light NP contributions to \( a_\mu \)

As stated in the introduction, our analysis is focused on NP models well described by the SMEFT. However, it is worth to point out that there are also explanations of the muon anomalous magnetic moment based on light new physics, i.e. in terms of new degrees of freedom with \( m_{\text{NP}} < v \) (see e.g. [51–55]).

For light NP the last two terms in eq. (3.22) are absent since no significant RG contributions are expected below the top quark mass scale \( (m_t \sim v) \). On the other hand, the first term in eq. (3.22) still needs to be aligned since also light NP contributions to \( a_\mu \) are subject to the strong flavour alignment conditions in eq. (2.7) and (2.10). The dipole operator flavour phase \( \theta_L^{\gamma} \) should be interpreted as the flavour phase of the effective photon di-lepton coupling (including short- and long-distance contributions) while the lepton Yukawa flavour phase \( \theta_L^Y \) is the angle in the charged lepton mass matrix. The flavour-alignment problem posed by \( \mu \rightarrow e\gamma \) on NP contributions to \( a_\mu \) is therefore a serious constraint also for the light NP explanations.

Moreover, LFV couplings of light NP can mediate additional LFV processes which are disconnected from the operators of our heavy NP analysis. For example, in the case of an axion-like particle explanation of \( \Delta a_\mu \), other processes such as \( \mu \rightarrow e + \text{invisible} \) and \( \mu \rightarrow eee \) provide additional constraints on the LFV couplings of the underlying model [52].

### 4 General alignment mechanisms

Barring accidental cancellations, the alignment in eq. (3.22) can be realised in general terms following two main strategies: either via dynamical assumptions, or by imposing appropriate flavour symmetries. In this section we illustrate these two possibilities in general terms, while in section 5 we discuss their explicit implementation in a specific NP model with scalar mediators.

#### 4.1 Dynamical conditions

The alignment via dynamical assumptions occurs if the UV dynamics is such that two or more SMEFT operators originate from the same short-distance structure and hence appear with the same orientation in flavour space. We should distinguish two main cases: i) the case \( \Delta_3 = \mathcal{O}(1) \), when the NP contribution \( \Delta a_\mu \) is dominated by the term induced by \( C_{\text{lequ}}^{(3)} \), and ii) the case \( |\Delta_{1,3}| \ll 1 \).
i) $\Delta_3 = \mathcal{O}(1)$. As shown in table 1, in this case we can identify three dynamical hypotheses which lead to specific alignments in flavour space in a wide class of explicit models. If all the three conditions in table 1 are satisfied we do not have a severe alignment problem in eq. (3.22).

| Case | Dynamical hypothesis | Alignment condition |
|------|----------------------|---------------------|
| I) $\Delta_3 = \mathcal{O}(1)$ | Dipole operator radiatively generated with $C_{\text{lequ}}^{(3)}$ | $\theta_{L}^{\gamma} = \theta_{L}^{u_3}$ |
| II) $C_{\text{lequ}}^{(1)}$ and $C_{\text{lequ}}^{(3)}$ from same UV dynamics | $\theta_{L}^{u_1} = \theta_{L}^{u_3}$ |
| III) $y_{u_{i}}$ radiatively generated with $C_{\text{lequ}}^{(1)}$ | $\theta_{L}^{Y} = \theta_{L}^{u_i}$ |
| $|\Delta_{1,3}| \ll 1$ | Dipole operator and lepton Yukawa radiatively generated by same UV dynamics | $\theta_{L}^{\gamma} = \theta_{L}^{Y}$ |

Table 1. Alignment conditions following from specific dynamical assumptions.

The condition I in table 1 is the most natural one: it consists in assuming that the dipole operator is not present in the UV theory, but is radiatively generated by the same dynamics giving rise to $C_{\text{lequ}}^{(3)}$ (e.g. via the tree-level exchange of a leptoquark (LQ) field). In this case we clearly have $\theta_{L}^{\gamma} = \theta_{L}^{u_3}$. Note that $\Delta_3$ is not necessarily one, since there can be finite UV matching contributions to $C_{\text{lequ}}^{(3)}$; however finite contributions and RG induced terms are, by hypothesis, flavour aligned. The condition II is already more restrictive: it implies that two semileptonic operators with different electroweak structures originate from the same underlying UV dynamics. More precisely, a unique combination of flavour symmetry breaking terms controls the orientation of the two operators in flavour space. Such condition can be realised assuming e.g. both $C_{\text{lequ}}^{(1)}$ and $C_{\text{lequ}}^{(3)}$ arise by the tree-level exchange of a single LQ field (as we discuss explicitly in section 5), but we must ensure they do not receive additional contributions (e.g. a heavy colourless scalar field would contribute to $C_{\text{lequ}}^{(1)}$ but not to $C_{\text{lequ}}^{(3)}$).

More challenging to realise is the condition III, which requires that not only the dipole operator is radiatively generated, but also the Yukawa coupling. This is more difficult to conceive since the Yukawa operator is a marginal operator ($d = 4$) which is naturally present in the theory at all scales, at least as long as the corresponding fields are well defined. One can forbid the Yukawa coupling at the tree-level assuming extra symmetries that, however, must be broken in other sectors of the theory. This should be contrasted to the $d = 6$ dipole operator, that is naturally absent in a well-behaved UV completion. A further complication arises by the fact that $C_{\text{lequ}}^{(1)}$ corresponds to a rank-one tensor in lepton space once we consider only the third-generation quarks (and trace over the quark-flavour indices). One can therefore generate only part of the full Yukawa coupling via $C_{\text{lequ}}^{(1)}$, ideally the leading entries in the $2 \times 2$ sub-block corresponding to the light families. The remaining terms need to be generated by additional operators (e.g. via semileptonic operators involving light quark families or exotic fermions), and the tuning problem is simply shifted to a different set of operators in the effective theory. Because of the difficulties associated to the condition III, the possibility of addressing the alignment problem in eq. (3.22) using only dynamical hypotheses appears to be rather unnatural.
ii) $|\Delta_{1,3}| \ll \mathcal{O}(1)$. If $\Delta_{1,3}$ are sufficiently small, the only condition we need to care about in order to satisfy eq. (3.22) is the flavour alignment of dipole operator and lepton Yukawa coupling. First, it is worth noting that we enter into this regime only if $|\Delta_{3}| \lesssim 10^{-5}$ that, by itself, is a rather unnatural condition. Second, we observe that in this case the problem is a genuine UV boundary condition in the SMEFT and is not related to the RG structure. This condition can be realised dynamically if dipole operator and lepton Yukawa are generated by the same UV dynamics (beyond the SMEFT). Models of this type have been proposed for instance in refs. [39, 56, 57] (see also refs. [58, 59]). As already commented for the case i), the fact that dipole operators and lepton Yukawa have different canonical dimension makes the flavour alignment in these constructions rather unnatural, unless supplemented by additional flavour symmetries (as e.g. in ref. [39]).

### 4.2 Flavour symmetries

Exact or approximate alignments in flavour space can be achieved by means of exact or approximate global symmetries. Here we discuss two representative cases: the individual lepton numbers, and the $U(2)_{L_L} \times U(2)_{E_R}$ flavour symmetry acting on the light lepton families.

#### 4.2.1 U(1) symmetries

The three individual lepton numbers,

$$U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} = U(1)_L \times U(1)_{L_e-L_\mu} \times U(1)_{L_e-L_\tau} \quad (4.1)$$

are, separately, exact accidental symmetries of the $d = 4$ operators in the SMEFT. As shown on the r.h.s. of eq. (4.1), we can rearrange these symmetries into total lepton number ($L = L_e + L_\mu + L_\tau$), and two $L$-conserving Abelian groups. Assuming the latter to be a good symmetry of the UV theory implies that all the flavour phases in (3.18) are zero. The symmetry (4.1) must be broken in the neutrino sector; however, the smallness of neutrino masses and their Majorana nature allows us to assume that the three individual lepton numbers are conserved to a high accuracy in all the operators preserving total lepton number.

An almost exact conservation of the individual lepton numbers is the assumption employed in many recent explicit models proposed to address the $(g-2)_\mu$ anomaly (see in particular [29, 32, 33, 37, 39]). Note in particular that any combination of $U(1)_{L_\mu}$ and $U(1)_{L_e}$ is sufficient to protect the tightly constrained $e-\mu$ mixing for heavy NP as well as for light NP.

#### 4.2.2 U(2)$_{L_L} \times U(2)_{E_R}$

The $U(2)$ flavour symmetries acting on the light families of each SM field, with small breaking terms, are introduced with a twofold purpose [60–62]: i) explaining the hierarchical structure of the SM Yukawa couplings (only the third generation couplings are allowed by the symmetry); ii) allowing TeV-scale NP coupled to the third generation, possibly addressing the Higgs hierarchy problem, in a way that is consistent with the tight bounds
from flavour violating processes (which necessarily involve the light families, and hence are suppressed by the symmetry). As proposed in refs. [63, 64] (see also [65–67]) this approach turns out to be very successful for a combined explanation of both charged- and neutral-current $B$ physics anomalies.

Focusing the attention to the lepton sector, the relevant symmetry is $U(2)_{L_L} \times U(2)_{E_R}$. This is assumed to be broken only by two spurions, $V_\ell = (2, 1)$ and $\Delta_e = (2, \bar{2})$, such that the charged-lepton Yukawa coupling assumes the form

$$Y_e = y_\tau \begin{pmatrix} \Delta_e & V_\ell \\ 0 & 1 \end{pmatrix}.$$

(4.2)

The two spurions can be parametrised by

$$V_\ell = \begin{pmatrix} 0 \\ \epsilon_\ell \end{pmatrix}, \quad \Delta_e = O_e^T \begin{pmatrix} \delta_e' & 0 \\ 0 & \delta_e \end{pmatrix}$$

(4.3)

with $|\delta_e'| \ll |\delta_e| \ll |\epsilon_\ell| \ll 1$ and where $O_e$ is a real orthogonal matrix ($[O_e]_{12} = \sin \theta_e \equiv s_e$). It must be stressed that, while we can deduce the size of $\delta_e$ and $\delta_e'$ by the eigenvalues of the charged-lepton Yukawa couplings, the size of $\epsilon_\ell$ and $s_e$ cannot be directly deduced from $Y_e$.

Here we assume $\epsilon_\ell = \mathcal{O}(10^{-1})$, which is the most natural choice for a similar treatment of quark and lepton sectors [62], and is also the value supported by the recent data on the $B$ physics anomalies [63, 64]. Similarly, we expect $s_e = \mathcal{O} \left( \sqrt{m_e/m_\mu} \right) \gtrsim 10^{-2}$ [65, 66].

The flavour orientation of all the effective operators relevant to our analysis is determined, up to $\mathcal{O}(1)$ coefficients, by the small breaking terms appearing in eq. (4.3). In practice, this characterisation is achieved constructing the effective operators via a perturbative expansion in terms of the spurions [68], requiring the theory to be invariant under $U(2)_{L_L} \times U(2)_{E_R}$.

**e − µ sector.** Non-vanishing entries for operators of the type (3.1) in the 12 sector appear at $\mathcal{O}(\Delta_e)$ in the spurion expansion. At this order all the flavour phases are aligned (to the phase of $\Delta_e$) and the condition (3.22) is satisfied automatically. However, this is not the case to higher orders in the spurion expansion.

At order $\mathcal{O}(V_\ell^2 \Delta_e)$ we can express the combination of spurions controlling the flavour structure of any operator of the type (3.1) as

$$X_{\alpha\beta}^n = a_n(\Delta_e)_{\alpha\beta} + b_n(V_\ell) a(V_\ell^+) \gamma(\Delta_e)_{\gamma\beta} = \begin{pmatrix} a_n c_\ell \delta_e' & -a_n s_\ell \delta_e \\ s_\ell \delta_e' (a_n + b_n \epsilon_\ell^2) \ c_\ell \delta_e (a_n + b_n \epsilon_\ell^2) \end{pmatrix}_{\alpha\beta},$$

(4.4)

where $a_n$ and $b_n$ are $\mathcal{O}(1)$ coefficients. The flavour matrix $X_{\alpha\beta}^n$ is defined such that the corresponding operator reads $X_{\alpha\beta}^n(\ell_\alpha \Gamma e_\beta) \eta^n$, where $\eta^n$ denotes the lepton independent structure $n \in \{ Y, eH, eG, u_3, u_1, d \}$. For each operator the $X^n$ are diagonalised by two unitary matrices, $O_{L,n}$ and $O_{R,n}$, whose rotation angles are

$$\theta^n_L \approx \frac{s_e}{c_\ell} \frac{1}{1 + \frac{b_n}{a_n} \epsilon_\ell^2} \approx \frac{s_e}{c_\ell} \left( 1 + \frac{b_n}{a_n} \epsilon_\ell^2 \right), \quad \theta^n_R \approx -\frac{s_e}{c_\ell} \delta_e' \frac{\delta_e}{\delta_e}.$$

(4.5)
Note that $\theta_R^n$ is independent of $n$, therefore all operators are aligned in the $U(2)_{E_R}$ space. This is a consequence of having assumed a single $U(2)_{E_R}$ breaking spurion. On the other hand, misalignments are possible in the $U(2)_{L_L}$ space. In such space, the difference in the flavour orientation between operators $m$ and $n$ is controlled by

$$
\Delta \theta_{L_L}^{nm} = \theta_{L_L}^m - \theta_{L_L}^n = \frac{s_e}{c_e} \epsilon^2 \left( \frac{b_n}{a_n} - \frac{b_m}{a_m} \right) = \frac{s_e}{c_e} \epsilon^2 (d_n - d_m),
$$

where $d_i \equiv b_i/a_i$. Looking in particular to the relative phase of the dipole operator and the Yukawa interaction, from the condition (3.22) and the bound on $|\epsilon_{12}^L|$ in (2.7) we obtain

$$
|\theta_{L_L}^e - \theta_{L_L}^Y| = \left| \frac{s_e}{c_e} \epsilon^2 |d_Y - d_{e\gamma}| \right| \leq 2 \times 10^{-5}. \tag{4.7}
$$

Setting $c_e = \mathcal{O}(1)$ and $\epsilon_{12} = \mathcal{O}(10^{-1})$ [67, 68], this leads to

$$
|s_e(d_Y - d_{e\gamma})| \lesssim 10^{-3}. \tag{4.8}
$$

Given the natural expectation $s_e = \mathcal{O}(\sqrt{m_e/m_\mu})$ [65, 66], this implies a tight alignment condition on the $\mathcal{O}(1)$ coefficients $d_i$.

**$\mu - \tau$ sector.** We can proceed in a similar way in the 23 sector. In this case off-diagonal coefficients in flavour space appear already at $\mathcal{O}(V)$ [72]. Denoting $Y_{a3}^n$ the combination of spurions controlling the flavour structure of the operators in the 23 sector we find

$$
Y_{a3}^n = \begin{pmatrix} 0 & f_n \epsilon \ell \\ 0 & g_n \end{pmatrix}, \tag{4.9}
$$

where $f_n, g_n = \mathcal{O}(1)$. Defining $h_i \equiv f_i/g_i$ and proceeding as in the 12 sector, taking into account the bound on $|\epsilon_{12}^{L_L}|$ in (2.10), leads to

$$
|\epsilon_{12}^{L_L} (h_Y - h_{e\gamma})| \lesssim 10^{-2} \quad \rightarrow \quad |h_Y - h_{e\gamma}| \lesssim 2 \times 10^{-1}. \tag{4.10}
$$

Similarly to the 12 sector, also in this case we find a non-trivial constraint on coefficients which are expected to be of $\mathcal{O}(1)$.

In summary, while a minimally broken $U(2)_{L_L} \times U(2)_{E_R}$ symmetry does provide a partial alignment in the flavour space of the operators contributing to the dipole and Yukawa couplings at low energies, this alignment is not enough to fully justify the smallness of $|\epsilon_{12}^L|$ and $|\epsilon_{23}^L|$.

## 5 Alignment in an explicit NP model

In order to illustrate the general mechanism discussed in the previous section in concrete cases, we analyse here a simplified model where both semileptonic operators are generated by the tree-level exchange of scalar mediators. To this purpose, we note that we can get rid of the tensor currents present in eq. (3.4) changing basis from $Q_{lequ}^{(1,3)}$ to $Q_{S_1}$ and $Q_\Phi$, defined as

$$
\begin{pmatrix} Q_{lequ}^{(1)} \\ Q_{lequ}^{(3)} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 8 & 4 \end{pmatrix} \begin{pmatrix} Q_{S_1} \\ Q_\Phi \end{pmatrix}, \quad Q_{S_1} = \epsilon_{jk} (\bar{q}^j q^c)^{ck}(\pi^c e), \quad Q_\Phi = \epsilon_{jk} (\bar{q}^j e)(\pi^k u). \tag{5.1}
$$
This implies that a $S_1$ scalar leptoquark transforming as $(\bar{3}, 1)_2$ under the SM gauge group, and a Higgs-like field $\Phi$ transforming as $(1, 2)_{\frac{1}{2}}$, allow us to generate generic tree-level matching conditions for $Q_{\text{lequ}}^{(1)}$. Interestingly enough, both these fields also lead to non-vanishing one-loop contributions to dipole amplitudes.

The simplified renormalizable model we consider contains these two exotic scalar fields, coupled to the SM via the following Lagrangian:

$$
\mathcal{L}_{S_1} = \mathcal{L}_{\text{SM}} + (D_\mu S_1)^\dagger (D^\mu S_1) - M_{S_1}^2 S_1^\dagger S_1 - \left[ \lambda_{ia}^L (\bar{q}_i E_\alpha) S_1 + \lambda_{ia}^R (\bar{u}_i E_\alpha) S_1 + \text{h.c.} \right] + (D_\mu \tilde{\Phi})^\dagger (D^\mu \tilde{\Phi}) - M_{\tilde{\Phi}}^2 \tilde{\Phi}^\dagger \tilde{\Phi} - \left[ \lambda_{a, \beta}^\dagger (\bar{E}_\alpha E_\beta) \tilde{\Phi} + \lambda_{ij}^R (\bar{q}_i u_j) \tilde{\Phi} + \text{h.c.} \right],
$$

where $\tilde{\Phi} \equiv \epsilon \Phi^*$. Integrating out the $S_1$ and $\Phi$ fields at the tree-level leads to the following matching conditions for the Wilson coefficients of $Q_{\text{lequ}}^{(1)}$ and $Q_{\text{lequ}}^{(3)}$:

$$
C_{\text{lequ}}^{(1)} = \frac{\lambda_{ia}^L \lambda_{j, \beta}^R}{2 M_{S_1}^2} - \frac{\lambda_{a, \beta}^\dagger \lambda_{ij}^R}{M_{\tilde{\Phi}}^2}, \quad C_{\text{lequ}}^{(3)} = -\frac{\lambda_{ia}^L \lambda_{j, \beta}^R}{8 M_{S_1}^2}.
$$

Given the interaction terms in (5.2), this model also yields a direct contribution to the dipole operators from the integration of $S_1$ at the one-loop level.\footnote{In principle, also $\Phi$ can generate a one-loop contribution to dipole amplitudes. However, this arises only if we add a quartic interaction between $\Phi$ and the SM Higgs field, and this is formally described by dimension-8 operators in the SMEFT expansion.} The latter yields:

$$
C_{\gamma}(\mu_H) = \frac{e}{16 \pi^2 M_{S_1}^2} \left\{ -\frac{1}{8} \left[ (\lambda^L)^\dagger \lambda^L Y_\gamma \right]_{\alpha \beta} - \frac{1}{8} \left[ Y_{\gamma} (\lambda^R)^\dagger \lambda^R \right]_{\alpha \beta} + \left( \frac{7}{4} + \log \frac{\mu_H^2}{M_{S_1}^2} \right) \left[ (\lambda^L)^\dagger Y^* \lambda^R \right]_{\alpha \beta} \right\}, \tag{5.4}
$$

which agrees with the results in refs. \[34, 69–71\]. We now have all the ingredients to determine the misalignment of the flavour phases entering eq. (3.22) in this model, but for $\theta_\ell^Y$ which is a free parameter. Setting $\mu_H = M_{S_1}$ we obtain the following expressions for the left-handed flavour phases in the 12 sector:

$$
\theta_\ell^{(1)} = \frac{\lambda_{31}^L \lambda_{32}^R + 2 \lambda_{12}^L \lambda_{33}^R M_{S_1}^2 / M_{\tilde{\Phi}}^2}{\lambda_{32}^L \lambda_{32}^R + 2 \lambda_{32}^L \lambda_{33}^R M_{S_1}^2 / M_{\tilde{\Phi}}^2}, \tag{5.5}
$$

$$
\theta_\ell^{(3)} = \frac{\lambda_{31}^L}{\lambda_{32}^L}, \tag{5.6}
$$

$$
\theta_\ell^{(y)} = \frac{(Y_{\gamma})_{1a} \lambda_{1a}^L \lambda_{32}^R \lambda_{32}^R + \lambda_{11}^L \lambda_{11}^L (Y_{\gamma})_{a1} - 14 y_t \lambda_{12}^L \lambda_{12}^R}{(Y_{\gamma})_{2a} \lambda_{2a}^L \lambda_{32}^R \lambda_{32}^R + \lambda_{22}^L \lambda_{22}^L (Y_{\gamma})_{a2} - 14 y_t \lambda_{22}^L \lambda_{22}^R}, \tag{5.7}
$$

where in eq. (5.7) we have neglected the light quark Yukawa couplings. As expected, in the general case we have three independent flavour phases, in addition to $\theta_\ell^{Y}$. This is representative of the result we expect in generic NP models, i.e. models with an extended sector containing more than one heavy field.
Starting from these expressions, we can analyse the specific implementation of some of the dynamical alignment mechanisms discussed in section 4.1 in the context of this model. The condition II in table 1, namely $\theta^{u_1}_L \approx \theta^{u_3}_L$, is easily obtained in the limit $M^2_\Phi \gg M^2_{S_1}$. As far as the condition I is concerned, we can achieve it neglecting the terms with two powers of $\lambda^R$, and assuming $\lambda_{L_{1i}} \ll \lambda^L_{3i}$ for $i = 1, 2$. Under reasonable dynamical assumptions it is therefore easy to reach the condition

$$\theta^{e\gamma}_L \approx \theta^{u_1}_L \approx \theta^{u_3}_L = \frac{\lambda^L_{31}}{\lambda^L_{32}}. \quad (5.8)$$

On the other hand, in this model (as in most models) it is hard to conceive a dynamical alignment of the flavour phase in eq. (5.8) with the phase of the Yukawa interaction ($\theta^Y_L$).

To further investigate the misalignment of $\theta^Y_L$ and the flavour phase in eq. (5.8) in this model, it is useful to investigate what happens if we supplement the dynamical assumptions so far discussed with the hypothesis of a minimally broken $U(2)_{LL} \times U(2)_{ER}$ symmetry (see section 4.2). To lowest order in the $U(2)_{LL} \times U(2)_{ER}$ breaking terms we get

$$\theta^{e\gamma}_L = \frac{\lambda^L_{31}}{\lambda^L_{32}} = \frac{(V_\ell)_1}{(V_\ell)_2} \xrightarrow{(4.3)} 0, \quad \theta^Y_L = \frac{(\Delta_e)_{12}}{(\Delta_e)_{22}} \xrightarrow{(4.3)} s_e, \quad (5.9)$$

which implies an (unnatural) limit of $O(10^{-5})$ on $|s_e|$ in order to satisfy the experimental bound in eq. (2.7). This constraint is much more stringent than the generic condition derived in section 4.2 for minimally broken $U(2)_{LL} \times U(2)_{ER}$ models. This can be understood by noting that after aligning $\theta^{e\gamma}_L$ and $\theta^{u_3}_L$, the leading $U(2)_{LL} \times U(2)_{ER}$ spurion contribution to $\theta^{e\gamma}_L$ is suppressed, which is equivalent to assuming a large $b_{e\gamma}/a_{e\gamma}$ ratio in eq. (4.5). What was beneficial to reach the dynamical alignment $\theta^{e\gamma}_L \approx \theta^{u_3}_L$, however has the drawback of making worse the misalignment between $\theta^{e\gamma}_L$ and $\theta^Y_L$ compared to the one obtained in a minimally broken $U(2)_{LL} \times U(2)_{ER}$ framework.

To summarise, in this simplified model it is easy to implement the dynamical hypotheses I and II in table 1. However, by doing so, it becomes more difficult to align the Yukawa and dipole interactions. The latter goal is attainable only with the further hypothesis of an (almost) exact conservation of the individual lepton numbers.

6 Conclusion

The stringent experimental bounds on lepton flavour violating processes involving charged leptons indicate that lepton flavour is approximately conserved above the electroweak scale. When considering only these bounds, we cannot exclude that this approximate symmetry arises accidentally in the SMEFT, being the consequence of an overall (flavour anarchic) suppression of $d = 6$ operators. The accidental lepton flavour conservation could also arise as the consequence of appropriate scale hierarchies, as in the general SM extensions proposed in [72–75], which provide an interesting explanation for the hierarchies observed in both charged-lepton and quark Yukawa couplings. However, the situation changes if we assume the $a_\mu$ anomaly is due to NP. As we have shown in this paper, in such case we are forced to assume that the conservation of lepton flavour, and in particular of the electron
flavour, is a key property of a large set of operators in the $d = 6$ sector of the SMEFT. This property is unlikely to arise accidentally.

The $a_\mu$ anomaly sets a well-defined reference scale (size) for the muon dipole operator, with respect to which a series of additional SMEFT operators need to be flavour aligned in order to satisfy the tight bounds from $\mu \to e\gamma$ and $\tau \to \mu\gamma$. A consistent treatment of the problem in the SMEFT requires addressing the RG evolution not only of the operators mixing into the dipoles, but also of those mixing into the effective Yukawa couplings. In general, the five independent flavour phases listed in eq. (3.18), describing the orientation in flavour space of the corresponding effective operators, need to be aligned. In the $e-\mu$ case, at least three of these phases (dipole, Yukawa, and semileptonic triplet operators) need to be aligned at the $10^{-5}$ level. As we have shown in section 4.1, and illustrated by means of a concrete example in section 5, dynamical mechanisms can force some of these alignments, but not all of them. Barring fine-tuned solutions, the required alignments require extra ingredients, such as exact (or almost exact) flavour symmetries ensuring the conservation of the electron flavour.

The most interesting conclusion of this study is that if the $a_\mu$ anomaly is a sign of NP, we are led to conclude that the quark and lepton sectors behave quite differently beyond the SM, at least in the few-TeV scale domain, with a lepton sector featuring enhanced symmetries which are not present in the quark sector.

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