Correspondence between Noncommutative Soliton and Open String/D-brane System via Gaussian Damping Factor

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Abstract

The gaussian damping factor (g.d.f.) and the new interaction vertex with the symplectic tensor are the characteristic properties of the $N$-point scalar-vector scattering amplitudes of the $p - p’ (p < p’)$ open string system which realizes noncommutative geometry. The g.d.f. is here interpreted as a form factor of the $Dp$-brane by noncommutative $U(1)$ current. Observing that the g.d.f. is in fact equal to the Fourier transform of the noncommutative projector soliton introduced by Gopakumar, Minwalla and Strominger, we further identify the $Dp$-brane in the zero slope limit with the noncommutative soliton state. It is shown that the g.d.f. depends only on the total momentum of $N - 2$ incoming/outgoing photons in the zero slope limit. In the description of the low-energy effective action (LEEA) proposed before, this is shown to follow from the delta function propagator and the form of the initial/final wave functions in the soliton sector which resides in $x^m (m = p + 1 \cdots p’)$ dependent part of the scalar field $\Phi(x^\mu, x^m)$. The three and four point amplitudes computed from LEEA agree with string calculation. We discuss related issues which are resummation/lifting of infinite degeneracy and conservation of momentum transverse to the $Dp$-brane.
I. Introduction

Notion of D-brane has led people to think what string theory ought to be beyond perturbation theory. Investigations of its spacetime properties are, however, so far limited to classical solutions to supergravity theory and its connection to configurations of branes. One reason to prevent a more direct study at string amplitudes is that, in the zero-slope limit, this object becomes too singular to study and gets simply removed from the low energy physics except that the momenta carried by the modes of a string are constrained. In an interesting setup of string theory with constant $B_{MN}$ background \[1\] realizing noncommutative geometry, it has been found \[2\] that distances at all scales can be kept finite. This offers a possibility to establish direct correspondence between string theory in the zero slope limit and the attendant local field theory: this time D-brane is present in both sides as physical degrees of freedom. We will accomplish this correspondence in an open string connecting a $Dp$-brane and a $Dp'$-brane with the $Dp$-brane inside, following the series of work \[3, 4\] which has uncovered a number of properties: these include 1) spectrum which contains a large number of light states and 2) the appearance in string amplitudes of a symplectic tensor $J$ and a multiplicative factor decaying exponentially with momenta (a gaussian damping factor). These are derived from several nontrivial worldsheet properties of system.

In quite different vein, it has been argued \[5\] that classical soliton solution is possible to construct in scalar noncommutative field theory, avoiding the no-go theorem of Derrick. We will find that the soliton solution of this type, in particular, the simplest projector soliton is just the right representation of the $Dp$-brane in field theory side in order to establish the correspondence.

For definiteness, let us first specify the process studied in this paper. At an initial state, we prepare a $Dp$-brane which is at rest and which lies in the worldvolume of a $Dp'$-brane. The $Dp'$-brane is regarded as entire space in this paper. We place the tachyon (the lowest mode) of a $p - p'$ open string which carries a momentum $k_{1\mu}$, $\mu = 0 \cdots p$ along the $Dp$-brane worldvolume. In addition, $N - 2$ noncommutative $U(1)$ photons carrying momenta $k_{aM}$ $a = 3 \cdots N$, $M = 0 \cdots p'$ in $p' + 1$ dimensions are present. They get absorbed into the $Dp$-brane. At a final state, the $Dp$-brane is found to be present and the momentum of the tachyon is measured to be $-k_{2\mu}$ along the $Dp$-brane worldvolume. We will examine the tree scattering amplitude of this process both from string perturbation theory of the D-brane/open string system in the zero slope limit and from perturbation theory of the field theory action proposed in \[4\]. We will find that computations from both sides in fact agree by identifying the $Dp$-brane with an initial/final configuration representing a noncommutative soliton.
The computation of the scattering amplitude of this process from string perturbation theory has been already carried out in [4]. In the next section, we will begin with recapitulating its properties, focusing upon the gaussian damping factor (g.d.f.) which is originally associated with each external vector leg. In the zero slope limit, the desirable cross terms develop and the g.d.f. is shown to be an overall multiplicative factor for any \( N \) which depends only upon the total momentum. An approximate resummation of infinitely many light states propagating in the \( t \)-channel is responsible for this phenomenon, which we will refer to as lifting of the infinite degeneracy. Finally in this section, we observe that the g.d.f. is in fact equal to the Fourier transform of the noncommutative projector soliton solution introduced by Gopakumar, Minwalla and Strominger in [5]. The g.d.f. is naturally interpreted as a form factor of the \( D_p \)-brane by noncommutative \( U(1) \) current. The \( D_p \)-brane in the zero slope limit is identified with the noncommutative soliton state.

In section three, we consolidate this identification and interpretation in the light of the low energy effective action (LEEA) which is proposed in [4]. The adequate description of the process above is given by perturbation theory of this LEEA which at the same time permits us to define a soliton sector residing in the \( x^m \) (\( m = p + 1 \cdots p' \)) dependent part of the scalar field \( \Phi(x^\mu, x^m) \). That the g.d.f. depends on the total photon momentum alone is found to be a simple consequence from the delta function propagator in perturbation theory and the form of the initial/final wave function given by Fourier transform of the projector soliton solution. The three and four point tree amplitudes agree with string calculation. It is satisfying to see that string theory realizing noncommutative geometry and the attendant local field theory in fact share the several interesting properties which are derived from two completely different lines of reasoning. Section four is devoted to outlook and a few comments which are more speculative. We basically follow the notation of [4]. With regard to the spacetime index, \( M, N \cdots \) run from 0 to \( p' \), \( \mu, \nu \cdots \) from 0 to \( p \) and \( m, n \cdots \) from \( p + 1 \) to \( p' \).

II. Gaussian damping factor of the scattering amplitude from the \( p - p' \) open string with constant \( B_{ij} \) field

Let us recall how the gaussian damping factor has been found in [4] from the scattering amplitude which involves two scalars and \( (N-2) \) massless vectors which are noncommutative \( U(1) \) photons. We begin with the integral representation of this amplitude. (See [4] for its derivation and more complete explanation of our notation.) The \( SL(2, \mathbb{R}) \) invariant integral (Koba-Nielsen) representation of this amplitude is

\[
A_N = c(2\pi)^{p+1} \prod_{\mu=0}^{p} \delta \sum_{c=1}^{N} k_{c\mu} \int dx_{a} \prod_{a=4}^{N} d\theta_{a'} d\eta_{a'} \exp \mathcal{C}_{a'}(\{\nu_I\})
\]
\[\times \prod_{c=4}^{N} \left[ x_c^{\alpha' s_c + \alpha' m^2} (1 - x_c) \right]^{2\alpha' k_3} \prod_{4 \leq c < c' \leq N} (x_c - x_{c'})^{2\alpha' k_3} \]

\[\times \prod_{3 \leq c < c' \leq N} \exp \left[ -2\alpha' \sum_{I,J} G^{IJ} \left( \kappa_{cI} \kappa_{cJ} \mathcal{H} \left( \nu_I; \frac{x_c}{x_c'} \right) + \kappa_{cJ} \kappa_{cI} \mathcal{H} \left( \nu_I; \frac{x_c'}{x_c} \right) \right) \right] \times \exp (\text{NC}) \exp \left( \left[ 0, 2 \right] + \left[ 2, 0 \right] + \left[ 1, 1 \right] + \left[ 2, 2 \right] \right) \bigg|_{x_1 = 0, x_2 = \infty, x_3 = 1} \]  

(2.1)

Here we have used the \(SL(2, \mathbb{R})\) invariance to fix the locations of two tachyon vertex operators and that of a massless vector vertex operator to be respectively at \(x_1 = 0, x_2 = \infty,\) and \(x_3 = 1\) \((x_a \equiv -\xi_a = e^{\tau_a}).\) From now on, we set \(c = 2i\) and multiply the expression by \((-\sqrt{2\alpha'})^{N-4}\).

We explain eq. (2.1) further:

1. We have employed the worldsheet superfield formalism. Eq. (2.1) involves integrations over fermionic variables \(\theta_a\) and \(\eta_a.\) The terms containing \(\theta_a\) and \(\eta_a\) are classified by the number of \(\eta_a\) and by the number of \(\theta_a,\) which we designate respectively by the first and by the second entry inside the bracket. These are given as \([0, 2], [2, 0], [1, 1],\) and \([2, 2].\) The explicit forms of these terms can be found in [4]. (See also [7] for a comparison to the more familiar \(Dp - Dp\) case with vanishing \(B.\))

2. The term denoted by \(\exp(\text{NC})\) originates from the noncommutativity of the worldvolume. It extends into all directions of \(Dp'\)-brane worldvolume:

\[(\text{NC}) = \sum_{1 \leq a < a' \leq N} \frac{i}{2} \epsilon(x_a - x_{a'}) \sum_{M,N=0}^{p'} \theta^{M,N} k_{aM} k_{a'N}, \quad (2.2)\]

where \(k_{1m} = k_{2m} = 0\) for \(m = p + 1, \ldots, p',\) and \(\theta^{2A-1,2A} = \frac{2\pi\alpha' b_A}{\varepsilon(1+b_A^2)}\) is the noncommutativity parameter.

3. The momentum dependent multiplicative factor \(\exp C_a(\{\nu_I\})\) with

\[C_a(\{\nu_I\}) = \alpha' \sum_{I,J} 2k_{aI} \kappa_{aJ} G^{IJ} \left\{ \gamma + \frac{1}{2} \left( \psi(\nu_I) + \psi(1 - \nu_I) \right) \right\} \]

(2.3)

comes from the subtracted Green function at a coincident point. This Green function has been introduced in [4] and is defined by the difference of the two distinct Green functions, namely, the one defined with respect to the \(SL(2, \mathbb{R})\) invariant vacuum and the other with respect to the oscillator vacuum. In eq. (2.3), \(\gamma, \psi(\nu_I)\) are the Euler constant and the digamma function respectively, and \(G^{IJ}\) is the inverse of the open string metric for the \(x^{p+1}, \ldots, x^{p'}\) directions

\[G^{IJ} = G^{JI} = \frac{2}{\varepsilon(1+b_A^2)} \delta^{IJ}. \quad (2.4)\]
We denote by \( \kappa_I, \kappa_J \) a set of momenta in complex notation defined as
\[
\kappa_I = \frac{1}{2} (k_{2I-1} - ik_{2I}) , \quad \kappa_J = \frac{1}{2} (k_{2J-1} + ik_{2J}) .
\] (2.5)

4. There are a few different ways in which an inner product of two vectors \( A_i \) and \( B_j \) is taken with respect to the open string metric. These are denoted by \( (p) \), \( (p') \) and \( (p,p') \), depending on the directions concerned with. For example,
\[
A_{(p)} B = \sum_{\sigma, \rho = 0}^{p} G^\sigma_{\rho} A_{\sigma} B_{\rho} , \quad \text{and}
\]
\[
k_{(p,p')} \zeta = \sum_{I, J = \pm \frac{1}{2}}^{p'} (G^{IJ} \kappa_I \kappa_J + G^{JI} \kappa_J \kappa_{I}) = \sum_{I, J = \pm \frac{1}{2}}^{p'} G^{IJ} (\kappa_I \kappa_J + \kappa_J \kappa_I) .
\] (2.6)

We also introduce \( \odot_{(p,p')} \) and \( \times_{(p,p')} \) to denote the "incomplete inner products"
\[
\left( k_{(p,p')} \odot \right)_I = \sum_{J} G^{IJ} (\kappa_I \kappa_J + \kappa_J \kappa_{I}) , \quad \left( k_{(p,p')} \times \right)_I = \sum_{J} \frac{2 \delta^{IJ} (\kappa_I \kappa_J - \kappa_J \kappa_{I})}{\varepsilon (1 + b_I^2)} ,
\] (2.7)

so that
\[
\sum_{I} \left( k_{(p,p')} \odot \right)_I = k_{(p,p')} \zeta , \quad \sum_{I} \left( k_{(p,p')} \times \right)_I = ik_{(p,p')} J \zeta .
\] (2.8)

Here the matrix \( J \) is a \((p' + 1) \times (p' + 1)\) antisymmetric matrix defined as
\[
J = (J_{\mu}^\rho) = \begin{pmatrix}
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & 1 & p + 1 \\
-1 & 0 & p + 2 \\
\vdots & \vdots & \ddots \\
0 & 1 & p' - 1 \\
-1 & 0 & p'
\end{pmatrix} .
\] (2.9)

5. The functions \( \mathcal{H}(\nu_I; \frac{z}{x_I}) \) or \( \mathcal{H}(\nu_I; \frac{z}{x_c}) \) is defined by the hypergeometric series as
\[
\mathcal{H}(\nu; z) = \begin{cases}
\mathcal{F} \left( 1 - \nu; \frac{1}{z} \right) - \frac{\pi}{2} b = \sum_{n=0}^{\infty} \frac{z^{-n-1+\nu}}{n+1-\nu} - \frac{\pi}{2} b & \text{for } |z| > 1 \\
\mathcal{F} (\nu; z) + \frac{\pi}{2} b = \sum_{n=0}^{\infty} \frac{z^{n+\nu}}{n+\nu} + \frac{\pi}{2} b & \text{for } |z| < 1 .
\end{cases}
\] (2.10)

The two infinite series in the above defining relation should be analytically continued to each other.
From now on, we will focus on the nontrivial zero slope limit of the amplitude. The zero slope limit is defined as
\[
\alpha' \sim \varepsilon^{1/2} \to 0 , \\
g \sim \varepsilon \to 0 , \\
|b_I| \sim \varepsilon^{-1/2} \to \infty .
\] (2.11)
This limit keeps $\alpha' b_I$ finite:
\[
\alpha' b_I \to \beta_I .
\] (2.12)
In terms of the open string metric and the noncommutativity parameter, this implies
\[
\frac{1}{2\pi} (JG\theta)_{2I-1} = \frac{1}{2\pi} (JG\theta)_{2I} = \beta_I .
\] (2.13)
In addition, the following limit is taken without loss of generality:
\[
\nu \equiv \nu_{\mu+2} \to 1 , \quad \nu_I \to 0 , \quad \text{for} \quad \tilde{I} \neq \frac{p+2}{2} ,
\] (2.14)
so that
\[
b_{\mu+2} \to +\infty , \quad b_{\tilde{I}} \to -\infty .
\] (2.15)
It is simple to obtain the three-point amplitude:
\[
A_3 = -i(2\pi)^{p+1} \prod_{\mu=0}^p \delta \left( \sum_{a=1}^3 k_{a\mu} \right) \left\{ (k_2 - k_1) \cdot \zeta_3 - ik_3(p,p') J\zeta_3 \right\} e^{C_3(\nu_I)} e^{i2\theta^j k_1 k_{2j}} .
\] (2.16)
The first term is the derivative coupling of a charged scalar with a massless vector and vanishes when there is no momentum transfer of the tachyon in the directions of $Dp$-brane worldvolume. The second term is the term found in [4]. Both are multiplied by the term representing noncommutativity $e^{i2\theta^j k_1 k_{2j}}$ as well as by the factor $\exp C(\nu_I)$.

This exponential multiplicative factor $\exp C(\nu_I)$ defined in eq. (2.3) becomes in the zero slope limit
\[
\exp C(\nu_I) \to \exp \left( -\pi \sum I,J |\beta_I| \kappa_1 \pi^- G^{I\tilde{J}} \right) = \exp \left( -\frac{\pi}{2} \sum I |\beta_I| \left( k_{(p,p')} k \right)_I \right) \equiv D(k_m) ,
\] (2.17)
where we have used $\psi(1) = -\gamma$ and eqs. (2.5), (2.12). This factor is associated with each vector propagating into the $x^{p+1} \sim x^{p'}$ directions. We will refer to this as gaussian damping factor (g.d.f.) in the rest of this paper. Inserting the initial and final wave functions of the tachyon and that of the noncommutative $U(1)$ photon, (which we should have inserted in the first place together with the vertex operators,) we obtain
\[
\lim A_3 = -i(2\pi)^{p+1} \prod_{\mu=0}^p \delta \left( \sum_{a=1}^3 k_{a\mu} \right) \frac{1}{(2\pi)^{p/2}} \frac{1}{\sqrt{2\omega_{k_2}}} \frac{1}{(2\pi)^{p/2}} \frac{1}{\sqrt{2\omega_{k_1}}} \frac{1}{(2\pi)^{p/2}} \frac{1}{\sqrt{2|k_3|}} \\
\times \left\{ (k_2 - k_1) \cdot \zeta_3 - ik_3(p,p') J\zeta_3 \right\} D(k_m) e^{i2\theta^j k_1 k_{2j}} .
\] (2.18)
Let us come back to eqs. (2.14), (2.15). It has been observed that, due to this fine tuning of the sign of \( b_I \), a large number of light states appear in the limit. To be more precise, these light states are obtained by acting the several low-lying fermionic modes on the oscillator vacuum and multiplying by an arbitrary polynomial consisting of the lowest bosonic mode. This latter bosonic mode is the one which has failed to become a momentum due to the boundary condition of the \( p - p' \) open string and is responsible for an infinite number of nearly degenerate low-lying states. We will see that the string amplitude in fact has resummed and lifted this approximate infinite degeneracy by evaluating its effect as an exponential factor and that this lifting renders the net g.d.f. of the amplitude to depend only upon the total momentum of the incoming photons.

In order to prove this last statement, let us recall that the contributions to the \( N \)-point amplitude surviving the zero slope limit come from the endpoints of the \( N - 3 \) integrations: at the endpoints \( x_c \) coalesces to either \( x_{c-1} \) or \( x_{c+1} \) and eventually gets close to either 0 or 1. We need only to analyse the behavior of the function \( \mathcal{H}(\nu; x) \) or \( \mathcal{H}(\nu; \frac{1}{x}) \) on the exponent near the \( x = 0 \) and the \( x = 1 \), paying attention to the order of the integrations and the limit. In the region \( \frac{x_c'}{x_c} \approx 0 \) for \( c' > c \), we find that the factor

\[
P_{c'} \equiv -2\alpha' \sum_{I,J} G_{I,J}^{c'} \left( \kappa_{c'} \pi_c \mathcal{H} \left( \nu_I; \frac{x_c'}{x_{c'}} \right) + \pi_c \kappa_{c'} \mathcal{H} \left( \nu_I; \frac{x_c'}{x_{c}} \right) \right)
\]

(2.19)

can be approximated by

\[
P_{c'} \approx -\pi \sum_I |\beta_I| (k_c \odot k_{c'})_I
\]

\[
-\alpha' \sum_I \left( (k_c \odot k_{c'}) + (k_c \times k_{c'}) \right)_I \lim_{I} \frac{(x_{c'} \nu_I - 1)}{1 - \nu_I}
\]

\[
-\alpha' \sum_I \left( (k_c \odot k_{c'}) - (k_c \times k_{c'}) \right)_I \lim_{I} \frac{(x_{c'} \nu_I - 1)}{\nu_I}
\]

(2.20)

Here we have exploited that the contribution to \( \mathcal{H}(\nu; \frac{1}{x}) \) from the modes other than the lowest bosonic mode is ignorable in the limit and that the constant piece is safe to evaluate in the limit. The limit in the last two terms of eq. (2.20) should be taken after the integration and will be discussed in the next paragraph. On the other hand, the factor \( P_{c'} \) in the region \( \frac{x_c'}{x_c} \approx 1 \) can be evaluated, using the expansion of \( \mathcal{H}(\nu; \frac{1}{x}) \) near \( x = 1 \). We find that \( P_{c'} \) can be approximated by

\[
P_{c'} \approx -\pi \sum_I |\beta_I| (k_c \odot k_{c'})_I + 2\alpha' \sum_I (k_c \odot k_{c'})_I \log(1 - \frac{x_{c'}}{x_c})
\]

(2.21)
We see that in either region the factor \( P^c_c \) contains the identical constant piece \(-\pi \sum_I |\beta_I (k_c \otimes k_{c'})_I|\) in the limit. Multiplying \( \prod_{3 \leq c < c' \leq N} \exp \left[ -\pi \sum_I |\beta_I | (k_c \otimes k_{c'})_I \right] \) by \( \prod_{a=3}^N D(k_{am}) \), (see eq. (2.17)), we find that the amplitude \( A_N \) contains an overall multiplicative factor

\[
D \left( \sum_{a=3}^N k_{am} \right) = \exp \left[ -\frac{\pi}{2} \sum_I |\beta_I | \left( \sum_{a=3}^N k_{a} \otimes (\sum_{b=3}^N k_{b})_I \right) \right], \tag{2.22}
\]

which depends upon the total photon momentum alone. This is what we wanted to show.

The singular behavior near \( \frac{x_c}{x_c'} \approx 1 \) of the second term of eq. (2.21) on the exponent of \( \exp P^c_c \) is responsible for the massless pole in the \( s \)-channel. It is instructive therefore to estimate near \( \frac{x_c}{x_c'} \approx 0 \) the second and the third terms of eq. (2.20) on the exponent of \( \exp P^c_c \), which we discuss qualitatively here. Expanding \( P^c_c \) in Taylor series, combining with the other factors in the integrand and integrating over \( x_{c'} \) near the origin, we find propagators of an infinite number of states corresponding to the light spectrum of a \( p - p' \) open string in the \( t \)-channel. We see that, in the treatment of eq. (2.20), we have taken care of this approximate infinite degeneracy on the exponent and its principal contribution is the cross term of the net g.d.f. It is this resummation of spectrum of states coming from the lowest bosonic mode that has provided this cross term. The infinite degeneracy has been lifted. A closer look at eq. (5.8) of [4] shows that ignoring the second and the third terms in eq. (2.20) corresponds to setting to zero the mass differences among the infinitely many light states due to the lowest bosonic mode.

Let us turn to the four point amplitude in the zero slope limit. After lifting the infinite degeneracy due to the lowest bosonic mode, we still have the contributions from several nearly degenerate states due to the lowest fermionic modes. In the subsequent section, we will discuss only those parts of the amplitude in which the state with the lowest mass (tachyon) participates. See eq. (5.8) of [4] for the complete formula in the zero slope limit which contains the above contributions as well. Taking into account what we have established above, we find

\[
\lim A_4 = -2i(2\pi)^{p+1} \delta \left( \sum_{a=1}^4 k_{a\mu} \right) D(k_{3m} + k_{4m}) \exp \left( i\theta^{\mu\nu} k_{1\mu} k_{2\nu} + i\theta^{MN} k_{3M} k_{4N} \right)
\]

\[
\left[ \frac{1}{t - m^2} \left\{ (k_2 - (k_1 + k_4)) (p) \zeta_3 - i(k_3 (p,p') \cdot J_{\zeta_3}) \right\} \right.
\]

\[
\left. \left\{ ((k_2 + k_3) - k_1) (p) \zeta_4 - i(k_4 (p,p') \cdot J_{\zeta_4}) \right\} \right]
\]
\[
\begin{align*}
+ \frac{1}{s} \left\{ \left( k_2 - k_1 \right) \zeta_3 - i(k_3 + k_4) J(\zeta_3) \right\} k_3 \zeta_4 \\
- \left( k_2 - k_1 \right) \zeta_4 - i(k_3 + k_4) J(\zeta_4) \right\} k_4 \zeta_3 \\
+ \left( \frac{1}{2} (k_3 - k_4) \zeta_4 - i(k_3 + k_4) Jk_4 \right) \zeta_3 \zeta_4 \right\} \\
+ (k_3 \leftrightarrow k_4; \zeta_3 \leftrightarrow \zeta_4).
\end{align*}
\] (2.23)

We end this section by giving an explicit connection and related remarks between the g.d.f. and the noncommutative projector soliton, which is a key observation we make to the remainder of this paper. Let us first rewrite the g.d.f. as

\[
D(k_m) = \exp \left( -\frac{1}{4} \sum_{I=\frac{d^2}{2}}^\nu | \theta^{2I-1,2I} | (k_{2I-1}k_{2I-1} + k_{2I}k_{2I}) \right) \\
= \prod_{I=\frac{d^2}{2}}^\nu \tilde{\phi}_0(k_{2I-1}, k_{2I}; \theta^{2I-1,2I}).
\] (2.24)

Observe that

\[
2\pi | \theta | \tilde{\phi}_0 = \int d^2x e^{ik_1x^1 + ik_2x^2} \phi_0(x^1, x^2; \theta),
\]

\[
\phi_0(x^1, x^2; \theta) = 2e^{-\frac{1}{\theta^2}((x^1)^2 + (x^2)^2)}.
\] (2.25)

Function \( \phi_0(x^1, x^2; \theta) \) is the projector soliton solution of the noncommutative scalar field theory discussed in [5]. It satisfies \( \phi_0 * \phi_0 = \phi_0 \) and is represented as a ground state projector \( |0\rangle \langle 0 | \) in the Fock space representation of noncommutative algebra \([x^1, x^2] = i\theta\). In [5], \( \phi_0 \) is discussed as a soliton solution of noncommutative scalar field theory in the large \( \theta \) limit. In our discussion, Fourier transform of \( \tilde{\phi}_0 \) is seen to appear for all values of \( \theta \).

There are a few points associated with this observation. From our discussion, it is natural to interpret that some of the physical degrees of freedom of the \( Dp \)-brane are participating in the process although the \( D \)-brane is introduced in the first quantized string through boundary conditions. Eq. (2.18) tells us rather obviously that the g.d.f. \( D(k_{3m}) \) is a form factor of the \( Dp \)-brane by noncommutative \( U(1) \) current, which can be written as

\[
\left( \Phi^\dagger \frac{\partial}{\partial \Phi}, -i\partial_n \left( \Phi^\dagger J^{mn} \Phi \right) \right),
\] (2.26)

using the scalar field \( \Phi(x^\mu, x^m) \) discussed in the next section.
classical profile of the $Dp$-brane/soliton or quantum mechanical wave function of this object in momentum space. Perturbation theory presented in the next section supports the latter point of view.

Another point on the degrees of freedom of the $Dp$-brane is the issue regarding with the conservation of momenta in the directions transverse to the $Dp$-brane worldvolume. In string calculation, there is no delta function which ensures the conservation of momenta in these directions as we place spin and twist fields on the worldsheet \[8, 9\]. On the other hand, our result eq. (2.22) does show that these momenta have been deposited in the $Dp$-brane. The momentum conservation holds for the combined system of tachyon, photons and the $Dp$-brane/soliton. The center of mass coordinate of the $Dp$-brane/soliton has become activated to receive the photon momenta. One way to interpret the absence of the delta function is that one is measuring in string calculation an inclusive process with regard to the $Dp$-brane: in the final state, we have confirmed its presence only and did not measure its momenta.

There is an alternative way to view the scattering process not by momentum eigenstates but by constructing wave packets as initial and final configurations. This point of view explains the absence of the delta function immediately. It involves instead integrations over both initial and final momenta of the $Dp$-brane/soliton. We will discuss the relationship of these two points of view in the next section.

### III. $Dp$-brane and the Projective Soliton of Noncommutative Scalar Field Theory

We now give a field theoretic derivation of the properties of the string amplitude in the zero slope limit given by eqs. (2.18), (2.22) and (2.23). We will show that an adequate description is given in perturbation theory of low energy effective action (LEEA) proposed in \[4\] by specifying proper initial and final states associated with the scalar field $\Phi(x^\mu, x^m)$.

In \[4\], the following action has been proposed:

$$
S = S_0 + S_1 ,
$$

with

$$
S_0 = \frac{1}{2g_Y^2} \int d^{p+1}x \sqrt{-G} \left\{ - (D_\mu \Phi)^\dagger \ast (D^\mu \Phi) - m^2 \Phi^\dagger \ast \Phi - \frac{1}{4} F_{MN} \ast F^{MN} \right\} ,
$$

$$
S_1 = \frac{1}{2g_Y^2} \int d^{p+1}x \sqrt{-G} \Phi^\dagger \ast F_{mn} J^{mn} \ast \Phi ,
$$

(3.1)

where

$$
D_\mu \Phi = \partial_\mu \Phi - i A_\mu \ast \Phi , \quad (D_\mu \Phi)^\dagger = \partial_\mu \Phi^\dagger + i \Phi^\dagger \ast A_\mu ,
$$

$$
F_{MN} = \partial_M A_N - \partial_N A_M - i [A_M, A_N] \ast , \quad [A_M, A_N] = A_M \ast A_N - A_N \ast A_M .
$$

(3.2)
The star product is defined by
\[
    f(x) \ast g(x) = e^{-\frac{i}{2} g^{MN} \partial_y^M \partial_z^N} f(y) g(z) \bigg|_{y,z \to x}. \tag{3.3}
\]

We denote by \( A_M(x^\mu, x^m) \) a \((p' + 1)\)-dimensional vector field which corresponds to noncommutative \( U(1) \) photon and by \( \Phi(x^\mu, x^m) \) a scalar field which corresponds to the ground state tachyon of the \( p - p' \) open string with \( m^2 = -\lim_{\alpha' \to 0} \left( 1 - \sum_{I} \nu_I \right) / 2\alpha' \). Reflecting the fact that the tachyon momenta are constrained to lie in \( p + 1 \) dimensions, the Lorentz index of the kinetic term for the scalar field runs from \( 0 \) to \( p \) and there is no kinetic term for the remaining \( p' - p \) directions. We set \( g_{YM} \) to 1 from now on.

Perturbation theory obtained from the action (eq.(3.1)) is elementary to carry out but we stop to explain here a few tricky points. The quantized scalar field \( \Phi(x^\mu, x^m) \) in the interaction picture obeys a free field equation
\[
    \left( \partial_\mu \partial^\mu - m^2 \right) \Phi(x^\mu, x^m) = 0. \tag{3.4}
\]

Expanding \( \Phi(x^\mu, x^m) \) in Fourier series, we find its mode expansion
\[
    \Phi(x^\mu, x^m) = \frac{1}{(2\pi)^{p'/2}} \int d^p k_i \int d^{p'-p} K_m \frac{1}{\sqrt{2\omega_k}} \left( a(k_i, K_m) e^{i k \cdot x} + b^\dagger(k_i, K_m) e^{-i k \cdot x} \right), \tag{3.5}
\]
where
\[
    \omega_k = \sqrt{k^2 + m^2}, \quad k = (k_0 = -\omega_k, k_i, K_m), \tag{3.6}
\]
and the equal time commutator is
\[
    [a(k_i, K_m), b^\dagger(k'_i, K'_m)] = \frac{1}{\sqrt{-G}} \delta^{(p)}(k_i - k'_i) \delta^{(p'-p)}(K_m - K'_m). \tag{3.7}
\]

A factorized expression is also permitted and we may write
\[
    a(k_i, K_m) = a(k_i) \alpha(K_m),
\]
\[
    [a(k_i), a^\dagger(k'_i)] = \delta^{(p)}(k_i - k'_i) \frac{1}{\sqrt{-G}},
\]
\[
    [\alpha(K_m), \alpha^\dagger(K'_m)] = \delta^{(p'-p)}(K_m - K'_m). \tag{3.8}
\]

Similar expressions hold for \( b(k_i, K_m) \). We will later use
\[
    \phi(x^m) \equiv \frac{1}{(2\pi)^{(p'-p)/2}} \int d^{p'-p} K_m \left( \alpha(K_m) e^{i K \cdot x} + \beta^\dagger(K_m) e^{-i K \cdot x} \right). \tag{3.9}
\]

Reflecting this factorization, we designate Fock space associated with \( a(k_\mu) \), \( b(k_\mu) \) by tach and the one with \( \alpha(K_m) \), \( \beta(K_m) \) by sol. Photon Fock space is denoted by vec.
It is clear that eq. (3.4) permits an arbitrary field configuration depending only upon \( x^m \) as a solution and this is reflected in the expansion eq. (3.5) or eq. (3.9). To say in a little different way, time evolution of the operator \( \Phi(x^\mu, x^m) \) in the interaction picture is independent of how it looks in the \( x^m \) direction. We will take advantage of this fact to accommodate the state representing one noncommutative projector soliton with momentum \( K_m \) shortly.

In perturbation theory, the scalar field propagator is

\[
G(x^M, y^M) = \langle 0 | T \Phi(x^M) \Phi^\dagger(y^M) | 0 \rangle, \\
\Delta_F(x^\mu - y^\mu; m^2) = \int \frac{d^d(p+1)k}{(2\pi)^{d+1}e} \frac{e^{-ik \cdot x}}{\sqrt{-G(k)}(k + m^2 - i\epsilon)} e^{ik \cdot y}, \\
\delta(p' - p)(x^m - y^m) = \int \frac{d^d(p')K}{(2\pi)^{d-1}e} e^{-iK \cdot (p', p')} \otimes e^{iK \cdot (p', p')}.
\] (3.10)

In eq. (3.10), \( x^M \) and \( y^M \) are two separate sets of integration variables and therefore represent two copies of noncommutative algebra. The delta function acts on this tensor product space.

Going to the momentum space, one can check

\[
g(x) \ast \int d^{(p' - p)}y \delta(p' - p)(x^m - y^m) \ast f(y^m) = g(x^m) \ast f(x^m); \\
g(\hat{x}) Tr_{y} \hat{\delta}(\hat{x}, \hat{y}) f(\hat{y}) = g(\hat{x}) f(\hat{x}),
\] (3.11)

which means

\[
g(\hat{x}) Tr_{y} \hat{\delta}(\hat{x}, \hat{y}) f(\hat{y}) = g(\hat{x}) f(\hat{x}),
\] (3.12)

as it should be. The Feynmann propagator \( \Delta_F(x^\mu - y^\mu) \) in the noncommutative space should be understood in the same way. We leave the discussion on noncommutative gauge fields to [10].

The interaction Lagrangian \( \mathcal{L}_{int}(\Phi, A_M) \) obtained from eq. (3.1) reads

\[
\mathcal{L}_{int}(\Phi, A_M) = \frac{1}{2} \Phi^\dagger \ast F_{mn} J^{mn} \ast \Phi - i\Phi^\dagger \left( \ast A_\mu \ast \partial^{\mu} - \partial^{\mu} \ast A_\mu \ast \right) \Phi - \Phi^\dagger \ast A_\mu \ast A_\mu \ast \Phi + i[A_M, A_N]_s \ast \partial^M A^N + \frac{1}{4}[A_M, A_N]_s \ast [A_M, A_N]_s.
\] (3.13)

We first determine the momentum space wave functions of the initial and final states by demanding that the three point amplitude agree with the string calculation given by eq. (2.18). Let the initial state associated with the scalar field carrying momentum \( (k_1^\mu, K_m^{(i)}) = 0 \) be

\[
| i \rangle = |k_1^\mu, K_m^{(i)} = 0 \rangle \otimes | k_3^M, \zeta_3^M, \cdots, k_{NM}, \zeta_{NM} \rangle_{vec}.
\] (3.14)
and the final state be
\[ | f \rangle = | - k_{2\mu}, -K_m^{(f)} \rangle \otimes | 0 \rangle_{\text{vec}} , \] (3.15)
where
\[ | k_\mu, K_m \rangle \equiv | k_\mu \rangle_{\text{tach}} \otimes | K_m \rangle_{\text{sol}} , \quad | K_m \rangle_{\text{sol}} = u(K_m) \alpha^+(K_m) | 0 \rangle_{\text{sol}} . \] (3.16)

The N-point amplitude is
\[ A_N = \int d^{(p'-p)} K_m \langle f | \hat{S} | i \rangle , \] (3.17)
with
\[ \hat{S} = T \exp \left[ i \int d^{(p'+1)} x M \sqrt{-G} L_{\text{int}}(\Phi, A_M) \right] . \] (3.18)

It is elementary to compute the three point tree amplitude from \( L_{\text{int}}(\Phi, A_M) \):
\[ A_3 = i \int d^{(p'-p)} K_m^{(f)} \int d^{(p'+1)} x M \sqrt{-G} \langle -K_m^{(f)} \rangle_{\text{sol}} \otimes \langle -k_{2\mu} \rangle \]
\[ \left\{ \begin{aligned}
&\left( \frac{1}{2} \Phi^+ \cdot \right. \left. \right. \\
&\left. \cdot \vec{\partial} \cdot \vec{\partial} \right|_{\text{vec}} \langle 0 | F_{mn} J^{mn} | k_{3M}, \zeta_{3M} \rangle_{\text{vec}}^* \Phi \\
&- i \Phi^+ \left. \left( \right. \cdot \vec{\partial} \cdot \vec{\partial} \right|_{\text{vec}} \langle 0 | A_\mu | k_{3M}, \zeta_{3M} \rangle_{\text{vec}}^* \Phi \right\} | k_{1\mu} \rangle_{\text{tach}} \otimes | 0 \rangle_{\text{sol}} \\
&= i (2\pi)^{p'+1} \delta^{(p+1)} \left( \sum_{a=1}^{3} k_{a\mu} \right) \epsilon^{\mu \nu \lambda} k_{1\mu} k_{2\nu} u^*(k_{3m}) u(K_m^{(i)} = 0) \prod_{a=1,2} \frac{1}{(2\pi)^{p+2} \omega_k} \\
&\frac{1}{\sqrt{-G}} \left( \frac{1}{2} \text{vec} \langle 0 | F_{mn}(0) J^{mn} | k_{3M}, \zeta_{3M} \rangle_{\text{vec}} + \text{vec} \langle 0 | A_\mu | k_{3M}, \zeta_{3M} \rangle_{\text{vec}} \right) \\
&= -i \left( \frac{1}{\sqrt{-G}} \right)^2 (2\pi)^{p'+1} \delta^{(p+1)} \left( \sum_{a=1}^{3} k_{a\mu} \right) \exp \left( i \epsilon_{\mu \nu \lambda} k_{1\mu} k_{2\nu} \right) u^*(k_{3m}) u(0) \] (3.19)
\[ \prod_{a=1,2} \frac{1}{\sqrt{(2\pi)^{p+2} \omega_k}} \sqrt{(2\pi)^{p+2} \omega_{k_3}} \left( (k_2 - k_1)_\mu (k_3 - k_1)_\nu \zeta_3 - i k_{3 (p, p')} J_\zeta \right) . \]

Eq. (2.18) from string theory and eq. (3.20) computed from eq. (3.1) agree completely provided
\[ u^*(k_m) u(0) = D(k_m) , \quad \text{or} \quad u(k_m) = D(k_m) . \] (3.20)

The momentum space wave function in soliton sector is identified with the g.d.f. and hence is equal to Fourier image of the distribution of the noncommutative soliton.

In the alternative point of view discussed in the end of section 2, the intial and final states are prepared as
\[ | u \rangle_{\text{sol}} \equiv \int d^{(p'-p)} K_m \langle K_m \rangle_{\text{sol}} = \int d^{(p'-p)} K_m u(K_m) \alpha^+(K_m) | 0 \rangle_{\text{sol}} . \] (3.21)
The projector soliton configuration is generated by
\[ \phi(x^m) \langle u \rangle_{\text{sol}}. \] (3.22)

This time, our calculation is different from the one at eq. (3.20) in that we must integrate over the initial soliton momentum \( K^{(i)}_m \) as well. We find that the following convolution property holds provided eq. (3.20) is satisfied:
\[ \int dK^{(i)}_m e^{i g_{\mu\nu} K^{(i)}_m P_\mu u * (P + K^{(i)})} u(K^{(i)}) = u(P). \] (3.23)

The two points of view to the scattering process thus give the same answer. We have checked that this conclusion also holds for those properties other than eq. (3.20) which will be discussed in the remainder of this section.

We now proceed to see that the N-point tree amplitude obtained from this field theory contains the g.d.f. whose argument is the total momentum. The amplitude is given by
\[ A_N \sim \int d^{(p'-p)} K^{(f)}_m \langle \langle - K^{(f)}_m \mid \otimes_{\text{tach}} \langle - k_{2\mu} \mid \otimes_{\text{vec}} \langle 0 \mid T \frac{j(N-2)}{(N-2)!} \] (3.24)
\[ \left( \int d^{(p'+1)} x^M L_{\text{int}}(\Phi, A_M) \right)^{N-2} \mid k_{3M}, \zeta_{3M}, \cdots k_{NM}, \zeta_{NM} \rangle_{\text{vec}} \otimes \mid k_{1\mu} \rangle_{\text{tach}} \otimes \mid K^{(i)}_m = 0 \rangle \rangle_{\text{sol}}. \]

Let us imagine that, in this expression, we first carry out the Wick contractions and compute the expectation value for the part associated with the vector Fock space. To each of the Feynman diagrams generated by this, the net effect to the scalar part of the Fock space is that there are \( L \) vector lines carrying momenta \( q_{aM}, a = 3 \sim L \) which are attached to the scalar line. These momenta satisfy
\[ \sum_{a=3}^{L} q_{aM} = \sum_{a=3}^{N} k_{aM}. \] (3.25)

The expression for the scalar part of the Fock space coming from this Feynman diagram is proportional to
\[ \int d^{(p'-p)} K^{(f)}_m \langle \langle - K^{(f)}_m \mid \otimes_{\text{tach}} \langle - k_{2\mu} \mid \] \[ T \prod_{a=3}^{L} \left( \int d^{(p'+1)} x^M \Phi^\dagger(x^M_a) \ast e^{iq_a(p',x^M_a)} \ast \Phi(x^M_a) \right) \mid k_{1\mu} \rangle_{\text{tach}} \otimes \mid K^{(i)}_m = 0 \rangle \rangle_{\text{sol}}. \] (3.26)

Carrying out the Wick contractions and using the propagator (eq. (3.10)), we find that eq. (3.26) in turn contains the following factor residing in the soliton sector:
\[ \int d^{(p'-p)} K^{(f)}_m \prod_{a=3}^{L} \left( \int d^{(p'-p)} x^M_a \right) \langle \langle - K^{(f)}_m \mid \phi^\dagger(x^M_3) \ast e^{iq_3(p',x^M_3)} \ast \delta(p'-p)(x_3 - x_4) \] \[ \ast e^{iq_4(p',x^M_4)} \ast \cdots \ast \delta(p'-p)(x_{L-1} - x_L) \ast e^{iq_L(p',x^M_L)} \ast \phi(x^M_L) \rangle_{\text{sol}}. \] (3.27)
Thanks to the delta function propagator, this equals

\[
= \int d^{p'-p} K^{(f)}_m \int d^{p'-p} x^m \exp \left( \frac{i}{2} \sum_{a,b=3}^L \sum_{m,n=p+1}^{p'} \theta_{mn} q_{am} q_{bn} \right)
\]

\[
\text{sol} \langle \langle -K^{(f)}_m | \phi^\dagger(x^m) \ast \exp \left( i \sum_{a=3}^L q_a \langle p', p \rangle x \right) \ast \phi(x^m) | K^{(i)}_m = 0 \rangle \rangle_{\text{sol}}
\]

\[
= \int d^{p'-p} K^{(f)}_m \delta^{(p'-p)} \left( K^{(f)}_m + \sum_{a=3}^L q_a \right) u^* (-K^{(f)}_m) u(0) \exp \left( \frac{i}{2} \sum_{a,b=3}^L \sum_{m,n=p+1}^{p'} \theta_{mn} q_{am} q_{bn} \right)
\]

\[
= D \left( \sum_{a=3}^L k_{am} \right) \exp \left( \frac{i}{2} \sum_{a,b=3}^L \sum_{m,n=p+1}^{p'} \theta_{mn} q_{am} q_{bn} \right) \cdot (3.28)
\]

This completes the demonstration.

Finally, let us check that the tree four point amplitude (the pole part) computed from \( S \) in fact agrees with string answer. The field theory amplitude is

\[
A_4 = \int d^{p'-p} K^{(f)}_m \text{sol} \langle \langle -K^{(f)}_m | \otimes \text{tach}(-k_{2\mu}) \otimes \text{vec}(0) \rangle \rangle_{\text{sol}} (3.29)
\]

\[
T \left\{ \frac{1}{2!} i \int d^{p'+1} x_1 \sqrt{-G} \mathcal{L}_{\text{int}}(x_1) i \int d^{p'+1} x_2 \sqrt{-G} \mathcal{L}_{\text{int}}(x_2) \right\}
\]

\[
|k_{3M}, \zeta_{3M}, k_{4M}, \zeta_{4M})_{\text{vec}} \otimes |k_{1\mu}) \otimes |K^{(i)}_m = 0 \rangle \rangle_{\text{sol}} .
\]

After the Wick contraction and the position space integration, we find

\[
A_4 = (2\pi)^{p+1} \delta^{p+1} \left( \sum_{a=1}^4 k_{a\mu} \right) \exp \left( \frac{i}{2} \theta^{\mu\nu} k_{1\mu} k_{2\nu} \right) u^* (k_{3M} + k_{4M}) u(0)
\]

\[
\left( \frac{1}{\sqrt{-G}} \right)^3 \prod_{a=1,2} \frac{1}{(2\pi)^2 \omega_{k_a}} \prod_{b=3,4} \frac{1}{(2\pi)^2 \sqrt{|k_b|}} \left( a^{(t,u)}_4 + a^{(s)}_4 \right) , (3.30)
\]

where

\[
a^{(t,u)}_4 = -\frac{i}{t - m^2} \left\{ (k_2 - (k_1 + k_4)) \langle p \rangle \zeta_3 - i k_3 \langle p, p' \rangle J \zeta_3 \right\}
\]

\[
\left\{ (k_2 + k_3) - (k_1 \rangle \langle p \rangle \zeta_4 - i k_4 \langle p, p' \rangle J \zeta_4 \right\} \exp \left( \frac{i}{2} \theta^{MN} k_{3M} k_{4N} \right)
\]

\[
+ (k_3 \leftrightarrow k_4; \zeta_3 \leftrightarrow \zeta_4) , (3.31)
\]

\[
a^{(s)}_4 = -\frac{i}{s} 2 \left[ (k_2 - k_1) \langle p \rangle \zeta_3 - i (k_3 + k_4) \langle p, p' \rangle J \zeta_3 \right] k_3 \langle p' \rangle \zeta_4
\]
\[-\left( (k_2 - k_1)_{(p)} \zeta_4 \right. \left. - i(k_3 + k_4)_{(p,p')} J \zeta_4 \right) k_4_{(p')} \zeta_3
\]
\[+ \left( \frac{1}{2}(k_3 - k_4)_{(p)}(k_1 - k_2) - ik_3_{(p,p')} Jk_4 \right) \zeta_3_{(p')} \zeta_4 \exp \left( \frac{i}{2} \delta^{MN} k_{3M} k_{4N} \right) \]
\[+ (k_3 \leftrightarrow k_4; \zeta_3 \leftrightarrow \zeta_4) \right]. \tag{3.32}

This expression agrees with eq. (2.23).

IV. Discussion and outlook

We have seen that perturbation theories of two different kinds in fact agree. We now make several remarks which are more speculative in nature. Our identification at eq. (3.20) tells that the initial and the final state wave functions in position space obey equation of motion of pure polynomial $\phi^3$ theory. This may indicate that initial/final state interactions are governed by $\phi^3$ dynamics. This point, together with the delta function propagator in perturbation theory, is in fact reminiscent of the pregeometric nature of string theory. In perturbation theory, we of course find no reason why the wave function of this form has appeared. Nonperturbative treatment of the $x_m$ part of the scalar field $\Phi(x_\mu, x_m)$ as a soliton operator may lead us somewhere beyond what we have accomplished in this paper. Finally, tachyon is still present in the spectrum. The system must find its ultimate stability. Our preliminary investigation shows relevance of a noncommutative soliton of a different kind, which is somewhat similar to the one of [11].

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