Characteristic energy of the nematic-order state and its connection to enhancement of superconductivity in cuprate superconductors

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The new development in sublattice-phase-resolved imaging of electronic structure now allow for the visualisation of the nematic-order state characteristic energy of cuprate superconductors in a wide doping regime. However, it is still unclear how this characteristic energy of the nematic-order state is correlated with the enhancement of superconductivity. Here the doping dependence of the nematic-order state characteristic energy in cuprate superconductors and of its possible connection to the enhancement of superconductivity is investigated within the framework of the kinetic-energy-driven superconductivity. It is shown that the characteristic energy of the nematic-order state is found to be particularly large in the underdoped regime, then it smoothly decreases upon the increase of doping, in full agreement with the corresponding experimental observations. Moreover, the characteristic energy of the nematic-order state as a function of the nematic-order state strength in the underdoped regime presents a similar behavior of the superconducting transition temperature. This suggests a possible connection between the nematic-order state characteristic energy and the enhancement of the superconductivity.

PACS numbers: 74.25.Jb, 74.25.Dw, 74.20.Mn, 74.72.-h

I. INTRODUCTION

In cuprate superconductors\textsuperscript{1}, the strongly correlated motion of the electrons is confined to the square-lattice CuO\textsubscript{2} planes\textsuperscript{21}. However, this strong electron correlation also induces the system to exhibit numerous ordering tendencies\textsuperscript{22}. In addition to superconductivity, a variety of spontaneous symmetry-breaking orders have been observed experimentally, indicating the coexistence and intertwining between these spontaneous symmetry-breaking orders and superconductivity\textsuperscript{23}. Among these spontaneous symmetry-breaking orders, the most distinct form of order is electronic nematicity,\textsuperscript{24,25} which corresponds to that the electronic structure preserves the translation symmetry but breaks the rotation symmetry of the underlying square-lattice CuO\textsubscript{2} plane. This is why in the common practice, the strength of the electronic nematicity is defined as the orthorhombicity of the electronic structure\textsuperscript{26}. As a natural consequence of a doped Mott insulator, the manipulation of the particular characteristics of the superconducting (SC) state with coexisting electronically nematic order through the control of the doping and strength of the electronic nematicity is hotly debated and has been believed to be key to the understanding of the problem of why cuprate superconductors exhibit a number of the anomalous properties\textsuperscript{27}.

Experimentally, the multiple measurement techniques have been used to elucidate the nature of the quasiparticle excitation and of its interplay with spontaneous symmetry-breaking orders and superconductivity\textsuperscript{10,20}, where it has been found that the electronically nematic order coexists with the translation symmetry breaking such as charge order (or equivalently charge density wave) in the well-defined regimes of the phase diagrams, appearing below the pseudogap crossover temperature $T^\ast$ in the underdoped regime, and coexists with charge order and superconductivity below the SC transition temperature $T_c$. It thus shows that the electronic nematicity is an integral part of the essential physics of cuprate superconductors. The temperature scale for the onset of the dynamical charge order may increase monotonically with the decrease of doping in the underdoped regime, however, the static charge order may exhibit a dome-like shape temperature dependence\textsuperscript{24,25}. In particular, this interplay of the electronically nematic order and charge order in the normal-state has been invoked recently to give a consistent explanation of the transport anisotropy\textsuperscript{27}. However, although a number of consequences from the electronic nematicity together with the associated fluctuation phenomena have been identified in the early experimental measurement\textsuperscript{10,20}, the evolution of the characteristic quantities of the electronic nematicity itself with doping in the entire range of the SC dome remains puzzling. Fortunately, the instrumentation for sublattice-phase-resolved imaging of electronic structure has improved dramatically in recent years, allowing this experimental technique to visualize simultaneously the doping and energy dependence of the quasiparticle scattering interference (QSI) in the SC-state with coexisting symmetry-breaking ordered states\textsuperscript{25}. In this case, as a compensation for the early scanning tunneling spectroscopy (STS) experimental studies\textsuperscript{15,21}, this experimental technique has been used to detect the doping and energy dependence of the tunneling conductance of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ over a large field of view, perform a Fourier transform, and analyze data from distinct regions of momentum space\textsuperscript{25}. Moreover, to establish the link between the pseudogap and electronic nematicity, the doping and energy dependence of the averaged density of states $\rho(E)$ and the doping and energy dependence...
dependence order parameter of the electronic conductance. This nematic-order spectrum $N$ can be described as the order parameter of the electronic nematicity. The pseudogap extracted directly from the measured data of the averaged density of states shows that the pseudogap smoothly decreases upon the increase of doping. On the other hand, the measured data of the order parameter of the electronic nematicity $N(E)$ show that $N(E)$ has a dome-like shape energy dependence, with the maximal $N(E)$ appearance at an energy $E_{\text{max}}^{(N)}$. This energy $E_{\text{max}}^{(N)}$ associated with the maximal $N(E)$ is so-called the nematic-order state characteristic energy. More importantly, the evolution of the nematic-order state characteristic energy with doping is identified, where measured on the samples whose doping spans the pseudogap regime, the nematic-state characteristic energy $E_{\text{max}}^{(N)}$ and pseudogap energy $\Delta_{\text{PG}}$ are, within the experimental error, identical. These experimental results therefore identify the electronically nematic order exists across the entire range of the SC dome. On the basis of these experimental results, it has been argued that the pseudogap is a consequence of a tendency towards an electronically ordered state that is a coexistence of the nematic order and charge orders and breaks both the translation and rotation symmetry.

Although the doping dependence of the nematic-order state characteristic energy in cuprate superconductors has been well-identified experimentally in the entire range of the SC dome, its full understanding is still a challenging issue. In particular, it is still unclear how this nematic-order state characteristic energy $E_{\text{max}}^{(N)}$ evolves with the strength of the electronic nematicity. Theoretically, the possible origins of the emergence of the electronic nematicity have been suggested: the electronically nematic order occurs upon melting of stripe order or charge order, or induces by the electron Fermi surface (EFS) instability, or is attributed to the incommensurate pair-density-wave. In particular, it has been proposed that the density of states near EFS and geometry of EFS (then the band structure) strongly affect the nematic-order formation. This follows a basic fact that in the square-lattice CuO$_2$ planes, both the density of states near EFS and geometry of EFS change significantly when the quasiparticle dispersion possesses a saddle point induced by the van Hove singularity. In our recent study, the intertwinement of the electronic nematicity with superconductivity in cuprate superconductors has been studied based on the kinetic-energy-driven superconductivity, where we have shown that the electronic nematicity enhances superconductivity. Moreover, we have also shown that the order parameter of the electronic nematicity achieves its maximum in the characteristic energy of the nematic-order state, and then decreases rapidly as the energy moves away from the characteristic energy of the nematic-order state, in agreement with the experimental observation. However, a natural question is whether the characteristic energy of the nematic-order state is correlated with the enhancement of superconductivity or not? In this paper, we study the doping dependence of the nematic-order state characteristic energy and of its possible connection to the enhancement of superconductivity along with this line, where one of our main results is that both the characteristic energy of the nematic-order state and enhancement of $T_c$ exhibit the same nematic-order state strength dependence, i.e., the maximal characteristic energy of the nematic-order state (then the maximal $T_c$) occurs at around the optimal strength of the electronic nematicity, and then decreases in both weak and strong strength regions. This suggests a possible connection between the characteristic energy of the nematic-order state and the enhancement of superconductivity.

This paper is organized as follows. We present the basic formalism in Sec. II and then discuss the doping dependence of the nematic-order state characteristic energy and of its possible connection to the enhancement of superconductivity in Sec. III, where we show that in a striking similar to the doping dependence of the pseudogap state, the nematic-order state is particularly obvious in the underdoped regime, i.e., the characteristic energy of the nematic-order state is particularly large in the underdoped regime, and then it monotonically decreases with the increase of doping, in full agreement with the corresponding experimental observation. Finally, we give a summary and discussions in Sec. IV.

II. MODEL AND THEORETICAL METHOD

When the quasiparticle scattering mixes the states $k$ and $k+q$, a QSI pattern with the wave vector $q$ appears in the norm of the quasiparticle wave function and the local density of states modulations with the wave length $\lambda = 2\pi/|q|$ appear, reflecting a basic fact that the QSI pattern manifests itself is an autocorrelation between the quasiparticle bands $E_k$ and $E_{k+q}$. In other words, the intensity in the QSI pattern is proportional to the spectral intensities of the single-particle excitation spectra at the momenta $k$ and $k+q$, while the sharp intensity peaks in the QSI pattern then are corresponding to the highest joint density of states. This is why the quasiparticle scattering processes, the quasiparticle momentum-space structure, and the dispersion of the peaks in the QSI pattern as a function of energy are interpreted in terms of the octet scattering model and yields the crucial information of the quasiparticle excitation. More importantly, by the analysis of the typical feature of the Bragg peaks in a QSI pattern, one is considering the phe-
where $N$ is the number of lattice sites, and $I_c(k, \omega)$ is the single-particle excitation spectrum, while the summation of momentum $k$ is extended up to the second Brillouin zone (BZ) for the discussion of QSI together with the Bragg scattering. This ARPES autocorrelation in Eq. (2) describes the correlation of the spectral intensities of the single-particle excitation spectra at two different momenta $k$ and $k + q$, separated by a momentum transfer $q$, at a fixed energy, and is effectively the momentum-resolved joint density of states. In particular, it has been demonstrated experimentally that the peaks, the momentum-space structure, and the dispersion of the peaks in the ARPES autocorrelation pattern are directly related to the peaks, the momentum-space structure, and the dispersion of the peaks in the QSI pattern, respectively, and can be also explained straightforwardly in terms of the octet scattering model. This is why the characteristic features of QSI can be also obtained in terms of the ARPES autocorrelation.

The single-particle excitation spectrum $I_c(k, \omega)$ in Eq. (2) is proportional to the electron spectral function $A_c(k, \omega)$ as,

$$I_c(k, \omega) \propto n_F(\omega)A_c(k, \omega),$$

where $n_F(\omega)$ is the fermion distribution, while the electron spectral function $A_c(k, \omega)$ in the SC-state with coexisting electronic nematicity can be obtained directly from the full electron diagonal propagator as $A_c(k, \omega) = -2\text{Im}G_c(k, \omega)$.

Now our goal is to evaluate this full electron diagonal propagator $G_c(k, \omega)$ starting from a microscopic SC theory. The strongly correlated motion of the electrons in cuprate superconductors is restricted to the square-lattice CuO$_2$ planes as mentioned above, and then the unconventional properties come from the strongly correlated motion of the electrons in these CuO$_2$ planes. In particular, as originally emphasized by Anderson, the essential physics of the strongly correlated motion of the electrons in a square-lattice CuO$_2$ plane can be described properly by the $t$-$J$ model,

$$H = -\sum_{i\tau\sigma} t_{\hat{x}} C^\dagger_{i\sigma} C_{i+\hat{x}\tau\sigma} + \sum_{i\tau\sigma} t'_{\hat{y}} C^\dagger_{i\sigma} C_{i+\hat{y}\tau\sigma} + \mu \sum_{i\sigma} C^\dagger_{i\sigma} C_{i\sigma} + \sum_{i\hat{n}} J_{\hat{n}} \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{n}},$$

where $\hat{n} = \hat{x}, \hat{y}$ represents the nearest neighbor (NN) sites of a given site $l$, $\hat{\tau} = \hat{x}, \hat{y}$ represents the next NN sites of a given site $l$, $C^\dagger_{l\sigma}$ and $C_{l\sigma}$ are the electron creation and annihilation operators, respectively, $\mathbf{S}_i$ is the spin operator with its components $S^x_i$, $S^y_i$, and $S^z_i$, while $\mu$ is the chemical potential. For the discussions of the exotic features of the SC-state with coexisting electronic nematicity, the next NN hoping amplitude in the $t$-$J$ model is chosen as $t' = t$, while the NN hoping amplitude $t_\parallel$ has the following form,

$$t_{\parallel} = (1 - \zeta) t, \quad t_{\perp} = (1 + \zeta) t,$$

which is strongly anisotropic along the $\hat{x}$ and $\hat{y}$ directions and follows from the previous analyses of the exotic features in the nematic-order state. In particular, this anisotropic NN hoping amplitude in Eq. (3) has been experimentally verified in terms of the standard tight-binding model to fit the ARPES spectrum in the nematic-order state. Concomitantly, this anisotropic NN hoping amplitude in Eq. (3) induces the anisotropic NN exchange coupling $J_2 = (1 - \zeta)^2 J$ and $J_0 = (1 + \zeta)^2 J$ in the $t$-$J$ model. Moreover, this anisotropic parameter in Eq. (3) represents the orthorhombicity of the electronic structure, and therefore can be defined as the strength of the electronic nematicity in the system. In this sense, the anisotropic NN hoping amplitudes in Eq. (3) also indicate that the rotation symmetry is broken already in the starting $t$-$J$ model. In cuprate superconductors, although the values of $J$, $t$, and $t'$ are believed to vary somewhat from compound to compound, the commonly used parameters in this paper are chosen as $t/J = 3$, $t'/t = 1/3$, and $J = 100$ meV as in our previous discussions. Moreover, the temperature $T$ is set at $T = 0.002J$. Unless otherwise indicated, the doping is fixed at $\delta = 0.06$ for a direct comparison with the corresponding experimental result.
fermion-spin representation (6), the original \( t-J \) model in Eq. (4) can be rewritten as,

\[
H = \sum_{\langle ij \rangle} t_{ij}(\hat{h}_{i+\bar{\eta}}^\dagger \hat{h}_{i+\bar{\eta}} \sigma_{-}^i \sigma_{+}^j + \hat{h}_{i+\bar{\eta}}^\dagger \hat{h}_{i+\bar{\eta}} \sigma_{+}^i \sigma_{-}^j) - \sum_{\langle ij \rangle} t'_{ij}(\hat{h}_{i+\bar{\eta}}^\dagger \hat{h}_{i+\bar{\eta}} \sigma_{-}^i \sigma_{+}^j + \hat{h}_{i+\bar{\eta}}^\dagger \hat{h}_{i+\bar{\eta}} \sigma_{+}^i \sigma_{-}^j) - \mu_{h} \sum_{i\sigma} \hat{h}_{i\sigma}^\dagger \hat{h}_{i\sigma} + \sum_{i\bar{\eta}} n_{\bar{\eta}}^\dagger(\hat{S}_{i}^\dagger \cdot \hat{S}_{i+\bar{\eta}}) - \sum_{\bar{\eta}} \gamma_{\bar{\eta}}(\sigma_{-}^i \sigma_{+}^j + \sigma_{+}^i \sigma_{-}^j),
\]

(7)

where \( \mu_{h} \) is the charge-carrier chemical potential, \( S_{i}^\dagger = S_{i}^\dagger - iS_{i}^\gamma \) and \( S_{i}^\dagger = S_{i}^\dagger + iS_{i}^\gamma \) are the spin-lowering and spin-raising operators for the spin \( S = 1/2 \), respectively, \( J_{\bar{\eta}}^{\dagger}(i = (1 - \delta)^2 J_{\bar{\eta}} \), and \( \delta = (\hat{h}_{i\sigma}^\dagger \hat{h}_{i\sigma}) \) is the charge-carrier doping concentration.

Within the \( t-J \) model in the fermion-spin representation, the kinetic-energy-driven SC mechanism has been developed in the case of the absence of the electronic nematicity, \( (39) \) where the interaction between the charge carriers directly from the kinetic energy of the \( t-J \) model by the exchange of a strongly dispersive spin excitation generates the d-wave charge-carrier pairing in the particle-particle channel, then the d-wave electron pairs originated from the d-wave charge-carrier pairing state are due to the charge-spin recombination, and their condensation reveals the d-wave SC-state. The typical features of the kinetic-energy-driven superconductivity can also be summarized as: (i) the mechanism of the kinetic-energy-driven superconductivity is purely electronic without phonons; (ii) the mechanism of the kinetic-energy-driven superconductivity shows that the strong electron correlation is favorable to superconductivity, since the bosonic glue is identified into an electron pairing mechanism not involving the phonon, the external degree of freedom, but the internal spin degree of freedom of the constrained electron; (iii) the SC-state is controlled by both the SC gap and quasiparticle coherence, which leads to that the maximal \( T_{c} \) occurs around the optimal doping, and then decreases in both the underdoped and the overdoped regimes. Very recently, the framework of the kinetic-energy-driven superconductivity has been generalized to discuss the intertwining of the electronic nematicity with superconductivity in cuprate superconductors \( (39) \) where the breaking of the rotation symmetry due to the presence of the electronic nematicity is verified by the inequivalence on the average of the electronic structure at the two Bragg scattering sites. Our following discussions builds on the work in Ref. \( (39) \) and only a short summary of the formalism is therefore given. In the recent discussions \( (59) \) the full electron diagonal and off-diagonal propagators of the \( t-J \) model \( (4) \) have been given explicitly as,

\[
G_{\bar{\eta}}(k, \omega) = \frac{1}{\omega - \kappa_{\bar{\eta}}^{(\bar{\eta})} - \Sigma_{\bar{\eta}}^{(\bar{\eta})}(k, \omega)}, \quad (8a)
\]

\[
\Sigma_{\bar{\eta}}^{(\bar{\eta})}(k, \omega) = \frac{L_{\bar{\eta}}^{(\bar{\eta})}(\omega)}{\omega - \kappa_{\bar{\eta}}^{(\bar{\eta})} - \Sigma_{\bar{\eta}}^{(\bar{\eta})}(k, \omega)}, \quad (8b)
\]

where the orthorhombic energy dispersion in the tight-binding approximation is obtained directly from the \( t-J \) model \( (4) \) as,

\[
\kappa_{\bar{\eta}}^{(\bar{\eta})} = -4t'((1 - \varsigma)\gamma_{k_{x}} + (1 + \varsigma)\gamma_{k_{y}}) + 4t'\gamma_{k_{z}} + \mu, \quad (9)
\]

with \( \gamma_{k_{x}} = \cos k_{x}/2, \gamma_{k_{y}} = \cos k_{y}/2, \gamma_{k_{z}} = \cos k_{z}\cos k_{y}, \) while the total self-energy \( \Sigma_{tot}^{(\bar{\eta})}(k, \omega) \) and weight function \( L_{\bar{\eta}}^{(\bar{\eta})}(\omega) \) are specific combinations of the normal self-energy \( \Sigma_{ph}^{(\bar{\eta})}(k, \omega) \) in the particle-hole channel and anomalous self-energy \( \Sigma_{pp}^{(\bar{\eta})}(k, \omega) \) in the particle-particle channel as,

\[
\Sigma_{tot}^{(\bar{\eta})}(k, \omega) = \Sigma_{ph}^{(\bar{\eta})}(k, \omega) + \frac{\Sigma_{pp}^{(\bar{\eta})}(k, \omega)}{\omega - \epsilon_{\kappa}^{(\bar{\eta})} + \Sigma_{pp}^{(\bar{\eta})}(k, -\omega)}, \quad (10a)
\]

\[
L_{\bar{\eta}}^{(\bar{\eta})}(\omega) = -\frac{\Sigma_{pp}^{(\bar{\eta})}(k, \omega)}{\omega + \epsilon_{\kappa}^{(\bar{\eta})} + \Sigma_{pp}^{(\bar{\eta})}(k, -\omega)}, \quad (10b)
\]

where the normal self-energy \( \Sigma_{ph}^{(\bar{\eta})}(k, \omega) \) and anomalous self-energy \( \Sigma_{pp}^{(\bar{\eta})}(k, \omega) \) have been obtained in Ref. \( (39) \) and can be expressed explicitly as,

\[
\Sigma_{pp}^{(\bar{\eta})}(k, \omega) = \frac{1}{2N^2} \sum_{pp'}(-1)^{p+p'} \Omega_{pp'}^{(\bar{\eta})}\left[1 + \frac{\epsilon_{p+k}^{(\bar{\eta})}}{E_{p+k}^{(\bar{\eta})}} \left( \frac{F_{1p}^{(\bar{\eta})}(p, p', k)}{\omega + \epsilon_{pp'}^{(\bar{\eta})} + E_{p+k}^{(\bar{\eta})}} - \frac{F_{2p}^{(\bar{\eta})}(p, p', k)}{\omega - \epsilon_{pp'}^{(\bar{\eta})} - E_{p+k}^{(\bar{\eta})}} \right) \right], \quad (11a)
\]

\[
\Sigma_{ph}^{(\bar{\eta})}(k, \omega) = \frac{1}{2N^2} \sum_{pp'}(-1)^{p+p'} \Omega_{pp'}^{(\bar{\eta})}\left[\sum_{\omega} \frac{\Delta_{\omega}^{(\bar{\eta})}(p + k)}{E_{p+k}^{(\bar{\eta})}} \left( \frac{F_{1p}^{(\bar{\eta})}(p, p', k)}{\omega + \epsilon_{pp'}^{(\bar{\eta})} + E_{p+k}^{(\bar{\eta})}} - \frac{F_{2p}^{(\bar{\eta})}(p, p', k)}{\omega - \epsilon_{pp'}^{(\bar{\eta})} - E_{p+k}^{(\bar{\eta})}} \right) \right], \quad (11b)
\]
where \( \nu = 1, 2 \), \( \Omega_{\nu p p'}^<(k) = Z_F^{(\nu)} [\Delta_0^{(\nu)}(p_p+p_k)^2 B_{p p'}^{(\nu)}(k)/(4 \omega_p^{(\nu)} \omega_{p p'}^{(\nu)}) \) with 
\( \Lambda_0^{(\nu)} = 4t(1-\gamma^2)/(1+\gamma^2) \), the SC quasiparticle energy spectrum \( E^{(\nu)}_{i k}(k) = \sqrt{\varepsilon_k^{(\nu)} + \Delta_0^{(\nu)}(k)^2} \) with the renormalized SC gap \( \Delta_0^{(\nu)}(k) = Z_F^{(\nu)} \Delta_0^{(\nu)}(k) \) and renormalized electron orthorhombic energy dispersion \( \varepsilon_k^{(\nu)} = Z_F^{(\nu)} \varepsilon_k \), \( \omega_{i p p'}^{(\nu)} = \omega_{p p'}^{(\nu)} - 1/2 \omega_p^{(\nu)} \), while the quasiparticle coherent weight \( Z_F^{(\nu)} \), the SC gap \( \Delta_0^{(\nu)}(k) \), the spin orthorhombic excitation spectrum \( \omega_k^{(\nu)} \), the weight function of the spin excitation spectrum \( B_k^{(\nu)} \), and the functions \( F_1^{(\nu)}(p, p', k) \) and \( F_2^{(\nu)}(p, p', k) \) have been given explicitly in Ref. 29. In particular, the sharp peak visible for temperature \( T \to 0 \) in the normal (anomalous) self-energy is actually a \( \delta \)-function, broadened by a small damping used in the numerical calculation at a finite lattice. The calculation in this paper for the normal (anomalous) self-energy is performed numerically on a \( 120 \times 120 \) lattice in momentum space, with the infinitesimal \( i0_+ \to i\Gamma \) replaced by a small damping \( \Gamma = 0.5 \). We are now ready to discuss the doping dependence of the nematic-order state characteristic energy and of its possible connection to the enhancement of superconductivity. In our previous studies of Ref. 28, the order parameter of the renormalized SC gap \( \bar{\Delta}_0^{(\nu)}(k) \) and the QSI pattern observed on BiSrCaCuO \( _{\delta+\delta} \) for the strength of the electronic nematicity \( \varsigma = 0.006 \). The corresponding experimental result of the quasiparticle scattering interference pattern of BiSrCaCuO \( _{\delta+\delta} \) in the binding-energy \( \omega = 94 \) meV at doping \( \delta = 0.06 \) taken from Fig. 1 is also shown in Fig. 1b. The results in Fig. 1 thus show that the momentum-space structure of the ARPES autocorrelation pattern is coexisting electronic nematicity is qualitatively consistent with the corresponding momentum-space structure of the QSI pattern observed on BiSrCaCuO \( _{\delta+\delta} \). Moreover, the characteristic features of two distinct classes of the broken-symmetry states have been summarized: (i) For the quasiparticle scattering processes with the corresponding wave vectors \( q_1, q_4 \), and \( q_3 \), the amplitudes of the quasiparticle scattering wave vectors are respectively inequivalent to their symmetry-coupled partners, while for the quasiparticle scattering process with the corresponding quasiparticle scattering wave vectors \( q_2, q_3, q_6 \), and \( q_7 \), the scattering wave vectors and their symmetry-equivalent partners occur with equal amplitudes. These results therefore indicate that the peaks at the corresponding scattering wave vectors \( q_1, q_4 \), and \( q_3 \) are the signatures of the electronically ordered states with broken both rotation and translation symmetries, while the peaks with the corresponding scattering wave vectors \( q_2, q_3 \), \( q_6 \), and \( q_7 \) are the signatures of the electronically ordered states with broken translation symmetry only; (ii) The intensity of the peak at the Bragg wave vector \( Q_B^{(1)} \) is different from that at the Bragg wave vector \( Q_B^{(0)} \). This difference leads to the inequivalence on the average of the electronic structure at the two Bragg scattering sites \( Q_B^{(1)} \) and \( Q_B^{(0)} \), and therefore shows that the Bragg peaks at the wave vectors \( Q_B^{(1)} \) and \( Q_B^{(0)} \) are the signature of the nematic-order state with the broken \( C_4 \) rotation symmetry.

III. QUANTITATIVE CHARACTERISTICS

In the presence of the electronic nematicity, the original electronic structure with the four-fold \( (C_4) \) rotation symmetry on the square lattice in the absence of the electronic nematicity is broken up into that with a residual two-fold \( (C_2) \) rotation symmetry, while such an aspect should be reflected in QSI. For convenience, we plot the ARPES autocorrelation \( C_i(q, \omega) \) in Fig. 1. We are considering the binding-energy \( \omega = 94 \) meV and the strength of the electronic nematicity \( \varsigma = 0.006 \). In Fig. 1a, the locations of the Bragg peaks \( Q_B^{(1)} = [\pm 2\pi, 0] \) along the \( \hat{x} \) axis and \( Q_B^{(0)} = [0, \pm 2\pi] \) along the \( \hat{y} \) axis are indicated by the circles, while \( q_1, q_2, q_3, q_4, q_5, \) and \( q_6 \) are different quasiparticle scattering wave vectors. For a better comparison, the corresponding experimental result of Ref. 28 of the QSI pattern observed on BiSrCaCuO \( _{\delta+\delta} \) in the binding-energy \( \omega = 94 \) meV at doping \( \delta = 0.06 \) is also shown in Fig. 1b. The results in Fig. 1 thus show that the momentum-space structure of the ARPES autocorrelation pattern in the SC-state with coexisting electronic nematicity is qualitatively consistent with the corresponding momentum-space structure of the QSI pattern observed on BiSrCaCuO \( _{\delta+\delta} \). Moreover, the characteristic features of two distinct classes of the broken-symmetry states have been summarized: (i) For the quasiparticle scattering processes with the corresponding wave vectors \( q_1, q_4 \), and \( q_3 \), the amplitudes of the quasiparticle scattering wave vectors are respectively inequivalent to their symmetry-coupled partners, while for the quasiparticle scattering process with the corresponding quasiparticle scattering wave vectors \( q_2, q_3, q_6 \), and \( q_7 \), the scattering wave vectors and their symmetry-equivalent partners occur with equal amplitudes. These results therefore indicate that the peaks at the corresponding scattering wave vectors \( q_1, q_4 \), and \( q_3 \) are the signatures of the electronically ordered states with broken both rotation and translation symmetries, while the peaks with the corresponding scattering wave vectors \( q_2, q_3 \), \( q_6 \), and \( q_7 \) are the signatures of the electronically ordered states with broken translation symmetry only; (ii) The intensity of the peak at the Bragg wave vector \( Q_B^{(1)} \) is different from that at the Bragg wave vector \( Q_B^{(0)} \). This difference leads to the inequivalence on the average of the electronic structure at the two Bragg scattering sites \( Q_B^{(1)} \) and \( Q_B^{(0)} \), and therefore shows that the Bragg peaks at the wave vectors \( Q_B^{(1)} \) and \( Q_B^{(0)} \) are the signature of the nematic-order state with the broken \( C_4 \) rotation symmetry.

FIG. 1: (Color online) (a) The ARPES autocorrelation pattern in momentum-space in the binding-energy \( \omega = 94 \) meV for the strength of the electronic nematicity \( \varsigma = 0.006 \). (b) The corresponding experimental result of the quasiparticle scattering interference pattern of BiSrCaCuO \( _{\delta+\delta} \) in the binding-energy \( \omega = 94 \) meV at doping \( \delta = 0.06 \) taken from Ref. 28.
the electronic nematicity has been given as,
\[ N(C_\omega)(\omega) = \frac{\bar{C}_x(\omega) - \bar{C}_y(\omega)}{\bar{C}_x(\omega) + \bar{C}_y(\omega)}, \]
where \( \bar{C}_x(\omega) = (1/N) \sum_{\mathbf{q} \in \{\mathbf{Q}^B_y\}} \bar{C}_x(\mathbf{q}, \omega) \) and \( \bar{C}_y(\omega) = (1/N) \sum_{\mathbf{q} \in \{\mathbf{Q}^B_x\}} \bar{C}_y(\mathbf{q}, \omega) \), with the summation \( \mathbf{q} \in \{\mathbf{Q}^B_x\} \) [\( \mathbf{q} \in \{\mathbf{Q}^B_y\} \)] that is restricted to the extremely small area \( \{\mathbf{Q}^B_x\} \) \( \{\mathbf{Q}^B_y\} \) at around \( \mathbf{Q}^B_x \) \( \mathbf{Q}^B_y \). This definition in Eq. (13) is confronted with the reduction of the size effect in a finite-lattice calculation. This follows a basic fact that the calculation for the normal and anomalous self-energies in Eq. (11) is performed numerically on a 120 lattice in momentum space as we have mentioned above, with the infinitesimal \( i\delta \rightarrow i\Gamma \) replaced by a small damping \( \Gamma = 0.05J \), which leads to that the peak weight of the ARPES auto-correlation \( \bar{C}_x(\mathbf{q}, \omega) \) in Eq. (2) at the Bragg wave vector \( \mathbf{Q}^B_x \) \( \mathbf{Q}^B_y \) spreads on the extremely small area \( \{\mathbf{Q}^B_x\} \) \( \{\mathbf{Q}^B_y\} \) at around \( \mathbf{Q}^B_x \) \( \mathbf{Q}^B_y \) point. The summation of these spread weights in \( \bar{C}_x(\omega) \) \( \bar{C}_y(\omega) \) at around this extremely small area \( \{\mathbf{Q}^B_x\} \) \( \{\mathbf{Q}^B_y\} \) can reduce the size effect in the finite-lattice calculation. If this order parameter \( N(C_\omega)(\omega) \) is non-zero, the break of the \( C_4 \) rotation symmetry is occurring. In Fig. 2, we plot the order parameter of the nematic-order state \( N(C_\omega)(\omega) \) as a function of energy for the strength of the electronic nematicity \( \zeta = 0.006 \). For a direct comparison, the corresponding experimental result\(^23\) of the energy dependence of the nematic-order state order parameter observed on \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) at doping \( \delta = 0.06 \) is also shown in Fig. 2. It thus shows clearly that the experimental result\(^23\) of the energy dependence of the nematic-order state order parameter is well reproduced, where \( N(C_\omega)(\omega) \) reaches its maximum in the characteristic energy \( \omega_{\text{max}} \). However, when the energy is turned away from this characteristic energy \( \omega_{\text{max}} \), \( N(C_\omega)(\omega) \) drops rapidly. Moreover, this anticipated characteristic energy \( \omega_{\text{max}} \sim 0.936J = 93.6 \text{ meV} \) is well consistent with the experimental result\(^23\) of \( E_{\text{max}}^{(N)} \sim 94 \text{ meV} \) observed on \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) at doping \( \delta = 0.06 \). This energy dependence of \( N(C_\omega)(\omega) \) with non-zero values further verifies the nematic-order state with broken \( C_4 \) rotation symmetry in a wide energy range.

As a natural consequence of the doped Mott insulators, the characteristic energy of the nematic-order state \( \omega_{\text{max}} \) also evolve strongly with doping. For a better understanding of the doping dependence of \( \omega_{\text{max}} \), we plot the result of \( \omega_{\text{max}} \) as a function of doping for the strength of the electronic nematicity \( \zeta = 0.006 \) in Fig. 3a, in comparison with the corresponding experimental result\(^23\) of the doping dependence of the nematic-order state characteristic energy observed on \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) in Fig. 3b. The result in Fig. 3a indicates clearly that \( \omega_{\text{max}} \) is particularly large in the underdoped regime, and then monotonically decreases as doping is increased, which is fully consistent with the corresponding result observed on \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \). Moreover, we have also compared the above result of the doping dependence of the nematic-order state characteristic energy with the experimental result of the doping dependence of the pseudogap\(^23\), and found that the nematic-order state characteristic energy and pseudogap energy are also identical. The pseudogap in the framework of the kinetic-energy-driven superconductivity originates from the electron self-energy resulting of the dressing of the electrons due to the electron interaction mediated by a strongly dispersive spin excitation\(^23\), and then it can be identified as being a region of the electron self-energy effect\(^23\) in which the pseudogap suppresses strongly the electronic density of states.

The characteristic energy of the nematic-order state in theory and experiment is virtually identical to each other and also to the corresponding pseudogap energy. These results therefore are important to confirm the nematic-order state characteristic energy at the \( C_4 \) rotation symmetry-breaking can be understood as the natural consequence of the electronic nematic-order state within the pseudogap of cuprate superconductors\(^23\).

In our recent studies\(^23\), the evolution of \( T_c \) with the strength of the electronic nematicity has been obtained.
within the framework of the kinetic-energy-driven superconductivity in terms of the self-consistent calculation at the condition of the SC gap $\Delta^{(c)} = 0$, where the optimized $T_c$ at the optimal doping $\delta = 0.15$ increases with the increase of the strength of the electronic nematicity, and reaches its maximum in the optimal strength of the electronic nematicity $\varsigma \approx 0.022$, subsequently, the optimized $T_c$ decreases with the increase of the strength of the electronic nematicity in the strong strength region. This dome-like shape nematic-order strength dependence of $T_c$ therefore shows clearly that superconductivity in cuprate superconductors is enhanced by the electronic nematicity. In particular, it has been shown that the energy in the SC-state with coexisting electronic nematicity is lower than the corresponding energy in the SC-state with the absence of the electronic nematicity. Moreover, the SC condensation energy as a function of the nematic-order state strength shows the same behavior of $T_c$. This same dome-like shape nematic-order strength dependence of the SC condensation energy thus leads to that superconductivity is enhanced by the electronic nematicity, and $T_c$ exhibits a dome-like shape nematic-order strength dependence.

Now we turn our attention to the possible connection between the nematic-order state characteristic energy and the enhancement of superconductivity. To show this possible connection more clearly, we plot (a) $\omega_{\text{max}}$ and (b) $T_c$ as a function of the strength of the electronic nematicity $\varsigma$ at the underdoping $\delta = 0.06$ in Fig. 4. In order to compare clearly the present results of the nematic-order state strength dependence of $\omega_{\text{max}}$ and $T_c$ with the corresponding results at different doping levels, the previous results of (c) $\omega_{\text{max}}$ and (d) $T_c$ as a function of the strength of the electronic nematicity at the optimal doping $\delta = 0.15$ are also shown in Fig. 4. Obviously, two characteristic features in Fig. 4 can be summarized as: (i) for the present case at the underdoping $\delta = 0.06$ (see Fig. 4a and Fig. 4b), the strength range together with the optimal strength in $\omega_{\text{max}}$ are the same with that in $T_c$. In particular, with the increase of the nematic-order state strength, $\omega_{\text{max}}$ (then $T_c$) is raised gradually in the weak strength region, and achieves its maximum at around the optimal strength $\varsigma = 0.022$. However, with the further increase of the strength, $\omega_{\text{max}}$ (then $T_c$) turns into a monotonically decrease in the strong strength region; (ii) In comparison with the result of $\omega_{\text{max}}$ at the optimal doping $\delta = 0.15$ (see Fig. 4c and Fig. 4d), $\omega_{\text{max}}$ in Fig. 4a ($T_c$ in Fig. 4b) for a given nematic-order state strength at the underdoping $\delta = 0.06$ is much larger (lower) than the corresponding $\omega_{\text{max}}$ in Fig. 4c ($T_c$ in Fig. 4d) at the optimal doping $\delta = 0.15$. However, the global dome-like shape of the nematic-order strength dependence of $\omega_{\text{max}}$ and $T_c$ at the underdoping $\delta = 0.06$ together with the magnitude of the optimal strength are the same with that at the optimal doping $\delta = 0.15$. Therefore the enhancement of superconductivity occurs at any given doping of the SC dome. This same strength range together with the same optimal strength in the characteristic energy $\omega_{\text{max}}$ and SC transition temperature $T_c$ therefore indicates firstly a possible connection between the nematic-order state characteristic energy and the enhancement of superconductivity.

![Figure 4](image-url)

**FIG. 4:** (Color online) (a) The nematic-order state characteristic energy as a function of the strength of the electronic nematicity $\varsigma$ at the underdoping $\delta = 0.06$. (b) The superconducting transition temperature as a function of the strength of the electronic nematicity at the underdoping $\delta = 0.06$. The corresponding results of (c) the nematic-order state characteristic energy as a function of the strength of the electronic nematicity at the optimal doping $\delta = 0.15$ and (d) the superconducting transition temperature as a function of the strength of the electronic nematicity at the optimal doping $\delta = 0.15$ taken from Ref. [39].

**IV. SUMMARY AND DISCUSSIONS**

Within the framework of the kinetic-energy-driven superconductivity, we have studied the doping dependence of the nematic-order state characteristic energy in cuprate superconductors and of its possible connection to the enhancement of superconductivity. Our results show clearly that the characteristic energy of the nematic-order state is particularly large in the underdoped regime, then it smoothly decreases as doping is increased, in full agreement with the corresponding to the STS experimental observations. More importantly, our results also indicate firstly that the characteristic energy of the nematic-order state as a function of the nematic-order state strength in the underdoped regime presents a similar behavior of the SC transition temperature. On the basis of these obtained results, the theory therefore predicts a possible connection between the nematic-order state characteristic energy and the enhancement of superconductivity.

Finally, it should be noted that apart from the emergence of the electronically nematic order in cuprate superconductors, the electronic nematicity has been observed across several families of...
strongly correlated electron systems, including iron-based superconductors, strontium ruthenate, kagome lattice materials, as well as nickel-based superconductors. In particular, in the context of iron-based superconductors, the experimental observations have shown a striking enhancement of nematic fluctuations centred at optimal tuning of superconductivity. Moreover, the nematic-fluctuation-enhanced superconductivity in nickel-based superconductors has been observed experimentally. In a strongly correlated electron system, the strong electron correlation induces the system to find new way to lower its total energy, often by spontaneous breaking of the native symmetries of the lattice. These experimental observations together with the experimental detection in cuprate superconductors therefore indicate that the electronic nematicity is a common phenomenon in strongly correlated electron systems, and then a characteristic feature in the complicated phase diagram is the interplay between the electronic nematicity and superconductivity.

Acknowledgements

ZC, XM, and SF are supported by the National Key Research and Development Program of China, and the National Natural Science Foundation of China (NSFC) under Grant Nos. 11974051 and 11734002. HG is supported by NSFC under Grant Nos. 11774019 and 12074022, and the Fundamental Research Funds for the Central Universities and HPC resources at Beihang University.
Lawler, A. P. Mackenzie, J. C. S. Davis, and K. Fujita, Proc. Natl. Acad. Sci. 116, 13249 (2019).
29 S. A. Kivelson, E. Fradkin, and V. J. Emery, Nature 393, 550 (1998).
30 J. Zaanan, Physica C 317-318, 217 (1999).
31 S. A. Kivelson, I. P. Bindfradkin, V. Oganesyan, J. M. Tranquada, A. Kapitulnik, and C. Howald, Rev. Mod. Phys. 75, 1201 (2003).
32 L. Nie, L. E. H. Sierens, R. G. Melko, S. Sachdev, and S. A. Kivelson, Phys. Rev. B 92, 174505 (2015).
33 C. J. Halboth and W. Metzner, Phys. Rev. Lett. 85, 5162 (2000).
34 M. Kitatani, N. Tsuji, and H. Aoki, Phys. Rev. B 95, 075109 (2017).
35 Z. Dai, Y.-H. Zhang, T. Senthil, and P. A. Lee, Phys. Rev. B 97, 174511 (2018).
36 W. L. Tu and T. K. Lee, Sci. Rep. 9, 1719 (2019).
37 N. Auvray, B. Loret, S. Benhabib, M. Cazayous, R. D. Zhong, J. Schneeloch, G. D. Gu, A. Forget, D. Colson, I. Paul, A. Sacuto, and Y. Gallais, Nat. Commun. 10, 5209 (2019).
38 S. Bultut, W. A. Atkinson, and A. P. Kampf, Phys. Rev. B 88, 155132 (2013).
39 Z. Cao, Y. Guo, and S. Feng, arXiv:2105.14494.
40 S. H. Pan, J. P. O’Neal, R. L. Badzey, C. Chamon, H. Ding, J. R. Engelbrecht, Z. Wang, H. Eisaki, S. Uchida, A. K. Gupta, K.-W. Ng, E. W. Hudson, K. M. Lang, and J. C. Davis, Nature 413, 282 (2001).
41 See, e.g., the review, Ø. Fischer, M. Kugler, I. Maggio-Aprile, C. Berthod, and C. Renner, Rev. Mod. Phys. 79, 353 (2007).
42 See, e.g., the review, J.-X. Yin, S. H. Pan, M. Z. Hasan, Nat. Rev. Phys. 3, 249 (2021).
43 D. Gao, Y. Mou, Y. Liu, S. Tan, and S. Feng, Phil. Mag. 99, 752 (2019).
44 Q.-H. Wang and D.-H. Lee, Phys. Rev. B 67, 020511 (2003).
45 U. Chatterjee, M. Shi, A. Kaminski, A. Kanigel, H. M. Fretwell, K. Terashima, T. Takahashi, S. Rosenkrantz, Z. Z. Li, H. Raffy, A. Santander-Syro, K. Kadowaki, M. R. Norman, M. Randeria, and J. C. Campuzano, Phys. Rev. Lett. 96, 107006 (2006).
46 Y. He, Y. Yin, M. Zech, A. Soumyanarayanan, M. M. Yee, T. Williams, M. C. Boyer, K. Chatterjee, W. D. Wise, I. Zeljkovic, T. Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, J. E. Hoffman, Science 344, 608 (2014).
47 P. W. Anderson, Science 235, 1196 (1987).
48 Ying-Jer Kao and Hae-Young Kee, Phys. Rev. B 72, 024502 (2005).
49 B. Edegger, V. N. Muthukumar, and C. Gros, Phys. Rev. B 74, 165109 (2006).
50 A. Wollny and M. Vojta, Physica B 404, 3079 (2009).
51 K. Lee, S. A. Kivelson, and E.-A. Kim, Phys. Rev. B 94, 014204 (2016).
52 S. Feng, J. B. Wu, Z. B. Su, and L. Yu, Phys. Rev. B 47, 15192 (1993).
53 See, e.g., the review, L. Yu, in Recent Progress in Many-Body Theories, edited by T. L. Ainsworth, C. E. Campbell, B. E. Clements, and E. Krotscheck (Plenum, New York, 1992), Vol. 3, p. 157.
54 See, e.g., the review, P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. 78, 17 (2006).
55 S. Feng, J. Qin, and T. Ma, J. Phys.: Condens. Matter 16, 343 (2004); S. Feng, Z. B. Su, and L. Yu, Phys. Rev. B 49, 2368 (1994).
56 See, e.g., the review, S. Feng, Y. Lan, H. Zhao, L. Kuang, L. Qin, and X. Ma, Int. J. Mod. Phys. B 29, 1530009 (2015).
57 S. Feng, Phys. Rev. B 68, 184501 (2003); S. Feng, T. Ma, and H. Guo, Physica C 346, 14 (2006).
58 S. Feng, H. Zhao, and Z. Huang, Phys. Rev. B 85, 054509 (2012); Phys. Rev. B 85, 099902(E) (2012).
59 S. Feng, L. Kuang, and H. Zhao, Physica C 517, 5 (2015).
60 See, e.g., the review, T. Timusk and B. Statt, Rep. Prog. Phys. 62, 61 (1999).
61 See, e.g., the review, S. H¨ufner, M. A. Hossain, A. Damascelli, and G. A. Sawatzky, Rep. Prog. Phys. 71, 062501 (2008).
62 T.-M. Chuang, M. P. Allan, J. Lee, Y. Xie, N. Ni, S. L. Budko, G. S. Boebinger, P. C. Canfield, J. C. Davis, Science 327, 181 (2010).
63 Y. Gallais, R. M. Fernandes, I. Paul, L. Chauvi`ere, Y.-X. Yang, M.-A. M´easson, M. Cazayous, A. Sacuto, D. Colson, and A. Forget, Phys. Rev. Lett. 111, 267001 (2013).
64 P. Massat, D. Farina, I. Paul, S. Karlsson, P. Strobel, P. Toulemonde, M.-A. M´easson, M. Cazayous, A. Sacuto, S. Kasahara, T. Shibatachi, Y. Matsuda, and Y. Gallais, Proc. Natl. Acad. Sci. 113, 9177 (2016).
65 R. A. Borzi, S. A. Grigera, J. Farrell, R. S. Perry, S. J. S. Lister, S. L. Lee, D. A. Tennant, Y. Maeno, and A. P. Mackenzie, Science 315, 214 (2007).
66 J. X. Yin, S. S. Zhang, H. Li, K. Jiang, G. Chang, B. Zhang, B. Lian, C. Xiang, I. Belopol’ski, H. Zheng, T. A. Cochran, S. Y. Xu, G. Bian, K. Liu, T. R. Chang, H. Lin, Z. Y. Lu, Z. Wang, S. Jia, W. Wang and M. Z. Hasan, Nature 562, 91 (2018).
67 R. Okazaki, T. Shibatachi, H. J. Shi, Y. Haga, T. D. Matsuda, E. Yamamoto, Y. Onuki, H. Ikeda, Y. Matsuda, Science 331, 439 (2011).
68 C. Eckberg, D. J. Campbell, T. Metz, J. Collini, H. Hodovanets, T. Drye, P. Zavalij, M. H. Christensen, R. M. Fernandes, S. Lee, P. Abbamonte, J. W. Lynn, and J. Paglione, Nat. Phys. 16, 346 (2020).