Probing microstructure of black hole spacetimes with gravitational wave echoes

Naritaka Oshita
Research Center for the Early Universe (RESCEU), Graduate School of Science,
The University of Tokyo, Tokyo 113-0033, Japan and
Department of Physics, Graduate School of Science,
The University of Tokyo, Tokyo 113-0033, Japan

Niayesh Afshordi
Department of Physics and Astronomy, University of Waterloo, Waterloo, ON, N2L 3G1, Canada and
Perimeter Institute for Theoretical Physics, 31 Caroline St. N., Waterloo, ON, N2L 2Y5, Canada

Quantum nature of black hole horizons has been a subject of recent interest and scrutiny. In particular, a near-horizon quantum violation of equivalence principle has been proposed as a resolution of the black hole information paradox. Such a violation may lead to a modified dispersion relation at high energies, which could become relevant due to the intense gravitational blueshift experienced by ingoing gravitational waves. We investigate the ringdown for a perturbed black hole with such a modified dispersion relation and find that infalling gravitational waves are partially reflected near the horizon. This results in the appearance of late-time echoes in the ringdown phase of black hole merger events, with similar properties to those (arguably) seen in the Advanced LIGO observations. Current measurements suggest a Lorentz-violation scale of $10^{13-14} \text{GeV}$ for gravitational waves, with comparable dissipation and dispersion. Therefore, if confirmed, black hole ringdown echoes probe the microstructure of horizons and thus can test Lorentz-violating UV completions, such as in Hofava-Lifshitz gravity.

Reconciliation of Einstein’s theory of general relativity with quantum mechanics is one of the deepest mysteries in theoretical physics, having lead to a multitude of proposals for a theory of quantum gravity. A quantum theory of gravity is expected to provide a microscopic picture of spacetime. One litmus test is an explanation of the black hole information paradox. Such a violation may lead to a modified dispersion relation at high energies, which could become relevant due to the intense gravitational blueshift experienced by ingoing gravitational waves. We investigate the ringdown for a perturbed black hole with such a modified dispersion relation and find that infalling gravitational waves are partially reflected near the horizon. This results in the appearance of late-time echoes in the ringdown phase of black hole merger events, with similar properties to those (arguably) seen in the Advanced LIGO observations. Current measurements suggest a Lorentz-violation scale of $10^{13-14} \text{GeV}$ for gravitational waves, with comparable dissipation and dispersion. Therefore, if confirmed, black hole ringdown echoes probe the microstructure of horizons and thus can test Lorentz-violating UV completions, such as in Hofava-Lifshitz gravity.

It has been recently proposed that ringdown gravitational waves (ringdown-GWs) may be useful to test the structure of black hole horizons [5-10]. This structure may arise in exotic compact objects such as wormholes [11], gravastars [12], fuzzballs [13], firewalls [14], 2-2 holes [15], and orbifold membranes [16]. In this manuscript, we study the possibility that microstructure on black hole horizons can lead to a modification of the dispersion relation (DR) of GWs, which can be probed with the observation of ringdown gravitational waves (ringdown-GWs) from merger events that lead to the formation of black holes.

Theories of quantum gravity generically modify the microstructure of spacetime on small scales comparable to Planck length $\ell_{Pl}$, which may lead to a modification of DR, or broken Lorentz invariance, as we approach Planck energy (e.g., [17, 18]). The broken Lorentz invariance requires existence of a preferred coordinate system (or spacetime foliation). For the purpose of this study, we shall assume that this preferred coordinate system is defined by the black hole spacetime Killing vectors, in terms of which the metric is static (or stationary) [12].

For simplicity, we shall focus on a non-spinning black hole, and calculate the ringdown-GWs in the Schwarzschild coordinate: \[ ds^2 = -F(r)dt^2 + F(r)^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2), \] where $F(r) \equiv 1 - r_g/r$ and $r_g$ is the Schwarzschild radius. The Bekenstein-Hawking entropy can be derived by using the (Euclidean) Schwarzschild coordinate that possesses the conical singularity on its horizon [4, 19, 20], and the derivation of the Wald entropy [21] is based on a Noether charge which is defined on a Cauchy surface not covering the interior region but only covering the exterior region. In this sense, the concept of the Bekenstein-Hawking entropy can be associated with the Schwarzschild coordinate (rather than e.g., Kruskal coordinates, which is seen by an infalling observer going across the horizon). As such, one may argue that the static Schwarzschild coordinates are the natural choice of coordinates for black hole microstates, which would break Lorentz symmetry.

In addition, arguments based on the black hole information paradox suggest that the black hole interior may not exist (e.g., [14, 22, 23]). According to this hypothesis, the “no drama” picture in which an observer freely falls across the black hole horizon is no longer valid, signalling a quantum violation of Einstein’s equivalence principle.

If a modified DR turns on at frequencies reach Planck energy $E_{Pl}$, the adiabatic approximation near the horizon (where frequencies diverge) is violated, and thus we
can convert ingoing to outgoing modes. This leads to a drastic change in the quasi-normal modes (QNMs) of black holes, and in particular, echoes could show up in the late-time tail of ringdown-GWs. Indeed, tentative evidence/detection of such echoes have been reported by several authors for all the gravitational wave merger events observed by the LIGO/Virgo collaboration [24–26]. Although the origin of the reported echoes is still controversial [27–30], we shall see that our results suggest such a signal could be supporting evidence for the modification of DR at the Planck scale, probing the microscopic structure of spacetime [17, 18].

The ringdown-GWs consist of the QNMs of a black hole, which are calculated by solving the Regge-Wheeler (RW) equation [31–33]

\[
\left[\frac{\partial^2}{\partial r^*^2} + \omega^2 - V_{t,s}(r^*)\right] \psi_s(r^*, \omega) = 0, \tag{1}
\]

with the appropriate boundary condition:

\[
\lim_{r^* \to -\infty} \psi_s \sim e^{-i kr^*}, \quad \lim_{r^* \to +\infty} \psi_s \sim e^{i kr^*}, \tag{2}
\]

where we used \(\ell_{\text{Pl}}/r_g \ll 1\). Here, we only include the radial derivative terms, the first and second term in (4), as modifications of RW equation since they dominate in the near-horizon limit because of the blueshift factor, \(1/F(r)\). All modification terms are suppressed by \((GM)^{-1}/\ell_{\text{Pl}}\) and are negligible far from the black hole. Then one can read the original modified DR, (3), from the modified RW equation, (4), by using \(K^2 \simeq k^2 F(r) \leftrightarrow -\partial^2 F^{-1}/\partial r^*^2\), near the horizon or at a distant region where the RW potential is negligible [43].

To compute QNMs, we can impose the outgoing modes at the distant region, \(\lim \psi_s \sim e^{i kr^*}\). On the other hand, imposing ingoing modes near the horizon is impossible since \(F(r)^{-1} = (1 - r_g/r)^{-1}\) in (4) diverges at the horizon and the first term becomes dominant, which, as we see below, gives a non-oscillatory solution near the horizon. The modified RW equation (4), is a fourth-derivative equation, and therefore, it gives four independent solutions. Using \(F(r) \simeq e^{r^*/r_g}^{-1}\) near the horizon, \(-r^* \gg r_g\), one obtains an analytic asymptotic solution:

\[
\psi_s \simeq \sum_{n=0}^{3} C_n \left(\frac{r^*}{r_g}\right)^n + \sum_{i=1}^{3} f_i(r^*, C_n) e^{i} + O(e^4), \tag{5}
\]

where \(f_i (i = 1, 2, 3)\) are fourth-order polynomials in \(r^*/r_g\) and an expansion coefficient, \(e\), is a function of \(r^*/r_g\), \(\omega\), and \(C\), which are the conjugates of the Schwarzschild time, \(t\), and tortoise coordinate, \(r^*\), respectively.

In the following calculation, as an example, we will use a quartic modified DR

\[
\Omega^2 - K^2 = -C^2 K^4/E_{\text{Pl}}^2 - i\gamma K^2 \Omega/E_{\text{Pl}}, \tag{3}
\]

where \(C\) and \(\gamma\) are arbitrary parameters, similar to “lattice size effects” and “viscosity” in ordinary material. \(\Omega \equiv \omega/\sqrt{F(r)}\) and \(K \equiv k/\sqrt{F(r)}\) are the proper frequency and wavenumber, respectively. Note that the wavenumber at which the group velocity becomes zero, \(K_{\text{max}}\), is estimated as \(K_{\text{max}} = E_{\text{Pl}}/\sqrt{\gamma^2/2 + 2C^2}\) (see Appendix A) and the dissipation effect is adjustable by changing the value of \(\gamma\). Then we have modification terms in RW equation corresponding to the right hand side in (3):

\[
\frac{C^2}{E_{\text{Pl}}^2} F^{-1}(r) \frac{\partial}{\partial r^*} \frac{\partial}{\partial r^*} + i\frac{\omega}{E_{\text{Pl}}} F^{-1/2}(r) \frac{\partial^2}{\partial r^*^2} + \frac{\partial^2}{\partial r^*^2} + \omega^2 - V_{t,s}(r^*) \psi_s(r^*, \omega) = 0, \text{ with } F(r) \simeq e^{r^*/r_g}^{-1}, \tag{4}
\]

where \(\ell\) is a spherical harmonics mode, \(s\) is the index of the spherical-harmonic expansion, \(V_{t,s}(r^*)\) is the RW potential. Here we define the Schwarzschild frequency, \(\omega\), and tortoise wavenumber, \(k\), which are the conjugates of the Schwarzschild time, \(t\), and tortoise coordinate, \(r^*\), respectively.

\[
\epsilon(r^* - r_g) \equiv e^{(r^* - r_g)/2r_g} \frac{r_g}{\ell_{\text{Pl}}} e^{(r^* - r_g)/2r_g}, \tag{6}
\]

where the sequential solutions, (5), are good approximations for \(|\epsilon(r^*)| < 1\) \((r^* < r_{\text{Pl}}) \equiv -r_g \ln \left[\frac{r_g^2}{(\ell_{\text{Pl}}^2)}\right]\). The explicit forms of \(f_i (i = 1, 2, 3)\), which has the dependence on the parameters \(C\) and \(\gamma\), are shown in Appendix B. As is shown in (5), in the vicinity of the horizon \((r^* \lesssim r_{\text{Pl}})\), the solution of modified RW equation is no longer oscil-
mimic the angular momentum barrier for a Schwarzschild

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tute boundary condition to obtain the QNMs of a black

hole with the assumed modified DRs. We will impose a

condition that there is no flux for \( r^* \to -\infty \), i.e. all the

ingoing energy is either reflected or absorbed by the hori-

zon microstructure. The no flux condition at the horizon

means that the mode function should be spatially

constant for \( r^* \to -\infty \), and on the other hand, we can

impose the outgoing condition for \( r^* \to +\infty \):

\[
\lim_{r^* \to -\infty} \psi_s = \text{const.}, \quad \lim_{r^* \to +\infty} \psi_s \sim e^{i\omega r^*}.
\]

(7)

The no flux condition at the horizon singles out the so-

olution with \( C_0 \neq 0 \) and \( C_1 = C_2 = C_3 = 0 \) in (5)

since the spatial derivative of the mode function ex-

ponentially approaches zero in the limit of \( r^* \to -\infty \). Set-

ting \( f = s = 2 \), the numerical solution for \( \psi_{s=2}(\omega, r^*) \)

is shown in (1). In the region between the hori-

zon \( (r^* = -\infty) \) and \( r^* \sim r^*_\text{Pl} \) (Region I in Fig. 1), the

mode function is almost spatially constant and there is

vanishing flux. On the other hand, in Region II in Fig.

(1), a long-lived mode, which is the superposition of an

outgoing and ingoing mode, is trapped. Denoting the

amplitudes of outgoing and ingoing modes in Region II

as \( A_{\text{out}} \) and \( A_{\text{in}} \), respectively, the several values of reflec-

tion rate, \( |A_{\text{out}}|/|A_{\text{in}}| \), are plotted with the various values of

frequency in Fig. (5). Finally, Region III is located

outside the angular momentum barrier, where we have

imposed a purely outgoing condition.

Numerically solving the modified RW equation (4),

with the boundary condition (7), we obtain the QNMs

that include highly long-lived modes (see Fig. 2). For

simplicity, we here use the Pöschl-Teller (PT) potential,

\( V_{\text{PT}}(r^*) \), which has the form [34, 35]

\[
V_{\text{PT}}(r^*) = \frac{V_0}{\cosh^2\left(\frac{\alpha(r^*-r^*_{\text{top}})/r_g}{2}\right)},
\]

(8)

where \( V_0 \), \( \alpha \) and \( r^*_{\text{top}} \) are constant parameters. To

mimic the angular momentum barrier for a Schwarzschild

black hole \( V_{2,2}(r^*) \), we set \( V_0 = V_{2,2}(r^*_{\text{top}}) \) and

\( \alpha^2 = -(2V_{2,2}(r^*_{\text{top}}))^{-1}\partial^2_r V_{2,2}(r^*)|_{r=r^*_{\text{top}}} \)

where \( r^*_{\text{top}} \) should be taken so that \( dV_{2,2}(r^*_{\text{top}})/dr^* = 0 \). In the fol-

lowing, we therefore choose \( V_0 = 0.605/r^2_g \), \( \alpha = 0.362 \) and

\( r^*_{\text{top}} = 1.195r_g \) to mimic the RW potential. We further

assume \( r_g/\ell_{\text{Pl}} = 10^{2.5} \) for illustrative purposes. Using

more realistic values of \( r_g/\ell_{\text{Pl}} \sim 10^4 \) would only loga-

rithmically increase echo time delays and the number of

trapped QNMs [10, 25].

One can see that the low-lying QNMs become less long-

lived as the dissipation factor \( \gamma \) becomes larger (solid

arrows in Fig. 2). The real parts of low-lying QNMs

become larger while their imaginary parts also become

slightly larger as the parameter \( C \) increases (dashed

arrows in Fig. 2).

We can confirm that the long-lived QNMs lead to the

appearance of echoes in the late-time tail of ringdown-

GWs by calculating the time-domain wave function,

which can be recovered via

\[
\Psi_{s=2}(t, r^*) = \frac{1}{\sqrt{2\pi}} \int d\omega e^{-i\omega t} \psi_s(\omega, r^*).
\]

(9)

Assuming a static initial data, \( \partial_t \Psi_s(0, r^*) = 0 \), the mode

function \( \psi_s \) can be obtained by the (retarded) Green’s

function, \( G(r^*, r^*, t) \), as [36]

\[
\psi_{s=2}(\omega, r^*) = \int dr^* \partial_t G(r^*, r^*, t) \psi_{s=2}(0, r^*)
\]

\[
= \frac{\psi_+}{2i\omega A_{\text{in}}} \int_{-\infty}^{r^*} dr^* I(\omega, r^*) \psi_-
\]

\[
+ \frac{\psi_-}{2i\omega A_{\text{in}}} \int_{r^*}^{\infty} dr^* I(\omega, r^*) \psi_+,
\]

(10)

where \( I(\omega, r^*) \) is a source term that includes an initial

data of GWs around the black hole. We here take the

static and Gaussian initial data which has a peak at

\( \sigma_s(t = 0, r^*) = e^{-(r^*-r_{\text{ini}})^2/\sigma^2} \) and \( \partial_t \Psi_{s}(t = 0, r^*) = 0 \), and the form of

the source term is given by

\[
I(\omega, r^*) = i\omega \Psi_{s=2}(t = 0, r^*).
\]

(11)

The real part of the mode function and time-domain

wave function, as seen by an observer at \( r^* = r^*_{\text{obs}} \),

\( \text{Re}[\psi_s(\omega, r^*_{\text{obs}})] \) and \( \text{Re}[\Psi_{s=2}(t, r^*_{\text{obs}})] \), are shown in Fig.

(3). One can find that the long-lived QNMs with \( n = 3, 4, 5 \)

are relatively excited compared to other QNMs (red line in Fig. 3a), which leads to the echoes in the late-time tail of ringdown-GWs. We find out that the phase of echoes depends on the parameter \( C \) (Fig. 4) and the time interval in which we observe the echo, \( \Delta t_{\text{echo}} \),

can be evaluated as (see Appendix C)

\[
\Delta t_{\text{echo}} \simeq -2 \times r^* |\kappa| K_{\text{max}} \simeq 2r_g \ln \left[ \frac{2(2C^2 + \gamma^2)E_{\text{Pl}}^2}{c(4C^2 + \gamma^2)^2 \omega^2} \right].
\]

(12)

On the other hand, the amplitude of echoes depends
on the dissipation factor $\gamma$, and the echoes disappear in the limit of $|\gamma| \gg |C|$ (Fig. 4). This is consistent with the dependence of QNMs on the parameters $C$ and $\gamma$; the real parts of QNMs (the phase of echoes) depend on $C$ and the imaginary parts (the dissipation effect on ringdown-GWs) largely depend on $\gamma$.

We also calculated the reflection rate of GWs near black hole horizon for several frequencies of GWs (Fig. 5) and confirmed that the reflection rate is consistent with the amplitude of echo (see Appendix D). A mode function $\psi_{s=2}(\omega, r^*)$ in the range of $r^*_p < r^* < 0$ (Region II in Fig. 1) can be decomposed into an out going and ingoing mode: $\psi_{s=2} = A_{\text{out}} e^{i\omega r^*} + A_{\text{in}} e^{-i\omega r^*}$. Then we can calculate the reflection rate, $|A_{\text{out}}|/|A_{\text{in}}|$, as a function of $\omega$, and one finds that the GWs is perfectly reflected at horizon for $\gamma = 0$, while the reflection rate drops as the dissipation term, $\gamma$, increases (Fig. 5a). Interestingly, the reflection rate of long wavelength GWs are higher compared to those of short wavelength GWs, which is consistent with Ref. [16]. We can find an analytic form of the reflection rate based on the WKB approximation (see Appendix C):

$$|A_{\text{out}}|/|A_{\text{in}}| \approx \exp \left[ -\sqrt{2 + 4C^2/\gamma^2} \left( \frac{\hbar \omega}{k_B T_H} \right) \right] , \quad (13)$$

in terms of Hawking temperature $T_H = \hbar k/(2\pi k_B)$ or surface gravity $\kappa$ for the BH horizon.

The zero flux boundary condition (7) is not appropriate for a modified DR with $\gamma = 0$ and with $C^2 < 0$ since it does not lead to zero group velocity of infalling GWs near horizon. However, the wavenumber at which the group velocity becomes zero is given by $K_{\text{max}} = E_{\text{Pl}}/\sqrt{\gamma^2/2 + 2C^2}$, and therefore, the negativity of $C^2$, as is suggested by e.g., Hořava-Lifshitz gravity [41], can be offset by the large value of $\gamma$ for which the zero flux condition is satisfied. Fixing $\gamma$ with a large value, we here investigate the dependence of reflection rate on the parameter $C$ (Fig. 5b). Since the term including
the parameter $C$ does not contribute to the reflection of GWs in the case of $C^2 < 0$, the reflection rate is smaller compared to the case of $C^2 > 0$ or $C = 0$. Furthermore, taking $C^2 < 0$ and $\gamma^2 \gg |C|^2$, suppresses the amplitude of echoes and shifts them to lower frequencies.

Let us now see how currently measured echo properties constrain the modified dispersion relation (3). For the binary NS merger event GW170817 [37], Ref. [26] reports repeating echoes with frequency $f = 72$ Hz that decay (in power) by a factor of 2 within 0.2 sec. This result is consistent with a BH remnant with $M = 2.6 - 2.7 M_\odot$ and spin $0.84 - 0.87$, fixing the exponent in (C4) to -2.4% for $\frac{\hbar}{2E_{\text{Pl}}} \approx 0.035$, which in turn implies $\gamma \approx C$.

Furthermore, Ref. [25] reports echoes in aLIGO BH mergers are consistent with: $2\Delta t_{\text{echo}}/r_g = 620 \pm 32$. For the aLIGO frequency range $100 - 200$ Hz, Eq. (12) implies $K_{\text{max}} \sim E_{\text{Pl}}/C \sim 10^{-6}\pm 2 E_{\text{Pl}} = 10^{13\pm 2}$ GeV, which is suggestively close to both the scale of grand unified theories (GUTs), as well as the tachyonic big bang models that seed cosmic structures [38, 39].

In this manuscript, we have proposed the possibility that the microstructure on black hole horizons cause a Planck scale modification of DR, which could be probed by observing the late-time tail of ringdown-GWs from formation of black holes. Assuming a simplified modified DR with the characteristic dissipation and dispersion scales, $\gamma$ and $C$, we have numerically calculated the QNMs of a Schwarzschild black hole. The modified DRs change the boundary condition for mode functions of GWs near the horizon and lead to the highly long-lived QNMs, resulting in the appearance of echoes at the late-time tail of ringdown-GWs. Although the echoes in ringdown-GWs have been studied in the context of the probe of exotic compact objects [11–16], our results suggest that the echoes could be a probe of quantum gravity theory itself, which would pinpoint the modification of DR in nature. In particular, current reported echo signals are consistent with a Lorentz-violation scale of $10^{13\pm 2}$ GeV, with comparable dissipation and dispersion. The future work will explore the ringdown-GWs with the DRs predicted by the loop quantum gravity [40] and the Hořava-Lifshitz gravity [41].

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**Appendix A: Derivation of $K_{\text{max}} = E_{\text{Pl}}/\sqrt{2C^2 + \gamma^2/2}$**

Starting with the modified DR,

$$\Omega^2 = K^2 - C^2 K^4 / E_{\text{Pl}}^2 - i \gamma K^2 / E_{\text{Pl}}, \quad (A1)$$

one has

$$\Omega = \pm K \sqrt{1 - (C^2 + \gamma^2/4)K^2 / E_{\text{Pl}}^2 - i \gamma K^2 / 2E_{\text{Pl}}^2}. \quad (A2)$$

Calculating $d\Omega/dK$, one obtains

$$\frac{d\Omega}{dK} = \pm K \sqrt{1 - (C^2 + \gamma^2/4)K^2 / E_{\text{Pl}}^2 - i \gamma K / E_{\text{Pl}}} \quad (A3)$$

where the first term, $\text{Re}(d\Omega/dK)$, is the group velocity and the second term, $\text{Im}(d\Omega/dK)$, gives dissipation rate. Finally, one finds that $K_{\text{max}}$, for which the group velocity becomes zero, takes the form of

$$K_{\text{max}} = \frac{E_{\text{Pl}}}{\sqrt{2C^2 + \gamma^2/2}} \quad (A4)$$

**Appendix B: asymptotic solutions for the modified RW equation**

In the near-horizon limit, $r^* \to -\infty$, the analytic asymptotic solutions for the modified RW equation can be derived. Since the term including a fourth derivative in Eq. (4) becomes the most dominant term in the near-horizon limit, the zeroth order of the asymptotic solutions should be a fourth-order polynomial. Furthermore, it is sequentially found that the first and second order of the asymptotic solutions should be suppressed by $e^{-r^*/2r_g}$ and by $e^{-r^*/r_g}$, respectively:

$$\psi_\text{s}(\omega, r^*) = \sum_{i=0}^{3} f_i(r^*) \epsilon^i + \mathcal{O}(\epsilon^4), \quad (B1)$$

where $\epsilon(r^*) \equiv r_g e^{(r^*-r_\text{min})/2r_g}/E_{\text{Pl}}$ and $f_i(r^*)$ ($i = 0, 1, 2, 3$) are fourth-order polynomials. Substituting the following ansatz into the modified RW equation, Eq. (4),

$$f_0(r^*) \equiv \sum_{n=0}^{3} C_n (r^*/r_g)^n, \quad (B2)$$

$$f_1(r^*) \equiv \sum_{n=0}^{3} D_n (r^*/r_g)^n, \quad (B3)$$

$$f_2(r^*) \equiv \sum_{n=0}^{3} E_n (r^*/r_g)^n, \quad (B4)$$

$$f_3(r^*) \equiv \sum_{n=0}^{3} F_n (r^*/r_g)^n, \quad (B5)$$

one can obtain all coefficients of the ansatz, $\{D_n\}_{n=0,1,2,3}$, $\{E_n\}_{n=0,1,2,3}$, and $\{F_n\}_{n=0,1,2,3}$ as follows:
where  and \( \{C_n\}_{n=0,1,2,3} \) are arbitrary constants to be determined by a boundary condition for the modified RW equation. Here we impose \( C_0 \neq 0 \) and \( C_1 = C_2 = C_3 = 0 \) as the no-flux condition near the horizon (\( r^* \ll r_{p1}^* \)). Finally, one has the asymptotic form of the mode function:

\[
\psi_{s=2}(\omega, r^*) \simeq C_0 \left( 1 - \frac{\bar{\omega}^2}{C^2} \epsilon^2 - \frac{16}{81} \frac{\bar{\omega}^3}{C^4} \epsilon^3 \right) \quad \text{for} \quad r^* < r_{p1}^*.
\]

One may find that the imaginary part of the asymptotic form of mode function is proportional to the dissipation factor, \( \text{Im}(\psi_{s=2}) \propto \gamma \), which suggests that the reflection rate may decrease as \( \gamma \) increases as is shown in the next section.

**Appendix C: reflection rate of GWs near the horizon**

In this section, we will derive the analytic form of the reflection rate of GWs near the horizon, \( |A_{\text{out}}|/|A_{\text{in}}| \), by using the WKB approximation.

To see this, we note that the imaginary part of the frequency in Eq. (A2) denotes the rate of decay in the amplitude of a wavepacket. As such, in the WKB limit, we can write:

\[
\log\left( \frac{|A_{\text{out}}|}{|A_{\text{in}}|} \right) \simeq - \int_{r_{\text{min}}}^{r_{p1}^*} dt \frac{\gamma K^2}{2E_{p1}} \int dt \sqrt{F(r^*)} \simeq - \frac{2\gamma \omega^2}{E_{p1} \sqrt{F(r_{\text{min}})}}.
\]

where \( \int dt \sqrt{F(r^*)} \) is the integral over the classical wavepacket trajectory with \( \Omega(r) = \omega/\sqrt{F(r)} \), and we only use the change in the dispersion relation to compute the reflection rate.
tion radius \[ r_{\text{min}} \equiv r_{\gamma} |K | \simeq r_{g} \ln \left( \frac{2(2C^{2} + \gamma^{2})E_{B}^{2}}{\epsilon(4C^{2} + \gamma^{2})^{2} \omega^{2}} \right). \] (C2)

Plugging this into Eq. (C1) yields

\[ |A_{\text{out}}|/|A_{\text{in}}| \simeq \exp \left[ -2r_{g} \sqrt{2 + 4C^{2} / \gamma^{2}} \right]. \] (C3)

The analytic formula (C3) and the numerically obtained reflection rates are compared in Fig. 6, which appear to agree for small values of \( \gamma_{\omega r} \), where adiabatic condition is satisfied.

We can also relate \( r_{g} \) to surface gravity \( \kappa \), or Hawking temperature of \( T_{H} \) via \( k_{B}T_{H} \equiv h \kappa / (2\pi) = h / (4\pi r_{g}) \). Therefore, Eq. (C3) can be written in terms of Hawking temperature as:

\[ |A_{\text{out}}|/|A_{\text{in}}| \simeq \exp \left[ -\sqrt{2 + 4C^{2} / \gamma^{2}} \left( \frac{\hbar}{2\pi(1 + 4C^{2} / \gamma^{2})} \right) \left( \frac{\omega}{k_{B}T_{H}} \right) \right]. \] (C4)

Given that the reflection only depends on the local blueshift near the horizon, we expect this formula to be valid for spinning BHs as well.

**Appendix D: Consistency between the reflection rate and amplitude of GW-echoes**

As a supporting evidence that the GW-echoes originate from the reflection of GWs at the horizon, we confirmed that the dependence of the amplitude of the first-echo on the dissipation effect \( \gamma \) (Fig. 7). As is shown in Fig. 3, the most excited QNM is around \( r_{g} \omega_{R} \sim 1 \) and this mode may be dominant in the GW-echoes. This is consistent with the comparison with the reflection rate for \( r_{g} \omega_{R} = 1 \) (red points) shown in Fig. 7 where one may find that the reflection rate with \( r_{g} \omega_{R} = 1 \) fits the regularized amplitude of the first-echo \( A_{\text{echo}} \). On the other hand, the reflection rate for the modes of \( r_{g} \omega_{R} = 0.5 \) (blue points) and 1.5 (green points), which are not well excited in the GW-echoes, do not fit the regularized amplitude of the echo.

![FIG. 7: The comparison between the regularized amplitude of the first-echo and the reflection rate with \( r_{g} \omega_{R} = 0.5, 1, \) and 1.5 (blue, red, green points, respectively).](image)

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[42] There is no local process that could set up this preferential stationary foliation. However, the non-perturbative quantum gravity effects that lead to the formation of the exotic compact object may also set up the preferential stationary foliation. Since \( F(r) \) and \( \partial_r \) do not commute, there is some arbitrariness in going from (3) to (4). Here, we make the simplest choice to place \( F(r) \)'s outside derivatives, but do not expect this to change our results significantly.

[43] Given that \( \Omega(K) \) in Eq. (A2) is complex, \( r_{\text{min}} \) resulting from \( \Omega(K_{\text{max}}) = \omega/\sqrt{F(r_{\text{min}})} \) is generally not real. As such, here we use \( |\Omega| (K_{\text{max}}) = \omega/\sqrt{F(r_{\text{min}})} \) to define a real reflection radius, \( r^*_{\text{min}} \).