Large Angle MSW Solution in Grand Unified Theories
with SU(3) × U(1) Horizontal Symmetry

Ryuichiro Kitano† and Yukihiro Mimura‡

*Theory Group, KEK, Oho 1-1, Tsukuba, Ibaraki 305-0801, Japan

†Department of Particle and Nuclear Physics, The Graduate University for Advanced Studies,
Oho 1-1, Tsukuba, Ibaraki 305-0801, Japan

Abstract

We construct a model with a SU(3) × U(1) horizontal symmetry in the context of Grand Unified Theories. In our models, the bi-maximal lepton mixing and suitable neutrino masses for the large angle MSW solution are obtained without any fine-tuning due to the spontaneously broken SU(3)H symmetry. The three generations of quarks and leptons are unified as members of the SU(3)H fundamental representation, and the U(1)H charge gives the origin of the fermion mass hierarchy and mixing angles. We present two explicit examples of SU(5)GUT and SO(10)GUT models, in which the Yukawa structures are given successfully.
1 Introduction

Flavor physics is a recent topic in particle physics. The Cabibbo-Kobayashi-Maskawa (CKM) parameters will be measured precisely in B-factory experiments. The recent SuperKamiokande data suggest that the mixing angles of the lepton sector are large in contrast to those in the quark sector. For atmospheric neutrinos, $\nu_\mu \to \nu_\tau$ oscillations are favored and the best fit values are $\sin^2 2\theta = 1.0$ and $\Delta m^2 = 3.5 \times 10^{-3} \text{ eV}^2$. For the solar neutrino problem, the only solution which is not disfavored at 95% confidence level is the large angle MSW solution i.e. $\sin^2 2\theta \sim 1$ and $\Delta m^2 \sim 10^{-5} - 10^{-4} \text{ eV}^2$. If we assume three flavor neutrino mixings, the solar neutrino deficit is explained by $\nu_e \to \nu_\mu$ oscillations and the CHOOZ experiment gives a severe constraint on the mixing angle between the electron neutrino and the heaviest neutrino, $\sin^2 2\theta_{e\delta} \lesssim 0.1$. These two nearly maximal mixings and a small mixing suggest that the lepton mixing matrix is the so-called bi-maximal type, which is a completely different structure to that of the CKM matrix. Understanding of the patterns of the mixing and fermion masses is one of the most challenging puzzles for particle physicists.

One approach to answer this puzzle is to consider a horizontal flavor symmetry. The horizontal symmetry provides the Yukawa coupling structure and enable us to predict the structure definitely.

The two generation model successfully explains the magnitude of the Cabibbo angle, namely

$$\sin \theta_C \sim \sqrt{\frac{m_d}{m_s}}. \quad (1)$$

We reproduce this relation by using a flavor symmetry. However the simple extension to three generations does not succeed. For example, the CKM matrix element $V_{cb}$ is not the same order of $\sqrt{m_s/m_b}$ but instead of order $m_s/m_b$.

The observation of the large mixing between $\nu_\mu - \nu_\tau$ plays an important role in understanding the 2-3 structure. The structure of the second and third generations are revealed to be non-trivial. The non-parallel family structure has been suggested, and this structure is also called lopsided family structure and can be organized by an Abelian symmetry.

The lopsided family structure may give us a neutrino mass matrix which leads to large angle MSW solution for solar neutrino problem. However the lopsided family structure does not always predict the large mixing angle unless the mass (squared) ratio is assumed
to take a particular value. There is no reason to obtain large MSW solution in the lopsided structure.

We will give a simple lopsided family structure by imposing a SU(3)$_H$×U(1)$_H$ horizontal symmetry. In our model, the fermion mass hierarchy is produced by the U(1)$_H$ symmetry. Contrary to the standard approach, we do not suppose that the vacuum expectation values (VEVs) of fundamental (anti-fundamental) representation of SU(3)$_H$ symmetry are non-hierarchical, and this feature provides large mixing angles in lepton sector.

In this paper, we assume a Supersymmetric Grand Unified Theory (SUSY GUT) [15] realized at the high energy scale ($M_G \sim 10^{16}$ GeV). The SUSY GUT is an attractive theory which explains many problems in the standard model. In particular, the tiny neutrino masses are elegantly explained by the seesaw mechanism [16] which requires heavy right-handed neutrinos with masses $M_N \sim 10^{14}$ GeV. This is just below the GUT scale. It is natural that the origin of the scale $M_N$ is GUT physics, so that it is important that we understand the flavor structures in the context of GUTs.

This paper is organized as follows. In section 2, we give the Yukawa structures which are realized in the model and show that they reproduce the correct mass ratios and mixing angles for the quarks and leptons. In section 3, we give an example of the SU(5) GUT model and explain how we can obtain the Yukawa matrices in section 3. In section 4, we construct the SO(10) GUT model in which the large MSW type mass structure is given. Section 5 is devoted to our conclusions.

2 Structure of the Model

We will construct the model to explain the CKM matrix, fermion masses, and lepton mixings suitable for the large angle MSW solution, by means of a horizontal symmetry.

The observed CKM matrix, the quark mass ratios and lepton mass ratios at GUT scale [17] are approximately expressed by powers of the Cabibbo angle $\lambda \sim 0.22$ as follows.

$$V_{\text{CKM}} \sim \begin{pmatrix}
1 & \lambda & \lambda^3 \\
-\lambda & 1 & \lambda^2 \\
-\lambda^3 & -\lambda^2 & 1
\end{pmatrix},$$

$$m_u : m_c : m_t \sim \lambda^7 : \lambda^4 : 1, \quad m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1,$$

$$m_e : m_\mu : m_\tau \sim \lambda^5 : \lambda^2 : 1.$$
For the neutrinos, the mass ratio is given by

\[ m_{\nu_2} : m_{\nu_3} \sim \lambda^{1-2} : 1. \]  

(3)

We obtain eq.(3) from the $\Delta m^2$ ratio of the atmospheric neutrino and large MSW solution, which is

\[ \frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim 10^{-2} \sim \lambda^3. \]  

(4)

The Maki-Nakagawa-Sakata (MNS) matrix \([18]\) is nearly bi-maximal,

\[ V_{\text{MNS}} \sim \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & \epsilon \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}, \]  

(5)

where the element $\epsilon$ is a small parameter constrained by the CHOOZ experiment.

We construct a supersymmetric SU(5) GUT model with a SU(3)$_H \times$U(1)$_H$ horizontal symmetry whose spontaneous breaking induces the generation mixing and the mass differences in eq.(2), (3), and (5). The quarks and leptons are unified as SU(3)$_H$ triplets, which explains why there are three generations.

In our model, the Yukawa couplings may be expressed in the following form:

\[ \sum_{i,j=1}^{3} \left( \frac{\Phi}{M_{\text{Pl}}} \right)^{x_i+x_j} \xi_i \xi_j^T \frac{1}{M_*^2}. \]  

(6)

The three $\xi$'s\(^3\) are SU(3)$_H$ anti-fundamental representations whose VEVs break the horizontal symmetry at the scale $M_*$. The $\Phi$ is SU(3)$_H$ singlet field, and has non-vanishing U(1)$_H$ charge. The VEV of $\Phi$ is $\lambda M_{\text{Pl}}$ and provides a hierarchy in the Yukawa coupling as follows.

With the appropriate U(1)$_H$ charge assignment of $\xi$ fields as shown later in section 3 and 4, the Yukawa coupling of up-type quark is given by

\[ Y_u \sim \begin{pmatrix} \langle \xi_3 \rangle & \langle \xi_2 \rangle & \langle \xi_1 \rangle \end{pmatrix} \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix} / M_*^2. \]  

(7)

The VEVs of $\xi$'s are given without loss of generality by

\[ \langle \xi_1 \rangle = M_* \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix}, \quad \langle \xi_2 \rangle = M_* \begin{pmatrix} 0 \\ b \\ c \end{pmatrix}, \quad \langle \xi_3 \rangle = M_* \begin{pmatrix} d \\ e \\ f \end{pmatrix}, \]  

(8)

\(^3\)In our models, we prepare three pairs of $\xi$'s, though minimal pairs to break SU(3)$_H$ is two. Of course, in absence of $\xi_3$, the exterior product $\bar{\xi}_1 \times \bar{\xi}_2$ plays a role of $\xi_3$ effectively. However, such a minimal choice has a difficulty in constructing realistic models.
where the $a \to f$ are parameters of the order unity. In this basis, $Y_u$ is given by

$$Y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (9)$$

where we omit the parameters of order unity. This Yukawa matrix gives the mass ratio for the up-type quarks as

$$m_u : m_c : m_t \sim \lambda^6 : \lambda^4 : 1, \quad (10)$$

which is consistent with eq.(2) except for the up quark but it is maybe within the uncertainty of parameters. The prediction for the magnitude of the top quark Yukawa coupling is also consistent with the experimental value $Y_t \sim 1$.

The Yukawa matrix for the down-type quarks and charged leptons are given by

$$Y_d = Y_e^T \sim \begin{pmatrix} \langle \xi_3 \rangle & \langle \xi_2 \rangle & \langle \xi_1 \rangle \end{pmatrix} \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda & \lambda \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle \\ \langle \xi_2 \rangle \\ \langle \xi_1 \rangle \end{pmatrix} / M^2 \sim \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda & \lambda \end{pmatrix}. \quad (11)$$

This Yukawa structure gives the down-type quark and charged lepton mass ratios as follows:

$$m_d : m_s : m_b = m_e : m_\mu : m_\tau \sim \lambda^4 : \lambda^2 : 1. \quad (12)$$

This is consistent with eq.(2). The difference between the charged lepton masses and down-type quark masses may be explained by the introduction of a new Higgs field such as the SU(5) 45 representation in the usual way [19].

Inputting the bottom quark mass, this model predicts the VEV ratio of the two Higgs doublets as

$$\tan \beta \equiv \frac{\langle H \rangle}{\langle H \rangle} \sim 17. \quad (13)$$

We obtain the correct CKM matrix which comes from Yukawa matrices (4) and (11) as

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 1 \end{pmatrix}. \quad (14)$$

The interesting point is that we can obtain the bi-maximal mixing of leptons without any fine-tuning in the following mass matrix. The neutrino mass matrix is proportional
\[
\langle \xi_3 \rangle \langle \xi_2 \rangle \langle \xi_1 \rangle \left( \begin{array}{ccc}
\lambda^2 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & \lambda^4 
\end{array} \right) \left( \begin{array}{c}
\langle \xi_3 \rangle^T \\
\langle \xi_2 \rangle^T \\
\langle \xi_1 \rangle^T 
\end{array} \right) \sim \left( \begin{array}{ccc}
\lambda^2 & \lambda & \lambda \\
\lambda & 1 & 1 \\
\lambda & 1 & 1 
\end{array} \right).
\]

We can see in eq.(15) that the SU(3)_H breaking VEVs give the large mixing for the 2\textsuperscript{nd} and 3\textsuperscript{rd} generations. Moreover, the 1\textsuperscript{st} and 2\textsuperscript{nd} generation mixing is also large. The easiest way to understand the large 1\textsuperscript{st} and 2\textsuperscript{nd} generation mixing is by changing the basis in eq.(8) to
\[
\langle \xi_1 \rangle \sim M_* \left( \begin{array}{c} 
0 \\
1 \\
1 
\end{array} \right), \quad \langle \xi_2 \rangle \sim M_* \left( \begin{array}{c}
0 \\
0 \\
1 
\end{array} \right), \quad \langle \xi_3 \rangle \sim M_* \left( \begin{array}{c}
1 \\
1 \\
1 
\end{array} \right).
\]

This change of basis corresponds to removing the large mixing of 2\textsuperscript{nd} and 3\textsuperscript{rd} generations from the mass matrix (15). In this basis, the neutrino mass matrix (15) is replaced by
\[
\left( \begin{array}{ccc}
\lambda^2 & \lambda^2 & \lambda \\
\lambda^2 & \lambda & \lambda \\
\lambda & \lambda & 1 
\end{array} \right).
\]

It turns out that this matrix gives the large 1\textsuperscript{st} and 2\textsuperscript{nd} generation mixing.

On the other hand, in this basis, the charged lepton Yukawa matrix is obtained from eq.(11) and eq.(16) as
\[
Y'_e \sim \left( \begin{array}{ccc}
\lambda^5 & \lambda^2 & \lambda^2 \\
\lambda^4 & \lambda & \lambda \\
\lambda^4 & \lambda & \lambda 
\end{array} \right),
\]

which gives the large mixing between the second and third generations. Therefore, the lepton mixing matrix is bi-maximal. The mass ratios of neutrinos are also suitable for the large MSW solution and are as follows:
\[
m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \lambda^4 : \lambda^2 : 1.
\]

The model predicts that the mixing between the first and the third generations is of order \(\lambda\) and thus small. It is interesting that the mixing angle is comparable to the CHOOZ bound, and will be observed in future long baseline experiments.

This natural derivation of the bi-maximal mixing is due to the SU(3)_H symmetry. The conventional way to obtain the bi-maximal mixing by an Abelian symmetry requires an accidental cancellation in the determinant. Consider the case that the neutrino mass matrix is given by
\[
\left( \begin{array}{ccc}
\lambda^4 & \lambda^2 & \lambda^2 \\
\lambda^2 & 1 & 1 \\
\lambda^2 & 1 & 1 
\end{array} \right).
\]
This type of mass matrix can be given by the Abelian flavor symmetry models in which the second and the third generations have the same charges. If there is an accidental cancellation in the determinant of the 2-3 submatrix so that the eigenvalues of this submatrix are of order $\lambda^2$ and unity, the matrix in which the 2-3 submatrix is diagonalized by the large mixing is given by

$$
\begin{pmatrix}
\lambda^4 & \lambda^2 & \lambda^2 \\
\lambda^2 & \lambda^2 & 0 \\
\lambda^2 & 0 & 1
\end{pmatrix}.
$$

(21)

This gives the large 1st and 2nd generation mixing if there is no cancellation in the (1,2) component while diagonalizing the 2-3 submatrix, and the correct mass ratio for the MSW solution is reproduced. However, without the accidental cancellation, the mixing angle between the 1st and 2nd generation is naturally of order $\lambda^2$ and the mass ratios are unacceptable, scaling as $\lambda^2 : 1 : 1$. As we have seen above, this cancellation can be controlled by the non-Abelian horizontal symmetry.

### 3 The Model

In this section, we present a SUSY SU(5) GUT model, in which the Yukawa structure in the previous section is reproduced.

The particle content of this model is listed in Tab.1. The U(1)$_H$ charge assignment is an example of realistic models. The U(1)$_H$ symmetry is anomaly free with respect to the SU(5)$_{GUT}$, therefore, the Nambu-Goldstone boson associated with U(1)$_H$ break down is purely massless and harmless [20].

The fields 10 and 5 are the usual matter fields as follows:

$$
10 : (q, u^c, e^c) , \quad 5 : (d^c, l).
$$

(22)

The fields $H$ and $\bar{H}$ are the Higgs fields. The SU(2)$_L$ doublet parts of these fields remain as the usual Higgs doublets at the electroweak scale and their VEVs give the masses of the quarks and leptons. The vector-like fields $T$, $F$, $G$ and $N$ are the Froggatt-Nielsen (FN) fields which generate the usual Yukawa interaction terms for quarks and leptons by being integrated out [3]. The SU(5) and SU(3)$_H$ singlet fields $\Phi$ and $\bar{\Phi}$ are the origin of the mass hierarchy and mixing angle of the quarks and leptons through the non-renormalizable interactions.

The superpotential which creates the Yukawa couplings for the matter fields is constructed as follows.

$$
W = W_{\text{matter}} + W_{\text{Higgs}} + W_{\text{mass}}.
$$

(23)
Table 1: The particle contents of the SU(5) model.

The matter part $W_{\text{matter}}$ contains SU(3)$_H$ non-singlet fields. It is given by

$$W_{\text{matter}} = \left\{ \xi_1 + \left( \frac{\Phi}{M_{\text{Pl}}} \right)^2 \xi_2 + \left( \frac{\Phi}{M_{\text{Pl}}} \right)^3 \xi_3 \right\} \cdot 10 \bar{T}$$

$$+ \left\{ \left( \frac{\Phi}{M_{\text{Pl}}} \right)^2 \xi_1 + \xi_2 + \left( \frac{\Phi}{M_{\text{Pl}}} \right)^2 \xi_3 \right\} \cdot \bar{5} F$$

$$+ \left\{ \left( \frac{\Phi}{M_{\text{Pl}}} \right) \xi_1 + \left( \frac{\Phi}{M_{\text{Pl}}} \right) \xi_2 + \left( \frac{\Phi}{M_{\text{Pl}}} \right)^2 \xi_3 \right\} \cdot \bar{5} G ,$$

where $\cdot$ is the inner product of the SU(3)$_H$ indices. We consider the case in which the fields $\Phi$ and $\bar{\Phi}$ acquire VEVs as follows:

$$\langle \Phi \rangle = \langle \bar{\Phi} \rangle \sim \lambda M_{\text{Pl}} .$$

In this case, the U(1)$_H$ symmetry breaks down at the scale of $\lambda M_{\text{Pl}}$ and the factor $\Phi/M_{\text{Pl}}$ in the superpotential can be replaced by the $\lambda \sim 0.22$ which originates the fermion masses and mixings.
The Higgs part, $W_{\text{Higgs}}$, contains the Yukawa interaction terms of the FN fields and Higgs fields $H$ and $\bar{H}$ and may be written as

$$W_{\text{Higgs}} = T H T + \left( \frac{\Phi}{M_{\text{Pl}}} \right) T \bar{H} \bar{F} + T \bar{H} \bar{G} + \bar{F} H N + \left( \frac{\Phi}{M_{\text{Pl}}} \right) \bar{G} H N .$$

(26)

The last term $W_{\text{mass}}$ in eq.(23) is the mass term for the FN fields and Higgs fields:

$$W_{\text{mass}} = M^\ast T \bar{T} + M^\ast F \bar{F} + M^\ast G \bar{G} + \frac{\Phi^5}{M_{\text{Pl}}} N N + M^\ast \sum_{i=1}^{3} \xi_i \bar{\xi}_i + M_{\text{GUT}} H \bar{H} ,$$

(27)

where $M^\ast$ is the FN scale which can be naturally of the order of $M_{\text{GUT}}$ and it can arise from the VEV of a singlet field, but we do not specify the scale and its origin. The Majorana mass for the FN field $N$ is given by $\lambda^5 M_{\text{Pl}} \sim 10^{14}$ GeV. It is suitable to give the neutrino masses by the seesaw mechanism. The Majorana mass can be given in another way. If $M^\ast$ is obtained by the singlet VEV and the Planck suppressed mass term in eq.(27) is forbidden by some symmetry like $Z_3$, the Majorana mass term can be given by $(\Phi/M_{\text{Pl}})^X M^\ast N N$ and the magnitude can be controlled by $M^\ast$ and $X$, which is determined by the U(1)$_H$ charge of $N$. Therefore, hereafter, we replace the Majorana mass term with $M_N N N$, where $M_N \sim 10^{14}$ GeV.

We can see that U(1)$_{\text{PQ}}$ symmetry exists in this superpotential, and the charge assignments are listed in Tab.1. The non-zero value of the Peccei-Quinn charge of the $\xi$’s indicates that the SU(3)$_H$ breaking simultaneously induces the U(1)$_{\text{PQ}}$ breaking, which solves the strong CP problem and creates the axion dark matter if the breaking scale $M^\ast$ is around $10^{12}$ GeV [21, 22, 23]. In this sense, $M^\ast \sim M_{\text{GUT}}$ may be unacceptable due to the creation of too many axions. A way out is to dilute out them by the inflation of the universe which is also needed by the GUT monopole dilution [24].

Now we show that the fermion masses and mixings are reproduced by the spontaneous breaking of the horizontal symmetry.

First, let us consider the up-type quarks. The up-type Yukawa couplings are given by the Feynman diagram in Fig.4. We can extract the Yukawa matrix from this diagram as follows:

$$Y_u \sim \frac{1}{M^2} \left( \lambda^3 \langle \xi_3 \rangle + \lambda^2 \langle \xi_2 \rangle + \langle \xi_1 \rangle \right) y_T \left( \lambda^3 \langle \xi_3 \rangle + \lambda^2 \langle \xi_2 \rangle + \langle \xi_1 \rangle \right)^T \sim \frac{1}{M^2} \left( \begin{array}{c} \langle \xi_3 \rangle \\ \langle \xi_2 \rangle \\ \langle \xi_1 \rangle \end{array} \right) y_T \left( \begin{array}{ccc} \lambda^3 & \lambda^2 & 1 \\ \lambda^2 & 1 & \lambda \\ 1 & \lambda & \lambda^3 \end{array} \right) \left( \begin{array}{c} \langle \xi_3 \rangle \\ \langle \xi_2 \rangle \\ \langle \xi_1 \rangle \end{array} \right)^T ,$$

(28)

where $y_T$ is the Yukawa coupling constant of $T H T$ in eq.(26). As we can see in eq.(28), the rank of the $Y_u$ matrix is one, which means that only one of the quarks can acquire
the non-vanishing mass while the others remain massless. This situation usually occurs
in the FN mechanism. In order to avoid this, more than three pairs of FN fields \( T \) and \( \bar{T} \)
are necessary. Then, the coupling \( y_T \) is a matrix and all the components are naturally of
order unity in the basis that the mass term \( M^*_T \bar{T}T \) is diagonal. In the case that there are
three pairs of \( T \) and \( \bar{T} \), the Yukawa couplings are given by

\[
Y_u \sim \frac{1}{M^*_e} \left( \begin{array}{cc}
\langle \xi_3 \rangle & \langle \xi_2 \rangle \\
\langle \xi_2 \rangle & \langle \xi_1 \rangle
\end{array} \right) \left( \begin{array}{ccc}
\lambda^3 & \lambda^3 & \lambda^3 \\
\lambda^2 & \lambda^2 & \lambda^2 \\
1 & 1 & 1
\end{array} \right) \left( \begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{array} \right) \left( \begin{array}{ccc}
\lambda^3 & \lambda^2 & 1 \\
\lambda^3 & \lambda^2 & 1 \\
\lambda^3 & \lambda^2 & 1
\end{array} \right) \left( \begin{array}{c}
\langle \xi_3 \rangle^T \\
\langle \xi_2 \rangle^T \\
\langle \xi_1 \rangle^T
\end{array} \right).
\]

The second and the fourth matrix in the first expression in eq.(29) is the coupling matrix
between 10 and \( \bar{T} \) fields and the center matrix is the \( y_T \) matrix. This \( Y_u \) matrix has the
same structure as eq.(7).

For the down-type quarks and charged leptons, there are two Feynman diagrams
because we introduce two kinds of SU(5)\(_{\text{GUT}}\) \( 5 \) representation FN fields \( F \) and \( G \), and
again more than three pairs of these fields are necessary. The introduction of two kinds of
\( 5 \) fields is the essential point of this model. The field \( G \) gives the down-type quarks and
charged leptons mass matrices and \( F \) gives the neutrino mass matrix suitably without
disturbing each other. The two diagrams are shown in Fig.2. From the upper diagram,

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\( ^4 \) We can obtain the same result for the case where there are more than three pairs of FN fields.
Figure 2: The Feynman diagrams for the down-type and charged lepton Yukawa interactions.
which is $G$ contribution, the Yukawa structure is given as

$$Y_d^G = (Y_e^G)^T$$

$$\sim \frac{1}{M_x^2} \left( \langle \xi_3 \rangle \langle \xi_2 \rangle \langle \xi_1 \rangle \right) \left( \begin{array}{ccc}
\lambda^3 & \lambda^3 & \lambda^3 \\
\lambda^2 & \lambda^2 & \lambda^2 \\
1 & 1 & 1 
\end{array} \right) \left( \begin{array}{ccc}
\lambda^2 & \lambda & \lambda \\
\lambda^2 & \lambda & \lambda \\
1 & 1 & 1 
\end{array} \right) \left( \begin{array}{ccc}
\langle \xi_3 \rangle^T \\
\langle \xi_2 \rangle^T \\
\langle \xi_1 \rangle^T 
\end{array} \right)$$

$$\sim \frac{1}{M_x^2} \left( \langle \xi_3 \rangle \langle \xi_2 \rangle \langle \xi_1 \rangle \right) \left( \begin{array}{ccc}
\lambda^5 & \lambda^4 & \lambda^4 \\
\lambda^4 & \lambda^3 & \lambda^3 \\
\lambda^2 & \lambda & \lambda 
\end{array} \right) \left( \begin{array}{ccc}
\langle \xi_3 \rangle^T \\
\langle \xi_2 \rangle^T \\
\langle \xi_1 \rangle^T 
\end{array} \right).$$

The $F$ contribution (the lower diagram) is

$$Y_d^F = (Y_e^F)^T$$

$$\sim \frac{1}{M_x^2} \left( \langle \xi_3 \rangle \langle \xi_2 \rangle \langle \xi_1 \rangle \right) \left( \begin{array}{ccc}
\lambda^3 & \lambda^3 & \lambda^3 \\
\lambda^2 & \lambda^2 & \lambda^2 \\
1 & 1 & 1 
\end{array} \right) \left( \begin{array}{ccc}
\lambda & \lambda & \lambda \\
\lambda & \lambda & \lambda \\
1 & 1 & 1 
\end{array} \right) \left( \begin{array}{ccc}
\lambda & 1 & \lambda^2 \\
\lambda & 1 & \lambda^2 \\
1 & 1 & \lambda^2 
\end{array} \right) \left( \begin{array}{ccc}
\langle \xi_3 \rangle^T \\
\langle \xi_2 \rangle^T \\
\langle \xi_1 \rangle^T 
\end{array} \right)$$

$$\sim \frac{1}{M_x^2} \left( \langle \xi_3 \rangle \langle \xi_2 \rangle \langle \xi_1 \rangle \right) \left( \begin{array}{ccc}
\lambda^5 & \lambda^4 & \lambda^4 \\
\lambda^4 & \lambda^3 & \lambda^3 \\
\lambda^2 & \lambda & \lambda 
\end{array} \right) \left( \begin{array}{ccc}
\langle \xi_3 \rangle^T \\
\langle \xi_2 \rangle^T \\
\langle \xi_1 \rangle^T 
\end{array} \right).$$

Then, the Yukawa couplings are given as follows:

$$Y_d = Y_e^T \sim Y_d^G + Y_d^F \sim \frac{1}{M_x^2} \left( \langle \xi_3 \rangle \langle \xi_2 \rangle \langle \xi_1 \rangle \right) \left( \begin{array}{ccc}
\lambda^3 & \lambda^3 & \lambda^3 \\
\lambda^2 & \lambda^2 & \lambda^2 \\
1 & 1 & 1 
\end{array} \right) \left( \begin{array}{ccc}
\lambda & \lambda & \lambda \\
\lambda & \lambda & \lambda \\
1 & 1 & 1 
\end{array} \right) \left( \begin{array}{ccc}
\lambda & 1 & \lambda^2 \\
\lambda & 1 & \lambda^2 \\
1 & 1 & \lambda^2 
\end{array} \right) \left( \begin{array}{ccc}
\langle \xi_3 \rangle^T \\
\langle \xi_2 \rangle^T \\
\langle \xi_1 \rangle^T 
\end{array} \right).$$

This reproduces the Yukawa structure in eq. (11).

Now, let us inspect the neutrino mass matrix whose entries originate from the Feynman diagrams shown in Fig. 3. The first diagram gives the main contribution:

$$m^{FF}_\nu \sim \frac{\langle H \rangle^2}{M_N} \left( \langle \xi_3 \rangle \langle \xi_2 \rangle \langle \xi_1 \rangle \right) \left( \begin{array}{ccc}
\lambda^2 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & \lambda^4 
\end{array} \right) \left( \begin{array}{ccc}
\langle \xi_3 \rangle^T \\
\langle \xi_2 \rangle^T \\
\langle \xi_1 \rangle^T 
\end{array} \right).$$

The others are given as follows.

$$m^{FG}_\nu \sim \frac{\langle H \rangle^2}{M_N} \left( \langle \xi_3 \rangle \langle \xi_2 \rangle \langle \xi_1 \rangle \right) \left( \begin{array}{ccc}
\lambda^4 & \lambda^3 & \lambda^3 \\
\lambda^3 & \lambda^2 & \lambda^2 \\
\lambda^3 & \lambda^2 & \lambda^2 
\end{array} \right) \left( \begin{array}{ccc}
\langle \xi_3 \rangle^T \\
\langle \xi_2 \rangle^T \\
\langle \xi_1 \rangle^T 
\end{array} \right),$$

$$m^{GF}_\nu \sim (m^{FG}_\nu)^T,$$

$$m^{GG}_\nu \sim \frac{\langle H \rangle^2}{M_N} \left( \langle \xi_3 \rangle \langle \xi_2 \rangle \langle \xi_1 \rangle \right) \left( \begin{array}{ccc}
\lambda^6 & \lambda^5 & \lambda^5 \\
\lambda^5 & \lambda^4 & \lambda^4 \\
\lambda^5 & \lambda^4 & \lambda^4 
\end{array} \right) \left( \begin{array}{ccc}
\langle \xi_3 \rangle^T \\
\langle \xi_2 \rangle^T \\
\langle \xi_1 \rangle^T 
\end{array} \right).$$
Figure 3: The Feynman diagrams for the neutrino masses.
The contributions from eq. (34–36) are small compared to $m_{\nu}^{FF}$ and can be safely ignored. The mass matrix $m_{\nu} \sim m_{\nu}^{FF}$ gives the bi-maximal neutrino mixing as shown in the previous section.

One may think that the $G$ contribution is not necessary if the $F$ contribution gives the neutrino mass matrix suitably. However the large mixing between the second and third generations does not occur in absence of the $G$ fields. This issue is easily seen in the basis of eq.(13). In this basis, the charged lepton mass matrix can be read off from eq.(31) as follows,

$$Y_e^{FF} \sim \begin{pmatrix} \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^4 & \lambda & \lambda \end{pmatrix}.$$  

(37)

This charged lepton mass matrix and the neutrino mass matrix in eq.(17) do not contain the large 2nd and 3rd generation mixing.

4 SO(10) Embedding

In this section, we construct the SO(10) GUT model in which the matter fields are unified in a 16 representation. This matter unification is an attractive feature but this is the difficulty of the SO(10) GUT model building simultaneously.

The minimal SO(10) GUT model is not realistic because the Yukawa couplings for the up-type and down-type quarks coincide. To avoid this, we extend the Higgs sector to introduce the 16 representation Higgs field $H_{16}$ whose SU(5) $\bar{5}$ component mixes with that of usual Higgs field $H_{10}$, i.e. the down type quark and charged lepton masses are both given by the VEV of $H_{16}$ and $H_{10}$. This situation can be realized by the introduction of $\chi$ and $\bar{\chi}$ fields which are 16 and $\bar{16}$ representations of SO(10), respectively, and break the SO(10) symmetry to SU(5) by the SU(5) singlet components acquire the VEV. This idea has been considered in Ref.[25, 26]. We use this mechanism and we can reproduce the appropriate quark masses, quark mixings, charged lepton masses, neutrino masses and mixings for the large angle MSW solution by imposing the SU(3) $\times$ U(1) horizontal symmetry.

The particle content and their horizontal charges are listed in Tab.2. In this model, the SO(10)$_{GUT}$ and U(1)$_H$ is broken spontaneously by the VEVs of the fields $\chi$, $\bar{\chi}$, $\Phi$ and $\bar{\Phi}$ at the order of $\lambda M_{Pl}$. Below this scale, the SU(5)$_{GUT} \times$ SU(3)$_H$ symmetry remains.

First, we consider the Higgs sector. The relevant superpotential for the Higgs fields
The particle contents of the SO(10) model are given by

\[ W = M_H H_{10} H_{16} + \left( \frac{\Phi}{M_{Pl}} \right) H_{10} H_{16} \bar{\chi} \]  

The breaking of SO(10)\textsubscript{GUT} and U(1)\textsubscript{H} leads to the term \( \lambda \langle \bar{\chi} \rangle H_{10} H_{16} \). If \( M_H \) is comparable to \( \lambda \langle \bar{\chi} \rangle \sim \lambda^2 M_{Pl} \sim 10^{17} \) GeV, the SU(5) \( \bar{5} \) component of \( H_{10} \) and that of \( H_{16} \) are strongly mixed, and the down-type quark and charged lepton masses are given by both \( H_{10} \) and \( H_{16} \). This assumption of the Higgs mass scale is suitable for suppressing proton decay \[27\], although this mass scale is slightly larger than the ordinary grand unified scale, which is about \( 10^{16} \) GeV. We assume such a mass scale \( M_H \sim 10^{17} \) GeV.

The matter interactions are given as follows:

\[ W_{\text{matter}} = \left\{ \xi_1 + \left( \frac{\Phi}{M_{Pl}} \right)^2 \xi_2 + \left( \frac{\Phi}{M_{Pl}} \right)^3 \xi_3 \right\} \cdot 16 \bar{S} \] 

\[ + \frac{\chi}{M_{Pl}} \left\{ \left( \frac{\Phi}{M_{Pl}} \right)^2 \xi_1 + \xi_2 + \left( \frac{\Phi}{M_{Pl}} \right) \xi_3 \right\} \cdot 16 J \]  

Below the SO(10) breaking scale, \( \chi/M_{Pl}, \Phi/M_{Pl} \) and \( \Phi/M_{Pl} \) can be replaced with \( \lambda \sim 0.22 \).

The Yukawa interaction terms of FN fields are given by

\[ W_{FN} = SSH_{10} + SJH_{16} \]  

Table 2: The particle contents of the SO(10) model.

|       | SO(10)\textsubscript{GUT} | SU(3)\textsubscript{H} | U(1)\textsubscript{H} | U(1)\textsubscript{PQ} |
|-------|--------------------------|------------------------|------------------------|------------------------|
| 16    | 16                       | 3                      | 0                      | 1                      |
| \( H_{10} \) | 10                       | 1                      | 11/2                   | 0                      |
| \( H_{16} \) | 16                       | 1                      | 13/4                   | 0                      |
| \( \xi_1 \) | 1                        | 3                      | 11/4                   | -1                     |
| \( \xi_2 \) | 1                        | 3                      | 3/4                    | -1                     |
| \( \xi_3 \) | 1                        | 3                      | -1/4                   | -1                     |
| \( \xi_1 \) | 1                        | 3                      | -11/4                  | 1                      |
| \( \xi_2 \) | 1                        | 3                      | -3/4                   | 1                      |
| \( \xi_3 \) | 1                        | 3                      | 1/4                    | 1                      |
| \( S \) | 16                       | 1                      | 11/4                   | 0                      |
| \( S \) | 16                       | 1                      | -11/4                  | 0                      |
| \( J \) | 10                       | 1                      | 1/2                    | 0                      |
| \( \chi \) | 16                       | 1                      | -5/4                   | 0                      |
| \( \bar{\chi} \) | 16                       | 1                      | 5/4                    | 0                      |
| \( \Phi \) | 1                        | 1                      | 1                      | 0                      |
| \( \bar{\Phi} \) | 1                        | 1                      | 1                      | 0                      |
Figure 4: The Feynman diagram for the up-type Yukawa interactions in the SO(10) model.

The other non-renormalizable interaction terms are small and they do not give the leading contribution to fermion masses and mixings. The mass terms of FN fields are given by

$$W_{\text{mass}} = \left( \frac{\bar{\Phi}}{M_{\text{Pl}}} \right)^2 S \chi + \left( \frac{\Phi}{M_{\text{Pl}}} \right) S \bar{J} + M_s \bar{S} \bar{S} + \left( \frac{\bar{\Phi}}{M_{\text{Pl}}} \right) M_s \bar{J} \bar{J}$$

$$+ \left( \frac{\Phi}{M_{\text{Pl}}} \right)^8 S S \bar{\chi} \bar{\chi} + \left( \frac{\Phi}{M_{\text{Pl}}} \right)^8 \bar{S} \bar{S} \chi \chi .$$

(41)

By the SO(10) breaking, the first two terms give the mass terms for SU(5) \(5\) component of \(\bar{S} (\bar{5})\) and \(J (\bar{5})\) and SU(5) \(\bar{5}\) component of \(S (5)\) and \(J (\bar{5})\) as follows:

$$\lambda^3 M_{\text{Pl}} \bar{5} S 5 J + \lambda^2 M_{\text{Pl}} \bar{5} S 5 J .$$

(42)

If \(M_s \sim M_{\text{GUT}}\), the contribution of the next two terms in eq.(41) is negligible compared to eq.(42) and the mass eigenstates for the \(5\) and \(\bar{5}\) FN fields are \((\bar{S}, J)\) and \((J, S)\) pair. It is true for sufficiently small \(M_s\) \((M_s \lesssim M_{\text{GUT}})\), but as we will show later small \(M_s\) leads to small Yukawa couplings for down-type quarks which needs small \(\tan \beta (M_s \gtrsim 10^{15} \text{ GeV})\).

The last two terms of eq.(41) give the Majorana mass terms for SU(5) singlet FN fields.

Now we construct the Yukawa structures. The Yukawa couplings for the up-type quarks are given by the Feynman diagram in Fig.4. Since this diagram is similar to the SU(5) case (Fig.2), the desired Yukawa structure is given as in eq.(29).

The down-type quark and the charged lepton Yukawa structures are given by the sum of the two contributions in Fig.4. The first diagram gives the Yukawa coupling as follows:

$$Y_d^{(1)} = (Y_e^{(1)})^T \sim M_s \frac{1}{\lambda^3 M_{\text{Pl}}} \left( \begin{array}{ccc} \langle \xi_3 \rangle & \langle \xi_2 \rangle & \langle \xi_1 \rangle \end{array} \right) \left( \begin{array}{ccc} \lambda^5 & \lambda^4 & \lambda^6 \\ \lambda^4 & \lambda^3 & \lambda^5 \\ \lambda^2 & \lambda & \lambda^3 \end{array} \right) \left( \begin{array}{ccc} \langle \xi_3 \rangle \\ \langle \xi_2 \rangle \\ \langle \xi_1 \rangle \end{array} \right) / M_s^2 .$$

(43)

Only with this matrix, it is unwanted type as in the case of \(F\) contribution (eq.(31)) in the SU(5)_{GUT} model because this type does not give the maximal mixing for atmospheric
Figure 5: The Feynman diagrams for the down-type quark and charged lepton Yukawa interactions in the SO(10) model.
neutrinos. However, by adding another contribution we can obtain the maximal mixing. It is given by the second diagram in Fig.5 as follows:

$$Y_d^{(2)} = (Y_e^{(2)})^T \sim \frac{M_*}{\lambda^2 M_{Pl}} (\langle \xi_3 \rangle \langle \xi_2 \rangle \langle \xi_1 \rangle) \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix} / M_*^2.$$ (44)

Then, the Yukawa couplings for the down-type quarks and the charged leptons are given by

$$Y_d = Y_e^T \sim Y_d^{(1)} + Y_d^{(2)} \sim \frac{M_*}{\lambda^3 M_{Pl}} (\langle \xi_3 \rangle \langle \xi_2 \rangle \langle \xi_1 \rangle) \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda & \lambda \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix} / M_*^2.$$ (45)

This matrix has the same structure as the SU(5)$_{GUT}$ case and gives the correct mass ratios and mixings. The difference is the pre-factor $M_* / (\lambda^3 M_{Pl})$. The prediction for $\tan \beta$ depends on the FN scale $M_*$ as follows:

$$\tan \beta \sim 16 \times \left( \frac{M_*}{M_{GUT}} \right).$$ (46)

The neutrino masses arise from a complicated mechanism. First, the $9 \times 9$ Majorana mass matrix for the SU(5) singlet components of $S$ ($1_S$), $\bar{S}$ ($1_{\bar{S}}$), and 16 ($1_{16}$) are given by

$$M_{\text{Maj}} = \begin{pmatrix} 1_S & 1_{\bar{S}} & 1_{16} \\ 1_S & \lambda^{10} M_{Pl} & M_* \\ 1_{\bar{S}} & M_* & \lambda^{10} M_{Pl} \\ 1_{16} & 0 & M_\xi \end{pmatrix} ,$$ (47)

where the submatrix $M_\xi$ is the Dirac mass terms from the first line of eq.(39):

$$M_\xi \sim M_* \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix}.$$ (48)

The $(1_{16}, 1_{16})$ components of eq.(47) are not exactly zero but are negligible ($\sim \lambda^{10} M_*^2 / M_{Pl}$). The Majorana masses for the FN fields $1_S$ and $1_{\bar{S}}$ are of the order of $M_*$ which is too large for the seesaw mechanism. However, the suitable Majorana masses for the $1_{16}$ fields are given by

$$M_R = \lambda^{10} M_{Pl} \cdot \frac{M_\xi M_\xi^T}{M_*^2} \sim \lambda^{10} M_{Pl} \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} ,$$ (49)

The Dirac mass terms for the neutrinos are given by the Feynman diagram in Fig.6. From
this diagram, the structure of the Dirac mass is as follows:

\[
m_D \sim \frac{\lambda M^2}{\lambda^2 M_{Pl}} \left( \begin{array}{c}
\langle \xi_3 \rangle \\
\langle \xi_2 \rangle \\
\langle \xi_1 \rangle 
\end{array} \right) \left( \begin{array}{ccc}
\lambda & \lambda & \lambda \\
1 & 1 & 1 \\
\lambda^2 & \lambda^2 & \lambda^2 
\end{array} \right) \left( \begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 
\end{array} \right)
\times \left( \begin{array}{ccc}
\lambda^3 & \lambda^2 & 1 \\
\lambda^3 & \lambda^2 & 1 \\
\lambda^3 & \lambda^2 & 1 
\end{array} \right) / M_*.
\]

Then the neutrino masses originating from the seesaw mechanism turn out to be

\[
m_\nu = m_D M_R^{-1} m_D^T
\]

\[
\sim \frac{M_*^2 \langle H \rangle^2}{\lambda^4 M_{Pl}^3} \left( \begin{array}{c}
\langle \xi_3 \rangle \\
\langle \xi_2 \rangle \\
\langle \xi_1 \rangle 
\end{array} \right) \left( \begin{array}{ccc}
\lambda^2 & \lambda^3 & \lambda^3 \\
\lambda & \lambda^2 & \lambda^2 \\
\lambda^3 & \lambda^2 & \lambda^2 
\end{array} \right) \left( \begin{array}{ccc}
\langle \xi_3 \rangle^T \\
\langle \xi_2 \rangle^T \\
\langle \xi_1 \rangle^T 
\end{array} \right) / M_*^2,
\]

where the matrix whose components are all unity in the second line represents the following calculation as

\[
\left( \begin{array}{ccc}
\lambda^3 & \lambda^2 & 1 \\
\lambda^3 & \lambda^2 & 1 \\
\lambda^3 & \lambda^2 & 1 
\end{array} \right) \left( \begin{array}{ccc}
\lambda^6 & \lambda^5 & \lambda^3 \\
\lambda^5 & \lambda^4 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1 
\end{array} \right)^{-1} \left( \begin{array}{ccc}
\lambda^2 & \lambda^3 & \lambda^3 \\
\lambda^2 & \lambda^3 & \lambda^3 \\
\lambda^2 & \lambda^3 & \lambda^3 
\end{array} \right) \sim \left( \begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 
\end{array} \right).
\]

The matrix \(m_\nu\) in eq.(51) also has the same structure as the SU(5)_{GUT} model (eq.(33)) which creates the suitable mass ratio and bi-maximal mixing for the large angle MSW solution. The pre-factor gives the appropriate order of magnitude for the masses.

The results of the mixing and mass ratio have been confirmed by the numerical analysis in which we diagonalize the whole mass matrix including FN fields.
5 Conclusions and Discussions

We have considered a horizontal SU(3)$_H \times U(1)_H$ symmetry to construct the appropriate mass matrices for the quarks and leptons. We concentrated on the large mixing angle MSW solution for the solar neutrino problem. The large mixing angle for $\nu_\mu \leftrightarrow \nu_\tau$ can be explained by the lopsided family structure. In such a building block, many models in the context of grand unified model predict that the ratio of mass squared differences $\Delta m^2_{\text{solar}} / \Delta m^2_{\text{atm}}$ is similar to $(m_c/m_t)^4$ [28, 29]; otherwise, fine-tuning is needed to explain the hierarchy.

In this paper, we have constructed realistic models in the context of grand unified models to explain the hierarchy naturally such that

$$\frac{\Delta m^2_{\text{solar}}}{\Delta m^2_{\text{atm}}} \sim 10^{-2}$$  \hspace{1cm} (53)

We emphasize that the unification of the three flavors enables us to reproduce a neutrino mass matrix consistent with the large angle MSW solution.

Horizontal unification provides interesting features in particle physics. In supersymmetric models, flavor unification is a motivation for the suppression of flavor changing neutral currents (FCNC) [29, 30]. However, it is well known that the simple models with a gauged horizontal symmetry does not suppress the FCNC in the gravity-mediated supersymmetry breaking scenario [31].

It is interesting that the strong CP problem can be solved in our models. The VEVs of the $\xi$ fields break the horizontal SU(3)$_H$ symmetry. The $\xi$’s have a Peccei-Quinn-like charge and the Peccei-Quinn mechanism will work well. The invisible axion will be created if the breaking scale $M_\star$ is of the order of $10^{12}$ GeV. The assumption of the horizontal scale ($M_\star \sim 10^{12}$ GeV) does not lead to gauge coupling divergence below the GUT scale, which is caused by the existence of the vector-like FN fields at the scale $M_\star$. Another attractive scale for $M_\star$ is the GUT scale. In the SO(10) unified model, the scale $M_\star$ is constrained to be $10^{15}$–$10^{16}$ GeV. If the scale $M_\star$ is at the GUT scale, too many axions will be created and our universe will be overclosed. In such a case, we need the axions to be diluted by the inflation of the universe.

In our formulation, the horizontal unification of SO(3)$_H$ instead of SU(3)$_H$ may be also available. The SU(3)$_H$ unification has the particularly interesting feature that the representation of SU(3) is complex and spontaneous CP violation can occur [32]. The origin of the KM phase might be in the fundamental representation of $\xi$ in our model.
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