Virtual electrical capacitance tomography sensor

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Abstract. Electrical capacitance tomography (ECT) is an effective technique for elucidating the distribution of dielectric materials inside closed pipes or vessels. This paper describes a virtual electrical capacitance tomography (VECT) system, which can simulate a range of sensor and hardware configurations and material distributions. A selection of popular image reconstruction algorithms has been made available and image error and capacitance error tools enable their performance to be evaluated and compared. Series of frame-by-frame results can be stored for simulating real-time dynamic flows. The system is programmed in Matlab with DOS functions. It is convenient to use and low-cost to operate, providing an effective tool for engineering experiment.

1. Introduction
Electrical capacitance tomography (ECT) is well developed for visualising dielectric material distribution inside closed pipes or vessels (typically circular or square), such as gas-solids flows in pneumatic conveyors (Yang and Liu 2000, Reinecke and Mewes 1996). It is based on the measurement of changes in capacitance between a set of electrodes. Using these data, image reconstruction algorithms are used to deduce the distribution of the dielectric materials. The most common image reconstruction algorithm for ECT is non-iterative linear back-projection (LBP). It is relatively simple and fast and suitable for on-line measurement. However, the quality of reconstructed images using LBP is poor because of over-simplified linear approximations. To obtain improved images, iterative algorithms have been developed, such as Newton-Raphson and Landweber (Yang et al. 1999, Yang and Peng 2003, Jiang and Wang 2003). A typical ECT system consists of three main units:

1. A multi-electrode sensor
2. Sensing electronics, including capacitance measuring circuits and a data acquisition unit
3. A computer for data acquisition and data processing and image reconstruction.

There is a demanding for develop virtual electrical capacitance tomography (VECT) systems because of the following reasons:

- An ECT hardware system is a very specialised instrument (Yang and York 1999) and relatively expensive.
- It would be expensive to carry out a feasibility test in industry for a particular process to be investigated.
• It is possible to simulate an industrial process or carry out research using software, which is cheaper and quicker than using a real hardware system.

In a VECT system, all three main units as mentioned above can be defined and operated within a software package. While a typical ECT sensor consists of 8 or 12 electrodes, the sensor geometry must be defined, a mesh then generated and capacitance between each electrode pair calculated using a finite element method (FEM) or finite difference method (FDM) (Mirkowski et al. 2004). By calling some DOS functions, which have been developed in the past, the software package would be able to calculate the capacitances and displaying sensitivity maps between each pair of electrodes (Yang and Conway 1998). After deriving the sensitivity maps, the image can be reconstructed by software using a selected algorithm and material distribution may be reconstructed and shown on a GUI of a VECT system.

2. Design of VECT system
Using Matlab and DOS functions, a VECT software package has been developed for simulating an ECT system. Any element of an ECT system has been implemented on this software platform, such as the sensor design, material permittivity distribution inside a closed pipe. FEM is used to calculate capacitance and sensitivity maps and different image reconstruction algorithms can be selected. Without building hardware, the software package can be used to test an ECT system and evaluate its performance. First, the virtual sensor geometry and other elements need to be defined (Figure 1). Then the mesh is generated by FEM with the imaging area divided into many equal pixels, and the potential distribution calculated (Figure 2). The sensitivity maps are also calculated (Figure 3).

![Figure 1. Part of definition for virtual sensor](image1.png)

![Figure 2. Potential distribution based on DOS](image2.png)

The inter-electrode capacitance values as well as the standing capacitance are calculated by FEM. All data are saved in files, so that they can be used later for image reconstruction. The DOS functions, which are in charge of calculating capacitance measurements and sensitivity maps are called by the Matlab software package. These programs are:

• Genmesh: used to generate the mesh data
• Ptube: used to calculate capacitance
• Ntube: used to calculate the sensitivity maps
• Conv_sen: used to convert the binary sensitivity map file to ASCII
• Ctube: used to calculate capacitance from distribution permittivity files.
The number of sensitivity maps is $N \times (N-1)/2$. Thus for an 8-electrode virtual sensor, there are 28 maps needed and these are combined into a matrix, $S$, which is used for image reconstructing. The software package can define different material distributions. The DOS function “Ctube” is used to calculate the measured capacitance based on different definition of distribution permittivity.

In order to visualise a high-permittivity component in a low-permittivity background, the values of permittivity need to be defined in the VECT system. Typically, high-permittivity is defined to be 2.1, which is similar to that of oil, and low-permittivity is 1.0, which is that of air. Capacitance measurements are taken from the virtual sensor uniformly filled with high-permittivity material and filled with low-permittivity material in sequence. Both the full capacitance matrix, $C_{full}$ and the empty capacitance matrix, $C_{low}$ are obtained. These are used to normalise the “measured” capacitance matrix.

3. Image reconstruction

Image reconstruction software has been developed to determine the permittivity distribution over the observed cross section from the capacitance measurements. In principle, it is impossible to guarantee a unique solution for each pixel as the number of pixels in the image exceeds the number of capacitance measurements. Furthermore, image distortion will occur because ECT is an inherently soft-field method, i.e. the relationship between permittivity distribution and capacitance is non-linear. In order to simplify image reconstruction, this relationship can be considered to be linear, giving LBP the advantage of speed. Hence, it is popular for on-line image reconstruction (Gomez et al. 2003, Ortiz-Aleman et al. 2004).

Using the VECT system, a phantom is chosen to define the material distribution inside a vessel. The low and high permittivity limits are selected, and an $800 \times 1$ (for 8-electrode sensor) or $1200 \times 1$ (for 12-electrode sensor) matrix generated. Four typical distributions can be chosen: (1) stratified, (2) core, (3) peripheral and (4) arbitrary. The number of rings can also be selected. The capacitance measurements matrix of $28 \times 1$ or $66 \times 1$ can be calculated by FEM, using “Ctube” (Figure 4). To reconstruct expediently, the high-permittivity is defined as 2.1, shown in red colour and the low-permittivity value is 1.0, shown in blue colour. For the 8-electrode virtual sensor, the imaging area is divided into 800 elements. Each electrode corresponds to 100 elements and the number of concentric rings is 10. Each element belongs to a fixed position, and image reconstruction for the distribution can be displayed on $32 \times 32$ pixel matrix.
Taking an \( N = 8 \) electrode virtual sensor system as an example, capacitance measurements can be obtained by FEM. There are \( M = N \times (N - 1)/2 \) independent capacitance measurements. The forward problem can be expressed as:

\[
C = S \times K
\]

where \( C \) is a capacitance matrix, \( M \) is the number of electrode pairs (28 for an 8-electrode sensor), \( K \) is an \( P \times 1 \) dimensioned matrix, containing the set of image pixel of permittivity values, which describe the distribution inside the sensor. If 100 elements are defined for each electrode, \( P \) is 800. \( S \) is the sensitivity-map, which has been generated by FEM.

In normalised form:

\[
\lambda = S \times g
\]

where \( \lambda \) is the normalised capacitance vector.

\[
\lambda = (C - C_j)/|C_j - C_j|
\]

and \( S \) is the Jacobian matrix of normalised capacitance and \( g \) is the permittivity vector, i.e. the grey level of pixels for visualization.

Having obtained the normalised capacitance vector, \( \lambda = 28 \times 1 \) matrix and the \( 800 \times 28 \) Jacobian matrix of the normalised sensitivity-maps matrix, the image reconstruction vector \( \hat{g} \) can be calculated by one of the image reconstruction algorithms.

There are three non-iterative image reconstruction algorithms that can be selected so far on GUI, namely LBP, TSVD and Tikhonov and four iterative methods, Newton-Raphson, Landweber, ART and SIRT. Image reconstruction of the peripheral phantom, using the Landweber iteration, is shown in Figure 5.

Three evaluation criteria have been developed to compare the performance of the different image processing algorithms:

(1) Relative image error: Image error = \[ \frac{\|\hat{g} - g\|}{\|g\|} \]

**Figure 4.** Definition of two typical distributions (MATLAB GUI)
(2) Relative capacitance residual: Capacitance residual = \[ \frac{\|A - S \cdot \hat{g}\|}{\|A\|} \]

(3) Correlation coefficient =
\[
\frac{\sum_{i=1}^{N} (\hat{g}_i \cdot g_i)(\hat{g}_i \cdot \hat{g})}{\sqrt{\sum_{i=1}^{N} (\hat{g}_i - g_i)^2 \sum_{i=1}^{N} (g_i - \bar{g})^2}}
\]

where \( g \) is the true permittivity distribution, \( \hat{g} \) is the reconstructed permittivity distribution and \( \bar{g} \) is the mean value of \( g \).

According to these three formulae, the image error produced by the non-iterative algorithms is around 38% for core, peripheral and arbitrary, and 21% image error for the stratified phantom. With the Landweber iterative algorithm, the image error for core is 11% and for the other three phantoms 20%-30%, clearly indicating the superiority of the iterative image reconstruction algorithms.

The VECT system can store up to 1000 frames of data and then replay the reconstructed images to simulate dynamic behaviour (Figure 6).

4. Conclusion

A virtual ECT system, including definition of a virtual sensor, permittivity distribution and capacitance measurements, can be used to simulate and test all aspects of a real ECT system. All single-electrode combinations can be calculated using FEM. Using these capacitance measurements and sensitivity maps, a range of algorithms for image reconstruction has been implemented. Noise and image errors cannot be avoided and reconstruction algorithms need to be further investigated and improved. New developments, like neural network and inheritance, may be incorporated into the VECT system. The definition of each part of the VECT system can be implemented by Matlab, such as the number of electrodes, lower and higher permittivity distribution and typical distributions. This investigation for integrated software using Matlab GUI for VECT system may be made publicly available as EIDORS has been for electrical resistance tomography (ERT). The functionality of the Matlab GUI for the VECT system will be improved and consolidated.
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