It was pointed out 20 years ago [1] that the loop-induced \(bsZ\) coupling is enhanced by large \(m_t\), which turns out to dominate \(b \to s \ell^+ \ell^- (\bar{B} \to X_s \ell^+ \ell^-)\) decay. The effective \(bs\gamma\) coupling gives a low \(q^2 \equiv m_t^2\) peak in the differential rate [2], while \(Z\) and \(\gamma\) induced amplitudes interfere across the \(q^2\) spectrum. One such effect is the forward-backward asymmetry [3], \(A_{FB}\), which is the asymmetry between forward and backward moving \(\ell^+\) versus the \(B\) meson direction in the \(\ell^+\ell^-\) frame.

The first measurement of \(A_{FB}\) in exclusive \(B \to K^* \ell^+ \ell^-\) decay was recently reported [4] by the Belle experiment, with 3.4 \(\sigma\) significance. The results are consistent with the Standard Model (SM), rules out the wrong handed \(\ell^+\ell^-\) current, but a sign flip of the \(bs\gamma\) coupling is still tolerated by the poor statistics of \(\sim 100\) signal events. However, taking the measured inclusive \(b \to s \gamma (\bar{B} \to X_s \gamma)\) and \(b \to s \ell^+ \ell^-\) rates together [3], the latter possibility is disfavored.

The relative insensitivity of \(A_{FB}\) to hadronic effects makes it an attractive probe for New Physics (NP) in the long run. For example, we expect a quantum jump in the number of events with the advent of LHC in 2008. A study by the LHCb experiment shows that \(\sim 7700\) \(B \to K^* \ell^+ \ell^-\) events are expected with 2 \(fb^{-1}\) data [5]. In this Letter we point out that the sensitivity of \(A_{FB}\) to NP is greater than previously thought. The complexity of the associated effective Wilson coefficients can be probed by \(dA_{FB}/dq^2\) as early as 2008 at the LHC.

The quark level decay amplitude is [1, 6]

\[
\mathcal{M}_{b \to s \ell^+ \ell^-} = - \frac{G_F \alpha}{\sqrt{2} \pi} V_{ts}^* V_{cb} \left\{ C_{7\text{eff}} \left[ \bar{s} \gamma_\mu L b \right] \left[ \ell \gamma^\mu \ell \right] \right.
+ C_{10} \left[ \bar{s} \gamma_\mu L b \right] \left[ \ell \gamma^\mu \gamma_5 \ell \right] \\
- 2 \frac{\hat{m}_b}{\hat{s}} C_{9\text{eff}} \left[ \bar{s} \sigma_{\mu \nu} q^\nu R b \right] \left[ \ell \gamma^\mu \ell \right] \left\} ,
\]

where \(s \equiv q^2\), and we normalize by \(m_B\), e.g. \(\hat{s} \equiv s/m_B^2\). We factor out \(V_{ts}^* V_{cb}\) instead of the usual \(V_{ts} V_{tb}\). Although trivial within SM, it has the advantage of being real and in terms of CKM elements that are already measured. Short distance physics, including within SM, are isolated in the Wilson coefficients \(C_{7\text{eff}}, C_{9\text{eff}}\) and \(C_{10}\).

Eq. (1) can be used directly for inclusive \(B\) decay. For \(B \to K^* \ell^+ \ell^-\), hadronic matrix elements of quark bilinears give well defined \(B \to K^*\) form factors. Thus, even for exclusive decay, the coefficients \(C_{7\text{eff}}, C_{10}\) and \(C_{9\text{eff}}\) can be viewed as physical measurable, hence scheme and scale independent, up to the definition of form factors. Indeed, \(C_{9\text{eff}}\) and \(C_{10}\) in Eq. (1) are at \(m_B\) scale, with \(C_7\) receiving large additive contributions from other Wilson coefficients through operator mixing [8].

\[
C_{9\text{eff}}(\hat{s}) = C_9 + Y(\hat{s}),
\]

where \(Y_i\) are QCD evolution factors. However,

\[
C_{7\text{eff}}(\hat{s}) = \xi_7 C_7 + \xi_8 C_8 + \sum_{i=3}^6 \xi_i C_i,
\]

is also a function of the dilepton mass through \(Y(\hat{s})\), the form of which can be found in Ref. [7], and depends on long distance (\(c\bar{c}\)) effects.

Within SM, because of the near reality of \(V_{ts}\) in the standard phase convention, \(C_{7\text{eff}}, C_9\) and \(C_{10}\) are practically real, with \(C_{9\text{eff}}(\hat{s})\) receiving slight complexity through \(Y(\hat{s})\). A widely invoked [8] “Minimal Flavor Violation” (MFV) scenario further asserts (usually assuming the operator structure of SM) that there are no further sources of flavor and \(CP\) violation, other than what is already present in SM. Indeed, many popular extensions of SM, such as minimal supersymmetric SM [9] or two Higgs doublet models [10], tend to follow this pattern. With MFV as the prevailing mindset, \(C_7, C_9\) and \(C_{10}\) are oftentimes taken as real [11] tacitly, hence the focus only on possible sign flips from large NP effects. For \(A_{FB}\), therefore, the main projection for the future has been the sensitivity of the zero to NP [8, 12].

As a quantum amplitude, however, there is no reason \(a\ priori\) why \(C_{7\text{eff}}, C_9\) and \(C_{10}\) in \(\mathcal{M}_{b \to s \ell^+ \ell^-}\) should be real. Despite the suggested reality from SM and MFV, whether they are real or complex should be \textit{measured experimentally}, and we will be able to do so in just a few
years! In fact, currently there are hints \[14\] for “anomalies” in time-dependent and direct CP violation (CPV) measurements of $b \to s q \bar{q}$ transitions. One possible explanation is NP in $b \to s q \bar{q}$ electroweak penguins \[13\], which are the hadronic cousins of $b \to s \ell^+ \ell^-$, but the latter is clearly much less plagued by hadronic effects.

Motivated by possible hints for New Physics CPV in hadronic $b \to s$ transitions, and in anticipation of major experimental progress in near future, we explore how much $\mathcal{A}_{\text{FB}}$ can differ from SM by allowing associated effective couplings to be complex. Constraints such as decay rates, of course, should be respected, and one should check whether models exist where $C_{7}^\text{eff}$, $C_{9}^\text{eff}$ and $C_{10}$ can be complex. We find, even without enlarging the operator basis, from a theoretical standpoint, MFV may be too strong an assumption.

Our insight comes as follows. Part of the impetus for MFV is the good agreement between theory and experiment for inclusive $b \to s \gamma$ rate, which provides a stringent constraint on NP. However, while depending on the existence of a third generation top quark, the $b \to s \gamma$ rate depends very little on the precise value of $m_t$ when it is large. For $m_t$ in the range of 150 to 300 GeV, the $b \to s \gamma$ rate changes by only $\sim 30\%$. In contrast, the $b \to s \ell^+ \ell^-$ rate depends very sensitively on $m_t$ through the effective $bsZ$ coupling, as we stated from the beginning, changing by a factor of $\sim 4$ in the same $m_t$ range.

Suppose there are extra SM-like heavy quarks. These could be the 4th generation, or could be vector-like quarks that mix with the top. Take the 4th generation as an example, the $b \to s \gamma$ rate is not sensitive to the existence of the $t'$ quark unless $|V_{ts'}V_{tb}|$ is very large \[10\]. However, “hard” (sensitive to heavy quark mass) amplitudes such as $b \to s \ell^+ \ell^-$, $B_s$ mixing \[17\] etc. would be easily affected by finite $V_{ts'}V_{tb}$, as $m_{t'} > m_t$ by definition. Since $V_{ts'}V_{tb}$ should be in general complex \[13\], so would $C_{9}^\text{eff}$ and $C_{10}$ (and $C_{9}^\text{eff}$). With this as an existence proof, we note further that the three $|s|b$ terms in Eq. (1) are 4-fermion operators. The possible underlying New Physics is precisely what we wish to probe at B factories and at the LHC. Thus, despite the apparent success of MFV, we find the usual assumption of near reality of $C_{7}^\text{eff}$, $C_{9}^\text{eff}$ and $C_{10}$ unfounded. When sufficient data comes, the experimenters are well advised to keep these parameters complex in doing their fit.

We remark that, in fact within SM, $B \to \rho \ell^+ \ell^-$ decay exhibits partially the physics we talk about. A complex factor $\delta_u \equiv (V_{ud}V_{ub})/(V_{ud}V_{tb})$ arises from the $u$-quark current-current operator, as well as top quark in the loop, making $C_9$ complex \[19\]. We will use this case at the end as an illustration within SM.

In this study we shall keep the operator set as in SM, since enlarging to include e.g. righthanded currents would not be profitably probed in early years of LHC. In the same vein, although inclusive $b \to s \ell^+ \ell^-$ (and $b \to s \gamma$) is theoretically cleaner, we focus on the experimentally more accessible $B \to K^* \ell^+ \ell^-$ (and $B \to K^* \gamma$). Experimental studies of inclusive processes usually apply cuts that complicate theoretical correspondence.

Having allowed $C_{7}^\text{eff}$, $C_{9}^\text{eff}$ and $C_{10}$ to be complex, we still need to consider the constraints. $C_{7}^\text{eff}$ is rather well constrained by $b \to s \gamma$ rate measurement. We take a one sigma experimental range \[20\] for $B \to K^* \gamma$ for our exclusive study. Likewise, inclusive $b \to s \ell^+ \ell^-$ measurement (by reconstructing a partial set of $X_s$ states), as well as the exclusive $B \to K^* \ell^+ \ell^-$, provide constraints on $C_{7}^\text{eff}$, $C_{9}^\text{eff}$ and $C_{10}$. At the moment, measurements are not precise enough, so we use only the integrated rate for the exclusive channel, again within one sigma experimental range. In the future, with high statistics, one could use the differential $d\mathcal{B}/ds$ rate, which is more powerful. We will plot $d\mathcal{B}/ds$ as an illustration.

Our main focus is the $\mathcal{A}_{\text{FB}}$ in exclusive $B \to K^* \ell^+ \ell^-$ decay. Assuming the form factors are real, we have

$$
\frac{d\mathcal{A}_{\text{FB}}}{ds} \propto \left\{ |\text{Re}(C_{7}^\text{eff}C_{10}^*)V A_1 + \frac{m_0}{s}|\text{Re}(C_{7}^\text{eff}C_{10}^* \{ (VT_2)_- + (A_1 T_1)_+ \}) \right\},
$$

where $(VT_2)_- = VT_2 (1 - \hat{m}_K)$, $(A_1 T_1)_+ = A_1 T_1 (1 + \hat{m}_K)$, and $V$, $A_1$, $T_1$ are form factors \[8\]. We use the light-cone sum rule (LCSR) \[21\] form factors in our numerical analysis. In Eq. (4) we have exhibited only the dependence on $C_{7}^\text{eff}$, $C_{10}^\text{eff}$ and $C_{7}^\text{eff}$, since it is customary to plot $d\mathcal{A}_{\text{FB}}/ds$ which is $d\mathcal{A}_{\text{FB}}/ds$ normalized by the differential rate $d\mathcal{B}/ds$. This reduces sensitivity to form factor models. The zero of $\mathcal{A}_{\text{FB}}$ is often considered quite stable against form factor variations \[7, 13\].

The Wilson coefficients are parameterized as,

$$
C_{7} \to C_{7}(1 + \Delta_7 e^{i\phi_7}),
$$

$$
C_{9} \to C_{9}(1 + \Delta_9 e^{i\phi_9}),
$$

$$
C_{10} \to C_{10}(1 + \Delta_{10} e^{i\phi_{10}}),
$$

with $\Delta_i = 0$ corresponding to SM. These Wilson coefficients are evaluated at the electroweak scale, then evolved down to the $m_B$ scale to be used in Eq. (4). We do not include any complexity from other Wilson coefficients. The tree level $C_1$ and $C_2$ are unchanged by NP, but as a simplifying assumption, we ignore possible NP induced complexities through the gluonic $C_{5-a}$ and $C_8$ coefficients, which enter $C_{7}^\text{eff}$ and $C_{9}^\text{eff}$ through operator mixing and long distance effects (see Eqs. (2) and (3)). In practice, this should not change our point.

Let us start with a SM-like framework, that is, viewing $B \to K^* \ell^+ \ell^-$ as induced by effective $bsZ$ and $bs\gamma$ couplings (and box diagrams). If such is the case, we expect $C_{9}$ and $C_{10}$ to be approximately the same, i.e.

$$
\Delta \equiv \Delta_9 \equiv \Delta_{10}, \quad \phi \equiv \phi_9 \equiv \phi_{10},
$$

in Eqs. (6) and (7), and one effectiively has the parameters $\Delta$, $\Delta_{7,9}$, $\phi$, and $\phi_7$, which covers the usual case of wrong sign $C_{7}^\text{eff}$. The 4th generation also belongs to this scenario, with $V_{ts'}V_{tb}$ bringing in complexity.

We plot $d\mathcal{A}_{\text{FB}}/ds$ and $d\mathcal{B}/ds$ in Figs. 1(a) and (b), respectively, for SM and 4th generation model (SM4).
For SM4, we take the CKM parameters which yield the correct $B_s-B_d$ mixing (17), predicts large time-dependent CPV in $B_s$ decay, as well as accommodating the NP hints in CPV in $b \to s\bar{q}q$ decays. We see that the zero of $d\bar{A}_{FB}/ds$ has shifted by a significant amount, with only a small positive value below the zero. These are due to the enrichment of (mostly) the $\phi$ phase. For larger $s$, one has little difference in $d\bar{A}_{FB}/ds$ from SM, as the effect of $C_7^{\text{eff}}$ has damped away, while $C_9^{\text{eff}}$ and $C_{10}$ carry almost the same phase. The general appearance of $d\bar{B}/ds$ for SM and SM4 is very similar.

A broader range is allowed by Eq. (8). Keeping $B \to K^*\gamma$ and $K^*\ell^+\ell^-$ rates in 1 $\sigma$ experimental range and exploring $\Delta$, $\Delta_7$, $\phi$ and $\phi_7$ parameter space, the results are plotted in Fig. 1 as the shaded area, which illustrates the range of variation allowed by $C_9 \simeq C_{10}$. This is just for illustration purpose and should not be taken as precise boundaries. For instance, we see that below the SM zero $\bar{A}_{FB}/ds$ could be very small, but the shaded region for $d\bar{B}/ds$ basically reflects the 1$\sigma$ constraints on $B \to K^*\gamma$ and $K^*\ell^+\ell^-$. $d\bar{B}/ds$ should also be fitted in the future, but it depends directly on $B \to K^*$ form factors, especially the overall scale.

We illustrate the power of early LHC data with the 2 fb$^{-1}$ study of LHCb, where $\sim 7700$ reconstructed $B \to K^*\ell^+\ell^-$ events are expected. We take the simulated errors (with signal events generated according to SM) for $\bar{A}_{FB}/ds$ from three bins, one around the SM zero, one below and one above, and plot in Fig. 1(a) to guide the eye. It should be clear that our suggestion can be tested early on in the LHC era.

The narrow, long “tail” at $\hat{s} \gtrsim 0.3$ for $\bar{A}_{FB}/ds$ indicates that Eq. (8) is probably too strong an assumption. Even if we keep the operator basis as in Eq. (1), treating these as 4-fermion interactions arising from possible NP at short distance (for instance, $Z'$ models [22]), one should keep the full generality of Eqs. (5)–(7). We proceed to explore the parameter space as before, keeping $B \to K^*\gamma$ and $K^*\ell^+\ell^-$ within 1 $\sigma$ constraint. Indeed we find much richer possibilities than Figs. 1(a) and (b).

As plotted in Figs. 2(a) and (b), we illustrate with the further cases of $c$, $d$ and $e$. The SM and SM4 are Cases a and b, respectively, as in Fig. 1. The $\Delta_i$ and $\phi_i$ values are given in Table I.

Case $d$ has wrong sign $C_{10}$, while Case $e$ has sign flip in both $C_7^{\text{eff}}$ and $C_{10}$ (equivalent to wrong sign $C_9$). Both are already ruled out [4] by Belle data. The possibility of flipping only the sign of $C_7^{\text{eff}}$ is ruled out by rate constraints [3], hence not plotted. Similar scenarios have been considered in the literature, and we give these cases to illustrate the versatility of Eqs. (5)–(7). Though ruled out, Case $e$ is illuminating. From Table I, $\Delta_7 \sim -4.8$ overwhelms the SM effect, flipping the sign of $C_7^{\text{eff}}$ (Eq. (2)). At the same time, $C_{10}$ also flips sign, with a much diminished $C_9$. However, even with no complex phases, the large effects from Case $e$ survives rate constraints, giving a more pronounced low $q^2$ peak for differential rate, as seen in Fig. 2(a), which lies outside of the boundary of the shaded region in Fig. 1(a). This is because Eq. (8) no longer holds. Similar cases may exist that remain to be probed.

An interesting new scenario is illustrated by Case $c$, where $d\bar{B}/ds$ and the zero of $d\bar{A}_{FB}/ds$ are hard to distinguish from SM, but $d\bar{A}_{FB}/ds$ above the zero reaches only half the SM value. Thus, a measurement of the zero does not pin down $C_9$. The scenario can be tested already with 1 ab$^{-1}$ data at B factories expected by 2008. If such phenomena are discovered with, e.g. LHCb data, it would imply NP that feed the $(s\gamma_{\mu}Lb)(\bar{e}\gamma^\mu\ell)$ and $(s\gamma_{\mu}Lb)(\bar{e}\gamma^\mu\gamma\ell)$ operators differently.

We mark the simulated errors from the 2 fb$^{-1}$ study at LHCb as before on Fig. 2(a), illustrating its power. The actual possibilities are far richer. The shaded area of Fig. 1(a) illustrates that, even with Eq. (8) imposed, a broad range is allowed for $\hat{s} < 0.2$. With the full freedom of Eqs. (5)–(7), the region allowed by rate constraint would likely cover a large part of Fig. 2(a), which is up to experiment to explore. One should keep the effective Wilson coefficients of Eq. (1) complex and use the general parametrization of Eqs. (5)–(7) to fit for $\Delta_i$ and $\phi_i$. Finite $\phi_i$ implies violation of MFV.
Our suggestion of keeping the Wilson coefficients complex is not just for NP. Even within SM, for the CKM quark and top contributions. We plot $\frac{dA_{FB}}{d\hat{s}}$ in Fig. 3 for $B \to K^*\ell^+\ell^-$ and $\rho\ell^+\ell^-$ in SM. Although the difference is not so great, especially since with 2 fb$^{-1}$ data at LHCb one expects less than 200 $B \to \rho\ell^+\ell^-$ events with larger background, the different behavior in $dA_{FB}/d\hat{s}$ can be tested with a larger dataset.

We offer some remarks before closing. We have focused on $A_{FB}$ for exclusive $B \to K^*\ell^+\ell^-$, mostly because of ease of experimental access, and with upgrade of statistics imminent. Second, it is usually stressed that the zero of $A_{FB}$ is insensitive to form factors. With NP sensitivities now going beyond the zero, form factor issues would have to be considered. One would have to combine the progress from form factor models, lattice, as well as experimental studies of $B \to \rho\ell\nu$. One can of course try to measure $A_{FB}$ in inclusive mode, and our discussion, starting with Eq. (1), can be easily employed and followed. Third, while the 4th generation model provides a good example, our approach is fully general and aimed at the experimental study, and does not depend on specific models. In fact, the 4th generation provides a good example against the prevailing MFV prejudice that limits the perspective for flavor and $CP$ violation expectations in $b \to s$ transitions. It is our opinion that the study of $b \to s$ transitions is still in its infancy, and is the least constrained. Imposing MFV may be overstretching our experience from other areas of flavor violation. It is up to experiment to reveal what may be in store for us in the excellent probe of $A_{FB}$. Finally, the 1 ab$^{-1}$ final data at B factories would only give limited improvement on the existing result. The next round of major improvement would come from LHC. The Super B factory upgrade in the future could bring back competitiveness of $e^+e^-$ machines, especially for inclusive studies.

In summary, we have explored the $CP$ conserving consequences of complex Wilson coefficients on the forward-backward asymmetry $A_{FB}$ in $B \to K^*\ell^+\ell^-$ decay. The possibilities are much broader than the usual consideration of sign flips under minimal flavor violation framework. In view of hints of $CP$ violation anomalies in $b \to s q\bar{q}$ decays, the large increase in statistics with the advent of LHC would make $A_{FB}$ one of the cleanest probes for New Physics in the near future.

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