String Corrected Supergravity; A Complete and Consistent Non-Minimal Solution

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Abstract

We complete the solution to string corrected (deformed), D=10, N=1 Supergravity as the non-minimal low energy limit of string theory. We reaffirm a previously given solution, and we make important corrections to that solution. We solve what was an apparently intractable Bianchi identity in superspace, and we introduce a new important modification to the known first order results. In so doing we show that this approach to string corrected supergravity is indeed a consistent approach and we pave the way for many applications of the results.
1 Introduction

The route to finding a manifestly supersymmetric theory of D=10, N=1 supergravity at second order in the string slope parameter has encountered many difficulties over the years. Some years ago a solution to D=10, N=1 Supergravity as the low energy limit of String Theory was given at first order in the string slope parameter, [1]. It was recently re-calculated [1]/(2004). In a sense this was a minimal solution. This approach was founded on what we now choose to call the scenario of Gates and collaborators, (see [1], [2], and references therein). Other varied approaches are nowadays pursued, however the power of this older approach is currently being vindicated,[1]. A partial second order solution was recently given in [3] and [4]. It was incomplete and therefore in doubt due to an unsatisfactory assumption in the curvature sector, as well as a calculational error. Here we reaffirm that that solution is correct up to a curvature. We then show that the results obtained satisfy the problem curvature, equation (3). We achieve this through introducing a new and important condition on $R^{(1)}_{\alpha\beta\gamma\delta}$, a quantity previously undefined. This result also modifies the old first order case. The difficulties that prevented completely closing the Bianchi identities at second order are fully overcome. We complete the set of equations that consistently satisfy all Bianchi identities. As the work in itself is lengthy we leave finding the equations of motion and other applications for another paper. We do not list results which are explicitly solved by Bianchi identities such as $H^{(2)}_{abc}$.

For this approach it is required that we solve the Bianchi identities for D=10 N=1 Supergravity in Superspace at second order in the slope parameter, in the presence of the Lorentz Chern Simmons Form, and the so called Beta Function Favored Constraints, ($\beta F F$). This approach has been detailed to first order in [1], and to second order in [3] and [4], so we will not recount it here. We show that all results fall neatly into place in a very elegant way, therefore further vindicating the whole original scenario. We note here that it appears also to work consistently at third order as we have proceeded to that order and that is left for another letter.

2 Review of Solution and Notation

The Bianchi identities in Superspace are as follows

$$[[\nabla_A, \nabla_B], \nabla_C] = 0$$

(1)

Here we have switched off Yang Mills fields and the commutator is given by

$$[\nabla_A, \nabla_B] = T_{AB}^\ C + \frac{1}{2}R_{ABd}^\ e M^d_e$$

(2)

This generates many identities, and a solution must be found in such a way that all of them are satisfied simultaneously. A small alteration in one solution will change the
whole picture. Most of the resulting identities are listed in [1] and [4], so we will not list them here. The second order solution given in parts in [3] and [4] was, to some extent based on an anzatz for the so called X tensor as well as extensive algebraic manipulations. The necessity for introducing the X tensor was predicted by Gates et. al., [1]. In [3], and [4], the following Bianchi identity was not properly solved.

\[ T_{(\alpha\beta|\lambda}R_{|\gamma)}\lambda_{de} - T_{(\alpha\beta}|\gamma}R_{|\gamma)gde} - \nabla_{(\alpha|R_{\beta|\gamma)de} = 0 \]  

(3)

\[ T_{AD}G = T^{(0)}_{AD}G + T^{(1)}_{AD}G + T^{(2)}_{AD}G + \ldots \]

The numerical superscript refers to the order of the quantity. In this work we make some improvements to the notation in references [3] and [4]. For convenience we also have the following quantity

\[ \Omega^{(1)}_{gef} = L^{(1)}_{gef} - \frac{1}{4}A^{(1)}_{gef} \]  

(4)

and its spinor derivative

\[ \Omega^{(1)}_{agef} = \nabla_{\gamma}\{L^{(1)}_{gef} - \frac{1}{4}A^{(1)}_{gef}\} \]  

(5)

A crucial input at first order is that for the super-current \( A^{(1)}_{gef} \). The choice made for on-shell conditions in [1] and hence also [3] and [4], is as follows

\[ A^{(1)}_{gef} = +i\gamma\sigma_{gef\tau}T^{mne}T_{mn}^{\tau} \]  

(6)
In [3] and [4], we proposed the form of the X tensor to be as follows

\[ T^{(2)}_{\alpha\beta} d = \sigma^{pqrf}_{\alpha\beta} X_{pqrefd} = -\frac{i\gamma}{6} \sigma^{pqrf}_{\alpha\beta} H^{(0)}_{d e f} A^{(1)}_{pqr} \tag{7} \]

Coupled with this we also have a conventional constraint which may or may not be imposed to all orders. We have

\[ T_{\alpha\beta}^\delta = -\frac{1}{48} \sigma_{\alpha\lambda} \sigma^{pq\lambda\delta} A_{pqr} \tag{8} \]

If we impose this at second order we have a result that relates this torsion at second order to the super current.

\[ T^{(2)}_{\alpha\beta} \delta = -\frac{1}{48} \sigma_{\alpha\lambda} \sigma^{pq\lambda\delta} A^{(2)}_{pqr} \tag{9} \]

However we may relax this constraint also. We will consider this option in reconsidering the solution to equation (13).

A fundamental result which was used in every Bianchi identity and which is very lengthy to derive is the following

\[ T^{(0)}_{(\alpha\beta|\gamma)\lambda A^{(1)}_{pqr}} H^{(0)}_{def} - \sigma^{pqrf}_{(\alpha\beta|} H^{(0)}_{d e f} \nabla_{(\gamma)} A^{(1)}_{pqr} = -24 \sigma^{g}_{(\alpha\beta|} H^{(0)}_{d e f} [\Omega^{(1)}_{(\gamma)g e f}] \tag{10} \]

We note however in this paper that this result can be arrived at indirectly by using the first order results found in [1], in conjunction with the Bianchi identity (3).

## 3 Torsions Solutions

We found from the H sector Bianchi identities that the following dimension one half torsion is given uniquely by

\[ T^{(2)}_{\alpha\beta \lambda} = -\frac{i\gamma}{12} \sigma^{pqrf}_{\alpha\beta} A^{(1)}_{pqr} T_{e f} \lambda \tag{11} \]

It was then shown that together with the proposed X tensor anzatz (7), as well as equation (8) and other observations and results, that the H sector Bianchi identities as listed in [1], [2] could be solved. Also solved was the torsions (10), below.

\[ T_{(\alpha\beta|\gamma)\lambda} d - T_{(\alpha\beta|\gamma)\lambda} d - \nabla_{(\alpha|T_{\beta\gamma}} d = 0 \tag{12} \]
These results also offer a solution to the following

\[ T(\alpha|\beta)_{\lambda}^{\delta} - T(\alpha|\beta)_{\gamma}^{\delta} + \nabla(\alpha)_{\beta\gamma}^{\delta} - \frac{1}{4} R(\alpha|\beta|de\sigma^{de})^{\delta}_{\gamma} = 0 \]  

(13)

However, consideration must be given here as to whether or not to impose the constraint (9). Either way we can solve the identity. It is important to note that imposing this constraint results in a null term in (13),

\[ T^{(0)}(\alpha|\beta)_{\gamma}^{\delta} = \frac{i}{48} \sigma(\alpha|\beta)_{\gamma}^{\delta} \sigma^{pq\lambda\delta} A^{(2)}_{pqrt} \]  

(14)

This is due to the fact that

\[ \sigma(\alpha|\beta)_{\gamma}^{\delta} = 0 \]  

(15)

We find the second order solutions to (12) to be given by (7) and the following

\[ \sigma(\alpha|\beta)_{\gamma}^{\delta} = 4 \gamma T^{(2)}(\gamma)_{ef \alpha\beta} \delta^{\mu\nu} \]  

(16)

The lengthy extracted and symmetrized equation is listed in [3], and [4]. In equation (13), we notice the occurrence of the term

\[ \nabla(\alpha)_{\beta\gamma}^{\delta} \]  

(17)

If we impose the constraint (14), then this term must be retained as being non-zero. If we relax the constraint (14), then we may include this as an extra constraint as follows,

\[ [2\delta(\alpha|\beta)_{\gamma}^{\delta} + \sigma(\alpha|\beta)_{\delta\gamma}^{\delta\lambda}] \nabla(\gamma)_{\lambda}^{\lambda(2)} = 0 \]  

(18)

Here, we choose to relax the constraint. Hence, in so doing, we find for the solution of (13) after some algebra and neat cancellations,

\[ T^{(2)}_{\gamma}^{\delta} = 2 \gamma T^{(0)}_{ef\gamma} \delta^{\alpha\beta} \Omega^{(1)}_{\gamma\alpha\beta} \]  

(19)

And

\[ R^{(2)}_{\alpha\beta} = - \frac{i \gamma}{12} \sigma^{pq\alpha\beta} A^{(1)}_{pqrt} R^{(0)}_{efde} \]  

(20)

We now must show that all of the above found results satisfy (3).
4 New Solution for $R^{(2)}_{\lambda gde}$

We must show that we can close equation (3) using the results (7), (11), and (20). As mentioned, in references [3] and [4] the curvature (3) was not properly solved. In fact there existed terms which seemed at first to predict serious problems for the entire scenario. The mentioned various approaches did not work, nor was there any way to manipulate the terms using the sigma matrix algebra. Eventually the following procedure provided a confident and elegant solution. At second order the Bianchi identity (3) becomes:

\[ T^{(0)}_{\langle \alpha \beta \rangle} \lambda R^{(2)}_{\gamma \lambda gde} + T^{(2)}_{\langle \alpha \beta \rangle} \lambda R^{(0)}_{\gamma \lambda gde} - T^{(0)}_{\langle \alpha \beta \rangle} g R^{(2)}_{\gamma gde} - T^{(2)}_{\langle \alpha \beta \rangle} g R^{(0)}_{\gamma gde} \\
\quad - \nabla_{\langle \alpha \rangle} [R^{(0)}_{\beta \gamma gde}^{\text{Order}(2)} + R^{(1)}_{\beta \gamma gde}^{\text{Order}(2)} + R^{(2)}_{\beta \gamma gde}^{\text{Order}(2)}] = 0 \]  

(21)

Using the results we found, (7), (11) and (20), we arrive at:

\[ -i\sigma g_{\langle \alpha \beta \rangle} R^{(2)}_{\gamma gde} + T^{(0)}_{\langle \alpha \beta \rangle} \lambda [-\frac{i \gamma}{12} \sigma_{pq rab}^{\gamma \lambda} A^{(1)}_{pq r} R^{(0)}_{abde}] \\
\quad - \frac{i \gamma}{12} \sigma_{pq rab}^{\alpha \beta} A^{(1)}_{pq r} T_{ab} \lambda R^{(0)}_{\gamma \lambda gde} + \frac{i \gamma}{6} \sigma_{pq rab}^{\alpha \beta} H^{(0) g ab} A^{(1)}_{pq r} R^{(0)}_{\gamma gde} \\
\quad - \nabla_{\langle \gamma \rangle} \{-2i\sigma g_{\langle \alpha \beta \rangle} \Pi^{(0) + (1)}_{gde} + \frac{i \gamma}{24} \sigma_{pq r}^{de \langle \alpha \beta \rangle} A^{(1)}_{pq r} \} = 0 \]  

(22)

Here we encounter second order contributions from zeroth order parts but in solvable form. We define:

\[ \Pi^e f_g = L^e f_g - \frac{1}{8} A^e f_g \]  

(23)

Now again using our key relation, (10) we obtain:

\[ -i\sigma g_{\langle \alpha \beta \rangle} R^{(2)}_{\gamma gde} + 2i \gamma \sigma g_{\langle \alpha \beta \rangle} R^{(0)}_{abde} \{\Omega^{(1)}_{\gamma gab} \} - \nabla_{\langle \gamma \rangle} \{-2i\sigma g_{\langle \alpha \beta \rangle} \Pi^{(0) + (1)}_{gde} \} \\
\quad - \frac{i \gamma}{12} \sigma_{pq rab}^{\alpha \beta} A^{(1)}_{pq r} T_{ab} \lambda R^{(0)}_{\gamma \lambda gde} + \frac{i \gamma}{6} \sigma_{pq rab}^{\alpha \beta} H^{(0) g ab} A^{(1)}_{pq r} R^{(0)}_{\gamma gde} \\
\quad + \frac{i \gamma}{12} \sigma_{pq rab}^{\alpha \beta} A^{(1)}_{pq r} \nabla_{\gamma} R^{(0)}_{abde} \} - \frac{i}{24} \sigma_{pq r}^{de \langle \alpha \beta \rangle} \nabla_{\gamma} A^{(1) \text{Order}(2)}_{pq r} \]  

(24)

Of particular concern and interest is the last term in (22). One possible approach to eliminating this term is that taken in [3] and [4]. However here we now disagree with that approach. Hence the problem terms will still remain. It was thought that a
possible modification of $A^{(1)}_{pqr}$, or a contribution from $A^{(2)}_{pqr}$ would be necessary. These approaches are now seen also to be unnecessary.

In advance we anticipate that the solution will be as follows

\[ +i\sigma^g(\alpha\beta)|R^{(2)}|\gamma)_{gde} = 2i\gamma\sigma^g(\alpha\beta)|R^{(0)}|_{gde} + \nabla(\gamma)\{2i\sigma^g(\alpha\beta)|\Pi^{(0)}(1)+1|_{gde}\} \text{Order}(2) \]  \tag{25}

And

\[ -\frac{i\gamma}{12}\sigma^{pqra}b(\alpha\beta)|A^{(1)}_{pqr}T_{ab}^\lambda R^{(0)}|\gamma)_{\lambda de} + \frac{i\gamma}{6}\sigma^{pqra}b(\alpha\beta)|H^{(0)}_{ab}A^{(1)}_{pqr}R^{(0)}|\gamma)_{gde} \]
\[ +\frac{i\gamma}{12}\sigma^{pqra}b(\alpha\beta)|A^{(1)}_{pqr}[\nabla(\gamma)R^{(0)}|_{abde}] - \frac{i}{24}\sigma^{pqra}b(\alpha\beta)|\nabla(\gamma)A^{(1)}_{Order(2)}|_{gde} = 0 \]  \tag{26}

We need to show that (26) does in fact vanish. We must begin with the Bianchi identity that gives the spinor derivative of $T_{kl}\tau$.

\[ \nabla(\gamma)T_{kl}\tau = T_{[kl]}^\lambda T_{\lambda\tau} + T_{[kl]}^\lambda T_{\lambda\tau} + T_{[kl]}^\lambda T_{\lambda\tau} - \nabla[kl]T_{[\tau]} - R_{kl\tau} \]  \tag{27}

At first order this simplifies to

\[ \nabla(\gamma)T_{kl}\tau^{Order(1)} = -R_{kl\tau} + T_{kl}^\lambda T_{\lambda\tau} + 1/48 [2H^{(0)}_{kl\gamma}\sigma^{pqra}b(\alpha\beta)|A^{(1)}_{pqr} - \sigma_{[\gamma|\gamma}\sigma^{pqra}b(\alpha\beta)|\nabla[\tau]A^{(1)}_{pqr}] \]  \tag{28}

We now write the last term in (26), using the ten dimensional metric so that the unsolved part becomes

\[ -\frac{i}{12}\sigma^{pqra}b(\alpha\beta)|\gamma A^{(1)}_{pqr}[T_{ab}^\lambda R^{(0)}|\gamma)_{\lambda de} + \nabla(\gamma)R^{(0)}|_{gde} - \nabla(\gamma)R^{(0)}|_{gde}] \]
\[ +\frac{1}{2}\eta_{ad}\eta_{be}\nabla(\gamma)A^{(1)}_{Order(2)}|_{pqr} = 0 \]  \tag{29}

Using the definition of $A^{(1)}_{pqr}$, (6), gives therefore

\[ +\frac{\gamma}{12}\sigma^{pqra}b(\alpha\beta)|T_{k}\ell^\gamma \{T_{ab}^\lambda R^{(0)}|\gamma)_{\lambda de} + T_{ab}^\lambda R^{(0)}|\gamma)_{gde} - \nabla(\gamma)R^{(0)}|_{gde}] \]
\[ +\eta_{ad}\eta_{be}\nabla(\gamma)T_{k}\ell^\gamma = 0 \]  \tag{30}

We now use equation (25) and the properties of the sigma matrices. After some algebra we obtain an extremely interesting condition on $R_{kl\gamma}$. We find
\begin{align*}
R^{(1)}_{kl\gamma \tau} &= \left\{ + \frac{\gamma}{100} T_{kl}^{\gamma \tau} \right. \\
&\quad \cdot [T_{mn}^{\lambda} R^{(0)}_{\gamma\lambda mn} + T^{(0)}_{mn} R^{(0)}_{\gamma g mn} - \nabla_{\gamma} R^{(0)}_{mn mn}] \\
&\quad \left. + T^{(1)}_{kl} T^{(0)}_{\lambda \tau} + 4i\gamma [T_{mn}^{\lambda} R^{mn\tau} H^{(0)}_{klg}\sigma g - \sigma_{[k|\gamma \lambda} T_{mn}^{\lambda} \nabla l|T^{mn\tau}]} \}
\end{align*}

(31)

This can now be added to the list of first order results quoted in [1]. It assumes a correction \( T^{(1)}_{kl} \lambda \) which may itself be complicated. \( R^{(1)}_{kl\gamma \tau} \) was not defined in [1]. Furthermore the curvature (3) is neatly solved. We obtain

\begin{align*}
R^{(2)}_{\gamma gde} &= 2\gamma R^{(0)}_{abde} [\Omega^{(1)}_{\gamma g} ab] + \nabla_{\gamma} \{ \Pi^{(0)}_{(0)+(1)} gde \}^{\text{Order}(2)}
\end{align*}

(32)

The following Bianchi identity also includes \( R^{(2)}_{abde} \).

\begin{align*}
\frac{1}{4} R_{(\alpha|\alpha mn|\beta}^{mn\beta|a} + T_{\alpha\beta}^{g T_{ga} \gamma} + T_{\alpha\beta}^{\lambda T_{\lambda a} \gamma} + T_{\alpha[a}^{\lambda T_{|\beta]} \gamma} - T_{a[a}^{\lambda T_{|\beta]} g \gamma} \\
- \nabla_{(\alpha|T_{\beta]a}^{\gamma} - \nabla a T_{\alpha\beta}^{\gamma} = 0
\end{align*}

(33)

Although not yet simplified this identity predicts the same term that we found exist in \( R^{(2)}_{\alpha amn} \). However it includes a great deal more information which we have included in another letter.

5 Conclusions

We have found a consistent solution to the manifestly supersymmetric equations of D=10, N=1 Supergravity, with string corrections to second order in the string slope parameter. We have reaffirmed the results of [3] and [4], and we have solved the remaining previously intractable curvature. We find a new and important modification to the first order case as in equation (31). We gave more careful consideration to the imposition of the constraint (9), and we note that imposing this constraint will modify the solution. However a solution can also be found by correspondingly modifying the constraint (18). This solution allows for flexibility in finding a suitable candidate for the supercurrent \( A^{(2)}_{pqr} \). Otherwise it is tied to the torsion \( T^{(2)}_{\alpha\beta} \).

6 Acknowledgement

I would like to acknowledge S. Bellucci for introducing me to the method of Bianchi identities and to recognize the founding work done in this area by S. J. Gates Jr.
Here for convenience we list the torsions curvatures and H sector results to second order. Other first order results listed in [1] also form part of the set.

\[ H_{\alpha\beta\gamma} = 0 + \text{Order}(\gamma^3) \] (34)

\[ H_{\alpha\beta\gamma} = \frac{i}{2} \sigma_{\alpha\beta} + 4i \gamma \sigma^g_{\alpha\beta} H_{\gamma}^{\sigma} H_{\delta}^{\sigma} \]

\[ \sigma_{\alpha\beta} = \left[ 8i \gamma H^{(0)}_{\delta\lambda\mu\nu} L^{(1)}_{\gamma \delta} - i \gamma H^{(0)}_{\delta\lambda\mu\nu} A^{(1)}_{\gamma \delta} \right] + \sigma^{pqref}_{\alpha\beta} \frac{i \gamma}{12} H^{(0)}_{\delta\lambda\mu\nu} A^{(1)}_{\gamma \delta} + \text{Order}(\gamma^3) \] (35)

\[ T_{\alpha\beta} = i \sigma_{\alpha\beta} - \frac{i \gamma}{6} \sigma^{pqref}_{\alpha\beta} H^{(0)}_{\delta\lambda\mu\nu} A^{(1)}_{\gamma \delta} + \text{Order}(\gamma^3) \] (37)

\[ T_{\alpha\beta} = -\delta_{(\alpha} \gamma \delta_{\beta)} + \sigma^g_{\alpha\beta} \sigma^g_{\gamma \delta} + \frac{i \gamma}{12} \sigma^{pqref}_{\alpha\beta} A^{(1)}_{\gamma \delta} + \text{Order}(\gamma^3) \] (38)

\[ T_{\alpha\gamma} = -\frac{1}{48} \sigma^g_{\alpha\beta} \sigma^{pqrf}_{\beta\gamma} \sigma^{\gamma \delta} + 2 \gamma T^{(0)\delta\lambda\mu\nu} H^{(0)\delta\lambda\mu\nu} A^{(1)}_{\gamma \delta} + \text{Order}(\gamma^3) \] (39)

\[ \sigma^g_{\alpha\beta} T^{(2)_{\gamma \delta}} = 4 \gamma \sigma^g_{\alpha\beta} \sigma^g_{\gamma \delta} H^{(0)\delta\lambda\mu\nu} - \frac{i \gamma}{6} \sigma^g_{\alpha\beta} \sigma^{pqref}_{\delta\gamma} \sigma^{\gamma \delta} A^{(1)}_{\gamma \delta} + \text{Order}(\gamma^3) \] (40)

Or symmetrized,

\[ T_{\gamma \delta} = +2 \gamma \left[ \Omega^{(1)}_{\gamma \delta} H^{(0)\delta\lambda\mu\nu} - \sigma_{ab} \gamma \phi \left[ \frac{\gamma}{3} \Omega^{(1)}_{\phi} H^{(0)\delta\lambda\mu\nu} \right] A^{(1)}_{\gamma \delta} \right] - \frac{\gamma}{6} \sigma^g_{\alpha\beta} \gamma \phi \left[ \Omega^{(1)}_{\phi\beta\gamma} H^{(0)\delta\lambda\mu\nu} + \Omega^{(1)}_{\phi\gamma\beta} H^{(0)\delta\lambda\mu\nu} \right] + \text{Order}(\gamma^3) \]
\[-i\gamma \frac{A^{(1)}_{pqr} \sigma^{pqrg}_{[a|\phi\lambda} T^{(0)}_{b]\phi\lambda \rho]} g}{12} \]
\[-i\gamma \frac{\sigma^{pqrg}_{[a|\phi\lambda} T^{(0)}_{b]\phi\lambda \rho]} g}{72} \phi^{pqreg} \phi^{pqreg} A^{(1)}_{pqr} T^{(0)}_{eg} \]

\[i\gamma \frac{A^{(1)}_{pqr} \sigma^{pqrg}_{[a|\phi\lambda} T^{(0)}_{b]\phi\lambda \rho]} g}{144} \gamma^\phi \left[ \sigma^{pqreg}_{[a\phi\lambda} T^{(0)}_{b]\phi\lambda \rho]} g + \sigma^{pqreg}_{[a\phi\lambda} T^{(0)}_{b]\phi\lambda \rho]} g \right] + \text{Order}(\gamma^3) + \ldots \]

\[R_{\alpha\beta de} = -2i\sigma^g_{\alpha\beta} \Pi^{(1)}_{gde} + \frac{i}{24} \sigma^{pqref}_{\alpha\beta} A^{(1)}_{pqr} \]
\[= -2i\sigma^g_{\alpha\beta} \Pi^{(1)}_{gde} + \frac{i}{24} \sigma^{pqref}_{\alpha\beta} A^{(1)}_{pqr} \]
\[= \frac{i\gamma}{12} \sigma^{pqref}_{\alpha\beta} A^{(1)}_{pqr} R_{efde} + \text{Order}(\gamma^3) \]

Where

\[\Pi^{(1)}_{gde} = L^{(1)}_{gde} - \frac{1}{8} A^{(1)}_{gde} \]

\[R_{\alpha gde} = -i\sigma_{[d|\alpha\phi T_{g]\phi}^{f]} + i\gamma \sigma_{[g|\alpha \phi T^{kl}_{kl de} + 2\gamma R^{(0)}_{abcd} [\Omega_{\alpha\beta}^{(1)} T_{gde}] + \nabla_{\alpha} \left\{ \Pi^{(0)+(1)}_{gde} \right\} \text{Order}(2) + \text{Order}(\gamma^3) \]

The spinor derivative of \( L_{abc} \) is solved and available from a Bianchi identity. We will list it in a later paper.

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