Numerical analysis of wet separation of particles by density differences

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**Abstract.** Wet particle separation is widely used in mineral processing and plastic recycling to separate mixtures of particulate materials into further usable fractions due to density differences. This work presents efforts aiming to numerically analyze the wet separation of particles with different densities. In the current study the discrete element method (DEM) is used for the solid phase while the smoothed particle hydrodynamics (SPH) is used for modeling of the liquid phase. The two phases are coupled by the use of a volume averaging technique. In the current study, simulations of spherical particle separation were performed. In these simulations, a set of generated particles with two different densities is dropped into a rectangular container filled with liquid. The results of simulations with two different mixtures of particles demonstrated how separation depends on the densities of particles.

**Keywords:** fluid-particle interaction, discrete element method, smoothed particle hydrodynamics, computational fluid dynamics

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**INTRODUCTION**

Wet particle separation is widely used in mineral processing and plastic recycling to separate mixtures of particulate materials into further usable fractions due to density differences. Despite its wide usage, the wet particle separation process is often attributed to operational problems. Difficulties arise in float-sink separation if density differences between plastic fractions become low [1] or in case of elevated feed rates. A review of varying separation technologies and their efficiencies can be found in [2]. However, a review of the state of the art indicates that numerical modeling has not yet been applied to wet separation processes due to the lack of applicable numerical schemes. The current research is aimed to fill this gap by developing the numerical tool and its application to wet separation of particles.

A smoothed particle hydrodynamics (SPH) method is used for modeling of the liquid phase. The principal idea of SPH is to treat the fluid in a completely mesh-free fashion, in terms of a set of sampling particles [3]. SPH particles represent a finite mass of the discretized fluid and carry all information about physical variables evaluated at their positions. Function values at a fluid particle are interpolated from function values at surrounding particles using a kernel function and its derivative. Because of the mesh-free nature of SPH, it can be used for dealing with problems where large displacements of the fluid-structure interface and a rapidly moving fluid free-surface are present [4, 5].

The discrete element method (DEM) is used for modeling of solid particles. The DEM is a Lagrangian method introduced by Cundall and Strack [6] to describe granular materials and is nowadays widely used for modeling particulate flows [7]. In this method, contact forces are calculated for particles interacting with their nearest neighbors and with the walls based on appropriate contact laws while the motion of the particles follows Newton’s second law. The interaction of the particles can be modeled using simple spring contact models, or using Hertz theory for modeling the normal interactions and the theory of Mindlin and Deresiewicz to model tangential interaction [8, 9]. Different particle shapes can be used in the DEM, however spherical particles dominate because these particles are easy to describe by their center of mass and radius and their use is computationally very efficient due to straightforward contact detection.

The volume-averaging technique is used for the coupling of the SPH with the DEM. This technique based on the locally averaged Navier-Stokes equations was first reported by Tsuji et al. [10] in their work where a finite volume method was coupled with the DEM. A two-way coupling scheme between the DEM and the SPH has been derived by Sun et al. [11] and Robinson et al. [12] which is applied here.

In the current study wet separation of particles is simulated numerically. The used numerical methodology, in which the DEM is used for the solid phase and the SPH is used for the liquid phase, is described in the following section. In the later section the numerical modeling is presented, where the separation of spherical particles with different densities is analyzed.
NUMERICAL METHODOLOGY

For modeling of the fluid the continuity equation and the momentum equation are used:

\[ \frac{d\rho_f}{dt} + \nabla \cdot (\rho_f \mathbf{u}_f) = 0, \quad \frac{d\rho_f \mathbf{u}_f}{dt} = -\nabla p + \nabla \cdot (\rho_f \mathbf{\tau}) - \mathbf{F}^{int} + \rho_f \mathbf{g}, \quad (1) \]

where \( \rho_f = \epsilon \rho \) is the superficial fluid density of the fluid, \( \epsilon \) is the local mean fluid volume fraction.

In the SPH the fluid is represented by separate particles. These particles carry variables such as velocity, pressure and mass. No connectivity is modeled between particles. The integral representation of the function is approximated by summing up the values of the neighboring particles using smoothing kernel functions.

The weekly compressible formulation of the SPH is used to simulate an incompressible fluid. In SPH the continuity equation and the momentum conservation equation (1) for the fluid particle \( a \) takes the form

\[ \frac{d\rho_a}{dt} = \sum_b m_b \mathbf{u}_{ab} \cdot \nabla \mathbf{W}_{ab}, \quad \frac{d\mathbf{u}_a}{dt} = -\sum_b \left( \frac{m_a}{\rho_a} + \frac{m_b}{\rho_b} \right) \nabla \mathbf{W}_{ab} + \mathbf{g} + \frac{\sum m_b \mathbf{v}(\mathbf{r}_{ab} + \mathbf{r}_b) \cdot \nabla \mathbf{W}_{ab}}{|\mathbf{r}_{ab}|^2} + \frac{\rho_a \mathbf{W}_{ab} \mathbf{u}_{ab} + \mathbf{F}^{int}}{m_a}, \quad (2) \]

where indexes \( a \) and \( b \) indicate fluid particles. \( m \) is the mass. \( \mathbf{W}_{ab} = \mathbf{u}_a - \mathbf{u}_b \) is the relative velocity between particles \( a \) and \( b \). \( \nabla \mathbf{W}_{ab} = \nabla \mathbf{W}(\mathbf{r}_a - \mathbf{r}_b, h) \) is the gradient of the kernel function. \( r_a \) and \( r_b \) are positions of the particles \( a \) and \( b \). The summation is performed over all neighboring particles \( b \) of particle \( a \). Here a viscous term introduced by Morris et al. [13] is used, where \( \nu \) is the kinematic viscosity. \( \delta \) is a small number used to keep the denominator non-zero.

\( \mathbf{F}^{int} \) in Eq. (2) is the solid-fluid interaction force acting on the fluid particle \( a \) due to the surrounding solid particles:

\[ \mathbf{F}^{int}_a = \sum_i \mathbf{F}^{int}_i = \sum_i -\nu_i \mathbf{W}_{ai}/(\sum_b \mathbf{W}_{bi}) \mathbf{F}^{int}_i, \quad (3) \]

where \( V_a \) and \( V_b \) are the volumes of fluid particles \( a \) and \( b \), while \( \mathbf{F}^{int}_i \) is the solid-fluid interaction force acting on solid particle \( i \). The fluid volume fraction \( \epsilon_i \) is calculated from the volumes of all solid particles \( i \) which are in the smoothing domain of the fluid particle:

\[ \epsilon_i = 1 - \sum_i \epsilon_i \mathbf{W}_{ai}, \quad (4) \]

where \( V_i \) is the volume of the solid particle \( i \), while \( \mathbf{W}_{ai} = \mathbf{W}(\mathbf{r}_a - \mathbf{r}_{i,k}) \) is the kernel function.

The solid phase is modeled using the discrete element method. The motion of solid particles is described by Newton’s second law:

\[ m_i \frac{d^2 \mathbf{r}_i}{dt^2} = \mathbf{F}^c_i + \mathbf{F}^g_i + \mathbf{F}^{int}_i, \quad (5) \]

where \( \mathbf{r}_i \) is the position of the solid particle, \( \mathbf{F}^c_i \) is the contact force, \( \mathbf{F}^g_i \) is the gravity force and \( \mathbf{F}^{int}_i \) is the solid-fluid interaction force. The contact force for particle \( i \) is obtained from all contact forces between \( i \) and neighboring particles:

\[ \mathbf{F}^c_i = \sum_{j=1}^{n} \mathbf{F}^c_{ij}, \quad (6) \]

A more detailed description of the used DEM model can be found in [14, 15].

The interaction force \( \mathbf{F}^{int}_i \) acting on solid particle \( i \) can consist from several individual solid-fluid forces [16]. Currently the drag force \( \mathbf{F}^D_i \) and the pressure gradient force \( \mathbf{F}^P_i \) are considered as the dominant interaction forces:

\[ \mathbf{F}^{int}_i = \mathbf{F}^D_i + \mathbf{F}^P_i. \quad (7) \]

Various models are available for the calculation of the drag force. In the current work the correlation proposed by Di Felice [17] is used:

\[ \mathbf{F}^D_i = \frac{1}{8} C_d \rho_f \pi d_i^2 |\mathbf{u}_{i,f} - \mathbf{v}_i| |\mathbf{u}_{i,f} - \mathbf{v}_i| \epsilon_i^{2-\chi}, \quad (8) \]

where \( \epsilon_i, C_d, d_i, \mathbf{u}_{i,f}, \mathbf{v}_i \) are the fluid fraction at solid particle \( i \) location calculated by Eq. 4, the drag coefficient, the solid particle diameter, the fluid velocity and the solid particle velocity correspondingly.
Figure 1. Particles in a rectangular container at different time instances (case S1): a) $t = 0 \text{ s}$, b) $t = 0.3 \text{ s}$ a) $t = 1 \text{ s}$

Figure 2. Evolution of averaged vertical particle coordinates depending on their densities during the separation process

**NUMERICAL ANALYSIS OF PARTICLE SEPARATION**

Wet separation of spherical particles with different densities is analyzed numerically. As a first step for the analyses of a wet separation problem, a set of generated particles with two different densities is dropped into a rectangular container filled with liquid. Depending on the density of the particles in comparison with the liquid density, they settle down or float. This process is simulated using DEM for the solid particles coupled with SPH for the fluid. Two simulations of the system are performed. In the first case (denoted as S1) particles with the densities $\rho_1 = 800 \text{ kg/m}^3$ and $\rho_2 = 1200 \text{ kg/m}^3$ are used. For the second case (denoted as S2), particles with the densities $\rho_1 = 900 \text{ kg/m}^3$ and $\rho_2 = 1100 \text{ kg/m}^3$ are used. In both cases the 1000 monosized particles (500 with $\rho_1$ and 500 with $\rho_2$) with diameter $d = 4 \text{ mm}$ are generated above the free surface of the liquid. Physical properties of water ($\rho_f = 1000 \text{ kg/m}^3$, $\mu = 0.001$) are used for the liquid.

The process of particle separation is presented in Fig. 1, where the particle positions at different time instances are shown. As can be seen, the particles with a higher density are settling to the bottom of the container, while the particles with a lower density than the density of the liquid are floating.

The evolution of averaged vertical coordinates of the particles is presented in Fig. 2. When the particles are dropped, both kinds of particles initially submerged and then start to separate. As can be expected, the particles in simulation S2 take longer to fully separate.

The presented simulations of particle separation in the rectangular container are used as an initial step for more sophisticated simulations of wet particle separation in a rotating cylindrical container. The initial result of a simulation
of such a system is shown in Fig. 3. Here, the mixture of the particles is dropped into the liquid. The sedimented particles are lifted from the bottom of the cylinder with the help of short lifters attached to the wall and are removed later.

CONCLUDING REMARKS

The presented work shows our efforts aiming to the numerical analysis of the wet separation of particles with different densities. DEM was used for the simulation of the solid particles, while SPH was used for the simulation of the liquid. Some details of the used coupled DEM-SPH method were presented. Simulations of particle separation were performed and results were presented. It can be concluded, that the used coupled DEM-SPH method is capable to simulate wet particle separation. The performed simulations showed how the numerical scheme is able to capture the dependency of the separation process on the different particle densities.

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