Ultrasensitive Refractive index detection with rotatory biased weak measurement

Chongqi Zhou and Zhangyan Li
Department of Physics, Tsinghua University, Beijing 100084, China and
Institute of Optical Imaging and Sensing, Shenzhen Key Laboratory for Minimal Invasive Medical Technologies,
Shenzhen International Graduate School, Tsinghua University, Shenzhen 518055, China

Yang Xu, Xiaonan Zhang, Tian Guan, and Yonghong He
Institute of Optical Imaging and Sensing, Shenzhen Key Laboratory for Minimal Invasive Medical Technologies,
Shenzhen International Graduate School, Tsinghua University, Shenzhen 518055, China

Yanhong Ji∗
School of Physics and Telecommunication Engineering,
South China Normal University, Guangzhou 510006, China

In this paper, we present an ultrasensitive reflection-type optical refractive index (RI) sensor with rotatory biased weak measurement. In our scheme, based on the optical dispersion effect, a bias phase is introduced by the postselection process. The bias phase without dispersion effect makes stronger postselection and smaller coupling strength accessible. Experimentally, our scheme obtains an enhanced sensitivity of 13605 nm/RIU while detecting fewer photons, which can effectively alleviate the saturation effect of the detector. We demonstrate a RI resolution of $1.8 \times 10^{-7}$ RIU. With response model and noise analysis of the CCD detector, we show that the performance of the RI sensor depends mainly on the long-time correlated noise of the light source and detector, and prove that our scheme gains advantages due to its high sensitivity. An ultimate resolution of $10^{-9}$ RIU can be attained for our system.

I. INTRODUCTION

Optical detection is widely used in the biological, chemical and food industries as RI sensors with the advantages of simple structure, non-marking, and high sensitivity [1–7]. Conventional optical sensors include total internal reflection (TIR) [1], surface plasmon resonance (SPR) [2], fiber interferometers [3] and metal-cladding waveguide [4]. Especially, reflective surface RI sensors, such as TIR and SPR, have attracted much attention due to their wide application scenarios. The conventional TIR has a simple configuration and can be integrated into various scenarios with a limited resolution of $10^{-6}$ RIU. The coupling between light and surface plasmon enables SPR to obtain a high RI sensitivity. Lots of methods of modulation are applied for SPR sensors, like amplitude, wavelength, and angle. Among them, wavelength interrogation SPR achieves the highest resolution of $2.5 \times 10^{-8}$ RIU so far [5]. Despite its high resolution, the SPR sensor generally requires a gold coating on the reflective interface, which greatly limits its promotions in practical applications.

For optical RI sensors with no fluorescent labeling required, there are many discussions about their limits. Generally, several parameters are utilized to characterize their performances [2]: RI sensitivity, figure of merit (FoM), and RI resolution. Work [2, 10] has shown that the resolution of RI sensors depends mainly on the noise properties of the light source and detector, like intensity noise and phase noise. So further improvement of RI resolution requires a light source with higher signal-to-noise (SNR). As the detector noise is limited by the shot noise corresponding to the number of detected photons, a light source with higher luminance would be helpful. However, it can also lead to the saturation of a commercial detector, and hinder its performance. On the other hand, RI resolution can be improved with higher sensitivity and narrower resonance. It can be achieved by a thin symmetric metal waveguide (a long-range surface plasmon [6]), a circular symmetric coating in the fiber taper [11] or the liquid core optical ring resonator [12]. But they all require complicated fabrications.

Weak value amplification (WVA) can greatly improve the sensitivity of the system by introducing the postselection process [13, 14]. A lot of experiments and theoretical work have verified the feasibility of WVA [13, 15]. Based on WVA technology, ultrasensitive measurements of tiny physical parameters have been experimentally achieved, like time delay [16], angular velocity [17] and optical nonlinearity [18]. With the properly chosen postselection, we can observe destructive interference in both frequency and time domains. It has been proposed that a reflection-type refractive index sensor with standard biased weak measurement (BWM) is achieved [22]. It is based on the TIR configuration and doesn’t require metal coating between the prism and the solution. Limited sensitivity of 1644 nm/RIU and resolution of $10^{-6}$ RIU have been achieved so far. For the standard biased weak measurement scheme, the relative phase difference between two orthogonal components of the pointer is set as $m \pi$ (m is an integer) [23], which sets a strict limitation on the performance of RI sensors.

In this work, we propose an ultrasensitive reflection-type RI sensor with rotatory biased weak measurement [24]. We break the limits on the sensitivity...
via the optical rotatory dispersion effect [25]. With noise analysis, we prove that our scheme can alleviate the saturation effect of the detector. The long-time correlated noise of the light source limits further improvements on the RI resolution of sensors. Rotatory biased weak measurement outperforms standard biased weak measurement in this case. We experimentally demonstrate an enhanced RI sensitivity of 13605 nm/RIU and a higher RI resolution of 1.8 × 10−7 RIU.

II. METHODS AND RESULTS

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{setup.png}
\caption{Experimental setup. P1 and P2, polarizers. FR, Fresnel rhomb retarder. QR, quartz rotator. The prism is fixed to the frame, and a flow cell is attached to the prism. The pre-selection process is accompanied by polarizer P1, prism, and FR; weak interaction and postselection process are realized by QR and P2, respectively. The output spectrum would be analyzed by a spectrograph (OCEANVIEW, HR4000, Shanghai, China), which is omitted here.}
\end{figure}

The experimental setup is depicted in Fig. 1. The broadband light source used here is a superluminescent laser diode (SLD, IPSDD0804, 5 mW, Inphenix). The beam transmitted through the optical fiber is collimated by the achromatic lens and preselected by polarizer P1 (Thorlabs Inc., 180 LPVIS050-MP, extinction ratio of 100 000:1). Then the beam is totally reflected on the inner surface of the ZF6 prism, resulting in a phase difference $\varphi$ between $s$ and $p$ polarizations. To transform the phase difference between $s$ and $p$ polarizations into the optical rotation, a Fresnel rhomb retarder FR (Thorlabs Inc., FR600Q) is placed after the prism. The rotation angle is proportional to the phase difference $\varphi$. After that, the beam passes through quartz crystal along the optical axis, which finally is postselected by the second polarizer P2. A spectrograph (OCEANVIEW, HR4000, Shanghai, China) is used for spectral analysis.

Normally, WVA technology consists of three parts: preselection, weak coupling, and postselection. The system, whose observable $A$ is to be measured, is coupled to the meter $\hat{\omega}$ of the measuring device by the quartz rotator with a Hamiltonian

$$H = \tau \delta(t-t_0) \hat{A} \hat{\omega},$$

where $\hat{\omega}$ is the angular frequency operator and $\delta(t-t_0)$ is Dirac delta function with $\int \tau \delta(t-t_0) \, dt = \tau$. Here real number $\tau$ represents the coupling strength of interaction. $A = | \sigma \rangle \langle \sigma | - | - \sigma \rangle \langle - \sigma |$, in which $| \sigma \rangle$ and $| - \sigma \rangle$ represent right-handed and left-handed circular polarizations, respectively. In this scheme, the wavelength spectrum of light is treated as the meter. The initial state $| \Phi \rangle$ of the measuring device can be expanded in terms of the eigenstates of $\omega$

$$| \Phi \rangle = \int f(\lambda)|\lambda\rangle d\lambda,$$

where $| \omega |\lambda\rangle = 2\pi/|\lambda |\lambda |$ with $c = 1$. We assume that wave function $f(\lambda) = (\sqrt{\pi} \sigma_0)^{-1/2} \exp\left[\frac{-(\lambda - \lambda_0)^2}{2\sigma_0^2}\right]$ satisfies normal distribution with center wavelength $\lambda_0$ and variance $\sigma_0$. In the preselection process, the system is prepared in the state $| \psi_{\text{pre}} \rangle$. Here, we regard the interaction between the light beam and the first polarizer, reflecting prism, and Fresnel retarder as preselection. The transmission axis of the first polarizer and the fast axis of the phase retarder are 45 degrees from the vertical direction. As the Fresnel rhomb retarder provides a uniform $\lambda/4$ retardance over a wide range of wavelengths, the preselection state can be expanded in terms of the eigenstates of $A$

$$| \psi_{\text{pre}} \rangle = \frac{1}{\sqrt{2}} \left( e^{i(\frac{\lambda_0 \varphi}{\lambda} - \frac{\lambda \varphi}{4\lambda_0})} | \sigma \rangle + e^{-i(\frac{\lambda_0 \varphi}{\lambda} - \frac{\lambda \varphi}{4\lambda_0})} | - \sigma \rangle \right).$$

Following the action of the coupling Hamiltonian, the whole system (measuring device and the system) will evolve to ($\hbar = 1$ throughout the paper)

$$\exp \left[ -i \int H \, dt \right] | \psi_{\text{pre}} \rangle | \Phi \rangle \approx \frac{1}{\sqrt{2}} \left( e^{i(\frac{\varphi}{\lambda_0} - \frac{\varphi}{4\lambda_0})} | \sigma \rangle + e^{-i(\frac{\varphi}{\lambda_0} - \frac{\varphi}{4\lambda_0})} | - \sigma \rangle \right).$$
Here Eq. 4 is derived with the approximate condition of \( \sigma_0 \ll \lambda_0 \).

The postselection process is performed with the second linear polarizer. The postselected state is adapted to nearly orthogonal to the preselected state, and it can be described as

\[
|\psi_{\text{post}}\rangle = \frac{1}{\sqrt{2}} (e^{i(\frac{\pi}{2} + \varepsilon)\sigma} |\varnothing\rangle + e^{-i(\frac{\pi}{2} + \varepsilon)\sigma} |\varnothing\rangle),
\]

where \( \varepsilon \ll 1 \). After the postselection procedure, the "measuring device" will be left in the state

\[
|\Phi_f\rangle = \langle \psi_{\text{post}} \rangle \exp \left[ -i \int \hat{H} \, dt \right] |\psi_{\text{pre}}\rangle |\Phi\rangle
= \left( \frac{1}{\sigma_0 \sqrt{\pi}} \right)^{1/2} \sin \left( \frac{2\pi \tau \lambda}{\lambda_0} - \frac{4\pi \tau}{\lambda_0} + \frac{\varphi}{2} - \varepsilon \right) e^{-i(\lambda - \lambda_0)^2 / 2\sigma_0^2}.
\]

And the probability distribution is

\[
p(\lambda) = |\delta(\lambda)|^2
= \frac{1}{\sigma_0 \sqrt{\pi}} \sin^2 \left( \frac{2\pi \tau \lambda}{\lambda_0} - \frac{4\pi \tau}{\lambda_0} + \frac{\varphi}{2} - \varepsilon \right) e^{-i(\lambda - \lambda_0)^2 / 2\sigma_0^2}.
\]

We employ centroid estimation to calculate the spectrum shift \( \delta \lambda \) caused by the phase difference \( \varphi \)

\[
\delta \lambda = \int p(\lambda) \lambda d\lambda - \lambda_0
= \int \frac{2\pi \tau \sigma_0^2 e^{-4\pi^2 \tau^2 \sigma_0^2} \sin^2 \left( \frac{2\pi \tau \lambda}{\lambda_0} + \frac{4\pi \tau}{\lambda_0} + \varepsilon - \frac{\varphi}{2} \right)}{1 - e^{-4\pi^2 \tau^2 \sigma_0^2} \cos^2 \left( \frac{2\pi \tau \lambda}{\lambda_0} + \frac{4\pi \tau}{\lambda_0} + \varepsilon - \frac{\varphi}{2} \right)} d\lambda
\approx -2\lambda_0 + \frac{(2\varepsilon - \varphi) \lambda_0^2}{2\pi \tau}, \quad |\frac{2\pi \tau \sigma_0^2}{\lambda_0^2}| \ll 2\frac{\pi^2 \lambda_0^2}{\lambda_0^2}.
\]

FIG. 2. Phase sensitivity \( S_p(\tau, \varepsilon) \) with respect to \( \tau, \varepsilon \) for RBWM scheme. We choose the RWVA regime as our working regime, in which we obtain the highest phase response.

In WM scheme, we will consider two specific parameter regimes: (1) the weak-value regime (SWVA)

\[
\frac{2\pi \tau \sigma_0^2}{\lambda_0^2} + \varepsilon - \frac{\varphi}{2} > \frac{2\pi \tau \sigma_0^2}{\lambda_0^2}, \quad (2) \text{the reverse weak-value regime (RWVA)} \quad \frac{2\pi \tau \sigma_0^2}{\lambda_0^2} + \varepsilon - \frac{\varphi}{2} < \frac{2\pi \tau \sigma_0^2}{\lambda_0^2}.
\]

These regimes are illustrated in Fig. 2. And we plot the phase sensitivity \( S_p(\tau, \varepsilon) = \frac{\partial \lambda}{\partial \varphi} \) with respect to \( \tau, \varepsilon \). It is shown that when \( |\frac{2\pi \tau \sigma_0^2}{\lambda_0^2} + \varepsilon - \frac{\varphi}{2}| < \frac{2\pi \tau \sigma_0^2}{\lambda_0^2} \), a much higher sensitivity is available for our system. And we choose it as our working regime. For standard biased weak measurement scheme, the spectral probability distribution can generally be written as \( p(\lambda) = \sin^2(\frac{2\pi \tau \lambda}{\lambda_0} + \varepsilon) |f(\lambda)|^2 \). Specially, the phase difference between two components of system pointer satisfies \( \frac{2\pi \tau \lambda_0^2}{\lambda_0^2} + \varepsilon \approx n \pi \tau R \) in our chosen regime. It imposes a rigorous restriction on the value of \( \tau \) when \( \varepsilon \) is generally much smaller than 1: \( \tau \approx \frac{\lambda_0}{2} \). By contrast, by introducing an bias phase \( \varphi \), rotatory biased weak measurement makes arbitrarily small \( \tau \) available. As shown in Fig. 2 there is always a \( \varepsilon \) satisfying \( \frac{2\pi \tau \lambda_0^2}{\lambda_0^2} + \varepsilon - \frac{\varphi}{2} \approx 0 \) when \( \tau \ll \frac{\lambda_0}{2} \). Similar to Eq. 5, the spectrum shift \( \delta \lambda \) for BWM is amplified with the factor of \( 1/\tau \). So the spectrum shift in rotatory biased weak measurement scheme can be much larger than that in standard biased weak measurement.

Here we demonstrate a rotatory biased weak measurement system with a 1-mm-thickness quartz crystal, in which the coupling strength \( \tau \) is determined as 0.06\( \lambda_0 \). The spectral probability distribution of light source is measured to be a superposition of two Gaussian distribution (not normalized) \( |f(\lambda)|^2 = e^{-(\frac{\lambda - \lambda_1}{\sigma_1})^2 + 1.035 \times e^{-(\frac{\lambda - \lambda_2}{\sigma_2})^2}} \), in which \( \lambda_1 = 821.1 \, \text{nm}, \, \sigma_1 = 7.55 \, \text{nm}, \, \lambda_2 = 845.8 \, \text{nm} \) and \( \sigma_2 = 19.58 \, \text{nm} \). A RI-dependent phase difference \( \varphi \) between \( p \) and \( s \) polarizations is added when the beam is reflected on the prism. According to Fresnel’s formula, we have

\[
\varphi = 2\tan^{-1} \sqrt{\frac{n_1^2 \sin^2 \theta - n_2^2}{n_1 \sin \theta \tan \theta}},
\]

where \( \theta \) is the incident angle. \( n_1 \) and \( n_2 \) are refractive indices of the prism and analytes, respectively.

In our system, the refractive indices of the ZF6 prism and solution are 1.75 and 1.33, respectively. The RI sensitivity \( S_{RI} \) could be written as \( S_{RI} = \frac{\partial \lambda}{\partial \delta n} \). In Fig. 3 (a), we characterize the performances of our sensor. In Fig. 3 (a), we measure the spectrum shift against the postselection angle \( \varepsilon \) by rotating the latter polarizer \( P2 \) in the chosen regime. Meanwhile, the theoretical results calculated from Eq. 7 are also plotted, which shows great correspondence with the experimental results. In our theoretical model, we ignore the small refractive index changes of prism and retarder varying with wavelength, which can result in a stronger coupling and a bigger \( \tau \). And the flatter experimental curve may result from the unavoidable dispersion effects of prism and retarder. A phase sensitivity \( \frac{\partial \lambda}{\partial \delta n} \) of 4478 nm/\text{rad} is achieved in our scheme, which is 9 times higher than that in previous work \( 22 \) .

Here, we focus on the RI sensitivity \( S_{RI} \) in our system. According to Eq. 9, \( \frac{\delta \lambda}{\delta n} \) is related to the incident angle

\[
\frac{\delta \lambda}{\delta n} = \frac{2\pi \tau \sigma_0^2}{\lambda_0^2} + \varepsilon - \frac{\varphi}{2} > \frac{2\pi \tau \sigma_0^2}{\lambda_0^2}, \quad (2) \text{the reverse weak-value regime (RWVA)} \quad \frac{2\pi \tau \sigma_0^2}{\lambda_0^2} + \varepsilon - \frac{\varphi}{2} < \frac{2\pi \tau \sigma_0^2}{\lambda_0^2}.
\]
\( \theta \). The critical angle is measured to be 49.6°. Here, we carry out a series of experiments to determine the precision of our system at three different angles, which is measured to be 50.8°, 51.5°, and 54.0°. Sodium chloride solutions of different concentrations are injected into the flow cell on the surface of the prism, and we measure the corresponding spectrum shift \( \delta \lambda \). The spectra are collected once per second with an integration time of 2 ms. The relationship between the refractive index \( n \) and concentration \( C \) of sodium chloride solution is given by \( n = 1.33 + 1.47 \times 10^{-3}C \), in which \( C \) is mass percent. As shown in Fig. 3(b), we obtain RI sensitivities of 13605 nm/RIU, 10799 nm/RIU and 4251 nm/RIU for incident angles of 50.8°, 51.5° and 54.0°, respectively. By adjusting the incident angle, our system has achieved various RI sensitivities with an identical coupling strength. Currently, a 9 times higher sensitivity of 13605 nm/RIU has been obtained in our system, and the variance of spectrum shift \( \Delta \lambda \) is measured to be 0.0024 nm. Here, we demonstrate a higher RI resolution of \( 1.8 \times 10^{-7} \) RIU.

### III. DISCUSSION

Compared to the standard biased weak measurement scheme, the rotatory biased weak measurement scheme provides an enhanced sensitivity, and allows our sensor to obtain more metrological information with fewer photons. These two characteristics endow us with the ability to attain better precision. Compared to conventional measurement, The postselection process in weak measurement results in a more distinct spectrum shift. However, this postselection is not arbitrarily strong, and only alleviates the detector’s saturation effect to a limited extent \[26\]. By contrast, rotatory biased weak measurement can attain further reduction of detection probability by introducing the optical rotatory dispersion effect, resulting in a better precision with fewer photons. Therefore, rotatory biased weak measurement can effectively eliminate the saturation effect. Besides, it is proposed that the ultimate performance of the RI sensor is limited by the noise of the light source. In this situation, a larger factor of magnification enables rotatory biased weak measurement to attain a much higher SNR.

According to the Cramér-Rao bound, the best precision of estimating RI \( n_{RI} \) from \( \nu \) times of repetitive measurement is given by

\[
\delta^2 n_{RI} = \frac{1}{\nu F},
\]

where \( \delta^2 n_{RI} \) is the variance of the estimator of \( n_{RI} \) and \( F \) is the Fisher information (FI) \[26\,27\]. The value of FI can be obtained from the probability distribution of the detector. The detector we used in the spectrometer is a 3648-element CCD (Toshiba, TCD 1304AP). When the total number of photons is \( \bar{n}_t \), the number \( N_j \) of photoelectrons at the \( j \)th pixel on the CCD follows Poisson distribution \( p(N_j|\bar{n}_t) = \exp(-\bar{n}_t)\bar{n}_t^{N_j}/N_j! \), where \( \bar{n} \) is the detection efficiency and \( \bar{n}_t(n_{RI}) = \int \bar{n}_t p(\lambda) d\lambda \), where \( p(\lambda) \) can be obtained from Eq. \[7\].

We have considered three effects in our response model of CCD. Firstly, we measure the dark noise of CCD by taking 300 frames without any incident light. The distribution of dark noise satisfies the normal distribution \( p_d(k_d) = N(\mu_d, \sigma_d^2) \), in which \( \mu_d=1040 \), \( \sigma_d=4.5 \). The second effect is the classical noise, which follows the normal distribution \( p_c(k_c) = N(0, \sigma_c^2) \), in which \( \ln \sigma_c = 0.46\ln N_f - 1.58 \). Thus, the conditional probability distribution of readout \( k_j \) is given by the convolution of...
these two effects \( R(k_j, N_j) = \sum_{k_d} p_4(k_d) p_c(k_j - k_d - N_j) \). The third effect we must consider is the detector’s saturation effect with saturation threshold \( k_s = 16384 \). Therefore, the response model is written as

\[
R_s(k_j, N_j) = \begin{cases} 
R(k_j, N_j), & k_j < 16384 \\
1 - \sum_{k_j < 16384} R(k_j, N_j), & k_j = 16384 \\
0, & k_j > 16384.
\end{cases}
\] (11)

And the readout \( k_j \) of the \( j \)th pixel can be calculated by

\[
p(k_j, n_{RI}) = \sum_{N_j} R_s(k_j, N_j) p(N_j | \eta n_j(n_{RI})).
\] (12)

With the probability distributions of the readout on every pixel, the value of FI can be calculated by summarizing each pixel’s FI

\[
F(n_{RI}) = \sum_j \sum_k \frac{1}{p(k_j, n_{RI})} \left( \frac{\partial p(k_j, n_{RI})}{\partial n_{RI}} \right)^2
\]

\[
= \sum_j \sum_k \sum_{N_j} R_s(k_j, N_j) p(N_j | \eta n_j) \left[ \sum_{N_j} R_s(k_j, N_j) \frac{-e^{-\eta n_j} \frac{\partial \eta n_j}{\partial n_{RI}} (\eta n_j)^{N_j}}{N_j!} + e^{-\eta n_j} (\eta n_j)^{N_j} - 1 \frac{\partial \eta n_j}{\partial n_{RI}} \right]^{2}
\]

\[
= \sum_j \frac{\eta}{\eta n_j} \left( \frac{\partial \eta n_j}{\partial n_{RI}} \right)^2 \sum_{N_j} R_s(k_j, N_j) p(N_j | \eta n_j) (\eta n_j - \eta \bar{n}_j)^2
\]

\[
= \sum_j \frac{\eta}{\eta n_j} \left( \frac{\partial \eta n_j}{\partial n_{RI}} \right)^2 \frac{\partial \phi}{\partial n_{RI}} \left( \Gamma(R_s, \bar{n}_j), \right)
\] (13)

In Fig. 4(a), we calibrate the gamma coefficient \( \Gamma(R_s, \bar{n}_j) \) for different \( \bar{n}_j \) according to our response model of CCD. When \( n < 1000 \), the value of \( \Gamma(R_s, \bar{n}_j) \) grows rapidly as the signal begins to suppress the dark noise. When \( n > 15000 \), the saturation effect leads to the decrease of \( \Gamma(R_s, \bar{n}_j) \).

Several experimental parameters must be determined to calculate FI. Here, coupling strengths are set as \( \tau = 0.52\lambda_0 \) for standard biased weak measurement and \( \tau = 0.27\lambda_0, 0.08\lambda_0, 0.04\lambda_0 \) for rotatory biased weak measurement, respectively. The incident angle is chosen as 50.8°, and the relationship between \( \varphi \) and \( n_{RI} \) can be obtained from Fig. 3 \( \varphi = 3.03n_{RI} + b \), in which \( b \) is a constant. By substituting it into Eq. 13, we plot calculated precision with respect to the total incident photons \( \bar{n}_t \). As shown in Fig. 4(b), the precision of both schemes improves with growing \( n \). When \( n \approx 1.7 \times 10^{10} \), the precision of the standard biased weak measurement scheme firstly reaches its extreme, then the saturation effect begins to hinder its performance. However, the rotatory biased weak measurement scheme can achieve a similar precision while detecting fewer photons, which makes it possible to avoid the saturation effect. By contrast, when \( \tau = 0.04\lambda_0 \), the precision remains enhanced until \( n \approx 1.2 \times 10^{12} \). Consequently, for \( n > 3 \times 10^{10} \), rotatory biased weak measurement achieves a better precision than that of standard biased weak measurement due to the saturation effect. For common SLD with output power in the order of 5 mW, a low-cost detector with a minimum integration time of 10 \( \mu \)s is at the edge of saturation for standard biased weak measurement. A higher-power light source is available up to now.

saturation effect of CCD is becoming an important factor limiting system performance. Thus, rotatory biased weak measurement turns to be a prospective solution for higher-power light sources.

Another common method to improve the performance of the system is temporal averaging. As indicated in Eq. 10, the precision is improved with the factor of \( 1/\sqrt{\nu} \), where \( \nu \) is the number of averaging time. However, if the ultimate performance of sensors is limited by the noise of the light source, it can’t be improved through temporal averaging. In this situation, rotatory biased weak measurement shows great advantages. In our system, the measurement is performed by a linear CCD. Light source noise affects all pixels in the same way, which produces a noise correlated in the spatial domain. Therefore, the readout \( k_j \) in the \( j \)th pixel is written as

\[
k_j(t) = \bar{n}_j + c_j(t) + s_j(t),
\] (14)

where \( \bar{n}_j \) is the mean of intensity, \( c_j(t) \) is the correlated noise and \( s_j(t) \) is the stochastic component of the noise. The measured spectrum shift is

\[
\delta \lambda(t) = \frac{\sum_{j}^{N} k_j(t) \lambda_j}{\sum_{j}^{N} k_j(t)} - \lambda_0
\]

\[
\approx \frac{\sum_{j}^{N} \bar{n}_j \lambda_j}{\sum_{j}^{N} \bar{n}_j} - \lambda_0 + \sum_{i}^{N} \frac{\sum_{j}^{N} \bar{n}_j (\lambda_i - \lambda_j)}{(\sum_{j}^{N} \bar{n}_j)^2} (c_i(t) + s_i(t))
\]

\[
= \bar{\lambda} + \sum_{j} a_j (c_j(t) + s_j(t))
\]

\[
= \bar{\lambda} + c(t) + s(t),
\] (15)
FIG. 4. Simulation results. (a) $\Gamma(R_s, \bar{n}_j)$ as a function of the average number of incident photons $\bar{n}_j$. The saturation effect leads to the decrease of $\Gamma$. (b) The precisions of RI for standard biased weak measurement and rotatory biased weak measurement schemes are obtained by varying the incident photon number $n_i$ respectively. Compared to standard biased weak measurement, rotatory biased weak measurement can mitigate the saturation effect and have the ability to achieve better precision.

FIG. 5. Correlated noise of the light source. (a) and (b) give the distribution of the correlated noise $\langle c_i c_j \rangle$ between channel $i$ and $j$ for $\nu = 20$ and $\nu = 80$, respectively. As $\nu$ increases from 20 to 80, the correlated noise remains constant. (c) We calculate the system noise against $\nu$. Through our calibrated response model of CCD, we distinguish the correlated components and stochastic components in the system noise. Results show that the system noise is mainly determined by the long-time correlated noise when $\nu > 40$. 
where $\lambda = \frac{\sum n_i(t)\lambda_i}{\sum n_i(t)} - \lambda_0$, $c(t) = \sum_i \frac{\sum n_i(t)\lambda_i}{\sum n_i(t)} c_i(t)$ is the correlated component and $s(t) = \sum_i \frac{\sum n_i(t)\lambda_i}{\sum n_i(t)} s_i(t)$ is the stochastic component of the noise. The uncertainty in this measured shift is given by $(\Delta \lambda)^2 = \sum_{ij}^N a_{ji} (\langle c_i(t) + s_i(t) \rangle (c_j(t) + s_j(t)))$, in which covariance $\langle ab \rangle = \text{cov}(a, b)$. In temporal averaging, a time sequence of $\nu$ times of measurement is averaged. For stochastic noise, the averaging reduce the noise $(s_i(t)s_j(t))$ as follows: $\langle s_i(t)s_j(t) \rangle \sum_\nu = \langle s_i(t)s_j(t) \rangle$. However, the correlated portion of noise $(\langle c_i(t)c_j(t) \rangle)$ is not affected by averaging. We characterize the correlation with the Pearson correlation coefficient $\rho = \frac{\sigma^2_{\rho}}{\sigma^2_{c} + \sigma^2_{s}}$, where $\sigma_{c}$ and $\sigma_{s}$ are standard deviations of $c(t)$ and $s(t)$, respectively. Consider the postselected case, where the signal $\varphi/\tau$ is amplified with the factor of $\tau$ and the postselected probability $N\tau^2$ is decreased by a factor of $\tau^2$. So the SNR scales like

$$\frac{\sqrt{\sigma^2_{c} + \sigma^2_{s}}}{\varphi/\tau} + \frac{\sigma_{s}}{\varphi} = \frac{\sigma_{s}}{\varphi} \sqrt{1 - \rho^2} + \tau^2 \rho$$

(16)

When $\rho = 0$, rotatory biased weak measurement shows no advantage when compared to standard biased weak measurement. However, for highly correlated noise which $\rho \approx 1$, the SNR shows an advantage over the standard biased weak measurement.

Here, we collect the spectrum of light source once a second and take 9600 times in total. We employ the readouts of 1500 pixels and divide them into 150 channels. The stochastic noise can be calibrated and distinguished with our response model of CCD. The correlated components of $\langle c_i c_j \rangle$ between channels for $\nu = 20$ and $\nu = 80$ are shown in Fig. 5(a) and (b), respectively. Unlike random noise, which decreases with temporal averaging, it is obvious that the correlated noise is barely affected by averaging. Therefore, as shown in Fig. 5(c), the remaining noise is mainly long-time correlated noise when $\nu > 40$. When $\nu = 100$, the Pearson correlation coefficient $\rho$ is 0.95. According to Eq. 16, temporal averaging won’t reduce the system noise. In this situation, rotatory biased weak measurement can further enhance the precision of the RI sensor. The RI resolution can be written as $\Delta n_{RI} = \frac{\Delta \lambda^2_{\rho}}{s_{RI}}$. For the BWM scheme, the RI sensitivity $S_{RI}$ is amplified with the factor of $1/\tau$. While the standard biased weak measurement is incapable of higher sensitivity, rotatory biased weak measurement provides a powerful solution to further promotion. When $\nu > 100$, the variance of spectrum shift is almost constant $\sigma_{s} = 0.00085$ nm. So an ultimate performance of $6.3 \times 10^{-8}$ RIU is expected for our scheme.

In Fig. 6 we summarize various representative RI sensors with the performances of RI sensitivity and RI resolution. The sensors under the same category are marked with the same color. The performance map in Fig. 6 has exhibited an interesting point. Despite the categories, different definitions of RI resolution (standard deviation, limit of detection, and resolution of spectrometer), and data process (temporal averaging), the RI resolution is generally improved with the RI sensitivity. Among them, the postselection process endows our scheme the ability to outperform most RI sensors even without complicated fabrication.

**IV. CONCLUSION**

Although some work [24, 28] has claimed that an ultrasensitive system can be achieved in their schemes theoretically, it is unrealistic to be applied in practice. In these proposals, the bias phase is introduced by a precoupling process, which consists of a series of wave plates. Generally, those wave plates will introduce additional dispersion effects, which are unavoidable and conspicuous. And it makes a small coupling strength $\tau \ll \lambda_0$ impossible to be attained in these schemes. By contrast, in our scheme, the bias phase is introduced by the polarizer. So the dispersion effect is eliminated and an ultrasensitive sensor is experimentally demonstrated.

In this work, we have discussed some common methods to improve the performance of RI sensors, like a light source with high luminance and higher sensitivity. To demonstrate the advantages of our proposed scheme, we calibrate the response model of the CCD, and prove that the correlated noise of the light source is the main factor that limits the performance of the system. Hence, our scheme can outperform previous schemes. Our proposed method is applicable to various optical measurements as the saturation effect and correlated noise can occur in many sensors. Our work also provides a pragmatic method to explore the ultimate performance of sensors.
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