IS B1422+231 A "GOLDEN LENS"?

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ABSTRACT

The object B1422+231 is a quadruply imaged QSO with an exceptionally large lensing contribution from
group galaxies other than main lensing galaxy. We have detected diffuse X-rays from the galaxy group in
archival Chandra observations; the inferred temperature is consistent with the published velocity dispersion.
We explore the range of possible mass maps that would be consistent with the observed image positions,
radio fluxes, and ellipticities. Under plausible but not very restrictive assumptions about the lensing galaxy,
predicted time delays involving the faint fourth image are fairly well constrained around 7 $h^{-1}$ days.

Key words: gravitational lensing — quasars: individual (B1422+231)

1. INTRODUCTION

One of the most attractive things about lensed quasars is
the possibility of measuring the distance scale without any
local calibrators. If the source is variable and the variation
is observed in different images with time delays, then a
formula of the type

$$\text{time delay} = h^{-1} z_L (1 + z_L) \times (1 \text{ month}) \times \text{image separation in arcsec}^2 \times \text{(lens-profile-dependent factor)}$$

(where $z_L$ is the lens redshift) applies. This formula depends
weakly on the source redshift (the time delay gets somewhat
shorter if the source is not very much farther than the lens)
and on cosmology (for the same $h$, an Einstein–de Sitter
cosmology gives long time delays and an open universe
gives short time delays, with the currently favored flat Λ-
cosmology being intermediate), but these dependencies are
at the 10% level or less. The troublesome term is the lens-
profile-dependent factor, which is of order unity but for a
given lens can be uncertain by a factor of 2 or more. So,
while the basic theory is well established (Refsdal 1964),
measuring $h$ from lensing is still problematic. Meanwhile,
the search for well-constrained lenses continues, and lensed-
quasar aficionados speak of them as “golden lenses”
(Williams & Schechter 1997).

In this paper, we study a particularly interesting lens:
B1422+231 was discovered in the Jodrell Bank–VLA
Astrometric Survey (Patnaik et al. 1992) and consists of
four images of a quasar at $z_S = 3.62$ lensed by an ellipti-
cal galaxy at $z_L = 0.334$ (Impey et al. 1996) with several
nearby galaxies at the same redshift (Kundić et al. 1997;
Tonry 1998). It is especially interesting for four reasons.
First, while external shear from other group galaxies is
important in most four-image lenses, unlike other well-
studied quadruple systems, B1422+231 (hereafter B1422)
is dominated by external shear. Second, the lensing-galaxy
group can be detected directly from its X-ray emission.
Third, VLBI imaging of the quasar core (Patnaik et al.
1999) gives information not just on the flux ratios, but
on the tensor magnification ratios. And fourth, a time
delay between two images has been reported (Patnaik &
Narasimha 2001).

2. SUMMARY OF PREVIOUS OBSERVATIONS

2.1. Image Configuration

The quadruple system has three images with $V = 16.5–17$
nearly in a straight line, and a fourth image with $V = 20$ on
the other side of the galaxy (see Fig. 1). The maximum
image separation is 1. The image configuration is well
established (Refsdal 1964), measuring $h$ from lensing is still problematic. Meanwhile,
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minimum estimates of the external shear are $\gamma = 0.67$ and $\gamma = 0.09$. Consistent with these bounds, other workers have fitted models that require $\gamma = 0.25$ (Hogg & Blandford 1994) and $\gamma \sim 0.1$ (Kormann, Schneider, & Bartelmann 1994). Using an elliptical potential model, Witt & Mao (1997) derive a lower limit to the external shear of 0.11. Thus, by all accounts, external shear plays a central role in B1422.

2.3. Magnification

The radio fluxes of the images 1, 2, 3, and 4 are in the ratio $16:30:33:1$ at 8.4 GHz and similar at other radio frequencies (Patnaik et al. 1999; Ros et al. 2001), whereas the flux ratios in the optical and near-infrared are $8:13:16:1$ (Yee & Bechtold 1996; Lawrence et al. 1992). Reddening by the lensing galaxy is unlikely because the flux ratios do not change much between different optical wave bands. Microlensing due to stars (Chang &Refsdal 1979) is also unlikely, because the flux ratio of image 2 to image 3 stayed the same while both images changed in brightness (Narasimha & Srianand 2000). This suggests that microlensing is unlikely because the flux ratios do not change much between different optical wave bands. Microlensing due to stars (Chang &Refsdal 1979) is also unlikely, because the flux ratio of image 2 to image 3 stayed the same while both images changed in brightness (Narasimha & Srianand 2000). 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We remark that since the unlensed flux is unknown, the absolute magnifications are also unknown. This unknown factor is a manifestation of the mass-disk degeneracy (see Saha 2000 and references therein).

3. CHANDRA X-RAY OBSERVATION OF THE GROUP

Redshifts of six galaxies, including that of the lens, indicate that the lens is a member of a compact group whose line-of-sight velocity dispersion is 550 km s\(^{-1}\) and median projected radius is 35 h\(^{-1}\) kpc (Kundic\' et al. 1997). A Faraday rotation corresponding to a rotation measure of 280 \(\pm 20\) rad m\(^{-2}\) (Patnaik et al. 1999) between images 2 and 3 is much larger than expected from an elliptical galaxy and further supports the presence of a group around the lens.

We show here the unambiguous detection of diffuse X-ray emission from hot gas belonging to this group in a Chandra observation. Previously, from a ROSAT HRI observation, Siebert & Brinkmann (1998) had found “no evidence for extended emission” over and above the 5” point-spread function of the HRI. Figure 2 shows the 28 ks Chandra ACIS-S observation represented as contours superposed on the Hubble Space Telescope (HST) image taken with the Near Infrared Camera and Multi-Object Spectrometer (NICMOS; H band) by the CASTLES team (Kochanek et al. 2000), where the four images, as well as the lens galaxy, are distinctly seen. There is clearly an astrometric offset between the HST and Chandra images, which we have made no attempt to correct for. However, we can safely assume that the two peaks in the X-ray distribution correspond to images 2 and 3 (brighter) and 1. The diffuse emission outside the source images is softer and extends to a scale of about 3", which corresponds to about 9 h\(^{-1}\) kpc at the lens redshift \(z_L = 0.334\).

B1422 was observed with the ACIS-S instrument on board the Chandra observatory on 2000 June 1 for 28.8 ks, the data being collected on the back-illuminated S3 CCD. We applied the standard data processing techniques recommended by the Chandra X-Ray Center (CXC), including the recent task ACISABS, to correct for the absorption, predominantly at softer energies, caused by molecular contamination of the ACIS optical blocking filters. Since the exposure time is long enough, we reprocessed the data without randomizing the event positions, which improved the point-spread function. Periods of high background were filtered out, leaving 28.4 ks of usable data.

To separate the spectrum of the source quasar from that of the lensing-galaxy group, we isolated circular areas of

![Fig. 2.—Chandra ACIS-S observation of B1422. The X-ray observation is represented as logarithmic contours superposed on the HST H-band NICMOS image, kindly supplied by the CASTLES team (J200 right ascension and declination). Images 2–3 and 1 are separately detected, and there is a hint of image 4 being detected as well. The pixel size of ACIS-S (0.49") is not enough to resolve images 2 and 3. There is a slight astrometric offset between the HST and Chandra images, which we have made no attempt to correct for.](image-url)
radius $0.7'$ around each of the three QSO images and extracted the spectrum of the quasar. The power-law fit (together with Galactic absorption) to it yielded a power law of slope 1.41 with a reduced $\chi^2$ statistic of 0.9. There was no evidence of extra absorption at the redshift of the quasar. We then fixed the slope of the power law and fitted a model of thermal bremsstrahlung plasma (Raymond-Smith) plus the quasar power-law spectrum to the rest of the X-ray emission, keeping the normalization of both components free but fixing elemental abundance to 0.3 times solar (reasonable for a group or cluster of galaxies; see Fig. 3). A fit to the data in the range 0.5–6 keV yielded a temperature of 0.71 keV for the group of galaxies, which translates into a velocity dispersion range of 150–450 km s$^{-1}$ (Helsdon & Ponman 2000), consistent with the observed dispersion of 550 or 240 km s$^{-1}$ (depending on whether G3 is included or not). The fits were performed with the CXC software package Sherpa.

The $M$-$T_X$ relation (mass vs. X-ray temperature) for virialized isothermal systems of galaxies, for mass within a radius $0.3 R_{\text{vir}}$ gives a mass of $3.6 \times 10^{12} M_\odot$ for the above temperature (Sanderson et al. 2003). The $V$ magnitude of the main lensing galaxy is $m_V = 21.5$ (Kundić et al. 1997), which after correction for color evolution (Bruzual 2003) and k-correction (Frei & Gunn 1994) yields a luminosity of $L_B = 1.9 \times 10^{40} \ h_{75}^2 \ L_{\odot}$ and a mass-to-light ratio of $M/L_B = 190 \ h_{75}$, far larger than that of a single galaxy. The bolometric X-ray flux of the diffuse emission is $L_X = 2 \times 10^{42}$ ergs s$^{-1}$, which means $\log (L_X/L_B) = 32$, also much larger than typical values for galaxies (Helsdon et al. 2001). We therefore conclude that the diffuse emission detected by Chandra belongs to the group of galaxies the lens is a part of.

### 4. A SIMPLIFIED MODEL

Significant external shear, combined with the fortuitous positioning of the QSO source along the same line as the group and the center of the main lensing galaxy (this alignment is a robust feature in all models for this system), transforms the case of images 3 and 4 of B1422 into a one-dimensional problem. As we now show, this makes the problem much better constrained than a generic 4 (+1) image system.

Consider a simplified analytic model of an axisymmetric lens, with the QSO source, lens, group center, and the images all lying on the $x$-axis. Let the mass distribution in the main galaxy lens around the image ring be given by a power law, $\rho \propto x^{-\alpha}$. Its contribution to the lensing potential is $\Phi_{\text{gal}} \propto x^{2-\alpha}$, and the deflection angle is $d\theta/dx = \pm Ax^{1-\alpha}$, where a plus (or minus) stands for the image formed on the positive (or negative) $x$-axis and $A$ lumps together all the constants. The total lensing potential is

$$\Phi = \frac{A}{2-\alpha} x^{2-\alpha} + \frac{\gamma}{2} x^{\gamma},$$

where we have discarded terms that are zero on the $x$-axis. The lens equation is

$$x_I (\gamma - 1) \pm Ax_I^{1-\alpha} + x_S = 0.$$

If we define $f_1 = x_I (\gamma - 1) + x_S$ and $f_2 = \mp Ax_1^{1-\alpha}$ and plot them against lens-plane coordinate $x$, then images are formed at the intersections. The four panels of Figure 4 show models with four different $(\gamma, \alpha)$-combinations: $(0.075, 0.48)$, $(0.2, 0.96)$, $(0.475, 1.36)$, and $(0.725, 1.9)$. All these models satisfy the locations of images 3 and 4 and the condition that image 4 arrive after 3. The vertical offset of $f_1$ is the source location, $x_S$, while external shear makes $f_2$ shallower. The difference in galactocentric distances of images 3 and 4 is given by $|x_3 - x_4|$.

Figure 4 graphically demonstrates that in a system where images are formed at very different galactocentric distances (such as 3 and 4 in B1422), there is a limit to the shallowness of the galaxy’s density profile index $\alpha$. It is clear from the figure that some nonzero $x_S$ is needed to produce large $|x_3 - x_4|$. For shallow density profiles (such as the one in the top left panel), $f_2$ tends to become the diagonal through quadrants II and IV. This makes it very difficult to have $f_2$ intersect $f_1$ while keeping $|x_3 + x_4|$ large. The lower limit on $\alpha$ is around $\sim 0.5$.

The figure also shows that $\alpha$ and $\gamma$ in B1422 are correlated: as profile slopes get steeper, shear must increase as well. For $\alpha > 1$ (profile steeper than isothermal), $f_2$ becomes hyperbolic, and a substantial flattening of $f_1$ (i.e., large external shear) is required to produce large $|x_3 + x_4|$. Note that large shear is necessary for this; a large $x_S$ by itself is not sufficient.

As long as profile is steeper than a minimum $\alpha$-value, the one-dimensional lensing case does not prefer large-$\gamma$, large-$\alpha$ solutions over small $\gamma$ and small $\alpha$: in Figure 4, $(\gamma, \alpha) = (0.075, 0.48)$ is as good a fit as $(0.725, 1.9)$. However, when we consider two dimensions, images 1 and 2 set an upper limit on the external shear because too large a shear will tend to place images 1, 2, and 3 in a straight line, perpendicular to the direction toward the external shear. Since observed images 1, 2, and 3 are not in a straight line, the shear is constrained from above. Furthermore, if the tensor magnification information is included in the modeling (as is done in
We can intuitively understand the effect of external shear on the time delays $\Delta r$ as follows: If the shear source is far from the image region, the shear can be represented by constant external shear, which provides linear deflection angles at the images. It corresponds to tilting the side of the arrival-time surface closest to the source of the shear upward, which moves the zero points in $\Delta r$ away from the shear source. Images lying on the line through the shear source, QSO source, and the main galaxy are most affected by this tilting (images 3 and 4 in B1422), while those lying perpendicular to that line are least affected (images 1 and 2). Thus images 3 and 4 give a well-constrained $\Delta r$, precisely because they are most sensitive to external shear.

To sum up, the fortuitous geometry of B1422 and large external shear imply a relation between profile slope and shear magnitude and impose a limit on the shallowness of the profile slope (through an upper limit on shear). As a result, $\Delta r_{34}$ should be well constrained.

5. PIXELATED MODELS

We now consider more detailed models of galaxy mass distribution in the lens plane. Given the difficulties caused to previous models of B1422 by the observed flux ratios, and especially since we now need to fit tensor magnifications, it is clear that more general models than those are necessary. Moreover, to properly estimate uncertainties, it is necessary to explore not just a few models but large ensembles of them. The pixelated lens reconstruction method, as developed in Saha & Williams (1997) and Williams & Saha (2000), was designed for this purpose. The idea is to model the lens as a sum of mass tiles or pixels, with two kinds of constraints: the “primary constraints” are that the lensing data should be fitted exactly; “secondary constraints” require the mass distribution to be centrally concentrated, $\Delta r_{34}$ should be well constrained. $\Delta r_{34}$ should be well constrained.

Fig. 4.—B1422 simplified: one-dimensional case showing the formation of images 3 and 4. Thick dashed lines represent $f_1 = x_2(\gamma - 1) + x_2$, and thick solid lines represent $f_2 = x_2\alpha$; they intersect at image locations. Four combinations of $\gamma$ and $\alpha$ are shown in the four panels, as labeled. The thin dotted lines show the axes, the diagonals, and the image locations.
not excessively elongated, and optionally inversion-symmetric. Here we follow the method of generating ensembles of models as detailed in the latter paper, but with two minor modifications necessitated by the type of data.

The first modification is needed because the time-delay measurements are currently very uncertain. So, instead of including them we run the reconstruction code with one time delay ($\Delta t_{34}$, say) set to a fiducial value $d_{\text{fid}}$. The code then generates an ensemble of models, each model having its own $h$ and its own set of $\Delta t_{ij}$. Because of the scaling properties of the lensing problem, the only dependence of the results on $d_{\text{fid}}$ will be that all the model $h^{-1}$ and $\Delta t_{ij}$ will be proportional to it. Hence $h \Delta t_{ij}$ will be independent of $d_{\text{fid}}$.

A second modification is needed to incorporate tensor magnifications. To do this, we first reconstruct the magnification matrix $M$ at each image position from the data in Table 1. We have

$$M = R(-\omega) \begin{pmatrix} \sqrt{M/r} & 0 \\ 0 & \sqrt{M/r} \end{pmatrix} R(\omega),$$

where $M$ is the scalar magnification (proportional to the flux), $r$ is the orientation (if we assume the source is circular), $\omega$ is the orientation (position angle $\text{P.A.} + 90^\circ$), and $R$ is a rotation matrix. The signs of the square roots in equation (2) depend on image parity: for minima, both roots will be positive; for saddle points, one will be negative. Having reconstructed $M$ at the image positions, we write the observed upper and lower bounds on the axis ratio as bounds on the ratio of the diagonal elements of $M$ (see AbdelSalam, Saha, & Williams 1998 for an application of this technique to arclets in cluster lensing). Having constrained the axis ratio, we...

Fig. 5.—Ensemble average of 100 mass maps including tensor magnification and assuming inversion symmetry. The pixel size is evident. Contours are in steps of one-third of the critical density for lensing, with the outermost contour at one-third of the critical density. The filled circles indicate the image, and dashed ellipses around them indicate the tensor magnification. Note that we have made the areas of the ellipses proportional to $\sqrt{M}$ rather than $M$.

Fig. 6.—Top, predicted time delays (in the form $h \Delta t$ in days) between images 3 and 4, for the same ensemble as in Fig. 5; other panels, predicted time delays between different images (going downward, 1–2, 2–3, and 3–4) for the same ensemble. Note that the horizontal scale varies between panels.

Fig. 7.—Same as Fig. 5, but without the inversion symmetry constraint.
set the flux ratio by introducing a fictitious quadruple system slightly displaced from the actual one; in particular, fictitious images displaced by

$$\begin{bmatrix}
\Delta \theta_x \\
\Delta \theta_y
\end{bmatrix} = \begin{bmatrix}
M \epsilon \\
0
\end{bmatrix} \quad (3)$$

correspond to a fictitious source displaced by $\epsilon$ along the $x$-axis of the source plane. The code is not told $\epsilon$ itself, just the displacements it maps to; in this way the magnification ratios are constrained but not the absolute magnifications.

We have generated three ensembles of models. Each ensemble contains 100 models sampling the allowed "model space" given a certain set of constraints.

The first ensemble uses the data from Table 1, together with the generic secondary constraints (Williams & Saha 2000), including inversion symmetry of the galaxy lens. Figure 5 shows the ensemble-average mass map, while Figure 6 shows the distribution of predicted time delays. (The "radial profile index" in Fig. 6 corresponds approximately to $-\alpha$ in the previous section.) The qualitative results are just as expected from our preceding discussion: (1) images 1, 2, and 3 are very close in $D_1$, with 4 arriving much later; (2) $\alpha$ is nearly confined to a narrow range, 0.6–0.8; and (3) while $\Delta t_{34}$ has a broad peak with median 7.3 $h^{-1}$ days, it is much better constrained than in models of other well-studied quadruple systems (Williams & Saha 2000).

For the second ensemble, we dropped the inversion symmetry constraint. Figures 7 and 8 show the results. Qualitatively, the results are similar, but the predicted $\Delta t_{34}$ shifts to a higher range (median 12.4 $h^{-1}$ days). One must keep in mind that observed image properties tightly constrain the position angle of the shear, not its radial location. So, non-inversion-symmetric modeling of lens systems with large external shear will produce mass profiles that are artificially elongated toward the source of the external shear. This is seen in Figure 7. The elongation results because the model cannot decide how far away to place the source of the shear. Thus, for systems with nondisturbed galaxy lenses and large external shear, the inversion-symmetric models are probably somewhat more trustworthy than non-inversion-symmetric ones.

For the third ensemble, we put back the inversion symmetry constraint but drop the magnification constraint. Figure 9 shows the time-delay predictions. (The ensemble-average mass map looks similar to Fig. 5, and we have not included it here.) Comparison of Figure 9 and Figure 6 shows what magnification information adds to modeling. Basically, magnification measurements put constraints on the mass distribution in the neighborhood of the images. Time delays between nearby images becomes better constrained. Thus, Figure 9 has a much larger spread in $\Delta t_{12}$ and $\Delta t_{23}$ than Figure 6. Without magnifications, some models' $\Delta t_{12}$ or $\Delta t_{23}$ can become zero: magnifications for the

![Fig. 8.—Same as Fig. 6, but for models without the inversion symmetry constraint.](image1)

![Fig. 9.—Same as Fig. 6, but for models without magnification constraints.](image2)
images in question can get arbitrarily large, effectively merging those images. Also, the upper bound on $\alpha$ that we argue comes from images 1 and 2 is greatly weakened. On the other hand, time delays between distant images are hardly affected: as anticipated in § 4, $\Delta t_{34}$ in Figure 9 has a median (7.8 $h^{-1}$ days) and range very similar to those in Figure 6.

6. CONCLUSIONS

In § 1, we mentioned four unusual features that make B1422 a particularly interesting lens. We now summarize what the results of this paper indicate about each of these features.

First, we have the surprising result that being dominated by external shear from group galaxies makes the lens better constrained. The predicted time delays, in particular the longest delay $\Delta t_{34}$, though it has a broad range, is narrower than in other comparable systems. The fortuitous alignment of the source displacement and the shear appears to help.

Second, one detail of the group contribution is important. Of two situations, (1) the lensing galaxy has $180^\circ$ rotation symmetry and the shear comes from relatively distant group members, and (2) the lensing galaxy is asymmetric and elongated along the group direction, the second case gives a $\Delta t_{34}$ that is 50% longer. Optical and X-ray images of the galaxy and group (see Fig. 2) are not conclusive on this point.

Third, tensor magnifications from fluxes and VLBI constrain time delays between nearby images, but remarkably, they have no discernible effect on longer time delays, $\Delta t_{34}$. The physical reason is not hard to appreciate: magnification is essentially the second derivative of the time delay, and hence time delays between widely separated images tend to wash out the sort of local variations of density that cause differences in magnifications.

Fourth, comparison with time-delay measurements is still problematic at present. Patnaik & Narasimha (2001) report $\Delta t_{12} = 7.6 \pm 2.5$ days, whereas our reconstructions predict $\Delta t_{12} \sim 0.4 h^{-1}$ days. The difficulty is that our predicted values would not be expected to show up in the Patnaik & Narasimha data, which sample at ~4 days; instead, aliases of our predicted values would show up. Even with closely sampled data, a $\Delta t_{12} \sim 0.4 h^{-1}$ days is unlikely to be measurable, because it would need intrinsic brightness variations on the scale of hours. On the other hand, the predicted $\Delta t_{34} \sim 5–10$ days is a convenient length for measurements, but unfortunately it involves image 4, whose flux is ~30 times smaller.

In summary, we conclude that B1422 is a very interesting lens, but probably not the sought-after golden lens.

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