Abstract

In this article, we solve the connection problem of the Hermite polynomials with the classical continuous orthogonal polynomials belonging to Askey scheme, using the hypergeometric functions method combined is with the work the Fields and Ismail.

Keywords: Orthogonal polynomials, Laguerre polynomials, Connection problems, hypergeometric functions

1. INTRODUCTION

The connection problem is to find the coefficients \( c_{nk} \) in the expansion of one polynomial \( P_n(x) \) in terms of an arbitrary sequence of orthogonal polynomials \( \{Q_k(x)\} \),

\[
P_n(x) = \sum_{k=0}^{n} c_{nk} Q_k(x).
\]
A wide variety of methods have been devised for computing the connection coefficients $c_{nk}$, either in closed form or by means of recurrence relations, usually in $k$. Lewanowicz [1] has shown that the connection problem (1) can, sometimes be solved by taking advantage of known results from the theory of generalized hypergeometric functions, derived by Fields and Wimp [2].

The hypergeometric functions method has been devised to solve the connection problem, that involving classical orthogonal polynomials [1, 3, 4, 5, 6, 7, 8]. These methods have also been used by Sánchez-Ruiz [7] to obtain the connection formulae involving squares of Gegenbauer Polynomials. In [8], authors seeking for solving (1) for a much wider class of polynomials, defined by terminating hypergeometric series, obtain connection formulae for Wilson and Racah polynomials with special parameter values. They also solve the connection problem for the families of generalized Jacobi and Laguerre polynomials defined by Sister Celine.

In [5] authors consider the expansion of arbitrary power series in various classes of polynomial sets. In particular, they obtain the connection formulae which generalize the expansions formulae Verma and Field and Wimp. (See Lema 2.1, Lema 2.2).

2. NOTATION AND PRELIMINARY RESULTS

The generalized hypergeometric function is defined by

$$\quad p F_q \left( \begin{array}{c} a_1, a_2, \ldots, a_p \\ b_1, b_2, \ldots, b_q \end{array} \bigg| x \right) = \sum_{k=0}^{\infty} \frac{(a_1)_k(a_2)_k \cdots (a_p)_k}{(b_1)_k(b_2)_k \cdots (b_q)_k} \frac{x^k}{k!}.$$

(2)
where \((a)_n\) represents the Pochhammer symbol and it used in the theory of special functions to represent the rising factorial.

\[
(a)_n := a(a+1)(a+2) \cdots (a+n-1) = \frac{\Gamma(a+n)}{\Gamma(a)} = \frac{(-1)^n \Gamma(1-a)}{\Gamma(1-a-n)},
\]

\((a)_0 = 1,
\]

where \(a_i \in C, 1 \leq i \leq p, b_j \in C, 1 \leq j \leq q, \) with \(b_j \notin N_0.\) Throughout this article, the letters \(p, q, r, s, t, u\) and \(n\) stand for nonnegative integers. We call \(x\) the argument of the function, and \(a_j, b_j\) the parameters. To shorten the notation for the left-hand side of (2), we will write it as

\[
_{pFq}\left(\begin{array}{l}
[a_p] \\
[b_q]
\end{array}\right| x) = \sum_{k=0}^{\infty} \frac{[a_p]_k}{[b_q]_k} \frac{x^k}{k!},
\]

where \([a_p]\) and \([b_q]\) represent the sets \(\{a_1, a_2, \ldots, a_p\}\) and \(\{b_1, b_2, \ldots, b_q\}\), respectively. We use the abbreviated notation

\[
[a_p]_k = \prod_{i=1}^{p} (a_i)_k, \quad [b_q]_k = \prod_{j=1}^{q} (b_j)_k.
\]

To prove the theorems in section 3, we use known results from the theory of generalized hypergeometric functions.

Lemma 2.1 (See [5], Formulae 1.3 and 3.2)

\[
\sum_{m=0}^{\infty} a_m b_m \frac{(zw)^m}{m!} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!(\gamma+n)n} \sum_{r=0}^{\infty} \frac{(-\gamma-2n+\theta+1)^r}{r!(\gamma+2n+1)} b_{n+r} z^r \\
\times \sum_{s=0}^{n} \frac{(-n-s)}{s!} \frac{(n+\gamma)}{(\mu_s(\theta))^s} a_s w^s.
\]

\[
\sum_{m=0}^{\infty} a_m b_m (zw)^m = \sum_{n=0}^{\infty} \frac{(c)_n (-x)^n}{n!} \sum_{j=0}^{n} \frac{(n+c)_j}{j!} b_{n+j} z^j \sum_{k=0}^{n} \frac{(-n)_k}{(c)_k} a_k w^k.
\]
Lemma 2.2 (See [2], [4, Vol. II, p. 7])

\[ p + r \sum_{n=0}^{\infty} \frac{(a_p)_n (c_r)_n (-z)^n}{(b_q)_n n!} F_{q+s} \left( \begin{array}{c} n + \alpha, \ n + a_p \\ \begin{array}{c} n + b_q \end{array} \end{array} \right | z \right) \\
\times r + 1 F_{s+1} \left( \begin{array}{c} -n, \ c_r \\ \begin{array}{c} c, \ d_s \end{array} \end{array} \right | \omega \right).
\]

(7)

\[ p + r + 1 \sum_{n=0}^{\infty} \frac{(a_p)_n (c_r)_n (-z)^n}{(b_q)_n n!} F_{q+s} \left( \begin{array}{c} n + \alpha, \ n + a_p \\ \begin{array}{c} n + b_q \end{array} \end{array} \right | z \right) \\
\times r + 1 F_{s+1} \left( \begin{array}{c} -n, \ c_r \\ \begin{array}{c} c, \ d_s \end{array} \right | \omega \right).
\]

(8)

Lemma 2.3 From the generalized hypergeometric definition, it is obtained:

\[ p F_{q+1} \left( \begin{array}{c} a_p \\ b_q \end{array} \right | z \right) = 2_p F_{2q+1} \left( \begin{array}{c} a_p, \ a_p + 1 \\ b_q, \ b_q + 1 \end{array} \right | 4^{p-q-1} z^2 \right) + \prod_{i=1}^{p} \frac{a_i}{a_i + 1} \frac{z}{1 - z} \]  

3. Results

In this section, we use the lemmas 2.1, 2.2, 2.3 and the different hypergeometric series representation the classical continuous orthogonal polynomials to solve the connection problem among them.

3.1. Theorem (Connection formula for Laguerre Polynomials in series of Hermite Polynomials)

\[ L_n(x) = \sum_{k=0}^{n} \frac{(-n)_k}{2^k (k)! (k)!} \left( \begin{array}{c} k \frac{a_p}{2} \ b_q \frac{a_p + 1}{2} \ k + \frac{1}{2} \ k + \frac{3}{2} \ \begin{array}{c} 1 \4 \end{array} \end{array} \right) \right) H_k(x). \]
Proof: Some of the hypergeometric representations of Laguerre and Hermite polynomials are given by:

$$L_n(x) = \sum_{k=0}^{n} \frac{(-n)_k x^k}{k!} = \binom{-n}{1} x_1 F_1 \left( -n \left| \frac{1}{x} \right. \right),$$  \hspace{1cm} (10)$$

$$H_{2m}(x) = (-1)^m 2^{2m} \left( \frac{1}{2} \right)_m \binom{-m}{\frac{1}{2}} x_1 F_1 \left( -m \left| \frac{1}{2} \right. \right),$$  \hspace{1cm} (11)$$

$$H_{2m+1}(x) = (-1)^m 2^{2m+1} \left( \frac{1}{2} \right)_m x_1 F_1 \left( -m \left| \frac{3}{2} \right. \right).$$  \hspace{1cm} (12)$$

Using formula (9) of Lemma 2.3 with the following identification

$$p = 1, \quad q = 1, \quad \{a_1\} = \{-n\}, \quad \{b_1\} = \{1\}, \quad z = x$$

it is obtained

$$1 F_1 \left( -n \left| x \right. \right) = 2 F_3 \left( -\frac{n}{2}, -\frac{n+1}{2} \left| \frac{1}{4} x^2 \right. \right) + x(-n) 2 F_3 \left( -\frac{n+1}{2}, -\frac{n+2}{2} \left| \frac{1}{4} x^2 \right. \right).$$  \hspace{1cm} (13)$$

If we call:

$$A_1(x) = 2 F_3 \left( -m, -\frac{2m+1}{2} \left| \frac{1}{4} x^2 \right. \right),$$  \hspace{1cm} (14)$$

$$A_2(x) = -2mx 2 F_3 \left( -(m-1), -\frac{2m+1}{2} \left| \frac{1}{4} x^2 \right. \right).$$  \hspace{1cm} (15)$$

and \( n = 2m \), then (13) can be written as:

$$L_n(x) = A_1(x) + A_2(x).$$  \hspace{1cm} (16)$$
Using Fields-Wimp formula (7) we connect $A_1(x)$ given in (14) with the Hermite polynomials of even degree, (11), making the following identification:

$$ [d_s] = \emptyset, \; s = 0, \; [c_r] = \emptyset, \; r = 0, \; c = \frac{1}{2} [a_p] = \left\{ -m, \frac{-2m + 1}{2} \right\} $$

$$ p = 2, \; [b_q] = \left\{ \frac{1}{2}, \frac{1}{2}, 1 \right\}, \; q = 3, \; w = x^2, \; z = \frac{1}{4}, $$

it is obtained

$$ 2F_3 \left( \begin{array}{c} -m, \frac{-2m + 1}{2} \\ \frac{1}{2}, \frac{1}{2}, 1 \end{array} \right| \frac{x^2}{4} \right) = \sum_{k=0}^{m} \frac{(-m)_k (\frac{-2m + 1}{2})_k (\frac{1}{2})^k}{(\frac{1}{2})_k (\frac{1}{2})_k k!} 3F_3 \left( \begin{array}{c} k + \frac{1}{2}, k - m, k + \frac{1 - 2m}{2} \\ k + \frac{1}{2}, k + \frac{1}{2}, k + 1 \end{array} \right| \frac{1}{4} \right) 1F_1 \left( \begin{array}{c} -k \\ \frac{1}{2} \end{array} \right| x^2 \right). $$

(17)

The left side of (17) is $A_1(x)$ given in (14) by reorganizing the terms of the right side of (17) the Hermite polynomials of even degree arise (11), and using the formula for duplicating the Pochhammer symbol, it results that:

$$ A_1(x) = \sum_{k=0}^{m} \frac{(-m)_{2k}}{2^{2k}(2k)!} 2F_2 \left( \begin{array}{c} \frac{2k-2m}{2}, \frac{2k+1-2m}{2} \\ \frac{2k+1}{2}, \frac{2k+2}{2} \end{array} \right| \frac{1}{4} \right) H_{2k}(x). $$

(18)

Likewise, polynomials $A_2(x)$ of (15) are connected with Hermite polynomials of odd degree, (12); making the following identification:

$$ [d_s] = \emptyset, \; s = 0, \; [c_r] = \emptyset, \; r = 0, \; c = \frac{3}{2} [a_p] = \left\{ -\frac{(2m - 2)}{2}, \frac{-2m + 1}{2} \right\}, \; p = 2 $$

$$ [b_q] = \left\{ \frac{3}{2}, \frac{3}{2}, 1 \right\}, \; q = 3, \; w = x^2, \; z = \frac{1}{4}, $$

$$ A_2(x) = \sum_{k=0}^{m-1} \frac{(-2m)_{2k+1}}{2^{2k+1}(2k+1)!(2k+1)} 2F_2 \left( \begin{array}{c} \frac{2k+1-2m}{2}, \frac{2k+1-2m+1}{2} \\ \frac{2k+1+2}{2}, \frac{2k+1+1}{2} \end{array} \right| \frac{1}{4} \right) H_{2k+1}(x). $$

(19)
Recalling $n = 2m$, and $L_n(x) = A_1(x) + A_2(x)$ then from (18) and (19) we conclude that

$$L_n(x) = \sum_{k=0}^{n} \frac{(-n)_k}{2^k(k)!} \binom{k}{n} \binom{k+1-n}{2} \binom{k+2}{2} H_k(x).$$

3.2. Theorem (Connection formula for Hermite polynomials in series of Laguerre Polynomials)

$$H_N(x) = N!2^N \sum_{m=0}^{N} \binom{2p}{m} \frac{(-1)^p}{(N-2k)!} (2x)^{N-2k},$$

**Proof:** Hermite polynomials can be expressed as:

$$H_N(x) = N! \sum_{k=0}^{[N/2]} \frac{(-1)^p}{k!(N-2k)!} (2x)^{N-2k},$$

If in the above expression we set up $N = 2p$, this is rewritten as:

$$H_{2p}(x) = (2p)! \sum_{k=0}^{p} \frac{(-1)^p}{k!(2p-2k)!} (2x)^{2p-2k}. \quad (20)$$

Let $m := 2p-2k$, for $k = 0, 1, 2, ..., p$ one realized that $m$ goes through the set \{2p, 2p - 2, ..., 0\} If in (20) we make the following substitution $m = 2p - 2k$, it is obtained:

$$H_{2p}(x) = (2p)! \sum_{m \in \{2p, 2p-2, ..., 0\}} \frac{(-1)^p}{m!(2p-m)!} \binom{2p-m}{2m} x^m \quad (21)$$

With the following identification $a_k = \frac{1}{k!}$, $c = 1$, $w = x$, $a_m = \frac{1}{m!}$ $z = 1$ and

$$b_m = \begin{cases} \frac{(-1)^p}{(2p-m)!} \binom{2p-m}{2m} x^m, & m \in \{2p, 2p-2, ..., 0\} \\ 0, & \text{otherwise} \end{cases} \quad (22)$$
the left side of formula (6) of Lemma 2.1, is precisely the expression for Hermite polynomials given in (21). By reorganizing the terms of the right side of (6) Laguerre polynomials result.

\[
H_{2p}(x) = \sum_{n=0}^{\infty} (-1)^n \sum_{j=0}^{\infty} \frac{(n + 1)_j}{j!} b_{n+j} L_n(x)
\]

Keeping in mind the definition given for coefficients \(b_m\), \(H_{2p}(x)\) it can be written:

\[
H_{2p}(x) = \left\{ \sum_{k=0}^{p} \sum_{s=0}^{p-k} (-1)^{2k} \frac{(2k+1)_{2s}}{(2s)!} b_{2k+2s} L_{2k}(x) \right\} + \left\{ \sum_{k=0}^{p-1} \sum_{s=0}^{p-k-1} (-1)^{2k+1} \frac{(2k+1+1)_{2s+1}}{(2s+1)!} b_{2k+1+2s+1} L_{2k+1}(x) \right\}.
\]

Let

\[
H_{2p}(x) := I + II.
\]

Making use of the expression of \(b_m\) given in (27), we have

\[
I = \sum_{k=0}^{p} \sum_{s=0}^{p-k} (-1)^{2k} \frac{(2k+1)_{2s}}{(2s)!} \frac{(-1)^{p-k-s} 2^{2k+2s} (2p)!}{(p-k-s)!} L_{2k}(x).
\]

\[
II = \sum_{k=0}^{p-1} \sum_{s=0}^{p-k-1} (-1)^{2k+1} \frac{(2k+2)_{2s+1}}{(2s+1)!} \frac{(-1)^{p-k-s-1} 2^{2k+2s+2} (2p)!}{(p-k-s-1)!} L_{2k+1}(x).
\]

By distributing some terms, the expression of \(I\) can be written as:

\[
I = (2p)! 2^{2p} \sum_{k=0}^{p} \left\\{ \sum_{s=0}^{p-k} \frac{(2k+1)_{2s} (-1)^{p-k-s} 2^{2k+2s}}{(2s)! (p-k-s)!} \frac{(2k)!}{2^{2p} (2p)! (2k)!} \right\} \frac{(-2p)_{2k} L_{2k}(x)}{(2k)!}.
\]

Using some properties of the Pochhammer symbol, the interior sum of (23), performed over \(s\), can be written as:

\[
\sum_{s=0}^{p-k} \frac{(k-p+\frac{1}{2})_{p-k-s} (k+s)! (p-k)! (-1)^{p-k-s} 2^{2k+2s} \sigma_{2s+1} (-\frac{1}{4})^s}{s! (p-k-s)! 2^{2p} (p-k-s)!} \frac{(-2p)_{2k} L_{2k}(x)}{(2k)!}.
\]
With the substitution 
\[ 2k + 2s = 2p - 2t, \]
the above expression can be written as:
\[
\sum_{t=0}^{p-k} \frac{(-p-k)_t(k-p + \frac{1}{2})_t(-1)^t}{(p)_t!(\frac{1}{2} - p)_t2^2t}.
\]

As \( N = 2p \), then (23) is transformed into:
\[
I = N!2^N \sum_{k=0}^{\frac{N}{2}} 2F_2 \left( \begin{array}{c} -\frac{1}{2}(N-2k), -\frac{1}{2}(N-2k-1) \\ -\frac{1}{2}(N), -\frac{1}{2}(N-1) \end{array} \right) \left| -\frac{1}{4} \right\} \frac{(-N)_{2k}}{(2k)!} L_{2k}(x).
\]

(25)

Likewise \( II \), turns out to be
\[
II = N!2^N \sum_{k=0}^{\frac{N-1}{2}} 2F_2 \left( \begin{array}{c} -\frac{1}{2}(N-2k-1), -\frac{1}{2}(N-2k-2) \\ -\frac{1}{2}(N), -\frac{1}{2}(N-1) \end{array} \right) \left| -\frac{1}{4} \right\} \frac{(-N)_{2k+1}}{(2k+1)!} L_{2k+1}(x).
\]

(26)

Recalling that
\[
H_N(x) = I + II.
\]

Thence, (25) and (26) allow us to write the connection of Hermite \( H_N(x) \) with Laguerre polynomials:

\[
H_N(x) = N!2^N \sum_{k=0}^{\frac{N}{2}} 2F_2 \left( \begin{array}{c} -\frac{1}{2}(N-2k), -\frac{1}{2}(N-2k-1) \\ -\frac{1}{2}(N), -\frac{1}{2}(N-1) \end{array} \right) \left| -\frac{1}{4} \right\} \frac{(-N)_{2k}}{(2k)!} L_{2k}(x)
\]

\[
+ N!2^N \sum_{k=0}^{\frac{N-1}{2}} 2F_2 \left( \begin{array}{c} -\frac{1}{2}(N-2k-1), -\frac{1}{2}(N-2k-2) \\ -\frac{1}{2}(N), -\frac{1}{2}(N-1) \end{array} \right) \left| -\frac{1}{4} \right\} \frac{(-N)_{2k+1}}{(2k+1)!} L_{2k+1}(x)
\]

\[
H_N(x) = N!2^N \sum_{m=0}^{N} 2F_2 \left( \begin{array}{c} -\frac{1}{2}(N-m), -\frac{1}{2}(N-m-1) \\ -\frac{1}{2}(N), -\frac{1}{2}(N-1) \end{array} \right) \left| -\frac{1}{4} \right\} \frac{(-N)_{m}}{(m+1)!} L_{m}(x),
\]

Note: Likewise the procedure can be applied when \( N \) is odd.
3.3. *Theorem (Connection formula for Hermite polynomials in series of Shifted Jacobi Polynomials)*

\[ H_n(x) = \frac{(-n)^n(2m+\lambda)(\alpha+1)}{(\alpha+1)n(\lambda+2n+1)} \quad _4\!F_2 \left( \begin{array}{c} \Delta(2,m-n), \Delta(2,-\lambda-n-m) \\ \Delta(2,-\alpha-n) \end{array} \left| \frac{1}{4} \right. \right) \]

\[ \times P_m^{(\alpha,\beta)}(1-x) \]

where, \( \lambda = \alpha + \beta + 1 \), \( y \Delta(r; \varphi) = \frac{(\varphi+j-1)}{r}, \quad j = 1, \ldots, r \).

**Proof**: Shifted Jacobi polynomials can be written as:

\[ R_n^{(\alpha,\beta)}(x) = \frac{(-1)^n(\beta+1)_n}{n!} \quad _2\!F_1 \left( \begin{array}{c} -n, n+\alpha+\beta+1 \\ \beta+1 \end{array} \left| x \right. \right) \]

\[ R_n^{(\alpha,\beta)}(x) = \frac{(-1)^n(\beta+1)_n}{n!} \sum_{s=0}^{n} \frac{(-n)_s(n+\alpha+\beta+1)_s}{(\beta+1)_s} \frac{x^s}{s!} \]

With the following substitution \( \gamma = \alpha+\beta+1, \theta = \beta+1, \mu = 1, a_s = s!, w = x, z = 1 \) and

\[ b_m = \begin{cases} \frac{(-1)^{2p-m}2^m(2p)!}{m!(2p-m)!}, & m \in \{2p, 2p-2, \ldots, 0\} \\ 0 & \text{otherwise} \end{cases} \] (27)

the left side of formula (5) of Lemma 2.1, is precisely the expression for Hermite polynomials given in (21). On the other hand in the right side (5) the Shifted Jacobi polynomials appear. Therefore (5) is transformed in

\[ H_{2p}(x) = \sum_{n=0}^{\infty} \frac{1}{(\beta+1)_n(\alpha+\beta+1)_n} \sum_{r=0}^{\infty} \frac{(n+r)!(\beta+1)_{n+r}}{r!(\alpha+\beta+2n+2)_r} b_{n+r} R_n^{(\alpha,\beta)}(x). \]

The result is also obtained by a similar procedure to the one followed in the proof of Theorem 3.2.
3.4. **Theorem (Connection formula for Shifted Jacobi Polynomials in series of Hermite polynomials)**

\[
R_n^{(\alpha, \beta)}(x) = \sum_{j=0}^{n} \frac{(-1)^n(-1)^j (\beta + 1)_n (n + \alpha + \beta + 1)_j}{(n - j)! 2^j (j)! (\beta + 1)_j} \\
\times \left. 4F_2 \left(\frac{i-n}{2}, \frac{i+1-n}{2}, \frac{i+n+\alpha+1}{2}, \frac{i+n+\alpha+2}{2}; \frac{i+\beta+1}{2}, \frac{i+\beta+2}{2}; 1 \right) \right| H_j(x).
\]

**Proof:** Using formula (9) of Lemma 2.3 with the following identification

\[ p = 2 \quad q = 1 \quad a_1 = \{-n\} \quad a_2 = \{n + \alpha + \beta + 1\} \quad b_1 = \{\beta + 1\} \quad z = x \]

it is obtained

\[
R_n^{(\alpha, \beta)}(x) = (-1)^n \frac{(\beta + 1)_n}{n!} 4F_3 \left(\frac{1-n}{2}, \frac{n+\alpha+\beta+1}{2}, \frac{1-n}{2}, \frac{n+\alpha+2}{2}; \frac{1}{2}, \frac{\beta+1}{2}, \frac{\beta+2}{2}; x^2 \right) \\
+ (-1)^n \frac{(\beta + 1)_n}{n!} x(n + \alpha + \beta + 1) \frac{1}{\beta + 1} 4F_3 \left(\frac{1-n}{2}, \frac{n+\alpha+\beta+3}{2}, \frac{2-n}{2}, \frac{n+\alpha+\beta+2}{2}; \frac{3}{2}, \frac{\beta+2}{2}, \frac{\beta+3}{2}; x^2 \right)
\]

If we note by

\[
A_1(x) = (-1)^n \frac{(\beta + 1)_n}{n!} 4F_3 \left(\frac{1-n}{2}, \frac{n+\alpha+\beta+1}{2}, \frac{1-n}{2}, \frac{n+\alpha+2}{2}; \frac{1}{2}, \frac{\beta+1}{2}, \frac{\beta+2}{2}; x^2 \right)
\]

\[
A_2(x) = (-1)^n \frac{(\beta + 1)_n}{n!} x(n + \alpha + \beta + 1) \frac{1}{\beta + 1} 4F_3 \left(\frac{1-n}{2}, \frac{n+\alpha+\beta+3}{2}, \frac{2-n}{2}, \frac{n+\alpha+\beta+2}{2}; \frac{3}{2}, \frac{\beta+2}{2}, \frac{\beta+3}{2}; x^2 \right)
\]

then (28) can be written as

\[
R_n^{(\alpha, \beta)}(x) = A_1(x) + A_2(x).
\]

Following a similar procedure to that used in Theorem 3.1 one end up with the expected result.
4. Summary and continuity of the work

In this work, we present the connections between the Polynomials of Laguerre in series of the polynomials of Hermite, between the polynomials of Hermite and the polynomials of Laguerre, between the polynomials of Hermite and the polynomials of Shifted Jacobi, between the polynomials of Shifted Jacobi and the polynomials of Hermite. He expects to present it in a new work the connection between the polynomials of Bessel and the polynomials of Hermite, between the polynomials of Hermite and the polynomials of Bessel.

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