Linear-Time Poisson-Disk Patterns

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February, 2011

Abstract

We present an algorithm for generating Poisson-disk patterns taking \(O(N)\) time to generate \(N\) points. The method is based on a grid of regions which can contain no more than one point in the final pattern, and uses an explicit model of point arrival times under a uniform Poisson process.

1 Introduction

There is a long-standing interest in Poisson-disk patterns in the graphics community, primarily for their use in sampling [Yel83, Coo86, Mit87]. There have been many algorithms for generating such patterns. Direct implementation of “dart-throwing” [Mit87, MF92] produces true Poisson-disk patterns, but is slow to converge. Approximations from relaxation [Jon83] or tiling [ODJ04, HDK01] can produce patterns similar to Poisson-disk patterns more efficiently. Recently, exact methods taking log-linear time (\(O(N \log N)\) where \(N\) is the total number of points) have been described [DH06, Jon06], as well as a method with empirical \(O(N)\) speed, but lacking a rigorous proof of this performance [WCE07].

2 Background

The Poisson Disk distribution can be defined as the limit of a uniform two-dimensional Poisson process with a minimum-distance rejection criterion. Successive points are independently drawn from the uniform distribution on \([0,1]^2\). If a new point is at least distance \(R\) from all points already accepted, it is also accepted. Otherwise, it is rejected. We call this the na"ıve algorithm. The choice of \(R\) controls the minimum distance between points (for \(N\) points in the unit square, \(\pi R^2 N / 4 \approx 0.548 \) as \(R \to 0\) [DWJ91]).

Efficient algorithms for Poisson-disk patterns rely on generating new points in regions where they are guaranteed (or highly probable) to be accepted [DH06, Jon06, WCE07]. In order to guarantee equivalence of results with the naïve algorithm, these methods have used \(O(\log N)\) area-weighted binary search to find where to insert a new point [DH06, Jon06], or weighted spatial indexing [WCE07] with theoretical \(O(\log N)\) but empirical \(O(1)\) cost.

3 Method

Our algorithm can be seen as an optimization of the naïve algorithm using a spatial data structure. We
Figure 1: A point $p$ is shown with its neighbor grid squares in grey. If $p$'s arrival time is earlier than any of its neighbors, it will be accepted and added to the output. The free regions of $p$'s neighbors will then be updated to the dark gray areas. This may result in points such as $q$ being invalidated. In $q$’s case, a replacement point $q'$ will be generated in its new free region, $A_{q'}$. The time of arrival of $q'$ will be $t_q + t_+$, where $t_+$ is drawn from an exponential distribution parameterized by the size of the updated free region.

To track which points are candidates for acceptance, we traverse the grid and identify every point that has a time of arrival earlier than any of its neighbors (ignoring neighbors that have already had their points accepted). We term these points locally early, and add them to a bucket (an unordered set). At each iteration, we can take any point from the bucket, and add it to the output pattern.

Accepting $p$ may lead to new points becoming locally early, which are then added to the bucket. Likewise, if a point $q$ is invalidated by $p$’s acceptance, points with $q$’s grid square in their neighbors may become locally early, as $q$’s replacement $q'$ will have $t_{q'} > t_q$.

Each iteration is $O(1)$ provided we can update, compute the area of, and sample uniformly from the free space of a grid square in $O(1)$ time. Previous
work has demonstrated specialized data structures [DH06] for exactly these purposes. In our reference implementation, we use a constructive planar geometry library and approximate disks with polygons for simplicity but without a loss of generality. We show the performance of our algorithm in terms of samples per second, as well as number of samples generated by the uniform Poisson arrival process versus accepted Poisson Disk samples (see figure 2). Since the size of grid squares is determined by the radius of the PD samples, the geometric complexity of the free space is $O(1)$.

4 Discussion

We have introduced an algorithm for generating Poisson-disk patterns in provable $O(1)$ time per generated sample. Our main insight, compared to recent $O(\log N)$ per point algorithms, is that rather than choosing the location for the next point based on area-weighted binary search, we can use an area-parameterized exponential distribution to order points in time under a uniform Poisson arrival process. While previous algorithms generate each point in sequence, with an implicit time linked to their sequential generation, we create many points with explicit arrival times and order them (in a local fashion) to find those that should be accepted.

Acknowledgements

The authors wish to thank Ron Perry, Peter-Pike Sloan, and the MIT CSAIL Computer Graphics Group for helpful comments on this paper.

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