Comparison of SUSY mass spectrum calculations

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Abstract

We provide a comparison of the results of four SUSY mass spectrum calculations in mSUGRA: Isajet, SuSpect, SoftSusy, and SPheno. In particular, we focus on the high tan $\beta$ and focus point regions, where the differences in the results are known to be large.

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1 Introduction

Many SUSY studies rely on computer codes that calculate the mass spectrum of the minimal supersymmetric standard model (MSSM), the couplings, branching ratios, etc., from given sets of model parameters. For the LHC, for instance, many simulations are done for particular benchmark scenarios or by mapping the $(m_0, m_{1/2})$ parameter plane. For such studies it is certainly important whether a particular decay channel is open or not and what branching ratio it has. Also, theoretically or experimentally excluded regions depend on the details of the spectrum. Studies for an $e^+e^-$ Linear Collider deal, in addition, with high precision measurements of (s)particle properties, with the determination of the underlying SUSY breaking parameters, their extrapolation to the GUT scale, model distinction, etc. Experimental accuracies of the per-cent or even per-mille level are expected. It is thus clear that we need theoretical predictions of a precision comparable to the experimental accuracy. However, it has been noticed that different programs can give quite different results for the same set of input parameters.

In this article, we compare the mass spectrum calculations of four public codes: Isajet 7.63, SuSpect 2.005, SoftSusy 1.4, and SPheno 1.0, in the minimal supergravity (mSUGRA) framework. We discuss the renormalization group (RG) running and the implementation of radiative corrections, concentrating on the parameter regions where the largest differences are encountered: large tan $\beta$ and large $m_0$. An overview of which corrections are implemented in each of the four programs is given in Table 1.

2 Large tan $\beta$

Large tan $\beta$ has always been recognized as a difficult case since it requires a thorough treatment of the bottom Yukawa coupling $h_b$. It is well known that $h_b$ gets large tan $\beta$ enhanced corrections from SUSY loops, the dominant contributions coming from $\tilde{b}\tilde{g}$ and $\tilde{t}\tilde{\chi}^+$ exchanges. These generate a $H_2^0bb$ coupling, which is forbidden at tree-level, $\mathcal{L} \sim h_b H_2^0bb + \Delta h_b H_2^0bb$. This modifies the tree-level relation between the bottom mass.
|                  | Isajet 7.63 | SuSpect 2.005 | SoftSusy 1.4 | SPheno 1.0 |
|------------------|------------|---------------|--------------|------------|
| **RGEs**         | 2-loop     | 2-loop        | 2-loop       | 2-loop     |
| gauge + Yuk.     | 2-loop     | 2-loop        | 2-loop       | 2-loop     |
| gaugino par.     | 2-loop     | 2-loop        | 2-loop       | 2-loop     |
| scalar par.      | 2-loop     | 1-loop        | 1-loop       | 2-loop     |
| **SUSY masses**  | some corr. for \(\tilde{\chi}_1^\pm\) | 1-loop approx. for \(\Delta M_1, \Delta M_2, \Delta \mu\) | full 1-loop | full 1-loop |
| \(\tilde{\chi}_1^\pm\) | \(\tilde{t}_g + \tilde{t}_g + \text{Yuk.}\) | \(\tilde{t}_g + \tilde{t}_g + \text{Yuk.}\) | full 1-loop | full 1-loop |
| \(\tilde{b}_g^+\) | \(\tilde{b}_g + \tilde{b}_g\) | \(\tilde{b}_g + \tilde{b}_g\) | full 1-loop | full 1-loop |
| \(\tilde{g} + \tilde{g}\) loops resummed | \(\tilde{g} + \tilde{g}\) loops resummed | \(\tilde{g} + \tilde{g}\) loops resummed | full 1-loop | full 1-loop |
| **Yukawa cpl.**  | full 1-loop resum. | full 1-loop resum. | full 1-loop resum. | full 1-loop resum. |
| \(h_t\)         | full 1-loop | \(t_g + \tilde{t}_g\) | full 1-loop | full 1-loop |
| \(h_b\)         | full 1-loop | \(b_g + \tilde{b}_g + \tilde{t}_g^\pm\) corr. resummed | full 1-loop | full 1-loop |
| **Higgs sector** | 3rd gen. (s)fermions | complete 1-loop corrections | 2-loop | 2-loop |
| \(h^0, H^0\)    | 1-loop \[3\] | 1-loop \[3\] | 2-loop \[10\] | 2-loop \[11\] |

Table 1: RGEs and radiative corrections implemented in Isajet, SuSpect, SoftSusy, and SPheno.

and Yukawa coupling, \(m_b = h_b v_1 \to m_b = h_b v_1 (1 + \Delta_b)\) with \(\Delta_b = (\Delta h_b / h_b) \tan \beta\). In the programs under discussion this is taken into account as

\[
    h_b(M_Z) = \tilde{m}^{\text{MSSM}}_b / v_1(M_Z), \quad \hat{m}^{\text{MSSM}}_b(M_Z) = \frac{\tilde{m}^{\text{SM}}_b(M_Z)}{1 + \Delta m_b / m_b}.
\]

Here \(\hat{m}^{\text{SM}}_b\) is the \(\overline{\text{DR}}\) bottom mass in the Standard Model and \(\Delta m_b = (\Delta m_b)^{\tilde{b}_g + \tilde{t}_g + \ldots}\) contains the SUSY-loop corrections. The complete 1-loop expression for \(\Delta m_b\) is given in \[1\]–\[6\]. Compared to the naive 1-loop expansion \(\hat{m}^{\text{MSSM}}_b = \hat{m}^{\text{SM}}_b (1 - \Delta m_b / m_b)\), eq. \([1]\) makes a numerical difference of about 10% in \(h_b\) and about 10–30% in \(m_A\) for large \(\tan \beta\). The resummation of SUSY threshold corrections \[13\] will be discussed elsewhere \[14\]. Although all four programs now apply eq. \([1]\), some numerical differences in \(h_b\) remain. These are partly due to differences in \(\alpha_s\): Suspect, SoftSusy and SPheno calculate \(\alpha_s\) in the \(\overline{\text{DR}}\) scheme, Isajet uses the \(\overline{\text{MS}}\) value. Another reason is that Isajet uses \(m_b = m_b(M_{\text{SUSY}})\) for the expression \(\Delta m_b / m_b\) in eq. \([1]\), while the other programs use \(m_b(M_Z)\) or the bottom pole mass; also the gluino masses differ by about 5%. Moreover, the vacuum expectation values \(v_{1,2}\) are not running in Isajet.

The bottom Yukawa coupling has its largest effect in the Higgs sector. Figure \[4\] shows the running of \(m_{H_{1,2}}^2\) for \(m_0 = 400\ GeV\), \(m_{1/2} = 300\ GeV\), \(A_0 = 0\), \(\mu > 0\), and the two cases \(\tan \beta = 10\) and \(\tan \beta = 50\). As one can see, there is good agreement for not too large \(\tan \beta\). However, for \(\tan \beta = 50\), quite different results are obtained for \(m_{H_1}^2\), whose evolution is driven by \(h_b\):

\[
    \frac{dm_{H_1}^2}{dt} \sim \frac{3}{8\pi^2} h_b X_b + \ldots, \quad X_b = (m_Q^2 + m_D^2 + m_{H_1}^2 + A_b^2).
\]

\[2\]Here note that \[3, 4, 5, 6\] and \[8\] partly have different conventions, e.g., for the ordering of the squark mass eigenstates and the sign of \(\mu\).
Note in particular the dotted line which shows the result obtained with Isajet 7.58. In this version, the SUSY corrections to $h_b$ were not yet resummed.

![Figure 1: Running of $m^2_{H_1}$ as a function of the scale $Q$, for $m_0 = 400$ GeV, $m_{1/2} = 300$ GeV, $A_0 = 0$, $\mu > 0$, $\tan \beta = \{10, 50\}$, $M_t = 175$ GeV. The full (dotted) lines are for Isajet 7.63 (7.58), the dashed lines are for SuSpect 2.005, the dash-dotted ones for SoftSusy 1.4, and the dash-dot-dotted ones for SPheno 1.0.](image)

The differences in $m^2_{H_1}$ directly translate into $m^2_A$ and thus into the physical Higgs boson masses, since

$$m_A^2 = \frac{1}{c^2_{\beta}} \left( m_{H_2}^2 - m_{H_1}^2 \right) + \frac{s_{\beta}^2 t_1}{v_1} + \frac{c_{\beta}^2 t_2}{v_2} - m_Z^2.\quad (3)$$

Here $m_{H_i}^2 = m_{H_i}^2 - t_i/v_i$, $i = 1, 2$, and $t_{1,2}$ are the tadpole contributions. The self energies of $Z$ and $A$ have been neglected in eq. (3). We note that including only the tadpoles from the third generation is in general a good approximation. The remaining 1-loop contributions account for an $\mathcal{O}(1\%)$ correction.

Figure 2 shows the Higgs boson masses obtained by the four programs as a function of $\tan \beta$. The new Isajet version 7.63 has led to a major improvement compared to the situation discussed in [2, 15] (the results obtained by Isajet 7.58 are again shown as dotted lines in Fig. 2). For $m_A$ and $m_{H^\pm}$ there is now agreement within $\sim 10\%$ up to $\tan \beta \sim 45$. Sources for the remaining differences are pointed out above. Moreover, it makes a difference whether one uses running couplings and/or masses for the tadpoles $t_{1,2}$. Here each program has a different approach. For the neutral scalars, however, the situation is not so good. Especially for $m_{h^0}$, a discrepancy of $\sim 4$ GeV is too large compared to the expected experimental accuracy. This discrepancy is mainly due to the different radiative corrections taken into account for the $(h^0, H^0)$ system. They vary between 1- and 2-loop, effective potential and diagramatic calculations, see Table 1. Given the expected experimental accuracy for $m_{h^0}$ it is clear that the best available calculation should be used.
Figure 2: Higgs boson masses as a function of \(\tan \beta\), for \(m_0 = 400\) GeV, \(m_{1/2} = 300\) GeV, \(A_0 = 0\), \(\mu > 0\), \(M_t = 175\) GeV; full (dotted) lines: Isajet 7.63 (7.58), dashed: SuSpect 2.005, dash-dotted: SoftSusy 1.4, dash-dot-dotted: SPheno 1.0.

3 Large \(m_0\)

For large \(m_0\), the running of \(m_{H_2}^2\) becomes very steep and very sensitive to the top Yukawa coupling \(h_t = \hat{m}_t / v_2\):

\[
\frac{d m_{H_2}^2}{dt} \sim \frac{3}{8\pi^2} h_t X_t + \ldots, \quad X_t = (m_Q^2 + m_U^2 + m_{H_2}^2 + A_t^2).
\]

(4)

As a result, the \(\mu\) parameter given by

\[
\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2
\]

(5)

becomes extremely sensitive to \(h_t\). This is visualized in Fig. 3 where we show in (a) the running of \(m_{H_{1,2}}^2\) for \(m_0 = 1450\) GeV, and in (b) \(\mu\) as a function of \(m_0\). The other
Figure 3: a) Running of $m_{H_1}^2$ for $m_0 = 1450$ GeV; b) $\mu$ as a function of $m_0$; for $m_{1/2} = 300$ GeV, $A_0 = 0$, $\mu > 0$, tan $\beta = 10$, and $M_t = 175$ GeV; full (dotted) lines: Isajet 7.63 (7.58), dashed: SuSpect 2.005, dash-dotted: SoftSusy 1.4, dash-dot-dotted: SPheno 1.0.

parameters are $m_{1/2} = 300$ GeV, $A_0 = 0$, $\mu > 0$, and tan $\beta = 10$. The large discrepancy in $\mu$ for $m_0 \gtrsim 800$ GeV lead to completely different chargino/neutralino properties and likewise to very different excluded regions in Isajet compared to the other programs. For instance, radiative EWSB breaks down in Isajet for $m_0 \gtrsim 1.5$ TeV. In SuSpect, SoftSusy, and SPheno, this happens only for $m_0 \gtrsim 2.5$–2.8 TeV.\

In order to understand the behaviour in Fig. 3b it is useful to write eq. (5) in the form

$$\mu^2 \simeq c_1 m_0^2 + c_2 m_{1/2}^2 - 0.5 M_Z^2.$$  

Approximate analytical expressions for $c_1$ and $c_2$ can be found e.g., in [16, 17]. For $A_0 = 0$ and tan $\beta = 10$ we get [17]

$$c_1 \sim \left( \frac{\hat{m}_t}{156.5 \text{ GeV}} \right)^2 - 1, \quad c_2 \sim \left( \frac{\hat{m}_t}{102.5 \text{ GeV}} \right)^2 - 0.52.$$  

Since the Higgs potential is minimized at $M_{SUSY} = \sqrt{\hat{m}_{t_1} \hat{m}_{t_2}}$, we take $\hat{m}_t$ in eq. (7) as $\hat{m}_t = \hat{m}_t(M_{SUSY})$. The $m_0$ dependence seen in Isajet is reproduced for $\hat{m}_t \sim 151$ GeV. The one of SuSpect, SoftSusy and SPheno is reproduced for $\hat{m}_t \sim 155$ GeV. Figure 4 shows a contour plot of $\mu$ in the $(m_0, \hat{m}_t)$ plane. Notice the fast increasing dependence on $\hat{m}_t$ for increasing $m_0$. Notice also that for $\hat{m}_t \sim 156$–157 GeV, $\mu$ becomes almost independent of $m_0$, which is the actual focus point condition.

There are some obvious differences in the calculations. For instance, $M_{SUSY}$, the scale where the SUSY parameters are frozen out and the Higgs potential is minimized, varies by about 100 GeV due to different radiative corrections to the stop masses, c.f. Table 1. In the loop corrections to $m_t$, analogous differences occur as discussed above for $\Delta m_b/m_b$.

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3After the conference, a sign error was corrected in SPheno. As a consequence, its results for large $m_0$ now nicely agree with those of SoftSusy and SuSpect (contrary to what was presented in the talk).
Figure 4: The parameter $\mu$ as given by eq. (6) in the $(m_0, \hat{m}_t)$ plane, for $m_{1/2} = 300$ GeV, $A_0 = 0$, and $\tan \beta = 10$.

Also the evolution of $h_t$ between $M_Z$ and $M_t$ and the inclusion of threshold effects are delicate points. However, this is not yet sufficient to explain the observed discrepancies. More work is needed to clarify the situation.

4 Conclusions

For the calculation of the SUSY mass spectrum from GUT scale boundary conditions, there are two particular difficult parameter regions where large numerical differences have been noticed: large $\tan \beta$ and large $m_0$. These regions are very sensitive to the bottom and top Yukawa couplings, respectively.

The inclusion of the SUSY 1-loop corrections to $h_b$ has led to a considerable improvement in the large $\tan \beta$ case. In particular, the four programs now agree on $m_A$ within $\lesssim 10\%$ for $\tan \beta \lesssim 45$. Further improvements are of course desirable.

For large $m_0$, there are still very large numerical discrepancies due to the corrections to $h_t$. As a matter of fact, $h_t$ is much smaller in Isajet than in the other programs. Some differences in the calculation of $h_t$ have been pointed out, but these do not satisfingly explain the observed discrepancies. Work is in progress to clarify the situation [14].

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