From real clusters to experimental nuclear matter phase diagram

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Abstract. We have determined the critical properties of infinite uncharged nuclear matter based on cluster emission data. To do this, three obstacles had to be overcome: finite size effects, the Coulomb interaction and appropriate physical picture (not particles coexisting in a box). That lead to the complete liquid to vapor phase diagram of neutral, symmetric, uncharged nuclear matter.

1. Introduction

As virtual clusters exist inside nuclei, real clusters are emitted by excited nuclei, carrying with them information about the nuclear liquid to vapor phase transition. After decades of theoretical and experimental studies, recent papers have published what can be considered a quantitative, credible liquid-vapor phase diagram containing the coexistence line up to the critical temperature [1, 2]. Somewhat unexpectedly, this diagram has not been obtained through the study of caloric curves [3, 4] or anomalous heat capacities [5, 6], but rather through the fitting of the charge distributions in multifragmentation by means of a Coulomb-corrected Fisher’s formula [1, 7] giving the cluster composition of a vapor:

\[ n_A(T) = q_0 A^{-\tau} \exp\left[ \frac{\Delta \mu_A}{T} - \frac{\epsilon A^{\sigma}}{T} \right] \]  \hspace{1cm} (1.1)

where \( q_0 \) is a normalization constant [7]; \( \tau \) is the critical exponent giving rise to a power law at criticality; \( A \) is the cluster number; \( \Delta \mu \) is the difference of chemical potentials between the liquid and the vapor; \( c_0 \) is the surface energy coefficient; \( T \) is the temperature; \( \epsilon = (T_C - T)/T_C \) is the distance from the critical temperature \( T_C \); \( \sigma \) is another critical exponent (expected to be approximately 2/3). For \( \Delta \mu = 0 \) the liquid and the vapor are in equilibrium and Eq. (1.1) can be taken to be the equivalent of the coexistence line. More conventionally, one can immediately obtain from Eq. (1.1) the usual \( p, T \) and \( p, \rho, T \) phase diagrams by recalling that in Fisher’s model, the clusterization is assumed to exhaust all the non-idealities of the gas. It then becomes an ideal gas of clusters. Consequently, the total pressure and density are:

\[ p(T) = \sum_A p_A(T) = T \sum_A n_A(T) \]  \hspace{1cm} (1.2)

\[ \rho = \sum_A A n_A(T) \]  \hspace{1cm} (1.3)

as well as the corresponding scaled quantities \( p/p_c \) and \( \rho/\rho_c \).

In order to make use of the Fisher description of the cluster concentrations there were three major obstacles to overcome: finite size effects, the problem of the long-range Coulomb interaction, and the (un)physical picture of particles in a box.
2. Finites size effects

Finite size effects are essential to the study of nuclei and other mesoscopic systems. We have developed a general method to deal with finite size effects in phase transitions [8]. In the case of liquid-vapor phase coexistence, a dilute nearly ideal vapor phase is in equilibrium with a dense liquid-like phase. The interesting case of finiteness is realized when the liquid phase is a finite drop[9]. We introduced the concept of the complement (the residual drop which remains after a cluster has been emitted) in order to further quantify finite size effects and to generalize Fisher’s theory to better describe the cluster yields from extremely small systems [8]. The complement approach consists of evaluating the change in free energy occurring when a particle or cluster is moved from one (finite) phase to another. In the case of a liquid drop in equilibrium with its vapor, this is done by extracting a vapor particle of any given size from the drop and evaluating the energy and entropy changes associated with both the vapor particle and the residual liquid drop (the complement).

Consider a vapor in equilibrium with a drop of its liquid. The system may be a physical system or the Ising realization of it. For each cluster of the vapor we can make the mental exercise of extracting it from the liquid, determining the change in entropy and energy of the drop and cluster system, and then putting it back in the liquid (the equilibrium condition), as if all other clusters of the vapor did not exist. Fisher’s expression can now be written for a drop of size $A0$ in equilibrium with its vapor as follows

$$n_A(T) = q_0 \left[ \frac{A(A_0-A)}{A_0} \right] \exp \left\{ [A^\gamma + (A_0 - A)^\gamma - A_0^\gamma] \frac{c_0 - c_A}{T_c - T} \right\}$$

(2.1)

In other words, we treat the “complement” $(A0 - A)$ in the same fashion as a cluster. While different than the standard Fisher expression, Eq. (1) still admits the same $T_c$ as that of the infinite system. This is because the $A0, A$ dependence of the surface energy finds its exact counterpart in that of the surface entropy.

We are now prepared to compare the Ising yields of the vapor concentrations to our modified Fisher’s droplet model. An example is given in Fig. 1. The yields in the upper panel show the linear Boltzmann behavior associated with a first order phase transition. In the lower panel of Fig. 1 is shown the scaling condition where the ordinate is scaled by the Fisher power law and the abscissa is scaled to reflect the total surface energy over the temperature.

From the Fisher droplet model fits to the yields comes an estimate of the critical temperature of $2.32 \pm 0.02$. This is to be compared to the known analytical solution of $T_c = 2/\ln(1 + \sqrt{2}) = 2.269$. Using that information, the coexistence pressure and density can be calculated. The agreement between the Fisher description, the Ising calculation and the known analytic solution of Onsager [16] is excellent.

Thus, we have obtained a practical way of moving from finite to infinite systems with the Fisher droplet model.

3. Coulomb

In ref. [10], we explored the effects of the Coulomb interaction upon the nuclear liquid phase transition. Because large nuclei are metastable objects, phases, phase coexistence, and phase transitions cannot be defined with any generality and the analogy to liquid vapor is ill-posed for these heavy systems. However, it is possible to account for the Coulomb interaction in the decay rates and obtain the coexistence phase diagram for the corresponding uncharged system [1].

4. Physical picture

We assume (just as in compound nuclear decay) that after prompt emission in the initial phase of the collision, the resulting system relaxes in shape and density and thermalizes on a time scale shorter
than its thermal decay. At this point, the excited nucleus emits particles in vacuum, according to standard statistical decay rate theory. In this picture, there is no surrounding vapor and no confining box; there is no need for either. By studying the outward flux of the first fragments emitted, we can study the nature of the vapor even when it is absent (the virtual vapor) because of the equivalence of the evaporation and condensation fluxes of a liquid in equilibrium with its saturated vapor. Quantitatively, the concentration $n_A(T)$ of any species $A$ in the vapor is related to the corresponding decay rate $R_A(T)$ (or to the decay width $\Gamma_A$) from the nucleus by matching the evaporation and condensation fluxes.

![Graph](image)

**Figure 1:** Upper panel: the yields of clusters of size one (open squares) up to clusters of size ten (open triangles) as a function of temperature from a two dimensional lattice gas calculation (lattice of 40 by 40) at constant density $\rho = 0.2$ (corresponding to a cold liquid drop of $A_0 = 80$) as a function of $1/T$. The lines are fits to the yields using the modified Fisher droplet model described in Eq. (2.1). Lower panel: the same yields scaled by the power law on the ordinate and the surface energy over temperature on the abscissa.

This is the fundamental and simple connection between the compound nucleus decay rate, and Eq. (2.1), the modified Fisher droplet model. In the latter, one immediately recognizes in the exponential the canonical expansion of the standard compound nucleus decay rate, namely, the Boltzmann factor. The surface part of the barrier is isolated from all other components, e.g., Coulomb [10], symmetry, and finite size [8]. Thus, the vapor phase in equilibrium can be completely characterized in terms of the decay rate.
Figure 2: The scaled charge yields for the six indicated reactions. Over 500 points are collapsed onto a single curve which describes the behavior of bulk nuclear matter. The color of the points shows the charge of the fragments. The solid line shows the liquid-vapor coexistence curve of bulk infinite nuclear matter.

Results are shown in Fig. 2 for the indicated reactions. When the yields are scaled by the Coulomb, rotational, symmetry energies, etc. (all terms that are not surface energy), and plotted as a function of the Boltzmann factor associated with surface energy, the different reactions show a similar scaling. This scaled line contains all the information of coexistence between the two phases for chargeless, symmetric, and infinite nuclear matter. The data can be fit to obtain an estimate of $T_c$ and such a procedure yields $T_c = 17.9 \pm 0.4$ MeV. Using the ideal gas approximations (1.2) and (1.3), estimates can be obtained for the critical density $\rho_c$ and critical pressure $p_c$. Such calculations give $\rho_c = 0.06 \pm 0.02$ fm$^{-3}$ and $p_c = 0.31 \pm 0.07$ MeV/fm$^3$.

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