Topology in the Little Higgs Models

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Abstract

We investigate the implications of the nontrivial vacuum structure of little Higgs models. In particular, focusing on the littlest Higgs model, we demonstrate the existence of three types of topological defects. One is a global cosmic string that is truly topological. The second is more subtle; a semilocal cosmic string, which may be stable due to dynamical effects. The final defect is a $Z_2$ monopole solution with an unusual structure. We briefly discuss the possible cosmological consequences of such nonperturbative structures, although we note that these depend crucially on the fermionic content of the models.

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I. INTRODUCTION

In the last few years, a new partial resolution of the hierarchy problem in particle physics has been proposed. This proposal, known as the little Higgs mechanism [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11], introduces new symmetries at the TeV scale, preventing the quadratic running of coupling constants and postponing the hierarchy problem by at least an order of magnitude in energy. Any potential solution to the hierarchy problem deserves serious attention and the little Higgs paradigm has therefore generated a lot of interest.

The basic idea is to make the electroweak Higgs particle a pseudo-Goldstone boson of a higher symmetry group. This construction means that quadratic divergences in the Higgs mass cancel to one loop. Thus, the Higgs remains weakly-coupled beyond the electroweak scale, perhaps up to 10 TeV.

Almost all work on this topic concerns the construction and phenomenological analysis of effective theories implementing the little Higgs idea. In this letter we adopt a different approach and focus on the nonperturbative structures that may be present in specific little Higgs models. Each little Higgs model introduces a new symmetry group at the TeV scale. Some of the new symmetries are local and some global and the group is spontaneously broken down to that of the standard model at electroweak scales. For us this raises the natural question of the possible existence of topological defect solutions in the low-energy theory.

There exists a wealth of literature concerning topological defects which is well summarized in [12, 13]. Here, for completeness, we merely provide a lightning review of the main concepts necessary for the analysis we perform in the rest of the paper.

Consider a field theory described by a continuous symmetry group $G$ which is spontaneously broken to a subgroup $H \subset G$. The space of all accessible vacua of the theory, the vacuum manifold, is defined to be the space of cosets of $H$ in $G$: $\mathcal{M} \equiv G/H$.

In general, the theory possesses a topological defect if some homotopy group $\pi_i(\mathcal{M})$ is nontrivial, where, in particular, $i = 0, 1, 2, 3$ lead to domain walls, cosmic strings, monopoles or textures respectively.

Applying these conditions to the electroweak theory, a particularly useful example here, the vacuum manifold is the space of cosets

$$\mathcal{M}_{EW} = \{[SU(2)_L \times U(1)_Y]/Z_2\}/U(1)_Q \ ,$$

(1)

which is topologically equivalent to the three-sphere, $S^3$, with vanishing zeroth, first and
second homotopy groups. Thus, the electroweak model does not lead to walls, strings, or
free magnetic monopoles. It does however, lead to texture, which exists because \( \pi_3(\mathcal{M}) \) is
nontrivial. It also contains confined magnetic monopoles \[13, 14\].

Further, while the topological structure of the models may be of intrinsic particle physics
interest, when one considers such models in the context of the expanding, cooling universe,
the possibility of the cosmic creation of topological relics of the little Higgs phase is raised.
Such structures may serve three possible purposes. They may, of course, have no observable
consequences in today’s universe. Alternatively, they may yield specific observable signatures
and indeed may even aid in the resolution of some cosmological problems. Finally, in the
most extreme case, it is possible that topological remnants of the little Higgs phase may
predict a cosmological catastrophe, which may be used to bound specific models. Historically
these considerations were considered natural in the case of new symmetry groups at the grand
unified scale and can have interesting effects \[15\] in the presence of the other approach to
the hierarchy problem - supersymmetry. Here we initiate an analogous program of study for
the little Higgs paradigm.

This paper is organized as follows. In section II we consider as a prototypical example
the littlest Higgs model and describe its symmetry groups. In section III we examine in
detail the symmetry breaking structure and in section IV we identify the topological and
embedded defects that result. Finally, in V we summarize our results and comment on the
possible cosmological consequences of the topological aspects of the little Higgs models.

II. THE LITTLEST HIGGS MODEL

We consider the “littlest Higgs” model \[4\]. The model is constructed by starting with a
purely global \( SU(5) \) theory for a scalar field \( \Sigma \) that is a \( 5 \times 5 \) symmetric, complex matrix.
The transformation law for \( \Sigma \) is

\[
\Sigma \rightarrow U \Sigma U^T , \quad U \in SU(5) .
\]  

(2)

Note that \( U^\dagger U = 1 \) and that \( \Sigma \) is multiplied on the right by \( U^T \), not \( U^\dagger \). The Lagrangian
for \( \Sigma \) is

\[
\mathcal{L} = \text{Tr} \left[ (\partial^\mu \Sigma)^* \partial^\mu \Sigma \right] - V(\Sigma) ,
\]  

(3)

3
where

\[ V(\Sigma) = \frac{1}{2} \mu^2 \Sigma^* \Sigma + \frac{\kappa_1}{4} (\Sigma^* \Sigma)^2 + \frac{\kappa_2}{4} (\Sigma^* \Sigma \Sigma^* \Sigma)^2 , \]  

(4)

where \( \mu \) has dimensions of mass and \( \kappa_1 \) and \( \kappa_2 \) are dimensionless coupling constants.

The full global symmetry group of this Lagrangian can be seen by noting that there is an extra set of transformations, corresponding to multiplication of \( \Sigma \) by \( e^{i\alpha}1 \), with \( \alpha \) a real parameter, under which \( \mathcal{L} \) is also invariant. Modding out by the discrete set of symmetries common to both \( SU(5) \) and to this new \( U(1) \) – namely by the center of \( SU(5) \) – reveals the full symmetry group to be

\[ U(5) \cong \frac{[SU(5) \times U(1)]}{Z_5} . \]  

(5)

For \( 5\kappa_1 + \kappa_2 > 0, \kappa_2 > 0 \), the vacuum expectation value (VEV) of \( \Sigma \) is \( \propto 1 \). Given the above analysis of the gauge groups, the subgroup left unbroken by the above VEV is then easily calculated, yielding the full symmetry breaking scheme as

\[ \frac{[SU(5) \times U(1)]}{Z_5} \rightarrow SO(5) \times Z_2 . \]  

(6)

where the left-over \( Z_2 \) is due to transformations with \( \text{Det}(U) = \pm 1 \). In other words, the symmetry breaking is \( U(5) \rightarrow O(5) \).

The \( U(5) \) model above is provided simply as a motivation for the construction of the actual model. The next step in constructing the littlest Higgs model is to introduce gauge fields. However, instead of gauging the full \( U(5) \) only a \( U(2)^2 \) subgroup is gauged. The Lagrangian (3) becomes

\[ \mathcal{L} = \text{Tr} \left[ (D_\mu \Sigma)^* D^\mu \Sigma \right] - V(\Sigma) - \frac{1}{4} \sum_{j=1}^{2} [(W^a_{j\mu \nu})^2 + (B^\mu_{j\nu})^2] , \]  

(7)

where the covariant derivative is given by

\[ D_\mu \Sigma = \partial_\mu \Sigma - \sum_{j=1}^{2} \left[ ig_j W^a_{j\mu} (Q^a_j \Sigma + \Sigma Q^a_j^T T) + ig'_j B_{j\mu} (Y_j \Sigma + \Sigma Y_j^T T) \right] . \]  

(8)

Here \( g_j \) are the coupling constants corresponding to the two \( SU(2) \) subgroups of \( SU(5) \) and \( g'_j \) are those corresponding to the two \( U(1) \) subgroups, and \( W^a_j \) and \( B_j \) are the \( SU(2) \) and \( U(1) \) gauge fields. Thus one has gauged a \( [(SU(2) \times U(1))/Z_2]^2 \equiv K \) subgroup of \( SU(5) \).

What remains of the global \( U(5) \) symmetry is merely the \( U(1) \) factor written explicitly in Eq. (5). The generators of the first \( SU(2) \), the second \( SU(2) \), the first \( U(1) \) and the second
$U(1)$, embedded into $SU(5)$, respectively are $^4_1$,

$$Q_1^a = \begin{pmatrix} \sigma^a/2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$  

(9)

$$Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sigma^a/2 \end{pmatrix},$$  

(10)

$$Y_1 = \frac{1}{10}\text{diag}(-3,-3,2,2,2),$$  

(11)

$$Y_2 = \frac{1}{10}\text{diag}(-2,-2,-2,3,3),$$  

(12)

where $\sigma^a$ are the Pauli spin matrices.

In this way the gauging has diminished the symmetry of the model from $U(5)$ to $K \times U(1)_g$ where the subscript $g$ denotes that the symmetry is global. Note that the potential of the model continues to be given by Eq. (4) and still carries the full $U(5)$ symmetry. This fact is important for us and we will describe its consequences more fully in the next section.

### III. VACUUM STRUCTURE OF THE MODEL

The Lagrangian $^4_1$ of the little Higgs model in has the peculiar feature that the symmetries of the gradient and potential terms are different. Before proceeding to elaborate this feature, let us consider the standard electroweak model where a similar feature is present.

In the electroweak model, the potential is

$$V_{ew}(\Phi) = \lambda(\Phi^\dagger\Phi - \eta^2)^2,$$  

(13)

where $\Phi^T = (\phi_1 + i\phi_2, \phi_3 + i\phi_4)$ is the standard model Higgs field and $\phi_i$ are its real-valued components. Therefore we can write

$$V_{ew}(\Phi) = \lambda \left(\sum_{i=1}^4 \phi_i^2 - \eta^2\right)^2,$$  

(14)

from which it is clear that the continuous symmetry of the potential is $SO(4)$, which is isomorphic to $SU(2)_L \times SU(2)_R$, with subscripts conforming to conventional notation$^1$. Of

$^1$ $SU(2)_R$ is often called the “custodial” symmetry.
course, in the electroweak theory only the $SU(2)_L$ factor and a $U(1)$ subgroup of the second $SU(2)_R$ factor are gauged. This diminishes the symmetry of the model to the electroweak symmetry group $[SU(2)_L \times U(1)_Y]/Z_2$. However, since the gauging affects only the gradient terms, the potential retains the full SO(4) symmetry. Once $\Phi$ gets a VEV, this symmetry is spontaneously broken down to $SU(2)_{L+R}$, while the gauged electroweak symmetry group is broken down to $U(1)_Q$. These considerations may be summarized in the following chart

$$
\text{Global} : \quad [SU(2)_L \times SU(2)_R]/Z_2 \rightarrow SU(2)_{L+R}
$$

$$
\downarrow \quad \downarrow \quad \downarrow
$$

$$
\text{Gauge} : \quad [SU(2)_L \times U(1)_Y]/Z_2 \rightarrow U(1)_Q
$$

(15)

Now consider the vacuum structure of the electroweak model. $V_{\text{ew}}(\Phi)$ is minimized on a three sphere, namely, $\sum \phi_i^2 = \eta^2$. The manifold obtained by symmetry considerations, $\{[SU(2)_L \times U(1)_Y]/Z_2\}/U(1)_Q$ is also three dimensional but it is important to realize that this is merely a coincidence. We shall see that in the littlest Higgs model, in which a similar structure exists, such a coincidence does not occur. The electroweak model does not contain any topological defects. However, if $SU(2)_L$ is left ungauged the theory does contain stable semilocal defects [13, 17, 18, 19] for the following reason. As above, the minimum of the potential remains an $S^3$, on which there are special orbits that are gauged (Fig. 1). (If there are gradients of $\Phi$ along these orbits, they can be completely compensated by a gauge field.) These orbits are $S^1$'s and hence are topologically non-trivial. A field configuration that lies on such a gauge orbit can be deformed to a constant field everywhere but the deformation costs gradient energy since the configuration needs to be lifted off the orbit and contracted on the $S^3$. If the scalar couplings are large compared to the gauge couplings, then this energy cost is large enough to stabilize the field configuration, as has been explicitly seen in the case of semilocal and electroweak strings. Since the defect is stable due to an interplay between global and gauge symmetries, such defects are called semilocal cosmic strings. For the electroweak model with $SU(2)_L$ left ungauged, the electroweak semilocal string is stable provided $m_H^2 < m_Z^2$ where $m_H$ and $m_Z$ are the masses of the Higgs scalar and the vector boson.

We now move on to the littlest Higgs model. Here the potential $V(\Sigma)$ is invariant under the full $U(5)$ group, while the Lagrangian as a whole respects a $K \times U(1)_g$ symmetry.

The potential is minimized on a $25 - 10 = 15$ dimensional manifold $M_V \equiv U(5)/O(5)$. However, the vacuum manifold $M_L$ corresponding to the full Lagrangian is isomorphic to a
FIG. 1: The vacuum manifold of the standard electroweak model is an $S^3$. If only the $U(1)$ group of the electroweak symmetry group is gauged, the manifold is better viewed as the Hopf fibration of $S^3$ \cite{20}. Then there are topologically non-trivial paths corresponding to orbits of the gauged $U(1)$. Field configurations that lie in the gauge orbit are trivial in the full $S^3$, yet to deform them to a trivial field configuration requires gradients that cannot be compensated by the gauge field. Hence the deformation costs energy and this is why topological defects corresponding to the gauged orbits can be stable even in the full model. Such defects are called “semilocal”.

coset space

$$M_L \cong \frac{K \times U(1)_g}{U(2) \times Z_2}.$$  

(16)

This is only a $9 - 4 = 5$ dimensional space. As depicted in Fig. 2 $M_V \supset M_L$ and so non-trivial topological features of $M_L$ need not be non-trivial in $M_V$. Therefore, in order to discuss topological defects in the model we need to (i) find any non-trivial topological features of $M_L$ and (ii) check if this non-trivial topology in $M_L$ can be trivialized in the full manifold $M_V$. Of course, if such a trivialization is not possible then the topological defect solution in $M_L$ is a genuine topological defect in the full theory. However, if the topology of the configuration is trivial in $M_V$, then the topological defect in $M_L$ may still be a semilocal defect of the full theory and may be stable over some parameter range. As we shall now see, the littlest Higgs model has both topological and semilocal strings.

The chart for this model corresponding to the one shown above for the electroweak model
is:
\[
\text{Global : } [SU(5) \times U(1)_{g}]/Z_{5} \rightarrow SO(5) \times Z_{2}
\]
\[
\downarrow \downarrow \downarrow
\]
\[
\text{Gauge : } K \times U(1)_{g} \rightarrow \{[SU(2)_{L} \times U(1)_{Y}]/Z_{2}\} \times Z_{2}
\]
where, as defined earlier,
\[
K = \{[SU(2) \times U(1)]/Z_{2}\}^{2}
\]

where, as defined earlier,
\[
K = \{[SU(2) \times U(1)]/Z_{2}\}^{2}
\]

FIG. 2: The minimum of the potential in the littlest Higgs model is a 15 dimensional space and the gauge orbits form a 5 dimensional subspace. The 5 dimensional subspace may have non-trivial topology but the corresponding semilocal defects will be stable only if the gradient cost in lifting the field configurations off the gauge orbit is significant.

IV. TOPOLOGICAL DEFECTS IN THE LITTLEST HIGGS MODEL

For the Lagrangian (7) the structure of the unbroken symmetry groups was best elucidated in a basis in which \(\langle \Sigma \rangle \propto 1\) (see the discussion below Eq. (5)). However, at this stage it is simpler to use a different basis in which the VEV is given by
\[
\langle \Sigma \rangle = \Sigma_{0} \equiv \begin{pmatrix}
0_{2 \times 2} & 0_{2} & 1_{2 \times 2} \\
0_{2} & 1 & 0_{2} \\
1_{2 \times 2} & 0_{2} & 0_{2 \times 2}
\end{pmatrix}
\]

The symmetry breaking scheme is still as shown in Eq. (17) and it is straightforward to check that the unbroken generators corresponding to the \(SU(2)_{L}\) remaining after the symmetry breaking are \(Q_{1}^{a} + Q_{2}^{a}\) (see Eq. (12)). Similarly the unbroken \(U(1)_{Y}\) has generator \(Y_{1} + Y_{2}\).
A. String Solutions

We are now in a position to find the topological defects in the littlest Higgs model. First consider the spontaneous breaking of the $U(1)_g$ factor in the symmetry breaking. The $U(1)_g$ factor spontaneously breaks to a $Z_2$ subgroup. Since $\pi_1(U(1)/Z_2) = \mathbb{Z}$, the group of integers under addition, it is clear that the model contains global $U(1)$ strings. An appropriate ansatz for these objects is given by

$$\Sigma_g(r, \theta) = f_g(r)e^{i\theta}\Sigma_0 ,$$

(20)

where $f_g(r)$ is a real profile function for the global string.

Similarly, in the gauged $U(1)$ sector there is a cosmic string configuration corresponding to the broken $U(1)$ symmetry whose generator is $Y_1 - Y_2$ which is orthogonal to $Y_1 + Y_2$, the generator of $U(1)_Y$. Let us define

$$\tilde{Y} = 10(Y_1 - Y_2) = \text{diag}(-1, -1, 4, -1, -1) .$$

(21)

Then an ansatz for the string is

$$\Sigma_l(r, \theta) = f_l(r)e^{i\tilde{Y}\theta}\Sigma_0 , \quad A_\theta = \frac{v_l(r)}{g' r} ,$$

(22)

where, in terms of the hypercharge gauge fields in Eq. (8),

$$A_\theta = \cos(\alpha)B_{2\theta} - \sin(\alpha)B_{1\theta}$$

(23)

and

$$g' \equiv \sqrt{g_1'^2 + g_2'^2} , \quad \tan(\alpha) \equiv \frac{g_1'}{g_2'} ,$$

(24)

with all other fields vanishing. Here $f_l(r)$ and $v_l(r)$ are real profile functions, obeying the usual second order ordinary differential equations satisfied by the Nielsen-Olesen vortex, with boundary conditions

$$f_l(0) = \frac{d v_l}{dr}(0) = v_l(\infty) = 0 , \quad f_l(\infty) = \left[\frac{1}{5\kappa_1 + \kappa_2}\right]^{1/2} \mu .$$

(25)

This string has a tension $T \sim (10 \text{ TeV})^2$ and carries a magnetic flux associated with the gauge field $A_\mu$ given by

$$\Phi_A = -\frac{2\pi}{g'} .$$

(26)
Note that these solutions may not be the least energy solutions for the given topology. For example, the presence of bosonic condensates could change the energy per unit length. However, the existence of a condensate is a model-dependent question.

This second string is not apparent in the global symmetry breaking shown in the top line of Eq. (17) since \( \pi_1(SU(5)/SO(5)) \) is trivial. Clearly this is because the string configuration can be deformed to the trivial configuration by using transformations belonging to the full \( SU(5) \). Thus the second string is a semilocal string.

The stability of the semilocal string depends on the relative importance of the potential energy to the gradient energy. If the scalar coupling constants are large, it is favorable to minimize the potential energy even at some cost of gradient energy. If the gauge coupling constant is large, it is most favorable to have vanishing gradient energy.

The stability condition that is analogous to the electroweak condition is that \( m_s^2 < m_v^2 \) where \( m_s \) and \( m_v \) are the scalar and vector masses in the model. In terms of the coupling constants:

\[
\frac{5}{2} \left( \frac{5\kappa_1 + \kappa_2}{g_1^2 + g_2^2} \right) < 1. \tag{27}
\]

B. A Monopole Solution

We now turn briefly to another defect solution in the littlest Higgs model. Although the symmetry breaking scheme is somewhat complicated, as described in [17], it is clear that a subset of the scheme involves \( SU(5) \to SO(5) \), effected by the symmetric tensor representation \( \Sigma(x) \). However, it is relatively simple to show that

\[ \pi_2[SU(5)/SO(5)] = Z_2, \]  

(28)

(for example by using the exact homotopy sequence which yields \( \pi_2[SU(5)/SO(5)] = \pi_1[SO(5)] \)). Hence, the theory contains a particularly interesting type of magnetic monopole – a \( Z_2 \) monopole (see for example [21]). Because these monopoles correspond to a \( Z_2 \) homotopy group, the monopole and the antimonopole are identical.

If the only symmetry breaking was the global \( SU(5) \to SO(5) \), then these monopoles would carry a purely global non-Abelian \( SO(5) \) charge. However, since the littlest Higgs model involves gauging subgroups of the \( SU(5) \), the resulting monopoles will be partially gauged. Consider an \( SU(3) \) subgroup of \( SU(5) \) that lies in the upper \( 3 \times 3 \) block of \( SU(5) \)
group elements. When $\Sigma$ gets a VEV proportional to $1$, the $SU(3)$ subgroup breaks down to $SO(3)$, and this symmetry breaking too leads to $Z_2$ monopoles. Therefore, to construct the monopoles, we need only consider the symmetry breaking $SU(3) \rightarrow SO(3)$. Now an $SU(2) \times U(1)$ subgroup of this $SU(3)$ is gauged. But this is the maximal non-trivial subgroup of $SU(3)$. The scalar field of a $Z_2$ monopole on the two sphere at infinity, is given by the action of elements of $SU(3)$ on some chosen VEV, say proportional to $1$. However, since none of the $SU(3)$ elements commutes with all the elements of the $SU(2) \times U(1)$ maximal subgroup, at least some of the angular gradients can be compensated by suitable gauge fields. This is what we mean by “partial gauging”. An explicit solution of the partially gauged $SU(3) \rightarrow SO(3)$ monopole is not known but would be very interesting to work out, especially if the monopole carries some electromagnetic field distribution after the electroweak symmetry breaking.

V. CONCLUSIONS AND COMMENTS

Little Higgs models provide a new logical possibility for explaining the stability of the weak scale. In such models the hierarchy problem is postponed by an order of magnitude, pushing the relevant scale up to around 10 TeV. In this paper we have investigated the nonperturbative structure of these models, in particular their topological defect structure, as a complement to detailed studies of their low-energy phenomenology. This seems to be a natural study to perform, since little Higgs models make use of new gauge and global symmetries at the TeV scale, and thus inherently involve new symmetry breaking schemes that one expects to be realized during the thermal evolution of the expanding universe.

Given the large number of possible ways to implement the little Higgs paradigm, we have chosen to focus on one of the simplest such examples, the littlest Higgs model. Our analysis demonstrates the existence of three distinct structures. The first is a global abelian cosmic string solution, topologically stable, arising from the breakdown of a global $U(1)$ symmetry. The second is a more subtle object; a semilocal gauge string, embedded in the larger group structure of the gauge sector of the theory. This object is not topologically stable, but may be stable dynamically, depending on the values of the coupling constants in the theory. The final object is what we describe as a partially gauged $Z_2$ monopole, which is also topologically stable.
We have constructed the appropriate ansätze for the scalar fields and the gauge fields making up these defects. In the case of the semilocal string we have also identified the gauge flux carried by the defect.

What remains to be done is a careful analysis of the possible cosmological implications of our findings. Clearly TeV-scale strings and monopoles will have negligible gravitational effects (for example, one does not expect them to play a role in structure formation.) However, the microphysics of such objects may be important in some circumstances. One example is their potential role in weak scale baryogenesis \cite{22, 23, 24, 25}. Another interesting possibility arises if the strings are superconducting. As originally pointed out by Witten \cite{26}, it is possible for cosmic strings to carry supercurrents along them. These may be due to the presence of a scalar condensate on the string, to fermion zero modes along it, or even more exotic types of superconductivity. The evolution of a network of such superconducting cosmic strings can differ from a nonsuperconducting one. In particular, the supercurrent along loops of string builds up as the loop radiates away its energy. This can affect the endpoint of loop evolution. In some cases the supercurrent can become large enough to destabilize the loop. In others, the current can compete with the tension of the string loop and result in stable remnants, known as vortons \cite{27} that constrain the theory \cite{28, 29} or even may act as dark matter \cite{28, 30}. These latter suggestions depend crucially on the fermionic content of the theory, and particularly on the potential existence of fermion zero modes on the strings, leading to superconductivity. Such effects require a model-specific analysis that is beyond the scope of this paper and which we therefore reserve for future work.

Although we have also demonstrated the existence of a partially gauged $\text{SU}(3) \rightarrow \text{SO}(3)$ monopole, as we have commented, the explicit solution is not known. An interesting future direction is to explicitly construct such a solution, since it is possible that the monopole carries some electromagnetic field distribution after the electroweak symmetry breaking, perhaps resulting in cosmological consequences.

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[1] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 513, 232 (2001) \texttt{arXiv:hep-ph/0105239}.

[2] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Rev. Lett. 86, 4757 (2001) \texttt{arXiv:hep-th/0104005}.

[3] C. T. Hill, S. Pokorski and J. Wang, Phys. Rev. D 64, 105005 (2001) \texttt{arXiv:hep-th/0104035}.

[4] N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 0207, 034 (2002) \texttt{arXiv:hep-ph/0206021}.

[5] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire and J. G. Wacker, JHEP 0208, 021 (2002) \texttt{arXiv:hep-ph/0206020}.

[6] I. Low, W. Skiba and D. Smith, Phys. Rev. D 66, 072001 (2002) \texttt{arXiv:hep-ph/0207243}.

[7] D. E. Kaplan and M. Schmaltz, JHEP 0310, 039 (2003) \texttt{arXiv:hep-ph/0302049}.

[8] S. Chang and J. G. Wacker, Phys. Rev. D 69, 035002 (2004) \texttt{arXiv:hep-ph/0303001}.

[9] W. Skiba and J. Terning, Phys. Rev. D 68, 075001 (2003) \texttt{arXiv:hep-ph/0305302}.

[10] S. Chang, JHEP 0312, 057 (2003) \texttt{arXiv:hep-ph/0306034}.

[11] E. Katz, J. y. Lee, A. E. Nelson and D. G. E. Walker, \texttt{arXiv:hep-ph/0312287}.

[12] A. Vilenkin and E.P.S. Shellard, \textit{Cosmic Strings and Other Topological Defects} (Cambridge: Cambridge Univ. Press, 1994).

[13] A. Achucarro and T. Vachaspati, Phys. Rept. 327, 347 (2000) [Phys. Rept. 327, 427 (2000)] \texttt{arXiv:hep-ph/9904229}.

[14] Y. Nambu, Nucl. Phys. B 130, 505 (1977).

[15] S. C. Davis, A. C. Davis and M. Trodden, Phys. Lett. B 405, 257 (1997) \texttt{arXiv:hep-ph/9702360}.

[16] L. F. Li, Phys. Rev. D 9, 1723 (1974).

[17] T. Vachaspati and A. Achucarro, Phys. Rev. D 44, 3067 (1991).

[18] M. Hindmarsh, Phys. Rev. Lett. 68, 1263 (1992).

[19] J. Preskill, Phys. Rev. D 46, 4218 (1992) \texttt{arXiv:hep-ph/9206216}.

13
[20] G. W. Gibbons, M. E. Ortiz, F. Ruiz Ruiz and T. M. Samols, Nucl. Phys. B 385, 127 (1992) arXiv:hep-th/9203023.

[21] J. Preskill, CALT-68-1287 Lectures presented at the 1985 Les Houches Summer School, Les Houches, France, Jul 1 - Aug 8, 1985

[22] R. H. Brandenberger, A. C. Davis and A. M. Matheson, Phys. Lett. B 218, 304 (1989).

[23] R. H. Brandenberger, A. C. Davis and M. Hindmarsh, Phys. Lett. B 263, 239 (1991).

[24] R. H. Brandenberger and A. C. Davis, Phys. Lett. B 308, 79 (1993) arXiv:astro-ph/9206001.

[25] R. H. Brandenberger, A. C. Davis and M. Trodden, Phys. Lett. B 335, 123 (1994) arXiv:hep-ph/9403215.

[26] E. Witten, Nucl. Phys. B 249, 557 (1985).

[27] R. L. Davis and E. P. S. Shellard, Nucl. Phys. B 323, 209 (1989).

[28] R. H. Brandenberger, B. Carter, A. C. Davis and M. Trodden, Phys. Rev. D 54, 6059 (1996) arXiv:hep-ph/9605382.

[29] B. Carter and A. C. Davis, Phys. Rev. D 61, 123501 (2000) arXiv:hep-ph/9910560.

[30] C. J. A. Martins and E. P. S. Shellard, Astrophys. Space Sci. 261, 325 (1999).