A parameter-selection method is proposed to improve the accuracy of the truncated singular value decomposition (TSVD) method, which is based on the Wilson-θ method and the principle of minimum response error, for dynamic load identification. First, using the Green kernel-function matrix, the dynamic load-identification model of the multi-degree-of-freedom system is established. Second, the response corresponding to the dynamic load is identified using the Wilson-θ method, and the minimum error between the response and the input response is obtained. Then, the best regularization parameters are obtained, and the dynamic load is identified using the TSVD regularization method. Finally, the SIMPACK dynamic model of a rail vehicle is established. Taking the German high-interference spectrum as the input, the axle-box displacement and the vertical wheel-rail force of each wheelset at speeds of 100, 160, and 200 km/h are simulated. Taking the simulated axle-box displacement response with 0%, 5%, and 10% noise as the input, the proposed load-identification model and regularization-parameter selection method are used to identify the vertical wheel-rail force of a rail vehicle. The effects of different track spectra on the identification results are considered. The results indicate that this method has a high identification accuracy for the wheel-rail vertical dynamic load. With an increase in the vehicle speed, the correlation coefficient for identifying the dynamic load decreases, but the correlation remains strong. At the speed of 200 km/h, when the input response noise level is 0%, the dynamic load identification correlation coefficient is 0.9556, which corresponds to extremely strong correlation. When the input response contains 5% noise, this method has stronger robustness than L-curve method, and the dynamic load identification correlation coefficient is 0.6354, which corresponds to strong correlation. The proposed load-identification model and regularization-parameter selection method have important theoretical and engineering application value for wheel-rail force monitoring and safety assessment of running trains.

1. Introduction

Dynamic load identification is the second kind of inverse problem in dynamic problems. The commonly used dynamic load-identification methods include the frequency-domain method, time-domain method, weighted acceleration method, series coefficient balance method, inverse system method, and wavelet analysis method [1]. Among them, the time-domain method is popular because of its intuitive calculation results and easy engineering application and has become a hotspot of dynamic load identification research. Because the dynamic load identification is an inverse problem, it often has ill-conditioned characteristics; thus, in the process of identification, it often fails because of small disturbances. The ill-conditioned properties of inverse problems can be reduced via regularization methods. The commonly used regularization methods include the Tikhonov regularization method, truncated singular value decomposition (TSVD) method, and iterative regularization method. In the regularization process, the selection of the regularization parameters plays a significant role in the success or failure of dynamic load identification. For the
Tychonov regularization method combined with the L-curve regularization-parameter selection method is low and it is often difficult to obtain accurate regularization parameters, proposed an ICAA regularization-parameter selection method based on Newmark-β. Compared with the L-curve method, this method has a higher calculation efficiency and accuracy and is more convenient and intuitive to use. Using the Green function method, Miao et al. [5] compared and analysed different regularization techniques combined with different regularization-parameter selection methods. Numerical examples and experiments indicate that the convergence and curve smoothness of TSVD regularization combined with the GCV method are better than those of Tikhonov regularization combined with the L-curve method. Li et al. [6] proposed a load-identification method to reduce the morbidity of the transfer function and weaken the influence of noise in two steps. According to the condition number of the transfer function, the optimal combination of measuring points for the structural response is obtained, and the transfer-function matrix with the lowest morbidity degree is obtained. Then, the Tikhonov regularization method combined with the B-spline function is used to interpolate the L-curve for obtaining more accurate regularization parameters. Simulation results indicated that this method can effectively reduce the error of load identification. Wang et al. [7] proposed a new regularization operator to identify the multi-source loads on the surfaces of composite laminated cylindrical shells, obtained the transient displacement response via the finite-element method, identified the multi-source dynamic loads on the composite surface, and verified the effectiveness and robustness of the method. Ma and Hua [8] proposed a homotopy regularization method based on the L-curve method and verified its reliability through a comparison with the load-identification results of the Tikhonov regularization method combined with L-curve method. Liu et al. [9] proposed a dynamic load-identification method based on interpolation, which involves dividing the load into a series of time elements and approximating the load of each time element using an interpolation function. Then, in the whole time domain, the dynamic response under the load of the interpolation function is calculated via a finite-element analysis, and the global kernel-function matrix is used to identify the load. Compared with the traditional Green kernel function method, this method significantly reduces the morbidity of the global kernel-function matrix. By analyzing the observability of augmented Kalman filter, Naets et al. [10] found that the estimation based on acceleration measurement is unobservable, which will lead to unreliable estimation. By adding virtual measurement at all degrees of freedom, the stability of the identification result is improved. Zhao et al. [11] proposed a new unconditionally stable dynamic load-identification method based on the Houbolt method and backward difference method, and the antinoise ability of the method was verified by a numerical example.

Wheel-rail force is an important factor to evaluate the stability and safety of railway vehicles, and it is of great significance to obtain the wheel - rail force in the process of vehicle operation. At present, the acquisition of wheel - rail force is mainly divided into two kinds: direct measurement (instrumented wheelset, force-measuring rail) and indirect measurement (load identification). The direct measurement method has some problems, such as high cost, complex calibration and follow-up maintenance, so it is a more economical and convenient way to apply load identification theory to wheel-rail force acquisition. Uhl [12] proposed a load identification method based on Bellman's principle, and applied this method to the identification of wheel - rail force, which can be applied to the test of railway equipment. Mehrpouya and Hamid[13] established the finite element model of a freight car, and the model is corrected using the measured mode of the car body. Then, they reduced the calculation scale by substructure method, established the frequency response function matrix of the system, and completed the identification of wheel - rail force of H665 bogie. Ronasi et al. [14] proposed an optimisation method for wheel - rail force identification. The wheel is regarded as a two-dimensional disc, and its finite element model is established. The wheel - rail force is identified through the radial strain time history of the wheel, and the robustness of the method is improved by adding regularization method. Wang et al. [15] proposed a time-domain load-identification method based on inverse structure filtering, which resolves the difficulty of load identification caused by non-parallel inverse system instability. The method was validated by rail vehicle dynamics simulations and experiments. Zhu et al. [16] developed a time-domain dynamic load-identification method based on the Duhamel integral, used it to identify the wheel-rail forces of rail vehicles, and compared the identification results with dynamic simulation results. The results indicated that the method has a high identification accuracy and is useful for vehicle operation safety assessment.

To identify the wheel-rail forces of rail vehicles, on the basis of reference [4], the dynamic load-identification model of the multi-degree-of-freedom system is established using the Green function method, and a TSVD regularization-parameter selection method based on Wilson-θ is proposed. Taking a rail vehicle as the research object, the vertical dynamic load of the rail vehicle was identified under a certain noise level, and compared with the SIMPACK simulation model having the same dynamic parameters, a good identification result was obtained.
2. Load-Identification Model Based on Green Function Method

In this study, the forward model of the system is established via the Green kernel function method. For the single-degree-of-freedom system, it is expressed as follows:

\[ x = HF, \]

where \( x = [x_1, x_2, \ldots, x_m]^T \) represents the response array of the system; \( F = [F_1, F_2, \ldots, F_m]^T \) represents the load array of the system; \( x_i \) and \( F_i \) represent the response and load values, respectively, of the \( i \)th moment; \( m \) represents the number of sampling points; and \( H \) represents the Green kernel-function matrix of the system. Formula (1) can be expanded as follows:

\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_m \\
\end{bmatrix} =
\begin{bmatrix}
  h_1 & h_1 & & \\
  h_2 & h_1 & & \\
  \vdots & \vdots & \ddots & \\
  h_m & h_{m-1} & \cdots & h_1 \\
\end{bmatrix}
\begin{bmatrix}
  F_1 \\
  F_2 \\
  \vdots \\
  F_m \\
\end{bmatrix} \Delta t.
\]

(2)

For the multi-degree-of-freedom system, the Green kernel-function matrix of the single-degree-of-freedom system can be used to obtain the forward model of the multi-degree-of-freedom system [17], as follows:

\[ h_i = \sum_{i=1}^{\infty} \frac{\Phi_i \Phi_i^T e^{-\omega_i \Delta t} \sin \omega_i \Delta t}{m_i \omega_i}, \]

(3)

\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_N \\
\end{bmatrix} =
\begin{bmatrix}
  H_1^1 & H_1^2 & \cdots & H_1^M \\
  H_2^1 & H_2^2 & \cdots & H_2^M \\
  \vdots & \vdots & \ddots & \vdots \\
  H_N^1 & H_N^2 & \cdots & H_N^M \\
\end{bmatrix}
\begin{bmatrix}
  F_1 \\
  F_2 \\
  \vdots \\
  F_M \\
\end{bmatrix}.
\]

(4)

where \( h_i \) represents the Green function matrix of the multi-degree-of-freedom system (\( h_i \) has different forms in equation (3), it is applicable to displacement response); \( \Phi_i \) represents the modal vector; \( m_i \) represents the modal mass; \( \omega_i \) represents the modal damping frequency; \( \zeta_i \) represents the modal damping ratio; \( \omega_i \) represents the modal frequency; \( x_i \) represents the response array of the \( i \)th position; \( F_i \) represents the load array of the \( i \)th position; \( H_i^j \) represents the Green function matrix between the load action point \( i \) and the response measurement point \( j \); and \( M \) and \( N \) represent the number of dynamic loads acting on the system and the number of response measurement points, respectively. To ensure that the linear equations are positive definite or hyper-positive definite, it is necessary to ensure that \( N \geq M \).

After the forward model is established, assuming that the kernel-function matrix \( H \) is invertible, the load can be identified by inverting the kernel-function matrix \( H \). For the single-degree-of-freedom model, the load-identification formula is given by (5). For the multi-degree-of-freedom model, the load-identification formula is given by (6).

\[ F = H^{-1} x, \]

(5)

\[
\begin{bmatrix}
  F_1 \\
  F_2 \\
  \vdots \\
  F_M \\
\end{bmatrix} =
\begin{bmatrix}
  H_1^1 & H_1^2 & L & H_1^M \sigma_n^{-1} \\
  H_2^1 & H_2^2 & L & H_2^M \\
  \vdots & \vdots & \ddots & \vdots \\
  H_N^1 & H_N^2 & \cdots & H_N^M \\
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_N \\
\end{bmatrix}.
\]

(6)

Because the kernel-function matrix \( H \) is usually ill-conditioned, the inverse problem is ill-posed. When the input response is polluted by a small amount of noise, the identified load fluctuates significantly, leading to the failure of load identification. Regularization methods are often employed to deal with ill-posed problems. The Tikhonov method and TSVD method are commonly used.

In this study, the TSVD regularization method is used to reduce the ill-posedness of kernel-function matrix \( H \) by truncating the small singular value of \( H \) to reduce the influence of noise pollution on the load-identification results.

The TSVD regularization solution is expressed as follows [18]:

\[ F_{TSVD} = \sum_{i=1}^{n} \sigma_i^{-1} (u_i^T x_i) v_i, \]

(7)

where \( \sigma_i \) represents the singular value of kernel-function matrix \( H \), \( u_i \) represents the left singular array of \( H \), \( x_i \) represents the system response with noise, and \( v_i \) represents the right singular array of \( H \). By obtaining the best truncation order \( n \) (n is the regularization parameter), the regularization solution of the TSVD regularization method is obtained, for achieving load identification.

3. Wilson-θ Method

The Wilson-θ method is a commonly used method for calculating the responses of dynamic systems and is an extension of the linear acceleration method. The Wilson-θ method assumes that the acceleration varies linearly in \( t + \theta \Delta t \), where \( \theta \) is an adjustable parameter (\( \theta \geq 1.0 \)). When \( \theta \geq 1.37 \), the method is unconditionally stable, and the optimal value of \( \theta \) is 1.420815 [19].

According to the assumption that the acceleration varies linearly in the range \( t + \theta \Delta t \), the acceleration expression between \( t \) and \( t + \theta \Delta t \) can be obtained, as given by equation (8). Through the integration of (8), the velocity and displacement expressions can be obtained. Using \( x = \theta \Delta t \), the velocity and displacement at time \( t + \theta \Delta t \) can be determined, as given by equations (9) and (10), respectively.

\[ \ddot{x}_{t+\theta t} = \dot{x}_t + \frac{t}{\theta \Delta t} (\ddot{x}_{t+\theta t} - \dot{x}_t), \]

(8)

\[ \dot{x}_{t+\theta t} = \dot{x}_t + \frac{\theta \Delta t}{2} (\ddot{x}_{t+\theta t} + \ddot{x}_t), \]

(9)

\[ x_{t+\theta t} = x_t + \theta \Delta t \dot{x}_t + \frac{\theta^2 \Delta t^2}{6} (\ddot{x}_{t+\theta t} + 2 \ddot{x}_t). \]

(10)
According to equations (9) and (10), $\ddot{x}_{t+\theta\Delta t}$ and $\ddot{x}_{t+\theta\Delta t}$ can be represented by $x_{t+\theta\Delta t}$, in accordance with (11) and (12).

$$\ddot{x}_{t+\theta\Delta t} = \frac{6}{\theta^2 \Delta t^2} (x_{t+\theta\Delta t} - x_t) - \frac{6}{\theta \Delta t} \dot{x}_t - 2\ddot{x}_t, \tag{11}$$

$$\ddot{x}_{t+\theta\Delta t} = \frac{3}{\theta \Delta t} (x_{t+\theta\Delta t} - x_t) - 2\dot{x}_t - \frac{\theta \Delta t}{2} \dddot{x}_t. \tag{12}$$

The system equilibrium equation of $t + \theta\Delta t$ moment can be obtained by bringing equations (11) and (12) into the dynamic (13), as given by (14).

$$M\ddot{x} + C\dot{x} + Kx = Q,$$ \tag{13}

$$M\ddot{x}_{t+\theta\Delta t} + C\dot{x}_{t+\theta\Delta t} + Kx_{t+\theta\Delta t} = Q_{\text{t+\theta\Delta t}}. \tag{14}$$

By solving $x_{t+\theta\Delta t}$ and bringing it into (11), $\ddot{x}_{t+\theta\Delta t}$ can be obtained.

Finally, let $\tau = \Delta t$. The acceleration, velocity, and displacement responses of moment $t + \Delta t$ can be obtained using equations (15)–(17), respectively.

$$\ddot{x}_{t+\Delta t} = \frac{6}{\theta (\theta \Delta t)^2} (x_{t+\theta\Delta t} - x_t) - \frac{6}{\theta (\theta \Delta t)} \dot{x}_t + \left( 1 - \frac{3}{\theta} \right) \dddot{x}_t, \tag{15}$$

$$\dot{x}_{t+\Delta t} = \dot{x}_t + \Delta t \dddot{x}_t + \frac{\Delta t^2}{2} (\dddot{x}_{t+\theta\Delta t} + \dddot{x}_t), \tag{16}$$

$$x_{t+\Delta t} = x_t + \Delta t \dot{x}_t + \frac{\Delta t^2}{6} (\dddot{x}_{t+\theta\Delta t} + 2\dddot{x}_t). \tag{17}$$

### 4. TSVD Regularization-Parameter Selection Method Based on Wilson-$\theta$

The use of conventional methods, such as the L-curve method, for selecting regularization parameters can lead to a nonideal regularization effect. Therefore, a parameter-selection method for TSVD regularization based on Wilson-$\theta$ is proposed herein.

A flowchart of the TSVD regularization-parameter selection method based on Wilson-$\theta$ is shown in Figure 1. The method can be divided into the following steps.

1. The external load $P$ to be identified is input ($P$ can be obtained via software simulation or generated by a function), the dynamic model of the system is established, and the response of the system is obtained (this method takes the displacement response as an example). If $P$ is obtained via simulation, the corresponding system response to $P$ is also obtained via simulation; if $P$ is generated by a function, the system response caused by $P$ is calculated via numerical methods.

2. Random noise is added to the response obtained from (1), and the response after the addition of noise is $x_{\text{noise}} = x + \delta \sum \frac{n_i}{N} |N$. Here, $\delta$ represents the noise level; $n_i$ represents the length of the response signal; and $N$ is a random vector with a mean value of 0, a deviation of 1, and the same dimension as $x$.

3. The load-identification model is established using the Green function method, and the response $x_{\text{noise}}$ after the addition of noise is taken as the input of the load-identification model.

4. Different regularization parameters $k$ within the range of a relatively high truncation order are entered (for example, the truncation order starts from $k = 0.8mN$ until $k = mN$).

5. The TSVD regularization method is used to calculate the identification load values $\tilde{P}$ corresponding to this series of truncation orders.

6. The Wilson-$\theta$ method is used to calculate the corresponding responses of this series of $\tilde{P}$.

7. Seeking the truncated order $n$ corresponding to the minimum response error $\| \tilde{x} - x \|_2$, $n$ is set as the optimised regularization parameter.

8. The TSVD regularization method is used to calculate the optimised load $P'$ when the regularization parameter is $n$, and the relative error (hereinafter referred to as "error") is calculated to determine whether the value satisfies the condition $\text{error} \leq \varepsilon$. If this condition is satisfied, the optimised load is output and taken as the final load-identification result; otherwise, the range of the regularization parameters is adjusted in the direction of the optimal regularization parameters, and better regularization parameters are input; that is, the truncation order is increased (the coefficient before $mN$ is increased). The optimised load $P'$ is recalculated from step (5) until the condition $\text{error} \leq \varepsilon$ is satisfied, and the output $P'$ is taken as the final load-identification result.

The relative error and Pearson correlation coefficient $\gamma$ reflect the accuracy of load identification and are given by equations (18) and (19), respectively [21].

$$\text{error} = \frac{\| P' - \tilde{P} \|^2_2}{\| P \|^2_2}. \tag{18}$$

$$\gamma = \frac{\sum_{i=1}^{mN} (P'_i - \bar{P})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^{mN} (P'_i - \bar{P})^2} \sqrt{\sum_{i=1}^{mN} (P_i - \bar{P})^2}}. \tag{19}$$

Here, $\bar{P}$ and $\bar{P}'$ represent the mean values of the external load $P$ and the optimised load $P'$, respectively.

Through the trial calculation of numerous examples, it was found that the best regularization parameter $n$ (truncation order) is always a large value close to $mN$. Therefore, in this study, the regularization parameter (truncation order) was limited to a small range close to $mN$ to avoid unnecessary calculations. The relationship between the relative error of the identification load and the truncation order is presented in Figure 2. As shown, the relative error of the identification load is large without regularization. With an increase in the truncation order, the relative error of the
identification load exhibits a downward trend, reaches its minimum value at a high truncation order, and increases with the continuous increase in the truncation order thereafter. Thus, according to the relationship between the relative error of load identification and the truncation order, a relatively large truncation order \( k \) is selected as the initial value to avoid unnecessary calculations, for achieving a good balance between the load-identification calculation accuracy and the calculation speed.

5. Identification of Wheel-Rail Vertical Force of Rail Vehicles

According to the book Vehicle-Track Coupled Dynamics [22] written by academician Wanning Zhai, the vehicle dynamics model is decoupled, and the vertical dynamics model is composed of the car body, the front and rear bogies, and four wheelsets. For the car body, as well as the front and rear frames, the upward and downward movements and nodding motions are considered. For the four wheelsets, the vertical vibration is considered. In total, the vertical dynamics model of the vehicle has 10 degrees of freedom. In this study, the dynamic modelling is performed according to this model for investigating the wheel-rail vertical force.

For the vertical 10-degree-of-freedom dynamic model of a train, the regularization parameters are selected using the proposed method and the traditional L-curve method, and the load-identification results are obtained and compared. The multi-body dynamic model of the train is established using SIMPACK software, as shown in Figure 3, and the vertical dynamic load of the wheel-rail system and the

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**Figure 1:** Flowchart of the parameter-selection method for TSVD regularization based on Wilson-\( \theta \).
vertical displacement response of the axle box of the simulation model are obtained with the German high-interference spectrum as the excitation.

The noise in the actual vehicle system arises from many physical interactions. It includes (1) the measurement noise of sensors; (2) the noise caused by abnormal vibrations resulting from rail wave grinding and wheel polygons; (3) the noise caused by local defects of wheels or rails; (4) the noise produced by the vehicle transmission system (traction motor, gearbox, bearing, etc.); (5) the noise caused by local vibration of the car body; and (6) the noise caused by vibration of the suspension equipment under the vehicle (fans, air conditioners, transformers, converters, etc.).

These noises may affect the accuracy of sensor measurement. In the simulation model and the numerical example of this study, the vehicle model is regarded as a multi-rigid-body vibration system, and the effects of the aforementioned factors on the sensor measurement are not considered. Therefore, it is necessary to add different levels of noise to the axle-box response calculated by the simulation model to simulate different situations. Considering that the situation can be complex, the general random noise model is used in this study to simulate the noise that may be generated in reality, and the model is validated under different noise levels. The random noise model was used in previous studies; thus, we consider its use to be reasonable.

Taking the simulated axle-box displacement response with 0%, 5%, and 10% noise as input, the identification of the wheel-rail vertical dynamic load (excluding static wheel load) by the proposed method and the traditional L-curve method at running speeds of 100, 160, and 200 km/h was investigated.

In addition to the L-curve method, there are many regularization-parameter selection methods, e.g. the GCV method, Morozov deviation principle, Engl criterion, and quasi-optimality criterion. The most commonly used method—the L-curve method—is sufficiently representative and is selected for comparison. Comparative studies on the identification performance of different regularization-parameter selection methods have been conducted [1, 5]; this is beyond the scope of the present paper.

The vertical 10-degree-of-freedom dynamic model of the train selected in this study is shown in Figure 4, where $M_c$ represents the body mass; $M_f$ represents the frame mass; $M_w$ represents the mass of the wheelset; $I_{cy}$ represents the moment of inertia of the car body around the $Y$ axis; $I_{fy}$ represents the moment of inertia of the frame around the $Y$ axis; $I_{wy}$ represents the moment of inertia of the wheelset around the $Y$ axis; $K_{sz}$ represents the vertical stiffness of the secondary suspension on one side of the bogie (N/m); $K_{pz}$ represents the vertical stiffness of the primary suspension of each axle box (N/m); $C_{sz}$ represents the vertical damping of the secondary suspension on one side of the bogie (N·s/m); $C_{pz}$ represents the vertical damping of the primary suspension of each axle box (N·s/m); $F_1$, $F_2$, $F_3$, and $F_4$ represent the vertical wheel-rail dynamic loads of the first, second,
third, and fourth wheelsets, respectively; \( l_c \) represents half of the length between truck centers; and \( l_t \) represents half of the fixed wheelbase of the vehicle.

Corresponding to Figure 4, the degrees of freedom considered in the train dynamic model are presented in Table 1, and the main dynamic parameters of the system are presented in Table 2.

### 5.1. Identification of Vertical Wheel-Rail Force at a Speed of 100 km/h

The noise level \( \sigma \) of the response is set as 0%, 5%, and 10%. The sampling frequency is set as 1000 Hz, and the sampling time is set as 2 s.

The identification results of the L-curve method and the proposed method for the second-wheelset vertical wheel-rail dynamic load \((F_2)\) and the fourth-wheelset vertical wheel-rail dynamic load \((F_4)\) under different noise levels are presented in Table 3.

As indicated by Table 3, when the input response noise level is 0%, the Pearson correlation coefficients of \( F_2 \) and \( F_4 \) obtained using the proposed method are 0.8873 and 0.8804, respectively, which are slightly smaller than those obtained using L-curve method (0.9005 and 0.9080, respectively). When the input response contains 5% noise, the Pearson correlation coefficients of \( F_2 \) and \( F_4 \) obtained using the proposed method are 0.8770 and 0.8684, respectively, which are larger than those obtained using the L-curve method (0.8367 and 0.8331, respectively). When the input response contains 10% noise, the Pearson correlation coefficients of \( F_2 \) and \( F_4 \) obtained using the proposed method are 0.8392 and 0.8285, respectively, which are larger than those obtained using the L-curve method (0.8136 and 0.8105, respectively).

The results indicate that when the train speed is 100 km/h and the input response noise level is 0%, the load-identification accuracy of L-curve method is slightly higher than that of the proposed method. At the noise levels of 5% and 10%, the proposed method has a higher recognition accuracy than the L-curve method.

To more intuitively compare the load identification between the two methods at different noise levels, the time-history diagram and frequency-domain amplitude diagram of the load identification for both methods with 0%, 5%, and 10% noise in the input response are examined, as shown in Figures 5–16.

As indicated by the figures, when the noise level is 0%, the identification accuracies of the two methods are high in both the time and frequency domains. At the noise levels of
Table 2: Dynamic parameters of the vehicle.

| Parameter                                      | Value |
|-----------------------------------------------|-------|
| Length between truck centers (m)              | 18    |
| Wheelbase (m)                                 | 2.5   |
| Rolling circle diameter of wheel (m)          | 0.92  |
| Lateral distance between primary spring/air spring axes (m) | 2     |
| Vehicle mass (kg)                             | 32000 |
| Height between vehicle’s barycentre and rail level (m) | 1.8   |
| Vehicle X-rotary inertia (kg\(\cdot\)m\(^2\)) | 560000|
| Vehicle Y-rotary inertia (kg\(\cdot\)m\(^2\)) | 2 \times 10^6 |
| Vehicle Z-rotary inertia (kg\(\cdot\)m\(^2\)) | 2 \times 10^6 |
| Framework mass (kg)                           | 3000  |
| Height between framework’s barycentre and rail level (m) | 0.6   |
| Framework X-rotary inertia (kg\(\cdot\)m\(^2\)) | 1500  |
| Framework Y-rotary inertia (kg\(\cdot\)m\(^2\)) | 2500  |
| Framework Z-rotary inertia (kg\(\cdot\)m\(^2\)) | 2800  |
| Wheelset mass (kg)                            | 1000  |
| Wheelset X-rotary inertia (kg\(\cdot\)m\(^2\)) | 1000  |
| Wheelset Y-rotary inertia (kg\(\cdot\)m\(^2\)) | 100   |
| Wheelset Z-rotary inertia (kg\(\cdot\)m\(^2\)) | 1000  |
| Longitudinal stiffness of primary suspension (every axle box) (N\(\cdot\)m\(^{-1}\)) | 1 \times 10^7 |
| Lateral stiffness of primary suspension (every axle box) (N\(\cdot\)m\(^{-1}\)) | 1 \times 10^7 |
| Vertical stiffness of primary suspension (every axle box) (N\(\cdot\)m\(^{-1}\)) | 6 \times 10^5 |
| Damping of primary vertical damper (N\(\cdot\)s\(\cdot\)m\(^{-1}\)) | 60000 |
| Longitudinal stiffness of secondary suspension (N\(\cdot\)m\(^{-1}\)) | 1.5 \times 10^5 |
| Lateral stiffness of secondary suspension (N\(\cdot\)m\(^{-1}\)) | 1.5 \times 10^5 |
| Vertical stiffness of secondary suspension (N\(\cdot\)m\(^{-1}\)) | 4.5 \times 10^5 |
| Damping of secondary vertical damper (N\(\cdot\)s\(\cdot\)m\(^{-1}\)) | 8 \times 10^4 |
| Damping of secondary lateral damper (N\(\cdot\)s\(\cdot\)m\(^{-1}\)) | 6 \times 10^4 |

Table 3: Comparison of the calculation results for \(F_2\) and \(F_4\) obtained using the L-curve method and the proposed method.

| Noise level (%) | \(F_2\) recognition results | \(F_4\) recognition results |
|-----------------|----------------------------|----------------------------|
|                 | Optimal regularization parameter | Pearson correlation coefficient | Optimal regularization parameter | Pearson correlation coefficient |
|                 | Proposed method | L-curve method | Proposed method | L-curve method | Proposed method | L-curve method | Proposed method | L-curve method |
| 0               | — | 6668 | 0.8873 | 0.9005 | — | 6668 | 0.8804 | 0.9080 |
| 5               | 7577 | 7722 | 0.8770 | 0.8367 | 7577 | 7722 | 0.8684 | 0.8331 |
| 10              | 7687 | 7802 | 0.8392 | 0.8136 | 7687 | 7802 | 0.8285 | 0.8105 |

Figure 5: Load time-history diagram of the two methods for \(F_2\) identification when the noise level is 0%.
5% and 10%, the L-curve method selects larger regularization parameters (truncation order) than the proposed method and filters more singular values of the kernel-function matrix. Therefore, the curve of dynamic load identification is smoother, and the accuracy is relatively low.

Regarding the frequency domain, in the first 50 Hz frequency range, when the noise level is 0%, the two methods have good recognition results for the external load. When the noise level is 5%, the proposed method better matches the external load than the L-curve method. When the noise level is 10%, the coincidence degree of the two methods for the frequency-domain amplitude curve of the proposed method is reduced. This may be because the noise frequency is close to part of the frequency of the external load. When the TSVD regularization is performed, the external load and noise in this frequency range are filtered together, reducing the recognition accuracy. However, compared with the L-curve method, the proposed method can identify the external load in a wider frequency range.

5.2. Identification of Vertical Wheel-Rail Force at a Speed of 160 km/h. The noise level of the response is set as 0%,
The sampling frequency is set as 1000 Hz, and the sampling time is set as 2 s. The recognition results of the L-curve method and the proposed method for $F_2$ and $F_4$ under different noise levels are presented in Table 4.

As indicated by Table 4, when the input response noise level is 0%, the Pearson correlation coefficients of $F_2$ and $F_4$ obtained using the proposed method are 0.9362 and 0.9452, respectively, which are larger than those obtained using the L-curve method (0.9118 and 0.9360, respectively). When the input response contains 5% noise, the Pearson correlation coefficients of $F_2$ and $F_4$ obtained using the proposed method are 0.7685 and 0.7543, respectively, which are larger than those obtained using the L-curve method (0.7199 and 0.7030, respectively). When the input response contains 10% noise, the Pearson correlation coefficients of $F_2$ and $F_4$ obtained using the proposed method are 0.7194 and 0.7034, respectively, which are larger than those obtained using the L-curve method (0.7041 and 0.6782, respectively).

Comparing the calculation results reveals that when the train running speed is 160 km/h and the input response noise level is 0%, 5%, and 10%, the proposed method has a higher identification accuracy than the L-curve method.
To more intuitively compare the load identification between the two methods at different noise levels, the time-history diagram and frequency-domain amplitude diagram of the load identification for both methods with 0%, 5%, and 10% noise in the input response are examined, as shown in Figures 17–28.

As indicated by the figures, when the noise level is 0%, the identification accuracies of the two methods are good in both the time and frequency domains, and the correlation coefficients are >0.91. At the noise levels of 5% and 10%, the regularization parameter (truncation order) selected by the proposed method is relatively small, and the small singular value of the kernel-function matrix is filtered less; thus, the proposed method can better approximate the peak value of the external load than the L-curve method, and its identification accuracy is higher.

Regarding the frequency domain, in the first 50 Hz frequency range, when the noise level is 0%, the two methods have good recognition results for the external load. At the noise levels of 5% and 10%, the coincidence degree of the two methods for the frequency-domain amplitude curve of the external load is reduced, possibly because the noise frequency is close to part of the frequency of the external load. Additionally, owing to the increase in the running speed, the components of the external load that are close to the noise frequency increase. Therefore, in the TSVD regularization, the external load and noise in this frequency range are filtered together, reducing the recognition accuracy. However, compared with the L-curve method, the recognition degree of the frequency-domain amplitude curve of the external load is better in the first 25 Hz frequency range when the vehicle speed is 160 km/h.

5.3. Identification of Vertical Wheel-Rail Force at a Speed of 200 km/h. The noise level del of the response is set as 0% and 5%. The sampling frequency is set as 1000 Hz, and the sampling time is set as 2 s.

The recognition results of the L-curve method and the proposed method for \( F_2 \) and \( F_4 \) at different noise levels are presented in Table 5.

As indicated by Table 5, when the input response noise level is 0%, the Pearson correlation coefficients of \( F_2 \) and \( F_4 \) obtained using the proposed method are 0.9491 and 0.9556, respectively, which are larger than those obtained using the L-curve method (0.8561 and 0.8619, respectively). When the input response contains 5% noise, the Pearson correlation coefficients of \( F_2 \) and \( F_4 \) obtained using the proposed method are 0.6354 and 0.6291, respectively, which are larger than...
Table 4: Comparison of the calculation results for $F_2$ and $F_4$ obtained using the L-curve method and the proposed method.

| Noise level (%) | $F_2$ recognition results | $F_4$ recognition results |
|-----------------|---------------------------|---------------------------|
|                 | Proposed method | L-curve method | Proposed method | L-curve method | Proposed method | L-curve method |
| 0               | — | 7023 | 0.9362 | 0.9118 | — | 7023 | 0.9452 | 0.9360 |
| 5               | 7570 | 7683 | 0.7685 | 0.7199 | 7570 | 7683 | 0.7543 | 0.7030 |
| 10              | 7645 | 7777 | 0.7194 | 0.7041 | 7645 | 7777 | 0.7034 | 0.6782 |

Figure 17: Load time-history diagram of the two methods for $F_2$ identification when the noise level is 0%.

Figure 18: Load time-history diagram of the two methods for $F_4$ identification when the noise level is 0%.

Figure 19: Amplitude-frequency curve of the two methods for $F_2$ identification when the noise level is 0%.

Figure 20: Amplitude-frequency curve of the two methods for $F_4$ identification when the noise level is 0%.
Comparing the calculation results reveals that when the train running speed is 200 km/h and the input response noise level is 0% or 5%, the proposed method has a higher identification accuracy than the L-curve method.

To more intuitively compare the load identification between the two methods at different noise levels, the time-history diagram and frequency-domain amplitude diagram of the load identification for both methods with 0% and 5% noise in the input response are examined, as shown in Figures 29–36.

As indicated by the figures, when the noise level is 0%, the identification accuracies of the two methods are good in both the time and frequency domains, and the correlation coefficients are >0.85. When the noise level is 5%, the regularization parameter (truncation order) selected by the proposed method is relatively small, and the small singular value of the kernel-function matrix is filtered less; thus, it can better approach the peak value of the external load, and the identification accuracy is relatively high.
Regarding the frequency domain, when the noise level is 0%, the two methods have good recognition results for the external load. When the noise level is 5%, the coincidence degree of the two methods for the frequency-domain amplitude curve of the external load is reduced. This may be because the noise frequency is close to some frequencies of the external load, and the component of the external load that is close to the noise frequency is increased owing to the further increase in the running speed of the model. The external load and the noise in this frequency range are filtered out together, reducing the recognition accuracy.

However, compared with the L-curve method, when the vehicle speed is 200 km/h, the proposed method can identify the external load in a wider frequency range in the first 30 Hz frequency range.

Taking the identification results of $F_2$ as an example, the relationships between the correlation coefficient with vehicle speed and noise level are shown in Figure 37.

5.4. Identification of Vertical Wheel-Rail Force Excited by Different Track Spectra. In the foregoing research, only the German high-interference spectrum was used as the input. We consider that it is necessary to use other track spectra as
Table 5: Comparison of the calculation results for $F_2$ and $F_4$ obtained using the L-curve method and the proposed method.

| Noise level (%) | $F_2$ recognition results | $F_4$ recognition results |
|-----------------|---------------------------|---------------------------|
|                 | Proposed method | L-curve method | Pearson correlation coefficient | Proposed method | L-curve method | Pearson correlation coefficient |
| 0               | —                | 7096           | 0.9491                      | —                | 7096           | 0.9556                      |
| 5               | 7523             | 7662           | 0.6354                      | 7523             | 7662           | 0.6291                      |

Figure 29: Load time-history diagram of the two methods for $F_2$ identification when the noise level is 0%.

Figure 30: Load time-history diagram of the two methods for $F_4$ identification when the noise level is 0%.

Figure 31: Amplitude-frequency curve of the two methods for $F_2$ identification when the noise level is 0%.

Figure 32: Amplitude-frequency curve of the two methods for $F_4$ identification when the noise level is 0%.
the input to further investigate the effect of track excitation on the vertical wheel-rail force identification. The American level 5 spectrum is used as the input here, and the vertical wheel-rail force of the second wheelset is taken as an example to compare the effects of different track excitations on the vertical wheel-rail force identification at a speed of 100 km/h.

Taking the American level 5 track spectrum as the excitation, the time-history diagram and frequency-domain amplitude diagram of the load identification for the two methods with 10% noise in the input are examined, as shown in Figures 38 and 39. A comparison of the calculation results is presented in Table 6.

As indicated by Table 6, under the same conditions, the identification accuracy of the vertical wheel-rail force excited by the American level 5 track spectrum is slightly higher than that for the German high-interference spectrum. Thus, the track excitation affects the identification of the vertical wheel-rail force. Regarding the frequency domain, at the speed of 100 km/h, the frequency range of the vertical wheel-rail force under the American level 5 track spectrum excitation is mainly below 20 Hz, whereas under the German high-interference spectrum excitation, the frequency range
of wheel-rail force is wider, reaching approximately 30 Hz. According to the frequency-domain amplitude diagrams in this paper, the two methods can identify the vertical wheel-rail force accurately at frequencies below 20 Hz, and the recognition accuracy decreases above 20 Hz, which explains why the recognition accuracies of the two methods were slightly higher for the American level 5 track spectrum than for the German high-interference spectrum. However, in general, regardless of whether the American level 5 track spectrum or the German high-interference spectrum is used for excitation, the Pearson correlation coefficients of the two methods for vertical wheel-rail force identification are both >0.8, corresponding to extremely strong correlation.
6. Conclusions

A load-identification model based on the Green function method is established, and the load identification of multi-degree-of-freedom system is realised by selecting TSVD regularization parameters based on Wilson-\(\theta\). The following conclusions are drawn:

1. Through the TSVD regularization-parameter selection method based on the Wilson-\(\theta\) method and the principle of minimum response error, the wheel-rail dynamic load of the train vertical 10-degree-of-freedom model is identified.

2. When the noise level of the input response is 0%, both the proposed method and the L-curve method have high identification accuracies for external loads, and the correlation coefficients are both >0.85.

3. When the noise level of the input response is 5% or 10%, the regularization parameter selected by the proposed method is better than that obtained using the L-curve method. In the time domain, the peak value of the identified load is closer to the external load and has stronger robustness. When the train speed is 200 km/h and the noise level is 5%, the identification correlation coefficient of the proposed method is 0.6354, corresponding to strong correlation. In the frequency domain, compared with the L-curve method, the proposed method can identify the amplitude-frequency curve of the external load more accurately in a certain frequency range and can identify the external load over a wider frequency range.

4. With an increase in the vehicle speed, the high-frequency component of the wheel-rail force increases. When the noise level of the input displacement response is the same, the proposed method can identify the load in a higher frequency range and has a higher identification accuracy than the L-curve method.

5. Differences in the track excitation can affect the vertical wheel-rail force identification results. However, for the German high-interference spectrum and American level 5 spectrum used in this study, when the vehicle speed is 100 km/h and the input response contains 10% noise, the Pearson correlation coefficients of the vertical wheel rail force are >0.8, indicating a high identification accuracy.

The proposed method takes the displacement as the input for system dynamic load identification. Considering the universality of using acceleration as the input to identify the dynamic load, the use of acceleration as the input for system dynamic load identification can be investigated in the future.

| Noise level (%) | Optimal regularization parameter | Pearson correlation coefficient | Optimal regularization parameter | Pearson correlation coefficient |
|-----------------|---------------------------------|-------------------------------|---------------------------------|-------------------------------|
|                 | Proposed method | L-curve method | Proposed method | L-curve method | Proposed method | L-curve method | Proposed method | L-curve method |
| 10              | 7687              | 7802              | 0.8392              | 0.8136              | 7736              | 7771              | 0.9135              | 0.9074              |

Figure 39: Amplitude-frequency curve of the two methods for \(F_2\) identification when the noise level is 10%.
Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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