Challenges in developing of 3D nonlinear viscoelastic models

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Abstract. Due to development of different material systems and structural elements, there is growing need for more complex material models, since linear elasticity or even more complicated linear viscoelasticity cannot capture the nuances of behaviour of these materials. This study describes the process and difficulties of development of 3D nonlinear viscoelastic model. Study proposes two models with different level of complexity and methodology for experimental parameter identification. The limitations of both of these models have been discussed.

1. Introduction
The advancement in materials, requires more and more complicated material models. For example, Bio-based materials offer an environmentally friendlier alternative to conventional composite materials [1]. However, it has been demonstrated that behaviour of bio-based composites and their constituents cannot be characterized by linear elasticity or even more complicated linear viscoelasticity (VE) [2-6]. There has been a great effort made to use full potential of different materials, thus significantly reducing weight, volume etc. of different components. This would require materials to function in more demanding environments e.g. elevated temperature and humidity, used at partially cured stages etc. Often these factors significantly contribute to nonlinear behaviour of materials. Thus, it is crucial to develop material models, that could capture all these nuances.

The most widely used nonlinear VE models have been developed by Schapery [7]. With advancement of structures, the complexity of elements also increases. VE analysis of structures using finite element modeling requires VE models in 3D formulation accounting for changes in material state and environmental conditions that can be easily implemented in computational codes. This paper is focused on the problems and difficulties of developing a 3D model and experimental parameter identification. Two models of different complexity have been proposed. The difficulties of experimental setup for parameter identification have been discussed.

2. Material models
Schapery developed stress formulation for nonlinear VE model based on thermodynamics [7-8]. This model was further modified to account for viscoplasticity (VP) and damage [9] and has been successfully applied to characterize different materials under load controlled tests [10-11]. In uniaxial loading it follows:
\[ \varepsilon = d(\sigma_{\text{max}}) \left( \varepsilon_{e1} + g_1 \int_0^t \Delta S (\psi - \psi') \frac{d(g_2\sigma)}{dt} d\tau + \varepsilon_{VP}(\sigma, t) \right) \]  

where \( d(\sigma_{\text{max}}) \) represents effect of damage, \( \varepsilon_{VP}(\sigma, t) \) is VP strain component and the rest is VE, where \( \varepsilon_{e1} \) is initial elastic response and \( g_1 \int_0^t \Delta S (\psi - \psi') \frac{d(g_2\sigma)}{dt} d\tau \) is time dependent part. In order to obtain all parameters a large number of time-consuming tests must be performed – tests introducing damage and measuring stiffness degradation, multiple-step creep tests at different stress levels for VP and VE.

2.1. Damage

In different material systems, such as composites, microcracks, cracks, delamination etc. can appear during service life. This can cause degradation of elastic modulus. This degradation can accumulate as a result of previous loading history to higher stresses/strains. In cases when the damage state is very complex, as it is in short fibre composites, a more pragmatic approach is to find the modulus reduction as a function of the applied strain (or stress) in tensile test.

Microdamage parameter \( d(\sigma_{\text{max}}) \) in [9] has been expressed as ratio between modulus in undamaged state \( E_0 \) and modulus in the damaged \( E(\sigma_{\text{max}}) \) state that usually corresponds to the highest stress \( \sigma_{\text{max}} \) experienced during the previous service life:

\[ d(\sigma_{\text{max}}) = \frac{E_0}{E(\sigma_{\text{max}})} \]  

\[ \text{Figure 1. Normalizes modulus } E/E_0 \text{ (Elastic modulus at damaged state versus modulus at undamaged state) as a function of stress (left) and strain (right) for Flax fiber/lignin and flax fiber/polylactic acid (PLA) composite. Data from [10,12,13]} \]

The tensile loading ramp for damage parameter identification in details has been described in [12,13]. The proposed tests consist of a sequence of four main steps:

1) new damage introduction by loading to certain (high) strain or stress level and unloading to “almost zero” stress;

2) recovery period at “almost zero” stress for a time at least 5 times longer than the length of the previous step for decay of all VE effects;
3) the elastic modulus determination applying high strain rate and low load level loading-unloading;

4) VE-strain recovery after modulus measurement for the time that is at least 5 times longer than the time interval for modulus measurement.

After the last step, the sequence is repeated with higher strain level than in the first step. The data of modulus obtained from these tests then is plotted and fitting curve for damage is obtained. An example of elastic modulus reduction is presented in Figure 1. VP strain may also develop during the first steps of the loading sequence thus resulting in situations where the same damage might be corresponding to different strain levels. Therefore, it is more suitable to analyze damage as a function of applied stress not strain.

![Figure 1. Elastic modulus reduction](image1)

**Figure 2.** VP strain development in regenerated cellulose fibers with respect to time (left) and stress (right). Data from [15]

2.2. Viscoplasticity
Term $\varepsilon_{VP}(\sigma, t)$ represents irreversible VP strains that develop at high stresses and increase with time of loading. In [14] a model for VP strains was suggested:

$$\varepsilon_{VP}(\sigma, t) = C_{VP} \left\{ \int_{t*}^{t} \left( \frac{\sigma(t)}{\sigma*} \right)^M d\tau \right\}^\alpha$$

(3)

$C_{VP}$, $M$ and $\alpha$ are material constants, that are dependent on different environmental conditions, temperature, moisture, degree of cure etc., $t/t*$ is normalized time where $t*$ is an arbitrary chosen characteristic time constant and $\sigma*$ is an arbitrary chosen reference constant stress value. The validity of this particular model has to be verified experimentally. This VP model was successfully used in [9-11,14] to characterize VP strain development.

In case of constant applied stress $\sigma = \sigma_0$, the VP strain dependency on the creep time $t$ becomes very simple:

$$\varepsilon_{VP}(\sigma, t) = C_{VP} \left( \frac{\sigma_0}{\sigma*} \right)^{M\alpha} \left( \frac{t}{t*} \right)^\alpha$$

(4)
Thus, if the model (3) is applicable, the time dependence of VP strains in constant stress (creep) test should follow a power function with respect to time:

\[ \varepsilon_{VP}(\sigma, t) = A \left( \frac{t}{t_*} \right)^{\alpha} \]  

Where \( A \) has power law dependence on the applied stress level in the creep test:

\[ A = C_{VP} \left( \frac{\sigma_0}{\sigma} \right)^{m \alpha} \]  

All parameters in the VP law can be determined from sequence of creep loading and VE strain recovery performed at different stress levels. After each recovery period, the remaining strain is considered as irreversible. Both equations (5)-(6) in ln-ln axis become linear, thus fitting experimental data in these axes are the easiest way to determine constants in VP law (see figure 2). However, this methodology requires specimen with no previous loading history – at each stress level different specimen must be used. This increases complexity of data analysis for materials with high scatter – the trends of VP can be lost in variations between specimens. Thus another, more complicated methodology has been developed for such cases, where the described tests at different loads are performed on the same specimen. Details of both methodologies can be found in [15].

2.3. Nonlinear viscoelasticity

In material model (1) for VE reduced time is used:

\[ \Psi(t) = \int_0^{t} \frac{dt}{a_\sigma} \quad \text{and} \quad \Psi'(t) = \int_0^{t} \frac{dt}{a_\sigma} \]  

In (1), the initial elastic response \( \varepsilon_{el} \), generally speaking, may be nonlinear function, \( g_1 \) and \( g_2 \) are nonlinearity functions and \( a_\sigma \) is the shift factor. Some materials behave or have a region of linear VE, where \( g_1 = g_2 = a_\sigma = 1 \). Schapery showed that the linear VE creep compliance can be written in form of Prony series [8]:

\[ \Delta S(\Psi) = \sum_i C_i \left( 1 - \exp \left( -\frac{\Psi}{\tau_i} \right) \right) \]  

where \( C_i \) are constants and \( \tau_i \) are retardation times. When damage and VP has been identified, they can be removed from the total strain to extract pure VE response. Afterwards the VE during creep and recovery can be characterized by equations:

\[ \varepsilon_{creep} = \varepsilon_0 + g_1 g_2 \sigma \sum_i C_i \left( 1 - \exp \left( -\frac{t}{a_\sigma \tau_i} \right) \right) \]  

\[ \varepsilon_{rec} = g_2 \sigma \sum_i C_i \left( 1 - \exp \left( -\frac{t_1}{a_\sigma \tau_i} \right) \right) \exp \left( -\frac{t-t_1}{\tau_i} \right) \]  

where \( t_1 \) is the length of the creep test. First the coefficients in Prony series are identified – this procedure is much simpler, if linear VE region can be found. Afterwards, one can move to higher stress regions and find nonlinearity functions. Figure 3 shows the pure VE strains in creep tests for bio-based resin “EpoBioX” and VE creep compliance curves for pure high-density polyethylene (HDPE). It can be seen that with increasing stress level the compliance increases, thus indicating that material is nonlinearly VE, even at relatively low stress levels. In Figure 4. Examples of the VE nonlinearity functions for resin “EpoBioX” and Lignin/flax fiber composite are presented.
2.4. Strain formulation

Although stress-formulation of the material model can be successfully applied to characterize nonlinear material behavior, there is growing need to simulate strain-controlled test. This is because most of the numerical structural analysis, analytical micromechanics models (rule of mixture, concentric cylinder assembly model etc.), classical laminate theory require model, where stresses are expressed as a function of strain, time etc. Moreover, many of experiments are performed in strain-controlled mode.
Similarly, to stress-formulation, Schapery [18,19] has developed nonlinear VE model where VE strain is used as an independent variable using expansion of Helmholtz’s free energy. In one-dimensional case, the constitutive law is in form:

\[
\sigma = E_r \varepsilon + h_1(\varepsilon) \int_0^{\psi_{\varepsilon}} \Delta E(\psi_{\varepsilon} - \psi_{\varepsilon}') \frac{d(h \varepsilon)}{d\psi_{\varepsilon}'} d\psi_{\varepsilon}' \tag{11}
\]

In (11) \( E_r \) is equilibrium modulus that, generally speaking, may be nonlinear function of strain. The “reduced time” \( \psi_{\varepsilon} \) is introduced in (11) as:

\[
\psi_{\varepsilon} = \int_0^t \frac{dt'}{a_{\varepsilon}} \quad \text{and consequently} \quad \psi_{\varepsilon}' = \int_0^t \frac{dt'}{a_{\varepsilon}} \tag{12}
\]

The transient part of the VE response is characterized by \( \Delta E(\psi_{\varepsilon}) \) which does not depend on stress and have a form of Prony series:

\[
\Delta E(\psi_{\varepsilon}) = \sum \lambda_i E_i \exp \left( -\frac{\psi_{\varepsilon}}{\tau_i} \right) \tag{13}
\]

**Figure 5.** Schematic drawing of VP and VE strain development during relaxation test with applied constant strain (left) and increasing applied strain to account for VP strain development (right). Red line represents pure VE strain.

In (13) \( E_i \) are constants and \( \lambda_i \) are relaxation times. Nonlinearity functions \( h_1 \) and \( h_2 \) and the shift factor \( a_{\varepsilon} \) are strain invariant dependent. If it is possible to find region where \( E_r \) is a linear function of the applied strain and \( h_1 = h_2 = a_{\varepsilon} = 1 \), equation (11) is reduced to stress-strain relationship for linear VE material. As demonstrated in [18,19], in case of nonlinear VE both forms of the material model are not compatible and cannot be directly inverted to each other. Moreover, in order to obtain parameters and functions for the model in strain formulation, relaxation tests where VE strain is kept constant need to be performed. However most of materials have also VP strain component, thus to perform such experiments becomes very complicated. If the strain is kept constant, during time, the VP strain develops, thus in reality, the VE strain is not constant but decreases. This would require the strain to be increased, to compensate for the VP strain. The visual description of this problem is presented in Figure 5, and discussed in more details in [20]. These issues can be resolved by using an inverted incremental form of the stress-formulation derived in [21]. This model is used to simulate pure VE relaxation tests using parameters that were obtained from creep tests, avoiding the problems with experimental relaxation test needed for parameter identification for strain formulation. Then these simulated relaxation tests are used to obtain parameters for strain formulation. This methodology was successfully used in [22] to simulate strain controlled tensile tests. The agreement between experiments and simulations was great if the characterization in creep was at stress levels close to rupture. However, in cases when it was not possible to characterize material behavior at high stresses due to creep rupture, the agreement between simulations and experiment was good only in the stress region where creep characterization was performed. At higher stresses, the method of extrapolation has significant effect on final curves. It was demonstrated that further adjustments of nonlinearity functions can be done with
help of simple tensile tests. In order to use models described in previous sections, a large number of time consuming tests and fittings needs to be performed. In case of more complex material systems such as composite materials, due to large number of available matrix and reinforcement (or filler) combinations, each time one component or morphology changes, the full characterization needs to be performed again. Thus, it would be more convenient to use models, where only the constituents have to be fully characterized in nonlinear VE. Such approach has been described in [16].

2.5. 3D nonlinear viscoelastic material model with parameters dependent on material state and temperature

In this Section the VE related Internal State Variables (ISV) have notation $\xi_m$. All matrixes, vectors (matrix-column) that have constant elements have upper index 0. Everything without “0” may be temperature and strain dependent. Indexes $i,j,l = 1,…,6$ are used for stress and strain and related functions. Indexes $m,n,k$ are used for ISV related functions. Free thermal expansion and curing shrinkage strains are not included in this derivation. This means that “strain” is actually “the applied strain minus the free expansion strain”. Following the scheme suggested by Schapery [8] the stress – strain relationship follows from:

$$\sigma_i = \frac{\partial H}{\partial \varepsilon_i}, \quad H = H_e(\alpha,T,\varepsilon) - \sum_m A_m(\alpha,T,\varepsilon)\xi_m + \frac{1}{2} \sum_{mn} B_{mn}(\alpha,T,\varepsilon)\xi_m\xi_n$$  \hspace{1cm} (14)

where $H$ is the Helmholtz free energy presented in form of expansion with respect to VE internal state variables (ISV) $\xi_m$, $m = 1, M$. Parameter $\alpha$ represents the effect on VE of the material state (degree of cure, aging etc.), $T$ is temperature and $\varepsilon$ strain invariant dependent function. We assume that in the initial state $\sigma(0) = \varepsilon(0) = \xi_m(0) = 0$. $B_{mn}$ is symmetric matrix with dependence on $\alpha, T, \varepsilon$ as follows:

$$B_{mn} = B_{mn}^0 a_2(\alpha,T,\varepsilon)$$  \hspace{1cm} (15)

Using (14) and neglecting quadratic terms with respect to ISV’s:

$$\sigma_i = \frac{\partial H}{\partial \varepsilon_i} - \sum_m \frac{\partial A_m}{\partial \varepsilon_i} \xi_m$$  \hspace{1cm} (16)

We assume that the evolution rate of ISV is linear with respect to conjugated thermodynamic forces

$$\frac{d\xi_m}{dt} = \frac{1}{a_1(\alpha,T,\varepsilon)} \sum_n c_{mn}^0 f_n$$, where $f_n = -\frac{\partial H}{\partial \xi_n} = A_n(\alpha,T,\varepsilon) - \sum_k B_{kn}^0 a_2(\alpha,T,\varepsilon)\xi_k$  \hspace{1cm} (17)

In (17) $c_{mn}^0$ is a positive definitive, symmetric matrix, $a_1 > 0$, can be assumed equal to 1 in the reference state. Using notations $[ ]$ for square-matrix and $\{ \}$ for a column and introducing reduced time $\psi$ as:

$$d\psi = \frac{a_2(\alpha,T,\varepsilon)}{a_1(\alpha,T,\varepsilon)} dt, \quad \psi(t) = \int_0^t \frac{a_2(\alpha,T,\varepsilon)}{a_1(\alpha,T,\varepsilon)} d\xi$$  \hspace{1cm} (18)

from (17) follows:

$$[c^0]^{-1} \frac{d}{d\psi} (\xi) + [B^0](\xi) = \frac{1}{a_2} [A]$$  \hspace{1cm} (19)
\[
\epsilon^0_m \frac{d \xi^m}{d \psi} + B^0_m \xi^m = \frac{A_m}{a_2} \quad m=1,2,3,\ldots
\]  

Solution (20) is as follows:

\[
\xi^m = \frac{A_m}{a_2} \beta_m^0 - \frac{1}{b_m^0} \int_0^l \exp \left( -\frac{\psi - \psi^\prime}{\tau_m} \right) \frac{d \Lambda_m}{d \psi} \frac{A_m}{a_2} \, d\psi', \text{ where } \tau_m = \frac{b_m^0}{\beta_m^0}
\]  

At this point, no assumptions have been made regarding \(A_m\). Two models will be presented now with different complexity regarding \(A_m\). The aim is to compare the resulting stress-strain equations, to see the differences and to discuss their potential regarding applicability and complexity.

First, following suggestion by Brouwer [23] we assume (referred as Model A) that for all strain components the dependence of \(A_m\) on \(T, \alpha\) and \(\dot{\epsilon}\) is the same:

\[
A_m = h(\alpha, T, \dot{\epsilon}) \sum_j D_{mj}^0 \dot{\epsilon}_j
\]  

\[
\frac{\partial A_m}{\partial \xi^l} = h(\alpha, T, \dot{\epsilon}) D_{mul}^0 + \frac{\partial h}{\partial \xi^l} \sum_j \xi^l \xi_j S_{il}
\]  

Equation (22) is substituted in (21) for \(\xi^m\). After that, the result and (23) are used in (16) obtaining the following stress-strain relationship:

\[
\sigma_i = \Gamma_i^e(\alpha, T, \dot{\epsilon}) + \frac{\partial}{\partial \xi^l} \sum_j S_{ij} \dot{\epsilon}_j + \sum_j \xi^l \sum_i S_{il} \xi_i
\]  

\[
\Delta E_{ij}(\psi - \psi') = \sum_m E_{ij}^{0(m)} \exp \left( -\frac{\psi - \psi'}{\tau_m} \right)
\]  

Obtaining these expressions, new unknown constants and functions were introduced instead of the used above, for example \(E_{ij}^{0(m)} = D_{mi}^0 \cdot D_{mj}^0 / B_{ma}^0\). This model, in addition to Prony coefficients and the equilibrium response \(\Gamma_i^e\), contains three unknown functions dependent on strain invariants, \(T\) and \(\alpha\): \(h(\alpha, T, \dot{\epsilon})\), \(a_{m\alpha}(\alpha, T, \dot{\epsilon})\) and \(a_{m\alpha}(\alpha, T, \dot{\epsilon})\) (in definition of \(\psi\)), the same number of functions as in the 1D model. However, the material class, where the assumption (22) is applicable, is a priori not known. Thus, the model’s applicability has to be verified in experiments.

The second model (Model B) has more “flexibility” in the sense that each strain component may have a different material state, environment and loading dependent function:

\[
A_m = \sum_j D_{mj}^0 f_j(\alpha, T, \dot{\epsilon}) \dot{\epsilon}_j
\]  

\[
\frac{\partial A_m}{\partial \xi^l} = D_{mi}^0 f_i + \sum_j D_{mj}^0 \frac{\partial f_j(\alpha, T, \dot{\epsilon})}{\partial \xi^l} \dot{\epsilon}_j
\]  

The stress-strain relationship is as follows:

\[
\sigma_i = \Gamma_i^e(\alpha, T, \dot{\epsilon}) + f_i \cdot \sum_j I_{ij} \dot{\epsilon}_j + \sum_j \frac{\partial f_j(\alpha, T, \dot{\epsilon})}{\partial \xi^l} \dot{\epsilon}_j I_{jl}
\]  

\[
I_{ij} = \int_0^l \Delta E_{ij}(\psi - \psi') \frac{d}{d\psi} \frac{f_j(\alpha, T, \dot{\epsilon})}{a_2} \, d\psi'
\]  

The eight unknown functions \(f_i, i = 1,2,\ldots, 6\), \(a_2\) and \(a_2\) have to be found experimentally. Hence, Model B requires larger amount of more complex tests for model identification than Model A. For example, one test has to be bi-axial creep test, hence significantly increasing the difficulty of
experimental parameter identification. On the other hand, this inconvenience may be justified for anisotropic materials. For example, for composites the stress nonlinearity effect can be very different in fiber direction than in shear loading. One solution for experimental parameter identification could be, that some of the parameters and functions are adjusted until simulations fit the experimental data. This, of course, would require several iterations, where parameters are changed. It is not clear that this identification procedure has to lead to unique solution. Unfortunately, it is possible that for certain loading ramps different combinations of parameters could lead to the same or very similar result.

3. Conclusions
The need for more complex 3D nonlinear model that in addition to viscoelasticity also accounts for viscoplasticity and damage has been demonstrated. The 1D model that includes all of the above-mentioned phenomena and that has been used successfully to characterize complex material behaviour has been used to illustrate the inelastic effects. Stress and strain formulations of the nonlinear viscoelastic model, developed by Schapery, have been discussed. The pros and cons of each approach has been considered.

Two 3D nonlinear viscoelasticity models have been introduced. Model A is simpler, due to fewer experimentally determined functions needed as input. The experimental procedure for parameter identification for this model is also relatively simple – the same number of tests as for 1D models’ case. However, it is unclear if the assumptions used developing this model would be valid for anisotropic materials. This model could be more suitable for isotropic materials. Model B is more complex with more functions needed for input as well as the experimental identification of parameters is more sophisticated. However, this model could be more accurate in predicting behavior of complex material systems.

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References
[1] S. V. Joshi, L. T. Drzal, A. K. Mohanty, S. Arora. “Are natural fiber composites environmentally superior to glass fiber reinforced composites?” Composites Part A: Applied Science and Manufacturing, vol. 35, pp. 371-376, 2004.
[2] E. Marklund, J. Varna and L. Wallström. “Nonlinear Viscoelasticity and Viscoplasticity of Flax/Polypropylene Composites,” Journal of Engineering Materials and Technology, vol. 128, pp. 527-536, 2006.
[3] J. Varna, E. Sparnins, R. Joffé, K. Nattinen and J. Lampinen. ”Time dependent behavior of flax/starch composites,” Mechanics of Time-Dependent Materials, vol. 16, pp. 47-70, 2012.
[4] K. M. Almgren, E. K. Gamstedt and J. Varna. “Contribution of wood fibres to the moisture-induced dimensional instability of composite plates,” Polymer Composites, vol. 31, pp. 762-771, 2010.
[5] L.-O. Nordin and J. Varna. “Nonlinear viscoelastic behavior of paper fiber composites,” Composite Science and Technology, vol. 65, pp. 1609-1625, 2005.
[6] C. Martins, V. Pinto, R. M. Guedes and A. T. Marques “Creep and stress relaxation behaviour of PLA-PCL fibres – A linear modelling approach” 1st International Conference on Structural Integrity, Procedia Engineering, vol. 114 pp. 768-775, 2015.
[7] Y. C. Lou and R. A. Schapery. “Viscoelastic Characterization of a Nonlinear Fiber-reinforced Plastic,” Journal of Composite Materials, vol. 5, pp. 208-234, 1971.
[8]  R. A. Schapery. “Nonlinear Viscoelastic and Viscoelastic Constitutive Equations Based on Thermodynamics”. *Mechanics of Time-Dependent Materials*, vol. 1, pp. 209-240, 1997.

[9]  E. Marklund, J. Eitzenberger and J. Varna. “Nonlinear viscoelastic viscoplastic material model including stiffness degradation for hemp/lignin composites,” *Composites Science and Technology*, vol. 68, pp. 2156-2162, 2008.

[10]  J. Varna, L. Rozite, R. Joffe and A. Pupurs. “Nonlinear behavior of PLA based flax composites” *Plastics, Rubber and Composites*, vol. 41, pp. 49-60, 2012.

[11]  K. Giannadakis, P. Mannberg, R. Joffe and J. Varna. “The source of inelastic behavior of Glass Fibre /Vinilester non-crimp fabric [±45]s laminates,” *Journal of Reinforced Plastics and Composites*, vol. 30, pp. 1015-1028, 2011.

[12]  L. Rozite, J. Varna, R. Joffe and A. Pupurs. “An analysis of nonlinear behaviour of lignin based flax composites,” *Mechanics of Composite Materials*, vol. 49, pp. 139-153, 2013.

[13]  L. Rozite, J. Varna, R. Joffe and A. Pupurs. “Nonlinear behavior of PLA and lignin-based flax composites subjected to tensile loading,” *Journal of thermoplastic composite materials*, vol. 26, pp. 476-496, 2011.

[14]  L. J. Zapas and J. M. Crissman. “Creep and Recovery Behavior of Ultra-high Molecular Weight Polyethylene in the Region of Small Uniaxial Deformations,” *Polymer*, vol. 25, pp. 57-62, 1984.

[15]  L. Pupure, J. Varna and R. Joffe. “On viscoplasticity characterization of natural fibers with high variability,” *Advanced composites letters*, vol. 24, pp. 125-129, 2015.

[16]  L. Pupure, J. Varna and R. Joffe. “Methodology for macro-modeling of bio-based composites with inelastic constituents,” *Composite Science and Technology*, vol. 163, pp. 41-48, 2018.

[17]  Z. Al-Maqdasi, L. Pupure, G. Gong, N. Emami and R. Joffe. “Time-dependent properties of graphene nanoplatelets reinforced high-density polyethylene”, Journal of Applied Polymer Science, vol. 138, 2021.

[18]  R. A. Schapery. “Further development of a thermodynamic constitutive theory: stress formulation,” *Purdue University report* No. 69-2, 1969.

[19]  R. A. Schapery. “On the Characterization of nonlinear viscoelastic materials,” *Journal of Polymer Engineering and Science*, vol. 9, pp. 295-310, 1969.

[20]  L. Pupure, R. Joffe, J. Varna and B. Nystrom. “Development of constitutive model for composites exhibiting time dependent properties,” *IOP Conference series: Materials Science and Engineering*, vol. 48, 2013.

[21]  J. Varna, L. Pupure and R. Joffe. “Incremental forms of Schapery’s model: convergence and inversion to simulate strain controlled ramps,” *Mechanics of Time-Dependent Materials*, vol. 20, pp. 353-552, 2016.

[22]  L. Pupure, J. Varna and R. Joffe. “Natural fiber composites: challenges simulating inelastic response in strain controlled tensile tests,” *Journal of Composite Materials*, vol. 50, pp. 575-587, 2016.

[23]  R. Brouwer. “Nonlinear viscoelastic characterization of transversely isotropic fibrous composites under biaxial loading”, *PhD thesis Free univ of Brussels*, 1986.