A Control Chart Approach to Power System Line Outage Detection Under Transient Dynamics

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Abstract—Online transmission line outage detection over the entire network enable timely corrective actions to be taken, which prevents a local event from cascading into a large scale blackout. Line outage detection aims to detect an outage as soon as possible after it happened. Traditional methods either do not consider the transient dynamics following an outage or require a full Phasor Measurement Unit (PMU) deployment. Using voltage phase angle data collected from a limited number of PMUs, we propose a real-time dynamic outage detection scheme based on alternating current (AC) power flow model and statistical change detection theory. The proposed method can capture system dynamics since it retains the time-variant and nonlinear nature of the power system. The method is computationally efficient and scales to large and realistic networks. Extensive simulation studies on IEEE 39-bus and 2383-bus systems demonstrated the effectiveness of the proposed method.

Index Terms—Anomaly detection, generalized likelihood ratio (GLR), line outage, outage localization, phasor measurement unit (PMU), transient dynamics.

I. INTRODUCTION

WITH the emergence and integration of distributed energy resources, there is increasing volatility in modern power systems. Ensuring a reliable electricity supply is an essential but challenging task for independent system operators (ISOs). To this end, fast anomalous event detection is necessary to contain system disruptions and minimize the potential impact. Power systems can experience numerous types of disruptions. Among them, power line outage receives a significant amount of attention from both the research community and industry. Power line outages can happen due to reasons like adverse weather conditions or component degradation. An outage, if not detected and addressed in time, could lead to severe disruptions and possibly cascading failures.

Real-time situational awareness about the system, e.g., changes in operating conditions and external system contingencies, enables ISOs to identify and respond to abnormal events promptly [1]. Without it, a local disruption can cascade into a large-scale blackout. One of the common contributing factors of the 2003 Northeast and 2011 Southwest blackout was that the ISOs were not alerted in time about external outage contingencies, e.g., tripping of a key transmission line [2]. One of the challenges about real-time monitoring is that outage dynamics can manifest in a time scale of milliseconds [3]. Traditional supervisory control and data acquisition (SCADA) system is not able to capture these dynamics since it reports at a rate of one measurement every several seconds [4]. On the other hand, the increasing penetration of Phasor Measurement Units (PMUs) makes many real-time monitoring, protection, and control applications possible. PMUs are devices installed at substations capable of recording high-fidelity GPS time-synchronized phasors. An industry-grade PMU can measure voltage and current phasors on the bus with a total vector error of less than 1%, and with a reporting rate of 30-60 samples per second. As an essential part of the wide-area management system, many consider PMU technology as the key to grid modernization. PMU technologies are actively studied for tasks such as power oscillation monitoring, abnormal event detection, and dynamic state estimation. For a comprehensive review of PMU applications in the power system, readers can refer to [5].

There is a growing body of work on power line outage detection leveraging on PMU data. Many of the detection schemes focus on the monitoring of bus voltage phase angles. They are implemented in real time, taking advantage of the high-reporting rate of PMUs. Consequently, a common challenge is to keep the computational cost low while still making sure that useful information can be extracted. Another challenge is that not all buses are equipped with a PMU device, making some parts of the system unobservable. Current related works of line outage detection using PMU data can be classified by the two approaches taken. One is a data-driven approach where no or very little physical knowledge about the system is required [6]–[8]. On the other hand, many take a hybrid approach where first-principle models are incorporated with data-driven methods [9]–[15].

1) Data-driven Approach: Using principal component analysis (PCA), Xie et al. monitor the reconstruction error of PMU measurements using a lower-dimensional representation obtained from data under a no-outage condition [6]. Similarly, using PCA of frequency measurements, Rafferty et al. design a control chart to detect and classify abnormal frequency events [7]. Hosur and Duan construct an observation matrix under a normal condition by modeling the network as a linear time-invariant (LTI) system [8]. An alarm is raised whenever the underlying null space of the observation matrix changes. The method requires a window of samples to reflect a null space change. Without a physical model, these data-
driven schemes are flexible enough to detect both outages and other abnormal events. However, they often face difficulties when the events have a low signal-to-noise ratio, e.g., outages with mild phase angle disturbances. The hybrid approach, on the other hand, augments PMU data with physical system information to improve the detection performance under such conditions.

2) Hybrid Approach: Using Ohm’s law, Jamei et al. show that the correlation matrix between voltage and current measurements of a pair of buses have rank one under normal condition [10]. An alarm is raised once this dependency changes. However, currents and voltages at both ends of the line are required. Another group of work assumes that the power system settles into a quasi-steady state immediately after an event. Ardakanian et al. monitor the discrepancy between measured and computed steady-state currents using recovered admittance matrix [12]. Using pre- and post-outage steady-state bus angles, outage detection is formulated as an optimization problem by Tate and Overbye [13], and a quickest change detection problem by Chen et al. [14]. This line of work does not require all buses to be monitored by a PMU. However, the steady-state approximation would not be sufficient at describing the actual system behavior following an outage. A later work by Rovatsos et al. attempts to incorporate transient dynamics using pre-determined participation factors (PF) matrix [15]. However, the adequacy of the chosen PF matrix at describing system response following an outage is not studied.

To the best of the authors’ knowledge, there is still minimal work on detection schemes that allow unobservable buses and consider system transient response to an outage. In this work, we take a hybrid approach where the power system model is the basis for the statistical detection method. We derive a time-variant small-signal relationship between net active power and nodal voltage phasor angles from the AC power flow model. Outage detection is then formulated as a statistical distribution change detection problem. A generalized likelihood ratio (GLR) detection scheme is implemented to detect the outage at a pre-specified false alarm rate.

The main contributions of our work can be summarized in two aspects. Firstly, our power system model retains the non-linearity and time-varying characteristics of transient response that follows after the outage. The system is not assumed a quasi-steady state immediately after the disruption. In fact, from our dynamic outage simulation, we observed that the transient response could last over 10 seconds. Secondly, the proposed GLR detection scheme can deal with the trade-off between system-wide false alarm rate and detection delay. The ability to decide among different detection thresholds gives ISOs the flexibility to cater to their system needs. The detection scheme is also computationally efficient, therefore friendly for online implementation in a large network.

The remainder of this paper is organized as follows. Section II describes the power system model and the statistical model used to characterize system behaviors before and after the outage. Then, dynamic detection and identification scheme is developed in Section III. Effectiveness of the proposed scheme on simulation data of two test power systems are reported and discussed in Section IV. In Section V we conclude the paper with two future research directions.

II. PROBLEM FORMULATION

A. Power System Model

Given a power network where \( N \) buses are connected by \( L \) power lines, power flows in the network can be characterized by a set of non-linear algebraic equations called the AC power flow equations. This set of equations describes the relationship between net active power (P), net reactive power (Q), nodal voltage magnitude (V) and phase angle (\( \theta \)) governed by Kirchhoff’s circuit laws. They can be written as:

\[
P_m = V_m \sum_{n=1}^{N} V_n Y_{mn} \cos(\theta_m - \theta_n - \alpha_{mn}),
\]

\[
Q_m = V_m \sum_{n=1}^{N} V_n Y_{mn} \sin(\theta_m - \theta_n - \alpha_{mn}),
\]

for bus \( m = 1, 2, \ldots, N \) [16]. \( Y_{mn} \) is the magnitude of the \((m,n)\)th element of the bus admittance matrix \( Y \) when the complex admittance is written in the exponential form, i.e.

\[
Y_{mn} e^{j\alpha_{mn}} = C_{mn} + jB_{mn}.
\]

For a bus equipped with PMU, V and \( \theta \) are measured and available. We also assume that the elements of bus admittance matrix are known. For a large system, \( Y \) is usually a sparse matrix since any single bus only has a few incident buses, i.e., \( Y_{mn} = 0 \) if bus \( m \) and bus \( n \) are not connected. The system topology is embedded in the admittance matrix \( Y \). In particular, the admittance matrix is constructed by

\[
Y = A|y|A^T
\]

where \( A \) is the bus to branch incidence matrix with columns representing lines and rows as buses. \( A^T \) is the transpose of \( A \). For the \( i_{th} \) line transmitting power from bus \( m \) to bus \( n \), the \( i_{th} \) column of the matrix \( A \) has 1 and -1 on the \( m_{th} \) and \( n_{th} \) position and 0 everywhere else. \( |y| \) is the diagonal matrix with individual line admittances on the diagonal.

Without the loss of generality, we assume bus 1 is the reference bus. This bus serves as the angular reference to all other buses, and its phase angle is set to 0°. Voltage magnitude at the reference bus is also set to 1.0 per unit (p.u.). Let \( P, Q, \theta, \) and \( V \) represent the \((N-1)\)-dimensional column vectors of net active power, net reactive power, voltage angles and magnitudes respectively at all buses except the reference bus. Taking a derivative with respect to time \( t \) on both sides of (1), we obtain

\[
\frac{\partial P}{\partial t} = \begin{bmatrix} J_1 & J_2 \end{bmatrix}, \quad \frac{\partial Q}{\partial t} = \begin{bmatrix} J_3 & J_4 \end{bmatrix},
\]

\[
\frac{\partial \theta}{\partial t} = \begin{bmatrix} J_1 \end{bmatrix}, \quad \frac{\partial V}{\partial t} = \begin{bmatrix} J_2 \end{bmatrix},
\]

where \( J_i, i = 1, \ldots, 4 \) are the four submatrices of the AC power flow Jacobian with

\[
J_1 = \frac{\partial P}{\partial \theta}, J_2 = \frac{\partial P}{\partial V}, J_3 = \frac{\partial Q}{\partial \theta}, J_4 = \frac{\partial Q}{\partial V}.
\]

In the usual operating range of relatively small angles, real power systems exhibit much stronger interdependencies between \( P \) and \( \theta \) and between \( Q \) and \( V \) than those between
\( \mathbf{P} \) and \( \mathbf{V} \) and between \( \mathbf{Q} \) and \( \theta \) \cite{17}. By neglecting \( \mathbf{J}_2 \) and \( \mathbf{J}_3 \), \cite{4} reduces to the decoupled AC power flow equations where the changes in voltage angles and magnitudes are not coupled, i.e., \( \mathbf{J}_2 = \mathbf{J}_3 = 0 \). Therefore, we obtain a small-signal time-variant model describing the relationship between active power mismatches and the changes in voltage angles:

\[
\frac{\partial \mathbf{P}}{\partial t} \approx \mathbf{J}_1(\theta) \frac{\partial \theta}{\partial t}. \tag{6}
\]

From here onwards, we drop the subscript 1 from \( \mathbf{J}_1 \). The off-diagonal and diagonal elements of the \( \mathbf{J} \) matrix can be derived from Eqn (1a) respectively:

\[
\frac{\partial P_m}{\partial \theta_n} = \mathbf{V}_m \mathbf{V}_n \mathbf{Y}_{mn} \sin(\theta_m - \theta_n - \alpha_{mn}) , m \neq n , \tag{7a}
\]

\[
\frac{\partial P_m}{\partial \theta_m} = - \sum_{n=1}^{N} \mathbf{V}_m \mathbf{V}_n \mathbf{Y}_{mn} \sin(\theta_m - \theta_n - \alpha_{mn}) . \tag{7b}
\]

Note that \( t \in [0, \infty) \) is implicit in the continuous-time quantities \( \mathbf{P}, \mathbf{V} \) and \( \theta \). Accordingly, we define their discrete counterparts as \( \mathbf{P}_k, \mathbf{V}_k \) and \( \theta_k \) at time \( t_k \) for \( k = 1, 2, \ldots \).

For PMU devices with a sampling frequency of 30 Hz, \( \Delta t = t_k - t_{k-1} = 1/30 \) s. A first-order difference discretization by Euler’s formula can approximate (6) by:

\[
\Delta \mathbf{P}_k = \mathbf{J}(\theta_{k-1}) \Delta \Theta_k , \tag{8}
\]

where \( \Delta \mathbf{P}_k = \mathbf{P}_k - \mathbf{P}_{k-1} \) and \( \Delta \Theta_k = \theta_k - \theta_{k-1} \), i.e., the active power mismatch and difference between two consecutive angle measurements. We have derived a time-variant relationship between variations in phasor angles and net active power on buses. The key feature of our model lies in the \( \mathbf{J} \) matrix in (8) which retains the non-linear and dynamic nature of the AC power system. The matrix changes with \( \theta \), which in turn changes with time.

Methods relying on a static relationship between \( \Delta \mathbf{P} \) and \( \Delta \Theta \) make three further assumptions about the system \cite{13, 14}: 1) flat voltage profile, i.e. \( \mathbf{V}_m \approx \mathbf{V}_n \approx 1.0 \) p.u.; 2) approximately homogeneous bus angles across the network, i.e. \( \cos(\theta_m - \theta_n) \approx 1, \sin(\theta_m - \theta_n) \approx 0 \); 3) reactive property of a line is much more significant than its resistive property, i.e. \( B_{mn} \gg G_{mn} \). Under these assumptions, (6) reduces to

\[
\frac{\partial \mathbf{P}}{\partial t} \approx -\mathbf{B} \frac{\partial \Theta}{\partial t} , \tag{9}
\]

where \( \mathbf{B} \) is the imaginary component of \( \mathbf{Y} \). While line resistances in transmission systems are generally one order of magnitude smaller than reactances, this is not usually the case for distribution systems \cite{18}. Also, a static model may not be accurate enough to reflect the transient behavior after an outage since the homogeneous angles assumption might be violated \cite{19}. We routinely encounter this phenomenon in our dynamic simulation. For example, in Fig. 1 the balance between voltage angles is severely distorted following an outage, e.g., at around \( t = 3.75 \) s. Furthermore, the duration of transient dynamics is non-negligible for real-time detection purposes. Therefore, to reflect the dynamic behavior in a timely and accurate manner, \( \mathbf{J} \) matrix in (8) is updated by real-time streaming PMU data.

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**B. Statistical Model**

For a balanced steady-state power system with no active power mismatch, we have \( \mathbf{P}_0 = 0 \). Within a short period of time, net active power fluctuates around zero as the generators respond to random changes in electricity demand. Therefore, we can model the trajectory of \( \mathbf{P} \) as a Brownian motion with drift \( 0 \) and variance \( \sigma^2 \mathbf{I} \) which is a continuous-time stochastic process: \( \{ \mathbf{P}_t : t \in [0, \infty) \} \). \( \sigma^2 \) is pre-determined and \( \mathbf{I} \) is an identity matrix of appropriate dimension. One of the implications of a Brownian motion is that their independent increment, i.e. \( \Delta \mathbf{P}_k = \mathbf{P}_{t_k} - \mathbf{P}_{t_{k-1}} \), follows a multivariate Gaussian distribution with mean \( 0 \) and variance \( \sigma^2 (t_k - t_{k-1}) \mathbf{I} \) \cite{20}. In particular, taking \( s = 1 \), we have \( t_k - t_{k-s} = \Delta t \) and

\[
\Delta \mathbf{P}_k \sim \mathcal{N}(0, \sigma^2 \Delta t \mathbf{I}). \tag{10}
\]

Since \( \sigma^2 \) is pre-determined, we can replace \( \sigma^2 \Delta t \) by \( \sigma^2 \) for notational simplicity. Rearranging the variables in (8), we have

\[
\Delta \Theta_k = \mathbf{J}(\theta_{k-1})^{-1} \Delta \mathbf{P}_k . \tag{11}
\]

Therefore, we can characterize bus angle variations by

\[
\Delta \Theta_k \sim \mathcal{N}(0, \sigma^2 (\mathbf{J}(\theta_{k-1})^{-1})^T \mathbf{J}(\theta_{k-1}))^{-1} . \tag{12}
\]

From (12), we see that the angle variations at time \( k \) are characterized by the structure of \( \mathbf{J} \) and the angle values at \( t = k-1 \). Let \( \mathcal{L} \) represent the set of all possible combinations of outages, e.g., single-line outage, double-line outage. When an outage \( \ell \in \mathcal{L} \) happens, the grid topology and the bus admittance matrix changes. The new bus admittance matrix \( \mathbf{Y}_\ell \) induces a new \( \mathbf{J}_\ell \) and therefore, a new distribution of \( \Delta \Theta_k \). There is a one-to-one correspondence between an outage scenario and a distribution of \( \Delta \Theta_k \). Furthermore, we assume that the outage is persistent, i.e., tripped lines are not restored in the time under consideration. We also assume that the outage would not result in any islanding in the network, i.e., no part of the system is isolated from the main grid.

In light of the above characterization, we adopt a hypothesis testing framework to detect the distribution change in \( \Delta \Theta_k \):

\[
H_0 : \Delta \Theta[k] \sim \mathcal{N}(0, \sigma^2 (\mathbf{J}_0^T \mathbf{J}_0)^{-1}) , \tag{13a}
\]

\[
H_1 : \Delta \Theta[k] \sim \mathcal{N}(0, \sigma^2 (\mathbf{J}_\ell^T \mathbf{J}_\ell)^{-1}) , \ell \in \mathcal{L} . \tag{13b}
\]
for \( k = 1, 2, \ldots \). The null hypothesis is that there is no outage, and the corresponding Jacobian is \( J_0 \). The alternative hypothesis is that there is an outage scenario \( \ell \), where the corresponding Jacobian is \( J_\ell \). If we reject the null hypothesis at time \( \tau \), then the distribution of \( \Delta \theta[k] \) has changed, and the outage is detected. The detailed procedure of real-time detection under this framework is described in Section III.

A common challenge for PMU deployment is that not all buses are equipped with a PMU. Here we adapt the previous formulations to a limited PMU deployment. Suppose \( K \) PMUs are installed where \( K < N \). Given a selection matrix \( S \in \{0, 1\}^{(K \times N)} \) that selects \( K \) observable buses from the complete set of \( N \) buses, observable bus angle data is

\[
\theta_k^o = S \theta_k^o, \tag{14}
\]

where \( S \) is a diagonal matrix of size \((K \times N)\) and entries equal to 0 or 1. The corresponding angle variations and Jacobian matrix are

\[
\Delta \theta_k^o = S \Delta \theta_k, \tag{15}
\]

\[
J^o(\theta_{k-1}^o) = S J(\theta_{k-1}) S^T. \tag{16}
\]

Therefore, \( \Delta \theta_k^o \) is a \( K \)-dimensional vector and \( J^o(\theta_{k-1}^o) \) is a \((K \times K)\)-dimensional matrix. To obtain the hypothesis testing framework in (13), we replace \( \Delta \theta_k, J_0 \), and \( J_\ell \) by \( \Delta \theta_k^o, J_0^o \), and \( J_\ell^o \) respectively.

Remark 1 (Correspondence Between Topology and Jacobian Matrix): The one-to-one correspondence between the Jacobian and grid topology can be established by looking at how the admittance matrix is constructed in (3). \( Y \) is constructed from the bus incidence matrix \( A \) and the line admittances. After an outage, \( A \) changes to \( A^\ell \) where entries in the columns corresponding to the tripped lines become 0. For example, if the \( l_{th} \)-th line connecting bus \( m \) and bus \( n \) were tripped, entries at the \( m_{th} \) and \( n_{th} \) position of the \( l_{th} \)-th column of \( A \) become 0 in \( A^\ell \), i.e. no active power flow between the two buses. The new bus admittance matrix \( Y^\ell \) is obtained by \( Y^\ell_{ij} = A^\ell_{ij} |Y^\ell_{ij}| A^\ell_{ij}^T \). Accordingly, the new Jacobian matrix \( J^\ell \) describing the post-outage system is obtained by

\[
J^\ell = A^\ell J_A^\ell. \tag{17}
\]

Remark 2 (Inaccuracy of Jacobian Due to Unobservable Neighbor Buses): For a limited PMU deployment, there may be some inaccuracies in the computed diagonal elements of \( J^o(\theta_{k-1}^o) \). In particular, if there is no PMU on bus \( n \), a neighbor of bus \( m \), measurements \( V_n \) and \( \theta_n \) would not be available. Therefore, the term, \( -V_m V_n m n \sin (\theta_m - \theta_n - \alpha_{mn}) \), would not be computable and is treated as 0 for the summation in (7). This issue could be alleviated by carefully designing the PMU placement, a problem that is beyond the scope of this paper. One possible design rule is to make sure that each observable bus has at least one observable neighbor.

III. OUTAGE DETECTION SCHEME

We have formulated the outage detection as a problem of distribution change detection under a hypothesis testing framework in Section II-B. In general, under normal condition, system outputs follow a common distribution with a probability density function \( f_0 \). At some unknown time \( \tau \), the system condition changes, and the density function changes to \( f_1 \). We wish to design a scheme where an alarm is raised once a monitoring statistic \( W(\cdot) \) crosses a pre-defined threshold of \( c \). The two key design aspects are: 1) how to compute the monitoring statistic, \( W(\cdot) \); 2) how to determine the detection threshold, \( c \). The monitoring statistic will be close to zero under a normal condition and increase unboundedly if a change happens. The detection threshold needs to be specified to meet a particular false alarm rate constraint.

We adopt a GLR approach originally proposed by [21] to design the detection scheme. The scheme repeatedly evaluates the likelihood of a normal condition against the likelihood of an abnormal condition. In our problem, bus angle variations are not independent samples since the distribution at time \( k \) is influenced by bus angles at time \( k - 1 \) as shown in (12).

\[
\Delta \theta_k = \theta_k - \theta_k^0, \quad \ell = \ell^o. \tag{18}
\]

\[
Z_k(\ell) = \ln \frac{f_\ell(\Delta \theta_k(\theta_{k-1}^o))}{f_0(\Delta \theta_k(\theta_{k-1}^o))} \tag{19}
\]

be the log-likelihood ratio of an outage scenario \( \ell \) at time \( k \). \( Z_k(\ell) \) is positive if the likelihood of a change is larger than that of a normal condition. Then the test statistic is:

\[
G_k = \max \left\{ 0, \max_{1 \leq j \leq k} \sum_{l=1}^{k \in L} Z_j(\ell) \right\} \tag{20}
\]

and the GLR detection scheme will raise an alarm at the time:

\[
D = \inf \left\{ k \geq 1 : G_k \geq c \right\}. \tag{21}
\]

Since the time and location of the outage are not known a priori, they are replaced by their maximum likelihood estimates. Schemes of the form involving searching through the maximum over time \((1 \leq i \leq k)\) and over likelihood \(\sum_{j=1}^{k} Z_j(\ell)\) are referred to as the GLR schemes. Such schemes have optimal properties in terms of their detection performance. Let \( E_{H_0}(D) \) be the expectation of time of alarm when there is no outage, i.e., mean time to a false alarm. Suppose \( c \) is chosen such that the scheme satisfies a certain false alarm rate, \( E_{H_0}(D) \geq \gamma (1 + o(1)) \). For conditionally independent data, Lai has proved that the detection rule (19) is asymptotically optimal in the sense that among all rules \( T \) with \( E_{H_0}(T) \geq \gamma (1 + o(1)) \), it minimizes the worst-case detection delay as defined by

\[
E_{H_1}(T) = \sup \sup E^{(T)}[T > \tau + 1] \tag{22}
\]

as the outage time \( \tau \to \infty \).

For the actual online implementation, we use an recursive formulation of the GLR scheme. Note that \( G_k \) in (18) can be
rewritten as
\[
G_k = \max \left\{ 0, \max_{\ell \in \mathcal{L}} \max_{1 \leq i \leq k} \sum_{j=1}^{k} Z_j(\ell) \right\},
\]
\[
= \max_{\ell \in \mathcal{L}} \max_{1 \leq i \leq k} \sum_{j=1}^{k} Z_j(\ell),
\]
\[
= \max_{\ell \in \mathcal{L}} W_{\ell,k}.
\]
(21)
where in the first step we have switched the position of the two inner max operators since the overall maximum is not affected \[23\]. Also, in the last step,
\[
W_{\ell,k} = \max \left\{ 0, W_{\ell,k-1} + Z_k(\ell) \right\},
\]
(22)
an equivalent recursive form of the term \( \max_{1 \leq i \leq k} \sum_{j=1}^{k} Z_j(\ell) \) in \( G_k \). Therefore, for every scenario \( \ell \), we just need to keep track of the monitoring statistic \( W_{k-1} \) at the previous time step and obtain the log-likelihood ratio \( Z_k(\ell) \) at the current time step. \( Z_k(\ell) \) can be found analytically by
\[
Z_k(\ell) = \log |\mathbf{J}_\ell| - \log |\mathbf{J}_0| + \frac{1}{2\sigma^2} \Delta \theta_k^T \left[ \mathbf{J}_0^T \mathbf{J}_0 - \mathbf{J}_\ell^T \mathbf{J}_\ell \right] \Delta \theta_k,
\]
(23)
based on the multivariate Gaussian distribution likelihood function. Using the recursive formulation, the stopping time is
\[
D = \inf \left\{ k \geq 1 : \max_{\ell \in \mathcal{L}} W_{\ell,k} \geq c \right\}.
\]
(24)
Intuitively, the threshold is crossed when the evidence against the normal condition, i.e., no outage, has accumulated to a significant level. \( c \) is a predefined threshold that controls the balance between the detection delay and the false alarm rate. A smaller \( c \) corresponds to a more sensitive scheme that may have a quicker detection but could potentially flag more normal fluctuations as outages. One advantage of using the GLR approach is that such tradeoff can be systematically quantified. Following \[14\], given a false alarm rate constraint, \( c \) could be approximated by
\[
c = \ln(ARL_0 \times p),
\]
(25)
where \( ARL_0 \) is the average run length to a false alarm of the scheme when no outage occurs. \( p \) is the number of PMUs installed. For example, \( c = 18.45 \) when \( ARL_0 = 1 \) day with 39 PMUs installed. With this detection delay and false alarm rate tradeoff in mind, ISOs can choose a desired level of sensitivity, catering to the individual system needs, and implement it in the detection scheme through parameter \( c \) and \( ARL_0 \). A flowchart summarizing the working of the detection and identification scheme outlined in this section is shown in Fig. 2.

Remark 3 (Identification of Tripped Lines): Following detection, the actual lines tripped need to be identified so that follow-up, potentially automatic, actions can be taken. Since we monitor and compare the likelihood of every outage scenario online, one way to identify the tripped line(s) without any extra computation is to search for the scenarios with the top three likelihoods at the time of detection. In particular,

\[
W_{\ell,(1)} \geq W_{\ell,(2)} \geq W_{\ell,(3)} \geq W_{\ell,D},
\]
(26)
for all \( \ell \in \mathcal{L} \).

IV. CASE STUDIES

A. Simulation Setting

We test our detection scheme on two IEEE standard test power systems, namely 39 bus New England system \[24\] and 2383 bus Polish system. System transient responses following an outage are simulated using the open-source dynamic simulation platform COSMIC \[25\] in which a third-order machine model is used. We conduct extensive single-line outage detection and identification analysis on the 39 bus system by comparing our method to two other methods. Outages on the 2383 bus system are simulated to show that the proposed scheme can be deployed on large-scale systems as well.

We assume that the sampling frequency of PMU is 30 Hz. For every new simulation, we vary the system loads by a random percentage between -5% and 5% from the baseline values. Each simulation runs for 10 seconds, and the line outage is takes place at the 3rd second. Active power fluctuations are assumed to be uncorrelated and have homogeneous variances where \( \sigma^2 = 0.005 \) in \[13\]. Artificial noise is added to all sampled bus angle data, \( \Delta \theta \), to account for system and measurement noise \[20\]. The noises are drawn from a normal distribution with mean 0 and standard deviation equivalent to 10% of the average value of sampled \( \Delta \theta \) on respective buses. Detection thresholds \( c \) in \[24\] corresponding to seven different false alarm rates are obtained by \[25\] and listed in Table 1.

B. Simulation Results

1) 39 Bus New England System: The 39 bus system has 39 buses, 10 generators, and 46 transmission lines. We conduct
Table I
Detection Thresholds Corresponding to Different Systems and False Alarm Rates

| Mean Time to False Alarm (day) | Detection Statistics | False Alarm Rate |
|-------------------------------|----------------------|------------------|
| 1/24                          | 0.00                 | 0.25             |
| 1/4                           | 0.00                 | 0.75             |
| 1/2                           | 0.00                 | 1.00             |
| 1                            | 0.00                 | 1.25             |
| 2                            | 0.00                 | 1.50             |
| 7                            | 0.00                 | 1.75             |
| 30                           | 0.00                 | 2.00             |

Fig. 3. Progression of monitoring statistics for line 10 outage.

Fig. 4. Comparison of the empirical distribution of detection delays in seconds under different false alarm rates. The number in the label is the number of days until a false alarm.

with $ARL_0 = 30$ days. These differences are not significant. Hence, the proposed scheme’s performance based on detection delay is not overly sensitive to different false alarm rates.

We have also studied the detection performance across different line outages. There are clear variations in terms of detection delay among those detected outages. These variations can be largely attributed to the PMU placement and the grid topology. For outages with almost zero detection delay, they are lines where either PMUs are installed on both ends of the line, e.g., line 3, 21, and 23, or one PMU is connected to the line, e.g., line 20, 25, and 27. Signals can be readily picked up by nearby PMUs. On the other hand, the absence of PMU nearby may have contributed to the longer detection delays. In particular, there are no PMUs available on either end of line 9, 10, 28, 32, 33, and 34. These outage signals have to be detected by sensors far away from the location. Fig. 5 summarizes the comparison.

Another factor is the power grid topology. The scheme recorded shorter delays for lines critical to the system. For example, line 2, 14, and 15 connect a generator bus to the system; line 30 and 31 are the only lines connecting a subnetwork to the main network. Transient signals from these outages were more severe and lasted longer. On the other hand, a non-critical line outage produces weak transient signals which are difficult to detect. These lines, e.g., line 5, 11, 13, and 26, are likely to have little effect on the active power transmitted in the system. See Fig. 5 for the comparison.

a) Detection Performance: The proposed method can detect outages instantaneously in most cases with a full PMU deployment. Due to the page constraint, we only present the detection results of a limited PMU deployment here. Fig. 4 shows the empirical distribution of detection delays under seven false alarm rates. A more stringent false alarm rate corresponds to a detection scheme with longer delays on average. For example, the scheme with an $ARL_0 = 1/24$ day detects much more outages within 0.25 second than the one with $ARL_0 = 30$ days. These differences are not significant. Hence, the proposed scheme’s performance based on detection delay is not overly sensitive to different false alarm rates.

b) Comparison with Other Methods: We compared the proposed method’s outage detection performance with two other methods. The line outages considered here are line 26, 27, and 34. Other methods considered here are the static detection method based on DC power flow model in [14], under a full and limited PMU deployment, and the CUSUM-type central rule based on Ohm’s law in [10], with a limited PMU deployment. The placement of 10 PMUs is the same for all methods. For the CUSUM scheme in [10], parameters are chosen to satisfy the same false alarm rates in Table I based on formula in [27]. The respective detection delays are...
In this work, we developed a real-time dynamic line outage detection and identification scheme based on the AC power flow model and GLR scheme. We derived a time-variant small-angle relationship between bus voltage angles and active power injections. We obtained the pre- and post-outage statistical models of the angle variations. The proposed scheme is effective in both detection and identification. It is also scalable as seen from the results in the 2383 bus system.

For further research, we would investigate the optimal number and placement of a limited number of PMUs. As seen from Section IV, there is a varying level of detection delays experienced are considerably longer than those in the 39 bus system. Therefore, delays experienced are considerably longer than those in the 39 bus system. There are also several undetected outages.

V. CONCLUSION

For further research, we would investigate the optimal number and placement of a limited number of PMUs. As seen from Section IV, there is a varying level of detection delays experienced are considerably longer than those in the 39 bus system. Therefore, delays experienced are considerably longer than those in the 39 bus system. There are also several undetected outages.

V. CONCLUSION
delays due to PMU placement. The number of PMUs needed to achieve a certain level of identification accuracy is also worth investigating. We would also consider incorporating generator dynamics into our system model, where we hope the detailed physical model could provide an even better direction for outage detection and identification.

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