Atomistics of Tensile Failure in Fused Silica: 
Weakest Link Models Revisited

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ABSTRACT

In weakest link models the failure of a single microscopic element of a brittle material causes the failure of an entire macroscopic specimen, just as a chain fails if one link fails. Pristine samples of glass, such as optical communications fiber, approach their ideal strength, and their brittle tensile failure has been described by this model. The statistics of weakest link models are calculable in terms of the statistics of the individual links, which, unfortunately, are poorly known. Use of the skewness of the failure distribution may permit simultaneous determination of the statistics of the individual weak links and of their number density, which indicates their physical origin. However, the applicability of weakest link models to real materials remains unproven.

Keywords: Fused silica, glass, fracture, weakest link models

1. INTRODUCTION

Leonardo da Vinci knew that longer wires fail at lower tensile loads than shorter ones. For a simple chain this result is so obvious it hardly bears mention: a chain is only as strong as its weakest link; if any link fails the chain fails. Adding links to a chain will weaken it if one of the added links is weaker than any of the initial links, or will leave its strength unchanged if all added links are stronger than the weakest initial link. Adding links to a given chain will inevitably weaken it if enough links are added, if the strengths of the links are drawn randomly from a non-singular distribution. The weakest link found in a large sample (a long chain) will on average be weaker than the weakest link found in a small sample (a short chain).

These results are obvious to us, but may have been less obvious in Leonardo’s day, before the invention of the science of mechanics (and statistics), when the mechanics of a chain was imperfectly understood. In fact, it is not completely trivial, for it depends on the loading conditions, the properties of the material of which the links are made, and the definition of failure. For example, if the links are capable of work-hardening and failure is defined as extension of the chain by a suitable fixed length, then a long chain may actually be able (depending on its detailed properties) to sustain a greater load than a short one.

2. WEAKEST LINK MODELS

From the picture of a chain in tension a simple idealized model, known as the weakest link model, is derived. In this model a fiber (wire, filament or rod) loaded in tension is described as a series of independent mechanical elements ordered in a single file, each of which transmits a load (force, or momentum flux) from its immediate neighbor on one side to its immediate neighbor on the other side. The model is quasi-static: all elements are assumed to be subjected to the same tensile load, which will be the case if there are no accelerations and no forces on the links except those exerted by their immediate neighbors. It is also local, in the sense that force is transmitted only across the boundaries between two elements; this assumption may fail if the elements are coupled by long range forces, such as electromagnetism, as in a current-carrying or charged chain or one made of piezoelectric material, although these effects are unlikely to be significant in practice. Failure of the entire chain is defined as the failure of a single element. Given these assumptions, the strength of a chain is the strength of its weakest link.

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The statistics of weakest link models were apparently first considered by Peirce, who worked for the British Cotton Industry Research Association and published in the Journal of the Textile Institute. This is curious because weakest link models are unlikely to be applicable to ropes and cables made up of many parallel strands. These strands divide the load amongst themselves, so that the failure of a single strand need not, and usually does not, imply the failure of the entire rope. We are all familiar with ropes that are partly frayed, but which still carry a load. Multi-stranded ropes are also non-local, because the shift of load from a failed strand to the remaining strands occurs by static friction over an extended length of the rope, and depends in a complex manner on the lay of the rope. In addition, load shifting occurs as a result of differential initial length, elastic stiffness and creep between strands, even when there has been no failure.

The statistics of weakest link models were reviewed by Epstein, whose paper remains useful today. He made a number of significant points:

1. If the links have a statistical distribution of strengths, then the mean strength of the entire chain (the strength of the weakest link) is a decreasing, albeit generally very slowly decreasing, function of the number of links in the chain.

2. The distribution of chain strengths is generally a skew function, with a longer “tail” in the direction of weaker chains; this is a consequence of the fact that a single unusually weak link makes a weak chain, while an unusually strong link has no effect on the strength of the chain.

3. For normally distributed link strengths, the dispersion in chain strengths is a slowly decreasing function of the number of links.

4. In many physically important problems the weakest link model is inapplicable; he cites notches on the surface of a bent beam as an example. A single notch serves as a site of stress concentration, while a large number of closely spaced notches (a roughened surface), if of equal depth, reduce the stress in the matter below them to low values, thus reducing the stress at the apex of any one notch.

Weakest link models are expected to be applicable to the tensile failure of brittle materials in many, but not all, circumstances. The reason for this, known since the work of Griffith, is that brittle materials usually fail in tension because of stress concentration at imperfections—surface scratches, voids, or heterogeneities in the bulk. Once local failure occurs, the size of the imperfection and its degree of stress concentration increase, and the object ruptures.

Suppose a macroscopic brittle object may be divided into small cells, each of which contains no more than one imperfection, and each imperfection is contained within a single cell. In most materials these cells must be very small because the density of imperfections is large, but in pristine glass fiber the imperfections may be so rare that a cell can encompass the entire diameter of the fiber and extend in length for many diameters. If such a division into cells is possible, with negligible interaction between the different cells and the imperfections within them, then we would expect a weakest link model to be applicable, because the failure of any one cell would lead to the failure of the entire object. This is true even though the cells need not be arrayed in a line like links in a chain; even cells side-by-side are independent potential sites of failure.

An essential assumption is that imperfections do not interact elastically. This is valid if the imperfections are small and well separated in all their dimensions, but may not hold if they are geometrically extended, such as grain boundaries in a ceramic or surface scratches. In such more complex geometries it may again be possible to use weakest link models if the cells are large enough to contain many imperfections, for then each cell may act as an independent link, even though the individual imperfections interact strongly. In such a case, the resulting statistics will be those of the cells, not those of the individual imperfections.

It is a curious feature of models of this kind that a renormalization group transformation is not possible (their statistics are not invariant under such a transformation) because the statistics of weakest link models depend on the total number of links, which is not invariant. This follows from the assumption that the flaws have a characteristic (small) size, and are not “fractal”. It also assumes that a single local failure is sufficient to produce general failure; in other words, that general failure is not a consequence of the interaction of many failures in the type of process now known as self-organized criticality.

The dependence of the statistics of weakest link models on the number of participating links means that it may be possible to learn something about the microphysics of failure from the statistics of the strength of macroscopic objects. That is the subject of this paper.
3. FUSED SILICA

Fused silica and related silica glasses are materials of enormous technological importance. They are also among the few materials (the others are carbon nanotubes and, when creep is negligible, metallic whiskers) which may approach their ideal limiting strengths because they can be produced apparently without volume or surface stress-concentrating flaws.

3.1. History

Griffith was apparently the first to realize the importance of flaws in reducing the strength of brittle materials. Griffith also found that finer glass fibers were systematically stronger than thicker ones, which may naturally be attributed (in a weakest link model) to the presence of more links in the thicker fibers. He compared this result to the similar results of Karmarsch from 1858 on metal wires, although we would now consider brittle (glass) and ductile (metallic) failure to be so different that a close comparison is not justified. For historical reviews see the work of Holloway and Kurkjian.

Griffith measured tensile strengths under ordinary room conditions of short lengths of very fine (3–4 µ diameter) glass fibers of about 500,000 psi, which are comparable to those measured under similar conditions today. He also noted the effect of aging and of static fatigue in reducing these strengths, which remain important subjects of research. He speculated that the strength of these fine fibers could be extrapolated to infinitesimal diameters to determine the limiting strength of ideal glass. This extrapolation is inconsistent with his hypothesis that he had produced flaw-free samples and not very meaningful because it was based on data obtained under ambient conditions for aged samples; we now know that ideal strength requires an inert environment. Despite this, his numerical value (1.6 × 10^6 psi) for the ideal strength of glass is quite close to modern values.

| Diameter   | Breaking Stress | Diameter   | Breaking Stress |
|------------|-----------------|------------|-----------------|
| 0.001 inch | ls. per sq. inch | 0.001 inch | ls. per sq. inch |
| 0.001 inch | 24,900          | 0.95       | 117,000         |
| 0.001 inch | 42,300          | 0.75       | 134,000         |
| 0.001 inch | 50,800          | 0.70       | 164,000         |
| 0.001 inch | 64,100          | 0.60       | 185,000         |
| 0.001 inch | 79,600          | 0.56       | 154,000         |
| 0.001 inch | 88,500          | 0.50       | 195,000         |
| 0.001 inch | 82,600          | 0.38       | 232,000         |
| 0.001 inch | 85,200          | 0.26       | 332,000         |
| 0.001 inch | 99,500          | 0.165      | 498,000         |
| 0.001 inch | 88,700          | 0.130      | 491,000         |

Some of Griffith’s data are also surprising, given modern understanding. He found (Table 1) a fairly smooth dependence of strength on diameter, while we would expect fibers to be either without mechanical damage and very strong, or damaged and very weak; finer fibers would be expected to be more frequently found undamaged, but not systematically stronger than undamaged thicker fibers. We would also expect the strength to be essentially independent of diameter for the finest (rarely damaged) fibers, so that no extrapolation to zero diameter would be
necessary or appropriate. After the lapse of 80 years it is impossible to interpret all his data, but some of his results may be a consequence of his testing of aged samples and his use of fiber diameter as an independent variable, in contrast to modern experiments which generally use communications fiber of a single diameter.

At the time of Griffith’s work no theory, reasonable in modern terms, of the ideal strength of materials existed. Approximate theories were soon developed. Polanyi used a simple energetic argument to predict the ideal strength of crystals from the energy required to form new surfaces by rupture. Although he did not refer to the (one year earlier) work of Griffith, his surface energy-based argument resembles Griffith’s argument for the growth of a crack, differing chiefly in that Griffith considered (using Inglis’s analytic solution) stress concentration at the crack tip while Polanyi made the naïve assumption of a cleavage with plane parallel faces. From this Polanyi correctly concluded that for most materials the measured strength is far below any reasonable theoretical estimate, but did not point out stress concentration as the explanation.

3.2. Properties

The development of quantum mechanics and the theory of interatomic forces made possible more quantitative models of the theoretical strength of materials, beginning with the work of Frenkel. Modern band structure theory makes reliable predictions of ideal strengths possible for crystalline materials, although these are generally unobservable because their much lower real strengths are determined by stress concentration at flaws for brittle materials and by dislocation motion for ductile materials. The modern methods are applicable to carbon nanotubes, which may perhaps be made free of defects because they are so small. They may also undergo plastic flow which may prevent them from reaching their ideal strength, even if free of defects, and they are known to buckle reversibly on bending.

Fused silica and silica glasses have apparently not been the subject of modern theoretical strength calculations, probably because it is difficult to specify their non-crystalline structure. This structure can be calculated by molecular dynamics methods, but the result is dependent on the model for the interatomic forces and represents only a small subvolume of the bulk material; extension to the bulk requires a questionable extrapolation (such as periodic boundary conditions). β-cristobalite, a high-temperature crystalline phase of silica with cubic symmetry and a density close to that of fused silica, may be a useful model, but its properties do not appear to have been calculated from first principles and even its elastic constants have not been measured.

The development of fiber optic communications produced a renewal of research on fused silica. Essentially defect-free fiber is now available in enormous quantity, generally encapsulated in a protective coating which prevents mechanical damage and reduces the rate of environmental chemical attack. The tensile strength of this material, directly measured at low temperature (77 °K) where chemical attack is believed to be negligible, may reasonably be taken to be its ideal strength, and is about 140 Kbar. Low-temperature measurements in which the stress is inferred from the strain in two-point bending give similar results, but are less direct because the reduction of the data depends on applying thin beam theory to a geometry in which it is not quantitatively applicable (because the radius of curvature is so small) and because the nonlinear stress-strain relation of fused silica is somewhat uncertain. The measured low-temperature strength is about 0.2 of the Young’s modulus, in accord with rough estimates of the strengths of the chemical bonds. Strengths measured under ambient conditions are about one third of those measured at low temperatures, and depend on the temperature, humidity, and duration of load, indicating complex processes of stress corrosion by water.

4. MICROSCOPIC PHYSICS

Now that the theory of strength-reducing cracks is well understood, and the strength of fused silica has been measured reasonably well, what more can be learned?

4.1. Mechanisms

It took many years before the importance of surface damage, as opposed to volume flaws, for the strength of glass was appreciated. However, the strength of an unflawed (pristine) sample of glass, its ideal strength, is likely determined in its bulk, by the strength of its network of covalent bonds. This does not answer the question of which component of the glass, or which physical process, actually determines that strength; it is possible that its strength is actually determined by the bonds at its surface.

A number of candidate strength-limiting components or processes need be considered. On the largest scale, a small heterogeneity may provide a slight degree of stress concentration and therefore determine the strength. The
extraordinarily small measured dispersion ($\ll 1\%$) of the strength of pristine glass and the at least semi-quantitative approach of the measured low-temperature strength to its estimated ideal value appear to make this implausible, unless the degree of stress concentration is very slight, as might be produced by heterogeneities of small amplitude (a small amplitude fluctuation in elastic constants). In optical communications fiber the germania-doped core provides such a fluctuation, but it is ordered rather than random and homogeneous (to good accuracy) along the length of the fiber.

Some heterogeneity is implicit in the fact that glasses are amorphous, and on the smallest spatial scale significant heterogeneity arises from the fact that Si—O bonds are not all equivalent; each has a slightly different length, and the O—Si—O and Si—O—Si bond angles differ from their ideal values. Thus, in a piece of glass subjected to tension in a specified direction different bonds will fail at different stresses.

In addition to SiO$_2$, real glass, even fused silica, contains other components. There are a variety of impurities, reduced to very low levels in communications-grade optical fiber, but present nonetheless. These introduce bonds other than Si—O bonds, and these other bonds will generally be weaker (as is shown by the fact that mixed silicate glasses are less strong than fused silica). Some of these impurities (such as Na$_2$O and CaO in soda-lime glass) are present at high abundances, while others (such as transition metal oxides in communications-grade fiber) are very rare, but all are sites of weakness. There may be other kinds of strength-limiting defects, such as non-stoichiometrically bonded atoms, interstitials, etc. In the core of optical communications fiber several percent of the Si has been replaced by Ge. Because Ge atoms are slightly larger than Si atoms the bond network is further distorted there, and at the boundary between the pure SiO$_2$ and the GeO$_2$-substituted regions there is systematic strain, analogous to imperfect lattice parameter matching in epitaxial growth of crystals. These effects are distinct from heterogeneity of bulk elastic constants referred to above.

Finding which of these mechanisms actually determines the strength of glass, particularly fused silica, is of both scientific interest and practical importance, for some mechanisms may be affected by changes in composition or processing, potentially leading to stronger fibers. Analogous considerations may apply to the more complicated problem of room-temperature strength, where the resistance to stress-assisted chemical attack determines the strength, rather than ideal mechanical failure alone.

### 4.2. Can Failure Statistics Help?

For each of the possible modes of failure a macroscopic (or even mesoscopic) piece of glass contains a large number of well-separated elastically independent regions. If the failure of a single region leads to general failure, a plausible but unproven assumption, then a weakest link model will apply. The specific mechanisms of failure differ greatly in the number $N$ of independent potential failure sites present per unit volume, or in a given sample. For example, in a 1 m length of 125 $\mu$m diameter fiber the number of elastic heterogeneities or geometrical irregularities may be very small ($N \sim 1–10^4$), the number of Si—O bonds is $N \sim 10^{21}$, and the number of rare impurities may (depending on the impurity) be in the range $0 < N < 10^{15}$.

One means of determining the sources of failure is to perform measurements on specimens in which the number of each type of potential failure site is varied. This is difficult in practice, partly because it may not be possible to control their number, and partly because each series of experiment would require the production of a batch of fiber of custom composition.

It is a general feature of weakest link models that as $N$ increases the strength of the specimen (as in the classical chain or wire) decreases. If the statistics of the strengths of the individual links were known this fact, by itself, would be used to determine $N$ from the dependence of strength on size but these statistics are rarely known, and not in the present case. It is also often difficult to perform experiments with a sufficiently large range of specimen sizes. Similarly, the dispersion of strength among specimens depends on $N$ and could be used to determine it, but again only if the statistics of the individual links are known. These two effects are closely related, and do not give independent information.

Quantitative analysis depends on the assumed statistics of the links, and many possible distributions have been considered. One possible model closely related to classical Weibull statistics, describes each link by a fracture readiness parameter $\alpha$, which may be considered a stress concentration factor, or the reciprocal of a link’s ideal strength. The values of $\alpha$ are distributed according to a distribution $f(\alpha)$, which is defined for $\alpha > 0$ and normalized

$$
\int_0^\infty f(\alpha') \, d\alpha' = 1.
$$

(1)
The sample fails if the largest fracture readiness parameter found among the $N$ links, $\alpha_{\text{max}}$, exceeds a value $\alpha_0$. The probability that it does not fail is, to good approximation if $N \gg 1$,

$$P(\alpha_0) \approx \exp \left(- \int_{\alpha_0}^{\infty} N f(\alpha') d\alpha' \right). \quad (2)$$

The probability that failure occurs for a value of $\alpha_0$ between $\alpha$ and $\alpha + d\alpha$ is $P(\alpha)d\alpha$, where

$$P(\alpha) = Nf(\alpha) \exp \left(- \int_{\alpha}^{\infty} N f(\alpha') d\alpha' \right). \quad (3)$$

The most useful way to parametrize the results is as the ratio of the width $w$ of $P(\alpha)$ to the value $\alpha_{\text{max}}$ at which $P(\alpha)$ is a maximum; in terms of the Weibull modulus $m$

$$\frac{w}{\alpha_{\text{max}}} \approx \frac{1}{m(m-1)^{1/2}}, \quad (4)$$

and the approximation is almost exact if $w$ is defined as the dispersion of $P(\alpha)$

$$w \equiv \left( \frac{d^2 \ln P(\alpha)}{d\alpha^2} \bigg|_{\alpha=\alpha_{\text{max}}} \right)^{-1/2} \quad (5)$$

and $m \gg 1$.

The stretched exponential function is defined

$$f(\alpha) = \frac{C(\nu)}{\alpha_0} \exp \left[ - \left( \frac{\alpha}{\alpha_0} \right)^\nu \right]. \quad (6)$$

This is a general form widely used when the actual functional form is unknown, and includes the simple exponential and Gaussian as special cases. The normalizing constant $C(\nu) \equiv \nu/\Gamma(1/\nu)$ and $N' \equiv NC(\nu)/\nu$. The distribution $P(\alpha)$ is shown in Figure 1 for $\nu = 2$ (a Gaussian).

By successive approximations,

$$\frac{w}{\alpha_{\text{max}}} \approx \frac{1}{\nu \ln N'} \left\{ 1 - \frac{1}{\nu \ln N'} \left[ \frac{\nu - 1}{\nu} - \left( \frac{\nu - 1}{\nu} \right)^2 \ln \ln N' \right] \right\} \approx \frac{1}{\nu \ln N'}. \quad (7)$$

It is evident that if $f(\alpha)$ is, or can be fitted to, a stretched exponential or to one of its special cases useful and plausible estimates of $N \approx \exp [\alpha_{\text{max}}/(w\nu)]$ can be obtained. However, the inferred value of $N$ depends very sensitively on $\nu$, and measurement of $m$ alone for a sample of test specimens of a single size does not determine $\nu$. Measurement of two or more populations of very different-sized test specimens of the same material (for which $N$ is proportional to the size) may determine both $N$ and $\nu$, and may be feasible; for example, in two-point bending experiments on optical fiber of $125 \mu$ diameter the number of atoms $N_a$ at significant risk of initiating fracture (those with stresses within about $1/m$ of the maximum, where the Weibull modulus $m > 100$, which are found only close to the outside of the sharpest part of the bend) is $N_a \sim 5 \times 10^{14}$, while in tensile loading of a $50 \text{ m}$ gauge length of fiber $N_a \sim 4 \times 10^{22}$ atoms are uniformly loaded. Two or more measurements of $m$ in which very different numbers of atoms are stressed permit simultaneous determination of both the ratio $N/N_a$ and $\nu$, although no extant data serve the purpose, in part because bending and tensile measurements are affected differently by variations in fiber diameter.

### 4.3. Skewness

In order to obtain useful results, even with data of unprecedented accuracy, it will probably be necessary to measure more than the dispersion of the strength. It has long been known, both theoretically within weakest link models and empirically, that distributions of failure strengths are strongly skewed, with outliers preferentially found in the direction of anomalously weak specimens. The reason for this is, of course, that an unusually weak link produces an
### Figure 1. Distribution $P(\alpha)$, normalized to $P(\alpha_{\text{max}})$, as a function of $(\alpha - \alpha_{\text{max}})/w$ for a Gaussian $f(\alpha)$. The substantial skewness is evident, as is the dependence on $N$.

unusually weak specimen, while an unusually strong link has no effect at all on the specimen strength. Quantitative measurements of the skewness of the strength distribution may constrain the statistics, and, more importantly, the number of individual links contributing.

The skewness of $P(\alpha)$ is defined:

$$s \equiv \frac{\int_{0}^{\infty} (\alpha - \alpha_{\text{max}})^3 P(\alpha) \, d\alpha}{w^3 \int_{0}^{\infty} P(\alpha) \, d\alpha}.$$  \hspace{1cm} (8)

The skewness is not small; see Fig. 1. It is also not readily estimated analytically because the Taylor expansion of $P(\alpha)$ does not converge sufficiently rapidly, but it may be calculated numerically. Values of the skewness, as a function of $\nu$ and $N$, are shown in Fig. 2.

The values plotted were computed using cutoffs on the integrals of $\pm 5w$ from $\alpha_{\text{max}}$, with $w$ defined self-consistently using the same cutoff. The reason for this is that $P(\alpha)$ has a long tail extending toward increasing $\alpha$ which contributes significantly to the skewness, but which is unlikely to be observed in a real experiment with a reasonable number of specimens because there will probably be no specimens that far out in the tail of the distribution of strengths. The skewness computed without this cutoff is significantly larger, typically by $O(10\%)$.

When comparing experimental data to Fig. 2 a similar cutoff must be applied to the data. This will, in addition, exclude samples which are anomalously weak because of mechanical damage or other gross flaws, which otherwise must be excluded ad hoc. If the number of measurements is not large it may be necessary to choose a narrower cutoff, and to recompute Fig. 2 accordingly.

### 4.4. Beyond Skewness

Skewness is not the only statistical parameter beyond dispersion which may be useful. In principle, there are an infinite number of moments of the strength distribution, but the higher moments (and the skewness, too) are difficult to calculate reliably and accurately, even from data of high quality, because they are very sensitive to outliers. A data set of reasonable size will not adequately sample rare outliers, and, in addition, there will generally be an additional population of outliers resulting from damaged specimens or errors of measurement. It is difficult, and perhaps impossible, to decide in an unbiased manner if an outlier should be discarded as a probably consequence of a bad specimen or error, or retained in calculating the skewness and higher moments, and even a single outlier
Figure 2. Contour graph of calculated skewnesses of $P(\alpha)$ as a function of $\nu$ and $N$. Contour intervals are 0.05, increasing downward. Integrations (to calculate both skewness and dispersion) were self-consistently truncated at $|\alpha - \alpha_{\text{max}}| = 5w$

may have a substantial effect on their calculated values. Therefore, it is better to truncate the data distribution deliberately in defining the moments (as was done for the skewness in Fig. 2).

Rather than use moments, it may be more informative (and robust) to compare the complete distributions of strength to models. This avoids the problem of outliers, but requires large data sets. While the truncated skewness can be determined to useful accuracy with comparatively few data (1000 measurements will typically determine it to $\pm < 0.1$ if outliers are excluded), comparing distributions divided among many bins may require more data, simply because with few data per bin statistical fluctuations are more important. However, optical communications fiber is cheap, and automated testing machines may permit the measurement of thousands of specimens.

5. DISCUSSION

In order to use successfully the methods discussed here it will be necessary to obtain strength measurements of unprecedented accuracy and number. For example, it is believed that the dispersion in existing measurements of the strength of fused silica optical fiber is largely the result of dispersion in fiber diameter, rather than dispersion in the actual strengths of the fiber. This problem is likely to occur whenever small dispersions in strength are measured, and may have contributed to Leonardo’s observation that long wires are weaker than short ones: a long wire is more likely to contain a region of narrower diameter, just as it is more likely to contain an intrinsic flaw, and it is not straightforward to distinguish the two effects.

Equation (7) indicates that the dispersion in intrinsic strengths is unlikely to be much less than 1% unless the individual links have a fractional dispersion in strength $\ll 1$. Well controlled fused silica fibers have a dispersion in diameter of 0.4% and in cross-section of 0.8%, and an apparent dispersion in strength comparable to that in cross-section. In order to measure the real dispersion in strength much tighter control of fiber diameters will be required. These data already hint that the intrinsic fractional dispersion in strength of individual links may be small, which gives significant information about the nature of the weakest links—perhaps surprisingly, it points towards the SiO$_2$ matrix, in which all chemical bonds are nearly the same, and away from rare impurities, which might be expected to involve a more heterogeneous range of bond strengths. It should be remembered, however, that these measurements were performed at room temperature, and reflect failure by stress corrosion rather than the ideal materials strength.
Weakest link models have been widely discussed for over half a century, but their applicability to real materials remains unproven. In part, this is because analysis of failure statistics rarely goes beyond calculation of the dispersion and Weibull modulus, and plotting on Weibull coordinates. There are few detailed quantitative studies of failure statistics, which would include measurement of the dependence of the dispersion and mean strength on specimen size, as well as quantitative study of the distribution of failure stresses. For specimens affected by heterogeneities and damage the data are unlikely to justify more careful analysis, but optical communications fiber offers both the opportunity to investigate the ultimate limit of ideal materials strength and the possibility of homogeneous data of sufficient quality to make such an investigation possible.

In fact, there are qualitative reasons for questioning the applicability of weakest link models to the ideal strength of glass. As discussed, extant data\[14\] indicate a surprisingly small dispersion in measured strength. The surprise is increased by the fact that no sample can be completely free of chemical impurities and radiation damage, which would be expected to introduce anomalously weak links. The three-dimensional network of Si—O bonds may not be described by weakest link models, but may instead show a well-defined collective failure limit which leads to homogeneous macroscopic behavior from a microscopically heterogeneous material\[4,5\].

Understanding the statistics of the ideal strength of glass at low temperatures is of interest to the physicist. Understanding its strength at ambient conditions is of much greater practical interest, and nearly all data have been obtained under these conditions. It is also unclear if weakest link models are applicable to stress corrosion; the simple picture of the failure of a single small link leading to a crack propagating across the entire specimen is not obviously applicable. The kinetics of stress corrosion of glass are not understood quantitatively, and the collection of more accurate statistics of its strength distribution may be a new and useful means of attacking that problem.

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