Experimental tests of coherence and entanglement conservation

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We experimentally demonstrate the migration of coherence between composite quantum systems and their subsystems. The quantum systems are implemented using polarization states of photons in two experimental setups. The first setup is based on linear optical controlled-phase quantum gate and the second scheme is utilizing effects of nonlinear optics. Our experiment allows to verify the relation between correlations of the subsystems and the coherence of the composite system, which was given in terms of a conservation law for maximal accessible coherence by Svozilík et al. [Phys. Rev. Lett. 115, 220501 (2015)]. We observe that the maximal accessible coherence is conserved for the implemented class of global evolutions of the composite system.

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I. INTRODUCTION

Fundamental laws in physics can often be formulated in terms of the conservation principles. These laws not only represent our understanding of the underlying physical phenomena but can also be used to predict time evolution of quantum correlations in the investigated systems [1–3].

Traditionally, the quantities conserved in the closed systems include overall energy or momentum. These conservation laws can be described even within the confines of classical mechanics. Quantum theory provides conservation laws for some additional, more abstract quantities. For example, it has been established by Englert, Greenberger and Yasin that the sum of interference pattern visibility and the coefficient of determination of the trajectory of a particle is also a conserved quantity [3–4]. This conservation law describes quantitatively our fundamental insight into wave-particle duality, i.e., the more we know about particle’s trajectory, the less it would manifest its wave properties – namely the interference pattern.

Coherence is a consequence of the principle of superposition which is a key property of quantum states. It typically manifests as interference patterns which have been observed in various physical systems including both strong and ultraweak optical fields [5], electrons [6, 7, 8], atoms or even fullerene molecules [9].

While coherence is also well known in classical physics, in quantum physics, this concept is further broadened to the coherence between two or more distinct parties also known as the entanglement [10]. The coherence of non-classical light (in terms of photon statistics) can be expressed as its ability to create entanglement [11]. Recently, theoretical research let to formulation of other measures of coherence that are invariant under coherence preserving evolutions [12–20]. In 2015, Svozilík et al. put forward the conservation law for the maximum accessible coherence under global unitary evolutions. They have shown how coherence migrates in multipartite quantum systems from the classical coherence of a given subsystem to the quantum correlations between subsystems [21]. In about the same time, other research groups investigated the coherence migration for Gaussian states [22], single-photon states [16] and subsequently in general [23].

Previously-mentioned theoretical results on coherence migration provide valuable insight into the relation between classical coherence and quantum correlations. So far, however, these results were not subjected to experimental verification. In this paper we report on such a test. We verify the conservation of the maximally accessible coherence while it migrates between classical coherence and quantum correlations as described in Ref [21]. For this purpose, we have selected two optical processes described by global unitary evolution. The first process, presented in Sec. I, involves a linear-optical controlled-phase (c-phase) gate. In the second experiment, we observe entanglement generation in the process of spontaneous parametric down-conversion from a partially coherent pump beam (see Sec. II). Finally in Sec. IV we discuss the results and conclude.

II. COHERENCE MIGRATION UNDER C-PHASE OPERATION

First, we analyze the migration of accessible coherence between classical coherence and quantum correlations in the experimental setup based on a tunable linear optical c-phase gate [24–25]. This setup is schematically depicted in Fig. 1 and described in detail in Ref. [24]. The c-phase gate performs the unitary input-output transformation:

\begin{align}
|HH\rangle_{1,2} &\rightarrow |HH\rangle_{A,B}, & |HV\rangle_{1,2} &\rightarrow |HV\rangle_{A,B}, \\
|VH\rangle_{1,2} &\rightarrow |VH\rangle_{A,B}, & |VV\rangle_{1,2} &\rightarrow e^{i\varphi}|VV\rangle_{A,B}. \end{align}

(1)
where the single-photon basis polarization states are horizontal ($|H\rangle$) and vertical ($|V\rangle$) linear polarizations. The phase $\varphi$ is a tunable parameter of this quantum gate. Qubits are encoded as polarization states of single photons. Thus, the classical coherence corresponds to the degree of polarization.

A separable two-photon state $|++\rangle$, where $|+\rangle = (|H\rangle + |V\rangle)/\sqrt{2}$ denotes diagonal linear polarization, is inserted at the input of the gate. Coherence of each photonic qubit can be calculated from its reduced density matrix $\hat{\rho}_i$ for $i = A, B$. Knowing the density matrix of the entire system $\hat{\rho}$, we can calculate the reduced matrices of individual photons as a partial trace, $\hat{\rho}_A = \text{Tr}_B[\hat{\rho}]$ for the first and $\hat{\rho}_B = \text{Tr}_A[\hat{\rho}]$ for the second photon.

Coherence proportional to the radius of a Poincaré sphere of each subsystem is calculated as

$$D_i^2 = \text{Tr}[\hat{\rho}_i^2] - \frac{1}{2}, \quad i = A, B; \quad (2)$$

where we use the notation identical to Ref. [21] up to a scaling factor of $\sqrt{2}$. We introduce a mean coherence of the two photons, $D^2 = (D_A^2 + D_B^2)/2$. For the considered input state $|++\rangle$, we have $D_A^2 = D_B^2 = D^2 = \frac{1}{2}$.

Mutual correlations between the two photons can be quantified by $T^2$ [21] given as

$$T^2 = \frac{1}{4} \left( 1 + \sum_{i,j=1}^3 t_{ij}^2 \right), \quad t_{ij} = \text{Tr}[\hat{\rho}\hat{\sigma}_i \otimes \hat{\sigma}_j], \quad (3)$$

where $\hat{\sigma}_i(i = 1, 2, 3)$ are Pauli matrices.

This quantity relates to various two-qubit entanglement witnesses based on eigenvalues of the Horodecki matrix $R_{ij} = R_{ij}^2$ (see Refs. [20, 27] and references therein), as $\text{Tr}R = 4T^2 - 1$. In particular, the maximal Bell-Clauser-Horne-Shimony-Holt quantity [20, 28] can be expressed as $B = 2\sqrt{\text{Tr}R - \text{min} \{\text{eig}(R)\}}$, $B \leq 2$ for spatially separated classical two-level systems and $B \leq 2\sqrt{2}$ for quantum systems. This quantity can be used for measuring the degree of nonlocality and to estimate various measures of quantum entanglement [27, 29]. It follows from $\text{min} \{\text{eig}(R)\} \leq \frac{1}{4} \text{Tr}R$ that $B \geq 2\sqrt{2\text{Tr}R/3} = 2\sqrt{(8T^2 - 2)/3}$. Hence, we witness nonlocality and entanglement ($B > 2$) if $T^2 > 5/8 = 0.625$.

The degree of correlation $T^2$ of the separable input state reaches its minimum value of 1/2. The c-phase gate is however capable to continuously change (in this case increase) the entanglement of the two-photon state from the minimum value of $T^2 = 1/2$ for a separable state to $T^2 = 1$ corresponding to a maximally entangled Bell state. This is achieved by tuning the phase $\varphi$ in the range from 0 to $\pi$.

In this experiment the classical coherence of the two-photon input state partially migrates to the degree of correlation $T^2$ at the output. In this process the conserved quantity is called the maximal accessible coherence $S^2$ of the two-photon system,

$$S^2 = D^2 + T^2. \quad (4)$$

The value of the conserved parameter $S^2$ equals to the purity $P = \text{Tr}[\hat{\rho}^2]$ of the composite system [21]. Ideally, purity of the two-photon state shall be equal to 1. Due to experimental imperfections, the observed output state purity typically fluctuates between 0.9 and 1.

The output state density matrix was reconstructed with optimal quantum state tomography and maximum likelihood estimation [30, 31]. This has been performed for seven phase shifts $\varphi$. Each time, we have calculated the values of $T^2$ and $D^2$ using the above mentioned formulas. We have also developed a theoretical model as-

![FIG. 1.](image_url) Conceptual scheme of the experimental setup with c-phase gate adjusted at phase $\varphi$.

![FIG. 2.](image_url) Parameters of the two-photon state at the output of the c-phase gate for seven values of the phase $\varphi$. $D_A, D_B$ - local coherence (degree of polarization) of individual photons; $T^2$ - degree of correlation; $S^2$ - maximal accessible coherence.

| $\varphi/\pi$ | $D_A$ | $\delta D_A$ | $D_B$ | $\delta D_B$ | $T^2$ | $\delta T^2$ | $S^2$ | $\delta S^2$ |
|---------------|-------|--------------|-------|--------------|------|--------------|------|--------------|
| 0             | 0.707 | 0.000000     | 0.707 | 0.000000     | 0.500| 0.000000     | 1.000| 0.000000     |
| 0.05          | 0.695 | 0.000003     | 0.701 | 0.000001     | 0.502| 0.000002     | 0.989| 0.000004     |
| 0.125         | 0.682 | 0.000002     | 0.681 | 0.000002     | 0.532| 0.000002     | 0.996| 0.000002     |
| 0.25          | 0.648 | 0.000004     | 0.652 | 0.000003     | 0.554| 0.000003     | 0.976| 0.000004     |
| 0.5           | 0.619 | 0.000003     | 0.607 | 0.000004     | 0.590| 0.000005     | 0.974| 0.000005     |
| 0.75          | 0.521 | 0.000004     | 0.518 | 0.000005     | 0.701| 0.000006     | 0.971| 0.000005     |
| 1             | 0.359 | 0.000006     | 0.330 | 0.000007     | 0.825| 0.000007     | 0.944| 0.000007     |

TABLE I. Parameters of the two-photon state at the output of the c-phase gate for seven values of the phase $\varphi$. $D_A, D_B$ - local coherence (degree of polarization) of individual photons; $T^2$ - degree of correlation; $S^2$ - maximal accessible coherence.
Theoretical model, the values of $\theta$ is scanned from 0 to $\pi$. Assuming perfect c-phase operation. This way we can predict the ratio of coherence migrated from the input state into the output state entanglement as the phase shift $\varphi$ is scanned from 0 to $\pi$. To compare our data to the ideal theoretical model, the values of $d^2$ and $T^2$ need to be normalized to the overall output state purity. The experimentally obtained values of $T^2$, $D^2$ and $S^2$ are summarized in Tab. I. In Fig. 2 we visualize both theoretical and experimental ratio of coherence migrated into entanglement. Up to experimental uncertainty, our data are in accordance with the theoretical prediction.

III. COHERENCE MIGRATION IN A NONLINEAR OPTICAL PROCESS

The second setup is a typical scheme for generation of two-photon states in the process of spontaneous parametric down-conversion (SPDC) in a nonlinear material. We use cascade of two BBO crystals also called the Kwiat configuration [32,34]. The optical axis of the first (second) crystal is in the vertical (horizontal) plane. This perpendicularly configuration of two Type I crystals allows direct generation of photon pairs correlated in polarization. Scheme of the second setup is drawn in Fig. 3. Here the input coherence is represented by coherent superposition of $H$ and $V$ polarization components of the pump beam. When these two polarization components are mutually delayed using a beam displacer assembly (BDA), the coherence of the pump beam is decreased. To inspect the value of the displacement we subject the pumping beam to a polarization Michelson interferometer (PMI) placed behind the crystal cascade (see Fig. 3). The value of the displacement is inferred from the position of autocorrelation function maxima.

The horizontal component of the pump beam leads to spontaneous parametric generation of a vertically polarized pair of photons $|VV\rangle$. Similarly, the vertically polarized pump beam is downconverted into a pair of horizontally polarized photons $|HH\rangle$. When the horizontal and vertical pump beam components are temporally overlapping, the overall two photon state generated by the crystals is a coherent superposition of the $|HH\rangle$ and $|VV\rangle$ states, aka. the Bell state $|\Phi^{+}\rangle = (|HH\rangle + |VV\rangle)/\sqrt{2}$. It should be emphasized, that the case of highest down-conversion coherence corresponds to exact time overlap of the $H$ and $V$ components of the pump beam at the interface between the two BBO crystals. In this case, the observed delay before or after the crystal cascade is 84 $\mu$m due to the birefringence of the BBO crystals. The value of 84 $\mu$m represents a constant bias to be subtracted in subsequent analysis.

The broad spectrum of the SPDC photons is narrowed using band-pass filters F (FWHM of 3 nm). Fitting the experimental autocorrelation function of two-photon coincidences with a Gaussian curve yields

$$V(d) = 0.029 + 0.945 e^{-(d/\sigma)^2},$$

where $d$ denotes the spatial displacement between the horizontal and vertical pump beam components. The corresponding FWHM is found to be FWHM = 2$\sqrt{\ln 2}\sigma = 142\mu$m. The coherence is proportional to value of this function in distance $d$ out of the maxima,

$$S_{in}^2 = \text{Tr}[\rho_{\text{pump}}^2] = (1 - V^2)/2.$$  

In the experiment, we have selected ten specific values of polarization components displacement of the pump beam $d$.

Simultaneously, the generated pairs of photons were collected into single-mode fibers leading to a balanced fiber coupler. Polarization state of the generated pair was transformed so that the two photons impinging on this fiber coupler are in a singlet-like state $|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|HV\rangle - |VH\rangle)_d$, where the subscript $d$ denotes that the state is displaced. The displacement between the two-photon state components can not be resolved and the resulting two-photon state has to be described in the form of a mixed state $\rho = p|\Psi^-\rangle\langle\Psi^-| + (1-p)|\Psi^+\rangle\langle\Psi^+|$, where $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}} (|HV\rangle \pm |VH\rangle)$ are two of the Bell states.

The value of $p$ can easily be measured as the probability of the two-photon state antibunching on the beam splitter (observing coincident detections behind the beam splitter, see Fig. 4). Note that only photons in the singlet state $|\Psi^-\rangle$ deterministically antibunch while photons in any of the triplet Bell state bunch on the beam splitter. Measuring $p$ allows one to reconstruct the density matrix $\rho$ and obtain the values of $T^2$ as defined in Sec. [4]. During the measurement we also tested the degree of polarization of individual photons from the SPDC.
pairs and found that $D^2$ is negligible ($\sim 4 \times 10^{-3}$). To compensate for any experimental imperfections, we have measured the bunching effect on the beam splitter for a triplet state $|\Psi^+\rangle$. For this state, the visibility reaches 94% which we use to correct the observed values of $p$. The output state coherence $S_{out}^2$ was calculated from $D^2$ and $T^2$ using Eq. (4).

We summarized the results in Tab. II and in Fig. 5. It is clear that in this case of nonlinear interaction, the input maximal accessible coherence is carried by the polarization of the pump beam. There is no entanglement at the input of the nonlinear crystals. At the output the generated photons seem unpolarized, corresponding to nearly zero degree of classical coherence. The whole input coherence of the pump is transferred to the degree of correlation between the SPDC-generated photons.

**IV. CONCLUSIONS**

In this paper, we have demonstrated transformation between coherence and correlations in two optical experiments. In the first experiment a linear-optical c-phase gate is used to tune the transfer of individual coherence of the input photons into their correlations. We have verified that during this operation the maximal accessible coherence $S^2$ is conserved.

In the second experiment we tune the coherence of the pump beam which is generating two-photon states in the nonlinear SPDC process. The degree of correlation of the generated states is limited by the coherence of the original pump beam. The maximal accessible coherence $S^2$ is conserved also in this process.

Our results experimentally prove that the conservation of accessible coherence over unitary operations belong to a broader set of conservation laws that are invaluable in predicting outcomes of complex physical phenomena.

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