FCNC $B$ and $K$ meson decays with light bosonic Dark Matter

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ABSTRACT: We consider decays of $B$ and $K$ mesons into a pseudo-scalar or vector meson plus missing energy. Within the SM, these modes originate from flavor changing neutral current (FCNC) processes with two neutrinos in the final state. In this paper we consider the experimental upper bounds on these modes and interpret the difference between these bounds and the SM prediction as a window into new light invisible particles. In particular we consider the case where some new symmetry requires the new particles to be produced in pairs. We first construct the general low energy effective Lagrangian coupling an FCNC with two dark sector particles of spin zero, one-half and one. We then present numerical estimates for the constraints that can be placed on these interactions, finding that an effective new physics scale from $\mathcal{O}(10)$-\(\mathcal{O}(10^{11})\) GeV can be probed, with the exact value strongly depending on the interaction structure as well as the mass of the invisible particle. For $K^+ \rightarrow \pi^+ E$ we incorporate into our constraints the effect of using only the signal regions of NA62, and for $B^+ \rightarrow K^+ E$ the $q^2$-dependent efficiency of Belle II.

KEYWORDS: New Light Particles, Rare Decays

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1 Introduction

It is well known that flavor changing neutral current (FCNC) processes are severely suppressed within the standard model (SM) due to both their loop origin and the GIM mechanism [1]. For this reason, these processes are very sensitive to new physics (NP) beyond the SM. We consider $B$ and $K$ decay modes with one meson and a neutrino pair in the final state because experimental upper bounds are available for them. As the neutrinos are not detected, these modes become decays with missing energy ($\not{E}$) in the final state from the experimental perspective. At the quark level, any transition of the form $b \rightarrow s(d) + \not{E}$ and $s \rightarrow d + \not{E}$, would contribute to these modes and $\not{E}$ can originate from any sufficiently light invisible particle. In addition to the SM neutrinos, the invisible particles can be any hypothetical neutral particle that escapes the current experimental detection. One well-motivated choice is to relate these light invisible particles to dark matter (DM) or other dark sectors. These rare meson decays can then be used to constrain light DM and are particularly important in the face of current stringent experimental constraints on heavy DM from direct detection experiments [2, 3].
Without knowledge of the fundamental interactions in dark sectors, it is suitable to study these FCNC processes in a model independent manner by incorporating new light, neutral, degrees of freedom with an effective field theory (EFT) approach. The new particles can be either scalar, fermion, or vector in nature if we limit our study to spin less than or equal to one. Well known examples for these invisible particles are the axion [4, 5], the sterile neutrino [6], and the dark photon [7]. Model independent studies of the FCNC $b \to s(d) + \bar{E}$ and $s \to d + \bar{E}$ transitions within the EFT framework have been carried out in the past for these three types of invisible particles for $B$ and $K$ meson decays. A summary of what has been done is shown in table 1. These processes are further divided into 2-body decays with a single new particle and 3-body decays with a pair of new particles. All the 2-body channels plus the 3-body decay with a pair of new fermions have been extensively studied before [8–14, 16–20].

The table indicates with a double-checkmark processes that have not received much attention, namely three body modes with a pair of vector particles. We are only aware of a classification of relevant SMEFT operators in [9]. The 3-body decays with a pair of scalar particles indicated with a checkmark in the table, have been partially studied before. For the $B \to K$ and $K \to \pi$ transitions: [12] has considered only two of the four possible operators we enumerate in eq. (2.2) (the scalar ones), whereas [15] has only considered one of them. For $s \to d + \phi\phi$ transitions, [21] has considered both kaon and hyperon decays and [22] has considered additional kaon decay modes. Our study of the kaon modes includes the new NA62 results for the relevant signal window. In this paper, we systematically investigate these channels using a general low energy effective theory (LEFT), and find constraints on all the relevant effective operators with the help of the most recent experimental results.

The observables we use to set the bounds are summarized in table 2. In the second column, we list the SM background with a pair of neutrinos. For most modes in the table, we obtain the SM prediction using the flavio package [27]. In some cases, the SM calculation of the form factors is not trivial and we use instead the predictions for $B \to \pi\bar{\nu}\nu$ from [26]. The kaon decay modes are very clean theoretically, but their parametric uncertainty due to CKM angles can be large, with central values changing by up to 20% [28], we quote the values from the PDG in this case [23].

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Table 1. List of possible $b \to s(d) + \bar{E}$ and $s \to d + \bar{E}$ modes related to $B$ and $K$ meson decays: a “✓✓” means that they have not been studied before while a “✓” indicates they have been partially studied, both are the subject of this paper.

| Particle Topology | Fermion ($\chi$) | Scalar ($\phi$) | Vector ($X$) |
|-------------------|------------------|----------------|-------------|
| 2-body decay      | $b \to s(d) + \chi$ [8] | $b \to s(d) + \phi$ [9, 10] | $b \to s(d) + X$ [9, 10] |
| 3-body decay      | $b \to s(d) + \chi\chi$ [9, 12–14] | $b \to s(d) + \phi\phi$ [12, 15] (✓) | $b \to s(d) + XX$ (✓✓) |
|                   | $s \to d + \chi\chi$ [9, 16–20] | $s \to d + \phi\phi$ [21, 22] (✓) | $s \to d + XX$ (✓✓) |

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1Note that the 2-body modes with a single new fermion are only possible when the meson decays into a baryon and $\chi$ [8]. For the 3-body decay with a pair of fermions, $\chi$ can be either the neutrino or some DM particle, the former is studied in [12, 14, 18–20], while the latter in [13].
Table 2. Summary on the status of FCNC $B \to (K, K^*, \pi, \rho)E$ and $K \to \pi E$ decays with missing energy.

| Observable | SM prediction ($\mathcal{E} = \nu \bar{\nu}$) | 90% C.L. upper bound | New physics bound $\mathcal{B}^{\text{UL}}$ |
|------------|-----------------------------------------------|----------------------|------------------------------------------|
| $B(B^+ \to K^+ \bar{\nu})$ | $(4.1 \pm 0.6) \times 10^{-9}$ | $1.6 \times 10^{-9}$ [23] | $1.3 \times 10^{-5}$ |
| $B(B^0 \to K^0 \bar{\nu})$ | $(4.1 \pm 0.6) \times 10^{-9}$ | $2.6 \times 10^{-7}$ [24] | $2.3 \times 10^{-5}$ |
| $B(B^+ \to K^+ \bar{\nu})$ | $(1.0 \pm 0.1) \times 10^{-5}$ | $4.0 \times 10^{-5}$ [25] | $3.1 \times 10^{-5}$ |
| $B(B^0 \to K^0 \bar{\nu})$ | $(9.5 \pm 1.0) \times 10^{-6}$ | $1.8 \times 10^{-5}$ [24] | $1.0 \times 10^{-5}$ |
| $B(B^+ \to \pi^+ \bar{\nu})$ | $(2.39^{+0.04}_{-0.23}) \times 10^{-7}$ [26] | $1.4 \times 10^{-5}$ [24] | $1.4 \times 10^{-5}$ |
| $B(B^0 \to \pi^0 \bar{\nu})$ | $(1.2^{+0.15}_{-0.114}) \times 10^{-7}$ [26] | $9.0 \times 10^{-6}$ [24] | $8.9 \times 10^{-6}$ |
| $B(B^+ \to \rho^+ \bar{\nu})$ | $(4.5 \pm 1.0) \times 10^{-7}$ | $3.0 \times 10^{-5}$ [24] | $3.0 \times 10^{-5}$ |
| $B(B^0 \to \rho^0 \bar{\nu})$ | $(2.0 \pm 0.4) \times 10^{-7}$ | $4.0 \times 10^{-5}$ [24] | $4.0 \times 10^{-5}$ |
| $B(K^+ \to \pi^+ \bar{\nu})$ | $(8.1 \pm 0.4) \times 10^{-11}$ [23] | $(1.14^{+0.40}_{-0.33}) \times 10^{-10}$ [23] | $1.1 \times 10^{-10}$ |
| $B(K_L \to \pi^0 \bar{\nu})$ | $(2.8 \pm 0.2) \times 10^{-11}$ [23] | $4.9 \times 10^{-9}$ [23] | $4.9 \times 10^{-9}$ |

The new physics we discuss, will always add incoherently to the SM di-neutrino background in $B(K) \to M\bar{\nu}$ processes (with $M$ representing the final state mesons shown in Table 2). In view of this we define the “room for new physics” in these modes as the difference between the experimental upper bound and the standard model prediction. We adopt the simple prescription of subtracting from the 90% experimental upper bound the lower limit of the 90% C.L. SM range and show this number in the last column. The constraints can be easily adapted for more sophisticated subtractions if desired. The mode $K^+ \to \pi^+ \bar{\nu}$ has been measured by both BNL787/949 [29, 30] and NA62 [31, 32], so in this case we use the upper limit of the 90% range quoted by PDG [23] as the experimental upper limit.

The paper is organized as follows. In section 2 we classify all the relevant effective interactions in the framework of low energy effective field theory. In section 3 we consider the FCNC $B$ meson decay $B \to (K, K^*, \pi, \rho)+DM+DM$ with a pair of scalar or vector DM, and use the current experimental bounds to constrain the relevant effective new physics scale involving $(bs)$ and $(bd)$ quark flavors. In section 4, we use chiral perturbation theory to analyze the FCNC kaon decay $K \to \pi+DM+DM$ to constrain the effective scale involving $(ds)$ quark flavors. In all cases we will refer to the invisible light particles as “DM” regardless of their origin. In section 5, we draw our conclusions. Supplementary material presented in the appendix includes: the phase space integration in A; the reduction of operators with vector DM fields in B; operators for lepton-DM interactions in C; a collection of form factors involving $B$ meson decays in D, and specific renormalizable model realizations for some scalar and vector DM operators as an illustration in appendix E.

2 Quark-DM interaction in LEFT

In this section we list the most general local quark-DM interactions in the framework of low energy effective field theory, with the FCNC interactions being a subset of these operators with appropriate flavor indices. In LEFT, only the unbroken $SU(3)_c \times U(1)_{\text{em}}$ symmetry of

\[2\text{Except for } K^+ \to \pi^+ \bar{\nu} \text{ channel, for which the } 1 \sigma \text{ measured value quoted by PDG [23] is used.}\]
the standard model is imposed. Operators at the weak scale in SMEFT have been listed before [33–35] along with their contributions to the LEFT operators [36, 37]. Since we are considering cases with light new particles it is more general to start from LEFT [36]. In this way we cover scenarios that contain both light DM as well as weak scale new mediators that can be integrated out to reach the LEFT description containing only the relevant degrees of freedom [38].

The scenario in which modes with two DM particles becomes relevant is that in which they must be pair produced due to underlying symmetries in the dark sector which can also guarantee that the DM is stable. In this case the relevant quark-DM interactions have to involve a pair of quark fields and a pair of DM fields. These interactions can be classified in terms of the spin of the DM particle, and we consider the cases of spin zero, one-half and one with corresponding scalar, fermion and vector DM fields. We denote the light SM quarks as $q \in \{d, s, b, u, c\}$, the fermionic DM particle as $\chi$ (Dirac or Majorana fermion), the scalar DM as $\phi$ (complex or real scalar), and the vector DM as $X$ (complex or real vector), respectively. For each type of DM particle, the relevant quark-DM interactions are enumerated below.

**Fermion case:** the leading dimension-6 (dim-6) operators for this case have been considered before [39, 40] and we list them (with slightly different convention) here for completeness. They are,

\[
O_{q\chi}^{\mathcal{S}1} = (\overline{q}q)(\chi \chi),
O_{q\chi}^{\mathcal{S}2} = (\overline{q}q)(\chi i \gamma_5 \chi),
O_{q\chi}^{\mathcal{P}1} = (\overline{q}i \gamma_5 q)(\chi \chi),
O_{q\chi}^{\mathcal{P}2} = (\overline{q}i \gamma_5 q)(\chi \gamma_5 \chi),
O_{q\chi}^{\mathcal{V}1} = (\overline{q} \gamma^\mu q)(\chi \gamma_\mu \chi),
O_{q\chi}^{\mathcal{V}2} = (\overline{q} \gamma^\mu q)(\chi \gamma_5 \gamma_\mu \chi),
O_{q\chi}^{\mathcal{A}1} = (\overline{q} \sigma^{\mu\nu} q)(\chi \sigma_{\mu\nu} \chi),
O_{q\chi}^{\mathcal{A}2} = (\overline{q} \sigma^{\mu\nu} q)(\chi \sigma_{\mu\nu} \gamma_5 \chi),
O_{q\chi}^{\mathcal{T}1} = (\overline{q} \sigma^{\mu\nu} q)(\chi \sigma_{\mu\nu} \chi),
O_{q\chi}^{\mathcal{T}2} = (\overline{q} \sigma^{\mu\nu} q)(\chi \sigma_{\mu\nu} \gamma_5 \chi),
\]

where the quark flavor indices have been omitted for notational simplicity, but are understood to be those required for the specific FCNC process throughout the paper. The “$(\times)$” indicates the accompanying operator vanishes for the Majorana DM case due to the fermion bilinear identity, $\chi \Gamma \chi = -\chi^c \Gamma \chi^c$ for $\Gamma \in \{\gamma_\mu, \sigma_{\mu\nu}, \sigma_{\mu\nu} \gamma_5\}$, where $\chi^c \equiv -i \gamma_2 \chi^*$ is the charge conjugation of $\chi$.

**Scalar case:** the leading order operators for the scalar DM appear at dimension 5 and 6,\(^3\) they can be parametrized in the following manner,

\[
O_{q\phi}^{\mathcal{S}1} = (\overline{q}q)(\phi^\dagger \phi),
O_{q\phi}^{\mathcal{S}2} = (\overline{q}i \gamma_5 q)(\phi^\dagger \phi),
O_{q\phi}^{\mathcal{P}1} = (\overline{q}i \gamma_5 q)(\phi^\dagger \phi),
O_{q\phi}^{\mathcal{V}1} = (\overline{q} \gamma^\mu q)(\phi^\dagger i \overleftarrow{\partial}_\mu \phi),
O_{q\phi}^{\mathcal{A}1} = (\overline{q} \sigma^{\mu\nu} q)(\phi^\dagger i \overleftarrow{\partial}_\mu \phi),
\]

\(^3\)They would all arise at dimension 6 in the SMEFT framework [9]. A specific renormalizable model realization for the operators $O_{q\phi}^{\mathcal{S}1}$ and $O_{q\phi}^{\mathcal{P}1}$ is given in appendix E. The realization for the operators $O_{qX}^{\mathcal{S}1}$ and $O_{qX}^{\mathcal{P}1}$ involving vector DM is also given there.
Once again the implicit quark flavor indices should be understood. The symbol "(...)" indicates the related operator vanishes for real scalar DM, and the double arrow derivative is defined as $A \overleftrightarrow{\partial}_\mu B \equiv A(\partial_\mu B) - (\partial_\mu A)B$.

**Vector case A:** for the vector DM, we consider separately two cases: when the DM field is represented by the four-vector potential $X_\mu$ (scenario A) or by the field strength tensor $X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$ (scenario B). For scenario A, we find there are 4 independent dim-5 operators with the field content $\bar{q}q X^\dagger X$ and 12 dim-6 operators with the content $\bar{q}q X^\dagger XD_\mu$.

By requiring the flavor diagonal operators to be self-conjugate, we can parametrize those operators in the following way,

\begin{align*}
O_{qX}^{S} &= (\bar{q}\gamma_\mu)(X^\dagger_\mu X_\mu), \quad (2.3a) \\
O_{qX}^{P} &= (\bar{q}\gamma_\mu q)(X^\dagger_\mu X_\mu), \quad (2.3b) \\
O_{qX}^{T1} &= \frac{i}{2} (\bar{q}\sigma^{\mu\nu} q)(X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu), \quad (\times) \quad (2.3c) \\
O_{qX}^{T2} &= \frac{1}{2} (\bar{q}\sigma^{\mu\nu} \gamma_5 q)(X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu), \quad (\times) \quad (2.3d) \\
O_{qX}^{V1} &= \frac{1}{2} [\gamma(\mu \overleftrightarrow{D}_\nu)] q (X^\dagger_\mu X_\nu + X^\dagger_\nu X_\mu), \quad (2.3e) \\
O_{qX}^{V2} &= (\bar{q}\gamma_\mu q) \partial_\nu (X^\dagger_\mu X_\nu + X^\dagger_\nu X_\mu), \quad (2.3f) \\
O_{qX}^{V3} &= (\bar{q}\gamma_\mu q) (X^\dagger_\mu \overleftrightarrow{D}_\nu X_\sigma) \epsilon^{\mu\nu\rho\sigma}, \quad (2.3g) \\
O_{qX}^{V4} &= (\bar{q}\gamma_\mu q)(X^\dagger_\mu \overleftrightarrow{\partial}_\nu X_\nu), \quad (\times) \quad (2.3h) \\
O_{qX}^{V5} &= (\bar{q}\gamma_\mu q) i \partial_\nu (X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu), \quad (\times) \quad (2.3i) \\
O_{qX}^{V6} &= (\bar{q}\gamma_\mu q) i \partial_\nu (X^\dagger_\mu \overleftrightarrow{D}_\nu X_\sigma) \epsilon^{\mu\nu\rho\sigma}, \quad (\times) \quad (2.3j) \\
O_{qX}^{A1} &= \frac{1}{2} [\gamma(\mu \overleftrightarrow{D}_\nu)] q (X^\dagger_\mu X_\nu + X^\dagger_\nu X_\mu), \quad (2.3k) \\
O_{qX}^{A2} &= (\bar{q}\gamma_\mu \gamma_5 q) \partial_\nu (X^\dagger_\mu X_\nu + X^\dagger_\nu X_\mu), \quad (2.3l) \\
O_{qX}^{A3} &= (\bar{q}\gamma_\mu \gamma_5 q)(X^\dagger_\mu \overleftrightarrow{D}_\nu X_\sigma) \epsilon^{\mu\nu\rho\sigma}, \quad (2.3m) \\
O_{qX}^{A4} &= (\bar{q}\gamma_\mu \gamma_5 q)(X^\dagger_\mu \overleftrightarrow{\partial}_\nu X_\nu), \quad (\times) \quad (2.3n) \\
O_{qX}^{A5} &= (\bar{q}\gamma_\mu \gamma_5 q) i \partial_\nu (X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu), \quad (\times) \quad (2.3o) \\
O_{qX}^{A6} &= (\bar{q}\gamma_\mu \gamma_5 q) i \partial_\nu (X^\dagger_\mu \overleftrightarrow{D}_\nu X_\sigma) \epsilon^{\mu\nu\rho\sigma}, \quad (\times) \quad (2.3p)
\end{align*}

where the current $\gamma(\mu \overleftrightarrow{D}_\nu) q \equiv \gamma(\mu \overleftrightarrow{D}_\nu) q + \mu \leftrightarrow \nu$, and similarly for the current $\gamma(\mu \overleftrightarrow{D}_\nu) q$. The symbol "(...)" indicates the corresponding operator vanishes for real vector DM. Other operators with different Lorentz contractions or derivatives are not independent and can always be reduced to those given above by using the Dirac gamma identities (DI), integration by parts (IBP), and equation of motions (EoM). The construction of the above dim-6 operators with a derivative is expanded upon in appendix B. We have also checked our result by using the Hilbert series method with a modified conformal representation for the vector DM field [41].

Using the above operators to calculate the amplitudes for physical processes results in rates that are divergent in the limit of massless DM particles. This divergence originates
from the longitudinal part in the polarization sum and is a well known problem. One way
to deal with this problem is to assume that the vectors are gauge bosons of some dark
symmetry and gauge invariance under that symmetry forbids the direct appearance of
the field \( X_\mu \). These would appear instead in covariant derivatives acting on other dark
matter fields which are not present in our effective Lagrangian. The net effect of such a scenario is
that \( X_\mu \) acquires mass by some Higgs mechanism \[9, 42\] and the effective operators inherit
a coefficient that vanishes in the limit of massless \( X_\mu \). An example of how this could work
in a specific model is given in appendix E. Operationally, for our numerical analysis, we
require the Wilson coefficients for the above operators to contain an explicit factor of the
DM mass to the minimum power necessary to cancel potential divergences as \( m \to 0 \).

**Vector case B:** with the vector DM entering through field strength tensors the minimal
dimensionality of the operators is 7.\(^4\) In this case two explicit DM fields are required again
for processes with two DM particles because a single field strength tensor referring to a
dark non-abelian symmetry could not couple to a quark current that is not charged under
the dark group. The operators in this scenario produce amplitudes that are well behaved
as \( m \to 0 \) and we find there are 6 operators as follows,

\[
\begin{align*}
\tilde{\mathcal{O}}_{qX1}^S &= (\bar{q}q) X_\mu^\dagger X_\nu, \\
\tilde{\mathcal{O}}_{qX2}^S &= (\bar{q}q) \tilde{X}_\mu^\dagger \tilde{X}_\nu,
\end{align*}
\]

\[
\begin{align*}
\tilde{\mathcal{O}}_{qX1}^P &= (\bar{q}i\gamma^5 q) X_\mu^\dagger X_\nu, \\
\tilde{\mathcal{O}}_{qX2}^P &= (\bar{q}i\gamma^5 q) \tilde{X}_\mu^\dagger \tilde{X}_\nu,
\end{align*}
\]

\[
\begin{align*}
\tilde{\mathcal{O}}_{qX1}^T &= \frac{i}{2} (\bar{q} \sigma^{\mu\nu} q)(X_\mu^\dagger X_\nu - X_\nu^\dagger X_\mu), (\times) \\
\tilde{\mathcal{O}}_{qX2}^T &= \frac{1}{2} (\bar{q} \sigma^{\mu\nu} \gamma^5 q)(X_\mu^\dagger X_\nu - X_\nu^\dagger X_\mu). (\times)
\end{align*}
\]

The dual field strength is defined as \( \tilde{X}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\rho\sigma} X_{\alpha\beta} \), and the symbol “(\times)” denotes
an operator that vanishes for real vector fields.

The method used to construct the LEFT quark-DM interactions given above can also
be used to obtain the corresponding lepton-DM interactions. We list those in appendix C
for completeness.

In the literature, ref. \[40\] provides a list of LEFT operators for the fermion, scalar, and
vector DM (scenario A) cases coupled to flavor diagonal currents. The fermion and scalar
cases agree with our list, but the vector case does not.\(^5\) Here we discuss the more general
case with flavor non-diagonal operators relevant to the FCNC processes we study.

As mentioned above, we only impose the unbroken \( SU(3)_c \times U(1)_{em} \) symmetry to obtain
the LEFT operators. If we instead assume that all mediators are far beyond the weak scale,
we can start from a SMEFT with the full SM gauge symmetry \( SU(3)_c \times SU(2)_L \times U(1)_Y \)
and obtain the corresponding LEFT in the Higgs phase. The relevant DM EFT operators in this

\(^4\)If working with SMEFT assumption, the resulting operators would be at dimension 8.

\(^5\)The list of dim-6 operators with a derivative in \[40\] is not complete. For example, it does not contain
operators corresponding to \( \tilde{\mathcal{O}}_{qX3}^{V(A)} \).
picture can be found in [9, 43–46]. The FCNC $B$ and $K$ meson decays into fermion DM (or similar invisible particles like sterile neutrinos) have been studied in [9, 13, 14, 18–20]. For this reason, in the following we restrict ourselves to the scalar and vector DM cases and investigate the experimental sensitivity to the interactions in eqs. (2.2)–(2.4) in detail. We will first consider $B$ meson decay in the next section, followed by the $K$ meson decay after that.

3 $B \to (K, K^*, \pi, \rho) + \text{DM} + \text{DM}$

To calculate the decay rate for the $B \to M$ transition (where $M$ denotes either a pseudo-scalar meson $P = K, \pi$ or a vector meson $V = K^*, \rho$) from the effective interactions in eqs. (2.2)–(2.4), we first need to know the hadronic transition matrix elements $\langle M|q\Gamma b|B\rangle$. These are usually parametrized in terms of scalar form factors associated with each possible allowed Lorentz structure. The Lorentz structures can be organized according to parity and charge conjugation. While some of the form factors can be determined from experimental data, others require theoretical models for the non-perturbative aspects of QCD. In the following subsections, we first collect the relevant form factors and their determination using light-cone sum rules [47–51]. We then consider the decay rates for both scalar and vector DM scenarios and the implications for the parameter space.

3.1 Form factors

We follow the parametrization of $B$ meson form factors in [47]. For the $B \to P(J^P = 0^-)$ transition with a final state pseudo-scalar $P = \pi, K$, the non-vanishing hadronic matrix elements from the scalar, vector, and tensor quark currents are parametrized by the form factors $f_0, f_+, f_T$.

$$
\langle P(k)|\bar{q}b|B(p)\rangle = \frac{m_B^2 - m_P^2}{m_b - m_q} f_0(q^2),
$$

$$
\langle P(k)|\bar{q}\gamma^\mu b|B(p)\rangle = \left( (p + k)^\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu \right) f_+(q^2) + \frac{m_B^2 - m_P^2}{q^2} q^\mu f_0(q^2),
$$

$$
\langle P(k)|\bar{q}\sigma^{\mu\nu} b|B(p)\rangle = \frac{2i}{m_B + m_P} (p^\mu q^\nu - p^\nu q^\mu) f_T(q^2),
$$

where $k$ and $p$ are the 4-momenta of $P$ and $B$ respectively, $q^\mu = p^\mu - k^\mu$, $m_B$ and $m_P$ are the masses of the initial $B$ meson and final state $P$ meson, and $m_b$ and $m_q$ are the masses of the quarks appearing in the currents. In the $q^2 \to 0$ limit, $f_+(0) = f_0(0)$, and we follow the light-cone sum rule (LCSR) methods [48] to parametrize the dependence on the momentum...
transfer $s \equiv q^2$ as,

$$f_0(s) = \frac{r_2}{1 - s/m_{\text{fit}}^2},$$

$$f_+^\sigma(T)(s) = \frac{r_1}{1 - s/m_{\text{fit}}^2} + \frac{r_2}{1 - s/m_{\text{fit}}^2},$$

$$f_+^K(T)(s) = \frac{r_1}{1 - s/m_{\text{fit}}^2} + \frac{r_2}{(1 - s/m_{\text{fit}}^2)^2}. \tag{3.2}$$

Above, $r_{1,2}$, $m_{R}^2$, and $m_{\text{fit}}^2$, are parameters with the preferred values given in [48] and collected in appendix D for reference.7

For the transition into a vector meson $V$, $B \rightarrow V(J^P = 1^{-})$ with $V = \rho, K^*$, the non-vanishing form factors $V_0$, $A_{0,1,2,3}$, $T_{1,2,3}$ are defined as

$$\langle V(k) | \bar{q} \gamma_5 b | B(p) \rangle = -i\epsilon_V^{\mu} q^{\nu} \frac{2m_V}{m_b + m_q} A_0, \tag{3.3a}$$

$$\langle V(k) | \bar{q} \gamma^{\mu} b | B(p) \rangle = \epsilon^{\mu\rho\sigma} c_{V,\rho} p_{\sigma} k_{\rho} \frac{2}{m_B + m_V} V_0, \tag{3.3b}$$

$$\langle V(k) | \bar{q} \gamma^{\mu} \gamma_5 b | B(p) \rangle = i\epsilon_V^{\mu} \left[ g^{\mu\nu} (m_B + m_V) A_1 - \frac{(p + k)^{\mu} q^{\nu}}{m_B + m_V} A_2 - q^{\mu} q^{\nu} \frac{2m_V}{q^2} (A_3 - A_0) \right], \tag{3.3c}$$

$$\langle V(k) | \bar{q} \sigma_{\mu\nu} b | B(p) \rangle = i\epsilon_{V,\rho\sigma} \epsilon_V^{\rho} \left\{ g^{\rho\nu} (p + k)^{\sigma} T_1 - g^{\rho\nu} q^{\sigma} \frac{m_B^2 - m_V^2}{q^2} (T_1 - T_2) + 2q^{\rho} q^{\nu} k_{\sigma} \left[ \frac{1}{m_B - m_V} T_3 - \frac{1}{q^2} (T_1 - T_2) \right] \right\}, \tag{3.3d}$$

where $\epsilon_V$ is the polarization vector of the spin-one meson and $m_V$ its mass.8 The axial-vector and pseudo-scalar current matrix elements can be related by EoM via the relation $i\partial_\mu \langle P | \bar{q} \gamma^{\mu} \gamma_5 b | B \rangle = \langle P | \bar{q} (\not D + \not D) \gamma_5 b | B \rangle$. Equivalently in momentum space, $q_\mu \langle P(k) | \bar{q} \gamma^{\mu} \gamma_5 b | B(p) \rangle = -(m_b + m_q) \langle P(k) | \bar{q} \gamma_5 b | B(p) \rangle$, which implies that the form factor $A_3$ is a redundant and can be expressed in terms of $A_1$ and $A_2$ as,

$$A_3 \equiv \frac{m_B + m_V}{2m_V} A_1 - \frac{m_B - m_V}{2m_V} A_2. \tag{3.5}$$

7After submitting the manuscript, we became aware of a recent lattice calculation of the $B \rightarrow K$ form factors [52]. We find that using these new lattice results has no significant impact on the sensitivity curves we obtained with the LCSR calculation of the form factors from [48].

8In literature, the tensor current is usually parametrized by multiplying by the four-momentum $q_\mu$. From the tensor current in eq. (3.3), they can be directly calculated to take the form,

$$\langle V(k) | \bar{q} \sigma^{\mu\nu} q_\mu b | B(p) \rangle = 2\epsilon^{\nu\rho\sigma} \epsilon_V^{\rho} p_{\sigma} T_1,$$

$$\langle V(k) | \bar{q} \sigma^{\mu\nu} \gamma_5 q_\mu b | B(p) \rangle = i\epsilon_V^{\mu} \left\{ \left[ g^{\mu\nu} (m_B^2 - m_V^2) - (p + k)^{\mu} q^{\nu} \right] T_2 + q^{\mu} \left[ q^{\nu} \frac{q^2 (p + k)^{\mu}}{m_B^2 - m_V^2} T_3 \right] \right\}. \tag{3.3d}$$

- 8 -
It is common practice to replace $A_2$ and $T_3$ by

\begin{align}
A_{12} &= \frac{(m_B + m_V)^2 (m_B^2 - m_V^2 - q^2)}{16m_Bm_V(m_B + m_V)} A_1 - \lambda(m_B^2, m_V^2, q^2) A_2, \\
T_{23} &= \frac{(m_B^2 - m_V^2)(m_B^2 + 3m_V^2 - q^2)}{8m_Bm_V^2(m_B - m_V)} T_2 - \lambda(m_B^2, m_V^2, q^2) T_3,
\end{align}

where the Källen function $\lambda(x, y, z)$ is the usual,

\begin{equation}
\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2(xy + yz + zx).
\end{equation}

In the $q^2 \to 0$ limit, some of form factors are related with as follows,

\begin{align}
A_0(0) &= A_3(0), \\
T_1(0) &= T_2(0), \\
A_{12}(0) &= \frac{m_B^2 - m_V^2}{8m_Bm_V} A_0(0).
\end{align}

Relabeling the form factors $\{A_0, A_1, A_{12}, V_0, T_1, T_2, T_{23}\}$ as $F_{1,2,3,4,5,6,7}$, their momentum transfer dependence can be parametrized as [49],

\begin{equation}
F_i(s) = \frac{1}{1 - s/m_{R,i}^2} \sum_k \alpha_i^k [z(s) - z(0)]^k, \\
z(s) \equiv \frac{\sqrt{s_+ - s - \sqrt{s_+ - s_0}}}{\sqrt{s_+ - s + \sqrt{s_+ - s_0}},
\end{equation}

where $s_+ \equiv (m_B \pm m_V)^2$ and $s_0 \equiv s_+ (1 - \sqrt{1 - s_+/s_+})$. $m_{R,i}$ are the resonance masses associated with the transition modes and are taken from table 3 in [49]. The parameters $\alpha_i^k$ are truncated at quadratic order in $z$, $k_{\text{max}} = 2$, so that three fit parameters $\alpha_0^1, \alpha_1^1, \alpha_2^1$ are needed for each form factor $i$. They are given in table 14 of [49] and we collect them in appendix D for convenience.

### 3.2 $B \to M + \phi\phi$ with scalar DM $\phi$

For the quark-scalar DM interactions in eq. (2.2), with the hadronic matrix elements given in eq. (3.1) and eq. (3.3), the non-vanishing amplitudes for the processes $B(p) \to P(k)\phi(k_1)\phi^*(k_2)$ and $B(p) \to V(k)\phi(k_1)\phi^*(k_2)$ take the following general form,

\begin{align}
iM_{B \to P\phi\phi} &= C_{q_0^F}^{S,\bar{x}b}(P(k)|\bar{q}_x b|B(p)) + C_{q_0^V}^{V,\bar{x}b}(k_1^\mu - k_2^\mu)(P(k)|\bar{q}_x \gamma_\mu b|B(p)), \\
iM_{B \to V\phi\phi} &= C_{q_0^F}^{P,\bar{x}b}(V(k)|\bar{q}_x i\gamma_5 b|B(p)) + C_{q_0^V}^{V,\bar{x}b}(k_1^\mu - k_2^\mu)(V(k)|\bar{q}_x \gamma_\mu b|B(p)) \\
&\quad + C_{q_0^V}^{A,\bar{x}b}(k_1^\mu - k_2^\mu)(V(k)|\bar{q}_x \gamma_\mu \gamma_5 b|B(p)),
\end{align}

where $x = d, s$ is a quark flavor label characterizing the final state meson $P(V) = \pi, K(\rho, K^*)$. Using the hadronic matrix elements in eq. (3.1) and eq. (3.3), with the help of FeynCalc [53],
the differential decay widths take the following compact form

\[
\frac{d\Gamma_{B\rightarrow P\phi\phi}}{dq^2} = \frac{(m_B^2 - m_p^2)^2}{256\pi^3 m_B^6 (m_B - m_q)e^2} \lambda^2 (m_B^2, m_B^2, s) \kappa^2 (m^2, s) f^2_0 |C_{q\phi}^{S,xb}|^2 \\
+ \frac{1}{768\pi^3 m_B^6} \lambda^2 (m_B^2, m_B^2, s) \kappa^2 (m^2, s) f^2_0 |C_{q\phi}^{V,xb}|^2,
\]

(3.11)

\[
\frac{d\Gamma_{B\rightarrow V\phi\phi}}{dq^2} = \frac{(m_B^2 - m_{q}^2)^2}{256\pi^3 m_B^6 (m_B + m_V)^2} \lambda^2 (m_B^2, m_B^2, s) \kappa^2 (m^2, s) A_0^2 |C_{q\phi}^{P,xb}|^2 \\
+ \frac{s}{384\pi^3 m_B^6 (m_B + m_V)^2} \lambda^2 (m_B^2, m_B^2, s) \kappa^2 (m^2, s) V_0^2 |C_{q\phi}^{V,xb}|^2 \\
+ \frac{1}{384\pi^3 m_B^6} \lambda^2 (m_B^2, m_B^2, s) \kappa^2 (m^2, s) [(m_B + m_V)^2 s A_0^2 + 32m_B^2 m_V A_{12}] |C_{q\phi}^{A,xb}|^2,
\]

(3.12)

where \( \kappa(m^2, s) \) is a kinetic “endpoint” function defined as

\[
\kappa(m^2, s) \equiv 1 - \frac{4m^2}{s},
\]

(3.13)

and the \( q^2 \) dependence of the form factors is left implicit for notational simplicity. In the above results, there are no interference effects between any pair of operators because the relevant hadronic matrix elements have different parity and/or charge conjugation properties and thus cannot mix. The dependence on the DM mass enters only through kinematics, unlike the vector DM case discussed below. For the case of real scalar DM, only the scalar and pseudo-scalar quark currents appear (due to some operators vanishing as noted in eq. (2.2)); there is also an additional factor of two in the decay width.

The different operators result in different \( q^2 \) distributions in the \( B \rightarrow ME \) decay. For example, we illustrate the normalized distributions in the \( b \rightarrow s \) transition for different operators in figure 1.9 In the left (right) panel, we consider scalar DM mass \( m = 100 \text{ MeV} (1 \text{ GeV}) \) and in both cases a solid (dashed) line is used for the \( B \rightarrow K^+ \phi \phi \) (\( K \rightarrow K^{*+} \phi \phi \)) channels. One can clearly see that the distribution varies significantly between operators and as a function of the DM mass. This feature could be exploited to differentiate the various cases in the upcoming experimental search from Belle II.

To quantify the constraints set on the parameter by the current experimental bounds listed in table 2, we use an effective scale \( \Lambda_{\text{eff}} \) associated with each operator from dimensional analysis: \( C_{q\phi}^{S(P)} \equiv \Lambda_{\text{eff}}^{-1} \) and \( C_{q\phi}^{V(A)} \equiv \Lambda_{\text{eff}}^{-2}. \) Figure 2 shows the current experimental sensitivity in the \( m-\Lambda \) plane for each operator with \((bs)\) and \((bd)\) flavor changing quark combinations. The solid (dashed) lines correspond to the constraints from charged (neutral) decay modes for both pseudo-scalar and vector final state mesons. The largest possible DM mass is restricted by the kinematic relation \( m \leq (m_B - m_M)/2 \) as reflected in each panel. Generally, for the scalar and vector current operators on the left two panels, the charged mode \( B^+ \rightarrow K^+ \) sets the stronger constraints for \((bs)\) transitions and the neutral

---

9In the figures we add a subscript to differentiate the type of DM in question, but in the text we refer generically to the mass of any DM particle as \( m \).
Figure 1. Normalized differential decay width for $B \to K^{(*)}\phi\phi$ decay from different types of operators. Left panel: $m = 100$ MeV; Right panel: $m = 1$ GeV.

Figure 2. Constraints on the effective new physics scale for each operator involving a $b$ quark as a function of the DM mass $m$ from all possible relevant $B$ meson decay channels.
Figure 3. Constraints on the effective new physics scale as a function of the DM mass \( m \) from the inclusive tag Belle II \( B^+ \rightarrow K^+ \nu \bar{\nu} \) search without (solid lines) and with (dashed lines) experimental efficiency (E.E.) included.

mode \( B^0 \rightarrow \pi^0 \) for the \((bd)\) transitions. For the pseudo-scalar and axial-vector quark current operators shown on the right two panels, the stronger bounds for the \((bs)\) and \((bd)\) transitions are set respectively by the neutral mode \( B^0 \rightarrow K^{*0} \) and the charged mode \( B^+ \rightarrow \rho^+ \). This feature is just a reflection of the current experimental bounds as can be seen in table 2.

The situation depicted in figure 2 will, of course, be modified by experimental considerations. For example, Belle II has reported with its current measurement of \( B^+ \rightarrow K^+ \bar{\nu} \) a signal efficiency that varies with \( q^2 \), peaking at low values and becoming very small for \( q^2 \lesssim 12 \text{ GeV}^2 \) [54]. Figure 1 then suggests that searches for very light DM will be more sensitive than searches for heavier DM. Inclusion of this experimental sensitivity changes the corresponding constraint, and we illustrate this in figure 3, where the left panel is for the scalar operator \( O^{S, sb}_{q \phi} \) while the right panel for the vector operator \( O^{V, sb}_{q \phi} \). The signal efficiency reported in [54] applies only to the search for \( B^+ \rightarrow K^+ \bar{\nu} \nu \) with an inclusive tag, and results in the 90\% confidence level limit \( B(B^+ \rightarrow K^+ \nu \bar{\nu}) \leq 4.1 \times 10^{-5} \), a few times weaker than the PDG value we quote in table 2. The solid lines in figure 3 show the constraints from figure 2 for the \( B^+ \rightarrow K^+ \phi \phi \) mode, but using the weaker upper limit from the inclusive tag Belle II search. When the experimental efficiency is included, these limits turn into the ones depicted by dashed lines. To estimate these corrections, we scale the Belle II upper limit by a ratio of normalized rates weighted by the reported efficiency,

\[
\omega(m) = \frac{\sum_i \tilde{\Gamma}_{i, \text{SM}} \epsilon_i}{\sum_i \tilde{\Gamma}_{i, \text{NP}}(m) \epsilon_i},
\]

where \( \tilde{\Gamma}_{i, \text{NP}}(m) \) is the normalized width from NP contribution in \( i \)-th bin, i.e.,

\[
\tilde{\Gamma}_{i, \text{NP}}(m) = \frac{1}{\Gamma_{\text{NP}}(m)} \int_{\text{bin}_i} dq^2 \frac{d\Gamma_{\text{NP}}(m)}{dq^2},
\]

and similarly for \( \Gamma_{i, \text{SM}} \).
### 3.3 \( B \to M + XX \) with vector DM \( X \): scenario A

For spin one DM, we first consider scenario A in which the operators were constructed using the vector field as given in eq. (2.3). Two of these operators involve a covariant derivative acting on the quark current, \( \mathcal{O}_{qX}^V \) and \( \mathcal{O}_{qX}^A \). To evaluate their contribution one needs form factors that have not been studied before. Within specific quark models for mesons one could estimate these form factors by replacing the derivatives with the corresponding quark momentum. As this is beyond the scope of this paper, we simply ignore these two operators in the following numerical study. The non-vanishing amplitudes for the processes \( B(p) \to P(k)X(k_1)X^*(k_2) \) and \( B(p) \to V(k)X(k_1)X^*(k_2) \) from the remaining operators in eq. (2.3) take the following form,

\[
\begin{align*}
    i\mathcal{M}_{B\to PXX}^A &= \epsilon^*_p(k_1)\epsilon^*_s(k_2) \left\{ g^{\rho\sigma} C_{qX}^{S,xb} \langle P(k)|q_x b|B(p) \rangle \\
    &+ \frac{i}{2} \left( 2g^{\rho\mu}g^{\nu\sigma} C_{qX}^{T,xb} + \epsilon^{\mu
u\rho\sigma} C_{qX}^{T,xb} \right) \langle P(k)|q_x \sigma_{\mu\nu} b|B(p) \rangle \\
    &+ \left[ i(g^{\mu\rho}k_1 + g^{\nu\sigma}k_2) C_{qX}^{V,xb} - i\epsilon^{\mu\nu\rho\sigma} (k_1 - k_2) C_{qX}^{V,xb} + g^{\sigma}(k_1 - k_2) C_{qX}^{V,xb} \right]
    \langle P(k)|q_x \gamma_\mu b|B(p) \rangle \right\},
\end{align*}
\]

(3.16a)

\[
\begin{align*}
    i\mathcal{M}_{B\to VXX}^A &= \epsilon^*_p(k_1)\epsilon^*_s(k_2) \left\{ g^{\rho\sigma} C_{qX}^{P,xb} \langle V(k)|q_x \gamma_5 b|B(p) \rangle \\
    &+ \frac{i}{2} \left( 2g^{\rho\mu}g^{\nu\sigma} C_{qX}^{T,xb} + \epsilon^{\mu
u\rho\sigma} C_{qX}^{T,xb} \right) \langle V(k)|q_x \sigma_{\mu\nu} b|B(p) \rangle \\
    &+ \left[ i(g^{\mu\rho}k_1 + g^{\nu\sigma}k_2) C_{qX}^{V,xb} - i\epsilon^{\mu\nu\rho\sigma} (k_1 - k_2) C_{qX}^{V,xb} + g^{\sigma}(k_1 - k_2) C_{qX}^{V,xb} \right]
    \langle V(k)|q_x \gamma_\mu b|B(p) \rangle \right\},
\end{align*}
\]

(3.16b)

For the final state with a pseudo-scalar meson \( P \), using the hadronic matrix elements in eq. (3.1), leads to the differential decay width

\[
\begin{align*}
\frac{d\Gamma_{B\to PXX}^A}{dq^2} &= \frac{(m_B^2 - m_p^2)^2(s^2 - 4m^2s + 12m^4)}{1024\pi^3m_B^6(m_B - m_q)^2m^2} \left[ \frac{1}{2} \lambda^2(m_B^2, m_p^2, s) \kappa^2(m^2, s) f_0^2 C_{qX}^{S,xb} \right]^2 \\
    &+ \frac{s(s + 4m^2)}{3072\pi^3m_B^6(m_B + m_P)^2m^2} \lambda^2(m_B^2, m_p^2, s) \kappa^2(m^2, s) f_1^2 C_{qX}^{T,xb} \\
    &+ \frac{s + 2m^2}{768\pi^3m_B^6(m_B + m_P)^2m^2} \lambda^2(m_B^2, m_p^2, s) \kappa^2(m^2, s) f_1^2 C_{qX}^{V,xb} \\
    &+ \frac{1}{3072\pi^3m_B^6m^2} \lambda^2(m_B^2, m_p^2, s) \kappa^2(m^2, s) \\
    &\left[ 3(s - 4m^2)(m_B^2 - m_p^2)^2f_0^2 + 4m^2\lambda(m_B^2, m_p^2, s) f_1^2 \right] C_{qX}^{V,xb} \\
    &+ \frac{1}{768\pi^3m_B^6m^2} \lambda^2(m_B^2, m_p^2, s) \kappa^2(m^2, s)
\end{align*}
\]
For the final state with a vector meson $V$, using the form factors in eq. (3.3), we obtain

$$
\frac{d\Gamma_A^{B \rightarrow V,XX}}{dq^2} = \frac{s^2 - 4m^2s + 12m^4}{1024\pi^3m_B^3(m_b + m_q)^2m^4} \lambda^2(m_B^2, m_V^2, s)\kappa^2(m^2, s)f_+^2 \bigg| C_{q,VX}^{A,xb} \bigg| ^2
$$

We have dropped interference between different operators (represented by “…” above) as our numerical study will deal only with one operator at a time.

\begin{align*}
&\times \left[ 6m^2(m_B^2 - m_P^2)f_0^2 + (s - 4m^2)\lambda(m_B^2, m_P^2, s)f_+^2 \right] C_{q,VX}^{A,xb}

&\quad+ \frac{s^2 - 4m^2s + 12m^4}{3072\pi^3m_B^4m^4} \lambda^2(m_B^2, m_P^2, s)\kappa^2(m^2, s) f_+^2 \bigg| C_{q,VX}^{A,xb} \bigg| ^2

&\quad+ \frac{s(s + 4m^2)}{3072\pi^3m_B^4m^4} \lambda^2(m_B^2, m_P^2, s)\kappa^2(m^2, s) f_+^2 \bigg| C_{q,VX}^{A,xb} \bigg| ^2

&\quad+ \frac{s + 2m^2}{768\pi^3m_B^2m^2} \lambda^2(m_B^2, m_P^2, s)\kappa^2(m^2, s) f_+^2 \bigg| C_{q,VX}^{A,xb} \bigg| ^2 + \ldots \cdot (3.17)
\end{align*}
\[ + \frac{s(s + 4m^2)}{1536\pi^2m_B^4m^4} \lambda^\frac{1}{2}(m_B^2, m_V^2, s)\kappa^\frac{3}{2}(m^2, s) \times \left[ (m_B + m_V)^2 sA_1^2 + 32m_B^2m_V^2A_1^2 \right] |C_{qX1}|^2 \]
\[ + \frac{s + 2m^2}{384\pi^3m_B^2m^2} \lambda^\frac{1}{2}(m_B^2, m_V^2, s)\kappa^\frac{3}{2}(m^2, s) \times \left[ (m_B + m_V)^2 sA_1^2 + 32m_B^2m_V^2A_1^2 \right] |C_{qX6}|^2 + \cdots, \] (3.18)

where again we have dropped interference between different operators.

It is evident that the above differential decay widths (and also decay widths) diverge in the limit of vanishing DM mass, \( m \to 0 \). As discussed above, we assume that each relevant Wilson coefficient also depends on the DM mass to some power determined by the number of independent vector four-potentials that cannot be reduced to field strength tensors. Operationally we use effective scales \( \Lambda_{\text{eff}} \) defined as follows,

\[ C_{qX}^{S,P} \equiv \frac{m^2}{\Lambda_{\text{eff}}^3}, \quad C_{qX1,2}^{T} \equiv \frac{m^2}{\Lambda_{\text{eff}}^3}, \quad C_{qX2,4,5}^{V,A} \equiv \frac{m^2}{\Lambda_{\text{eff}}^3}, \quad C_{qX3,6}^{V,A} \equiv \frac{m}{\Lambda_{\text{eff}}^3}. \] (3.19)

In eq. (3.19), \( C_{qX3,6}^{V,A} \) depends linearly on \( m \) because one of two vector fields in the operators \( O_{qX3,6}^{V,A} \) can be rewritten as a field strength tensor.

The possible origin of these mass factors is illustrated with an example in appendix E. The divergence as \( m \to 0 \) also affects the kaon decay mode, \( K \to \pi XX \), where we use a parametrization similar to eq. (3.19). As already mentioned, the problem with the \( m \to 0 \) limit can be avoided by assuming these vector particles are gauge bosons of a dark gauge symmetry and requiring them to enter the LEFT as field strength tensors as in eq. (2.4). We elaborate on this second scenario for vector DM in the next subsection.

Figure 4 shows the current experimental sensitivity in the \( m-\Lambda_{\text{eff}} \) plane for the 4 dim-5 operators \( O_{qX}^{S,P} \) and \( O_{qX1,2}^{T} \) following eq. (3.19). For the quark scalar current operator \( O_{qX}^{S,P} \) (left upper panel), the charged mode \( (B^+ \to K^+ XX) \) gives the strongest constraint for \( (sb) \) flavor indices in the whole DM mass range. Similarly, for \( (db) \) flavor indices, the strongest constraint arises from the neutral mode \( B^0 \to \pi^0 XX \). For the quark pseudo-scalar current operator \( O_{qX}^{P} \) (right upper panel), the constraints also exhibit a similar behavior to those for the pseudo-scalar DM operator \( O_{qX}^{P} \). However, due to the different dimensionality of the quark pseudo-scalar current operators in the two cases, the numerical results are very different for the two cases, as clearly seen in figure 2 and figure 4.

For the tensor operator \( O_{qX1}^{T}(O_{qX2}^{T}) \) (lower two panels), the neutral mode \( B^0 \to K^{*0} XX \) gives a stronger bound for DM mass \( m \lesssim 2.2(1.9) \text{ GeV} \) and the charged mode \( B^+ \to K^+ XX \) for \( m \gtrsim 2.2(1.9) \text{ GeV} \) for \( (sb) \) flavor indices. For \( (db) \) flavor indices instead, the neutral mode \( B^0 \to \pi^0 XX \) gives a stronger bound in the full DM mass range for \( O_{qX1}^{T} \). For \( O_{qX2}^{T} \) with \( (db) \) flavor indices, it is the charged mode \( B^+ \to \rho^+ XX \) that gives a stronger bound for \( m \lesssim 1.5 \text{ GeV} \) and the neutral mode \( B^0 \to \pi^0 XX \) for \( m \gtrsim 1.5 \text{ GeV} \). The different behavior of the operator \( O_{qX2}^{T} \) from the \( B \to K^{(\pi)XX} \) modes is due to a quadratic (rather than quartic) inverse dependence on \( m \) in the decay widths. In all the four cases, the effective scale is constrained to be above a few hundreds of GeV, validating our use of an EFT framework for this discussion.
Figure 4. Constraints on the effective new physics scale for the 4 dim-5 operators $O_{qX}^{S,P}$ and $O_{qX}^{T}_{1,2}$ as a function of the DM mass $m$ from $B \to K(\pi)E$ channels.

Figure 5 shows the constraints for the 4 dim-6 operators with quark (axial-)vector currents, $O_{qX}^{V,A}$, and following eq. (3.19). It can be seen that these constraints observe a similar behavior to those for the quark (axial-)vector current operators $O_{q\phi}^{V,A}$ in the scalar DM case (figure 2). The limit on $\Lambda_{\text{eff}}$ is weaker by roughly an order of magnitude, due to the different dimensionality of the operators. Figure 6 shows the results for the remaining dim-6 operators, $O_{qX}^{V,A}$. The constraints for $O_{qX}^{V}$ from the $B \to K^+(\rho)XX$ exhibit similar behavior to those for $O_{qX}^{T}$ in figure 4. They are much weaker due to the higher dimensionality and result in limits on $\Lambda_{\text{eff}}$ of order a few tens of GeV. These are still much larger than the $B$ meson mass implying that the LEFT framework is valid. On the other hand, it may be difficult to interpret them within SMEFT. It would also be difficult to UV complete these operators, as UV completions would very likely predict new states with collider accessible masses.

Following the discussion on the scalar DM case with the inclusion of the Belle II experimental efficiency in figure 3, we show the similar plots for the vector DM of scenario A from $B^+ \to K^+XX$ mode in figure 7. It can be seen that the sensitivity on $\Lambda_{\text{eff}}$ for $m = 0$ is weaker by a factor of about 1.2–1.5 (with the specific value depending on the operator) when the efficiency is included, and then gradually decreases as $m$ increases. Furthermore, the sensitivity in this case is limited to $m \lesssim 2\text{ GeV}$ by $4m^2 \lesssim q^2_{\text{max}}$, with the maximum $q^2_{\text{max}} \approx 16\text{ GeV}^2$ corresponding to the non-vanishing signal efficiency region in Belle II.
Figure 5. Constraints on the effective new physics scale for the 4 dim-6 operators $O^{VA}_{q\chi,6}$ as a function of the DM mass $m$ from $B \to K(\pi)/E$ channels.

\subsection*{3.4 $B \to M + XX$ with vector DM $X$: scenario B}

For these operators there is no issue with the $m \to 0$ limit, and in addition all the form factors needed have been estimated in the literature before. The non-vanishing amplitudes for the two processes $B(p) \to P(k)X(k_1)X^*(k_2)$ and $B(p) \to V(k)X(k_1)X^*(k_2)$ take the following form,

\begin{align}
 iM^B_{B \to PXX} &= \epsilon^*_\rho(k_1)\epsilon^*_\sigma(k_2) \left\{ \left[ 2(k_1^2 k_2^2 - k_1 \cdot k_2 g^{\rho\sigma}) \tilde{C}_{qX1}^{S,xb} + 2\epsilon^{\mu\rho\sigma\nu} k_1 \iota_\nu k_2 \tilde{C}_{qX2}^{S,xb} \right] (P(k)|\bar{q}_x b|B(p)) \\
 &\quad + \frac{i}{2} \left[ (k_1^2 k_2^2 g^{\rho\sigma} + k_1 \cdot k_2 g^{\rho\sigma} - k_1^2 k_2^2 g^{\beta\sigma} - k_1 k_2 g^{\rho\sigma}) \left( 2g_{\mu\nu} g_{\rho\beta} \tilde{C}_{qX1}^{T,xb} + \epsilon_{\rho\nu\beta\sigma} \tilde{C}_{qX2}^{T,xb} \right) \right] \times (P(k)|\bar{q}_x \sigma^{\mu\nu} b|B(p)) \right\}, \\
 &\quad (3.20a)
\end{align}

\begin{align}
 iM^B_{B \to VXX} &= \epsilon^*_\rho(k_1)\epsilon^*_\sigma(k_2) \left\{ \left[ 2(k_1^2 k_2^2 - k_1 \cdot k_2 g^{\rho\sigma}) \tilde{C}_{qX1}^{P,xb} + 2\epsilon^{\mu\rho\sigma\nu} k_1 \iota_\nu k_2 \tilde{C}_{qX2}^{P,xb} \right] (V(k)|\bar{q}_x i\gamma_5 b|B(p)) \\
 &\quad + \frac{i}{2} \left[ (k_1^2 k_2^2 g^{\rho\sigma} + k_1 \cdot k_2 g^{\rho\sigma} - k_1^2 k_2^2 g^{\beta\sigma} - k_1 k_2 g^{\rho\sigma}) \left( 2g_{\mu\nu} g_{\rho\beta} \tilde{C}_{qX1}^{T,xb} + \epsilon_{\rho\nu\beta\sigma} \tilde{C}_{qX2}^{T,xb} \right) \right] \times (V(k)|\bar{q}_x \sigma^{\mu\nu} b|B(p)) \right\}.
 &\quad (3.20b)
\end{align}
Figure 6. Constraints on the effective new physics scale for the 6 dim-6 operators $O_{2,4,5}^{V,A}$ as a function of the DM mass $m$ from $B \to K(\pi)\ell \nu$ channels.
Figure 7. Constraints on the effective new physics scale as a function of the DM mass \( m \) from the inclusive tag Belle II \( B^+ \rightarrow K^+ \nu \bar{\nu} \) for the vector DM of scenario A.

From these amplitudes, the differential decay widths result in the following compact forms,

\[
\frac{d\Gamma_{B \rightarrow PXX}}{dq^2} = \frac{(m_B^2 - m_P^2)^2(s^2 - 4m^2s + 6m^4)}{128\pi^3 m_B^4(m_b - m_q)^2} \lambda^2 (m^2_B, m^2_P, s) \kappa^2 (m^2, s) f_0^2 \left| \tilde{C}_{qX1}^{P,P} \right|^2 \\
+ \frac{(m_B^2 - m_P^2)^2 s^2}{128\pi^3 m_B^4(m_b - m_q)^2} \lambda^2 (m^2_B, m^2_P, s) \kappa^2 (m^2, s) f_0^2 \left| \tilde{C}_{qX1}^{S,P} \right|^2 \\
+ \frac{s(s + 2m^2)}{1536\pi^3 m_B^5(m_b + m_P)^2} \lambda^2 (m^2_B, m^2_P, s) \kappa^2 (m^2, s) f_0^2 \left| \tilde{C}_{qX1}^{T,P} \right|^2 \\
+ \frac{s^2 - 2m^2s + 4m^4}{1536\pi^3 m_B^5(m_b + m_P)^2} \lambda^2 (m^2_B, m^2_P, s) \kappa^2 (m^2, s) f_0^2 \left| \tilde{C}_{qX1}^{V,P} \right|^2,
\]

\[
(3.21)
\]

\[
\frac{d\Gamma_{B \rightarrow VXX}}{dq^2} = \frac{s^2 - 4m^2s + 6m^4}{128\pi^3 m_B^4(m_b + m_q)^2} \lambda^2 (m^2_B, m^2_V, s) \kappa^2 (m^2, s) A_0^2 \left| \tilde{C}_{qX1}^{P,V} \right|^2 \\
+ \frac{s^2 - 2m^2s + 4m^4}{128\pi^3 m_B^4(m_b + m_q)^2} \lambda^2 (m^2_B, m^2_V, s) \kappa^2 (m^2, s) A_0^2 \left| \tilde{C}_{qX1}^{S,V} \right|^2 \\
+ \frac{1}{768\pi^3 m_B^4 m_b} \lambda^2 (m^2_B, m^2_V, s) \kappa^2 (m^2, s) \lambda^2 (m^2_B, m^2_V, s) \kappa^2 (m^2, s) f_0^2 \left| \tilde{C}_{qX1}^{T,V} \right|^2 \\
+ \frac{1}{768\pi^3 m_B^4 m_b} \lambda^2 (m^2_B, m^2_V, s) \kappa^2 (m^2, s) \lambda^2 (m^2_B, m^2_V, s) \kappa^2 (m^2, s) f_0^2 \left| \tilde{C}_{qX1}^{V,V} \right|^2 \\
\]

\[
(3.22)
\]

Similarly to the case of scalar DM, there is no interference between the different operators due to their different parity and/or charge conjugation. Interestingly, in the above expressions, the contributions from each pair of operators with similar quark Lorentz structure (characterized by the same superscript “S/P/T”\(^\text{\textsuperscript{10}}\)) become the same in the limit of \( m \rightarrow 0.\)

\(^{10}\)Note that the two tensor currents are “similar” to each other because \( \bar{q} \gamma^{\mu} \gamma^5 q = \frac{i}{2} \epsilon^{\mu \nu \rho \sigma} q_{\rho} \sigma \).
Figure 8. Normalized differential decay width for $B \to K^{(*)}+XX$ from different operators in the second scenario for vector DM. Left panel: $m = 0$; Right panel: $m = 1$ GeV.

Figure 8 shows the normalized distributions for $B \to K^{+}XX$ (solid lines) and $B \to K^{*+}XX$ (dashed lines) for different operators with $(sb)$ flavor indices. In the left (right) panel, we set the vector DM mass to $m = 0$ and $1$ GeV respectively. One can see in the left panel that the distributions are the same for the two operators with the same superscript “S/P/T”. Even for $m = 1$ GeV as shown in the right panel, this degeneracy still holds except for a rather narrow range of $q^2$ value, which implies the $d\tilde{\Gamma}/dq^2$ observable alone cannot be used to distinguish these interactions.

To set numerical constraints we include the new physics scale in the Wilson coefficients as $\tilde{C}_{qX1,2} \equiv \Lambda_{\text{eff}}^{-3}$. Figure 9 shows the bounds on the $m$-$\Lambda_{\text{eff}}$ plane for each operator. These bounds are comparable to those shown in figure 4 for operators with the same quark current as they have the same dependence on $\Lambda_{\text{eff}}$. The effective scale $\Lambda_{\text{eff}}$ which can be probed depends on the value of DM mass, and is above a few hundreds of GeV for $m \lesssim 2$ GeV. Finally, figure 10 shows the sensitivity after including the experimental efficiency for $B^{+} \to K^{+}XX$ from Belle II. Similar to the other two DM cases, the constraint is weaker by a factor of 1.5 (1.4) for $\tilde{O}_{qX1,2}^{S,\text{sb}}$ ($\tilde{O}_{qX1,2}^{T,\text{sb}}$) at $m = 0$ when the efficiency is included, with a sensitivity limit for $m$ around 2 GeV.

4 $K \to \pi^{+}\text{DM}+\text{DM}$

Since the kaon decay process $K \to \pi E$ involves only the light quarks $q = u, d, s$, its transition matrix element due to effective interactions in LEFT can be evaluated by matching onto chiral perturbation theory ($\chi$PT), which is the low energy effective field theory of QCD. $\chi$PT is based on the fact that the QCD Lagrangian has the approximate chiral symmetry $SU(3)_L \times SU(3)_R$ for the three light quarks which is spontaneously broken by the quark condensate $\langle \bar{q}q \rangle$ to the diagonal flavor $SU(3)_V$. The symmetry breakdown results in eight pseudo-Nambu-Goldstone bosons (pNGBs), which are identified with the octet of the lowest-lying pseudoscalars $\pi^{\pm}$, $\pi^0$, $K^{\pm}$, $K^0$, $\bar{K}^0$, $\eta$. In the $\chi$PT formalism they are represented by the element in the coset space $SU(3)_L \times SU(3)_R/SU(3)_V$ and take the matrix form,

$$U(x) = \exp \left( i \frac{\sqrt{2} \Pi(x)}{F_0} \right), \quad \Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- - \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 & \bar{K}^0 \\ K^- & K^0 & -\frac{2}{3} \eta \end{pmatrix}, \quad (4.1)$$
Figure 9. Constraints on the effective new physics scale for each operator as a function of the DM mass $m$ from $B \to K(\pi)\bar{E}$ channels.
where $F_0$ is the pion decay constant in the chiral limit. Corresponding to the chiral transformations of quarks $q_L \rightarrow Lq_L$ and $q_R \rightarrow Rq_R$, $U$ transforms as $U \rightarrow LUR^\dagger$ with $L \in SU(3)_L$ and $R \in SU(3)_R$.

The interactions of pNGBs with DM due to the effective operators in eqs. (2.2)–(2.4) can be realized through the external source method in which the global chiral symmetry is promoted to a local one [55–57]. At the quark-gluon level, the QCD Lagrangian with all possible external sources associated with quark bilinear currents is parameterized as follows,

$$
\mathcal{L} = \mathcal{L}_{\text{QCD}} + \bar{\psi} \gamma^\mu q + \bar{\eta} \gamma^\mu \gamma_5 q - \bar{s} \gamma q + \bar{p} \gamma^5 q + \bar{\chi} \gamma_{\mu \nu} \sigma^{\mu \nu} q
$$

$$
= \mathcal{L}_{\text{QCD}} + \bar{\psi} L \gamma^\mu q_L + \bar{\eta} R \gamma^\mu q_R - \left[ \bar{s} \gamma (s + i\gamma_5) q - \bar{p} \gamma_{\mu \nu} \gamma_5 \sigma_{\mu \nu} q + \text{h.c.} \right],
$$

where $\mathcal{L}_{\text{QCD}}$ is the QCD Lagrangian for $u$, $d$, $s$ quarks in the massless limit. The external sources, $\bar{\psi} = \bar{\psi}^\dagger$, $\bar{\eta} = \bar{\eta}^\dagger$, $\bar{s} = \bar{s}^\dagger$, $\bar{p} = \bar{p}^\dagger$, $\bar{\chi} = \bar{\chi}^\dagger$, are $3 \times 3$ Hermitian traceless matrices in flavor space. In the second line, we rewrite the Lagrangian in terms of chiral quark fields to make the chiral symmetry manifest. The external sources in the second line are related to those in the first line by the relations,

$$
l_\mu = \hat{v}_\mu - \hat{a}_\mu, \quad r_\mu = \hat{v}_\mu + \hat{a}_\mu, \quad \chi = 2B(\hat{s} - i\hat{p}), \quad t_{L}^\mu = P_{L}^{\mu \alpha \beta} \hat{a}_\alpha \hat{a}_\beta, \quad t_{R}^\mu = P_{R}^{\mu \alpha \beta} \hat{a}_\alpha \hat{a}_\beta = t_{L}^{\mu \dagger},
$$

where the chiral projection for the tensor currents is defined by $P_{R,L}^{\mu \alpha \beta} = \frac{1}{4}(g^{\mu \alpha}g^{\nu \beta} - g^{\mu \beta}g^{\nu \alpha} + i\epsilon^{\mu \alpha \beta \gamma})$ [57], with the property $t_{L}^{\mu \nu} = t_{R}^{\mu \nu} + t_{R}^{\nu \mu}$. Under chiral transformations, $l_\mu \rightarrow Lt_\mu L^\dagger + iL\partial_\mu L^\dagger$, $r_\mu \rightarrow Rr_\mu R^\dagger + iR\partial_\mu R^\dagger$, $\chi \rightarrow L\chi R^\dagger$, and $t_{L}^{\mu \nu} \rightarrow Rt_{L}^{\mu \nu} L^\dagger$, respectively. One should note that the tensor external sources have mass dimension one in our convention.\(^{11}\)

The constant $B$ is related to the quark condensate and $F_0$ by the relation $B = -\langle \bar{q}q \rangle / (3F_0^2)$. For numerical estimates, we use $F_0 = 87$ MeV \([58]\) and $B \approx 2.8$ GeV.

\(^{11}\)Our convention for $U$ and $\chi$ are equivalent to $U^\dagger$ and $\chi^\dagger$ in [57].
By comparing the external sources in eq. (4.2b) with the effective interactions in L\(ET\) in eqs. (2.2)–(2.4) we see that the correspondence needed to calculate the \(K \rightarrow \pi\) transition with scalar DM is,
\[
(\hat{\sigma})_{ds} = C_{q\bar{q}}^{V,ds} \phi^\dagger i\bar{d}_d^\dagger \sigma_{\bar{d}_d} \phi, \quad (\hat{\sigma}^\dagger)_{ds} = C_{q\bar{q}}^{A,ds} \phi^\dagger i\bar{d}_d^\dagger \sigma_{\bar{d}_d} \phi, \quad (\hat{s})_{ds} = -C_{q\bar{q}}^{S,ds} \phi^\dagger \phi, \quad (\hat{p})_{ds} = C_{q\bar{q}}^{P,ds} \phi^\dagger \phi,
\]
and the corresponding Hermitian conjugates interchanging labels \(s\) and \(d\). For the vector DM case, it is also easily to identify the relevant external sources from the operators in eqs. (2.3)–(2.4). For the second scenario of eq. (2.4), the non-vanishing sources are
\[
(\hat{s})_{ds} = -C_{q\bar{q}}^{S,ds} X^\dagger_{\mu\nu} X^{\mu\nu} - C_{q\bar{q}}^{S,ds} \tilde{X}^\dagger_{\mu\nu} \tilde{X}^{\mu\nu}, \quad (\hat{p})_{ds} = C_{q\bar{q}}^{P,ds} X^\dagger_{\mu\nu} + C_{q\bar{q}}^{P,ds} \tilde{X}^\dagger_{\mu\nu},
\]
\[
(\hat{P}^{\mu\nu})_{ds} = \frac{i}{2} C_{q\bar{q}}^{T,ds}(X^\dagger_{\mu\nu} - X^{\mu\nu}) + \frac{i}{2} C_{q\bar{q}}^{T,ds}(\epsilon^{\mu\nu\rho\sigma} X^\dagger_{\rho\sigma} X^{\rho\sigma}),
\]
and the corresponding Hermitian conjugates exchanging the \(s\) and \(d\) labels.

In \(\chi PT\) the vector and scalar sources first appear at order \(\mathcal{O}(p^2)\) in the chiral power counting scheme \([55, 56]\)
\[
\mathcal{L}^{(2)}_{\chi PT} = \frac{F_0^2}{4} \text{Tr} \left[ D_\mu U(D^\mu U)^\dagger + \frac{F_0^2}{4} \text{Tr} \left[ \chi U^\dagger + U \chi \right] \right], \quad D_\mu U = \partial_\mu U - i \ell_\mu U + i U r_\mu,
\]
whereas the tensor sources first appear at \(\mathcal{O}(p^4)\) \([57]\)
\[
\mathcal{L}^{(4)}_{\chi PT} \ni i \Lambda_2 \text{Tr} \left[ \rho^{\mu\nu}(D_\mu U)^\dagger U(D_\nu U)^\dagger + i \rho^{\mu\nu} D_\mu U^\dagger D_\nu U \right],
\]
with \(\Lambda_2\) being a new low energy constant with mass dimension one. For our numerical estimates we follow \([59]\) and use \(|\Lambda_2| \approx 0.018\) GeV, noting this is comparable with naive dimensional analysis estimates which find \(\Lambda_2 \sim \Lambda_{\chi} \sim 0.008\) GeV with the chiral symmetry breaking scale \(\Lambda_{\chi} \sim 1.2\) GeV. To find the relevant local interactions mediating \(K \rightarrow \pi E\) in question from the \(\chi PT\) formalism, it suffices to expand the above Lagrangian in eq. (4.6) and eq. (4.7) to linear order in the kaon and pion fields as well as each external source. This leads to the following local interactions mediating \(K \rightarrow \pi E\),
\[
\mathcal{L}_{K \rightarrow \pi} = -B \hat{s}_{sd} \pi^- K^+ + \frac{B}{\sqrt{2}} \hat{s}_{sd} \pi^0 K^0 \quad \text{(4.8a)}
\]
\[
+ i (\hat{\pi}_{sd}) \left( \partial^\mu \pi^- K^+ - \partial^- \partial^\mu K^+ \right) - \frac{i}{\sqrt{2}} (\hat{\pi}_{sd}) \left( \partial^\mu \pi^0 K^0 - \partial^\mu \partial^\mu K^0 \right) - \frac{i \Lambda_2}{F_0^2} \hat{t}_{sd} \left( \partial^\mu \pi^- \partial^\mu K^+ - \partial^- (\partial^\mu \partial^\mu K^+) \right) + i \frac{\Lambda_2}{\sqrt{2} F_0^2} \hat{t}_{sd} \left( \partial^\mu \pi^0 \partial^\mu K^0 - \partial^\mu \partial^\mu K^0 \right) + \text{h.c.}
\]
\[
+ \frac{B}{2} (\hat{s}_{sd} + \hat{s}_{ds}) \pi^0 K L \quad \text{(4.8b)}
\]
\[
+ i (\hat{\pi}_{sd}) \left( \partial^\mu \pi^- K^+ - \partial^- \partial^\mu K^+ \right) - i \left[ (\hat{\pi}_{sd}) - (\hat{\pi}_{ds}) \right] \left( \partial^\mu \pi^0 K_L - \partial^\mu \partial^\mu K_L \right) - \frac{\Lambda_2}{F_0^2} \hat{t}_{sd} \left( \partial^\mu \pi^- \partial^\mu K^+ - \partial^- (\partial^\mu \partial^\mu K^+) \right) + i \frac{\Lambda_2}{2 F_0^2} \left( \hat{t}_{sd} - \hat{t}_{ds} \right) \left( \partial^\mu \pi^0 \partial^\mu K_L - \partial^\mu \partial^\mu K_L \right).
\]
In the second equation we ignore CP violation in kaon mixing to write $K^0(\bar{K}^0) \approx \frac{1}{\sqrt{2}} (K_L \pm K_S)$. It can be seen in the above effective Lagrangian that the neutral mode $K_L \to \pi^0E$ can be obtained from the charged mode by replacing the relevant Wilson coefficients $C_i^{sd}$ with $(C_i^{sd} \pm C_i^{ds})/2 \sim \Re(C_i^{ds})(3|C_i^{ds}|)$. The plus sign (real) applies to the scalar current while the minus sign (imaginary) applies to the vector and tensor currents. The above Lagrangian leads to the following form factors

$$
\langle \pi^+(k) | \bar{s}d | K^+(p) \rangle \simeq B,
$$

$$
\langle \pi^+(k) | \bar{s}\gamma^\mu d | K^+(p) \rangle \simeq (p + k)^\mu,
$$

$$
\langle \pi^+(k) | \bar{s}\sigma^{\mu\nu} d | K^+(p) \rangle \simeq \frac{A_2}{F_0} (p^\mu k^\nu - p^\nu k^\mu).
$$

These matrix elements have exactly the same Lorentz structure as those for $B \to P$ transitions given in eq. (3.1) in the limit of $q^2 \to 0$ and noticing that $f_+(0) = f_0(0)$. Thus, the differential decay width for $K \to \pi E$ can be obtained directly from the result for $B$ decay with suitable replacements of variables. The differential decay width for the charged mode and scalar DM is then

$$
\frac{d\Gamma_{K^+ \to \pi^+ \phi}}{dq^2} = \frac{B^2}{256\pi^3 m_K^2} \lambda^2(m_K^2, m_\pi^2, s)\kappa^2(m^2, s) \left| C_{q\phi}^{S,ds} \right|^2 + \frac{1}{768\pi^3 m_K^2} \lambda^2(m_K^2, m_\pi^2, s)\kappa^2(m^2, s) \left| C_{q\phi}^{V,ds} \right|^2.
$$

For the two vector DM scenarios, using eqs. (2.2)–(2.4) and eq. (4.9), the differential decay widths are

$$
\frac{d\Gamma_{K^+ \to \pi^+ X}}{dq^2} = \frac{B^2}{1024\pi^3 m_K^4} \lambda^2(m_K^2, m_\pi^2, s)\kappa^2(m^2, s) \left| C_{qX}^{S,ds} \right|^2 + \frac{\Lambda_2^2 s(s + 4m^2)}{12288\pi^3 F_0^4 m_K^2 m_\pi^2} \lambda^2(m_K^2, m_\pi^2, s)\kappa^2(m^2, s) \left| C_{qX}^{T,ds} \right|^2 + \frac{s^2}{3072\pi^3 F_0^4 m_K^2 m_\pi^2} \lambda^2(m_K^2, m_\pi^2, s)\kappa^2(m^2, s) \left| C_{qX}^{V,ds} \right|^2

\times \left[ 3(s - 4m^2)(m_K^2 - m_\pi^2)^2 + 4m^2\lambda(m_K^2, m_\pi^2, s) \right] \left| C_{qX}^{V,ds} \right|^2 + \frac{1}{768\pi^3 m_K^2 m_\pi^2} \lambda^2(m_K^2, m_\pi^2, s)\kappa^2(m^2, s)

\times \left[ 6m^2(m_K^2 - m_\pi^2)^2 + (s - 4m^2)\lambda(m_K^2, m_\pi^2, s) \right] \left| C_{qX}^{V,ds} \right|^2 + \frac{s^2 - 4m^2 s + 12m^4}{3072\pi^3 m_K^4 m_\pi^2} \lambda^2(m_K^2, m_\pi^2, s)\kappa^2(m^2, s) \left| C_{qX}^{V,ds} \right|^2 + \frac{s(s + 4m^2)}{3072\pi^3 m_K^4 m_\pi^2} \lambda^2(m_K^2, m_\pi^2, s)\kappa^2(m^2, s) \left| C_{qX}^{V,ds} \right|^2

+ \frac{s + 2m^2}{768\pi^3 m_K^2 m_\pi^2} \lambda^2(m_K^2, m_\pi^2, s)\kappa^2(m^2, s) \left| C_{qX}^{V,ds} \right|^2 + \cdots,
$$

(4.11)
The difference, a factor of $\Gamma_\pi \sim 150$, only two signal regions corresponding to $q \to d,s$ quarks as a function of the DM mass $m$ from $K \to \pi E$ channel.

$$\frac{d\Gamma_{K^+ \to \pi^+ X X}}{dq^2} = \frac{B^2(s^2 - 4m^2s + 6m^4)}{128\pi^3m_K^3} \lambda^2(m_K^2, m_s^2, s) \kappa^2(m_s^2, s) \left| c_{qX1}^{S,ds} \right|^2$$

$$+ \frac{B^2s^2}{128\pi^3m_K^3} \lambda^2(m_K^2, m_s^2, s) \kappa^2(m_s^2, s) \left| c_{qX2}^{S,ds} \right|^2$$

$$+ \frac{\lambda_2^2(s + 2m^2)}{6144\pi^3F_0^4m_K^5} \lambda^2(m_K^2, m_s^2, s) \kappa^2(m_s^2, s) \left| c_{qX1}^{T,ds} \right|^2$$

$$+ \frac{\lambda_2^2(s - 2m^2 + 4m^4)}{6144\pi^3F_0^4m_K^5} \lambda^2(m_K^2, m_s^2, s) \kappa^2(m_s^2, s) \left| c_{qX2}^{T,ds} \right|^2. \quad (4.12)$$

The corresponding results for the neutral kaon mode $K_L \to \pi^0 E$ are obtained from these ones replacing $C_i^{S,ds}$ by their real parts $\Re[C_i^{S,ds}]$ for scalar quark currents and $C_i^{V(T),ds}$ by their imaginary parts $\Im[C_i^{V(T),ds}]$ for vector and tensor quark currents, respectively.

Figure 11 shows the bounds obtained on $\Lambda_{\text{eff}}$ from $K \to \pi \phi \phi$ for scalar DM. Kaon decays only cover the low mass region, $m \lesssim 180 \text{ MeV}$, but the constraints on $\Lambda_{\text{eff}}$ in this region are much stronger than the corresponding ones in $B$ meson decay shown in figure 2.

The difference, a factor of $\mathcal{O}(10^4)$ ($\mathcal{O}(10)$) for scalar (vector) current operators, is due both to the much longer kaon lifetime and the much stronger experimental bounds on kaon modes.

Figure 12 shows the constraints for the vector DM cases: the upper four panels for scenario A and the lower two panels for scenario B. Unlike the case of scalar DM, the constraints for vector DM case from kaon and $B$ meson decays are similar as seen in figures 4, 5, 6, and 9. This can be understood from dimensional arguments: the decay width from vector DM operators scales approximately as $\Gamma \sim m_K^7/B_{\text{eff}}/\Lambda_{\text{eff}}^8(m_K^7/B_{\text{eff}}/\Lambda_{\text{eff}}^8)$ for operators $O_{qX, T_{X1,2}}, O_{V,V_T, S, T, S, T_{X1,2}}$. This large enhancement of $(m_B/m_K)^7$ for $B$ mesons compensates for its shorter lifetime and weaker experimental bounds, resulting in comparable constraints on $\Lambda_{\text{eff}}$ (of course, for different flavor indices).

In practice, the decay-in-flight search of $K^+ \to \pi^+ E$ by NA62 experiment [31, 60] has only two signal regions corresponding to $0 < q^2 < 0.01 \text{ GeV}^2$ (region 1) and $0.026 \text{ GeV}^2 < q^2 < 0.068 \text{ GeV}^2$ (region 2), with both signal regions also being constrained in the pion.
Figure 12. Constraints on the effective new physics scale for the vector DM operators with $d, s$ quarks as a function of the DM mass $m$ from $K \rightarrow \pi \ell \nu$ channel.
Table 3. The strongest bounds on the effective scale $\Lambda_{\text{eff}}$ associated with all FCNC operators with two representative DM masses: $m = 0$ and $m = 2 \text{ GeV}$ (or $m = 150 \text{ MeV}$ for $(ds)$-flavor). For the $(ds)$-flavor, all the constraints in the table come from the charged channel $K^+ \rightarrow \pi^+ \ell^\mp$.

momentum by $15 \text{ GeV} < |p_{\pi}| < 35 \text{ GeV}$ (equivalently, the pion energy by $15 \text{ GeV} < E_{\pi} < 35 \text{ GeV}$). When restricting the phase space to these kinematic windows, the corresponding sensitivity bounds on the parameter space shift from the solid color lines into the solid gray lines in figures 11 and 12. The bound weakens by factors of a few relative to what could be obtained from the entire phase space. The sensitivity drops quickly for heavier DM masses and vanishes around $m \approx 130 \text{ MeV}$, the cutoff value being determined by the largest $q^2$, $q_{\text{max}}^2 \approx 0.068 \text{ GeV}^2$, in the NA62 signal region.

Finally, in table 3, we summarize the strongest constraints on $\Lambda_{\text{eff}}$ for all FCNC interactions with two representative DM masses: $m = 0$ and $m = 2 \text{ GeV}$ (or $m = 150 \text{ MeV}$ for kaon decays). In the second column, we show the scaling behavior of the Wilson coefficients employed to obtain the bounds. One should note that all results we present correspond to the case of complex DM fields. For the case of real DM fields, the operators $O^V_{q\phi}$ in eq. (2.2), $O^T_{qX,1,2}$, $O^V_{qX,4,5,6}$ in eq. (2.3), and $\hat{O}^T_{qX,1,2}$ in eq. (2.4) do not exist. For
the remaining ones, the bounds on \( \Lambda_{\text{eff}} \) will be enhanced by a factor of \( 2^{1/2n} \) with \( n \) being the power of \( \Lambda_{\text{eff}}^{-1} \) in the corresponding Wilson coefficient, shown in the second column of table 3.

5 Summary and conclusions

In this paper we have carried out a systematic study of possible flavor changing neutral current \( B \) and \( K \) meson decays with a pair of light scalar or vector invisible particles in the final state using the effective field theory approach. This completes the existing studies of \( B(K) \to M E \) transitions where the missing energy is attributed to a pair of new invisible particles in the context of effective field theory. The case of two invisible fermions was studied before, and we have now addressed the cases of two invisible scalars or vectors. This study is particularly relevant when new symmetries forbid the appearance of single DM particles.

We first constructed the local quark-DM interactions relevant to these processes in the low energy effective field theory framework at leading order. We tabulated results for invisible scalar, fermion, or vector particles completing and correcting existing lists in the literature. We then used the effective interactions to consider \( B \to (K, \pi, K^*, \rho) + \text{DM} + \text{DM} \) transitions using the form factor formalism, followed by \( K \to \pi + \text{DM} + \text{DM} \) transitions using chiral perturbation theory. We describe the different characteristic features of each operator in the differential decay rate, which could be exploited in future detailed experimental searches to differentiate between various possibilities of new physics. Finally, with the help of the most recent experimental results on these modes, we set constraints on all the relevant effective operators. The sensitivity to the new physics scale strongly depends on the operator structure as well as the DM mass. In the massless limit, for the \( B \) meson decay with scalar (vector) DM, we find that the current experimental data can probe the effective new physics scale up to \( O(10^7)(O(10^3)) \) GeV for some operators, while for the kaon decays \( O(10^{11})(O(10^3)) \) GeV can be reached.

In two cases, \( B^+ \to K^+ E \) and \( K^+ \to \pi^+ E \), we considered the effect that experimental efficiency affects the theoretical constraints. For the former we relied on the current Belle II sensitivity to bins of different \( q^2 \) and for the latter on the signal window in NA62.

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A Differential decay rate

For reference we detail here the kinematics relevant for the modes discussed in this paper. For the three-body decay, \( B(p) \rightarrow M(k) + \text{DM}(k_1) + \text{DM}(k_2) \), there are two independent Mandelstam variables describing the kinematics that are denoted as \( s \equiv (p-k)^2 = (k_1 + k_2)^2 \) and \( t \equiv (p-k_1)^2 = (k_2 + k)^2 \). Then the scalar product of any pair of four vectors can be expressed in terms of the two Mandelstam variables and masses as follows,

\[
p \cdot k_1 = \frac{m_B^2 + m_1^2 - t}{2}, \quad p \cdot k_2 = \frac{s + t - m_1^2 - m_M^2}{2}, \quad p \cdot k = \frac{m_B^2 + m_M^2 - s}{2}, \quad (A.1)\]
\[
k_1 \cdot k_2 = \frac{s - m_1^2 - m_2^2}{2}, \quad k_1 \cdot k = \frac{m_B^2 + m_2^2 - s - t}{2}, \quad k_2 \cdot k = \frac{t - m_2^2 - m_M^2}{2}, \quad (A.2)\]

where \( m_B \) is the initial \( B \) meson mass while \( m_M \) for the mass of final state meson; \( m_1 = m_2 \equiv m \) is the mass of the DM particle. For the process to happen, the largest possible DM mass is restricted by kinematics to be \( m \leq (M_B - m_M)/2 \).

The differential decay width can be expressed as

\[
\frac{d\Gamma}{dq^2} = \frac{1}{S} \frac{1}{256\pi^4 m_B^2} \int_{t_{-}}^{t_{+}} dt |\mathcal{M}|^2, \quad (A.3)
\]

where \( S \) is a possible symmetry factor for identical DM particles at the final state, and \( |\mathcal{M}|^2 \) is the spin-averaged squared matrix element for the relevant decay process. The integration domain for \( t \) is

\[
t_{\pm} = (E_2^s + E_3^s)^2 - \left( \sqrt{E_2^s - m_2} + \sqrt{E_3^s - m_M^2} \right)^2, \quad E_2^s = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}}, \quad E_3^s = \frac{m_B^2 - s - m_M^2}{2\sqrt{s}}. \quad (A.4)
\]

To extract the constraints we take one operator at a time, therefore ignoring interference between different operators. As the DM are not SM particles, there is never interference with the SM. If these interactions exist, they thus contribute additively to the SM and can be directly constrained by the “room for new physics” of the last column in table 2. The branching ratio can be written as a sum of numerical coefficients times the squares of the couplings of the new operators, schematically

\[
B_{\text{NP}} = \frac{1}{\Gamma_B} \int dq^2 \frac{d\Gamma}{dq^2} = \sum_i \hat{B}_i(m) |C_i|^2, \quad (A.5)
\]

where the generic range of \( q^2 \) goes from \((m_1 + m_2)^2\) to \((m_B - m_M)^2\).

We allow for two modifications to the integration range to accommodate reported details of existing experiments. For \( B^+ \rightarrow K^+ \pi^- \) we take into account the experimental efficiency as a function of \( q^2 \) reported in figure 3 (supplementary material) of Belle II [54], with the details of our analysis being given at the end of subsection 3.2. For \( K^+ \rightarrow \pi^+ \pi^- \), we limit the integration region to the two signal regions of NA62 [31, 60]

\[
15 \leq |p_\pi| \leq 35 \text{ GeV} \quad \text{for the pion momentum in the NA62 rest frame},
\]
\[
0 \leq q^2 \leq 0.1 \text{ GeV}^2 \quad \text{or} \quad 0.026 \leq q^2 \leq 0.068 \text{ GeV}^2 \quad \text{for the two signal regions}. \quad (A.6)
\]

In this case we work on the NA62 lab frame, with the kaon momentum \( |p_K| = 75 \text{ GeV} \). 

\[\text{JHEP03(2023)037}\]
Requiring that the NP contribution of each operator to the branching ratio does not exceed the value $B_{UL}$ given in table 2, we set the constraints

$$|C_i|^2 \leq \frac{B_{UL}}{B_i(m)}. \quad (A.7)$$

To interpret this as a bound on new physics we then write $C_i \equiv \Lambda_{\text{eff}}^{-n}$ (with the power $n$ depending on the dimension of the corresponding operator), leading to

$$\Lambda_{\text{eff}}(m) \geq \left( \frac{\hat{B}_i(m)}{B_{UL}} \right)^{\frac{1}{2n}}. \quad (A.8)$$

### B Detailed analysis for the operators with a vector DM

Here we describe in detail how to obtain the operators with a vector DM in eq.(2.3) for scenario A. First, we can always choose the operators to be self-conjugate for the flavor diagonal case as given in eq.(2.3), and we denote the Hermitian and anti-Hermitian combination of vector DM fields as,

$$S_{\mu\nu} \equiv X_{\mu}^\dagger X_{\nu} + X_{\nu}^\dagger X_{\mu}$$

and

$$A_{\mu\nu} \equiv X_{\mu}^\dagger X_{\nu} - X_{\nu}^\dagger X_{\mu}.$$  

It can be seen that they are automatically symmetric and anti-symmetric in their two Lorentz indices, respectively. For a combination of the vector quark current $q\gamma_{\mu}q$ with two DM fields $X_{a}^\dagger X_{b}$ to form dim-6 operators by attaching an additional derivative, we have the following possibilities

$$\begin{align*}
(q\gamma_{\mu}q)(X_{a}^\dagger X_{b}^\dagger i\partial_{\mu}X_{\nu}) & \Rightarrow (q\gamma_{\mu}q)X_{\nu}^\dagger i\partial_{\mu}X_{a}^\dagger X_{b}, \\
(q\gamma_{\mu}q)S_{\mu\nu} i\partial_{\nu} & \Rightarrow (q\gamma_{\mu}q)\partial_{\nu}S_{\mu\nu}, \quad (q\gamma_{\mu}q)i\partial_{\nu}S_{\mu\nu}, \quad (q\gamma_{\mu}q)i\partial_{\nu}A_{\mu\nu}, \quad (q\gamma_{\mu}q)i\partial_{\nu}A_{\mu\nu}, \quad \text{(B.1b)} \\
(q\gamma_{\mu}q)A_{\rho\sigma} i\partial_{\nu} & \Rightarrow (q\gamma_{\mu}q)\partial_{\nu}A_{\rho\sigma}, \quad (q\gamma_{\mu}q)A_{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}, \quad (q\gamma_{\mu}q)\partial_{\nu}A_{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}, \quad (q\gamma_{\mu}q)\partial_{\nu}A_{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}. \quad \text{(B.1d)}
\end{align*}$$

where we have used IBP and the on-shell condition $\partial_{\mu}X_{\mu} = 0$. For the quark axial-vector current $q\gamma_{\mu}\gamma_{5}q$, a similar operator realization can be obtained by replacing the vector gamma matrix $\gamma_{\mu}$ in the quark current by $\gamma_{\mu}\gamma_{5}$.

Above, we have written 8 possible operators. However, the two operators in a “□” are redundant and they can be transformed into others appearing in our basis given in eq. (2.3).

Using the Dirac gamma matrix identities (DIs),

$$\gamma_{\mu}\gamma_{\nu} = g_{\mu\nu} - i\sigma_{\mu\nu}, \quad \gamma_{\mu}\gamma_{\nu} \gamma_{\rho} = g_{\mu\nu}\gamma_{\rho} + g_{\nu\rho}\gamma_{\mu} - g_{\mu\rho}\gamma_{\nu} + i\epsilon_{\mu\nu\rho\sigma}\gamma_{\sigma}\gamma_{5}, \quad \sigma_{\mu\nu}\gamma_{5} = \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}\sigma_{\rho\sigma}. \quad (B.2)$$
the two operators can be manipulated as follows,

\[
2(\bar{q}_a \gamma_\mu \vec{D}_\nu q_b) A^{\mu\nu} = \left[ \bar{q}_a (\gamma_\mu \gamma_\nu \partial - i \gamma_\mu \gamma_\nu \partial) q_b + \bar{q}_a (\gamma_\mu \gamma_\nu \partial - i \gamma_\mu \gamma_\nu \partial) q_b \right] A^{\mu\nu} \\
= - \left[ \bar{q}_a (\sigma_{\mu\nu} i \partial + i \partial_{\mu\nu}) q_b + \bar{q}_a (\gamma_\mu \gamma_\nu \partial + \gamma_\mu \gamma_\nu \partial) q_b \right] A^{\mu\nu} \\
= (m_a - m_b) (\bar{q}_a \sigma_{\mu\nu} q_b) A^{\mu\nu} - \partial^{\alpha} (\bar{q}_a \epsilon_{\mu\nu\alpha\beta}) \gamma_5 q_b) A^{\mu\nu} \\
= (m_a - m_b) (\bar{q}_a \sigma_{\mu\nu} q_b) A^{\mu\nu} + (\bar{q}_a \gamma_\mu \gamma_5 q_b) i \partial^{\alpha} A^{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}, \quad \text{(B.3a)}
\]

In the reduction of the second operator, we used the identity \( \epsilon^{\mu\nu\alpha\beta} \epsilon_{\mu\nu\rho\sigma} = -2(\partial_\rho \delta_\sigma - \delta_\rho \partial_\sigma) \). Since the final 4 operators are already in our basis, we conclude that the two operators in a “\( \Box \)” are redundant. Similarly, for the axial-vector quark current, the two corresponding operators are redundant and can be transformed into those in our basis as follows,

\[
2(\bar{q}_a \gamma_\mu \vec{D}_\nu \gamma_5 q_b) A^{\mu\nu} = (m_a + m_b) (\bar{q}_a \sigma_{\mu\nu} \gamma_5 q_b) A^{\mu\nu} + (\bar{q}_a \gamma_\mu \gamma_5 q_b) i \partial^{\alpha} A^{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}, \quad \text{(B.4a)}
\]

\[
(\bar{q}_a \gamma_\mu \vec{D}_\nu \gamma_5 q_b) A^{\rho\sigma} \epsilon_{\mu\nu\rho\sigma} = -i (m_a + m_b) (\bar{q}_a \sigma_{\mu\nu} q_b) A^{\mu\nu} - 2(\bar{q}_a \gamma_\mu q_b) i \partial \nu A^{\mu\nu}. \quad \text{(B.4b)}
\]

In conclusion, there are 6 independent operators for each quark current and they are listed in eq. (2.3) after normalization and (anti-)symmetrization.

When using the Basisgen package [61], we only find 4 dim-6 operators containing a derivative, the remaining 8 operators are missing. The reason for this undercounting is an oversimplified treatment of the equation of motion of the vector field in that package.\(^\text{12}\)

Since the partial derivative \( \partial_\mu \) and the vector field \( X_\nu \) both belong to the vector representation \( (1/2, 1/2) \) of the Lorentz algebra \( (su(2), su(2)) \), the general irreducible representation decomposition of their product \( \partial_\mu X_\nu \) under Lorentz algebra is (for example, see eq. (34.31) in [62])

\[
\left( \begin{array}{c} 1/2 \ \ 1/2 \\ -1/2 \ -1/2 \end{array} \right) \otimes \left( \begin{array}{c} 1 \ \ 1 \\ 1 \ -1 \end{array} \right) = \{(0, 0)_S \oplus (1, 1)_S \oplus (1, 0)_A \oplus (0, 1)_A, \quad \text{(B.5)}
\]

or in terms of fields and derivatives,

\[
\partial_\mu X_\nu = \frac{1}{4} g_{\mu\nu} \partial_\alpha X^\alpha + \frac{1}{2} \left( \partial_\mu X_\nu + \partial_\nu X_\mu - \frac{1}{2} g_{\mu\nu} \partial_\alpha X^\alpha \right) + \frac{1}{4} X_\mu^+ + \frac{1}{4} X_\mu^-.
\text{(B.6)}
\]

\(^{12}\)We thank J. C. Criado for confirming this.
as implemented in its python script. The last two components \((1, 0)_A\) (self-dual 2-form field) and \((0, 1)_A\) (anti-self-dual 2-form field), are missing in the package and lead to the difference. After including these two components, using the method outlined in section 2.2 in [61], we indeed obtain the same total number of operators, i.e., 12.

C Lepton-DM interaction in LEFT

For completeness we list here the independent operators involving a lepton current and two dark sector particles. Denoting the charged leptons as \(\ell \in \{e, \mu, \tau\}\), the charged lepton-DM interactions can be directly obtained from the quark-DM interactions given in section 2 by exchanging the quark flavor label \(q\) by the lepton label \(\ell\). Following the conventions for quark-DM interactions in section 2, they are:

Fermion case:

\[
\begin{align*}
\mathcal{O}^{S}_{\ell X 1} &= (\overline{\ell} \ell) (\overline{\chi} \chi), \\
\mathcal{O}^{S}_{\ell X 2} &= (\overline{\ell} \ell) (\overline{\chi} i \gamma_5 \chi), \\
\mathcal{O}^{P}_{\ell X 1} &= (\overline{\ell} i \gamma_5 \ell) (\overline{\chi} \chi), \\
\mathcal{O}^{P}_{\ell X 2} &= (\overline{\ell} \gamma_5 \ell) (\overline{\chi} \chi), \\
\mathcal{O}^{V}_{\ell X 1} &= (\overline{\ell} \gamma_{\mu} \ell) (\overline{\chi} \gamma_{\mu} \chi), (\times) \\
\mathcal{O}^{V}_{\ell X 2} &= (\overline{\ell} \gamma_{\mu} \ell) (\overline{\chi} \gamma_{\mu} \gamma_5 \chi), (\times) \\
\mathcal{O}^{A}_{\ell X 1} &= (\overline{\ell} \gamma_{\mu} \gamma_5 \ell) (\overline{\chi} \gamma_{\mu} \chi), (\times) \\
\mathcal{O}^{A}_{\ell X 2} &= (\overline{\ell} \gamma_{\mu} \gamma_5 \ell) (\overline{\chi} \gamma_{\mu} \gamma_5 \chi), (\times)
\end{align*}
\]

Scalar case:

\[
\begin{align*}
\mathcal{O}^{S}_{\ell \phi} &= (\overline{\ell} \ell) (\phi^\dagger \phi), \\
\mathcal{O}^{P}_{\ell \phi} &= (\overline{\ell} i \gamma_5 \ell) (\phi^\dagger \phi), \\
\mathcal{O}^{V}_{\ell \phi} &= (\overline{\ell} \gamma_{\mu} \ell) (\phi^\dagger \gamma_{\mu} \phi), (\times) \\
\mathcal{O}^{A}_{\ell \phi} &= (\overline{\ell} \gamma_{\mu} \gamma_5 \ell) (\phi^\dagger \gamma_{\mu} \phi), (\times).
\end{align*}
\]

Vector case A:

\[
\begin{align*}
\mathcal{O}^{A}_{\ell X} &= (\overline{\ell} \ell) (X^\dagger_{\mu} X^\mu), \\
\mathcal{O}^{P}_{\ell X} &= (\overline{\ell} i \gamma_5 \ell) (X^\dagger_{\mu} X^\mu), \\
\mathcal{O}^{V}_{\ell X 1} &= \frac{i}{2} (\overline{\ell} \sigma^\mu_{\nu} \ell) (X^\dagger_{\mu} X^\nu - X^\dagger_{\nu} X^\mu), (\times) \\
\mathcal{O}^{V}_{\ell X 2} &= \frac{i}{2} (\overline{\ell} \sigma^\mu_{\nu} \gamma_5 \ell) (X^\dagger_{\mu} X^\nu - X^\dagger_{\nu} X^\mu), (\times) \\
\mathcal{O}^{V}_{\ell X 3} &= \frac{1}{2} \overline{\ell} (\gamma_{\mu} i D^\nu \ell) (X^\dagger_{\mu} X^\nu + X^\dagger_{\nu} X^\mu), \\
\mathcal{O}^{V}_{\ell X 4} &= (\overline{\ell} \gamma_{\mu} \ell) (X^\dagger_{\mu} X^\mu), \\
\mathcal{O}^{V}_{\ell X 5} &= (\overline{\ell} \gamma_{\mu} \ell) (X^\dagger_{\mu} X^\mu), (\times) \\
\mathcal{O}^{V}_{\ell X 6} &= (\overline{\ell} \gamma_{\mu} \ell) (X^\dagger_{\mu} X^\mu), (\times)
\end{align*}
\]
Table 4. Parameters appearing in the form factors of the $B \to K(\pi)$ transitions [48]. $f_T$ is a scale-dependent quantity and the value is given at $\mu = 4.8$ GeV.

| Form factor | $r_1$   | $r_2$   | $m^2_{fit}$ (GeV$^2$) | $m_R$ (GeV) |
|-------------|---------|---------|-----------------------|------------|
| $f_T^+$     | 0.744   | -0.486  | 40.73                 | 5.32       |
| $f_T^0$     | 0       | 0.258   | 33.81                 | —          |
| $f_T^-$     | 1.387   | -1.134  | 32.22                 | 5.32       |
| $f_T^K$     | 0.162   | 0.173   | —                     | 5.41       |
| $f_T^K$     | 0       | 0.330   | 37.46                 | —          |
| $f_T^K$     | 0.161   | 0.198   | —                     | 5.41       |

In addition to the usually considered electron-DM scattering process for direct DM detection, the above lepton-DM interactions can induce lepton flavor violating transitions, $\ell_j \to \ell_i + \text{DM} + \text{DM}$. The charged leptons can be replaced by SM neutrinos leading to exotic interactions between neutrinos and DM particles. We defer a study of these possibilities to a future publication.

D Form factors for the $B \to P(V)$ transitions

In tables 4 and 5 we collect the fitted parameters for the form factor parameterizations with $q^2 \neq 0$ as given in eq. (3.2) and eq. (3.9) relevant to $B \to P$ and $B \to V$ transitions respectively.
Table 5. Parameters in the form factors of the $B \to \rho(K^*)$ processes with $k_{\text{max}} = 2$ [49].

E Specific renormalizable models to illustrate a possible origin of the LEFT operators

Here we illustrate with two simple renormalizable models a possible origin for the LEFT operators. First for the case of scalar DM generating the operators with a scalar mediator. Then for the case A of vector DM illustrating a possible origin for the additional mass factors that we argued should accompany the operators in eq. (2.3) when $X_\mu$ is regarded as a dark sector gauge boson.

In these examples we will employ three dark sector fields: a light real scalar $\phi$ and a real vector gauge particle $X_\mu$ from a U(1)$_X$ gauge group, and a second scalar $\Delta$ that gives mass to $X$. These three particles are singlets under the SM gauge group. To generate the FCNC in the quark sector we introduce two Higgs doublets $H_1$ and $H_2$. Under the full gauge group SU(3)$_c \times$ SU(2)$_L \times U(1)_Y(U_X(1))$, these new scalars have the following charge assignments,

$$H_i(1,2,1/2)(0) = \left( \frac{h_i^+}{\sqrt{v_i + h_i^+ i I_i}} \right), \quad \Delta(1,1,0)(1) = \frac{v_\Delta + h_\Delta + i I_\Delta}{\sqrt{2}}. \quad (E.1)$$
Scalar case: the relevant Lagrangian is given by

\[ \mathcal{L}_{\text{scalar}} \ni \lambda_{ij}^{H \phi} H_i^\dagger H_j \phi + \left( \bar{Q}_L Y_1 H_1 D_R + \bar{Q}_L Y_2 H_2 D_R + \text{h.c.} \right), \]  

(E.2)

where \( Q_L \) and \( D_R \) are the SM left-handed quark doublet and right-handed down-type quark singlet, respectively. After spontaneous symmetry breaking and rotating back to the physical states, \( h_1 \) and \( h_2 \) will mix giving rise to a SM Higgs \( (h_1^m) \) and a heavy Higgs \( (h_2^m) \), \( h_i = \alpha_{ij} h_j^m \). Integrating out the physical Higgs bosons, we generate the operators \( O_{q\phi}^S \) and \( O_{q\phi}^P \).

Vector case: the relevant interactions are given by

\[ \mathcal{L}_{\text{vector}} \ni (D^\mu \Delta)^\dagger (D_\mu \Delta) + \lambda_{ij}^{H \Delta} H_i^\dagger H_j \Delta + \left( \bar{Q}_L Y_1 H_1 D_R + \bar{Q}_L Y_2 H_2 D_R + \text{h.c.} \right), \]  

(E.3)

where \( D_\mu = \partial_\mu - ig X_\mu \). After \( \Delta \) develops a vev, the \( \Delta \) kinetic term will induce a mass for \( X \) given by \( m = g_X v_\Delta \) and a \( \Delta-X-X \) vertex with a coupling \( g_X^2 v_\Delta \). The Higgs quartic interaction leads to a mixing of \( h_\Delta \) and \( h_i \) with a mixing parameter \( \frac{1}{2} \lambda^{H \Delta} v_\Delta \). Then integrating out \( h_\Delta \) and \( h_i \) we can obtain the operator \( O_{qX}^S \) and \( O_{qX}^P \), with coefficients proportional to \( m^2 \).

One should note that if only one Higgs doublet is introduced, the diagonalization of the mass matrix also diagonalizes the Yukawa matrix \( Y \) and there are no tree-level FCNC interactions. Introducing two Higgs doublets, where \( Y_1 \) and \( Y_2 \) cannot be diagonalized simultaneously, generates tree-level FCNC interactions. Since exchange of one of these Higgs bosons would generate undesirable FCNC interactions purely within the quark sector, this simple model should be considered as an existence proof. A realistic model would have to introduce many of the features that resolve this issue in two Higgs doublet models [63].

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References

[1] S.L. Glashow, J. Iliopoulos and L. Maiani, *Weak Interactions with Lepton-Hadron Symmetry*, Phys. Rev. D 2 (1970) 1285 [inSPIRE].

[2] L. Roszkowski, E.M. Sessolo and S. Trojanowski, *WIMP dark matter candidates and searches — current status and future prospects*, Rept. Prog. Phys. 81 (2018) 066201 [arXiv:1707.06277] [inSPIRE].

[3] S. Bottaro et al., *Closing the window on WIMP Dark Matter*, Eur. Phys. J. C 82 (2022) 31 [arXiv:2107.09688] [inSPIRE].

[4] S. Weinberg, *A New Light Boson?*, Phys. Rev. Lett. 40 (1978) 223 [inSPIRE].

[5] F. Wilczek, *Problem of Strong P and T Invariance in the Presence of Instantons*, Phys. Rev. Lett. 40 (1978) 279 [inSPIRE].
[6] B. Dasgupta and J. Kopp, Sterile Neutrinos, Phys. Rept. 928 (2021) 1 [arXiv:2106.05913] [inSPIRE].

[7] M. Fabbrichesi, E. Gabrielli and G. Lanfranchi, The Dark Photon, arXiv:2005.01515 [DOI:10.1007/978-3-030-62519-1] [inSPIRE].

[8] C.O. Dib et al., Probing R-parity violation in B-meson decays to a baryon and a light neutralino, JHEP 02 (2023) 224 [arXiv:2208.06421] [inSPIRE].

[9] J.F. Kamenik and C. Smith, FCNC portals to the dark sector, JHEP 03 (2012) 090 [arXiv:1111.6402] [inSPIRE].

[10] G. Li, T. Wang, J.-B. Zhang and G.-L. Wang, The light invisible boson in FCNC decays of B and B_s mesons, Eur. Phys. J. C 81 (2021) 564 [arXiv:2103.12921] [inSPIRE].

[11] X.-G. He, X.-D. Ma, J. Tandean and G. Valencia, Eviding the Grossman-Nir bound with $\Delta I = 3/2$ new physics, JHEP 08 (2020) 034 [arXiv:2005.02942] [inSPIRE].

[12] W. Altmannshofer, A.J. Buras, D.M. Straub and M. Wick, New strategies for New Physics search in $B \rightarrow K^{*}\nu\bar{\nu}$, $B \rightarrow K\nu\bar{\nu}$ and $B \rightarrow X_s\nu\bar{\nu}$ decays, JHEP 04 (2009) 022 [arXiv:0902.0160] [inSPIRE].

[13] G. Li et al., Spin-1/2 invisible particles in heavy meson decays, Phys. Rev. D 102 (2020) 095019 [arXiv:2004.10942] [inSPIRE].

[14] T. Felkl, S.L. Li and M.A. Schmidt, A tale of invisibility: constraints on new physics in $b \rightarrow s\nu\bar{\nu}$, JHEP 12 (2021) 118 [arXiv:2111.04327] [inSPIRE].

[15] C. Bird, P. Jackson, R.V. Kowalewski and M. Pospelov, Search for dark matter in $b \rightarrow s$ transitions with missing energy, Phys. Rev. Lett. 93 (2004) 201803 [hep-ph/0401195] [SPIRE].

[16] J. Tandean, Rare hyperon decays with missing energy, JHEP 04 (2019) 104 [arXiv:1901.10447] [inSPIRE].

[17] J.-Y. Su and J. Tandean, Exploring leptoquark effects in hyperon and kaon decays with missing energy, Phys. Rev. D 102 (2020) 075032 [arXiv:1912.13507] [inSPIRE].

[18] T. Li, X.-D. Ma and M.A. Schmidt, Implication of $K \rightarrow \pi\nu\bar{\nu}$ for generic neutrino interactions in effective field theories, Phys. Rev. D 101 (2020) 055019 [arXiv:1912.10433] [inSPIRE].

[19] F.F. Deppisch, K. Fridell and J. Harz, Constraining lepton number violating interactions in rare kaon decays, JHEP 12 (2020) 186 [arXiv:2009.04494] [inSPIRE].

[20] X.G. He and G. Valencia, $R_{K^{(*)}}$ and non-standard neutrino interactions, Phys. Lett. B 821 (2021) 136607 [arXiv:2108.05033] [inSPIRE].

[21] G. Li, J.-Y. Su and J. Tandean, Flavor-changing hyperon decays with light invisible bosons, Phys. Rev. D 100 (2019) 075003 [arXiv:1905.08759] [inSPIRE].

[22] C.-Q. Geng and J. Tandean, Probing new physics with the kaon decays $K \rightarrow \pi\pi\bar{\nu}\bar{\nu}$, Phys. Rev. D 102 (2020) 115021 [arXiv:2009.00608] [inSPIRE].

[23] Particle Data Group collaboration, Review of Particle Physics, PTEP 2022 (2022) 083C01 [inSPIRE].

[24] Belle collaboration, Search for $B \rightarrow h\nu\bar{\nu}$ decays with semileptonic tagging at Belle, Phys. Rev. D 96 (2017) 091101 [Addendum ibid. 97 (2018) 099902] [arXiv:1702.03224] [inSPIRE].

[25] Belle collaboration, Search for $B \rightarrow h^{(*)}\nu\bar{\nu}$ with the full Belle $\Upsilon(4S)$ data sample, Phys. Rev. D 87 (2013) 111103 [arXiv:1303.3719] [inSPIRE].
[26] C. Hambrock, A. Khodjamirian and A. Rusov, *Hadronic effects and observables in $B \to \pi\ell^+\ell^-$ decay at large recoil*, *Phys. Rev. D* **92** (2015) 074020 [arXiv:1506.07760] [inSPIRE].

[27] D.M. Straub, *flavio: a Python package for flavour and precision phenomenology in the Standard Model and beyond*, arXiv:1810.05132 [inSPIRE].

[28] A.J. Buras, D. Buttazzo, J. Girrbach-Noe and R. Knegjens, $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ in the Standard Model: status and perspectives, *JHEP* **11** (2015) 033 [arXiv:1503.02693] [inSPIRE].

[29] E949 collaboration, *New measurement of the $K^+ \to \pi^+\nu\bar{\nu}$ branching ratio*, *Phys. Rev. Lett.* **101** (2008) 191802 [arXiv:0808.2459] [inSPIRE].

[30] BNL-E949 collaboration, *Study of the decay $K^+ \to \pi^+\nu\bar{\nu}$ in the momentum region $140 < P_\pi < 199$ MeV/c*, *Phys. Rev. D* **79** (2009) 092004 [arXiv:0903.0030] [inSPIRE].

[31] NA62 collaboration, *An investigation of the very rare $K^+ \to \pi^+\nu\bar{\nu}$ decay*, *JHEP* **11** (2020) 042 [arXiv:2007.08218] [inSPIRE].

[32] NA62 collaboration, *Measurement of the very rare $K^+ \to \pi^+\nu\bar{\nu}$ decay*, *JHEP* **06** (2021) 093 [arXiv:2103.15389] [inSPIRE].

[33] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, *JHEP* **10** (2010) 085 [arXiv:1008.4884] [inSPIRE].

[34] L. Lehman, *Extending the Standard Model Effective Field Theory with the Complete Set of Dimension-7 Operators*, *Phys. Rev. D* **90** (2014) 125023 [arXiv:1410.4193] [inSPIRE].

[35] Y. Liao and X.-D. Ma, *Renormalization Group Evolution of Dimension-seven Baryon- and Lepton-number-violating Operators*, *JHEP* **11** (2016) 043 [arXiv:1607.07309] [inSPIRE].

[36] E.E. Jenkins, A.V. Manohar and P. Stoffer, *Low-Energy Effective Field Theory below the Electroweak Scale: Operators and Matching*, *JHEP* **03** (2018) 016 [arXiv:1709.04486] [inSPIRE].

[37] Y. Liao, X.-D. Ma and Q.-Y. Wang, *Extending low energy effective field theory with a complete set of dimension-7 operators*, *JHEP* **08** (2020) 162 [arXiv:2005.08013] [inSPIRE].

[38] B.V. Lehmann and S. Profumo, *Cosmology and prospects for sub-MeV dark matter in electron recoil experiments*, *Phys. Rev. D* **102** (2020) 023038 [arXiv:2002.07809] [inSPIRE].

[39] A. Badin and A.A. Petrov, *Searching for light Dark Matter in heavy meson decays*, *Phys. Rev. D* **82** (2010) 034005 [arXiv:1005.1277] [inSPIRE].

[40] J. Kumar and D. Marfatia, *Matrix element analyses of dark matter scattering and annihilation*, *Phys. Rev. D* **88** (2013) 014035 [arXiv:1305.1611] [inSPIRE].

[41] B. Henning, X. Lu, T. Melia and H. Murayama, *Operator bases, $S$-matrices, and their partition functions*, *JHEP* **10** (2017) 199 [arXiv:1706.08520] [inSPIRE].

[42] M. Williams, C.P. Burgess, A. Maharana and F. Quevedo, *New Constraints (and Motivations) for Abelian Gauge Bosons in the MeV-TeV Mass Range*, *JHEP* **08** (2011) 106 [arXiv:1103.4556] [inSPIRE].

[43] J. Brod, A. Gootjes-Dreesbach, M. Tammaro and J. Zupan, *Effective Field Theory for Dark Matter Direct Detection up to Dimension Seven*, *JHEP* **10** (2018) 065 [arXiv:1710.10218] [inSPIRE].

[44] J.C. Criado, A. Djouadi, M. Perez-Victoria and J. Santiago, *A complete effective field theory for dark matter*, *JHEP* **07** (2021) 081 [arXiv:2104.14443] [inSPIRE].
[45] C. Arina, J. Hajer and P. Klose, Portal Effective Theories. A framework for the model independent description of light hidden sector interactions, JHEP 09 (2021) 063 [arXiv:2105.06477] [inSPIRE].

[46] J. Aebischer, W. Altmannshofer, E.E. Jenkins and A.V. Manohar, Dark matter effective field theory and an application to vector dark matter, JHEP 06 (2022) 086 [arXiv:2202.06968] [inSPIRE].

[47] N. Gubernari, A. Kokulu and D. van Dyk, B → P and B → V Form Factors from B-Meson Light-Cone Sum Rules beyond Leading Twist, JHEP 01 (2019) 150 [arXiv:1811.00983] [inSPIRE].

[48] P. Ball and R. Zwicky, New results on B → π, K, η decay formfactors from light-cone sum rules, Phys. Rev. D 71 (2005) 014015 [hep-ph/0406232] [inSPIRE].

[49] A. Bharucha, D.M. Straub and R. Zwicky, B → Vℓ+ℓ− in the Standard Model from light-cone sum rules, JHEP 08 (2016) 098 [arXiv:1503.05534] [inSPIRE].

[50] C.-D. Lü, Y.-L. Shen, Y.-M. Wang and Y.-B. Wei, QCD calculations of B → π,K form factors with higher-twist corrections, JHEP 01 (2019) 024 [arXiv:1810.00983] [inSPIRE].

[51] J. Gao et al., Precision calculations of B → V form factors from soft-collinear effective theory sum rules on the light-cone, Phys. Rev. D 101 (2020) 074035 [arXiv:1907.11092] [inSPIRE].

[52] HPQCD collaboration, B → K and D → K form factors from fully relativistic lattice QCD, Phys. Rev. D 107 (2023) 014510 [arXiv:2207.12468] [inSPIRE].

[53] V. Shtabovenko, R. Mertig and F. Orellana, New Developments in FeynCalc 9.0, Comput. Phys. Commun. 207 (2016) 432 [arXiv:1601.01167] [inSPIRE].

[54] Belle II collaboration, Search for B+ → K+νν̅ Decays Using an Inclusive Tagging Method at Belle II, Phys. Rev. Lett. 127 (2021) 181802 [arXiv:2104.12624] [inSPIRE].

[55] J. Gasser and H. Leutwyler, Chiral Perturbation Theory to One Loop, Annals Phys. 158 (1984) 142 [inSPIRE].

[56] J. Gasser and H. Leutwyler, Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark, Nucl. Phys. B 250 (1985) 465 [inSPIRE].

[57] O. Cata and V. Mateu, Chiral perturbation theory with tensor sources, JHEP 09 (2007) 078 [arXiv:0705.2948] [inSPIRE].

[58] G. Colangelo and S. Durr, The Pion mass in finite volume, Eur. Phys. J. C 33 (2004) 543 [hep-lat/0311023] [inSPIRE].

[59] S.-Z. Jiang, Q. Wang and Y. Zhang, Computation of the p\textsuperscript{6} order low-energy constants with tensor sources, Phys. Rev. D 87 (2013) 094014 [arXiv:1203.0712] [inSPIRE].

[60] NA62 collaboration, First search for K+ → π+νν̅ using the decay-in-flight technique, Phys. Lett. B 791 (2019) 156 [arXiv:1811.08508] [inSPIRE].

[61] J.C. Criado, BasisGen: automatic generation of operator bases, Eur. Phys. J. C 79 (2019) 256 [arXiv:1901.03501] [inSPIRE].

[62] M. Srednicki, Quantum field theory, Cambridge University Press (2007) [ISBN: 9780521864497, 9780511267208].

[63] A.L. Foguel, G.M. Salla and R.Z. Funchal, (In)Visible signatures of the minimal dark abelian gauge sector, JHEP 12 (2022) 063 [arXiv:2209.03383] [inSPIRE].