Gyroscope with two-dimensional optomechanical mirror

Sankar Davuluri\textsuperscript{1,4}, Kai Li\textsuperscript{1} and Yong Li\textsuperscript{1,2,3}

\textsuperscript{1} Beijing Computational Science Research Center, Beijing 100193, People’s Republic of China
\textsuperscript{2} Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei 230026, People’s Republic of China
\textsuperscript{3} Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha 410081, People’s Republic of China
\textsuperscript{4} Author to whom any correspondence should be addressed.

E-mail: sd3964@csrc.ac.cn

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Abstract

We propose an application of two-dimensional optomechanical oscillator as a gyroscope by detecting the Coriolis force which is modulated at the natural frequency of the optomechanical oscillator. Dependence of gyroscope’s sensitivity on shot noise, back-action noise, thermal noise, and input laser power is studied. At optimal input laser power, the gyroscope’s sensitivity can be improved by increasing the mass or by decreasing the temperature and decay rate of the mechanical oscillator. When the mechanical oscillator’s thermal occupation number, $n_{\text{th}}$, is zero, sensitivity improves with decrease in frequency of the mechanical oscillator. For $n_{\text{th}} \gg 1$, the sensitivity is independent of the mechanical oscillator’s frequency.

1. Introduction

Detection of absolute rotation \cite{1, 2} has significant importance in fundamental physics for testing gravitation theories \cite{3, 4} and in practical applications for improving navigation systems \cite{5, 6}. The Sagnac effect \cite{7–11} and detection of rotation induced pseudo forces \cite{12, 13} are the two most commonly used methods for absolute rotation detection. Pseudo forces that arise due to rotation are centrifugal force, Coriolis force, and force due to angular acceleration. The angular acceleration force disappears in a system rotating with constant angular velocity. Estimating centrifugal force requires additional procedures to locate the axis of rotation \cite{14}. Hence, Coriolis force \cite{15} measurement is more suitable for absolute rotation detection. In this work, we propose an optomechanical gyroscope by detecting the Coriolis force which is modulated at the natural frequency of the optomechanical oscillator.

Optomechanical cavities \cite{16, 17} are very sensitive to any external forces acting on them, and hence they were extensively studied for gravitational wave detection \cite{18, 19}, weak force detection \cite{20–25}, and displacement sensors \cite{20, 26, 27}. In general, the freely oscillating mirror of the optomechanical cavity (OMC) is subject to the radiation pressure force \cite{16, 17, 28} of the intra-cavity field. This radiation pressure force changes the length of the cavity and thus the optical response of the cavity itself. In this work, we consider a two dimensional optomechanical mirror \cite{29}, which is free to oscillate along $x$-axis and $y$-axis. Such an oscillator can be realized by connecting the one-dimensional oscillator, which oscillates along the $x$-axis, to an electrical circuit. The electrical circuit oscillates the mirror along the direction perpendicular to $x$-axis. The two dimensional oscillator is placed on a rotating table and driven by a laser field along the $x$-axis. When the table rotates, oscillator is driven by radiation pressure force and Coriolis force. The frequency of Coriolis force acting on the mirror is equal to the frequency of optomechanical mirror’s oscillation along the $y$-axis.

2. Optomechanical gyroscope

Figure 1 shows the schematics of the two-dimensional optomechanical gyroscope. The two-dimensional optomechanical mirror has mass $m$ and angular frequency $\omega_{\text{m}}$ along $x$-axis and $y$-axis. The OMC, which is placed...
in arm 2 of the interferometer, is driven by an optical field \( \hat{E} \). Arm-2 is parallel to the x-axis of the rotating table. When the table is not rotating, the equation of motion of the optomechanical mirror oscillating along the x-axis is given as

\[
\hat{b}_x = \left( -i \omega_m - \frac{\gamma_m}{2} \right) \hat{b}_x - \frac{i}{\chi_p} \hat{c} \hat{c}^\dagger + \hat{F},
\]

where \( \gamma_m \), \( \hat{b}_x \), and \( \hat{F} \) are the decay rate of the mechanical oscillator, cavity field annihilation operator, optomechanical mirror annihilation operator, and thermal noise operator, respectively. \( \chi_p = \sqrt{2 h m \omega_m} \), and \( \omega_m = -h \omega_c / l \) with \( h \) as Planck constant, \( l \) as the length of OMC and \( \omega_c \) is the eigen frequency of the cavity. The OMC is mounted on a stage which is driven co-sinusoidally so that the instantaneous velocity of the mechanical mirror is

\[
\text{When the table starts to rotate with angular velocity } \dot{\theta}, \text{ then fictitious forces begin to act on the optomechanical mirror. In presence of the fictitious forces, equation (1) is given as [15]}
\[
\hat{b}_x = \left( -i \omega_m - \frac{\gamma_m}{2} \right) \hat{b}_x - \frac{i}{\chi_p} \hat{c} \hat{c}^\dagger + \hat{F} + \frac{i}{\chi_p} (2m \dot{\theta} + m \dot{\theta}^2 + m \ddot{\theta}).
\]

In equation (2), \( \dot{\theta} \) term represents Coriolis force, \( \dot{\theta}^2 \) term represents centrifugal force and \( \ddot{\theta} \) term represents angular acceleration force. We set \( \dot{\theta} = 0 \) by assuming that the table is rotating with uniform angular velocity. We are interested in detecting the rotation rates that are much smaller than 1 Hz. Hence the \( \dot{\theta} \) contribution to the centrifugal force, which has \( \dot{\theta}^2 \) dependence, is negligible in comparison with the Coriolis force term which has \( \dot{\theta} \) dependence. With these simplifications, the final simplified form of equation (2) can be written as

\[
\hat{b}_x = \left( -i \omega_m - \frac{\gamma_m}{2} \right) \hat{b}_x - \frac{i}{\chi_p} \hat{c} \hat{c}^\dagger + \hat{F} + \frac{i}{\chi_p} (2m \dot{\theta}).
\]

The cavity field equation of motion is given as

\[
\hat{c} = \left( i \omega_m - \omega_0 - \frac{\chi_p}{\chi_p} \left( \hat{b}_x + \hat{b}_x^\dagger \right) \right) - \frac{\kappa}{2} \hat{c} + \sqrt{\kappa} \hat{E}.
\]

We linearize equations (4) and (3) by writing \( \hat{c} = \hat{c} + \hat{\delta}_c \), \( \hat{b}_x = \hat{b} + \hat{\delta}_b \), \( \hat{\dot{\theta}} = \hat{\dot{\theta}} + \hat{\delta}_{\dot{\theta}} \) and \( \hat{E} = \hat{E} + \hat{\delta}_E \), where \( \hat{c} \), \( \hat{b} \), \( \hat{\dot{\theta}} \) and \( \hat{E} \) are the zero order classical variables of \( \hat{c} \), \( \hat{b}_x \), \( \hat{\dot{\theta}} \) and \( \hat{E} \), respectively. \( \hat{\delta}_c \), \( \hat{\delta}_b \), \( \hat{\delta}_{\dot{\theta}} \), and \( \hat{\delta}_E \) are the first-order fluctuations in \( \hat{c} \), \( \hat{b}_x \), \( \hat{\dot{\theta}} \), respectively. \( \delta_{\dot{\theta}} \) is the zero-mean fluctuation in the optical field \( \hat{E} \). We treat the Coriolis force term in equation (3) as a small perturbation and include it in the first-order fluctuation equations. Hence the zero-order equations of motion do not have any contribution from the Coriolis force and \( \langle \hat{\delta}_{\dot{\theta}} \rangle = 0 \), \( \langle \hat{\delta}_c \rangle = 0 \) in presence of the Coriolis force. Moreover, the noise term in the Coriolis force, which is given by \( 2m \dot{\delta}_{\dot{\theta}} \), contributes to the second order and can be neglected in this work.

Figure 1. Optomechanical gyroscope. FM: rigidly fixed mirror, OMC: optomechanical cavity, L: laser source, D1 and D2 are the photo-detectors. The OMC is mounted on a stage which is driven co-sinusoidally so that the instantaneous velocity of the mechanical oscillator along y-axis is \( \dot{y} \). When the table rotates with angular velocity \( \dot{\theta} \), the Coriolis force acts on the mechanical oscillator along the x-axis and is equal to \( 2m \dot{\theta} \dot{y} \). The phase shift, corresponding to the change in the optomechanical cavity length due to the action of Coriolis force, is measured at the detectors D1 and D2 to estimate the angular velocity of the table.
By setting \( \omega_d - \omega_e - \frac{\chi}{\lambda_p} (\vec{b} + \vec{b}^*) = 0 \), we can write the solutions of \( \vec{b} \) and \( \vec{c} \) as

\[
\vec{b} = -i \frac{\chi_0}{\lambda_p} \left( |\vec{c}|^2 \right) \left( \frac{1}{\Gamma_3} - \frac{1}{\Gamma_4} \right); \quad \vec{c} = \vec{E} - \frac{2}{\sqrt{\kappa}}. \tag{5}
\]

The mean field at the output of the OMC, represented by \( \vec{c}^o \), can be obtained by using input–output formalism [30, 31] as \( \vec{c}^o = \vec{E} - \sqrt{\kappa} \vec{c} = -\vec{E} \). The solution for the equation of motion of \( \dot{\vec{b}} \) can be obtained in the Fourier frequency space by using the transform function \( \mathcal{F}(f(t)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt \). Then the fluctuation in the cavity field output, denoted by \( \delta(\omega) \), can be obtained by using input–output formalism as

\[
\delta(\omega) = (1 - \kappa b_{13}) \delta^o(\omega) - \kappa b_{12} \delta E^o(\omega) - \sqrt{\kappa} (b_{13} \dot{\vec{E}}(\omega) + b_{14} \dot{\vec{E}}^o(\omega)) \tag{6a}
\]

\[
\delta^o(\omega) = (1 - \kappa b_{23}) \delta^o E^o(\omega) - \kappa b_{21} \dot{\delta} E^o(\omega) - \sqrt{\kappa} (b_{23} \dot{\vec{E}}(\omega) + b_{24} \dot{\vec{E}}^o(\omega)) \tag{6b}
\]

where \( b_{mn} \) represents the element in \( m \)th row and \( n \)th column of the matrix \( B \), and

\[
B = \begin{bmatrix}
\Gamma_3 \Gamma_4 + \frac{\lambda^2}{\lambda_p} |\vec{c}|^2 (\Gamma_3 - \Gamma_4) & -\frac{\lambda^2}{\lambda_p} |\vec{c}|^2 (\Gamma_4 - \Gamma_3) & -i \frac{\chi_0}{\lambda_p} \vec{c} \Gamma_4 & -i \frac{\chi_0}{\lambda_p} \vec{c} \Gamma_3 \\
-\frac{\lambda^2}{\lambda_p} (\vec{c}^o)^2 (\Gamma_3 - \Gamma_4) & \Gamma_3 \Gamma_4 + \frac{\lambda^2}{\lambda_p} |\vec{c}^o|^2 (\Gamma_4 - \Gamma_3) & i \frac{\chi_0}{\lambda_p} \vec{c}^o \Gamma_4 & i \frac{\chi_0}{\lambda_p} \vec{c}^o \Gamma_3 \\
-i \frac{\chi_0}{\lambda_p} \vec{c} \Gamma_4 & -i \frac{\chi_0}{\lambda_p} \vec{c} \Gamma_3 & \Gamma_4 & 0 \\
i \frac{\chi_0}{\lambda_p} \vec{c}^o \Gamma_4 & i \frac{\chi_0}{\lambda_p} \vec{c}^o \Gamma_3 & 0 & \Gamma_3 \\
\end{bmatrix}
\]

where \( \Gamma = (\kappa/2 - i\omega) \), \( \Gamma_3 = \gamma_m/2 + i(\omega_m - \omega) \), and \( \Gamma_4 = \gamma_m/2 - i(\omega_m + \omega) \).

2.1. Signal evaluation

We add a \( \pi/2 \) phase to the field coming out of the OMC so that the difference in the photo-detector reading is given as

\[
\vec{I}_2 - \vec{I}_1 = \vec{c}^o \vec{E}^o + \vec{c}^o \vec{E}^o, \tag{8}
\]

where \( \vec{I}_2, \vec{I}_1, \vec{c}^o \) and \( \vec{E}^o \) are the intensity at detector D2, intensity at detector D1, output field from FM and output field from OMC, respectively. We linearize the equation (8) by writing \( \vec{c}^o = \vec{c}^o + \vec{\delta} \) and \( \vec{E}^o = \vec{E}^o + \vec{\delta} \). The \( \vec{c}^o \) and \( \vec{\delta} \) represent the mean value of \( \vec{c}^o \), mean value of \( \vec{\delta} \), and input fluctuation in arm-1, respectively. After linearization, equation (8) can be approximated as

\[
\vec{I}_2 - \vec{I}_1 \approx \vec{c}^o \vec{E}^o + \vec{c}^o \vec{E}^o + \vec{\delta} \vec{c}^o + \vec{\delta} \vec{E}^o + \vec{\delta} \vec{\delta} \vec{E}^o. \tag{9}
\]

Assuming an ideal 50:50 beam splitter, fields entering into arm-2 and arm-1 are given as

\[
\vec{E} = (i E_0 + E_0)/\sqrt{2} \quad \text{and} \quad \vec{E}_1 = (i E_0 - E_0)/\sqrt{2}, \text{respectively. Where } E_0, E_0 \text{ and } E_0 \text{ are the laser field from 'L.' (see figure 1), vacuum field, and the field entering into arm-1, respectively. Mean values of } \vec{E} \text{ and } \vec{E}_1 \text{ are given as } \vec{E} = iE_0/\sqrt{2} \text{ and } \vec{E}_1 = E_0/\sqrt{2}, \text{respectively. } E_0 \text{ is the mean value of } \vec{E}_0 \text{ and is a real quantity. Since the field in arm-1 is the simply reflected field from the FM, we can write } \vec{c}^o = - \vec{E}_1 = - E_0/\sqrt{2}. \text{ By substituting } \vec{c}^o = - \vec{E}_0/\sqrt{2} \text{ and } \vec{E}^o = - \vec{E} = -i \vec{E}_0/\sqrt{2}, \text{ we can write } \vec{c}^o \vec{E}^o = \vec{c}^o \vec{E}^o = 0 \text{ in equation (9). Hence, in Fourier frequency domain, equation (9) is given as}
\]

\[
\vec{I}_2(\omega) - \vec{I}_1(\omega) \approx \vec{c}^o(\omega) \vec{E}^o + \vec{\delta}(\omega) \vec{E}^o + \vec{c}^o \vec{\delta}(\omega) + \vec{\delta} \vec{E}^o + \vec{\delta} \vec{\delta} \vec{E}^o. \tag{10}
\]

Using equation (6), the mean value of equation (10) is given as

\[
\langle \vec{I}_2(\omega) - \vec{I}_1(\omega) \rangle = -\sqrt{\kappa} \mathcal{F}(2m^0 \theta) \frac{\chi_0}{\lambda_p} \vec{c}^o \vec{E}^o - \vec{c} \left( \frac{1}{\Gamma_4} - \frac{1}{\Gamma_3} \right). \tag{11}
\]

Equation (11) represents the signal from the gyroscope. The velocity of the optomechanical mirror along the y-axis of the rotating table, shown by \( \dot{\vec{y}} \) in equation (11), is treated as a general function of time. We fix the optomechanical mirror to a co-sinusoidal oscillator such that the optomechanical mirror is being oscillated with
an angular frequency of $\omega_m$ along the $y$-axis of the table. Then $\dot{\phi} = \dot{\gamma}_m \cos(\omega_m t)$, where $\dot{\gamma}_m$ is the maximum velocity of the optomechanical mirror along the $y$-axis of the rotating table. Hence the Coriolis force acting on the mechanical oscillator becomes $2m \dot{\theta}_m \dot{\gamma}_m \cos(\omega_m t)$. This technique of modulating the Coriolis force periodically at the natural frequency of mechanical oscillator enhances the responsiveness of the mechanical oscillator. Hence, we can write

$$F(2m \dot{\theta}) = \sqrt{\frac{\pi}{2}} 2m \dot{\gamma}_m \dot{\theta}[\delta(\omega - \omega_m) + \delta(\omega + \omega_m)]. \quad (12)$$

The signal in time domain, represented by $S(t)$, can be obtained by inverse Fourier transforming the equation (11) as shown below

$$S(t) = \langle \dot{I}_2 - \dot{I}_1 \rangle \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t} \langle \dot{I}_2(\omega) - \dot{I}_1(\omega) \rangle d\omega. \quad (13)$$

Substituting equations (11) and (12) in (13), we can write

$$S(t) = \frac{(16m \dot{\gamma}_m \dot{\theta}) \lambda_I \omega_m}{\gamma_m \sqrt{(\omega_m^2 + \kappa^2/4)(4\omega_m^2 + \gamma_m^2/4)}} \Re(e^{-i\omega_m t - \phi}), \quad (14)$$

where $2I_l = |\dot{E}_l|^2$ is the laser intensity, $\Re$ represents real part, and the phase $\phi$ is given as

$$\tan \phi = \frac{\omega_m^2 \kappa + \omega_m \gamma_m/2}{\kappa \gamma_m/4 - 2\omega_m^2}. \quad (15)$$

The signal in equation (14) oscillates with angular frequency $\omega_m$, phase $\phi$ and amplitude

$$S_0 = 4\dot{\theta} \omega_m I_l \lambda_I / (\omega_m \gamma_m \sqrt{\omega_m^2 + \kappa^2}).$$

We assumed that $\omega_m \gg \gamma_m$ in writing $S_0$. The presence of $\omega_m t$ term in equation (14) indicates that the signal is due to the Coriolis force which is modulated at angular frequency $\omega_m$.

2.2. Noise evaluation

The fluctuation, represented by $\Delta(\dot{I}_2 - \dot{I}_1)$, in equation (9) is given as $\Delta(\dot{I}_2 - \dot{I}_1) = \dot{I}_2 - \dot{I}_1 - \langle \dot{I}_2 - \dot{I}_1 \rangle$. Hence, in the Fourier frequency domain, the fluctuation is given as

$$\Delta(\dot{I}_2(\omega) - \dot{I}_1(\omega)) = \dot{I}_2(\omega) - \dot{I}_1(\omega) - \langle \dot{I}_2(\omega) - \dot{I}_1(\omega) \rangle. \quad (16)$$

Using equations (6), (10) and (11) the variance of $\Delta(\dot{I}(\omega))$ is evaluated as

$$\langle \Delta(\dot{I}_2(\omega) - \dot{I}_1(\omega)) \Delta(\dot{I}_2(\omega') - \dot{I}_1(\omega')) \rangle = \left[ 2I_l + a(\omega)I_l^2 + b(\omega)I_l^2 \right] \delta(\omega - \omega'),$$

where $a(\omega) = 256 \dot{\gamma}_m^2 \lambda_I^2 / [\Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma \Gamma 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KT reduces the restoring force and hence the optomechanical mirror becomes more sensitive to the Coriolis force. The sensitivity can also be improved by increasing the mass of the oscillator. Increasing the mass of the mechanical oscillator modifies the Coriolis force by decreasing the value of \( \gamma_m \) and the sensitivity given in equation (19) can be approximated as

\[
\dot{\theta}_s = \frac{\gamma_m \gamma_w}{K_w} \left( \frac{2}{h} \frac{\omega_m^2}{m} \right) \theta_s.
\]

(19)

The optimum power \( I_o \) corresponding to \( I_s \) is given as

\[
I_o = \frac{2}{\sqrt{a(\omega_m)}} = \frac{(\kappa^2/4 + \omega_m^2)\gamma_m \omega_m^3 m}{\sqrt{32 \omega_m^2 h}}.
\]

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\]

Sensitivity in equation (20) is in the same order of magnitude as the sensitivity achieved in the squeezed-vacuum gyroscope [15]. In the squeezed-vacuum gyroscope, squeezed vacuum field is injected into two optomechanical cavities to suppress the shot noise and back-action noise and thus improving the sensitivity of the gyroscope. In the present work, the frequency of the Coriolis force is modulated such that it is on resonance with the optomechanical mirror’s natural frequency and hence the mechanical oscillator more responsive to the Coriolis force. As a consequence, without using squeezed vacuum field, the gyroscope in the present work is able to attain the similar sensitivity as that in the squeezed-vacuum gyroscope. It is important to note that the contributions from shot noise, back-action noise and mirror noise are almost equal in equation (20), hence the sensitivity given in equation (20) can not be improved significantly by suppressing shot noise and back-action noise, at optimal input laser intensity, by using the squeezed light technique described in [15]. Equation (20) implies that the sensitivity can be improved by decreasing \( \omega_m \). Decreasing \( \omega_m \) reduces the restoring force and hence the optomechanical mirror becomes more sensitive to the Coriolis force. The sensitivity can also be improved by increasing the mass of the oscillator. Increasing \( m \) leads to increase in \( I_s \) according to equation (19), but this can be compensated by decreasing \( \gamma_m \) such that the product \( m \gamma_m \) remains constant in equation (19). So increasing \( m \) and decreasing \( \gamma_m \) such that the product \( m \gamma_m \) remains constant can enhance the gyroscopes sensitivity without changing \( I_s \).

The thermal occupation number, represented by \( n_{th} \), of the mechanical oscillator is assumed to be zero in equation (20). Given the recent progress in cooling [33, 34] the mechanical oscillator to its ground state [35], the result shown in equation (20) is practically relevant. However, in many practical cases we have \( n_{th} \gg 0 \) and hence it is necessary to consider the temperature dependence. Taking \( T \) as the temperature and \( K_b \) as the Boltzmann constant, temperature dependence of the rotation detection sensitivity is given as

\[
\dot{\theta}_s = \frac{1}{2} \sqrt{\frac{2}{h} \frac{\gamma_m \gamma_w}{m} \left( \sqrt{2} + (1 + 2n_{th}) \right)}.
\]

(21)

For \( n_{th} \approx K_b T / \hbar \omega_m \gg 1 \), equation (21) can be approximated as

\[
\dot{\theta}_s \approx \frac{1}{2} \sqrt{\frac{2}{h} \frac{\gamma_m \gamma_w}{m} \frac{2 K_b T}{\hbar \omega_m}} = \frac{1}{2} \sqrt{\frac{2 K_b T \gamma_m}{m}}.
\]

(22)

Hence for \( n_{th} \gg 1 \), \( \dot{\theta}_s \) is independent of \( \omega_m \) and the sensitivity can not be improved indefinitely, unlike equation (20), by decreasing the value of \( \omega_m \). Dependence of \( \dot{\theta}_s \) on \( \omega_m \) for \( n_{th} = 0 \) and \( n_{th} = 0 \) is shown in figure 3. The Purple line, drawn at \( T = 0 \) K, or at \( n_{th} = 0 \), shows that the sensitivity can be improved indefinitely.
by decreasing \( \omega_m \). The Green line, for \( T = 10 \) K, and the Blue line, for \( T = 300 \) K, shows the same behavior as the Purple line for \( \omega_m \geq 10^{13} \) rad s\(^{-1}\). But with decrease of \( \omega_m \), the Blue line and the Green line become flat indicating that the sensitivity is independent of \( \omega_m \). For \( n_{th} \gg 1 \), even though the sensitivity is independent of \( \omega_m \), it still can be improved by increasing \( m \) or \( \dot{\gamma}_s \), or by decreasing \( \gamma_m \).

For simulation purpose, we choose parameters from [36] \( m = 69 \times 10^{-5} \) Kg, \( \omega_c = 18 \times 10^{14} \) Hz, \( \omega_m = 532.5 \) rad s\(^{-1}\), \( \kappa = 15.4 \times 10^{6} \) Hz, \( \gamma_m = 1.9 \times 10^{-3} \) Hz, \( l = 12.3 \times 10^{-3} \) m, \( \dot{\gamma}_s = 0.1 \) m s\(^{-1}\). Using these parameters the gyroscope’s sensitivity is estimated as \( 7 \times 10^{-15} \) rad s\(^{-1}\) Hz\(^{-1/2}\) at 0 K temperature and \( 1.5 \times 10^{-9} \) rad s\(^{-1}\) Hz\(^{-1/2}\) at 300 K temperature.

Other prominent method for rotation detection uses Sagnac effect [37]. Improving the performance of fiber optic Sagnac-gyros requires increasing the length of the optical fiber through which light travels. Such a design leads to increased scattering and absorption of light by the optical fiber. Sagnac based ring laser gyros, which detect rotation by measuring frequency difference at the output, are limited by laser line-width. Moreover, the sensitivity of the Sagnac effect based gyroscopes is limited by the area [38] of the closed loop. The method described in this work do not depend on the dimensions of the optical path. Hence this scheme can be realized in miniature structures more easily when compared with Sagnac gyroscopes. The two dimensional optomechanical mirror can be realized by coupling a one-dimensional oscillator to an electrical circuit or by designing two dimensional oscillators as described in [39, 40].

4. Conclusion

We propose an application of two-dimensional optomechanical oscillator as a gyroscope to detect absolute rotation. We showed that the two-dimensional optomechanical gyroscopes’s sensitivity is better than one-dimensional optomechanical gyroscopes [15] by a factor of \( \sqrt{\gamma_m/\omega_m} \). Using the parameters from [36], the sensitivity of the gyroscope is estimated as \( 7 \times 10^{-15} \) rad s\(^{-1}\) Hz\(^{-1/2}\) at 0 K temperature. Temperature dependence of the gyroscope’s sensitivity is studied and at 300 K temperature, the sensitivity is approximately equal to \( 1.5 \times 10^{-9} \) rad s\(^{-1}\) Hz\(^{-1/2}\). Given the recent progress in the field of optomechanics, the work described in this manuscript can be used to design very sensitive gyroscopes.

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