Calculation of total reaction cross sections of $^6$He, $^8,^9$Li on nuclei within the microscopic optical potential model

K.V. Lukyanov, Yu.G. Sobolev, Yu.E. Penionzhkevich, E.V. Zemlyanaya
Joint Institute for Nuclear Research, Dubna 141980, Russia
E-mail: luky@jinr.ru

Abstract. Experimental data on total reaction cross sections of exotic nuclei $^6$He, $^8,^9$Li interaction with the $^{181}$Ta, $^{59}$Co, $^{28}$Si and $^9$Be targets in a wide energy range are analyzed in the framework of a hybrid model of microscopic optical potential (OP). The real part of OP is obtained on the basis of the double folding model, while the imaginary part of OP is calculated within the high energy approximation. This theoretical approach demonstrates the fairly well agreement with experimental data at the wide energy regions.

1. Introduction
In last few decades the nuclear structure of light exotic nuclei is intensively investigated both theoretically and experimentally. In particular, experimental data of the total reaction cross section $\sigma_R$ of interaction of stable nuclei with the exotic helium and lithium isotopes in a wide energy range are of interest for their physical properties study.

Current calculations continues the previous one [1], made for $^4,^6$He and $^6,^7$Li reactions with the $^{28}$Si target, where we considered a possibility of the microscopic folding potentials [2] to obtain the total reaction cross sections. It was shown, that such optical potentials based on the density distributions of the nuclei-participants give rather good description of experimental data. In this work we apply the same microscopical optical potentials to study the total reaction cross sections of the $^6$He and $^8,^9$Li on $^9$Be, $^{28}$Si, $^{50}$Co and $^{181}$Ta targets. The motivation is that the microscopic models in their origin do not contain free parameters and thus provide the possibility to test the models of nuclear structure of exotic nuclei.

The experimental data used in this calculations were obtained at the cyclotron U-400M FLNR JINR with ACCULINA beam-line. The energy of $^6$He and $^8,^9$Li particles has been changed in an energy range $\sim 10 - 25$ AMeV [3]-[6].

2. Calculation technique
2.1. Microscopic optical potential
In our calculations we use the microscopic optical potential (OP) of the form

$$U_{opt}(r) = \left[ N_r V^{DF} - N_r^{(1)} r \frac{dV^{DF}}{dr} \right] + i \left[ N_{im} W^H - N_{im}^{(1)} r \frac{dW^H}{dr} \right], \quad (1)$$
where $V^{DF}$ is the double-folding potential with exchange effects included, and $W^H$ is obtained within the so-called high-energy approximation (HEA) model, $N_{r,im}$ are fitted renormalization coefficients. The ”surface terms” of OP $dV^{DF}/dr$ and $dW^H/dr$ describe effect of nuclear collective modes on the mechanism of nucleus-nucleus scattering. It was shown in [1] that this form of OP allows one to get the fairly well agreement with the experimental data comparing to the other potential forms of OP.

### 2.2. Double-folding potential

The real part $V^{DF}$ of constructed OP is the double-folding nucleus-nucleus potential (see [7],[2]) consists of the direct and exchange parts:

$$V^{DF} = V^D + V^EX,$$

$$V^D(r) = \int d^3r_p d^3r_t \rho_p(r_p) \rho_t(r_t) v^D_{NN}(s), \quad s = r + r_t - r_p,$$

$$V^EX(r) = \int d^3r_p d^3r_t \rho_p(r_p, r_p + s) \rho_t(r_t, r_t - s) v^EX_{NN}(s) \exp \left[ \frac{iK(r) \cdot s}{M} \right],$$

where $\rho_p, \rho_t$ are the one-particle nuclear densities of the projectile (p) and target (t) nuclei. In our calculations we apply the effective Paris nucleon-nucleon CDM3Y6 potential $v_{NN}$ having the form

$$v_{NN}(E, \rho, s) = g(E) F(\rho) v(s),$$

where

$$g(E) = 1 - 0.003E/A_p,$$

$$F(\rho) = C \left[ 1 + \alpha \exp(-\beta \rho) - \gamma \rho \right],$$

$$v(s) = \sum_{i=1,2,3} N_i \exp(-\mu_i s) \mu_i s,$$

$$\rho = \rho_p + \rho_t.$$

The parameters $C$, $\alpha$, $\gamma$, $N_i$ and $\mu_i$ are known and presented, for example, in [7].

The energy dependence of $V^{EX}$ arises primarily from the exponential function in the integrand, where

$$K(r) = \{2Mm/h^2[E - V^{DF}(r) - V_c(r)]\}^{1/2}$$

is the local nuclear collision momentum, $M = A_p A_t/(A_p + A_t)$, and $m$ is the nucleon mass. One can see that $K(r)$ depends on $V^{DF}$, and thus reveals the typical non-linear problem. The iteration procedure is described in [8] and implemented here.

### 2.3. HEA microscopical potential

When constructing microscopic OP, many authors usually apply only the real double-folding potential $V^{DF}$ without accounting for the exchange term (4), and the imaginary part is taken in a phenomenological form with free parameters fitted to experimental data for each specific energy. In some cases they also use their form to be proportional to the form of $V^{DF}$. In our calculations, we use the imaginary part $W^H$ obtained in [2] within the high-energy theory of nuclear scattering [9, 10]:

$$U^{H}_{opt} = V^H + W^H,$$

$$V^H = -\frac{2E}{k(2\pi)^2} \sigma_{NN}^\alpha \int_0^\infty dq q^2 j_0(qr) \rho_p(q) \rho_t(q) f_N(q),$$

$$W^H = -\frac{2E}{k(2\pi)^2} \sigma_{NN}^{\alpha} \int_0^\infty dq q^2 j_0(qr) \rho_p(q) \rho_t(q) f_N(q).$$
Here $\rho(q) = \int d^3r \ e^{iqr} \rho(r)$ is the form factor of a point like nuclear density; $\sigma_{NN}$ and $\alpha_{NN}$ are the total $NN$ cross section and the ratio of real to imaginary part of $NN$-amplitude of scattering respectively; $f_N(q)$ corresponds to the form of $NN$-amplitude of scattering.

2.4. Density models

It was shown in [1] that the behavior of total cross sections are sensitive to the form of densities of interacting nuclei in their peripheral region. Generally, microscopical optical potentials do not contain free parameters and thus provide the possibility to test the models of nuclear structure. Figure 1 demonstrates how the density models of the $^6$He projectile affects the resulting $^6$He+$^{28}$Si total reaction cross section. As a result of studies in [1] it was concluded, that the densities with the realistic exponential behavior at large distances are preferable.

![Density models](image)

**Figure 1.** Different models of density distributions of $^6$He (solid curves: Tanihata model [11]; dash-dotted: LSSM [12]; dashed - COSMA [13]) and the resulting total reaction cross section calculations for $^6$He+$^{28}$Si (solid curves: Tanihata model; dashed curves: LSSM; dash-dotted curves: COSMA). Reproduced from [1].

Therefore in our calculations we used the large-scale shell-model (LSSM) densities [12] for the helium projectile and Table densities [14] for other nuclei.

3. Results of calculations

Numerical calculations of cross sections were made basing on the given potential (1) by using the DWUCK4 code [15]. The renormalization coefficients $N_{r,im}$ of microscopic OPs (1) were introduced to get a better agreement to the data at larger energies by fitting their strengths $V^{DF}$ and $W^H$. Two other parameters $N_{r,im}^{(1)}$ are responsible to the contribution of collective terms.

|         | $^6$He | $^8$Li | $^9$Li |
|---------|--------|--------|--------|
|         | $^9$Be | $^{28}$Si | $^{59}$Co | $^{181}$Ta | $^9$Be | $^{28}$Si | $^{59}$Co | $^{181}$Ta | $^9$Be | $^{28}$Si | $^{59}$Co | $^{181}$Ta | $^9$Be | $^{28}$Si | $^{59}$Co | $^{181}$Ta |
| $N_r$   | 0.5    | 0.7    | 0.7    | 0.9    | 0.1    | 0.85   | 1.0    | 1.0    | 0.7    | 1.0    | 1.0    | 1.0    | 0.7    | 1.0    | 1.0    | 1.0    |
| $N_{im}$| 0.3    | 0.5    | 0.5    | 0.9    | 0.1    | 0.75   | 1.0    | 1.0    | 0.5    | 1.0    | 1.0    | 1.0    | 0.5    | 1.0    | 1.0    | 1.0    |
| $N_r^{(1)}$| 0.4 | 0.4 | 0.7 | 0.9 | 0.1 | 0.5 | 0.4 | 0.2 | 0.4 | 0.6 | 0.6 | 0.2 |
| $N_{im}^{(1)}$| 0.03 | 0.03 | 0.035 | 0.03 | 0.01 | 0.07 | 0.08 | 0.35 | 0.03 | 0.12 | 0.13 | 0.35 |

**Table 1.** Renormalization coefficients of the OP (1)
Basically we obtain qualitative agreement with experimental data for $^6$He and $^8,^9$Li reactions on different targets, shown on Figure 2. The corresponding fitted coefficients of the depths of OPs are represented in Table 1. We mention that there exist some kind of ambiguity when comparing calculated cross sections with the experimental data. Usually one performs the $\chi^2$ fitting procedure to obtain the best set of adjusted parameters, and such a procedure assumes enough experimental points in a wide energy range. Unfortunately, in our case one sees that the number of experimental data is very small. It means that one can fit several lines with similar $\chi^2$ values, but with difference in their behavior out of the energy range $\sim 10 - 25$ AMeV. To exclude this uncertainty more experimental data are needed.

According to Table 1, one sees that the $N_r$ and $N^{(1)}_m$ coefficients for the volume parts of OPs are close to 1 for reactions with $^{28}$Si, $^{58}$Co and $^{181}$Ta. Also, influence of renormalization on the surface terms (derivatives) is less significant, especially of its imaginary part, represented by $N^{(1)}_m$. On the other hand, one can see very low $N$-values for reactions with $^9$Be, especially in case of the $^9$Li projectile. The possible reason is the structure of $^9$Be, which could be represented by cluster model, for example of two $\alpha$-particles and neutron in periphery. From this point of view, it can be also considered as an exotic nuclei. In this case, the used gaussian form of density distribution doesn’t corresponds to it real form. Therefore the other density models need to be checked, and interaction between two exotic nuclei needs in more detailed investigations too.

4. Conclusion

Microscopic model of the nucleus-nucleus OP has been constructed by using physical characteristics of structure of densities of colliding nuclei, and the effective nucleon-nucleon forces in nuclear medium. It was shown that a renormalization of strengths of this potentials by introducing the respective renormalization parameters allow us to explain the experimental data. We obtain the qualitative agreement to the most energy range.

Simultaneously, there are only few experimental points for each reaction (except $^6$He on $^{28}$Si target), which sets the ambiguity of parameters at low and high energy regions. More
experimental data (also data on differential cross sections) are required to reduce this ambiguity. We need the further developing of the theory and model to understand the inconsistency with experimental data.

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