Sensorless position estimation of Permanent-Magnet Synchronous Motors using a saturation model

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Abstract—Sensorless control of Permanent-Magnet Synchronous Motors (PMSM) at low velocity remains a challenging task. A now well-established method consists in injecting a high-frequency signal and use the rotor saliency, both geometric and magnetic-saturation induced. This paper proposes a clear and original analysis based on second-order averaging of how to recover the position information from signal injection; this analysis blends well with a general model of magnetic saturation. It also proposes a simple parametric model of the saturated PMSM, based on an energy function which simply encompasses saturation and cross-saturation effects. Experimental results on a surface-mounted PMSM and an interior magnet PMSM illustrate the relevance of the approach.

Index Terms—Permanent-magnet synchronous motor, sensorless position estimation, signal injection, magnetic saturation, energy-based modeling, averaging.

I. INTRODUCTION

P ERMANENT-Magnet Synchronous Motors (PMSM) are widely used in industry. In the so-called “sensorless” mode of operation, the rotor position and velocity are not measured and the control law must make do with only current measurements. While sensorless control at medium to high velocities is well understood, with many reported control schemes and industrial products, sensorless control at low velocity remains a challenging task. The reason is that observability degenerates at zero velocity, causing a serious problem in the necessary rotor position estimation.

A now well-established method to overcome this problem is to add some persistent excitation by injecting a high-frequency signal [11] and use the rotor saliency, whether geometric for Interior Permanent-Magnet machines or induced by main flux saturation for Surface Permanent-Magnet machines [2]–[10]. Signal injection is moreover considered as a standard building block in hybrid control schemes for complete drives operating from zero to full speed [11]–[15].

However to get a good position estimation under high-load condition it is important to take cross-saturation into account [16]–[20]. It is thus necessary to rely on a model of the saturated PMSM adapted to control purposes, i.e. rich enough to capture in particular cross-saturation but also simple enough to be used in real-time and to be easily identified in the field; see [27]–[32] for references more or less in this spirit.

The contribution of this paper, which builds on the preliminary work [15], is twofold: on the one hand it proposes a clear and original analysis based on second-order averaging of how to recover the position information from signal injection; this analysis can accommodate to any form of injected signals, e.g. square signals as in [34], and blends well with a general model of magnetic saturation including cross-saturation. On the other hand a simple parametric model of the saturated PMSM, well-adapted to control purposes, is introduced; it is based on an energy function which simply encompasses saturation and cross-saturation effects.

The paper runs as follows: section II presents the saturation model. In section III position estimation by signal injection is studied thanks to second-order averaging. Section IV is devoted to the estimation of the parameters entering the saturation model using once again signal injection and averaging. Finally section IV.C experimentally demonstrates on two families of motors (with interior magnets and surface-mounted magnets) the relevance of the approach and the necessity of considering saturation to correctly estimate the position.

II. AN ENERGY-BASED MODEL OF THE SATURATED PMSM

A. Notations

In the sequel we denote by \( x_{ij} := (x_i, x_j)^T \) the vector made from the real numbers \( x_i \) and \( x_j \), where \( i,j \) can be \( dq, \alpha\beta \) or \( \gamma\delta \). We also define the matrices

\[
M_\mu := \begin{pmatrix} \cos \mu & -\sin \mu \\ \sin \mu & \cos \mu \end{pmatrix} \quad \text{and} \quad K := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},
\]

and we have the useful relation

\[
\frac{dM_\mu}{d\mu} = KM_\mu = M_\mu K.
\]

B. Energy-based model

The model of a two-axis PMSM expressed in the synchronous \( d-q \) frame reads

\[
\frac{d\phi_{dq}}{dt} = u_{dq} - R i_{dq} - \omega K (\phi_{dq} + \phi_m) \tag{1}
\]

\[
\frac{J}{n^2} \frac{d\omega}{dt} = \frac{3}{2} i_{dq}^T K (\phi_{dq} + \phi_m) - \frac{\tau_L}{n} \tag{2}
\]

\[
\frac{d\omega}{dt} = \omega, \tag{3}
\]

with \( \phi_{dq} \) flux linkage due to the current; \( \phi_m := (\lambda, 0)^T \) constant flux linkage due to the permanent magnet; \( u_{dq} \) impressed voltage and \( i_{dq} \) stator current; \( \omega \) and \( \theta \) rotor (electrical) speed and position; \( R \) stator resistance; \( n \) number of pole pairs; \( J \) inertia moment and \( \tau_L \) load torque. The physically impressed voltages are \( u_{\alpha\beta} := M_\theta u_{dq} \) while the physically measurable
curtains are $i_{αβ} := M_{αβ}dγd$. The current can be expressed in function of the flux linkage thanks to a suitable energy function $H(ϕ_d, ϕ_q)$ by

$$i_{dq} = I_{dq}(ϕ_{dq}) := \frac{∂_1 H(ϕ_d, ϕ_q)}{∂_2 H(ϕ_d, ϕ_q)}, \quad (4)$$

where $∂_k H$ denotes the partial derivative w.r.t. the $k$th variable $[35], [36]$ without loss of generality $H(0, 0) = 0$. Such a relation between flux linkage and current naturally encompasses cross-saturation effects.

For an unsaturated PMSM this energy function reads

$$H_i(ϕ_d, ϕ_q) = \frac{1}{2L_d}ϕ^2_d + \frac{1}{2L_q}ϕ^2_q,$$

where $L_d$ and $L_q$ are the motor self-inductances, and we recover the usual linear relations

$$i_d = ∂_1 H_i(ϕ_d, ϕ_q) = \frac{ϕ_d}{L_d},$$

$$i_q = ∂_2 H_i(ϕ_d, ϕ_q) = \frac{ϕ_q}{L_q}.$$

Notice the expression for $H_i$ should respect the symmetry of the PMSM w.r.t. the direct axis, i.e.

$$H(ϕ_d, -ϕ_q) = H(ϕ_d, ϕ_q), \quad (5)$$

which is obviously the case for $H_i$. Indeed (1)-(3) is left unchanged by the transformation

$$(u_d, u_q, ϕ_d, ϕ_q, i_d, i_q, ω, θ, τ_L) \rightarrow (u_d, -u_q, ϕ_d, -ϕ_q, i_d, -i_q, -ω, -θ, -τ_L).$$

C. Parametric description of magnetic saturation

Magnetic saturation can be accounted for by considering a more complicated magnetic energy function $H_i$, having $H_i$ for quadratic part but including also higher-order terms. From experiments saturation effects are well captured by considering only third- and fourth-order terms, hence

$$H(ϕ_d, ϕ_q) = H_i(ϕ_d, ϕ_q) + \sum_{i=0}^{3} α_{3-i}ϕ^3_i + \sum_{i=0}^{4} α_{4-i}ϕ^4_i.$$  

This is a perturbative model where the higher-order terms appear as corrections of the dominant term $H_i$. The nine coefficients $α_{ij}$ together with $L_d$, $L_q$ are motor dependent. But (3) implies $α_{21} = α_{03} = α_{31} = α_{13} = 0$, so that the energy function eventually reads

$$H(ϕ_d, ϕ_q) = H_i(ϕ_d, ϕ_q) + α_{30}ϕ^3_d + α_{12}ϕ_dϕ^2_q + α_{40}ϕ^4_d + α_{22}ϕ^2_dϕ^2_q + α_{04}ϕ^4_q.$$

From (4) and (6) the currents are then explicitly given by

$$i_d = \frac{ϕ_d}{L_d} + 3α_{30}ϕ^2_d + α_{12}ϕ^2_d + 4α_{40}ϕ^3_d + 2α_{22}ϕ_dϕ^2_q \quad (7)$$

$$i_q = \frac{ϕ_q}{L_q} + 2α_{12}ϕ_dϕ_q + 2α_{22}ϕ_dϕ_q + 4α_{04}ϕ^3_q. \quad (8)$$

which are the so-called flux-current magnetization curves.

To conclude, the model of the saturated PMSM is given by (1)-(3) and (7)-(8), with $ϕ_d, ϕ_q, ω, θ$ as state variables. The magnetic saturation effects are represented by the five parameters $α_{30}, α_{12}, α_{40}, α_{22}, α_{04}$.

D. Model with $i_d, i_q$ as state variables

The model of the PMSM is usually expressed with currents as state variables. This can be achieved here by time differentiating $i_{dq} = I_{dq}(ϕ_{dq})$.

$$\frac{dϕ_{dq}}{dt} = DI_{dq}(ϕ_{dq})\frac{dϕ_{dq}}{dt},$$

with $\frac{dϕ_{dq}}{dt}$ given by (1). Fluxes are then expressed as $ϕ_{dq} = I_{dq}(i_{dq})$ by inverting the nonlinear relations (7)-(8); rather than performing the exact inversion, we can take advantage of the fact the coefficients $α_{ij}$ are experimentally small. At first order w.r.t. the $α_{ij}$ we have $ϕ_{dq} = L_{dq}i_{dq} + O(|α_{ij}|)$ and $ϕ_q = L_qi_q + O(|α_{ij}|)$; plugging these expressions into (7)-(8) and neglecting $O(|α_{ij}|^2)$ terms, we easily find

$$ϕ_d = L_d(i_d - 3α_{03}L_d^2i_d^2 - α_{12}L^2_i q^2_d$$

$$- 4α_{40}L_d^4i_d^4 - 2α_{22}L^2_dL^2_dq^2_i d_q^2)(9)$$

$$ϕ_q = L_q(i_q - 2α_{12}L_dL_dq i_d q_d - 2α_{22}L^2_dL^2_dq^2_i d_q^2 - 4α_{04}L^2_q q^2_d). \quad (10)$$

Notice the matrix

$$\begin{pmatrix} G_{dd}(i_{dq}) & G_{dq}(i_{dq}) & G_{qq}(i_{dq}) \end{pmatrix} := DI_{dq}(I_{dq}^{-1}(i_{dq})), \quad (11)$$

with coefficients easily found to be

$$G_{dd}(i_{dq}) = \frac{1}{L_d} + 6α_{30}L_d^2i_d^2 + 12α_{22}L^2_dL^2_dq^2_i d_q^2$$

$$G_{dq}(i_{dq}) = 2α_{12}L_dL_dq_i q_d + 4α_{22}L^2_dL^2_dL_dL_dq_i q_d$$

$$G_{qq}(i_{dq}) = \frac{1}{L_q} + 2α_{12}L_dL_dq_i q_d + 2α_{22}L^2_dL^2_dL^2_dq^2_i d_q^2 + 12α_{04}L^2_q q^2_d,$$

is by construction symmetric; indeed

$$DI_{dq}(ϕ_{dq}) = \begin{pmatrix} ∂_1 H(ϕ_d, ϕ_q) & ∂_2 H(ϕ_d, ϕ_q) & ∂_3 H(ϕ_d, ϕ_q) \\ \partial_1 H(ϕ_d, ϕ_q) & ∂_2 H(ϕ_d, ϕ_q) & ∂_3 H(ϕ_d, ϕ_q) \end{pmatrix}$$

and $∂_1 H = ∂_2 H = ∂_3 H$. Therefore the inductance matrix

$$\begin{pmatrix} L_{dd}(i_{dq}) & L_{dq}(i_{dq}) & L_{qq}(i_{dq}) \end{pmatrix} := \begin{pmatrix} G_{dd}(i_{dq}) & G_{dq}(i_{dq}) & G_{qq}(i_{dq}) \end{pmatrix}^{-1}.$$

is also symmetric, though this is not always acknowledged in saturation models encountered in the literature.

III. POSITION ESTIMATION BY HIGH FREQUENCY VOLTAGE INJECTION

A. Signal injection and averaging

A general sensorless control law can be expressed as

$$u_{αβ} = M_{αβ} u_{γδ} \quad (12)$$

$$\frac{dθ_c}{dt} = ω_c \quad (13)$$

$$\frac{dθ_c}{dt} = a(M_{αβ} i_{γδ}, ω_c, η, t) \quad (14)$$

$$ω_c = Ω_e(M_{αβ} i_{γδ}, η, t) \quad (15)$$

$$u_{γδ} = U_δ(M_{αβ} i_{γδ}, η, t), \quad (16)$$
where the measured currents $i_{\alpha\beta} = M_{\alpha\beta} i_{\gamma\delta}$ are used to
calculate $u_{\gamma\delta}$, $\omega_c$, and the evolution of the internal vector
variable $\eta$; $\eta$ is due to the controller; $\theta_c$ and $\omega_c$ are known by
design.

It will be convenient to write the system equations (11)–(13)
in the $\gamma - \delta$ frame defined by $x_{\gamma\delta} := M_{\gamma\delta} x_{dq}$, which gives
\[
\frac{d\bar{\phi}_{\gamma\delta}}{dt} = \bar{\omega}_{\gamma\delta} - R_1 \bar{\phi}_{\gamma\delta} - \omega_c \bar{K}_{\gamma\delta} - \omega \bar{K} M_{\gamma\delta} \bar{\phi}_m \tag{17}
\]
\[
\frac{J}{n^2} \frac{d\bar{\omega}}{dt} = \frac{3}{2} \bar{T}_2 \bar{K} (\bar{\phi}_{\gamma\delta} + M_{\gamma\delta} \theta_3 \bar{\phi}_m) - \frac{\tau_L}{n} \tag{18}
\]
\[
\frac{d\bar{\theta}}{dt} = \bar{\omega}, \tag{19}
\]
where from (17)–(19) currents and fluxes are related by
\[
i_{\gamma\delta} = M_{\gamma\delta} x_{dq} (M_{T \gamma\delta} \bar{\phi}_{\gamma\delta}). \tag{20}
\]

To estimate the position we will superimpose on some
desirable control law (16) a fast-varying pulsating voltage,
\[
u_{\gamma\delta} = \mathcal{U}_{\gamma\delta} (M_{\gamma\delta} i_{\gamma\delta} + \eta, t) + \bar{\nu}_{\gamma\delta} f (\Omega t), \tag{21}
\]
where $f$ is a $2\pi$-periodic function with zero mean and $\bar{\nu}_{\gamma\delta}$ could like
\[
\bar{\nu}_{\gamma\delta} \sim \mathcal{U}_{\gamma\delta} (M_{\gamma\delta} i_{\gamma\delta} + \eta, t) \quad \text{and} \quad \bar{\nu}_{\gamma\delta} \sim f (\Omega t)
\]
also taken constant in the sequel). The constant pulsation
\[
\Omega \sim \frac{1}{T}, \quad \sigma := \frac{t}{T}, \quad \text{and} \quad x := (\bar{\phi}_{\gamma\delta}, \omega, \theta_3, \bar{\eta}), \tag{17} - (19)
\]
acted upon by the modified control law (12)–(15) and (21) is in
the so-called standard form for averaging (with slow-time dependance)
\[
\frac{dx}{d\sigma} = \varepsilon f_1 (x, \varepsilon \sigma, \sigma) := \varepsilon (\mathcal{F}_1 (x, \varepsilon \sigma) + \tilde{f}_1 (x, \varepsilon \sigma) f (\sigma)),
\]
with $f_1$ $T$-periodic w.r.t. its third variable ($T = 2\pi$ in our case)
and $\varepsilon$ as a small parameter. Therefore its solution can be approximated as
\[
x (\sigma) = z (\sigma) + \varepsilon (u_1 (z (\sigma), \varepsilon \sigma, \sigma) + O (\varepsilon^2)),
\]
where $z (\sigma)$ is the solution of
\[
\frac{dz}{d\sigma} = \varepsilon g_1 (z, \varepsilon \sigma) + \varepsilon^2 g_2 (z, \varepsilon \sigma)
\]
and
\[
g_1 (y, \varepsilon \sigma) := \frac{1}{T} \int_0^T f_1 (y, \varepsilon \sigma, s) ds = \mathcal{F}_1 (y, \varepsilon \sigma) \tag{22}
\]
\[
v_1 (y, \varepsilon \sigma) := \frac{1}{T} \int_0^T (f_1 (y, \varepsilon \sigma, s) - g_1 (y, \varepsilon \sigma)) ds
\]
\[
= \tilde{f}_1 (y, \varepsilon \sigma) \int_0^\sigma f (s) ds \tag{23}
\]
\[
u_1 (y, \varepsilon \sigma, \sigma) := \varepsilon \int_0^\sigma v_1 (y, \varepsilon \sigma, \sigma) \frac{1}{T} \int_0^T \v_1 (y, \varepsilon \sigma, s) ds
\]
\[
= \tilde{f}_1 (y, \varepsilon \sigma) F (\sigma) \tag{24}
\]
\[
K_2 (y, \varepsilon \sigma, \sigma) := \varepsilon \int_0^\sigma \int_0^\sigma f_2 (y, \varepsilon \sigma, \sigma) ds \tag{25}
\]
\[
g_2 (y, \varepsilon \sigma) := \frac{1}{T} \int_0^T K_2 (y, \varepsilon \sigma, s) ds = 0 \tag{26}
\]
Notice this slowly-varying system is exactly the same as (17)–(19) acted upon by the unmodified control law (12)–(15). In other words adding signal injection:

- has a very small effect of order $O (\frac{1}{\Omega^2})$ on the mechanical variables $\theta, \omega$, and the controller variables $\theta_3, \eta$.
- has a small effect of order $O (\frac{1}{\Omega})$ on the flux $\phi_{\gamma\delta}$; this effect will be used in the next section to extract the position information from the measured currents.

The proof relies on a direct application of second-order averaging of differential equations, see [17] section 2.9.1 and for the slow-time dependence section 3.3. Indeed setting $\varepsilon := \frac{1}{\Omega}, \sigma := \frac{\tau}{T},$ and $x := (\bar{\phi}_{\gamma\delta}, \omega, \theta_3, \bar{\eta}), (17) - (19)$ acted upon by the modified control law (12)–(15) and (21) is in
the so-called standard form for averaging (with slow-time dependence)
\[i_{\gamma\delta} = M_{\gamma\delta} \sigma_e + O \left( \frac{1}{\Omega^2} \right)\]

\[\mathcal{I}_{dq} \left( M_{\gamma\delta}^T \sigma_e + O \left( \frac{1}{\Omega^2} \right) \left( \bar{\tau}_{\gamma\delta} + \bar{u}_{\gamma\delta} F(\Omega t) + O \left( \frac{1}{\Omega^2} \right) \right) \right)\]

\[= \bar{\tau}_{\gamma\delta} \bar{u}_{\gamma\delta} + i_{\gamma\delta} F(\Omega t) + O \left( \frac{1}{\Omega^2} \right), \quad (28)\]

where we have used (27) and performed a first-order expansion to get

\[\tilde{\tau}_{\gamma\delta} := M_{\gamma\delta} \sigma_e D L_{dq} \left( M_{\gamma\delta}^T \bar{\sigma}_e \gamma_{\delta} \right) M_{\gamma\delta}^T \bar{u}_{\gamma\delta} \]

\[= M_{\gamma\delta} \sigma_e D L_{dq} \left( I_{dq}^{-1} \left( M_{\gamma\delta}^T \bar{\sigma}_e \gamma_{\delta} \right) \right) M_{\gamma\delta}^T \bar{u}_{\gamma\delta}. \quad (29)\]

We will see in the next section how to recover \(\tilde{\tau}_{\gamma\delta}\) and \(\bar{\tau}_{\gamma\delta}\) from the measured currents \(i_{\gamma\delta}\). Therefore (29) gives two (redundant) relations relating the unknown angle \(\bar{\theta}\) to the known variables \(\bar{\sigma}_e, i_{dq}, \bar{\tau}_{\gamma\delta}, \bar{u}_{dq}\), provided the matrix

\[S(\mu, \bar{\tau}_{\gamma\delta}) := M_{\mu} D L_{dq} \left( I_{dq}^{-1} \left( M_{\mu} \bar{\tau}_{\gamma\delta} \right) \right) M_{\mu}^T\]

effectively depends on its first argument \(\mu\). This “saliency condition” is what is needed to ensure nonlinear observability. The explicit expression for \(S(\mu, \bar{\tau}_{\gamma\delta})\) is obtained thanks to (11). In the case of an unsaturated magnetic circuit this matrix boils down to

\[S(\mu, \bar{\tau}_{\gamma\delta}) = M_{\mu} \left( \begin{array}{cc} \frac{L_d}{L_q} + L_q & 0 \\ 0 & \frac{L_q}{L_d} \end{array} \right) M_{\mu}^T\]

\[= \frac{L_d + L_q}{2L_d L_q} \left( 1 + \frac{L_d - L_q}{L_d L_q} \cos 2\mu + \frac{L_d - L_q}{L_d L_q} \sin 2\mu \right) \]

\[\left( 1 - \frac{L_d - L_q}{L_d L_q} \sin 2\mu - 1 \right) \cdot \frac{L_d - L_q}{L_d + L_q} \cos 2\mu \]

and does not depend on \(i_{\gamma\delta}\); notice this matrix does not depend on \(\mu\) for an unsaturated machine with no geometric saliency. Notice also (29) defines in that case two solutions on \([-\pi, \pi]\) for the angle \(\bar{\theta}\) since \(S(\mu, \bar{\tau}_{\gamma\delta})\) is actually a function of \(2\mu\); in the saturated case there is generically only one solution, except for some particular values of \(\bar{\tau}_{\gamma\delta}\).

There are several ways to extract the rotor angle information from (29), especially for real-time use inside a feedback law. In this paper we just want to demonstrate the validity of (29) and we will be content with directly solving it through a nonlinear least square problem; in other words we estimate the rotor position as

\[\bar{\theta} = \theta_c + \arg \min_{\mu \in [-\pi, \pi]} \left\| \tilde{\tau}_{\gamma\delta} - S(\mu, \bar{\tau}_{\gamma\delta}) \bar{u}_{\gamma\delta} \right\|^2. \quad (30)\]

\[C. \text{Current demodulation}\]

To estimate the position information using e.g. (30) it is necessary to extract the low- and high-frequency components \(\tilde{\tau}_{\gamma\delta}\) and \(\bar{\tau}_{\gamma\delta}\) from the measured current \(i_{\gamma\delta}\). Since by (28)

\[i_{\gamma\delta}(t) \approx \tilde{\tau}_{\gamma\delta}(t) + i_{\gamma\delta}(t) F(\Omega t) \]

we may write

\[\tilde{\tau}_{\gamma\delta}(t) = \frac{1}{T} \int_{-T}^{t} i_{\gamma\delta}(s) ds\]

\[\bar{\tau}_{\gamma\delta}(t) = \frac{1}{T} \int_{-T}^{t} i_{\gamma\delta}(s) F(\Omega s) ds\]

where \(T := \frac{2\pi}{\Omega}\). Indeed as \(F\) is \(2\pi\)-periodic with zero mean,

\[\int_{-T}^{t} i_{\gamma\delta}(s) ds \approx \tilde{\tau}_{\gamma\delta}(t) + \bar{\tau}_{\gamma\delta}(t) F(\Omega s) ds\]

\[= T \tilde{\tau}_{\gamma\delta}(t) + \bar{\tau}_{\gamma\delta}(t) F(\Omega s) ds\]

\[\int_{-T}^{t} i_{\gamma\delta}(s) F(\Omega s) ds \approx \tilde{\tau}_{\gamma\delta}(t) F(\Omega s) ds\]

\[+ \bar{\tau}_{\gamma\delta}(t) \int_{-T}^{t} F^2(\Omega s) ds\]

\[= \bar{\tau}_{\gamma\delta}(t) \int_{0}^{T} F^2(\Omega s) ds.\]

![Fig. 1. Experimental time response of \(i_{dq}\) in (31)-(32)](image)

### IV. ESTIMATION OF MAGNETIC PARAMETERS

The seven parameters in the saturation model (7)-(8) must of course be estimated. This can be done with a rather simple procedure also relying on signal injection and averaging.

\[A. \text{Principle}\]

The rotor is locked in the position \(\theta := 0\), hence the model (11)-(13) reduces to \(\omega = 0\) and

\[\frac{d\phi_{dq}}{dt} = u_{dq} - R i_{dq}, \quad (31)\]

with \(i_{dq} = \mathcal{I}_{dq}(\phi_{dq})\). Moreover \(u_{dq}\) can now be physically impressed and \(i_{dq}\) physically measured.

As in section III-A but now working directly in the \(d - q\) frame, we inject a fast-varying pulsating voltage

\[u_{dq} = \overline{u}_{dq} + \overline{u}_{dq} f(\Omega t), \quad (32)\]

with constant \(\overline{u}_{dq}\) and \(\overline{u}_{dq}\). The solution of (31)-(32) is then

\[\phi_{dq} = \phi_{dq} + \overline{u}_{dq} F(\Omega t) + O \left( \frac{1}{\Omega^2} \right)\]
TABLE I
RATED AND ESTIMATED MAGNETIC PARAMETERS OF TEST MOTORS

| Motor   | IPM     | SPM     |
|---------|---------|---------|
| Rated power | 750 W   | 1500 W  |
| Rated current $I_a$ (peak) | 4.51 A   | 5.19 A   |
| Rated voltage (peak per phase) | 110 V   | 245 V   |
| Rated speed | 1800 rpm | 3000 rpm |
| Rated torque | 3.98 Nm | 5.19 A   |
| $n$ | 3       | 5       |
| $R$ | 1.52 Ω | 2.1 Ω |
| $\lambda$ (peak) | 196 mWb | 155 mWb |
| $L_{dq}$ | 9.15 mH | 7.86 mH |
| $L_d$ | 13.58 mH | 8.18 mH |
| $\alpha_{1,2}L_dI_a i_a$ | 0.039   | 0.056   |
| $\alpha_{1,2}L_d I_a i_a$ | 0.053   | 0.055   |
| $\alpha_{1,2}L_d L_d I_a i_a$ | 0.0051 | 0.0164 |
| $\alpha_{1,2}L_d L_d I_a i_a$ | 0.0171 | 0.027   |
| $\alpha_{1,2}L_d I_a i_a$ | 0.0060 | 0.0067 |

where $\phi_{dq}$, the “slowly-varying” component of $\phi_{dq}$, satisfies

$$ \frac{d\phi_{dq}}{dt} = \tau_{dq} - R\tau_{dq}, $$

with $\tau_{dq} = I_{dq}(\phi_{dq})$. Moreover (29) now boils down to

$$ \tau_{dq} = DL_{dq}(I_{dq}^{-1}(\tau_{dq})) \frac{\tilde{u}_{dq}}{\Omega}. $$

Since $\tau_{dq}$ is constant (32) implies $R\tau_{dq}$ tends to $\tau_{dq}$, hence after an initial transient $\tilde{\tau}_{dq}$ is constant. As a consequence $i_{dq}$ is by (34) also constant. Fig. 1 shows for instance the time response of $i_d$ for the SPM motor of section IV-C starting from $i_d(0) = 0$ and using a square function $f$; notice the current ripples seen on the scope are $\max_{\tau \in [0,2\pi]} F(\tau) = \pi$ (since $f$ is square with period $2\pi$) smaller than $i_{dq}$.

The magnetic parameters can then be estimated repeatedly using (34) with various values of $\tau_{dq}$ and $\tilde{u}_{dq}$, as detailed in the next section.

B. Estimation of the parameters

From (11) the entries of $DL_{dq}(I_{dq}^{-1}(\tau_{dq}))$ are given by

$$ G_{dd}(\tilde{i}_{dq}) = \frac{1}{L_d} + 6\alpha_{3,0}L_d \tilde{i}_{d} + 12\alpha_{4,0}L_d^2 \tilde{i}_{d}^2 + 2\alpha_{2,2}L_d^2 \tilde{i}_{q}^2 $$

$$ G_{dq}(\tilde{i}_{dq}) = 2\alpha_{1,2}L_d \tilde{i}_{q} + 4\alpha_{2,2}L_d \tilde{i}_{d} \tilde{i}_{q} $$

$$ G_{qq}(\tilde{i}_{dq}) = \frac{1}{L_q} + 2\alpha_{1,2}L_d \tilde{i}_{d} + 2\alpha_{2,2}L_d^2 \tilde{i}_{d}^2 + 12\alpha_{4,2}L_d^2 \tilde{i}_{q}^2. $$

Since combinations of the magnetic parameters always enter linearly those equations, they can be estimated by simple linear least squares; moreover by suitably choosing $\tau_{dq}$ and $\tilde{u}_{dq}$, the whole least squares problem for the seven parameters can be split into several subproblems involving fewer parameters:

- with $\tau_{dq} = 0$, hence $\tilde{\tau}_{dq} = 0$, and $\tilde{u}_{dq} = 0$ (34) reads

$$ \tilde{i}_{d} = \frac{\tilde{u}_{d}}{\Omega} \left( \frac{1}{L_d} + 6\alpha_{3,0}L_d \tilde{i}_{d} + 12\alpha_{4,0}L_d^2 \tilde{i}_{d}^2 \right) $$

$$ \tilde{i}_{q} = 0 $$

- with $\tau_{dq} = 0$, hence $\tilde{\tau}_{dq} := 0$, and $\tilde{u}_{dq} = 0$ (34) reads

$$ \tilde{i}_{d} = \frac{\tilde{u}_{d}}{\Omega} \left( \frac{1}{L_d} + 2\alpha_{2,2}L_d \tilde{i}_{q}^2 \right) $$

$$ \tilde{i}_{q} = \frac{2\tilde{u}_{d}}{\Omega} \frac{\tilde{u}_{q}}{\alpha_{1,2}L_d \tilde{i}_{q}} $$

Fig. 2. IPM: fitted values vs measurements for (37) and (39)

Fig. 3. SPM: fitted values vs measurements for (37) and (39)
with \( u_d = 0 \), hence \( \tilde{r}_d := 0 \), and \( \tilde{u}_d = 0 \) (34) reads

\[
\tilde{r}_d = \frac{2u_q}{\Omega} \alpha_{1,2} L_q i_q
\]

(40)

\[
\tilde{r}_q = \frac{u_q}{\Omega} \left( \frac{1}{L_q} + 12\alpha_{0,4} L_q^{-2} \right).
\]

(41)

\( L_d \) and \( L_q \) are then immediately determined from (35) and (36); \( \alpha_{3,0} \) and \( \alpha_{4,0} \) are jointly estimated by least squares from (37); \( \alpha_{2,2}, \alpha_{1,2} \) and \( \alpha_{0,4} \) are separately estimated by least squares from respectively (38), (39), (40) and (41).

C. Experimental setup

The methodology developed in the paper was tested on two types of motors, an Interior Magnet PMSM (IPM) and a Surface-Mounted PMSM (SPM), with rated parameters listed in the top part of table I.

The experimental setup consists of an industrial inverter (400 V DC bus, 4 kHz PWM frequency), an incremental encoder, a dSpace fast prototyping system with 3 boards (DS1005, DS5202 and EV1048), and a host PC. The measurements are sampled also at 4 kHz, and synchronized with the PWM frequency. The load torque is created by a 4 kW DC motor.

D. Estimation of the magnetic parameters

We follow the procedure described in section IV with the rotor locked in the position \( \theta := 0 \), a square wave voltage with frequency \( \Omega := 2\pi \times 500 \) rad/s and constant amplitude \( \tilde{u}_d \) or \( \tilde{u}_q \) (15 V for the IPM, 14 V for the SPM) is applied to the motor; but for the determination of \( L_d, L_q \) where \( \tilde{u}_d = \tilde{u}_q := 0 \), several runs are performed with various \( u_d \) (resp. \( u_q \)) such that \( \tilde{i}_d \) (resp. \( \tilde{i}_q \)) ranges from \(-200\%\) to \(+200\%\) of the rated current. The magnetic parameters are then estimated by linear least squares according to section IV-B yielding the values in the bottom part of table II. Notice the SPM exhibits as expected little geometric saliency \( (L_d \approx L_q) \) hence the saturation-induced saliency is paramount to estimate the rotor position. Notice also the cross-saturation term \( \alpha_{12} \) is as expected quantitatively important for both motors.

The good agreement between the fitted curves and the measurements is demonstrated for instance for (37) and (39).
on Fig. 2-3 notice (37) corresponds to saturation on a single axis while (39) corresponds to cross-saturation.

E. Validation of the rotor position estimation procedure

The relevance of the position estimation methodology developed in section III is now illustrated on the two test motors, using the parameters estimated in the previous section. Since the goal is only to test the validity of the angle estimation procedure, a very simple $V/f$ open-loop (i.e. $\Omega_c$ and $U_{\alpha\delta}$ do not depend on $i_{\gamma\delta}$) control law is used for (12)–(16); a fast-varying ($\Omega := 2\pi \times 500 \text{ rad/s}$) square voltage with constant amplitude is added in accordance with (21), resulting in

$$\frac{d\theta_c}{dt} = \omega_c(t)$$

$$u_{\gamma\delta} = u_{\gamma\delta}^d(t) + \omega_c(t)\phi_m + \tilde{u}_{\gamma\delta}f(\Omega t).$$

Here $\omega_c(t)$ is the motor speed reference; $u_{\gamma\delta}^d(t)$ is a filtered piece-wise constant vector compensating the resistive voltage drop in order to maintain the torque level and the motor stability; finally $\tilde{u}_{\gamma\delta} := (\tilde{u}, 0)^T$ with $\tilde{u} := 15 \text{ V}$.

The rotor position $\hat{\theta}$ is then estimated according to (30).

1) Long test under various conditions, Fig. 4-5: Speed and torque are changed over a period of 210 seconds; the speed remains between $\pm 5\%$ of the rated speed and the torque varies from $0\%$ to $180\%$ of the rated torque. This represents typical operation conditions at low speed.

When the saturation model is used the agreement between the estimated position $\hat{\theta}$ and the measured position $\theta$ is very good, with an error always smaller than a few (electrical) degrees. By contrast the estimated error without using the saturation model (i.e. with all the magnetic saturation parameters $\alpha_{ij}$ taken to zero) can reach up to $40^\circ$ for the IPM and $70^\circ$ the SPM. This demonstrates the importance of considering an adequate saturation model including in particular cross-saturation.

2) Slow speed reversal, Fig. 6-7: This is an excerpt of the long experiment between 35 s and 55 s. The speed is slowly changed from $-0.2\%$ to $+0.2\%$ of the rated speed at $150\%$ of the rated torque. This is a very demanding test since the motor always remains in the poor observability region, moreover under high load. Once again the estimated angle closely agrees with the measured angle.

![Fig. 6. Slow speed reversal for IPM: (a) measured $\theta$, estimated $\hat{\theta}$; (b) measured speed $\omega$, reference speed $\omega_c$; (c) load torque $\tau_L$; (d) voltages $u_{\gamma\delta}^d$](image)

![Fig. 7. Slow speed reversal for SPM: (a) measured $\theta$, estimated $\hat{\theta}$; (b) measured speed $\omega$, reference speed $\omega_c$; (c) load torque $\tau_L$; (d) voltages $u_{\gamma\delta}^d$](image)
3) Load step at zero speed, Fig. 8-9. This is an excerpt of the long experiment around $t = 125\text{s}$. The load is suddenly changed from 0\% to 100\% of the rated torque while the motor is at rest. This test illustrates the quality of the estimation also under dynamic conditions.

V. Conclusion

We have presented a simple parametric model of the saturated PMSM together with a new procedure based on signal injection for estimating the rotor angle at low speed relying on an original analysis based on second-order averaging. This is not an easy problem in view of the observability degeneracy at zero speed. The method is general in the sense it can accommodate virtually any control law, saturation model, and form of injected signal. The relevance of the method and the importance of using an adequate magnetic saturation model has been experimentally demonstrated on a SPM motor with little geometric saliency as well as on an IPM motor.

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