The Effects of Convolution Geometry and Boundary Condition on the Failure of Bellows

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Abstract

Bellows have wide applications in automobile, aerospace, piping systems and industrial systems. An optimised shape for the bellows is expected to give good guidelines for practical design. Metallic bellows expansion joints are subject to displacement loading, this frequently results in cyclic strains. The effects of convolution geometry and boundary conditions on the failure of bellows are analysed in this study. Lateral deflection is considered as a boundary condition and a FEM solution for bellows is obtained. The stress distribution and the number of cycles to failure in the conditions of lateral deflection are obtained. Comparing results, the relation between the number of load cycles to failure and the flexible tube radius is obtained. In addition, the relation between stress and boundary conditions for the model in this study is also obtained.

Keywords: Bellows, Finite Element Analysis, Number of Load Cycles to Failure, Principal Stress, S/N Curve

1. Introduction

A flexible connection between the exhaust system and the manifold, as shown in Figure 1, is necessary because of the rolling of engines. Some torsion takes place because of the curved path of the exhaust system and considerable axial and bending deflections must be allowed for. Using a rigid joint would cause severe vibration of the exhaust system, with noise and early failure due to exceeding the material strength. Proper dimensioning requires deep understanding of the characteristics of the flexible tube and their interaction with the rest of the exhaust system. Off-the-shelf products seldom fit specific applications, this was experienced when flexible tubes were first introduced into exhaust systems. Failures, as shown in Figure 2, took place after rather short operation times and substitution of stronger and much more expensive materials did not solve the problem.

Unlike most piping components in use, the flexible tube consists of a thin walled shell with a corrugated meridian, in order to provide the flexibility needed to absorb mechanical movements. Due to its geometric complexity, it is difficult to analyse the behavior of the flexible tube. The axi-symmetrical deformation problems of the flexible tube have been discussed\(^1\,^2\). These problems have been investigated by the finite difference method\(^3\). Flexible metal tubes have been used for a considerable time in other applications. Numerous papers deal with various aspects of flexible tubes, such as stresses due to internal pressure and axial deflection, fatigue life estimations\(^4\), column instability and scrim. A good grasp of flexible tube research can also be gained from the conference proceedings of the 1989 ASME Pressure Vessels and Piping Conference\(^5\). Andersson\(^6\) derived correction factors relating the behavior of the flexible tube convolution to that of a simple strip beam. This approach has subsequently been the basis of standards and other publications presenting formulae for hand-calculation for flexible tube design.

Some formulae have been included in national pressure vessel codes, among which the ASME code is the most well known. The most comprehensive and widely accepted text on flexible tube design is, however, the Standards of the Expansion Joint Manufacturers
A comparison of the ASME code and the EJMA standards is given by Hanna, concluding that the two conform quite well in most aspects. In addition, the EJMA standards were compared with finite element and experimental analyses in other papers.

Even though, EJMA is useful for the design of flexible tubes, it is difficult to analyse the fatigue failure of flexible tubes because of its complex geometry. As fatigue failure is generally an important aspect of design for metallic flexible tubes, these components are subject to displacement loading that frequently results in cyclic strains, this study aims to represent the effect of geometric parameters and lateral deflection on fatigue failure of flexible tubes. The finite element method was used to analyse the mechanical behavior of flexible tubes.

2. Simulation Model

To obtain a flexible tube profile, it was modeled with finite element code. The flexible tube was meshed with 8 node shell elements and elastic - plastic non-linear analysis is performed. Figure 3 displays the geometry profile for the analysis model. The mesh consists of 100,000 nodes and lateral deflection is loaded at the end of the boundary conditions. Material properties used in analysis and analysis parameters are described in Table 1. ANSYS is used as the solver for stress analysis and the stresses and other results are imported from ANSYS into FEMFAT for fatigue analysis.

3. Results and Discussions

Figure 4 represents the stress distribution and deformed shape in the condition of lateral deflection. The maximum stress occurs around the secondary convolution root from the end cap, as shown in Figure 4. The number of load cycles to failure, as shown in Figure 5, is obtained at this position. The yield stress of the model is 350Mpa and the flexible tube yields at 8mm deflection.

When the flexible tube is deformed by only the lateral deflection, stress concentrations arise from four corners, as shown in Figure 4. As the lateral deflection increases from 7mm to 13mm, the stress increases and the stress concentration area at the four corners gradually increases.

Figure 5 represents the S/N curve for the model of 9 convolutions with a 20mm inner radius and 1.7mm radius of convolution. The lower line represents the S/N curve for the base material from the dumbbell type specimen test and the upper line represents the S/N curve obtained by FEMFAT for the actual flexible tube model in the study. The obtained principal stress from FEM is 320MPa and the number of load cycles to failure of the model in Figure 5 is calculated as 4.89 x 10^5 cycles. Experimental
As the inner radius of the flexible tube increases, the principal stress increases linearly as shown Figure 7(a). The number of load cycles to failure decreases linearly from 945,000 cycles to 857,000 cycles according to the increase of the flexible tube radius from 20mm to 32.5mm as shown in Figure 7(b). It is caused by the increase of the convolution crown: 2mm).

**Figure 4.** Stress distribution and deformed shape in the condition of lateral deflection.

**Figure 5.** S/N for flexible tube with 9 convolutions (inner radius of tube: 20mm, radius of Convolution: 1.7mm) - Number of cycles to failure: 4.89 x 10^5 cycles, stress: 320MPa. The effect of the convolution geometry and boundary conditions on the failure of bellows examined in this study can be summarised as follows;

- Comparing the stress variation with a flexible tube length, it varies almost linearly as the inner radius of the bellows increases.

**Figure 6.** Number of load cycles to failure (170mm length).

(a) Principal stress

(b) Number of cycles to failure

**Figure 7.** The principal stress and number of cycles to failure versus inner radius of flexible tube (Numbers of Convolution: 19ea, Meridional radius of the convolution crown: 2mm).
bending moment due to the increase of the flexible tube radius and the section modulus in the boundary condition of the same bending deflection.

As the bending moment increases, the principal stress increases as shown in Figure 7(a) and the number of load cycles to failure decreases as shown in Figure 7(b).

Figure 8 represents the Von-Mises stress according to the deflection with the variation of the angle of rotation. The Von-Mises stress increases linearly at the 0° angle of rotation as the deflection increases. If the deflection is 0mm, the Von-Mises stress increases as the angle of rotation increases. In addition, the Von-Mises stress decreases to a minimum value and the stress exhibits an upward trend after the minimum stress with the increase of deflection for each angle of rotation.

The number of load cycles to failure increases to 1,320,000 cycles at a 1.7mm meridional radius of the convolution crown and the load cycles exhibit a downward trend after the maximum cycles is reached, as shown in Figure 9. As the meridional radius of the convolution crown increases, the stress concentration effect decreases and the number of cycles to failure increases. After the meridional radius of the convolution crown becomes 1.7mm, the flexible tube radius increases and the bending moment increases. As the bending stress increases with an increment of the bending moment, the number of load cycles to failure decreases. Figure 10 represents the principal stress and number of load cycles to failure versus the deflection of the flexible tube. The real line in Figure 10(a) represents the principal stress of the 170mm long flexible tube and the dashed line represents the principal stress of the 180mm long flexible tube. The principal stress increases as the deflection of the flexible tube end increases.

The stress of a 170mm long flexible tube is higher than the stress of a 180mm long flexible. Comparing the stress variation with a flexible tube length, it varies almost linearly for a 170mm long flexible tube but non-linearly for a 180mm long flexible tube. It is assumed that the flexibility of a flexible tube increases as the length of flexible tube increases and the stress varies non-linearly.

![Figure 8](image1.png)  
*Figure 8.* Von-Mises stress according to deflection at the end.

![Figure 9](image2.png)  
*Figure 9.* The number of load cycles to failure versus radius of convolution.

![Figure 10](image3.png)  
*Figure 10.* Principal stress number of load cycles to failure versus deflection of flexible tube (t = 0.2mm, D = 55mm).
The number of load cycles to failure versus the deflection of a flexible tube is shown in Figure 10(b). The real line represents the number of load cycles for a 170mm long flexible tube and the dashed line represents the number of load cycles for a 180mm long flexible tube. The number of load cycles to failure decreases as the deflection of the end cap increases, it is higher for a 170mm long flexible tube length than a 180mm long flexible tube.

4. Conclusion

The effect of the convolution geometry and boundary conditions on the failure of bellows examined in this study can be summarised as follows;

1. The Von-Mises stress decreases to minimum value and the stress exhibits an upward trend after the minimum stress with an increase of deflection for each angle of rotation.
2. The principal stress increases and the number of load cycles to failure decreases linearly as the inner radius of the bellows increases.
3. The principal stress increases and the number of load cycles to failure decreases with the increase of deflection at the bellows end.

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6. References

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