New look at the $[70, 1^-]$ nonstrange and strange baryons in the $1/N_c$ expansion

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Abstract. The masses of excited nonstrange and strange baryons belonging to the multiplet $[70, 1^-]$ are calculated in the $1/N_c$ expansion to order $1/N_c$ with a new method which allows to considerably reduce the number of linearly independent operators entering the mass formula. This study represents an extension to SU(6) of our work on nonstrange baryons, the framework of which was SU(4).

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In the $1/N_c$ expansion method [1, 2], when SU(3) is broken, the mass operator takes the following general form, as first proposed in Ref. [3] for the symmetric baryon multiplet $[N_c]$

$$M = \sum_i c_i O_i + \sum_i d_i B_i,$$

where $O_i$ are formed of SU($N_f$) invariant operators combined with SO(3) and SU(2)-spin generators and $B_i$ break SU(3) explicitly. Presently we are studying its applicability to orbitally excited states of symmetry $[N_c - 1, 1]$. The standard approach to applying the $1/N_c$ expansion method to baryons described by mixed symmetric states $[N_c - 1, 1]$, both in the orbital and flavor-spin degrees of freedom, is to decouple the system of $N_c$ quarks into a ground state core of $N_c - 1$ quarks and an excited quark [4]. This implies that each generator of SU(2$N_f$) and SO(3) has to be written as a sum of two terms, one acting on the excited quark and the other on the core. As a consequence, the number of linearly independent operators $O_i$ in the mass formula increases tremendously and the number of the coefficients $c_i$ and $d_i$ encoding the quark dynamics and the flavor symmetry breaking, to be determined in a numerical fit, becomes much larger than the experimental data available, as for example for the lowest negative parity nonstrange baryons [4] where $d_i = 0$. Accordingly, the choice of the most dominant operators in the mass formula becomes out of control which implies that important physical effects can be missed.

In a previous work [5] we have proposed a new method where the core + quark separation is avoided. Then we deal with SU(2$N_f$) generators acting on the whole system and the number of linearly independent operators turns out to be considerably smaller than the number of data. All these operators can be included in the fit to clearly find out the most dominant ones up to order $1/N_c$. The knowledge of matrix elements of SU(2$N_f$) generators between mixed symmetric states $[N_c - 1, 1]$ is necessary.

In this approach we have first analyzed the nonstrange $[70, 1^-]$ multiplet where the algebraic work was based on Ref. [6] which provided the matrix elements of SU(4) generators in terms of isoscalar factors of SU(4), initially derived in the context of nuclear physics but quite easily applicable to a system of $N_c$ quarks. In this way we have shown that in the mass formula the flavor (in this case the isospin) term becomes as dominant in $\Delta$ resonances as the spin term in $N$ resonances. This means that the isoscalar factors of the mass formula (1) have comparable values and contribute dominantly to the flavor-spin breaking. Note that the flavor operator was neglected in Ref. [4].

Due to this interesting physical implication, presently we extend the method of Ref. [5] to incorporate the strange baryons. This means that we need the matrix elements of SU(6) generators between mixed symmetric $[N_c - 1, 1]$ states. According to the generalized Wigner-Eckart theorem described in Ref. [6] this amounts at finding the corresponding isoscalar factors. The algebraic work has been performed in two steps. First we have obtained the isoscalar factors of all SU(6) generators for symmetric $[N_c]$ states [7] and next the isoscalar factors for mixed symmetric $[N_c - 1, 1]$ states [8]. The latter work has been completed in Ref. [9]. This report represents a summary of Ref. [9].

The coefficients $c_i$ and $d_i$ of the mass formula (1) have been obtained using the experimental masses of nonstrange and strange baryons from PDG [10]. In the numerical fit we considered the 17 resonances with a sta-
Experimentally one finds the operator coefficients indicating only a small \text{SO}(3) breaking. In the fit, the angular momentum components have small coefficients, although in the fit 1 they are ample in the fit 1 they are nearly equal to the mass formula. The spin and flavor operators contribute nearly equally to the mass formula. For example in the fit 1 they are \( c_3 = 129 \pm 19 \) MeV and \( c_4 = 167 \pm 12 \) MeV respectively and they contribute dominantly to the flavor-spin breaking. The matrix elements of \( O_3 \) and \( O_4 \) are of order \( 1/N_c \) except for \( O_4 \) in flavor singlets when they become of order \( \mathcal{O}(N_c^0) \) [9]. The expression of \( O_4 \) was introduced in Ref. [12] where it was shown that it recovers the expectation value of the isospin operators when \( N_f = 2 \). The operators containing the angular momentum components have small coefficients indicating only a small \text{SO}(3) breaking. In the fit 2 the operator \( O_7 \), which has a rather complex form, is removed and the \( \chi^2_{\text{ dof}} \) slightly improves. The \( \text{SU}(3) \) breaking is dominated by \( B_1 \) where \( \mathcal{J} \) is the strangeness.

The total mass and the partial contributions of the operators considered in the fit 2 are indicated in Table 2. The total masses have been obtained by including the mixing of \( N_f' \) and \( N_f (j = 1/2, 3/2) \) from Eq. (2) and the transformation matrix of \( \Lambda(S01) \) resonances defined by Eq. (3). It turns out that \( O_3 \) is dominant in the \( ^8 \text{S}i \) and \( O_4 \) in the decuplets and the flavor singlets. In the latter case the quantity \( c_4 \) is large and negative giving to \( N_f'_{1/2} \) a mass practically identical to the experimental value of \( \Lambda(1520) \). However this contribution is not enough for \( N_f'_{3/2} \) to be identified with \( \Lambda(1405) \). The situation is similar to all constituent quark models where \( \Lambda(1405) \) still raises serious problems having a mass at least 150 MeV above the experimental value [13].

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| Operator | \( O_1 = N_1 \) | \( O_2 = \partial x \) | \( O_3 = \frac{1}{2} S \vec{S} \) | \( O_4 = \frac{1}{2} (T^a T^a - \frac{1}{12} N_c (N_c + 6)) \) | \( O_5 = \frac{1}{2} \overleftrightarrow{L} T G_{1a} \) | \( O_6 = \frac{1}{2} \overleftrightarrow{L} G_{1a} \{ S^j, G_{1a} \} \) | \( B_1 = -J \) | \( B_2 = \frac{1}{2} S G_{18} - \frac{1}{2} \sqrt{3} O_3 \) |
|-----------------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( O_1 \) | 498 \pm 6 | 498 \pm 6 | | | | | 146 \pm 10 | 146 \pm 15 |
| \( O_2 \) | 5 \pm 6 | 6 \pm 6 | | | | | 78 \pm 69 | 74 \pm 69 |
| \( O_3 \) | 129 \pm 19 | 129 \pm 19 | | | | | | |
| \( O_4 \) | 167 \pm 12 | 165 \pm 11 | | | | | | |
| \( O_5 \) | 10 \pm 8 | 5 \pm 3 | | | | | | |
| \( O_6 \) | 9 \pm 1 | 9 \pm 1 | | | | | | |
| \( O_7 \) | -24 \pm 34 | | | | | | | |
| \( B_1 \) | | | | | | | | |
| \( B_2 \) | | | | | | | | |
| \( \chi^2_{\text{ dof}} \) | 1.42 | 1.33 | | | | | | |

### Table 1. Operators and their coefficients in the mass formula obtained from numerical fits. The values of \( c_i \) and \( d_i \) are indicated under the heading Fits \((n=1,2,3)\), in each case.
TABLE 2. The partial contribution and the total mass (MeV) predicted by the $1/N_c$ expansion obtained from the Fit 2. The last two columns give the empirically known masses [10] and the resonance name and status.

| Part. contrib. (MeV) | Total (MeV) | Exp. (MeV) | Name, status |
|----------------------|-------------|------------|--------------|
| $N^+_1$ | 1467 -2 32 41 -5 0 0 0 | 1513 ± 22 | 1538 ± 18 | $S_{11}(1535)$*** |
| $N^+_3$ | 1467 1 32 41 2 0 0 0 | 1542 ± 20 | 1523 ± 8 | $D_{13}(1520)$*** |
| $N^+_5$ | 1467 -5 162 41 -12 -18 0 0 | 1656 ± 22 | 1660 ± 20 | $S_{11}(1650)$*** |
| $N^+_7$ | 1467 -2 162 41 -5 15 0 0 | 1681 ± 20 | 1700 ± 50 | $D_{13}(1700)$*** |
| $N^+_9$ | 1467 3 162 41 7 -4 0 0 | 1677 ± 14 | 1678 ± 8 | $D_{15}(1675)$*** |
| $N^+_11$ | 1467 2 32 206 -10 0 0 0 | 1697 ± 18 | 1645 ± 30 | $S_{31}(1620)$*** |
| $N^+_13$ | 1467 -1 32 206 -10 0 0 0 | 1709 ± 19 | 1720 ± 50 | $D_{33}(1700)$*** |
| $N^+_15$ | 1467 -6 32 -124 0 0 0 0 | 1547 ± 41 | 1407 ± 4 | $S_{01}(1405)$*** |

$N_{3/2} - N^*_{3/2}$

| Part. contrib. (MeV) | Total (MeV) | Exp. (MeV) | Name, status |
|----------------------|-------------|------------|--------------|
| $N_{3/2} - N^*_{3/2}$ | 0 -2 0 0 -55 -2 0 0 | -55 | 0 0 18 | $S_{01}(1405)$*** |