An activity-based bottleneck model with stochastic capacity

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Abstract—This study investigated commuters’ departure time choice with stochastic capacity. It has typically been solved by using Vickrey’s bottleneck model which doesn’t take the impact of commuters’ activity on decision-making into account. We added activity utility parameters to Vickrey’s bottleneck model. Travel costs became negative utility. The model considered home-activity utility before work and work-activity utility in the morning rush. Simultaneously, the capacity of the bottleneck changed randomly between a designed value (good condition) and a degraded one (bad condition). We derived six reasonable equilibrium patterns by maximizing the commuters’ activity utility and closed form solutions to all possible equilibrium patterns. The departure rate curve for each equilibrium pattern was made within a reasonable parameter range.

1. INTRODUCTION
Urban congestion has become a serious problem all over the world, and in-depth study of traveler behavior characteristics plays a very important role in alleviating congestion. In 1969, Vickrey[1] first proposed the famous bottleneck model. When equilibrium, all people’s travel costs are equal, and no one can reduce the cost by changing his or her departure time. In 50 years of development of the bottleneck model, it has been applied in many ways, such as route choice[2,3], parking[4,5] and so on.

In real life, traffic is stochastic and dynamic due to various factors such as traffic accident, bad weather or other emergencies. Therefore, it is necessary to consider the influence of randomness on the traveler’s choice of departure time. A probability distribution function need to be selected to describe the uncertainty of the transportation system, such as uniform distribution[6-8], Gumbel distribution[9] and exponential distribution[10].

Most of studies above use trip-based models which simply think of commuting behavior as discrete trips. However, trip-based models cannot explain the motivations for trips and the time allocation for travel and activities. The departure-time choices commuters made in the morning and evening are usually associated to the utilities of home and work activities besides schedule delay and bottleneck congestion[11-14]. Activity-based models consider travel as the demand of participating in activities at different time and activity-travel patterns are the results of time-use decisions for a duration[15-17]. Based on these researches, this paper studies the relevant properties of the activity-based bottleneck model with stochastic capacity and derives the analytical solutions in reasonable equilibrium patterns and make the corresponding departure rate curve.

2. THE MATHEMATICAL MODEL
The travel cost of the classic bottleneck model can be expressed as,

\[ C(t) = \alpha T(t) + \beta SDE(t) + \gamma SDL(t) \] (1)
Which is composed of travel cost and schedule delay cost.

2.1. The activity-based bottleneck model
We extend the classic bottleneck model by considering commuters’ activity schedule throughout the morning. The new activity-based bottleneck model consists of two parts. One is the utility of travel (includes home-activity utility and work-activity utility) which is positive, the other is the disutility of travel (exactly the trip cost) which is negative. For derivation, we define home-activity utility and work-activity utility as constants and the activity-based bottleneck model is defined as,

\[
U(t) = \int_{T_0}^{T_1} \hat{g} dt + \int_{t(T)}^{T} \hat{k} dt - \alpha T(t) - \beta \max\{0, \omega - T(t) - t\} - \gamma \max\{0, T(t) + t - \omega\}
\]

(2)

2.2. Symbol definition
- \(T_0\): starting time of morning activity
- \(T_1\): ending time of morning activity
- \(\omega\): expected arrival time
- \(\hat{g}\): the value of home-activity utility
- \(\hat{k}\): the value of work-activity utility
- \(\alpha\): the unit cost of travel time
- \(\beta\): the unit cost of schedule delay early
- \(\gamma\): the unit cost of schedule delay late
- \(\theta\): the degradation ratio of capacity (\(0 < \theta \leq 1\))
- \(\pi\): the design capacity
- \(\pi\): the degradation probability of capacity (\(0 \leq \pi \leq 1\))
- \(r_i(t)\): the departure rate in the \(i^{th}\) situation at time \(t\)
- \(U(t)\): the total utility

3. Equilibrium Departure Rate Of Each Situation

3.1. Situations in the stochastic bottleneck
Because of the capacity of the bottleneck may change day by day with probability \(1 - \pi\) in good condition (the capacity is \(\pi\)) and probability \(\pi\) in bad condition (the capacity is \(\theta \pi\)), commuters may experience different schedule delay and queuing types in spite of they depart at the same time of day. Table 1 gives three types of schedule delay and queuing types in spite of they depart at the same time of day. Commuters experience six situations by combining schedule delay and queuing experience which are shown in Table 2.

| Schedule Delay Experience Types | Queuing Experience Types |
|---------------------------------|-------------------------|
| Always experience schedule delay early (SDE) | Always experience queuing (AQ) |
| Possibly experience schedule delay either early or late (SDE/L) | Possibly experience queuing (PQ) |
| Always experience schedule delay late (SDL) | |

Table 1 schedule delay and queuing types in the model
Table 2: Six possible situations in the model

| Situation | Combination |
|-----------|-------------|
| S1        | SDE+AQ      |
| S2        | SDE/L+AQ    |
| S3        | SDL+AQ      |
| S4        | SDL+PQ      |
| S5        | SDE/L+PQ    |
| S6        | SDE+PQ      |

3.2. The departure rate of each situation under equilibrium

The UE condition for the activity-based bottleneck model with stochastic capacity could be defined as follows: no commuter can reduce his/her total utility by changing his or her departure time at equilibrium.

The departure rates of situations in Table 2 are calculated as follows:

3.2.1. Commuters always experience schedule delay early and always experience queuing (SDE+AQ)

In this situation, commuters always arrive early and experience queuing regardless of the capacity of the bottleneck. The total utility of this situation is formulated as follows:

\[ U(t) = \left( \hat{g} + \alpha \right) t - \left( \hat{k} + \alpha - \beta \right) R(t) + \left( \hat{k} + \alpha - \beta \right) t_s - \hat{g} T_0 - \hat{k} T_1 - \beta \omega, \text{ or } \pi \]

The mean utility can be formulated as follows:

\[ EU(t) = \left( \hat{g} + \alpha \right) t - \left( \hat{k} + \alpha - \beta \right) R(t) \left( \frac{1 - \pi}{\pi} + \frac{\pi}{\theta} \right) - \left( \hat{k} + \alpha - \beta \right) t_s - \hat{g} T_0 - \hat{k} T_1 - \beta \omega \]

According to the UE condition, when \( \frac{dEU(t)}{dt} = 0 \), we can obtain the equilibrium departure rate, given as follows:

\[ r_1(t) = \frac{\left( \hat{g} + \alpha \right) \theta \pi}{\left( \hat{k} + \alpha - \beta \right) \left( \theta(1 - \pi) + \pi \right)} \]

3.2.2. Commuters possibly experience schedule delay either early or late, and always experience queuing (SDE/L+AQ)

Identically, we can obtain the equilibrium departure rate given as follows:

\[ r_2(t) = \frac{\left( \hat{g} + \alpha \right) \theta \pi}{\pi \left( \gamma + \alpha + \hat{k} \right) \theta(1 - \pi) \left( \beta - \alpha - \hat{k} \right)} \]

3.2.3. Commuters always experience schedule delay late and always experience queuing (SDL+AQ)

The equilibrium departure rate is

\[ r_3(t) = \frac{\left( \hat{g} + \alpha \right) \theta \pi}{\left( \alpha + \gamma + \hat{k} \right) \theta(1 - \pi) \left( \theta(1 - \pi) + \pi \right)} \]

3.2.4. Commuters always experience schedule delay late and possibly experience queuing (SDL+PQ)

The equilibrium departure rate is

\[ r_4(t) = \frac{\left[ \hat{g} + \pi \alpha - (1 - \pi) \left( \hat{k} + \gamma \right) \right] \theta \pi}{\pi \left( \hat{k} + \alpha + \gamma \right)} \]

3.2.5. Commuters possibly experience schedule delay either early or late, and possibly experience queuing (SDE/L+PQ)

The equilibrium departure rate is
\[ r_k(t) = \frac{\hat{g} + \pi \alpha - (1 - \pi)(\hat{k} - \beta)}{\pi(\hat{k} + \alpha + \gamma)} \]  

(9)

3.2.6. Commuters always experience schedule delay early, and possibly experience queuing (SDE+PQ)

The equilibrium departure rate is

\[ r_0(t) = \frac{\hat{g} + \pi \alpha - (1 - \pi)(\hat{k} - \beta)}{\pi(\hat{k} + \alpha - \beta)} \]  

(10)

4. EQUILIBRIUM DEPARTURE PATTERNS

Although there are six situations that commuters will experience, these situations could not be combined in one particular equilibrium pattern at the same time. In fact, there are only six realistic equilibrium patterns exist, which are shown in Table 3.

| Pattern | 1 | 2 | 3 | 4 | 5 | 6 |
|---------|---|---|---|---|---|---|
| S1      | S1 | S6 | S1 | S1 | S6 |
| S2      | S2 | S5 | S2 | S2 | S5 |
| S3      | S5 | S4 | S3 | S5 |
| S4      | S4 |   |   |   |

Table 3: Six patterns in the model

By calculating the critical time points of these six patterns, we can draw the curve of departure rate of each pattern, which are shown in Figure 1.
5. **Boundary Conditions for Equilibrium Departure Patterns**

Each equilibrium departure pattern meets different constraints, we call them boundary conditions to divide the patterns.

Take Pattern 1 as an example, it must meet the following four conditions. Condition 1: the equilibrium departure rate \( r_1(t) \geq \Xi \). Condition 2: the critical time point \( t_{12} \leq t_{13} \). Condition 3: the critical time point \( t_{13} \leq t_e \). Condition 4: the equilibrium departure rates \( r_1(t), r_2(t), r_3(t) \) and \( r_4(t) > 0 \). Through these four conditions, the boundary condition of Pattern 1 is

\[
\hat{g} - \frac{\hat{k} + \beta}{\hat{k} + \alpha + \gamma} \leq \pi \leq \frac{\hat{g} - \hat{k} + \beta}{\hat{k} + \alpha + \gamma}(1 - \theta).
\]

Similarly, we can get other patterns’ boundary conditions. For easy expression, we define several composite parameters:

\[
\pi_1 = \frac{\hat{k} - \hat{g} + \gamma}{\hat{k} + \alpha + \gamma}; \quad \pi_2 = \frac{\hat{g} - \hat{k} + \beta}{\hat{k} + \alpha + \gamma}(1 - \theta); \quad \pi_3 = \frac{\hat{g} - \hat{k} + \beta}{\hat{k} + \alpha + \gamma}(1 - \theta); \quad \pi_4 = \frac{\theta}{\hat{k} + \alpha + \gamma - \theta(\hat{k} + \alpha - \beta)}.
\]

Figure 2 shows the typical diagram which exhibits the six equilibrium patterns.
6. CONCLUSION
This paper establishes an activity-based bottleneck model with stochastic capacity to study the
departure time choice behavior of commuters in the morning peak. Analytical solutions of the
departure rate of each situation under equilibrium are calculated and the departure rate curves for six
equilibrium patterns are made. In the future, the utility of the evening peak can be further considered
into the model.

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