Multiparty Dependent Session Types
(Extended Abstract)

Hanwen Wu and Hongwei Xi
Boston University, Boston MA 02215, USA
\{hwwu, hwxi\}@bu.edu

Abstract. Programs are more distributed and concurrent today than ever before, and structural communications are at the core. Constructing and debugging such programs are hard due to the lack of formal specification/verification of concurrency. This work formalizes the first multiparty dependent session types as an expressive and practical type discipline for enforcing communication protocols. The type system is formulated in the setting of multi-threaded \(\lambda\)-calculus with inspirations from multirole logic, a generalization of classical logic we discovered earlier. We prove its soundness by a novel technique called deadlock-freeness reducibility. The soundness of the type system implies communication fidelity and absence of deadlock.

Keywords: session types · applied type system · dependent types · linear types · multirole logic · concurrency · deadlock-free

1 Introduction

Session type [10,15,11] is a typed formalism for concurrency. A session is an abstraction of structured communication among two or more logical parties connected by a communication channel. Session types denote the structures of communications, or protocols, and are assigned to communication channels. In a typical session type system, subjection reduction ensures session fidelity and progress property ensures absence of deadlock. As a result, well-session-typed programs cannot make communication errors.

In this work, we present our research results on a practical multiparty dependent session type system. It is a system that can describe more than two participants (multiparty), that supports quantification and polymorphism in the session type (dependent), and that it is formulated in the settings of multi-threaded \(\lambda\)-calculus (practical). Other features include higher-order sessions (sending channels over channels), forwarding (connecting channels with channels), recursive sessions (as an extension), etc. A well-session-typed program strictly follows the session protocol (subject reduction) and is absent of deadlock (progress). To the best knowledge of the authors, this is the first formulation of a multiparty dependent session type system.
1.1 Simple Examples

We first fix some terminologies. A *session* has several *parties* connected via a *channel*. *Session types* encode communication structures *globally*. A channel has many endpoints. Each party is (typically) implemented as a thread, and each thread function is given an *endpoint*. Endpoints are assigned an *endpoint type* representing the *local protocol* from the perspective of this particular party.

*Example 1.* A simple “Hello World” protocol `hello` between server `S` and client `C`, can be described using session types as follows.

```
hello ::= msg(C, string) :: msg(S, string) :: end(C)
```

This protocol specifies the communications globally, where `C` first sends a message of type `string`, followed by `S` sending also a `string`, followed by `C` terminating the session while `S` waits for the termination. Locally at each party, `C` holds an endpoint of type `chan(C, hello)` while `S` holds an endpoint of type `chan(S, hello)` where the linear type constructor `chan` combines `hello` with a role `C` or `S` to form local endpoint types. A program for `C` can be written as `cli`, while the server is `srv`.

```
cli ::= lam c. let c = bsend(c, 'hello') in let ⟨c, rpl⟩ = brecv(c) in close(c)
```

```
srv ::= lam s. let ⟨s, req⟩ = brecv(s) in let s = bsend(s, 'world') in wait(s)
pool ::= let s = fork(cli) in app(srv, s)
```

`bsend/brecv` etc., are Session APIs provided by our type system used to realize the dynamic semantics of session types by interpreting them locally at each party. The type system will guarantee that the correct API is invoked at the correct stage of protocol in the correct order and that all endpoints are invoking *dual/compatible* APIs in order to make progress. Finally in `pool`, `fork` spawns a new thread with thread function `cli`, and connects to that thread with a session typed channel while returning the other endpoint `s` to the caller.

*Example 2.* With quantification in the session types, one can safely send an array by firstly sending a length `n` followed by `n` repeated messages for `n` elements of the array.

```
array(τ:type) ::= quan(C, λn:int.msg(C, int(n)) :: repeat(τ, n))
```

In the above definition, `int(n)` is a singleton dependent type for an integer of value `n`, `quan` is a session type constructor that represents a quantifier, where `λn:int` is the actual binder. The quantifier in this case will be interpreted as universal by `C`, and existential by all others. For instance, the endpoint at `C` will have linear type `chan(C, array(τ))`, and after invoking API `unify` on this endpoint, the type becomes `∀n:int.chan(C, msg(C, int(n)) :: repeat(τ, n))`. The bound variable `n` ensures that the length of the array equals the number of repeated messages that follows.
1.2 Contributions

The main contribution lies in the formalization of multiparty dependent session types and its deadlock-freeness proof via a novel technique named deadlock-freeness reducibility. We summarize the contributions as follows.

- Formalized the first multiparty dependent session type system ($\lambda^\forall_{\psi, \exists}$) and proved its soundness.
- Formalized deadlock-freeness reducibility. It is a pool reduction invariant, even in the presence of higher-order sessions and various forms of forwarding. The progress property directly depends on the df-reducibility of thread pools.
- Discovered and formulated classical multirole logics (MRL) and linear multirole logic (LMRL) as generalizations of classical logic (LK) and classical linear logic (CLL). We proved the admissibility of a cut rule that combines more than two sequents in both MRL and LMRL, thus generalizing Gentzen’s celebrated results of cut-elimination.
- We report that we have work-in-progress implementations. Due to space limits, we omit this part and please see http://multirolelogic.org.

1.3 Overview

This work has three parts, and we focus mostly on the third part below. First, our technical foundation is \textit{ATS} [20]. We very briefly mention the approaches to formulating types, reasoning about resources, and adding pre-defined functions in \textit{ATS}. Second, we mention the intuitions of MRL/LMRL and present the generalized cut rule combining more than two sequents. We discuss how LMRL deeply influenced the design of $\lambda^\forall_{\psi, \exists}$. Third, we formulate $\lambda^\forall_{\psi, \exists}$ in \textit{ATS} and show the deadlock-freeness reducibility proof.

2 Applied Type System

$\lambda^\forall_{\psi, \exists}$ is built on Applied Type System (\textit{ATS}), a multi-threaded $\lambda$-calculus with advanced types. \textit{ATS} [20,22] is the successor of Dependent ML [21,19,23]. As a general framework for formalizing type systems, \textit{ATS} supports dependent types of DML-style, linear types, theorem proving, general recursion, among other features. Due to space limits, we will only cover selected features of \textit{ATS} to prepare for the development of $\lambda^\forall_{\psi, \exists}$. Please refer to [20,22] for a full treatment.

2.1 Preview

The key salient feature of \textit{ATS} lies in the complete separation between \textit{statics}, where types are formed and reasoned about, and \textit{dynamics}, where programs are constructed and evaluated.

Types in the dynamics are terms of the statics, where statics can be regarded as a simply-typed $\lambda$-calculus whose “types” are called \textit{sorts}. Types can depend
on static terms (hence dependent types). In $\lambda^\pi\forall_3\exists$, we will be formulating session types $\pi$ in the statics, so that it can be used as an index for the endpoint type, i.e., $\text{chan}(rs, \pi)$. After having terms of endpoint types, we will provide programmers with pre-defined functions called Session APIs to manipulate them, like $\text{bsend/brecv}$. They are formulated in the dynamics as constant functions $\text{dcf}$ in $\mathcal{ATS}$. Their pre-defined types, called dc-types (dynamic constant types), will be carefully designed and recorded in a pre-defined context called signature $\mathcal{S}$ to perform type-checking. Their reductions in a thread pool are also formulated and reasoned about.

Endpoints are resources, meaning they cannot be randomly copied or discarded. Resource ownership has to be tracked by the type system to prevent bugs like memory leaks, use-after-free, etc. The support for linear types in $\mathcal{ATS}$ provides a mechanism for reasoning about resources. We will define endpoints as dynamic resources $\text{dcr}$, define the function $\rho(\cdot)$ (Figure 3) to collect resources, define its consistency conditions to prevent ill-formed channels (e.g., missing endpoints, duplicated endpoints), and prove that resource consistency is an invariant during reduction. Linear types also provide a way to track the progress of protocols. For instance, given an endpoint of $\text{chan}(C, \text{msg}(C, \text{int}(C)) :: \pi)$, invoking $\text{bsend}$ at party $C$ will send a message, consume this linear endpoint, and return the endpoint with a new type $\text{chan}(C, \pi)$. And only this endpoint with the continuation of the protocol is available for use in the typing context.

### 2.2 $\mathcal{ATS}$

The development of $\mathcal{ATS}$ is fairly standard. It has an ML-like syntax (Figure 1), non-linear/linear split context typings (Figure 2), and call-by-value, left-to-right, reduction semantics (Figure 4). Since $\mathcal{ATS}$ is not the contribution of this work, we will only illustrate some concepts using examples.

**Example 3.** Consider the dynamic term (in red) and its type (in blue) below.

$$\text{lam} x. \text{lam} y.x/y : \forall m:\text{int}.\forall n:\text{int}(n \neq 0) \supset (\text{int}(m), \text{int}(n)) \rightarrow \text{int}(m/n)$$

The term represents a function that does integer division (with the result rounded to the nearest integer.) Given a static integer $i$, $\text{int}(i)$ is a singleton type representing a dynamic integer whose value equals to $i$, where $\text{int}$ is a type constructor of sort $\text{int} \Rightarrow \text{type}$. We use $P$ for propositions, i.e. static terms of sort $\text{bool}$. Given $P$, $P \supset \tau$ is a guarded type. Intuitively, if a value $v$ is assigned a guarded type $P \supset \tau$, then $v$ can be used only if the guard $P$ is satisfied. The whole type is a universally quantified, guarded, function type that reads, for all (quantified) static integers $m$ and $n$ where $n \neq 0$ (guarded), we form a function type where given two integers whose values are $m$ and $n$, returns a integer whose value is $m/n$ where $\div$ should be interpreted as a static integer division with the result rounded to the nearest integer. Importantly, division-by-zero will be an type error.

Thread pool $\Pi$ is a collection of mappings $t:e$ from thread id to closed expressions. Typing judgement is of the form $\Sigma; \hat{P}; \Gamma; \Delta \vdash e : \hat{\tau}$ where $\Sigma$ is the sorting
Syntax of Statics

- base sorts: \( b ::= int \ | \ bool \ | \ type \ | \ vtype \)
- sorts: \( \sigma ::= b \ | \sigma_1 \rightarrow \sigma_2 \)
- static constants: \( \text{sex} ::= \text{sec} \ | \ \text{scf} \)
- static terms: \( s ::= a \ | \text{sex}(\tilde{x}) \ | \lambda x.\sigma.s \ | s_1(s_2) \)

Syntax of Types

- types: \( \tau ::= a \ | \delta(\tilde{x}) \ | \text{unit} \ | \tau_1 \times \tau_2 \ | \tau_1 \rightarrow \tau_2 \ | \bigwedge P \ | \bigvee P \ | \forall \alpha.\sigma.\tau \ | \exists \alpha.\sigma.\tau \)
- linear types: \( \hat{\tau} ::= a \ | \delta(\tilde{x}) \ | \tau \ | \hat{\tau}_1 \rightarrow \hat{\tau}_2 \ | \hat{\tau}_1 \rightarrow \tau_2 \ | \bigwedge P \hat{\tau} \ | \bigvee P \hat{\tau} \ | \forall \alpha.\sigma.\hat{\tau} \ | \exists \alpha.\sigma.\hat{\tau} \)

Syntax of Dynamics

- constants: \( \text{dxc} ::= \text{dec} \ | \ \text{def} \ | \ \text{dcr} \ | \ \text{lam} \ x.e \ | \ \text{app}(e_1, e_2) \ | \ \text{snd}:\text{fst}(e) \ | \ \text{snd}(e) \ | \ \text{let}(x_1, x_2) = e_1 \ \text{in} \ e_2 \ | \bigvee^+(v) \ | \bigvee^-(v) \ | \bigwedge(v) \ | \bigvee^+(v) \ | \exists(v) \)

Some Signatures of Dynamic Constants

\[
\text{fork} : (\text{unit} \rightarrow \text{unit}) \Rightarrow \text{unit}
\]

Fig. 1. Selected Syntax of ATS

\[\Sigma; \overline{P}; \Gamma; \Delta \vdash e : \tau\]

\[
\begin{array}{c}
\Sigma; P; F; \Delta \vdash \text{dec} : \tilde{x} \\
\Sigma; \overline{P}; F; \Delta \vdash \text{dec} : \tilde{x}
\end{array}
\]

\[\Sigma; P; F; \Delta \vdash \text{dcr} : \tilde{x} \]

\[\Sigma; P; F; \Delta \vdash \text{dcr} : \tilde{x}
\]

\[\Sigma; P; F; \Delta \vdash \lambda x.\sigma.s : \forall \alpha.\sigma.\tau
\]

\[\Sigma; P; F; \Delta \vdash \forall^+(v) : \forall \alpha.\sigma.\hat{\tau}
\]

\[\Sigma; P; F; \Delta \vdash \forall^-(v) : \forall \alpha.\sigma.\hat{\tau}
\]

\[\Sigma; P; F; \Delta \vdash \exists(v) : \exists \alpha.\sigma.\hat{\tau}
\]

\[\Sigma; P; F; \Delta \vdash \text{let } \exists(x) = e_1 \ \text{in} \ e_2 : \hat{\tau}_2
\]

\[\Sigma; P; F; \Delta \vdash \text{let } \exists(x) = e_1 \ \text{in} \ e_2 : \hat{\tau}_2
\]

Fig. 2. Selected Typing Rules of ATS

- \(\rho(\text{dcr}) = \{ \text{dcr} \}
- \rho(x) = \emptyset
- \rho(()) = \emptyset
- \rho((e_1, e_2)) = \rho(e_1) \cup \rho(e_2)
- \rho(\text{fst}(e)) = \rho(e)
- \rho(\text{snd}(e)) = \rho(e)
- \rho(\text{app}(e_1, e_2)) = \rho(e_1) \cup \rho(e_2)

Fig. 3. Selected Definition of \(\rho(\cdot)\)
Redexes

pure redex ::= \( \text{app}(\text{lam} \, x, \, e, \, v) \mid \text{let} \, \langle x_1, \, x_2 \rangle = \langle v_1, \, v_2 \rangle \, \text{in} \, e \mid \text{fst}(\langle v_1, \, v_2 \rangle) \mid \text{let} \, \exists (x) = \exists (v) \, \text{in} \, e \mid \text{snd}(\langle v_1, \, v_2 \rangle) \mid \text{\&}(\text{let} \, x = \text{\&}(v) \, \text{in} \, e) \mid \forall (\text{let} \, x = \forall (v) \, \text{in} \, e) \mid \text{dcf}((\vec{v})) \)

ad-hoc redex ::= \( \text{dcf}(\vec{v}) \)

Contractums

(tr-beta) \( \text{app}(\text{lam} \, x, \, e, \, v) \xrightarrow{v} e[x \mapsto v] \)
(tr-guard) \( \nabla (\Delta^+(v)) \xrightarrow{v} v \)
(tr-adhoc) \( \text{dcf}(\vec{v}) \xrightarrow{v} v' \) if def is defined at \( \vec{v} \) and the result is \( v' \)

Reductions

\( E[e] \xrightarrow{e} E[e'] \) if \( e \xrightarrow{v} e' \)

Pool Reductions

(pr-fork) \( \Pi, \, t:E[\text{fork}(\text{lam} \, x, \, e)] \xrightarrow{P} \Pi, \, t:E[\langle \rangle], \, t':e[x \mapsto \langle \rangle] \)
(pr-gc) \( \Pi, \, t:\langle \rangle \xrightarrow{P} \Pi \) if \( t > 0 \)
(pr-lift) \( \Pi, \, t:e \xrightarrow{P} \Pi, \, t':e' \) if \( e \xrightarrow{v} e' \)

Fig. 4. Selected Reductions in \( \mathcal{ATS} \)

column, \( \vec{P} \) is a set of propositions representing constraints, \( \Gamma \) is the non-linear typing context, and \( \Delta \) is the linear typing context. We may simply write \( \vdash e : \hat{\tau} \) if all contexts are empty. Type equality is defined in terms of subtyping relations. Type-checking is reduced into constraint-solving in some constraint-domain.

Lastly, we mention that \( \mathcal{ATS} \) is sound. Please refer to [20] for the proof of subject reduction and the progress property.

3 Multirole Logic

Along the lines of [1,2,4,17,5] that interpret cut reductions (i.e., cut-elimination steps) as communications between two parties in some session-typed process calculi, we seek to find a logic that admits a cut rule combining more than two sequents as a foundation for multiparty session types. The notion of duality, which is an inexplicit side condition for the traditional cut rule, has to be generalized to account for the coherence/compatibility of multiple sequents in a cut, too. This led us to the discovery of multirole logic. Giving a full account of MRL in this work is infeasible. We only present our insights relevant to \( \lambda^{\forall,\exists} \). In short, all the propositions in the guarded types of Session APIs are directly influenced by the side conditions of inference rules of MRL/LMRL. Please refer to [25] for details.

The intuition is best summarized in Figure 5 with selected rules from two-sided, one-sided, and “many-sided” sequent calculus for classical logic. The rules
for the two-sided sequent calculus is well-known to be symmetric. The \(\neg L\) and \(\neg R\) rules provide a way to move a formula from one side to the other while remembering how many times a formula has been moved. Due to involutive negation, we have the equivalent one-sided presentation, where formulas are identified up to de Morgan duality. One possible explanation for this duality is to think of the availability of two roles 0 and 1 s.t. the left side of \(\vdash\) plays role 1 while the right side does role 0. Negation is still about changing roles/sides.

With this explanation, we can write the \(\text{id}\) rule in the following ways in Figure 5. In \(\text{id-two-sided}\) and \(\text{id-one-sided}\), the subscript 0 and 1 denotes sides, and \(\vdash\) both separate sides and denotes derivability. Note how negation changes the side of \(A\) from 1 to 0 while remembering it has been moved once. In \(\text{id-two-sided-on-one-side}\), we still have two sides, denoted by the subscripts 0 and 1, except that we write them both on the right of \(\vdash\). In this case, \(\vdash\) no longer separate sides. It merely is a meta-symbol denoting the derivability of formulas on its right. Using the style of \(\text{id-two-sided-on-one-side}\), it seems entirely natural for us to introduce more roles into classical logic like in this three-sided sequent \(\vdash [A]_0, [A]_1, [A]_2\). This leads us to multirole logic.

Figure 6 presents the syntax and inference rules of (classical) MRL. A formula, combined with a role set \(R\) (represented as a set of integers) within which the formula should be interpreted, is an \(i\)-formula. Note that MRL is parameterized by some underlying full set of roles \(\mathcal{R}\), potentially infinite. Negation is generalized into endomorphism \(f\) whose rules involves changing roles \(R\), that corresponds to our intuition of changing sides. Connectives \(\land\) and \(\lor\) are generalized into ultrafilters \(\mathcal{U}\), where \(\mathcal{U}\) are interpreted based on roles, as illustrated in rule \(\lor_1, \lor_2\), and \(\land\) where the rules are named after the connectives’ intended meanings. Similarly, quantifiers \(\forall\) and \(\exists\) are generalized into ultrafilters, this time written as \(\mathcal{U}^\lambda\) to avoid conflicts.

Among all the results, the most important ones are the admissibility of the followings. Note that \(2\)-cut-residual is by far the most general form of cut, which cannot be formulated in (traditional) classical logic. In this rule, a cut may not be a complete cut in that it leaves a residual \(i\)-formula whose roles are the intersection of the original cut formulas. In LK/CLL with only two roles available, the intersection is always empty, thus making this the same as a regular cut. Multirole reveals this subtlety hidden in plain sight!

**Lemma 1 (Admissibility of Cut).** The following rules are admissible.
Formulas

\[ A, B ::= a | f(A) | A \cup B | \forall \alpha x. A \]

\[ \alpha \text{-formulas} \]

\[ [A]_R \]

\[ \begin{align*}
R \cup \cdots \cup R_n &= \overline{\text{id}} \\
\vdash [a]_{R_1}, \ldots, [a]_{R_n} \quad &\vdash \Gamma, [A]_{R_1} \quad &\vdash \Gamma, [\forall \alpha x. A]_R \quad &\vdash \Gamma, [f(A)]_R
\end{align*} \]

\[ \begin{align*}
R \notin \forall 1 &\vdash \Gamma, [A \cup B]_R \\
R \notin \forall 2 &\vdash \Gamma, [A \cup B]_R \\
R \notin \forall &\vdash \Gamma, [\forall \alpha x. A]_R
\end{align*} \]

Fig. 6. Multirole Logic

We omit the discussion of linear multirole logic (LMRL). In MRL, formulas (including connectives) are “global,” \( \alpha \)-formulas are “local,” and inference rules interpret connectives locally. As mentioned in the beginning, session types are global, endpoint types are local, and session APIs interpret global session types locally. The design of \( \lambda^\exists_{q, \exists} \) comes from this insights of MRL/LMRL.

4 Multiparty Dependent Session Types

In this section, we first introduce the syntax and semantics of \( \lambda^\exists_{q, \exists} \), mention some extensions and examples, then prove its soundness via deadlock-freeness reducibility.

Syntax and Static Semantics The syntax is listed in Figure 7. We add \textit{static type} to the statics as a new base sorts. Static terms of sort \textit{static type} are session types. We add \textit{set} as a new base sort for static integer sets to represent the roles of a party. We use \( \emptyset \) for empty set and \( \overline{\emptyset} \) for full set. We assume the existence of static constant functions for basic set operations, e.g. \( \forall \) for disjoint union, \( \exists \) for complement w.r.t. \( \emptyset \), and \( \setminus \) for set minus.

We use \( \pi \) for session types. We add session type constructors \textit{end}, \textit{msg}, and \textit{quan}. Their \( \alpha \)-sorts are also given in the signature \( \mathcal{S} \). Again, \( \pi \) describe protocols \textit{globally}. \( \text{end}(r) \) means party \( r \) is to close the channel, while other parties will wait. \( \text{msg}(r, \tau) :: \pi \) means party \( r \) is to broadcast a value of non-linear type \( \tau \), then proceed according to \( \pi \), while others are to receive a value of that type, then proceed according to \( \pi \). We overload the name \text{msg} and use \text{msg}(r_1, r_2, \tau) :: \pi for sending \textit{point-to-point} \textit{linear messages}, from \( r_1 \) to \( r_2 \), while others will skip the message and continue the session following \( \pi \). \textit{quan} has sort scheme \( \text{int}, \sigma \rightarrow \text{static type} \Rightarrow \text{static type} \). It takes a role, and a \textit{static function} denoting the binder of the quantification from sort \( \sigma \) to a session type, to construct a
Additional Syntax of Statics

- **Base sorts**: 
  - $b ::= \cdots \mid \text{set} \mid \text{stype}$
- **Roles**: 
  - $r ::= \cdots \mid -1 \mid 0 \mid 1 \mid \cdots$
- **Role sets**: 
  - $rs ::= \emptyset \mid \{ r_1, \ldots, r_n \} \mid rs_1 \uplus rs_2 \mid rs_1 \cup rs_2 \mid rs_1 \cap rs_2 \mid \cdots$
- **Session types**: 
  - $\pi ::= \text{end}(r) \mid \text{msg}(r, \tau) ::= \pi \mid \text{msg}(r_1, r_2, \tau) ::= \pi \mid \text{quan}(r, \lambda \sigma. \pi)$
- **Linear base types**: 
  - $\delta(\pi) ::= \cdots \mid \text{chan}(rs, \pi)$
- **Dynamic constant resources**: 
  - $\text{dcr} ::= \cdots \mid c^{rs}$
- **Signature**: 
  - $S ::= \cdots \mid S, c : \pi$

Additional Signature of Static Constants

- $\text{chan} : (\text{set}, \text{stype}) \Rightarrow v\text{type}$
- $\text{end} : (\text{int}) \Rightarrow \text{stype}$
- $\text{msg} : (\text{int}, \text{type}, \text{stype}) \Rightarrow \text{stype}$

Additional Typings

$$S, c : \pi \vdash c^{rs} : \text{chan}(rs, \pi)$$

Additional Signature of Dynamic Constants

- Fork: $\forall r_1, r_2 : \text{set}. \forall \pi : \text{stype}. (r_1 \uplus r_2 = \emptyset) \Rightarrow (\text{chan}(r_1, \pi) \rightarrow \text{unit}) \Rightarrow \text{chan}(r_2, \pi)$
- Cut: $\forall r_1, r_2 : \text{set}. \forall \pi : \text{stype}. (r_1 \uplus r_2 = \emptyset) \Rightarrow (\text{chan}(r_1, \pi), \text{chan}(r_2, \pi)) \Rightarrow \text{chan}(r_1 \cap r_2, \pi)$
- Elim: $\forall \pi : \text{stype}. \text{chan}(\emptyset, \pi) \Rightarrow \text{unit}$
- Split: $\forall r_1, r_2 : \text{set}. \forall \pi : \text{stype}. (r_1 \uplus r_2 = \emptyset) \Rightarrow (\text{chan}(r_1 \uplus r_2, \pi), \text{chan}(r_1, \pi) \rightarrow \text{unit}) \Rightarrow \text{chan}(r_2, \pi)$
- Send: $\forall r : \text{set}. \forall \pi : \text{stype}. (r \in \pi \Rightarrow \text{chan}(r, \text{msg}(\tau, \pi)) : \tau) \Rightarrow \text{chan}(r, \pi)$
- Recv: $\forall r : \text{set}. \forall \pi : \text{stype}. (r \notin \pi \Rightarrow \text{chan}(r, \text{msg}(\tau, \pi))) \Rightarrow \text{chan}(r, \pi)$
- Skip: $\forall r_1, r_2 : \text{set}. \forall \pi : \text{stype}. (r_1 \notin \pi \Rightarrow \text{chan}(r_1, \text{msg}(\tau, \pi))) \Rightarrow \text{chan}(r_2, \pi)$
- Close: $\forall r_1, r_2 : \text{set}. \forall \pi : \text{stype}. (r_1 \in \pi \Rightarrow \text{chan}(r_1, \text{end}()) \Rightarrow \text{unit})
- Wait: $\forall r_1, r_2 : \text{set}. \forall \pi : \text{stype}. (r_1 \notin \pi \Rightarrow \text{chan}(r_1, \text{end}()) \Rightarrow \text{unit})$
- Unify: $\forall r_1, r_2 : \text{set}. \forall \pi : \text{stype}. (r_1 \in \pi \Rightarrow \text{chan}(r_1, \text{quan}(\tau, f))) \Rightarrow \text{chan}(r_2, \text{f}())$
- End: $\forall r_1, r_2 : \text{set}. \forall \pi : \text{stype}. (r_1 \notin \pi \Rightarrow \text{chan}(r_1, \text{quan}(\tau, f))) \Rightarrow \exists \text{chan}(r_2, \text{f}())$

Fig. 7. Additional Syntax and Typings of $\lambda^{rs}_{n, \beta}$

Session type that represents a global quantifier in the session type. The first argument denotes a role of a party who will interpret the quantification as universal, while the others will interpret the quantification as existential as in Example 2.

Linear base type constructor $\text{chan}$ are for endpoint types. Given a set $rs$ representing the roles played by this endpoint, and a session type $\pi$, $\text{chan}(rs, \pi)$ is the linear type we assign to the endpoint of roles $rs$ in the session $\pi$. In LMRL syntax, $\text{chan}(rs, \pi)$ is an analogy to $[\pi]_r$ with a formula $\pi$ and a role set $rs$. Note that we inexplicit assume some underlying full set $\forall \pi$ and omit it for brevity.

Endpoints are resources. We use $c$ for channels, and $c^{rs}$ for an endpoint of $c$ with roles $rs$. We classify $c^{rs}$ as $\text{dcr}$. To facilitate presentation, we define the followings.
Definition 1. Given a multiset of resources $R$, i.e. the result of $\rho(\cdot)$ on some term, we define the following functions.

All channels in $R$ 
$\text{channels}(R) := \{c | c^{rs} \in R\}$ set

All endpoints in $R$ 
$\text{endpoints}(R) := \{c^{rs} | c^{rs} \in R\}$ multiset

All endpoints of $c$ in $R$ 
$\text{endpoints}(R, c) := \{c^{rs}_0 | c^{rs}_0 \in R, c_0 = c\}$ multiset

We also write $\text{endpoints}(c)$ to mean the set of all endpoints of channel $c$ where the disjoint union of their roles is the underlying full set for this session. Note that if $\text{consistent}(R)$, then all of these functions result in $\text{sets}$. From now on we simply assume that they are sets.

Definition 2 (Consistency of Resources). Given a multiset of resources $R$, we define the consistency of $R$ as,

- $\text{consistent}(\emptyset)$
- $\text{consistent}(R \uplus \text{endpoints}(c))$ iff $\text{consistent}(R)$, $\text{endpoints}(R, c) = \emptyset$, and roles of all endpoints of $c$ forms a partition of $\emptyset$.

To assign linear types to endpoints, we add a new constant typing rule $\text{sig-chan}$ that says, if $c$ has session type $\pi$ in the signature, then its endpoint $c^{rs}$ has linear type $\text{chan}(rs, \pi)$. Note that when a protocol advances, the signature will change accordingly. All other aspects of the static semantics are the same as in $\text{ATS}$.

Session APIs

Session APIs provide local interpretations of global session types. The dc-types assigned to them (Figure 7) ensure correct and coherent local interpretations.

Session type for broadcasting, $\text{msg}(r, \tau) :: \pi$, is interpreted by a pair $\text{bsend}$ and $\text{brecv}$. The dc-type for $\text{bsend}$ says, given roles $rs$, a role $r$, a session type $\pi$, and a type $\tau$, if $r \in rs$, then given an endpoint of roles $rs$ whose type is $\text{chan}(rs, \text{msg}(r, \tau) :: \pi)$, and a message of type $\tau$, $\text{bsend}$ will broadcast the message, and return the endpoint indexed by the continuation of the protocol, which is $\pi$. This is shown in reduction rule $\text{pr-bmsg}$. Note that the $\text{msg}(r, \tau)$ part is consumed. The type system mandates that the head of the protocol must be $\text{msg}(r, \tau)$ and that this $r$ in the protocol must belongs to the roles of the endpoint, $r \in rs$. Only when this proposition of the guarded type can be proven true, that this function invocation is well-typed. Correspondingly, only when $r \notin rs$ that one can invoke $\text{brecv}$ to receive such broadcasting messages from party $r$.

It can be proven that given a role $r$ as in $\text{msg}(r, \tau) :: \pi$, given a consistent collection of endpoints of $c$ whose roles should form a partition of some full set, there will be exactly one endpoint whose roles contains $r$. Therefore, in a well-typed pool, there can be only one thread invoking $\text{bsend}$ at this point, while all other threads connected by this channel can only invoke $\text{brecv}$. Also, since only $\text{bsend}$ and $\text{brecv}$ can deal with protocols starting with $\text{msg}(r, \tau)$, any other session APIs invoked for this protocol will be ill-typed. Since endpoint types are linear, one can only invoke these functions once, and then must proceed to the continuation of the protocol, denoted by $:: \pi$. All combined, each endpoint is
guaranteed to follow the protocol strictly, and all endpoints are guaranteed to be coherent/compatible with others in the same session.

Session type \( \text{msg}(r_1, r_2, \hat{r}) :: \pi \) is interpreted by \texttt{send}, \texttt{recv}, and \texttt{skip} for sending (at party \( r_1 \)), receiving (at party \( r_2 \)), or simply ignoring (at all others), as in \texttt{pr-msg}. Self-looping messages are disallowed. \texttt{skip} is a proof function, meaning it simply changes the type of the endpoint and has no runtime effects. It can be eliminated safely after type-checking. Note that since there is only one sending party and one receiving party, it is now possible to exchange linear data. This means that endpoints as linear data can be exchanged, resulting in higher-order sessions. This is one of the factors that makes the proof of deadlock-freeness much more involved. Session type \texttt{end}(r) is interpreted by \texttt{close} at party \( r \) and \texttt{wait} at all other parties, as in \texttt{pr-end}.

Quantification in the session types has never been treated in the \( \lambda \)-calculus setting before. This work is the first of its kind to support quantification and polymorphism in the session types. \texttt{quan}(r, \lambda a. \sigma. \pi) is a quantifier that needs interpretation, just as \( H^\lambda \) in LMRL. It is interpreted by \texttt{unify} at party \( r \) as universal quantification and \texttt{exify} at all other parties as existential quantification, shown in \texttt{pr-quan} and Example 2. They are proof functions as well. Note that if we quantify over session types, we obtain polymorphic sessions. See Example 4.

Given a thread function, \texttt{fork} creates a new thread and connects to it with a fresh channel \( c \). The dc-type of \texttt{fork} mandates that \( rs_1 \) and \( rs_2 \) form a partition of the full set, ensuring the consistency of channel endpoints. \texttt{cut} is for connecting two channels of the same session type, i.e., forwarding. It loosely corresponds to the 2-cut-residual rule of LMRL. Given one endpoint from each of the two channels, \texttt{cut} connects the two channels into a single channel and returns a residual endpoint of roles \( rs_1 \cap rs_2 \) to the caller, also connected in this channel. The admissibility of cut in LMRL means connecting multiple channels of the same type via forwarding is equivalent to establishing just a single channel connecting all parties from the very beginning. \texttt{elim} is for eliminating an endpoint with empty role sets. \texttt{split} is for splitting an endpoint into two disjoint endpoints, in separate threads.

Dynamic Semantics Term reductions model normalizations in a single thread. In a thread pool \( \Pi \), we use pool reductions. Note that our formulation is for synchronous communications for simplicity, although our implementations fully support asynchronous communications. We define partial ad-hoc redexes in Figure 8. Only matching partial redexes can reduce according to some pool reduction rules.

**Definition 3 (Matching Partial Redexes).** For any channel \( c \),

\[
\begin{align*}
&\text{match}\{\{\texttt{bsend}(c^{rs_1}, v), \texttt{brecv}(c^{rs_2}), \ldots, \texttt{brecv}(c^{rs_n})\}\} \\
&\text{match}\{\{\texttt{send}(c^{rs_1}, v), \texttt{recv}(c^{rs_2}), \texttt{skip}(c^{rs_3}), \ldots, \texttt{skip}(c^{rs_n})\}\} \\
&\text{match}\{\{\texttt{close}(c^{rs_1}, v), \texttt{wait}(c^{rs_2}), \ldots, \texttt{wait}(c^{rs_n})\}\} \\
&\text{match}\{\{\texttt{unify}(c^{rs_1}, v), \texttt{exify}(c^{rs_2}), \ldots, \texttt{exify}(c^{rs_n})\}\} \\
\end{align*}
\]

where endpoints\((c) = \{c^{rs_1}, \ldots, c^{rs_n}\} \). We write \text{match}\((\{e_1, \ldots, e_n\}\) for \text{match}(e_1', \ldots, e_n') if \( e_1 = E[e_1'], \ldots, e_n = E[e_n'] \).
Redexes

Partial (ad-hoc) redex: \( \text{send}(c^e, v) \mid \text{recv}(c^e) \mid \text{bsend}(c^e, v) \mid \text{brecv}(c^e) \mid \text{beecv}(c^e) \mid \text{close}(c^e) \mid \text{wait}(c^e) \mid \text{unify}(c^e) \mid \text{exify}(c^e) \)

Pool Reductions

\[
\begin{align*}
\text{(pr-fork)} & \quad H, t. E[\text{fork}(x, y)] \xrightarrow{c, f} H, t. E[c^{x}], t'. E[x \mapsto c^y] \\
\text{(pr-cut)} & \quad H, t. E[\text{cut}(c^1, c^2)] \xrightarrow{c, f} H[c_1, c_2 \mapsto e, c]. t. E[c^{e}] \\
\text{(pr-elim)} & \quad H, t. E[\text{elim}(c^e)] \xrightarrow{c, f} H, t. E[[]] \\
\text{(pr-split)} & \quad H, t. E[\text{split}(c^{e_1}, \text{lam } x. E[[]]) \xrightarrow{c, f} H, t. E[c^e], t'. E[x \mapsto c^y] \\
\text{(pr-bmsg)} & \quad H, t_1. E[\text{bsend}(c^{e_1}, v)], t_2. E[\text{recv}(c^{e_2})], \ldots, t_n. E[\text{bsend}(c^{e_n}, v)] \xrightarrow{c, f} H, t_1. E[c^{e_1}], t_2. E[c^{e_2}], \ldots, t_n. E[c^{e_n}] \\
\text{(pr-end)} & \quad H, t_1. E[\text{send}(c^{e_1}, v)], t_2. E[\text{recv}(c^{e_2})], \ldots, t_n. E[\text{recv}(c^{e_n})] \xrightarrow{c, f} H, t_1. E[c^{e_1}], t_2. E[c^{e_2}], \ldots, t_n. E[c^{e_n}] \\
\text{(pr-msg)} & \quad H, t_1. E[\text{send}(c^{e_1}, v)], t_2. E[\text{recv}(c^{e_2})], \ldots, t_n. E[\text{recv}(c^{e_n})] \xrightarrow{c, f} H, t_1. E[c^{e_1}], t_2. E[c^{e_2}], \ldots, t_n. E[c^{e_n}] \\
\end{align*}
\]

For the last four pool reduction rules, we assume \( \text{endpoint}(c) = (c^{e_1}, \ldots, c^{e_n}) \).

Pool Equivalences

\[
\text{(pe-cut)} \quad H, t. E[\text{cut}(x, y)] = H, t. E[\text{cut}(y, x)]
\]

\textbf{Extensions} Some common features are intentionally left out for brevity. We mention some very briefly. Branching in the session types can be supported by adding a new session type constructor, \( \text{branch}(r, \pi_1, \pi_2) \) and a pair of corresponding session APIs \textit{offer} and \textit{choose}. Party \( r \) will be offering the choice, and all other parties need to choose. Recursive sessions can be supported by adding \( \text{fix}(\lambda x. \text{stype}. \pi) \). A session API \textit{recurse} can be added to unroll \( \text{chan}(r, \text{fix}(f)) \) into \( \text{chan}(r, f(\text{fix}(f))) \) for any \( r \) and \( f. \) \textit{fork} only forms sessions locally. One can introduce \( \text{init}(r, \pi) \) for forming sessions distributedly. A pair of APIs, \textit{accept} (at \( r \)) and \textit{request} (at all others) can be provided. Note that after \textit{request}, a new thread is created and the new endpoint is passed to the thread. \textit{request} can not return the endpoint to the current thread as it breaks relaxation, thus df-reducibility, and will cause a loop. See [18] for more possible constructs.

\textbf{Example 4.} We model a cloud service. When the provider \( P \) gives the server \( S \) a function that serves \textit{any} protocol \( x \) \textit{once}, the server \( S \) will repeatedly serving that protocol to client \( C \). As a syntactical sugar, we will write \( e_1; e_2 \) as a shorthand for \( \text{snd}(e_1, e_2) \). We also write \( \text{let } x = e_1 \text{ in } e_2 \) for \( \text{app}(\text{lam } x. e_2, e_1) \). The session type is

\[\text{quau}(P, \lambda x. \text{stype}. \text{msg}(P, S, \text{chan}(S, x)) \to \text{unit}) :: \text{fix}(\lambda y. \text{stype}. \text{msg}(C, S, \text{chan}(S, x)) :: y)\]

This is a polymorphic, higher-order session type, that involves both a 3-party session among \( P/C/S \), and a 2-party session (of session type \( x \)) between \( S/C \). The program of \( S \) can be implemented below. The annotations on the right denote the endpoint’s type \textit{after} the invocation of the session API on that line.

\[
\begin{align*}
\text{lam } c. \text{let } c = \text{exify}(c) \text{ in } c : \text{chan}(S, \text{msg}(P, S, \text{chan}(S, x)) \to \text{unit}) :: \text{fix}(\ldots) \\
\text{let } (c, f) = \text{recv}(c) \text{ in } c : \text{chan}(S, \text{fix}(\lambda y. \text{stype}. \text{msg}(C, S, \text{chan}(S, x)) :: y)) \\
\text{let } \text{loop} = \text{fix } g. \text{lam } x. \text{let } x = \text{recurse}(x) \text{ in } x : \text{chan}(S, \text{msg}(C, S, \text{chan}(S, x)) :: \text{fix}(\ldots)) \\
\text{let } (x, y) = \text{recv}(x) \text{ in } \text{app}(f, y); \text{app}(g, x) y : \text{chan}(S, x) \\
\end{align*}
\]
On the first line, `exify` interprets `quan` as existential, and then immediately `ty-exists-elim` (Figure 2) is used to eliminate the quantifier. On the second line, `S` receives the thread function that `P` wants to let `S` repeatedly serve. The function can serve protocol `x`, for any `x`. On the third and fourth line, we define a recursive function (a feature in `ATS` not covered here) called `loop`. The function will, on the third line, unroll the endpoint once, and on the forth line, call `recv` to receive an endpoint of type `chan(S,x)` created by and sent from `C`, followed by invoking `P`-supplied function `f` on `C`-supplied endpoint `y`, providing service `x`. On the fifth line, `S` invokes the `loop` function with endpoint `c`.

The network topology is shown here. In (1), `P/C/S` is connected in a 3-party session described above. Then client `C` invokes `fork`, spawning a new thread with a new channel, sending one endpoint to `S`, and starts a new 2-party session with protocol `x` between `S` and a child thread of `C`, denoted as `C'`.

This example can be successfully type-checked in our current implementation of $\lambda_{v,\exists}$.

### 4.1 Deadlock-freeness Reducibility

The technique of df-reducibility is introduced in our early work [24] for binary session types. It is adapted for multiparty session types. The notion captures the invariance of pool reduction that, at any time, there are no loops or self-loops in all endpoint connections. The proof of Lemma 3 carried out as follows. First, we show that reduction preserves df-reducibility (Lemma 2). Then df-reducibility implies relaxation (Proposition 2) which in turn means reducible (Proposition 1). Notably, relaxation (Definition 7) is not an invariant during reduction, e.g., for the case of `pr-end`, and that is why we strengthened it and formalized df-reducibility (Definition 6). This proof technique, and particularly Proposition 2, guided the choices of when to return an endpoint to the caller and when to spawn a new thread, like `accept/request` discussed before.

**Definition 4 (Abstract Collections of Endpoints).** We use $M$ to denote a finite set of endpoints and $\mathcal{M}$ to denote a finite set of $M$ where all $M \in \mathcal{M}$ are pair-wise disjoint. We use $\bigcup \mathcal{M}$ to mean the (disjoint) union of all $M \in \mathcal{M}$. For any channel `c`, we assume either `endpoints(c) ⊂ \bigcup \mathcal{M}` or `endpoints(c) ∩ \bigcup \mathcal{M} = \emptyset`.

We lift the definition of `endpoints(·)` and `channels(·)` onto $M$ and $\mathcal{M}$ to collect endpoints and channels from them.

**Definition 5 (Deadlock-free Reduction $\rightsquigarrow$).** We write $\mathcal{M} \rightsquigarrow \mathcal{M}'$ if there exists a channel `c`, some sets of endpoints $M_i$ s.t.

\[
\text{endpoints}(c) = \{c^{r_1}, \ldots, c^{r_n}\} \text{ and } c^{r_i} \in M_i \in \mathcal{M} \text{ for } 1 \leq i \leq n
\]

\[
\mathcal{M}' = (\mathcal{M} \setminus \{M_1, \ldots, M_n\}) \cup \{M'\} \text{ and } M' = (M_1 \cup \cdots \cup M_n) \setminus \text{endpoints}(c)
\]
We also write $\mathcal{M} \rightsquigarrow \mathcal{M}'$ if there exists some $c$ s.t. $\mathcal{M} \overset{\rho}{\rightsquigarrow} \mathcal{M}'$. We say $\mathcal{M}$ is df-normal if it can not be further df-reduced, denoted as $\mathcal{M} \not\rightsquigarrow$.

**Definition 6 (Deadlock-freeness Reducibility).** We write df-reducible$(\mathcal{M})$ if

- each $\mathcal{M} \in \mathcal{M}$ is an empty set, or
- $\mathcal{M}$ is not df-normal, and for any $\mathcal{M}'$ where $\mathcal{M} \rightsquigarrow \mathcal{M}'$ holds, df-reducible$(\mathcal{M}')$.

There are certain properties about df-reducibility that are easy to prove. We omit them and refer readers to [24]. We only note that empty elements can always be removed from $\mathcal{M}$ without breaking df-reducibility.

**Definition 7 (Relaxed).** Let $|\mathcal{M}|$ be the number of non-empty elements.

\[
\text{relaxed}(\mathcal{M}) := \begin{cases} 
|\mathcal{M}| \geq \text{endpoints}(\mathcal{M}) - \text{channels}(\mathcal{M}) + 1 \\
|\mathcal{M}| = 0
\end{cases}
\]

**Proposition 1 (Pigeonhole).** If relaxed$(\mathcal{M})$ and $|\mathcal{M}| > 0$ where $|\mathcal{M}|$ is the number of non-empty elements, then $\mathcal{M} \overset{\rho}{\rightsquigarrow} \mathcal{M}'$ for some $c$ and $\mathcal{M}'$. Namely, $\mathcal{M}$ is not df-normal.

**Proposition 2.** $\neg$relaxed$(\mathcal{M})$ implies $\neg$df-reducible$(\mathcal{M})$.

We now make $\mathcal{M}$ and $\mathcal{M}$ concrete. Let consistent$(\rho(\Pi))$, we define $M(\Pi(t))$ as endpoints$(\rho(\Pi(t)))$, $M(\Pi)$ as $\bigcup \{ M(\Pi(t)) \}$, relaxed$(\Pi)$ as relaxed$(M(\Pi))$, and df-reducible$(\Pi)$ as df-reducible$(M(\Pi))$. We also define blocked$(e)$ as $e = E[e']$ for some evaluation context $E$ and partial redex $e'$. A blocked expression is blocked on some endpoint $e'$ of some channel $c$. We write blocked$(e, c')$ or blocked$(e, c)$ to make it explicit.

**Lemma 2 (Reducibility of Pools).** Pool reduction preserves df-reducibility, consistent$(\rho(\Pi))$, df-reducible$(\Pi)$, and $\Pi \overset{\rho}{\rightarrow} \Pi'$ implies df-reducible$(\Pi')$.

**Lemma 3 (Deadlock-free).** Let $\Pi$ be a well-typed pool s.t. $\Pi(0)$ is either a value $v$ without endpoints or blocked$(\Pi(0))$, and blocked$(\Pi(t))$ for $0 < t \in \text{dom}(\Pi)$. If $\Pi$ is obtained from evaluating an initial pool without any channels, then there exist $t_1, \ldots, t_n \in \text{dom}(\Pi)$ s.t. match$(\{ \Pi(t_1), \ldots, \Pi(t_n) \})$.

**Proof.** The initial pool is consistent by Definition 2 and df-reducible by Definition 6 since it contains no endpoints. Therefore df-reducible$(\Pi)$ by Lemma 2 and consistent$(\Pi)$ by Theorem 1. By Proposition 2 we have relaxed$(\Pi)$. Parallel to Proposition 1, by the Pigeonhole Principle, there exist $t_1, \ldots, t_n \in \text{dom}(\Pi)$ s.t. blocked$(\Pi(t_1), c^{rs_1}), \ldots$, and blocked$(\Pi(t_n), c^{rs_n})$ for some channel $c$ where endpoints$(c) = \{ c^{rs_1}, \ldots, c^{rs_n} \}$. Since $\Pi$ is well-typed and consistent, these endpoints are assigned coherent types by rule sig-chan. Therefore we have match$(\{ \Pi(t_1), \ldots, \Pi(t_n) \})$ by straightforwardly examining the typing derivations of the partial redexes.
4.2 Soundness

Theorem 1 (Subject Reduction). Assume $\vdash \Pi_1 : \hat{\tau}$ under some signature $S_1$, consistent($\rho(\Pi_1)$), and $\Pi_1 \xrightarrow{P} \Pi_2$ for some $\Pi_2$. Then $\vdash \Pi_2 : \hat{\tau}$ under a corresponding signature $S_2$ and consistent($\rho(\Pi_2)$).

Proof. Proof by induction on the derivation of $\Pi_1 \xrightarrow{P} \Pi_2$.

Theorem 2 (Progress). Assume $\vdash \Pi : \hat{\tau}$ and consistent($\Pi$). We have the following possibilities:

- \( \Pi \) is a singleton mapping $0 \mapsto v$.
- $\Pi \xrightarrow{P} \Pi'$ holds for some $\Pi'$.

Proof. In the case where all threads are blocked expressions, Lemma 3 is needed.

5 Related Works and Conclusions

To address a problem found in higher-order sessions that breaks type preservation theorem [27], polarity and balanced typing [9,13] are introduced to distinguish the two ends of a channel syntactically, and to ensure their typing duality, respectively. The advantage of our formulation is that endpoint types are inherently balanced by sig-chan, given that $S$ only records global session types, and that all roles of endpoints of a channel should form a partition of $\emptyset$ due to Definition 2. Also, we found our approach much cleaner for generalizing into multiparty session types compared to the polarized approach. [12,3,26] explored multiparty session types, [16,14] explored binary dependent session types. However, the present work is the first to combine dependent types with multiparty session types. [1,2,4] explored the connection of process calculi with linear logic. However, the present work is the first to combine dependent types with multiparty session types. [16,14] explored the connection of process calculi with linear logic. However, the present work is the first to combine dependent types with multiparty session types. [1,2,4] explored the connection of process calculi with linear logic. However, the present work is the first to combine dependent types with multiparty session types.

[5,17] established a Curry-Howard correspondence between session typed process calculi with propositions in linear logic. Later works [7,6,8] developed a generalized notion of duality called coherence, to correspond multiparty session types with propositions in linear logic equipped with a separate proof system for coherence. We consider their formulation as an extension instead of a generalization, since the coherence rule is a separate proof system, and the well-known duality of the axiom rule and the cut rule is lost. We consider our work as a formal generalization, with LK/CLL being a special case of MRL/LMRL. Also, results like 2-cut-residual are made possible only via multirole.

We have demonstrated the first formulation of multiparty dependent session types, the df-reducibility proof technique, and the intuitions behind multirole logic. We point out that by representing session types as program terms, by implementing propositions in the guarded types of the APIs as runtime assertions, one can still greatly benefit from the ability to deterministically know in advance whether the system is deadlock-free, even in a language without dependent types and linear types. This is precisely the significance of our practical system. With the help of such formal reasoning, concurrency can be made better, safer, and more accessible.
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