Scaling in Modeling of Core Losses in Soft Magnetic Materials Exposed to Nonsinusoidal Flux Waveforms and DC Bias Condition

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Assuming that core loss data of Soft Magnetic Materials obey scaling relations, models describing the power losses in materials exposed to nonsinusoidal flux waveforms and DC Bias conditions have been derived. In order to test these models, the measurement data for two materials have been collected and the core losses calculated. Agreement between the experimental data and the model predictions is satisfactory.

INTRODUCTION

The usual waveforms in power electronics are square waves or a superposition of square waves rather than sine waves. Some magnetic components also operate under a DC bias, which has a significant influence on the core losses. During the last two decades significant progress in modeling the dependencies of core losses both due to square wave parameters and DC bias has been achieved. The derived methods for handling the core losses in soft magnetic materials (SMM) exposed to nonsinusoidal flux waveforms have been based on the Steinmetz Equation [1]. These methods include the Modified Steinmetz Equation (MSE) [2], [3], Generalized Steinmetz Equation (GSE) [4], Improved Generalized Steinmetz Equation (iGSE) [5], [6] and Improved-improved Generalized Steinmetz Equation (i2GSE) [7]. Methods and models exist, however, based on the assumption that the shape of the waveform does not matter and as a result only look at peaks: Scaling of power Losses in SMM [11]-[13] and Field-Extrema Hysteresis Model (FHM), [14]. Both do not capture the effects of the waveform. Is this feature a disadvantage or an advantage? All the models we are talking about have been derived for prediction of losses for the designing processes. If one assumes that the waveform does not vary in such a process then the model is universal and can be applied to any waveform possessing a constant period. Moreover, if experimental data obey a scaling law then the data collapse is possible. In practise the collapse leads to a reduction of the number of independent variables [11]-[13] which makes a model or a method much simpler. For these reasons we will apply the scaling theory in order to describe the core losses in soft magnetic materials exposed to any periodic flux waveform. In order to take into account DC Bias we apply the scaling approach to modeling of core losses for any periodic flux waveform. In order to take into account DC bias we combine the scaling with the Van den Bossche method. Section II is devoted to experiment, measurement data, estimation of the model parameters and comparison of theoretical model with the measurement data. Section III contains conclusions.

1. APPROACH TO SCALING OF CORE LOSSES DATA

Instead of an analysis based on Maxwell’s equations [18],[16] we have assumed that the SMM is a complex system in which the function of power losses obeys the scaling law. This assumption leads to the formula for total power loss in the form of general homogenous function [9],[10]:

$$\exists a, b, c \in \mathbb{R} :$$

$$\forall \lambda \in \mathbb{R}^+ \quad P_{\text{tot}}(\lambda^a f, \lambda^b (\Delta B)) = \lambda^c P_{\text{tot}}(f, (\Delta B)),$$

where we have derived the general form of $P_{\text{tot}}$:

$$P_{\text{tot}}(f, (\Delta B)) = (\Delta B)^\beta F\left(\frac{f}{(\Delta B)\alpha}\right),$$

where $F(\cdot)$ was an arbitrary function, $\alpha = \frac{a}{3}$, and $\beta = \frac{c}{3}$. This function depends on the features of the phenomena to be described. Since our measurement data are unable to consider quasi-static losses we choose for $F(\cdot)$ the power series as a rough description of the power losses $P_{\text{tot}}$: [11]-[13]:

$$\frac{P_{\text{tot}}(f, \Delta B)}{(\Delta B)^\beta} = \Gamma_1 \left(\frac{f}{(\Delta B)^\alpha}\right)^2 + \Gamma_2 \left(\frac{f}{(\Delta B)^\alpha}\right)^3 + \Gamma_3 \left(\frac{f}{(\Delta B)^\alpha}\right)^4 + ...][.$$

Note that the left hand side of ($\Delta B$) is a function of the one effective variable $\sqrt[3]{\Delta B}$. It is easy to recognize in ($\Delta B$) the hysteresis losses $P_h$ and the eddy current losses $P_e$ by the powers...
\( f \) and \( f^2 \), respectively. All higher terms correspond to excess losses \( P_{\text{ex}} \). The formula (5) is the mathematical model, ready to apply. Values of \( \alpha, \beta \) and amplitudes \( \Gamma_n \) have been estimated for the ten selected soft magnetic materials [11]-[13].

Very recently [3] has been applied to Soft Magnetic Composites [19]. Obtained results confirm the good quality of this model. Unfortunately, the model fails when a DC Bias is present. In this situation we propose to keep (3) as a limit for \( H_{\text{DC}} \to 0 \) and add the second term of a similar form to (3) with \( \Gamma \)'s dependent on \( H_{\text{DC}} \). These we take from Van den Bossche, et al. [6], [15]. They have proposed mapping \( H_{\text{DC}} \) to the corresponding primary magnetization curve which works very well. In the case of a lack of appropriate experimental data it is possible to use a mathematical model for \( H_{\text{DC}} \). For instance:

\[
H_{\text{DC}} \to \tanh(H_{\text{DC}} \cdot c),
\]

or more generally:

\[
H_{\text{DC}} \to \{ \tanh(H_{\text{DC}} \cdot c_1), \tanh(H_{\text{DC}} \cdot c_2), \ldots, \tanh(H_{\text{DC}} \cdot c_p) \},
\]

where \( c_i \) are free parameters to be determined [6]. By this way we derive the final model’s form:

\[
P_{\text{tot}}(f, \Delta B, H_{\text{DC}}) = \sum_{i=1}^{p} \Gamma_i (f/\Delta B^\alpha)^{(i-1)} \tanh(H_{\text{DC}} \cdot c_i),
\]

where \( x \) is a small phenomenological tuning parameter. This parameter is derived in the following way. (3) arises as the Maclaurin series of (2). This is why expansion of (3) possesses only integer exponents. However, in nature this need not be so. Therefore by the small \( x \) we introduce a natural correction of the series’ exponents.

Equation (6) has been successfully applied to SIFERRIT N87, however playing with data for VITROPERM 500F 18K and model (6) we have got an experience which suggested the following modifications: 1) to split the \( x \) parameter into two independent tuning parameters \( x \) and \( y \), where \( x \) tunes the bias-independent term, whereas \( y \) tunes the part describing the influence of the bias on the losses, and 2) to distribute functions \( \tanh \) along the \( H_{\text{DC}} \) axis. These requirements lead to the following model:

\[
P_{\text{tot}}(f, \Delta B, H_{\text{DC}}) = \sum_{i=1}^{p} \Gamma_i (f/\Delta B^\alpha)^{(i-1)} \tanh(H_{\text{DC}} \cdot c_i - r_i),
\]

where \( r_i \) describes the position of the \( i \)-th function (5).

**II. EXPERIMENTAL DATA AND PARAMETERS’ ESTIMATIONS**

The B-H Loop measurements have been performed for two materials: SIFERRIT N87 and VITROPERM 500F 18K. The

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**FIG. 1.** SIFERRIT N87. Projection of measurement points and scaling theory points [5] in \(( f/(\Delta B)^\alpha )^{1-x}, P_{\text{tot}}/(\Delta B)^\beta \) plane.

**FIG. 2.** SIFERRIT N87. Projection of measurement points and scaling theory points [5] in \( \{ \tanh(H_{\text{DC}} \cdot c_1), P_{\text{tot}}/(\Delta B)^\beta \} \) plane.

Core Losses per unit volume have been calculated as the enclosed area of the B-H loop, multiplied by the frequency \( f \). The following factors influence the accuracy of measurements: 1) Phase Shift Error of Voltage and Current \( \leq 4\% \), 2) Equipment Accuracy \( \leq 5.6\% \), 3) Capacitive Couplings negligible (capacitive currents are relatively lower compared to inductive currents), and 4) Temperature \( \leq 4\% \). For details of the applied measurement method and the errors of factors we refer to [7], [8]. In order to enable readers to compare their own results with this work we enclose measurement data for SIFERRIT N87 (TABLE I), SIFERRIT N87 (TABLE I), VITROPERM 500F 18K (TABLE II), and SIFERRIT N87 (TABLE III). The parameter values of (3) have been estimated for each sample’s measurement data by minimization of \( \chi^2 \) using the Simplex method of Nelder and Mead [20], see TABLE II and TABLE III.

Both measurement series consist of 30 points. All measurement points have been taken in temperature 27°C ± 0.5°C. Standard deviations per point are equal \( \pm 12\% \) and \( \pm 15\% \) for VITROPERM 500F 18K and SIFERRIT N87, respec-
Applying the formulae (6) and (8) and the estimated values of parameters we have drawn scatter plots which compare estimated points with the experimental ones. Each sample is presented in the two projections. Note that the estimated model parameters are universal, which means that they work for each \( f \) and \( \Delta B \). Attention, in order to prevent generation of large numbers in the estimation process the unit of frequency was kHz while other magnitudes were expressed in SI unit system.

**III. CONCLUSIONS**

Agreement between the models of core losses (6), (8) and the measurement data confirms the hypothesis that these data obey scaling relations. Now we can construct estimators for core loss in soft magnetic materials exposed to nonsinusoidal periodic flux waveforms and DC Bias condition. Moreover, thanking to data collapse the space of independent variables is reduced from the three-dimensional one spaned by \([f, \Delta B, H_{DC}]\) to the two-dimensional one spaned by \([f/(\Delta B)^\alpha, P_{tot}/(\Delta B)^\beta]\). The consequence which results from this reduction is very significant: both models (6) and (8) describe the core loss problem completely (for each \( f \), for each \( \Delta B \) and for each \( H_{DC} \)). The question is then how many model parameters must one introduce in order to completely describe the core loss in the 3-dimensional space.

| \( \Delta B [T] \) | \( f [kHz] \) | \( H_{DC} [\frac{\Delta B}{T}] \) | \( P_{tot} [\frac{W}{m^2}] \) |
|------------------|-----------|-----------------|-----------------|
| 0.201            | 50        | 0.070           | 13155.0         |
| 0.202            | 50        | 4.699           | 13384.6         |
| 0.204            | 50        | 10.70           | 14043.9         |
| 0.201            | 50        | 14.75           | 14338.9         |
| 0.204            | 50        | 19.83           | 15295.3         |
| 0.414            | 50        | 4.920           | 52552.5         |
| 0.411            | 50        | 9.96            | 54106.4         |
| 0.416            | 50        | 15.11           | 56641.6         |
| 0.411            | 50        | 20.18           | 60464.5         |
| 0.618            | 50        | 4.662           | 118206          |
| 0.600            | 50        | 10              | 121104          |
| 0.619            | 50        | 15.03           | 127222          |
| 0.613            | 50        | 19.85           | 135366          |
| 0.807            | 1         | 4.810           | 582.66          |
| 0.814            | 1         | 10.29           | 582.38          |
| 0.812            | 1         | 15.29           | 618.87          |
| 0.804            | 1         | 19.81           | 682.53          |
| 0.810            | 1         | 25.26           | 829.74          |
| 0.804            | 2         | 0.067           | 1127.81         |
| 0.802            | 2         | 4.780           | 1036.27         |
| 0.805            | 2         | 10.04           | 1010.43         |
| 0.805            | 2         | 14.78           | 1059.07         |
| 0.803            | 2         | 19.81           | 1161.91         |
| 0.808            | 5         | 4.818           | 4066.07         |
| 0.808            | 5         | 15.27           | 4098.72         |
| 0.807            | 5         | 19.59           | 4418.86         |
| 0.806            | 10        | 5.077           | 12875.9         |
| 0.806            | 10        | 10.16           | 12931.8         |
| 0.805            | 10        | 14.78           | 13593.5         |
| 0.806            | 10        | 19.70           | 14891.1         |
TABLE II. The set of estimated model’s parameters for VITROPERM 500F 18K, formula (8)

| $\alpha$ | $\beta$ | $x$ | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | $\Gamma_4$ | $\Gamma_5$ | $\Gamma_6$ |
|----------|---------|-----|------------|------------|------------|------------|------------|------------|
| $\Gamma_7$ | $y$ | $c_1$ | $c_2$ | $c_3$ | $r_1$ | $r_2$ | $r_3$ | $-$ |
| 7.846801 | 13.40684 | 0.3529973 | -101.7297 | 2390.43 | -334.3942 | 121.0903 | 7.88E+00 | 1,29E+03 | - |
| -7.66917 | 9.77E-01 | 4.668659 | 9.52E-02 | 43.98191 | 0.2228664 | 3.042717 | -2.016617 | - | - |

TABLE III. The set of estimated model’s parameters for SIFERRIT N87, formula (6)

| $\alpha$ | $\beta$ | $x$ | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | $\Gamma_4$ | $\Gamma_5$ | $\Gamma_6$ | $\Gamma_7$ | $\Gamma_8$ | $\Gamma_9$ | $\Gamma_{10}$ | $\Gamma_{11}$ | $\Gamma_{12}$ | $\Gamma_{13}$ | $\Gamma_{14}$ | $\Gamma_{15}$ |
|----------|---------|-----|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| $\Gamma_{16}$ | $c_1$ | $c_2$ | $c_3$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ |
| -7.7413 | -11.375 | -0.1712 | 285.292 | 118.459 | -27.1697 | -2.2167 | -5.35E-01 | -3.03E+01 | 5.8086 | -5.750 | -0.05529 | 8.0579 | 0.1102 | |

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