Predictions of the causal entropic principle for environmental conditions of the universe

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The causal entropic principle has been proposed as a superior alternative to the anthropic principle for understanding the magnitude of the cosmological constant. In this approach, the probability to create observers is assumed to be proportional to the entropy production $\Delta S$ in a maximal causally connected region — the causal diamond. We improve on the original treatment by better quantifying the entropy production due to stars, using an analytic model for the star formation history which accurately accounts for changes in cosmological parameters. We calculate the dependence of $\Delta S$ on the density contrast $Q = \delta \rho/\rho$, and find that our universe is much closer to the most probable value of $Q$ than in the usual anthropic approach and that probabilities are relatively weakly dependent on this amplitude. In addition, we make first estimates of the dependence of $\Delta S$ on the baryon fraction and overall matter abundance. Finally, we also explore the possibility that decays of dark matter, suggested by various observed gamma ray excesses, might produce a comparable amount of entropy to stars.

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I. INTRODUCTION

The nature of the cosmological constant problem has changed since it was fully recognized in the 1980s. At that time, a seemingly natural solution was to find a mechanism which would explain why $\Lambda$ should be zero. Among the most promising such ideas were those of Hawking [1], Coleman [2], and the Euclidean path integral for gravity was interpreted as a wave function for the universe, shown to be strongly peaked at $\Lambda = 0$. However we now know with a high degree of confidence that $\Lambda$ is not zero, due to observations of distant type Ia supernovae [3] combined with constraints on the flatness of the universe from the CMB [4] and complementary information from the x-ray baryon fraction in galactic clusters [5].

Among the most promising such ideas were those of Brown and Teitelboim (BT) [6] (building on work in [7]), involving tunneling between false vacua with different values of $\Lambda$, and the anthropic argument of Weinberg [8], which predicted that

$$-\Lambda_{\text{obs}} \lesssim \Lambda \lesssim 100 \Lambda_{\text{obs}}$$

(1)

based on the requirement that galaxies should be able to form and thus give rise to observers before the universe collapsed or expanded too quickly. These approaches have recently gained support in a number of ways from the string theory community: by the explicit realization of the BT mechanism within heterotic M-theory and type II string theory [9] and the realization that the vacuum structure of string theory is a vast landscape [10] encompassing more than $10^{600}$ possible values of $\Lambda$ [11]. The picture that has emerged is that this multitude of vacua can be simultaneously realized through the process of eternal inflation [12]. Tunneling à la BT from the eternally inflating regions would populate universes like ours with small $\Lambda$, as well as those with much larger $\Lambda$. The latter would be incompatible with life, and we should disregard them if we are interested in explaining properties of universes which can admit the existence of observers. This is the weakest form of the anthropic principle; it does not insist that physics must be compatible with our existence, only that we take our existence as a data point in putting experimental constraints on the landscape of vacua.

This point of view has been strongly criticized on the grounds that it admits no real predictions, only postdictions. Moreover, the anthropic explanation of $\Lambda$ has been weakened by the discovery of dwarf galaxies at $z \sim 10$, proving that structure formed earlier than $z \sim 4$ as was supposed in [8]. Since the bound (1) scales like $(1+z)^3$, the anthropic constraint is weakened by an order of magnitude. To counteract this, it has been argued that only galaxies of a certain minimum size are capable of retaining the heavy elements needed to sustain life [13], but such assumptions seem questionable and make the anthropic principle appear to be increasingly arbitrary. The anthropic bound on $\Lambda$ is further weakened by allowing other cosmological parameters to vary at the same time. For example, the primordial density contrast $Q = \delta \rho/\rho \sim 2 \times 10^{-5}$ in our universe, but, if $Q$ were larger, structure could form earlier despite a larger expansion rate. The anthropic bound on $\Lambda$ scales like $Q^3$, so increasing $Q$ can relax the bound by many orders of magnitude [14, 15]. Similarly, [16] showed that allowing the effective Planck scale of a tensor-scalar theory of gravity to vary also weakens the anthropic bound on $\Lambda$;
this result is potentially important for the string theory landscape.

Despite these shortcomings, one should keep in mind that the anthropic approach was the only one to predict the range of \( \Lambda \) before its nonzero observation, and the value \( \Lambda_{\text{obs}} \approx 10^{-123} M_\odot^2 \), is so peculiar from the particle physics perspective that it seems exceedingly unlikely that a dynamical mechanism could by itself explain the observed value of \( \Lambda \). Something like the anthropic principle therefore appears to be necessary for understanding the magnitude of the dark energy. One is thus motivated to search for some improvement which retains the virtues of the anthropic principle while getting rid of overly specific assumptions about the nature of observers. Such an idea, dubbed the "causal entropic principle," has recently been proposed by [17].

It assumes that a good tracer of the potential for forming structure, and hence observers, is the amount of entropy produced within a causally connected region (the causal diamond) defined to start at some early initial time \( t_i \), such as reheating, and ending in the infinite future, which corresponds to a finite conformal time \( \tau \) if \( \Lambda > 0 \). The created entropy, \( \Delta S \), does not include entropy already present at the moment of reheating but only that which is created after \( t_i \).

The causal entropic principle has several virtues which make it worthy of further investigation. First, it predicts a probability distribution \( dP/d\log \Lambda \) such that \( \Lambda_{\text{obs}} \) is within 1 \( \sigma \) of the most likely value of \( \Lambda \), unlike the anthropic approach for which \( P(\Lambda \leq \Lambda_{\text{obs}}) < 10^{-3} \). Second, it makes minimal assumptions about the detailed nature of observers or life. It only assumes that free energy (leading to increased entropy) is available, which is a fundamental thermodynamical requirement for making any measurement. Third, it provides a specific proposal to separate potentially divergent volume factors (for example, from eternal inflation) into the prior probability distribution. (For recent discussions of volume-weighted measures, see [18].)

In their paper, the authors of [17] understandably restricted their attention to the determination of \( \Lambda \). To carry the idea further, we are interested in exploring the probability distributions for other cosmological parameters, for example \( Q \). This requires knowing how the rate of entropy production depends on these parameters. In particular, since [17] found that starlight is the main source of entropy, a major result of the present work is to show how the rate of star formation depends on various cosmological parameters. This was somewhat roughly estimated in [17]; here we have tried to be more quantitative. The most straightforward quantity to vary in fact is \( Q \), and we have studied this dependence to show that the predicted value of \( \Lambda \) in the causal entropic approach is much less sensitive to \( Q \) than in the anthropic approach. Furthermore, the entropic prediction for \( Q \) is much closer to the observed value than is the anthropic prediction. Similarly, we find that the prediction for the baryon fraction is within a factor of a few of the observed value.

An important aspect of [17] is the realization that the typical timescale of star formation and galaxy evolution is of the same order of magnitude as the age of the universe, providing another coincidence problem. The causal entropic view effectively elevates this coincidence to a principle, with potentially testable predictions. A process that generates a large amount of entropy on a characteristic timescale would make more likely, by this principle, a cosmological constant that becomes dynamically important on a comparable timescale. This suggests that there is not a large source of entropy that is acting on a timescale significantly different than the current age of the universe. If we discover a new important entropy source, we should expect its timescale to be the cosmological one. In fact, such an entropy source would make the observed value of \( \Lambda \) more likely.

Beyond the consideration of universes very similar to our own, the entropic approach has the capability of quantifying the likelihood of universes with quite different properties. This leads one to ask whether alternative universes with much greater entropy production than our own can be constructed, addressing the critique of [21] that existing work has focused too much on universes similar to our own. If so, it might call into question the validity of entropic principle, since our universe would then appear to be relatively unlikely. One way in which entropy might be copiously generated is through the decay of dark matter. Interestingly, hints of excess gamma rays at various energies has led to speculation that the dark matter in our universe indeed is unstable. One goal of this work is to determine whether the entropy released by such decays can be competitive with that due to starlight; if so, this could be an indication that the entropic principle can explain properties of dark matter as well as dark energy.

We will review the causal entropic approach in section II, including our results for \( dP/d\log \Lambda \), and motivate the improved star formation rate (SFR) which we develop in section III. This will be followed in IV by our results for the probability distribution for \( Q \) and for the baryon fraction. Section V will discuss the production of entropy from decaying dark matter. We give conclusions in section VI. Throughout, except when units are given explicitly, we work with \( h = c = 1 \). In formulae, we keep factors of \( M_\odot \) and \( G \) explicit, but we omit them.

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1 An interesting paper [19] has recently analyzed the joint probability of observing a universe with a given \( \Lambda \) at a given CMB temperature using star formation and evolution as the key indicator of observers, following the original ideas of [20]. Even though the spirit of [19] is not as general as that of [17], the results are, unsurprisingly, similar.

2 Also, like [21], one might ask whether we should be interested in the conditional probability of observing \( \Lambda_{\text{obs}} \) given the existence of observers or given the existence of human-like life. While more traditional approaches focus on human-like life, the causal entropic principle allows us to consider more generic observers.
for brevity in figures and when writing \( dp/d\log \Lambda \) (which should be read as \( dp/d\log(M_f^3) \)).

II. THE CAUSAL ENTROPIC PRINCIPLE

In this section, we briefly review the basic ideas and results of Ref. [17]. The fundamental assumption is that the probability distribution for \( \Lambda \) is proportional to entropy production in the causal diamond,

\[
\frac{dP}{d\Lambda} \propto \Delta S = \int_{t_i}^{\infty} dt V_c(t) \frac{dS}{dV_c dt} ,
\]

where \( V_c = (4\pi/3)t^3 \) is the comoving volume in the diamond at a given time, and \( dS/dV_c dt \) is the rate of entropy production per comoving volume. The causal diamond is defined in conformal time, \( ds^2 = a^2(\tau)(-d\tau^2 + dx^2) \), as the region shown in figure 1. The initial time is taken to be the time of reheating, although results are quite insensitive to this choice since \( V_c \) is negligible at such early times. In conformal time, the final time is finite (in fact, it is defined to be 0) since \( \tau = \int dt/a(t) \) converges for any \( \Lambda > 0 \); recall that \( a(t) \propto \exp(t\sqrt{\Lambda/3}) \) asymptotically. The initial time is \(-\tau_{\text{max}} \approx -2.8t^{1/3}_\Lambda \equiv 2.8 (3/\Lambda)^{1/6} \). The diamond achieves a maximum volume at the intermediate time \(-\tau_{\text{max}}\).

![FIG. 1: The causal diamond](image)

Ref. [17] estimated the amount of entropy produced by various astrophysical sources, including active galactic nuclei, supernovae, and cooling of galaxies, and determined that starlight, inelastically scattered by dust, is the largest source of entropy.\(^3\) The rate of entropy production per comoving volume can be written as

\[
\frac{dS}{dV_c dt} = \left( \frac{\Lambda}{4} \right)^3 \left( \frac{3}{\Lambda} \right)^{2/3} \times 8 \left( \frac{\Lambda}{4} \right)^{-1/3} \frac{\dot{\rho}_*}{T} .
\]

where \((d^2S/dM_dt)(t-t')\) is the rate of entropy production per stellar mass at time \( t \) due to stars born at time \( t' \), while \( \dot{\rho}_*(t') \) is the rate of stellar mass production (star formation rate, or SFR) at time \( t' \). \((d^2S/dM_dt)(t-t')\) in turn is given by

\[
\frac{d^2S}{dM_* dt}(t-t') = \frac{1}{\langle M \rangle} \int_{0.08M_\odot}^{M_{\text{max}}(t-t')} dM \xi_{\text{IMF}}(M) \frac{d^2s}{dN_* dt} ,
\]

in terms of the initial mass function \( \xi_{\text{IMF}}(M) \) (equal to \( 0.105 M^{-2.35} \) for \( M \geq 0.5M_\odot \) and \( 0.189 M^{-1.5} \) for \( M < 0.5M_\odot \)). The entropy production rate for a single star, \( d^2s/dN_* dt \), is its luminosity over an effective temperature, \( L_*/T_{\text{eff}} \approx (M/M_\odot)^{3.5} \times 10^{54} \text{yr}^{-1} \). The effective temperature \( T_{\text{eff}} \approx 20 \text{meV} \) reflects the fact that half of all starlight, originally UV, is scattered into the IR by dust and its associated entropy is thereby multiplied. The integration limits in [17] are the minimum and maximum masses of stars of a given age, where \( M_{\text{max}}(t-t') = \max(100M_\odot, 10 \text{Gyr}/(t-t'))^{2/3} \), reflecting the fact that the largest stars burn out more quickly than the smallest ones.

The total entropy production is thus the convolution of a cosmological factor, \( V_c(t) \), with an astrophysical one, \((dS/dV_c dt)(t)\). The former gives a preference to small values of \( \Lambda \) since \( V_c(t) \propto 1/\sqrt{\Lambda} \) and \( \int_{t_i}^{\infty} dt V_c(t) \propto 1/\Lambda \). The latter function is peaked at a time \( 2-3 \) Gyr, depending on the choice of the SFR, \( \dot{\rho}_* \), of which there are several somewhat different determinations in the literature [22, 23, 24]. The fact that star formation tapers off in the late universe softens the preference for small \( \Lambda \) such that the total entropy production (and hence by assumption the probability distribution for \( \Lambda \)) approaches a constant as \( \Lambda \rightarrow 0 \), but falls off rapidly as \( \Lambda \) becomes much greater than the observed value. To better visualize the range of likely \( \Lambda \) values, it is useful to consider the distribution in \( \log \Lambda \), \( dp/d\log \Lambda = \Delta dP/d\Lambda \), which is peaked at a nonvanishing value of \( \Lambda \). We have reproduced (and extended) the calculations of Ref. [17] to obtain \( dp/d\log \Lambda \), using three different determinations for the SFR, Nagamine et al. [22] (N), Hopkins-Beacom [22] (HB), and Hernquist-Springel [22] (HS). The distributions (not normalized) are shown in fig. 2.

Although there is some uncertainty in the prediction for \( dp/d\log \Lambda \) due to differences in the estimates for the SFR, all three curves in fig. 2 show that the observed value of \( \Lambda \) is within \( 1\sigma \) of the most likely value, which is a much better agreement than that obtained using the anthropic approach. In Ref. [17], only the the HB and N SFRs were considered, which are phenomenologically derived for our universe and in particular for the observed value of \( \Lambda \). In order to estimate the effect of varying \( \Lambda \), Ref. [17] multiplied the entire SFR by a correction factor \( F(t, M) \), taken to be the Press-Schechter fraction [22] of matter collapsed into halos at a typical galactic mass scale \( 10^7 M_\odot \) and the time \( t \sim 2-3 \) Gyr when the SFR is maximized. (We discuss the Press-Schechter formalism in some detail in the appendix.) This is a reasonable, though somewhat

\(^3\) The horizon entropy of black holes is a special case which is excluded; a single black hole’s entropy far outnumbers that of other entropy sources in the universe, but a universe filled with black holes would seem to be inimical to observers.
crude approximation, since the true answer must depend on the details of gravitational collapse over a range of mass and time scales. A major goal of the present work is to improve on this approximation by developing an SFR which has the correct quantitative dependences on $\Lambda$ and other cosmological parameters. We have carried this out, based on an analytical model due to Hernquist and Springel [24] (henceforth HS), which is the subject of the next section. Our result for $dP/d\log \Lambda$ on the HS SFR, as shown in fig. 2, is in reasonable agreement with the earlier results. More importantly, it allows us to consider with greater confidence the probability distributions for other parameters, as a way of further testing the causal entropic principle.

III. THE HERNQUIST-SPRINGEL SFR

As mentioned above, the N [22] and HB [23] SFRs are simple phenomenological formulae derived from observations in our universe, so they cannot tell us how the variation of cosmological parameters will affect the SFR. Fortunately, Hernquist & Springel [24] have developed a simple analytic model for the SFR based on more detailed numerical simulations [26]. Because the HS model can be written explicitly in terms of cosmological parameters and other physical constants, it is possible to vary the SFR in response to changes in, for example, the cosmological constant or the amplitude of cosmological perturbations. In this section, we will briefly describe our implementation of the HS SFR model, leaving a more detailed discussion of the model for the appendix.

HS start with the formula

$$\dot{\rho}_*(t) = \int \frac{dF}{d\ln M} (M, t) s(M, t) d\ln M . \tag{5}$$

Here, $F(M, t)$ is the Press-Schechter fraction (the fraction of matter collapsed into clouds of mass $M$ or less) and $s = \langle \dot{\rho}_* \rangle$ is the averaged rate of star formation in collapsed haloes.

The collapsed fraction for a given scale as a function of time can be simply calculated from the statistics of Gaussian random fields for an assumed input matter power spectrum with a power spectrum amplitude that evolves with time as expected from linear theory.

The scale at which the collapsed fraction should be calculated is set by atomic physics: one can collisionally excite hydrogen atomic transitions when kinetic temperatures exceed $\sim 10^4$ K. The virial relation between mass and temperature at time $t$ is found to be

$$M = \frac{(2kT/\mu_m)^{3/2}}{\rho_c} \approx \frac{(2kT/\mu_m)^{3/2}}{10GH(t)} , \tag{6}$$

where $k$ is Boltzmann’s constant, $\mu_m$ is the mean mass per particle, and $M$ depends on the Hubble parameter at the given time.

Using constants $\delta_c = 1.6868$ and $a = 0.707$, numbers found to provide good agreement between theory and large N-body simulations for the statistics of collapse, a simple version of the SFR suggested by HS is

$$\dot{\rho}_* = q(t) \left( 1 - \text{erf} \left( \frac{\sqrt{3} \delta_c}{2 \sigma_4} \right) \right) , \tag{7}$$

where $\sigma_4$ stands for fluctuations at the mass scale which virializes at temperature $10^4$ K, and the physics of cooling is parameterized in the prefactor $q(t)$.

The star formation rate within a collapse object is regulated by the rate of radiative cooling and the efficiency of radiative cooling is difficult to calculate; HS report that a good approximation to the star formation efficiency can be characterized by

$$q(t) \propto \left[ \chi(t)\tilde{\chi}/(\chi(t)^m + \tilde{\chi}^m)^{1/m} \right]^{\eta/2} , \tag{8}$$

where $\chi = (H/H_0)^{2/3}$, and $\tilde{\chi} = 4.6$, $\eta = 1.65$, and $m = 6$ provide good fits to numerical simulations ($H_0$ is the observed value of the Hubble rate taken as a reference value). Because we are interested in relative probabilities, we do not need to normalize the star formation rate.

To illustrate the parametric dependences of our modified SFR, we plot $\dot{\rho}_*(t)$ for several values of $\Lambda$ in figure 3. We also show there a naive approximation to the HS result, which consists of the analytic fit which HS made to their result,

$$\dot{\rho}_{*, \text{approx}}(t) \propto \frac{\chi^2}{(1 + \alpha(\chi - 1)^3 \exp(\beta \chi^{7/4}))} . \tag{9}$$
Here all dependence on the cosmological parameters is due to $\chi$, which depends upon $\Lambda$ through the time-dependent Hubble rate, and the parameter $\beta$, which depends on the density contrast $Q$ as $\beta \sim Q^{-2}$ via the Press-Schechter formalism. We have normalized (9) to agree with the more exact result at late times. The rough approximation tends to overestimate or underestimate the peak value of the more accurate SFR.

Let us also contrast our treatment with that of [17], where the shape of the SFR was assumed to remain constant, and only its overall normalization depended on $\Lambda$. Our results indicate a significant change in shape with $\Lambda$, with the peak even disappearing at large $\Lambda$. The reader may be surprised that the SFR increases with $\Lambda$, with the peak even disappearing at large $\Lambda$. The most straightforward cosmological parameter to vary is the density contrast $Q$, since it appears only via the time-dependent Hubble rate, and the parameter $\beta$, which depends on the density contrast $Q$ as $\beta \sim Q^{-2}$ via the Press-Schechter formalism. We have normalized (9) to agree with the more exact result at late times. The rough approximation tends to overestimate or underestimate the peak value of the more accurate SFR.

In the above we have given the simplest reasonable implementation of the HS model, and there is room for improvement, as outlined in the appendix. However, this is a physically motivated model for how star formation depends on cosmological parameters, and it captures the important elements that any such model must have.

IV. PREDICTIONS FOR OTHER PARAMETERS

As long as the properties of the universe are such that starlight continues to be the dominant source of entropy production, we now have the tools to compute $\Delta S$ as a function of cosmological parameters beyond just $\Lambda$. These include the density contrast $Q$, the ratio of baryonic matter to dark matter, the ratio of matter to photons, and possibly the spatial curvature. The latter is more subtle to try to quantify because it is time-dependent. One must specify the curvature at some reference time, for example at matter-radiation equality. However, there are good reasons for believing that the curvature of our universe was determined by inflation rather than by environmental considerations. We therefore will confine our investigations to the explicitly time-independent quantities.

A. Density contrast

The most straightforward cosmological parameter to vary is the density contrast $Q$, since it appears only through $\Lambda, Q$. It is also interesting because of the great sensitivity of the anthropic prediction for $\Lambda$ on $Q$. This can be understood from (A.10) and (A.24), which show that $Q$ and $\Lambda$ appear in the Press-Schechter fraction through the combination $Q\Lambda^{1/3}$. In the anthropic approach, structure formation is the most important effect, and this causes the anthropic upper bound on $\Lambda$ to scale like $Q^3$. In contrast, the entropic approach gives less weight to very early structure formation due to larger $Q$, because the volume of the causal diamond is smaller at earlier times. One can thus anticipate less sensitivity to $Q$ in the entropic approach.

This expectation is borne out in Figure 5, where $Q$ is varied by 4 orders of magnitude above the observed value, $Q = Q_{\text{obs}}$. In fact, the curves for $dP/d\log \Lambda$ saturate near the highest one shown as $Q \to \infty$. The position of the peak of the distribution shifts by only an order of magnitude, which is a much weaker dependence than the $\Lambda \sim Q^3$ dependence due to structure formation alone.
As $Q$ is varied, the epoch of star formation moves to earlier times, but the characteristic time of 1 Gyr remains relevant. It matters little whether stars form at $10^6$ years or $10^8$ years since their characteristic lifetime is $10^9$ years. At extremely low $Q$, though, the $\Lambda \sim Q^3$ dependence should reemerge.

We also plot $\Delta S(Q)$ for fixed $\Lambda = \Lambda_{\text{obs}}$ in figure 8 and a contour plot varying both $\Lambda$ and $Q$ in figure 7. Although our universe is not at the peak of the distribution, the probability of $Q_{\text{obs}}$ is only a factor of 8 smaller than the most likely value. For comparison, the usual $\chi^2$ statistic is of order $-2\ln P$, so a factor of 8 corresponds to a change in the effective $\chi^2$ on the order of 4.2.

One effect which has not been quantified in our approach is the cutoff on $Q$ which would result from overproduction of black holes; beyond some large value of $Q$, formation of stars (and observers) would be impeded by the loss of material to black holes. A discussion of this issue can be found in [13].

The reader should note that we have only calculated the entropic weighting factor for the probability, independent of any prior distribution for $Q$. What our results show is that, first, the observed value of $\Lambda$ is still near the most likely value, even for large values of $Q$, and also that entropic effects will not modify the prior distribution for $Q$ by more than an order of magnitude across a large range. It is reasonable to expect the prior distribution for $Q$ to be $dP/dQ \propto Q^n$ with $-1 \lesssim n \lesssim 1$ [15]; the smaller value works well with our results.

### B. Baryon fraction

Another cosmological parameter which can obviously affect star formation is the ratio of baryonic to dark matter, or baryon fraction $f_b$. Since stars are baryonic, the SFR should have an overall factor of $f_b$ to account for the availability of baryonic material for star formation, as appears in (A.16).

The details of structure formation are also affected by $f_b$; at very large values, gravitational collapse is impeded by pressure; this is known as Silk damping. Because the baryonic matter cannot collapse easily due to pressure, the matter power spectrum in (A.4) scales as $(1-f_b)$ for a fixed initial amplitude $Q$, so the r.m.s. fluctuation $\sigma$ scales like $(1-f_b)^{1/2}$.

Since our universe has $f_b \sim 1/6$, we define the relative baryon fraction as $r_b = 6f_b$. The dependence of $\dot{\rho}_\star$ on $r_b$ as described above can be summarized as making the replacement

$$\dot{\rho}_\star(\mu, \sigma) \rightarrow r_b \dot{\rho}_\star \left( \mu, \frac{6-r_b}{5} \sigma \right).$$

The entropic distribution for $r_b$ is shown in figure 8. Similarly to the distributions for $\Lambda$ and $Q$, it indicates that the measured value of $f_b$ is not quite the optimal one,
but also not very far from being so: the probability of $f_b_{\text{obs}}$ is $\sim 1/2$ of that for the most likely value.

The reader should note that, as with $Q$, we have not attempted to calculate the prior probability distribution for the baryon fraction. Our results show that the entropic weighting does not change the probability distribution by more than a factor of 2 for $1/6 \lesssim f_b \lesssim 5/6$. Therefore, the value taken in our universe is reasonably probable unless the prior distribution is strongly peaked at some other value.

\[ \frac{\text{entropy production}}{\text{relative baryon fraction}} \]

\[ \text{FIG. 8: Unnormalized probability distribution for relative baryon fraction } r_b = 6f_b \text{ at fixed } \Lambda = \Lambda_{\text{obs}}. \]

C. Matter abundance

A third quantity which can strongly affect star formation is the overall abundance $\xi$ of matter relative to photons, for fixed baryon fraction, $Q$ and $\Lambda$. In a universe with larger $\xi$, matter domination will occur earlier, and the CMB temperature at a given time will be reduced. In our analysis, radiation does not play a significant role in the evolution of the scale factor of the universe, so we can again focus on how $\xi$ affects the SFR. The dependence of the SFR on $\xi$ has two origins: first, $\mu$ scales as $\xi^2$ by definition. Second, the linear growth factor for perturbations, $G(a_{eq})$ in [\ref{eq:linear_growth}] depends on $\xi$ simply because it is evaluated at matter-radiation equality. As long as matter dominates over curvature and $\Lambda$ at matter-radiation equality, it is straightforward to find [\ref{eq:xi_dependence}] that $G(a_{eq}) \sim \xi^{-4/3}$. From those arguments, we can infer that the $\xi$ dependence comes in the form

\[ \dot{\rho}_* (\mu, \sigma) \rightarrow \dot{\rho}_* \left( r_m^2 \mu, r_m^{4/3} \sigma \right), \quad (11) \]

where the relative matter abundance $r_m$ is the rescaling of $\xi$ such that $r_m = 1$ for our universe. Unlike the case of varying $f_b$, there is no reduction in the matter power spectrum as $r_m$ is increased, so star formation and entropy production only become more efficient as $\xi$ increases but in a manner qualitatively similar to the dependence on the density contrast $Q$. This is shown in figure \ref{fig:entropy_matter} Similarly to the case of $Q$, the relative probability of the observed value of $\xi$ is $\sim 1/8$.

Again, we have not attempted to calculate the prior probability distribution for $\xi$, but our results show that the entropic weighting factor itself does not disfavor our universe greatly.

\[ \frac{\text{entropy production}}{\text{relative matter abundance}} \]

\[ \text{FIG. 9: Unnormalized probability distribution for relative matter abundance } r_m \text{ at fixed } \Lambda = \Lambda_{\text{obs}}. \]

V. DECAYING DARK MATTER

In the previous section we confined our attention to the variation of standard cosmological parameters. More generally, one could imagine universes with properties rather different from our own, requiring for their description other parameters than those characterizing the standard cosmological model. A possible new source of entropy is the decay of dark matter particles. We shall show in this section that one can design universes not so different from our own where entropy production is actually dominated by dark matter decays.

Interestingly, there are experimental hints from observations of gamma rays that dark matter could be unstable. The EGRET collaboration observes $\gamma$-rays in the $2 - 10$ GeV range which are unaccounted for by standard mechanisms [33]. It has been suggested that these excess $\gamma$s are due to the annihilation [34] or decays [35] of dark matter. At lower energies, the COMPTEL experiment observed an excess in the $1 - 5$ MeV range. Ref. [36] has proposed the decays of Kaluza-Klein (KK) or supersymmetric (SUSY) dark matter to explain this anomaly. In a similar energy range, numerous experiments, including the SPI spectrometer on the INTEGRAL observatory [37] have observed 511 keV photons from the galactic center, indicative of an excess of positrons annihilating nearly at rest. Astrophysical explanations have not convincingly accounted for this excess, leading to suggestions
that the positrons are a result of dark matter annihilations \[38\] or decays \[39\].

If indeed dark matter is shown to be unstable, it is curious that the MeV-scale anomaly requires a lifetime which is within a few orders of magnitude of the age of the universe. This could then pose a new coincidence problem. Interestingly, the entropic principle could explain such a coincidence, as we now discuss. The rate of entropy production from dark matter (DM) decays is much simpler to calculate than that from stars; it is given (proportionality).

\[\frac{dS_d}{dV_c dt} = g_s \Gamma n e^{-\Gamma t} ,\]

where \(\Gamma\) is the DM decay rate, \(n\) the DM density, and \(g_s\) the entropy increase per particle decay. We find analytically that the entropy produced by decays has the functional form

\[\Delta S_d = t_A f(\Gamma t_A) ,\]

where we recall that \(t_A \equiv \sqrt{3}/\Lambda\). The function \(f\), calculated numerically, is shown in figure 10. It is closely fit by the analytic approximation

\[\ln f \approx c_1 + c_2 x - \ln(1 + c_3 e^{c_4 x}) ,\]

where \(x = \ln \Gamma t_A\) and the constants are \(c_1 = 1.164\), \(c_2 = 0.0512\), \(c_3 = 0.889\), and \(c_4 = 1.921\). The distribution is peaked for a DM lifetime given by

\[\tau = \frac{1}{\Gamma} \approx \frac{t_A}{4.7} .\]

In our universe, \(t_A = 16.7\) Gyr, corresponding to a preferred lifetime of 3.6 Gyr. This could then explain why dark matter should be decaying on a timescale comparable to the age of the universe.

![Figure 10: Entropy produced by dark matter decays as a function of the dimensionless combination \(\Gamma t_A\) (upto an overall proportionality).](image)

The above argument might only be valid if the entropy produced by decays is greater than or equal to that coming from starlight. We must therefore compare the magnitudes of \(\Delta S\) from decays and from stars. Ref. \[17\] found that the peak rate of entropy production by stars was of the order

\[\frac{dS_s}{dV_c dt} = 10^{63} \text{ Mpc}^{-3} \text{ yr}^{-1} .\]

Using the known energy density of dark matter, we find that the corresponding expression from decays is

\[\frac{dS_d}{dV_c dt} \approx 10^{66} g_s \left(\frac{\Gamma}{\text{Gyr}^{-1}}\right) \left(\frac{eV}{m}\right) \left(\frac{\rho_{DM}}{(10^{-3}eV)^4}\right) \text{ Mpc}^{-3} \text{ yr}^{-1} ,\]

where \(m\) is the DM mass. Therefore the requirement for entropy from DM to dominate is

\[\frac{m}{g_s} \leq \text{ keV} .\]

This can obviously be satisfied even if \(g_s \approx 1\) if dark matter is sufficiently light.\(^{5}\) However, to explain the 511 keV signal, for example, one needs \(m \sim \text{ MeV}\), which would require \(g_s \sim 1000\). At first sight, this would seem like an unreasonably large increase in entropy per decay. However, it might actually be easy to achieve, when one considers that electrons are typically produced along with the positrons, since we know that dark matter is charge-neutral, and that these electrons, if even mildly relativistic, must produce a great number of lower-energy photons as they thermalize.

Let us consider how an electron of energy \(\sim \text{ MeV}\) thermalizes within a galaxy. Synchrotron radiation is kinematically blocked because the cyclotron frequency is only \(eB/m_e \sim 10^{-13}\) eV for \(B \sim 10^{-9}\) T, while photons have a plasma mass of \(m_\gamma \sim (\alpha n_e/m_e)^{1/2} \sim 10^{-12}\) eV, since \(n_e \sim 1\) cm\(^{-3}\). Instead, the electron loses energy to the galactic medium by Coulomb interactions; using the formalism developed by \[40\], we can see that an electron would lose all its kinetic energy over \(\sim 10^5\) yr and \(\sim 3\) kpc. Thermalization thus takes place much faster than the age of the universe. The final energy is of the order \(E \sim 1 - 100\) eV, resulting in an entropy production of \(\Delta S \sim \text{ MeV}/E \sim 10^4 - 10^5\). For electrons produced in the galactic halo, thermalization will be less efficient. Nevertheless, from this number we see that it is possible for DM decays which produce mildly relativistic \(e^+e^-\) pairs to result in entropy production which is similar in magnitude to that produced by stars.

It might be objected that the existence of such low energy photons should be irrelevant for creating observers, even if they far outnumber photons originating from stars. However we wish to avoid making any assumptions about the detailed nature of observers, since this was one of the arbitrary features of the anthropic principle which one would like to overcome with the entropic principle.

\(^{5}\) Notice that \(g_s\) is quantized, so it cannot be arbitrarily small and still nonzero.
VI. CONCLUSIONS

Using the analytic star formation model of HS, we have improved upon and extended the calculations of [17] to test the causal entropic principle further. First, we recalculated the probability distribution for the cosmological constant $\Lambda$ alone, finding good agreement with the approximations of [17]. We subsequently extended the original analysis to allow for simultaneous variation of $\Lambda$ with the density perturbation amplitude $Q$, finding that the most probable value of $\Lambda$ changes by less than an order of magnitude as $Q$ varies by 5 orders of magnitude. In addition, the width of the probability distribution grows as $Q$ grows, so the observed value of $\Lambda$ remains reasonably probable even at large $Q$. The observed value of $Q$ (at fixed $\Lambda$) has about 1/8 the probability of the peak values, which may or may not be compensated by the prior probability distribution. Due to the flexibility of the HS SFR, we have also been able to calculate the entropic weighting factor for varying baryon fraction and matter abundance, finding that the observed values are not unreasonable.

We have moreover demonstrated the adaptability of the causal entropic principle to universes which could be qualitatively different from our own, by considering entropy produced by the decay of massive particles. If entropy due to particle decay is the dominant source of entropy produced by the decay of massive particles. In this appendix, we review in more detail the HS SFR, which was developed based on more detailed numerical simulations [20] combined with analytic reasoning.

We remind the reader of the basic formula

$$\dot{\rho}_\ast(t) = \int \frac{dF}{d\ln M}(M, t) s(M, t) d\ln M.$$  \hspace{1cm} (A.1)

Here, $F(M, t)$ is the Press-Shechter fraction (the fraction of matter collapsed into clouds of mass $M$ or less) [23], and $s = \langle \dot{\rho}_\ast \rangle$ is the averaged rate of star formation in collapsed haloes. We can now break down the individual parts of this formula.

1. Virialization

An important issue for star formation is the virialization of collapsing gas, which we here review, following the discussion of [28] on the spherical collapse model in a matter-dominated FRW universe.

Consider a universe of critical density in matter with a spherical overdensity (and a corresponding underdense shell). There is an exact solution for the evolution of this universe starting from $a = 0$ at proper time $t = 0$ (its precise form is unimportant for us). The overdense sphere reaches its turn-around radius at time $t_{ta}$ and then recollapses completely (to a local value $a = 0$) in time $2t_{ta}$. At the time of complete recollapse, the overdensity in the linearized theory is $\delta_c = 1.686$, which is known as the critical overdensity. Any fluctuation in the initial density field with a larger linearly evolved overdensity would have collapsed at an earlier time.

Of course, the overdense sphere does not recollapse completely, due to the effective pressure supplied by the random velocities that the particles would inevitably acquire during the collapse when deviations from strict spherical symmetry are allowed. From the virial theorem, energy redistributes among the dust particles such that the total kinetic energy becomes $-1/2$ the total potential energy in the final state. This suggests that the virial radius $R_v$ be $1/2$ the turn-around radius in the matter-dominated universe. Therefore, between turn-around and virialization, the density of the sphere increases by 8. Meanwhile, since the time doubles, the
background density decreases by a factor of 4. At turnaround, the density contrast is \( \rho / \bar{\rho} = 9\pi^2 / 16 \), so the virial density becomes \( \rho_v = 18\pi^2 \rho_m \).

Other virial quantities can then be determined in terms of the virial density and the total mass of the collapsed cloud. Ignoring order-unity numerical factors, the virial temperature

\[
T_v = \frac{\mu_m V_v^2}{2k} \approx \frac{\mu_m G}{2k} (M^2 \rho_v)^{1/3} 
\]

where \( \mu_m \approx 0.6m_p \) is the (appropriately averaged) molecular mass in the cloud. These are all time dependent through the evolution of the background densities, and the last relationship gives rise to \( 4 \).

A slight generalization of the above argument allows the calculation of the virial density in other backgrounds. For instance, the same collapsing solution can be used to determine the virial density when the background universe has curvature; however, the virial density is determined by comparison to the spatially curved background. An analytic approximation to the exact virial density is given in \( 24 \), based on a derivation in \( 30 \). In the case of a background universe with a cosmological constant (but no spatial curvature), the evolution of the overdense sphere must be calculated numerically, but the virial density is derived in much the same way. One caveat is that the cosmological constant modifies the gravitational potential, so the virial radius is no longer half of the maximum radius of the overdense cloud. Based on calculations in \( 31 \), \( 24 \) found an approximate analytical formula for the virial density in terms of the background densities. We have calculated the virial density in terms of the background densities for a universe with three components, matter, curvature, and \( \Lambda \).

In all the cases, however, the virial density is within two orders of magnitude of the dominant component of the background density. Since the virial density only enters our calculations through a fractional power, the precise value is not terribly important, and we simply use the result for the matter-dominated universe. (We plan to correct this detail in future work.)

\[ \text{2. Press-Schechter Collapsed Fraction} \]

The Press-Schechter collapsed fraction \( 25 \) is the fraction of matter bound in halos of mass \( M \) or less, which one can approximate as the fraction of density fluctuations at length scale \( R \) with density contrast greater than the critical density \( \delta_c \). It is given by

\[
F(M, t) = \text{erf} \left( \frac{\delta_c}{\sqrt{2\sigma(M, t)}} \right),
\]

where \( \sigma \) is the root-mean-square density fluctuation with a wavelength given by the comoving radius associated with the mass scale \( M \): \( R^3 = (3/4\pi)(M/a^3 \rho_m) \).

Therefore another quantity needed for determining the SFR is the linear growth of fluctuations in the argument of the Press-Schechter function. In linear theory, the variance in overdensity in spheres that enclose a mass \( M \) is

\[
\sigma^2(M, a) = D(a)^2 \int_0^\infty \frac{dk}{2\pi^2} k^2 P(k) \left( \frac{3j_1(kR)}{kR} \right). \tag{A.4}
\]

This form factorizes the variance into an overall linearized growth factor and a matter power spectrum, which is valid after the end of radiation domination. It does not work during radiation domination because modes inside and outside the horizon evolve differently during that time. After the radiation era, \( P \) takes into account the different evolutions. There are additional small effects after radiation domination but before decoupling that we will neglect here.

One can then integrate

\[
\int_0^\infty \frac{dk}{2\pi^2} k^2 P(k) \left( \frac{3j_1(kR)}{kR} \right) = Q^2 \Sigma(\mu)^2, \tag{A.5}
\]

where the dimensionless parameter \( \mu = \xi^2 M/M_0^3 \) is proportional to the mass in terms of the horizon mass at matter-radiation equality \( 13 \). Here \( \xi = \rho_m / n_\gamma \) is a time-independent measure (apart from decays which increase the density of photons) of the amount of nonrelativistic matter in the universe. Empirically, the spectrum is found to be well approximated by \( 13 \)

\[
\Sigma(\mu) = \left[ (9.1\mu^{-2/3})^{-0.27} + (50.5 \log(834 + \mu^{-1/3}) - 92)^{-0.27} \right]^{-1/0.27} \tag{A.6}
\]

and \( Q = \delta \rho / \rho \) is the amplitude of fluctuations at horizon entry, which coincides with the value at the end of inflation.

Next, we compute the linear growth factor, assuming that matter dominates over both curvature and \( \Lambda \) during the radiation era. If there were a single overall growth factor during the radiation era, we would have for subhorizon scales at sufficiently early times that \( a < a_{eq} \)

\[
D(a) = 1 + 3 \frac{a}{2 a_{eq}}. \tag{A.7}
\]

This is a well-known result (see for example \( 28 \)). After the radiation dominated era \( (a > a_{eq}) \), \( A.7 \) generalizes to \( 13 \)

\[
D(a) = 1 + 3 \frac{G(a)}{2G(a_{eq})} \tag{A.8}
\]

where the spectrum \( \Sigma(\mu) \) accounts for the differential growth of different wavelengths during the radiation era. Shortly after the end of the radiation era, the first term becomes unimportant. The well-known result for matter, curvature, and/or \( \Lambda \)-dominated universes is

\[
G(t) = H(t) \int_0^{a(t)} \frac{da}{(aH)^3} \tag{A.9}
\]
as long as the universe is not pure de Sitter \cite{28} or dominated by a form of dark energy that is not a simple cosmological constant. The authors of \cite{24} obtain an analytic approximation for \( \Lambda \) for their SFR, but that approximation is not valid for values of \( \Lambda \) much different that in our universe. Therefore we evaluate the integral numerically.

Nevertheless, it is enlightening to have an analytic formula to get some intuition for parametric dependences. Ref. \cite{13} finds that the growth factor for a flat universe is approximated by

\[
D(t) \approx 0.2 \frac{e^{4/3}}{\rho_\Lambda^{1/3}} x^{1/3} \left[ 1 + \left( \frac{x}{1.44} \right)^{\alpha} \right]^{-1/3\alpha}, \tag{A.10}
\]

where \( \alpha = 0.795 \), \( \rho_\Lambda \) is the vacuum energy density, and \( x = \rho_\Lambda / \rho_\Lambda(t) \) serves as a dimensionless “time” variable.

### 3. Cooling and Star Formation Efficiency

The other key factor in the HS analysis is the star formation rate in a collapsed halo of mass \( M \), denoted by \( s(M, t) \). HS propose a simple model for gas cooling, which matches well with numerical studies of a detailed multi-phase model \cite{26}. They find that the star formation rate is proportional to the radiative cooling rate. This model ignores metal-line cooling and the slow change of elemental abundances due to star burning. For our purposes, metal-line cooling is sufficiently small to neglect, while the change of abundances (nuclear fuel depletion) is important only for special values of the cosmological parameters, so we will ignore both of these effects in this paper. This also neglects cooling through \( H_2 \) molecules, a mechanism which may be important in forming the first stars in our universe but is believed to be currently negligible.

From their numerical studies, HS argue for a factorization

\[
s(M, t) = q(t) \times \begin{cases} 0, & T_v < 10^4 \text{K} \\ 1/3, & 10^4 \text{K} < T_v < 10^{6.5} \text{K} \\ 1, & 10^{6.5} \text{K} < T_v \end{cases} \tag{A.11}
\]

and calculate the cooling rate at \( 10^7 \) K. Based on numerical results and physical reasoning about winds in star forming regions, HS assume a simple form for the proportionality factor,

\[
q(t) \propto \min \left( \frac{1}{T_v}, \bar{q}(t) \right). \tag{A.12}
\]

The idea is that at high densities, the star formation rate saturates at (slightly less than or equal to) the gas consumption time scale \( t^* \), approximately 2.1 Gyr. Then \( \bar{q} \) is the star formation rate due to radiative processes, calculated without regard to this upper limit.

To compute the cooling rate, consider the cooling time at a radius \( r \) from the center of the cloud. This is

\[
t(r) = \left( \frac{3}{2} kT / \rho_b(r) \mu \right) \left( \frac{1}{n^2 H \Omega(T)} \right), \tag{A.13}
\]

where \( \rho_b \) is the baryon density within the cloud, \( n_H \) is the hydrogen number density, and \( \Omega(T) \) is the cooling function. We can understand this formula as follows: \( 3kT / 2 \) is the energy per particle, \( \rho_b / \mu \) is the particle number per volume, \( n_H^2 / \Omega(T) \) is proportional to the rate of hydrogen collisions, and \( \Omega(T) \) is proportional to the energy loss per collision. To phrase this in terms of more fundamental parameters, we write \( n_H = X \rho_b / m_b \), where \( X \) is the hydrogen mass fraction. Following HS, we take the density of the cloud to be a power law

\[
\rho_b(r) = \frac{(3 - \eta) f_b M}{4\pi R_c^3 \eta \eta}, \tag{A.14}
\]

where \( f_b = M_b / M \) is the baryon mass fraction. HS note that \( \eta = 2 \) is the prediction for a thermal distribution, but matching to numerical results gives \( 1.5 < \eta < 2 \) with \( \eta = 1.65 \) as the best fit value.

From the above, we can show that

\[
\frac{dr}{dt}_{\text{cool}} = \frac{1}{\eta T_v}. \tag{A.15}
\]

Moreover, HS argue that the cooling time should be the natural timescale for the gas to virialize, \( t = R_v / V_e \). Combining these results leads to

\[
\bar{q}(t) \propto \frac{3 - \eta}{\eta} f_b \left( \frac{3 - \eta}{4\pi G} \bar{f}(T) \right)^{(3-\eta)/\eta} (G \rho_v(t))^{3/2\eta}. \tag{A.16}
\]

For shorthand, we have written

\[
f(T) = \frac{3}{2} \frac{kT m_H^2}{\mu X^2 \Omega(T)}. \tag{A.17}
\]

From \cite{32}, the cooling function is known to be

\[
\Omega(10^7 \text{K}) = 10^{-23} \text{erg} \cdot \text{cm}^3 \cdot \text{s}^{-1} \tag{A.18}
\]

with little variation over several orders of magnitude in the temperature.

It is not, however, necessary for our purposes to compute the cooling and star formation rates to this level of detail. The cooling rate \( \bar{f}(T) \) depends on time only through the virial density \( \rho_v \), which is, to within an order of magnitude, well-approximated by \( 10 M_b^2 / H^2 \). Therefore, it is reasonable to fit the star formation rate \( \bar{f}(T) \) by a function of \( H \), which HS give as

\[
\bar{q}(t) \propto H^{3/\eta} \left( \frac{\bar{f}^{3/\eta}}{H^{2m/3} + H^{2m/3})^{3/2m\eta}} \right), \tag{A.19}
\]

with \( m \sim 6 \) and \( H \sim 10 H_0 \) providing good fits to simulations. This can be rewritten in the form \( \bar{q}_H \). Again, we hope to correct this inaccuracy in future work.

### 4. Synthesis

The final expression for the HS star formation rate can be simplified by the approximation in terms of elementary functions of the error function which appears in the
Press-Schechter theory,
\[ \text{erf}(y) \approx 1 - \frac{1}{1 + \sqrt{\pi} ye^{y^2}}. \quad (A.20) \]
Using constants \( \delta_c = 1.6868 \) and \( a = 0.707 \), HS find that \( \rho_\star = q(t) \left( \frac{1}{3} \text{erf} \left( \frac{\sqrt{a} \, \delta_c}{2 \, \sigma_3} \right) - \frac{2}{3} \text{erf} \left( \frac{\sqrt{a} \, \delta_c}{2 \, \sigma_3} \right) \right) \), \quad (A.21)
where \( \sigma_n \) stands for fluctuations at the mass scale which virializes at temperature \( 10^n \) K. The relation between mass and temperature at time \( t \) is found to be
\[ M = \frac{(2kT/\mu_m)^{3/2}}{\rho_v} \approx \frac{(2kT/\mu_m)^{3/2}}{10GH(t)}, \quad (A.22) \]
where \( k \) is Boltzmann's constant and \( M \) depends on the Hubble parameter at the given time. Using the measured value \( \xi/M_p = 3.3 \times 10^{-28} \) for the matter abundance, the dimensionless mass parameter \( \mu_n \) corresponding to a \( 6 \) The constant \( a \) is introduced to modify the Press-Schechter fraction along the lines of the Sheth-Tormann fraction.

-virialization temperature \( T = 10^n \) K is found to be
\[ \mu_n = 1.1 \times 10^{3+1.5n} \left( \frac{\text{Gyr}}{H(t)} \right). \quad (A.23) \]
Using (A.6,A.9), the corresponding fluctuation amplitude is given by
\[ \sigma_n = Q\Sigma(\mu_n)D(t). \quad (A.24) \]
A simpler version of the SFR is also suggested by HS, namely
\[ \dot{\rho}_\star = q(t) \left( 1 - \text{erf} \left( \frac{\sqrt{a} \, \delta_c}{\sqrt{2} \, \sigma_3} \right) \right). \quad (A.25) \]
Our numerical comparisons indicate that this differs very little from the version (A.21), so we adopt (A.25) for our subsequent analysis. This is our final SFR, given earlier in (7).

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