ABSTRACT. New systematic classification of cosmological models of the present Universe is introduced. After making the comparison of these models with all existing observational data three viable models remain: the cold dark matter model with the cosmological constant (which becomes the most reasonable one if the Hubble constant $H_0 > 60$ km/s/Mpc); the mixed cold-hot dark matter model (models with two and especially three types of neutrinos with equal masses are in a slightly better agreement with observational data than the model with one massive neutrino); the pure cold dark matter model with a step-like initial spectrum of perturbations. The two latter models require $H_0 \leq 60$ km/s/Mpc.

Looking at the enormous amount of papers in which different cosmological models of the present Universe are studied and compared with observational data, the first impression might be that there exists a great degree of freedom and arbitrariness in such models. However, actually this is not so. First, using a natural fundamental classification described below, it appears that all currently popular models follow some simple logic. It becomes clear also what are the natural directions of generalization of these models. Second, if all existing observational data are used (and not some arbitrarily chosen tests only), then a number of viable cosmological models reduces to a few - we shall argue below that only three models remain viable at present. Would we know the Hubble constant $H_0$ with a better accuracy, we could further reduce this number and might even finish with a unique preferred one.

To introduce this classification, let us remind the logical structure of cosmology. The aim of theoretical cosmology, as of any other branch of science, is to produce definite predictions basing upon as minimum number of assumptions as possible (“Occam’s razor”) which can be tested by observations. To make these predictions, first, we need equations (or Lagrangians). It is generally accepted now that the Universe is described by a space-time metric satisfying the Einstein equations with some matter source. Moreover, it was shown that a broad class of theories more general than the Einstein gravity (e.g., scalar-tensor theories of gravity, the $R + f(R)$ theory where $R$ is the trace of the Ricci tensor, etc.) can be represented in the Einsteinian form after some conformal transformation. So, the left-hand side of the equations may be taken as $R_{ik} - \frac{1}{2}Rg_{ik}$.

However, the equations are not defined if their right-hand side is not completely specified. So, we should assume some matter content of the Universe (in particular, at present
time). It is remarkable that all viable cosmological models require the main part of matter in the present Universe to be the dark non-baryonic matter. It is dark in the direct sense of the word, i.e. not participating in electromagnetic interactions. Moreover, all candidates for this dark matter are not yet discovered in laboratory experiments (or not yet discovered are the specific properties of particles making them important for cosmology, if speaking, e.g., about neutrino masses). Thus, modern cosmology strongly suggests and awaits great laboratory discoveries in the field of particle physics!

The evidence for dark matter follows from the following numbers.

1. The energy density of luminous matter in the Universe (stars, hot gas in galaxies and between them) in terms of the critical one $\varepsilon_c = 3H_0^2/8\pi G$ (we assume $c = h = 1$): $\Omega_{\text{lum}} < 0.01$.
2. The primordial (Big Bang) nucleosynthesis (BBN) prediction for the energy density of baryons in the Universe based on observed abundances of light elements ($^4\text{He}, \text{D}, ^3\text{He}, ^7\text{Li}$) is $\Omega_b = (0.01 - 0.02)h^{-2}$ where $h = H_0/100$.
3. The virial energy density of matter gravitationally clustered at scales $R = (5 - 25)h^{-1}\text{Mpc}$ is $\Omega_{\text{vir}} = 0.2 - 0.5$.
4. Finally, the theoretical expectation in most inflationary models is $\Omega_{\text{tot}} = 1$.

Comparing 2 with 1, we see that some amount of baryon dark matter should exist (e.g., in the form of faint stars). However, from the comparison of 3 to 1 we see that much more matter should be in a non-baryonic dark form. Note that this conclusion is not based on any theoretical assumptions about a cosmological model. If we compare 4 with 3 (using now some hypothesis about the early Universe), then the further conclusion follows that there may exist different types of dark non-baryonic matter in the Universe.

The main 3 types of dark matter (further, we shall speak about non-baryonic dark matter only) are:

1. Hot dark matter (HDM) - neutrinos with restmasses of the order of few eV.
2. Cold dark matter (CDM) - supersymmetric particles with masses $\sim 100\text{ Gev}$ or axions with $m_a \sim 10^{-5}\text{ eV}$.
3. Ultra-cold (vacuum-like) matter - the cosmological constant.

However, we understand now that there are no impenetrable barriers between these 3 types. In more exotic models, intermediate types of dark matter may appear between these ones. In particular, warm dark matter (e.g., thermal particles with masses of about 1 keV or non-equilibrium neutrinos) fills the gap between 1 and 2, and a scalar field with the exponential potential can be used to produce an arbitrary equation of state of matter between 2 and 3 (actually, even in the range from $p = \varepsilon$ to $p = -\varepsilon$ where $p$ is the pressure of matter).

So, if the dark matter content is specified, the equations are fully determined. However, to solve them we need initial conditions (mostly, for perturbations). The observational evidence from $\Delta T/T$ angular anisotropy of the cosmic microwave background (CMB) proves that the Universe was FRW-like (isotropic and homogeneous) at the recombination time (at the redshift $z \approx 1100$). The BBN theory shows that the Universe was FRW-like in the much earlier period ($t = (1 - 100) s$ after the cosmological singularity, $z = 10^8 - 10^9$). The absence of primordial black holes with $M = 10^{15} - 10^{17}\text{ g}$ which would evaporate now through the Hawking radiation suggests the isotropy of the Universe at even earlier times, and so on. However, even if we assume that the Universe was FRW-like just from the very beginning, i.e. from times close to the Planck time $t_P = \sqrt{G}$, we still remain with three arbitrary functions of space coordinates (or 3 arbitrary functions of the wave
vector $\mathbf{k}$ in the Fourier representation) specifying initial amplitudes of the quasi-isotropic modes (called so because they do not destroy local isotropy and homogeneity at arbitrarily early times). One of these functions refers to scalar (adiabatic) perturbations and two others - to the quasi-isotropic mode of tensor perturbations (primordial gravitational waves). In more complicated models, additional functions giving initial values for non-decreasing isocurvature modes can appear. Therefore, generally we have a large functional arbitrariness in initial conditions.

The second main advantage of the inflationary scenario of the early Universe, after the elegance and beauty of its main assumption that our Universe was in the maximally symmetric (de Sitter, or inflationary) state during some period in the past, is that it predicts these initial conditions in terms of a small number of fundamental parameters of an effective Lagrangian describing an inflationary stage realized in any concrete version of this scenario. Thus, the arbitrariness reduces to a few (minimum one) parameters. From all this discussion, a natural and fundamental classification of cosmological models of the present Universe follows (including non-inflationary models, too, but I shall discuss only inflationary ones further) [1] (see also [2]): let us classify them by their level of complexity number which is equal to a number of

- a) new (not known before, e.g., $H_0$ and $\Omega_\gamma$ are not counted),
- b) fundamental (appearing in basic equations, not in initial conditions),
- c) significant (more than $\sim 10\%$ effect, e.g. if $|n_s - 1| \leq 0.1$, it is counted as $n_s = 1$),
- d) dimensionless (this can be always achieved, e.g., working in the Planck units)

Constants introduced in any particular models in order to explain all observational data. These constants may refer either to the present dark matter content or to the initial spectrum of perturbations. Note that the complexity level of a given model may decrease as a result of future discoveries in laboratories (e.g., measurements of neutrino restmasses) or due to a progress in an underlying unified physical theory (e.g., a derivation of inflaton parameters from the superstring theory).

Distinguishing features of this classification which radically discriminate it from all previous attempts in this direction are, first, the fact that the number of assumptions using no numbers (numerical constants) is not counted at all. Thus, this classification is favourable even for very "strange", "crazy" models if they are internally consistent and do not introduce a large number of free parameters. Second, it counts assumptions referring to the present dark matter content and to the initial spectrum of perturbations on equal footing. Thus, if we want to complicate a model under the pressure of observational data and shift to the next level of complexity, we may add one parameter either to the effective Lagrangian describing an inflationary stage (and then we get one more parameter in the initial spectrum of perturbations), or to the description of the dark matter content (e.g., by introducing one new type of dark matter). Then the logical order of the development is to begin with the lowest complexity level, compare it with the data, and if there is no agreement, move to successive higher levels until the agreement will be reached.

In the Table 1 below a very brief sketch of existing cosmological tests is presented with an approximate range of scales to which they are sensitive (or of scales for which data exist at present). Here $\Phi$ is the gravitational potential and $h_{\alpha\beta}$ are the tensor perturbations (gravitational waves). The Table 2 gives a list of models with their complexity level numbers, initial (fundamental) parameters, observational parameters which can be unambiguously expressed through the fundamental ones, and the result of comparison with observational data. $A$ is the rms amplitude of the spectrum of adiabatic perturba-
tions, $A_{\text{iso}}$ is the same for isocurvature perturbations, $\sigma_8$ is the total rms top-hat matter perturbation at the scale $R = 8h^{-1}$ Mpc. For the model 6, $\phi_0$ is the value of the inflaton field at the moment of bubble formation, $\phi_f$ is its value at the end of inflation. For the model 10, $\phi_s$ is the value of the inflaton field at the beginning of the last stage of inflation. For the model 11, $\nu_1$ denotes an unstable neutrino while $\nu$ denotes the stable one. The latter model (which is actually a class of different models) is still largely unexplored, so no conclusion about its viability is drawn.

All details about the confrontation of the models with observational data can be found in [3,2] for the models 3 and 9, in [4,5] for the model 4, in [6] for the model 5 and in [7,8] for the model 10. The list of references is, of course, very incomplete, but references to other papers can be found in the given ones.

So, the first remarkable conclusion is that it is possible to explain all existing cosmological observational data using 2 or 3 parameters only (4 in the case of three neutrinos with comparable masses). There no necessity to go to higher complexity levels at present.

The second conclusion shows the dependence of results on the value of $H_0$ (still not determined with the desired degree of accuracy). If $H_0 > 60$ km/s/Mpc, then the best (and probably the only possible) model is the model 5: CDM+$\Lambda$ with $n_s \approx 1$. So, a reliable observational proof that $H_0 > 60$ should be considered as a very strong argument for the positive cosmological constant. Note, however, another prediction that $H_0 < 80$ because in the opposite case no reasonable model exists (at least, at the complexity levels considered). On the other hand, for $H_0 \leq 60$ we can avoid introducing the cosmological constant, and then the choice is between the mixed CDM+HDM models 3 or 4 and the pure CDM model 10. They can be most easily discriminated by the abundance of compact objects at large redshifts.

The third conclusion is that if the mixed CDM+HDM model is the right one, then it is very interesting that the model 4 having several neutrino species with comparable masses produces a fit to all data slightly better than the model 3 with one massive neutrino. So, in this case cosmology provides some support (though not the final evidence, of course) for strong neutrino mixing.

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| Name                                          | Type of perturbation | Scales ($h^{-1}$ Mpc) |
|----------------------------------------------|----------------------|-----------------------|
| Large-angle $\Delta T/T$ ($\theta > 2^\circ$) | $\Phi, h_{\alpha\beta}$ | 200-6000             |
| Intermediate angle $\Delta T/T$ ($10^\prime < \theta < 2^\circ$) | $\Phi, \nabla \Phi, \Delta \Phi$ | 20-200               |
| Peculiar (bulk) velocities                    | $\nabla \Phi$       | 10-75                 |
| Galaxy-galaxy correlations                   | $\Delta \Phi$       | 0.1-200               |
| Large-scale structure                         | $\frac{\partial^2 \Phi}{\partial x^\alpha \partial x^\beta}$ | 5-100                |
| Cluster-cluster correlations                  | $\Delta \Phi$       | 5-150                 |
| Cluster abundance                             | $\Delta \Phi$       | 5-10                  |
| Galaxy and quasar abundance at large $z$     | $\Delta \Phi$       | 0.5-1                 |
| $Ly - \alpha$ clouds                          | $\Delta \Phi$       | 0.1-1                 |
Table 2. Classification of cosmological models

| No. | Models | Level No. | Fundamental parameters | Observational parameters | Agreement with observational data |
|-----|--------|-----------|------------------------|-------------------------|----------------------------------|
| 1   | Standard CDM ($n_s = 1$) | 1 | ($H^2/\phi)_{inf}$ | A | No |
| 2   | CDM, $n_s = -3$ isocurvature | 1 | $A_{iso}$ | No |
| 3   | CDM+HDM, 1$\nu$, $n_s = 1$, standard concentration | 2 | ($H^2/\dot{\phi})_{inf}$, $m_\nu/M_p$ | $A, \Omega_\nu$ | Marginally good if $H_0 \simeq 50$, $\Omega_\nu \simeq 0.2$ |
| 4   | CDM+HDM 2$\nu$ or 3$\nu$, $n_s = 1$, standard concentration | 3,4 | ($H^2/\dot{\phi})_{inf}$, $m_\nu = m_{\nu_1}$ ($= m_{\nu_2}$) | $A, \Omega_{\nu_i}$ Good if $2 \nu$ or $3 \nu$, $m_\nu 1 \nu = m_\nu 2 \nu$ $H_0 \leq 60$, $\sum m_{\nu_i} \simeq 5(H_0/50)^3 \text{ eV}$ |
| 5   | CDM+Λ, $\Omega_{tot} = 1$, $n_s = 1$ | 2 | ($H^2/\dot{\phi})_{inf}$, $\Lambda M_p^2$ | $A, \Omega_\Lambda$ Good, $H_0 = 50 - 80$, $\Omega_m = 0.5 - 0.2$ |
| 6   | CDM curved, $\Omega_m < 1$, $n_s = 1$ | 2 | ($H^2/\dot{\phi})_{inf}$, $\phi_0/\dot{\phi}_f$ | $A, \Omega$ Worse than CDM+Λ, $\Omega_m > 0.3$, $H_0 < 70$ |
| 7   | CDM tilted adiabatic | 2 | $V(\phi) = V_0 e^{-\alpha \phi/M_p}$, $\alpha, V_0$, $n_s \neq 1$ | $A(k_{hor})$, (if $\sigma_8 \leq 0.7$) No |
| 8   | CDM tilted isocurvature | 2 | $A_{iso}(k_{hor})$, $n_s \neq -3$ | No |
| 9   | CDM+HDM tilted, 1$\nu$, standard concentration | 3 | $\alpha, V_0$, $m_\nu/M_p$ | $A(k_{hor})$, $n_s \neq 1$, $\Omega_\nu$ No, if $|n_s - 1| > 0.1$ |
| 10  | CDM with a step-like spectrum | 3 | $(H^2/\dot{\phi})_+ />, (H^2/\dot{\phi})_-$, $\phi_s/\dot{\phi}_f$, $k_s$ | $A_+, A_-$, $k_s$ Good if $H_0 \leq 60$ |
| 11  | CDM+HDM, decaying $\nu$, $n_s = 1$ | 3 | ($H^2/\dot{\phi})_{inf}$, $m_{\nu_1}, \tau_{\nu_1}$ | $A, \Omega_\nu$, $m_\nu$ |