COLLABORATIVE MISSION OPTIMIZATION FOR SHIP RAPID SEARCH BY MULTIPLE HETEROGENEOUS REMOTE SENSING SATELLITES

QIAN ZHAO, BITAO JIANG*, XIAOGANG YU AND YUE ZHANG
Beijing Institute of Remote Sensing Information
Beijing 100089, China

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ABSTRACT. Multiple heterogeneous satellites mission optimization is a typical kind of non-deterministic polynomial-time hard (NP-hard) problem, and some complicated scenarios bring new challenges. A novel method of missing ship rapid search using multiple grouped heterogeneous satellites is introduced in this paper. The focus is on optimization of collaborative mission optimization for various satellites including low-earth orbit (LEO) satellite and geostationary orbit (GEO) satellites. A fast coverage of the wide sea area using imaging satellites with narrow coverage range has become the most important part to tackle this problem. However, due to different imaging mechanisms of heterogeneous satellites and other constraints, it brings a great challenge to construct the optimization model. A constrained optimization problem model considering the cooperation between LEO and GEO satellites is constructed. A solution strategy based on bi-level metaheuristic algorithm is designed. The time optimal solution of the collaborative task planning between LEO and GEO satellites can be obtained based on the optimal attitude maneuver path of GEO satellites. Thus, wide-area search for missing ships can be conducted in an effective way. The effectiveness of the proposed method is verified by an example.

1. Introduction. Remote sensing satellite is a kind of satellite that uses remote sensing technology and equipment to observe the surface cover and natural phenomena. The demand for remote sensing satellite data in various fields has been growing rapidly, and the task demand presents a diversified and complicated development trend. Satellite images can be exploited in many fields of human life, including meteorology, agriculture, environmental monitoring, energy exploration, iceberg monitoring, disaster relief and so on.

Satellite mission planning plays an important role in the process of remote sensing satellite observation. Its results directly affect the mission execution and efficiency of remote sensing satellite system. Remote sensing satellites usually use optical cameras or synthetic aperture radar sensors to acquire images. Mission planning of satellite was originally performed manually, but with the increasing
demand for satellite images, researchers and task managers have successfully improved the utilization of single satellite and small satellite constellation by applying operational research and optimization methods to satellite business [16, 6, 7]. With the increasing number and variety of remote sensing satellites, the task planning of multi-satellite cooperation has been paid more and more attention by researchers. This problem has been proved to be a NP-Hard problem, which is usually solved by metaheuristic algorithms such as simulated annealing and genetic algorithm. At present, there are many researches on collaborative mission planning for a single class of multiple low-earth orbit (LEO) satellites. Bianchesi et al. [1] put forward a tabu search element heuristic algorithm to solve the problem of multi-satellite image acquisition and scheduling for optical agile satellites, which is simulated and verified by the French Pleiades constellation. Hwang et al. [8] proposed a genetic algorithm for scheduling multiple agile satellites. Malladi et al. [10, 11] studied the task scheduling of synthetic aperture radar satellite constellation, which consists of three identical non-agile satellites. Zhang et al. [18] solved the multi-satellite mission planning problem based on genetic algorithm. Liu et al. [17] designed a scheduling algorithm for multiple satellites based on merging mechanism.

Heterogeneous satellites refer to satellite clusters composed of satellites with different orbits, different functions and different imaging mechanisms. Collaborative task planning of heterogeneous satellites brings greater challenges to the design of planning models and algorithms. Snezana [13] et al. built a task planning model for multi-regional target coverage for different types of LEO satellites, including agile satellites and non-agile satellites.

High-orbit satellite is usually referred to geostationary orbit (GEO) satellite, which is a kind of satellite with an orbit height of about 36000km and a relatively fixed sub-satellite point. GEO remote sensing satellite mainly relies on attitude maneuver to image different places, which is different from LEO satellite imaging process. Although researchers have studied the task planning of single or multiple GEO satellites [20, 9], there are few studies considering the cooperative tasks of LEO and GEO satellites. Most of the existing researches focus on guiding cooperation in time series [19]. To solve the problem of wide-area search for missing ships on the sea surface, a fast coverage of the wide sea area using imaging satellites with narrow coverage range has become the most important part to tackle this problem. However, due to different imaging mechanisms of heterogeneous satellites and other constraints, it brings a great challenge to construct the optimization model and algorithms. In this paper, a constrained optimization model considering the cooperation between LEO and GEO satellites is constructed. The optimization variables are constructed as meta-task selection sequence of LEO satellites and observation sequence of GEO satellite. The objective function is mission duration of LEO and GEO satellite. Complex constrains are considered, such as contain on-board energy, image interval, attitude angular velocity and so on. A solution method based on bi-level hybrid metaheuristic algorithm is hereby designed, which consists of genetic algorithm and ant colony optimization. In the inner layer, ant colony algorithm is used to solve the optimal maneuvering path problem of GEO satellite at a given target point. In the outer layer, genetic algorithm is used to solve the optimal time required for parallel implementation of observation tasks by LEO and GEO satellites. The time optimal solution of the collaborative task planning between LEO and GEO satellites can be obtained based on the optimal attitude maneuver path.
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of GEO satellites, thus providing an effective way for the wide-area rapid search of lost ships.

The rest of this paper is as follows. The basic model of the system is presented in Section 2, including satellite orbit and attitude model, satellite observation model. Then in Section 3, the optimization problem model is constructed, which includes optimization variables, objective functions, constraints and so on. In Section 4, the structure of solution method is introduced, which constructs a hybrid solution strategy based on genetic algorithm and ant colony algorithm. The results are given in Section 5, and effectiveness of the method is verified. The conclusion and future research are finally shown in Section 6.

2. System model.

2.1. Satellite orbit and attitude model. In this paper, the collaborative mission optimization of LEO and GEO satellites is studied. Firstly, the orbit and attitude dynamic models of satellites are given. In order to calculate the access time to the observation area. This paper adopts the orbit dynamics model considering the $J_2$ perturbation is used[2]. The formula is given as follows

\[
\begin{align*}
\dot{x} &= v_x \\
\dot{y} &= v_y \\
\dot{z} &= v_z \\
\dot{v}_x &= -\frac{\mu}{r^3} \left( 1 + \frac{3}{2} J_2 \left( \frac{R_e}{r} \right)^2 \left( 1 - z^2 r^2 \right) \right) \\
\dot{v}_y &= -\frac{\mu y}{r^3} \left( 1 + \frac{3}{2} J_2 \left( \frac{R_e}{r} \right)^2 \left( 1 - z^2 r^2 \right) \right) \\
\dot{v}_z &= -\frac{\mu z}{r^3} \left( 1 + \frac{3}{2} J_2 \left( \frac{R_e}{r} \right)^2 \left( 3 - 5 z^2 r^2 \right) \right)
\end{align*}
\]

where $\mu$ is the constant of gravity, $x$, $y$ and $z$ are the position of the satellite in the J2000 geocentric inertial coordinate system, $v_x$, $v_y$ and $v_z$ are the velocity of the satellite in the J2000 geocentric inertial coordinate system, $r$ is the geocentric distance, $R_e$ is earth radius, and $J_2$ is the perturbation constant of the earth’s oblateness.

The kinematic equation of satellite defined by Modified Rodrigues parameter (MRP) [14], which described the attitude with respect to the orbit frame, is given by

\[
\dot{\sigma} = G(\sigma) \left( \omega - \omega_0(\sigma) \right)
\]

where $\omega$ is the angular rate of satellite, $\omega_0(\sigma)$ is the orbit angular rate. $G(\sigma)$ is defined as

\[
G(\sigma) = \frac{1}{2} \left( 1 - \frac{\sigma^T \sigma}{2} E_3 + \sigma^T \sigma + [\sigma^\times] \right)
\]

where $[\sigma^\times]$ represents the skew cross product matrix, and $E_3$ is $3 \times 3$ identity matrix.

2.2. Satellite observation model. LEO satellites are usually deployed at an orbital altitude of several hundred kilometers, thus the orbital period is usually less than 2 hours. The orbit of the sub-satellite point is affected by the rotation of the satellite around the earth and the rotation of the earth, and presents a continuous trace on the earth surface. Due to the influence of orbital dynamics, LEO satellites are usually unable to observe designated ground task areas immediately. The accurate access time to the task area can be calculated based on formula (1). In order to increase the imaging range of LEO Earth observation satellites, satellites usually have the ability of attitude maneuver. LEO satellite usually images the area within
a certain field of view through attitude maneuver in pitch and roll directions. Due to the limitation of satellite attitude maneuverability, and considering the influence of satellite yaw on imaging quality, the yaw angle is usually limited in a certain range. Usually, multiple satellites work together to achieve regional coverage as shown in Figure 1.

![LEO satellite observation](image)

**Figure 1. LEO satellite observation**

According to the number of initial orbit of satellites, payload capacity constraints and the position of target points, the access window of satellites to targets can be calculated. The meta-task model of satellites can be obtained, as shown in the following formula

\[
m_k = \langle s(k), g(k), t_0(k), t_f(k), t_m(k), \phi(k) \rangle
\]

where \( s(k) \) indicates the satellite executing the meta-task, \( g(k) \) indicates the target point code, \( t_0(k) \) indicates the start time of the meta-task, \( t_f(k) \) indicates the end time of the meta-task, \( t_m(k) \) indicates the center time of the meta-task, and \( \phi(k) \) is the attitude side swing angle. The value of \( m_k \) is 0 or 1. If \( m_k=1 \), it means the meta-task \( m_k \) will be executed. If \( m_k=0 \), it means the meta-task \( m_k \) won’t be executed.

GEO satellites are usually deployed in geostationary orbit with an altitude of about 36000 km. The orbital period of this kind of orbit is consistent with the earth rotation period, so its sub-satellite point is fixed. The GEO satellite can observe the craft through continuous attitude maneuver, as shown in figure 2.

The maneuver angle of GEO satellite can be approximately calculated by cosine formula

\[
\theta_1 = \arccos \frac{l_{OA}^2 + l_{OB}^2 - l_{AB}^2}{2l_{OA}l_{OB}}
\]

\[
\theta_2 = \arccos \frac{l_{OB}^2 + l_{OC}^2 - l_{BC}^2}{2l_{OB}l_{OC}}
\]
where $l_{OA}$ is the distance between the GEO satellite and location A. $l_{OB}$ is the distance between the GEO satellite and location B. $l_{OC}$ is the distance between the GEO satellite and location C. $l_{AB}$ is the distance between location A and location B. $l_{BC}$ is the distance between location B and location C.

3. Optimization problem. In this paper, focusing on solving the problem of rapid coverage of target areas, a mission optimization model for cooperative observation of LEO and GEO satellites is established.

3.1. Optimization variables. For the collaborative observation mission of the LEO and GEO satellites, the optimization variables are different because of the different imaging process of the LEO and GEO satellites. For the LEO satellites, according to the calculation results of satellite target area access, whether each meta-task is selected as an optimization variable. Assuming that there are $p$ meta-tasks in a given time period, then the meta-task selection sequence of LEO satellites is shown as follows

$$M_{LEO} = [m_1 \ m_2 \ \ldots \ \ldots \ m_k \ \ldots \ m_p] \quad (7)$$

In which $m_k$ is a variable from 0 to 1. If $m_k=1$, it means that the $k$ meta-task will be executed. If $m_k=0$, it means that the $k$ meta-task will not be executed.

For GEO satellites, it is assumed that there are $q$ meta-areas to be observed, and the observation sequence of each grid in the target area is selected as the optimization variable, as shown in the following formula

$$M_{GEO} = [s_1 \ s_2 \ \ldots \ s_q] \quad (8)$$

Figure 2. GEO satellite observation
3.2. **Objective function.** In order to cover the target area rapidly, the total time to complete the task is selected as the objective function as shown

\[
\min f(x) = \text{Max } (T_{\text{LEO}}, T_{\text{GEO}})
\]

where \( T_{\text{LEO}} \) is the mission time of the LEO satellites, and \( T_{\text{GEO}} \) is the mission time of the GEO satellites.

For the meta-task selection sequence, the duration of LEO satellite tasks can be obtained by the following formula, which depends on the earliest start time and the latest end time of all executed meta-tasks

\[
T_{\text{LEO}} = \max t_f(k) - \min t_0(k)
\]

The time for the GEO satellite to complete the whole task areas can be estimated by the following formula

\[
T_{\text{GEO}} = q \times d_{\text{single}} + \frac{q-1}{\omega_{\text{max}}} \sum \alpha_i
\]

where \( q \) is the number of areas, \( d_{\text{single}} \) is the time required for GEO satellite to complete imaging once, \( \omega_{\text{max}} \) is the maximum angular velocity of satellite attitude rotation, \( \alpha_i \) is the attitude maneuver angle after completing the \( i \)th imaging action, which is determined by the target observation sequence of GEO satellite.

3.3. **Constraints.** Limited by the on-board energy constraint, the imaging times of LEO satellite in a period of time are limited. For all the meta-tasks of the \( i \)th satellite, the following constraint should be satisfied

\[
k=p,s(k)=i \sum m_k \leq \frac{E_{\text{max}}}{E_{\text{single}}} \quad (i = 1 \ldots n)
\]

where \( E_{\text{max}} \) is the maximum energy constraint of a single satellite, and \( E_{\text{single}} \) is the energy required for imaging once.

There is a certain interval constraint between the two imaging action of LEO satellite, which depends on the attitude angle of the two imaging action. The constraint can be expressed as follows

\[
|t_m(i) - t_m(j)| \geq f_\phi(|\phi(i) - \phi(j)|) \quad (s(i) = s(j))
\]

where \( f_\phi(x) \) is a nonlinear function, which depends on the attitude maneuver speed of LEO satellite.

In order to ensure that observation resources are not wasted, all LEO satellites only observe the grid to be observed once at most as given

\[
k=p,g(k)=i \sum m_k \leq 1 \quad (i = 1 \ldots n)
\]

The attitude angular velocity constraint of GEO satellite is

\[
\omega \leq \omega_{\text{max}}
\]

Considering the motion of moving ships at sea, in order to ensure the capture of ships, two adjacent mosaic images should meet the following constraints. In this way, even if the craft sails at the maximum speed, as long as the craft is within the range of two images, the satellite can certainly capture it.

\[
L_{\text{blend}} \geq v_{\text{max}} \cdot d_{\text{image}}
\]
where \( L_{blend} \) is the overlapping width of mosaic area, \( v_{max} \) is the maximum speed of the ships, and \( dt_{image} \) is the time interval between two image action.

Figure 3. The relationship between ship speed and coverage area

Furthermore, the above formula is also applicable to the task of regional coverage in large sea areas. For spliced images, the overlapping area and imaging time interval of any two adjacent images satisfy the above formula, which means the ships in this area can be captured. In order to make the grid division more standardized, the problem can be simplified furthermore. Set the total time to complete all grid imaging as \( T_{dur} \), and the overlapping interval of all spliced images can be calculated by the following formula.

\[
L_{blend} \geq v_{max} \cdot T_{dur} \tag{17}
\]

Then, the distance between the center points of adjacent regional grids can be calculated according to the following formula

\[
D_{interval} \equiv D_{image} - v_{max} \cdot T_{dur} \tag{18}
\]

where \( D_{image} \) is the imaging width of the satellite.

4. Solution method.

4.1. General structure. In this paper, a bi-level hybrid solution method based on ant colony algorithm and genetic algorithm is proposed to solve the large-scale search problem of ships. In the inner layer, ant colony algorithm is used to solve the optimal maneuvering path problem of GEO satellite for given areas. In the outer layer, genetic algorithm is used to solve the optimal time required for parallel implementation of observation tasks by LEO and GEO satellite. Considering the image width of satellite and the maximum speed of moving target at sea, the calculation formula (18) is used to divide the regional grid. Based on the LEO satellite orbit dynamics model of formula (1), the access calculation is carried out for each meta-area of the whole task area. The access time and attitude angle of
Figure 4. Mesh generation considering ship moving

all LEO satellites to the each meta-area in a given time can be obtained, thus the observation meta-tasks of LEO satellites can be obtained.

4.2. Outer layer: Genetic algorithm optimization. The core idea of genetic algorithm is to select the gene sequence with the best fitness function by using the strategies of selection, crossover and mutation[15, 12]. Genetic algorithm usually uses 0-1 coding gene sequence as optimization variable, which is consistent with LEO satellite’s choice of meta-task as optimization variable.

It is assumed that within a given planning period $H$, there are $p$ meta-tasks, and the sequence of meta-tasks is $M_{LEO} = [m_1 \ m_2 \ ... \ m_p]$. For the meta-task selection sequence, the duration of LEO satellite task can be calculated by formula (10). All meta-areas of the LEO satellites is

$$M_{LEO} = unique ([s(1) \ s(2) \ ... \ s(p)])$$

(19)

where $k_{forbid}$ is the weight coefficient of penalty function. Then, all the solutions in the population are selected, crossed and mutated, and the new population is obtained by iterative update. Further, the unselected remaining areas are taken as the meta-areas to be observed for GEO satellites. $T_{GEO}$ will be determined by inner layer optimization calculation.

$$f(x) = Max ([T_{LEO}, T_{GEO}]) + k_{forbid} \left( \frac{\sum_{i=1}^{s(k)} (m_k - \frac{E_{max}}{E_{single}})}{T} \sum_{i=1}^{s(k)} f_0 (|\phi(i) - \phi(j)| - |t_m(i) - t_m(j)| + \sum_{i=1}^{s(k)} (m_k - 1)) \right)$$

(20)
Figure 5. General structure of solution method
4.3. Inner layer: Ant colony optimization. Ant colony optimization (ACO) was first proposed by Marco Dorigo in 1992, which is a typical metaheuristic algorithm[5, 3, 4]. Its core idea is to determine the probability of selecting optimized variables based on constantly updated pheromones, and it has been widely used in various optimization scheduling problems. In this paper, ant colony algorithm is used to solve the inner optimization problem, which means the optimal attitude maneuver path of GEO satellite can be observed.

For a given meta-areas, ants are randomly placed on the center points of meta-areas. Assume that the number of meta-areas is $N$, and the number of ants is $M$. The probability of moving from point $i$ to point $j$ can be calculated by the following formula

$$P_{ij}(t) = \begin{cases} \frac{\left[\tau_{ij}(t)^\alpha [dt_{single} + \theta_{ij}(t)/\omega_{max}]^2\right]}{\sum_{j \in allowed_k(i)} \left[\tau_{ij}(t)^\alpha [dt_{single} + \theta_{ij}(t)/\omega_{max}]^2\right]^{\beta} j \in allowed_k(i)} \\ 0 \end{cases}$$

When the next point $j$ is in the taboo table, this probability is 0 below the formula. When $j$ is in the allowable table, probability can be calculated as the formula. In the molecule, $\tau_{ij}$ is the pheromone from meta-area $i$ to meta-area $j$, and $\theta_{ij}(t)$ is the attitude angle of GEO satellite from center point of meta-area $i$ to center point of meta-area $j$. $\theta_{ij}(t)/\omega_{max}$ is the maneuvering time from center point of meta-area $i$ to center point of meta-area $j$. $dt_{single}$ is the single imaging time of GEO satellite.

And so on, ants can finish accessing meta-areas. After all of the ants finish moving, it forms a set of access sequences, and the iteration of this generation is finished. Then the pheromones can be updated. The updated value of pheromone is calculated from the maneuver time of each ant in the ant colony and then summed up.

For the $k_{th}$ ant in the ant colony, if he passes the $\theta_{ij}(t)$, then the pheromone increment from $i$ to $j$ caused by the $k_{th}$ ant is the total pheromone of an ant divided by the total angle of the ant. If not, the increment caused by the ant is 0.

$$\tau_{ij}(t + 1) = (1 - \rho)\tau_{ij}(t) + \Delta \tau_{ij}(t)$$

$$\Delta \tau_{ij}(t) = \sum_{k=1}^{m} \Delta \tau^k_{ij}(t)$$

$$\Delta \tau^k_{ij}(t) = \begin{cases} Q\omega_{max}/\theta_{ij}(t) & \text{The } k_{th} \text{ ant has passed the } \theta_{ij} \\ 0 & \text{The } k_{th} \text{ ant has not passed the } \theta_{ij} \end{cases}$$

where $\rho$ is the volatilization coefficient of pheromone. $Q$ is a constant and can be taken as 1. pheromone update matrix, add the attenuated value of the current pheromone matrix, which is the pheromone matrix used in the next iteration. This is repeated until the maximum number of iterations is reached or the optimal value converges.

Then, after the optimization is completed, the optimal maneuver time $T_{OptGEO}$ of GEO satellite can be obtained. It is not difficult to see that the optimal maneuver time $T_{OptGEO}$ depends on the number and distribution of $M_{GEO}$. $T_{OptGEO}$ can be regarded as the function of $M_{GEO}$, which is shown in the following formula

$$T_{OptGEO} = f_{opt}(M_{GEO})$$

Under the framework of the algorithm, the inner optimization can be calculated in parallel, and the number of calculation nodes is set to be consistent with the
population number of each generation of genetic algorithm, so as to improve the
calculation efficiency.

5. Results.

5.1. Problem. The scenario is that a transport ship is lost of communication in
the Pacific Ocean due to fault, and the satellite cluster is needed to search the area
where the ship appeared last time rapidly.

The mission scenario includes one GEO satellite and 24 LEO satellites. The
satellites orbit parameters are generated randomly. The orbit parameters are shown
in Table 1. The semi-major axis of the satellite is denoted by \( a \). The eccentricity of
satellite is denoted by \( e \), and the inclination of orbit is denoted by \( i \). Raan means
right ascension of ascending node, and \( w \) is the argument of perigee. These orbital
parameters can be converted by the position and speed of the satellite. The search
area is about 4000km \(^2\) 4000km.

Table 2 contains the constant parameter settings, mainly including orbit pertur-
bation constant \( J_2 \), earth sea level neutral acceleration \( g_n \), earth gravity constant \( \mu \),
earth radius \( R_e \), maximum speed of ship, imaging width of LEO satellite, imaging
width of GEO satellite, maximum angular velocity of GEO satellite.

| Table 1. Satellite Orbit Parameters |  |
|-------------------------------------|--|
| \( a (\text{km}) \) | \( e \) | \( i (\text{rad}) \) | \( \text{raan} (\text{rad}) \) | \( w (\text{rad}) \) |
| GEO | 42166.3 | 0 | 0 | 2.6180 | 0 |
| LEO-1 | 6978 | 0 | 0.6981 | 0.7854 | 2.0944 |
| LEO-2 | 6978 | 0 | 0.6981 | 0.7854 | 4.1888 |
| LEO-3 | 6978 | 0 | 0.6981 | 0.7854 | 6.2832 |
| LEO-4 | 6978 | 0 | 0.6981 | 1.5708 | 2.0944 |
| LEO-5 | 6978 | 0 | 0.6981 | 1.5708 | 4.1888 |
| LEO-6 | 6978 | 0 | 0.6981 | 1.5708 | 6.2832 |
| LEO-7 | 6978 | 0 | 0.6981 | 2.3562 | 2.0944 |
| LEO-8 | 6978 | 0 | 0.6981 | 2.3562 | 4.1888 |
| LEO-9 | 6978 | 0 | 0.6981 | 2.3562 | 6.2832 |
| LEO-10 | 6978 | 0 | 0.6981 | 3.1416 | 2.0944 |
| LEO-11 | 6978 | 0 | 0.6981 | 3.1416 | 4.1888 |
| LEO-12 | 6978 | 0 | 0.6981 | 3.1416 | 6.2832 |
| LEO-13 | 6978 | 0 | 0.6981 | 3.9270 | 2.0944 |
| LEO-14 | 6978 | 0 | 0.6981 | 3.9270 | 4.1888 |
| LEO-15 | 6978 | 0 | 0.6981 | 3.9270 | 6.2832 |
| LEO-16 | 6978 | 0 | 0.6981 | 4.7124 | 2.0944 |
| LEO-17 | 6978 | 0 | 0.6981 | 4.7124 | 4.1888 |
| LEO-18 | 6978 | 0 | 0.6981 | 4.7124 | 6.2832 |
| LEO-19 | 6978 | 0 | 0.6981 | 5.4978 | 2.0944 |
| LEO-20 | 6978 | 0 | 0.6981 | 5.4978 | 4.1888 |
| LEO-21 | 6978 | 0 | 0.6981 | 5.4978 | 6.2832 |
| LEO-22 | 6978 | 0 | 0.6981 | 6.2832 | 2.0944 |
| LEO-23 | 6978 | 0 | 0.6981 | 6.2832 | 4.1888 |
| LEO-24 | 6978 | 0 | 0.6981 | 6.2832 | 6.2832 |
Table 2. Constant Parameters

| Parameters                      | Value                  | Unit   |
|---------------------------------|------------------------|--------|
| Orbit perturbation constant $J_2$ | 0.0010826298989052     |        |
| Gravity acceleration of earth’s sea level $g_e$ | 0.00980665             | km/s²  |
| Gravitational constant $\mu$    | 3.98600.4418           | km³/s² |
| Radius of the earth $R_e$       | 6.378137e3             | km     |
| Ship maximum speed $v_{max}$    | 20                     | km/hour|
| Imaging width of LEO satellite $D_{LEO}$ | 250km                  | km     |
| Imaging width of GEO satellite $D_{GEO}$ | 250km                  | km     |
| Maximum angular velocity of GEO satellite $w_{max}$ | 1e-4                  | deg/hour|
| Single imaging time of GEO satellite $t_{single}$ | 20                   | s      |

According to the satellite imaging width and the maximum speed of the ship, the center point distance of the grid can be estimated, which is about 220km. Then, the task area can be divided into 10 *10 meta-areas. Table 3 shows the access calculation results of 24 LEO satellites to 100 meta-areas. The table only gives the information of the first 20 meta tasks. See the appendix for the detailed calculation results.

Table 3. Access calculation results

| Meta Mission No. | Satellite No. | Grid No. | Observation Time (hour) |
|------------------|---------------|----------|-------------------------|
| 1                | 4             | 1        | 0.119444                |
| 2                | 4             | 2        | 0.113889                |
| 3                | 4             | 3        | 0.108333                |
| 4                | 4             | 11       | 0.125                   |
| 5                | 4             | 12       | 0.122222                |
| 6                | 4             | 21       | 0.13333                 |
| 7                | 7             | 7        | 1.625                   |
| 8                | 7             | 8        | 1.619444                |
| 9                | 7             | 16       | 1.636111                |
| 10               | 7             | 17       | 1.63333                 |
| 11               | 7             | 18       | 1.627778                |
| 12               | 7             | 26       | 1.641667                |
| 13               | 7             | 27       | 1.641667                |
| 14               | 7             | 35       | 1.655556                |
| 15               | 7             | 36       | 1.652778                |
| 16               | 7             | 37       | 1.644444                |
| 17               | 7             | 45       | 1.663889                |
| 18               | 7             | 46       | 1.658333                |
| 19               | 7             | 54       | 1.675                   |
| 20               | 7             | 55       | 1.669444                |

5.2. Result analysis. As shown in Figure 6, the coverage ratio of the craft area over time when only LEO satellite constellation is used. When the mission time reaches about 3 hours, LEO satellite constellation has completed 100% coverage of the craft area.

As shown in Figure 7, the optimization result of the outer layer optimization problem is shown. Using genetic algorithm, the initial population is 50, and after about 30 generations of iteration, the objective function converges. The optimal
value of the objective function is 0.766692 (hour), which means that LEO satellite and GEO Satellite complete the task at the same time, taking about 46.0015 minutes. According to the craft’s maximum speed of 25km/h, the maximum sailing distance is 19.17km, which meets the observation grid overlap constraint and can achieve the acquisition of moving ships.

As shown in Figure 8, the grid observation task of LEO satellite constellation under the optimal outer layer optimization problem contains 63 grids. Then the other 37 grids are observed by GEO satellite.

The iterative process of the inner optimization problem is optimized by ant colony algorithm. The number of ants is 50, the pheromone factor is $\alpha = 1$, the heuristic factor is $\beta = 5$, the information evaporation coefficient is $\rho = 0.5$, and the pheromone enhancement coefficient is $q = 100$. In the 200 iterations, the objective function converges as shown in Figure 9.
Figure 8. LEO Mission Selected

As shown in Figure 10, it is the optimal observation sequence of the inner optimization problem. Under this observation sequence, the attitude maneuver time of GEO satellite is the shortest. According to formula (11), it can be calculated that the attitude maneuver time required by GEO satellite to complete all areas is 0.766692 hours, which is consistent with the mission duration of LEO satellites. It indicates that GEO satellite and LEO satellite have completed observation at the same time according to their respective selected meta-areas. Compared with using all LEO satellite constellations, the coverage time is 2.99 hours, and the time
consumption is reduced by 74.36%. Compared with only using GEO satellite, the coverage time is 1.21 hours, and the time consumption is reduced by 36.11%. There are as many as 24 LEO satellites in total. However, due to the orbital characteristics of LEO satellites, it is difficult to achieve rapid coverage of large areas, and the task takes a long time. Collaborative mission optimization can bring the comprehensive benefits of LEO satellite constellation and GEO satellite into full play, and realize the complementary advantages of different types of satellites. The example shows that the collaborative task planning is effective, and it can achieve more efficient observation of wide area coverage.

Based on the results and analysis of the numerical example, we can see that the proposed cooperative mission planning model and algorithm for moving ships can effectively solve the problem of rapid coverage of ship areas. In addition, the problem is calculated repeatedly, and the results are consistent, which shows good robustness.

6. Conclusion. This paper introduces a novel method of multiple heterogeneous satellite missions for the purpose of ship rapid search. A collaborative mission optimization model for area coverage using LEO and GEO satellites is established. Considering the characteristics of the problem, a novel task planning solution algorithm based on hybrid solution strategy is designed, which contains two-layer optimization model. Using this method, the optimal solution of space heterogeneous satellite coordination mission planning time can be obtained based on the optimal attitude maneuver path of GEO satellite. The results analysis shows that the model and algorithm can solve the ship rapid searching problem effectively.

In later research works, the impact of cloud and other factors should be considered, and the synthetic aperture radar satellite should be considered into the mission planning model. Thus the mission optimization model of multi-source information fusion can be established, so as to solve the problem of ship search in complex environment more effectively.
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E-mail address: zhaoqian_nudt@163.com
E-mail address: jiangbitao@bjirs.org.cn
E-mail address: yuxiaogangfly@126.com
E-mail address: zhangyue@bjirs.org.cn
### Appendix. Data

| Meta-task No. | Satellite No. | Grid No. | Observation Time (hour) |
|---------------|---------------|----------|-------------------------|
| 1             | 4             | 1        | 0.119444                |
| 2             | 4             | 2        | 0.113889                |
| 3             | 4             | 3        | 0.108333                |
| 4             | 4             | 11       | 0.125                   |
| 5             | 4             | 12       | 0.122222                |
| 6             | 4             | 21       | 0.133333                |
| 7             | 7             | 7        | 1.625                   |
| 8             | 7             | 8        | 1.619444                |
| 9             | 7             | 16       | 1.636111                |
| 10            | 7             | 17       | 1.633333                |
| 11            | 7             | 18       | 1.627778                |
| 12            | 7             | 26       | 1.641667                |
| 13            | 7             | 27       | 1.641667                |
| 14            | 7             | 35       | 1.655556                |
| 15            | 7             | 36       | 1.652778                |
| 16            | 7             | 37       | 1.644444                |
| 17            | 7             | 45       | 1.663889                |
| 18            | 7             | 46       | 1.658333                |
| 19            | 7             | 54       | 1.675                   |
| 20            | 7             | 55       | 1.669444                |
| 21            | 7             | 56       | 1.666667                |
| 22            | 7             | 64       | 1.680556                |
| 23            | 7             | 65       | 1.675                   |
| 24            | 7             | 73       | 1.694444                |
| 25            | 7             | 74       | 1.688889                |
| 26            | 7             | 75       | 1.686111                |
| 27            | 7             | 83       | 1.702778                |
| 28            | 7             | 84       | 1.697222                |
| 29            | 7             | 92       | 1.713889                |
| 30            | 7             | 93       | 1.708333                |
| 31            | 8             | 2        | 2.780556                |
| 32            | 8             | 3        | 2.775                   |
| 33            | 8             | 4        | 2.772222                |
| 34            | 8             | 8        | 1.055556                |
| 35            | 8             | 9        | 1.052778                |
| 36            | 8             | 12       | 2.786111                |
| 37            | 8             | 13       | 2.780556                |
| 38            | 8             | 18       | 1.063889                |
| 39            | 8             | 19       | 1.058333                |
| 40            | 8             | 21       | 2.8                     |
| 41            | 8             | 22       | 2.797222                |
| 42            | 8             | 23       | 2.791667                |
| 43            | 8             | 28       | 1.069444                |
| 44            | 8             | 29       | 1.066667                |
| 45            | 8             | 31       | 2.808333                |
|   |   |   |   |
|---|---|---|---|
| 46 | 8 | 32 | 2.802778 |
| 47 | 8 | 37 | 1.080556 |
| 48 | 8 | 38 | 1.075 |
| 49 | 8 | 41 | 2.813889 |
| 50 | 8 | 47 | 1.086111 |
| 51 | 8 | 48 | 1.086111 |
| 52 | 8 | 56 | 1.1 |
| 53 | 8 | 57 | 1.094444 |
| 54 | 8 | 58 | 1.091667 |
| 55 | 8 | 66 | 1.108333 |
| 56 | 8 | 67 | 1.102778 |
| 57 | 8 | 75 | 1.119444 |
| 58 | 8 | 76 | 1.113889 |
| 59 | 8 | 77 | 1.111111 |
| 60 | 8 | 85 | 1.125 |
| 61 | 8 | 86 | 1.119444 |
| 62 | 8 | 94 | 1.136111 |
| 63 | 8 | 95 | 1.133333 |
| 64 | 8 | 96 | 1.127778 |
| 65 | 9 | 5 | 2.197222 |
| 66 | 9 | 6 | 2.191667 |
| 67 | 9 | 9 | 0.486111 |
| 68 | 9 | 10 | 0.483333 |
| 69 | 9 | 14 | 2.211111 |
| 70 | 9 | 15 | 2.202778 |
| 71 | 9 | 16 | 2.2 |
| 72 | 9 | 19 | 0.491667 |
| 73 | 9 | 20 | 0.491667 |
| 74 | 9 | 24 | 2.216667 |
| 75 | 9 | 25 | 2.211111 |
| 76 | 9 | 29 | 0.502778 |
| 77 | 9 | 30 | 0.497222 |
| 78 | 9 | 33 | 2.227778 |
| 79 | 9 | 34 | 2.225 |
| 80 | 9 | 39 | 0.508333 |
| 81 | 9 | 40 | 0.505556 |
| 82 | 9 | 43 | 2.236111 |
| 83 | 9 | 44 | 2.230556 |
| 84 | 9 | 48 | 0.510667 |
| 85 | 9 | 49 | 0.513889 |
| 86 | 9 | 52 | 2.247222 |
| 87 | 9 | 53 | 2.241667 |
| 88 | 9 | 58 | 0.525 |
| 89 | 9 | 59 | 0.519444 |
| 90 | 9 | 61 | 2.261111 |
| 91 | 9 | 62 | 2.255556 |
|   |   |   |   |
|---|---|---|---|
| 92| 9 | 63| 2.25 |
| 93| 9 | 68| 0.530556 |
| 94| 9 | 69| 0.527778 |
| 95| 9 | 71| 2.269444 |
| 96| 9 | 72| 2.261111 |
| 97| 9 | 77| 0.544444 |
| 98| 9 | 78| 0.541667 |
| 99| 9 | 81| 2.275 |
|100| 9 | 87| 0.552778 |
|101| 9 | 88| 0.547222 |
|102| 9 | 96| 0.563889 |
|103| 9 | 97| 0.558333 |
|104| 9 | 98| 0.552778 |
|105| 10| 10| 1.45 |
|106| 10| 20| 1.458333 |
|107| 10| 30| 1.463889 |
|108| 10| 70| 1.497222 |
|109| 10| 80| 1.502778 |
|110| 10| 90| 1.511111 |
|111| 10| 100| 1.516667 |
|112| 11| 10| 0.886111 |
|113| 11| 20| 0.891667 |
|114| 11| 30| 0.9 |
|115| 11| 40| 0.905556 |
|116| 11| 50| 0.913889 |
|117| 11| 60| 0.919444 |
|118| 11| 69| 2.630556 |
|119| 11| 70| 2.627778 |
|120| 11| 79| 2.638889 |
|121| 11| 80| 2.636111 |
|122| 11| 89| 2.647222 |
|123| 11| 90| 2.641667 |
|124| 11| 99| 2.652778 |
|125| 11| 100| 2.65 |
|126| 11| 10| 2.583333 |
|127| 11| 20| 2.591667 |
|128| 11| 30| 2.597222 |
|129| 11| 40| 2.608333 |
|130| 11| 50| 2.613889 |
|131| 11| 60| 2.619444 |
|132| 12| 9 | 0.313889 |
|133| 12| 10| 0.319444 |
|134| 12| 19| 0.322222 |
|135| 12| 20| 0.325 |
|136| 12| 30| 0.330556 |
|137| 12| 40| 0.341667 |
|   |   |   |   |
|---|---|---|---|
| 138 | 12 | 50 | 0.347222 |
| 139 | 12 | 60 | 0.352778 |
| 140 | 12 | 70 | 0.361111 |
| 141 | 12 | 80 | 0.366667 |
| 142 | 12 | 90 | 0.377778 |
| 143 | 12 | 100 | 0.383333 |
| 144 | 12 | 30 | 2.030556 |
| 145 | 12 | 40 | 2.038889 |
| 146 | 12 | 50 | 2.044444 |
| 147 | 12 | 60 | 2.052778 |
| 148 | 12 | 70 | 2.058333 |
| 149 | 12 | 80 | 2.069444 |
| 150 | 12 | 90 | 2.075 |
| 151 | 12 | 100 | 2.080556 |
| 152 | 13 | 3 | 1.258333 |
| 153 | 13 | 4 | 1.263889 |
| 154 | 13 | 9 | 2.980556 |
| 155 | 13 | 10 | 2.986111 |
| 156 | 13 | 14 | 1.269444 |
| 157 | 13 | 15 | 1.275 |
| 158 | 13 | 19 | 2.991667 |
| 159 | 13 | 20 | 2.991667 |
| 160 | 13 | 24 | 1.277778 |
| 161 | 13 | 25 | 1.283333 |
| 162 | 13 | 35 | 1.291667 |
| 163 | 13 | 36 | 1.294444 |
| 164 | 13 | 45 | 1.297222 |
| 165 | 13 | 46 | 1.302778 |
| 166 | 13 | 56 | 1.308333 |
| 167 | 13 | 57 | 1.313889 |
| 168 | 13 | 66 | 1.316667 |
| 169 | 13 | 67 | 1.319444 |
| 170 | 13 | 77 | 1.327778 |
| 171 | 13 | 78 | 1.333333 |
| 172 | 13 | 87 | 1.336111 |
| 173 | 13 | 88 | 1.338889 |
| 174 | 13 | 97 | 1.341667 |
| 175 | 13 | 98 | 1.347222 |
| 176 | 13 | 99 | 1.35 |
| 177 | 14 | 1 | 0.680556 |
| 178 | 14 | 2 | 0.688889 |
| 179 | 14 | 7 | 2.408333 |
| 180 | 14 | 8 | 2.413889 |
| 181 | 14 | 11 | 0.691667 |
| 182 | 14 | 12 | 0.697222 |
| 183 | 14 | 18 | 2.419444 |
|   |   |   |   |
|---|---|---|---|
| 184 | 14 | 19 | 2.422222 |
| 185 | 14 | 22 | 0.702778 |
| 186 | 14 | 23 | 0.708333 |
| 187 | 14 | 28 | 2.427778 |
| 188 | 14 | 29 | 2.430556 |
| 189 | 14 | 32 | 0.711111 |
| 190 | 14 | 33 | 0.713889 |
| 191 | 14 | 34 | 0.722222 |
| 192 | 14 | 38 | 2.433333 |
| 193 | 14 | 39 | 2.438889 |
| 194 | 14 | 43 | 0.719444 |
| 195 | 14 | 44 | 0.725 |
| 196 | 14 | 49 | 2.447222 |
| 197 | 14 | 50 | 2.447222 |
| 198 | 14 | 54 | 0.733333 |
| 199 | 14 | 55 | 0.738889 |
| 200 | 14 | 59 | 2.452778 |
| 201 | 14 | 60 | 2.455556 |
| 202 | 14 | 64 | 0.741667 |
| 203 | 14 | 65 | 0.747222 |
| 204 | 14 | 69 | 2.458333 |
| 205 | 14 | 70 | 2.463889 |
| 206 | 14 | 75 | 0.752778 |
| 207 | 14 | 76 | 0.758333 |
| 208 | 14 | 79 | 2.466667 |
| 209 | 14 | 80 | 2.469444 |
| 210 | 14 | 85 | 0.761111 |
| 211 | 14 | 86 | 0.763889 |
| 212 | 14 | 90 | 2.477778 |
| 213 | 14 | 96 | 0.772222 |
| 214 | 14 | 97 | 0.777778 |
| 215 | 14 | 100 | 2.486111 |
| 216 | 15 | 5 | 1.833333 |
| 217 | 15 | 6 | 1.836111 |
| 218 | 15 | 7 | 1.841667 |
| 219 | 15 | 16 | 1.844444 |
| 220 | 15 | 17 | 1.847222 |
| 221 | 15 | 26 | 1.85 |
| 222 | 15 | 27 | 1.855556 |
| 223 | 15 | 31 | 0.138889 |
| 224 | 15 | 37 | 1.863889 |
| 225 | 15 | 38 | 1.866667 |
| 226 | 15 | 41 | 0.144444 |
| 227 | 15 | 42 | 0.152778 |
| 228 | 15 | 47 | 1.869444 |
| 229 | 15 | 48 | 1.875 |
|     |     |     |      |
|-----|-----|-----|------|
| 230 | 15  | 51  | 0.152778 |
| 231 | 15  | 52  | 0.158333 |
| 232 | 15  | 58  | 1.880556 |
| 233 | 15  | 59  | 1.886111 |
| 234 | 15  | 62  | 0.163889 |
| 235 | 15  | 63  | 0.169444 |
| 236 | 15  | 68  | 1.886111 |
| 237 | 15  | 69  | 1.891667 |
| 238 | 15  | 72  | 0.172222 |
| 239 | 15  | 73  | 0.177778 |
| 240 | 15  | 74  | 0.183333 |
| 241 | 15  | 78  | 1.894444 |
| 242 | 15  | 79  | 1.897222 |
| 243 | 15  | 83  | 0.186111 |
| 244 | 15  | 84  | 0.191667 |
| 245 | 15  | 88  | 1.902778 |
| 246 | 15  | 89  | 1.905556 |
| 247 | 15  | 90  | 1.908333 |
| 248 | 15  | 93  | 0.191667 |
| 249 | 15  | 94  | 0.197222 |
| 250 | 15  | 95  | 0.202778 |
| 251 | 15  | 99  | 1.913889 |
| 252 | 15  | 100 | 1.916667 |
| 253 | 16  | 51  | 2.019444 |
| 254 | 16  | 61  | 2.825   |
| 255 | 16  | 62  | 2.830556 |
| 256 | 16  | 71  | 2.836111 |
| 257 | 16  | 72  | 2.841667 |
| 258 | 16  | 82  | 2.847222 |
| 259 | 16  | 83  | 2.852778 |
| 260 | 16  | 92  | 2.852778 |
| 261 | 16  | 93  | 2.858333 |
| 262 | 16  | 94  | 2.863889 |
| 263 | 17  | 91  | 2.283333 |