Multiscale Edge Analysis of Gravity Data and Its Applications

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Abstract  In order to improve the processing and interpretation of gravity data, multiscale edge theory in image processing is introduced into the study of gravity field. Multiscale edges of gravity anomaly are analyzed based on a special wavelet. It shows that the multiscale edges are the extrema points of the horizontal gravity gradient at different heights, which are related to the sharp discontinuities of underground sources. The applications of multiscale edge in downward continuation and gravity inversion are discussed. The simulated examples show that the multiscale edges can be applied to stabilize the downward continuation operator when the continuation height is low. The multiscale edges also have a convenient application to infer the geometry of the underground source. Moreover, the gravity inversion algorithm based on the multiscale edges has a good antinoise property.

Keywords  multiscale edge; gravity anomaly; downward continuation; gravity inversion

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Introduction

During the past decades, the great progress has been made in gravimetry techniques. Satellite altimetry has efficiently improved our knowledge of marine gravity field\[^{1,2}\]. Airborne gravimetry can be used to fill in gravity data gaps of traditional gravimetry and satellite gravimetry\[^{3-5}\]. Currently, a first generation of dedicated satellite gravity missions (CHAMP, GRACE, and GOCE) provides us with huge amount of information on the dynamics of planet earth\[^{6,7}\], in particular, on the mass distribution in the Earth’s interior, the entire water cycle, and on changes in the mass distribution. Technical progress in gravimetry will allow more detailed results in the future. However, the analysis and interpretation of these rich gravity data is still a challenge for geodesy and geophysics communities.

In this study, we explored the ability of multiscale edge analysis to process gravity data. Multiscale edge analysis, which was proposed based on wavelet transform by Mallat and Zhong\[^{8}\], has become a powerful signal or image processing technique. It can be used to sharpen, denoise, or enhance the data in order to facilitate the interpretation. In geophysics, edges have precise mathematical meaning within potential field theory and are usually used to infer geophysical interpretation. Recently, Hornby et al.\[^{9}\] derived a family of wavelets from the Green’s function of the Poisson equation and its spatial derivatives and extended the multiscale edge theory for potential field
data analysis. Sequentially, potential applications were demonstrated such as the stabilization of the downward continuation operator, the isolation, and removal of individual anomalies \cite{10, 11}.

1 Multiscale edge analysis of gravity anomaly

In a Cartesian coordinate system, gravity anomaly due to an underground source can be represented as

\[
\Delta g_z(x,y) = -2\pi G \int_{-\infty}^0 \rho(x,y,z') K_z(x,y) \, dz'
\]

where \( * \) denotes convolution operator, \( \Delta g_z(x,y) \) is gravity anomaly at point \((x,y,z)\), \( G \) is Newtonian gravitational constant, and \( \rho(x,y,z) \) is the density distribution that is assumed zero for \( z > 0 \). The convolution kernel \( K_z(x,y) \) is the Green’s function for gravity anomaly, that is,

\[
K_z(x,y) = \frac{1}{2\pi} \frac{z}{(x^2 + y^2 + z^2)^{3/2}}
\]

It is also an upward continuation operator, which is defined by

\[
\Delta g_z(x,y) = \Delta g_0 * K_z(x,y)
\]

where \( \Delta g_0 \) is the gravity anomaly at ground level \((z = 0)\).

It is obvious that \( K_z(x,y) \) is non-negative for \( z > 0 \) and differentiable. Furthermore, the integral of the Green’s function can be proven to be unity for all \( z > 0 \). Thus, \( K_z(x,y) \) is admissible as a smoothing function for a wavelet, with a corresponding scaled version

\[
K_{sz}(x,y) = s^{-2} K_z(x/s, y/s)
\]

where \( s \) is the scale.

Defining the smoothing function \( \theta(x,y) \) to be the Green’s function associated with height \( z = 1 \), that is,

\[
\theta(x,y) = K_{sz}(x,y)
\]

and

\[
\theta_{sz}(x,y) = K_z(x,y) = s^{-2} \theta(x/s, y/s)
\]

The corresponding ‘mother’ wavelet is a vector-valued function, with two components given by

\[
\psi_i(x,y) = \frac{\partial \theta(x,y)}{\partial x} \quad \text{and} \quad \psi_{s+1}(x,y) = \frac{\partial \theta(x,y)}{\partial y}
\]

The components

\[
\psi_i(x,y) = s^{-2} \psi_i(x/s, y/s) \quad i = 1, 2
\]

form a set of self consistent dilation equations for this wavelet. Hence, the 2-D wavelet transform of gravity anomaly \( \Delta g_z \) is given by

\[
\hat{W}[\Delta g_z](s,x,y) = \left( \Delta g_z * \psi_i \right) = s \hat{V}(\Delta g_z * \theta_i)
\]

where \( \hat{V} \) denotes the 2-D gradient operator in the \((x, y)\) plane, and \( \hat{W}[\Delta g_z] \) denotes the 2-D wavelet transform of \( \Delta g_z \).

From Eq.(9), we can see that the wavelet transform of gravity anomaly is the combination of upward continuation and horizontal gradient. In the multiscale wavelet edge detection theory \cite{8}, edges are defined as points where the wavelet transform modulus are locally extrema. Therefore, the multiscale edges of gravity anomaly \( \Delta g_z \) can be calculated as follows: (1) upward continue the measured \( \Delta g_z \) to a level \( z + s \), (2) compute the 2-D gradient of \( \Delta g_{sz} \) in the \((x, y)\) plane and multiply the resulting vector by the scale \( s \), and (3) calculate the local extrema along each gradient streamline. The scaling parameter can be thought as upward continuation height.

The multiscale edges of gravity anomaly are local extrema points of horizontal gradients of gravity anomaly at all scales, where the gravity anomaly varies sharply. Obviously, these edges correspond to sharp discontinuities of underground mass or interfaces between contrasting rock materials such as faults, unconformities, or intrusive contacts. This relationship implies that multiscale edges are helpful in analyzing and interpreting gravity data.

2 Applications

2.1 Stabilization of downward continuation

The positions and shapes of multiscale edges are strongly related to the locations and shapes of individual features in gravity image. This suggests that the features can be modified or removed by manipulating its corresponding edges, leaving the rest features minimally perturbed. The property can be ex-
ploited to stabilize the downward continuation process by removing the edges that are due to the spurious oscillations caused by noise amplification. In order to discriminate between edges due to the features in the signal and edges caused by the spurious noise, we assume that multiscale edges of the measured gravity and downward continued gravity have the same positions and shapes. Although this assumption is contestable, it is acceptable when the continuation height is low such as for airborne gravity. Wang has shown that the similarity of edges at two levels is more than 95% when the height difference is lower than 10 km\(^2\). Based on this assumption, the algorithm to stabilize downward continuation follows the steps:

1. Perform traditional downward continuation;
2. Calculate the multiscale edges of the measured gravity data and downward-continued gravity data, respectively;
3. Modify the edges in the downward-continued data and reconstruct the downward-continued gravity from the modified edges.

The criterion to modify the edges is to keep the ones corresponding to the edges present in the measured gravity and to abandon the others which have no matching in the measured gravity. The algorithm has been verified for 1-D profiles\(^{10}\). Here, we extend to 2-D case with a simulation example.

Fig.1 shows gravity anomalies due to a simulated spherical source at ground level and 10 km height, respectively. The measurements are the sum of the anomalies at 10 km height and a Gaussian noise which variance is equal to 1 mGal\(^2\). The measurements are downward continued to the ground using iteration method based on planar Poisson formula\(^{13}\), and the aforementioned algorithm is applied to the downward continued gravity. The results are illustrated in Fig.2, and the comparisons are listed in Table 1. The standard error (2.1 mGal) of the downward continued gravity anomalies after stabilization is smaller than the one before stabilization. Also, the signal-noise-ratio (SNR) is significantly improved from 31.15 dB to 41.03 dB. Therefore, the multiscale edges can be used as a constraint to suppress the amplified noise in the downward continued gravity data.

| Table 1 | Statistics of the downward continued results before and after stabilization |
|---------|--------------------------------------------------|
|         | min/mGal | max/mGal | mean/mGal | std/mGal | SNR/dB  |
| Before stabilization | -11.86 | 15.64 | 1.08 | 3.79 | 31.15 |
| After stabilization  | -4.64  | 14.66 | 1.08 | 2.10 | 41.03 |

2.2 **Gravity inversion based on multiscale edges**

The multiscale edges of gravity anomaly represent the location of the local extrema of horizontal gradient of gravity at different altitudes. The multiscale edges behave differently depending on the shape of the source. Thus, the edges contain information about the geometry and type of discontinuity in the source and can be applied to recover the position and shape, as well as density contrast of a source. In order to explore this application, a simple example of a 2-D horizontal cylinder is demonstrated below.

For a 2-D horizontal cylinder, its gravity anomaly at height \(z\) can be represented by

\[
A_z = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y, z) \, dx \, dy
\]

where \(g(x, y, z)\) is the gravitational field due to the cylinder.
\[ \Delta g_c(x) = -2G \rho \frac{z - D}{(x - C)^2 + (z - D)^2} \]  

where \( C \) is the horizontal distance from the center of cylinder to the origin, and \( D \) is the depth of the center, and \( \rho \) is the density contrast of the cylinder. The depth \( D \) must be a negative value because positive \( z \) is upward. The anomaly can be thought as a 1-D signal along \( x \)-axis. In one dimension, the wavelet in Eq.(7) becomes

\[ \psi(x) = -\frac{2}{\pi} \frac{x}{(1 + x^2)^{3/2}} \]  

Therefore, the wavelet transform of the anomaly at ground level is

\[ W[\Delta g_n](s,x) = -4G \rho \frac{s(x - C)(z - D)}{[(x - C)^2 + (s - D)^2]^2} \]  

From Eq. (12), the location of the extrema of the wavelet transform can be deduced, that is,

\[ x - C = \pm \frac{\sqrt{3}}{3} (s - D) \]  

It can be seen that the multiscale edges for a 2-D horizontal cylinder are two straight lines whose slopes are equal to \( \pm \sqrt{3}/3 \). The two lines intersect at the center of the cylinder. Therefore, we can determine the location of the center using the multiscale edges.

Fig.3(a) shows the gravity anomaly profile of a 2-D horizontal cylinder whose center is buried at location \( (x = 128\text{m}, z = -10\text{m}) \). Fig.3(b) illustrates the corresponding wavelet transform. The two white lines in Fig.3(b) are the multiscale edges. The center of the cylinder can be exactly calculated in the noisy free case by computing the crosspoint of the two edges. Fig.4 is the noisy case. The energy of the noise in the anomaly profile is a quarter of the energy of the gravity signal. Although the noise produces lots of edges (black lines), the two edges (white lines) corresponding to the feature are hardly affected. The location of the center determined from the two edges is \((128.06\text{m}, -10.43\text{m})\). It shows a good anti-noise property.
3 Conclusion

In this paper, multiscale edge analysis of gravity data is discussed. The multiscale edges are the extrema points of the horizontal gravity gradient at different heights. The edges can be used to stabilize the downward continuation when the continuation height is low. This technique is suitable for the processing of airborne gravity data, because the flying height is often lower than 10 km.

The multiscale edges also have a potential application to infer the geometry of underground sources. The gravity inversion algorithm based on the multiscale edges has a good antinoise property, since the local extrema have large SNR. Another advantage is that no iterative calculation is needed.

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