DUST FILTRATION BY PLANET-INDUCED GAP EDGES: IMPLICATIONS FOR TRANSITIONAL DISKS

Zhao-huan Zhu$^{1,2}$, Richard P. Nelson$^3$, Ruobing Dong$^1$, Catherine Espaillat$^4$, and Lee Hartmann$^2$

$^1$ Department of Astrophysical Sciences, 4 Ivy Lane, Peyton Hall, Princeton University, Princeton, NJ 08544, USA; zhzhu@astro.princeton.edu, rdong@astro.princeton.edu
$^2$ Department of Astronomy, University of Michigan, 500 Church St., Ann Arbor, MI 48109, USA; lhartm@umich.edu
$^3$ Astronomy Unit, Queen Mary University of London, Mile End Road, London E1 4NS UK; r.p.nelson@qmul.ac.uk
$^4$ Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA; cespaillat@cfa.harvard.edu

Received 2012 March 6; accepted 2012 June 1; published 2012 July 19

ABSTRACT

By carrying out two-dimensional two-fluid global simulations, we have studied the response of dust to gap formation by a single planet in the gaseous component of a protoplanetary disk—the so-called dust filtration mechanism. We have found that a gap opened by a giant planet at 20 AU in an $\alpha = 0.01$, $M = 10^{-8} M_\odot$ yr$^{-1}$ disk can effectively stop dust particles larger than 0.1 mm drifting inward, leaving a submillimeter (submm) dust cavity/hole. However, smaller particles are difficult to filter by a gap induced by a several $M_\oplus$ planet due to (1) dust diffusion and (2) a high gas accretion velocity at the gap edge. Based on these simulations, an analytic model is derived to understand what size particles can be filtered by the planet-induced gap edge. We show that a dimensionless parameter $T_f/\alpha$, which is the ratio between the dimensionless dust stopping time and the disk viscosity parameter, is important for the dust filtration process. Finally, with our updated understanding of dust filtration, we have computed Monte Carlo radiative transfer models with variable size distributions to generate the spectral energy distributions of disks with gaps. By comparing with transitional disk observations (e.g., GM Aur), we have found that dust filtration alone has difficulties depleting small particles sufficiently to explain the near-IR deficit of moderate $M$ transitional disks, except under some extreme circumstances. The scenario of gap opening by multiple planets studied previously suffers the same difficulty. One possible solution is to invoke both dust filtration and dust growth in the inner disk. In this scenario, a planet-induced gap filters large dust particles in the disk, and the remaining small dust particles passing to the inner disk can grow efficiently without replenishment from fragmentation of large grains. Predictions for ALMA have also been made based on all these scenarios. We conclude that dust filtration with planet(s) in the disk is a promising mechanism to explain submm observations of transitional disks but it may need to be combined with other processes (e.g., dust growth) to explain the near-IR deficit of some systems.

Key words: accretion, accretion disks – astroparticle physics – planet–disk interactions – stars: formation – stars: pre-main sequence

Online-only material: color figures

1. INTRODUCTION

The transitional and pre-transitional disks are protoplanetary disks around young stars which exhibit strong dust emission at wavelengths $\gtrsim 10\mu m$, while showing significantly reduced fluxes relative to typical T Tauri disks at shorter wavelengths (e.g., Strom et al. 1989; Calvet et al. 2002, 2005; D’Alessio et al. 2005; Espaillat et al. 2007, 2008). Pre-transitional disks still have some infrared emission from warm, optically thick dust near the star (Espaillat et al. 2007, 2008, 2010), while transitional disks only have near- and mid-infrared emission from optically thin dust (Calvet et al. 2002, 2005; Espaillat et al. 2010). The depletion of near- to mid-infrared emission is generally interpreted as being due to evacuation of the disk interior to scales of 5 to 50 AU (Marsh & Mahoney 1992; Calvet et al. 2002, 2005; Rice et al. 2003; Schneider et al. 2003; Espaillat et al. 2007, 2008, 2010; Hughes et al. 2009), an interpretation confirmed in some cases via direct submillimeter (submm) imaging (e.g., Piétu et al. 2006; Brown et al. 2007, 2009; Hughes et al. 2009; Andrews et al. 2009, 2011).

These cavities need to be both large and deep concerning the dust surface density (e.g., Espaillat et al. 2010). Here, deep means the gap is optically thin at mid-IR. In the case of the transitional disks, the optically thin region must extend from radii as large as tens of AU all the way into the central star, forming a dust hole. Even the pre-transitional disks, which display evidence of optically thick dust emission in the innermost regions, must have large disk gaps from $\sim$AU scales to tens of AU. The requirement that the gaps/holes are optically thin implies that the population of dust in sizes of the order of a micron or less must be extremely small, due to the large opacity of these small dust grains.

Furthermore, many of these objects also exhibit gas accretion rates comparable to but slightly smaller than T Tauri disk accretion rates ($\sim 10^{-8} M_\odot$ yr$^{-1}$; Hartmann et al. 1998) onto their central stars (e.g., Calvet et al. 2002, 2005; Espaillat et al. 2007, 2008; Najita et al. 2007). Maintaining this accretion rate requires a significant mass reservoir in the inner disk. The conflict between the moderate disk accretion rate (which means a high gas surface density) and the near-infrared deficit (which means a low dust density) puts strong constraints on any theoretical attempts to explain these objects (Zhu et al. 2011).

In summary, there are three observational constraints for (pre)transitional disks: (1) a wide gap/hole extending over one order of magnitude in radii, (2) a deep gap/hole that is optically thin, and (3) a moderate gas accretion rate.

Gap opening by multiple giant planets (Zhu et al. 2011; Dodson-Robinson & Salyk 2011) has been proposed to explain the wide gaps in (pre)transitional disks. However, Zhu et al. (2011) also pointed out that even with four giant planets, the opened gap is not deep enough to explain transitional disks, unless there are massive giant planets close to the central star. Zhu et al. (2011) concluded that dust depletion/growth is still required in the inner disk to be consistent with observations. On
the other hand, Dodson-Robinson & Salyk (2011) speculated that dust may be significantly confined in the spiral density wakes or streamers. Both of these works imply that dust dynamics may need to be considered along with the gas dynamics.

By considering dust dynamics independently, one promising mechanism to explain these wide and deep (dust) gaps is dust filtration by the gap outer edge opened by planet(s) (Paardekooper & Mellema 2006; Rice et al. 2006). This mechanism is intrinsically due to the dust being a pressureless system, while the gas orbital dynamics is affected by the gas pressure. Dust particles orbit the central star at the Keplerian speed, but the gas rotates slightly faster or slower than this depending on the radial pressure gradient. In this case, the dust particles feel either a headwind or tailwind from the gas particles and can lose or gain angular momentum to the gas and start drifting radially in the disk. The net effect is that dust particles tend to drift toward gas pressure maxima. Thus, at the outer edge of a planet-induced gap where the radial density/pressure gradient is positive, dust particles drift outward, possibly overcoming their coupling to the inward gas accretion process. Dust particles will then stay at the gap outer edge while the gas flows through the gap. This process is called “dust filtration” (Rice et al. 2006), and it depletes the disk interior to the radius of the planet-induced gap of dust, forming a dust-depleted inner cavity.

However, this filtration process sensitively depends on the pressure gradient at the planet-induced gap outer edge and it only operates for large particles whose drift velocities are significant. Small particles strongly coupled to the gas flow can still penetrate the planet-induced gap to the inner disk. More importantly, dust diffusion may significantly reduce the filtration efficiency even for the big particles (Ward 2009). Thus, it is crucial to understand what sized particles can be filtered by a realistic planet-induced gap. In this paper, we will show only 0.1 mm particles and above can be filtered by a realistic gap if the disk viscosity parameter $\alpha = 0.01$ and accretion rate $M = 10^{-8} M_\odot$ yr$^{-1}$. This size limit may decrease to 0.01 mm if the planet is very massive and the disk is less turbulent accreting at a low accretion rate. Although this can explain the cavity from submm observations of transitional disks, it cannot explain the near-IR deficit of these disks, given that significant small dust is still present in the inner disk. However, on the other hand, if dust filtration is combined with dust growth in the disk, then the near-IR deficit of transitional disks can be very well reproduced.

In Section 2, we introduce the two-dimensional (2D) two-fluid simulations. In Section 3, we construct a simpler one-dimensional (1D) model. Our results are presented in Section 4. A short discussion is presented in Section 5. Various scenarios trying to explain transitional disks are tested with the Monte Carlo radiative transfer model in Section 6, and conclusions are drawn in Section 7.

2. 2D TWO-FLUID SIMULATIONS

In this section, we will first introduce our two-fluid numerical algorithms, and then introduce our special inner boundary condition to study long timescale disk evolution. After a short discussion on the numerical challenge of simulating dust fluid, we set up our gas/dust two-fluid simulations.

2.1. Numerical Algorithms

Gaseous and dust components of the disk are simulated separately in our simulations.

2.1.1. Gas Component

The gas disk evolution is simulated with FARGO (Masset 2000), a 2D hydrodynamic code which utilizes a fixed grid in cylindrical polar coordinates $(R, \phi)$. FARGO uses finite differences to approximate derivatives, and the evolution equations are divided into source and transport steps, similar to those of ZEUS (Stone & Norman 1992). However, an orbital advection scheme has been incorporated which reduces the numerical diffusivity and significantly increases the allowable time step as limited by the Courant–Friedrichs–Lewy condition. Thus, FARGO enables us to study the interaction between the disk and embedded planets over a full viscous timescale for disks. The setup of the gaseous disk and the planet is discussed in Section 2.4.1.

2.1.2. Dust Component

Beyond the gas component, we have implemented an additional fluid in FARGO to simulate the dust’s response to the gas. The dust is treated as a low pressure fluid and couples with the gas via drag terms. No feedback from the dust on the gas is simulated, since dust filtration normally takes place when the gas density dominates the dust density. However, this assumption may be violated under some circumstances as discussed in the Appendix. The drift terms are added as an additional source step for the dust fluid

$$\frac{\partial v_{r,d}}{\partial t} = -\frac{v_{r,d} - v_{r,g}}{\tau_s},$$  \hspace{1cm} (1)

$$\frac{\partial v_{\phi,d}}{\partial t} = -\frac{v_{\phi,d} - v_{\phi,g}}{\tau_s},$$  \hspace{1cm} (2)

where all other terms/steps are the same as the gaseous fluid (Stone & Norman 1992). Since we will focus on dust particles with radii smaller than 1 mm, these particles are in the Epstein regime (Whipple 1972; Weidenschilling 1977), so that the dust stopping time (Takeuchi & Lin, 2002) is

$$\tau_s = \frac{s \rho_p c_s}{\rho_g v_T},$$  \hspace{1cm} (3)

where $\rho_g$ is the gas density, $s$ is the dust particle radius, $\rho_p$ is the dust particle density (we chose $\rho_p = 1$ g cm$^{-3}$), $v_T = \sqrt{8/\pi c_s}$, and $c_s$ is the gas sound speed. Considering the mean disk density is $\rho_g = \Sigma_g/\sqrt{2\pi H}$, it can also be written as

$$\tau_s = \frac{\pi s \rho_p}{2 \Sigma_g \Omega},$$  \hspace{1cm} (4)

In a dimensionless form, the stopping time can be written as

$$T_s = \tau_s \Omega,$$  \hspace{1cm} (5)

where $\Omega$ is the Keplerian angular velocity.

When the dust stopping time is far shorter than the hydro time step, the drift terms become stiff. With an explicit method, the time step $\Delta t$ needs to be smaller than both the stopping time $\tau_s$ and the hydro time step constraint $\delta t$ in order to be numerically stable. However, $\tau_s$ is very small, with micron size

---

5 With our disk parameters, the mean free path of the molecule is 0.1 mm at 0.1 AU and 5 m at 10 AU, which is larger than the dust size. At the very inner disk where the mean free path is small and the particles are no longer in the Epstein regime, the viscous velocity dominates the drift velocity so that whether the particles are in the Epstein regime or not is no longer important.
particles, thus an implicit method is desired. Considering that the ZEUS/FARGO scheme is first-order accurate in time, the first-order implicit scheme is

\[ v_{r,d}^{n+1} = v_{r,d}^n - \frac{v_{r,d}^n - v_{r,d}^n}{t_s + \Delta t} \Delta t, \]

where \( n + 1 \) denotes the quantities at the new time step. Since the drift term is stiff, it may be helpful if we go to a higher order scheme. It turns out that the second-order scheme just requires replacing the \( \Delta t \) in the denominator with \( \Delta t/2 \).

Although the implicit method ensures a stable scheme no matter what the time step we use, Equation (6) suggests it merely adds \( \Delta t \) to the stopping time. The new equivalent stopping time is \( \Delta t + \frac{t_s}{2} \), which can be significantly larger than the real \( t_s \) if the numerical time step is much larger than \( t_s \). Thus, in order to accurately study the dust drift which is controlled by \( t_s \), the hydro time step needs to be comparable to \( t_s \). By numerical testing, we found that the radial drift velocity is close to the theoretical value only if the time step is close to \( t_s \). We tried different schemes as mentioned above, but they differ little.

Thus, to simulate the dynamics of dust particles much smaller than 1 mm correctly, the time step is set by the dust stopping time and is inversely proportional to the particle size. Simulating dust drift for 0.01 mm particles is thus very computationally expensive since it is 100 times more expensive than simulating 1 mm particles.

On the other hand, we are not interested in dust dynamics happening on the dust stopping timescale. We are only interested in the dust’s final response to the gas flow, which changes on a much longer timescale than the stopping time. In other words, we can use the dust’s terminal velocity to represent its final response to the gas. This is introduced as the “Short Friction Time Approximation” (SFT) in Johansen & Klahr (2005), where the dust velocity is related with the gas velocity as

\[ v_d = v_r + t_s \frac{\nabla P}{\rho}. \]

This approximation is valid only if \( t_s \) is much smaller than the dynamical timescale of the gas flow. More accurately, \( t_s \) needs to be smaller than the hydrodynamic time step. In our simulation setup, even with 1 mm particles, \( t_s \) is at least one order of magnitude smaller than the hydrodynamic time step. In the Appendix, we compare the SFT approximation with the two-fluid simulation and show good agreement between them.

Furthermore, dust can diffuse in the gaseous disk due to turbulence. In this work, dust diffusion is implemented in the operator split fashion in the source step for the dust fluid (Clarke & Pringle 1988)

\[ \frac{\partial \Sigma_d}{\partial t} = \nabla \cdot \left( D \nabla \left( \frac{\Sigma_g}{\Sigma_g} \right) \right), \]

where \( D \) is the turbulent diffusivity which relates with the turbulent viscosity \( \nu \) through

\[ D = \frac{\nu}{Sc}. \]

where \( Sc \) is the Schmidt number defined as the ratio between the total accretion stress and particle mass diffusivity. Note that \( \nu \) here is the viscosity of the “gaseous” disk, representing the efficiency of the angular momentum transport experienced by the gas disk due to disk turbulence. When the disk orbital time (which is close to the turbulent eddy turnover time) is much larger than the dust stopping time (which is always true for particles smaller than \( \sim 1 \) mm), \( Sc \sim 1 \) (Johansen & Klahr 2005; Carballido et al. 2011) and dust will not settle significantly so that 2D approximation is better justified. The above formula for dust diffusion has been confirmed by both analytic work and numerical simulations including both magnetorotational instability turbulence and particle dynamics under the circumstances that the background gas surface density is uniform (Youdin & Lithwick 2007; Carballido et al. 2011).

In Section 4, we discuss how diffusion plays a significant role in the dust filtration process.

2.2. Inner Boundary Conditions

For the gaseous fluid, as discussed by Crida et al. (2007), a standard open inner boundary condition (Stone & Norman 1992) in a fixed 2D grid can produce an unphysically rapid depletion of material through the inner boundary in the presence of the planets. There are two reasons for this. First, due to waves excited by the planet, the gas in the disk can have periodic inward and outward radial velocities larger than the net viscous velocity of accreting material. Thus, with the normal open boundary, material can flow inward while there is no compensating outflow allowed. Second, the orbit of the gas at the inner boundary is not circular due to the gravitational potential of the planets; again, as material cannot pass back out through the inner boundary, rapid depletion of the inner disk material is enhanced. As we are interested in the amount of gas depletion in the disk inward of the planet-induced gap(s) over substantial evolutionary timescales, it is important to avoid or minimize this unphysical mass depletion.

Crida et al. (2007) were able to ameliorate this problem by surrounding the 2D grid by extended 1D grids (FARGO2D1D, see their Figure 5). We follow Pierens & Nelson (2008), who found reasonable agreement with the Crida et al. results while using a 2D grid only by limiting the inflow velocities at the inner boundary to be no more than three times larger than the viscous radial velocity in a steady state,

\[ v_{rs} = -\frac{3v_{in}}{2R_{in}}, \]

where \( v_{in} \) and \( R_{in} \) are the viscosity and radius at the inner boundary. This scheme shows good agreement with the analytic estimate and the results from the FARGO2D1D model (Pierens & Nelson 2008; Zhu et al. 2011).

For the dust fluid, we adopt a similar approach by limiting the radial velocity of the dust fluid to be no more than three times larger than the dust drift speed in a viscous disk,

\[ v_{rs,d} = -\frac{(3v_{in}/2R_{in})T_s^{-1} - \eta v_d}{T_s + T_s^{-1}}, \]

where \( \eta \) is the ratio between the pressure gradient and gravitational force \( \eta = -(R\Omega^2 \rho_d)^{-1} \partial P/\partial R \), and it is equal to \( 3/2(H/R)^2 \) in our setup below.

2.3. Fluids with and without Pressure

A zero pressure fluid means zero scale height, which implies any velocity disturbance in the disk will quickly sharpen and shock, known as the delta shock.\(^6\) In reality, a shock will not

\(^6\) If a delta shock forms in the disk, then the fluid treatment needs to be replaced by a particle treatment since particles will cross orbits changing their distribution function in phase space.
form in the particle fluid if there is a strong coupling between gas and dust. Numerically, due to the inability to simulate coupling on scales smaller than the grid, a zigzag shaped density profile forms grid by grid in our simulations and we rely on the artificial viscosity to stabilize the shock. To minimize this effect, we set the dust scale height to be \( H/R = 0.0044 \) for the dust fluid, which is one order of magnitude smaller than the gaseous fluid (\( H/R = 0.044 \)). \( H_d \) is close to our grid spacing, while significantly smaller than the gas disk scale height. Since dust drift is intrinsically due to the gaseous disk pressure gradient, adding 10% pressure to the dust fluid means the drift speed is only affected by 10%. But if the dust fluid has a sudden density change (e.g., 10 times steeper than the gaseous fluid), then the effect of the small pressure can be amplified and becomes erroneous.

### 2.4. Model Setup

#### 2.4.1. Gas Component

Similar to Zhu et al. (2011), we assume a central stellar mass of \( 1 M_\odot \) and a fully viscous disk. We further assume a radial temperature distribution \( T = 221(R/\text{AU})^{-1/2} \) K, which is roughly consistent with typical T Tauri disks in which irradiation from the central star dominates the disk temperature distribution (e.g., D’Alessio et al. 2001). The disk is vertically isothermal. The adopted radial temperature distribution corresponds to an implicit ratio of disk scale height to cylindrical radius \( H/R = 0.029(R/\text{AU})^{0.25} \). This differs from the \( H/R = \) constant assumption used in many previous simulations, which implies a temperature distribution \( T \propto R^{-1} \), which is inconsistent with observations. Consequently, our assumed temperature distribution with a constant viscosity parameter \( \alpha (\nu = \alpha c_s^2/\Omega, \quad \nu = \nu_{\text{SFT}}) \), where \( \nu \) is the kinematic viscosity, \( c_s \) is the sound speed, \( \Omega \) is the angular velocity) leads to a steady-disk surface density distribution \( \Sigma \propto R^{-1} \) instead of the \( \Sigma \propto R^{-1/2} \) which would result from either assuming that both \( H/R \) and \( M \) are constant or both \( \nu \) and viscous torque \(-2\pi R\Sigma \nu^2 d\Sigma /dR\) are constant. This makes a significant difference in the innermost disk surface densities, and thus the implied inner disk optical depths in our models will be larger.

We set \( \alpha = 0.01 \) for the standard cases. Given our assumed disk temperature distribution, we set the initial disk surface density to be \( \Sigma = 178 \) (\( R/\text{AU} \)) \( -1 \) \( \exp(-R/100 \text{ AU}) \) g cm\(^{-2}\) from \( R \approx 8–300 \text{ AU} \), which yields a steady disk solution with an accretion rate \( M \approx 10^{-8} M_\odot \text{ yr}^{-1} \), typical of T Tauri disks (Gullbring et al. 1998; Hartmann et al. 1998).

#### 2.4.2. Dust Component

The dust surface density is assumed to be 0.01 of the gas surface density initially. With this setup, the dust stopping time \( t_s \propto 1/(\Sigma_\nu \Omega) \propto R^{5/2} \). Motivated by (pre-)transitional disk observations, we place the planet at 20 AU.

We have carried out two-fluid simulations with a 1 \( M_J \) planet in the gaseous and dust disk (1 mm particles) to \( 2.5 \times 10^4 \) years, using two methods for the dust fluid: (1) self-consistently solving the dust fluid equations together with the gas equations, or (2) using the SFT approximation (Equation (7)). In the following, we will refer to the former as two-fluid simulations and the second method as the SFT approximation (although the second method is also a two-fluid method). Good agreement has been found between these two methods (see Appendix). Since the SFT approximation is not limited by the dust stopping time, we use this approximation for the dust component in all the other runs.

Finally, gaseous and dust disks including three different size particles (1, 0.1, and 0.03 mm) and three different mass planets

### Table 1

| Case Name | Method   | Diffusion | Planet Mass \((M_J)\) | Dust Size (mm) | Evolution Time (yr) |
|-----------|----------|-----------|-----------------------|----------------|---------------------|
| 1 \( M_J \) |          |           |                       |                |                     |
| 1J1mm2F   | Two-fluids | No Diff   | 1                     | 1              | \( 5 \times 10^3 \) yr |
| 1J1mm2FD  | Two-fluids | Diff      | 1                     | 1              | \( 5 \times 10^3 \) yr |
| 1J1mm     | SFT approx. | No Diff   | 1                     | 1              | \( 1 \times 10^3 \) yr |
| 1J0p1mm   | SFT approx. | No Diff   | 1                     | 0.1            | \( 2.5 \times 10^3 \) yr |
| 1J0p03mm  | SFT approx. | No Diff   | 1                     | 0.03           | \( 5 \times 10^3 \) yr |
| 1J1mmD    | SFT approx. | Diff      | 1                     | 1              | \( 1 \times 10^3 \) yr |
| 1J0p1mmD  | SFT approx. | Diff      | 1                     | 0.1            | \( 2.5 \times 10^3 \) yr |
| 1J0p03mmD | SFT approx. | Diff      | 1                     | 0.03           | \( 5 \times 10^3 \) yr |

| 3 \( M_J \) |          |           |                       |                |                     |
| 3J1mm     | SFT approx. | No Diff   | 3                     | 1              | \( 1 \times 10^5 \) yr |
| 3J0p1mm   | SFT approx. | No Diff   | 3                     | 0.1            | \( 2.5 \times 10^3 \) yr |
| 3J0p03mm  | SFT approx. | No Diff   | 3                     | 0.03           | \( 5 \times 10^3 \) yr |
| 3J1mmD    | SFT approx. | Diff      | 3                     | 1              | \( 1 \times 10^3 \) yr |
| 3J0p1mmD  | SFT approx. | Diff      | 3                     | 0.1            | \( 2.5 \times 10^3 \) yr |
| 3J0p03mmD | SFT approx. | Diff      | 3                     | 0.03           | \( 5 \times 10^3 \) yr |

| 6 \( M_J \) |          |           |                       |                |                     |
| 6J1mm     | SFT approx. | No Diff   | 6                     | 1              | \( 1 \times 10^5 \) yr |
| 6J0p1mm   | SFT approx. | No Diff   | 6                     | 0.1            | \( 2.5 \times 10^3 \) yr |
| 6J0p03mm  | SFT approx. | No Diff   | 6                     | 0.03           | \( 5 \times 10^3 \) yr |
| 6J1mmD    | SFT approx. | Diff      | 6                     | 1              | \( 1 \times 10^3 \) yr |
| 6J0p1mmD  | SFT approx. | Diff      | 6                     | 0.1            | \( 2.5 \times 10^3 \) yr |
| 6J0p03mmD | SFT approx. | Diff      | 6                     | 0.03           | \( 5 \times 10^3 \) yr |
(1, 3, 6 $M_J$) have been simulated, respectively. All the runs are summarized in Table 1 and discussed in Section 4.3.

3. 1D SIMULATION

Because the dust filtration process is mainly determined by the planet-induced gap structure which is quite axisymmetric except in the region close to the planet, it is useful to construct simpler azimuthally averaged 1D models; by comparing 1D models with 2D simulations, we can separate effects due to axisymmetric and non-axisymmetric features. For axisymmetric flows, the dust surface density evolution is governed by the continuity equation (Takeuchi & Lin 2005)

\[ \frac{\partial \Sigma_d}{\partial t} + \frac{1}{R} \frac{\partial}{\partial R} [R(F_{\text{diff}} + \Sigma_d v_d)] = 0, \tag{12} \]

where $F_{\text{diff}}$ is the dust mass flux due to diffusion,

\[ F_{\text{diff}} = -D \Sigma_g \frac{\partial}{\partial R} \left( \frac{\Sigma_d}{\Sigma_g} \right), \tag{13} \]

and $v_d$ is the dust radial velocity in the rest frame due to the gas-drag

\[ v_d = \frac{v_g T_s^{-1} - \eta v_K}{T_s + T_s^{-1}}, \tag{14} \]

where $T_s$, $\eta$ are defined above. When $T_s \ll 1$ the above equation can be approximated by

\[ v_d = v_g - \eta v_K T_s. \tag{15} \]

We can also incorporate the diffusion term in Equation (12) into the dust velocity so that the equivalent total dust velocity is

\[ v_{d,t} = v_g - \eta v_K T_s - \frac{D}{R} \frac{d \ln (\Sigma_d/\Sigma_g)}{d \ln R}, \tag{16} \]

where the first term on the right-hand side is the dust radial velocity induced by the gas radial velocity (normally it is negative due to the accretion process), the second term is the dust drift velocity with respect to the gas due to the gas pressure gradient (at the gap outer edge $\eta$ is negative), and the last term is the dust velocity due to dust diffusion. The addition of the first two terms is the dust drift velocity in the rest frame without diffusion.

The only quantities which cannot be calculated in the 1D models are the gaseous disk surface density with a gap opened by a planet ($\Sigma_g$, which is needed to calculate $\eta$) and the gas radial velocity $v_g$. Here, we adopt the gaseous disk surface density ($\Sigma_g$) from 2D simulations, and the initial dust surface density is chosen as $1/1000$th of the gas surface density. We also assume that the gas radial mass flux at each radius is a constant (equal to $M$), giving

\[ v_g = \frac{M}{2\pi R \Sigma_g}. \tag{17} \]

This implies that the gas velocity can be quite large deep inside the gap where the gas surface density is very low. This high velocity has significant impact on the dust drift as discussed in Rice et al. (2006) and in Section 4.3. In 1D, the radial velocity advection is ignored and the azimuthal velocity is assumed to be Keplerian. Thus, we can simulate the pressureless fluid without worrying about shock formation. Furthermore, in our setup, $v_g$ and $\Sigma_g$ are fixed, so that $v_g$ is also fixed and the part of Equation (12) without diffusion is a linear equation for $\Sigma_d$.

Due to the simplicity of the 1D model, it not only extracts the essential physics but can also serve as a sanity check for 2D simulations.

4. RESULTS

Before we present any results, we want to emphasize that Equation (16) guides all our discussions below. Dust filtration happens when Equation (16) (or Equation (15) if there is no dust diffusion) becomes positive so that the total dust velocity is outward. In the following, we will first present the simplest case without dust diffusion and then we will present results with dust diffusion considered.

4.1. Dust Filtration without Diffusion

Although a 1 $M_J$ planet can only open a shallow gap in the gas disk in our setup, 1 mm dust particles can be effectively trapped at the gap outer edge. As shown in the left panel of Figure 1, the gaseous gap is barely apparent and the depth of the gap is half of the unperturbed disk. However, if dust diffusion is ignored, such a shallow gaseous gap can effectively stop 1 mm particles at the gap outer edge due to the dust drift. Then, without replenishment, 1 mm dust particles in the inner disk quickly move inward to the central star, leaving a large mm dust cavity/hole in the disk (the second to left panel of Figure 1).

The cavity is highly depleted by four orders of magnitude. Thus, the planetary wake is less apparent within the cavity, but it can still be seen outside the cavity. One feature which can be seen but is not very apparent in the dust image is the dust ring in the horseshoe region (the ring can also be observed in Figure 3). The ring forms because the gas pressure has a local maximum in the center of the horseshoe region, effectively trapping dust particles.

On the other hand, smaller particles (0.1 mm, the second to right panel) can still penetrate the planet-induced gap to the inner disk. This is because, without dust diffusion, the dust outward drift velocity is less apparent due to the gas pressure gradient and the accretion velocity, the structure of the dust disk looks exactly like that of the gaseous disk with a perfectly circular cavity/hole.

For smaller particles (e.g., 0.03 mm in the rightmost panel of Figure 1), $T_s$ decreases, and the dust disk behaves more similarly to the gaseous disk. In the extreme limit that dust particles are very small so that the drift velocity is negligible compared with the accretion velocity, the structure of the dust disk looks exactly like that of the gaseous disk.

4.2. Dust Filtration with Diffusion

The dust filtration process can be significantly hindered by dust diffusion, as originally pointed out by Ward (2009).
Dust diffusion tries to smooth any density feature of the dust relative to the gas. Thus, at the gap outer edge, dust tries to diffuse inward, leading to an additional inward dust speed (adding the third term in Equation (16)), which lowers the outward drift speed. The diffusion velocity (Equation (12)) is

$$v_{\text{diff}} = \frac{-D}{R} \frac{d\ln(\Sigma_d/\Sigma_g)}{d\ln R}. \quad (18)$$

Unlike the gas diffusion velocity

$$v_{g} = -\frac{3}{R^{1/2}\Sigma_g} \frac{d}{dR}(R^{1/2}v\Sigma_g), \quad (19)$$

which depends on the gas surface density, the dust diffusion velocity depends on the dust concentration relative to the gas rather than on its absolute abundance.

At the beginning of the simulation, the abundances of dust and gas do not vary much so that the concentration of dust $\Sigma_d/\Sigma_g \sim 0.01$ everywhere and dust diffusion can be ignored. However, as shown in the last section, the dust quickly concentrates at the gap outer edge and dust diffusion can play a significant role. The timescale for dust diffusion becoming important is the dust radial drift timescale, assuming the gaseous disk is in steady state,

$$\tau_{\text{dust}} = \frac{R_{\text{gap}}}{v_d} \quad (20)$$

where $R_{\text{gap}}$ is the position of the planet-induced gap and $v_d$ is defined in Equation (14). With our disk parameters ($M = 10^{-8}M_\odot$ yr$^{-1}$, $\alpha = 0.01$, $R_{\text{gap}} = 20$ AU), $\tau_{\text{dust}} = 9 \times 10^4$ yr for 1 mm particles, and $2 \times 10^5$ yr (the gas viscous timescale) for 0.1 mm and smaller particles. Thus, to study dust filtration, we have to evolve the disk long enough. For small dust particles ($\leq 0.1$ mm), this means to simulate the disk evolution at the disk viscous timescale at the position of the gap ($\sim 2 \times 10^5$ yr). In this work, we managed to carry out such simulations by using the SFT approximation.

Figure 2 shows that dust diffusion significantly hinders dust filtration. Compared with the right panels of Figure 1, the dust gap is much shallower and the dust distribution is a lot smoother. 1 mm dust particles can still penetrate the gap opened by a 1 $M_J$ planet, unlike the case without diffusion. However, dust diffusion cannot stop dust filtration indefinitely. If the gap is steeper (e.g., a gap opened by a 3 $M_J$ planet, the lower panels of Figure 2), then 1 mm dust particles can still be filtered by the gap, which will be discussed below.

### 4.2.1. Varying Gap Structure and Dust Sizes

Figure 3 shows both gas and dust surface density profiles for various planets in the disk (1, 3, 6 $M_J$ from top to bottom) with and without dust diffusion (right and left panels, respectively). The solid curves are the gas surface densities divided by 100. At the end of the simulation, if the dust surface density is depleted by more than 1000 at the inner boundary (three orders of magnitude smaller than the solid curve), we say that the dust is significantly depleted and filtered.

The top panels show the disk surface densities with a 1 $M_J$ planet at 20 AU. As discussed above, the gaseous gap is quite shallow, approximately half of the unperturbed disk surface density. Without dust diffusion, such a shallow gap is capable of filtering 1 mm dust particles, but unable to filter smaller particles. However, with dust diffusion, all the dust particle sizes we considered ($0.03 \leq s \leq 1$ mm) can pass through this 1 $M_J$ planet-induced gap. Dust diffusion also changes the dust distribution in the outer disk: dust can diffuse outward, slowing down the shrinkage of the dust disk and making the decline of the dust surface density with increasing radius more gradual.

The middle panels show the disk surface densities with a 3 $M_J$ planet at 20 AU. The gaseous gap is one order of magnitude deep. Without dust diffusion, both 1 mm and 0.1 mm particles are filtered, while 0.03 mm particles can pass through. With dust diffusion, 1 mm particles are filtered, while 0.1 mm and 0.03 mm particles pass through.

The lower panels show the disk surface densities with a 6 $M_J$ planet at 20 AU. The gaseous gap is almost two orders of magnitude deep. Without dust diffusion, all kinds of particles are filtered. With dust diffusion, 1 mm and 0.1 mm particles are filtered.
To emphasize the effect of dust diffusion, we use the simplest 1D model to show various components of the dust velocity in Figure 4 for 1 mm particles in a gaseous disk with a 1 \( M_J \) mass planet. The left panel shows the case without dust diffusion, while the right panel includes dust diffusion. The gas velocity is the green curve. The total dust velocity is the solid black curve, including the dust drift velocity in the rest frame (the addition of the first and second terms of Equation (16), dotted curve) and the diffusion velocity (the third term of Equation (16), dashed curve). As shown in this figure, dust drift is fully capable of filtering the 1 mm particles (the dust drift velocity in the rest frame is positive at the gap outer edge). However, dust diffusion adds an additional negative velocity component. The net effect is that the total dust velocity is still inward (dust filtration fails) when dust diffusion is considered.

In summary, dust diffusion hinders the dust filtration process. 1 mm sized particles will be filtered by a gap one order of magnitude deep and particles with sizes \( \geq 0.1 \) mm will be filtered by a gap that is two orders of magnitude deep.

4.3. High Gas Velocity at the Gap Edge

Another effect hindering filtration is the high gas velocity at the gap edge. Although this effect is self-consistently treated in simulations, it needs special attention in analytical approaches as noted in Rice et al. (2006). We have identified and verified this effect in our simulations.

Considering the gas accretion rate is constant across the planet-induced gap (if we allow the planet to accrete, this is still true at the outer edge of the gap), the radial velocity has to increase in the gap since the disk surface density decreases in the gap (according to \( M = 2\pi R \Sigma_g v_g \)). With a two orders of magnitude deep gaseous gap, the gas velocity can be amplified by two orders of magnitude within the gap. Thus, the first term of Equation (16) is significantly increased. Please note that this argument is built on the assumption that the velocity is axisymmetric inside the planet-induced gap. Although this assumption is incorrect deep inside the gap around the horseshoe orbits (see the Appendix), the flow is quite axisymmetric at the edge of the gap where dust filtration takes place. We have checked 2D simulations directly and confirmed that the radial velocity increases by a factor that is close to the gaseous gap depletion factor. This effect becomes increasingly important for a deeper gaseous gap.

To illustrate this effect, we again use the simplest 1D model to show various components of the dust velocity, as shown in Figure 5. Even with dust diffusion considered, by ignoring the amplification of the gas radial velocity across the gap (green curve in the left panel is flat), 0.03 mm particles will be filtered in the 6 \( M_J \) gap. However, considering the gas velocity is amplified (the right panel), 0.03 mm particles can pass through the gap, consistent with the results from 2D simulations.

4.4. What Size Particles will be Filtered? Analytical Approach

Although our simulations have suggested that 1 mm particles will be filtered by a gap one order of magnitude deep and \( \geq 0.1 \) mm particles will be filtered by a gap that is two orders of magnitude deep, it would be insightful to have a simple analytic model to show how the critical filtration particle size depends on the disk parameters.

Both dust diffusion and high gas velocity at the gap edge are important as pointed out above. Thus, we will consider them separately and then combine their effects.

First, we will assume the radial gas flow velocity is zero, and only consider the dust diffusion process to balance the dust drift. We are trying to find the marginal state between the dust being filtered and drifting inward. This marginal state can be derived...
Figure 3. Azimuthal-averaged gas (solid curves) and dust surface densities (1 mm: dotted curves, 0.1 mm: dashed curves, 0.03 mm: long dashed curves) if the planet is at 20 AU with different masses (1 $M_J$ upper panels, 3 $M_J$ middle panels, and 6 $M_J$ lower panels). The left panels are without dust diffusion, while the right panels are with dust diffusion considered. 1 mm particles are shown at $10^9$ yr which is longer than its radial drift timescale, while 0.1 mm and 0.01 mm particles are shown at $2.5 \times 10^9$ yr and $5 \times 10^9$ yr since their drift timescales are longer. The gas surface densities are divided by 100 to scale with the dust surface densities. Smaller particles can be filtered by a gap induced by a more massive planet. However, particles smaller than 0.1 mm cannot be filtered even with a 6 $M_J$ planet if dust diffusion due to disk turbulence is properly included.
by assuming that the diffusion velocity equation (18) balances the drift velocity

\[ -v \frac{d}{dR} \ln \left( \frac{\Sigma_d}{\Sigma_g} \right) = \frac{\eta V_K}{T_s + T_s^{-1}} \]  

(21)

Since \( T_s \ll 1 \), and plugging in \( \eta = -(R \Omega^2 \Sigma_g)^{-1} \partial P/\partial R \) and \( v = \alpha c_s^2/\Omega \), we derive

\[ \frac{d}{dR} \ln \left( \frac{\Sigma_d}{\Sigma_g} \right) = \frac{T_s}{\alpha} \frac{\partial \ln \Sigma_g}{\partial R} \]  

(22)

Here, we have assumed that the sound speed varies slowly compared with \( \Sigma_g \) so that it can be treated as a constant, which is a good approximation at the gap edges.

In order to proceed, we need to assign a planet-induced gap shape \( \Sigma_g \), which can be derived by balancing the planetary torque density and the gradient of the viscous torque (Ward 2009)

\[ 3 \pi v R^2 \Omega \frac{\partial \Sigma_g}{\partial R} = \mu^2 R^2 \Omega^2 \Sigma_g \frac{R}{x^4} \]  

(23)

where \( \mu = M_p/M_\ast \) and \( x = (R - R_p)/R_p \). Assuming \( R, \Omega, \) and \( v \) are constant (which is a good approximation in the gap region compared with the factor \( x^{-4} \)), this equation can be integrated to obtain the gap profile

\[ \Sigma_g = \Sigma_{g,0} e^{-|W/x|} \]  

(24)

where \( \Sigma_{g,0} \) is the ambient unperturbed disk surface density.
Plugging Equation (24) into Equation (22) and noticing that \( T_s = \rho_p s \pi / (2 \Sigma_g) \), we obtain

\[
\frac{d \ln(\Sigma_g / \Sigma)}{d R} = \frac{3(W / x)^3 \rho_p s \pi e^{W / x}}{2 \Sigma_{g,0} \alpha R_p W}. \tag{25}
\]

Note that \( T_s \) is also a function of \( \Sigma_g \), suggesting that dust and gas are more decoupled within the gap. Integrating this equation with respect to \( x \) and assigning \( T_{s,0} = \rho_p s \pi / (2 \Sigma_{g,0}) \), we derive

\[
- \frac{T_{s,0}}{\alpha} e^{W / x} = \ln \left( \frac{\Sigma_g / \Sigma}{\Sigma_{g,0} / \Sigma_{g,0}} \right) = \ln \left( \frac{\gamma}{\gamma_0} \right). \tag{26}
\]

where \( \gamma \) is the dust to gas mass ratio, \( \Sigma_g / \Sigma_g \), at position \( x \) within the gap.

Then, \( x / W \) can be translated back to \( \Sigma / \Sigma_g \) with Equation (24), giving

\[
- \frac{T_{s,0}}{\alpha} \left( \frac{\Sigma_g}{\Sigma_{g,0}} \right)^{-1} = \ln \left( \frac{\gamma}{\gamma_0} \right). \tag{27}
\]

The relationship between the dust depletion factor in the gap \((\gamma / \gamma_0\), the depletion of the dust/gas mass ratio inside the gap compared with that outside the gap\) and the gaseous gap depth \((\Sigma_g / \Sigma_{g,0})\) is shown in Figure 6, with the assumption that \( \alpha = 0.01 \) and \( M = 10^{-8} M_\odot \) yr\(^{-1}\). As in Section 4.2.1, if the dust depletion factor is smaller than 0.001, then we consider that the particles are filtered efficiently. From the hydrodynamic simulations, we know the gaseous gap opened by a planet that is a few times more massive than Jupiter is on the order of 0.01 of the unperturbed disk surface density (this is clearly shown in the bottom panel of Figure 3, where the gaseous gap opened by a 6 \( M_J \) planet is two orders of magnitude deep). Thus, Figure 6 shows that particles smaller than 100 \( \mu \)m will penetrate through such a planet-induced gap and not be filtered.

We note that, with a given gap depth and structure, the dust depletion within the gap only depends on one-dimensionless parameter \( T_{s,0}/\alpha \). Since \( T_{s,0} = \rho_p s \pi / (2 \Sigma_{g,0}) \), this parameter is \( \propto s / (\Sigma_{g,0} \alpha) \). Considering that the quantity \( \Sigma_{g,0} \alpha \) is proportional to the disk mass accretion rate \((\dot{M})\), \( T_{s,0}/\alpha \) depends only on the particle size over the disk accretion rate \((\dot{M}/M)\). The curve labeled with 10 \( \mu \)m in our Figure 6 assuming \( M = 10^{-8} M_\odot \) yr\(^{-1}\) can also represent 100 \( \mu \)m particles in an \( M = 10^{-7} M_\odot \) yr\(^{-1}\) disk or 1 \( \mu \)m particles in an \( M = 10^{-9} M_\odot \) yr\(^{-1}\) disk. Thus, if the disk accretion rate is very low, then smaller dust can be filtered. The reason can be directly seen from Equation (21). With the same \( \Sigma_g \) and \( T_s \), smaller \( \alpha \) means less dust diffusion so that the critical particle size for which diffusion balances outward drift decreases. One caution is that the gap density (Equation (24)) diverges to infinity when \( x = 0 \), and in reality the gap density profile truncates at some radius \( x \). Here, we use the gas component from our numerical simulations (Section 2) to determine where the gap truncates and the depth of the gaseous gap \((\Sigma / \Sigma_g)\).

Now, we consider the second effect: amplified gas radial velocity at the gap edge. The particle drift velocity due to the pressure gradient (the second term of Equation (16)) not only needs to be larger than the dust depletion velocity as above, but also needs to counter the gas radial velocity \((v_g)\) within the gap. Here, we compare the dust radial drift velocity derived above with the gas radial velocity from the gas accretion. The dust drift velocity (the right side of Equation (21)) at the gap outer edge (Equation (24)) is

\[
v_{\text{drift}} = -\eta V_K T_s = \frac{3 c_s^2 T_{s,0} W^3}{R_p \Omega x^4} e^{W / x}. \tag{28}
\]

Considering that the gas flow velocity \( v_g \) outside the gap gives \( v_{g,0} = -3 v / (2 R) \), and the gas velocity is amplified by \( v_g = v_{g,0} \times \Sigma_{g,0} / \Sigma_g \) at the gap edge to maintain a constant accretion rate, we derive

\[
\left| \frac{v_{\text{drift}}}{v_g} \right| = 2 \frac{T_{s,0}}{\alpha} (W / x)^4 \frac{1}{W}. \tag{29}
\]

Again, we can relate \( W / x \) with the gaseous gap depth \( \Sigma_g / \Sigma_{g,0} \) from Equation (24) and obtain

\[
\left| \frac{v_{\text{drift}}}{v_g} \right| = 2 \frac{T_{s,0}}{\alpha} \left[ \ln \left( \frac{\Sigma_g}{\Sigma_{g,0}} \right) \right]^{4/3} \frac{1}{W}. \tag{30}
\]

This is plotted in Figure 7. When \|v_{\text{drift}}/v_g\| is smaller than 1, dust can pass through the gap with the gas due to the amplified gas velocity. Thus, with a gap at a two orders of magnitude deep, 10 \( \mu \)m particles can pass through the gap with the gas. Again, the dimensionless parameter \( T_{s,0}/\alpha \) is present in this analysis. Here, lower \( \alpha \) means lower radial velocity and easier particle filtering. But besides that, another dimensionless parameter \( W \) is also present, and \( W \) depends on \( \alpha, \mu \), and \( R \). At smaller radii, dust is easier to filter since the outward drift velocity is larger. For the fixed radii, the dependence of \( W \) on \( \alpha \) is rather weak.

When \|v_{\text{drift}}/v_g\| < 1, more dust particles can penetrate the gap than that estimated by Equation (27) which just considers dust diffusion. Thus, when the amplified gas velocity
This dust filtration process by the planet-induced gaseous gap outer edge was first studied by Rice et al. (2006) with an analytical approach, and they concluded that micron-sized particles can be filtered by the planet-induced gap. In detail, they suggested that a gap opened by a 5 $M_J$ planet might be able to filter 1 $\mu$m particles and above. Dust diffusion due to disk turbulence is ignored in their calculations. Beyond the analytical approach, numerical simulations are difficult to carry out to study dust filtration since small particles have very short stopping times which limit the numerical time step. In this work, we use the SFT approximation which allows us to study this problem using numerical simulations. We have found that the critical size for dust filtration is larger than that estimated by Rice et al. (2006). In our simulations, the gap outer edge near a 6 $M_J$ planet can only filter 100 $\mu$m particles and above. The difference partly comes from a lower accretion rate in their models (the discussion of the critical particle size on disk accretion rate is in Section 4.4). But more importantly, the difference is due to dust diffusion from disk turbulence (Ward 2009).

The larger critical particle size for filtration poses challenges to explain transitional disks with dust filtration alone, as discussed in Section 6.

5.2. Steady State, Feedback, and Outer Disk

Without considering dust feedback and dust diffusion, the dust velocity is fully determined by the gas surface density (Equation (14)). Thus, due to the lack of feedback between the dust concentration and its velocity, the dust disk cannot achieve a steady state. Dust will continue to pile up at the outer edge of the gap (e.g., the upper left panel of Figure 3), and eventually dust feedback on the gas through drag forces becomes important there, as pointed out by Ward (2009).

However, with dust diffusion considered, the dust concentration can affect the dust velocity (Equation (15)), and a steady state can be reached on the dust drift timescale. This leads to a smoother and lower dust surface density (e.g., the upper right panel of Figure 3), which weakens the dust feedback. Furthermore, with smaller and smaller particles, dust diffusion is much more important than the dust drift velocity so that the dust surface density is smoothed even more.

The dust disk steady state can be calculated by assuming that the product of the dust velocity and density is a constant. At the outer disk and for 1 $\mu$m particles, dust drift and diffusion velocities are far larger than the gas velocity. Thus, we can seek a solution for which the last two terms of Equation (16) balance each other. It can be easily derived that a gaseous disk surface density varying as $\Sigma_d \propto R^\beta$ requires the dust surface density to vary as $\Sigma_d \propto R^{2\beta-4.5}$ if outward diffusion balances inward drift. If $\beta = 1$, $\Sigma_d \propto R^{-2.5}$, which is consistent with our simulations with dust diffusion at the outer disk (the left panels of Figure 3). This property has important implications for submm observations in protoplanetary disks. It suggests the dust disk will not shrink indefinitely if there is dust diffusion and we can use the dust surface density structure to imply the gas surface density structure.

5.3. How Much Dust can be Filtered by a Gap in a Protoplanetary Disk?

Since our simulations have determined the critical dust size due to gap edge filtration, we can estimate the fraction of dust mass being filtered by the gap in a protoplanetary disk.

The exact mass fraction of dust larger than some size depends on the dust size distribution function $n(s) \propto s^{-\beta}$, where $s$ is...
the dust size. Dust in the diffuse interstellar medium (ISM) is thought to have a size distribution $\beta = 3.5$ from 0.005 to 1 $\mu$m (Mathis et al. 1977). In protoplanetary disks, the size distribution function can be flatter, possibly due to dust growth, with $\beta = 2.5$ (D’Alessio et al. 2001). The total dust mass fraction for particles smaller than $s_p$ is

$$\frac{m(s < s_p)}{m(\text{total})} = \frac{s_p^{-\beta} - s_{\text{min}}^{-\beta}}{s_{\text{max}}^{-\beta} - s_{\text{min}}^{-\beta}} \quad (\text{if } \beta \neq 4),$$

where $s_{\text{max}}$ and $s_{\text{min}}$ are the maximum and minimum sizes of the dust with distribution $n(s) \propto s^{-\beta}$. If both $s_{\text{max}}$ and $s_p$ are far larger than $s_{\text{min}}$, then this reduces to $(s_p/s_{\text{min}})^{\beta-3}$. Our simulations above suggest that only dust equal or larger than 0.01–0.1 mm can be filtered. Thus, if we adopt $s_{\text{max}} = 1$ mm (D’Alessio et al. 2001), then the dust smaller than 0.1 mm only accounts for 10% of the total dust mass with $\beta = 3$ or 1% of the total dust mass with $\beta = 2$, and dust smaller than 0.01 mm only accounts for 1% of the total dust mass with $\beta = 3$. Thus, the dust mass fraction is decreased significantly when dust crosses the planet-induced gap. With only 1% dust mass passing through the gap, the dust to gas ratio within the gap decreases to $10^{-4}$. However, since all the micron-sized dust grains can pass through the gap freely, the near-IR spectral energy distribution (SED) of the disk is hardly affected by the filtration process, which will be emphasized in Section 6.

6. TRANSITIONAL DISKS

As summarized in the introduction, (pre-)transitional disks have wide and deep gaps, and a moderate gas accretion rate onto the star. Gap opening by planet(s) is an intriguing possibility. To produce these wide and deep gaps with planets, two main scenarios have been proposed: gap opening by multiple planets (e.g., Zhu et al. 2011; Dodson-Robinson & Salyk 2011) and dust filtration (Rice et al. 2006).

Since both gap opening by multiple planets (Zhu et al. 2011) and dust filtration (this work) hardly affect the micron-sized particle distribution within AU scales, the near-IR SEDs of these disks should look similar to those of classical T Tauri disks. Thus, both scenarios can explain pre-transitional disk near-IR SEDs, especially for those having the same SED as classical T Tauri disks (Andrews et al. 2011). However, both of the scenarios have difficulties in reproducing transitional disk SEDs which have strong near-IR deficits.

In the following, we will use a Monte Carlo radiative transfer model to demonstrate this difficulty with both (1) gap opening by multiple planets, (2) dust filtration, and provide one possible solution with (3) filtration+grain growth to reproduce the transitional disk GM Aur’s SED.

6.1. Monte Carlo Radiative Transfer Setup

The Monte Carlo radiative transfer code was developed by Whitney et al. (2003a, 2003b), Robitaille et al. (2006), and B. A. Whitney et al. (2012, in preparation), while for the disk structure see Whitney et al. (2003a) for references. This entire disk is composed of two dust components: a thick disk with small (i.e., ISM-like $\mu$m-sized) grains and a thin disk with large (mm-sized) grains. We use the standard ISM dust model for the small dust (Kim et al. 1994) and use Model 3 in Wood et al. (2002) for the large dust ($\beta = 3$ with the maximum particle size 1 mm). Both disk components are isothermal in the vertical direction, and their respective scale heights $h_{\text{thin}}$ and $h_{\text{thick}}$ obey a simple power law $h \propto R^\eta$, with $h_{\text{thin}}$ fixed to be 0.2 $\times$ $h_{\text{thick}}$. The disk extends from the sublimation radius (self-consistently determined by the dust sublimation temperature) to 200 AU.

To reproduce a classical T Tauri disk SED for comparison, we set up a full disk model, where the gas surface density profile is $\Sigma_R = \Sigma_0 R_e R_c^{-\eta}/R$, and $R_c$ is the scaling length fixed to be 100 AU. This is the same as our hydrodynamic simulations. Dust mass is 1% of the disk mass. Among all the dust, 1% is in the small dust, while the rest is in the large dust, which is equivalent to saying that the small dust depletion and settling factor is 0.01. This choice is based on both our argument in Section 5.3 (if the dust distribution function is $n(s) \propto s^{-\beta}$ from submicron sizes to 1 mm, 99% of the dust will be in the grains larger than 0.01 mm) and observational constraints (Furlan et al. 2006). Note that this implies dust has already grown to mm sizes in the outer disk and the small dust is only 1% of the total dust mass. The full disk model has a total mass of 0.1 $M_\odot$, and a scale height profile with $h/R = 0.075$ at 100 AU and $\Psi = 1.2$. The accretion rate is assumed to be $10^{-5} M_\odot$ yr$^{-1}$. These are nominal values for a classical T Tauri disk, and they are consistent with our hydrodynamical simulations. Since we are trying to fit the SED of GM Aur, the central source is assumed to be a 5730 K pre-main star with radius 1.5 $R_\odot$ and mass 1.2 $M_\odot$ (Calvet et al. 2005) and the inclination angle of this system is 55 deg.

The full disk’s SED is shown as the dotted curves in the bottom panels of Figure 8. It produces a strong near-IR flux, similar to a classical T Tauri disk SED. In the following, we will modify this full disk model based on results from hydrodynamic simulations trying to reproduce transitional disk GM Aur’s SED (the red curves in Figure 8).

6.2. Gap Opening by Multiple Planets?

To simulate gap opening by multiple planets, we cut a wide gap in the disk for both small and large dust (the left panel of Figure 8). In Zhu et al. (2011), with the same disk structure, using hydrodynamic simulations, we found that four giant planets can open a gap from 2 to 20 AU with the gap depth 1/1000th of the unperturbed disk surface density. Thus, here we cut the gap from 2 to 25 AU and both small and large dust surface densities are decreased by a factor of 1000 compared with the unperturbed disk. The gap is clearly seen in the dust surface density contours in the left panels of Figure 8. However, the near-IR SED from the modeled disk with four planets produces an SED more similar to that of classical T Tauri disks, rather than transitional disks (the left bottom panel). This is because the inner disk within 2 AU is the same as a full disk.

This similar SED as that of a full disk is not surprising, since previous work has already suggested that the near-IR deficit of a transitional disk requires small dust in the inner disk, on scales less than 1 AU, to be depleted by many orders of magnitude (Espaillat et al. 2010; Zhu et al. 2011). To be more specific, considering those transitional disks having accretion rates $\dot{M} \sim 10^{-8} M_\odot$ yr$^{-1}$ onto the star, and using

$$\Sigma_R = \frac{\dot{M}}{3\pi v},$$

$\Sigma_R$ is 10$^2$ g cm$^{-2}$ at 0.1 AU for $\alpha = 0.01$. Considering that the nominal opacity of ISM dust at 10 $\mu$m is 10 cm$^2$ g$^{-1}$, the optical depth at 0.1 AU is 10$^3$. But SED modeling of transitional disks (e.g., GM Aur) requires the disk to be optically thin at 0.1 AU (Calvet et al. 2005); thus, the micron-sized dust, which
Figure 8. Various scenarios to explain transitional disks GM Aur. (1) A wide gap opened by multiple planets (left panels), (2) a deep gap opened by one planet which can filter large dust particles (middle panels), and (3) after big particles are filtered by the gap, small particles can grow (right panels). The upper panels show the dust density distribution for large particles ($\gtrsim 10\ \mu m$) in the disk (big particles are totally filtered in the second scenario and some new big particles are generated in the third scenario), while the middle panels show the dust density distribution for small particles ($\lesssim 10\ \mu m$) in the disk (small dust is continuous in the second scenario). The lower panels show the SED from these models. The dotted curve is the SED from a full disk. Photometric (red, open symbols) and IRS data (green) for GM Aur are from Espaillat et al. (2011); refer to that work for more details. As clearly shown, either gap opening by multiple planets or dust filtration (left and right panels) has little effect on the SED compared with a full disk (solid curves overlap with the dotted curves). This is because small dust particles in the inner disk ($< 1\ AU$) which produce most of the optical and IR flux) still have similar abundance as the full disk model. But dust filtration plus dust growth can explain transitional disk SED, since small dust particles in the inner disk ($< 1\ AU$) are depleted significantly in this scenario.

(A color version of this figure is available in the online journal.)

contributes most to the near-IR flux, needs to be depleted at least by three orders of magnitude. However, gap opening by multiple planets from 2–20 AU has little effect on the dust distribution within AU scales, and thus it will not prevent the near-IR SED from being similar to a full disk model.

6.3. Dust Filtration by the Gap?

To simulate the effect of dust filtration, large dust is absent within 25 AU compared with the full disk model (the middle panel of Figure 8). All large dust is assumed to be filtered. However, the small dust can pass the planet-induced gap and has the same distribution as a full disk (middle panels of Figure 8). Again, although the mid-IR SED changes a little bit due to the depletion of the large dust, the optical to near-IR SED looks similar to the full disk (the middle bottom panel) since small dust dominates the optical to near-IR opacity.

This demonstrates that although dust filtration is efficient at reducing the total dust mass, since most of the dust mass resides in large dust particles (Section 5.3), it has little effect on the near-IR SED due to the fact that the near-IR SED is determined by the micron-sized particles which cannot be filtered by the gap. Although Dodson-Robinson & Salyk (2011) speculated that dust can be trapped in the spiral wakes, trapping in spiral wakes is only a second-order effect compared with the gap filtration, since the density wakes have far smoother density profiles than the gap edge. If the gap edge fails to trap micron-sized particles, then the density wakes will not be able to trap these particles. Our two-fluid simulations (Figure 2 and 11) also do not show dust pile up in the density wakes.

However, under some extreme circumstances (e.g., $\dot{M} < 10^{-9} M_\odot\ yr^{-1}$, the presence of a 10 $M_\oplus$ planet), the critical filtration size may decrease to 10 $\mu$m. With a flat dust size distribution (e.g., $\beta = 2$ in Section 5.3), dust smaller than 10 $\mu$m could have a mass less than $10^{-4}$ of the total dust mass. Thus, after filtration, small dust could have a depletion factor equal to $10^{-4}$, which is close to the small dust depletion factor ($10^{-5}$) required to explain the near-IR deficit of GM Aur.8

8 If the gas surface density is very low (e.g., 2 gcm$^{-3}$), then the dust depletion factor can be $10^{-3}$ for the disk to be optically thin (Salyk et al. 2007). In this case, a large disk viscosity parameter $\alpha$ is required to explain the observed gas accretion rates.
In Figure 9, we have calculated two cases with a small 
(<10 \mu m)/large(>10 \mu m) dust mass ratio \( \sim 10^{-5} \) at the outer disk and only small dust existing in the inner disk. As expected, in this case, the near-IR deficit can be reproduced due to the tiny amount of small dust in the disk. However, this tiny amount of small dust makes not only the inner disk optically thin, but also the outer disk’s atmosphere optically thin (assuming large dust grains have settled to the midplane forming a dust layer whose thickness is only 20% of the gas disk). The mid-IR flux is very weak too and unable to reproduce the Infrared Spectrograph (IRS) observations (dotted curve in Figure 9). In order to reproduce the mid-IR flux, the large dust grains have to remain suspended in the atmosphere of the disk and not settle (maintaining the same thickness as the gaseous disk) to intercept the stellar radiation, which is shown by the dashed curve in Figure 9.

The dashed curve in Figure 9 suggests that it is not impossible for dust filtration alone to explain GM Aur. But several conditions have to be met: (1) dust grows significantly in the outer disk (flat dust size distribution, and the small dust (<10 \mu m) to big dust mass ratio is \( \sim 10^{-5} \)); (2) a very massive planet forms in a disk with a low accretion rate; and (3) large grains in the outer disk cannot settle. These conditions are not easy to satisfy since a normal T Tauri disk has a small dust depletion factor of 20\% (accounting for 99\% of the total dust mass) which can reproduce the near-IR flux, whereas the small dust mass ratio is 1:999. Since the total dust mass is decreased by 1\% after dust filtration and this new distribution further reduces the small dust by a factor of 1000, the small dust is depleted by a factor of 10^3, and the mass ratio of small dust to gas is \( 10^{-7} \). This leads to an optically thin inner disk. The disk’s SED fits the GM Aur SED quite well in the bottom right panel.

Note that in this scenario small particle growth is a gradual process. Although we deplete the dust in the inner disk uniformly for computational convenience (upper two panels in the right column of Figure 8, or in other words we assume small particles grow instantaneously after they cross the gap), in reality the abundance for small particles may change gradually and join the outer disk abundance smoothly.

The readers may notice that the 10 \mu m silicate feature is not fitted well by our simple models. But we want to point out that the strength of the silicate emission depends not only upon the total amount of dust inside the gap, but upon dust size distribution, especially for small dust (<10 \mu m). The dust size distribution sensitively depends on growth, fragmentation, and differential drift of particles of differing size. Predicting the strength of the silicate emission feature will require addressing the very complex processes involved in grain evolution in concert with calculations of drift similar to those in the present

\( <0.01 \) mm abundance in the inner disk within AU scales. One solution is considering the growth of small dust particles after large particles are filtered by the planet-induced gap. Dust can grow quite rapidly in the inner disk. It may only take \( 10^{3} \) yr for dust particles to grow from submicron sizes to 1000 \mu m at 1 AU (e.g., models S3 and S4 in Dullemond & Dominik 2005). One way to stop the rapid dust growth is by collisional fragmentation (Dullemond & Dominik 2005; Dominik & Dullemond 2008), in which case large particles are shattered to replenish small dust grains. Thus, the growth and fragmentation maintains a quasi-stationary dust size distribution function. In disks with gaps, if all big grains (>10–100 \mu m) are filtered across the gap (accounting for 99\% of the total dust mass) that manage to pass through the gap to the inner disk can grow quite efficiently without replenishment from fragmentation of large particles. Although the dust growth time is 100 times longer than the timescale for a non-filtered disk (the dust growth time is inversely proportional to the dust abundance), it is still comparable to the disk viscous timescale at 20 AU with \( \alpha = 0.01 \). Eventually, a new balance is made and the quasi-stationary dust size distribution is established again. At this time, due to grain growth, the small dust is only 1\% of the abundance that was present after it had passed through the planet’s orbit location. In other words, small dust is depleted indirectly due to dust growth and the net depletion factor for small particles in the inner disk is \( 10^{-4} \) (1\% after filtration \times 1\% mass fraction in the new size distribution). Such a scenario of “double depletion” may explain transitional disks, but requires further study to place it on a firmer foundation.

To simulate this scenario, we assume that small dust grows in the inner disk and reestablish the dust size distribution after large particles are filtered by the planet-induced gap at 25 AU. This is illustrated in the right panels of Figure 8. In the inner disk, we assume that the new dust size distribution has a small to large dust mass ratio 1:999. Since the total dust mass is decreased by 1\% after dust filtration and this new distribution further reduces the small dust by a factor of 1000, the small dust is depleted by a factor of 10^3, and the mass ratio of small dust to gas is \( 10^{-7} \). This leads to an optically thin inner disk. The disk’s SED fits the GM Aur SED quite well in the bottom right panel.

Note that in this scenario small particle growth is a gradual process. Although we deplete the dust in the inner disk uniformly for computational convenience (upper two panels in the right column of Figure 8, or in other words we assume small particles grow instantaneously after they cross the gap), in reality the abundance for small particles may change gradually and join the outer disk abundance smoothly.

The readers may notice that the 10 \mu m silicate feature is not fitted well by our simple models. But we want to point out that the strength of the silicate emission depends not only upon the total amount of dust inside the gap, but upon dust size distribution, especially for small dust (<10 \mu m). The dust size distribution sensitively depends on growth, fragmentation, and differential drift of particles of differing size. Predicting the strength of the silicate emission feature will require addressing the very complex processes involved in grain evolution in concert with calculations of drift similar to those in the present

\( \alpha = 0.01 \).

\( \beta = 2 \) or the critical size < 0.01 mm abundance in the inner disk within AU scales. One solution is considering the growth of small dust particles after large particles are filtered by the planet-induced gap. Dust can grow quite rapidly in the inner disk. It may only take \( 10^{3} \) yr for dust particles to grow from submicron sizes to 1000 \mu m at 1 AU (e.g., models S3 and S4 in Dullemond & Dominik 2005). One way to stop the rapid dust growth is by collisional fragmentation (Dullemond & Dominik 2005; Dominik & Dullemond 2008), in which case large particles are shattered to replenish small dust grains. Thus, the growth and fragmentation maintains a quasi-stationary dust size distribution function. In disks with gaps, if all big grains (>10–100 \mu m) are filtered across the gap (accounting for 99\% of the total dust mass) that manage to pass through the gap to the inner disk can grow quite efficiently without replenishment from fragmentation of large particles. Although the dust growth time is 100 times longer than the timescale for a non-filtered disk (the dust growth time is inversely proportional to the dust abundance), it is still comparable to the disk viscous timescale at 20 AU with \( \alpha = 0.01 \). Eventually, a new balance is made and the quasi-stationary dust size distribution is established again. At this time, due to grain growth, the small dust is only 1\% of the abundance that was present after it had passed through the planet’s orbit location. In other words, small dust is depleted indirectly due to dust growth and the net depletion factor for small particles in the inner disk is \( 10^{-4} \) (1\% after filtration \times 1\% mass fraction in the new size distribution). Such a scenario of “double depletion” may explain transitional disks, but requires further study to place it on a firmer foundation.

To simulate this scenario, we assume that small dust grows in the inner disk and reestablish the dust size distribution after large particles are filtered by the planet-induced gap at 25 AU. This is illustrated in the right panels of Figure 8. In the inner disk, we assume that the new dust size distribution has a small to large dust mass ratio 1:999. Since the total dust mass is decreased by 1\% after dust filtration and this new distribution further reduces the small dust by a factor of 1000, the small dust is depleted by a factor of 10^3, and the mass ratio of small dust to gas is \( 10^{-7} \). This leads to an optically thin inner disk. The disk’s SED fits the GM Aur SED quite well in the bottom right panel.

Note that in this scenario small particle growth is a gradual process. Although we deplete the dust in the inner disk uniformly for computational convenience (upper two panels in the right column of Figure 8, or in other words we assume small particles grow instantaneously after they cross the gap), in reality the abundance for small particles may change gradually and join the outer disk abundance smoothly.

The readers may notice that the 10 \mu m silicate feature is not fitted well by our simple models. But we want to point out that the strength of the silicate emission depends not only upon the total amount of dust inside the gap, but upon dust size distribution, especially for small dust (<10 \mu m). The dust size distribution sensitively depends on growth, fragmentation, and differential drift of particles of differing size. Predicting the strength of the silicate emission feature will require addressing the very complex processes involved in grain evolution in concert with calculations of drift similar to those in the present

\( \alpha = 0.01 \).
paper. For example, when small dust particles grow to big particles, big particles will quickly drift to the central star, and if the drift timescale is much shorter than the dust growth timescale, only small particles will exist in the inner disk. A careful dust evolutionary model combined with dust filtration is important to test this scenario. Nevertheless, small dust needs (<10 \text{ \mu m}) to be depleted more than what dust filtration predicts to explain near-IR deficit of transitional disks.

In this work we only put one planet in the disk to study dust filtration, because the dust filtration process does not sensitively depends on the number of the planets in the disk since most dust will be filtered by the outermost planet gap or the deepest gap. Thus most of our results can also be applied to the gap opened by multiple planets in the disk, which is complimentary to Zhu et al. (2011). We note, however, that a multiple planet system may need to be invoked to dynamically clear planetesimals from the inner disk region as the presence of such bodies is likely to provide a source for small dust grains through their mutual collisions.

6.5. Observational Implications for ALMA

Dust filtration has other observational implications besides SEDs. Dust filtration by the planet-induced gap differentiates various particles. Thus, if we observe transitional disks at various wavelengths the gap/cavity should be more distinctive at longer wavelengths (Fouchet et al. 2010 and our Figures 1–3).

Dust growth as argued above will not happen instantaneously as the flow passes through the planet-induced gap. Thus at shorter wavelengths, the cavity which is found by submm observations is less apparent or even disappears (e.g., Figure 1). Andrews et al. (2011) notice that there are (pre-)transitional disks with classical T-Tauri disk SEDs but that show gaps in submm interferometry. More directly, recent Subaru observations have found that a lot of (pre-)transitional disks having submm cavities (Andrews et al. 2011) do not show cavities in near-IR scattered light images (Dong et al. 2012). Both of these findings seem to agree with the dust filtration scenario.

However, after the big grains are filtered, whether and how much small grains can grow is a difficult issue; it requires a dust evolutionary model combined with dust dynamics. The complexity of the problem is illustrated by, for example, Birnstiel et al. (2011) and references therein. In this work, based on transitional disk SEDs, we suggest that small dust needs to grow in the inner disk. Figure 10 shows both big dust (larger than the critical filtration size) surface density and the 850 \text{ \mu m} opacity in the filtration+grain growth scenario.10 Both the large-grain surface density and the submm opacity change at the gap edge due to dust filtration. The submm opacity decreases by a factor >100 at the gap edge, which is consistent with submm constraints from Andrews et al. (2011). In the near future, ALMA can determine how sharp the opacity decreases in greater detail, which is important to distinguish a pure grain growth scenario (Section 6.6) and scenarios involving a gaseous gap.

More generally, ALMA can test all the three scenarios above. For gap opening by multiple planets without dust filtration, both gas (probed by molecular lines) and dust (probed by dust continuum) inside the gap should be equally depleted. Furthermore, in this scenario, the sharpness of the outer gap edge and inner gap edge should look similar due to the symmetric nature of the gaseous gap shape. If dust filtration is at work, the mm-sized dust will be more depleted inside the gap than the gas, and the inner gap edge in submm images should be smoother, or even disappear, than the outer gap edge (Figure 3).

To test whether there is grain growth inside the gap, we can use multiple wavelength observations to see how the slope of the opacity changes. However, the dust radial drift may change the big dust distributions in the inner disk.

ALMA can also test if the dust and gas have similar density profiles at the outer disk beyond the gap. Our simulations suggest that, combining dust drift and diffusion, the dust and gas surface density profiles in the outer disk can be quite different (Section 5.2). Although submm observations (Andrews et al. 2012) have indeed suggested that the dust disk is more compact than the gaseous disk, the different surface density slopes between the dust and gas disks predicted in Section 5.2 need to be tested by future ALMA observations. But we note that dust growth and fragmentation can potentially change this relationship (e.g., Birnstiel et al. 2012).

6.6. Other Possibilities

As illustrated above, the key ingredient to reproduce the GM Aur SED is reducing the abundance of micron-sized dust particles by five orders of magnitude in the inner disk while maintaining a sufficient gas accretion. Due to the good coupling between micron-sized particles and the gas, any theory only considering gap formation in a gaseous disk is not enough, such as photoevaporation, gap opening by planets, etc. Dust growth

---

10 To calculate the opacity, we assume the gas surface density is unaffected by the presence of the planet. The gaseous disk is the same as a constant $M$ accretion disk.
and settling have to be considered. Besides filtration combined with dust growth, there are several other possibilities.

The first alternative is purely dust growth. If dust can grow significantly in the inner disk to make the inner disk optically thin, then it can explain GM Aur’s SED. In this scenario, it may suggest that transitional disks are older than CTTS, which has not been suggested by observations. Furthermore, both SED fitting and submm observations suggest the gap edge is very sharp, inconsistent with the pure dust growth model. However, with CARMA, Isella et al. (2012) have suggested that LkCa 15 can be explained by the pure dust growth model. Thus, pure dust growth could still be a possible solution.

The second alternative is a large gas mass reservoir close to the central star with a wide and deep gap beyond. The wide and deep gap can be caused by a very massive planet (or even a star) or several planets so there is no accretion flow from the outer disk to the inner disk. The accretion onto the star is sustained by the mass reservoir close to the planet (e.g., a dead zone). However, this mass reservoir needs to be very narrow or depleted in small dust to not produce too much near-IR flux.

The third alternative is the dust being held back by the radiation pressure (Chiang & Murray-Clay 2007). However, the efficiency of the radiation pressure is questioned by Dominik & Dullemond (2011).

The fourth alternative is planet gap opening in layered disks. The pros and cons of this model will be discussed in a forthcoming paper.

However, there is one challenge to all the scenarios trying to explain transitional disks with gap opening by planet(s) located at a few tens of AU from the central star. In the core-accretion scenario, the solid component in the inner disk is likely to have undergone significant growth to form planetesimals and planetary embryos within the time taken to form the putative planet at ~20 AU. These planetesimals should collide and continuously regenerate small dust grains. As we have alluded to earlier in this paper, one way round this problem is to invoke the presence of multiple planets in the inner disk to clear these planetesimals. But this hypothesis clearly requires further investigation.

6.7. Transitional versus Pre-transitional Disks

Dust filtration also suggests that more massive planets can lead to stronger dust filtration and depletion. Thus, transitional disks may have higher mass planet(s) than pre-transitional disks have. Since a higher mass planet exerts a stronger torque on the outer disk, it may slow down the accretion flow passing the planet and lead to a lower disk accretion rate onto the star. This is consistent with observations that transitional disks have lower accretion rates than pre-transitional disks (Espaillat et al. 2012).

Regarding dust growth, transitional disks put strict constraints on the dust abundance in the inner disk since the dust is optically thin. If these systems have moderate $M$, then we know they have a significant amount of gas, and thus the dust to gas ratio needs to be depleted. On the other hand, for traditional disks with little $M$, the disk may harbor a massive companion (e.g. brown dwarf). In this case, both gas and dust components of the inner disk are significantly reduced below the detection limit and we know little about their dust to gas ratio, and thus dust growth/depletion is not necessary.

For pre-transitional disks, the dust abundance in the inner disk is difficult to constrain since it is optically thick. Thus, it is possible to explain pre-transitional disks without invoking dust growth (e.g., dust filtration, multiple-planets, photoevaporation; Alexander & Armitage 2007). However, it is also possible that the planet(s) in a pre-transitional disk is/are less massive (making the gap less sharp) so that more dust passes through the planet-induced gap and the inner disk remains optically thick.

7. CONCLUSION

In this paper, we have used 2D two-fluid simulations, a 1D model, and analytic arguments to study dust filtration by the tidally induced gap outer edge. We have found that dust diffusion and the high gas velocity at the gap edge significantly lower the dust filtration efficiency. Only particles equal or larger than 0.1 mm can be filtered by a planet-induced gap if the disk has $\alpha = 0.01$, $M = 10^{-6} M_\odot \text{yr}^{-1}$ and the planet mass is a few Jupiter masses. These results can be partly scaled to disks having different mass accretion rates with one-dimensionless parameter $T_i/\alpha$. With better understanding of the disk and gap structure, we may be able to constrain the planet mass with future multi-wavelength observations (optical/near-IR scattered images, ALMA, etc.).

We have applied this dust filtration threshold (0.1 mm) to transitional disks, and by using a Monte Carlo radiative transfer model, we have shown that dust filtration alone has difficulties in explaining transitional disk observations, especially for systems with moderate $M$ (e.g. GM Aur). The same difficulty is suffered by the multiple planet scenario. One possible solution is combining dust filtration with dust growth in the inner disk, although we have also discussed other possibilities. We conclude that dust filtration is a natural consequence of gap opening in protoplanetary disks, and although it has some difficulties to explain the near-IR deficit of transitional disks (which may require additional processes such as dust growth), it has important implications for future observations.

This work was supported in part by NASA grant NNX08AI39G from the Origins of Solar Systems program, and in part by the University of Michigan. Z.Z. and R.D. were also supported by NSF grant AST-0908269 and Princeton University. C.E. was supported by the National Science Foundation under Award No. 0901947. Z.Z. thanks Xuening Bai, Jim Stone, Roman Rafikov, Steve Lubow, and Ruth Murray-Clay for helpful discussion. Z.Z. also thanks Eugene Chiang for suggesting the layer accretion scenario during the International Summer Institute for Modeling in Astrophysics (ISIMA) in Beijing organized by Pascale Garaud and Doug Lin. The authors thank the referee for significantly improving this paper.

APPENDIX

TWO-FLOW SIMULATIONS VERSUS SFT APPROXIMATION VERSUS 1D MODELS

We will compare the dust distribution using three different methods in this section. We expect SFT approximation should give similar results as two-fluid simulations for particles smaller than mm, since the dust stopping time for 1 mm particles is 1/10th of the hydrodynamic time step and the SFT approximation is valid.

The comparison among two-fluid simulations, the SFT approximation, and 1D models is shown in Figure 11 with (the right panel) and without considering dust diffusion (the left panel). A 1 $M_J$ planet and 1 mm particles are considered and the simulations have been run to $5 \times 10^4$ yr.

As shown in Figure 11, the two-fluid simulations and the SFT approximation agree with each other quite well in both
In 1D, with the assumption that the flow is axisymmetric and the radial velocity is given by Equation (17), material in the horseshoe region will be quickly depleted. However, slightly outside the horseshoe region, even at the edge of the gap the flow is still quite axisymmetric. Considering the gap outer edge is where dust filtration takes place, the 1D model is still capable to study dust filtration by the gap outer edge, although the dust surface density at the bottom of the gap is incorrect.

**REFERENCES**

Andrews, S. M., Wilner, D. J., Hughes, A. M., Qi, C., & Dullemond, C. P. 2009, ApJ, 700, 1502

Andrews, S. M., Wilner, D. J., Hughes, A. M., et al. 2012, ApJ, 744, 162

Alexander, R. D., & Armitage, P. J. 2007, MNRAS, 375, 500

Birnstiel, T., Klahr, H., & Ercolano, B. 2012, A&A, 539, A148

Birnstiel, T., Omel, C. W., & Dullemond, C. P. 2011, A&A, 525, A11

Brown, J. M., Blake, G. A., Dullemond, C. P., et al. 2007, ApJ, 664, L107

Brown, J. M., Blake, G. A., Qi, C., et al. 2009, ApJ, 704, 496

Calvet, N., D’Alessio, P., Hartmann, L., et al. 2002, ApJ, 568, 1008

Calvet, N., D’Alessio, P., Watson, D. M., et al. 2005, ApJ, 630, L185

Carballido, A., Bai, X.-N., & Cuzzi, J. N. 2011, MNRAS, 415, 93

Chiang, E., & Murray-Clay, R. 2007, Nat. Phys., 3, 604

Clarke, C. J., & Pringle, J. E. 1985, MNRAS, 213, 365

Crida, A., Morbidelli, A., & Masset, F. 2007, A&A, 461, 1173

D’Alessio, P., Calvet, N., & Hartmann, L. 2001, ApJ, 553, 321

D’Alessio, P., Hartmann, L., Calvet, N., et al. 2005, ApJ, 621, 461

Dodson-Robinson, S. E., & Salyk, C. 2011, ApJ, 738, 131

Dullemond, C. P., & Dominik, C. 2005, A&A, 434, 971

Dominik, C., & Dullemond, C. P. 2008, A&A, 491, 663

Dominik, C., & Dullemond, C. P. 2011, A&A, 531, A101

Dong, R., Rafikov, R., Zhu, Z., et al. 2012, ApJ, 750, 161

Espaillat, C., Calvet, N., D’Alessio, P., et al. 2007, ApJ, 670, L135

Espaillat, C., Calvet, N., Luhan, K. L., Muzerolle, J., & D’Alessio, P. 2008, ApJ, 682, L125

Espaillat, C., D’Alessio, P., & Hernández, J., et al. 2010, ApJ, 717, 441

Espaillat, C., Furlan, E., D’Alessio, P., et al. 2011, ApJ, 728, 49

Espaillat, C., Ingleby, L., Hernandez, J., et al. 2012, ApJ, 747, 103

Fouchet, L., Maddison, S. T., Gonzalez, J.-F., & Murray, J. R. 2007, A&A, 474, 1037

Fouchet, L., Gonzalez, J.-F., & Maddison, S. T. 2010, A&A, 518, A16

Furlan, E., Hartmann, L., Calvet, N., et al. 2006, ApJS, 165, 568

Gullbring, E., Hartmann, L., Briceno, C., & Calvet, N. 1998, ApJ, 492, 323

Hartmann, L., Calvet, N., Gullbring, E., & D’Alessio, P. 1998, ApJ, 495, 385

Hughes, A. M., Andrews, S. M., Espaillat, C., et al. 2009, ApJ, 698, 131

Isella, A., Perez, L. M., & Carpenter, J. M. 2012, ApJ, 747, 136

Johansen, A., & Klahr, H. 2005, ApJ, 634, 1353

Kim, S.-H., Martin, P. G., & Hendry, P. F. 1994, ApJ, 422, 164

Marsh, K. A., & Mahoney, M. J. 1992, ApJ, 395, L115

Masset, F. 2000, A&A, 141, 165

Mathis, J. S., Rumpl, W., & Nordsieck, K. H. 1977, ApJ, 217, 425

Najita, J. R., Strom, S. E., & Muzerolle, J. 2007, MNRAS, 378, 369

Paardekooper, S.-J., & Mellema, G. 2006, A&A, 453, 1129

Pierens, A., & Nelson, R. P. 2008, A&A, 482, 333

Pietu, V., Dutrey, A., Guilloteau, S., Chapillon, E., & Pety, J. 2006, A&A, 460, L43

Rice, W. K. M., Armitage, P. J., Wood, K., & Lodato, G. 2006, MNRAS, 373, 1619

Rice, W. K. M., Wood, K., Armitage, P. J., Whitney, B. A., & Bjorkman, J. E. 2003, MNRAS, 342, 79

Robitaille, T. P., Whitney, B. A., Indebetouw, R., Wood, K., & Denzmore, P. 2006, ApJS, 167, 256

Schneider, G., Wood, K., Silverstone, M. D., et al. 2003, AJ, 125, 1467

Salyk, C., Blake, G. A., Boogert, A. C. A., & Brown, J. M. 2007, ApJ, 655, L105

Stone, J. M., & Norman, M. L. 1992, ApJS, 80, 753

Strom, K. M., Strom, S. E., Edwards, S., Cabrit, S., & Skrutskie, M. F. 1989, AJ, 97, 1451

Takeuchi, T., & Lin, D. N. C. 2002, ApJ, 581, 1344

Takeuchi, T., & Lin, D. N. C. 2005, ApJ, 623, 482
Youdin, A. N., & Lithwick, Y. 2007, ICARUS, 192, 588
Ward, W. R. 2009, in Lunar and Planetary Institute Science Conference Abstracts, Particle Filtering by a Planetary Gap (the Woodlands, Texas: Lunar and Planetary Science), 40, 1477
Weidenschilling, S. J. 1977, MNRAS, 180, 57
Whipple, F. L. 1972, in From Plasma to Planet, ed. A. Evlilus (New York: Wiley), 211
Whitney, B. A., Wood, K., Bjorkman, J. E., & Cohen, M. 2003a, ApJ, 598, 1079
Whitney, B. A., Wood, K., Bjorkman, J. E., & Wolff, M. J. 2003b, ApJ, 591, 1049
Wood, K., Wolff, M. J., Bjorkman, J. E., & Whitney, B. 2002, ApJ, 564, 887
Zhu, Z., Nelson, R. P., Hartmann, L., Espaillat, C., & Calvet, N. 2011, ApJ, 729, 47