MESOSCOPIC TRANSPORT: THE ELECTRON-GAS SUM RULES IN A DRIVEN QUANTUM POINT CONTACT

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1. INTRODUCTION

The electron gas is characterized, above all, by its response as a correlated many-particle system. In that setting the conserving sum rules have been thoroughly studied and exhaustively applied for many years [1]. The sum rules and their centrality in the electron-gas problem are widely understood throughout the many-body community – if not elsewhere.

At any scale, from the bulk to the near-microscopic, metallic conduction processes are dominated by the physical constraints encoded within the electron-gas sum rules. Thus they apply, with perfect rigor, to mesoscopic devices such as quantum point contacts. They also govern nonequilibrium transport and fluctuations, as they do the much more familiar linear-response limit.

In this paper we review the structure of the conserving sum rules in the mesoscopic regime, from the weakest driving fields to the strongest. As compact quantitative expressions of the microscopic conservation laws, the sum rules entail a set of straightforward criteria to be met by proper theoretical descriptions of metallic-electron behavior. As we discuss below, these criteria are quite enough to rule out models of mesoscopic processes that fail to uphold the conservation laws.

We recall the leading sum rules for mesoscopic conduction. In particular we apply one of them, the compressibility sum rule, to strongly driven, open electronic systems. Despite their fundamental importance, very little is known about the role of the sum rules in mesoscopic electronics. Since the latter is so deeply rooted in the electron gas, one expects that those relations, as canonical constraints, carry significant information about the way in which the processes underlying transport and its associated current noise are deployed at short scales.

We first discuss the intimate relationship between the transport response of an
open mesoscopic device, connected to macroscopic metallic leads, and the structure of the microscopic excitations, or fluctuations, within the device. This relationship is strongly conditioned by the stabilizing effect of the bounding leads on the internal dynamics of the device. Next, we describe how the microscopic form of the fluctuations in the conductor develops away from equilibrium. This leads, in particular, to the sum rules for perfect screening and compressibility [1]. Then we address the physical implications of the compressibility sum rule, whose conceptual and practical import for mesoscopics is on a par with the familiar fluctuation-dissipation theorem (FDT).

A major, and thus far unexplored, experimental application of the compressibility sum rule is to mesoscopic systems with built-in nonuniformity of their conduction bands. In that context we review some surprising Coulomb effects. These should be clearly manifested through the behavior of the electronic compressibility of inhomogenous quantum wires.

2. BASICS

Sum rules express the microscopic conservation laws at the level of what is measurable in a system. The physical worth of a theory of conduction can be judged, practically and objectively, by the degree to which it satisfies them. Approximate descriptions of an electron system should respect at least the principal ones:

(a) the fluctuation-dissipation relation, expressing energy conservation,
(b) the perfect-screening sum rule (gauge invariance), and
(c) the compressibility sum rule (number conservation).

The importance of (a) in mesoscopic transport is well known. Unfortunately, that of (b) and (c) is not well known at all. All three rules are equally pivotal; all three must be satisfied. None of the three is optional; if even one of them is violated by a candidate model of electron transport, that model is manifestly nonviable. Whatever special pleading may be made for it (for example, some fancied phenomenological simplicity), it cannot compensate for basic failures of this order.

Each of the electron-gas sum rules follows a common pattern. Each is an identity in which a measurable function of the one-particle distribution for the system is equated to an integral over a microscopic, two-particle correlation function. Let us briefly examine the FDT as a starting point. In the linear-response limit the FDT provides an expression for the average power $P$ in steady state, supplied by the external current generator to the driven system [2,3]:

$$
P = VI = -\lim_{t\to\infty} \left\{ \int_0^L E(x)dx \langle j(x,t)/L \rangle I \right\}$$

$$
\equiv \lim_{t\to\infty} \left\{ \int_0^t dt' \int_0^L dx \int_0^L dx' (-E(x)) \frac{\langle [j(x,t)/L, j(x',t')/L]_0 (-E(x')) \rangle_0}{k_B T} \right\}
$$

\begin{align}
= \frac{1}{k_B T} \int_0^\infty dt'' \int_0^L dx \int_0^L dx' (-E(x)) \langle [j(x,t''), j(x',0)/L]_0 (-E(x')) \rangle_0 (-E(x')) \tag{1}
\end{align}
where we specialize to one dimension. (This will be appropriate to a quantum point contact, in which a quasi-one-dimensional wire is attached to macroscopic metallic leads [2].) The thermal energy is $k_B T$, the electromotive force across the conductor of length $L$ is $V = -\int_0^L E(x) dx$ and $I$ is the current. The brackets $\langle \ldots \rangle$ represent a trace over the many-body density matrix for the carriers, subject to transport (for $I = 0$ it is the equilibrium trace), and $j(x,t) = -e v(x,t)$ is the particle-flux operator (to be evaluated in the mean over the channel length, following the Ramo-Shockley theorem [4]).

The form of Eq. (1) is paradigmatic. On the left we have the experimentally accessible total power absorbed by the system. On the right, we have the auto-correlation function of the microscopic power-loss density $-E(x) j(x,t)/L$, which is not directly accessible. The velocity-velocity correlation $\langle [v(t), v(0)] \rangle / k_B T$ is a measure of the excursions of carrier energy taken up by electron-hole pair fluctuations. As the excited pair states propagate, scatter and decay, their excess energy dissipates. Conservation requires that these losses add up to the total electrical power continuously supplied by the generator. Equation (1) expresses this.

In a uniform system, the local electric field $E = -V/L$ is constant. We obtain the standard Kubo conductance formula [5]

$$G = \frac{P}{V^2} = \frac{1}{k_B T} \int_0^\infty dt \int_0^L dx \frac{E(x)}{V} \int_0^L dx' \frac{E(x')}{V} \langle [j(x,t), j(x',0)] \rangle / L^2$$

$$\rightarrow \frac{1}{L^2 k_B T} \int_0^\infty dt \langle [j(t), j(0)] \rangle.$$

For inhomogeneous systems the sum rule for conductance has a more general, but conceptually identical, structure. This is covered more closely in Ref. [3].

The demonstration of the Kubo formula’s gauge invariance for an open system was given in explicit detail by Sols [6]. Having been established, it furnishes necessary and sufficient criteria for the system’s theoretical description to be microscopically and globally conserving. This contrasts with the status of gauge invariance within Landauer-Büttiker-Inuy approaches; see, for example, the commentary in Ref. [7] following Eq. (51) therein. Eq. (2) leads directly to the quantized Landauer formula as a (very particular) limiting case whose microscopic proof is free of the customary Landauer phenomenology [8]. As Eqs. (9)–(11) below demonstrate, the latter badly compromises both gauge invariance and the compressibility sum rule.

3. CONSERVATION IN OPEN CONDUCTORS

The FDT’s far-reaching importance derives from the following. The conductance $G$, which is directly measurable, can be computed as a one-particle object within a given transport model. If, using the same model, one computes the two-body current auto-correlation (which also determines the measurable Johnson-Nyquist noise), then it must yield precisely the same value of $G$. That value may, or may not, turn out to be a good fit to experiment. For the model’s integrity, however, the essential point is that the consistency of the FDT has to be guaranteed internally. Otherwise the
physical basis of such a description is flawed, since it fails to conserve energy. It would then be difficult to take such a description seriously.

A similarly fundamental and normative significance attaches to the compressibility sum rule for open conductors, which we now derive. Consider a conductor in electrical contact with a pair of large metallic reservoirs, so that electrons are freely exchangeable. At equilibrium, the whole assembly is strictly neutral. We attach a current generator across the interfaces between the leads and the device, to drive a current $I$ through the structure; from the viewpoint of the nonequilibrium carriers within the conductor, the interfaces appear as an external source and sink for the current.

In this nonequilibrium situation, three boundary conditions apply without qualification:

- **Thermodynamic equilibrium of the reservoirs.** Away from the disturbed device and its interface regions, the electron population is equilibrated and stable at any value of $I$. Thus the local electron density $n(\mu, T)$ in each lead is unchanged as a function of the equilibrium chemical potential $\mu$ and bath temperature $T$.

- **Charge neutrality of the reservoirs.** Within a well-defined finite range beyond the interfaces, strong metallic screening ensures the neutrality of the reservoir carriers with their ionic background, once again independently of $I$.

- **Charge neutrality of the driven device.** Since global neutrality and that of the individual reservoir leads are each preserved, the neutrality of the intervening, current-bearing conductor is secured for all values of the current. (Note: an independent formal proof of this criterion comes from the global gauge-invariance theorem for open conductors [6].)

Suppose that the bounded region in which the electrons are appreciably disturbed (namely the device with its interfaces) has volume $\Omega$ and contains $N$ mobile electrons on average. The electrons are fully compensated at all times. Hence the neutrality conditions are equivalent to the statement that

$$\frac{\delta N}{\delta I} = 0 = \frac{\delta \Omega}{\delta I}$$

for all $I$.

In a time-dependent situation, the electron distribution function is the trace of the local-number quantum operator $\rho_k(r, t)$ with the nonequilibrium density matrix in the presence of the current:

$$f_k(r, t; I) = \langle \rho_k(r, t) \rangle_I$$

in which $k$ and $r$ are, respectively, the wave-vector and real-space labels (spin and valley indices are subsumed in $k$, and we assign the effective volume normalization $(2\pi)^{-\nu}$ to it). It follows that

$$\int \Omega dr \int \frac{dk}{(2\pi)^\nu} f_k(r, t; I) = N = \int \Omega dr \int \frac{dk}{(2\pi)^\nu} f_k^\text{eq}(r).$$

Since absolute neutrality holds, the number of carriers in the active region cannot change from the equilibrium value determined by the distribution $f^\text{eq} \equiv \langle \rho \rangle_{I=0}$. 
An immediate result of this identity is that any fluctuation, \( \delta N \), in \( N \) is determined equally well by the corresponding fluctuation in \( f_k(r, t; I) \) out of equilibrium, as it is by that in \( f_k^{eq}(r) \) at equilibrium. Therefore

\[
\int_\Omega dr \int \frac{dk}{(2\pi)^\nu} \delta f_k(r, t; I) = \delta N = \int_\Omega dr \int \frac{dk}{(2\pi)^\nu} \delta f_k^{eq}(r).
\]

This establishes the perfect-screening sum rule in its most general form.

4. NONEQUILIBRIUM COMPRESSIBILITY SUM RULE

We may now vary the population \( N \) systematically with respect to the chemical potential, a procedure that takes its shape from the response of the equilibrium state. In the important case of the free electron gas, we know the resulting analytical expressions:

\[
f_k^{eq}(r) = \frac{1}{1 + \exp[(\varepsilon_k + U_0(r) - \mu)/k_B T]};
\]

\[
\frac{\partial f_k^{eq}(r)}{\partial \mu} = \frac{1}{k_B T} f_k^{eq}(r)(1 - f_k^{eq}(r)).
\]

Note that we incorporate the static mean-field potential \( U_0(r) \) at equilibrium. This is set up whenever the system has inhomogeneities built into it.

Recalling that \( \Delta f^{eq} = k_B T \partial f^{eq} / \partial \mu \) defines the mean-square thermal fluctuation of the equilibrium occupation number [1], the corresponding nonequilibrium fluctuation \( \Delta f(t) \) must satisfy

\[
\int_\Omega dr \int \frac{dk}{(2\pi)^\nu} \Delta f_k(r, t; I) = \Delta N = \int_\Omega dr \int \frac{dk}{(2\pi)^\nu} \Delta f_k^{eq}(r)
\]

where the mean-square total number fluctuation is \( \Delta N = k_B T \partial N / \partial \mu \). This identity, emerging straight out of the perfect-screening rule, is the heart of the compressibility sum rule.

What does it mean, physically, to subject \( f_k(r, t; I) \) – an inherently nonequilibrium object – to a variation with respect to \( \mu \), an equilibrium parameter? The answer is straightforward. Whether at equilibrium or not, the asymptotic density of carriers in the macroscopic leads remains unchanged. They sense nothing of the conditions (possibly extreme) that dominate the driven region. On the other hand, the density in the leads is directly controlled by the chemical potential local to each reservoir. This local value is unique, completely unaffected by the current, and proper to the local conduction band. Changing \( \mu \) is equivalent to replacing the neutral reservoirs at electron density \( n(\mu) \), say, with neutral reservoirs at the new density \( n(\mu + \delta \mu) \). At equilibrium, that is the variation’s physical meaning. Because of perfect screening, the variational procedure remains well defined even when a current is exciting the sample, all the while connected to its large stabilizing leads. Perfect screening confines such a disturbance absolutely to the finite region \( \Omega \). Nothing else is touched.
The compressibility sum rule for an open mesoscopic system can now be formulated. At equilibrium, the rule states that the compressibility $\kappa$ for $N$ mobile particles in volume $\Omega$ is given by \[ \kappa = \frac{\Omega}{N^2} \frac{\partial N}{\partial \mu} = \frac{\Omega}{Nk_B T} \frac{\Delta N}{N}. \] (8)

Note especially that $\kappa$ is and must always remain independent of any transport parameter. This distinguishes Eq. (8) from the fluctuation-dissipation relation and Kubo formula.

The compressibility is the inverse of the stiffness (bulk modulus) of a many-particle system, which determines its sound velocity. Hence, customarily, $\kappa$ has been investigated through sound-velocity measurements. The pattern typical of the right-hand side of Eq. (2) is repeated here; in analogy with the FDT, it is determined by an integral (equation (7)) of the microscopic electron-hole pair correlation, whose static limit is $-\partial f^{eq}/\partial \mu$. For a careful exposition, see Ref. [1].

It is clear from Eqs. (3), (4) and (7) that, since all the quantities on the right-hand side of the compressibility sum rule are independent of $I$,

- the compressibility of an open mesoscopic conductor is strictly invariant under transport.

This is the principal result that we wish to recall. It articulates the conservation of particle number, just as perfect screening expresses the general gauge invariance of a conducting system.

5. APPLICATIONS

a. Violations of the compressibility sum rule

We are ready to revisit and judge some theoretical assumptions that have gained currency in recent mesoscopic research [9]. Though plausible at first blush, they turn out to vitiate the sum-rule structure of any transport model relying on them. In place of the gauge-invariant and microscopically canonical prescription of current-driven transport presented above, let us instead posit ad hoc that the current in a narrow mesoscopic conductor is sustained purely by a difference of chemical potentials between an upstream and a downstream electron reservoir [7],[9],[10].

Assume, for argument, that the density differential $n(\mu_{up}) - n(\mu_{dn})$ induced across the sample by the upstream and downstream chemical potentials, $\mu_{up}$ and $\mu_{dn}$, causes a diffusive-like current. Assume too that the electromotive force measured across the interfaces is $V = -(\mu_{dn} - \mu_{up})/e$ (and note that it makes no difference at all to the generality of the argument, whether $V$ is static or time-dependent; cf Eqs. (4) & (5)).

The density profile along the conductor will be some function $n(\mu_{up} - eV(x))$, taking the boundary values $n(\mu_{up})$ and $n(\mu_{dn})$ at the ends of the sample. According to such an account, in the linear-response limit the total number of carriers in the active structure of length $L$ changes, under transport, by an amount
\begin{align*}
N(V) - N(0) &= \int_0^L [n(\mu_{up} - eV(x)) - n(\mu_{up})] dx \\
&\rightarrow \int_0^L \frac{dn}{d\mu_{up}} (-eV(x)) dx \\
&\sim -\Delta N(0) \frac{eV}{2k_B T}.
\end{align*}

The result is patently counter to Eq. (4), and consequently also breaks the compressibility sum rule.

If the mooted pseudo-diffusive arrangement is not to violate the mandatory invariance of total particle number for any driven, necessarily neutral, mesoscopic structure, the nominal density difference \(n(\mu_{up}) - n(\mu_{dn})\) which causes the current (and so the current itself) cannot be accorded a standard physical meaning. This makes any conclusions drawn from such a picture hard – if not impossible – to interpret in a rational way.

The only escape from that unpalatable outcome is to allow effectively arbitrary inflows and outflows of charge between the device and its leads. In the event, the device cannot stay neutral under transport; there is \textit{always} an uncompensated excess (or deficit) of carriers. Such a dilution of the perfect screening sum rule immediately destroys gauge invariance \[6\]. In any case, one or more of the sum rules is countermanded. Transport arguments based on this scenario are unphysical.

At this stage we have addressed only a single-electron view of the compressibility within pseudo-diffusive transport. However, at the level of the electron-hole pair correlations, the sum-rule violation is far worse (if that is possible). To make the point we take a representative pseudo-diffusive fluctuation analysis, that of Martin and Landauer \[10\]. In their theory, the current-current correlation function is synthesized from the set of all possible quantum-transmission outcomes that involve a single electron scattering off the core region of a one-dimensional quantum point contact. Its (constant) transmission probability is taken to be \(T\), where \(0 \leq T \leq 1\). Applying Eqs. (2.6)–(2.15) of Ref. \[10\], the total one-electron “fluctuation”, say \(dN\), can be obtained. This determines every quantity of experimental interest. We apply it now to the compressibility within the model.

Before doing so, it is vital to distinguish \(dN\) from the microscopically prescribed number fluctuation

\[\delta N \sim (-\partial f^eq/\partial \mu)\delta \mu\]

appearing in the perfect-screening sum rule, Eq. (5). \(\delta N\) represents the \textit{intrinsically correlated electron-hole excitations}: the conservation laws that govern electron-hole pair processes means that these are irreducible two-body objects \[1\]. On the other hand, \(dN\) is strictly constructed as a sum of one-body quantities.

The model at hand, and its closely related cousins \[7\],\[9\] take the single-carrier \(dN\) – not the two-body \(\delta N\) – as the fundamental fluctuation. Therefore the true pair correlations, crucial to the fluctuation structure of the electron gas, are missing.
Here the tacit assumption is, instead, that a fermionic particle-hole pair is always reducible to two kinematically uncorrelated, quite independent, single-particle excitation factors. Such an assumption is entirely contrary to basic Fermi-liquid theory [1],[3].

Following construction of the single-electron dN after Ref. [10], we arrive at the mean-square expectation $\Delta N$ over the quantum point contact:

$$\Delta N \equiv \langle (dN)^2 - (dN)^2 \rangle = L \frac{n(\mu_{up})}{2\varepsilon_F} \left[ T^2 k_B T + T (1 - T) \frac{\mu_{up} - \mu_{dn}}{2} \coth \left( \frac{\mu_{up} - \mu_{dn}}{2k_B T} \right) \right]$$

$$= N \frac{k_B T}{2\varepsilon_F} \left[ T + \frac{T (1 - T)}{3} \left( \frac{eV}{2k_B T} \right)^2 + O \left( \frac{(eV/2k_B T)^4}{2} \right) \right], \quad (10)$$

where $\varepsilon_F$ is the Fermi energy at density $n(\mu_{up})$ in the uniform wire (the Martin-Landauer model works only in the degenerate-electron limit $\varepsilon_F \gg k_B T$). To leading order in the voltage, the theory duly predicts the transport-dependent ratio

$$\frac{\Delta N}{N} = \left[ \frac{\Delta N}{N} \right]_{CSR} \left[ T + \frac{T (1 - T)}{3} \left( \frac{eV}{2k_B T} \right)^2 \right], \quad (11)$$

in which

$$\left[ \frac{\Delta N}{N} \right]_{CSR} = \frac{k_B T}{2\varepsilon_F}$$

is the equilibrium ratio of the total mean-square number fluctuation to total carrier number as it appears in the transport-invariant compressibility sum rule, Eq. (8).

Equation (11) clearly contradicts the invariance of Eq. (8). Even at equilibrium, the total mean-square fluctuation $\Delta N$ in pseudo-diffusive theories depends on the transport parameter $T$. In the strong backscattering limit it will vanish with $T$.

The electronic compressibility cannot depend on any transport parameter. As we saw in the previous section, $\kappa$ is an equilibrium property completely insensitive to external sources of elastic scattering (such as potential barriers) which determine the transmission fraction $T$. Thus Eq. (11) fails to recover the physically required compressibility at equilibrium, much less away from it. Nor can one appeal to Coulomb effects here, since this spurious result is derived for noninteracting one-dimensional conduction electrons.

**b. Electronic compressibility in nonuniform quantum channels**

We turn our attention to some unexplored mesoscopic implications of the compressibility sum rule. When the conduction band of a quantum point contact, or similar device, differs from that of its bulk leads, there will be contact potentials at the interfaces. In Eq. (6) the mean-field potential $U_0(r)$, which vanishes when there is no band mismatch between device and leads, becomes a nontrivial function of the electron density. The complete variation of $f_k^{\rho_1}(r)$ is
\[
\frac{\delta f_k^{\text{eq}}(r)}{\delta \mu} = \left(1 - \frac{\delta U_0(r)}{\delta \mu}\right) \left. \frac{\partial f_k^{\text{eq}}(r)}{\partial \mu} \right|_{U_0},
\]
(12)

Since
\[
\frac{\delta n(r)}{\delta \mu} = \int \frac{d k}{(2\pi)^3} \frac{\delta f_k^{\text{eq}}(r)}{\delta \mu},
\]
the implicit Eq. (12) can be solved by integrating over \( k \) on both sides, and feeding back into it the leading right-hand factor in closed form. The result is
\[
\frac{\delta f_k^{\text{eq}}(r)}{\delta \mu} = \frac{1}{1 + \frac{d U_0}{d n} \frac{\partial f_k^{\text{eq}}(r)}{\partial \mu}}.
\]
(13)

An easy case to analyze is a uniform wire. Then \( U_0 \) is constant over most of the wire’s length, except in the neighborhood of the interfaces where the band mismatch forces \( U_0 \) to vary. The leading factor on the right-hand side of Eq. (13) is nearly constant everywhere. Denote it by \( \gamma \). It appears as an additional factor in the total compressibility for our inhomogeneous system:
\[
\kappa \equiv \frac{\Omega}{N^2} \frac{\delta N}{\delta \mu} = \frac{\Omega}{N k_B T} \frac{\gamma \Delta N}{N}.
\]
(14)

The self-screening response of the contact potential substantially modifies the compressibility that would otherwise be observed in the absence of band inhomogeneity. Such effects are particularly strong in III-V heterojunction quantum-well structures [3].

We state the corollary of the perfect-screening Eq. (5) in this situation. It is the nonequilibrium compressibility sum rule for inhomogeneous mesoscopic systems (see Ref. [3] for its mathematical grounding).

- The compressibility of a nonuniform mesoscopic conductor, driven at any current, is invariant and its form is given by Eq. (14).

This is a surprising result, for it asserts that, even with the large changes in the carrier distribution of a device subjected to high current, there is no change at all in its electronic compressibility. That quantity is fixed, once and for all, by the electrostatic response of the contact potential at equilibrium.

The Coulomb-induced suppression of compressibility correlates closely with the analogous suppression theoretically anticipated for high-field current noise in heterojunction based, quasi-two-dimensional wires [3]. The nonequilibrium excess noise of such a structure is accessible [11]. It could be systematically measured, and direct comparison made with studies of the static compressibility in the same sample.

5. SUMMARY

The main result in this work is the “rigidity” of the electronic compressibility in a mesoscopic wire, in the sense that it cannot respond in any way to the internal
Coulomb effects that alter the nonequilibrium arrangements inside the conductor. That rigidity, coming from the utter dominance of the boundary conditions in mesoscopic transport, is striking evidence of the power and reach of the conservation laws. Novel experiments in this context, especially the comparative examination of static and fluctuation responses, would therefore be of interest.

We recapitulate our findings. The conserving sum rules are universal constraints that apply to any system of mobile electrons. They express the basic conservation laws through a set of relations between, on the one hand, one-body properties that are measurable (conductance, particle number, compressibility) and, on the other hand, expectation values of microscopically calculable, two-body correlation functions.

Sum rules embody the unified origin of single- and many-particle behavior in the electron gas. We have shown that the principal sum rules for energy dissipation, charge neutrality and number conservation are core properties of a mesoscopic conductor. Their consequences are directly observable in the laboratory. A key quantity for experimental investigation is the compressibility.

The implications of the compressibility sum rule have not previously been studied in the mesoscopic setting. In sharp contrast with its crucial and long-established role in electron-gas physics [1], this property has not had any attention paid to it by more popularly accepted phenomenologies of mesoscopic transport and noise [7],[9],[10]. It is not so astonishing, then, to find that the compressibility sum rule is violated by such approaches.

Through the gauge invariance of the boundary conditions and their overriding influence on internal carrier dynamics, it may indeed be said that the conservation laws and their leading sum rules instill the very meaning of “mesoscopic” conduction. Therefore a microscopically credible description of mesoscopics must satisfy all these rules, not just a subset, else the description is faulty. Its reliability is void of any guarantee – notwithstanding any perceived empirical or cosmetic merits.

We appear to be enjoying a phase of development in mesoscopics wherein heuristically dominated theory-making is much to be preferred over labor-intensive, assiduously orthodox and microscopically correct analyses of electron transport. In such a climate, Sir Francis Bacon’s old warning against subjective confections of knowledge remains apt today:

God forbid that we should give out a dream of our imagination for a pattern of the world.

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