Secrecy-Capacity Bounds for Visible Light Communications With Signal-Dependent Noise

Jin-Yuan Wang, Member, IEEE, Peng-Fei Yu, Xian-Tao Fu, Jun-Bo Wang, Member, IEEE, Min Lin, Member, IEEE, Julian Cheng, Fellow, IEEE, and Mohamed-Slim Alouini, Fellow, IEEE

Abstract—In physical-layer security, secrecy capacity is an important performance metric. This work aims to determine the secrecy capacity for an indoor visible light communication system consisting of a transmitter, a legitimate receiver and an eavesdropping receiver. In such a system, both signal-independent noise and signal-dependent noise are considered. Under nonnegativity and average optical intensity constraints, lower and upper bounds on secrecy capacity are derived by the variational method, the dual expression of the secrecy capacity, and the concept of “the optimal input distribution that escapes to infinity”. By an asymptotic analysis at large optical intensity, there is a small gap between the asymptotic upper and lower bounds. Then, by adding a peak optical intensity constraint, we further analyze the exact and asymptotic secrecy-capacity bounds. For practical considerations, the effects of imperfect channel state information, multi-photodiode eavesdropper, and artificial noise on secrecy performance are also discussed. Finally, the derived secrecy-capacity bounds are verified by numerical results.

Index Terms—Physical-layer security, secrecy capacity, signal-dependent noise, signal-independent noise, visible light communications.

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Jin-Yuan Wang is with the Key Laboratory of Broadband Wireless Communication and Sensor Network Technology, Nanjing University of Posts and Telecommunications, Nanjing 210003, China, and also with the Chuan and Zang Smart Tourism Engineering Research Center of Colleges and Universities of Sichuan Province, Sichuan Tourism University, Chengdu 610100, China (e-mail: jywang@njupt.edu.cn).
Peng-Fei Yu, Xian-Tao Fu, and Min Lin are with the College of Communication and Information Engineering, Nanjing University of Posts and Telecommunications, Nanjing 210003, China (e-mail: 1020010321@njupt.edu.cn; 1019010206@njupt.edu.cn; linmin@njupt.edu.cn).
Jun-Bo Wang is with the National Mobile Communications Research Laboratory, Southeast University, Nanjing 21111, China (e-mail: jbwang@seu.edu.cn).
Julian Cheng is with the School of Engineering, The University of British Columbia, Kelowna, BC V1V 1V7, Canada (e-mail: julian.cheng@ubc.ca).
Mohamed-Slim Alouini is with the Computer, Electrical and Mathematical Science, and Engineering Division, King Abdullah University of Science and Technology, Thuwal 23955, Saudi Arabia (e-mail: slime.alouini@kaust.edu.sa). Color versions of one or more figures in this article are available at https://doi.org/10.1109/TWC.2023.3249222.
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I. INTRODUCTION

With the rapid development of the fifth generation wireless network, new transmission technologies have been developed to meet the demands of explosive growth in data traffic. One such new transmission technology is indoor visible light communication (VLC), which uses visible light generated by light-emitting diodes (LEDs) for communication and is considered as a promising solution for the pressing data traffic demands [1].

Different from conventional radio frequency wireless communications (RFWC), indoor VLC has several distinctive properties. First, the optical intensity in VLC is controlled to convey wireless information. Second, the indoor VLC performs communication and illumination simultaneously. The average optical intensity should not fluctuate with time to satisfy the indoor illumination requirement. Third, since the optical intensity cannot be negative, the input signal in indoor VLC should be nonnegative. Consequently, the developed theory and analysis in traditional RFWC are not directly applicable to indoor VLC.

For conventional RFWC, the performance is usually evaluated by the classic concept of “Shannon capacity” [2]. Unfortunately, such a concept is unsuitable for indoor VLC. What is the channel capacity of indoor VLC? Recently, the channel capacity of VLC was analyzed by the inverse source coding approach [3], but the derived expression was untractable. While closed-form expressions of channel-capacity bounds were derived [4], [5], the LED’s peak optical intensity, which reflects the LED’s maximum luminous ability, was not considered. With an additional peak constraint, channel-capacity bounds were derived [6], [7]. However, these bounds ignore the modulation schemes. Employing pulse amplitude modulation [8], orthogonal frequency division multiplexing [9], and color shift keying [10], the channel capacities were studied. With random receiver [11] and interference [12], realistic channel capacities were evaluated. All these works [3], [4], [5], [6], [7], [8], [9], [10], [11], [12] employ signal-independent noise. In practical VLC, common indoor environments desire high optical intensity [13], [14]. In this case, the noise strength relies on the input signal. A Poisson model that reflects the particle nature of optical signal was proposed [15], where the noise was modeled as a discrete Poisson variable using a signal-dependent rate. To better reflect the physical properties of optical communication, a more accurate Gaussian noise
model was proposed [16], where the variance of the Gaussian noise was assumed to be dependent on the input signal. Under such signal-dependent noise [16], we further analyzed channel-capacity bounds of VLC [17]. In short, all the existing bounds [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17] provide theoretical references for designing practical VLC systems having high data rates [18].

Compared with conventional RFWC, VLC not only has higher data rate, but also can provide better security because light cannot penetrate through walls. Despite better signal confinement, the VLC channels still have broadcast feature [19]. As a result, information security issue has become a major concern of network administrators. Conventional security schemes are generally performed via password protection, access controls, and end-to-end encryption. The safety of these schemes depends on the restricted storage capacity and computational power of eavesdroppers. As opposed to traditional network security, physical-layer security (PLS) has been proposed recently. The basic principle of PLS is to exploit the wireless channel characteristics to ensure the successful decoding of the legitimate receiver and prevent the eavesdropper from doing so. Similar to Shannon capacity, the PLS theory was also first developed by Shannon [20]. After that, the widely used concept “secrecy capacity” was proposed [21], and then the research was extended to many RFWC scenarios [22], [23], [24]. Although much work has been done, the derived PLS results in RFWC cannot be directly applied to VLC.

To determine the PLS performance of VLC, we derived tight secrecy capacity bounds [25]. However, the channel model considers only the specular reflection. With consideration of both specular and diffusive reflections, a ray tracing channel model was proposed [26]. The secrecy capacity analysis of VLC was also extended to the multiple-input single-output scenario [27] and the spatially random transceiver scenario [28]. Moreover, the PLS of VLC was comprehensively discussed [29]. In these works [25], [26], [27], [28], [29], the noise is signal-independent. Under signal-dependent noise, the optimal input distribution and the asymptotic secrecy capacities were discussed [30], [31]. However, the derived results depend on the assumption that the signal-dependent noise of the main channel and the eavesdropping channel are identical. Moreover, exact expressions of secrecy-capacity bounds have not yet been derived.

In this paper, we further analyze the secrecy capacity of a classic three-node indoor VLC system having signal-dependent noise. Without assuming identical signal-dependent noise variances at the legitimate receiver and the eavesdropper [30], [31], we consider arbitrary noise variances. The main contributions are summarized as follows:

1) We analyze secrecy-capacity bounds for VLC having a nonnegativity and an average optical intensity constraint. By the variational method, we first derive a lower bound on secrecy capacity. Applying the dual expression of secrecy capacity and the concept of “the optimal distribution that escapes to infinity”, we then provide a secrecy-capacity upper bound.

2) By adding an additional peak optical intensity constraint, we further investigate the secrecy-capacity bounds for the VLC. Based on similar approaches, we derive novel lower and upper bounds on secrecy capacity.

3) We analyze the asymptotic behaviors at high optical intensity. Based on the exact secrecy-capacity bounds, we derive asymptotic secrecy-capacity bounds. Theoretical analysis shows that a small gap exists between the asymptotic upper and lower bounds on secrecy capacity.

4) For practical considerations, we extend the derived secrecy-capacity results. We discuss the effects of imperfect channel state information (CSI), multi-photodiode (PD) eavesdropper, and artificial noise (AN) on security performance of VLC.

The reminder of this paper is organized as follows. Section II details the system model. Sections III and IV derive the secrecy-capacity bounds when considering different constraints. Section V discusses some extensions. Numerical results are provided in Section VI. Finally, conclusions are presented in Section VII.

Notation: In this paper, \( \mathcal{N}(\mu, \sigma^2) \) denotes a Gaussian distribution having mean \( \mu \) and variance \( \sigma^2 \); \( f_X(x) \) denotes the probability density function (PDF) of \( X \); \( E_X(\cdot) \) denotes the expectation about \( X \); \( \text{var}(\cdot) \) denotes the variance. We use \( I(\cdot;\cdot) \) for the mutual information; \( H(\cdot) \) for the entropy; \( D(\cdot|\cdot) \) for the relative entropy; \( E(e^x) \) for the exponential integral function; \( Q(\cdot) \) for the Gaussian-Q function; \( |\cdot| \) for the determinant of a matrix; and \( \text{diag}(\cdot) \) for the diagonal matrix.

II. SYSTEM MODEL

We consider an indoor VLC system consisting of a transmitter (Alice), a legitimate receiver (Bob) and an eavesdropper (Eve), as shown in Fig. 1. In such a system, Alice is equipped with a single LED to transmit optical intensity signals to Bob in the presence of Eve. Both Bob and Eve are equipped with one PD individually to perform the optical-to-electrical conversion. At Bob or Eve, the main noise includes signal-independent noise and signal-dependent noise [25]. The received signals at Bob and Eve can be expressed as

\[
\begin{align*}
Y_B &= H_B X + \sqrt{H_B} X Z_{B,1} + Z_{B,0} \\
Y_E &= H_E X + \sqrt{H_E} X Z_{E,1} + Z_{E,0}
\end{align*}
\]

(1)

where \( X \) is the transmit optical signal from Alice. \( H_k \) denotes the channel gains of the main channel (\( k = B \)) or the eavesdropping channel (\( k = E \)). \( Z_{k,0} \sim \mathcal{N}(0, \sigma^2_{k,0}) \) and \( Z_{k,1} \sim \mathcal{N}(0, \xi_k^2 \sigma^2_{k,0}) \) are the signal-independent noise and signal-dependent noise at Bob (\( k = B \)) or Eve (\( k = E \)), where the positive factor \( \xi_k^2 \) denotes the ratio of signal-dependent noise variance to signal-independent noise variance. We also assume \( Z_{B,0} \) and \( Z_{E,0} \) to be independent.

For indoor VLC, we consider the following three signal constraints [25]:

1) Nonnegativity: In (1), the input signal \( X \) is a nonnegative random variable representing the intensity of the optical signal. Therefore, we have

\[
X \geq 0.
\]

(2)
2) **Peak optical intensity constraint:** Because of the practical and safety restrictions, the input signal is constrained by a peak optical intensity, i.e.,

\[ X \leq A \]  

where \( A \) is the peak optical intensity of the LED at Alice.

3) **Average optical intensity constraint:** Due to the illumination requirement in VLC, the average optical intensity cannot fluctuate with time but can be adjusted according to the users’ requirement. Therefore, the average optical intensity constraint is given by

\[ E_X (X) = \xi P \]  

where \( \xi \in (0,1] \) is the dimming target, \( P \in (0, A] \) is LED's nominal optical intensity.

In VLC, the received optical intensity of the line-of-sight (LoS) path dominates that of the reflection paths, and thus we consider only the LoS path and ignore reflections from surrounding surfaces [32]. In (1), the LoS channel gain \( H_k \) (\( k = B \) or \( E \)) can be expressed as

\[ H_k = \begin{cases} \frac{(m+1)A}{2\pi D_k^2} g \cos^m (\varphi_k) \cos(\psi_k), & 0 \leq \psi \leq \Psi \\ 0, & \text{otherwise} \end{cases} \]  

(5)

where \( m \) is Lambertian emission order; \( A \) is the PD’s physical area; \( T_s \) and \( g \) are PD’s optical filter gain and concentrator gain; \( \Psi \) is the field of view of the PD; \( D_k, \varphi_k \) and \( \psi_k \) are the link distance, irradiance angle and incidence angle from Alice to Bob (\( k = B \)) or Eve (\( k = E \)).

III. **Secrecy Capacity for VLC Having Nonnegativity and Average Optical Intensity Constraints**

In this section, we analyze the secrecy capacity of the VLC system having nonnegativity in (2) and average optical intensity constraint in (4). Specifically, we will provide secrecy-capacity bounds and investigate the asymptotic behavior in the high optical intensity regime.

The secrecy capacity represents the maximum transmission rate at which the legitimate receiver can reliably decode the transmitted message, while the eavesdropper cannot infer information at any positive rate [33]. If the main channel is worse than the eavesdropping channel, then the secrecy capacity \( C_s \) is zero. Otherwise, we can derive the secrecy capacity \( C_s \) by solving the following optimization problem

\[
C_s = \max_{f_X(x)} [I (X; Y_B) - I (X; Y_E)]
\]

s.t. \[
\int_0^\infty f_X(x) \, dx = 1 \]

\[
E_X (X) = \int_0^\infty x f_X (x) \, dx = \xi P. \quad (6)
\]

There exists a unique discrete PDF \( f_X^* (x) \) [30] that achieves the secrecy capacity of problem (6). However, the exact expression of \( f_X^* (x) \) is still unknown. Consequently, it is challenging to obtain a closed-form expression of secrecy capacity for problem (6). Alternatively, we will derive lower and upper bounds on the secrecy capacity.

A. **Lower Bound on Secrecy Capacity**

In the optimization problem (6), by choosing an arbitrary input PDF \( f_X(x) \) satisfying constraints (2) and (4), we can obtain a lower bound on the secrecy capacity as

\[
C_s \geq [I (X; Y_B) - I (X; Y_E)] |_{f_X(x) \text{ satisfies } (2) \text{ and } (4)}
\]

\[
= \mathcal{H} (Y_B) - \mathcal{H} (Y_B | X) - \mathcal{H} (Y_E) + \mathcal{H} (Y_E | X). \quad (7)
\]

According to the received signal model (1), the conditional PDF \( f_{Y_k | X} (y | X) \) is given by

\[
f_{Y_k | X} (y | X) = \frac{\exp \left[ - \frac{(y-H_kX)^2}{2(1+H_kX\sigma_k^2)^2\sigma_k^2} \right]}{\sqrt{2\pi (1+H_kX\sigma_k^2)^2\sigma_k^2}}, \quad k = B \text{ or } E. \quad (8)
\]

Then, the conditional entropy \( \mathcal{H} (Y_k | X) \) in (7) is written as

\[
\mathcal{H} (Y_k | X) = \frac{1}{2} \ln (2\pi e \sigma_k^2) + \frac{1}{2} E_X \left[ \ln \left( 1 + H_k\sigma_k^2 X \right) \right]. \quad (9)
\]

Furthermore, \( \mathcal{H} (Y_E) \) is upper-bounded by the differential entropy of a Gaussian random variable having a variance \( \sigma (Y_E) \) [34], i.e.,

\[
\mathcal{H} (Y_E) \leq \frac{1}{2} \ln [2\pi e \sigma (Y_E)]. \quad (10)
\]

According to Proposition 11 in [35], we know that the output entropy is always larger than the input entropy (OELIE). As a result, \( \mathcal{H} (Y_B) \) can be lower-bounded by

\[
\mathcal{H} (Y_B) \geq \mathcal{H} (X) + f_{\text{low}} (H_B, \xi, P) \quad (11)
\]

where \( f_{\text{low}} (H_B, \xi, P) \) is given by [35]

\[
f_{\text{low}} (H_B, \xi, P) = \frac{1}{2} \ln \left( H_B^2 + \frac{2H_B \sigma_B^2 \sigma_B^2}{\xi P} \right) - H_B \xi P + \frac{\xi P}{2} \sigma_B^2 + \frac{\sigma_B^2}{2} \left( \frac{2}{\xi P} \right). \quad (12)
\]

Substituting (9), (10) and (11) into (7), we can rewrite the lower bound as

\[
C_s \geq \mathcal{H} (X) + f_{\text{low}} (H_B, \xi, P) + \frac{1}{2} \ln \left( \frac{\sigma_B^2}{\xi P} \right) + \frac{1}{2} E_X \left[ \ln \left( 1 + H_E \frac{\sigma_B^2}{\sigma_B^2} \right) X \right] - \frac{1}{2} \ln [2\pi e \sigma (Y_E)]. \quad (13)
\]
As can be seen from (13), we can obtain a secrecy-capacity lower bound by choosing an arbitrary input PDF satisfying the constraints in problem (6). However, we should choose a good PDF to obtain a tight lower bound. By using the variational method, we derive the following theorem.

**Theorem 1:** For indoor VLC having constraints (2) and (4), we derive a lower bound on the secrecy capacity as

\[
C_{\text{Low}} = \frac{1}{2} \ln \left[ \frac{e^{\xi^2 P^2 \sigma_k^2}}{2\pi \sigma_k^2 (H^2 \xi^2 P^2 + \sigma_k^2)} \right] + f_{\text{low}}(H_B; \xi, P) - e^{\xi^2 P^2 \sigma_k^2} e^{-\xi^2 P^2 \sigma_k^2} e^{H_B \xi^2 P^2 + \sigma_k^2} \right].
\]

\[\text{Corollary 1:} \quad \text{In Theorem 1, when we ignore the signal-dependent noise (i.e., } \xi_B \to 0 \text{ and } \sigma_k \to 0), \text{ the secrecy-capacity lower bound in (14) reduces to}
\]

\[
\lim_{\xi_B \to 0, \sigma_k \to 0} C_{\text{Low}} = \frac{1}{2} \ln \left[ \frac{e^{H^2 \xi^2 P^2 \sigma_k^2}}{2\pi \sigma_k^2 (H^2 \xi^2 P^2 + \sigma_k^2)} \right].
\]

**Proof:** Its proof is straightforward and so is omitted.

**Remark 1:** For VLC having only signal-independent noise, we derive the lower bound (15) based on the principle of OELIE, while a previous lower bound [25, eq. (8)] was derived by the entropy power inequality (EPI). It is shown that the lower bound (15) is smaller than the previous lower bound [25, eq. (8)]. This finding suggests that the EPI is more efficient than the OELIE when considering only the signal-independent noise. Unfortunately, the EPI approach is inapplicable to the PLS analysis of VLC having signal-dependent noise.

**B. Upper Bound on Secrecy Capacity**

In this subsection, the derivation of the upper bound is based on the dual expression of the secrecy capacity. For an arbitrary \( g_{Y_B|Y_E}(y_B|y_E) \), the following inequality holds [25]

\[
I(X; Y_B|Y_E) \leq \mathbb{E}_{XY_E} \left[ D \left( f_{Y_B|XY_E}(y_B|X,Y_E) \mid g_{Y_B|Y_E}(y_B|Y_E) \right) \right]
\]

where \( D \left( f_{Y_B|XY_E}(y_B|X,Y_E) \mid g_{Y_B|Y_E}(y_B|Y_E) \right) \) denotes the relative entropy, and it is given by

\[
D \left( f_{Y_B|XY_E}(y_B|X,Y_E) \mid g_{Y_B|Y_E}(y_B|Y_E) \right) = \int f_{Y_B|XY_E}(y_B|X,Y_E) \ln \frac{f_{Y_B|XY_E}(y_B|X,Y_E)}{g_{Y_B|Y_E}(y_B|Y_E)} dy_B.
\]

In the inequality (16), selecting any \( g_{Y_B|Y_E}(y_B|Y_E) \) will result in an upper bound of \( I(X; Y_B|Y_E) \). To obtain a good upper bound, we have

\[
I(X; Y_B|Y_E) = \min_{g_{Y_B|Y_E}(y_B|Y_E)} \mathbb{E}_{XY_E} \left[ D \left( f_{Y_B|XY_E}(y_B|X,Y_E) \mid g_{Y_B|Y_E}(y_B|Y_E) \right) \right].
\]

From problem (6), the secrecy capacity can be rewritten as

\[
C_s = \max_{f_X(x)} I(X; Y_B|Y_E).
\]

In (19), there exists a unique input PDF \( f_X(x) \) that maximizes \( I(X; Y_B|Y_E) \) subject to the constraints in problem (6). Therefore, we have the dual expression of secrecy capacity as

\[
C_s = \min_{g_{Y_B|Y_E}(y_B|Y_E)} E_{X,Y_E} \left[ D \left( f_{Y_B|XY_E}(y_B|X,Y_E) \mid g_{Y_B|Y_E}(y_B|Y_E) \right) \right]
\]

\[\text{Corollary 2:} \quad \text{In Theorem 2, when we ignore the signal-dependent noise (i.e., } \xi_B \to 0 \text{ and } \sigma_k \to 0), \text{ the secrecy-capacity upper bound (21) reduces to}
\]

\[
\lim_{\xi_B \to 0, \sigma_k \to 0} C_{\text{Up}} = \frac{1}{2} \ln \left( \frac{4e^{H^2 \xi^2 P^2 \sigma_k^2}}{\pi e^{H^2 \xi^2 P^2 \sigma_k^2}} + \frac{2e^{H^2 \xi^2 P^2 \sigma_k^2}}{M e^{H^2 \xi^2 P^2 \sigma_k^2}} \right),
\]

where \( M = H^2 \xi^2 P^2 \sigma_k^2 / H_B + H^2 \xi^2 P^2 \sigma_k^2 \).

**Proof:** Its proof is straightforward and so is omitted.

**Remark 2:** Under constraints (2) and (4), a secrecy-capacity upper bound [25, eq. 16] for VLC having only signal-independent noise was derived. As can be seen, the derived upper bound (22) in Corollary 2 is the same as the previous bound [25, eq. 16], which is a special case of Theorem 2.

**C. Asymptotic Behavior Analysis**

For indoor VLC, we are more interested in the secrecy performance in the high optical intensity regime [17]. Therefore, we let the nominal optical intensity \( P \) approach infinity. According to Theorem 1 and Theorem 2, the asymptotic performance is derived in the following corollary.

**Corollary 3:** For indoor VLC having constraints (2) and (4), we derive asymptotic lower and upper bounds on secrecy capacity as

\[
\begin{align*}
\lim_{P \to \infty} C_{\text{Low}} &= \frac{1}{2} \ln \left( \frac{e^{H^2 \xi^2 P^2 \sigma_k^2}}{2\pi e^{H^2 \xi^2 P^2 \sigma_k^2}} \right) \times \\
\lim_{P \to \infty} C_{\text{Up}} &= \frac{1}{2} \ln \left( \frac{4e^{H^2 \xi^2 P^2 \sigma_k^2}}{\pi e^{H^2 \xi^2 P^2 \sigma_k^2}} \right). 
\end{align*}
\]
Proof: See Appendix C.

Remark 3: From Corollary 3, both the asymptotic lower and upper bounds do not scale with P but converge to real and positive constants. Moreover, the gap between the asymptotic upper and lower bounds is $C_{\text{gap}} = \frac{1}{2} \ln \left( \frac{2}{\pi} \right) \approx 0.4674 \text{nat/ transmission}$, which is small.

IV. Secrecy Capacity for VLC Having Nonnegativity, Average and Peak Optical Intensity Constraints

In this section, we further derive exact and asymptotic secrecy-capacity bounds for the VLC system when an additional peak optical intensity constraint is imposed on the channel input. By considering constraints (2), (3) and (4), we can obtain the secrecy capacity as

$$C_s = \max_{f_X(x)} \left[ I(X; Y_B) - I(X; Y_E) \right]$$

s.t. \( \int_0^A f_X(x) \, dx = 1 \)

$$\int_0^A x f_X(x) \, dx = \xi P. \quad \text{(24)}$$

Similar to problem (6), it is challenging to obtain the exact secrecy capacity expression for problem (24). We will derive secrecy-capacity bounds in the following two subsections.

A. Lower Bound on Secrecy Capacity

By choosing an arbitrary input PDF in problem (24) satisfying constraints (2), (3) and (4), we can obtain a lower bound on the secrecy capacity as

$$C_s \geq \mathcal{H}(Y_B) - \mathcal{H}(Y_B | X) - \mathcal{H}(Y_E | X) + \mathcal{H}(Y_E | X). \quad \text{(25)}$$

In this case, the secrecy-capacity lower bound (13) also holds. Define the average to peak optical intensity ratio as $\alpha = (\xi P/A) \text{, we derive a lower bound in the following theorem.}$

Theorem 3: For VLC having constraints (2), (3) and (4), we derive a lower bound on the secrecy capacity as

$$C_{\text{Low}}' = \begin{cases} C_1, & \text{if } \alpha = 0.5 \\ C_2, & \text{if } \alpha \neq 0.5 \end{cases} \quad \text{(26)}$$

where $C_1$ and $C_2$ are defined as

$$C_1 = f_{\text{low}}(H_B, \xi, P) + \frac{1}{2} \ln \left( \frac{1 + H_{E_S}^2 A}{1 + H_{B_{S}}^2 A} \right)$$

$$+ \frac{1}{2} \ln \left( \frac{6A^2 \sigma_E^2}{\pi e \sigma_B^2 (H_B^2 A^2 + 6AH_{E_S}^2 \sigma_E^2 + 12\sigma_E^2)} \right)$$

$$- \ln \frac{1 + H_{B_{S}}^2 A}{2AH_{B_{S}}^2} + \ln \frac{1 + H_{E_S}^2 A}{2AH_{E_S}^2} \quad \text{(27)}$$

and

$$C_2 = f_{\text{low}}(H_B, \xi, P) - \xi \xi P + \frac{1}{2} \ln \left( \frac{\sigma_B^2 (e^{\xi P} - 1)}{e^2} \right)$$

$$+ \frac{1}{2} \left\{ \ln \left( \frac{1 + H_{E_S}^2 A}{1 + H_{B_{S}}^2 A} e^{\xi P} \right) \right\}$$

$$- e^{-H_{B_S}^2} \left[ Ei \left( \frac{c}{H_{E_S}^2} (1 + H_{E_S}^2 A) \right) - Ei \left( \frac{c}{H_B^2} \right) \right]$$

$$+ e^{-H_{B_S}^2} \left[ Ei \left( \frac{c}{H_{B_S}^2} (1 + H_{B_S}^2 A) \right) - Ei \left( \frac{c}{H_B^2} \right) \right]$$

$$- \frac{1}{2} \ln \left[ 2\pi e \left( H_E^2 \left( \frac{A (cA - 2)}{c (1 - e^{cA})} + \frac{2}{c^2} - \xi^2 P^2 \right) + H_E \xi P_{E_S}^2 \sigma_E^2 + \sigma_E^2 \right) \right] \quad \text{(28)}$$

where $c$ in (28) is the solution to the following equation

$$\alpha = \frac{1}{1 - e^{-cA}} - \frac{1}{ca}. \quad \text{(29)}$$

Proof: See Appendix D.

From Appendix D, the maxentropic input PDF is used to obtain the secrecy-capacity lower bound in Theorem 3. For different $\alpha$ values, the maxentropic input PDFs are different. The properties of such PDFs are provided in Theorem 4.

Theorem 4: If $\alpha = 0.5$, then the maxentropic input PDF (D.1) is uniformly distributed in $[0, A]$; if $\alpha \in (0.5, 1)$, then $c > 0$ in (29), and the maxentropic input PDF (D.3) is a monotonically increasing function of $x \in (0, A]$; if $\alpha \in (0, 0.5)$, then $c < 0$ in (29), and the maxentropic input PDF (D.3) is a monotonically decreasing function of $x \in (0, A]$. Moreover, the curves of maxentropic input PDF (D.3) with $\alpha$ and $1 - \alpha$ are symmetric with respect to $X = A/2$.

Proof: See Appendix E.

Corollary 4: In Theorem 3, when we ignore the signal-dependent noise (i.e., $\sigma_B \rightarrow 0$ and $\sigma_E \rightarrow 0$), the secrecy-capacity lower bound (26) reduces to

$$\lim_{\sigma_B \rightarrow 0, \sigma_E \rightarrow 0} C'_{\text{Low}} = \begin{cases} \frac{1}{2} \ln \left( \frac{3H_B^2 \sigma_E^2 A^2}{2 \pi e H_B^2 (\xi^2 P^2 + \sigma_E^2)} \right), & \text{if } \alpha = 0.5 \\ \frac{1}{2} \ln \left( \frac{\sigma_B^2 A^2 e^{-\xi P}}{2 \pi e H_B^2 (\xi^2 P^2 + \sigma_E^2)} \right), & \text{if } \alpha \neq 0.5 \end{cases} \quad \text{(30)}$$

Proof: According to Theorem 3, the proof of Corollary 4 is straightforward.

Remark 4: Under constraints (2)-(4), a secrecy-capacity lower bound (20) for VLC having signal-independent noise was derived [25]. The lower bound (30) in Corollary 4 using the OELIE approach is smaller than the lower bound using the EPI approach [25, eq. (20)], indicating that the EPI approach is more efficient for PLS analysis of VLC with signal-independent noise.

B. Upper Bound on Secrecy Capacity

When adding an additional peak optical intensity constraint, the secrecy capacity’s dual expression (20) also holds. According to (20) and Theorem 2, we have the following theorem.
Theorem 5: For VLC having constraints (2), (3) and (4), we derive an upper bound on secrecy capacity as
\[
C_{Upp}' = \frac{1}{2} \ln \left( \frac{H^2}{\sigma^2_E} \left( H_B A + \frac{\varepsilon}{N} \sigma^2_B \right) \right) - \frac{1}{2} \ln \left( \frac{H^2}{\sigma^2_E} \left( \frac{H_B A + \frac{\varepsilon}{N} \sigma^2_B}{H^2 B + H^2 E \sigma^2_E} \right) \right)
\] (31)
where \( M = \frac{H^2}{\sigma^2_E} \sigma^2_B / H_B + H^2 E \sigma^2_E \).

Proof: See Appendix F.

Corollary 5: In Theorem 5, when we ignore the signal-dependent noise (i.e., \( \sigma_B \to 0 \) and \( \xi \to 0 \)), the secrecy-capacity upper bound (31) reduces to
\[
\lim_{\xi, \sigma_B \to 0} C_{Upp}' = \frac{1}{2} \ln \left( \frac{H^2}{\sigma^2_E} \left( \frac{H_B A \xi + \frac{\varepsilon}{N} \sigma^2_B \xi}{H_B A \xi + \frac{\varepsilon}{N} \sigma^2_B \xi + \frac{\varepsilon}{N} \sigma^2_B} \right) \right) \] (32)

Remark 5: Under constraints (2)-(4), a secrecy-capacity upper bound (26) for VLC without signal-dependent noise was derived [25]. The derived upper bound (32) in Corollary 5 is the same as a previous upper bound [25, eq. (26)]. This also indicates that the upper bound [25, eq. (26)] is just a special case of Theorem 5.

C. Asymptotic Behavior Analysis

According to Theorem 3 and Theorem 5, when \( A \) tends to infinity, we can obtain the asymptotic secrecy-capacity bounds.

Corollary 6: For VLC having constraints (2), (3) and (4), we derive asymptotic lower and upper bounds on secrecy capacity as (33) shown at the bottom of the next page.

Proof: See Appendix G.

Remark 6: In Corollary 6, when \( \alpha = 0.5 \), the gap between the asymptotic upper and lower bounds is \( C_{gap}' = \frac{1}{2} \ln \left( \frac{\pi e}{\varepsilon} \right) \approx 0.1765 \) nat/transmission, which is small. When \( \alpha \neq 0.5 \), the asymptotic gap can be evaluated by numerical approach.

V. Extensions and Practical Considerations

In this section, we will discuss some extension results by considering practical scenarios, such as imperfect CSI, multi-PD eavesdropper, and AN.

A. Imperfect CSI

In the aforementioned sections, perfect CSI is employed. However, perfect CSI may not be realistic. To show the impacts of imperfect CSI, we consider an ellipsoidal-approximation-based bounded channel error model [36], i.e.,
\[
H_k = \hat{H}_k 10^{\frac{\mu_k}{10}}, \mu_k \in [-\varepsilon, \varepsilon], k \in \{B, E\}
\] (34)
where \( \hat{H}_k \) and \( H_k \) are the estimated channel gain and the actual channel gain, \( \mu_k \) is the uncertain parameter, and \( \varepsilon \) is the nonnegative uncertainty bound. By substituting (34) into Theorems 1, 2, 3, and 5, we can obtain the secrecy-capacity bounds with imperfect CSI.

B. Multi-PD Eavesdropper

In (1), we consider a single-PD eavesdropper (Eve). However, from a security perspective, it is better to assume that Eve has multiple PDs. Let the number of PDs at Eve be \( N \), the received signal vector \( Y_E \in \mathbb{R}^{N \times 1} \) at Eve becomes
\[
Y_E = H_E X + \sqrt{\text{diag}(H_E)^T} Z_{E,1} + Z_{E,0}
\] (35)
where \( H_E = [\theta H_{E,1,1}, H_{E,1,2}, \cdots, H_{E,N}]^T \in \mathbb{R}^{N \times 1} \) denotes the channel gain vector of the eavesdropping channel, \( Z_{E,0} \sim \mathcal{N}(0, \sigma^2_{E,0} I_N) \) and \( Z_{E,1} \sim \mathcal{N}(0, \sigma^2_{E,1} I_N) \) are the channel-independent noise vector and channel-dependent noise vector.

Due to page limitation, we only extend the result in Theorem 1 by considering the multi-PD Eve. In this case, \( \mathcal{H}(Y_B) \) and \( \mathcal{H}(Y_B|X) \) are the same as that in Theorem 1. Moreover, \( \mathcal{H}(Y_E) \) and \( \mathcal{H}(Y_E|X) \) become
\[
\begin{align*}
\mathcal{H}(Y_E) & \leq \frac{1}{2} \ln \left( \frac{(2\pi e)^N \cdot e^{\xi^2 P^2 H_E^T H_E^T}}{\sigma^2_E} \right) \\
+ \xi P \text{diag}(H_E)^T \sigma^2_E I_N + \xi P \text{diag}(H_E)^T \sigma^2_E I_N \right) \right) \\
- \frac{1}{2} \sum_{i=1}^{N} e^{\frac{\mu_E}{\sqrt{2} \varepsilon P}} E_i \left( \frac{1}{H_E, i} \right). \tag{37}
\end{align*}
\]
Therefore, we can derive the secrecy-capacity lower bound as
\[
C_{Low, MPD} = f_{low} \left( H_B, \xi, P \right) + \frac{1}{2} e^{\frac{1}{\sqrt{2} \varepsilon P}} E_i \left( \frac{1}{H_B, \xi, \sigma^2_E} P \right)
\]

C. AN

To further improve PLS, a jammer is added to the original system model to transmit the AN \( W \). Assume that the jammer only induces interference to Eve but not to Bob. In this case, the received signal at Eve in (1) becomes
\[
Y_E = H_E X + \sqrt{H_E X Z_{E,1}} + H J W + Z_{E,0}
\] (38)
where \( J \) is channel gain between jammer and Eve. Assume that \( W \) follows a uniform distribution, i.e., \( f_W(x) = 1/P_3, x \in [0, P_3] \), where \( P_3 \) is the jammer’s peak optical intensity.

Due to page limitation, we only extend the result in Theorem 1 by considering the AN. In this case, \( \mathcal{H}(Y_B) \) and \( \mathcal{H}(Y_B|X) \) are the same as that in the system without AN. However, \( \mathcal{H}(Y_E) \) and \( \mathcal{H}(Y_E|X) \) become
\[
\begin{align*}
\mathcal{H}(Y_E) & \leq \frac{1}{2} \ln \left( \frac{2\pi e H_E^2 \xi^2 P^2 + H_E \xi P \sigma^2_E + H_E^2 P^2 - \pi e \sigma^2_E}{2 \sigma^2_E} \right) \\
\mathcal{H}(Y_E|X) & \geq \frac{1}{2} \ln \left( H_E^2 P^2 + 2\pi e \sigma^2_E \right) \\
- \frac{1}{2} e^{\frac{1}{\sqrt{2} \varepsilon P}} E_i \left( \frac{1}{H_E, \xi, \sigma^2_E} P \right). \tag{39}
\end{align*}
\]
Then, the secrecy-capacity lower bound can be written as

\[ C_{\text{Low,AN}} = f_{\text{low}} (H_B, \xi, P) + \frac{1}{2} e^{\frac{1}{2}} \left( -\frac{1}{H_B \xi} \right) \left( \frac{3\xi^2 \sigma_B^2 (H_B^2 P^2 + 2\pi e \sigma_B^2)}{\pi^2 \sigma_B^2 (12H_B^2 \xi^2 P^2 + 12H_B \xi \sigma_B^2 + H_B^2 P^2 + 12\sigma_B^2)} \right) \]

\[ = \frac{1}{2} \ln \left( \frac{6H_B \xi \sigma_B^2}{\pi e \sigma_B^2} \right) \left( -\frac{1}{H_B \xi} \right) \]

When the AN is not considered, eq. (40) reduces to (14).

VI. NUMERICAL RESULTS

In this section, numerical results are provided. The accuracy of the derived theoretical bounds is verified. Unless stated otherwise, we set \( \sigma_B^2 = \sigma_E^2 = 1 \) and \( \xi = 1.5 \) [37].

A. Results of VLC Having Nonnegativity and Average Optical Intensity Constraints

Fig. 2 shows the secrecy-capacity bounds versus the nominal optical intensity \( P \) with different \( H_B/H_E \) when \( \xi = 0.3 \). At low optical intensity, the secrecy-capacity bounds approach zero, and the PLS performance is bad. However, at high optical intensity, all bounds increase rapidly first and then tend to stable values with \( P \). Moreover, for a fixed \( P \), the secrecy-capacity bounds increase with \( H_B/H_E \). This indicates that the larger the difference between \( H_B \) and \( H_E \) is, the PLS performance becomes better. Quantitatively, all performance gaps at high optical intensity are about 0.4674 nat/transmission. This conclusion coincides with that in Remark 3.

Fig. 3 plots the secrecy-capacity bounds versus the dimming target \( \xi \) with different \( P \) when \( H_B/H_E = 1000 \). As can be seen, all secrecy-capacity bounds are monotonically non-decreasing functions of \( \xi \). At small optical intensity (e.g., \( P = 20 \) dB), the secrecy-capacity bounds increase rapidly with \( \xi \). However, at large optical intensity (e.g., \( P = 40 \) dB and \( P = 60 \) dB), the secrecy-capacity bounds increase first and then tend to constant values. This indicates that large dimming target has a strong impact on PLS performance when \( P \) is small, but has a weak impact when \( P \) is large. Moreover, the capacity bounds when \( P = 40 \) dB are almost the same as that when \( P = 60 \) dB. This indicates that for fixed \( H_B/H_E \) and \( \xi \) values, enlarging the optical intensity cannot enhance the secrecy performance of VLC without limitation at all.

Fig. 4 shows the effects of noise and eavesdropper on capacity performance when \( \xi = 0.3 \) and \( H_B/H_E = 1000 \). In this figure, the secrecy-capacity bounds and channel-capacity bounds for VLCS with different noise scenarios are provided. As can be observed, all secrecy-capacity bounds increase and then tend to stable values as the increase of \( P \). However, all channel-capacity bounds monotonously increase with \( P \). For a fixed type of noise, the channel-capacity bounds always larger than the secrecy-capacity bounds. This indicates that the existence of the eavesdropper degrades the information transmission ability. Compared with the signal-independent noise, the signal-dependent noise decreases the channel capacity and secrecy capacity of VLC.

\[
\lim_{A \to \infty} C'_{\text{Low}} = \begin{cases} 
\frac{1}{2} \ln \left( \frac{6H_B \xi \sigma_B^2}{\pi e \sigma_B^2} \right), & \text{if } \alpha = 0.5 \\
\frac{1}{2} \ln \left( \frac{6H_B \xi \sigma_B^2}{\pi e \sigma_B^2} \right), & \text{if } \alpha = 0.5
\end{cases}
\]

\[
\lim_{A \to \infty} C'_{\text{Upp}} = \frac{1}{2} \ln \left( \frac{6H_B \xi \sigma_B^2}{\pi e \sigma_B^2} \right).
\]
Fig. 5. Secrecy-capacity upper bound (21) versus Eve’s position when $\xi = 0.3$ and $H_{B}/H_{E} = 1000$.

Moreover, we consider a VLC system within a $10 \times 10 \times 3$ m room, and set the coordinates of Alice, Bob and Eve as $(a, b, c)$, $(d, e, f)$, and $(x, y, f)$. To show the secrecy levels by moving Eve, Fig. 5 shows secrecy-capacity upper bound (21) versus Eve’s position when $\xi = 0.3$, $P = 50$ dB, and $(a, b, c) = (3\ m, 3\ m, 3\ m)$. In Fig. 5(a), when Bob is placed underneath Alice, secure transmission can always be guaranteed. Moreover, a larger the horizontal distance between Eve and Bob can improve the PLS performance. In Fig. 5(b), when Bob is not located underneath Alice, the region with zero secrecy capacity enlarges. Such a region is insecure for data transmission. To guarantee secure transmission, system designers should prevent Eve from entering this region by using various detecting devices or make Bob closer to Alice.

B. Results of VLC Having Nonnegativity, Average and Peak Optical Intensity Constraints

To verify Theorem 4, Fig. 6 shows the maxentropic input PDFs for different $\alpha$ values when $A = 10^6$ W. As can be seen, the PDF curve when $\alpha = 0.5$ does not vary with $X$. In other words, the input signal follows a uniform distribution in $[0, A]$, i.e., (D.1). When $\alpha < 0.5$, the PDF is a monotonically decreasing function of $X$. When $\alpha > 0.5$, the PDF becomes a monotonically increasing function of $X$. Moreover, it can be observed that the PDF curves with $\alpha$ and $1 - \alpha$ are symmetric with respect to $X = A/2$. Therefore, all conclusions in Theorem 4 hold.

Figs. 7(a) and (b) plot the secrecy-capacity bounds versus $A$ with different $H_{B}/H_{E}$ when $\alpha = 0.2$ and 0.5. Similar to Fig. 2, the secrecy-capacity bounds increase with $H_{B}/H_{E}$ for a fixed $A$. This indicates that the larger the difference between the main channel and eavesdropping channel is, the better the secrecy performance becomes. Moreover, when $A$ is small, all bounds are almost zero. As the increase of $A$, the secrecy-capacity bounds increase rapidly. When $A$ is large, all secrecy-capacity bounds tend to constants, which indicates that the secrecy-capacity bounds are not affected by $A$ at large optical intensity. Moreover, when $\alpha = 0.2$, the asymptotic performance gaps for different $H_{B}/H_{E}$ are 0.36 nat/transmission, which is small. When $\alpha = 0.5$, the gaps are 0.1765 nat/transmission, which agrees with Remark 6.

Fig. 8 plots the secrecy-capacity bounds versus $A$ for different VLC scenarios when $\xi = 0.3$, $A/P = 1$, and $H_{B}/H_{E} = 1000$. Similar to Fig. 4, the secrecy-capacity bounds and channel-capacity bounds for VLCs with different noise scenarios are also provided here. As can be observed, the trends of secrecy-capacity bounds and channel-capacity bounds are different. Specifically, no matter the noise is signal-dependent or signal-independent, the channel-capacity bounds in [6] and [17] increase approximately linearly with $A$. However, as the increase of $A$, the secrecy-capacity bounds in this paper and [25] increase first and then tend to stable values. Moreover, the signal-dependent noise degrades the channel capacity or secrecy capacity. Furthermore, at large $A$, the channel-capacity bounds are always larger than the secrecy-capacity bounds, this is because the system performance degrades due to the wiretap of Eve.

C. Extension Results

To show the effect of imperfect CSI, Fig. 9 plots the secrecy-capacity bounds versus $A$ for different channel uncertainty bounds $\varepsilon$ when $\xi = 0.5$, $A/P = 1$, and $H_{B}/H_{E} = 1000$. As can be observed, secrecy-capacity bounds decrease with the uncertainty bound $\varepsilon$. This indicates that imperfect CSI has a strong effect on system secrecy performance. Specifically, the system with perfect CSI (i.e., $\varepsilon = 0$) achieves the best secrecy performance. When the uncertainty bound $\varepsilon$ enlarges, the secrecy performance degrades.
To show the effect of the multi-PD eavesdropper, Fig. 10 shows the secrecy-capacity lower bound (37) versus the number of PDs at Eve $N$ with different $H_{B}/H_E$ when $\xi = 0.3$ and $P = 60$ dB. As can be seen, all curves of the secrecy-capacity lower bound rapidly decrease with $N$, suggesting that the PLS performance degrades with the number of PDs at Eve.

When $N$ is sufficiently large, all curves tend to zero. In this case, secure transmission cannot be guaranteed, and novel transmission scheme should be developed to combat this adverse effect.

To show the effect of AN, Fig. 11 plots the secrecy-capacity lower bound (40) versus $P_J$ for different $H_I/H_E$ when $P = 40$ dB, $H_{B}/H_E = 1000$, and $\xi = 0.3$. As can be observed, the secrecy-capacity lower bound for the system without AN is independent of $P_J$. When considering AN, the secrecy-capacity lower bound increases with $P_J$, which indicates that a stronger AN can improve the secrecy performance. Moreover, with the increase of $H_I/H_E$, the secrecy-capacity lower bound also increases. This suggests that better channel conditions of the jammer can also significantly improve secrecy performance.

To show the effect of the signal-dependent noise, Fig. 12 plots the secrecy capacity bounds versus the factor $\varsigma^2 = \varsigma^2_B = \varsigma^2_E$ when $H_{B}/H_E = 1000$, $\xi = 0.5$, and $A = P = 60$ dB. The secrecy-capacity results for the signal-independent noise case (i.e., $\varsigma^2 = 0$) and signal-dependent noise case (i.e., $\varsigma^2 > 0$) are obtained according to [25] and this paper.
respectively. As can be seen, the secrecy-capacity bounds for the signal-independent noise case are always much larger than that for the signal-dependent noise case, which indicates that the signal-dependent noise significantly degrades the PLS performance of VLC. Moreover, for the signal-dependent noise case (i.e., \( \varsigma^2 > 0 \)), the secrecy-capacity bounds only slightly decrease with \( \varsigma^2 \). This indicates that the specific positive value of \( \varsigma^2 \) has little effect on the secrecy-capacity performance.

VII. CONCLUSION

We have derived the secrecy-capacity bounds for VLC having signal-dependent noise. Numerical results have shown that the signal-dependent noise has a strong impact on secrecy capacity, thereby degrading the PLS performance. We have derived asymptotic secrecy-capacity bounds. We have observed that these asymptotic bounds do not increase with \( P \) or \( A \), but keep stable values. Such an observation of secrecy capacity is different from that of channel capacity. We have also discussed the effects of imperfect CSI, multi-PD eavesdropper and AN. It is shown that increasing the uncertainty bound, decreasing the number of PDs at Eve, or increasing the jammer’s optical intensity can improve the PLS performance.

In this study, the derived secrecy-capacity bounds provide fundamental performance limits for VLC, which can be employed to rapidly evaluate the PLS performance without time-consuming simulations. These bounds show how key parameters affect the PLS performance, thus providing some important engineering insights for real system design. These bounds are also useful for parameter optimizations to further improve the PLS performance. After deriving these bounds, our future work will focus on establishing experimental platforms to validate the derived results.

APPENDIX A
PROOF OF Theorem 1

In (13), we obtain a lower bound by maximizing the source entropy under constraints (2) and (4), i.e.,

\[
\max_{f_X(x)} \mathcal{H}(X) = -\int_0^\infty f_X(x) \ln[f_X(x)] \, dx
\]

s.t. \( \int_0^\infty f_X(x) \, dx = 1 \)

\( \int_0^\infty x f_X(x) \, dx = \xi P. \) \( \quad (A.1) \)

By the variational method, we obtain the input PDF as

\[
f_X(x) = \frac{1}{\xi P} e^{-\frac{\varsigma^2 x}{\xi P}}, x \geq 0.
\]

Then, we obtain

\[
\begin{align*}
\mathcal{H}(X) = & \ln(\xi P) \\
E_X(X) = & \xi P \\
\var(X) = & \xi^2 P^2
\end{align*}
\]

\( \quad (A.3) \)

\[
\begin{align*}
E_{X,Z_E,1}(\sqrt{X}Z_{E,1}) = & E_X(\sqrt{X})E_{Z_E,1}(Z_{E,1}) = 0 \\
\var(Y_E) = & H_{E|B}^2 P^2 + H_{E|B}^2 \varsigma^2 + \sigma_E^2.
\end{align*}
\]

Moreover, \( E_X \left[ \ln \frac{1 + H_{E|B}^2 X}{1 + H_{E|B}^2} \right] \) can be obtained as

\[
E_X \left[ \ln \frac{1 + H_{E|B}^2 X}{1 + H_{E|B}^2} \right] = \left[ e^{\frac{1}{H_{E|B}^2 \varsigma^2}} E_i \left( -\frac{1}{H_{E|B}^2 \varsigma^2} \right) - e^{\frac{1}{H_{E|B}^2 \varsigma^2}} E_i \left( -\frac{1}{H_{E|B}^2 \varsigma^2} \right) \right].
\]

\( \quad (A.4) \)

Substituting (A.3) and (A.4) into the lower bound (13), we obtain Theorem 1.

\[\blacksquare\]

APPENDIX B
PROOF OF Theorem 2

According to (20), the secrecy capacity is upper-bounded as (B.1) shown at the bottom of the next page, where \( I_1 \) can be written as

\[
I_1 = -[\mathcal{H}(Y_B|X^*) + \mathcal{H}(Y_E|X^*, Y_B) - \mathcal{H}(Y_E|X^*)]
\]

\( \quad (B.2) \)

where \( \mathcal{H}(Y_k|X^*) \) is given by

\[
\mathcal{H}(Y_k|X^*) = E_{X^*} \left\{ \frac{1}{2} \ln \left[ 2\pi e \left( 1 + H_k X^* \varsigma_k^2 \right) \sigma_k^2 \right] \right\}.
\]

\( \quad (B.3) \)
Moreover, the conditional PDF \( f_{Y|X,Y_{b}}(y_{E}|X^*, Y_{b}) \) is given by

\[
f_{Y|X,Y_{b}}(y_{E}|X^*, Y_{b}) = e^{-\frac{(y_{E} - \mu_{E} + s_{b})^2}{2\sigma_E^2}} \frac{1}{\sqrt{2\pi}(M^* + N)}
\]

(B.4)

where \( M = H_{E}^2C_{b}\sigma_E^2/H_{B} + H_{E}\sigma_E^2 \) and \( N = H_{E}^2\sigma_B^2/H_{B} + \sigma_E^2 \). According to (B.4), we derive

\[
\mathcal{H}(Y_{E}|X^*, Y_{b}) = E_{X^*}\left\{ \frac{1}{2}\ln [2\pi e(M^* + N)] \right\}.
\]

(B.5)

Substituting (B.3) and (B.5) into (B.2), we can write \( I_1 \) as

\[
I_1 = -\frac{1}{2}\ln\left(2\pi e\frac{\sigma_B^2}{\sigma_E^2}\right) - \frac{1}{2}E_{X^*}\left\{ \ln (M^* + N) + \ln \left(1 + H_{B}X^*\frac{s^2}{2}\right) \right\}.
\]

(B.6)

To obtain \( I_2 \) in (B.1), we select \( g_{Y|Y_{b}}(y_{E}|y_{B}) \) as

\[
g_{Y|Y_{b}}(y_{E}|y_{B}) = \frac{1}{2\pi e^{-\frac{(y_{E} - \mu_{E} - s_{b})^2}{2\sigma_E^2}}}
\]

(B.7)

where \( \mu \) and \( s \) are free parameters. Substituting (B.7) into (B.1), we can obtain \( I_2 \) as

\[
I_2 = E_{X^*, Y_{b}}\left\{ \ln(2s^2) + \frac{1}{s^2} \right\} - \frac{1}{2}\int_{-\infty}^{\infty} e^{-\frac{(y_{E} - \mu_{E} + s_{b})^2}{2\sigma_E^2}} |Y_{b} - \mu_{y_{b}}| \, dy_{E}.
\]

(B.8)

Let \( t = y_{E} - H_{E}Y_{b}/H_{B} \) and then use the fact that \(|a - b| \leq |a| + |b|\), \( C_1 \) in (B.8) is expressed as

\[
C_1 \leq -\frac{1}{2}\ln\left[ \frac{M^* + N}{2\pi} \right] + \ln\left[ 1 - \mu_{H_{E}/H_{B}} \right] |Y_{b}|. \tag{B.9}
\]

Substituting (B.9) into (B.8), we have

\[
I_2 = E_{X^*, Y_{b}}\left\{ \ln(2s^2) + \frac{1}{s^2} \right\} - \frac{1}{2}\int_{-\infty}^{\infty} e^{-\frac{(y_{E} - \mu_{E} - s_{b})^2}{2\sigma_E^2}} |y_{b}| \, dy_{b}.
\]

(B.10)

Using the triangle inequality \(|a + b| \leq |a| + |b|\), we can upper-bound \( C_2 \) by

\[
C_2 \leq 2\sqrt{\left(1 + H_{B}X^*\frac{s^2}{2}\right)\sigma_B^2} + H_{B}X^*. \tag{B.11}
\]

To obtain a tight upper bound, we perform the first partial derivative of (B.10) with \( s^2 \) to obtain a minimum upper bound of \( I_2 \). Therefore, the minimal point is \( s^2 = \frac{2|\mu|\sqrt{(M^* + N)/(2\pi)} + 1 - \mu_{H_{E}/H_{B}}}{C_2} \). Then, \( I_2 \) is upper-bounded by

\[
I_2 \leq E_{X^*}\left\{ \ln\left[ 4e\left(\frac{|\mu|\sqrt{M^* + N}}{2\pi} + 1 - \mu_{H_{E}/H_{B}}\right) \right] \times \left(\frac{1 + H_{B}X^*\frac{s^2}{2}C_{b}}{2\pi(M^* + N)} + \frac{H_{B}X^*}{2\pi(M^* + N)} \right) \right\}. \tag{B.12}
\]

Substituting (B.6) and (B.12) into (B.1), we can upper-bound \( C_s \) by

\[
C_s \leq -\frac{1}{2}\ln\left(2\pi e\frac{\sigma_B^2}{\sigma_E^2}\right) - \frac{1}{2}E_{X^*}\left\{ \ln\left[ 1 + H_{B}X^*\frac{s^2}{2}\right] \right\} + E_{X^*}\left\{ \ln\left[ 2e\left(\frac{|\mu|\sqrt{M^* + N}}{2\pi} + 1 - \mu_{H_{E}/H_{B}}\right) \right] \times \left(\frac{1 + H_{B}X^*\frac{s^2}{2}C_{b}}{2\pi(M^* + N)} + \frac{H_{B}X^*}{2\pi(M^* + N)} \right) \right\}. \tag{B.13}
\]

By \( \frac{H_{B}X^*}{2\sqrt{M^* + N}} \leq \frac{H_{B}X^*}{2\sqrt{M}} \) and the Jensen’s inequality, eq. (B.13) is further written as

\[
C_s \leq -\frac{1}{2}\ln\left(2\pi e\frac{\sigma_B^2}{\sigma_E^2}\right) - \frac{1}{2}E_{X^*}\left\{ \ln\left[ \frac{1 + H_{B}X^*\frac{s^2}{2}}{1 + H_{B}X^*\frac{s^2}{2}} \right] \right\} + \ln\left[ 4e\left(\frac{|\mu|\sqrt{M^* + N}}{2\pi} + 1 - \mu_{H_{E}/H_{B}}\right) \right] \times \left(\frac{1 + H_{B}X^*\frac{s^2}{2}C_{b}}{2\pi(M^* + N)} + \frac{H_{B}X^*}{2\pi(M^* + N)} \right). \tag{B.14}
\]

In VLC, we are more interested in the performance at large optical intensity. As \( P \to \infty \), the optimal input PDF escapes to infinity \[17, 35]. Therefore, when \( P \to \infty \), we have

\[
E_{X^*}\left[ \frac{1 + H_{B}X^*\frac{s^2}{2}}{1 + H_{B}X^*\frac{s^2}{2}} \frac{P}{2\pi(M^* + N)} \right] = \frac{1 + H_{B}X^*\frac{s^2}{2}}{2\pi(M^* + N)}. \tag{B.15}
\]
Moreover, by using L’Hospital’s rule for (B.15), we have
\[
\lim_{P \to \infty} \left\{ \begin{array}{l}
\frac{1 + H_B^2 B^2}{2} \sigma_B^2 \\
\frac{(1 + H_B^2 B^2)}{2} \sigma_B^2 \frac{1}{2 \pi M} \sigma_E^2
\end{array} \right\} = \frac{H_B B^2 \sigma_B^2}{2 \pi M}.
\]
(B.16)
Substituting (B.15) and (B.16) into (B.14), we have
\[
C_\sigma \leq -\frac{1}{2} \ln \left( 2\pi e \frac{H_B B^2 \sigma_B^2}{H_B B^2 + \frac{H_B^2 B^2}{2} \sigma_E^2} \right) + \ln \left( 2 e \right)
\]
(\text{B.17})
\[
\times \left( \frac{B}{\sqrt{2\pi}} + 1 - \frac{H_B}{B} \left( \frac{H_B B^2 \sigma_B^2}{2 \pi M} + \frac{\xi P}{M} \right) \right).
\]
(B.17)
\[
\text{In (B.17), } C_3 \text{ is the function of } \mu. \text{ To obtain a tight upper bound, we should determine a small } C_3 \text{ value. Here, three cases are considered: Case 1: when } \mu \leq 0, \text{ we have } C_3 \geq \frac{H_B B^2 \sigma_B^2}{H_B B^2 \sqrt{2\pi M}} + \frac{H_B^2 B^2}{2 \pi M} \sigma_E^2. \text{ Case 2: when } \mu \geq H_B/H_E, \text{ we have } C_3 \geq \frac{H_B B^2 \sigma_B^2}{H_B B^2 \sqrt{2\pi M}} \text{ case 3: when } 0 \leq \mu \leq H_B/H_E, \text{ we have } C_3 \geq \frac{H_B B^2 \sigma_B^2}{H_B B^2 \sqrt{2\pi M}}. \text{ According to the above three cases, we have}
\]
\[
C_3 \geq \left\{ \begin{array}{l}
\frac{H_B B^2 \sigma_B^2}{2 \pi M} + \frac{H_B^2 B^2}{2 \pi M} \sigma_E^2 \\
\frac{H_B B^2 \sigma_B^2}{2 \pi M} + \frac{H_B^2 B^2}{2 \pi M} \sigma_E^2 \\
\frac{H_B B^2 \sigma_B^2}{2 \pi M} \sqrt{2\pi M} \\
\end{array} \right\} \text{ (B.18)}
\]
\[
\text{Substituting (B.19) into (B.17), we obtain Theorem 2.}
\]
\text{APPENDIX C}
\text{PROOF OF COROLLARY 3}
\text{In Theorem 1, the term } f_{low}(H_B, \xi, P) \text{ is a monotonically decreasing positive function of } H_B, \xi \text{ and } P. \text{ When } P \to \infty, \text{ we can obviously obtain}
\]
\[
\lim_{P \to \infty} f_{low}(H_B, \xi, P) = \ln \left( H_B \right)
\]
\[
\lim_{P \to \infty} \left\{ \begin{array}{l}
ed^{\frac{1}{H_B B^2 \sigma_B^2}} E_i \left( \frac{1}{H_B B^2 \sigma_B^2} \right) - e^{\frac{1}{H_B B^2 \sigma_B^2 \xi^2}} E_i \left( \frac{1}{H_B B^2 \sigma_B^2 \xi^2} \right) \\
= \ln \left( \frac{H_B B^2 \sigma_B^2}{H_B B^2} \right)
\end{array} \right\}
\]
(C.1)
\text{Substituting (C.1) into Theorem 1, we obtain an asymptotic lower bound as}
\]
\[
\lim_{P \to \infty} C_{low} = \frac{1}{2} \ln \left( \frac{e H_B B^2 \sigma_B^2}{2 \pi H_B B^2 \sigma_E^2} \right).
\]
(C.2)
\text{For the asymptotic upper bound, the case of } \frac{1}{\sqrt{2\pi}} \geq \frac{H_B}{H_B B^2} \left( \frac{H_B B^2 \sigma_B^2}{2 \pi M} + \frac{H_B^2 B^2}{2} \sqrt{\frac{M}{P}} \right) \text{ does not occur when } P \to \infty. \text{ Therefore, we can obtain the asymptotic upper bound as}
\]
\[
\lim_{P \to \infty} C_{up} = \frac{1}{2} \ln \left( \frac{4e H_B B^2 \sigma_B^2}{\pi^2 H_B B^2 \sigma_E^2} \right).
\]
(C.3)
Combining (C.2) with (C.3), we prove Corollary 3.

\text{APPENDIX D}
\text{PROOF OF THEOREM 3}
\text{Replacing the integral upper limit } \infty \text{ with } A \text{ in (A.1), we can obtain the functional optimization problem in this case. By employing the variational method, we can derive the input PDF as } f_X(x) = e^{cx+b-1}, \text{ } x \in [0, A] \text{ [25], where } b \text{ and } c \text{ are two free parameters.}
\text{When } c = 0, \text{ substitute } f_X(x) = e^{b-1} \text{ into the constraints, we can obtain [25]}
\]
\[
f_X(x) = \frac{1}{A}, \text{ } x \in [0, A].
\]
(D.1)
\text{In this case, } E_X(X) = A/2 = \xi P, \text{ we have } \alpha = 0.5. \text{ Then, we can obtain } \mathcal{H}(X) = \text{ln}(A) \text{ and } \text{var}(Y_E) = \frac{H_B^2 A^2}{12} + \frac{2 e^{\xi P} H_B B^2 \sigma_E^2}{\sigma_E^2}. \text{ Moreover, } E_X \left[ \text{ln} \left( \frac{1 + H_B B^2 \sigma_E^2}{1} X \right) \right] \text{ in (13) can be derived as}
\]
\[
E_X \left[ \text{ln} \left( \frac{1 + H_B B^2 \sigma_E^2}{1} X \right) \right] = \text{ln} \left( \frac{1 + H_B B^2 \sigma_E^2 A}{1 + H_B B^2 \sigma_E^2 A} \right) - \frac{\text{ln} (1 + H_B B^2 \sigma_E^2 A)}{AH_B B^2} + \frac{\text{ln} (1 + H_B B^2 \sigma_E^2 A)}{AH_B B^2}.
\]
(D.2)
\text{Substituting } \mathcal{H}(X), \text{ var}(Y_E) \text{ and (D.2) into (13), we obtain the lower bound for } \alpha = 0.5.
\text{When } c \neq 0, \text{ we have } \alpha \neq 0.5. \text{ Substituting } f_X(x) = e^{cx+b-1} \text{ into the constraints, we derive the input PDF as [25]}
\]
\[
f_X(x) = \frac{e^{cx}}{e^{cx} - 1}, \text{ } x \in [0, A]
\]
(D.3)
\text{where } c \text{ is the solution to (29). Then, we obtain } \mathcal{H}(X) = \ln \left( \frac{e^{c-1}}{c} \right) - \xi P. \text{ Moreover, we have}
\]
\[
\text{var}(Y_E) = H_B^2 \left[ \frac{A (e^A - 2)}{e (1 - e^{-cA})} + 2 e^2 - \xi^2 P^2 \right] + H_B \xi P^2 e^{-2 \xi P^2}
\]
(D.4)
\text{and}
\[
E_X \left[ \text{ln} \left( \frac{1 + H_B B^2 \sigma_E^2}{1 + H_B B^2 \sigma_E^2} X \right) \right] = \frac{1}{e^A - 1} \left\{ \text{ln} \left( \frac{1 + H_B B^2 \sigma_E^2 A}{1 + H_B B^2 \sigma_E^2 A} \right) e^{cA} - \frac{e^{cx}}{e^{cx} - 1} \right\} + \text{Ei} \left( \frac{c (1 + H_B B^2 A)}{H_B B^2} \right) - \text{Ei} \left( \frac{c}{H_B B^2} \right)
\]
+ \text{Ei} \left( \frac{c}{H_B B^2} \right) - \text{Ei} \left( \frac{c (1 + H_B B^2 A)}{H_B B^2} \right) - \text{Ei} \left( \frac{c}{H_B B^2} \right)
\]
(D.5)
Substituting $\mathcal{H}(X)$, (D.4) and (D.5) into (13), we obtain the lower bound for $\alpha \neq 0.5$.

**APPENDIX E**

**PROOF OF THEOREM 4**

1) Proof of monotonicity: When $\alpha = 0.5$, the conclusion is obvious. When $\alpha \neq 0.5$, we let $F(c) = \frac{1}{(e^{-cA} - e^{-cA}) - \frac{1}{c^2}}, c \neq 0$ in (29). Taking the first-order derivative of $F(c)$, we can obtain

$$
\frac{dF(c)}{dc} = \left(1 - e^{-cA} + cAe^{-\frac{cA}{2}} \right) \left(1 - e^{-cA} - cAe^{-\frac{cA}{2}} \right) c^2 (1 - e^{-cA})^2.
$$

(E.1)

When $c > 0$, we have $1 - e^{-cA} + cAe^{-\frac{cA}{2}} > 0$ and $c^2 (1 - e^{-cA})^2 > 0$, then, we get $G(c) = 1 - e^{-cA} - cAe^{-\frac{cA}{2}}, c > 0$, and take the first-order derivative of $G(c)$ as

$$
\frac{dG(c)}{dc} = A e^{-\frac{cA}{2}} \left(e^{-\frac{cA}{2}} + \frac{cA}{2}\right).
$$

(E.2)

Let $K(c) = e^{-\frac{cA}{2}} - 1 + \frac{cA}{2}, c > 0$, we have $\frac{dK(c)}{dc} = \frac{1}{2} e^{-\frac{cA}{2}} + \frac{cA}{2} > 0$. This indicates that $K(c)$ is an increasing function of $c$. As a result, $K(c) > K(0) = 0, \forall c > 0$. Then, we have $dG(c)/dc > 0$, and thus $G(c) > G(0), \forall c > 0$. After that, we have $dF(c)/dc > 0$ in (E.1), which indicates that $F(c)$ is an increasing function of $c$ when $c > 0$. Moreover, by using L'Hospita’s rule, we have $\lim_{c \to 0} F(c) = 0.5$. Therefore, we have

$$
F(c) = \alpha > 0.5, \text{ if } c > 0.
$$

(E.3)

According to (E.3), we know that $c > 0$ when $\alpha \in (0.5, 1]$, and thus the PDF (D.3) is an increasing function of $x$ in $[0, A]$. By using the similar approach, we can also prove that $c < 0$ when $\alpha \in (0, 0.5]$, and thus the PDF (D.3) is a decreasing function of $x$ in $[0, A]$.

2) Proof of symmetry: For a fixed $\alpha$ value, the input PDF in (D.3) is re-denoted by $f_X(x, \alpha)$ to facilitate the notation. When $x \in [0, A]$, we have

$$
f_X(A - x, 1 - \alpha) = \frac{(c_2) e^{(c_2) A}}{e^{(c_2) A} - 1},
$$

where $c_2$ is the solution to $1 - \alpha = \frac{\frac{1}{1-e^{-(-\alpha)cA}}}{\frac{1}{1-e^{-(-\alpha)cA}} + \frac{1}{(-\alpha)cA}}$, and such an equality is equivalent to

$$
\alpha = \frac{1}{1 - e^{c_2 A}} - \frac{1}{(-c_2) A}.
$$

(E.5)

According to (29) and (E.5), we have $c_2 = -c$. Then, eq. (E.4) can be further expressed as

$$
f_X(A - x, 1 - \alpha) = \frac{c e^{-cX}}{c e^{cA} - 1} = f_X(x, \alpha).
$$

(E.6)

This indicates that the curves of (D.3) with $\alpha$ and $1 - \alpha$ are symmetric with $X = A/2$.

**APPENDIX F**

**PROOF OF THEOREM 5**

In this scenario, eq. (B.1) also holds, where $I_1$ can also be expressed as (B.6). To obtain $I_2$ in (B.1), we select $g_{\alpha I_1} (y_B | y_E) = e^{-\left(\frac{y_B - \mu y_E}{\sigma^2 y_E}\right)^2}/\sqrt{2\pi y_E^2}$, where $\mu$ and $s$ are free parameters to be determined. Then, $I_2$ is given by

$$
I_2 = E_X \cdot Y_B \left[\frac{1}{2} \ln (2\pi s^2) + \frac{1}{2s^2} \right]
$$

$$
\times \int_{-\infty}^{\infty} e^{-\left(\frac{y_B - \mu y_E}{\sqrt{2\pi s^2 y_E}}\right)^2} (y_B - \mu y_E)^2 dY_E
$$

(F.1)

where $C_4$ can be expressed as

$$
C_4 = \left(1 - \frac{H_E}{H_B}\right)^2 Y_B^2 + \mu^2 (M X^* + N).
$$

(F.2)

Substituting (F.2) into (F.1), we can obtain $I_2$ as (F.3) shown at the top of the next page, where the inequality holds because $X^* \leq A$. Taking the first partial derivative of $C_5$ with $s^2$ and letting it be zero, we obtain the minimum point as $s^2 = (1 - \mu H_E / H_B)^2 (H_B X^* + H_B X^* s_B^2 + \sigma_B^2 + \sigma_B^2) + \mu^2 (M X^* + N)$. Then, substituting $s^2$ into (F.3), we can rewrite $C_5$ as

$$
C_5 = \frac{1}{2} \ln \left\{ \left(1 - \frac{H_E}{H_B}\right)^2 \right\}
$$

$$
\times \left\{ H_B^2 X^* + H_B X^* s_B^2 + \sigma_B^2 + \mu^2 (M X^* + N) \right\}.
$$

(F.4)

Substituting (B.6), (F.3) and (F.4) into (B.1), and using the Jensen's inequality for the convex function $\ln(\cdot)$, we can obtain the upper bound as

$$
C_s \leq \frac{1}{2} \ln \left\{ 2 \pi \left(\frac{\sigma_B^2 \sigma_B^2}{H_B^2 X^* + H_B X^* s_B^2 + \sigma_B^2} \right) + \mu^2 (M X^* + N) \right\}
$$

$$
\times E_X \cdot \left( \left(1 + \frac{H_B X^* s_B^2}{1 + H_E X^* s_B^2} \right)^2 \right) + \frac{1}{2} \ln \left\{ \left(1 - \frac{H_E}{H_B}\right)^2 \right\}
$$

(F.5)

Because we are more interested in the performance at large optical intensity, the optimal input PDF escapes to infinity when $A \to \infty$ [17], [35]. Therefore, we have

$$
E_X \cdot \left( \left(1 + \frac{H_B X^* s_B^2}{1 + H_E X^* s_B^2} \right)^2 \right) = H_B X^* A^2 + H_B X^* A \sigma_B^2 + \sigma_B^2
$$

(F.6)

By using the L’Hospital’s rule, we have

$$
\lim_{A \to \infty} \frac{H_B X^* A^2 + H_B X^* A \sigma_B^2 + \sigma_B^2}{A^2} = H_B A + H_B A \sigma_B^2 + \sigma_B^2.
$$

(F.7)
\[ I_2 \leq E_X \cdot \left\{ \frac{1}{2} \ln (2\pi s^2) + \frac{1}{2s^2} \left[ (1 - \frac{H_E}{H_B})^2 \left( H_B^2 A X^* + H_B X^* \sigma_B^2 + \sigma_B^2 \right) + \mu^2 (M X^* + N) \right] \right\} \]  

(F.3)

According to (F.5), (F.6) and (F.7), the upper bound can be asymptotically expressed as

\[ C_s \leq \frac{1}{2} \ln \left( \frac{H_B^2 A + H_B \sigma_B^2 + \sigma_B^2}{M} \right) + \frac{1}{2} \ln \left[ 2 \pi e \left( 1 - \frac{H_E}{H_B} \right)^2 \frac{H_B^2 A + H_B \sigma_B^2 + \sigma_B^2}{M} + \mu^2 \right]. \]  

(F.8)

Taking the first partial derivative of \( C_s \) with respect to \( \mu \), and letting it to be zero, we have

\[ \mu = \frac{H_B^2 A + H_B \sigma_B^2 + \sigma_B^2}{1 + \left( \frac{H_E}{H_B} \right)^2 \frac{H_B^2 A + H_B \sigma_B^2 + \sigma_B^2}{M}}. \]  

(F.9)

Substituting (F.9) into (F.8), we can derive (31).

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**Jin-Yuan Wang** (Member, IEEE) received the B.S. degree in communication engineering from the Shandong University of Science and Technology, Qingdao, China, in 2009, the M.S. degree in electronic and communication engineering from the Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2012, and the Ph.D. degree in information and communication engineering from Southeast University, Nanjing, in 2015. From January 2016 to June 2019, he was a Lecturer with the Nanjing University of Posts and Telecommunications, Nanjing, where he has been an Associate Professor since July 2019. He has authored/coauthored over 120 journals/conference papers. His current research interests include visible light communications. He is serving as a Youth Editorial Board Member for Journal of Electronics and Information Technology and Aerospace Technology, the Topic Editor of Sensors, and the Guest Editor of *Frontiers in Signal Processing*. He has been the track chair, the workshop chair, or a TPC member of many conferences. He also serves as a reviewer for many international journals.

**Peng-Fei Yu** received the B.S. degree in communication engineering from the Anhui University of Science and Technology, Huainan, China, in 2020. He is currently pursuing the M.S. degree in communication and information system with the Nanjing University of Posts and Telecommunications, Nanjing, China. His current research interests include visible light communications.

**Xian-Tao Fu** received the B.S. degree in optoelectronic information science and engineering from HuaiBei Normal University, Huaibei, China, in 2019. He is currently pursuing the M.S. degree in communication and information system with the Nanjing University of Posts and Telecommunications, Nanjing, China. His current research interests include visible light communications.
Jun-Bo Wang (Member, IEEE) received the B.S. degree in computer science from the Hefei University of Technology, Hefei, China, in 2003, and the Ph.D. degree in communications engineering from the National Mobile Communications Research Laboratory, Southeast University, Nanjing, China, in 2008. From October 2008 to August 2013, he was with the Nanjing University of Aeronautics and Astronautics, China. From March 2011 to February 2013, he was a Post-Doctoral Fellow with the National Laboratory for Information Science and Technology, Tsinghua University, Beijing, China. From September 2016 to August 2018, he was a Marie Skłodowska-Curie Visiting Scholar with the University of Kent, Kent, U.K. He is currently an Associate Professor with the National Mobile Communications Research Laboratory, Southeast University. His current research interests include wireless communications, signal processing, and information theory and coding.

Min Lin (Member, IEEE) received the B.S. degree in electrical engineering from the National University of Defense Technology, Changsha, China, in 1993, the M.S. degree in electrical engineering from the Nanjing Institute of Communication Engineering, Nanjing, China, in 2000, and the Ph.D. degree in electrical engineering from Southeast University, Nanjing, in 2008. From April 2015 to October 2015, he has visited the University of California at Irvine, Irvine, CA, USA, as a Senior Research Fellow. He is currently a Professor and a Supervisor of Ph.D. and graduate students with the Nanjing University of Posts and Telecommunications, Nanjing. He has authored or coauthored over 130 articles. His current research interests include wireless communications and array signal processing. He has served as a TPC Member for many IEEE sponsored conferences, such as IEEE ICC and IEEE GLOBECOM; and the Track Chair for Satellite and Space Communications (SSC) of IEEE ICC 2019.

Julian Cheng (Fellow, IEEE) received the B.Eng. degree (Hons.) in electrical engineering from the University of Victoria, Victoria, BC, Canada, in 1995, the M.Sc. (Eng.) degree in mathematics and engineering from Queens University, Kingston, ON, Canada, in 1997, and the Ph.D. degree in electrical engineering from the University of Alberta, Edmonton, AB, Canada, in 2003. He was with Bell Northern Research and NORTEL Networks. He is currently a Full Professor with the Faculty of Applied Science, School of Engineering, The University of British Columbia, Kelowna, BC, Canada. His research interests include machine learning and deep learning for wireless communications, wireless optical technology, and quantum communications. He was the Co-Chair of the 12th Canadian Workshop on Information Theory in 2011, the 28th Biennial Symposium on Communications in 2016, and the Sixth EAI International Conference on Game Theory for Networks (GameNets 2016). He was the General Co-Chair of the 2021 IEEE Communication Theory Workshop. He is the Chair of the Radio Communications Technical Committee of the IEEE Communications Society. He served as the President for the Canadian Society of Information Theory (2017–2021). He was an Associate Editor of the IEEE TRANSACTIONS ON COMMUNICATIONS, the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, the IEEE COMMUNICATIONS LETTERS, and IEEE ACCESS; and an Area Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS. He served as a Guest Editor for a Special Issue of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS on optical wireless communications. He is a registered Professional Engineer in BC, Canada.

Mohamed-Slim Alouini (Fellow, IEEE) was born in Tunis, Tunisia. He received the Ph.D. degree in electrical engineering from the California Institute of Technology (Caltech), Pasadena, CA, USA, in 1998. He worked as a Faculty Member of the University of Minnesota, Minneapolis, MN, USA, then with Texas A&M University at Qatar, Education City, Doha, Qatar, before joining the King Abdullah University of Science and Technology (KAUST), Thuwal, Mecca, Saudi Arabia, as a Professor of electrical engineering, in 2009. His current research interests include the modeling, design, and performance analysis of wireless communication systems.