Emission altitudes in young and old radio pulsars

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ABSTRACT
The paper presents a comparison of emission altitudes in very young and very old radio pulsars. The author confirms that altitudes at which radio emission is generated depend on pulsar period and on pulsar age, although the latter dependence is quite weak.

Key words: pulsars; general - radiation mechanisms: non-thermal - stars: neutron - stars: magnetic fields

1 INTRODUCTION
The radial location of the radio emission regions in pulsar magnetospheres was discussed by several authors. Using various methods reviewed by Cordes (1992, 1993), constraints on the emission radii were obtained in a number of papers. These estimates show that the pulsar emission region is relatively compact and lies near the surface of the neutron star, apparently closer than about 10 per cent of the light-cylinder radius (Cordes 1978; Blaskiewicz, Cordes & Wasserman, apparently closer than about 10 per cent of the light-cylinder radius (Cordes 1978; Blaskiewicz, Cordes & Wasserman 1991; Phillips 1992; Gil & Kijak 1993; Hoensbroech & Xilouris 1997; Kijak & Gil 1997, 1998; Kramer et al. 1997; Gupta, Bhat & Rao 1999).

Assuming that pulsar is a rotating magnetic dipole we can calculate its characteristic age $\tau = P/2P$ (Manchester & Taylor 1977), which is a good estimate of the true age providing that $P \gg P_0$, where $P$ and $P_0$ are current and initial spin periods, respectively. Most pulsars are $10^6 - 10^7$ years old, however, one separates young and old objects with characteristic ages $\tau$ about $10^4$ yr and $10^9$ yr, respectively. It is important to note that the group of old pulsars contains not only millisecond pulsars, but also pulsars with typical period $P \sim 0.5$ s (e.g. PSR B1848+04, B1952+29) as well as recently discovered very old pulsar J2144–3933 with the longest measured period 8.5 seconds (Young, Manchester, Johnston 1999). It is commonly believed that pulsars form two distinct populations: high-magnetic field objects that are radio pulsars for relatively short times and recycled objects which can be very old.

The millisecond pulsars (MSPs) have different evolutionary history compared to normal pulsars. They are recycled pulsars, i.e. they acquired their rapid rotation rates ($\sim$ a few ms) due to accretion-driven spin-up (Bhattacharya 1996). Consequently, their magnetic dipole field strength at the surface $B_d = 3.2 \times 10^{19} (PP)^{1/2} G$ (Manchester & Taylor 1977) is relatively low ($\sim 10^8 - 10^9 G$), as compared with the whole population of radio pulsars ($\sim 10^{13} G$). Previously, it was thought that MSPs also have different characteristics of radio emission from those of normal pulsars. However, it has been recently argued (Kramer et al. 1998, 1999; Gil & Sendyk 2000) that MSPs emission properties do not differ from typical pulsars.

The semi-empirical relationship proposed by Kijak & Gil (1997) describe the dependence of emission altitude on pulsar period $P$ (in seconds), observing frequency $\nu$ (in GHz) and characteristic age $\tau$ (in $10^6$ yr)

$$r_{KG} = (55 \pm 5) R_{GHz}^{-0.21 \pm 0.07} \nu_{GHz}^{-0.07 \pm 0.03} \tau^{0.33 \pm 0.05},$$

where $R = 10^6$cm is the neutron star radius. This was obtained from analysis of emission altitudes for a number of pulsars at two frequencies. Emission altitudes were estimated using the pulse width method (see Section 2) based on a number of straightforward assumptions: (i) the pulsar radiation is narrow-band with a radius-to-frequency mapping operating in the emission region, (ii) pulsar emission is relativistically beamed tangentially to dipolar magnetic field lines and (iii) the extreme profile wings originate at or near the last open field lines. This, of course, involves an implicit assumption that the magnetic field in the radio emission region is purely dipolar (see Section 3 for discussion). The uncertainties given for various quantities in equation (1) follow from systematic errors in the pulse width $W$ measurements and random errors in estimations of the inclination angle $\alpha$ between the rotation and magnetic axes and the impact angle $\beta$ of the closest approach of the observer to the magnetic axis.

First suggestion that radio emission altitudes at a given frequency depend on the pulsar period ($r_{em} \propto P^{-0.5}$) was published by Gil & Kijak (1993). This period dependence was further carefully investigated by Kijak & Gil (1997, 1998; hereafter Papers I and II) and more precise relationship $r_{em}(P) \propto P^{-0.33}$ was obtained. Anyway, the emission region is apparently located close to the neutron star in short-period pulsars and further away in longer-period pulsars.
Figure 1. Emission altitudes at 1.4 GHz versus pulsar period for young (circles) and old (squares) pulsars (data taken from Table 2 and for old PSRs: B1855+09, J1022+1001 and J1713+0747 from Paper II). Formal weighted fit for pulsars with $\tau_6 < 0.1$ gives $r_{\text{em}} = R(55 \pm 8)P^{0.25 \pm 0.08}$ and for $\tau_6 > 1000$ gives $r_{\text{em}} = R(35 \pm 6)P^{0.38 \pm 0.04}$.

Alternatively, the ratio $r_{\text{KC}}/r_{\text{LC}}$ decreases with increasing period (where $r_{\text{LC}}$ is the light cylinder radius), what was quantitatively demonstrated in Papers I (Fig. 3b) and II (Fig. 4a and 4b). The equation (1) describes also a radius-to-frequency mapping (RFM) $r(\nu) \propto \nu^{-p}$ (where $p \sim 0.21$), which means that different frequencies are generated at different altitudes above the magnetic polar cap. The RFM models of pulsar emission region provide an attractive explanation of why profile widths (usually) increase with decreasing frequency. Kijak & Gil (1998) studied the concept of RFM using the multifrequency data of low-intensity pulse widths for normal and millisecond pulsars. They obtained RFM with the average exponent $\bar{p} = 0.26 \pm 0.09$ (for 16 pulsars), consistent with previous estimates using different methods (see Table 4 in Paper II). They ultimately confirmed that the emission altitude depends on frequency $\nu$, pulsar period $P$ as well as on the pulsar age $\tau$, although the latter dependence was weak. In this paper we investigate a putative weak dependence of emission altitudes on the characteristic age indicated in earlier work (equation 1), by selecting a sample of very young and very old radio pulsars and applying to them the same analysis methods as used in Papers I and II.

2 PULSAR CHARACTERISTIC AGE AND EMISSION ALTITUDE

In this section we calculate emission altitudes for very young and very old pulsars using 1.4 GHz data from Effelsberg Observatory. In addition, data for two southern pulsars B1727–33 (Gould & Lyne 1998) and B0833–45 (Johnston, Nicastro & Koribalski 1998) were included because of their young age. All these pulsars (Table 1 and Fig. 1) were not analysed in previous work (Papers I and II). For estimations of the emission altitude we use the geometrical method (pulse profile width $W$ measurements at low-intensity level of the maximum intensity) described in Papers I and II.

Assuming that the magnetic field in the emission region has a dipolar form, the emission altitude $r_{\text{em}}$ (at which radi-
Table 1. Emission altitudes for old and young pulsars at 1.4 GHz. We calculated \( r_{\text{em}} \) from Eq. 2 and \( r_{\text{KG}} \) from Eq. 1. Pulse width measurements \( W_0 \) at low-intensity level (i.e. \( \approx 0.1 \) per cent) and calculations of opening angle \( \rho_0 \) are presented. References are marked for \( \alpha \) and \( \beta \) angles (see Paper I).

| PSR      | \( \tau_0 \) (10\(^6\) yr) | \( P \) (sec) | \( r_{\text{em}} \) (\( r/R \)) | \( r_{\text{KG}} \) (\( r/R \)) | \( W_0 \) (yr) | \( \rho_0 \) (deg) | Ref. |
|----------|-----------------|--------------|-----------------|-----------------|-------------|---------------|-----|
| B0329+54 | 0.011           | 0.089        | 30±3            | 32              | 39.0±1.4    | 22.8±1.3      | R93 |
| B0540+23 | 0.016           | 0.133        | 39±11           | 35              | 137.±8.0    | 21.2±4.1      | G94 |
| B1823+13 | 0.022           | 0.101        | 30±7            | 31              | 120.±4.0    | 21.3±3.6      | G94 |
| B1727+33 | 0.025           | 0.139        | 31±6            | 34              | 86.0±7.0    | 18.5±2.7      | G94 |
| B2334+61 | 0.040           | 0.495        | 59±7            | 51              | 34.8±2.6    | 13.5±1.3      | G94 |
| B1916+14 | 0.089           | 1.180        | 50±10           | 68              | 16.2±2.5    | 8.0±1.2       | G94 |
| B1848+04 | 2.818           | 0.284        | 21±5            | 19              | 107.±9.0    | 10.7±2.1      | R93 |
| B1952+29 | 4.168           | 0.426        | 26±6            | 17              | 35.0±3.0    | 9.7±1.6       | LM88 |

\( r_{\text{em}} = \left( \frac{\rho}{1.24 \cdot s} \right)^2 P \),

where \( \rho \) is the opening angle [in degrees] between the magnetic axis and the tangent to dipolar magnetic field lines at points where the emission corresponds to the apparent pulse-width originates, and \( s \) is the mapping parameter \( 0 \leq s \leq 1 \) describing the locus of field lines on the polar cap \( s = 0 \) at the pole and \( s = 1 \) at the edge of the Goldreich-Julian (1969) circular polar cap. Opening angles \( \rho = \rho(W; \alpha, \beta) \) are calculated using the knowledge on the viewing geometry (represented by the inclination angle \( \alpha \) between the rotation and the magnetic axes and the impact angle \( \beta \) of the closest approach of the observer to the magnetic axis; see Paper I and references therein for more detailed explanation), as well as measurements of the pulse width \( W \). Estimates of emission altitude (for \( s \approx 1 \)) from equation (2) are listed in Table 1.

In Figure 1, we present a plot of emission altitude \( r_{\text{em}} \) versus period \( P \) for very young and very old pulsars corresponding to pulse width measurements at 1.4 GHz. The data have been taken from Table I and from the Paper II (for MSPs: J1022+1001, J1713+0747 and B1855+09). It is noticeable that emission altitudes differ slightly between very young and very old pulsars. A formal weighted fits to very young and very old pulsars yield \( r_{\text{em}} = R(55±8)P^{-0.25±0.08} \) and \( r_{\text{em}} = R(35±6)P^{-0.38±0.04} \), respectively. We can now gather all estimates of emission altitudes from Papers I, II and from this paper (Table 1) to obtain a plot of emission altitude versus characteristic age for 37 pulsars (see Fig. 2 and Table 2). The periods of normal pulsars were divided into six groups: \( (0.05 s - 0.2 s), (0.2 s - 0.4 s), (0.4 s - 0.6 s), (0.6 s - 0.8 s), (0.8 s - 2.0 s), (2.0 s - 4.0 s) \) and data belonging to each group were represented by different size of the circle (with biggest circles corresponding to longest periods in our sample). This is consistent with equation (1), showing that for a given characteristic age, pulsars with longer periods emit their radio emission from higher altitudes than those with shorter periods. A formal weighted fit to all data points in Fig. 2 is \( r_{\text{em}} = R(33±1)\tau_0^{-0.07±0.02} \), confirming a putative weak dependence \( r_{\text{em}} \) on pulsar age \( \tau = \tau_0 \cdot 10^6 \) years (equation 1).

3 DISCUSSION

In Papers I and II the radius-to-frequency mapping and the period dependence of emission altitude have been discussed in details. Here, a detailed analysis of age dependence on emission altitude is carried out. Results presented in Figs. 1 and 2 indicate that emission altitudes depend on the characteristic age \( r_{\text{em}}(\tau_0) \propto \tau_0^{-0.07} \), which is consistent with equation (1). In this analysis we used 37 pulsars at 1.4 GHz for which all necessary information were available, that is: very good quality profiles (pulse width measurements) as well as reliable estimates of \( \alpha \) and \( \beta \) angles.

It is important to notice that the radio emission region for a group of very old pulsars (containing millisecond and normal pulsars) is located at correspondingly lower altitudes than that of very young objects (see Figs. 1 and 2). Three points in Fig. 2 deviate significantly from the rest of distribution. These points correspond to 3 millisecond pulsars in our sample. One could think that this is an effect of nondipolar magnetic field in the radio emission region of millisecond pulsars. However, we do not regard these outlier points as strong indication of deviation from dipolar fields. In fact, the emission altitude at a given frequency is determined more strongly by the value of pulsar period than by...
Figure 2. Emission altitudes versus characteristic age $\tau$ for 37 pulsars. The size of circles represents increasing period from 0.1 s to 4.0 s (see text for explanation). The filled dots correspond to millisecond pulsars. A formal weighted fit gives $r_{\text{em}} = R(33 \pm 1)\tau_6^{-0.07\pm0.02}$. The surface magnetic field is estimated using the assumption that the main rotational energy loss is in the form of electromagnetic dipole radiation. With a few further assumptions about the pulsar spindown mechanism (Manchester & Taylor 1977; Camilo, Thorsett & Kulkarni 1994), the surface magnetic dipole field component of a pulsar (inferred from its values of $P$ and $\dot{P}$) can be expressed in terms of pulsar characteristic age $\tau = 0.5 \cdot P/\dot{P}$ s as follows

$$B_{d} = 4 \times 10^{12} \tau_6^{-1/2} P \quad [\text{G}],$$

where $\tau_6 = \tau_{\text{years}}/10^6$. If the magnetic field in the emission region is dipolar, one can calculate its values at the emission altitude $r_{\text{em}}$ as

$$B_{r} = B_{d} \left(\frac{r_{\text{em}}}{R}\right)^{-3}. \quad (4)$$

From equations (1), (3) and (4), one obtains

$$B_{r} \approx 2 \times 10^7 \nu_{0.63}^{0.29} \text{GHz} \tau_6^{-0.29} [\text{G}]. \quad (5)$$

The above equation indicates that the magnetic field in the emission region may depend on the characteristic pulsar age. In typical pulsars with $\tau_6 \sim 1$, the magnetic field in the emission region corresponding to 1.4 GHz should be about $10^7$ G.

Figure 3 presents a plot of emission altitude versus the dipolar component of surface magnetic field (calculated from equation 3) and the magnetic field in the emission region (calculated from equation 4). The magnetic field in the emission region at 1.4 GHz for normal pulsars ($10^6$ - $10^7$ yr) has values of about $10^7$ - $10^8$ G. Gil & Kijak (1993) suggested that the magnetic field in the emission region $B_{r}$ is almost independent of the pulsar period and characteristic age. This analysis clearly shows that very old pulsars (filled circles) and very young pulsars (circles with stars) have quite dif-
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![Graph of emission altitude vs. magnetic field](image)

Figure 3. Plot of the emission altitude $r_{em}$ versus two kinds of magnetic field, the surface magnetic dipolar field component $B_d$ (squares) and the field strength at the estimated emission altitude $B_r$ (circles). The filled circles correspond to very old pulsars. Circles with star correspond to very young pulsars. The PSRs B1534+12 and B1913+16 are marked by the circles with horizontal arrows (see Section 3).

3, these two pulsars seem to have $B_r$ values considerable lower than the rest of the group of normal pulsar (between two dashed vertical lines). It is worthy of noticing, that even their characteristic age (not to mention the “true age”) is much smaller than rest of the old pulsars (filled circles). However, if we use “true age” (after ACW) in equation (5) then the verified values of $B_r$ increase such that they fall into a region between dashed vertical lines (which is marked by horizontal arrows). This indicates that the emission altitudes $r_{em}$ and hence the magnetic field $B_r$ in the emission region depends on the true age rather than on the characteristic age. Of course, in most cases, when the actual period $P$ is much larger than the initial period, these values are the same.

Finally, let us discuss a possibility that the magnetic field lines in the emission region are not purely dipolar. The observational opening angle $\rho = \rho(W; \alpha, \beta)$ is calculated using geometrical equation proposed by Gil, Gronkowski & Rudnicki (1984), which holds on the pulsar centered celestial sphere, independently of geometry of magnetic field.
lines in the emission region. This observationally inferred opening angle $\rho = \rho_{\text{obs}}(W; \alpha, \beta)$ is also the angle between magnetic axis and the tangent to magnetic field lines at points where the emission correspondig to the apparent pulse width $W$ originates. For dipolar field lines $\rho$ is described by equation (2), with $s = 1$ for last open field lines corresponding to low level emission below 1% of the maximum intensity (e.g. Table 1). For a simple model of star-centered quadrupole field in the radio emission region, the opening angles $\rho_q = 0.8s(r_q/R)P^{-1/2}$ deg (where $s < 1$), as compared with $\rho_d = 1.2(r_d/R)^{1/2}P^{-1/2}$ deg for dipolar geometry. Assuming that $\rho_d = \rho_q = \rho_{\text{obs}}(W; \alpha, \beta)$ we get $r_d^{1/2}/r_q = s/R^{1/2}$, where $r_d$ and $r_q$ represent the emission altitudes for dipole and quadrupole field lines, respectively. Thus, if one allows nondipolar magnetic field in the emission region, then equation (1) represents the upper limit for emission altitudes. Moreover, the radius-to-frequency mapping would change from $r \propto \nu^{-p}$ for dipolar field to $r \propto \nu^{-p/2}$ for quadrupole field, where the index $p$ is constrained observationally as 0.2±0.1 (see Table 4 in Paper II). Unfortunately, this constraint is too weak to differentiate between dipole and quadrupole field lines.

Theoretically, a deviations from dipolar form can be caused by (i) outflow of relativistic currents, (ii) field lines sweep-back due to fast rotation, and (iii) currents flowing in the thin layer of the neutron stars outer crust. Let us briefly discuss these three cases:

(i) The outflow of the charged component of the magnetospheric plasma cannot change a structure of strong magnetic field generated inside the neutron star. In fact, using the Ampere’s law $\oint B \cdot ds = (4\pi/e)J_p$, where $B_p$ is the toroidal component generated by the maximum available poloidal current through the polar cap $J_p = 4\pi e \rho_{cJ} r_p^2 (r_{em}/R)^{3/2}$, where $r_{em} \geq R$ is the emission altitude, $r_p \approx 10^4 P^{-1/2} \text{cm}$ is the polarp cap radius, $\rho_{cJ} = B/cP$ is the Goldreich-Julian (1969) corotational charge density, $c$ is the speed of light, $B$ is a poloidal (dipolar) pulsar magnetic field and $P$ is the pulsar period. One can easily show that $B_p/B \approx 10^{-6} P^{-3/2} (r_{em}/R)^{3/2}$. Thus, for typical pulsar $B_p$ is absolutely negligible, while for the millisecond pulsars the toroidal contribution to dipolar magnetic field can achieve a few percent at most.

(ii) The sweep-back of field lines by corotation with neutron star can occur near the light cylinder, where the corotation velocity approaches the speed of light. However, it is generally accepted that the radio emission originates at altitudes much smaller than the light cylinder radius (Cordes 1978, BCW, Kramer et al. 1997).

(iii) The currents in the crust are most probably sources of multipolar components of the pulsar magnetic field outside the neutron star. However, these contributions decay rapidly with increasing distance from the star surface, and at radio emission altitudes only dipolar field can be expected, at least in normal pulsars. In the millisecond pulsars, in which the emission region can be as close as few stellar radii from the polar cap, the magnetic field lines can, in principle, deviate significantly from dipolar form. However, Arons (1993) has shown that the location of the spin-up line in the $P - \dot{P}$ diagram constrains the surface strength of multipole components to no more than 40% of the dipole field in millisecond pulsars.

## 4 CONCLUSIONS

The main conclusions from this paper and from previous work (Papers I and II) on emission regions in radio pulsars are following:

(1) A radius-to-frequency mapping operates in the emission region and has a form of $r_{em}(\nu) \propto \nu^{-0.21±0.07}$.

(2) Pulsar radio emission is typically generated at altitudes smaller than a few per cent of the light-cylinder radius $r_{LC}$ and the ratio $r_{KG}/r_{LC}$ decreases with increasing period. The total extent of the emission region is smaller than 250$R$ in longer-period pulsars and correspondingly less in shorter-period pulsars.

(3) The emission region in old pulsars is located at correspondingly lower altitudes than in young pulsars with approximately the same period.

In summary, pulsar emission altitudes depend on frequency $\nu$, pulsar period $P$ and pulsar age $\tau$, as described by equation (1).

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