Domination and 2-packing numbers in graphs

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Abstract

A dominating set of a graph $G$ is a set $D \subseteq V(G)$ such that every vertex of $G$ is either in $D$ or is adjacent to a vertex in $D$. The domination number of $G$, $\gamma(G)$, is the minimum order of a domination set. On the other hand, a 2-packing set of a graph $G$ is a set $R \subseteq E(G)$ such that if three edges are chosen in $R$ then they are not incidents in a common vertex. The 2-packing number of $G$, $\nu_2(G)$, is the maximum order of a 2-packing set. It can be proved that for any connected graph $G$ satisfies $\gamma(G) \leq \nu_2(G) - 1$.

In this paper, a characterization of simple connected graphs $G$ will be given to satisfy $\gamma(G) = \nu_2(G) - 1$.

Keywords. Domination, 2-packing

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1 Introduction

Throughout this paper $G$ is a finite, undirected, simple and connected graph with the vertex set $V(G)$ and the edge set $E(G)$. For $A \subseteq V(G)$, $G[A]$ is denoted to the induced graph by $A$. The open neighborhood of a vertex $u \in V(G)$, denoted by $N(u)$, is the set of vertices of $V(G)$ adjacent to $u$ in $G$, and the closed neighborhood of a vertex $u \in V(G)$, denoted by $N[u]$, is defined as $N[u] = N(u) \cup \{u\}$. The degree of a vertex $u \in V(G)$, denoted by $\text{deg}(u)$, is defined as $\text{deg}(u) = |N(u)|$, and it is denoted by $\delta(G)$ and $\Delta(G)$ to

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be the minimum and maximum degree of the graph $G$, respectively. Let $H$ be a subgraph of $G$. The restricted open neighborhood for a vertex $u \in V(H)$, denoted by $N_H(u)$, is defined as $N_H(u) = \{v \in V(H) : uv \in E(H)\}$, the restricted closed neighborhood for a vertex $u \in V(H)$, denoted by $N_H[u]$, is defined as $N_H[u] = N_H(u) \cup \{u\}$, and the restricted degree of a vertex $u \in V(H)$, denoted by $\text{deg}_H(u)$ is defined as $\text{deg}_H(u) = |N_H(u)|$.

A dominating set of a graph $G$ is a set $D \subseteq V(G)$ such that each vertex $u \in V(G) \setminus D$ satisfies $N(u) \cap D = \emptyset$. The domination number of $G$, denoted by $\gamma(G)$, is the minimum order of a domination set. An independent set of a graph $G$ is a set $I \subseteq V(G)$ such that any two vertices of $I$ are not adjacent. The independent number of $G$, denoted by $\alpha(G)$, is the maximum order of an independent set. A covering set of a graph $G$ is a set $T \subseteq V(G)$ such that every edge of $G$ has at least one end in $T$. The covering number of $G$, denoted by $\beta(G)$, is the minimum order of a covering set. It is well-known that if $G$ is a graph with no isolated vertices then

$$\gamma(G) \leq \beta(G). \quad (1)$$

In [4] a characterization of simple graphs was given which attains the inequality of the Equation (1) (see also [5, 6]). A 2-packing set of a graph $G$ is a set $R \subseteq E(G)$ such that if three edges are chosen in $R$ then they are not incidents in a common vertex. The 2-packing number of $G$, denoted by $\nu_2(G)$, is the maximum order of a 2-packing set. In [2] and [3] the following inequalities holds for a connected graph $G$

$$\left\lceil \nu_2(G)/2 \right\rceil \leq \beta(G) \leq \nu_2(G) - 1. \quad (2)$$

In [2] a characterization of simple connected graphs $G$ was given which attains the upper and lower inequality of the Equation (2). There are interesting results related to the covering and 2-packing numbers in a more general context, see [1, 3].

By Equations (1) and (2)

$$\gamma(G) \leq \nu_2(G) - 1 \quad (3)$$

is obtained.

In this paper a characterization of simple connected graphs is given which attains the inequality of the Equation (3). It is important to say that there is not a lower bound for the domination number of a graph in terms of the 2-packing number, since it is easy to see that $\gamma(K_n) = 1$ and $\nu_2(K_n) = n$, for all $n \geq 3$. 

2
2 Graphs with $\gamma(G) = \nu_2(G) - 1$

To begin with, some terminology is introduced in order to simplify the description of the simple connected graphs $G$ which satisfy $\gamma(G) = \nu_2(G) - 1$.

**Definition 2.1.** Let $P_4$ be a path of length 4, say $P_4 = v_0 \cdots v_4$. We define the tree $T_{s,t} = (V, E)$, with $s + 4 = r$, as follow:

- $V = V(P_4) \cup \{p_1, \ldots, p_s\} \cup \{q_1, \ldots, q_s\} \cup \{w_1, \ldots, w_t\}$,
- $E = E(P_4) \cup \{p_i q_i : i = 1, \ldots, s\} \cup \{v_2 w_i : i = 1, \ldots, t\} \cup \{v_2 p_i : i = 1, \ldots, s\}$,

where $s, t \in \mathbb{N}$ and depicted in Figure 2.1.

**Proposition 2.1.** $\gamma(T_{s,t}) = r - 1$.

**Proof.** It is not difficult to see that $D = (V(P_4) \setminus \{v_0, v_4\}) \cup \{p_1, \ldots, p_s\}$ is a covering set of $T_{s,t}$ of cardinality $s + 3 = r - 1$, which implies that $r - 1 \geq \gamma(T_{s,t})$. On the other hand, it is easy to see that if $u \in D$ then there is a $v_u \in N(u)$ such that $\deg(v_u) = 1$, that is $D = \{u \in V(G) : uv \in E(G) \text{ with } \deg(v) = 1\}$. Hence $\gamma(T_{s,t}) \geq |D| = r - 1$. Therefore $\gamma(T_{s,t}) = r - 1$, and the statement holds.

**Proposition 2.2.** $\nu_2(T_{s,t}) = r$.

**Proof.** It is not difficult to see that $R = E(P_4) \cup \{p_i q_i : i = 1, \ldots, s\}$ is a 2-packing of $T_{s,t}$ of cardinality $s + 4 = r$, which implies that $\nu_2(T_{s,t}) \geq r$. On the other hand, suppose that there is a maximum 2-packing of $T_{s,t}$ of cardinality $r + 1$, then there are edges $v_2 w_i, v_2 w_j \in E(T_{s,t})$, with $i \neq j$. 

![Figure 2.1: Tree $T_{s,t}$](image)
\{1, \ldots, t\}, necessarily. This implies necessarily that
\[ R = \{p_iq_i : i = 1, \ldots, s\} \cup \{v_0v_1, v_3v_4\} \cup \{v_2w_i, v_2w_j\}, \]
which is a contradiction, since \(|R| = r + 1\). Hence \(\nu_2(T_{s,t}^r) \leq r\). Therefore \(\nu_2(T_{s,t}^r) = r\), and the statement holds. 

**Theorem 2.1.** \(\gamma(T_{s,t}^r) = \nu_2(T_{s,t}^r) - 1\).

**Proof.** Immediate consequence of Proposition 2.1 and 2.2.

In the next section it will be proven that if \(\gamma(G) = \nu_2(G) - 1\), then \(G \simeq T_{s,t}^{\nu_2}\), where \(\nu_2 = \nu_2(G)\).

### 3 Results

It is considered simple connected graphs \(G\) with \(|E(G)| > \nu_2(G)\) due to the fact \(|E(G)| = \nu_2(G)\) if and only if \(\Delta(G) \leq 2\). Moreover, it is assumed that \(\nu_2(G) \geq 5\), since in \cite{3} the following was proven:

**Proposition 3.1.** \cite{3} Let \(G\) be a simple connected graph with \(|E(G)| > \nu_2(G)\), then \(\nu_2(G) = 2\) if and only if \(\beta(G) = 1\).

**Proposition 3.2.** \cite{3} Let \(G\) be a simple connected graph with \(|E(G)| > \nu_2(G)\). If \(\nu_2(G) = 3\) then \(\beta(G) = 2\).

**Theorem 3.1.** \cite{3} Let \(G\) be a simple connected graph with \(|E(G)| > \nu_2(G)\). If \(\nu_2(G) = 4\) then \(\beta(G) \leq 3\).

Hence, since \(\gamma(G) \leq \beta(G)\), for a connected graph \(G\), then the bipartite graph \(K_{1,n}\), for \(n \geq 3\), is the unique graph with \(\gamma(G) = 1\) and \(\nu_2(K_{1,n}) = 2\). On the other hand, it is not difficult to prove, see \cite{2, 3}, the families of graphs of Figure 3.1 are the unique families of graphs which satisfy \(\nu_2 = 3\) and \(\beta = 2\), incise \((a)\), and \(\nu_2 = 4\) and \(\beta = 3\), incise \((b)\).

**Lemma 3.1.** Let \(G\) be a connected graph with \(|E(G)| > \nu_2(G)\) and let \(R\) be a maximum 2-packing of \(G\). If \(G[R]\) is a connected graph then \(\gamma(G) \leq \nu_2(G) - 2\).
Case (i): $I = \emptyset$, that is $V(G[R]) = V(G)$. It is well-known that either $G[R]$ is a path or a cycle. Suppose that $G[R]$ is a cycle, that is $G[R] = v_0v_1 \cdots v_{\nu_2}v_0$. If there are not adjacent vertices $v_i, v_j \in V(G[R])$, then $D = V(G[R]) \setminus \{v_i, v_j\}$ is a dominating set of $G$ of size $\nu_2(G) - 2$, otherwise the graph $G$ is a complete graph of order $\nu_2$, which implies that $\gamma(G) \leq \nu_2(G) - 2$, since $\gamma(G) = 1$.

On the other hand if $G[R] = u_0u_1 \cdots u_{\nu_2}$ is a path, then trivially $D = V(G[R]) \setminus \{v_0, v_{\nu_2}\}$ is a dominating set of cardinality $\nu_2(G) - 1$. It is assumed that either $u_0u_j \in E(G)$ or $u_\nu_2u_j \in E(G)$, for some $u_j \in D^* = D \setminus \{u_1, u_{\nu_2+1}\}$, otherwise $D \setminus \{u_j\}$ is a dominating set of $G$, which implies that $\gamma(G) \leq \nu_2(G) - 2$, and the statement holds. Without loss of generality, if there is $u_i \in D^{**} = D^* \setminus \{u_2, u_{\nu_2-2}\}$ (since $\nu_2(G) \geq 5$) such that $u_0u_i \in E(G)$ then $u_{i+1}u_{\nu_2} \notin E(G)$, otherwise $\hat{R} = (R \setminus \{u_iu_{i+1}\}) \cup \{u_0u_i, u_{i+1}u_{\nu_2}\}$ is a 2-packing of $G$ of cardinality $\nu_2(G) + 1$, which is a contradiction. Similarly if there is $u_i \in D^{**}$ such that $u_0u_i \in E(G)$ then $u_{i-1}u_{\nu_2} \notin E(G)$, otherwise $\hat{R} = (R \setminus \{u_{i-1}u_i\}) \cup \{u_0u_i, u_{i-1}u_{\nu_2}\}$ is a 2-packing of $G$ of cardinality $\nu_2(G) + 1$, which is a contradiction. Hence if there is $u_i \in D^{**}$ such that $u_0u_i \in E(G)$ then either $(D \setminus \{u_{i-1}, u_i, u_{i+1}\}) \cup \{u_0\}$ is a dominating set of cardinality $\nu_2(G) - 3$ or $(D \setminus \{u_i, u_{i+1}\}) \cup \{u_0\}$ is a dominating set of cardinality $\nu_2(G) - 2$ or $(D \setminus \{u_{i-1}, u_i\}) \cup \{u_0\}$ is a dominating set of cardinality $\nu_2(G) - 2$, and the statement holds.

Figure 3.1: In (a) the only family of graphs which satisfies $\nu_2 = 3$ and $\gamma = 2$ is shown. On the other hand, in (b) the only family of graphs which satisfies $\nu_2 = 4$ and $\gamma = 3$ is shown.

**Proof.** Let $R$ be a maximum 2-packing of $G$ such that $G[R]$ is a connected graph, and let $I = V(G) \setminus V(G[R])$. Hence, if $I \neq \emptyset$ then it is an independent set.
Case(ii): \( I \neq \emptyset \). Suppose that \( G[R] \) is a path, say \( G[R] = u_0u_1 \cdots u_{\nu_2} \). Note that if \( u \in I \) then \( u_0, u_{\nu_2} \not\in N(u) \), otherwise either \( R \cup \{u_0u\} \) or \( R \cup \{uu_{\nu_2}\} \) is a 2-packing of cardinality \( \nu_2(G) + 1 \), which is a contradiction. If there is \( u_i \in D^{**} = V(G[R]) \setminus \{u_0, u_1, u_{\nu_2-1}, u_{\nu_2}\} \) (since \( \nu_2(G) \geq 5 \)) such that \( u_i \not\in N(u) \), for all \( u \in I \), then \( \hat{D} = V(G[R]) \setminus \{u_0, u_i, u_{\nu_2}\} \) is a dominating set of cardinality \( \nu_2(G) - 2 \), and the statement holds.

Suppose that for every \( u_i \in D^{**} \) there is \( u \in I \) such that \( u_i \in N(u) \). If \( u_i \in N(u) \), for some \( u \in I \), then \( u_{\nu_2} \not\in N(u) \), otherwise the following set \( \hat{R} = (R \setminus \{u_0u_i\}) \cup \{uu_i, uu_{\nu_2}\} \) is a 2-packing of \( G \) of cardinality \( \nu_2(G) + 1 \), which is a contradiction. Therefore, if \( u_i, u_{\nu_2} \in D^{**} \) then there are \( u, u' \in I \) such that \( u_i \in N(u) \) and \( u_{\nu_2} \in N(u') \). Hence the following set \( \hat{R} = (R \setminus \{u_0u_i\}) \cup \{uu_i, uu_{\nu_2}\} \) is a 2-packing of \( G \) of cardinality \( \nu_2(G) + 1 \), which is a contradiction.

To end, suppose that \( G[R] \) is a cycle, say \( G[R] = u_0u_1 \cdots u_{\nu_2-1}u_0 \). Similarly than before, it is assumed that \( u_i, u_{\nu_2} \not\in N(u) \), for every \( u_i, u_{\nu_2} \in D^{**} \) and \( u \in I \). Moreover if there are \( u, u' \in I \), with \( u_i \in N(u) \) and \( u_{\nu_2} \in N(u') \), for some \( u_i \in D^{**} \), then the following set \( \hat{R} = (R \setminus \{u_0u_i\}) \cup \{uu_i, uu_{\nu_2}\} \) is a 2-packing of \( G \) of cardinality \( \nu_2(G) + 1 \), which is a contradiction. Therefore if \( u_i \in D^{**} \) then there is \( u \in I \) such that \( u_i \in N(u) \), and hence \( \hat{D} = V(G[R]) \setminus \{u_{i-1}, u_{i+1}\} \) is a dominating set of \( G \) of cardinality \( \nu_2(G) - 2 \), and the statement holds.

\[ \blacksquare \]

**Lemma 3.2.** Let \( G \) be a connected graph with \( |E(G)| > \nu_2(G) \) and let \( R \) be a maximum 2-packing of \( G \). If \( \gamma(G) = \nu_2(G) - 1 \) then \( G[R] \) is a forest.

**Proof.** Let \( R \) be a maximum 2-packing of \( G \). The induced graph \( G[R] \) is not a connected graph (by Lemma 3.1), hence let \( R_1, R_2, \ldots, R_k \), with \( k \leq \nu_2(G) - 1 \), be the components of \( G[R] \) with \( k \) as small as possible. Suppose that \( R_1, \ldots, R_s \), are the components of \( G[R] \) with only one edge (that is \( R_i \simeq K_2 \), for \( i = 1, \ldots, s \), that is \( R_i = p_iq_i \), for \( i = 1, \ldots, s \), and \( R_{s+1}, \ldots, R_k \) are the components with at least two edges. Trivially

\[
D = \{u \in V(G[R]) : \text{deg}_R(u) = 2\} \cup \{p_i \in V(R_i) : i = 1, \ldots, s\}
\]

is a dominating set of cardinality at most \( \nu_2(G) \).

Let \( I = V(G) \setminus V(G[R]) \) then:
Case (i): $I = \emptyset$. Suppose that $R_{s+1}$ is a cycle and there is an edge $uv \in E(G)$ such that $u \in V(R_{s+1})$ and $v \in V(R_j)$, for some $j \in \{1, \ldots, k\} \setminus \{s + 1\}$. Then $\hat{D} = D \setminus N_{R_{s+1}}(u)$ is a dominating set of $G$ of cardinality $\nu_2(G) - 2$, which is a contradiction. Therefore, there are not cycles as components of $G[R]$.

Case (ii): $I \neq \emptyset$. Suppose that $R_{s+1}$ is a cycle and there is $u \in I$ such that $uv_{s+1}, uw_j \in E(G)$, where $v_{s+1} \in V(R_{s+1})$ and $v_j \in V(R_j)$ for some $j \in \{1, \ldots, k\} \setminus \{s + 1\}$.

Remark 3.1. If $v, w \in V(R_{s+1})$ are such $vw \in E(G[R_{s+1}])$ then $v, w \notin N(u)$, otherwise the following set $\tilde{R} = (R \setminus \{vw\}) \cup \{uv, uw\}$ is a 2-packing of $G$ of cardinality $\nu_2(G) + 1$, which is a contradiction. Similarly, if $v, w \in V(R_{s+1})$ are such $vw \in E(G[R_{s+1}])$ then there are not $u, u' \in I$ such that $v \in N(u)$ and $w \in N(u')$, otherwise the following set $\tilde{R} = (R \setminus \{vw\}) \cup \{uv, u'w\}$ is a 2-packing of $G$ of cardinality $\nu_2(G) + 1$, which is a contradiction.

By Remark 3.1 it is implied that $\hat{D} = D \setminus N_{R_{s+1}}(v_{s+1})$ is a dominating set of $G$ of cardinality $\nu_2(G) - 2$, which is a contradiction. Therefore there are not cycles as components of $G[R]$.

\[\blacksquare\]

**Theorem 3.2.** Let $G$ be a connected graph with $|E(G)| > \nu_2(G)$. If $\gamma(G) = \nu_2(G) - 1$ then $G \simeq T_{s,t}^{\nu_2}$, where $\nu_2 = \nu_2(G)$.

**Proof.** Let $R$ be a maximum 2-packing of $G$ and $R_1, \ldots, R_k$ be the components of $G[R]$, with $2 \leq k \leq \nu_2(G) - 1$ (by Lemma 3.1). By Lemma 3.2 each component of $G[R]$ is a path. Suppose that $R_1, \ldots, R_s$, are the components of $G$ with only one edge, that is $R_i = p_iq_i$, for $i = 1, \ldots, s$, and $R_{s+1}, \ldots, R_k$ are the components with at least two edges. Trivially

$$D = \{u \in V(G[R]) : \deg_R(u) = 2\} \cup \{p_i \in V(R_i) : i = 1, \ldots, s\},$$

is a dominating set of cardinality at most $\nu_2(G) - 1$. This implies either there is at most one component of length greater or equal than 2 and the rest of the components have only one edge, or all components of $G[R]$ have only one edge. Suppose that $k$ is as small as possible. Hence $R_1, \ldots, R_{k-1}$ are the components with only one edge and $R_k$ is a path of length greater or equal than 2.
Remark 3.2. If \( u \in I = V(G) \setminus V(G[R]) \) then either \( u_\pi \not\in E(G) \) or \( u_\eta \not\in E(G) \), for all \( i = 1, \ldots, k - 1 \), otherwise either \( \hat{R} = R \cup \{u_\pi\} \) or \( \hat{R} = R \cup \{u_\eta\} \) is a 2-packing of \( G \) of cardinality \( \nu_2(G) + 1 \), which is a contradiction.

It is assumed that \( |E(R_k)| \geq 4 \), otherwise if \( R_k = v_0 \cdots v_s \), with \( s = 2, 3 \), then there is an edge \( v_3w \in E(G) \) (by Remark 3.2), with \( w \in V(R_t) \), for some \( t \in \{1, \ldots, k-1\} \), since \( G \) is a connected graph. Hence the following set \( \hat{R} = (R \setminus \{v_0v_1\}) \cup \{v_1w\} \) is a maximum 2-packing of \( G \) with less components than \( R \), which is a contradiction. Therefore, \( R_k = v_0v_1 \cdots v_l, \) where \( l \geq 4 \).

Let \( V^*_k = V(R_k) \setminus \{v_0, v_1, v_{l-1}, v_l\} \). If \( v lp \in E(G) \), with \( v_i \in V^*_k \) and \( p \in V(R_t) \), for some \( t \in \{2, \ldots, k\} \), then \( \deg(q_t) = 1 \), otherwise either \( v_3q_t \in E(G) \) or \( v_3q_t \in E(G) \), for some \( v_j \in V^*_j \setminus \{v\} \). In both cases either \( \hat{R} = (R \setminus N_{R_k}(v)) \cup \{v lp, v'q_t\} \) or \( \hat{R} = (R \setminus \{v_{l-1}v, v_jv_{j+1}\}) \cup \{v lp, v_3q_t\} \) is a maximum 2-packing of \( G \) with a cycle as a component, which is a contradiction (by Lemma 3.2). Therefore, for all \( i \in \{1, \ldots, k-1\} \) there is \( v \in V^*_k \) such that \( p v \in E(G) \) and \( \deg(v) = 1 \).

Remark 3.3. If \( u \in I \) then \( uv \in E(G) \) for some \( v \in V^*_k \), otherwise either \( \hat{R} = R \cup \{v_0u\} \) is a 2-packing of \( G \) of cardinality \( \nu_2(G) + 1 \), which is a contradiction, or \( \hat{R} = (R \setminus \{v_1v_2\}) \cup \{v_1u\} \) is a 2-packing of \( G \) with two components as paths of length greater than 2, which is a contradiction.

Now, it will be proven that \( l = 4 \), that is \( R_k = v_0 \cdots v_4 \). Suppose that \( R_k = v_0 \cdots v_i \), with \( i \geq 5 \). If \( v_i, p_j \in E(G) \), for some \( v_i \in D^{**} \) and \( j \in \{1, \ldots, s\} \), then either \( \hat{R} = (R \setminus \{v_iv_{i+1}\}) \cup \{v_ip_j\} \) or \( \hat{R} = (R \setminus \{v_{i-1}v_i\}) \cup \{v_ip_j\} \) is a maximum 2-packing of \( G \) with two components as paths of length greater than 2, which is a contradiction. Therefore \( l = 4 \), \( v_2p_1 \in E(G) \) and \( v_2u \in E(G) \), for all \( i = 1, \ldots, s \) and \( u \in I \).

Concluding, to show that \( G \simeq T_{s,i}^{v_2} \) it shall be verified that \( v_2v_4 \not\in E(G) \) and \( I \neq \emptyset \). If \( v_2v_4 \in E(G) \) then \( \hat{R} = (R \setminus \{v_1v_2, v_3v_4\}) \cup \{v_1v_3, v_2p_1, \} \) (since \( \nu_2(G) \geq 5 \) is a maximum 2-packing with a path of length 5 as a component, which is a contradiction. Thus, the graph \( G \) does not contain a cycle as a subgraph of \( G \). On the other hand, if \( I = \emptyset \), then \( D^{**} \setminus \{v_2\} \) is a dominating set of \( G \) of cardinality \( v_2(G) - 2 \), which is a contradiction. Therefore \( G \simeq T_{s,i}^{v_2} \). \( \blacksquare \)
Theorem 3.3. Let $G$ be a connected graph with $|E(G)| > \nu_2(G)$. Then $\gamma(G) = \nu_2(G) - 1$ if and only if $G \simeq T_{s,t}^{\nu_2}$.

Proof. Immediate consequence of Theorem 2.1 and 3.2. □

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