Hole spin qubits in Si FinFETs with fully tunable spin-orbit coupling and sweet spots for charge noise

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The strong spin-orbit coupling in hole spin qubits enables fast and electrically tunable gates, but at the same time enhances the susceptibility of the qubit to charge noise. Suppressing this noise is a significant challenge in semiconductor quantum computing. Here, we show theoretically that hole Si FinFETs are not only very compatible with modern CMOS technology, but they present operational sweet spots where the charge noise is completely removed. The presence of these sweet spots is a result of the interplay between the anisotropy of the material and the triangular shape of the FinFET cross-section, and it does not require an extreme fine-tuning of the electrostatics of the device. We present how the sweet spots appear in FinFETs grown along different crystallographic axes and we study in detail how the behaviour of these devices change when the cross-section area and aspect ratio are varied. We identify designs that maximize the qubit performance and could pave the way towards a scalable spin-based quantum computer.

I. INTRODUCTION

Strong spin-orbit coupling [1] is a key ingredient to build a scalable spin-based quantum computer [2, 3], enabling fast and fully electrical manipulations of quantum bits [4–6]. Suitable platforms to reach large values of spin-orbit interactions are p-doped semiconductor nanowires, where the charge carriers are holes rather than electrons [7–11]. A great advantage of holes is their small susceptibility to hyperfine noise [12–14], which is a leading decoherence mechanism for electron spin qubits. This effect is a consequence of the p-type wavefunction of the holes and makes them promising candidates for quantum information processing [15–20].

When holes are strongly confined in two directions, an externally tunable electric field generates a large effective spin-orbit field [21, 22] that results in ultrafast Rabi frequencies, larger than 400 MHz [23, 24], and in spin-orbit lengths of tens of nanometers [25–31], shorter than typical interdot distances. The regime of strong coupling between spins and photons in microwave resonators [32, 33] has been predicted in these systems [34], which could enable long-range coupling between distant qubits. Because of the large spin-orbit interaction, hole-semiconductor heterostructures have attracted also much interest as platforms to detect and manipulate Majorana bound states [35–37].

On the other hand, large interactions between the spin and the charge degrees of freedom render the system strongly susceptible to charge noise, reducing the qubit lifetime [23, 38–40]. Efforts to find operational sweet spots where charge noise is reduced have been focusing on planar Ge qubit designs [41] or considering single atoms [42], as well as artificial spin-orbit fields [43]. The appearance of operational sweet spots depending on the direction of the applied magnetic field has also been analyzed [44, 45]. In these studies, the focus is to find working points where the qubit is not susceptible to small changes of the spin-orbit coupling, i.e. where its first derivative as a function of an external homogeneous electric field vanishes. However, in electrostatically defined hole quantum dots, the large value of spin-orbit coupling makes the qubit energy fluctuate also as a function of the dot size, which is directly dependent on the external gate potential and can thus create an additional noise channel that is not suppressed at these sweet spots. Consequently, convenient systems to perform robust spin-based quantum computations need the ability to on-demand fully switch ON and OFF the spin-orbit interaction depending on whether the qubit is operational or idle.

The tunability of the spin-orbit coupling by varying the electric field is a well-known feature of semiconductor nanowires [9, 10]. In many typical geometries, such as wires with rectangular or circular cross-sections, that have been extensively studied both theoretically [21, 22, 34] and experimentally [23, 30, 46, 47], the spin-orbit interaction is only fully removed when the external electric field is strictly zero, and it remains finite otherwise. While completely removing the electric field can be possible for etched [48] or self-assembled [49] quantum dots, in electrostatically defined nanostructures, an external gate potential is required to delimit the dot and the electric field cannot be easily set to zero, resulting in a residual spin-orbit interaction which degrades the qubit performance.

In this context, we find that Silicon Fin Field Effect Transistors (FinFETs) [50–52] are not only appealing because of their compatibility to modern semiconductor industry, but they naturally present operational sweet spots where charge noise completely vanishes. In these devices, the spin-orbit interactions can be exactly switched off at finite values of the electric field, and thus FinFET qubits are ideal candidates to reliably store quantum information. By driving the qubit away from the sweet spot, large spin-orbit interactions are restored, re-enabling ultrafast qubit gates, and making these devices particularly well-suited for quantum computing. Silicon is one of the frontrunner materials for scalable large scale quantum computers. High fidelity two-qubit gates can be executed in electron Si qubits [53–57], and
reliable qubit operations can be performed at temperatures higher than 1 K [58, 59]. Singlet-triplet qubit operations [60] and control over higher spin states [61] have also been demonstrated, and, in addition, Silicon is also a good platform to implement different scalable read-out schemes [62–65], including parity measurements [66]. In terms of noise, Si offers the unique possibility to drastically reduce the effect of hyperfine noise by isotopic purification, [67, 68], leading to spin qubits whose performances are essentially limited only by the charge noise [38].

Comparing to the many possible architectures such as planar FETs [69] or square FinFETs [46, 47], a crucial feature of the Si FinFETs examined here is their nearly triangular cross-section, which is not inversion symmetric, and has a qualitatively different behaviour compared to symmetric wires. In particular, holes confined in wires with triangular cross-sections present extremely large intrinsic spin-orbit interactions even without external fields that result in a spin-orbit length of the same order of the side of the triangle and smaller than the typical interdot distance. Depending on the orientation of the wire with respect to the crystallographic axes, an external gate potential can enhance or suppress this intrinsic spin-orbit field, potentially leading to a complete removal of the spin-orbit interaction at a finite value of the electric field. The strong dependence of the properties of the nanostructures on the growth direction is a characteristic feature of anisotropic semiconductors such as Si [22, 70–72], where the kinetic energy is not rotationally invariant, but only preserves the lower four-fold rotational symmetry of the cubic lattice [1].

A convenient orientation for the FinFET is the one where the wire extends along the [001] crystallographic axis and the height of the fin is directed along the [110] axis. This orientation is known to guarantee the largest mixing of heavy and light holes in Si and to maximise the spin-orbit interactions in Si wires with inversion-symmetric cross-sections [22] and rectangular quantum dots [70]. We find that for a wide range of device designs, it also permits the complete removal of charge noise. A similar suppression of charge noise can occur when the wire is grown along different directions. For example, convenient sweet spots appear in Silicon-on-Insulator (SOI) FinFETs [50] or hut-wires [28] when the wire extends along the [110] direction, the standard growth direction used in experiments [28, 46, 51, 52]; in this case, however, the spin-orbit interactions away from the sweet spots tend to be smaller.

By analysing the response of different state-of-the-art FinFET designs [50–52] to realistic electric fields with an inhomogeneous profile, we identify qualitatively distinct mechanisms that can switch off the spin-orbit interactions. When the triangular cross-section of the fin has a side larger than ∼35 nm, the intrinsic spin-orbit interactions are cancelled out by a conventional direct Rashba-like component [21, 22] that is driven by a homogeneous electric field perpendicular to the substrate and that is easily generated in FinFETs. In contrast, the response of smaller FinFETs to the electric field changes drastically as a result of the spin-orbit split-off hole band. This band is energetically separated from heavy and light holes by the bulk Si spin-orbit gap [1] and becomes relevant in small wires. In these devices, these states tend to remove the sweet spot and a more challenging device design is required to counteract their action. In SOI-FinFETs, the spin-orbit coupling can be suppressed by an inhomogeneous electric field, whose effect is enhanced when the fin is squeezed, or by inverting the polarity of the gate potential and pushing the hole wavefunction towards the bottom of the fin. In bulk FinFETs, the thick Si substrate strongly coupled to the fin naturally restores the spin-orbit sweet spot but at the same time it decreases the overall spin-orbit interaction by an order of magnitude compared to the SOI isolated nanowire.

The present paper is organized as follows. In Sec. II, we first introduce the different FinFET designs and the theoretical model that we use in our analysis.

In Sec. III, we consider an ideal fin with a large equilateral triangular cross-section and study the long wavelength dynamics of the holes confined there. We first restrict ourselves to the analysis of heavy holes and light holes, and we pay particular attention to how different growth directions affect the effective spin-orbit interactions. We divide the spin-orbit interactions in two terms: an isotropic direct Rashba-like term [21, 22] vanishing without an external electric field, and an anisotropic term dependent on the orientation of the wire. The latter component includes intrinsic spin-orbit interactions that are large even without electric field and arise because of the lack of inversion symmetry of the cross-section. At particular growth directions, the competition between these spin-orbit terms generates sweet spots where charge noise is suppressed. We examine this effect in detail by considering realistic inhomogeneous electric field profiles and we identify different mechanisms that cause it. Here, we also summarize the differences between triangular and square wires and we comment on the effect of a possible moderate strain on the spin-orbit switch.

In Sec. IV, we study how the spin-orbit interactions change when the spin-orbit split-off hole band are included. This band strongly affects Si FinFETs with small fins, with side length ∼35 nm, and can even remove the charge noise sweet spot. In Sec. IV A, we present FinFET designs that counteract the action of the SOHs. We focus on both SOI and bulk FinFETs. In SOI FinFETs, the charge noise sweet spot is restored by appropriately squeezing the fin and pushing the hole wavefunction towards the apex of the fin, or by growing the wire along the [110] direction, as standardly done in experiments [28, 46, 51, 52], and pushing the holes towards the bottom of the fin. In bulk FinFETs, the Si substrate below the wire tends to decrease the spin-orbit interactions, but at the same time it re-establishes the spin-orbit sweet spot.
even when the fin is small.

Finally, in Sec. V, we examine the charge noise in FinFET spin qubits. We find that working close to the spin-orbit sweet spot drastically suppresses the influence of charge noise on the qubit lifetime, strongly improving the dephasing time. By including in the analysis the fluctuations of the g-factor as a function of the electric field, the exact position of the sweet spot is slightly shifted, but the charge noise can still be exactly cancelled, resulting in a system fully resilient against small charge fluctuations.

II. MODEL

In this paper, we analyze the hole Si FinFET sketched in Fig. 1. The fin extends in the z-direction and it defines a nanowire with an isosceles triangular cross-section with equal sides \( L_y \) and base \( L_x \). We study two different FinFET designs: Silicon-on-Insulator and bulk FinFETs. In SOI FinFETs, the triangular fin lies on top of a dielectric material, while in bulk FinFETs, it lies on top of a Si substrate. The apex of the fin is covered by a dielectric material, while in bulk FinFETs, it lies on top of a Si substrate. The apex of the fin is covered by a dielectric with an ideal metallic gate placed on top. The top gate is fixed at a potential \( V_g \) measured with respect to a grounded back-gate at a distance \( d_B \) from the bottom of the wire. In bulk FinFET, a negative gate potential \( V_g \) is required to localize the hole wavefunction inside the fin, while in SOI FinFETs, the holes are confined in the wire by the dielectric and \( V_g \) can attain positive values, too.

The dynamics of this system is accurately described by the Hamiltonian

\[
H = H_{\text{LK}} + V_{\text{HW}}(x, y) + V_E(x, y),
\]

which comprises the hole kinetic energy \( H_{\text{LK}} \) and two distinct potential energies \( V_{\text{HW}} \) and \( V_E \).

The potential \( V_{\text{HW}} \) captures the abrupt interfaces between the semiconductor and the dielectric and because of the large energy gap between the materials, we model it by requiring the wavefunction to vanish at the edges of the system (hard-wall boundary conditions). In contrast, \( V_E \) describes the smoother and externally tunable electrostatic potential generated by \( V_g \). In the cross-section of the wire, this term is well-approximated by the multipole expansion

\[
V_E(x, y) = -e\mathbf{E} \cdot \mathbf{r} - \frac{e}{2} \mathbf{r} \cdot \delta \mathbf{E} \cdot \mathbf{r},
\]

that includes a homogeneous electric field vector \( \mathbf{E} = (E_x, E_y) \) and a tensor modelling the inhomogeneous component of the electric field \( (\delta \mathbf{E})_{ij} \approx \delta E_{ij} \delta_{ij} \); this tensor is approximately diagonal in the systems studied here. The coordinate system \( \mathbf{r} = (x, y) \) is centred in the center of mass of the wire. Importantly, both the homogeneous and inhomogeneous components of the electric field depend linearly on the external gate potential \( V_g \) and their strength can be tuned by the gate design. More details on this approximation and a thorough discussion on the

values of \( E_i \) and \( \delta E_{ij} \) in the FinFETs analyzed in this paper can be found in App. A.

A precise description of the kinetic energy of the holes in the valence bands of semiconductors includes the strong bulk spin-orbit interactions and is modelled by the \( 4 \times 4 \) Luttinger-Kohn (LK) Hamiltonian \( [1, 73] \)

\[
H_{\text{LK}} = \left( \gamma_1 + \frac{5}{2} \gamma_2 \right) \frac{p^2}{2m} - \frac{\gamma_2}{m} p^2 \cdot (\mathbf{J'})^2 - \frac{2\gamma_3}{m} \mathbf{p}_i \mathbf{p}_j \{ J'_i, J'_j \} + \epsilon_p,
\]

Figure 1. Sketch of a Si FinFET. In a), the blue and light-blue areas represent the semiconductor and the dielectric, respectively, while the gray areas are the metallic gates. The orientation of the Si FinFET with respect to the crystallographic axes (blue axes) depends on the angles \( \theta \) and \( \varphi \). The wire extends along the \( z \)-direction and the fin has an isosceles triangular shape with base \( L_y \), equal sides \( L_x \) and height parallel to the \( y \)-direction. The apex angle of the triangle is \( \Theta \). The dashed line at the bottom of the fin indicates the lower boundary of the fin. In a SOI FinFET there is a clear physical separation between the Si substrate and the wire, which is provided by a thick dielectric layer at the position of the dashed line. In contrast, in bulk Si FinFET, there is no physical separation between substrate and the fin. In this case, the holes are localized in the fin by negative values of the gate potential \( V_g \) applied with respect to a grounded back gate at a distance \( d_B \) from the bottom of the triangle. In b), we show the main orientations of the axes of confinement with respect to the crystallographic axes. The angles \( \theta \) and \( \varphi \) parametrize the rotation of the coordinate systems and their geometrical meaning is shown in a).
which describes the mixing of heavy holes (HHs) and light holes (LHs) with spin 3/2 and 1/2, respectively. Here, \( m \) is the bare electron mass, we use the anticommutator \( \{A, B\} = (AB + BA)/2 \), and \( \epsilon_p \) stands for cyclic permutations. Also, we defined \( p_i^2 = p_{x_i}^2 + p_{y_i}^2 + p_{z_i}^2 \) and the vectors \( p_i^2 = (p_{x_i}^2, p_{y_i}^2, p_{z_i}^2) \) and \( (J_i^2) = (J_{x_i}^2, J_{y_i}^2, J_{z_i}^2) \), where \( p_i = -i\hbar \partial_i \) are canonical momenta and the four-dimensional matrices \( J_i \) are spin 3/2 matrices. The primed coordinate system is aligned to the main crystallographic axes, i.e. \( x' \parallel [100], y' \parallel [010] \) and \( z' \parallel [001] \).

The LK Hamiltonian is parametrized by three material-dependent dimensionless quantities: \( \gamma_1, \gamma_2, \gamma_3 \). Here, we use the values of \( \gamma_i \) given in Ref. [1]. The parameters \( \gamma_2 \) and \( \gamma_3 \) account for the spin-orbit interactions and because in isotropic semiconductors such as Germanium they are roughly equal, these materials are usually described within an isotropic approximation \( \gamma 

\approx \gamma_3 \) [1]. In contrast, in anisotropic semiconductor such as Silicon, \( \gamma_2 \) and \( \gamma_3 \) are quite different and the isotropic approximation does not describe the system accurately. In particular, in this case, the low-energy description of the system strongly depends on the orientation of the nanowire with respect to the crystallographic axes [22, 70–72]. In our convention, the nanowire alignment with respect to the crystallographic axes is specified by the unitary matrix \( U \), i.e. \( x' \parallel [100], y' \parallel [010] \) and \( z' \parallel [001] \). The LK Hamiltonian \( H_{\text{LK}} \) is given in Eq. (C1).

In particular, we perform a rotation of \( \theta \) around the \( y' \parallel [010] \) crystallographic axis and a subsequent rotation of \( \varphi \) around the rotated \( z' \parallel [001] \) axis, see Fig. 1. This transformation aligns the coordinate system and the spin-matrices to the axes \( (x, y, z) \) in the figure. The most relevant orientations of the wire for this paper are summarized in Fig. 1b). In particular, it is instructive to study the behaviour of wires where the coordinate system is aligned to the crystallographic axes, and of wires grown along the [110] direction, as standardly done in experiments [28, 46, 51, 52]. We also consider the growth direction that maximizes the direct Rashba spin-orbit interactions in inversion symmetric Silicon nanowires [22] and quantum dots [70]. We will refer to these orientations as to Crystallographic Axes (CA), Standard Axes (SA), and Direct Rashba Axes (DRA), respectively.

The 4 × 4 LK Hamiltonian in Eq. (3) captures accurately the physics of Si nanowires with large cross-sections, however to describe smaller wires one needs to include the contribution of the spin 1/2 spin-orbit split-off holes (SOHs), that are gapped from the HHs and LHs by a material-dependent spin-orbit energy \( \Delta_0 \). The SOHs are only negligible when \( \Delta_0 \) is much larger than the confinement energy, such that the HH-LH subspace is well-separated in energy from the subspace of the SOHs. In Silicon, \( \Delta_0 \approx 44.1 \text{ meV} \) is about an order of magnitude smaller than in other semiconductors [1], and the confinement energy can easily become comparable to \( \Delta_0 \) in narrow wires, leading to a strong influence of the SOHs on the response, especially in the presence of an electric field. To take the SOHs fully into account in narrow wires, we compare our results obtained with the 4 × 4 Hamiltonian (3) to more accurate results obtained with the 6 × 6 extension of the LK Hamiltonian, the complete form of which can be found for example in Appendix C of Ref. [1]. The conduction electrons are separated by a much larger gap from the holes, and so their contribution is always neglected in the present analysis.

Without magnetic field, the eigenvalues of the Hamiltonian in Eq. (1) are doubly-degenerate Kramers partners. When the wavelength of the hole wavefunction along the nanowire is much larger than the confinement length in the cross-section, the low-energy physics of the system is well described by an effective nanowire Hamiltonian \( H_{\text{NW}} \) that only acts on the lowest pair of eigenstates. To second order in the momentum \( p_z \) along the wire, we obtain

\[
H_{\text{NW}} = \frac{p_z^2}{2m^*} + \mathbf{v} \cdot \mathbf{\sigma} p_z ,
\]

where \( \mathbf{\sigma} \) is a vector of Pauli matrices acting on the subspace of the lowest Kramers partners. The nanowire Hamiltonian is parametrized by an effective mass \( m^* \) and a spin-orbit velocity vector \( \mathbf{v} \) that can be found in perturbation theory. In particular, one can decompose \( H \) into powers of \( p_z \) as \( H = H_0 + H_1 p_z + H_2 p_z^2 \), with \( H_0 = H_{\text{LK}}(p_z = 0) + V_{\text{HW}}(x,y) + V_E(x,y) \), and introduce the unitary matrix \( M_E \) that diagonalizes \( H_0 \), i.e. \( (M_E^\dagger H_0 M_E)_{nm} = \epsilon_{E}^2 \delta_{nm} \). To obtain accurate results, we compute the eigenvectors \( M_E \) and the eigenenergies \( \epsilon_{E} \) numerically by discretizing the Hamiltonian \( H_0 \), the different discretization schemes used depend on the geometry of the fin and are described in more detail later in the text. By standard perturbation theory, it is straightforward to find

\[
(v \cdot \sigma)_{ij} = (M_E^\dagger H_1 M_E)_{ij} ,
\]

\[
\frac{1}{2m^*} = (M_E^\dagger H_2 M_E)_{ii} + \sum_{k \neq i,j} \left| \frac{(M_E^\dagger H_1 M_E)_{ik}}{\epsilon_{E}^2 - \epsilon_{E}'^2} \right|^2 ,
\]

where \( i, j \) only act on the lowest pair of Kramers partners and the terms proportional to \( \mathbf{\sigma} \mathbf{p}_z^2 \) vanish because of time-reversal symmetry. On the left-hand side of Eq. (5b), we omitted the index \( i \) because the effective mass of each Kramers partner is equal, i.e. \( m_{E}^* = m_{E}'^* = m^* \). The subscripts \( E \) in the eigensystem \( M_E \) and \( \epsilon_E \) emphasizes the dependence on the electric field. For this reason, the spin-orbit interactions and the effective mass are externally tunable by the gate potential \( V_g \), which controls the electrostatic potential \( V_E(x,y) \). Another convenient
parameter that characterizes the spin-orbit interactions relative to the inertia of the particle is the spin-orbit length
\[ l_{so} = \frac{\hbar}{m^*|v|}, \]
i.e. the length over which a particle flips its spin.

To define a spin qubit, we also include an external, homogeneous magnetic field \( B \). For weak values of the magnetic field, typically below one Tesla [74], we can safely neglect the orbital contribution of the magnetic field and only focus on the coupling of the magnetic field to the spin-degree of freedom via the Zeeman energy, which in the \( 4 \times 4 \) Luttinger-Kohn Hamiltonian is
\[ H_Z = \frac{-2B}{\kappa} \cdot (\kappa \sigma + q J^3). \]
Here, \( \kappa \) and \( q \) are material-dependent parameters for the magnetic interactions. The magnetic interactions when SOHs are included, as well as the precise value of \( \kappa \) and \( q \) for Si, can be found in Ref. [1]. Projecting the Zeeman Hamiltonian onto the groundstate of the wire, to linear order in \( B \), the effective Hamiltonian in Eq. (4) acquires the correction
\[ H_{SW}^Z = \frac{1}{2} \Delta \cdot \sigma, \]
where we introduce the vector \( \Delta = \mu_B \vec{g} \cdot \vec{B} \). Here, \( \mu_B \) is the Bohr magneton and \( \vec{g} \) is a dimensionless \( 3 \times 3 \) matrix of \( g \)-factors.

From perturbation theory, we obtain
\[ \left( \Delta \cdot \sigma \right)_{ij} = 2 \left( M_E^i H_Z M_E^j \right)_{ij}. \]

Importantly, the Zeeman interactions are also electric field dependent, and in FinFET qubits this dependence causes an extra source of charge noise that is not directly related to the effective spin-orbit coupling in the wire.

### III. TRIANGULAR NANOWIRES

To have a simple model of FinFETs, we consider first a nanowire with a triangular cross-section and we require that the hole wavefunction vanishes at the boundaries of the triangle, see Fig. 1. The hard-wall boundary condition is justified at the interfaces between Si and the dielectric because of the large energy gap between the materials. This model provides an accurate description of SOI FinFETs, but it is questionable in bulk Si FinFETs, where there is no sharp interface at the bottom of the fin and the wavefunction can leak into the bulk. In this case, however, the hard-wall approximation still provides a good qualitative understanding of the system, especially when the hole wavefunction is strongly confined inside the fin by a large negative gate potential \( V_g \). The effect of the substrate in a bulk Si FinFET is discussed in Sec. IV A.

The choice of a triangular fin is crucial in our analysis. In fact, compared to more conventional rectangular, circular, or hexagonal nanowires, a triangular cross-section lacks inversion symmetry in the \((x,y)\) plane, i.e. \( V_{HW}(x,y) \neq V_{HW}(-x,-y) \), and consequently the triangular nanowire can present large intrinsic spin-orbit interactions without external homogeneous electric fields, i.e. \( \vec{v} \neq 0 \) when \( \vec{E} = 0 \) [75].

In the following, we examine how the spin-orbit interaction varies as a function of the gate potential and of the growth direction in an ideal case, where the cross-section is an equilateral triangle of side \( L \). In Si FinFETs, the triangular cross-section can be made rather equilateral, see e.g. [52], however it is often the case that the fin resembles more an acute [51] or obtuse [28] isosceles triangle. The effect of the squeezing of the cross-section is analyzed in Sec. IV A. A convenient orthonormal basis to describe this system comprises the eigenfunctions of the two-dimensional Laplace operator \( p_x^2 + p_y^2 \) vanishing at the boundary of the triangle. Because of the highly symmetric geometry, the eigenfunctions can be expressed in terms of trigonometric functions, see App. B and Ref. [76] for more details. A natural energy scale for this problem is the confinement energy
\[ \epsilon_c = \frac{16\hbar^2 \pi^2}{3mL^2} \gamma_1, \]
which characterizes the energy gap between different orbital states, see Eq. (B3). This energy is quite large, for example in a Silicon wire with side \( L = 35 \) nm, the quantization energy is \( \epsilon_c \approx 14 \) meV, approximately \( 30\% \) the gap to the split-off band \( \Delta_0 \approx 44.1 \) meV [1]. While when \( L \geq 35 \) nm, the \( 4 \times 4 \) LK Hamiltonian in Eq. (3) is valid, in smaller wires, such a strong quantization results in a large contribution of the SOHs, which demands a more detailed analysis that fully includes these states. In the following, we will refer to small (large) wires when the side \( L \) is smaller (larger) than \( 35 \) nm. To gain a qualitative understanding of the system, we begin our analysis by studying large nanowires by using the \( 4 \times 4 \) LK Hamiltonian; a detailed analysis of the effect of the SOHs is postponed to Sec. IV.

We first compute the intrinsic spin-orbit velocity \( \vec{v}_o \) in the absence of electric fields, focusing on its dependence on the growth direction. Then, we separately describe the effect of homogeneous and inhomogeneous electric fields. We highlight the main differences between triangular and square cross-sections and we discuss when the electric field-induced spin-orbit coupling compensates for the intrinsic spin-orbit interactions, yielding convenient operational sweet spots where spin-orbit effects vanish.

#### A. Intrinsic spin-orbit velocity

Without external fields, a satisfactory description of the system can be obtained by including the lowest three orbital eigenstates in Eqs. (B1) that satisfy the Laplace equation and vanish at the boundary of the equilateral triangle. These states comprise a three-fold rotationally symmetric ground state and two degenerate excited states that can be taken to be even or odd with respect
to the height of the fin. The wavefunction of these states is shown in Fig. 13. Projecting the rotated \(4 \times 4\) LK Hamiltonian in Eq. (C1) onto this subspace, we obtain a \(12 \times 12\) reduced Hamiltonian which parametrically depends on the azimuthal \(\varphi\) and elevation \(\theta\) angles of the nanowire, see Fig. 1. While we do not report the explicit expression of the reduced Hamiltonian for arbitrary angles \(\theta\) and \(\varphi\), we describe in more detail how to obtain it in App. C.1.

With a second order Schrieffer-Wolff transformation \([1, 77]\), we find that the ground state of the wire has an intrinsic spin-orbit coupling that can be rewritten as

\[
\mathbf{\psi}_0 = \frac{-i}{\hbar} (\gamma_3 - \gamma_2) \mathbf{a}_0(\theta, \varphi),
\]

where \(\mathbf{a}_0(\theta, \varphi)\) is a dimensionless three-dimensional vector that characterizes strength and direction of the spin-orbit field and depends on the growth direction and on the Luttinger parameters. We emphasize once again that this effect, materials such as Silicon, where the anisotropy is large, are a convenient choice to study this phenomenon. Isotropic semiconductors, such as Ge or GaAs where \(\gamma_2 \approx \gamma_3\), can have a finite intrinsic spin-orbit coupling when the cross-section is less symmetric, e.g. in isosceles triangles.

Importantly, for an equilateral triangle, the intrinsic spin-orbit interaction at zero electric field is a result of the anisotropy of the semiconductor and it vanishes when \(\gamma_2 = \gamma_3\). For this reason, materials such as Silicon, where the anisotropy is large, are a convenient choice to study this effect. Isotropic semiconductors, such as Ge or GaAs where \(\gamma_2 \approx \gamma_3\), can have a finite intrinsic spin-orbit interaction when the cross-section is less symmetric, e.g. in isosceles triangles.

More details on the general dependence of these quantities on the growth directions in Silicon are provided in App. C.1, see, in particular, Fig. 14. From the analysis, we observe that the maximal value of \(|\mathbf{\psi}_0| \approx 2.91\hbar/(mL)\) is reached when the wire extends along one of the crystallographic axes, i.e. when \(z \parallel [100]\), [010], or [001]. In this case, we find that there is no spin-orbit coupling in the direction of the wire, i.e. \(|\mathbf{\psi}_0\rangle = 0\), and we define the complex quantity \(a_0(\theta) = (a_0)_x(\theta, \varphi) + i(a_0)_y(\theta, \varphi)\), where to simplify the notation we suppress the explicit dependence of \(a_0(\theta)\) on \(\varphi\). In particular, we find

\[
\alpha_0(0) = 9.34 \frac{\gamma_3}{\gamma_1 + (5/2)\gamma_2} e^{-i\varphi} \approx 2.631 e^{-i\varphi},
\]

\[
\alpha_0(\pi/2) = \frac{a_1 - a_2 \cos(4\varphi) + a_3 (\gamma_3 - \gamma_2) \cos(8\varphi)}{1 - b_1 (\gamma_3 - \gamma_2) \cos(4\varphi) + b_2 (\gamma_3 - \gamma_2)^2 \cos(8\varphi)},
\]

where \(a_i\) and \(b_i\) are real functions of the Luttinger parameters, whose values for Silicon are shown in Table I. The numerical value 2.631 in Eq. (11a) is obtained by using the Luttinger parameters of Si and describes the CA and the DRA growth directions. From Eq. (11b), we estimate \(\alpha_0(\pi/2) \approx 2.354\) at \(\varphi = -3\pi/4\), corresponding to the SA. When \(\theta = -\pi/2\) and \(\varphi = 0\), then \(z \parallel [100]\) and \(x \parallel [001]\), and we recover the value \(\alpha_0(\pi/2) = |\alpha_0(0)| \approx 2.631\) obtained when the confinement and the crystallographic axes are aligned (CA growth direction).

A comparison between the perturbative results in Eq. (10) and the exact spin-orbit velocity computed numerically by using Eq. (5a) and a larger number \(N = 200\) of the orbital basis states in Eq. (B1) is shown in Fig. 2. When the elevation angle is \(\theta = 0\), the wire extends along the [001] direction, and the spin-orbit vector has a roughly constant amplitude, up to fast oscillations with a small amplitude, but as a consequence of the four-fold rotational symmetry of the Luttinger-Kohn Hamiltonian, its direction oscillates as a function of the azimuthal angle \(\varphi\) with period \(\pi/2\). In contrast, when \(\theta = \pm\pi/2\), the intrinsic spin-orbit vector has a constant direction \((\mathbf{\psi} \parallel \mathbf{e}_x)\) and an oscillating amplitude that reaches its maximum when the wire is aligned to the main crystallographic axes [100] or [010]. The minimal spin-orbit coupling occurs when the wire extends along the [110] direction, which corresponds to the standard experimental growth direction (SA), see Fig. 1b). These oscillations of the spin-orbit amplitude are a result of the large anisotropy of the material, which enhances the spin-orbit coupling when the material is grown along a main crystallographic axis.

| \(a_1\) | \(a_2\) | \(a_3\) | \(b_1\) | \(b_2\) |
|--------|--------|--------|--------|--------|
| 2.468  | 0.683  | 0.013  | 0.298  | 0.011  |

Figure 2. Intrinsic spin-orbit velocity \(|\mathbf{\psi}_0|\) without external electric fields. We compute \(|\mathbf{\psi}_0|\) as a function of the azimuthal angle \(\varphi\) for elevation angle \(\theta = 0\) (red lines) and \(\theta = -\pi/2\) (blue lines). The dashed lines are obtained by the approximate Eqs. (10) and (11), while the solid lines are obtained numerically by using Eq. (5a) and including 200 orbital eigenstates in Eq. (B1). We mark with black, orange, and purple circles the results obtained for the relevant orientations of axes CA, DRA, and SA, respectively, see Fig. 1b).

Table I. Parameters for the intrinsic spin-orbit vector \(\mathbf{\psi}_0\) in Eq. (11b) when \(\theta = -\pi/2\). These parameters describe the SA orientation when \(\varphi = -3\pi/4\) and allow to recover the results for the CA growth direction when \(\varphi = 0\).
While Eq. (11a) agrees well with the numerically computed spin-orbit coupling, Eq. (11b) is not quantitatively accurate. The disagreement is a consequence of the large anisotropy of Silicon. In fact, because of the second order perturbation theory, only terms proportional to $e^{\pm 4i\varphi}$ and $e^{\pm 8i\varphi}$ appear in Eq. (11b); these terms only affect the amplitude of the spin-orbit interaction at $\theta = \pm \pi/2$, and not at $\theta = 0$. The numerical solution can be better approximated by including higher harmonics, that capture the fast decrease of spin-orbit interaction at the SA growth direction.

The effective masses are also straightforwardly obtained by perturbation theory and their explicit expressions in Silicon and at $\theta = 0$ and $\theta = \pm \pi/2$ is given in Eq. (C5). The perturbative analysis captures the qualitative behaviour of the masses, but the results are not in excellent quantitative agreement with the numerics, see Fig. 15a). From the full numerical analysis and by using Eq. (6), we extract the intrinsic spin-orbit length

$$l_{so}(\theta = 0) \approx 0.83L \tag{12a}$$

$$l_{so}(\theta = \pm \pi/2) \in [0.83L, 2.89L]. \tag{12b}$$

In analogy to the spin-orbit velocity, the spin-orbit length is constant when $\theta = 0$, but when $\theta = -\pi/2$, it is an oscillating function of $\varphi$ with period $\pi/2$ and it attains the maximum and minimum values at $\varphi = \pi n/2$ and $\varphi = \pi(2n + 1)/4$, respectively. A more detailed analysis of the intrinsic spin-orbit length in the nanowire is given in App. C1, see in particular Fig. 15b). Importantly, we observe that the spin-orbit length is extremely short, and is of the order of the side $L$ of the triangle, which is typically much smaller than the lateral size of the dot. Consequently, we expect hole Si Finfets to show effects due to large spin-orbit interaction such as renormalization of the $g$-factor [27, 30, 31, 46, 47, 69] and ultrafast Rabi oscillations [23].

While our quantitative analysis here is limited to Si, our results apply also to other semiconductors. For example, in [28], the spin-orbit interaction in hole Ge hut-wires is studied and an intrinsic spin-orbit field of $h|v_0| \sim 10$ meV · nm was measured for a triangular device with width $L_x = 80$ nm and height $H = 4$ nm. An estimate of the amplitude of the intrinsic spin-orbit interaction related to the triangular cross-section can be found by using the equilateral triangle equation for the crystallographic growth direction,

$$|v_0| = \frac{9.34}{mL} \frac{\hbar \gamma_3 (\gamma_3 - \gamma_2)}{(\gamma_1 + (5/2)\gamma_2)}, \tag{13}$$

obtained by combining Eqs. (10) and (11a). To better compare with the experiment, we consider a fictitious equilateral triangle of side $\tilde{L}$ having the same area of the cross-section of the hut-wire: from the condition $\sqrt{3}\tilde{L}^2/4 = L_x H/2$, we find that the effective side of the fictitious equilateral triangle is $\tilde{L} \approx 19$ nm. Using the Luttinger parameters of Ge [1], we estimate an intrinsic spin-orbit field $h|v_0| \approx 12.65$ meV · nm, in very good agreement with the experiment [28].

Figure 3. Spin-orbit velocity $|v|$ as a function of the homogeneous electric field $E_y$. We neglect the inhomogeneous contribution to the electric field profile and we compute $|v|$ for the growth directions shown in Fig. 1b). At negative (positive) electric fields, i.e. when the hole wavefunction is pushed to the bottom (apex) of the triangle, the spin-orbit can be switched off when the wire is grown along the SA (DRA) orientation. We show the points $E_{SW}^{SA}$ and $E_{SW}^{DRA}$, where the switching occurs, see Eq. (14), with purple and orange circles, respectively.

B. Homogeneous electric field

We now analyze the effect of the electrostatic potential $V_E$ in Eq. (2) on the spin-orbit velocity.

For large Si wires, there are two distinct mechanisms that suppress the spin-orbit interactions and eventually remove them entirely. These mechanisms are the application of a homogeneous electric field $E_y$ pointing along the $y$ direction and the application of inhomogeneous electric fields $\delta E_{xx}$ and $\delta E_{yy}$ that harmonically confine the wavefunction inside the fin. Both these fields are controllable by the external potential, and their strengths strongly depend on the gate design, see App. A for a more detailed analysis.

Let us first consider the effect of homogeneous electric fields, which models setups where the non-linearities of the fields are suppressed. In realistic devices the top gate covers the wire rather symmetrically with respect to the $y$-direction. For this reason, the homogeneous electric field $E_y$ in the $x$-direction is zero and we will neglect it in the present analysis [78].

In Fig. 3, we show the results of a numerical analysis showing the dependence of the spin-orbit velocity on the dipole energy $eE_yL$ for the growth directions given in Fig. 1b). These results are obtained by projecting the Hamiltonian (1) onto the first 200 orbital states in Eq. (B1) and using Eq. (5a). We observe that the electric field strongly influences the spin-orbit field and it can increase it or decrease it depending on the orientation of the wire. In particular, for the DRA and the SA orientations, the spin-orbit field can be exactly switched off when the dipole energy $eE_yL$ becomes comparable to
the confinement energy $\epsilon_c$. For the DRA, the spin-orbit coupling is removed when the electric field is positive and the holes are pushed to the apex of the fin, while for the SA growth direction, a negative field is required and the holes are pushed to the bottom of the triangle. Consequently, the SA is convenient in SOI FinFETs, where the wire is separated from the substrate by an oxide and the hard-wall boundary condition is a good approximation also at the bottom of the fin. In contrast, the DRA can also be suitable in bulk FinFETs because the large positive electric field confines the hole wavefunction in the fin and suppresses the leakage of the wavefunction into the substrate. The spin-orbit interaction is completely removed at

$$E_{\text{DRA}}^{SW} \approx 1.13 \frac{\epsilon_c}{eL} \approx 19.42 \times \frac{10^3 \text{nm}^3}{L^3} \text{V/um}, \quad (14a)$$

$$E_{\text{SA}}^{SW} \approx -4.3 \frac{\epsilon_c}{eL} \approx -73.9 \times \frac{10^3 \text{nm}^3}{L^3} \text{V/um}, \quad (14b)$$

for the DRA and SA case, respectively. Note that the switching field scales as $1/L^3$ and for realistic cross-section with sides of a few tens of nanometers it is of the order $V/\mu m$, easily reachable in state-of-the-art devices. For example, for the DRA and in a fin with side length $L = 20 \text{ nm}$, $E_{\text{DRA}}^{SW}$ corresponds to a dipole energy of $\epsilon E_{\text{DRA}}^{SW} \approx 4.3 \text{ meV}$.

The suppression of the spin-orbit interaction is a result of the anisotropy of Si and can be qualitatively explained by a fourth order Schrieffer-Wolff transformation in the reduced Hilbert space spanned by the first three orbital eigenstates in Eq. (B1), whose wavefunctions are shown in Fig. 13. Let us focus on wires where $z || [001]$, i.e. $\theta = 0$. In this limit, the explicit form of the reduced $12 \times 12$ Hamiltonian is given in App. C2, see in particular Eq. (C7). We now examine the dependence of $v$ on $\varphi$. In this case, $v_z = 0$, and, generally, we can write the complex off-diagonal matrix element of the spin-orbit velocity $v = (v)_x + iv(y)$ as the sum of an anisotropic term $a_A$ and an isotropic direct Rashba-like term $a_I$, i.e.

$$v = \frac{\hbar}{mL} \left[(\gamma_3 - \gamma_2)\alpha_A(\varphi) + (\gamma_3 + \gamma_2)\alpha_I\right]. \quad (15)$$

A similar theory, although with more complicated equations, can be developed generally. Importantly, $\alpha_I$ is a real function of $eE_yL$ and it does not depend on the azimuthal growth angle $\varphi$, while $\alpha_A$ is a complex function of $eE_yL$ and oscillates as a function of $\varphi$. In particular, for small electric fields and in Silicon we find

$$\alpha_A(\varphi) \approx \alpha_0 - 0.367e^{-4i\varphi} (eE_yL/\epsilon_c)^2, \quad (16a)$$

$$\alpha_I \approx 0.35 (eE_yL/\epsilon_c) + 0.246 (eE_yL/\epsilon_c)^2, \quad (16b)$$

where $\alpha_0$ is the zero field result in Eq. (11a) and in $\alpha_A$ we also neglected quantitatively small corrections linear in $eE_yL/\epsilon_c$ and proportional to different powers of $e^{-4i\varphi}$. For the general dependence of these parameters on the Luttinger parameters, see Eq. (C10).

The spin-orbit interaction is fully switched off when $|v| = 0$. From Eq. (16), it follows that this cancellation can only occur at the growth angles $\varphi = (2n+1)/4$ where $\alpha_A$ is a real-valued function, and where $\alpha_0$ has a sign opposite to all the electric field-dependent terms. From Eqs. (15) and (16), we estimate that the switch in Silicon occurs at $E_{\text{DRA}}^{SW} \approx 1.52\epsilon_c/(eL)$. Note that our perturbative analysis provides good qualitative insights into the switching mechanism, and in addition, the numerical prefactor $1.52$ is reasonably close to the prefactor $1.13$, derived from the detailed numerical analysis including higher orbital states.

C. Comparison with square cross-section

We summarize here the key qualitative differences between Si FinFETs with triangular cross-section and Si wires with an inversion symmetric cross-section. In particular, here we focus on wires with a square cross-section; a detailed analysis of the direct Rashba spin-orbit coupling in these wires can be found in Ref. [22]. The first key difference is that without external fields, the inversion symmetry of a square cross-section prohibits the presence of an intrinsic spin-orbit coupling, and $\mathbf{v}_n^0 = 0$. Another important difference between the two systems is that the amplitude of the spin-orbit velocity in an square wire is a symmetric function of the homogeneous electric field, and $|\varphi_{sq}(E_y)| = |\varphi_{sq}(-E_y)|$. In contrast, in a triangular wire, the spin-orbit coupling does not need to be symmetric and the spin-orbit velocity $v$ is modified in different ways when the hole wavefunction is pushed to the bottom or to the apex of the triangle, see Fig. 3. We notice however that the amplitude of the spin-orbit velocity in triangular wire is still a symmetric function of the homogeneous field $E_x$.

To make a more quantitative comparison, we consider a Si wire with side $L$ grown along the $z || [001]$ direction. To linear order in the electric field, the direct Rashba spin-orbit coupling can be written as [22]

$$v_{sq} \approx \frac{\hbar}{mL} \left[-0.41(\gamma_3 - \gamma_2)e^{-4i\varphi} + 0.38(\gamma_3 + \gamma_2)\right] eE_yL/\epsilon_c, \quad (17)$$

where $\epsilon_c = \hbar^2 \pi^2 \gamma_1/mL^2$ is the characteristic confinement energy for a particle in a square cross-section. To obtain this expression, we combined Eqs. (77), (78), (79) and (80) in Ref. [22] and used the Luttinger parameter of Si. To facilitate the comparison with our Eqs. (15) and (16) obtained for an equilateral triangle, we also introduced the imaginary spin-orbit velocity $v_{sq} = \mathbf{v}_{sq0} + iv_{sq}$, we expanded the function $\chi(\varphi) = 0.36/\left[1 + 0.16((\gamma_3 - \gamma_2)\cos(4\varphi))\right]$ [directly related to the function in Eq. (79) of [22]] to linear order in $(\gamma_3 - \gamma_2)\cos(4\varphi)$, and we discarded the small terms oscillating as $e^{4i\varphi}$ and $e^{-4i\varphi}$. The overall minus sign of the spin-orbit velocity here compared to Eq. (80) in Ref. [22] is a
result of the field being applied in the $y$-direction instead of the $x$-direction.

Comparing Eqs. (15) and (17), we observe that in square and triangular wires the spin-orbit coupling is a sum of an isotropic term and an anisotropic term, proportional to $\gamma_3 + \gamma_2$ and to $\gamma_3 - \gamma_2$, respectively. To linear order in $E_y$, the isotropic contributions in both cross-sections are in good quantitative agreement, but the anisotropic terms are qualitatively different, see Eq. (16). In fact, while in square wires the anisotropic term varies linearly with $E_y$ and its contribution to the overall direct Rashba spin-orbit velocity is roughly equal to the isotropic contribution, in a triangular wire $\alpha_A(\varphi)$ comprises a constant intrinsic term and has a negligible linear dependence on $E_y$.

Including higher powers in the electric field, we find an additional qualitative difference between the spin-orbit coupling in the two different cross-sections. In fact, in a square wire, the spin-orbit velocity has no corrections quadratic in $E_y$ and the next order corrections are proportional to $E_y^3$. In contrast, in a triangular FinFET, both $\alpha_I$ and $\alpha_A$ present quadratic terms proportional to $E_y^3$, that make the spin-orbit velocity asymmetric in $E_y$, see Eq. (16).

These qualitative differences are crucial here because, as discussed in Sec. III.B, in triangular wires the presence of a spin-orbit switch at a finite value of the electric field is a result of the competition between the intrinsic spin-orbit coupling and the direct Rashba-like spin-orbit interaction dependent on the electric field. In a square Si wire there is no intrinsic spin-orbit velocity and the spin-orbit coupling only vanishes when $E_{x,y} = 0$ [or when $E_{x,y} \to \infty$, see Eq. (86) in Ref. [22]], a much inconvenient working point for electrostatically defined quantum dots.

Finally, we anticipate that another difference between triangular and square cross-sections comes from the sensitivity of the spin-orbit coupling to the quadratic potential $\delta E_{ij} r_i r_j / 2$, see Eq. (2). Without an homogeneous electric field, such a potential is inversion symmetric and does not produce spin-orbit coupling in square wires. In contrast, in the next section, we show how this potential can produce another spin-orbit switch in triangular FinFETs, where the inversion symmetry is broken by the cross-section.

### D. Inhomogeneous electric field

The electric field profile in a triangular FinFET comprises a large inhomogeneous component that significantly alters the spin-orbit velocity. When the back gate is far from the wire, the inhomogeneous component of the electric field in the cross-section can be well-approximated by a linearly varying electric field parametrized by the tensor $\delta E_{ij}$, and results in a harmonic potential for the holes. As discussed in App. A, in typical devices, the gradient fields $\delta E_{xx}$ and the $\delta E_{yy}$ are large and they vary linearly with the top gate potential $V_g$, with slopes that are comparable in absolute value, but have opposite signs, see Eqs. (A1) and (A2). The $\delta E_{xy}$ term is generally much smaller and we will neglect it at first. In this section, we restrict ourselves to the analysis of Si FinFETs where a negative gate potential $V_g$ is applied, such that the hole wavefunction is pushed to the apex of the triangle. Because of the inhomogeneous electric field, negative values of $V_g$ generate a saddle potential, with a harmonic confinement in the $y$-direction that further confines the holes in the fin and an inverted parabolic potential that pushes the wavefunction to the sides of the triangle.

Let us first neglect the different amplitudes of the slopes of $\delta E_{xx}$ and $\delta E_{yy}$ as a function of $V_g$ by considering the potential energy

$$V_E(x, y) = e\delta E(y^2 - x^2)/2.$$  \hspace{1cm} (18)

This approximation is justified by the simulation in App. A, where we estimate $\delta E_{xx}/\delta E_{yy} \approx -1.01$, see Eqs. (A1) and (A2); negative values of $V_g$ correspond to positive values of the parameter $\delta E$. We are also studying separately the effect of the inhomogeneous coupling, and for now we set the homogeneous electric field to zero, i.e. $E_y = 0$. We remark that for inversion symmetric cross-sections such as cylindrical or rectangular wires, the potential in Eq. (18) does not induce any spin-orbit interactions because $V_E(x, y) = V_E(-x, -y)$, and the results obtained in this section are specific for triangular wires.

In Fig. 4, we show the spin-orbit interactions as a function of $\delta E$ and for the orientations in Fig. 1b). We observe a qualitatively similar picture as discussed in Sec. III.B for the homogeneous electric field: depending on the growth direction, the spin-orbit velocity varies with $\delta E$, and while it increases in the CA and SA growth direction, it decreases for the DRA, resulting in an operational sweet spot where $|u|$ vanishes. This sweet spot is at

$$\delta E^{SW} \approx 20.5 \frac{e \epsilon_0}{cL^2} \approx 35 \times 10^3 \times \frac{10^4 \text{nm}^4}{L^4} \text{V/\mu m}^2,$$  \hspace{1cm} (19)

This switching field scales as $1/L^4$, in contrast to the homogeneous switching field in Eq. (14) and it also requires a higher value of potential, in fact, when $L = 20$ nm, the switch occurs when the harmonic potential is $e\delta E^{SW} L^2 \approx 0.88$ eV.

This spin-orbit sweet spot remains present also when non-idealities of the electric fields are included. To account for the possible variations from the case $\delta E_{xx} = -\delta E_{yy}$, we examine in more detail the dependence of the spin-orbit velocity on arbitrary $\delta E_{ij}$. The results for different growth directions are shown in App. D. For the DRA growth direction, the spin-orbit velocity can be tuned exactly to zero for a wide range of $\delta E_{xx}$ and $\delta E_{yy}$, see Fig. 16a), and, as a consequence, the sweet spot driven by inhomogeneous fields can be found in a rather general FinFET design.

So far we neglected the cross-coupling field $\delta E_{xy}$. This approximation is justified in the simple electrostatic
model of the FinFET used here, see App. A, however, a small but finite value of $\delta E_{xy}$ can be present when the top gate is not exactly symmetric with respect the fin. The effect of $\delta E_{xy}$ on the spin-orbit velocity is shown in the inset of Fig. 4a). By including the cross-coupling $\delta E_{xy}$, we find that the spin-orbit switch is removed. However, we expect that in most cases the cross coupling remains a few orders of magnitude smaller than $\delta E$ and thus at the switch the spin-orbit velocity remains orders of magnitude smaller than the intrinsic spin-orbit velocity in Eq. (10), still providing a good working point where charge noise is strongly reduced.

To verify whether it is the inhomogeneous or the homogeneous component of the electric field that drives the switching off of the spin-orbit interaction in realistic devices, we study the simultaneous effect of $\delta E$ and $E_y$. In the simple FinFET design analyzed in App. A, $\delta E$ and $E_y$ change linearly with the gate potential $V_y$ and their values are constrained to the line $E_y = c_y \delta E L/c_{xx}$, with $c_y/c_{xx} \approx 0.25$, see Eqs. (A1) and (A2). In Fig. 4b), we show the spin-orbit velocity as a function of both the homogeneous and inhomogeneous fields for the DRA growth direction. In the whole parameter space, the spin-orbit velocity vanishes along a curve (blue) that intersects the (purple) line $E_y = 0.25 \delta E L$ at the point $(e \delta E L^2, e E_y L) = (7.8 \epsilon_c, 1.95 \epsilon_c)$. Combining with Eqs. (A1) and (A2), this point corresponds to the potential $V_y^{SW} = -111.7 \times n^2 (d_B/L)^3$ V and when $L = 20$ nm and $d_B = 100$ nm, one obtains the working point $V_y^{SW} = -1.4$ V. In the design studied here, the switching-off of the spin-orbit velocity is driven by the homogeneous electric field $E_y$ and the harmonic potential $\delta E$ only renormalizes the value of $E_y$ required to compensate for the intrinsic spin-orbit interaction.

The general dependence of the spin-orbit velocity on the homogeneous and inhomogeneous electric fields can be qualitatively described by a fourth order Schrieffer-Wolff transformation on the reduced 12 Hamiltonian [81], that generalizes the treatment in Sec. III B. General expressions for the spin-orbit velocity as a function of the homogeneous and inhomogeneous electric fields are given in App. C 2, see Eq. (C10).

Finally, we now comment on the effect of a moderate strain of the Silicon wire. When the dielectric is very thin, Si could be strained by the presence of the nearby metallic gates. Alternatively, in small wires, strain can be caused by the incoherent interface between SiO$_2$ and Si [79, 80]. While a detailed analysis of strain is outside the scope of the present paper, we can obtain a rough estimate of its effect on the spin-orbit switch, by using the isotropic Bir-Pikus Hamiltonian [81] $H_{BP} = \epsilon_{strain} J_z^2$, which well-captures the strain in Ge/Si core/shell nanowires [21]. The energy $\epsilon_{strain}$ depends strongly on the specific design of the device and can be engineered, for example, by tuning the thickness of the dielectrics. By adding $H_{BP}$ to the wire Hamiltonian (1), we compute the spin-orbit velocity in a DRA wire with side $L = 20$ nm in the presence of strain. We observe that for $|\epsilon_{strain}| \lesssim 1.5$ meV, positive (negative) values of $\epsilon_{strain}$ only shift $\delta E^{SW}$ to higher (lower) values, while the homogeneous electric field switch $E_y^{SW}$ is not altered significantly. Hence, we expect the spin-orbit switch to be robust against moderate strain.

IV. SPIN-ORBIT IN SMALL WIRES

In Sec. III, the effective spin-orbit velocity is computed by using the $4 \times 4$ LK Hamiltonian in Eq. (3), which describes the mixing of heavy and light holes. In
this case, nanowires with equilateral triangular cross-sections having different sides $L$ show the same qualitative behaviour, and $L$ only sets the scale of the spin-orbit velocity $v \propto \hbar/(mL)$ and of the confinement energy $\epsilon_c \propto \hbar/(mL^2)$, see Eqs. (15) and (9), respectively. This model is valid for wires with a large cross-section, where $\epsilon_c$ is the smallest energy scale and one can neglect the coupling to the split-off holes, gapped by a large energy $\Delta_0$. In contrast, for small Si wires, $\Delta_0$ is comparable with $\epsilon_c$ and because of the influence of the SOHs, the ground-state dynamics of the wire depends non-trivially on the side $L$ of the cross-section [82]. As anticipated in Sec. II, to study this dependence we use the $6 \times 6$ LK Hamiltonian [1], fully accounting for the SOHs.

In Fig. 5, we show how the intrinsic spin-orbit velocity $|v_0|$ is modified by the SOHs in wires with different cross-section sides $L$. For large wires, with $L \gtrsim 35$ nm, the contribution of the SOHs is small and $|v_0|$ approaches the value in Eq. (10) obtained with the $4 \times 4$ LK Hamiltonian (dashed lines). In contrast, for very small wires, with $L \lesssim 10$ nm, the SOHs strongly suppress $|v_0|$. For moderately small wires, because of the anisotropy of Si, the effect of the SOHs strongly depends on the growth direction. In fact, while for the SA growth direction $|v_0|$ decreases monotonically, when $\theta = 0$, the spin-orbit velocity overshoots and reaches a maximum at $L \sim 20$ nm, where $|v_0|$ is larger than the value obtained for the $4 \times 4$ LK Hamiltonian. In addition, at $L \sim 10$ nm we observe that for the DRA growth direction, the intrinsic spin-orbit interaction can be exactly cancelled, restoring the usual direct Rashba spin-orbit coupling typical of inversion symmetric cross-sections [21, 22].

The SOHs strongly affect the response of the system to external electric fields. For example, we analyze here the spin-orbit velocity in wires with the DRA orientation, where in Sec. III, we predict that the intrinsic spin-orbit interaction can be exactly cancelled either by a positive homogeneous electric field $E_y$ or by an inhomogeneous field $\delta E$. In Fig. 6, we show how the spin-orbit switch-off mechanism discussed in Sec. III B and Sec. III D are modified by the SOHs. In particular, in Fig. 6a) we show the combined effect of $E_y$ and $\delta E$ on the spin-orbit velocity $v$ when the SOHs are accounted for. We study here an equilateral triangle of side $L = 20$ nm, which maximises the intrinsic spin-orbit coupling, see Fig. 5, and is easily achievable in state-of-the-art devices [51, 52]. Comparing to Fig. 4b), where $v$ is obtained by using the $4 \times 4$ LK Hamiltonian, we observe that the SOHs drastically alter the response of the wire and they remove the charge noise sweet spot produced by the homogeneous electric field $E_y$, while maintaining the sweet spot resulting from the inhomogeneous field $\delta E$. This latter sweet spot persists also when a strong homogeneous electric field $E_y$ is present and $E_y$ only pushes the switch-off field $\delta E_{SW}$ to larger values. In the plot, we show with a dashed line the curve along which the spin-orbit vanishes when the SOHs are neglected. Importantly, for the simple gate design studied here, where $E_y$ and $\delta E$ are constrained along the purple line in the figure, the charge noise sweet spot is removed by the SOHs. Some possible ways to circumvent this problem and to restore the sweet spot in realistic SOI and bulk Si FinFETs are discussed in Sec. IV A.

To have a better understanding of the system, we study the two different mechanisms separately for wires with different cross-sections and grown in the DRA direction. In Fig. 6b), we analyze the effect of a homogeneous electric field $E_y$ on the spin-orbit velocity $v$ in wires with different cross-section sides $L$. For small-wires, the SOHs tend to remove the spin-orbit switch at $E_{SW}^{DRA}$, and when $L \lesssim 35$ nm this leads to a minimal spin-orbit velocity with a finite value that increases as $L$ decreases. In contrast, in Fig. 6c), we show the dependence of $v$ on $\delta E$ including the SOHs. Wires with different cross-section areas have a qualitatively similar response to the inhomogeneous electric field and $v$ always vanishes at a switch-off field $\delta E_{SW}$, whose precise value now depends weakly on $L$. In particular, the SOHs push $\delta E_{SW}$ to lower values and a lower value of gate potential is required to remove the spin-orbit coupling.

A detailed analysis of the effect of the SOHs on the electric field response including the general dependence on $\delta E_{xx}$ and $\delta E_{yy}$ for different growth directions is presented in App. D, see in particular Fig. 16. Interestingly, from this analysis, we observe that the SOHs push the switching field to lower gate potentials for the crystallographic growth direction, and in this case the sweet spot occurs roughly at a constant values of $c\delta E_{xx}L^2 \approx 30\epsilon_c$, see Fig. 16d).
how the SOHs affect the dependence of $\delta E$ only because of the inhomogeneous field $E$, a function of homogeneous and inhomogeneous electric field, $E_y$. When the SOHs are included, the spin-orbit switch driven by $E_y$ is removed and $|\psi|$ vanishes only because of the inhomogeneous field $\delta E$. To facilitate the comparison with Fig. 4b, we show with a dashed blue line the curve along which $|\psi|$ vanishes in large wires. Importantly, for the device geometry studied in App. A, where $\delta E$ and $E_y$ are constrained to lie on the purple line, the SOHs completely remove the spin-orbit switch at $V_{g}^{SW}$. In b) and c) we study how the SOHs affect the dependence of $|\psi|$ on $E_y$ and $\delta E$ when the cross-section side $L$ is varied. While the homogeneous spin-orbit switch $E_{\text{DRA}}^{SW}$ is removed for wires with $L \lesssim 35$ nm, the inhomogeneous switch $\delta E_{\text{DRA}}^{SW}$ remains and is pushed to lower values as the side length decreases. In the units used, the results obtained without the SOHs are independent of $L$.

Table II. Examples of FinFETs designs where the spin-orbit sweet spot is restored in the small cross-section limit. To estimate the values of minimal spin-orbit length we consider devices with $L = L = 20$ nm. For the SOI designs, we consider a back gate at a distance $d_B = 100$ nm, while for the bulk FinFET, we consider a cross-section 120 nm wide and 60 nm high. Increasing the distance $d_B$ of the top gate or decreasing the cross-section area $L$ and $L$ increases the value of $V_{g}^{SW}$.

| FinFET     | Orientation | Aspect ratio $r$ | $L_{\text{min}}^{\text{SOI}}$ | $V_{g}^{SW}$ |
|------------|-------------|-----------------|------------------|-------------|
| SOI        | DRA         | $r = 1.2$       | $1.5L$           | $-3.12$ V   |
| SOI        | SA          | $r = 1$         | $4L$             | $+0.72$ V   |
| Bulk       | DRA         | $r = 1$         | $10L$            | $-0.28$ V   |

A more involved device design can minimize the effect of the SOHs. We now describe possible ways to restore the charge noise sweet spot in SOI and bulk FinFETs. In bulk FinFETs, the spin-orbit can be switch-off by considering wires with a squeezed cross-section that enhances the effect of the inhomogeneous electric field or by using wires grown along the [110] direction (SA direction) and by pushing the hole wavefunction at the bottom of the triangles. In bulk FinFETs, the spin orbit switch is naturally restored by considering the leakage of the hole wavefunction into a thick Si substrate. In Table II, we summarise the different FinFETs designs where the spin-orbit sweet spot is restored in the small cross-section limit. To study these devices, we perform numerical simulations of the system by discretizing in the appropriate cross-section the full 6 $\times$ 6 LK Hamiltonian; the spin-orbit velocity $\psi$ is then found by using Eq. (5a).

We begin by analyzing the spin-orbit coupling in a SOI FinFET grown in the DRA growth direction and with an isosceles triangular cross-section with base $L_y$ and equal sides $L_x$, see Fig. 1. To parametrize the deviation from the equilateral triangle, we introduce the dimensionless aspect ratio $r = L_x/L_y \in (0,2)$; equivalently, in terms of the apex angle $\Theta$ of the triangle, $r = 2\sin(\Theta/2)$. In particular, $r < 1$ when the triangle is equilateral, and $r < 1$ ($r > 1$) for acute (obtuse) triangles.

To have a fair comparison with the the results in the previous sections, we define an effective side length $\tilde{L} = L_y\sqrt{\sin(\Theta)/\sin(\pi/3)} = 3^{-1/4}L_y\sqrt{7(4-r^2)^{1/4}}$, that is the side of an ideal equilateral triangle with the same area of the isosceles triangle. We now redefine the confinement energy $\epsilon_c$ in Eq. (9) by the substitution $L \rightarrow \tilde{L}$.

In Fig. 7a), we focus on cross-sections with the same effective length $\tilde{L} = 20$ nm and we examine how the spin-orbit dependence on the inhomogeneous field $\delta E$ varies when different aspect ratios $r$ are considered. We observe that squeezing the triangle decreases the intrinsic spin-orbit field compared to the equilateral triangle, and that when the triangle is acute ($r < 1$), the spin-orbit coupling does not vary monotonously for small $\delta E$ but it reaches a maximal value before decreasing. Importantly, the spin-
homogeneous field $E$ and the effect of the inhomogeneous field $\delta E$ driven by the spin-orbit switch. In small SOI FinFET, the spin-orbit velocity can also be switched-off for a wide range of aspect ratios and in obtuse (acute) triangles, the required field $\delta E^{SW}$ is lower (higher).

Here, we used the approximate potential in Eq. (18) and neglected the effect of different values of $\delta E_{xx}$ and $\delta E_{yy}$; the spin-orbit coupling as a function of these individual components is discussed in App. D. We observe from Fig. 17 that for acute triangles there is a region in parameter space where the switch-off does not occur, and this region becomes larger the more squeezed the triangle is; for $r \lesssim 0.7$ it becomes more challenging to find a switch-off point in this geometry because the gates have to be more carefully designed to achieve a regime where $|\delta E_{xx}| < |\delta E_{yy}|$. In contrast, for squeezed obtuse triangles, with $r > 1$, the switching field is pushed towards small values of $\delta E$; the effect of the inhomogeneous field is enhanced in obtuse triangles and a lower value of gate potential is required to compensate for the intrinsic spin-orbit velocity.

This enhancement is crucial to restore the spin-orbit sweet spot. In particular, in Fig. 7b), we show the simultaneous effect of the inhomogeneous field $\delta E$ and of the homogeneous field $E_g$ when $r = 1.2$ and $\bar{L} = 20$ nm, i.e. $L_x = 22.8$ nm and $L_y = 19$ nm. Using the same gate design discussed in App. A, the components of the field are constrained to lie on the line $E_y = 0.25E\bar{L}$ and using Eqs. (A1) and (A2), we find that the spin-orbit coupling can be switched off by the inhomogeneous electric field at the gate potential $V_g^{SW} \approx -249 \times 10^{-3} \times d/2\bar{L}^2 V$, which corresponds to $V_g^{SW} \approx -3.12$ V when $\bar{L} = 20$ nm and the back gate is $d_B = 100$ nm apart from the center of mass of the wire. The total hole density $|\psi|^2$ at $V_g^{SW}$ is shown in the inset of Fig. 7b). This gate potential is rather large, but it can be reduced by placing a back gate closer to the SOI FinFET. We also expect that the value of the working potential can be decreased by optimizing the electrostatic design of the device, but we do not investigate this aspect further. In this setup and for the range of parameters considered, the spin-orbit length can be pushed down to a minimal value of $L_{\text{min}} \approx 1.5\bar{L}$ by reducing the amplitude of the gate potential. Also, from Fig. 7b), we note that as $r$ is increased, the effect of the SOHs is reduced and at $\bar{L} = 20$ nm, we recover also the spin-orbit sweet spot $E_g^{SW}^{DRA}$ driven by the homogeneous field $E_g$, see the dashed line in the figure. We stress that while these results are obtained for the specific aspect ratio $r = 1.2$ and effective side length $\bar{L} = 20$ nm, by squeezing the triangle the spin-orbit switch can be recovered for a wider range of $r$ and $\bar{L}$.

Figure 7. Spin-orbit velocity $|\psi|$ in a squeezed SOI FinFET grown along the DRA direction. In a) we show how the dependence of $|\psi|$ on the inhomogeneous electric field $\delta E$ varies as a function of the aspect ratio $r = L_x/L_y$ of the triangular cross-section. We compare triangles with the same area $\sqrt{3}L^2/4$, with effective length $\bar{L} = 20$ nm. When $1 < r < 2$ ($0 < r < 1$) the triangle is obtuse (acute). In b) we show $|\psi|$ as a function of $E_g$ and $\delta E$ for a device with $r = 1.2$ and $\bar{L} = 20$ nm. The solid blue line shows the spin-orbit switch driven by the inhomogeneous field $\delta E$. Because of the squeezing of the triangle, the spin-orbit switch driven by the homogeneous field $E_g$ is restored and $|\psi|$ vanishes also along the dashed blue curve. For the electrostatic design discussed in App. A, $\delta E$ and $E_g$ are constrained along the purple line $E_y = 0.25E\bar{L}$, see Eqs. (A1) and (A2), and in this setup the spin-orbit switch is driven mostly by the inhomogeneous field. In the inset, we show the total density $|\psi|^2$ of the hole wavefunction in the cross-section at the switching point $V_g^{SW}$ where purple and blue lines intersect. The hole density vanishes in the blue region and attains maximal value in the red region.

orbit interaction can be switched-off for a wide range of aspect ratios and in obtuse (acute) triangles, the required field $\delta E^{SW}$ is lower (higher).
Figure 8. Spin-orbit velocity $|v|$ in an equilateral SOI FinFET grown along the SA direction. In a), we show how the dependence of $|v|$ on the homogeneous electric field $E_g$ varies as a function of the cross-section side $L$. The switching field $E_{SW}$ is pushed towards lower values as the triangle becomes smaller. In this device, the spin-orbit coupling is removed by a negative electric field generated by a positive gate potential $V_g$ that pushes the hole wavefunction to the bottom of the triangle. In the units used, the results obtained without including the SOHs are independent of $L$. In b) we show $|v|$ as a function of $E_g$ and $\delta E$ for a device with $L = 20$ nm. The spin-orbit coupling vanishes along the blue line. The purple line indicates the constraint $E_g = 0.25\delta E L$ set by the electrostatic design, see App. A. The total density $|\psi|^2$ of the hole wavefunction in the cross-section at $V_g^{SW}$ where purple and blue lines intersect is shown in the inset. The hole density vanishes in the blue region and attains maximal value in the red region.

electric field $\delta E$ is included in an equilateral wire with side $L = 20$ nm. In this case, we observe that in the simple gate design considered here, i.e. along the purple line $E_g = 0.25\delta E L$, the spin-orbit vanishes at the gate potential $V_g^{SW} \approx 57.3 \times nm^2 d_B/L^3$ V, corresponding to $V_g^{SW} \approx 0.72$ V when $L = 20$ nm and $d_B = 100$ nm, easily achievable in state-of-the-art devices. The total hole density $|\psi|^2$ at the switching potential $V_g^{SW}$ is shown in the inset of Fig. 8b). Comparing to the squeezed SOI DRA setup, we find that in the regime of parameters examined, the SA device has a larger minimal spin-orbit length $l_{so}^{\min} \approx 4L$, leading to smaller spin-orbit interactions when the qubit is operational.

So far, we focused on SOI FinFETs, where the triangular fin is well-separated from the bulk. We modelled these systems by using hard-wall boundary conditions at the edges of the triangle and a rather simple electric field profile comprising homogeneous and linearly varying terms. This approximation is valid as long as the hole wavefunction is well-confined inside the fin, such that the substrate and higher order terms of the inhomogeneous electric field can be neglected. In bulk Si FinFETs, there is a thick substrate that is strongly coupled to the holes in the wire and the confinement potential that localizes the holes in the fin is provided by the negative potential $V_g$ applied to the top gate. By fully simulating this cross-section, in Fig. 9, we show that a convenient working point where the spin-orbit velocity can be completely removed emerges naturally also in these systems when the wire is grown in the DRA orientation. To obtain this result, we simulate a cross-section composed of an equilateral triangular fin with side $L = 20$ nm symmetrically placed on top of a rectangular substrate 120 nm wide and 60 nm high. For this simulation, we used the $6 \times 6 LK$ Hamiltonian and the electrostatic potential generated by the top gate is calculated by solving the Laplace equation with the boundary conditions described in App. A. When $|V_g| \gtrsim 0.1$ V, the holes are confined in the fin and the spin-orbit velocity shows a behaviour that is in qualitative agreement with our treatment, see e.g. Fig. 3. The results obtained for lower values of the gate potential $|V_g| < 0.1$ V, where the hole wavefunction is largely spread also in the substrate are inaccurate and have been removed from the figure. In this system, the spin-orbit velocity vanishes at $V_g^{SW} = -0.28$ V, where the wavefunction is strongly localized into the fin, see the inset of the figure; in larger devices, we expect again that the potential will scale roughly as $V_g^{SW} \propto d_B/L^3$. While the spin-orbit can be conveniently switched off in this setup, we also note that the absolute value of $|v|$ is an order of magnitude smaller than in a SOI FinFET with an equivalent design. In the range of parameters considered, this effect results in a larger minimal spin-orbit length $l_{so}^{\min} \sim 10L$.

V. SUPPRESSING CHARGE NOISE IN FINFET QUBITS

We now study the susceptibility to charge noise of an elongated quantum dot that defines a spin 1/2 qubit [2]. All the results discussed in this section take fully into account the SOHs. To define the dot, we include a confining potential in the direction of the wire $eBz z^2/2$, which is assumed to be much smoother than the cross-
whose ground state wavefunction is a gaussian with spatial renormalization of the \( g \) along the wire. When projected onto the groundstate of a magnetic field that oscillates as a function of the position \( B \), we show that while here we discuss only elongated quantum dots, a finite value of the gate potential \( V_g \) is simulated by solving the Laplace equation as discussed in App. A. In the inset, we show the total density \( |\psi|^2 \) of the hole wavefunction at \( V_g = -0.28 \) V, where the spin-orbit coupling vanishes. The hole density vanishes in the blue region and attains maximal value in the red region. At this value of potential, the holes are completely localized in the fin.

Figure 9. Spin-orbit velocity \( |v| \) in a bulk FinFET grown along the DRA direction. We show \( |v| \) as a function of the gate potential \( V_g \) in a device where an equilateral triangle with side \( L = 20 \) nm is placed on top of a substrate \( 6L \) wide and \( 3L \) high. The electrostatic potential in this calculation is simulated by solving the Laplace equation as discussed in App. A. In the inset, we show the total density \( |\psi|^2 \) of the hole wavefunction at \( V_g = -0.28 \) V, where the spin-orbit coupling vanishes. The hole density vanishes in the blue region and attains maximal value in the red region. At this value of potential, the holes are completely localized in the fin.

section side \( L \); in this way, the effective wire Hamiltonian in Eq. (4) accurately describes the system. We emphasize that while we discuss only elongated quantum dots, where \( l \gg L \), our results apply also to FinFET-based spin qubits where \( l \approx L \) [83], such as the ones in [52]. A potential \( \epsilon \delta E_{zz} z^2/2 \) is typically generated by terminating the top gate above the FinFET such that it has a finite value in the red region. At this value of potential, the holes are completely localized in the fin.

Without an external magnetic field \( B \), one can exactly gauge the spin-orbit coupling away by the unitary transformation \( S = e^{-i \mathbf{n} \cdot \mathbf{r} \sigma_z / \omega_z} \) [84], where \( \mathbf{n} \) is the direction of the spin-orbit vector \( \mathbf{v} \), and the Hamiltonian reduces to a harmonic oscillator with frequency \( \omega_z = \sqrt{\epsilon |\delta E_{zz}| / m^*} \), whose ground state wavefunction is a gaussian with standard deviation

\[
l = \sqrt{\frac{\hbar}{m^* \omega_z}} = \sqrt{\frac{\hbar^2}{em^*|\delta E_{zz}|}}.
\]

Because \( \delta E_{zz} \propto V_g \), the harmonic length \( l \) depends on the gate potential as \( l \propto \sqrt{|V_g|^{-1/4}} \); the divergence of \( l \) for \( V_g \to 0 \) is a consequence of the fact that in electrostatically defined quantum dots a finite value of the gate potential \( V_g \) is required to confine the particles.

At finite values of \( B \), the unitary \( S \) leads to an effective magnetic field that oscillates as a function of the position along the wire. When projected onto the groundstate of the dot, these oscillations cause a spin-orbit coupling dependent renormalization of the \( g \)-tensor in the direction perpendicular to \( \mathbf{n} \) [27, 30, 31]. In particular, decomposing the vector \( \Delta \) defined in Eq. (8) into the sum of the two vectors \( \Delta_{\perp} \) and \( \Delta_{\parallel} \) that are parallel and perpendicular to the spin-orbit vector \( \mathbf{n} \), respectively, we obtain the qubit Hamiltonian

\[
H_q = \frac{1}{2} \left( \Delta_{\parallel} + e^{-i \frac{\omega_z}{2} \Delta_{\perp}} \right) \cdot \sigma.
\]
To study the effect of charge noise, we consider small fluctuations $\delta V$ of the gate potential around the fixed working point $V_g$. To linear order in $\delta V$ the parameters of $H_q$ modify as $\Delta \rightarrow \Delta + \Delta' \delta V$, $l \rightarrow l + l' \delta V$ and $l_{so} \rightarrow l_{so} + l'_{so} \delta V$, leading to

$$H_q \rightarrow H_q + \frac{\delta V}{2} Q \cdot \sigma ,$$

(22)

where we define the vector with the units of charge

$$Q = \Delta' l + e \frac{\delta V}{2} l_{so} \Delta' \frac{l'_{so} l}{l_{so} l'} + 2 l^2 \frac{\delta V}{2} e \frac{l'_{so} l}{l_{so} l'} \left( \frac{l'_{so}}{l_{so}} - \frac{l'}{l} \right) \Delta_\perp .$$

(23)

Unless the device is operated at the sweet spot, in elongated hole quantum dots, $l$ is comparable with the spin-orbit length $l_{so}$, and the last term in Eq. (23) dominates. In addition, we stress that in these systems the requirement of a vanishing first derivative of the dot size $l_{so}$, leading to a spin-orbit-caused charge noise [41, 42], because there is an additional large contribution coming from the variation of the dot size $l$ [85]. In contrast, at the spin-orbit switch point, where $l_{so} \rightarrow \infty$, the last term in Eq. (23) vanishes exactly and charge noise only affects the qubit by the fluctuations of the $g$-factor.

We now restrict ourselves to the analysis of the squeezed SOI FinFET grown along the DRA orientation, which presents the largest spin-orbit interactions, see Table II. In this case, a qubit basis for holes is well-defined only if the magnetic field is applied along a high-symmetry axis of the system [72]. The system studied here allows for three magnetic field directions where the qubit is symmetry protected. There is a mirror symmetry plane if $B \parallel e_x$ or $B \parallel e_z$, and a twofold rotation axis for $B \parallel e_y$. The qubit basis is then provided by the corresponding angular momentum component (mod(2)).

With this choice of the qubit basis, the $g$-tensor is diagonal and the $i$th component of the Zeeman energy is $\Delta_i = g_i \mu_B B_i$. Here, we consider a magnetic field pointing in the $y$-direction, i.e. $B = B_y e_y$. This choice maximizes the Rabi frequency in electric spin dipole resonance experiments [86] because the spin-orbit vector points in the $x$-direction, see Eq. (C6). In addition, this magnetic field direction minimizes the noise and provides the largest Zeeman energy gap. The dependence of the elements of the $g$-factor matrix on the gate potential $V_g$ for this device design is shown in Fig. 10a). The value of the $g$-factors computed here and the strong anisotropy of the Zeeman energy depending on the direction of the magnetic field is in reasonable agreement with experiments [19, 46, 52].

In general, the vector $Q$ can have a component pointing along the vector $\Delta$ and a component perpendicular to it; these components cause the dephasing and the relaxation of the qubit, respectively. However, when the matrix of $g$-factors is diagonal and $B = B_y e_y$, the vector $Q$ is aligned to $\Delta$ and the qubit is only subjected to dephasing. From a Bloch-Redfield analysis and for $1/f$-type noise with spectrum $S(\omega) = (\delta V^2)/|\omega|$, the dephasing rate $1/T_2^\ast$ is given by [87, 88]

$$\frac{1}{T_2^\ast} = \frac{1}{\hbar} |Q| \sqrt{\langle \delta V^2 \rangle} \left[ \frac{1}{2\pi} \log \left( \frac{|Q| \sqrt{\langle \delta V^2 \rangle}}{\hbar \omega_{ir}} \right) \right] ,$$

(24)

where $\omega_{ir} \sim 1$ Hz is a cut-off frequency depending on the experiment. Here, we only consider free induction decay, and do not account for echo pulses that can further improve the qubit lifetime.

The dependence of the dephasing rates due to charge noise on the gate potential for the squeezed SOI FinFET qubit with $L = 20$ nm and $r = 1.2$ is shown in Fig. 10b). For the plot, we consider a magnetic field $B_y = 100$ mT, which leads to a Zeeman energy of tens of $\mu eV$, comparable to the values measured in [23]. Also, we consider dots with a fixed value of $\delta E_{zz}/V_g$, chosen such that the lateral size of the dot is $l = 30$ nm at the spin-orbit switching point $V_g = V_{g}^{SW} = -3.12$ V. In addition, at $V_g = V_{g}^{SW}$, the longitudinal confinement energy is $\hbar \omega_z = 0.14$ meV, an order of magnitude larger than the Zeeman gap $\Delta_z = 11 \mu eV$ and an order of magnitude smaller than the transverse subband gap $\Delta E = 1.7$ mV; we then conclude that our approach is valid in this regime. To estimate the fluctuations of the gate potential, we consider that the typical fluctuations of the energy levels are $\sqrt{\langle \hbar^2 \omega_z^2 \rangle} \sim 5 \mu eV$ [89] and are connected to the fluctuations of the gate potential by the dimensionless lever arm $\alpha \equiv |\hbar \partial \omega_z/(\epsilon \partial V_g)|$, i.e. $\sqrt{\langle \delta V^2 \rangle} = \alpha \sqrt{\langle \hbar^2 \omega_z^2 \rangle}/e$. At $V_g = V_{g}^{SW}$, choosing $l = 30$ nm, we obtain $\alpha \approx 65$, and we estimate $\sqrt{\langle \delta V^2 \rangle} \approx 0.3$ mV. We note that if $l = 15$ nm at $V_g = V_{g}^{SW}$, the lever arm is $\alpha \approx 16$, in reasonable agreement with recent experiments where the lever arm is about $20$ [90].

The black solid line in Fig. 10b) represents the total dephasing rate of the FinFET qubit. We observe that charge noise leads to dephasing times $T_2^\ast$ of hundreds of nanoseconds, in agreement with recent experimental data [90], that can be pushed to infinity when the devices are tuned to work at the sweet spots. Importantly, the sweet spot is close to the spin-orbit switching point $V_{g}^{SW} = -3.12$ V, but it does not exactly coincide with it. To have a better understanding of this shift, we show with dashed red and gray lines the dephasing rates $1/T_2^\ast$ obtained by considering only the terms of the vector $Q$ in Eq. (23) that are related respectively to the fluctuations of the $g$-factor, i.e. $\propto \Delta$, and to the spin-orbit coupling, i.e. $\propto l^2/l_{so}^2$. Because these different contributions in Eq. (23) can have a different sign depending on whether the $g$-factor and the spin-orbit coupling increases or decreases as a function of $V_g$, the small shift of the sweet spot is a result of the interference between the $g$-factor fluctuations and the spin-orbit coupling contribution to dephasing. In the squeezed SOI DRA device, where the spin-orbit length is very short, the spin-orbit coupling contribution to dephasing is dominating and the $g$-factor...
fluctuations are relevant only very close to the spin-orbit switch. Working at the sweet spot leads to a clear practical advantage, completely removing the charge noise when the qubit is idle.

VI. CONCLUSION

In this work, we present ways of suppressing charge noise in hole Si FinFET qubits. The advantage of these structures compared to other nanowires is their triangular cross-section, which by symmetry permits large intrinsic spin-orbit interactions without external electric fields. When the device has a simple equilateral triangular cross-section, we find this effect to be a result of the interplay between the low symmetry of the cross-section and the anisotropy of Si. When an external gate potential is applied an extra tunable contribution to the spin-orbit coupling arises and depending on the growth direction of the wire, it can enhance or suppress the total spin-orbit interactions. This suppression leads to points where the spin-orbit velocity can be tuned exactly to zero, dramatically boosting up the coherence times of spin-1/2 qubits.

We study in detail the dependence of spin-orbit coupling on the gate potential by considering an inhomogeneous electric field profile, which matches numerical simulations of the electrostatics of realistic FinFET devices. We distinguish between different mechanisms that drive the switching-off of the spin-orbit coupling and that have a different behaviour as the cross-section area becomes smaller as a result of the spin-orbit split-off hole band. These states generally degrade the performance of the FinFET and can even remove the spin-orbit switch. More involved FinFET designs are required to compensate for their effect and we discuss and compare possible realistic designs where the spin-orbit switch is restored.

When a small external magnetic field is applied, hole nanowire qubits become susceptible also to fluctuations of the Zeeman energy caused by an electrically tunable g-factor. By analyzing the response of FinFET devices to small magnetic fields, we find sweet spots where the charge noise can be completely removed to linear order in the fluctuations of the gate potential, providing an ideal working point where quantum information can be reliably stored in charge noise resilient spin qubits.

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Figure 11. Electrostatic potential $V_{E}^{N}$ in the cross-section of the Si FinFET shown in Fig. 1. For the simulation, we used $d_B = 15L$ and a substrate $30L$ wide with an equilateral triangular fin of side $L$ placed on top of it. The top gate covers the whole upper part of the device and is fixed at the potential $V_g$; the rest of the boundary is grounded.

Appendix A: Electric field simulation

Here, we present the model of the electric field profile in Si FinFETs and we compare the approximate potential in Eq. (2) used in the main text to the electrostatic potential obtained by solving the Laplace equation in the FinFET sketched in Fig. 1. We study a fin with an equilateral triangular cross-section with side $L$. For simplicity, we neglect the effect of the dielectric on the electric field lines. This approximation describes well devices that use thin high-k dielectric materials, with a dielectric constant similar to Si, but we expect our results to be at least qualitatively correct for a wider range of devices, including devices made with SiO$_2$. In addition, we assume that the top and back gates extend to infinity along the wire (in the $z$-direction), such that we can restrict our analysis to a cross-section of the FinFET in the $(x,y)$ plane. We solve the Laplace equation considering that the top gate fixes the electrostatic potential of Si to $V_g$ and that the back gate is grounded to zero potential. To model the lateral sides of the substrate, we consider a wide substrate, that extends symmetrically up to $x = \pm 15L$ from the position of the fin at $x = 0$, and we set the potential at the sides below the top gate to ground. This approximation describes well the potential in the fin as long as the distance $d_B$ of the back gate from the fin is $\lesssim 30L$. In Fig. 11, we show the potential energy $V_{E}^{N}$ simulated in this setup when the back gate is at $d_B = 15L$ from the bottom of the fin. Note that the potential in this simple design varies linearly with $V_g$ and that the lengths are normalized against the side $L$ of the fin.

We compare the numerical solution $V_{E}^{N}$ of the Laplace equation to the approximate potential in Eq. (2); in this section, we call the approximate potential $V_{E}^{A}$ to distinguish it from the numerical solution $V_{E}^{N}$. To find the parameters $E_i$ and $\delta E_{ij}$, we compute the appropriate derivatives of the potential and find their average in
the fin. For example, $\delta E_{xx} = -1/A_{\text{fin}} \int_{\text{fin}} d\mathbf{r} \partial_{xx} V_N^N(x,y)$, with $A_{\text{fin}}$ being the area of the triangle, where the averaging is performed. Because the device is symmetric around $x = 0$, the homogeneous electric field in the $x$-direction vanishes and $E_x = 0$. Simulating devices with different substrate thickness $d_B$, we find that the homogeneous and inhomogeneous components of the electric field can be written to good approximation as

$$E_y = \frac{c_y V_g}{d_B} \quad \text{and} \quad \delta E_{ij} = \frac{c_{ij} V_g}{d_B L}, \quad (A1)$$

where the dimensionless coefficients $c$ are of order one. By fitting these formulas against the results of the simulation, we find a good fit when

$$c_y = -0.3, \quad c_{xx} = -1.21, \quad c_{yy} = 1.2, \quad (A2)$$

see Fig. 12a). Note that $\delta E_{xx} \approx -\delta E_{yy}$, justifying the approximation in Eq. (18), used in the main text. We also find that $c_{xy} = 0$ in this geometry. When accounting for the finite size of the top gate, $c_{xy}$ acquires a finite value, however, it is reasonable to assume that $|c_{xy}| \ll |c_{ii}|$ as long as the top electrode fully covers the fin. A comparison between the approximate potential $V_E^A$ and $V_E^N$ simulated from the Laplace equation for $d_B = 15L$ is shown in Fig. 12b) and c). From the comparison, we observe that the electrostatic potential in the fin $V_E^N$ is reasonably well approximated by $V_E^A$.

We remark that the precise values of the coefficients $c$ can change for different devices, e.g. by including dielectric materials, changing the aspect ratio of the fin or including additional gates. While we do not expect drastic changes of our model, we do not investigate these effects in detail here.

Figure 12. Comparison between the electrostatic potential $V_E^N$ simulated from the Laplace equation and the approximate potential $V_E^A$ in Eq. (2). In a) we show the dependence of the parameters $\delta E_{ii}$ and $E_y$ defining $V_E^A$ on the substrate thickness $d_B/L$. The dots are the results of the simulation, while the solid lines are obtained by combining Eqs. (A1) and (A2). In b) and c), we show a comparison between the potential in the fin computed numerically (b) and its approximation (c) when $d_B = 15L$. To facilitate the comparison, in b), we subtract the constant potential $V_0 = 0.996$ obtained by averaging the electrostatic potential in the fin.

Appendix B: Orbital eigenstates

A convenient basis to analyze wires with triangular cross-sections comprises the eigenstates of the two-dimensional Laplace operator $p_x^2 + p_y^2$ vanishing at the boundary of an equilateral triangle of side $L$ [76]. These solutions can be chosen to be even ($e$) or odd ($o$) with respect to the height of the triangle at $x = 0$ and they can be written compactly as

$$\psi_e(x, y) = c_e f_e(x/L) \cdot g(y/L), \quad (B1a)$$

$$\psi_o(x, y) = c_o f_o(x/L) \cdot g(y/L), \quad (B1b)$$

where $c_{e/o}$ are normalization constants and we define the three-component vectors
\[ f_e(x) = \left( \cos \left[ \frac{2\pi(3m+l)}{3} x \right], \cos \left[ \frac{2\pi(3m+2l)}{3} x \right], -\cos \left[ \frac{2\pi lx}{3} \right] \right), \quad (B2a) \]

\[ f_o(x) = \left( -\sin \left[ \frac{2\pi(3m+l)}{3} x \right], \sin \left[ \frac{2\pi(3m+2l)}{3} x \right], \sin \left[ \frac{2\pi lx}{3} \right] \right), \quad (B2b) \]

\[ g(y) = \left( \sin \left[ \frac{2\pi(m+l)}{3} \left( \sqrt{3}y - 1 \right) \right], \sin \left[ \frac{2\pi m}{3} \left( \sqrt{3}y - 1 \right) \right], \sin \left[ \frac{2\pi(2m+l)}{3} \left( \sqrt{3}y - 1 \right) \right] \right). \quad (B2c) \]

The quantum numbers \( m, l \) are integers satisfying the conditions \( m \geq 1, l \geq 0 \) and \( m \geq 1, l \geq 1 \) for the even and odd solutions, respectively. These quantum numbers label the eigenvalues of the Laplace operator according to

\[ \left( p_x^2 + p_y^2 \right) \psi_\lambda(x,y) = \frac{\hbar^2}{L^2} \frac{16\pi^2}{3} \left( m^2 + lm + \frac{l^2}{3} \right) \psi_\lambda(x,y), \quad (B3) \]

where \( \lambda = e,o \) indicates the even or odd parity of the function, respectively. The states characterized by \( l = 0 \) are three-fold rotationally symmetric and even with respect to the height of the triangle; no odd solutions with \( l = 0 \) are allowed. The remaining even and odd solutions labelled by the same quantum numbers \( m \) and \( l \neq 0 \) are degenerate. The degeneracy follows from the fact that the \( l \neq 0 \) states are constructed to be even or odd only with respect to one particular inversion axis of the triangle and a linear combination of the degenerate solutions is required to find solutions that satisfy the three-fold rotational symmetry of the problem. The lowest three normalized eigenfunctions are shown in Fig. 13. These three states are the ones that used in App. C to find an effective analytical model for the triangular FinFET.

**Figure 13.** First three orbital eigenstates in Eq. (B1) of the Laplace equation in an equilateral triangle. We consider the even solution with \( (l,m) = (0,1) \) (a) and the solutions with \( (l,m) = (1,1) \) and even (b) and odd (c) symmetry. The wavefunctions in this plot are normalized. These states are the ones used in App. C to find simple analytical formulas for the triangular FinFET.
Appendix C: Spin-orbit interaction in perturbation theory

1. Intrinsic spin-orbit velocity and length

In this section, we show the general dependence of the intrinsic spin-orbit vector $\mathbf{v}_0$ in Eq. (10) and of the spin-orbit length $l_{so}$ in Eq. (6) on the orientation of the wire. To account for the different growth directions, we rotate the $4 \times 4$ LK Hamiltonian $H_{LK}'$ in Eq. (3) by the unitary operator $U = e^{i\Phi F^\prime} e^{i\Phi F^\prime}$, i.e. $H_{LK}' = U^\dagger H_{LK} U$, where $\Phi$ is the total angular momentum and $\theta$ and $\varphi$ are the elevation and the azimuthal angles between the (primed) crystallographic axes and the coordinate system of the nanowire, respectively, see Fig. 1. This unitary rotation aligns the coordinate system and the direction of the spin-matrices to the axes $x$, $y$ and $z$. More explicitly, the Hamiltonian in the rotated coordinate system is given by

$$H_{LK} = \left( \gamma_1 + \frac{5}{2} \gamma_2 \right) \frac{p_x^2}{2m} - \frac{\gamma_2}{m} J^2 - \frac{2\gamma_2}{m} p_i p_j \{ J_i, J_j \} + \mathcal{C}_0 \; ,$$

where the rotated momenta and spin 3/2 matrices are

$$\mathbf{p} = \begin{pmatrix} \cos(\theta) \cos(\varphi) p_{x'} + \cos(\theta) \sin(\varphi) p_{y'} - \sin(\theta) p_{z'} \\ \cos(\varphi) p_{y'} - \sin(\varphi) p_{x'} \\ \sin(\theta) \left( \cos(\varphi) p_{x'} + \sin(\varphi) p_{y'} \right) + \cos(\varphi) p_{z'} \end{pmatrix} \; , (C2a)$$

$$\mathbf{J} = \begin{pmatrix} \cos(\theta) \cos(\varphi) J_{x'} + \cos(\theta) \sin(\varphi) J_{y'} - \sin(\theta) J_{z'} \\ \cos(\varphi) J_{y'} - \sin(\varphi) J_{x'} \\ \sin(\theta) \left( \cos(\varphi) J_{x'} + \sin(\varphi) J_{y'} \right) + \cos(\varphi) J_{z'} \end{pmatrix} \; , (C2b)$$

and $\mathbf{p}^2 = (p_x^2, p_y^2, p_z^2)$, $\mathbf{J}^2 = (J_x^2, J_y^2, J_z^2)$, and $p^2 = p_x^2 + p_y^2 + p_z^2$.

We do not include electric or magnetic fields at the moment and we focus on the intrinsic spin-orbit coupling. To obtain simple equations, we restrict the orbital space to the space spanned by the lowest three eigenstates of the Laplace equation in an equilateral triangle, see Eq. (B1) and Fig. 13. By including the spin-orbit interactions between heavy and light holes via the rotated Hamiltonian $H_{LK}$, we obtain a $12 \times 12$ Hamiltonian matrix that models the kinetic energy of the holes confined in the fin and that parametrically depends on the angles $\varphi$ and $\theta$ and on the momentum along the wire $p_z$. The approximate Hamiltonian is found from $H_{LK}$ in Eq. (C1) by separating the different powers of $p_j p_j$, i.e.

$$H_{LK} = \sum_{i,j} p_i p_j H_{ij} ,$$

and using the matrix representation of the momenta operators in the basis $| m = 1, l = 0, \lambda = c \rangle$, $| m = 1, l = 1, \lambda = c \rangle$, $| m = 1, l = 1, \lambda = o \rangle$, where the quantum numbers $m, l, \lambda$ are defined in Appendix B, see Eq. (B2). Explicitly, the matrices of the momenta squared are given by

$$p_x^2 = \frac{\hbar^2}{L^2} \begin{pmatrix} \frac{8\pi^2}{3} & \frac{k}{\sqrt{2}} & 0 \\ \frac{k}{\sqrt{2}} & \frac{56\pi^2}{9} - \frac{21}{2} k & 0 \\ 0 & 0 & \frac{56\pi^2}{9} + \frac{21}{2} k \end{pmatrix} , (C3a)$$

$$p_y^2 = \frac{p_z^2}{k \rightarrow -k} , (C3b)$$

$$p_z^2 = p_z^2 I_3 \; , (C3c)$$

Figure 14. Components of the intrinsic spin-orbit vector $\mathbf{v}_0$ as a function of the angles $\theta$ and $\varphi$ that parametrize the orientation of the wire with respect to the crystallographic axes, see Fig. 1. The results shown here are obtained by using a second order Schrieffer-Wolff transformation and including only the lowest 3 orbital eigenstates in Eq. (B1). The wavefunction of these states is shown in Fig. 13. We indicate with red and blue lines the cuts $\theta = 0$ and $\theta = -\pi/2$, respectively. These two cases are studied extensively in the main text. The black circle marks the point $\varphi = \pi/4$ and $\theta = -\arctan \sqrt{2 + \sqrt{3}}$, where the wire is grown along the $z \parallel [111]$ direction, and where $\langle \mathbf{v}_0 \rangle_{z,y} = 0$ and $\langle \mathbf{v}_0 \rangle_z$ is maximal.
and the matrices of the cross-momenta are

\[
p_{x}p_{y} = \frac{\hbar^{2}}{L^{2}} \begin{pmatrix} 0 & 0 & -\frac{k}{2} \\ 0 & 0 & -\frac{25k}{28} \\ -\frac{k}{\sqrt{2}} & -\frac{25k}{28} & 0 \end{pmatrix}, \quad (C4a)
\]

\[
p_{x}p_{z} = \frac{9\hbar kp_{z}}{10\sqrt{2}\pi L} \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad (C4b)
\]

\[
p_{y}p_{z} = \frac{9\hbar kp_{z}}{10\sqrt{2}\pi L} \begin{pmatrix} 0 & i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (C4c)
\]

Here, \( k = 2187/112 \approx 19.53 \) and \( I_{3} \) is the 3-dimensional identity matrix. We use a under bar here to denote this matrix representation of the momentum operators. The approximate \( 12 \times 12 \) Hamiltonian is then obtained by a tensor product between orbital and spin-degrees of freedom, i.e. \( H_{\text{LK}} = \sum_{i,j} p_{i}p_{j} \otimes H_{ij} \). The expression for general values of \( \theta \) and \( \phi \) is lengthy and we do not report it explicitly here. In the simple case \( \theta = 0 \), the \( 12 \times 12 \) Hamiltonian, also including the electric field, is explicitly given in Eq. (C7).

We resort to perturbation theory on this \( 12 \times 12 \) Hamiltonian to derive a low-energy description of the system. With a second order Schrieffer-Wolff transformation, see e.g. Appendix B of [1] or [77], we obtain a \( 2 \times 2 \) effective Hamiltonian that acts on the ground state subspace of the wire. Expanding the matrix elements of this Hamiltonian in power series of the momentum along the wire \( p_{z} \), up to second order, we find an effective wire Hamiltonian as in Eq. (4). The terms linear in \( p_{z} \) are related to the spin-orbit velocity \( \mathbf{v}_{0} \), while the effective mass \( m^{*} \) is twice of the inverse of the diagonal term quadratic in \( p_{z} \).

For a Si wire, the dependence of the components of the vector \( \mathbf{v}_{0} \) on the growth angles obtained in this way is shown in Fig. 14. Importantly, we find that the \( \mathbf{v}_{0} \) is in general proportional to the anisotropy of the material \( \gamma_{3} \sim \gamma_{2} \) and to the velocity \( \hbar/mL \), and thus it can be written as in Eq. (10). In addition, there is no spin-orbit coupling in the direction along the wire when \( \theta = 0 \) and \( \theta = \pm \pi/2 \) (red and blue lines, respectively), and in these cases \( (\mathbf{v}_{0})_{z} = 0 \). While these results were found by perturbation theory, we find numerically that they hold generally. Interestingly, the off-diagonal components of the spin-orbit velocity \( (\mathbf{v}_{0})_{x,y} \) vanish e.g. when \( \varphi = \pi/4 \) and \( \theta = -\arctan \sqrt{2 + \sqrt{3}} \), where the diagonal spin-orbit \( (\mathbf{v}_{0})_{z} \) is maximal. This orientation corresponds to \( z \parallel [111] \), along which the Si lattice has a three-fold rotational symmetry, and is marked with black circles in the figures.

By taking the limits \( \theta = 0 \) and \( \theta = \pm \pi/2 \), the expressions of the spin-orbit vector simplify notably and are given in Eqs. (11). When \( \theta = \pm \pi/2 \), the spin-orbit vector points along the \( x \)-direction and it has an oscillating amplitude, while when \( \theta = 0 \), the direction is oscillating as a function of \( \varphi \). A plot of the absolute values of \( \mathbf{v}_{0} \) in these cases and a comparison between the perturbation theory and a more detailed numerical solution comprising 200 orbital eigenstates in Eq. (B1) is shown in Fig. 2.

With this approach, we also find the effective mass \( m^{*} \) from the diagonal elements of the effective Hamiltonian. Along the \( \theta = 0 \) and \( \theta = \pm \pi/2 \) directions, \( m^{*} \) can be compactly written as
where in the second equation we discarded higher harmonic components oscillating with a small amplitude and a fast period in \( \varphi \); \( m \) is the bare electron mass. In Fig. 15a), we show the effective mass as a function of the azimuthal angle \( \varphi \), comparing the approximate Eq. (C5) with the more precise numerical result obtained by using Eq. (5b) and extending the Hilbert space to accommodate 200 orbital states given in Eq. (B1). We observe that the simple analytical results capture well the oscillating behaviour of the mass as a function of \( \varphi \), but they underestimate the amplitude of the oscillations, leading to a smaller mass, especially when the wire extends along a crystallographic axis, e.g. when \( \theta = 0 \) or when \( \theta = -\pi/2 \) and \( \varphi = 0 \). For the SA direction, where \( \theta = -\pi/2 \) and \( \varphi = -3\pi/4 \), Eq. (C5b) works well and gives an effective mass \( m_{SA}^* \approx 0.17m \), while for the CA and DRA directions, the numerical analysis is more precise and it gives an effective mass \( m_{CA}^* \approx m_{DRA}^* \approx 0.41m \).

Analytical expressions for the intrinsic spin-orbit length \( l_{so} \) defined in Eq. (6) can be obtained when \( \theta = 0 \) and \( \theta = \pi/2 \) by combining Eqs. (10), (11) and (C5). A comparison between these expressions and the numerically computed values of \( l_{so} \) is shown in Fig. 15b). Because of the underestimation of the effective mass, the spin-orbit length predicted by perturbation theory is larger than the numerical values when the wire extends along a crystallographic axis. In contrast, for the SA growth direction, the perturbative result is smaller than the numerical result because of the overestimation of the spin-orbit velocity, see Fig. 2. Importantly, the intrinsic spin-orbit length is always of the order of the side of the triangle, typically much shorter than the confinement length of the quantum dot along the wire, leading to large spin-orbit interactions.

2. Electric field dependence

Here, we focus on wires with elevation angle \( \theta = 0 \), i.e. wires grown along the [001] crystallographic direction, and we study the dependence of the spin-orbit velocity on the electric field. In this case, the spin-orbit velocity vector has no component along the \( z \)-direction and is off-diagonal. We define then the complex off-diagonal component of the spin-orbit velocity \( v = (v_x, v_y) \). This quantity can be decomposed into the sum of an isotropic direct Rashba-like component that vanishes when the external electrostatic potential in Eq. (2) is turned off, and an anisotropic component that varies as a function of the azimuthal angle \( \varphi \), i.e.

\[
v = \frac{\hbar}{mL} \left[ (\gamma_3 - \gamma_2)\alpha_A(\varphi) + (\gamma_3 + \gamma_2)\alpha_I \right].
\]

At finite values of the electrostatic potential, the spin-orbit depends on the homogeneous electric field \( E_y \) and on the inhomogeneous fields \( \delta E_{xx} \) and \( \delta E_{yy} \). Here, we do not account for the effect of the terms \( \delta E_x \) and \( \delta E_{xy} \) and we introduce the sum and difference of the inhomogeneous fields \( \delta E = (\delta E_{xx} - \delta E_{yy})/2 \) and \( \Sigma E = (\delta E_{xx} + \delta E_{yy})/2 \). For simplicity of notation, we rescale the electric field by the confinement energy to obtain dimensionless quantities, i.e. \( eE_yL/\epsilon_c \rightarrow E_y, e\delta EL^2/\epsilon_c \rightarrow \delta E \) and \( e\Sigma EL^2/\epsilon_c \rightarrow \Sigma E \), and we define the vector \( \mathbf{e} = \left( E_y, \delta E, \Sigma E \right) \).

We neglect the SOHs and in analogy to the treatment in Sec. C1, we study the \( 12 \times 12 \) Hamiltonian obtained from the \( 4 \times 4 \) total Hamiltonian in Eq. (1) rotated by the angle \( \varphi \) and projected onto the subspace spanned by the lowest three orbital eigenstates in Eq. (B1). Explicitly,

\[
H_{12} = \frac{\hbar^2}{mL^2} \begin{pmatrix}
H_{00} & H_{0e} & H_{0o} \\
H_{0e}^\dagger & H_{ee} & H_{eo} \\
H_{0o}^\dagger & H_{eo}^\dagger & H_{oo}
\end{pmatrix},
\]

where the dimensionless blocks are given by
\[ H_{00} = \text{diag}\left( \frac{8\pi^2}{3} \gamma_1^+ + \frac{p_z^2}{2} \gamma_1^-, \frac{8\pi^2}{3} \gamma_2^+ + \frac{p_z^2}{2} \gamma_2^-, \frac{8\pi^2}{3} \gamma_3^+ + \frac{p_z^2}{2} \gamma_3^-, \frac{8\pi^2}{3} \gamma_4^+ + \frac{p_z^2}{2} \gamma_4^- \right), \tag{C8a} \]

\[ H_{ee} = \begin{pmatrix}
\frac{56\pi^2}{9} \gamma_1^+ + \frac{p_z^2}{2} \gamma_1^- & 0 & -\frac{21}{25} C_- & 0 \\
0 & \frac{56\pi^2}{9} \gamma_2^+ + \frac{p_z^2}{2} \gamma_2^- & 0 & -\frac{21}{25} C_- \\
-\frac{21}{25} C_- & 0 & \frac{56\pi^2}{9} \gamma_3^+ + \frac{p_z^2}{2} \gamma_3^- & 0 \\
0 & -\frac{21}{25} C_- & 0 & \frac{56\pi^2}{9} \gamma_4^+ + \frac{p_z^2}{2} \gamma_4^- \\
\end{pmatrix} - I_4 \left( \frac{21}{25} a_E E_y - a_\delta \delta E + \frac{16}{27} \gamma_1 \Sigma E \right), \tag{C8b} \]

\[ H_{oo} = H_{ee}|_{C_- \to -C_-}, \delta E \to -\delta E, E_y \to -E_y, \]

\[ H_{oe} = \begin{pmatrix}
0 & \frac{1}{\sqrt{2}} C_- & 0 \\
-\frac{1}{\sqrt{2}} C_- & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{2}} C_- \\
\end{pmatrix} - I_4 \left( \sqrt{2} a_E E_y - b_\delta \delta E \right), \tag{C8d} \]

\[ H_{oo} = \begin{pmatrix}
0 & -i K p_z & 0 \\
-i K p_z & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{2}} C_+ \\
\end{pmatrix}, \tag{C8e} \]

\[ H_{eo} = \begin{pmatrix}
0 & 0 & -\frac{21}{25} i C_+ \\
0 & 0 & 0 \\
\frac{21}{25} i C_+ & 0 & 0 \\
\end{pmatrix}. \tag{C8f} \]

Here, the momentum \( p_z \) is measured in units of \( h/L \), \( I_4 \) is a \( 4 \times 4 \) identity matrix and we define

\[ C_\pm = \frac{\sqrt{3}}{2} k \left[ e^{i\phi} (\gamma_3 - \gamma_2) \pm (\gamma_3 + \gamma_2) \right], \tag{C9a} \]

\[ a_E = \frac{59049}{4480\pi} \gamma_1 \approx 4.186\gamma_1, \tag{C9b} \]

\[ a_\delta = \left( \frac{3969}{2000\pi} - \frac{7\sqrt{3}}{25} \right) a_E \approx 0.616\gamma_1 \tag{C9c} \]

\[ b_\delta = \left( \frac{1107}{280\sqrt{2}\pi} - \frac{\sqrt{3}}{3} \right) a_E \approx 0.308\gamma_1 \tag{C9d} \]

\[ K = \frac{9}{10\pi} \sqrt{\frac{3}{2}} k \gamma_3 \approx 6.851\gamma_3. \tag{C9e} \]

Also, we introduce the bulk heavy and light hole inverse effective masses in the direction perpendicular and parallel to the wire as \( \gamma_1^\pm = \gamma_1 \pm \gamma_2 \) and \( \gamma_2^\pm = \gamma_1 \mp 2\gamma_2 \).

We use \( k = 2187/112 \approx 19.53 \), as in Eqs. (C3) and we subtract a global energy shift linear in \( \Sigma E \).

In contrast to Sec. C 1, a second-order Schrieffer-Wolff transformation does not capture accurately the dependence of the spin-orbit on the electric field and thus we increase the accuracy of our calculation by using a fourth-order Schrieffer-Wolff transformation. To obtain compact equations, we also Taylor expand the spin-orbit velocity to second order in the vector \( \epsilon \), leading to
\[ \alpha_I \approx \frac{\gamma_3}{\gamma_1 + 5\gamma_2/2} \left( \frac{\gamma_1}{\gamma_1 + 5\gamma_2/2} \alpha_I^{(1)} \cdot e + \frac{\gamma_1^2}{(\gamma_1 + 5\gamma_2/2)(\gamma_1 - \gamma_2)} e \cdot \Omega_{A}^{(2)} \cdot e \right) \],
\[ \alpha_A(\varphi) \approx \frac{\gamma_3}{\gamma_1 + 5\gamma_2/2} e^{-4i\varphi} \left( \alpha_A^{(0)} + \frac{\gamma_1}{\gamma_1 + 5\gamma_2/2} \alpha_A^{(1)} \cdot e + \frac{\gamma_1^2}{(\gamma_1 + 5\gamma_2/2)(\gamma_1 - \gamma_2)} e \cdot \Omega_{A}^{(2)} \cdot e \right) \],
\[ \alpha_I^{(1)} \approx \left( \begin{array}{c}
0.938(\gamma_1 + 6\gamma_2) \\
0.066(\gamma_1 - 0.407\gamma_2) \\
0
\end{array} \right),
\[ \Omega_{I}^{(2)} \approx \left( \begin{array}{ccc}
0.817\gamma_1 + 0.938\gamma_2 \\
-0.107\gamma_1 + 0.155\gamma_2 \\
0.0034\gamma_1 + 0.0055\gamma_2
\end{array} \right)
\left( \begin{array}{ccc}
\gamma_1 - \gamma_2 \\
\gamma_1 - \gamma_2 \\
\gamma_1 - \gamma_2
\end{array} \right)
\left( \begin{array}{c}
0.0317\gamma_1^2 + 0.2692\gamma_2\gamma_1 + 0.0871\gamma_2^2 \\
0.0022\gamma_1^2 - 0.0217\gamma_2\gamma_1 - 0.0006\gamma_2^2 \\
0
\end{array} \right),
\[ \alpha_A^{(0)} = \frac{14}{45} (k/2\pi)^3 \approx 9.338,
\[ \alpha_A^{(1)} \approx \left( \begin{array}{c}
0, 
0, 
0.158
\end{array} \right),
\[ \Omega_{A}^{(2)} \approx \left( \begin{array}{ccc}
-1.251\gamma_1^2 + 4.231\gamma_2\gamma_1 + 1.604\gamma_2^2 \\
0 \\
0
\end{array} \right)
\left( \begin{array}{c}
\gamma_1 - \gamma_2 \\
\gamma_1 - \gamma_2 \\
\gamma_1 - \gamma_2
\end{array} \right)
\left( \begin{array}{c}
0.13\gamma_1^2 + 0.52\gamma_2\gamma_1 + 0.0855\gamma_2^2 \\
0.0019\gamma_1^2 + 0.0185\gamma_2\gamma_1 - 0.0013\gamma_2^2 \\
0.0027\gamma_1^2 + 0.005\gamma_2\gamma_1 - 0.0002\gamma_2^2
\end{array} \right).
\]

To simplify further the expressions, we kept only the terms with the lowest possible power in $1/\gamma_1$. This approximation allows for a good qualitative understanding of the system while keeping the equations short. As a result of this approximation, the $\alpha$ parameters are independent of $\gamma_3$ and we discard small terms -at least one order of magnitude smaller than the dominant terms- in $\alpha_A$ that do not oscillate as $e^{-4i\varphi}$, i.e. terms proportional to $e^{+4i\varphi}$ and to $e^{(\gamma_3 - 2\gamma)} e^{-8i\varphi}$. To specify where these terms have been neglected in Eq. (C10), we use the notation $0$ and $0$ to distinguish between terms that are negligibly small but finite (0.) from terms that are exactly zero (0). For Si, Eq. (C10) reduces to

\[ \alpha_I \approx 0.35E_y + 0.246E_y^2 + 0.0086\delta E + 0.0011\delta E^2 - 0.0331E_y\delta E + 0.0124E_y\Sigma E + 0.00011\delta E\Sigma E ,
\[ \alpha_A(\varphi) \approx e^{-4i\varphi} \left( 2.631 - 0.367E_y^2 - 0.00078\delta E^2 + 0.0372\Sigma E + 0.00050.35\Sigma E^2 + 0.0398\delta E E_y \right) .
\]

These expressions give valuable insights into the dependence of the spin-orbit coupling on the external fields and allow for a qualitative understanding of the numerical results presented in Sec. III B and III D. For example, let us take the limit $\Sigma E = 0$ as in the main text. To switch off the spin orbit we require $v = 0$. For this to happen, the large intrinsic spin-orbit need to be compensated for by the electric field induced spin-orbit, which needs to have opposite sign. By looking at Eq. (C11), it is clear that the DRA growth direction, with $\varphi = \pi/4$, can drive the switch because in this case all the terms varying with the electric field have the opposite sign with respect to the intrinsic spin-orbit coupling. In this case, when $\delta E = 0$, we find that $v = 0$ at $E_{SW}^{SW} \approx 1.52$ and when $E_y = 0$, the spin-orbit vanish at $\delta E^{SW} \approx 29.5$. Quantitatively, the values of these critical fields are only accurate up to prefactors of order one, see Eqs. (14a) and (19).

The spin-orbit vanishes also when the fields $E_y$ and $\delta E$ are both present. The precise shape of the curve along which this occurs strongly depends on the numerical values of the $\alpha$ parameters. Using Eq. (C11), one predicts $v = 0$ along two separate lines that do not intersect, while numerically we observe that the two lines merge together, see the blue curve in Fig. 4b). This discrepancy is a consequence of the numerical inaccuracy of the approximation used: slight variations of the cross-coupling terms $\times E_y\delta E$ can drastically change the behaviour of the switching curve. We note that the correct qualitative behaviour of the spin-orbit is restored by including higher powers of $e$ in the expansion in Eq. (C10), but we do not give explicit expressions for these terms here.
Appendix D: Spin-orbit coupling against $\delta E_{xx}$ and $\delta E_{yy}$

In the main text, we assume that the inhomogeneous electric field component has the form given in Eq. (18). Here, we show that the presence of the spin-orbit switch is not related to this approximation by examining separately the effect of the fields $\delta E_{xx}$ and $\delta E_{yy}$ on the spin-orbit velocity. We restrict ourselves to the analysis of FinFETs grown along the [001] direction, with $\theta = 0$. In this section, we consider $E_y = 0$.

In Fig. 16, we study the spin-orbit velocity in equilateral triangles. In the top figures 16a) and 16b), we show the results obtained for large wires neglecting the SOHs and show results obtained for the DRA and the CA orientations, respectively. In c) and d) we include the SOHs and simulate a wire with cross-section of side $L = 20$ nm. We show results obtained for the DRA and CA orientations, respectively. The purple lines show the constraint $\delta E_{xx} = -\delta E_{yy}$ for the inhomogeneous fields, corresponding to the approximation used in the main text, see Eq. (18).

Figure 16. Spin-orbit velocity $|v|$ as a function of $\delta E_{xx}$ and $\delta E_{yy}$ when $\theta = 0$. Here, we consider an equilateral triangular cross-section. In a) and b) we neglect the SOHs and show results obtained for the DRA and the CA orientations, respectively. In c) and d) we include the SOHs and simulate a wire with cross-section of side $L = 20$ nm. We show results obtained for the DRA and CA orientations, respectively. The purple lines show the constraint $\delta E_{xx} = -\delta E_{yy}$ for the inhomogeneous fields, corresponding to the approximation used in the main text, see Eq. (18).
the SOHs and using the rotated \( 4 \times 4 \) LK Hamiltonian in Eq. (C1). The two figures correspond to the DRA and CA orientations, respectively. The purple line marks the approximation \( \delta E_{xx} = -\delta E_{yy} \) used in the main text. We observe that in the DRA orientation, the vanishing spin-orbit coupling is not a just result of the approximation \( \delta E_{xx} = -\delta E_{yy} \), but one can find an operational sweet spot where \( |v| = 0 \) also by considering a more general relation between the inhomogeneous fields \( \delta E_{yy} = c_{yy} \delta E_{xx} / c_{xx} \), see Eq. (A1). The spin-orbit coupling is not suppressed only when \( |c_{yy} / c_{xx}| \ll 1 \). We remark that the parameters \( c_{yy} \) and \( c_{xx} \) can vary depending on the device design. In the same regime of parameters, we do not find a comparable sweet spot for the crystallographic orientation. In the bottom figures 16c) and 16d), we show the results obtained by including the SOHs for an equilateral triangular cross-section of side \( L = 20 \) nm. For the DRA growth direction, the spin-orbit coupling vanishes for any value of the ratio \( |c_{yy} / c_{xx}| \) and interestingly also when \( |c_{yy} / c_{xx}| \ll 1 \), in contrast to when the SOHs are neglected. In addition, the SOHs modify the response when the wire is grown along the crystallographic orientation. In fact, in 16d) we observe that the spin-orbit velocity vanishes along the vertical line \( \delta E_{xx} \approx 30 e_c / (e L^2) \). While this result shows that also this orientation might present suitable working points where charge noise is suppressed, we do not investigate this possibility in more details.

We now study how these results change when we consider wires with an isosceles triangular cross-section as the ones studied in Sec. IV A. We consider a SOI FinFET in the DRA orientation and in Fig. 17, we compare the spin-orbit velocity in isosceles triangles with the same area \( \sqrt{3} L^2 / 4 \) and different aspect ratios \( r = L_x / L_y \). Here, we fully account for the SOHs and we use an effective side length of \( L = 20 \) nm. In the top figures 17a) and 17b), we show the spin-orbit velocity when the triangle is acute. When \( r = 0.8 \), the spin-orbit coupling is still suppressed when \( \delta E_{yy} = c_{yy} \delta E_{xx} / c_{xx} = -\delta E_{xx} \) (purple line), but when the ratio \( |c_{yy} / c_{xx}| \ll 1 \), the spin-orbit coupling does not vanish and the charge noise sweet spot is removed. Even worse, when \( r = 0.7 \), the spin-orbit coupling vanishes only when the ratio \( |c_{yy} / c_{xx}| \gg 1 \), away from the limit studied in the main text. In the bottom figures 17c) and 17d), we show the spin-orbit velocity when the triangle is obtuse. In contrast to the acute triangle, here the spin-orbit coupling vanishes for any values of the ratio \( |c_{yy} / c_{xx}| \). By increasing \( r \), we observe that the line where \( |v| = 0 \) is pushed towards lower values of \( \delta E \) and when \( r = 1.5 \), an additional line where the spin-orbit coupling vanishes appears. However, we also note that the maximal amplitude of the spin-orbit velocity in these devices decreases compared to the equilateral triangle case, see Fig. 16c).
Figure 17. Spin-orbit velocity $|\mathbf{v}|$ as a function of $\delta E_{xx}$ and $\delta E_{yy}$ for isosceles triangles. Here, we consider wires grown along the DRA orientation whose cross-sections have an effective length $\tilde{L} = 20$ nm and different aspect ratios $r = L_y/L_x$. The SOHs are fully included in these results. In a) and b) we show the spin-orbit coupling obtained for acute triangles ($r < 1$), while in c) and d) we show the results obtained for obtuse triangles ($r > 1$). The purple lines show the constraint $\delta E_{xx} = -\delta E_{yy}$ for the inhomogeneous fields, corresponding to the approximation used in the main text, see Eq. (18).

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[74] More precisely, in a wire, the orbital effects caused by a magnetic field directed along the wire begin to be relevant when the area $h/(eB)$ occupied by a single flux quantum is comparable or smaller than the area of the cross-section, $\sqrt{3}L^2/4$ for an equilateral triangle. In an elongated quantum dots, characterized by a length $l$ in the direction along the wire, orbital effects causes by in-plane fields matter when $h/(eB) \lesssim l$.

[75] If $\mathbf{E} = 0$ and the cross-section is inversion symmetric, then $V(x, y) \equiv V_{\text{NS}}(x, y) + V_{\text{K}}(x, y) = TV(x, y)^{1} = V(-x, -y)$, where $\mathbf{E} = e^{-i\mathbf{F}z}$ is the inversion operator in the $(x, y)$ plane; $F_z$ is the component parallel to the wire of the total angular momentum. It follows that the ground subspace of $H_{\text{K}}(p_z = 0) + V(x, y)$ can be labelled by $|i, \uparrow\rangle$ and is composed of a degenerate Kramers doublet $|\uparrow\rangle$ that is also an eigenstate of $\mathbf{E}$ to eigenvalue $i$. Let us consider the correction to the LK Hamiltonian $H_{\text{K}}p_z$ to linear order in $p_z$, such that the states $|i, \uparrow\rangle$ are still approximate eigenstates. Because $H_{\text{K}}$ anticommutes with $\mathbf{E}$, i.e. $iH_{\text{K}}\mathbf{E}^1 = -H_{\text{K}}\mathbf{E}^1$ is an block off-diagonal matrix in the basis of the eigenstates of $\mathbf{E}$. Consequently, in the groundstate subspace there cannot be spin-orbit interactions linear in $p_z$ and because time-reversal symmetry prohibits different mass for the Kramers partners, the first possible spin-orbit interactions are $\propto p_z^2$.

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[78] The results presented in Sec. IIIIB are qualitatively valid even in the presence of asymmetries that lead to a finite $E_z$. In particular, we find that the value of the spin-orbit velocity at $E_y = 0$ is increased by $E_z$ and consequently the spin-orbit switch is pushed to higher values of $E_y$.

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Explicitly, the component of the charge noise related to the spin-orbit coupling can be expanded by considering independent variations of the spin-orbit strength $|v| \rightarrow |v(V_g)| + |v|^\prime \delta V$, of the effective mass $m^\ast \rightarrow m^\ast(V_g) + (m^\ast)^\prime \delta V$ and the variation of harmonic potential, $\delta E_{zz} \rightarrow \delta E_{zz}^\prime (V_g + \delta V)$, leading to $l^\prime - l = \frac{1}{4V_g} - \frac{3(m^\ast)^\prime}{4m^\ast} - \frac{|v|^\prime}{|v|}$. While the variation of the effective mass relative to the mass is typically small, the variations of the confinement potential and of the spin-orbit amplitude are comparable.

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