CMS ridge effect at LHC as a manifestation of bremsstrahlung of gluons due to the quark-anti-quark string formation

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Abstract
The recently reported effect of long-range near-side angular correlations at LHC occurs for large multiplicities of particles with $1\text{ GeV} < p_T < 3\text{ GeV}$. To understand the effect several possibilities have been discussed. In the letter we propose a simple qualitative mechanism which corresponds to gluon bremsstrahlung of quarks moving with acceleration appropriate to the quark–anti-quark string. The smallness of azimuthal angle difference $\Delta\phi$ along with large $\Delta\eta$ at large multiplicities in this interval of $p_T$ are natural in the mechanism. The mechanism predicts also bremsstrahlung photons with mean values of $p_T \approx 2.9$ and $0.72\text{ GeV}$.

Keywords: CMS ridge effect, quark-anti-quark string, gluon bremsstrahlung

In paper [1] the effect in proton-proton collisions at LHC is reported for existence of a ridge in the plot of data for two-particle correlations versus pseudo-rapidity difference $\Delta\eta$ and azimuthal angle $\Delta\phi$ plane. This ridge means essential excess of events with $\Delta\phi$ close to zero and large $\Delta\eta$. It is important to emphasize, that the effect is observed under condition that the accompanying charged particles have multiplicity $> 100$ and are each situated in restricted region of transverse momentum $1\text{ GeV} < p_T < 3\text{ GeV}$.

The result already causes discussion devoted to possible interpretation of the data [2] – [13]. In note [2], possibilities of interpretation of the effect in terms of quark-anti-quark strings are discussed. The first point is that a string is formed being stretched close to the direction of $p\bar{p}$ collision and thus might decay with $\Delta\phi \approx 0$ and $\approx \pi$. However it is emphasized in [2], that there is no reason, why the effect is observed only for very high multiplicity $N > 100$, while multiple production of hadrons via string decomposition leads to randomization and thus simulations show no ridge.

In the present letter we would try to explain why even for high multiplicity the correlation persists when we take into account radiation of gluons in the process of QCD quark-anti-quark string formation in proton-proton collision at very high energies. We mean formation of a string between either quark from the first proton and anti-quark from the second one or vice versa. The direction of the string is evidently close to the direction of momenta of the colliding protons. The string has overall motion with some momentum.

Each quark (anti-quark) moves considerable time in a very strong color field which we describe in terms of the string tension. In this case an acceleration has almost the same direction as the velocity and thus we may use the well-known classical expression for dipole electromagnetic radiation of electric charge $e$ in electric field parallel to velocity of the motion [14]

$$\frac{dE}{dt} = \frac{2\alpha}{3} \left( \frac{A\hbar^2}{m} \right)^2;$$

(1)

where $m$ is a light quark mass, $\hbar^2$ is the string tension and $\alpha$ is the fine structure constant. We take initial expression (1) in an accompanying reference frame. In view to make estimates we take the following values for these fundamental quantities

$$m_u = 2.5\text{ MeV}; \ m_d = 5\text{ MeV}; \ \hbar = 420\text{ MeV};$$

(2)

where light quark masses are chosen to be in the middle of interval of their possible values: $1.7\text{ MeV} < m_u < 3.3\text{ MeV}; \ 4.1\text{ MeV} < m_d < 5.8\text{ MeV}$ [15].

So quarks are moving with acceleration and thus radiate gluons. Let us obtain simple quasi-classical estimate of the mean energy of radiated gluons. In view of this we rewrite expression (1) in the form

$$\frac{\Delta E}{\Delta t} = \frac{\alpha\hbar^2}{9} \left( \frac{A\hbar^2}{m} \right)^2;$$

(3)
where we change in \( \alpha \to \alpha_s \) and introduce the evident color factor. Now to obtain the quasi-classical estimates we use the well-known uncertainty relation

\[
\Delta E \Delta t = 1. \tag{4}
\]

Finally we have for the mean energy of a gluon

\[
\Delta E = \sqrt{\frac{\alpha_s}{9} \frac{\hbar^2}{m}}. \tag{5}
\]

Then we use the standard one loop expression for \( \alpha_s \) at scale \( \Delta E \)

\[
\alpha_s(\Delta E) = \frac{12\pi}{(33 - 2N_f) \ln \left( \frac{\Lambda_{QCD}^2}{\Delta E} \right)}; \tag{6}
\]

We have for one loop expression \( \alpha_s \) with \( N_f = 4, \Lambda_{QCD} \approx 190\, MeV \). Then with this result the solution of relations \( \eqref{5} \) under conditions \( \eqref{2} \) gives us the following estimates for radiation off quarks \( u \) and \( d \)

\[
\Delta E_u \approx 11.2\, GeV; \quad \Delta E_d \approx 5.6\, GeV. \tag{7}
\]

The result \( \eqref{7} \) gives an estimate for mean energies of the bremsstrahlung gluons.

First of all let us consider an explanation of large differences in pseudo-rapidity \( \Delta \eta \) along with small differences in azimuthal angle \( \Delta \phi \). Here we are to take into account both quarks constituting the string. Namely let the string be produced with some overall momentum \( k \) while its direction remains being (almost) parallel to the line of \( p p \) collision. Such situation is presented in Fig.1.

![Fig.1. The string moving with momentum \( k \) from the point of collision of two protons, \( \psi_1, \psi_2 \) are angles in Eq.\( \eqref{6} \) and \( q_1, q_2 \) are momenta of the quarks.](image)

Then velocities of quarks are not parallel to the direction of acceleration, but constitute some angles \( \psi_1, \psi_2 \) with this direction. When a velocity and an acceleration are not parallel \( \forall a = v a \cos \psi \) and there are two accelerated quarks we have the following angular distribution\footnote{We normalize \( \alpha_s \) at the point of \( \tau \)-lepton mass due to better precision of data here.}

\[
\frac{dE}{d\tau'} = \frac{\alpha_s}{24\pi} \left( \frac{\hbar^2}{m} \right)^2 \times
\]

\[
\Phi(\psi_1, \theta, \phi, v_1) + \Phi(\psi_2, \theta, \phi, v_2) \, d \Omega; \tag{8}
\]

\[
\Phi(\psi, \theta, \phi, v) = \frac{X + v^2 Y}{Z^3} \cos \psi \sin \theta \cos \phi
\]

\[
X = \sin^2 \theta - 2v \sin \psi \sin \theta \cos \phi
\]

\[
Y = \cos^2 \theta \sin^2 \psi + \sin^2 \theta \sin^2 \psi \cos^2 \phi
\]

\[
Z = 1 - v(\cos \psi \cos \theta + \sin \psi \sin \theta \cos \phi);
\]

where \( \tau' \) is a time with account of a retardation \( \eqref{14} \), \( \psi_1, \psi_2 \) are respectively angles for the first and the second quark. Small \( \Delta \phi \) along with the wide spread in pseudo-rapidity in the effect \( \eqref{11} \) is connected with the same sign of \( \sin \psi \), because quarks are directed to one side from the line of collision (see Fig.1). Then the distribution in polar angle \( \theta \) has two maxima divided by some significant interval \( \Delta \theta \), while distribution in azimuthal angle \( \phi \) is again close to zero. After integration of \( \Phi \) by \( \phi \) and \( \theta \) correspondingly we have these distributions. The situation is illustrated in Fig.2 and Fig.3, in which we present normalized distribution in rapidity \( \eta \) and normalized angular distribution in \( \phi \) in \( \Phi(\psi_1 = 0.1, \psi_2 = \pi - 0.1, v_1 = v_2 = 0.999).\]

![Fig.2. Behaviour of \( \Phi(\eta), v = 0.999, \psi_1 = 0.1, \psi_2 = \pi - 0.1, \eta < 5 \).](image)

\[
\frac{dE(\eta)}{d\tau'} = \frac{\alpha_s}{24\pi} \left( \frac{\hbar^2}{m} \right)^2 \Phi(\eta) \frac{d\eta}{\cosh^2 \eta};
\]
\[ \Phi(\eta) = \int_0^\pi \Phi_1(\psi_i, v_i, \phi, \theta, \phi) \cos \theta = f(\eta) \, d\phi; \quad (9) \]

\[ f(\eta) = \frac{\sinh \eta}{\cosh \eta}; \]

\[ dE(\phi) = \frac{\alpha_s}{24 \pi} \left( \frac{k^2}{m} \right)^2 \Phi(\phi) \, d\phi; \]

\[ \Phi(\phi) = \int_0^\pi \Phi_1(\psi_i, v_i, \phi, \theta, \phi) \sin \theta \, d\theta; \quad (10) \]

\[ \Phi_1(\psi_i, v_i, \phi, \theta, \phi) = \Phi(\psi_1, \theta, \phi) + \Phi(\psi_2, \theta, \phi, v_2). \]

\[ \Phi_N(\phi) \]

\[ \Phi_{\phi}(\phi) \]

\[ -3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3 \]

\[ \Phi \]

Fig. 3. Behaviour of \( \Phi_N(\phi) \), \( \nu = 0.999 \), \( \psi_1 = 0.1 \), \( \psi_2 = \pi - 0.1 \), for \( -\pi < \phi < \pi \). From Fig.2, Fig.3 we see that \( \Delta \eta \) may be quite significant while \( \Delta \phi \) is small. One should note that the peaks in Fig.2 and Fig.3 become narrower with increasing of speed and with increasing of \( \psi \).

Now let us consider properties of gluon radiation of a single quark. For this purpose we approximately assume the same direction of the velocity and of the acceleration in the actual reference frame of LHC. Angular distribution is described by the following expression \[ \Phi \]

\[ \frac{dE}{dt'} = \frac{\alpha_s}{24 \pi} \left( \frac{k^2}{m} \right)^2 \sin^2 \theta \frac{d\Omega}{1 - \nu \cos \theta}; \]

\[ \Phi_0(\theta') = \Phi(0, \theta', \phi, v); \]

where \( v \) is a velocity of a quark, \( \theta' \) is a polar angle and \( d\Omega = \sin \theta \, d\theta' \, d\phi' \). Using angular distribution of the radiation \[ \int \]

we estimate the mean \( p_T \) of the radiated gluon

\[ \begin{align*}
\langle p_T^u \rangle &= \frac{\Delta E}{v} \frac{I_1}{\sqrt{1 - \nu^2}} \frac{I_2}{\nu \sin \theta' \cos \theta} ; \\
I_1 &= \int \Phi_0(\theta') A(\nu, \theta') \sin \theta' \, d\Omega; \\
A(\nu, \theta') &= 1 + \cos \theta' \left( 1 - \nu^2 \right) - \nu \sin^2 \theta'.
\end{align*} \]

where \( \Phi_0(\theta') \) is defined in (11). Calculating integrals in (12) with the aid of the following relation valid for \( \nu \to 1 \) and \( \rho > \frac{1}{2} \)

\[ \int_0^\pi \frac{\sin^{\nu-1} \theta \, d\theta}{1 - \nu \cos \theta} = \frac{2^{\nu-1} \Gamma \left( \frac{1}{2}, \frac{1}{2} \right) \Gamma(2 \rho - \mu)}{(1 - \nu^2)^{\nu/2} \Gamma(\nu) \Gamma \left( \nu + \frac{1}{2}, \nu + \frac{1}{2} \right)}; \]

we obtain for quark \( u \) and \( d \) respectively with \( \nu \to 1 \)

\[ \begin{align*}
\langle p_T^u \rangle &= \frac{9 \pi \Delta E_u}{32} \approx 9.9 \text{ GeV}; \\
\langle p_T^d \rangle &= \frac{9 \pi \Delta E_d}{32} \approx 4.95 \text{ GeV}. \quad (13)
\end{align*} \]

We have to bear in mind also that in the process of hadronization a gluon give few ordinary hadrons. Their multiplicity we estimate by the following expression valid in the region of few GeV for charged multiplicity \[ \int \]

\[ \begin{align*}
\langle N_{ch} \rangle &= a + b \ln \sqrt{s}; \\
a &= -0.43 \pm 0.09; \quad b = 2.75 \pm 0.06. \quad (14)
\end{align*} \]

Neutral particles have to be also taken into account. In view to estimate the total multiplicity we multiply expression \[ \begin{align*}
\end{align*} \]

by \( \frac{1}{2} \) (isotopic spin of a gluon is zero). Then we estimate \( \sqrt{s} = \sqrt{2\Delta E M_P + M_G^2} \) and corresponding mean multiplicity

\[ \begin{align*}
u : & \quad \sqrt{s} = 4.15 \text{ GeV}; \quad \langle N \rangle = 5.2; \\
d : & \quad \sqrt{s} = 3.37 \text{ GeV}; \quad \langle N \rangle = 4.3. \quad (15)
\end{align*} \]

We take values \[ \int \]

with spread \( \pm 2 \) and thus obtain estimate for transverse momenta of hadrons \( p_T = p_T^u/N \)

\[ \begin{align*}
u : & \quad 1.3 \text{ GeV} < p_T < 3.0 \text{ GeV}; \\
d : & \quad 0.8 \text{ GeV} < p_T < 2.0 \text{ GeV}. \quad (16)
\end{align*} \]

Estimates \[ \int \]

just correspond to the interval of the ridge effect \[ \int \]. Of course, by changing light quark
masses in their allowable regions we can move boundaries in (16). However order of magnitude of the effect remains the same.

Next point of our interpretation is that gluons are flying in the narrow cone in the directions of a quark and the angular spread for the multiple gluon radiation is estimated to be

$$\Delta \hat{\theta} = \frac{< p_T^g > \sqrt{N_g}}{E_g N_g};$$  \hspace{1cm} (17)

where $N_g$ is the multiplicity of bremsstrahlung gluons in the event. Obtaining (17) we take into account that average transversal momentum squared for $N_g$ produced gluons

$$< p_T^g(N_g) > = < p_T^g >^2 N_g$$

due to statistical nature of the multiple radiation. We take for transversal momentum of a gluon $p_T^g$ estimates (15) and $< E_g >$ is a mean energy of a gluon. From (17) we see, that for small multiplicity of gluons $\Delta \hat{\theta}$ increases and this explains why the effect is absent in this case. For estimation of the real experimental situation (1) we replace the denominator in (17) by the energy of partons collision

$$\Delta \hat{\theta} = \frac{2 < p_T^g > \sqrt{N_g}}{\sqrt{\lambda_1 \lambda_2 s}};$$  \hspace{1cm} (18)

where $\lambda_1, \lambda_2$ are values of $\lambda$ for quark in the first proton and the anti-quark in the second one. Number of radiated gluons $N_g$ depends on angle $\psi$ and velocity $v$. Using again formulas from (14) we have the following estimate

$$N_g = \frac{\lambda_1 \lambda_2 s}{\sqrt{\Delta E \left(1 + m_u^2 / m_T^2\right)}};$$  \hspace{1cm} (19)

For example with $\psi = 0.1$ and $v = 0.999$, average $\Delta E = (\Delta E_u + \Delta E_d)/2 = 8.4 \text{ GeV}$, $\sqrt{s} = 7 \text{ TeV}$ (11) and with average of the product $< x_1 x_2 > = 0.01$ (see, e.g. (17) and references therein) we have $N_g \approx 17$. Bearing in mind, that in our interpretation one bremsstrahlung gluon gives average number of charged hadrons $N_{ch} \approx 3.2$, with $N_g = 17$ we have total number of charged particles produced by a quark $N_{ch}^g = 54$ that gives just multiplicity $\geq 100$ for two radiating quarks. So our mechanism does not contradict to real experimental situation (1).

Now with $N_g = 17$, $\sqrt{s} = 7 \text{ TeV}$, average $< p_T^g > = 7.4 \text{ GeV}$ and $< x_1 x_2 > = 0.01$ we have from (15)

$$\Delta \hat{\theta} \approx 0.09;$$  \hspace{1cm} (20)

This angular spread (20) actually gives widening of distributions (9) (10) in $\eta$ and $\phi$. The resulting $\Delta \phi$ is to be obtained by simultaneous account of (20) and of $\Phi(\phi)$ width (10). Let us also draw attention to widening of the ridge with $\sqrt{s}$ decreasing. E.g. for $\sqrt{s} = 0.9 \text{ TeV}$ in accordance with (18) $\Delta \hat{\theta} = 0.8$, that means vanishing of the effect.

Thus one can conclude the simple mechanism of gluon bremsstrahlung off quarks moving in a strong string field describes qualitatively the CMS ridge effect without adjusting parameters. Of course, a real situation could be much more involved. In particular, other colour configurations, as was pointed out in various studies (see, for example, (11), may play a significant role. Our consideration based on simple quasi-classical estimations shows that constituting string configurations may lead to basic features of the ridge effect, namely, correlations in particular kinematical region at very high multiplicities. Obviously, in order to show more accurate properties of proposed mechanism one should elaborate in more detail corresponding model and develop corresponding event generator to perform more realistic simulations.

Let us note, that the accelerated quarks radiate photons as well. The same quasi-classical estimate gives for radiated photons two values of mean $p_T$ for two values of (anti-)quark charge (shown in brackets)

$$\left(\frac{2e}{3}\right) : p_T \approx 2.9 \text{ GeV} \times \frac{2.5}{m_{q}(\text{MeV})};$$

$$\left(\frac{e}{3}\right) : p_T \approx 0.72 \text{ GeV} \times \frac{5}{m_{d}(\text{MeV})}.\hspace{1cm} (21)$$

It seems to be interesting to check these predictions with CMS data in the region of the ridge. In case of confirmation of the effect, measurement of $p_T$ of the photons may give useful information on current masses of light quarks $m_u, m_d$. Let us remind, that for the moment these parameters are known with considerable uncertainty.

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