Phase transition and spin-wave dispersion in quantum Hall bilayers at filling factor $\nu = 1$

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Abstract

We present an effective Hamiltonian for a bilayer quantum Hall system at filling factor $\nu = 1$ neglecting charge fluctuations. Our model is formulated in terms of spin and pseudospin operators and is an exact representation of the system within the above approximation. We analyze its low-lying excitations in terms of spin-wave theory. Moreover we add to previous first-principle exact-diagonalization studies concentrating on the quantum phase transition seen in this system.

73.43-f,7321-b

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I. INTRODUCTION

At small layer separations, the ground state of a $\nu = 1$ bilayer quantum Hall system exhibits spontaneous interlayer phase coherence. Interest in this issue has been renewed by intriguing tunneling transport measurements by Spielman et al. showing a very pronounced conductance peak at zero bias voltage [1] for sufficiently small values of the ratio of layer separation to magnetic length.

The quantum phase transition underlying this phenomenon has been investigated recently in an exact diagonalization study using the spherical geometry [2]. The results suggest that a single phase transition, likely of first order, separates incompressible states with strong interlayer correlations from compressible states with weak interlayer correlations.

In the present work we report on a study of $\nu = 1$ quantum Hall bilayer using a different approach introducing an effective spin-pseudospin model on an imaginary lattice in the lowest Landau level (LLL) orbital space (von Neumann lattice). We also add further exact diagonalization results concerning the position and order of the compressible-incompressible transition.

II. EFFECTIVE SPIN-PSEUDOSPIN-MODEL

A single-particle state of electron in a $\nu = 1$ bilayer is specified by three quantum numbers: lowest Landau level (LLL) orbit-center quantum number $i$, spin $\sigma$ and pseudospin $\tau$, describing the layer degree of freedom. However, in the incompressible state bilayer has a gap for charged excitations and therefore one can assume that only the spin and pseudospin degrees of freedom are relevant for the low temperature physics. The microscopic Hamiltonian
reads $H = H_{1p} + H_{\text{Coul}}$, where $H_{\text{Coul}} = V^S + V^D +$ represents the usual Coulomb interaction within $(V^S)$ and between layers $(V^D)$, and the single-particle Hamiltonian $H_{1p}$ implements tunneling of electrons between the layers with an amplitude $\Delta_t$ and couples electron spins to the perpendicular magnetic field with amplitude $\Delta_z$. One would like to eliminate the irrelevant charge degree of freedom from the microscopic Hamiltonian and arrive at an effective model containing only spin and pseudospin variables. The most convenient and mathematically rigorous way to do this is provided by the functional integral approach. One notices that the no-charge-fluctuations subspace of the system is spanned by single Slater determinant many body wave functions of the form $|\Psi[z]\rangle = \prod_i \left( \sum_{k=1}^{4} z_{ik} c_{ik}^\dagger \right) |0\rangle$. Here $i$ is the LLL orbital quantum number and $k$ is a 4-component spinor index describing the combined spin-pseudospin degree of freedom. $k = 1$ means electron in the top layer with an up spin, $k = 2$—top layer down spin, $k = 3$—bottom layer up spin, $k = 4$—bottom layer down spin. $c_{ik}^\dagger$ is the creation operator for an electron in the LLL orbital state $i$ and with 4-spinor index $k$. $z_{ik}$ are complex amplitudes, satisfying the normalization condition $\sum_{k=1}^{4} |z_{ik}|^2 = 1$.

One can therefore formally write the partition function of the system as a functional integral over this overcomplete set of Slater determinants. After considerable algebra one arrives at the following effective Hamiltonian

$$H = - \sum_i \left[ \Delta_t T_i^x + \Delta_z S_i^z \right] + \sum_{ij} \left[ (2H_{ij} - \frac{1}{2} F_{ij}^S ) T_i^z T_j^z - \frac{1}{2} F_{ij}^D T_i^\perp T_j^\perp \right. \\
- \frac{1}{2} F_{ij}^S S_i S_j 2 F_{ij}^S (S_i S_j) T_i^z T_j^z \\
\left. - 2 F_{ij}^D (S_i S_j) (T_i^\perp T_j^\perp) \right]$$  

(2.1)

Here $F_{ij}^{S,D} = F_{ij}^+ \pm F_{ij}^-$, and $H_{ij} = \langle ij | V_- | ij \rangle$, $F_{ij}^\pm = \langle ij | V_\pm | ji \rangle$ are the direct and exchange terms.
matrix elements of the Coulomb interaction $V^\pm = (V^S \pm V^D)/2$, and $S_i$ and $T_i$ are local spin and pseudospin operators, respectively.

Our spin-pseudospin model is defined on a “lattice” with “sites” labeled by the LLL orbital quantum numbers $i$. So far we have not specified the orbital basis we are using. It is obvious that the usually used orbit-center quantum numbers in Landau or symmetric gauges are not good choices here. Both of these basis sets introduce an artificial gauge-dependent asymmetry into the problem. One would like to use a basis more appropriate for a spin model: that of Wannier-like functions localized at the sites of imaginary square lattice (von Neumann lattice) with lattice constant $\sqrt{2\pi l^2}$ to accommodat exactly one electron or corresponding spin-pseudospin pair per site at filling factor $\nu = 1$. It is not obvious in advance that such a basis exists. Strong magnetic field imposes certain restrictions on the localization properties of magnetic orbitals [4,5], and it is well established for example that a set of linearly independent and exponentially localized single-particle orbitals in the LLL does not exist. However, it turns out to be possible [6,7] to construct a complete orthonormal set of Wannier-like eigenfunctions, which, although not exponentially localized, have a well defined Gaussian core with power law falloff at large distances. Following [6,7] we will call them magnetic Wannier functions. The procedure one uses to construct such a basis set is very much like the one used to construct the usual Wannier functions in a crystal. One starts from the set of minimum uncertainty wavepackets for electrons in the LLL, centered at the sites of the square lattice described above. The difference from the case of a crystal here is that this set is overcomplete, as was shown by Perelomov [8]. One then constructs Bloch functions from linear combinations of the minimum uncertainty wavepackets and Fourier transforms them to obtain the Wannier functions. There are subtleties in this procedure
and we refer the reader to the original papers [6,7] for further details.

In this paper we will present only the simplest linear spin wave results for our spin-pseudospin model, which are applicable at zero temperature. A detailed study of finite temperature properties of (2.1) will be presented in a forthcoming publication [9]. As evident by direct inspection, the interaction part of (6) has the correct $SU(2) \times U(1)$ spin×pseudospin symmetry. Correspondingly there are two Goldstone modes associated with the spontaneous breaking of this symmetry. Their dispersions are given by

$$E_{k}^{\text{spin}} = \Delta_z + F_0^+ - F_k^+$$

$$E_{k}^{\text{pseudospin}} = \left( (\Delta_t + F_0^D + H_k - F_k^+)^2 ight)^{1/2}$$

$$-(H_k - F_k^-)^2 \right)^{1/2}$$

(2.2)

In Fig.1 we show dispersions (2.2) evaluated for a 20×20 square lattice using realistic values for the tunneling and Zeeman splittings at interlayer separation $d = 1.4l_B$ with $l_B$ being the magnetic length. One can see that the spin dispersion is quadratic and the pseudospin one is linear at small values of the wavevector in accordance with broken $SU(2) \times U(1)$ symmetry. Another thing to notice is that the pseudospin gap is appreciably larger than the spin gap even though the bare values of tunneling and Zeeman coupling are the same. This makes invalid (at finite temperature) the usual argument about spin fluctuations being frozen out by the magnetic field [10]. The dip in the pseudospin mode dispersion at the Brillouin zone boundary signals softening of the pseudospin mode due to the development of antiferromagnetic instability, which at large enough interlayer separations ($\sim 1.45$ in our model) leads to the transition to the compressible state.
III. RESULTS ON THE PHASE TRANSITION

Our exact diagonalization results are obtained for a bilayer quantum Hall system in the spherical geometry containing spin-polarized electrons. We will consider both the case of zero and finite width $w$ of the two quantum wells. In the exact diagonalization data a transition between a compressible ground state with weak interlayer correlations to an incompressible strongly correlated phase is signalled by a maximum of the fluctuation
\[ \Delta T_x = \sqrt{\langle T_x^2 \rangle - \langle T_x \rangle^2} \]
of the ground state pseudospin magnetisation along with its susceptibility $\chi = d\langle T_x \rangle / d\Delta$. These maxima grow rapidly with increasing system indicating a quantum phase transition. The positions of these maxima as a function of tunneling gap and layer separation (measured in units of the magnetic length) define finite-size phase boundaries (cf. figure 3 in Ref. [2]). The spectacular phenomenon found by Spielman et al. [1] occurred in samples with extremely small tunneling amplitude close to the limit of vanishing tunneling where spontaneous interlayer phase coherence arises. Therefore, the critical layer separation at zero tunneling is of particular interest. The values of this quantity obtained for finite systems with up to twelve electrons form a rapidly converging data series (cf figure 4 in Ref. [2]). The critical values $d_c$ for the layer separation at zero tunneling are shown in figure 4 as a function of the ratio $w/d$ of well width to layer separation. A value of $w/d = 0.65$ corresponds to the samples used in Refs. [1], where the experimental value of $d = 1.83$ for the critical layer separation agrees very well with the exact diagonalization result of $d = 1.81$.

In order to further investigate the order of the quantum phase transition, we introduce the ratio
\[ \omega_N = \frac{2 \langle \Delta T^x \rangle_N^2}{(d\langle T^x \rangle/d\Delta t)_N}, \]  

(3.1)

where the subscript \( N \) refers to the system size. As discussed in Ref. [2], this type of ratio defines a characteristic energy scale of the system at the phase boundary and should prove to be a powerful general tool in the analysis of any quantum phase transition.

For a continuous phase transition one would clearly expect \( \omega_N \) to vanish at the phase boundary for an infinite system, while a finite limit \( \lim_{N \to \infty} \omega_N \) is indicative of a finite energy scale, i.e. a first order transition. From our finite-size data for \( \omega_N \) (evaluated at vanishing tunneling and \( d = d_c(N) \)) shown in figure 3 we conclude that this quantity extrapolates for \( N \to \infty \) to a rather substantial non-zero value of order \( 0.05e^2/\epsilon l_B \sim 5K \) for all values of \( w \) considered here. Along with the arguments and experimental findings given so far, this result strongly suggests that the bilayer quantum Hall system at filling factor \( \nu = 1 \) undergoes a single first order phase transition as a function of the ratio of layer separation and magnetic length at all values of the tunneling amplitude. The phase boundary separates a phase with strong interlayer correlation (and a finite gap for charged excitations) from a phase with weak interlayer correlations and vanishing gap for charged excitations.

**ACKNOWLEDGEMENTS**

This work was supported by the Deutsche Forschungsgemeinschaft, the National Science Foundation, and the Welch Foundation.
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FIG. 1. Spin and pseudospin mode dispersions in the (100) direction for $\Delta_t = \Delta_z = 0.01$ in units of $e^2/\ell$ and $d = 1.4l_B$.

FIG. 2. Critical layer separation (extrapolated in the thermodynamic limit) at zero tunneling as a function of the ratio of well width to layer separation.
FIG. 3. The averaged excitation energy $\omega_N$ as a function of the system size for various ratios of well width $w$ to layer separation $d$. Assuming that the these data curves remain of a convex shape also for larger $N$, one concludes that $\omega_N$ extrapolates to finite values (being of order 0.05 in units of the Coulomb energy scale) for all values of $w/d$. 