Numerical investigation of mechanical losses of a tilting-pad journal bearing

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Abstract. Mechanical losses of large tilting-pad journal bearings are increasing with higher rotational speed. However, mechanical losses of loaded and unloaded pads vary largely. This study analyzes the reason of varying mechanical losses in different pads. A three dimensional model is built for a full large tilting-pad journal bearing. The real backflow environment of journal bearings can be an important factor influencing lubricant viscosity and bearing mechanical loss. Thus, two boundaries, full-oil backflow and full-air backflow in two sides of the bearing, are simulated and compared with the experimental data of S.Taniguchi. Apart from the backflow lubricant, the gaseous cavitation and SST turbulence model are included in bearing simulations. Compared with experimental data, SST model is proved to be more suitable than laminar model for simulations. While the mechanical loss simulated by SST model with oil backflow is much higher than experimental data, the mechanical loss of the journal bearing by SST model with air backflow has small difference with experimental data. The pressure distributions in the bearing centerline simulated by oil backflow and air backflow are the same. Considering the full air backflow is in consistent with the real air environmental working condition, the air backflow is a suitable boundary for journal bearing simulations.

1. Introduction

With rotating speed increasing, the mechanical losses of large tilting-pad journal bearings increase. Accurate predictions of the mechanical losses are important for bearing analysis and design. There are several critical factors influencing the predictions of mechanical losses, like cavitation. This factor can affect the material volume fraction and shear stress, influencing the mechanical losses.

Many researches has focused on bearing oil cavitation in the past years and built different cavitation models to predict the material fraction and mechanical losses accurately. One of the most widely used cavitation models is the half-Sommerfeld model. This model was firstly used and validated for an approximation of finite journal bearings by Fedor [1] in 1963. Based on this study, the half-Sommerfeld model has been applied for bearing simulations in many researches, such as tilt stiffness of finite journal bearings [2]. Apart from this model, the Rayleigh–Plesset cavitation model has been developed. This model is based on the vaporous cavitation mechanism [3], validated by Someya et al. [4] and Gehannin et al. [5]. Based on bubble dynamics, the Rayleigh–Plesset cavitation model has been optimized for cavitation in lubricant contacts [6].

The Jakobsson–Floberg–Olsson (JFO)-type cavitation models follow the JFO cavitation theory, including film rupture and reformation conditions. Various algorithms, such as Elrod’s algorithm [7] and the algorithm of Kumar and Booker [8], have been applied for solving the JFO-type cavitation
Based on the theory and algorithms, complex applications, like cavitation in dimples, can be realized [9,10]. The JFO cavitation condition can be combined with other models, which has been realized by Murty [11] and Bonneau et al. [12].

In addition to the vaporous cavitation model, gaseous cavitation models have been built for bearing simulations. Based on air solubility, a single-phase homogeneous mixture gaseous cavitation model has been built and validated for bearing oil films [13]. The vaporous and gaseous cavitation models have different mechanisms. In large tilting-pad journal bearings, the lubricant pressure is often higher than the vaporous pressure (0.4 Pa for VG22 at 100 °C). Therefore, the bearing oil cavitation generally depends on the dissolved air and oil. This model has been applied in many complex simulations, like analysing non-equilibrium dissolution effect in thrust bearings [14-16].

In above cavitation researches, the backflow material is set to be oil and the effect of backflow boundary on cavitation is not taken into consideration. Under working condition, the oil from the outlets join in the bottom of bearing houses and the oil outlets of bearings can be in air environment. Thus, the backflow material can be oil or air. Considering there is no enough work about the backflow material in bearings, this work takes one large tilting-pad journal bearing as a subject and compares the simulation results of oil backflow and air backflow with experimental data.

The outline of the present work is as follows. In Section 2, the governing equations of the one-phase homogeneous mixture models are given. The comparison between air backflow and oil backflow is illustrated in Section 3; moreover, gaseous cavitation is used to predict the bearing oil cavitation. Section 4 is devoted to the conclusions.

2. Governing Equations

2.1. Cavitation Model

A Gaseous cavitation model based on air solubility [13] is included in this study. Assuming that air dissolution in oil approaches balance, a reduction of oil-film pressure can break the balance and lead to air emission. The standard status of air emission is chosen at $T_0 = 273.16K$ and $p_0 = 101.325kPa$.

Under this status, the decreasing value of solubility and oil volume ($V_{oil}$) are used to calculate the emission volume which can be expressed below:

$$V_{air1} = (\delta_{surround} - \delta_{local})V_{oil}$$  \hspace{1cm} (1)

where, $\delta_{surround}$ and $\delta_{local}$ mean the air Bunsen solubility under environmental and local situations, respectively.

The Bunsen solubility is based on the unit volume of solvent. Its value for air can be calculated by [17]:

$$\delta_{surround} = \frac{p_{surround}}{p_0} \exp(-2.476 + 250.8/T_{surround})$$  \hspace{1cm} (2)

Where, $p$ and $T$ are decided using the dissolution status.

Considering that the local situation for air is different from the standard status, the local air emission volume can be calculated using the standard volume, as shown below:

$$V_{air1} = \frac{p_{local}T_{0}}{p_{local}T_{0}} V_{air1}$$  \hspace{1cm} (3)

Based on the above equations, the air emission volume fraction can be expressed as below:

$$f_{air1} = \frac{V_{air1}}{V_{air1} + V_{oil}} = \frac{A}{A + 1}$$  \hspace{1cm} (4)

where, $A = \max\left\{\frac{T_{0}}{T} \left[p_{surround} \exp(-2.476 + 250.8/T_{surround}) - \exp(-2.476 + 250.8/T)\right], 0\right\}$. 


The air emitted from the lubricant oil and the lubricant oil are considered to be mixed fully. Thus, for simplicity, the lubricant oil and the released air from the oil can be calculated as a mixture, the cavitation oil. The density and laminar viscosity of the cavitation oil can be calculated as:

\[
\rho_{\text{cavoil}} = (1 - f_{\text{air}}) \rho_{\text{oil}} + f_{\text{air}} \rho_{\text{air}} \\
\mu_{\text{cavoil}} = (1 - f_{\text{air}}) \mu_{\text{oil}} + f_{\text{air}} \mu_{\text{air}}
\]

where, \( \mu_{\text{cavoil}} \) can be calculated by the turbulence models.

2.2. Navier-Stokes Equations
In the example of the oil backflow boundary, the backflow oil and the cavitation oil are taken as a mixture. For the homogeneous bearing lubrication model, the Navier–Stokes equations can be written as:

\[
\frac{\partial \rho_{\text{cavoil}}}{\partial t} + \nabla \cdot (\rho_{\text{cavoil}} \bar{v}) = 0
\]

\[
\frac{\partial (\rho_{\text{cavoil}} \bar{v})}{\partial t} + \nabla \cdot (\rho_{\text{cavoil}} \bar{v} \otimes \bar{v}) = -\nabla p + \nabla \cdot \left[ \mu_{\text{cavoil}} (\nabla \bar{v} + (\nabla \bar{v})^T) \right]
\]

where, \( \mu_{\text{cavoil}} = \mu_{l, \text{cavoil}} + \mu_{t, \text{cavoil}} \) and \( \mu_{t, \text{cavoil}} \) can be calculated by the turbulence model.

In the example of the air backflow boundary, the backflow air and the cavitation oil are taken as two phases and the two-phase homogeneous mixture model is used. In the two-phase mixture model, the cavitation oil and backflow air phases are calculated separately. Assuming that the cavitation oil and backflow air share a common flow field and the two-phase flow is homogenous, \( \bar{v}_{\text{air}} = \bar{v}_{\text{cavoil}} \) and the pressure field is common. The continuity and momentum equations for the two-phase homogeneous mixture model can be written as follows.

For the backflow air phase, the continuity equation is:

\[
\frac{\partial (f_{\text{air2}} \rho_{\text{air}})}{\partial t} + \nabla \cdot (f_{\text{air2}} \rho_{\text{air}} \bar{v}) = 0
\]

For the cavitation oil phase, the continuity equation is:

\[
\frac{\partial [(1 - f_{\text{air2}}) \rho_{\text{cavoil}}]}{\partial t} + \nabla \cdot [(1 - f_{\text{air2}}) \rho_{\text{cavoil}} \bar{v}] = 0
\]

The momentum equation is:

\[
\frac{\partial}{\partial t} \left\{ [(1 - f_{\text{air2}}) \rho_{\text{cavoil}} + f_{\text{air2}} \rho_{\text{air}}] \bar{v} \right\} + \nabla \cdot \left\{ [(1 - f_{\text{air2}}) \rho_{\text{cavoil}} + f_{\text{air2}} \rho_{\text{air}}] \bar{v} \otimes \bar{v} \right\} =
\]

\[
-\nabla p + \nabla \cdot \left\{ [(1 - f_{\text{air2}}) \mu_{\text{cavoil}} + f_{\text{air2}} \mu_{\text{air2}}] (\nabla \bar{v} + (\nabla \bar{v})^T) \right\}
\]

where, \( f_{\text{air2}} = \frac{V_{\text{air2}}}{V_{\text{air2}} + V_{\text{cavoil}}} = \frac{V_{\text{air2}}}{V_{\text{air2}} + (V_{\text{oil}} + V_{\text{air}})} \) and \( \mu_{\text{air2}} = \mu_{l, \text{air2}} + \mu_{l, \text{air2}} \cdot \mu_{t, \text{air2}} \) can be calculated by the turbulence model.

The air volume is defined as the sum of the air emission volume and the backflow air volume. The air volume fraction can be calculated as:

\[
f_{\text{air}} = \frac{V_{\text{air1}} + V_{\text{air2}}}{V_{\text{air1}} + V_{\text{air2}} + V_{\text{oil}}}
\]

3. Numerical Examples
3.1. Simulation Object

The oil backflow and air backflow boundaries are compared using a tilting-pad journal bearing at 3000 rpm rotation speed and under 180 kN load. The simulated bearing specifications [18] are listed in table 1.

| Bearing Type | Tilting Pad |
|--------------|-------------|
| Bearing Diameter [mm] | 479 |
| Bearing Length [mm] | 300 |
| Number of Pads | 4 |
| Pad Arc [deg] | 80 |
| Pivot Offset | 0% |
| Preload Factor | 0 |
| Load Angle | Between Pads |
| Radial Clearance [mm] | 0.612 |
| Pad Thickness [mm] | 121 |

Apart from the above specifications, the experimental oil film thickness at 3000 rpm and under 180 kN are shown in the figure 1. Considering this bearing has four pads, these four pads are numbered as Nos. 1, 2, 3 and 4 from the 0° to 360°. Based on the measured film thickness, Nos. 3 and 4 pads have large film thicknesses and cannot bear the bearing load. Nos. 1 and 2 pads have relatively small film thicknesses and can bear the bearing load. Thus, Nos 1 and 2 pads are loaded pads.

**Figure 1.** Film thickness of tested bearing at 3000 rpm and under 180 kN.

Based on the film thickness and bearing specifications, the three-dimensional bearing geometry can be drawn with a structured mesh. The film part mesh is a 480 × 80 × 20 (tangential by axial by radial) mesh system. The oil hole part mesh is four 20 × 80 × 80 (tangential by axial by radial) mesh systems.
Figure 2. Mesh of tested bearing.

The CFD code [19, 20] is adopted with all-speed Roe scheme [21–23] and discontinuous Galerkin reconstruction methods [24–26]. The residuals of continuity, velocity, energy, and turbulent kinetic energy are below $10^{-5}$ for convergence.

The lubricant of this journal bearing is ISO VG32. The physical properties are listed in table 2.

| Physical property                      | Value |
|----------------------------------------|-------|
| Density [kg/m$^3$]                     | 861   |
| Thermal conductivity [W/(m·K)]         | 0.135 |
| Viscosity [kg/(m·s)]                   | 0.017221 |
| Specific heat [J/(kg·K)]               | 2009.28 |

Based on experimental conditions [18], table 3 lists the boundaries, with the reference pressure set as 1 bar. Apart from the listed boundaries, other boundaries are all no-slip stationary walls. The heat transfer coefficient value recommended by [18] is 115 W/(m$^2$·K). Assumed to be uniformly distributed, the inlet mixture temperature is set to be the experimental mixture inlet temperature (i.e., 315.15 K).

The entrainment boundary is set for the backflow from the outlet. The entrainment absolute pressure and temperature for the backflow material are set as 1 bar and 313.15 K, respectively. The backflow material is set as air or oil for the air backflow example and the oil backflow example, respectively.

| Boundary | Boundary Type          | Boundary details            |
|----------|------------------------|-----------------------------|
| Inlet    | Total pressure inlet   | 0.1 MPa; 315.15 K           |
| Outlet   | Static pressure outlet | 0 Pa                        |
The oil flow in this bearing has been studied and found to be superlaminar and the most suitable turbulence model for the simulations is SST model [27]. Therefore, this study includes the SST model in simulations.

For proving the mesh independence, the pressure simulation results of two meshes are compared in table 4. Mesh 1 is the mesh shown in figure 2. The element number of each direction in Mesh 2 is 1.3 times that of Mesh 1. Based on the same pressure distribution results, Mesh 1 is independent.

| Bearing Type                  | Mesh 1   | Mesh 2   |
|-------------------------------|----------|----------|
| Element number of fluid domain| 1280000  | 2812160  |
| Highest pressure value [MPa]  | 4.11     | 4.11     |
| Highest pressure angle [deg]  | 47.2     | 47.2     |


### 3.2. Simulated Mechanical Loss

Mechanical loss is one of the most important parameters in bearing simulations. Four examples are simulated and compared with experimental data. These four examples are laminar model with air backflow, laminar model with oil backflow, SST model with air backflow and SST model with oil backflow. The simulation results are listed in table 5.

| Angle [deg] | Air backflow (laminar) | Oil backflow (laminar) | Air backflow (SST) | Oil backflow (SST) |
|-------------|-------------------------|------------------------|--------------------|--------------------|
| 0-10        | 661                     | 1876                   | 4613               | 7513               |
| 10-90       | 30898                   | 32372                  | 62948              | 67484              |
| 90-100      | 1972                    | 3302                   | 7887               | 9537               |
| 100-180     | 33794                   | 33952                  | 66865              | 69273              |
| 180-190     | 2323                    | 3434                   | 8168               | 9744               |
| 190-270     | 9163                    | 12183                  | 23686              | 33020              |
| 270-280     | 554                     | 2489                   | 5134               | 8411               |
| 280-360     | 3786                    | 11843                  | 23204              | 34757              |
| Sum         | 83000                   | 101000                 | 203000             | 240000             |

As shown in table 5, the simulation results of SST with air backflow is closest to experimental data and its error is about 2.01%, accurate enough for bearing mechanical loss predictions.

The simulation results of laminar with air backflow and oil backflow are both about 50% lower than the experimental mechanical loss. It indicates that the lubricant flow is not laminar and the turbulence effect should be taken into consideration, which is inconsistent with another research [27].

Compared with experimental data, the SST model with oil backflow gets a 20% higher mechanical loss. Taking SST model with air backflow into consideration, the high mechanical loss of SST model with oil backflow might be caused by unsuitable oil backflow.

When the backflow is oil, the air volume fraction in lubricants decreases and the high-viscosity material, oil, takes more volume. The entire viscosity of mixture increases. Based on the relationship between viscosity and shear stress (Equation 10), the shear stress on rotor-side wall increases. In
consequence, the mechanical loss increases linearly with viscosity increasing. Therefore, based on above comparison, air backflow is closer to the real working condition than oil backflow.

\[
\tau = \mu \frac{\partial (v)}{\partial y}
\]  

(13)

Thus, among all four examples, the SST model with air backflow is the best. The oil backflow and laminar are not suitable for the simulations of this tilting-pad journal bearing. The air backflow is a reasonable boundary for journal bearing simulations.

3.3. Simulated Air Volume Fraction

As noted above, the air volume fraction influences the mechanical loss and the backflow material is one method of changing air volume fraction. Therefore, it becomes important to make research on air volume fraction in details. As the flow is not laminar, the contours of air volume fraction simulated by SST model are shown in figure 3.

![Figure 3. Contours of air volume fraction: (a) SST model with air backflow; (b) SST model with oil backflow.](image)
As shown in figure 3, the contour of air volume fraction simulated by SST model with air backflow has wider high-air-volume-fraction area than SST model with oil backflow. Around the outlet, the air volume fraction simulated by SST model with oil backflow is especially lower than SST model with air backflow. It is consistent with analyzed influence of backflow material in section 3.1.

3.4. Simulated Pressure

Hydrodynamic characteristics of bearings play an important role in bearing simulations and design [28–30]. Among hydrodynamic characteristics, pressure distribution is often studied. Considering that Nos. 1 and 2 pads are loaded pads, Nos. 3 and 4 pads are unloaded pads and have low pressure distributions. Thus, the hydrodynamic characteristics of this entire bearing can be caught by the pressure distributions of Nos. 1 and 2 pads. Based on experimental pressure distributions [18], No.2 pad has the similar pressure distribution with No. 1 pad. To simplify the pressure distribution analysis, No. 2 pad are chosen to be studied on behalf of the whole bearing. The pressure contours of No. 2 pad are shown in figure 4.
As shown in figure 4, the pressure distributions of SST model with air backflow and oil backflow are same and best in comparison with experimental data. It means that there is no significant effect of backflow in hydrodynamic characteristics of loaded pads and SST model can catch the hydrodynamic characteristics accurately. The error of highest pressure value simulated by SST model is 6.19%. The difference between experimental angle and highest pressure angle of SST model is 0.2 degrees.

The simulated highest pressure values of laminar model are more than 38.66% lower than the experimental data, indicating that the laminar model is unsuitable for the bearing simulations. The
simulated highest pressure angle of laminar model with air backflow is different from laminar model with oil backflow. The laminar model with oil backflow cannot get neither right highest pressure value nor angle and performs the worst in four examples. It indicates that the oil backflow boundary is not as suitable for bearing simulations as the air backflow boundary.

In general, the SST model and air backflow is more suitable than laminar model and oil backflow for bearing simulations. The air backflow boundary takes the real air environment around the outlet into consideration and leads the air volume fraction to be enough high during the simulation process.

4. Conclusions

In this paper, a three-dimensional model with structured mesh for a tilting-pad journal bearing has been built. With SST model and the gaseous cavitation model based on air solubility included, the air backflow boundary and oil backflow boundary are used for bearing simulations. In comparison with experimental data, the conclusions which may be drawn from this work are summarized below:

For mechanical loss, the SST model with air backflow performs the best. The air backflow boundary leads the air volume fraction to increase and the entire material viscosity decreases. In consequence, the simulated mechanical loss is lower than the oil backflow boundary and is closer to the experimental data. Compared with the poor performance of laminar model, SST model is more proper for bearing simulations.

The contours of air volume fraction are in consistent with the change of backflow material. Air backflow takes the real air environment around the outlet into consideration and leads the air volume fraction to be high enough like real working condition, decreasing the mechanical loss.

The change of backflow material has no significant influence on the simulation results of loaded pads, especially for the hydrodynamic characteristics. Combined with mechanical loss and air volume fraction analysis, the air backflow boundary is more suitable than oil backflow boundary for bearing simulations.

Nomenclature

\[
\begin{align*}
  f & = \text{air volume fraction in the lubricant} \\
  p & = \text{absolute pressure} \\
  p_0 & = \text{standard atmospheric pressure: } 101.325 \text{ kPa} \\
  T & = \text{temperature} \\
  T_0 & = \text{standard atmospheric temperature: } 23.16 \text{ K} \\
  v & = \text{velocity} \\
  V & = \text{specific volume at } p \text{ and } T \\
  \bar{V} & = \text{specific volume at } p_0 \text{ and } T_0 \\
  \delta & = \text{Bunsen solubility of air} \\
  \rho & = \text{density} \\
  \nu & = \text{Molecular viscosity} \\
  \mu & = \text{dynamic viscosity} \\
  \tau & = \text{shear stress} \\
  \theta & = \text{coordinate in the circumferential direction}
\end{align*}
\]

Indexes

\[
\begin{align*}
  \text{air} & = \text{refers to air} \\
  \text{air}1 & = \text{refers to air emission from the gaseous cavitation} \\
  \text{air}2 & = \text{refers to backflow air} \\
  \text{cavoil} & = \text{refers to the mixture of the air emission and oil} \\
  \text{local} & = \text{refers to local parameters}
\end{align*}
\]
Development of negative pressure in oil film and the characteristics of dynamic

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