Learning the Compositional Spaces for Generalized Zero-shot Learning

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ABSTRACT
This paper studies the problem of Generalized Zero-shot Learning (G-ZSL), whose goal is to classify instances belonging to both seen and unseen classes at the test time. We propose a novel space decomposition method to solve G-ZSL. Some previous models with space decomposition operations only calibrate the confident prediction of source classes (W-SVM [46]) or take target-class instances as outliers [49]. In contrast, we propose to directly estimate and fine-tune the decision boundary between the source and the target classes. Specifically, we put forward a framework that enables to learn compositional spaces by splitting the instances into Source, Target, and Uncertain spaces and perform recognition in each space, where the uncertain space contains instances whose labels cannot be confidently predicted. We use two statistical tools, namely, bootstrapping and Kolmogorov-Smirnov (K-S) Test, to learn the compositional spaces for G-ZSL. We validate our method extensively on multiple G-ZSL benchmarks, on which it achieves state-of-the-art performances. The codes are available on https://github.com/hendrydong/demo_zsl_domain_division.

1. Introduction

Zero-shot learning (ZSL) has attracted extensive attention from various research areas of computer vision. It aims at recognizing novel target classes that are unseen at the training stage by transferring knowledge from observed source (or auxiliary) categories with many labeled instances. To enable the knowledge transfer, semantic representations, auxiliary information such as visual attributes [28] and word embeddings [18] are used to relate target classes with source classes. Typically, most approaches formulate ZSL as a visual-semantic alignment problem: an embedding space is learned on source classes by transforming their instances from the visual to semantic space [17, 20, 28, 37], or vice versa [8, 41, 25]; in the learned embedding space, such a transformation is applied to project unseen data onto the space for classification.

In the general experimental setting of ZSL, test instances only come from target unseen classes. However, this is an unrealistic simplification of the object categorization tasks in the wild. In consequence, a more realistic setting – Generalized Zero-Shot Learning (G-ZSL) where test instances come from both source and target classes, is considered as a more realistic benchmark of ZSL performance [9, 54, 15].

The G-ZSL is still addressed in the form of learning a visual-semantic alignment with an assumption that the distributions of classes in the semantic and visual spaces are relatively similar [15]. In contrast to ZSL, approaches of G-ZSL are prone to be biased toward target classes, resulting in poor classification accuracy, especially for the target classes [9, 60]. As demonstrated in Fig. 1(a), the decision boundary between source and target classes will be inevitably shifted to source classes in the learned visual embedding space, as target data are unavailable during the optimization stage of these approaches; thus, a large portion of target data will be misclassified as source classes at the test phase, which can also be recognized as a sort of overfitting.

To alleviate such a bias, Long et al. [34] optimized the semantic spaces to learn a better transformation from visual to semantic space. Besides, Generative Adversarial Networks (GANs) based approaches [54, 59] synthesized visual representations of target classes. Liu et al. [30] calibrated both confidence of source classes and uncertainty of target classes. Notably, most of these previous works are built upon the assumption that the distributions of classes should be similar in the semantic and visual spaces.

Fundamentally, we argue that source and target classes may follow largely disparate visual/semantic distributions. Therefore, it is unnecessary to predict source classes in the semantic space. Such an intuition can be incorporated in addressing the prediction bias in G-ZSL. It is thus surprising to note that there is little if any existing work that fully and explicitly explores different distributions between source and the target classes. Is
It because there is a trivial extension of encoding such a new idea? By training a supervised mapping from the visual and semantic space, it is indeed possible to implicitly take the G-ZSL as an outlier detection task [50]: determining whether a given test instance is on the manifold of source classes – if it is of a source class (in source space), a supervised classifier is applied; otherwise, it is in the target space and labeled as one of the nearest class prototypes. To allow this, it is essential to learn the compositional spaces which separate the instances either from source and target classes.

However, there are still two key problems remaining. First, visual or semantic features alone may not be discriminative enough in differentiating the source and the target classes. It is thus imperative to fully combine the information of both visual and semantic spaces. Second, it is notoriously difficult in tuning the model parameters for outlier detection in judging whether an instance is from source or target classes. Critically, a portion of instances can potentially be misclassified regarding parameter tuning. As is illustrated in Fig. 1(a), we can easily find the overlapping region where the instances may be categorized as either the source or the target classes, depending on the model hyperparameters. This results in the misclassification in the final prediction. Even worse, due to the aforementioned biased problem, instances of target classes may still be inclined to be categorized as one of source classes.

To tackle the issues, our key insight is to learn to categorize test instances into the compositional spaces: source, target, and uncertain spaces. The uncertain spaces are newly introduced here to contain the test instances that cannot be confidently classified into source or target space, as visualized as in Fig. 1(a). Particularly, the source and the target space can be implicitly learned due to the different visual/semantic distributions of source and target classes. The recognition algorithms are applied in each space. The uncertain space enables us to analyze the instance distribution from a statistical perspective and we can thus categorize the class ambiguous instances more accurately.

Formally, we propose exploiting the distributions of source and target classes to efficiently learn the compositional spaces from a statistical perspective according to Fig. 1(b). With the extracted feature representations of images [52], our framework computes the extreme values of training instances as confidence scores. Specifically, in term of extreme value theory [46], the maximum/minimum confidence scores predicted by the classifier of each class are drawn from extreme value distributions. Unfortunately, we do not have prior knowledge of the underlying data distributions of each class; thus, bootstrapping is utilized as an asymptotically consistent method in estimating an initial boundary of source classes and in dividing the embedding space into the source and the target space. Nevertheless, the initial boundary estimated by bootstrapping is too relaxed to include novel testing instances as is illustrated in Fig. 1. Furthermore, we introduce K-S test [35, 36, 56] to validate whether the learned predictors on source classes are trustworthy, to define the uncertain space to include instances predicted by unreliable predictors. Intrinsically, we can take this process as the recalibration over source space. Finally, recognition can be conducted in each learned space.

**Beyond G-ZSL.** One can find that our algorithm can be easily generalized to Open-Set Learning (OSL), which breaks the closed set assumption in supervised learning and recognizes the testing instances from one of source classes (i.e., source space), or from the novel class (i.e., target space). The novel class includes the test instances which have different distributions from that of the source ones. In contrast to G-ZSL, OSL only categorizes those instances not in source space as the novel class rather than a specific class.

**Contributions.** The main contribution of this paper is to present a systematic framework in learning compositional spaces by configuring probabilistic distributions of instances, which is capable of addressing the G-ZSL. Towards this goal, we firstly integrate the bootstrapping and the Kolmogorov-Smirnov test algorithms into G-ZSL framework. In particular,
we introduce a uncertain space, which encloses the instances which cannot easily be classified into source or target with high confidences. We extensively evaluate the importance of compositional spaces on several G-ZSL benchmarks and achieve significant improvement over existing G-ZSL approaches. Additionally, our framework can be easily generalized to OSL and also achieve state-of-the-art performances on several datasets.

2. Related Work

2.1. Generalized Zero-Shot Learning

The goal of ZSL is to recognize the instances that have never been trained before. Typically, it requires knowledge transfer from source to target classes where the knowledge is given in the form of semantic attributes [14, 19, 27], semantic word vector [16, 20, 37, 63], or ontology [40]. Many researchers [8, 61, 42, 29, 42, 52] recently extended ZSL to a more general setting – G-ZSL, where test instances can be from either source or target classes. A thorough evaluation of G-ZSL is further conducted by Xian et al. [60]. Their results show that the existing ZSL algorithms do not perform well when directly applied to G-ZSL. The predicted results are inclined to be biased towards source classes. This is because samples of target categories have never been reflected in the optimization of loss function, so the model inevitably over-fits to source categories during training time. Recent work in G-ZSL puts forward generative models to create target instances artificially. Generative Adversarial Network (GANs) based models [10, 15, 59] and Variational Auto-Encoders (VAE) based models [33, 47, 54] can be used for this purpose to generate examples of target classes. However, the synthesized pseudo-samples are still not drawn from the true sample distribution, which may interfere with both source and target sample judgment.

Some work [19, 26, 39, 51] introduced the idea of transductive learning, which utilized unlabeled test data to help build the classification model. Particularly, these work fine-tuned the mapping from feature space to semantic space and update the parameters of classification models accordingly. In contrast, though the unlabeled data are queried at the K-S test stage, note that our framework and classification models are not updated by the features of unlabeled test instances. Critically, the K-S Test is a parameter-free process for statistical hypothesis testing.

2.2. Open set Learning (OSL)

Open set Learning [16] focuses on the judgment whether instances are belonging to known classes [43, 46, 53, 4], given the testing instances. Typically, by reverting to the probability from known categories, it can judge whether the instances belong to unknown category by directly employing the OSL algorithms, such as One-class Support Vector Machine (OCSVM) [44], Local Outlier Factor (LOF) [7]. These algorithms utilize different statistics to help the inference. Inspired by the W-SVM [46] in OSL, our framework firstly conducts multi-class SVM on known categories, and further utilizes extreme value theory to complete the task. Recently, there are many works on multi-class SVM algorithms, such as the plain multi-class SVM [57]. One-against-all SVM [31], Platt’s sigmoid thresholding SVM [38]. However, the open set learning can only detect the instances from some unknown classes, rather than identifying the exact class label of instances from unknown classes. This significantly restricts its usage in real world applications. Unfortunately, it is very non-trivial to infer the labels of instances in unknown classes. Semantic knowledge should be transferred from known to unknown categories as has done in ZSL algorithms. Critically, the semantic knowledge should in principle further improve the performance of OSL in separating known from unknown data. To this end, our division algorithm steps forward to recognize the classes of both known and unknown domains, whilst one of most important novelties comes from the newly introduced uncertain domain, with the aid of attributes to categorize confusing ones in feature space.

2.3. Bootstrapping and K-S Test

In statistics, bootstrapping refers to random sampling with replacement [12]. Bootstrapping has been widely used in machine learning, especially for bootstrap aggregating [6], which is a strategy to avoid overfitting. Bootstrapping can be used for estimating statistical properties, such as mean, variance, etc. In our model, bootstrapping is introduced to estimate the quantile of W-SVM confidence scores, which is more robust than a hard threshold. K-S Test [35, 36, 56, 22] is one of the most well-known test to examine whether two samples are drawn from the same distribution. It validates the distance between the Cumulative Distribution Functions (CDF), which can be recognized as a metric distance. In transfer learning, such a distance is essential for domain adaptation. Long et al. [32] firstly introduced two-sample test in transfer learning, which aims to shorten the distance between source and target space. In our setting, we would like to identify the distance and classify them respectively, which is different from previous work.

3. Learning Compositional Spaces for G-ZSL

**Problem setup.** In our learning task, we have a training dataset, i.e., source classes, with \( n_s \) instances, \( D_s = \{x_i, y_i, l_i\}_{i=1}^{n_s}; x_i \in \mathbb{R}^p \) is the feature of \( i_{th} \) instance with the class label \( l_i \in C_s \), where \( C_s \) is the source class set; \( n_t \) is the number of instances in source class \( c \). Analogous to standard ZSL setting, we introduce target label classes \( C_t \) with \( C_s \cap C_t = \emptyset \) and the full class label set \( C = C_s \cup C_t \). \( y_i \) is the semantic vector of instance \( x_i \). According to Lampert et al. [28], we assume a class-level semantic vector profile existed: \( y_i \) is denoted as the semantic prototype for all the instances in class \( c \). Given one test instance \( x_i \), our goal is to predict its class label \( c_i \). In G-ZSL tasks, we target at learning to predict \( c_i \in \{C_s, C_t\} \). The semantic prototype is predefined for each class in \( C \). Additionally, our framework can also address the OSL task by predicting
$c_i \in \{C_s, \text{ novel class}\}$, where the novel class is an umbrella term referring to any class not in $C_s$.

### 3.1. Extreme Values as Confidence Scores

One can conduct the G-ZSL by directly learning compositional spaces of source and target classes. Such an idea has been well explored in the CMT [49], which employed Local Outlier Factor (LOF) to measure the degree a data point is outlying in target space. Thus, CMT relies on the density of clusters of source classes. In contrast, our framework of learning compositional space is developed upon the extreme value theory [46].

Given a test instance $x_i$, the supervised predictor can compute the confidence score $\zeta_i = f_c(x_i)$, which indicates the certainty of $x_i$ belonging to the class $c$. In our experiments, we follow the setting of W-SVM, which utilize the output of Weibull-based SVM as the confidence predictor. Therefore, we define two events,

\[
E_1: \quad x_i \text{ belonging to class } c; \\
E_2: \quad x_i \text{ belonging to the other classes}
\]

In term of extreme value theory [46], the maximum/minimum confidence scores predicted by the classifier of each class can be taken as one of extreme value distributions (i.e., Weibull distribution $G(z; \lambda_c, \nu_c, \kappa_c)$ and reverse Weibull distributions $rG(z; \lambda_c', \nu_c', \kappa_c')$ respectively). Note that the parameters of weibull/reverse-weibull density $\lambda_c, \nu_c, \lambda_c', \nu_c', \kappa_c$ can be obtained by MLE, which was introduced by W-SVM. We can thus estimate the upper/lower extremes of instance $x_i$ for event $E_1/E_2$ individually. Accordingly, given $x_i$ and $\zeta_i$, we can estimate the probability of these two events as $P_G(E_1) = 1 - \exp \left( - \left( \frac{\zeta_i - \nu_c}{\lambda_c} \right)^\kappa \right) \text{ and } P_G(E_2) = \exp \left( - \left( \frac{\zeta_i - \nu_c'}{\lambda_c'} \right)^\kappa \right)$. Note that $G$ and $rG$ refer to the weibull distribution and reverse weibull distribution, $P_G$ and $P_{rG}$ represent their probability.

We introduce the calibrated extreme values $m_c(x_i)$ [46] as the confidence scores in measuring the confidence that $x_i$ belonging to class $c$ as,

\[
m_c(x_i) = P_{rG}(\sim E_2 \mid f_c(x_i)) \cdot P_G(E_1 \mid f_c(x_i)) \quad (1)
\]

To determine if one testing instance $x_i$ belongs to class $c$, Weibull-calibrated SVM (W-SVM) [46] introduced a threshold $\delta_c$ as

\[
c_i = \begin{cases} 
  c & m_c(x_i) > \delta_c \\
  \sim c & \text{otherwise}
\end{cases} \quad (2)
\]

where $\delta_c$ is a fixed value [46] in determining whether the instance $i$ belongs to the class $c$. The instance $x_i$ rejected by all source classes in Eq (2) should be labeled as the target space. Generalizing to $C_s$ class is straightforward by training multiple prediction functions $\{f_c(x_i), c = 1, \ldots, |C_s|\}$.

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3In our setting, we use W-SVM to get the score, whose input is the neural network pre-trained feature. Note that the score is the value before softmax prediction.

### Input:
Confidence score set on training data $[\zeta_{tr}]$

### Output:
Threshold $\delta_c$.

1. We sample from $[\zeta_{tr}]$ for $n$ times (with replacement), producing a sampling set $[\zeta_{tr}^{i}]_{i=1}^{n}$, where $\zeta_{tr}$ indicates the $k_{th}$ sampled instance;
2. We also choose the significance level $\alpha$, and generate the $\alpha$ quantile $\zeta^*_{tr}$ from $[\zeta_{tr}^{i}]_{i=1}^{n}$. Particularly, we sort $\zeta_{tr}$ with an ascending order and extract $(\max \{\text{Round}\{\alpha n\}, 1\})_{th}$ value as $\zeta^*_{tr}$. We repeat it for $n$ times over $[\zeta_{tr}]$ to get $[\zeta^*_{tr}]_{i=1}^{n}$.
3. The threshold of Eq (3) can thus be computed as the mean of these values, i.e., $\delta_c = \frac{1}{n} \sum_{i=1}^{n} \zeta^*_{tr}$.

Algorithm 1: Determining the initial threshold

### Limitations
We argue that there are several key limitations indirectly utilizing Eq (1) and Eq (2) of learning compositional spaces: First, it is undesirable to have a fixed threshold $\delta_c$ which is empirically pre-defined for any data distributions in the source space, since the score is not an invariant. Essentially, the instances may derive from many different source classes. Such a fixed threshold cannot account the versatile data distribution in practice. For example, the instances in our uncertain space (Fig. 1) may be predicted by a wrong source/target space label; these wrongly labeled instances will never be correctly categorized. Furthermore, Eq (1) directly multiply two terms, which presumes a potential hypothesis that no correlation exists between $E_1$ and $E_2$, which is not the case in reality.

### 3.2. Model Selection by Bootstrapping

Rather than using a fixed $\delta_c$, it can be taken as a model selection task in estimating the $\delta_c$ for Eq (1) and Eq (2). Therefore, we tackle this task with the bootstrapping approach [11]. Different from the bootstrapping (i.e., self-training) in computer vision [49], bootstrapping is a statistical strategy in estimating the statistics of sampling distributions. Particularly, it estimates the standard errors and the confidence intervals of parameters of the underlying distributions. Such a procedure essentially enables model selection to determine the parameters.

Its procedures are closely related to the other methods such as cross-validation and jackknife sampling. The whole algorithm is shown in Alg. 1. To facilitate the discussion, we denote the training set of class $c$ as $\{x_{tr}^c\}$; the testing set whose instances are mostly confidently predicted as class $c$, as $\{x_{te}^c\}$. Thus, the corresponding confidence score set on training and testing data are $[\zeta_{tr}^c] = m_c(\{x_{tr}^c\})$ and $[\zeta_{te}^c] = m_c(\{x_{te}^c\})$ respectively. We can sample the $[\zeta_{tr}^c]$ (with replacement) to obtain the quantile of confidence scores for each class, then the threshold can be defined directly.

Till now, we had a sketch of our algorithm. Specifically, the training instances of source classes are utilized to get the $m_c^*(\cdot)$, $c \in C_s$; For any given testing instance $x_i$, we compute its confidence score $m_c(x_i), c \in C_s$. To determine whether an instance $x_i$ is in source or target classes, we calculate the statistic $m_c(x_i)$ in Eq (1) with the threshold $\delta_c$ estimated by the bootstrapping algorithm in Alg. 1. The instances computed in the source
and target space will be categorized by supervised, or zero-shot classifiers respectively. Once the $\delta_c$ is estimated, we can have the boundary between source and target spaces.

Note that the whole framework relies on the classifier $m_c(\cdot)$, $c \in C$, which is supposed to be robust and well-trained. Unfortunately empirically, we cannot always train good classifiers for all classes, e.g., the class with insufficient training samples. It is also nontrivial to tune the hyper-parameters of these predictors, especially, the deep network based predictors.

3.3. Learning Uncertain Space by K-S Test

The bootstrapping in Alg. 1 only give a good approximation of the distributions of empirical quantiles in practice [13]. In G-ZSL task, we observe that the estimated $\delta_\epsilon$ may be consistently too relaxed to determine the boundary of the source distribution of each class $c \in C$. We are certain that a classifier $m_c(\cdot)$ should produce similar confidence score distributions in training and testing instance $x_i$ of class $c$. The Kolmogorov-Smirnov (K-S) test is an efficient, straightforward, and qualified choice method for comparing distributions [35][36][50]. Remarkably, K-S test is a distribution-free test and the statistics of K-S test is effortless to compute. We define the null and alternative hypothesis as

$$H_0: \{z_{tr}^c\} \text{ and } \{z_{te}^c\} \text{ are from the same distribution.}$$

$$H_1: \{z_{tr}^c\} \text{ and } \{z_{te}^c\} \text{ are from different distributions.}$$

We define $K^c = \sup \|F_{tr}^c(z) - F_{te}^c(z)\|$ as the distance measure, where $F_{tr}^c = \text{ecdf}(\{z_{tr}^c\})$ and $F_{te}^c = \text{ecdf}(\{z_{te}^c\})$; ecdf ($\cdot$) is the empirical distribution function. The null hypothesis would be rejected at the significant level $\alpha$ when,

$$K^c(\alpha) > \sqrt{\frac{\|z_{tr}^c\| + \|z_{te}^c\|}{2\|z_{tr}^c\| \cdot \|z_{te}^c\|}} \cdot \log \left(\frac{\alpha}{2}\right).$$

When $H_0$ is accepted, it indicates that the $m_c(\cdot)$ is trustworthy, and the confidence scores of training and testing instances in class $c$ come from the same distribution. We are certain that a large portion of testing instances $\{z_{te}^c\} = m_c([x_i^c])$ should be in the class $c$. On the other hand, when $H_0$ is rejected, we are not sure whether $m_c(\cdot)$ is well learned; the class labels of these testing instances are uncertain. To this end, we introduce a new compositional space – uncertain space to include these instances.

Uncertain Space. Labels of instances in the uncertain space should be labeled as the most likely source class or one of target classes. Specifically, we can compute the $\{z^c = m_c(x)\}_{c=1}^{\left|C\right|}$ over all $C$ classes; and we can obtain,

$$c^* = \arg \max_{c \in C_t} \{z^c\}$$

$$z^c = \max_{c \in C_t} \{z^c\}$$

The mapping function $g(\cdot)$ is learned on the source space from features $x_i$ to its corresponding semantic vector $y_i$. Given one testing instance $x_i$; if $c^*_i$ is very high, we can confidently predict $x_i$ belonging to one of source classes; otherwise, the label of $x_i$ is either in the uncertain or target space. We thus have,

$$c^*_i = \arg \min_{c \in C_t \cup \{c^*\}} \|g(x_i) - y_i\|$$

where $y_c$ is the semantic prototype of class $c$; $c^*$ is the most likely source class to which $x_i$ belongs to.

Sample Selections. In general, we have three spaces so far: source, target, and uncertain. We firstly use the superviser predictor $f_t$ to obtain the confidence score $z^c_i$; then use initial threshold determined by Alg. 1 to split rough source and target space. Finally, we use K-S test to bring uncertain space to fine-tune the embedding space.

4. Recognition in Compositional Spaces

With the learned compositional spaces, we can make predictions in source, target, and uncertain space. Formally, we make the prediction as,

$$c^*_i = \begin{cases} \arg \max_{c \in C_t} m_c(x_i) & \text{Source space} \\ \arg \min_{c \in C_t} \|g(x_i) - y_i\| & \text{Target space} \\ \arg \min_{c \in C_t \cup \{c^*\}} \|g(x_i) - y_i\| & \text{Uncertain space} \end{cases}$$

Thus, we use $m_c(\cdot)$ to determine which space $x_i$ belongs to. If $x_i$ is from the source space, the label can be directly got by $m_c(x_i)$; if $x_i$ is from the target space, we should draw support from $g(x_i)$ and $y_i$ to predict the label. Moreover, the search space of target space differs, due to the dissimilar likelihood for $x_i$ from source classes. Additionally, a state-of-the-art supervised classifier $f(\cdot)$ and a zero-shot learner $g(\cdot)$ are orthogonal and potential useful here, since our work is a general framework and we do not define the specific forms of these classifiers.

Target versus Uncertain Spaces. We highlight the differences between these two spaces. Particularly, by using the learned embedding $g(\cdot)$, the class labels of instances can be inferred as,

$$\begin{cases} c \in C_t & \text{Target space} \\ c \in C_t \cup \{c^*\} & \text{Uncertain space} \end{cases}$$

where $c^*$ is the most likely source class for $x_i$, which is computed by the supervised classifier. Therefore, the search space of our framework is $|C_t|$ (target space) or $|C_t| + 1$ (Uncertain space), rather than $|C_t| + |C_t|$ in [9].
4.1. Recognition in Source Space

As the \( m_c(x_i) \) is available for any \( c \in \mathcal{C}_s \), we can directly find the most likely class for \( x_i \) as our prediction. In particular, we employ the argmax over \( m_c(x_i) \) as prediction of our model in the source space. SVM is used here.

4.2. Recognition in Target and Uncertain Spaces

The predictions in target/uncertain space entail a good embedding from feature space to semantic space, i.e., \( g(\cdot) \). In general, \( g(\cdot) \) should be flexible to use any ZSL algorithm. Particularly, we adopt the linear and non-linear embedding ZSL algorithms in our framework.

**Linear Embedding:** The linear model is utilized in learning the embedding. Impressively, such a plain model can achieve remarkable results compared against the other G-ZSL algorithms, as shown in the experimental section. Particularly, we propose the direct mapping model (D-M) which just employs the feature prototypes of each class and a linear predictor in predicting the attribute/word vector \( g(x) = w^T \cdot x \). The feature prototype embedding is computed as,

\[
\mathbf{w} = \arg \min _{\mathbf{w}} \sum _{c \in \mathcal{C}_t} \| g(\mathbf{x}_c) - \mathbf{y}_c \|^2 + \lambda \| \mathbf{w} \|^2
\]  

(9)

where \( \mathbf{x}_c \) is prototype feature of class \( c \) computed by averaged the instance features of source class \( c; y_c \) is the semantic prototype of class \( c \), with the computed embedding weight \( \mathbf{w} \).

**Non-linear Embedding:** To further show the efficacy of our framework, we also consider the non-linear embedding model. The Adversarial Generative Model (A-G) is investigated here, since the generative models can better learn the feature embedding. Particularly, we implement the \( f\text{-CLSWGAN} \) as the algorithm for target domain.

**Beyond G-ZSL:** Our framework can be generalized to Open-Set Learning, by predicting the labels as,

\[
\hat{c}^* = \begin{cases} 
\hat{c} & \text{if } \hat{c} \in \mathcal{C}_s \\
\text{novel class} & \text{otherwise}
\end{cases}
\]  

(10)

where \( \hat{c} \) is the predicted class label in source space, and we denote the instances not belonging to any source class as the novel class.

5. Experiments

5.1. Datasets and settings

**Datasets.** Animal with Attribute (AwA) Dataset \([28]\) has 50 classes and 30,475 images in total, with 85 class-level attributes annotated. We use 40 source training classes (including 13 classes as validation); the rest are for testing. CUB Dataset \([55]\) includes 200 classes and 11,788 fine-grain images with 312 class-level attributes annotated. The training set has 150 classes (including 50 classes as validation). (3) aPY Dataset \([14]\) has 15,339 images in 32 classes with 64 class-level annotated attributes. We use 20 classes for training (including 5 validation classes). For the AwA, CUB and aPY, we use ResNet-101 features and the class split contributed by Xian et al. \([60]\). (4) ImageNet 2012/2010 dataset is proposed by Fu et al. \([20]\). As the large-scale dataset, we use the VGG-19 feature and split as Fu et al. \([20]\): 1000 training classes with full training instances in ILSVRC 2012; and 360 testing classes in ILSVRC 2010, non-overlapped with ILSVRC 2012 classes. Notably, the attribute of ILSVRC dataset is the word2vec vectors provided by \([20]\).

**Experimental settings.** Our model is validated in standard G-ZSL settings as \([60]\). G-ZSL gives the class label of testing instances either from source or target classes. We set the significance level \( \alpha = 0.05 \) to tolerate 5% Type-I error. By default, we use SVM with RBF kernel with parameter cross-validated, unless otherwise specified. The code will be available once acceptance.

5.2. Results of Generalized Zero-Shot Learning

**Settings:** We first compare the experiments on G-ZSL by using the settings by Xian et al. \([60]\). The results are summarized in Tab. 1. In particular, we further compare the separate settings; and top-1 accuracy in (%) is reported here: (1) \( \text{Acc}_{s \rightarrow t} \): Test instances from source classes, the prediction candidates include both source and target classes; (2) \( \text{Acc}_{u \rightarrow t} \): Test instances from target classes, the prediction candidates include both source and target classes. (3) We employ the harmonic mean as the main evaluation metric to further combine the results of both \( S \rightarrow T \) and \( U \rightarrow T \), as \( H = 2 \cdot \text{Acc}_{u \rightarrow t} \cdot \text{Acc}_{s \rightarrow t} / (\text{Acc}_{u \rightarrow t} + \text{Acc}_{s \rightarrow t}) \).

**Competitors.** We compare several competitors. (1) DAP \([28]\), trains a probabilistic attribute classifier and utilizes the joint probability to predict labels; (2) ConSE \([27]\), maps features into the semantic space by convex combination of attributes; (3) CMT \([49]\), projects features into unsupervised semantic space and uses LOF to detect novel classes; (4) SSE \([64]\), regards novel classes as mixtures of source proportions to measure the instance similarity. (5) Latem \([58]\), is a novel latent embedding for ZSL and G-ZSL. (6) ALE \([1]\), embeds labels into the attribute space by learning a function to rank the likelihood of each class. (7) DeviSe \([16]\), uses both unsupervised information and annotated attributes to classify classes in an embedding model; (8) SIE \([2]\) is a hierarchical embedding to learn an inner product gram matrix between features and attributes. (9) ESZSL \([41]\), focuses on the regularization term in the projection from features to semantic space. (10) SYNC \([8]\), aligns the semantic space to feature space by manifold learning. (11) SS-VOC \([20]\), optimizes the triplet loss to learn the projection from features to semantic space. (12) SAE \([25]\) is an auto-encoder to combine feature and semantic space. (13-15) PTMCA & SE-GZSL & CADA-VAE \([33, 47, 54]\) leverages VAE \([24]\) as the generator of pseudo instances to train the mapping. (16-18) SP-AEN & cycle-CLSWGAN & \( f\text{-CLSWGAN} \) \([10, 15, 59]\) use GAN \([3]\) to reconstruct features to balance the target space. (19) CDE \([23]\) aligns semantic and feature space with dictionary learning.

**Results.** We use SVM/D-M and SVM/A-G to indicate the recognition models in source and target/uncertain spaces. As in Tab. 1, our harmonic mean results are significantly better than all the competitors on almost all datasets. This shows that
ours can effectively address G-ZSL tasks. Particularly – (1) Our SVM/D-M results can outperform other competitors on a large margin on AwA and aPY dataset, due to the efficacy of our compositional space learning algorithm. Further, with the non-linear embedding model – A-G, our SVM/A-G results are even better on both CUB and AwA dataset. We argue that the key advantage of our framework comes from the recognition in the compositional spaces. (3) The good results of both SVM and SVM/A-G indicate that our framework is a general framework. In other words, those previous recognition models are orthogonal and potentially be useful in each learned compositional space.

Our framework is also applied to a large-scale dataset as Tab. 3. We compare several state-of-the-art methods that address G-ZSL on the large-scale dataset. We use the SVM with the linear projection on this dataset, due to the huge computational cost on the large-scale dataset. Our harmonic mean results surpass the other competitors with a very significant margin. We notice that other algorithms have very poor performances on \( U \rightarrow T \). This indicates the intrinsic difficulty of G-ZSL on large-scale datasets. In contrast, our algorithm can better separate the testing instances into different spaces, achieving better recognition performance. Additionally, we found that the prediction of ConSE is heavily biased towards source classes which is consistent with the results in small datasets. This is due to the probability of target classes are expressed as the convex combination of source classes.

5.3. Ablation study

**Open-Set Learning.** Our framework is also evaluated on the task of OSL. Critically, we compare against the competitors, including Attribute Baseline (Attrb), W-SVM, One-class SVM, Binary SVM, OSDN and LOF. The attribute baseline is the variant of our task without using compositional space learning algorithm. Particularly, the Attrb uses the same semantic space and embedding as our model without using the compositional space learning step, i.e., using negative samples and prototypes to identify projected instances directly. F1-measure is used here as the harmonic mean of source class accuracy (specific class) and target prediction accuracy (unnecessary to predict the specific class). We summarize the results in Tab. 4. The accuracy here denotes open class detection accuracy (%), which is \( \frac{\text{Correct classified samples}}{\text{Wrong samples}} \). Significant performance gain over existing approaches has been observed, in particular for AwA, aPY, and ImageNet. This validates the effectiveness of our framework. We attribute the improvement to

![Table 1. G-ZSL Results on AwA, CUB and aPY. (Acc refers to accuracy (%), H is the harmonic mean)](attachment:image.png)

| Type       | Method   | AwA          |      | CUB          |      | aPY          |      |
|------------|----------|--------------|------|--------------|------|--------------|------|
|            |          | Acc\(U\rightarrow T\) | Acc\(S\rightarrow T\) | H   | Acc\(U\rightarrow T\) | Acc\(S\rightarrow T\) | H   | Acc\(U\rightarrow T\) | Acc\(S\rightarrow T\) | H   |
| G-ZSL Models | Chance   | 2.0          | 2.0  | -            | 0.5  | 0.5          | -    | 3.1          | 3.1  | -            |
|            | DAP      | 0.0          | 88.7 | 0.0          | 1.7  | 67.9         | 3.3  | 4.8          | 78.3 | 9.0          |
|            | ConSE    | 0.4          | 88.6 | 0.8          | 1.6  | 72.2         | 3.1  | 0.0          | 91.2 | 0.0          |
|            | CMT      | 8.4          | 86.9 | 15.3         | 4.7  | 60.1         | 8.7  | 10.9         | 74.2 | 19.0         |
|            | SSE      | 7.0          | 80.6 | 12.9         | 8.5  | 46.9         | 14.4 | 0.2          | 78.9 | 0.4          |
|            | LateM    | 7.3          | 71.7 | 13.3         | 15.2 | 57.3         | 24.0 | 0.1          | 73.0 | 0.2          |
|            | ALE      | 16.8         | 76.1 | 27.5         | 23.7 | 62.8         | 34.4 | 4.6          | 73.7 | 8.7          |
|            | DeVISE   | 13.4         | 68.7 | 22.4         | 23.8 | 53.0         | 32.8 | 4.9          | 76.9 | 9.2          |
|            | SJE      | 11.3         | 74.6 | 19.6         | 23.5 | 59.2         | 33.6 | 3.7          | 55.7 | 6.9          |
|            | ESZSL    | 6.6          | 75.6 | 12.1         | 12.6 | 63.8         | 21.0 | 2.4          | 70.1 | 4.6          |
|            | SYNC     | 8.9          | 87.3 | 16.2         | 11.3 | 70.9         | 19.8 | 4.7          | 66.3 | 13.3         |
|            | SAE      | 1.1          | 82.2 | 2.2          | 7.8  | 54.0         | 13.6 | 0.4          | 80.9 | 0.9          |
| G-ZSL Models | SE-GZSL  | 56.3         | 67.8 | 61.5         | 41.5 | 53.3         | 46.7 | -            | -    | -            |
|            | CADA-VAE | 57.3         | 72.8 | 64.1         | 51.6 | 53.5         | 52.4 | -            | -    | -            |
|            | PTMCA    | 22.4         | 80.6 | 35.1         | 23.0 | 51.6         | 31.8 | 15.4         | 71.3 | 25.4         |
|            | SP-AEN   | 23.3         | 90.9 | 37.1         | 34.7 | 70.6         | 46.6 | 13.7         | 63.4 | 22.6         |
|            | cycle-CLSWGAN | 56.9     | 64.0 | 60.2         | 45.7 | 61.0         | 52.3 | -            | -    | -            |
|            | f-CLSWGAN | 57.9         | 61.4 | 59.6         | 43.7 | 57.7         | 49.7 | -            | -    | -            |
|            | f-CLSWGAN* | 57.8        | 72.4 | 64.2         | 43.4 | 58.3         | 49.8 | 16.8         | 45.7 | 24.6         |
|            | CDL      | 28.1         | 71.5 | 40.6         | 23.5 | 55.2         | 32.9 | 19.8         | 48.6 | 28.1         |
| Ours       | SVM/D-M  | 53.6         | 90.4 | 67.3         | 37.2 | 45.2         | 40.8 | 44.0         | 89.2 | 58.9         |
|            | SVM/A-G  | 66.0         | 91.2 | 76.6         | 53.1 | 59.4         | 56.1 | 22.4         | 81.3 | 35.1         |

**Table 2. G-ZSL on the large-scale dataset – ImageNet 2012/2010. (Acc refers to accuracy (%), H is the harmonic mean)**

|          | SS-Voc | SAE | ESZSL | DeVISE | ConSE | Chance | Ours (SVM/D-M) |
|----------|--------|-----|-------|--------|-------|--------|----------------|
| Acc\(U\rightarrow T\) | 2.3    | 0.2 | 0.5   | 0.4    | 0.0   | <0.1  | 5.7            |
| Acc\(S\rightarrow T\) | 33.5   | 32.8| 38.1  | 24.7   | 56.2  | <0.1  | 54.1           |
| H        | 4.3    | 0.5 | 0.9   | 0.8    | 0.0   | -     | 10.3           |
the newly introduced uncertain space which helps better differentiate whether testing instances derive from source or target space.

### Table 3. Ablation Study. √/× indicate using/not using the corresponding step respectively. Numerical results refer to accuracy (%).

| Dataset | AwA | aPY | CUB |
|---------|-----|-----|-----|
| K-S test | √   | √   | ×   | ×   |
| Bootstrapping | √   | ×   | √   | ×   |
| OSL      | 93.7| 85.6| 37.1| 80.2|
| G-ZSL    | 67.3| 63.5| 11.4| 61.7|

### Table 4. Comparison of Open-Set recognition algorithms

| Method / Accuracy | AwA | CUB | aPY | ImageNet |
|-------------------|-----|-----|-----|----------|
| Attrb             | 33.8| 18.7| 5.1 | 3.7      |
| Binary SVM        | 57.7| 29.8| 66.6| 24.6     |
| W-SVM             | 80.2| 58.6| 78.6| 50.1     |
| One-Class SVM     | 58.9| 27.6| 57.1| 23.4     |
| OSDN              | 49.9| 36.7| 41.5| –        |
| LOF               | 60.0| 54.5| 49.1| 38.0     |
| Ours              | 93.7| 59.5| 94.3| 67.6     |

**Importance of model selection by bootstrapping.** We introduce a variant $A$ (K-S test (✓) and Bootstrapping (✗)) by replacing bootstrapping step as in Sec. 3.2 and using Eq (1) and Eq (2) to fix the threshold (i.e., W-SVM [46]). As in Tab. 3 the results of variant $A$ are significantly lower than ours on all datasets. This validates the importance of determining the initial threshold by bootstrapping.

**Importance of K-S test.** We define variant $B$ (K-S test (✗) and Bootstrapping (✓)) as the step without using K-S Test, and compare the results in Tab. 3. In particular, we note that variant $B$ has significant lower results on OSL and G-ZSL than variant $A$ and our framework. One reason is that our bootstrapping step actually learns to determine a very wide boundary of the source space, to make sure the good results in labeling testing instances as target space samples (as illustrated in Fig. 1). The K-S test will further split the initial source space into source/uncertain space by shrinking the threshold. Without such a fine-tuning step, variant $B$ may wrongly categorize many instances from target classes as one of the source classes. Thus, we can show that the two steps of our framework are very complementary to each other. They work as a whole to enable good performance on OSL and G-ZSL. Finally, we introduce the variant $C$ (K-S test (✓), and Bootstrapping (✓)) in Tab. 3 by using W-SVM to do OSL, and then use our ZSL model for G-ZSL. The performance of variant $C$ is again significantly lower than that of ours, and this demonstrates the efficacy of our model.

**Visualization of each space.** We use the t-SNE visualization as Fig. 2 to show each learned space. Critically, the bicycle and motorbike, are one of the source and the target classes in aPY dataset respectively. The testing instances of motorbike can easily be categorized as one of source classes (in the initial boundary estimated by bootstrapping), due to the visual similarity to bicycle. With our K-S test, the instances of motorbike are labeled into uncertain space and finally correctly classified by our framework.

### 6. Conclusion

This paper proposes a method that learns to divide the instances into source, target and uncertain spaces for the recognition tasks from a probabilistic perspective. The compositional space procedure consists of bootstrapping and K-S Test steps. We use the bootstrapping to set an initial threshold for each class in the source space. The K-S test is further employed to fine-tune the boundary between spaces. Our proposed framework is validated for G-ZSL tasks over many benchmark datasets and achieves notable results.

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