Associated $Z^0H^0$ production with leptonic decays at LHC in next-to-leading order QCD

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Abstract

In this work we investigate the effects of the littlest Higgs model (LHM) up to the QCD next-to-leading order (NLO) on the $Z^0H^0$ associated production at the CERN Large Hadron Collider (LHC). We study the dependences of the leading order and NLO QCD corrected integrated cross sections for this process on the factorization/renormalization scale and the LHM parameters. We also provide the distributions of the transverse momenta of final decay products $\mu^-$ and $\tau^-$. Our results show that the heavy neutral gauge bosons $Z_H$ and $A_H$ could induce significant discrepancies from the standard model predictions. It is found that when the LHM parameters are taken as $c = 0.5$, $c' = 0.22$, $f = 4 \text{ TeV}$ and $\mu = (M_H + M_Z)/2$, the effects at the $\sqrt{s} = 14 \text{ TeV}$ LHC from the heavy neutral gauge boson are about 12.83% and 10.37% to the leading order and NLO QCD corrected integrated cross sections, respectively. We also conclude that the NLO QCD corrections at the $\sqrt{s} = 14 \text{ TeV}$ LHC can obviously reduce the scale uncertainty of the integrated cross section, and significantly enhance the differential cross sections of $p_T^{\mu^-}$ and $p_T^{\tau^-}$. It demonstrates that the precision measurement of the $Z^0H^0$ associated production process at the LHC could provide the clue of the LHM physics.

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I. Introduction

The CDF and D0 experiments ruled out the standard model (SM) \[1, 2\] Higgs boson with mass between 156 GeV and 177 GeV at 95% confidence level (CL) \[3\]. Recently, the ATLAS and CMS experiments at the CERN Large Hadron Collider (LHC) have excluded most of the Higgs mass ranges of 146 – 466 GeV and 145 – 400 GeV at 95% CL in their reports of \[4\] and \[5\], respectively. Currently, the ATLAS and CMS groups exclude a substantial region of the possible Higgs boson mass range, and find several Higgs like events around the locations of $M_H \sim 126$ GeV (ATLAS) and $M_H \sim 124$ GeV (CMS) \[6, 7\]. Further searching for Higgs boson and studying its properties are still the important tasks for the present and upcoming high energy colliders.

Despite the tremendous success of the SM in describing the high energy physics at the energy scale up to several hundred GeV, the instability of the Higgs boson mass leads to the “hierarchy problem” \[8\] which comes from the quadratic loop corrections to the Higgs boson mass. In order to give a proper electroweak symmetry breaking (EWSB) scale, the Higgs boson mass needs unnatural fine-tuning when it gets a radiative correction with the cutoff scale about 10 TeV. In order to solve the “hierarchy problem”, physicists developed several new particle models such as supersymmetry \[9\], extra dimensions \[10\], little Higgs \[11, 12\], technicolor and so on. Among these theories, the little Higgs models are proposed as one kind of models without fine-tuning in which the Higgs boson is naturally light as a result of nonlinearly realized symmetry. The littlest Higgs model (LHM) \[13, 14, 15\] is the most economical model of them and a phenomenological viable model.

There are an $SU(5)$ global symmetry and a locally gauged subgroup $G_1 \otimes G_2 = [SU(2)_1 \otimes U(1)_1] \otimes [SU(2)_2 \otimes U(1)_2]$ in the LHM. At the scale $\Lambda_S$, the global symmetry $SU(5)$ is broken into its subgroup $SO(5)$. At the same time, the local gauge symmetry $[SU(2) \otimes U(1)]^2$ is also spontaneously broken into its diagonal subgroup $SU(2)_L \otimes U(1)_Y$, which is identified as the SM gauge group. In the LHM, a set of new heavy gauge bosons ($W^\pm_H$, $Z_H$ and $A_H$) and a new heavy-vector-like quark ($T$) are introduced to cancel the quadratic divergence induced
by SM gauge boson loops and the top quark loop, respectively. These new gauge bosons might provide the significant signatures at the present and future high energy colliders. The $pp \rightarrow Z^0 H^0 + X$ process is one of the main production mechanisms of Higgs boson with moderate mass at the LHC, which gives a very distinctive signature. This process could be used to measure the Higgs mass and the couplings between Higgs boson and gauge bosons and determine the quantum numbers of the Higgs boson. Therefore, investigating the process $pp \rightarrow Z^0 H^0 + X$ at the LHC in the context of the LHM is necessary for probing the LHM physics [16]. We find that the SM and minimal supersymmetric standard model (MSSM) analyses to the $pp \rightarrow Z^0 H^0 + X$ process at the LHC have been already existed in Ref.[17].

In this work we study the effects of the LHM on neutral Higgs boson production associated with $Z^0$ boson up to the QCD next-to-leading order (NLO) at the CERN LHC. In the LHM the new neutral gauge bosons, such as $Z_H$ and $A_H$, give additional contributions to this process. The paper is constructed as follows: In section II, we provide related theory of the LHM to our calculations. In section III, we describe the calculations at the leading order (LO) and the QCD NLO for the $pp \rightarrow Z^0 H^0 + X$ process. The numerical results and discussions are presented in section IV. Finally, a short summary is given.

II. Related theory of LHM

The LHM is based on an $SU(5)/SO(5)$ nonlinear $\sigma$ model. The vacuum expectation value (VEV) breaks the $SU(5)$ global symmetry into its subgroup $SO(5)$ and breaks the local gauge symmetry $[SU(2) \otimes U(1)]^2$ into its diagonal subgroup $SU(2)_L \otimes U(1)_Y$ at the same time, which is identified as the SM electroweak gauge group. The gauge fields $W'^\mu$ and $B'^\mu$ associated with the broken gauge symmetries are related to the SM gauge fields by

$$W'^\mu = -c W_1^\mu + s W_2^\mu, \quad W^\mu = s W_1^\mu + c W_2^\mu,$$

$$B'^\mu = -c' B_1^\mu + s' B_2^\mu, \quad B^\mu = s' B_1^\mu + c' B_2^\mu,$$
with mixing angles of

\[ c = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad c' = \frac{g'_1}{\sqrt{g'_1^2 + g'_2^2}}. \]  

At the scale \( f \) the SM gauge bosons remain massless, while the heavy gauge bosons acquire masses of order \( f \). The \( W \) and \( B \) are identified as the SM gauge bosons, with couplings of \( g = g_1s = g_2c \) and \( g' = g'_1s' = g'_2c' \). The electroweak symmetry breaking (EWSB) gives the masses for the SM gauge bosons and induces further mixing between the light and heavy gauge bosons. We denote the light gauge boson mass eigenstates as \( W^\pm, Z_0 \) and the new heavy gauge boson mass eigenstates as \( W^\pm_H, Z_H, A_H \). The masses of the charged and neutral gauge bosons to the order of \( v^2/f^2 \) are given by [13]

\[ M^2_{W^\pm} = m_w^2 \left[ 1 - \frac{v^2}{f^2} \left( \frac{1}{6} + \frac{1}{4}(c^2 - s^2)^2 \right) + 4\frac{v'^2}{v^2} \right], \]  

\[ M^2_{W_H^\pm} = m_w^2 \left( \frac{f^2}{s^2c^2v^2} - 1 \right), \]  

\[ M^2_\gamma = 0, \quad M^2_{A_H} = m_\omega^2 s^2w^2 \left( \frac{f^2}{5s^2c^2v^2} - 1 + \frac{\chi H S^2_W}{4s^2c^2S^2_W} \right), \]  

\[ M^2_Z = M^2_{Z_L} = m_\omega^2 \left\{ 1 - \frac{v^2}{f^2} \left[ \frac{1}{6} + \frac{1}{4}(c^2 - s^2)^2 + \frac{5}{4}(c^2 - s^2)^2 - \chi^2 \right] \right\}, \]  

\[ M^2_{Z_H} = m_\omega^2 C^2_W \left( \frac{f^2}{s^2c^2v^2} - 1 - \frac{\chi H S^2_W}{s^2c^2C^2_W} \right), \]  

with

\[ \chi = \frac{4fv'}{v^2}, \quad \chi H = \frac{5S_W C_W s c s' c'}{2} \frac{s^2 c^2 + s^2 c'^2}{C^2_W - S^2_W}, \]  

where \( m_\omega \equiv g v/(2C_W) \), \( C_W \equiv \cos \theta_W = \frac{m_\omega}{m_\omega} \), \( \theta_W \) is the Weinberg angle, \( v' \) and \( v \) are the VEV's of the scalar \( SU(2)_L \) triplet and doublet, respectively. In the following numerical calculations we take \( v = 246 \text{ GeV} \) and \( \chi = 0.5 \).
The couplings of the neutral gauge bosons to quarks are expressed in the form as $i\gamma_\mu (g_L P_L + g_R P_R)$ where $P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)$. The explicit expressions are given below.

$$g^{ZUU}_L = -\frac{e}{2SW_CW} \left\{ 1 - \frac{4}{3} \frac{S^2_W}{f^2} + \frac{v^2}{f^2} \left[ \frac{c^2}{2} (c^2 - s^2) - \frac{5}{2} (c^2 - s^2) \left( \frac{8}{15} - \frac{1}{3} c^2 \right) \right] \right\}, \quad (2.10)$$

$$g^{ZUU}_R = -\frac{e}{2SW_CW} \left\{ \frac{4}{3} \frac{S^2_W}{f^2} - \frac{v^2}{f^2} \left[ \frac{5}{2} (c^2 - s^2) \left( \frac{2}{15} + \frac{2}{3} c^2 \right) \right] \right\}, \quad (2.11)$$

$$g^{ZDD}_L = -\frac{e}{2SW_CW} \left\{ 1 + \frac{2}{3} \frac{S^2_W}{f^2} - \frac{v^2}{f^2} \left[ \frac{c^2}{2} (c^2 - s^2) + \frac{5}{2} (c^2 - s^2) \left( -\frac{2}{15} + \frac{1}{3} c^2 \right) \right] \right\}, \quad (2.12)$$

$$g^{ZDD}_R = -\frac{e}{2SW_CW} \left\{ \frac{2}{3} \frac{S^2_W}{f^2} - \frac{v^2}{f^2} \left[ \frac{5}{2} (c^2 - s^2) \left( \frac{4}{15} - \frac{2}{3} c^2 \right) \right] \right\}, \quad (2.13)$$

$$g^{A_{\mu}UU}_L = \frac{e}{2s'c'C_W} \left( \frac{2}{15} - \frac{1}{3} c^2 \right), \quad g^{A_{\mu}UU}_R = \frac{e}{2s'c'C_W} \left( \frac{8}{15} - \frac{8}{6} c^2 \right), \quad (2.14)$$

$$g^{A_{\mu}DD}_L = \frac{e}{2s'c'C_W} \left( \frac{2}{15} - \frac{2}{6} c^2 \right), \quad g^{A_{\mu}DD}_R = \frac{e}{2s'c'C_W} \left( \frac{4}{15} + \frac{4}{6} c^2 \right), \quad (2.15)$$

$$g^{Z_{\mu}UU}_L = \frac{ec}{2sS_W}, \quad g^{Z_{\mu}UU}_R = 0, \quad g^{Z_{\mu}DD}_L = -\frac{ec}{2sS_W}, \quad g^{Z_{\mu}DD}_R = 0, \quad (2.16)$$

where $U$ and $D$ represent the up-type ($U = u, c, t$) and down-type ($D = d, s, b$) quarks, respectively. The couplings between neutral gauge boson and Higgs boson are expressed as

$$g^{HZZ} = \frac{i e v g_{\mu \nu}}{2S_W C_W^2} \left\{ 1 - \frac{v^2}{f^2} \left[ \frac{1}{3} - \frac{3}{4} \chi^2 + \frac{1}{2} (c^2 - s^2)^2 + \frac{5}{2} (c^2 - s^2)^2 \right] \right\}, \quad (2.17)$$

$$g^{HZA_H} = -\frac{i e v g_{\mu \nu}}{2S_W C_W^2} \frac{c^2 - s^2}{2s'c'}, \quad g^{HZZ_H} = -\frac{i e v g_{\mu \nu}}{2S_W C_W^2} \frac{c^2 - s^2}{2sc}. \quad (2.18)$$

The heavy neutral gauge boson $V_H$ ($V_H = Z_H, A_H$) can decay into a fermion pair and $Z^0 H^0$. We obtain the partial decay rates expressed below [18].

$$\Gamma(V_H \rightarrow f \bar{f}) = \frac{N_c}{12\pi} \left[ (g^V_H f_f)^2 (1 + 2r_f) + (g^V_H f_f)^2 (1 - 4r_f) \right] \sqrt{1 - 4r_f} M_{V_H}, \quad (2.19)$$

$$\Gamma(V_H \rightarrow Z^0 H^0) = \frac{(g^V_H)^2}{192\pi} \sqrt{\lambda} \left[ (1 + r_Z - r_H)^2 + 8r_Z \right] M_{V_H}, \quad (2.20)$$
where \( N_c = 3 \) is the color factor, 
\[
g_v^{VHff} = (g_R^{VHff} + g_L^{VHff})/2, \quad g_a^{VHff} = (g_R^{VHff} - g_L^{VHff})/2,
\]
\[
g^{A_H} = g'(c'^2 - s'^2)/(2c's'), \quad g^{Z_H} = g(c'^2 - s'^2)/(2cs), \quad \lambda = 1 + r_Z^2 - 2r_Z - 2r_H - 2r_Zr_H,
\]
and
\[
r_i = X_i^2/M_{VH}^2, \quad X_i = m_f, M_Z, M_H.
\]
Since in our investigated parameter space the \( V_H \to T\bar{T} \) and \( V_H \to T\bar{t}(T\bar{t}) \) decays are kinematically forbidden, we assume that the total decay width \( \Gamma_{V_H} (V_H = Z_H, A_H) \) is the sum of \( \Gamma(V_H \to f\bar{f}) \) and \( \Gamma(V_H \to Z^0H^0) \), where \( f = u, d, c, s, b, t, e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau \).

### III. Analytical calculations

#### III.1 LO calculations

We generate the Feynman diagrams and their corresponding amplitudes by using FeynArts3.5 package \[19\], and apply FormCalc5.4 package \[20\] to implemented the amplitude simplification. The LO contribution to the \( pp \to Z^0H^0 + X \) process comes from \( q\bar{q} \) annihilation \( (q = u, d, c, s, b) \). We denote the partonic process as

\[
q(p_1) + \bar{q}(p_2) \to Z^0(p_3) + H^0(p_4), \quad (q = u, d, c, s, b),
\]

where \( p_1, p_2, p_3 \) and \( p_4 \) represent the four-momenta of incoming partons, the outgoing \( Z^0 \)- and \( H^0 \)-boson, respectively. We use the ’t Hooft-Feynman gauge throughout our calculations. Comparing with the partonic colliding energy at the LHC, the quark masses, \( m_q \) \( (q = u, d, c, s, b) \), are relatively small. We neglect their masses in our further calculations. The Feynman diagram for the \( q\bar{q} \to Z^0H^0 \) subprocess in the SM is shown in Fig.\( \Pi \)\( (a) \). The amplitudes corresponding to Figs.\( \Pi \)\( (a) \), \( (b) \) and \( (c) \) without introducing the decay widths in propagators are denoted as \( M^Z_{LO}(\Gamma_Z = 0) \), \( M^{Z_H}_{LO}(\Gamma_{Z_H} = 0) \) and \( M^{A_H}_{LO}(\Gamma_{A_H} = 0) \), respectively.

As shown in Eq.\( (2.10) \)-Eq.\( (2.13) \), the coupling between \( Z^0 \) and quarks in the LHM can be obtained from the SM one with a correction of \( \mathcal{O}(v^2/f^2) \). The \( q\bar{q} \to Z^0H^0 \) subprocess in the LHM obtains additional contributions coming from the diagrams with the exchange of heavy gauge bosons \( Z_H \) and \( A_H \) shown in Fig.\( \Pi \)\( (b)-(c) \). These two heavy neutral gauge bosons, \( Z_H \) and \( A_H \), are potentially resonant. For disposal of the singularities due to \( V_H \)
The LO Feynman diagrams for the $q\bar{q} \rightarrow Z^0 H^0$ ($q = u, d, c, b$) partonic process.

$(V_H = Z_H, A_H)$ resonances in the calculations, we have to introduce the decay widths of $Z_H$ and $A_H$ by doing the following replacements in the resonance propagators of the amplitudes $\mathcal{M}^{Z_H}_{LO}(\Gamma_{Z_H} = 0)$ for Fig.1(b) and $\mathcal{M}^{A_H}_{LO}(\Gamma_{A_H} = 0)$ for Fig.1(c),

$$\frac{1}{\hat{s}_{12} - M^2_{V_H}} \rightarrow \frac{1}{\hat{s}_{12} - M^2_{V_H} + i M_{V_H} \Gamma_{V_H}},$$

(3.2)

where $\Gamma_{V_H}$ ($V_H = Z_H, A_H$) represents the decay width of $V_H$. Then we get the LO amplitudes for Fig.1(b) and (c) at the tree-level respectively expressed as

$$\mathcal{M}^{Z_H}_{LO} = \frac{\hat{s}_{12} - M^2_{Z_H}}{\hat{s}_{12} - M^2_{Z_H} + i M_{Z_H} \Gamma_{Z_H}} \mathcal{M}^{Z_H}_{LO}(\Gamma_{Z_H} = 0),$$

$$\mathcal{M}^{A_H}_{LO} = \frac{\hat{s}_{12} - M^2_{A_H}}{\hat{s}_{12} - M^2_{A_H} + i M_{A_H} \Gamma_{A_H}} \mathcal{M}^{A_H}_{LO}(\Gamma_{A_H} = 0).$$

(3.3)

The modified amplitudes $\mathcal{M}^{Z_H}_{LO}$ and $\mathcal{M}^{A_H}_{LO}$ are safe amplitudes being free of the $Z_H$ and $A_H$ resonance singularities. Since the $\mathcal{O}(\alpha_s)$ corrections do not contribute to the LO $Z_H$ and $A_H$ decay widths, these replacements cannot induce the double-counting problem in our NLO calculations.

The LO cross section for the subprocess $q\bar{q} \rightarrow Z^0 H^0$ is expressed as

$$\hat{\sigma}^{q\bar{q}}_{LO} = \frac{11}{4} \frac{(2\pi)^4}{28^2} \int \sum_{\text{spin}} |\mathcal{M}_{LO}|^2 d\Omega_2, \quad (q = u, d, c, s, b)$$

(3.4)

where the factors $\frac{1}{4}$ and $\frac{11}{28}$ come from the averaging over the spins and colors of the initial partons respectively, $\hat{s}$ is the partonic center-of-mass energy squared, and $\mathcal{M}_{LO}$ is the amplitude of all the LO diagrams shown in Fig.1. The summation is taken over the spins and colors of all the relevant particles in the $q\bar{q} \rightarrow Z^0 H^0$ subprocess. The integration is performed over
the two-body phase space of the final particles $Z^0$ and $H^0$. $d\Omega_2$ is the two-body phase space element expressed as

$$d\Omega_2 = \delta^{(4)} \left( p_1 + p_2 - \sum_{i=3}^{4} p_i \right) \prod_{j=3}^{4} \frac{d^3 p_j}{(2\pi)^3 2E_j}. \quad (3.5)$$

Within the framework of the QCD factorization, the LO cross section for the process $pp \rightarrow Z^0H^0 + X$ at the LHC can be obtained by performing the following integration of the cross section for the subprocess $q \bar{q} \rightarrow Z^0H^0$ over the partonic luminosities (see Eq.(3.6)).

$$\sigma_{LO} = \sum_{ij=ss,cc,bb,uu,dd} \int_{0}^{1} dx_1 \int_{0}^{1} dx_2 \left[ G_{ij/P_1} (x_1, \mu_f) G_{ij/P_2} (x_2, \mu_f) + (x_1 \leftrightarrow x_2, P_1 \leftrightarrow P_2) \right] \hat{\sigma}_{LO}^{ij}(\hat{s} = x_1x_2s), \quad (3.6)$$

where $G_{ij/A} (x, \mu_f)$ ($i = u, d, s, c, b$) is parton distribution function (PDF) of proton $A (= P_1, P_2)$ which describes the probability to find a parton $i$ with momentum $xp_A$ in proton $A$, $s$ is defined as the total colliding energy squared in proton-proton collision, $\hat{s} = x_1x_2s$, and $\mu_f$ is the factorization scale. In our LO calculations, we adopt the CTEQ6L1 [21] PDFs.

### III..2 Virtual and real emission corrections

The QCD one-loop vertex correction diagrams for the partonic process $q \bar{q} \rightarrow Z^0H^0$ with nonzero contribution are presented in Fig[2]. There exist both ultraviolet (UV) and soft/collinear infrared (IR) singularities in the one-loop diagrams. We regularize all the singularities by using the dimensional regularization method in $D = 4 - 2\epsilon$ dimensions, and apply the modified minimal subtraction (MS) scheme to renormalize the relevant fields. The UV divergence of the virtual corrections are removed by renormalized wave functions of the relevant quarks.

We define the renormalization constants of the relevant quark fields as

$$\psi_q^{0,L,R} = \left( 1 + \frac{1}{2} \delta Z_q^{L,R} \right) \psi_q^{L,R}, \quad (3.7)$$

where $\psi_q^{L,R}$ denotes the field of the SM quark. Its renormalization constant are expressed as

$$\delta Z_q = \delta Z_q^L = \delta Z_q^R = -\frac{\alpha_s(\mu_f)}{3\pi} \left[ \Delta_{UV} - \Delta_{IR} \right]. \quad (3.8)$$
The notations used in above equation are defined as $\Delta_{UV} = 1/\epsilon_{UV} - \gamma_E + \ln(4\pi)$ and $\Delta_{IR} = 1/\epsilon_{IR} - \gamma_E + \ln(4\pi)$.

Although the total NLO QCD amplitude of subprocess $q\bar{q} \rightarrow Z^0H^0$ is UV finite after performing renormalization procedure, it still contains soft/collinear IR singularities. The soft IR singularity can be completely canceled by the contribution of real gluon emission subprocess $q\bar{q} \rightarrow Z^0H^0g$, while the collinear singularity is eliminated partially by the light-quark emission subprocesses $q(\bar{q})g \rightarrow Z^0H^0q(\bar{q})$. The remaining collinear IR divergence can be absorbed by the counterterms of PDFs. We adopt the analytical expressions for IR-singular parts of loop integrals from Ref. [22], and use the expressions in Refs. [23, 24, 25] to implement the numerical evaluations of IR-safe $N$-point ($N \leq 4$) integrals. The Feynman diagrams for real gluon/light-quark emission are depicted in Fig. 3 and Fig. 4, respectively.

We apply the two cutoff phase space slicing (TCPSS) method [26] to isolate the soft and collinear IR singularities of the real emission correction from the IR-safe region. In performing the calculations with the TCPSS method, we should introduce arbitrary small soft cutoff $\delta_s$ and collinear cutoff $\delta_c$. The phase space of the $q(p_1)\bar{q}(p_2) \rightarrow Z^0(p_3)H^0(p_4)g(p_5)$ partonic process can be split into two regions, $E_5 \leq \delta_s\sqrt{s}/2$ (soft gluon region) and $E_5 > \delta_s\sqrt{s}/2$ (hard gluon region) by soft cutoff $\delta_s$. The hard gluon region is separated as hard collinear (HC) and hard non-collinear (HNC) regions by cutoff $\delta_c$. The HC region is the phase space where $-\hat{t}_{15}(\text{or} \ -\hat{t}_{25}) < \delta_c\hat{s}$ ($\hat{t}_{15} \equiv (p_1 - p_5)^2$ and $\hat{t}_{25} \equiv (p_2 - p_5)^2$). The phase space of light-quark emission $q(p_1)[(\bar{q}(p_1)]g(p_2) \rightarrow Z^0(p_3)H^0(p_4)q(p_5)[\bar{q}(p_5)]$ is split into hard collinear (HC) region and hard non-collinear (HNC) region by introducing a cutoff $\delta_c$. The real gluon
emission corrections over the HC region are finite and can be calculated numerically with general Monte Carlo method [27]. Finally, the cross section for the real emission partonic process can be written as

$$\hat{\sigma}_R = \hat{\sigma}_S + \hat{\sigma}_H = \hat{\sigma}_S + \hat{\sigma}_{HC} + \hat{\sigma}_{\overline{HC}}.$$  \hfill (3.9)

With the NLO correction components given above, the full QCD NLO corrected cross section for the $Z^0H^0$ production at the LHC can be formally obtained by the QCD factorization formula as

$$\sigma_{NLO}(pp \to Z^0H^0 + X) = \int dx_{P_1}dx_{P_2} \left\{ \sum_{ij} [G_{i/P_1}(x_{P_1},\mu_f)G_{j/P_2}(x_{P_2},\mu_f)\hat{\sigma}_{NLO}^{ij}(x_{P_1}x_{P_2}s,\mu_r)] + (P_1 \leftrightarrow P_2) \right\},$$ \hfill (3.10)

where $i$ and $j$ run over all possible initial partons contributing to the $pp \to Z^0H^0 + X$ process up to the QCD NLO, and the notations of $\mu_f$, $x_{P_1}$, $x_{P_2}$ are the same as in Eq. (3.6). We adopt the CTEQ6m PDFs for $G_{i/P_1}(x_{P_1},\mu_f)$ and $G_{j/P_2}(x_{P_2},\mu_f)$ in the NLO calculations.
Figure 4: The Feynman diagrams for the real light-quark emission subprocess \( q(\bar{q})g \rightarrow Z^0H^0q(\bar{q}) \) \((q = u,d,c,s,b)\).

[21] The total QCD NLO corrected cross section for partonic process \( q\bar{q} \rightarrow Z^0H^0 \) can be expressed as

\[
\hat{\sigma}^{ij}_{NLO} = \hat{\sigma}^{ij}_{LO} + \Delta\hat{\sigma}^{ij}_{NLO} = \hat{\sigma}^{ij}_{LO} + \hat{\sigma}^{ij}_R + \hat{\sigma}^{ij}_V,
\]

(3.11)

where \( \hat{\sigma}^{ij}_{LO}, \hat{\sigma}^{ij}_R \) and \( \hat{\sigma}^{ij}_V \) denote the cross sections for tree level, real emission and virtual corrections for parton level process, respectively.

IV. Numerical results and discussions

In this section we provide and discuss the numerical results for the \( pp \rightarrow Z^0H^0 + X \) process in the LHM up to the QCD NLO. In order to make a cross check with previous work on the \( Z^0H^0 \) associated production in the SM at the LHC, we take \( \mu = \mu_f = \mu_r = \sqrt{s_{ZH}} \), and the input parameters and PDFs being the same as used in Ref.[17], and calculate the LO and NLO QCD corrected total cross sections for \( pp \rightarrow Z^0H^0 + X \) at the \( \sqrt{s} = 14 \) TeV LHC in the SM. We get the total cross sections for \( M_H = 140 \) GeV as \( \sigma_{LO} = 0.46827(3) \) pb and \( \sigma_{NLO} = 0.5770(4) \) pb, separately. The corresponding results can be read out from Table 8 of Ref.[17]: \( \sigma_{LO} = 0.4684(2) \) pb and \( \sigma_{NLO} = 0.5768(2) \) pb, which are coincident with ours within the calculation errors.
In our following numerical calculations we take the colliding energy in proton-proton center-of-mass system as $\sqrt{s} = 8\ TeV$ for the early LHC and $\sqrt{s} = 14\ TeV$ for the future LHC. We use one- and two-loop running $\alpha_s(\mu)$ by taking $\Lambda_{\text{LO}}^5 = 165\ MeV$ and $\Lambda_{\text{NLO}}^5 = 226\ MeV$ for the LO and NLO calculations, respectively [28]. The factorization and the renormalization scales are set to be equal for simplicity ($\mu \equiv \mu_f = \mu_r$). We take $\mu = \mu_0 = (M_H + M_Z)/2$ in default unless otherwise stated. We neglect the masses of $u$-, $d$-, $c$-, $s$-, and $b$-quark, and take $G_\mu = 1.6637 \times 10^{-5}\ GeV^{-2}$, $M_W = 80.399\ GeV$, $M_Z = 91.1876\ GeV$, $m_t = 171.2\ GeV$, $M_H = 125\ GeV$. (4.1)

The $G_\mu$ scheme is adopted, i.e., the electromagnetic coupling $\alpha$ is derived from $\alpha_{G_\mu} = \sqrt{2}G_\mu M_W^2 (1 - M_W^2/M_Z^2)/\pi$. Considering the constraints of the electroweak precision data on LHM parameters [29], we assumed that $0.1 < c < 0.5$, $0.1 < c' < 0.9$ and $3\ TeV < f < 7\ TeV$, and take the representative input LHM parameter set as $c = 0.5$, $c' = 0.22$ and $f = 4\ TeV$ in our numerical calculations if there is no other statement. From Eqs. (2.5), (2.6) and (2.8) with this input parameter set the masses of the heavy gauge bosons $M_{A_H}$, $M_{W_H}^{\pm}$ and $M_{Z_H}$ are obtained as $1.461\ TeV$, $3.025\ TeV$ and $3.025\ TeV$ respectively, where the mass values of heavy vector gauge bosons $Z_H^0$ and $W_H^{\pm}$ are beyond the corresponding experimental lower mass limits [30].

In order to verify the independence of the total NLO QCD corrections on the introduced arbitrary cutoff values of $\delta_s$ ($\delta_c$), we depict the $\Delta\sigma_{\text{NLO}}$ for the $pp \to u\bar{u} \to Z^0 H^0 + X$ process in the LHC at the $\sqrt{s} = 14\ TeV$ LHC as the functions of $\delta_s$ in Figs. 5(a,b), where we take $c = 0.2$, $c' = 0.7$, $f = 2\ TeV$, $\delta_c = \delta_s/100$ and $\mu = \mu_0$. The amplified curve for the total NLO QCD correction ($\Delta\sigma_{\text{NLO}}$) for the process $pp \to u\bar{u} \to Z^0 H^0 + X$ is shown in Fig. 5(b). We can see in Figs. 5(a,b) that the total QCD correction to the $pp \to Z^0 H^0 + X$ process does not depend on the arbitrarily chosen value of the cutoffs $\delta_s$ and $\delta_c$. The two-body correction ($\Delta\sigma^{(2)}$) and three-body correction ($\Delta\sigma^{(3)}$) and the total QCD correction ($\Delta\sigma_{\text{NLO}} = \Delta\sigma^{(2)} + \Delta\sigma^{(3)}$) for the $pp \to u\bar{u} \to Z^0 H^0 + X$ process at the LHC are depicted as
the functions of the soft cutoff $\delta_s$ in Figs. 5(a). The curve for $\Delta\sigma_{NLO}$ is presented in Fig. 5(b) together with calculation errors. We adopt also the dipole subtraction (DPS) method to deal with the IR singularities for further verification. The $\Delta\sigma_{NLO}$ results from the DPS method including $\pm 1\sigma$ statistic errors are plotted as the shadowing region in Fig. 5(b). It shows that the results by using both the TCPSS method and the DPS method are in good agreement. In further numerical calculations we adopt the TCPSS method and fix $\delta_s = 1 \times 10^{-5}$ and $\delta_c = 1 \times 10^{-7}$.

![Figure 5](image)

Figure 5: (a) The NLO QCD corrections to the $pp \to u\bar{u} \to Z^0H^0 + X$ process in the LHM at the $\sqrt{s} = 14$ TeV LHC as the functions of the soft cutoff $\delta_s$, where we take $c = 0.2$, $c' = 0.7$ and $f = 2$ TeV, $\delta_c = \delta_s/100$ and $\mu = \mu_0$. (b) The amplified curve for the NLO QCD correction to the cross section $\Delta\sigma_{NLO}$.

We show the integrated LO, NLO QCD corrected cross sections and the corresponding K-factor ($K(\mu) \equiv \sigma_{NLO}(\mu)/\sigma_{LO}(\mu)$) at the $\sqrt{s} = 14$ TeV ($\sqrt{s} = 8$ TeV) LHC for the process $pp \to Z^0H^0 + X$ as the functions of the factorization/renormalization scale $(\mu/\mu_0)$ in Figs. 6(a) (Figs. 6(c)), where we set $\mu \equiv \mu_f = \mu_0 \equiv (M_H + M_Z)/2$, $c = 0.5$, $c' = 0.22$ and $f = 4$ TeV. If we define the scale uncertainty for the $pp \to Z^0H^0 + X$ process as $\eta = |\sigma(\mu=5\mu_0) - \sigma(\mu=0.2\mu_0)|/\sigma(\mu=\mu_0)$, from the curves in Figs. 6(a,c) we can figure out the corresponding uncertainties at the $\sqrt{s} = 14$ TeV LHC being $\eta_{LO}^{SM} = 0.251$, $\eta_{NLO}^{SM} = 0.034$, $\eta_{LO}^{LHM} = 0.176$ and $\eta_{NLO}^{LHM} = 0.045$, and at the $\sqrt{s} = 8$ TeV LHC $\eta_{LO}^{SM} = 0.080$, $\eta_{NLO}^{SM} = 0.081$, $\eta_{LO}^{LHM} = 0.033$ and
\[ \eta_{NLO}^{LHM} = 0.081, \] respectively. We can see that at the \( \sqrt{s} = 14 \text{ TeV} \) LHC the LO cross sections are strongly related to the scale in the plotted \( \mu \) range, and the NLO QCD corrections significantly reduce the scale uncertainties. But at the \( \sqrt{s} = 8 \text{ TeV} \) LHC there is no distinct improvement for the scale dependence when the QCD NLO corrections are involved. Fig.6(b) and Fig.6(d) present the relative deviations defined as \( \delta(\mu) \equiv \frac{[\sigma_{LHM}(\mu) - \sigma_{SM}(\mu)]}{\sigma_{SM}(\mu)} \), as the functions of \( \mu/\mu_0 \), which correspond to Fig.6(a) and Fig.6(c), respectively. The two figures demonstrate that the NLO QCD corrections obviously reduce the relative deviation \( \delta \) in our plotted \( \mu/\mu_0 \) range. The theoretical relative deviations including the NLO corrections are above 9.75% and 2.67% at the \( \sqrt{s} = 14 \text{ TeV} \) and \( \sqrt{s} = 8 \text{ TeV} \) LHC, respectively. We can read out from the figures that the effects \( (\delta) \) from the heavy neutral gauge boson interactions at the \( \sqrt{s} = 14 \text{ TeV} \) LHC in the vicinity of \( \mu = \mu_0 \) can be about 12.83% for \( \sigma_{LO} \) and 10.37% for \( \sigma_{NLO} \).

In following analysis we show the influence of the LHM parameters \( c, c' \), and the global symmetry breaking scale \( f \). In Figs.7(a,b,c,d) we assume \( \mu = \mu_0, \ c = 0.5 \) and \( c' = 0.22 \), and depict the plots for the LO and NLO QCD corrected cross sections and the corresponding K-factors for the \( pp \to Z^0H^0 + X \) process in both the SM and the LHM as the functions of the global symmetry breaking scale \( f \) at the \( \sqrt{s} = 14 \text{ TeV} \) and \( \sqrt{s} = 8 \text{ TeV} \) LHC in Figs.7(a) and (c), separately. The corresponding relative deviations of the cross sections in the LHM from those in the SM, \( \delta(f) \equiv \frac{[\sigma_{LHM}(f) - \sigma_{SM}(f)]}{\sigma_{SM}(f)} \), are shown in Figs.7(b) and (d), respectively. We can see from Figs.7(a,b,c,d) that when \( f \to \infty \), the relative deviations at both the LO and the NLO tend to be vanished, and the relative deviations become to be less than 5% for the \( \sqrt{s} = 14 \text{ TeV} \) LHC and the \( \sqrt{s} = 8 \text{ TeV} \) LHC in the ranges of \( f > 5 \text{ TeV} \) and \( f > 4 \text{ TeV} \), respectively. We find also that the deviations are sensitive to the scale \( f \) in the range of \( f < 5 \text{ TeV} \) in both Figs.7(b) and (d).

From Eq.(2.8) we can conclude that the mass of the heavy gauge boson \( Z_H \) is mostly related with the scale \( f \) and the mixing angle parameter \( c \) between two SU(2) gauge bosons, but not sensitive to the parameter \( c' \). For demonstrating the effects from the interactions
Figure 6: In these four plots we take $c = 0.5$, $c' = 0.22$ and $f = 4 \text{ TeV}$. (a) The dependence of the LO and the QCD corrected cross sections and the corresponding K-factor for the process $pp \rightarrow Z^0 H^0 + X$ on the factorization/renormalization scale ($\mu/\mu_0$) at the $\sqrt{s} = 14 \text{ TeV}$ LHC. (b) The corresponding relative deviation of the integrated cross sections in the LHM from those in the SM, as the functions of $\mu/\mu_0$ at the $\sqrt{s} = 14 \text{ TeV}$ LHC. (c) The LO and the QCD corrected cross sections and the corresponding K-factor for the process $pp \rightarrow Z^0 H^0 + X$ versus the scale $\mu/\mu_0$ at the $\sqrt{s} = 8 \text{ TeV}$ LHC. (d) The corresponding relative deviation of the integrated cross sections in the LHM from those in the SM, as the functions of $\mu/\mu_0$ at the $\sqrt{s} = 8 \text{ TeV}$ LHC.
Figure 7: We take $c = 0.5$, $c' = 0.22$ and $\mu = \mu_0$. (a) The LO and NLO QCD corrected cross sections and the corresponding K-factors for the $pp \rightarrow Z^0H^0 + X$ process at the $\sqrt{s} = 14$ TeV LHC in both the SM and LHM as the functions of scale $f$. (b) The relative deviations of the cross sections in the LHM from those in the SM corresponding to Fig. 7(a) as the functions of scale $f$. (c) The LO and NLO QCD corrected cross sections at the $\sqrt{s} = 8$ TeV LHC in both the SM and the LHM as the functions of scale $f$. (d) The relative deviations corresponding to Fig. 7(c) as the functions of scale $f$. 
involving $Z_H$ boson, we take $\mu = \mu_0$, $f = 4$ TeV and $c' = 1/\sqrt{2}$ in Figs. 8(a,b,c,d), in which case the contributions from the $A_H$ exchange diagrams are vanished (see Eqs. (2.18)). We plot the LO and NLO QCD corrected cross sections and the corresponding K-factors as the functions of the parameter $c$ at the $\sqrt{s} = 14$ TeV and $\sqrt{s} = 8$ TeV LHC in Figs. 8(a) and (c), separately. The relative deviations of the cross sections in the LHM from those in the SM, $\delta(c) \equiv \frac{[\sigma_{LHM}(c) - \sigma_{SM}(c)]}{\sigma_{SM}(c)}$, are shown in Figs. 8(b) and (d). Figs. 8(a) and (c) show that the K-factors in the LHM and SM are beyond 1.29 for both the $\sqrt{s} = 14$ TeV and $\sqrt{s} = 8$ TeV LHC. We can see from Figs. 8(b,d) that the difference between the relative deviations of $\delta_{LO}(c)$ and $\delta_{NLO}(c)$ goes up with the increment of the mixing angle parameter $c$ in the range of $c \in [0.1, 0.5]$, and the LO and NLO deviations in the LHM and SM are all sensitive to the mixing angle parameter $c$. We see also that in the range of $c < 0.2$ the LO and NLO relative deviations between the two models are nearly the same for both the $\sqrt{s} = 14$ TeV and $\sqrt{s} = 8$ TeV LHC.

Eq. (2.6) tells us that the heavy photon mass $M_{A_H}$ mainly depends on the scale $f$ and the mixing angle parameter $c'$ between two $U(1)$ gauge fields, but is insensitive to the mixing parameter $c$. In order to investigate and discuss the contributions of $A_H$ exchange diagrams to the $Z^0H^0$ associated production, we present the LO and NLO QCD corrected cross sections and the corresponding K-factors as the functions of the mixing angle parameter $c'$ at the $\sqrt{s} = 14$ TeV and $\sqrt{s} = 8$ TeV LHC in Figs. 9(a) and (c), separately. In Figs. 9(a,b,c,d) we take $\mu = \mu_0$, $f = 4$ TeV and $c = 1/\sqrt{2}$, in this case there is no contribution from the $Z_H$ exchange diagrams (see Eq. (2.18)). The corresponding relative deviations of the cross sections in the LHM from those in the SM, $\delta(c') \equiv \frac{[\sigma_{LHM}(c') - \sigma_{SM}(c')]}{\sigma_{SM}(c')}$, are demonstrated in Figs. 9(b) and (d), respectively. We can see from Figs. 9(a) and (c) that the LO and NLO QCD corrected total cross sections in the LHM at the early and future LHC are obviously related to the mixing angle parameter $c'$ in the range of $c' \in [0.10, 0.65]$, and the K-factors in the LHM are sensitive to $c'$ in the range of $c' \in [0.10, 0.65]$. Figs. 9(b) and (d) demonstrate that the difference of $\delta_{LO}(c') - \delta_{NLO}(c')$ becomes smaller when $c'$ increases from 0.40 to 0.65,
Figure 8: We take $c' = 1/\sqrt{2}$, $f = 4\ TeV$ and $\mu = \mu_0$. (a) The LO and NLO QCD corrected cross sections and the corresponding K-factors for the $pp \to Z^0H^0 + X$ process at the $\sqrt{s} = 14\ TeV$ LHC in both the SM and LHM as the functions of parameter $c$. (b) The relative deviations of the cross sections in the LHM from those in the SM corresponding to Fig.8 versus parameter $c$. (c) The LO and NLO QCD corrected cross sections at the $\sqrt{s} = 8\ TeV$ LHC in both the SM and LHM as the functions of $c$. (d) The relative deviations corresponding to Fig.8(c) versus $c$. 
while in the range of \( c' \in [0.65, 0.90] \) the NLO and LO relative deviations, \( \delta_{NLO}(c') \) and \( \delta_{LO}(c') \), have almost the same values for the \( \sqrt{s} = 14 \text{ TeV} \) and \( \sqrt{s} = 8 \text{ TeV} \) LHC.

![Diagram](image)

Figure 9: We take \( c = 1/\sqrt{2}, f = 4 \text{ TeV} \) and \( \mu = \mu_0 \). (a) The LO and NLO QCD corrected cross sections and the corresponding K-factors for the \( pp \rightarrow Z^0 H^0 + X \) process at the \( \sqrt{s} = 14 \text{ TeV} \) LHC in both the SM and LHM as the functions of the parameter \( c' \). (b) The relative deviations of the cross sections in the LHM from those in the SM corresponding to Fig.9(a) versus parameter \( c' \). (c) The LO and NLO QCD corrected cross sections at the \( \sqrt{s} = 8 \text{ TeV} \) LHC in both the SM and LHM as the functions of \( c' \). (d) The relative deviations corresponding to Fig.9(c) versus parameter \( c' \).

As we know, the final \( Z^0 \) and \( H^0 \) bosons are unstable and can be detected experimentally via the subsequential leptonic decays of \( Z^0 \rightarrow \mu^+\mu^- \) and \( H^0 \rightarrow \tau^+\tau^- \). We employ the SM leptonic decay branch ratios of \( Z^0 \) and \( H^0 \) boson in further numerical calculations, i.e., \( Br(Z^0 \rightarrow \mu^+\mu^-) = 3.366\% \) and \( Br(H^0 \rightarrow \tau^+\tau^-) = 6.5\% \) \[^{28}\]. Since the transverse momentum distributions of \( \mu^+ \) and \( \tau^+ \) of the process \( pp \rightarrow Z^0 H^0 \rightarrow \mu^+\mu^-\tau^+\tau^- + X \) should be the same as those of \( \mu^- \) and \( \tau^- \) correspondingly, we present only those of \( \mu^- \) and \( \tau^- \).
We depict the LO and QCD NLO corrected transverse momentum distributions of final $\mu^-$ and the corresponding relative deviations of the cross sections in the LHM from those in the SM at the $\sqrt{s} = 14$ TeV LHC in Figs.10(a) and (b) separately, where we take $M_H = 125$ GeV, $c = 0.5$, $c' = 0.22$ and $f = 4$ TeV. All the curves in Figs.10(a) go down with the increment of the $\mu^-$ transverse momentum within the plotted $p_T^{\mu^-}$ range. The differential cross sections, $d\sigma_{\text{LO,NLO}}/dp_T^{\mu^-}$, and the corresponding relative deviations of the cross sections at the $\sqrt{s} = 14$ TeV LHC in the LHM from those in the SM as the functions of $p_T^{\mu^-}$ are shown in Figs.10(c) and (d), respectively. There we adopt again $M_H = 125$ GeV, $c = 0.5$, $c' = 0.22$ and $f = 4$ TeV. In Fig.10(c) we see that the curves for the LO and QCD NLO distributions of $p_T^{\mu^-}$ in both the SM and the LHM frameworks fall down when the transverse momentum $p_T^{\mu^-}$ goes up. Figs.10(a,c) demonstrate that the LO differential cross sections of $p_T^{\mu^-}$ and $p_T^{\tau^-}$ ($d\sigma_{\text{LO}}/dp_T^{\mu^-}$, $d\sigma_{\text{LO}}/dp_T^{\tau^-}$) are significantly enhanced by the QCD corrections. We can see from Fig.10(b) and Fig.10(d) that the corresponding relative deviations between the two models are significantly suppressed by the QCD NLO corrections, and in the ranges of $p_T^{\mu^-} > 130$ GeV and $p_T^{\tau^-} > 150$ GeV the QCD NLO corrected relative deviations can exceed 10%, separately.

Similar with Figs.10(a,b,c,d) we plot the corresponding distributions of final $\mu^-$ and $\tau^-$ in the $pp \to Z^0 H^0 \to \mu^+\mu^-\tau^+\tau^- + X$ process at the $\sqrt{s} = 8$ TeV LHC in Figs.11(a,b,c,d). From Fig.11(a) and Fig.11(c) we can see that for the $\sqrt{s} = 8$ TeV LHC all the curves for both the LO and QCD NLO distributions of $p_T^{\mu^-}$ and $p_T^{\tau^-}$ decrease with the increment of the corresponding transverse momentum, which are similar with the curves for the $\sqrt{s} = 14$ TeV LHC. Again, we see that both the LO differential cross sections of $p_T^{\mu^-}$ and $p_T^{\tau^-}$ ($d\sigma_{\text{LO}}/dp_T^{\mu^-}$, $d\sigma_{\text{LO}}/dp_T^{\tau^-}$) are significantly enhanced by the QCD corrections. We can see from Fig.11(b) and Fig.11(d) that the relative deviations between the two models are significantly suppressed by the QCD NLO corrections, and in the ranges of $p_T^{\mu^-} > 155$ GeV and $p_T^{\tau^-} > 170$ GeV the QCD NLO corrected deviations at the $\sqrt{s} = 8$ TeV LHC can exceed 10%, separately.
Figure 10: The LO and NLO QCD corrected distributions of the transverse momenta of final leptons and the corresponding K-factors for the $pp \rightarrow Z^0 H^0 \rightarrow \mu^+ \mu^- \tau^+ \tau^- + X$ process at the $\sqrt{s} = 14$ TeV LHC in both the SM and LHM, where we take $\mu = \mu_0$, $c = 0.5$, $c' = 0.22$ and $f = 4$ TeV. (a) The distributions of $p_T^\mu$. (b) The relative deviations of the cross sections in the LHM from those in the SM corresponding to Fig.10(a) as the functions of $p_T^\mu$. (c) The distributions of $p_T^\tau$. (d) The relative deviations corresponding to Fig.10(c) as the functions of $p_T^\tau$. 

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Figure 11: The LO and NLO QCD corrected distributions of the transverse momenta of final leptons and the corresponding K-factors for the $pp \to Z^0 H^0 \to \mu^+ \mu^- \tau^+ \tau^- + X$ process at the $\sqrt{s} = 8$ TeV LHC in both the SM and LHM, where we take $c = 0.5$, $c' = 0.22$ and $f = 4$ TeV. (a) The distributions of $p_T^\mu$. (b) The relative deviations of the differential cross sections in the LHM from those in the SM corresponding to Fig.11(a) as the functions of $p_T^\mu$. (c) The distributions of $p_T^\tau$. (d) The relative deviations corresponding to Fig.11(c) as the functions of $p_T^\tau$. 
V. Summary

In this paper we investigate the phenomenological effects induced by the new heavy neutral
gauge bosons in the LHM up to QCD NLO on the $Z^0H^0$ associated production at the early
($\sqrt{s} = 8$ TeV) and future ($\sqrt{s} = 14$ TeV) LHC. We study the dependences of the LO
and NLO QCD corrected cross sections on the factorization/renormalization scale $\mu$, the
LHM parameters $c$, $c'$ and $f$, and present the LO and NLO QCD corrected distributions
of the transverse momenta $p_T^\mu$ and $p_T^\tau$. It demonstrates that the new neutral gauge bosons
could induce significant discrepancies to the kinematic observables from the standard model
predictions for this process at both LO and up to QCD NLO. Our results show that when
we take the $c = 0.5$, $c' = 0.22$, $f = 4$ TeV and $\mu = \mu_0$, the effects from the heavy neutral
gauge boson interactions can make the relative deviations to be about 12.83% and 10.37%
at the LO and up to QCD NLO, respectively. We find that the QCD corrections at the
$\sqrt{s} = 14$ TeV LHC can obviously make the cross section being mildly related to the $\mu$ scale,
and significantly enhance the differential cross sections of the transverse momenta of the
final decay products $\mu$ and $\tau$. We also find the LO relative deviations of the integrated cross
sections are significantly suppressed by the NLO QCD corrections. We conclude that the
precision measurement of the $Z^0H^0$ associated production process at the LHC could provide
the clue of the LHM physics.

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