Concatenated Permutation Codes for Indoor MIMO Visible Light Communication

Oluwafemi Kolade, Ling Cheng

Abstract—In this paper, concatenated permutation codewords are used to improve the data rate of permutation-aided space-time shift keying in multiple input multiple output (MIMO) visible light communication. Soft-decision detection schemes are then designed for the receiver and compared with the maximum likelihood (ML) detection method. Bit error rate (BER) results show the soft decision detection algorithm is able to detect the transmitted information without knowledge of the channel state information. The BER results also show a close match with the ML detection in some codebooks.

Index Terms—Assignment problem, Permutation codes, Soft-decision, Space-time shift keying, Visible light communication.

I. INTRODUCTION

The concept of using existing illumination for Visible Light Communication (VLC) is becoming attractive for low-cost communication with potential to deliver information to the last mile. Since lighting infrastructures usually consist of multiple Light Emitting Diodes (LEDs), a natural Multiple Input Multiple Output (MIMO) setup is established. MIMO schemes provide several advantages such as higher data rates, spatial diversity and improvement of the error rate performance using the receiver diversity. For example, Space-Time Block Codes (STBC) [1] provide transmit diversity such as the Alamouti STBC [2]. The single receiver’s Maximum Likelihood (ML) detection method requires the Channel State Information (CSI) but with reduced detection complexity.

Permutation Modulation (PM) [3] is one of several methods of generating space-time codes used for antenna transmit matrices [4]-[6] and is preferred mainly because of its ability to mitigate the effect of Inter-Channel Interference (ICI). In [4], information bits are mapped on to PM-aided matrices which are then differentially modulated. In [5], the PM-aided matrices are used to design antenna dispersion matrices. The main drawback of PM-aided MIMO schemes is that they transmit less information when compared with conventional MIMO schemes such as generalized spatial modulation [7], [8]. Moreover, decoding PM-aided matrices becomes too complex as the number of antennas increase. A low-complexity decoder in [9], [10] combines optimization algorithms in [11] and [12] to decode permutation block codes. A similar concept in [8] decodes PM-aided transmit matrices at the receiver with and without the knowledge of the CSI at the receiver. In order to increase the information rate of permutation-coded systems, concatenated permutation block codes [13] have been used while simultaneously increasing the code’s minimum distance. However, the concatenated permutations presented in [13] do not construct equal weight matrices which is required for the MIMO scheme presented in this paper.

In this paper, a higher rate PM-aided MIMO VLC scheme is presented at the transmitter and low-complexity detection schemes at the receiver. The first contribution of this paper introduces a class of concatenated permutation codes which generate equal weight space-time codes in order to increase the number of bits conveyed by the transmit matrix of the PM-aided MIMO VLC scheme. The second contribution designs low-complexity, iterative soft-decision (SD) detection methods using optimization algorithms. The SD methods are capable of decoding the likely transmitted bits without knowledge of the channel matrix. The BER expression for the ML decoder is also derived with the SD decoders matching the ML performance for some setups.

The following notations are used in this paper. $\text{tr}[\cdot]$ is the trace operation, $\| \cdot \|_F$ is the Frobenius norm, $(\cdot)^H$ is the conjugate transpose operation and the Q-function is defined as $Q(r) = \frac{1}{\sqrt{2\pi}} \int_{r}^{+\infty} \exp\left(-\frac{x^2}{2}\right) dx$.

II. SYSTEM MODEL FOR PERMUTATION-AIDED MIMO FOR VLC

A. Permutation-Aided STSK Scheme

Consider a permutation codebook $C$ with $Q$ codewords. Each codeword $c_q$ at row $q$ ($q = 1, 2, \ldots, Q$) is a row vector consisting of a unique permutation of integers $(c_1, c_2, \ldots, c_L)$. The information bits to be transmitted are uniquely mapped onto one of $Q$ permutation matrices. Each matrix $P_q$ is equivalent to the codeword at row $q$ and the index of the non-zero element in each row of $P_q$ corresponds to each integer in $c_q$. Each integer indicates the column with a 1 while other columns are 0. We define the weight $w$ of the matrix as the number of non-zero elements in its rows and columns. Hence, a $\{0, 1\}^{L \times L}$ permutation matrix $P_q$, having columns and rows with weight $w = 1$ is transmitted within a block of multiple time slots. Using $P_q$, an activation pattern per transmit block is created with each non-zero element representing the activated LED index. As a result, each transmit block conveys $\log_2 Q$ bits from the incoming message bits. The power constraint of
the transmitted block is given as \( \text{tr} [P_q H P_q^H] \). Additional \( \log_2 M \) information bits, mapped onto a unipolar \( M \)-ary modulation scheme such as Pulse Amplitude Modulation (PAM) can be transmitted over the active LED in order to increase the information rate. Hence, a total of \( \log_2 (Q \cdot M) \) bits can be transmitted at each transmit block. The resulting transmitted matrix \( S = [s_{ij}] \)

\[
S = a \cdot P_q, 
\]

represents a space-time code where \( a \) takes any of the \( M \) intensities

\[
I_m = \frac{2I}{w(M + 1)} m \quad \text{for} \quad m = 1, 2, \ldots, M, 
\]

where \( I \) is the mean optical transmit power. The \( M \)-PAM symbol is transmitted over each activated LED index such that

\[
\sum_{i=1}^{L} s_{ij} = \sum_{j=1}^{L} s_{ij} = a. 
\]

As a result, the number of bits per symbol transmitted at each sampling period is given as \( \frac{\log_2 (Q \cdot M)}{L} \).

**B. MIMO VLC Channel Model**

A line-of-sight (LOS) channel model between \( L \) LEDs as transmitters and \( L \) photodiodes (PD) as receivers are assumed. The transmitters employ intensity modulation while direct detection is employed at each receiver. Assuming Lambertian LEDs and PDs, the channel gain between an LED and a PD detection is employed at each receiver. Assuming Lambertian field of view of the receiver. The received matrix at the \( \Phi \) where

\[
\Theta \in \mathbb{C}^{R \times \Phi}, \quad \text{transmitters and LEDs and PDs, the channel gain between an LED and a PD is taken.}
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**C. Concatenated Permutations for STSK**

In order to increase the data rate of the permutation-aided scheme, multiple permutation codewords are combined to activate more than one LED at each transmit time. First, we denote \( c^w_q \) as the codeword in codebook \( \Pi \) (\( 1 \leq \Pi \leq w \)) at row \( q \) and \( P_w^q \) is the equivalent permutation matrix. The concatenated codewords \( (c^1_q, c^2_q, \ldots, c^w_q) \) are chosen such that the Hamming distance between any two codewords at row \( q \) of the codebook

\[
d_m(c^1_q, c^2_q) = L. 
\]

This means any \( w \) concatenated codewords differ in as many places as the length of a codeword. The number of codewords with Hamming distance \( L \) from a given codeword \( c^m_q \in C_w \) is

\[
|K| = L! \sum_{k=0}^{L} \frac{-1}{k!}. 
\]

and the codewords can be concatenated in order to produce unique codeword transmit matrices. Each matrix has a weight \( w \) on the columns and rows, same as the number of concatenated codewords. The transmitted matrix of the concatenated codewords is given as

\[
P_q = P^1_q + P^2_q + \cdots + P^w_q. 
\]

A simple construction of codewords which satisfy (7) is achieved by selecting from \( L \)-order Latin squares [16], each consisting of an \( L \times L \) integer matrix, derived from a set of \( L \) codewords. In each matrix, an integer occurs only once in each row and column and each row and column is a permutation. The permutations that form the Latin square can be constructed for example, by performing a cyclic shift on a codeword. For example, in codebook \( C_1 \), a cyclic shift on a codeword \( c^1_q \) of \( (c^1_q, c^2_q, \ldots, c^L_q) \) produces \( c^1_q = (c^2_q, \ldots, c^{L-1}_q, c^0_q) \). Assuming \( c^1_q \) is (2314) and is cyclically shifted to (3142), \( c^1_q \) and \( c^2_q \) satisfy (7) and the concatenated codewords produce a matrix

\[
P_q = \begin{bmatrix}
0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 
\end{bmatrix},
\]

with \( w = 2 \). This enables \( w \) unique LED combinations in each time slot in a block using codewords which belong to one of \( w \) codebooks \( \{C_1, C_2, \ldots, C_w\} \) for \( 1 < w < L \). The message symbols are then mapped onto \( w \)-tuple codewords \( (e^1_q, e^2_q, \ldots, e^w_q) \) which belong to the same row \( q \) in \( C_1, C_2, \ldots, C_w \) respectively.

The concatenated codewords can be added to the conventional \( w = 1 \) codewords in order to increase the number of codewords. Hence, the total number of codewords achievable is

\[
Q = Q_1 + Q_2 + \cdots + Q_{L-1},
\]

where \( Q_w \) is the number of codewords available for a given weight \( w \). An example is shown in Table I where the bits per symbol is increased to \( 2L+1 \) without increasing the size of \( L \). The increase in the number of active indices also enables higher order constellation points to be modulated on top of
TABLE I: Mapping between modulation scheme and codes with $w = 1$ and $w = 2$.

| Input Bits | $Q = 32$, $M = 1$ | Codewords | $m$ | $Q = 32$, $M = 2$ | Codewords | $m$ |
|------------|-------------------|-----------|-----|-------------------|-----------|-----|
| 00000      | 1234              | 1         | 1234| 1                 |
| 00001      | 1243              | 1         | 1243| 2                 |
| 00010      | 1243              | 1         | 1243| 1                 |
| 00011      | 1243              | 1         | 1243| 2                 |
| ...        | ...               | ...       | ... | ...               | ...       | ... |
| 11100      | 1234, 2143        | 1         | 1234| 1                 |
| 11101      | 1324, 2413        | 1         | 1324| 2                 |
| 11110      | 3241, 4132        | 1         | 3241| 1                 |
| 11111      | 3124, 4312        | 2         | 3124| 2                 |

the active LED index, hence increasing the data rate when compared to the $w = 1$ permutation-aided scheme. Using (1), the transmitted matrix is given as

$$s_{ij} = \sum_{j=1}^{M} s_{ij} = w \cdot a. \quad (11)$$

Given equiprobable symbols with unique one-to-one mapping onto codewords, the pairwise error probability of receiving a codeword signal $\hat{S}$ when $S$ is transmitted is given as

$$P(\hat{S} \rightarrow S|H) \leq Q\left(\frac{E_s}{2N_0}||H(S - \hat{S})||_F^2\right). \quad (12)$$

Within a transmit block, the symbol to noise ratio is $E_s/N_0 = (c_l^2)/T_s$ for sampling time $T_s$ and an optical-to-electrical conversion coefficient $r$. By using the union bound method, the upper bound of the Bit Error Rate (BER) in (13) compares all possible $M \cdot Q$ codeword matrices and intensity combinations. The term $d_{m_1}(q_{m_1}, q_{m_2})$ denotes the Hamming distance between the received bits $b_{m_2}$ when actually, $b_{m_1}$ was transmitted.

III. PROPOSED OPTIMIZATION ALGORITHMS FOR
SOFT-DECISION DETECTION

From the received matrix $Y \in [y_{ij}]$ at each block, the detector processes $\hat{Y} \in [\hat{y}_{ij}]$ such that $\hat{y}_{ij} = -y_{ij}$. For $w = 1$, the decoder finds the corresponding codeword $c$ at row $q$ that produces the cost

$$g_1 = \sum_{i=1}^{L} \sum_{j=1}^{L} \hat{y}_{ij}s_{ij}, \quad (14)$$

that minimizes (14). For $1 < w < L$, the decoder finds the set of $w$-tuple codewords $\{c_1^w, c_2^w, \ldots, c_w^w\}$ which belong to the same row $q$ in $\{C_1, C_2, \ldots, C_w\}$ that produce the cost

$$g_1 = \arg\min_{c \in C} \left(\sum_{i=1}^{L} \sum_{j=1}^{L} \hat{y}_{ij}s_{ij} (c_1) + \sum_{i=1}^{L} \sum_{j=1}^{L} \hat{y}_{ij}s_{ij} (c_2) + \ldots + \sum_{i=1}^{L} \sum_{j=1}^{L} \hat{y}_{ij}s_{ij} (c_w), \quad \text{for } q = 1, 2, \ldots, Q, \right) \quad (15)$$

that minimize (15). Here, $s_{ij}(c_q)$ values are the elements in $S_q$, produced by a codeword at row $q$. Each codeword $c_1^q, c_2^q, \ldots, c_w^q$ belongs to codebooks $C_1, C_2, \ldots, C_w$ respectively.

A. Brute Force Soft-Decision Receiver

The brute force (BF) of the soft-decision decoders solves

$$g_{BF} = \arg\min_{c \in C} \left(\sum_{i=1}^{L} \sum_{j=1}^{L} \hat{y}_{ij}s_{ij} (c_q), \quad \text{for } q = 1, 2, \ldots, Q, \right) \quad (16)$$

where $s_{ij}(c_q)$ is each element in the permutation matrix produced by a codeword $c$. The BF SD receiver ranks the costs $g_1, g_2, \ldots, g_Q$ and chooses the lowest cost $g_{BF}$ that produces the permutation matrix $P_{BF}$ and whose corresponding codeword $c_{BF} \in C$.

B. Branch and Bound

The branch and bound algorithm (B&B) [17] can be used to solve an $w = 1$ received matrix in (14) using the tree-based method. $L + 1$ levels defined as $\epsilon_1, \epsilon_2, \ldots, \epsilon_L$ are created from $\hat{Y}$. The initial node is a permutation of all integers $\{c_1, c_2, \ldots, c_L\}$. The surviving node at $\epsilon_1$ is determined by finding the node that satisfies

$$\hat{\delta}_{\epsilon_1} = \sum_{i=1}^{L} \min_{1 \leq j \leq L} (\hat{y}_{ij}). \quad (17)$$

At $\epsilon_2$, $L - \epsilon_1$ branches are created and the surviving node satisfies

$$\hat{\delta}_{\epsilon_2} = \sum_{i=2}^{L} \min_{1 \leq j \leq L} (\hat{y}_{ij}). \quad (18)$$

This process is then repeated until the $L - 1$ node and at such point, a permutation codeword is created.

C. Iterative Soft-Decision Detection

The iterative decoder finds the maximum cost of $\hat{Y}$ that produces a codeword $c_q^w$ in each codebook $\{C_1, C_2, \ldots, C_w\}$ at each iteration $e$. Assuming $\hat{c}_1^1$ is the decoded codeword for $C_1$ and $\hat{c}_2^2$ is decoded for $C_2$, then the decoder chooses between the codeword pairs $\{\hat{c}_1^1, \hat{c}_1^2\}$ and $\{\hat{c}_2^1, \hat{c}_2^2\}$ having the highest cost.

At iteration $e = 1$, the Hungarian algorithm finds the minimum cost $g_1$ using steps described in [11]. This produces a row-column pair $\{(1, \hat{c}_1), (2, \hat{c}_2), \ldots, (Q, \hat{c}_L)\}$ which corresponds to a codeword $\hat{c} = (\hat{c}_1 \hat{c}_2 \ldots \hat{c}_L)$ with the minimum cost $g_1$. If $\hat{c} \notin \{C_1, C_2, \ldots, C_w\}$, then at $e = 2, 3, \ldots, Q$, are found using Murty’s [12]. The solution matrix from the Hungarian algorithm at $e = 1$ is used to create $L - 1$ nodes or subsets $U_1, U_2, \ldots, U_{L-1}$. The nodes are created by partitioning the solution matrix at $e = 1$ such that $U_1 = \{(1, j_1), \ldots, \}, \ldots, U_{L-1} = \{(1, j_1), \ldots, (L-1, j_{L-1})\}$. The items with a bar ($\bar{x}$) in $U_1, U_2, \ldots, U_{L-1}$ are replaced with $\infty$ while the other items are excluded. By solving for the minimum cost of each derived node, the next assignment is then derived from the node with the lowest cost. This process can be iterated from $e = 2, 3, \ldots, Q$ until $\hat{c} \in \{C_1, C_2, \ldots, C_w\}$. 

BER ≤ \frac{1}{M \cdot Q \log_2(M \cdot Q)} \sum_{m_1=1}^{M} \sum_{q_1=1}^{Q} \sum_{m_2=1}^{M} \sum_{q_2=1}^{Q} d_m(b_{m_1}^{q_1}, b_{m_2}^{q_2}) Q \left( \sqrt{\frac{E_b}{2N_0}}|H(I_{m_1}S^{q_1} - I_{m_2}S^{q_2})|^2 \right). \quad (13)

IV. SIMULATION RESULTS

The error rate performance of the space-time schemes are evaluated for an indoor environment. The transmitter units are assumed to be placed at the top of the room while the receiver units are placed on a table in the room, 0.75 m from the transmitters. The channel matrix used for the numerical simulations is adopted from [8]

\[ H = \begin{bmatrix}
1.0708 & 0.9937 & 0.9937 & 0.9226 \\
0.9937 & 1.0708 & 0.9226 & 0.9937 \\
0.9937 & 0.9226 & 1.0708 & 0.9937 \\
0.9226 & 0.9937 & 0.9937 & 1.0708 
\end{bmatrix} \times 10^{-4}, \]

with values obtained using (4) with 0.2m and 0.1m spacing between the transmitters and receivers respectively. \( A_{pd} \) is assumed to be unity while both \( \Phi_2 \) and \( \Psi \) are set to 15° [18], [19].

A. BER Analysis

The simulated BER of the ML performance of the scheme with perfect knowledge of the CSI is compared with the theoretical BER defined in (13). For the scheme using the concatenated codebook, equal total transmit power is maintained for all codewords by transmitting intensity \( I \) over each active transmitter. We shorten the simulation properties by describing the schemes using \( P(L, Q, M, w) \).

The BER performance of the ML decoder is also compared with the BER of the SD decoders. In the results, the SD decoders decode the channel output directly, hence the CSI is not required. In Fig. 1 the B&B as an SD decoder without CSI has close BER performance with the BF performance of the SD detection. B&B without CSI also shows similar performance with a 3 dB difference with the ML performance. While the SD which uses the BF detection matches the ML BER performance, the SD decoder differs with 3 dB from the ML due to no CSI. The theoretical BER are also seen to match the ML performance especially at higher SNR regions as shown in Figs. 1-3.

B. Data Rate Analysis

The combination of codebooks with different weights makes more codewords available in order to increase the bits per symbol. Consider the codebook with \( L = 4 \) which has 24 \((L!)\) matrices for \( w = 1 \). An exhaustive search of unique codewords that satisfy (7) produces 90 matrices for \( w = 2 \). For the same \( L, 24 \) matrices are found for \( w = 3 \). If the codebooks with the 3 weights are combined, 7 bits per symbol can be transmitted by each matrix compared to the possible 4 bits per symbol achievable for the \( w = 1 \) codebook.

C. Complexity Analysis of Soft-Decision Decoders

The B&B decoder in Fig. 1 uses a complexity of \( O(L^3 \ln^2(L)) \) while the iterative soft-decision decoder in Figs. 1 and 2 uses a worst case complexity of \( O(L^3) \) [20]. The BF soft-decision decoder in Figs. 1 and 3 operates a \( O(L \cdot Q!) \) [21] complexity in order to find the optimal cost that corresponds to the decoded codeword.

V. CONCLUSION

Concatenated permutations are used to increase the data rate of the PM-aided MIMO scheme in VLC by combining...
permutation matrices of different weights. Low-complexity, SD techniques are also used to detect the transmitted signals without the knowledge of the CSI. The results show the soft-decision decoder can match the ML decoder in some codebooks and the decoding complexities are also analyzed. Future work will require low-complexity code construction techniques and decoding techniques with close performance to the ML performance.

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