BULK COUPLINGS TO NONCOMMUTATIVE BRANES

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ABSTRACT. We propose a way to identify the gauge invariant operator in noncommutative gauge theory on a D-brane with nonzero B field which couples to a specific supergravity mode in the bulk. This uses the description of noncommutative gauge theories in terms of ordinary $U(\infty)$ gauge theories in lower dimensions. The proposal is verified in the DBI approximation. Other authors have shown that the proposal is also consistent with explicit string worldsheet calculations. We comment on implications to holography.

1. Introduction and Overview

The study of coupling of bulk modes with usual D-branes have played a crucial role in our understanding of black holes, holography and many other features of string theory. Recent developments indicate that D-branes with constant NSNS $B$-fields probe a very fruitful corner of string theory. The low energy theory of such D-branes is noncommutative gauge theory (NCGT)\(^1,2,3\). In this talk I will discuss how to couple such D-brane to bulk fields and its implications to holography, based on work with S.J. Rey and S.P. Trivedi.

Bulk modes must, of course, couple to gauge invariant operators. However in NCGT, gauge transformations and translations are intertwined. For example, consider an adjoint field $\phi(x)$. Under a gauge transformation with gauge parameter $\Lambda(x) = b_i x^i$ one has

\begin{equation}
\phi(x) \to e^{ib_i x^i} \star \phi(x) \star e^{-ib_i x^i} = \phi(x^i + \theta^{ij} b_j)
\end{equation}

This shows that such a gauge transformation is equivalent to a translation by the parameter $\theta^{ij} b_j$ and implies that there are no gauge invariant operators which are local in position space.

Since the theory has translation invariance, gauge invariant operators with definite momentum must exist. Such operators were constructed by Iso, Ishibashi, Kawai and Kitazawa (IIKK)\(^4\). These are fourier transforms of open Wilson lines, with the separation between the base points being proportional to the momentum. In fact, noncommutative gauge theories have supergravity duals\(^5,6\) - this implies that there must be operators of definite momentum which are dual to the supergravity modes\(^7,9\). In\(^8\) it was proposed that these operators are in fact the
operators constructed by IIKK and the fact that their size increases with momentum is reflected in the dual supergravity. In [9] it was proposed that the relevant operators are in fact Wilson lines which are straight, with operator insertions at the end point and in [10] other classes of open Wilson lines with operator insertions along it were considered. Various properties of these operators have been studied in [14]. Correlation functions of Wilson tails have an interesting universal high energy behavior [8, 11, 12]. It was, however, unclear how to obtain the precise form of such Wilson lines with operator insertions which couples to a specific supergravity mode.

In [13] we proposed that the way to do this is to use the construction of NCGT from ordinary $U(\infty)$ gauge theories in lower dimensions or matrix models [15, 16] which was used to write down these operators in the first place. The latter is the theory of a large number of lower dimensional branes with no $B$ field. Suppose we know the linearized couplings of a set of ordinary $D_p$ branes to supergravity backgrounds. Then the proposal is to use the above construction to find the couplings of these backgrounds to noncommutative $D(p+2n)$ branes with noncommutativity in $2n$ of the directions. We find that the resulting operators are straight Wilson lines, which we call Wilson tails, with local operators smeared along them. By the nature of this construction we obtain the operator in the $\Phi = -B$ description [8]. Such Wilson tails with smeared operators were constructed in [17] and argued to be the correct operators from a rather different point of view. Generalized star products [18] which have appeared in various contexts in NCGT make an appearance here as well.

The form of the coupling of supergravity modes to a large number of ordinary $D_p$ branes is not known in general - it is known in the low energy limit [19] or in the DBI approximation [20, 19]. The couplings are consistent with the symmetrized trace prescription of [21]. To test our proposal we work in the DBI approximation and construct the couplings to noncommutative branes. For a single brane in the DBI approximation, one can also write down the coupling in terms of ordinary gauge fields and the $B$ field on the brane by standard methods. For dilaton couplings, we show, to second order in the noncommutative gauge potential but to all orders in the noncommutativity parameter, that the coupling we propose is in fact identical to that in terms of ordinary gauge fields and transformed by the Seiberg-Witten map. It has been subsequently shown that the proposal is also consistent with worldsheet calculations of couplings to other NSNS fields in the low energy limit [22], and to couplings of RR fields [23].

2. NCGT from large-N Yang-Mills

The fact that space-time can be encoded in $N = \infty$ matrix models living at a single point has been known for some time: this is the basis for Eguchi-Kawai models [24]. In the twisted Eguchi-Kawai model [25] the space-time which emerges is in fact noncommutative. However since planar diagrams of commutative and noncommutative gauge theories are identical, the twisted model indeed describes Yang-Mills theory at large $N$. In the present context, we want to describe finite $N$ noncommutative gauge theories in terms of $N = \infty$ matrix models, or $N = \infty$ gauge theories in lower dimensions. The fact that the twisted Eguchi-Kawai model has noncommutative space built in it comes as a bonus [15]. In fact this is how branes arise in matrix models [26], both in the BFSS [27] and IKKT [28] versions.
Consider a $U(\infty)$ ordinary gauge theory in $(p + 1)$ dimensions with the usual gauge fields $A_\mu(\xi), \mu = 1, \cdots p + 1$ and $(9-p)$ scalar fields $X^I(\xi), I = 1, \cdots (9-p)$ in the adjoint representation, together with their fermionic partners. In this paper we will restrict ourselves to bosonic components of operators. Consequently, fermions will not enter the subsequent discussion. The bosonic part of the action is

\begin{equation}
S = \text{Tr} \int d^{p+1}\xi \left[ F_{\mu\nu} + D_i X^i D^\mu X^j g_{IJ} + [X^I, X^J][X^K, X^L]_{g_{IK}g_{JL}} \right]
\end{equation}

where $g_{IJ}$ are constants and the other notations are standard. Boldface has been used to denote $\infty \times \infty$ matrices.

The action has a nontrivial classical solution

\begin{equation}
X^i(\xi) = x^i (i = 1, \cdots 2n); \quad X^a = 0 (a = 2n + 1 \cdots 9 - p); \quad A_\mu = 0
\end{equation}

where the constant (in $\xi$) matrices $x^i$ satisfy

\begin{equation}
[x^i, x^j] = i\theta^{ij}I
\end{equation}

The antisymmetric matrix $\theta^{ij}$ has rank $p$ and $I$ stands for the unit $\infty \times \infty$ matrix. The inverse of the matrix $\theta^{ij}$ will be denoted by $B_{ij}$.

The idea is then to expand the various fields as follows.

\begin{equation}
C_i = B_{ij} X^j = p_i + A_i; \quad X^a = \phi^a; \quad A_\mu = A_\mu
\end{equation}

where $p_i = B_{ij} x^j$. We will expand any matrix $O(\xi)$ as follows

\begin{equation}
O(\xi) = \int d^{2n}k \exp[i\theta^{ij}k_ip_j] O(k, \xi)
\end{equation}

where $O(k, \xi)$ are ordinary functions. Regarding these $O(k, \xi)$ as fourier components of a function $O(x, \xi)$, where $x^i$ are the coordinates of a $2n$ dimensional space we then get the following map between matrices and functions.

\begin{equation}
\text{Tr}O(\xi) = \frac{1}{(2\pi)^n}[\text{Pf} \ B] \int d^{2n}x \ O(x, \xi)
\end{equation}

The product of two matrices $O_1(\xi)$ and $O_2(\xi)$ is then mapped to a star product

\begin{equation}
O_1(\xi)O_2(\xi) \rightarrow O_1(x, \xi)* O_1(x, \xi),
\end{equation}

\begin{equation}
O_1(x, \xi)* O_2(x, \xi) \equiv \exp \left[ \frac{\theta^{ij}}{2i} \frac{\partial^2}{\partial s^i \partial s^j} \right] O_1(x + s, \xi)O_2(x + t, \xi) \mid_{s=t=0}
\end{equation}

With these rules, one can easily verify that

\begin{align}
F_{\mu\nu} &\rightarrow \partial_\mu A_\nu - \partial_\nu A_\mu - iA_\mu * A_\nu + iA_\nu * A_\mu \equiv F_{\mu\nu} \\
D_\mu X^i &\rightarrow \theta^{ij}(\partial_\mu A_j - \partial_j A_\mu - iA_\mu * A_j + iA_j * A_\mu) \equiv \theta^{ij} F_{\mu j} \\
D_\mu X^a &\rightarrow \partial_\mu \phi^a - iA_\mu * \phi^a + i\phi^a * A_\mu \equiv D_\mu \phi^a \\
[X^i, X^j] &\rightarrow i\theta^{ij}(E_{kl} - B_{kl}) \\
[X^i, X^a] &\rightarrow i\theta^{ij}(\partial_j \phi^a - iA_j * \phi^a + i\phi^a * A_j) \equiv i\theta^{ij} D_j \phi^a
\end{align}

where we have defined

\begin{equation}
F_{ij} = \partial_i A_j - \partial_j A_i - iA_i * A_j + iA_j * A_i
\end{equation}

In the above equations the quantities appearing in the right hand side are ordinary functions of $(x, \xi)$.

The action $[\Sigma]$ becomes the action of $U(1)$ noncommutative gauge theory in the $p + 2n + 1$ dimensions spanned by $x, \xi$. The noncommutativity is entirely
in the $2n$ directions. In addition to the gauge fields we also have $(9 - p - 2n)$ “adjoint” scalars $\phi^a$. The gauge field appears in the combination $F_{AB} - B_{AB}$ where $B_{AB}$ is an antisymmetric matrix whose $(ij)$ components are $B_{ij}$ and the rest zero. This corresponds to a specific choice of “description” in the N CYM theory \[3\]. Furthermore the upper and lower indices of various quantities some contracted with the “open string metric” whose components in the noncommutative directions are

$$G^{ij} = -\theta^{ik} g_{kl} \theta^{lj}$$

The components of the open string metric in the commutative directions are the same as the original metric $g_{ab}$. Finally the coupling constant which appears in front is the open string coupling $G_s$ which is related to the closed string coupling $g_s$ by

$$G_s = g_s \left( \frac{\det(G - B)}{\det(g + B)} \right)^{1/2} = g_s \left( \frac{\det B}{\det g} \right)^{1/2}$$

It may be also easily verified that

$$\frac{1}{G - B} = -\theta + \frac{1}{g + B}$$

(Recall that $\theta^{-1} = B$ as matrices.)

It is straightforward to extend the above construction to obtain a nonabelian noncommutative theory. The classical solution which one starts with is now

$$X^i(\xi) = x^i \otimes I_M$$

where $I_M$ denotes the unit $M \times M$ matrix. Now the various $\infty \times \infty$ matrices map on to $M \times M$ matrices which are functions of $x$, in addition to $\xi$. With this understanding the formulae above can be almost trivially extended. The star product would now include matrix multiplication and the map for the trace becomes

$$\text{Tr} O(x, \xi) = \frac{1}{(2\pi)^n} \text{Pf} B \int d^{2n} x \text{ tr} O(x, \xi)$$

where $\text{tr}$ now denotes trace over $M \times M$ matrices. Instead of obtaining a $U(1)$ noncommutative theory one now obtains a $U(n)$ noncommutative theory.

### 3. Open Wilson lines

Consider the following gauge invariant operator in the $Dp$-brane theory \[4\]

$$W(C, k) = \int d^{p+1} \xi \lim_{M \to \infty} \text{Tr} \left[ \prod_{n=1}^M U_j \right] e^{ik \cdot \xi^n}$$

$$U_j = \exp \left[ i \hat{\mathcal{G}} \cdot (\hat{\Delta} d)_n \right]$$

where $\Delta d_n$ denotes the $n$-th infinitesimal line element along the contour $C$. The momentum components $k_n$ along the commutative directions appear explicitly in \[3.1\], while the components along the noncommutative directions $k^i$ are given by

$$k_i = B_{ji} d^j$$

where $d^j$ are the components of the vector $\vec{d} = \sum_{n=1}^M \hat{\Delta} d$. Applying the above dictionary it is easy to see that this operator becomes the following expression in
terms of the noncommutative gauge field

\[ W(k, C) = \int d^{d+1}x \text{tr} \left[ P_e \exp \left[ i \int_C dy A^A(x + y(\lambda)) \right] * e^{ik B x^B} \right] \]

where we have used the indices \( A, B = 1, \ldots, (p+2n+1) \) to denote all the directions collectively. The trace in \( (\text{3.3}) \) is over the nonabelian gauge group. \( \lambda \) is a parameter that increases along the path. In our conventions the path ordering is defined so that points at later values of \( \lambda \) occur successively to the left. Equation (3.2) implies that the end points of the contour are separated by an amount \( \Delta x^A = k_B \theta^{BA} \). Clearly the separation is nonzero only along the noncommutative directions.

In a similar way consider the operator

\[ O(k) = \int d^{d+1}x e^{ik \cdot \xi} \text{Tr} \left[ e^{ik \cdot X} O(X, A, \xi) \right] \]

where \( O(X, A, \xi) \) is some operator in the \( Dp \) brane gauge theory which transforms according to the adjoint representation. In terms of noncommutative gauge fields and star products, this becomes

\[ \hat{O}(k) = \int d^{d+1}x \text{tr} O(x + k \cdot \theta) * P_e \exp[i \int_0^1 d \lambda \, k_A \theta^{AB} A_B(x + k \cdot \theta \lambda)] * e^{ik \cdot x}. \]

The contour is now a \textit{straight} path transverse to the momentum along the direction, \( \eta^A = k_B \theta^{BA} \). \( O(x + k \cdot \theta) \) is a local operator constructed from the fields which is inserted at the endpoint of the path, and \( (k \cdot \theta)^A \equiv k_B \theta^{BA} \). This is the operator defined in [9]. We will call the straight Wilson line a Wilson tail.

The operator \( O(X, A, \xi) \) can be a composite operator made of field strengths, \( F_{\mu \nu} \), the covariant derivatives of the scalar fields \( D_{\mu} X^I \) and \([X^I, X^J]\). That is

\[ O(X, A, \xi) = \prod_{\alpha=1}^n O_{\alpha}(X, A, \xi) \]

where each of the \( O_{\alpha} \) denotes a \( F_{\mu \nu} \), \( D_{\mu} X^I \) or a \( [X^I, X^J] \). A symmetrized trace (denoted by the symbol “STr”) of the expression in (3.4) is then defined as follows. Imagine expanding the exponential in \( e^{ik \cdot X} \). For some given term in the exponential we thus have a product of a number of \( X \)'s and \( O_{\alpha} \)'s. We symmetrize these various factors of \( X \)'s and \( O_{\alpha} \)'s and average. The rules derived in the previous section then lead to the following expression for the symmetrized trace version of (3.4):

\[ \hat{\hat{O}}(k) = \int d^{d+1}x \frac{d}{d \tau} \prod_{\alpha=1}^n \text{tr} \left[ \prod_{\alpha=1}^n O_{\alpha}(x^i + \theta^{ij} \tau_{\alpha}) W_i(k, A, \phi, x) \right] * e^{ik \cdot x^i} \]

where \( W_i \) denotes the Wilson tail

\[ W_i(k, A, \phi, x) = \exp[i \int_0^1 d \lambda \, k_A \theta^{AB} A_B(x + k \cdot \theta \lambda)] \]

Thus the Wilson tail now has operators which are smeared over it. This is the operator considered in [17].
4. The proposal

Consider a large number of coincident $p$ branes with no $B$ field in the presence of a weak supergravity background. Let us denote a supergravity mode in momentum space by $\Phi(k, k_\mu)$ where $k_\mu$ denotes the momentum along the brane and $k_I$ denotes the momentum transverse to the brane. Let $X^I$ denote the transverse coordinate and $A_\mu$ the gauge field on the brane. Then in the brane theory, the transverse coordinates are represented by scalar fields $X^I(\xi)$. Then the results of \cite{19, 20} show that the linearized coupling of some supergravity mode $\Phi(k, k_\mu)$ to this set of branes is of the form

\begin{align}
\Phi(k, k_\mu) \int e^{ik_\mu \xi^\mu} \text{Str} \left[ e^{ik_I X^I} O_\phi(X, A, \xi) \right]
\end{align}

(4.1)

The exponential now contains the transverse matrices $X^I$ as well.

Our proposal is the following \cite{13}. Once we know the coupling (4.1) we can obtain the coupling of the same supergravity mode to a $(p + 2n)$ dimensional noncommutative brane by simply expanding the matrices which appear around the relevant classical solution as in (2.4). Using the results of the previous section, this coupling then becomes

\begin{align}
S_{int} = \Phi(k) \int \frac{d\xi dx}{(2\pi)^n} \text{Pf} B \int \prod_{\alpha=1}^n d\tau_\alpha e^{ik_\mu \xi^\mu} \text{tr} P_\alpha \left[ \prod_{\alpha=1}^n O_\alpha(x^i + \theta^{ij} k_j \tau_\alpha) W_i \right] e^{ik_I x^I}
\end{align}

(4.2)

\begin{align}
W_i(k, A, \phi) = \exp \left[ i \int_0^1 d\lambda \ k_i \theta^{ij} A_j(x + \eta(\lambda)) + i \int_0^1 d\lambda \ k_a \phi^a(x + \eta(\lambda)) \right]
\end{align}

(4.3)

and $\eta^i(\lambda) = \theta^{ij} k_j \lambda$. This is the generalization of the Wilson line with Higgs fields considered in \cite{8}.

5. Tests of the proposal : DBI approximation

The proposal described above is quite general and makes no reference to any approximation. However, the exact form of the operator $O_\phi$ which appears in (4.1) is not known. These operators are known in the low energy limit or in the DBI approximation. We now perform a test of our proposal by considering the dilaton coupling in the DBI approximation. For simplicity we consider a single noncommutative euclidean $D(2n-1)$ brane (with $2n$ dimensional worldvolume) with noncommutativity in all the directions. We will construct this from a large number $N$ of $D(-1)$ branes. Following \cite{19, 20} we will assume that the action in the presence of a dilaton field $D(k)$ with momentum $k$ (with all backgrounds trivial) is given by

\begin{align}
S_{int} = \frac{D(k)}{g_s} \text{Str} e^{ik X} \sqrt{\det(\delta^I_J - i[X^I, X^K] g_{KJ})}
\end{align}

(5.1)

The notation is the same as in the previous sections. We then expand around the classical solution as in (2.2) and (2.3) to obtain a single noncommutative $(p + 2n)$ brane. To simplify things further we will take $k_a = 0$ and also set $\phi^a = 0$. A nonzero $k_a$ or $\phi^a$ can be easily incorporated.

In the following we will be interested in terms upto $O(A^2)$ in the noncommutative gauge fields. In the language of matrices we will be interested in terms which
contain at most two matrices. For such terms there is no distinction between the symmetrized trace and ordinary trace. We will therefore replace \( \text{STr} \) in (5.1) with \( \text{Tr} \). Using the results of the previous section, this interaction is then written in terms of noncommutative gauge fields \( F_{ij} \)

\[
S_{\text{int}} = \frac{D(k)}{G_s} \int d^{2n} x \ e^{ikx} \ P_* \left[ \exp \left( i \int d\eta^i A_i(x + \eta(\lambda)) \right) \right] \sqrt{\det(G + F - B)}
\]

where in (5.2) the quantities \( \theta, F, B, g \) are written as \( (2n) \times (2n) \) matrices and \( I \) stands for the identity matrix, in a natural notation. In the following whenever these quantities appear without indices they denote these matrices. We have used (2.10) and (2.11) to write the expression in terms of the open string metric \( G_{ij} \) and the open string coupling \( G_s \). Here the path used is given by (??) and all products are star products.

In terms of the ordinary gauge fields \( f_{ij} \), the closed string metric and the closed string coupling, the interaction may be read off from the standard Dirac-Born-Infeld action

\[
\hat{S}_{\text{int}} = \frac{D(k)}{g_s} \int d^{2n} x \ e^{ikx} \sqrt{\det(g + f + B)}
\]

The strategy is now to express (5.3) in terms of the noncommutative gauge field \( F_{ij} \) using the Seiberg-Witten map in a series involving powers of the potential \( A_i \) and compare the result with (5.2) which is also expanded in a similar fashion.

For zero momentum operators this is the comparison done in [3], where it is shown that

\[
\frac{1}{g_s} \sqrt{\det(g + f + B)} = \frac{1}{G_s} \sqrt{\det(G - B + F)} + O(\partial F) + \text{total derivatives}
\]

which shows the equivalence of the two actions in the presence of constant backgrounds. The crucial aspect of our comparison is the presence of these total derivative terms in (5.4) which cannot be ignored if \( k \neq 0 \). We will find that these total derivative terms are in precise agreement with similar terms coming from the expansion of the Wilson tail in (5.2) up to \( O(A^2) \).

Since we are using the DBI action, the field strengths should be really treated as constant. In carrying out the comparison, however, some caution must be exercised. Since the Seiberg-Witten map contains gauge potentials as well as field strengths a term containing a derivative of a field strength multiplied by a gauge potential without a derivative on it, cannot be set automatically to zero, as emphasised in [3].

The details of this comparison is given in [13]. The fact that operators with Wilson tails can be written in terms of generalized star products [18] becomes very useful in the calculations. In particular the expansion of the expression (5.7) to \( O(A^2) \) is

\[
\hat{O}(k) = \int d^{p+1} \xi \frac{d^{2n} \eta}{(2\pi)^n} e^{ik\eta} (\text{PfB}) \text{tr} [O(x, \xi)] + \theta^{ij} \partial_j (O *' A_i) \\
+ \frac{1}{2} \theta^{ij} \theta^{kl} \partial_i \partial_j [O(A_i A_k) *_{3}] * e^{ikx}
\]

The various generalized star products are defined in [18].

Consider first the comparison to \( O(A) \). To this order the Seiberg-Witten map simply reduces to \( f_{ij} = \partial_i A_j - \partial_j A_i + O(A^2) \). Thus it is sufficient to expand the
determinant in (5.3) to linear order in \( f \). One obtains
\[
\tilde{S}^{(1)}_{\text{int}} = \frac{D(k)}{g_s} \sqrt{\det(g + B)} \int d^{2n} x \ e^{ikx} \left[ 1 + \frac{1}{2} \left( \frac{1}{g + B} \right)^{ij} (\partial_j A_i - \partial_i A_j) + O(A^2) \right]
\]
(5.6)

Using (2.11) and (2.12) this may be written as
\[
\tilde{S}^{(1)}_{\text{int}} = \frac{D(k)}{G_s} \sqrt{\det(G - B)} \int d^{2n} x \ e^{ikx} \left[ 1 + \frac{1}{2} \left( \frac{1}{G - B} + \theta \right)^{ij} (\partial_j A_i - \partial_i A_j) + O(A^2) \right]
\]
(5.7)

The products in all expressions which involve \( A \) and \( F \) rather than \( a \) and \( f \) are star products. We have to compare this with the expansion of the expression (5.2) to \( O(A) \). To do this we can use (5.5) with the function \( O \) being replaced by the quantity \( \sqrt{\det(G + F - B)} \). To linear order in \( A \) we have
\[
\sqrt{\det(G + F - B)} = \sqrt{\det(G - B)} \left[ 1 + \frac{1}{2} \left( \frac{1}{G - B} \right)^{ij} (\partial_j A_i - \partial_i A_j) + O(A^2) \right]
\]
(5.8)

The various products appearing on the left hand side of the above equation are star products. However to this order these collapse to ordinary products since \( G, B \) etc. are constants. Also to this order one has
\[
\theta^{ij} \partial_j (\sqrt{\det(G + F - B)} \star^j A_i) = \frac{1}{2} \sqrt{\det(G - B)} \theta^{ij} (\partial_j A_i - \partial_i A_j)
\]
(5.9)

Using (5.8) and (5.9) it is easy to see that \( S_{\text{int}} \) agrees with (5.7) to \( O(A) \). Note that the term proportional to \( \theta \) in (5.7) came because of the relation (2.12) while the corresponding term on the noncommutative side came from the “Wilson tail” involved in the gauge invariant operator. To this order one is sensitive only to the linear term of the Seiberg Witten map. However the agreement of the two derivations of the interaction term is still nontrivial and the importance of the open Wilson line is evident.

The nontriviality of the Seiberg-Witten map enters at \( O(A^2) \). Using the various properties of generalized star products in [18] and following the same strategy as above we have checked that the operator we propose is indeed identical to the standard operator in terms of ordinary gauge fields. See [13] for details. While we have not carried out the calculation explicitly to \( O(A^3) \) we are fairly confident that the agreement will persist. At this order the nontriviality of the symmetrized trace will be important.

6. Other developments

Another test of our proposal can be performed in the Seiberg-Witten low energy limit, where the results can be compared with explicit string worldsheet calculations. This has been done in [22], and the results are in agreement for superstrings, though not for bosonic strings. For bosonic theory, however, we do not expect the above procedure to work since the nontriviality of the matrix model measure will swamp whatever classical solution we start with.

A recent interesting development is the determination of the operators which couple to RR fields [23], generalizing earlier results on couplings to constant RR fields [31]. One of the outcomes is an exact Seiberg-Witten map. Another recent development has been a proposal that closed string fields can be reconstructed out of open Wilson lines of arbitrary shapes by a suitable harmonic expansion [32].
7. Holography

So far we have considered the linearized couplings of noncommutative branes to supergravity modes. Another context in which gauge invariant operators should arise is in holography. Supergravity duals of noncommutative gauge theories are known \[5, 6\]. Supergravity modes in these backgrounds should be dual to momentum space gauge invariant operators in the gauge theory \[7\]. Generally, these operators are not identical to the operators we have considered, though for some cases they can be obtained by linearizing e.g. the DBI action around the background geometry \[29\]. Moreover, as argued in \[19\], it is possible to obtain the correlation functions of these operators from those of the operators which couple to linearized backgrounds by solving the scattering problem in the full geometry.

The identification of the holographically dual operators in this context is an open problem. However there is one observation which may be useful. The asymptotic geometry for the \(p+2n+1\) dimensional non-commutative theory is identical to that for the \(p+1\) dimensional ordinary theory at a particular point in the Coulomb branch where the \(p\)-branes are spread out uniformly along the \(2n\) directions. This is in fact the dual manifestation of the relationship between commutative and non-commutative Yang-Mills theories discussed above \[30\]. This connection may be possibly used to tackle the problem of mode mixing in such supergravity backgrounds. We do not have definitive results about this at present.

These holographically dual operators should also involve Wilson tails. A good indication is the fact that the linear relationship between the size of these operators and the momentum is something which is also visible in the dual supergravity theory \[8\]. In the AdS/CFT correspondence, the size of the hologram of an object in the bulk decreases monotonically as the object moves closer to the boundary. In the supergravity duals of noncommutative gauge theories, something interesting happens. In these backgrounds, the region deep into the bulk is AdS space-time and the above IR/UV relationship holds. However near the boundary, the relationship is opposite \[3\]. In this region the size of the hologram increases as the object moves closer to the boundary. A similar relationship was found in the full \(D3\) brane geometry in \[33\]. The latter is known to be a special case of the former \[7\]. The relationship between the hologram size and the momentum scale of the NCGT is in fact almost the same as that between the size of the Wilson tails and the momentum: they differ by a factor of the square root of the 't Hooft coupling. The latter factor, however, is always present in the relationship between the noncommutativity scale of the gauge theory and the scale observed in the supergravity dual and is presumably a strong coupling effect.

While this is a good indication, the precise form of the dual operators remains to be discovered. A knowledge of these operators should throw valuable light on holography in backgrounds which are not asymptotically AdS.

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