Two cavity modes in a dissipative environment: cross decay rates and robust states

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Abstract

We investigate the role of the cross decay rates in a system composed by two electromagnetic modes interacting with the same reservoir. Two feasible experiments sensitive to the magnitudes and phases of these rates are described. We show that if the cross decay rates are appreciable there are states less exposed to decoherence and dissipation, and in limit situations a decoherence free subspace appears.

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It has become increasingly important to understand how the environment acts over a system we are interested in. The promise of Quantum Computation often has as its main obstacle the entanglement of the variables of interest with the reservoir degrees of freedom [1]. Also, in the investigation of Foundations of Quantum Mechanics, the environment plays a central role [2].

A very successful model for the environment is the Caldeira-Legget model, where one harmonic oscillator linearly coupled to a bath of oscillators is considered [3]. The investigation of bipartite quantum systems interacting with the environment usually displays amazing effects [4, 5]. In [6], the Caldeira-Legget model was extended for two oscillators subjected to the same bath, leading to a possible communication between the oscillators mediated by the reservoir. This extension may be used to study two electromagnetic modes constructed in one or two cavities.

In [7], an experiment dealing with two modes constructed in a single cavity was described. The experimental findings were compared to theoretical results in [8], with satisfactory agreement. However, due to the large detuning of the modes, the experiment is not sensitive to the cross decay rates: the parameters related to the their communication through the environment.

Cross decay rates and their interference effects over quantum systems are well-known for a long time [9]. Some results related to them are population trapping [3, 10] and decoherence free subspaces (DFS) [11, 12]. The extension of the Caldeira-Legget model for two oscillators predicts the possibility of DFS if the cross decay rates are large enough [8]. Given the growing interest in DFS, the knowledge about this topic is accordingly important. Thus, we propose, here, feasible experiments to investigate these rates in the cavity Quantum Electrodynamics context, and also identify states that are more resistant against decoherence and dissipation when the cross decay rates are not zero.

The Hamiltonian we use to model two electromagnetic modes subjected to the same environment is

\[
H = H_0 + H_{int},
\]

\[
H_0 = \hbar \Omega_1 a_1^\dagger a_1 + \hbar \Omega_2 a_2^\dagger a_2 + \hbar \sum_k \omega_k c_k^\dagger c_k,
\]

\[
H_{int} = \hbar \sum_k \left( \alpha_{1k} c_k^\dagger a_1 + \alpha_{1k}^* a_1^\dagger c_k \right) + \hbar \sum_k \left( \alpha_{2k} c_k^\dagger a_2 + \alpha_{2k}^* a_2^\dagger c_k \right),
\]
where \( a_1, a_2, a_1^\dagger \) and \( a_2^\dagger \) are annihilation and creation bosonic operators for the modes of interest, \( M_1 \) and \( M_2 \), with frequencies \( \Omega_1 \) and \( \Omega_2 \). The annihilation and creation operators \( c_k \) and \( c_k^\dagger \) are used to model the environment by a set of harmonic oscillators linearly coupled to \( M_1 \) and \( M_2 \) modes. This Hamiltonian shall be used for two modes in different cavities or in the same cavity. Under the usual approximations, Hamiltonian (1) leads to the master equation

\[
\frac{d}{dt} \rho_S (t) = \mathcal{L} \rho_S (t),
\]

with the Liouvillian superoperator (an operator which acts on operators)

\[
\mathcal{L} = k_{11} \left( 2a_1 \bullet a_1^\dagger - \bullet a_1^\dagger a_1 - a_1^\dagger a_1 \bullet \right) + i \left( \Delta_{11} - \Omega_1 \right) \left[ a_1^\dagger a_1, \bullet \right] + \]

\[
k_{22} \left( 2a_2 \bullet a_2^\dagger - \bullet a_2^\dagger a_2 - a_2^\dagger a_2 \bullet \right) + i \left( \Delta_{22} - \Omega_2 \right) \left[ a_2^\dagger a_2, \bullet \right] + \]

\[
k_{12} \left( a_1 \bullet a_2^\dagger + a_2 \bullet a_1^\dagger - \bullet a_2^\dagger a_1 - a_1^\dagger a_2 \right) + \]

\[
k_{21} \left( a_2 \bullet a_2^\dagger + a_1 \bullet a_1^\dagger - \bullet a_1^\dagger a_2 - a_2^\dagger a_1 \right) + \]

\[
i \left( \frac{\Delta_{12} - \Delta_{21}}{2} \right) \left( a_1 \bullet a_2^\dagger - a_2 \bullet a_1^\dagger - \bullet a_2^\dagger a_1 + a_1^\dagger a_2 \bullet \right) + \]

\[
i \left( \frac{\Delta_{21} - \Delta_{12}}{2} \right) \left( a_2 \bullet a_1^\dagger - a_1 \bullet a_2^\dagger - \bullet a_1^\dagger a_2 + a_2^\dagger a_1 \bullet \right) + \]

\[
i \left( \frac{\Delta_{12} + \Delta_{21}}{2} \right) \left[ a_1^\dagger a_2 + a_2^\dagger a_1, \bullet \right],
\]

for zero temperature, where

\[
k_{ij} + i\Delta_{ij} = \sum_k \alpha_{ik} \alpha_{jk}^* \int_0^\tau e^{i(\omega_k - \Omega_j)\tau} d\tau,
\]

and we used the conventional notation for superoperators [13] (the dot sign (\( \bullet \)) indicates the place to be occupied by \( \rho_S (t) \), where the superoperator acts). A discussion about the Hamiltonian (1) and a detailed derivation of the master equation (2) from (1) may be found in [6]. The coupling between the environmental modes and the modes of interest may occur by complicated processes, e. g., a photon may be scattered from a mode of interest to an environmental mode by an atom. Of course, \( H_{int} \) does not come from a first principles analysis, and, thus, we do not know much about the coupling constants \( \alpha_{1k} \) and \( \alpha_{2k} \). However, as we have said, this model presents results that agree with experimental ones. The cross decay rates are \( k_{12} + i\Delta_{12} \) and \( k_{21} + i\Delta_{21} \), and will assume relevant values if the summations involving \( \alpha_{1k} \alpha_{2k}^* \) and \( \alpha_{2k} \alpha_{1k}^* \) are not null. This demands that the \( \alpha_{1k} \)
and the $\alpha_{2k}$ must have some correlation, *i. e.*, the way the modes of interest interact with the environment must be microscopically correlated. This may be achieved if the modes of interest are close to each other in the scale of the wavelength of the environmental modes that effectively interacts with them. These are the ones with frequencies around $\Omega_1$ or $\Omega_2$, as may be seen in Eq. (1). If the environmental modes are electromagnetic modes, the modes of interest must be close in the scale of their proper wavelengths. An interesting case is two modes with orthogonal polarizations constructed in the same cavity. They occupy essentially the same positions in space, but maybe the difference in the polarizations spoils the correlation.

In Fig. 4, we sketch an experiment where circular Rydberg atoms $A_s$ and $A_p$, with relevant levels $e$ and $g$, are produced in box $B$, cross the cavities $C_1$ and $C_2$ and are detected in detector $D$. The energy of level $e$ is higher than the energy of level $g$ by $\hbar \Omega_a$. The cavities are macroscopically identical: $\Omega_1 = \Omega_2 = \Omega$ and $k_{11} = k_{22} = k$. We will assume a huge number of environmental modes, with random distribution in the frequencies around $\Omega$, what leads to $\Delta_{11} = \Delta_{22} \approx 0$ and $k_{12} + i \Delta_{12} \approx (k_{21} + i \Delta_{21})^*$, as may be seen using expression (4). We may adjust the atom-fields interaction time using the Stark effect: when an appropriate voltage is applied across the mirrors of the cavities, the $e \leftrightarrow g$ transition becomes resonant with the modes $M_1$ and $M_2$; when no voltage is applied, the detuning $\Delta = \Omega_a - \Omega$ turns negligible the atom-fields interaction.

In the analysis below, global phases will be often disregarded. The cavities are initially in vacuum state, and $A_s$ is sent in state $e$:

$$|\psi(t = 0)\rangle_{M_1,M_2,A_s} = |0, 0, e\rangle.$$  

Atom $A_s$ enters cavity $C_1$, and is put in resonance with mode $M_1$ by a time $t_{1s}$, yielding

$$|\psi(t = t_{1s})\rangle_{M_1,M_2,A_s} = \cos (Gt_{1s}) |0, 0, e\rangle - i \sin (Gt_{1s}) |1, 0, g\rangle,$$

where we used the RWA (rotating wave approximation) Hamiltonian

$$H_{1s} = \hbar \Omega \left( a_1^\dagger a_1 + a_2^\dagger a_2 + \frac{\sigma_z}{2} \right) + \hbar G \left( a_1^\dagger \sigma_- + a_1 \sigma_+ \right),$$

with $\sigma_z = |e\rangle \langle e| - |g\rangle \langle g|$, $\sigma_- = |g\rangle \langle e|$, $\sigma_+ = |e\rangle \langle g|$. During a time $t_{0s}$, $A_s$ is put far of resonance with the modes. Considering $|\Delta| \gg G$, the Hamiltonian may be written

$$H_{0s} = \hbar \Omega \left( a_1^\dagger a_1 + a_2^\dagger a_2 \right) + \frac{\hbar \Omega_a \sigma_z}{2}.$$
and the system evolves to
\[ |\psi(t = t_{1s} + t_{0s})\rangle_{M_1,M_2, A_s} = \cos(Gt_{1s})|0,0,e\rangle - ie^{i(\Omega_a - \Omega)t_{0s}}\sin(Gt_{1s})|1,0,g\rangle. \]

Next, \( A_s \) enters \( C_2 \), and is put in resonance with \( M_2 \) by \( t_{2s} = \pi/(2G) \). Considering that \( M_1 \) and \( M_2 \) have the same polarizations, the Hamiltonian will be
\[ H_{2s} = \hbar \Omega \left(a_1^\dagger a_1 + a_2^\dagger a_2 + \frac{\sigma_z}{2}\right) + \hbar G \left(a_2^\dagger \sigma_- + a_2 \sigma_+\right), \]
what leads to
\[ |\psi(t = t_{1s} + t_{0s} + t_{2s})\rangle_{M_1,M_2} = \cos \theta |0,1\rangle + e^{i\phi} \sin \theta |1,0\rangle, \tag{5} \]
where \( \theta = Gt_{1s} \) and \( \phi = (\Omega_a - \Omega)t_{0s} \). The \( A_s \) state ends up in \( g \), factorized, and needs not to be considered anymore.

During the time \( T \) (\( T \gg t_{1s} + t_{0s} + t_{2s} \)), no atom interacts with the modes. Using Eq. (2) to compute the action of the environment in this period, we get, for the state of \( M_1 \leftrightarrow M_2 \) system,
\[ \rho_S(t = T + t_{1s} + t_{0s} + t_{2s}) = \{ u_1(T)|1,0\rangle + u_2(T)|0,1\rangle \} \{ HC \} + |0,0\rangle \langle 0,0| \{ 1 - |u_1(T)|^2 - |u_2(T)|^2 \}, \tag{6} \]
where HC stands for Hermitian conjugate, and
\[
\begin{align*}
u_1(t) & = \frac{e^{-kt}}{2} \left\{ (e^{-rt} + e^{rt}) e^{i\phi} \sin \theta + re^{-i\gamma} (e^{-rt} - e^{rt}) \cos \theta \right\}, \\
u_2(t) & = \frac{e^{-kt}}{2} \left\{ (e^{-rt} + e^{rt}) \cos \theta + re^{i\gamma} (e^{-rt} - e^{rt}) e^{i\phi} \sin \theta \right\}, \\
r e^{i\gamma} & = k_{12} + i\Delta_{12}.
\end{align*}
\]

In this calculation, we used the parameter differentiation technic, explained in [6]. At time \( t = t_{1s} + t_{0s} + t_{2s} + T \), atom \( A_p \), initially in state \( g \), starts to interact with mode \( M_1 \). Considering that \( A_p \) interacts with \( M_1 \) during a time \( t_{1p} = 3\pi/(2G) \), with \( M_2 \) during \( t_{2p} = (2\theta - \pi)/(2G) \), and a time \( t_{0p} = t_{0s} \) between these interactions, the probability to find the atom \( A_p \) in state \( e \) at \( t = t_{1s} + t_{0s} + t_{2s} + T + t_{1p} + t_{0p} + t_{2p} \) will be
\[ P_e = \frac{e^{-2kT}}{4} \left| (e^{-rT} + e^{rT}) + \sin 2\theta \cos (\gamma + \phi) (e^{-rT} - e^{rT}) \right|^2. \]

This is the main result of the present contribution. Observe that \( P_e = 1 \) if there is no environment, since in this case \( k = r = 0 \).
The probability $P_e$ depends on $\theta$ and $\phi$, which may be freely chosen by varying the times $t_{1s}$, $t_{0s}$ and $t_{2s}$. For fixed $T$ and $\theta$, a $P_e \times \phi$ plot constructed with experimental data tells us about the cross decay rates: if any sinusoidal pattern is observed, $k_{12} + i\Delta_{12}$ is non zero, the amplitude of the curve being related to $r$; the position of the maximum of the curve may be used to find $\gamma$ (this maximum occurs for $\gamma + \phi = \pi$). This is exemplified in Fig. 2. The larger visibility is achieved for $\theta = \pi/4$, i.e., when the state (3) is maximally entangled.

Since the magnitudes of the cross decay rates are related to correlations between the way each mode interacts with the environment, we expect that the amplitudes of the curves grow when the cavities get closer.

In order to get better insight into how the robust states arise in the model, let us rewrite Eq. (3) in the form

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2,$$

$$\mathcal{L}_1 = (k - r) \left(2A_1 \cdot A_1^\dagger - \cdot A_1^\dagger A_1 - A_1^\dagger \cdot \right) + i\Omega \left[ A_1^\dagger A_1, \cdot \right],$$

$$\mathcal{L}_2 = (k + r) \left(2A_2 \cdot A_2^\dagger - \cdot A_2^\dagger A_2 - A_2^\dagger \cdot \right) + i\Omega \left[ A_2^\dagger A_2, \cdot \right],$$

where the operators $A_1$ and $A_2$, given by

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\exp(-i\gamma) \\ \exp(i\gamma) & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix},$$

obey the usual commutation relations for bosons. Notice that the decay rate related to $\mathcal{L}_1$ is smaller than the one related to $\mathcal{L}_2$. Thus, states which may be written as

$$\rho_S = \sum_{m,n} c_{m,n} \left( A_1^\dagger \right)^n |0,0\rangle \langle 0,0| A_1^m + HC$$

(7)

are less exposed to the environment. When $r = k$, these states are decoherence-free, a probably very difficult condition to reach. If we learn the value of $\gamma$, by performing the experiment just described, we may chose to work with the states less exposed to the environment (7). Relevant examples for Quantum Information and Quantum Optics are the maximally entangled state

$$\rho_S = (|1,0\rangle - e^{i\gamma} |0,1\rangle) (\langle 1,0| - e^{-i\gamma} \langle 0,1|)$$

and the coherent state

$$\rho_S = |v, -e^{i\gamma} v\rangle \langle v, -e^{i\gamma} v|.$$
As discussed above, the physical distance between the cavities may lead to the destruction of the microscopic correlations important for the appearing of DFS and robust states. There is, however, another possibility, which circumvents this problem: one can use two modes differing by their polarization in a single cavity. Is this a better proposition than the first one? In order to investigate this, let us consider two degenerate modes with orthogonal polarizations and equal dissipation rates. A circular Rydberg atom with $e \leftrightarrow g$ transition resonant with the modes will interact simultaneously with them via the Hamiltonian

$$H_{12a} = \hbar \Omega \left( a_1^\dagger a_1 + a_2^\dagger a_2 + \frac{\sigma_z}{2} \right) + \hbar G \left( i a_1^\dagger \sigma_- - i a_1 \sigma_+ + a_2^\dagger \sigma_- + a_2 \sigma_+ \right).$$

The phase difference in the coupling constants of the atom to each mode are due to the polarization orthogonality. Consider that one atom in state $e$ begins to interact with the modes, initially in vacuum state, at time $t = 0$, and this interaction lasts until the time $t_{12a} = \pi/(2\sqrt{2}G)$. This atom creates the entangled state

$$\rho_S (t = t_{12a}) = \frac{1}{\sqrt{2}} \left( |0, 1\rangle + i |1, 0\rangle \right) (\text{HC})$$

for the modes in the cavity, and ends up in state $g$. Taking into account the action of the environment in the period between $t = t_{12a}$ and $t = t_{12a} + T$ ($T \gg \pi/(2\sqrt{2}G)$), the state of the fields will be given by Eq. (6) with $u_1 (t)$ and $u_2 (t)$ calculated using $\theta = \pi/4$ and $\phi = \pi/2$. Then, if another atom, initially in state $g$, starts its interaction with the modes at time $t = t_{12a} + T$, and this interaction lasts until $t = 2t_{12a} + T$, we get

$$P_{e,r} = \frac{e^{-2kT}}{4} \left| \left( e^{-rT} + e^{rT} \right) - \sin (\gamma) \left( e^{-rT} - e^{rT} \right) \right|^2$$

for the probability to find this second atom in state $e$. Notice that $P_{e} = 1$ if we disregard the environment ($k = r = 0$). Another way to have $P_e = 1$ is $k = r$ and $\gamma = \pi/2$: in this case we have a DFS.

If the cavity is subjected to a pressure, in such a way that it becomes slightly elliptical, the orthogonal modes become non resonant. For a large enough detuning, we may work with the Hamiltonian

$$H_{12a} = \hbar \Omega \left( a_2^\dagger a_2 + \frac{\sigma_z}{2} \right) + \hbar G \left( a_2^\dagger \sigma_- + a_2 \sigma_+ \right),$$

related to an atom interacting with just one resonant mode. Now, an atom, initially in state $e$, interacts with the field mode, initially in vacuum state, between $t = 0$ and $t = \pi/2G$,
yielding the field state $|1\rangle$ and ending up in state $g$. If we take into account the action of the environment between $t = \pi/2G$ and $t = \pi/2G + T$ ($T \gg \pi/2G$), and let a second atom, initially in $g$ state, to interact with the field in the period between $t = \pi/2G + T$ and $t = \pi/G + T$, we get

$$P_{e,nr} = e^{-2kT}$$

for the probability to find this atom in state $e$.

The difference between probabilities $D = P_{e,r} - P_{e,nr}$ must be a sign of significative cross decay rates. In the experiment with two cavities, we can maximize the effects of the parameter $r$ by choosing the state built by the first atom; in this experiment with a single cavity, the first atom always constructs the state $|\rangle$, and the effects of $r$ will be maximized for $\gamma = \pi/2$ only. This is the condition that makes $D$ more easy to detect, and it was used in Fig. 3. Since $\gamma$ depends on the environment, it is not trivial to control it.

Cross decay is related to interference: if the environment acts over both modes in a microscopic correlated way, this action may be canceled (at least partially) for a set of states. For the ideal case of perfect correlation, the cross decay rates are large, and a DFS appears. If we get some correlation (even non perfect), the knowledge about the cross decay rates may be useful, since some states will be more resistant to the degradation produced by the environment. The experiments we proposed here are feasible with the present technological state, and the confirmation of these interference effects in Cavity Quantum Electrodynamics systems could encourage a search of analogous behavior in other contexts, maybe in scalable systems, which would be important for Quantum Information implementation.

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FIG. 1: Sketch of the experiment with two cavities.
FIG. 2: Probability $P_e$ for $k = 1000 \, s^{-1}$, $\theta = \pi/4$, $\gamma = \pi/2$, $T = 500 \, \mu s$ and various values for $r$: $r = 500 \, s^{-1}$, $r = 750 \, s^{-1}$ and $r = 1000 \, s^{-1}$. Higher $r$ values correspond to higher amplitudes in the oscillation of the curve.
FIG. 3: Difference between probabilities $D = P_{e,r} - P_{e,nr}$ for $k = 1000 \text{ s}^{-1}$, $\gamma = \pi/2$ and various values for $r$: $r = 500 \text{ s}^{-1}$, $r = 900 \text{ s}^{-1}$ and $r = 1000 \text{ s}^{-1}$. Higher $r$ values correspond to higher curves.