New Results on Stability Analysis for Sampled-Data Control Systems With Nonuniform Sampling and Communication Delays

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ABSTRACT This paper investigates the stability analysis of sampled-data control systems with nonuniform sampling and communication delays. First, by taking full advantage of effective information of the sampling intervals \([t_k, t_{k+1}]\) and \([t_k - \tau, t_{k+1} - \tau]\), we propose a new looped Lyapunov functional called two-sided sampling-interval-dependent looped Lyapunov functional, which is tailored for the use of generalized free-matrix-based inequality and canonical Bessel-Legendre inequality. Second, by using the proposed looped Lyapunov functional and the integral inequality technique, some novel \(N\)-dependent stability criteria with less conservatism are derived for the considered systems. The obtained stability criteria are shown to have a hierarchical character: the higher level of hierarchy, the less conservatism of the derived result. Finally, two examples are provided to illustrate the effectiveness of the proposed approaches.

INDEX TERMS Stability, sampled-data control systems, nonuniform sampling, communication delays, looped Lyapunov functional, integral inequality.

I. INTRODUCTION

Sampled-data control systems have been extensively applied in digital control systems and networked control systems because of their high reliability, efficient control and low-cost implementation [1]–[4]. In real applications, larger sampling period of systems means more relaxed operating conditions, such as communication capacity, limitation of load, computational burden, and then the cost of sampling device might be saved. Therefore, to make the sampling period as large as possible, it is necessary to derive some less conservative stability conditions for sampled-data control system.

Considering the types of communication channels, sampled-data control systems can be classified into two cases, i.e., the case with and without communication delays [5]. Many significant and valuable results have been obtained for sampled-data control systems until now [5]–[23]. However, the corresponding results on sampled-data control systems considering communication delays are relatively few [5]–[8], which has much room to be improved. Recalling some existing literatures, there are three types of methods to stability analysis and synthesis for sampled-data control system: discrete-time method [9]–[11], impulsive systems method [12], [13], and input delay method [14], [15].

Because of the wide application of the direct Lyapunov approach in studying stability of linear systems, the input delay method has been used widely. In this framework, various Lyapunov-based methods have been proposed to investigate the stability of sampled-data systems, such as time-dependent Lyapunov functional method [16], discontinuous Lyapunov functional approach [17], [18], looped-functional-based Lyapunov technique [22], [23], and so on. Some examples are given below. In [16], a time-dependent Lyapunov functional method is proposed to
study the stability issue of sampled-data control systems. Improved conditions are established in [17] by employing time-dependent discontinuous Lyapunov functional method based on Wirtinger inequality. In [18], a free-matrix-based discontinuous Lyapunov functional method is developed for sampled-data control system and extended to investigate the sampled-data synchronization control for neural networks with time-varying delays [19]–[21]. By employing looped-functional-based Lyapunov technique studied in [22], the stability on sampled-data control systems with incremental delay has been studied. In addition, in [23], two-side looped-functional-based Lyapunov approach has been used to improve stability criteria.

The Lyapunov-based approach is a powerful tool for analyzing the stability problem of sampled-data and time-delay systems [24]–[27]. In this method, the treatment of integral term $\ell(t) = -\int_{s}^{t} x^{T}(s) R_{s} x(s) ds$ is closely related with its less conservative stability criterion. Hence, in order to derive less conservative stability results, various inequality approaches have been proposed to treat the integral term $\ell(t)$ appeared in the derivative of Lyapunov functional, for instance, Wirtinger inequality method [15], auxiliary function-based inequality method [28], reciprocally convex inequality method [29], [30], Bessel-Legendre-based type inequality method [31], [32], free-matrix-based type inequality method [33]–[36], and so on. Some examples are given below. In [25], less conservative stability conditions for delayed neural networks have been derived through the use of the free-matrix-based inequality. By employing the Bessel-Legendre-based inequality and reciprocally convex inequality, the stability and stabilization have been derived in [26] for event-triggered nonlinear networked control system with time delay. It is found that the canonical N-order Bessel-Legendre inequality method presented in [32] and generalized N-order free-matrix-based inequality method proposed in [35] can provide a lower bound for $\ell(t)$ as tight as possible if N goes to infinity, which can greatly decrease the conservatism of the derived stability criterion based on the proper Lyapunov functional.

On the other hand, it can be seen that the theoretical research for sampled-data control systems has attracted much attention of many researchers. In most related literatures [5]–[23], the comparison of the conservatism between different stability conditions is generally discussed by employing several simple numerical examples, while the practical importance in terms of reducing conservatism is hardly illustrated just from the derived results. A practical systems is more helpful to investigate the conservativeness of stability criteria [37]. Thus, in [38], an electric power market system is modeled by a sampled-data control system to study the impact the marker clearing time (i.e. the updating period of power price signals) and communication delays on the system stability. In [39], the system is further discussed to investigate the practical importance of the conservativeness-reducing of the stability condition. Notice that the literature [39] mainly focuses on the information of intervals $x(t_k)$ to $x(t)$ and $x(t_k - \tau)$ to $x(t - \tau)$, while the information of intervals $x(t)$ to $x(t_{k+1})$ and $x(t-\tau)$ to $x(t_{k+1}-\tau)$ are not taken into account. Hence, the results derived may be conservative, which should be studied further.

In this paper, the problem of stability for sampled-data control systems with nonuniform sampling and communication delays is further investigated. A two-side sampling-interval-dependent looped Lyapunov functional tailored for the use of generalized N-order free-matrix-based inequality and canonical N-order Bessel-Legendre inequality is constructed, in which the information of intervals $x(t_k)$ to $x(t)$, $x(t_k - \tau)$ to $x(t-\tau)$, $x(t)$ to $x(t_{k+1})$ and $x(t-\tau)$ to $x(t_{k+1}-\tau)$ are taken fully into account. By utilizing the above mentioned inequalities to estimate the derivative of this looped Lyapunov functional, some novel N-dependent and less conservative stability criteria are derived for sampled-data systems. It is shown that these stability criteria form a hierarchy of linear matrix inequality conditions: the larger $N$, the larger sampling or delay upper bound, which means that the conservativeness of the obtained results will be decreased with $N$ increasing. The validity of the presented methods is demonstrated through a numerical example and a practical electric power market.

Notation: Throughout this paper, $\mathbb{R}^{n}$ and $\mathbb{R}^{n \times m}$ stand for the n-dimensional vectors and the set of all $n \times m$ matrices, respectively; $\mathbb{N}_+$ represents the sets of non-negative scalars and integers; $P > 0$ means that the matrix $P$ is symmetric and positive definite; the superscripts ‘T’ and ‘−1’ denote the transpose and inverse of a matrix, respectively; $I$ and $0$ denote the identity matrix and the zero-matrix with appropriated dimensions, respectively; the symbol ‘*’ represents the symmetric terms in symmetric block matrices and $\text{Sym}(M) = M + M^{T}$; $\text{diag}(\cdot \cdot \cdot)$ stands for a block-diagonal matrix; $p!/(p-q)q!$.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following linear control system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in \mathbb{R}^n$ and $u(t) = [u_1(t), u_2(t), \cdots, u_m(t)]^T \in \mathbb{R}^m$ are the state vector and the control input of the system, respectively; $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are known system matrices. The sequence of sampling instants, $t_k$, satisfies $0 = t_0 < t_1 < \cdots < t_k < \cdots$. The nonuniform sampling intervals are defined as follows

$$t_{k+1} - t_k = h_k \in [h_1, h_2], \quad \forall k \in \mathbb{N}$$

where $h_1$ and $h_2$ indicate the upper bound and lower bound of sampling periods, respectively. The control signal $u(t_k)$ between sampling instant $t_k$ and $t_{k+1}$ is maintained to be constant by zero-order holder (ZOH).

When the control and measurement signals transmitted through the use of a practical network-based communication channel (for instance, wide-area network [40]), the communication delays during the signals transmission from the sampler to the controller or from the controller to ZOH should
be taken into consideration, and then the control law can be expressed as the following form [5]

\[ u(t) = Kx(t_k - \tau), \quad t \in [t_k, t_{k+1}) \]  

(3)

where \( K \) is the gain matrix of the sampled-data controller and \( \tau \) denotes the communication delays which may be smaller or bigger than the sampling period. Thus, the sampled-data control system (1) can be converted to the following closed-loop system

\[ \dot{x}(t) = Ax(t) + A_dx(t_k - \tau), \quad t \in [t_k, t_{k+1}) \]  

(4)

where \( A_d = BK \).

Our aim is to find the acceptable maximal sampling period (or communication delays) under preset communication delays (or maximal sampling period) such that system (4) with a given controller \( K \) is asymptotically stable. In other words, the objective of this paper is to investigate the impact of the sampling period and communication delays for the stability of system (4).

Now, we introduce the following lemmas, which are indispensable to establish some less conservative stability criteria.

**Lemma 1** (Canonical Bessel-Legendre inequality) [32]: For a given integer \( N \in \mathbb{N}_+ \), two scalars \( b \) and \( a \) with \( b > a \), a symmetric matrix \( R \in \mathbb{R}^{n \times n} > 0 \), and a vector-valued differentiable function \( x : [a, b] \rightarrow \mathbb{R}^n \) such that the integrations below are well defined, the following inequality holds

\[-\int_a^b x^T(s)R\dot{x}(s)ds \leq -\frac{1}{b-a}\sigma_{1N}^T(t)\Gamma_{1N}^{-1}R_N\Gamma_{1N}\sigma_{1N}(t) \]  

(5)

where \( i = 1, 2 \) and

\[ \Gamma_{1N} = \Pi_{1N} \Sigma_{1N}, \quad (i = 1, 2) \]  

(6)

\[ R_N = \text{diag}\{R, 3R, \ldots, (2N + 1)R\}, \]  

(7)

\[ \Pi_{1N} = \begin{bmatrix} I & (1-I)(1+1)I & \cdots & 0 \\ I & (1-I)(1+1)I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I & (1-I)(1+1)I & \cdots & (1-N)\binom{N+1}{N}I \end{bmatrix} \]  

(8)

\[ \Pi_{2N} = \text{diag}\left\{ (1-I)^2, (1-I)^3, \ldots, (1-N)^N \right\} \]  

(9)

\[ \Sigma_{1N} = \begin{bmatrix} I & -I & 0 & 0 & \cdots & 0 \\ 0 & -I & I & 0 & \cdots & 0 \\ 0 & -I & 0 & 2I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -I & 0 & 0 & \cdots & N! \end{bmatrix} \]  

(10)

\[ \Sigma_{2N} = \begin{bmatrix} I & -I & 0 & 0 & \cdots & 0 \\ 0 & -I & I & 0 & \cdots & 0 \\ 0 & -I & 0 & 2I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ I & 0 & 0 & 0 & \cdots & -N! \end{bmatrix} \]  

(11)

\[ \sigma_{1N}(t) = \begin{bmatrix} x^T(b), x^T(a), \varphi_{11}^T(t), \varphi_{12}^T(t), \cdots, \varphi_{1N}^T(t) \end{bmatrix}^T \]  

(12)

\[ \varphi_{1p}(t) = \frac{1}{(b-a)^p} \int_a^b (s-a)^{p-1} x(s)ds, \quad (p = 1, 2, \cdots, N) \]  

(13)

\[ \varphi_{2p}(t) = \frac{1}{(b-a)^p} \int_a^b (s-a)^{p-1} x(s)ds, \quad (p = 1, 2, \cdots, N) \]  

\[ \varphi_{1p}(t) = \frac{1}{(b-a)^p} \int_a^b (s-a)^{p-1} x(s)ds, \quad (p = 1, 2, \cdots, N) \]  

\[ \varphi_{2p}(t) = \frac{1}{(b-a)^p} \int_a^b (s-a)^{p-1} x(s)ds, \quad (p = 1, 2, \cdots, N) \]  

**Lemma 2** (Generalized Free-Matrix-Based Inequality) [35]: For a vector \( \xi_N \in \mathbb{R}^m \), two scalars \( b \) and \( a \) with \( b > a \), a real symmetric matrix \( R \in \mathbb{R}^{n \times n} > 0 \) and any matrix \( L = \left[ L_1^T, L_2^T, \ldots, L_N^T \right]^T \), and a vector-valued differentiable function \( x : [a, b] \rightarrow \mathbb{R}^n \) such that the integrations below are well defined, the following inequality holds

\[-\int_a^b \dot{x}(t)R_Nx(t)ds \leq (b-a)\xi_N^T L_N R_N L_N^T \xi_N \]  

(14)

where \( R_N, \Gamma_{1N}, (i = 1, 2) \) and \( \sigma_{1N}(t), (i = 1, 2) \) are defined in Lemma 1

**III. MAIN RESULTS**

In this section, some \( N \)-dependent stability criteria for sampled-data control system will be established. For brevity, let us define the following nomenclatures for \( i = 1, 2, \cdots, N \), where \( N \) is a positive integer.

\[ \alpha_i(t) = \frac{1}{(t-t_k)^i} \int_{t_k}^t (t-s)^{-1} x(s)ds \]  

(15)

\[ \beta_i(t) = \frac{1}{(t-t_k)^i} \int_{t_k}^{t_{k+1}} (s-t)^{-1} x(s)ds \]  

(16)

\[ \gamma_i(t) = \frac{1}{(t-t_k)^i} \int_{t_k}^t (t-s)^{-1} x(s)ds \]  

(17)

\[ \delta_i(t) = \frac{1}{(t_{k+1}-t_k)^i} \int_{t_k}^{t_{k+1}} (s-t)^{-1} x(s)ds \]  

(18)

\[ \vartheta_i(t) = \frac{1}{(t_{k+1}-t_k)^i} \int_{t_k}^{t_{k+1}} (s-t)^{-1} x(s)ds \]  

(19)

\[ \psi_{1N}(t) = \left[ \alpha_1^T(t), \alpha_2^T(t), \alpha_3^T(t), \cdots, \alpha_N^T(t) \right]^T \]  

(20)

\[ \psi_{2N}(t) = \left[ \beta_1^T(t), \beta_2^T(t), \beta_3^T(t), \cdots, \beta_N^T(t) \right]^T \]  

(21)

\[ \psi_{3N}(t) = \left[ \gamma_1^T(t), \gamma_2^T(t), \gamma_3^T(t), \cdots, \gamma_N^T(t) \right]^T \]  

(22)

\[ \psi_{4N}(t) = \left[ \delta_1^T(t), \delta_2^T(t), \delta_3^T(t), \cdots, \delta_N^T(t) \right]^T \]  

(23)

\[ \psi_{5N}(t) = \left[ \vartheta_1^T(t), \vartheta_2^T(t), \vartheta_3^T(t), \cdots, \vartheta_N^T(t) \right]^T \]  

(24)

\[ \eta_1(t) = \left[ x^T(t), x^T(t-\tau), \tau \vartheta_1^T(t), \tau^2 \vartheta_1^T(t), \cdots, \tau^N \vartheta_1^T(t) \right]^T \]  

(25)

\[ \eta_2(t) = \left[ x^T(t) - x^T(t_k), x^T(t-\tau) - x^T(t_k - \tau) \right]^T \]  

(26)

\[ \eta_3(t) = \left[ x^T(t) - x^T(t_{k+1}), x^T(t-\tau) - x^T(t_{k+1} - \tau) \right]^T \]  

(27)
\[ \eta_4(t) = (t - t_k) \left[ \begin{array}{c} \psi_{1N}^T(t) \\ \psi_{2N}^T(t) \end{array} \right]^T \]

\[ \eta_5(t) = (t_{k+1} - t) \left[ \begin{array}{c} \psi_{3N}^T(t) \\ \psi_{4N}^T(t) \end{array} \right]^T \]

\[ \eta_{6k} = \left[ x^T(t_k), x^T(t_{k+1}), x^T(t_k - \tau), x^T(t_{k+1} - \tau) \right]^T \]

\[ \eta_7(t) = \left[ \eta_1^T(t), \eta_2^T(t) \right]^T, \quad \eta_8 = \left[ x^T(t), \dot{x}^T(t) \right]^T, \]

\[ \eta_9(t) = \left[ \eta_3^T(t), \eta_4^T(t), \psi_{2N}(t), \psi_{3N}(t) \right]^T \]

\[ e_j = \left[ 0_{n \times (j - 1)n}, I, 0_{n \times (5N + 8 - j)n} \right], \quad (j = 1, 2, \ldots, 5N + 8) \]

Now, by utilizing two-sided sampling-interval-dependent looped Lyapunov functional method and integral inequality technique, an \(N\)-dependent stability condition for sampled-data systems will be established as follows.

**Theorem 1**: For given scalars \(\tau \geq 0, h_1, h_2\) with \(0 < h_1 \leq h_2\), and a positive integer \(N\), (4) \(\land\) (5) is asymptotically stable if there exist matrices \(P_N > 0, R_i > 0, (i = 1, 2, 3, 4), S_1 > 0, S_2 > 0, Q_1, Q_2N, M = M^T, Z_i, L_{jN}, H_j, (j = 1, 2, 3, 4), U_1, U_2\) such that, for all \(\forall h_k \in [h_1, h_2]\)

\[
\begin{bmatrix}
\Phi_{1N} + h_k \Phi_{2N} & \sqrt{h_k} \Sigma_{16N}^T L_{3N}^T \\
* & -R_{3N}^T
\end{bmatrix}
\begin{bmatrix}
\sqrt{h_k} \Sigma_{16N}^T L_{3N} \\
0
\end{bmatrix} < 0
\]

\[
\begin{bmatrix}
\Phi_{1N} + h_k \Phi_{3N} & \sqrt{h_k} \Sigma_{14N}^T L_{1N}^T \\
* & -R_{1N}^T
\end{bmatrix}
\begin{bmatrix}
\sqrt{h_k} \Sigma_{14N}^T L_{1N} \\
0
\end{bmatrix} < 0
\]

where

\[ \Phi_{1N} = \text{Sym} \left[ \Sigma_{1N}^T P_N \Sigma_{2N} + \Sigma_{3}^T Q_1 \Sigma_{3} + \Sigma_{4}^T Q_1 \Sigma_{4} \right. \\
- \Sigma_{1}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) + \Sigma_{1}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) \\
+ \Sigma_{3}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) + \Sigma_{1}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) \\
+ \Sigma_{3}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) + \Sigma_{1}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) \\
- \Gamma_{2N} \Sigma_{16N} - H_1 \Sigma_{20} - H_2 \Sigma_{21} - H_3 \Sigma_{22} - H_4 \Sigma_{25} \\
\left. + \Sigma_{28} \Sigma_{29} + \Sigma_{29} \Sigma_{17} - \Sigma_{16} \Sigma_{18} + \tau^2 e_3 s_2 e_3 \right] - \Sigma_{19N} \Gamma_{2N} s_{2N} \Sigma_{19N} \]

\[ \Phi_{2N} = \text{Sym} \left[ \Sigma_{8N}^T Q_{2N} \Sigma_{7N} + \Sigma_{5}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) \\
+ \Sigma_{5}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) + \Sigma_{5}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) \\
+ \Sigma_{5}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) + \Sigma_{5}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) \right] \\
e_3 R_1 e_3 + e_4 R_2 e_4 + \Sigma_{10}^T M_{10} \\
\Phi_{3N} = \text{Sym} \left[ \Sigma_{6N}^T Q_{2N} \Sigma_{9N} + \Sigma_{4}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) \\
+ \Sigma_{4}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) + \Sigma_{4}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) \\
+ \Sigma_{4}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) + \Sigma_{4}^T (Z_1 \Sigma_{10} + Z_2 \Sigma_{11}) \right] \\
e_3 R_1 e_3 + e_4 R_2 e_4 - \Sigma_{10}^T M_{10} \]

with \(\Gamma_{IN}, (i = 1, 2)\) are defined in (6), and

\[ R_{IN} = \text{diag} \left[ R_1, 3R_1, \ldots, (2N + 1)R_1 \right], \quad I = 1, 2, 3, 4 \]

\[ S_{2N} = \text{diag} \left[ S_2, 3S_2, \ldots, (2N + 1)S_2 \right] \]
Proof: For system (4), we construct the following two-side sampling-interval-dependent looped Lyapunov functional

\[ V(t) = \sum_{i=1}^{n} V_i(t), \quad t \in [t_k, t_{k+1}) \]  

(19)

where

\[
\begin{align*}
V_1(t) &= \eta_1^T(t)P_N \eta_1(t) \\
V_2(t) &= 2 \left[ \eta_2^T(t)Q_1 \eta_3(t) + \eta_4^T(t)Q_{2N} \eta_5(t) \right] \\
V_3(t) &= (t_{k+1} - t)(t - t_k)N_{10k}^T M_{10k} \\
V_4(t) &= 2(t_{k+1} - t) \eta_2^T(t) [Z_1 \eta_6 + Z_2 \eta_7(t)] \\
V_5(t) &= 2(t - t_k) \eta_3^T(t) [Z_3 \eta_6 + Z_4 \eta_7(t)] \\
V_6(t) &= (t_{k+1} - t) \int_{t_k}^{t} \dot{x}^T(s)R_1 \dot{x}(s) ds \\
V_7(t) &= (t_{k+1} - t) \int_{t_k}^{t} \dot{x}^T(s)R_2 \dot{x}(s) ds \\
V_8(t) &= -(t - t_k) \int_{t_k}^{t} \dot{x}^T(s)R_3 \dot{x}(s) ds \\
V_9(t) &= -(t - t_k) \int_{t_k}^{t} \dot{x}^T(s)R_4 \dot{x}(s) ds \\
V_{10}(t) &= \int_{t_k}^{t} \eta_5^T(s)S_1 \eta_8(s) ds \\
&\quad + \tau \int_{t_k}^{t} \int_{s}^{t} \dot{x}^T(u)S_2 \dot{x}(u) du ds
\end{align*}
\]

The looped Lyapunov functional \( V(t) \) is continuous in time because the functional terms \( V_p(t), (p = 2, 3, \ldots, 9) \) vanish at the sampling time before and after \( t_k \), i.e., \( \lim_{t \to t_k} V(t) = V(t_k) \geq 0 \). We can also see that \( V_p(t_k) = V_p(t_{k+1}) = 0, (p = 2, 3, \ldots, 9) \), which means that the functional \( \sum_{p=2}^{9} V_p(t) \) satisfies the looped-functional condition in [41] and [42]. In addition, it is worth noting that the functional \( V(t) \) does not demand to be positive at all times, but is only required to be positive definite at sampling instants. Since this kind of looped Lyapunov functional can decrease the conservatism of the results by relaxing matrix constraints in the Lyapunov functional, they have widely used to sampled-data systems [18]–[23], [39].

Remark 1: The looped Lyapunov functional (19) has three conspicuous characters compared to the existing ones. First, it includes not only some \((t_{k+1} - t)\)-dependent terms (i.e., \( V_2(t), V_4(t), V_6(t), V_7(t) \)) but also some \((t - t_k)\)-dependent terms (i.e., \( V_5(t), V_8(t), V_9(t) \)), which fully utilizes the state information of the sampling intervals \( x(t_k) \) to \( x(t) \), \( x(t_k) \) to \( x(t_{k+1}) \) and \( x(t - \tau) \) to \( x(t_{k+1} - \tau) \). Second, it forms a hierarchy due to that it depends on the positive integer \( N \). For example, the functional term \( V_1(t) = \eta_1^T(t)P \eta_1(t) \) is dependent on the augmented vector \( \eta_1(t) = [x^T(t), \dot{x}^T(t - \tau), \tau \theta_1^T(t), \tau^2 \theta_1^T(t), \cdots, \tau^N \theta_1^T(t)]^T \) that includes the integral of the states \( \theta_1(t), k = 1, 2, \cdots, N \), which means that the system information of \( V_1(t) \) will be increased if \( N \) is increased. Thus, it is dependent on the positive integer \( N \). Third, the proposed functional (19) is tailored for the generalized \( N \)-order free-matrix-based inequality and canonical \( N \)-order Bessel-Legendre inequality.

Before computing the derivative of the Lyapunov functions (19), let us make a simple calculation as follows

\[
\frac{d}{dt} [(t - t_k) \alpha_j(t)] = \begin{cases} x(t), & j = 1 \\
(j - 1) [\alpha_{j-1}(t) - \alpha_j(t)], & j \geq 2 \end{cases}
\]

\[
\frac{d}{dt} [(t_{k+1} - t) \beta_j(t)] = \begin{cases} -x(t), & j = 1 \\
(j - 1) [\beta_{j+1}(t) - \beta_j(t)], & j \geq 2 \end{cases}
\]

\[
\frac{d}{dt} [(t - t_k) \gamma_j(t)] = \begin{cases} x(t - \tau), & j = 1 \\
(j - 1) [\gamma_{j-1}(t) - \gamma_j(t)], & j \geq 2 \end{cases}
\]

\[
\frac{d}{dt} [(t_{k+1} - t) \delta_j(t)] = \begin{cases} -x(t - \tau), & j = 1 \\
(j - 1) [\delta_{j+1}(t) - \delta_j(t)], & j \geq 2 \end{cases}
\]

\[
\frac{d}{dt} [\tau \dot{\theta}_j(t)] = \begin{cases} x(t - x(t - \tau), & j = 1 \\
\tau^{j-1} [x(t - (j - 1)) \theta_{j-1}(t)], & j \geq 2 \end{cases}
\]

Thus, we have

\[
\begin{align*}
\eta_1(t) &= \mathcal{E}_{1N} \xi(t), \\
\eta_2(t) &= \mathcal{E}_{3N} \xi(t), \\
\eta_3(t) &= \mathcal{E}_{4N} \xi(t), \\
\eta_4(t) &= (t - t_k) \mathcal{E}_{6N} \xi(t), \\
\eta_5(t) &= (t_{k+1} - t) \mathcal{E}_{7N} \xi(t) \\
\eta_6(t) &= \mathcal{E}_{10} \xi(t), \\
\eta_7(t) &= \mathcal{E}_{11} \xi(t), \\
\eta_8(t) &= \mathcal{E}_{2N} \xi(t), \\
\eta_9(t) &= \mathcal{E}_{5N} \xi(t), \\
\eta_{10}(t) &= \mathcal{E}_{8N} \xi(t), \\
\eta_{11}(t) &= \mathcal{E}_{9N} \xi(t), \\
\eta_{12}(t) &= \mathcal{E}_{12} \xi(t)
\end{align*}
\]

Then, taking the time derivative of looped Lyapunov functional \( V(t) \) along the trajectory of system (4) yields

\[
\begin{align*}
\dot{V}_1(t) &= 2 \eta_1^T(t)P \eta_1(t) \\
\dot{V}_2(t) &= 2 [\eta_2^T(t)Q_1 \eta_3(t) + \eta_4^T(t)Q_2N \eta_5(t) + \eta_6^T(t)Q_{2N} \eta_7(t)] \\
&= 2 \xi^T(t) \mathcal{E}_{7N} \xi + \mathcal{E}_{3N}^T \mathcal{E}_{4N} + (t_{k+1} - t) \mathcal{E}_{5N} \mathcal{E}_{6N} \\
&= \mathcal{E}_{10}^T(t) \mathcal{E}_{11} \xi(t) \\
\dot{V}_3(t) &= [(t_{k+1} - t) - (t - t_k)] \eta_{10} \mathcal{E}_{6N} M_{10k} \eta_{10} \\
\dot{V}_4(t) &= -2 \eta_2^T(t) [Z_1 \eta_6 + Z_2 \eta_7(t)] + 2(t_{k+1} - t) \\
&\times \eta_2^T(t) [Z_1 \eta_6 + Z_2 \eta_7(t)] + \eta_2^T(t)Z_2 \eta_7(t) \\
&= 2 \xi^T(t) \mathcal{E}_{7N} \xi + \mathcal{E}_{3N}^T \mathcal{E}_{4N} + (t_{k+1} - t) \\
&\times \eta_2^T(t) [Z_3 \eta_6 + Z_4 \eta_7(t)] + \eta_2^T(t)Z_4 \eta_7(t) \\
\dot{V}_5(t) &= \eta_4^T(t) \mathcal{E}_{9N} \mathcal{E}_{10N} \xi(t) + 2(t - t_k)^T \eta_4^T(t) \\
&\times \eta_4^T(t) \mathcal{E}_{9N} \mathcal{E}_{10N} \xi(t) + \eta_4^T(t) \mathcal{E}_{9N} \mathcal{E}_{10N} \xi(t) \\
&= 2 \xi^T(t) \mathcal{E}_{7N} \xi + \mathcal{E}_{3N}^T \mathcal{E}_{4N} + (t - t_k) \\
&\times \eta_4^T(t) \mathcal{E}_{9N} \mathcal{E}_{10N} \xi(t) + \mathcal{E}_{9N}^T \mathcal{E}_{10N} \xi(t) \\
\dot{V}_6(t) &= \xi^T(t) \mathcal{E}_{12} \xi(t) \\
&\quad - \int_{t_k}^{t_{k+1}} \dot{x}^T(s)R_1 \dot{x}(s) ds \\
\dot{V}_7(t) &= \xi^T(t) \mathcal{E}_{12} \xi(t) \\
&\quad - \int_{t_k}^{t_{k+1}} \dot{x}^T(s)R_2 \dot{x}(s) ds
\end{align*}
\]
\[
\dot{V}_8(t) = \xi^T(t)\left[(t - t_k)e_3^T R_3 e_3\right] \xi(t) - \int_{t_{k-1}}^{t_k} \dot{x}^T(s) R_1 \dot{x}(s) ds \\
\dot{V}_9(t) = \xi^T(t)\left[(t - t_k)e_4^T R_4 e_4\right] \xi(t) - \int_{t_{k-1}}^{t_k} \dot{x}^T(s) R_2 \dot{x}(s) ds \\
\dot{V}_{10}(t) = \xi^T(t)\left[\Xi_{17}^T S_{17} \Xi_{17}^{-1} - \Xi_{18}^T S_{18} + \tau_2^2 S_{e3} e_3\right] \xi(t) - \tau \int_{t_{k-1}}^{t_k} \dot{x}^T(s) S_2 \dot{x}(s) ds
\]

Applying the generalized N-order free-matrix-based inequality (14) with \(\xi_N = \sigma_{1N}(t)\) or \(\xi_N = \sigma_{2N}(t)\), we get
\[
- \int_{t_{k-1}}^{t_k} \dot{x}^T(s) R_1 \dot{x}(s) ds \\
\leq \xi^T(t)\left[\text{Sym}\left(\Xi_{13N}^T L_{13N}^T \Gamma_{1N} \Xi_{13N}\right) + (t - t_k)\Xi_{13N}^T L_{13N}^T R_{13N}^{-1} L_{13N} \Xi_{13N}\right] \xi(t)
\]

\[
- \int_{t_{k-1}}^{t_k} \dot{x}^T(s) R_2 \dot{x}(s) ds \\
\leq \xi^T(t)\left[\text{Sym}\left(\Xi_{14N}^T L_{14N}^T \Gamma_{1N} \Xi_{14N}\right) + (t - t_k)\Xi_{14N}^T L_{14N}^T R_{14N}^{-1} L_{14N} \Xi_{14N}\right] \xi(t)
\]

\[
- \int_{t_{k-1}}^{t_k} \dot{x}^T(s) R_3 \dot{x}(s) ds \\
\leq \xi^T(t)\left[\text{Sym}\left(\Xi_{15N}^T L_{15N}^T \Gamma_{2N} \Xi_{15N}\right) + (t - t_k)\Xi_{15N}^T L_{15N}^T R_{15N}^{-1} L_{15N} \Xi_{15N}\right] \xi(t)
\]

\[
- \int_{t_{k-1}}^{t_k} \dot{x}^T(s) R_4 \dot{x}(s) ds \\
\leq \xi^T(t)\left[(t_{k+1} - t)\Xi_{16N}^T L_{16N}^T R_{16N}^{-1} L_{16N} \Xi_{16N} + \text{Sym}\left(\Xi_{16N}^T L_{16N}^T \Gamma_{2N} \Xi_{16N}\right)\right] \xi(t)
\]

where \(\Gamma_{1N}, \Gamma_{2N}, \text{ and } R_{IN}, (I = 1, 2, 3, 4)\) are defined in (6) and (17), respectively. From Lemma 1, we have
\[
- \int_{t_{k-1}}^{t_k} \dot{x}^T(s) S_2 \dot{x}(s) ds \leq -\xi^T(t)\left[\Xi_{19N}^T \Gamma_{2N} \Xi_{19N}\right] \xi(t)
\]

where \(S_{2N}\) is defined in (18).

Integrating both sides of system (4) on sampling intervals \([t_k - \tau, t - \tau]\) and \([t - \tau, t_{k+1} - \tau]\), respectively, we obtain
\[
x(t - \tau) - x(t_k - \tau) = (t - t_k)A \gamma_1(t) + A_d x(t_k - \eta) \tag{35}
\]
\[
x(t_{k+1} - \tau) - x(t - \tau) = (t_{k+1} - t)A \delta_1(t) + A_d x(t - \eta) \tag{36}
\]

Thus, from (35) and (36), the following equalities hold for any matrices \(H_1\) and \(H_2\) of appropriate dimensions
\[
0 = 2\xi^T(t)H_1\left[(t - t_k)\Xi_{22} - \Xi_{20}\right] \xi(t) \tag{37}
\]
\[
0 = 2\xi^T(t)H_2\left[(t_{k+1} - t)\Xi_{23} - \Xi_{21}\right] \xi(t) \tag{38}
\]

Similarly, integrating system (4) on sampling intervals \([t_k, t]\) and \([t, t_{k+1}]\), respectively, it can be derived that
\[
0 = 2\xi^T(t)H_3\left[(t - t_k)\Xi_{26} - \Xi_{24}\right] \xi(t) \tag{39}
\]
\[
0 = 2\xi^T(t)H_4\left[(t_{k+1} - t)\Xi_{27} - \Xi_{25}\right] \xi(t) \tag{40}
\]

Besides, for any matrices \(U_1\) and \(U_2\), it can be obtained from (4) that
\[
0 = 2[x^T(t)U_1 + \dot{x}^T(t)]U_2[\dot{x}(t) + A_d x(t_k - \tau) - \dot{x}(t)]
\]
\[
= 2\xi^T(t)\Xi_{28} \Xi_{29} \xi(t). \tag{41}
\]

From (20)-(34) and (37)-(41), the upper bound of \(\dot{V}(t)\) can be obtained as
\[
\dot{V}(t) = \xi^T(t)\left[\frac{(t_{k+1} - t)}{h_k} \Delta_1(h_k) + \frac{(t - t_k)}{h_k} \Delta_2(h_k)\right] \xi(t) \tag{42}
\]

where
\[
\Delta_1(h_k) = \Phi_{1N} + \Phi_{2N} + h_k \Xi_{15N}^T L_{15N}^T R_{15N}^{-1} L_{15N} \Xi_{15N} + h_k \Xi_{16N}^T L_{16N}^T R_{16N}^{-1} L_{16N} \Xi_{16N}
\]
\[
\Delta_2(h_k) = \Phi_{1N} + \Phi_{3N} + h_k \Xi_{14N}^T L_{14N}^T R_{14N}^{-1} L_{14N} \Xi_{14N} + h_k \Xi_{14N}^T L_{14N}^T R_{14N}^{-1} L_{2N} \Xi_{14N} \tag{43}
\]

According to the Schur complement, \(\Delta_1(h_k) < 0\) and \(\Delta_2(h_k) < 0\) are equivalent to (15) and (16), respectively, which lead to \(\dot{V}(t) < 0\). Thus, it follows from Theorem 1 in [41] that system (4) is asymptotically stable. The proof is completed.

Remark 2: The augmented term \(V_1(t)\) is dependent on the integral terms \(\varphi_p(t), (p = 1, 2, \ldots, N)\), which is inherited from canonical N-order Bessel-Legendre inequality. Similarly, the term \(V_2(t)\) is dependent on the integral terms \(\alpha_p(t), \beta_p(t), \gamma_p(t),\) and \(\delta_p(t), (p = 1, 2, \ldots, N)\), which is inherited from generalized N-order free-matrix-based inequality. That is to say, the looped Lyapunov functional \(V(t)\) is tailored for the use of the above two inequalities. As stated in [24] and [32], if these integral terms of the state are not introduced in Lyapunov functional \(V(t)\), they will not appear in the derivative of the Lyapunov functional \(V(t)\). Thus, the above two inequalities are not so useful for decreasing conservativeness of the derived results, even though they offer a tighter bound for the single integral term.

Remark 3: Theorem 1 is derived through the use of the N-order-based inequalities (5), (14), and the looped Lyapunov functional (19) that depends on the positive integer \(N\), that is, it is also dependent on the integer \(N\). In other words, Theorem 1 is an N-dependent stability criterion, which form a hierarchy of linear matrix inequality conditions: the larger \(N\), the less conservatism of the results, which is demonstrated through two examples in Section IV. Besides, recalling some existing results (see [5]–[22], [39]), most stability criteria for sampled-data system are established based first-order (\(N = 1\)) free-matrix-based inequality or first-order (\(N = 1\)) Bessel-Legendre inequality or their equivalent versions. It is obvious that Theorem 1 in this paper provides a more general form.

Remark 4: The use of the inequality (14) introduces some slack matrix variables \(L_{1N}, L_{2N}, L_{3N}\), and \(L_{4N}\), which fundamentally enhance the relationships among various extra-states to achieve more flexibility of the linear matrix inequalities. In the process of proof, the four zero-value equations, (37)-(40), are introduced in Theorem 1, which further
utilize the information on the realistic sampling intervals \([t_k - \tau, t - \tau], [t - \tau, t_{k+1} - \tau], [t_k, t],\) and \([t, t_{k+1})\). The above-mentioned two features are also important for decreasing conservatism of the obtained stability criterion.

**Remark 5:** Notice that the literatures [5], [6], [39] only employ the system information on the sampling intervals \([t_k, t)\) and \([t_k - \tau, t - \tau)\), while the system information on the sampling intervals \([t, t_{k+1})\) and \([t - \tau, t_{k+1} - \tau)\) are completely ignored. Similar to [7], the proposed approach makes full use of the realistic information about the whole sampling intervals \([t_k, t_{k+1})\) and \([t_k - \tau, t_{k+1} - \tau)\), that is, it considers not only the information about the sampling intervals \([t_k, t)\) and \([t_k - \tau, t)\) through the introduction of the terms \(V_2(t), V_4(t), V_6(t),\) and \(V_7(t)\), but also the information about the sampling intervals \([t, t_{k+1})\) and \([t - \tau, t_{k+1} - \tau)\) through the use of the new terms \(V_3(t), V_5(t), V_8(t),\) and \(V_9(t)\). Therefore, the stability criterion obtained in this paper is less conservative than [5], [6], [39].

In order to show the advantage of the zero-value equations, (37)-(40), the following corollary is derived.

**Corollary 1:** (Without the Four Zero-Value Equations (37)-(40)): For given scalars \(\tau \geq 0, h_1, h_2\) with \(0 < h_1 \leq h_2\), and a positive integer \(N\), system (4) is asymptotically stable if there exist matrices \(P_N > 0, R_i > 0, (i = 1, 2, 3, 4), S_1 > 0, S_2 > 0, Q_1, Q_{2N}, M = M^T, Z_j, L_j, (j = 1, 2, 3, 4), U_1, U_2\) such that, for \(\forall h_k \in \{h_1, h_2\}\)

\[
\begin{bmatrix}
P_{1N} + h_kP_{2N} & \sqrt{h_k}z_{15N}^TL_{3N}^T & \sqrt{h_k}z_{16N}^TL_{4N}^T \\
* & -R_{3N} & 0 \\
* & * & -R_{4N}
\end{bmatrix} < 0
\]

(43)

\[
\begin{bmatrix}
P_{1N} + h_kP_{3N} & \sqrt{h_k}z_{13N}^TL_{1N}^T & \sqrt{h_k}z_{14N}^TL_{2N}^T \\
* & -R_{1N} & 0 \\
* & * & -R_{2N}
\end{bmatrix} < 0
\]

(44)

where

\[
P_{1N} = \Phi_{1N}|_{(h_1=0, h_2=0, h_3=0, h_4=0)}
\]

\[
P_{2N} = \Phi_{2N}|_{(h_1=0, h_2=0)}, \quad \Psi_{3N} = \Phi_{3N}|_{(h_1=0, h_3=0)}
\]

\[
\Phi_{1N}, \Phi_{2N} \text{ and } \Phi_{3N} \text{ are defined in Theorem 1.}
\]

**Remark 6:** Theorem 1 and Corollary 1 require \((8.5N + 39N + 98.5)n^2 + (0.5N + 6.5)n\) and \((8.5N + 19N + 66.5)n^2 + (0.5N + 6.5)n\) decision variables, respectively. Theorem 1 and Corollary 1 in Section IV shows that a larger \(N\) will lead to a larger the admissible upper bound \(\tau\) or \(h_2\), while a larger number decision variables and calculation complexity will be required. However, Theorem 1 and Corollary 1 can adjust the size of \(N\) to achieve tradeoff between conservativeness and calculation complexity for the derived results.

**IV. NUMERICAL EXAMPLES**

In this section, two examples are provided to show the validity and the superiority of the presented approaches.

**TABLE 1.** The maximum allowable upper bound of \(h_2\) under aperiodic sampling with \(h_1 = 10^{-5}\) for different \(\tau\).

| Methods | \(h_2=0.1\) | \(h_2=0.3\) | \(h_2=0.5\) | \(h_2=0.7\) | \(h_2=0.9\) |
|---------|-------------|-------------|-------------|-------------|-------------|
| Cor.1 \((N=1)\) | 1.022 | 0.9175 | 0.8344 | 0.6896 | 0.5369 |
| Cor.1 \((N=2)\) | 1.036 | 0.9371 | 0.8370 | 0.6931 | 0.5395 |
| Th.1 \((N=1)\) | 1.177 | 1.0158 | 0.9095 | 0.7963 | 0.6739 |
| Th.1 \((N=2)\) | 1.207 | 1.0170 | 0.9108 | 0.7968 | 0.6742 |

**Example 1:** Consider system (4) with the following matrix values

\[
\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} \begin{bmatrix} 3.75 \\ 11.5 \end{bmatrix} x(t_k - \tau)
\]

For different communication delays \(\tau\), the maximum allowable upper bound of \(h_2\) calculated by Theorem 1, Corollary 1 and the criteria in [5], [6], [39] are listed in Table 1. As shown in Table 1, it is observed that the proposed methods in this paper are less conservative than the methods in [5], [6], [39]. It is also revealed from Table 1 that Theorem 1 provides less conservative results than Corollary 1. This is due to the fact that the zero-value equations, (37)-(40), are introduced in the proof of Theorem 1.

In addition, for different values of \(h_2\) with \(h_1 = 10^{-5}\), by applying Theorem 1 and Corollary 1 with \(N = 1\) and \(N = 2\), the maximum allowable delay upper bounds \(\tau\) can be obtained, which are summarized in Table 2. It is clear to see that Theorem 1 can provide a larger delay upper bound \(\tau\) than Corollary 1. Furthermore, the derived stability criteria are dependent on the positive integer \(N\). As we can see in Table 1 and Table 2, these \(N\)-dependent stability criteria have a hierarchical feature: the larger \(N\), the lower conservativeness of the results.

Choose the initial condition \(x_0 = [2, -1.8]\), the sampling upper bound \(h_2 = 0.9\) and the communication delay \(\tau = 0.6742\), and then the state trajectories of this system are depicted in Figure 1. It is observed from Figure 1 that this system is stable, which verifies the effectiveness of the presented approaches.

**Example 2:** In this example, we will show the practical application of the developed methods in a practical systems. Thus, let us consider the following electric power market model given in [38].

\[
\begin{align*}
\tau_g \dot{P}_g(t) &= \lambda(t_k - \tau) - b_g - c_g P_g(t) - kE(t), \\
\tau_d \dot{P}_d(t) &= b_d + c_d P_d(t) - \lambda(t_k - \tau), \\
\dot{E}(t) &= P_g(t) - P_d(t), \\
\tau_d \dot{\lambda}(t) &= -E(t)
\end{align*}
\]
where \( P_g(t), P_d(t), E(t) \) and \( \lambda(t) \) are the amount of generated power, the amount of consumed power, the time integral of the difference in supply and demand and the observed power price, respectively; \( \tau_g, \tau_d \) and \( \tau_c \) are the time constant, which denote the change rate of supply, demand, and price in response to market perturbations, respectively; \( b_g + c_g P_g(t) \) is the marginal production cost; \( kE(t) \) is the additional cost paid by the producer when there is a history of excess supply; \( b_d + c_d P_d(t) \) presents the marginal benefit function; \( b_g \) and \( b_d \) are the fixed cost and benefit; \( c_g \) and \( c_d \) are the fixed coefficient; \( t_k \) denotes the updating instant of the price satisfying

\[ t_{k+1} - t_k = T_{mct_k} \in [T_{mct_1}, T_{mct_2}], \quad \forall k \in \mathbb{N} \]

with \( T_{mct_1} \) and \( T_{mct_2} \) indicate the upper bound and lower bound of market clearing time, i.e. the updating period of price power, and \( \tau \) represents a communication delay.

The model parameters are given as \( \tau_g = 0.2, \tau_d = 0.1, b_g = 2, \tau_c = 0.1, c_d = -0.2, b_d = 10, \tau_l = 100 \) and \( k = 0.1 \). Then, the power market model (45) can be represented by the following sampled-data control system:

\[ \dot{z}(t) = Az(t) + A_d z(t - \tau) + J, \quad t \in [t_k, t_{k+1}) \quad (46) \]

where \( t_{k+1} - t_k = T_{mct_k} \in [T_{mct_1}, T_{mct_2}] \) and

\[
\begin{align*}
A(z) &= \begin{bmatrix} P_g(t) \\ P_d(t) \\ E(t) \\ \lambda(t) \end{bmatrix}, & A = \begin{bmatrix} -0.5 & 0 & -0.5 & 0 \\ 0 & -2 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -0.01 & 0 \end{bmatrix} \\
A_d &= \begin{bmatrix} 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & -10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, & J = \begin{bmatrix} -10 \\ 100 \\ 0 \\ 0 \end{bmatrix}
\end{align*}
\]

There exists an equilibrium \( z^* \). By defining vector \( x = z - z^* \), the system (46) can be transformed into

\[ \dot{x}(t) = Ax(t) + A_d x(t - \tau) \quad (47) \]

where \( A, A_d \) and \( \tau \) have same physical meanings in model (46). Therefore, the proposed approaches in this paper can be also used to analyze the stability of the electric power markets system for different communication delay \( \tau \) or market clearing time \( T_{mct} \).

By employing Theorem 1 with \( T_{mct} = 10^{-5} \) and the methods in [5], [39], the acceptable maximum values of market clearing time \( T_{mct} \) with respect to different \( \tau \) are obtained and are listed in Table 3, while the acceptable maximal communication delay \( \tau \) with respect to different \( T_{mct} \) are also calculated and are summarized in Table 4. From the results, it is obvious that our methods provide larger market clearing time \( T_{mct} \) (or communication delay \( \tau \)) when compared with that obtain in [5], [39], which means that our approaches are less conservative than the approaches obtained in [5], [39].

The equilibrium points of system (46) can be obtained as \( z^* = [26.68, 26.68, 0, 4.66]^T \). Choose the initial condition \( z_0 = [20, 20, 0, 0]^T \) and \( T_{mct} = 2 \), the state trajectories of system (46) for various delay \( \tau \) are given in Figure 2. It is shown in Figure 2 that the acceptable maximal communication delay \( \tau \) of system (46) is about 8.4. The result is a little larger than ours 8.1712, which implies our methods are still existing conservatism. However the conservatism is very small.
Next, we discuss the significance for the reduction of the conservativeness. The obtained results by Theorem 1 can be utilized as a guideline to select appropriate operation environment such as choosing the capacity and type of network channels, and determining the frequency of the collector. Obviously, Theorem 1 can provide less conservative results. Therefore, a looser operation environment for ensuring the energy balance for the electric power grid is allowable based on our methods than the methods proposed in [5], [39]. Consider the following cases to illustrate its importance:

Selecting the networks channel with same communication delay (for instance, $\tau = 4$) to transmit signals, the obtained results in [39] shows that the maximal market clearing time $T_{\text{mat}}$ for maintaining system (46) stable should be less than 5.0083, while a larger market clearing time, 7.2991, can be tolerated based on our method. The larger market clearing time means that we can use less time to collect the price signals in a fixed sampling period and then they can focus on improving the power supply quality. Besides, the lower frequency of collector can save the cost.

Thus, a relaxed operation environment with a larger communication and market clearing time tolerance is important for helping to achieve the operation of power market system. This illustrates the importance of the reduction of conservativeness.

V. CONCLUSION

The problem of stability of sampled-data control systems with nonuniform sampling and communication delays has been investigated. A novel two-sided sampling-interval-dependent looped Lyapunov functional have been proposed to establish less conservative stability criteria for sampled-data system. It has been proven that the derived stability conditions in this work exhibit a hierarchical feature: with $N$ increasing, the conservatism of the results decreases. Finally, two illustrative examples have been taken to show the effectiveness and superiority of the presented methods.

In addition, the proposed looped Lyapunov functional in this paper can be also extended to study the control synthesis problem for other various dynamical system such as networked control system [26], neural networks [43], [44], T-S fuzzy systems [45], and so on. By employing the Lyapunov functional obtained from the method, future work will focus on the research of stability, stabilization and synchronization control for the above mentioned systems.

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