Threshold Resummation for Higgs Production in Effective Field Theory

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We present an effective field theory approach to resum the large double logarithms originated from soft-gluon radiations at small final-state hadron invariant masses in Higgs and vector boson (γ∗, W, Z) production at hadron colliders. The approach is conceptually simple, independent of details of an effective field theory formulation, and valid to all orders in sub-leading logarithms. As an example, we show the result of summing the next-to-next-to-next-to leading logarithms is identical to that of the standard pQCD factorization method.

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1. Introduction. In hadron colliders such as Tevatron and LHC, the rates on Higgs boson and Drell-Yan pair production demand reliable perturbative quantum chromodynamics (pQCD) calculations. When the final-state invariant mass of hadrons is small, a fixed-order pQCD calculation yields large threshold double logarithms in the coefficient functions

\[ \alpha_s^k \left[ \ln^{n-1} (1-z) \right], \quad (m \leq 2k) \]  

which must be resummed to all orders in \( \alpha_s \), where \( 1-z \) is the fraction of center-of-mass energy of the initial partons going into soft radiations. In moment space, these large logarithms appear in the form, \( \alpha_s^k \ln^{n-1} N \), where \( N = N \exp(\gamma_E) \) with \( N \) the order of moment and \( \gamma_E \), the Euler constant. In the past decade, a standard method based on pQCD factorization has been established to perform the resummation \([1, 2, 3]\), and it has been carried out to next-to-next-to-next to leading logarithms (N³LL) for the above processes \([1, 2, 3, 4, 5, 6, 7, 8, 9, 10]\).

In this paper, we present an alternative, effective field theory (EFT) resummation of these large threshold logarithms. The EFT approach is conceptually simple, and is readily extended to other processes. It is motivated by the recent development of soft-collinear effective field theory \([11, 12]\) and its applications to threshold resummation \([13, 14]\). The major steps described here are standard in EFT methodology and are similar to those in Refs. \([13, 14]\). However, there are also significant differences: First, we work in the limit of \( Q = M_H \) (Higgs mass) going to infinity first (twist expansion), and \( z \to 1 \) subsequently. The kinematic region \( (1-z) \sim \Lambda_{QCD}/Q \), where there might be subtleties in formulating an effective theory \([13]\), is avoided. Second, our approach uses the full QCD results in the soft-collinear limit, and thus an actual formulation of an effective lagrangian is unnecessary. Finally, our resummation formula is valid to all orders in sub-leading logarithms and fully equivalent to the standard resummation. As an example, we demonstrate that the EFT resummation to N³LL agrees completely with the latest result in the literature using the traditional approach \([9, 10]\).

Let us consider the standard model Higgs production in hadron-hadron collisions. An almost identical discussion applies to Drell-Yan production of \( \gamma^*, W \) and \( Z \) bosons. At high energy, the Higgs cross section is dominated by gluon-gluon fusion through the top quark loop. For a large range of Higgs mass, the top quark can be considered as heavy and can be integrated out first \([10]\).

The effective lagrangian density for Higgs production is

\[ \mathcal{L} = -\frac{1}{4} C_\phi (M_t, \mu_R) \phi G^{\mu\nu} G_{\mu\nu}(\mu_R), \]  

where \( \phi \) is the scalar field, \( G^{\mu\nu} \) is the gluon field strength, \( C_\phi \) is the effective coupling \([17]\), and \( \mu_R \) is a renormalization scale. We focus on the kinematic limit in which the final-state hadron invariant mass \( (1-z)M_H \) is small in the sense that \( (1-z) \ll 1 \). The scale \( (1-z)M_H \), however, is still perturbative and is in principle much larger than \( \sqrt{\Lambda_{QCD}/M_H} \). In the above kinematic limit, the pQCD process is dominated by soft and collinear gluon radiations.

Renormalization scale \( \mu_R \) is arbitrary in principle, but we will set it to \( M_H \) in the following discussion. The process then has three independent scales: \( M_H, (1-z)M_H \) and \( \mu_F \), last of which is related to factoring the collinear divergences into Feynman parton distributions. To calculate the Higgs production cross section reliably in the threshold region, we shall study the physics at different scales separately using EFT techniques \([13]\).

2. Physics at scale \( M_H \), and between \( M_H \) and \( (1-z)M_H \). Momentum scale \( M_H \) is confined to the gluon-Higgs-ghon vertex region. To account for their contribution, we introduce the scalar current \( J_{QCD}(M_H) = G^{\mu\nu} G_{\mu\nu}(M_H) \) and make the operator expansion,

\[ J_{QCD}(M_H) = C_g (M_H/\mu, \alpha_s(\mu)) J_{\text{eff}}(\mu) + \ldots, \]  

where coefficient function \( C_g \) contains the perturbative contribution between momentum scale \( M_H \) and \( \mu \).
\( J_{\text{eff}}(\mu) \) contains the soft and collinear contributions below scale \( \mu \). To calculate the coefficient function, we take the matrix element of the above equation between gluon states—the left-hand side defines the gluon form factor \( F_g(M_H) \).

In dimensional regularization and minimal subtraction scheme, the gluon form factor can be calculated as a power series in the renormalized strong coupling,

\[
F_g(M_H) = C_g(M_H/\mu, \alpha_s(\mu)) S_g(M_H/\mu, \alpha_s(\mu), 1/\epsilon)
\]

where \( S_g \) contains the pole contributions only and can be regarded as the gluon matrix element of effective current \( J_{\text{eff}}(\mu) \). Expanding the coefficient function at \( \mu = M_H \) as \( C_g(1, \alpha_s(M_H)) = \sum a_i \mu^i C_{g(i)} \), and using the form factor calculated to the second order in \( \alpha_s \), we obtain

\[
C_{g(1)} = 7C_A \zeta_2
\]

\[
C_{g(2)} = C_A^2 \left( \frac{5105}{162} + \frac{335}{6} \zeta_2 - \frac{143}{9} \zeta_3 + \frac{125}{10} \zeta_2^2 \right)
\]

\[
+ \frac{C_A n_f}{6} \left( \frac{916}{81} - \frac{25}{3} \zeta_2 - \frac{46}{9} \zeta_3 \right)
\]

\[
+ \frac{C_F n_f}{6} \left( -\frac{67}{6} + 8 \zeta_3 \right)
\]

where \( \zeta_n \) is the Riemann zeta-function, \( n_f \) is the number of massless quark flavors, \( C_A = N_c \) and \( C_F = (N_c^2 - 1)/2N_c \) and \( N_c = 3 \) is the number of color.

The physics between \( M_H \) and \( M_H(1 - z) \) can be accounted for by integrating over the running scale \( \mu \) between them. This requires an anomalous dimension,

\[
\gamma_{1,g}(\alpha_s) = \mu \frac{d \ln C_g}{d \mu} = -\mu \frac{d \ln S_g}{d \mu} = \sum a_i \gamma_{1,g}^{(i)}
\]

A simple way to calculate this is to start with the following representation of the form factor [14],

\[
\ln F_g(\alpha_s) = \frac{1}{2} \int_0^{M_H/\mu^2} \frac{d \xi}{\xi} \left( K_g(\alpha_s(\mu), \epsilon) + G_g(1, \alpha_s(\xi \mu, \epsilon) + \int_1^{\xi} \frac{d \lambda}{\lambda} A_g(\alpha_s(\lambda \mu, \epsilon)) \right)
\]

where \( A_g \) is the cusp anomalous dimension of Wilson lines in adjoint representation [24], \( A_g = \sum a_i A_g^{(i)} \), and has been calculated to three-loops recently [21]. The soft function \( K_g \) contains only the infrared pole terms and can be constructed from \( A_g \) through \( \mu^2 dK_g/d\mu^2 = -A_g \).

The perturbative function \( G_g = \sum a_i G_g^{(i)} \) contains both finite terms and those vanishing when \( \epsilon \to 0 \). According to Ref. 22, \( G_g^{(i)} \) has a simple representation

\[
G_g^{(i)} = 2B_{2,g}^{(i)} - 2i\beta_{i-1} + f_{g}^{(i)} + \Delta G_g^{(i)}
\]

for \( i = 1, 2, 3 \), where \( \Delta G_g^{(i)} \) can be constructed from those vanishing terms in lower-order \( G_g^{(i)} \). \( B_{2,g} = \sum a_i B_{2,g}^{(i)} \) is the coefficient of \( \delta(1 - x) \) term in the gluon splitting function, with \( B_{2,g}^{(i)} = 11C_A/3 - 2n_f/3 \), etc [21]. The QCD \( \beta \)-function is defined as \( \beta(\alpha_s) = -d \ln a_s/d \ln \mu^2 = \beta_0 \alpha_s + \beta_1 \alpha_s^2 + ..., \) with \( \beta_0 = 11C_A/3 - 2n_f/3 \). The functions \( f_{g}^{(i)} \) are universal in the sense that the corresponding quark expressions are obtained by replacing the overall factor of \( C_A \) by \( C_F \).

The pole terms in Eq. (8), the \( S_g \) terms, will be used to calculate the anomalous dimension by employing the last two relations of Eq. (7). After taking into account the renormalization of the strong coupling constant \( \alpha_s(\mu) \), we get the anomalous dimension of the gluon current as

\[
\gamma_{1,g}(\alpha_s) = A_g^{(i)} \ln (M_H^2/\mu^2) + B_{1,g}^{(i)} + 2i\beta_{i-1},
\]

where

\[
B_{1,g}^{(i)} = -2B_{2,g}^{(i)} - f_{g}^{(i)}
\]

Since \( A_g^{(i)}, B_{2,g}^{(i)}, \) and \( f_{g}^{(i)} \) are known to three loops [22, 24], the anomalous dimension is now known to the same order.

Using the evolution equation, one can summarize the physics between scales \( M_H \) and \( \mu_1 \sim (1 - z)M_H \) in \( C_g(M_H/\mu_1, \alpha_s(\mu)) \) with a coefficient \( C_g(1, \alpha_s(M_H)) \) and running

\[
C_g \left( \frac{M_H}{\mu}, \alpha_s(\mu) \right) = C_g(1, \alpha_s(M_H)) \times \exp \left( -\int_{\mu}^{M_H} \frac{d \mu}{\mu} \gamma_{1,g}(\mu) \right)
\]

where the exponent contains Sudakov double logarithms.

3. **Physics at scale \((1 - z)M_H\)**: At this scale, one must consider soft-gluon radiations from the initial gluon partons. In principle, one should formulate a soft-collinear effective theory to calculate these contributions, as was done in Ref. 13. However, this is unnecessary in practice and the result can simply be obtained from a full QCD calculation at the appropriate kinematic limit.

The real emission diagrams without any internal radiative corrections contain both soft and collinear divergences. The infrared contributions from diagrams with internal radiative corrections (just the pole terms in dimensional regularization) serve to cancel the soft divergences. The collinear divergences can be factorized into
a standard Feynman parton distributions. The finite remainder is the coefficient function (or matching coefficient in the sense that we match a product of effective gluon currents onto a product of gluon light-cone distribution operators) which is what we are interested in. To see that the relevant physics happens around the scale \( \mu_I \) is to take Mellin transform of the coefficient function, which contains logarithms of type \( \alpha_s^2 \ln^m M_F^2/\mu^2 \). If \( \mu \) is set as \( \mu = M_H/\sqrt{N} \), the large logarithms disappear. Expanding the matching coefficient \( M_N = \sum \alpha_s^i(\mu_I) M_N^{(i)} \), the full pQCD calculation \( \text{in the soft limit yields} \)

\[
M_N^{(1)} = 2C_A \zeta_2
\]

\[
M_N^{(2)} = C_A^2 \left[ \frac{242}{81} + \frac{67}{9} \zeta_2 - \frac{22}{9} \zeta_3 - 10 \zeta_2^2 \right]
+ C_A N_F \left[ -\frac{328}{81} - \frac{10}{9} \zeta_2 + \frac{4}{9} \zeta_3 \right],
\]

(13)

at scale \( \mu_I = M_H/\sqrt{N} \). Note that the coupling constant \( \alpha_s \) is also evaluated at the intermediate scale \( \mu_I \).

4. **Physics between scales** \( \mu_I \) and \( \mu_F \), and at \( \mu_F \). In the previous section, QCDF factorization produces gluon distributions at scale \( \mu_I = M_H/\sqrt{N} \). We can bring the distributions to an arbitrary scale \( \mu_F \) using the standard DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) evolution. This introduces an evolution factor,

\[
\exp \left( 2 \int_{\mu_F}^{\mu_I} \frac{d\mu}{\mu} \gamma_{2,g}^N \right),
\]

(14)

where the twist-two anomalous dimension \( \gamma_{2,g}^N \) has the following large \( N \) behavior:

\[
\gamma_{2,g}^N = -A_g \ln N^2 + 2B_{2,g},
\]

(15)

where \( A_g \) and \( B_{2,g} \) are the same as those in Eqs. (8) and (9), respectively. The moment of gluon distributions introduces a factor,

\[
g(\mu_F, N)g(\mu_F, N) .
\]

(16)

To simplify the result, the factorization scale \( \mu_F \) is henceforth chosen to be \( M_H \).

5. **Resummation to all orders in sub-leading logarithms.** Putting all factors together, the \( M_H^2/\sqrt{s} \)-moment of the cross section is (\( s \) is the total center-of-mass energy squared) \( \text{in} \)

\[
\sigma_N = \sigma_0 \cdot G_N(M_H) \cdot g(M_H, N)g(M_H, N) ,
\]

(17)

where \( \sigma_0 \) is a reference cross section and

\[
G_N(M_H) = |C(\alpha_s(M_H))|^2 e^{I_1(M_H, \mu_I)} \times M_N(\alpha_s(\mu_I))e^{I_2(\mu_I, M_H)}
\]

(18)

is a pQCD factor, where \( I_1 = 2 \int_{M_H}^{\mu_I} \frac{d\mu}{\mu} \zeta_{1,g} \) with \( \zeta_{1,g} = \gamma_{1,g} - 2i\beta_{-1} \) is the anomalous dimension for \( C = C_\phi \times C_q \), and \( I_2 = 2 \int_{M_H}^{\mu_I} \frac{d\mu}{\mu} \zeta_{2,g} \). To capture all large \( \ln N \), we translate the dependence on the intermediate scale \( \alpha_s(\mu_I) \) to \( \alpha_s(M_H) \), using

\[
M_N(\alpha_s(\mu_I)) = M_N(\alpha_s(M_H)) e^{I_3},
\]

(19)

where \( I_3 = -2 \int_{\mu}^{M_H} \frac{d\mu}{\mu} \Delta B_1 \) and \( \Delta B_1 \) is defined as

\[
\Delta B_1 = -\beta(\alpha_s) d \ln M_N/d \ln \alpha_s .
\]

The final form of resummation is

\[
G_N(M_H) = F(\alpha_s(M_H))e^{I(\lambda, \alpha_s(\mu), \Delta C)}
\]

(20)

where \( F = |C(\alpha_s(M_H))|^2 M(\alpha_s(M_H)) \) depends only on \( \alpha_s(M_H) \). In fact, it is just the standard full-QCD coefficient function in the soft-gluon approximation, without the large logarithms. \( I = I_1 + I_2 + I_3 \) is a function of \( \lambda = \beta_0 \ln N \alpha_s(M_H) \) and \( \alpha_s(M_H) \) with all leading and sub-leading large logarithms resummed.

The above result can be related to the conventional expression if one writes

\[
I = I_\Delta + \ln \Delta C ,
\]

(21)

where

\[
I_\Delta = \int_0^1 dz \frac{N-1 - 1}{1 - z} \left[ 2 \int_{M_H^2}^{(1-z)^2 M_H^2} \frac{d \mu^2}{\mu^2} A_g(\alpha_s(\mu^2)) \right] + D_g(\alpha_s((1-z)^2 M_H^2)) ,
\]

(22)

and \( \Delta C \) is just a function of \( \alpha_s(M_H) \), serving to cancel the non-logarithmic terms in \( I_\Delta \). Using similar methods as the ones in Ref. \( \text{in} \)

\[ \text{it is a matter of some technical steps to get,} \]

\[
D_g(\mu^2) = 2(B_{1,g} + \Delta B_1 + 2B_{2,g}) - \partial_{\alpha_s} \left. C_2(\alpha_s) \right| A_{g_2}(\alpha_s) - \partial_{\alpha_s} D_g(\alpha_s) \right| ,
\]

(23)

where \( C_2(\epsilon) = 1/\epsilon^2 [1 - e^{-\mu^2 s} \Gamma(1 - \epsilon)] = -\zeta_2^2 / 2 - 3 \zeta_3 / 3 + \ldots \) and \( \partial_{\alpha_s} = 2 \beta(\alpha_s) \partial / \partial \alpha_s \). The above equations are our main result connecting the EFT resummation to the conventional approach, valid to all orders in leading and sub-leading logarithms. Similar results have been obtained for deep-inelastic scattering and Drell-Yan processes \( \text{in} \)

\[ \text{6. Results up to } N^3 \text{LL and Conclusion.} \]

The function \( I \) has a perturbative expansion in \( \alpha_s(M_H) \),

\[
I = \ln N g_1(\lambda) + g_2(\lambda) + \alpha_s(M_H) g_3(\lambda) + \alpha_s^2(M_H) g_4(\lambda) + \ldots ,
\]

(24)

where \( g_1(\lambda) \) can be found from Ref. \( \text{in} \)

\[ \text{They are functions of } A_g \text{ and } D_g; \]

\[ \text{g}_1(\lambda) \text{ sums over the leading logarithms, depending on } A_{g_1}^{(g)} \text{ and } g_2(\lambda) \text{ sums over next-to-leading logarithms, depending on } A_{g_2}^{(g)} \text{ and } D_g^{(g)} \text{, etc.} \]

The \( D_g^{(2)} \) coefficients can be solved iteratively from Eq.
\[ D_g^{(1)} = 0 \]
\[ D_g^{(2)} = -2 f_g^{(2)} + 4 \beta_0 \zeta_2 A_g^{(1)} - 2 \beta_0 M_N^{(1)} \]
\[ D_g^{(3)} = -2 f_g^{(3)} + 4 \zeta_2 \beta_1 A_g^{(1)} + 8 \zeta_2 \beta_0 A_g^{(2)} + \frac{32}{3} \zeta_3 \beta_2 A_g^{(1)} - 2 \beta_1 M_N^{(1)} - 2 \beta_0 \left[ 2 M_N^{(2)} - \left( M_N^{(1)} \right)^2 \right], \]  
(25)

and so on.

As an example to demonstrate the equivalence to the conventional formalism, we calculate \( D_g^{(3)} \) using the known result \( f_g^{(3)} \), \( M_N^{(1),(2)} \) (Eq. (13)), and \( A_g^{(1),(2)} \) \[21\]. The answer is,

\[ D_g^{(3)} = C_A^3 \left[ -\frac{594058}{729} + \frac{98224}{81} \zeta_2 + \frac{40144}{27} \zeta_3 - \frac{2992}{15} \zeta_2^2 - \frac{352}{3} \zeta_2 \zeta_3 - 384 \zeta_5 \right] + C_A^2 n_f \left[ \frac{125252}{729} - \frac{29392}{81} \zeta_2 - \frac{2480}{9} \zeta_3 + \frac{736}{15} \zeta_2^2 \right] + C_A C_F n_f \left[ \frac{3422}{27} - 32 \zeta_2 - \frac{608}{9} \zeta_3 - \frac{64}{5} \zeta_2^2 \right] + C_A n_f^2 \left[ \frac{3712}{729} + \frac{640}{27} \zeta_2 + \frac{320}{27} \zeta_3 \right], \]  
(26)

which agrees completely with the recent calculations \[4\].

To conclude, we have presented an effective field theory method to resum large threshold double logarithms in standard model Higgs production. The approach is simple conceptually and uses the full QCD calculation in the soft limit. The result is valid to all orders in leading and sub-leading logarithms, and reproduces the known answer to N^3LL order.

It has been shown that the inclusion of threshold resummation effects helps to reduce the theoretical uncertainties in the prediction of Higgs production rates at hadron colliders \[2\]. It will be interesting to see if we can get even better results when the newly calculated \( D_g^{(3)} \) term being included. We note that the other sources of theoretical uncertainties, such as those stemming from heavy quark loop approximation \[10, 17\], and those from parton distribution parameterizations, need to be considered in a detailed phenomenological studies of the Higgs production.

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