BHAGEN-1PH: A Monte Carlo event generator for radiative Bhabha scattering.

M. Caffo $^{ab}$ and H. Czyż $^c$ *

$^a$ INFN, Sezione di Bologna, I-40126 Bologna, Italy
$^b$ Dipartimento di Fisica, Università di Bologna, I-40126 Bologna, Italy
$^c$ Institute of Physics, University of Silesia, PL-40007 Katowice, Poland

E-mail:
caffo@bologna.infn.it
czyz@usctoux1.cto.us.edu.pl

Abstract

BHAGEN-1PH is a FORTRAN program providing fast Monte Carlo event generation of the process $e^+e^- \rightarrow e^+e^-\gamma$, within electroweak theory, for both unpolarized beams and also for the longitudinally polarized electron beam. The program is designed for final leptons outside a small cone around the initial leptons direction and has a new algorithm allowing also for a fast generation of non collinear initial and final emission, as well as for asymmetric and different angular cuts for final leptons.

PACS: 12.15.-y, 12.15.Ji, 13.40.Ks, 13.88.+e

* Partly supported by USA-Poland Maria Sklodowska-Curie Joint Fund II (grant MEN/NSF-93-145) and the Polish Committee for Scientific Research (grant no PB659/P03/95/08).
PROGRAM SUMMARY

Title of program: BHAGEN-1PH

Program obtainable from: the authors on request
(E-mail: caffo@bologna.infn.it, czyz@usctoux1.cto.us.edu.pl).

Computer/Operating system: any supporting FORTRAN 77.

Programming language used: FORTRAN 77.

Memory required to execute with typical data: about 1.6 MB

No. of bits in a word: 32

No. of lines in distributed program: 3607 lines.

Subprograms used: RANLUX [9] (included in the source code).

Keywords: radiative Bhabha scattering, polarized beam, longitudinal polarization, photon emission, event generator.

Nature of physical problem: The process is measured in $e^+e^-$ experiments with $e^-$ beam longitudinally polarized or not, and is of interest for tests of the Standard Model (QED at low energies), as a background estimation for other processes, and as a radiative correction contribution to small angle Bhabha scattering for luminosity monitoring.

Method of solution: a Monte Carlo event generation with importance sampling method.

Typical running time: to generate $10^4$ unweighted events are requested 10-20 sec for typical event selection on an AlphaVAX DEC 3000 M700 with OpenVMS 6.2.
1. Introduction.

The cross section for $e^+e^- \rightarrow e^+e^-\gamma$ in QED has been calculated with different approximations for various purposes, producing distributions to which we refer in [1]. At high energies (TRISTAN, LEP, SLC) it is necessary to include the weak-boson $Z^0$ exchange and with a longitudinally polarized electron beam (SLC) the polarization effects. For the unpolarized case a relatively simple expression for the square of the matrix element is obtained in [2,3], for the longitudinally polarized electron beam in [4] is given a reasonably compact expression for the square of the matrix element, and in [1] that expression is improved with some other terms relevant on the $Z$ boson peak and with all the relevant mass-corrections for the configurations in which the final fermions have angles with the initial direction larger than, say, 1 mrad. For extremely forward final fermions the mass-corrections reported there are not sufficient.

Because of the experimental interest in this process we have implemented it within a fast Monte Carlo event generator. The main target is to obtain a fast generation procedure for the mentioned radiative Bhabha scattering configurations, where final leptons are outside a small cone around the initial direction (of a semi opening angle of 1 mrad) and allowing also for the generation of a non collinear initial or final photon without losing the efficiency. At variance from the existing Monte Carlo event generators for Bhabha scattering [5,6,7], which include the radiative Bhabha scattering as a correction, we keep it as a distinct process and the developed method of generation allows to choose asymmetrical and different cuts for electron and positron angular variables, notably in t-channel.

We do not report here the squared matrix element for the cross-section relevant in the region of applicability of the program, which can be found in literature for the unpolarized case in [1,3,4] and for the longitudinally polarized case in [1]. Here we present the details of the structure of the program BHAGEN-1PH, while some results and the numerical comparisons of BHAGEN-1PH with other existing programs (for the unpolarized case and special configurations) are presented in [1].

2. Monte Carlo Algorithm.

The importance sampling method [8] is used as basic Monte Carlo algorithm.

We denote the differential cross-section as

$$d\sigma = \Sigma \, d\Omega_1 d\Omega_\gamma d\omega.$$  \hspace{1cm} (2.1)

where $d\Omega_1 = d\phi_1 d\cos\theta_1$ ($d\Omega_\gamma = d\phi_\gamma d\cos\theta_\gamma$) is the final electron (photon) solid angle, $\omega$ is the photon energy and $\Sigma$ is the distribution.

Moreover we use the following notation: $d\Omega_2 = d\phi_2 d\cos\theta_2$ is the final positron solid angle; $E_1(E_2)$ is final electron (positron) energy; $E_b$ is the initial electron and positron energy; $m_e$ is the electron mass; $s = 4E_b^2$; $\alpha$ is fine-structure constant. We work in the CM frame of initial electron and positron with the $z$-axis chosen along initial electron direction and $x$-$z$-plane given by momenta of initial and final electron. The last choice is convenient due to the fact that the matrix element for the unpolarized cross section, as well as the matrix element for the cross section with a longitudinally polarized beam, depends only
on one azimuthal angle, while the second one is evenly distributed. As a consequence one can actually generate in the chosen frame only four variables and the evenly distributed (in the laboratory frame) azimuthal angle of the final electron can be generated at the end with the subsequent rotation of all momenta.

The distribution $\Sigma$ is approximated by $\Sigma_A$, which consists of 10 parts, in the following called channels, related to all possible combinations of enhancements due to $Z^0$ resonance, collinear and soft photon emission and $t$-channel photon exchange

$$\Sigma \simeq \Sigma_A = \Sigma_1 + \Sigma_3 + \Sigma_6 + \Sigma_7 + \Sigma_{10} + \left(\frac{E_1}{E_2}\right)^2 (\Sigma_2 + \Sigma_4 + \Sigma_5 + \Sigma_8 + \Sigma_9). \quad (2.2)$$

With the proper choice of the channel approximants $\Sigma_i$, $i = 1, \ldots, 10$, described below, all the relevant peaks in the distribution $\Sigma$ can be reproduced. As each peak can be expressed in a more simple way using different set of angular variables, the cross-section (and its channel contributions exact $d\sigma_i$ and approximate $d\sigma_i^A$) is rewritten as

$$d\sigma = \sum_{i=1}^{10} d\sigma_i = \sum_{i=1}^{10} \frac{\Sigma}{\Sigma_A} d\sigma_i^A$$

$$= \frac{\Sigma}{\Sigma_A} d\omega \left[ \Sigma_1 d\Omega_1 d\Omega_{1\gamma} + \Sigma_2 d\Omega_2 d\Omega_{2\gamma} \right. \quad (2.3)$$

$$\left. + (\Sigma_3 + \Sigma_6 + \Sigma_7 + \Sigma_{10}) d\Omega_1 d\Omega_{\gamma} + (\Sigma_4 + \Sigma_5 + \Sigma_8 + \Sigma_9) d\Omega_2 d\Omega_{\gamma} \right],$$

where $d\Omega_{1\gamma}$ ($d\Omega_{2\gamma}$) is the photon solid angle in the frame where $z$-axis is chosen along final electron (positron). The ratio $w \equiv (\Sigma/\Sigma_A)$, called weight, is a relatively flat function. This fact and the absorption of all the peaks in $\Sigma_1, \ldots, \Sigma_{10}$ through an appropriate change of variables, allow for a fast generation. Besides of the most relevant peaks, all the other angular and energy distributions are absorbed, whenever possible in a simple way.

In the following are given explicit expressions for the approximants $\Sigma_i$, $i = 1, \ldots, 10$, which define probability density in each channel. The integration for each of them is examined and the proper form for $d\sigma_i^A$ is obtained. The form integrated over one of the azimuthal angles is used, due to its plain distribution as previously discussed.

We select channels according to:

1) $s$-channel with final lepton emission (Channels 1-2);
2) $s$-channel with initial lepton emission (Channels 3-8);
3) $t$-channel with photon emission (Channels 9 and 10).

For all the channel approximants we succeeded to obtain analytical integration (fast generation) and analytically invertible variables.

1) $s$-channel with final lepton emission (Channels 1-2).

- Channel 1 - final electron emission.

To approximate well the peak in the cross-section coming from the $s$-channel contribution (from both photon and $Z^0$ boson exchange), when the photon is almost collinear
to the final electron, we choose

\[
d\sigma_1^A = \int_0^{2\pi} d\phi_1 \Sigma_1 d\cos\theta_1 d\Omega_{1\gamma} d\omega \\
= \frac{\alpha^3}{4\pi s} F(s) (1 + \cos^2 \theta_1) \frac{1}{a + 2 \sin^2 \frac{\theta_{1\gamma}}{2}} \frac{E_b - \frac{\omega}{2}}{\omega(E_b - \omega)} d\cos\theta_1 d\Omega_{1\gamma} d\omega , \tag{2.4}
\]

where

\[
F(s) = 2 + 4c_V^2 \frac{s(s - M_Z^2)}{D(s)} + 2 \left[ (c_V^2 + c_A^2)^2 - 2P_L c_V c_A (c_V^2 + c_A^2) \right] \frac{s^2}{D(s)} , \tag{2.5}
\]

\[
D(s) = (s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 , \tag{2.6}
\]

\[
c_A = \frac{-1}{4 \sin \theta_W \cos \theta_W} , \quad c_V = \frac{-1 + 4 \sin^2 \theta_W}{4 \sin \theta_W \cos \theta_W} , \tag{2.7}
\]

and

\[
a = \frac{m_e^2}{2E_b^2} , \tag{2.8}
\]

\(\theta_W\) is Weinberg angle, \(M_Z\) (\(\Gamma_Z\)) is \(Z^0\) boson mass (width) and \(P_L\) is the electron longitudinal polarization.

Moreover the expression in (2.4) mimics the electron angular (\(\theta_1\)) and the photon energetic (\(\omega\)) behaviours of this part of the cross-section. The photon energy dependence is well approximated at both ends of its spectrum.

By performing the change of variables

\[
x(\omega) = \ln \left( \frac{\omega}{\sqrt{E_b^2 - \omega E_b}} \right) , \tag{2.9}
\]

\[
y(\theta_{1\gamma}) = -\ln \left( a + 2 \sin^2 \frac{\theta_{1\gamma}}{2} \right) , \tag{2.10}
\]

\[
z(\theta_1) = \cos \theta_1 + \frac{1}{3} \cos^3 \theta_1 \tag{2.11}
\]

the expression in (2.4) becomes

\[
d\sigma_1^A = \frac{\alpha^3}{4\pi s} F(s) d\phi_{1\gamma} dx(\omega) dy(\theta_{1\gamma}) dz(\theta_1) . \tag{2.12}
\]

It is now easy to obtain analytically the integral of (2.12) over the allowed region of the variables, which is a key step in the importance sampling method for the choice of the proper channel for generation.
Also the relations (2.9)-(2.11) are analytically invertible:

\[ \omega(x) = E_b \frac{2 \exp(x)}{\exp(x) + \sqrt{\exp(2x) + 4}}, \]

\[ \sin^2 \frac{\theta_1 \gamma(y)}{2} = \frac{\exp(-y) - a}{2}, \]

\[ \cos \theta_1(z) = \frac{3z}{1 + \left[ \frac{3}{2}z + \sqrt{1 + \frac{9}{4}z^2} \right]^\frac{3}{2} + \left[ -\frac{3}{2}z + \sqrt{1 + \frac{9}{4}z^2} \right]^\frac{3}{2}}. \]

What exposed provides a fast generation in this channel. 

- Channel 2 - final positron emission.

This channel approximates well the peak in the cross-section which appears in the \( s \)-channel contribution (from both photon and \( Z^0 \) boson exchange diagrams), when photon is almost parallel to the final positron. The situation is analogous to the channel 1, with the electron variables substituted by the positron ones. We use both channels to allow for asymmetrical and different cuts for electron and positron.

The formulae are identical to the previous ones (with the change of subscript 1 into 2) and the approximate expression for channel 2 is

\[ d\sigma_2^A = \int_0^{2\pi} d\phi_2 \Sigma_2 d \cos \theta_2 d\Omega_2 \gamma d\omega = \frac{\alpha^3}{4\pi s} F(s) d\phi_2 \gamma d\omega d\theta_2. \]

The generation is done for convenience in the frame were \( x \)-\( z \)-plane is given by initial electron and final positron momenta so after generation the momenta are rotated into our frame of reference (\( x \)-\( z \)-plane given by initial and final electron).

2) \( s \)-channel with initial lepton emission (Channels 3-8).

The next six channels are used to approximate the initial emission spectrum. The situation here is slightly complicated due to the presence of \( Z^0 \) boson resonance. We choose to use more channels and simpler formulae, instead of a numerical approximant, avoiding accuracy adjustments and getting faster procedure.

Channels 5-8 are used only when the total energy is above \( Z^0 \)-boson mass, due to the fact that in the photon spectrum there is a peak, connected with the \( Z^0 \)-resonance, which is located in the hard part of the spectrum. A posteriori is found that the efficiency of the generator improves using these channels only when \( \tilde{\omega} = (s - M_Z^2)/(4E_b) > 1 \) GeV. Covering the phase space with additional four channels allows us to get a proper shape of all angular distributions, avoiding the difficulty of matching more than one peak with the same variable. Furthermore all the variables used are analytically invertible, so the generation is fast.

- Channel 3 - initial electron emission.
In this channel the influence of the $Z^0$-boson resonance on the photon energy distribution is not taken into account. The approximant is therefore very similar to the approximants used in channels 1 and 2 and approximates well the peak in the cross-section, which appears in the $s$-channel when the photon is collinear to the initial electron

$$d\sigma_3^A = \int_0^{2\pi} d\phi_1 \Sigma_3 d\cos\theta_1 d\Omega_\gamma d\omega$$

$$= \frac{\alpha^3}{4\pi s} F(s) \left(1 + \cos^2 \theta_1\right) \frac{1}{a + 2\sin^2 \frac{\theta_2}{2}} \frac{E_b - \omega}{\omega(E_b - \omega)} d\cos\theta_1 d\Omega_\gamma d\omega,$$

and the peaks are absorbed in a way analogous to channel 1, with the obvious substitution $\theta_1 \gamma \to \theta_\gamma$.

- Channel 4 - initial positron emission.

Similarly to channel 3, the peak is approximated in the $s$-channel contribution, when photon is collinear to the initial positron, leaving the $Z^0$ resonance as a spectator

$$d\sigma_4^A = \int_0^{2\pi} d\phi_2 \Sigma_4 d\cos\theta_2 d\Omega_\gamma d\omega$$

$$= \frac{\alpha^3}{4\pi s} F(s) \left(1 + \cos^2 \theta_2\right) \frac{1}{a + 2\cos^2 \frac{\theta_2}{2}} \frac{E_b - \omega}{\omega(E_b - \omega)} d\cos\theta_2 d\Omega_\gamma d\omega.$$

Note however the change in the photon angular dependance, due to the fact that the $z$-axis is chosen along initial electron direction. Now for peak absorption and photon polar angle generation, instead of the variable $y(\theta_\gamma)$ in (2.10), we use the variable $\tilde{y}(\theta_\gamma)$ and its inverse

$$\tilde{y}(\theta_\gamma) = \ln \left(a + 2\cos^2 \frac{\theta_\gamma}{2}\right), \quad \cos^2 \frac{\theta_\gamma(\tilde{y})}{2} = \frac{\exp(\tilde{y}) - a}{2}.$$ (2.19)

Generation is done in a frame were $x$-$z$-plane is given by initial electron and final positron momenta so again at the end we perform the rotation into the proper frame.

- Channel 5 - initial electron emission, $Z^0$ resonance and $\theta_2$ angular distribution.

The influence of the $Z^0$ resonance on the photon energy is accounted for, in conjunction with the positron angular distribution and the photon emission collinear to the initial electron. As mentioned this channel is complementary to the channel 3 and of relevance for total energy well above the mass of the $Z^0$.

$$d\sigma_5^A = \int_0^{2\pi} d\phi_2 \Sigma_5 d\cos\theta_2 d\Omega_\gamma d\omega$$

$$= \frac{\alpha^3 c}{4\pi s} \left(1 + \cos \theta_2\right)^2 \frac{1}{a + 2\sin^2 \frac{\theta_2}{2}} \frac{1}{\tilde{\omega}} \frac{16E_b^2}{16(\tilde{\omega} - \omega)^2 + M_Z^2 \Gamma_Z^2 d\cos\theta_2 d\Omega_\gamma d\omega},$$

(2.20)
where we introduced the notations

\[ c_{\pm} = (c_v^2 + c_A^2)^2 \pm 4c_v^2c_A^2 \quad \text{and} \quad \tilde{\omega} = \frac{s - M_Z^2}{4E_b}. \]  

(2.21)

With the change of variable in (2.10), with the substitution \( \theta_1 \gamma \rightarrow \theta_\gamma \), and the following ones

\[ \tilde{z}(\theta_2) = \frac{1}{3} (1 + \cos \theta_2)^3, \]  

(2.22)

\[ r(\omega) = -\frac{4E_b^3}{M_Z \Gamma_Z \omega} \arctan \left[ \frac{4(\tilde{\omega} - \omega)E_b}{M_Z \Gamma_Z} \right], \]  

(2.23)

the following easily integrable expression can be obtained

\[ d\sigma_A^6 = \frac{\alpha^3}{4\pi s} c_Z d\phi_\gamma d\gamma(\theta_\gamma) d\tilde{z}(\theta_2) d\omega. \]  

(2.24)

These relations have the inverted expressions

\[ \cos \theta_2(\tilde{z}) = \sqrt[3]{3\tilde{z}} - 1, \]  

(2.25)

\[ \omega(r) = \tilde{\omega} - \frac{M_Z \Gamma_Z}{4E_b} \tan \left( -r \frac{\tilde{\omega} M_Z \Gamma_Z}{4E_b^3} \right), \]  

(2.26)

allowing for simple generation procedure.

- Channel 6 - initial electron emission, \( Z^0 \) resonance and \( \theta_1 \) angular distribution.

The influence of the \( Z^0 \) resonance on the photon energy is accounted for this time in conjunction with the electron angular distribution and with the photon emission collinear to the initial electron. Together with channels 3 and 5 gives a complete approximation of the \( s \)-channel cross-section with photon emission collinear to the initial electron, when the total energy is above the \( Z^0 \) mass.

\[ d\sigma_6^A = \int_0^{2\pi} d\phi_1 \Sigma_6 d\cos \theta_1 d\Omega_\gamma d\omega \]

\[ = \frac{\alpha^3}{4\pi s} c_Z^+ (1 + \cos \theta_1)^2 \frac{1}{a + 2 \sin^2 \frac{\theta_1}{2}} \frac{1}{\tilde{\omega}} \frac{16E_b^2}{16(\tilde{\omega} - \omega)^2 + M_Z^2 \Gamma_Z^2} d\cos \theta_1 d\Omega_\gamma d\omega \]

\[ = \frac{\alpha^3}{4\pi s} c_Z^+ d\phi_\gamma d\gamma(\theta_\gamma) d\tilde{z}(\theta_1) d\omega. \]  

(2.27)

- Channel 7 - initial positron emission, \( Z^0 \) resonance and \( \theta_1 \) angular distribution.

Now the photon emission collinear to the initial positron is considered, accounting for the influence of the \( Z^0 \) resonance on the photon energy and in conjunction with the
electron angular distribution, of relevance for total energy above the $Z^0$ mass

\[
d\sigma_A^7 = \int_0^{2\pi} d\phi_1 \Sigma_7 d\cos\theta_1 d\Omega_\gamma d\omega
\]

\[
= \frac{\alpha^3}{4\pi s} c^{-}_Z (1 - \cos\theta_1)^2 \frac{1}{a + 2\cos^2 \frac{\theta_1}{2}} \frac{1}{\omega} \frac{16E_b^2}{16(\tilde{\omega} - \omega)^2 + M_Z^2\Gamma_Z^2} d\cos\theta_1 d\Omega_\gamma d\omega
\]

\[
= \frac{\alpha^3}{4\pi s} c^{-}_Z d\phi_1 d\gamma(\theta_1) d\tilde{z}(\theta_1) d\tilde{y}(\theta_1) d\tilde{z}(\theta_2) dr(\omega)
\]

where the variables in (2.19), (2.23) are used with the new variable

\[
\tilde{z}(\theta_1) = -\frac{1}{3} (1 - \cos\theta_1)^3
\]

(2.28)

again invertible into

\[
\cos\theta_1(\tilde{z}) = -\sqrt{-3\tilde{z}} + 1
\]

(2.29)

- Channel 8 - initial positron emission, $Z^0$ resonance and $\theta_2$ angular distribution.

Again the photon emission is collinear to the initial positron, accounting for the influence of the $Z^0$ resonance on the photon energy and in conjunction with the positron angular distribution. This channel with channels 4 and 7 gives a complete approximation of the $s$-channel cross-section, when the photon is emitted collinear to the initial positron and the total energy is above the $Z^0$ mass.

\[
d\sigma_A^8 = \int_0^{2\pi} d\phi_2 \Sigma_8 d\cos\theta_2 d\Omega_\gamma d\omega
\]

\[
= \frac{\alpha^3}{4\pi s} c^{+}_Z (1 - \cos\theta_2)^2 \frac{1}{a + 2\cos^2 \frac{\theta_2}{2}} \frac{1}{\omega} \frac{16E_b^2}{16(\tilde{\omega} - \omega)^2 + M_Z^2\Gamma_Z^2} d\cos\theta_2 d\Omega_\gamma d\omega
\]

\[
= \frac{\alpha^3}{4\pi s} c^{+}_Z d\phi_1 d\gamma(\theta_1) d\tilde{z}(\theta_1) d\tilde{y}(\theta_1) d\tilde{z}(\theta_2) dr(\omega)
\]

(2.31)

Here the variables in (2.19), (2.23) and (2.29) are used with the substitution $\theta_1 \rightarrow \theta_2$.

3) $t$-channel with photon emission (Channels 9 and 10).

Even if for $s$-channel generation we do not follow any existing solution, the generation there is relatively simple. The complexity of the overlapping peaks in $t$-channel makes the problem much more complicated and the only satisfactory solution to this problem existing up to now [5-7], is not efficient enough, when one is interested in generating photons outside small angular regions around final and initial lepton directions, which is of experimental relevance, when the photons are detected separately from leptons. We present the method we use for this case in more detail. We split the approximant of that part of the cross-section into two parts, which are naturally separated in the exact form of the cross-section and related to the emission from the electron or the positron line.
- Channel 9 - electron emission.

This part approximates the peaks coming from $t$-channel emission from electron line, when photon is mostly collinear to initial or final electron.

\[
d\sigma^A_9 = \frac{2\pi}{\pi s} \int_0^{2\pi} d\phi_2 \sum_\gamma d\cos \theta_2 d\Omega_\gamma d\omega
\]

\[
d\sigma^A_9 = \frac{\alpha^3}{\pi s} A \frac{E_b - \omega}{\omega(E_b - \omega)} \frac{1}{1 + \cos \theta_2} \frac{1}{a + 1 + \cos \theta_2\gamma} \frac{1}{a + 1 - \cos \theta_\gamma} d\cos \theta_2 d\Omega_\gamma d\omega ,
\]

(2.32)

where

\[
\cos \theta_2\gamma = \cos \theta_2 \cos \theta_\gamma + \sin \theta_2 \sin \theta_\gamma \cos \phi_\gamma .
\]

(2.33)

The factor $A$ will be defined later when its origin will become clear.

With the change of variables (2.9) the approximate expression (2.32) becomes

\[
d\sigma^A_9 = \frac{\alpha^3}{\pi s} A \frac{1}{1 + \cos \theta_2} \frac{1}{a + 1 + \cos \theta_2\gamma} \frac{1}{a + 1 - \cos \theta_\gamma} d\cos \theta_2 d\Omega_\gamma d\omega x(\omega) .
\]

(2.34)

Splitting the angular range of $\phi_\gamma$ into two parts $[0, \pi)$ and $[\pi, 2\pi)$ the variable

\[
v(\phi_\gamma) = \frac{2}{\sqrt{c^2 - b^2}} \arctan \left( \frac{\sqrt{c^2 - b^2} \tan \frac{\phi_\gamma}{2}}{c + b} \right) ,
\]

(2.35)

can be used to absorb one of the peaks. Here the following notation is used

\[
c = a + 1 - \cos \theta_2 \cos \theta_\gamma , \quad b = - \sin \theta_2 \sin \theta_\gamma .
\]

(2.36)

The region $[0, \pi)$ is mapped into $v \in [0, v_{max})$, while $[\pi, 2\pi)$ into $v \in [-v_{max}, 0)$, where

\[
v_{max} = \frac{\pi}{\sqrt{c^2 - b^2}} .
\]

(2.37)

All that allows to generate $v \in [-v_{max}, v_{max})$ with the inverse relations

\[
\phi_\gamma = 2 \arctan \left[ \frac{c + b}{\sqrt{c^2 - b^2}} \tan \left( \frac{v \sqrt{c^2 - b^2}}{2} \right) \right] \quad \text{for} \quad v \geq 0
\]

(2.38)

and

\[
\phi_\gamma = 2\pi + 2 \arctan \left[ \frac{c + b}{\sqrt{c^2 - b^2}} \tan \left( \frac{v \sqrt{c^2 - b^2}}{2} \right) \right] \quad \text{for} \quad v < 0 .
\]

(2.39)

To generate in the usual interval the variable $\rho(\phi_\gamma) \in (0, 1]$ is introduced

\[
\rho(\phi_\gamma) = \frac{v + v_{max}}{2v_{max}} ,
\]

(2.40)
so the approximate channel 9 contribution to the cross-section becomes

\[
\frac{d\sigma_A^9}{s} = \frac{2\alpha^3}{s} A \frac{1}{1 + \cos \theta_2} \frac{1}{a + 1 - \cos \theta_\gamma} \sqrt{\frac{1}{a^2 + 2a(1 + \cos \theta_2 \cos \theta_\gamma) + (\cos \theta_2 + \cos \theta_\gamma)^2}}.
\]

(2.41)

To absorb the remaining peaks and to eliminate the square root in the denominator the following change of variables is done

\[z_1(\theta_\gamma) = a + 1 - \cos \theta_\gamma,\]

(2.42)

and

\[z_2(\theta_2) = 1 + \cos \theta_2,\]

(2.43)

so to have

\[
\frac{d\sigma_A^9}{s} = -\frac{2\alpha^3}{s} A \frac{1}{z_2(\theta_2)} \frac{1}{z_1(\theta_\gamma)} \sqrt{\frac{z_1^2(\theta_\gamma) + z_2^2(\theta_2) - 2(a + 1)z_2(\theta_2)z_1(\theta_\gamma) - 2z_2(\theta_2) + 2(a + 1)^2z_2(\theta_2)}{D^2}}.
\]

(2.44)

With the further change of variables

\[t(\theta_\gamma, \theta_2) = \frac{D + z_1(\theta_\gamma) + \sqrt{(D + z_1(\theta_\gamma))^2 + 4P}}{2\sqrt{P}},\]

(2.45)

whose inverse is

\[z_1(\theta_\gamma) = \sqrt{P}\left(t - \frac{1}{t}\right) - D,\]

(2.46)

with

\[D = -(a + 1)z_2(\theta_2), \quad P = \frac{1}{4}a(2 + a)z_2(\theta_2)(2 - z_2(\theta_2)),\]

(2.47)

the approximate expression becomes

\[
\frac{d\sigma_A^9}{s} = -\frac{2\alpha^3}{s} A \frac{1}{z_2(\theta_2)} \sqrt{\frac{1}{P t^2 - D t - \sqrt{P}}} \frac{1}{z_1(\theta_\gamma)} \frac{dz_2(\theta_2)dt(\theta_\gamma, \theta_2)d\rho(\phi_\gamma)dx(\omega)}{dz_2(\theta_2)d\rho(\phi_\gamma)dx(\omega)dt(\theta_\gamma, \theta_2)}.
\]

(2.48)

The last change of variables is

\[r_1(\theta_\gamma, \theta_2) = \frac{r - r_{\text{min}}}{r_{\text{max}} - r_{\text{min}}},\]

(2.49)

and

\[r_2(\theta_2) = \frac{1}{z_2(\theta_2)},\]

(2.50)
where
\[ r = \frac{1}{\sqrt{\Delta}} \ln \left| \frac{2\sqrt{Pt} - D - \sqrt{\Delta}}{2\sqrt{Pt} - D + \sqrt{\Delta}} \right|, \tag{2.51} \]
and
\[ \Delta = z_2^2(\theta_2) + 2a(2 + a)z_2(\theta_2), \tag{2.52} \]
so that the minimum and maximum values of \( r_m \) (\( m \) stands for \( \min \) or \( \max \)) are
\[ r_m = \frac{1}{\sqrt{\Delta}} \ln \left| \frac{2z_{1m}[z_{1m} - (a + 1)z_2(\theta_2) + \sqrt{z_{1m}^2 - 2z_{1m}(a + 1)z_2(\theta_2) + \Delta}]}{(z_{1m} + \sqrt{\Delta} + \sqrt{z_{1m}^2 - 2z_{1m}(a + 1)z_2(\theta_2) + \Delta})^2} \right|, \tag{2.53} \]
and the approximate expression is now
\[ d\sigma_9^A = \frac{2\alpha^3}{s} A (r_{\text{max}} - r_{\text{min}}) z_2 d\theta_2 d\psi(\theta_2, \theta_2) d\rho(\phi, \delta) d\chi(\omega). \tag{2.54} \]
All the variables used are analytically invertible
\[ \cos \theta_2 = -1 + \frac{1}{r_2}, \]
\[ r = r_{\text{min}} + r_1 (r_{\text{max}} - r_{\text{min}}), \tag{2.55} \]
\[ \cos \theta_1 = a + 1 - \frac{2\Delta e^{rv\sqrt{\Delta}}}{(1 - e^{rv\sqrt{\Delta}}) [\sqrt{\Delta} (e^{rv\sqrt{\Delta}} + 1) + D (1 - e^{rv\sqrt{\Delta}})]}. \]
A convenient choice for the factor \( A \) is then
\[ A = \frac{400}{(r_{\text{max}} - r_{\text{min}}) z_2(\theta_2)}, \tag{2.56} \]
so to have at last the very simple approximate expression
\[ d\sigma_9^A = \frac{800\alpha^3}{s} dr_2(\theta_2) dr_1(\theta_1, \theta_2) d\rho(\phi, \delta) d\chi(\omega), \tag{2.57} \]
The choice for \( A \neq 1 \) in (2.56) implies that the approximation of the peaks in \( t \)-channel is not perfect, but nevertheless is good enough (logarithmic in \( z_2(\theta_2) \)) to allow a fast generation. Moreover it is chosen to fix properly the relative strength respect to the other channels subgenerators.

- Channel 10 - positron emission.

This part is specular to the channel 9 and approximates the peaks coming from \( t \)-channel emission of a photon mostly collinear to initial or final positron
\[
d\sigma_{10}^A = \int_0^{2\pi} d\phi_1 \Sigma_{10} d\cos \theta_1 d\Omega_\gamma d\omega
= \frac{\alpha^3}{\pi s} \frac{E_b - \omega}{\omega(E_b - \omega)} \frac{1}{1 - \cos \theta_1} \frac{1}{a + 1 + \cos \theta_1} \frac{1}{a + 1 + \cos \theta_\gamma} d\cos \theta_1 d\Omega_\gamma d\omega
= \frac{800\alpha^3}{s} dr_2(\theta_1) dr_1(\theta_1, \theta_1) d\rho(\phi, \delta) d\chi(\omega) , \tag{2.58} \]
where now

\[ z_2(\theta_1) = 1 - \cos \theta_1 , \]  
(2.59)

and

\[ z_1(\theta_\gamma) = a + 1 + \cos \theta_\gamma , \]  
(2.60)

with

\[ c = a + 1 - \cos \theta_1 \cos \theta_\gamma , \]  
(2.61)

and

\[ b = - \sin \theta_1 \sin \theta_\gamma , \]  
(2.62)

while all the other expressions remain valid. \( \tilde{A} \) has the same functional form of \( A \), but as \( z_2, z_1, c \) and \( b \) are now different, it is different from \( A \).

Having so given the separate description of various channels, obtaining a good approximant of the cross-section, the events can be generated in an easy way according to the approximated forms in each channel.

The pseudo random number generator used to produce the equidistributed random numbers between \([0,1)\) is the CERNLIB generator (V115) RANLU [9], which source code is added into our program.

The algorithm used to select the channel for generation is the following: the \( i \)-th channel is chosen randomly with the probability

\[ P_i = \frac{v_i}{\sum_{j=1}^{10} v_j} , \]  
(2.63)

where

\[ v_i = \int_{V_i} d\sigma^A_i , \]  
(2.64)

and \( V_i \) is the allowed region of the variables appearing in \( d\sigma^A_i \).

A random generation of 1000 events is done at the beginning and the maximum-weight \( w_S \) is taken to be 2.5 times the largest of the generated weights \( w = \frac{\sigma}{\sigma_A} \). As the \( w \) function is relatively flat the method works well and in all tests an event with the weight \( w > w_S \) was never generated. Nevertheless this eventuality is checked automatically in the program and in case events with \( w > w_S \) appear the cross-section corresponding to them is given in the output. However the cross-section and its error calculated with the weighted event sample is not affected by the appearance of the events with weight \( w > w_S \).

The cross-section \( \sigma \) integrated over the allowed phase space is calculated as a sum of all channels contributions, according to the stratification method described in [8],

\[ \sigma = \sum_{i=1}^{10} \sigma_i = \sum_{i=1}^{10} \int d\sigma_i = \sum_{i=1}^{10} \int \frac{\Sigma}{\Sigma_A} d\sigma^A_i , \]  
(2.65)

and its statistical error \( \Delta \sigma \) is calculated as

\[ \Delta \sigma = \sqrt{\sum_{i=1}^{10} (\Delta \sigma_i)^2} , \]  
(2.66)
where $\Delta \sigma_i$ is the statistical error of the \(i\)-th channel contribution $\sigma_i$.

A more complicated event selection, then that allowed by choosing the range of the generation variables, can be implemented by simply setting the weight of the generated event to zero in case it is not allowed by the requested cuts (this can be easily done by the user in the subroutine TRIGGER).

Finally we remark that a big effort was devoted to assure that the accuracy of the calculations is not lost due to possible cancellations in formulae.

3. **Routines in the Generator.**

**BHAGEN-1PH**

It is the main program and calls the subroutines PARAMET, VOLUME, MAXEST, GENER and CROSS. It calculates and writes the total cross section for both weighted and unweighted events sample.

**PARAMET(i1,i2,i3)**

The argument $i1$ is the logical number of the input file, from which all relevant parameters are read (see input file description in the Users Guide section); $i2$ is the logical number of the first output file (BHAGEN-1PH.RES), where parameters and cross-section values are reported; $i3$ is the logical number of the second output file (BHAGEN-1PH.EVN), where generated events are listed (if requested). The values, which are constant during all the run, are calculated (such as coupling constants, boundaries of the integration regions etc.) and the random number generator is initialized.

**VOLUME**

The values of $v_i$ and $P_i$ ($i = 1, \ldots, 10$) are calculated (see algorithm description).

**MAXEST**

It generates 1000 events in the same way as in GENER and on its basis estimates the maximal weight $w_S$.

**GENER**

It chooses randomly (RANDOMN1) with the probability $P_i$ a given channel and calls a proper generation procedure in each channel (GENER1, \ldots, GENER10). It writes the generated events (kinematics of the outgoing particles), if requested. It stops the generation, when the unweighted event sample is equal to the requested number.

**CROSS**

It calculates the integrated cross-sections for each channel, for both weighted and unweighted event sample.

**GENER1, ..., GENER10**

They are master programs for each separate channel. They generate (RANDOMN) the kinematical variables, which are chosen as independent (see algorithm description). The subroutines GENER3, \ldots, GENER10 call the subroutines PHASESP1 or PHASESP2 to calculate the rest of the necessary kinematics (energies and angles of the final particles) and reject the event if it is generated outside the allowed phase space. In GENER1 and GENER2 the same is done inside the subroutines, as the calculations are different from other channels. All the subroutines call FRAME, which generate (RANDOMN1) the azimuthal angle of the final electron and rotates the event (up to this call the event is given in a frame where the $x$-$z$-plane is given by initial and final electron momenta). They also call TRIGGER, which is the event selection subroutine to be supplied by the
It gives one random number evenly distributed on (0,1] used to choose the channel or to generate the azimuthal angle of the final electron.

It gives a sequence of 5 random numbers evenly distributed on (0,1]: 4 to generate the event variables and 1 for the selection of the unweighted event.

They calculates energies and angles of the final particles, given $\phi_1, \theta_1, \phi_\gamma, \theta_\gamma$ and $\omega$ (PHASESP1), or $\phi_2, \theta_2, \phi_\gamma, \theta_\gamma$ and $\omega$ (PHASESP2).

The azimuthal angle of the electron is generated randomly (RANDOMN1) and the event (momenta of the final particles) is rotated around z-axis by this angle. This allows the use of an arbitrary frame with z-axis along the initial electron. Before that rotation the x-z-plane is the plain given by initial and final electron momenta.

This is an event selection subroutine, to be supplied by the user, allowing for rejection (k=0 has to be set inside the subroutine) or acceptance (k=1 has to be set inside the subroutine) of the generated event. The final particles energies and angles are stored in the "ev" vector. They are given in CM frame of initial electron and positron, with the z-axis along initial electron momentum. If left as it is given in the distributed version, the trigger has no effect on the generation. The reading of the event vector is as follow:

- ev(1) - the electron energy in GeV
- ev(2) - the electron polar angle in rad.
- ev(3) - the electron azimuthal angle in rad.
- ev(4) - the positron energy in GeV
- ev(5) - the positron polar angle in rad.
- ev(6) - the positron azimuthal angle in rad.
- ev(7) - the photon energy in GeV
- ev(8) - the photon polar angle in rad.
- ev(9) - the azimuthal angle in rad.

It calculates parameters which enter in both, the exact and the approximate cross-sections. It calculates the weight ($w$) of the event and it provides the book keeping for the calculation of the cross-section for the weighted sample, adding $w$ to $\sum_i w_i$ and $w^2$ into $\sum_i w_i^2$ for the proper channel. It checks if $w$ is smaller of the estimated maximum-weight ($w_S$), and if not, it sums the weight $w \equiv w_i^o$ into the separate sums $\sum_i w_i^o$ and $\sum_i (w_i^o)^2$ allowing for the calculation of a separate cross-section of such overweighted events, writing also identifying parameters. The unweighted events sample can still be used, if the contribution to the cross-section due to the overweighted events is within the errors of the total cross-section. However it never happened to find an event with weight $w > w_S$ in the test runs, so the maximum estimation is pretty good. The hit or miss method is
also applied here: the random number \( r_5 \) is generated (RANDOMN), \( w_t = r_5 \) \( w_S \) is calculated and the event is accepted if \( w > w_t \).

RFULL

It calculates the exact differential cross-section; some factors which are the same in both exact and approximate form are omitted, as only the ratio of the two is used in calculations.

CHANNEL1, ..., CHANNEL10

They calculate the approximate differential cross-sections for each channel (but the factors which are the same in both exact and approximate form, as only the ratio of the two is used in calculations).

FACT

It is an auxiliary function used by CHANNEL9 and CHANNEL10.

4. Users Guide.

In the version installed on VAX with VMS system the parameters are given in the BHAGEN-1PH.COM file (the same file where are the system commands running the program). In the version installed on IBM 340 (RISC/6000) with the system AIX 3.2, the only differences can be in the syntax of the OPEN statement (depending on the FORTRAN 77 version installed) and in the fact that the file BHAGEN-1PH.COM contains only parameters. The user has to supply the following data:
- The mass of the \( Z^0 \) boson and its width.
- The mass of the \( W \) boson and the value of \( \sin^2 \theta_W \). Actually only one of this parameters is used by the program. If \( \sin^2 \theta_W \) is set to zero the program uses the expression \( \sin^2 \theta_W = 1 - (M_W^2/M_Z^2) \) to calculate it, while if it is given different from zero, \( M_W \) is not used by the program. Up to now no radiative corrections are included into the program.
- The beam energy.
- The degree of longitudinal polarization of the electron beam.
- The number of requested events in the unweighted events sample.
- The minimal and maximal allowed energy of the final particles. The maximal allowed energy of the photon can be modified by the program (and notified in the output file) to \( E^{\gamma}_{\text{max}} = 2E_b - E^1_{\text{min}} - E^2_{\text{min}} \leq E_b(1 - m_e^2/E_b^2) \) if the given maximum is too high to be consistent with the other cuts supplied by the user.
- The minimal and maximal polar angles (in radians) of the final electron (\( \theta_\pm \equiv \theta_1 \)) and positron (\( \theta_+ \equiv \pi - \theta_2 \)) with their initial directions.
- The minimal angle between final lepton and photon (it can be set to zero).
- The minimal angle between initial lepton and final photon (it can be set to zero).
- The maximal allowed acollinearity (acollinearity = \( | \pi - \theta_1 - \theta_2 | \), where \( \theta_1 \) and \( \theta_2 \) are the angles between final electron and positron momenta with the initial direction of the electron) and acoplanarity (acoplanarity = \( | \pi - (\phi_2 - \phi_1) | \), \( \phi_1 \) and \( \phi_2 \) are the azimuthal angles of the final electron and positron.
- A flag which allows writing (1) or no writing (0) the generated events into the created file BHAGEN-1PH.EVN.
- A flag which allows writing (1) or no writing (0) the separate channel contributions.
- A flag which allows to take into account separate contributions from \( s- \) and \( t- \)channel, which can be useful for tests, but require the switch to be set to 0, for comparing the results...
with experiment. If the flag is equal to 0 full version of the program is used; if it is equal to 1 only $t$-channel photon exchange contribution is included (with up-down interference neglected); if it is equal to 2 only $s$-channel contribution is included (from both $\gamma$ and $Z^0$ exchange diagrams).

- A seed to initialize the pseudo random number generator.

Two output files are created. The first one BHAGEN-1PH.RES contains the calculated cross-sections for both weighted and unweighted event sample and if requested the contributions from all ten channels. The second one BHAGEN-1PH.EVN contains the generated events (if the proper flag was set to 1). Both files report the parameters, which are read from the input file.

Acknowledgments

Useful discussions with E. Remiddi are gratefully acknowledged. One of us (HC) is grateful to the Bologna Section of INFN and to the Department of Physics of Bologna University for support and kind hospitality.
References.

[1] M. Caffo, H. Czyż and E. Remiddi, Phys. Lett. B 378 (1996) 357.
[2] F.A. Berends et al., Phys. Lett. B 103 (1981) 124.
[3] F.A. Berends et al., Nucl. Phys. B206 (1982) 61.
[4] M. Caffo, R. Gatto and E. Remiddi, Nucl. Phys. B286 (1987) 293.
[5] F.A. Berends, R. Kleiss, Nucl. Phys. B228 (1983) 537.
[6] S. Jadach et al., Phys. Lett. B 253 (1991) 469.
[7] F.A. Berends, R. Kleiss and W. Hollik, Nucl. Phys. B304 (1988) 712.
[8] J.M. Hammersley and D.C. Handscomb, Monte Carlo Methods, London: Methuen (1964),
F. James, Rep. Prog. Phys. 43 (1980) 1145.
[9] F. James, Comp. Phys. Comm. 79 (1994) 111.
TEST RUN INPUT.

Input file to run with VMS system:

$run bhagen-1ph.exe
91.1887 Z mass (GeV)
2.49661 Z width (GeV)
80.304 W mass (GeV) (used only if sin**2(thW)=0.)
0.224882 sin**2(thW)(if=0 then it is calculated =1-(M_W/M_Z)**2
47.5 e_beam (GeV)
1.0 longitudinal polarization of the initial electron
10000 nevent - numb. of events in unweighted event sample
4.75 23.75 eg min,eg max (GeV) see comment (1)
23.75 47.5 e1_min,e1_max (GeV) see comment (2)
23.75 47.5 e2_min,e2_max (GeV) see comment (3)
0.7 2.3 th1_min,th1_max (rad) see comment (4)
0.8 2.2 th2_min,th2_max (rad) see comment (5)
0.01 th1g_min=th2g_min (rad) see comment (6)
180. 180. acolcut(deg),acoplcut(deg) see comment (8)
1 printing(1) no printing(0) generated events
0 s-t channel switch see comment (9)
123456789 seed to the random number generator
$exit

Comments:

(1) - min. and max. photon energy in GeV. The minimum photon energy cannot be set to 0 !!!! The maximum photon energy is set to
eg_max = e_beam * (1 - me**2/e_beam**2) in case it is given bigger then that value. This is the biggest value of the photon energy allowed by four momentum conservation.

(2) - min. and max. final electron energy in GeV. In case e1_min=0. it is set to the electron mass by the program.

(3) - min. and max. final positron energy in GeV. In case e2_min=0. it is set to the electron mass by the program.

(4) - min. and max. electron polar angle measured from initial electron direction.
A user is kindly requested not to use this program with th1_min < 0.001 rad. as the formulae used are not adequate for that region.

(5) - min. and max. positron polar angle measured from initial positron direction.
A user is kindly requested not to use this program with th2_min < 0.001 rad. as the formulae used are not adequate for that region.
(6) - minimal angle between final electron(positron) and the photon. In case they are not equal the smaller should be given. It can be set to 0.
(7) - minimal angle between initial electron(positron) and the photon. In case they are not equal the smaller should be given. It can be set to 0.
(8) - maximal acolinearity and acoplanarity allowed. Where acolinearity is defined as \(|(\pi - \theta_1 - \theta_2)|\), and \(\theta_1(\theta_2)\) is the angle between final electron(positron) and initial electron. While acoplanarity is defined as \(|(\phi_2-\phi_1-\pi)|\), where \(\phi_1\) and \(\phi_2\) are respectively the electron and positron azimuthal angles.
(9) - 0: all included; 1: t-channel QED only with up-down interference neglected suitable for tests at small angles; 2: s-channel only, both photon and Z boson exchange include

Additional, more detailed, event selection can be done by modification of the subroutine TRIGGER in the FORTRAN code. The cross section given in the output corresponds to the complete event selection e.g. with the cuts imposed by both the input parameters in this file and the cuts imposed in the subroutine TRIGGER. A simple example how to use this subroutine is given below. An event with electron having energy between 0.1 GeV and 10 GeV is rejected and accepted if the electron energy is outside this region.

```
c * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
c c This is a trigger which is to be defined by user allowing for the c rejection (k=0 has to be set) or acceptance (k=1 has to be set) c of the generated event. c The final particles energies and angles are stored in the 'ev' vector. c They are given in CM frame of initial electron and positron, with c the z-axis along initial electron momentum.
c c c ev(1) - electron energy in GeV c ev(2) - theta (electron) in rad. c ev(3) - phi (electron) in rad. c ev(4) - positron energy in GeV c ev(5) - theta (positron) in rad. c ev(6) - phi (positron) in rad. c ev(7) - photon energy in GeV c ev(8) - theta (photon) in rad. c ev(9) - phi (photon) in rad. c```

20
subroutine trigger(ev,k)
  c
  implicit real*8 (a-h,o-z)
  dimension ev(9)
  c
  if((ev(1).gt.0.1d0).and.(ev(1).lt.10.d0))then
    k=0
  else
    k=1
  endif
  c
  return
end
  c * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
TEST RUN OUTPUT.

Output file BHAGNE-1PH.RES.

BHAGEN-1PH, VERSION OF 22-JAN-1996
Z mass (GeV) : 91.188700D+00
W mass (GeV) : 80.304000D+00
\( \sin^2(\theta_w) = 0.224482000000000 \)
Total Z width (GeV) = 2.4966100D+00
E_{\text{beam}} = 47.5000 GeV
Longitudinal polarization of electron = 1.0000
23.7500D+00 GeV <= electron energy <= 47.5000D+00 GeV
23.7500D+00 GeV <= positron energy <= 47.5000D+00 GeV
4.75000D+00 GeV < photon energy < 2.37500D+01 GeV
7.00000D-01 rad < \theta (electron) < 2.30000D+00 rad
9.41593D-01 rad < \theta (positron) < 2.34159D+00 rad
Minimal allowed angle between photon and final lepton = 1.00000D-02 rad.
Minimal allowed angle between photon and initial lepton= 1.00000D-01 rad.
acollinearity < 3.14159D+00 rad.
acoplanarity < 3.14159D+00 rad.

* * * * * * * * * * * * * * * * * * * * * * *
no. of events with weights > max.weight: 0
Total cross section of the events with weights > max. weight:
cross section (nb) = 0.0000000D+00 +- 0.00D+00
* * * * * * * * * * * * * * * * * * * * * * *
Number of generated (unweighted) events in the sample: 10000
Number of hits (weighted events) in the sample: 130202
Summary :
Total cross section obtained using weighted events :
total cross section1 (nb) = 1.3745087D-02 +- 3.61D-05
Total cross section obtained using unweighted events:
total cross section2 (nb) = 1.3874919D-02 +- 1.39D-04
BHAGEN-1PH,  VERSION OF 22-JAN-1996

Z mass (GeV) : 91.188700D+00
W mass (GeV) : 803.04000D-01
sin**2 (th_w) = 0.224482000000000
Total Z width (GeV) = 2.4966100D+00
E_beam = 47.5000 GeV

Longitudinal polarization of electron = 1.0000
23.7500D+00 GeV <= electron energy <= 47.5000D+00 GeV
23.7500D+00 GeV <= positron energy <= 47.5000D+00 GeV
4.75000D+00 GeV < photon energy < 2.37500D+01 GeV
7.00000D-01 rad < theta (electron) < 2.30000D+00 rad
9.41593D-01 rad < theta (positron) < 2.34159D+00 rad

Minimal allowed angle between photon and final lepton = 1.00000D-02 rad.
Minimal allowed angle between photon and initial lepton= 1.00000D-01 rad.
acollinearity < 3.14159D+00 rad.
acoplanarity < 3.14159D+00 rad.

Structure of the event:
Electron energy (GeV); theta(electron) (rad.); phi(electron) (rad.)
Positron energy (GeV); theta(positron) (rad.); phi(positron) (rad.)
Photon energy (GeV); theta(photon) (rad.); phi(photon) (rad.)
3.83488D+01  1.33469D+00  1.53144D+00
4.26590D+01  2.13715D+00  4.65588D+00
1.39922D+01  1.02513D-01  5.11897D+00
4.35498D+01  1.14156D+00  8.91211D-01
4.54257D+01  2.12109D+00  3.98290D+00
6.02442D+00  3.63628D-01  5.15577D+00