Dimensional structural constants from chiral and conformal bosonization of QCD

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Abstract
We derive the dimensional non-perturbative part of the QCD effective action for scalar and pseudoscalar meson fields by means of chiral and conformal bosonization. The related structural coupling constants $L_5$ and $L_8$ of the chiral lagrangian are estimated using general relations which are valid in a variety of chiral bosonization models without explicit reference to model parameters. The asymptotics for large scalar fields in QCD is elaborated, and model-independent constraints on dimensional coupling constants of the effective meson lagrangian are evaluated. We determine also the interaction between scalar quarkonium and the gluon density and obtain the scalar glueball-quarkonium potential.

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1. Introduction

The low-energy effective action for light mesons [1] describes their strong and electroweak interactions by means of several structural constants [2] which contain the information about the dynamics of quark and gluon interaction – Quantum Chromodynamics (QCD). These constants can be evaluated from the analysis of experimental data [2, 3], thereby giving a possibility to examine QCD at low energies.

For the calculation of coupling constants of the chiral lagrangian direct bosonization methods of low-energy QCD [4]–[10] and quark models [11]–[20] have been developed (see reviews [17, 21]). These methods give good and stable numerical estimates for certain dimensionless chiral coefficients, namely for \(L_1, L_2, L_3, L_4, L_6, L_9, L_{10}\) [4], [15]–[19] in the \(SU_F(3)\) case and for \(L_{16}, L_{17}, L_{19}\) in the \(U_A(1)\) extension [22] of the chiral lagrangian. Meantime the chiral coefficients \(L_5, L_7, L_8, L_{14}, L_{15}, L_{18}\) depending explicitly on QCD order parameters (such as the quark and gluon condensate [24]) were not so far calculated in a model-independent way (see however large-\(N_c\) relations between them in [22]). Thus, it is one of the goals of our paper, to find new constraints on those constants, in particular \(L_5\) and \(L_8\), from basic properties of the QCD vacuum [25]. We analyze the common structure of effective lagrangians for pseudoscalar and scalar mesons as they are derived in general by the low-energy QCD bosonization procedure, and impose on such lagrangians the conditions to reproduce the behavior of the non-perturbative QCD vacuum energy in the large mass limit. These conditions together with a minimal number of physical inputs allow us to find the dimensional chiral coefficients, in particular \(L_5\) and \(L_8\), in Chiral Perturbation Theory (ChPT) the constant \(L_5\) enters the determination of the ratio of weak decay constants \(F_\pi : F_K : F_\eta\) of pseudoscalar mesons. The constant \(L_8\) characterizes the \(K^0 - K^+\) meson mass difference as well as the current quark mass ratio \((2m_s - m_d - m_u) : (m_d - m_u)\) [2].

In a general QCD bosonization approach the dynamical chiral symmetry breaking (DCSB) is implemented in order to derive the effective meson lagrangian. The DCSB is typically simulated by means of a quark momentum cutoff \(\Lambda\) and a quark spectrum asymmetry \(M\) in QCD [4, 7] or a dynamical quark mass \(M_d\) in chiral quark models [11, 12, 18] and quark models of Nambu-Jona-Lasinio type [15–17], [19], [20]. The low-energy meson lagrangian is obtained from the bosonization of the quark determinant by application of the derivative expansion combined with the expansion in inverse powers of \(M\) or \(M_d\). It contains functionals of meson fields and external sources of different canonical dimension with coupling constants proportional to the DCSB order parameters. We find that the very structure of the above-mentioned functionals is universal in respect to different bosonization schemes and the model-dependent information is collected in structural constants. Just this observation together with QCD vacuum energy constraints entails the definite predictions on chiral
coefficients. As an interesting result, in addition we derive the scalar glueball-quarkonium potential directly from the QCD generating functional.

The starting point of our analysis is the conventional QCD generating functional for colorless quark and gluon currents,

\[
Z(\rho, V, A, S, P) \equiv \int DG \, Dq \, D\bar{q} \, \exp \left\{ -\int d^4x \left[ \left( \frac{1}{g^2} + \rho \right) \frac{1}{4} (G^a_{\mu\nu})^2 + \bar{q}(i\not{D} + iS + \gamma_5 P)q \right] \right\}. \tag{1}
\]

Throughout, we will work in the Euclidean space. The covariant derivative is given by

\[ \not{D} = i\gamma^\mu (\partial^\mu - ig_{\mu\lambda}^a \lambda^a + V^\mu + \gamma_5 A^\mu) \]

with \( \text{tr} \left[ \lambda^a \lambda^b \right] = \frac{1}{2} \delta^{ab} \) in color space. The external isoscalar and isovector colorless sources \( \rho(x), V^\mu(x), A^\mu(x), S(x), P(x) \) serve for the building of Green functions of colorless currents as usual. The gauge fixing and ghost terms are not quoted explicitly but are understood to be taken into account in perturbative calculations.

The derivation of the scalar-pseudoscalar meson lagrangian from QCD by direct bosonization \([4]–[10]\) or by bosonization of a quark model \([11]–[20]\) can be described schematically as follows,

\[
Z(\rho, V, A, S, P) = \int DG \, Dq \, D\bar{q} \, D\delta(h) \, \delta(h - (G^a_{\mu\nu})^2) \exp(-S(\bar{q}, q, G; \rho, V, A, S, P)) \approx \int Dh \, D\Sigma \, DU \exp(-S_{\text{eff}}(h, \Sigma, U; \rho, V, A, S, P)), \tag{2}
\]

where the averaging over gluons \( G_{\mu} \) and quarks \( q, \bar{q} \) is approximately replaced by the averaging over chiral fields \( U \) as well as over scalar glueball \( h \) and quarkonium \( \Sigma \) fields. Next the effective action is expanded in derivatives of meson fields and external sources. The model design consists in the choice of collective bosonic fields describing light mesons and of non-perturbative parameters specifying the DCSB (the type and magnitude of a momentum cutoff, the spectral asymmetry or the dynamical mass).

The paper is organized as follows: In the next section we discuss the relations between the structure constants of the phenomenological low-energy meson lagrangian with those of model calculations. We will show that the values of dimensional structural constants, especially, of \( L_5 \) and \( L_8 \) are rather model-dependent. In section 3 we therefore extend the QCD bosonization approach to the scalar sector including all possible dimensional coupling constants. Comparing with the large scalar field asymptotics of the QCD vacuum energy, which is derived in section 4, we find additional constraints on the chiral structure constants which are discussed in section 5. In addition, this yields the scalar glueball-quarkonium potential. A summary of the results and a conclusion are given in section 6.
2. QCD bosonization in the pseudoscalar sector

In the chiral field sector \((h, \Sigma \simeq \text{const})\) the effective lagrangian has the following structure,

\[
S_{\text{eff}} = \int d^4x (\mathcal{L}_2 + \mathcal{L}_4) + S_{\text{WZW}},
\]

(3)

Here, the Weinberg lagrangian which is of chiral dimension 2 is given by

\[
\mathcal{L}_2 = \frac{F_0^2}{4} \text{tr} \left[(D_\mu U)\dagger D^\mu U - (\chi\dagger U + U\dagger \chi)\right].
\]

(4)

where \(F_0 \simeq 90\) MeV is the (bare) pion-decay constant, \(U\) is the usual \(SU_F(3)\)-chiral field describing pseudoscalar mesons, \(UU\dagger = 1, \det U = 1\). The external sources are assembled in the covariant derivative, \(D_\mu = \partial_\mu + [V_\mu, \ast] + \{A_\mu, \ast\}\) with vector sources \(V_\mu\) and axial-vector sources \(A_\mu\) and in the complex density

\[
\chi = 2B_0(S + iP),
\]

(5)

with scalar sources \(S\) and pseudoscalar sources \(P\). The current quark mass \(m_q\), as usual, is included in the scalar source. The constant \(B_0\) is related to the quark condensate, \(i\langle \bar{q}q \rangle = -B_0 F_0^2\), the order parameter of DCSB in QCD and appears in the Gell-Mann–Oakes–Renner relation for pseudoscalar meson masses [2].

In the large-\(N_c\) bosonization approach, i.e. when restricting oneself to the calculation of diagrams with only one quark-loop\(^1\), the leading contribution to the effective lagrangian \(\mathcal{L}_4\) of chiral dimension 4 is parametrized [22] by nine structural constants \(I_k\):

\[
\mathcal{L}_4^{\text{eff}} = \text{tr} \left[-I_1 (D_\mu U)(D_\nu U)\dagger D_\mu U(D_\nu U)\dagger - I_2 (D_\mu U)(D_\nu U)\dagger D\nu U(D_\nu U)\dagger

-I_3 (D_\mu U)\dagger D^2 U + I_4 (D_\mu \chi)\dagger D_\mu U + D_\mu \chi(D_\mu U)\dagger

+I_5 D_\mu U(D_\mu U)\dagger (U\chi\dagger + U\dagger \chi) - I_6 (U\chi\dagger U\chi\dagger + U\dagger \chi U\dagger)

-I_7 \left(\chi\dagger U - U\dagger \chi\right) \text{tr} \left[\chi\dagger U - U\dagger \chi\right]

-I_8 \left[F_{\mu\nu}^R D_\mu U(D_\nu U)\dagger + F_{\mu\nu}^L (D_\mu U)\dagger D_\nu U\right] - I_9 U\dagger F_{\mu\nu}^R U F_{\mu\nu}^L + \mathcal{L}_4^{\text{inv}}\right]
\]

(6)

in Euclidean denotations. Herein \(F_{\mu\nu}^L = \partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu]\) and \(L_\mu = V_\mu + A_\mu; R_\mu = V_\mu - A_\mu\).

The constants \(I_i\) \((i \neq 7)\) arise from the bosonization of one-loop quark diagrams in the soft-momentum expansion. The coefficient \(I_7\) is essentially saturated [26] by the non-perturbative correlator of gluon pseudoscalar densities (see [22] and references therein). The

\(^1\)This yields only terms with one connected trace operation.
bosonization yields also vertices of dimension 4 in $L_4^{\text{inv}}$ which are invariant under chiral transformations of external sources,

$$L_4^{\text{inv}} = -H_1 \text{tr} \left[ F_{\mu}^L F_{\mu}^L + F_{\mu}^R F_{\mu}^R \right] - H_2 \text{tr} \left[ \chi^\dagger \chi \right].$$

The Wess-Zumino-Witten action $S_{\text{WZW}}$ includes the so-called anomalous vertices $^2, ^3$ and it is not displayed here.

The low-energy phenomenology of pseudoscalar mesons is described by the standard Gasser-Leutwyler lagrangian $^2$ which contains ten structural chiral constants $L_i$,

$$L_i^{GL} = -L_1 \left( \text{tr} \left[ (D_{\mu} U)^\dagger D_{\mu} U \right] \right)^2 - L_2 \text{tr} \left[ (D_{\mu} U)^\dagger D_{\nu} U \right] \text{tr} \left[ (D_{\mu} U)^\dagger D_{\nu} U \right] - L_3 \text{tr} \left[ (D_{\mu} U)^\dagger D_{\mu} U + D_{\nu} U \right] + L_4 \text{tr} \left[ (D_{\mu} U)^\dagger D_{\mu} U \right] \text{tr} \left[ \chi^\dagger U + U^\dagger \chi \right]
+ L_5 \text{tr} \left[ (D_{\mu} U)^\dagger D_{\mu} U + D_{\nu} U \right] + L_6 \left( \text{tr} \left[ \chi^\dagger U + U^\dagger \chi \right] \right)^2
+ L_7 \left( \text{tr} \left[ \chi^\dagger U - U^\dagger \chi \right] \right)^2 - L_8 \text{tr} \left[ \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi \right]
- L_9 \text{tr} \left[ F_{\mu}^R D_{\mu} U (D_{\nu} U)^\dagger + F_{\mu}^L (D_{\mu} U)^\dagger D_{\nu} U \right] - L_{10} \text{tr} \left[ U^\dagger F_{\mu}^R U F_{\mu}^L \right]
+ L_4^{\text{inv}}.$$  

We will neglect meson-loop corrections in the following$^4$, keeping our attention to the leading order in $1/N_c$. Then the (tree-level) relations between coefficients $L_i$ and $I_j$ are derived following the usual on-shell scheme of Chiral Perturbation Theory, i.e. imposing the equations of motion from the Weinberg lagrangian $L_2$,

$$U^\dagger D_{\mu}^2 U - (D_{\mu}^2 U)^\dagger U - \chi^\dagger U + U^\dagger \chi = \frac{1}{N_F} \text{tr} \left[ U^\dagger \chi - \chi^\dagger U \right],$$

where $N_F = 3$ is the number of light flavors. One finds,

$$2L_1 = L_2 = I_1, \quad L_3 = I_2 + I_3 - 2I_1, \quad L_4 = L_6 = 0, \quad L_5 = I_4 + I_5,$$

$$L_7 = I_7 - \frac{1}{6} I_4 + \frac{1}{12} I_3, \quad L_8 = -\frac{1}{4} I_3 + \frac{1}{2} I_4 + I_6, \quad L_9 = I_8, \quad L_{10} = I_9.$$  

Thereby the predictions of a bosonization scheme can be directly referred to physical characteristics of pseudoscalar mesons.

The low-energy QCD bosonization yields the chiral lagrangian in the soft momentum expansion whereas the ChPT concept is based upon the soft momentum and light meson mass expansion $\text{dim}[p^2] = \text{dim}[m_p^2] = 2$. Therefore chiral coefficients in the bosonized action have different status in the canonical and chiral dimensional analysis. Namely, the canonical dimension of vertices containing powers of the field $\chi$ (see eq. $^3$) is always less than the

$^2$ Consequently, we will not achieve higher precision than the usual logarithmic uncertainty due to the choice of renormalization scale.
chiral one because the canonical dim[χ] = dim[S] = 1 and the chiral dim[χ] = dim[m^2_π] = 2. The corresponding structural constants I_{4,5,6,7} or L_{4,5,6,7,8} and H_2 are in fact dimensional in the canonical sense that becomes evident when to supplement them with factors of B_0 from χ. Thus these constants as well as the coefficients of L_2 are essentially non-perturbative as they are proportional to powers of the basic QCD scale
\[ \Lambda_C \simeq \mu \exp \left( -\frac{1}{b_0 g^2(\mu)} \right); \quad b_0 = \frac{11 N_c - 2 N_F}{24 \pi^2}, \] (11)
where \( g^2(\mu) \) is a QCD gauge coupling constant measured at a scale \( \mu \).

It happens to be that the dimensionless structural constants \( I_{1,2,3,8,9} \) or \( L_{1,2,3,9,10} \) are not sensitive to a DCSB model and take universal values \[ (4, 15), \]
\[ 4 I_1 = -2 I_2 = 2 I_3 = I_8 = 2 I_9 = \frac{N_c}{48 \pi^2}. \] (12)
These constants are in a satisfactory agreement with phenomenological values obtained from the strong and electromagnetic interactions of pseudoscalar mesons \[ (4, 13, 17, 19) \].

On the other hand, the values of other, dimensional chiral constants (except for \( L_4 = L_6 = 0 \) which are suppressed in the leading 1/N_c order \[ (3) \]) are strongly sensitive to modeling the DCSB by means of a momentum cutoff \( \Lambda \) and a spectrum asymmetry \( M \) or a dynamical mass \( M_d \). In order to estimate dimensional constants in a model independent way we extend the chiral lagrangian by introducing in the next section scalar glueball, \( h \) and quarkonium, \( \Sigma \) variables so that their v.e.v.’s provide the required constants. Further on, we will normalize the corresponding lagrangian to reproduce the QCD motivated expansion of the vacuum energy for small and large external scalar sources. It will provide the model-independent constraints on coupling constants of the joint scalar-pseudoscalar lagrangian. Finally, the large mass reduction of scalar mesons in the latter one yields the required estimates on the chiral constants \( L_5, L_8 \).

Let us start this derivation with preparing the concise form of the dimensional part of \( \mathcal{L}_{ch} \),
\[ \mathcal{L}_{ch}^{dim} = \text{tr} \left[ -F_0^2 \left( \hat{A}_\mu^2 + B_0 \hat{S} \right) - 8 B_0 I_4 i \hat{A}_\mu \tilde{D}_\mu \tilde{P} - 16 B_0 (I_4 + I_5) \hat{A}_\mu^2 \hat{S} \right. \]
\[ -8 B_0^2 I_6 \left( \tilde{S}^2 - \tilde{P}^2 \right) - 4 B_0^2 H_2 \left( \tilde{S}^2 + \tilde{P}^2 \right) \] + \( \Delta \mathcal{L}_{ch}^{dim} \), (13)
where \( \tilde{D}_\mu \equiv \partial_\mu + [\hat{V}_\mu, \star] \). Here and in what follows we omit the \( I_7 \) term as we do not discuss the \( U(1)_A \) terms in this paper. The chiral fields \( U(x) \) are encoded in rotated external sources by means of the following chiral bosonization rules,
\[ \hat{V}_\mu + \hat{A}_\mu = U^\dagger (V_\mu + A_\mu) U + U^\dagger \partial_\mu U; \quad \hat{V}_\mu - \hat{A}_\mu = V_\mu - A_\mu; \]
\[ \hat{S} + i \hat{P} = U^\dagger (S + iP); \quad \hat{S} - i \hat{P} = (S - iP)U. \] (14)
With these rules one can easily reproduce the chiral lagrangian (13). The terms in (13) are ordered in accordance with the ChPT count and $\Delta L_{\text{dim}}^{\text{ch}}$ stands for vertices of dim-6 and higher. Meanwhile from the canonical dimensional analysis it follows that two more vertices, namely

$$\Delta L_{\text{dim}}^{\text{ch}} = 8B_0I_{10}\tilde{S}^3 + 8B_0I_{11}\tilde{S}\tilde{P}^2 + \ldots,$$  

have the comparable dim-3 and they should be retained in the extended scalar-pseudoscalar lagrangian (see below).

The bosonization models with DCSB introduced by an asymmetric cutoff $\Lambda$, $M$ [11, 14, 19] or by a dynamical mass $M_d$ [11, 19] yield in fact more relations between the above constants. Let us derive them first in the framework of the chiral quark model [11, 12]. We impose that the quark loop effective action in external fields were regularized (see [19, 28]) by means of a momentum cutoff $\Lambda_d$ so that it is left (and right) invariant under local chiral transformations of external sources like in (14) (we ignore here the P-odd Wess-Zumino-Witten action). It corresponds to the definition of chirally invariant quarks. The leading low energy contribution to this action in the large cutoff approximation $\Lambda_d \gg |S|, |P|, |V|, |A|$ is given by divergent parts of loop integrals [17, 19, 28],

$$\Gamma_+ \simeq \frac{N_c}{16\pi^2} \int d^4x \text{tr} \left[ -2c\Lambda_d^2 (S^2 + P^2) + \ln \frac{\Lambda_d^2}{\mu^2} \left\{ ((S^2 + P^2)^2 - [S, P]^2) + \left( (D^V_\mu S)^2 + (D^P_\mu P)^2 - \{A_\mu, S\}^2 - \{A_\mu, P\}^2 - 2iD^V_\mu P\{A_\mu, S\} + 2iD^P_\mu S\{A_\mu, P\} \right) - \frac{1}{3} ((F^L_\mu)^2 + (F^R_\mu)^2) \right\} + \ldots \right],$$  

It consists of four independent chiral invariants corresponding to the four lines of (16). For an $O(4)$ invariant momentum cutoff one has $c = 1$ but it is regularization dependent. Meanwhile, the logarithmically divergent coefficients are unique.

The coupling of chirally invariant quarks to pseudoscalar fields is provided by the dynamical mass term introduced in such a way that the designed chiral invariance of (the P-even part of) the fermion determinant is preserved,

$$(S - i\gamma_5 P) \longrightarrow (S - i\gamma_5 P + M_d (U_L^\dagger P_L + P_R)) \longrightarrow (\tilde{S} - i\gamma_5 \tilde{P} + M_d) ,$$  

following the notations of (14), and as well $V_\mu, A_\mu \rightarrow \tilde{V}_\mu, \tilde{A}_\mu$. The equivalence is provided by the invariance under quark field rotations $q_L \rightarrow Uq_L$. The shifted effective action (16) yields the dimensional part (13) of the chiral lagrangian $L_{\text{dim}}^{\text{ch}}$ in terms of the parameters $\Lambda_d, M_d$, which simulate the DCSB, with the following predictions for the coefficients,

$$F_0^2 \simeq \frac{N_c M_d^2}{4\pi^2} \ln \frac{\Lambda_d^2}{\mu^2}, \quad B_0 F_0^2 \simeq \frac{N_c M_d}{4\pi^2} \left( c\Lambda_d^2 - M_d^2 \ln \frac{\Lambda_d^2}{\mu^2} \right); \quad \mu \simeq M_d;$$  

(18)
\[ I_4 = I_{10} = I_{11} = \frac{F_0^2}{8B_0 M_d}; \quad I_5 = 0; \quad I_6 = -\frac{F_0^2}{16B_0^2}; \] (19)

\[ H_2 = \frac{F_0^2}{8B_0 M_d} - \frac{F_0^2}{8B_0^2} = I_4 + 2I_6. \] (20)

Inspecting other bosonization models [4]–[10] we find that the relations for \( I_j \) displayed in (19) are also valid in the large-cutoff approximation. For instance, in the QCD chiral bosonization model [4] the DCSB is introduced by means of an asymmetric regulator for the quark determinant, \( \Theta \left( \Lambda^2 - (\not{D} - iM)^2 \right) \) which is non-invariant under local chiral rotations. The pseudoscalar meson fields arise from local chiral rotations of the quark fields \( q(x) = (P_L U(x) + P_R) q_{inv}(x) \), and the chiral lagrangian is made from the chirally non-invariant part,

\[ Z = \int DU \frac{Z_q(V, A, S, P)}{Z_q(V, A, S, P)} \langle Z_{inv} \rangle_G \equiv \int DU \exp(-S_{eff}(U; V, A, S, P)) \langle Z_{inv} \rangle_G, \] (21)

where \( \langle \ldots \rangle_G \) stands for the averaging over the gluon vacuum. \( S_{eff} \) does not contain any gluon fields. The role of gluons is reduced only to the formation of the dimensional parameters of the theory, \( (\Lambda, M) \), on the base of the equation of stability of the low-energy region, supported by the gluon condensate [4] (see the next section). After calculation of the chirally non-invariant part of the quark determinant one comes to the chiral lagrangian (6) with dimensional vertices collected in (13). The description of the main parameters and the structural constants in the minimal version of the model in [4] has the following form:

\[ F_0^2 \simeq \frac{N_c}{4\pi^2} \left( \Lambda^2 - M^2 \right); \quad B_0 F_0^2 \simeq \frac{N_c M}{2\pi^2} \left( \Lambda^2 - \frac{1}{3} M^2 \right); \] (22)

\[ I_4 = I_{10} = I_{11} = \frac{N_c M}{16\pi^2 B_0}; \quad I_5 = 0; \quad I_6 = \frac{F_0^2}{16B_0^2}; \] (23)

\[ H_2 = -\frac{F_0^2}{4B_0^2} = 4I_6. \] (24)

When comparing with (19) one can see that the following relations between the constants \( I_i \),

\[ I_4 = I_{10} = I_{11}; \quad I_5 = 0; \quad I_6 = -\frac{F_0^2}{16B_0^2}; \] (25)

are the same in both models, but the analytic dependence on model parameters and the quantitative predictions for the coefficients are (of course) model-dependent. In particular, the values for \( H_2 \) do not coincide for any choice of parameters. As to the other constants \( F_0, B_0, I_i \), their values may be adjusted to the same values in both models but for an unnatural choice of the constituent mass parameters: \( M_d \approx 100 \text{ MeV} \) in the chiral quark model and \( M \approx 600 \text{ MeV} \) in the chiral bosonization model [4], respectively. In the following, we will therefore only use the general and model-independent relations (25) to reduce the number
of unknown parameters in the chiral lagrangian (13) or its extensions including heavy scalar fields [27] (see below).

For illustration, let us however check the consistency of these predictions with the phenomenology of pseudoscalar mesons (10) for \( M_d \simeq 250 \text{ MeV} \) and \( B_0 \simeq 1.3 \text{ GeV} \) (\( \Lambda_d \sim 1 \text{ GeV} \)):

\[
L_5 = \frac{F_0^2}{8B_0M_d} \simeq 3 \cdot 10^{-3} \quad (\text{cf. } (2.26 \pm 0.14) \cdot 10^{-3} \text{ from [29]}); \\
L_8 = -\frac{1}{128\pi^2} + \frac{1}{2}L_5 - \frac{F_0^2}{16B_0^2} \simeq 0.5 \cdot 10^{-3} \quad (\text{cf. } (0.9 \pm 0.4) \cdot 10^{-3} \text{ from [21]}); 
\]

where \( N_c = 3 \) is taken. Thus the chiral quark model displays a satisfactory agreement with experiment. Note that whereas \( L_5 = I_4 \) rather serves as an input parameter, which may be tuned by a suitable choice of model parameters, the value of \( L_8 \) is well calculated from all the inputs. A similar situation arises in the chiral bosonization model.

3. QCD bosonization in the scalar sector

In the large \( N_c \) limit the main contribution of the scalar glueball sector to the effective quarkonium lagrangian is provided [24] by the v.e.v. of the glueball field \( \langle h \rangle = \langle \left( G_{\mu\nu} \right)^2 \rangle_{\text{n.p.}} \), i.e. the gluon condensate (see (2)),

\[
C_g \equiv \frac{1}{4\pi^2} \langle \left( G_{\mu\nu} \right)^2 \rangle_{\text{n.p.}} \approx (400 \text{ MeV})^4. 
\]

(27)

Here \( n.p. \) means that the perturbative part has been subtracted. Let us associate the scalar quarkonium fields with the radial colorless fluctuations of light quark fields, \( q(x) = \exp \left\{ \frac{1}{2} \Sigma(x) \right\} q_{\text{inv}}(x) \). Herein \( q_{\text{inv}} \) are fields invariant under dilatations. In general, the dilatations mix flavor numbers, \( \Sigma(x) = \sigma I + \sigma_i T_i \) where \( T_i \) are generators of the \( SU(N_F) \) group. This factorization can be used in the conformal bosonization procedure to derive the effective scalar-pseudoscalar lagrangian [7, 27]. We evaluate it in the soft momentum region neglecting the kinetic terms and higher-order derivatives of (heavy) scalar meson fields.

The dimensional part of the lagrangian for the flavor-singlet (dimensionless) scalar field \( \sigma \) then has the general form:

\[
L_\sigma^\text{dim} = \text{tr} \left[ \frac{\sigma}{48\pi^2} \langle \left( G_{\mu\nu}^a \right)^2 \rangle_{\text{n.p.}} + \frac{1}{4}a_4 e^{-4\sigma} + a_3 \tilde{S} e^{-3\sigma} \\
+ \left( a_{21} \tilde{S}^2 + a_{22} \bar{P}^2 - a_{23} \tilde{A}_\mu^2 \right) e^{-2\sigma} \\
+ \left( a_{11} \tilde{S}^3 + a_{12} \bar{S} \bar{P}^2 - a_{13} i \tilde{A}_\mu \bar{D}_\mu \bar{P} - 2a_{14} \tilde{A}_\mu^2 \tilde{S} \right) e^{-\sigma} \right] + \ldots 
\]

(28)

Let us understand this form of the lagrangian in more detail: The first term simply reproduces correctly the scale anomaly of the quark determinant [30]–[32]. The following terms
are ordered according to their canonical dimension, i.e. they have dimensional vertices with coupling constants which are polynomials in the QCD scale $\Lambda_C$, $a_{jk} \sim \Lambda_C^d$ (the dim-4 term is a purely scalar vertex). According to the dimensionality of the coupling constants the vertices have been dilated with appropriate powers of $\exp(-\sigma)$. We have again neglected dim-4 perturbative contributions and higher dimensional vertices proportional to inverse powers of $\Lambda_C$. It will be shown in the next section that such an expansion can be made consistent with the basic properties of the QCD vacuum energy.

Let us first normalize the lagrangian such that the minimum of $L_{\text{dim}}^\sigma$ for vanishing external sources is just reached at $\sigma = 0$. It corresponds to

$$a_4 = \frac{1}{48\pi^2} \langle (G^a_{\mu\nu})^2 \rangle_{\text{n.p.}} = \frac{C_g}{12}. \quad (29)$$

In order to find the chiral structural constants $I_{4,5,6}$ one should perform the saddle point approximation for the effective scalar $\Sigma$-field lagrangian [27] and develop the heavy scalar mass and ChPT expansion. Of course, when the external sources are present, the lagrangian (28) does no longer possess the minimum at $\Sigma = 0$. The saddle point then is rather given by

$$\Sigma_{\text{min}} = \frac{3a_3}{4a_4} \tilde{S} + \left( \frac{a_{21}}{2a_4} - \frac{9a_3^2}{16a_4^2} \right) \tilde{S}^2 + \frac{1}{2a_4} \left( a_{22} \tilde{P}^2 - a_{23} \tilde{A}_\mu^2 \right) + \cdots; \quad \Sigma \equiv \Sigma_{\text{min}} + \bar{\Sigma}. \quad (30)$$

Now, in terms of the shifted variable $\bar{\Sigma}$, the minimum occurs at $\bar{\Sigma} = 0$. Let us therefore define

$$L_{\sigma}^\text{dim}(\Sigma = \Sigma_m) \equiv L_{\sigma}^\text{dim}(\sigma = 0) \bigg|_{a_{i(j)} \rightarrow \tilde{a}_{i(j)}}, \quad (31)$$

which should be then identified with the dimensional part of the chiral lagrangian (13), $L_{\sigma}^\text{dim}(\Sigma = \Sigma_m) = L_{\sigma}^\text{ch}$. Consequently, the coefficients of the chiral lagrangian (13) can be matched to (28) in the following way:

$$a_3' = a_3 = -B_0 F_0^2;$$
$$a_{21}' = a_{21} - \frac{9a_3^2}{8a_4} = -4B_0^2(2I_6 + H_2); \quad a_{22}' = a_{22} = 4B_0^2(2I_6 - H_2); \quad a_{23}' = a_{23} = F_0^2;$$
$$a_{11}' = a_{11} - \frac{3a_3a_{21}}{2a_4} + \frac{45a_3^3}{32a_4^2} = 8B_0 I_{10}; \quad a_{12}' = a_{12} - \frac{3a_3a_{22}}{2a_4} = 8B_0 I_{11};$$
$$a_{13}' = a_{13} = 8B_0 I_4; \quad a_{14}' = a_{14} - \frac{3a_3a_{23}}{4a_4} = 8B_0(I_4 + I_5). \quad (32)$$

Just these coupling constants should be compared with the chiral bosonization predictions. In particular, the quark model lagrangian (16)-(24) (or the bosonization model (21) - (24)) is induced by the choice:

$$a_{21}' = F_0^2 \left( 1 - \frac{B_0}{2M_d} \right) \quad \text{[or]} \quad = \frac{3}{2} F_0^2 \quad (1 - \frac{B_0}{2M_d}). \quad (32)$$
\[ a'_{22} = a'_{21} - a'_{23} = -\frac{F_0^2 B_0}{2M_d} \] \[ a'_{11} = a'_{12} = a'_{13} = a'_{14} = \frac{F_0^2}{M_d} \]

\[ \text{or } = \frac{N_c M}{2\pi^2} \] . \hspace{1cm} (33)

In the next section we will remind basic properties of the QCD quark vacuum energy as a function of external scalar fields and further derive the additional constraints on some of the \( a_{jk} \). It will allow us to reduce the number of input parameters in any model building.

4. QCD vacuum energy and large scalar fields

We go back to the QCD generating functional [1] and consider nearly constant scalar gluonium \( \rho \) and quarkonium \( S \) sources. The renormalized coupling constant at a given scale \( \mu \) for \( N_F \) active quarks obeys the following RG equation

\[ \partial_\tau g^2 = \beta(g^2) \simeq -b_0 g^4; \quad \frac{1}{g^2(\mu)} \simeq b_0 \ln \frac{\mu}{\Lambda_C} \simeq \frac{11N_c - 2N_F}{24\pi^2} \ln \frac{\mu}{\Lambda_C}; \] \hspace{1cm} (34)

in the 1-loop approximation. Herein \( \partial_\tau \equiv \partial/\partial \ln(\mu/\Lambda_C) \).

From eq. (1) it is obvious, that the field \( \rho \) can be reabsorbed into the definition of the coupling constant, i.e. the definition of the basic QCD scale \( \Lambda_C \),

\[ \frac{1}{g^2} \equiv \frac{1}{g^2} + \rho = b_0 \ln \frac{\mu}{\Lambda_C e^{-\rho/b_0}} , \quad \tilde{\Lambda}_C \equiv \Lambda_C e^{-\rho/b_0} . \] \hspace{1cm} (35)

Now let us perform an expansion of the vacuum energy in \( N_F \), the number of (light) flavors,

\[ \mathcal{E}_{\text{vac}}(\rho, S) = \mathcal{E}_{\text{vac}}^{(0)}(\rho) + N_F \mathcal{E}_{\text{vac}}^{(1)}(\rho, S) + O(N_F^2) . \] \hspace{1cm} (36)

The leading term for \( N_F = 0 \) is purely gluonic. The non-perturbative part of the QCD gluonic vacuum energy density \( \mathcal{E}_{\text{vac}}^{(0)}(\rho) \) is then defined as

\[ Z(\rho, S) \big|_{N_F=0} \equiv \exp \left\{ -\Omega \mathcal{E}_{\text{vac}}^{(0)}(\rho) \right\} , \] \hspace{1cm} (37)

with \( \Omega = \int d^4x \) being the space-time volume.

From dimensional arguments we obtain that

\[ \mathcal{E}_{\text{vac}}^{(0)}(\rho = 0) = \langle \Theta_{44} \rangle_{\text{n.p.}} = \frac{1}{4} \langle \Theta_{\mu\nu} \rangle_{\text{n.p.}} \simeq -\Lambda_C^4 , \] \hspace{1cm} (38)

where \( \Theta_{\mu\nu} \) stands for the energy-momentum tensor. One therefore arrives at the RG equation for the energy density \( \partial_\tau \mathcal{E}_{\text{vac}}^{(0)}(0) = -4 \mathcal{E}_{\text{vac}}^{(0)}(0) \) which has the following (RG invariant) solution \[ \text{[31]} \] (see eqs.(1),(34) ) given by the trace anomaly \[ \text{[30]} \]-\[32\]:

\[ \mathcal{E}_{\text{vac}}^{(0)}(0) = \frac{1}{16g^4} \beta(g^2) \langle (G^a_{\mu\nu})^2 \rangle_{\text{n.p.}} \bigg|_{N_F=0} \simeq -\frac{\bar{b}_0}{16} \langle (G^a_{\mu\nu})^2 \rangle_{\text{n.p.}} , \] \hspace{1cm} (39)
where $b_0 \equiv b_0 \mid_{N_F=0}$. When $\rho \neq 0$ one simply has to replace $\Lambda_C$ with $\bar{\Lambda}_C$ from (33)

$$\mathcal{E}_\text{vac}^{(0)}(\rho) = e^{-4\rho/b_0} \mathcal{E}_\text{vac}^{(0)}(0) = -\frac{\pi^2 b_0}{4} C_g e^{-4\rho/b_0},$$

in terms of (27).

In order to evaluate the effective potential for the scalar glueball field $h(x)$ we follow the bosonization ansatz (2) which can be rewritten in the form of a functional Legendre transform with respect to the (nearly constant) field $\tilde{\rho}$,

$$\tilde{Z}(h) = \exp \left\{ -\Omega V_g(h) \right\} = \int \mathcal{D}\tilde{\rho} \exp \left\{ -\Omega \left[ \mathcal{E}_\text{vac}^{(0)}(\tilde{\rho}) - \frac{1}{4} \tilde{\rho} h \right] \right\}. \quad (41)$$

For $\Omega \to \infty$ the saddle point configuration,

$$\tilde{\rho} = \bar{\rho}(h) = -\frac{\bar{b}_0}{4} \ln \frac{h}{\langle (G^a_{\mu\nu})^2 \rangle_{n.p.}},$$

delivers the required effective potential:

$$V_g(h) = \mathcal{E}_\text{vac}^{(0)}(0) \frac{h}{h_0} \left( 1 - \ln \frac{h}{h_0} \right), \quad h_0 = \langle (G^a_{\mu\nu})^2 \rangle_{n.p.}, \quad (43)$$

which coincides with the expression in [32].

Now we proceed to the analysis of the quark-loop contribution in (1) to the vacuum energy and its dependence on the external scalar source $S(x) \simeq \text{const}$ (other external sources are not shown for the time being),

$$Z(\rho, S) \equiv \exp \left\{ -\Omega \mathcal{E}_\text{vac}(\rho, S) \right\} = \int \mathcal{D}G \exp \left\{ -\int d^4x \frac{1}{4g^2} (G^a_{\mu\nu})^2 + N_F \frac{1}{2} \text{Tr} \left[ \ln \left( \frac{D^2 + \partial S + S^2}{\mu^2} \right) \right] R \right\}_{n.p.}, \quad (44)$$

where the quark fields are integrated out leading to the quark determinant. Here $R = R(D^2, \Lambda^2_{UV})$ is a gauge invariant regulator. In the following we restrict ourselves to slowly varying sources $\partial S \simeq \tilde{\partial} \rho \simeq 0$.

The first-order term $\mathcal{E}_\text{vac}^{(1)}$ gets two contributions [25] from both the $\beta$-function,

$$\partial_{N_F} g^{-2} \simeq -\frac{1}{24\pi^2} \ln \frac{\mu^2}{\Lambda^2_C}, \quad (45)$$

and the quark determinant. In addition, all dimensional operators are supplemented with scaling factors $\exp(-\rho/b_0)$ so that the higher-dimensional condensates $\bar{C}_{2n+4} \sim$\footnote{We retain the 1-loop contributions only and therefore neglect the anomalous dimension of $\bar{q}q(x)$, i.e. of $S(x)$.}

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\[ \langle \text{Tr} [D^{2n}R] \rangle_{\text{n.p.}} \sim \Lambda^{2n+4}_C \]

are rescaled according to their canonical dim = 2n + 4 and (33). Thereby one derives that

\[ E^{(1)}_{\text{vac}}(\rho, S) = e^{-4\rho/b_0} \left\langle -\frac{1}{96\pi^2} \ln \frac{\mu^2}{\Lambda^2_C} (G_{\mu\nu}^a)^2 - \frac{1}{2} \text{tr} \left[ \langle x | \ln \left( \frac{D^2 e^{-2\rho/b_0} + S^2}{\mu^2} \right) R | x \rangle \right] \right\rangle_{\text{n.p.}} . \]  

(46)

It is a RG invariant quantity [25],

\[ \frac{\partial E^{(1)}_{\text{vac}}}{\partial \ln \mu^2} = e^{-4\rho/b_0} \left\langle -\frac{1}{96\pi^2} (G_{\mu\nu}^a)^2 + \frac{1}{2} \text{tr} \left[ \langle x | R | x \rangle \right] \right\rangle_{\text{n.p.}} = 0 , \]

which is provided by the Fujikawa’s theorem [30],

\[ \langle \text{tr} \left[ \langle x | R | x \rangle \right] \rangle_{\text{n.p.}} = \frac{1}{48\pi^2} \langle (G_{\mu\nu}^a)^2 \rangle_{\text{n.p.}} , \]

(47)

as long as the regulator obeys the conditions \( R(D^2 = 0) = 1, \ R(D^2 \to \infty) = 0 \) (in fact any momentum cut-off scheme).

Let us introduce a fermionic reference scale \( \Lambda_F \) [24] for the Dirac operator \( D \) as a normalization at \( S = 0 \) in the following way:

\[ \left\langle \text{tr} \left[ \langle x | \ln \left( \frac{D^2}{\Lambda^2_F} \right) R | x \rangle \right] \right\rangle_G \equiv 0 , \]

(48)

leading to

\[ E^{(1)}_{\text{vac}}(\rho, S = 0) \equiv -e^{-4\rho/b_0} \frac{1}{24} \left( \ln \frac{\Lambda^2_F}{\Lambda^2_C} - \frac{2\rho}{b_0} \right) C_g . \]

(49)

Presumably, this (fermionic) scale is of the order of 1 GeV, \( \Lambda_F > \Lambda_C \). Then we can redefine the quark vacuum energy using this scale,

\[ E^{(1)}_{\text{vac}}(\rho, S) = E^{(1)}_{\text{vac}}(\rho, 0) + e^{-4\rho/b_0} W_F(\bar{S}) , \]

\[ W_F(\bar{S}) \equiv -\frac{1}{2} \left\langle \text{tr} \left[ \langle x | \ln \left( \frac{D^2 + S^2}{\Lambda^2_F} \right) R | x \rangle \right] \right\rangle_{\text{n.p.}} , \]

(50)

where we have defined \( \bar{S} \equiv S e^{\rho/b_0} \). When other external sources are present, they should be rescaled in the same way, according to their canonical dimension, \( (\bar{P}, \bar{D}_\mu^V, \bar{A}_\mu) = (P, D^V_\mu, A^V_\mu) e^{\rho/b_0} \).

The glueball potential is again given by the Legendre transformation (41), and to the first order in \( N_F \) the quark contribution is described by (50) where for the field \( \rho \) the saddle point value \( \rho := \bar{\rho}(h) \) (12) is taken. These rules can be straightforwardly applied to QCD bosonization models in order to derive the joint scalar glueball-quarkonium potential,

\[ \exp \left[ -\Omega W_F(\bar{S}) e^{-4\bar{\rho}(h)/b_0} \right] \simeq \int \mathcal{D}\sigma \exp \left[ -\Omega \left( \mathcal{L}_\sigma(\bar{S}) - \frac{N_F C_g}{48} \right) e^{-4\bar{\rho}(h)/b_0} \right] . \]

(51)
The constant in the exponential on the r.h.s. is fixed by the normalization $W_F(\bar{S} = 0) = 0$ provided at the saddle point $\sigma = 0$, and for our purposes $\mathcal{L}_\sigma$ is taken from eq. (28). After substituting (51) into the QCD generating functional (44) and performing the shift $\sigma \rightarrow \sigma - \rho/\bar{b}_0$, the glueball-quarkonium potential may be eventually expressed in the following form:

$$\mathcal{L}_{h,\sigma} = V_g(h) + \frac{N_F}{48\pi^2} h \left( \sigma - \ln \frac{\Lambda_F}{\Lambda_C} - \frac{1}{4} \right) + \mathcal{L}_{\sigma}^{\text{dim}}(S) - \frac{N_F C_g}{12} \sigma, \quad (52)$$

We see that in (52) the coupling between glueball and quarkonium fields is fixed unambiguously.

Let us examine the dependence of the QCD vacuum energy on slowly varying scalar fields, $S(x) \simeq \text{const}$. The quark vacuum energy is connected to the quark condensate (at 1-loop approach) [24]

$$\partial S W_F(\bar{S}) = i \langle \bar{\Psi} \Psi \rangle = - \left\langle \text{tr} \left[ \langle x | \bar{\Psi} \Psi \rangle \right] \right\rangle \quad (53)$$

Let us focus our attention on the case $S \gg \Lambda_C$. From eq. (44) we have

$$Z(\rho, S \rightarrow \infty) \rightarrow \int \mathcal{D}G \exp \left\{ - \int d^4x \frac{1}{4} \left( \frac{1}{\bar{g}^2} - N_F \frac{1}{24\pi^2} \ln \frac{S^2}{\mu^2} \right) (G^a_{\mu \nu})^2 + O(N_F)^2 \right\} \quad (55)$$

As in (33) one can make the RG improvement by redefining the strong coupling constant,

$$\bar{g}^{-2} = \bar{g}^{-2} - N_F \frac{1}{24\pi^2} \ln \frac{S^2}{\mu^2} \simeq \frac{11N_c}{48\pi^2} \ln \frac{\mu^2}{\Lambda_C^2} - \frac{2N_F}{48\pi^2} \ln \frac{S^2}{\Lambda_C^2} \equiv \frac{11N_c}{48\pi^2} \ln \frac{\mu^2}{\Lambda_C^2}, \quad (56)$$

so that

$$\bar{\Lambda}_C = \Lambda_C \left( \frac{|S|}{\Lambda_C} \right)^{\frac{2N_F}{11N_c}} \quad (57)$$

Consequently, the leading contribution into vacuum energy for large $S$ factorizes,

$$\varepsilon_{\text{vac}}(S \rightarrow \infty, \rho) = \varepsilon_{\text{vac}}^{(0)}(\rho) \left( \frac{|S|}{\Lambda_C} \right)^{\frac{8N_F}{11N_c}} + O(N_F^2) < 0 \quad (58)$$

In all cases quark polarization effects lower the QCD vacuum energy.

Passing on we remark that this result is related to conclusions in ref. [33]. Indeed let us assume the value of the strong coupling constant and therefore of $\Lambda_C$ is fixed at a scale much larger than the masses of all $N_F = 6$ participating quarks (say e.g. a GUT scale $M_U$). Let
us denote this scale by $\Lambda_C$. If one likes to know how this is connected to the low-energy $\Lambda_C$ with $N_F = 3$ dynamical quarks, one has to integrate out the heavy quarks step by step. In this way one extracts the large mass logarithms in a similar manner. The result is

$$\Lambda_C = \hat{\Lambda}_C \left( \frac{m_c m_b m_t}{\hat{\Lambda}_C^3} \right)^{2/27} > \hat{\Lambda}_C; \quad 27 = 11N_c - 2N_F^{\text{light}}.$$ (59)

Thus in this Sec. the structure of bosonized action for a scalar glueball and its coupling to quark degrees of freedom is determined as well as the asymptotic conditions on the behavior of quark vacuum energy for very large but constant scalar sources $S$ are formulated. We would like to stress that this limit belong to the soft momentum region and it is safe being related to the decoupling of heavy quarks, i.e. one does not expect any phase transition or strong coupling phenomena when $S \to \infty$. Hence one should use this conditions to obtain model independent constraints on parameters of effective meson lagrangian.

5. Asymptotic constraints on chiral constants

Let us impose the QCD asymptotics (55) to be fulfilled also in the extended meson lagrangian (28) for scalar and pseudoscalar fields. We normalize $a_4$ according to (29) and $a_3$ as in (32). First we switch off all the external sources but the scalar one, i.e. we retain the scalar field vertices with constants $a_{21}, a_{11}$.

For these unknown parameters, we examine the asymptotics $S \to \infty$ ($V, A, P = 0, U = 1$). It is convenient to introduce a new variable $y$ such that

$$S y := e^{-\sigma} \leftrightarrow \sigma = -\ln S - \ln y.$$ (60)

We then obtain for the effective potential

$$W(y, S) = -a_4 \ln S - a_4 \ln y + \left( \frac{1}{4} a_4 y^4 + a_3 y^3 + a_{21} y^2 + a_{11} y \right) S^4.$$ (61)

The first term ($-a_4 \ln S$) already reproduces the leading logarithmic behaviour (55) of the QCD vacuum energy with the correct coefficient. Consequently, after integrating out the field $y$ from the saddle point approximation, the remaining terms should at most behave like $(\text{const.} + O(1/S^2))$.

Thus the term in front of $S^4$ has to be of higher order in $1/S^2$ for $y$ taking its saddle point value $y_{min}$

$$y_{min} \left( \frac{1}{4} a_4 y_{min}^3 + a_3 y_{min}^2 + a_{21} y_{min} + a_{11} \right) = O \left( \frac{1}{S^4} \right); \quad y_{min} \neq 0.$$ (62)

For $y_{min}$ being the saddle point, the first derivative of $W(y, S)$ with respect to $y$ has to vanish,

$$W'(y) = a_4 y_{min}^3 + 3a_3 y_{min}^2 + 2a_{21} y_{min} + a_{11} - \frac{a_4}{S^2} \frac{1}{y_{min}} = 0.$$ (63)
Let us expand the last relation in \(1/S^2\), with \(y_{\text{min}} = y_0 + \delta/S^2 + O(1/S^4)\). We obtain

\[
W'(y) = a_4 y_0^3 + 3 a_3 y_0^2 + 2 a_{21} y_0 + a_{11} + \frac{\delta (3 a_4 y_0^2 + 6 a_3 y_0 + 2a_{21})}{S^2} + O\left(\frac{1}{S^4}\right).
\]

(H64)

Hence one arrives to the second constraint,

\[
a_4 y_0^3 + 3 a_3 y_0^2 + 2 a_{21} y_0 + a_{11} = 0,
\]

(H65)

Two constraints make it possible to find the saddle point value \(y_0\) and to estimate one of the coefficients, \(a_{11}\):

\[
y_0 = \frac{-4a_3 \pm \sqrt{16a_3^2 - 12a_{21}a_4}}{3a_4};
\]

\[
a_{11} = y_0 \frac{8a_3^2 - 6a_{21}a_{21}}{9a_4} \frac{4a_3a_{21}}{9a_4}.
\]

(H66)

In order to obtain information for the chiral lagrangian, namely the constants \(L_5, L_8\), we perform the large scalar mass reduction \(a_{i(j)} \rightarrow a'_{i(j)}\), see eq. (32), and express the chiral constants in terms of \(B_0, F_0, C_g\) and \(a'_{21}\). Together with the universal relations (B3) \(a'_{11} = a'_{12} = a'_{13} = a'_{14} = 8B_0I_4\), \(I_5 = 0\) one finds the parametrization of the chiral constant \(L_5\)

\[
L_5^\pm = \frac{B_0^2 F_0^6}{3C_g^2} \left[ \frac{3}{4} A - \frac{29}{16} \left(\frac{5}{2} - A\right)^{3/2} \right]; \quad A \equiv \frac{C_g a'_{21}}{B_0^2 F_0^4}.
\]

(H67)

The chiral constant \(L_8\) is supposed to obey (29) and will be in agreement with the experimental estimates for the choice of \(L_5 \sim 3 \cdot 10^{-3}\). Note that the pre-factor in (67) already sets the right scale \(B_0^2 F_0^6/3C_g^2 \sim 10^{-3}\). Therefore the remaining numerical factor in parentheses is expected to be of order 3, which is in fact only achieved for the choice \(L_5^+\). It can be achieved for a value of \(a'_{21} \sim -\frac{1}{2} F_0^2 < 0\) which just lie inbetween the predictions of the chiral quark model \(a'_{21} = F_0^2 (1 - (B_0/2M_d)) \sim -(2 \div 3) F_0^2\) and the chiral bosonization model \(a'_{21} = 3/2 F_0^2\), respectively. Thus we see that the asymptotic constraints impose rigid conditions on the choice of a bosonization model.

The next-to-leading term of the asymptotics (H64) needs a special care. If \(\delta = 0\) then the next term of order \(1/S^4\) contributes to the quark vacuum energy (H61) only corrections of the same order in virtue of (H63). Then \(O(1/S^2)\) terms in the effective potential do not appear. That would correspond to the identically vanishing dim-6 gluon condensate [24] \(C_6^g \sim \langle G^3\rangle\). As there is no reason \textit{a priori} to neglect the latter one we obtain the third constraint for \(\delta \neq 0\),

\[
3 a_4 y_0^2 + 6a_3 y_0 + 2 a_{21} = 0.
\]

(H68)
Together with (62), (65) we thereby have 3 constraints for the variables $a_{21}, a_{11}, y_0$. The analysis of these 3 constraints shows that their compatibility holds only if $a_{21}, a_{11} \neq 0$. One then has,

$$y_0 = \frac{16B_0F_0^2}{C_g}; \quad a_{21} = \frac{16B_0^2F_0^4}{C_g}; \quad a_{11} = -\frac{256B_0^3F_0^6}{3C_g^2};$$

which corresponds to

$$L_5 = L_5^+ = L_5^- = \frac{B_0^2F_0^6}{48C_g^2} \sim O(10^{-4}); \quad A = \frac{5}{2}. \tag{70}$$

However, the amount of terms in the polynomial expansion of (28) may still not be enough to make a numerically reliable interpolation, and it will be necessary to include terms of $O(S^4)$ into the scalar lagrangian (28). (Note that the unnaturally large value of $a_{11}$ which is the last coefficient in the polynomial expansion of (28) will presumably reduce then, leading also to more realistic values for $L_5$.) Therefore, the third constraint (68) cannot be taken too seriously for a given interpolation, and neglecting the dim-6 condensate corresponding to the solution $\delta = 0$ may well be consistent with the precision of the expansion of the bosonization model (28).

Nevertheless, we have seen that the asymptotic constraints can be embedded in the meson scalar-pseudoscalar lagrangian, giving a powerful tool to estimate some of the chiral coefficients.

Evidently the constants in other vertices of the lagrangian (28) can be treated in the same manner. For instance, one may expect the mutual cancellation of large $S$ contributions in certain combinations of vertices $a_{22}, a_{12}$ or $a_{23}, a_{14},$

$$a_{22}y_{\text{min}} + a_{12} \simeq 0; \quad a_{23}y_{\text{min}} + 2a_{14} \simeq 0, \tag{71}$$

which however is not anticipated to be accurate as only two terms of corresponding expansions are included.

6. Summary and conclusions

In this paper we have developed the modified bosonization approach to construct meson lagrangians from QCD where the large field but low-energy asymptotics in QCD is embedded after bosonization for scalar field vertices.

First, it was shown that the chiral bosonization in different approaches results in the same structure of vertices (in the large-cutoff approximation), encoding the information about the QCD dynamics in a few chiral constants $I_i$ which can be related to the phenomenological structure constants $L_i$ and obey certain model-independent relations among each other.

Next, we have determined the vacuum properties of the QCD quark energy in the presence of
large external scalar fields and applied the results to estimate some of the coupling constants in the extended effective meson lagrangian, including scalar and pseudoscalar fields. These ones in turn have been used to obtain an unambiguous estimate of the phenomenologically interesting (but generally rather model-dependent) structure constants $L_5$ and $L_8$.

Moreover, by employing conformal bosonization methods one gets a scalar meson lagrangian which satisfies all QCD motivated requirements and asymptotics and thereby gives a direct way to extend it unambiguously, such that the coupling to glueball fields can be described. As an interesting result, we have given a path-integral derivation of the effective glueball potential from the generating functional of QCD using $\delta$-functional constraints.

There are a few improvements to be done in order to reach better precision, and one of the most important is to include higher-dimensional terms into the polynomial expansion of the lagrangian (28). In a sense this will lead to an infinite number of asymptotic sum rules for an infinite number of unknown constants, relating them to gluon condensates of increasing dimension.

Finally, note that in the conformal bosonization approach dynamical scalar mesons are treated as intrinsic dilatational modes. It is a challenge to introduce them in the QCD effective action as true collective variables, in analogy to chiral rotation modes describing pseudoscalar mesons. This program will be realized elsewhere.

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