Effective Theories for Exclusive and Seminclusive Processes and Factorization

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The effective theories for massless quarks describing exclusive and seminclusive processes are discussed, considering in particular the factorization problem.

1. Introduction

Factorization is the property that a hadronic matrix element can be written as the product of simpler matrix elements. For example, the decay

$$B \rightarrow D + \pi$$

has amplitudes which can be written, if factorization holds, as

$$g^{\mu\nu} \langle D, \pi | J_{(b \rightarrow c)_{\mu,i}} J_{(u \rightarrow d)_{\nu,i}} | B \rangle$$

$$= g^{\mu\nu} \langle D | J_{(b \rightarrow c)_{\mu,i}} | B \rangle \langle \pi | J_{(u \rightarrow d)_{\nu,i}} | 0 \rangle$$

where $J^{\mu}_{i}$ is the color singlet (octet) weak current for $i = 1(8)$. The above property implies that the rate of (1) can be expressed in terms of the rate of the semileptonic decay

$$B \rightarrow D + l + \nu,$$

whose amplitude is proportional to the matrix element

$$\langle D | J^{(b \rightarrow c)_{\mu,i}} | B \rangle,$$

and of the (purely) leptonic decay rate of

$$\pi \rightarrow l + \nu,$$

whose amplitude is proportional to

$$\langle 0 | J^{(u \rightarrow d)_{\nu,i}}_{\mu,1} | \pi \rangle = i f_{\pi} p_{\nu}.$$
In sec.2 we sketch the resulting ‘Large Energy effective Theory’ (LEET) \[6\]. The main property of the limit \(\hat{\mathcal{P}}\) is that quantum fluctuations in the motion are suppressed and the quark ‘resembles’ a classical, in analogy with the limit of an infinite mass in the HQET \[5\]. In particular, the transverse motion of the quark with respect to the line of flight (the direction \(\mathbf{n}\)) is completely neglected. This implies that quarks created in the same point do not separate from each other at any time. There is not any dipole color grow (as it does occur in Bjorken picture) to account for hadronization into a quark, the effect of the ‘valence’ quarks, there is not any dynamics of the ‘valence’ quarks, to which quarks move on a prescribed trajectory.

In sec.3 we show that the LEET can describe seminclusive decays, such as for example

\[
B \to D + \text{jet}
\]

and can be used to prove factorization for this class of processes.

In sec.4 we come back to the problem of describing an exclusive process such as \(D/\pi\) in the effective theory framework and we introduce a new effective theory containing the required transverse momentum terms.

Sec.5 contains the conclusions and an outlook to future developments.

2. The LEET and its failure to describe exclusive processes

Let us briefly review the derivation of the LEET propagator \[6,7\]. The momentum \(P\) of a massless quark is decomposed into a ‘classical part’ \(E\mathbf{n}\) and a fluctuation \(k\):

\[
P = E\mathbf{n} + k.
\]

\(n\) is a light-like vector, \(n^2 = 0\), normalized by the condition \(v \cdot n = 1\), where \(v\) is a reference time-like vector, \(v^2 = 1\), with positive time component, \(v_0 > 0\). \(E\) is the classical, ‘primordial’, energy of the quark in the rest frame of \(v\) and has to be considered large,

\[
|k_\mu| \ll E.
\]

The propagator is given by

\[
iS_0(E\mathbf{n} + k) = i\frac{E\hat{n} + \hat{k}}{(E\mathbf{n} + k)^2 + i\epsilon}
\]

\[
= i\frac{\hat{n}/2 + \hat{k}/(2E)}{n \cdot k + k^2/(2E) + i\epsilon}
\]

\[
\approx \frac{i}{2} \frac{\hat{n}}{n \cdot k + i\epsilon}.
\]

where in the last line the limit \(\hat{\mathcal{P}}\) has been taken. The energy-momentum relation is

\[
\epsilon = \hat{u} \cdot \hat{k} = k_z,
\]

where \(\hat{u} \equiv \hat{n}/n_0\) is the kinematical velocity, with \(|\hat{u}| = 1\). In the last term a motion along the \(z\)-axis has been assumed.

The propagator in configuration space is the scalar density of a particle moving along a ray with the velocity of light:

\[
iS_0(x) = \frac{\hat{n}}{2} \theta(t) \frac{\delta^{(3)}(\hat{x} - \hat{u}t)}{n_0}.
\]

The interaction with the gauge field produces a P-line factor joining the origin with the point \(x\) along the light-like trajectory specified by \(n\):

\[
iS(x) = iS_0(x)P \exp\left[ig \int_0^{t/n_0} ds \mathcal{A}_\mu^\nu(\mathbf{n}s)\right].
\]

Note the factorization of both spin and color degrees of freedom. As it is clear from the derivation, the LEET propagator describes massless particles suffering soft interactions only. In other words, the effect of the quark on the gluon field is fully taken into account, while the reaction of the field on the particle is completely neglected. The limit \(\hat{\mathcal{P}}\) is a systematic no recoil approximation and is called (massless) eikonal model in perturbative QCD \[8\].

We present now a physical argument of the failure of the LEET to describe an exclusive decay such as \(D/\pi\). Since the \(\bar{\pi}\) quark is approximated by a LEET quark, the effect of the \(d\) on the \(\bar{\pi}\) is completely neglected. Analogously, since the \(d\) quark also is taken in the LEET, the effect of the \(\bar{\pi}\) on the \(d\) is neglected. As a consequence, there is not any dynamics of the ‘valence’ quarks, and consequently not a chance of bounding them together into a \(\pi\). This conclusion is clear also from the form \(\hat{\mathcal{P}}\) of the propagators, according to which quarks move on a prescribed trajectory.
The irrelevance of the LEET in the exclusive domain can be analytically confirmed with a spectral decomposition of the correlation functions associated to the decay \( \Pi \) or with a study of the one-loop pinch singularities of Feynman diagrams \([7,8]\).

3. Seminclusive decays and factorization by means of the LEET

The argument of the previous section is centered on the fact that the \( \bar{u} \) and \( d \) quarks are required to combine into a single meson state in the decay \([1,2]\): they are the relevant degrees of freedom of the system, the meson valence quarks. If we consider instead the seminclusive decay \([8]\), many other degrees of freedom are involved, participating to the jet development. In the latter case, we can identify the \( \bar{u} \) and \( d \) quarks with partons, i.e. with high energy excitations, emitting (softer) secondary quarks and gluons in the parton cascade. If we neglect the reaction of the secondary particles, we can legitimately replace the \( \bar{u}d \) pair with LEET quarks. Assuming a jet with a small angular width \([1,2]\) we can take equal velocities for the quarks. Apart from irrelevant constants, the dynamics of the pair is represented, before functional integration over the gluon field, by the factor

$$T \left[ P e^{i\alpha} \int_0^1 A_\mu \, dx^\mu \, P e^{i\beta} \int_0^1 A_\mu \, dx^\mu \right] = T \left[ \xi_i \right],$$

(15)

where the weak current is taken in the origin, the light pair interpolating field is a color singlet bilinear placed in \( x, \xi_i = 1, t_a \) for \( i = 1, 8 \) respectively and \( T \left[ \xi_i \right] = N_C \delta_{i,1} \). The above factor is independent of the gluon field, implying the absence of any interaction of the ‘light’ system with the ‘heavy’ system. This implies factorization for the seminclusive decay in the spirit of Bjorken color screening argument.

4. A new effective theory

Let us consider a meson in the infinite momentum frame moving along the \( z \)-axis: that is equivalent to the limit \([1,2]\) discussed in sec.2. We observe the valence quarks exchanging transverse momenta of order of the hadronic scale \( \Lambda_{QCD} \): \( P_T \sim \Lambda_{QCD} \). These interactions are necessary for the bound state dynamics, while they are corrections of order \( \Lambda_{QCD}^2 / E \) to the leading term \( P_z \sim E \), and are neglected in the limit \([7]\). Therefore the failure of the LEET to describe exclusive processes is of kinematical origin. Eq.\([12]\) is the lowest order term in the expansion of the energy-momentum relation \( P^2 = 0 \) with \( P \) given in eq.\([13]\):

$$E + \epsilon = \sqrt{(E + k_z)^2 + k_T^2} = E + k_z + \frac{k_T^2}{2E} + \ldots$$

(16)

We can ‘correct’ the LEET by including the term of order \( 1/E \) into the propagator:

$$iS(k) = \frac{\tilde{n}}{2} \frac{k}{n \cdot k - k_T^2 / 2E + i\epsilon}$$

(17)

where \( n^\mu = (1; 0, 0, 1) \), \( k_T^\mu = (0; k_T, 0) \).

Eq.\([17]\) defines an effective theory: the propagator contains the 4-velocity \( n \) as an external vector, so there is a formal breaking of Lorentz invariance as it happens in the LEET or in the HQET \([3]\). Furthermore \( iS(k) \) is forward in time, so that antiparticles are removed and the vacuum is consequently trivial. This is expected from an effective theory describing hard partons, because particle-antiparticle pairs have a threshold energy of order \( 2E \) and cannot be excited with soft interactions. This new effective theory, which we called LEET, is much more complicated than the LEET. In particular, the hard scale \( E \) is still present in the theory, i.e. it cannot completely be removed.

The propagator is given in configuration space by

$$iS(t, \vec{x}) = \frac{n}{2} \theta(t) \delta(z - t) \frac{E}{2\pi i t} e^{iE^2 / (2t)}$$

(18)

As it is well known, a proper definition of a jet involves also an energy resolution parameter in order to cancel infrared singularities. We do not consider this problem explicitly, which deserves further investigation. We wish to thank Prof. G. Altarelli for having brought this point to our attention.

\[5\] We neglect perturbative corrections coming from hard gluons producing a broad tail in the quark transverse momentum distribution of the form \( \sigma_s \, dk_T^2 / k_T^2 \).
where \( b = |\vec{x}_T| \) is the impact parameter. The effect of the transverse momentum term is factorized (compare with eq. (13)) and produces fast oscillations of the wavefunction with \( b \) at small times. The larger the energy \( E \), the faster the oscillations are. The transition from the \( \text{LEET} \) to the \( \text{LEET} \) is analogous to the transition from geometrical optics to physical optics, the latter taking into account diffraction in first approximation. It is interesting to note that after euclidean continuation, \((t \rightarrow -it)\), the last two terms in eq. (18) represent a gaussian diffusion in the impact parameter space.

At \( b = 0 \) we have:

\[
S(t, \vec{x}) \simeq \frac{\hat{n}}{2} \theta(t) \delta(z-t) \frac{E}{2\pi it}. \tag{19}
\]

There is a diffusion normal to the classical particle trajectory \( z = t \) produced by transverse momentum fluctuations, which is instead absent in the \( \text{LEET} \). The amplitude for the particle to remain into the classical trajectory decays like \( 1/t \), so the probability decays like \( 1/t^2 \).

We believe that the \( \text{LEET} \) is the correct effective theory for massless particles as long as exclusive processes are concerned.

It is hard to reach a conclusion about factorization in the exclusive decay (1) on the basis of simple analytical computations with the \( \text{LEET} \). That is because of diffusion in the impact parameter space, according to which light quark dynamics is represented by a superposition of non trivial Wilson loops instead of a single (trivial) one.

5. Conclusions and Outlook

The conclusions of our analysis consists of various statements, both of negative and positive character, which can be organized in the following way.

\textit{ii)} Exclusive decays such as (1) cannot be described by the \( \text{LEET} \), which is a too rude approximation to describe individual hadron properties. The problem of proving factorization inside the \( \text{LEET} \) therefore is not a well posed one.

\textit{ii)} The \( \text{LEET} \) is capable of describing \textit{seminclusive} reactions, such as (4), in which high energy partons, evolving into hadronic jets, are replaced by effective quarks. For the latter processes factorization can be proved in a non perturbative and gauge invariant way with the theory of Wilson loops. The proof is a formalization of the Bjorken color dipole argument presented in the introduction.

\textit{iii)} Exclusive decays can be studied in the framework of a more complicated effective theory than the \( \text{LEET} \), which accounts for transverse momentum dynamics in first approximation.

\textit{iii)} It is not clear at present if Bjorken idea can be formalized for exclusive decays in the framework of the new effective theory. That is because transverse momentum terms produce a diffusion of Wilson lines off the classical quark trajectory. The only clue is that the form of the propagator (18) shows large interference effects in the quark wave function away from the classical trajectory. That gives perhaps some indication of a small effective color dipole interaction of the system of light quarks in the decay (\[\]).

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