Modelling and operation analysis of autonomous electromechanical power supplies with a commensurate power load

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Abstract. Nowadays, diesel power plants (DPS) are widely used as sources of primary, backup or emergency power supply for various facilities. Mathematical modeling and optimization techniques are widely used for studies in order to improve energy and economic efficiency of diesel power plants. The article describes the modeling of a stand-alone electromechanical power system consisting of a Diesel Power Plant and an ESS (Energy Storage System) based on supercapacitors with a load of comparable power of a diesel motor to increase its energy efficiency.

1. Introduction
Starting asynchronous motors in isolated electrical systems of limited power is a highly dynamic process that can lead to motor and load damage, as well as to fluctuations in the electrical network parameters. The mechanical damage is a result of the high starting current consumed by the induction motor when its speed changes. As the load increases, the electrical system’s voltage decreases. Various start-up methods may be used in the electrical system to reduce these and other start-up problems.

Nowadays, diesel power plants are widely used as primary sources, backup or emergency power supply for various facilities. They are also suitable for organizing mobile and autonomous power supply and they are commonly used at the mineral resource complex facilities, that are remote from centralized energy systems. The diesel engine used as the primary source for generating electricity, is one of the main components in diesel power plants.

In [1] it is said that the transient voltage deviation during a drop-surge of 100% of symmetrical load should not exceed:
1- $\pm 30\%$ for consumers having only important, the main characteristics of voltage and frequency.
2- $\pm 20\%$ for consumers having the voltage requirements of electrical units corresponding to voltage characteristics of power supply systems of commercial enterprises.
3- $\pm 10\%$ for consumers that show strict demand for voltage, frequency, and voltage waveforms.

At the same time, it is specified in [2] that the transient voltage deviation during a dump-surge of a symmetrical load of 100% of nominal rated power shouldn’t be more than $\pm 20\%$, with a recovery time not exceeding 2.3 s; the transient voltage deviation during a dump-surge of a symmetrical load of 50% of nominal rated power should not be more than $\pm 10\%$, with a recovery time not exceeding 1.2 s.
At the same time, the electric generating sets of the three-phase alternating current with a frequency
of 50 Hz (at no load) must ensure the start of an asynchronous squirrel-cage motor with a starting
current multiplicity of up to 7 and a power not less than what is indicated in table 1.

| Table 1. The ratio of rated power of generating set and power of an asynchronous squirrel-cage motor |
|-----------------------------------------------|
| Rated power of generating set, kWt | Power of an asynchronous squirrel-cage motor as a percentage of the rated power of the generating set |
|-----------------------------------------------|
| Up to 60 included | 70% |
| From 100 up to 200 included. | 60% |
| More than 200 up to 500 included. | 50% |
| More than 500 up to 1000 included. | 35% |
| Over 1000 | Set in standards or specifications for specific types of power plants |

According to the table 1. It can be seen that the higher the generating set’s rated power, the lower
the squirrel-cage motor’s power as a percentage of the rated power of the generating set, the direct
start of which they must provide.

Ensuring the operation of the diesel generator set while maintaining the voltage and frequency
values within the established norms, with an increase in the power of the simultaneously switched-on
load, during load shedding, is achieved by increasing the power of the generator set. However, this is
not effective and economically feasible.

Thus, the study of autonomous electromechanical power systems in isolated limited power
electrical systems with diesel generator sets with commensurate power load in order to reduce the
power of the primary motor and optimize their parameters is an urgent task.

The behavior of a diesel engine was a well-defined and thoroughly studied subject [3, 4] based on
various models that were used with either a thermodynamic or a control-oriented point of view. Also,
a diesel engine can be represented approximated first-order transfer functions [5]. A number of studies
have been devoted to the design of mathematical models for the analysis of transient phenomena in
power systems, including diesel power plants and an asynchronous drive as a load [6]. [7] describes an
algorithm for modeling a variable speed diesel engine to increase its energy efficiency. To simulate the
performance of an internal combustion engine, a functional model was built using the Model-Based
Calibration Toolbox ™ (MBC Toolbox), followed by export (followed by export) to Simulink
(MATLAB).

Article [8], presents a method for combining the starting device of asynchronous drives of
submersible electric centrifugal pumps with an autonomous power supply system, so that the starting
modes of the pump drives do not affect the operation of the power system.

A prototype module was developed for Beam Pumps and Progressive Cavity Pump (PCP) using
power electronic components and supercapacitors.

In [9] a practical ride through scheme is presented based on supercapacitor energy storage system
for Adjustable Speed Drive equipment. The topology adopts AC-DC-AC structure and 12 pulses diode
rectifier as input interface.

However, in [8, 9] the possibility of voltage drop and duration of voltage dips during start-up of
electric drives is not analyzed, this possibility may cause the operation of autonomous power supply
system to be disrupted.

In [10], dynamic models of diesel engine and synchronous generator were used. The article shows
that a complete dynamic model of a diesel generator set requires simulation of a diesel engine with
speed control, with a voltage control system. However, the article does not take into account the effect
of load changes on the characteristics of a diesel generator set.
In [11, 12], experimental studies have been made on the energy efficiency of a parallel hybrid drive of a marine vessel, including diesel generator sets. However, the article does not take into account the load distribution between the diesel generator sets and the load.

In [13, 14], existing technical means and solutions for mining enterprises power supply backup were analyzed, including online operating modern uninterruptible power supplies, as well as voltage distortion dynamic compensations. However, the possibility of using a common starting device, incorporating supercapacitor based ESS, for implementing successive start-up of electric drives with their subsequent switching to the Diesel Power Plant was not considered.

The article [15] presents justification for adjusting the approved method of calculating electrical loads in order to increase accuracy significance which affects technical and economic indicators. However, the case of standalone energy systems operation of limited power, with diesel generator sets, with a comparable power load aimed to reduce the power of the primary engine, was not considered.

The article [16, 17] presents the modeling and analysis of active compensators functional modes in the conditions of distributed generation. During simulation, the relationship helping to properly select the structure and main parameters of active compensator, for the implementation of uninterruptible power supply, harmonic compensation and power factor correction, was obtained. Based on the simulation results, the active compensation structure for the requirements of distributed generation and combined power supply systems, is proposed and justified. However, the possibility of using an active compensator that incorporates supercapacitor based ESS has not been considered. On the other hand, a method for determining the capacity of ESS at the DC link for the UPS implementation, harmonics compensation, and power factor correction has not been considered also.

The article [18] provides an overview of various equivalent circuits of supercapacitor. It highlights the concept of the unbalance in series-connected ultracapacitors and various balancing options. However, determining the capacitance of supercapacitors in the implementation of distributed generation systems with diesel generator sets and load of comparable power has not been considered.

In [19, 20, 21, 22], power supply systems based on transformers having various voltages operating at full load and supplying linear and nonlinear loads and a capacitor bank has been modeled. The relationship between the capacitor overload current and the ratio of voltage harmonic components on the capacitor's capacitance were obtained. However, the method of sizing the capacitance of the capacitor banks and accounting for the required charge level reduction, to feed the load is not considered.

2. Modelling approach

2.1. Diesel Generator Set Modeling

To determine the dynamic parameters of a diesel engine, we construct the static combined characteristics of the "diesel-load" complex, having a diesel electric power station of 100 kWt.

Calculation of characteristic points will be performed according to the empirical formula:

$$N_e(\omega) = N_{en}(0.5 \frac{\omega}{\omega_n} + 1.5 \left(\frac{\omega}{\omega_n}\right)^2 - \left(\frac{\omega}{\omega_n}\right)^3),$$  \hspace{1cm} (1)

where $N_{en}$ is the nominal effective diesel power, [kWt]; $\omega_n$ is the nominal angular speed of rotation of the diesel shaft, [rad / s].

The following assumptions are used in the calculations:

1 kWt = 1.36 hp; $\omega = \frac{\pi n}{30} = \frac{3.14 \cdot 1500}{30} = 157.08$ rad/s where $n$ is the rotation frequency [rpm].

To calculate the characteristic of diesel power, the frequency $\omega$ varies from $0.2\omega_n$ to $1.1\omega_n$ in increments of $0.05\omega_n$. 

\hspace{1cm}
Table 2. Static characteristics of diesel power

| ω, rpm | Nc(ω), kWt | h₀ | 0.75-h₀ | 0.5-h₀ | 0.25-h₀ | Nc(ω), kWt |
|--------|------------|----|---------|--------|---------|------------|
| 300    | 15.2       | 12.91 | 10.26  | 6.95   | 0.8     |
| 375    | 20.3125    | 17.27 | 13.75  | 9.30   | 1.5625  |
| 450    | 25.8       | 21.95 | 17.46  | 11.76  | 2.7     |
| 525    | 31.5875    | 26.85 | 21.32  | 14.23  | 4.2875  |
| 600    | 37.6       | 31.92 | 25.25  | 16.66  | 6.4     |
| 675    | 43.7625    | 37.06 | 29.17  | 18.95  | 9.1125  |
| 750    | 50         | 42.21 | 33.01  | 21.04  | 12.5    |
| 825    | 56.2375    | 47.29 | 36.70  | 22.86  | 16.6375 |
| 900    | 62.4       | 52.23 | 40.16  | 24.32  | 21.6    |
| 975    | 68.4125    | 56.95 | 43.31  | 25.36  | 27.4625 |
| 1050   | 74.2       | 61.37 | 46.09  | 25.89  | 34.3    |
| 1125   | 79.6875    | 65.43 | 48.40  | 25.85  | 42.1875 |
| 1200   | 84.8       | 69.04 | 50.19  | 25.15  | 51.2    |
| 1275   | 89.4625    | 72.14 | 51.38  | 23.73  | 61.4125 |
| 1350   | 93.6       | 74.64 | 51.88  | 21.50  | 72.9    |
| 1425   | 97.1375    | 76.47 | 51.63  | 18.39  | 85.7375 |
| 1500   | 100        | 77.56 | 50.55  | 14.34  | 100     |
| 1575   | 102.1125   | 77.83 | 48.57  | 9.25   | 115.7625|
| 1650   | 103.4      | 77.21 | 45.60  | 3.06   | 133.1   |

2.1.1. Building load characteristic

The dependence of the load resistance power on the angular speed of rotation of the diesel shaft is called its load characteristic. This characteristic is given by:

\[ N_c(\omega) = K_N \omega^3. \] (2)

To construct the load characteristics, it is necessary to determine the load coefficient by taking \( N_c = N_{cn}, \omega = \omega_n \).

\[ K_{Nn} = \frac{N_{cn}}{\omega_n^3} = \frac{1000000}{157.083} = 0.026. \]

Then, the points of the load characteristic of the resistance power are calculated by the formula:

\[ N_c(\omega) = K_{Nn} \cdot \omega^3. \] (3)

The calculation results are entered in the corresponding column of the table-2, and then a graph of the load resistance power is plotted, which is combined with a graph of the rated effective power.

On the graphs, the point \( D \) is determined, where the characteristic of the effective power and the load characteristic intersect. This point corresponds to the full load mode at the nominal position of the fuel pump rail \( h = h_n \).

2.1.2. Construction of partial characteristics of the effective power

On the graph of combined static characteristics, we draw horizontal lines parallel to the abscissa axis at the levels of \( 0.75N_{cn}, 0.5N_{cn}, 0.25N_{cn} \).

The intersection of these lines with the load characteristic will determine the modes of operation with partial diesel power when the fuel rail (rail of the high-pressure fuel pump) moves \( 0.75h_n, 0.5h_n, \) and \( 0.25h_n \), respectively. Intersection points (operating points) are denoted by \( C, B \) and \( A \), respectively. For each point, the graph, the rotational angular speed of the diesel shaft in the corresponding mode \( (\omega_C, \omega_B \) and \( \omega_A) \) is determined. Thus, the nominal parameters of the three diesel
Engines with partial power are determined. For each of these modes, it is necessary to construct the effective power partial characteristic.

Partial power characteristic points are calculated according to the following relationship:

\[
N_{el}(\omega) = N_{epi}[0.5 \frac{\omega}{\omega_{pi}} + 1.5\left(\frac{\omega}{\omega_{pi}}\right)^2 - \left(\frac{\omega}{\omega_{pi}}\right)^3],
\]

where \(N_{epi}\) and \(\omega_{pi}\) are the operating point coordinates of the i-th diesel operation mode.

For the mode where \(i=2\) with a nominal effective power of \(0.75N_{en}\), \(N_{ep2} = 0.75N_{en}\) and \(\omega_{p2} = \omega_{C}\) are taken, and then the \(N_{el}(\omega)\) dependence points are calculated.

The calculation results are entered in the table. A graph is constructed according to the table. This graph will go through point C and refers to the position of the rail of the diesel fuel pump \(h = 0.75h_n\). Similarly, power characteristics are constructed for the \(0.5h_n\) and \(0.25h_n\) modes, which will pass through points B and A.

![Figure 1. Partial characteristics of the effective power of Diesel Power Station.](image)

### 2.1.3. Determination of the stability factor of a diesel engine

The parameters of the diesel engine depend on its operating mode, therefore they must be separately determined for operating points D, C, B, and A. For all operating points, the diesel parameters are determined similarly.

The stability factor of a diesel engine characterizes its ability to automatically compensate for changes in the shaft load due to changes in rotation speed and torque with a constant fuel supply:

\[
F_D = \frac{1}{\omega_{pi}} \left[ \frac{\partial N_{el}}{\partial \omega} \right]_p - \left( \frac{\partial N_{el}}{\partial \omega} \right)\omega_{pi}. \quad (5)
\]

The stability factor for the 4 power diesel operation modes is determined. Below, the diesel stability factor determination for the nominal mode (operating point D) is described. For other modes, this parameter is defined similarly.

At point D, tangents are drawn to the curves \(N_{el}(\omega, h_n)\) and \(N_{el}(\omega)\) and the slope is determined for them.
where $\Delta \omega$ is a small set change in frequency at the operating point $\Delta N_c$. $\Delta N_c$ are the corresponding power changes to the frequency changes $\Delta \omega$. When calculating the slope angles, the power should be substituted in watts.

The stability factor of the diesel engine: $F_D = \frac{\alpha - \beta}{\omega_p}$. The magnitude of $F_D$ is expressed in $Wt\cdot s^2$.

\[
\Delta \omega = \frac{200\pi}{30} = 20.94 \text{ s}^{-1}, \quad \omega = \frac{1500\pi}{30} = 157.08 \text{ s}^{-1};
\]

\[
\alpha = \frac{40000}{20.94} = 1910.22, \text{ Wt}\cdot s;
\]

\[
\beta = \frac{6000}{20.94} = 286.53, \text{ Wt}\cdot s.
\]

Stability factor of a diesel engine at effective power:

\[
F_D = \frac{1910.22 - 286.53}{157.08} = 10.33, \text{ Wt}\cdot s^2.
\]
2.1.4. Determination of the time constant of a diesel engine

To determine the time constant of a diesel engine, it is necessary to know the dynamic moment of inertia $J$ of the rotating parts connected to the diesel shaft. The moment of inertia is expressed in $kg\ m^2$ and approximately can be determined by the formula:

$$J = 8.733 \cdot 10^3 \frac{N_{el}}{\omega_1} = 8.733 \cdot 10^3 \cdot \frac{100}{157.083} = 0.225 \ kg\ m^2,$$

where $\omega = \frac{\omega_n}{i_p}$ is the angular nominal rotation speed of the load;

$i_p$ – gear ratio of the gearbox, taken equal to 1;

$N_{en}$ – rated effective power supplied to the load (kWt).

The diesel engine’s time constant is determined by the formula:

$$T_D = \frac{J}{P_D}.$$  \hspace{1cm} (7)

The calculation of the diesel time constant at point D: $T_D = \frac{0.225}{10.33} = 0.022$ s.

The calculation of the diesel time constant at point C: $T_D = \frac{0.225}{9.13} = 0.025$ s.

The calculation of the diesel time constant at point B: $T_D = \frac{0.225}{4.99} = 0.045$ s.

The calculation of the diesel time constant at point A: $T_D = \frac{0.225}{7.81} = 0.029$ s.

2.1.5. Determination of the gain in fuel supply

Diesel gain by the position of the fuel rail is:

$$K_{hl} = \frac{(\frac{\partial N_e}{\partial h})_{\omega_p \const}}{\omega_p F_D}.$$ \hspace{1cm} (8)

It is calculated for each operating mode.

To determine this coefficient (gain), it is necessary to construct an auxiliary graph of the dependence of $N_e(h)$ at $\omega_p \const$.

![Figure 3. Diesel gain determination for various modes](image_url)
To determine the coordinates of the auxiliary graph \( N_e(h) \) on the graph of combined static characteristics, draw a vertical line through the operating point \( D \) corresponding to the frequency \( \omega_D = \omega_n \), and determine the intersection points of this line with the partial characteristics of the effective power for \( h = h_n, \ h = 0.75h_n, \ h = 0.5h_n, \ h = 0.25h_n \). The results are summarized in Table 4. The constructions are repeated for operating points A, B, C.

Table 4. The dependency of the power on the position of the rail fuel pump

| Fuel rail position | Power in kWt at angular speed |
|-------------------|-------------------------------|
|                   | \( \omega_A = 98.95 \)       | \( \omega_B = 124.67 \)       | \( \omega_C = 142.72 \)       | \( \omega_D = 157.08 \)       |
| 0.25\( h_n \)     | 25                            | 26                            | 22                            | 14                            |
| 0.5\( h_n \)      | 43                            | 50                            | 52                            | 50                            |
| 0.75\( h_n \)     | 55                            | 68                            | 75                            | 78                            |
| \( h_n \)         | 66                            | 84                            | 94                            | 100                           |

According to the table 3 plots are drawn for operating points A, B, C, D.

The values of \( h \) are plotted along the horizontal axis of the auxiliary plot, and the effective power value is plotted along the vertical axis for each value of \( h \). The resulting points are connected by a smooth curve.

For each curve obtained, the tangent to this curve is determined at the operating point. For the curve \( D \), we take \( h = h_n, \ \omega_p = \omega_n \) and a tangent to the curve is drawn at this point. For a tangent, the slope’s angle of inclination is determined:

Calculation of the diesel gain at point \( D \)

\[
\left( \frac{\partial N_e}{\partial h} \right)_{h=h_n} = \frac{\Delta N_e}{\Delta h} = \gamma, \quad (9)
\]

\[
\gamma = \frac{8000}{0.1} = 80000.
\]

When determining \( \gamma \), the power should be expressed in watts. From a certain value of \( \gamma \) and the diesel engine stability factor, we determine the gain in fuel supply:

\[
K_h = \frac{\gamma}{\omega_pF_D} = \frac{80000}{157.08 \cdot 10.33} = 49.3, \text{ s}^{-1}.
\]

Calculation of the diesel gain at point C:

\[
\left( \frac{\partial N_e}{\partial h} \right)_{h=h_n} = \frac{\Delta N_e}{\Delta h} = \gamma, \quad (9) \]

\[
\gamma = \frac{90000}{142.72 \cdot 9.13} = 69.07, \text{ s}^{-1}.
\]

Calculation of the diesel gain at point B:

\[
\left( \frac{\partial N_e}{\partial h} \right)_{h=h_n} = \frac{\Delta N_e}{\Delta h} = \gamma, \quad (9) \]

\[
\gamma = \frac{140000}{124.67 \cdot 7.81} = 143.79, \text{ s}^{-1}.
\]

Calculation of the diesel gain at point A:

\[
\left( \frac{\partial N_e}{\partial h} \right)_{h=h_n} = \frac{\Delta N_e}{\Delta h} = \gamma, \quad (9) \]

\[
\gamma = \frac{150000}{98.95 \cdot 4.99} = 303.79, \text{ s}^{-1}.
\]
2.1.6. The diesel equation

Since the dynamic parameters of the diesel engine depend on its operation mode, the differential equation of the diesel engine will be written as follows: 

\[(T_{di} \cdot p + 1) \cdot \omega(t) = K_{hi} \cdot h(t)\],

where \(i\) is the symbol of the diesel operating mode.

The parameters of the diesel equation obtained as a result of the above calculations are summarized in Table 5.

The nominal mode of the diesel engine corresponds to the mode with the nominal position of the rail of the fuel pump \(h_n\) (nominal fuel supply) and the nominal effective power on the shaft \(N_{en}\).

### Table 5. Dynamic parameters of the "diesel - load" complex

| Parameter | 0.25\(h_n\) | 0.5\(h_n\) | 0.75\(h_n\) | \(h_n\) |
|-----------|-----------|-----------|-----------|---------|
| \(T_{di}\), s | 0.045 | 0.029 | 0.025 | 0.022 |
| \(K_{hi}\), s\(^{-1}\) | 303.79 | 143.79 | 69.07 | 49.30 |

Input parameters for engine-generator transient study:
Nominal power: 100 kVA;
line-to-line voltage: 400 V;
frequency: 50 Hz;
\(X_d = 1.305\) pu;
\(X_d' = 0.296\) pu;
\(X_d'' = 0.252\) pu;
\(X_q = 0.474\) pu;
\(X_q'' = 0.243\) pu;
\(X_i = 0.18\) pu.

2.2. Modeling of Supercapacitor Module

Add a supercapacitor to the DC link. Fig. 4 shows the equivalent circuit of a supercapacitor.

The output voltage of the supercapacitor is expressed using the Stern equation as:

\[V_{sc} = \frac{N_s Q_T d}{N_p N_c e_{sc} A_i} + \frac{2 N_c N_s R_T}{F} \cdot \text{arsinh} \left( \frac{Q_T}{N_p N_c^2 A_i \sqrt{BRT e_{sc}}} \right) - R_{sc} \cdot i_{sc}. \] (10)
Where $A_i$ – Contact surface between electrodes and electrolyte ($m^2$); $c$ – molar concentration (mol/m$^3$) considering: $c = 1 / (8N_Ar^3)$; $r$ – molecular radius (m); $F$ – Faraday constant; $i_{sc}$ – supercapacitor current (A); $V_{sc}$ – supercapacitor’s voltage (V); $C_T$ – capacity (F); $R_{sc}$ – Resistance (Ω); $N_e$ – number of electrode layers; $N_A$ – Avogadro constant; $N_p$ – number of parallel supercapacitors; $N_s$ – number of supercapacitors in series; $Q_T$ – electric charge (C); $R$ – ideal gas constant; $d$ – molecular radius; $T$ – working temperature (K); $\varepsilon$ – Permittivity of material; $\varepsilon_0$ –Permittivity of free space.

$$Q_T = \int i_{sc} \, dt.$$  \hspace{1cm} (11)

To represent self-discharge, the electric charge of the supercapacitor changes as follows (at $isc = 0$):

$$Q_T = \int i_{self, dis} \, dt.$$  \hspace{1cm} (12)

Where

$$i_{self, dis} = \begin{cases} \frac{C_T \alpha_1}{1 + sR_{sc}C_T}, & \text{если } t - t_{oc} \leq t_3; \\ \frac{C_T \alpha_2}{1 + sR_{sc}C_T}, & \text{если } t_3 < t - t_{oc} \leq t_4; \\ \frac{C_T \alpha_3}{1 + sR_{sc}C_T}, & \text{если } t - t_{oc} > t_4. \end{cases} \hspace{1cm} (13)$$

The constants $\alpha_1$, $\alpha_2$, and $\alpha_3$ are the rates of change of the voltage of the supercapacitor during the time intervals $(t_{oc}, t_3)$, $(t_3, t_4)$ and $(t_4, t_5)$, respectively, as shown in Fig. 5. The constants $\alpha_1$, $\alpha_2$, and $\alpha_3$ are the rates of change of the voltage of the supercapacitor during the time intervals $(t_{oc}, t_3)$, $(t_3, t_4)$ and $(t_4, t_5)$, respectively, as shown in Fig. 5.

**Figure 5.** Voltage changes of the supercapacitor.

Supercapacitor parameters used in the model [23]:

- Rated capacitance (F) : 99.5;
- Equivalent DC series resistance (Ohms), internal resistance of the supercapacitor: 8.9e-3;
- Rated Voltage (V): Typical rated voltage is equal to 2.7 V. We consider a supercapacitor module, which includes 18 series-installed capacitor elements. Therefore, the nominal voltage is 48 V;
- Number of series capacitors: 18;
- Number of parallel capacitors: 1;
- Initial voltage (V): 0;
- Operating temperature (°C): 25;
- Number of layers, related to the Stern model: 1;
- Molecular radius related to the Stern model: 1e-9;
- Permittivity of the electrolyte material (F/m): 6.0208 e-10;
- Charge current during a constant current charge test (A): 10;
• Supercapacitor voltage (V), at 0 s, 20 s и 60 s, when the supercapacitor is charged with a constant current equal to the value provided in the charge current parameter (A): respectively, 0.161, 2.7, 7.8;
• Current prior to an open-circuit event (A): 10;
• Supercapacitor voltage, (V), at 0 s, 10 s, 100 s, and at 1000 s, when the supercapacitor is open-circuit. The corresponding current prior to open-circuit is given in the Current prior open-circuit parameter (A): respectively, 48, 47.8, 47.06, 44.65.

These parameter values were determined from experimental tests, and they can be used as default values to represent a common supercapacitor. An experimental verification of the model showed a maximum error of 2% for charge and discharge using the given parameters.

The State of Charge (SOC) varies from 0 to 100%. The SOC for a fully charged supercapacitor is 100%, and for an empty supercapacitor SOC is 0%. SOC is calculated as:

\[
SOC = \frac{q_{\text{in}} - \int_{0}^{t} i(t) dt}{q_{T}} \cdot 100.
\]

Assumptions made when simulating a supercapacitor:
• The internal resistance is considered constant during charge and discharge cycles;
• The model does not take into account the temperature effect on the electrolyte material;
• The effect of aging is not taken into account;
• The charge redistribution is the same for all voltage values;
• Cell balancing is not taken into account, the voltage on individual elements of the supercapacitor module has the same value;
• The current through the supercapacitor is assumed to be continuous.

3. Results and discussion
Using the developed model of the diesel generator set, studies were conducted of direct and frequency starts of an induction motor. Figures 6 and 7 show the supply voltage, stator current, rotation speed, and electromagnetic torque when starting the electric motor in direct and frequency starting modes.

Figure 6. Plots of direct start of asynchronous motor of 37 kWt when powered by Diesel Power Station.
By analyzing the results, we conclude that the voltage drop during the direct start-up of a 10 kW asynchronous motor is 22% of $U_{\text{ном}}$; with direct start-up of a 20 kW asynchronous motor is 37% of $U_n$ and with a direct start-up of a 30 kW asynchronous motor is 47% of $U_{\text{ном}}$. At a frequency start-up of a 80 kW asynchronous motor the voltage drop is 8% of $U_{\text{ном}}$. The use of a frequency converter made it possible to reduce the load in the start-up mode for a diesel power station. This can be used as follows:

1. To increase in the power of the motor load supplied by diesel power station.
2. To reduce the power of the diesel generator feeding the motor load.

However, when feeding several electric drives from a diesel generator, the use of a frequency converter as a starting device on each of them may not be a rational solution. Let us consider the possibility of using a common starting device, which incorporates energy storage to implement an alternate start-up of several electric drives being subsequently connected to the diesel generator. Fig. 8 shows a schematic diagram of a power supply having a diesel generator (G) and a supercapacitor module (C).

Figure 7. Frequency start plots of asynchronous motor of 80 kWt when powered by Diesel Power Station.

Figure 8. Schematic diagram of power supply with a starting device from supercapacitor modules.
Figure 9. Dependences of active power and energy on time with a frequency start of an 80 kWt motor under load.

It is also worth to note that supercapacitor modules in this circuit can be used not only as a starting device, but also as a dynamic compensator for voltage distortions in emergency situations.

To determine the capacitance of supercapacitors needed to start the engine, we will measure the active power necessary for the start-up. Fig. 9 shows the dependences of active power and energy on time at a frequency start of a motor of 80 kW under load. This consumes about $8.65 \times 10^4 \text{ J}$.

Figure 10. Dependence of engine power on the capacity of supercapacitors, $\Delta t = 5 \text{ s}$. 
Let us determine the capacity of the supercapacitor module for which the module voltage will drop to given value (threshold value). This value will be the target voltage drop.

\[ C = \frac{2S\Delta t}{U_1^2 - U_2^2}, \quad C = \frac{2\Delta W}{U_1^2 - U_2^2}, \]

(15)

where \( S \) is the total engine power, VA; \( \Delta t \) is the discharge time of supercapacitors, s; \( U_1 \) is the initial value of the voltage of the supercapacitors’ module, V; \( U_2 \) is the value of the threshold voltage of the supercapacitor’s module, V; \( \Delta W \) is the value of the discharged energy of the supercapacitor’s module, J.

Fig. 10 shows the dependence of engine power on the capacitance of supercapacitors for various values of the threshold voltage \( U_2 \), \( \Delta t = 5 \) s.

Figure 11 shows that the discharged energy of supercapacitors depends on their capacitance at various values of the residual voltage \( U_2 \).

From fig. 11 it can be seen that when the discharge energy of the supercapacitor module is \( \Delta W = 8.65 \times 10^4 \) J, the capacitance value is \( C = 12 \) F at a threshold voltage level of 95% of the nominal voltage.

Also, to ensure the absence of current and torque surges when switching to the main power source, the following synchronization requirements must be taken into account:

- The difference between the inverter’s output voltage powered by supercapacitors, and the generator set voltage should not exceed 1%;
- The phase angle difference of the inverter’s output voltage, powered by supercapacitors, and the voltage of the generator set should not exceed 10 degrees;
- The frequency deviation should not exceed not more than 0.1%.

Let us perform a simulation of motor starting when powered by a supercapacitor module with subsequent switching to the main power source with a supercapacitor module of capacitance \( C = 12 \) F.
Figures 12 and 13 show the voltage dependencies of the supercapacitor module, (V), the state of charge (%), the supply voltage (V), the stator current (A), the rotor speed (rad / s) and electromagnetic moment (Nm) at \( C = 12 \, \text{F} \).

To evaluate the dimensions of the supercapacitor module used to start the engines, the Maxwell supercapacitor K2 BCAP0650 series is used as an example. The technical parameters of this supercapacitor are given in table 6.

| Technical Parameters | BCAP0650 |
|----------------------|----------|
| Rated voltage, V     | 2.7      |
| Minimum capacity, F  | 650      |
| Maximum capacity, F  | 780      |
| Internal resistance (ESR DC), mOhm | 0.8 |
| Maximum voltage, V   | 2.85     |
| Maximum current, A   | 680      |
| Leakage current at 25°C, less than, mA | 1.5 |
| Specific working power, kW / kg | 6.8 |
| Specific maximum power, kW / kg | 14 |
| Specific Energy, Wh / kg | 4.1 |
| Stored energy, Wh    | 0.66     |

**Figure 12.** The voltage of the supercapacitors module, (V) and the state of charge (%) at \( C = 12 \, \text{F} \).

BCAP0650 rated voltage is 2.7 V. With a DC link voltage of 537 V, the number of capacitors installed in series will be

\[ n = \frac{537}{2.7} \approx 200 \, \text{pcs}. \]

For the calculated capacity, we take the average capacity between the minimum and maximum initial capacities

\[ C = \frac{650 + 780}{2} = 715 \, \text{F}. \]
Considering a series connection of 200 pieces of capacitors, the capacity will be

\[ C = \frac{715}{200} = 3.575 \, F. \]

To ensure a capacity of 12 F for the supercapacitor module, parallel connection of the capacitors is necessary. The number of parallel rows will be:

\[ \frac{12}{3.575} = 3.4 \approx 4 \, \text{rows}. \]

Figure 13. Dependencies of the supply voltage, stator current, rotor speed and electromagnetic moment at C = 12 F.

Thus, the total number of supercapacitors of the required voltage and capacitance will be:

\[ 200 \times 4 = 800 \, \text{pcs}. \]

Table 7. Supercapacitor linear dimensions

| Supercapacitor         | Dimensions, mm |
|------------------------|----------------|
| BCAP0650 P270 K04/05  | L (±0.3)       |
|                        | D1 (±0.2)      |
|                        | D2 (±0.7)      |
|                        | 51.5           |
|                        | 60.4           |
|                        | 60.7           |

The height of a 4-rows block of BCAR0650 (BCAPxxxxP270 K05) having a height of 51.5 mm + 2 x 3.18 = 57.86 mm will be 57.86 x 4 = 240 mm.

The length and width of the block at 200 series-connected capacitors with 40 elements in length and 5 in width will be:
- Length: D2 x 40 = 60.7 x 40 = 2428 mm;
- Width: D2 x 5 = 60.7 x 5 = 303.5 mm.

Of course, the dimensions of the module can be selected based on specific conditions, arranging the elements of the capacitors uniformly in length and width. For example,
15x15 (225 pcs $U = 607.5$ V) or 14x14 (196 pcs $U = 529.2$ V).

4. Conclusion

- A mathematical model of a diesel generator set has been developed; it provides power to a frequency converter vector controlled asynchronous motor. The diesel generator set has the ability to work in parallel with a supercapacitor module.
- It was revealed that the voltage drop during direct start-up of the asynchronous motor with a power of 10 kW is 22% of $U_{nom}$; 37% of $U_{nom}$ for a 20 kW motor; 47% of $U_{nom}$ for a 30 kW motor; With a frequency start-up, the voltage drop for an asynchronous motor of 80 kW power was 8% of $U_{nom}$. Thus, the use of a frequency converter allows to reduce the load in the start-up mode for a diesel power station, which can be taken into account when designing a diesel power station as of increasing the power of the motorized load supplied by this station or decreasing the power needed from the diesel power station to supply the motor load.
- It was shown that the use of several electric drives fed from a diesel power station, having a frequency converter as a starting device for each one of them may not be a rational solution. The possibility of using a common starting device, incorporating supercapacitor modules as energy storage devices, to perform an alternating startup of the electric drives with their subsequent switching to the genset is shown. The supercapacitor modules in this circuit can be used not only as a starting device, but also as a dynamic compensator for voltage distortions during emergency conditions.
- The dependency of motor power on the supercapacitors’ capacitance are obtained for various values of the threshold voltage $U_2$, $\Delta t = 5$ s. They show that the higher the level of threshold voltage $U_2$, the higher the value of the capacitance of supercapacitors $C$. For example, for a 80 kVA motor power and a threshold voltage $U_2$ of 95% of the initial voltage $U_1$, the capacitance of supercapacitors is 63.1 F. As for when the threshold voltage $U_2$ is 90% of the initial voltage $U_1$, the capacitance of supercapacitors is 32.4 F.
- The dependencies of the voltage of the supercapacitors module, (V) and the state of charge SOC (%), the supply voltage (V), the stator current (A), the speed (rad/s) and the electromagnetic moment (Nm) at $C = 12$ F were obtained. Analyzing these dependencies, it is clear that the motor normally starts. In this case, the voltage and charge of supercapacitors is reduced by 2.5%. When switching to a power source, there are no current or torque surges.
- A method has been developed for determining the capacitance of a capacitor module, which is necessary and sufficient for starting electric motor drives having a commensurate capacity to the supply source power and subsequent surge-free switching to this source.

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