Dynamic Modeling and Analysis of a Freight Train Vertical Vibration Reduction System

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Abstract. Based on wheel-rail impact vibration and considering the body stiffness and natural damping, this paper builds a three-degree-of-freedom vibro-impact system model for freight train’s vertical vibration reduction system. The dynamic behavior of the system is analyzed. The Poincaré map of the system is determined by the analytic solution of the system derived from the motion differential equation of the multi-degree-of-freedom vibro-impact system combined with Newton’s second law. It is found that the fork bifurcation, Hopf bifurcation and other dynamical behavior leading to Chaos when the system parameters are changed. In the process diagram, fork bifurcation is easier to be observed by engineers than Hopf bifurcation and can be easily applied to the control strategy of semi-active suspension. The dynamic parameters of the train are optimized to avoid chaos in the train operation, reduce the vertical vibration of the train, improve the stability of the train operation, and provide the theoretical basis for the vibration reduction design of the train.

1. Introduction

A gap in the mechanical system will make the system produce collision and impact, which will affect the performance and safe operation of the mechanical system. For example, the impact between the wheels and the track in the running of high-speed trains intensifies the vibration of the trains and affects the running stability and comfort of the trains. Therefore, the study of impact vibration is of great significance to reduce the collision, impact and abrasion of mechanical system, and improve the safety, lifetime and efficiency of the mechanical system. In recent years, the theory and application of impact vibration system and gap system have made rapid progress. The study found that the train in actual operation, the wheel pair often arise more cyclical acceleration and vibration amplitude changes (especially in train deceleration or through the Bridges and tunnels), and then to the wheels overhaul, found no wheel injury problems, but by changing the spring stiffness and damping of shock absorber can effectively improve the situation.

The existing vehicle vertical vibration reduction system is mainly designed according to the track (road surface) irregularity [1,2]. Some papers study the effect of vertical vibration reduction by field test [3-5]. However, due to the existence of the wheel and rail gap, even if the train is running on completely smooth track, Chaos vibration will occur. There are many studies on wheel-rail impact [6,7]. These
papers give full consideration to the rail damping [8,9] and the three-dimensional structure of the wheel [10,11]. However, the effects of body stiffness and natural damping on train vibration are rarely considered in these papers. In recent years, there are much theoretical research on impact vibration [12]. These papers will select appropriate parameters to find out the different bifurcation behavior of the system [13-15]. However, few of them are combined with engineering practice. Therefore, in this paper, based on the wheel-rail impact vibration, the stiffness and natural damping of the vehicle body are considered. The dynamic model of the vertical vibration damping system of the freight train is built. The influence of different damping system parameters on the running vibration of the train is studied.

2. System model
Assuming that the freight train’s wheel-rail impact is a rigid impact and only considers the motion in the vertical direction, the model of a wheel vibro-impact system is shown in Fig.1. The meanings of the symbols in Fig.1 are shown in Table 1. Natural damping and body stiffness are connected to imaginary inertial Spaces. When the displacement of the wheel minus the displacement of the rail is equal to the gap, wheel and rail impact. After impact, wheel and rail get new speed and then impact after impact.

![Figure 1. Dynamic model of a Freight Train Vertical Vibration Reduction System.](image)

As shown in Fig.1. The dimensionless differential equation where the system does not impact with each other can be written as:

$$\begin{bmatrix} 1 & 0 \\ 0 & \mu_{m2} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + 2\mu_2 \begin{bmatrix} 1 + \mu_{c2} - \mu_{c2} \\ -\mu_{c2} + \mu_{c2} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 1 + \mu_{k2} - \mu_{k2} \\ -\mu_{k2} + \mu_{k2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_{10} \\ f_{20} \end{bmatrix} \sin(\omega T + \tau)$$  \tag{1}

$$\mu_{m3} \ddot{x}_3 + 2\mu_2 \mu_{c3} \dot{x}_3 + \mu_{k3} x_3 = f_{30} \sin(\omega\tau + \tau)$$  \tag{2}

In the formula, dimensionless quantities are:

$$\mu_{mi} = \frac{M_i}{M_1}, \quad \mu_{kr} = \frac{K_i}{K_1}, \quad f_{i0} = \frac{P_i}{P_0}, \quad \mu_{c} = \frac{C_i}{C_1}, \quad \omega = \sqrt{\frac{M_1}{K_1}}, \quad t = T \sqrt{\frac{K_1}{M_1}}, \quad \zeta = \frac{C_i}{2M_1 K_1}$$  \tag{3}

$$x_i = \frac{X_i K_1}{P_0}, \quad i = 1,2,3, \quad b = \frac{B K_1}{F_0}, \quad P_0 = P_1 + P_2 + P_3$$
Table 1. The symbolic definition in Figure 1.

| Symbol | Definition | symbol | meaning |
|--------|------------|--------|---------|
| $M_1$  | Bogie mass | $C_1$  | Suspension damping |
| $M_2$  | Partial body mass | $C_2$  | Natural damping |
| $M_3$  | Track mass | $C_3$  | Track damping |
| $K_1$  | Suspension stiffness | $P_i \sin(\Omega T + \tau)$ | Harmonic force |
| $K_2$  | Vehicle body stiffness | $X_i$ | Vertical displacement |
| $K_3$  | Track stiffness | $B$    | Wheel/rail gap |

When the gap is 0, the motion equation of the wheel and rail is:

\[
\dot{x}_{1-} + \mu_{m3} \dot{x}_{3-} = \dot{x}_{1+} + \mu_{m3} \dot{x}_{3+}
\]

\[
R = \frac{\dot{x}_{3+} - \dot{x}_{3-}}{\dot{x}_{1-} - \dot{x}_{3-}}
\]

The dimensionless instantaneous velocity before and after the impact of the wheel can be represented by $\dot{x}_{1+}$ and $\dot{x}_{3+}$ respectively. The dimensionless instantaneous velocity before and after the rail impact can be expressed by $\dot{x}_{4-}$ and $\dot{x}_{3+}$ respectively. Equation (4) and equation (5) can be obtained:

\[
\dot{x}_{1+} = \frac{1-R\mu_{m3}}{1+\mu_{m3}} \dot{x}_{1+} + \frac{\mu_{m3}(1+R)}{1+\mu_{m3}} \dot{x}_{3-}
\]

\[
\dot{x}_{3+} = \frac{1+R}{1+\mu_{m3}} \dot{x}_{3+} + \frac{\mu_{m3}-R}{1+\mu_{m3}} \dot{x}_{3-}
\]

For freight trains, the quality of the system will change due to the different load. Due to the difference of environment (temperature, humidity, altitude) and operation line (through tunnel and bridge), the rigidity, damping of track and vehicle body will also change. Due to the contact friction between wheel and rail for a long time, the impact clearance and collision recovery coefficient will also change. Therefore, the parameters of the system are infinite.

When two groups of data in Table 2 are selected for simulation, the Poincaré section diagram of the system obtained is shown in Fig.2 and Fig.3 respectively.

Table 2. Simulation parameters

| Symbol | The first set of values | The second set of values |
|--------|-------------------------|--------------------------|
| $\mu_{m2}$, $\mu_{m3}$ | 1.65, 2.65 | 1.12, 1.73 |
| $\mu_{k2}$, $\mu_{k3}$ | 0.8, 2.32 | 0.86, 2.2 |
| $\mu_{c2}$, $\mu_{c3}$ | 0.01, 0.2 | 0.03, 0.3 |
| $\zeta$, $R$, $b$ | 0.25, 0.6, 0.1 | 0.11, 0.8, 0.03 |
Choosing the first set of parameters to the simulation, when \( \omega < \omega_c = 2.212 \), the system has a stable periodic motion \( q = 1/1 \), as shown in Fig.2(a). As \( \omega \) increases, the system times tumble bifurcation for \( q = 2/2 \) periodic motion, as shown in Fig.2(b), further \( \omega \) in the \( \omega = 2.236 \), the fixed point bifurcation of periodic motion \( q = 4/4 \), as shown in Fig.2(c). As \( \omega \) increases further, Hopf bifurcation occurs in the system, for example, \( \omega = 2.237 \), as shown in Fig.2(d). Then the system goes into phase lock, as shown in Fig.2(e), and finally with \( \omega \) continues to increase, the system finally goes into chaos motion, as shown in Fig.2(f).

**Figure 2.** Poincaré maps of the first set of parameters
Choosing the second set of parameters to the simulation, from the Poincaré maps, when $\omega = 1.82$, the system has a stable periodic motion $q = 1/1$, as shown in Fig.3(a). As $\omega$ increases, the system’s Hopf bifurcation occurs, attract the same ring form, as shown in Fig.3(b). As $\omega$ increases, the same ring form deforms, as shown in Fig.3(c). As $\omega$ continues to increase, the system finally goes into chaos motion, as shown in Fig.3(d).

![Poincaré maps](image1.png)

(a) $\omega = 1.82$

![Poincaré maps](image2.png)

(b) $\omega = 1.898$

![Poincaré maps](image3.png)

(c) $\omega = 2.016$

![Poincaré maps](image4.png)

(d) $\omega = 2.32$

**Figure 3.** Poincaré maps of the second set of parameters

3. **Engineering application and discussion**

During the operation of the train, the mass of the whole train will change. Track elasticity and damping will change as road conditions change. As the temperature changes, the damping and stiffness of the system will change. The initial sensitivity of a chaotic system can cause a completely different vibration effect.
Figure 4. Process diagram of the first set of parameters

(a) \( \omega = 2.212 \) (b) \( \omega = 2.23 \)

(c) \( \omega = 2.236 \) (d) \( \omega = 2.237 \)

(e) \( \omega = 2.243 \) (f) \( \omega = 2.247 \)
As shown in Fig.4 is the process diagram of the first set of parameters for the simulation of the system, when $\omega = 2.212$, the system has a stable periodic motion, when $\omega = 2.23$, the system has a stable two periodic motion, when $\omega = 2.236$, the system has a stable four periodic motion, when $\omega = 2.237$, the system of periodic no longer obvious, when $\omega = 2.243$ and $\omega = 2.247$, the system enter a state of chaos. Process diagram of the second set of parameters is shown in Fig.5, when $\omega = 1.82$, the system has a stable periodic motion, when $\omega = 1.898$, the periodicity of the system is not obvious, when $\omega = 2.016$, the periodicity of the system is also not obvious, when $\omega = 2.32$, the system enter a state of chaos.

In engineering application, for the vertical vibration of the train, the engineers generally use the acceleration sensors for data acquisition. Due to track irregularity and the existence of random vibration source, vibration signals are often disorganized, so for the system via Hopf bifurcation going into chaotic motion process engineers can not observe in process diagram, but for the system through the fork bifurcation going into chaotic state, the engineers can capture. For freight trains, the semi-active suspension is adopted to control system parameters, can effectively avoid the chaotic vibration, so as to achieve the protection of the train.

4. Conclusion
This paper analyzes the vertical vibration damping system of the freight train. The dynamic model of the freight train’s vertical vibration damping system is established as a three-degree-of-freedom vibro-impact system model. Different system parameters are selected, and the different paths to enter the
chaotic state are qualitatively analyzed. The train may enter into a chaotic state in the actual operation process, and the database of various vibration parameters of the train in the actual operation can be established later, and the vertical acceleration and displacement of the train can be monitored in real time. The train semi-active suspension adjusts the stiffness and damping of the train suspension in real time according to the road excitation and the vertical vibration acceleration of the train, so as to avoid the chaotic state of the train in operation, and thus restrain the unnecessary vibration.

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References
[1] Saglam, Ferhat, Unlusoy, Y. and Samim. Adaptive ride comfort and attitude control of vehicles equipped with active hydro-pneumatic suspension. Int J Vehicle Des. 2016; 71: 31-51.
[2] Wang, Qunsheng, Zeng, et al. Reduction of vertical abnormal vibration in carbodies of low-floor railway trains by using a dynamic vibration absorber. P I Mech Eng F-J Rai. 2018; 232: 1437-47.
[3] G.R. Mettam, How to prepare an electronic version of your article, in: B.S. Jones, R.Z. Smith (Eds.), Introduction to the Electronic Age, E-Publishing Inc., New York, 1999, pp. 281-304.
[4] Wei K, Zhao Z, Du X, Li H and Wang P. A theoretical study on the train-induced vibrations of a semi-active magneto-rheological steel-spring floating slab track. Constr Build Mater. 2019; Vol.204: 703-15.
[5] Cai X, Li D and Zhang Y. Experimental Study on the Vibration Control Effect of Long Elastic Sleeper Track in Subways. Shock Vib. 2018; 2018: 1-13.
[6] Torstensson PT, Squicciarini G, Krüger M, Pålsson BA, Nielsen JCO and Thompson DJ. Wheel–rail impact loads and noise generated at railway crossings – Influence of vehicle speed and crossing dip angle(Article). J Sound Vib. 2019; Vol.456: 119-36.
[7] Li Q, Thompson DJ and Toward MGR. Estimation of track parameters and wheel–rail combined roughness from rail vibration. P I Mech Eng F-J Rai. 2017; 232: 1149-67.
[8] Bian J, Gu Y and Murray MH. A dynamic wheel–rail impact analysis of railway track under wheel flat by finite element analysis. Vehicle Syst Dyn. 2013; 51: 784-97.
[9] Shadfar, Morad, Naeimi, et al. 3D dynamic model of the railway wagon to obtain the wheel-rail forces under track irregularities. P I Mech Eng K-J Mul. 2015; 229: 357-69.
[10] Choi and Jungyoul. Prediction of displacement induced by tilting trains running on ballasted tracks through measurement of track impact factors. Engineering Failure Analysis. 2013; 31: 360-74.
[11] Meng J, Xu R, Li D and Chen X. Combining the Matter-Element Model with the Associated Function of Performance Indices for Automatic Train Operation Algorithm(Article). IEEE T Intell Transp. 2019; Vol.20: 253-63.
[12] Luo G and Lv X. Dynamics of a plastic-impact system with oscillatory and progressive motions. Int J Nonlin Mech. 2008; 43: 100-10.
[13] Yue Y, Miao P and Xie J. Coexistence of strange nonchaotic attractors and a special mixed attractor caused by a new intermittency in a periodically driven vibro-impact system(Article). Nonlinear Dynam. 2017; Vol.87: 1187-207.
[14] Li S, Yan L, Li W, Zhao H and Ling X. Research on Energy Response Characteristics of Rock under Harmonic Vibro-Impacting Drilling. J Vib Eng Technol. 2019; 7: 487-96.
[15] Hu H. Controlling chaos of a periodically forced nonsmooth mechanical system. Acta Mech Sinica-Prc. 1995; 11: 251-8.