LENSING DISPERSION OF SNIa AND SMALL SCALES OF THE PRIMORDIAL POWER SPECTRUM

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Probing the primordial power spectrum at small scales is crucial for discerning inflationary models, especially if BICEP2 results are confirmed. We demonstrate this necessity by briefly reviewing single small field models that give a detectable gravitational waves signal, thus being degenerate with large field models on CMB scales. A distinct prediction of these small field models is an enhancement of the power spectrum at small scales, lifting up the degeneracy. We propose a way to detect this enhancement, and more generally, different features in the power spectrum at small scales $1 \lesssim k \lesssim 10^2 - 10^3 \ Mpc^{-1}$ by considering the existing data of lensing dispersion in Type Ia supernovae. We show that for various deviations from the simplest $n_s \simeq 0.96$ the lensing dispersion cuts considerably into the allowed parameter space by PLANCK and constrains the spectrum to smaller scales beyond the reach of other current data sets.

1 Introduction

State of the art CMB and Lyα measurements probe only about 8 e-folds, $(H_0 \lesssim k \lesssim 1 \ Mpc^{-1})$ out of the expected 60 e-folds of observable inflation, rendering a huge degeneracy between inflationary models. Even a confirmation of the BICEP2 measurement, will not break all the degeneracy. For example, small field models with a non-monotonic $\epsilon$ reproduce a spectrum similar to that of a monomial $V \sim \phi^n$ for a limited range of wave numbers. Even within the class of large field models there is a degeneracy that can only be lifted by probing enough e-folds of the power spectrum. The answer lies in probing smaller scales of the power spectrum. In we proposed using the lensing dispersion of Type Ia supernovae as a novel cosmological probe and specifically as a constraint on the primordial power spectrum at small scales. See also. The lensing dispersion, $\sigma_{\mu}$ is sensitive to $0.01 \lesssim k \lesssim 10^2 - 10^3 \ Mpc^{-1}$, thus giving access to $2 - 3$ more decades (4 - 7 e-folds) of the spectrum, even using only current data.

In the next section, we review the non-monotonic $\epsilon$ idea. In section 3 we present how the lensing dispersion probes the primordial power spectrum on small scales. Section 4 describes the results for various parameterizations of the spectra, complementing, and some discussion.

2 Small field models and large $r$

Consider canonically normalized, single field models $V(\phi) = \Lambda^4 \sum_{n=0} a_n \phi^n$, assuming CMB scales are at $\phi \simeq 0$. Generically $a_0$ sets the scale of inflation, $a_1$ sets the tensor to scalar ratio $r = 16 \epsilon$, $a_2$ sets $n_s$, etc. A small field model $\Delta \phi < 1$ requires parametric tuning of a few parameters for a successful model of inflation, i.e. $\epsilon, |\eta| \ll 1$, to get 60 e-folds and $n_s \approx 0.96$. This generically means $a_1 \ll 1$ and hence $r \ll 0.01$ in odds with the BICEP2 result. A large field generically means functional tuning, for
example $a_n=2=0$, for all $n$, which gives a free massive inflaton. Such models give $r \sim 0.1$, in accord with the BICEP2 findings. One would like a UV theory that explains the functional tuning we use. Moreover, taken at face value, the $r=0.2$ BICEP2 result is in tension with PLANCK, unless the cosmological model is further extended to include primordial Helium, additional light degrees of freedom or a scale dependent spectral index $n_s(k)$, in a way that suppresses the power on intermediate scales. Regardless, hints of "running" $\alpha (k_o) \equiv d n_s/d \ln k$ have been around since WMAP1.\(^{10}\)

Several years prior to PLANCK and BICEP2, in\(^3\) we demonstrated the key idea, that non-monotonic $\epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2$ allows small field models to have $r \sim 0.1$, avoiding the need of functional tuning, which is especially interesting in light of BICEP2\(^{11}\). If at CMB scales, $\epsilon$ is rather large, then from $r=16\epsilon$ we get detectable signal, $r \sim 0.1$. However, away from the CMB scales, $\epsilon$ decreases, giving many e-folds of inflation, $N = \int d\phi/\sqrt{2\epsilon}$. In Figure 1, reproduced from\(^3\), we demonstrated the behaviour of $\sqrt{2\epsilon} = |V'|/V$ and the potential $V$ as a function of the inflation. One can have an arbitrary number of e-folds in a very small interval $\Delta \eta \sim 1^{3,12}$. Therefore, the main limitation is the scale dependence of the power spectrum, $P \sim V/\epsilon$, since by now about 8 e-folds have been measured with limited amount of scale dependence parameterized by $\alpha (k_o) \equiv d n_s/d \ln k, \beta (k_o) \equiv d^2 n_s/d \ln k^2$.

Because of the non-monotonic $\epsilon$, a distinct prediction of the models, which was made prior to PLANCK and BICEP2 results, is the enhancement of the power spectrum at small scales. In\(^3\) the spectrum was calculated numerically by solving the Mukhanov-Sasaki equation, and in\(^{13}\) it was argued that a spectrum with a bend at some $k_i$ is a good approximation of the model, which we will use in section 4. Knowing the power spectrum at smaller scales is interesting by itself for a better understanding of inflation. Specifically, it can break the degeneracy between the above models and the monomial ones, being an important feature of a UV theory that explains the functional tuning we use. Moreover, one would like a UV theory that explains the functional tuning we use. Moreover, this is an important feature of a UV theory that explains the functional tuning we use.

### 3 Lensing Dispersion of SNIa

Using the light-cone averaging approach up to second order in the Poisson (longitudinal) gauge\(^{16}\), a simple expression for the lensing dispersion in\(^4\) was derived:

$$\sigma^2_{\mu} \approx \left( \frac{5}{\ln 10} \right)^2 \frac{\pi}{\Delta \eta^2} \int_{\eta_s(0)}^{\eta_0} \frac{dk}{k} P_\psi(k, \eta_1) k^3 (\eta_1 - \eta_s(0))^2 (\eta_0 - \eta_1)^2, \quad (1)$$

$$\approx \left( \frac{5}{\ln 10} \right)^2 \frac{\pi}{\Delta \eta^2} \left( \frac{k_{eq}}{H_0} \right)^3 \int d\eta_1 dp P_\psi(p, \eta_1) P^2(\eta_1 - \eta_s(0))^2 (\eta_0 - \eta_1)^2. \quad (2)$$

where $\eta_0$ is the observer conformal time, $\eta_s(0)$ is the conformal time of the source with unperturbed geometry, $\Delta \eta(z) = \eta_o(z) - \eta_s(0)(z) = \int_0^z dz$ $H_0 \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{c0}}$, and $P_\psi$ is the linear (LPS, $P_L$) or non-linear dimensionless power spectrum (NLPS, $P_{NL}$) of the gravitational potential. In the second line we switched
to dimensionless variables, $\bar{\eta} = H_0 \eta$ and $p = k/k_{eq}^d$. Equation (2) demonstrates the relevant physical scales $H_0$ and $k_{eq}$, the sensitivity to scales smaller than the equality scale $p > 1$, and the expected enhancement pattern $(k_{eq}/H_0)^3$ in the linear regime and potentially additional $(k_{NL}/k_{eq})^3$ at a redshift dependent non-linearity scale, $k_{NL}$. So we have a direct probe of the integrated late-time power spectrum and of the cosmological parameters.

At redshift $z \sim 1$ the dispersion, $\sigma_\mu$, grows approximately linearly with redshift, so the best constraints will be obtained from the maximal available redshift of current data, $z = 1$. We do not have a definite detection, but a conservative 2-sigma upper bound $\sigma_\mu(z = 1) \leq 0.12^{17}$. It is conservative because all analyses$^{16,17,18,19}$ point to a lower value of the dispersion, at most $\sigma_\mu(z) \approx 0.093z^{18}$. Moreover, the most up-to-date JLA analysis uses the actual value from$^{17}$, $\sigma_\mu = 0.055z$ and still sees a decrease as a function of redshift in the left over ‘coherent’ or ‘intrinsic’ dispersion, suggesting that even the total dispersion at $z = 1$ is only $\sigma_\mu^{tot} \leq 0.12^{19}$. Additionally, partial sky coverage and higher redshift SN, which have already been used for cosmological parameter inference, will increase the dispersion, making our analysis even more conservative.

The main limitation of (2) is the validity of the spectrum$^{16}$, because for $k \gg H_0$ standard cosmological perturbation theory breaks down, and one has to resort to numerical simulations to get an approximate fitting formula for the power spectrum. We use the HaloFit model$^{20}$ with $k_{UV} \approx 320h \text{Mpc}^{-1}$. For the standard case $P_k = A_s(k/k_0)^{\alpha(k)} - 1$, $\sigma_\mu(z = 1, k_{UV} = 320h \text{Mpc}^{-1}) \approx 0.08$. Within a certain range, varying $H_0, \Omega_m, k_{UV}$ can account at most for 15% difference$^4$. Hence the bound cannot be saturated by varying the background parameters and/or integrating up to arbitrarily small scales. Hence, it can be used for probing small scales of the power spectrum. After fixing all the background parameters, including $A_s, n_s(k_0 = 0.05 \text{Mpc}^{-1})$ to the most likelihood value of$^1$, we achieve accuracy of about 20%.

### 4 Results

We analyze four different, more general parameterizations of the spectrum and the corresponding panel in Figure 2:

\[
P_k = A_s \left( \frac{k}{k_0} \right)^{n_s(k_0) - 1} \left[ 1 + \frac{B}{A_s} \Theta(k - k_i) \right] \left( \frac{k}{k_i} \right)^{n_s(k_i) - 1} \Theta(k - k_i), \quad \text{bottom left panel (5)}
\]

where $\Theta$ is the Heaviside function, $\alpha(k_0) \equiv dn_s/d \ln k, \beta(k_0) \equiv d^2n_s/d \ln k^2$ are the “running” and “running of running” of the spectral index, (4) is a typical parameterization of one episode of particle production$^{21}$, (5) describes a step in the power spectrum, for instance due to several episodes of inflation$^{22}$, and (6) describes an enhancement which is not necessarily captured just by running. The models discussed in section 2 fit the latter parameterization.

For the above parameterizations, the HaloFit formula is not reliable anymore due to its sensitivity to initial conditions. It is nevertheless obvious that the non-linear evolution causes clustering and enhances the power spectrum. We therefore define a ratio, $F(k, z) \equiv P_{NL}(k, z)/P_{lin}(k, z)$, where $P_{NL}$ is the non-linear power spectrum, $P_L = (3/5)^2P_LT(k)g^2(z)$ is the linear spectrum, $g(z)$ is the growth factor and $T(k)$ is the transfer function with baryons$^{16}$, in the standard scenario $n_s \approx 0.96$. We take the enhancement into $^4$The choice of the equality scale $p = k/k_{eq}$ is because we know the general behaviour of $P_L$, or more precisely, its transfer function $T(k)$ which is constant for $p < 1$ and scales like $p^{-2} \ln p$ for $p \gg 1$
account by substituting in (1):

\[ P_\Psi \rightarrow P_L(k,z)(1 - c + cF(k,z)), \]

and evaluate \( \sigma_\mu \) with \( c = 0, 0.01, 0.1, 0.5, 1 \). \( c = 0 \) corresponds to computing the dispersion with the linear power spectrum only, while \( c = 1 \) corresponds to exactly following the HaloFit enhancement pattern. Except \( c = 1 \) all values of \( c \) are underestimates\(^7\). The results are presented in Figure 2. In all panels, coloured regions give \( \sigma_\mu(z = 1) \geq 0.12 \) for \( c = 0, 0.01, 0.1, 0.5, 1 \) from dark to light and are disfavoured.

From Fig. 2, it is obvious that the lensing dispersion or its absence is an extremely powerful cosmological probe. Even if a scale dependent spectral index induces clustering which is an order of magnitude smaller than the standard constant \( n_s \) scenario, some of the parameter space allowed by PLANCK is ruled out. Moreover, the analysis probes the spectrum up to \( k \sim 320h \text{ Mpc}^{-1} \), more than two orders of magnitude beyond PLANCK’s lever arm (\( \sim 5 \) e-folds more). Calling \( c = 0.1 \) ‘realistic’ and \( c = 1 \) ‘optimistic’,

\(^7\)In \( \Psi \), we also considered a step function of the sort \( P_\Psi \rightarrow P_L(k,z)(1 + b \Theta(k - k_{NL})) \), for \( b = 0, 3, 10, 50 \) with corresponding \( k_{NL} = 1, 1.2, 1.5 \text{ Mpc}^{-1} \) always underestimating the ratio \( F \). The results are very similar to that of (7).
the spectrum never goes above \((6, 3.7) \times 10^{-7}\) up to \(k \leq 320 h \text{Mpc}^{-1}\) respectively for features up to \(k_i \leq 100 \text{Mpc}^{-1}\). The only exception is (6) which gives \(P_k(320 h \text{Mpc}^{-1}) = 2.3 \times 10^{-6}\) in the realistic case for \(k_i \leq 50 \text{Mpc}^{-1}\). This is due to a slow enhancement and on smaller scales so it is quickly erased via Silk damping. We are currently analyzing numerical simulations that will test our claims. Combining SN lensing in analyses (present and forthcoming missions), will undoubtedly allow a much better determination of the cosmological parameters.

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