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Guowu Zhang, Student Member, IEEE
Odile Liboiron-Ladouceur, Senior Member, IEEE

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Guowu Zhang, Student Member, IEEE, and Odile Liboiron-Ladouceur, Senior Member, IEEE

Department of Electrical and Computer Engineering, McGill University, Montreal, QC H3A 0E9, Canada

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Abstract: In this paper, we design and experimentally demonstrate a two-mode and three-mode mode exchanger (ME) using an inverse design method. The designed MEs provide more flexibility for mode division multiplexing (MDM) system links. The optimized designs are compact, 16 $\mu$m$^2$ and 24 $\mu$m$^2$ for the two-mode and three-mode ME, respectively. During the optimization process, the fabrication imperfection tolerance, insertion loss (IL), and crosstalk performance are optimized. Considering the symmetry of these devices, some forward and adjoint simulations are reused. Thus, only N simulations per iteration are required for N mode ME even considering crosstalk in the figure of merit (FOM). The fabricated two-mode ME exhibits IL less than 0.52 dB within the wavelength range from 1.5 $\mu$m to 1.6 $\mu$m. The corresponding crosstalk is at most $-18.5$ dB within the same wavelength range. The 2 $\times$ 10 Gbps non-return to zero (NRZ) PRBS-31 payload transmission shows clear and open eye diagrams for all output modes. A three-mode ME is also experimentally demonstrated validating the scalability using inverse design for adaptable ME devices. The fabricated three-mode ME exhibits an IL less than 0.85 dB and 0.9 dB for the conversions from TE0 to TE1 and from TE1 to TE0, while the TE2 to TE2 transmission has an IL of 1.7 dB within the same wavelength range. The crosstalk is less than $-16.5$ for all three output modes with 3 $\times$ 10 Gbps NRZ open eye diagrams.

Index Terms: Silicon nanophotonics, waveguide devices, optical interconnects.

1. Introduction

Mode division multiplexing (MDM) has been investigated intensively in recent years. It has the potential to further increase the capacity of existing communication systems when combined with polarization division multiplexing (PDM) and wavelength division multiplexing (WDM) [1]. The idea of transmitting multiple modes in a single multimode waveguide using only one light source not only provides another freedom for multiplexing, but also reduces the power consumption and footprint of the system [2], [3]. The silicon-on-insulator (SOI) has been regarded as an optimum platform for MDM photonic devices due to the large high index contrast between the silicon and the surrounding silicon dioxide. Several key components are proposed and demonstrated for realizing an on-chip MDM system on SOI, such as mode (de)multiplexers [4], [5], multimode
waveguide bends [6]–[11], multimode waveguide crossings [12]–[14], multimode 3 dB splitters [15]–[19], higher order mode pass filters [20], multimode switches [21]–[25], and mode exchangers (ME) [26], [27]. Among these, ME enables exchanging data between different mode channels for an MDM link, similar to the wavelength conversion in WDM systems and polarization rotation in PDM systems. Such a device is inevitable for realizing flexible and advanced optical MDM systems.

A mode exchanger can be realized using demultiplexing, exchanging, and multiplexing strategies where all higher order modes are demultiplexed into fundamental modes, then exchanged to be multiplexed back to the corresponding higher order modes again [26]. This approach increases the footprint of the whole MDM link especially when more modes are required. More recently, significant research effort has been invested in designing ME as a single component [27]–[32]. In [27]–[29], ME is theoretically and experimentally proposed using subwavelength grating (SWG) based on an SOI platform. In this method, the conversion/exchanging efficiency between two modes can be engineered by tuning the parameters of the SWG meta structures. However, these demonstrations are mainly focusing on the data exchanging for only two modes as it is not straightforward to maintain mode conversion/exchanger efficiency for more than three modes in such SWG-based metastructure. Scalability to more mode count for MDM system with existing ME devices remains to be demonstrated. Recently, an ultra-compact two-mode ME has been also demonstrated using metasurfaces [30], [31] on SOI platform. However, the crosstalk performance is limited to approximately −8 dB. The best crosstalk for previous demonstrations on SOI is limited to approximately less than −12 dB. Other demonstrations of ME devices are based on silica-PLC leading to relatively larger footprints compared to SOI and lower crosstalk performance mitigated using digital signal processing [32]–[34].

In this paper, high performance and flexible MEs are demonstrated using the adjoint method based inverse design on an SOI platform. Adjoint method-based optimization was introduced in optimizing WDM (De)MUX [35], [36], grating coupler [37], [38], MDM (De)MUX [39], but have not yet been explored for MDM-based building blocks. There are two main advantages for introducing this method into ME design. On one hand, the crosstalk performance, one key performance for ME device, can be optimized while not affecting the overall number of simulations needed in each iteration. Indeed, only N simulations are required in each iteration for N mode exchanger device regardless of crosstalk been taken into account. In our experimental validation, the measured crosstalk is below −18.5 dB within an optical bandwidth range from 1.5 \( \mu \)m to 1.6 \( \mu \)m for two mode ME (2ME) device after crosstalk is taken into account during the optimization. To show the scalability of the proposed design, a three-mode ME (3ME) is also designed, fabricated and characterized. The crosstalk is less than −16.5 dB for all three modes.

The remainder of this paper is organized as follows. In Section 2, we detail the design methodology along with the desired input and output relation of the devices. In Section 3, fabrication details of the devices are discussed along with the continuous wave (CW) and payload transmission measurement results. Finally, we conclude and discuss the scalability of the designs in Section 5.

2. Design and Modeling of the Proposed Mode Exchanger

2.1 Principle of the Adjoint Method-Based Optimization

The optimization is applied on a rectangular surface region onto a standard SOI technology platform with an optical waveguide thickness of 0.22 \( \mu \)m. The optimization includes three stages. In the first stage, a continuous optimization, also referred as gray scale optimization method, is employed where all the permittivity inside the design region can be optimized to fully explore the design space. The three-field topology optimization scheme [40] is used to represent permittivity distribution inside the design region which consists of a design field \( \rho \), a filtered field \( \tilde{\rho} \) and a physical field \( \tilde{\rho} \). The relation between these three fields are defined as follow,
where \( D \) represents the design region, \( w \) is the two-dimension linear hat-shape filter [40] with its coefficients defined in (2). \( R \) is the radius of the linear hat-shape filter (set to 300 \( \mu \)m). \( \beta \) is the parameter that controls the strength of projection (set to 100). \( \eta \) is a parameter between 0 and 1 that defines the mid-point of the projection (set to 0.5 and referred to the projection threshold). \( \varepsilon_{Si} \), \( \varepsilon_{SiO_2} \) represents the permittivity of silicon (\( \varepsilon_{Si} = n_{Si}^2 = 12.11 \)) and silicon dioxide (\( \varepsilon_{SiO_2} = n_{SiO_2}^2 = 2.075 \)). At the end of the first stage, the structure is binarized by changing the distribution of \( \rho \) ranges from 0 to 1 using (5). Here, \( k \) is set so that the minimum and maximum value of \( \rho \) is 0 and 1 after binarization.

\[
\rho = k \times (\rho - 0.5) + 0.5
\]  

In the second stage, the generated designs become robust to fabrication imperfections. Random fabrication errors caused by over-etch and under-etch are considered in the design optimization by modelling the projection threshold \( \eta \) as a random variable. In other words, instead of fixing \( \eta \) to 0.5 in the first stage, \( \eta \) is generated randomly within the pre-defined range from \( \eta_{under} \) to \( \eta_{over} \) in this stage. The relation between the projection threshold \( \eta \) and manufacturing error magnitude \( \Delta \) can be derived as (6) following the method detailed in [41]. However, (6) only give a lower bound of the fabrication error at a given \( \eta \). To illustrate this, Fig. 1(b) shows the boundary of ME devices for different \( \eta \) values where it becomes apparent that the distribution of manufacturing error magnitude at different positions is nonuniform.

\[
\frac{\Delta}{R} = \begin{cases} 
1 - \sqrt{2\eta}, & \eta < 0.5 \\
\sqrt{2(1-\eta)} - 1, & \eta \geq 0.5 
\end{cases}
\]  

Finally, a level-set method [42] is employed to fine-tuning the boundary of the device from the previous step. The filtered field \( \hat{\rho} \) in the previous step is used as the level-set function in this stage.
The $\tilde{\rho}$ is interpolated using a cubic interpolation method in Matlab to generate a smooth boundary. The permittivity distribution is defined as below in (7). Additionally, the crosstalk is added as a design objective during this step to further improve the performance of the ME device.

$$\varepsilon(x, y) = \begin{cases} \varepsilon_{\text{Si}}, & \tilde{\rho}(x, y) \geq \eta \\ \varepsilon_{\text{SiO}_2}, & \tilde{\rho}(x, y) < \eta \end{cases}$$ (7)

### 2.2 Adjoint Method-Based Optimization for Two-Mode Mode Exchanger (ME)

For 2ME, the optimization objective is to exchange or map the fundamental transverse electric (TE0) input mode at the input waveguide onto the TE1 mode at the output waveguide. The optimization is carried out over the $4 \mu m \times 4 \mu m$ design region shown in Fig. 2(a). The figure of merit (FOM) is defined as the transmission onto the TE1 mode when the input is the fundamental mode TE0. From symmetry, the output should ideally be TE0 for TE1 input. Thus, the TE0 to TE1 conversion is optimized and only two simulations are required in each of the iteration. According to the desired input to output relation, the FOM in the first and second design steps can be expressed as,

$$FOM_{2\text{ME}} = T_{01} = \frac{P_{\text{out,TE}_1}}{P_{\text{in,TE}_0}}$$ (8)

where $T_{ij}$ represents the transmission of the input mode TE$_i$ onto the output mode TE$_j$. $P_{\text{in,TE}_i}$ represents the optical input power of TE$_i$ mode and $P_{\text{out,TE}_j}$ represents the optical output power of TE$_j$ mode. The gradient of the FOM with respect to permittivity can be calculated efficiently using the adjoint method detailed in [43]. To calculate the gradient of the FOM versus the permittivity distribution, the adjoint method requires running two simulations. One referred as the forward simulation which solves Maxwell equations with sending the desired input mode at the input waveguide. The field obtained for forward simulation is $E_{\text{forward}}$. The second simulation is in the backward direction and is referred as the adjoint simulation. In this case, the Maxwell equations are solved for the cases where the desired output mode is injected at the output waveguide. This obtained field distribution, $E_{\text{backward}}$, is then by the following phase factor $A$ to generate the adjoint field distribution $E_{\text{adjoint}}$.

$$A = \frac{\varepsilon_0 \Delta V (\int E_{\text{forward}} \times H_m^* \cdot dS + \int E_m^* \times H_{\text{forward}}^* \cdot dS)^*}{4 \int \text{Re}(E_m \times H_m^*) \cdot dS}$$ (9)

where $\ast$ represents the conjugation operation, $E_m$ and $H_m$ represent the electrical and magnetic field of the desired output mode of mode order $m$. $\Delta V = \Delta x \Delta y \Delta z$ where $\Delta x$, $\Delta y$ and $\Delta z$ are simulation grid spacings in the $x$, $y$ and $z$ directions. The integration is calculated over the output surface $S$ where $S$ is the cross-section of the simulation domain at the desired output position. After obtaining the forward and adjoint fields, the gradient of the FOM with respect to the permittivity distribution is
calculated as follow,

$$\frac{\partial FOM}{\partial \epsilon_i} = \text{Re} \left[ E_{\text{adjoint}}^i \cdot E_{\text{forward}}^i \right]$$

(10)

where the subscript $i$ represents the calculated $i$-th element of the design region. In our optimization, the forward and adjoint fields are obtained using a commercial simulation tool Lumerical FDTD. After the FOM gradient with respect to the permittivity is calculated, the FOM gradient with respect to design parameter $\rho$ inside the design region can be calculated using chain rules as,

$$\frac{\partial FOM}{\partial \rho_i} = \sum_{j \in D} \frac{\partial FOM}{\partial \epsilon_j} \frac{\partial \epsilon_j}{\partial \rho_i}$$

(11)

To summarize, the optimization algorithm consists of a main loop with the following three steps,

1) generate the permittivity distribution inside the design domain according to (1)–(4); 2) obtain through simulation the forward and adjoint field distribution $E_{\text{forward}}$ and $E_{\text{adjoint}}$; 3) calculate the gradient of the FOM as a function to the design parameter $\rho$ according to (9)–(11).

The FOM for the final step takes the crosstalk into account and is defined as,

$$FOM_{2ME} = T_{01} - \tau(T_{00} + T_{11}) = \frac{P_{\text{out}TE1}}{P_{\text{in}TE0}} - \tau \left( \frac{P_{\text{out}TE0}}{P_{\text{in}TE0}} + \frac{P_{\text{out}TE1}}{P_{\text{in}TE1}} \right)$$

(12)

where $\tau$ is a penalty weight that is used to balance the importance between transmission and crosstalk optimization. It is set to 5 which is sufficient to ensure $-20$ dB crosstalk over the entire targeted bandwidth. The FOM gradient with respect to the design parameter $\rho$ can be calculated separately for each of the terms in the expression above. For each term, one forward and one adjoint simulation is required. Considering the symmetry of the design, some forward and adjoint simulations can be reused. This has the advantage of only two simulations being required even when adding crosstalk into the FOM. More generally, for N mode ME devices, only N simulations are required even when crosstalk is considered. This improves computation time considerably.

After the FOM gradient is obtained with respect to the design variable $\rho$, a gradient-based algorithm adaptive moment estimation (Adam) is employed to update the design variable [44]. To get a broadband response, the optimization is carried out over five equally spaced wavelength points from 1.5 $\mu$m to 1.6 $\mu$m. During the optimization process, the FOM at these five wavelengths are optimized by making it as large as possible. The initial permittivity distribution inside the design region is chosen as $(\epsilon_{Si} + \epsilon_{SiO2})/2$. The final optimized design is shown in Fig. 2(b). Fig. 2(c) shows how the FOM at 1.55 $\mu$m changes with iteration. For the grayscale design optimization step, the FOM increases and saturates after 80 iteration. During the second design step, the FOM improves and shows random behaviour since fabrication error is modelled as a random parameter. Fig. 2(d) and (e) show the simulated transmission spectrum of the optimized design. The simulated magnetic field $H_z$ components for different input modes are shown in the inserts of Fig. 2(d) and (e). The result shows that the simulated IL is less than 0.44 dB (0.38 dB) for TE0 (TE1) to TE1 (TE0) conversion within the wavelength range from 1.5 $\mu$m to 1.6 $\mu$m. The corresponding crosstalk is less than $-29$ dB for all these two input modes.

To further investigate the benefits of considering fabrication imperfections during the optimization, Fig. 3 shows the transmission and crosstalk of the optimized device at 1.55 $\mu$m as a function of different projection threshold with and without fabrication robustness optimization. The results show that with robustness optimization, the final optimized device can be tolerant to a larger range of fabrication error at the expense of a 0.2 dB peak transmission degradation. In our optimization, we set $\eta_{\text{under}}$ and $\eta_{\text{over}}$ to 0.47 and 0.53, respectively, which corresponds to at least $\pm10$ nm shifts in the designed vertical edges according to (6) and Fig. 1(a).
2.3 Adjoint Method-Based Optimization for Three-Mode ME

Another advantage of using an adjoint method-based optimization for ME devices is its scalability. In contrast, it is not straightforward to manipulate the mode conversion/exchange efficiency for more than two modes in SWG based ME devices. To demonstrate the scalability of our designs, a three-mode ME is optimized using the same method as presented in the previous section where all three input modes at the input multimode waveguide are mapped onto one of the modes at the output waveguide. Without assuming symmetry along the transmission direction of the mode converter, there are \( A_3^3 = 3! = 6 \) mapping scenarios. With symmetry, as it is the case in our device, there are only three possible scenarios can exist (Table 1). We detail scenario A as the other two scenarios can be obtained by changing the FOM according to the input/output relation without any other modification. Thus, the optimization objective is to exchange the TE0 input mode onto the TE1 mode at the output port while TE2 input mode is expected to be transmitted without conversion. The optimization is carried out over the \( 6 \mu \text{m} \times 4 \mu \text{m} \) design region shown in Fig. 4(a). The 3ME FOM is defined in (13) according to its input to output relation.

\[
FOM_{3\text{ME}} = T_{01/10} + T_{22} = \frac{P_{\text{out,TE1/TE0}}}{P_{\text{in,TE0/TE1}}} + \frac{P_{\text{out,TE2}}}{P_{\text{in,TE2}}}
\]  

(13)
Fig. 4. (a) Illustration of the input/output relation for the 3ME (b) Optimized 3ME design layout; Simulated transmission spectrum of first scenario A for (c) TE0 input (d) TE1 input and (d) TE2 input.

The optimized results for the 3ME are shown in Fig. 4. The optimized design is shown in Fig. 4(b). Fig. 4(c)–(e) show the simulated transmission spectrum of the optimized design for different input modes. The corresponding simulated magnetic field $H_z$ components are shown in the inserts of Fig. 4(c)–(e). The result shows that the IL is less than 0.79 dB, 0.8 dB and 1.4 dB for TE0, TE1 and TE2 input modes respectively within the wavelength of 1.5 $\mu$m to 1.6 $\mu$m. The IL performance is relatively large compared to the 2ME but can be improved by increasing the size of the design region. The corresponding crosstalk is less than $-20$ dB for all these three input modes.

3. Fabrication and Characterization

3.1 Fabrication and Characterization Method

The design is fabricated on a SOI wafer through Applied Nanotool Inc. (ANT). The silicon device layer is patterned using a 100 keV electron-beam lithography (EBL) followed by an inductively coupled plasma-induced reactive ion etching (ICP-RIE) process. A 2.2 $\mu$m SiO$_2$ layer is deposited by chemical vapor deposition (CVD) to protect the device. To characterize the performance of the 2ME and 3ME, an adiabatic directional coupler (ADC) based on-chip mode multiplexer and demultiplexer is used to generate the required TE1 and TE2 modes. Likewise, an on-chip demultiplexer at the output of the MEs demultiplex the high order modes to fundamental modes. Fig. 5(a) and (b) illustrates the schematic of the two-mode multiplexer and three-mode multiplexer. The input fundamental modes from the left will be multiplexed onto corresponding high order modes at the output multimode waveguide, and vice versa. The parameters of these mode multiplexers are selected according to our previous work [23] and further optimized according to [5]. We illustrate the optical microscope image of test structure and indicate the inputs and outputs of our designs for 2ME in Fig. 5(c) and 3ME in Fig. 5(d). The measurements of the MDM links are conducted with
Fig. 6. Experimental setup for validation of the proposed MEs (a) for the 2ME and (b) for the 3ME. The black solid, black dotted, and red line represents electrical, clock, and optical signals, respectively. PC: polarization controller; PS: power splitter; SSMF: standard single-mode fiber; FA, multi-channel fiber array; 2ME, two mode exchanger; 3ME, three mode exchanger; EDFA, erbium-doped fiber amplifier; TF, tunable filter; DCA, digital communication analyzer; PPG, pulse pattern generator.

A test bed comprising a tunable laser (Keysight 8164B), a polarization controller, a 12-channel fiber array which is used for coupling multiple off chip inputs from the fiber into the chip and for coupling multiple outputs from the chip out to the fiber, fiber alignment stages, and an optical power meter. A pair of grating couplers is used to receive/transmit the light from/into the single-mode fibers. All CW measurements are normalized to a loop back structure that connects two grating couplers with a short waveguide. The measured IL for the loop back structure is approximately $-13$ dB at 1.55 $\mu$m.

The payload transmission experiment is conducted to evaluate the performance of the fabricated devices for a practical MDM data transmission link. Here, both single mode channel transmission and MDM transmission are conducted. For single mode transmission, only one input of the MDM link is validated at a time, corresponding to one mode channel transmission. All inputs of the MDM link are simultaneously used to show the impact of crosstalk between different mode channels. Fig. 6 shows the experimental setup of payload transmission. The laser output is passed through a polarization controller (PC), then modulated using a modulator with a 3 dB bandwidth of 12.5 GHz. The output power of the laser is fixed to 13 dBm. Then, a 1 by 2 (1 by 3 for 3ME) power splitter is used to generate two (three) modulated signals. To decorrelate the three input data streams, one of the channels is delayed by 2 km standard single-mode fiber (SSMF), while the other channel is delayed by 0.5 km SSMF. Each channel passes through another PC to ensure on-chip TE mode. After the PCs, the light is coupled onto the chip, the device under test (DUT), using a vertical grating coupler. The output from the DUT is amplified using an erbium-doped fiber amplifier (EDFA) to compensate for the coupling loss between the fiber and the chip. A tunable optical filter is employed after the EDFA to filter the generated ASE out-band noise. A variable optical attenuator (VOA) is then used to control the optical power at the photodetector (PD). For eye diagram measurements, the output optical power is fixed to $-4$ dBm after the VOA. The signal is detected using the optical module of the digital communication analyser (DCA 86100C), and the eye diagrams are recorded. All payload transmission measurements are conducted at 1.55 $\mu$m wavelength considering the peak transmission efficiency of the grating couplers.

3.2 Two-Mode ME

Fig. 7(a) shows the normalized transmission spectrum for a reference design of a 2-mode MDM link consisting of a 2-mode MUX and DEMUX. This MDM link is used to show the crosstalk and IL performance from mode MUX and DeMUX only to isolate the performance of the proposed ME device. For 2ME characterization, the MDM link includes a 2-mode MUX, 2ME, and a 2-mode DEMUX. The effectiveness of fabrication tolerant and crosstalk optimization is initially investigated. The normalized transmission spectrum for a device without the fabrication robustness optimizations is given in Fig. 7(b). For this device, the measured IL and the crosstalk are less than 2 dB and $-12$ dB, respectively, within the wavelength range from 1.5 $\mu$m to 1.6 $\mu$m. As a comparison,
Fig. 7. Measured normalized transmission spectrum for (a) the two-mode MDM link including a mode MUX and DEMUX, (b) 2ME without fabrication tolerant and crosstalk design optimization and (c) 2ME with fabrication tolerant and crosstalk optimization; (d) Measured transmission spectrum for two, four, six and eight 2MEs (e) Measured transmission at 1.55 μm as a function of the number of 2 MEs.

Fig. 7(c)–(e) show the measured results for the 2ME with these fabrication tolerance design optimizations. Specifically, Fig. 7(c) shows that the measured crosstalk is less than $-18.5$ dB within the same wavelength range. The IL of this 2ME is measured by cascading two, four, six, and eight 2MEs in a 2-mode multimode waveguide. In Fig. 7(d), we show the normalized transmission spectrum these. The total IL at 1.55 μm as a function of the number of 2MEs is shown in Fig. 7(e). From this result, the average IL at 1.55 μm is approximately 0.34 dB. The IL is less than 0.52 dB within the wavelength range from 1.5 μm to 1.6 μm. The IL and crosstalk performance improvements after these optimizations validate the effectiveness of our proposed inverse design optimization. Payload transmission is also conducted for the optimized 2ME. The corresponding eye diagrams for 10 Gbps non-return to zero (NRZ) PRBS-31 payload transmission are shown in Fig. 8(a)–(d). For signal mode transmission which is shown in Fig. 8(a) and (b), clear eye diagrams are observed. For simultaneous two modes transmission, the eye diagrams shown in Fig. 8(c) and (d) are deteriorated slightly by the crosstalk from other modes. Specifically, the corresponding measured SNR penalties are 8.7 dB and 9.3 dB for TE0 and TE1 output mode. Nevertheless, open and clear eyes are observed even with this deterioration. Another finding from the eye diagrams is that the crosstalk has a greater impact on the digital level 1 since the signal beating is stronger. This was also observed in [2]. To illustrate this source of noise, the photocurrent can be expressed as,

$$i_{\text{total}} \propto \left( E_{\text{signal}} + E_{\text{crosstalk}} \right)^2 = E_{\text{signal}}^2 + E_{\text{crosstalk}}^2 + 2E_{\text{signal}}E_{\text{crosstalk}}$$

(14)

where $i_{\text{total}}$ is the total photocurrent generated by the signal and the crosstalk, $E_{\text{signal}}$ is the signal field and $E_{\text{crosstalk}}$ represents the field generated by crosstalk. At level 0, $E_{\text{signal}}$ is approximately 0, thus the mixing term $2E_{\text{signal}}E_{\text{crosstalk}}$ will not have an impact. The dominating noise term is only $E_{\text{crosstalk}}^2$ for level 0. However, for level 1, two terms, $E_{\text{crosstalk}}^2$ and $2E_{\text{signal}}E_{\text{crosstalk}}$, will both degrade the performance. Due to the fact that the amplitude of $E_{\text{signal}}$ is usually much larger than the amplitude of $E_{\text{crosstalk}}$, thus the mixing term $2E_{\text{signal}}E_{\text{crosstalk}}$ is the dominating noise term at level 1.
3.3 Three-Mode ME

Fig. 9(a)–(c) show the normalized transmission spectrum for a reference 3-mode MDM link consisting of a 3-mode MUX and DEMUX. For 3ME characterization, the MDM link includes a 3-mode MUX, a 3ME, and a 3-mode DEMUX. For TE1 to TE0 conversion, the measured IL and crosstalk are less than 0.85 dB and $-16.5$ dB within the wavelength range from 1.5 $\mu$m to 1.6 $\mu$m. For TE0 to TE1 conversion, the measured IL is less than 0.9 dB. The corresponding crosstalk for TE1 output mode is less than $-16.4$ dB. Both the IL and crosstalk performances are worse than the measured results for 2ME in the previous section which agree with the simulation results. To reduce IL and crosstalk further, a larger design region can be used as shown in [47]. For TE2 to TE2 transmission, the measured IL is less than 1.7 dB. The crosstalk from TE0 and TE1 input modes to TE2 output modes are both less than $-18$ dB. The results validate that the fabricated devices realize what was defined at the beginning of the optimization process.

Payload transmission is also conducted for the optimized 3ME. The corresponding eye diagrams for 10 Gbps NRZ PRBS-31 transmission are also shown in Fig. 9(g)–(l). For signal mode transmission which is shown in Fig. 9(f)–(i), clear eye diagrams are observed. The SNR performance for single mode transmission is worse than that in 2ME. This is due to the larger loss of the link...
as a 1 by 3 splitter is used and the IL of 3ME is larger than 2ME. For simultaneous transmission which is shown in Fig. 9(j)–(l), the eye diagrams are deteriorated by the crosstalk from other modes. Specifically, the corresponding measured SNR penalties are 7 dB, 6.7 dB and 6.3 dB for TE0, TE1 and TE2 output mode, respectively. Compared with 2ME, the SNR degrade 1.3 dB and 0.7 dB for TE0 and TE1 mode because the crosstalk comes from two modes rather than one mode as in 2ME. However, as the crosstalk is low enough, clear eyes are obtained even with this deterioration for all three output modes. Again, the crosstalk has stronger impact for level 1 because of the reason we have explained above.

### 3.4 Investigation for Fabrication Error Through SEM Images

To investigate the fabrication error on our optimized device, scanning electron microscope (SEM) images for the fabricated 2ME and 3ME are shown in Fig. 10. Here the optimized boundary is shown using a red line in the same image to illustrate the difference between optimized boundary and fabricated boundary. From the zoom in images which are shown in Fig. 10(c) and (d), the under-etch dominates in our fabrication which agrees with recently published findings [46]. Specifically, for 2ME, the optimized design is properly transferred to the silicon layer through fabrication process. We attribute the discrepancies between the optimized device in simulation (Fig. 2) and the measured results (Fig. 7) to the under-etch fabrication error during the fabrication process. For 3ME, most of the features are well reproduced by the fabrication process, except for two small holes close to the input and output ports as shown in Fig. 10(d). The dimension of these two holes is approximately 78 nm × 90 nm. These fabrication imperfections cause some performance degradation between the optimized design and the fabricated device. However, as we take into

| Ref. | Year | Structure | Length (μm) | BW (nm) | XT (dB) | IL (dB) | Modes |
|------|------|-----------|-------------|---------|---------|---------|-------|
| [25] | 2015 | Micro-Ring | NA          | NA      | -20.0   | 2.50    | TE0 to TE1 |
| [28] | 2016 | Dielectric Meta surface | 23.0 | Not Reported | -12.8   | 0.78    | TE0 to TE1 |
| [30] | 2018 | Inverse design | 4.0 | 40 (IL < 2.2 dB) | -9.1    | 1.34    | TE0 to TE1 |
| [45] | 2020 | SWG | 2.3 | >> 80 (system limited) | -12.0   | 0.23    | TE0 to TE1; TE0 to TE2 |
| This work | 2020 | Inverse design | 4.0 | 100 (IL≈0.42, XT<18.5) | -22.0   | 0.34    | TE0 to TE1 |
| This work | 2020 | Inverse design | 6.0 | 100 (IL=1.7, XT<16.5) | -21.0   | 0.85; 0.90; 1.70 | TE0 to TE1; TE1 to TE0; TE2 to TE2 |
consider under-etch and over-etch during the optimization process, these discrepancies remain small, and the fabricated devices show high-performance as presented in the previous section.

3.5 Comparison to Other Works

To compare our experimental results with the performance of state-of-art MEs, we summarize the performance of our device and previous reported MEs. As shown in the Table 2, most previous demonstrations are focusing on data exchanging between two modes. In contrast, scaling strategy can be used in our work with ME for a large mode count. Our optimized MEs also exhibit broader optical response and comparable IL and lower crosstalk performance with previous reported works.

4. Conclusion

We design and experimentally demonstrate a compact and broadband two-mode and three-mode exchangers using an inverse design method adapted to improve crosstalk and fabrication robustness tolerance. Both the design and experimental results are presented in detail. As the IL, crosstalk and robustness to fabrication imperfection are considered at the same time during the optimization process, the experimental results for final optimized design shows appealing performance compared with other works. Specifically, the fabricated two-mode ME exhibits IL less than 0.52 dB within the wavelength range from 1.5 \( \mu \)m to 1.6 \( \mu \)m. The corresponding crosstalk is at most \(-18.5\) dB within the same wavelength range. The scalability of the method to more modes is valid by optimizing a 3ME. The fabricated three-mode ME exhibits an IL less than 0.85 dB and 0.9 dB for the conversions from TE0 to TE1 and from TE1 to TE0, while the TE2 to TE2 transmission has an IL of 1.7 dB within the same wavelength range. The crosstalk is less than \(-16.5\), \(-16.5\), and \(-18\) dB for the TE0, TE1 and TE2 output, respectively. The corresponding 2 \times 10\ Gbps and 3 \times 10\ Gbps NRZ PRBS-31 payload transmission show clear and open eye diagrams for all output modes.

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