Constraining new physics in $B_s^0$ meson mixing

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Abstract
Neutral mesons exhibit a phenomenon called flavour mixing. As a consequence of a second order weak process the flavour eigenstates corresponding to the meson and its anti-meson are superpositions of two mass eigenstates. A meson produced in a flavour state changes into an anti-meson and back again as a function of time. Such flavour oscillations are considered sensitive probes of physics beyond the Standard Model. In this brief review I summarize the status of experimental constraints on mixing parameters in the $B_s^0$ meson system.

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1 Introduction

Quarks are the fundamental fermions that make up baryonic matter in the universe. In the Standard Model (SM) of elementary particles quarks come in six flavours, organized in three families,

$$
\begin{pmatrix}
\text{up (u)} \\
\text{down (d)}
\end{pmatrix}
\begin{pmatrix}
\text{charm (c)} \\
\text{strange (s)}
\end{pmatrix}
\begin{pmatrix}
\text{top (t)} \\
\text{beauty (b)}
\end{pmatrix}.
$$

The up-type quarks (top row) have charge $\frac{2}{3}e$ while the down-type quarks (bottom row) have charge $-\frac{1}{3}e$. We denote these quarks with the symbol $q_i$ where the index refers to the flavour. Quarks have mirror images called anti-quarks, which we denote with the symbol $\bar{q}_i$. Their physical properties are identical to those of the quarks, except that they have opposite quantum numbers for charge and flavour.

In the SM only the charged weak interaction, mediated by the charged $W$ boson, can change quark flavour. It leads to couplings of the form $u \rightarrow W^+ d$, where the transition is always between an up-type and a down-type quark. The strength of the coupling is proportional to the weak coupling constant and to the elements of a complex unitary matrix that is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix and is usually represented as

$$
V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}.
$$

(1)
The off-diagonal elements of $V_{\text{CKM}}$ are responsible for transitions between the three quark families. They are small compared to the on-diagonal elements, which are all close to unity. For three generations the $V_{\text{CKM}}$ matrix can be parametrized by four real numbers, namely three rotation angles and one phase. The non-zero value of this phase is the single source of $CP$ violation (a difference between matter and anti-matter) in the quark sector of the SM. In 2008 Kobayashi and Maskawa were awarded a Nobel prize for their explanation of $CP$ violation with this mechanism. In a sense they predicted the existence of the charm, beauty and top quark well before their discovery.

Mesons are bound states of a quark $q_i$ and an anti-quark $\bar{q}_j$. (The top quark does not appear in bound states as it decays too quickly.) The lowest-energy states of mesons with quarks with different flavour — those with no net orbital angular momentum — can only decay via the weak interaction and are therefore meta-stable, with lifetimes in the range of $10^{-13}$ to $10^{-7}$ seconds.

If the $q_i\bar{q}_j$ combination is neutral, a phenomenon occurs that is called mixing: the mass eigenstates of such mesons are quantum-mechanical superpositions of the flavour eigenstate $q_i\bar{q}_j$ and the $CP$-conjugate flavour eigenstate $\bar{q}_i q_j$. As a result a meson created in a $q_i\bar{q}_j$ state may decay as a $\bar{q}_i q_j$ state with a probability that changes as a function of time. Table 1 lists the average decay times and oscillation period of the four mesons that are subject to mixing.

Table 1: Neutral charm and beauty mesons with approximate mass, lifetime and mixing period. In the $K^0$ system there are two states with very different lifetime. Below only the lifetime of the short-lived state, the $K_{\text{short}}$, is shown.

| name | quark content | mass/MeV | lifetime/ps | oscillation period/ps |
|------|---------------|----------|-------------|-----------------------|
| $K^0$ | $sd$          | 498      | 90          | 1190                  |
| $D^0$ | $cu$          | 1865     | 0.41        | 440                   |
| $B^0_d$ | $db$         | 5280     | 1.5         | 12.4                  |
| $B^0_s$ | $sb$         | 5367     | 1.5         | 0.36                  |

The transition amplitude that governs neutral meson mixing is an example of a so-called flavour-changing neutral current (FCNC). In the SM neutral meson mixing occurs via a second order weak amplitude, depicted for $B^0_s$ mesons in Fig. 1. As such processes are heavily suppressed, they are considered to be very sensitive to contributions from physics beyond the SM. Mixing of $B^0_s$ mesons is particularly interesting for two reasons. First, the heavy mass of the $b$ quark allows for relatively reliable calculations of mixing parameters. Second, it is sensitive to new contributions in $b \to s$ transitions, which are until now relatively poorly constrained.

The subject of this review is the status of experimental constraints on $B^0_s$ mixing. We start with a summary of the neutral meson mixing phenomenology in order to introduce the experimental observables and SM predictions. Subsequently, experimental techniques and existing measurements are discussed. We finish with a conclusion and a brief outlook.
2 Beauty mixing phenomenology in a nutshell

Excellent pedagogical introductions to neutral meson mixing can be found in textbooks, recent reviews, and lecture notes. An up-to-date review of experimental constraints on $B$ meson mixing can also be found in the PDG. The following discussion applies to neutral mesons of any kind. However, we shall denote the flavour eigenstate with the symbol $B^0$ for beauty meson and use numerical estimates that apply to $B_s^0$ and $B_d^0$.

2.1 Time-evolution of the $B^0$-$\bar{B}^0$ system

Consider the wave function $B^0(t)$ for a neutral meson that is the superposition of flavour eigenstates $B^0_s$ and $B^0_d$. The time-evolution of its projections into flavour eigenstates is given by a Schrödinger equation

$$i \frac{d}{dt} \left( \begin{pmatrix} \langle B^0_s | B(t) \rangle \\ \langle B^0_d | B(t) \rangle \end{pmatrix} \right) = \left( \begin{array}{cc} H_{11} & H_{12} \\ H_{21} & H_{22} \end{array} \right) \left( \begin{pmatrix} \langle B^0_s | B(t) \rangle \\ \langle B^0_d | B(t) \rangle \end{pmatrix} \right).$$

(2)

Since the meson decays and we do not consider the wave function of final states, the Hamiltonian $H$ is not hermitian. However, like any other complex matrix, it can be decomposed in terms of two hermitian matrices, which we label by $M$ and $\Gamma$,

$$H = M - \frac{i}{2} \Gamma.$$  

(3)

Since $M$ and $\Gamma$ are hermitian, their diagonal elements are real and we have $M_{21} = M_{12}^*$ and $\Gamma_{21} = \Gamma_{21}^*$. CPT invariance requires $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. Ignoring for the moment the interference with phases in the final state, the common phase of $B^0_s$ and $\bar{B}^0_d$ is arbitrary such we can choose either the phase of $M_{12}$ or $\Gamma_{12}$ and only their phase difference matters. Consequently, the mixing can be parametrized by five real parameters, which are conventionally chosen to be

$$M_{11}, \quad \Gamma_{11}, \quad |M_{12}|, \quad |\Gamma_{12}| \quad \text{and} \quad \phi_{12} = \text{arg} \left( -\frac{M_{12}}{\Gamma_{12}} \right).$$

(4)

The mass $M_{11}$ is determined by the quark masses and strong interaction binding energy. In the $B$ system it is about 5 GeV and more than ten orders of magnitude larger than the size of the other elements, which all involve the weak interaction.

The time-evolution of the meson-anti-meson system is described in terms of the eigenstates of the Hamiltonian. The two mass eigenstates can be written as

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure1.png}
\caption{Leading order diagrams for neutral meson mixing in the SM.}
\end{figure}
linear combinations of the flavour eigenstates,

\[ |B_L⟩ = p |B^0⟩ + q |\overline{B}^0⟩ \]
\[ |B_H⟩ = p |B^0⟩ - q |\overline{B}^0⟩ \]  \hspace{1cm} (5)

where the subscripts \(H\) and \(L\) stand for ‘heavy’ and ‘light’, \(q\) and \(p\) are complex numbers and normalization requires \(|p|^2 + |q|^2 = 1\). For \(q/p = 1\) the mass eigenstates correspond to \(CP\) eigenstates. On the other hand, if \(q/p \neq 1\), \(CP\) is not conserved in the time-evolution of the \(B^0-\overline{B}^0\) system.

The eigenvalues corresponding to the heavy and light states are written as

\[ \omega_{L,H} ≡ m_{L,H} - \frac{i}{2} \Gamma_{L,H} \]  \hspace{1cm} (6)

and are usually recast in terms of the observables

\[ m ≡ \frac{1}{2} (m_H + m_L) = M_{11} \]
\[ \Gamma ≡ \frac{1}{2} (\Gamma_H + \Gamma_L) = \Gamma_{11} \]
\[ \Delta m ≡ m_H - m_L \]
\[ \Delta \Gamma ≡ \Gamma_L - \Gamma_H . \]  \hspace{1cm} (7)

Note that the two eigenstates can have both different mass and lifetimes. By convention the heavy and light solutions are labeled such that \(\Delta m\) is positive. Using that in the \(B\) meson systems \(|\Gamma_{12}| \ll |M_{12}|\) one finds

\[ \Delta M ≈ 2|M_{12}| \quad \text{and} \quad \Delta \Gamma ≈ 2|\Gamma_{12}| \cos \phi_{12} . \]  \hspace{1cm} (8)

As we shall see later, \(\Delta m\) can be measured by observing an asymmetry in the decay time distribution of \(B^0\) and \(\overline{B}^0\), while \(\Delta \Gamma\) is obtained by combining lifetime measurements of decays to final states with different \(CP\) content.

The ratio \(q/p\) can be written as

\[ \frac{q}{p} = e^{-i\phi_M} \sqrt{\frac{|M_{12}| + \frac{1}{2} |\Gamma_{12}| e^{i\phi_{12}}}{|M_{12}| + \frac{1}{2} |\Gamma_{12}| e^{-i\phi_{12}}}} \]  \hspace{1cm} (9)

where \(\phi_M \equiv \arg(M_{12})\). For \(\phi_{12} \neq 0, \pi\) the absolute value of \(q/p\) is different from unity, a case we refer to as \(CP\) violation in mixing. Using \(|\Gamma_{12}| \ll |M_{12}|\) we have for the difference of \(|q/p|^2\) with one

\[ 1 - \left| \frac{q}{p} \right|^2 \approx \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin \phi_{12} \]  \hspace{1cm} (10)

Since the time-evolution of the mass eigenstates follows \(|B_{H,L}(t)⟩ = \exp(-i\omega_{H,L}) |B_{H,L}(0)⟩\), the time-evolution of a \(B^0\) meson produced in a \(B^0\) or \(\overline{B}^0\) flavour eigenstate at \(t = 0\) can now be written as

\[ |B^0(t)⟩ = g_+(t) |B^0⟩ + \frac{q}{p} g_-(t) |\overline{B}^0⟩ \]
\[ |\overline{B}^0(t)⟩ = g_+(t) |\overline{B}^0⟩ + \frac{p}{q} g_-(t) |B^0⟩ \]  \hspace{1cm} (11)
with the functions $g_\pm(t)$ defined as
\[
g_\pm(t) = \frac{1}{2} \left( e^{-i\omega_L} \pm e^{-i\omega_H} \right) \tag{12}
\]
The probability to observe at time $t$ the decay into a state with a flavour that is the same as (plus sign) or opposite to (minus sign) the flavour with which it was produced is then proportional to
\[
|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2} \left[ \cosh \left( \frac{1}{2} \Delta \Gamma t \right) \pm \cos \left( \Delta m t \right) \right], \tag{13}
\]
while the time-integrated oscillation probability is given by
\[
\chi = \frac{1}{2} \frac{4 \Delta m^2 + \Delta \Gamma^2}{4 \Delta m^2 + \Gamma^2}. \tag{14}
\]

2.2 Including decay amplitudes

The formalism above only describes the time-evolution of the $B^0 - \bar{B}^0$ system and not yet the decay to an observable final state $f$. For a given final state we define two transition amplitudes
\[
A_f \equiv \langle f | \mathcal{H} | B^0 \rangle \quad \text{and} \quad \bar{A}_f \equiv \langle f | \mathcal{H} | \bar{B}^0 \rangle, \tag{15}
\]
where $\mathcal{H}$ is the weak interaction Hamiltonian responsible for the decay. For a meson produced in an initial flavour eigenstate $B^0$ the decay width to the final state $f$ receives two contributions, namely one from $A_f$ and another one from $\bar{A}_f$ where the $B^0$ first oscillated to a $\bar{B}^0$, schematically depicted in Fig. 2. The partial decay rate can be written as
\[
\Gamma_{B^0 \to f}(t) = |A_f|^2 \left( 1 + |\lambda_f|^2 \right) \frac{e^{-\Gamma t}}{2} \left[ \cosh \left( \frac{1}{2} \Delta \Gamma t \right) + D_f \sinh \left( \frac{1}{2} \Delta \Gamma t \right) + C_f \cos \left( \Delta m t \right) - S_f \sin \left( \Delta m t \right) \right] \tag{16}
\]
where we defined
\[
\lambda_f = \frac{q \bar{A}_f}{p A_f} \tag{17}
\]
and
\[
C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad S_f \equiv \frac{2 \Im(\lambda_f)}{1 + |\lambda_f|^2}, \quad D_f \equiv -\frac{2 \Re(\lambda_f)}{1 + |\lambda_f|^2}. \tag{18}
\]
The decay rate for $\bar{B}^0 \to f$ is obtained from this expression by changing the sign of $C_f$ and $S_f$ and multiplying by an overall factor $|p/q|^2$. Similar expressions can be derived for the decay of $B^0$ to the $CP$ conjugate state $\bar{f}$ by a suitable redefinition of $\lambda$. It is important to note that, in contrast to $M_{12}$, $\Gamma_{12}$ and the elements of the CKM matrix, $\lambda_f$ is a phase convention-independent physical observable.
We now consider two cases relevant for this review. Flavour-specific final states are final states for which $|A_f| \ll |\bar{A}_f|$ such that $|\lambda| \simeq 0$. Important examples are the tree-level transitions $B_0 \to D_s^+ \pi^-$ and $B_0 \to D_s^+ \mu^- \bar{\nu}_\mu$ shown in Fig. 3. Under the assumption of no $CP$-violation in mixing ($|q/p| = 1$) and no $CP$-violation in the decay ($|A_f| = |\bar{A}_f|$), we can derive the following expression for the so-called time-dependent oscillation (or ‘mixing’) probability,

$$A_{\text{mix}}(t) \equiv \frac{\Gamma_{B_0 \to f} + \Gamma_{\bar{B}_0 \to f}}{2} \left[ 1 + \frac{\Gamma_{\bar{B}_0 \to f} - \Gamma_{B_0 \to f}}{\Gamma_{B_0 \to f} + \Gamma_{\bar{B}_0 \to f}} \right] \cosh \frac{\Delta m t}{2} \Delta \Gamma t. \quad (19)$$

The two terms in the definition of the asymmetry are usually called the ‘unmixed’ and ‘mixed’ contribution, respectively. In section 4.1 we discuss measurements of the oscillation probability with flavour-specific final states.

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1There exist alternative notations for these three quantities, in particular $A_f^{\text{dir}} \equiv C_f$, $A_f^{\text{mix}} \equiv -S_f$ and $A_f^{\Delta \Gamma} \equiv D_f$. However, be aware that different conventions are used regarding the signs of these quantities.
Alternatively, dropping the requirement on $q/p$ we can form the following $CP$ asymmetry

$$a_{fs} \equiv \frac{\Gamma_{B^0 \to f} - \Gamma_{B^0 \to \bar{f}}}{\Gamma_{B^0 \to f} + \Gamma_{B^0 \to \bar{f}}} = \frac{1 - |q/p|^4}{1 + |q/p|^2}$$

which is notably time-independent. This asymmetry is called the flavour-specific or semi-leptonic asymmetry. (HFAG denotes this quantity with $A_{sl}$.) Note the particle and anti-particle labels in this definition: both are ‘mixed’ contributions. In the $B$ meson system $|q/p|$ is close to one. To translate the measurement of $a_{fs}$ into constraints on $\phi_{12}$ one often uses the relation

$$a_{fs} \approx 1 - |q/p|^2 \approx \frac{\Delta \Gamma}{\Delta m} \tan \phi_{12} \tag{21}$$

One way to measure $a_{fs}$ is by counting the number of positive like-sign and negative like-sign muon pairs in events in which both $b$ quarks decay semi-leptonically,

$$a_{fs} = \frac{N(\mu^+\mu^+) - N(\mu^-\mu^-)}{N(\mu^+\mu^+) + N(\mu^-\mu^-)} \tag{22}$$

As we shall see in section 4.4 the measurement of flavour-specific asymmetry with semi-leptonic and $B \to D\pi$ decays provides the best constraints on $|q/p|$ in the $B^0_s$ system.

Next we consider decays to final states $f$ that are (mixtures of) $CP$ eigenstates, most notably $B^0_d \to J/\psi K^0_S$ and $B^0_s \to J/\psi \phi$. Figure 3c and d show tree- and penguin-level contributions to the $B^0_s \to J/\psi \phi$ amplitude. If a final state $f$ is accessible to both the $B^0$ and the $\bar{B}^0$ meson, then interference between decay via mixing and decay without mixing gives rise to $CP$-violation. More specifically, if all contributing decay amplitudes carry the same weak phase, then the decay amplitude ratio can be written as $A_f/\overline{A}_f = \eta_f e^{2i\phi_f}$, where $\eta_f = \pm 1$ is the $CP$-eigenvalue of the final state and $\phi_D = \arg(A_f)$. If one further assumes that $CP$ violation in mixing is small ($|q/p| \approx 1$), then

$$\lambda = \eta_f e^{-i\phi_M} e^{2i\phi_D} \tag{23}$$

where the phase $\phi_M \equiv \arg(M_{12})$ enters via Eq. 9. The time-dependent $CP$-asymmetry can then be written as

$$A_{CP}(t) \equiv \frac{\Gamma_{B^0 \to f} - \Gamma_{B^0 \to \bar{f}}}{\Gamma_{B^0 \to f} + \Gamma_{B^0 \to \bar{f}}} = \frac{\eta_f \sin \phi_f \sin \Delta mt}{\cosh \frac{1}{2} \Delta \Gamma t - \eta_f \cos \phi_f \sinh \frac{1}{2} \Delta \Gamma t} \tag{24}$$

where we introduced the commonly used $CP$ violating phase

$$\phi_f \equiv -\arg(\lambda_f) = \phi_M - 2\phi_D \tag{25}$$

By measuring the amplitude of the sinusoid in the time-dependent asymmetry we constrain the phase $\phi_f$. Since this phase is related to the phase of $M_{12}$, the $CP$ asymmetry is a direct probe of new contributions to $M_{12}$.
For decays to \( CP \)-eigenstates with a single contributing amplitude the phase \( \phi_f \) can be directly expressed in terms of elements of \( V_{\text{CKM}} \). In particular, we have for the so-called ‘golden modes’, that occur through a tree-level \( b \to c\bar{c}s \) transition,

\[
\begin{align*}
B^0_d &\to J/\psi K_S^0 : \quad \phi^c_{d\bar{c}s} = 2\beta \\
B^0_s &\to J/\psi \phi : \quad \phi^c_{s\bar{c}s} = -2\beta
\end{align*}
\]

where the CKM phases \( \beta \) and \( \beta_s \) are defined by

\[
\begin{align*}
\beta &\equiv \arg \left( -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right) \quad \text{and} \quad \beta_s &\equiv \arg \left( -\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right)
\end{align*}
\]

(27)

We discuss the measurement of \( \phi^c_{s\bar{c}s} \) with \( B^0_s \to J/\psi \phi \) and \( B^0_s \to J/\psi f^0 \) decays in section 4.2.

Finally, we consider lifetimes. The ‘untagged’ decay time distribution for a final state \( f \) can be obtained from Eq. 16 by setting \( C = S = 0 \). The average decay time (sometimes called the ‘effective lifetime’) is given by

\[
\tau_f = \frac{(1-D_f)/\Gamma_L + (1+D_f)/\Gamma_H}{(1-D_f)/\Gamma_L + (1+D_f)/\Gamma_H} = \frac{1}{\Gamma} \frac{1+2D_f y^2+y^4}{(1-y^2)(1+D_f y)}
\]

(28)

where \( D_f \) was defined above and \( y = \Delta \Gamma/2\Gamma \). For flavour-specific modes \( D_f = 0 \) while for decays to \( CP \)-eigenstates with a single contribution amplitude it is \( D_f = -\eta_f \cos \phi_f \). In section 4.3 we shall discuss constraints on \( \Gamma_s \) and \( \Delta \Gamma_s \) from various final states.

### 2.3 Standard Model predictions

In the SM the computation of \( \bar{B}^0 \) mixing parameters is performed by evaluating the amplitudes corresponding to the box diagrams shown in Fig. 1. Since quarks are not free particles, these amplitudes need to be corrected for hadronization effects. The calculations have been the cumulative effort of many people, over a period of over twenty years. (See Refs. 14, 15, 16, 17 and references therein.)

The latest complete computation of the mixing observables can be found in Ref. 16 with an update of numerical estimates in Ref. 18. The value of \( M_{12} \) is obtained from a calculation of the box diagram in Fig. 1 with a virtual top quark in the loop. The result can be expressed as

\[
M_{12}^2 = \frac{G_F^2 m_W^2}{12\pi^2} \left( V_{tq}^* V_{tb} \right)^2 S_0 \left( \frac{m_t^2}{m_W^2} \right) \eta_B \hat{B}_{B_d} f^2_{B_d} m_{B_d}
\]

(29)

The part of this expression to the left of \( \eta_B \) follows from a computation of the box diagram for free quarks in perturbation theory. It depends on parameters of the SM, such as the Fermi coupling constant \( G_F \), the CKM matrix elements \( V_{ij} \) and the top quark and \( W \) boson masses. The function \( S_0(x) \) is a known kinematic function, called the Inami-Lim function, and \( S_0(m_t^2/m_W^2) \approx 2.3 \). The numerical factor \( \eta_B \approx 0.55 \) accounts for QCD corrections. The factors to its right account for the fact that the quarks are confined in hadrons. While the \( B \) meson mass \( m_B \)
is just taken from measurements, the decay constant $f_B$ and the bag factor $\hat{B}_B$ are computed using Lattice gauge theory. (For a recent review see Ref. 20). The uncertainty on the prediction of $M_{12}$ is dominated by the theoretical uncertainty in $\hat{B}_B f_B^2$.

The computation of $\Gamma_{12}$ involves the evaluation of the box diagram with ‘on-shell’ internal quarks, the dominant contribution coming from the $b \to c\bar{s}s$ transition. Since the latter is a tree-level transition, $\Gamma_{12}$ is expected to be less sensitive to new physics than $M_{12}$ is. It can be written as

$$
\Gamma_{12}^q = -\frac{G_F^2 m_b^2}{8\pi^2} \left[ (V_{tq}^* V_{tb})^2 + V_{tq}^* V_{tb} V_{cq}^* V_{cb} \mathcal{O} \left( \frac{m_c^2}{m_b^2} \right) + (V_{cq}^* V_{cb})^2 \mathcal{O} \left( \frac{m_c^2}{m_b^2} \right) \right] \eta_B \hat{B}_B f_B^2 m_B \eta_B^f$$

where the QCD correction factor $\eta_B^f$ is of order unity. Note that $\Gamma_{12}$ is proportional to $\hat{B}_B f_B^2$ as well, such that predictions for the ratio $\Gamma_{12}/M_{12}$ have smaller theoretical uncertainty than those for $M_{12}$ and $\Gamma_{12}$ separately. Furthermore, since the ratio is proportional to $m_c^2/m_W^2 \approx 0.005$, we expect $|\Gamma_{12}| \ll |M_{12}|$ in the SM.

The SM predictions are summarized in Table 2. Note that, for some results in the table, experimental measurements are used to reduce the uncertainties. For example, the ratio $\Delta \Gamma_d/\Delta m_d$ has smaller uncertainties than $\Delta \Gamma_d$ by itself. Consequently, for the prediction of $\Delta \Gamma_d$, the measured value of $\Delta m_d$ was used.

Likewise, the computation of $\Delta m_s$ requires an estimate of $V_{ts}$. Since the computation of the perturbative parts of the amplitude is identical for $B_s^0$ and $\bar{B}_d$, the
ratio $\Delta m_s/\Delta m_d$ can be written as

$$\frac{\Delta m_s}{\Delta m_s} = \xi^2 \frac{m_{B_0^s}}{m_{B_0^d}} \left| \frac{V_{ts}}{V_{td}} \right|^2,$$

where the so-called $SU(3)$-breaking ratio is defined as

$$\xi \equiv \frac{f_{B_0^s} \sqrt{B_{B_0^s}}}{f_{B_0^d} \sqrt{B_{B_0^d}}}.$$  \hspace{1cm} (31)

The computation of $\xi$ with lattice QCD has meanwhile reached a precision of a few percent. \cite{25} With these expressions one can either test the prediction of $\Delta m_s$ by using other constraints to estimate $V_{ts}/V_{td}$ (as was done to obtain the results in the table), or use the measurement of $\Delta m_s/\Delta m_d$ to obtain a precise determination of $V_{ts}/V_{td}$.

There exists no SM predictions for the $B$ meson masses as these are essentially determined by the quark masses. Although the total decay widths cannot be computed reliably either, their ratio is well constrained and is almost unity in the SM \cite{26,27,18}.

$$0 \leq \frac{\Gamma_s}{\Gamma_d} - 1 \leq 4 \cdot 10^{-4}.$$ \hspace{1cm} (33)

As we shall see, this prediction agrees well with experimental data. Since uncertainties in this computation rely on similar assumptions as those for $\Gamma_{12}$, the agreement with data is sometimes taken as a sign that the computation of $\Gamma_{12}$ is reliable.

The interpretation of the observable $\phi_f$ in $B_d^0 \to J/\psi K^0_S$ and $B_s^0 \to J/\psi \phi$ decays in terms of the tree-level quantity $\phi^{cs}$ ignores a small contribution from penguin decays with a different weak phase, such as depicted in Fig. 3d. These contributions may change $\phi_f$ by up to a few degrees and can be constrained using measurements from other decay modes that are related by flavour symmetries. \cite{28}

### 2.4 Beyond the Standard Model

Many viable TeV-scale extensions of the SM predict new flavour-changing neutral couplings, which in turn affect the mixing parameter $M_{12}$ and for instance the rare decay $B_d^0 \to \mu^+\mu^-$. As different models affect these quantities differently, it is through a combination of flavour measurements that one hopes to identify the correct theory. (For an overview, see Ref. \cite{29}.)

The effect of new contributions to $M_{12}$ is usually parametrized by introducing a complex parameter $\Delta_q$ such that \cite{30,31,16}

$$M_{12}^q \equiv M_{12}^{SM} \Delta_q.$$ \hspace{1cm} (34)

If the magnitude of $\Delta_q$ differs from unity, this affects the observed mixing frequency. If its phase differs from zero, this affects $\phi_{12}$ and thereby $\Delta \Gamma$, $a_{\phi}$ and the $CP$ phases extracted from measurements of time-dependent $CP$ violation. For recent evaluations on constraints on $\Delta_q$ from $B_d^0$ mixing measurements, see for instance Ref. \cite{32}. A recent overview of implications and relations to other flavour physics observables can be found in Refs. \cite{33,34}.
3 Experimental facilities and techniques

The discovery of the Υ (a $b\bar{b}$ bound state) in decays to $\mu^+\mu^-$ by Lederman and collaborators in 1977 marks the onset of beauty-quark physics. Although the $b$ quark was first found in a fixed-target experiment, most experimental knowledge comes from two other types of facilities. The first are $e^+e^-$ colliders with a centre-of-momentum energy tuned to the $\Upsilon(4S)$ resonance, a $b\bar{b}$ state with a mass just above the $B_0\bar{B}_0$ and $B^+B^-$ threshold. These facilities are usually called $e^+e^-$-$B$-factories. The $b\bar{b}$ cross-section at the $\Upsilon(4S)$ is about 1 nb. The fact that this is about one fourth of the total hadronic cross-section at this energy allows for very clean studies of $B$ meson properties.

Early $e^+e^-$-$B$-factories (DORIS, CESR) were operated with symmetric energy beams. As the $\Upsilon(4S)$ resonance is just above the $B\bar{B}$ meson threshold, $B\bar{B}$ pairs are produced practically at rest in the centre-of-momentum system. Investigations at $\Upsilon(4S)$ facilities took a big step forward with the advance of the asymmetric-energy $e^+e^-$ colliders KEKB and PEP-II. These accelerators operated at the $\Upsilon(4S)$ resonance as well, but with a positron beam energy roughly half that of the electron beam. The resulting $B$-meson boost, approximately $\gamma\beta = 0.5$ at both colliders, allows for a measurement of the decay time with sufficient precision to resolve $B_0\bar{B}_0$ flavour oscillations. Thanks to the unprecedented instantaneous luminosity, the asymmetric $B$-factories have collected samples of approximately $10^9 B\bar{B}$ events.

The next generation $e^+e^-$-$B$-factory, super-KEK, is expected to start operation in 2015, allowing for an increase in statistics with another factor 50.

Beauty hadrons have also been extensively studied at high-energy colliders, both at $e^+e^-$ machines (SLD, LEP) and at hadron colliders (SPS, Tevatron, LHC). At these facilities one profits from a larger cross-section, albeit at the expense of a poorer signal-to-background ratio. For example, with a cross-section of about 300 $\mu$b, a total of approximately $10^{12}$ $b\bar{b}$ pairs have been produced in collisions in the first LHC run (2010-2012). However, these need to be extracted from an inelastic background that is a factor 200 larger. As only a fraction of events can be written to permanent storage, the experiments rely on signatures such as $J/\psi \rightarrow \mu^+\mu^-$ decays, detached muons or detached high-$p_T$ hadrons to select the events of interest.

Besides the larger cross-section the high-energy facilities have two advantages: First, all species of $b$ quark hadrons are produced, not only the $B_0^0$ and $B^+_u$ mesons found at the $\Upsilon(4S)$ resonance. Approximately 10% of all $b$ ($\bar{b}$) quarks hadronize into $B^0_0$ ($B^0_\bar{u}$) mesons. Although studies of $B^0_0$ mesons have been performed by operating at higher $e^+e^- \rightarrow \Upsilon$ resonances, conditions at these higher resonances are not favourable enough to produce competitive $B^0_0$ samples. Second, thanks to the much larger $b$ quark boost, the decay time resolution in high energy colliders far exceeds that at the $B$-factories. As we shall see below, a good decay time resolution is essential to resolve $B^0_0$ flavour oscillations. Consequently, the study of $B^0_0$ oscillations is (for now) only performed at high-energy colliders.

Figure 4 shows schematically the production and decay of a neutral $B$ meson in a high-energy collider experiment. Due to the finite decay time of the $B$, its decay vertex is displaced with respect to the collision point. Silicon vertex trackers
Experimental observables: mixing frequency

time

or

to measure mixing frequency, look at “flavour specific” decays, e.g.
decay time allows to observe time-evolution

“tagging B”

“signal B”:

flavour specific decay

or CP eigenstate

“tagging B”:
determines flavour of

signal B at t = 0

Figure 4: Schematic representation of the production and decay of a B meson in a high-energy collision.

have sufficient position accuracy to measure the decay length \( L \), which is typically a few hundred micron at the \( e^+e^- \) B-factories and up to centimeters at the hadron colliders. The decay time in the rest frame of the particle is then obtained from the observed decay length and momentum \( p \) in the detector frame,

\[
t = \frac{mL}{p},
\]

where \( m \) is the rest mass of the particle. At the \( e^+e^- \) B-factories the production point is not reconstructed and one measures the difference between the decay time of the “signal” \( B \) and the “tagging” \( B \) instead.

The mixing process described above changes the ‘flavour’ of the meson as a function of its decay time with a frequency governed by \( \Delta m \). If one can measure both the flavour at the time of production and the flavour at the time of decay, the decay time distributions for ‘mixed’ (equal flavour) and ‘unmixed’ (opposite flavour) events are given by Eq. 13. The observed distributions for \( B_s^0 \rightarrow D_s^+ \pi^- \) decays in an actual experiment – the LHCb experiment at CERN – are shown in Fig. 5. The mixing frequency \( \Delta m_s \) is extracted from the oscillation that modulates the decay time distribution.

The distribution in Fig. 5 differs in several aspects from the function in Eq. 13. First, at small decay times, the exponential shape is distorted by inefficiencies in the event selection. In this particular case, the drop in efficiency at small \( B \) decay times arises from a requirement on the minimum distance between the final state tracks and the primary vertex. This selection was applied in order to remove a large fraction of the prompt (zero lifetime) background in an early stage of the event selection. Though less important for the determination of the mixing frequency, calibration of the decay time acceptance is crucial to obtain an unbiased estimate of the lifetime parameters \( \Gamma \) and \( \Delta \Gamma \).

Second, the observed amplitude of the oscillation is much smaller than that expected from Eq. 13. This ‘dilution’ of the oscillation amplitude is caused by the imperfect determination of the flavour of the initial state and by decay time resolution effects, both of which we now discuss in more detail.

For flavour-specific final states, such as the \( B_s^0 \rightarrow D_s^+ \pi^- \) decays used for Fig. 5, the flavour at the time of decay follows from the charge of the final state particles:
The charge of the $D_s$ meson (reconstructed in decays to charged kaons and pions) uniquely determines whether the $b$ quark was a $b$ or an anti-$b$.

The determination of the flavour at the time of the production of the $B^0$ is performed with a procedure that is called ‘flavour tagging’. Two methods of flavour tagging are used. The first method relies on the fact that $b$ quarks are produced in $b\bar{b}$ pairs. Consequently, at the time of production there are two $B$ hadrons with opposite $b$ flavour in the event. Assuming that the flavour of the other $B$ — called the tag-side or tagging $B$ — can be inferred from its decay products, the initial flavour of the ‘signal’ $B$ follows. The tagging $B$ is usually not fully reconstructed such that only part of its decay products can be used to identify its flavour. The charge of high $p_\perp$ leptons and kaons can be used, as well as the total charge of an inclusively reconstructed vertex. This procedure is called ‘opposite-side tagging’. Note that one inherent limitation to this method is that approximately half the tagging $B$ hadrons are neutral mesons and hence subject to flavour oscillations as well, leading to mistakes in the flavour tag.

The second method of flavour tagging exploits that the spectator quark (the $s$ ($d$) quark for $B^0_s$ ($B^0_d$) mesons) also originates from a quark-anti-quark pair, leading to a flavour correlation between the $B$ meson and a light meson close to the $B$ meson in the fragmentation. The algorithm selects a charged kaon (or pion, for $B^0_d$) that is near in phase space to the $B$ meson. The charge of the kaon, in case of $B^0_s$, or the pion, in case of $B^0_d$, reveals the sign of the $b$ quark. This procedure is usually referred to as ‘same-side tagging’.

Mistakes in flavour tagging lead to a dilution of the observed oscillation asymmetry,

$$A_{\text{mix}}^{\text{observed}}(t) = D A_{\text{mix}}(t).$$

In practice, the dilution factor $D$ appears in front of all $\sin \Delta m t$ and $\cos \Delta m t$ terms in the differential decay rates, because it is exactly those terms that change sign between the expressions for an initial $b$ or initial anti-$b$ state. The dilution
factor due to flavour tagging is equal to
\[ D_{\text{tag}} = 1 - 2w \]  
(37)

where \( w \) is the fraction of events in which the tag is wrong. Flavour tagging performance is expressed as the so-called effective tagging efficiency or tagging power \( P = \epsilon_{\text{tag}} D_{\text{tag}}^2 \) where \( \epsilon_{\text{tag}} \) is the fraction of events for which a flavour tag could be obtained. Typical values for the tagging power are 30\% at the \( e^+e^- \) B-factories and a few percent at hadron colliders.

Besides the imperfect flavour tagging also the effects of finite decay time resolution lead to a dilution effect. For a Gaussian resolution \( \sigma_t \) the dilution is given by
\[ D_{\text{reso}} = \exp \left( -\frac{1}{2} \sigma_t^2 \Delta m^2 \right) \]  
(38)

The decay time resolution at high-energy machines is in general better than at the B-factories due to the larger boost and reduced effects of multiple scattering. The resolution depends on the final state and is substantially worse for partially reconstructed decays than for fully reconstructed decays. For the latter, it ranges from about 0.6\( \text{ps} \) at the asymmetric-energy \( e^+e^- \) factories down to about 0.05\( \text{ps} \) at the LHCb experiment with its forward geometry. With these numbers the resolution dilution factor for \( B_s^0 \) oscillations is 0.7 at the latter experiment and negligibly small at the \( \Upsilon(4S) \) factories, illustrating why the measurement of \( \Delta m_s \) is the exclusive domain of experiments at high-energy machines.

For measurements of the \( CP \) violating phase \( \phi_f \) a proper calibration of dilution factors is essential. The resolution function can be obtained from simulations or by taking a process with a known ‘zero’ decay time distribution, such as prompt \( J/\psi \) production. The flavour tagging performance is calibrated exactly by measuring the size of the amplitude observed in the decay time distribution of flavour-specific final states, such as shown in Fig. 5.

4 Status of experimental constraints on \( B_s^0 \) mixing

4.1 Mixing frequency

The first evidence of mixing in neutral \( B \) mesons was obtained by the Argus experiment in 1987: by counting the relative fraction of mixing and unmixed events the value of the \( B_d^0 \) mixing frequency \( \Delta m_d \) could be extracted using the expression for the integrated oscillation probability in Eq. 14. As this was an integrated rate the decay time of the candidates did not need to be reconstructed.

For \( B_s^0 \) mesons the oscillation period is so small compared to the lifetime, that the integrated oscillation rate does not provide a meaningful constraint on the mixing frequency. Rather, the measurement of \( \Delta m_s \) is extracted from a fit to the decay time distribution. The first evidence of mixing in \( B_s^0 \) mesons was obtained by the CDF experiment in 2006, using a combination of fully reconstructed \( B_s^0 \to D_s^+\pi^- \) and \( B_s^0 \to D_s^+\pi^-\pi^+\pi^- \) and partially reconstructed semi-leptonic \( B_s^0 \to \)
\( D_s^+ \ell^- \bar{\nu}_\ell X \) decays.\(^{10}\) The current world average value is dominated by the latest LHCb result,

\[
\Delta m_s = 17.768 \pm 0.023 \text{ (stat)} \pm 0.006 \text{ (syst) [ps}^{-1}] \tag{39}
\]

obtained from the decay time distribution of \( B_s^0 \rightarrow D_s^+ \pi^- \) events shown in Fig.\(^{3}\).\(^{36}\) The systematic uncertainty is determined by the uncertainties in the length scale and the momentum scale, which enter the measurement of the oscillation frequency through the decay time measurement in Eq.\(^{35}\).

The current value of \( \Delta m_s \) is in good agreement with the SM predictions presented in section \(2\). Assuming the validity of the SM, the value of \( \Delta m_s/\Delta m_d \) provides the best constraint on \( V_{ts}/V_{td} \).

### 4.2 Measurements of the mixing phase through time-dependent \( CP \) violation

Measurements of time-dependent \( CP \) violation give access to the mixing phase \( \phi_M \). The first such measurements were performed by Babar and Belle in the golden mode \( B_d^0 \rightarrow J/\psi K_S^0 \).\(^{40,41}\) Their observation of a large \( CP \) violation, in accordance with the prediction in Table \(2\), established the CKM mechanism of the SM as the dominant source of \( CP \) violation in the quark sector.

In the \( B_s^0 \) system the best accessible decay channels for this type of measurement are the decays \( B_s^0 \rightarrow J/\psi \phi(1020) \) (with \( \phi(1020) \rightarrow K^+ K^- \)) and \( B_s^0 \rightarrow J/\psi f^0(980) \) (with \( f^0 \rightarrow \pi^+ \pi^- \)). The leading order decay diagrams are shown in Fig.\(^3\). As explained in Section \(2\), the \( CP \) violating phase \( \phi_s^{\text{obs}} \) is extracted from the amplitude of an oscillation in the flavour-tagged decay time distribution. Starting from Eq.\(^{16}\) and assuming \( |\lambda_f| = 1 \) the latter takes the form

\[
\frac{dN_+}{dt} = N_f e^{-\Gamma t} \left[ \cosh \left( \frac{1}{2} \Delta \Gamma t \right) - \eta_f \cos \phi_f \sinh \left( \frac{1}{2} \Delta \Gamma t \right) \right] \tag{40}
\]

where the plus (minus) sign on the left-hand-side holds for mesons produced in a \( B_s^0 \) (\( \bar{B}_s^0 \)) flavour eigenstate, \( \phi_f \) was defined in Eq.\(^{25}\) and \( \eta_f \) is the \( CP \) eigenvalue of the final state. Final states with \( \eta_f = 1 \) are called \( CP \)-even, and those with \( \eta_f = -1 \) are called \( CP \)-odd. For \( B_s^0 \rightarrow J/\psi \phi \) and \( B_s^0 \rightarrow J/\psi f^0 \) the observable phase is usually denoted with the shorthand \( \phi_s \), to distinguish it from the theoretical, tree-level quantity \( \phi_s^{\text{theo}} = -2\beta_s \).

The \( J/\psi f^0(980) \) final state is \( CP \)-odd.\(^{42,43}\) A recent Dalitz analysis by LHCb has shown that this holds to a good extend for the entire \( J/\psi \pi^+ \pi^- \) final state.\(^{44}\) The phenomenology of the \( B_s^0 \rightarrow J/\psi \phi(1020) \) decay is more complicated. Since it concerns a decay to two vector mesons (both the \( J/\psi \) and the \( \phi \) have spin one) the final state is a superposition of states with different angular momentum quantum numbers, leading to different values of \( \eta_f \). In order to extract \( \phi_s \) these contributions need to be statistically disentangled using the observed decay angles, requiring a so-called ‘time-dependent angular analysis’ of the data.
Near the $\phi(1020)$ resonance the $B^0_s \to J/\psi \phi$ final state receives contributions from four amplitudes, namely three P-wave amplitudes that belong to the spin-one $\phi \to K^+K^-$ decays and a small S-wave component, that is partially from $f^0(980) \to K^+K^-$. The total decay time distribution can be written as the sum of 10 equations reminiscent of Eq. 40 above, one for each of the four amplitudes, and another six for their interference terms.

Although the angular analysis certainly makes the analysis of the data more complicated, it also has some notable advantages: First, because of the mixture of odd and even final states, the average observed decay time is sensitive to both $\Gamma_H$ and $\Gamma_L$ (or, equivalently, $\Gamma$ and $\Delta\Gamma$). Second, as the interference terms contains flavour-dependent $\sin \Delta m t$ terms with approximately unit amplitude, the decay is to a certain extent self-tagging: In principle one can extract the tagging dilution from the fit to the data without the use of a tagging control channel. In practice, smaller uncertainties are obtained if the tagging dilution is constrained to a control sample. Finally, thanks to these same interference terms, the value of $\Delta m_s$ can be measured. Note that this allows to extract all mixing parameters from just $B^0_s \to J/\psi \phi$ decays alone.

The most precise determination of $\phi_s$ with $B^0_s \to J/\psi \phi$ and $B^0_s \to J/\psi f^0$ events has been reported by LHCb. The two final states yield compatible values, with uncertainties of about 0.09 and 0.17 respectively. The combined result is

$$\phi_s = 0.01 \pm 0.07 \text{ (stat)} \pm 0.01 \text{ (syst) [rad]},$$

and dominates the current world average. The main contributions to the systematic uncertainty are from the decay angle acceptance for $B^0_s \to J/\psi \phi$ and from the background model in $B^0_s \to J/\psi f^0$.

The expression for the partial width Eq. 40 remains invariant under the substitution $(\Delta \Gamma_q, \phi_q) \mapsto (-\Delta \Gamma_q, \pi - \phi_q)$, leaving room for a discrete ambiguity in the result extracted from the data. The SM predicts $\Delta \Gamma_s > 0$ and $\phi_s$ close to zero. In the expressions for $B^0_s \to J/\psi \phi$ decays, involving both P-wave and S-wave amplitudes, the ambiguity persists, but also involves the (strong) phase differences between the amplitudes. In particular, the relative phase between the P-wave and S-wave amplitude changes sign. Although the relative phase cannot cleanly be predicted, the variation of the phase difference with the $K^+K^-$ invariant mass, which varies rapidly across the $\phi(1020)$ resonance, is well known, allowing the ambiguity to be resolved. Using this technique measurements by the LHCb collaboration have shown that only the solution with $\Delta \Gamma_s > 0$ and $\phi_s \approx 0$ is viable, in agreement with the prediction.

### 4.3 Lifetimes

Constraints on the average lifetime $\Gamma_s$ and the lifetime difference $\Delta \Gamma_s$ are obtained by combining information from states with different $CP$ content, c.f. Eq. 28. As indicated above the analysis of the vector-vector final state $B^0_s \to J/\psi \phi$ allows for the extraction of both $\Gamma_s$ and $\Delta \Gamma_s$. In the absence of $CP$ violation the lifetime of a $CP$-odd final decay like $B^0_s \to J/\psi f^0$ is $1/\Gamma_{H,s}$, while that of the $CP$-even final
state $B_s^0 \to K^+ K^-$ is $1/\Gamma_{L,s}$. The lifetime measured in flavour specific decays is equal to $1/\Gamma_s \times (4\Gamma_s^2 + \Delta\Gamma_s^2)/(4\Gamma_s^2 - \Delta\Gamma_s^2)$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{dll_contours}
\caption{Constraints on $\Delta\Gamma$ and $1/\Gamma_s$ shown as $\Delta \log L$ contours obtained from the analysis of $B_s^0 \to J/\psi \phi$ decays (author’s average from D0 [51], CDF [52], ATLAS [53], CMS [54] and LHCb [55]). $B_s^0 \to J/\psi f^0$ [65], $B_s^0 \to K^+ K^-$ [55] and the HFAG average for flavour specific decays [56]. The constraints from $B_s^0 \to J/\psi f^0$ and $B_s^0 \to K^+ K^-$ are obtained under the assumption that there is no CP violation in these decays. The SM prediction (see text) is shown in black.}
\end{figure}

These different experimental constraints, shown in Fig. 6, are currently in good agreement. The combined average of the $B_s^0 \to J/\psi \phi$ and $B_s^0 \to J/\psi f^0$ results is

$$\Delta\Gamma_s = 0.081 \pm 0.011 \text{[ps}^{-1}]$$
$$1/\Gamma_s = 1.519 \pm 0.010 \text{[ps]}$$

The SM prediction, using for $\Delta\Gamma$ the value in Tab. 2 and for $\tau(B_s^0)$ the predicted ratio in Eq. 33 and the world average value of $\tau(B_d^0)$, is in good agreement with the measurements. It is noteworthy that the non-zero value of $\Delta\Gamma$ leads to subtleties in the definition of branching fractions, affecting for instance the prediction of the $B_s^0 \to \mu^+ \mu^-$ branching fraction by about 10% [57].

### 4.4 Flavour-specific asymmetry

Figure 7 summarizes experimental constraints on CP violation in mixing obtained from measurements of the flavour-specific asymmetry $a_{fs}$ (defined in Eq. 20). A measurement of $a_{fs}$ requires both an understanding of the relative production rate of $B^0$ and $\bar{B}^0$ and of the relative reconstruction efficiencies for the final states $\bar{f}$ and $f$.

At the $e^+ e^-$ $B$-factories the best constraints in the $B_d^0$ system have been obtained using same-sign di-lepton events,

$$a_{fs}^d = \frac{\Gamma(\Upsilon(4S) \to \ell^+ \ell^+) - \Gamma(\Upsilon(4S) \to \ell^- \ell^-)}{\Gamma(\Upsilon(4S) \to \ell^+ \ell^+) + \Gamma(\Upsilon(4S) \to \ell^- \ell^-)}.$$
Figure 7: Constraints on on the flavour-specific asymmetry in the $B^0_d$ and $B^0_s$ mixing shown as $\Delta \log L$ contours using data from Babar\cite{58}, Belle\cite{59}, D0\cite{60,61} and LHCb\cite{62}. The SM prediction is shown in black.

The result, $a_{fs}^d = -0.0005 \pm 0.0056$, from a combination\cite{58,59} of BaBar\cite{58} and Belle\cite{59} measurements, is perfectly compatible with the expectation. A production asymmetry is not a concern at the $B$ factories, but an asymmetry in the efficiency is. This is why these measurements are systematics dominated, even though they have been performed with only a fraction of the $B$-factory data set.

As explained above high statistics measurements in the $B^0_d$ system can only be performed at high-energy colliders. Two types of probes have been used. In the same-sign di-lepton analysis $B^0_d$ and $B^0_s$ decays cannot be distinguished, such that at a high-energy collider one measures a linear combination of the $a_{fs}(B^0_d)$ and $a_{fs}(B^0_s)$, 

$$A_{fs}^d = C_d a_{fs}^d + (1 - C_d) a_{fs}^s$$  \hspace{1cm} (44) 

where the coefficient for $B^0_d$ is approximately $C_d = 0.59$ at the Tevatron\cite{25}. To reduce the uncertainty from an eventual tracking efficiency asymmetry D0 regularly reverses the magnetic field, a strategy that is also applied by LHCb. Perhaps the most tantalizing sign of physics beyond the SM in $B^0$ mixing comes from the observation of a non-zero value for this effective asymmetry by the D0 collaboration\cite{63}, shown by the diagonal band in Fig. 7.

An important concern in the same-sign di-lepton analysis is the understanding of asymmetries in the non-$B$ backgrounds, for instance in muons from kaon and pion decays in flight. The purity for the signal can be substantially be improved with a more exclusive reconstruction, that also allows for a separation of the $B^0_d$ and $B^0_s$ contribution: the asymmetry in $B^0_d \rightarrow D^+_s \mu^- \bar{\nu}_\mu X$ production has been studied by both D0\cite{60} and LHCb\cite{62}, while D0 has looked in addition at the $B^0_d \rightarrow D^+ \mu^- \bar{\nu}_\mu X$ asymmetry\cite{61}.

In these analyses the opposite side $b$ hadron is not tagged, leading to a slightly different relation to $a_{fs}$. Including the effect of an eventual production asymmetry,
defined as

\[ a_{\text{prod}} = \frac{N(B^0(t = 0)) - N(\bar{B}^0(t = 0))}{N(B^0(t = 0)) + N(\bar{B}^0(t = 0))} \tag{45} \]

the observed asymmetry is related to \( a_{fs} \) by

\[ \frac{N(D_q^- \mu^+) - N(D_q^+ \mu^-)}{N(D_q^- \mu^+) + N(D_q^+ \mu^-)} = \frac{a_{fs}}{2} + \left( \frac{a_{fs}}{2} - a_{\text{prod}} \right) \int_0^\infty dt \, e^{-\Gamma t} \cos(\Delta m t) - \int_0^\infty dt \, e^{-\Gamma t} \cosh(\frac{1}{2} \Delta \Gamma t) \tag{46} \]

At the Tevatron, a proton-anti-proton collider, the production asymmetry is zero. At the LHC it is expected to be at the percent level, but with large uncertainty. As a consequence of the rapid oscillations, the integral on the right hand side of Eq. 46 is of the order of 1 per mille for \( B_s^0 \) mesons, which strongly dilutes any contribution from the production asymmetry. This is not the case for \( B_d^0 \) mesons and explains why LHCb can measure \( a_{fs} \) but not \( a_d^{fs} \) with this method. As shown in Fig. 7 the semi-exclusive \( D_q^+ \mu^- \) analyses are competitive with the di-lepton analysis. So far they are in good agreement with the SM.

5 Concluding remarks and Outlook

I have presented a brief review of experimental constraints of mixing phenomena in \( B_0 \) decays. These phenomena are interesting because they are cleanly predicted in the SM and sensitive to physics at higher mass scales. While mixing in the \( B_0 \) system has been extensively studied at the \( e^+e^- \) B-factories, the \( B_0 \) system is the exclusive domain of high-energy colliders. The Tevatron experiments CDF and D0 have shown the potential of this research, being the first experiments to observe \( B_0^0 \) oscillations and rule out leading order new physics effects. The LHC experiments, in particular LHCb, have caught on quickly, providing even stronger constraints on the oscillation frequency, lifetimes and time-dependent \( CP \) violation.

With only a subset of the data from the first LHC run analysed, and more luminosity expected in 2015 and beyond, we can expect significantly tighter constraints in the near future. The increase in available statistics also allows the study of mixing-induced \( CP \) violation in more decay modes, such as \( B_s^0 \rightarrow \phi \phi \), \( B_s^0 \rightarrow h^+h^- \) and \( B_s^0 \rightarrow D^{(*)} \bar{D}^{(*)} \). Clearly, as statistical precision increases, controlling systematic uncertainties, both experimental and theoretical, becomes more important. Experimental uncertainties in lifetime and \( a_s^{fs} \) are expected to be dominated by detector effects rather soon. On the other hand, combinations of \( CP \) violation and branching fraction measurements for different channels, will help to reduce theoretical uncertainties due to subdominant amplitudes and non-perturbative effects.

Finally, we can expect another jump in precision near the end of the decade. The upgrade of the KEKB accelerator and Belle detector are well under way, with the start of data taking planned for 2015. The LHCb collaboration is preparing an upgrade that allows for a 10-fold increase in integrated luminosity, to be collected in a five year period starting approximately in 2016. These flavour physics facilities are both competitive with and complementary to the direct searches for new forces and particles in high energy collisions.
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