Motion planning of a space robot

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Abstract. The plane problem of motion control of a free-floating space robot is investigated. The robot is supposed to be in a state of passive flight. It is assumed that the vectors of momentum and kinetic moment of the robot equal to zero. The movement of the arm relative to the robot body changes the position of the center of masses of the body and leads to its rotation around the center of masses. There are constraints on the range of variation of the manipulator length and the angle of its rotation relative to the body. It is shown that while carrying out special movements of the manipulator, it is possible to ensure the movement of robot gripper from arbitrary initial to an arbitrary final position, if they are located inside the workspace, which is a ring with a centre in the centre of masses of the robot. Moreover, it is possible to get the desired value of the angle between the manipulator and the body in the final position.

1. Introduction
Space robots, consisting of a main body and provided with one or more manipulators, are promising for various type of works in outer space and orbiting Earth satellite to conduct of repair, maintenance and construction of various objects (space stations, orbital telescopes, etc.), and also for carrying out the work of removing space debris. Survey of work in this direction is given in [1, 2]. On Figure 1 is shows the reusable spacecraft "Buran" (USSR) with the robotic arm in the cargo bay [3]. On Figure 2 is shows astronaut in open space, which is similar to anthropomorphic space robot. On Figure 3 is shows a project of Japanese free-flying of space robot [4].
In this work we consider planar motion of free-floating space robot flight. The robot consists of a body and manipulator arm. It does not have or does not use propulsions that control movement and orientation of the robot body.
The movement of the manipulator has a significant influence on the movement of the robot body due to the theorems about the change of quantity of motion and kinetic moment about the center of masses [1, 2, 5-9].
**Figure 1.** The reusable spacecraft "Buran" with the manipulator in the cargo bay

**Figure 2.** Astronaut in open space (anthropomorphic space robot)

**Figure 3.** Project of the free-flying space robot ETS-VII (Japan)
In [1, 2, 5-7] considered the plane problem of the motion control free-floating robot consisting of the body and the manipulator arm. The manipulator arm has kinematic redundancy and consists of at least three hinged unit links. There are no constraints on the limits of variation of the deviation angles of the manipulator arm. It is assumed that the vector of quantity of motion of the robot and its kinetic moment relative the center of masses are zero.

In [1, 2] the motion control of the robot (the movement of the gripper of the robot from the given start position to the specified final position) is constructed locally from the current position determined by the small increments of the coordinates in the robot degrees of freedom with the aim to achieve the desired final position of the robot gripper. As a result, despite the efforts of the authors, failed to move a gripper in a significant portion of kinematically achievable final positions located in the robot's workspace.

In [5-7] this problem is solved by methods of the theory of motion control without considering the features of the robot dynamics associated with the existence of laws of conservation of the quantity of motion and kinetic moment. Control the movement of the gripper of the robot is constructed using feedback-based information on the position of the final goal point robot. The motion of robot body caused by the motion of the manipulator arm are treated as unknown perturbation. In the same way as in [1, 2] it is impossible to move the gripper of the manipulator from a given initial to an arbitrary final position inside its workspace.

In [8-9], it is shown that this result appears to be erroneous and caused by the methods used to control the motion of the robot based on the local principle of their formation. It takes transition to a global principle of constructing programmed motion of the robot, which would require the performance of special, not obvious movements in advance for the realization of this goal. On a simple model example, of planar motion of the robot with telescopic manipulator analytically shown that it is possible to provide the motion of the gripper of the robot from an arbitrary initial to an arbitrary kinematically achievable final position. The problem was solved under the assumption that there are constraints on the range of varying of the length of manipulator arm and there are no constraints on the range of varying of the angle of manipulator arm rotation relative to robot body. In this work, which is the evolution [8-9], this problem is solved if there are constraints as on varying the length of the manipulator arm, and so on the limits of varying of the angle of manipulator arm rotation relative to robot body. It is shown that the workspace of the robot significantly more than that of a similar robot with a fixed body. By special motions of the manipulator arm can ensure a reversal of space robot and move the gripper of the robot from an arbitrary initial to an arbitrary final position, if they are located inside the ring with the center in the center of masses of the robot is bounded by circles with radii equal to the minimum and maximum distance from the gripper of the robot to its center of masses. Furthermore, it is shown that it is possible to provide the required (the most convenient for the execution of works) a value of the angle between the robot body and manipulator arm in the final position.

2. Statement of the problem
Consider planar motion of a free-floating space robot. In Figure 4 shows a schematic diagram of the robot, which consists of the body with mass $m_1$ and the telescopic manipulator arm attached to the body at its center of mass $B$. The moment of inertia of the body about its center of mass equal $J_1$.

The manipulator arm consists of a cylinder that rotates relative to the point of his suspension to the body, and the rod is moved along the telescopic link of the arm. We denote $m_2,m_3,J_2,J_3$ respectively the mass of the cylinder and the rod of the manipulator arm and their moments of inertia about their centers of mass $C_2$ and $C_3$.

When the robot moves on it, only internal forces act in the degrees of freedom of the manipulator arm. There are a conservation laws of the robot's quantity of motion and its kinetic moment about the center of mass. Consider the problem of motion control of the robot at zero values of the vector of the of robot's quantity of motion and its kinetic moment about the center of mass. The center of mass of the
robot $C$ remains stationary and its position is taken as the beginning of inertial (orbital) coordinate system $C\eta_\zeta$.

![Diagram of the space robot](image)

**Figure 4.** The constructive scheme of the space robot

The coordinate system $B\eta\zeta$ associated with the robot body, the axis $B\zeta$ is constructive vertical and the axis $B\eta$ – the longitudinal axis of the body. The position of the robot in the absolute coordinate system $C\eta_\zeta$ is defined by the coordinates of the body center of mass, the angle of body rotation $\Theta$, the angle, the angle between the axis $B\eta$ and manipulator arm $\alpha$ and the length of the arm $l = BS$. Point $S$ – gripper of the manipulator. The angle $\varphi$ specifies the orientation of the manipulator arm in absolute space:

$$\varphi = \Theta + \alpha \quad .$$  \hspace{1cm} (1)

The center of mass of the cylinder $C_2$ lies on the axis of the telescopic link at the distance $r_2 = BC_2$ from the point of its suspension to the body. The center of mass of the rod $C_3$ lies on the axis of the telescopic link at the distance $r_3 = SC_3$ from the gripper $S$. The center of mass of the robot $C$ lies on the axis of telescopic link at the distance $\rho_B = BC$ from the point of suspension of the manipulator arm to robot body [7, 8]:

$$\rho_B = \frac{m_1r_2 + m_3(l - r_3)}{m_1 + m_2 + m_3} \quad ,$$

then length of the manipulator arm $l$ and the distance from the center of mass of the robot to the gripper $\rho = l - \rho_B$ are related by

$$\rho_B = \frac{m_1r_2 + m_3(l - r_3)}{m_1 + m_2 + m_3} \quad ,$$ \hspace{1cm} (2)

The kinetic moment conservation law about the center of mass has the form [7]
\[ k \dot{\varphi} + J_1 \dot{\Theta} = k \dot{\alpha} + (k + J_1) \dot{\Theta} = 0 \],

where

\[ k = k(l) = \mu_1 (l - \lambda)^2 + \mu_2 \], \quad \mu_1 = \frac{(m_1 + m_2)m_3}{m_1 + m_2 + m_3}, \quad \lambda = \frac{m_2 r_2}{m_1 + m_2}, \]

\[ \mu_2 = J_2 + J_3 + \frac{m_1^2 r_3^2}{m_1 + m_2 + m_3} + \frac{m_1 m_2 r_3^2}{m_1 + m_2}. \]  

Suppose that there are constraints on the limits of the length of the manipulator arm and the angle of its rotation of regarding the robot body:

\[ l \in [l_{\text{min}}, l_{\text{max}}], \quad \alpha \in [\alpha_{\text{min}}, \alpha_{\text{max}}]. \]  

The purpose of control is to move the gripper of the robot from any arbitrary start position \((\rho_0, \varphi_0)\) to arbitrary final position \((\rho_D, \varphi_D)\) inside the workspace. The workspace is limited by two circles with center in the center of mass of the robot \(C\) and the radii \(\rho_{\text{min}} = \rho(l_{\text{min}})\) and \(\rho_{\text{max}} = \rho(l_{\text{max}})\). In addition, we require that the angle between the robot body and its arm in final position was a predetermined value \(\alpha_D\) (for example, the most convenient to perform the work). From (1), (2) it follows that for this it is necessary and sufficient to translate the robot from initial state \(\Theta_0, \alpha_0, l_0\) to final state \(\Theta_D, \alpha_D, l_D\) — where \(\Theta_D = \varphi_D - \alpha_D, \quad l_0 = l(\rho_0), \quad l_D = l(\rho_D)\).

3. Planning of the programmed motion

The motion of the robot will be implemented in such a way that it was divided into time intervals (stages), during each of which operates only one actuator of manipulator arm. Either changes the length of the manipulator arm under a fixed angle \(\alpha\), in this case the angle \(\dot{\Theta}\) remains constant due to (3), or change the angle of arm rotation relative to the robot body at a fixed length of the manipulator arm, then due to (3)

\[ \dot{\Theta} = -b(l) \dot{\alpha}, \]

where

\[ b(l) = \frac{k(l)}{k(l) + J_1}. \]

Statement 1. The ratio \(b(l)\) is an increasing function of \(l\). 

By (4) \(k(l) > 0\) is a monotonically increasing function in the domain (5), as \(l_{\text{min}} > r_2 > \lambda\). Therefore, if \(l_2 > l_1\), then
$$b(l_2) - b(l_1) = \frac{J_1(k(l_2) - k(l_1))}{k(l_1) + J_1(k(l_2) + J_1)} > 0.$$ \[ (6) \]

Integrating (6) for \( l = \text{const} \) from the initial position \( \tilde{\alpha}, \tilde{\theta} \) obtained

$$\theta = f(\tilde{\theta}, l, \alpha, \tilde{\alpha}) = \tilde{\theta} - b(l)(\alpha - \tilde{\alpha}) \quad , \quad (7)$$

then a turn of the manipulator arm on the angle \( \Delta \alpha = \alpha - \tilde{\alpha} \) relative of the robot body leads to its rotation at the angle \( \Delta \theta = 0 - \tilde{\theta} \). On the plane of variables \( \alpha \) and \( \theta \) this dependence \( \theta(\alpha) \) has the form of the family of straight lines, the slope of which depends at \( l \). At the maximum length value of the manipulator arm \( l = l_{\text{max}} \), its rotation produces a maximum effect on the motion of the robot body, so the straight line (7) in this case has the greatest slope. With a minimum length value of the manipulator arm \( l = l_{\text{min}} \), its rotation has a minimal effect on the motion of the robot body, straight line (7) has the smallest slope. The rest of the lines passing through the point \( \tilde{\alpha}, \tilde{\theta} \) are implemented with intermediate values \( l \in (l_{\text{min}}, l_{\text{max}}) \).

**Figure 5.** The attainability domain of the position \( \tilde{\alpha}, \tilde{\theta} \)

Let’s call the shaded domain and together with its boundaries in Figure 5 the attainability domain from the position \( \tilde{\alpha}, \tilde{\theta} \) at the fixed \( l \in [l_{\text{min}}, l_{\text{max}}] \). Note that at each stage the robot motion its final position does not depend on the law of variation of the manipulator arm coordinates \( l \) and \( \alpha \), but determine only by the initial and final value of this coordinate.

**Statement 2.** The robot can move from start position \( \alpha_0, \theta_0, l_0 \) to terminal position \( \alpha_D, \theta_D, l_D \), if they satisfy the constraints (5) and represents the point \( \alpha_D, \theta_D \) lies in the attainability domain of point \( \alpha_0, \theta_0 \).

In this case, from (7) it follows that

$$\theta_D = \theta_0 - b(l^*)(\alpha_D - \alpha_0) \quad , \quad (8)$$
where the value $l^*$ determined from relations (4), (8)

$$l^* = \lambda + \frac{1}{\mu_1} \left( \frac{1}{\mu_2} \frac{J_1(\theta_D - \theta_0)}{\theta_D - \theta_0 + \alpha_D - \alpha_0} \right),$$

and in accordance with the statement 1 $l^* \in [l_{\min}, l_{\max}]$.

**Remark.** Here and further on instead $l_{\min}$ and $l_{\max}$ it is possible use other two values $\hat{l}_{\min}$ and $\hat{l}_{\max}$ that satisfy the conditions $l_{\min} \leq \hat{l}_{\min} < \hat{l}_{\max} \leq l_{\max}$.

**Statement 3.** The robot can move from any arbitrary start position $\alpha_0, \theta_0, l_0$ to arbitrary final position $\alpha_D, \theta_D, l_D$ if they satisfy the constraints (5).

If the final position of the imaging point $\alpha_D, \theta_D$ lies inside the attainability domain from its initial position, the programmed motion is constructed in accordance with the statement 2.

If the final point $\alpha_D, \theta_D$ lies below the attainability domain from the initial position $\alpha_0, \theta_0$, there are two ways of planning a programmed motion. In Figure 6 shows the corresponding trajectory of the imaging point on the plane $\alpha, \theta$.

When using the first method (Figure 6.a) the gripper of the manipulator arm in the relative coordinate system $B_{yz}$ connected with the robot body moves along the trajectory shown in Figure 7. The initial position of the gripper is indicated $S_0$, and the final $S_D$. If the final position of the imaging point $\alpha_D, \theta_D$ lies near the attainability domains from the initial position $\alpha_0, \theta_0$, the gripper of the manipulator arm moves along the path $S_0 S_2 S_3 S_3 S_3_4 S_4 S_D$. Otherwise, the gripper of the manipulator arm moves along a trajectory $S_0 S_1 S_2 S_3 S_3 S_3 S_3 S_3_4 S_4 S_D$, and allocated closed trajectory of the gripper $S_3 S_3 S_3 S_3 S_3 S_3$ can be repeated several times.

In this cyclic motion of the gripper of the manipulator arm in the sections $S_2 S_3$ and $S_3 S_6$ change only the length of the arm and the angle of rotation of the robot body in absolute space remains unchanged. In the section $S_3 S_5$ of the manipulator arm rotated relative to the body clockwise under $l = l_{\min}$ and in accordance with (7), the robot body is rotated counterclockwise by the angle modulo equal to

$$|\Delta \theta_1| = b(l_{\min}) (\alpha_{\max} - \alpha_{\min}),$$

In the section $S_6 S_2$ the manipulator arm rotated relative to the body counterclockwise under $l = l_{\max}$ and in accordance with (7), the robot body is rotated clockwise by the angle is modulo equal to

$$|\Delta \theta_2| = b(l_{\max}) (\alpha_{\max} - \alpha_{\min}).$$

Because of statement 1 $b(l)$ is a monotonically increasing function of the manipulator arm length $l$, and, consequently,
Figure 6. The trajectory of the representative point for the case when its final position lies below the attainability domain from the initial position.

Figure 7. The trajectory of the manipulator arm gripper in relative coordinate system connected with the robot body.
\[ |\Delta \theta_2| - |\Delta \theta_1| = (b(l_{\max}) - b(l_{\min}))(\alpha_{\max} - \alpha_{\min}) > 0 \]

As a result of each cycle of motion of the gripper of manipulator arm on closed section \( S_2S_3S_5S_6S_2 \), robot body turned clockwise at the angle \( |\Delta \theta_2| - |\Delta \theta_1| > 0 \).

If the final position of the imaging point \( \alpha_D, \theta_D \) lies above the attainability domain from the initial position \( \alpha_0, \theta_0 \), the programmed motion is planning analogous.

4. Conclusion
The planar motion of space robot in a state of passive flight under the assumption that the vector of momentum and kinetic moment of the robot equal to zero is investigated. Considered robot consist of a body and telescopic manipulator arm. It assumed that there are the constraints as to the limits of the length of the manipulator arm, and so at the its rotation angle above the body. It is shown that due to special cyclic motions of the manipulator arm relative to the body it is possible to turn the robot body to an arbitrary angle. In the result, the workspace of a free-floating space robot (the possible final position of the robot gripper) is more larger than the workspace of the robot with a fixed body. The workspace of the robot in absolute space is a ring bounded by two circles centered with the center of mass of the robot and the radii equal to the minimum and maximum distance from the center of mass of the robot to gripper. Moreover, in programmed motion planning can be provided not only the release of the gripper of the robot to a given final position, but also the required (convenient for work) the value of the angle between the robot body and manipulator arm in the end position.

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