Discrete Time Crystal Made of Topological Edge Magnons

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We report the emergence of time-crystalline behavior in the $\pi$-Berry phase protected edge states of a Heisenberg ferromagnet in the presence of an external driving field. The magnon amplification due to the external field spontaneously breaks the discrete time-translational symmetry, resulting in a discrete time crystal with a period that is twice that of the applied EM field. We discuss the nature and symmetry protection of the time crystalline edge states and their stability against various perturbations that are expected in real quantum magnets. We propose an experimental signature to unambiguously detect the time crystalline behavior and identify two recently discovered quasi-2D magnets as potential hosts. We present a first-of-its-kind realization of time crystals at topological edge states, which can be generalized and extrapolated to other bosonic quasi-particle systems that exhibit parametric pumping and topological edge states.

I. INTRODUCTION

Symmetries and symmetry breaking underlie many interesting phases and phenomena in condensed matter physics. A crystal with a periodic array of atoms/molecules is a simple example where continuous symmetry in space is spontaneously broken. Based on Lorentz invariance that puts spatial and temporal coordinates on equal footing, Wilczek in 2012 proposed the idea of a time crystal \cite{Struck2012}, where time translation symmetry can also be spontaneously broken in the ground state of a quantum many body system – local observables oscillate in time with fixed periodicity, analogous to the spatial modulation in crystalline solids. However, despite Lorentz invariance, space and time are not completely interchangeable, as evidenced by their different signs in the metric tensor. Moreover, by its very own definition, the ground state or any equilibrium state of a closed quantum system does not vary with time and Wilczek’s original idea was shown to be unfeasible \cite{2012PhysRevLett.109.180404,2012PhysRevB.86.134443,2012PhRvL.109i7203G,2014PhRvX...4d1062S,2015PhRvX...5d1045S}. Nevertheless the idea of time crystals as new phase of matter, has generated much interest over the past decade. More recent studies have established that time crystals can emerge under proper conditions. It is now widely accepted that time crystals can be realized in out-of-equilibrium systems \cite{2015PhRvL.115w6802M,2016Sci...353.7850A,2016Sci...353.7851L,2016NatPh..12..544B,2016PhRvX...6e1029B,2017NatPh..13..479B,2018NatPh..14..562B} and particularly in the presence of a periodic driving field.

Time crystals have been theoretically studied and experimentally reported in a range of systems, including magnons \cite{2012PhRvL.109.220401J,2013PhRvB..88k1123J}, ultracold atoms \cite{2014PhRvL.112n0404C,2016PhRvL.116w5301R}, superfluid quantum gas \cite{2015PhRvL.114x5303C,2016PhRvB..93d5119C} and qubits \cite{2017Sci...358.1620D,2017Sci...358.1624D,2018Sci...361.1280C}. Different time crystals can be broadly categorized in two categories, continuous time crystal \cite{2015PhysRevX...5b1084H,2016PhRvX...6b1042B,2017Sci...358.1620D,2018PhRvL.121c0501B} and discrete time crystal \cite{2018NatPh..14..562B,2018NatPh..14..659B,2019NatPh..15..649B}. Discrete time crystalline behavior in a periodically driven system is characterized by the local properties that oscillate in time with a period which is a multiple of that of the driving field \cite{2012PhRvL.109.220401J,2012PhRvX...2b1003S,2016NatPh..12..544B,2016PhRvX...6e1029B}. In many cases, the driving field injects energy into the system that eventually leads to thermalization. The periodic behaviors before thermalization are known as pre-thermal time crystals \cite{2014NatPh...10...49A,2015Sci...348.1286A,2016Sci...354.1526A,2016PhRvL.117p6804A,2017NatPh..13..357M,2017PhRvB..96h5131B}. Conversely, if the driving frequency is much larger than the local energy scales or if heat generated during thermalization can be dissipated, a driven dissipative time crystal can form because thermalization takes a long time \cite{2016Sci...353.7850A,2016PhRvX...6e1029B,2017Sci...358.1624D}. For Floquet many body localized (MBL) systems \cite{2015Sci...353.7850A,2016Sci...353.7851L}, where absence of coupling between different energy eigenstates prevent thermalization of the states, a more robust long-lived time crystal can be realized. However, MBL phase requires a strong disorder, which is practically not feasible to prepare experimentally. Thus, a research effort is still ongoing to find new more efficient and effective ways for stabilizing the manybody phase time crystal which is much more unstable than its’ counterpart space crystals.

In this work, we show that a discrete time crystal can emerge in the topological $\pi$-Berry phase protected magnon edge state of a quantum magnet driven by a periodic field in absence of any time reversal symmetry breaking interactions (see Fig. 1). The topological protec-
II. DISCRETE TIME CRYSTAL

We consider the ferromagnetic Heisenberg model on a kagome lattice,
\[ H_0 = -J \sum_{ij} \hat{S}_i \cdot \hat{S}_j. \]  

The low energy magnon excitations above the ferromagnetic ground states are described by the linear spin-wave theory: \( \hat{S}^+_i = \sqrt{2S} \hat{a}_i \), \( \hat{S}^-_i = \sqrt{2S} \hat{a}_i^\dagger \), \( \hat{S}^z_i = S - \hat{a}_i^\dagger \hat{a}_i \), where \( S \) denotes the magnitude of the spin, and \( \hat{a}_i^\dagger (\hat{a}_i) \) creates (annihilates) a magnon at site \( i \). Application of the above transformation to \( H_0 \) yields a tight binding magnon Hamiltonian where the interactions are neglected. The resulting band structure for a ribbon geometry (Fig. 2(a)) is shown in Fig. 2(b). The bulk bands carry a non-trivial quantized \( \mathbb{Z}_2 \) topological invariant (Zak-phase or \( \pi \)-Berry phase), and contain nearly flat topological edge states between the projected Dirac points \([75, 76]\). Any effective time reversal symmetry breaking terms in the Hamiltonian, such as the Dzyaloshinskii-Moriya interaction(DMI), would open up a gap in the magnon spectrum at the Dirac points \([77]\) and imparts dispersion to the edge states at \( k = \pi \), destroying discrete time crystalline behavior that is discussed later in Section III.

As bosons not subject to the Pauli exclusion principle, magnons normally populate the bottom of the band, far from the edge states. However, recent studies have shown that edge state magnons can be controllably amplified at arbitrary energies by tailored EM waves \([78]\). The EM field with amplitude \( E \) and frequency \( \Omega \) couples to the magnetic insulators via polarization \([79–81]\) as

\[ H_c = \cos(\Omega t) E \cdot \sum_{(i,j)} P_{ij} \]  

where \( P_{ij} \) is the polarization operator of magnetic insulator. The relevant terms in \( P_{ij} \) that contribute to magnon amplification consists of bilinear spin operators on the nearest neighbor bonds (see Appendix A).

\[ P_{ij} \approx p_{0,ij} (S_i \cdot Q_{ij}) (S_j \cdot Q_{ij}) . \]  

Other polarization terms are not important for this study as they will be neglected in the rotating wave approximation \([78]\). Furthermore, we have neglected the effect of the magnetic field component of electromagnetic wave for the following reasons. The magnetic field couples with the system in a form of Zeeman coupling which carries a term proportional to \( \cos(\Omega t) \) and only able to couple with the magnons of energy \( \Omega \). Thus the edge magnons remain unaffected which has a frequency of \( \Omega/2 \) (see later in this section). Bulk magnons are also less affected by magnetic field, as discussed in Appendix F.

The equation of motion for the magnetic field \( \delta \alpha_k \) is given...
by (see Supplementary Section B for more details),
\[
\frac{d}{dt} \langle \hat{a}_{n,k} \rangle = i \left( \hat{\epsilon}_k - \frac{[\hat{H}]_{12}^2}{2} - \frac{[\hat{H}]_{21}^2}{2} - \hat{\epsilon}_k \text{ and } -\hat{\epsilon}_k \right) \left( \langle \hat{\alpha}_{n,k} \rangle \right),
\]
(4)

where \( \langle \hat{a}_{n,k} \rangle \) represent the magnon fields \( \langle \hat{a}_{n,k} \rangle \); \( \hat{\epsilon}_k \) is a diagonal matrix with elements \( \epsilon_{n,k} \), is the energy eigenvalue; \( \gamma \) and \( \eta \) are phenomenological linear and non-linear damping constants; \( I \) is the identity matrix and \( |\alpha_k|^2 \) is the diagonal matrix with entries
\[
|\hat{\alpha}_{n,k}|^2.\]
The subscripts \( n \) and \( k \) are band-index and reciprocal space point respectively. Moreover, \( \left[ \hat{H}_r \right]_{12} \) is the off-diagonal elements of the coupling matrix in eigenbasis (see Appendix. B). The square matrix on the right hand side of Eq. 4 is the dynamical matrix with complex eigenvalues (for \( \eta = 0 \)). The real and imaginary parts of the eigenvalues represent the energy and lifetime of the magnon respectively. In absence of EM coupling \( \left[ \hat{H}_r \right]_{12} \approx O_{N \times N} \), the imaginary part of eigenvalues is negative indicating magnon decay. However as the amplitude of EM field increases the imaginary part of some of the eigenvalues satisfying \( \epsilon_{n,k} - 2\gamma |\alpha_k|^2 \approx \Omega \) become positive. This indicates the onset of spontaneous amplification of magnons. The yellow dots in the band structure (Fig. 2(b)) are the eigenvalues with positive imaginary part.

The solution of Eq. 4 describes amplified coherent magnons above a cutoff amplitude of EM field \([15, 16, 82, 83]\). Fig. 2(c) shows the amplified coherent magnons population for the edge states of upper and lower edges at \( k = \pi \) as a function of time. The presence of the non-linear damping suppresses the exponential increase of the magnon number and the system reaches a steady state.

While the number of magnons \( \langle |\hat{a}_{n,k}|^2 \rangle \) are identical in the rotating and the lab frames in the steady state (see Fig. 2), the field \( \langle \hat{a}_{n,k} \rangle \) oscillates in time. Specifically when the pair of amplified magnons satisfy \( \epsilon_{n,k} = -\epsilon_{n,-k} = \Omega/2 \), the steady state expectation values for the field in the rotating frame is independent of time i.e. \( \langle \hat{a}_{n,k}(t) \rangle^s_{rot} = \langle \hat{a}_{n,k} \rangle^s_{rot} \) (see Appendix. D). Thus the fields in the two frames are related as,
\[
\langle \hat{a}_{n,k}(t) \rangle^s_{lab} \approx \langle \hat{a}_{n,k} \rangle^s_{rot} \exp(i \frac{\Omega}{2} t),
\]
(5)

where the superscript “s” denotes steady state expectation value.

The equation of motion Eq. 4 has a \( \mathbb{Z}_2 \) symmetry \( \hat{a}_{n,k} \rightarrow -\hat{a}_{n,k} \). Above a critical amplitude of the EM field, the amplified magnon field at the edges the system spontaneously breaks the \( \mathbb{Z}_2 \) symmetry by acquiring a finite, non-zero \( \langle \hat{a}_{n,k} \rangle^s_{lab} \) that oscillates in time with a period which is twice that of the driving EM field. Thus a discrete time crystal of edge state magnons is formed via amplification \([15, 16]\) that breaks the discrete time translational symmetry spontaneously.

This time crystalline behavior can be experimentally observed by measuring the transverse magnetization at the edges, i.e., the spin components \( S_{i}^x \) and \( S_{i}^y \) – the spin component \( S_{i}^z \) is constant, because it is related to the number of magnons \( \langle |\hat{a}_{n,k}|^2 \rangle \), which is invariant in time in the steady state. The x-component of the spin, \( \langle \hat{S}_{i}^x \rangle \), is given in terms of the fields \( \langle \hat{a}_{n,k} \rangle^s_{rot} \) as
\[
\langle \hat{S}_{i}^x \rangle = \sqrt{S} \sum_{k > 0, n} \left( U_1^T \right)_{in} \langle \hat{a}_{n,k} \rangle^s_{rot} e^{i k x} e^{-ikx_i} + \sum_{k > 0, n} \left( U_2^T \right)_{in} \langle \hat{a}_{n,-k} \rangle^s_{rot} e^{i k x} e^{ikx_i} + \left( U_1^T \right)_{in} \langle \hat{a}_{n,0} \rangle^s_{rot} e^{i k x} + \text{H.C.}
\]
(6)

Fig. 2(d) demonstrate the oscillation of \( S_{i}^x \) at a site at the upper edge (blue) and the lower edge (red). The different \( k \)-points arrive at steady state at a different time (see Appendix. C) and so \( S_{i}^x \) in Fig. 2(d) modulates transiently and reaches a steady state when all the amplified \( k \)-points do. The oscillation amplitudes of \( \langle \hat{S}_{i}^x \rangle \) at both edges are nearly identical, but they are not exactly equal, since it is a superposition of several fields \( \langle \hat{a}_{n,k} \rangle^s_{rot} \) (see Eq. 6); while the magnitude of the fields \( \langle \hat{a}_{n,k} \rangle^s_{rot} \) at two edges are the same, the phases are not. Moreover, the amplitude of oscillation varies with different simulations due to the random starting conditions representing the vacuum fluctuations \([82, 83]\). Finally, time crystalline behavior also holds for the long-range order in spatial directions due to the coherence of the pumped magnon at \( k = \pi \). Since the amplification of magnons extends over a finite momentum range around \( k = \pi \), a spatial modulation in the amplitude of oscillation is expected.

III. SYMMETRY PROTECTION

Although Heisenberg ferromagnet breaks time-reversal symmetry \( \tau \), the system still preserves effective time reversal symmetry \( \tau_{\text{eff}} = R_{\pi}(n)\tau \), where \( R_{\pi}(n) \) is \( \pi \)-
we get dispersive edge magnon state. Thus the amplification is not only limited to the $k = \pi$ but other momentum points such that the relationship $\omega_k + \omega_{-k} = \Omega$ is satisfied, which is shown in Fig. 3(a). Moreover the spin oscillation at the edges become chaotic in nature as shown in Fig. 3(b). The reason behind this chaotic oscillation can be understood from the following simple model with only two coupled modes with energies $\omega_k$ and $\omega_{-k}$,

$$\mathcal{H} = \omega_k a_k^\dagger a_k + \omega_{-k} a_{-k}^\dagger a_{-k} + 2\epsilon \cos(\Omega t) \left( a_k^\dagger a_{-k} + a_k a_{-k}^\dagger \right).$$

(8)

It can be shown that the fields in lab frame have the following time dependence (see Appendix. D),

$$a_{\pm,k}^{\text{lab}}(t) = A_{\pm}(t) e^{\mp (\omega_k - \omega_{-k}) t} e^{-i\frac{\Omega}{2} t},$$

(9)

where $A_{\pm}(t)$ is amplitude modulation due to amplification which becomes time independent in the steady state. From this equation, we can conclude the oscillation frequency due to parametric amplification is half of the driving field only when the coupled state which is amplified have equal energies $\omega_k = \omega_{-k}$, otherwise the frequency of oscillation is different. Moreover, when effects of oscillation from many coupled oscillators with $\omega_k \neq \omega_{-k}$ are superimposed as in equation Eq. 6, then the resultant oscillation will be chaotic as in Fig. 3(b).

IV. STABILITY OF THE TIME CRYSTAL

Discrete time crystals of the amplified edge magnons are stable unless bulk eigenmodes with a significant overlap with the edge modes is amplified. The choice of the Kagome ferromagnet is preferable because the energy spectrum around edge state is asymmetrical (see Fig. 2(b)), resulting in suppression of bulk magnon amplification (see Appendix. E, where a comparison with honeycomb ferromagnet is provided). However, edge magnon scattering can excite other bulk magnon eigenmodes and contribute to the oscillation of spin according to Eq. 6, which may become the reason of instability of time crystal.

The Hamiltonian $\mathcal{H}_0$ does not contain any magnon non-conserving terms, but such terms may arise in the presence of spin anisotropy in many real quantum magnets. We have calculated the bulk band structure and momentum resolved two-magnon density of states, 

$$D_k(\omega) = \frac{1}{N} \sum_{n,n' = \Delta} \delta(\omega - \omega_{n,\Delta} - \omega_{n',k-\Delta})$$

(10)

for a system with toroidal boundary condition (Fig. 4(a), (b)). When the energy of an eigenstate matches that of the two magnon continuum, it can decay into two magnons with lower energies. The two magnon continuum energy scales as twice the negative of Zeeman term $(2B_z S)$ of a longitudinal magnetic field $B_z$, while that
of the magnon bands scale as $B_z S$. Hence for magnetic fields $B_z > E_{\text{edge}}$, the edge states are energetically separated from the 2-magnon continuum and cannot decay via this channel. At even higher fields, $B_z > 6J$ the two-magnon continuum gets separated in energy from the magnon band structure, implying that two edge magnons cannot combine and produce a higher energy magnon. As the magnetic field increases, the higher order magnon non-conserving processes will disappear faster than the two magnon decay processes. Thus an external magnetic field can suppress the magnon non-conserving scattering in the system.

Magnon conserving scattering processes cannot be eliminated by an external field, and are always present in any spin Hamiltonian. We calculated the scattering rate of magnons due to two-magnon scattering (quartic terms of magnon Hamiltonian) using Fermi golden rule,

$$s_2 = \frac{2\pi}{\hbar} \sum f \sum i \left| \left\langle f \left| H^{(4)}_{0,\text{int}} \right| i \right\rangle \right|^2 \delta(E_f - E_i),$$  

where $|f\rangle$ and $E_f$ ($|i\rangle$ and $E_i$) denote the state and energy of final (initial) state respectively. $H^{(4)}_{0,\text{int}}$ is the quartic term of the magnon Hamiltonian which is obtained by Taylor series expansion of Holstein-Primakoff transformation,

$$H^{(4)}_{0,\text{int}} = -\frac{J\hbar^2}{4} \sum_{ij} \left\{ 4\hat{a}^\dag_i \hat{a}^\dag_j \hat{a}_i \hat{a}_j + \hat{a}^\dag_j \hat{a}^\dag_i \hat{a}_j \hat{a}_i + \hat{a}^\dag_j \hat{a}^\dag_i \hat{a}_j \hat{a}_i + \hat{a}^\dag_i \hat{a}^\dag_j \hat{a}_i \hat{a}_j \right\},$$  

where higher order terms of the expansion are neglected which have amplitudes of order $O\left( \frac{1}{\delta k} \right)$, $n \in \mathbb{N} \geq 1$. For simplicity, we have restricted our calculation to the upper-edge states and considered only the scattering from the points $k = \pm \pi$ and $k = \pm \pi \pm \delta k$ (where $\delta k = 0.0628$). To numerically calculate the scattering rate $s_2$, we have considered a finite width of energy levels $\delta E$, which physically implies band broadening. The scattering rate as a function of band broadening is plotted in a logarithmic scale in Fig. 4(c) for different system sizes.

It is observed that the scattering rate decreases rapidly as the band broadening decreases. The band broadening results from magnon interactions and the only way to control the bandwidth is by controlling the density of the amplified magnons. This is achieved by working at low temperature and low EM field intensity. Even if the scattering rate is high, as long as the life-time of magnons at the scattered state is small the behavior of the system is governed by only the selectively amplified edge state magnons [12]. For example, the band broadening of an ideal magnon band structure $\delta E \leq 10^{-2}JS$, corresponding to scattering rate $s_2 \leq 10^{-4}ps^{-1}a^{-1} = 10ns^{-1}a^{-1}$, can be interpreted as maximum one edge magnon is scattered to the bulk magnon every $10ns$, so secondary amplification of bulk magnon is not possible since their lifetimes are usually less than $1ns$. Even in the worst case scenario of large band broadening, a small density of amplified edge state magnons is preferable to create a stable time crystal. These conditions also help minimise magnon decay due to magnon-phonon scattering. Finally, edge imperfections cause broadening of the edge states, and reduce the yield of coherent magnons [84, 85].

The reduced scattering is a consequence of the topological protection of the edge state. The presence of chiral symmetry induces a flat edge mode at zero energy due to finite $\pi$-Berry phase. Absence of chiral symmetry can result in a dispersive edge state at non-zero energy in the presence of a quantized non-zero $\pi$-Berry phase [75, 76]. Interestingly, the $\pi$-Berry phase protected topological edge states are robust against impurities with or without chiral symmetry [76], providing additional stability to the time crystalline behavior.

### V. EXPERIMENTAL REALISATION

The edge-magnon time-crystals can be observed using direct spatial and temporal imaging of spin-wave dynamics via multiple recently developed techniques, such as, Kerr microscopy [86, 87], Brillouin light scattering spectroscopy (BLS) [88–90] and time resolved scanning.
transmission x-ray microscopy (TR-STXM) [13, 91–101]. BLS is useful to detect magnons at a fixed frequency and wave vector [102–107] and has recently been used to detect the space-time crystal in the ferromagnetic insulator YIG [12]. Additionally, theoretically proposed spin Hall noise spectroscopy is a promising technique to detect the space-time crystal in the ferromagnetic insulator YIG [12]. The TR-STXM, in particular, is promising for directly imaging spin dynamics at the edge due to its high accuracy in detecting magnon dynamics with a spatial and temporal resolution of 20 nm and 50 ps respectively [13, 91–93]. Recently, this method has been used to observe the dynamics of space-time crystal of bulk magnons in permalloy strips [13].

The required estimated electric field amplitude to amplify topological edge magnons is in between $10^6$-10$^{12}$ V/m, depending on damping of edge magnons – weaker damping requires lower intensity, (see Appendix. F).

We propose the spin-$\frac{1}{2}$ kagome ferromagnets haydeeite [110] and Cu(1,3–bdc) [111] as possible hosts of the discrete time crystals of edge magnons as discussed in this work. While the Haydeeite has experimental evidence for the absence of DMI [110], the Cu(1,3-bdc) contains out-of-plane DMI that does not break any effective time reversal symmetry [77] for the ferromagnetic ground state with in-plane magnetization [112]. The period of oscillations for the materials haydeeite and Cu(1,3–bdc) are calculated to be 0.05ps and 0.25ps respectively, which are estimated using experimentally determined Heisenberg exchange interactions [110, 111]. Thus these quantum magnets are perfect candidates for realizing discrete time crystal of edge magnons.

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Appendix A: Polarization Operator

The form of the polarization operator depends on the lattice symmetries and independent of the magnetic ground state. However, for the estimation of the coefficient of the polarization operator, one requires a more fundamental electronic model. There are various possible electronic model for the spin exchange interactions in different materials. One of the simplest electronic model is the Hubbard model. Despite of its limitations, the simplistic Hubbard model is used to give an estimation of the coefficients of polarization operator. The Hubbard model is given by,

$$H_{\text{Hubbard}} = - \sum_{ij} \left[ (\hat{c}^\dagger_{i\uparrow} \hat{c}^\dagger_{j\downarrow} (t \cos(\theta) + it \mathbf{n} \cdot \mathbf{\sigma} \sin(\theta)) (\hat{c}_{j\uparrow} \hat{c}_{i\downarrow}) + \text{H.c.} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} \right]$$  \hspace{1cm} (A1)

where $t \cos(\theta)$ and $it \mathbf{n} \sin(\theta)$ ($\mathbf{n}$ is an unit vector) are the real and complex hopping amplitude of electrons on nearest neighbour bonds respectively. $U$ is the onsite Coulomb repulsion. The polarization operator corresponding to the Hubbard model is [78],

$$P_{ij} \approx p_{0,ij} (S_i \cdot Q_{ij}) (S_j \cdot Q_{ij}),$$  \hspace{1cm} (A2)

where the $p_{0,ij}$ and $Q_{ij}$ are given by,

$$p_{0,ij} = -16t^3 \frac{a}{(e \mathbf{\sigma})^3} (e_{jk} - e_{ki}) = p_0(e_{jk} - e_{ki}),$$

$$Q_{ij} = \mathbf{n} - n^2 \hat{z}$$  \hspace{1cm} (A3)

where $e$ and $a$ are the electron charge and lattice constant respectively. $e_{jk}$ is a vector on nearest neighbour bonds from site-$j$ to site-$k$. The sites $i$, $j$ and $k$ are the sites on the same triangle of the kagome lattice. The polarization terms other than the terms in Eq. A2 are not important for this study because in the diagonal basis in the rotating frame those terms are time dependent and so those terms are dropped in rotating wave approximation in Eq. 4. Thus the polarization operator is proportional to $\frac{t^3}{U \tau}$. 
Appendix B: Derivation of equation of motion

The Hamiltonian describing a kagome ferromagnet on a cylindrical geometry, coupled to an external EM field is given by

\[
\mathcal{H} = \frac{1}{2} \sum_k \left( \Psi_k^\dagger \mathcal{H}_k \Psi_k \right) + \frac{1}{2} \cos(\Omega t) \sum_k \left( \Psi_k^\dagger \mathcal{H}_1 \Psi_k \right),
\]

where \(\epsilon_k\) and \(\epsilon_{-k}\) are the diagonal matrices of eigenvalues of matrices \(H_0(k)\) and \(H_0(-k)^T\) respectively. The matrix \(H_{12} = U_1 H_{12} U_2^\dagger\) is the coupling matrix in the diagonal basis. Only the off-diagonal terms \([H]_{12}\) and \([H]_{21}\) of the coupling Hamiltonian appear in \(\mathcal{H}_{\text{eff}}\) due to the rotating wave approximation. The equation of motion of field \(\hat{\Psi}_k = \left\langle \hat{a}_{1,k}, \hat{a}_{2,k}, ..., \hat{a}_{N,k} \right\rangle^T\) is given by,

\[
\frac{d}{dt} \left( \hat{\Psi}_k \right) = i \left( \hat{\epsilon}_k - \frac{\Omega}{2} \right) \left( \hat{\Psi}_k \right),
\]

where \(\hat{\epsilon}_k\) is a diagonal matrix with elements \(\epsilon_{n,k} - \frac{\Omega}{2}\), where \(\epsilon_{n,k}\) is the energy eigenvalue; \(\gamma\) and \(\eta\) are phenomenological linear and non-linear damping constants; \(I\) is identity matrix and \(|\alpha_k|^2\) is diagonal matrix with entries \(\left| \left\langle \hat{a}_{n,k} \right| \right|^2\).

Appendix C: Equilibration of spin oscillation

In this section, we show that the oscillation at different \(k\)-points equilibrate at different times (see Fig. 5(a),(b)) resulting in a transient modulation of amplitude before reaching the steady state as in Fig. 2(d) in main text. The figure also shows that the amplitude of oscillation decreases rapidly away from \(k = \pi\).

Appendix D: A toy-model for analytical calculation of dynamics

In the main text we showed that the time-crystal is not stable in absence of effective time reversal symmetry. The reason behind this chaotic oscillation can be understood from the following simple model with only two coupled modes with energies \(\omega_k\) and \(\omega_{-k}\),

\[
\mathcal{H} = \omega_k \hat{a}_k^\dagger \hat{a}_k + \omega_{-k} \hat{a}_{-k}^\dagger \hat{a}_{-k} + 2\epsilon \cos(\Omega t) \left( \hat{a}_k^\dagger \hat{a}_{-k}^\dagger + \hat{a}_k \hat{a}_{-k} \right)
\]
Next we rotate the basis states with the unitary operator $U = e^{i\Omega(n_{k} + n_{-k})t}$, where $n_{k} = \hat{a}_{k}^\dagger \hat{a}_{k}$. The Hamiltonian in rotating frame is,

$$
H' = U(t)HU^\dagger(t) + i\hbar U(t)U^\dagger(t)
$$

$$
= \hat{\omega}_{k}\hat{a}_{k}^\dagger \hat{a}_{k} + \hat{\omega}_{-k}\hat{a}_{-k}^\dagger \hat{a}_{-k} + \epsilon \left( e^{2i\Omega t} + 1 \right) \hat{a}_{k}^\dagger \hat{a}_{-k} + \epsilon \left( 1 + e^{-2i\Omega t} \right) \hat{a}_{k}\hat{a}_{-k},
$$

where $\omega_{k} = \omega_{-k} = \Omega/2$. Neglecting fast rotations (terms with $e^{\pm 2i\Omega t}$), the effective Hamiltonian in rotating wave approximation becomes,

$$
H_{\text{eff}} = \hat{\omega}_{k}\hat{a}_{k}^\dagger \hat{a}_{k} + \hat{\omega}_{-k}\hat{a}_{-k}^\dagger \hat{a}_{-k} + \epsilon \hat{a}_{k}^\dagger \hat{a}_{-k} + \epsilon \hat{a}_{k}\hat{a}_{-k}
$$

By defining the field $\alpha_{k} = \langle \hat{a}_{k} \rangle$ and using the equation of motion $i\frac{d}{dt} \langle \hat{O} \rangle = \langle [\hat{O}, H_{\text{eff}}] \rangle$, we get the following coupled differential equation,

$$
i\frac{d}{dt} \left( \begin{array}{c} \alpha_{k} \\ \alpha_{-k}^\dagger \end{array} \right) = \left( \begin{array}{cc} \hat{\omega}_{k} - i\frac{\gamma}{2} & \epsilon \\ -\epsilon & -\hat{\omega}_{-k} - i\frac{\gamma}{2} \end{array} \right) \left( \begin{array}{c} \alpha_{k} \\ \alpha_{-k}^\dagger \end{array} \right),
$$

where the phenomenological damping $\gamma$ is added. The coupled differential equation can be transformed into the following second order differential equation,

$$
\frac{d^{2}\alpha_{k}}{dt^{2}} - (\hat{\omega}_{k} - \hat{\omega}_{-k} - i\gamma) \frac{d\alpha_{k}}{dt} + \left[ \epsilon^{2} - \left( \hat{\omega}_{k} - i\frac{\gamma}{2} \right) \left( \hat{\omega}_{-k} + i\frac{\gamma}{2} \right) \right] \alpha_{k} = 0
$$

Now using the ansatz $\alpha_{k} = A_{1} e^{-i\omega_{k}t}$ we get an quadratic equation in $\omega$ and the solution of which is given by,

$$
\omega = \frac{\omega_{k} - \omega_{-k}}{2} - i\frac{\gamma}{2} \pm i\sqrt{\epsilon^{2} - \left( \hat{\omega}_{k} + \hat{\omega}_{-k} \right)^{2}/4}
$$

The solution with positive sign is the only physical solution, because in the limit $\epsilon \to 0$ the frequency should be $\omega \to \omega_{k}$. Thus the solution for the field,

$$
\alpha_{k} = A_{1} e^{-i\frac{(\omega_{k} - \omega_{-k})t}{2}} e^{-\frac{\gamma}{2} t} e^{i\sqrt{\epsilon^{2} - \left( \hat{\omega}_{k} + \hat{\omega}_{-k} \right)^{2}/4}},
$$

where $A_{1}$ is a constant.
FIG. 6. The magnon band structure is shown in blue, whereas the red dots denote eigenstate with eigenvalues with positive imaginary part for electric fields (a) $E_0^y = 0.015$ and (b) $E_0^x = 0.015$. Maximum of absolute values of matrix elements $\tilde{H}_{12}$ is plotted as a function of momentum for electric field amplitudes (c) $E_0^y = 0.001$ and (d) $E_0^x = 0.002$. The other parameters used for all the plots are $J = 1.0, S = 1, B_z \to 0^+, \gamma = 2.5 \times 10^{-3}, \Omega = 3JS, K_{x,xy} = K_{y,xx} = K_{y,yy} = 1.0$.

Thus the condition for amplification is given by,

$$\sqrt{\epsilon - \frac{(\tilde{\omega}_k + \tilde{\omega}_{-k})}{4}} > \frac{\gamma}{2}$$

(D8)

From now on, we focus on the oscillatory part of the solution, by taking the exponential decay or amplification into the amplitude,

$$\alpha_k = A_{+}(t)e^{-i\frac{(\omega_k - \omega_{-k})t}{2}}$$

(D9)

From the Eq.D4, we get, $\alpha_{-k} = A_{-}(t)e^{-i\frac{(\omega_k - \omega_{-k})t}{2}}$, and thus we have,

$$\alpha_{-k} = A_{-}(t)e^{i\frac{(\omega_k - \omega_{-k})t}{2}}$$

(D10)

It can be shown that the relation between the fields in lab and rotating frame is given by $\alpha_{\pm k}^{\text{lab}} = e^{-i\frac{\Omega t}{2}}\alpha_{\pm k}$, thus the fields in lab frame is given by,

$$\alpha_{\pm k}^{\text{lab}} = A_{\pm}(t)e^{i\frac{(\omega_k - \omega_{-k})t}{2}}e^{-i\frac{\Omega t}{2}}.$$  

(D11)

From this equation, we can conclude the oscillation frequency due to parametric amplification is half of the driving field only when the coupled state which is amplified have equal energies $\omega_k = \omega_{-k}$, otherwise the frequency of oscillation is different. Moreover, when effects of oscillation from many coupled oscillators with $\omega_k \neq \omega_{-k}$ are superimposed as in equation Eq. 6 in main text, then the resultant oscillation will be chaotic, which is visible in Fig. 3(b).

**Appendix E: Comparision between Kagome lattice and Honeycomb lattice**

In this section we show that Kagome lattice structure is a suitable lattice structure for achieving magnon time crystal, by comparing the results with honeycomb lattice. We have studied a similar model in honeycomb lattice to
FIG. 7. The magnon band structure is shown in black; the yellow and blue dots denote eigenstates with eigenvalues with positive imaginary part for electric fields (a) $E_0^y = 0.015$ and (b) $E_0^x = 0.015$. Maximum of absolute values of matrix elements $\left| \tilde{H}_{12} \right|$ is plotted as a function of momentum for electric field amplitudes (c) $E_0^y = 0.015$ and (d) $E_0^x = 0.015$. The other parameters used for all the plots are $J = 1.0$, $D = 0.00$, $B_z \to 0^+$, $\gamma = 5 \times 10^{-4}$, $\Omega = 5.1716$, $p_0 = 1.0$.

show the advantage of the Kagome lattice structure. The model Hamiltonian we take on Honeycomb lattice is given as,

$$
\mathcal{H} = -J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j + E^x(t) \sum_i K^{xy}_i \left( \hat{S}^x_{j,A} \hat{S}^y_{j,B} - \hat{S}^x_{j,B} \hat{S}^y_{j,A} \right) + E^y(t) \sum_i \left[ K^{yx}_j \left( \hat{S}^x_{j,A} \hat{S}^y_{j,B} + \hat{S}^x_{j,B} \hat{S}^y_{j,A} \right) + K^{yy}_j \left( \hat{S}^y_{j,A} \hat{S}^y_{j,B} + \hat{S}^y_{j,B} \hat{S}^y_{j,A} \right) \right],
$$

where the polarization terms are considered respects the symmetry of the lattice and the terms which do not contribute to the amplification is already discarded (mathematically those terms will be discarded in rotating wave approximation). By diagonalizing the dynamical matrix as discussed in the main text we achieved the band structure as shown in the Fig. 6(a) and (b) for different polarization. It can be noticed that the electromagnetic field amplifies the bulk magnon bands instead of edge magnon bands, which is backed up by the results in Fig. 6(c) and (d) showing maximum coupling with electromagnetic field occurs with the bulk magnon states.

Whereas, the Kagome lattice structure is useful for amplification of edge magnons without amplifying the bulk magnons. The reason behind not amplifying the bulk magnons is for making the system more stable. Fig. 7(a) and (b) shows that the magnon amplification in the Kagome lattice is confined to the edge states only. Moreover, choice of polarization of electromagnetic field should be in y-direction to specifically amplify the edge magnons. The Fig. 7(a) and (b) shows that the imaginary parts of bulk magnon modes zero and non-zero for y-polarized and x-polarized electromagnetic field. The reason of this discrepancy is due to the difference in coupling terms as shown in Fig. 7(c) and (d). The y-polarized EM-field strongly couples with edge magnons, whereas the x-polarized EM-field strongly couples with bulk magnons.

**Appendix F: Required intensity of light and the effect of magnetic field of electromagnetic wave**

In this section, we discuss about the effect of magnetic field and intensity of light that is required for amplification procedure. The intensity of light that is required to get amplification is unknown and depends on damping of magnon states. The lifetime of magnons can be $1\mu s$, $1\text{ ns}$, $1\text{ ps}$ etc. Depending on that the energy-state broadening according to uncertainty principle would be $10^{-9} \text{eV}$, $10^{-8} \text{eV}$, $10^{-3} \text{eV}$ respectively. Thus, the damping parameters should be $\lambda = 10^{-9} \text{eV}$, $10^{-8} \text{eV}$, $10^{-3} \text{eV}$ respectively. Now to have amplification, one requires to have electric field amplitude $E_c$ (V/µm) which should follow the inequality relationship for a system with damping $\lambda$, (considering the following
ideal system parameters, lattice constant \(a=1\text{nm}, t/U = 10^{-2}\)

\[
E_cP \geq \lambda[eV] = e(t/U)^3 \geq \lambda[eV] \left[ \text{where, } P \approx a \frac{t^3}{U^3} \right]
\]

\[
E_c \times 10^{-6} \geq \lambda V/\text{nm}
\]

\[
E_c \geq 10^6\lambda V/\text{nm}
\]

\[
E_c \geq 10^{15}\lambda V/\text{m}
\]

Thus the electric field required for the amplification should be \(10^6V/m, 10^9V/m, 10^{12}V/m\) respectively. Based on the relation \(B=E/c\), the magnetic field amplitude should be, \(B = 0.01\text{T}, 1\text{T}, 1000\text{T}\) respectively. The magnetic field 0.01T and 1T are still very negligible for a magnetic insulator, because the Heisenberg exchange interaction is 1meV whereas the energy equivalent to 1T is \(\mu_B B = 0.01\text{meV}\).

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