Model of heat and mass transfer in heated premises

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Abstract. The heat loss model identifying inverse problem for a building through building envelopes is considered. The heat balance linear equations are used for heated rooms. These equations take into account the thermal conductivity through the walls and the glazing of the openings, two-way convective heat exchange between the room air, heating devices and enclosing structures, as well as convective heat dissipation into the environment.

1. Introduction
Maintaining the temperature in heated buildings requires operational status of heating systems. The temperature of the heating convectors and the condition of the external walls have serious influence on the temperature regime. The factors of humidity and wind speed are taken into account since they also affect the heat loss [1].

Analysis of heat losses is important when putting a building into operation and developing a thermal passport, as well as during its long-term operation. In the first case, the coefficients of thermal resistance of enclosing structures are measured at actual thermal characteristics of the materials at the building breaking-in. In the second case, the degradation of thermal insulation properties over time is taken into account. The temperature in the building is estimated with varying accuracy according to the intended purpose. To obtain predicted values, the heat balance is estimated either for the building as a whole or for individual premises [2-6].

A significant number of publications deal with more detailed models of heating systems. There are works studying the heat loss through enclosing structures [7], and those in detail analysing the supply of thermal energy to the premises [8].

In this regard, the task of modeling heat exchange processes in the room is relevant.

2. Problem Statement
The heat exchange processes in a heated building are presented in a diagram of figure 1. Let us consider these processes in more detail.
Figure 1. Scheme of heat exchange processes.

The temperature in the room is evenly distributed; so the heat flux density from the convector surface into the room is determined by the physical law of convection:

\[ q = h(T_0 - T_1), \]  

(1)

- \( h \) – free convection coefficient to air, W/(m\(^2\)K);
- \( T_0 \) – average integral temperature of the convector;
- \( T_1 \) – air temperature in the room.

The free convection coefficient is constant, so the power delivered by the convector is found by integrating over the area of the heating convector \( S_e \):

\[ N = h \int_{S_e} (T_0 - T_1) dS_e = h \cdot S_e (T_0 - T_1), \]  

(2)

where the average integral temperature of the convector is:

\[ T_0 = \frac{1}{S_e} \int_{S_e} T dS_e. \]

Heat flux is transmitted from the room to the inner wall of the building, which can be defined similar to (2):

\[ N = h \cdot S_{\text{wall}} \cdot (T_1 - T_2), \]  

(3)

where \( S_{\text{wall}} \) is the area of the room wall through which the heat transfer occurs.

The process of heat conduction through the wall of a building (III in figure 1) is determined by the Fourier law. The density of the heat flux through the wall is equal to:

\[ q = a \frac{\partial T}{\partial x}, \]  

(4)

where \( a \) is the coefficient of thermal conductivity, W/(m⋅K).

Let us find the power transmitted through the wall of thickness \( H \), integrating over the area of the wall \( S \):

\[ N = a \int_{S_{\text{wall}}} \frac{\partial T}{\partial x} dS_{\text{wall}} = a \int_{S_{\text{wall}}} \frac{T_2 - T_3}{H} dS_{\text{wall}} = a \cdot S_{\text{wall}} \frac{T_2 - T_3}{H}. \]  

(5)

where \( S_{\text{wall}} \) is the wall area of the building.

The final stage (IV in figure 1) will be the process of heat transfer from the external wall of the building to the environment. The amount of heat released outside will be equal to
The system of equations for heat balance can be represented as a set of systems for each riser separately: the general system of equations is split into systems of heat balance equations for individual risers with coefficient matrices and right part. The required temperatures are divided into six groups: room temperature $T_I$, temperature of the nodes on the inner surface of the walls $T_{II}$, temperature of the outer surface of the walls $T_{III}$ and temperature of the internal nodes of the enclosing structures $T_{IV}$, temperature of heating radiators $T_V$, and temperature of the outside air $T_{VI}$. For convenience let us add the seventh group of variables $T_{VII}$, heat-transfer temperature at the entrance to the radiators and at the exit of the last radiator of the riser, and the eighth group of variables, the temperature at the entrance to each of the risers $T_{VIII}$.

3. Complete model of heat and mass transfer in heated rooms

Let us integrate the models of heat balance of heating convectors and enclosing structures into a common model describing the heat balance of a heated building together with the heating system. We write the system of equations, combining them into one system of equations:

\[
N = h \cdot S_{ex \_wall} \cdot (T_3 - T_w), \quad (6)
\]

where $S_{ex \_wall}$ is the outer wall area of the building.

This system of equations should be supplemented by a linear relationship for variables of the 7th group (riser temperatures) and variables of the 5th group (average temperatures of radiators). However, without taking these ratios into account, the system of equations (7) apparently contains all the variables only in the left part and gives an asymmetric matrix. This complicates the solution and does not prevent from rounding errors. It is more rational to focus on the iterative process, in which two systems of equations are solved: the thermal balance of the building and the heat and mass transfer in the heating pipes. However, it is necessary to ensure the convergence of this iterative process.

4. Algorithm for solving the heat balance system

The algorithm for jointly solving the equations of heat balance of a building with a heating system can be built as follows.

Step 0. Accepted initial values of radiator temperatures are equal to the temperature of the heat-transfer at the entrance to the corresponding heating riser.

Step 1. Before reaching convergence:

- Step 1.1. The system of equations for the thermal balance of enclosing structures and premises is solved;
- Step 1.2. According to the calculated values of temperatures in the premises, the right-hand sides of the system of equations are calculated for all convectors;
- Step 1.3. The system of equations is solved and the temperature distribution over each riser is determined;
- Step 1.4. Average integral temperatures for each radiator are calculated;
- Step 1.5. If all the differences in the calculated temperatures of the radiators and the same values at the previous iteration do not exceed the specified error threshold value, finish the calculation (convergence is achieved), otherwise proceed to step 1.1.
5. Calculation example
The above technique was used in the heat loss diagnostics in a seven-floor educational building. Figure 2 shows a fragment of a finite element model for a facade wall of the building. When identifying the model, the characteristics of the heating devices were set in accordance with the actually measured average temperatures of the radiators, and the variable factors were the effective thermal conductivities of the window openings and walls. These values were determined from the minimum condition of a sum of squares of deviations between the calculated indoor temperatures and the measured ones.

![Figure 2. Fragment of the building front wall with openings for windows and balconies.](image)

Figure 3a shows the calculated temperatures in rooms adjacent to the building facade before setting up the model (at standard values of thermal conductivity coefficients), and figure 3, b stands for the values of the coefficients found by identification. A rectangle indicates a room, the number of which is indicated at the top. The difference between the calculated and measured temperatures before setting up the model reaches 11°C, while after setting it does not exceed 1.5°C.

![Figure 3. Calculated temperatures in rooms adjacent to the facade: a) before identification, b) after identification of the model.](image)

6. Conclusions
After the completion of the algorithm, the achieved approximation is an assessment of heat-transfer temperatures in risers, radiator temperatures, indoor temperatures and temperatures at all nodes of the
enclosing structures. The adequacy of the simulation should be ensured by the reasonable setting of the thermal parameters of the model. This requires adjustment of the model according to direct measurements obtained with the help of thermal imaging equipment.

The adjusted balance model of building heat loss allows predicting a heat regime change in the premises after scheduled maintenance work (thermal insulation of building envelopes, repair of heating appliances) and determining rational options for routine repair with limited resources of the operating organization.

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