A Gravitational non-Radiative Memory Effect

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We revisit the issue of memory effects, i.e. effects giving rise to a net cumulative change of the configuration of test particles, using a toy model describing the emission of radiation by a compact source and focusing on the scalar, hence non-radiative, part of the Riemann curvature. Motivated by the well known fact that gravitational radiation is accompanied by a memory effect, i.e. a permanent displacement of the relative separation of test particles, present after radiation has passed, we investigate the existence of an analog effect in the non-radiative part of the gravitational field. While quadrupole and higher multipoles undergo oscillations responsible for gravitational radiation, energy, momentum and angular momentum are conserved charges undergoing non-oscillatory change due to radiation emission. We show how the source re-arrangement due to radiation emission produce time-dependent scalar potentials which induce a time variation in the scalar part of the Riemann curvature tensor. As a result, on general grounds a velocity memory effect appears, depending on the inverse of the square of the distance of the observer from the source, thus making it almost impossible to observe, as shown by comparison to the planned gravitational detector noise spectral densities.

Keywords: Gravitational potential, memory effect, General Relativity

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I. INTRODUCTION

The recent detections of gravitational waves (GWs) \[1\textsuperscript{-}5\], beside marking the beginning of the new science dubbed GW Astronomy, have triggered scientific interest over all aspects of GW production and detection.

The gravitational field sourced by an astrophysical compact object, with mass and velocity multipoles changing with time, includes both a radiative and a non-radiative (or longitudinal) part, the former being the gravitational radiation with \(1/r\) behavior (being \(r\) the distance between source and observer), the latter being sourced by conserved charges, mass and angular momentum, which in the static limit enters the Riemann curvature tensor components as \(1/r^3\) (in the standard 3+1 dimensional space). Gravitational radiation carries the fingerprints of source, its frequency evolution being determined by the internal dynamics of the source constituents. Gravitational non-radiative modes undergo secular changes as energy and angular momentum (and momentum) is dragged out of the system by radiation.

The focus of present work is studying the non-radiative part of the gravitational field and related observational effects which manifest as a different status of test particles between before and after the passage of radiation, thus producing a cumulative change in suitable observables which does not vanish at late times.

The presence of memory effects was first noticed in linearized gravity already in \[25\], where the passage of GWs sourced by moving massing objects was identified to cause a permanent displacement between test particles, not fading away after the gravitational perturbation as gone quiet, which was later quantified in \[20, 24\] to leading order \(1/r\) in linearized gravity in an equation relating the difference in the gravitational radiative field at early and late times to the source velocities at early and late times.

In literature it is usual to distinguish between the linear and a non-linear Christodoulou effect, due to the seminal analysis of \[8\], where the energy momentum tensor of GW radiation is considered as an additional source of GW radiation: being the energy momentum tensor quadratic in GW amplitude this is a non-linear effect. Another peculiarity of the non-linear Christodoulou memory effect is that the source is neither slowly moving (being made of radiation it moves at the speed of light) nor confined to a small region of space (the energy momentum tensor of GWs falling off as \(1/r^2\)). Actually an energy momentum tensor made
of radiation as a source of GWs was already studied in [10, 23] (see also the more recent [18]), in the context of anisotropic neutrino emission by supernovae (at the time neutrinos were compatible with having zero mass) leading as well to memory terms.

Memory effects can also be considered as part of a wider category of hereditary effects, so named as their value at any instant of time depends on the history of the source rather than source status at retarded time. The term hereditary has been used to indicate the correlation in the dynamics of a gravitating system between arbitrarily large time spans for the first time in in [6]. There, within the context of the post-Newtonian approximation to General Relativity, the tail effect onto the metric due to scattering of gravitational radiation with the background curved by the same source emitting GWs were studied. Subsequently [7] showed another type of hereditary effect due to the radiation of GWs sourced by the stress-energy tensor of GWs, causing a cumulative change in the waveform, i.e. a memory effect that does not vanish after the passage of the radiation. ¹

More recently the issue of memory effects was revisited in [21, 22] where it has been shown that in the case of a source involving the emission of a massless particle there is a memory effect of the “Christodoulou” type, and in the toy model case of decay of initial source into two massive particles the standard “linear” memory effect is recovered. Considering the geodesic deviation equation for two nearby geodesic separated by space vector with components $D^i$

$$\ddot{D}^i = R^i_{\ 0j0}D^j, \quad (1)$$

a distinctive feature of the memory effect is the presence of a $\dot{\delta}(t)$ (time derivative of a delta function) term in the Riemann tensor which, in the language of [21], gives displacement kick, or a permanent displacement once integrated twice over time in eq. (1).

We will find in this work that non-radiative part of the metric imparts a velocity kick, i.e. it involves a $\delta(t)$ term in the Riemann curvature tensor, which has a $1/r^2$ fall-off at infinity and which does not generate a displacement memory effect.

The outline of this paper is as follows. In sec. II we present the toy model for the source of gravitational field and how the linearized Einstein equations are going to be solved, reviewing

¹ Whether this neutrino radiation is actually of memory or tail type is discussed in ch. 10.5.4 of [16], where it is shown that on a very long time scale $t \gg r$ the GW amplitude decays, thus not leaving a permanent displacement characteristic of a memory term. However the such test-particle displacement after the passage of the GWs, even it not permanent, is present for $t \gtrsim r$, hence it can well be seen as “permanent” on the human observation time scale when $r$ is an astronomical scale.
the Scalar-Vector-Tensor (SVT) formalism developed in [12]. In sec. III we show the results of our analysis, and in sec. IV we interpret them and investigate the detectability of the non-radiative gravitational modes. We finally conclude in sec. V.

II. METHOD

To consider the asymptotic behaviour of metric component in linearized General Relativity, we find convenient to split the metric according to the SVT decomposition introduced in [12]. After modelling the source with an astrophysically realistic dynamics of emission of particles/radiation, the metric components are obtained by solving linearized Einstein equations. Then we concentrate our analysis on the scalar, non-radiative degrees of freedom of the metric, looking for effects present even after the passage of radiation and their observational signature and observability will be studied in sec. IV.

According to the standard SVT decomposition, we parametrize the metric via irreducible fields according to

\[
ds^2 = -dt^2 (1 + 2\phi) + 2dt dx^i (\beta_i + b_i) \\
+ dx^i dx^j \left[ \delta_{ij} (1 - 2\psi) + \frac{1}{2} (F_{i,j} + F_{j,i}) + \left( \partial_i \partial_j - \frac{1}{3} \delta_{ij} \nabla^2 \right) e + h_{ij} \right],
\]

with constraints

\[
\beta_{i,i} = 0, \\
F_{i,i} = 0, \\
h_{i} = 0 = h_{i,j,j},
\]

so that the fields \(\phi, \psi, b, e, \beta_i, F_i, h_{ij}\) have 10 independent components in total parametrizing the 10 metric components. Note that the constraints imply that the irreducible fields above are non-local combinations of the original metric components \(g_{\mu\nu}\), but the tidal Riemann fields can be written in terms of irreducible fields derivatives, which are local expression of the derivative of the original metric.

Consistently we apply the SVT decomposition to source components as well to obtain

\[
T_{00} = \rho, \\
T_{0i} = S_i + \partial_i S, \\
T_{ij} = P\delta_{ij} + \sigma_{ij} + \frac{1}{2} (\sigma_{i,j} + \sigma_{j,i}) + \left( \partial_i \partial_j - \frac{1}{3} \delta_{ij} \nabla^2 \right) \sigma,
\]

with
where \( S_{i,i} = 0 = \sigma_{i,i} = \sigma_{ij,j} \). Using now the energy momentum conservation, with \( T_{\mu \nu} = 0 \) one has
\[
\dot{\rho} = \nabla^2 S \quad \Rightarrow \quad S = \frac{\dot{\rho}}{\nabla^2},
\]
and from \( T_{\mu \nu} = 0 \)
\[
\dot{S}_i + \partial_i \dot{S} = p_i + \frac{1}{2} \nabla^2 \sigma_i + \frac{2}{3} \nabla^2 \sigma_{,i}
\]
\[
\quad \Rightarrow \quad \left\{ \begin{array}{l}
p + \frac{2}{3} \nabla^2 \sigma = \dot{S} = \frac{\ddot{\rho}}{\nabla^2}, \\
\sigma_i = 2 \frac{\dot{S}_i}{\nabla^2}
\end{array} \right.
\]
(6)

Following a standard procedure, see app. A for details, one can define the gauge invariant SVT-decomposed metric components
\[
\Phi \equiv \phi + \dot{b} - \frac{\ddot{e}}{2},
\]
\[
\psi^e \equiv \psi + \frac{1}{6} \nabla^2 e,
\]
\[
V_i \equiv \beta_i - \frac{1}{2} \dot{F}_i,
\]
\[
h_{ij},
\]
(7)

and obtain the following expressions for the Einstein equations:
\[
\nabla^2 \psi^e = 4\pi G_N \rho, \quad S : 00, 0i,
\]
\[
\nabla^2 \Phi = 4\pi G_N \left( \rho + 3p - 3\dot{S} \right), \quad S : ij,
\]
\[
\nabla^2 V_i = -16\pi G_N S_i, \quad V : 0i, ij,
\]
\[
\Box h_{ij} = -16\pi G_N \sigma_{ij}, \quad T : ij,
\]
(8)

Although the gauge invariant variables \( \Phi, \psi^e, V_i, h_{ij} \) are non-local quantities of the metric component \( g_{\mu \nu} \), they appear in a local combinations in the Riemann components, e.g. at linear order
\[
R^i_{\ 0j0} = \delta_{ij} \psi^e + \Phi_{,ij} + \frac{1}{2} \left( \dot{V}_{i,j} + \dot{V}_{j,i} \right) - \frac{1}{2} \ddot{h}_{ij}.
\]
(9)

As long as observables depend on Riemann tensor components, they can be expressed indifferently in terms of \( g_{\mu \nu} \) or \( \psi^e, \Phi, V_i, h_{ij} \).

The physical situation we are going to consider corresponds to a source whose constituents change suddenly from a static object following a time-like geodesic to an expanding shell of radiation, plus the massive remnant (eventually in a new dynamical state). Such system
arises naturally in a variety of astrophysical situation, hence represent a training ground of phenomenological interest to test how hereditary effects can be recorded in the non-radiative part of the metric perturbations.

III. RESULTS

To illustrate our result we consider as a source a massive object at rest at the origin of coordinates for $t < 0$ which turns into an expanding shell of radiation for $t > 0$. This example is simple enough to be treated completely analytically and at the same time, as it will be shown, it contains the hearth of the physical result we are going to investigate.

We consider a source for which the initial rest mass is entirely converted into an isotropically expanding shell of radiation:

$$T_{00}(t, \vec{x}) = (1 - \Theta(t)) M \delta^{(3)}(\vec{x}) + \frac{M}{4\pi r^2} \Delta(t - r),$$

with $r = |\vec{x}|$, where with $\Theta(t)$ we denote a generic monotonic (non necessarily even) smooth function interpolating between 0 for $t \to -\infty$ to 1 for $t \to \infty$ and $\Delta(x) \equiv \frac{d\Theta(x)}{dx}$. We assume

$$p = \frac{M}{12\pi r^2} \Delta(t - r),$$

so that the scalar components of the energy-momentum tensor are completely specified: the scalar part of eq. (6) is solved by

$$S = \frac{M}{4\pi} \int_r^\infty dr' \frac{\Delta(t - r')}{r'^2},$$

and see app. A for the derivation of the remaining scalar component in eq. (11) $\sigma$, which does not enter explicitly the Riemann tidal components. The resulting scalar metric fields are

$$\psi^e = -\frac{G_N M}{r} + G_N M \int_r^\infty \frac{\Theta(t - r')}{r'^2} dr',$$

$$\Phi = -\frac{G_N M}{r} + G_N M \int_r^\infty \frac{\Theta(t - r')}{r'^2} \left(2 - 3 \frac{r^2}{r'^2}\right) dr',$$

or equivalently

$$\psi^e = -\frac{G_N M}{r} + \Theta(t - r) \frac{G_N M}{r} - G_N M \int_r^\infty dr' \frac{\Delta(t - r')}{r'^2},$$

$$\Phi - \psi^e = -G_N M \int_r^\infty dr' \frac{\Delta(t - r')}{r'} \left[1 - \left(\frac{r}{r'}\right)^2\right].$$
Note that while the source undergoes a discontinuity at \( t = 0 \), fields \( \Phi, \psi^e \) change only at time \( t \sim r \), as required by causality.

We observe that \( \psi \) is continuous at \( t = 0 \) since it is obtained by directly convolving the Green function with \( \rho \), which is continuous function of time. The combination \( \phi - \psi \) is proportional to the convolution of the Green function with \( p - \dot{S} \), both \( p \) and \( \dot{S} \) individually change discontinuously at \( t = 0 \) for \( r = 0 \) but their combined contribution to the potential is also constant around \( t = 0 \).

In particular their early/late time behaviour

\[
\psi^e(t \ll r) = \Phi(t \ll r) = -\frac{G_N M}{r},
\]

\[
\psi^e(t \gg r) = \Phi(t \gg r) = -\frac{G_N M}{t},
\]

(15)

resemble the Newtonian potential \( \phi_N \) for the same physical source.

\[
\phi_N(t \ll r) = -\frac{G_N M}{r},
\]

\[
\phi_N(t \gg r) = -\frac{G_N M}{t}.
\]

(16)

The scalar part of the Riemann component entering the geodesic deviation equation reads

\[
R^i_{\ 0j0}|_S = \ddot{\psi}^e \delta^i_j + \dot{\Phi}^i_j
= \left( \delta^i_j - \frac{x^i x_j}{r^2} \right) \frac{G_N M}{r^2} \Delta(t - r) + \left( \delta^i_j - 3 \frac{x^i x_j}{r^2} \right) \frac{G_N M}{r^3} \left( 1 - \Theta(t - r) \right),
\]

(17)

which does not have a displacement memory type term (that would look like \( G_N M \Delta(t-r)/r \)) able to give a displacement kick to test particles, but rather a term \( \propto G_N M \Delta(t-r)/r^2 \) which gives a transverse velocity kick to test particles, see discussion in the next section.

Note also the presence of standard \( 1/r^3 \) tidal terms for the static source for early enough time which vanish when the expanding sphere of radiation pass through the test particle. Finally we observe that \( \delta^i_j R^i_{\ 0j0} \) correctly vanishes outside the sources.\(^2\)

IV. DISCUSSION

To interpret the first term on the right hand side of eq. (17) one can integrate the geodesic deviation equation in a small time interval around \( t = r \) for two test masses at distance \( D \)

\(^2\) A similar setup was studied in [13] where a potential analog to our \( \psi^e \) was erroneously found to be time-independent, by first assuming that \( t/r \ll 1 \) and then taking the limit \( t \to \infty \), see eqs. (22), (24) and (25) there. See later this section for a general derivation of the gravitational scalar potentials.
from each other to get
\[
\Delta \dot{D}^i = \left( \delta^i_j - \frac{x^i x_j}{r^2} \right) \frac{G_N M}{c r^2} D^j,
\]
where the speed of light \( c \) has been re-instated. Eq. (18) shows that two test masses with transverse separation \( D \) to with respect to the source-observer direction will experience a velocity drift \( \sim G_N M D/(c r^2) \).

The detectability of a (Fourier-transformed) metric perturbation \( \tilde{h}(f) \) by a detector with (single-sided) noise spectral density \( S_n(f) \) is quantified \(^3\) by its Signal-to-Noise-Ratio (SNR), see e.g. the standard textbook [16] at ch. 7:
\[
\text{SNR}^2 = 4 \int_0^\infty d(\log f) \frac{|\tilde{h}(f)|^2}{S_n(f)},
\]
where a threshold for detection of \( \text{SNR} \geq 8 \) is usually assumed. From the second derivative of the strain responsible for the velocity kick \( \propto \Delta(t - r) \), using that \( |\tilde{h}(f)| = |\tilde{\nabla h}/(2\pi f)^2| \) on gets a velocity kick strain \( \tilde{h}_{vk}(f) \)
\[
\tilde{h}_{vk}(f) \sim \frac{G_N M}{c (2\pi f r)^2},
\]
from which one can estimate a dimension-less strain \( h_c \) given by
\[
h_c = f \tilde{h}_{vk} \approx 2 \times 10^{-27} \left( \frac{M}{M_\odot} \right) \left( \frac{f}{10^{-4} \text{Hz}} \right)^{-1} \left( \frac{r}{r_{GC}} \right)^{-2},
\]
with \( r_{GC} \) the distance to the galactic center \( r_{GC} \sim 8 \) kpc.

Considering the possibility to observe this effect by the future GW detectors, fig. I shows the square root of \( S_n(f) \) (i.e. the denominator in the SNR integrand in eq. (19)) for various future GW detectors, and \( \sqrt{f |\tilde{h}_{vk}(f)|} \) (i.e. the square root of the numerator of the SNR integrand), considering an optimistic mass release \( M = M_\odot \) for a source located at the galactic center.

**V. CONCLUSION**

By analyzing a toy model of emission of generic radiation by a compact astrophysical object we find a simple expression for the time behaviour of the scalar part of the Riemann curvature.

\(^3\) The single-sided noise spectral density is defined in terms of the average of the Fourier transform of the dimension-less strains \( \tilde{n}(f) \) via \( \langle \tilde{n}(f) \tilde{n}^*(f') \rangle = \frac{1}{2} S_n(f) \delta(f - f') \).
Figure 1: Square root of the noise spectral density of several planned GW detectors compared to the velocity-kick amplitude $\sqrt{|\dot{\tilde{h}}_{vk}|}$ for $M = M_\odot$ at $r = r_{GC} = 8$ kpc, which is the approximate distance to the galactic center. Also shown for comparison spectral noise densities for different detectors: SKA [17], LISA [19], DECIGO [15], BBO [9], ET [11].

We find that the scalar potential and curvature at the observer site undergo variations only at the time when the perturbations reach the observers, as expected by causality. Beside the standard $1/r^3$ almost-static tidal term, the scalar part of the curvature contains a term producing a “step function” in the relative velocity of test masses between immediately before and after he passage of radiation through detector test masses, like mirrors in gravitational observatories. The resulting velocity kick amplitude varies with the inverse of the square of the source-detector distance.

From the observational point of view the dependence on the inverse square distance, in sharp contrast with the inverse distance dependence of the gravitational waves, suppresses such effect making virtually impossible to be detect with GW detectors.

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Appendix A: Einstein equations in the SVT decomposition

Defining $\psi^e \equiv \psi + 1/6 \nabla^2 e$ and $V_i \equiv \beta_i - 1/2 \dot{F}_i$, one has for the Christoffel coefficients

$$\Gamma^{00}_0 = \dot{\phi},$$
$$\Gamma^{0i}_0 = \phi_{,i},$$
$$\Gamma^{0i}_j = -\delta_{ij} \psi^e + \frac{1}{2} \dot{\psi}_{,ij} - b_{,ij} - \frac{1}{2} (\dot{V}_{i,j} + \dot{V}_{j,i}) + \frac{1}{2} \ddot{h}_{ji},$$
$$\Gamma^{k0}_0 = \delta^{km} (\phi_{,m} + \dot{b}_{,m} + \dot{\beta}_{,m}),$$
$$\Gamma^{ki}_0 = -\delta_{ki} \dot{\psi}^e + \frac{1}{2} \delta^{km} \dot{\psi}^e_{,im} + \frac{1}{2} (\dot{V}_{k,i} - \dot{V}_{i,k}) + \frac{1}{2} \delta^{km} \dot{h}_{mi},$$
$$\Gamma^{kj}_i = \delta^{km} \delta_{ij} \psi^e_{,m} - \delta_{ki} \dot{\psi}^e_{,j} - \delta_{kj} \dot{\psi}^e_{,i} + \frac{1}{2} \delta^{km} e_{,ijk} + \frac{1}{2} \delta^{km} F_{l,im} + \frac{1}{2} \delta^{km} (h_{im,j} + h_{jm,i} - h_{ij,m}).$$

\text{(A1)}

and for Riemann tensor components

$$R^{00}_{0i} = -\delta_{ij} \ddot{\psi}^e - \left( \phi + \dot{b} - \frac{1}{2} \dot{e} \right)_{,ij} - \frac{1}{2} (\dot{V}_{i,j} + \dot{V}_{j,i}) + \frac{1}{2} \ddot{h}_{ij},$$
$$R^{k0}_{ij} = \delta^{km} \left( \delta_{ij} \dot{\psi}^e_{,ml} - \delta_{il} \dot{\psi}^e_{,mj} \right) + \delta^{km} \dot{\psi}^e_{,il} + \frac{1}{2} \delta^{km} (h_{jm,il} + h_{im,jl} - h_{ij,lm} - h_{lm,ij}),$$
$$R^{0i}_{jk} = \delta_{ij} \dot{\psi}^e_{,k} - \delta_{ik} \dot{\psi}^e_{,j} + \frac{1}{2} (\dot{V}_{j,ik} - \dot{V}_{k,ij}) + \frac{1}{2} (\dot{h}_{ijk} - \dot{h}_{ij,k}).$$

\text{(A2)}

Now defining $\Phi \equiv \phi + \dot{b} - \ddot{e}/2$ one has

$$R_{00} = 3 \ddot{\psi}^e + \nabla^2 \Phi,$$
$$R_{0i} = 2 \dot{\psi}^e_{,i} - \frac{1}{2} \nabla^2 V_i,$$
$$R_{ij} = \delta_{ij} \left( -\ddot{\psi}^e + \nabla^2 \dot{\psi}^e \right) + \dot{\psi}^e_{,ij} - \Phi_{,ij} - \frac{1}{2} \left( \dot{V}_{i,j} + \dot{V}_{j,i} \right) + \frac{1}{2} \ddot{h}_{ij} - \frac{1}{2} \nabla^2 h_{ij},$$

hence

$$R = -6 \ddot{\psi}^e - 2 \nabla^2 \Phi + 4 \nabla^2 \dot{\psi}^e.$$

\text{(A3)}

\text{(A4)}

The Einstein’s equations can be split according to their representation under rotation:

- **Scalar**

  $$G_{00} = 2 \nabla^2 \dot{\psi}^e,$$
  $$G_{0i} = 2 \dot{\psi}^e_{,i},$$
  $$G_{ij} = \delta_{ij} \left( 2 \ddot{\psi}^e + \nabla^2 (\Phi - \psi^e) \right) + (\psi^e - \Phi)_{,ij}.$$

\text{(A5)}

- **Vector**

  $$G_{00} = 0,$$
  $$G_{0i} = -\frac{1}{2} \nabla^2 V_i,$$
  $$G_{ij} = -\frac{1}{2} \left[ \dot{V}_{i,j} + \dot{V}_{j,i} \right].$$

\text{(A6)}
Tensor

\[ G_{00} = G_{0i} = 0, \]
\[ G_{ij} = \frac{1}{2} h_{ij} - \frac{1}{2} \nabla^2 h_{ij}, \]  

(A7)

The action of a coordinate transformation

\[ x^0 \rightarrow x'^0 = x^0 + \xi^0, \]
\[ x^i \rightarrow x'^i = x^i + \xi^i + a^i, \]  

(A8)

with \( \xi^i = 0 \), has the following effects on SVT fields:

\[ \phi \rightarrow \phi' = \phi - \dot{\xi}^0, \]
\[ b \rightarrow b' = b + \xi^0 - \dot{a}, \]
\[ \beta_i \rightarrow \beta'_i = \beta_i - \dot{\xi}^i, \]
\[ \psi \rightarrow \psi' = \psi + \frac{1}{3} \nabla^2 a. \]
\[ F_i \rightarrow F'_i = F_i - 2\xi_i, \]
\[ e \rightarrow e' = e - 2a, \]  

(A9)

from which one can verify that fields in eq. (7) are coordinate transformation invariant, at linear order.

We complement the results of subsec. (III) by providing the explicit solution for \( \sigma \), the remaining scalar component of the energy-momentum tensor:

\[ \sigma = \frac{Mr^2}{8\pi} \int_r^\infty \Delta(t - r') \left( 3 \frac{r^2}{r'^2} - 1 \right) dr', \]

(A10)

which can be checked for consistency from \( \psi^e - \Phi = 8\pi G_N \sigma \).

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