MINIJETS AS A PERTURBATIVE PROBE OF COLOUR CHARGE

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Abstract

We motivate the study of “Minijets” (that is jets soft with respect to the hard scattering, but hard with respect to $\Lambda_{QCD}$) as a means to study the underlying QCD colour flow in events. We discuss, with the aid of a simplistic model, currently available events in which minijets are observed, that is the events have high multiplicity, as a means of understanding the physics of minijets. We encourage both theoreticians and experiments to continue the study of minijets.
In the past the concept of QCD colour flow has been an interesting prediction, and hence test, of QCD. One of the more surprising examples of QCD colour flow has been the rapidity gaps seen at HERA. In the majority of jet events observed at HERA the incoming electron/positron radiates a photon, which scatters off one of the coloured constituents of the proton, to produce the observed jet. This hard scattering leaves the proton remnant and the pre-jet in a non-colour singlet state, during hadronization the colour of these objects is rearranged to form colourless hadrons, and these hadrons fill the gap between the produced jet and the proton remnant with an underlying event. However among the HERA events are some with an observed jet produced at central rapidities; i.e. there is no underlying event, and this means that the mechanism for producing this jet can not be that just described. The lack of hadrons between the observed jet and the proton remnant means that there is no QCD colour is exchanged between the proton and the photon, and hence that photon must have scattered off a colour singlet object in the proton, the pomeron. So the flow of hadrons in an event has told us something about the structure of the proton, that the struck object inside the proton that took part in the hard scattering is a QCD colour singlet.

This ability to probe the QCD colour of the object that takes part in the hard scattering is, in the future, likely to provide a strong experimental probe. A prime example is in the detection of a heavy (∼ 500 GeV) Higgs boson at the LHC, such a Higgs is produced via two different mechanisms. Approximately two thirds of heavy Higgs are produced from $gg$ fusion via a top quark loop, with the remaining third produced via vector boson fusion, show in Fig.1a. Such a heavy Higgs typically decays into two massive vector bosons, either $W$ or $Z$ bosons, and the back ground from continuum diboson production, shown in Fig.1b, can be large. However the QCD colour flow in the diboson fusion signal process is very different from the background continuum diboson production, the leading colour flow is shown in Fig.1 as a dotted line. For the diboson fusion process QCD colour flows between the proton remnant and the parton that emits the vector boson, this parton travels in very much the same direction as the proton remnant. This means that the hadrons associated with the underlying event will travel in a similar direction to the proton remnant, i.e. at very large rapidities. For the diboson continuum background (as well as the Higgs signal

![Figure 1: Typical Feynman diagrams for $pp$ scattering to a pair of vector bosons. The a) signal, vector boson scattering via a Higgs boson, b) background, divector boson production. Show, dotted, is the leading QCD colour flow.](image-url)
from $gg$ fusion) QCD colour is exchanged between the two protons, and this means that the underlying event will produce hadrons at all rapidities. This difference in the flow of hadrons gives us a method of separating the vector boson fusion Higgs signal from the continuum background, effectively the Higgs signal has a double rapidity gap on both sides of the event, with the Higgs decay taking place at central rapidities [2].

However this method for separating signal from background by the flow of hadrons has many difficulties. On the theoretical side we make predictions and the rate of the signal and background based upon perturbative QCD (pQCD). pQCD’s fundamental fields are partons (quarks and gluons) and it is about these fields that pQCD makes predictions. By local parton hadron duality we expect that the produced partons will hadronize into jets; however this tells us nothing about individual hadrons and it is in terms of individual hadrons that rapidity gaps are defined. This means that we can make no absolute predictions based upon pQCD of the signal rate of events where a rapidity gap is observed, or indeed the background rate where a fluctuation leads to an observed rapidity gap, despite QCD colour being exchanged between the protons. On the experimental side the cross section for Higgs productions drops very rapidly as the Higgs gets heavy, and for very heavy Higgs the cross section is small enough that it is only with very high luminosities that we produce any Higgs events at all. However at very high luminosities there are likely to be many interactions per bunch crossing, and each of these interactions will produce its own underlying event, which will wash out the rapidity gap at central rapidities.

One possible solution to these problems is rather that look for events with a rapidity gap, i.e. no hadrons at central rapidities, instead to look for events with no soft “minijets” at central rapidities [3]. Naively we expect additional jets to be suppressed by a factor of $\alpha_s$, however there can often be a large volume of phase space in which additional jets can be radiated, and this can enhance the additional jet rate by a large logarithm such that multijet emission is not suppressed. In particular jet emission that is soft with respect to the hard scattering is enhanced by such a large logarithm. If we define soft jets in such a way that the background di-vector boson continuum (which has more QCD colour acceleration, and hence more jet activity) typically has additional jet emission, while the vector boson fusion signal (with less QCD colour accelerated) typically has no additional jet activity; then the difficulties with using hadrons to define a rapidity gap are overcome. Additional soft jet production we expect to be predictable by pQCD as long as the scale for soft jet emission is far above $\Lambda_{QCD}$. On the experimental side we expect that the majority of interactions will be very soft, and although producing an underlying event the majority of interactions will not produce observed jets, and so not interfere with tagging a Higgs signal event. This means that firm theoretical predictions can, in principle, be made about a minijet tag of a Higgs signal, and that a minijet tag can be used at high luminosities where there are overlapping events.

However before we can use minijet emission as a probe of QCD colour charge it is necessary to understand minijet emission both on a theoretical and experimental level. In order for multiple jet emission to be important we need a large phase space for emission of those jets, and this means that we require the hard scattering to be as energetic as possible. Currently the most energetic events are to be found at the TeVatron, and CDF have studied the
multijet distributions in their most energetic events [5]. They apply the cuts,

\[ E_{T_{\text{jet}}} > 20 \text{ GeV}, \quad |\eta| < 4.2 \]
\[ \sum E_T > 420 \text{ GeV}, \quad m_{\text{jets}} > 600 \text{ GeV} \]
\[ |\cos \theta^*| < \frac{2}{3} \]

(1)

where \( \theta^* \) is the scattering angle of the highest \( E_T \) jet in the multijet centre of mass frame. Due to the high invariant mass of these events the events are forced to be fairly central, and this means that at least two jets are always observed. In figure 2 we show the multiplicity distribution observed in the CDF events, although 3rd and additional jets are naively produced at a higher order in perturbation theory because of the large volume of phase space such additional jet activity is not suppressed, with typical events with 3 or 4 jets observed.

The high multiplicity of these events means that pQCD in its lowest order is not not reliable. The large logs that give rise to the high multiplicity need to be dealt with in the theoretical calculation. Now most of the additional jet activity takes place at low \( E_T \), and these soft jets typically originate from gluons. Now for soft gluons the matrix element factorises, the probability to radiate \( n \) additional soft gluons is given by,

\[ |\mathcal{M}_{n_g}^2| = |\mathcal{M}_1|^n. \]

(2)

The phase space to radiate \( n \) additional gluons is given by,

\[ d(LIPS_n) = \frac{1}{n!} \prod_{i=1}^{n} \frac{d^3}{(2\pi)^3} E_i \delta^4(P - \sum p) H(\text{observed properties of the gluons}) \]

(3)
where the $1/n!$ term arises from the symmetry factor for integrating $n$ gluons over the same phase space. Now if $H = 1$, that is we make no experimental requirements about the observed properties of the gluons, then in moment space we find,

$$d(LIPS) = 1/n! \prod d(LIPS) .$$

This means that the probability to see $n$ gluon jets is given by,

$$\mathcal{P}(n \text{ gluons}) = \mathcal{P}(1 \text{ g})^n/n!$$

and so,

$$\mathcal{P}(\text{any number of gluons}) = \exp(\mathcal{P}(1 \text{ g}) .$$

However for the case of jets defined by the CDF algorithm $H \neq 1$, also not all jets are soft, and not all jets are gluon initiated. This means that this simple factorisation of the additional jet rate does not hold, however it is a reasonable ansatz to assume equation 5.

This is not the full story though, equation 5 does not conserve probability, this happens because the rate to radiate an addition gluon $\mathcal{M}_1$ is an inclusive quantity, whereas if we are to make predictions about the specific multiplicity distributions then we require the exclusive rate to produce exactly $n$ additional jets. We can obtain the exclusive jet rate by adding a Sudakov suppression factor $\exp(-\mathcal{P}(1 \text{ g}))$ that excludes all additional jet activity, and then equation 5 produces exactly $n$ exclusive jets. This gives the ansatz for additional jet activity over the hard scattering as,

$$\mathcal{P}(\text{exactly } n \text{ additional jets}) = \mathcal{P}(1 \text{ additional jet})^n/n! \exp(-\mathcal{P}(1 \text{ additional jet}))$$

where $\mathcal{P}(1 \text{ jet})$ is the average multiplicity $\bar{n}$, and at the leading logarithmic level is given by,

$$\mathcal{P}(1 \text{ jet}) = \frac{\sigma(3 \text{ jet production})^{\text{LO}}}{\sigma(\text{total 2 jet production})} .$$

This formula has the advantage over strictly tree level pQCD of being still valid even when $\mathcal{P}(1 \text{ jet}) > 1$. Also we have,

$$\sigma_3 = \sigma_2 \mathcal{P} \exp(-\mathcal{P}) = \sigma_3^{\text{LO}}(1 + \mathcal{O}(\alpha_s))$$

and so the 3 jet cross section has the same accuracy as at leading order.

If we compare equation 6 with the measured CDF multiplicity distribution, fitting only $\mathcal{P}$ to the observed average excess multiplicity over the 2 hard jets always observed we find the curve shown in figure 2. Although the point for multiplicity 4 is 3.5 standard deviations above the fitted curve, and thus the equation 6 has limitations, it is clear that the fit has got the gist of the multiplicity distribution correct and gives a place to start when studying event multiplicities, and hence the physics of minijets.

If we now try to predict the value for $\mathcal{P}$ from pQCD using equation 6 with a tree level calculation for $\sigma_3$ and a next-to-leading-order calculation for $\sigma_2$ then we find the results shown in figure 3 plotted against the minimum $E_T$ used to define a jet.

For the leading order calculation of $\sigma_3$ we have several different choices of scale. For the renormalisation scale of $\alpha_s$; this can either be some multiple of the hard scattering scale,
Figure 3: Ratio of the tree level 3-jet cross section to the NLO cross section for 2-jet inclusive events within the CDF acceptance cuts, equation 1. The cross section ratio $\bar{n} = \sigma_3(E_{T,\text{min}})/\sigma_{2,\text{incl}}$, with $\sigma_{2,\text{incl}} = 33$ pb, is shown as a function of the transverse energy threshold, $E_{T,\text{min}}$, of the third jet. Results are given for four different scale choices in $\sigma_3$: $\mu_R = \mu_F = \sum E_T/4$ (solid line), $\mu_R = \mu_F = E_{T,3}$ (dashed line), and $\mu_F = \xi E_{T,3}$, $\alpha_s^3 = \prod_{i=1}^3 \alpha_s(\xi E_{T,i})$ with a scale factor $\xi = 1$ (dash-dotted line) and $\xi = 1/2$ (dotted line). The CDF value for the average minijet multiplicity, $\bar{n} = 1.57$, is given by the diamond.
\( \sum E_T \), or some multiple of the soft scale at which the jet is emitted \( E_{T,3} \). For the factorisation scale at which the parton densities are evaluated we similarly can choose a hard scale related to \( \sum E_T \), if we feel we should sum up the initial state radiation, or a soft scale related to \( E_{T,3} \) if we feel that explicitly calculating high order emission in \( \sigma_3 \) means that we should only generate higher order corrections below that scale. A case can be made for each of these choices, and from figure 3 we can see the prediction for \( \bar{n} \) changes by a factor of 3 for the experimental value of \( E_{T,\text{min}} \). This happens because we have two natural scales in the calculation of \( \sigma_3 \) the hard scattering scale, and the \( E_T \) of the emission, and as these have very different values the predictions for \( \bar{n} \) vary accordingly. The theoretical predictions for \( \bar{n} \) span the experimental measurement, but we can not make an accurate theoretical prediction at this time.

Without improvements to the theoretical predictions we do not currently have the ability to accurately predict the rate of minijet emission, and this means that we do not currently have the technology to use minijets as a absolute theoretical prediction to probe QCD colour charge. On the theoretical side the large scale dependence of \( \sigma_3 \) can be cured by working at higher order in perturbation theory \( \mathbb{8} \); however this would necessitate an improvement in the simple minijet model, equation \( \mathbb{8} \), which is non consistent we such a higher order calculation. Alternatively it may be possible to show which logarithmic terms need to be resummed in order to remove the large scale dependence, without a full calculation of the higher order finite corrections. In addition equation \( \mathbb{8} \) does not take advantage of our knowledge of the tree level rate for high jet multiplicities \( \mathbb{9} \). Clearly there is much theoretical work that remains to be done. There is also much work to be done on the experimental side to help our understanding of minijets, for example the different theoretical scale choices not only affect the average multiplicity, but also affect the shape of distributions. This can be seen in figure 3 where a renormalisation scale related to the \( E_T \) of the emitted jet has the average multiplicity growing far more rapidly as the \( E_T \) cut used to define a jet is decreased, than a harder renormalisation scale. If the experimenter were able to measure this distribution then this information can be used to motivate different choices for the scale choice.

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