Maximum Displacement Profiles of Reinforced Concrete Frames

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Abstract

New expressions to estimate maximum seismic lateral displacement profiles of regular plane RC frames at three different damage levels, slight, moderate, and severe damage, are developed in this study for the purpose of displacement-based seismic design. These expressions relating the maximum floor displacements with the maximum inter-story drift ratio over the height and the main structural characteristic of the frame are based on statistical analysis of the results of hundreds of nonlinear time history analysis conducted on a set of 25 plane RC frames subjected to a set of 16 physical accelerograms containing different frequency spectrum. The fundamental period, column-to-beam strength ratio, and damage level were identified as the main structural characteristic having significant effects on the maximum displacement profiles. A case study was conducted on a 12-story RC frame model tested on the shaking table before and shows good agreement between the estimated profiles and test results. The developed profiles are independent of sections and reinforcement of the structure so that they can be used as the starting design variables in displacement-based seismic design.

Keywords: plane reinforced concrete frame; maximum displacement profile; displacement-based seismic design

1. Introduction

After a great deal of damage caused by earthquakes in the last two decades, there is a general agreement in the world that the current seismic design processes need to be upgraded so as to be capable of designing a structural system with predictable seismic performance in the future earthquake. Up to now, the philosophy of performance-based seismic engineering (PBSE) has been sufficiently developed. In the development of PBSE, displacement rather than force has been recognized as the most suitable and direct performance or damage indicator. The most suitable approach to achieve the objectives of performance-based seismic design appears to be the displacement-based method to which the most attentions have been paid by researchers so far (Panagiatakos et al., 1999; Kowalsky, 2002; Xue et al., 2003). These research works highlight the importance of employing displacement as a performance quantifier. To some extent, performance-based design and displacement-based design have been used interchangeably.

Since damage is mainly produced by deformation induced within a structural system, estimation of seismic deformation demands is of primary importance and regarded as the fundamental concern in a displacement-based seismic design when damage control is of interest. Capacity spectrum method is one of the most representative and well accepted procedures to estimate the maximum displacements of multi-degree-of-freedom (MDOF) building structures (Fajfar, 1999). This method requires that both the capacity curve and the demand curve be represented in response spectral ordinates. The capacity curve is developed from pushover curve by using the concept of equivalent single-degree-of-freedom (SDOF) system. This method consists of three stages, pushover analysis of an MDOF structure, production of capacity spectrum and estimation of the maximum displacement of the equivalent SDOF system, and translation of the maximum displacement of the equivalent SDOF to the maximum floor displacements of the initial MDOF system. In recent years, a lot of research efforts have been devoted to develop equivalent linearization method (Miranda et al., 2002; Chopra et al., 2004), inelastic design spectra (Tena-Colunga, 2001; Chai, 2005), and improved pushover analysis procedures accounting for higher mode effects (Chopra et al., 2002; Kalkan et al., 2006). Since a pre-design structure is needed in pushover analysis, the abovementioned procedures are suitable for seismic evaluation of
existing structures or for the performance check after the initial design of new structures.

The maximum displacement associated with particular performance or damage level should be afforded as the starting design variable in the direct displacement-based seismic design. For an MDOF building system, the maximum floor displacement profile is usually needed at the beginning of the design (Priestley et al., 2000). However, few researchers have studied the maximum floor displacement profiles. To the authors’ knowledge, in the literature only Loeding et al. (1998) proposed the maximum displacement profiles of plane RC frames based on elastic time history analysis and Karavasilis et al. (2006) developed the maximum displacement profiles of plane steel moment resisting frames based on nonlinear time history analysis respectively. The expressions proposed by Loeding et al. only depend on the number of stories, ignoring the influence of other structural characteristic. The proposed displacement profiles in the above literature are somewhat limited in application. In this work the maximum floor displacement profiles are studied for regular plane RC frames undergoing elastic and inelastic response on the basis of hundreds of nonlinear time history analysis of a set of 25 RC frames designed with different structural characteristic. The expressions to estimate the maximum displacement profiles at three different damage levels, slight, moderate, and severe damage, are developed for the purpose of direct displacement-based seismic design of RC frames.

2. Generic RC Frame Models
2.1 Design Parameters

The deformation distribution in a frame is closely related with the mechanism of formation and development of plastic hinges in structural members. It has been well recognized that the strength distribution in structural members has considerable influence on the sequence of occurrence and distribution of plastic hinges. The column-to-beam strength ratio in a joint, defined as below, is used here to reflect the relative strength relationship between beams and columns connected to the same joint:

$$\eta_c = \frac{\sum M_{Rc}}{\sum M_{Rb}}$$

(1)

where $\sum M_{Rc}$ is the sum of the flexural strengths of all columns framing into a joint, and $\sum M_{Rb}$ is the sum of the flexural strengths of all beams framing into that joint.

Five values, 0.8, 1.0, 1.2, 1.6, and 2.0, were specified in the design of structural members. The subject structures range from 3 to 15 stories in three-story increments, covering the ordinary scope of number of stories appropriate to RC frames. A set of 25 frames was designed.

The number of bays, spans of bays, story height, and cross sections of beams are identical for all frames. The cross sections and reinforcement of columns and the reinforcement of beams vary every three stories. For the frames with same number of stories, the reinforcement of beams is identical while the reinforcement of columns, determined by the column-to-beam strength ratio and the reinforcement of beams, is different. The dimensions of a 15-story frame are illustrated in Fig. 1. The initial design parameters were defined as follows: dead load 6kN/m², live load 2kN/m², seismic protection intensity VIII, site soil class IV, design group 1, yield stresses of longitudinal and transverse steel 300 and 210 MPa, and concrete compressive strength 30 MPa according to current Chinese seismic design code for buildings. The reinforcement was determined by strength-based seismic design method. The required strength of structural members was computed by modal response spectrum analysis. The distribution of the horizontal seismic forces along the height was determined by the modal combination procedure. The response of all modes of vibration contribution significantly to the global response was taken into account.

Fig. 1. Dimensions of a 15-story Plane Frame (mm)

2.2 Analytic Model and Ground Motions

The basic analysis approach consists of performing nonlinear time history analysis for a given structure and ground motion, using three-dimensional nonlinear analysis computer program, Canny Structural Analysis
Program (Li, 2006). Uniaxial spring model was adopted for beams, using two nonlinear rotational springs at element ends, and elastic shear and axial springs located in mid span. The cross-peak trilinear hysteretic model as shown in Fig.2. was used for moment-rotation relationship. Multi-axial spring model, which considers the interaction between the biaxial bending moments and the varying axial load, was employed for columns. The beam-column joint was modeled by partial rigid zone. A set of 16 ground motions containing different frequency spectrum and duration was used as the seismic excitations. The elastic spectrum acceleration with the same peak ground acceleration of 0.2g is presented in Fig.3.

It has been realized that the damage level of structural and nonstructural components is closely related with inter-story drift ratio which is often used as damage indicator in building structures. In this study the maximum inter-story drift ratio was employed as the performance quantifier to measure the damage degree in a frame after the first occurrence of yielding in structural members. Three damage states were defined: slight damage (indicated by the first occurrence of yielding in structural members), moderate damage (the maximum inter-story drift ratio just reaches 0.01), severe damage but collapse prevention (the maximum inter-story drift ratio just reaches 0.02). The maximum floor displacement profiles corresponding to the above three damage states were analyzed. To identify the maximum displacement response of the frames just arriving at an individual damage state, iterative computations were required. In total 1200 maximum floor displacement profiles were produced.

3. Computational Results and Statistical Analysis
3.1 Computational Results

For each frame at an individual damage state, 16 maximum floor displacement profiles were obtained under the seismic excitations. The median of the profiles and coefficient of variance (COV) were calculated on the basis of the appropriate assumption that the earthquake response follows a log-normal distribution. The median is the central value of 16 maximum displacements, determined by the following formula:

$$D_{m,j} = \exp \left( \frac{\sum_{i=1}^{n} \ln D_{i,j}}{n} \right)$$

where $D_{m,j}$ is the central value of the maximum displacement of the $j$th floor, $D_{i,j}$ is the maximum displacement of the $j$th floor subjected to the $i$th seismic excitation, and $n$ is the number of seismic excitations, here $n$ is equal to 16.

The standard deviation of the logarithm of the sample values for the $j$th floor and COV are defined as

$$\sigma_{m,j} = \sqrt{\frac{\sum_{i=1}^{n} (\ln D_{i,j} - \ln D_{m,j})^2}{n-1}}$$

$$COV = \sqrt{\exp(\sigma_{m,j}^2) - 1}$$

To be convenient for analysis, the maximum floor displacement is normalized as below:

$$D_{nor,j} = \frac{D_{m,j}}{H \theta_{s,max}}$$

where $D_{nor,j}$ is the normalized maximum displacement of the $j$th floor, $H$ is the total height of the frame, and $\theta_{s,max}$ is the maximum inter-story drift ratio over the height.

Figs. 4. and 5. show the median of normalized maximum floor displacement and the corresponding COV for 6-story frames having different column-to-beam strength ratios at each damage level. From the figures it can be concluded that for the frames having same stories but different column-to-beam strength ratio, the displacement profiles are roughly identical at slight damage state, somewhat different at moderate damage state, and considerably different at severe damage state. With the increase of column-to-beam
strength ratio, the inter-story deformation distributes more uniformly along the height, and the value of normalized maximum floor displacement at upper floors increases.

Figs. 6. and 7. compares the median of normalized maximum floor displacement and the corresponding COV for 9-story frames at different damage state. It is obvious that the shape of the profile is clearly different at different damage state. With the increase of damage degree, the value of normalized maximum floor displacement at upper floors decreases and the deformation inclines to concentrate at lower stories. With the decrease of column-to-beam strength ratio, the difference of the shape of the profile between

![Diagram](image1)

Fig. 4. Median of Normalized Maximum Floor Displacement for 6-Story Frames

![Diagram](image2)

Fig. 5. Coefficient of Variation of Normalized Maximum Floor Displacement for 6-Story Frames

![Diagram](image3)

Fig. 6. Median of Normalized Maximum Floor Displacement for 9-Story Frames
Moderate and severe damage state becomes more distinct. Dispersion is small at slight damage state. With the increase of damage degree, the larger value of dispersion appears at the lower stories where the nonlinear response is more notable.

Fig. 8 compares the median of normalized maximum floor displacement having the same column-to-beam strength ratio of 1.2 but different number of stories. The shape of the profile is substantially affected by the number of stories. With the increase of number of stories, the vibration period is prolonged so that the higher mode effect becomes more evident, and the value of normalized maximum floor displacement at most floors decreases. This tendency becomes more obvious when the damage degree is increased.

Based on the above discussion, the number of stories, damage level, and column-to-beam strength ratio were identified as the main factors influencing the normalized maximum floor displacement.

### 3.2 Statistical Analysis

On the basis of above analysis of computational results, the least-squares regression analysis of the maximum displacement profile was conducted for frames at individual damage state. The elastic fundamental period was employed to represent the effect of number of stories. The following equation was generated:

\[ D_j = (P_1 x_j + P_2 x_j^2 + P_3 x_j^3) H \theta_{x, \text{max}} \]  

where \( D_j \) is the maximum floor displacement at the \( j \)th floor, \( x_j \) is the relative height of the \( j \)th floor normalized by the total height, \( \xi = H_j / H \), in which \( H_j \) is the height of the \( j \)th floor measured from the ground level. \( P_1, P_2, \) and \( P_3 \) are the parameters dependent on the damage state as below:

For slight damage,

\[ P_1 = 0.851 - 0.175 T_1 \]  
\[ P_2 = 0.528 + 0.077 T_1 \]  
\[ P_3 = -0.513 \]  

For moderate damage,

\[ P_1 = 1.563 - 0.456 \eta_c + \left( \frac{0.246}{\eta_c + 1.155} \right) T_1 \]  
\[ P_2 = -0.888 + 0.853 \eta_c + \left( 0.414 - 0.217 \eta_c \right) T_1 \]  
\[ P_3 = 0.066 - 0.322 \eta_c \]  

For severe damage,

\[ P_1 = 1.698 - 0.449 \eta_c + \left( -0.203 - \frac{0.060}{\eta_c - 0.553} \right) T_1 \]  

Fig. 7. Coefficient of Variation of Normalized Maximum Floor Displacement for 9-Story Frames

Fig. 8. Median of Normalized Maximum Floor Displacement for Frames with Different Stories (\( \eta_c = 1.2 \))

**Fig.8.** Coefficient of Variation of Normalized Maximum Floor Displacement for Frames with Different Stories (\( \eta_c = 0.8 \)), (\( \eta_c = 1.2 \)), (\( \eta_c = 1.6 \))
where $T_i$ is the elastic fundamental period of the frame in seconds. The second and third terms in the right-hand side of Eq.6 consider the effect of the higher modes on the displacement profile.

The fitting accuracy of Eq.6 at the slight damage state is good. With the increase of damage degree, the fitting accuracy was reduced.

4. Case Study

The expressions developed above were employed to estimate the maximum displacement profiles of a 12-story RC frame model tested on shaking table in State Key Laboratory for Disaster Reduction in Civil Engineering of Tongji University before (Zhou, 2004). The 1/10-scale model with the span of 0.6m and story height of 0.3m shown in Fig.9. was constructed in line with dynamic similitude theory. The model was made of micro-concrete and mild steel wires. The structural layout in two directions are exactly identical. The Similitude scale factors for the test model are presented in Table 1. The elastic fundamental periods of the first three modes were measured as 0.268s, 0.067s, and 0.036s. In the test the model, subjected to a series of base excitations with gradually increased peak ground acceleration (PGA) ranging from 0.09g to 0.904g, underwent different damage state, cracking, yielding, and ultimate limit state. Plastic hinges formed at the ends of the beams from Floor 3 to 8 at last. El Centro wave (1940), Kobe wave (1995), Shanghai artificial wave, and Shanghai bedrock wave with two or one horizontal component were used as input motions. The model responded much more severely under Shanghai artificial wave.

The seismic response in x direction was larger than y direction since the amplitude of the input motion component in x direction was larger than y direction. The maximum displacement profiles in x and y directions under individual seismic wave were derived by Eq.6 for comparison. Due to the length limitation, only the results in x direction are presented in Fig.10. The comparison is similar in y direction. The results illustrated in Fig.10. show good agreement between the test and estimated profiles. The estimation at slight damage state agrees better with test results than other state. The estimated displacements at upper floors are generally larger than the test results.

5. Conclusions

The research work and main conclusions are summarized as follows:
(1) New expressions are generated to estimate the maximum displacement profiles of regular plane RC frames at slight, moderate, and severe damage state individually on the basis of the statistical analysis of the results obtained by nonlinear time history analysis conducted on 25 frames with different structural characteristic.

(2) The developed expressions associate the maximum floor displacement with the maximum inter-story drift ratio over the height, the elastic fundamental period, and the column-to-beam strength ratio. The shape of the profile is quite different at different damage state.

(3) The dispersion of the maximum floor displacement was measured by the coefficient of variance. Dispersion is comparatively small at slight damage state. With the increase of damage degree, the dispersion grows, and the larger value appears at the lower stories.

(4) The estimated maximum displacement profiles of a tested 12-story RC frame model are verified to be close to the test results. The estimation at slight damage state agrees better with test results than other state.
Fig. 10. Comparison of Maximum Displacement Profiles

(a) El Centro wave (PGA=0.09g, $\theta_{max}=0.0007609$)

(b) Kobe wave (PGA=0.09g, $\theta_{max}=0.00103$)

(c) Shanghai bedrock wave (PGA=0.09g, $\theta_{max}=0.000569$)

(d) Shanghai artificial wave (PGA=0.09g, $\theta_{max}=0.00160$)

(e) El Centro wave (PGA=0.517g, $\theta_{max}=0.00931$)

(f) Kobe wave (PGA=0.517g, $\theta_{max}=0.00615$)

(g) Shanghai bedrock wave (PGA=0.517g, $\theta_{max}=0.00738$)

(h) Shanghai artificial wave (PGA=0.517g, $\theta_{max}=0.0166$)
Acknowledgements

The authors are grateful to the support from National Natural Science Foundation of China under grant No.50708081 and 90815029, Shanghai Pujiang Program under grant No.08PJ14099, and Innovation Program of Shanghai Municipal Educational Committee under grant No.09ZZ32.

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