Effects of Vector Coupling on Chiral and Color-superconducting Phase Transitions – interplay among the scalar, pairing and vector interaction –
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We investigate effects of the vector interaction appearing in chiral effective Lagrangians on the chiral and color superconducting (CSC) phase transitions using Nambu-Jona-Lasinio model. It is shown that the repulsive density-density interaction coming from the vector term enhances competition between the chiral symmetry breaking ($\chi$SB) and CSC phase transition: When the vector coupling is increased, the first order transition between the $\chi$SB and CSC phase becomes weaker, and the coexisting phase in which both the chiral and color-gauge symmetries are dynamically broken comes to exist in a wider region in the $T$-$\mu$ plane. We find that the critical line of the first order transition can have two endpoints for an intermediate range of the vector coupling.

1. Introduction

It is one of the central issues in hadron physics to determine the phase diagram of strongly interacting matter in finite temperature $T$ and density. Recent renewed interest in color superconductivity (CS) \cite{1} has stimulated intensive studies in this field, which have been revealing a rich phase structure at finite density quark matter \cite{2}.

However, although many works on CS have been carried out with the use of effective chiral models, an important interaction in the vector channel

$$L_V = -G_V (\bar{\psi} \gamma^\mu \psi)^2$$

has been scarcely considered. The purpose of this article is to reveal new characteristics of the chiral to CSC transition based on a simple effective model incorporating $L_V$ and explore how the vector interaction affect the phase structure\cite{3}.

The significance to incorporate $L_V$ may be understood as follows. First of all, the instanton-anti-instanton molecule model, as well as the renormalization-group equation, show that $L_V$ appears as a part of the effective Lagrangians. Furthermore, since the vector interaction $L_V$ includes the term $(\bar{\psi} \gamma^0 \psi)^2$, it gives rise to a repulsive energy proportional to the density squared. Notice that the chiral transition at finite density is necessarily accompanied by a change in quark density. Then one expects naturally that $L_V$ causes large effects on the chiral transition; in fact, $L_V$ is known to weaken the chiral phase transition and postpone the transition to a larger chemical potential $\mu$. 
2. Thermodynamic Potential

To investigate the effects of the vector interaction, we use a simple Nambu-Jona-Lasinio model with two flavors and three colors,

\[ \mathcal{L} = \bar{\psi}(i\gamma \cdot \partial - \mathbf{m})\psi + \mathcal{L}_V \\
+ G_S \{ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2 \} + G_C (\bar{\psi}i\gamma_5\tau_2\lambda_2\psi^C)(\bar{\psi}^C i\gamma_5\tau_2\lambda_2\psi). \]  \( \text{(2)} \)

Here, \( \mathbf{m} = \text{diag}(m_u, m_d) \) is the current quark mass of up and down quarks, \( \psi^C \equiv C\bar{\psi}^T \), with \( C = i\gamma^2\gamma^0 \) being the charge conjugation operator, and \( \tau_2 \) and \( \lambda_2 \) are the second component of the Pauli and the Gell-Mann matrices, representing the flavor SU(2)\(_F\) and the color SU(3)\(_C\), respectively.

The scalar coupling constant \( G_S \) and the three momentum cutoff \( \Lambda \) are chosen so as to reproduce the pion mass \( m_\pi \) and the pion decay constant \( f_\pi \) with the current quark mass \( m_u = m_d = 5.5 \text{ MeV} \). The \( G_C/G_S = 0.6 \) is adopted, which gives a similar results for CSC obtained with the instanton-induced interaction [3]. As for the vector coupling, we shall vary it as a free parameter within a range given in the literature to clarify the effects of the vector coupling on the phase diagram.

To determine the phase diagram, we shall calculate the thermodynamic potential \( \Omega \) in the mean-field approximation (MFA), assuming the chiral condensate, \( M_D = -2G_S\bar{\psi}\psi \) and diquark condensate, \( \Delta = \langle \bar{\psi}^C i\gamma_5\tau_2\lambda_2\psi \rangle \). The optimal values of these condensates at given \( T \) and \( \mu \) are determined so that \( \Omega \) has the minimum value. We notice that \( \mathcal{L}_V \) in MFA reduces to \( -2G_V\bar{\psi}\gamma^0\psi\langle \bar{\psi}\gamma^0\psi \rangle + G_V\langle \bar{\psi}\gamma^0\psi \rangle^2 \). Owing to the first term, it is found convenient to introduce a shifted chemical potential as \( \tilde{\mu} = \mu - 2G_V\rho_q \). Here, \( \rho_q \equiv \langle \bar{\psi}\gamma^0\psi \rangle \) is the quark number density: In the following, we shall show the results in terms of the baryon density \( \rho_B = 1/3 \cdot \rho_q \) and the baryon chemical potential \( \mu_B = 3\mu \).

3. Phase Diagrams

As preliminary to the discussion on the effects of the vector interaction, we first present the phase structure without the vector interaction. In Fig. 1(a), the phase diagram in the \( T-\mu_B \) plane is shown. One can see that there are four different phases, i.e. the \( \chi\text{SB} \) phase, the Wigner phase, the CSC phase, and the coexisting phase of \( \chi\text{SB} \) and CS\(_1\). as seen from the upper small panel, which is an enlargement of the part around the solid line near \( T = 0 \). The figure also shows that the chiral transition is first-order at low temperatures and there is an endpoint at finite \( T \) and \( \mu_B \). The phase transition from the CSC to the Wigner phase is second order in our model. These features are qualitatively the same as that in [3] except for the existence of the coexisting phase. We have checked that the appearance of the coexisting phase with vanishing \( G_V \) is sensitive to the parameters; if a slightly larger \( G_S \) is used, the coexisting phase disappears.

The corresponding phase diagram in the \( T-\rho_B \) plane is shown in Fig. 1(b).
Figure 1. The phase diagrams in $T$-$\mu_B$ (a) and $T$-$\rho_B$ (b) plane with $G_V = 0$. The solid line represents the critical line of a first-order phase transition, the dashed line a second-order transition and the dot-dashed line a crossover.

Now, we switch on the vector interaction and discuss the effects of $G_V$. In Fig. 2(a)-(c), we show the phase diagram with $G_V = 0.2, 0.35$, and $0.5$ in the $T$-$\mu_B$ plane. From these figures, one can see that the phase structure is strongly affected by $G_V$ especially near the critical line between the $\chi_{SB}$ and CSC phases:

1. The endpoint of the first-order transition moves toward a lower temperature as $G_V$ is increased, and disappears eventually (Fig. 2(c)). One can also see that the vector coupling postpones the chiral restoration toward larger $\mu_B$ at low temperatures.

2. The region of the coexisting phase becomes broader with finite $G_V$. We have checked that even with the large $G_S$ as used in [4], the coexisting phase comes to exist as $G_V$ is sufficiently increased. Intuitively, this is because the chiral restoration is shifted toward larger $\mu_B$ as $G_V$ is increased, then the system can have a large Fermi surface even with the large constituent quark mass $M$ owing to the large chiral condensate; the larger the Fermi surface, the larger the diquark condensate.

3. In Fig. 2(b), there appear two endpoints at both sides of the critical line of the first order transition. In our calculation, such two-endpoint structure gets to exist in a range of $0.33 \leq G_V/G_S \leq 0.38$.

It is also found that the thermodynamic potential has a shallow minimum in a wide region of the $M_D$-$\Delta$ plane near the two endpoints, which implies the large fluctuations of the chiral and diquark condensates. The underlying mechanism to make the first order transition to the second order in the low $T$ region is understood as follows: First of all, an enhanced coexisting phase is realized due to $G_V$. Then in the high $\mu$ region, the chiral condensate is suppressed owing to the CSC that makes the Fermi surface diffused. This works to weaken the phase transition as $T$ does with the Fermi-Dirac distribution function.

4. Summary

We have investigated effects of the vector interaction (VI) on the chiral and color superconducting phase transitions at finite density and temperature. We have shown that
VI enhances the interplay between the $\chi_{SB}$ and CS phases and that the phase structure is strongly affected by VI especially near the critical line between the $\chi_{SB}$ and CS phases. Although our analysis is based on a simple model, the nontrivial interplay between the $\chi_{SB}$ and CSC phases induced by VI is expected to be a universal phenomenon and should be confirmed and further studied with more realistic models including the random matrix model and on the lattice QCD. As a future task, the color neutrality condition (CNC) should be taken into account; although we believe that the present results for the effects of VI will not change qualitatively with CNC incorporated, the two-endpoint structure realized by a delicate interplay between $\chi_{SB}$ and CS through VI may disappear or persist in the mean-field approximation we employed.

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