Non-equilibrium phase transition in the kinetic Ising model driven by a propagating magnetic field wave

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Abstract
The two-dimensional ferromagnetic Ising model in the presence of a propagating magnetic field wave (with a well-defined frequency and wavelength) was studied by Monte Carlo simulation. This study differs from similar earlier studies, because in earlier studies the oscillating magnetic field was considered to be uniform in space. The time averaged magnetization over a full cycle (the time period) of the propagating magnetic field acted as the dynamic order parameter. The dynamical phase transition was observed. The temperature variation of the dynamic order parameter, the mean square deviation of the dynamic order parameter, the dynamic specific heat and the derivative of the dynamic order parameter were studied. The mean square deviation of the dynamic order parameter and the dynamic specific heat show sharp maxima near the transition point. The derivative of the dynamic order parameter shows a sharp minimum near the transition point. The transition temperature is found to depend also on the speed of propagation of the magnetic field wave.

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1. Introduction
The non-equilibrium response of the Ising ferromagnet in the presence of a time-varying magnetic field has been widely studied [1]. Among all the dynamical responses (e.g. hysteretic response, dynamic phase transition and stochastic resonance), non-equilibrium dynamical phase transition is an important phenomenon [1] and it became an interesting field of research recently. This dynamic phase transition bears several similarities to equilibrium thermodynamic phase transition. Efforts in studying the invariance of time scale (i.e. critical slowing down) [2], divergence of specific heat [2], divergence of critical fluctuations in energy [3], divergence of length scale near the transition point [4] and the order of the transition [5] established the dynamic transition as an interesting non-equilibrium phase transition. This dynamic transition is closely related to the hysteretic loss [6] and stochastic resonance [7]. The existence of dynamic transition was found experimentally [8] in Co film on a Cu surface (at room temperature) by using the surface magneto-optic Kerr effect. Recently, evidence for dynamic phase transition was found experimentally [9] in the [Co(4 Å)Pt(7 Å)]₉ multilayer system with strong perpendicular anisotropy by applying a time-varying (sawtooth-type) out-of-plane magnetic field in the presence of a small additional constant magnetic field. In this study, the dynamic phase boundary was drawn and found to be similar to that obtained from a simulation in the kinetic Ising model in analogous conditions.

The dynamic phase transition was also observed previously [10–13] in other ferromagnetic models. It was studied [14] in the Ginzburg–Landau model of anisotropic XY ferromagnet and different types of chaotic behaviour were observed. Recently, multiple dynamic transitions were observed [15, 16] in the anisotropic Heisenberg model. These studies were reviewed recently [17].

However, all these earlier studies on the dynamic phase transition were carried out with a time-varying magnetic field that was uniform in space. No attempt has been made to study the dynamic phase transition with a magnetic field depending on both space and time. In this paper, the dynamic phase transition in the Ising ferromagnet in the presence of a propagating magnetic field wave is studied by Monte Carlo simulation [18].

This paper is organized as follows. Section 2 is devoted to describing the model and the Monte Carlo simulation method.
The simulation results are reported in section 3 and the paper ends with a summary in section 4.

2. The model and simulation

The Hamiltonian of an Ising model (with ferromagnetic nearest-neighbour interaction) defined in two dimensions (square lattice) in the presence of a propagating magnetic field wave can be represented as

\[ H = -J \sum_{\langle ij \rangle} s_i s_j - \sum_i h(\vec r, t) s_i. \]  

(1)

Here, \( s_i \) (\( \pm 1 \)) is the Ising spin variable, \( J > 0 \) is the ferromagnetic interaction strength and \( h(\vec r, t) \) is the value of the propagating magnetic field wave at any time \( t \) and position \( \vec r \). The propagating magnetic field \( h(\vec r, t) \) is represented as

\[ h(\vec r, t) = h_0 \cos(\omega t - K y), \]  

(2)

where \( h_0 \) is the amplitude, \( \omega (= 2\pi f) \) is the angular frequency of the oscillating field and \( K (= 2\pi / \lambda) \) is the wave vector. Here, \( f \) is the frequency and \( \lambda \) the wavelength (measured in units of lattice spacing) of the propagating magnetic field wave. The wavelength considered is smaller than and commensurate with the lattice size \( L \). The direction of propagation of the magnetic field wave \( \left( h(y, t) \right) \) is taken along the \( y \)-direction only. It may be noted here that all earlier studies of the dynamical phase transitions were carried out with an oscillating (in time) but uniform (over the space) magnetic field. The boundary condition is taken as periodic in all directions. This completes the description of the model.

In the simulation, the system is cooled gradually from a high temperature. Randomly selected 50% up \( (s_i = +1) \) spins are taken as the initial configuration. Physically, this corresponds to the high-temperature configuration of spins. In the cooling process, the last spin configuration corresponding to a particular temperature was used as the initial configuration of the next lower temperature. At any finite temperature \( T \), the dynamics of this system is studied here by Monte Carlo simulation using the Metropolis single spin-flip rate [18]. The transition rate is specified as

\[ W(s_i \rightarrow -s_i) = \text{Min}[1, \exp(-\Delta H/k_B T)], \]  

(3)

where \( \Delta H \) is the change in energy due to spin flip \( (s_i \rightarrow -s_i) \) and \( k_B \) is the Boltzmann constant. Any lattice site is chosen randomly and the spin variable \( (s_i) \) is updated according to the Metropolis spin-flip probability. \( L^2 \) such updates constitute the unit (Monte Carlo step per spin (MCSS)) of time. The instantaneous bulk magnetization (per site), \( m(t) = (1/L^2) \sum_i s_i \), has been calculated. The time-averaged (over the complete cycle of the propagating magnetic field wave) magnetization,

\[ Q = \frac{1}{\tau} \int m(t) \, dt, \]  

(4)

defines the dynamic order parameter [1].

The frequency is \( f = 0.01 \) (kept fixed throughout the study). Hence, one complete cycle of the propagating field takes 100 MCSS (time period \( \tau = 1/f = 100 \) MCSS). A time series of magnetization \( m(t) \) has been generated up to \( 2 \times 10^3 \) MCSS. This time series contains \( 2 \times 10^3 \) (since \( \tau = 100 \) MCSS) cycles of the oscillating field. Here, the first \( 10^3 \) of such transient values are discarded to get the stable values of the dynamical quantities. The dynamic order parameter \( Q \) has been calculated over \( 10^3 \) values. It has been checked (for a few values) that this number of samples \( (N_Q) \) is sufficient to get the stable values of the dynamical quantities. Hence, the statistics (distribution of \( Q \)) is based on \( N_Q = 10^3 \) different values of \( Q \). To have confidence with this number of samples, the mean square deviation (i.e. \( \langle (\delta Q)^2 \rangle = \langle Q^2 \rangle - \langle Q \rangle^2 \) of \( Q \) is also calculated and studied as a function of temperature. It may be noted that values of the dynamic order parameter (at lower temperatures) become both positive and negative with equal probability. Here, only the positive values of \( Q \) are shown. The statistical error \( (\Delta) \) in calculating \( Q \) may be defined as the square root of \( \langle (\delta Q)^2 \rangle \). The maximum error \( (\Delta_{\text{max}}) \) occurs near the transition point and this reasonably indicates the critical fluctuations.

The time-averaged dynamic energy is defined as

\[ E = \frac{1}{\tau} \int H \, dt. \]  

(5)

The dynamic specific heat \( (C = \frac{dE}{dT}) \) is also calculated. The temperature variations of all these (above-mentioned) quantities are studied. The temperature \( T \) is measured in units of \( J/k_B \), and the field amplitude \( h_0 \) and energy \( E \) are measured in units of \( J \).

3. Results

To investigate the nature of the spatio-temporal variations of the field \( h(y, t) \) and the local ‘strip magnetization’, \( m(y, t) = \int_{-L/2}^{L/2} m(x, y, t) \, dx \), where \( s(x, y) = \pm 1 \) is the spin variable at the position \( (x, y) \), they are studied as a function of the coordinate \( y \) (along the direction of propagation of the field wave) for different times \( (t) \). Figure 1 shows such plots. From the figure, the propagating nature of the field wave and the ‘strip magnetization’ is clear. It may be noted that, for a particular instant of time, the magnetic field and the ‘strip magnetization’ differ by a phase. It is observed that this phase difference depends on the temperature of the system, the wavelength and the frequency of the propagating magnetic field wave. A systematic study of this dependence requires considerable computational effort and time.

The temperature variation of the dynamic order parameter is studied. This is shown in figure 2. For fixed values of the amplitude, the frequency and the wavelength of the propagating magnetic field wave, it is observed that below a certain temperature the dynamic ordering develops \( (Q \neq 0) \) and vanishes \( (Q = 0) \) above it. Keeping the values of the frequency \( (f) \) and the wavelength \( (\lambda) \) of the propagating magnetic field wave fixed, if the amplitude \( (h_0) \) of the field increases the dynamic phase transition occurs at lower temperature. For comparison, similar studies are carried out for a non-propagating (sinusoidally oscillating in time but uniform in space) magnetic field with the same frequency and amplitude. This clearly indicates that the dynamic transition occurs at different higher temperatures than those observed in the case of a propagating field.
Figure 1. Spatio-temporal variations of the propagating magnetic field wave \((h(y, t))\) and ‘strip magnetization’ \((m(y, t))\) for \(h_0 = 0.5\), \(T = 1.50\) and \(\lambda = 25\). The magnetic field and magnetization are represented by open circles and bullets, respectively. Continuous lines joining the data points are a guide to the eye. The plots for different times \((t)\) are shown: (a) \(t = 100001\) MCSS, (b) \(t = 100025\) MCSS, (c) \(t = 100050\) MCSS and (d) \(t = 100075\) MCSS.

Figure 2. Temperature \((T)\) variations of the dynamic order parameter \(Q\) for different types of fields. Symbols: (o) for a propagating wave field with \(h_0 = 0.5\), \(\lambda = 25\) (and \(\Delta_{\text{max}} = 0.216\)), (●) for a propagating wave field with \(h_0 = 0.3\), \(\lambda = 25\) (and \(\Delta_{\text{max}} = 0.197\)), (○) for a non-propagating field with \(h_0 = 0.5\) (and \(\Delta_{\text{max}} = 0.167\)) and (□) for a non-propagating field with \(h_0 = 0.3\). \(f = 0.01\) for both types of fields. Continuous lines just join the data points.

Figure 3. Temperature \((T)\) variations of mean square deviation \((\langle\delta Q^2\rangle)\) of the dynamic order parameter \(Q\) for different types of fields. Symbols: (o) for a propagating wave field with \(h_0 = 0.5\) and \(\lambda = 25\), (●) for a propagating wave field with \(h_0 = 0.3\) and \(\lambda = 25\), (○) for a non-propagating field with \(h_0 = 0.5\) and (□) for a non-propagating field with \(h_0 = 0.3\). \(f = 0.01\) for both types of fields. Continuous lines just join the data points.

These dynamic transition temperatures can be estimated by studying the temperature variations of the mean square fluctuations \((\langle\delta Q^2\rangle)\) of the dynamic order parameter \(Q\). These results are shown in figure 3. Here, the \((\delta Q^2)\) shows a very sharp maximum, indicating the dynamic transition temperature. From this one can estimate the maximum error \((\Delta_{\text{max}})\) involved in statistical calculation of the dynamic order parameter \(Q\). For a propagating magnetic field wave of \(f = 0.01\) and \(\lambda = 25\), the dynamic phase transitions (indicated by the maxima of \((\delta Q^2)\)) occur at \(T = 1.50\) and \(T = 1.88\) for the field amplitudes \(h_0 = 0.5\) and \(h_0 = 0.3\), respectively. Also, for comparison, similar studies were carried out in the case of a non-propagating magnetic field. For \(f = 0.01\) the dynamic phase transitions occur at \(T = 1.68\) and \(T = 1.94\) for \(h_0 = 0.5\) and \(h_0 = 0.3\), respectively.

The derivative \((\frac{dQ}{dT})\) of the dynamic order parameter \(Q\) is calculated by the central difference formula [19]

\[
\frac{dQ}{dT} = \frac{Q(T + \Delta T) - Q(T - \Delta T)}{2\Delta T}.
\]  

In the simulation, the system was being cooled from a high temperature (random spin configuration) to a certain temperature slowly in steps of \(\Delta T = 0.02\). It may be noted that the error in calculating the derivative numerically by this central difference formula is \(O((\Delta T)^2)\) [19]. Hence, the error involved is of the order of 0.0004. The temperature variation of the derivative of the dynamic order parameter was studied, and the results are shown in figure 4. Here, the derivative shows a very sharp minimum, indicating the dynamic phase transition temperature. For a propagating magnetic field wave of \(f = 0.01\) and \(\lambda = 25\), the dynamic phase transitions (indicated by very sharp minima of \((\Delta Q^2)\)) are observed to occur at \(T = 1.50\) and \(T = 1.88\) for the field amplitudes \(h_0 = 0.5\) and \(h_0 = 0.3\), respectively. For comparison, similar studies were carried out in the case of a non-propagating magnetic field. For \(f = 0.01\) the dynamic phase transitions occur at \(T = 1.68\) and \(T = 1.94\) for \(h_0 = 0.5\) and \(h_0 = 0.3\), respectively.
heat becomes maximum near the dynamic transition point indicating the dynamic transition independently. Here, the term independently means the following: the dynamic phase transition was studied and the transition temperature was estimated from two types of quantities. One is the dynamic order parameter $Q$ and its derivatives ($\frac{dQ}{dT}$), moments ($\langle \delta Q^2 \rangle$), etc. These depend directly on $Q$. Another quantity is the dynamic specific heat ($C = \frac{dE}{dT}$), which is not directly related to $Q$. For a propagating magnetic field wave of $f = 0.01$ and $\lambda = 25$, the dynamic phase transitions (indicated by the maxima of $C = \frac{dE}{dT}$) occur at $T = 1.50$ and $T = 1.88$ for the field amplitudes $h_0 = 0.5$ and $h_0 = 0.3$, respectively. Also, for comparison, similar studies were carried out in the case of a non-propagating magnetic field. For $f = 0.01$, the dynamic phase transitions occur at $T = 1.68$ and $T = 1.94$ for $h_0 = 0.5$ and $h_0 = 0.3$, respectively.

The dependence of the dynamic phase transition on the speed of propagation of the propagating magnetic field was studied briefly. For $f = 0.01$, $h_0 = 0.5$ the temperature variations of the dynamic order parameters for $\lambda = 25$ and $\lambda = 50$ were studied. The results are shown in figure 6. It is observed that dynamic transition occurs at higher temperature for higher speed ($v = f\lambda$) of propagation of the propagating magnetic field.

The dynamical transition temperature $T_c$ was measured for a system of linear size $L = 100$. A systematic finite size analysis has not yet been performed. However, a few results were checked for smaller (say $L = 50$) system sizes. No appreciable change in $T_c$ was observed.

4. Summary

The dynamical response of the two-dimensional Ising ferromagnet in the presence of a propagating magnetic field wave was studied by Monte Carlo simulation. A dynamical phase transition was observed. This dynamical phase transition was observed from studies of the temperature variations of the dynamic order parameter, the derivative of the dynamic order parameter, the mean square deviation of the dynamic order parameter and the dynamic specific heat. All these studies indicated the dynamic phase transition, and the transition temperatures were estimated.

For comparison, the dynamic transition was studied also for a non-propagating (sinusoidally oscillating in time but uniform in space) magnetic field. It is observed that
the dynamic transition temperatures are different from that observed in the case of a propagating magnetic field wave. It is observed, from figures 3–5, that the propagating field wave causes a dynamical phase transition at lower temperature than that obtained from a non-propagating field of the same amplitude. One may argue that since the propagating magnetic field makes the strip magnetization a wave-like structure, the value of $Q$ will be less than that for a non-propagating field of the same amplitude and frequency at the same temperature. This would cause the transition to take place at lower temperature.

The dependence of the transition temperature on the speed of propagation of the propagating magnetic field wave was studied briefly and it was observed that the transition takes place at higher temperature for higher values of the speed of propagation. One may try to understand this fact in the following way: increasing wavelength (or speed for a fixed frequency) simply makes the field more homogeneous, approaching the infinite wavelength spatially homogeneous limit. It does appear that the transition for a propagating field (with $h_0 = 0.5$) has shifted from $T = 1.50$ to that obtained for the approximately spatially homogeneous case, i.e. $T = 1.68$.

Observations, based on the Monte Carlo simulation, were reported here briefly. The dynamical phase boundary for a propagating magnetic field wave is yet to be sketched and the dependence of the phase boundary on the frequency and wavelength of the propagating wave is yet to be determined. A finite size analysis and a detailed study of the behaviour of the phase difference between a propagating magnetic field wave and ‘strip magnetization’ remain to be done; they require considerable computational effort and will be reported later. The non-equilibrium dynamic phase transition in the Ising ferromagnet in the presence of a propagating magnetic field wave will be tackled in the near future.

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