Violations of Bell Inequalities for Measurements with Macroscopic Uncertainties: What does it Mean to Violate Macroscopic Local Realism?

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Abstract

We suggest to test the premise of “macroscopic local realism” which is sufficient to derive Bell inequalities when measurements of photon number are only accurate to an uncertainty of order \( n \) photons, where \( n \) is macroscopic. Macroscopic local realism is only sufficient to imply, in the context of the original Einstein-Podolsky-Rosen argument, fuzzy “elements of reality” which have a macroscopic indeterminacy. We show therefore how the violation of local realism in the presence of macroscopic uncertainties implies the failure of “macroscopic local realism”. Quantum states violating this macroscopic local realism are presented.

I. INTRODUCTION

There is increasing evidence for the failure of “local realism” as defined originally by Einstein, Podolsky and Rosen [1], Bohm [2] and Bell [3,4]. For certain correlated quantum systems, Einstein, Podolsky and Rosen argued that “local realism” is sufficient to imply that the results of measurements are predetermined. These predetermined “hidden variables” (called “elements of reality” by Einstein, Podolsky and Rosen) exist to describe the value of a physical quantity, whether or not the measurement is performed, and as such are not part of a quantum description. Bell later showed that the predictions of quantum mechanics for certain ideal quantum states could not be compatible with such hidden variable theories.

It is now widely accepted therefore, as a result of Bell’s theorem and related experiments [3], that local realism must be rejected. However the rejection of local realism implied by these results is at the most microscopic level of a single photon, in the sense that the “hidden variables” (or “elements of reality”) and the experimental results of measurements involved must be defined to the precision of one photon or better in order to prove local realism invalid. The failure of local realism in microscopic systems has long been associated with the existence of entangled quantum superposition states. In microscopic systems there can only be superpositions of states microscopically distinct.
Little is known about the validity of local realism in more “macroscopic experiments”, where experimental uncertainties are larger (becoming macroscopic) in size, in an absolute sense. At least there has to our knowledge been no formulation of a Bell-type theorem or related experimental investigation for such a situation. Previous works \[6\text{–}8\] suggestive of incompatibilities of local realism in macroscopic systems have considered the case where the measurements are performed with perfect accuracy and are thus not examples of macroscopic experiments as we have defined them here.

Generally it is thought that true macroscopic quantum effects come about from quantum superpositions of states which are macroscopically distinguishable \[9,10\], often referred to as “Schrodinger cat” states. This point has been much discussed. Leggett and Garg \[10\] have shown incompatibility of such macroscopic quantum states with the combined premises of “macroscopic realism” and “macroscopic noninvasive measurability”. They however considered macroscopic quantum superposition states at a single location only, and did not introduce the premise of locality.

There has been much interest and debate over whether or not “Schrodinger cat” states can truly exist. The existence, for which there is now experimental evidence \[11\text{–}13\], of “Schrodinger cat” states would appear to be closely linked to the question of the validity of local realism at the more macroscopic level we have described. One would suspect that a violation of local realism evident in an experiment where uncertainties are large would be due more to entangled macroscopic superpositions than microsuperpositions.

In this paper we begin (in Section 2) by defining the physical premise of “macroscopic local realism” \[14\] so as to identify the peculiar features of the macroscopically entangled quantum states in a way which is independent of the quantum formulation. Macroscopic local realism is only sufficient to assign, to a system, predetermined “elements of reality” (or hidden variables) which are intrinsically macroscopic, in that they have a macroscopic indeterminacy in their values.

Suppose our “Schrodinger’s cat” is correlated with a second system spatially separated from the cat, for example a gun used to kill the cat. Let us suppose a gun that has been fired implies a dead cat; a gun that has not been fired implies a cat that is alive. We can predict the result for a measurement of the cat (whether dead or alive), without disturbing the cat, by measurement on the gun. Macroscopic local realism is the premise used to imply the existence of an element of reality for the cat. The element of reality in this case is a variable that assumes one of two values, one value corresponding to the “dead” state and the other value corresponding to the “alive” state. The assignment of this element of reality then means that the cat is always either “dead” or “alive”, regardless of whether or not it is being observed or measured. Macroscopic local realism is used in this case, as opposed to local realism as used originally by Bell and Einstein et al, because the two possible results of measurement of the cat, “alive” and “dead”, are macroscopically distinct. To summarise, the rejection of macroscopic local realism in this example means that we cannot think of the cat as being either dead or alive, even though we can predict the “dead” or “alive” result of “measuring” the cat, without disturbing the cat, by measuring the correlated spatially-separated second system, which in this case is the gun. The rejection of macroscopic local realism is a more startling result than, and is not implied by, the rejection of local realism as indicated by Bell’s theorem.

In Section 3 we point out that for situations where the possible results are all macroscopic-
ically distinguishable, we need only assume the strong premise of macroscopic local realism in order to derive the Bell inequalities. We then focus attention on the more general case where the results of measurement may be microscopically separated. We show that with the addition of macroscopic classical noise sources which model a macroscopically imprecise measurement, one may derive the Bell inequalities using only the premise of macroscopic local realism. Thus if a violation of a Bell inequality is maintained in the presence of macroscopic uncertainties in the measurement process, we have direct evidence for an incompatibility with macroscopic local realism.

In Section 4 quantum states are presented which show a violation of Bell’s inequality with such macroscopic noise, thus indicating an incompatibility of the predictions of quantum mechanics with the very strict form of macroscopic local realism we have defined. We believe this is the first such result although some preliminary results presented in this paper have been published previously \[15\]. Such a test of macroscopic local realism provides an avenue to focus the peculiar macroscopic nonlocal aspects of the “macroscopic entangled quantum state”.

The application of Bell inequality theorems, and the effect of noise on the violations predicted, to situations where many photons fall on a detector is relevant to the question of whether or not tests of local realism can be conducted in the experiments such as those performed by Smithey et al \[16\]. Here correlation of photon number between two spatially separated but very intense fields is sufficient to give “squeezed” noise levels. In these high flux experiments detection losses can be relatively small, allowing for the possibility of violation of a strong Bell inequality, but noise which limits the resolution of the photon number measurement can be large in absolute terms, as compared to traditional Bell inequality experiments which involve photon counting with low incident photon numbers.

II. DEFINITIONS OF MACROSCOPIC LOCAL REALISM

In the original argument of Einstein, Podolsky and Rosen \[1\] (EPR), “local realism” is defined in the following way. “Realism” is sufficient to state that if one can predict with certainty the result of a measurement of a physical quantity at \( A \), without disturbing the system \( A \), then the results of the measurement were predetermined and one has an “element of reality”, corresponding to this physical quantity. The element of reality is a variable which assumes one of the set of values which are the predicted results of the measurement. Locality postulates that measurements at \( B \) cannot disturb \( A \) in any way. Taken together with realism then, as defined above, “local realism” is sufficient to imply that, if one can predict the result of a measurement at \( A \), by making a simultaneous measurement at \( B \), then the result of the measurement at \( A \) is a predetermined property of the system \( A \). In the case of perfect correlation and perfect measurements, the predetermined value (the element of reality) for any individual system \( A \) will have zero uncertainty, since we can determine it precisely by measurements on \( B \), and because all orders of change to system \( A \) as a result of the measurement at \( B \) are excluded by locality.

Macroscopic local realism may be defined as a premise stating the following. This meaning and definition of “macroscopic local realism” has been previously introduced in references \[14\] and \[15\] in line with the original EPR argument, and its experimental realisation for
continuous variables by Ou et al. If one can predict the result of a measurement at \( A \) by performing a simultaneous measurement on a spatially separated system \( B \), then the result of the measurement at \( A \) is predetermined but described by an element of reality which has an indeterminacy in each of its possible values, so that only values macroscopically different to those predicted are excluded. We note that the meaning of “predict” in the above definition could be loosened to allow an uncertainty in the prediction, as one would have in macroscopic experiments which incorporate measurement uncertainties.

Macroscopic local realism incorporates two assumptions. We define a “macroscopic locality”, which states that measurements at a location \( B \) cannot instantaneously induce changes of a macroscopic magnitude (for example the dead to alive state of a cat, or a change between macroscopically different photon numbers) in a second system \( A \) spatially separated from \( B \). Locality in its entirety, as used originally by EPR and Bell, postulates that measurements at \( B \) cannot disturb \( A \) in any way. We expect that our definition of a macroscopic order of locality is equivalent to postulating that locality will always be appear to be satisfied in experiments where measurement uncertainties do not enable resolution of results different by a microscopic or mesoscopic number of photons.

The second assumption incorporated by macroscopic local realism is the assumption of a “macroscopic realism”, since macroscopic local realism implies elements of reality with (up to) a macroscopic indeterminacy. Suppose an element of reality may be symbolised by the variable \( x \), where \( x \) can take on numerical values \( x_1, x_2, ... \). For microscopic realism these values are specified to a microscopic level. For “macroscopic realism” these values have a macroscopic indeterminacy, by this meaning that one can only exclude values for the associated physical variable which are macroscopically different to the values \( x_1, x_2, ... \). We see that if \( x_1, x_2 \) are only microscopically distinct, they are in this case no longer distinguished by different hidden variable values.

The notion of realism is exclusive of “quantum superposition states” in the following sense. If a physical quantity for an ensemble of systems is attributed an element of reality \( x \) as above, then the element of reality for each individual system will take on one of the values \( x_1, x_2, ... \). This value is the result of the measurement of the physical quantity, should it be performed. This element of reality picture is different to the standard quantum picture of a system being in a “quantum superposition” of two states of different \( x_i \). According to standard quantum mechanics interpretation, an individual system described by such a superposition cannot be thought of as being in one or other of the two states prior to measurement. If the values of the element of reality are defined with zero uncertainty then the element of reality picture excludes (or is different in its interpretation to) a “quantum superposition” of states \( x_i \) and \( x_i + \delta \) where \( \delta \) is nonzero.

We consider the existence of an element of reality which is only macroscopically specified, having values that can only be specified not to be macroscopically different to a value \( x \). This macroscopic realism description says nothing about the possibility of superpositions of states microscopically or mesoscopically different to \( x \). Macroscopic local realism cannot exclude the possibility of “quantum superpositions” of states micro- or meso-scopically different, with respect to the physical quantity represented by the element of reality. We can however exclude the possibility of quantum superpositions of states with macroscopically different values for the physical quantity concerned.
Since it says nothing about microscopic systems, macroscopic local realism is a less restrictive premise than “local realism” used in its entirety. Local realism in its full sense can define elements of reality with values having no uncertainty and therefore can exclude the possibility of quantum superpositions of states with all separations (micro to macro inclusive) in the relevant variable.

III. BELL INEQUALITIES WITH NOISE: TESTS OF MACROSCOPIC LOCAL REALISM

Our proposed experiment to test macroscopic local realism is depicted in Figure 1, where \( \hat{a}_\pm \) and \( \hat{b}_\pm \) are boson operators for outgoing fields, generated from a suitable source to be discussed in Section 4, at the spatially separated locations \( A \) and \( B \) respectively. We define the Schwinger spin operators

\[
\hat{S}_x^A = \frac{(\hat{a}_+^\dagger \hat{a}_- + \hat{a}_-^\dagger \hat{a}_+)}{2} \\
\hat{S}_y^A = \frac{(\hat{a}_+^\dagger \hat{a}_- - \hat{a}_-^\dagger \hat{a}_+)}{2i} \\
\hat{S}_z^A = \frac{(\hat{a}_+^\dagger \hat{a}_- - \hat{a}_-^\dagger \hat{a}_+)}{2}.
\]

(1)

Similar operators \( \hat{S}_x^B, \hat{S}_y^B, \hat{S}_z^B \) are defined for the modes at \( B \). We measure simultaneously at \( A \) and \( B \) the Schwinger spin operators

\[
\hat{S}_\theta^A = \hat{S}_x^A \cos \theta + \hat{S}_y^A \sin \theta
\]

(2)

and

\[
\hat{S}_\phi^B = \hat{S}_x^B \cos \phi + \hat{S}_y^B \sin \phi
\]

(3)

respectively.

In Figure 1a the measurement at \( A \) is performed with phase shift \( \theta \) and beam splitter to produce \( \hat{c}_\pm' = (\hat{a}_+ \pm \hat{a}_- \exp(-i\theta))/\sqrt{2} \), followed by photodetection. At \( B \) modes \( \hat{d}_\pm' = (\hat{b}_+ \pm \hat{b}_- \exp(-i\phi))/\sqrt{2} \) are similarly generated. The possible outcomes for the photon number \( \hat{c}_\pm' \hat{c}_\pm' \) (and \( \hat{d}_\pm' \hat{d}_\pm' \)) are 0, 1, ... in integer steps. The spin values for \( \hat{S}_\theta^A \) and \( \hat{S}_\phi^B \) are then given by the photon number differences \( \hat{n}_\theta^A = 2\hat{S}_\theta^A = \hat{c}_+^\dagger \hat{c}_+ - \hat{c}_-^\dagger \hat{c}_- \) and \( \hat{n}_\phi^B = 2\hat{S}_\phi^B = \hat{d}_+^\dagger \hat{d}_+ - \hat{d}_-^\dagger \hat{d}_- \).

Alternatively in Figure 1b, the \( \hat{a}_\pm \) are first combined through a beam splitter, and phase shifted, to give outgoing fields \( \hat{a}_- = (\hat{a}_- - \hat{a}_+)/\sqrt{2} \) and \( \hat{a}_+ = i(\hat{a}_+ + \hat{a}_-)/\sqrt{2} \). These may now be considered system fields, upon which the measurement \( \hat{n}_\theta^A = 2\hat{S}_\theta^A = \hat{c}_+^\dagger \hat{c}_+ - \hat{c}_-^\dagger \hat{c}_- \) is made through the transformation (with polariser or beam splitter) \( \hat{c}_+ = \hat{a}_+^\dagger \cos \theta/2 + \hat{a}_-^\dagger \sin \theta/2 \) and \( \hat{c}_- = \hat{a}_+^\dagger \sin \theta/2 - \hat{a}_-^\dagger \cos \theta/2 \) followed by photodetection. Figure 1b depicts a measurement \( \hat{S}_z^A \cos \theta + \hat{S}_y^A \sin \theta \) made on system operators \( \hat{a}_\pm' \), but is the same measurement depicted in 1a for the fields \( \hat{a}_\pm \). We use similar definitions \( \hat{S}_x^A, \hat{S}_y^A, \hat{S}_z^A \) for the Schwinger operators in terms of \( \hat{a}_\pm' \). Similar transformations are defined for the measurement at \( B \). We present this scheme because, for the particular choice of quantum state discussed in Section 4, it ensures both fields \( \hat{a}_\pm \) incident on the measurement apparatus (polarizer) can
be macroscopic. This arrangement then is crucial in providing a test of macroscopic local realism.

We classify the result of our measurement as +1 if the result for the photon number difference measurement $\hat{n}_A^\theta$ or $\hat{n}_B^\phi$ is positive or zero, and −1 otherwise. The results at $B$ are classified similarly. We build up the following probability distributions: $P_+^A(\theta)$ for obtaining + at $A$; $P_+^B(\phi)$ for obtaining + at $B$; and $P_{++}(\theta, \phi)$ the joint probability of obtaining + at both $A$ and $B$.

We first consider the predictions as given by the original definition of local realism (local hidden variables) used by Einstein-Podolsky-Rosen, Bell and Clauser-Horne\(^3,4\). The probability of obtaining +1 for $S_A^\theta$ is expressed as

$$P_+^A(\theta) = \int \rho(\lambda) \ p_+^A(\theta, \lambda) \ d\lambda. \hspace{1cm} (4)$$

The probability of obtaining ‘+1’ for $S_B^\phi$ is

$$P_+^B(\phi) = \int \rho(\lambda) \ p_+^B(\phi, \lambda) \ d\lambda. \hspace{1cm} (5)$$

The joint probability for obtaining ‘+1’ for both of two simultaneous measurements with $\theta$ at $A$ and $\phi$ at $B$ is

$$P_{++}(\theta, \phi) = \int \rho(\lambda) \ p_+^A(\theta, \lambda)p_+^B(\phi, \lambda) \ d\lambda \hspace{1cm} (6)$$

Here $p_+^A(\theta, \lambda)$ is the probability for getting the result +1 given the hidden variables $\lambda$; $p_+^B(\phi, \lambda)$ is the probability for getting the result +1 given $\lambda$; while $\rho(\lambda)$ is the probability distribution for the hidden variables $\lambda$.

It is well known\(^5\) that one can derive the following “strong” Bell-Clauser-Horne inequality from the assumptions of local realism made so far.

$$S = \frac{P_{++}(\theta, \phi) - P_{++}(\theta, \phi') + P_{++}(\theta', \phi) + P_{++}(\theta', \phi')}{P_+^A(\theta') + P_+^B(\phi')} \leq 1 \hspace{1cm} (7)$$

To date this “strong” inequality has not been violated in any experiment, because of the poor detection inefficiencies which occur in photon counting experiments. It is well documented that it is possible to derive, with the assumption of additional premises, a weaker form of the Bell inequality which has been violated in photon counting experiments where detection losses are high. In this paper however we restrict attention to the strong inequalities which do not require additional assumptions. Our proposed experiments involve photodiode detectors which have high efficiencies and therefore allow the possibility of a strong violation of local realism.

In deriving the Bell inequalities, one specifies a probability $p_+^A(\theta, \lambda)$ for getting the result +1 as opposed to −1 given the hidden variables $\lambda$. If the results +1 and −1 are always macroscopically different, it becomes apparent that one need only assume “macroscopic local realism” as opposed to local realism in its entirety to obtain the Bell inequalities. This is because in assuming the independence of this probability $p_+^A(\theta, \lambda)$ on $\phi$, we need only assume a macroscopic locality, that the measurement at $B$ does not disturb the system at $A$.
in a macroscopic way to make the change from $+1$ to $-1$. The elements of reality need only be specified “macroscopically”, that is they can have a macroscopic indeterminacy in their values, and still adequately represent the distinct outcomes of measurement. We can add certain (though not all) perturbations of a macroscopic size (in photon number) to the values predicted by the “elements of reality” and not change the final form of the Bell inequality.

The violation of the Bell inequality (7), where the possible results of all relevant measurements (for all relevant angles $\theta$ and $\phi$) are macroscopically distinct, would be firm confirmation of an incompatibility with macroscopic local realism. To our knowledge no such violation has yet been demonstrated.

In order to test conclusively for macro forms of local realism in more general situations (where the possible results are not always macroscopically separated), we propose to add local noise sources to the final readout stage of each of the measurement processes, at $A$ and $B$. We will assume that the result for the photon number difference $\hat{n}_A^\theta$ or $\hat{n}_B^\phi$ at each of $A$ and $B$ respectively is of the form $n + \text{noise}$, where $n$ is the result of the measurement in the absence of the noise and $\text{noise}$ is a local classical noise term. The noise terms at $A$ and $B$ are independent, modeling a local physical source of noise, and as such always satisfy locality, the noise added at $A$ for example being independent of the experimental choice of the angle $\phi$ at $B$.

We will derive a Bell inequality based on the premise of macroscopic local realism alone, by showing that the addition of this classical noise to the final measurement result can alter the premises needed to derive the Bell inequality.

We first define the probability $P_{ij}^{0,AB}(\theta, \phi)$ for obtaining results $i/2$ and $j/2$ respectively upon joint measurement of $S_A^\theta$ at $A$, and $S_B^\phi$ at $B$, in the absence of the applied noise. The $i$ and $j$ are then results for the photon number differences $\hat{n}_A^\theta$ or $\hat{n}_B^\phi$ respectively. In terms of a local hidden variable description, this probability is given by

$$P_{ij}^{0,AB}(\theta, \phi) = \int \rho(\lambda) \ p_i^A(\theta, \lambda)p_j^B(\phi, \lambda) \ d\lambda$$ (8)

We next outline how the assumption of local realism, as defined originally by EPR, implies the hidden variable description (8) above. This is in order to postulate how the above expression is modified if one makes only the macroscopic local realism assumption.

A perfect correlation between measurement results at $A$ and $B$ is predicted possible for some quantum states. For such situations, it is possible to predict precisely the result of a measurement at $A$ by performing a particular measurement at $B$. We are able to deduce $\Box$, assuming local realism and following the reasoning of EPR as outlined in Section 2, the existence of a set of “elements of reality”, $m_A^\theta$ and $m_B^\phi$, one for each subsystem at $A$ and $B$, and one for each choice of measurement angle, $\theta$ or $\phi$, at $A$ or $B$ respectively. The $m_A^\theta$ assumes one of a set of definite values, this value giving the result of the measurement $\theta$ at $A$ should it be performed. The set $m_A^\theta, m_B^\phi$ forms a set of hidden variables $\lambda$ for the system.

More generally there will be a reduced correlation between measurements performed at $A$ and $B$. This is generally so for the case where measurements incorporate macroscopic uncertainties. Local realism still allows us to deduce the existence of an element of reality (we will call it $m_A^\theta$) for the photon number difference at $A$, with measurement angle $\theta$ at $A$, since we can make a prediction of the result at $A$, without disturbing the system at $A$, under the locality assumption. This prediction is based on a measurement performed at $B$. 7
In this case however the element of reality \( m^A_\theta \) becomes “fuzzy”. The “values” which the element of reality can assume do not form a set of definite numbers with zero uncertainty, but rather a set of distributions, one for each possible result \( m \) at \( B \), which we label by \( m^A_\theta = m \). The distribution labeled by the element of reality \( m^A_\theta \) assuming the value \( m \) gives the probability of a result for the measurement \( \theta \) at \( A \) should it be performed. It is independent of \( \phi \) the experimenter’s choice of angle at \( B \) if a simultaneous measurement at \( B \) should be performed. One can apply similar reasoning to deduce the existence of a set of indeterminate elements of reality \( m^B_\phi \).

The assumption of “local realism” then justifies the local hidden variable description used in (8), and (4)-(6), above. Local realism implies that the system is always in a state corresponding to a particular value for each of the elements of reality \( m^A_\theta \) and \( m^B_\phi \). The whole set of “elements of reality” \( m^A_\theta \) and \( m^B_\phi \) form a set of “hidden variables” which can be attributed to the system at a given time. Common notation symbolises the complete set of hidden variables by \( \lambda \), and the underlying joint probability distribution \( p(m^A_\theta, m^B_\phi) \) becomes \( \rho(\lambda) \). The probabilities \( \rho(\lambda) \) for the hidden variables are predetermined, and do not depend on the experimental choice of \( \theta \) and \( \phi \). For each such state \( \lambda \) there is a probability \( p^A_\theta(\theta, \lambda) \) that the result of a \( \theta \) measurement at \( A \) will be \( n \). In the case with perfect correlation the “elements of reality” give precise values for the result of the photon number measurement. Suppose the result \( m \) at \( B \) correlates with \( n \) at \( A \). Then we have \( p^A_\theta(\theta, \lambda) = 1 \) if \( \lambda = m^A_\theta = m \), and is zero otherwise. More generally we have imperfect correlation and “fuzzy” elements of reality, meaning that this \( p^A_\theta(\theta, \lambda) \) assumes a finite variance as discussed above.

We focus attention on the distribution \( p^A_\theta(\theta, \lambda) \), the probability of getting a photon number \( i \) for measurement at \( A \) with angle \( \theta \), given that the system is in a hidden variable state \( \lambda \). The independence of \( p^A_\theta(\theta, \lambda) \) on \( \phi \) is based on the locality assumption used in its entirety, that the experimenter’s choice of measurement angle at \( B \) cannot (instantaneously) change the result of the measurement at \( A \) in any way. With macroscopic local realism the locality condition is relaxed, allowing the conditional distributions \( p^A_i(\theta, \lambda) \) to become nonlocal, that is to have an explicit dependence on the experimental angle \( \phi \). The locality condition is relaxed however only up to the level of \( M \) photons, where \( M \) is not macroscopic, by maintaining that the measurement at \( B \) cannot instantaneously change the result at \( A \) by an amount exceeding \( M \) photons.

By relaxing the locality assumption up to \( M \) photons, the elements of reality \( m^A_\theta \) (deduced by way of the EPR argument) even in situations of perfect correlation will automatically have a distribution \( p^A_i(\theta, \phi, \lambda) \) which is no longer a delta function, though the distribution will be zero for values of \( i \) exceeding the value of \( m^A_\theta \) by greater than \( M \) photons. This is because we can no longer exclude the possibility of changes to the result of photon number measurements at \( A \) by an amount of up to \( M \) photons, due to the measurement at \( B \).

Similarly in the case of imperfect correlation the “fuzziness” of the elements of reality as given by the conditional distribution \( p^A_i(\theta, \lambda) \) is increased, by an amount whose upper limit is determined by the value of \( M \) and which may depend on \( \phi \).

Now we must consider the prediction for equation (8) as given by macroscopic local realism. The elements of reality deduced using macroscopic local realism cannot give predictions for the results of measurement which are macroscopically different to those predicted from the elements of reality deduced using local realism. Where our predicted result for a measurement at \( A \) is \( i' \) using local realism, macroscopic local realism allows the result to
be $i' + m_A$ where $m_A$ can be any number not macroscopic. Importantly, while $i'$ is not dependent on the choice $\phi$ for a simultaneous measurement at $B$, the value $m_A$ can be.

We therefore introduce the macroscopic locality assumption into the expression (8) for the probabilities in terms of the hidden variables in the following manner. We assume that the conditional probability $p_i^A(\theta, \lambda)$ in equation (8) takes the form of the following convolution (where $M$ is a integer which is not macroscopic).

$$p_i^A(\theta, \phi, \lambda) = \sum_{m_A=-M}^{+M} p_{m_A}^{A,NL}(\theta, \phi, \lambda)p_{i'=-m_A}^{A,L}(\theta, \lambda)$$

(9)

(We similarly relax the locality assumption for $p_i^B(\phi, \lambda)$, allowing for a dependence on $\theta$, and introduce a $p_i^B(\phi, \theta, \lambda)$ defined in a similar fashion.) The original local probability distribution $p_{\theta,\lambda}^A(\theta, \lambda)$, as would be specified through local realism, may be convolved with a microscopic or mesoscopic nonlocal probability function $p_{m_A}^{A,NL}(\theta, \phi, \lambda)$. The local specification, which is not dependent on the experimental choice of angle $\phi$ at $B$, gives a (local) probability distribution $p_{\theta,\lambda}^A(\theta, \lambda)$ for obtaining $i'$ photons at $A$, but the prediction is only correct to within $\pm M$ photons. These (local) distributions form the fuzzy “macroscopic elements of reality”. The probability distribution for an actual result $i = i' + m_A$ at $A$ is determined by the further nonlocal perturbation term $p_{m_A}^{A,NL}(\theta, \phi, \lambda)$, which gives the probability of a further change of $m_A$ photons. The nonlocal term is necessary because macroscopic local realism allows for the possibility that the measurement at $B$ instantaneously changes the result at $A$ by $M$ or less photons, where $M$ is not macroscopic. The only restriction is that the nonlocal distribution does not provide macroscopic perturbations, so that the probability of getting a nonlocal change outside the range $m_A = -M, \ldots, +M$ is zero. Equivalently we must have (and similarly for terms with $B$)

$$\sum_{m_A=-M}^{M} p_{m_A}^{A,NL}(i', \theta, \phi, \lambda) = 1.$$  

(10)

We now wish to obtain an expression for the measurable probabilities $P_{++}^{AB}(\theta, \phi)$ in the presence of the local noise terms, in terms of the $P_{ij}^{0,AB}(\theta, \phi)$. We introduce noise distribution functions at each of $A$ and $B$, and define probabilities such as $P^{A}(\text{noise} \geq x)$, that the noise at $A$ is greater than or equal to the value $x$. A probability $P^{B}(\text{noise} \geq x)$ is defined similarly, for the noise term at $B$. The final measured probability in the presence of noise is expressible as

$$P_{++}^{AB}(\theta, \phi) = \sum_{i,j=-\infty}^{\infty} P_{ij}^{0,AB}(\theta, \phi)P^{A}(\text{noise} \geq -i)P^{B}(\text{noise} \geq -j)$$

(11)

We write the predictions for this expression in terms of the hidden variable theory by substituting the macroscopic locality assumption (9) into the hidden variable prediction (8) for $P_{ij}^{0,AB}(\theta, \phi)$. We get

$$P_{++}^{AB}(\theta, \phi) = \sum_{i,j=-\infty}^{\infty} \int p(\lambda) [\sum_{m_A=-M}^{M} p_{m_A}^{A,NL}(i', \theta, \phi, \lambda)p_{i'=-m_A}^{A,L}(\theta, \lambda)] \times \sum_{m_B=-M}^{M} p_{m_B}^{B,NL}(j', \phi, \theta, \lambda)p_{j'=-m_B}^{B,L}(\phi, \lambda)] d\lambda \quad P^{A}(\text{noise} \geq -i)P^{B}(\text{noise} \geq -j)$$

(12)
Recalling \(i = i' + m_A\) and \(j = j' + m_B\) we change the \(i, j\) summation to one over \(i', j'\) to get

\[
P^{AB}_{++}(\theta, \phi) = \sum_{i',j'=-\infty}^{\infty} \int \rho(\lambda)p^{AL}_{i'}(\theta, \lambda) \left[ \sum_{m_A=-M}^{M} p^{A, NL}_{m_A}(i', \theta, \phi, \lambda)P^A(\text{noise} \geq -(i' + m_A)) \right] 
\times p^{BL}_{j'}(\phi, \lambda) \left[ \sum_{m_B=-M}^{M} p^{B, NL}_{m_B}(j', \phi, \theta, \lambda)P^B(\text{noise} \geq -(j' + m_B)) \right] d\lambda \tag{13}
\]

At this point we introduce the following assumption regarding the macroscopic nature of the noise term \(P^A(\text{noise} \geq x)\), that the increase or decrease of \(x\) by an amount of up to \(M\) photons gives only a negligible change to the probability that the noise is of size \(x\) or greater, \(P^A(\text{noise} \geq -(i' + m_A)) \approx P^A(\text{noise} \geq -i')\) and similarly for the noise term at B. This gives us

\[
\sum_{m_A=-M}^{M} p^{A, NL}_{m_A}(i', \theta, \phi, \lambda)P^A(\text{noise} \geq -(i' + m_A)) \approx P^A(\text{noise} \geq -i') \sum_{m_A=-M}^{M} p^{A, NL}_{m_A}(i', \theta, \phi, \lambda). \tag{14}
\]

Clearly this is only valid for noise which is macroscopic in size (recalling \(M\) is a number which is not macroscopic). With assumption (10) we get simplification to obtain a final form

\[
P^{AB}_{++}(\theta, \phi) = \sum_{i',j'} \int \rho(\lambda)p^{AL}_{i'}(\theta, \lambda)p^{BL}_{j'}(\phi, \lambda) d\lambda \times P^A(\text{noise} \geq -i')P^B(\text{noise} \geq -j'). \tag{15}
\]

This prediction of the hidden variable theory is now given in a (local) form like that of (6). Similar study of the expressions for the marginal probabilities lead to (local) expressions like that of (4) and (5), and the Bell inequalities (7) therefore readily follow. The noise terms \(\text{noise}\) which add a macroscopic uncertainty to the photon number result alter the premises needed to derive the Bell inequality. We need only assume macroscopic local realism to derive the inequalities (7) in the presence of macroscopic noise terms. Violation therefore of these Bell inequalities in the presence of the macroscopic noise terms would be evidence of a failure of macroscopic local realism.

**IV. QUANTUM STATES VIOLATING BELL INEQUALITIES WITH MACROSCOPIC NOISE: PREDICTED FAILURE OF MACROSCOPIC LOCAL REALISM**

We present a quantum state which shows violations of Bell inequalities in the presence of macroscopic noise. By the above arguments, this state then is evidence of a failure of macroscopic local realism.

\[
|\psi> = |I_0(2r_0^2)|^{-1/2} \sum_{n=0}^{\infty} \left(\frac{r_0^2}{n!}\right)^n |n >_{a_-} |n >_{b_-} |\alpha >_{a_+} |\beta >_{b_+} \tag{16}
\]

Here \(I_0\) is a modified Bessel function. The fields \(\hat{a}_+\) and \(\hat{b}_+\) are in coherent states \(|\alpha >_{a_+}\) and \(|\beta >_{b_+}\) respectively and we allow \(\alpha, \beta\) to be real and large. \(|n >_k\) is a Fock state.
for field $k$. The fields $\hat{a}_-$ and $\hat{b}_-$, often referred to as signal and idler fields respectively, are microscopic and are generated in a pair-coherent state with $r_0 = 1.1$. Pair-coherent states were considered originally by Agarwal [11]. They might potentially be generated using nondegenerate parametric oscillation (as suggested by Krippner and Reid [12] and explored in the recent work by Gilchrist and Munro [13]) in a limit where one-photon losses are negligible, or some similar process, as modelled by the following Hamiltonian in which coupled two-photon signal-idler loss dominates over linear single-photon loss:

$$H = i\hbar E(\hat{a}_-^{\dagger}\hat{b}_- - \hat{a}_-\hat{b}_-^{\dagger}) + \hat{a}_-\hat{b}_-\hat{\Gamma} + \hat{a}_-^{\dagger}\hat{b}_-^{\dagger}\hat{\Gamma}$$

(17)

The coherent states for $\hat{a}_+$ and $\hat{b}_+$ would be derived from the laser pump for the oscillator. Here $E$ represents a coherent driving parametric term which generates signal-idler pairs, while $\hat{\Gamma}$ represents reservoir systems which give rise to the coupled signal-idler loss. The Hamiltonian preserves the signal-idler photon number difference operator $\hat{a}_-^{\dagger}\hat{a}_- - \hat{b}_-^{\dagger}\hat{b}_-$, of which the quantum state (16) is an eigenstate, with eigenvalue zero. We note the analogy here to the single mode “even” and “odd” coherent superposition states $N^{\pm1/2}(\alpha > \pm|\alpha >)$ (where $\alpha$ is real and $N^{\pm1/2} = 2(1 \pm \exp(-2|\alpha|^2))$) which are generated by the degenerate form (put $\hat{a}_- = \hat{b}_-$) of the Hamiltonian (17). These states for large $\alpha$ are analogous to the famous “Schrodinger-cat” states [12] and have been recently experimentally explored [13].

We point out later other choices of $|\psi>$ possible.

To model noise we allow noise to be a random noise term with a gaussian distribution of standard deviation $\sigma$. An example of a noisy photon number measurement is photodiode detection of very large intensities, such as used in the experiments of Smithey et al [14]. The photocurrent is processed electronically in a way that adds noise to the final output current, giving a final imprecision in the photon number measurement. Although percentage detection efficiencies are high for diode detectors, detection inefficiencies can also create a potentially large absolute noise term which also limits the resolution of the photon number measurement.

Violations of the Bell inequality (7), for the state (16), in the absence of noise are shown in Figure 2, curve (a). The effect of adding increasing noise is to reduce the value of $S$ until eventually the violation is lost, at a cut-off noise value $\sigma_c$, as shown in Figure 3. Figure 2, curve (b) shows this cut-off value $\sigma_c$ (the maximum noise still allowing a violation of the Bell inequality) versus $\alpha$. We note the linear dependence of $\sigma_c$ on $\alpha$ ($\sigma_c = .26\alpha$). In the limit of larger $\alpha$ this cut-off noise $\sigma_c$ then becomes macroscopic. Violations of fixed magnitude ($S \to 1.0157$ as $\alpha \to \infty$) are still possible for increasingly larger absolute noise, simply by increasing $\alpha$.

The asymptotic behavior in the large $\alpha,\beta$ limit is crucial in determining whether macroscopic local realism will be violated, and is understood by replacing the boson operators $\hat{a}_+$ and $\hat{b}_+$ by classical amplitudes $\alpha$ and $\beta$ respectively. We see that $\hat{S}_\theta^A$ from equation (2) can be expressed as $\hat{S}_\theta^A = (\hat{a}_+^{\dagger}\hat{a}_-\exp(-i\theta) + \hat{a}_+\hat{a}_-^{\dagger}\exp(i\theta))/2 = \alpha \hat{X}_\phi^A/2$, and similarly $\hat{S}_\phi^B = \beta \hat{X}_\phi^B/2$, where $\hat{X}_\phi^A = \hat{a}_-\exp(-i\phi) + \hat{a}_+\exp(i\phi)$ and $\hat{X}_\phi^B = \hat{b}_-\exp(-i\phi) + \hat{b}_+^{\dagger}\exp(i\phi)$. The $\hat{X}_\phi^A$ and $\hat{X}_\phi^B$ are the quadrature phase amplitudes of the fields $\hat{a}_-$ and $\hat{b}_-$ respectively. We see then that the photon number measurements $2\hat{S}_\theta^A$ and $2\hat{S}_\phi^B$ give results in the large $\alpha,\beta$ limit corresponding numerically to the scaled quadrature phase amplitudes $\alpha \hat{X}_\phi^A$ and
β\hat{X}_\phi^B$ respectively. Figure 1a in fact shows for large $\alpha, \beta$ the experimental arrangement for balanced homodyne detection \cite{20}, a technique commonly used to measure quadrature phase amplitudes. In Figure 1a the homodyne scheme measures the quadrature phase amplitudes $\hat{X}_\theta^A$ and $\hat{X}_\phi^B$, of the fields $\hat{a}_-$ and $\hat{b}_-$. The large intensity fields $\hat{a}_+$ and $\hat{b}_+$ are the “local oscillator” fields usually considered to be classical amplitudes $\alpha, \beta$. Violations of Bell inequalities (7) (failure of local realism) for precisely these asymptotic quadrature phase amplitude measurements have recently been shown by Gilchrist et al \cite{21}, the value of $S = 1.0157$ presented in these quadrature phase amplitude calculations indeed corresponding to our large $\alpha$ limit (Figure 2).

Calculations \cite{22} which model the addition of noise to the quadrature phase amplitude measurements $\hat{X}_\theta^A, \hat{X}_\phi^B$ reveal violations of the Bell inequality to be lost at the cutoff value of $\sigma_0 = 0.26$. This asymptotic result allows us to make a prediction of the effect of noise (in the large $\alpha$ limit) on the full photon number calculation presented in Figure 2. The detected photon number difference is given as

$$\hat{n}_\theta^A = 2\hat{S}_\theta^A = \hat{c}^+_i \hat{c}^+_i - \hat{c}^-_j \hat{c}^-_j = \alpha \hat{X}_\theta^A \tag{18}$$

Noise of size $\text{noise}$ added to the photon number difference $\hat{n}_\theta^A$ result is equivalent to noise of size $\text{noise}/(\alpha)$ added to the signal quadrature phase amplitude $\hat{X}_\theta$ result. The noise in the photon number difference is scaled by a factor of $\alpha$, the local oscillator amplitude. Therefore the cut-off value $\sigma_0 = 0.26$ will correspond to a cut-off noise value of $\sigma_c = \alpha \sigma_0$ in the measurement of photon number difference $\hat{n}_\theta^A = 2\hat{S}_\theta^A$, confirming the linear behavior shown in Figure 2, and the prediction that from this that macroscopic noise values are possible while still obtaining a contradiction with local realism. This property then is a predicted contradiction of quantum mechanics with macroscopic local realism as we have defined it.

The tolerance of the bell inequality violations to increasing noise as $\alpha$ increases can be understood as follows. For large $\alpha$, the shape (envelope) of the joint probability $P_{i,j}^{\theta,AB}(\theta, \phi)$, for obtaining results $i$ and $j$ upon measurements of $\hat{n}_\theta^A$ and $\hat{n}_\phi^B$ respectively, is determined by $P_{x,y}(\theta, \phi)$ (where $x = i/\alpha$ and $y = j/\beta$), the joint probability distribution for results $x$ and $y$ upon measurements $\hat{X}_\theta^A$ and $\hat{X}_\phi^B$ respectively. As the scale factor $\alpha$ linking result $x$ to result $i$ increases, the probability for obtaining a result in a region between two fixed, yet macroscopically- distinct photon numbers, will become small. In this limit we have macroscopically distinct outcomes, and measurements can tolerate a macroscopic noise without loss of violation of the Bell inequality.

Detection inefficiencies will also contribute to a noise in the final result for the measurement, though in this case the noise will not be gaussian. Noise caused by detector losses is often modeled by a beam splitter interaction immediately prior to photodetection. The field to be detected, $\hat{c}'_+$ say, is taken to be an input to a beam splitter. The second input to the beam splitter $\hat{a}_{\text{vac} +}$ is considered to be a vacuum. The output

$$\hat{c}'_{L+} = \sqrt{\eta} \hat{c}'_+ + \sqrt{1 - \eta} \hat{a}_{\text{vac} +}, \tag{19}$$

where $\eta$ is the overall efficiency factor, is then taken to be the effective detected field. A similar effective field $\hat{c}'_{L-}$ is constructed for the second detector, used to measure $\hat{c}'_-$, at
location $A$ and a second vacuum input $\hat{a}_{\text{vac}^{-}}$ defined. The detected photon number difference is now given as

$$\hat{n}_A^{\theta} = \hat{c}_{L+}^\dagger \hat{c}_{L+} - \hat{c}_{L-}^\dagger \hat{c}_{L-} = \eta \alpha \hat{X}_{L\theta}^A$$

(20)

where

$$\hat{X}_{L\theta}^A = \eta \hat{X}_\theta^A + (\sqrt{1 - \eta}/\sqrt{2})(X_{\theta,\text{vac}+} + X_{\theta,\text{vac}^{-}})$$

(21)

and the terms $\hat{X}_{\theta,\text{vac}^\pm}$ are quadrature phase amplitudes for the independent + and $-$ vacuum modes representing the input fields $\hat{a}_{\text{vac}+}$ and $\hat{a}_{\text{vac}^{-}}$ respectively. Additional terms which give negligible contributions with large $\alpha$ have been omitted. We see how loss (described by $\eta$ less than 1) causes a noise term $(\sqrt{1 - \eta}/\sqrt{2})(X_{\theta,\text{vac}+} + X_{\theta,\text{vac}^{-}})$ in the signal quadrature phase amplitude. Because of the factor $\eta \alpha$ this term can be large to give potentially macroscopic absolute noise values in photon number for the photon number difference measurement. Violations of the Bell inequality considered by Gilchrist et al [21] have been shown obtainable in the presence of detector losses ($\eta \approx .98$). We see from the above analysis that this will correspond for sufficiently large $\alpha$ to a macroscopic absolute noise term in the photon number measurements. Thus we have a second situation where violations of a Bell inequality are predicted possible in the presence of large absolute detector noise, this prediction indicating an incompatibility of quantum mechanics with macroscopic local realism.

We can deduce from our asymptotic (large $\alpha, \beta$) study other states $|\psi\rangle$ which will give a failure of macroscopic local realism. Any state $|\psi\rangle$ which shows a failure of local realism for measurements $\hat{X}_\theta^A$ and $\hat{X}_\theta^B$ on fields $\hat{a}_{\text{vac}+}$ and $\hat{b}_{\text{vac}^{-}}$ will also show a violation of macroscopic local realism, provided $\alpha, \beta$ are large. This follows because there will always be a finite noise cutoff $\sigma_0$, meaning that a failure of local realism is possible for noise values less than $\sigma_0$. For large enough $\alpha, \beta$ this cut-off will correspond to a macroscopic noise cut-off value $\sigma_c = \alpha \sigma_0$ in the photon number measurement $\hat{n}_\theta^A$ (and similarly for measurement $\hat{n}_\phi^B$). This is an important point since other states violating local realism for quadrature phase amplitude measurements, either by way of a Bell inequality or by way of the Greenberger-Horne-Zeilinger phenomenon, have been recently predicted [21]. This greatly increases the scope for a practical violation of macroscopic local realism.

A failure of local realism in the presence of macroscopic noise terms (as we have predicted here for states showing failure of local realism for quadrature phase amplitude measurements) is not typical. Consider as a source for the outgoing fields $\hat{a}_\pm'$, pictured in Figure 1b, the following higher spin state which has been studied in much detail by Mermin and Drummond and others [6–8]. It is well known that this state gives a violation of Bell inequalities for large $N$, and is often considered to be an example of a violation of a “macroscopic local realism”.

$$|\varphi\rangle = \frac{1}{N! (N + 1)^{1/2}} (\hat{a}_{\text{vac}+}^\dagger \hat{b}_{\text{vac}+}^\dagger + \hat{a}_{\text{vac}^{-}}^\dagger \hat{b}_{\text{vac}^{-}}^\dagger)^N |0\rangle |0\rangle$$

(22)

Yet a study of the behaviour of the violation of the Bell inequality (7) with respect to noise added to the final photon number measurements gives a cut-off noise limit which is microscopic for large incident photon number $N$. This effect is plotted in Figure 4. This is
in contrast with our state (16) which gives a macroscopic cut-off noise value in the limit of large $\alpha$.

It may be asked how a macroscopic claim can be made from the predictions discussed in this paper, given that the signal field $\hat{a}_-$ is microscopic. It is noted in response to this question that, although the field $\hat{a}_-$ is itself microscopic, the physical quantity measured, and to which the elements of reality relate, is the combined Schwinger operator $\hat{S}_A^\theta$. The results for this measurement have a macroscopic range and can tolerate increasing levels of (absolute) noise.

However it is crucial that the macroscopic nature of our result is clarified in the arrangement of Figure 1b. Here the field $\hat{a}_-$ is combined with field $\hat{a}_+$, to produce macroscopic fields $\hat{a}_\pm'$, prior to the experimenter’s selection of the angle $\theta$. These outgoing macroscopic fields $\hat{a}_\pm'$ may then be regarded as the system at $A$. In this situation both fields $\hat{a}_\pm'$ incident on the measurement apparatus, depicted by a polariser (or beam splitter) with the choice of $\theta$ in the Figure 1b, are macroscopic. (A similar description applies to the fields at $B$).

That a microscopic state was involved in the preparation of the spatially separated and propagating fields $\hat{a}_\pm'$ and $\hat{b}_\pm'$ does not affect the macroscopic nature of our work. It is important to realise that the measurement events at $A$ and $B$ must be causally separated, and that by the time the fields reach the measurement destinations $A$ and $B$, the original apparatus used to prepare the fields need no longer exist.

The important point is that this combining of fields which comes about as part of the state preparation can be clearly distinguished from amplification which comes after the selection of $\theta$, as part of the measurement process. This second-mentioned amplification comes about in all experiments, but does not imply that one can deduce “macroscopic elements of reality” as we have defined it here. The “element of reality” is a variable whose values refer to a physical quantity defined for a system, for example, the position of a particle. In the context of Einstein-Podolsky-Rosen and Bell arguments the system (for example the particle or photon field) has a well-defined meaning independent of the measuring apparatus (polariser or beamsplitter phase-shift combinations) and associated amplification. A macroscopic element of reality is a variable whose possible values are defined only with a macroscopic uncertainty. The value for the element of reality and its associated uncertainty have a clear meaning, and can be readily classified as macroscopic or not macroscopic. For example the uncertainty in the measured value for the position of a particle can be microscopic regardless of an amplified final readout value. In this work the “element of reality” refers to a photon number (actually a photon number difference) and “macroscopic” means a large photon number.

V. CONCLUSIONS

Our claim therefore is that earlier work suggestive of violations of local realism at a macroscopic level must be interpreted carefully before claiming a loss of local realism at a “macroscopic” level. The failure of a Bell inequality in cases where the photon number can be macroscopic but where measurement resolution is perfect may not automatically imply the failure of a macroscopic local realism, as we have defined it.

In summary we have considered the concept of orders of local realism, from macro-
through meso- to microscopic, which apply to experiments with increasing precision of measurement. Macroscopic local realism excludes the possibility of macroscopic changes to a system $B$ occurring as a result of events which occur simultaneously at a spatially separated system $A$. This is as opposed to local realism used in its entirety, right down to the most microscopic level, which excludes all orders of change.

We have derived Bell inequalities which, if violated in experiments with limited resolution of photon number, will imply a failure of these less restrictive forms of local realism. We claim that the proven failure, if ever achievable, of this macroscopic local realism is conclusive evidence that the “startling” properties apparently attributed to “entangled Schrödinger cat” states are inescapable. A class of quantum states (those showing a violation of local realism for quadrature phase amplitudes) with this property has been proposed.
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FIG. 1. Schematic representation of our proposed test of macroscopic local realism. (a) Measurement of spin operators $\hat{S}_A^\theta$ and $\hat{S}_B^\phi$. This measurement scheme is equivalent to balanced homodyne detection of the quadrature phase amplitudes $\hat{X}_A^\theta$ and $\hat{X}_B^\phi$ of the fields $\hat{a}_-, \hat{b}_-$, in the limit of large $\alpha, \beta$. In the proposed experiment $\hat{a}_-, \hat{b}_-$ are of low intensity while $\hat{a}_+, \hat{b}_+$ are intense coherent-state $|\alpha>$ “local oscillator” fields. In this experiment large intensities are incident on each of the photodiode detectors. (b) Importantly in this alternative arrangement the fields $\hat{a}_\pm$ are first combined using a beam splitter and phase shift so that both outgoing fields $\hat{a}_\pm'$ incident on the measuring apparatus are macroscopic. The measurement apparatus is depicted here by the beam splitter with variable angle $\theta$, although a polariser may also be possible for suitable states. A similar arrangement occurs at $B$. In this experiment the entire boxed apparatus may be considered the source. The measured quantity in terms of the $\hat{a}_\pm, \hat{b}_\pm$ fields is still $\hat{S}_A^\theta$ and $\hat{S}_B^\phi$ as above in (a).
FIG. 2. $S$ versus $\alpha$, for $\theta = 0, \phi = -\pi/4, \theta' = \pi/2, \phi' = -3\pi/4, \alpha = \beta$ for the quantum state (16) with no noise present. The dashed line gives the maximum noise $\sigma_c$ still giving a violation of the Bell inequality (7) for the above parameters, versus $\alpha$. Macroscopic values are possible with increasing $\alpha$.

FIG. 3. $S$ versus the noise parameter $\sigma$, for $\theta = 0, \phi = -\pi/4, \theta' = \pi/2, \phi' = -3\pi/4, \alpha = \beta$ for the quantum state (16), where $\alpha = 10$. 
FIG. 4. Line (a) gives $S$ versus $N$, for the quantum state (22) with no noise present. Here we have selected the following relation between the angles: $\phi - \theta = \theta' - \phi = \phi' - \theta' = \psi$ and $\phi' - \theta = 3\psi$ and optimised $S$ with respect to $\psi$. Line (b) gives the maximum noise $\sigma_c$ still giving a violation of the Bell inequality (7) for the above parameters. In this case the cut-off noise $\sigma_c$ remains microscopic for large $N$. 