**F-Term Hybrid Inflation Followed by Modular Inflation**

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**Abstract.** We consider the well motivated model of the (standard) supersymmetric (SUSY) F-term hybrid inflation (FHI) which can be realized close to the grand unification (GUT) scale. The predicted scalar spectral index $n_s$ cannot be smaller than 0.98 and can exceed unity including corrections from minimal supergravity (SUGRA), if the number of e-foldings corresponding to the pivot scale $k_*=0.002$ Mpc is around 50. These results are marginally consistent with the fitting of the five-year Wilkinson microwave anisotropy probe (WMAP5) data by the standard power-law cosmological model with cold dark matter and a cosmological constant, $\Lambda$CDM. However, $n_s$ can be reduced by restricting the number of e-foldings that $k_*$ suffered during FHI. The additional e-foldings required for solving the horizon and flatness problems can be generated by a subsequent stage of fast-roll [slow-roll] modular inflation (MI) realized by a string modulus which does [does not] acquire effective mass ($m_s|_{\text{eff}}$) before the onset of MI.

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**INTRODUCTION**

We focus on the model of the (standard) SUSY FHI [1] which can be realized [2] adopting the superpotential $W = \kappa S (\Phi \Phi - M^2)$ where $\Phi, \bar{\Phi}$ is a pair of left handed superfields belonging to non-trivial conjugate representations with dimensionality $N$ of a gauge group $G$, and reducing its rank by their vacuum expectation values (VEVs), $S$ is a $G$ singlet left handed superfield and the parameters $\kappa$ and $M$ can be made positive.

$W$ leads to the spontaneous breaking of $G$ since from the emerging scalar potential $V_F = \kappa^2 M^4 \left( (\Phi - 1)^2 + 2 S^2 \Phi^2 \right)$ where $\Phi = |\Phi|/M$ and $S = |S|/M$ (we use the same symbol for the superfields and their scalar components) we can deduce that the vanishing of the F-terms gives the VEVs of the fields in the SUSY vacuum, $\langle S \rangle = 0$ and $\langle \Phi \rangle = 1$ (the vanishing of the D-terms implies that $|\bar{\Phi}| = |\Phi|$). $W$ gives also rise to FHI. This is due to the fact that, for $S > 1$, the direction with $\Phi = 0$ is a valley of local minima with constant $V_F$. The general form of the potential which can drive FHI reads

$$V_{\text{HI}} \simeq \kappa^2 M^4 + \frac{\kappa^4 M^4 N^3}{32 \pi^2} \left( 2 \ln \frac{\kappa^2 \sigma^2}{2 Q^2} + 3 \right) + \kappa^2 M^4 \frac{\sigma^4}{8 m_p^4}, \quad \text{with} \quad \sigma = \sqrt{2} S.$$  \hspace{1cm} (1)

Here, the 1st term is the dominant contribution to $V_{\text{HI}}$, the 2nd term (with $Q$ being an arbitrary renormalization scale) is the contribution to $V_{\text{HI}}$ due to logarithmic radiative corrections originating from the SUSY breaking on the inflationary valley and the 3rd term (with $m_p \simeq 2.44 \times 10^{18}$ GeV) is the SUGRA correction [3] to $V_{\text{HI}}$, assuming minimal Kähler potential.
Under the assumption that the cosmological scales leave the horizon during FHI and are not reprocessed, we can extract:

- The number of e-foldings $N_{\text{HI}*} = \frac{1}{m_p} \int_{\sigma_f}^{\sigma_s} d\sigma \frac{V''_{\text{HI}}}{V_{\text{HI}}} |_{\sigma=\sigma_s}$ that $k_*$ suffered during FHI, where prime means derivation with respect to $\sigma$, $\sigma_s$ is the value of $\sigma$ when the scale $k_*$ crossed outside the horizon of FHI and $\sigma_f$ is the value of $\sigma$ at the end of FHI, which coincides practically with the end of the phase transition $\sigma_*= M/\sqrt{2}$.

- The power spectrum of the curvature perturbations at $k_*$, $P_{s*} = \frac{1}{2\sqrt{3} \pi m_p} \left[ \frac{V^{3/2}}{V_{\text{HI}}} \right] |_{\sigma=\sigma_s}$.

- The spectral index $n_s = 1 - 6\varepsilon_s + 2\eta_s$ and its running $\alpha_s = \frac{2(4n_s^2-(n_s-1)^2)}{3} - 2\xi_s$, where $\varepsilon \simeq \frac{m_s^2}{2\left(\frac{V''_{\text{HI}}}{V_{\text{HI}}}\right)^2}$, $\eta \simeq m_s^2 \frac{V''_{\text{HI}}}{V_{\text{HI}}}$ and $\xi \simeq m_s^4 \frac{V''_{\text{HI}}}{V_{\text{HI}}}$ and the subscript $*$ means that the quantities are evaluated for $\sigma = \sigma_*$.

If FHI is to produce the total amount of e-foldings, $N_{\text{tot}}$, needed for the resolution of the horizon and flatness problems of standard cosmology, i.e., $N_{\text{tot}} = N_{\text{HI}*} \simeq 50$, we get $n_s \sim 0.98 - 1$ which is just marginally consistent with the fitting of the WMAP5 data [4] by the standard power-law cosmological model $\Lambda$CDM, according to which

$$n_s = 0.963^{+0.016}_{-0.015} \Rightarrow 0.931 \lesssim n_s \lesssim 0.991$$

at 95% confidence level with negligible $\alpha_s$. However, for $\kappa \simeq (0.01 - 0.1)$ and $N_{\text{HI}*} \sim (15 - 20)$ we can obtain $n_s \simeq 0.96$. $N_{\text{tot}} - N_{\text{HI}*}$ can be produced during another stage of (complementary) inflation, realized at a lower scale. In this talk, which is based on Ref. [6], we show that MI can successfully play this role.

**THE BASICS OF MODULAR INFLATION**

After the gravity mediated soft SUSY breaking, the potential which can support MI has the form [5] $V_{\text{MI}} = V_{\text{MI}0} - m_s^2 s^2/2 + \cdots$, with $V_{\text{MI}0} = v_s (m_{3/2} m_p)^2$ and $m_s \sim m_{3/2}$ where $m_{3/2} \sim 1$ TeV is the gravitino mass, the coefficient $v_s$ is of order unity and the ellipsis denotes terms which are expected to stabilize $V_{\text{MI}}$ at $s \sim m_p$ with $s$ being the canonically normalized string modulus. In this model, inflation can be of the slow or fast-roll type [7] depending on whether $|\eta_s| = m_s^2 [d^2 V_{\text{MI}}/ds^2]/V_{\text{MI}} = m_s^2/3H_s^2$ is lower or higher than unity, respectively. In both cases the solution of the equation of motion of $s$ during MI is

$$s = s_{\text{MI}} e^{F_s \Delta N_{\text{MI}}} \quad \text{with} \quad F_s \equiv \sqrt{9/4 + (m_s/H_s)^2} - 3/2,$$

with $H_s \simeq \sqrt{V_{\text{MI}0}/3 m_p}$, $s_{\text{MI}}$ the value of $s$ at the onset of MI and $\Delta N_{\text{MI}}$ the number of the e-foldings obtained from $s = s_{\text{MI}}$ until a given $s$. Through the use of Eq. (3) and considering that the final value of $s$, $s_f$, is close to its VEV, $s_f \sim m_p$, we can estimate the total number of e-foldings during MI, which is $N_{\text{MI}} \simeq \frac{1}{F_s} \ln \left( \frac{m_p}{s_{\text{MI}}} \right)$. We observe that MI can not play successfully the role of complementary inflation for $s_{\text{MI}}/m_p \lesssim 0.1$. 


OBSERVATIONAL CONSTRAINTS

Our double inflationary model needs to satisfy a number of constraints which arise from:

1. The normalization of $P_{\delta s}$: We require $[4] P_{\delta s}^{1/2} \simeq 4.86 \times 10^{-5}$.

2. The resolution of the horizon and flatness problems: We entail $N_{HI^*} + N_{MI} \simeq 22.6 + \frac{2}{3} \ln \frac{V^{1/4}_{H0}}{1 \text{ GeV}} + \frac{1}{3} \ln \frac{\delta_{s0}}{1 \text{ GeV}}$, where we assumed that there is matter domination in the inter-inflationary era ($T_{Mrh}$ is the reheat temperature after MI).

3. The Low Enough Value of $T_{\text{eff}}$. For natural MI we need: $0.5 \leq v_s \leq 10 \Rightarrow 2.45 \geq \frac{m_s}{T_{\text{eff}}} \geq 0.55$.

4. The Nucleosynthesis Constraint. This constraint dictates $T_{Mrh} > 1 \text{ MeV}$. In the absence of other specified interactions, $s$ has just gravitational interactions. Therefore, $\Gamma_s \sim m_s^2/m_p^3$, and since $T_{Mrh} \sim \sqrt{\Gamma_s m_p}$, we need $[6] m_s \simeq m_{3/2} ^{3/2} \geq 100 \text{ TeV}$.

5. The evolution of the cosmological scales. We have to ensure that the cosmological scales leave the horizon during FHI and do not re-enter the horizon before the onset of MI. This can be achieved $[8]$ if $N_{HI^*} \geq N_{HI}^{\min} \simeq 3.9 + \frac{1}{6} \ln \frac{V_{MIO}}{V_{HI}} \simeq 10$.

6. The homogeneity of the present universe. If $\delta s_{|_{\text{MI}}} [\delta s_{|_{\text{HMI}}}]$ are the quantum fluctuations of $s$ during MI [FHI which enter the horizon of MI], we require $s_{\text{Mi}} > \delta s_{|_{\text{MI}}} \simeq H_s/2 \pi$ and $s_{\text{Mi}} > \delta s_{|_{\text{HMI}}}$.

7. The evolution of $s$ before the onset of MI. (i) If $m_s|_{\text{eff}} = 0$, we assume that FHI lasts long enough so that $s$ is completely randomized. We further require that all $s$'s belong to the randomization region $[9]$ with equal possibility, i.e., $V_{MIO} \lesssim H^4_{\text{HI0}}$ where $H_{\text{HI0}} = \sqrt{V_{HI0}}/\sqrt{3}m_p$. (ii) If $m_s|_{\text{eff}} \neq 0$, we assume that $s$ is decoupled from the visible sector superfields both in Kähler potential and superpotential and has canonical Kähler potential, $K_s = s^2/2$. Therefore the value $s_{\text{min}}$ at which the SUGRA potential has a minimum is $s_{\text{min}} = 0$. We obtain for the value of $s$ at the onset of MI: $s_{\text{MI}} \simeq m_p (V_{MIO}/V_{HI0})^{1/4} e^{-3N_{\text{HI}}/2}$ where $s_{\text{HI}} \simeq m_p$ is the value of $s$ at the onset of FHI and $N_{\text{HI}}$ the total number of e-foldings during FHI.

8. The homogeneity of the present universe. If $\delta s_{|_{\text{MI}}} [\delta s_{|_{\text{HMI}}}]$ are the quantum fluctuations of $s$ during MI [FHI which enter the horizon of MI], we require $s_{\text{Mi}} > \delta s_{|_{\text{MI}}} \simeq H_s/2 \pi$ and $s_{\text{Mi}} > \delta s_{|_{\text{HMI}}}$. (i) If $m_s|_{\text{eff}} = 0$, $\delta s_{|_{\text{HMI}}} \sim H_{HI}/2 \pi \Rightarrow \delta s_{|_{\text{MI}}}$. (ii) If $m_s|_{\text{eff}} \neq 0$, $\delta s_{|_{\text{HMI}}} \sim H_s/3^{1/4} 2 \pi < \delta s_{|_{\text{MI}}}$ and so, $s_{\text{Mi}} > \delta s_{|_{\text{MI}}} \sim H_s/2 \pi \Rightarrow N_{\text{HI}} \leq N_{\text{HI}}^{\max}$ where $N_{\text{HI}}^{\max} = -\frac{2}{3} \ln \frac{(V_{H0}V_{MIO})^{1/4}}{2^{\sqrt{3}m_p^2}} \sim (15 - 18)$. This result signifies an ugly tuning since it would be more reasonable FHI has a long duration due to the flatness of $V_{HI}$. This could be evaded if we had $s_{\text{min}} = 0$ (as in Ref. [10]).

RESULTS AND CONCLUSIONS

In our numerical investigation, we take $N = 2$ and $m_{3/2} = m_s = 100 \text{ TeV}$ which results to $T_{Mrh} = 1.5 \text{ MeV}$. Our results are displayed in Table 1 for $n_s = 0.963$, $m_s|_{\text{eff}} = 0$ or $m_s|_{\text{eff}} \neq 0$ and selected $\kappa$'s which delineate the allowed regions. For $m_s|_{\text{eff}} = 0$ we place $s_{\text{Mi}}/m_p = 0.01$. This choice signalizes a very mild tuning (see point 7). For $m_s|_{\text{eff}} \neq 0$, $s_{\text{Mi}}$ is evaluated dynamically (see point 7). However, due to our ignorance of $N_{\text{HI}}$, we can derive a maximal [minimal] $m_s/H_s$ which corresponds to $N_{\text{HI}} = N_{\text{HI}}^{\max} [N_{\text{HI}} = N_{\text{HI}}^+]$. 
TABLE 1. Input and output parameters of our scenario which are consistent with the requirements 1-8 for $n_s = 0.963$ and selected $\kappa$’s, when the inflaton of MI does [does not] acquire effective mass ($m_s|_{\text{eff}} \neq 0$ [$m_s|_{\text{eff}} = 0$]).

| $m_s|_{\text{eff}} = 0$ | $m_s|_{\text{eff}} \neq 0$ |
|------------------------|------------------------|
| $\kappa$               | 0.04       | 0.09       | 0.14       | 0.0028     | 0.006      | 0.085      | 0.14       |
| $M/10^{16}$ GeV        | 0.87       | 0.98       | 1.07       | 0.74       | 0.8        | 0.97       | 1.07       |
| $\sigma_0/10^{10}$ GeV | 12.1       | 20.93      | 25.88      | 1.56       | 2.26       | 20.1       | 25.88      |
| $N_{\text{HI}}$        | 22.6       | 16.12      | 11.9       | 8.4        | 17.4       | 16.5       | 11.9       |
| $-\omega_0/10^{-3}$    | 2          | 5          | 10         | 2.4        | 1.5        | 4.8        | 10         |
| $N_{\text{MI}}$        | 21.2       | 28         | 32.5       | 34.1       | 25.7       | 27.6       | 32.5       |
| $m_s/H_s$              | 0.8        | 0.72       | 0.67       | 1.44 - 1.96| 2.35       | 2.25       | 1.78 - 2.02|

We observe that (i) for $m_s|_{\text{eff}} = 0$ [$m_s|_{\text{eff}} \neq 0$], the lowest $\kappa$’s are derived from the condition 7 [6] and therefore, lower $\kappa$’s are allowed for $m_s|_{\text{eff}} \neq 0$; (ii) the upper $\kappa$’s come from the condition 3; (iii) for $m_s|_{\text{eff}} = 0$ [$m_s|_{\text{eff}} \neq 0$], MI is of slow [fast]-roll type since $m_s/H_s \sim (0.6 - 0.8)$ [$m_s/H_s \sim (1.4 - 2.35$)]; (v) for $m_s|_{\text{eff}} \neq 0$ FHI is constrained to be of short duration since $N_{\text{HI}} \leq N_{\text{H}}^{\text{max}} \simeq (16 - 17)$ and as a consequence, the region $0.006 \lesssim \kappa \lesssim 0.085$ is disallowed; (vi) in both cases, the allowed $M$’s increase with $\kappa$’s but remain slightly below the GUT scale, $M_{\text{GUT}} \simeq 2.86 \cdot 10^{16}$ GeV. In total, we obtain $0.04 \lesssim \kappa \lesssim 0.14$ [$0.0028 \lesssim \kappa \lesssim 0.006$ and $0.085 \lesssim \kappa \lesssim 0.14$] for $m_s|_{\text{eff}} = 0$ [$m_s|_{\text{eff}} \neq 0$].

In conclusion, we showed that the results on $n_s$ within FHI can be reconciled with data if FHI is followed by MI realized by a string modulus $s$. Acceptable $n_s$’s can be obtained by restricting $N_{\text{HI}}$. The most natural version of this scenario is realized when $s$ remains massless before MI.

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