The Rate for $e^+e^- \rightarrow BB^{\pm}\pi^{\mp}$
and its Implications for the Study of CP Violation, $B_s$ Identification, and the Study of $B$ Meson Chiral Perturbation Theory *

Laurent Lellouch,† Lisa Randall‡ and Eric Sather

Center for Theoretical Physics
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology
Cambridge, MA 02139

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Abstract

H. Yamamoto [1] has proposed employing $B$ mesons produced in conjunction with a single charged pion at an $\Upsilon$ resonance for studies of CP violation in the neutral $B$ meson system at a symmetric $e^+e^-$ collider. The sign of the charged pion would tag the neutral $B$ meson. We estimate this branching ratio, employing the heavy meson chiral effective field theory. We find a negligible branching ratio to $BB^{\pm}\pi^{\mp}$ at the $\Upsilon(5S)$ and a branching ratio of only a few percent at the $\Upsilon(6S)$. However, if nonresonant studies of neutral $B$ mesons should prove feasible, Yamamoto’s proposal could be a good method for tagging neutral $B$’s for the study of CP violation at a symmetric collider.

We also explore the possibility of studying $B_s$ at the $\Upsilon(5S)$. The rate is low but depends sensitively on the precise value of the mass of the $B_s$. The background we compute is comparable to the rate at the largest allowed value of the $B_s$ mass.

Finally, we discuss the extraction of the axial pion coupling to $B$ mesons from measurement of the $BB\pi$ branching fraction in a restricted region of phase space, where chiral perturbation theory should work well.

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† Present address: Department of Physics, University of Southampton, Highfield, Southampton, SO9 5NH, U.K.

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1. Introduction

It is well known that CP violation studies are difficult at a symmetric collider. In $e^+e^- \rightarrow B^0\bar{B}^0$, the heavy mesons are produced in a $C$-odd state, so that the time integrated asymmetry vanishes unless the time ordering of the signal and tag can be measured. An alternative tagging method for which the time integrated asymmetry does not vanish has been proposed by H. Yamamoto [1]. His suggestion is to study $B$ mesons produced in conjunction with a single charged pion at an $\Upsilon$ resonance, so that the sign of the charged pion tags the single neutral $B$ meson as $B^0$ or $\bar{B}^0$. Neutral $B^*$ mesons can also be used, since they decay immediately via photon emission into pseudoscalars, before weak mixing or decays have occurred. This method should provide a simple and efficient tag of the neutral $B$ meson: In addition to the decay products of the neutral $B$, one only needs to detect the additional soft pion. The charged $B$ is then tagged by the invariant mass of the missing four momentum, so it need not be reconstructed. Unlike many conventional proposals for the study of CP violation in the neutral $B$ system, essentially all the events are tagged.

The utility of this method depends on the event rate. We calculate this rate, employing the heavy meson chiral effective theory, in order to evaluate the potential of Yamamoto’s proposal. Although it is difficult to do a reliable calculation for all relevant kinematic regions, we can nevertheless do a calculation which incorporates propagator enhancements, phase space, and derivative couplings. We find that the branching ratio should be negligible at the $\Upsilon(5S)$ and only a few percent at the $\Upsilon(6S)$. However, Yamamoto’s proposal could prove a competitive method for tagging neutral $B$’s for the
study of CP violation at a symmetric collider at slightly higher center of mass energy, about 12 GeV.

Our calculation is also useful because the $BB\pi$ mode is a potential background to $B_s$ identification (as discussed in section 8). We find the $BB\pi$ background could be comparable to the rate for $B_s$ production at the largest experimentally allowed value of the $B_s$ mass, but is most likely small compared to the rate if the $B_s$ mass is near the central value of the experimentally allowed range.

Finally, we discuss the extraction of the axial coupling constant of the $B$ mesons from a measurement of the $BB\pi$ branching fraction in a restricted region of phase space, where the heavy meson chiral lagrangian should apply.

We begin in section 2 by describing the heavy-meson effective theory, in order to review the assumptions and establish notation. In section 3, we describe how heavy quark ideas apply to the process $e^+e^- \rightarrow BB^\pm\pi^\mp$, and in section 4 we construct the effective lagrangian. We then calculate the cross section for $e^+e^- \rightarrow BB^\pm\pi^\mp$ in section 5. We discuss the implications of our calculation for CP violation studies and $B_s$ identification in sections 6 and 7. In section 8, we discuss the regime of validity of the results, and the extraction of $g$. Conclusions follow in the final section.

2. Review of the Heavy Meson Theory

In this section, we review the treatment of heavy meson fields in the heavy meson chiral effective theory. It is convenient to describe the $\bar{B}$ and $\bar{B}^*$ mesons which contain a bottom quark of velocity $v$ and a light antiquark, $\bar{q}$ by a Dirac tensor field of the form [2]:

$$B(v) = \frac{1 + \gamma^5}{2}(-b\gamma_5 + \sum_\varepsilon b\varepsilon\slash{\varepsilon}).$$

(2.1)
Here $b$ and $b_\varepsilon$ are the destruction operators for $\bar{B}$ and $\bar{B}^*$ mesons. The field of the $B$ mesons, which we denote as $\mathbf{B}(v)$, can be obtained from $\mathbf{B}(v)$ by using the charge-conjugation properties of the $B$ mesons. One obtains [3]

$$\mathbf{B}(v) = C (\mathbf{C} \mathbf{B}(v) C^{-1})^T C^T$$

$$= (-b_\gamma + \sum_\varepsilon b_\varepsilon \varepsilon) \frac{1 - f^\dagger}{2},$$

where $C$ is the charge conjugation operator and $C = i \gamma^2 \gamma^0$.

We also require fields which create $B$ mesons, obtained from the destruction fields by Dirac conjugation:

$$\mathbf{\bar{B}}(v) = \gamma^0 \mathbf{B}(v)^\dagger \gamma^0$$

$$\mathbf{\bar{B}}(v) = \gamma^0 \mathbf{B}(v)^\dagger \gamma^0.$$  \hspace{1cm} (2.3)

Under a heavy-quark symmetry transformation the mesons transform as follows:

$$\mathbf{B}(v) \to S_\nu \mathbf{B}(v),$$

$$\mathbf{\bar{B}}(v) \to \mathbf{\bar{B}}(v) S_\nu^\dagger,$$  \hspace{1cm} (2.4)

where $S_\nu$ is an element of the spin-1/2 representation of the little group of a particle of velocity $v$.

We implement the chiral symmetry in the usual way [4] by introducing the nonlinear field $\xi = e^{i \pi^a T^a / f^*}$. Under an $\text{SU}(3)_L \times \text{SU}(3)_R$ chiral transformation, $\xi \to L_\xi U^\dagger = U_\xi R^\dagger$. This defines the $\text{SU}(3)$ matrix $U$ as a nonlinear function of $L$, $R$ and $\pi(x)$. From $\xi$ one can construct an axial vector field,

$$A^\mu = \frac{i}{2} (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger) = -\partial^\mu \pi / f_\pi + \ldots,$$  \hspace{1cm} (2.5)

which transforms under the chiral symmetry as $A^\mu \to U A^\mu U^\dagger$. One can also construct a vector field from $\xi$,

$$V^\mu = \frac{1}{2} (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger) = \pi^0 \partial^\mu \pi / 2 f_\pi^2 + \ldots,$$  \hspace{1cm} (2.6)
which functions as a connection term in a covariant derivative: under a chiral transformation, $\partial^\mu + V^\mu \equiv D^\mu \rightarrow UD^\mu U^\dagger$.

According to the usual convention [5], the field $B_a$, which destroys a $\bar{B}$ meson containing a light antiquark of flavor $a$, transforms under the chiral symmetry as

$$B_a(v) \rightarrow B_a(v)U^\dagger_{ab}. \quad (2.7)$$

Similarly, the field $\bar{B}_a$ which destroys a $B$ meson containing an light quark of flavor $a$ transforms as

$$\bar{B}_a(v) \rightarrow U_{ab}\bar{B}_b(v). \quad (2.8)$$

3. Applicability of the Chiral Heavy Meson Effective Theory to $e^+e^- \rightarrow BB^\pm \pi^\mp$

In this section we discuss the energy regimes in which $B$ meson production can be reliably described using the chiral heavy meson effective theory. We argue that although such a calculation is not reliable to better than an order of magnitude within the resonance region, heavy quark methods should apply at higher center of mass energy. Because the calculation also requires that the pion energy be sufficiently low for chiral perturbation theory to work well, the calculation with the heavy meson chiral lagrangian is trustworthy only over a restricted region of phase space.

In order to become familiar with the energy scales involved in $BB$ and $BB\pi$ production, we begin by listing the masses of the $B$ mesons and the $\Upsilon$ resonances and also the total final-state kinetic energy, or $Q$ value, for $\Upsilon$ decay into $B\bar{B}$ and $BB^\pm\pi^\mp$: The charged pseudoscalar $B$ mesons have mass $M_B = 5278.6 \pm 2.0$ MeV (all values for
Table 1. Masses and $Q$ Values

| Resonance   | $\sqrt{s}$ (MeV) | $Q_{BB}$ (MeV) | $Q_{BB\pm\pi \mp}$ (MeV) |
|-------------|------------------|----------------|--------------------------|
| $\Upsilon(4S)$ | 10580            | 23             | —                        |
| $\Upsilon(5S)$ | 10865            | 308            | 168                      |
| $\Upsilon(6S)$ | 11020            | 463            | 323                      |
| Off Resonance | 12000            | 1443           | 1303                     |

Particle masses in this section are taken from ref. [6]); the neutral pseudoscalar mass is essentially the same. The vector $B^*$ meson mass is $M_{B^*} = 5324.6 \pm 2.1$ MeV. In Table 1 we list the central values for the masses of the $\Upsilon$ resonances and the $Q$ values for decay into two heavy pseudoscalars with and without a charged pion:

$$Q_{BB} \equiv M_\Upsilon - 2M_B,$$

$$Q_{BB\pm\pi \mp} \equiv M_\Upsilon - 2M_B - m_{\pi \pm}.$$  \hspace{1cm} (3.1)

For final states including heavy vector mesons, the $Q$ values are reduced by one or two times the heavy meson hyperfine splitting, $M_{B^*} - M_B = 46.0 \pm 0.6$ MeV. We also list the $Q$ values for a center of mass energy of 12 GeV, which is above the resonance regime.

We see that pions produced at a resonance have small momentum and energy relative to the chiral scale. At 12 GeV, the most energetic pions which are produced can probably not be treated as soft in a chiral expansion; however, we will see in section 8 that even at this energy the pion can be treated as soft over a substantial portion of the phase space. A naive estimate of the relative rate of $BB\pm\pi \mp$ to $B\bar{B}$ production would say that the first is suppressed by a factor of $(p_\pi/4\pi f_\pi)^2$ relative to the latter, so that it will only be produced significantly at center of mass energy approximately 1 GeV above threshold, which is why we investigate not only the resonance regime, but higher center of mass energy as well.
In our calculation, we need both the chiral and heavy quark approximations to be valid. Some care must be taken in the application of the heavy quark effective theory to a process with a $B$ and a $\bar{B}$ meson in the final state. The problem is that the Isgur–Wise function (or its analytic continuation) is not useful in the resonance regime, where the matrix element $\langle B(p)\bar{B}(p')|\bar{b}\gamma^{\mu}b|0\rangle$ varies rapidly as a function of $p \cdot p'$. Because of this rapid variation, the form factor is not well described in the resonance region as a function of $v \cdot v'$, where $v$ and $v'$ are the heavy quark velocities. Furthermore, it is not normalized, and drops rapidly to zero beyond the resonance region. This behavior was studied by Jaffe in ref. [7], and is expected on the basis of general QCD considerations, since the states which can decay into $B$ mesons are not well described as simple Coulomb bound states.

This has several important consequences. First, in the resonance region the calculation must be considered as, at best, an order of magnitude estimate. In the language of the heavy meson effective theory, this is because there is no well-defined derivative expansion; the rapid variation of the cross section with momentum in the resonance region is reflected in the heavy meson lagrangian by the presence of higher derivative operators “suppressed” only by the QCD scale. This is generally not the case in heavy quark calculations because of reparameterization invariance[8][9]. However, here, although the total cross section for $e^+e^- \rightarrow BB\pi$ respects a reparameterization invariance since the cross section depends only on the $B$ meson momenta ($p$ and $p'$) and not separately on the heavy quark velocities ($v$ and $v'$) and the residual momenta ($k$ and $k'$), the rate for “decay” of the source is not reparameterization invariant. This is because in the effective lagrangian, we work with heavy quark fields at fixed values of $v$ and $v'$, so
that we necessarily assume a resonance produced at fixed $v \cdot v'$ (not $p \cdot p'$). Therefore, the effective lagrangian must contain nonreparameterization invariant derivative terms to reproduce the full cross section, which does respect reparameterization invariance. Since the momentum of a $B$ meson is of order $\sqrt{T_B M_B}$, where $T_B$ is the kinetic energy of the $B$ meson, the higher derivative terms are always large, even in the heavy quark limit so that the derivative expansion is not reliable.

It is clear that the relative rate predictions for decay to two pseudoscalars, vector and pseudoscalar, and two vectors, considered in refs. [10], [11] and [12] are only appropriate beyond the resonance regime, as has been emphasized by these authors. The measured [13] branching ratios for the decay of the $\psi(3S)$ into $D$ mesons of different spins strongly disagree with those predicted by a naive application of the results of the heavy quark theory to the resonance regime [14]. The explanation of ref. [15] is that the nodes in the $\psi(3S)$ momentum-space wavefunction result in almost no overlap with the final state $D$ mesons except when both are vector particles. In our approach, we attribute the large discrepancy between the prediction and the experimental results to the presence of higher dimension operators, suppressed only by the QCD scale, which give different contributions for decays to heavy mesons of different spins because of the significant mass splitting.

We conclude that the only regime where one would trust the calculation of $B$ meson production to better than an order of magnitude is beyond the resonance regime. Fortunately, this is the region where the result is most interesting since the rate in the resonance regime is too small. When we calculate in the resonance region where heavy quark relations are untrustworthy, we view the calculation as a model which should
reproduce important qualitative features of the true rate. These include the relevant kinematic features, phase space, propagator enhancements, and derivative couplings.

4. Effective Theory for $e^+e^- \to BB^{\pm}\pi^\mp$

We proceed to the construction of the effective lagrangian for $BB$ and $BB\pi$ production, keeping in mind the above caveats regarding the application of the heavy meson and chiral effective theory to these processes.

It is useful to divide the range of possible center of mass energies into three regions: the resonance regime, above the resonance regime but with the $B$ mesons nonrelativistic, and high energy, with the $B$ meson fully relativistic. The heavy quark theory is straightforward to construct in the third regime, and has been treated in previous work [10][11]. One couples the current to a heavy quark and antiquark of velocities $v$ and $v'$. The heavy quark operator is then evaluated between heavy meson states and the appropriate spin symmetry relations between amplitudes can be deduced.

However, the regions of greatest physical relevance are those at low center of mass energy, since at high energy the rate for exclusive production of $BB$ or $BB\pi$ is very low. Moreover, the experiments of interest are conducted at or near threshold. Since the kinetic energies of the $B$ and $\bar{B}$ are on the order of the QCD scale, both the $B$ and $\bar{B}$ mesons have essentially timelike four velocities in the center of mass frame: $v^\mu \approx v'^\mu \approx (1,0,0,0)$.

In the second region — beyond the resonances, but with the $B$ mesons still nonrelativistic — the effective theory is constructed as at high energy, but with the velocities $v$ and $v'$ equal to $(1,0,0,0)$. As before, the virtual photon produces a $b$ and a $\bar{b}$ quark,
each of velocity $v^\mu = (1, 0, 0, 0)$. In the heavy quark theory, one can couple a source $S^\mu$ to the $b$-quark current as $\bar{b}_v S^\mu \gamma_\mu b_v$ (i.e., with the heavy quark spins coupled to the spin of the source). When we match onto the heavy meson theory, we can couple the source to the mesons as $\bar{B}(v') S^\mu \gamma_\mu B(v)$. This gives the same matrix elements as if we had coupled a source to heavy quarks, and then evaluated the heavy quark matrix elements.

Finally, we consider the resonance regime. If only hard gluons were relevant to the binding potential, it would be clear how to construct such an effective theory. The matching would again proceed in two steps. First, one would match onto the heavy quark theory, and then match the $b$ quark operator onto the heavy meson effective lagrangian. However, for a resonance which can decay into $B$ mesons, the binding is sufficiently weak that both hard and soft gluons play a role. Hence, it might instead be appropriate to match directly onto the low energy heavy quark chiral lagrangian. While we still expect that interactions with the light quarks cannot flip a heavy quark spin, the interactions between the heavy quarks can flip their spins. The total spin of the heavy quarks is conserved, however, and hence the spin of the source is transferred entirely to the heavy quark spins. This can be incorporated in the heavy meson theory by again coupling the source to the mesons as $\bar{B}(v') S^\mu \gamma_\mu B(v)$. Fortunately, the same heavy meson lagrangian describes the matrix elements in both scenarios, since the diagonal subgroup of the heavy quark spin symmetry, where both heavy quark spins are rotated simultaneously, is sufficient to determine the form of the coupling of the heavy mesons to the source.
The lagrangian applicable to low-energy production of $B$ and $\bar{B}$ meson is

$$L_{\text{eff}} = -i\text{tr}\{\overline{B}_a(v)\gamma^\mu \partial_\mu B_a(v)\} - i\text{tr}\{\overline{B}_a(v)\gamma^\mu \partial_\mu B_a(v)\}$$

$$+ g\text{tr}\{\overline{B}_a(v)B_b(v)A_{ba}^\nu \gamma_\nu \gamma_5\} + g\text{tr}\{\overline{B}_a(v)B_b(v)A_{ba}^\nu \gamma_\nu \gamma_5\} + L_S,$$  

(4.1)

$$L_S = -\frac{i\lambda}{2} S_\mu \text{tr}\{\gamma_\mu \overline{B}_a(v) \overline{D}_{ab}^\nu \gamma_\nu B_b(v)\}$$

$$+ \lambda g' S_\mu \text{tr}\{\gamma_\mu \overline{B}_a(v) A_{ab}^\nu \gamma_\nu \gamma_5 B_b(v)\}.$$  

Here $D = \partial + V$ is the chiral covariant derivative incorporating the pion fields. As usual, a factor of $\sqrt{M_B}$ has been absorbed into the heavy meson fields along with the position-dependent phase corresponding to the momentum of the heavy quark (so that a derivative acting on these fields only gives a factor of the residual momentum), in order to suppress the appearance of the heavy quark mass and emphasize the heavy quark symmetry. Because this is the low energy theory, no large momentum transfers are permitted. At higher energies, the appropriate lagrangian would be the heavy meson lagrangian with velocities $v \neq v'$. The result for two meson production matches smoothly, as the difference in residual momenta in the amplitude gets replaced by the difference in heavy meson velocities.

The kinetic and axial coupling terms for the $B$ mesons have been discussed previously and result from the straightforward application of heavy quark and chiral effective field theories [5]. $L_S$ is the new term and follows from the assumptions described above. Note that with the trace the heavy quark spin labels are coupled to $S_\mu \gamma_\mu$.

The coupling $\lambda$ corresponds for a crossed process, e.g., $B^* \rightarrow B\gamma$, to the Isgur–Wise form factor. Here, we treat the ratio $\sigma(e^+e^- \rightarrow BB\pi)/\sigma(e^+e^- \rightarrow BB)$ as independent of $\lambda$, although this is strictly true only when beyond the resonance region. (In a full
We therefore consider the ratios \( \sigma_{\text{mesons}}, \sigma_{\text{BB}} \) together with the \( B \) meson through the heavy-meson axial coupling. Or, the pion can be produced “directly”, production. The pion can be produced “indirectly” by being emitted from a virtual \( B \) meson through the heavy-meson axial coupling. Or, the pion can be produced “directly”, together with the \( B \) mesons at a single vertex. This diagram comes from the contact term in the lagrangian in which the source couples directly to the \( B \) meson and axial fields. Examples of both types of diagrams are shown in Figure 1a. We will see that the direct contribution is much the smaller of the two, and so most of our discussion concentrates on the indirect contribution.

In order to compare \( BB^\pm \pi^\mp \) with \( B \bar{B} \) as sources of neutral \( B \) mesons we normalize the \( BB^\pm \pi^\mp \) cross sections by dividing them by the cross section for \( e^+e^- \rightarrow \) neutral \( B \) mesons, \( \sigma_0 \),

\[
\sigma_0 \equiv \sigma(e^+e^- \rightarrow B^0\bar{B}^0, B^0\bar{B}^{*0}, B^{*0}\bar{B}^0, \text{or } B^{*0}\bar{B}^{*0}).
\]

We therefore consider the ratios

\[
R_{PP} = \frac{1}{\sigma_0} \sum_{\pm} \sigma(e^+e^- \rightarrow BB^\pm\pi^\mp),
\]

\[
R_{PV} = \frac{1}{\sigma_0} \sum_{\pm} \{ \sigma(e^+e^- \rightarrow BB^{*\pm}\pi^\mp) + \sigma(e^+e^- \rightarrow B^{*}\bar{B}^{\pm}\pi^\mp) \},
\]

\[
R_{VV} = \frac{1}{\sigma_0} \sum_{\pm} \sigma(e^+e^- \rightarrow B^{*}\bar{B}^{*\pm}\pi^\mp),
\]

5. Calculation of \( \sigma(e^+e^- \rightarrow BB^\pm\pi^\mp)/\sigma(e^+e^- \rightarrow B\bar{B}) \)

From the lagrangian (4.1) we see that two types of diagrams contribute to \( BB\pi \) production. The pion can be produced “indirectly” by being emitted from a virtual \( B \) meson through the heavy-meson axial coupling. Or, the pion can be produced “directly”, together with the \( B \) mesons at a single vertex. This diagram comes from the contact term in the lagrangian in which the source couples directly to the \( B \) meson and axial fields. Examples of both types of diagrams are shown in Figure 1a. We will see that the direct contribution is much the smaller of the two, and so most of our discussion concentrates on the indirect contribution.

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\]

\[
R_{VV} = \frac{1}{\sigma_0} \sum_{\pm} \sigma(e^+e^- \rightarrow B^{*}\bar{B}^{*\pm}\pi^\mp),
\]

(5.2)
and also their sum,

\[ R = R_{PP} + R_{PV} + R_{VV}. \] (5.3)

The subscripts denote the heavy-meson content of the \( BB^{\pm}\pi^{\mp} \) final state, with a \( P \) for each pseudoscalar and a \( V \) for each vector. In these ratios of cross sections, \( R_\alpha \), kinematic factors associated with the initial state and also the unknown coupling of the source to the \( B \) mesons, \( \lambda \), cancel out.

The cross sections for two-body \( B\bar{B} \) final states are, apart from a common factor, each given by the product of a \( p \)-wave phase-space factor and a spin-counting factor \cite{12}. The total cross section for producing neutral \( B \) mesons, \( \sigma_0 \), is then proportional to a sum of such products:

\[ \sigma_0 \propto r_{PP}^{3/2} + 4r_{PV}^{3/2} + 7r_{VV}^{3/2} \equiv P(s). \] (5.4)

Here \( r \) is the center-of-mass energy that remains after supplying the rest-mass energies of the heavy-mesons. For a two-body \( B\bar{B} \) final state, \( r^{3/2} \propto |\Delta \vec{k}|^3 \), where \( \Delta \vec{k} \) is the relative three-momentum of the heavy mesons, which is the familiar \( p \)-wave phase-space factor. The different values of \( r \) that correspond to the various \( B\bar{B} \) final states are then given by

\[ r_{PP} = \sqrt{s} - 2M_B \]
\[ r_{PV} = \sqrt{s} - M_B - M_{B^*} \]
\[ r_{VV} = \sqrt{s} - 2M_{B^*} \] (5.5)

For \( B\bar{B} \) final states, the \( r_\alpha \) coincide with the \( Q \) values for the decay of the source into heavy mesons; however, we will also use the \( r_\alpha \), as defined above, when we consider \( BB^{\pm}\pi^{\mp} \) final states. Note that we are including violation of the heavy-quark symmetry

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as it enters through the $B-B^*$ mass splitting, $\Delta M = 46.0 \pm 0.6$ MeV; this splitting significantly affects the available phase space and the off-shellness of intermediate $B$ mesons.

The density of states for a final state of $B$ mesons and a pion simplifies for nonrelativistic $B$ mesons to

$$D_0 = \frac{1}{256\pi^5 E_\pi} d^3\overrightarrow{k} d^3\Delta\overrightarrow{k} \delta\left(r - E_\pi - \frac{1}{M_B} \left(\frac{1}{4} \Delta \overrightarrow{k}^2 + \overrightarrow{k}^2\right)\right). \quad (5.6)$$

Here $\overrightarrow{k} = (k + k')/2$ is the average of the heavy-meson residual momenta and $\Delta k^\mu = (k - k')^\mu$ is their difference. We have included a factor of $(\sqrt{M_B})^2$ for each of the two heavy mesons in the final state in order to compensate for the rescaling of the heavy meson fields in the amplitudes. If we perform the trivial angular integrals as well as the integral over $|\Delta \overrightarrow{k}|$ (using the $\delta$-function), we obtain the (integrated) density of states,

$$D = \frac{1}{16\pi^3} \frac{M_B}{E_\pi} |\Delta \overrightarrow{k}| |\overrightarrow{k}|^2 d|\overrightarrow{k}| dE_\pi d(\cos \theta). \quad (5.7)$$

Here $\theta$ is the angle between $\overrightarrow{k}$ and $\Delta \overrightarrow{k}$. Then, expressing $|\overrightarrow{k}|$ and $|\Delta \overrightarrow{k}|$ in terms of the pion energy and momentum,

$$p_\pi = -2\overrightarrow{k} \Rightarrow |\overrightarrow{k}| = \frac{p_\pi}{2},$$

$$|\Delta \overrightarrow{k}| = \sqrt{4M_B\left(r - E_\pi - \frac{p_\pi^2}{4M_B}\right)} \approx 2\sqrt{(r - E_\pi)M_B}, \quad (5.8)$$

the density of states becomes

$$D = \frac{1}{64\pi^3} M_B |\Delta \overrightarrow{k}| |\overrightarrow{k}| dE_\pi d(\cos \theta)$$

$$= \frac{1}{64\pi^3} M_B^{3/2} p_\pi (r - E_\pi)^{1/2} dE_\pi d(\cos \theta). \quad (5.9)$$

As discussed in the last section, we treat the $B$ mesons as nonrelativistic in the laboratory frame, working to lowest nonvanishing order in the heavy-meson three-momenta.
Implicit in our approximations is the realization that the kinetic energy is fairly evenly shared between the pion and the heavy mesons: Consider the density of states just above (eq. (5.9)). It can be reexpressed in terms of the total kinetic energy, \( T = r - m_\pi \), and the kinetic energy of the pion, \( T_\pi \), as

\[
D \propto T_\pi^{1/2}(T - T_\pi)^{1/2}(T_\pi + m_\pi)^{1/2}dT_\pi
\]

\[
= T^{5/2} x^{1/2}(1 - x)^{1/2}(x + m_\pi/T)^{1/2}dx
\]

(5.10)

where \( x = T_\pi/T \) is the fraction of the kinetic energy given to the pion. We find that as \( T \) runs from 0 to \( \gg m_\pi \), the phase-space average of \( x \) runs from \( 1/2 \) to \( 4/7 \), i.e., on average the pion kinetic energy about equals the sum of the \( B \) meson kinetic energies.

In the differential cross section, where the density of states is multiplied by the squared amplitude, which includes such factors as \( |\Delta k|^2 \) and \( |p_\pi|^2 \), the powers of \( x \) or \( (1 - x) \) are increased, shifting the average value of \( x \) up or down, but the kinetic energy remains fairly evenly distributed between the pion and the heavy mesons.

Therefore, when we work at energies where the heavy mesons are nonrelativistic, there is a hierarchy of energy scales,

\[
T_B, E_\pi, p_\pi (\propto M_B^0) \ll |\vec{k}_B| (\propto M_B^{1/2}) \ll M_B, \quad (5.11)
\]

where \( T_B \) is the kinetic energy of the \( B \) mesons. Since the dimensionful quantities that compensate the different powers of \( M_B \) here are of order \( r \), we are essentially working to lowest order in \( r/M_B \). Taking sums and differences of the \( B \) meson momenta and using eq. (5.8), this hierarchy can be reexpressed as

\[
\vec{k}^0, \Delta k^0, |\vec{k}| (\propto M_B^0) \ll |\Delta \vec{k}| (\propto M_B^{1/2}) \ll M_B. \quad (5.12)
\]
In calculating a given amplitude, we retain only the leading term according to this hierarchy. Specifically, we drop $k^0$ and $\Delta k^0$ compared to $M_B$, incurring errors of order $r/M_B$. We also drop $|\vec{k}|$ compared to $|\Delta \vec{k}|$ and $|\Delta \vec{k}|$ compared to $M_B$. This would seem to mean dropping terms of order $\sqrt{r/M_B}$. However, once an amplitude is squared, averaged/summed over initial/final polarizations, and integrated over $\cos \theta$, the result depends only on $\vec{k}^2$ and $\Delta \vec{k}^2$ ($\vec{k} \cdot \Delta \vec{k}$ vanishes in the angular integration). Hence all the dropped terms are smaller by a factor of $r/M_B$.

Later, in section 8, we will consider the contribution to $BB^{\pm} \pi^\mp$ production from a restricted region of phase space where the pion energy is less than a given bound. By imposing this cutoff the average value of the pion momentum, and therefore of $|\vec{k}|$, will be reduced. This only improves our approximation of neglecting $|\vec{k}|$ compared to $|\Delta \vec{k}|$.

We will illustrate these approximations in the simplest case, the calculation of the indirect contribution to $R_{PP}$. The two graphs that contribute are shown in Figure 1b. The graph on the left in Figure 1b corresponds to the process $S \to BB^* \to B(\bar{B}\pi)$. Consider the propagator for the intermediate $\bar{B}^*$ state, which is described by the velocity $v^\mu = (1, 0, 0, 0)$ and a residual momentum $q^\mu = k^\mu + p^\mu_{\pi}$ and has a mass that is $\Delta M$ greater than that of the $\bar{B}$ in the final state. The propagator is then

$$\frac{i}{2(v \cdot q + q^2/2M_B - \Delta M)} \approx \frac{i}{2(v \cdot p_\pi + k \cdot p_\pi/M_B + m_{\pi}^2/2M_B - \Delta M)} \approx \frac{i}{2(v \cdot p_{\pi} - \Delta M)} \approx \frac{i}{2(E_{\pi} - \Delta M)}. \quad (5.13)$$

The first equality comes from the on-shell condition for the $\bar{B}$ meson, $(v + k/M_B)^2 = 1$, which is just the nonrelativistic formula $T_B \approx k^2/2M_B$. The next line results from
neglecting terms of order $k \cdot p_\pi / M_B$ and $m_\pi^2 / M_B$ compared to $E_\pi$, in accordance with the hierarchy in eq. (5.12). The propagator we have obtained, which is inversely proportional to the difference in energy between the intermediate and final states, is simply that prescribed by nonrelativistic, time-ordered perturbation theory.

Now consider the part of the amplitude coming from the $S \rightarrow B \bar{B}^*$ vertex. From our effective lagrangian (4.1) we see that it is proportional to $q - k' = \Delta k + p_\pi$. Then according to the hierarchy (5.12), we can drop $p_\pi = -2\overline{k}$. The amplitude for $S \rightarrow B \bar{B}^* \rightarrow B(B\pi)$ is then given by

$$A_1(\varepsilon; v, k, k') \approx -\frac{ig\lambda\sqrt{2}}{2(E_\pi - \Delta M)f_\pi} \epsilon_{\alpha\beta\gamma\delta}v^\alpha \Delta k^\beta p_\pi^\gamma\varepsilon^\delta,$$

(5.14)

where $\varepsilon$ is the (purely spatial) polarization of the source. The second graph, shown on the right in Figure 1b, is related to the first by charge conjugation and isospin, so that their sum is given by

$$A = A_1(\varepsilon; v, k, k') - A_1(-\varepsilon; v, k', k) \approx 2A_1(\varepsilon; v, k, k')$$

$$\approx -\frac{ig\lambda\sqrt{2}}{(E_\pi - \Delta M)f_\pi}(\Delta k \times p_\pi) \cdot \varepsilon.$$

(5.15)

Squaring and summing over the averaging over initial polarizations we obtain

$$\langle |A|^2 \rangle_{\varepsilon} \approx \frac{2}{3} \left( \frac{g\lambda}{(E_\pi - \Delta M)f_\pi} \right)^2 \Delta k^2 p_\pi^2 (1 - \cos^2 \theta).$$

(5.16)

If we substitute for $|\Delta k|$, integrate over the density of states, and divide by $\sigma_0$ (not including the common kinematical factors corresponding to the initial state) we obtain $R_{PP}$. 

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The cross-section ratios for the various $BB^{\pm \pi^\mp}$ final states are given by

$$R_i^\alpha(s) = \frac{2g^2}{3\pi^2 f_\pi^2} \frac{1}{P(s)} \int_{m_\pi}^{r_\alpha} dE_\pi p_\pi^2(r_\alpha - E_\pi)^{3/2} \times \left\{ \begin{array}{ll} \frac{1}{(E_\pi - \Delta M)^2}, & \alpha = PP; \\ \frac{7/4}{(E_\pi - \Delta M)^2} + \frac{1/2}{E_\pi^2 - (\Delta M)^2} + \frac{1}{E_\pi} + \frac{3/4}{(E_\pi^2 + \Delta M)^2}, & \alpha = PV; \\ \frac{5}{E_\pi^2} + \frac{2}{(E_\pi + \Delta M)^2}, & \alpha = VV. \end{array} \right. \quad (5.17)$$

Here the $i$ signifies that these are the indirect contributions to $R_\alpha$. The upper integration limit is the value of $r$ appropriate to the heavy-meson content of the $BB^{\pm \pi^\mp}$ final state, as given by eq. (5.5).

At a fixed center of mass energy, the pion attains its maximum energy when the heavy meson pair is produced with zero relative momentum ($\Delta k = 0$), back to back with the pion. From eq. (5.8) we see that, to the accuracy we are are working to, this occurs when $E_\pi = r_\alpha$. Hence the upper limit of the phase-space integral over pion energy is $r_\alpha$.

As already discussed, there is also a direct vertex that describes the production of $B$ mesons and a pion at a single vertex. For $BB^{\pm \pi^\mp}$ final states that include at least one heavy vector, it is possible to produce the heavy mesons in an $s$ wave. Then, the amplitudes are proportional to $E_\pi/(M_B f_\pi)$ while the amplitudes for the indirect graphs considered above are proportional to $|\Delta k| p_\pi/(M_B f_\pi E_\pi)$. At large $r$, where the cross section is dominated by contributions with $E_\pi \gg m_\pi$, the ratio of direct and indirect amplitudes is proportional to $E_\pi/|k_B|$ which is of order $r/M_B$. Therefore, the $s$-wave contributions of the direct graphs are suppressed by a factor of $r/M_B$, which is small for $r$ inside the region of validity of the chiral expansion. The $s$-wave direct graphs,
Table 2. The ratios $\sigma(e^+e^- \rightarrow BB^{\pm}\pi^{\mp})/\sigma(e^+e^- \rightarrow B\bar{B})$ expressed as percentages.

| $\sqrt{s}$(MeV) | $R_{\text{PP}}^d$ | $R_{\text{PV}}^i$ | $R_{\text{VV}}^i$ | $R_i^d$ | $R_{\text{PV}}^d$ | $R_{\text{VV}}^d$ | $R_d$ | $R$ |
|-----------------|-----------------|-----------------|-----------------|--------|-----------------|-----------------|------|-----|
| 10865           | 0.12            | 0.12            | 0.02            | 0.26   | 0.03            | 0.01            | 0.04 | 0.30|
| 11020           | 0.53            | 0.99            | 0.70            | 2.2    | 0.19            | 0.10            | 0.29 | 2.5 |
| 11200           | 1.3             | 3.1             | 3.1             | 7.6    | 0.59            | 0.40            | 0.99 | 8.6 |
| 11500           | 3.3             | 9.2             | 11.2            | 24.2   | 1.6             | 3.7             | 27.  |     |
| 12000           | 8.2             | 26.0            | 36.0            | 70.0   | 7.8             | 6.7             | 14.  | 85. |

which cannot interfere with the $p$-wave indirect graphs, contribute to the $R_\alpha$ as

$$R_{\alpha}^d(s) = \frac{3g'^2}{2\pi^2 f_\pi^2 P(s)} \int_{m_\pi}^{r_\alpha} dE_\pi E_\pi^2 P_\pi(r_\alpha - E_\pi)^{1/2} \times \begin{cases} 0, & \alpha=\text{PP}; \\ 1/M_B, & \alpha=\text{PV}; \\ 1/M_B, & \alpha=\text{VV}. \end{cases}$$ (5.18)

The $d$ stands for direct. Note that the source cannot decay into two pseudoscalars in an $s$-wave and a pion without violating either angular momentum or parity conservation. Hence $R_{\text{PP}}^d$ is zero to this order in $1/M_B$.

The axial coupling of the heavy mesons, $g$, has been bounded above by $g^2 \leq 0.5$ [16] using the experimental upper limit for the $D^{*+}$ width [17] which is dominated by $D^{*+} \rightarrow D^0\pi^+$ and $D^{*+} \rightarrow D^+\pi^0$. Using the maximum allowed value of $g^2$ and taking $g'^2 = 1$, the numerical values of the ratios $R_\alpha$ (expressed as percentages) are given in Table 2 for various values of the center-of-mass energy, $\sqrt{s}$, and are plotted in Figure 2. The sum of the direct contributions to $R$ is also displayed in Figure 2. Because they are so small, we neglect them in the rest of our discussion. The overall sum is uncertain due to the uncertainties in both the direct and indirect contributions.

At the $\Upsilon(6S)$ resonance, the $B$ mesons are produced with a charged pion only a few percent as often as they are produced alone. Below this resonance, for example at the $\Upsilon(5S)$, this ratio is negligible, but above it grows with increasing energy as the limited
three-body phase space for pion production is overcome. For center of mass energy of 12 GeV, the rate is almost comparable to the rate without a pion.

Returning to the results for the $R^i_\alpha$ given in eq. (5.17), we see that in the limit of exact heavy quark symmetry, $\Delta M = 0$, the ratios $R_\alpha$ are proportional to one another as

$$R^{i}_{\text{PP}} : R^{i}_{\text{PV}} : R^{i}_{\text{VV}} :: 1 : 4 : 7 \quad (\Delta M = 0). \quad (5.19)$$

This limit is approximately realized for $\Delta M \ll r \ll M_B$, where the effects of $\Delta M \neq 0$ are minimized and the nonrelativistic treatment still applies. For such large values of $r$, it is also a good approximation to take $m_\pi = 0$. Then we find $R^i_\alpha \propto r^{7/2}$. The two extra powers of $r$ relative to the $r^{3/2}$ scaling of the $B\bar{B}$ cross sections reflect the derivative coupling and additional phase space factor for the pion.

The proportions 1 : 4 : 7 are the same as was found for the production rates of heavy mesons without an accompanying pion in the various spin states (PP, PV and VV) using simple spin counting [12] (see eq. (5.4)). We can use the same method to understand the persistence of the proportions 1 : 4 : 7 when a pion is emitted from one of the heavy mesons.

In $S \to B\bar{B}$ (pseudoscalars and vectors), the heavy quark spins are fixed so as to carry the spin of the source. Charge-conjugation invariance then determines the spin state of the light quarks: Consider the $B\bar{B}$ final state from the point of view of heavy quark symmetry, with each heavy meson described by a bispinor. Under charge conjugation, the positions and the heavy- and light-quark spins labels of the $B$ and $\bar{B}$ mesons are interchanged. Since the heavy mesons are in a $p$ wave and the heavy quark spin state is symmetric (spin one), the light quark spin state must also be symmetric.
if the final state is to be charge-conjugation odd like the source. Hence the total spin of the light quarks must be one. Because the spin of the source is carried by the heavy quarks, after the angular positions of the heavy mesons have been integrated over (but with the source spin fixed), the total light quark spin points with equal probability in all directions. Adding the light and heavy quark spins in each heavy meson to find the meson spins, one finds the ratios $1 : 4 : 7$.

Now consider the case where a pion is emitted from a heavy meson. Because of isospin invariance, we need only consider the case where the pion is neutral and hence self-conjugate. Since the $\pi^0$ is charge-conjugation even, the total spin of the light quarks in the heavy mesons must still be one. After integrating over the angular positions of all the particles, the light quark spin distribution will again be isotropic. Hence in the heavy quark limit, the heavy meson spin proportionalities do not change when a pion is emitted from one of the heavy mesons.

6. Implications for CP Violation Studies

We have calculated the $BB\pi$ branching ratio primarily in order to assess the prospects for studying CP violation in the $B^0-\bar{B}^0$ system using Yamamoto’s pion-tagging method [1]. Let us now see if the $BB^\pm\pi^\mp$ branching ratios obtained in the previous section are large enough to produce the statistics required to resolve the small, CP-violating asymmetries in the decays of neutral $B$ mesons. The very small values found for $R$ at the $\Upsilon(5S)$ and $\Upsilon(6S)$ resonances indicate that the pion-tagging method will not be useful in the resonance regime, but the rapid increase in the rate with center of mass energy suggests that the method could prove useful at higher energies, above the resonances.
The analysis of this section is due to Yamamoto [1], but uses our results for the $BB^\pm\pi^\mp$ branching ratios. We follow him in introducing a figure of merit which measures the statistical power of a given method of measuring CP violation,

$$f = \sigma \varepsilon_{\text{tag}} d^2.$$  \hspace{1cm} (6.1)

It is proportional to the cross section, $\sigma$, for the process in which the $B$ mesons are produced. There is a factor of the tagging efficiency, which is taken to include the number of neutral $B$ mesons produced in the process. Finally, it includes the square of the “dilution factor”, $d$, which is given by

$$A^\text{CP} = d \sin 2\beta,$$  \hspace{1cm} (6.2)

where $\beta$ is a CP-violating angle that appears in the CP-violating asymmetry, $A^\text{CP}$, measured in neutral $B$ decay. For example, using the pion-tagging method one measures the asymmetry

$$A^\text{CP}_{BB^\pm\pi^\mp} = \frac{N(B \to \psi K_S) - N(\bar{B} \to \psi K_S)}{N(B \to \psi K_S) + N(\bar{B} \to \psi K_S)} = \frac{x}{1 + x^2} \sin 2\beta,$$  \hspace{1cm} (6.3)

where $N$ is the number of events from a given decay process, $x = \delta M/\Gamma \approx 0.73$ ($\delta M$ is the mass difference between the CP-even and -odd linear combinations of $B^0$ and $\bar{B}^0$ and $\Gamma$ is their common lifetime), and $\psi K_S$ is a representative CP eigenstate. The utility of the figure of merit is that it is inversely proportional to the integrated luminosity required in order to measure a CP-violating angle; the larger the figure of merit, the easier it is to measure the angle to a given precision.

In Table 3 we compare the figure of merit for the pion-tagging method, at the $\Upsilon(6S)$ and at $12\text{ GeV}$, with the familiar methods of generating neutral $B$ mesons for
CP-violation studies: $\Upsilon(4S) \to B^0 \bar{B}^0$ at an asymmetric collider and $\Upsilon(5S) \to B^0 \bar{B}^0 \gamma$ at a symmetric collider. Although the asymmetric collider provides the best figure of merit, it is worthwhile to determine alternative methods at a symmetric collider in case either such a machine is not constructed or the required luminosity is not obtained. The relevant benchmark for us is therefore comparison with the figure of merit for studies using $\Upsilon \to B \bar{B} \gamma$, which are feasible at a symmetric collider.

The $BB(X)$ cross section is approximately $0.3 \text{nb}$ at the $\Upsilon(5S)$ and $\Upsilon(6S)$ resonances. Using the value of $R = 2\%$ at the $\Upsilon(6S)$ found above, not surprisingly we find that the pion-tagging method is not competitive at the $\Upsilon(6S)$.

To estimate the $BB^\pm \pi^\mp$ cross section at 12 GeV, beyond the resonances, we take the $(BB)X$ cross section as $1/2$ of its on-resonance value at the $\Upsilon(6S)$. We further assume that $(BB)X$ is dominated by pair production of strange and nonstrange $B$ mesons and $B$ mesons accompanied by a pion (charged or neutral). We expect that because of the sharing of kinetic energy discussed earlier, multipion production will be much smaller than single pion production at this energy. To estimate the contribution of $BBK$ and $BB\eta$ production to $(BB)X$, we repeated our calculation of $BB^\pm \pi^\mp$ production above but with $K$ and $\eta$ mesons radiated from the heavy mesons, and including SU(3) breaking via the $K$ and $\eta$ masses and decay constants and the $B_s$-$B$ mass splitting. We found that $BBK$ and $BB\eta$ together are about $1/4$ $BB\pi$, so we neglect their contribution to $(BB)X$. Estimating $B_s$ meson production, as discussed in the following section, by using eq. (5.4) but with the values of $r_\alpha$ determined by the $B_s$ meson masses, we find that $B_s$ mesons are produced about 80% as often as $B^0 \bar{B}^0$. We therefore estimate that the
$BB^{\pm} \pi^{\mp}$ cross section is $\approx 0.15 \text{nb} \times R/(2.8 + 3R/2) \approx 0.027 \text{nb}$. We find a figure of merit of 0.005.

If we extrapolate our results for $BB^{\pm} \pi^{\mp}$ production to even higher energies, we find that at about 12.5 GeV, $R$ is about twice its value at 12 GeV, so that the figure of merit is also roughly doubled. We conclude that the figure of merit can be comparable to the figure of merit for $\Upsilon(5S) \to B^0 \bar{B}^0 \gamma$ at a symmetric collider. Because our calculation is not exact, it is conceivable that the figure of merit is even higher. This can only be determined experimentally.

We emphasize that it is unlikely that we have vastly overestimated the rate, since it is clear that as one approaches higher center of mass energy that pion production will be less suppressed. So long as the center of mass energy is not too high, single pion production will dominate. Moreover, we show in section 8 that even if we restrict our integral over the phase space to pion energies for which the chiral results are certainly reliable, we still get a substantial fraction the total cross section determined from the full phase space integration.

We conclude that Yamamoto’s pion-tagging method should be competitive. An advantage of running at a higher center of mass energy would be that it could be possible to use both $BB^*$ production and $BB^{\pm} \pi^{\mp}$ production simultaneously. This might augment statistics, and provide a useful check on both methods.

7. Implications for $B_s$ Identification

Two methods have been used for identifying $B_s$ mesons at a resonance: One is to scan about the resonance and look for an increased production of strange mesons.
Table 3. Comparison of the pion-tagging method with standard methods of studying CP violation in the $B$ system.

| Mode                   | $\sigma$ | $\varepsilon_{\text{tag}}$ | $d$               | $\sigma \varepsilon_{\text{tag}} d^2$ |
|------------------------|----------|----------------------------|-------------------|----------------------------------------|
| $B^0 \bar{B}^0$ at $\Upsilon(4S)$ | 0.5 nb   | $2 \times 0.4$            | $x/(1 + x^2)$     | 0.092                                  |
| $B^0 \bar{B}^0 \gamma$ at $\Upsilon(5S)$ | 0.05 nb  | $2 \times 0.4$            | $2x/(1 + x^2)^2$  | 0.015                                  |
| $BB^\pm \pi^\mp$ at $\Upsilon(6S)$ | 0.004 nb | 0.8                       | $x/(1 + x^2)$     | 0.0006                                 |
| $BB^\pm \pi^\mp$ at 12 GeV      | 0.0027 nb| 0.8                       | $x/(1 + x^2)$     | 0.005                                  |

Another method is to identify a $B_s$ meson by the lower endpoint of the spectrum of a lepton produced in its decay; $B_s$ mesons are heavier than nonstrange $B$ mesons and are therefore produced with smaller momenta; consequently, a lepton from $B_s$ decay will have smaller momentum than a lepton from $B$ decay. However, nonstrange $B$ mesons produced with a pion also have less momentum than those produced alone, and therefore $BB\pi$ is a potential source of background for $B_s$ identification.

We normalize $B_s$ production to nonstrange $B$ meson production,

$$R_s \equiv \frac{\sigma(e^+e^- \rightarrow B_s\bar{B}_s, B_s\bar{B}_s^*, B_s^*\bar{B}_s, \text{ or } B_s^*\bar{B}_s^*)}{\sigma_0},$$

(7.1)

where $\sigma_0$ is the cross section for production of neutral, nonstrange $B$ mesons of spin 0 and 1 defined above in eq. (5.1). If we assume that the coupling of the source to the heavy mesons is SU(3) symmetric, then up to a common factor both the strange and nonstrange cross sections are given by eq. (5.4) but with different values of $r_\alpha$ because of the $B_s$-$B$ mass difference. (The hyperfine splittings of the strange and nonstrange $B$ mesons are essentially equal, however: $M_{B_s^*} - M_{B_s} \approx M_{B_s^*} - M_B$.) To determine the importance of the $BB\pi$ background we compare $R_s$ with $3R/2$; $R$ must be multiplied by 3/2 in order to include $BB\pi^0$ production.
Table 4. Comparison of $R_s$ (in %), for various values of $M_{B_s} - M_B$, with $\frac{3}{2} R$

| $\sqrt{s}$ | 80 MeV | 105 MeV | 130 MeV | $\frac{3}{2} R$ (%) |
|------------|--------|--------|--------|------------------|
| $\Upsilon(5S)$ | 20. | 5.7 | .76 | .39 |
| $\Upsilon(6S)$ | 46. | 32. | 20. | 3.3 |

Because of the large uncertainty in the experimental value for the strange-nonstrange mass splitting, between 80 and 130 MeV [18], we calculate $R_s$ for three different values of the splitting that span the allowed range: 80, 105, and 130 MeV. The results, expressed as percentages, are shown for center of mass energies corresponding to the $\Upsilon(5S)$ and $\Upsilon(6S)$ resonances in Table 4.

At the $\Upsilon(5S)$ resonance, just above $B_s \bar{B}_s$ threshold, we find that the $B_s \bar{B}_s$ cross section depends strongly on the $B_s$-B mass splitting which determines the size of the p-wave, $B_s \bar{B}_s$ phase space. As a result, the relative size of the $BB\pi$ background also depends strongly on this splitting. For the largest allowed value of the $B_s$-$B$ mass splitting, the $BB\pi$ cross section is about 1/2 of the $B_s \bar{B}_s$ cross section while at low end it is only a few percent. From the application of heavy-quark symmetry at leading order, we expect the $B_s$-$B$ mass difference to be close to the $D_s$-$D$ mass difference. This is measured as $99.5 \pm 0.6$ MeV [6], not far from the center of the allowed range of the $B_s$-$B$ splitting. At this point $BB\pi$ is about 10% of $B_s \bar{B}_s$. At the $\Upsilon(6S)$, the $B_s$-$B$ mass splitting is less important, and the $BB\pi$ background is of order 10% of the $B_s \bar{B}_s$ cross section. We conclude that the $BB\pi$ background is probably not a problem for $B_s$ identification at either of the $\Upsilon(5S)$ and $\Upsilon(6S)$ resonances.

Of course, it should be borne in mind that this calculation assumed SU(3) symmetry and was based on heavy meson effective chiral theory applied in the resonance regime. However, this naive calculation indicates that $BB\pi$ background should not be a problem.
8. Measurement of $g$

So far, we have integrated over the entire phase space because both CP violation studies and the identification of $B_s$ mesons rely on the total rate. In this section, we instead focus on the region of phase space where we expect the calculation to be reliable. This serves two purposes. First, it allows for the possible extraction of the axial coupling constant of the $B$ mesons, $g$. It would be useful to directly extract $g$ in this way and compare to the bounds on $g$ in the $D$ system. This method is probably not useful for extracting $g$ in the $D$ system however as we show below. Second, we can establish a reliable lower limit on the branching fraction to $BB\pi$ for center of mass energies beyond the resonance regime.

Because we want a reliable prediction, we focus on energies beyond the resonance region. We also need to determine when the derivative expansion of the heavy quark chiral lagrangian is sufficiently reliable that we can trust the leading order (in derivatives) result. As stated in ref. [5], we need both $v \cdot p_\pi$ and $v' \cdot p_\pi$ to be small. How small depends on the cutoff for the theory; this may be the chiral symmetry breaking scale, $\Lambda_\chi$ or it may be smaller (see ref. [19] for a discussion). We call the cutoff $\Lambda$.

Notice that in order for both $v \cdot p_\pi$ and $v' \cdot p_\pi$ to be small, $v \cdot v'$ and $v^0$ are restricted. We see this by adding both constraints together, which yields $\vec{v}^0 E_\pi < \Lambda$. The lowest possible $E_\pi$ is $m_\pi$, implying $\vec{v}^0 < \Lambda/m_\pi$ which in turn implies $v \cdot v' < 2(\Lambda/m_\pi)^2 - 1$.

This means that even for the pion emitted at threshold, the momentum of the heavy mesons is constrained if the chiral expansion is to be valid, and furthermore, that the best region to apply the heavy meson lagrangian will be in the region where $\vec{v}^0 \approx 1$ but above the resonance region, where the calculation will be valid over the
maximum possible range of pion energies. Therefore, we will concentrate on extracting $g$ in the regime where the $B$ mesons are nonrelativistic. This is clearly the best place from an experimental vantage point, and as we have argued, is probably also the region where the branching fraction to $B$ mesons and a single pion is maximal. One might also hope to extract $g$ for $D$ mesons. However, at CLEO, where $D$ mesons are copiously produced, $v_0^D \approx 3$. Therefore, even with a high cutoff for $\Lambda$ of order $1$ GeV, one could only integrate to pion energies less than $2m_\pi$. Even with this limited region of phase space, $v \cdot p_\pi$ and $v' \cdot p_\pi$ are very close to the cutoff so that the extraction of $g$ for $D$ mesons is probably not reliable. So we focus on the extraction of $g$ in the $B$ system.

Recall that our expression for $R$ — the ratio of the cross section for production of the various $BB^{\pm}_\pi\mp$ final states divided by the cross section for $B\bar{B}$ production — was expressed as an integral over pion energies (see eq. (5.17)). We now integrate the differential cross section over $E_\pi$ only up to some maximum pion energy, $E_{\pi}^{\text{max}}$. We will refer to the result as $R(s; E_{\pi}^{\text{max}})$. Ultimately one wants to choose $E_{\pi}^{\text{max}}$ as the maximum pion energy for which we expect the cross section to be reliable. The allowed range of $E_{\pi}^{\text{max}}$ is between $m_\pi$ and $r_{PP}$.

In Figure 3, we plot $R(s; E_{\pi}^{\text{max}})$ as a function of $E_{\pi}^{\text{max}}$ for $\sqrt{s}$ of $11.5$ GeV, $11.75$ GeV and $12$ GeV. Note that for $E_{\pi}^{\text{max}} \lesssim 500$ MeV, $R(s; E_{\pi}^{\text{max}})$ is roughly independent of $\sqrt{s}$ in this range. This is because at these energies, the $p$-wave phase-space factor for the heavy mesons is not much different for heavy mesons produced with a soft pion than for heavy mesons produced alone, and approximately cancels out in the ratio $R(s; E_{\pi}^{\text{max}})$. Accordingly, for $E_{\pi}^{\text{max}} = 400$ MeV we find that $R(s; E_{\pi}^{\text{max}})$ is roughly $10\%$ for any value
of $\sqrt{s} \gtrsim 11.5$ GeV; for $E_{\pi}^{\text{max}} = 500$ MeV we find $R(s; E_{\pi}^{\text{max}})$ slowly varies from about 15% to 20% as $\sqrt{s}$ runs from 11.5 to 12 GeV.

For larger values of $E_{\pi}^{\text{max}}$, the dependence of $R(s; E_{\pi}^{\text{max}})$ cannot be ignored. If we allow a large value of the cutoff, $E_{\pi}^{\text{max}} = 800$ GeV, we find that we gain rather little at $\sqrt{s} = 11.5$ GeV, moving up to a value of $R(s; E_{\pi}^{\text{max}})$ just above 20%, whereas at 11.75 GeV we have increased to over 35%, and at 12 GeV to over 45%.

This analysis also allows us to conclude that we can reliably predict a large value of $R_{\alpha}$, since within the regime of reliability of our calculation, the ratio $R_{\alpha}$ is very large, greater than 45% at $\sqrt{s} = 12$ GeV if we allow a cutoff of 800 MeV. It is encouraging that this rate is so large.

It is important to recognize that one can first test that the heavy quark symmetry relations apply, by studying the relations among pseudoscalar and vector $B$ meson production without a pion. If these prove valid, one can then proceed to measure the cross section with a sufficiently small cut on the pion energy so that the chiral theory is applicable. By varying the cutoff, one could test where the results disagree with our prediction, indicating that the derivative expansion of the heavy meson effective theory has broken down. This should permit a reliable extraction of $g$.

9. Conclusion

It is clear that the proposal of Yamamoto is quite interesting. It appears that there might be a sufficiently large rate for self-tagging $B$ meson events for this to be a viable method of studying CP violation for neutral mesons at a symmetric collider. Despite the limitations of our calculation, we can nevertheless establish several interesting results.
Within the resonance regime, the process we consider will probably not occur at a sufficiently large rate to compete with the more conventional proposal for the study of CP violation at a symmetric collider, namely $BB^*$ production followed by $B^* \to B\gamma$. At center of mass energy of about 12000 MeV, pion emission from a $B$ meson pair could occur as often as not. Although the calculation here is reliable only over about half the range of pion energies, it is clear that the rate for a single pion accompanying the $B$ mesons is large, even from this restricted phase space.

It could therefore be useful to run at center of mass energy above the resonance region. In this region, one has the advantage that $BB^*$ and $BB^\pm \pi^\mp$ production should both be large. With two different methods of looking for CP violation, more reliable results might be obtained. And of course, the $BB^\pm \pi^\mp$ process has a much cleaner tag and employs known technology.

Furthermore, it is likely that we have been conservative in our evaluation of the utility of Yamamoto’s method. We assumed a drop in cross section by a factor of 2 at center of mass energy 12 GeV. In addition, we have neglected production of resonant $B$ meson states which would decay primarily to $B\pi$. Because the production of such a resonance with a low-lying $B$ meson state is not $p$-wave suppressed, there could be a larger rate for $BB^\pm \pi^\mp$ than we computed.

From our calculation, we have also established that at the $\Upsilon(5S)$, the production of $BB\pi$ is probably not a large background.

Finally, we have shown that the rate for $BB^\pm \pi^\mp$ outside the resonance regime should allow for a reliable extraction of the axial pion heavy meson coupling constant,
$g$. In general, running above resonance but in the regime where $B$ mesons are nonrelativistic could be the best place from the point of view of testing heavy quark relations at energies sufficiently high that they should be reliable, but not so large that exclusive modes are suppressed. It is encouraging that the rate is largest in the range of energies where the calculation should be most reliable.

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Figure Captions

Figure 1. Diagrams that contribute to $BB\pi$ production. The blob represents the source $S^\mu$. (a) Examples of the two kinds of graph: indirect and direct. (b) The two indirect graphs that contribute to the production of two pseudoscalar $B$ mesons and a pion.

Figure 2. The ratio, $R$, of the $BB^{\pm}\pi^{\mp}$ and $B^{0}\bar{B}^{0}$ cross sections as a function of the center of mass energy. Both indirect and direct contributions are shown. Also shown are the individual indirect contributions, $R^\alpha$, coming from specific $BB^{\pm}\pi^{\mp}$ final states. The couplings were taken as $g^2 = 0.5$ and $g'^2 = 1$.

Figure 3. The (indirect) contribution to $R$ for pion energies less than a cutoff $E^\pi_{\text{max}}$ for center of mass energies of 11.5, 11.75 and 12 GeV.