On the Relation between the True Directions of Neutrinos and the Reconstructed Directions of Neutrinos in L/E Analysis Performed by Super-Kamiokande Collaboration Part 2
— Four Possible L/E Analyses for the Maximum Oscillations by the Computer Numerical Experiment—

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Abstract

In the previous paper (Part 1), we have verified that the SK assumption on the direction does not hold in the analysis of neutrino events occurred inside the SK detector, which is the cornerstone for their analysis of zenith angle distributions of neutrino events. Based on the correlation between \( L_\nu \) and \( L_\mu \) (Figures 16 to 18 in Part 1) and the correlation between \( E_\nu \) and \( E_\mu \) (Figure 19 in Part 1), we have made four possible \( L/E \) analyses, namely \( L_\nu/E_\nu \), \( L_\nu/E_\mu \), \( L_\mu/E_\mu \) and \( L_\mu/E_\nu \). Among four kinds of \( L/E \) analyses, we have shown that only \( L_\nu/E_\nu \) analysis can give the signature of maximum oscillations clearly, not only the first maximum oscillation but also the second and third maximum oscillation and etc., as they should be, while the \( L_\mu/E_\mu \) analysis which are really done by Super-Kamiokande Collaboration cannot give any maximum oscillation at all. It is thus concluded from those results that the experiments with the use of the cosmic-ray beam for neutrino oscillation, such as Super-Kamiokande type experiment, are unable to lead the maximum oscillation from their \( L/E \) analysis, because the incident neutrino cannot be observed due to its neutrality. Therefore, we would suggest Super-Kamiokande Collaboration to re-analyze the zenith angle distribution of the neutrino events which occur inside the detector carefully, since \( L_\nu \) and \( L_\mu \) are alternative expressions of the cosine of the zenith angle for the incident neutrino and that for the emitted muon, respectively.

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1. Introduction

The specification of the oscillation parameters for neutrino oscillation is entirely based on the survival probability for a given flavor (Eq.(1)) in which two physical quantities to be measured, namely, the directions of incident neutrinos and their energies are included. However, these two quantities cannot be measured directly due to their neutrality and vival probability for a given flavor (Eq.(1)) in which two physical quantities to be measured, namely, the directions of incident neutrinos and their energies are included. However, these two quantities cannot be measured directly due to their neutrality and colors with colors are strongly impressive compare with those with monochrome. In the figures with colors, we classify neutrino events by blue points and anti-neutrino events by orange ones.

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Super-Kamiokande Collaboration are forced to introduce the SK assumption on the direction.

As shown in Figures 11 to 13 and/or Figures 16 to 18 of the preceding paper, we have shown that the SK assumption on the direction requiring that the directions of the incident neutrinos are the same as those of the emitted muons does not hold in the case of the neutrino events with the highest quality, namely the single ring muon events due to quasi-elastic scattering (QEL) among Fully Contained Events.

Also, in Figure 19 of the same paper, we have shown that the energies of the incident neutrinos cannot be determined uniquely from those of the emitted muons.

Compared Figures 11 to 18 with Figures 19 in the preceding paper, it is easily understood that the approximated $E_{\nu}$ by Eq. (9) in \cite{1} does not bring a fatal error into the survival probability for a given flavor (Eq. (1) in the present paper) in spite of the unsuitable theoretical treatment, while the SK assumption on the direction introduces the fatal error into the $L/E$ analysis.

In the present paper, we examine how does the variable $L/E$ in the survival probability for a given flavor influences over the results around $L/E$ analyses. In our computer numerical experiments, it is possible to analyze four different kinds of $L/E$ distributions, namely, $L_{\nu}/E_{\nu}$, $L_{\mu}/E_{\nu}$, $L_{\mu}/E_{\mu}$ and $L_{\mu}/E_{\nu}$ distributions, where $L_{\nu}$, $L_{\mu}$, $E_{\nu}$ and $E_{\mu}$ denote the flight length of neutrino, the corresponding flight length of emitted muon, the incident neutrino energy and the emitted muon energy in QEL, respectively.

2. $L/E$ Distributions in Our Computer Numerical Experiment

Here, we explain the procedure of our computer numerical experiment roughly (See details in the Appendices in the preceding paper \cite{1}). At first we construct a hypothetical SK detector in the computer. We randomly sample a neutrino event with both certain energy and zenith angle from neutrino spectra at the opposite side of the Earth, imaging the injection of the neutrino concerned into the detector. We pursue the neutrino concerned up to the inside detector where the neutrino interaction is expected. In the interaction (QEL) occurred in

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2 Exactly speaking, instead of the description in the text, we adopt to sample the neutrino events from the neutrino

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Figure 1: $L_{\nu}/E_{\nu}$ distribution without oscillation for 1489.2 live days (one SK live day).

Figure 2: $L_{\nu}/E_{\nu}$ distribution without oscillation for 37230 live days (25 SK live days).

Figure 3: Survival probability of $P(\nu_\mu \rightarrow \nu_\mu)$ as a function of $L_{\nu}/E_{\nu}$ under the neutrino oscillation parameters obtained by Super-Kamiokande Collaboration. A, B and C represent the first, the second and the third maximum oscillation, respectively.
side the detector, we "measure" the muon energy from the random sampling of $Q^2$ for the neutrino concerned and we pursue the muon concerned up to the end of the detector, taking into account of all physical processes due to the muon and judging whether the muon concerned belongs to **Fully Contained Event or Partially Contained Events**. There, we adopt **Fully Contained Events** only. As the result of a series of these procedure, we are able to know a series of a pair of the neutrino with the known primary energy, $E_\nu$, and the zenith angle, $\cos\theta_\nu$ or $L_\nu$, and the produced muon with the energy $E_\mu$ and the zenith angle, $\cos\theta_\mu$ or $L_\mu$. In our computer numerical experiments, every physical process is treated stochastically and physical results are thus obtained, taking account of the stochastic characters inherent in their processes exactly. Namely in this sense, there is one-to-one correspondence between "measured" neutrinos and "measured" their daughters muons, while one cannot generally specify the parent neutrinos from the measured muons in the real experiment.

Our computer numerical experiments are carried out in the unit of 1489.2 days. The live days of 1489.2 is the total live days for the analysis of the neutrino events generated inside the detector used by Super-Kamiokande Collaboration\[^2\]. Hereafter, we call 1489.2 live days as one SK live day. We repeat one SK live day experiment as much as 25 times, namely, the total live days for our computer numerical experiments is 37230 live days (25 SK live days).

2.1. $L_\nu/E_\nu$ distribution

2.1.1. For null oscillation

In Figure 1, we show $L_\nu/E_\nu$ distribution without oscillation for one experiment (1489.2 live days) among twenty five computer numerical experiments (25 SK live days). In those numerical experiments, there are statistical uncertainties only which are due to both the stochastic character in the physical processes concerned and the geometry of the detector. Therefore we add the standard deviation as for the statistical uncertainty around their average in the forthcoming graphs, if necessary. In Figure 2, we show the statistical uncertainty, the standard deviations around their average values through twenty five experiments. Similarly for other possible combinations of $L$ and $E$ ( $L_\nu/E_\mu$, $L_\mu/E_\nu$ and $L_\nu/E_\nu$) for 37230 live days (25 SK live days) we did so.

In Figures 1 and 2, both distributions show the sinusoidal-like character for $L_\nu/E_\nu$ distribution, namely, the appearance of the top and the bottom, even for null oscillation. In this case, it should be noticed that their distributions are expressed in a logarithmic scale. The uneven histograms in Figure 1 comparing with those in Figure 2 show that the statistics of Figure 1 is not enough compared with that of Figure 2. Roughly speaking, smaller parts of $L_\nu/E_\nu$ correspond to the contribution from downward neutrinos, larger parts of $L_\nu/E_\nu$ correspond to those from upward neutrinos and $L_\nu/E_\nu$ near the minimum correspond to the horizontal neutrinos, although the real situation is more complicated, because the backscattering effect in QEL as well as the azimuthal angle effect in QEL could not be neglected as shown in the preceding paper\[^1\]. From Figure 2, we understand that the bottom around 70 km/GeV corresponds to the contribution mainly from the horizontal-like direction and has no relation with neutrino oscillation in any sense.

2.1.2. For oscillation (SK oscillation parameters)

The survival probability of a given flavor, such as $\nu_\mu$, is given by

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \cdot \sin^2 (1.27\Delta m^2 L_\nu/E_\nu). \quad (1)$$

Then, for maximum oscillations under SK neutrino oscillation parameters\[^2\], we have

$$1.27\Delta m^2 L_\nu/E_\nu = (2n + 1) \times \frac{\pi}{2} \quad (2)$$

where $\sin^2 2\theta = 1.0$ and $\Delta m^2 = 2.4 \times 10^{-3}\text{eV}^2$. From Eq.(1), we have the following values of $L_\nu/E_\nu$ for maximum oscillations:

$$L_\nu/E_\nu = 515\text{km}/\text{GeV} \quad \text{for } n = 0 \ (3 - 1)$$
$$= 1540\text{km}/\text{GeV} \quad \text{for } n = 1 \ (3 - 2)$$
$$= 2575\text{km}/\text{GeV} \quad \text{for } n = 2 \ (3 - 3)$$

and so on.

In Figure 3, we give the survival probability $P(\nu_\mu \rightarrow \nu_\mu)$ as a function of $L_\nu/E_\nu$ under the neutrino oscillation parameters obtained by Super-Kamiokande Collaboration. In cosmic ray experiments, the energy spectrum of the incident neutrinos is convoluted into the survival probability.

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interaction spectra inside the detector, which is mathematically equivalent to the procedure described in the text.
In Figure 4, we give one example of our $L_\nu/E_\nu$ distribution for one SK live day (1489.2 live days) among twenty five sets of the computer numerical experiments in the unit of one SK live day. In Figure 5, we give another example for one SK live day. Arrows A, B and C represent locations for the first, the second and the third maximum oscillation which are given in Eqs. (3-1), (3-2) and (3-3), respectively. By the definition of our computer numerical experiments, there are no experimental error bars in $L_\nu/E_\nu$ distributions in Figures 4 and 5.

In Figure 6, we show the $L_\nu/E_\nu$ distribution for 14892 live days (10 SK live days). Compared Figure 6 with Figures 4 and 5, it is clear that $L_\nu/E_\nu$ distribution in Figure 6 becomes smoother due to larger statistics.

In Figure 7, we give $L_\nu/E_\nu$ distribution with and without oscillation for 14892 live days (10 SK live days). Figure 8: $L_\nu/E_\nu$ distribution with standard deviations with oscillation for 37230 live days (25 SK live days).
of frequency indicated by arrows A, B and C are almost same. It should be noticed from Figure 7 that \( L_\nu/E_\nu \) distribution without oscillation represents the envelop of the corresponding distribution with oscillation. Namely, \( L_\nu/E_\nu \) distribution with oscillation is equivalent to the \( L_\nu/E_\nu \) distribution without oscillation multiplied by the survival probability for a given flavor (Eq.(6)).

We have repeated the computer numerical experiment for one SK live day as much as twenty five times independently, in both cases with oscillation and without oscillation. In Figure 8 we can add the statistical uncertainty (standard deviation in this case) around their average, because every one SK live day experiment among twenty five sets of the experiments fluctuates one by one due to their stochastic character in their physical processes and geometrical conditions of the detectors concerned.

In order to make the image of the maximum oscillations in \( L_\nu/E_\nu \) distributions clearer, we show the correlations between \( L_\nu \) and \( E_\nu \) in Figures 9 and 10 which correspond to Figures 4 and 6, respectively. In Figure 9 for one SK live day, we can observe vacant regions for the events concerned assigned by A, B and C. In Figure 10 for ten SK live days, the existence of the vacant regions for the events concerned becomes clearer due to larger statistics.

We give ratios of \((L_\nu/E_\nu)\) distribution with oscillation to that without oscillation for 1489.2 live days (one SK live day) in Figure 11 and for 14892 live days (10 SK live days) in Figure 12 respectively.

The situation shown in Figures 4 to 10 shows definitely that our computer numerical experiments carried out exactly from the view point of the stochastic treatment to the matter, if neutrino oscillation parameters obtained by Super-Kamiokande are correct.

2.2. \( L_\nu/E_\mu \) distribution

2.2.1. For null oscillation

In Figure 13 we give \( L_\nu/E_\mu \) distribution without oscillation for 37230 live days (25 SK live days) of Super-Kamiokande Experiment to consider the statistical fluctuation effect as precisely as possible.

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3 Superkamiokande collaboration mention that the second or more higher maximum oscillations are not observed due to their experimental condition. However, according to our results, they could have observed the second well and even higher maximum oscillations, if they could have really occurred like the first maximum oscillation. See arrows A, B and C in Figure 11.
It is seen from the comparison of Figure 15 with Figure 2 for $L_\nu/E_\mu$ distribution that there is no appreciable difference between them and it denotes that the transform from $E_\mu$ to $E_\nu$ (see Figures 19 in [1]) does not cause any appreciable change in $L/E$ distribution. In other words, the appreciable change are caused by the transform of $L_\nu$ to $L_\mu$ (see Figures 11 to 13, and/or Figures 16 to 18 in [1]).

2.2.2. For oscillation (SK oscillation parameters)

Here, we compare Figure 13 $L_\nu/E_\mu$ distribution for 37230 live days (25 SK live days), with Figure 8 corresponding $L_\mu/E_\mu$ distribution. Combined Figure 13 with Figure 14 we give $L_\nu/E_\mu$ distributions with and without oscillation are given in a linear scale in Figure 15. Being different from Figure 7 for $L_\nu/E_\mu$ distribution, $L_\mu/E_\mu$ distribution with oscillation in Figure 15 begins to make the maximum oscillation pattern collapse after the first maximum oscillation. This fact corresponds to the situation that in $L_\nu/E_\mu$ distributions, the transform of $E_\mu$ from $E_\nu$ makes it difficult to form the "envelope-like" relation between $L_\nu/E_\mu$ distributions with and without oscillation after the first maximum oscillation. It is clear from the comparison of these figures that $L_\nu/E_\mu$ distribution coincides almost with $L_\mu/E_\mu$ distribution around the first maximum oscillation, but the former becomes to deviate from the latter after the second maximum oscillation. It reflects the correlation between $E_\mu$ and $E_\nu$ (see Figures 19 in [1]). The situation that the first maximum oscillation can be "observed" is understandable from the existence of the vacant region indicated by arrow A in Figure 11 too. However, it is needless to say that both $L_\nu/E_\mu$ and $L_\mu/E_\mu$ distributions cannot be observed because of the neutrality of $L_\nu$.

2.3. $L_\mu/E_\mu$ distribution

The physical quantities measured by Super-Kamiokande Collaboration are $L_\mu$ and $E_\mu$, but neither $L_\nu$ and $E_\nu$. In this sense, we carefully examine the validity of the survival probability for a given flavor whose variables are $L_\mu$ and $E_\mu$, but neither $L_\nu$ and $E_\nu$. In other words, we examine whether we can find the maximum oscillations on $L_\mu/E_\mu$ distribution or not, because the existence of the maximum oscillation in $L_\mu/E_\mu$ distribution is exactly the same as the existence of the survival probability for a given flavor whose variable are $L_\mu$ and $E_\mu$ and vice-versa.
2.3.1. For null oscillation

In Figure 17 we give one sample for one SK live day (1489.2 live days) from the totally 37230 live days (25 SK live days) events, each of which has 1489.2 live days. Figure 18 shows the average distribution accompanied by the statistical uncertainty bar (not experimental error bar). It is clear from these figures that the existence of the dip or bottom, namely the sinusoidal character, means the contribution merely from horizontal-like contribution, having no relation with any neutrino oscillation character, as they must be.

2.3.2. For oscillation (SK oscillation parameters)

In Figures 19 and 20, we give the $L_{\mu}/E_{\mu}$ distributions with oscillation for 1489.2 live days (one SK live day) and 37230 live days (25 SK live days), respectively. In Figure 19 we may observe the uneven histogram, something like curious bottoms coming from neutrino oscillation. However, in Figure 20 where the statistics is 25 times as much as that of Figure 19 the histogram becomes smoother and such bottoms disappear, which turns out finally for the bottoms to be pseudo.

In order to examine the existence of the maximum oscillations in $L/E$ distribution, it is better to express $L/E$ distribution in a linear scale, but not in a logarithmic scale, as shown in Figure 7 for $L_{\nu}/E_{\nu}$ distribution. In Figure 21, we give $L_{\mu}/E_{\mu}$ distribution with and without oscillation for 10 SK live days (14892 live days). It is clear from the comparison of Figure 21 with Figure 7 for $L_{\nu}/E_{\nu}$ distribution that we cannot find any maximum oscillation in $L_{\mu}/E_{\mu}$ distribution with oscillation. Also, we find the $L_{\nu}/E_{\nu}$ distribution without oscillation forms an envelop of the corresponding one with oscillation in Figure 7, as it must be, while we cannot find such a relation on $L_{\mu}/E_{\mu}$ distribution. In order to confirm the lack of the maximum oscillations in $L_{\mu}/E_{\mu}$ distribution, we give a correlation diagram between $L_{\mu}$ and $E_{\mu}$ for one SK live days in Figure 22 and that for 10 SK live days in Figure 23. However, we cannot find anything like vacant regions in Figures 22 and 23 at all.
In Figure 21 we show one example of \( \frac{L_\mu/E_\mu}{L_\nu/E_\nu} \) osc for one SK live day among all possible sets of ratios. We may find pseudo dips in the figure. In Figure 22 we give those for ten SK live days, whose statistics is larger than that of Figure 21 by 10 times, such pseudo dips disappear here. Thus the histogram becomes a rather decreasing function of \( L_\mu/E_\mu \) in Figure 23. When we further make statistics higher, the survival probability for \( L_\mu/E_\mu \) distribution should be a monotonously decreasing function of \( L_\mu/E_\mu \), whithout showing any characteristics of the maximum oscillation, which is in remarkable contrast to Figures 11 or 12 for \( L_\nu/E_\nu \) distribution.

Summarized Figures 19 to 25, we say that \( L_\mu/E_\mu \) distribution cannot give the maximum oscillations in any sense. This denotes that \( L_\mu/E_\mu \) distributions are not constructed based on the survival probability for a given flavor which is the fundamental principle for neutrino oscillation.

2.4. \( L_\mu/E_\nu \) distribution

Instead of analyzing \( L_\mu/E_\mu \) distribution, Super-Kamiokande Collaboration have analyzed \( L_\mu/E_\nu \) distribution where \( E_\nu \) is approximated as the polynomial of \( E_\nu \) (See, Eq.(7) in the preceding paper[1]). Consequently, we examine the \( L_\mu/E_\nu \) distribution.

2.4.1. For null oscillation

In Figures 26 and 27, we give \( L_\mu/E_\nu \) distributions without oscillation for 1489.2 live days (one SK live day) and 37230 live days (25 SK live days), respectively. Comparing Figure 26 with Figure 27, the larger statistics makes the distribution smoother. Also, there is a sinusoidal-like bottom expressed in a logarithmic scale which has no relation with neutrino oscillation.

2.4.2. For oscillation (SK oscillation parameters)

In Figures 28 and 29, we give the \( L_\mu/E_\nu \) distribution with oscillation for 1489.2 live days (one SK live day) and 37230 live days (25 SK live days), respectively. In Figure 28, the larger statistics makes the distribution smoother. Also, there is a sinusoidal-like bottom expressed in a logarithmic scale which has no relation with neutrino oscillation.

At any rate, we cannot find any indication on the maximum oscillation from these figures.
2.4.3. $L_\mu/E_{\nu,SK}$ distribution for the oscillation

Instead of $E_\nu$ which is correctly sampled from the corresponding probability functions, let us utilize $E_{\nu,SK}$ which is obtained from the "approximate" formula (Eq.(6)) in the preceding paper[I].

We express $E_\nu$ described in Eq.(6) utilized by Super-Kamiokande Collaboration as $E_{\nu,SK}$ to discriminate from our $E_\nu$ obtained in the stochastic manner correctly.

In Figure 29, we give $L_\mu/E_{\nu,SK}$ distribution for 14892 live days (10 SK live days), comparing with $L_\mu/E_\nu$ distribution. It is understood from the comparison that there is no significant difference between $L_\mu/E_{\nu,SK}$ distribution and $L_\mu/E_\nu$ one in a logarithmic scale. This fact tells us that the "approximate" formula for $E_\nu$ used by Super-Kamiokande Collaboration does not produce so significant difference in the logarithmic scale, which may be accidental. Although the approximated formula is not suitable for the treatment of stochastic quantities (see discussion in 3.3 [I]), the result is understandable from Figure 14 in the preceding paper[I], because there is no significant difference between the real distribution (correlation) and the "approximate" formula apparently. Also, we can conclude that we do not find any hole corresponding to the maximum oscillation in $L_\mu/E_{\nu,SK}$ distributions. The reason why the Figure 29 can not show such dip structure as shown in Figures 4 and 5 comes from the situation that the role of $L_\nu$ is much more crucial than that of $E_\nu$ in the $L/E$ analysis. Namely, $L_\nu$ cannot be replaced by $L_\mu$ at all.

In Figure 30 we give the comparison of $L_\mu/E_\nu$ distribution with $L_\mu/E_{\nu,SK}$ one. The apparent small difference between $L_\mu/E_{\nu,SK}$ distribution and $L_\mu/E_\nu$ one in Figure 30 may come from the situation that there are non-negligible differences between $E_\nu$ or $E_{\nu,SK}$, in spite of the situation that $L_\mu$ plays an effective role in comparison with $E_\nu$ or $E_{\nu,SK}$.

In Figure 31 we compare $L_\mu/E_\nu$ distribution with $L_\mu/E_{\nu,SK}$ one. The pretty overlapping between them in a logarithmic scale denotes that $L_\mu$ play an important role while the energies concerned only play the secondary role. The similar situation is expected in $L_\nu$. In Figure 32 we compare $L_\nu/E_\mu$ distribution with $L_\nu/E_\nu$ one. It is clear from the figure that we find the first maximum oscillations

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in both distributions on nearly same locations. Another clearer situation is found in the comparison of both distributions expressed in a linear scale shown as Figures 7 and 15. It become clear from the comparison of Figures 30, 31 and 32 that the flight length, either $L_\nu$ or $L_\mu$, plays a decisively important role in any $L/E$ distribution, compared with the energies concerned, either $E_\nu$ or $E_\mu$, as it should be.

3. Comparison of $L/E$ Distribution in the Super-Kamiokande Experiment with our Results

In our classification, $L/E$ distribution by Super-Kamiokande Collaboration should be compared with our $L_\mu/E_\mu$ distribution because they measure $L_\mu$ and $E_\mu$ directly, or it should be compared with our $L_\mu/E_\nu$ distribution because they get $E_\nu$ through the transform from $E_\mu$. However, they assert that they measure $L_\nu$ on the SK assumption on the direction. Consequently, we compare here their $L/E$ distribution with our $L_\nu/E_\nu$ one at first. In Figure 33, we compare our $L_\nu/E_\nu$ distribution with their single ring muon events among Fully Contained Events should be compared as corresponding ones.

There are two important matters to be examined in the $L/E$ distribution related to the shapes between ours and theirs which can be discussed without entering the details for technical and experimental condition or criteria around their experiments. The first one is related to the location and its shape for the first maximum oscillation. And the second one is related to the location which give the maximum frequency of the events concerned. In the first one, we can observe the first maximum oscillation at $L_\nu/E_\nu = 515$ km/GeV (see Figures 4 to 12) exactly in our computer numerical experiment under the oscillation parameters obtained by Super-Kamiokande Collaboration. Furthermore, we can observe clearly the second, the third or more higher order maximum oscillations at the anticipated locations (see Eq.(3) and Figure 7) in our numerical experiment. Also, the shapes of those maximum oscillations are rather sharp which comes from the

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4We read out Fully Contained Events among total events from Super-Kamiokande Collaboration [2, 4]. Since we are only interested in single ring muon events, events with the highest quality, excluding Partially Contained Events for our analysis.
Figure 31: Comparison between $L_\mu/E_\mu$ distribution and $L_\nu/E_\mu$ distribution with oscillation for 37230 days (25 SK live days).

Figure 32: Comparison between $L_\nu/E_\nu$ distribution and $L_\mu/E_\mu$ distribution with oscillation for 37230 days (25 SK live days).

Figure 33: The comparison of $L/E$ distribution for single-ring muon events due to QEL among Fully Contained Events with the corresponding one by the Super-Kamiokande Experiment.

specified oscillation parameters obtained by Super-Kamiokande Collaboration. On the other hand, they obtain a broader region for the absence of the neutrino events as the result of the first maximum oscillation such as $100 < L/E < 800$ (km/GeV). Such a broader region may contradict the concept of the survival probability for a given flavor under the specification of their oscillation parameters, taking account of the results from the analysis of the single ring muon events among Fully Contained Events, the highest quality events among all events to be analyzed by them.

Now, we examine the remarkable difference between ours and theirs as for the locations of the maximum frequencies for the events. As shown in Figure 34, we give it as $1.0 < L_\nu/E_\nu < 1.26$ (km/GeV), while they give $20 < L_\nu/E_\nu < 25$ (km/GeV) which is larger than ours by one order of the magnitude.

Here, at first, in our computer numerical experiment, we discuss the correlation between $L$ and $E$ at the location for the maximum frequency for the events and the similar correlation at the corresponding location where Super-Kamiokande Collaboration give their maximum frequency. Next, we clarify what happen at the location for the maximum frequency for the events in their experiment.

In the following discussion, we designate neutrino events with $135^\circ < \theta_\nu < 180^\circ$ as vertical like events, events with $90^\circ < \theta_\nu < 135^\circ$ as horizontal like events and events with $88^\circ < \theta_\nu < 92^\circ$ as exclusively horizontal events, respectively.

In Figures 34 to 36, we give the correlations for the maximum frequency for the events in our $L_\nu/E_\nu$ distribution. Also, in Figures 37 to 39, we give the similar correlations for the location in our $L_\nu/E_\nu$ distribution which correspond to the maximum frequency of the events obtained by Super-Kamiokande Collaboration. In Figure 34, we give the correlation between $L_\nu$ and $E_\nu$ for the interval $1.0 < L_\nu/E_\nu < 1.26$ (km/GeV) at the maximum frequency for the events in our computer numerical experiment. It is clear from the figure that all incident neutrino events have the values of $L_\nu$ less than 10 km, corresponding to the vertical-like, horizontal like and the exclusively horizontal downward neutrinos, taking account of the transform between $L_\nu(L_\mu)$ and $\cos\theta_\nu(\cos\theta_\mu)$, as they must be. All these neutrinos cover all incident directions as the downward and they correspond to the maximum frequency for the events. It is clear from Figure 34 that the incident neutrino events are concentrated.
Figure 34: Correlation diagram between $L_{\nu}$ and $E_{\nu}$ for $1.0 < L_{\nu}/E_{\nu} < 1.26$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for $L_{\nu}/E_{\nu}$ distribution in our computer numerical experiment for 14892 live days (10 SK live days).

Figure 35: Correlation diagram between $L_{\nu}$ and $L_{\mu}$ for $1.0 < L_{\nu}/E_{\nu} < 1.26$ (km/GeV) under the neutrino oscillation parameters obtained by Super-Kamiokande Collaboration for 14892 live days (10 SK live days).

Figure 36: Correlation diagram between $L_{\mu}$ and $E_{\mu}$ for $1.0 < L_{\nu}/E_{\nu} < 1.26$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for $L_{\nu}/E_{\nu}$ distribution in our computer numerical experiment for 14892 live days (10 SK live days).

Figure 37: Correlation diagram between $L_{\nu}$ and $E_{\nu}$ for $20 < L_{\nu}/E_{\nu} < 25$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for $L_{\nu}/E_{\nu}$ distribution in our computer numerical experiment for 14892 live days (10 SK live days).

Figure 38: Correlation diagram between $L_{\nu}$ and $L_{\mu}$ for $20 < L_{\nu}/E_{\nu} < 25$ (km/GeV) under the neutrino oscillation parameters obtained by Super-Kamiokande Collaboration for 14892 live days (10 SK live days).

Figure 39: Correlation diagram between $L_{\mu}$ and $E_{\mu}$ for $20 < L_{\nu}/E_{\nu} < 25$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for $L_{\nu}/E_{\nu}$ distribution in our computer numerical experiment for 14892 live days (10 SK live days).
into the interval of from 20 to 100 km for the value of $L_\nu$ which correspond to the horizontal like events and the exclusively horizontal events, not including vertical like events, taking the account of the relation between $L_\nu(L_\mu)$ and $\cos\theta_\nu(\cos\theta_\mu)$. It is quite natural that events number there is smaller than that for the maximum frequency which includes the vertical like events.

In Figure 55, we give the correlation between $L_\nu$ and $L_\mu$ for the same intervals as in Figure 54. It is clear from the figure that the majority of the events is concentrated into the squared region with $L_\nu < 10$ km and $L_\mu < 10$ km. This denotes that the downward incident neutrinos produce muons in the forward direction irrespective of scattering angles. At the same time, it should be noticed from the figure that the non-negligible parts of the downward incident neutrino events produce the upward muons ($\sim$1000 to 10000 km) due to the backscattering as well as the azimuthal angle effect due to QEL for both horizontal like and exclusively horizontal events which is clearly shown in Figure 39 too. These upward muons may be surely identified as the products of the upward neutrinos in the analysis performed by the Super-Kamiokande Collaboration.

Furthermore, from the comparisons of Figures 55 with Figure 58 and of Figure 56 with Figure 39 it is easily understood that exclusively horizontal neutrinos which occupy the majority in Figures 55 to 58 are more influenced by the effects of both the backscattering and the azimuthal angle in QEL, compared with the cases in Figures 55 and 56. This denotes exclusively horizontal like neutrino (downward) produce upward muons (backward direction) with higher probability compared with both vertical like events and horizontal like events. Thus, from the comparison of Figures 55 to 56 with Figures 57 to 58 it is concluded that there is no contradiction for the interpretation of all figures between Figures 37 to 39 are more influenced by the effects of both the backscattering and the azimuthal angle in QEL, compared with the cases in Figures 55 and 56. This denote exclusively horizontal like neutrino (downward) produce upward muons (backward direction) with higher probability compared with both vertical like events and horizontal like events. Thus, from the comparison of Figures 55 to 56 with Figures 57 to 58 it is concluded that there is no contradiction for the interpretation of all figures between the maximum frequency for the event in our $L_\nu/E_\nu$ distribution and the corresponding distribution at the location where Super-Kamiokande Collaboration give their maximum frequency.

Now, we examine the reliability of the maximum frequency of the events obtained by Super-Kamiokande collaboration as shown in Figure 38. They assert that they measure the directions of the incident neutrinos by measuring those of muons under the SK assumption on the direction. However, what they measure really are the directions of the muons, but not those of the corresponding
neutrinos due to their neutrality. Consequently, here, we examine the $L_\mu/E_\mu$ distribution in detail, not $L_\nu/E_\nu$ distribution for checking the experimental data obtained by Super-Kamiokande Collaboration. Strictly speaking, they measure $L_\mu$ and $E_\mu$, not $L_\nu$ and $E_\nu$. However, as they transform the original $E_\nu$ to $E_\nu$ (Eq.(7) in Part1[1]), we interpret they “measure” $L_\mu$ and $E_\nu$. Thus, we examine our $L_\mu/E_\nu$ distribution for the interval $15.8 < L_\mu/E_\nu < 31.6$ (km/GeV) where Super-Kamiokande Collaboration give their maximum frequency of the events.

In Figures 10 to 12, we give our correlations for the events at the location $15.8 < L_\mu/E_\nu < 31.6$ (km/GeV) where Super-Kamiokande Collaboration give their maximum frequency for the events. In Figure 10, we give the correlation between $L_\mu$ and $E_\nu$. In Figure 11, we give the correlation between $L_\nu$ and $L_\mu$. In Figure 12, we give the correlation between $L_\nu$ and $E_\mu$.

It is understood from these figures that here, neutrino events produce exclusively the downward muons which consist of horizontal like events and exclusively horizontal events, but not vertical like events, taking account of the transform from $L_\nu(L_\mu)$ to $\cos\theta_\nu(\cos\theta_\mu)$. Also, it is easily seen that such the muons are produce by the parent neutrinos whose zenith angle distribute over downward to upward widely. This fact shows without doubt that one cannot decide the direction of the incident neutrino even the maximum frequency for the events from the measurement of the produced muons.

Here, we comment to the recent work on $L/E$ analysis by Dufours[6], a member of Super-Kamiokande Collaboration[3]. In her paper, she has carried out the Monte Carlo simulation with neutrino oscillation around $L/E$ analysis and has obtained a beautiful agreement between the experimental data and her Monte Carlo results. This seems to be the first Monte Carlo simulation with oscillation in Super-Kamiokande Collaboration, since before this work, Super-Kamiokande Collaboration have been comparing their experimental results with their Monte Carlo simulation without oscillation and have estimated neutrino oscillation parameters from the difference between them. In order to keep the consistency with the usual Monte Carlo simulations without oscillation performed by Super-Kamiokande Collaboration, she must have been carried out her Monte Carlo simulation with oscillation under the SK assumption on the direction, because the SK assumption on the direction is the cornerstone throughout their analysis on neutrino oscillation. It is too clear that the results obtained by us contradict her result, even if considering the difference that we examine the Fully Contained Events only, while she has examined the Partially Contained Events in addition to the Fully Contained Events. Furthermore, it may be unnatural that she has obtained extremely beautiful agreement with the experimental data.

Finally, we examine the data selection procedure made by Super-Kamiokande (hereafter called as SK data selection procedure) which is imposed upon their Fully Contained Events in the single ring muon events. They introduce such a procedure to exclude ambiguity mainly coming from horizontal like events. They exclude single ring muon events as Fully Contained Events which exist within the region described in Figure 2(a) in their paper[1]. We reproduce it in Figure 43. The region enclosed by two lines is the excluded region in their analysis. Here, it should be emphasized that we need not exclude the horizontal like neutrino events at all in our $L/E$ analysis against the SK analysis, because we have not any experimental errors even in our horizontal-like neutrino events due to the nature of our computer numerical experiment. However, it is not vain to examine whether the SK data selection procedure is appropriate or not for our analysis in our computer numerical experiment. In Figure 44.

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5One may take a notice that it is not appropriate to cite Ph.D due to their nature of “unpublished”. However, many Ph.D thesis around Super-Kamiokande have been published and there are no any contradictions between their Ph.D theses and SK papers and the detailed descriptions are exclusively found in these Ph.D theses.
we give correlation diagram between $L_{\nu}$ and $E_{\nu}$ with oscillation together with the lines for exclusion given by Supper-Kamiokande Collaboration. The lines for exclusion in Figure 44 are transformed from the lines in Figure 43. The region enclosed by two lines in Figure 44 are the excluded region. Namely, the events concerned within the excluded region should be subtracted from the original distribution. Thus, $L_{\nu}/E_{\nu}$ distribution of the resultant events after subtraction procedure (with restriction in the figure) is shown in Figure 45 together that without subtraction (no restriction in the figure).

It is clear from the figure that $L_{\nu}/E_{\nu}$ distribution with the SK data selection procedure keeps still the characteristics of a series of the maximum oscillations in spite of the decrease of the events in smaller value of $L_{\nu}/E_{\nu}$. Here, we show $L_{\nu}/E_{\nu}$ distribution under the SK data selection procedure in a linear scale to make their physical image clearer. It is needless to say that we need not introduce the SK data selection procedure into our computer numerical experiment due to the “complete experiment” by the definition, but even if we introduce it, Figure 45 shows that the essential characteristics of our $L_{\nu}/E_{\nu}$ distribution is never changed.

Similarly we examine the case of $L_{\mu}/E_{\mu}$ distribution. Figure 46 for $L_{\mu}/E_{\mu}$ distribution corresponds to Figure 44 for $L_{\nu}/E_{\nu}$ distribution. Also in Figure 45, $L_{\mu}/E_{\mu}$ distribution corresponds to Figure 45 for $L_{\nu}/E_{\nu}$ distribution.

It is clear from Figure 47 that the $L_{\mu}/E_{\mu}$ distribution with the SK data selection procedure do not show anything like the maximum oscillation in the same way as the original $L_{\mu}/E_{\mu}$ distribution.

Summarized from Figures 43 to 47, we could not extract the neutrino oscillation parameters from $L_{\mu}/E_{\mu}$ distribution, even if we apply the SK data selection procedure to the original $L_{\mu}/E_{\mu}$ distribution, while we can keep the essential character of the maximum oscillations in $L_{\mu}/E_{\mu}$ analysis, even if we apply to the SK data selection procedure to the original $L_{\mu}/E_{\mu}$ distribution. Finally, we should say again that we need not introduce the SK data selection procedure into our computer numerical experiment due to the nature of no error experiment.

4. Conclusion

The determination of the neutrino oscillation parameters is entirely based on the survival probability for a given flavor. Consequently, it is inevitable to decide $L_{\nu}$ and $E_{\nu}$ as precisely as pos-
sible, when one wants specify neutrino oscillation parameters. However, in cosmic ray experiments, one may not measure both $L_\nu$ and $E_\nu$ due to their neutralities, in addition to because of the incapability for the determination of the directions of the incident neutrinos due to the essential nature of cosmic ray beams. Therefore, cosmic ray physicists are forced to assume the direction of incident neutrinos a priori, when they do not carry out computer numerical experiment as their second experiment. In the case of Super-Kamiokande Collaboration, they assume that the direction of incident neutrino is approximately the same as that of emitted lepton (the SK assumption on the direction). In the preceding paper [1], we have verified that the SK assumption on the direction does not hold even if approximately. The essential conclusion obtained by the present paper is that in principle one may not specify the neutrino oscillation parameters from the cosmic ray experiments due to the unknown directions of the incident neutrinos. Our verification that Super-Kamiokande Collaboration cannot specify the neutrino oscillation parameters through their $L/E$ analysis at least, and consequently, our conclusion can be applied to any type of experiment of cosmic ray physics where the direction of the neutrino, in principle, cannot be determined. Our conclusion tells us that only accelerator physics can specify the neutrino oscillation parameters reliably, if the neutrino oscillation really exists.

Deduction of our conclusion is as follows:

(1) There are much uncertainty factors in cosmic ray physics, compared with the accelerator physics due to the original nature of cosmic ray. Consequently, in spite of such a difficulty, if one wants to carry out the experiment with high precision on neutrino oscillation, we should focus the simplest and clearest "target" by which one get high quality information on neutrino oscillation. With such a motivation, we choose the single ring muon events due to QEL which they occur inside the detector and terminate inside the detector (Fully Contained Events). Here, the kind of neutrino concerned is clear (electron neutrino or muon neutrino). The energy of emitted lepton and its direction can be estimated reliably ( from the standpoint of Super-Kamiokande at least). The circumstance around our computer numerical experiment is modeled after real Super-Kamiokande experiment in essential points. We have analyze the single ring muon events due to QEL as Fully Contained Events obtained by our computer numerical experiment.

(2) We have carefully and in detail examined the validity of the SK assumption on the direction which is the "cornerstone stone" for their analysis around neutrino oscillation. As the result, we have clarified that the SK assumption on the direction does not hold even if approximately. Also, we examine the validity of the unique relation between $E_\nu$ and $E_\mu$ expressed in the polynomial obtained by Super-Kamiokande Collaboration and we have clarified that the unique relation between them does not hold. These two improper treatments originate from the situation that they do not consider characteristics of the stochastic processes concerned seriously. However, the unreliability on the directions of the incident neutrinos influences final result in a fatal manner, while unreliability on the energy estimation does not provide the significant errors compared with the former. The concrete summaries are given in (3).

(3) Due to the nature of the computer numerical experiment, assuming neutrino oscillation parameters obtained by Super-Kamiokande Collaboration, we carry out all possible combination of $L/E$ analysis, namely, $L_\nu/E_\nu$ analysis, $L_\nu/E_\mu$ analysis, $L_\mu/E_\nu$ analysis and $L_\mu/E_\mu$ analysis, based on the survival probability for a given flavor whose variables are $L_\nu$ and $E_\nu$. Among four $L/E$ analyses, only $L_\nu/E_\nu$ analysis has reproduced the existence of the maximum oscillations, not only the first maximum oscillation but also the second, the third, the fourth and so on. The confirmation of a series of the maximum oscillations, such as the first, the second, the third and so on in $L_\nu/E_\nu$ analysis shows that our computer numerical experiments have been carried out in a correct manner. The $L_\nu/E_\mu$ analysis has reproduced the first maximum oscillation roughly, but cannot reproduce the maximum oscillation after the second. Both $L_\mu/E_\nu$ and $L_\mu/E_\mu$ analyses cannot reproduce any characteristics of the maximum oscillation at all. Notice that Super-Kamiokande Collaboration have carried out either $L_\mu/E_\nu$ analysis or $L_\mu/E_\mu$ analysis, neither $L_\nu/E_\nu$ analysis nor $L_\mu/E_\mu$ analysis. Combined with the item (1), these facts tell us that the
decisive variable in the survival probability for neutrino oscillation is $L_\nu$ but neither $L_\mu$, nor $E_\nu$, nor $E_\mu$. Thus, our verification that the SK assumption on the direction does not hold even approximately requests urgently Super-Kamiokande Collaboration to re-analyse their results around the zenith angle distributions for neutrino events which have been regarded as the establishment of the existence of the neutrino oscillation, because their analysis on neutrino oscillation (atmospheric neutrino) is entirely based on the survival probability and the SK assumption on the direction.

Finally, we would like to emphasize the importance of the cosmic ray study in order to avoid any misunderstandings. This characteristics of the cosmic ray study never make it lose its raison-d’etre. The main role of cosmic ray physics is to grasp qualitatively the essential of something like new. Up to now, cosmic ray study has been contributing to find new indications in fundamental physics and from now on, it will be continue to fulfill its role.

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