Source Counts Spanning Eight Decades of Flux Density at 1.4 GHz

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Abstract

Brightness-weighted differential source counts \( S^2 n(S) \) spanning the eight decades of flux density between 0.25 \( \mu Jy \) and 25 Jy at 1.4 GHz were measured from (1) the confusion brightness distribution in the MeerKAT DEEP2 image below 10 \( \mu Jy \), (2) counts of DEEP2 sources between 10 \( \mu Jy \) and 2.5 mJy, and (3) counts of NVSS sources stronger than 2.5 mJy. We present our DEEP2 catalog of 1.7 \( \times 10^4 \) discrete sources complete above \( S = 10 \mu Jy \) over \( \Omega = 1.04 \deg^2 \). The brightness-weighted counts converge as \( S^2 n(S) \times S^{1/2} \) below \( S = 10 \mu Jy \), so \( >99\% \) of the \( \Delta T_b \sim 0.06 \mathrm{K} \) sky brightness produced by active galactic nuclei and \( \approx 96\% \) of the \( \Delta T_b \sim 0.04 \mathrm{K} \) added by star-forming galaxies has been resolved into sources with \( S > 0.25 \mu Jy \). The \( \Delta T_b \approx 0.4 \mathrm{K} \) excess brightness measured by ARCADE 2 cannot be produced by faint sources smaller than \( \approx 50 \mathrm{kpc} \) if they cluster like galaxies.

Unified Astronomy Thesaurus concepts: Galaxy evolution (594); Galaxy counts (588); Star formation (1569); Radio galaxies (1343)

Supporting material: figure set, machine-readable tables

1. Introduction

There have been persistent discrepancies in the faintest direct source counts at \( S_{1.4 \mathrm{GHz}} < 100 \mu Jy \) (see de Zotti et al. 2010, for a review and compilation of previous source counts), far exceeding the errors caused by Poisson fluctuations and clustering uncertainties (Owen & Morrison 2008; Heywood et al. 2013). Direct counts of faint radio sources rely primarily on high angular-resolution images and must account for possible “missing” resolved sources whose peak flux density falls below the surface brightness sensitivity of the image. Corrections of the source counts due to these missing sources depend on the highly uncertain intrinsic angular size distribution of faint radio sources, according to Bondi et al. (2008). The large uncertainty in these resolution corrections propagates into the integrated flux measurements, source counts, and number of missing sources due to limited surface brightness sensitivity. This effect is further magnified by the steep slope of differential source counts \( n(S) \propto S^{-5/2} \), which exacerbates flux density overestimates and leads to higher counts at faint flux densities.

Following the pioneering \( P(D) \) method of Scheuer (1957), radio astronomers have used confusion to measure accurate source counts (see Condon et al. 2012; Vernstrom et al. 2014, for recent examples). A low-resolution image ensures that all faint radio galaxies appear as point sources—eliminating the need for uncertain resolution corrections. The term “confusion” means fluctuations in sky brightness caused by multiple faint sources inside the point-source response. Historically, confusion was described by the probability distribution \( P(D) \) of pen deflections of magnitude \( D \) on a chart-recorder plot of detected power (Scheuer 1957). The analog of the deflection \( D \) in a modern image is the sky brightness expressed as a peak flux density \( S_p \) in units of flux density per beam solid angle, so a \( P(D) \) distribution is the same as a \( P(S_p) \) distribution.

Low-resolution confusion-limited images offer an independent way of measuring faint radio source counts and are free from uncertain angular size corrections. While unable to determine properties of individual galaxies, confusion studies are able to constrain source counts of the radio population far below the noise and do not require multiwavelength cross-identifications as priors (unlike source counts measured from “stacking,” e.g., Mitchell-Wynne et al. 2014).

The differential source count \( n(S)dS \) at frequency \( \nu \) is the number of sources per steradian with flux densities between \( S \) and \( S + dS \). The Rayleigh–Jeans sky brightness temperature \( dT_b \) per decade of flux density added by these sources is

\[
\left[ \frac{dT_b}{d \log(S)} \right] = \frac{\ln(10)e^2}{2k_B \nu^2} S^2 n(S),
\]

where \( k_B \approx 1.38 \times 10^{-23} \mathrm{J} \mathrm{K}^{-1} \). Can one or more “new” populations of radio sources fainter than 0.25 \( \mu Jy \) make comparable contributions to the sky brightness at 1.4 GHz? The ARCADE 2 instrument measured the absolute sky temperature at frequencies from \( \nu = 3–90 \mathrm{GHz} \), and Fixsen et al. (2011) reported finding an excess power-law brightness temperature

\[
\left( \frac{T_b}{K} \right) = (24.1 \pm 2.1) \left( \frac{\nu}{\nu_0} \right)^{-2.599 \pm 0.036}
\]

from 22 MHz to 10 GHz, where \( \nu_0 = 310 \mathrm{MHz} \). Removing the contribution from known populations of extragalactic sources leaves

\[
\left( \frac{\Delta T_b}{K} \right) = (18.4 \pm 2.1) \left( \frac{\nu}{\nu_0} \right)^{-2.57 \pm 0.05},
\]

(Seiffert et al. 2011). Possible explanations for this large excess fall into three categories: (1) the excess was overestimated owing to the limited sky coverage of ARCADE 2 and the zero-point levels of low-frequency radio maps may be inaccurate, (2) the excess is primarily smooth emission from our Galaxy, (3) the excess is primarily extragalactic, making it the only photon background that does not agree with published source
counts dominated by radio galaxies and star-forming galaxies. Vernstrom et al. (2011) and Seiffert et al. (2011) explored the possibility of a new source population contributing an additional bump to the source counts at flux densities <10 μJy. In order to match the ARCADE 2 excess background, this hypothetical new population must add ∆T_b ~ 0.4 K to the sky brightness at 1.4 GHz. Condon et al. (2012) showed that the brightness-weighted counts S^2 n(S) of this new population must peak at flux densities below S_{1.4 GHz} = 0.1 μJy to be consistent with their observed P(D) distribution.

This paper presents 1.4 GHz brightness-weighted source counts S^2 n(S) covering the eight decades of flux density between S = 0.25 μJy and S = 25 Jy based on the very sensitive ν = 1.266 GHz MeerKAT DEEP2 sky image (Mauch et al. 2020) confusion brightness distribution between S = 0.25 μJy and S = 10 μJy, the DEEP2 discrete-source catalog from S = 10 μJy to S = 2.5 mJy, and on the 1.4 GHz NRAO VLA Sky Survey (Condon et al. 1998, NVSS) catalog above S = 2.5 mJy. Nearly all of these sources are extragalactic. We present the first complete catalog of discrete sources with S > 10 μJy in the DEEP2 field. While Mauch et al. (2020) derived the best-fitting power-law source counts to describe the DEEP2 P(D) distribution, the actual source counts do not follow a simple power law. To improve upon the Mauch et al. (2020) fit, we allowed the source counts to be any continuous function. We further explore the possibility of new populations of faint extragalactic sources contributing to the total radio background, and constrain the lower limit to the number of such sources adding ∆T_b ~ 0.4 K to the sky brightness at 1.4 GHz, remaining consistent with the DEEP2 P(D) distribution.

The data used to construct the source counts across eight decades are presented in Section 2. The radio sky simulations needed to constrain the source counts from the P(D) distribution and derive confusion corrections to the discrete counts is detailed in Section 3. Statistical source counts with 0.25 μJy < S < 10 μJy estimated from the P(D) confusion distribution are reported in Section 4. Section 5 presents the complete S > 10 μJy discrete-source catalog from the DEEP2 field, and Section 5.4 presents the source counts derived from this catalog. Differential source counts for the NVSS catalog are calculated in Section 6. The contributions of star-forming galaxies (SFGs) and active galactic nuclei (AGNs) to the 1.4 GHz sky background and constraints on new populations of faint sources explaining the ARCADE 2 radio excess are described in Section 7. Section 8 summarizes this work.

Absolute quantities in this paper were calculated for a ΛCDM universe with H_0 = 70 km s^{-1} Mpc^{-1} and Ω_m = 0.3 using the equations in Condon & Matthews (2018). Our spectral-index sign convention is α ≡ +d ln S/d ln ν.

2. Data

2.1. The MeerKAT DEEP2 Field

The 1.266 GHz DEEP2 image (Mauch et al. 2020) covers the θ_{1/2} = 69/2 diameter half-power circle of the MeerKAT primary beam centered on J2000 α = 04^h 13^m 26.4^s, δ = −80° 00’ 00’’.

Its point-source response is a θ_{1/2} = 7”6 full width at half maximum (FWHM) Gaussian, and the rms noise is σ_n = 0.56 ± 0.01 μJy beam^{-1} at the pointing center (Table 1). The wideband DEEP2 image is the average of 14 narrow subband images weighted to maximize the signal-to-noise ratio (S/N) of sources with spectral index α = −0.7 (Table 2). The dirty DEEP2 image was CLEANed down to a residual peak flux density S_p = 5 μJy beam^{-1}.

The DEEP2 image is strongly confusion limited, so we could not treat its position and flux-density error distributions analytically. Therefore we created radio sky simulations (Section 3) to model the statistical source counts consistent with the confusion brightness distribution, to refine our catalog of discrete DEEP2 sources, and to correct our counts of the faintest sources (Section 5).

2.2. NRAO VLA Sky Survey

The 1.4 GHz NRAO VLA Sky Survey (NVSS; Condon et al. 1998) imaged the entire sky north of J2000 δ = −40° with θ_{1/2} = 45” FWHM resolution and σ_n ≈ 0.45 mJy beam^{-1} rms noise. The NVSS catalog lists source components as Gaussian fits to significant peaks in the NVSS images. From it we selected the 1,117,067 components with S ≥ 2.5 mJy in the Ω ≈ 7,016 sr solid angle with absolute Galactic latitude |b| > 20°. In Section 6 we detail how we derived the NVSS direct-source counts above 2.5 mJy.

3. The Radio Sky Simulations

We produced computer simulations of the 1.266 GHz DEEP2 image (along with mock catalogs) to calculate source

| Parameter | Value |
|-----------|-------|
| R.A. (J2000) | 04:13:26.4 |
| Decl. (J2000) | −80:00:00 |
| Primary FWHM θ_{1/2} | 69’ |
| Solid angle Ω_{1/2} | 1.04 deg^2 |
| Synthesized FWHM θ_{1/2} | 7’6 |
| Central rms noise σ_n | 0.56 ± 0.01 μJy beam^{-1} |
| P(D) on-sky noise σ_n | 0.57 ± 0.01 μJy beam^{-1} |

Note. Column 1 is the subband number i, Column 2 the subband central frequency ν_i, Column 3 the rms noise σ_n in the subband image, and Column 4 is the subband image weight w_i used to produce the wideband DEEP2 image with the highest S/N for sources with spectral index α = −0.7.
counts below 10 μJy, assess the quality of the algorithms (e.g., our source-finding algorithm) used on the real data, and to derive corrections and uncertainties for the discrete-source catalog between 10 μJy < S < 2.5 mJy.

The confusion brightness distribution can be calculated analytically only for scale-free power-law differential source counts of the form n(S) ∝ S^{−α} (Condon 1974). Likewise, population-law biases in counts of faint discrete sources can easily be estimated only in the power-law count approximation (Murdoch et al. 1973). The actual source counts are not well approximated by a single power law near S ~ 10 μJy because this flux density corresponds to the bend in the SFG luminosity function of sources at z ~ 1 (Condon et al. 2012, Figure 11), so we used computer simulations of the 1.266 GHz DEEP2 image to estimate statistical source counts below 10 μJy from the DEEP2 image brightness distribution and to correct for biases in the DEEP2 discrete-source counts above 10 μJy. We simulated only point sources because the measured median angular diameter ⟨ϕ⟩ ≈ 0.′3 of real μJy sources (Cotton et al. 2018) is much smaller than the DEEP2 restoring beam FWHM and only ~0.2% of the DEEP2 sources stronger than S = 10 μJy are clearly resolved (Section 5.3). The simulated sources all have spectral index α = −0.7, the median spectral index of extragalactic sources (Condon 1984). Varying α by ±0.14, the rms width of the observed spectral-index distribution of faint sources, changes the 1.4 GHz flux densities of the simulated sources by only ±1%.

The input for each simulation is an arbitrary user-specified 1.266 GHz source count n(S). In every flux-density bin of width ∆log(S) = 0.001, the actual number of simulated sources is chosen by a random-number generator sampling the Poisson distribution whose mean matches the input n(S). The sources are scattered randomly throughout the DEEP2 half-power circle. The real μJy sources in DEEP2 are nearly all extragalactic and very distant (median redshift ⟨z⟩ ~ 1), so they are spread out over a radial distance range ∆z ~ 1 much larger than the galaxy correlation length, and their sky distribution is quite random and isotropic (Benn & Wall 1995; Condon & Matthews 2018), unlike the visibly clustered sky distribution of nearby optically selected galaxies. In addition, clustering appears to have little effect on far-IR (FIR), millimeter, and radio confusion brightness distributions observed with resolutions close to the DEEP2 restoring beam diameter (Béthermin et al. 2017).

The simulations also reproduce the DEEP2 observational effects and imaging processes described by Mauch et al. (2020). The simulated image replicates CLEANing by representing each source as the sum of two components: (1) a component whose brightness distribution is the DEEP2 dirty beam and whose peak flux density is the lesser of either the input source flux density or subband CLEAN threshold, plus (2) a CLEAN component whose brightness distribution matches the circular Gaussian restoring beam and whose amplitude is the difference between the input source flux density and the subband CLEAN threshold. The subband CLEAN threshold was determined from the wideband value of 5 μJy and scaled to the subband central frequency assuming a spectral index of α = −0.7. The dirty beam used for each subband of the simulation is the actual DEEP2 subband dirty beam, which is nearly circular and does not have strong diffraction spikes since the MeerKAT antennas are not distributed along straight arms. The first negative sidelobe of the weighted dirty beam is at the ~5% level (Figure 1). A 5 μJy residual leaves a ~0.25 μJy negative ring in the image, which is less than half the rms sky noise in the P(D) region. The first positive sidelobe of the dirty beam is at the ~1% level.

The simulation generates sources with spectral index α = −0.7 and combines the subband images with the weights listed in Column 4 of Table 2. To incorporate the DEEP2 primary-beam attenuation, the simulated subband images were multiplied by the frequency-dependent MeerKAT primary beam specified by Equations (3) and (4) in Mauch et al. (2020). After multiplying by the primary-beam attenuation, the simulation adds to each pixel of the wideband image a randomly generated sample of Gaussian noise. The noise in an aperture-synthesis image has the same (u,v)-plane coverage as the signal, so the DEEP2 image noise is smoothed by the same dirty beam (Figure 1). To duplicate this smoothing, the simulation convolved the pixel noise distribution with the dirty beam. The rms amplitude of the convolved noise was set to match the observed rms noise in the actual DEEP2 image prior to correction for primary-beam attenuation.

Finally, this simulated image must be divided by primary-beam attenuation to yield a simulated sky image. Figure 2 compares 4′2 × 4′2 (200 × 200 square pixels, each 1′25 on a side) cutouts from one simulated sky image with the actual DEEP2 sky image to show that the simulated image looks like the real image.

4. The DEEP2 P(D) Distribution

4.1. Observed P(D) Distribution

The peak flux density Sp at any point in an image is the sum of contributions from noise-free source confusion and image noise. Confusion and noise are independent, so the observed P(D) distribution is the convolution of the confusion and noise distributions. The noise amplitude distribution in an aperture-synthesis image is easy to deconvolve because it is extremely stable, Gaussian, and uniform across the image prior to correction for primary-beam attenuation (Condon et al. 2012), unlike the noise in a single-dish image, which usually varies significantly with position and time during an observation. Consequently, we were able to measure the DEEP2 rms noise and confirm its Gaussian amplitude distribution with very small uncertainties. The noise distribution is narrower than the noiseless P(D) distribution in the very sensitive DEEP2 image,
so we could deconvolve the Gaussian noise distribution from
the observed $P(D)$ distribution to calculate the desired noiseless
$P(D)$ distribution with unprecedented accuracy and sensitivity.

Following the same procedure as was used in Mauch et al.
(2020), we extracted the $P(D)$ distribution from the circle of
radius $r = 500''$ covering solid angle $\Omega = 1.85 \times 10^{-5}$ sr
centered on the S/N-weighted 1.266 GHz DEEP2 image corrected
for primary-beam attenuation and shown in Figure 11 of Mauch
et al. (2020). The $P(D)$ circle is small enough ($2r \ll \Theta_{1/2}$)
that the mean primary-beam attenuation is 0.98 inside the circle
and 0.96 at the edge, so its rms noise after correction for primary-
beam attenuation is only

$$\sigma_n = (0.56 \pm 0.01 \mu Jy \, beam^{-1})/0.98 = 0.57 \pm 0.01 \mu Jy \, beam^{-1}.$$ 

The $P(D)$ circle is still large enough to cover $N_b = 1.20 \times 10^4$ restoring beam solid angles
$\Omega_b = \pi\theta_{1/2}^2/(4 \ln 2) = 1.54 \times 10^{-9}$ sr. The solid angle of the square
of the restoring-beam attenuation pattern determines the number
of independent samples per unit solid angle of sky (Condon et al.
2012). For a Gaussian restoring beam, the solid angle of the
beam squared is half the beam solid angle, so the observed
DEEP2 $P(D)$ distribution shown by the large black points in
Figure 3 actually contains $2N_b = 2.40 \times 10^4$ statistically inde-
pendent samples. The observed $P(D)$ distribution is the
convolution of the noiseless sky $P(D)$ distribution (black curve) with
the $\sigma_n = 0.57 \pm 0.01 \mu Jy \, beam^{-1}$ Gaussian noise distribution
accurately represented by the parabolic dotted curve in the
semilogarithmic Figure 3.

The 1.266 GHz DEEP2 $P(D)$ distribution shown in Figure 3
is four times as sensitive to point sources with $\alpha \approx -0.7$ as the
most sensitive published 3 GHz $P(D)$ distribution (Condon et al.
2012). Such sources are 1.83 times stronger at 1.266 GHz than at 3 GHz, so the rms noise $\sigma_n = 1.255 \mu Jy \, beam^{-1}$ of the
3 GHz $P(D)$ distribution is equivalent to $\sigma_n = 2.30 \mu Jy \, beam^{-1}$
at 1.266 GHz. The peak of the DEEP2 noise distribution is higher
than the peak of the noiseless $P(D)$ distribution (Figure 3), indicating that DEEP2 is strongly confusion
limited, while the peak of the 3 GHz noise distribution is only
half as high as the peak of the 3 GHz noiseless $P(D)$ distribution.

The DEEP2 $P(D)$ distribution also has smaller statistical uncertainties because it includes 3.1 times as many
independent samples. Finally, the DEEP2 $P(D)$ distribution
was extracted from the very center of the image where the
primary-beam attenuation $>0.96$, so systematic errors caused
by antenna pointing fluctuations or primary-beam attenuation
corrections are negligible.

4.2. $P(D)$ Statistical Counts of $S < 0.25 \mu Jy \leq 10$ Sources

The noiseless confusion $P(D)$ distribution is sensitive to the
differential counts $n(S)$ of sources more than 10 times fainter
than the usual $S_n$ detection limit for individual sources. In
terms of the number $N(S) = n(S)$ of sources per steradian stronger
than $S$, the mean number of sources stronger than $S$ per beam
solid angle $\Omega_b$ is $\mu = [N(S) \Omega_b]$ and the Poisson probability
that all sources in a beam are weaker than $S$ is $P_0 = \exp(-\mu)$. The
DEEP2 image $\Omega_b \approx 1.54 \times 10^{-9}$ sr, and the best-fit source counts from Mauch et al. (2020) imply $P_0 \approx 0.4$ at
$S = 0.25 \mu Jy \, beam^{-1}$. The DEEP2 $P(D)$ distribution was extracted from a solid angle containing $2.40 \times 10^4$ independent samples of the sky, so changes in the source count down to

![Figure 2](image1)

![Figure 3](image2)
$S = 0.25 \mu\text{Jy beam}^{-1}$ can be detected statistically from the 10\(^4\) independent samples that contain only fainter sources if the rms noise is lower than the rms confusion. We used the radio sky simulations described in Section 3 to constrain the source counts consistent with the observed $P(D)$ distribution. The DEEP2 $P(D)$ distribution is smoothed by Gaussian noise with rms $\sigma_n = 0.57 \pm 0.01 \mu\text{Jy beam}^{-1}$, which degrades its sensitivity to significantly fainter sources. To estimate the sensitivity of DEEP2 to faint sources in the presence of noise, we simulated noisy $P(D)$ distributions using a variety of differential source counts below $S = 10 \mu\text{Jy}$. Above $S = 10 \mu\text{Jy}$, we used the direct-source counts from DEEP2 and the NVSS presented in Sections 5.4 and 6. The simulation accepts brightness-weighted differential source counts $S^2 n(S)$ specified in bins of width $\Delta \log(S) = 0.2$. Directly binning counts $n(S)$ that vary rapidly with $S$ can introduce a significant bias (Jauncey 1968). We mitigated this bias by binning the quantity $S^2 n(S)$, which changes little across a flux-density bin.

The source counts $S^2 n(S)$ in the flux range $-8 < \log(S(\text{Jy})) < -4.9$ are well fit by a cubic polynomial. To measure the goodness of fit for each input source count, we defined a statistic that quadratically combines the reduced $\chi^2$ from differences between the simulated and observed $P(D)$ distributions for all $S_n < 15 \mu\text{Jy beam}^{-1}$ with the $\chi^2_{\text{DC}}$ of the differences between the simulated counts and the direct counts of DEEP2 sources in bins centered on $\log S = -4.9$ and $-4.7$:

$$\chi^2 = \sqrt{(\chi^2_{P(D)})^2 + (\chi^2_{\text{DC}})^2}.$$  

We optimized the parameters for the cubic polynomial by minimizing Equation (4). We inspected the residuals $\Delta N/\sigma_N$, where $N$ is the number of independent samples per bin and $\sigma_N$ is the Poisson error per bin associated with the observed $P(D)$ distribution, of the resulting best fits for the presence of correlations as a function of brightness $D$. The existence of a signal similar to red noise in our residuals would imply that our counts under- or overestimate the counts of sources in specific flux-density ranges.

The third-degree polynomial source count

$$\log[S^2 n(S)] = 2.718 + 0.405(\log S + 5) - 0.020(\log S + 5)^2 + 0.019(\log S + 5)^3,$$  

where $S$ is the 1.266 GHz flux density in Jy, gave the simulated $P(D)$ distribution (red curve in Figure 3) that best fits the observed distribution (black points) while maintaining continuity in the transition from the $P(D)$ to direct counts at $S = 10 \mu\text{Jy}$.

We converted the 1.266 GHz flux densities and brightness-weighted source counts in Equations (5), (9), and (10) to the common source-count frequency $\nu = 1.4$ GHz for sources with spectral index $\alpha = -0.7$ using

$$\log(S_{1.4\text{GHz}}) = \log(S_{1.266\text{GHz}}) + \alpha \log \left( \frac{1.4}{1.266} \right) \approx \log(S_{1.266\text{GHz}}) - 0.0306$$  

and

$$\log[S^n(S)]_{1.4\text{GHz}} = \log[S^n(S)]_{1.266\text{GHz}} + \alpha \log \left( \frac{1.4}{1.266} \right) \approx \log[S^n(S)]_{1.266\text{GHz}} - 0.0306.$$  

We also calculated the commonly used static-Euclidean source counts from the brightness-weighted source counts via

$$\log[S^{1/2}(S)] = 0.5 \log(S) + \log(S^n(S)).$$  

Our 1.4 GHz differential source counts with both normalizations are plotted—along with the discrete-source counts calculated in Sections 5.4 and 6—in Figure 11. In the following subsections, we describe our accounting of the various biases and uncertainties in our statistical fit of the source counts.

4.3. Zero-level Offset

Before calculating the value of this statistic for a given simulation, we removed the brightness zero-point offset between the simulated and observed $P(D)$ distributions. Numerous faint radio sources produce a smooth background, which is invisible to MeerKAT and other correlation interferometers lacking zero-spacing data. Thus the brightness zero level of our observed $P(D)$ distribution is unknown and must be fitted out. We minimized the zero-level offset by comparing the observed and simulated $P(D)$ distributions shifted in steps of 0.001 \mu Jy beam\(^{-1}\), this step size being smaller than the rms noise divided by the square root of the number 2.40 \times 10^4 of independent samples in the DEEP2 $P(D)$ area.

4.4. DEEP2 rms Noise Uncertainty

Although the simulation includes sources as faint as $S = 0.01 \mu\text{Jy}$, the DEEP2 image is not sensitive to the counts of such faint sources. The strongest cause of uncertainty in our sub-\mu Jy source counts is the 0.01 \mu Jy beam\(^{-1}\) uncertainty in the DEEP2 rms noise. To estimate the flux density of the faintest sources that we can usefully count, we set the rms noise to $\sigma_n = 0.57 \mu\text{Jy beam}^{-1}$ and iteratively removed the lowest flux-density bin before using that subsample of bins to produce the DEEP2 simulation. After repeating this process for a total of 50 trials, we found that removing the source-count bin at $\log(S(\text{Jy})) = -6.6$ increases the simulation $\chi^2$ by more than $1\sigma$ above the mean minimum $\chi^2$, as shown by the black points above the dotted line in Figure 4. The same process was repeated for for $\sigma_n = 0.56 \mu\text{Jy beam}^{-1}$ (blue points) and $0.58 \mu\text{Jy beam}^{-1}$ (red points). The results are consistent with a count sensitivity limit $\log[S(\text{Jy})] \approx -6.6$ or $S \approx 0.25 \mu\text{Jy}$ in the presence of $\sigma_n = 0.57 \pm 0.01 \mu\text{Jy beam}^{-1}$ noise.

To determine the sensitivity of our best-fitting $P(D)$ distribution above $S = 0.25 \mu\text{Jy}$ to small changes in the rms noise, we ran 1000 simulations of the DEEP2 $P(D)$ distribution using the counts given by Equation (5) and varying the $\sigma_n = 0.57 \mu\text{Jy beam}^{-1}$ noise by adding values drawn randomly from a Gaussian distribution of rms width 0.01 \mu Jy beam\(^{-1}\). The range of $P(D)$ containing 68% of these simulations best fitting (according to Equation (4)) the average of all 1000 simulations defines the $\pm \sigma$ uncertainty region of our model $P(D)$. Figure 3 includes dotted red lines showing this narrow
uncertainty region, which is easily visible only in the $S_p > 10 \mu$Jy beam$^{-1}$ tail of the distribution.

4.5. Estimating the Source-count Uncertainty

To estimate the ±σ source-count errors resulting from the above $P(D)$ distribution range, we ran 500 simulations with the following variations: (1) the noise was drawn randomly from Gaussian distributions with mean $\sigma_n = 0.57 \mu$Jy beam$^{-1}$ and scatter 0.01 μJy beam$^{-1}$, (2) the input source counts were modeled with a fourth-degree polynomial to allow for the rapidly growing count uncertainty at the lowest flux densities caused by noise as well as by a possible new population of very faint radio sources, and (3) the coefficients of the fourth-degree polynomials were drawn randomly from Gaussian distributions centered on the best-fitting values given in Equation (5) with an rms of 0.1 (the unknown fourth-degree coefficient was initially centered on zero).

Each combination of coefficients and rms noise was repeated an additional six times to determine the effects of noise on the goodness of fit. We considered a set of coefficients to be in agreement with the DEEP2 $P(D)$ distribution if at least five of the total seven simulations fell within the ±σ uncertainty region determined from the original, unaltered 1000 simulations. The subset of the 500 coefficient-varying simulations that satisfied this criterion define the statistical uncertainty of the measured source counts. Then we added quadratically a 3% count uncertainty to absorb possible 3% systematic flux-density calibration errors. In the flux-density range $-6.6 < \log[S(Jy)] < 5$, the 1σ lower limit of the 1.266 GHz source-count error region is

$$
\log[S^2n(S)] = 2.677 + 0.489(\log S + 5)
+ 0.077(\log S + 5)^2
+ 0.061(\log S + 5)^3
- 0.058(\log S + 5)^4
$$

and the 1σ upper limit is

$$
\log[S^2n(S)] = 2.768 + 0.367(\log S + 5)
- 0.076(\log S + 5)^2
- 0.009(\log S + 5)^3
+ 0.023(\log S + 5)^4.
$$

5. The DEEP2 Source Catalog and Direct Counts

We used the attenuation-corrected sky image to search for discrete sources. The effective frequency of the wideband S/N-weighted DEEP2 image for sources with median spectral index $<\nu_o = 1.266$ GHz (Mauch et al. 2020). Even after correction for primary-beam attenuation, the DEEP2 image is strongly confusion limited with rms noise $\sigma_n < 1.12 \mu$Jy beam$^{-1}$ everywhere inside the primary-beam half-power circle. Consequently, our catalog brightness sensitivity limit $S_p(1.266$ GHz) = 10 μJy beam$^{-1}$ is uniform over the whole primary half-power circle, unlike the variable sensitivity limit of a deep source catalog extracted from an image that is still attenuated by the primary beam. Nearly all μJy radio sources are unresolved by the $\theta_{P2} = 7^{\circ}6$ DEEP2 restoring beam, so the DEEP2 catalog should be nearly complete for sources with total flux densities just above $S(1.266$ GHz) = 10 μJy. The relatively large DEEP2 restoring beam is actually advantageous because incompleteness corrections for partially resolved sources can be large and uncertain when the beam size is not much larger than the median source size (Morrison et al. 2010; Owen 2018).

5.1. The DEEP2 Component Catalog

We applied the Obit (Cotton 2008) source-finding task FndSou to the DEEP2 sky image inside the DEEP2 primary-beam half-power circle. FndSou searches for islands of contiguous pixels and decomposes each island into elliptical Gaussian components as faint as $S_p = 10 \mu$Jy beam$^{-1}$. Most radio sources with $S(3$ GHz) ≳ 5 μJy (equivalent to $S > 9 \mu$Jy at 1.266 GHz for $<\nu_o = -0.7$) have angular diameters $\phi < 0^{\times}66$ (Cotton et al. 2018) and would be completely unresolved in the DEEP2 image. This point-source approximation is supported by the qualitative similarity of our point-source simulation and the actual DEEP2 image shown in Figure 2. A small fraction of the DEEP2 sources stronger than ~100 μJy are clearly resolved jets or lobes driven by unresolved central AGNs, and they can be represented by combining multiple components as described in Section 5.3.

The sky density of sources reaches one per 25 restoring beam solid angles at $S(1.266$ GHz) ≈ 17 μJy (Mauch et al. 2020), so a significant fraction of our $S > 10 \mu$Jy components partially overlap, and our catalog accuracy, completeness, and reliability are limited more by confusion than by noise. To optimize the DEEP2 component catalog and understand its limitations, we used FndSou to extract catalogs of components from simulated images and compared these catalogs with the simulation input source lists. We compared catalogs in which

![Figure 4. The reduced $\chi^2$ statistic from 1.266 GHz DEEP2 simulations is shown as a function of the starting source-count bin increasing from the nominal $\log[S(Jy)] = -8$ to -6.1. At $\log[S(Jy)] = -6.5$, the $\chi^2$ statistic of the $P(D)$ distribution from DEEP2 simulations with $\sigma_n = 0.57 \mu$Jy averaged over the $r = 500^\circ$ $P(D)$ circle (black points) exceeds the value of the mean minimum $\chi^2$ plus 1σ (dotted black line). This indicates that we are sensitive to changes in the source count down to $\log[S(Jy)] = -6.6$. The $\chi^2$ values for DEEP2 simulations with ±1σ in rms noise are shown in blue and red for 0.56 and 0.58 μJy beam$^{-1}$, respectively.](image-url)
the fitted elliptical Gaussians were allowed to vary in width to
point-source catalogs in which they were not and found that
forcing point-source fits generally gave better matches to
the true simulation input catalogs. Therefore we forced point-
source fits to make the DEEP2 component catalog, and we later
combined components as needed to represent multicomponent
extended radio sources.

In a few crowded regions, FndSou reported spurious faint
components very close to much stronger sources. To decide
which components to reject from our catalog, we generalized
the original Rayleigh criterion for resolving two equal point
sources observed with an Airy pattern point-spread function
(PSF): the peak of one lies on or outside the first zero of the
second, which ensures that the total response has a minimum
between them.

The total image response $R$ at position $x$ between unequal
components $S_1$ at $x_1 = 0$ and $S_2 < S_1$ at $x_2 = \Delta$ to a Gaussian
PSF with FWHM $\theta_{1/2}$ is

$$ R = S_1 \exp\left( -\frac{4 \ln 2}{\theta_{1/2}^2} x^2 \right) + S_2 \exp\left( -\frac{4 \ln 2}{\theta_{1/2}^2} (x - \Delta)^2 \right). $$ (11)

For $R$ to have a minimum between the components, $dR/dx = 0
for some $0 < x < \Delta$. The continuous curve in Figure 5 shows
the required component separation $\Delta/\theta_{1/2}$ as a function of the
flux-density ratio $S_1/S_2$. All DEEP2 catalog components
stronger than $S > 10 \mu Jy$ have $S/N > 9$, so the requirement in
Equation (11) is stricter than necessary. By comparing 10
catalogs of components extracted from simulated images with
the true input components used to generate the simulated images, we found that faint components near stronger
components are reliable if they satisfy the weaker criterion

$$ \frac{\Delta}{\theta_{1/2}} \geq 0.574 + 0.357[\log(S_1/S_2) + 0.01]^{1/2} $$

$$ +0.082 \log(S_1/S_2) \quad \text{if } (S_1/S_2) < 9 $$

$$ \frac{\Delta}{\theta_{1/2}} \geq 1 \quad \text{if } (S_1/S_2) \geq 9 $$ (12)

shown by the dashed curve in Figure 5. We rejected the 334
probably spurious DEEP2 components ($<2\%$ of the total)
failing to satisfy Equation (12).

To estimate the effects of confusion and noise on the
completeness, reliability, positions, and flux densities of the
surviving 17,350 DEEP2 components, we ran 10 independent
simulations of the DEEP2 field out to the half-power circle of
the primary beam using input source counts consistent with the
differential source counts in Table 5 and the 1.4 GHz statistical
count $S^2_{\text{int}}(S) = 1.07 \times 10^{-5}S^{-0.48} \text{Jy sr}^{-1}$ of fainter
sources from Mauch et al. (2020). For each simulated image, we used
FndSou to find all components stronger than $10 \mu Jy$ and
rejected the components that did not satisfy the resolution
criterion in Equation (12). The positions and flux densities of
the resulting 10 catalogs were compared with the true
simulation input positions and flux densities of all simulated
sources stronger than $5 \mu Jy$.

We matched a simulated source to a cataloged component if
(1) its position was within $\theta_{1/2}/2 = 3 \arcsec 8$ of the
cataloged position, and (2) the catalog-to-true flux ratio satisfied
$0.5 \leq S_{\text{cat}}/S_{\text{true}} \leq 2$. If there were two or more matches, only
the strongest simulated source was matched with the cataloged
component. If there were no simulated sources that satisfied
these criteria, the cataloged component was rejected as
spurious. The 10 simulations yielded $\sim 1.6 \times 10^5$ matches.
Only $\sim 0.5\%$ of the cataloged components had no simulated-
source counterpart, for a catalog reliability $\sim 99.5\%$.

FndSou measures intensities relative to the image zero level.
The DEEP2 interferometric image is insensitive to the smooth
background of very faint radio sources. Our simulations of the
radio sky brightness include such a background, so the average
confusion $P(D)$ distribution from 10 simulations of DEEP2
appears shifted by $\Delta D = +0.28 \mu \text{Jy beam}^{-1}$ compared with the
$P(D)$ distribution of the real DEEP2 image. The final flux
densities of components in the DEEP2 source catalog were
corrected for the zero-level offset by subtracting $0.28 \mu \text{Jy beam}^{-1}$
from the peak flux densities reported by FndSou.

### 5.2. DEEP2 Catalog Position Uncertainties

The random position errors of DEEP2 source components are
dominated by confusion errors whose non-Gaussian
distributions are difficult to calculate analytically, so we
estimated the random position errors from the differences
$\Delta \alpha$, $\Delta \delta$ between the cataloged and true input positions
of source components in our 10 simulations. The normalized
probability distributions $P(\Delta)$ are shown separately for R.A. $\alpha$
(red histogram) and decl. $\delta$ (blue histogram) in Figure 6.
The distributions of $\Delta \alpha$ and $\Delta \delta$ are indistinguishable, as expected
for a circular PSF. Also as expected, the distributions of random
positions errors are symmetrical about $\Delta = 0$ and have
long non-Gaussian tails—a Gaussian distribution would look
like a parabola in the semilogarithmic Figure 6. The formal
width of $P(\Delta)$ is dominated by these tails and is not a stable
measure of the position error distribution. In a Gaussian
distribution with rms $\sigma$, $68\%$ of the sources would lie within
the range $-\sigma < \Delta < +\sigma$, so we used the range of actual
position offsets $\Delta \alpha \approx \Delta \delta$ and defined the rms position errors
$\sigma_{\alpha}$ and $\sigma_{\delta}$ such that $68\%$ of the components lie in the
range $-\sigma_{\alpha} < \Delta \alpha < +\sigma_{\alpha}$ or $-\sigma_{\delta} < \Delta \delta < +\sigma_{\delta}$. The DEEP2
random position errors vary with component flux density $S$. For bins of width 0.2 in $\log(S)$ centered on $\log[S(\mu \text{Jy})] =$
and used the NASA DEEP2 images. The small peaks at integer multiples of $Δ = 1''25$ are artifacts from measuring distances in simulated images composed of $1''25$ pixels but do not affect the rms position errors.

Figure 6. The distributions $P(\text{arcsec}^{-1})$ of differences in R.A. $Δ\alpha$ (red curve) and decl. $Δδ$ (blue curve) between the cataloged and true simulation positions for all sources with $S \geq 10 \mu Jy$ from 10 simulated DEEP2 images. The small peaks at integer multiples of $Δ = 1''25$ are artifacts from measuring distances in simulated images composed of $1''25$ pixels but do not affect the rms position errors.

1.1, 1.3, ..., 2.9, we determined the distributions of position offsets $Δ\alpha$ and $Δδ$ in the 10 simulations. Figure 7 shows $σ_{α,\text{cat}} \approx σ_α$ as a function of log($S$).

The relation between this error limit and flux density is well described by a broken power law of the form

$$\frac{Δ\alpha}{\text{arcsec}} = C \left[ \left( \frac{S_α}{S} \right)^{R/2} + \left( \frac{S_α}{S} \right)^R \right]^{1/R},$$

(13)

where the parameter $R$ controls the sharpness of the break at $S = S_α$. A nonlinear least-squares fit to Equation (13) yields $R = -11.97$, $C = 0.219$, and $S_α = 78.6 \mu Jy$. As the catalogs were made for simulated images, there are no systematic position errors included in Equation (13). The dashed black curve in Figure 7 shows the random rms errors $σ_{α,\text{cat}} \approx σ_α$ as a function of component flux density. The slope of the dashed curves in Figures 7 and 9 changes from $−1$ to $−0.5$ below $S_α$, because the DEEP2 image is strongly limited by confusion, the source-count slope changes by $γ \approx 1$ near $S = S_α$, and the rms confusion from weaker sources is proportional to $S^{3/2}$ (see Equation (20) in Condon et al. 2012). No such break occurs in noise-limited images.

To estimate the DEEP2 systematic position uncertainties and offsets, we selected the 268 strong components with calculated random errors $σ_{α,\text{cat}} = σ_α < 0''05$ and used the NASA/IPAC Infrared Science Archive (IRSA) to find eight identifications with Gaia DR2 sources whose position errors are much smaller than $0''05$. Their DEEP2 minus Gaia offsets have $σ_α = σ_δ = 0''12 \pm 0''04$, an insignificant mean offset $+0''03 \pm 0''04$ in R.A., and a $3σ$ significant mean decl. offset $-0''12 \pm 0''04$. We therefore added $-0''12$ to our fitted DEEP2 declinations and added the $0''12$ systematic position errors to the random errors in quadrature to obtain the total DEEP2 position error shown by the continuous curve in Figure 7. The total position errors reported in the final component catalog (Table 3) reflect this quadrature sum and the corrected declinations.

We compared the flux densities calculated from the forced point-source fits of cataloged components in the 10 simulated images with their true input flux densities. The distribution of these differences $ΔS = S_{\text{cat}} - S_{\text{true}}$ is shown in Figure 8 for four flux-density ranges: $10 \mu Jy < S < 10^{1.2} \sim 16 \mu Jy$, $10^{1.2} \mu Jy < S < 10^{1.4} \sim 25 \mu Jy$, $10^{1.4} \mu Jy < S < 10^{1.6} \sim 40 \mu Jy$, and $S > 10^{1.6} \mu Jy$. The flux-density error distributions all peak near $ΔS = 0 \mu Jy$ but have positive tails that grow with flux density because stronger components are able to obscure stronger confusing components.

The fractional flux-density errors $σ_S/S$ were calculated for all 10 catalogs of the simulated images in bins of width 0.2 dex centered on log($S(\mu Jy)$) = 1.1, 1.3, ..., 2.9 and are shown in Figure 9. In the ideal case of uncorrelated Gaussian noise, high $S/N S/σ_S$, and a circular Gaussian beam of FWHM $θ_1/2$, Equation (21) of Condon (1997) gives

$$\frac{σ_S}{S} = \sqrt{8 \ln 2} \left( \frac{σ_α}{θ_1/2} \right) = \sqrt{8 \ln 2} \left( \frac{σ_δ}{θ_1/2} \right),$$

(14)

Table 3

| R.A. (J2000) | Decl. (J2000) | $S(1.266 \text{ GHz})$ (μJy) | $G$-Group Code |
|--------------|--------------|-------------------------------|----------------|
| 04:08:34.781 ± 0.170 | $-79:51:43.08 \pm 0.45$ | 20.0 ± 2.1 | $...$ |
| 04:08:34.897 ± 0.089 | $-80:20:26.14 \pm 0.22$ | 87.2 ± 5.3 | G90 |
| 04:08:34.956 ± 0.166 | $-79:35:02.48 \pm 0.45$ | 20.0 ± 2.2 | $...$ |
| 04:08:34.986 ± 0.160 | $-80:24:19.47 \pm 0.40$ | 25.5 ± 2.5 | $...$ |
| 04:08:35.066 ± 0.051 | $-79:47:01.95 \pm 0.13$ | 278.0 ± 10.5 | $...$ |

**Note.** The quoted uncertainties are similar to rms errors in that they encompass 68% of the sources, but they are insensitive to the long tails of confusion-limited error distributions. There are 35 multicomponent sources labeled by their component group numbers G01 through G35, as described in Section 5.3. (This table is available in its entirety in machine-readable form.)
so for either $\alpha$ or $\delta$,

$$
\log \left( \frac{\alpha \mu}{S} \right) = \log(8 \ln 2)/2 + \log(\sigma_\alpha) - \log(\theta_{1/2}) \\
= \log(\sigma_\alpha) - 0.51, \tag{15}
$$

for $\theta_{1/2} = 7.56^\circ$. This gives conservative flux-density fitting errors. To them we add in quadrature a 2% uncertainty for telescope-pointing errors and primary attenuation uncertainty inside the primary-beam half-power circle plus a 3% for the absolute flux-density uncertainty of the gain calibrator PKS B1934–638 (Mauch et al. 2020) to obtain the total fractional uncertainty

$$
\frac{\sigma_\delta}{S} = \left( 8 \ln 2 \frac{\sigma_\alpha^2}{\theta_{1/2}^2} + 0.036^2 \right)^{1/2}, \tag{16}
$$

The fractional flux-density errors calculated from Equation (16) are shown by the solid black curve in Figure 9. This method yields more conservative error estimates than directly fitting the measured flux differences from the simulated images with a broken power law for flux densities $\log S < 2.5$ when these measured differences are added in quadrature with the cumulative calibration uncertainties (shown as the dotted line in Figure 9).

The 10 simulated images were corrected for primary-beam attenuation before the catalog was created, so the noise contribution increases with the distance $r$ from the pointing center as

$$
\sigma_n(r) = 0.56 \mu\text{Jy beam}^{-1}, \tag{17}
$$

where

$$
a(r) = \exp\left( -4 \ln 2 \frac{r^2}{\theta_{1/2}^2} \right), \tag{18}
$$

is the primary-beam attenuation. The simulation placed sources in the sky randomly but uniformly, so we subtracted the average noise variance within the half-power circle $\langle \sigma_n \rangle = 0.824 \mu\text{Jy beam}^{-1}$ from the total average flux-density variance calculated from Equation (16). We added back the distance-dependent rms noise variance to obtain a better estimate of errors on the individual source flux densities at various distances from the pointing center.

### 5.3. Multicomponent Sources

We visually inspected the DEEP2 image and found 35 groups of components that appear to comprise multicomponent radio sources. We labeled these components by their group numbers G01 to G35. For an extended source well approximated by a collection of Gaussian components, we summed the individual component flux densities to determine the source flux density. For a source containing diffuse emission regions, we estimated the flux density of such regions by directly summing over the pixel brightness distribution. Table 4 lists the 35 multicomponent sources, the number of components in each source group, our best estimate of the source core position, and the source flux density. Figure 10 shows the contour map of multicomponent source G01 with crosses marking the positions of its three components. Similar contour maps of all multicomponent sources appear in the appendix.

### 5.4. DEEP2 Direct Source Counts

We counted DEEP2 sources in bins of width 0.2 dex centered on 1.266 GHz flux densities $\log[S(\mu\text{Jy})] = -4.9, -4.7, \ldots, -2.5$. Component groups comprising an extended source were counted as a single source whose flux density is the sum of its individual component flux densities. For the few extended sources with diffuse emission regions, we estimated the flux densities of these regions by directly summing over their pixel brightness distributions.
Table 4
DEEP2 Multicomponent Sources

| Group | Code | R.A. (J2000) | Decl. (J2000) | S (μJy) |
|-------|------|--------------|--------------|---------|
| G01   | 3    | 04:00:47.04  | −79:51:31.4  | 817     |
| G02   | 3    | 04:01:31.81  | −79:59:08.6  | 159     |
| G03   | 8    | 04:03:54.19  | −80:08:49.1  | 757     |
| G04   | 4    | 04:05:51.81  | −79:58:56.2  | 709     |
| G05   | 3    | 04:04:14.84  | −79:56:21.5  | 13536   |
| G06   | 3    | 04:06:15.50  | −80:10:57.5  | 2742    |
| G07   | 3    | 04:06:26.05  | −79:38:01.0  | 168     |
| G08   | 7    | 04:06:27.83  | −80:18:48.0  | 19200   |
| G09   | 14   | 04:08:42.38  | −80:20:49.0  | 885     |
| G10   | 12   | 04:08:47.70  | −80:24:02.4  | 1569    |
| G11   | 5    | 04:11:32.60  | −79:48:41.3  | 766     |
| G12   | 3    | 04:11:38.97  | −79:48:17.5  | 2867    |
| G13   | 11   | 04:11:59.21  | −80:14:54.8  | 1422    |
| G14   | 7    | 04:12:16.93  | −79:46:33.2  | 6772    |
| G15   | 5    | 04:12:32.00  | −79:34:36.4  | 509     |
| G16   | 8    | 04:13:24.93  | −79:49:21.2  | 5455    |
| G17   | 9    | 04:13:41.22  | −79:46:34.9  | 6680    |
| G18   | 4    | 04:13:58.03  | −79:42:19.2  | 1369    |
| G19   | 18   | 04:14:17.93  | −80:11:38.4  | 5455    |
| G20   | 5    | 04:14:58.85  | −80:29:08.4  | 1261    |
| G21   | 3    | 04:16:10.11  | −80:03:31.9  | 169     |
| G22   | 6    | 04:16:23.34  | −80:20:54.5  | 4699    |
| G23   | 3    | 04:16:47.09  | −79:48:50.3  | 54268   |
| G24   | 5    | 04:16:58.32  | −79:54:46.2  | 1953    |
| G25   | 9    | 04:17:02.19  | −80:12:33.9  | 6115    |
| G26   | 3    | 04:17:06.86  | −79:51:28.6  | 3682    |
| G27   | 7    | 04:18:58.15  | −79:51:23.5  | 1603    |
| G28   | 10   | 04:19:10.77  | −80:30:32.4  | 3051    |
| G29   | 4    | 04:20:03.10  | −80:27:11.3  | 2769    |
| G30   | 14   | 04:22:05.41  | −80:03:30.1  | 10882   |
| G31   | 5    | 04:23:19.81  | −79:51:10.6  | 11408   |
| G32   | 8    | 04:25:02.63  | −80:14:15.9  | 26271   |
| G33   | 6    | 04:25:18.38  | −79:52:22.3  | 15983   |
| G34   | 9    | 04:25:51.51  | −79:54:38.7  | 731     |
| G35   | 8    | 04:26:07.75  | −80:09:23.7  | 766     |

(This table is available in machine-readable form.)

Sources near the catalog lower limit \(S(1.266\,\text{GHz}) = 10\,\mu\text{Jy}\) may be biased up by confusion or biased down and missed entirely. We estimated the effects of confusion on the direct-source counts by comparing the measured counts in the 10 simulated images with their true input counts. Their differences in each flux-density bin were calculated individually for the 10 simulations. We added the mean differences \(\Delta\) in \(\log(J_0^2/S)\) from the simulations to the raw DEEP2 counts to yield more accurate counts of radio sources with \(-5.0 < \log(S(\text{Jy})) < -2.5\).

Table 5 shows the \(v = 1.266\,\text{GHz}\) corrected counts based on the 17,350 DEEP2 components with \(S > 10\,\mu\text{Jy}\) inside the half-power circle of the primary beam. For the 13 flux-density bins of width 0.2 in \(\log(S)\), Column 1 lists the bin center \(\log(S(\text{Jy}))\) and Column 2 lists the number \(N_{\text{bin}}\) of sources in the bin. The corrections \(\Delta\) in Column 3 were added to the values of \(\log(S^2/n(S)\text{ (Jy sr}^{-1}\text{)})\) in Column 4. Columns 5 and 6 are the rms positive and negative uncertainties in \(\log(S^2/n(S))\). These uncertainties are the quadratic sum of the Poisson uncertainties in samples of size \(N_{\text{bin}}\), the count-correction uncertainties that we conservatively estimate to be \(\Delta/2\), and a 3% overall flux-density scale uncertainty.

### 6. NVSS Source Counts

The NVSS catalog reports flux densities rounded to the nearest multiple of 0.1 mJy. For example, all NVSS components with fitted flux densities 2.45 ≤ \(S(\text{mJy})\) < 2.55 are listed as having \(S = 2.5\,\text{mJy}\). We separated the NVSS components into flux-density bins of nearly constant width 0.2 in \(\log(S)\) whose exact boundaries \(S_{\text{min}}\) and \(S_{\text{max}}\) are midway between multiples of 0.1 mJy. Thus the lowest flux-density bin covers 2.45 ≤ \(S(\text{mJy})\) < 3.95 and includes all NVSS components listed with \(S(\text{mJy})\) = 2.5, 2.6, 2.7, ..., 3.9. The first column of Table 6 lists the bin centers, the second lists the numbers \(N_{\text{bin}}\) of components in each bin, and the third column shows the brightness-weighted counts \(\log(S^2/n(S))\) at \(\log(S(\text{Jy})) = -2.5, -2.3, ..., +1.3\). The fourth and fifth columns are the total rms uncertainties in \(\log(S^2/n(S))\).

Complex radio sources significantly more extended than the 45° FWHM NVSS restoring beam may be represented by two or more catalog components, and such large multicomponent sources are more common at flux densities \(S \gtrsim 1\,\text{Jy}\). To
estimate the fraction of components comprising strong extended sources, we compared the NVSS component catalog with the low-resolution 1.4 GHz Bridle et al. (1972) catalog of 424 sources having $S > 1.7$ Jy and equivalent angular diameters $\phi \lesssim 10'$ in the area defined by $-5^\circ < \delta < +70^\circ$, $|b| > 5^\circ$. We combined NVSS components within $\sim 5'$ of each Bridle et al. (1972) source, after excluding those that appeared to be unrelated background sources. In the six flux-density bins centered on $\log(S_{\text{1.4 GHz}}) = +0.3$ through $+1.3$, grouping NVSS components into sources changed the brightness-weighted source count $\log[S^2 n(S)\text{(Jy sr}^{-1})] \times -0.013$, $+0.013$, $+0.109$, $+0.193$, $+0.133$, and $0.000$, respectively.

The differential source count $n(S)$ is a rapidly declining function of flux density, so simply counting the number of sources in each fairly wide flux-density bin throws away flux-density information and can bias the resulting estimate of $n(S)$. If $n(S)dS$ is the number of sources per steradian with flux densities between $S$ and $S + dS$ and $\eta(S)d\ln(S)$ is the number per steradian with flux densities between $S$ and $S + d\ln(S)$, then $n(S)dS = \eta(S)d\ln(S)$. We added the flux density of each source to its bin of logarithmic width $\Delta \approx \exp(0.2)$ to calculate the more nearly constant quantity

$$S^2 n(S) = S\eta(S) = \left[ \frac{1}{\ln(\Delta)} \right] \sum_{j=1}^{n_{\text{bin}}} S_j^{1/2}$$

directly.

Finally, counts in the faintest bins must be corrected for population-law bias (Murdoch et al. 1973): faint sources outnumber strong sources, so noise moves more faint sources into a bin than it moves strong sources out. We used their Table 2 and the cumulative source-count approximation $N(S > S_0) \approx \int_S^\infty n(S)dS \propto S^{-1}$ for $S \geq 2.5$ mJy to calculate the required corrections to $\log[S^2 n(S)]$. They are $-0.030$, $-0.012$, and $-0.004$ in bins centered on $\log(S_{\text{1.4 GHz}}) = -2.5$, $-2.3$, and $-2.1$, respectively.

The rms statistical uncertainty in $S^2 n(S)$ for each bin with $n_{\text{bin}} \gg 1$ is

$$\sigma_{\text{stat}} \approx \left[ \frac{1}{\Omega \ln(\Delta)} \right] \left( \sum_{j=1}^{n_{\text{bin}}} S_j^{1/2} \right)^{1/2}.$$

There are only 6 sources in the $\log[S_{\text{1.4 GHz}}] = +1.1$ bin and 3 sources in the $\log[S_{\text{1.4 GHz}}] = +1.3$ bin, so we replaced their rms statistical errors in $\log[S^2 n(S)]$ by the Gehrels (1986) 84% confidence-level errors $+0.203$, $-0.219$ and $+0.295$, $-0.341$, respectively. To these statistical uncertainties we added quadratically the 3% error in $S^2 n(S)$ caused by the 3% NVSS flux-density scale uncertainty (Condon et al. 1998) and systematic uncertainties equaling half the corrections for component grouping and population-law bias.

7. Discussion

Figure 11 shows our 1.4 GHz differential source counts with traditional static-Euclidean weighting $S^2 n(S)$ and with brightness weighting $S^2 n(S)$. Counts from $S = 0.25$ mJy to $S = 10$ mJy were derived statistically from the DEEP2 confusion $P(D)$ distribution extracted from solid angle $\Omega = 0.061$ deg$^{-2}$. Individual sources uniformly covering solid angle $\Omega = 1.04$ deg$^{-2}$ between 10 mJy and 2.5 mJy were counted directly, as were NVSS sources above 2.5 mJy in solid angle $\Omega = 7.016$ sr (0.56 of the sky). Together these counts span the eight decades in flux density from $\log[S_{\text{1.4 GHz}}] = -6.6$ to $\log[S_{\text{1.4 GHz}}] = +1.4$. Their largest fractional uncertainties are caused by the rms noise $\sigma_n = 0.57 \pm 0.01$ mJy beam$^{-1}$ and finite resolution $\theta_{1/2} = 7.96$ just above $S = 0.25$ mJy, by statistical fluctuations in the small numbers of sources in the DEEP2 half-power circle between 0.5 mJy and 2.5 mJy, and by cosmic variance in the NVSS counts above $S \approx 3$ Jy.

Figure 12 compares our 1.4 GHz direct counts (black points) of sources fainter than 10 mJy with those of Hopkins et al. (2003; red triangles), Prandoni et al. (2018; filled red points), Heywood et al. (2020; open red points), plus the Smolčić et al. (2017; filled blue points) and Van der Vlugt et al. (2021; blue triangles) 3 GHz counts converted to 1.4 GHz assuming the median spectral index is $\alpha = -0.7$. The Smolčić et al. (2017) counts have small ($\sim 10\%$) uncertainties because they are based on the large (10,830 sources) noise-limited (median $\sigma_n = 2.3$ mJy beam$^{-1}$ at 3 GHz) VLA-COSMOS catalog. They are $\sim 20\%$ to $2\sigma$ lower than most other counts and $\sim 30\%$ lower than the Van der Vlugt et al. (2021) counts, possibly because resolution corrections for the small ($\theta_{1/2} = 0.75$) VLA COSMOS beam were insufficient or the median spectral index of $\mu$Jy sources is more negative than the assumed $-0.7$. In any case, the agreement among all of these $\mu$Jy source counts is much better than other counts have agreed in the recent past (Heywood et al. 2013), suggesting that the large earlier discrepancies were caused by observational and analysis errors, not by surprisingly strong source clustering.

7.1. Resolving the AGN and SFG Backgrounds

The 1.4 GHz brightness-weighted counts $S^2 n(S)$ shown in Figure 11 have two broad peaks. The peak at $S \sim 0.1$ Jy is dominated by AGNs and the peak at $S \approx 3 \times 10^{-3}$ Jy by SFGs. If the counts below $S = 0.25$ mJy do not exceed the extrapolation with slope $d \log[S^2 n(S)]/d \log(S) = +0.5$, sources stronger than $S = 0.25$ mJy resolve $> 99\%$ of the AGN contribution.
$\Delta T_b \approx 0.06 \text{ K}$ to the sky brightness temperature and $> 96\%$ of the $\Delta T_b \approx 0.04 \text{ K}$ SFG contribution. Thus most of the stars in the universe were formed in SFGs stronger than $0.25 \mu\text{Jy}$. For example, our fairly typical Galaxy currently has $1.4\text{ GHz}$ spectral luminosity $L_\nu \approx 2.5 \times 10^{21} \text{ WH} \text{ z}^{-1}$. With 10 times luminosity evolution (Madau & Dickinson 2014), it would be a $1.2 \mu\text{Jy}$ source around cosmic noon at $z \sim 2$ and a $0.25 \mu\text{Jy}$ source even at $z = 4$. 

### 7.2. $P(D)$ Limits on New Source Populations

The MeerKAT correlation interferometer used to make the DEEP2 image does not respond to backgrounds smooth on angular scales $\gg \theta_{1/2} = 7\arcsec 6$, and the resolution of ARCADE 2 is too coarse to detect individual $< 0.1 \mu\text{Jy}$ sources, so there is actually no observational tension between our results in Section 7.1 and the ARCADE 2 background. However, the DEEP2 $P(D)$ distribution can set a lower limit to the number of faint sources not much larger than $\theta_{1/2} = 7\arcsec 6 \approx 50 \text{ kpc}$ in the redshift range $0.5 < z < 5$ that can produce a $\Delta T_b \sim 0.4 \text{ K}$ background smooth enough to be consistent with the DEEP2 $P(D)$ distribution. Very numerous faint sources contribute a nearly Gaussian $P(D)$ distribution similar to the instrumental noise distribution. A source population with rms confusion not much larger than $\sigma_z \approx (0.58^2 - 0.57^2)^{1/2} \mu\text{Jy beam}^{-1} \sim 0.1 \mu\text{Jy beam}^{-1}$ is consistent with the uncertainty in the measured DEEP2 rms noise. Figure 13 plots the brightness-weighted source counts $S^2 n(S)$ as a function of $\log(S)$. The two broad peaks

![Image](image-url)
log log log2 pk 2

Source counts at 1.4 GHz consistent with the DEEP2 P(D) distribution are shown by the thin black line surrounded by its ±1 σ error region. A hypothetical new population with logarithmic FWHM A located at log(Spk) = −0.5 nJy (dashed black parabola) to remain consistent with our DEEP2 P(D) observation.

corresponding to star-forming galaxies and AGN are well represented by the approximation (Condon et al. 2012)

\[
\log[S^2n(S)] \approx a - b[\log(S) - \log(S_{pk})]^2
\]

Figure 13. Source counts at 1.4 GHz consistent with the DEEP2 P(D) distribution are shown by the thin black line surrounded by its ±1 σ error region. A hypothetical new population with logarithmic FWHM A located at log(Spk) = −0.5 nJy (dashed black parabola) to remain consistent with our DEEP2 P(D) observation.

Equation (22) is valid for any count peak flux density S_{pk}. Because \( \Delta T_b \) fixes the product \( A\phi \), a new population with \( T_b \) either has a small number of sources N with a narrow FWHM \( \phi \) but large amplitude \( A \), or a broad peak \( \phi \) with a larger number of sources per square arcmin and a fainter peak flux density.

The known AGN and SFG populations are well characterized by Gaussians of FWHM \( \phi = 2 \), so we assumed \( \phi = 2 \) for the hypothetical new population. In order to explain the excess \( \Delta T_b = 0.4 \) K, the amplitude of this population must be A \( \sim 5000 \) Jy sr\(^{-1}\).

For any \( S_{pk} \) we can find the maximum value of A that is consistent with the DEEP2 P(D). Inserting a new population of sources with \( \phi = 2 \) and log(S_{pk}) = −0.5 nJy (dashed black parabola) to remain consistent with our DEEP2 P(D) observation.

where \( \phi \) is the logarithmic FWHM and \( S_{pk} \) is the flux density of the \( S^2n(S) \) peak; log(S_{pk}) \( \sim -5 \) and log(S_{pk}) \( \sim -1 \) for SFGs and AGNs, respectively, while both populations are well described by \( \phi = 2 \). Inserting Equation (21) into Equation (1) and integrating over flux density determines the peak amplitude A for FWHM \( \phi \) for a new population adding \( \Delta T_b \) to the total sky brightness:

\[
A\phi = \frac{4\kappa_B \nu^2}{\ln(10) c^2} \left[ \frac{\ln(2)}{\pi} \right]^{1/2} \Delta T_b.\tag{22}
\]

For rms noise values \( \sigma_n = 0.56 \) and 0.57 Jy beam\(^{-1}\), the maximum amplitude consistent with the observed DEEP2 P(D) distribution is A \( \sim 50000 \) Jy sr\(^{-1}\). Simulations assuming rms noise \( \sigma_n = 0.55 \) Jy beam\(^{-1}\) allow A \( < 150 \). The final 1σ upper bound on the counts of nJy sources is the combination of Equation (9) and the curve representing the sum of the new population and the measured counts from DEEP2 (dotted curve in Figure 13).
adds only $\Delta T_s \sim 0.01$ K to the total radio source background at 1.4 GHz, yet it must contain at least 3,000 sources per arcmin$^2$, exceeding by a factor of 30 the sky density of galaxies brighter than $m_{AB} + 29$, the magnitude of the Large Magellanic Cloud at redshift $z = 2$) in the Hubble Ultra Deep Field (Beckwith et al. 2006).

A hypothetical new population contributing the full 0.4 K ARCADE 2 excess background is consistent with the narrow DEEP2 $P(D)$ distribution only if $\log(A) \approx 3.7$, the sources are randomly distributed on the sky, and $S_{pk} < 0.5$ nJy at 1.4 GHz, as indicated by the dashed parabola in Figure 13. This upper limit to $S_{pk}$ is a factor of 10 lower than the Condon et al. (2012) limit and more strongly excludes any bright population of numerous faint sources that cluster like galaxies or parts of galaxies.

8. Summary

In this work, we presented source counts in the eight decades of flux density from $S = 0.25$ $\mu$Jy to $S = 25$ $\mu$Jy using the MeerKAT DEEP2 field and archival NVSS data.

1. Statistical source counts between $S = 0.25$ $\mu$Jy and $S = 10$ $\mu$Jy (Section 4) were estimated from the confusion $P(D)$ distribution within 500$''$ of the DEEP2 pointing center. Simulations of the radio sky were developed and used to constrain the counts and their uncertainties.

2. We constructed a uniformly sensitive catalog of $\approx 17,000$ discrete sources stronger than $S = 10$ $\mu$Jy at 1.266 GHz in $\Omega_{1/2} = 1.04$ deg$^2$ (Section 5) and used it to count discrete sources in the flux-density range $10$ $\mu$Jy $\leq S < 2.5$ mJy (Section 5.4).

3. The NVSS catalog of radio source components was used to determine 1.4 GHz source counts between $S = 2.5$ mJy and $S = 25$ Jy in $\Omega \approx 7.016$ sr (Section 6).

We find good agreement with previously published 1.4 GHz counts, but report higher source counts (at the 2$\sigma$ level) than previously published 3 GHz counts from the VLA-COSMOS catalog. The agreement among all $\mu$Jy source counts is much improved from past studies.

Sources stronger than our lower limit of $S = 0.25$ $\mu$Jy resolve >99% of the AGN contribution and >96% of the SFG contribution to the sky brightness temperature. The maximum source-count amplitude for a hypothetical new population explaining the ARCADE 2 excess radio background is $\log(A) \approx 3.7$, and the peak of the distribution must be fainter than $S_{pk} \sim 0.5$ nJy to remain consistent with the DEEP2 $P(D)$ distribution. In a second paper (A. M. Matthews et al. 2021, in preparation), we will use the results from this paper to estimate the star formation history of the Universe.

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