NNLO corrections to the total cross section for Higgs boson production in hadron-hadron collisions

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Abstract

We present the next-to-next-to-leading order (NNLO) corrections to the total cross section for (pseudo-) scalar Higgs boson production using an alternative method than those used in previous calculations. All QCD partonic subprocesses have been included and the computation is carried out in the effective Lagrangian approach which emerges from the standard model by taking the limit $m_t \to \infty$ where $m_t$ denotes the mass of the top quark. Our results agree with those published earlier in the literature. We estimate the theoretical uncertainties by comparing the $K$-factors and the variation with respect to the mass factorization/renormalization scales with the results obtained by lower order calculations. We also investigate the dependence of the cross section on several parton density sets provided by different groups.

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Further we study which part of the coefficient functions dominates the cross section. This is of interest for the resummation of large corrections which occur near the boundary of phase space. It turns out that depending on the definition of the total cross section the latter is dominated by the soft-plus-virtual gluon corrections represented by \( \delta(1-x) \) and \( \ln^i(1-x)/(1-x)_+ \) terms.

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1 Introduction

The Higgs boson, which is the cornerstone of the standard model, is the only particle which has not been discovered yet. Its discovery or its absence will shed light on the mechanism how particles acquire mass as well as answer questions about supersymmetric extensions of the standard model or about compositeness of the existing particles and the Higgs boson. The LEP experiments [1] give a lower mass limit of about $m_H \sim 114 \text{ GeV}/c^2$ and fits to the data using precision calculations in the electro-weak sector of the standard model indicate an upper limit $m_H < 200 \text{ GeV}/c^2$ with 95% confidence level. After the end of the LEP program the search for the Higgs will be continued at hadron colliders in particular at the TEVATRON and the LHC.

In this paper we concentrate on Higgs production where the lowest order reaction proceeds via the gluon-gluon fusion mechanism. In the standard model the Higgs boson couples to the gluons via heavy quark loops. Since the coupling of the scalar Higgs boson $H$ to a fermion loop is proportional to the mass of the fermion (for a review see [2]), the top-quark loop is the most important. The latter contribution is also dominant for the pseudo-scalar Higgs boson $A$ provided $\tan \beta$ is small where $\beta$ denotes the mixing angle in the Two-Higgs-Doublet model (2HDM). In lowest order (LO) the gluon-gluon fusion process $g + g \rightarrow B$ with $B = H, A$, represented by the top-quark triangle graph, was computed in [3]. The two-to-two body tree graphs, given by gluon bremsstrahlung $g + g \rightarrow g + B, g + q(\bar{q}) \rightarrow g(q) + B$ and $g + \bar{q} \rightarrow g + B$ were computed for $B = H$ in [4] and for $B = A$ in [5]. From these reactions one can derive the transverse momentum ($p_T$) and rapidity ($y$) distributions of the (pseudo-) scalar Higgs boson. The total integrated cross section, which also involves the computation of the QCD corrections to the top-quark loop, has been calculated in [5]. This calculation is rather cumbersome since it involves the computation of two-loop triangular graphs with massive quarks. Furthermore also the two-to-three parton reactions have been computed in [6] using helicity methods. From the experience gained from the next-to-leading (NLO) corrections presented in [5] it is clear that it will be very difficult to obtain the exact NLO corrections to one-particle inclusive distributions as well as the NNLO corrections to the total cross section.

Fortunately one can simplify the calculations if one takes the large top-quark mass limit $m_t \rightarrow \infty$. In this case the Feynman graphs are obtained from an effective Lagrangian describing the direct coupling of the (pseudo-)}
scalar Higgs boson to the gluons. The LO and NLO contributions to the total cross section in this approximation were computed in [7]. A thorough analysis [5], [8] reveals that the error introduced by taking the \( m_t \to \infty \) limit is less than about 5% provided \( m_H \leq 2 m_t \). The two-to-three body processes were computed with the effective Lagrangian approach for the scalar and pseudo-scalar Higgs bosons in [9] and [10] respectively using helicity methods. The one-loop corrections to the two-to-two body reactions above were computed for the scalar Higgs boson in [11] and the pseudo-scalar Higgs boson in [12]. These matrix elements were used to compute the transverse momentum and rapidity distributions of the scalar Higgs boson up to NLO in [13], [14] and the pseudo-scalar Higgs boson in [12]. The effective Lagrangian method was also applied to obtain the NNLO total cross section for scalar Higgs production by the calculation of the two-loop corrections to the Higgs-gluon-gluon vertex in [15], the soft-plus-virtual gluon corrections in [16], [17] and the computation of the two-to-three body processes in [18], [19]. These calculations were repeated for pseudo-scalar Higgs production in [20], [21].

In this paper we recalculate the NNLO corrections to the total cross section for (pseudo-) scalar Higgs production which have been computed recently in [18]-[21] and we found complete agreement with their analytic results when the number of colours is set \( N = 3 \). However our method differs from the one presented in [18], [20] and the approach followed in [19], [21]. The authors in the latter references compute the total cross section using the Cutkosky [22] technique where one- and two-loop Feynman integrals are cut in certain ways. These Feynman integrals can be computed using various techniques (for more details see the references in [19]). Furthermore this method been also applied to compute rapidity distributions in NLO which is presented in [23]. The authors in [18] choose a more conventional method which was already used in [24], [25], [26] to compute the coefficient functions for the Drell-Yan process. However instead of an exact computation of the \( 2 \to 2 \) and \( 2 \to 3 \) body phase space integrals they expand them around \( x = 1 \). Here \( x = m^2/s \) where \( m \) and \( \sqrt{s} \) denote the (pseudo-) scalar Higgs mass and the partonic centre-of-mass energy respectively. Since the coefficient functions can be expressed into a finite number of Polylogarithms of the types \( \text{Li}_n(x) \), \( S_{n,p}(x) \) (see e.g. [27]) and logarithms of the types \( \ln^i x \ ln^j(1 - x) \), which are all multiplied by polynomials in \( x \), one can expand them in the limit \( x \to 1 \) and match them with the expressions coming from the phase space integrals. In this way the coefficients of the above functions are determined.
However as is shown in [26] this procedure is not necessary since one can obtain the Polylogarithms in a direct way by expanding the hypergeometric functions [28] in $\varepsilon = n - 4$ where $n$ is the number of dimensions characteristic of $n$-dimensional regularization. The point is that there is no essential difference between Higgs production in the effective Lagrangian approach and the Drell-Yan production mechanisms except that the matrix elements in the former case contain higher powers of the invariants in the numerators. In [26] a program was made using the algebraic manipulation program FORM [29] to evaluate the phase space integrals analytically. For this calculation we have extended the program so that it can accommodate integrals having higher powers of invariants in the numerator. Our approach can be also used for differential distributions in particular for jet production (see e.g. [30]). Here one cannot avoid multi-particle phase space integrals. Of course two- to-four body processes are even more cumbersome to deal with but one can at least try to compute the pole terms $(1/\varepsilon)^k$, which arise from infrared and collinear divergences, analytically or numerically (for numerical methods see [31]). Another difference with the previous results is that we present the radiative parts of the coefficient functions for general colour factors of the local gauge group SU(N). The latter is important if one wants to resum the dominant contributions. (see [8], [32]).

Our paper will be organized as follows. In Section 2 we give definitions and discuss the regularization method to compute the partonic cross sections in the effective Lagrangian approach. In Section 3 we present the calculation of the coefficient functions. In particular we discuss the various frames in which the three-particle phase space integrals are computed. In Section 4 we present the total cross sections for scalar Higgs production at the LHC and the TEVATRON. The long expressions for the coefficient functions expressed in the various colour factors are presented in Appendix A (scalar Higgs) and Appendix B (pseudo-scalar Higgs).
2 Application of the effective Lagrangian approach to Higgs production

In the large top-quark mass limit the Feynman rules (see e.g. [9]) for scalar Higgs production \( (H) \) can be derived from the following effective Lagrangian density

\[
\mathcal{L}^H_{\text{eff}} = G_H \Phi^H (x) O (x) \quad \text{with} \quad O (x) = - \frac{1}{4} G^a_{\mu \nu} (x) G^{a, \mu \nu} (x) , \tag{2.1}
\]

whereas pseudo-scalar Higgs \( (A) \) production is obtained from

\[
\mathcal{L}^A_{\text{eff}} = \Phi^A (x) \left[ G_A O_1 (x) + \tilde{G}_A O_2 (x) \right] \quad \text{with}
\]

\[
O_1 (x) = - \frac{1}{8} \epsilon_{\mu \nu \lambda \sigma} G^\mu_\nu a G^{\lambda \sigma} a (x) ,
\]

\[
O_2 (x) = - \frac{1}{2} \partial^\mu \sum_{i=1}^{n_f} \bar{q}_i (x) \gamma_\mu \gamma_5 q_i (x) , \tag{2.2}
\]

where \( \Phi^H (x) \) and \( \Phi^A (x) \) represent the scalar and pseudo-scalar fields respectively and \( n_f \) denotes the number of light flavours. Furthermore the gluon field strength is given by \( G^\mu_\nu a \) and the quark field is denoted by \( q_i \). The factors multiplying the operators are chosen in such a way that the vertices are normalised to the effective coupling constants \( G_H , G_A \) and \( \tilde{G}_A \). The latter are determined by the top-quark triangular loop graph, including all QCD corrections, taken in the limit \( m_t \to \infty \) which describes the decay process \( B \to g + g \) with \( B = H , A \) namely

\[
G_B = - 2^{5/4} a_s (\mu_r^2) G_F^{1/2} \tau_B F_B (\tau_B) C_B \left( a_s (\mu_r^2), \frac{\mu_r^2}{m_t^2} \right) ,
\]

\[
\tilde{G}_A = - \left[ a_s (\mu_r^2) C_F \left( \frac{3}{2} - 3 \ln \frac{\mu_r^2}{m_t^2} \right) + \cdots \right] G_A , \tag{2.3}
\]

and \( a_s (\mu_r^2) \) is defined by

\[
a_s (\mu_r^2) = \frac{\alpha_s (\mu_r^2) }{4\pi} , \tag{2.4}
\]

\[6\]
where $\alpha_s(\mu_r^2)$ is the running coupling constant and $\mu_r$ denotes the renormalization scale. Further $G_F$ represents the Fermi constant and the functions $F_B$ are given by

$$F_H(\tau) = 1 + (1 - \tau) f(\tau), \quad F_A(\tau) = f(\tau) \cot \beta,$$

$$\tau = \frac{4 m_t^2}{m^2},$$

$$f(\tau) = \arcsin^2 \frac{1}{\sqrt{\tau}}, \quad \text{for} \quad \tau \geq 1,$$

$$f(\tau) = -\frac{1}{4} \left( \ln \frac{1 - \sqrt{1 - \tau}}{1 + \sqrt{1 - \tau}} + \pi i \right)^2 \quad \text{for} \quad \tau < 1,$$

(2.5)

where $\cot \beta$ denotes the mixing angle in the Two-Higgs-Doublet Model. Further $m$ and $m_t$ denote the masses of the (pseudo-) scalar Higgs boson and the top quark respectively. In the large $m_t$-limit we have

$$\lim_{\tau \to \infty} F_H(\tau) = \frac{2}{3 \tau}, \quad \lim_{\tau \to \infty} F_A(\tau) = \frac{1}{\tau} \cot \beta.$$

(2.6)

The coefficient functions $C_B$ originate from the corrections to the top-quark triangular graph provided one takes the limit $m_t \to \infty$. We have presented the Born level couplings $G_B$ in Eq.(2.3) for general $m_t$ for on-shell gluons only in order to keep some part of the top-quark mass dependence. This is an approximation because the gluons which couple to the H and A bosons via the top-quark loop in partonic subprocesses are very often virtual. The virtual-gluon momentum dependence is neither described by $F_B(\tau)$ nor by $C_B$. However for total cross sections the main contribution comes from the region where the gluons are almost on-shell so that this approximation is better than it is for differential cross sections with large transverse momentum. The coefficient functions are computed up to order $\alpha_s^2$ in [8], [33] for the H and in [34] for the A. They read as follows

$$C_H \left( a_s(\mu_r^2), \frac{\mu_r^2}{m_t^2} \right) = 1 + a_s^{(5)}(\mu_r^2) \left[ 5 C_A - 3 C_F \right] + \left( a_s^{(5)}(\mu_r^2) \right)^2 \left[ \frac{27}{2} C_F^2 - \frac{100}{3} C_A C_F + \frac{1063}{36} C_A^2 - \frac{4}{3} C_F T_f - \frac{5}{6} C_A T_f + \left( 7 C_A^2 \right) \right]$$

7
\[-11 C_A C_F \ln \frac{\mu_r^2}{m_t^2} + n_f T_f \left( -5 C_F - \frac{47}{9} C_A + 8 C_F \ln \frac{\mu_r^2}{m_t^2} \right) \]  

\[ C_A \left( a_s(\mu_r^2), \frac{\mu_r^2}{m_t^2} \right) = 1 , \]  

(2.7)

where \( a_s^{(5)} \) is presented in the five-flavour number scheme. Notice that the coefficient function in Eq. (2.7) is derived for general colour factors of the group SU(N) from Eq. (6) in [33]. These factors are given by

\[ C_A = N, \quad C_F = \frac{N^2 - 1}{2N}, \quad T_f = \frac{1}{2}. \]  

(2.8)

Notice that \( T_f \) is also incorporated into \( G_B \) in Eq. (2.3) where it is set to the value \( T_f = 1/2 \).

Using the effective Lagrangian approach we will calculate the total cross section of the reaction

\[ H_1(P_1) + H_2(P_2) \to B(-p_5) + 'X', \]  

(2.10)

where \( H_1 \) and \( H_2 \) denote the incoming hadrons and \( X \) represents an inclusive hadronic state. The total cross section is given by

\[ \sigma_{\text{tot}} = \frac{\pi G_B^2}{8(N^2 - 1)} \sum_{a,b=q,\bar{q},g} \int x_1 \int x_2 f_a(x_1, \mu^2) f_b(x_2, \mu^2) \]  

\[ \times \Delta_{ab,B} \left( \frac{m^2}{x_1 x_2 \mu^2} \right) , \]  

(2.11)

with \( x = \frac{m^2}{S} \), \( S = (P_1 + P_2)^2 \), \( p_5^2 = m^2 \),

where the factor \( 1/(N^2 - 1) \) originates from the colour average in the case of the local gauge group SU(N). Further we have assumed that the (pseudo-) scalar Higgs boson is mainly produced on-shell i.e. \( p_5^2 \approx m^2 \). The parton densities denoted by \( f_a(y, \mu^2) \) \((a,b = q, \bar{q}, g)\) depend on the mass factorization/renormalization scale \( \mu \). The same scales also enter the coefficient functions \( \Delta_{ab,B} \) which are derived from the partonic cross sections

\[ \sigma_{ab,B} \left( z, \frac{m^2}{\mu^2} \right) = \frac{\pi}{s} \int \frac{d^4p_5}{(2\pi)^n} \delta^+(p_5^2 - m^2) T_{ab,B}(p_5, p_1, p_2) , \]  

(2.12)
where the incoming parton momenta, denoted by $p_1$ and $p_2$, are related to the hadron momenta by

$$p_1 = x_1 P_1, \quad p_2 = x_2 P_2,$$

$$s = (p_1 + p_2)^2, \quad \implies \quad s = x_1 x_2 S, \quad z = \frac{m^2}{s}. \quad (2.13)$$

The amplitude $T_{ab,B}$ can be written as

$$T_{ab,B}(p_5, p_1, p_2) = G_H^2 \int d^4 y e^{i p_5 \cdot y} \langle a, b | O(y) O(0) | a, b \rangle, \quad (2.14)$$

$$T_{ab,A}(p_5, p_1, p_2) = \int d^4 y e^{i p_5 \cdot y} \langle a, b | (G_A O_1(y) + \tilde{G}_A O_2(y)) \times (G_A O_1(0) + \tilde{G}_A O_2(0)) | a, b \rangle. \quad (2.15)$$

The expressions above for the amplitude $T_{ab}$ are similar to those given for the Drell-Yan process except that the conserved electroweak currents are replaced by the operators $O$ and $O_1, O_2$. The latter are not conserved so that they acquire additional renormalization constants defined by

$$O(y) = Z_O \hat{O}(y), \quad O_i(y) = Z_{i j} \hat{O}_j(y). \quad (2.16)$$

where the hat indicates that the quantities under consideration are unrenormalized. Insertion of the above equations into Eqs. (2.14),(2.15) leads to the renormalized expressions

$$T_{ab,H}(p_5, p_1, p_2) = G_H^2 Z_O^2 \int d^4 y e^{i p_5 \cdot y} \langle a, b | \hat{O}(y) \hat{O}(0) | a, b \rangle, \quad (2.17)$$

$$T_{ab,A}(p_5, p_1, p_2) = \int d^4 y e^{i p_5 \cdot y} \left[ G_A^2 Z_{11}^2 + \tilde{G}_A^2 Z_{21}^2 + 2 G_A \tilde{G}_A Z_{11} Z_{21} \right]$$

$$\times \langle a, b | \hat{O}_1(y) \hat{O}_1(0) | a, b \rangle + \left[ G_A^2 Z_{12}^2 + \tilde{G}_A^2 Z_{22}^2 + 2 G_A \tilde{G}_A Z_{12} Z_{22} \right]$$

$$\times \langle a, b | \hat{O}_2(y) \hat{O}_2(0) | a, b \rangle + \left[ G_A Z_{11} Z_{12} + \tilde{G}_A Z_{22} Z_{21} + G_A \tilde{G}_A \right.$$

$$\times \left( Z_{11} Z_{22} + Z_{12} Z_{21} \right) \langle a, b | \hat{O}_1(y) \hat{O}_2(0) + \hat{O}_2(y) \hat{O}_1(0) | a, b \rangle \bigg]. \quad (2.18)$$
The operator renormalization constants depend on the regularization scheme in particular on the prescription for the $\gamma_5$-matrix and the Levi-Civita tensor in Eq. (2.2). The computation of $T_{ab,\Gamma}$ will be carried out by choosing n-dimensional regularization where in the case of $T_{ab,\Lambda}$ we adopt the HVBM prescription [35], [36] for the $\gamma_5$-matrix. For this choice the contraction of the Levi-Civita tensors proceeds as

$$
\varepsilon_{\mu_1\nu_1\lambda_1\sigma_1} \varepsilon^{\mu_2\nu_2\lambda_2\sigma_2} = \begin{vmatrix}
\delta_{\mu_1}^{\mu_2} & \delta_{\nu_1}^{\nu_2} & \delta_{\lambda_1}^{\lambda_2} & \delta_{\sigma_1}^{\sigma_2} \\
\delta_{\nu_1}^{\nu_2} & \delta_{\lambda_1}^{\lambda_2} & \delta_{\nu_1}^{\nu_2} & \delta_{\lambda_1}^{\lambda_2} \\
\delta_{\lambda_1}^{\lambda_2} & \delta_{\sigma_1}^{\sigma_2} & \delta_{\lambda_1}^{\lambda_2} & \delta_{\sigma_1}^{\sigma_2} \\
\delta_{\sigma_1}^{\sigma_2} & \delta_{\lambda_1}^{\lambda_2} & \delta_{\sigma_1}^{\sigma_2} & \delta_{\lambda_1}^{\lambda_2}
\end{vmatrix},
$$

(2.19)

where all Lorentz indices are taken to be n-dimensional. To facilitate the calculation one can replace $\gamma_\mu \gamma_5$ in Eq. (2.2) by (see [37])

$$
\gamma_\mu \gamma_5 = \frac{i}{6} \varepsilon_{\mu\rho\sigma\tau} \gamma^\rho \gamma^\sigma \gamma^\tau,
$$

(2.20)

which is equivalent to the HVBM scheme. Choosing the $\overline{\text{MS}}$ subtraction scheme the renormalization constant corresponding to the operator $O$ becomes [38]

$$
Z_O = 1 + a_s(\mu_r^2) S_\varepsilon \left[ \frac{2}{\varepsilon} \beta_0 + a_s^2(\mu_r^2) S_\varepsilon \left[ \frac{4}{\varepsilon^2} \beta_0^2 + \frac{2}{\varepsilon} \beta_1 \right] \right] + \cdots
$$

(2.21)

where $S_\varepsilon$ denotes the spherical factor characteristic of n-dimensional regularization. It is defined by

$$
S_\varepsilon = \exp \left\{ \frac{\varepsilon}{2} \left[ \gamma_E - \ln 4\pi \right] \right\}.
$$

(2.22)

The lowest order coefficients $\beta_0$ and $\beta_1$ originate from the beta-function given by

$$
\beta(\alpha_s) = -a_s^2(\mu_r^2) \beta_0 - a_s^3(\mu_r^2) \beta_1 + \cdots,
$$

$$
\beta_0 = \frac{11}{3} C_A - \frac{4}{3} n_f T_f, \quad \beta_1 = \frac{34}{3} C_A^2 - 4 n_f T_f C_F - \frac{20}{3} n_f T_f C_A.
$$

(2.23)
The operator renormalization constants corresponding to $O_1$ and $O_2$ are computed in [39] and they read

$$Z_{11} = Z_{\alpha_s} = 1 + a_s(\mu_r^2) S_\epsilon \frac{2}{\epsilon} \beta_0 + a_s^2(\mu_r^2) S_\epsilon^2 \left[ \frac{4}{\epsilon^2} \beta_0^2 + \frac{1}{\epsilon} \beta_1 \right] + \cdots$$  \hspace{1cm} (2.24)

where $Z_{\alpha_s}$ denotes the coupling constant renormalization factor defined by

$$\hat{a}_s = Z_{\alpha_s} a_s(\mu_r^2).$$  \hspace{1cm} (2.25)

The remaining constants are

$$Z_{21} = 0,$$  \hspace{1cm} (2.26)

$$Z_{12} = a_s(\mu_r^2) S_\epsilon \frac{1}{\epsilon} \left[ -6 C_F \right],$$  \hspace{1cm} (2.27)

$$Z_{22} = Z_{\text{MS}}^s Z_{5}^s,$$  \hspace{1cm} (2.28)

where $Z_{\text{MS}}^s$ and $Z_{5}^s$ are the constants characteristic of the HVBM scheme. They are given by

$$Z_{\text{MS}}^s = 1 + a_s^2(\mu_r^2) S_\epsilon \frac{1}{\epsilon} \left[ -\frac{44}{3} C_A C_F - n_f T_f C_F \frac{20}{3} \right],$$

$$Z_{5}^s = 1 + a_s(\mu_r^2) \left[ -4 C_F \right] + a_s^2(\mu_r^2) \left[ 22 C_F^2 - \frac{107}{9} C_A C_F + \frac{31}{9} n_f T_f C_F \right].$$  \hspace{1cm} (2.29)

The latter renormalization constant is determined in such a way that the Adler-Bell-Jackiw anomaly [40]

$$O_2(y) = 4 a_s(\mu_r^2) n_f T_f O_1(y),$$  \hspace{1cm} (2.30)

is preserved in all orders in perturbation theory according to the Adler-Bardeen theorem [41].
3 Computation of the partonic cross section up to NNLO

In this section we will give a short outline of the computation of the partonic cross sections $\sigma_{ab,B}$ in Eq. (2.12) up to next-to-next-leading order (NNLO). Insertion of a complete set of intermediate states and using translational invariance in Eq. (2.13) we obtain for scalar Higgs production

$$
\sigma_{ab,H} = K_{ab} \frac{\pi G_H^2}{s} Z_O^2 \int d^4p_5 \delta^+(p_5^2 - m^2) \sum_{m=3}^\infty \prod_{i=3, i\neq 5}^m \int \frac{d^4p_i}{(2\pi)^{n-1}} \delta^+(p_i^2) 
\times \delta^{(n)}(\sum_{j=1}^m p_j) |M_{ab\rightarrow X,H}|^2,
$$

with

$$
M_{ab\rightarrow X,H} = \langle p_1, p_2 | \hat{O}(0) | X, p_5 \rangle \quad \text{with} \quad |X, p_5\rangle = |p_3, p_4, p_6 \cdots p_m, p_5\rangle, \quad (3.1)
$$

where $K_{ab}$ represents the spin and colour average over the initial states. Further one can write down a similar expression for the pseudo-scalar Higgs (see Eq. (2.15)). Up to NNLO we have to compute the following subprocesses. On the Born level we have the reaction

$$
g + g \rightarrow B. \quad (3.2)
$$

In NLO we have in addition to the one-loop virtual corrections to the above reaction the following two-to-two body processes

$$
g + g \rightarrow B + g \quad , \quad g + q(\bar{q}) \rightarrow B + q(\bar{q}) \quad , \quad q + \bar{q} \rightarrow B + g. \quad (3.3)
$$

In NNLO we receive contributions from the two-loop virtual corrections to the Born process in Eq. (3.2) and the one-loop corrections to the reactions in Eq. (3.3). To these contribution one has to add the results obtained from the following two-to-three body reactions

$$
g + g \rightarrow B + g + g \quad , \quad g + g \rightarrow B + q_i + \bar{q}_i. \quad (3.4)
$$

$$
g + q(\bar{q}) \rightarrow B + q(\bar{q}) + g, \quad (3.5)
$$
\[ q + \bar{q} \rightarrow B + g + g \quad , \quad q + \bar{q} \rightarrow B + q_i + \bar{q}_i \]  
(3.6)

\[ q_1 + q_2 \rightarrow B + q_1 + q_2 \quad , \quad q_1 + \bar{q}_2 \rightarrow B + q_1 + \bar{q}_2 \]  
(3.7)

\[ q + q \rightarrow B + q + q \]  
(3.8)

In the case of pseudo-scalar Higgs production one also has to add the contributions due to interference terms coming from the operators \( \hat{O}_1 \) and \( \hat{O}_2 \) in Eq. (2.18). Up to order \( \alpha_s^2 \) we have to compute the following expression for the reactions in (3.4)

\[
\sigma_{gg,A} = K_{ab} \frac{\pi}{s} \int d^n p_5 \delta^+(p_5^2 - m^2) \left\{ G_A^2 Z_{11}^2 \sum_{m=3}^{4} \prod_{i=3}^{m} \int \frac{d^n p_i}{(2\pi)^{n-1}} \delta^+(p_i^2) \times \delta^{(n)}(\sum_{j=1}^{m} p_j) \right\}.
\]

The Feynman graphs for \( \langle p_1, p_2|\hat{O}_1|p_5 \rangle \) and \( \langle p_1, p_2|\hat{O}_2|p_5 \rangle \) can be found in Fig. 1a and Fig. 2a of [20] respectively. From the above equation one infers that the interference term contributes as a delta-function \( \delta(1 - z) \) to \( \sigma_{gg,A} \). Furthermore since \( G_A \) is proportional to \( G_A \) (see Eq. (2.3)) one can extract the latter constant as an overall factor from the above equation. For the reactions in Eq. (3.5) we have to compute

\[
\sigma_{gq,A} = K_{ab} \frac{\pi G_A^2}{s} \int d^n p_5 \delta^+(p_5^2 - m^2) \left\{ Z_{11}^2 \sum_{m=3}^{4} \prod_{i=3}^{m} \int \frac{d^n p_i}{(2\pi)^{n-1}} \delta^+(p_i^2) \times \delta^{(n)}(\sum_{j=1}^{m} p_j) \right\}.
\]

The Feynman graphs for \( \langle p_1, p_2|\hat{O}_1|p_5 \rangle \) and \( \langle p_1, p_2|\hat{O}_2|p_5 \rangle \) can be found in Fig. 1a and Fig. 2a of [20] respectively. From the above equation one infers that the interference term contributes as a delta-function \( \delta(1 - z) \) to \( \sigma_{gg,A} \). Furthermore since \( G_A \) is proportional to \( G_A \) (see Eq. (2.3)) one can extract the latter constant as an overall factor from the above equation. For the reactions in Eq. (3.5) we have to compute

\[
\sigma_{gq,A} = K_{ab} \frac{\pi G_A^2}{s} \int d^n p_5 \delta^+(p_5^2 - m^2) \left\{ Z_{11}^2 \sum_{m=3}^{4} \prod_{i=3}^{m} \int \frac{d^n p_i}{(2\pi)^{n-1}} \delta^+(p_i^2) \times \delta^{(n)}(\sum_{j=1}^{m} p_j) \right\}.
\]

The Feynman graphs for \( \langle p_1, p_2|\hat{O}_1|p_5 \rangle \) and \( \langle p_1, p_2|\hat{O}_2|p_5 \rangle \) can be found in Fig. 1a and Fig. 2a of [20] respectively. From the above equation one infers that the interference term contributes as a delta-function \( \delta(1 - z) \) to \( \sigma_{gg,A} \). Furthermore since \( G_A \) is proportional to \( G_A \) (see Eq. (2.3)) one can extract the latter constant as an overall factor from the above equation. For the reactions in Eq. (3.5) we have to compute

\[
\sigma_{gq,A} = K_{ab} \frac{\pi G_A^2}{s} \int d^n p_5 \delta^+(p_5^2 - m^2) \left\{ Z_{11}^2 \sum_{m=3}^{4} \prod_{i=3}^{m} \int \frac{d^n p_i}{(2\pi)^{n-1}} \delta^+(p_i^2) \times \delta^{(n)}(\sum_{j=1}^{m} p_j) \right\}.
\]
The Feynman graphs for $\langle p_1, p_2 | \hat{O}_1 | p_3, p_5 \rangle$ and $\langle p_1, p_2 | \hat{O}_2 | p_3, p_5 \rangle$ can be found in Fig. 1b and Fig. 2b of [20] respectively. In the HVBM-scheme the latter matrix element is proportional to $\varepsilon = n - 4$. However this contribution does not vanish in the limit $\varepsilon \to 0$ because of the single pole term present in $Z_{12}$ in Eq. (2.27). The same expression as in Eq. (3.10) also exists for $\sigma_{q\bar{q},A}$ originating from the first reaction in Eq. (3.6). In this case the Feynman graphs corresponding to $\langle p_1, p_2 | \hat{O}_1 | p_3, p_5 \rangle$ and $\langle p_1, p_2 | \hat{O}_2 | p_3, p_5 \rangle$ are shown in Fig. 1d and Fig. 2d of [20] respectively. Notice that these contributions also survive in the calculation of differential distributions, see [12].

The computation of the phase space integrals proceeds as follows. First we take over the expressions from the one- and two-loop corrections to the Born reaction in Eq. (3.2) which are computed in [7] (one-loop) and [15] (two-loop). Unfortunately the two-loop result is not presented for arbitrary colour factors so that we cannot distinguish between $n_f C_A T_f$ and $n_f C_F T_f$. Hence we have shuffled all contributions proportional to $n_f$ into the term proportional to $n_f C_A T_f$. To compute the two-to-two body processes including the virtual corrections we have chosen the centre-of-mass frame of the incoming partons given by

\begin{align}
  p_1 &= \frac{1}{2} \sqrt{P_{12}} (1, 0, \cdots, 0, 1), \\
  p_2 &= \frac{1}{2} \sqrt{P_{12}} (1, 0, \cdots, 0, -1), \\
  -p_3 &= \frac{P_{12} - m^2}{2\sqrt{P_{12}}} (1, 0, \cdots, -\sin \theta, -\cos \theta), \\
  -p_5 &= \frac{1}{2\sqrt{P_{12}}} (P_{12} + m^2, 0, \cdots, (P_{12} - m^2) \sin \theta, (P_{12} - m^2) \cos \theta),
\end{align}

where we have introduced the invariants

\begin{equation}
  P_{ij} = (p_i + p_j)^2, \quad P_{12} = s.
\end{equation}

In this frame the two-to-two body phase space integral becomes

\begin{equation}
  \sigma_{ab\to c H} = \frac{K_{ab} Z_O^2 \pi G_H^2}{P_{12}} \frac{2^{3-n}}{(4\pi)^{n/2}} \frac{1}{\Gamma(n/2 - 1)} \left( \frac{P_{12}}{\mu^2} \right)^{1-n/2} \left( \frac{P_{12} - m^2}{\mu^2} \right)^{n-3}
\end{equation}
Here $\mu$ indicates the dimension of the strong coupling constant $g \rightarrow g \mu^{(4-n)/2}$ which is characteristic of $n$-dimensional regularization. In order to make all ratios dimensionless we have already included the factor $\mu^{(4-n)/2}$ in the phase space integrals. Notice that in principle the scale $\mu$ has nothing to do with the factorization or renormalization scale. However for convenience one puts the latter scales equal to $\mu$. The evaluation of the integral in Eq. (3.13) is rather easy even when $|M_{ab\rightarrow c B}|$ contains virtual contributions. Since the integrals can be evaluated analytically we have made a routine using the algebraic manipulation program FORM which provides us with the results. A similar routine is made for the two-to-three body phase space integrals but here the computation is not so easy unless one chooses a suitable frame. Since we integrate over the total phase space the integrals are Lorentz invariant and therefore frame independent. The matrix elements of the partonic reactions can be partial fractioned in such a way that maximally two factors $P_{ij}$ in Eq. (3.12) depend on the polar angle $\theta_1$ and the azimuthal angle $\theta_2$. Furthermore one factor only depends on $\theta_1$ whereas the other one depends both on $\theta_1$ and $\theta_2$. Therefore the following combinations show up in the matrix elements

\[ P^k_{ij} P^l_{mn} , P^k_{ij} P^l_{m5} , P^k_{i5} P^l_{m5} \quad , \quad 4 \geq k \geq -2 , 4 \geq l \geq -2 , \]

\[ p_2^i = p_2^j = p_2^m = p_2^p = 0 \quad , \quad p_2^5 = m^2 . \]  

For the first combination it is easy to perform the angular integration since all momenta represent massless particles and one obtains a hypergeometric function (see e.g. Eq. (4.19) in [14]). The angular integral of the second combination is more difficult to compute because one particle is massive and the result is an one dimensional integral over a hypergeometric function which however can be expanded around $\varepsilon$. Examples of these types of integrals can be found in Appendix C of [42]. The last combination is very difficult to compute in $n$ dimensions because both factors contain the massive particle indicated by $p_5$. Therefore one has to avoid this combination at any cost. This is possible if one chooses the following three frames. The first one is the centre-of-mass frame of the incoming partons. The momenta are

\[ p_1 = \frac{1}{2} \sqrt{P_{12} (1,0,\cdots,0,0,1)} , \]
\[
p_2 = \frac{1}{2} \sqrt{P_{12}} (1, 0, \cdots, 0, 0, -1),
\]
\[
-p_3 = \frac{P_{12} - P_{45}}{2 \sqrt{P_{12}}} (1, 0, \cdots, 0, \sin \theta_1, \cos \theta_1),
\]
\[
-p_4 = \frac{P_{12} - P_{35}}{2 \sqrt{P_{12}}} (1, 0, \cdots, \sin \psi \sin \theta_2, \cos \psi \sin \theta_1 + \sin \psi \cos \theta_2 \cos \theta_1,
\]
\[
\cos \psi \cos \theta_1 - \sin \psi \cos \theta_2 \sin \theta_1),
\]
\[
\sin^2 \frac{\psi}{2} = \frac{P_{12} P_{34}}{(P_{12} - P_{35}) (P_{12} - P_{45})},
\]
\[
\sigma_{ab \rightarrow cd \ H} = K_{ab} Z_O^2 \frac{G_H^2}{2 P_{12}} \frac{1}{(4\pi)^n} \frac{1}{\Gamma(n - 3)} \left( \frac{P_{12}}{\mu^2} \right)^{1-n/2} \int_{m^2}^{P_{12}} dP_{35}
\]
\[
\times \int_{P_{12}^2 + m^2 - P_{35}}^{P_{12}^2} dP_{45} \left( \frac{P_{35} P_{45} - P_{12} m^2}{\mu^4} \right)^{n/2 - 2}
\]
\[
\times \left( \frac{P_{12} + m^2 - P_{35} - P_{45}}{\mu^2} \right)^{n/2 - 2} \int_0^\pi d\theta_1 \sin^{n-3} \theta_1
\]
\[
\times \int_0^\pi d\theta_2 \sin^{n-4} \theta_2 |M_{ab \rightarrow cd \ B}|^2.
\]

(3.15)

For the next frame we choose the centre-of-mass frame of the two outgoing particles indicated by the momenta \(p_3\) and \(p_4\).

\[
p_1 = \omega_1 (1, 0, \cdots, 0, 0, 1),
\]
\[
p_2 = (\omega_2, 0, \cdots, 0, |\vec{p}_5| \sin \psi, |\vec{p}_5| \cos \psi - \omega_1),
\]
\[
-p_3 = \frac{1}{2} \sqrt{P_{34}} (1, 0, \cdots, \sin \theta_1 \sin \theta_2, \sin \theta_1 \cos \theta_2, \cos \theta_1),
\]
\[
-p_4 = \frac{1}{2} \sqrt{P_{34}} (1, 0, \cdots, - \sin \theta_1 \sin \theta_2, - \sin \theta_1 \cos \theta_2, - \cos \theta_1),
\]

(3.16)
\[
-p_5 = (\omega_5, 0, \cdots, 0, |p_5\rangle \sin \psi, |p_5\rangle \cos \psi),
\]
\[
\omega_1 = \frac{P_{12} + P_{15} - m^2}{2 \sqrt{P_{34}}}, \quad \omega_2 = \frac{P_{12} + P_{25} - m^2}{2 \sqrt{P_{34}}}
\]
\[
\omega_5 = \frac{P_{15} + P_{25}}{2 \sqrt{P_{34}}},
\]
\[
\cos \psi = \frac{(P_{34} - m^2)(P_{15} - m^2) - P_{12}(P_{25} + m^2)}{(P_{12} + P_{15} - m^2)\sqrt{(P_{15} + P_{25})^2 - 4 m^2 P_{34}}}, \quad (3.17)
\]
\[
\sigma_{ab \rightarrow cd \ H} = K_{ab} Z^2 \frac{G^2_H}{2 P_{12}} \frac{1}{(4\pi)^n} \frac{1}{\Gamma(n - 3)} \left( \frac{P_{12}}{\mu^2} \right)^{1-n/2} \int_{m^2-P_{12}}^0 dP_{25}
\]
\[
\times \int_{m^2-P_{12}-P_{25}}^{m^2+P_{12} m^2/(P_{25}-m^2)} dP_{15} \left( \frac{(P_{15} - m^2)(P_{25} - m^2) - P_{12} m^2}{\mu^4} \right)^{n/2-2}
\]
\[
\times \left( \frac{P_{12} + P_{15} + P_{25} - m^2}{\mu^2} \right)^{n/2-2} \int_0^\pi d\theta_1 \sin^{n-3} \theta_1
\]
\[
\times \int_0^\pi d\theta_2 \sin^{n-4} \theta_2 |M_{ab \rightarrow cd \ B}|^2. \quad (3.18)
\]

For the last frame we choose the centre-of-mass frame of one of the outgoing partons and the (pseudo-)scalar Higgs boson indicated by the momenta \( p_4 \) and \( p_5 \) respectively
\[
p_1 = \omega_1 (1, 0, \cdots, 0, 0, 1),
\]
\[
p_2 = (\omega_2, 0, \cdots, 0, \omega_3 \sin \psi, \omega_3 \cos \psi - \omega_1),
\]
\[
-p_3 = \omega_3 (1, 0, \cdots, 0, \sin \psi, \cos \psi),
\]
\[
-p_4 = \omega_4 (1, 0, \cdots, \sin \theta_1 \sin \theta_2, \sin \theta_1 \cos \theta_2, \cos \theta_1)
\]
\[
-p_5 = (\omega_5, 0, \cdots, -\omega_4 \sin \theta_1 \sin \theta_2, -\omega_4 \sin \theta_1 \cos \theta_2, -\omega_4 \cos \theta_1),
\]
\[ \omega_1 = \frac{P_{12} + P_{13}}{2 \sqrt{P_{45}}}, \quad \omega_2 = \frac{P_{12} + P_{23}}{2 \sqrt{P_{45}}} \]

\[ \omega_3 = \frac{P_{13} + P_{23}}{2 \sqrt{P_{45}}}, \quad \omega_4 = \frac{P_{45} - m^2}{2 \sqrt{P_{45}}} \]

\[ \omega_5 = \frac{P_{45} + m^2}{2 \sqrt{P_{45}}}, \quad \cos \psi = \frac{P_{12} P_{23} - P_{45} P_{13}}{(P_{12} + P_{13})(P_{13} + P_{23})}, \]

(3.19)

\[ \sigma_{ab \rightarrow cd \ H} = K_{ab} Z_O^2 \frac{G_H^2}{2P_{12}} \frac{1}{(4\pi)^n} \frac{1}{\Gamma(n-3)} \left( \frac{P_{12}}{\mu^2} \right)^{1-n/2} \int_{m^2 - P_{12}}^0 dP_{23} \]

\[ \times \int_{m^2 - P_{12} - P_{23}}^0 dP_{13} \left( \frac{P_{13} P_{23}}{\mu^4} \right)^{n/2-2} \left( \frac{P_{12} + P_{13} + P_{23} - m^2}{\mu^2} \right)^{n-3} \]

\[ \times \left( \frac{P_{12} + P_{13} + P_{23}}{\mu^2} \right)^{1-n/2} \int_0^\pi d\theta_1 \sin^{n-3} \theta_1 \int_0^\pi d\theta_2 \sin^{n-4} \theta_2 \]

\[ \times |M_{ab \rightarrow cd \ H}|^2. \]

(3.20)

The integration over the angular independent invariants \( P_{ij} \) can be performed by rescaling them with respect to \( P_{12} \) (see Appendix E in [25]). The integrals are such that they can be performed in an algebraic way by a program based on FORM [29] which has been also used to compute the NNLO coefficient functions of the Drell-Yan process in [24], [25], [26] (for more details see [43]).

The partonic cross sections Eq. (2.12) can be written as follows

\[ \sigma_{ab, H} \left( z, \frac{m^2}{\mu^2} \right) = \frac{\pi G_H^2}{8(N^2 - 1)} \frac{1}{1 + \varepsilon/2} Z_O^2 \hat{W}_{ab, H} \left( z, \frac{m^2}{\mu^2} \right) \]

\[ \sigma_{ab, A} \left( z, \frac{m^2}{\mu^2} \right) = \frac{\pi G_A^2}{8(N^2 - 1)} \frac{1 + \varepsilon}{1 + \varepsilon/2} Z_{11}^2 \hat{W}_{ab, A} \left( z, \frac{m^2}{\mu^2} \right) \]

(3.21)

\[ z^{-1} \hat{W}_{gg, B} = \delta(1 - z) + \hat{a}_s S_\varepsilon \left( \frac{m^2}{\mu^2} \right)^{\varepsilon/2} \left[ \frac{2}{\varepsilon} \left( P_{gg}^{(0)} - 2 \beta_0 \right) + w_{gg, B}^{(1)} + \varepsilon \tilde{w}_{gg, B}^{(1)} \right] \]

\[ \sigma_{gg, B} = -1 \frac{\pi G_A^2}{8(N^2 - 1)} \frac{1 + \varepsilon}{1 + \varepsilon/2} Z_{11}^2 \hat{W}_{gg, B} \left( \frac{m^2}{\mu^2} \right)^{\varepsilon/2} \left[ \frac{2}{\varepsilon} \left( P_{gg}^{(0)} - 2 \beta_0 \right) + w_{gg, B}^{(1)} + \varepsilon \tilde{w}_{gg, B}^{(1)} \right] \]

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\[ + \hat{a}_s^2 S_\varepsilon \left( \frac{m^2}{\mu^2} \right)^\varepsilon \left[ \frac{1}{\varepsilon} \left( 2 P_{qq}^{(0)} \otimes P_{gg}^{(0)} + P_{qq}^{(0)} \otimes P_{qq}^{(0)} - 10 \beta_0 P_{gg}^{(0)} \right) \right. \\
\left. + 12 \beta_0^2 \varepsilon + \frac{1}{\varepsilon} \left( P_{gg}^{(1)} - 6 \beta_0 w_{gg,B}^{(1)} + 2 P_{gg}^{(0)} \otimes w_{gg,B}^{(1)} \right) \right] - 6 \beta_0 w_{gg,B}^{(1)} + 2 P_{gg}^{(0)} \otimes \bar{w}_{gg,B}^{(1)} \\
\left. + 2 \bar{w}_{gg,B}^{(1)} \otimes P_{gg}^{(0)} + w_{gg,B}^{(2)} \right] \\
\text{with } z_H = -4 \beta_1, \quad z_A = -2 \beta_1, \quad (3.22) \]

The coefficients \( z_B \), multiplying the single pole terms in the equations above, originate from the operators \( O \) in Eq. (2.1) and \( O_1 \) in Eq. (2.2) and they are removed via the operator renormalization constants \( Z_O \) in Eq. (2.21) and \( Z_{11} \) in Eq. (2.24).

\[
z^{-1} \hat{W}_{gg,B} = \hat{a}_s S_\varepsilon \left( \frac{m^2}{\mu^2} \right)^{\varepsilon/2} \left[ \frac{2}{\varepsilon} \left( P_{gg}^{(0)} + w_{gg,B}^{(1)} \right) \right. + \left. \varepsilon \bar{w}_{gg,B}^{(1)} \right] + \hat{a}_s^2 S_\varepsilon \left( \frac{m^2}{\mu^2} \right)^\varepsilon \\
\times \left[ \frac{1}{\varepsilon} \left( -\frac{3}{2} P_{gg}^{(0)} \otimes P_{gg}^{(0)} + \frac{1}{2} P_{qq}^{(0)} \otimes P_{qq}^{(0)} - 5 \beta_0 P_{gg}^{(0)} \right) \right. \\
\left. + \frac{1}{\varepsilon} \left( \frac{1}{2} P_{qq}^{(1)} - 6 \beta_0 w_{gg,B}^{(1)} + \frac{1}{2} P_{qq}^{(0)} \otimes w_{qq,B}^{(1)} + P_{qq}^{(0)} \otimes w_{qq,B}^{(1)} \right) \right] \\
\left. + \frac{1}{\varepsilon} \left( P_{gg}^{(0)} + P_{qq}^{(0)} \otimes w_{qq,B}^{(1)} \right) - 6 \beta_0 \bar{w}_{gg,B}^{(1)} + \frac{1}{2} P_{qq}^{(0)} \otimes \bar{w}_{qq,B}^{(1)} \\
\left. + P_{qq}^{(0)} \otimes \bar{w}_{qq,B}^{(1)} + (P_{gg}^{(0)} + P_{qq}^{(0)}) \otimes \bar{w}_{gg,B}^{(1)} + w_{gg,B}^{(2)} \right], \quad (3.23) \]

\[
z^{-1} \hat{W}_{qq,B} = \hat{a}_s S_\varepsilon \left( \frac{m^2}{\mu^2} \right)^{\varepsilon/2} \left[ w_{qq,B}^{(1)} + \varepsilon \bar{w}_{qq,B}^{(1)} \right] + \hat{a}_s^2 S_\varepsilon \left( \frac{m^2}{\mu^2} \right)^\varepsilon \left[ \right. \\
\left. \frac{1}{\varepsilon} P_{gg}^{(0)} \otimes P_{gg}^{(0)} + \frac{1}{\varepsilon} \left( -6 \beta_0 w_{gg}^{(1)} + 2 P_{gg}^{(0)} \otimes w_{gg,B}^{(1)} \right) \right. \\
\left. + \frac{1}{\varepsilon} \left( -6 \beta_0 w_{gg}^{(1)} + 2 P_{gg}^{(0)} \otimes w_{gg,B}^{(1)} \right) \right], \quad (3.23) \]
\[ +2 P_{qq}^{(0)} \otimes w_{qq,B}^{(1)} + 6 \beta_0 w_{qq,B}^{(1)} + 2 P_{gq}^{(0)} \otimes w_{gq,B}^{(1)} \]

\[ +2 P_{qq}^{(0)} \otimes w_{qq,B}^{(1)} + w_{qg,B}^{(2)} \], \quad (3.24) \]

\[ z^{-1} \hat{W}_{q_1q_2,B} = z^{-1} \hat{W}_{q_1q_2,B}, \]

\[ = \hat{a}_s^2 S_\varepsilon \left( \frac{m_t^2}{\mu^2} \right)^\varepsilon \left[ \frac{1}{\varepsilon^2} P_{gq}^{(0)} \otimes P_{gq}^{(0)} + \frac{2}{\varepsilon} P_{gq}^{(0)} \otimes w_{gq,B}^{(1)} \right. \]

\[ + 2 P_{gq}^{(0)} \otimes w_{gq,B}^{(1)} + w_{q_1q_2,B}^{(2)} \], \quad (3.25) \]

where \( \otimes \) denotes the convolution symbol defined by

\[ f \otimes g(z) = \int_0^1 dz_1 \int_0^1 dz_2 \delta(z - z_1 z_2) f(z_1) g(z_2). \quad (3.26) \]

For identical quark-quark scattering we have the same formula as in Eq. (3.25) except that \( w_{qq,B}^{(2)} \neq w_{q_1q_2,B}^{(2)} \). The expressions above follow from the renormalization group equations. They are constructed in such a way that become finite after coupling constant renormalization, operator renormalization and mass factorization are carried out. The splitting functions \( P_{ab}(z) \) and the coefficients \( w_{ab}(z) \) with \( z = m^2/s \) also occur in the coefficient functions given below except for the NLO terms \( \bar{w}_{ab}^{(1)}(z) \), which are proportional to \( \varepsilon \). They are given by

\[ \bar{w}_{gg,H}^{(1)} = C_A \left[ 8 D_2(z) - 6 \zeta(2) D_0(z) + \left( \frac{8}{z} - 16 + 8 z - 8 z^2 \right) \left( \ln^2(1 - z) \right. \right. \]

\[ - \frac{3}{4} \zeta(2) \left. + \left( \frac{8}{1 - z} + \frac{8}{z} - 16 + 8 z - 8 z^2 \right) \left( \frac{1}{4} \ln^2 z \right. \right. \]

\[ - \ln z \ln(1 - z) \right. - \frac{22}{3} \left( \frac{(1 - z)^3}{z} \right) \left( \ln(1 - z) - \frac{1}{2} \ln z \right) - \frac{55}{3} + \frac{67}{9z} \]

\[ + \frac{55}{3} z - \frac{67}{9} z^2 + 2 \delta(1 - z) \]. \]
\[ \bar{w}_{gq,H}^{(1)} = C_F \left[ \left( \frac{4}{z^2} - 4 + 2 z \right) \left( \ln^2(1 - z) - \ln z \ln(1 - z) + \frac{1}{4} \ln^2 z \right) \right. \\
\left. - \frac{3}{4} \zeta(2) + \left( - \frac{3}{z} + 6 - z \right) \left( \ln(1 - z) - \frac{1}{2} \ln z \right) + \frac{7}{2z} - 5 + \frac{3}{2} \right], \]

\[ \bar{w}_{qg,H}^{(1)} = C_F^2 \left[ \frac{8}{3} \frac{(1 - z)^3}{z} \left( \ln(1 - z) - \frac{1}{2} \ln z + \frac{1}{6} \right) \right], \] (3.27)

where \( D_i \) denotes the distribution

\[ D_i = \left( \frac{\ln^i(1 - z)}{1 - z} \right)_+. \] (3.28)

The difference between scalar and pseudo-scalar Higgs production is given by

\[ \bar{w}_{gq,A-H}^{(1)} = C_A \left[ - 8 (1 - z) - 14 \delta(1 - z) \right], \]

\[ \bar{w}_{gq,A-H}^{(1)} = 0, \]

\[ \bar{w}_{qg,A-H}^{(1)} = 0. \] (3.29)

To render the partonic cross sections finite one has first to perform coupling constant renormalization. This is done by replacing the bare coupling constant by the renormalized one see Eq. (2.25). Then one has to carry out operator renormalization which is achieved by multiplying the expressions in Eqs. (3.22)-(3.25) with the operator renormalization constants in Eqs. (2.21), (2.24). The remaining divergences, which are of collinear origin, are removed by mass factorization

\[ z^{-1} Z_\Omega^2 \hat{W}_{ab,B} \left( \frac{1}{\varepsilon^2}, \frac{m^2}{\mu^2} \right) = \sum_{c,d=q,q,g} \Gamma_{ca} \left( \frac{1}{\varepsilon^2} \right) \otimes \Gamma_{db} \left( \frac{1}{\varepsilon^2} \right) \otimes z^{-1} \Delta_{cd,B} \left( \frac{m^2}{\mu^2} \right), \] (3.30)

where \( \Gamma_{ca}(z) \) denote the kernels containing the splitting functions which multiply the collinear divergences represented by the pole terms \( 1/\varepsilon^k \). Note that
in the case of pseudo-scalar Higgs production the $Z_O$ (see Eq. (2.21)) in the above equation has to be replaced by $Z_{11}$ in Eq. (2.24). For the four different subprocesses the mass factorization relations in Eq. (3.30) become equal to

\[ z^{-1} Z_O^2 \hat{W}_{gg,B} = \Gamma_{gg} \otimes \Gamma_{gg} \otimes z^{-1} \Delta_{gg,B} + 4 \Gamma_{gg} \otimes \Gamma_{gg} \otimes z^{-1} \Delta_{gg,B} , \]

(3.31)

\[ z^{-1} Z_O^2 \hat{W}_{qq,B} = \Gamma_{qq} \otimes \Gamma_{gg} \otimes z^{-1} \Delta_{gg,B} + \Gamma_{qq} \otimes \Gamma_{gg} \otimes z^{-1} \Delta_{gg,B} + 2 \Gamma_{gg} \otimes \Gamma_{qq} \otimes z^{-1} \Delta_{gg,B} , \]

(3.32)

\[ z^{-1} Z_O^2 \hat{W}_{qg,B} = \Gamma_{qq} \otimes \Gamma_{gg} \otimes z^{-1} \Delta_{gg,B} + 2 \Gamma_{qq} \otimes \Gamma_{gg} \otimes z^{-1} \Delta_{gg,B} + \Gamma_{qq} \otimes \Gamma_{qq} \otimes z^{-1} \Delta_{qg,B} , \]

(3.33)

\[ z^{-1} Z_O^2 \hat{W}_{q1q2,B} = \Gamma_{qq} \otimes \Gamma_{gg} \otimes z^{-1} \Delta_{gg,B} + 2 \Gamma_{qq} \otimes \Gamma_{gg} \otimes z^{-1} \Delta_{gg,B} + \Gamma_{qq} \otimes \Gamma_{qq} \otimes z^{-1} \Delta_{q1q2,B} , \]

(3.34)

where we have identified

\[ \Gamma_{qu} = \Gamma_{qu} , \quad \Gamma_{qu} = \Gamma_{qu} , \]

\[ \hat{W}_{qq,B} = \hat{W}_{qg,B} = \hat{W}_{gq,B} = \hat{W}_{qg,B} , \]

\[ \Delta_{gg,B} = \Delta_{gg,B} = \Delta_{gg,B} = \Delta_{gg,B} . \]

(3.35)

Since we need the finite expressions up to order $\alpha_s^2$ it is sufficient to expand the kernels $\Gamma_{ca}$ up to the following order in the renormalized coupling constant

\[ \Gamma_{qq} = \delta(1-z) + a_s(\mu^2) S_\epsilon \left[ \frac{1}{\epsilon} P_{qq}^{(0)} \right] , \]

(3.36)

\[ \Gamma_{qq} = a_s(\mu^2) S_\epsilon \left[ \frac{1}{2\epsilon} P_{qq}^{(0)} \right] , \]

(3.37)

\[ \Gamma_{qq}^{PS} = a_s^2(\mu^2) S_\epsilon^2 \left[ \frac{1}{4\epsilon^2} P_{qq}^{(0)} \otimes P_{qq}^{(0)} + \frac{1}{4\epsilon^2} P_{qq}^{(1),PS} \right] , \]

(3.38)
\[ \Gamma_{qq} = a_s(\mu^2) S_\varepsilon \left[ \frac{1}{\varepsilon} P_{qq}^{(0)} \right] + a_s^2(\mu^2) S_\varepsilon^2 \left[ \frac{1}{\varepsilon^2} \left( \frac{1}{2} P_{qq}^{(0)} \otimes P_{qq}^{(0)} \right) \right. \\
+ \frac{1}{2} P_{qq}^{(0)} \otimes P_{qq}^{(0)} + \beta_0 P_{qq}^{(0)} \left] \right] \frac{1}{2} P_{qq}^{(1)} \right] \]  

(3.39)

\[ \Gamma_{gg} = \delta(1-z) + a_s(\mu^2) S_\varepsilon \left[ \frac{1}{\varepsilon} P_{gg}^{(0)} \right] + a_s^2(\mu^2) S_\varepsilon^2 \left[ \frac{1}{\varepsilon^2} \left( \frac{1}{2} P_{gg}^{(0)} \otimes P_{gg}^{(0)} \right) \right. \\
+ \frac{1}{2} P_{gg}^{(0)} \otimes P_{gg}^{(0)} + \beta_0 P_{gg}^{(0)} \left] \right] \frac{1}{2} P_{gg}^{(1)} \right] \]  

(3.40)

(PS denotes pure-singlet). After renormalization and mass factorization the coefficient functions have the following algebraic form

\[ z^{-1} \Delta_{gg,B} = \delta(1-z) + a_s(\mu^2) \left[ P_{gg}^{(0)} \ln \frac{m^2}{\mu^2} + w_{gg,1B}^{(1)} \right] + a_s^2(\mu^2) \left[ \right. \\
+ \left\{ \frac{1}{2} P_{gg}^{(0)} \otimes P_{gg}^{(0)} + \frac{1}{4} P_{gg}^{(0)} \otimes P_{gg}^{(0)} - \beta_0 P_{gg}^{(0)} + 3 \beta_0^2 \right] \ln 2 \frac{m^2}{\mu^2} \\
+ \left( P_{gg}^{(1)} + 3 \beta_0 w_{gg,B}^{(1)} \right) + P_{gg}^{(0)} \otimes w_{gg,B}^{(1)} + w_{gg,B}^{(1)} \otimes P_{gg}^{(0)} \\
+ z_B \left] \right] \ln \frac{m^2}{\mu^2} + w_{gg,1B}^{(2)} \left] \right] \]  

(3.41)

\[ z^{-1} \Delta_{qq,B} = a_s(\mu^2) \left[ \frac{1}{2} P_{qq}^{(0)} \ln \frac{m^2}{\mu^2} + w_{qq,1B}^{(1)} \right] + a_s^2(\mu^2) \left[ \right. \\
+ \left\{ \frac{3}{8} P_{gg}^{(0)} \otimes P_{gg}^{(0)} + \frac{5}{4} \beta_0 P_{gg}^{(0)} \right] \ln 2 \frac{m^2}{\mu^2} + \left( \frac{1}{2} P_{qq}^{(1)} - 3 \beta_0 w_{qq,B}^{(1)} \\
+ \frac{1}{4} P_{qq}^{(0)} \otimes w_{qq,B}^{(1)} + \frac{1}{2} P_{qq}^{(0)} \otimes w_{qq,B}^{(1)} \\
+ \frac{1}{2} \left( P_{qq}^{(0)} + P_{qq}^{(0)} \right) \otimes w_{qq,B}^{(1)} \right] \ln \frac{m^2}{\mu^2} + w_{qq,B}^{(2)} \left] \right] \]  

(3.42)

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\[ z^{-1} \Delta_{qq,B} = a_s(\mu^2) \left[ w_{qq,B}^{(1)} \right] + a_s^2(\mu^2) \left[ \frac{1}{4} P_{gg}^{(0)} \otimes P_{gg}^{(0)} \ln^2 \frac{m^2}{\mu^2} + \left\{ -3 \beta_0 w_{qq,B}^{(1)} + P_{gg}^{(0)} \otimes w_{qq,B}^{(1)} + P_{qq}^{(0)} \otimes w_{qq,B}^{(1)} \right\} \ln \frac{m^2}{\mu^2} + w_{qq,B}^{(2)} \right], \] 

\[ z^{-1} \Delta_{q_1\bar{q}_2,B} = z^{-1} \Delta_{q_1\bar{q}_2,B}, \]

\[ = a_s^2(\mu^2) \left[ \frac{1}{4} P_{gg}^{(0)} \otimes P_{gg}^{(0)} \ln^2 \frac{m^2}{\mu^2} + P_{gg}^{(0)} \otimes w_{qq,B}^{(1)} \ln \frac{m^2}{\mu^2} + w_{q_1\bar{q}_2,B}^{(2)} \right]. \] 

In Appendix A and B we give the explicit expressions for the coefficient functions so that one can determine the coefficients \( P_{ab}^{(k)} \) and \( w_{ab,B}^{(k)} \). Our results which are expressed in the colour factors \( C_A, C_F \) and \( T_f \) agree with those published in [20], [19], [21] for \( N = 3 \). In the representation of the coefficient functions above we have put the renormalization scale \( \mu_r \) equal to the mass factorization scale \( \mu \). If one wants to distinguish between both scales one can make the simple substitution

\[ \alpha_s(\mu^2) = \alpha_s(\mu_r^2) \left[ 1 + \frac{\alpha_s(\mu_r^2)}{4\pi} \beta_0 \frac{\mu_r^2}{\mu^2} \right]. \]
4 Total cross sections for the process 
\[ p + p \to H +' X' \]

In this section we will present total cross sections (see Eq. (2.1)) for Higgs-boson production in proton-proton collisions at the LHC and in proton-anti-proton collisions at the TEVATRON. The cross section can be written in two ways (for the definitions see [16], [44])

\[ \sigma_{\text{tot}} = \frac{\pi G_B^2}{8(N^2 - 1)} \sum_{a,b=q,\bar{q},g} \int_x^1 dy \tilde{\Phi}_{ab}(y, \mu^2) \Delta_{ab}\left(\frac{x}{y}, \frac{m^2}{\mu^2}\right) , \quad (4.1) \]

where \( x = m^2/S \) and \( \tilde{\Phi}_{ab} \) is the momentum fraction luminosity defined by

\[ \tilde{\Phi}_{ab}(y, \mu^2) = \int_y^1 \frac{du}{u} f_a(u, \mu^2) f_b\left(\frac{y}{u}, \frac{\mu^2}{\mu^2}\right) . \quad (4.2) \]

However Eq. (4.1) can be also cast in the form

\[ \sigma_{\text{tot}} = \frac{\pi G_B^2}{8(N^2 - 1)} \sum_{a,b=q,\bar{q},g} x \int_x^1 dy \Phi_{ab}(y, \mu^2) \left(\frac{y}{x} \Delta_{ab}\left(\frac{x}{y}, \frac{m^2}{\mu^2}\right)\right) , \]

with \( \Phi_{ab}(y, \mu^2) = y^{-1} \tilde{\Phi}_{ab}(y, \mu^2) , \quad (4.3) \]

where \( \Phi_{ab} \) denotes the parton luminosity. For the exact cross section the expressions in Eqs. (4.1) and (4.3) lead to the same results but if one wants to study the various approximations in the literature like the soft-plus-virtual (SV) gluon approximation then it makes a difference if \( \Delta_{ab}(z) \) or \( z^{-1} \Delta_{ab}(z) \) will be expanded around \( z = 1 \). This difference will show up in the less singular terms as we will show at the end of this section. In the subsequent part of this section we study the dependence of the cross section on the input parameters like the QCD scale \( \Lambda \), the renormalization/factorization scale \( \mu \), the mass \( m \) of the Higgs boson and the input parton densities. We are also interested in the theoretical uncertainty of the NNLO cross section. One uncertainty is due to the corrections which show up beyond NNLO. They can be guessed from the rate of convergence represented by the \( K \)-factor and the variation with respect to the scale \( \mu \). We will show that the coefficient functions \( \Delta_{ab}(z) \) in Eq. (4.1) are dominated by the logarithmic terms of the
type $\ln^i(1-z)$. A determination of these logarithms is of importance to estimate the higher order corrections. Another uncertainty is caused by the input parton densities. Parton density sets differ from each other in several aspects. They are based on fitting data from different experiments. Furthermore the shapes of the densities at the specific input scale chosen for $\mu$ differ from each other. Finally these sets are based on different theoretical assumptions, for instance in the treatment of heavy flavours. In some sets the heavy flavour is treated as a massless particle which is described by a parton density similar to the treatment of the light quarks. In other sets the mass of the heavy flavour is considered to be on the same order of magnitude as the other large scales. Then the heavy flavour production is described by exact perturbation theory at a certain order so that threshold effects are fully taken into account. Although the coefficient functions in the effective Lagrangian approach are computed exactly in NNLO this is not the case for the parton densities because the exact three-loop splitting functions (anomalous dimensions) are not known yet. Until now only a finite number of moments are available (see [45]) which are used in [46] together with other constraints to approximate the splitting functions. These approximations are very reliable as long as $y > 10^{-4}$ in Eq. (4.1). The approximated splitting functions were used in [47] to obtain NNLO parton density sets. One of them called MRST02 (see Table 1) will be used in our paper. For the approximations in LO and NLO we use the sets in [47] and [48] respectively. For the LO, NLO and NNLO plots we employ the one-, two-, and three-loop asymptotic forms of the running coupling constant as given in Eq. (3) of [49]. In order to make a comparison with other parametrizations for the parton densities we also choose the sets made by CTEQ [50] and GRV [51]. However these sets do not contain NNLO versions so that we can present the NLO results only. All sets are listed in Table 1 together with the corresponding QCD scale $\Lambda_{n_f}$ determined for $n_f = 5$. The same number of flavours is also chosen for the coefficient functions. Notice that the GRV sets do not contain densities for charm and bottom quarks. For simplicity the factorization scale is set equal to the renormalization scale $\mu_r$. For our plots we take $\mu = m$ and use the MRST sets for the LO, NLO and NNLO computations (see Table 1) unless mentioned otherwise. Here we want to emphasize that the magnitude

\[\text{\footnote{Notice that according to the prescription of the CTEQ group [50] also the LO corrected quantities have to be computed with the NLO running coupling constant.}}\]
of the $\sigma_{\text{tot}}$ is extremely sensitive to the renormalization scale because the effective coupling constant $G_B \sim \alpha_s(\mu_r)$, which implies that $\sigma^{\text{LO}} \sim \alpha_s^2$. The sensitivity to the factorization scale is much smaller.

For the computation of the effective coupling constant $G_B$ in Eq. (2.3) we choose the top quark mass $m_t = 173.4$ GeV/$c^2$ and the Fermi constant $G_F = 1.16639$ GeV$^{-2} = 4541.68$ pb. In this paper we will only study scalar Higgs production and omit pseudo-scalar Higgs production. The cross sections of the latter are about 9/4 $\cot^2\beta$ larger than those for the standard model Higgs so that all our conclusions also apply to pseudo-scalar Higgs production. We give results for both proton-proton collisions at the centre-of-mass energy $\sqrt{S} = 14$ TeV and proton-anti-proton collisions at $\sqrt{S} = 2$ TeV.

In Fig. 1 we have plotted the contributions coming from the various subprocesses up to NLO. From this figure we infer that $\sigma_{\text{tot}}$ is completely dominated by the $gg$ reaction whereas the other contributions are down by two ($gq + g\bar{q}$-subprocess) or even by three ($q\bar{q}$-subprocess) orders of magnitude. This picture is different from the behaviour of the transverse momentum $p_T$- and rapidity $y$-distributions in [14] where the $qq + g\bar{q}$-reaction amounted to about 1/3 of that for the $gg$-reaction when $p_T > 30$ GeV/$c$. This is because $\sigma_{\text{tot}}$ receives all its contribution from the small $p_T$-region ($p_T < 30$ GeV/$c$) where the $gg$ subprocess overwhelms all other reactions. This picture is not changed when we study the cross section in NNLO (see Fig. 2) where new subprocesses contribute given by the $qq$, $q\bar{q}$ and $\bar{q}\bar{q}$ reactions. Notice that the incoming (anti-) quarks can be identical and non-identical. The relative order of magnitude is the same as in NLO and the new reactions are down

| Parton Density Set | $\Lambda^{\text{LO}}$ | $\alpha_s^{\text{LO}}(M_Z)$ |
|-------------------|------------------------|---------------------------|
| MRST02 (LO, lo2002.dat) | 167 MeV | 0.130 |
| MRST01 (NLO, alf119.dat) | 239 MeV | 0.119 |
| MRST02 (NNLO, vnvalf1155.dat) | 176 MeV | 0.115 |
| CTEQ6 (LO, cteq6l.tbl) | 226 MeV | 0.118 |
| CTEQ6 (NLO, cteq6m.tbl) | 226 MeV | 0.118 |
| GRV98 (LO, grv98lo.grid) | 132 MeV | 0.125 |
| GRV98 (NLO, grvnlm.grid) | 174 MeV | 0.114 |

Table 1: Various parton density sets with their values for the QCD scale $\Lambda$ and $\alpha_s(M_Z)$.
by three orders of magnitude. In Fig. 3 we have plotted $\sigma_{\text{tot}}$ in LO, NLO and NNLO as a function of $m$. The cross section decreases when $m$ increases due to a reduction in the available phase space. In NNLO the curve shows a steeper decrease than in NLO and LO. Furthermore the growth of the cross section slows down while going to higher orders. To show more clearly how the cross sections change with respect to $m$ we have put them in Table 2. We have also shown them for the TEVATRON (proton-anti-proton collisions at $\sqrt{S} = 2$ TeV) in Table 3 where they are appreciably smaller than in the case of the LHC.

The NNLO cross sections in Tables 2, 3 have some theoretical errors which can be estimated in various ways. First one can study their variation with respect to the scale $\mu$. This can be achieved by computing the ratio

$$N \left( \frac{\mu}{\mu_0} \right) = \frac{\sigma_{\text{tot}}(\mu)}{\sigma_{\text{tot}}(\mu_0)},$$

(4.4)

where $\mu_0 = m$ and $\mu$ is varied in the range $0.1 < \mu/\mu_0 < 10$. In Fig. 4 a plot of $N$ is shown for $m = 100$ GeV/$c^2$. Here one observes a clear improvement while going from LO to NNLO. In particular the curve for NNLO is flatter than that for NLO. The improvement becomes even better when we choose a larger Higgs mass see e.g. Fig. 5 ($m = 200$ GeV/$c^2$). However for still larger masses there is hardly any improvement anymore (see Fig. 6 where $m = 300$ GeV/$c^2$). This is in contrast to the NNLO corrected cross section for vector boson production in [26] which is insensitive to the choice of scale chosen in the range above.

A second way to study the reliability of our prediction is to study the rate of convergence of the perturbation series, which is represented by the $K$-factor. We choose the following definitions

$$K^{\text{NLO}} = \frac{\sigma_{\text{tot}}^{\text{NLO}}}{\sigma_{\text{tot}}^{\text{LO}}}, \quad K^{\text{NNLO}} = \frac{\sigma_{\text{tot}}^{\text{NNLO}}}{\sigma_{\text{tot}}^{\text{NLO}}},$$

(4.5)

Notice that in the above expression the definition of $K^{\text{NNLO}}$ differs from the usual one which is given by $K^{\text{NNLO}} = \sigma_{\text{tot}}^{\text{NNLO}}/\sigma_{\text{tot}}^{\text{LO}}$. We have chosen this definition because the rate of convergence can be shown in a better way. In Fig. 7 one observes that both $K$-factors vary slowly as $m$ increases. Moreover there is considerable improvement in the rate of convergence if one goes from NLO to NNLO. At $m = 100$ GeV/$c^2$ we have $K^{\text{NLO}} \sim 1.74$.
| mass (GeV) | LO (pb) | NLO (pb) | NNLO (pb) |
|-----------|---------|----------|-----------|
| 100       | 30.35   | 52.75    | 60.84     |
| 110       | 25.53   | 44.75    | 51.80     |
| 120       | 21.77   | 38.43    | 44.62     |
| 130       | 18.77   | 33.37    | 38.85     |
| 140       | 16.36   | 29.27    | 34.17     |
| 150       | 14.38   | 25.88    | 30.29     |
| 160       | 12.74   | 23.06    | 27.07     |
| 170       | 11.37   | 20.69    | 24.36     |
| 180       | 10.23   | 18.69    | 22.09     |
| 190       | 9.254   | 17.00    | 20.10     |
| 200       | 8.425   | 15.53    | 18.43     |
| 210       | 7.714   | 14.28    | 16.97     |
| 220       | 7.102   | 13.20    | 15.71     |
| 230       | 6.573   | 12.26    | 14.62     |
| 240       | 6.115   | 11.45    | 13.69     |
| 250       | 5.718   | 10.74    | 12.86     |
| 260       | 5.375   | 10.13    | 12.16     |
| 270       | 5.080   | 9.604    | 11.55     |
| 280       | 4.827   | 9.152    | 11.03     |
| 290       | 4.615   | 8.772    | 10.59     |
| 300       | 4.441   | 8.468    | 10.24     |

Table 2: Total cross sections in pb for Higgs masses between 100 GeV and 300 GeV at the LHC. The LO, NLO and NNLO results are generated with the MRST parton densities listed in Table 1.
| mass | LO   | NLO  | NNLO |
|------|------|------|------|
| 100  | 0.616| 1.351| 1.781|
| 110  | 0.461| 1.025| 1.363|
| 120  | 0.353| 0.791| 1.060|
| 130  | 0.275| 0.619| 0.835|
| 140  | 0.216| 0.491| 0.666|
| 150  | 0.172| 0.393| 0.538|
| 160  | 0.139| 0.318| 0.438|
| 170  | 0.112| 0.259| 0.359|
| 180  | 0.092| 0.213| 0.293|
| 190  | 0.076| 0.177| 0.248|
| 200  | 0.063| 0.147| 0.207|
| 210  | 0.053| 0.124| 0.176|
| 220  | 0.044| 0.105| 0.149|
| 230  | 0.037| 0.089| 0.127|
| 240  | 0.031| 0.076| 0.109|
| 250  | 0.027| 0.065| 0.095|
| 260  | 0.023| 0.056| 0.082|
| 270  | 0.020| 0.049| 0.072|
| 280  | 0.018| 0.043| 0.063|
| 290  | 0.015| 0.038| 0.055|
| 300  | 0.013| 0.033| 0.049|

Table 3: Total cross sections in pb for Higgs masses between 100 GeV and 300 GeV at the TEVATRON. The LO, NLO and NNLO results are generated with the MRST parton densities listed in Table 1.
whereas \( K^{\text{NNLO}} \sim 1.15 \). Still the corrections for Higgs boson production are larger than those obtained from \( Z \)-boson production at the LHC where one gets \( K^{\text{NLO}} \sim 1.22 \) and \( K^{\text{NNLO}} \sim 0.95 \) (see [26]). In Fig. 8 we compare the \( K \)-factor in NLO obtained from the MRST with the results computed by using the parton density sets given by the CTEQ [50] and GRV [51] collaborations (see Table 1). From this figure we infer that the \( K \)-factor computed using GRV98 and CTEQ6 is larger than the one obtained from MRST01(NLO) at \( m = 100 \text{ GeV}/c^2 \). However at large \( m \) there is a cross over and we observe \( K^{\text{CTEQ}} > K^{\text{MRST}} > K^{\text{GRV98}} \). The large result shown by the CTEQ6-set is due to the behaviour of their gluon density in LO. In NLO this density behaves in the same way as those in the MRST and GRV sets. We also investigated older sets provided by the CTEQ-collaboration, namely CTEQ5 and CTEQ6. It turns out that all these LO CTEQ sets lead to the same large value for \( K^{\text{NLO}} \) as shown by Fig. 8. A study of \( K^{\text{NLO}} \), using the most recent parton densities, was also made in [44] but now for the TEVATRON.

After the \( K \)-factor we now make a comparison between \( \sigma_{\text{tot}} \) computed from the various sets which leads to the third uncertainty in the theoretical prediction. For that purpose we plot the ratios

\[
R^{\text{CTEQ}} = \frac{\sigma_{\text{tot}}^{\text{CTEQ}}}{\sigma_{\text{tot}}^{\text{MRST}}} \quad \text{and} \quad R^{\text{GRV}} = \frac{\sigma_{\text{tot}}^{\text{GRV}}}{\sigma_{\text{tot}}^{\text{MRST}}}, \tag{4.6}
\]

which are shown in NLO in Fig. 9. In this figure we see that the CTEQ6 set leads to the same cross section as the one obtained by MRST01. The cross section given by the GRV98 is 20\% above the MRST01 result and the ratio decreases with increasing \( m \) to 5\% above at \( m = 300 \text{ GeV}/c^2 \). The reason that the CTEQ6 result is closer to the MRST sets can be attributed to the fact that their parton density sets are more recent and are constructed to fit the same experiments. Notice that a study of the ratios in Eq. (4.6) was also made for the TEVATRON in [44] using the most recent parton densities.

The factorization scale dependence in Figs. 4-6 and the \( K \)-factor in Fig. 7 of the NNLO cross section can be used to give an error estimate of the latter for the LHC. Since \((\sigma^{\text{NNLO}} - \sigma^{\text{NLO}})/\sigma^{\text{NLO}} = K^{\text{NNLO}} - 1\) the error on \( \sigma^{\text{NNLO}} \) is equal to 15\%, 19\% and 21\% for \( m = 100 \text{ GeV}/c^2 \), \( m = 200 \text{ GeV}/c^2 \) and \( m = 300 \text{ GeV}/c^2 \) respectively (see Fig. 7). This is corroborated by the scale variation if we choose the range \( 0.25 < \mu/\mu_0 < 4.0 \). For \( \mu/\mu_0 = 0.25 \) the errors above become 18\%, 15\% and 14\% respectively. In the case \( \mu/\mu_0 = 0.25 \)
they are given by 16% for all \( m \) given above. Notice that the error due the
\( K \)-factor increases when \( m \) gets larger whereas the opposite happens if the
error estimate is derived from the mass factorization/renormalisation scale.
On top of this we have to add the error due to the chosen parton density. For
this we choose the NLO plots in Fig. 9 where the cross section predicted by
CTEQ6 hardly differs from the one given by MRST01 but the GRV98 plot
deviates from the latter by 20% for small \( m \) to 5% for large \( m \).

An important feature of the Higgs cross section, which is very often em-
phasised in the literature, is that the integral in Eq. (4.1) is dominated by
\( y \sim x = m^2/S \). Since \( x \) is small the \( gg \) part of \( \sigma_{\text{tot}} \) dominates the contribu-
tions coming from the other partonic subprocesses because of the steep rise
of the gluon-gluon luminosity in Eq. (4.2) at small \( y \). The importance of the
small \( y \)-region is revealed when we impose an upper cut on the integral in
Eq. (4.1). Hence we compute

\[
\sigma_{\text{tot}}(x_{\text{max}}) = \frac{\pi G_B^2}{8(N^2 - 1)} \left[ \Phi_{gg}(x, \mu^2) + \sum_{a,b=q,g} \sum_{i=1}^{\infty} a_s^i(\mu^2) \int_x^{x_{\text{max}}} dy \tilde{\Phi}_{ab}(y, \mu^2) \times \Delta_{\text{ab}}^{(i)} \left( \frac{x}{y}, \frac{m^2}{\mu^2} \right) \right],
\]

and plot the ratio

\[
R(x_{\text{max}}) = \frac{\sigma_{\text{tot}}(x_{\text{max}})}{\sigma_{\text{tot}}(1)} , \quad \sigma_{\text{tot}}(1) = \sigma_{\text{tot}}^{\text{EXACT}},
\]

which is shown for NLO and NNLO in Fig. 10a for the choice \( x_{\text{max}} = 5 \ x \). The figure reveals that more than 95% of the cross section comes
from the integration region \( x \leq y \leq 5 \ x \) where \( x < 5.1 \times 10^{-5} \). When \( m \)
gets larger \( R(x_{\text{max}}) \) will increase because the available phase space for Higgs
boson production decreases. The latter also happens when the centre-of-mass
energy decreases like in the case for the TEVATRON where \( \sqrt{S} = 2 \) TeV
(Here \( x < 2.5 \times 10^{-3} \) see Fig. 10b. (Note Figs. 10a and 10b have different
scales.) In the latter case more than 99% of the cross section receives its
contribution from \( x \leq y \leq 5 \ x \) which becomes 100% when \( m = 300 \) GeV/c².

Finally we study the validity of the soft-plus-virtual gluon approximation
including subleading terms. The result of this approximation depends on the
definitions for the total cross section given in Eq. (4.1) and Eq. (4.3) as has
been pointed out in [44]. If we follow the definition in [8], [17], where one choses the expression in Eq. (4.1), the sof-plus-virtual coefficient function becomes

\[
\Delta_{ab}^{SV}(z, \frac{m^2}{\mu^2}) = \delta(1-z) + \sum_{i=1}^{\infty} a_i^s(\mu^2) \left[ \sum_{j=0}^{2i-1} a_{i,j} D_j + b_i \delta(1-z) \right].
\] (4.9)

where the distributions \( D_i(z) \) are defined in Eq. (3.28). Furthermore the logarithmic terms \( \ln^j(1-z) \) including the constants \( (j=0) \) are important. The latter are denoted by

\[
\Delta_{ab}^L(\frac{m^2}{\mu^2}) = \sum_{i=1}^{\infty} a_i^s(\mu^2) \sum_{j=0}^{2i-1} (a_{i,j} + c_{i,j}) \ln^j(1-z).
\] (4.10)

Notice that the coefficients \( a_{i,j}, b_i, c_{i,j} \) also contain terms in \( \ln m^2/\mu^2 \). The soft-plus-virtual gluon contributions only emerge from the \( gg \)-subprocess and they can be found in Eqs. (A.2) and (A.6). This subprocess also leads to the logarithms of the type \( c_{i,j} \ln^j(1-z) \). The latter also show up in the \( gg + g\bar{g} \)-channel but they are absent in the \( q\bar{q} \) and \( qq \) subprocesses provided one neglects the terms \( (1-z)^k \) for \( k > 0 \). The coefficients \( c_{i,j} \) can be found in Eqs. (A.26)-(A.32) (see also [18]) (scalar Higgs) and Eqs. (B.21), (B.22) (pseudo-scalar Higgs). If we use the definition of the cross section in Eq. (4.3) advocated in [16], [44] the expansion of \( z^{-1}\Delta_{ab}^{SV} \) will not differ from the one shown in Eq. (4.9) but \( \Delta_{ab}^L \) will change due to contribution of subleading logarithms and it becomes equal to

\[
\Delta_{ab}^L(\frac{m^2}{\mu^2}) = \sum_{i=1}^{\infty} a_i^s(\mu^2) \sum_{j=0}^{2i-1} \left( a_{i,j} + c_{i,j} \right) \ln^j(1-z).
\] (4.11)

Because of the different luminosities \( \tilde{\Phi}(z) \) and \( \Phi(z) \) in Eqs. (4.1) and (4.3) respectively the value of the soft-plus-virtual cross section \( \sigma_{tot}^{SV} \) will change. This will be only partially compensated by \( \sigma_{tot}^L \) coming from Eqs. (4.10) and (4.11). In fact the two results for \( \sigma_{tot}^{SVL} \) coming from definitions (4.1) and (4.3) will differ by terms in \( (1-z)^k \ln^j(1-z) \) for \( k \geq 1 \) convoluted by one of the luminosities. In order to study the effect of the two definitions on the approximation we will plot

\[
R^{SV} = \frac{\sigma_{tot}^{SV}}{\sigma_{tot}}, \quad R^{SVL} = \frac{\sigma_{tot}^{SVL}}{\sigma_{tot}}.
\] (4.12)
The same study is made in [8], [16], [17] and [44]. However in these papers the complete result for the NNLO coefficient function was not known yet. Therefore one could only study the leading logarithmic terms in Eqs. (4.10), (4.11) which are of collinear origin. In Fig. 11a and Fig. 11b we have shown the ratios above for the LHC in NLO and NNLO respectively. The figures reveal that $\sigma_{SV}^{\text{tot}}$ using Eq. (4.3) gives a much better approximation than Eq. (4.1). This also holds if we include the logarithms in Eqs. (4.10) and (4.11) denoted by $\sigma_{SVL}^{\text{tot}}$. The latter overestimates the exact cross section a little bit. Further we observe that both SV approximations become worse in NNLO with respect to NLO. In Figs. 12a,b we have performed the same analysis but now for the TEVATRON. The features are the same as in Figs. 11a,b for LHC except that for the TEVATRON the SV approximations derived from Eq. (4.1) and Eq. (4.3) become better. This is no surprise because for the TEVATRON the Higgs mass is much closer to the boundary of phase space. This also explains why at larger masses the SV approximation improves. From this study we infer that the soft-plus-virtual approximation works much better for Eq. (4.3) than for Eq. (4.1). The reason is that the cross section is dominated by the small $x$-region (see Eq. (4.7) and Fig. 10). By choosing the gluon luminosity in Eq. (4.3) this region will become even more important than the gluon luminosity chosen in Eq. (4.2) which is used in Eq. (4.7). For NLO this was already shown in [44] but it is now also confirmed in NNLO. Further this approximation becomes worse in NNLO which holds for both definitions. In spite of this drawback one should still choose Eq. (4.3) to perform the soft-plus-virtual gluon resummation (see [52]) in order to get a better estimate of the cross section. We also noticed that the contribution of logarithmic terms in Eqs. (4.10), Eqs. (4.11) is substantial. The determination of the coefficient of the leading term, which is of collinear origin, is done in [8], [16]. A systematic approach to determine $c_{i,j}$ can be found in [53].

Summarising our results we have recalculated the NNLO corrections to the total cross section for (pseudo-) scalar Higgs production using a different method than those presented in [20] and [21]. In particular our approach differs from the one given in [20] since we directly evaluated the multi-particle phase space integrals without fitting the coefficient functions to an expansion in the terms $(1 - z)^k \ln^l(1 - z)$. Further we presented the radiative parts of the coefficient functions for general colour factors and checked that for $N = 3$ our answers are in agreement with the literature. It turns out that the total
cross section is almost completely determined by the \(gg\)-subprocess which is in contrast to the differential cross section where also the \(gq + g\bar{q}\)-channel gives a considerable contribution. We studied the \(K\)-factors and the dependence of the cross sections on the chosen mass factorization/renormalization scale and the adopted parton density set from which one can infer an error estimate of the cross section. If we exclude the GRV98 parton density the error can be mainly attributed to the missing higher order contributions to the coefficient functions which we estimate to lie between 14% and 21%. Finally we studied the validity of the soft-plus-virtual gluon approximation. Depending on the definition of the total cross section (Eq. (4.1) versus Eq. (4.3)) this approximation is excellent in NLO but it becomes less good in NNLO but Eq. (4.3) is the best way to resum the soft-plus-virtual corrections.

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Appendix A

In this Appendix we present the coefficient functions for scalar Higgs boson production. In the case of the $gg$ subprocess one can split the corresponding coefficient function into

$$\Delta_{gg,B} \left( z, \frac{m^2}{\mu^2} \right) = \Delta_{gg,B}^{S+V} \left( z, \frac{m^2}{\mu^2} \right) + \Delta_{gg,B}^{H} \left( z, \frac{m^2}{\mu^2} \right), \quad (A.1)$$

where the superscripts $S + V$ and $H$ denote the soft-plus-virtual and hard gluon parts respectively. The former contains the distributions $\delta(1 - z)$ and $D_i(z)$ where the latter is defined in Eq.(3.28). We find

$$\Delta_{gg,H}^{(1),S+V} = C_A \left[ 8 \mathcal{D}_0(z) \ln \left( \frac{m^2}{\mu^2} \right) + 16 \mathcal{D}_1(z) + 8 \zeta(2) \delta(1 - z) \right], \quad (A.2)$$

$$\Delta_{gg,H}^{(1)} = C_A \left[ \left\{ -16 z + 8 z^2 - 8 z^3 \right\} \ln \left( \frac{m^2}{\mu^2} \right) + \left( -32 z + 16 z^2 - 16 z^3 \right) \times \left( \ln(1 - z) - \frac{1}{2} \ln z \right) - \frac{8}{1 - z} \ln z - \frac{22}{3} (1 - z)^3 \right], \quad (A.3)$$

$$\Delta_{gg,H}^{(1),q\bar{q}} = C_F \left[ \left\{ 4 - 4 z + 2 z^2 \right\} \ln \left( \frac{m^2}{\mu^2} \right) + \left( -32 z + 16 z^2 - 16 z^3 \right) \left( \ln(1 - z) \right) \right.$$  
$$\left. - \frac{1}{2} \ln z \right] - 3 + 6 z - z^2 \right], \quad (A.4)$$

$$\Delta_{gg,H}^{(1),q\bar{q}} = C_F^2 \left[ \frac{8}{3} (1 - z)^3 \right], \quad (A.5)$$

$$\Delta_{gg,H}^{(2),S+V} = C_A^2 \left[ \left\{ 64 \mathcal{D}_1(z) - \frac{44}{3} \mathcal{D}_0(z) - 32 \zeta(2) \delta(1 - z) \right\} \ln^2 \left( \frac{m^2}{\mu^2} \right) \right.$$  
$$\left. + \left\{ 192 \mathcal{D}_2(z) - \frac{176}{3} \mathcal{D}_1(z) + \left( \frac{536}{9} - 80 \zeta(2) \right) \mathcal{D}_0(z) \right\} \right],$$

36
\[+\delta(1 - z) \left( -24 - \frac{88}{3} \zeta(2) + 152 \zeta(3) \right) \ln \left( \frac{m^2}{\mu^2} \right)\]

\[+128 \mathcal{D}_3(z) - \frac{176}{3} \mathcal{D}_2(z) + \left( \frac{1072}{9} - 160 \zeta(2) \right) \mathcal{D}_1(z)\]

\[+ \left( - \frac{1616}{27} + \frac{536}{9} \zeta(2) + 312 \zeta(3) \right) \mathcal{D}_0(z)\]

\[+\delta(1 - z) \left( 93 + \frac{536}{9} \zeta(2) - \frac{220}{3} \zeta(3) - \frac{4}{5} \zeta^2(2) \right)\]

\[+n_f T_f C_A \left[ \left\{ \frac{16}{3} \mathcal{D}_0(z) \right\} \ln^2 \left( \frac{m^2}{\mu^2} \right) + \left\{ \frac{64}{3} \mathcal{D}_1(z) - \frac{160}{9} \mathcal{D}_0(z) \right\} \ln \left( \frac{m^2}{\mu^2} \right) + \frac{64}{3} \mathcal{D}_2(z) - \frac{320}{9} \mathcal{D}_1(z)\right.\]

\[+\left( \frac{448}{27} - \frac{64}{3} \zeta(2) \right) \mathcal{D}_0(z) + \delta(1 - z) \left( - \frac{2024}{27} - \frac{160}{9} \zeta(2)\right.\]

\[+^8 \frac{80}{9} \zeta(3)\]

\[+n_f T_f C_F \left[ 8 \delta(1 - z) \ln \left( \frac{m^2}{\mu^2} \right) + 4 \delta(1 - z) \right]. \quad (A.6)\]

Notice that the colour decomposition of the \(\delta(1 - z)\) term into \(n_f T_f C_A\) and \(n_f T_f C_F\) is arbitrary due to our ignorance of the same parts appearing in the two-loop virtual corrections computed in [15]. They are only correct when \(N = 3\).

\[\Delta_{gg,H}^{(2)} = C_A^2 \Delta_{gg,H}^{(2),\Lambda} + n_f T_f C_A \Delta_{gg,H}^{(2),\Lambda} + n_f T_f C_F \Delta_{gg,H}^{(2),\Lambda T_f}, \quad (A.7)\]

\[\Delta_{gg,H}^{(2),\Lambda} = \left\{ \left( -128 z + 64 z^2 - 64 z^3 \right) \ln(1 - z) + \left( -96 z^2 + 32 z^3\right.\right.\]

\[\left. - \frac{32}{1 - z} \ln z - \frac{352}{3} + \frac{376}{3} z - \frac{332}{3} z^2 + 132 z^3 \right\} \ln^2 \left( \frac{m^2}{\mu^2} \right)\]
\[\begin{align*}
&+ \left\{ \left( 64 z + 32 z^2 + 32 z^3 + \frac{32}{1 + z} \right) \left( \text{Li}_2(-z) + \ln z \ln(1 + z) \right) \\
&- \left( 256 z + 256 z^2 \right) \text{Li}_2(1 - z) + \left( -384 z + 192 z^2 - 192 z^3 \right) \\
&\times \ln^2(1 - z) + \left( 192 z - 480 z^2 + 224 z^3 - \frac{224}{1 - z} \right) \\
&\times \ln z \ln(1 - z) + \left( 128 z^2 - 48 z^3 + \frac{40}{1 - z} - \frac{8}{1 + z} \right) \ln^2 z \\
&+ \left( -528 + \frac{2032}{3} z - \frac{1856}{3} z^2 + \frac{1760}{3} z^3 \right) \ln(1 - z) + \left( 176 z - 248 z^2 - \frac{248}{3} z^3 \right) \\
&\times \ln z + \left( 192 z - 64 z^2 \\
&+ 96 z^3 + \frac{16}{1 + z} \right) \zeta(2) + \frac{1262}{3} - \frac{3562}{9} z + \frac{3026}{9} z^2 - \frac{4322}{9} z^3 \right\} \\
&\times \ln \left( \frac{m^2}{\mu^2} \right) \\
&+ \left( 128 z + 64 z^2 + 64 z^3 + \frac{64}{1 + z} \right) \left( \ln z \ln(1 - z) \ln(1 + z) \right) \\
&+ \ln(1 - z) \text{Li}_2(-z) + \text{Li}_3 \left( -\frac{1 - z}{1 + z} \right) - \text{Li}_3 \left( \frac{1 - z}{1 + z} \right) \\
&+ \left( -24 + 8 z - 32 z^2 - 32 z^3 + \frac{16}{1 + z} \right) \left( 2 S_{1,2}(-z) \right) \\
&+ \zeta(2) \ln(1 + z) + 2 \ln(1 + z) \text{Li}_2(-z) + \ln z \ln^2(1 + z) \right) \\
&+ \left( -8 + 24 z - 64 z^3 + \frac{16}{1 + z} \right) \text{Li}_3(-z) + \left( -16 - 208 z \\
&- 624 z^2 + 112 z^3 - \frac{144}{1 - z} - \frac{64}{1 + z} \right) S_{1,2}(1 - z) + \left( -16 \right) \right\}.
\end{align*}\]
\[ +672 z + 536 z^2 + 88 z^3 - \frac{8}{1-z} + \frac{64}{1+z} \log_2(1-z) + (24 \]
\[ -104 z - 16 z^2 + 16 z^3 - \frac{64}{1+z} \log z \log_2(-z) + \left( 8 + 8 z \]
\[ -88 z^2 + 8 z^3 - \frac{56}{1-z} - \frac{48}{1+z} \log z \log_2(1-z) \]
\[ -512 z \left( 1 + z \right) \log(1-z) \log_2(1-z) + (240 z - 504 z^2 \]
\[ +248 z^3 - \frac{248}{1-z} \log z \log^2(1-z) + \left( -96 z + 304 z^2 - 144 z^3 \]
\[ + \frac{128}{1-z} - \frac{16}{1+z} \right) \log^2 z \log(1-z) + \left( 20 - 92 z - 16 z^2 - 16 z^3 \]
\[ - \frac{56}{1+z} \right) \log^2 z \log(1+z) + \left( -256 z + 128 z^2 - 128 z^3 \right) \]
\[ \times \log^3(1-z) + \left( \frac{8}{3} + \frac{16}{3} z - \frac{136}{3} z^2 + 24 z^3 - \frac{56}{3} \frac{1}{1-z} \]
\[ + \frac{16}{3} \frac{1}{1+z} \right) \log^3 z + \left( 384 z - 128 z^2 + 192 z^3 + \frac{32}{1+z} \right) \]
\[ \times \zeta(2) \log(1-z) + \left( 8 - 72 z + 392 z^2 - 168 z^3 + \frac{144}{1-z} \]
\[ - \frac{24}{1+z} \right) \zeta(2) \log z + \left( - \frac{112}{3} - 32 z + 56 z^2 + \frac{88}{3} z^3 \right) \log_2(-z) \]
\[ + \log z \log(1+z) + \left( - \frac{1112}{3} + \frac{464}{3} z + \frac{548}{3} z^2 - \frac{572}{3} z^3 \right) \]
\[ - \frac{44}{3} \frac{1}{1-z} \right) \log_2(1-z) + \left( \frac{1232}{3} - \frac{2144}{3} z + \frac{2848}{3} z^2 - \frac{2992}{3} z^3 \right) \]
\[ + \frac{176}{3} \frac{1}{1-z} \right) \log z \log(1-z) + \left( - 528 + \frac{2032}{3} z - \frac{1856}{3} z^2 \right) \]
\[ 39 \]
\[
\Delta_{gg,H}^{(2),C_A T_f} = \left[ \left\{ -\frac{32}{3} z^3 + \frac{16}{3} z + \frac{16}{3} z^3 \right\} \ln^2 \left( \frac{m^2}{\mu^2} \right) + \left\{ -\frac{128}{3} z + \frac{64}{3} z^2 - \frac{64}{3} z^3 \right\} \ln(1 - z) + \left( \frac{32}{3} z - \frac{64}{3} z^2 \right) \right.
\]
\[
\left. + \left\{ \frac{32}{3} z^3 - \frac{32}{3} \frac{1}{1 - z} \right\} \ln z - \frac{296}{9} z - \frac{728}{9} z^2 + \frac{568}{9} z^3 \right] + \left( \frac{152}{3} z^3 \right) \ln \left( \frac{m^2}{\mu^2} \right) + \left( -16 + 16 z - 8 z^2 \right) \left( S_{1,2}(1 - z) + \text{Li}_3(1 - z) \right) + \left( \frac{16}{3} \right.
\]
\[
\left. + \left\{ -\frac{160}{3} z + \frac{8}{3} z^2 - \frac{32}{3} z^3 + \frac{16}{3} \frac{1}{1 - z} \right\} \text{Li}_2(1 - z) + \left( -\frac{128}{3} z \right.
\]
\[
\left. + \frac{64}{3} z^2 - \frac{64}{3} z^3 \right\} \left( \ln^2(1 - z) - \zeta(2) \right) + \left( \frac{64}{3} \frac{z^2}{z - \frac{128}{3} z^2} \right.
\]
\[
\Delta_{99,\text{H}}^{(2),\text{C}_{\text{F}}\text{T}_T} = \\
\left\{16 z \left(1 + z\right) \ln z + \frac{32}{3} + 8 z - 8 z^2 - \frac{32}{3} z^3\right\} \ln^2\left(\frac{m^2}{\mu^2}\right) + \left\{32 z \left(1 + z\right) \left(2 \text{Li}_2(1 - z) + 2 \ln z \ln(1 - z) - \ln^2 z\right) + \left(\frac{128}{3} + 32 z - 32 z^2 - \frac{128}{3} z^3\right) \ln(1 - z) + \left(- \frac{64}{3} - 64 z - 48 z^2 + \frac{128}{3} z^3\right) \ln z - \frac{256}{9} - \frac{448}{3} z + \frac{352}{3} z^2 + \frac{544}{9} z^3\right\} \ln\left(\frac{m^2}{\mu^2}\right) + \left(32 + 32 z + 80 z^2\right) S_{1,2}(1 - z) + \left(32 - 160 z - 112 z^2\right) \times \text{Li}_3(1 - z) + \left(\frac{64}{3} - 48 z - 160 z^2 + \frac{64}{3} z^3\right) \text{Li}_2(1 - z) + z \left(1 + z\right) \left(128 \ln(1 - z) \text{Li}_2(1 - z) - 32 \ln z \text{Li}_2(1 - z) - 64 \zeta(2) \ln z + 64 \ln z \ln^2(1 - z) - 64 \ln^2 z \ln(1 - z)\right)
\]
\[ + \frac{40}{3} \ln^3 z + \left( \frac{128}{3} + 32 z - 32 z^2 - \frac{128}{3} z^3 \right) \left( \ln^2 (1 - z) \right) \\
- \zeta(2) + \left( - \frac{128}{3} - 128 z - 96 z^2 + \frac{256}{3} z^3 \right) \ln z \ln(1 - z) \\
+ \left( \frac{32}{3} + 52 z + 28 z^2 - 32 z^3 \right) \ln^2 z + \left( - \frac{512}{9} - \frac{896}{3} z \right) \\
+ \left( \frac{704}{3} z^2 + \frac{1088}{9} z^3 \right) \ln(1 - z) + \left( \frac{256}{9} + 192 z - 128 z^2 \\
- \frac{1088}{9} z^3 \right) \ln z - \frac{608}{27} \frac{4144}{9} - \frac{3280}{9} z^2 - \frac{1984}{27} z^3 \right) \] (A.10)

\[ \Delta^{(2)}_{gq,H} = C_F \Delta^{(2),C^2_F} + C_A C_F \Delta^{(2),C^A_C_F} + n_f T_f C_F \Delta^{(2),C^F_T} \] , \hspace{1cm} (A.11)

\[ \Delta^{(2),C^2_F} = \left\{ \left( 8 - 8 z + 4 z^2 \right) \ln(1 - z) + \left( 4 z - 2 z^2 \right) \ln z + 4 z - z^2 \right\} \\
\times \ln^2 \left( \frac{m^2}{\mu^2} \right) \\
+ \left\{ 16 \text{Li}_2(1 - z) + \left( 8 - 8 z + 4 z^2 \right) \left( 3 \ln^2 (1 - z) - 4 \zeta(2) \right) \\
+ \left( - 16 + 32 z - 16 z^2 \right) \ln z \ln(1 - z) + \left( - 8 z + 4 z^2 \right) \ln^2 z \\
+ \left( - 36 + 64 z - 28 z^2 \right) \ln(1 - z) + \left( - 8 z + 38 z^2 + \frac{16}{3} z^3 \right) \\
\times \ln z + \frac{106}{9} - 12 z - 4 z^2 - \frac{124}{9} z^3 \right\} \ln \left( \frac{m^2}{\mu^2} \right) \\
+ \left( - 32 + 48 z - 24 z^2 \right) \left( S_{1,2}(1 - z) - \zeta(2) \ln z \right) \\
+ \left( - 16 - 16 z + 8 z^2 \right) \left( \text{Li}_3(1 - z) - \ln(1 - z) \text{Li}_2(1 - z) \right) \]
\[ \Delta_{gq, H}^{(2), CA CP} = \left\{ \left( 24 - 24 z + 12 z^2 \right) \ln(1 - z) + \left( -24 - 24 z - 24 z^2 \right) \ln z \right. \\
- \frac{230}{3} + \frac{188}{3} z - \frac{4}{3} z^2 + 8 z^3 \} \ln^2 \left( \frac{m^2}{\mu^2} \right) \\
+ \left\{ \left( -48 - 144 z - 72 z^2 \right) \text{Li}_2(1 - z) + \left( 16 + 16 z + 8 z^2 \right) \ln(1 - z) \right. \\
\times \ln^2 z \ln(1 - z) + \left( -24 + 40 z - 20 z^2 \right) \ln z \ln^2(1 - z) \\
\times \left( \frac{10}{3} z - \frac{5}{3} z^2 \right) \ln^3 z + \left( \frac{32}{3} + 16 z + 16 z^2 + \frac{32}{3} z^3 \right) \ln^3 z + 2 \ln \left( \frac{m^2}{\mu^2} \right) \right. \\
\times \left( \text{Li}_2(-z) + \ln z \ln(1 + z) \right) + \left( -\frac{92}{3} + 56 z + 42 z^2 \right) \ln^2(1 - z) \\
\times \left( \text{Li}_2(1 - z) + \left( -60 + 94 z - 45 z^2 \right) \ln^2(1 - z) \right. \\
\left. + \frac{16}{3} z^3 \right) \left( \frac{36 - 56 z + 88 z^2 + \frac{32}{3} z^3}{3} \right) \ln z \ln(1 - z) + \left( -14 z \right. \\
- \frac{63}{2} z^2 - \frac{40}{3} z^3 \} \ln^2 z + \left( \frac{232}{3} - 104 z + 60 z^2 + \frac{16}{3} z^3 \right) \ln(1 - z) + \left( -\frac{106}{9} \right. \\
\left. + \frac{878}{9} z + 48 z^2 - \frac{248}{9} z^3 \right) \ln(1 - z) + \left( -\frac{106}{9} \right. \\
\left. + 69 z - \frac{214}{9} z^2 + \frac{112}{9} z^3 \right) \ln z - \frac{1393}{54} - \frac{130}{9} z + \frac{17}{18} z^2 \\
\left. + \frac{1304}{27} z^3 \right) , \] (A.12)
\begin{equation}
\times (\text{Li}_2(-z) + \ln z \ln(1+z)) + \left(72 - 72z + 36z^2\right)
\times \ln^2(1 - z) + \left(-128 - 64z - 112z^2\right) \ln z \ln(1 - z)

+ \left(24 + 32z + 28z^2\right) \ln^2 z + 16z \zeta(2) + \left(-\frac{868}{3} + \frac{736}{3} z\right)

+ \left(\frac{40}{3} z^2 + 32z^3\right) \ln(1 - z) + \left(124 - \frac{448}{3} z + \frac{8}{3} z^2 - 32z^3\right)

\times \ln z + \frac{2422}{9} - \frac{1724}{9} z - \frac{362}{9} z^2 - \frac{32}{9} z^3\right) \ln \left(\frac{m^2}{\mu^2}\right)

+ \left(-136 - 248z - 148z^2\right) S_{1,2}(1 - z) + \left(104 + 344z + 148z^2\right) \text{Li}_3(1 - z) + \left(8 + 8z + 4z^2\right) \left(4 \text{Li}_3\left(-\frac{1 - z}{1 + z}\right)\right)

- 4 \text{Li}_3\left(-\frac{1 - z}{1 + z}\right) + 2 \text{Li}_3(-z) - 4 \ln z \text{Li}_2(-z)

+ 4 \ln(1 - z) \text{Li}_2(-z) + 4 \ln z \ln(1 - z) \ln(1 + z)

- 3 \ln^2 z \ln(1 + z) + \left(-80 - 304z - 136z^2\right) \ln(1 - z)

\times \text{Li}_2(1 - z) + \left(-48 - 32z - 32z^2\right) \ln z \text{Li}_2(1 - z) + \left(\frac{140}{3} \right.

\left.- \frac{140}{3} z + \frac{70}{3} z^2\right) \ln^3 (1 - z) + \left(-8 - 12z - 10z^2\right) \ln^3 z

+ \left(64 + 48z + 64z^2\right) \ln^2 z \ln (1 - z) + \left(-132 - 60z - 114z^2\right) \ln z \ln^2 (1 - z) + 32z \zeta(2) \ln(1 - z) + \left(64 + 112z + 80z^2\right) \zeta(2) \ln z + \left(136 - 112z + 68z^2\right) \zeta(3) + \left(-\frac{538}{3}\right)

\end{equation}
\[+40z + 34z^2 - \frac{40}{3}z^3\text{Li}_2(1 - z) + \left(- \frac{88}{3} - 40z - 12z^2\right)\]
\[-\frac{16}{3}z^3\left(\text{Li}_2(-z) + \ln z \ln(1 + z)\right) + \left(- \frac{784}{3} + \frac{634}{3}z\right)\]
\[+\frac{97}{3}z^2 + 32z^3\ln^2(1 - z) + \left(\frac{692}{3} - \frac{832}{3}z - \frac{40}{3}z^2\right)\]
\[-64z^3\ln z \ln(1 - z) + \left(- 62 + \frac{344}{3}z - \frac{10}{3}z^2 + \frac{68}{3}z^3\right)\]
\[\times \ln^2 z + \left(\frac{574}{9} - 128z - 58z^2 - \frac{104}{3}z^3\right)\zeta(2) + \left(\frac{4342}{9}\right)\]
\[-\frac{904}{3}z - 116z^2 - \frac{64}{9}z^3\ln(1 - z) + \left(- \frac{2402}{9} + \frac{2660}{9}z\right)\]
\[+\frac{1079}{9}z^2 + \frac{224}{9}z^3\ln z + \frac{21539}{54} + \frac{9962}{27}z + \frac{1171}{54}z^2\]
\[-\frac{238}{27}z^3\right], \quad (A.13)\]

\[\Delta^{(2)}_{\text{gT,HT}} = \left\{\left\{\frac{16}{3} - \frac{16}{3}z + \frac{8}{3}z^2\right\} \ln^2 \left(\frac{m^2}{\mu^2}\right)\right\}
\[+\left\{\left(\frac{32}{3} - \frac{32}{3}z + \frac{16}{3}z^2\right) \left(\ln(1 - z) - \ln z\right) - \frac{232}{9} + \frac{304}{9}z\right\}\]
\[-\frac{152}{9}z^2\right\} \ln \left(\frac{m^2}{\mu^2}\right)\]
\[+\left(\frac{8}{3} - \frac{8}{3}z + \frac{4}{3}z^2\right) \left(\ln^2(1 - z) + 2\ln^2 z - 4\ln z \ln(1 - z)\right)\]
\[+\left(- \frac{104}{3} + \frac{128}{3}z - 24z^2\right)\ln(1 - z) + \left(\frac{232}{9} - \frac{304}{9}z\right)\]
\[
\Delta_{q_1,q_2,H}^{(2)} = \Delta_{q_1,q_2,H}^{(2)} = C_F^2 \Delta_{q_1,q_2,H}^{(2),C_F^2}, \\
\Delta_{q_1,q_2,H}^{(2),C_F^2} = \left\{ \left( -16 - 16z - 4z^2 \right) \ln z - 24 + 16z + 8z^2 \right\} \ln^2 \left( \frac{m^2}{\mu^2} \right) \\
+ \left\{ \left( 16 + 16z + 4z^2 \right) \left( -4 \text{Li}_2(1-z) - 4 \ln z \ln(1-z) \right. \\
+ \ln^2 z \left. \right) + \left( -96 + 64z + 32z^2 \right) \ln(1-z) + (48 - 32z \\
-20z^2) \ln z + 102 - 72z - 30z^2 \right\} \ln \left( \frac{m^2}{\mu^2} \right) \\
+ \left( 16 + 16z + 4z^2 \right) \left( -8 \text{S}_{1,2}(1-z) + 8 \text{Li}_3(1-z) - \frac{1}{3} \ln^3 z \\
-8 \ln(1-z) \text{Li}_2(1-z) - 2 \ln z \text{Li}_2(1-z) - 4 \ln z \ln^2(1-z) \right. \\
+ 2 \ln^2 z \ln(1-z) + 4\zeta(2) \ln z \right) + \left( -48 + 32z + 8z^2 \right) \\
\times \text{Li}_2(1-z) + \left( -96 + 64z + 32z^2 \right) \left( \ln^2(1-z) - \zeta(2) \right) \\
+ \left( 96 - 64z - 40z^2 \right) \ln z \ln(1-z) + \left( -24 + 32z \\
+ 8z^2 \right) \ln^2 z + \left( 204 - 144z - 60z^2 \right) \ln(1-z) + \left( -118 \\
+ 88z + 58z^2 \right) \ln z - 210 + 188z + 22z^2 \right\}, \\
\Delta_{q_2,H}^{(2)} = C_A C_F^2 \Delta_{q_2,H}^{(2),C_A C_F^2} + C_F^3 \Delta_{q_2,H}^{(2),C_F^3} + C_F^3 \Delta_{q_2,H}^{(2),C_F^3}, \\
\Delta_{q_2,H}^{(2),C_F^3} = \left\{ \left( -16 - 16z - 4z^2 \right) \ln z + \frac{1060}{27} - \frac{1672}{27}z + \frac{716}{27}z^2 \right\},
\]
\[
\Delta^{(2),CA,C^2_F}_{q\bar{q},H} = \left[ \left( -16 + 16z - 8z^2 \right) S_{1,2}(1 - z) + \left( 8z + 12z^2 \right) \ln^2 z \\
+ \left( -16z - 36z^2 \right) \ln z - 10 - 32z + 42z^2 \right], \quad (A.18)
\]

\[
\Delta^{(2),C^2_F}_{q\bar{q},H} = \left[ \left( 32 - 32z + 16z^2 \right) S_{1,2}(1 - z) + \left( -16z - 24z^2 \right) \ln^2 z \\
+ \left( 32z + 72z^2 \right) \ln z + 20 + 64z - 84z^2 \right], \quad (A.19)
\]

\[
\Delta^{(2),C^2_F}_{q\bar{q},H} = \Delta^{(2),C^2_F}_{q_1q_2,H}, \quad (A.20)
\]

\[
\Delta^{(2),CA,C^2_F}_{q\bar{q},H} = CA C_F^2 \Delta^{(2),CA,C^2_F}_{q\bar{q},H} + C_F^3 \Delta^{(2),C^3_F}_{q\bar{q},H} + C_F^2 \Delta^{(2),C^2_F}_{q\bar{q},H} \\
+ n_f T_f C_F^2 \Delta^{(2),C^2_F T_f}_{q\bar{q},H}, \quad (A.21)
\]

\[
\Delta^{(2),CA,C^2_F}_{q\bar{q},H} = \left[ \left\{ -\frac{88}{3} + 88z - 88z^2 + \frac{88}{3}z^3 \right\} \ln \left( \frac{m^2}{\mu^2} \right) \\
+ \left( 8 + 8z + 4z^2 \right) \left( -4S_{1,2}(-z) - 6Li_3(-z) \\
- 4 \ln(1+z) Li_2(-z) + 6 \ln z Li_2(-z) - 2 \ln z \ln^2(1+z) \\
+ 3 \ln^2 z \ln(1+z) - 2\zeta(2) \ln(1+z) - 4 \zeta(3) \right) + \left( \frac{16}{3} - 16z \\
+ 16z^2 - \frac{16}{3}z^3 \right) \left( 5Li_2(1-z) + 2 \ln^2(1-z) - 8 \ln(1-z) \right) \\
+ \left( 32z + 16z^2 + \frac{32}{3}z^3 \right) \left( Li_2(-z) + \ln z \ln(1+z) \right) \\
+ \left( -\frac{176}{3} + 192z - 168z^2 + 64z^3 \right) \zeta(2) + \left( -40z - \frac{32}{3}z^3 \right) \right]
\]
\[\Delta^{(2)}_{\bar{q}q,H}^{(3)} = \left[ \left( \frac{64}{3} - 64z + 64z^2 - \frac{64}{3}z^3 \right) \ln(1-z) + \left( 32z - 32z^2 + \frac{64}{3}z^3 \right) \ln z + \frac{64}{3}z - \frac{64}{3}z^2 \right] \ln \left( \frac{m^2}{\mu^2} \right) \]

\[+ \left( 16 + 16z + 8z^2 \right) \left( 4S_{1,2}(-z) + 6\text{Li}_3(-z) + 4\zeta(3) \right) \]

\[+ 4 \ln(1+z)\text{Li}_2(-z) - 6 \ln z \text{Li}_2(-z) + 2 \ln z \ln^2(1+z) \]  

\[-3 \ln^2 z \ln(1+z) + 2 \zeta(2) \ln(1+z) \]  

\[+ \left( -32 + 128z - 128z^2 + \frac{160}{3}z^3 \right) \text{Li}_2(1-z) + \left( -64z - 32z^2 \right) \left( \text{Li}_2(-z) + \ln z \ln(1+z) \right) + \left( -32 + 128z - 128z^2 + \frac{160}{3}z^3 \right) \ln^2(1-z) + \left( -64z - 32z^2 - \frac{32}{3}z^3 \right) \ln z + \frac{64}{3}z - 96z + 96z^2 + \frac{128}{3}z^3 \]

\[\times \ln z \ln(1-z) + \left( 40z + 48z^2 - \frac{16}{3}z^3 \right) \ln^2 z + \left( \frac{224}{3} - 256z + 208z^2 - \frac{224}{3}z^3 \right) \zeta(2) + \left( -32 + \frac{352}{3}z - \frac{352}{3}z^2 + 32z^3 \right) \]

\[\times \ln(1-z) + \left( -\frac{256}{3}z - \frac{200}{3}z^2 - \frac{32}{3}z^3 \right) \ln z - \frac{164}{3}z - 64z + \frac{260}{3}z^2 + 32z^3 \] \quad \text{(A.23)}
\[ \Delta^{(2),C_2}_q \]  
\[ = \Delta^{(2),C_2}_q, \]  
(A.24)

\[ \Delta^{(2),C_2}_qT_f \]  
\[ = \left[ \left\{ \frac{32}{3} - 32 z + 32 z^2 - \frac{32}{3} z^3 \right\} \ln \left( \frac{m^2}{\mu^2} \right) + \left( \frac{64}{9} - \frac{64}{3} z + \frac{64}{9} z^2 - \frac{64}{9} z^3 \right) \ln(1 - z) + \left( - \frac{32}{3} + \frac{160}{3} z ight) - \frac{128}{3} z^2 + \frac{128}{9} z^3 \right] \ln z - \frac{368}{27} + \frac{592}{9} z - \frac{688}{9} z^2 + \frac{656}{27} z^3, \]  
(A.25)

As discussed below Eq. (4.10) the corrections are dominated by the terms \( c_{i,j} \) \( \ln^j(1 - z) \) in the coefficient functions. They are obtained by taking the limit \( z \to 1 \) in the expressions above. Most of them vanish. Those which survive are given by

\[ \lim_{z \to 1} \Delta^{(1),C_A}_q \]  
\[ = \left[ -16 \ln \left( \frac{m^2}{\mu^2} \right) - 32 \ln(1 - z) + 8 \right], \]  
(A.26)

\[ \lim_{z \to 1} \Delta^{(2),C_A}_q \]  
\[ = \left\{ -128 \ln(1 - z) + \frac{184}{3} \right\} \ln^2 \left( \frac{m^2}{\mu^2} \right) + \left\{ -384 \ln^2(1 - z) + \frac{1024}{3} \ln(1 - z) + 160 \zeta(2) - \frac{1336}{9} \right\} \ln \left( \frac{m^2}{\mu^2} \right) - 256 \ln^3(1 - z) + \frac{1096}{3} \ln^2(1 - z) + \left( 320 \zeta(2) - \frac{2660}{9} \right) \ln(1 - z) - 624 \zeta(3) - \frac{784}{3} \zeta(2) + \frac{4228}{27} \right], \]  
(A.27)

\[ \lim_{z \to 1} \Delta^{(2),C_A}_q T_f \]  
\[ = \left[ -\frac{32}{3} \ln^2 \left( \frac{m^2}{\mu^2} \right) + \left\{ -\frac{128}{3} \ln(1 - z) + \frac{416}{9} \right\} \ln \left( \frac{m^2}{\mu^2} \right) \right] \]  

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\[ -\frac{128}{3} \ln^2(1-z) + \frac{808}{9} \ln(1-z) + \frac{128}{3} \zeta(2) - \frac{1232}{27} , \]

(A.28)

\[
\lim_{z \to 1} \Delta_{gg,H}^{(1)} \frac{C_F}{C_A} = \left[ 2 \ln \left( \frac{m^2}{\mu^2} \right) + 4 \ln(1-z) + 2 \right],
\]  

(A.29)

\[
\lim_{z \to 1} \Delta_{gg,H}^{(2)} \frac{C_F^2}{C_A} = \left[ \left\{ 4 \ln(1-z) + 3 \right\} \ln^2 \left( \frac{m^2}{\mu^2} \right) + \left\{ 12 \ln^2(1-z) - 16 \zeta(2) - 18 \right\} \ln \left( \frac{m^2}{\mu^2} \right) + \frac{26}{3} \ln^3(1-z) - 11 \ln^2(1-z) + \left( -32 \zeta(2) - 26 \right) \ln(1-z) + 16 \zeta(3) + 12 \zeta(2) + 9 \right],
\]  

(A.30)

\[
\lim_{z \to 1} \Delta_{gg,H}^{(2)} \frac{C_A \cdot C_F}{C_T} = \left[ \left\{ 12 \ln(1-z) - \frac{22}{3} \right\} \ln^2 \left( \frac{m^2}{\mu^2} \right) + \left\{ 36 \ln^2(1-z) + \frac{4}{3} \ln(1-z) + \frac{8}{3} \zeta(2) + \frac{304}{9} \right\} \ln \left( \frac{m^2}{\mu^2} \right) + \frac{70}{3} \ln^3(1-z) + \frac{43}{3} \ln^2(1-z) + \left( -8 \zeta(2) + 58 \right) \ln(1-z) + 62 \zeta(3) + 14 \zeta(2) - \frac{460}{27} \right],
\]  

(A.31)

\[
\lim_{z \to 1} \Delta_{gg,H}^{(2)} \frac{C_F \cdot T_T}{C_T} = \left[ \frac{8}{3} \ln^2 \left( \frac{m^2}{\mu^2} \right) + \left\{ \frac{16}{3} \ln(1-z) - \frac{80}{9} \right\} \ln \left( \frac{m^2}{\mu^2} \right) + \frac{4}{3} \ln^2(1-z) - 16 \ln(1-z) + \frac{104}{27} \right].
\]  

(A.32)
The nonvanishing coefficients $c_{i,j}$ in Eq. (4.9) can be read off from the equations above.
Appendix B

In this Appendix we present the coefficient functions for pseudo-scalar Higgs boson production. To shorten the expressions we give the difference between the coefficient functions originating from pseudo-scalar and scalar production only, namely,

\[ \Delta^{(1)}_{gg, A-H} = C_A \left[ 8 \delta(1 - z) \right], \quad (B.1) \]

\[ \Delta^{(1)}_{gg, A-H} = 0, \quad (B.2) \]

\[ \Delta^{(1)}_{gq, A-H} = 0, \quad (B.3) \]

\[ \Delta^{(1)}_{q\bar{q}, A-H} = 0. \quad (B.4) \]

In the NNLO corrections below we indicate by \( O_{12} \) the contribution coming from the interferences between the operators \( O_1 \) and \( O_2 \) in Eqs. (3.9), Eqs. (3.10). In the coefficient functions one has to put \( O_{12} = 1, \)

\[ \Delta^{(2), S+V}_{gg, A-H} = C_A^2 \left[ \left\{ 64 D_0(z) - \frac{20}{3} \delta(1 - z) \right\} \ln \left( \frac{m^2}{\mu^2} \right) \right. \]

\[ + 128 D_1(z) + \delta(1 - z) \left( \frac{215}{3} + 64 \zeta(2) \right) \]

\[ + C_A T_f n_f \left[ \left\{ - \frac{8}{3} \delta(1 - z) \right\} \ln \left( \frac{m^2}{\mu^2} \right) \right. \]

\[ + \delta(1 - z) \left( - \frac{196}{9} + O_{12} \left( \frac{32}{3} \ln \left( \frac{\mu_r^2}{m_t^2} \right) - \frac{16}{3} \right) \right) \]

\[ + C_F T_f n_f \left[ \left\{ - 8 \delta(1 - z) \right\} \ln \left( \frac{m^2}{\mu^2} \right) - 4\delta(1 - z) \right]. \quad (B.5) \]
For the colour decomposition of the $n_f$-part of the expression above see the remark below Eq. (A.6). Also

\[
\Delta^{(2),C^2_A}_{gg,A-H} = \left\{ -128 z + 64 z^2 - 64 z^3 \right\} \ln \left( \frac{m^2}{\mu^2} \right) - 16 z \ln^2 z + \left( -256 z + 128 z^2 - 128 z^3 \right) \ln(1-z) + \left( 32 + \frac{440}{3} z - 64 z^2 + 64 z^3 - \frac{64}{1-z} \right) \ln z + \frac{44}{3} + \frac{232}{3} z - \frac{452}{3} z^2 + \frac{176}{3} z^3 \right]
\]  (B.6)

\[
\Delta^{(2),C_AT_{f}}_{gg,A-H} = \left[ \frac{32}{3} z \ln z + \frac{16}{3} + \frac{16}{3} z - \frac{32}{3} z^2 \right]
\]  (B.7)

\[
\Delta^{(2),C_F T_{f}}_{gg,A-H} = \left[ 16 z \ln^2 z - 16 + 32 z - 16 z^2 \right]
\]  (B.8)

\[
\Delta^{(2),C^2_F}_{gg,A-H} = \left[ 8 z \ln^2 z - 24 z \ln z - 4 + 16 z - 12 z^2 + O_{12} ( -12 + 12 z^2 ) \right]
\]  (B.9)

\[
\Delta^{(2),C_A C_F}_{gg,A-H} = \left\{ 32 - 32 z + 16 z^2 \right\} \ln \left( \frac{m^2}{\mu^2} \right) - 16 z \ln^2 z + \left( 64 - 64 z + 32 z^2 \right) \ln(1-z) + \left( 80 z - 16 z^2 \right) \ln z + 60 - 48 z + 4 z^2 \right]
\]  (B.10)

\[
\Delta^{(2),C_F T_{f}}_{gg,A-H} = 0
\]  (B.11)
\[
\Delta^{(2),C_2^F}_{q_1,q_2,A-H} = \left[ -16 z \ln^2 z + \left( 32 + 48 z \right) \ln z + 88 - 96 z + 8 z^2 \right], \tag{B.12}
\]

\[
\Delta^{(2),C_A C_2^F}_{q_1,q_2,A-H} = \left[ -16 z \ln^2 z + 32 z \ln z + 32 - 32 z \right], \tag{B.13}
\]

\[
\Delta^{(2),C_3^F}_{q_1,q_2,A-H} = \left[ 32 z \ln^2 z - 64 z \ln z - 64 + 64 z \right], \tag{B.14}
\]

\[
\Delta^{(2),C_2^F}_{q_1,q_2,A-H} = \Delta^{(2),C_2^F}_{q_1,q_2,A-H}, \tag{B.15}
\]

\[
\Delta^{(2),C_2^F}_{q_1,q_2,A-H} = \Delta^{(2),C_2^F}_{q_1,q_2,A-H}, \tag{B.16}
\]

\[
\Delta^{(2),C_A C_2^F}_{q_1,q_2,A-H} = \left[ 16 z \ln^2 z + \frac{80}{3} z \ln z + \frac{8}{3} + \frac{56}{3} z^2 - \frac{64}{3} z^3 \right], \tag{B.17}
\]

\[
\Delta^{(2),C_3^F}_{q_1,q_2,A-H} = \left[ -16 z \ln^2 z - 32 z \ln z + 64 - 160 z + 96 z^2 + O_{12} \left( -48 + 96 z - 48 z^2 \right) \right], \tag{B.18}
\]

\[
\Delta^{(2),C_2^F}_{q_1,q_2,A-H} = \Delta^{(2),C_2^F}_{q_1,q_2,A-H}, \tag{B.19}
\]

\[
\Delta^{(2),C_2^F}_{q_1,q_2,A-H} = \left[ -\frac{64}{3} z \ln z - \frac{32}{3} + \frac{32}{3} z^2 \right]. \tag{B.20}
\]

The nonvanishing coefficients \( c_{i,j} \) in Eq. (4.9) appearing in the difference between the pseudo-scalar and scalar coefficient functions are determined from the expressions above by taking the limit \( z \to 1 \). Those which survive in this limit are given by

\[
\lim_{z \to 1} \Delta^{(2),C_2^F}_{g g,A-H} = \left[ -128 \ln \left( \frac{m^2}{\mu^2} \right) - 256 \ln(1 - z) + 64 \right], \tag{B.21}
\]

\[
\lim_{z \to 1} \Delta^{(2),C_A C_2^F}_{g g,A-H} = \left[ 16 \ln \left( \frac{m^2}{\mu^2} \right) + 32 \ln(1 - z) + 16 \right]. \tag{B.22}
\]
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Figure Captions

**Fig. 1** The total cross section $\sigma_{\text{tot}}$ plotted as a function of the Higgs mass at $\sqrt{S} = 14$ TeV with $\mu = m$. The NLO plots are presented for the total (solid line) and the subprocesses $gg$ (long-dashed line), $10 \times \text{abs}(gq + g\bar{q})$ (dot-dashed line) and $100 \times (q\bar{q})$ (short-dashed line). Also shown is the LO result (dotted line).

**Fig. 2** The total cross section $\sigma_{\text{tot}}$ plotted as a function of the Higgs mass at $\sqrt{S} = 14$ TeV with $\mu = m$. The NNLO plots are presented for the total (solid line) and the subprocesses $gg$ (long-dashed line), $\text{abs}(gq + g\bar{q})$ (dot-dashed line), $100 \times (q\bar{q})$ (dotted line) and $100 \times (qq + q\bar{q})$ (short-dashed line).

**Fig. 3** The total cross section $\sigma_{\text{tot}}$ with all channels included plotted as a function of the Higgs mass at $\sqrt{S} = 14$ TeV with $\mu = m$; NNLO (solid line), NLO (dashed line) and LO (dotted line).

**Fig. 4** The quantity $N(\mu/\mu_0)$ (see Eq. (4.3)) at $\sqrt{S} = 14$ TeV plotted in the range $0.1 < \mu/\mu_0 < 10$ with $\mu_0 = m$ and $m = 100$ GeV/c$^2$. The results are shown for LO (dotted line), NLO (dashed line) and NNLO (solid line).

**Fig. 5** Same as Fig. 4 but now for $m = 200$ GeV/c$^2$.

**Fig. 6** Same as Fig. 4 but now for $m = 300$ GeV/c$^2$.

**Fig. 7** The $K$-factors in NLO and NNLO at $\sqrt{S} = 14$ TeV as a function of the Higgs mass using the MRST-sets; $K_{\text{NNLO}}$ (solid line), $K_{\text{NLO}}$ (dot-dashed line).

**Fig. 8** The $K$-factors in NLO at $\sqrt{S} = 14$ TeV as a function of the Higgs mass using the following parton density sets MRST01 (solid line), GRV98 (dashed line) and CTEQ6 (dotted line).

**Fig. 9** The ratios $R = \sigma_{\text{tot}}/\sigma_{\text{tot}}^{\text{MRST}}$ (Eq.(4.5)) in NLO at $\sqrt{S} = 14$ TeV and $\mu = m$ as a function of the Higgs mass; $R_{\text{GRV}}$ (solid line), $R_{\text{CTEQ}}$ (dotted line).
Fig. 10a The ratio $R(x_{\text{max}})$ (see Eq. (4.7)) for proton-proton collisions (LHC) where $x_{\text{max}} = 5 \times x$ with $x = m^2 / S$. The CM energy and scale are given by $\sqrt{S} = 14$ TeV and $\mu = m$ respectively; NNLO (solid line), NLO (dashed line).

Fig. 10b Same as in Fig. 10a but for proton-anti-proton collisions (TEVATRON) at $\sqrt{S} = 2$ TeV and $\mu = m$.

Fig. 11a The ratios in Eq. (4.12) in NLO for proton-proton collisions (LHC) as a function of the Higgs mass at $\sqrt{S} = 14$ TeV and $\mu = m$; $R^{SV}(\text{Eq. (4.1)})$ (dot-dashed line), $R^{SVL}(\text{Eq. (4.1)})$ (dashed line), $R^{SV}(\text{Eq. (4.3)})$ (dotted line), $R^{SVL}(\text{Eq. (4.3)})$ (solid line).

Fig. 11b The ratios in Eq. (4.12) in NNLO for proton-proton collisions (LHC) as a function of the Higgs mass at $\sqrt{S} = 14$ TeV and $\mu = m$; $R^{SV}(\text{Eq. (4.1)})$ (dot-dashed line), $R^{SVL}(\text{Eq. (4.1)})$ (dashed line), $R^{SV}(\text{Eq. (4.3)})$ (dotted line), $R^{SVL}(\text{Eq. (4.3)})$ (solid line).

Fig. 12a Same as in Fig. 11a but for proton-anti-proton collisions (TEVATRON) at $\sqrt{S} = 2$ TeV and $\mu = m$.

Fig. 12b Same as in Fig. 11b but now for proton-anti-proton collisions (TEVATRON) at $\sqrt{S} = 2$ TeV and $\mu = m$. 
The graph shows the cross-section (σ) in pb as a function of the invariant mass (m) in GeV. The lines represent different processes:

- **NLO(sum)**: The total NLO cross-section.
- **gg**: The cross-section for gg production.
- **10*abs(gq+gq)**: A linear combination of gg and gq production.
- **100*(q̅q)**: The cross-section for q̅q production.
- **LO**: The lowest-order cross-section.

The cross-section decreases with increasing mass, consistent with the expected behavior of these processes.
The graph shows the variation of $R$ with $m$(GeV) for different models:

- **GRV** (solid line)
- **CTEQ** (dotted line)

The $R$ value decreases as $m$ increases, with GRV showing a steeper decrease compared to CTEQ.
\begin{align*}
R(x_{\text{max}}) & \propto m(GeV) \\
\text{NNLO} & \quad \text{NLO}
\end{align*}
\[ R^{SVL} (Eq.(4.1)) \]
\[ R^{SV} (Eq.(4.1)) \]
\[ R^{SVL} (Eq.(4.3)) \]
\[ R^{SV} (Eq.(4.3)) \]
\begin{align*}
R^{SVL} \text{ (Eq. (4.1))} \\
R^{SV} \text{ (Eq. (4.1))} \\
R^{SVL} \text{ (Eq. (4.3))} \\
R^{SV} \text{ (Eq. (4.3))}
\end{align*}
\[ R_{SVL}^{\text{Eq.(4.1)}} \]
\[ R_{SV}^{\text{Eq.(4.1)}} \]
\[ R_{SVL}^{\text{Eq.(4.3)}} \]
\[ R_{SV}^{\text{Eq.(4.3)}} \]