Stress-strain state of a rock mass with a fracture in opening roof

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Abstract. The influence of a fracture formed in the opening roof on stress-strain state of the rock mass is considered. The relationships determining displacement and stress components at the boundary of an opening with rectangular shape, on contact lines of the layers and on the line containing fracture are written. The examples of numerical implementation are given. The analysis of the obtained results is carried out.

1. Introduction
Evaluation of the condition of the roof of underground mine workings (opening) has been one of the priority concerns of the geological risk assessment. In practice, this problem is solved by various methods, and some of them are often preferred due to their high efficiency and capability of estimating the internal structure, properties and state of rock mass from surface measurements. At this, their reliable geological and geomechanical interpretation, along with determination of the structure and degree of rock massif jointing, and evaluation of the opening roof stability are related to solution of the inverse problems, which in itself is rather problematic.

The opening roof stability is dictated by a combination of the mining and technological and geological conditions, of which only the former can be classified as controllable. Notwithstanding the fact that our knowledge about them is always limited, they largely determine the methods both for maintaining the roof stability and evaluating its prediction quality. Labor safety and technical and economic indicators of the underground coal mining depend on how effectively the roof management issues are solved at the working face, which is a pivotal link in the technological scheme of mine operations. For prevention of mine accidents and hazardous events, it is equally important to have information about the roof failure potential and also to know when this is going to happen. Given that the catastrophic roof failure phenomenon is a combination of processes of fracture formation and propagation, the study of the laws of roof collapsing should essentially focus on their understanding.

Fracturing can also have a positive meaning for coal mining, because (i) it can facilitate the process of coal extraction from coal seams; (ii) rational orientation in relation to the fractured zones associated with blastholes drilling and blasting works favors growth rate of the broken rock mass. However, in itself, the fracturing often triggers both mining and geological processes and phenomena which have long proven to be adverse to mining operations (rock motion, rock impacts, caving, etc.). The fractures’ orientation, density, type and geometry have material influence on major physical and mechanical properties of rocks that determine both stability of mine workings and developability of coal deposits. Destruction of structures and natural objects is usually triggered or accompanied by the growth of fractures or fracture-like defects, in themselves representing the most hazardous stress
concentrators. It stands to reason that, conversely, the stress state of rocks is a major control of fractures inception and propagation in the roof of mine workings. Therefore, prior to studying the process of roof failure, the stress state of rocks should be analyzed, given that it changes with advancement of the working face. The method of singular integral equations is widely applicable to studies of the stress-strain state of fractured media. It is found to be most advantageous for solving plane problems of the elasticity theory for areas containing inclusions, holes, and fractures of arbitrary shape.

This paper raises the question of determining the stress-strain state of a rock massif weakened by a rectangular mine opening containing a crack in the roof [1, 2]. The study is underpinned by a system of singular integral equations connecting all the boundary values of the components of stresses and displacements [3].

2. Mathematical model and results
Consider a rectangular mine workings with a fracture in the roof rock mass (Figure 1). Different variants of the boundary conditions are possible at the boundary of the presented region of interest (ROI) within frames of the three fundamental problems of the elasticity theory. Let us consider the case when the fracture is located asymmetrically (highlighted with a bold line).

In this case, the ROI is split into the corresponding parts, with their the contact lines having the form: \( y = 0 \ (|x| \geq a) \), \( y = -2h \ (|x| \geq a) \), \( x = a \) (\( h_1 \leq y \leq H \)). Let the line \( l \) be containing the fracture. In this formulation, the ROI consists of five parts, with each of them singly connected. As such, this separation also allows, when necessary, to obtain additional information about the values of normal and tangential components of stresses and displacements at the contact between the corresponding parts with line \( l \) containing the fracture, without calculating the stress-strain state in the entire ROI.

![Figure 1. Calculation scheme of mine workings with a crack in the roof.](image)

By varying the position of the line \( l \) and of the fracture itself we can consider the problems of the stress-strain state calculating for both homogeneous and inhomogeneous rock massifs with a fracture located at any point of ROI, including the cases when it reaches the boundary or contact line. A system of singular integral equations connecting the boundary values of the components of stresses and displacements for an arbitrary singly connected region is given in [3], where \( \mu = \frac{E}{2(1 + \nu)} \), \( \nu \) is Poisson’s ratio, \( E \) is Young’s modulus of elasticity containing function \( f(t) \):
\[ f(x) = i \int_{0}^{t} (X_n + iY_n) \, ds. \]  
(1)

Here, \( X_n, Y_n \) are the forces on the contour \( \Gamma \) in the direction of the x and y axes, respectively; \( t \in \Gamma, i \) is an imaginary unit. The research does not introduce an a priori assumption about the process of the rock massif deformation, relying however on the continuity of normal and tangential components of stresses and displacements at the contact between different media. This means that on the contact lines of the corresponding parts of ROI and partly on the line \( l \) \((x = a_1, h_1 \leq y \leq H)\) the following conditions occur:

\[ \sigma_n^+ = \sigma_n^- , \quad \tau_n^+ = \tau_n^- , \quad u^+ = u^- , \quad v^+ = v^- , \]  
(2)

where the subscript indicates the membership of a specific part of the rock massif as it tends to line \( l \) or contact lines (Figure 1).

On the basis of [3] and conditions (2), we can write relations connecting the boundary values of the stress and displacement components on the workings contours and contact lines. They are similar to those obtained in [4–6], but are not cited here because of the record’s awkwardness. As an example, we chose the case of a symmetrically located crack whose edges are free of stress. The boundary conditions for the rest of the contour are formulated as:

\[
\begin{align*}
\sigma_x &= \sigma_{x0} = \text{const} , \quad \tau_n = 0 \quad \text{in the boundaries} \quad -2h \leq y \leq 0 , \quad x = a \quad \text{and} \quad x = -a , \\
\sigma_y &= \sigma_{y0} = \text{const} , \quad \tau_n = 0 \quad \text{in the boundaries} \quad -a \leq x \leq a , \quad y = 0 \quad \text{and} \quad y = -2h , \\
0 &= u, \quad 0 = v \quad \text{if} \quad y = -H_1 ; \\
0 &= \sigma_y, \quad \tau_n = 0 \quad \text{if} \quad y = H .
\end{align*}
\]  
(3)

Then, given the problem symmetry, consider half of the ROI (Figure 2), for which the boundary conditions can be formulated as:

\[
\begin{align*}
\sigma_n &= \sigma_{y0} , \quad \tau_n = 0 \quad \text{in} \quad \Gamma_{11} \quad \text{and} \quad \Gamma_{13} , \\
\sigma_n &= \sigma_{x0} , \quad \tau_n = 0 \quad \text{in} \quad \Gamma_{12} , \\
0 &= \sigma_n , \quad \tau_n = 0 \quad \text{for} \quad x = 0 , \quad 0 \leq y \leq h_1 \quad \text{and} \quad y = H , \\
0 &= u, \quad 0 = v \quad \text{for} \quad x = 0 , \quad h_1 \leq y \leq H ; \quad x = 0 , \quad -H_1 \leq y \leq -2h \quad \text{and} \quad y = -H_1 ,
\end{align*}
\]  
(4)

where \( \sigma_n \), \( \tau_n \) are normal and shear stresses; \( u, v \) are the horizontal and vertical displacement components.

![Figure 2](image-url)  
**Figure 2.** Calculation scheme for half of the workings with a fracture symmetrically located in the roof.
For the numerical implementation of the obtained expressions, we proceed to dimensionless values, taking the values that have the dimension of length, and to thickness that feature the layer of rocks $2h$, while the dimension of stresses refers to the stresses at a characteristic depth of virgin massif $\gamma H$. Calculations of the results presented here were carried out if $v = 0.25$, $E = 10^4$ for the cases:

1) $a = 0.5$, 2) $a = 1.0$; 3) $a = 1.5$, (5)

with the variants of different lengths of the crack considered for each of them.

Figure 3 shows the configuration of the opening contour considered after deformation of the workings and part of the contact boundaries for the case $a = 1$, $h_1 = 0.2$. Note that for clarity, we show here the demonstration (not real) values of displacements.

Figure 3. Deformations of the mine workings contour, contact lines and edges of the crack.

Figure 4 shows results of the calculations of the vertical component of displacements $\nu(x)$ and the real part of function $f(t)$ on the line $y = 0$, $x \geq 0$. Curves 1, 2, 3 correspond to the cases described in (5) for $h_1 = 0.2$.

Figure 4. Boundary values of the vertical components of displacements and the real part of the function $f(x)$ on the line $y = 0$, $x \leq 0$ for the cases: 1—$a = 0.5$; 2—$a = 1.0$; 3—$a = 1.5$ if $h_1 = 0.2$.

The results obtained reveal the complex structure of the stress-strain state and a number of parameters critical for the problem solution. Analysis of function (1) has provided insights about the corresponding stress components both on the contact lines and on other parts of the ROI boundary. The obtained ratios contain therefore necessary information on all the components of normal and shear
stresses and displacements in the entire mine workings contour, including additional information on these components on the contact lines and the line containing the fracture, by splitting the ROI into its constituent parts.

3. Conclusions
The relations obtained on the basis of a system of singular integral equations determine the components of displacements and stresses in the boundary of a rectangular opening, on the contact lines between layers and on edges of the fracture and on the line of its continuation. The results obtained allow considering the problem of calculating the stress-strain state near the mine workings with a fracture located arbitrarily in the region of interest.

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