A Short Note on Collecting Dependently Typed Values

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Within dependently typed languages, such as Idris, types can depend on values. This dependency, however, can limit the collection of items in standard containers: all elements must have the same type, and as such their types must contain the same values. We present two dependently typed data structures for collecting dependent types: DList and PList. Use of these new data structures allow for the creation of single succinct inductive ADTs whose constructions were previously verbose and split across many data structures.

Additional Key Words and Phrases: Dependent Types

1 INTRODUCTION

Dependently typed languages such as Idris [2] and Agda [8], provide programmers with a rich and expressive type-system that facilitates greater precision when reasoning about our software programs. However, this expressiveness comes with a cost when collecting values within a container. Take, for example, ‘cons’-style lists that provides an inductive Algebraic Data Type (ADT) for collecting values:

```
data List a = Nil | (::) a (List a)
```

The type List is indexed by the type of the list’s elements, and the constructors Nil and (::) allow lists to be constructed by appending elements to an initially empty list. For example:

```
hoi : List String
hoi = "H" :: "o" :: "i" :: Nil
```

With the list hoi, each element has the type String: This is fine, and by design, as the type List is indexed by a single value. Suppose, however, we are to work with a dependently typed value and collect several different values together using List. For instance, suppose we are modelling a TODO list in which each TODO item has a type parameterised by the item’s TODO state.

```
data Status = TODO | STARTED | DONE
```

```
data Item : Status -> Type where
  MkItem : (state : Status) -> (title : String) -> Item state
```

A question arises concerning how we are to use List to contain a list of TODO items that may have differing TODO states.

```
items : List ?myTypeIs
items = MkItem STARTED "Write Paper"
  :: MkItem TODO "Write Introduction"
  :: Nil
```

The List data type is not able to collect elements from the same indexed families. Each element must have the same type, and in a dependently typed language this also means the types must also depend on the same value. Lists of type List can be made for TODO items, STARTED items, and DONE items, but not for a mixture of such items.

A natural solution would be to represent items as a list of dependent pairs:
items : List ( ty : Status ** Item ty )
items = (STARTED ** MkItem STARTED "Write Paper")
:: (TODO ** MkItem TODO "Write Introduction")
:: Nil

With this construction the type-level values are also represented at the value level for each element. While we can now have a list of dependently typed values, working with the resulting data structure is cumbersome. We have to take into account type-level only information at the value level when operating on individual elements within the list.

We can do better!

1.1 Contributions

This paper presents two dependently typed list data structures that allows for values from the same dependent type to be collected in the same container.

(1) Section 3 presents DList a dependently typed list that collects, at the type level, a single value from a dependent type.

(2) Section 4 introduces PList an extended definition of DList where the collected values must also satisfy a provided predicated.

Both data structures have been made available as part of the idris-containers library [6]. Section 2 further motivates the need for these data structures by examining the specification of an ADT for JavaScript Object Notation (JSON) documents. While such ADTs are naturally inductive, we further motivate the paper by examining a version of JSON in which the root element must be a key-value store. This provides a minimum motivating example suitable for examination in the paper. Section 5 discusses the limitations of the data structures presented in this paper; discusses similar structures; and other larger examples in which these structures prove useful.

2 MOTIVATING EXAMPLE

JSON is a well known serialisation format. JSON documents can contain elements that are either objects or values. An object is either a key value store (associative array) mapping String values to other JSON elements, or an array or elements. JSON values are either: String, Double, Bool, or Null. The natural shape of a JSON document makes it ideally suited for modelling as an inductive ADT. For example:

data JSONDoc =
| JStr String | JNum Double | JBool Bool
| JNull | JArray JSONDoc
| JMap (List (String, JSONDoc))
| JDoc JSONDoc

This is fine. Suppose, however, that the root element in a JSON document must be a key value store. With this restriction our once reasonable data structure becomes problematic in its use. First, it is not trivial to declare a function that, through its type signature, is guaranteed to accept or return a complete JSON document. For example, take the following type signatures for reading and writing JSON documents.

writeDoc : String -> JSONDoc -> IO ()
readDoc : String -> IO (Either JSONDoc Error)

The second argument passed in to writeDoc is only guaranteed to be of type JSONDoc and not necessarily a value constructed using JDoc. Likewise, when using readDoc, we are not guaranteed...
to return a complete JSON document. To provide such guarantees, one has take a defensive pro-
gramming stance and ensure that the functions works with full documents only or fail gracefully.

Secondly, how can the internal structure of a document be specified such that only valid doc-
uments are created. For example, the constructors JArray, JMap, and JDoc can take in any valid value or document. This violates the requirement that the root of a JSON document is an associative array.

A natural way to address these concerns is to introduce more data types to model specific sub-
sections of a JSON document. For example:

```haskell
data JVal = JStr String | JNum Float | JBool Bool | JNull

data JObj = JDict (List String, JObj) | JArray (List JObj) | JValue JVal

data JRoot = JMap (List (String, JObj))
data JSONDoc = JDoc JRoot
```

However, with this approach the natural inductive structure of the original attempt has been lost. Further, the JSON document is no longer a single data type, it is now made up of four distinct ones. It is now no longer possible to write simple recursive functions that traverse or query JSON documents. Multiple functions and instances must now be created to work with each different data type used to model the document.

Following from the Well-Typed Interpreter [1], dependent types can capture the shape of individual sections within a JSON document directly within the document’s type. We begin by defining the following enumerated type JTy.

```haskell
data JTy = DOC | OBJECT | VALUE
```

Values of JTy allow us to distinguish between a complete JSON document itself, and the objects and values contained therein. By indexing JSONDoc with JTy the allowed structure of a JSON document can be capture more accurately.

```haskell
data JSONDoc : JTy -> Type where
    JStr : String -> JSONDoc VALUE
    JNum : Float -> JSONDoc VALUE
    JBool : Bool -> JSONDoc VALUE
    JNull : JSONDoc VALUE
    JArray : List (JSONDoc VALUE)
             -> JSONDoc OBJECT
    JMap : List (JSONDoc VALUE)
             -> JSONDoc OBJECT
    JDoc : JSONDoc OBJECT -> JSONDoc DOC
```

With JSONDoc, we are now able to specify functions that operate on documents and not values. For example:

```haskell
writeDoc : JSONDoc DOC -> IO ()
readDoc : String -> IO (JSONDoc DOC)
```

However, the internal structure of a document is not well-formed. We need to be able to specify that: (a) the constructor JDoc takes a map as its input; and (b) that both JArray and JMap have elements that are either values or objects. A naive attempt to address these issues would be to introduce a fourth constructor to JTy, MAP to represent associative arrays, and introduce versions of JMap and JMap that collects objects. This doubles the number of duplicate data constructors, and

As with our introductory example, A standard list construct is not sufficient; all contents of the list must have the same type and in a dependently typed language, the same values. We need to
data DList : (aTy : Type)
  -> (elemTy : aTy -> Type)
  -> (as : List aTy)
  -> Type
where
Nil : DList aTy elemTy Nil
(::) : (elem : elemTy x)
  -> (rest : DList aTy elemTy xs)
  -> DList aTy elemTy (x :: xs)

Fig. 1. A Cons-Style ADT for collecting Collectively Typed Values.

head : (xs : DList aTy eTy (a :: as))
  -> {auto ok : NonEmpty xs}
  -> eTy a
  -> DList aTy eTy as

head (y :: rest) (ok = IsNonEmpty) (ok' = IsNonEmpty) = y

(a) Head
tail : (xs : DList aTy eTy (a :: as))
  -> {auto ok : NonEmpty xs}
  -> DList aTy eTy as

(a) Tail
drop : (n : Nat)
  -> DList aTy eTy as
  -> DList aTy eTy as

(a) Drop
take : (n : Nat)
  -> DList aTy eTy as
  -> DList aTy eTy as

(a) Take
take Z rest = Nil
take (S k) Nil = Nil
take (S k) (e :: rest) = e :: take k rest

(b) Take
drop Z rest = rest
drop (S k) Nil = rest
drop (S k) (e :: rest) = drop k rest

(b) Drop

Fig. 2. Example functions operating on DList instances.

be able to construct a list that contains elements from the same dependent type but whose type level values differ.

3 THE DLIST CONTAINER

Christiansen [3] presented the UList a dependently typed ADT for encoding lists of values encoded using a Universe Pattern. UList is a generalised cons-style ADT that allows for a value contained within the type of a dependent type to be collected at the type-level. All elements within the list come from the same family of indexed types and that the index within the type of the element can differ. With UList, the family of indexed types is constrained to a singular instance.

Although, UList is useful for encoding constraints on types, the pattern can be used more generally and be used for collecting elements of a dependent type regardless using a cons-style ADT. This was observed in de Muijnck-Hughes [4, Chapter 9] in which the author developed (independently) DList that was designed for collecting type-level information.

Figure 1 presents the definition for DList. In this definition: aTy is the type of the value contained within the list element type; elemTy is the type of the elements within the list; and xs is the List containing the collected values. DList data structure only collects a single value from the type. Dependent types that are parameterised using multiple elements must ensure that all required values are collected. Structurally, UList and DList are the same\(^1\). Using DList a single library of operations operating on generic instances can now be specified. For example Figure 2 presents several common functions on lists as replicated for DList. Notice how the actions performed at the value level are mirrored at the type-level.

\(^1\)For the remainder of the paper, we will use DList to distinguish from the original use of UList
DList allows us to collect dependently typed values. Returning to the introductory example of TODO lists, we can use DList to collect the individual TODO items.

```haskell
items : DList Status Item [STARTED, TODO]
items = MkItem STARTED "Write Paper"
    :: MkItem TODO "Write Introduction"
    :: Nil
```

Notice, how the structure of `items` mirrors that of a standard list. Values are appended to the list, and at the type level the values indexing each element are collected.

Using DList a more accurate description of the internal shape of our running JSONDoc example can be attempted. First we extend JTy with constructors to differentiate between associative arrays and arrays using ARRAY and MAP.

```haskell
data JTy = DOC | ARRAY | MAP | VALUE
```

Secondly, we change the definition of JArray and JMap to use DList. For JMap, we also introduce an anonymous function to ensure that the correct value is collected at the type-level.

```haskell
data JSONDoc : JTy -> Type where
JStr : String -> JSONDoc VALUE
JNum : Float -> JSONDoc VALUE
JBool : Bool -> JSONDoc VALUE
JNull : JSONDoc VALUE

JArray : DList JTy JSONDoc ts -> JSONDoc ARRAY

JMap : DList JTy
    (\ty => (String, JSONDoc ty)) ts
    -> JSONDoc MAP
JDoc : JSONDoc MAP -> JSONDoc DOC
```

Unfortunately, use of DList here has resulted in too permissive a collection of JSONDoc elements and as such a Json object can not contain any JSON value or object. To address this permissiveness we need to be able to constrain the types in a DList to have the following values of type JTy: MAP, ARRAY, or VALUE.

One solution would be to introduce a predicate (JPred) that models a constraint on instances of JTy that are allowed JSON values. For example:

```haskell
data JPred : JTy -> Type where
JMap : JPred MAP
JArr : JPred ARRAY
JVal : JPred VALUE
```

Such a predicate can be enforced using Idris’ proof search mechanism and construction of a helper constructor JNode. This constructor would contain an instance of a JSONDoc value and proof that the predicate applied to the type-level value holds. For instance JSONDoc can be extended as follows:

```haskell
data JSONDoc : JTy -> Type where
...
JNode : JSONDoc ty
    -> {auto prf : JPred ty}
    -> JSONDoc NODE
JArray : List (JSONDoc NODE)
However, this approach requires that we needless increase the verbosity of our documents representation using JNode, that adds a layer of indirection to access values. We will see in the next section how we can remove the need for an explicit constructor to contain items.

4 PREDICATED LISTS

data PList : (aTy : Type) 
  -> (elemTy : aTy -> Type) 
  -> (predTy : aTy -> Type) 
  -> (as : List aTy) 
  -> (prf : DList aTy pred as) 
  -> Type

where
Nil : PList aTy elemTy predTy Nil Nil
(::__) : (elem : elemTy x) 
  -> {prf : predTy x} 
  -> (rest : PList aTy elemTy predTy xs ps) 
  -> PList aTy elemTy predTy (x::xs) (prf::ps)

Fig. 3. Definition for a Predicated List.

Figure 3 presents the definition of PList. PList is a variant of DList that constrains the elements within a list by reasoning about the allowed values in each element’s type. Much of the definition of PList follows that of DList. At the type level, we collect the values that index the list’s elements. PList differs such that the ‘cons’ constructor, (::__), requires implicit proof that the element to be added also satisfies the given predicate. Proof of predicate satisfaction is collected in the type for each element in prf using a DList instance. Predicates are themselves dependently typed values.

To illustrate how PList works, let us revisit the introductory example of modelling a list of TODO items, and how to model a list of complete items. First we define a predicate IsComplete as:

data IsComplete : Status -> Type where
  IsDone : IsComplete DONE

With IsComplete we can now specify a list of completed items:

items : PList Status Item IsComplete
[DONE,DONE] [IsDone,IsDone]

items = MkItem DONE "Write Paper"
  :: MkItem DONE "Proof Read"
  :: Nil

For each element in items, the value parameterising the type, and proof that the collected value satisfies IsComplete are collected. If we were to add a non-complete item (i.e. TODO or STARTED) Idris’ will fail to compile as neither item’s status satisfies IsComplete.

The type of the ‘cons’ constructor for PList uses an implicit argument (prf) to establish proof that the element to be added satisfies the list’s predicate. Here use of Idris’ proof search mechanism is not suitable to construct the proof. If we wish to construct arbitrary predicates over PList instances Idris’ proof search will not be able to construct the proof. There is not enough concrete information. Take for example, the definition of NonEmpty presented in Figure 4.
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data NonEmpty : PList aTy eTy pTy as prfs -> Type
where
  IsNonEmpty : NonEmpty (x::rest)

Fig. 4. The NonEmpty Predicate for PList.

head : (xs : PList aTy eTy pTy (a::as) prfs) -> {auto ok : NonEmpty xs} -> (xs : PList aTy eTy pTy (a::as) (p::prfs))
tail : (xs : PList aTy eTy pTy (a::as) (p::prfs)) -> {auto ok : NonEmpty xs} -> {auto ok' : NonEmpty (a::as)}
-> NonEmpty prfs
-> PList aTy eTy pTy as prfs

drop : (n : Nat) ->
  (xs : PList aTy elemTy predTy as prfs) ->
  (xs : PList aTy elemTy predTy as prfs)
  -> PList aTy elemTy predTy (take n as) (drop n prfs)

Figure 5 illustrates how common list operations can be defined for PList are constructed. Note their similarity to the implementations for DList in Figure 2, and those for standard lists. Note again that the operations performed at the value level for each element are also mirrored in the values collected at the type level.

The benefit of using PList is that no secondary data types are required to describe the structure of a JSON document, the inductive structure of ADT is kept.

5 DISCUSSION

5.1 Alternative Approaches

There are several alternative methods with which dependently typed values can be collected.
5.1.1 List of Dependent Pairs. Dependent pairs allow one to specify a dependency between the second element in the pair to the value presented as the first element. Dependent Pairs would allow us to collect dependently typed values much the same as \texttt{DList}. However, this requires that the value in the type is presented at the value level, making programming with such lists more cumbersome due to extra information. \texttt{DList} is a formulation of a list of dependent pairs in which the depended upon value is hidden away at the type level.

Further, one can constrain the elements in the list of dependent pairs using a nested tuple. Using the example of a predicated list for TODO items from Section 4 an alternative construction would be:

```haskell
items : List (ty : Status ** IsComplete ty
    ** Item ty)
items = (DONE ** IsDone
        ** MkItem DONE "Writing Paper")
:: Nil
```

the contents of the list are not constrained. One will need to introduce a predicate to constrain the contents.

5.1.2 Heterogeneous Vectors. Another approach would be to use Heterogeneous vectors: Lists with a prescribed length whose elements can be of any type. However, there are no restrictions on the types that can be listed within such vectors.

5.1.3 Using Custom Lists. Idris allows list syntax to be provided for data structures that overrrive the \texttt{Nil} and \texttt{::} constructors. A common idiom within Idris is the creation of bespoke lists using this syntax. However, a custom list is required to collect each different dependent type. Operations on lists are not generic and for each dependent type all operations on list like structures have to be written for each list. \texttt{PList} and \texttt{DList} provide generic structures and operations on those structures.

5.2 Relation to List Quantifiers

Dependently typed languages provide a means to existentially quantify proof that a predicate holds over a list of values using parameterised types. Two such examples are the \texttt{All} and \texttt{Any} data types. \texttt{DList} and \texttt{PList} are two comparable structures. However, \texttt{Any} and \texttt{All} are concerned with presenting proofs that a list of homogeneously typed values satisfy some predicate. Further, these data structures present data structure that are a collection of proofs that the values satisfy the predicate. With \texttt{DList} and \texttt{PList}, the proofs are the values in the type.
5.3 Real-World Uses

The variant of JSON, used as a running example, provides an exemplar of the limitations of simple types to accurately capture the inductive structure of some real world data structures. Modelling these documents using a dependent type and dependently typed containers shows how succinct and accurate data structures can be constructed. **DList** and **PList** have been used in several existing Idris packages to provide such succinct data structures.

**idris-xml**. Presents a library for working with XML documents, and allows for simple queries using an XPath like language de Muijnck-Hughes [7]. Here **PList** is used to capture the list of elements presented at each node in the document. Using **PList** facilitates the construction of a single ADT to represent the structure of an XML document in its entirety.

**idris-commons**. Presents a library collecting 'common' modules for Idris whose size does not merit distinct Idris packages [5]. Within **idris-commons** is a module for working with JSON. The ADTs for the JSON format utilises our dependent list structure (**PList**) as a proof-of-concept. Future work will be to include data types for YAML, INI, TOML, and CONF that also use **PList**.

6 CONCLUSION

**DList** and **PList** are dependently typed containers to collect dependently typed values as a 'cons'-style list. For both of these data structures a library of generic operations can be defined and reused, where once bespoke structures and operations were created.

These structures are useful when constructing inductive ADTs that are dependently typed. This was demonstrated through specification of a data structure for JSON documents, and links to other real-world uses.

Both **DList** and **PList** have been made available online for use by others when programming in Idris [6].

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