Variable stiffness stability analysis based on time-domain method and experimental verification

Min Wang, Quankai Yang *, Peng Qin, Tiewei Sun, Chen Wang
School of Beijing University of Technology, Beijing, China

*Corresponding author e-mail: 15611635512@163.com

Abstract. During the turning process, if the feed rate too high, or the feed rate too large, chattering will be occurred. The variable stiffness tip can change the system stiffness during the machine work, so as to increase the limit depth, but the stability of time-varying systems is difficult to predict. This article will be explained in as follows: Firstly, in this paper, the turning stiffness is studied, and the turning dynamic model is established. Secondly, it predicts the limit depth of cut by changing the time domain model parameters, and using the variable stiffness method to optimize the ultimate depth of cut. Finally, verify the correctness of the limit depth of cut through experiments.

1. Introduction
Improving productivity is an eternal task in the field of machinery manufacturing, which is also a common concern in the current manufacturing field. However, high-speed and high-efficiency turning often uses a larger velocity (feed rate) and cutting depth. This will bring about dynamic cutting force, and then cause regenerative chatter. Flutter will reduce the efficiency of cutting, can reduce the machining accuracy, and reduce the life of the tools. So in the course of working, it’s indispensable to avoid the occurrence of flutter. However, it contradicts the high processing efficiency, so a method is proposed to adjust the stiffness of the system to change the natural frequency of the system. Due to the presence of the vibration pattern on the surface of the workpiece, the flutter frequency is relatively stable in a short period of time. If the natural frequency is continuously changed and the amplitude of the change causes the vibration frequency to fall into the stable region, the occurrence of cutting flutter may be avoided.

At present, research on the time domain methods of cutting stability is springing up. Smith and Tlusty[1] proposed a method to estimate time domain by addressing the difference between the maximum and minimum as a parameter criterion to judge whether flutter occurs. Altintas and Campomanes[2] have made a improvement. They judged the flutter base on the cutting thickness and the theoretical thickness of the transient cutting; Shin and Li[3] judge the stability of the cutting system of the thin-walled part by using the extreme value in the time domain value. All of the above methods use time domain parameters to determine flutter, but the ultimate depth of cut cannot be predicted. It is a common method to draw the stability image and the predict the ultimate depth of cutting by using ZOA method[4]. However, the ZOA method is used to study the time-invariant system, and the system stiffness is time-varying when use the variable stiffness tip. Therefore, the stability image cannot be drawn by the ZOA method, and then the cutting stability cannot be predicted. Hence, this article uses time domain analysis method to analyze the stability of time-varying systems.
In controlling flutter, a variable stiffness tip is used, which can be electrostrictive and changes the stiffness of the system, and then suppresses chatter. In this article, by simulating the turning system and adjusting the time domain parameters to draw a stability image. Then verify its correctness through the experiment. Long-axis workpieces are weak in rigidity and difficult to cut. As a result, it is more important to determine the ultimate depth of cut.

2. The establishment of turning dynamics model

2.1. Turning model establishment

When turning the outer circumference of the slender shaft, the slender shaft is regarded as the ideal rigid body, turning in the direction of the turning tool, and the machining process is as shown in Figure 1: the dotted line is the last machining position. In the turning process, the actual cutting thickness is determined by the previous vibration pattern and the current vibration pattern, but only the overlapping parts of the last vibration pattern has impacts. The product of the cutting thickness $ap$ and the feed amount $h$ is the contact area of the turning tool, so as to solving the cutting force. So the tool and machine can be simplified to a single degree of freedom system, thus creating the single degree of freedom system as follows:

$$m\ddot{x}(t) + c\dot{x}(t) + k(t)x = F_f(t) \cdot \cos \gamma$$  \hspace{1cm} (1)

$$F_f(t) = K_f \left[ a \cdot h(t - T) \right]$$  \hspace{1cm} (2)

Where: $m$, $c$, $k$—— modal parameters in cutting;
$\mu$—— coincidence coefficient;
$F_f$—— cutting force;
$K_f$—— cutting force coefficient;
$\gamma$—— The angle between the cutting force direction and the main vibration direction.

![Figure 1. Turning legend](image)

2.2. Stability analysis of time-varying system

The ZOA method can be used to solve the stability of the steady system. If the system is time-varying, the ZOA method cannot be used. Because in the ZOA method the stiffness $k$ and the rotational speed $n$ are constant. But the ZOA method can be used to contrast with other methods, it is an important means to verify other methods. This paper proposes a time domain solving method, which is solving the response of the dynamic differential equation of the system. Within a certain depth of cut, it solve
equations in turn and makes a judgment according to the response, and repeat front steps in the speed range, then draw a stability image.

Using the tip displacement statistic as a stability criterion. The method is to collect the vibration of the tool during one rotation of the spindle. Using the collected sample variance as a criterion for determining flutter, as in the formula (3):

$$\sigma^2 = \frac{\sum_{i=1}^{N_f} (x_i - \bar{x})^2}{N_f - 1}$$  \hspace{1cm} (3)

Where: $N_f$ —— the number of samples,
\(\bar{x}\) —— Sample mean.

When $\sigma^2 > 1.0 \mu m^2$, the flutter is appearing, use this as a limit criterion. Using this criterion as the stability solution, when solve the stability of the time-varying system, taking the data of the tool tip turning through the workpiece as statistic, Solving system stability based on sample variance. This method is also called the time domain method.

3. Time domain method for predicting system stability and parameter optimization

3.1. Time domain methodology modeling

Time domain method is to solve dynamic equations based on dynamic differential equations, to get the output response of the system. According to the dynamic characteristics of tool machine system, using Matlab/Simulink to create the model shown in Figure 3.

![Figure 2. Time Domain Simulation Based on Matlab/Simulink](image)

| Cutting parameters | System Parameters |
|--------------------|-------------------|
| Feed rate h | Tool Angle | Component cos\(\gamma\) | System stiffness k | Equivalent mass m | Damping ratio \(\zeta\) |
| 0.2mm/s | Kr45°Kr′8° | 0.6 | 1000N/mm | 100g | 0.02 |

In the time domain model, the relevant parameters can be assigned to the values of Table 1. The first feedback is the effect of the vibration caused by the current cutting force on the cutting thickness, and the regenerative feedback has the effect on the thickness of the corrugation of the last cutting. The delay time is the time per revolution of the spindle, and the coincidence coefficient is the proportion of the last cut to the current cut.

Using the time domain model criterion, the sample variance of the vibration amount of the tool tip turning over the main axis is counted as a criterion to determine whether it is in a stable state at a certain depth of cut. At this time, the cutting depth is taken in a range, and the stability is calculated separately,
and the critical depth of the stable solution and the unstable solution is found as the critical depth of cut at a certain speed. The above steps are repeated in a range of rotational speeds to obtain a turning stability lobes.

### 3.2. Optimization of time domain method

In the steady system, the stable image which obtained based on the time domain method is consistent with that obtained by the ZOA method, as shown in Figure 3.

![Figure 3. Comparison between time domain method and ZOA method](image)

Figure 4, Figure 5 show the variation of the ultimate depth of cut with frequency. According to the picture, the higher the frequency, the lower the highest limit depth of cut, and the lowest limit depth of cut does not change much, which is shown in Figure 6.

![Figure 4. Stability of 1Hz, 5Hz, 10Hz frequency](image)

![Figure 5. Stability of 1Hz, 0.1Hz frequency](image)

![Figure 6. Comparison chart of 1Hz, 5Hz, 10Hz frequency](image)

Figure 7 shows the variation of the ultimate depth of cut with amplitude. According to the picture, the higher the amplitude, the higher the highest limit depth of cut, and the higher the lowest limit depth of cut, which is shown in Figure 8.
4. Test verification

4.1. Test conditions
In order to achieve the changing stiffness, an telescopic tip is designed (shown in Figure 9). The test was carried out on a common lathe whose model is CY62328/750. The 45# steel rod with a length of 1000 mm and a diameter of 35 mm, it was used as the processing material. Using the variable stiffness tip for support, the turning tool model is SDJCR2525M11. The speed is 375r/min, the feed is 0.2mm/s, and the cutting depth is 0.5mm. The experimental setup is shown in Figure 10.

4.2. Test process and results analysis
Using two identical bars as controls, the scale cinder should be removed with a depth of 0.1 mm. The first set of experiments is a non-variable stiffness test. The other set of tests is processed by variable stiffness method. Variable stiffness control frequency is 1Hz, control amplitude is 1%. The processing process is shown in Figure 11.

As is shown in figure 12. Observing the shape, it can be seen that non-variable stiffness will cause chatter vibration, and adding variable stiffness can suppress flutter.
5. Conclusion
(1) During the turning process, the use variable stiffness tip will change the stiffness of the tool machine system.

(2) In the cutting process, using the variable stiffness tip can improve the system's ultimate depth of cut and improve the stability of the turning system compared with the normal tip. This will increase the processing efficiency.

(3) In the process of solving the variable stiffness tip using the time domain method, a large number of iterations are needed to find the critical depth of cut. This process is complicated and needs to improve its processing efficiency.

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