Reliability Analysis of Engine Shell Strength Under Rocket Sledge Vibration Environment

Jun Xiao*, Wei-wei Zhang, Qiang Xue, Lin-rui Zhang, Xuan-bo Zhou
Huayin Ordnance Test Center of China, HuaYin, China, 714200
Corresponding author e-mail: 50310644@qq.com

Abstract. The strength reliability of a solid rocket motor shell is analyzed. The maximum entropy test method, which is based on information theory, improves the test entropy of a single test sample by reducing the load capacity of the engine shell design under the condition of keeping the load of the engine housing unchanged, so that the total sample size that is required for the test is greatly reduced. The results show that test method of maximum entropy can solve the problem of shell strength reliability assessment when the engine sample size is small. This method can also be applied to small sample reliability assessment in other fields.

1. Introduction

The rocket sled test is a test method that simulate the flight speed and acceleration characteristics of the tested products. When the rocket sled is moving at high speed in the track, the vibration environment of the sled is bad because of the track irregularity and the gap between the slipper and the track, which affects the safety of the engine. The study shows that the average vibration acceleration of the sled car is 74.6g when the speed of [1] is running at the speed of 100m/s. The acceleration will be greater if the speed is further improved. The structural reliability of the newly developed rocket engine is not only related to the success or failure of the test, but also to the personal safety of the test personnel.

The rocket engine is one of the disposable products, and the price is expensive. The new evaluation methods of the reliability of the new rocket engine include the Bayes method [2], the safety factor method [3,4] and the classical statistical method. On the basis of the information theory, the maximum entropy test method keeps the engine shell from bearing the load, and reduce the load capacity of the engine shell design as far as possible, thus reducing the test times and solving the less sample size of the engine.

2. Maximum entropy test method

In information theory, information source is the source of information and the source of producing news or message sequence. In communication system, the source can be described by a sample space and its probability measure - probability space. They are \(X: [a_1, a_2, \ldots, a_n] \) and \(P: [P_1, P_2, \ldots, P_n] \). Among them, \(P_i\) is the probability of sending messages to a source which is called self information, and its value is expressed by the negative value of its probability logarithm, that is [7]:

\[
H = -\log_a P
\]
In the form: H-Entropy, P-The probability of the information source to transmit the signal successfully, a-At the bottom of the logarithm.

Similar to information theory, the reliability test information obtained from each sample test can be defined as information generated by sources.

\[ TH = - \ln R = -\ln(1 - Q) \]  \hspace{1cm} (2)

In the form: TH-Test entropy, for computing convenience, R - the probability of success in testing, that is, product reliability, Q - the probability of failure in testing, that is, product reliability.

\[ C_D \] is the carrying capacity of product design, \[ C_E \] is the actual carrying capacity of the product, as shown in Figure 1 \[ 8 \]. \( M = C_D / P_B \) is called the function margin coefficient.

**Fig. 1** the relationship between the unreliability of the product and the load and carrying capacity.

In Figure 1, the success and failure (F=0) tests at B point (using design load PB) and at A point (using intensified load PA) are analyzed and compared.

\[ N_A(\ln R_A) = N_B(\ln R_B) = -\ln(1 - \gamma) \]  \hspace{1cm} (3)

In the form: R-for reliability, \( \gamma \)-for confidence, N-for all successful samples.

As a result, the entropy of a single test sample can be greatly increased by strengthening the load (or reducing the load capacity), so that the total sample size required for the success or failure type (F=0) test under the strengthened load is greatly reduced. The basic formula of the maximum entropy test method is derived from Figure 1 below.

The coordinate values of the load \( P_A \) and \( P_B \) are first converted into the standard positive and state distribution \[ 9 \], and can be obtained from Fig. 1:

\[ \frac{P_A - \mu}{\sigma} = \Phi^{-1}[1 - (1 - \gamma)^{\frac{1}{2}}] = \frac{P_A - \mu}{\sigma} \]  \hspace{1cm} (4)

\[ \frac{P_B - \mu}{\sigma} = \Phi^{-1}[1 - R] = \frac{P_B - \mu}{\sigma} \]  \hspace{1cm} (5)

After finishing:

\[ N = \frac{\ln(1 - \gamma)}{\ln \{ 1 - \Phi \left[ (K - 1) \frac{\mu}{\sigma} - K\Phi^{-1}(R) \right] \}} \]  \hspace{1cm} (6)

In the form: R - reliability of products, \( \gamma \)-the confidence required, \( \mu \)-the mean of product function parameters (ability), \( \sigma \)-the mean variance of product function parameters (ability), \( K \)-entropy strengthening coefficient, N- the sample size required for maximum entropy test, \( \Phi \)-the normal distribution symbol.

After the maximum entropy test is carried out, the above formula can be used to evaluate the reliability of the product.
3. The method of determining the influence factors of reliability assessment

Through the product reliability calculation formula of maximum entropy test method, entropy strengthening coefficient $K^{[10]}$ and material strength variation coefficient ($D=\sigma/\mu$) can be seen$^{[11]}$.

### 3.1. determination of entropy strengthening coefficient $K^{[10]}$

In the test of the structural strength of the rocket motor, the bearing capacity is minimized when the load remains unchanged. The load capacity decreases, the load capacity is constant, the load increases. The decrease of load capacity is equal to the increase of load when the carrying capacity is constant, $C_D - C_E = P_A - P_B$.

The definition of entropy strengthening coefficient $9$ and the function margin coefficient:

$$K = 1 + \frac{C_D - C_E}{P_B} = 1 + \frac{C_D - C_E}{C_D}M$$

(8)

In order to guarantee the success probability of test method of maximum entropy, the structure strength for skid engine is $C_E \geq P_B$, so the value range of entropy strengthening coefficient is $1 < K < M$.

In practice, the experimenter is not aware of the design margin (or design load) of the tested product $M$ is unknown. In this case, the entropy enhancement factor $K$ can be estimated using the average load ratio of each unit carrying capacity.

$$\hat{K} = \frac{P_B / C_E}{P_B / C_D} = \frac{C_D}{C_E} \approx K$$

(9)

$\hat{K}$’s point estimation is biased estimation, but the estimated value is small, biased to conservative estimation, which will not affect the reliability test and evaluation result, which will only lead to the increase of test samples. Therefore, in the case that the function margin $M$ is unknown (or the design load $P_B$ is unknown), the entropy strengthening coefficient $F$ in the basic formula of test method of maximum entropy can be replaced as $\hat{K}$ value.

### 3.2. determination of variation coefficient $D = \sigma / \mu$

The variation coefficient of the structural strength of the engine shell can be easily calculated by using the total coefficient of variation$^{[11]}$. Its basic method is based on the characteristic value of the sample mean and sample standard deviation, mathematical statistics method, calculate the confidence lower limit such as sample mean type (13) and the sample standard deviation confidence limit such as type (14), and that is equal to the population mean $J$ and the overall standard deviation of the characteristic value of $\mu_i$, to calculate the characteristic value of the total variation coefficient.

$$P[(\bar{x} - \mu_i)S_i \leq \mu_i < \infty] = \gamma$$

(10)

$$P(\sigma_i \leq \sqrt{K_2S_i}) = \gamma$$

(11)

In the form: $K_1 - K_1 = t_{2\alpha} / \sqrt{n}$, $K_2 = (n-1) / \chi^2_{\alpha(n-1)}$, $\gamma$ -confidence, $\alpha$ -risk ratio, $n$ -the number of samples is $24 \sim 50$$^{[12]}$. 

$$R = \Phi\left\{\frac{(K-1)\frac{\mu}{\sigma} - \Phi^{-1}\left\{1 - \exp\left[\frac{\ln(1-\gamma)}{\ln K}\right]\right\}}{K}\right\}$$

(7)
4. Reliability analysis of engine structure

4.1. Shell thickness design

Rocket engine is for rocket skiing sports power components, in order to make the whole vehicle in meeting the requirements of the technical indicators of quality of speed, load, etc, requirement of rocket engine thrust \(3231 \text{ kgf}\), hours of work for \(3.5 \text{s}\). In propellant performance parameters, charging and motor structure parameters must be the case, using the zero dimension interior ballistic equations \(^{[13]}\) to calculate the maximum pressure of gas in the engine \(12.1 \text{MPa}\), propellant in \(50 ^\circ\text{C}\) when the balance of pressure, maximum pressure by the calculation formula of \(^{[14]}\):

\[
p_m = p_{eq}(+50 ^\circ\text{C}) \cdot k_p \cdot r_p
\]

In the form: \(k_p\) - pressure jump coefficient \(1 \sim 1.25\), \(r_p\) - pressure peak ratio \(1 \sim 1.2\).

The calculation can be obtained after substituting in the above formula. Calculation formula of minimum wall thickness of chamber shell \(^{[14]}\):

\[
\delta_{\text{min}} = \frac{\varphi p_m D_i}{2.3 \xi \sigma} - \varphi p_m
\]

In the form: \(\varphi\) - pressure fluctuation coefficient \(1.1 \sim 1.2\), \(1.1\), \(\xi\) - weld strength coefficient \(0.9 \sim 1.0\), \(D_i\) - outer diameter of the shell and \(b\), \([\sigma]\) - the shell material \(30\text{CrMnSiA}\) allowable stress, according to the material manual \([\sigma] = 833 / 1.15 = 768 \text{MPa}\), and substitute the values of \(D_i, p'_m\) and \([\sigma]\) into the above formula to obtain \(\delta_{\text{min}} = 3.512 \text{mm}\). The wall thickness of the actual product is 4.0mm.

4.2. Shell strength test

Stage of product development, manufacturing of shell thickness of 3.6 mm 30 motor shell sample, test its structural strength is \(825 \text{MPa}\), to calculate the allowable stress sample mean for \(85 \text{MPa}\), allowable stress sample standard deviation is \(812 \text{MPa}\), allowable stress are obtained by type (13), the overall average for \(m\), allowable stress are obtained by type (14), the overall standard deviation for \(n\), the resulting: the coefficient of variation \(D = \sigma / \mu = 83 / 812 = 0.1022\).

4.3. Ground test test.

Install 5 before the rocket engine on the static test bench for the ignition experiment \(^{[15]}\), by pressure sensors and force sensors, will be in the process of combustion pressure and thrust into electrical signal, the data acquisition system for processing, to obtain parameters such as pressure, thrust can satisfy the requirements of indicators, including measurement, 5 times the minimum value of maximum pressure is \(22.1 \text{MPa}\).

4.4. Reliability evaluation

Test, in the case of the load unchanged (maximum pressure according to the requirements of thrust calculation), to minimize the carrying capacity after (test product processing when the wall thickness for \(3.6 \text{mm}\) is less than the actual product is 4.0 mm thickness), entropy coefficient of reinforcement are obtained by type (12):

\[
K = \frac{C_D}{C_E} = \frac{22.1 \times 252 / 3.6 \times 2}{22.7 \times 252 / 4.2 \times 2} = 1.1358
\]

The reliability of the product is obtained by equation (10) at the request of the confidence \(\gamma = 0.8\).
Based on the above calculation, the maximum entropy test of the five ground tests is obtained, and the results of the evaluation of the structural strength of the engine shell are obtained by 0.9548. If the same reliability is assessed using conventional test methods of success or failure, it is necessary to do at least $u$ test of ground test $N = \frac{\ln(1-\gamma)}{\ln 0.9548} = 35$. It can be seen from this that it is very effective to solve the problem of small sample reliability test and evaluation with maximum entropy test.

5. conclusion

Using the maximum entropy method based on information theory, the structural reliability of engine test, under the condition of constant maintain engine load, reduce engine design carrying capacity, improve the individual test samples passed the test information, thus greatly reducing the reliability evaluation of the sample, solved the engine sample size is small, difficult to evaluate structural reliability problems.

References

[1] Zhang Ligan Zong-cai deng, Guo Fumin, Yan Jin. Skid high-speed motion dynamic effects and slides smooth degree of the relationship between research [J]. Journal of ballistic, 2011, 23 (2) : 106-110
[2] Shi Ling, Zu-xiang Zhao. Method of reliability evaluation of solid rocket motor structure [J]. Journal of Shanghai aerospace, 1999, (4) : 37 ~ 42
[3] Zhang Jun-hua. Guide for reliability design of structural strength. Beijing: aerospace publishing house, 1989.
[4] Zhang Ji-ru et al. Design of solid rocket motors [M]. Beijing: the second institute of weapons industry, 1982.10.
[5] Zhao Yan-fan, Song Ming. Application and discussion of reliability method in pressure vessel design [J]. Chemical engineering, 2002, 12(5): 24-16.
[6] Chen Lian, Fan Yan. Optimization design of pressure vessel reliability [J]. Journal of engineering design, 2005, 12(1): 39 ~ 43.
[7] Cai Rui-jiao, Dong Hai-ping. Reliability test information entropy of certain conditions [J]. Energy materials, 2008, 16(5): 550 ~ 552.
[8] Wen Yuan-quan. Reliability assessment. Beijing: science press, 1990.
[9] Liu Bing-zhang, Ding Tong-cai. The reliability evaluation method and its application of small sample verification of high reliability [J]. Quality and reliability, 2004, (1): 19 ~ 23.
[10] Liu Jie, Wang Pu, Liu Bing-zhang. Maximum entropy test method and its application [J]. Journal of automation, 2007, 33(11): 1226 ~ 1228.
[11] Ren Guo-zhou. Analysis of reliability calculation method of solid rocket engine structure [J]. Propulsion technology, 1995(1): 41 ~ 45.
[12] Lapin L. Probability and statistics for modern engineering. Technometrics, 1999, 33(4): 490.
[13] Guan Ying-zi. Rocket engine tutorial. Harbin: Harbin industrial university press, 2006.
[14] Kui Ying, Hu Ke-xian. Solid rocket motor [M]. Beijing: Beijing university of science and technology press, 1990.
[15] Л • H • lavrovi. Solid rocket motor structure [M]. Beijing: China aerospace publishing house, 2006.