Surveying the Scope of the $SU(2)_L$ Scalar Septet Sector

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ABSTRACT: Extending the Standard Model by adding a scalar field transforming as a septet under $SU(2)_L$ preserves the $\rho$ parameter at tree level and can satisfy experimental constraints on the electroweak parameters $S$ and $T$. This work presents the first fully general phenomenological study of such an extension. We examine constraints on the septet model couplings based on electroweak and Higgs observables, and use LHC searches for new physics to bound the mass of the septet to be above $\sim 400$ GeV at a 95% CL.
1 Introduction

The discovery of the Higgs boson begins a new era in particle physics. We now know many general details about the Higgs, although more precise measurements are needed. Many models of physics beyond the Standard Model (SM) contain more than one scalar which can contribute to breaking the electroweak symmetry. The amount that the scalars mix to create the physical particles can have a large impact on the couplings of the Higgs to both gauge bosons and fermions. Thus, measurements of the production cross section and decay channels of the Higgs offer new constraints on BSM models.

In addition to the new constraints provided by Higgs observables, models are still subject to the electroweak precision observables. The $\rho$ parameter ($\rho \equiv m_W^2/m_Z^2 \cos^2 \theta_W$) measures the ratio of the $W$ and $Z$ boson masses, which depend on the properties of the Higgs sector. If the scalar sector is comprised only of singlets and doublets, the $\rho$ parameter is equal to one. However, a scalar field transforming as a triplet under $SU(2)_L$ does not preserve $\rho = 1$ at tree level, so experimental constraints force a small vacuum expectation value (vev) of the triplet. For a review of exotic scalar fields see [1, 2].

The general form of the $\rho$ parameter for an $SU(2)_L$ multiplet with weak isospin $j_\phi$ and hypercharge $Y_\phi$ is given by

$$\rho = \frac{\sum_\phi (j_\phi(j_\phi + 1) - Y_\phi^2)v_\phi^2}{\sum_\phi (2Y_\phi^2)v_\phi^2}. \quad (1.1)$$

As $\rho$ is measured to be close to 1, the examination of models which protect the value of $\rho$ at tree level is well motivated. After the doublet ($j = 1/2, Y = 1/2$), the next $SU(2)_L$...
multiplet that leaves $\rho$ unity at tree level is the septet ($j = 3, Y = 2$). In fact, [3] showed that the doublet and the septet are the only multiplets which maintain $\rho = 1$ and preserve perturbative unitarity of scattering amplitudes involving transverse W pairs and pairs of scalars. Interest in the septet has increased since the discovery of the Higgs. Since the septet adds many charged scalars to the Standard Model, it could be used to explain any discrepancy in the observed $h \to \gamma\gamma$ rate. The authors of [1, 4] examined the production and decays of the observed Higgs for the septet, while [5] explored how the electroweak quantum numbers of an additional scalar field, such as the septet, could be determined through a measurement of the Higgs-Higgs-vector-vector coupling. The sum rules for general scalar representations of $SU(2)_L \times U(1)_Y$ were studied in [6] using perturbative unitarity, including the septet model as a specific example.

One artifact that arises in models containing a scalar with weak isospin $j > 2$ is an accidental $U(1)$ symmetry at the renormalizable level. If the scalar is to develop a vev to contribute to the breaking of the electroweak symmetry, the accidental $U(1)$ symmetry is broken as well. This accidental symmetry cannot be spontaneously broken without generating a phenomenologically unacceptable extra Nambu-Goldstone boson. All such models preserving the $U(1)$ are excluded by dark matter cosmological relic densities and direct-detection cross section via Z exchange [7]. In this paper we are not concerned with dark matter and will thus explicitly break the accidental symmetry with a non-renormalizable operator as done in [4].

This work presents the first constraints on a fully generic septet model with a survey of the parameter space of all possible couplings. Earlier works used simplified scenarios, ignoring certain parameters in the potential. We use the full potential and explicitly calculate electroweak parameters and Higgs observables from the masses and mixings between the doublet and the septet. Using these calculations and the most current experimental results from ATLAS and CMS searches for new physics, we are able to place a general bound on the mass of the septet.

The organization of the paper is as follows. In Section 2 we declare our notation and introduce the model Lagrangian and parameters. Section 3 examines constraints on the septet model from Higgs observables and the $S$ and $T$ parameters. Using LHC searches for new physics, we place bounds on the mass of the septet in Section 4. Section 5 presents our conclusions.

2 The Model

In this section we present an overview of the septet model. This model includes the familiar scalar doublet field, $\Phi$, with quantum numbers $(1/2, 1/2)$ under $SU(2)_L \times U(1)_Y$. In addition, there is a second scalar field, $\chi$, with quantum numbers $(3, 2)$. This is a 7-dimensional representation of $SU(2)$, hence the designation “septet.” These two scalar
fields can be explicitly represented as

\[ \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^+ \\ \chi^0 \\ \chi^0 \\ \chi^0 \\ \chi^0 \end{pmatrix}. \quad (2.1) \]

It is important to note that \( \chi^+_1 \) is not the antiparticle of \( \chi^-_2 \). When the neutral components of the scalar fields develop vevs, \( \langle \phi^0 \rangle \equiv v_2 \) and \( \langle \chi^0 \rangle \equiv v_7 \), mass is given to the \( W \) and \( Z \) according to the covariant derivatives:

\[ m_W^2 = \frac{g^2}{2} (v_2^2 + 16v_7^2), \quad m_Z^2 = \frac{g^2 + g'^2}{2} (v_2^2 + 16v_7^2). \quad (2.2) \]

At tree level, the vev of the septet does not alter the mass of the gauge bosons as long as

\[ v^2 = (174 \text{ GeV})^2 = v_2^2 + 16v_7^2, \quad v_2 = v \sin \beta, \quad v_7 = \frac{1}{4} v \cos \beta. \quad (2.3) \]

To remain gauge invariant, the septet field cannot couple to fermions with renormalizable operators. It can also only couple to the doublet through quartic interactions containing each field multiplied by its conjugate. The most general potential is easiest to see using tensor methods [4] and is given by

\[ V = m_1^2 \Phi^2 + m_2^2 \chi^2 + \lambda (\Phi^\dagger \Phi)^2 - \frac{1}{\Lambda^4} \{(\chi^* \Phi \Phi^* \Phi^*) + \text{H.C.}\} + \sum_{A=1}^4 \lambda_A (\chi^\dagger \chi^\dagger \chi^\dagger \chi)_A + \sum_{B=1}^2 \kappa_B (\Phi^\dagger \Phi \chi^\dagger \chi)_B + \frac{1}{\Lambda^2} \sum_{C=1}^3 \eta_C (\Phi^\dagger 2 \Phi^2 \chi^\dagger \chi)_C. \quad (2.4) \]

The tensor structure of the potential, with explicit forms for the terms labeled with indices \( A, B, \) and \( C \) is shown in Appendix A. The \( \lambda_A \) term is a sum over four sub-terms with slightly different tensor structures that collectively give rise to quartic self-interactions of the septet field. Similarly, the \( \kappa_B \) term in (2.4) is a sum over two sub-terms; in this case allowing 4-point interactions between the septet and the doublet. Finally, the \( \eta_C \) term contains three different tensor configurations of a dimension six operator which affects the mass relations of the septet. The dimension-6 \( \eta_C \) term is necessary to maintain consistency of the effective field theory, since we are including the dimension-7 term to break the accidental \( U(1) \) symmetry. There are no possible dimension-5 terms.

It will be shown in Section 3 that the LHC Higgs observations force a small value of \( v_7 \). Since \( v_7 \) must be small, the septet quartic couplings \( \lambda_A \) will contribute negligibly to the mass of the septet, and the septet three-point interactions will be small. The exact values of the \( \lambda_A \) will thus not have a large effect on this study, as very precise measurements would be needed to determine these values. Another result of \( v_7 \) being small is that the possible dimension-6 operators other than \( \eta_C \) either have negligible effects or can be absorbed into \( \lambda_A \) or \( \kappa_B \).
Keeping in mind that we will later show $v_7$ to be small, we here show the masses of the septet particles, neglecting the contribution from $v_7$ and keeping the contributions from $m_2$ and $v_2$.

\begin{align}
m_2^2 &= m_2^2 + v_2^2 (\kappa_1 + \eta_1 \frac{v_2^2}{\Lambda^2}) \\
m_3^2 &= m_2^2 + v_2^2 (\kappa_1 + \eta_1 \frac{v_2^2}{\Lambda^2}) + v_2 \frac{1}{6} (\kappa_2 + \eta_2 \frac{v_2^2}{\Lambda^2}) \\
m_4^2 &= m_2^2 + v_2^2 (\kappa_1 + \eta_1 \frac{v_2^2}{\Lambda^2}) + v_2 \frac{1}{3} (\kappa_2 + \eta_2 \frac{v_2^2}{\Lambda^2}) + \eta_3 \frac{v_2^4}{15 \Lambda^2} \\
m_5^2 &= m_2^2 + v_2^2 (\kappa_1 + \eta_1 \frac{v_2^2}{\Lambda^2}) + v_2 \frac{1}{2} (\kappa_2 + \eta_2 \frac{v_2^2}{\Lambda^2}) + \eta_3 \frac{v_2^4}{5 \Lambda^2} \\
m_6^2 &= m_2^2 + v_2^2 (\kappa_1 + \eta_1 \frac{v_2^2}{\Lambda^2}) + v_2 \frac{2}{3} (\kappa_2 + \eta_2 \frac{v_2^2}{\Lambda^2}) + \eta_3 \frac{2 v_2^4}{3 \Lambda^2} \\
m_7^2 &= m_2^2 + v_2^2 (\kappa_1 + \eta_1 \frac{v_2^2}{\Lambda^2}) + v_2 \frac{5}{6} (\kappa_2 + \eta_2 \frac{v_2^2}{\Lambda^2}) + \eta_3 \frac{2 v_2^4}{5 \Lambda^2} \\
m_8^2 &= m_2^2 + v_2^2 (\kappa_1 + \eta_1 \frac{v_2^2}{\Lambda^2}) + v_2^2 (\kappa_2 + \eta_2 \frac{v_2^2}{\Lambda^2}) + \eta_3 \frac{v_2^4}{\Lambda^2} \tag{2.5}
\end{align}

Equations (2.4) and (2.5) show that the parameters $\kappa_1$ and $\eta_1$ couple the septet and doublet equally for each particle. However, the contributions from $\kappa_2$, $\eta_2$, and $\eta_3$ increase for lower components of the septet representation. This trend will have a direct impact on the $S$ and $T$ parameters as well as the $h \to \gamma \gamma$ coupling. We also see that $\eta_1$ ($\eta_2$) can be reabsorbed into $\kappa_1$ ($\kappa_2$).

In order to not have the charged components of the septet develop a vev, we do not allow for a negative $m_2^2$; all of the vev of the septet thus comes from the tadpole term of the dimension-7 operator in the first line of (2.4). The fact that this term contains only a single septet field allows for the tadpole and explicitly breaks the accidental $U(1)$ symmetry mentioned in the introduction, thereby preventing a massless Nambu-Goldstone boson. Reference [4] has a more detailed analysis of a possible UV completion leading to this dimension-7 term. Momentarily setting $\lambda_A$, $\kappa_B$, and $\eta_C$ in (2.4) to zero gives a septet vev of

\begin{equation}
\varepsilon = \frac{v_2^6}{\sqrt{6 m_2^2 \Lambda^3}}. \tag{2.6}
\end{equation}

As the mass of the septet is decoupled or $\Lambda$ becomes large, the vev of the septet goes to zero and the Standard Model is recovered. From this point on, we allow $\lambda_A$, $\kappa_B$, and $\eta_C$ to vary.

The septet contains two distinct singly charged particles as well as a neutral one. These will mix with the charged and neutral components of the doublet to form physical particles. We define $\Phi_0 = v_2 + (\phi^0_R + i \phi^0_I)/\sqrt{2}$ and $\chi^0 = v_7 + (\chi^0_R + i \chi^0_I)/\sqrt{2}$ with the rotations given
Table 1: Higgs signal strengths from ATLAS [9] and CMS [10]. The signal strength $\mu_{xx}$ is defined as the bound on the Higgs decay rate $\Gamma(h \rightarrow xx)$ divided by the SM expectation.

| Signal     | ATLAS  | CMS    |
|------------|--------|--------|
| $\mu_{WW}$ | $1.25 \pm 0.43$ | $0.68 \pm 0.20$ |
| $\mu_{ZZ}$ | $1.20 \pm 0.58$ | $0.92 \pm 0.28$ |
| $\mu_{\gamma\gamma}$ | $1.76 \pm 0.50$ | $0.77 \pm 0.27$ |
| $\mu_{tb}$  | $0.47 \pm 2.17$ | $1.15 \pm 0.62$ |
| $\mu_{tt}$  | $0.44 \pm 1.55$ | $1.10 \pm 0.41$ |

3 Constraints from observables

To study the phenomenology of this model we first examine the couplings of the Higgses. The new couplings between the CP-even Higgses and vector bosons are

$$g_{hVV} = \frac{g^2 v}{\sqrt{2}} (4 \cos \beta \sin \alpha + \cos \alpha \sin \beta) = \frac{g^2 v}{\sqrt{2}} (4 \cos \beta \sin \alpha + \cos \alpha \sin \beta),$$

$$g_{HHVV} = \frac{g^2 v}{\sqrt{2}} (4 \cos \alpha \cos \beta - \sin \alpha \sin \beta) = \frac{g^2 v}{\sqrt{2}} (4 \cos \alpha \cos \beta - \sin \alpha \sin \beta).$$

Of course, the septet field cannot couple directly to Standard Model fermions. So the Higgs-fermion coupling is only through the doublet:

$$g_{hf} = \frac{g_f}{\sqrt{2}} \cos \alpha = \frac{-m_f}{v_2} \cos \alpha = g_{hf}^{SM} \cos \alpha,$$

$$g_{Hf} = \frac{g_f}{\sqrt{2}} \sin \alpha = \frac{-m_f}{v_2} \sin \alpha = g_{hf}^{SM} \sin \alpha.$$
Some bounds on the Higgs signal strengths for ATLAS [9] and CMS [10] are shown in Table 1. The signal strengths are the observed rates divided by the SM expectation. In these categories, one experiment measures a value above the SM expectation and the other experiment a value below the SM expectation. We make the conservative assumption that the tree level processes will not deviate from the SM by more than 15 percent. Using this assumption, we make the following cut for gluon fusion with decays to fermions:

$$0.85 \leq \left( \frac{g_{hf^f}/g_{hf^f}^{SM}}{4} \right) \leq 1.15.$$  \hspace{1cm} (3.3)

A similar cut is made for gluon fusion followed by decays to gauge bosons:

$$0.85 \leq \left( \frac{g_{hf^f gVV}/g_{hf^f gVV}^{SM}}{2} \right)^2 \leq 1.15.$$  \hspace{1cm} (3.4)

The left panel of Figure 1 shows the effect of the fermion decay constraints in blue and red, while the gauge boson decays are constrained in green and yellow.

These constraints have come only from the production and decay of the Higgs. These couplings compared to the SM are only determined by the mixing angles of the neutral Higgses and the vevs, with no other assumptions about the masses or couplings in the model. To further constrain the model, we generate random model points which pass constraints (3.3) and (3.4), yield the observed Higgs mass, do not allow for stable charged
scalars, and have perturbative couplings. The model points chosen then have the following form.

\[ -2 \leq \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \kappa_1, \kappa_2, \eta_3\} \leq 2, \quad 0 \leq \cot \beta \leq 0.3, \quad \text{and} \quad 0 \leq m_2 \leq 2 \text{ TeV}. \tag{3.5} \]

At each point, the values of \(m_1\) and \(\Lambda\) are used to minimize the potential and \(\lambda\) sets \(m_{h_0} = 125.5\) GeV. The neutral Higgs rotation angle, \(\alpha\) is an output of the model. We only keep points which generate \(\alpha\) reproducing the allowed Higgs couplings. To satisfy charged dark matter bounds, we assume the mass hierarchy \(m_{\chi^5} > m_{\chi^4} > m_{\chi^3} > m_{\chi^2}\). \(\tag{3.6}\)

This and the mass relations in (2.5) lead to the assumption that \(\kappa_2 \leq 0\). It is possible that three body (or higher) decays could be used to satisfy dark matter bounds if this mass hierarchy is not used. Relaxing this assumption would require an in-depth analysis of actual decay widths and possible charge injection into the early universe. While this is beyond the scope of this paper, a detailed examination of \(\kappa_2 > 0\) would be a rich topic of study.

In the right panel of Figure 1 the model points are plotted in the same window as the \((\alpha, \cot \beta)\) tree-level Higgs coupling constraints. It is not possible to generate the observed Higgs mass for all values of \((\alpha, \cot \beta)\). The constraint that the gauge boson signal strengths are not greater than the SM expectation by more than 15\% forces \(\cot \beta \lesssim 0.14\), or \(v_7 < 6\) GeV. \(\tag{3.7}\)

This small value of \(v_7\) justifies dropping \(\mathcal{O}(q^2 v_7^2)\) contributions to the masses given in (2.5).

### 3.1 \(S\) and \(T\) Parameters

One of the main motivations of the septet model is that it does not affect the \(T\) parameter at tree level \((\alpha T = \rho - 1)\). However, it can induce changes at loop level. The scalar contributions to the vacuum polarization at one loop are given by

\[
\Pi^{AB}(q^2) = \frac{T^{A,B}_{ij}}{16 \pi^2} \left[(m_i^2 + m_j^2)(1 + \xi) - 2f_2(m_i, m_j) + 2m_i^2(\log m_i^2 - \xi - 1) + \frac{q^2}{3} \right] + \mathcal{O}(q^4), \tag{3.8}\]

where

\[
f_1(m_i, m_j) = \int_0^1 \, dx \, x(1-x) \log(xm_i^2 + (1-x)m_j^2),
\]

\[
f_2(m_i, m_j) = \int_0^1 \, dx \, (xm_i^2 + (1-x)m_j^2) \log(xm_i^2 + (1-x)m_j^2),
\]

\[
\xi = \lim_{\epsilon \to 0} \frac{2}{\epsilon - \gamma} + \log 4\pi.
\]

The masses, \(m_(i,j)\), are given in (2.5) and the \(SU(2)_L\) generators for the septet, \(T_{ij}^{A}\), are given in Appendix B. The \(S\) and \(T\) parameters are then

\[
S = 16\pi \left[\Pi^{33}_3(0) - \Pi^{33}_Q(0)\right], \tag{3.9}\]

\[
T = \frac{4\pi}{s_W c_W m_Z^2} \left[\Pi^{11}(0) - \Pi^{33}(0)\right]. \tag{3.10}\]
Figure 2: The left (right) panel shows the septet contributions to the $S$ ($T$) parameter plotted against the septet mass parameter $m_2$. The model points are randomly chosen according to equations (3.3), (3.4), and (3.5) such that the observed Higgs mass is generated and the tree-level Higgs couplings do not deviate by more than 15% from the SM value. The shaded area is the allowed region for $S$ and $T$. At large values of $m_2$ the contributions to these parameters are small. However, there are still many model points with small values of $S$ and $T$ and low values of $m_2$. At low masses the $S$ and $T$ parameters are controlled by $\kappa_2$ and $\eta_3$, so these parameters are forced to take values close to zero.

Note that $\xi$ cancels in the calculation so that $S$ and $T$ are finite. The oblique parameters are larger when the mass of different parts of a multiplet are non-degenerate. As $v_7$ is small from the tree level constraints (3.7), the mass splitting of the septet comes from $\kappa_2$ and $\eta_3$, although the $\eta_3$ contribution is suppressed by an extra factor of $(v_2/\Lambda)^2$. The contributions to $S$ and $T$ should go to zero as the septet is decoupled. The randomly generated model points contributions to $S$ and $T$ are plotted against the common mass parameter of the septet, $m_2$, in figure 2. The shaded box shows the allowed regions [8] where

$$|\Delta S| \leq 0.12 \quad \text{and} \quad |\Delta T| \leq 0.10.$$  \hspace{1cm} (3.11)

The septet protects the $T$ parameter at tree level, and the loop contributions are smaller for $T$ than $S$. For points with $m_2 \gtrsim 600$ GeV the $T$ parameter is small whereas the $S$ parameter is not necessarily small until $m_2 \gtrsim 1$ TeV.

Figure 3 shows the same model points as before with the added condition that $m_2 < 500$ GeV, such that the contributions to $S$ and $T$ can be substantial. If $\kappa_2$ is large, the mass splitting among different components of the septet is large, leading to greater contributions to $S$ and $T$. The mass hierarchy chosen so as to not have a relic abundance of charged septet states has already forced $\kappa_2 \leq 0$ and now the electroweak observables force small values of $\kappa_2$ when the mass parameter $m_2$ is small.

3.2 Higgs to $\gamma\gamma$

The septet model contains many more charged particles coupling to the Higgs than does the SM. The experimental observations of the rate of $h \to \gamma\gamma$ leave room for non-SM-like behavior. However, CMS measures the rate to be below the SM rate whereas ATLAS measures it to be above, as shown in Table 1. With this in mind, we allow a greater range
Figure 3: The left (right) panel shows the septet contributions to the $S$ ($T$) parameter plotted against $\kappa_2$ which couples the septet to the doublet in the potential shown in equation (2.4). The model points are randomly chosen according to equations (3.3), (3.4), and (3.5) such that the observed Higgs mass is generated and the tree-level Higgs couplings do not deviate by more than 15\% from the SM value. In addition, the model points in this figure have $m_2 < 500$ GeV to allow for large contributions to $S$ and $T$. The coupling $\kappa_2$ alters the mass of the septet differently for each component as seen in equation (2.5). Large absolute values of $\kappa_2$ lead to greater mass splitting and thus more contribution to $S$ and $T$.

for this signal than we did for the gluon fusion signals by only requiring

$$0.5 \leq \Gamma(h \rightarrow \gamma\gamma)/\Gamma(h \rightarrow \gamma\gamma)_{SM} \leq 2.0.$$  \hfill (3.12)

The partial width of $h \rightarrow \gamma\gamma$ is given by [11] as

$$\Gamma(h \rightarrow \gamma\gamma) = \frac{\alpha^2 m_h^3}{512\pi^3} \left| g_{hVV} Q_v^2 A_1(\tau_V) + \frac{2g_{hf} T}{m_f} N_{c,f} Q_f^2 A_{1/2}(\tau_f) + N_{c,s} Q_s^2 \frac{g_{hSS}}{m_S^2} A_0(\tau_S) \right|^2,$$  \hfill (3.13)

where $v = 174$ GeV, $Q$ is the charge of the particle, $N_c$ is the number of colors, $\tau_x = 4m_x^2/m_h^2$, and the $A_S$ functions are given by

$$A_1(x) = -x^2 \left( 2x^{-2} + 3x^{-1} + 3(2x^{-1} - 1)f(x^{-1}) \right),$$
$$A_{1/2}(x) = 2x^2 \left( x^{-1} + (x^{-1} - 1)f(x^{-1}) \right),$$
$$A_0(x) = -x^2 \left( x^{-1} - f(x^{-1}) \right),$$
$$f(x) = \arcsin^2(\sqrt{x}).$$  \hfill (3.14)

For the $W$ and top quark, $A_1(\tau_W) = -8.3$ and $A_{1/2}(\tau_t) = 1.4$. If the scalar mass is greater than the Higgs mass $A_0(\tau_S) \sim 1/3$. Thus the scalar contributions work against the SM dominant $W$ boson contributions if the $g_{hSS}$ couplings are positive. This allows the septet to decrease and potentially flip the sign of the $g_{h\gamma\gamma}$ coupling with positive Higgs-septet couplings. If the couplings are negative then the contributions increase the partial decay width. Figure 4 shows the random model points passing the tree-level coupling constraints. Again, the blue shaded region is the allowed region defined by (3.12). As the septet decouples the SM expectation is recovered as shown in the left panel. The tree-level
Higgs couplings force a small value of the neutral mixing angle, $\alpha$. This implies that the observed Higgs is mostly doublet like, therefore the couplings $g_{hSS}$ come from the $\kappa_1$, $\kappa_2$, and $\eta_3$ terms. At low masses, $S$ and $T$ force small $\kappa_2$ and $\eta_3$. Thus the common coupling of $h_0$ to the septet, $\kappa_1$, has the most effect on the partial decay width to two photons. A negative value of $\kappa_1$ increases $\Gamma(h \rightarrow \gamma\gamma)$ and a positive value decreases it. The right panel shows this trend, where the only model points shown have $m_2 < 500$ GeV. As $\kappa_1$ does not affect $S$ and $T$, any future precise measurement of $h \rightarrow \gamma\gamma$ can be accommodated by the septet model.

4 Bounds from the LHC

After taking all of the constraints from Section 3 into account, the allowed parameter space contains $\cot \beta < 0.14$ and $\alpha < 0.25$. The constraints leading to these bounds all came from electroweak observables or properties of the observed Higgs. The question still remains: how do direct searches for new particles at the LHC constrain the septet model? To answer this question, we create a grid of model points with $0 \leq \cot \beta \leq 0.14$ and $100 \leq m_2 \leq 700$ GeV, with the rest of the model parameters following (3.5). The particular points are chosen to sample the parameter space at very small $\cot \beta$ and provide good coverage for $m_2$ between 300 and 500 GeV, where the inclusive cross section ranges between hundreds of fb to tens of fb. The grid values are given by

\[
m_2 = \{100, 200, 300, 350, 400, 450, 500, 600, 700\} \text{ GeV}
\]

\[
\cot \beta = \{0.001, 0.005, 0.01, 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14\}.
\]

Each grid point is also forced to follow all of the constraints discussed in Section 3.
As previously noted, the septet model offers a rich landscape of multiply charged particles that can be phenomenologically useful. However, this relatively large number of particles makes a direct search method difficult. For our search, we implemented the septet model in FeynRules [12]. The events were then simulated with MadGraph 5 [13], using Pythia 6.4 [14] for hadronization and PGS [15] for detector simulation.

Using these tools, we considered the production of any new particle pair at the LHC with $\sqrt{s} = 8$ TeV. As the septet does not couple directly to SM fermions or gluons, the production mechanism must be electroweak, going through either a neutral or charged current Drell-Yan type process. It is also possible to create the new particles through VBF leading to four-point interactions with two gauge bosons and two new scalars. The cross section for the VBF process is less than 10% of the total new scalar cross section. The presence of so many new states makes the VBF simulations extremely computationally demanding, so we do not include them in this analysis. Our results are thus conservative, but a dedicated search could find the VBF process useful.

The production of the multiply charged scalars depends only on their quantum numbers, whereas production of the singly charged and neutral components depends on the mixing between the septet and doublet. The $W$ boson couples most strongly to the middle of the representation, while the $Z$ couples most strongly to the top and bottom of the representation. The coupling to the photon is strongest for the particles with the highest charge. With these observations and the mass hierarchy from Eq. (3.6), the order of the cross sections tends to be

$$
\sigma(pp \to H_{1,2}^{-}, H_{1,2}^{+}) > \sigma(pp \to \chi^{+3}\chi^{-}) > \sigma(pp \to \chi^{+5}\chi^{-}) > \sigma(pp \to \chi^{+4}\chi^{3-}) > \sigma(pp \to \chi^{+4}\chi^{4-}) > \ldots
$$

(4.2)

where the choice of $(1, 2)$ depends on the mixing involved. Due to the mixing in both the singly charged and the neutral components, (4.2) is not completely general. The ordering of the cross sections of the neutral and singly charged particles may slightly change among themselves as well compared to the ordering with the multiply charged states. However, the ordering of the multiply charged states with respect to other multiply charged states will remain constant. The large quantum numbers and sheer number of states in the septet leads to relatively large inclusive cross sections. For example, with $m_2 \sim 250$ GeV the cross section is on the order of a pb. In comparison the production of the much lighter $W^+W^-$ at $\sqrt{s} = 8$ TeV is $\sim 35$ pb. The total cross section in fb of all combinations of septet pairs is shown in Figure 5.

The decay of a multiply charged state goes to the next lowest charged state and either a $W$ or a singly charged Higgs. However, the septet states tend to be nearly degenerate so the $W$ or charged Higgs will be off-shell. The doubly charged scalar can decay to two $W$ bosons, or a $W$ and a singly charged Higgs. The dominant decay depends on the masses as well as $\alpha$ and $\beta$. Once the decay chain reaches the neutral septet state, the decay will have to continue through mixing with the neutral doublet. As $\cot \beta \to 0$ and $\alpha \to 0$ the $H_0$ couplings to fermions and gauge bosons go to zero as seen in (3.1) and (3.2), leading to a stable particle at the end of the decay chain. As mentioned before, [7] shows that models
such as these can be ruled out from a dark matter perspective. However, even at our smallest value of \( \cot \beta = 0.001 \), the width of \( H_0 \) is large enough to decay instantaneously. We again ignore the dark matter constraint and only study the collider results.

Due to the relative strength of the production of multiply charged particles and their decays, the search strategy will be to look for many \( W \)s. If the \( W \) decays leptonically, the lepton is easy to identify, and the neutrino leads to missing energy. The other option is to have hadronic \( W \) decays, producing many jets.

For each of the 90 model points in the grid, we simulate proton-proton collisions at \( \sqrt{s} = 8 \) TeV. We then employ the same search strategies as done by the CMS and ATLAS experiments. The searches used are: the CMS search for anomalous production of events with three or more leptons [16]; the ATLAS search for strongly produced SUSY particles in final states with two same sign leptons and jets [17]; the ATLAS search for charginos and neutralinos in events with three leptons and missing transverse momentum [18]; and the ATLAS search for squarks and gluinos in final states with jets and missing transverse momentum and no leptons [19].

We use the experimental search data to calculate the 95\% CL limit on the number of allowed events in a particular bin, denoted by \( s_{i,95} \). To compute \( s_{i,95} \), we use the SM expected number of events \( (b_i) \), the error in the SM expected number \( (\sigma_{b,i}) \), and the number of events observed by the experiment \( (n_i) \). These values are plugged into the following equation, which is then numerically solved for \( s_{i,95} \):

\[
\int \delta b_i \text{Gaus} \left( \delta b_i, \frac{\sigma_{b,i}}{b_i} \right) \times \text{Pois} \left( n_i | b_i (1 + \delta b_i) + s_{i,95} \right) = 0.05. \tag{4.3}
\]

To compute the expected limit instead of the observed limit, \( b_i \) is used in place of \( n_i \). Using the Gaussian distribution is an attempt to take into account the uncertainties. However, many of the ATLAS papers present their own values for \( s_{i,95} \), which include systematics and other uncertainties. In all such cases, our calculation of \( s_{i,95} \) resulted in a larger value than that given by ATLAS. A larger value means that more signal events are required to

\[\text{Figure 5: The total cross section in fb of the production of all possible pairs of septet particles.}\]
Table 2: Experimental search bins which are the most constraining bin for the greatest number of model points. These two bins were the most significant for \( \sim 2/3 \) of the model points tested. The transverse mass is defined as \( m_T = \sqrt{2 \cdot E_{T}^\text{miss} \cdot p_T \cdot (1 - \cos \Delta \phi_{\ell,E_{T}^\text{miss}})} \). The variable \( m_{\text{eff}} \) is the scalar sum of the transverse momentum of the two leptons. For the three lepton search, \( m_{\text{SFOS}} \) is the invariant mass of the same flavor opposite sign lepton pair which is closest to the \( Z \) mass.

| Signal Region | Cuts | Observed | SM Expected | Error | Expected \( s_{i,95} \) | Observed \( s_{i,95} \) |
|---------------|------|----------|-------------|-------|-----------------|-----------------|
| ATLAS-CONF    | 0 b-jets |          |             |       |                 |                 |
| 2013-007      | \( N_{\text{jets}} \geq 3 \) |          |             |       |                 |                 |
| SR0b          | \( E_{T}^\text{miss} > 150 \) GeV |          |             |       |                 |                 |
| Two same sign leptons | \( m_T > 100 \) GeV |          |             |       |                 |                 |
|               | \( m_{\text{eff}} > 400 \) GeV |          |             |       |                 |                 |
| ATLAS-CONF    | \( m_{\text{SFOS}} < 81.2 \) GeV |          |             |       |                 |                 |
| 2013-035      | or \( m_{\text{SFOS}} > 101.2 \) GeV |          |             |       |                 |                 |
| SRnoZc        | \( E_{T}^\text{miss} > 75 \) GeV |          |             |       |                 |                 |
|               | \( m_T > 100 \) GeV |          |             |       |                 |                 |
| Three leptons | \( p_T^{3^{\text{rd}}} \ell > 30 \) GeV |          |             |       |                 |                 |

The observed number of events passing the two lepton search is less than expected, which leads to a lower allowed cross section for new physics. Thus, when computing the observed...
limits, the two same sign lepton search is most limiting for most of the events, followed by the three lepton search.

![Exclusion Region](image)

**Figure 6**: A combination of ATLAS and CMS experimental searches for BSM physics have been used to constrain the mass of the septet for fixed values of the mixing angle ($\beta$) of the vevs of the septet and the doublet. All model points follow the constraints from section 3, specifically the tree level Higgs couplings do not change by more than 15%, the $h \to \gamma\gamma$ rate is consistent with observation, and the one loop contributions to the oblique $S$ and $T$ parameters are within experimental limits. The limits are calculated by considering allowed cross section for every bin and taking the smallest for each model point. The allowed cross section is obtained using equation (4.4) where $s_{x,95}$ comes from equation (4.3). The dashed line represents the expected limit while the solid line is the observed limit. The model points to the left of the lines have a larger cross section than allowed by the experimental searches and are thus excluded. From this we have shown that LHC searches rule out a mass of the septet below about 400 GeV.

Figure 6 shows the limits coming from the most limiting bin at each model point. The dashed line is the expected limit, and follows the expected limit from the three lepton search. The solid line is the observed limit, which follows that of the two same sign lepton search. The area to the left of the lines has a larger cross section than allowed by the searches and is excluded. From this we rule out a mass parameter for the septet ($m_2$) below $\sim 400$ GeV at the 95% CL.

5 Conclusion

We have presented a phenomenological study of a general scalar septet extension to the Standard Model, and shown that septet masses are excluded below about 400 GeV at the 95% CL. This constraint came from using ATLAS and CMS searches for BSM physics. The search for two same sign leptons gave the best observed limit followed closely by a three lepton search. The three lepton search gave the best expected limit. The 400 GeV bound on $m_2$ is still low enough that $S$ and $T$ are not guaranteed to be small, which happens closer to 1 TeV. This in turn forces small values of $\kappa_2$, which is responsible for separating
the mass of the different components of the septet. Having a small value of $\kappa_2$ then allows
the other doublet-septet coupling, $\kappa_1$ to determine the $h \to \gamma\gamma$ rate. For $m_2 \sim 400$ GeV, a
choice of $\kappa_1$ between -2 and 2 allows for a range of $h \to \gamma\gamma$ from 0 to above 10 times
the standard model rate. Even if the mass of the septet is pushed to 1 TeV, the septet model
has enough charged particles coupling to the Higgs that $\kappa_1$ has freedom to match a future
more precise measurement of the diphoton coupling.

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**Appendix A** Expanded Potential

As a reminder, the potential is given by

$$
V = m_1^2 \Phi^2 + m_2^2 \chi^2 + \lambda(\Phi^\dagger \Phi)^2 - \frac{1}{\Lambda^3} \left\{ (\chi^\dagger \Phi^5 \Phi^*) + \text{H.C.} \right\} 
+ \sum_{A=1}^4 \lambda_A (\chi^\dagger \chi \chi^\dagger \chi)_A + \sum_{B=1}^2 \kappa_B (\Phi^\dagger \Phi \chi^\dagger \chi)_B 
+ \frac{1}{\Lambda^2} \sum_{C=1}^3 \eta_C (\Phi^\dagger 2 \Phi^2 \chi^\dagger \chi)_C.
$$

(A.1)

For completeness, we here show the explicit tensor structure of the terms (summation over
repeated indices is assumed).

\[
\begin{align*}
(\chi^\dagger \chi)^2_1 &= \chi^*_{ijklmn} \chi_{ijklmn} \chi_{abcdef} \chi_{abcdef}, \\
(\chi^\dagger \chi)^2_2 &= \chi^*_{ijklmn} \chi_{ijklmn} \chi^*_{abcdef} \chi_{abcdef}, \\
(\chi^\dagger \chi)^2_3 &= \chi^*_{ijklmn} \chi_{ijklmn} \chi_{abcdef} \chi_{abcdef}, \\
(\chi^\dagger \chi)^2_4 &= \chi^*_{ijklmn} \chi_{ijklmn} \chi^*_{abcdef} \chi_{abcdef}, \\
(\Phi^\dagger \Phi^\dagger \chi^\dagger \chi)_1 &= \Phi^*_{i} \Phi^*_{j} \chi_{abcdef} \chi_{abcdef}, \\
(\Phi^\dagger \Phi^\dagger \chi^\dagger \chi)_2 &= \Phi^*_{i} \Phi^*_{j} \chi^*_{abcdef} \chi_{abcdef}, \\
(\Phi^\dagger \Phi^\dagger \chi^\dagger \chi)_3 &= \chi_{abcdef} \chi_{abcdef}, \\
(\Phi^\dagger \Phi^\dagger \chi^\dagger \chi)_4 &= \chi_{abcdef} \chi_{abcdef}.
\end{align*}
\]

(A.2)

The field definitions in the tensor method are as follows.

\[
\begin{align*}
\chi^{111111} &= \chi^+ \\
\chi^{211111} &= \chi^4 / \sqrt{6} \\
\chi^{221111} &= \chi^3 / \sqrt{15} \\
\chi^{222211} &= \chi^{++} / 2 \sqrt{15} \\
\chi^{222221} &= \chi^*_1 / \sqrt{15} \\
\chi^{222222} &= \chi^0 / \sqrt{6} \\
\chi^{222222} &= \chi^+_2
\end{align*}
\]

(A.3)
Appendix B  \( SU(2) \) Septet Generators

The septet is a (3,2) under \( SU(2)_L \times U(1)_Y \). For convenience, the generators for the 7-dimensional representation of \( SU(2) \) are listed below, along with \( Q = T_3 + Y \).

\[
T_1 = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & \sqrt{5} & 0 & 0 & 0 & 0 \\
0 & \sqrt{5} & 0 & \sqrt{6} & 0 & 0 & 0 \\
0 & 0 & \sqrt{6} & 0 & \sqrt{5} & 0 & 0 \\
0 & 0 & 0 & \sqrt{5} & 0 & \sqrt{3} & 0 \\
0 & 0 & 0 & 0 & \sqrt{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & 0
\end{pmatrix}
\] (B.1)

\[
T_2 = \frac{i}{\sqrt{2}} \begin{pmatrix}
0 & -\sqrt{3} & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3} & 0 & -\sqrt{5} & 0 & 0 & 0 & 0 \\
0 & \sqrt{5} & 0 & -\sqrt{6} & 0 & 0 & 0 \\
0 & 0 & \sqrt{6} & 0 & -\sqrt{5} & 0 & 0 \\
0 & 0 & 0 & \sqrt{5} & 0 & -\sqrt{3} & 0 \\
0 & 0 & 0 & 0 & \sqrt{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{3} & 0
\end{pmatrix}
\] (B.2)

\[
T_3 = \begin{pmatrix}
3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -3 & 0
\end{pmatrix}
\] (B.3)

\[
Q = \begin{pmatrix}
5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1
\end{pmatrix}
\] (B.4)

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