Mathematical model for dynamic characteristics of automatic electrohydraulic drive for technological equipment

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Abstract. The paper is devoted to the development mathematical model of automatic rotary motion electrohydraulic drive for technological equipment and structural scheme of mathematical model for drive as object of automatic control. The hydraulic drive of rotary motion with volume regulation is considered. The drive contains regulated pump and unregulated hydraulic motor. The regulation of the working volume of the pump is carried out by the electrohydraulic amplifier. The transfer function of the regulating process of the pump working volume is represented by aperiodic link of the first order. The adequacy of mathematical model for dynamic characteristics of electrohydraulic drive has been confirmed experimentally by the Fisher criterion. To verify the adequacy, we compared the experimental and calculated transient processes for the shaft angular rotation velocity of hydraulic motor when applying the control signal. The mathematical model of automatic rotary motion electrohydraulic drive for technological equipment as object of automatic control was used as basis for further research on the synthesis of automatic control systems for equipment.

1. Introduction
Expanding the functionality and improving the efficiency of technological equipment for materials machining, can be achieved by developing and using automatic control systems [1-5]. The achievement of arbitrary kinematics for the working body and possibility for software implementation of the optimal laws for its movement is ensured by the use of automatic electrohydraulic drives (EHD), in particular, EHD with volume regulation for equipment with power over $10\, kW$. The use of hydraulic drive allows improving accuracy and reliability of technological equipment and the level of automation, simplifying the kinematics and reducing metal consumption.

In this regard, actual task for the synthesis and study of automatic control systems is the development of reliable and simple mathematical models of workflows that take place in the EHD and take into account the technical features of equipment.

The mathematical simulation problems for the automatic EHD characteristics of technological equipment is widely represented in the literature. The research of working processes in equipment is based on the fundamental equations of hydromechanics, solid mechanics and electrical engineering. To study the stability and control quality of such systems and to correct them, apply the methods of the theory of automatic control and regulation [5-10].

In mathematical simulation of the dynamic characteristics of technological equipment drives with hydraulic drive, certain difficulties arise with the description of nonstationary hydromechanical processes occurring in them. In the dynamics of hydraulic systems there are features due to the mutual
influence of hydraulic elements, presence of fluctuations for pressure, flow rates, individual parts due to compressibility of the working fluid, the working fluid influence on regulating devices, etc. This leads to complex non-stationary processes that need to be considered [11-15].

At the same time, the expansion of functional capabilities and the efficiency increase for technological equipment through the development and use of automatic control systems require simple reliable mathematical models of working processes in EHD and taking full enough into account the technical features of equipment [16-20].

The purpose of this paper is the development of mathematical model of automatic rotary motion EHD with volume regulation for technological equipment and the structural scheme for mathematical model of EHD as object of automatic control.

2. Dynamic characteristics of automatic EHD for technological equipment

In figure 1 shows typical principal scheme of the power part for hydraulic drive of rotary motion with volume regulation, containing the main axial-piston pump 2 and the axial-piston hydraulic motor 3. The shaft of pump is driven by electric motor 1. The flow rate of pump is regulated by changing the inclination angle of the titling washer (or the cylinder block) by means of the device 4, which can be hydraulic amplifier. The pump with two pipelines 5 is connected with the hydraulic motor. Through the gearbox 6, the motor shaft is connected with the working body 7. The leakages of the working fluid are compensated by auxiliary pump 8, which is driven from the main pump shaft. The inclination angle of the titling washer (or the cylinder block) for the main pump is regulated by the hydraulic amplifier and the auxiliary pump is applied to supply the hydraulic amplifier with pressurized fluid. The pressure for pressure line of this pump is maintained by the overflow valve 9. This line is connected through two check valves 10 to the pipelines connecting the main pump and the hydraulic motor. Sub-supply valves must provide minimum pressure in the pipeline so that cavitation does not occur in the main pump.

The hydraulic lines are protected from excessive high pressure by two safety valves 11. In the pressure line of the auxiliary pump, there is also a safety valve 12. These valves protect the pump from high pressure when the filter 13 is clogged.

![Figure 1. The scheme of the power part for drive with volume regulation: HM – hydraulic motor; P1 and P2 – pumps; SV1 ... SV4 – safety valves; CV1, CV2 – check valves; T – tank; F – filter; M – electric motor; R - gearbox; O – working body.](image)

When developing scheme solutions of automatic EHD for technological equipment, it is important to use the right choice of control system for pump flow rate to take advantage of the volumetric method.
for regulation. Analysis of systems for various purposes shows [21, 22] that the electrohydraulic amplifiers (EHA) application allows you to fully implement the functional requirements for equipment.

Compared to other types of amplifiers, such as electromechanical, hydraulic amplifiers have several advantages: better dynamic properties, greater reliability, very high-power gains (over $3 \times 10^6$) and simplicity of design. It is enough to note that for the hydraulic amplifier the mass to power ratio reaches $0.04 \text{kg/kW}$, while in magnetic amplifiers this indicator is up to $70 \text{kg/kW}$, and electromechanical - about $20 \text{kg/kW}$.

EHA can be used with almost any type of devices and machines. In mechanical engineering, axial-piston machines are widely distributed, which have good weight characteristics (they are 2-3 times lighter than other types of machines with equal power), response speed, etc. [23-25].

The regulation scheme of the inclination angle for the titling washer of pump by single-stage EHA is shown in figure. 2. Here the single-stage EHA 1 is used, in which the rod of the proportional electromagnet is connected directly to the control spool of the throttling valve. The four-slot throttling valve delivers fluid to the plungers, which turn the tilting washer 2 of pump.

![Figure 2. The inclination angle control for the tilting washer of pump.](image)

The regulated axial-piston pumps with tilting washer, most often in the nomenclature of produced hydraulic machines are presented as hydraulic units with integrated EHA [10, 21]. Using passport data, it is permissible to construct the second-order dynamic model of the regulation process for the inclination angle of pump titling washer $\gamma$ by the control voltage $U$:

$$T_{2\gamma} \frac{d^2 \gamma}{dt^2} + T_{1\gamma} \frac{d\gamma}{dt} + \gamma = k_{\gamma U} U,$$

where $k_{\gamma U}$ – transfer coefficient, which can be determined by the nominal control voltage $U_{\text{nom}}$ and the nominal inclination angle for the titling washer of pump $\gamma_{\text{nom}}$:

$$k_{\gamma U} = \frac{\gamma_{\text{nom}}}{U_{\text{nom}}}.$$

The time constants $T_{2\alpha}, T_{1\alpha}$ in known manner are determined from the frequencies $\nu_1, \nu_2$ of the phase shift, respectively, at -45° and -90° [3, 10]:

$$T_{2\alpha} = \frac{1}{2\pi \nu_2}; T_{1\alpha} = \frac{1}{2\pi \nu_1} - \frac{2\pi \nu_1}{(2\pi \nu_2)}.$$

This approach to developing the dynamic model of automatic control for the inclination angle of the pump titling washer is also applicable in the case of EHA application as independent device, which in
the nomenclature of produced hydraulic devices are presented as throttling valves or distributors with proportional control [26-28].

It should be noted that the regulation of the pump flow can be carried out not only by changing the angle of the washer, but in other ways, for example, by changing the angle of the cylinder block. It is easy to verify that in this case the dynamic model of the regulation process for the inclination angle of the cradle for the cylinder block will have similar appearance.

Thus, the transfer function of the regulation process for the inclination angle of the pump titling washer (or cylinder block) according to equation (1) has the form:

\[ W_{rp}(s) = \frac{k_{s}}{T_{rp}s + 1} \]  

(4)

where \( s \) – the Laplace variable.

As confirmed by further experimental researches, as well as analysis of the characteristics of EHA used in adjustable pumps, the transfer function of the regulation process for the pump working volume can be considered as an aperiodic link of the first order:

\[ W_{rp}(s) = \frac{k_{s}}{T_{rp}s + 1} \]  

(5)

where \( T_{rp} \) – time constant of the regulation process:

\[ T_{rp} = \frac{1}{2\pi v_{l}} \]  

(6)

In the work [3, 10], the transfer function of the power part of the EHD with volume regulation was obtained for the shaft rotation angle \( \alpha \) of the hydraulic motor by the inclination angle of the titling washer (or cylinder block) \( \gamma \):

\[ W_{pd}(s) = \frac{\alpha(s)}{\gamma(s)} = \frac{1}{T_{hd}s(T_{hd}^{2} + 2\zeta_{m}T_{hd}s + 1)} \]  

(7)

where \( T_{hd} \) – time constant of the hydraulic drive; \( T_{m} \) – time constant of the hydraulic motor; \( \zeta_{m} \) – relative damping coefficient for the hydraulic motor.

To build a mathematical model of EHD with volume regulation as object of automatic control, the transfer function (7) for the rotation angle of the shaft by the inclination angle of the titling washer (or cylinder block) is rewritten as:

\[ W_{pd}(s) = \frac{\alpha(s)}{\gamma(s)} = \frac{\alpha(s) \Omega(s)}{\Omega(s)} \frac{1}{s} = \frac{k_{pd}}{s(T_{hd}^{2} + 2\zeta_{m}T_{hd}s + 1)} \]  

(8)

where \( W_{pd} \) – transfer function for the drive power part, i.e. transfer function for the angular rotation velocity of the hydraulic motor shaft by the inclination angle of the titling washer (or cylinder block):

\[ W_{pd}(s) = \frac{\Omega(s)}{\gamma(s)} = \frac{k_{pd}}{T_{hd}s^{2} + 2\zeta_{m}T_{hd}s + 1} \]  

(9)

\( k_{pd} \) – transfer coefficient for the drive power part, which in accordance with [3, 10] can be determined by the expression:

\[ k_{pd} = \frac{2\pi k_{Q_{p}}}{q_{m}} \]  

(10)

\( q_{m} \) – working volume of hydraulic motor; \( k_{Q_{p}} \) – transfer coefficient for the flow rate of pump by the inclination angle of the titling washer (or cylinder block), which for the axial-piston pump...
\[ k_{\Omega p} = \frac{F_p z_p D_p \Omega_p}{2\pi}; \]  

\( F_p \) – working area of the single piston (plunger) of the pump; \( z_p \) – number of pistons; \( D_p \) – diameter of the circle on which the axes of the pump pistons are located; \( \Omega_p \) – shaft angular velocity of pump.

Note the following. The shaft angular rotation velocity of hydraulic motor, in general, is determined not only by the pump working volume, but also by the magnitude of the loading moment \( M \) on the motor shaft. The degree of influence is established for a particular drive, and increase of the loading moment unambiguously leads to a decrease of the angular rotation velocity of, which in linear approximation can be reflected by the transfer function:

\[ W_{\text{OM}}(s) = \frac{\Omega(s)}{M(s)} = -k_{\text{OM}}, \]

where \( k_{\text{OM}} \) – transfer coefficient for the angular rotation velocity by the loading moment, the value of which can be estimated from the static characteristics for the volumetric hydraulic drive [23, 29, 30].

3. Result and discussion

Thus, given the principle of superposition

\[ \Omega(s) = W_{\text{pd}}(s)\gamma(s) + W_{\text{OM}}(s)M(s), \]

the mathematical model of automatic rotary motion EHD with volume regulation for technological equipment as object of automatic control can be represented by the structural scheme shown in figure 3.

![Figure 3. The structural scheme for the EHD mathematical model as object of automatic control.](image)

![Figure 4. The transient response for the shaft angular rotation velocity of hydraulic motor.](image)
To verify the adequacy of the proposed mathematical model for the regulation process of the pump working volume, as well as the drive as a whole, the automated experimental research bench and technique for processing experimental data were developed. The pump type NAS 0.04/20 with the maximum working volume $0.04\, l$ and hydraulic motor type PM N2.5A with the working volume $0.0317\, l$ were used on bench. The total dimensionless error during dynamical tests did not exceed 0.04. In figure 4 compared the experimental and calculated transient characteristics for the angular rotation velocity of the hydraulic motor shaft $\Delta \Omega$ when the control voltage is applied. The experimental values of the Fisher criterion (less than 1.2) did not exceed the table values $\approx 1.8$ at confidence level $\alpha = 0.95$. Therefore, the developed model of dynamic characteristics can be considered adequate.

4. Summary

Thus, mathematical model of automatic rotary motion EHD with volume regulation for technological equipment have been developed. The drive contains regulated pump and unregulated hydraulic motor. The pump working volume is regulated by EHA. The transfer function of the regulating process of the pump working volume is represented by aperiodic link of the first order. The structural scheme of mathematical model of EHD as object of automatic control is presented. The adequacy of mathematical models for dynamic characteristics of electrohydraulic drive has been confirmed experimentally by the Fisher criterion. To verify the adequacy, we compared the experimental and calculated transient processes for the shaft angular rotation velocity of hydraulic motor when applying the control signal.

The mathematical model of automatic rotary motion electrohydraulic drive for technological equipment as object of automatic control was used as basis for further research on the synthesis of automatic control systems for equipment.

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