Anomalous CMB polarization and gravitational chirality

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We consider the possibility that gravity breaks parity, with left and right handed gravitons coupling to matter with a different Newton’s constant and show that this would affect their zero-point vacuum fluctuations during inflation. Should there be a cosmic background of gravity waves, the effect would translate into anomalous CMB polarization. Non-vanishing $TB$ (and $EB$) polarization components emerge, revealing interesting experimental targets. Indeed if reasonable chirality is present a $TB$ measurement would provide the easiest way to detect a gravitational wave background. We speculate on the theoretical implications of such an observation.

Introduction

The fact that the standard model is chiral, allied with the belief that all the forces of nature should be unified, leads to the natural suggestion that gravity itself might be chiral \cite{1,2}. Furthermore, the formulation of relativity due to Cartan and Kibble \cite{3}, as well as its development in the Ashtekhar formalism \cite{4}, shows that gravity has a definite propensity for chirality due to the presence of terms in the action exhibiting odd parity. The extent of parity violation, however, remains unresolved, both at the classical and quantum levels \cite{5,6,7}. In this letter we explore some implications of gravitational chirality that would improve the prospects of gravitational wave detection in CMB experiments.

Following the successes in CMB temperature anisotropy mapping, the future of CMB physics now lies in improving polarization measurements \cite{8,9,10}. Polarization can be decomposed into electric ($E$) and magnetic ($B$) modes, with positive and negative parities respectively. Correlators between these modes and with the temperature ($T$) may be obtained, and in the absence of parity violation the only non-vanishing quadratic correlators are $TT$, $TE$, $EE$ and $BB$. Scalar perturbations (i.e. density fluctuations) are known to seed $E$-mode polarization only, and so affect $TT$, $TE$ and $EE$ correlators. Tensor modes, the hallmark of a gravitational wave background, are needed in order to generate $B$-mode polarization. For this reason it has been suggested that a $BB$ measurement would be a choice method for a first detection of gravitational waves. The effect, however, is predicted to be very small, presenting a major experimental challenge, particularly when galactic foregrounds are considered.

But what if parity is violated by gravity? Then one could expect a non-vanishing $TB$ correlator. This correlation may provide the easiest way to detect $B$-mode polarization—and by implication gravitational waves—for the same reason that $TE$ correlators are easier to measure than $EE$ ones \cite{11}: they correlate something big ($T$) with something small ($E$ or $B$), rather than two smaller quantities ($EE$ or $BB$). The proposed $TB$ measurement means nothing short of catching two pigeons with one stone. Should gravity be chiral and should there be a gravitational wave background it would be easier to detect them together, via their combined effects, rather than separately. The catch: if at least one of these two premises is violated then no effect is expected. Either premise on its own would not lead to $TB$, and a lack of $TB$ measurement would not disprove either. What is at stake, however, is of such importance that we believe the issue deserves to be investigated further, experimentally and theoretically. We speculate towards the end of the paper on the theoretical implications of gravitational parity violation.

Parity breaking in linearized gravity and the implications for inflation

We first consider the implications of chiral symmetry breaking in linearized gravity for the production of gravitational waves during inflation. We shall show that a chiral imprint would be left in the gravitational wave background, with dramatic implications for CMB polarization. Consider a metric of the form $g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu})$. Usually the second order action may be written as

$$S = \frac{1}{64\pi G} \int a^2(h^{ij}h_{ij} - h^{ij,k}h_{ij,k}) \, d^3 x, \quad (1)$$

and with expansion

$$h_{ij} = \int \frac{d^3 k}{(2\pi)^3 2k^0} \sum_{r=L,R} A^r(k) \epsilon_{ij}^r h(k, \eta)e^{ik\cdot x} + h.c., \quad (2)$$

the action and Hamiltonian break into left and right components, each real on its own. Circular polarization states in a frame aligned with $\mathbf{k}$ have tensors:

$$\epsilon_{ij}^{R/L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \pm i \\ 0 & \pm i & -1 \end{pmatrix}, \quad (3)$$

and the split in $S$ can be traced to the orthonormality conditions for these tensors.

For $k\eta \ll 1$ we choose boundary condition $h \to e^{-i\omega \eta}$, but more generally in an expanding universe we have equation:

$$v'' + \left(k^2 - \frac{a''}{a} \right) v = 0, \quad (4)$$

with $v(k) \propto ah$. The usual calculation of inflationary vacuum quantum fluctuations relies on the fact that the
action becomes a regular scalar field action with normalizations: \( v = a h / \sqrt{32\pi G} \). Canonical quantization inside the horizon supplies a vacuum fluctuation that can then be followed outside the horizon with the textbook result\(^{16,17}\).

There is nothing in the linearized theory that prevents us from attributing a different gravitational constant to R and L gravitons. They are genuinely independent degrees of freedom and (1) could be replaced by

\[
S = \frac{s^R}{6\pi G^R} + \frac{s^L}{6\pi G^L},
\]
leading to a Hamiltonian:

\[
H = \frac{1}{64\pi} \int \frac{d^3k}{(2\pi)^3 2k^0} \sum_{r=L,R} \frac{k^0}{G_r} \left( A_r^\dagger(k) A_r(k) + \frac{1}{2} \right).
\]

Canonical quantization can be studied as usual and a vacuum fluctuation found, from the boundary condition inside the horizon, and followed until it freezes out. We find the straightforward modification of the standard result:

\[
P_{R/L}(k)k^3 = \frac{4G^{R/L}}{\pi} H^2_{k=aH},
\]
which is scale-invariant when the background is close to de Sitter with H constant. A crucial modification in the normalizations, however, slips into the CMB polarization calculation. For reasons to be made obvious later, we shall parametrize this asymmetry by:

\[
G^{R/L} = \frac{G}{1 \mp i \gamma}
\]

A large \( |\gamma| \) means no measurable chirality. The sign of \( \gamma \) matters and \( \gamma > 0 \) means stronger gravity for R gravitons. If \( |\gamma| < 1 \) then gravity becomes repulsive for one of the R/L modes. For \( |\gamma| = 1 \) the coupling constant for one of the modes diverges and we can ignore the other mode.

It is important to note that for linear polarization the basis change induced by the transformation \( \mathbf{k} \rightarrow -\mathbf{k} \) is such that \( \epsilon^{\dagger}_{ij}(-\mathbf{k}) = \epsilon^{+}_{ij}(\mathbf{k}) \) and \( \epsilon^{\dagger}_{ij}(-\mathbf{k}) = -\epsilon^{+}_{ij}(\mathbf{k}) \). This results in reality conditions \( h_{+}(\mathbf{k}) = h^{+}(\mathbf{k}) \) and \( h_{\times}(\mathbf{k}) = h^{\times}(\mathbf{k}) \). Using the relation \( h_{R/L} = (h^{\pm} \pm i h^{\times})/\sqrt{2} \), this implies the separate reality conditions \( h_{R}(\mathbf{k}) = h^{+}_{R}(\mathbf{k}) \) and \( h_{L}(\mathbf{k}) = h^{+}_{L}(\mathbf{k}) \). In the inflationary setting we have described we therefore have:

\[
\begin{align*}
\langle h_{R}(\mathbf{k})h^{\dagger}_{R}(\mathbf{k}') \rangle &= \delta(\mathbf{k} - \mathbf{k}') P_{R}(k), \\
\langle h_{L}(\mathbf{k})h^{\dagger}_{L}(\mathbf{k}') \rangle &= \delta(\mathbf{k} - \mathbf{k}') P_{L}(k), \\
\langle h_{R}(\mathbf{k})h_{L}(\mathbf{k}') \rangle &= 0
\end{align*}
\]

and obviously \( P_{R}(k) \neq P_{L}(k) \) does not contradict the reality conditions.

**CMB polarization in chiral gravity** We now examine the impact of such chirality upon the CMB. Linear polarization of the radiation is described by the three Stokes parameters \( I, Q, \) and \( U \). The \( I \) component is invariant under right handed rotations \( \psi \) about the line of sight vector \( \mathbf{n} \) while \( Q \) and \( U \) components transform as \( Q' = Q \cos 2\psi + U \sin 2\psi \) and \( U' = -Q \sin 2\psi + U \cos 2\psi \). We can construct two fields with spin-2 symmetry on the sky from the \( Q \) and \( U \) parameters and rotate as

\[
(Q \pm iU)(\mathbf{n}) = e^{\pm 2i\psi}(Q \pm iU)(\mathbf{n})
\]

On the sky the spin-2 fields can be decomposed onto the basis of spin-2 weighted spherical harmonic functions \( \pm Y_{l,m}(\mathbf{n}) \).

\[
(Q \pm iU)(\mathbf{n}) = \sum_{l,m} a_{l\pm2,lm} \pm Y_{lm}(\mathbf{n})
\]

Polarization induced by Thomson scattering of a plane wave perturbation is best considered in the local frame \( \mathbf{\hat{e}}_{1}, \mathbf{\hat{e}}_{2}, \mathbf{\hat{e}}_{3} \) whose axis \( \mathbf{\hat{e}}_{3} \) is aligned with plane wave vector \( \mathbf{k} \)\(^{19,20}\). In the aligned frame only the Q Stokes parameter is generated and its magnitude is proportional to \( (1 - \mu^2)e^{\pm i\phi} \) where \( \mu \) is the angle between the plane wave and the outgoing photon momentum \( \mu = \mathbf{k} \cdot \mathbf{\hat{n}} \), \( \phi \) the azimuthal angle of the wavevector and \( m = 0, 1, 2 \) for scalar, vector and tensor sources respectively. The induced polarization can then be rotated and averaged over the whole sky to obtain the tensor generated spin-2 fields\(^{19}\) e.g. for the tensor sources:

\[
(Q \pm iU)(\mathbf{n}, k) = [(1 \mp \mu^2)e^{\pm i\phi}h_{T}(k) + (1 \mp \mu^2)e^{-i\phi}h_{L}(k)] \Delta_{P}(\mu, k)
\]

where \( \Delta_{P}(\mu, k) \) are the tensor polarization source functions for the perturbation to the photon phase space density\(^{21}\) and are obtained as solutions of the full Einstein-Boltzmann system.

It is convenient to define two rotationally invariant, spin-0 fields by acting twice on the spin-2 fields with spin raising and lowering operators \( \hat{\sigma} \) and \( \hat{\bar{\sigma}} \):\(^{8}\)

\[
E(\mathbf{n}) = -\frac{1}{2} [\hat{\sigma}^2(Q + iU)(\mathbf{n}) + \hat{\bar{\sigma}}^2(Q - iU)(\mathbf{n})]
\]

\[
B(\mathbf{n}) = \frac{i}{2} [\hat{\sigma}^2(Q + iU)(\mathbf{n}) - \hat{\bar{\sigma}}^2(Q - iU)(\mathbf{n})]
\]

The two rotationally invariant fields have opposite parity with respect to fields with opposite spin. Scalar perturbations give \( \hat{\sigma}^2(Q + iU) = \hat{\bar{\sigma}}^2(Q - iU) \) since the spin-2 fields generated have no parity dependence. Thus scalars do not source the B field. Tensors however generate the parity sensitive B-field due to the extra \( e^{\pm i2\phi} \) dependence.

Solving for the \( E, B \) and \( T \) fields at late time and expanding onto spherical harmonic coefficients \( a_{lm}^{T,E,B} \) one can define the present day angular power spectra \( C_{\ell}^{XY} = 1/(2\ell + 1) \sum_{m} (a_{\ell m}^{X} a_{\ell m}^{Y}) \) for all correlations of the three fields. Here we list only the cross-correlation spectra which are of particular interest

\[
C_{\ell}^{TE} = 8\pi \int dk P^{+}(k) \Delta_{\ell}^{T}(k, \eta_0) \Delta_{\ell}^{E}(k', \eta_0),
\]

\[
C_{\ell}^{TB} = 8\pi \int dk P^{+}(k) \Delta_{\ell}^{T}(k, \eta_0) \Delta_{\ell}^{B}(k', \eta_0),
\]

\[
C_{\ell}^{EB} = 8\pi \int dk P^{-}(k) \Delta_{\ell}^{E}(k, \eta_0) \Delta_{\ell}^{B}(k', \eta_0)
\]
where the $\Delta_L^X(k, \eta_0)$ are the Legendre expanded radiation transfer functions integrated to the present time. The functions $P^+(k) = P_R(k) + P_L(k)$ and $P^-(k) = P_R(k) - P_L(k)$ are the sum and difference of the $R$ and $L$ mode power spectra under the assumption of isotropy. Following (7) we can write

$$P^+(k) = \frac{P_R(k)}{1 - \frac{1}{\gamma^2}} \quad \text{and} \quad P^-(k) = \frac{P_R(k)}{2\gamma (1 - \frac{1}{\gamma^2})},$$

where $P_R(k)$ is a reference spectrum for the combination of the two gravitational modes for the standard case ($\gamma \rightarrow \infty$). As shown in (12) any tensor contribution to the $TB$ and $EB$ cross-correlation spectra vanishes for the standard parity invariant case. Thus any non-zero $TB$ and $EB$ signal would be an unambiguous indication of new parity breaking physics either in the primordial gravitational wave spectrum or from effects along the line of sight that rotate the polarizations.

Results

Standard line of sight, Einstein-Boltzmann codes (e.g. CAMB) can be easily modified to include the calculation of the $TB$ and $EB$ spectra and in Fig. 1 we show the tensor sensitive combinations obtained for a model with $\gamma = 10$ and tensor to scalar ratio $r = 0.1$. Searching for such a unique signal in the cross-correlation spectra offers some observational advantages. As mentioned previously the $TB$ signal is larger than the pure $BB$ correlation but also does not suffer from noise bias in the absence of noise correlations between total intensity and polarization sensitive measurements. In addition the $TB$ spectrum is free of any ambiguities induced by the coupling of $E$ and $B$-modes due to cut-sky effects in multipole space.

Observationally, the strength of the effect is determined by both the amplitude of the gravitational wave background, usually denoted by the ratio of primordial tensor-to-scalar normalization $r = A_h/A_S$, and the value of our parity breaking measure $\gamma$. The ratio of quadrupole power of the two, opposite parity tensor contributions can be approximated as $C_2^{TB}/C_2^{BB} \approx \alpha_2/\gamma$, where $\alpha_2$ is a depends on the exact cosmology and $\alpha_2 \approx 200$ for a standard $\Lambda$CDM model. In this case the $TB$ contribution will be larger than the $BB$ one for $\gamma < 102$. Alternatively we can examine the overall amplitude of the effect by comparing to the scalar contribution to the total intensity spectrum

$$\frac{C_2^{TB}}{C_2^{TT(s)}} \approx \beta_2 \frac{r}{\gamma} \frac{1}{1 - \frac{1}{\gamma^2}} \sim 1 \times 10^{-3} \frac{r}{\gamma},$$

for $\gamma \gg 1$ and where $\beta_2 \sim 1 \times 10^{-3}$ is again a reference value for a standard $\Lambda$CDM model.

CMB results have not yet reached the sensitivity required to impose interesting limits but most polarisation experiments are now reporting the parity violating spectra $TB$ and $EB$ in addition to the usual four since these also provide useful consistency checks on instrumental and analysis methods. The best constraint so far are from the latest WMAP 5-year results which observed a $TB$ quadrupole $\ell(\ell + 1)C_4^{TB}/2\pi = 1.26 \pm 0.87\mu K^2$. This can be interpreted broadly as a $3\sigma$ upper bound of $-1.5 < C_2^{TB} < 4\mu K^2$ which translates into a limit of $\gamma - 1 > 0.4r$ and $\gamma + 1 < -0.15r$ (where for simplicity we ignored the $|\gamma| > 1$ possibility). We are still in the regime $|\gamma| > 1$, but future data will give much more stringent constraints of $\gamma > 1$; or else provide a detection.

Motivating chiral gravity

What would be the theoretical implications of such an observation? While linearized gravity is all that is needed to deduce a spectrum for tensor fluctuations during inflation, it is generally assumed that this theory is a linearization of a classical non-linear gravitational theory, which is general relativity or a closely related modification. General relativity is parity symmetric, so it is pertinent to ask how radical the modifications of its principles need be to allow parity asymmetry in the form of $G_L \neq G_R$.

Chiral gravitation has been associated with a Chern-Simons term coupled to a dilaton, or the presence of spinors. Note also that in the linearized calculation presented above all...
that may be needed from the full theory is parity violation in the action, as opposed to the field equations, since we’re only concerned with the quantum zero-point fluctuations. Several actions in use, such as the JSS and Holtz actions, already have this property.

We sketch how this may come about, leaving details to [12]. Let us consider the Euclidean action:

\[
S = \frac{1}{32\pi G} \int \left( e^{abcd} e^a \wedge e^b \wedge R^{cd} - \frac{2}{\gamma} e^a \wedge e^b \wedge R^{ab} \right),
\]

where \( e^a \) is the tetrad, \( R_{ab} \) the curvature and \( \gamma \) is the Immirzi parameter (we’ll use latins for the SO(4) group index). Introducing the area form \( \Sigma_{ab} = e^a \wedge e^b \) the action can be written in terms of 2-forms as

\[
S = \frac{1}{16\pi G} \int \Sigma_{ab} \wedge \left( * R_{ab} - \frac{1}{\gamma} R_{ab} \right),
\]

where \( * \) represents the dual form. Splitting into self-dual and anti-self-dual components, we have \( S = S^+ + S^- \) with the illuminating result

\[
S^\pm = \frac{1}{16\pi G} \int \Sigma_{ab}^{\pm} \wedge R_{ab}^{\pm} \left( \pm 1 - \frac{1}{\gamma} \right),
\]

(here \( F^{\pm} = (F \pm *F)/2 \) and \( *F^{\pm} = \pm F^{\pm} \)). Thus, if \( \gamma \) is real, we find a shift in the gravitational constant for + and - with a formula identical to (8) used in our phenomenology. It is tempting to translate this argument into SO(3,1) to conclude that a pure imaginary Immirzi parameter would shift \( G \) for + and - modes (see [6] for closely related work). However this is where we lose connection with our work, because the + and - modes are no longer R and L (which are real). We are currently working on a suitable modification of the standard theory [15] that does connect with this phenomenology in this Letter.

Should chirality be required to appear in the classical field equations the implications could be even more dramatic. In [15] we shall demonstrate the following lemma. Consider theories of gravity in 3+1 dimensions which are:

i) Functions of frame fields \( e^a_\mu \) and a lorentzian connection \( \omega_{\mu}^{ab} \) plus ordinary matter degrees of freedom. ii) Diffemorphism invariant. ii) Invariant under local lorentz transformations. iii) The field equations expressed in terms of \( e^a_\mu \) and \( \omega_{\mu}^{ab} \) contain terms at most first order in derivatives. These in general have parity asymmetric actions, nonetheless, the linearization of the field equations around deSitter spacetime are those of general relativity (with \( G_L = G_R \)). The implication is that if \( TB \) were observed and this were due to a chiral effect in the linearized classical field equations, then one of the very reasonable assumptions in this lemma would have to be violated.

**Conclusion** From the point of view of grand unification it would make much more sense if gravity were chiral [1]. We have shown that should gravity be chiral at leading order in the linearized approximation, it would be easiest to detect gravitational waves precisely by making use of their chirality. Current observations are not yet suitably sensitive, yet the future is bright. But what other effects might gravitational chirality have? One should bear in mind that the parameter \( \gamma \) could be dynamical, with chirality active during inflation (when gravity waves were produced) but switched off nowadays. TB observations would then be the only way in which the theory could be constrained. If, however, gravitational chirality is still present nowadays other interesting observational targets emerge, which we mention in closing. The effect would appear in the quadrupole formula for gravity wave emission, leading to different intensities for L and R. In the case of the millisecond pulsar by symmetry any polarization bias in one direction would be matched by the reverse bias in the opposite direction. The total power emitted would therefore be the same, but a small “rocket effect” would be present. The issue of chirality in a gravity wave background has also been discussed in the context of direct gravitational wave detection [23]. Most existing experiments are polarization myopic, but this could change in future experiments. Other effects on the CMB should also be studied, in the context of specific quantum gravity theories, such as the emergence of circular polarization.

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