Reanalysis of Azimuthal Spin Asymmetries of Meson Electroproduction

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Abstract

The azimuthal spin asymmetries for pion production in semi-inclusive deep inelastic scattering of unpolarized charged lepton beams on longitudinally polarized nucleon targets, are reanalyzed by taking into account an important sign correction to previous formulas. It is found that different approaches of distribution functions and fragmentation functions may lead to distinct predictions on the azimuthal asymmetries measured in the HERMES experiments, thus the available data cannot be considered as a direct measurement of quark transversity distributions, although they still can serve to provide useful information on these distributions and on T-odd fragmentation functions. Predictions of the azimuthal spin asymmetries for kaon production are also presented, with different approaches of distribution and fragmentation functions. The unfavored fragmentation functions cannot be neglected for $K^-$ and $K^0_S$ production in semi-inclusive deep inelastic processes.

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The HERMES collaboration reported the observation of single-spin azimuthal asymmetries for pion production in semi-inclusive deep inelastic scattering (DIS) of unpolarized positron beam on the longitudinally polarized nucleon target [1, 2]. Such azimuthal asymmetries are important because they could provide information on the chiral-odd transversity distributions and T-odd fragmentation functions, which are less known than the usual distribution functions and fragmentation functions. The experimental measurements of quark transversity distributions are difficulty, since the transversity is not directly observable in inclusive DIS processes. It has been proposed that the transversity can manifest itself through the Collins effect [3] of nonzero production between a chiral-odd structure function and a T-odd fragmentation function, which is accessible in some specific semi-inclusive hadron production experiments [3, 4, 5, 6, 7, 8, 9]. There have been a number of studies [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23] to show that the azimuthal asymmetries measured by HERMES are related to the quark transversity distributions of the nucleon. However, for the azimuthal spin asymmetries of meson production in DIS processes of unpolarized charged lepton beams on longitudinally polarized nucleon targets, it just became clear recently [23] that there was an error in the formulas used in previous calculations [11, 17, 18, 19, 21, 22]. It is thus necessary to reanalyze and check the influences on the calculated results and conclusions from such a correction. It is the purpose of this note to update the analysis of our results presented in Ref. [22], and to extend them in order to also predict azimuthal spin asymmetries for kaon production.

The analyzing power of azimuthal spin asymmetry measured by HERMES is defined as

\[
A_{UL}^{W} = \frac{\int [d\phi] W(\phi) \{N^+(\phi) - N^-(\phi)\}}{\int [d\phi] \{N^+(\phi) + N^-(\phi)\}},
\]

where \(UL\) denotes unpolarized beam on a longitudinally polarized target, \(W(\phi) = \sin \phi\) or \(\sin 2\phi\) is the weighting function for picking up the Collins effect, and \(N^+(\phi) (N^-(\phi))\) is the number of events for meson production, as a function of \(\phi\), when the target is positively (negatively) polarized. The azimuthal angle \(\phi\) is the angle between the meson emitting plane and the lepton scattering plane, with the lepton scattering
plane determined by the incident and scattered leptons, and the meson emitting plane determined by the final detected meson and the virtual photon. The virtual photon acts as the common axis of both planes. The analyzing powers of azimuthal asymmetries for pions have been measured by the HERMES collaboration [1, 2], and there is clear evidence for non-zero values of $A^\sin\phi_{UL}$ for $\pi^+$ and $\pi^0$ production, which indicates the existence of azimuthal asymmetries.

One can relate the theoretical calculations with the azimuthal spin asymmetry by

$$A^\sin\phi_{UL} = \left\langle \frac{|P_{h_L^\perp}|}{M_h} \sin \phi \right\rangle = \frac{\Sigma_2}{\Sigma_1}, \quad (2)$$

where a sum over all quark flavors, $\Sigma_i = \sum_q e_q^2 \Sigma_{i_q}^q$, is implicitly assumed, and will be assumed from now on. In the case of unpolarized beam and longitudinally polarized target, $\Sigma_1$ and $\Sigma_2 = \Sigma_{2L} + \Sigma_{2T}$ are given [4, 6, 18, 19] by

$$\Sigma_1 = \left[ 1 + (1 - y)^2 \right] f_1(x) D_1(z), \quad (3)$$

$$\Sigma_{2L} = 4 S_L M \frac{M}{Q} (2 - y) \sqrt{1 - y} \left[ x h_L(x) z H_1^\perp(1)(z) - h_{1L}^\perp(1)(x) \tilde{H}(z) \right], \quad (4)$$

$$\Sigma_{2T} = -2 S_{Tz} (1 - y) h_1(x) z H_1^\perp(1)(z). \quad (5)$$

We should emphasize here that the “−” sign in front of the right side of (5) is the correction [23] to Ref. [22], and it brings significant changes to the numerical results presented there. In the above formulas, the twist-2 distribution functions and fragmentation functions have a subscript “1”: $f_1$ and $D_1$ are the usual unpolarized distribution and fragmentation function; $h_{1L}^\perp(1)(x)$ and $h_1(x)$ are the quark transverse spin distribution functions of longitudinally and transversely polarized nucleons, respectively; $h_L(x)$ is the twist-3 distribution function of a longitudinally polarized nucleon, and it can be split into a twist-2 part, $h_{1L}^\perp(1)(x)$, and an interaction dependent part, $\tilde{h}_L(x)$:

$$h_L(x) = -2 \frac{h_{1L}^\perp(1)(x)}{x} + \tilde{h}_L(x). \quad (6)$$

The fragmentation function $\tilde{H}(z)$ is the interaction dependent part of the twist-3 fragmentation function:

$$H(z) = -2 z H_1^\perp(1)(z) + \tilde{H}(z). \quad (7)$$
The functions with superscript “(1)” denote $p^2_\perp$- and $k^2_\perp$-moments, respectively:

$$h_{1L}^{(1)}(x) \equiv \int d^2 p_\perp \frac{p^2_\perp}{2M^2} h_{1L}(x, p^2_\perp),$$  

(8)

$$H_{1L}^{(1)}(z) \equiv z^2 \int d^2 k_\perp \frac{k^2_\perp}{2M^2} H_{1L}(z, z^2 k^2_\perp),$$  

(9)

where $p_\perp$ and $k_\perp$ are the intrinsic transverse momenta of the initial and final parton in the target and produced hadrons, respectively.

To calculate the spin asymmetries and compare them with experiments, we need the quark distribution functions: $f_1(x), h_1(x), \tilde{h}_L(x),$ and $h_{1L}^{(1)}(x),$ and the fragmentation functions: $D_1(z), H_{1}^{(1)}(z),$ and $\tilde{H}(z).$ Most of the distribution functions and fragmentation functions in these expressions are not known a priori, since they have not been measured yet. Thus we have to make some assumptions and approximations, and this leads to different approaches for the distribution functions and fragmentation functions:

**Leading Approach** is to neglect the $1/Q$ term $\Sigma_{2L}$ in $\Sigma_2$, i.e., we neglect both the $\tilde{h}_L(x)$ and $h_{1L}^{(1)}(x)$ terms in the spin asymmetry $\langle |\vec{P}_{h}\sin \phi| / M_h \rangle$. Then we find immediately that [11, 17]

$$\langle |\vec{P}_{h}\sin \phi| / M_h \rangle = \frac{\Sigma_{2T}}{\Sigma_1} \propto -\frac{h_1(x)H_{1}^{(1)}(z)}{f_1(x)D_1(z)}.$$  

(10)

We emphasize here that the “−” sign has been added, and it brings an opposite trend for the azimuthal asymmetries in comparison with the earlier predictions given in [11, 17].

**Approach 1** is to assume that the twist-2 quark transverse spin distribution function of longitudinally polarized nucleon, $h_{1L}^{(1)}(x),$ is zero [19]. Then it follows that

$$h_L(x) = \tilde{h}_L(x) = h_1(x).$$  

(11)

Notice that in this approach, the spin asymmetry is directly related to the quark transversity distribution $h_1(x),$ without any additional terms.

**Approach 2** is to assume that the interaction dependent twist-3 part, $\tilde{h}_L(x),$ is zero, thus we can also assume that $\tilde{H}(z)$ is zero [10]. Then by neglecting the term
proportional to the current quark mass, one can obtain a Wandzura-Wilczek type relation [24, 25]

$$h_{1L}^{(1)}(x) = -x^2 \int_x^1 \frac{d\xi}{\xi^2} h_1(\xi).$$

It follows, from Eq. (6), that

$$h_L(x) = 2x \int_x^1 \frac{d\xi}{\xi^2} h_1(\xi).$$

Now we need the distribution functions $f_1(x)$ and $h_1(x)$ of the target, and the fragmentation functions $D_1(z)$ and $H_{1\perp}^{(1)}(z)$ for the produced meson. The usual distribution functions $f_1(x)$ have been known with rather high precision, and we adopt two model parametrizations: a quark-diquark model and a pQCD based analysis [22]. The transversity distributions $h_1(x)$ have not been measured yet, but can be roughly predicted, and we adopt those given in the two models, with different flavor and spin structure [22]. Please see Ref. [22] for detailed descriptions and references. For the fragmentation functions of the pion, we adopt the new parametrization presented in [26], with a complete set of both favored and unfavored fragmentation functions.

The so-called Collins fragmentation function $H_{1\perp}^{(1)}(z)$, which describes the transition of a transversely polarized quark into a pion, has not been systematically measured yet, and it is also theoretically less known. There has been a so-called Collins parametrization [3] of this fragmentation function,

$$A_C(z, k_\perp) = \frac{|k_\perp| H_{1\perp}^{(1)}(z, z^2 k_\perp^2)}{M_C D_1^{(1)}(z, z^2 k_\perp^2)} = \frac{M_C}{M_C^2 + |k_\perp|^2},$$

with $M_C$ being a typical hadronic scale around $0.3 \rightarrow 1$ GeV. Assuming a Gaussian type of the quark transverse momentum dependence in the unpolarized fragmentation function

$$D_1^{(1)}(z, z^2 k_\perp^2) = D_1^{(1)}(z) \frac{R^2}{\pi z^2} \exp(-R^2 k_\perp^2),$$

one obtains

$$H_{1\perp}^{(1)}(z) = D_1^{(1)}(z) \frac{M_C}{2M_h} \left( 1 - M_C^2 R^2 \int_0^\infty \frac{dx}{x + M_h^2 R^2} \right),$$

where $R^2 = z^2 / \langle P_{h\perp}^2 \rangle$, an $\langle P_{h\perp}^2 \rangle = z^2 \langle k_\perp^2 \rangle$ is the mean-square momentum that the hadron acquires in the quark fragmentation process. We will set the parameters
$M_C = 0.7$ GeV and $\langle P_{h\perp}^2 \rangle = (0.44)^2$ GeV$^2$ as they are consistent with the spin asymmetry measured at HERMES [20]. It has been recently indicated by Efremov, Goeke, and Schweitzer [23], that the uncertainties related to the magnitude of the Collins fragmentation functions are still rather big, and can vary within a factor of 2. So we consider a further option for Approach 1 and Approach 2 by simply multiplying by a factor 2 to the Collins parametrization (14) and (16).

In Figs. 1-3, we present predictions for the azimuthal asymmetries $A_{UL}^{\sin \phi}$ for semi-inclusive pion production in deep inelastic scattering of unpolarized charged lepton on the longitudinally polarized proton target, with polarization $S = 0.86$ [20]. The upper row corresponds to (a) $\pi^+$, (b) $\pi^0$, and (c) $\pi^-$ production with the distribution functions in the quark diquark model, and the lower row corresponds to (d) $\pi^+$, (e) $\pi^0$, and (f) $\pi^-$ production with the distribution functions in the pQCD based analysis. The thick dotted, dashed, and solid curves correspond to the calculated results for Leading Approach, Approach 1, and Approach 2, and the thin dashed and solid curves correspond to the calculated results for Approach 1 and Approach 2 with an additional factor 2 in the Collins fragmentation functions. Both the favored and unfavored fragmentation functions for the pions are included in the calculation.

In Figs. 1-3, we present predictions for the azimuthal asymmetries $A_{UL}^{\sin \phi}$ for pion production in proton, deuteron, and neutron targets respectively. It can be seen that
Figure 2: The same as Fig. 1, but for the deuteron target with polarization $S = 0.75$.

Figure 3: The same as Fig. 1, but for the neutron target with polarization $S = 0.75$. 
the predictions are significantly different for the five cases: Leading Approach, Approach 1, Approach 2, and two further options for Approach 1 and Approach 2 with the Collins parametrization multiplied by a factor 2. By comparison with the experimental data, we find that the Leading Approach seems to have opposite trend with the data, except that for $\pi^-$. This means that the available data on the azimuthal asymmetries by HERMES collaboration cannot be interpreted as a direct measurement of the transversity distributions of the nucleon, and/or the $1/Q$ term is not negligible at the HERMES energy. Predictions of Approach 1 and Approach 2 can be compatible with the available experimental data by including the uncertainties in the Collins parametrization of T-odd fragmentation functions, but the two approaches differ significantly at large $x$. Both the quark diquark model and the pQCD base analysis give similar predictions for the proton target. So the HERMES observation of the azimuthal asymmetries may still provide information on the transversity distributions of the nucleon at small $x$, as well as on the Collins fragmentation functions. To check the effects from the unfavored fragmentation functions, we also present in Fig. (4) the predictions on the azimuthal asymmetries with only favored fragmen-
tation functions included. We find big difference for $\pi^-$ production by comparing Fig. (4) with Fig. (1). This implies that the unfavored fragmentation functions are important for $\pi^-$ production from a proton target, as the $u$ quark content is dominant at large $x$ in the target, and consequently, the unfavored $u \to \pi^-$ fragmentation is important. Further theoretical and experimental studies are still needed to discriminate between different distribution functions and fragmentation functions as reflected in Approach 1 and Approach 2.

We also mention here that an alternative mechanism for the azimuthal asymmetries has been proposed [27, 28, 29]. Here it was shown that the QCD final-state interactions (gluon exchange) between the struck quark and the proton spectators in semi-inclusive deep inelastic lepton scattering can produce single-spin asymmetries which survive in the Bjorken limit. This provides a physical explanation, within QCD, of these asymmetries, and it also predicts that the initial-state interactions from gluon exchange between the incoming quark and the target spectator system lead to leading-twist single-spin asymmetries in the Drell-Yan process $H_1H_2 \to \ell^+\ell^-X$ [28, 30]. So our study suggests that we also need to consider the new mechanism as a plausible source for the azimuthal asymmetries observed by HERMES.

The HERMES collaboration will also measure the azimuthal asymmetries for the kaon production. Thus it is necessary to make predictions of azimuthal asymmetries $A_{UL}^{\sin \phi}$ for $K^+$, $K^0$, and $K^-$ production from proton, deuteron, and neutron targets respectively. We present our numerical results with different approaches and options for the distribution functions and fragmentation functions in Figs. 5-7. In principle, the flavor structure of the kaon fragmentation functions is more complicated than that of the pion, but the available parametrizations [31, 32, 33] only make distinction between the favored fragmentation functions $D^K$, which are related to the valence quarks of the kaon, and the unfavored fragmentation functions $\hat{D}^K$, which are related to the light-flavor sea quarks of the kaon. For $K^\pm$ we have [31]

$$D^{K^\pm}(z) = 0.31z^{-0.98}(1 - z)^{0.97},$$
$$\hat{D}^{K^\pm}(z) = 1.08z^{-0.82}(1 - z)^{2.55},$$

(17)
and for $K_0^S$ we have \[32\]

\[
D^{K_0^S}(z) = 0.53 z^{-0.57} (1 - z)^{1.87}, \\
\hat{D}^{K_0^S}(z) = 1.45 z^{-0.62} (1 - z)^{3.84}.
\]

The predictions given in Fig. 5-7 are with both favored and unfavored fragmentation functions included. To reflect the influence from the unfavored fragmentation functions for kaon production, we give in Fig. 8 also predictions for the azimuthal asymmetries of kaon production, for a proton target, and with only favored fragmentation functions $D^K$ included. We find, by comparing Fig. 5 with Fig. 8, that the unfavored fragmentation functions also play an important role in $K^-$ and $K_0^S$ production. This is easy to understand, since the favored fragmentation should be $u \rightarrow K^-$ and $s \rightarrow K^-$ for $K^-$ production, and both the $u$ and $s$ quarks belong to the sea content in the nucleon target. The sea in both the quark diquark model and pQCD based analysis is assumed to be unpolarized. Thus the azimuthal asymmetries for $K^-$ production should have zero values in the two models with only favored fragmentation functions included. But the situation will be different if both favored and unfavored fragmentation functions are included, as the unfavored $u$ fragmentation will be enhanced due to the dominant $u$ quark content at large $x$ in the proton. The valence $u$ quark is positively polarized in both the quark diquark model and the pQCD based analysis, thus the $K^-$ production is sensitive to the unfavored fragmentation functions. Therefore we conclude that the unfavored fragmentation functions cannot be neglected for $K^-$ and $K_0^S$ production in semi-inclusive DIS process, and this is different from the predictions in [23].

In summary, we reanalyzed the azimuthal spin asymmetries for pion production in deep inelastic scattering of unpolarized charged lepton beam on the longitudinally polarized nucleon target, by taking into account an important sign correction to previous formulas. We find that this sign correction causes significant changes to the predictions, and we also find that the predictions are very different with different approaches for distribution functions and fragmentation functions. With the result that now it is difficult to use this process for a direct measurement of transversity
Figure 5: The azimuthal asymmetries $A_{UL}^{\text{pin} \phi}$ for semi-inclusive kaon production in deep inelastic scattering of unpolarized charged lepton on the longitudinally polarized proton target, with polarization $S = 0.86$. The upper row corresponds to (a) $K^+$, (b) $K^0_S$, and (c) $K^-$ production with the distribution functions in the quark diquark model, and the lower row corresponds to (d) $K^+$, (e) $K^0_S$, and (f) $K^-$ production with the distribution functions in the pQCD based analysis. The thick dotted, dashed, and solid curves correspond to the calculated results for Leading Approach, Approach 1, and Approach 2, and the thin dashed and solid curves correspond to the calculated results for Approach 1 and Approach 2 with an additional factor 2 in the Collins fragmentation functions. Both the favored and unfavored fragmentation functions for the kaons are included in the calculation.

at large $x$. As a result of similarity between the results of Approach 1 and Approach 2 at small $x$, one can still attribute the recent HERMES measurements as a rough estimate of the transversity distributions of the nucleon at small $x$, thus the HERMES data can be used to provide some useful constraints on the transversity distributions and on the T-odd Collins fragmentation functions. Further theoretical and experimental studies are still necessary to reveal various distribution functions and fragmentation functions. This includes taking into account new physical mechanisms in these processes [27, 28, 29, 30]. Predictions for the azimuthal spin asymmetries of kaon productions are also presented with different approaches and options for the distribution functions and fragmentation functions, and we conclude that the unfa-
Figure 6: The same as Fig. 5, but for the deuteron target with polarization $S = 0.75$.

Figure 7: The same as Fig. 5, but for the neutron target with polarization $S = 0.75$. 
Figure 8: The same as Fig. 5, but with only favored fragmentation functions for the kaons being included.

Favored fragmentation functions cannot be neglected for $K^-$ and $K^0_S$ productions in semi-inclusive deep inelastic scattering.

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