Uncertain KOL Selection with Multiple Constraints in Advertising Promotion

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ABSTRACT Social media marketing is a new mode of marketing industry. KOL (Key Opinion Leader) marketing is a popular way of social media marketing, which is profit-oriented. During the brand building in the early stage of marketing, the product side generally carries out corresponding advertising promotion, so as to achieve the purpose of promoting marketing. As decision-makers, different KOLs selection affects the final promotion effect. Therefore, to understand the advertising promotion effect of social media, this paper considers the instability of the network environment and the uncertainty of a KOL’s promotion ability, solves the advertising promotion problem in the absence of historical data, and provides meaningful insights for decision-makers. First, this paper takes the advertising promotion effect of the KOL belonging to different levels and the cost of advertisers as uncertain variables and constructs an uncertain KOL selection model considering the constraints of income (promotion effect), cost and risk. Second, based on the relevant algorithm of uncertainty theory, the uncertainty of the model is eliminated, the uncertainty model is transformed into a corresponding clear model, and the KOL’s optimal choice at each level is calculated. Finally, the effectiveness and practicability of the model and the algorithm are verified.

INDEX TERMS Advertising promotion, Key Opinion Leader (KOL), Uncertain variable, Uncertain programming

I. INTRODUCTION

In the digital economy, social networks have become the largest information portal [1]. Users can share information with friends and fans through these various social platforms, such as Facebook and YouTube, and each platform has its own unique value [2]. Social media marketing is a way to use social media for marketing. Among the most popular marketing methods is key opinion leader (KOL) marketing. The KOL includes internet celebrities and stars, which refers to those with strong influence and scope in social networks. KOLs generally realize traffic by means of product endorsements, live broadcasts, and so on. Because of the adage "those who get traffic get the world", the number of KOL fans determines a KOL’s income. Therefore, the fan economy has arisen at a historic moment.

With the rapid development of mobile Internet in recent years, especially under the COVID-19 epidemic in 2020, the acceleration of online economy makes social media marketing be the first choice for enterprise marketing. It is found that whether it is film industry [3], smart phone industry [4], tourism industry [5] or fast fashion industry [6], its marketing behavior on social media has a significant positive impact on consumers' purchase intention.

At present, most of the research on social media marketing is to build stochastic programming models. For example, Cohen [7] expressed the multi-project promotion optimization problem as a nonlinear integer programming and took the business rules as constraints. Baard [8] studied the problem of how to maximize the profit of vehicle sales promotion. The author regards this problem as a nonlinear dichotomy matching problem, establishes a related model, and proposes a polynomial of approximate integer programming. The research results show that the promotion method based on this model can improve sales profit. Deng [9] studied the promotion optimization problem of single fast-moving consumer goods (FMCGs). The author regards the problem as a quadratic programming problem with binary variables, and the main problem is to deal with binary decision variables, so as to obtain the global optimal solution. Numerical simulations are carried out to verify the feasibility of the model.

In addition, KOL marketing is a popular marketing method in network marketing, which realizes commercial
value through fans' emotional dependence on KOL [10]. KOL's product recommendation tendency has measurable influence on users' purchase intention [11]. Traffic determines income, so most scholars focus on the diffusion range of marketing content. Scholars mainly focus on network structure characteristics [12], user characteristics [13-16], network user roles [17-18], and influences [19-20]. network diffusion characteristics [21], and driving factors of information diffusion [22-23], etc., and then analyze the information diffusion mechanism, and finally promote marketing. In addition, considering the traffic and marketing effect, scholars have qualitatively analyzed the marketing content, and put forward corresponding promotion strategies, such as sharing brand information [24], intervening in competition information [25], keeping up with market trends [26], predicting marketing objects [27], and recommending interest perception [28]. In particular, Chen [29] studied the selection and scheduling of KOLs in advertising promotion, and analyzed its promotion effect according to the change of audience’s clicks on advertising video links with time, and finally put forward corresponding management suggestions. Research shows that KOL selection and scheduling optimization have great influence on the final promotion effect.

It is not difficult to find that there are all kinds of uncertainties in social media marketing, such as the uncertainty of the network environment. Li [30] carries out optimization research from the point of view of high uncertainty of advertising environment, and puts forward a stochastic programming model for grouping advertising keywords, which groups search advertising words in order to achieve the optimal grouping of keywords, and puts forward meaningful opinions for advertisers' decision-making. On the other hand, Shin [31] solves the high uncertainty of the revisit date in the use of an application launched by a retail company. The author generalizes the optimization from the point of view of optimizing the application, and proposes a robust multi-period inventory model by approximately regarding the problem as a multi-stage random variable. The model provides a robust and stable solution to the worst case. In this direction, most scholars consider the uncertainties in social media marketing as random variables. However, only when the sample size is large enough, can the probability of the event be close to the real frequency. And then, we can determine the probability distribution. Therefore, it is reasonable to use stochastic theory. For example, when we roll the dice, if we roll the dice 4 times, it is difficult for us to ensure that the number of times face up is 2. At this point, we can consider this event as uncertain rather than random. Only when we roll the dice enough times can we make the probability of facing up be close to the real frequency.

Uncertainty theory has been widely used in finance [33-38], the vehicle routing problem [39-40], the intensive production plan [41] and other fields, in order to solve the uncertainty influence. In social media marketing, the products we want to advertise are usually new products. At this time, we do not have enough historical data to determine the probability distribution, so it is incomplete for us to consider uncertain factors as random variables. The uncertainty theory is precisely a theory that uses reliability to measure the possibility of events when the sample size is small. At this point, we can determine the uncertain distribution that the variable satisfies. Therefore, in this paper, the uncertain factors in advertising promotion are considered as uncertain variables.

KOL traffic realization methods mainly include product sales, advertising endorsements, and fan rewards. Different marketing categories have different profit models. In response to the content of the advertising, a company's market department must make decisions, with new product promotion as a background, to release KOL advertising videos for product promotion, thereby achieving the purpose of increasing new product exposure and visibility. The main contributions of this article are as follows. First, this paper solves the most important planning issues in advertising promotion in the absence of historical data. Second, this paper studies the problem of KOL selection in advertising promotion in uncertain environments. The rest of this article is as follows. In the second section, we review some basic concepts of uncertain theory. The third section establishes an uncertain KOL selection model considering promotional income, promotional costs and promotional risks. In the fourth section, the uncertain model is converted to an equivalent clear model. The fifth section verifies the validity and practicability of the model by numerical simulation. Finally, some conclusions and prospects are given.

II. Preliminary

In this section, we will introduce the uncertainty theory. Then, we mainly review some basic definitions and theorems of the uncertainty theory.

In order to rationally deal with the likelihood that something will happen, there exist two axiomatic mathematical systems, one is probability theory and the other is uncertainty theory. Probability theory is a branch of mathematics concerned with the analysis of frequency, while uncertainty theory is a branch of mathematics concerned with the analysis of belief degree. In order to use them to handle a quantity (e.g., stock price, market demand, and product lifetime) in practice, the first action we take is to produce a distribution function representing the likelihood that the quantity falls into the left side of the current point. If you believe the distribution function is to close enough to the real frequency, then you should use probability theory. Otherwise, you have to use uncertainty theory.
Definition 1 (Liu [32]): Setting $\Gamma$ is a non-empty collection. A collection of $L$ consisting of a subset of $\Gamma$ can be called $\sigma$ algebra. If it meets:

1. $\Gamma \subseteq L$;
2. If $\Lambda \in L$, so there is $\Lambda^c \in L$;
3. If $\Lambda_1, \Lambda_2, \ldots, \Lambda_n \in L$, so there is $\Lambda_1 \cup \Lambda_2 \cup \cdots \cup \Lambda_n \in L$ .

The elements in collection $L$ are called events. The uncertain measure is a function mapped from $L$ to $[0, 1]$. In order to ensure the mathematical nature of the uncertain measurement $M[\Lambda]$ , Liu [22] puts forward the following four axioms:

Axiom 1 (Normality Axiom): $M[\Gamma] = 1$ for the universal set $\Gamma$.

Axiom 2 (Duality Axiom): $M[\Lambda] + M[\Lambda^c] = 1$ for any $\Lambda \in L$.

Axiom 3 (Subadditivity Axiom): For any countable sequence of events $\{\Lambda_i\}$, there is

$$M\left(\bigcup_{i=1}^{\infty} \Lambda_i\right) \leq \sum_{i=1}^{\infty} M\{\Lambda_i\}.$$  

Axiom 4 (Product Axiom): Let $(\Gamma_k, L_k, M_k)$ be uncertainty spaces for $k = 1, 2, \ldots$. The product uncertain measure $M$ is an uncertain measure satisfying

$$M\left(\prod_{k=1}^{n} \Lambda_k\right) = \bigwedge_{k=1}^{n} M_k\{\Lambda_k\}$$

where $\bigwedge_{k}$ are arbitrarily chosen events from $L_k$ for $k = 1, 2, \ldots$, respectively.

If $M$ satisfies the above four axioms, the set function $M$ is called the uncertainty measure.

Definition 2 (Liu [32]): An uncertain variable is a function $\xi$ from an uncertainty space $(\Gamma, L, M)$ to the set of real numbers such that $\{\xi \in B\}$ is an event for any Borel set $B$ of real numbers. Note that the event $\{\xi \in B\}$ is a subset of the universal set $\Gamma$, i.e.,

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}.$$  

We know that under the system of probability theory, random variables are characterized by probability distribution function or probability density function. By the same token, in the uncertain theoretical system, uncertain variables are described by uncertain distribution functions.

Definition 3 (Liu [32]): The uncertain distribution function of the uncertain variable $\xi$ is

$$\Phi(x) = M\{\xi \leq x\}.$$  

Among them, $x$ is a real number.

Definition 4 (Liu [32]): An uncertain variable $\xi$ is called normal if it has a normal uncertainty distribution

$$\Phi(x) = (1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3\sigma}}\right))^{-1}, \quad x \in R$$
denoted by $N(e, \sigma)$ where $e$ and $\sigma$ are numbers with $\sigma > 0$. A normal uncertainty distribution is called standard if $e = 0$ and $\sigma = 1$.

Example 1 (Liu [32]): The inverse uncertainty distribution of normal uncertain variable $N(e, \sigma)$ is

$$\Phi^{-1}(\alpha) = e + \sqrt{\frac{3}{\pi}} \ln \frac{\alpha}{1-\alpha}.$$  

According to the properties of uncertain variables, if $\xi_1, \xi_2, \ldots, \xi_n$ are normal uncertain variables, the variable

$$\eta = \sum_{i=1}^{n} \lambda_i \xi_i$$

is still normal uncertain variables. There is

$$\eta \sim N\left(\sum_{i=1}^{n} \lambda_i \xi_i, \sum_{i=1}^{n} \lambda_i \sigma_i\right), \lambda_i > 0, i = 1, 2, \ldots, n.$$  

Theorem 1 (Liu [32]): It is assumed that $\xi_1, \xi_2, \ldots, \xi_n$ are independent uncertain variables, and their corresponding uncertain distribution functions are $\Phi_1, \Phi_2, \ldots, \Phi_n$. Suppose $f(r_1, r_2, \ldots, r_n)$ is a strictly increasing function for $r_1, r_2, \ldots, r_n$. Then, $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ is an uncertain variable, and the inverse function of the uncertain distribution function at $\alpha$ is

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)).0 < \alpha < 1.$$  

Definition 5 (Liu [32]): Expected value is the average value of uncertain variables at the level of uncertain measure, which is used to describe the size of uncertain variables. Set $\xi$ as an uncertain variable, then the expected value of the uncertain variable $\xi$ is

$$E[\xi] = \int_{-\infty}^{\infty} M\{\xi \leq t\} dt - \int_{-\infty}^{0} M\{\xi \leq t\} dt.$$  

When $\xi \sim N(e, \sigma)$, there is $E[\xi] = e$.

Definition 6 (Liu [32]): Set $\xi$ as an uncertain variable, then the expected value of the uncertain variable $\xi$ is $e$, then the variance of the uncertain variable $\xi$ is

$$V[\xi] = E[(\xi - e)^2].$$  

When $\xi \sim N(e, \sigma)$, there is $V[\xi] = \sigma^2$.

III. Uncertain KOL Selection model

Decision-makers typically choose to use advertisements for product promotion. When a company promotes new products, to increase product exposure, increase product popularity and further increase sales, relevant departments will choose different social platforms for product promotion. Different platforms have different marketing strategies. Taking "Tik Tok" as an example, since December 2020, "Tik Tok Star Map" has set the marketing strategy for KOL marketing as follows:

Step 1: Decision makers publish tasks on the platform and set marketing objectives and requirements.

Step 2: KOLs sign up, and bid for the task.
Step 3: Decision makers choose KOLs to promote their products and compensate KOLs.

Step 4: The KOL produces and publishes advertising videos.

Step 5: The “Tik Tok” platform publishes the video.

Among them, step 3 is the core issue in the decision-making process. How to choose the KOL reasonably? The marketing department, as the decision maker, classifies KOLs according to the number of fans, the type of audience, the degree of matching between the content to be promoted and the style of the KOL, and the marketing ability of the KOL. For example, "VIPSHOP" is a platform for selling clothing and cosmetics, and its customers are mainly women. Therefore, decision makers will rank the KOL according to the number of their female fans, the degree of matching between the style of the KOL and promotional products, and the promotion data of previous products of the same type.

The main goal of this study is to show that to increase the exposure and popularity of new products, the products are promoted by advertising content distribution, and the promotion effect of products is measured by the dissemination data from advertising videos. It is assumed that the KOL’s levels are from level 1 to level n. Because the promotion ability of the KOL is affected by uncertain factors such as the network environment, the previous promotion data are not enough to explain the promotion effect in the future. Therefore, in this paper, the expected promotion effects of different levels of the KOL (the promotion scope of advertising videos) are expressed as the uncertain variable \( \xi_1, \xi_2, \ldots, \xi_n \).

Before making a decision, the decision-maker will have a certain expectation of income. At this time, the expected promotion effect of advertising video is the expected income of decision makers. Decision makers should ensure that the average expected return reaches at least one level, which is set as \( \alpha \). Therefore, the constraints to be satisfied by the expected return are

\[
E[\xi_1 x_1 + \xi_2 x_2 + \cdots + \xi_n x_n] \geq \alpha .
\]

Among them, \( E \) represents the expected value of uncertain variable. The proportion of the KOL from the level \( i \) is \( x_i (x = 1,2,\ldots, n) \), which satisfies \( x_1 + x_2 + \cdots + x_n = 1 \). \( x_i \) is the decision variable and \( \alpha \) is the lowest income expectation given by the decision maker in advance.

Advertisers are handed over to the KOL for advertising promotion, and also should pay corresponding wages. As for the salary of the KOL, one is to pay according to the actual effect, and the other is to pay according to the fixed quotation of the talent. It is not difficult to find that the first charging method is more reasonable. At the same time, this method is also a common charging method at present. Therefore, in order to grasp the cost better, the decision-makers need to budget at the early stage of promotion.

It is assumed that the KOL of the same level has the same advertising promotion ability, and the cost for decision-makers to choose the KOL of the same level is the same. That is, the wages paid by decision-makers to the KOL of the same level are the same. According to the rule of "more work, more pay", the cost of decision makers is directly proportional to the promotion ability of the KOL. In addition, the promotion ability of the KOL is an uncertain variable, so the cost that the decision maker has to pay should also be an uncertain variable. Let’s assume that the cost of the decision maker for KOLs at level \( i \) is \( c_i, i = 1,2,\ldots,n \). Therefore, the average cost constraints of decision makers are

\[
M \{c_1 x_1 + c_2 x_2 + \cdots + c_n x_n \geq L \} \leq \gamma .
\]

Among them, \( L \) is the ratio of the maximum cost that decision makers can pay to the total number of the KOL, that is, the highest average cost from the decision maker. \( \gamma \) indicates the maximum tolerance of the decision maker for exceeding the cost budget.

Decisions are accompanied by risks. It is assumed that the fluctuation of the expected return of each decision is a risk. Decision makers should ensure that the risk of product promotion is not too large, that is, it does not exceed the level \( \beta \). So,

\[
V[\xi_1 x_1 + \xi_2 x_2 + \cdots + \xi_n x_n] \leq \beta .
\]

Among them, \( V \) indicates the variance of the expected communication effect. \( \beta \) is the maximum average risk tolerance given by the decision maker in advance.

How to meet the expectation of publicity effect to a greater extent and reduce the risk of breaking the contract is a problem that decision makers need to ponder deeply. The goal of decision makers is to maximize incomes and minimize risks. However, because the units of returns and risks are inconsistent, decision makers often consider minimizing risks on the basis of unit returns. Therefore, this paper takes minimizing \( V/E \) as the goal of decision-making, that is, minimizing the risk on the basis of unit income, which also means maximizing the income when the risk is fixed. Therefore, the objective function is

\[
\min \frac{V[\xi_1 x_1 + \xi_2 x_2 + \cdots + \xi_n x_n]}{E[\xi_1 x_1 + \xi_2 x_2 + \cdots + \xi_n x_n]} .
\]

Based on the above conditions, aiming at the product promotion mode of advertising promotion, the decision-maker constructs an uncertain model considering the expected promotion effect, promotion cost and promotion risk of advertising video, which is INcomes - Costs – Risks (ICR) model, is
IV. Model transformation

The model (5) is a general form of the uncertain KOL selection model, and the decision-makers can’t get the best choice of the KOL of each level according to the model (5). In order to solve the model (5) conveniently, this paper gives the equivalent form of the model (5) according to the uncertainty theory. It is assumed that the advertising promotion ability of the KOL in level \(i\) is an independent normal uncertain variable, so, \(\xi_i \sim N(\mu_i, \sigma_i), i = 1, 2, \ldots, n\). And, it’s uncertain distribution function \(\Phi_i, i = 1, 2, \ldots, n\) is continuous monotonically increasing. It is assumed that the cost paid \(c_i\) by the decision maker to KOL at the level \(i\) is an independent normal uncertain variable, \(c_i \sim N(e_i, \delta), i = 1, 2, \ldots, n\). Also, it’s uncertain distribution function \(\Psi_i, i = 1, 2, \ldots, n\) is continuous monotonically increasing.

According to Definition 5, \(\xi_i\) is an uncertain variable, \(\xi_i \sim N(\mu_i, \sigma_i), i = 1, 2, \ldots, n\). Therefore,

\[
E[\sum_{i=1}^{n} x_i \xi_i] = \sum_{i=1}^{n} x_i E[\xi_i] = \sum_{i=1}^{n} x_i \mu_i .
\]

In the same way, according to Definition 6, there are

\[
\sqrt{\sum_{i=1}^{n} x_i^2 \sigma_i^2 + x_2 \sqrt{\sum_{i=1}^{n} \sigma_i^2} + \ldots + x_n \sqrt{\sum_{i=1}^{n} \sigma_i^2}}
= x_1 \sqrt{\sum_{i=1}^{n} \xi_i^2} + x_2 \sqrt{\sum_{i=1}^{n} \xi_i^2} + \ldots + x_n \sqrt{\sum_{i=1}^{n} \xi_i^2}
= x_1 \sigma_1 + x_2 \sigma_2 + \ldots + x_n \sigma_n
= \sum_{i=1}^{n} x_i \sigma_i .
\]

Therefore, the constraint (III) in model (5) is

\[
V[\sum_{i=1}^{n} \xi_i] = \left(\sum_{i=1}^{n} \sigma_i \right)^2 \leq \beta .
\]

\(c_i\) is a normal uncertain variable, \(c_i \sim N(e_i, \delta), i = 1, 2, \ldots, n\). According to Definition 3, the uncertain inverse distribution function of \(\sum_{i=1}^{n} c_i x_i\) at \(\gamma\) is:

\[
\sum_{i=1}^{n} x_i ^{\Psi^{-1}_i}(\gamma) .
\]

According to the monotonicity of uncertain variable \(c_i\), the constraint (II) in the model (5) can be written as follows:

\[
\sum_{i=1}^{n} x_i \Psi^{-1}_i(\gamma) \leq L .
\]

That is,

\[
\sum_{i=1}^{n} x_i (e_i + \frac{\sqrt{3\delta}}{\pi} \ln \frac{\gamma}{1-\gamma}) \leq L .
\]

Therefore, the model (5) can be transformed into the following form:

\[
\min \left(\sum_{i=1}^{n} \sigma_i x_i \right)^2 / \sum_{i=1}^{n} \mu_i x_i
\]

\[s.t.
\sum_{i=1}^{n} \mu_i x_i \geq \alpha \quad (I)
\]

\[\sum_{i=1}^{n} x_i \left(e_i + \frac{\sqrt{3\delta}}{\pi} \ln \frac{\gamma}{1-\gamma}\right) \leq L \quad (II)
\]

\[\left(\sum_{i=1}^{n} \sigma_i x_i \right)^2 \leq \beta \quad (III)
\]

\(x_1 + x_2 + \ldots + x_n = 1\)

\(x_i \geq 0\)

\(i = 1, 2, \ldots, n\).

V. Numerical example

In this section, this paper will verify the effectiveness of the model and algorithm through numerical experiments.

A. Numerical experiment

To allow decision makers to better apply the uncertain KOL selection model with income, cost and risk constraints, this study uses a numerical example. The advertising promotion effect of KOLs on new products is influenced by many factors, which makes a future promotion effect different from a historical situation. Therefore, it will be inconclusive to predict a future promotion effect based on past advertising data. Therefore, this paper adopts the expert experience method [28] in uncertain programming theory to predict a final promotion effect and then obtains the corresponding uncertain distribution. Thus, the decision makers utilize "Tik Tok" as a publicity platform and divide the KOL into eight levels according to the number of fans, audience types, the matching degree between the promotion content and the style of the KOL and the comprehensive evaluation results of the KOL. For example, if the product to be publicized and promoted is cosmetics, its audience type is...
mainly women. Therefore, decision makers should comprehensively evaluate female fans of KOLs, the matching degree between products and the KOL’s style, and the promotion effect of the KOL’s previous products and further grade them.

Due to the instability of social networks and the uncertainty of KOLs’ promotion ability, KOLs’ previous promotion data cannot explain the promotion effect in the future. Because of the lack of historical data at this time, we use uncertainty theory to solve this problem. According to the expert experience method in uncertainty theory [42], the advertising promotion effect and promotion cost of KOLs estimated by experts according to their own knowledge, ability and social network environment are subject to the normal uncertainty distribution. Then, the least square method [42] in uncertainty theory is used to estimate the parameters. The final parameter results are shown in Table 1.

**TABLE I**

| Level | μ_i | σ_i | ε_i | δ_i |
|-------|-----|-----|-----|-----|
| 1     | 26000 | 50  | 3000 | 33  |
| 2     | 51000 | 100 | 4500 | 87  |
| 3     | 100000| 150 | 9900 | 57  |
| 4     | 340000| 300 | 19000| 75  |
| 5     | 1100000| 350 | 120000| 89 |
| 6     | 8700000| 500 | 780000| 96 |
| 7     | 15000000| 800 | 1400000| 125 |
| 8     | 370000000| 1000 | 290000000| 200 |

The strong fans effect can double the advertising promotion effect of the KOL. Therefore, suppose the decision maker decides to select a total of 20 KOLs for advertising promotion, the expected publicity effect of the KOL (the sum of the number of likes and retweets of the advertising video) is 600 million, and the expected total cost is 80 million RMB.

Suppose the decision maker uses the model (6) to deal with the above problems. The specific model is

\[
\min \left( \frac{\sum_{i=1}^{8} \sigma_i x_i}{\sum_{i=1}^{8} \mu_i x_i} \right)^2
\]

s.t.

\[
\sum_{i=1}^{8} \mu_i x_i \geq 30000000
\]

\[
\sum_{i=1}^{8} x_i (\epsilon_i + \frac{\sqrt{3} \delta_i}{\pi} \ln \frac{0.1}{1-0.1}) \leq 4000000
\]

\[
\left( \sum_{i=1}^{8} \sigma_i x_i \right)^2 \leq 3000000
\]

\[
x_i + x_2 + \ldots + x_8 = 1
\]

\[
x_i \geq 0, i = 1, 2, \ldots, 8.
\]

Among them, the lowest limit of average expectation is 30000000. The maximum tolerance of average risk is 3000000. The cost constraint threshold value acceptable to decision makers is 4000000, and the reliability that exceeds the threshold value cannot exceed 0.1.

The following results are programmed in Matlab R2020a and fmincon algorithm is used. The optimal solution of the model (15) is 0.0249. At this point, the distribution ratio of different levels is shown in Table 2.

**TABLE 2**

| Level | Allocation ratio |
|-------|------------------|
| 1     | 0.0268           |
| 2     | 0.0261           |
| 3     | 0.0255           |
| 4     | 0.0239           |
| 5     | 0.0240           |
| 6     | 0.0308           |
| 7     | 0.0349           |
| 8     | 0.8079           |

At this time, when the number of the KOL selected by the decision maker is \( (1, 1, 1, 1, 1, 1, 1, 13) \), the decision can well meet the leaders' expectations of the advertising effect, the cost is small enough and the risk is low enough. The results in Table 2 show that the highest level KOL should be the main choice for advertising promotion candidates and should involve multiple levels. This can not only ensure the final promotion effect, but also diversify KOL’s grades without causing aesthetic fatigue of the audience.

**B. Effectiveness of the model and the algorithm**
When the set parameters are the same as those in Section B, the iterative convergence diagram of objective function value (OFV) is shown in Fig 1. It can be easily known from the Fig. 1 that the convergence speed of the fmincon algorithm is very fast. In addition, it stops when the number of iterations reaches 25, which shows that the fmincon algorithm is an effective algorithm for solving ICR model. Then, this paper describes the different tendencies of decision makers by changing the values of $\alpha, \beta$ and $L$ respectively, such as the tendency of expected promotion effect, risk tendency and cost expenditure tendency.

Table 3 describes the best choice under different expected popularization tendencies, and calculates the corresponding objective function values. As shown in Fig 2, with the increase of the expected promotion effect of decision makers, the objective function value also increases. And in reality, high income is accompanied by high risk. The higher the income is, the greater increase of the risk is. So, the larger the value of $\alpha$, the larger the objective function value. Therefore, the results of the examples are consistent with the actual ones.

### Table 3

| $\alpha$ | Optimal selection strategy $x^*$ | OFV  |
|---------|------------------|------|
| 3000000 | $x_1 = 0.0268, x_2 = 0.0261$ | 0.0249 |
| 3200000 | $x_1 = 0.0175, x_2 = 0.0174$ | 0.0256 |
| 3400000 | $x_1 = 0.0173, x_4 = 0.0172$ | 0.0262 |
| 3600000 | $x_1 = 0.0175, x_6 = 0.0226$ | 0.0266 |
| 3800000 | $x_1 = 0.0289, x_5 = 0.8616$ | 0.0292 |
| 4000000 | $x_1 = 0.0102, x_7 = 0.0102$ | 0.0336 |

### Table 4

In addition, Table 4 describes the optimal KOL selection for advertising promotion under different risk tendencies. Under a certain risk threshold, the higher the risk threshold that decision makers can bear, the greater the risk increase under unit income, which is consistent with the target value results shown in Table 4. As shown in Table 4 and Fig. 3, when the risk threshold is below 2,400,000, with the increase of the risk threshold, the risk value under unit income is also greater. However, when the risk threshold is set too high, the risk value under unit income will not change basically. Therefore, ICR model is reasonable and effective.
When the cost threshold is high enough, some risks can be avoided, so the risks will not change significantly. As shown in Table 5 and Fig. 4, when \( L < 3000000 \), the objective function value increases with the increase of \( L \). When \( L > 3000000 \), the target value has no obvious change. This shows that the simulation results are consistent with the reality.

| \( \alpha = 30000000 & \beta = 3000000 \) | Optimal selection strategy \( x^* \) | OFV |
|---|---|---|
| \( \beta \) | \( x_1 = 0.0527, x_2 = 0.0422 \) | 0.0239 |
| 1500000 | \( x_3 = 0.0151, x_6 = 0.0187 \) | |
| | \( x_7 = 0.0123, x_9 = 0.8095 \) | |
| | \( x_8 = 0.0462, x_2 = 0.0383 \) | |
| | \( x_1 = 0.0323, x_4 = 0.0190 \) | |
| | \( x_5 = 0.0181, x_8 = 0.0254 \) | |
| 1800000 | \( x_1 = 0.0156, x_8 = 0.8052 \) | 0.0241 |
| | \( x_7 = 0.0391, x_2 = 0.0434 \) | |
| | \( x_3 = 0.0246, x_4 = 0.0244 \) | |
| | \( x_5 = 0.0213, x_6 = 0.0172 \) | |
| | \( x_9 = 0.0183, x_3 = 0.8117 \) | |
| | \( x_1 = 0.0248, x_2 = 0.0247 \) | |
| 2100000 | \( x_3 = 0.0245, x_9 = 0.0241 \) | 0.0242 |
| | \( x_5 = 0.0245, x_6 = 0.0316 \) | |
| | \( x_2 = 0.0393, x_4 = 0.8065 \) | |
| | \( x_1 = 0.0248, x_2 = 0.0247 \) | |
| 2400000 | \( x_3 = 0.0245, x_9 = 0.0241 \) | 0.0250 |
| | \( x_5 = 0.0245, x_6 = 0.0316 \) | |
| | \( x_2 = 0.0393, x_4 = 0.8065 \) | |
| | \( x_1 = 0.0268, x_2 = 0.0261 \) | |
| 2700000 | \( x_3 = 0.0255, x_9 = 0.0239 \) | 0.0250 |
| | \( x_5 = 0.0240, x_6 = 0.0308 \) | |
| | \( x_2 = 0.0349, x_8 = 0.8079 \) | |

**FIGURE 3.** The graph of OFV changing with \( \beta \).

Generally speaking, high investment can bring high income. Moreover, when the cost is high enough, risks can be avoided to a certain extent. Therefore, under a certain cost threshold, the value at risk under unit income increases.

| \( \alpha = 30000000 & \beta = 3000000 \) | Optimal selection strategy \( x^* \) | OFV |
|---|---|---|
| \( \beta \) | \( x_1 = 0.0423, x_2 = 0.0397 \) | 0.0239 |
| 2500000 | \( x_3 = 0.0364, x_4 = 0.0237 \) | |
| | \( x_5 = 0.0209, x_6 = 0.0285 \) | |
| | \( x_7 = 0.0016, x_8 = 0.8069 \) | |
| | \( x_8 = 0.0270, x_2 = 0.0263 \) | |
| | \( x_9 = 0.0257, x_4 = 0.0241 \) | |
| | \( x_1 = 0.0250, x_2 = 0.0248 \) | |
| | \( x_2 = 0.0348, x_3 = 0.8069 \) | |
| 3000000 | \( x_3 = 0.0242, x_9 = 0.0309 \) | 0.0249 |
| | \( x_5 = 0.0348, x_3 = 0.8057 \) | |
| | \( x_7 = 0.0393, x_4 = 0.8057 \) | |
| | \( x_8 = 0.0268, x_2 = 0.0261 \) | |
| 3500000 | \( x_3 = 0.0247, x_9 = 0.0318 \) | 0.0250 |
| | \( x_5 = 0.0393, x_4 = 0.8057 \) | |
| | \( x_7 = 0.0393, x_4 = 0.8057 \) | |
| | \( x_8 = 0.0268, x_2 = 0.0261 \) | |
| 4000000 | \( x_3 = 0.0255, x_9 = 0.0239 \) | 0.0249 |
| | \( x_5 = 0.0240, x_6 = 0.0308 \) | |
| | \( x_7 = 0.0349, x_4 = 0.8079 \) | |
| | \( x_8 = 0.0249, x_2 = 0.0247 \) | |
| 4500000 | \( x_3 = 0.0245, x_9 = 0.0241 \) | 0.0250 |
| | \( x_5 = 0.0245, x_6 = 0.0316 \) | |
| | \( x_7 = 0.0391, x_9 = 0.8067 \) | |

**FIGURE 4.** The graph of OFV changing with \( L \).
To sum up, the experimental results shown in this section confirm the rationality and effectiveness of the ICR model and the fmincon algorithm.

VI. Summary and Prospect

The decision makers of advertising promotions should minimize the promotion risk and make the optimal promotion decision reasonably on the basis of meeting the expectations of advertisers. Considering the instability of the network environment and the uncertainty of the advertising promotion effect, this paper innovatively introduces uncertainty theory and solves the optimization problem during advertising promotion in the absence of historical data. Therefore, aiming at advertising promotion behavior in the stage of brand building, this paper uses "Tik Tok" as the promotion platform, utilizes KOL's advertising promotion ability and advertiser's cost as uncertain variables, constructs an uncertain KOL selection model in advertising promotion, and then eliminates the uncertainty by the laws of operation. Finally, the optimal combination of numerical examples is calculated by running the program, and the scientificity and effectiveness of the model and the algorithm are verified. The research results in this paper can provide a theoretical basis for scientific decision making regarding advertising promotion and guide the rational decision making process for advertising promotion on the "Tik Tok" platform.

There are many ways of social media marketing, such as advertising content distribution, live broadcast or video delivery, brand endorsement and so on. Different marketing methods have different marketing characteristics and are restricted by different factors. Different social platforms also have different marketing strategies. Taking "Tik Tok" as the background, this paper optimized the advertising promotion decision in the stage of brand building. In the following research, the marketing strategies of different platforms and different marketing methods will be carefully studied in order to put forward the optimal marketing plan.

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