Adapted wavelets by parameterization of wavelet bases for estimates of feature extraction of signals

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Abstract- The work demonstrates the parameterization of filters coefficients of compactly supported wavelet to implement in the design of the best wavelet selection. The technique determines the best wavelet by means of a mathematical description and gives a representation in a single parameter for wavelets of finite length. The approach takes an input wavelet that is approximated to the best match. Through a choice of the parameter to adapt to the wavelet coefficients the perfect adaptation of the wavelet is achieved. For wavelet selection, the adaptive approximation carried out through parametrization addresses the challenge of visualization by setting up a Matlab programme that relies on the best selection with high potential to feature extraction of arbitrary signals.

1. Introduction

In this time, the primary reason of death around the globe is malignant growth or carcinoma. The central problem in approximation theory is to represent a function by convenient family of functions. Wavelets are well suited to approximate one-dimensional piece-wise signals when non-linear approximation is allowed. As the original signal can be represented in terms of a wavelet expansion by using wavelet coefficients as a linear combination of wavelet functions, data operations can be performed using just the corresponding wavelet coefficients. Moreover if a best wavelet is chosen so as to adapt to the data, then it gives the best results. The fundamental idea of decomposition of $\mathcal{L}^2(\mathbb{R})$ into approximation or detail spaces is that a function can be decomposed in to an approximation of the original and the details needed to recover the original from the approximate. Wavelets are compactly supported in a diadic space and so it is possible to generate a set of parametric coefficients. The study of parameterization of trigonometric functions were conducted by many authors [7][3][9][10]. These formulae are a convenient way to generate a collection of wavelets with varying approximation properties. The explicit formulae for parameterization can be found in that were constructed for orthogonal wavelets for various lengths. Constrained solution for filter length eight and ten was developed in [5]. Pollen parametrization of wavelet systems is found in [2]. Parameterization of four, six, eight and ten are presented in [4].
As wavelet analysis deals with the expansion of functions in terms of basic functions of a fixed function called mother wavelet, they permit a closer connection between the function being represented and their coefficients. This powerful wavelet basis function created a new approach for mathematicians to answer some of the difficult problems. It has further helped for efficient methods of computation in many applications. Here we study the fault detection of a very important component of all machines - the roller bearings. To know the status of a machine, it is important to monitor the component which has great influence on the accuracy, reliability and life dependency of the machine. Studies [6] have shown that 90 percentage of the faults in roller bearings are related to either an inner or outer race fault. Such defects generate a series of impact vibrations every time a running roller passes over the surfaces of the defects. Hence we focus on the above two faults and their combinations. The construction of optimal wavelets using parameterization and its application to biomedical signal compression, pattern recognition can be found in [8][1][4].

Vibration signals are seen to be a preferable source in discovering the patterns of defects. In other words, any defect away from the normal or perfect condition will result in a change in waveform. Our interest lies in this changed waveform. Therefore we look forward to mathematically describe or model the problem. As long as the pattern is identified, the machine fault associated with the pattern can also be identified. Hence wavelet stands best that can fit a close connection to any other waveforms. Our interest lies in this changed waveform. Therefore we look forward to mathematically represent and their coefficients. This powerful wavelet basis function is a sequence satisfying

\[
\sum_{k \in \mathbb{Z}} |h(k)| < \infty \text{ for some } \epsilon > 0.
\]

The finite length sequence is a sequence satisfying

\[
\sum_{k \in \mathbb{Z}} h_k = \sqrt{2}
\]

Then \( \prod_{j=1}^N H(\omega_{2j}) \) converges uniformly on bounded subsets of \( \mathbb{R} \) to a function \( \tilde{\varnothing} \in L^2(\mathbb{R}) \), where

\[
H(\omega) = \frac{1}{\sqrt{2}} \sum_{k \in \mathbb{Z}} h_k e^{-ik\omega}.
\]

Let \( \varnothing = (\tilde{\varnothing}_n) \). Then for each \( j \in \mathbb{Z} \), \( V_j \) defined by

\[
V_j = \{ (0, \varnothing) \varnothing_{j,k} : z = z(k) k \in \mathbb{Z} \} \in L^2(\mathbb{Z})
\]

is a multi resolution analysis with scaling function \( \varnothing \) and scaling sequence \( h \).

2. Algorithm for the construction of Wavelets from Finite length Filters

The finite length sequence is \( h = \{ h_k \}_{k \in \mathbb{Z}} \), \( h = (h(0), h(1), \ldots, h(N - 1)) \) and \( (h_k) \) satisfies the following

(i) \( \sum_{k \in \mathbb{Z}} h_k = \sqrt{2} \)

(ii) \( \{ R_{2k} h \}_{k=0}^{N/2} \) is orthonormal in \( L^2(\mathbb{Z}) \).

Suppose \( H(\omega) = \frac{1}{\sqrt{2}} \sum_{k \in \mathbb{Z}} h_k e^{-ik\omega} = \frac{1}{\sqrt{2}} \sum_{k=0}^{N-1} h_k e^{-ik\omega} \)

Then \( \prod_{j=1}^N H(\omega_{2j}) \) converges uniformly on \( \mathbb{R} \) to a function \( \tilde{\varnothing} \in L^2(\mathbb{R}) \).

The steps listed out in the setting up of a wavelet system in \( L^2 \) is as follows:

Step 1: Fix a finite sequence \( (h_k) \) satisfying the conditions

1. \( \sum_{k=0}^{N-1} h_k = \sqrt{2} \)
2. \( \{ R_{2k} h \}_{k=0}^{N-1} \) is orthonormal in \( L^2(\mathbb{Z}) \)
Step 2: Determine \( \hat{\theta}(\omega) = \frac{1}{\sqrt{2}} \prod_{j=1}^{\infty} H\left(\frac{\omega}{2j}\right) \) where \( H(\omega) = \frac{1}{\sqrt{2}} \sum_{k=0}^{N-1} h_k e^{-ik\omega} \)

Step 3: Determine \( \theta = (\hat{\theta})^\vee \)

Step 4: Determine \( \psi(t) = \sqrt{2} \sum_{k=0}^{N-1} g_k \, \theta(2t - k) \) where \( g_k = (-1)^k h_{1-k}, k = 0 \text{ to } N-1 \)

This gives the wavelet function \( \psi \) and thus the wavelet basis \( \psi_{1,k} \) can be determined.

3. Construction of Pattern Adapted Wavelet

We consider the function

\[
f(x) = \sin x, \quad f(x) = \cos x, \quad \text{and the triangular wave } f(x) = \begin{cases} x \text{ for } 0 \leq x < \frac{1}{2} \\ 1-x \text{ for } \frac{1}{2} \leq x < 1 \end{cases}
\]

As examples and determine the best approximation wavelets for various choices of the number of terms in the series for approximation.

The program is presented for the case of filter length 4. The following are the steps of the computation.

The codes generated are named as Code 1, NwCode 1, Code 2, Code 3 and Code 4.

Code 1: Generation of filter sequence

Fix a value for the parameter \( \alpha \in [0,2\pi] \). Then the filter coefficients are given by

\[
h_0 = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\alpha, \quad h_1 = \frac{1}{2\sqrt{2}} + \frac{1}{2}\sin\alpha,
\]

\[
h_2 = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\alpha, \quad h_3 = \frac{1}{2\sqrt{2}} - \frac{1}{2}\sin\alpha
\]

The Code contains the formulae for the corresponding filter coefficients of length 4. The file contains \( H = [h_0, h_1, h_2, h_3] \).

If an arbitrary value for the argument \( \alpha \in [0,2\pi] \) is given in Code 1, it returns four values. For the present calculation we consider the argument \( \alpha \) to vary in \([0,\pi]\).

NwCode 1: Construction of Scaling Function and Wavelet Function

The sequence \( \{h_0,h_1,h_2,h_3\} \) so generated is then used for the construction of the trigonometric polynomial \( H(\omega) \) given by the algorithm

\[
H(\omega) = \frac{1}{\sqrt{2}} \sum_{k=0}^{\infty} h_k e^{-ik\omega}
\]

\[
= \frac{1}{\sqrt{2}} (h_0 + h_1 e^{-i\omega} + h_2 e^{-2i\omega} + h_3 e^{-3i\omega})
\]

Our construction of the scaling function \( \theta \) proceeds from the Fourier transform \( \hat{\theta} \) given by Theorem 1.1

\[
\hat{\theta}(\omega) = \frac{1}{\sqrt{2}} \prod_{j=1}^{\infty} H\left(\frac{\omega}{2j}\right)
\]

\( \hat{\theta}(\omega) \) is constructed using the infinite product in \( j \). We can restrict the range of \( j \) to a chosen value \( p \) so that

\[
\hat{\theta}(\omega) = \frac{1}{\sqrt{2}} \prod_{j=1}^{p} H\left(\frac{\omega}{2j}\right)
\]

The programme is carried out using NwCode 1. The value of \( \hat{\theta} \) is obtained by the function HH4. For the present programme \( p = 100 \).

Further we find \( \theta \) by finding the inverse Fourier Transform. It is given by the function ifftHH4. Thus NwCode 1 gives the scaling function \( \theta(\omega) \).
Verification for Scaling function: We check if \( \varnothing(x) = \sqrt{2} \sum_{k=0}^{\infty} h_k \varnothing(2x - k) \) to confirm that \( \varnothing(x) \) is the solution of the scaling equation. If not, increase the value of \( p \) in (6) and get a new value for \( \varnothing \). This is continued till the verification is satisfactory. In the present programme this value is obtained as \( p = 100 \).

In continuation to this, the next programme generates \( \psi \).

The construction of the wavelet function \( \psi \) is carried out by

\[
\psi(x) = \sqrt{2} \sum_{k=0}^{\infty} g_k \varnothing(2x - k)
\]

where \( g_k = (-1)^k h_{1-k} \).

\[
= (h_1, -h_0, h_3, -h_2)
\]

The values of \( g \) and \( \psi \) are displayed.

This completes the construction of the wavelet function of filter length 4 for an arbitrary angular parameter \( \alpha \).

The following step gives the approximation of an input function using the wavelet produced. Code2: Fixing input function

Three input functions are considered. They are the sine wave named as input 1, cosine wave named as input 2, and a triangular wave named as input 3. For convenience of computation all input functions are considered as defined in [0,1]. When \( f(x) = \sin x \) is considered in \([0,2\pi]\), we regard this as the function \( f(x) = \sin 2\pi x \) in [0,1]. Similarly cosine function is considered as \( f(x) = \cos 2\pi x \) in [0,1].

Here we have Code2(inp_number,n) where inp_number represent the number given for the input and \( n \) represents the number of values of the input function.

For example Code2(inp_number 1,10) represents the function \( f(x) = \sin 2\pi x \) represented by 1024 values spread at equal intervals in the domain.

Code3: ErrorEstimation

When a function \( f \) is approximated by wavelet basis \( \psi_{j,k} \), the error is

\[
\left\| f - \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} c_{j,k} \psi_{j,k}(t) \right\|^2
\]

For convenience of computation we consider the square of the norm

\[
\left\| f - \sum_{j=0}^{n} \sum_{k=0}^{2^j-1} c_{j,k} \psi_{j,k}(t) \right\|^2\text{ to represent the error.}
\]

Now the norm in (9) is approximated by the mean square error (mse) which is given by

\[
mse = \frac{1}{N} \sum_{i=1}^{N} [f(t_i) - \tilde{f}(t_i)]^2
\]

where

\[
\tilde{f} = \sum_{j=0}^{n} \sum_{k=0}^{2^j-1} c_{j,k} \psi_{j,k}
\]

We determine the value of \( \alpha \) for which the mse is minimum.

Code3 is used for approximating the signal.

From the fact that wavelet basis \( \psi_{j,k} \) is

\[
\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k) \text{ for integers } j, k \text{ and } k = 0,1,2, \ldots, 2^j - 1
\]

and

\[
c_{j,k} = \int_{-\infty}^{\infty} f(t) \psi_{j,k}(t) dt
\]

and

\[
f = \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} c_{j,k} \psi_{j,k}
\]
In order to carry out the procedure, we need to fix the number of terms in eqn(10) by fixing the maximum of $j$ to be $s$.

Now the signal $f$ has the approximation

$$f(t) \approx \sum_{j=0}^{s} \sum_{k=0}^{2^j-1} c_{j,k} \psi_{j,k}(t) = \tilde{f}(t) \quad (11)$$

In this sum for $j = 0, k = 0$

when $j = 1, k = 0$ to $1$

when $j = 2, k = 0,1,2,3$

and when $j = s, k = 0,1,2, \ldots 2^s - 1$

Then $N = 2^{s+1}$ is the number of terms in the sum

$$\text{(14)}$$

For convenience of presentation, we consider the number of terms in (11) as $N = 2^s - 1$ by assuming that an initial term of value 0 is added in the sum.

Here in Code3 we may fix the pattern, and select the number $N$ of coefficients $c_{j,k}$ for approximation for any argument $\alpha$.

The mean square error is taken as a measure in approximating the signal.

Let $n = n_f$ be the number of points in $[0,1]$ at which the values of the input signal $f$ is given. Let $f_j = f(t_j), j = 1,2, \ldots, n$

then $\text{mse} = \frac{1}{n} \sum_{j=1}^{n} (\tilde{f}_j - f_j)^2 \quad \text{[19]}$

where $\tilde{f}_j = \sum_{j=0}^{s} \sum_{k=0}^{2^j-1} c_{j,k} \psi_{j,k}(t_j)$ and $N = 2^s - 1$

Thus Code3 outputs the mean square error when approximating the pattern with $N$ number of coefficients and $n$ number of input values for the input function.

Code4: Optimization

The value for the parameter $\alpha \in [0,1]$ that gives the best approximation is intended in Code4. For a fixed number of coefficients $N$, in the wavelet approximation, the best value of the parameter say $\alpha_{\text{best}}$ is extracted, the value of $\alpha$ that gives the minimum error when approximating with $N$ terms. The code optimization can be used for this purpose. An initial argument for $\alpha$ and a desired step size are included in the code. In this code it is fixed as $\alpha = 0.001$ and step size = 0.001. The code namely Code4 is called to execute the program. In Code4 we select the input function and the number of sample points of the input function, that is $n$. In addition, the number of coefficients chosen for approximation, that is $N$, is also provided.

The objective function is minimize (mse) of the selected input function given by Eqn (12). As the initial value for the argument $\alpha$ is 0.001, the mse is calculated for the above fed data to generate the error. In the next step, to the initial argument, the step size is added to get the new argument to calculate the new mse. The newly obtained mse is compared with the earlier mse and the lower of the two is now stored as the error minimum. The code runs from the initial value $\alpha = 0.001$ till $\alpha = 3.14$, since the interval considered is $[0,1]$. The mse values so obtained are compared every time and the final value of the mse gives the lowest of all the error values. This is the minimised mse and the argument corresponding to this is given as $\alpha_{\text{best}}$.

4. Illustration and Results:

Consider the input function $\sin(x), f(t) = \sin2\pi t, t \in [0,1]$. The values of $f(t)$ are stored in $n = 1024$ values at points $t_j, j = 1,2\ldots 1024$ spread at equal intervals in $[0,1]$. The number of terms in
the approximation sum in eqn (13) is taken as N. Various values of \( N = 2, 4, 8, \ldots, 1024 \) are considered. Note that when \( n = 1024 \), the indexing variable \( j \) varies from 0 to 9.

The parameter \( \alpha \in [0,1] \) is considered with initial value \( \alpha = 0 \) and incremented by .001. In Code4, input 1 is chosen as the sine wave. It fetches a mean square error for some arbitrary value of the argument \( \alpha \in [0,1] \). Here the input function \( f \) and the approximated function \( \tilde{f} \) are shown at certain values of \( t \). The wavelet is considered with argument \( \alpha = 0.8 \). The mean square error is calculated for the points of the input function taken as \( n = 1024 \). In Table 1, \( N = 3 \) and in Table 2, \( N = 5 \).

| \( t \) | \( f(t) \) | \( \tilde{f}(t) \) | error |
|---|---|---|---|
| 0  | 0   | -0.3960 | 0.3960 |
| 0.1429 | 0.7818 | 0.3800 | 0.4018 |
| 0.2857 | 0.9749 | 0.7051 | 0.1698 |
| 0.4286 | 0.4339 | 0.7156 | -0.2817 |
| 0.5714 | -0.4339 | -0.6869 | 0.1530 |
| 0.7143 | -0.9749 | -0.7075 | -0.2674 |
| 0.8571 | -0.7818 | -0.3966 | -0.3852 |
| 1  | 0   | 0.3908 | -0.3908 |

The \( \text{mse} = 9.8134 \times 10^{-5} \)

| \( t \) | \( f(t) \) | \( \tilde{f}(t) \) | error |
|---|---|---|---|
| 0  | 0   | 0   | 0   |
| 0.9680 | 0.5713 | 0.8109 | -0.2396 |
| 0.2581 | 0.9987 | 0.8232 | -0.1755 |
| 0.4194 | 0.4853 | 0.7180 | -0.2327 |
| 0.5806 | -0.4853 | -0.8121 | 0.3368 |
| 0.7419 | -0.9987 | -0.8417 | -0.1570 |
| 0.9032 | -0.5713 | -0.3286 | -0.2427 |
| 0.9355 | -0.3944 | -0.3285 | -0.0759 |
| 0.9677 | -0.2013 | -0.3250 | 0.1237 |
| 1  | 0   | -0.3249 | 0.3249 |

The \( \text{mse} = 1.6213 \times 10^{-5} \)

Table 3 gives the result of the calculation of the best wavelet with parameter \( \alpha_{\text{best}} \). This is given for various choices of the number of terms in the wavelet approximation series which is denoted as \( N \). In all these cases the input function \( f \) is specified by values of 1024 points in \([0,1]\).
Table 3

| N  | α  | MSE            |
|----|----|----------------|
| 2  | .8970 | .0658         |
| 4  | .8750 | .0350         |
| 8  | .8140 | .0126         |
| 16 | .8740 | .0045         |
| 32 | .7800 | .0012         |
| 64 | .7940 | 3.3707x10^{-4} |
| 128| .7880 | 8.416x10^{-5} |
| 256| .7860 | 2.0031x10^{-5} |
| 512| .7850 | 3.8900x10^{-6} |
| 1024| .7850 | 1.0507x10^{-8} |

Table 4 gives the values of the filter coefficients and mean square errors for different values of parameter α

Table 4

| N  | α  | H = (h0, h1, h2, h3) | Mse       |
|----|----|---------------------|-----------|
| 32 | .5 | (0.7923, 0.5933, -0.0852, 0.1138) | 0.0072 |
| 64 | .5 | (0.7923, 0.5933, -0.0852, 0.1138) | 0.0041 |
| 32 | .75 | (0.7194, 0.6944, -0.0123, 0.0127) | 0.0016 |
| 64 | .75 | (0.7194, 0.6944, -0.0123, 0.0127) | 4.9813x10^{-4} |

Table 5, Table 6, Table 7 and Table 8 gives the corresponding results for the cosine wave f(t) = cos2πt for t ∈ [0,1].

Table 5

| t   | f(t) | f̃(t) | error |
|-----|------|-------|-------|
| 0   | 0    | 0.3974| 0.3974|
| 0.1429 | 0.6235 | 0.4091| 0.2144|
| 0.2857 | -0.2225 | -0.4805| 0.2580|
| 0.4286 | -0.9010 | -0.4935| -0.4175|
| 0.5714 | -0.9010 | -0.4942| -0.4068|
| 0.7143 | -0.2225 | -0.4943| 0.2718|
| 0.8571 | 0.6235 | 0.8799| 0.2564|
| 1   | 1    | 0.9001| 0.9001|
Table 6

| t    | f(t) | \( \hat{f}(t) \) | error |
|------|------|-------------------|-------|
| 0    | 1    | -0.0031           | 0.0031|
| 0.9680 | 0.8208 | 0.4201          | 0.4007|
| 0.2581 | -0.0506 | 0.4264          | 0.3758|
| 0.4194 | -0.8743 | -0.7927         | 0.2339|
| 0.5806 | -0.8743 | -0.6404         | -0.2239|
| 0.7419 | 0.0506  | 0.0838           | 0.0332|
| 0.9032 | 0.8208  | 0.6469           | 0.1739|
| 0.9355 | 0.9190  | 0.6470           | 0.2720|
| 0.9677 | 0.9795  | 0.6562           | 0.3233|
| 1     | 1     | 0.6564           | 0.3436|

Table 7

| N   | \( \alpha \) | MSE |
|-----|--------------|-----|
| 2   | 0.8910       | 0.0863|
| 4   | 1.000        | 0.0432|
| 8   | 1.010        | 0.0146|
| 16  | 0.8880       | 0.0060|
| 32  | 0.7820       | 0.0018|
| 64  | 0.7880       | 4.3822\times10^{-4}|
| 128 | 0.7810       | 9.9225\times10^{-5}|
| 256 | 0.7830       | 2.2395\times10^{-5}|
| 512 | 0.7810       | 4.2584\times10^{-6}|
| 1024| 0.7800       | 8.3015\times10^{-9}|

Table 8

| N   | \( \alpha \) | \( H = (h_0, h_1, h_2, h_3) \) | Mse |
|-----|--------------|---------------------------------|-----|
| 32  | 0.5          | (0.7923, 0.5933, -0.0852, 0.1138)| 0.0034|
| 64  | 0.5          | (0.7923, 0.5933, -0.0852, 0.1138)| 0.0052|
| 32  | 0.75         | (0.7194, 0.6944, -0.0123, 0.0127)| 0.0023|
| 64  | 0.75         | (0.7194, 0.6944, -0.0123, 0.0127)| 5.812\times10^{-4}|

For triangular wave, \( f(x) = \begin{cases} x & \text{for } 0 \leq x < \frac{1}{2} \\
1 - x & \text{for } \frac{1}{2} \leq x < 1 \end{cases} \)

the table 9 gives the validation of the mean square errors for various approximations that gives the best value of the parameter \( \alpha \).
The table 10 gives the filter coefficients and the corresponding mean square errors for a few approximations.

### Table 10

| N  | α   | H = (h₀, h₁, h₂, h₃)                  | Mse              |
|----|-----|--------------------------------------|------------------|
| 32 | 0.5 | (0.7923, 0.5933, -0.0852, 0.1138)     | 4.8678x10⁻⁴      |
| 64 | 0.5 | (0.7923, 0.5933, -0.0852, 0.1138)     | 2.2792x10⁻⁴      |
| 32 | 0.75| (0.7194, 0.6944, -0.0123, 0.0127)    | 2.3678x10⁻⁵      |
| 64 | 0.75| (0.7194, 0.6944, -0.0123, 0.0127)    | 1.2174x10⁻⁵      |

5. Application of approximation using parameters

Effective utilization of the vibration signals helps in identifying and estimating the performance in working of the machine. In comparison to graph plot study, wavelets provide a flexible tool, while graphs do not take into consideration the particular class of signals to be processed. Moreover, wavelets are robust to moderate changes, small shifts and deformations that can reflect a great variation in graphs.

The main advantage of time frequency analysis is discovering the patterns of frequency changes, which usually represent the nature of the signal. Hence any defect away from normal or perfect condition will result in a change of waveform, be it the ECG Signal, or EEG signal or the bearing signals [18][21]. As long as this pattern is identified, the machine fault associated with this pattern can also be identified. Misinterpretation of results may lead to false alarms or the failure to detect anomalous signals. Such negligence may cause fatal breakdown of machines, which could interrupt production and services. An experiment has been designed for use in vibration-based machine fault diagnosis [11][12][13].

5.1 Experimental Setup

In the application considered [11], the vibration signal from the piezoelectric pickup mounted on the test bearing was taken, after allowing for initial running of the bearing. The sampling frequency was 12000 Hz and sample length was 8192 for all speeds and all conditions. The sample length is chosen long enough to ensure data consistency. Sample length of around 10000 was chosen in each case. In techniques like wavelet based feature extraction, the number of samples is preferably 2n. The nearest 2n to 10000 is 8192 and hence, it was taken as sample length. The vibration signal was stored in the data file.
Four cases were considered –

i) Normal bearing (without any fault),
ii) Bearing with inner race defect,
iii) Bearing with outer race defect
iv) Bearing with both inner race and outer race defects.

The data file containing the sample signals of each of the four classes are collected. One sample of each is taken to fit as the pattern for our approximation study.

5.2. Implementation:

Signal transmission is based on transmission of a series of numbers. The series representation of a function is important in all types of signals. The data samples are collected for the four classes of faulty bearings. Consider one single sample in each for our study. The first sample is the good ones (GO) coded as input 4, the inner race (IR) is input 5, the outer race (OR) is input 6, the inner outer race (IOR) is named as input 7 in Code2.

In Signal processing, sampling is the reduction of a continuous signal to discrete signal [20]. A sample refers to a value or set of values at a point in time and/or space. Sampling frequency/rate defines the number of samples per second taken from a continuous signal to a discrete signal. For functions varying with time, if \( f(t) \) is a continuous function(or signal) to be sampled and if sampling be performed by measuring the values of \( f(t) \) every \( T \) seconds, then the sampled function is given by the sequence \( f(nT) \), for every integer value of \( n \).

In our experiment the sampling frequency is 12,000 Hz and so the sampling time is \( T = \frac{1}{12000} \) seconds. In Code2, the sampling sequence \( nT \) is taken along the \( x \)-axis and the data values are plotted along the \( y \)-axis by taking the corresponding values from the data set to obtain the patterns in each classes of data.

Consider one sample signal each from the four classes mentioned above. By selecting inputs 4,5,6,7 respectively inCode 4, the corresponding wavelet can be generated that outputs the mean square error between the sample and the wavelet \([14][15][16]\). The mean square error between the wavelet and each pattern needs to be optimized. The optimization.m file could be used to further obtain the least error for a wavelet of filter length 4 thus giving the parameter \( \alpha \). Let \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) be the corresponding parameters obtained in that order. They are assumed to be the standard values to discriminate the faults for any later study [17]. On applying the code optimization.m we obtained

\[
\alpha_1 = 0.6010, \alpha_2 = 0.4650, \alpha_3 = 0.5000, \alpha_4 = 0.001
\]

6. Conclusion

The samples GO, IR, OR, IOR, is now representable in terms of the parameters \( \alpha_i \), \( i = 1,2,3,4 \)which is the feature extracted for the detection of faulty bearings. The above obtained values of \( \alpha_i \) can be taken as a measure. For any arbitrary pattern to be verified for faultiness the parametric feature has to be extracted. Subsequently a comparative study with the above values identifies the class in which this pattern falls. A pre assigned limit of variation may be fixed to discriminate the amount of fault in the bearing. The procedure presented here bares the simplicity in its representation in terms of a single parameter. This has met with the challenge of the apparent lack of quantitative results in wavelet analysis. Thus the paper could give an easy to use wavelet analysis tool kit for feature extraction.
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