Multi-timescale Distributed Model Predictive Control for Large-Scale Systems and a Case Study

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Abstract. To solve the control problem effectively of complicated large scale system with obvious difference in the dynamic response at each channel, a strategy based on multi-timescale and distributed communication mode is presented. These systems can be regarded as combinations of fast system and slow system, the response speeds of which are in two-time scale. The algorithm takes into account the fast and slow characteristics and the coupling relationship of each subsystem, uses the Nash optimal idea and the multi-time standard information prediction method to realize the optimization control of the whole system. A simulation example is given to illustrate the effectiveness.

1. Introduction
The large-scale interconnected system consisting of a large number of dynamic unit systems is a common system form in the industrial process of chemical, petroleum, metallurgy and other industrial production processes. Its optimization control plays an important role in improving the economic efficiency[1-2]. However, the nonlinear and uncertain characteristics of the components and the complex correlation between each other, as well as the difference in the time scale of the dynamic characteristics of each subsystem make the conventional and single small scale control means difficult to achieve the desired control effect[3].

As we all know, model predictive control based on model prediction, rolling optimization and feedback correction is one of the effective methods to solve the optimal control of complex dynamic systems. And MPC systems are widely used in the manufacturing industries. The ability to incorporate complex objectives as well as constraints in a unified framework makes it an extremely attractive and handy tool[4-5]. To date, several control structures have been presented: including centralized, decentralized, and distributed control structures. The centralized model predictive control (CMPC) is not practical in terms of computational burden, organizational complexity, and fault tolerance. Distributed model predictive control (DMPC) decomposes the design and implementation of optimal control strategy into parallel subsystems of an interconnected large-scale system, which significantly reduces computational complexity. Therefore, it is an effective solution to the optimal control problem of the above interconnected large-scale systems[6-10].

However, because the dynamic characteristics of the interconnection subsystems in the actual objects are often quite different, and all of the above algorithms are based on the same time standard algorithm, which will still be limited to the slow sampling process in the decision making process, and
it is difficult to handle the control of the complex large system with great disparity in the difference between fast and slow dynamic characteristics[11-12]. On the other hand, time-scale multiplicity is a common feature of many systems. For chemical processes, it usually arises due to the strong coupling of physicochemical phenomena[13-14]. A direct application of standard control without taking into account time-scale multiplicity to systems with different time scales may lead to ill-conditioning even the loss of closed-loop stability. It is effective to control the performance based on distributed predictive control combined with the time scale of dynamic behavior of subsystems.

Therefore, the multi-timescale prediction is supposed to cooperate with the dynamics and coupled relation of each subsystem, and the distributed control scheme based on Nash optimization is used to control the entire system in the algorithm. This method takes account of the differences between the dynamic characteristics of the fast and slow subsystem, and takes into account the optimization of system performance and the reduction of computational complexity. It effectively degrades the scale of solving problems and improves efficiency.

2. The concept of multi-time scale

In the actual multivariable large system, it often contains a number of dynamic processes that vary greatly on the time scale, which is called a multi time scale system. It exists in a variety of natural models in nature, such as the transient process of the voltage and frequency of the power system from a few seconds to a few minutes. In the chemical process, the chemical vapor deposition reactor has a different reaction rate of reaction rate. In general, this practical system is described by differential equations, the linear continuous time invariant system is set as:

\[
\dot{x}(t) = Ax(t) + Bu(t)
\]

(1)

Where \(x(t)\) and \(u(t)\) are state variables and control vectors respectively, \(A\) is a constant value matrix of \(n\times n\), \(B\) is a constant value matrix of \(m\times m\).

Assuming that the system (1) is asymptotically stable and contains \(n_1\) small eigenvalues and \(n_2\) large eigenvalues, the formula (1) can be rewritten as follows:

\[
\begin{pmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{pmatrix} =
\begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix}
\begin{pmatrix}
x_1(t) \\
x_2(t)
\end{pmatrix} +
\begin{pmatrix}
B_1 \\
B_2
\end{pmatrix}u(t)
\]

(2)

Set \(\lambda_1\) and \(\lambda_2\) are the maximum values of the small eigenvalue group and the minimum value of the large eigenvalue group, respectively. If

\[
\mu = \left| \frac{\lambda_1}{\lambda_2} \right| < 1
\]

(3)

The system (1) is a dual time scale system. The separation degree of the eigenvalue group of system (2) on time scale is expressed by small parameter \(\mu\). If there are other eigenvalues "cracks" in the eigenvalue spectrum of the system, then the system (1) is the multi time scale system.

3. Multi-timescale Distributed Predictive Control Algorithm

3.1 Model Predictive Control

Model predictive control is formulated as resolving an on-line open loop optimal control problem in moving horizon style. Suppose that the prediction output model of the whole system is described as[15]

\[
Y(k + j|k) = f \left( Y(k), \Delta u_{mm}(k|k) \right) \quad (j = 1, \cdots, P)
\]

(4)

Where \(\Delta u_{mm}(k|k) = [\Delta u_{m1}(k|k), \Delta u_{m2}(k|k), \cdots, \Delta u_{mm}(k|k)]^T\) is the increment of the manipulated variables of the system, \(P\) denotes the prediction horizon, \(M\) denotes the control horizon, \(f\) is the mapping function vector, where the element \(f_i\) satisfied some smooth condition. The performance index of the whole system is
\[
\min_{\Delta u_i(k)} J = \sum_{j=1}^{p} L \left[ y(k+j|k), \Delta u_{m,l}(k|k) \right]
\]

where \(L\) is the nonlinear function of input and output variables. The objective of the whole system is to regulate the system output to the expected values while keeping the performance index minimal. For the large-scale discrete systems composed of \(M\) subsystems, the local performance index for the \(i\)th subsystems can be expressed as

\[
\min_{\Delta u_i,k} J_i = \sum_{j=1}^{p} L_i \left[ y_i(k+j|k), \Delta u_{i,l,M}(k|k) \right]
\]

where \(L_i\) is the nonlinear function of \(\Delta u_{i,l,M}(k|k)\) and \(y_i(k+j|k)\). This indicates the global performance index of the whole system is

\[
\min J = \sum_{i=1}^{m} J_i
\]

Because the dynamic characteristics of the interconnected subsystems in the actual object are often different, the weak role of other subsystems can be ignored, and the \(q\) association effects obtained from other subsystems through the network are considered. At time instant \(k\), the future predictive output of the \(i\)th subsystem can be expressed as

\[
y_{i,PM}(k) = f_i \left( y_{i,PO}(k), \Delta u_{i,l,M}(k); \{ \Delta U_{q,M}(k) \} \right)
\]

where

\[
\{ \Delta U_{q,M}(k) \} = \begin{cases} \Delta u_{i,l,M}(k), \ldots, \Delta u_{i,l,M}(k), & q \leq N - 1 \\ \Delta u_{j,l,M}(k) & j = 1, \ldots, N, j \neq i \end{cases}
\]

It can be seen that the global performance index can be decomposed into a number of local performance indexes, but the output of each subsystem is still related to all the input variables due to the input coupling. The distributed control problem with different goals can be resolved by means of Nash optimal concept. Concretely speaking, the group of control decisions \(u^N = (u_1^N, \ldots, u_m^N)\) is called to be the Nash optimal solution if for all \(u_i, i = 1, \ldots, m\), the following relations are held

\[
J^*_i \left( u_1^N, \ldots, u_i^N, \ldots, u_m^N \right) \leq J_i \left( u_1^N, \ldots, u_{i-1}^N, u_i^N, \ldots, u_m^N \right)
\]

If the Nash optimal solution is adopted, each subsystem doesn’t change its control decision \(u_i\) because it has achieved the locally optimal objective under the above condition, otherwise the local performance index \(J_i\) will degrade. Each subsystem optimizes its objective only using its own control decision assuming that other subsystems’ Nash optimal solutions have been known, that is

\[
\min_{\Delta u_i,k} J_i \left|_{\Delta u^*_i,k} \right|
\]

\[
y_{i,PM}(k) = f_i \left( y_{i,PO}(k), \Delta u_{i,l,M}(k); \{ \Delta U^*_i,M(k) \} \right)
\]

s.t.

\[
\Delta u_{i,min}(k) \leq \Delta u(\cdot) \leq \Delta u_{i,max}(k)
\]

\[
u_{i,min}(k) \leq u_i(\cdot) \leq u_{i,max}(k)
\]

\[
y_{i,min}(k) \leq y_i(\cdot) \leq y_{i,max}(k)
\]

\[
\{ \Delta U_{q,M}(k) \} = \{ \Delta u_{i,l,M}(k), \ldots, \Delta u_{i,l,M}(k) \}, j = 1, \ldots, N, j \neq i, q \leq N - 1
\]
3.2 multi-timescale information prediction

Because the internal structure of the large-scale system is very complex, it is necessary to adopt the corresponding sampling time for the different fast and slow characteristics of each subsystem. The fast system takes a small step and a long time interval. The slow system takes a large time interval. But because of the different time standard, the fast system will adopt the large sampling step for the slow system. It is necessary to take appropriate prediction measures to compensate for the loss of the actual state information of slow systems when the time scales of speed and time do not coincide.

In model prediction, it is assumed that the control amount no longer changes after the control of a \( u(k) \) at a certain time, then the predicted value of the model output of the future \( P \) time is the result of the cumulative effect of the control action at all times in the past.

\[
y(k + i|k) = y_o(k + i|k) + a \Delta u(k) \quad i = 1, \ldots, P
\]  

(11)

Then, the large step size of the slow system can be divided into a series of time steps with the same step length of the fast system. Control increments at each time point \( \Delta u^{\text{wd}}(k) \) approximated to a linear function of a point with a large step

\[
\Delta u^{\text{wd}}(k) = \frac{\Delta u(k) \cdot d}{D}, d = 1, \ldots, D - 1
\]  

(12)

Among them, \( D \) is the sampling time ratio between fast and slow systems. The predictive control value of slow system in virtual time scale is obtained through communication.

Assuming that each subsystem has multiple time scales, and the input function is additive and separable, then the fast system can be divided into two items:

\[
y_{i,pu}(k) = f^1_i(y_{i,pu}(k), \Delta u_{i,m}(k); \Delta u_{i,M}(k), \ldots, \Delta u_{N,M}(k)) + f^2_i(y_{i,pu}(k), \Delta u_{i,M}(k); \Delta u_{i,M}(\bar{k}), \ldots, \Delta u_{N,M}(\bar{k}))
\]

\[
= y^1_{i,pu}(k) + y^2_{i,pu}(k), j = 1, \ldots, q, j \neq i.
\]  

(13)

Among them, \( k \) is the sampling time of fast system, and \( \bar{k} \) is the sampling time of slow system. The first item of the right type represents the control increment sequence of the \( Q \) subsystem with its coupling relation at the actual time, and the second part represents the virtual time. For the slow system, the predicted output value remains unchanged.

\[
y_{r,pu}(k) = f_r(y_{r,pu}(\bar{k}), \Delta u_{r,M}(\bar{k}); \Delta u_{i,M}(\bar{k}), \ldots, \Delta u_{N,M}(\bar{k}))
\]

\[
f = 1, \ldots, q, j \neq \bar{r}.
\]  

(14)

In the decision time of fast and slow time standard reclosing, each fast and slow subsystem notifies each other’s control strategy through the network, and forecasts the control results of the first and formula (12) of the output (11) respectively. At the same time, the fast system obtains the control strategy of the slow system in the virtual time through the multi time standard prediction, and the control of the second items in the composition type (11). As a result, the fast system updates the control strategy that the slow system continues at the previous moment and performs its own optimization solver in the decision time that does not coincide with the fast and slow time. The steps are as follows:

Step 1: Initialization: according to the different fast and slow characteristics of each subsystem, the corresponding sampling time is adopted respectively, and the fast system with the minimum sampling time is selected as the datum time;

Step 2: Communication and multi time standard prediction;

Step 3: Each subsystem collects the required neighborhood information from other subsystems;

Step 4: At each sampling decision time, the fast subsystem is independently solved the respective predictive control optimization subproblems, and the optimal solution of this iteration is obtained;

Step 5: Assignment and implementation;
Step 6: Receding horizon: move horizon to the next sampling time, that is, $k + 1 \rightarrow k$, go to Step 1, and repeat the above steps.

4. CASE STUDY

Then, we consider a plant composed of two continuous stirred-tank reactors in a cascade. The plant consists of two constant volume reactors cooled by a single coolant stream flowing in a cocurrent fashion. An irreversible, exothermic reaction, $A \rightarrow B$, occurs in the two tanks. In this model, it is required to maintain the concentration of liquid in the second tank at a desired level, in spite of variation of inlet concentration to the first tank, by the addition of reactant through a control valve. It is required to control the liquid concentration of two reaction tanks to reach their respective set values[16-17].

Because there are some differences in the response speed of the two reaction tanks, and there is a coupling between the fast and slow models of the system itself, so it is necessary to use the method described above to control it. The process model consists of four nonlinear ordinary differential equations as Eq.(15). The schematic of the system is shown in Figure 1. The operating conditions and parameters for the two-level CSTR are given in Tables 1.

![Continuous stirred tank reactor system](image)

### Figure 1. Continuous stirred tank reactor system.

\[
\begin{align*}
\dot{C}_1 &= \frac{q}{V_1} (C_f - C_1) - k_0 C_1 e^{\frac{E}{RT_f}} \\
\dot{T}_1 &= \frac{q}{V_1} (T_f - T_1) + \frac{(-\Delta H) k_0 C_1}{\rho c_p} e^{\frac{E}{RT_f}} + \frac{\rho c_p}{\rho c_p V_1} q_f \left(1 - e^{-\frac{UA}{\rho c_p V_f}}\right)(T_f - T_1) \\
\dot{C}_2 &= \frac{q}{V_2} (C_1 - C_2) - k_0 C_2 e^{\frac{E}{RT_f}} \\
\dot{T}_2 &= \frac{q}{V_2} (T_1 - T_2) + \frac{(-\Delta H) k_0 C_2}{\rho c_p} e^{\frac{E}{RT_f}} + \frac{\rho c_p}{\rho c_p V_2} q_f \left(1 - e^{-\frac{UA}{\rho c_p V_f}}\right) \times \left[T_1 - T_2 + e^{-\frac{UA}{\rho c_p V_f}} (T_f - T_1)\right]
\end{align*}
\]

| Notation | Value | Description |
|----------|-------|-------------|
| $q$      | 100/ min | flow rate |
| $V_1=V_2$ | 100/ l  | volume of ith tank |
| $C_f$    | 1 mol/l | feed concentration |
| $T_f$    | 350$^\circ$K | feed temperature |
| $T_c$    | 350$^\circ$K | coolant flow rate |
| $k$      | $7.2 \times 10^{10}$ min$^{-1}$ | reaction rate constant |
| $E/R$    | 10000$^\circ$K | E: activation energy R: gas constant |
| $\Delta H$ | $-4.78 \times 10^4$ J/mol | heats of reaction |
| $\rho=p_c$ | 1000 g/l | solution density |
| $C_p=C_p$ | 0.239 J/g$^\circ$K | specific heat |
| $UA_1=UA_2$ | $1.67 \times 10^4$ J/min$^\circ$K | U: heat transfer coefficient A: heat transfer area |
It was found from analyses that the state variables are defined as $x=[C_1, T_1, C_2, T_2]^T$, the disturbances are defined as $d=[T_{cf_1}, T_{cf_2}]^T$, the input is $q_c$, and outputs are defined as $y=[C_1, C_2]^T$. The system satisfies the following physical constraints: $0\leq q_c \leq 130$, $400 \leq T \leq 500$, $0 \leq C \leq 1$. Then, the open-loop responses are shown in Figure 9. The dynamic behavior also illustrates the effect of the nonlinearity, as exemplified by the oscillation for the ±10% change.

Figure 2. System open loop response

From the above simulation results, we can see that this process has strong nonlinearity. When the amount of +10% changes, the system even oscillates, which brings difficulties to the design of the controller. The DMC algorithm is suitable for linear systems, which can be approximated by an approximate linearization method on the equilibrium state of the system (that is, a certain equilibrium point) to form a linear model required by the DMC algorithm. Finally, the system is analyzed and designed according to the linear control theory, and then linearized.

| Table 2 Regulation parameters of the system |
|--------------------------------------------|
| $T(s)$ | $P$ | $M$ | $Q$ | $R$ | $\epsilon$ |
| fast   | 20  | 30  | 6   | 1   | 1           | 0.01   |
| slow   | 40  | 20  | 5   | [1.5 1] | [0.8 1]     | 0.01   |

Figure 3. Exportation concentration $C_1$ and $C_2$
As can be seen from, each quantity gradually reaches the steady state value with the passage of time. The whole range is steadily running at the specified set value, that is, the specifications and quality of the product can be guaranteed. In order to further verify the feasibility and effectiveness of the method, the export concentration C2 is compared in two ways, and random disturbance and noise are added to the simulation process. The real line in Figure 5 is the control effect combined with the algorithm in this paper, and the virtual line is neglecting the effect of the multi time scale prediction algorithm. From this, it can be seen that the results of the algorithm without considering the multi time scale prediction algorithm have a large amplitude and have not been well controlled. On the contrary, the algorithm in this paper has been effectively suppressed to make it more stable, and the control effect has been greatly improved.

5. Conclusion
The multi time standard distributed predictive control algorithm proposed in this paper can be effectively applied to the control of complex large-scale systems. The decomposed subsystems can take the corresponding control cycle and strategy independently according to the fast and slow characteristics of their own systems, and have good control performance.

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