Simultaneous Wireless Information and Power Transfer Under Different CSI Acquisition Schemes

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Abstract—In this work, we consider a multiple-input single-output system in which an access point (AP) performs a simultaneous wireless information and power transfer (SWIPT) to serve a user terminal (UT) that is not equipped with external power supply. In order to assess the efficacy of the SWIPT, we target a practically relevant scenario characterized by imperfect channel state information (CSI) at the transmitter, the presence of penalties associated to the CSI acquisition procedures, and nonzero power consumption for the operations performed by the UT, such as CSI estimation, uplink signaling and data decoding. We analyze three different cases for the CSI knowledge at the AP: no CSI, and imperfect CSI in case of time-division duplexing and frequency-division duplexing communications. Closed-form representations of the ergodic downlink rate and both the energy shortage and data outage probability are derived for the three cases. Additionally, analytical expressions for the ergodically optimal duration of power transfer and channel estimation/feedback phases are provided. Our numerical findings verify the correctness of our derivations, and also show the importance and benefits of CSI knowledge at the AP in SWIPT systems, albeit imperfect and acquired at the expense of the time available for the information transfer.

Index Terms—Simultaneous wireless information and power transfer (SWIPT), energy harvesting, wireless power transfer, TDD, FDD, analog feedback.

I. INTRODUCTION

In conventional wireless systems, the limited battery capacity of mobile devices typically affects the overall network lifetime. Increasing the size of the battery might not be a feasible solution to address this problem, due to a consequent reduction of the portability and increase in the cost of the equipment. For these reasons, the study of novel techniques to prolong the lifetime of the battery has triggered an increased interest in the wireless communications community. In this context, the study of the so-called wireless power transfer (WPT) has recently gained prominence as means to implement a cable-less power transfer between devices [1], either by resonant inductive coupling [2] or by far-field power transfer [3]. The latter approach has seen increasing momentum in recent years, due to its promising potential for longer range transfers. A fundamental breakthrough in this context has been the design of rectifying antennas (rectennas) for microwave power transfer (MPT), key component to achieve an efficient radio frequency to direct current (RF-to-DC) conversion. This has brought to several technological advances, e.g., the design of flying vehicles powered solely by microwave [4], which confirmed that RF-to-DC conversion can not only be performed but also achieve remarkable efficiency. In this regards, commercial products exhibiting an efficiency larger than 50% are already available on the market [5]. Remarkably, performance of state-of-the-art RF-to-DC converters can be even higher, i.e., more larger than 80%, resulting in an impressive potential DC-to-DC efficiency of 45% [6], [7].

A. Related Works

Recent advances in signal processing and microwave technology have shown that far-field power transfer can offer interesting perspectives also in the context of traditional wireless communication systems. For instance, the same electromagnetic field could be used as a carrier for both energy and information, realizing the so-called simultaneous wireless information and power transfer (SWIPT). The potential of this approach has been first highlighted by the studies proposed in [8] and [9]. Therein the trade-off between the two transfers within SWIPT was investigated, in the case of both flat fading and frequency-selective channels. After these pioneering works, several studies have been performed to assess the implementability of receivers that can make use of RF signals to both harvest energy and decode information [10], and analyze the performance of SWIPT in many scenarios, e.g., multi-antenna systems [11], opportunistic networks [12], wireless sensor networks [13]. In particular, the aforementioned trade-off is investigated in multiple-input multiple-output (MIMO) systems for two information and power transfer architectures, i.e., time-switching and power-splitting, for both perfect [11] and imperfect [14] channel state information (CSI). A different approach is considered in [15], where the trade-off between the information and the energy transfer is investigated in a multi-user network under two different constraints, i.e., a constraint expressed in terms of secrecy rate for the former and amount of harvested energy at the receiver for the latter.

A line of work considering orthogonal frequency division multiplexing (OFDM) systems is presented in [16], [17], [18],
where the resource allocation policy at the transmitter is studied for both single and multi-user scenarios, in which the receiver adopts a power-splitting strategy and considers undesired interference as an additional energy resource. Finally, ad-hoc scenarios departing from the standard SWIPT paradigm have been analyzed in works such as [19], where it is assumed that the energy and information transfers are performed by two different devices, operating in frequency-division-duplexing (FDD) mode. More precisely, in [19] the time allocation policy for the two transmitters is studied, under the assumption that the efficiency of the energy transfer is maximized by means of an energy beamformer that exploits a quantized version of the CSI received in the uplink by the energy transmitter. Along similar lines, [20] investigates the effect of imperfect CSI acquisition, training, feedback as well as the resource allocation problem have been thoroughly analyzed [21]. Departing from these observations, in this work we aim at investigating the efficacy of SWIPT for practically relevant scenarios, by relaxing the three aforementioned ideal assumptions. In particular, we target a multi-input single-output (MISO) system consisting of a multi-antenna access point (AP) which transfers both information symbols and energy to a single user terminal (UT) that does not have access to any external power source. Accordingly, in contrast to the previous contributions on this topic, in this work the power harvested by means of the WPT is used by the UT to perform all the necessary signal processing operations for both information decoding and uplink communications. Additionally, we adopt a systematic approach and consider the three main possible scenarios for an AP that engages in a downlink transmission in modern networks, to provide a more complete characterization of the considered system. Consequently, in this work, the performance and feasibility of the SWIPT in a MISO system is studied for the three following cases:

- No CSI available at the AP.
- Time-division duplexing (TDD) communications and CSI acquisition at the AP by means of training symbols.
- FDD communications and CSI acquisition at the AP by means of analog symbols feedback.

We compare these three scenarios for three performance metrics of interest, namely, ergodic downlink rate, energy shortage probability, and data outage probability. Our contributions in this work are as follows:

- We derive closed-form representations for the three performance metrics of interest in all three scenarios and match them to the numerical results.
- We derive the approximations of the ergodically optimal duration of the WPT phase in all the three scenarios as a portion of the channel coherence time.
- Additionally, for the TDD and the FDD scheme, we derive closed-form approximations for the ergodically optimal duration of the channel training/feedback phases, to maximize the downlink rate.
- We show that the TDD scheme can outperform the FDD scheme in SWIPT systems in terms of both downlink rate and data outage probability.

Our numerical findings verify the correctness of our derivations. More specifically, concerning the downlink rate, we show that the performance gap between the numerical optimal solutions and the results obtained by means of our approximations is very small for low to mid signal-to-noise ratio (SNR) values and negligible for high SNR values. Moreover, we show that both TDD and FDD outperform the non-CSI case at any SNR value in terms of downlink rate. This confirms that CSI knowledge at the AP is always beneficial for the information transfer in SWIPT systems, despite both its imperfectness and the resources devoted to the channel estimation/feedback procedures. The correctness of our derivations is further verified when numerically evaluating both the energy shortage and data outage probability of the considered MISO system adopting SWIPT, for which a perfect match of analytic and numerical results is achieved. Finally, it is worth noting that throughout our study TDD consistently outperforms FDD in terms of both downlink rate and data outage probability, confirming the potential of this duplexing scheme for the future advancements in modern networks.

The rest of the paper is organized as follows. In Sec. II we specify the system model. In Sec. III-A, III-B and III-C we specify the system model and derive the downlink rate for the non-CSI, the TDD, and the FDD scheme. In Sec. IV we derive the energy shortage and data outage probability for each scheme. In Sec. V we show and discuss the numerical results. Finally, we conclude in Sec. VI.

Notations: In this paper, we denote matrices as boldface upper-case letters, vectors as boldface lower-case letters. Additionally, we let $[·]^\dagger$ be the conjugate transpose of a vector. All vectors are columns, unless otherwise stated. Furthermore, for a scalar $c \in \mathbb{C}$ we note by $c^*$ its complex conjugate. We use $x \perp y$ to express the orthogonality between vectors $x$ and $y$. We denote a circular symmetric Gaussian random vector with mean $\mu$ and covariance matrix $\Sigma$ as $\mathcal{CN}(\mu, \Sigma)$. The chi-squared distribution with $K$ degrees of freedom is denoted by $\chi^2_K$ and its probability density function (PDF) is given by $f_{\chi^2_K}(x) = \frac{1}{\Gamma(K/2)\sqrt{2^K K!}} x^{K/2-1} e^{-x/2}$ where $\Gamma(q) = \int_0^\infty u^{q-1} e^{-u} du$ is the Gamma function. The non-central chi-squared distribution with $K$ degrees of freedom and non-central parameter $\nu$ is denoted by $\chi^2_{\nu}(\nu)$ and its PDF is given by $f_{\chi^2_{\nu}}(x) = e^{-\nu x/2} I_{\nu/2-1}(\sqrt{\nu x})$, modified Bessel function of the first kind. $\Gamma(q, r) = \int_r^\infty u^{q-1} e^{-u} du$ is the upper incomplete Gamma function.
Q_M(q,r) = \int_{q}^{\infty} \frac{e^{\xi}}{\xi^{M}} \exp\left(\frac{-\xi^2_+ + \xi^2_-}{2}\right) I_{M-1}(q\xi) \, d\xi \text{ is the generalized Marcum Q-function [23].}

II. SYSTEM MODEL

We consider a point-to-point communication system consisting of an AP with L antennas and a UT with single antenna. We denote the downlink channel (from the AP to the UT) as \( h = [h_1, \ldots, h_L]^T \). The channel is assumed to be block fading, with independent fading from block to block. The entries of the channel vector are complex Gaussian (Rayleigh fading), hence \( h \sim \mathcal{CN}(0, I_L) \). Let \( T_C \) be the coherence time length. For simplicity in the notation, we assume that the total number of symbols that can be transmitted within the coherence time is \( T_C \). The AP transmits the symbol \( x \in \mathbb{C}^{L \times 1} \) with a transmit power \( P \), i.e., \( \mathbb{E}[\|x\|^2] = P \). The received signal at the UT is given by \( y = h^\dag x + n \), where \( n \sim \mathcal{CN}(0, N_0) \) is the thermal noise, modeled as a complex additive white Gaussian noise (AWGN). We assume the UT does not have any external power source (such as the battery) and all the power required for the operations to be performed at the UT is provided by the AP through the WPT component of the SWIPT. Accordingly, the UT is equipped with a circuit that can perform two different functions: a) harvest energy from the received RF signal, b) information decoding. As considered in previous literature [11], we assume that the UT cannot harvest energy and decode information from the same signal, at the same time. Hence, a time switching strategy is adopted under which the AP transmits the signals in two phases: the signal sent during the first phase has WPT purposes and is used by the UT to harvest energy, whereas the signal sent in the second phase has information transfer purposes. Note that, throughout this work, we assume that the energy harvested in the first phase (power transfer phase) is the sole source of power for all the subsequent operations performed by the UT (the exact details of these operations will be specified in the following sections).

1) Details of the Power Transfer Phase: During each coherence time interval, the AP first transmits the power wirelessly to the UT for \( \epsilon < T_C \) time slots. First, the AP divides its power \( P \) equally between its L transmit antennas to perform the WPT. Hence the \( L \)-sized transmit symbol during this phase, denoted by \( x_{EH} \), is given by \( x_{EH} = \sqrt{\frac{P}{L}} s \), where \( s \) is a random vector with zero mean and covariance matrix \( \mathbb{E}[ss^\dag] = I_L \). Thus, the power harvested at the UT is given by

\[
P_H = \frac{\beta P\|h\|^2}{L},
\]

where \( \beta \in [0, 1] \) is a coefficient that measures the efficiency of the RF to direct current (RF-to-DC) power conversion [11, 13].

2) Details of the Information Transmission Phase: For the second phase, namely information transmission phase, we adopt a systematic approach and consider the three scenarios. In the first one, the AP transmits the information symbols without the knowledge of CSI (we will refer to this approach as non-CSI scheme). In the second scheme, we consider a TDD communication, in which the downlink and uplink communications are performed over the same bandwidth. Accordingly, first the AP acquires the CSI by evaluating a pilot sequence transmitted by the UT in the uplink, and then engages in the downlink transmission. In the last case, we consider an FDD communication, in which the downlink and uplink communications are performed over two separate bandwidths. Consequently, in this case, UT sends an analog feedback signal in the uplink, carrying the downlink channel estimation, to allow the AP to acquire the CSI and subsequent transmit information symbols.

Under the aforementioned settings, we analyze the performance of the system for two different metrics of interest, namely the downlink rate and outage probability. We provide a detailed analysis of these two metrics in the rest of the paper.

III. ANALYSIS OF THE DOWNLINK RATE

In this section, we analyze the downlink rate for the three considered schemes.

A. Non-CSI Scheme

We first consider the case where the AP transmits the information symbols without the knowledge of CSI. The schematic diagram of this scenario is shown in Fig. 1. Under

\[
\begin{align*}
\alpha_N T_C & \quad \text{Power transfer} \\
(1 - \alpha_N) T_C & \quad \text{Information transfer}
\end{align*}
\]

(a) Operations of the AP.

\[
\begin{align*}
\alpha_N T_C & \quad \text{Energy harvesting} \\
(1 - \alpha_N) T_C & \quad \text{Data decoding}
\end{align*}
\]

(b) Operations of the UT.

Figure 1. Operations of the AP and the UT during the coherence time in the non-CSI scheme.

this scheme, the system utilizes \( \epsilon = \alpha_N T_C \) symbols to transfer power and the remaining symbols to transmit information symbols, where \( 0 < \alpha_N < 1 \). The received signal during the information transmission phase is given by \( y = h^\dag x + n \), where the \( x = \sqrt{\frac{P}{L}} s \) and \( s \sim \mathcal{CN}(0, I_L) \). Note that in the absence of CSI, the AP performs equal power allocation over all its antennas to transmit the information symbol. For the information decoding at the UT, we consider that the power consumption of the circuit components devoted to the decoding is proportional to the number of received symbols (as typically considered in previous works on the subject [23]). Accordingly, we denote the power consumption per decoded symbol at the UT as \( P_D \).

Since the power harvested in the first phase must be sufficient to decode all the information symbols, we have that \( \alpha_N T_C P_H = (1 - \alpha_N) T_C P_D \). Now, if we plug \( T_C \) into this

\[1\]Note that, the words energy and power are used interchangeably in this paper, for the sake of simplicity, in spite of their conceptual difference.

\[2\]The exact value of \( \epsilon \) will be specified later depending upon the mode of operation.
equation then, after some manipulations, we have that the minimum fraction of time that should be devoted to the power transfer, i.e., $\alpha_N$, given by

$$\alpha_N = \frac{\beta P \| \mathbf{h} \|^2 + LP_D}{\beta P \| \mathbf{h} \|^2 + LP_D}.$$  \hspace{1cm} (2)

We now analyze the downlink rate for this scheme. We recall that the AP can transmit $1 - \alpha_N$ symbols for the information transfer. Accordingly, using (4), the downlink rate obtained for the non-CSI scheme is given by

$$R_{NC} = R_{NC}(\alpha_N) = (1 - \alpha_N) \log_2 \left( 1 + \frac{P \| \mathbf{h} \|^2}{N_0 L} \right) = \frac{\beta P \| \mathbf{h} \|^2}{\beta P \| \mathbf{h} \|^2 + LP_D} \log_2 \left( 1 + \frac{P \| \mathbf{h} \|^2}{N_0 L} \right).$$  \hspace{1cm} (3)

**B. TDD Scheme**

We switch our focus to the TDD scheme, whose schematic diagram is shown in Fig. 2. We recall that, in this case, the AP

| Power transfer | Channel estimation | Information transfer |
|----------------|--------------------|----------------------|
| $\alpha_T T_C$ | $\eta_T T_C$ | $(1 - \alpha_T - \eta_T) T_C$ |

(a) Operations of the AP.

| Energy harvesting | Pilots transmission | Data decoding |
|------------------|--------------------|--------------|
| $\alpha_T T_C$ | $\eta_T T_C$ | $(1 - \alpha_T - \eta_T) T_C$ |

(b) Operations of the UT.

Figure 2. Operations of the AP and the UT during the coherence time in the TDD scheme.

should provide the UT with sufficient energy for the latter to be able not only to decode the received data but also to perform all the operations related to the uplink signaling inherent to the TDD scheme. Accordingly, the system utilizes $\epsilon = \alpha_T T_C$ symbols for power transfer, where $0 < \epsilon < 1$. This is followed by the CSI acquisition phase. Since we assume that all the operations are performed within the coherence time, the AP can exploit the reciprocity of the downlink and uplink channels, inherent feature of the TDD scheme. This way, the channel estimated in the uplink can be used to design the beamformer for the downlink transmission. Accordingly, the UT transmits uplink pilots with power $P_E$ for the next $\eta_T T_C$ symbol periods, with $0 < \eta_T < 1$ and $0 < \alpha_T + \eta_T < 1$. The signal received by the AP during the $i$th symbol period (in the uplink pilot transmission phase) is given by $y_E[i] = \sqrt{P_E} \mathbf{h}^* + \mathbf{w}[i]$, where $\mathbf{w}[i] \sim \mathcal{CN}(0, N_0 I_L)$ is the Gaussian noise at the AP. The AP estimates the channel by a minimum variance unbiased (MVU) based estimator [24]. Thus, the channel estimate at the AP is given by

$$\hat{\mathbf{h}} = \frac{1}{\sqrt{P_E \eta_T T_C}} \sum_{i=1}^{\eta_T T_C} \left( \sqrt{P_E} \mathbf{h}^* + \mathbf{w}[i] \right) = \mathbf{h} + \tilde{\mathbf{w}},$$  \hspace{1cm} (4)

where $\tilde{\mathbf{w}} \sim \mathcal{CN}(0, N_0 \eta_T T_C P_E I_L)$ denotes the estimation error.

This is followed by the information transmission phase. The focus of this work is on the performance of the SWIPT under non-ideal system assumptions and practical transmit schemes. Therefore, for simplicity we will assume that the AP beamforms the signal carrying the information symbols with a matched filter precoder (MFP) [25], optimal linear filter for maximizing the SNR. Accordingly, the AP exploits the CSI estimate to design the desired beamforming vector, obtained as $m_T = \hat{\mathbf{h}}/\| \hat{\mathbf{h}} \|$. Then, the received signal at the UT is given by

$$y_T^s = \frac{\hat{\mathbf{h}}^\dagger \mathbf{s} + n}{\| \hat{\mathbf{h}} \|},$$  \hspace{1cm} (5)

where $s$ is the information symbol, with $\mathbb{E} \| s \|^2 = P$. As in the previous case, the UT consumes a power $P_D$ to decode every received information symbol. Thus, since the harvested power by the UT must be sufficient to send the pilot symbols and decode information at the UT, the condition $\alpha_T T_C P_H = \eta_T T_C P_E + (1 - \alpha_T - \eta_T) T_C P_D$ must be satisfied for the TDD scheme. Now, if we plug (1) into this condition then, after some manipulations, we have that the minimum fraction of time that should be devoted to the power transfer, i.e., $\alpha_T$, is given by

$$\alpha_T = \frac{\eta_T LP_E - \eta_T LP_D + LP_D}{\beta P \| \mathbf{h} \|^2 + LP_D}.$$  \hspace{1cm} (6)

Accordingly, we can use (5) to compute the downlink rate for the TDD scheme as

$$R_T = R_T(\alpha_T, \eta_T) = (1 - \alpha_T - \eta_T) \log_2 \left( 1 + \frac{P \| \hat{\mathbf{h}} \|^2}{N_0 \| \mathbf{h} \|^2} \right) = (1 - \eta_T) \frac{\beta P \| \mathbf{h} \|^2 - \eta_T LP_E}{\beta P \| \mathbf{h} \|^2 + LP_D} \log_2 \left( 1 + \frac{P \| \hat{\mathbf{h}} \|^2}{N_0 \| \mathbf{h} \|^2} \right).$$  \hspace{1cm} (7)

Note that the channel training time $\eta_T T_C$ impacts both the accuracy of the estimated channel vector and the remaining available time for information transfer. As a consequence, let $\eta_T^*$ be the duration of the portion of coherence time devoted to the channel training/estimation that maximizes the ergodic downlink rate, defined as

$$\eta_T^* = \arg\max_{\eta_T} \mathbb{E}_{\mathbf{h}, \tilde{\mathbf{w}}} \left[ \left( 1 - \eta_T \right) \frac{\beta P \| \mathbf{h} \|^2 - \eta_T LP_E}{\beta P \| \mathbf{h} \|^2 + LP_D} \log_2 \left( 1 + \frac{P \| \hat{\mathbf{h}} \|^2}{N_0 \| \mathbf{h} \|^2} \right) \right].$$  \hspace{1cm} (8)

The derivation of the exact value of $\eta_T^*$ is very complicated. However, two approximations of this value, valid for high and low SNR, respectively, can be derived as stated in the following result.

**Lemma 1.** At high SNR, $\eta_T^*$ can be approximated as

$$\eta_T^* \approx \frac{N_0 \log_2 e}{B_1 T_C P_E (L - 1)},$$  \hspace{1cm} (9)

where $B_1 = \mathbb{E}_{\mathbf{h}} \left[ \left( 1 + \frac{P \| \mathbf{h} \|^2}{N_0 \mathbb{E} \| \mathbf{h} \|^2} \right) \log_2 \left( 1 + \frac{P \| \mathbf{h} \|^2}{N_0 \mathbb{E} \| \mathbf{h} \|^2} \right) \right]$. At low SNR, it can be approximated as

$$\eta_T^* \approx \frac{N_0}{T_C P_E} \left( -1 + \frac{1}{\log_2 \left( 1 + \frac{(L - 1) \beta P T_C P_E}{LN_0 (\beta P + P_E) - \frac{1}{L}} \right) \right).$$  \hspace{1cm} (10)

**Proof:** See Appendix A
Lemma 1 provides a result whose interpretation is not trivial. In fact, several parameters are present in (9) and (10), thus understanding their impact on the accuracy of the proposed approximations is rather complex. However, some interesting insights can be drawn from Lemma 1, if we focus on the approximations that are introduced in order to derive the final results. First, we note that the impact of $P_D$ on the accuracy of the results is likely negligible, due to the fact that $P \gg P_D$ by construction. Subsequently, let us focus on the quantity $\lambda = \frac{2\eta_F T_C P_F ||h||^2}{N_0}$, introduced in (39). If we fix $N_0$ at the denominator of $\lambda$ then it is straightforward to see that the latter increases with an increase in $P_F$ and $L$. Now, consider the low SNR case. In this case, $N_0$ is very large, thus the approximation $\lambda \approx 0$ is adopted. In practice, the accuracy of this approximation depends on the value of $L$ and $P_F$, i.e., the lower those values are, the more accurate the approximation is. Switching our focus to the high SNR analysis, we observe a phase takes place, in which the AP sends pilot sequences of $\eta_F T_C \in \mathbb{Z}^+$ symbols with power $P$ to the UT for estimating the downlink channel, with $0 < \eta_F < 1$. Finally, the UT feeds back in the uplink the estimated CSI in analog form over the subsequent $\tau_F T_C \in \mathbb{Z}^+$ symbols, where $0 < \tau_F < 1$ and $0 < \alpha_F + \eta_F + \tau_F \leq 1$. Note that, in this work we adopt a simplified model for the uplink communication, for the sake of simplicity of the analysis, and matters of space economy. Specifically, we assume that the feedback signal sent by the UT to the AP experiences an AWGN channel. We note that, this follows the typical approach proposed in the literature for first studies on CSI acquisition schemes based on analog feedback signals [21, 26].

Now, let us analyze the aforementioned steps in detail. Consider the $l$th antenna. We denote the pilot sequence sent over it as $e_l = [e_l[1], \ldots, e_l[\eta_F T_C]]^\top \in \mathbb{C}^{\eta_F T_C}$, $l \in [1, L]$. Naturally, the sequences adopted in this phase are known at both ends of the communications. In particular, without loss of generality, we assume orthogonality between pilot sequences sent over different antennas, i.e., $e_i \perp e_j$, for $i \neq j$. Thus, in order to guarantee their orthogonality, and estimate $L$ independent channel coefficients, a lower bound on the minimum sequence size must be satisfied, i.e., $\eta_F T_C \geq L$. Moreover, the AP equally divides the power $P$ among its $L$ antennas, yielding $||e_l||^2 = \frac{P}{L}$ and thus $||e_l||^2 = \cdots = ||e_L||^2 = \frac{\eta_F T_C P}{L}$. Then, the signal received by the UT during the downlink channel training phase is given by $y^d_{UT,F} = e_1 h_1^* + \cdots + e_L h_L^* + w_{UT}$, where $w_{UT} \sim \mathcal{CN}(0, N_0 \eta_F T_C)$ is the thermal noise at the UT. The UT in turn multiplies the received signal $y^d_{UT,F}$ by $e_l^\top / ||e_l||^2$ to estimate the $l$th channel coefficient, $h_l$. Similar to the previous section, the $L$ downlink channel coefficients are estimated by an MVU based estimator. The estimated channel vector at the UT can be written as $\hat{h}_{UT} = [h_{UT,1}, \ldots, h_{UT,L}]^\top = \mathbf{h} + \hat{w}_{UT}$, with $\hat{w}_{UT} \sim \mathcal{CN}(0, N_0 L \eta_F T_C)$. The estimation error vector at the UT. The power consumption at UT to decode a pilot sequence sent from one of the $L$ transmit antennas is modeled similarly to the previous case, i.e., proportional to $P_D$. Accordingly, the total power consumed in decoding the pilot symbols is given by $\eta_F T_C P_D$.

At this stage, the UT encodes each coefficient by means of a sequence $f_l = [f_l[1], \ldots, f_l[\tau_F T_C]]^\top \in \mathbb{C}^{\tau_F T_C}$, $\forall l \in [1, L]$, such that the $L$ sequences form an orthogonal set, i.e., $f_i \perp f_j$, for $i \neq j$, and $||f_i||^2 = \cdots = ||f_L||^2 = \frac{\tau_F T_C P}{L}$. As before, the adopted sequences are known at both ends of the communications. In particular, in order to guarantee their orthogonality and encode $L$ independent channel coefficients, a lower bound on the minimum sequence size must be satisfied, i.e., $\tau_F T_C \geq L$.

After the encoding, the signal to be fed back by the UT to the AP is obtained as the sum of all the obtained sequences at the previous step, i.e., $x_F = f_1 h_{UT,1} + \cdots + f_L h_{UT,L}$. Consequently, its transmission requires a power given by

$$P_F \frac{L}{L} \left( ||\hat{h}_{UT,1}||^2 + \cdots + ||\hat{h}_{UT,L}||^2 \right) = \frac{P_F ||\hat{h}_{UT}||^2}{L}.$$

(11)

Then, the received signal by the AP is given by $y^d_{AP,F} = f_1 h_{UT,1} + \cdots + f_L h_{UT,L} + w_{AP}$, where $w_{AP} \sim$
\( \mathcal{C}N \left( 0, N_0 I_T T_C \right) \) is the thermal noise at the AP. Now, the latter multiplies the received sequence by \( f_k^2 / |f|^2 \) to estimate \( h_k \). Thus, the estimated channel vector at the AP is obtained as \( \hat{h}_{AP} = h + \hat{w}_{UT} + \hat{w}_{AP} \), where \( \hat{w}_{AP} \sim \mathcal{C}N \left( 0, \frac{N_0 L}{T_C} I_L \right) \).

In particular, we note that \( \hat{w}_{UT} \) and \( \hat{w}_{AP} \) are independent by definition.

Finally, the AP can exploit the knowledge of \( \hat{h}_{AP} \) to derive the desired MFP as before, given by \( \hat{h}_{AP} / \| \hat{h}_{AP} \| \), and use it as beamforming vector while transmitting the information symbols for the remaining \( (1 - \alpha_F - \eta_F - \tau_F) T_C \) symbols.

The received information symbol at the UT is given by

\[
y_{UT,F}^* = \frac{\hat{h}^\dagger \hat{h}_{AP}}{\| \hat{h}_{AP} \|} s + n, \tag{12}
\]

where \( s \) is the information symbol, with \( \mathbb{E} \left[ |s|^2 \right] = P \).

Concerning the energy required to perform all the operations at the UT, as a matter of fact, since the harvested energy must be sufficient to decode the received pilot sequences, feedback the estimated CSI, and decode the subsequent information, we have that the condition \( \alpha_F T_C P_H = \eta_F T_C P_D + \tau_F T_C P_F \| \hat{h}_{UT} \|^2 / L + (1 - \alpha_F - \eta_F - \tau_F) T_C P_D \) must be satisfied. Therefore, if we plug (11) into this condition then, after some manipulations, we have that the minimum duration of the energy transfer/harvesting phase for this case, i.e., \( \alpha_F \), should be

\[
\alpha_F = \frac{\tau_F P_F \| \hat{h}_{UT} \|^2 - \tau_F \beta P \| \hat{h}_{AP} \|^2 + LP_D}{\beta P \| \hat{h}_{AP} \|^2 + LP_D}. \tag{13}
\]

Now, we can use (13) to compute the downlink rate for the FDD scheme as

\[
R_F = R_F(\alpha_F, \eta_F, \tau_F) = (1 - \alpha_F - \eta_F - \tau_F) \log_2 \left( 1 + \frac{P \| \hat{h}^\dagger \hat{h}_{AP} \|^2}{N_0 \| \hat{h}_{AP} \|^2} \right)
= (1 - \eta_F - \tau_F) \beta P \| \hat{h} \|^2 - \tau_F \beta P \| \hat{h}_{UT} \|^2 - \eta_F LP_D
\times \log_2 \left( 1 + \frac{P \| \hat{h}^\dagger \hat{h}_{AP} \|^2}{N_0 \| \hat{h}_{AP} \|^2} \right). \tag{14}
\]

In this case, two parameters describe the duration of the channel estimation phase, i.e., \( \eta_F \) and \( \tau_F \), related to the channel estimation procedures at the UT and the AP, respectively. In practice, these parameters impact both the accuracy of the estimated channel vectors and the remaining available time for information transfer at the AP. As a consequence, let \( (\eta^*_F, \tau^*_F) \) be the optimal couple of parameters that maximizes the ergodic downlink rate, defined as

\[
(\eta^*_F, \tau^*_F) = \arg \max_{\eta_F, \tau_F} \mathbb{E}_{h, \hat{h}} \left[ \log_2 \left( 1 + \frac{P \| \hat{h}^\dagger \hat{h}_{AP} \|^2}{N_0 \| \hat{h}_{AP} \|^2} \right) \times \frac{(1 - \eta_F - \tau_F) \beta P \| \hat{h} \|^2 - \tau_F \beta P \| \hat{h}_{UT} \|^2 - \eta_F LP_D}{\beta P \| \hat{h} \|^2 + LP_D} \right], \tag{15}
\]

where \( \hat{w} = (\hat{w}_{AP}, \hat{w}_{UT}) \). As before, the derivation of the exact value of \( \eta^*_F \) and \( \tau^*_F \) is very complicated. Nevertheless, two approximations of this value, valid for high and low SNR, respectively, can be derived as stated in the following result.

**Lemma 2.** At high SNR, \( \eta^*_F \) and \( \tau^*_F \) can be approximated as

\[
\eta^*_F \approx \sqrt{\left( 1 + \frac{P_F}{\beta P} \right) \frac{P_F}{P} \times \tau^*_F}, \tag{16}
\]

\[
\tau^*_F \approx \frac{N_0 L^2 \log_2 e}{B_5 T_C (L - 1) \frac{P}{(1 + \frac{\beta P}{P})}}, \tag{17}
\]

where \( B_5 = \mathbb{E}_{h} \left[ \log_2 \left( \frac{P \| h \|^2}{N_0} \right) \right] \). At low SNR, they can be approximated as

\[
\eta^*_F \approx \frac{P_F N_0 L}{PT_C \left( \beta P + P_F \right)} \left( -1 + \sqrt{1 + \frac{1 + (1 - \frac{\beta P}{P})}{T_C P (\beta P + P_F)}} \right), \tag{18}
\]

\[
\tau^*_F \approx \frac{2T_C}{\beta P + P_F} \left( \frac{P_F N_0 L}{PT_C (\beta P + P_F)} - 1 + \sqrt{1 + \frac{1 + (1 - \frac{\beta P}{P})}{T_C P (\beta P + P_F)}} \right). \tag{19}
\]

**Proof:** See Appendix B.

Despite the complexity of (16), (17), (18) and (19), some interesting insights can be drawn from the approximations adopted in the derivation in Appendix B following an approach similar to what has been done for Lemma 1. As before, the impact of \( P_D \) on the accuracy of the results is likely negligible, due the fact that \( P \gg P_D \) by construction. Now, consider the quantities \( \lambda_1 = \frac{2T_C \| h \|^2}{N_0 L \| \hat{h}_{AP} \|^2} \) and \( \lambda_2 = \frac{2NP_F \| h \|^2}{N_0 L \| \hat{h}_{AP} \|^2} \), introduced in (49) and (50) respectively. We first focus on the low SNR case. Therein, the approximations \( \lambda_1, \lambda_2 \approx 0 \) are adopted. In this case, a smaller \( P_F \) improves the accuracy of these approximations, whereas no clear insight can be drawn for \( L \). Conversely, in the high SNR case, the approximation \( \lambda_1 \gg 0 \) is adopted. Differently from the previous case, the accuracy of this approximation increases with \( P_F \). A further approximation is introduced in this part of the study, i.e., \( \eta^*_F \approx 0 \) in (50). Accordingly, an additional insight on the impact of the number of antennas on the accuracy of the result in Lemma 2 can be drawn, i.e., the smaller \( L \) the larger the accuracy. Interestingly, this is in contrast with the impact of the same parameter in the TDD case and highlights the expected larger penalty for CSI acquisition that FDD pays w.r.t. TDD as the number of antennas grows. A more detailed discussion on its impact on the accuracy of the results in Lemma 2 is referred to Sec. IV where a comparative study of the downlink rate of the three considered schemes is provided.

**IV. Analysis of the Outage Probability**

In this section, we will study the outage probability for the considered system as a function of the parameters introduced so far, and the downlink rate. In the considered practical SWIPT implementation two possible outage events can occur:

- The harvested energy is not sufficient for all the operations at the UT (channel estimation, pilot transmission/CSI feedback and information decoding), i.e., the UT experiences an energy shortage.
• The harvested energy is sufficient to perform all the operations at the UT, but the achieved downlink rate is smaller than a target value, i.e., the UT experiences a data outage.

We first focus on the case for which energy shortage occurs. Subsequently, we analyze the case for which the harvested energy is sufficient for all the operations at the UT, and compute the data outage probabilities for the three transmit schemes considered in this work. Before we proceed, we remark that, the analytic expressions derived in this section for the outage probabilities as a function of the system parameters are very complicated, and straightforward inference on their behavior is difficult to be drawn. Consequently, as before we defer the discussion on the outage as a function of the system parameters for all the cases considered in this work to Sec. V.

A. Energy Shortage Probability

1) Non-CSI Scheme: Referring to (2), for any given value for $\alpha_N$, the energy shortage probability for the non-CSI case can be expressed mathematically as

$$\mathcal{P}_N^{E,\text{out}}(\alpha_N) = \Pr \left\{ \frac{\alpha_N \beta P \|\hat{w}\|^2}{L} < (1 - \alpha_N) P_D \right\} = \frac{\gamma(L, \frac{(1-\alpha_N)LP_D}{\alpha_N \beta P})}{\Gamma(L)}. \quad (20)$$

where $\gamma(q,r) = \int_0^\infty u^{q-1}e^{-ru}du$ is the lower incomplete Gamma function. The closed-form expression of this probability is derived by considering the cumulative distribution function (CDF) of $\chi^2_{2L}$ if we note that $2\|\hat{w}\|^2 \sim \chi^2_{2L}$.

2) TDD Scheme: Referring to (3), for any given value for $\alpha_T$ and $\eta_T$, the energy shortage probability for the TDD case, denoted by $\mathcal{P}_T^{E,\text{out}}(\alpha_T, \eta_T)$, can be expressed mathematically as

$$\mathcal{P}_T^{E,\text{out}}(\alpha_T, \eta_T) = \Pr \left\{ \frac{\alpha_T \beta P \|\hat{w}\|^2}{L} < (1 - \alpha_T - \eta_T) P_D + \eta_T P_E \right\} = \frac{\gamma(L, \frac{\eta_T LP_E + (1-\alpha_T - \eta_T)LP_D}{\alpha_T \beta P})}{\Gamma(L)}. \quad (21)$$

Using the same approach as for (20), the closed-form expression of the probability in (21) is computed.

3) FDD Scheme: Consider (13). For any given value for $\alpha_F, \eta_F$ and $\tau_F$, the energy shortage probability for the FDD case can be stated mathematically as

$$\mathcal{P}_F^{E,\text{out}}(\alpha_F, \eta_F, \tau_F) = \Pr \left\{ \frac{\alpha_F \beta P \|\hat{w}\|^2}{L} < \frac{\tau_F P_F \|\hat{w}\|^2}{L} + (1 - \alpha_F - \tau_F) P_D \right\}. \quad (22)$$

The following result provides a closed-form expression of (22). However, for the sake of the simplicity of the representation of the result, let us denote $\sigma_1 = \sqrt{\frac{N_0 L}{2P_{F,C} T_F}}$, $\rho_1 = \sqrt{\frac{\tau_F P_F}{\alpha_F \beta P - \tau_F P_F}}$, $\rho_2 = \sqrt{\frac{\tau_F P_F}{\alpha_F \beta P - \tau_F P_F} + \frac{2\tau_F P_F}{\alpha_F \beta P - \tau_F P_F}}$, and $\rho_3 = \sqrt{\frac{2(1-\alpha_F - \tau_F)LP_D}{\alpha_F \beta P - \tau_F P_F}}$.

**Lemma 3.** The energy shortage probability for the FDD scheme, as in (22), can be computed as

$$\mathcal{P}_F^{E,\text{out}}(\alpha_F, \eta_F, \tau_F) = 1 - \int_{\theta_4=0}^{\infty} \frac{Q_L \left( \sqrt{\rho_1^2 \sigma_4^2 \theta_4}, \sqrt{\rho_2^2 \sigma_2^2 \theta_4 + \rho_3^2} \right)}{e^{\theta_4 \frac{\sigma_4^2}{\alpha_F \beta P}}} d\theta_4. \quad (23)$$

**Proof:** The outage probability can be evaluated as follows. First, applying the law of total probability, i.e., given a random variable $A$, $\Pr (\cdot | C) = \mathbb{E}_A [\Pr (\cdot | A)]$, we have

$$\mathcal{P}_F^{E,\text{out}}(\alpha_F, \eta_F, \tau_F) = \mathbb{E}_{\hat{w}_{UT}} \left[ \Pr \left\{ \sqrt{\eta_T \|\hat{w}\|^2} - \rho_1 \hat{w}_{UT} \|\hat{w}\|^2 < \rho_2 \|\hat{w}\|^2 + \rho_3 \hat{w}_{UT} \right\} \right]. \quad (24)$$

From (24), it can be easily deduced that $\|\hat{w}\|^2 - \rho_1 \hat{w}_{UT} = \chi^2_{2L} (\rho_2^2 \|\hat{w}\|^2 + \rho_3^2)$, where $\Theta_1 \sim \chi^2_{2L}$. Substituting the PDF of $\|\hat{w}\|^2$ into (24), we can rewrite (24) as

$$\mathcal{P}_F^{E,\text{out}}(\alpha_F, \eta_F, \tau_F) = 1 - \mathbb{E}_{\hat{w}_{UT}} \left[ Q_L \left( \rho_1 \|\hat{w}\|^2, \sqrt{\rho_2^2 \|\hat{w}\|^2 + \rho_3^2} \right) \right]. \quad (25)$$

Since $\hat{w}_{UT} \sim \mathcal{N} (0, \sigma^2_{\hat{w}})$, we have $\|\hat{w}\|^2 = \sigma^2_{\hat{w}} \Theta_1$, where $\Theta_1 \sim \chi^2_{2L}$. Substituting the PDF of $\|\hat{w}\|^2$ into (25), we derive the RHS of (25), and this concludes the proof.

At this stage, if we focus on (20), (21), and (23), we note that the energy shortage probability in the three considered cases clearly depends on the values of $\alpha_N$, $(\alpha_T, \eta_T)$, and $(\alpha_F, \eta_F, \tau_F)$ respectively. However, drawing meaningful insights from these results is extremely difficult, due to their complexity. Accordingly, we will investigate this aspect in Sec. V by means of suitable numerical analyses.

B. Data Outage Probability for the Non-CSI Scheme

We now compute the data outage probability for the non-CSI scheme. Given $\alpha_N$ and a specific target downlink rate $R_{NC}$, the data outage probability can be stated mathematically as

$$\mathcal{P}_N^{D,\text{out}}(\alpha_N, R_{NC}) = \Pr \left\{ \frac{\alpha_N \beta P \|\hat{w}\|^2}{L} \geq (1 - \alpha_N) P_D, \frac{(1 - \alpha_N) \log_2 \left( 1 + \frac{P \|\hat{w}\|^2}{N_0 L} \right) - R_{NC}}{N_0 L} \right\}, \quad (26)$$

that is the probability that the harvested energy is sufficient for the decoding operations at the UT, but the achieved downlink rate is smaller than $R_{NC}$. Now, let us rewrite $\mathcal{P}_N^{D,\text{out}}$ as

$$\Pr \left\{ \frac{(1 - \alpha_N) \log_2 \left( 1 + \frac{P \|\hat{w}\|^2}{N_0 L} \right) - R_{NC}}{N_0 L} \right\} \leq \frac{(1 - \alpha_N) \log_2 \left( 1 + \frac{P \|\hat{w}\|^2}{N_0 L} \right) - R_{NC}}{N_0 L} \right\} \leq (21-\alpha_N) \frac{L}{P} \right\}. \quad (26)$$

The intersection between the two events in (26) is non-empty when

$$\frac{(1 - \alpha_N) \log_2 \left( 1 + \frac{P \|\hat{w}\|^2}{N_0 L} \right) - R_{NC}}{N_0 L} \leq \frac{(1 - \alpha_N) \log_2 \left( 1 + \frac{P \|\hat{w}\|^2}{N_0 L} \right) - R_{NC}}{N_0 L} \right\} \leq (21-\alpha_N) \frac{L}{P} \right\}. \quad (27)$$

If this condition is not satisfied, then (26) in this case is equal to 0. In is worth noting that, assuming $R_{NC} \neq 0$, the data outage probability would be 0 only in case of extremely low value of $N_0$, given that typically $P \gg P_D$, as previously
discussed. This is in line with what could be expected in a wireless communication system, in which the data outage probability tends to 0 as the SNR at the receiver increases. If this is not the case, and (27) is satisfied, then (26) can be computed as
\[
P_{N,\text{out}}^{\text{D}}(\alpha_N, R_N) = \gamma \left( L, \frac{2^{\beta P_{\text{T}}D}}{\alpha T} - 1 \right) - \Gamma \left( \frac{(1-\alpha_N)L P_D}{\alpha T P}, \frac{\alpha T P}{\alpha T P} \right),
\]
where we made use of the CDF of the $\chi^2_{2L}$ distribution.

C. Data Outage Probability for the TDD Scheme

We switch our focus back to the TDD scheme. For given values of $\alpha_T$, $\eta_T$, and a target downlink rate $R_T$, the data outage probability is expressed as
\[
P_T^{\text{D, out}}(\alpha_T, \eta_T, R_T) = \Pr \left\{ \frac{P_T^2 T P}{L} \geq (1 - \alpha_T - \eta_T) P_D \right\} + \eta_T P_E, (1 - \alpha_T - \eta_T) \log_2 \left( 1 + \frac{P_T^2 T P^2}{N_0 L} \right) < R_T \},
\]
that is the probability that the harvested energy is sufficient to engage in the pilots transmission and decode the received data, but the achieved downlink rate is smaller than $R_T$. The following result provides a closed-form expression for $\beta_T$, and concludes the study of the TDD case. However, before proceeding, let us denote $b_3 = \frac{N_0}{P_T} \left( 2^{\frac{\beta T P}{\alpha T P} - 1/2} - 1 \right)$, $b_4 = \frac{N_0}{P_T} \left( 2^{\frac{\beta T P}{\alpha T P} - 1} - 1 \right)$, $b_5 = \frac{N_0}{\eta_T} \frac{P_T}{L_D}$ and $b_6 = \frac{N_0}{\eta_T} \frac{P_T}{L_P}$, for the sake of the simplicity of the representation of the result.

Lemma 4. When $\frac{N_0}{P_T} \left( 2^{\frac{\beta T P}{\alpha T P} - 1} - 1 \right) < \frac{N_0}{\eta_T} \frac{P_T}{L_D} (1 - \eta_T - \eta_T) L_P D$, then the data outage probability for the TDD scheme, as in (29), can be computed as
\[
P_T^{\text{D, out}} = \int_{\theta_3 = 0}^{\infty} \int_{\theta_3 = 0}^{\theta_3} \frac{1}{L} \left( 1 - b_5 \theta_3 - \frac{\theta_3^2}{2} \right) \theta_3^{L-1} \times I_0 \left( \frac{\theta_3}{b_6} \right) e^{-\left( \frac{\theta_3^2}{b_6} + \frac{\theta_3}{b_6} + \frac{\theta_3}{b_6} \right)} d\theta_3 d\theta_3.
\]
Conversely, when $\frac{N_0}{P_T} \left( 2^{\frac{\beta T P}{\alpha T P} - 1} - 1 \right) \geq \frac{N_0}{\eta_T} \frac{P_T}{L_D} (1 - \eta_T - \eta_T) L_P D$, it can be computed as
\[
P_T^{\text{D, out}} = \int_{\theta_3 = 0}^{\infty} \int_{\theta_3 = 0}^{\theta_3} \frac{1}{L} \left( 1 - b_5 \theta_3 - \frac{\theta_3^2}{2} \right) \theta_3^{L-1} \times I_0 \left( \frac{\theta_3^2}{b_6} \right) \sqrt{2 \theta_3^2 b_6} \times Q_1 \left( \frac{\theta_3^2}{b_6}, \sqrt{2 \theta_3^2 b_6} \right) d\theta_3 d\theta_3.
\]

Proof: See Appendix C

D. Data Outage Probability for the FDD Scheme

We conclude our study on the data outage probability by considering the FDD case. For given $\alpha_F$, $\eta_F$, $\tau_F$, and a specific target downlink rate $R_F$, the data outage probability can be stated mathematically as
\[
P_F^{\text{D, out}}(\alpha_F, \eta_F, \tau_F, R_F) = \Pr \left\{ \frac{\alpha_F^2 T_P^2}{L} \geq (1 - \alpha_F - \eta_F) P_D, (1 - \alpha_F - \eta_F - \tau_F) \log_2 \left( 1 + \frac{\alpha_F^2 T_P^2}{N_0 L} \right) < R_F \right\},
\]
that is the probability that the harvested energy is sufficient to estimate the downlink channel, feed back its estimated version in the uplink, and decode the received data, but the achieved downlink rate is smaller than $R_F$. The following result provides a closed-form expression of (32) and concludes the study of the FDD case. However, as before, let us introduce some new notation to further simplify representation of the results. Accordingly, we let $\sigma_2 = \frac{N_0}{\eta_T} \frac{P_T}{L_D}$, $\sigma_3 = \frac{N_0}{\eta_T} \frac{P_T}{L_P}$, $\sigma_4 = \frac{N_0 L}{\alpha_F^2 T_P}$, $\sigma_5 = \frac{N_0 L}{\alpha_F^2 T_P}$, $b_7 = \frac{N_0}{\alpha_F^2 T_P}$, and $b_8 = (1 - \frac{\alpha_F - \tau_F}{\eta_T}) \frac{P_T}{L_D}$, $\eta_T$.

Lemma 5. The data outage probability for the FDD scheme, as in (32), can be computed as
\[
P_F^{\text{D, out}} = \int_{\theta_3 = 0}^{\infty} \int_{\theta_3 = 0}^{\theta_3} \frac{1}{L} \left( 1 - b_5 \theta_3 - \frac{\theta_3^2}{2} \right) \theta_3^{L-1} \times I_0 \left( \frac{\theta_3}{b_6} \right) \sqrt{2 \theta_3^2 b_6} \times Q_1 \left( \frac{\theta_3^2}{b_6}, \sqrt{2 \theta_3^2 b_6} \right) d\theta_3 d\theta_3.
\]
of clarity, with a little abuse of notation, we denote computed by extensive Monte-Carlo simulations. For the sake of the coherence time devoted to the channel training/estimation, according to Lemma 1. For the FDD scheme, a similar notation optimal parameter of interest for the TDD scheme, computed numerically solving the problems in (8) and (15), by means of an exhaustive search whose complexity and time requirements are not suitable for realistic implementations. Subsequently, we model the ratio between the available power budgets at both sides of the communication in modern networks, which is roughly 20 dB. Accordingly, we set a distance between the AP and the UT in the order of meters such that we can ensure that the latter is situated in the far-field region of the radiating AP. This reduces the accuracy of our closed-form representation consequence, a greater number of approximations is necessary. In this section, we evaluate the performance of SWIPT for MISO systems, to assess its merit under the transmit schemes considered in this work. The parameters used in our numerical results are as follows. We consider $\beta = 0.5$, which is a good approximation of the performance delivered by state-of-the-art commercial products [5], and typically adopted value in the literature on this subject [11], [12], [27]. We consider $P = 1$ and $T_C = 1000$ for simplicity, and $L \in \{3, 6\}$. We assume that the system operates in the industrial, scientific and medical (ISM) band, i.e., carrier frequency of 2.4 GHz. We let the power budgets available at the AP and the UT following the same logic. We let $P_A = P_T = 1000$, in accordance with the typical ratio between the available power budgets at both sides of the communication in modern networks, which is roughly 20 dB [28].

A. Downlink Rate

Now, we focus on the ergodic downlink rate. First, we compute the optimal numerical performance of the system by numerically solving the problems in (8) and (15), by means of an exhaustive search whose complexity and time requirements are not suitable for realistic implementations. Subsequently, we evaluate the accuracy of our theoretical results by comparing them to the numerical performance results. Throughout this section, we will refer to the derived approximated parameters in Lemma 1 and Lemma 2 as analytic results, for the sake of clarity. Now, for the TDD scheme, let $R^T_\gamma$ and $\eta^T_\gamma$ be the optimal downlink rate and the optimal duration of the portion of the coherence time devoted to the channel training/estimation, computed by extensive Monte-Carlo simulations. For the sake of clarity, with a little abuse of notation, we denote $\hat{R}^T_\gamma$ as the optimal parameter of interest for the TDD scheme, computed according to Lemma 1. For the FDD scheme, a similar notation is defined. Now, we define $\zeta_T = \frac{R^T_\gamma[\alpha_T, \delta_T^T, \xi^T_T]}{R^T_\gamma} \in [0, 1]$, for TDD, and $\zeta_F = \frac{R_F[\alpha_F, \delta_F^T, \xi^T_F]}{R_F} \in [0, 1]$, for FDD, as the ratio between the downlink rate obtained with the analytic and optimal numerical results.\footnote{Note that, $\alpha_T$ and $\alpha_F$ are computed according to [6] and [13] respectively.} We use $\text{SNR} \in [0, 30]$ dB and compute $\zeta_T$ and $\zeta_F$ for both $L = 3$ and $L = 6$ in Fig. 4, Fig. 5, Fig. 6 and Fig. 7. Quantitatively, if we focus on the best performer for each of the considered SNR values, the gap between $\zeta_T$ ($\zeta_F$ in the FDD case) and 1 is remarkably small. Thus, the accuracy of our derivations is confirmed. If we focus on the impact of $L$ on the analytic results, we note that they confirm the intuitions provided in Sec. III-B and Sec. III-C. However, the difference in terms of the best $\zeta_T$ (and $\zeta_F$) between the two antenna configurations is rather small. This shows that the impact of the number of antennas at the AP on the accuracy of the analytic results is not very significant. Furthermore, we see that $\zeta_F \leq \zeta_T$, $\forall \text{SNR} \in [0, 30]$ dB and $\forall L \in \{3, 6\}$. This is due to the two-step channel estimation process that is needed in the FDD scheme for the CSI acquisition at the AP. As a consequence, a greater number of approximations is necessary. This reduces the accuracy of our closed-form representation
high SNR analytic
Low SNR analytic
High SNR analytic
Low SNR analytic

Figure 6. $\zeta_F$ for analytic and numerical parameters, FDD and $L = 3$ antennas.

Figure 7. $\zeta_F$ for analytic and numerical parameters, FDD and $L = 6$ antennas.

Figure 8. Ratio between the ergodic downlink rate for the CSI acquisition schemes and the non-CSI case.

outperform the non-CSI approach in terms of downlink rate. This shows that, despite the penalties incurred to acquire the CSI, evident downlink rate enhancements are experienced by the AP, thanks to presence of the CSI, however imperfect the latter might be. The result in Fig. 8 is even more remarkable, considering that therein the two duplexing schemes always outperform the non-CSI approach, regardless of the antenna configuration and the SNR value. Furthermore, the largest advantage over the non-CSI performance is obtained in the low-to-mid SNR regime. In this regard, we first focus on the TDD case. In both cases, i.e., $L = 3$ and $L = 6$, $\frac{R}{R_{NC}}$ is a monotonically decreasing function of the SNR, confirming that the MFP performs better for low than for high SNR values $[25], [29]$. In particular, this shows that the availability of the...
CSI at the AP, albeit imperfect, is sufficient to achieve a much larger downlink rate as compared to the non-CSI approach. Furthermore, the performance for \( L = 6 \) is strictly larger than for \( L = 3 \), showing that, as in the case of traditional wireless communications, the SWIPT can effectively exploit the transmit diversity gain delivered by a MISO system as \( L \) grows. Interestingly, the same is true for the FDD scheme. The CSI acquisition procedure in this case is more complex and prone to a higher uncertainty, especially at low SNR. This impacts the behavior of \( R_{NC} \) that presents a maximum at \( \text{SNR} = 10 \text{ dB} \), for both the considered antenna configurations. On one hand, the gain brought by the FDD scheme over the non-CSI approach is dominated by the power gain at the UT, brought by a more accurate beamformer design at the AP, for \( \text{SNR} \leq 10 \text{ dB} \). On the other hand, the reduction of the multiplexing gain due to the increasing impact that both the channel estimation and feedback phases have on available time for information transfer, as the quality of the CSI increases, determines the decreasing behavior of \( R_{NC} \) for \( \text{SNR} > 10 \text{ dB} \).

Finally, we note that the difference at high SNR between the values of \( R_{NC} \) for TDD and FDD is very low, but increases with the \( L \). In fact, when the SNR is high, the channel estimation/feedback phases are very short, thus the difference in the amount of time available for the information transfer in both cases is small. Nevertheless, a bigger \( L \) entails a larger \( \tau_F \) (thus \( \alpha_F \)) and, in turn, increases the difference between the values of \( R_{NC} \) for TDD and FDD at high SNR as well.

**B. Outage Probability**

We switch our focus to the analysis of the energy shortage and the data outage probability. A set of Monte-Carlo simulations is performed to obtain the numerically computed probabilities. Subsequently, we set the values of \( \eta_T, \eta_F, \text{ and } \tau_F \) according to Lemma 1 and Lemma 2 and compute the exact value of both metrics by means of the analytic results in Sec. [IV] At this stage, we only consider the case \( L = 3 \) owing to space economy. In the previous subsection, we verified that the impact of change in the number of antennas on the accuracy of the analytic results on the downlink rate is rather small. Accordingly, a robustness of the accuracy of our results to a change in the number of antennas could be conjectured. For the sake of clarity we let \( p^{E,\text{out}} \) and \( p^{D,\text{out}} \) be the energy shortage probability and the data outage probability when no energy shortage occurs, respectively. Furthermore, we let \( R_{NC} = R_T = R_F = 6 \) (bit/s/Hz)\(^3\) be the target rate for the considered system. Finally, we depict \( p^{E,\text{out}} \) for \( \text{SNR} \in [0,30] \text{ dB} \) in Fig. [9] and Fig. [10] and \( p^{D,\text{out}} \) for \( \text{SNR} \in [0,15] \text{ dB} \) in Fig. [12]. As shown in these figures, the numerical results perfectly match the analytic results derived in Sec. [IV] for all the three schemes. This perfect match verifies the correctness of our derivations.

We start by noting that the energy shortage probability strongly depends on the considered parameters. Thus, a comparison between schemes could have limited interested w.r.t. a comparison between the results obtained for each scheme, as the duration of the energy transfer phase varies. Accordingly, we restrain our focus to the latter aspect. As expected, the energy shortage probability is independent of the SNR, regardless of the value of \( \alpha_N \). However, for both the TDD and the FDD scheme, the energy shortage probability decreases with the SNR, regardless of the value of \( \alpha_T \) and \( \alpha_F \). In these cases, a larger SNR reduces the optimal time for both devices to perform the operations intrinsic to the CSI acquisition and achieve accurate channel estimations. In other words, the channel estimation accuracy increases with the SNR value, thus the CSI acquisition requires less time. Therefore, the energy consumption at the UT is lower when the SNR is large. Now, if the duration of the energy transfer phase is doubled or tenfold, a reduction of the energy shortage probability from almost one to three orders of magnitude is

\[ \begin{align*}
\text{Figure 9. Energy shortage probability, non-CSI and } L = 3 \text{ antennas.} \\
\text{Figure 10. Energy shortage probability, TDD and } L = 3 \text{ antennas.}
\end{align*} \]

---

\(^3\)Referring to Sec. [IV-B] [IV-C] and [IV-D] we note that \( R_{NC}, R_T, \) and \( R_F \) are specified values.
observed, depending on the considered scheme. In practice, if the coherence time is long enough, even a rather small increase of the duration of the energy transfer phase can positively impact the energy shortage probability.

We now switch our focus to the data outage probability illustrated in Fig. 12. We start by noting that, to compute the numerical data outage probability in this case, the duration of the energy transfer phase for the three considered schemes, i.e., $\alpha_N$, $\alpha_T$, and $\alpha_F$, is chosen at each iteration of the simulations such that the harvested energy at the UT is sufficient to perform the receiver operations intrinsic to each scheme. As a matter of fact, the obtained quantitative results for a study of this kind are not extremely relevant, in fact they clearly depend on the selected target rate. In practice, their qualitative behavior is definitely more interesting. In this regard, the lowest data outage probability is experienced by the considered system in the case of the TDD scheme. This could have been expected after our findings on the downlink rate in the previous section, in which the TDD scheme resulted as the best performer out of the three considered cases.

VI. Conclusion

In this work, we have examined the efficacy of SWIPT in a MISO system consisting of an AP and a single UT. In particular, the latter is not equipped with any local power source, but instead harvests the necessary energy for its operations from the received RF signals. The performance of the considered system has been analyzed under realistic and practically relevant system assumptions. Three practical cases have been considered: a) absence of CSI at the AP, b) imperfect CSI at the AP acquired by means of pilots estimation (TDD), c) imperfect CSI at both the UT and the AP acquired by means of analog CSI feedback in the uplink (FDD). We have compared the considered scenarios by means of three performance metrics of interest, i.e., the ergodic downlink rate, the energy shortage probability, and the data outage probability. Accordingly, we have derived closed-form expressions for each metric, and for the ergodically optimal duration of both the WPT and the channel training/feedback phases, to maximize the downlink rate in all the three scenarios. The accuracy of our derivations has been verified by an extensive numerical analysis. First, it is worth noting that TDD has consistently been the best performer for each considered metric, confirming the potential of this duplexing scheme for the future advancements in modern networks. More specifically, concerning the downlink rate, our findings show that CSI knowledge at the AP is always beneficial for the information transfer in SWIPT systems, despite the resources devoted to the channel estimation/feedback procedures and the presence of estimation errors. In a follow-up of this work, we will study both strategies to maximize the efficiency of the WPT, in the case of the availability of CSI knowledge at the AP prior to the WPT phase (or part of it), and their impact on the energy shortage probability. Additional subject of future investigation will be the extension of the considered set-up to a multi-user scenario.

APPENDIX A

Proof of Lemma 1

To evaluate (3), we use the law of iterated expectations, i.e.,

$$\mathbb{E}_{h, w} [\bar{w}] = \mathbb{E}_h [\mathbb{E}_{w} [\bar{w} | h]].$$

Furthermore, we neglect $LP_D$ in (3) since, in practice, $P_D \ll P$ generally [28]. We proceed by first computing the following expression:

$$\left(1 - \eta_T - \frac{\eta_T LP_E}{\beta P \|h\|^2}\right) \mathbb{E}_{\bar{w}} \left[\log_2 \left(1 + \frac{P\|h\|^2}{N_0 \|\bar{h}\|^2}\right)\right],$$

for a given channel realization $h$. In order to compute (36), the following straightforward results can be derived:

$$\|h^\dagger \bar{h}\|^2 = \frac{N_0 \|\bar{h}\|^2}{2\eta_T T_C P_E} \Psi_1 \text{ and } \|\bar{h}\|^2 = \frac{N_0}{2\eta_T T_C P_E} \Psi_2.$$
where $Ψ_1 \sim \chi^2 \left( \frac{2\eta_T L \nu P}{N_0} \right)$ and $Ψ_2 \sim \chi^2 \left( \frac{2\eta_T L \nu P}{N_0} \right)$. We break up the subsequent analysis into two cases, namely, the high SNR and low SNR cases.

First, we consider the analysis at high SNR. In this case, applying the approximation, $\log_2(1 + SNR) \approx \log_2(NR)$ when $SNR \gg 0$, and (37) to (38), we can derive

$$ \mathbb{E}_h \left[ \log_2 \left( 1 + \frac{P|h|^2}{N_0} \right) \right] \approx \mathbb{E}_h \left[ \log_2 \left( \frac{P|h|^2}{N_0} \Psi_1 \right) \right]. \quad (38) $$

Subsequently, using the Taylor series expansion of $\log_2 \Psi_1$ and $log_2 \Psi_2$ at their respectively mean values (i.e. $2 + \lambda$ and $2L + \lambda$ respectively, where $\lambda = \frac{2\eta_T L \nu P}{N_0}$), we have

$$ (38) = log_2 \frac{P|h|^2}{N_0} + log_2 e $$

$$ \times \mathbb{E}_{\Psi_1, \Psi_2} \left[ \ln (2 + \lambda) + \frac{(\Psi_1 - 2 - \lambda)}{2 + \lambda} \right] $$

$$ \times \ln (2 + \lambda) - \frac{(\Psi_1 - 2 - \lambda)^2}{2 + \lambda} \right] + \cdots $$

$$ - \ln (2 + \lambda) - \frac{(\Psi_1 - 2 - \lambda)^2}{2 + \lambda} \right] + \cdots $$

$$ = log_2 \frac{P|h|^2}{N_0} + log_2 e \left( \ln (2 + \lambda) \right) $$

$$ - \ln (2 + \lambda) - \frac{(\Psi_1 - 2 - \lambda)^2}{2 + \lambda} \right] + \cdots $$

$$ \approx \log_2 \frac{P|h|^2}{N_0} (2L + \lambda) = \log_2 \frac{P|h|^2}{N_0} + \log_2 \left( 1 + \frac{2L}{\lambda} \right), \quad (40) $$

where (a) in (41) is derived by noting that $\lambda$ is large (at high SNR), and hence we can neglect the higher order fractional terms in (40). Moreover, $2$ in the $\ln(2 + \lambda)$ term is neglected owing to $\lambda \gg 2$. Further, we use (41) in (39) and take the expectation over $h$. Moreover, since $L/\lambda$ is small at high SNR, we use the approximation, $\log_2 (1 + x) \approx x \log_2 e$ when $x \approx 0$, in the derivation. By some straightforward computations, we can rewrite (38) as

$$ \eta_T^* \approx \operatorname{argmax}_{\eta_T} \left( B_2 - \eta_T B_1 - \frac{N_0 L \log_2 e}{\eta_T T C \nu P (L - 1)} \right), \quad (42) $$

where $B_1 = E_h \left[ \left( 1 + \frac{L \nu P}{2T C |h|^2} \right) \log_2 \left( \frac{P|h|^2}{N_0} \right) \right]$, and $B_2$ is a constant value. In order to derive $\eta_T^*$, we differentiate (42) with respect $\eta_T$ and set it equal to 0. By some straightforward computations, we can derive the result (9).

We now move to the analysis at low SNR. Before deriving, we first note that $E[X] = E[\exp(\ln(X))]$. Subsequently, using the approximation, $\log_2 (1 + SNR) \approx SNR \log_2 e$ when $SNR \approx 0$, and Jensen’s inequality, i.e., $E[\exp(\ln(X))] \geq \exp(E[\ln(X)])$, we have

$$ (43) \geq \left( 1 - \eta_T - \frac{\eta_T L \nu P}{\beta P |h|^2} \right) \frac{P \log_2 e}{N_0} $$

$$ \times \mathbb{E}_h \left[ \ln \left( \frac{\hat{h}|T| \nu P}{|h|^2} \right) \right]. \quad (43) $$

Once again, we apply Taylor series expansion to $\mathbb{E}_h \left[ \log_2 \left( \frac{\hat{h}|T| \nu P}{|h|^2} \right) \right]$, i.e., steps (38), (39), (40), and (41). In this case, since $\lambda$ is small at low SNR, we approximate the higher order terms, such as $\frac{\eta_T L \nu P}{\beta P |h|^2}$ in (40), by a constant value $\kappa_1$. Using this, we can rewrite (43)

$$ \kappa_1 \left( 1 - \eta_T - \frac{\eta_T L \nu P}{\beta P |h|^2} \right) \frac{P |h|^2 \log_2 e}{N_0} \times \frac{2 + \lambda}{2L + \lambda}. \quad (44) $$

Finally, we take the expectation over $h$. By using Jensen’s inequality (similar approaches in (43)) and applying Taylor series expansion for the logarithm term at the mean value $|h|^2$ (similar approaches in (39), (40), and (41)), we rewrite (43) as

$$ \eta_T^* \approx \operatorname{argmax}_{\eta_T} \left( 1 + \frac{\eta_T L \nu P}{\beta P} \right) \left( L - \eta_T L \eta_T - \frac{\eta_T L \nu P}{\beta P} \right) $$

where $\kappa_2$ is a constant. In order to derive $\eta_T^*$, we differentiate the above formula with respect $\eta_T$ and set it equal to 0. By some straightforward computations, we can derive the result (10).

APPENDIX B

PROOF OF LEMMA 2

To evaluate (15), we use the similar approaches as in (36). Hence, we first compute

$$ \mathbb{E}_h \left[ \left( 1 - \frac{1}{\tau_F} \frac{1}{\beta P |h|^2} \right) \mathbb{E}_h \left[ \hat{h} \mid \nu P \right] \right] \times \log_2 \left( 1 + \frac{P|h|^2 \hat{h} \mid \nu P \right) $$

$$ \left( \frac{N_0}{N_0} \right)^2 \right]. \quad (45) $$

To compute (45), the following results can be derived (details omitted for lack of space):

$$ \hat{h} \hat{h} \mid \nu P \right] \approx \frac{N_0 L |h|^2}{2T_C \nu P } \left( \frac{\frac{1}{\eta_T} + \frac{1}{\nu P \beta P} \right) \Phi_1, \quad (46) $$

$$ \hat{h} \hat{h} \mid \nu P \right] \approx \frac{N_0 L |h|^2}{2T_C \nu P } \left( \frac{\frac{1}{\eta_T} + \frac{1}{\nu P \beta P} \right) \Phi_2, \quad (47) $$

$$ \hat{h} \hat{h} \mid \nu P \right] \approx \frac{N_0 L |h|^2}{2T_C \nu P } \Phi_3, \quad (48) $$

where $\Phi_1 \sim \chi^2 \left( \frac{2T_C |h|^2}{N_0 L (\frac{1}{\eta_T} + \frac{1}{\nu P \beta P}) \right)$, $\Phi_2 \sim \chi^2 \left( \frac{2T_C |h|^2}{N_0 L (\frac{1}{\eta_T} + \frac{1}{\nu P \beta P}) \right)$, and $\Phi_3 \sim \chi^2 \left( \frac{2T_C |h|^2}{N_0 L (\frac{1}{\eta_T} + \frac{1}{\nu P \beta P}) \right)$.

Before proceeding, we note that the pre-log term and the term inside the logarithm in (45) are correlated, due to the presence of $\hat{w} \hat{h} \mid \nu P \right]$ in both terms. However, at high SNR, the variance of $\hat{w} \hat{h} \mid \nu P \right]$ will be small. Thus, we assume that the pre-log term and term inside the logarithm are approximately independent (and hence we can take the expectations of these two terms in (45) separately). For the term inside the logarithm, using (46), (47), and (48) and by Taylor series expansion (similar approaches in (39), (40), and (41)), we obtain

$$ \mathbb{E}_h \left[ \log_2 \left( 1 + \frac{P|h|^2 \hat{h} \mid \nu P \right) \right] \approx \log_2 \frac{P|h|^2 \lambda_1}{N_0 (2L + \lambda_1)}, \quad (49) $$
where $\lambda_1 = \frac{2\tau F_1 ||h||^2}{N_0 L \left( \frac{1}{\beta_F} + \frac{1}{\tau F_1} \right)}$. For the pre-log term, we have

$$
\mathbb{E}_{\hat{h}} \left[ 1 - \tau F - \frac{\tau F P_F ||\hat{h}_{UT}||^2}{\beta P ||h||^2} - \eta F \right] \approx 1 - \tau F - \frac{\tau F P_F}{\beta P} - \eta F.
$$

(50)

The approximation in (50) is derived using $\mathbb{E}_{\hat{h}} [||\hat{h}_{UT}||^2] = ||h||^2 + \frac{N_0 L^2}{\eta_F \bar{P} P}$ (since at high SNR, $\frac{N_0 L^2}{\eta_F \bar{P} P}$ is small enough to be neglected). Finally, we substitute (49) and (50) into (45) and take the expectation over $h$. Using the approximation $\log_2 (1 + x) \approx x \log_2 e$ when $x \approx 0$ and by some straightforward computations, we can rewrite (15) as

$$(\eta_F^*, \tau_F^*) \approx \argmax_{\eta_F, \tau_F} \frac{1}{1 - \eta_F - \frac{\tau F P_F}{\beta P} - \eta F} \times \left( B_0 - \frac{N_0 L^2 \log_2 e}{\bar{P}_C (L - 1)} \left( \frac{1}{\eta_F P} + \frac{1}{\tau F P F} \right) \right),$$

(51)

where $B_0 = \mathbb{E}_{h} \left[ \log_2 \left( \frac{P ||h||^2}{N_0} \right) \right]$. In order to find $\eta_F^*$ and $\tau_F^*$, we differentiate (51) with respect $\eta_F$ and $\tau_F$ and equate them to 0. By some straightforward computations, we can derive the results (46) and (47).

We now turn to the analysis at low SNR. Herein, we follow the similar derivations as in Appendix A. First, we use the same approach as in (43). Further, we apply (45), (47), and (48) and use the Taylor series expansion (similar to the approach in (44)). Thus, we can derive

$$45 \geq \left( 1 - \eta_F - \tau_F \right) \frac{\beta P ||h||^2}{\lambda_2} - \frac{\tau F P F N_0 L}{2 \eta_F \bar{P}_C (L - 1) \left( 2L + \lambda_2 \right)} \times \frac{\kappa_3 \log_2 e \left( 2 + \lambda_2 \right)}{N_0 \left( 2L + \lambda_2 \right)},$$

(52)

where $\lambda_2 = \frac{2 \eta F \bar{P}_C P ||h||^2}{N_0 L}$, and $\kappa_3$ is a constant value. Finally, as before we take the expectation over $h$. By using Jensen’s inequality (similar approaches in (43)) and applying Taylor series expansion of the logarithm term at the mean value $||h||^2$ (similar approaches in (43), (46), and (47)), we can rewrite the expected value of (52) over $h$ as

$$\kappa_4 \left( 1 - \eta_F - \tau_F \right) \frac{\beta P ||h||^2}{\lambda_2} - \frac{\tau F P F N_0 L}{2 \eta_F \bar{P}_C \left( L + \frac{\bar{P}_C \left( \frac{1}{\eta F} + \frac{1}{\tau F} \right)}{N_0 \left( \frac{1}{\eta F} + \frac{1}{\tau F} \right)} \right)} \tau_F,$$

(53)

where $\kappa_4$ is a constant value. Now, we differentiate (53) with respect $\eta_F$ and $\tau_F$ and equate them to 0. Using some straightforward computations, we obtain the result (18). When $x \approx 0$, $\sqrt{1 + x} \approx 1 + \frac{x}{2}$. By utilizing this approximation in (18) since $N_0$ is large at low SNR, we can derive $\tau_F^* \approx \left( \frac{\eta F_0 N_0 L}{\beta P} \right)^{1/2}$.

Substituting the latter approximated $\tau_F^*$ into the differentiated equation $\frac{\partial}{\partial \eta_F} = 0$, we can also derive the result (19).

APPENDIX C

PROOF OF LEMMA 4

We now proceed with the proof. In equation (29), we have two random variables, i.e., $\frac{||h||^2}{||h||^2}$ and $||h||^2$. To evaluate (29), we first express $||h||^2$ as the sum of $\frac{||h||^2}{||h||^2}$ and another independent random variable (see the steps below). This step simplifies the evaluation as shown in the following. In order to do so, first we project the row vector $h^T$ onto an orthonormal set of vectors $\Omega = \{ v_1, v_2, \cdots, v_L \}$, where $v_2, \cdots, v_L$ are chosen arbitrarily such that the vectors in $\Omega$ span the complex $L$ dimensional space. Recall that $h^T | h \sim \mathcal{CN} \left( \frac{1}{1 + b_3} \hat{h}^T, \frac{1}{b_3} I_L \right)$. Since the distribution of a Complex Gaussian random vector (with distribution $\mathcal{CN}(0, I)$) projected onto an orthonormal set remains unchanged (29), we can conclude that $\frac{||h||^2}{||h||^2} \sim \mathcal{CN} \left( \frac{1}{1 + b_3} \hat{h}^T, \frac{1}{b_3} I_L \right)$ and $h^T g \sim \mathcal{CN} \left( 0, b_3 I \right)$, $l = 2, \cdots, L$. Additionally, $h^T g_l = 0$, where $l = 2, \cdots, L$ are independent random variables. Thus we can conclude the following:

$$\left( \frac{||h||^2}{||h||^2} \right) = 1 - \frac{1}{2b_3} \Theta_1,$$

(54)

$$\left( h^T g \right)^2 = 1 - \frac{1}{2b_3} \Theta_2,$$

(55)

where $\Theta_1 \sim 2 \sum_{l=2}^{L} \left| h^T g_l \right|$, and $\Theta_2 \sim 2 \sum_{l=2}^{L} \left| h^T g_l \right|^2$. Furthermore, $\Theta_1$ and $\Theta_2$ are independent and $\Theta_1 + \Theta_2 = 2b_3 ||h||^2$. Now, applying the same approach as in (24) to (29) and using (54) and (55), we can derive the following: $P_T^{D, \text{out}} = \mathbb{E}_h \left[ \Pr \left( \Theta_1 < 2b_3 \beta_3, \Theta_1 + \Theta_2 \geq 2b_3 \beta_4 \right) \right]$. Let us focus on the relationship between $b_3$ and $\beta_3$ and consider two possible cases, i.e., $b_3 < \beta_4$ and $b_3 \geq \beta_4$. We start from the former. In this case, denoting the PDF of $\Theta_1$ as $f_{\Theta_1}(\theta)$ for $i \in \{1, 2\}$, we have,

$$P_T^{D, \text{out}} = \mathbb{E}_h \left[ \int_{\theta_1=0}^{2b_3 \beta_3} \Gamma \left( L - 1, b_3 \left( b_3 - \frac{\theta_1}{2} \right) \right) \left( \frac{\theta_1}{2} \right) \left( \frac{\beta_3}{b_3 \beta_4} \right) e^{\frac{\beta_3}{b_3 \beta_4} \theta_1} d\theta_1 \right] \times 2 \Gamma \left( L - 1, e^{\frac{\beta_3}{b_3 \beta_4} \theta_1} \right),$$

(56)

Since $h \sim \mathcal{CN}(0, I)$, it follows that $||h||^2 = \mathcal{CN}(1 + b_0, \Theta_3)$, where $\Theta_3 \sim \chi^2_L$. Substituting the PDF of $||h||^2$ into (56), we obtain our result (30). In the second case $b_3 \geq \beta_4$, following the same approaches as in (56), we obtain our result (31) and conclude the proof.

APPENDIX D

PROOF OF LEMMA 5

Adopting a similar approach to what has been adopted in (24) and denoting the conditional distribution of $\hat{h}_{UT}$ given $\hat{h}_{AP}$ as $f(\hat{h}_{UT} | \hat{h}_{AP})$, the analytic expression for the outage probability, i.e., $P_T^{D, \text{out}}$, in (32), can be written as

$$\mathbb{E}_{\hat{h}_{AP}} \left[ \int \Pr \left( \frac{||\hat{h}_{UT}||^2}{||\hat{h}_{AP}||^2} < b_7, ||\hat{h}_{UT}||^2 \geq b_8 \right) + b_9 ||\hat{h}_{UT}||^2 f(\hat{h}_{UT} | \hat{h}_{AP}) d\hat{h}_{UT} \right].$$

(57)

To compute (57), we use the projection approach that we have used in Appendix C. First, we project the row vector $h^T$ onto an orthonormal set of vector $\{ v_1, v_2, \cdots, v_L \}$, (such
that these vectors span the $L$ dimensional complex space). Since $h_{AP, UT} \sim \mathcal{CN} \left( \frac{1}{2} \hat{h}_{AP}, \frac{1}{2} I_L \right)$, following the same approach as in Appendix C, we obtain

$$\left| h_{AP, UT}^T \hat{h}_{AP} \right|^2 = \frac{1}{2} \Theta_5, \quad (58)$$

$$\left( |h_2|^2 + \cdots + |h_L|^2 \right)_{AP, UT} = \frac{1}{2} \Theta_2, \quad (59)$$

where $\Theta_5 \sim \chi_2^2 \left( \frac{2}{\sigma_2^2} \left| \hat{h}_{AP, UT}^T \hat{h}_{AP} \right|^2 \right)$ and $\Theta_6 \sim \chi_{2L-2} \left( \frac{2}{\sigma_2^2} \left( \left| \hat{h}_{UT} \right|^2 - \left| \hat{h}_{AP, UT} \right|^2 \right) \right)$. Moreover, $\Theta_5$ and $\Theta_6$ are independent and $\Theta_5 + \Theta_6 = 2\sigma_2 \| h \|^2$. Using (58) and (59), we rewrite (57) as

$$E_{AP} \left[ \int \Pr \left( \Theta_5 + \Theta_6 \geq 2\sigma_2 \| \hat{h}_{UT} \|^2 \right) \right],$$

$$\Theta_5 < 2\sigma_2 b_7 \right] f \left( \hat{h}_{UT} \left| \hat{h}_{AP} \right) d\hat{h}_{UT} \right]. \quad (60)$$

To compute the double integration over $\theta_5$ and $\theta_6$ in (60), we have two cases, i.e., $b_7 < b_8 + b_9 \| \hat{h}_{UT} \|^2$ and $b_7 \geq b_8 + b_9 \| \hat{h}_{UT} \|^2$. In the first case, the probability term in (60) can be computed as

$$\int_{\theta_5=0}^{2\sigma_2 b_7} \frac{1}{2} Q_{L-1} \left( \frac{2}{\sigma_2^2 \sigma_3^2} \left( \left| \hat{h}_{UT} \right|^2 - \left| \hat{h}_{AP, UT} \right|^2 \right)^2 \right),$$

$$\sqrt{2\sigma_2 \left( b_8 + b_9 \| \hat{h}_{UT} \|^2 \right) - \theta_5} I_0 \left( \frac{2\Theta_5}{\sigma_2^2 \sigma_3^2} \left| \hat{h}_{UT} \hat{h}_{AP}^T \right|^2 \right)$$

$$\times e^{-\frac{\theta_5}{\sigma_2^2 \sigma_3^2} \left( \left| \hat{h}_{AP, UT} \right|^2 \right)} d\theta_5. \quad (61)$$

Since $\hat{h}_{UT}^T \hat{h}_{AP} \sim \mathcal{CN} \left( \frac{\sigma_2^2 \hat{h}_{AP}, \sigma_3^2 I_L \right)$, using the projection approach once again, we can derive

$$\left| \hat{h}_{UT}^T \hat{h}_{AP} \right|^2 = \frac{\sigma_5}{2} \Theta_7, \quad (62)$$

$$\left( \left| \hat{h}_{UT} \right|^2 - \left| \hat{h}_{AP, UT} \right|^2 \right)_{AP, UT} = \frac{\sigma_5}{2} \Theta_8, \quad (63)$$

where $\Theta_7 \sim \chi_2^2 \left( \frac{2\sigma_2}{\sigma_5} \| \hat{h}_{AP} \|^2 \right)$ and $\Theta_8 \sim \chi_{2L-2}^2$. Furthermore, $\Theta_7$ and $\Theta_8$ are independent and $\Theta_7 + \Theta_8 = \frac{2}{\sigma_5} \| \hat{h}_{UT} \|^2$. Lastly, we note that since $b_7 < b_8 + b_9 \| \hat{h}_{UT} \|^2$, we have $b_7 < (b_8 + b_9) \| \hat{h}_{AP, UT} \|^2 / 2(b_8 - b_9)$. Now, applying (61), (62), and (63) to (60), the integral of (60) can be computed as

$$\int_{\theta_7+\theta_8 > 2(b_8-b_9) \sigma_5 / \sigma_7} I_0 \left( \frac{2\sigma_5 \| \hat{h}_{AP} \|^2}{\sigma_7^2} \right) \frac{\theta_7}{2 L+1} \times Q_{L-1} \left( \frac{2\sigma_5 \| \hat{h}_{AP} \|^2}{\sigma_7^2} \right) 2\sigma_2 \left( b_8 + b_9 \left( \frac{\theta_7 + \theta_8}{\sigma_7^2} \right) \right) - \theta_5 \right)$$

$$\times I_0 \left( \frac{2\sigma_5 \| \hat{h}_{AP} \|^2}{\sigma_7^2} \right) \frac{\theta_5}{2 L+1} \times \Gamma \left(L-1 \right) e^{\frac{2\sigma_5 \| \hat{h}_{AP} \|^2}{\sigma_7^2} + \frac{\sigma_7^2}{2} \left( \theta_7 + \theta_8 \right) / \sigma_7^2} \frac{d\theta_7 d\theta_8}{\sigma_7^2}. \quad (64)$$

We note that, since $\hat{h}_{AP} \sim \mathcal{CN} \left( 0, \left(1 + \sigma_3 + \sigma_4 \right) I_L \right)$, we have that $\| \hat{h}_{AP} \|^2 = \frac{1}{2} \sigma_3^2 + \sigma_4^2 \Theta_9$, where $\Theta_9$ is $\chi_2^2$. Using this fact and (64) in (60), we obtain (33) in Lemma 5.

We now consider the second aforementioned case, i.e., $b_7 \geq b_8 + b_9 \| \hat{h}_{UT} \|^2$. Following similar steps as in the first case, we obtain (34) and (35) in Lemma 5 (the detailed steps have been omitted for matters of space economy). At this stage, the outage probability in (32) is obtained as the sum of (33), (34), and (35), and this concludes the proof.
[17] D. W. K. Ng, E. S. Lo, and R. Schober, “Energy-efficient resource allocation in multiuser OFDM systems with wireless information and power transfer,” in Proc. IEEE Wireless Commun. and Netw. Conf., Apr. 2013, pp. 3823–3828.

[18] ———, “Wireless information and power transfer: energy efficiency optimization in OFDMA systems,” IEEE Trans. Wireless Commun., vol. 12, no. 12, pp. 6352–6370, Dec. 2013.

[19] X. Chen, C. Yuen, and Z. Zhang, “Wireless energy and information transfer tradeoff for limited feedback multi-antenna systems with energy beamforming,” IEEE Trans. Veh. Technol., vol. 63, no. 1, pp. 407–412, Jan. 2014.

[20] G. Yang, C. K. Ho, and Y. L. Guan, “Dynamic resource allocation for multiple-antenna wireless power transfer,” IEEE Trans. Signal Process., vol. 62, no. 14, pp. 3565–3577, Jul. 2014.

[21] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, “Multiuser MIMO achievable rates with downlink training and channel state feedback,” IEEE Trans. Inf. Theory, vol. 56, no. 6, pp. 2845–2866, Jun. 2010.

[22] J. I. Marcum, “Table of Q functions,” Project RAND Research Memorandum M-339, ASTIA Document AD 1165451, Rand Corporation, Santa Monica, CA, vol. abs/1302.0585, Jan. 1950.

[23] W. R. Heinzelman, A. Chandrakasan, and H. Balakrishnan, “Energy-efficient communication protocol for wireless microsensor networks,” in Proc. 33rd Annu. Hawaii Int. Conf. Syst. Sci., Jan. 2000, pp. 1–10.

[24] S. M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory. Prentice Hall PTR, 1993, ch. 3.4, pp. 30–35.

[25] R. de Miguel and R. R. Muller, “Vector precoding for a single-user MIMO channel: matched filter vs. distributed antenna detection,” in Proc. 1st Int. Symp. Appl. Sci. Biomed., and Commun. Tech., Oct. 2008, pp. 1–4.

[26] D. Samardzija and N. Mandayam, “Unquantized and uncoded channel state information feedback in multiple-antenna multiuser systems,” IEEE Trans. Commun., vol. 54, no. 7, pp. 1335–1345, Jul. 2006.

[27] L. Liu, R. Zhang, and K.-C. Chua, “Wireless information and power transfer: a dynamic power splitting approach,” IEEE Trans. Commun., vol. 61, no. 9, pp. 3990–4001, Sep. 2013.

[28] 3GPP, “TR 36.814, further advancements for E-UTRA physical layer aspects, v.9.0.0,” 3GPP, Tech. Rep., Mar. 2010.

[29] D. Tse and P. Viswanath, Fundamentals of Wireless Communication. Cambridge University Press, Jun. 2005, ch. A.1.3, pp. 501–502.