Toward the existence of odderon as a three-gluon bound state

Hua-Xing Chen1,*, Wei Chen2,† and Shi-Lin Zhu3,4,‡

1 School of Physics, Southeast University, Nanjing 210094, China
2 School of Physics, Sun Yat-Sen University, Guangzhou 510275, China
3 School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
4 Collaborative Innovation Center of Quantum Matter, Beijing 100871, China
5 Center of High Energy Physics, Peking University, Beijing 100871, China

Inspired by the evidence of the odderon exchange recently observed by the D0 and TOTEM Collaborations, a QCD sum rule investigation is performed to study the odderon as a three-gluon bound state. There may exist six lowest-lying three-gluon odderons with the quantum numbers \( J^{PC} = 1/2/3^{\pm -} \). We systematically construct their interpolating currents, and calculate their mass spectra. To verify their existence, we propose to search for the spin-3 odderons in their VVV and VVP decay channels directly at LHC, with V and P light vector and pseudoscalar mesons respectively.

Keywords: odderon, glueball, exotic hadron, QCD sum rules

Introduction.— Very recently, the D0 and TOTEM Collaborations compared their \( pp \) and \( \bar{p}p \) [1] cross sections, and found they differ with a significance of 3.4σ [2]. This leads to the evidence of a \( t \)-channel exchanged odderon [3, 4], i.e., a colorless C-odd gluonic compound. They further combined their previous result of Ref. [5], and increased the significance of this evidence to 5.2σ–5.7σ. Accordingly, the D0 and TOTEM Collaborations claimed that they have accomplished the first experimental observation of the odderon!

Actually, in the previous TOTEM experiment [5] they had attempted to probe the existence of the odderon, and there had been some discussions on it [6–9]. However, these discussions are mainly on the contribution of the odderon exchange in certain collisions, so it is crucial and important to directly study the odderon itself. Recall that the odderon was first introduced in 1973 [10], and later reintroduced in QCD in the 1980’s [11–13]. There had been several attempts to prove its existence, and we refer to the review [14] for detailed discussions.

The odderon is probably/predominantly a \( C \)-odd three-gluon bound state, that is one category of glueballs. Glueballs are important for the understanding of non-perturbative QCD, but there is currently no definite experimental evidence for their existence, although there have been tremendous of discussions on them using various theoretical methods and models, such as the MIT bag model [15, 16], the flux tube model [17], the Coulomb Gauge model [18, 19], Lattice QCD [20–24], and QCD sum rules [25–28], etc. Especially, mass spectra of the spin-0 odderons were systematically studied in Refs. [29–33] by using QCD sum rules. However, in Ref. [34] the authors studied glueball trajectories, and their results suggest that the odderon is essentially an object in the Regge pole model, and any derivation of its finite mass from massless gluons meets conceptual difficulties in QCD, preventing reliable calculations of its spectrum.

The lowest-lying two-, three-, and four-glueballs were systematically investigated in Ref. [35], where the authors classified altogether six lowest-lying odderons with the quantum numbers \( J^{PC} = 1/2/3^{\pm -} \), and constructed their corresponding non-relativistic low-dimension operators. Some of these operators have been successfully used in Lattice QCD calculations. This inspires us to construct their corresponding relativistic odderon currents, which can be further applied to perform QCD sum rule analyses and calculate masses of the odderon, and introduced in QCD in the 1980’s [11–13]. There had been several attempts to prove its existence, and we refer to the review [14] for detailed discussions.

As the first step, we use the gluon field strength tensor \( G^a_{\mu\nu} \) to construct relativistic odderon currents. Here \( a \) is a color index, and \( \mu, \nu \) are Lorentz indices. Besides, we use \( d^{abc} \) to denote the totally symmetric \( SU_c(3) \) structure constant, and \( G^a_{\mu\nu} \) to denote the dual gluon field strength tensor, defined as \( G^a_{\mu\nu} = G^{a,\rho\sigma} \times \epsilon_{\mu\nu\rho\sigma}/2 \).

In Ref. [35] the chromoelectric and chromomagnetic fields \((i, j = 1, 2, 3)\),

\[
E_i = G_{i0} \quad \text{and} \quad B_i = -\frac{1}{2} \epsilon_{ijk} G^{jk},
\]

were used to write down the non-relativistic low-dimension odderon operators:

\[
\begin{align*}
1^+ & \quad d^{abc}(\vec{E}_a \cdot \vec{B}_b) \vec{B}_c, \\
1^- & \quad d^{abc}(\vec{B}_a \cdot \vec{E}_b) \vec{E}_c, \\
2^+ & \quad d^{abc} [E^i_a (\vec{E}_b \times \vec{B}_c)_i], \\
2^- & \quad d^{abc} [B^i_a (\vec{B}_b \times \vec{E}_c)_i], \\
3^+ & \quad d^{abc} [B^i_a B^j_b B^k_c],
\end{align*}
\]
where $S$ denotes symmetrization and subtracting trace terms in the set $\{ij\}$ or $\{ijk\}$.

To perform QCD sum rule analyses, we further construct their corresponding relativistic currents:

$$J^\alpha_1 = d^{abc} G^\mu_1 G_{b\mu} G^\alpha_3,$$

$$J^\alpha_2 = d^{abc} S' [G^\alpha_2 G_{b\mu} G^\beta_3 - \{\alpha_2 \leftrightarrow \beta_2\}],$$

$$J^\alpha_3 = d^{abc} S' [G^\alpha_3 G_{b\mu} G^{\beta_2} G_{c\mu} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

where $S'$ denotes symmetrization and subtracting trace terms in the two sets $\{\alpha_1, \ldots, \alpha_f\}$ and $\{\beta_1, \ldots, \beta_f\}$ simultaneously.

The first current $J^\alpha_1$ contains both $J^{PC} = 1^+\pm$ and $1^-\pm$ components, and it couples to the $1^+\pm$ and $1^-\pm$ odderon simultaneously:

$$\langle 0 | J^\alpha_1 | X^\pm \rangle = i f_{1^+\pm} - e_\pm G_{\mu\nu} p_\mu p_\nu,$$

$$\langle 0 | J^\alpha_1 | X^\pm \rangle = i f_{1^-\pm} - (p^\mu e^\pm - p^\pm e^\mu),$$

where $f_{1^+\pm}$ and $f_{1^-\pm}$ are decay constants, and $e_\pm$ is the polarization vector. This current has a partner

$$\tilde{J}^\alpha_1 = d^{abc} \tilde{G}_a G_{b\mu} \tilde{G}^\alpha_3,$$

which just has the opposite couplings:

$$\langle 0 | \tilde{J}^\alpha_1 | X^\pm \rangle = i f_{1^+\pm} - (p^\mu e^\pm - p^\pm e^\mu),$$

Therefore, we can use either $J^\alpha_1$ or $\tilde{J}^\alpha_1$ to investigate the $1^+\pm$ and $1^-\pm$ odderon at the same time. In the present study we use $J^\alpha_1$ to investigate the $1^+\pm$ odderon and $\tilde{J}^\alpha_1$ to investigate the $1^-\pm$ one, because the couplings given in Eqs. (6) and (10) can be more easily calculated.

The currents $J^\alpha_2$ and $J^\alpha_3$ and their partner

$$\tilde{J}^\alpha_2 = d^{abc} S' [G^\alpha_2 G_{b\mu} G^{\beta_3} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

couple to the $2^+\pm$ and $2^-\pm$ odderon through:

$$\langle 0 | J_2^- | X^\pm \rangle = i f_{2^+\pm} - S' [e_{\alpha_1, \beta_2, \mu \nu} G_{\mu\nu} p_\mu p_\nu]^2,$$

$$\langle 0 | \tilde{J}_2^- | X^\pm \rangle = i f_{2^-\pm} - S' [e_{\alpha_1, \beta_2, \mu \nu} G_{\mu\nu} p_\mu p_\nu]^2,$$

where

$$[\cdots]^J = e_{\alpha_1, \beta_2, \mu \nu} \cdots e_{\alpha_f, \beta_f, \mu \nu}.$$}

The current $J^\alpha_3$ and its partner

$$\tilde{J}^\alpha_3 = d^{abc} S' [G^\alpha_3 G_{b\mu} G^{\beta_3} G_{c\mu} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

couple to the $3^+\pm$ and $3^-\pm$ odderon through:

$$\langle 0 | J_3^- | X^\pm \rangle = i f_{3^+\pm} - S' [e_{\alpha_1, \beta_2, \mu \nu} G_{\mu\nu} p_\mu p_\nu]^3,$$

$$\langle 0 | \tilde{J}_3^- | X^\pm \rangle = i f_{3^-\pm} - S' [e_{\alpha_1, \beta_2, \mu \nu} G_{\mu\nu} p_\mu p_\nu]^3.$$

It is interesting to notice that all the above currents have $N = 2J$ Lorentz indices with certain symmetries, so that they can couple to the positive- and negative-parity odderon simultaneously. For example, the spin-2 current $J^\alpha_2$ and $J^\alpha_3$ have four Lorentz indices, satisfying

$$J_2^\alpha_2 = - J_2^\beta_2, \quad J_3^\alpha_3 = J_3^\beta_3.$$

**QCD sum rule analyses.** In this section we use the current $J^\alpha_1$ as an example to perform QCD sum rule analyses, which method has been widely applied in the study of hadron phenomenology [36, 37]. We study the two-point correlation function

$$\Pi^{\alpha, \alpha'}(q^2) = i \int d^4 x e^{i q x} (0 | [J^\alpha_1(x) J^\beta_1(0) | 0]) = (g^{\alpha\alpha'} - g^{\beta\beta'}) \Pi(q^2),$$

at both hadron and quark-gluon levels.

At the hadron level we use the dispersion relation to express Eq. (17) as

$$\Pi(q^2) = \int_0^\infty \frac{\rho(s)}{s - q^2 - i\varepsilon} ds,$$

where $\rho(s) = \Im \Pi(s)/\pi$ is the spectral density. We parameterize it using one pole dominance for the ground state $X$ together with the continuum contribution,

$$\rho(s) = \sum_n \delta(s - M_n^2) |\langle n | J | 0 \rangle|^2$$

$$= f_X^2 \delta(s - M_X^2) + \text{continuum}.$$
Parameter Set–I \([\alpha_s(G^2G)]\), and their combination \((g_s^2GG)^2\):

\[
\rho_{1^+}(s) = \frac{4\alpha_s^3}{81\pi} s^4 + \frac{10\alpha_s^2}{9} (g_s^2GG) s^2 + \frac{35\alpha_s^3}{36\pi} (g_s^2GG)^2 s^2 - \frac{205\alpha_s^2}{54} (g_s^3G^3) s,
\]

\[
\rho_{1^-}(s) = \frac{4\alpha_s^3}{81\pi} s^4 - \frac{10\alpha_s^2}{9} (g_s^2GG) s^2 + \frac{25\alpha_s^3}{36\pi} (g_s^2GG)^2 s^2 + \frac{5\rho_3^2}{27} (g_s^2GG) s^2,
\]

\[
\rho_{2^+}(s) = \frac{\alpha_s^3}{81\pi} s^4 + \frac{5\alpha_s^2}{27} (g_s^2GG) s^2 + \frac{15\alpha_s^3}{32\pi} (g_s^2GG)^2 s^2 - \frac{20\rho_3^2}{27} (g_s^3G^3) s,
\]

\[
\rho_{2^-}(s) = \frac{\alpha_s^3}{81\pi} s^4 - \frac{5\alpha_s^2}{27} (g_s^2GG) s^2 + \frac{15\alpha_s^3}{32\pi} (g_s^2GG)^2 s^2 - \frac{10\rho_3^2}{27} (g_s^3G^3) s,
\]

\[
\rho_{3^+}(s) = \frac{5\alpha_s^3}{2016\pi} s^4 + \frac{\alpha_s^2}{16} (g_s^2GG) s^2 - \frac{59\alpha_s^3}{512\pi} (g_s^2GG)^2 s^2 + \frac{13\rho_3^2}{384} (g_s^3G^3) s,
\]

\[
\rho_{3^-}(s) = \frac{5\alpha_s^3}{2016\pi} s^4 - \frac{\alpha_s^2}{16} (g_s^2GG) s^2 - \frac{49\alpha_s^3}{1536\pi} (g_s^2GG)^2 s^2 + \frac{157\rho_3^2}{3456} (g_s^3G^3) s.
\]

Especially, we find all the \(D = 8\) terms proportional to \((g_s^2GG)^2\) vanish, so the convergence of the above OPE series is quite good.

We need the strong coupling constant to perform numerical analyses [38]:

\[
\alpha_s(Q^2) = \frac{4\pi}{11\ln(Q^2/L^2_{\text{QCD}})}.
\]

with the QCD scale at \(L_{\text{QCD}} = 300\) MeV.

We shall see that the odderon mass \(M_X\) depends significantly on the gluon condensate \((g_s^2GG)^2\). However, this parameter is still not well known, so we use altogether two sets of parameters:

- Parameter Set–I [39]:

\[
\begin{align*}
(\alpha_s(G^2G)) &= (0.005 \pm 0.004) \times \pi \text{ GeV}^4, \\
(g_s^2G^3) &= (\alpha_s(G^2G)) \times (8.2 \pm 1.0) \text{ GeV}^2.
\end{align*}
\]

- Parameter Set–II [40, 41]:

\[
\begin{align*}
(\alpha_s(G^2G)) &= (6.35 \pm 0.35) \times 10^{-2} \text{ GeV}^4, \\
(g_s^2G^3) &= (\alpha_s(G^2G)) \times (8.2 \pm 1.0) \text{ GeV}^2.
\end{align*}
\]

As shown in Eq. (21), the mass \(M_X\) depends on two free parameters, the threshold value \(s_0\) and the Borel mass \(M_B\). We use two criteria to determine the Borel window. Firstly, we investigate the convergence of OPE, which is the cornerstone of a reliable sum rule analysis. Because the \(D = 8\) term proportional to \((g_s^2GG)^2\) vanishes, this convergence is already quite good, while we further require the \(\alpha_s^{n>3}\) terms \(\alpha_s^3(g_s^2GG)^2\) and \(\alpha_s^2(g_s^3G^3)\) to be less than 5%:

\[
\text{CVG} \equiv \left| \frac{\Pi(s_0, M_B^2)}{\Pi(s_0, M_B^2)} \right| \leq 5\%.
\]

Secondly, we investigate the one-pole-dominance assumption, and require the pole contribution (PC) to be larger than 40%:

\[
\text{PC} \equiv \left| \frac{\Pi(s_0, M_B^2)}{\Pi(s_0, M_B^2)} \right| \geq 40\%.
\]

Altogether we can determine the Borel window for a fixed \(s_0\). Then we change \(s_0\) and redo the same procedures, so that we can find the lower bound of \(s_0\).

Take the current \(J_{1}^{\alpha_3}\) as an example. When using the Parameter Set–I, there exist non-vanishing Borel windows as long as \(s_0 > 14.1\) GeV\(^2\), and the Borel window is determined to be 3.67 GeV\(^2\) \(< M_B^2 < 4.13\) GeV\(^2\) for \(s_0 = 16\) GeV\(^2\). Accordingly, we choose the working regions to be 14.0 GeV\(^2\) \(< s_0 < 18.0\) GeV\(^2\) and 3.67 GeV\(^2\) \(< M_B^2 < 4.13\) GeV\(^2\), and calculate the mass of the \(1^{++}\) odderon to be

\[
M_{X,1^{++}} = 2.87^{+0.17}_{-0.20} \text{ GeV}.
\]

Its uncertainty is quite large, mainly coming from the uncertainty of the gluon condensate \((g_s^2GG)^2\) given in Eqs. (29).

Similarly, we use the currents \(J_{1/2}^{\alpha_1\cdots\alpha_J,\beta_1\cdots\beta_J}\) and \(J_{1/2}^{\alpha_1\cdots\alpha_J,\beta_1\cdots\beta_J}\) to perform numerical analyses, and calculate masses of the \(J^{PC} = 1/2^{3^{++}}\) odderon. The obtained results are summarized in Table I. Note that the OPE convergence is sometimes so good that the lower bound of \(M_B\) can not be well determined, for which cases we need to properly choose \(s_0\) according to their partner states, e.g., see the result of \(X_{2^+}\) using the Parameter Set–II. We clearly see from Table I that our QCD sum rule results depend significantly on the gluon condensate \((g_s^2GG)^2\), which is currently not well known and still waiting to be clarified.

Summary and discussion.— In this letter we apply the method of QCD sum rules to study the odderon as a three-gluon bound state. There may exist six lowest-lying odderon with the quantum numbers \(J^{PC} = 1/2^{3^{++}}\). We systematically construct their interpolating currents using the gluon field strength tensors \(G_{\mu\nu}^a\) and \(\tilde{G}_{\mu\nu}^a\). All these currents have certain symmetries, so that they couple to both the positive- and negative-parity odderon, which need to be further separated at the hadron level. The construction of such currents is quite general and may be applied in fields other than hadron physics.
TABLE I: Masses of the $J^{PC} = 1/2/3^{±−}$ odderons, extracted from the currents $J_{1/2/3}^{α_1,⋯,α_J,β_1⋯β_J}$ and $\tilde{J}_{1/2/3}^{α_1,⋯,α_J,β_1⋯β_J}$. In the Parameter Set–I we choose the gluon condensate to be $\langle α_s GG \rangle = (0.005 \pm 0.004) \times π$ GeV$^4$ [39], and in the Parameter Set–II we choose it to be $\langle α_s GG \rangle = (6.35 \pm 0.35) \times 10^{-2}$ GeV$^4$ [40].

| Odderon     | Current    | $s_0^{min}$ [GeV$^2$] | Working Regions | Pole [%] | Mass [GeV] |
|-------------|------------|------------------------|-----------------|----------|------------|
|             |            | $s_0$ [GeV$^2$] | $M_P^2$ [GeV$^2$] |          |            |
| $X_{1−−}$   | $J_1^{β̃}$  | 14.1                   | 16.0 ± 2.0      | 3.67–4.13 | 40–50      | 2.87 ± 0.17 |
| $X_{2−−}$   | $J_2^{α_1α_2β_1β_2}$ | 10.6               | 16.0 ± 2.0      | 2.76–4.07 | 40–73      | 2.85 ± 0.16 |
| $X_{3−−}$   | $J_3^{α_1α_2α_3β_1β_2β_3}$ | 8.9                | 16.0 ± 2.0      | 2.60–4.23 | 40–81      | 2.78 ± 0.18 |
| $X_{1++}$   | $J_1^{β̃}$  | 15.1                   | 17.0 ± 2.0      | 2.93–3.52 | 40–54      | 3.29 ± 1.49 |
| $X_{2++}$   | $J_2^{α_1α_2β_1β_2}$ | 15.3               | 17.0 ± 2.0      | 3.29–3.74 | 40–50      | 3.16 ± 3.33 |
| $X_{3++}$   | $J_3^{α_1α_2α_3β_1β_2β_3}$ | 15.0               | 17.0 ± 2.0      | 2.55–3.36 | 40–58      | 3.47 ± 0.50 |

We construct altogether six relativistic low-dimension odderon currents with the quantum numbers $J^{PC} = 1/2/3^{±−}$. We use them to perform QCD sum rule analyses, and calculate masses of the $J^{PC} = 1/2/3^{±−}$ odderons. The results are summarized in Table I, sometimes with quite large uncertainties coming from the gluon condensates $⟨g_s^2 GG⟩$ and $⟨g_s^3 G^3⟩$. It is interesting to compare our results with the Lattice QCD results [21–24] obtained using non-relativistic odderon operators, as given in Table II.

From the above comparison, we can see how we poorly understand the odderon. Recall that there is currently no definite experimental evidence for the existence of any glueball yet, we quickly realize how important is the evidence of the odderon exchange recently observed by D0 and TOTEM [2]. Since this is still an indirect evidence, we propose to directly search for the odderon at LHC.

From the viewpoint of quark model, the odderon can decay after exciting three quark-antiquark pairs, and recombine into three mesons. Generally speaking, its width can be quite large, preventing it to be easily observed. We use $P$ and $V$ to denote the light vector and pseudoscalar mesons respectively, and its possible decay patterns are:

$1^{−−} \rightarrow VPP, VVP, VVP, VVV$ (S-wave),  
$1^{++} \rightarrow PPP, VPP, VVP, VVV$ (P-wave),  
$2^{−−} \rightarrow VVP, VVV$ (S-wave),  
$2^{++} \rightarrow VPP, VVP, VVV$ (P-wave),  
$3^{−−} \rightarrow VVV$ (S-wave),  
$3^{++} \rightarrow VVP, VVV$ (P-wave).

Due to their limited decay patterns, the spin-3 odderons have relatively smaller widths probably, and so we propose to search for them in their VVV and VVP decay channels directly at LHC.

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[1] V. M. Abazov et al. [D0 Collaboration], Measurement of the differential cross section $dσ/dt$ in elastic $p\bar{p}$ scattering at $\sqrt{s} = 1.96$ TeV, Phys. Rev. D 86, 012009 (2012).
[2] V. M. Abazov et al. [D0 and TOTEM Collaborations], Comparison of $pp$ and $p\bar{p}$ differential elastic cross sections and observation of the exchange of a colorless C-odd gluonic compound, arXiv:2012.03981 [hep-ex].
[3] J. R. Cudell et al. [COMPETE Collabora-
TABLE II: Masses of the $J^{PC} = 1/2^+/-$ odderon, in units of GeV. Our QCD sum rule results are obtained using the Parameter Set–I and Set–II, and the Lattice QCD results are taken from Refs. [21–23] (quenched) and Ref. [24] (unquenched).

| Odderon | Set–I | Set–II | Ref. [21] | Ref. [22] | Ref. [23] | Ref. [24] |
|---------|-------|--------|-----------|-----------|-----------|-----------|
| $X_{1}^{+}$ | 2.85$^{+0.10}_{-0.20}$ | 3.27$^{+0.15}_{-0.17}$ | 2.98$^{+0.03}_{-0.17}$ | 2.94$^{+0.03}_{-0.17}$ | 2.67$^{+0.07}_{-0.12}$ | 3.27$^{+0.34}_{-0.34}$ |
| $X_{2}^{-}$ | 2.85$^{+0.10}_{-0.20}$ | 3.28$^{+0.14}_{-0.19}$ | 4.23$^{+0.05}_{-0.20}$ | 4.14$^{+0.05}_{-0.20}$ | – | – |
| $X_{3}^{+}$ | 2.75$^{+0.18}_{-0.23}$ | 3.30$^{+0.15}_{-0.17}$ | 3.60$^{+0.04}_{-0.17}$ | 3.55$^{+0.04}_{-0.17}$ | 3.27$^{+0.09}_{-0.15}$ | 3.85$^{+0.35}_{-0.35}$ |
| $X_{1}^{-}$ | 3.29$^{+1.49}_{-0.32}$ | 5.05$^{+0.17}_{-0.14}$ | 3.83$^{+0.04}_{-0.19}$ | 3.85$^{+0.05}_{-0.19}$ | 3.24$^{+0.33}_{-0.15}$ | – |
| $X_{2}^{-}$ | 3.16$^{+0.33}_{-0.23}$ | 4.72$^{+0.15}_{-0.17}$ | 4.01$^{+0.05}_{-0.20}$ | 3.93$^{+0.04}_{-0.19}$ | 3.66$^{+0.13}_{-0.17}$ | 4.59$^{+0.74}_{-0.74}$ |
| $X_{3}^{-}$ | 3.47$^{+0.31}_{-0.50}$ | 5.45$^{+0.32}_{-0.21}$ | 4.20$^{+0.05}_{-0.20}$ | 4.13$^{+0.09}_{-0.20}$ | 4.33$^{+0.26}_{-0.20}$ | – |
[38] P. A. Zyla et al. [Particle Data Group], Review of Particle Physics, PTEP 2020, 083C01 (2020).

[39] B. L. Ioffe, QCD (Quantum chromodynamics) at low energies, Prog. Part. Nucl. Phys. 56, 232 (2006).

[40] S. Narison, QCD parameter correlations from heavy quarkonia, Int. J. Mod. Phys. A 33, 1850045 (2018) Addendum: [Int. J. Mod. Phys. A 33, 1892004 (2018)].

[41] S. Narison, Gluon condensates and precise $\overline{m}_{c,b}$ from QCD-moments and their ratios to order $\alpha_s^3$ and $\langle G^2 \rangle$, Phys. Lett. B 706, 412 (2012).