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Massive parallel generation of indistinguishable single photons via the polaritonic superfluid to Mott-insulator quantum phase transition

Neil Na\textsuperscript{1,4} and Yoshihisa Yamamoto\textsuperscript{2,3}
\textsuperscript{1} Intel Corporation, 2200 Mission College Blvd, Santa Clara, CA 95054, USA
\textsuperscript{2} E L Ginzton Laboratory, Stanford University, Stanford, CA 94305, USA
\textsuperscript{3} National Institute of Informatics, Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan
E-mail: yun-chung.n.na@intel.com

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Abstract. We study the possibility of utilizing the superfluid to Mott-insulator quantum phase transition in an array of quantum well exciton–polariton traps to generate indistinguishable single photons in a massive parallel fashion. By means of analytical and numerical methods, the device operations and system properties are examined using realistic experimental parameters. Such a deterministic, massive parallel generation may find new applications in photonic quantum information processing.

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\textsuperscript{4} Author to whom any correspondence should be addressed.
1. Introduction

Generation of indistinguishable single photons from a large number of independent emitters is essential in many recent proposals of scalable quantum information processing [1]. Preliminary experimental effort has demonstrated the feasibility of it in systems such as trapped atoms and ions [2], as well as impurity-bound excitons [3]. However, in these works where single photons are generated by the radiative decay of spatially independent emitters pumped via incoherent optical excitation, at most two independent single-photon sources could be prepared. Collective generation of many indistinguishable single photons simultaneously remains out of reach.

An approach that is less frequently discussed in the literature can be found in the context of polaritonic quantum phase transition (QPT) [4]. Such a polaritonic QPT from a superfluid (SF) to a Mott-insulator (MI) state was predicted in a variety of solid-state systems, such as a cavity QED array containing either four-level atomic ensembles in an EIT configuration or single/many two-level atoms in the strong coupling regime. The existence of the Bose-glass phase due to system disorder was also examined [5]. In this paper, we study specifically the possibility of utilizing quantum well (QW) exciton–polaritons [6] to generate indistinguishable single photons in a massive parallel fashion. The basic idea is to load a dilute gas of QW exciton–polaritons into periodic potential traps and drive the system across the SF–MI QPT by modulating the photon–exciton frequency detuning. Indistinguishable single photons can then be triggered deterministically in the MI phase, thereby obtaining many of them simultaneously. More importantly, a selective switching scheme is introduced so that the quantum efficiency of single-photon emissions is dramatically enhanced, making our non-classical light source truly useful. We also show that the proposed system features a stronger polaritonic nonlinearity compared to the previous proposals, and the impact of inevitable fabrication disorder is of limited importance. These two ingredients are essential for overcoming the major difficulties in implementing the theoretical concept in a solid-state system, and therefore provide a practical path toward experimental demonstration.

2. Experimental setup

Figure 1 shows a schematic diagram of the proposed device. A single GaAs QW is embedded in a half-wavelength Al$_x$Ga$_{1-x}$As optical cavity layer, which is sandwiched in between the upper and the lower distributed Bragg reflectors (DBRs). The thickness of the optical cavity layer is spatially modulated by etching/oxidizing small mesas that serve as photon-trapping centers [7]. Metal gates are fabricated on the top to apply a vertical electric field so that the photon–exciton frequency detuning can be controlled by the quantum confined Stark effect (QCSE) [8]. Photons and excitons in this system are strongly coupled to each other, and their normal modes are defined as polaritons. Details of the formation of these three-dimensionally confined polaritons can be found in [9], and here we take advantage of the result that they can be treated as single-mode systems. The dynamics of such an array of QW exciton–polariton traps can be described by using the Bose–Hubbard model (BHM) with a system–reservoir coupling, which will be discussed in depth in the next section. The lower DBR is made thicker than the upper DBR to enforce single-side cavity emission. The modulated planar microcavities inherit circular symmetry and are suitable for coupling to downstream fiber optics with high collection efficiency.
3. The system Hamiltonian

The system Hamiltonian is given by

\[ H = \sum_{c=a,b} \int dr \left( \frac{-\nabla^2}{2m_c} + V_c(r) \right) \Psi_c(r) + g' \int dr \Psi_a(r) \Psi_b(r) + \text{H.c.} \]

\[ + \frac{u'}{2} \int dr \Psi_a(r) \Psi_b(r) \Psi_b(r) \Psi_a(r) - \Delta g' \int dr \Psi_a(r) \Psi_b(r) \Psi_b(r) \Psi_a(r) + \text{H.c.} \]

\[ + \int dr f(r,t)e^{-i\nu t} \Psi_a(r) + \text{H.c.}, \]

(1)

where the field operators \( \Psi_a \) and \( \Psi_b \) refer to a cavity photon and a QW exciton. The first term in (1) represents the free Hamiltonians of trapped photons and excitons. The second to fifth terms correspond to photon–exciton coupling, exciton–exciton repulsion, reduction of excitonic dipole moment and external laser coupled to cavity mode, respectively. Since the effective mass of a QW exciton is much larger than that of a cavity photon, it would be appropriate to define a single-mode exciton operator \( b_i \) that features the same wavefunction as that of a single-mode photon operator \( a_i \) [10]. By doing so, (1) can be rewritten as

\[ H = \omega_a \sum_i a_i^\dagger a_i + t \sum_{\langle ij \rangle} a_i^\dagger a_j + \omega_b \sum_i b_i^\dagger b_i + g \sum_i (a_i^\dagger b_i + \text{H.c.}) \]

\[ + \frac{u}{2} \sum_i b_i^\dagger b_i^\dagger b_i b_i - \Delta g \sum_i (b_i^\dagger a_i^\dagger b_i b_i + \text{H.c.}) \]

\[ + f(t) \sum_i (a_i^\dagger e^{-i\nu t} + \text{H.c.).} \]

(2)

Here \( \omega_a \) and \( \omega_b \) are the site cavity photon and QW exciton energies; \( t \) is the photon tunneling energy determined by the overlapping of nearest-neighbor cavity fields; \( g \) is the photon–exciton coupling constant; \( u \) and \( \Delta g \) are energies that correspond to the exciton–exciton repulsion and
the reduction of excitonic dipole moment; and \( f(t) \) and \( v \) are the external laser amplitude and energy. Next, we define the upper polariton (UP) and lower polariton (LP) operators \( q_i \) and \( p_i \) as a linear superposition of \( a_i \) and \( b_i \) with appropriate Hopfield coefficients \( A \) and \( B \).\(^5\) The system master equation for LPs in the rotating frame of the external laser is derived as

\[
\frac{d\rho}{dt} = \frac{1}{i}[\hat{H}, \rho] - \frac{\Gamma}{2} \sum_i (\rho p_i^+ p_i + p_i^+ p_i \rho - 2p_i \rho p_i^+),
\]

under rotating wave approximation, where

\[
\hat{H} = -\Delta \sum_i p_i^+ p_i + J \sum_{(ij)} p_i^+ p_j + \frac{U}{2} \sum_i p_i^+ p_i p_i + F(t) \sum_i (p_i^+ + p_i).
\]

The UP dynamics are discarded because the external laser selectively pumps the LPs. \( \Delta \) is the energy difference between the external laser and the trapped LPs; \( J \) is the LP tunneling energy and is equal to \( t A^2 \); \( U \) is the LP–LP interaction energy and is equal to \( u B^4 + 4\Delta g B^3 \). Assuming an infinite potential barrier with area \( S \) for polariton trapping, \( u \) can be calculated by \( \sim 2.2 E_B \cdot \pi a_B^2 / S \) due to fermionic exchange interaction, and \( \Delta g \) can be calculated by \( \sim 4 g \cdot \pi a_B^2 / S \) due to phase space filling and fermionic exchange interaction \( [11] \). \( E_B \) and \( a_B \) are the 1s exciton binding energy and Bohr radius, respectively. Let \( S = \pi (\lambda / 2)^2 \), where \( \lambda = 211 \text{ nm} \) (the emission wavelength of a 10 nm GaAs QW divided by the GaAs refractive index at 4 K), then \( u \) and \( \Delta g \) are derived as 200 and 90 \( \mu eV \), given \( E_B = 10 \text{ meV} \), \( a_B = 10 \text{ nm} \) and \( g = 2.5 \text{ meV} \). \( F(t) \) is equal to \( f(t)A \); \( \Gamma \) is the LP decay rate and is equal to \( A^2 \omega_a / Q + B^2 / \tau_p \). A cavity \( Q \) factor equal to \( 10^6 \) is used\(^6\) and a QW exciton lifetime \( \tau_p \) equal to 0.5 ns is used. Note that because the acoustic phonon–polariton scattering time exceeds 1 ns for zero in-plane momentum regime at 4 K, and the polariton–polariton scattering is negligible for LP densities smaller than \( 10^{10} \text{ cm}^{-2} \), our system decoherence is expected to be limited by the radiative process.

For an ideal one-dimensional (1D) system with unit filling, the critical point of the BHM calculated by quantum Monte-Carlo simulation is \( U/J_c \approx 2.04 \) [13]. If we assume that the polariton lifetime is long enough compared to all other time scales, this condition of QPT can be reasonably applied to our system [4, 5]. In figure 2, we plot \( U/J \) as a function of photon–exciton frequency detuning \( \delta \), given different \( t \) values that are determined by the inter-cavity distance \( d \). It is found that the critical point can be reached by modulating a negative \( \delta \) (red-detuning) into a positive \( \delta \) (blue-detuning), i.e. changing from a photon-like polariton into an exciton-like polariton. This is physically expected, as an exciton-like polariton features larger \( U \) (due to exciton nonlinearity) and at the same time smaller \( J \) (due to photon tunneling). Note that the coherence length of a polariton condensate has been experimentally measured to be \( \sim 20 \mu m \), limited only by the pump spot size [14]. In this case, we can prepare roughly 2000

\(^5\) Here we define \( q_i = -B a_i - A b_i \) and \( p_i = -A a_i + B b_i \) so that \( A = 2g \sqrt{\frac{\delta + \sqrt{\delta^2 + 4g^2}}{2} + 4g^2} \) and \( B = \sqrt{\frac{\delta + \sqrt{\delta^2 + 4g^2}}{2} + 4g^2} \) are both greater than zero. \( \delta = \omega_a - \omega_b \) is the photon–exciton frequency detuning. Note that the LPs are photon-like, i.e. \( A >> B \), when \( \delta < 0 \) and \( |\delta| >> g \) (large red-detuning); they are exciton-like, i.e. \( B >> A \), when \( \delta > 0 \) and \( |\delta| >> g \) (large blue-detuning).

\(^6\) Note that \( Q \) close to \( 10^6 \) has been demonstrated in similar microcavity systems, see [12]. While more engineering effort is needed for the devices in [7] (\( Q \) close to \( 10^5 \)) to reach this goal, there are no fundamental hurdles to closing the technology gap.
sites given $t = 20 \text{GHz}$, which corresponds to $d \sim 0.45 \mu\text{m}$ presuming a cavity field consisting of a horizontal Gaussian function with $1/e$ half-width equal to $\lambda/2$ and a vertical rectangle function with full-width equal to $\lambda/2$. The relationship between $t$ and $d$ is calculated by using $t = \omega_a \exp(-2d^2/\lambda^2)/2$, which is derived from equation (9) of [5]. In the following simulations, $t = 20 \text{GHz}$ will be used.

4. Device operations and system properties

To study the system dynamics that cannot be captured by the calculations shown in figure 2, numerical simulations are performed by discretizing (3) in the time domain where matrix representations of all operators are constructed. Due to the rapid increase of Hilbert space size with cavity number, we choose six 1D coupled cavities with periodic boundary conditions. The sharp SF–MI QPT is smeared in such a finite number of cavities, but suffices to prove the operation principle of our proposal.

The device operation procedures are shown in figures 3(a) and (b) for the odd- and even-numbered cavities, respectively. The system is initially (at 0 ps) prepared in a photon-like SF state where $U/J \sim 0.13$, which is realized by a large red photon–exciton frequency detuning $\delta = -3g$ and a numerical excitation condition

$$\frac{1}{N!} \left( \frac{1}{\sqrt{N}} \sum_{i=1}^{N} p_i^\dagger \right)^n \rho_0 \left( \frac{1}{\sqrt{N}} \sum_{i=1}^{N} p_i \right)^n . \quad (5)$$

Here $N$ is the cavity number, $n$ is the polariton number and $\rho_0$ is the density matrix of vacuum, respectively. Note that (5) excites on average one polariton per cavity that hops randomly in the coupled cavities. Experimentally, this can be achieved by controlling the external laser coupled to the cavity mode with appropriate pulse amplitude and width, because a coherent field excitation can mimic the initialization condition (5) in the thermodynamic limit, i.e. $N \gg 1$. Then, by using QCSE (from 0 to 200 ps), the QW exciton energy is lowered by the applied vertical electric field so that $\delta$ is switched from $-3g$ to $4g$, i.e. into an exciton-like MI state where $U/J \sim 35$. A single LP is localized in each cavity due to the dominance of polariton–polariton interaction over nearest-neighbor tunneling. The shape of the electrical...
Figure 3. The device operation procedures and dynamics in the odd (a, c) and even (b, d) numbered cavities plotted as a function of time. In (a, b), the solid blue line corresponds to the electrical switch pulse. In (c, d), the solid blue, dashed green and dotted red lines correspond to the average LP number, average photon number and LP second-order coherence, respectively.

switch pulse follows a hyperbolic tangential function with switching speed equal to 10 GHz, which is chosen to perform an adiabatic transition during this time window. Finally, while $\delta$ of the even-numbered cavities stay at $4g$, $\delta$ of the odd-numbered cavities are switched rapidly at the speed of 1 THz back to $-4g$ (at 200 ps). Single-photon emissions are now triggered from the odd-numbered cavities, and the purpose of such a selective switching will be explained shortly. Note that $\tau_b$ and $g$ are independent of $\delta$ because the lifetime and oscillator strength of a QW exciton barely change for the range of vertical electric fields used in the above $\delta$ switching [8].

The use of selective switching at 200 ps is for three considerations. Firstly, the quantum efficiency of generating single photons [15]

$$\eta = \int \left\langle p_i^\dagger(t)p_i(t)\right\rangle A^2(t) \frac{\alpha_g}{Q} dt$$

should be maximized, so that a polariton decay is mostly directed to the cavity mode. Switching the exciton-like LPs back to the photon-like LPs enables achieving this goal. Secondly, if all of the cavities are switched back to a large red-detuning regime, the rapid tunneling process with $J/\Gamma \sim 194$ readily destroys the deterministic single-polariton decay from individual sites (simulation results not shown). Instead, in the present selective switching, only the LPs in the odd-numbered cavities are switched back to a large red-detuning regime so that the neighboring site energy mismatch effectively cuts off the unwanted tunneling events. Finally, the frequency of the emitted single photons is tuned away from that of the external laser. Using a narrow band-pass frequency filter, a clean output signal can be selected.

The dynamics of the odd- and even-numbered cavities are shown in figures 3(c) and (d), respectively. During the adiabatic $\delta$ switching, the normalized zero-delay second-order
coherence function $g^{(2)}(0)$ starts at $\sim 0.81$ at 0 ps because of injecting six photon-like LPs that hop randomly in the coupled cavities and subsequently drops to $\sim 0.01$ at 200 ps because of localizing one exciton-like LP in each cavity. This strong antibunching behavior indicates the crossing of the SF–MI boundary. The effect of selective switching can be seen from the sharp increase of the average photon number $\langle N_a \rangle$ in the odd-numbered cavities. The $\eta$ of the single-photon emissions is as high as $\sim 79.5\%$ in figure 3 by calculating (6) starting at 200 ps, and can be further maximized by carefully designing the switch pulse shape. The ultimate physical limit of $\eta$ comes from how large $U$ or $J_c$ can be and therefore how fast is the adiabatic $\delta$ switching used.

To further understand the system dynamics, we define two parameters: the far-field optical interference visibility [16]

$$V(t) = \frac{\langle n_a(t) \rangle_{\text{max}} - \langle n_a(t) \rangle_{\text{min}}}{\langle n_a(t) \rangle_{\text{max}} + \langle n_a(t) \rangle_{\text{min}}},$$

where

$$n_a(t) = \frac{1}{N} \sum_{m,n \in \mathbb{Z}} a_m^\dagger(t)a_n(t)e^{i(m-n)\phi},$$

and the single-photon indistinguishability [17]

$$I(t) = 1 - \frac{\langle c_1^\dagger(t)c_3^\dagger(t)c_3(t)c_1(t) \rangle}{\langle c_1^\dagger(t)c_1(t) \rangle \langle c_3^\dagger(t)c_3(t) \rangle},$$

where

$$\begin{pmatrix} c_1 \\ c_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_3 \end{pmatrix}.$$
Figure 4. The far-field optical interference visibility (solid blue line) and the single-photon indistinguishability (dashed green line) plotted as a function of time. Only odd-numbered cavities are taken for calculations. The slight oscillations in both parameters indicate the weak non-adiabaticity due to the use of a moderately fast $\delta$ switching.

... manually addressed and compensated for by QCSE, which alleviates the inhomogeneity seen by LPs. Finally, since the system is prepared initially in an SF state, the site energy disorder is effectively reduced by roughly a factor of $d/J$. Compared to other deterministic generation schemes such as the photon blockade (PB) effect [18], where the bandwidth of a pumped $\pi$ pulse cannot spectrally overlap the inhomogeneous LP site energies well, the initialization of LP population in our scheme can be much more uniform. Note that an increase of a pumped $\pi$ pulse bandwidth in the PB scheme to improve the spectral coupling is not allowed because a second LP is then excited and breaks down the PB principle. Based on these benefits, our proposal can largely overcome the site energy disorder and therefore promises to be a practical approach toward massive parallel generation of indistinguishable single photons.

The required temperature for our proposal determines the feasibility of a laboratory demonstration. To avoid particle–hole excitation in an MI state, we need the thermal energy $KT$ to be much smaller than $U$. Suppose $KT$ is an order of magnitude smaller than $U$, then $T \sim 0.2$ K is in general needed. This increases the experimental difficulties because a dilute refrigerator must be used. Nevertheless, the proposed system operation is based on a coherent spectroscopic technique and a serious thermalization effect kicks in only when the LPs are exciton-like, which lasts shorter than 100 ps during the device operation procedure (see figures 3(c) and (d)). Such a value is smaller than the typical thermalization time in an exciton–polariton system at 4 K, and in this sense we may really probe the zero-temperature quantum dynamics shown above.

6. Generation of polarization-entangled photon pairs

So far we have neglected the spin of an LP by assuming that a circularly polarized external laser is used for optical pumping. It is possible to generate polarization-entangled photon pairs via the QPT from an SF to an MI state if the two spin species are simultaneously injected. Our scheme is illustrated in figure 5. Initially, a linearly polarized external laser injects on average two LPs per cavity at a large $J/U$, which forms a photon-like SF. Subsequent adiabatic $\delta$ switching sweeps the system into an exciton-like MI with two localized LPs in each cavity. While more studies...
are required for establishing the exact phase diagram of spin-dependent interacting polaritons, we argue that the ground state of the proposed scenario is a collection of two opposite-spin LPs occupying the same site. This is due to the fact that the electron component of an exciton must satisfy the Pauli exclusion principle, so that the following emissions are similar to those of biexcitons in a semiconductor quantum dot [19]. By using the selective switching as described above, two-photon cascaded emission is triggered where the anticorrelation of LP spins is translated into the circularly polarized states of photons. A maximally polarization-entangled photon pair \( |\sigma^+\rangle_1 |\sigma^-\rangle_2 + |\sigma^-\rangle_1 |\sigma^+\rangle_2 \)/\( \sqrt{2} \) can be obtained (subscripts 1 and 2 refer to the first and second photons emitted that have an energy difference equal to \( U \)).

7. Summary

We have shown how to harness the QPT from an SF to an MI state in a QW exciton–polariton system to deterministically generate indistinguishable single photons in a massive parallel fashion. More importantly, a selective switching scheme is introduced for significantly increasing the quantum efficiency of single-photon emissions. The strong polaritonic nonlinearity and the robustness against site energy disorder together pave a practical route for laboratory studies. A variety of other applications such as the photon number eigenstate interferometer [20] and subwavelength quantum lithography [21] can benefit from our proposal.

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