Requirements for CubeSat on-board sensors for identification of design and inertial characteristics

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Abstract. In this paper, we discuss requirements formulation for CubeSat on-board sensors characteristics for solving identification problem. The problem is to estimate design and inertial parameters of CubeSat-format nanosatellite using on-board sensor measurements. Identification problem for CubeSats becomes more and more actual every year because CubeSat design becomes more complicated due to complex missions requirements. Modern CubeSats can be equipped with deployable solar panels, propulsion units and different types of transformable structures. This fact makes it necessary to study CubeSats as bodies with variable inertial and design characteristics. In this article the design parameter is fuel level in the CubeSat propulsion system and inertial parameters are inertial coefficients (inertia moments analog). We study requirements formulation for magnetometer and angular rate sensor, as this combination of sensors is the most popular in CubeSat development. The main idea of this paper is to study the sensitivity of measurements to estimated parameters. This sensitivity depends on time and measurement noise, large sensitivity leads to low estimation error. We conducted a numerical study and provide an algorithm which allows to define sensitivity-time-noise dependence for each type of sensors and formulate the requirements for their characteristics.

1. Introduction

Today the number of nanosatellites (nanosats) which are involved in complex space missions has significantly increased. These missions are creation of global communication systems, remote Earth sensing, space debris observing, geophysical fields research and etc [1]. Achieving novel results in these branches of science is possible with nanosatellite constellations. Spatial geometry of constellation is constantly changing due to gravitational and aerodynamics disturbances so nanosat should be able to perform orbital maneuvers.

Thus nanosat should be equipped with propulsion unit (PU). During orbital operation fuel mass decreases so inertial and design characteristics of nanosat change. These changes influence on the effectiveness of attitude control system (ACS). To increase efficiency of ACS it is necessary to identify these variations and take into account in ACS.

Moreover nanosats are still not reliable enough comparing with large spacecraft [2]. This is due to commercial electronic components that are widely used in CubeSat development. So it may be an emergency situation on board the nanosatellite. In this case it is useful to have special identification on-board algorithms, which can help mission operator to determine cause of this situation and make a decision how to safe efficiency of nanosat. Identification procedure is based on measurements processing so it is necessary to provide recommendations and special algorithm for choosing on-board sensors characteristics.
2. Problem formulation

Low-orbital nanosatellite dynamics and kinematics are presented by differential equations systems [3]. These systems behavior depend on different parameters such as initial conditions of angular rate $\mathbf{\omega}(t_0)$ and orientation quaternion $\mathbf{q}(t_0)$, nanosat inertial coefficients $\mathbf{K}_K = [\lambda, \mu]$ and vector of aerodynamic coefficients $\mathbf{K}_A$; these parameters are directly included in nanosat model of angular motion and are combined into vector of estimated motion model parameters $\mathbf{b}^{est} = [\mathbf{\omega}(t_0), \mathbf{q}(t_0), \mathbf{K}_K, \mathbf{K}_A]$. In some cases it is necessary to identify specific nanosat parameters that are not contained in motion model for example fuel level in PU or solar panels deployment position. These parameters values affect such nanosat parameters as $(\mathbf{K}_K$ and $\mathbf{K}_A)$ and are combined into vector of nanosatellite specific parameters $\chi$. So it is useful to study dependence between $\mathbf{b}^{est}$ and $\chi$ by obtaining direct $\mathbf{b}^{est}(\chi) = f(\chi)$ and inverse $\chi(\mathbf{b}^{est}) = F(\mathbf{b}^{est})$ models, which are used for estimation $\chi$ after identification of $\mathbf{b}^{est}$.

Identification process is based on the idea of minimizing objective function which has variable parameters [6]. Parameters values which provide minimum of differences are the most close to real. This process can be described by equation (3).

$$\min_{\mathbf{b}^{est} \in R^n} I(\mathbf{b}^{est}, T) = \frac{1}{2} \sum_{i=1}^{N} \mathbf{r}_i(\mathbf{b}^{est})^T \mathbf{r}_i(\mathbf{b}^{est}) = \frac{1}{2} \left\| \mathbf{r}_i(\mathbf{b}^{est}) \right\|^2,$$

where $n$ is the number of parameters, $R^n$ - “observation space”, $N = T / \Delta t$ - number of measurements, $\Delta t$ - measurement time step, $T$ - time interval of data collection, $\mathbf{r}_i(\mathbf{b}^{est}) = \mathbf{g}(t_i, \mathbf{b}^{est}) - \mathbf{y}(t_i)$ - are auxiliary residual functions, $\mathbf{g}(t_i, \mathbf{b}^{est})$ - measurement model, $\mathbf{y}(t_i)$ - observed measurement values.

Effectiveness of identification directly depends on sensor characteristics. Attitude sensors used on nanosats can be divided into two groups according to the physics of measurement: inertial sensors and vector sensors. The inertial sensors include gyroscope and accelerometer. The vector ones include three-axis magnetometer, horizon sensor, solar sensor, and star tracker, etc. We provide a brief statistical overview on attitude sensors that are used on nanosats according to [4].

![Figure 1. Attitude sensor comparison.](image-url)

Attitude determination systems based on a combination of gyroscopes (GYR), magnetometers (MAG) and sun sensors (SS) received the greatest application in nanosat design because of low cost,
acceptable accuracy and small dimensions. In general, measurement model of these sensors can be described as follows:

\[ g(t_i, b^{est}) = \xi(t_i, b^{est}) + w(t_i, \sigma), \]

where \( \xi(t_i, b^{est}) \) - measurement function \( (\xi(t_i, b^{est}) = A(t_i, b^{est})v) \) in case of use vector sensor and \( \xi(t_i, b^{est}) = \omega(t_i, b^{est}) \) in case of use GYR), \( w(t_i) \) - measurement noise, \( \sigma \) - sensor noise standard deviation \( A(t_i, b^{est}) \) - rotation matrix [5], \( v \) - model vector of measured physical value.

It can be seen that \( b^{est} \) consists of different-type parameters so for desired error vector \( \Delta b^{des} \) and given \( \sigma \) time interval \( T \) will be different (if nanosat is under development both \( \sigma \) and \( T \) should be chosen). To choose them we use equation:

\[ \Delta b_i^{max} = 3\sigma^2 \left( \frac{\partial l(b^{est}, T)}{\partial b_i} \right)^{-1}, \quad i = 1,n. \]  

(2)

If \( \Delta b_i^{max} < \Delta b_i^{est} \) so it is a time \( T \) that allows to identify \( b \) with given error for particular sensor. If nanosat has some specific features parameters \( \chi \) of which should be estimated then we should choose \( b_j^{est} \) that allows to estimate \( \chi_j \) with minimum error. Choice is made by analysis of derivatives \( \frac{\partial b_j^{est}}{\partial \chi_j} \cdot i = 1,n \), \( j = 1,k \). For each \( \chi_j \) we should chose those \( b_j^{est} \) which have larger derivative.

Corresponding element of \( b^{est} \) with maximum derivative for each \( \chi_j \) is defined as \( b_j^{\delta} \). Similarly to the case of \( \Delta b_i^{max} \), we can write:

\[ \Delta \chi_j^{max} = 3 \sigma \left( \frac{\partial b_j^{\delta}}{\partial \chi_j} \right)^{-1} \left( \frac{\partial l(b, T)}{\partial b_j^{\delta}} \right)^{-1}, \quad j = 1,k. \]  

and choose \( T \) and \( \sigma \) in the same way. In general derivatives \( \frac{\partial \chi_j^{max}}{\partial \chi_j} \) and \( \frac{\partial l(b^{est}, T)}{\partial b_i} \) are calculated numerically due to non-linear behavior of nanosat dynamics.

2.1. Algorithm for requirements formulation

We provide an algorithm that allows to choose characteristics of attitude sensors \( \sigma \) and time interval of data collection for both vectors \( b^{est} \) and \( \chi \). Algorithm consists of the following steps:

- Define vector \( b^{est} \) according to mission specific;
- Define vector \( \chi \) according to nanosat specific;
- Obtain dependence models \( b^{est}(\chi) = f(\chi) \) and \( \chi(b^{est}) = F(b^{est}) \);
- Define desired errors \( \Delta \chi_j^{des} \) and \( \Delta b_i^{des} \);
- Define range of \( \sigma \) according to available sensors;
- For each \( \sigma \) and varying \( T \) calculate \( \Delta \chi_j^{max} \) and \( \Delta b_i^{max} \), using (2) and (3);
- For each \( \sigma \) find such \( T \) that \( \Delta \chi_j^{max} \leq \Delta \chi_j^{des} \) and \( \Delta b_i^{max} < \Delta b_i^{des} \).

3. Requirements for SamSat-M sensors for identification its inertial and design parameters

This section illustrates algorithm application for SamSat-M nanosat which is being developed in Samara University. The mission goal of SamSat-M is testing of orbital maneuver technology and propulsion unit [6]. SamSat-M has design parameters that are shown in Table 1.
Table 1. SamSat-M design parameters.

| Parameter     | Min. Value | Max. Value |
|---------------|------------|------------|
| $\mu$         | -0.81      | -0.78      |
| $\lambda$     | 0.979      | 0.982      |
| $m_f \text{ (kg)}$ | 0          | 0.4        |
| $h_f \text{ (m)}$ | 0          | 0.07       |
| $M_s \text{ (kg)}$ | 3.17      | -          |

In Table 1 $m_f$ is the fuel mass, $h_f$ is the fuel level and $M_s$ is SamSat-M mass without fuel. In the mission of flight PU testing it is necessary to monitor fuel level so $\chi = [h_f]$. Inertial characteristics $K_f$ of low-orbit nanosat SamSat-M are described by dimensionless inertia coefficients $\lambda = I_x/I_z$ and $\mu = (I_y - I_z)/I_z$ [7], aerodynamic moment coefficients $K_A$ also contain inertial characteristics. Thus vector $b^ext$ is written in the following form:

$$b^ext = [K_f \quad K_A].$$

Derivatives have a form:

$$\frac{\partial b^ext}{\partial \chi} = \begin{bmatrix} \frac{\partial \lambda}{\partial h_f} & \frac{\partial \mu}{\partial h_f} & \frac{\partial K_x}{\partial h_f} & \frac{\partial K_y}{\partial h_f} & \frac{\partial K_z}{\partial h_f} \end{bmatrix}.$$

We obtain model $b(\chi) = f(\chi)$ in the analytical form (in this case) and calculate values of derivatives:

$$\frac{\partial b^ext}{\partial \chi} = [0.05 \quad 0.41 \quad 0.01 \quad 0.08 \quad 0.02],$$

as it can be seen for SamSat-M $b^\delta = \frac{\partial \mu}{\partial h_f} = 0.41$. We also obtain models $b^ext(\chi) = f(\chi)$, for example model $\mu(h_f)$ is written as follows:

$$\mu(h_f) = \frac{I_{xx} - I_{yy} + M_x \left( X_{cm} - \frac{M_s X_{cm}}{Q_7} \right) + Q_6 - Q_1 + 0.5m_f r_f^2 + m_f \left( \frac{M_s^2 X_{cm}^2}{(M_s + m_f)^2} - Q_4 \right) - Q_2 - Q_3}{I_{xx} + Q_1 + Q_2 + Q_3}$$

where $Q_1 = \frac{m_f (h_f^2 + 3r_f^2)}{12}, Q_2 = m_f \left( \frac{h_f^2}{2} - Q_5 + L_4 \right)^2 + Q_4, Q_3 = M_s (Q_6 + Y_{cm}), Q_4 = \frac{Q_7}{Q_5}, Q_5 = 1, Q_2 = m_f (0.5h_f + L_4) + M_s Y_{cm}, Q_6 = (Z_{cm} - \frac{M_s Z_{cm}}{Q_7})^2, Q_7 = m_f + M_s, m_f = \pi h_f r_f^2 \rho_f$ - fuel mass, $h_f$ - fuel level, $\rho_f$ - fuel density, $r_f$ - fuel tank radius, $[I_{xx} \quad I_{yy} \quad I_{zz}]$ - satellite inertia moments without fuel, $L_4$ - distance between satellite front edge and satellite fuel tank front edge, dependence $\mu(h_f)$ is shown in figure 2.
4. Results
We studied algorithm application for different sensors characteristics. We simulated MAG measurements with \( \sigma_h \) noise 8nT, 20nT and 50nT, for GYR measurements \( \sigma_\omega \) 0.025 deg/sec, 0.1 deg/sec and 0.25 deg/sec. Results for GYR sensor for \( \mathbf{b}^{\text{est}} \) vector are shown in Figure 3 and Figure 4.

![Figure 2. Dependence \( \mu(h_f) \).](image)

**Figure 2.** Dependence \( \mu(h_f) \).

![Figure 3. \( \mathbf{K}_A \) estimation error for different GYR sensors.](image)

**Figure 3.** \( \mathbf{K}_A \) estimation error for different GYR sensors.
Figure 4. $K_i$ estimation error for different GYR sensors. Results for MAG sensor for $b^{est}$ vector are shown in Figure 5 and Figure 6.

Figure 5. $K_A$ estimation error for different TAM sensors.

Figure 6. $K_f$ estimation error for different TAM sensors.
Results for vector $\chi$ are presented in figures 7 and 8.

Figure 7. $\chi$ estimation error for different TAM sensors.

Figure 8. $\chi$ estimation error for different GYR sensors.

Figures 3-8 show that time increasing of data collection leads to more accurate solution, this happens due to increase of information content of measurements. We used decimal logarithm in these figures because estimation error decreases rapidly in small time intervals.

5. Discussion and Conclusion
The results of numerical simulation (Figures 3-8) are used to choose the duration of data collection and sensor characteristics for the on-board implementation of identification procedure. For example if we consider acceptable relative error for $\chi$ and $b^{\text{fmr}}$ as 5% ($\Delta l^{\text{acc}} = 0.0035m$, $\Delta b^{\text{acc}} = [0.0045 \ 0.0015]$) we achieve following results (Table 2).
Table 2. Time interval for data collection.

| Sensor type | Sensor characteristic | T (sec) | $b^{ea}$ | $\chi$ |
|-------------|-----------------------|--------|----------|--------|
| TAM         | $\sigma_a = 8nT$     | 5000   | 4000     | 8000   |
|             | $\sigma_b = 20nT$    | 10000  | 4500     | 13000  |
|             | $\sigma_a = 50nT$    | 14000  | 5300     | 23000  |
|             | $\sigma_a = 0.025\text{deg/ sec}$ | 6000 | 7500 | 10000 |
|             | $\sigma_a = 0.1\text{deg/ sec}$ | 12000 | 10000 | 17000 |
|             | $\sigma_a = 0.25\text{deg/ sec}$ | 17000 | 13000 | >25000 |

Results (Table 2) show that $K$ parameters can be estimated faster using TAM measurements, time differences relative to GYR measurements are approximately 20%. Estimation of $P$ parameters is more efficient using TAM sensor, time differences relative to GYR measurements are approximately 50%. Fuel level is more efficient to identify by TAM measurements.

Thus the developed algorithm can provide recommendations (for sensors characteristics and time interval of data collection) both for operating nanosat with defined on-board sensors and for nanosat which is under development.

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References
[1] Villela T, Costa C, Brandao A and Leonardi R 2019 Towards the thousandth CubeSat: A statistical overview Int. J. of Aerosp. Eng. 3 pp 1-13
[2] Langer M, Bouwmeester J 2016 Reliability of CubeSats – statistical data, developers beliefs and the way forward Proc. of the AIAA/USU Conf. on Small Sat.
[3] Belokonov I, Lomaka I 2018 In-flight calibration of nanosatellites inertia tensor: The algorithm and requirements for on-board sensors Proc. of the Int. Astr. Cong
[4] Bouwmeester J, Guo J 2010 Survey of worldwide pico- and nanosatellite missions, distributions and subsystem technology Acta Astronautica 67 pp 854-862
[5] Belokonov I, Kramlikh A, Lomaka I and Nikolaev P 2019 Reconstruction of a spacecraft’s attitude motion using the data on the current collected from solar panels J. of Comp. and Syst. Sciences Int. 57 pp 286-296
[6] Belokonov I, Ivliev A, Bogatyrev A, Kumarin A, Lomaka I and Simakov S 2019 Selection of project structure for nanosatellite propulsion system Vestnik of Samara University. Aerospace and Mech. Eng. 18 pp 29-37
[7] Belyaev M, Volkov O, Monakhov M and Sazonov V 2017 Estimating the accuracy of the technique of reconstructing the rotational motion of a satellite based on the measurements of its angular velocity and the magnetic field of the earth Cosmic Research 55 pp 345-360