Saturation Physics on the Energy Frontier

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Saturation and pA Collisions

Saturation

$\ln \frac{1}{x} = 1$

$\ln \frac{Q^2}{Q_0^2}$

saturation

x = 1

small $x$

ln $\frac{1}{x}$

small $Q$

large $Q$

small $Q$

$\ln \frac{Q^2}{Q_0^2}$
Advantages of pA

\[ Q^2 \lesssim Q_s^2 = cA^{1/3}Q_0^2 \left( \frac{x_0}{x} \right)^\lambda \]

- Heavy target: large A
- Light projectile: no medium
Advantages of pA

\[ Q^2 \lesssim Q_s^2 = cA^{1/3}Q_0^2 \left( \frac{x_0}{x} \right)^\lambda \]

- Heavy target: large \( A \)
- Light projectile: no medium
Cross section in the hybrid formalism:

\[
\frac{d^3 \sigma}{dY d^2 \vec{p}_\perp} = \sum_i \int \frac{dz}{z^2} \frac{dx}{x} x f_i(x, \mu) D_{h/i}(z, \mu) F\left(x, \frac{p_\perp}{z}\right) \mathcal{P}(\xi)(\ldots)
\]

- Parton distribution (initial state projectile)
- Dipole gluon distribution (initial state target)
- Fragmentation function (final state)
- Perturbative factors

figure adapted from Dominguez 2011.
History of the pA Calculation

- Dumitru and Jalilian-Marian (2002)
- Dumitru, Hayashigaki, et al. (2006)
- Fujii et al. (2011)
- Albacete et al. (2013)
- Rezaeian (2013)
- Staśto, Xiao, and Zaslavsky (2014)
- Kang et al. (2014)
- Staśto, Xiao, Yuan, et al. (2014)
- Altinoluk et al. (2014)
- Watanabe et al. (2015)
Dumitru and Jalilian-Marian (2002)

No numerical results
Dumitru, Hayashigaki, et al. (2006)

Prefactor $K = 1.6$
Inclusive Cross Section

Inelastic Diagrams

Leading:

Next-to-leading:
Inclusive Cross Section

Inelastic NLO Terms

Albacete et al. (2013)

\( dN/d\eta/dp_t \) (GeV\(^{-2}\))

**dAu @ 200 GeV**

\( g = 1.119 \) i.c

**Prefactor**

\( K = 1 \) for charged hadrons

\( K = 0.4 \) for neutral hadrons

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Inclusive Cross Section

NLO Diagrams

Leading:

Next-to-leading:

Chirilli et al. 2012, 1203.6139.
Staśto, Xiao, and Zaslavsky (2014)

Includes virtual corrections

$K = 1$
Staśto, Xiao, Yuan, et al. (2014)

First use of kinematical constraint

more on constraints: Beuf 2014, 1401.0313.
Staśto, Xiao, Yuan, et al. (2014)

$k_\perp \lesssim Q_s$: saturation formalism

more on constraints: Beuf 2014, 1401.0313.
Inclusive Cross Section

Matching to Collinear

Staśto, Xiao, Yuan, et al. (2014)

BRAHMS $\eta = 3.2$

\[ \frac{d^3N}{d\eta d^2p_\perp} \text{ [GeV}^{-2}] \]

$\mathbf{k_\perp \gg Q_s}$: no subtraction of divergence

more on constraints: Beuf 2014, 1401.0313.
Inclusive Cross Section

Kinematical Constraint

Watanabe et al. (2015)

First LHC numerical results

Alternate derivation: Altinoluk et al. 2014, 1411.2869.
New terms improve matching at low $p_\perp$

data: Arsene et al. 2004, nucl-ex/0403005.
plots: Watanabe et al. 2015, 1505.05183.
New terms improve matching at low $p_\perp$

data: Arsene et al. 2004, nucl-ex/0403005.
plots: Watanabe et al. 2015, 1505.05183.
Expanding to Higher Energy

Large Phase Space

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\[ x_g \geq \frac{p_\perp}{\sqrt{s_{NN}}} e^{-y} \]
\[ \sim 10^{-3} \]
Expanding to Higher Energy

Large Phase Space

\[ x_g \geq \frac{p_\perp}{\sqrt{s_{NN}}} e^{-y} \]
\[ \sim 10^{-4} \]
Expanding to Higher Energy

Large Phase Space

\[ x_g \geq \frac{p_{\perp}}{\sqrt{s_{NN}}} e^{-y} \sim 10^{-5} \]
Challenges for Numerical Calculation

Singularities

\[
\int_{\tau}^{1} d\tau \int_{z}^{1} d\xi \left[ \frac{F_s(z, \xi)}{(1 - \xi)_+} + F_n(z, \xi) + F_d(z, \xi)\delta(1 - \xi) \right]
\]
Expanding to Higher Energy

Challenges for Numerical Calculation

Singularities

\[ \int_{\tau}^{1} dz \int_{\tau}^{1} d\xi \left[ \frac{F_s(z, \xi)}{(1 - \xi)_+} + F_n(z, \xi) + F_d(z, \xi) \delta(1 - \xi) \right] \]

Fourier integrals

\[ \int d^2 \vec{s}_\perp d^2 \vec{t}_\perp e^{i \vec{l}_\perp \cdot \vec{s}_\perp} e^{i \vec{l}'_\perp \cdot \vec{t}_\perp} (\ldots) \]
Challenges for Numerical Calculation

Singularities

\[
\int_{\tau}^{1} dz \int_{\frac{\tau}{z}}^{1} d\xi \left[ \frac{F_s(z, \xi)}{(1 - \xi)_+} + F_n(z, \xi) + F_d(z, \xi) \delta(1 - \xi) \right]
\]

Fourier integrals

\[
\int d^2 \vec{s}_\perp d^2 \vec{t}_\perp e^{i\vec{l}_\perp \cdot \vec{s}_\perp} e^{i\vec{l}_\perp' \cdot \vec{t}_\perp} (\ldots)
\]

Leading Order Cancellations

\[
\mathcal{O}(k_{\perp}^{-2}) - \mathcal{O}(k_{\perp}^{-2}) \rightarrow \mathcal{O}(k_{\perp}^{-4})
\]

...plus Monte Carlo statistical error
Fourier Integrals

Fourier integrals are highly imprecise

\[ \int d^2\vec{r}_\perp S_{Y}^{(2)}(r_\perp) e^{i\vec{k}_\perp \cdot \vec{r}_\perp} (\ldots) \]

\[ \int d^2\vec{s}_\perp S_{Y}^{(4)}(r_\perp, s_\perp, t_\perp) e^{i\vec{k}_\perp \cdot \vec{r}_\perp} (\ldots) \]

Easiest solution: transform to momentum space

\[ F(k_\perp) = \frac{1}{(2\pi)^2} \int \int d^2\vec{r}_\perp S_{Y}^{(2)}(r_\perp) e^{i\vec{k}_\perp \cdot \vec{r}_\perp} \]

\[ = \frac{1}{2\pi} \int_0^\infty dr_\perp S_{Y}^{(2)}(r_\perp) J_0(k_\perp r_\perp) \]

and compute \( F \) directly
Expanding to Higher Energy

New Fourier Transforms

\[ \int \frac{d^2 x_\perp}{(2\pi)^2} S(x_\perp) \ln \frac{c_0^2}{x_\perp^2 \mu^2} e^{-ik_\perp \cdot x_\perp} \]

\[ = \frac{1}{\pi} \int \frac{d^2 \vec{l}_\perp}{l_\perp^2} \left[ F(\vec{k}_\perp + \vec{l}_\perp) - J_0\left(\frac{c_0}{\mu} l_\perp\right) F(\vec{k}_\perp) \right] \]

\[ \int \frac{d^2 r_\perp}{(2\pi)^2} S(r_\perp) \left( \ln \frac{r_\perp^2 k_\perp^2}{c_0^2} \right)^2 e^{-ik_\perp \cdot r_\perp} \]

\[ = \frac{2}{\pi} \int \frac{d^2 \vec{l}_\perp}{l_\perp^2} \ln \frac{k_\perp^2}{l_\perp^2} \left[ \theta(k_\perp - l_\perp) F(k_\perp) - F(\vec{k}_\perp + \vec{l}_\perp) \right] \]

All terms now use momentum-space gluon distribution

Watanabe et al. 2015, 1505.05183.
Expanding to Higher Energy

rcBK in Momentum Space

$F_{xg}(k)$

Numerical instability at high $k_{\perp}$
Predictions at high energy require accurate high-$k_{\perp}$ gluon distribution

- Use symbolic model: GBW, MV, AAMQS/MV$^\gamma$

  or

- Use formula to extrapolate: $k_{\perp}^{-4}$ or $\alpha_s^2 k_{\perp}^{-4}$

  or

- More precise numerical Fourier transform algorithms (promising, work in progress)

- Improve accuracy of position space solution at low $r_{\perp}$
rcBK calculation matches neatly up to $p_\perp \approx 6$ GeV

data: Milov 2014, 1403.5738.
plots: Watanabe et al. 2015, 1505.05183.
Expanding to Higher Energy

Importance of Higher Rapidity

Higher rapidity alters low-$p_{\perp}$ result

![Graphs showing GBW and rcBK predictions](image)

- GBW
- rcBK $\Lambda_{QCD}^2 = 0.01$

$y = 1.75$

$d^3N/d^2p_{\perp}$ [GeV$^{-2}$] vs. $p_{\perp}$ [GeV]
Expanding to Higher Energy

Importance of Higher Rapidity

Higher rapidity alters low-$p_\perp$ result
Expanding to Higher Energy

Importance of Higher Rapidity

Higher rapidity alters low-$p_\perp$ result
Complete numerical implementation of NLO $pA \rightarrow h + X$

What we need

- More forward-rapidity data from LHC experiments
- Improved position space BK solution
Supplemental Slides

- full expressions
- additional history
  - rcBK
  - rapidity divergence
  - Ioffe time
- numerical challenges
  - singularities
  - Fourier integrals
  - new Fourier transforms
  - other numerical errors
- sources of negativity
- kinematical constraint
- beam direction
- LHC results
Complete NLO corrections to the cross section for $pA \rightarrow h + X$:

$$\frac{d^3\sigma}{dY d^2\vec{p}_\perp} = \int \frac{dz d\xi}{z^2} [x q_i(x, \mu) \ x g(x, \mu)] \begin{bmatrix} S_{qq} & S_{qg} \\ S_{gq} & S_{gg} \end{bmatrix} \begin{bmatrix} D_{h/q_i}(z, \mu) \\ D_{h/g}(z, \mu) \end{bmatrix}$$

$$S_{jk} = \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) \mathcal{H}^{(0)}_{2jk} \quad \text{LO dipole}$$

$$+ \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) \mathcal{H}^{(1)}_{2jk} \quad \text{NLO dipole}$$

$$+ \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{s}_\perp d^2 \vec{t}_\perp}{(2\pi)^2} S_Y^{(4)}(r_\perp, s_\perp, t_\perp) \mathcal{H}^{(1)}_{4jk} \quad \text{NLO quadrupole}$$

$$+ \cdots \quad \text{etc.}$$

Note: we also use $S_Y^{(4)}(r_\perp, s_\perp, t_\perp) \rightarrow S_Y^{(2)}(s_\perp) S_Y^{(2)}(t_\perp)$

Chirilli et al. 2012, 1203.6139.
Quark-Quark Channel

\[
S_{qq} = \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) \mathcal{H}_{2qq}^{(0)} + \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) \mathcal{H}_{2qq}^{(1)}
\]

\[
+ \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{s}_\perp d^2 \vec{t}_\perp}{(2\pi)^2} S_Y^{(4)}(r_\perp, s_\perp, t_\perp) \mathcal{H}_{4qq}^{(1)}
\]

\[
\mathcal{H}_{2qq}^{(0)} = e^{-i\vec{k}_\perp \cdot \vec{r}_\perp} \delta(1 - \xi)
\]

\[
\mathcal{H}_{2qq}^{(1)} = C_F P_{qq}(\xi) \left( e^{-i\vec{k}_\perp \cdot \vec{r}_\perp} + \frac{1}{\xi^2} e^{-i\vec{k}_\perp \cdot \vec{r}_\perp / \xi} \right) \ln \frac{c_0^2}{r_\perp^2 \mu^2} - 3C_F e^{-i\vec{k}_\perp \cdot \vec{r}_\perp / \xi} \delta(1 - \xi) \ln \frac{c_0^2}{r_\perp^2 k_\perp^2}
\]

\[
\mathcal{H}_{4qq}^{(1)} = -4\pi N_c e^{-i\vec{k}_\perp \cdot r_\perp} \left\{ e^{-i\frac{1-\xi}{\xi} k_\perp \cdot (x_\perp - b_\perp)} \frac{1 + \xi^2}{1 - \xi} \frac{1}{\xi} \frac{x_\perp - b_\perp}{(x_\perp - b_\perp)^2} \cdot \frac{y_\perp - b_\perp}{(y_\perp - b_\perp)^2}
\]

\[
- \delta(1 - \xi) \int_0^1 d\xi' \frac{1 + \xi'^2}{(1 - \xi')} \left\{ e^{-i(1-\xi) k_\perp \cdot (y_\perp - b_\perp)} \frac{1}{(b_\perp - y_\perp)^2} - \delta(2)(b_\perp - y_\perp) \int d^2 r_\perp' e^{-i\vec{k}_\perp \cdot \vec{r}_\perp' / r_\perp'} \right\}
\]

where

\[
P_{qq}(\xi) = \frac{1 + \xi^2}{(1 - \xi)_+} + \frac{3}{2} \delta(1 - \xi)
\]
\[
S_{gg} = \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) S_Y^{(2)}(r_\perp) H_{2gg}^{(0)} + \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) H_{2gg}^{(1)} + \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) H_{2q\bar{q}}^{(1)} \\
+ \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{s}_\perp d^2 \vec{t}_\perp}{(2\pi)^2} S_Y^{(2)}(s_\perp) S_Y^{(2)}(t_\perp) H_{\delta gg}^{(1)}
\]

\[
H_{2gg}^{(0)} = e^{-i\vec{k}_\perp \cdot \vec{r}_\perp} \delta(1 - \xi)
\]

\[
H_{2gg}^{(1)} = N_c \left[ \frac{2\xi}{(1 - \xi)_+} + \frac{2(1 - \xi)}{\xi} + 2\xi(1 - \xi) + \left( \frac{11}{6} - \frac{2N_f T_R}{3N_c} \right) \delta(1 - \xi) \right] \\
\times \ln \frac{c_0^2}{\mu^2 r_\perp^2} \left( e^{-i\vec{k}_\perp \cdot \vec{r}_\perp} + \frac{1}{\xi^2} e^{-i\frac{\vec{k}_\perp \cdot \vec{r}_\perp}{\xi}} \right) - \left( \frac{11}{3} - \frac{4N_f T_R}{3N_c} \right) N_c \delta(1 - \xi) e^{-i\vec{k}_\perp \cdot \vec{r}_\perp} \ln \frac{c_0^2}{r_\perp^2 k_\perp^2}
\]

\[
H_{2q\bar{q}}^{(1)} = 8\pi N_f T_R e^{-i\vec{k}_\perp \cdot (\vec{y}_\perp - \vec{b}_\perp)} \delta(1 - \xi) \\
\times \int_0^1 d\xi' \left[ \xi'^2 + (1 - \xi')^2 \right] \left[ \frac{e^{-i\xi' \vec{k}_\perp \cdot (\vec{x}_\perp - \vec{y}_\perp)}}{(\vec{x}_\perp - \vec{y}_\perp)^2} - \delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp) \int \frac{d^2 \vec{r}'}{r'^2} \frac{e^{i\vec{k}_\perp \cdot \vec{r}'}}{r'^2} \right]
\]
Gluon-Gluon Channel

\[
S_{gg} = \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) S_Y^{(2)}(r_\perp) \mathcal{H}_{2gg}^{(0)} + \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) \mathcal{H}_{2gg}^{(1)} + \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) \mathcal{H}_{2q\bar{q}}^{(1)}
\]

\[
+ \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{s}_\perp d^2 \vec{t}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) S_Y^{(2)}(s_\perp) S_Y^{(2)}(t_\perp) \mathcal{H}_{6gg}^{(1)}
\]

\[
\mathcal{H}_{6gg}^{(1)} = -16\pi N_c e^{-i \vec{k}_\perp \cdot \vec{r}_\perp} \left\{ e^{-i \frac{\vec{k}_\perp}{\xi} \cdot (\vec{y}_\perp - \vec{b}_\perp)} \left[ \frac{1 - \xi(1 - \xi)}{(1 - \xi)_+} \frac{1}{\xi^2 (\vec{x}_\perp - \vec{y}_\perp)^2} \cdot \frac{\vec{b}_\perp - \vec{y}_\perp}{(\vec{b}_\perp - \vec{y}_\perp)^2} \right]
\]

\[
- \delta(1 - \xi) \int_0^1 d\xi' \left[ \frac{\xi'}{(1 - \xi')}_+ + \frac{1}{2} \xi'(1 - \xi') \right] \left[ e^{-i \xi' \vec{k}_\perp \cdot (\vec{y}_\perp - \vec{b}_\perp)} \frac{\vec{b}_\perp - \vec{y}_\perp}{(\vec{b}_\perp - \vec{y}_\perp)^2} \right]
\]

\[
- \delta^{(2)}(\vec{b}_\perp - \vec{y}_\perp) \int d^2 \vec{r}'_\perp \frac{e^{i \vec{k}_\perp \cdot \vec{r}'_\perp}}{r'_\perp^2} \right\}
\]
Quark-Gluon Channel

\[ S_{gq} = \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp)[\mathcal{H}_{2gq}^{(1,1)} + S_Y^{(2)}(r_\perp)\mathcal{H}_{2gq}^{(1,2)}] + \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{s}_\perp d^2 \vec{t}_\perp}{(2\pi)^2} S_Y^{(4)}(r_\perp, s_\perp, t_\perp)\mathcal{H}_{4gq}^{(1)} \]

\[ \mathcal{H}_{2gq}^{(1,1)} = \frac{N_c}{2} \frac{1}{\xi^2} e^{-i \vec{k}_\perp \cdot \vec{r}_\perp / \xi} \frac{1}{\xi} \left[ 1 + (1 - \xi)^2 \right] \ln \frac{c_0^2}{r_\perp^2 \mu^2} \]

\[ \mathcal{H}_{2gq}^{(1,2)} = \frac{N_c}{2} e^{-i \vec{k}_\perp \cdot \vec{r}_\perp} \frac{1}{\xi} \left[ 1 + (1 - \xi)^2 \right] \ln \frac{c_0^2}{r_\perp^2 \mu^2} \]

\[ \mathcal{H}_{4gq}^{(1)} = 4\pi N_c e^{-i \vec{k}_\perp \cdot \vec{r}_\perp / \xi - i \vec{k}_\perp \cdot \vec{t}_\perp} \frac{1}{\xi} \left[ 1 + (1 - \xi)^2 \right] \frac{\vec{r}_\perp^2}{r_\perp^2} \cdot \frac{\vec{t}_\perp^2}{t_\perp^2} \]
Hard Factors

Gluon-Quark Channel

\[ S_{qg} = \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{r}_\perp}{(2\pi)^2} S_Y^{(2)}(r_\perp) [H_{2qg}^{(1,1)} + S_Y^{(2)}(r_\perp) H_{2qg}^{(1,2)}] \]

\[ + \frac{\alpha_s}{2\pi} \int \frac{d^2 \vec{s}_\perp d^2 \vec{t}_\perp}{(2\pi)^2} S_Y^{(4)}(r_\perp, s_\perp, t_\perp) H_{4qg}^{(1)} \]

\[ H_{2qg}^{(1,1)} = \frac{1}{2} e^{-i\vec{k}_\perp \cdot \vec{r}_\perp} \left[ (1 - \xi)^2 + \xi^2 \right] \left( \ln \frac{c_0^2}{r_\perp^2 \mu^2} - 1 \right) \]

\[ H_{2qg}^{(1,2)} = \frac{1}{2\xi^2} e^{-i\vec{k}_\perp \cdot \vec{r}_\perp / \xi} \left[ (1 - \xi)^2 + \xi^2 \right] \left( \ln \frac{c_0^2}{r_\perp^2 \mu^2} - 1 \right) \]

\[ H_{4qg}^{(1)} = 4\pi e^{-i\vec{k}_\perp \cdot \vec{r}_\perp - i\vec{k}_\perp \cdot \vec{t}_\perp / \xi} (1 - \xi)^2 + \xi^2 \left( \ln \frac{c_0^2}{r_\perp^2 \mu^2} - 1 \right) \]

Saturation Physics on the Energy Frontier

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Additional History

Incorporating rcBK

Fujii et al. (2011)

Prefactor \( K = 1.5 \) for charged particles
\( K = 0.5 \) for neutral particles
Rapidity Divergence

Rapidity divergence in gluon distribution\(^1\)

\[
\mathcal{F}(x_g, k_{\perp}) = \mathcal{F}^{(0)}(x_g, k_{\perp}) - \frac{\alpha_s N_c}{2\pi^2} \int_0^1 \frac{d\xi}{1 - \xi} \times \int \frac{d^2 \vec{x}_{\perp} d^2 \vec{y}_{\perp} d^2 \vec{b}_{\perp}}{(2\pi)^2} e^{-i\vec{k}_{\perp} \cdot (\vec{x}_{\perp} - \vec{y}_{\perp})} \frac{(\vec{x}_{\perp} - \vec{y}_{\perp})^2}{(\vec{x}_{\perp} - \vec{b}_{\perp})^2 (\vec{y}_{\perp} - \vec{b}_{\perp})^2} \times \left[ S_Y^{(2)}(\vec{x}_{\perp}, \vec{y}_{\perp}) - S_Y^{(4)}(\vec{x}_{\perp}, \vec{b}_{\perp}, \vec{y}_{\perp}) \right]
\]

Upper limit:

- \(\xi_{\text{max}} = 1\) in \(\sqrt{s} \to \infty\) limit
- \(\xi_{\text{max}} = 1 - \frac{k_{\perp}}{\sqrt{s}} e^{-y} = 1 - e^{-Y}\) in exact kinematics
- \(\xi_{\text{max}} = 1 - e^{-Y_0}\) with rapidity cutoff?\(^2\)

\(^1\)Chirilli et al. 2012, 1203.6139, eq. (21).
\(^2\)Kang et al. 2014, 1403.5221.
Rapidity correction (believed unphysical) (by us)
Altinoluk et al. (2014)

\[
\frac{2(1 - \xi)\xi x_g P^+}{k_{\perp}^2} > \tau
\]

No numerical results
Analysis of Negativity

Breakdown by Channel

Plot shows magnitude of channel contribution

Coloring indicates where value is Negative Positive

Negativity comes from NLO diagonal channels: qq and gg
Analysis of Negativity

Breakdown by Channel

Plot shows magnitude of channel contribution

Coloring indicates where value is Negative Positive

Negativity comes from NLO diagonal channels: qq and gg
Negativity comes from NLO diagonal channels: qq and gg

Plot shows magnitude of channel contribution

Coloring indicates where value is Negative Positive
Analysis of Negativity

Breakdown by Channel

Plot shows magnitude of channel contribution

Coloring indicates where value is Negative Positive

Negativity comes from NLO diagonal channels: qq and gg
Analysis of Negativity

Breakdown by Channel

Plot shows magnitude of channel contribution

Coloring indicates where value is Negative Positive

Negativity comes from NLO diagonal channels: qq and gg
Negativity comes from plus prescription

\[
\int_{\tau/z}^{1} \frac{d\xi}{(1 - \xi)^+} f(\xi) = \int_{\tau/z}^{1} d\xi \frac{f(\xi) - f(1)}{1 - \xi} + f(1) \ln \left( 1 - \frac{\tau}{z} \right)
\]

- First term negative because \( f(\xi) < f(1) \)
- Second term negative because \( \tau/z < 1 \)

\[
qq \quad f(\xi) \sim 1 + \xi^2
\]

\[
gg \quad f(\xi) \sim \xi
\]

\[
gg \quad f(\xi) \sim (1 - \xi + \xi^2)^2
\]
Eliminate delta functions and plus prescriptions

\[
\int_{\tau}^{1} dz \int_{\tau}^{1} d\xi \left[ \frac{F_s(z, \xi)}{(1 - \xi)_+} + F_n(z, \xi) + F_d(z, \xi) \delta(1 - \xi) \right]
\]

\[
= \int_{\tau}^{1} dz \int_{\tau}^{1} dy \frac{z - \tau}{z(1 - \tau)} \left[ \frac{F_s(z, \xi) - F_s(z, 1)}{1 - \xi} + F_n(z, \xi) \right]
\]

\[
+ \int_{\tau}^{1} dz \left[ F_s(z, 1) \ln \left( 1 - \frac{\tau}{z} \right) + F_d(z, 1) \right]
\]

\[
\delta^2(\vec{r}_\perp) \int \frac{d^2\vec{r}_\perp'}{r_\perp'^2} e^{i\vec{k}_\perp \cdot \vec{r}_\perp'} - \frac{1}{r_\perp^2} e^{-i\xi' \vec{k}_\perp \cdot \vec{r}_\perp}
\]

\[
= \frac{1}{4\pi} \int d^2\vec{k}_\perp e^{-i\vec{k}_\perp \cdot \vec{r}_\perp} \ln \left( \frac{(\vec{k}_\perp' - \xi' \vec{k}_\perp)^2}{k_\perp^2} \right)
\]
Numerical Challenges

Remaining Evaluation Errors

- Inaccuracy of Fourier integrals
- Monte Carlo statistical error
- Cancellation of large terms

Multiple runs to improve statistics
Numerical Challenges

Remaining Evaluation Errors

- Inaccuracy of Fourier integrals
- Monte Carlo statistical error
- Cancellation of large terms

Two parallel implementations of selected parts:
- Mathematica, for rapid prototyping
- C++, for execution speed
Derivation of the Kinematical Constraint

\[ p \rightarrow (x_p P^+, 0, 0_\perp) \rightarrow A \rightarrow (1 - \xi) x_p P^+, l^-, l_\perp \]

\[ \times (\xi x_p P^+, k^-, k_\perp) \]

\[ (0, x_g P^-, k_\perp + l_\perp) \]

\[ x_g P^- = \frac{l^2_\perp}{2(1 - \xi)x_p P^+} + \frac{k^2_\perp}{2\xi x_p P^+} \leq P^- \]

\[ x_g \leq 1 \]

\[ \xi \leq 1 - \frac{l^2_\perp}{x_p s} \]

figure adapted from Watanabe et al. 2015, 1505.05183.
The Beam Direction Problem

$p, d$ \rightarrow \begin{cases} y < 0 \\ y = 0 \\ (LHC) \\ y > 0 \end{cases} \rightarrow A

forward hadron production

figure adapted from Watanabe et al. 2015, 1505.05183.
Saturation Physics on the Energy Frontier

GBW

rcBK $\Lambda_{QCD}^2 = 0.01$

data: Milov 2014, 1403.5738; Abelev et al. 2013, 1210.4520.

plots: Watanabe et al. 2015, 1505.05183.
Saturation Physics on the Energy Frontier

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Additional LHC Results

LHC Predictions for Run II

GBW

rcBK $\Lambda_{QCD}^2 = 0.01$

$y = 1.75$, $p_\perp \,[\text{GeV}]$

$\frac{d^3N}{dydp_\perp d^2p_\perp} \, [\text{GeV}^{-2}]$

$y = 1.75$

$p_\perp \,[\text{GeV}]$

$\frac{d^3N}{dydp_\perp d^2p_\perp} \, [\text{GeV}^{-2}]$

$y = 1.75$

$p_\perp \,[\text{GeV}]$