Grand Unified origin of gauge interactions and families replication in the Standard Model

António P. Morais¹, Roman Pasechnik² and Werner Porod³

¹ Departamento de Física, Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal; aapmorais@ua.pt
² Department of Astronomy and Theoretical Physics, Lund University, SE-223 62 Lund, Sweden; Roman.Pasechnik@thep.lu.se
³ Institut für Theoretische Physik und Astrophysik, Uni Würzburg, Germany; porod@physik.uni-wuerzburg.de

* Correspondence: Roman.Pasechnik@thep.lu.se

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Abstract: The tremendous phenomenological success of the Standard Model (SM) suggests that its flavor structure and gauge interactions may not be arbitrary but should have a fundamental first-principle explanation. In this work, we explore how the basic distinctive properties of the SM dynamically emerge from a unified New Physics framework tying together both flavour physics and Grand Unified Theory (GUT) concepts. This framework is suggested by a novel anomaly-free supersymmetric chiral $E_6 \times SU(2)_F \times U(1)_F$ GUT containing the SM. Among the most appealing emergent properties of this theory is the Higgs-matter unification with a highly-constrained massless chiral sector featuring two universal Yukawa couplings close to the GUT scale. At the electroweak scale, the minimal SM-like effective field theory limit of this GUT represents a specific flavored three-Higgs doublet model consistent with the observed large hierarchies in the quark mass spectra and mixing already at tree level.

Keywords: Grand Unified theories; supersymmetry; phenomenology of New Physics

With a handful of physical parameters such as fermion masses and gauge couplings the Standard Model (SM) explains a huge variety of collider and low energy data spanning over several orders of magnitude for the corresponding energy scales. Its success builds strongly on the gauge principle. However, it is fundamentally incomplete as it leaves the cosmological Dark Matter and baryon asymmetry of the universe unexplained. Moreover, it neither contains mechanisms for generating the tiny neutrino masses nor explains the structure of the SM fermion families. This suggests that the SM is not the ultimate theory but an excellent effective field theory (EFT) of the subatomic world.

Since the birth of the SM in mid-1970, there have been numerous attempts to come up with a consistent first-principle explanation of the well-measured but yet totally arbitrary and rather odd properties of the SM. Among these are the remarkable proton stability, the specific structure of gauge and Yukawa interactions and the properties of the Higgs and Yukawa sectors which are intimately connected to the rather peculiar observed patterns in the neutrino and charged fermion mass spectra and generation mixings (the so-called flavor problem).

It is fairly easy to achieve unification of the gauge couplings at higher energy scales by postulating the existence of additional scalars and/or fermions belonging to incomplete representations of $SU(5)$ [1]. This is for example realized in supersymmetric (SUSY) extensions of the SM [2]. This unification is a necessary requirement to embed the SM into a larger gauge group such as $SU(5)$, $SO(10)$, or $E_6$ [3–13], so-called grand unified theories (GUTs). For a recent thorough discussion of theoretical features and most important phenomenological implications of the $E_6$ GUTs,
see e.g. Refs. [14–21]. Remarkably, in Ref. [22] it has been demonstrated that the well-known hierarchy and doublet-triplet splitting problems appear to be naturally resolved in the framework of SUSY SU(6) GUT (see also Refs. [23,24]). This model also provides means for explanation of the origin of the fermion mass hierarchy, i.e. why the 3rd fermion family in the SM is heavier than first two [25]. Other promising scenarios designed to address the flavor problem invoke new “horizontal” symmetries at high energies; for recent studies, see e.g. Refs. [26–32]. For original works, where it was suggested, in particular in the context of SU(3)$_F$, that the observed fermion spectrum with its hierarchies of masses and mixing angles are due to horizontal symmetry breaking hierarchy, i.e. by the flavon VEV breaking it, see Refs. [33–36].

In general, it is rather difficult to combine both gauge symmetries and horizontal symmetries without a proliferation of unknown parameters in the Yukawa sector. Our main goal here is to find a consistent GUT framework in four spacetime dimensions, with both types of unification realised dynamically in the gauge and Yukawa sectors. For this purpose, we would like to study high-scale SUSY-based framework binding together both observed fermion families’ replication in the SM and grand unification. For this purpose, let us consider the $\mathcal{N} = 1$ SUSY $E_6 \times SU(2)_F \times U(1)_F$ GUT in four dimensions where the gauge symmetries of the SM originate from $E_6$ whereas additional group factors $SU(2)_F \times U(1)_F$ conveniently represent a “horizontal” gauge symmetry distinguishing the fermion families, i.e. the family symmetry$^1$.

Note that a further constrained scenario based on SU(3)$_F$ has been previously studied by some of the authors in Refs. [39,40]. Indeed, promoting $SU(2)_F \times U(1)_F$ to SU(3)$_F$, the model seems to become more compact and natural. However, it was found that the top and charm quark tree-level masses are degenerate and a strong fine-tuning in the soft SUSY breaking sector is necessary in order to induce a realistic mass splitting at one-loop level. Such a fine-tuning implies a rather strong hierarchy between different soft SUSY breaking parameters that has prompted the search for a less constrained fully-gauged family symmetry such as the one considered in this work.

The subsequent symmetry breaking steps can be realised by means of the Higgs mechanism as follows:

\[
E_6 \times SU(2)_F \times U(1)_F \xrightarrow{M_6} [SU(3)]^3 \times SU(2)_F \times U(1)_F \\
\xrightarrow{M_3} SU(3)_C \times [SU(2) \times U(1)]^2 \\
\times SU(2)_F \times U(1)_F \xrightarrow{M_3} \ldots .
\]

where $[SU(3)]^3 \equiv SU(3)_C \times SU(3)_L \times SU(3)_R$ is the trinification group and $[SU(2) \times U(1)]^2 \equiv SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$. We adopt at this stage that $E_6 \times SU(2)_F \times U(1)_F$ theory originates from a certain large gauge group $G$ at the upper-most GUT-scale $M_{GUT}$ (with a single universal gauge coupling) by means of some unknown dynamics and formulate the basic criteria for phenomenological consistency of such a scenario.

The mass scales of the rank-preserving symmetry breaking steps in Eqs. (1) and (2) are given by the sizes of the superpotential quadratic terms implying a nearly-compressed scale hierarchy $M_{GUT} \gtrsim M_6 \gtrsim M_3$. The . . . in Eq. (3) represent the subsequent low-scale breaking steps down to the SM gauge group triggered by soft-SUSY breaking interactions at the soft scale $M_S$. The latter can be decoupled from the trinification breaking scale, i.e. $M_S \ll M_3$, in consistency with the low-scale electroweak spontaneous symmetry breaking (EW-SSB) in the SM.

In this work, our main goal is to briefly discuss the main features of the SUSY $E_6 \times SU(2)_F \times U(1)_F$ GUT with the symmetry breaking pattern given in Eqs. (1) - (3) and with a particular anomaly-free superfield content summarised in Table 1. In order to build a minimal working GUT

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\[\text{\footnotesize 1 For a previous discussion of implications of non-abelian family symmetries in supersymmetric GUT model building, see e.g. Refs. [37,38].}\]
Table 1. Fundamental superfield content of the anomaly-free 4-dimensional SUSY $E_6 \times SU(2)_F \times U(1)_F$ GUT. Here, $i = 1, 2$ and $k = 1, 2, 3$.  

| $Z_2$-even          | $Z_2$-odd          |
|----------------------|---------------------|
| $\psi^{\mu^1} = (27, 2)_{(1)}$, $\psi^{\mu^3} = (27, 1)_{(-2)}$ | $\mathcal{L}_k = (1, 2)_{(-1)}$ |
| $\mathcal{H}_U = (1, 2)_{(-1)}$, $\mathcal{H}_D = (1, 2)_{(+1)}$ | $\mathcal{E}_k = (1, 1)_{(+2)}$ |
| $\mathcal{A} = (78, 1)_{(0)}$ | $\mathcal{N}_k = (1, 1)_{(0)}$ |
| $\Sigma, \Sigma' = (650, 1)_{(0)}$ |                     |
| $\Psi = (2430, 1)_{(0)}$ |                     |

Hence, an analogue of right-handed neutrino, $\mathcal{N}'$, receives Majorana mass at some large scale $\mu_{\mathcal{N}'},$ while other additional superfields $\mathcal{H}_{U,D}, \mathcal{L}$ and $\mathcal{E}$ acquire their masses upon breaking of $SU(2)_F \times U(1)_F$ symmetry through VEVs in scalar components of $SU(2)_F \times U(1)_F$ doublets. This resembles the Higgsino, (s)neutrinos’ and (s)leptons’ mass generation via the EW symmetry breaking mechanism in the conventional MSSM framework, but for the family $SU(2)_F \times U(1)_F$ SSB and at a larger scale, $\mu_{\mathcal{H}} \gtrsim M_S \gg M_{EW}$.

Provided that $\mathcal{H}_{U,D}$ are $Z_2$-even, their VEVs are not affecting $Z_2$ symmetry which is therefore preserved in this model and survives down to low scales. Similarly to $R$-parity, this symmetry provides a possible way to stabilise the lightest state among the $Z_2$-odd components of $\mathcal{L}$, $\mathcal{E}$ and $\mathcal{N}'$ superfields. Whether or not such a state can play a role of a Dark Matter candidate remains one of the interesting topics for further studies in this model. Besides, neutrino-like states may receive a relatively small mass scale due to a seesaw-type mixing with the Majorana fermion from $\mathcal{N}'$. Communication of such $Z_2$-odd sector with the SM sectors would be suppressed due to a large mass scale of family $SU(2)_F \times U(1)_F$ gauge bosons at tree level and due to a small loop-generated coupling to the SM Higgs boson. Apart from potentially light and decoupled $Z_2$-odd states, all the other fields in $W_{\mathcal{H}\mathcal{N}'}$ are well above the EW scale and their impact on the low-scale phenomenology is expected to be strongly suppressed. Thus, they can be safely integrated out below $M_S$ scale.

It is worth noticing here that the particle content of the considered $E_6 \times SU(2)_F \times U(1)_F \times Z_2$ GUT and charge assignments in Table 1 enable the superpotential mass terms to all the fundamental $Z_2$-even superfields except for $\psi^{\mu^1}$ and $\psi^{\mu^3}$, providing a novel GUT framework manifestly free of the gauge and Witten anomalies.

The large $E_6$ representations $\Sigma, \Sigma'$ and $\Psi$ trigger (through their scalar VEVs) the spontaneous rank- and SUSY-preserving breaking of $E_6$ symmetry at $M_6$ scale in Eq. (1), while the components of the $Z_2$-even $E_6$-adjoint representation $\mathcal{A}$ play a critical role in triggering the subsequent trinification SUSB in Eq. (2). All these fields conveniently received large masses and can be integrated out below either $M_6$ (large $E_6$ reps) or $M_3$ (adjoint $E_6$ rep) scale in the considered SUSY GUT. Taking the trinification breaking at $M_3$ scale as an example, the SU(3)$_{L,R}$-adjoint superfields $\Delta_{L,R}^6 \subset (78, 1)$ (see Table 2) originating from $\mathcal{A}$ superfield have a universal mass term $\mu_{78} \sim M_{GUT}$ in the superpotential, which together with a cubic term triggers a rank- and SUSY-preserving VEV in one of its scalar components $\langle \Delta_{L,R}^{68} \rangle \equiv M_3$ [40]. The SUSY-preserving breaking of a gauge symmetry implies that the $D$- and $F$-terms have to vanish separately. This means that the scalar potential has zero value in
both the SU(3)$_{L,R}$-symmetric and SU(3)$_{L,R}$-broken vacua in the exact SUSY case. Thus, the presence of even a tiny soft-SUSY breaking effect already at the $M_{\text{GUT}}$ scale is needed to make these vacua non-equivalent [41], hence, enabling the trinification SSB in Eq. (2). As a result, all the components of $\Delta_{\text{L,R}}^a$ acquire a universal mass, $M_{\Delta_{\text{L,R}}} \sim M_3$ due to $D$-terms and thus are integrated out below the $M_{\Delta_{\text{L,R}}} \sim M_3$ scale.

The remaining massless superfields $\psi^{\mu \nu \lambda}$ and $\psi^{\mu \nu \lambda}$ neatly unite the SM Higgs and matter (neutrino, charged lepton and quark) sectors hence imposing very specific constraints on the structure of the resulting low-energy EFT below $M_6$. How does such Higgs-matter unification comply with observations? Notably, the considered $E_6 \times SU(2)_{F} \times U(1)_{F}$ SUSY GUT enables for a phenomenologically consistent splitting between the second- and third-generation quark masses already at tree level with only two distinct quark Yukawa couplings below $M_6$. Let us explore this interesting phenomenon in more detail.

**Table 2.** Upper part: fundamental chiral superfields in the $[SU(3)]^3 \times SU(2)_F \times U(1)_F$ theory – components of the massless $\psi^{\mu \nu \lambda}$ and $\psi^{\mu \nu \lambda}$ superfields of $E_6 \times SU(2)_F \times U(1)_F$ [42]. Lower part: the corresponding components of the massive superfield $(78,1)_0$. Accidental symmetries’ charges are shown in last two columns.

|       | SU(3)$_{L}$ | SU(3)$_{R}$ | SU(3)$_{C}$ | SU(2)$_{F}$ | U(1)$_{F}$ | U(1)$_{W}$ | U(1)$_{B}$ |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $L_i^i$ | 3           | $\bar{3}$   | 1           | 2           | 1           | 1           | 0           |
| $L_i^3$ | 3           | $\bar{3}$   | 1           | 1           | $-2$        | 1           | 0           |
| $Q_i^L$ | $\bar{3}$   | 1           | 3           | 2           | 1           | $-1/2$      | 1/3         |
| $Q_i^R$ | 3           | 1           | 3           | 1           | $-2$        | $-1/2$      | 1/3         |
| $Q_i^\nu$ | 1          | 3           | 3           | 2           | 1           | $-1/2$      | $-1/3$     |
| $\Delta_L$ | 8          | 1           | 1           | 1           | 0           | 0           | 0           |
| $\Delta_R$ | 1          | 8           | 1           | 1           | 0           | 0           | 0           |
| $\Delta_C$ | 1          | 1           | 8           | 1           | 0           | 0           | 0           |
| $\Xi$ | 3           | $\bar{3}$   | 3           | 1           | 0           | 0           | 0           |
| $\Xi'$ | $\bar{3}$   | 3           | $\bar{3}$   | 3           | 1           | 0           | 0           |

First of all, the SUSY $E_6 \times SU(2)_F \times U(1)_F$ theory below $M_{\text{GUT}}$ features a vanishing dim-3 superpotential $d_{\mu \nu \lambda} \varepsilon_{ijk} \psi^{\mu \nu \lambda} / \psi^{\mu \nu \lambda} = 0$, caused by anti-symmetry of family index contractions, where $d_{\mu \nu \lambda}$ is a completely symmetric $E_6$ tensor [43,44], and $\varepsilon_{ijk}$ is the totally anti-symmetric SU(2) Levi-Civita pseudotensor. Since the renormalisable $E_6$ interactions cannot generate a non-trivial Yukawa structure in this theory, the effects of high-dimensional operators become important. In particular, the relevant part of the superpotential below $M_{\text{GUT}} \sim M_6$ scale that contains the SM matter and Higgs sectors reads

$$W_\psi = \frac{\varepsilon_{ijk} \psi^{i j} \psi^{i j} / \psi^{i j}}{2M_{\text{GUT}}} \left[ \bar{\chi}_1 \Sigma_{\mu}^{\alpha} d_{\nu \lambda} + \bar{\chi}_2 \Sigma_{\nu}^{\beta} d_{\lambda \mu} + \bar{\chi}_3 \Sigma_{\gamma}^{\kappa} d_{\mu \lambda} + \bar{\chi}_4 \Sigma_{\alpha}^{\mu} d_{\nu \lambda} + \bar{\chi}_5 \Sigma_{\nu}^{\alpha} d_{\mu \lambda} \right],$$  \hspace{1cm} (5)

where the minimal superfield content necessary to generate two distinct Yukawa couplings below $M_6$ requires the presence of two different bi-fundamental $650$-superfields of $E_6$ [45], $\Sigma^\mu$ and $\Sigma^\nu$. Their VEVs, $\langle \Sigma \rangle \propto k_\Sigma M_6$ and $\langle \Sigma' \rangle \propto k_{\Sigma'} M_6$, trigger subsequent breaking of $E_6$ [46] down to the trinification symmetry (1). As a result, an EFT superpotential of massless fields below $M_6$ reads

$$W_{\text{eff}} = \varepsilon_{i j} (\gamma_1 L^i \cdot Q_j^L - \gamma_2 L^i \cdot Q_j^L + \gamma_2 L^i \cdot Q_j^L + \gamma_3 Q_j^L \cdot Q_j^L)$$  \hspace{1cm} (6)
in terms of the massless trinification leptonic \( L^{i3} \) and quark \( Q^{i3}_{L,R} \) superfields – components of the original massless \( \psi^{\mu\nu} \) and \( \psi^{\mu3} \) superfields of \( E_6 \times SU(2)_F \times U(1)_F \) described in Table 2. Next, it is convenient to perform the following decomposition

\[
(L^{i3})^r = \left( \begin{array}{c} \chi^r \\ \ell_{R} \\ \phi \end{array} \right)^{i3}, \quad (Q^{i3}_{L,R})^x = \left( \begin{array}{c} q^x_{L} \\ D^x_L \\ \phi \end{array} \right)^{i3}, \quad (Q^{i3}_{R})^x = \left( \begin{array}{c} q^x_{R} \\ D^x_R \end{array} \right)^{i3},
\]

such that, upon further splitting into \( SU(2)_{L,R} \) representations, one has

\[
\chi^r = \left( H^i_0, H^i_3 \right)^{i3}, \quad \ell_{R} = \left( e_R, v_R \right)^{i3}, \quad q^x_{R} = \left( u_{Rx}, d_{Rx} \right)^{i3}.
\]

Above, \( l, r \) and \( x \) represent \( SU(3)_L, SU(3)_R \) and \( SU(3)_C \) triplet indices, \( l \) and \( r \) denote \( SU(2)_L \) and \( SU(2)_R \) doublet indices, respectively, \( i \) is the \( SU(2)_F \) index, while the labels \( L (R) \) should not be identified with left (right) chiralities at this stage (all fermionic components are L-handed Weyl spinors). In Eq. (8) one can see that the model offers three up specific \( H^i_0 \) and three down specific \( H^i_3 \) Higgs doublet candidates. While by no means unique, and in fact not a preferred scenario according to the discussion in [47], a MSSM-like Higgs sector is a possible low-scale limit of the model. However, due to a non-trivial mixing structure, such doublets result from a linear combination of those in Eq. (8) and cannot be promptly identified at the level of unbroken trinification. The Higgs-matter unification here implies that the Higgs doublet superfields of the EW theory are unified together with the lepton and quark \( SU(3)_{L,R} \)-superfields in the \( \psi^{\mu\nu} \) and \( \psi^{\mu3} \). Such a unification is thus enforced by the gauge symmetry of the high-scale theory and that cannot be consistently realised in the MSSM.

The effective trinification superpotential (6) contains two universal Yukawa couplings

\[
\mathcal{Y}_1 = \frac{\zeta}{\sqrt{6}} (\tilde{\lambda}_{45} - 1), \quad \mathcal{Y}_2 = \frac{\zeta}{2\sqrt{2}} (\tilde{\lambda}_{45} - \tilde{\lambda}_{21}),
\]

where \( \tilde{\lambda}_{ij} \equiv \tilde{\lambda}_i - \tilde{\lambda}_j \) and \( \zeta \approx M_6/M_{\text{GUT}} \). As we will demonstrate below, due to a very steep Renormalisation Group (RG) evolution of the gauge couplings in the \( E_6 \times SU(2)_F \times U(1)_F \) theory at high scales and the required matching of the SM gauge couplings to their measured values at the electroweak (EW) scale, one has \( \zeta \sim 1 \) and \( k_E \sim - k_{W} \). On another hand, a possible common origin of the dim-4 operators from yet unknown \( M_{\text{GUT}} \)-scale dynamics in the superpotential (5) and a compressed hierarchy \( M_{\text{GUT}} \gtrsim M_6 \) imply that \( \tilde{\lambda}_{21} \approx \tilde{\lambda}_{45} \) suggesting the following hierarchy \( \mathcal{Y}_2 \ll \mathcal{Y}_1 \sim 1 \). It turns out that such an emergent hierarchy is consistent with the existence of an order-one top-quark Yukawa coupling given by \( \mathcal{Y}_1 \). Besides, it leads to the observed top-charm and bottom-strange quark mass hierarchies in the SM as well as to the down-type vector-like quark mass hierarchy already at tree level, namely,

\[
\frac{\mathcal{Y}_1}{\mathcal{Y}_2} = \frac{m_t}{m_c} \approx \frac{m_b}{m_s} \approx \frac{m_B}{m_{D,S}} \sim \mathcal{O}(100),
\]

implying also a possibility for two light vector-like \( D, S \)-quark species potentially within the reach of the LHC or future collider measurements.

The superpotential (6) possesses an accidental Abelian \( U(1)_W \times U(1)_B \) symmetry whose charges, \( W \) and \( B \), are summarised in Table 2. Furthermore, the theory has an extra \( Z_2 \) parity denoted as \( \mathbb{P}_B \)-parity defined as

\[
\mathbb{P}_B = (-1)^{2W+2S} = (-1)^{3B+2S},
\]

where \( S \) is the spin. In the considered GUT theory, the \( \mathbb{P}_B \)-parity replaces the conventional R-parity and forbids triple-squark or quark-quark-squark trilinear interactions in the soft-SUSY breaking...
sector capable of destabilising the proton at the soft scale. Together with the baryon-number
U(1)_B-symmetric Yukawa sector, this ensures that only E_6 gauge interactions can trigger the proton
decay, highly suppressed by a large M_6 close to M_{GUT}.

The dim-3 superpotential of Ξ, Ξ′ and Δ_{L,R,C} superfields – components of the massive chiral A
superfield (see Table 2, also Ref. [42]) reads

\[ W_{78} = \sum_{A=L,R,C} \left[ \frac{1}{2} \mu_{78} \text{Tr} A_A^2 + \frac{1}{3} \nu_{78} \text{Tr} A_A^3 \right] + \mu_{78} \text{Tr}(ΞΞ′) + \sum_{A=L,R,C} \nu_{78} \text{Tr}(ΞΞ′ A_A), \]  

with the universal μ_{78} \simeq M_{GUT}. As was mentioned above, the last rank/SUSY-preserving breaking
step in Eq. (2) represents the trinification breaking by means of degenerate VEVs at M_3 \leq M_6 in the
SU(3)_L, SU(3)_R octet superfields Δ_L, Δ_R, respectively [39/40]. In this case, all the Δ_{L,R,C} components
acquire large masses M_{Δ_{L,R,C}} \sim M_3 and hence are integrated out leaving no heavy fields in the
resulting left-right (LR) symmetric SUSY EFT [47]

\[ W_{LR} = 3 \nu_{ij} [χ^1 ⋅ q_L^3 ⋅ q_R^1 + ℓ_L^1 ⋅ D_R^3 ⋅ q_R^1 + ℓ_L^1 ⋅ q_L^3 ⋅ D_R^1 + φ^i ⋅ D_L^3 ⋅ D_R^1] \]

with two unknown M_{GUT} _-scale effects may induce

\[ \delta C = - \frac{1}{\sqrt{2}} k_Σ - \frac{1}{\sqrt{26}} k_Ψ, \]

written in terms of the massless components of trinification bi-triplets introduced in Eq. (7). The
further symmetry breaking steps down to the SM and hence the masses/mixings of the L^3 and
Q^3_R components are controlled by the structure of the soft-SUSY breaking mass terms and tri-linear
interactions as well as by the tree-level Yukawa hierarchy (10).

In the LR symmetric SUSY theory the largest amount of free parameters comes from the
soft-SUSY breaking sector, namely, 17 trilinear couplings (5 involving sleptons and 12 – squarks),
16 soft LL- and QQ-type mass terms, 2 high-scale gaugino mass parameters in (E_6 and gauge-family
sectors). In addition, there are 4 gauge couplings in the gauge sector whereas all the low-scale Yukawa
couplings are matched to two universal high-scale ones defining the strongest hierarchies (10) already
at tree level. The loop corrections to the Yukawa sector are controlled by the soft-SUSY breaking
parameters and gauge couplings, whose number is sufficient to accommodate the measured values of
the SM fermion masses and mixing angles.

Let us now investigate how strong the hierarchy between the soft and trinification breaking
scales, M_S \ll M_3, can be – the question of primary importance for a realistic low-energy theory.
Provided the compressed hierarchy M_{GUT} _> M_6, the unknown M_{GUT}-scale effects may induce
significant threshold corrections to the trinification gauge couplings at M_6 scale. Indeed, the relevant
gauge-kinetic dim-5 operators [46]

\[ \mathcal{L}_{SD} = - \frac{1}{4C} \text{Tr}(F_{\mu\nu} \cdot \Phi_{E_6} \cdot F^{\mu\nu}) \]  

where C is the charge normalization, F_{\mu\nu} is the E_6 field strength tensor, ξ \sim 1 is a non-renormalisable
coupling constant, and \Phi_{E_6} is a linear combination of the scalar fields originating from the symmetric
product of two E_6 adjoint representations \Phi_{E_6} ∈ \{78 ⊗ 78\}_{sym} = 1 ⊕ 650 ⊕ 2430, with two 650-reps
Σ_\nu^\mu and Σ_\nu^\mu already utilised above. The E_6-breaking VEVs in these fields modify the gauge coupling
unification condition at M_6 scale via dim-5 threshold corrections from Eq. (14) [46]

\[ \begin{align*}
\alpha_{3C}^{-1}(1 + ζC) &= \alpha_{3L}^{-1}(1 + ζL) - \alpha_{3R}^{-1}(1 + ζR)^{-1}, \\
\alpha_{3A}^{-1} &= \frac{4π}{8A}, \\
\delta C &= - \frac{1}{\sqrt{2}} k_Σ - \frac{1}{\sqrt{26}} k_Ψ, \\
\delta_{LR} &= \frac{1}{2\sqrt{2}} k_Σ \pm \frac{3}{2\sqrt{2}} k_Ψ - \frac{1}{\sqrt{26}} k_Ψ, \\
&\quad k_Ψ \sim k_Ψ = \langle 2430 \rangle / M_6, \\
\end{align*} \]  

(15)
where $a_{3A}^{-1}$, $A = L, R, C$, are the inverse trinification structure constants, $k_{\Sigma'\Sigma}$ and $\zeta \sim 1$ were defined above. Provided that the family U(1)$_T$ and the hypercharge U(1)$_Y$ gauge groups remain unbroken above the EW scale, their the $T$- and $Y$-charges are related to the high-scale ones as $T_T = 6T_R^3 - 4T_F^3 + \frac{1}{\sqrt{3}}(T_L^8 - T_R^8 - 2T_T^8)$ and $T_Y = 2T_R^3 + \frac{1}{\sqrt{3}}(T_L^8 + T_T^8)$, respectively. The corresponding inverse structure constants are matched (at tree level) to the high-scale ones below $M_3$-scale as follows: $a_T^{-1} = \frac{1}{2} (a_2F + \frac{1}{12} [a_L^{-1} + a_R^{-1} + 4a_F^{-1}]) + a_{2R}^{-1}$ and $a_Y^{-1} = \frac{1}{2} (a_L^{-1} + a_R^{-1}) + a_{2R}^{-1}$, respectively. (Here, $a_{2A}$ and $a_A$ are the structure constants for SU(2)$_A$ and U(1)$_A$, respectively.)

We have performed a sophisticated numerical analysis of the one-loop RG flow of gauge couplings between $M_{\text{GUT}}$ and $M_{\text{EW}}$ scales accounting for tree-level matching at intermediate scales as well as the matching to their measured counterparts at $M_{\text{EW}}$. We have demonstrated that the presence of threshold corrections $\delta_A$ to the gauge couplings at $M_6$ enables a perturbative universal hierarchy with $\xi \simeq 1$ as well as $k_\Sigma \simeq -k_{\Sigma'}$. One particular example for such RG flow for a valid parameter space point is shown in Fig. 1. The $E_6$ gauge coupling evolves very fast as indicated by a steep line stretched between $M_{\text{GUT}}$ and $M_6$ scales. The threshold corrections are quite sizable in this example. We give the corresponding parameters in Table 3. Note, in an unrealistic case of a strong $M_6 - M_{\text{GUT}}$ hierarchy $\xi \ll 1$, one could recover an approximate unification $a_{3C}^{-1} \simeq a_{3L}^{-1} \simeq a_{3R}^{-1}$ corresponding to $\Sigma_3$-permutation symmetry in the trinification gauge sector, originally imposed in Ref. [48]. However, a small $\xi \ll 1$ implies unacceptably small Yukawa couplings (see Eq. (9)).
Table 3. Benchmark points used for the running of the gauge couplings in Fig. 1. The top line corresponds to a parameter space point where $\delta_i$ differ considerably whereas in the bottom line their absolute values are of the same order. Here, $t_i = \log_{10} \frac{M_i}{\text{GeV}}$.

| $t_8$ | $t_3$ | $t_s$ | $\zeta$ | $a_s^{-1}(M_S)$ | $a_t^{-1}(M_S)$ | $\delta_l$ | $\delta_R$ | $\delta_C$ | $k_Y$ | $k_C$ | $k_{\Sigma}$ | $k_V$ |
|-------|-------|-------|---------|-----------------|-----------------|-----------|-----------|-----------|-------|-------|-----------------|-------|
| 17.42 | 16.53 | 5.455 | 0.844   | 5.067           | 82.70           | -0.622    | -0.161    | -0.737    | 0.862 | 0.326 | -0.377          | 0.844 |

A simple configuration of soft-scale induced VEVs breaking the symmetry in Eq. (3) down to $\text{SU}(3)_C \times \text{U}(1)_{\text{EM}}$ reads

$$\langle L^k \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u_k & 0 & 0 \\ 0 & d_k & 0 \\ 0 & 0 & \sim M_S & \sim M_S \end{pmatrix}, \quad k = 1, 2, 3, \quad (16)$$

where $u_k$ and $d_k$ represent the EW-breaking Higgs up-type and down-type VEVs in the low-energy SM-like EFT, respectively, while all other VEVs are considered to be of order $M_S$, for simplicity. With such a VEV setting, we can have up to six Higgs doublets in the low-energy limit of the theory. However, for the sake of simplicity, here we consider that only five Higgs doublets acquire VEVs. In this case we found that there is only one particular configuration of these VEVs with $d_1 = 0$ that provides the Cabibbo–Kobayashi–Maskawa (CKM) matrix and the quark mass spectrum compatible with those in the SM, i.e.

$$V_{\text{CKM}} = \begin{pmatrix} d_{2u_1}Y_1^2 + d_{3u_2}Y_2^2 & -u_1Y_1 & (d_{2u_1} - d_{3u_2})Y_1Y_2 \\ \sqrt{AB} & \sqrt{C} & \sqrt{D} \\ (d_{2u_2} - d_{3u_1})Y_1Y_2 & -u_2Y_2 & \sqrt{E} \\ \sqrt{F} & \sqrt{G} & \sqrt{H} \end{pmatrix}$$

$$A = C \frac{Y_1^2}{\sqrt{A}B}, \quad B = d_1^2 Y_1^2 + d_3^2 Y_2^2, \quad C = u_1^2 + u_2^2, \quad m_c^2 = \frac{1}{6} \left( Y_1^2 (u_1^2 + u_2^2) + \frac{1}{2} \sqrt{A} \left( u_1^2 + u_2^2 + 3b \right) \right),$$

$$m_t^2 = \frac{1}{6} \left( d_3^2 Y_2^2 + \frac{1}{2} \sqrt{D} \left( d_3^2 Y_2^2 + 3b \right) \right),$$

while the $u$- and $d$-quarks as well as charged leptons and light neutrinos do not acquire masses at tree level. In the limit of small $\frac{Y_2}{Y_1} \ll 1$ suggested by the second- and third-generation mass hierarchies (10), an approximate Cabibbo mixing arises, with the angle $\theta_C \approx \arctan(u_1/u_2)$. Indeed, in this limit the top-bottom mixing element $V_{tb} \approx 1 - (Y_2/Y_1)^2 \sim O(1)$ is well under control. Moreover, the same ratio (10) provides a strong suppression for $V_{td}, V_{ts}, V_{bu}$ and $V_{bc}$ CKM elements, in agreement with measurements. In addition, small tree-level contributions to the masses and CKM from a seesaw-type mixing with the heavy vector-like quarks are present. This is similar to the model presented in Ref. [47] which we refer to for further details. Furthermore, loop contributions generate additional (small) terms to the CKM mixing entries and fermion masses. Note, the minimal scenario that is compatible with the CKM quark mixing and mass spectrum corresponds to a Three Higgs Doublet Model with $d_{1,3} = u_3 = 0$; a suitable benchmark scenario for detailed phenomenological explorations.

Note, as an interesting possibility for future studies, the fields $\Sigma, \Sigma'$ can break $E_6$ not only to the trinification group but also to $\text{SU}(6) \times \text{SU}(2)$. The two scenarios have an intersection and the presence of adjoint $\mathcal{A}$ which breaks trinification at a lower scale $M_3$ makes a difference in the associated symmetry breaking pattern and in the corresponding low-energy SM-like EFT limit. A further analysis would be necessary to conclude on which of these two $E_6$ breaking schemes is favoured by the vacuum structure of the theory, as well as by phenomenology. Such studies should include the full RG flow analysis and the matching to the SM Higgs and flavor sectors in both scenarios.
In summary, the suggested flavored SUSY-GUT framework exhibits two-fold unification in the gauge and Yukawa sectors as a consequence of the Higgs-matter and the SM gauge couplings’ Grand Unification under $E_6$. While higher-dimensional $E_6$ operators in the $E_6 \times SU(2)_F \times U(1)_F$ GUT theory generate the necessary splittings in the Yukawa and gauge sectors, the gauge couplings’ RG flow suggests a strongly-decoupled energy scale for the soft-SUSY breaking sector, giving rise to a consistent low-scale SM-like EFT. The latter exhibits the minimum of three light Higgs doublets for the model to be generically compatible with SM quark masses and CKM mixing. The main features of the SM fermion spectra such as the observed top-charm and bottom-strange mass hierarchies as well as a Cabibbo-type mixing in the quark sector are generated already at tree level. Other parameters of the light fermion spectra such as the small CKM mixing elements, $u, d$-quark and charged lepton masses, neutrino masses and mixing should be established at higher-loop orders via a mixture of different-type seesaw mechanisms which is planned for further studies. But it is already clear that vast phenomenological prospects offered in the proposed framework by a rich scalar, neutrino and heavy vector-like fermion sectors as well as by the gauge family interactions can be expected in the reach of future collider experiments. One of the remaining key theoretical goals for further studies is to explore the potential of the ultimate Left-Right-Color-Family gauge and Yukawa couplings’ unification through a dynamical origin of $E_6 \times SU(2)_F \times U(1)_F$ from yet unknown large gauge group $G$ at the $M_{GUT}$ scale.

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