

\textit{g}_{\text{ND} \Lambda_c} \textit{ from QCD Sum Rules}

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Abstract

The $\text{ND} \Lambda_c$ coupling constant is evaluated in a full QCD sum rule calculation. We study the Borel sum rule for the three point function of one pseudoscalar one nucleon and one $\Lambda_c$ current up to order seven in the operator product expansion. The Borel transform is performed with respect to the nucleon and $\Lambda_c$ momenta, which are taken to be equal, whereas the momentum $q^2$ of the pseudoscalar vertex is taken to be zero. This coupling constant is relevant in the meson cloud description of the nucleon which has been recently used to explain exotic events observed by the H1 and ZEUS Collaborations at HERA.

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Many explanations have been proposed for the recent observation by H1 and ZEUS Collaborations at HERA of an excess of events at large $x$ and $Q^2$ in deep inelastic scattering (DIS). Whereas most explanations have concentrated on physics beyond the standard model, some of them were based on more conventional mechanisms. One of these was suggested by Melnitchouk and Thomas \cite{1}. They pointed out that an enhanced charm quark distribution in the nucleon sea at large $x$ could account for the observed effect. This hard component of the $c$ distribution would have to be generated non-perturbatively, since the usual charm quark distribution arising from gluon bremsstrahlung alone looks like a typical soft sea distribution, namely peaks at $x \to 0$ and is negligible beyond $x \approx 0.4$. In the works \cite{2,3} it was suggested that the charmed sea may arise from the quantum fluctuation of the nucleon to a virtual $D^- \Lambda_c$ configuration. A natural prediction of this picture are non-symmetric $c$ and $\bar{c}$ distributions, the latter being much harder. Following this approach Melnitchouk and Thomas studied the effects of this charmed meson cloud on the DIS cross sections measured at HERA.

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In the framework of the standard meson cloud model the anti-charm quark distribution in the nucleon, $\bar{c}_N$, is given by the convolution of the valence $\bar{c}$ momentum distribution in the $D^-$ meson, $\bar{c}_D(x)$, with the momentum distribution of this meson in the nucleon $f_D(y)$:

$$\bar{c}_N(x) = \int_x^1 dy f_D(y) \frac{1}{y} \bar{c}_D\left(\frac{x}{y}\right). \quad (1)$$

The virtual $D^-$ meson distribution in the nucleon cloud which characterizes its probability of carrying a fraction $y$ of the nucleon momentum in the infinite momentum frame is given by

$$f_D(y) = \frac{g_{ND\Lambda_c}^2}{16\pi^2} \int_0^\infty dk_\perp^2 \frac{F^2(k_\perp^2, y)}{y (1 - y) (s_{D\Lambda_c} - M_N^2)^2} \left(\frac{k_\perp^2 + [M_{\Lambda_c} - (1 - y)M_N]^2}{1 - y}\right) \quad (2)$$

where $s_{D\Lambda_c} = (k_\perp^2 + m_D^2)/y + (k_\perp^2 + M_{\Lambda_c}^2)/(1 - y)$. In the above equation $F(k_\perp^2, y)$ is a form factor at the $DN\Lambda_c$ vertex.

In the heavy quark effective theory it is assumed that the heavy quark interacting with light constituents, inside a hadronic bound state, exchanges momenta much smaller than its mass. Therefore, to a good approximation, the heavy quark moves with the velocity of the charmed hadron. Being almost on-shell, the heavy quark carries almost the whole momentum of the hadron. These considerations suggest that the $\bar{c}$ distribution in the $D^-$ meson is expected to be quite hard and may be approximated by a delta function:

$$\bar{c}_D\left(\frac{x}{y}\right) \simeq \delta\left(\frac{x}{y} - 1\right) = x \delta (x - y). \quad (3)$$

From eqs. (1), (2) and (3) we obtain

$$\bar{c}_N(x) = f_D(x) \quad (4)$$

Similar expressions can be found for the charm quark distribution in the nucleon, $c_N(x)$. The above equation shows that the charm quark distributions in the nucleon are directly proportional to $g_{ND\Lambda_c}^2$. The value used in ref. was $g_{ND\Lambda_c}^2/4\pi = -3.795$.

Plugging the above calculated $\bar{c}_N(x)$, together with parton distributions of all flavors, in the standard DIS cross sections expressions, $d\sigma / dx dQ^2$, Melnitchouk and Thomas were able to reproduce the observed rise of the cross sections at large values of $M (M \simeq 200 GeV)$ and $Q^2$.

In this work we shall concentrate our attention on the coupling constant $g_{ND\Lambda_c}$, since it enters directly in the calculation of the DIS cross section. The value used to reproduce the HERA events in , while a reasonable guess, has neither a solid theoretical justification nor a phenomenological basis. It may, however, be calculated in the framework of QCD Sum Rules (QCDSR). In the literature there are many calculations of coupling constants with QCDSR. We shall make reference to two of them: to , which describes the general method and to , which is recent and estimates $g_{NK\Lambda}$. In order to calculate the $N D^-\Lambda_c$ coupling constant using the QCD sum rule approach we consider the three-point function

$$A(p, p', q) = \int d^4x d^4y \langle 0 | T\{\eta_{\Lambda_c}(x) j_5(y)\eta_N(0)\} | 0 \rangle e^{i p' \cdot x} e^{-i q \cdot y} \quad (5)$$
constructed with two baryon currents, \( \eta_{\Lambda_c} \) and \( \eta_N \), for \( \Lambda_c \) and the nucleon respectively, and the pseudoscalar meson \( D \) current, \( j_5 \), given by \[8,9\]

\[
\eta_{\Lambda_c} = \varepsilon_{abc}(u^T_a C \gamma_5 d_b) Q_c ,
\]

\[
\eta_N = \varepsilon_{abc}(u^T_a C \gamma_\mu u_b) \gamma_\mu \gamma_5 d_c ,
\]

\[
j_5 = \bar{Q} i \gamma_5 u ,
\]

where \( Q, u \) and \( d \) are the charm, up and down quark fields respectively, \( C \) is the charge conjugation matrix.

Due to restrictions from Lorentz, parity and charge conjugation invariance the general expression for \( A(p, p', q) \) in Eq.(5) has the form

\[
A(p, p', q) = \frac{F_1(p^2, p'^2, q^2) i \gamma_5}{i \gamma_5} + \frac{F_2(p^2, p'^2, q^2) i \gamma_5 u}{i \gamma_5} + \frac{F_3(p^2, p'^2, q^2) p i \gamma_5}{i \gamma_5} + \frac{F_4(p^2, p'^2, q^2) \sigma^{ \mu \nu} \gamma_5 p_\mu p'_\nu}{i \gamma_5} ,
\]

where \( q = p' - p \) and \( P = (p + p')/2 \).

In the phenomenological side the different Lorentz structures appearing in Eq.(9) are obtained by the consideration of the \( \Lambda_c \) and \( N \) states contribution to the matrix element in Eq. (5):

\[
\langle 0 | \eta_{\Lambda_c} | \Lambda_c(p') \rangle \langle \Lambda_c(p') | j_5 | N(p) \rangle \langle N(p) | \bar{\pi}_N | 0 \rangle ,
\]

where the matrix element of the pseudoscalar current defines the pseudoscalar form-factor

\[
\langle \Lambda_c(p') | j_5 | N(p) \rangle = g_P(q^2) \pi(p') i \gamma_5 u(p) ,
\]

where \( u(p) \) is a Dirac spinor and \( g_P(q^2) \) is related to \( g_{ND\Lambda_c} \) through the relation \[5\]

\[
g_P(q^2) = \frac{2m_D^2 f_D}{m_u + m_c} \frac{g_{ND\Lambda_c}}{q^2 - m_D^2} ,
\]

where \( m_D \) and \( f_D \) are the meson \( D \) mass and decay constant and \( m_u(c) \) is the \( u(c) \) quark mass.

The other matrix elements contained in Eq.(10) are of the form

\[
\langle 0 | \eta_{\Lambda_c} | \Lambda(p') \rangle = \lambda_{\Lambda_c} u(p')
\]

\[
\langle N(p) | \bar{\pi}_N | 0 \rangle = \lambda_N \pi(p) ,
\]

where \( \lambda_{\Lambda_c} \) and \( \lambda_N \) are the couplings of the currents with the respective hadronic states.

Saturating the correlation function Eq.(3) with \( \Lambda_c \) and \( N \) intermediate states, and using Eqs.(10), (11), (12), (13) and (14) we get

\[
A^{(phen)}(p, p', q) = \lambda_{\Lambda_c} \lambda_N \frac{2m_D^2 f_D}{m_u + m_c} \frac{g_{ND\Lambda_c}}{q^2 - m_D^2} \frac{(p' + M_{\Lambda_c}) i \gamma_5 (p + M_N)}{p^2 - M_{\Lambda_c}^2} + \text{higher resonances} ,
\]

where \( \lambda_{\Lambda_c} \) and \( \lambda_N \) are the couplings of the currents with the respective hadronic states.
which can be rewritten as

\[
A^{(\text{phen})}(p,p',q) = \lambda_{\Lambda_c} \Lambda_c \frac{2m_D^2f_D}{(m_u + m_c)} \frac{g_{NDc}}{q^2 - m_D^2p'^2 - M_{\Lambda_c}^2p'^2 - M_N^2} \left[(M_{\Lambda_c} - p.p')i\gamma_5
\right.
\]

\[
+ \frac{M_{\Lambda_c} + M_N}{2} \Phi_i\gamma_5 + (M_{\Lambda_c} - M_N)\Pi\gamma_5 - \sigma^{\mu\nu}\gamma_5\sigma_{\mu\nu}' \left]\right] + \text{higher resonances}, \quad (16)
\]

where we clearly see all the Lorentz structures present in Eq. (9). We will follow refs. [5,6] and write a sum rule for the structure \( \Phi_i\gamma_5 \). As we are interested in the value of the coupling constant at \( q^2 = 0 \), we will make a Borel transform to both \( p^2 = p'^2 \to M^2 \). The contribution from excited baryons will be taken into account as usual through the standard form of ref. [10].

As in any QCD sum rule calculation, our goal is to make a match between the two representations of the correlation function (5) at a certain region of \( M^2 \): the OPE side and the phenomenological side. The contributions to the OPE side are represented in figures 1, 2 and 3.

In the OPE side only odd dimension operators contribute to the \( \Phi_i\gamma_5 \) structure, since the dimension of Eq. (5) is four and \( \Phi \) takes away one dimension. Therefore, the perturbative diagram in Fig. 1 contributes through the \( m_c \) operator. To evaluate the perturbative contribution we write a double dispersion relation to the amplitude \( F_2 \) in Eq. (9) and use the Cutkosky’s rules to evaluate the double discontinuity (see ref. [11]). After Borel transforming with respect to \( P^2 = -p^2 = -p'^2 \), and subtracting the continuum contribution, we get

\[
\left[ \tilde{F}_2(M^2, q^2) \right]_1 = -\frac{1}{4\pi^2} \int_{m_c^2}^{u_0} du \int_0^{u-m_c^2} ds \rho(u, s, q^2) \frac{1}{s-u} (e^{-u/M^2} - e^{-s/M^2}), \quad (17)
\]

with

\[
\rho(u, s, q^2) = \frac{3}{2} \frac{m_c}{(2\pi)^2} \frac{1}{\sqrt{\lambda(s, u, q^2)}} \int_0^s dm^2 \left\{ m^2 \left( \frac{1}{4} + \frac{\sqrt{sp}_0}{2\lambda(s, u, q^2)}(u - p_0\sqrt{s} - m_c^2) \right) \right.
\]

\[
- \frac{m^4}{2} \left[ \frac{1}{2s} + \frac{s}{\lambda(s, u, q^2)} \left( 1 - \frac{p'_0}{\sqrt{s}} \right) \right]^2 \right\} \Theta((\cos \theta_{\Lambda_c})^2 - 1), \quad (18)
\]

where \( p'_0 = (s + u - q^2)/(2\sqrt{s}) \) and

\[
\cos \theta_{\Lambda_c} = 2s \frac{u + m^2 - m_c^2 - p'_0(s + m_c^2)/\sqrt{s}}{(s - m_c^2)\sqrt{\lambda(s, u, q^2)}}. \quad (19)
\]

In Eq. (17) \( \tilde{F}_2 \) stands for the Borel transformation of the amplitude \( F_2 \), the subscript 1 refers to the diagram 1 and \( u_0 \) gives the continuum threshold for \( \Lambda_c \).

The next lowest dimension operator is the quark condensate with dimension three (Fig. 2). In ref. [3] only the diagram shown in Fig. 2a was considered because the pion mass was neglected and, therefore, only terms proportional to \( 1/q^2 \) contributed. Since in our case \( m_D \) will not be neglected we have also to consider the diagrams in Figs. 2b and 2c. Their contributions are given by
\[ \left[ \tilde{F}_2(M^2, q^2) \right]_{2a} = \frac{\langle \bar{q}q \rangle}{8\pi^2} \frac{1}{q^2 - m_c^2} M^4 E_1, \]  

with \( E_1 = 1 - e^{-u_0/M^2} (1 + u_0/M^2). \)

\[ \left[ \tilde{F}_2(M^2, q^2) \right]_{2b+2c} = \frac{\langle \bar{q}q \rangle}{8\pi^2} \int_{m_c^2}^{u_0} du \int_0^{u-m_c^2} ds \frac{(2 - \delta + \alpha)s + (1 + \alpha)u}{(u-s)^2} (e^{-u/M^2} - e^{-s/M^2}), \]  

where \( \delta = 1/2 - \alpha(u+s)/(2s) \) and

\[ \alpha = \frac{2s}{(u-s)^2} \left( m_c^2 + \frac{s-u}{2} \right). \]  

The last class of diagrams which we will consider is the dimension 7 operators of the type \( m_c \langle \bar{q}q\bar{q}q \rangle \) \( q \) being a \( u \) or \( d \) quark) shown in Fig. 3. Other dimension 7 operators come from graphs which contain at least one loop and are suppressed by factors \( 1/4\pi^2 \). The expressions for these contributions are

\[ \left[ \tilde{F}_2(M^2, q^2) \right]_{3a+3b} = \frac{\langle \bar{q}q\bar{q}q \rangle}{6} \frac{m_c}{q^2 - m_c^2}, \]  

\[ \left[ \tilde{F}_2(M^2, q^2) \right]_{3c} = - \frac{\langle \bar{q}q\bar{q}q \rangle}{6} \frac{1 - e^{-m_c^2/M^2}}{m_c}. \]  

The Borel transformation of the phenomenological side gives

\[ \left[ \tilde{F}_2(M^2, q^2) \right]_{\text{phen}} = \lambda_{\Lambda_c} \lambda_N \frac{m_D^2}{(m_u + m_c)} \frac{g_{NDA_c}}{2} M_{\Lambda_c} + \frac{M_N}{2} (e^{-M_{\Lambda_c}^2/M^2} - e^{-M_c^2/M^2}). \]  

For \( \lambda_{\Lambda_c} \) and \( \lambda_N \) we use the values obtained from the respective mass sum rules for the nucleon \( \bar{\Xi} \) and for \( \Lambda_c \) \( \bar{\Upsilon} \):

\[ |\lambda_{\Lambda_c}|^2 e^{-M_{\Lambda_c}^2/M^2} = \frac{m_c^4}{512\pi^2} \int_{m_c^2}^{u_0} e^{-u/M^2} \left[ \left( 1 - \frac{m_c^4}{u^2} \right) \left( 1 - \frac{8u}{m_c^2} + \frac{u^2}{m_c^4} \right) - 12 \ln \left( \frac{m_c^2}{u} \right) \right] \]

\[ + \frac{\langle \bar{q}q\bar{q}q \rangle}{6} e^{-m_c^2/M^2}, \]  

\[ |\lambda_N|^2 e^{-M_N^2/M^2} = \frac{M_N^6}{32\pi^4} E_2 + \frac{2}{3} \langle \bar{q}q\bar{q}q \rangle, \]  

where \( E_2 = 1 - e^{-s_0/M^2} (1 + s_0/M^2 + s_0^2/(2M^4)) \) accounts for the continuum contribution with \( s_0 \) being the continuum threshold for the nucleon. We have neglected the contribution of the gluon condensate in the mass sum rules and in the three point function since it is of little influence.

In order to obtain \( g_{NDA_c} \) we identify eq. (25) with the sum of eqs. (17, 20, 21, 23, 24) and using eqs. (23, 27) we solve the resulting equation for \( g_{NDA_c} \) as a function of the Borel mass

\[ \frac{m_c^4}{512\pi^2} \int_{m_c^2}^{u_0} e^{-u/M^2} \left[ \left( 1 - \frac{m_c^4}{u^2} \right) \left( 1 - \frac{8u}{m_c^2} + \frac{u^2}{m_c^4} \right) - 12 \ln \left( \frac{m_c^2}{u} \right) \right] \]

\[ + \frac{\langle \bar{q}q\bar{q}q \rangle}{6} e^{-m_c^2/M^2}, \]  

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At first sight, since the normalization of the hypothesis \([12]\) we introduce a correction factor \(K\). There are many other possible intermediate states apart from mesonic clouds, specially in view of the small value found here for events. Quantitative calculations should, however, include the contribution of other charm generated charm sea remains qualitatively a good candidate to explain the observed exotic states to be even more important. Our conclusion is therefore, that the non-perturbatively quark momentum distributions. In the charm sector we expect the contribution of these may give important contributions for the evaluation of strange form factors and strange neglected in some calculations but, as shown in ref. \([3]\), already in the strange sector they to the square of \(g\).

The resulting curves are shown in Figs. 4 and 5. Since eqs. \((26, 27)\) determine only the absolute value of \(\lambda_N\) and \(\lambda_{\Lambda_c}\) we can not determine the sign of \(g_{NDA_c}\).

In this calculation there is no free parameter. There are, instead, some numbers which are heavily constrained by other theoretical or phenomenological analyses. They are the sources of uncertainties in our result. The quark condensate was taken to be \(\langle \bar{q}q \rangle = -(0.23)^3\) GeV\(^3\). The continuum thresholds appearing in \(E_1\) and \(E_2\) were chosen to be respectively \(u_0 = (M_{\Lambda_c} + 0.7)^2\) GeV\(^2\) and \(s_0 = (M_N + 0.7)^2\) GeV\(^2\). The hadron masses are \(M_N = 0.938\) GeV, \(M_{\Lambda_c} = 2.285\) GeV and \(m_D = 1.8\) GeV. The charm quark mass was taken to be \(m_c = 1.3\) and 1.5 GeV. In order to take into account possible deviations from the factorization hypothesis \([12]\) we introduce a correction factor \(K\) in the formula

\[
\langle \bar{q}q \rangle = K \langle \bar{q}q \rangle^2 \tag{28}
\]

and consider the cases \(K = 1\) and \(K = 2\). Finally we need the \(D\) meson decay constant \(f_D\). This constant will be probably evaluated in the next generation of experiments on fully leptonic \(D\) decays. So far it has been usually estimated \([13]\) to be \(f_D = 200 \pm 30\) MeV. On the other hand, new measurements of \(f_{D_s}\) \([14]\) by the CLEO Collaboration point to \(f_{D_s} = 344 \pm 37\) MeV. Considering that \(f_D\) and \(f_{D_s}\) are related by \(f_{D_s}/f_D \simeq 1.15 - 1.2\) \([13]\) this indicates that \(f_D \simeq 300\) MeV. In view of this we take \(f_D\) in the interval 200 – 300 MeV.

The relevant Borel mass here is \(M \simeq \frac{M_N + M_{\Lambda_c}}{2}\) and therefore we analyse the sum rule in the interval \(1.5 \leq M^2 \leq 3.5\) GeV\(^2\). In Fig. 4 we plot \(g_{NDA_c}\) as a function of \(M^2\) for different values of \(f_D\) and \(K\). As it can be seen, the curve is rather flat showing that, for QCD sum rules standards, this sum rule result is stable. Moreover, the uncertainties of 50% both in \(f_D\) and \(K\) lead to a spread in the value of \(g_{NDA_c}\). In Fig. 5 we fix \(f_D = 250\) MeV and \(K = 1\) and vary \(m_c\) from 1.3 (solid line) to 1.5 GeV (dashed line). The corresponding variation in \(g_{NDA_c}\) is of the order of 15%. We have also varied the continuum thresholds in the interval \(M_N + 0.5 \leq s_0 \leq M_N + 1.0\) GeV and \(M_{\Lambda_c} + 0.5 \leq u_0 \leq M_{\Lambda_c} + 1.0\) GeV. Under these variations the final result does not change appreciably. This indicates that the continuum contribution is under control.

Considering all the uncertainties discussed above our final result is

\[
\frac{|g_{NDA_c}|}{\sqrt{4\pi}} = 1.9 \pm 0.6 \tag{29}
\]

This value is smaller than the one used in the calculations of ref. \([1]\) by a factor two. At first sight, since the normalization of the \(\bar{c}\) distribution in the nucleon is proportional to the square of \(g_{NDA_c}\), using our value would significantly reduce the contribution of the non-perturbative nucleon charm sea employed to study the HERA events. However there are many other possible intermediate states apart from \(D - \Lambda_c\) which contribute to generate this charm sea like, for example, \(D^* - \Lambda_c, D - \Sigma\) and \(D^* - \Sigma\). These higher mass states are neglected in some calculations but, as shown in ref. \([1]\), already in the strange sector they may give important contributions for the evaluation of strange form factors and strange quark momentum distributions. In the charm sector we expect the contribution of these states to be even more important. Our conclusion is therefore, that the non-perturbatively generated charm sea remains qualitatively a good candidate to explain the observed exotic events. Quantitative calculations should, however, include the contribution of other charm mesonic clouds, specially in view of the small value found here for \(g_{NDA_c}\).
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Figure Captions

**Fig. 1** Perturbative contribution to the OPE.

**Fig. 2** Diagrams that contribute to the Wilson coefficient of the operator $\bar{q}q$.

**Fig. 3** Diagrams that contribute to the Wilson coefficient of the operator $\bar{q}q\bar{q}q$.

**Fig. 4** $g_{NDc}$ as a function of the squared Borel mass $M^2$. The different lines correspond to different choices for the $D$ meson decay constant, $f_D$, and for the factorization correction factor $K$ as indicated in the legends.

**Fig. 5** The same as Fig. 4 for different values of the $c$ quark mass. $f_D = 250$ MeV and $K = 1$. 

fig. 1
(a) (b) (c)

fig. 2
fig. 3
fig. 5

\[g_{N\Delta_c}^{\text{m.c.}}\] vs. \(M^2\) (GeV\(^2\)) for two values of \(m_c\):
- Solid line: \(m_c = 1.3\) GeV
- Dashed line: \(m_c = 1.5\) GeV