Scour due to turbulent wall jets downstream of low-/high-head hydraulic structures

Youssef I. Hafez

Cogent Engineering (2016), 3: 1200836
Scour due to turbulent wall jets downstream of low-/high-head hydraulic structures

Youssef I. Hafez*

Abstract: To overcome over-prediction of the scour depths by existing methods, a mathematical model is developed based on a work transfer theory. This model predicts the equilibrium scour hole's depth and length due to two-dimensional turbulent wall jets downstream of low-/high-head hydraulic structures. The work transfer theory states that the work done by the attacking jet flow is transferred to work done to remove the volume of the scoured bed material out of the scour hole. This results in an analytical nonlinear equation for predicting the equilibrium scour depth. This unique feature of the nonlinearity of the developed equation shows the mutual dependence of the scour geometry and flow hydrodynamics on each other. Non-circulating wall jet flows and re-circulating jet flows within the scour hole are both considered. A separate equation is developed for predicting the length of the scour hole. Field data at the Nile Grand Barrages in Egypt and the Shimen Arch Dam in China are used to validate the developed model. The developed equation was checked against laboratory data for scour downstream of a spillway for which a complete data-set exists. For scour downstream of grade control structures, an equation was derived from the low-head hydraulic structure equation and tested against laboratory data.

Keywords: low-/high-head hydraulic structures; barrages; grade control structures; apron; sluice gates; spillways; turbulent wall jets; work transfer theory; scour hole; scour hole depth; scour hole length; Egypt's Grand Barrages; China's Shimen Arch Dam

ABOUT THE AUTHOR

Youssef I. Hafez graduated in 1984 from Cairo University, Egypt, Civil Engineering Department. Soon after, he joined the Nile Research Institute, Egypt. In 1990, he got his Master of Science at Colorado State University, USA, in Civil Engineering, Hydraulics. In 1995, he got his PhD form Colorado State University, USA, in turbulence modeling in closed and open channels using the Finite Element numerical method. In 2000, he became an associate professor at the Nile Research Institute. In 2004 and till now, Youssef has joined the Royal Commission Colleges and Institutes in Yanbu in Saudi Arabia. This paper presents a novel mathematical modeling approach for scour hole predictions downstream of barrages and low-head hydraulic structures. The mathematical model adopts work transfer theory that will open new doors to many analytical developments in the field of scour predictions at hydraulic structures (weirs, spillways, abutments, bridge piers, plunge pool scour, and grade control structures).

PUBLIC INTEREST STATEMENT

It is well known that downstream of hydraulic structures with apron such as low-dams, barrages, spillways, and weirs local scour occurs due to the nature of high velocity of the outflow water jet from the structure gates. These high velocities can destroy the bed's protective layer and erode the unprotected bed downstream of the hydraulic structure. The existence of significant scour holes downstream of a hydraulic structure causes undermining of the bed material below the foundation with a consequent collapse of the hydraulic structure which may be worth millions of US Dollars. The present study provides estimates to the expected maximum scour depth for the safe design and operating purposes.
1. Introduction

It is well known that downstream of low-/high-head hydraulic structures with apron such as barrages, low-head dams, spillways, grade control structures, and weirs local scour occurs due to the nature of high velocity of the outflow water jet from the structure gates (Figure 1). The water jet is controlled by the head difference on the hydraulic structure, gate opening dimensions (width and height), stilling basin characteristics, and the downstream water level. Typical schemes of economic design in which the total gates' width is considerably less than the river width and sometimes gate operations concentrate the whole river discharge to a few gates located close together and accordingly, the resulting narrow flow width induces high outlet jet velocities. These high velocities can destroy the bed's protective layer and erode the unprotected bed downstream of the hydraulic structure. The existence of significant scour holes downstream of hydraulic structures causes undermining of the bed material below the foundation with a consequent collapse of the hydraulic structure. The development of large scour holes downstream of three major barrage structures in Egypt (Isna, Naga Hamadi, and Assiut Barrages) necessitated their replacement with new barrages costing hundred million US dollars. Therefore, it is important to estimate the expected maximum scour depth downstream of low-head hydraulic structures for design purposes and for operating or emergency conditions that might endanger the structure.

Scour investigations can be classified mainly into two approaches, namely: experimental and mathematical approaches. In the experimental approach, flume experiments are used to measure the scour’s most influencing factors (Chiew & Lim, 1996; Dietz, 1969; Eggenberger, 1943; Hamidifar, Omid, & Nasrabadi, 2011; Lim & Yu, 2002; Schoklitsch, 1932; Shalash, 1959; Shawky, 2008). Then, dimensional analysis is applied to the experimental data to deduce relations between the equilibrium scour hole's depth and length, on the one hand, and the flow hydrodynamic variables and sediment characteristics, on the other hand. It is noted that most flume experiments have been run under constant discharges and nearly uniform bed sediments. Breusers and Raudkivi (1991) indicated that several of the (scour) equations cannot readily be extended to prototype scale. They further believe that even those which are dimensionally consistent lack corroboration from full scale tests, and the variability of the predictions point at considerable inconsistency even at model scale.

![Figure 1. Definition sketch and schematic representation of the scour hole mechanism downstream of low-head hydraulic structures such as barrages.](image-url)
In the second approach, mathematical models are developed which are solved analytically or numerically. The mathematical treatment to the subject was adopted using often the concept of incipient motion of bed sediment particle (Dey & Sarkar, 2006; Hogg, Huppert, & Dade, 1997; Hopfinger et al., 2004). In this approach, it is assumed that the longitudinal distribution of the excess bed shear stress (local bed shear stress—critical shear stress for sediment particles) is responsible for the motion of the sediment particles out of the scour hole. The shape of the scour hole follows the assumed model for the longitudinal distribution of this excess bed shear stress. Several constants appearing in the mathematical formulations are determined using laboratory data. In the experimental and mathematical approaches, the resulted prediction equations are tested against laboratory flume data often with uniform bed sediments.

1.1. Experimental methods

Schoklitsch (1932) proposed an equation based on model tests for the underflow case with short horizontal sill (which is relevant to the case of the Nile Barrages) as

\[
D_s = 0.378 H^{0.5} q^{0.35} + 2.15 a
\]

(1)

where \( D_s \) is the equilibrium or maximum scour depth below the original river bed (m), \( H \) is the difference between upstream water level and sill level (m), \( q \) is the flow discharge per unit width (m\(^2\)/s), and \( a \) is the level of the downstream riverbed below the sill level (m). This equation is dimensionally inconsistent.

Eggenberger (1943) performed tests with combined flow over a weir and flow under the weir acting as a sluice gate. If the overflow is zero, the scour resulting from the submerged horizontal jet is

\[
D_s + y_o = 7.255 H^{0.5} q^{0.6} d_{90}^{-0.4}
\]

(2)

where \( y_o \) is the downstream flow depth (m), \( H \) is the head on the structure (m), and \( D_s \) is sediment size for which 90% of the bed sediment is finer, expressed in mm throughout all the cited equations in this section.

Shalash (1959) included the effect of the apron length with a horizontal end sill as

\[
D_s + y_o = 9.65 H^{0.5} q^{0.6} d_{90}^{-0.4} (L_{min}/L)^{0.4}
\]

(3)

where \( L \) is the apron length and \( L_{min} = 1.5 H \).

Among the few reported formulas for fine sediment scour formulas is that by Dietz (1969) as

\[
\frac{D_s}{y_o} = \frac{U_{max} - U_c}{U_c}
\]

(4)

where \( U_{max} \) is the maximum jet velocity and \( U_c \) is the critical mean velocity from Shields’ curve. Breusers and Raudkivi (1991) reported that Equation (4) predicts very large equilibrium scour depths as, for example, when \( U_{max} = 6 U_c \), the equilibrium scour depth is \( 5y_o \). This value is used throughout this study for lack of the information needed in determining values of \( U_c \).

Chiew and Lim (1996) developed empirical equations by considering the densimetric Froude number \( F_o \) as the main characteristic parameter to estimate scour dimensions caused by circular wall jet. The densimetric Froude number is given as

\[
F_o = \frac{U_o}{\sqrt{(\rho_s/\rho - 1) g D_{50}}}
\]

(5)

where \( U_o \) is the jet velocity, \( \rho_s \) is the sediment density, \( \rho \) is the fluid density, \( g \) is the gravitational acceleration, and \( D_{50} \) is the median sediment diameter. The scour equations are given as:
Lim and Yu (2002) applied regression technique on the database of 161 flume experiments, out of which 61 data-sets were from Nanyang Technological University. Their equation for predicting the maximum scour depth in case of existing apron is

\[
\frac{D_s}{b_o} = 1.04 \sigma_g^{-0.69} F_o^{1.47} \left( \frac{b_o}{d_{50}} \right)^{-0.33} e^{-0.044 \beta_1 L/b_o} \tag{8}
\]

and \( \beta_1 = \sigma_g^{-0.5} F_o^{-0.35} \left( \frac{b_o}{d_{50}} \right)^{0.5} \tag{9} \)

where \( \sigma_g \) is the sediment gradation. They reported that comparison between the calculated and measured scour depths indicates that 76% of the data were within ±20% error band and 91% were within ±30% error band. They indicated that the scour depth would decrease by about 50% if \( \sigma_g \) were increased from 1.2 to 3.13, consistent with the findings of Aderibigbe and Rajaratnam (1998). When there is no apron, Equation (8) becomes

\[
\frac{D_s}{b_o} = 1.04 \sigma_g^{-0.69} F_o^{1.47} \left( \frac{b_o}{d_{50}} \right)^{-0.33} \tag{10}
\]

Shawky (2008), in his investigation of scour due to high floods, used a scale model with a length scale ratio of 1:32 to study scour at Rossetta Barrage on the Nile River, near Cairo, Egypt (Figure 2). The model was based on Froude number, similarity with the prototype, i.e. assuming only inertial and gravitational forces as the only forces determining the hydraulic scour phenomenon. Due to model space availability and economic factors, only 5 vents (gates) were modeled out of 46 vents which are the total vents of Rossetta Barrage. Shawky (2008) modeled 100 m of the river section upstream the Rossetta Barrage, the stilling basin, and 150 m of the river section downstream of the barrage with the final dimensions of the model as 15 m length and 1.75 m width. He plotted the data of scour areas and length vs. the scour levels and obtained equations with the best fit. These equations could predict the scour area and length downstream of Rossetta Barrage in all cases of discharge (but valid only for Rossetta Barrage). The scour hole’s depth or length was not given as an explicit function of the scour influencing factors as in typical scour equations. This approach definitely lacks generality, though it confirmed some useful findings such as the increase of the scour depth due to increase of the jet unit discharge and that the upstream slope of the scour hole is more steeper than the downstream slope.

Hamidifar et al. (2011) examined experimentally the effects of bed roughness on the local scour downstream of an apron. Their results showed that the main characteristics of the scour holes, such as the maximum scour depth and the maximum extension of the scour hole, were much smaller for rough than smooth aprons. A minimum reduction of 60% was obtained for the maximum scour depth with respect to the smooth aprons. A total of 33 experiments were carried out with five different roughness conditions and combined plots of smooth and rough conditions were used to show roughness effects. No explicit scour equations were given and it is noted that the maximum densimetric Froude number reached 13 while Table 1 shows field values of the densimetric Froude number (18th row) higher than 40.
Bombardelli and Gioia (2005) investigated a closely related problem: the jet-driven turbulent cauldron that plunges into a pool of water and scour a pothole in a cohesionless granular bed. They assumed that the dynamic equilibrium is attained between a localized turbulent flow and a granular bed. They made the following observations on past empirical formulas for predicting the depth of pothole: (1) the formulas often lack dimensional homogeneity; (2) the formulas have often been the product of mangled attempts at dimensional analysis; (3) the formulas have often been predicated on limited experimental data; (4) the formulas have sometimes disregarded important parameters such as the diameter of the grains of the bed; and (5) the formulas do not provide much insight into the interaction between the granular bed and the turbulent cauldron. The author agrees with these observations except of the importance of the diameter of grains as it influences the rate of scour not the magnitude of the equilibrium scour depth and scour geometry in general.
It is noted that most scour lab experiments used bed materials (usually sands) with grain sizes comparable to those in field conditions while the flow discharges and flow depths in these experiments are significantly less than their corresponding field values. For example, Schoklitsch (1932) used sediments sizes in the range from 1.5 to 12 mm, Eggenberger (1944) used $D_{90}$ from 1.2 to 7.5 mm, Muller (1947) used $D_{50}$ from 0.43 to 3.67 mm, and Shalash (1959) used $D_{50}$ from 0.7 to 2.65 mm, which are typical sediment sizes found in natural rivers. In lab flumes, usually flow depths are predetermined by the construct of the flume which is in the order of few centimeters and flow discharges by the pump capacity which is in the order of few cubic meters per second. For example, Eggenberger (1944) used unit discharges (q) ranging from 0.006 to 0.024 m$^2$/s and Shalash (1959) had q values from 0.011 to 0.027 m$^2$/s. There is no guarantee that labs’ hydrodynamic and sediment variables are matching their corresponding field values. This, of course, will induce significant scale effects in lab experiments and cause lab-based prediction equations to deviate much when applied to field cases. In lab experiments, small relative flows cause significant scour depths and when scaling these conditions using lab-based equations to field conditions, it results in higher than actually predicted scour depths, thus creating the over-prediction of scour depth problem. Rajaratnam (1981) data indicate upstream slope of the scour holes as 0.5 while field data (Table 1) show values around 0.2. This confirms that scour holes in lab experiments are significantly much steeper than field scour holes. This induces more turbulence structure in lab experiments. Also that field densimetric Froude numbers are more than their lab values points to scale effects.

### Table 1. Field data of scour holes at the Nile Grand Barrages, Egypt

| Variable | NEBTG | NEBSG | ONHB | ABV58 | RBV23 | DBV24 | ZB |
|----------|-------|-------|------|-------|-------|-------|----|
| $H_1$ (m) | 13.0  | 13.0  | 4.9  | 5.25  | 5.5  | 4.5  | 5.7 |
| $a$ (m)    | 3.0   | 3.0   | 0.0  | 0.0   | 0.0  | 0.5  | 0.0 |
| $L$ (m)    | 50.0  | 50.0  | 72.0 | 50.0  | 65.0 | 60.0 | 45.0|
| $H$ (m)    | 3.89  | 3.89  | 4.5  | 4.2   | 3.8  | 3.8  | 3.9 |
| $y_1$ (m)  | 15.11 | 15.11 | 5.8  | 2.75  | 3.2  | 4.09 | 1.8 |
| $D_{\text{90}}$ (mm) | 1.0  | 1.0   | 1.162| 0.822 | 0.92 | 0.772| 1.0 |
| $\theta$   | 0.4   | 0.4   | 0.4  | 0.4   | 0.4  | 0.4  | 0.4 |
| $S_o$      | 2.65  | 2.65  | 2.65 | 2.65  | 2.65 | 2.65 | 2.65|
| $\tan \varphi_1$ | 0.11 | 0.16  | 0.20 | 0.20  | 0.15 | 0.2  | 0.2 |
| $\tan \varphi_2$ | 0.045| 0.05  | 0.10 | 0.10  | 0.08 | 0.1  | 0.1 |
| $\alpha_\infty$ | 2.64 | 3.2   | 2.0  | 2.0   | 1.9  | 2.0  | 2.0 |
| $\sin \varphi_1$ | 0.11 | 0.158 | 0.20 | 0.20  | 0.2  | 0.2  | 0.2 |
| $q$ (m$^2$/s) | 27.77| 20.50 | 36.15| 27.24 | 10.00| 10.00| 10.64|
| $H_1 = b_1$ (m) | 3.0   | 3.0   | 3.0  | 3.0   | 1.92 | 1.92 | 2.0 |
| $U_o$ (m/s) | 9.26  | 6.83  | 12.05| 9.08  | 5.20 | 5.20 | 5.32|
| $F_1$ | 1.71  | 1.26  | 2.22 | 1.67  | 1.20 | 1.20 | 1.20 |
| $F_2$ | 72.76 | 53.71 | 137.46| 78.72 | 42.68| 46.52| 41.80|
| $D_s$ (m) | 6.5–8.0 | 6.0–6.5 | 10.0 | 7.5–8.0 | 3.5 | 3.25 | 3.5 |
| $L_s$ (m) | 205–213 | 98   | 150  | 100   | 63  | 63   | 63  |
| Aspect ratio$^c$ | 1/25 | 1/15 | 1/15 | 1/15 | 1/12.5 | 1/18 | 1/18 |

Notes: ABV58 = Assiut Barrage Vent No. 58; DBV24 = Damietta Barrage Vent No. 24; NEBSG = New Isna Barrage Sluice Gate; NEBTG = New Isna Barrage Turbine Gate; ONHB = Old Naga Hamadi Barrage; RBV23 = Rosetta Barrage Vent No. 23; and ZB = Zifta Barrage.

$^a\alpha_\infty = \frac{\tan \alpha_1}{\tan \alpha_2}$

$^bF_o = U_o/\sqrt{gH_j}$

$^c$Aspect ratio $= (D_s/L_s)$. 
In agreement with the above statement, despite it is in the context of the closely related bridge scour subject, Jones and Sheppard (2000) stated that “Due to the complexity of the flow and sediment transport associated with local scour processes there are a number of dimensionless groups needed to fully characterize the scour. Many of these groups, such as the ratio of water depth to structure diameter, can be maintained constant between the laboratory model and the prototype structure. However, since there is a lower limit on the sediment particle size before cohesive forces become important, those groups involving sediment size cannot be maintained constant between the model and the prototype. In fact, most laboratory experiments are performed with near prototype scale sediment. If the sediment to structure length scales are not properly accounted for in the predictive equations then problems arise when the equations are applied to situations different from the laboratory conditions on which they are based.”

1.2. Mathematical modeling methods

As laboratory experimental investigations are time-consuming and expensive, mathematical/numerical modeling has become very popular lately. Hogg et al. (1997) developed an analytical framework to model the progressive erosion of an initially flat bed of grains by a turbulent wall jet. In their model, the grains are eroded if the shear stress, exerted on the grains at the surface of the bed, exceeds a critical value which is a function of the physical characteristics of the grains (this is why $D_{50}$ or some characteristic particle size appears in all the scour formulae adopting the incipient motion concept). They balanced the mobilizing and resisting moments on the particles at the surface of the sloping bed to obtain critical conditions for incipient particle motion. This enables the prediction of the characteristic dimensions of the steady-state scour pit. They indicated that after the wall jet has been flowing for a sufficiently long period, the boundary attains a steady state, in which the mobilizing forces associated with the jet are insufficient to further erode the boundary. They calculated the profile of the scour pit by considering the shear stress distribution along the surface of the bed. They assumed the following:

(a) The boundary shear stress for the flow of a two-dimensional jet over an erodible boundary is equivalent to the flow of a two-dimensional jet over fixed rough boundary. (b) The boundary profile has negligible influence on the jet flow; or equivalently, that the aspect ratio of the eroded profile is small. However, they admitted that this is a crude approximation because any bed topography will exert an additional drag on the flow, leading to a more rapid attenuation of the boundary shear stress. (c) The mean flow does not separate from the boundary at any downstream location to avoid the need to introduce models of regions in which the flow recirculates. (d) The shear stress along the scour pit boundary varies with Gaussian-like characteristics. They emphasized that this is a somewhat arbitrary model of the shear stress distribution and that any other shape function could be used. However, since the shape of the scour hole was assumed by them similar to the Gaussian-like curve, this in a sense is similar to assuming the bed profile right at the start. Using the aforementioned assumptions, the profile of the boundary is given according to them by integrating the following equation

\[
\frac{d\eta}{d\zeta} = \tan \beta = \frac{-\tan \alpha + \left[\tan^2 \alpha - (f_b^2 - 1)(f_b^2 - \tan^2 \alpha)\right]^{1/2}}{f_b^2 - 1}
\]

(11)

where $\eta$ is a non-dimensional vertical distance, $\zeta$ is a non-dimensional downstream distance, $\beta$ is the angle of inclination of the bed, $f_b$ is a non-dimensional bed shear stress, and $\alpha$ is the angle of repose of the granular material of the bed. Equation (11) is solved subjected to the following two boundary conditions: (1) that far from the jet source, erosion is absent and (2) that the volume of the particles is conserved.

They reported that the condition of a fixed volume of particles may not be entirely appropriate for comparison with experiments in which particles are swept downstream into a “sand trap;” however, it is appropriate for geophysical applications. They introduced a relaxation or adjustment process in
which the gradient of the upward slopes never exceeds the tangent of the angle of repose. They showed that agreement between the predicted eroded profiles with the experimental data of Rajaratnam (1981) is fairly good except around the downstream locations where the profiles attain a maximum value (dune region). This difference in their opinion is because the model has neglected flow separation over the crest. They obtained the shape of the eroded boundary at intermediate times, before the steady state is attained, by the application of a sediment–volume conservation equation.

It can be seen that some uncertainties exist about Hogg et al.’s (1997) formulation for the boundary shear stress which rests on many empirical constants (up to seven constants were used) and the values of these constants were derived based on laboratory experiments which might differ under field conditions. In addition, scour in field conditions extends to depths of the order of several meters. The one grain size for the bed used in their equation cannot be assumed to represent the grains along the depth of the scour hole. In addition, they adopted a critical shear stress value of 0.05 according to Nielsen (1992). They found from the data of Rajaratnam (1981) that the ratio of the maximum eroded depth, \( \varepsilon_m \), to the downstream distance to the position of maximum eroded depth, \( \chi_m \), to be constant, \( \sigma \), where \( \sigma = 0.5 \). According to field values for \( \tan \varphi_1 \) in Table 1 and keeping in mind that \( \sigma = \varepsilon_m/\chi_m \approx \tan \varphi_1 \), \( \tan \varphi_2 \) has a maximum value of about 0.2. In addition, their assumption that the downstream slope of the scour hole is close to the angle of repose which might be true in flume experiments is far from field values for \( \tan \varphi_2 \) as seen in Table 1. For example, if the angle of repose is 30° or 35°, then its tangent value is 0.58 or 0.7, respectively, while field values for \( \tan \varphi_2 \) in Table 1 have a maximum value of 0.1.

Hopfinger et al. (2004) adopted the model developed by Hogg et al. (1997) but increased the effective shear stress by a constant factor to account for effects of Görtler vortices which were found from observing loose sediment streaks or longitudinal ridges on the upstream-facing sediment slope of the scour hole (that also supports that sediment particles are creeping along the slope of the scour hole). They explained that the contribution of Görtler vortices to bed shear stress is likely to be of the same form as normal turbulent shear stress and is therefore additive, i.e. \( \tau_b = \tau_t + \tau_G \) where \( \tau_t \) is the effective bed shear stress, \( \tau_G \) is the turbulent shear stress, and \( \tau_G \) is the shear stress due to Görtler vortices. They showed numerically that Görtler vortices can increase the effective shear stress by an order of magnitude. In addition, they stated that Görtler vortices cause strong up-slope sediment transport and, in turn, strong avalanching which intermittently destabilizes the sediment hill. They emphasized that at least two scouring regimes must be distinguished: a short time regime after which a quasi-steady state is reached, followed by a long time regime, leading to an asymptotic state of virtually no sediment. The quasi-steady-state scour hole’s depth, \( h_s \), is given as

\[
\frac{h_s}{b_o} = D_s \frac{b_o}{b_o} = B_1 \left( \frac{b_o}{d_{50}} \right)^{-0.11} F_o^{1.1} - B_2
\]  

(12)

here \( b_o \) is sluice gate opening, \( D_{so} \) is the densimetric particle Froude number, and the constants \( B_1 \) and \( B_2 \) are given as \( B_1 = 0.43 \) and \( B_2 = 0.2 \). They stated that because of the narrow range of the experimental conditions, the exponent 1.1 of the densimetric particle Froude number should be taken with some caution. There is also weak dependency of \( h_s/b_o \) on \( b_o/D_{so} \), but they argued that this is a must for two data points to collapse onto a straight line. They used only five data points to build Equation (12).

Dey and Sarkar (2006) computed the scour profiles downstream of a smooth apron due to submerged wall jets from the threshold condition of the sediments particles on the scour bed which is expressed by the following differential equation:

\[
(\Omega^2 - 1) \frac{d\hat{y}}{d\hat{x}} = \mu \pm \left[ \mu^2 - (\Omega^2 - 1)((\Omega^2 - \mu^2) \right]^{0.5}
\]  

(13)
where

$$\Omega = \frac{0.0081 e^{0.5/\delta} + 0.04 (c y^2/\delta)^2}{\left[ f_{c}^{2} \right]_{\bar{y}=0}} \left( 2 \frac{\delta u_{o} d u_{o}}{d x} + \frac{\delta}{b_{o}} d \delta \right) o^{n} (\psi^2 + \Phi_{11} - \Phi_{22}) d \eta. \quad (14)$$

Here, $\bar{y} = y/b_{o}$, $\bar{x} = x/b_{o}$ are the non-dimensional vertical and streamwise distances, respectively, $b_{o}$ is the sluice gate opening, $\mu$ is the Coulomb frictional coefficient of sediment, $c$ is a parameter being a function of $\bar{x}$ and $D_{50}^{y/\delta}$, $u_{o}$ is local maximum velocity, $U_{o}$ is the issuing jet velocity, $\delta$ is the boundary layer thickness, and $[f_{c}^{2}]_{\bar{y}=0}$ is the critical bed shear stress on a horizontal bed ($\bar{y} = 0$).

In addition, $\psi$, $\Phi_{11}$, and $\Phi_{22}$ are functional relationships given by Dey and Sarkar (2006). They state that Equation (13) is a first-order differential equation which can be solved numerically by the fourth-order Runge–Kutta method to determine the variation of $\bar{y}$ with $\bar{x}$. It is clear from the complex nature of Equations (13) and (14) that this approach is not practical for the design engineer. In addition, some uncertainty appears about determining the constants needed in this formulation. Uncertainty also appears in defining incipient motion for sediment particles where uniform sediments are assumed with $D_{50}$ representing the bed sediments.

Several numerical models dealing with scour prediction due to turbulent wall jets are summarized in Balachandar and Reddy (2013). A good example is the work of Abdelaziz, Bui, and Rutschmann (2010) who developed a bed load sediment transport module and integrated into FLOW-3D. This model was tested and validated by simulations for turbulent wall jet scour in an open channel flume. Effects of bed slope and material sliding were also taken into account. The hydrodynamic module was based on the solution of the three-dimensional Navier–Stokes equations, the continuity equation, and $k$-$\varepsilon$ turbulence model (which has several constants not accurately determined for re-circulating flows). The rough logarithmic law of the wall equation was iterated in order to compute the shear velocity and consequently the bed shear stress necessary for bed load computations. The predicted local scour profile fitted well with experimental data; however, the maximum scour depth was slightly underestimated and the slope downstream of the deposition dune was overestimated.

In the author’s view, such numerical models though highly structured and complex proved somehow successful in fitting the numerical model to certain observed scour data from laboratory flume experiments; however, they cannot be reliable in predicting scour under field conditions. Several constants needed for determining the bed shear stress distribution, as seen before, can have different values under field conditions. In addition, the reliability of these numerical models rests on the reliability of the selected sediment transport formula. How much confidence can be on any sediment transport formula? The need to use a sediment transport formula in any numerical model for scour puts some doubt on this approach due to the fact that there is no reliable sediment transport formula. The presented approach avoids use of sediment transport formulas.

Breusers and Raudkivi (1991) stated that existing equations for predicting the scour depths downstream of low-head structures are limited to laboratory studies using coarse sediments. They report that a lack of verification by field data limits the usefulness of the results. The same authors report that for fine sediments, no general expressions are available for predicting the equilibrium scour depth. Measured field values of scour depths deviate very much from values obtained from existing formulas (Hafez, 2004b) as will be seen in this study. This finding is behind the motivation for developing a novel analytical equation herein for predicting scour downstream of low-head structures. Oliveto and Comuniello (2010) state that “Despite several studies on local scour below low-head spillways have been made, the results appear still inconclusive.” In this study, testing is made of the available scour prediction equations along with the developed one in the present study using valuable observed field data at the Grand Nile Barrages in Egypt and the Shimen Arch Dam in China in addition to lab scour data at grade control structures.
2. Mathematical model for scour downstream of low-head hydraulic structures
   based on work transfer theory

2.1. Case of no flow separation

Following the main lines of the energy balance theory by Hafez (2004a) that was used for modeling and predicting bridge pier scour, an analytical equation for predicting equilibrium scour depth downstream of low-head hydraulic structures is developed herein. However, the concept used here is termed “the work transfer theory” which is found to be more appropriate. Scope of the work here does not include the time development of the profile of scour holes and the three-dimensional aspects of the scour process. The principles of the work transfer theory state that the work done by the attacking fluid flow or jet flow is transferred to the work done in removing the volume of the scoured bed material out of the scour hole. In other words, as the work is equal to the potential energy, the potential energy contained in the attacking fluid flow is converted to a potential energy consumed in removing or transporting the sediment out of the scour hole. The mechanics of energy exchange between the jet flow and the sediment particles are complex and beyond the scope of this work. The basic assumptions or limitations are: (1) two-dimensional steady horizontal wall jets, i.e. the analysis is done in the longitudinal plane that bisects the scour hole as this plane contains the maximum scour depth (Figure 1), (2) granular non-cohesive sand bed porous material or rock bed non-porous type can both be considered, and (3) small aspect ratio of the scour hole dimensions which results in an attached jet to the river bed. The aspect ratio is defined here as the scour hole’s depth over the scour hole’s length. Small aspect ratios are defined to be less than 1/10. This indicates that no flow separation in the scour hole occurs (however, case of flow separation will be dealt with later), (4) the shape of the scour hole in its longitudinal bisecting plane can be assumed triangular in form with upstream and downstream inclination angels of \( \varphi_1 \) and \( \varphi_2 \), respectively, as seen in Figure 1, (5) the whole scour hole can be considered as a mega porous sediment particle, i.e. one unit or one porous big sediment particle, (6) assuming unlimited bed material along its depth and no armoring, and (7) there is sufficient time for scour formation to reach equilibrium. Katoulas (1967) found that in case of coarse sand, about 64% of the final scour occurred in the first 20 s, and about 97% of scour depth was attained in 2 h. This leads to assuming in the hypothetical model herein that scour can be considered to occur instantaneously or in a very short time. The first three assumptions are also assumed in the work of Dey and Sarkar (2006), Hogg et al. (1997), and Hopfinger et al. (2004). The triangular shape of the scour hole is evident from many fully developed scour hole profiles as seen in Breusers and Raudkivi (1991) and Hafez (2004b).

The fluid flow force, \( F_j \) or jet flow force per unit width (depth-averaged and assumed acting at the jet flow half-depth) is expressed according to fluid mechanics basics as

\[
F_j = \rho U_1^2 H_j \cos \varphi_1
\]

where \( \rho \) is the fluid (usually water) density, \( U_1 \) is the jet flow velocity, and \( H_j \) is the jet depth. When the bed is initially flat, scour is initiated by the tangential boundary shear stress which results in formation of small scour hole. Once the scour hole is formed with its associated upstream and downstream slopes, the jet flow starts to flow along these slopes as an attached jet and quickly the scour hole grows to its equilibrium profile.

The jet flow force is assumed to flow as an underflow jet that is responsible for eroding the bed material and thus forming the equilibrium scour hole. The jet is assumed to creep along the upstream slope of the scour hole giving its energy to the bed sediment particles. The component of force that is creeping downward along the upstream slope is given as

\[
F_j \cos \varphi_1 = \rho U_1^2 H_j \cos \varphi_1 \left( \frac{H_j}{2D_s} \right)
\]

The distance that the jet force is assumed to move along the upstream slope can be assumed as \( \sin \varphi_1 \) (Figure 1) where \( D_s \) is the maximum or equilibrium scour depth. Gravity exerts work on this down flow creeping jet and this work or potential energy is the cause of the equilibrium scour hole formation. Thus, the work done (work = force \times distance) by gravity on the attacking jet flow force is:

\[
\rho U_1^2 H_j \cos \varphi_1 \frac{H_j}{2D_s} \sin \varphi_1 = \frac{\rho U_1^2 H_j^2 \cos \varphi_1}{\sin \varphi_1} \left( \frac{1}{2} + \frac{D_s}{H_j} \right).
\]

\[ (15) \]
Now, this work done by the jet flow is converted to work done in raising or moving the weight of the volume of the scoured bed material out of the scour hole to the original bed level before occurrence of scour where it is transported by the flow further downstream. The weight of the volume of the material in the scour hole per unit width (for unit width analysis, the volume is equal to the area of the triangular shape of the scour hole that was filled with sediments) is

\[
(\gamma_s - \gamma)(1 - \theta) \frac{D_s}{2} \left( \frac{D_s}{\tan \phi_1} + \frac{D_s}{\tan \phi_2} \right).
\]  

(16)

where \(\gamma_s\) is the bed material unit weight, \(\gamma\) is the fluid (water) unit weight, and \(\theta\) is the bed material or sediment porosity. The weight of the volume of the scoured bed material as given in Equation (16) can be assumed to be concentrated at the scour hole's center of gravity which is at \(D_s/3\) for the triangular-shaped scour hole. The work done in removing the scoured material out of the scour hole to the original bed level can be assumed to be the weight force given by Equation (16), times the distance of its center of gravity from the original bed level \(D_s/3\). Thus, the work done in removing the bed material out of the scour hole is

\[
(\gamma_s - \gamma)(1 - \theta) \frac{D_s}{2} \left( \frac{D_s}{\tan \phi_1} + \frac{D_s}{\tan \phi_2} \right) \frac{D_s}{3} = \rho g (S_o - 1)(1 - \theta) \frac{D_s^3}{6} \left( \frac{1}{\tan \phi_1} + \frac{1}{\tan \phi_2} \right).
\]  

(17)

where \(g\) is the gravitational acceleration and \(S_o\) is the sediment-specific gravity. According to the principles of the work transfer theory mentioned above, the two work done expressions in Equations (15) and (17) are assumed equal at equilibrium conditions. Appendix A explains that Equations (15) and (17) represent the work done during the entire scouring process.

Equality of the right sides of Equations (15) and (17), and after rearranging, yields the following analytically based equation for predicting the scour depth downstream of low-head structures for the case of no flow separation as:

\[
\left( \frac{D_s}{H_j} \right)^3 = \frac{6 \cos \phi_1}{(S_o - 1)(1 - \theta)} \left( \frac{1}{\tan \phi_1} + \frac{1}{\tan \phi_2} \right) \sin \phi_1 \frac{U_o^2}{gH_j} \left( \frac{1}{2} + \frac{D_s}{H_j} \right).
\]  

(18)

Equation (18), which is dimensionless in form, expresses the general mathematical model for modeling and predicting the maximum or equilibrium scour depth due to turbulent wall jets downstream of low-head hydraulic structures, especially at barrages under the assumptions made above. It is noted that the unknown scour depth exists in both sides of Equation (18) which indicates that the equation is nonlinear in form, thus reflecting the complexity of the phenomenon, the interaction between the variables, and the mutual dependence of the scour geometry and flow hydrodynamics on each other. Also, data are needed when applying Equation (18) about the upstream and downstream slopes of the scour hole which are not known before scour occurrence.

Equation (18) can be simplified and expressed in a more attractive form if the following additional assumptions are made: (1) \(\tan \phi_1 = \alpha_s \tan \varphi_1\) where \(\alpha_s\) is the ratio of the upstream to downstream side slope of the scour hole and (2) \(\varphi_1\) is a small angle for which it can be assumed that: \(\tan \varphi_1 = \sin \varphi_1\) and \(\cos \phi_1 = 1.0\) as seen in Table 1. With reference to point (2) above, it is noted from the data of scour holes at Egyptian Barrages in Table 1 that if \(\tan \varphi_1 = 0.2\), then \(\sin \varphi_1 = 0.196\), i.e. \(\tan \varphi_1 = \sin \varphi_1\) and \(\cos \phi_1 = 0.981 \approx 1.0\). In fact, the assumption \(\tan \varphi_1 \approx \sin \varphi_1\) is valid for angles less than 15° where \(\tan (15^\circ) \approx 0.268\). The simplifications made in (2) when inserted into Equation (18) yield:

\[
\left( \frac{D_s}{H_j} \right)^3 = \frac{6}{(S_o - 1)(1 - \theta)} \left( \frac{1}{\tan \phi_1} + \frac{1}{\tan \phi_2} \right) \frac{U_o^2}{gH_j} \left( \frac{1}{2} + \frac{D_s}{H_j} \right).
\]  

(19)
Using $\tan \varphi_1 = \alpha \varphi$ in Equation (19) yields

$$
\left( \frac{D_s}{H_j} \right)^3 = \frac{6}{(S_G - 1)(1 - \theta)} \left( \frac{1}{1 + \alpha} \right) \frac{U_o^2}{gH} \left( \frac{1}{2} + \frac{D_s}{H_j} \right)
$$

(20)

Now assuming that $\tan \varphi_1 = 2 \tan \varphi_2$ or simply that $\alpha = 2$ (which can be seen from the field data of the scour holes downstream of the Egyptian Barrages in Table 1) in Equation (20) and after simplification yields:

$$
\left( \frac{D_s}{H_j} \right)^3 = \frac{2}{(S_G - 1)(1 - \theta)} \frac{U_o^2}{gH} \left( \frac{1}{2} + \frac{D_s}{H_j} \right)
$$

(21)

Equation (21) has the advantage over Equation (18) in that no information is needed about the shape of the scour hole through its slopes. The jet flow depth $H_j$ is assumed to be usually as the sluice gate opening, $b_o$, i.e. $H_j = b_o$ if no data are available. Moreover, Equation (21) can be made more tractable by casting it in terms of the unit width discharge if expressing the unit width discharge of the jet flow as $q = U_o H_j$ and substituting $S_G = 2.65$, $\theta = 0.4$ and $g = 9.81 \text{ m/s}^2$ to obtain the following simplified equation:

$$
D_s^3 = 0.206 q^2 \left( \frac{1}{2} + \frac{D_s}{b_o} \right)
$$

(22)

Care must be taken when using Equation (22), which is dimensional, as $D_s$ will be in m, $q$ in m$^3$/s, and $b_o$ in m. Either of Equations (21) or (22) can be used to obtain the maximum scour depth. Equations (21) or (22) can be solved easily by successive iterations or trials where in the first iteration, $D_s$ is set to zero in the right-hand side of the equation to obtain the first value for $D_s$ in the left-hand side. In the second iteration, the first calculated value of $D_s$ is substituted back in the right-hand side of the equation and accordingly, a new value for $D_s$ is obtained from the left-hand side. This process is continued until the difference between the two values for $D_s$ from the left- and right-hand sides is nearly equal or within a specified tolerance value (assumed here 0.001 m). A simple FORTRAN code was developed to implement this iterative solution process. However, solving Equation (22) iteratively can be easily done with a pocket scientific calculator or any spreadsheet program as well.

In case of unavailability of data about $q$ and $U_o$, the maximum jet velocity as an under flow could be calculated from the following formula (Hafez, 2004b; Hopfinger et al., 2004)

$$
U_o = \sqrt{2gh}
$$

(23)

where $H$ is the head on the hydraulic structure, i.e. difference between the upstream and downstream water levels. With the jet velocity calculated from Equation (23) and with the jet depth $H_j$ or gate opening $b_o$ known, the maximum unit depth discharge can be calculated and used in the calculations. This jet could be an underflow jet that in the absence of a weir, downstream of the barrage keeps its momentum till it meets the unprotected bed.

The length of the scour hole along its longitudinal axis can be derived once the scour depth was obtained as follows. From Figure 1, the length of the scour hole, $L_s$, can be stated as

$$
L_s = \left( \frac{D_s}{\tan \varphi_1} + \frac{D_s}{\tan \varphi_2} \right) = (1 + \alpha) D_s \tan \varphi_1
$$

(24)

Using the assumption stated earlier that $\alpha = 2$ or $\tan \varphi_1 = 2 \tan \varphi_2$ in Equation (24) and after simplification yields
\[ L_s = \frac{3D_s}{\tan \varphi_1} \]  

Equation (25) predicts the length of the scour hole in the main flow direction in terms of the (predicted) depth of scour and the slope of the upstream face of the scour hole, \( \tan \varphi_1 \). In case of unavailability of data about \( \tan \varphi_1 \), it can be assumed from 0.10 to 0.2 (Table 1) with a conservative value of 0.1 or assuming \( \phi_2 \) equal to the angle of repose of bed sediments and calculate \( \phi_1 \) from \( \tan \phi_1 = 2 \tan \phi_2 \).

### 2.2. Case of flow separation

Scour holes with large aspect ratios (greater than 1/10) are often associated with flow separation. In this case, the jet flow hits the downstream slope of the scour hole, bends down along the downstream slope of the hole, and circulates inside the hole. It transfers its work or potential energy to the sediment particles on the downstream face. The sediment particles fall down along with the creeping jet and circulate inside the whole till they move up leaving the hole. Owing to the work or energy gained from the down jet flow, the sediment particles are able to move upward out of the scour hole. It is assumed that a portion of the work or energy in the jet flow is transferred to jet flow circulating inside the scour hole through a coefficient \( \eta_2 \) (where \( \eta_2 < 1 \)). Since the jet flow force is proportional to the square of the velocity, \( \eta_2 \) is therefore used. The jet flow force per unit width is thus assumed equal to \( \rho \eta^2 U_o^2 H_j \) and the distance of motion along the downstream slope is \( \frac{(H_j/2+D_s)}{\sin \phi_2} \).

Now the work done by gravity on the attacking jet flow in case of flow separation and circulation is given as

\[ \frac{\rho \eta^2 U_o^2 H_j^2}{\sin \phi_2} \left( \frac{1}{2} + \frac{D_s}{H_j} \right) \]

(26)

The work done in removing the bed material out of the scour hole is still given by Equation (17). Equality of Equations (26) and (17) gives (after rearranging and assuming \( \phi_2 \) is small, i.e. \( \sin \phi_2 = \tan \phi_2 \)):

\[ \left( \frac{D_s}{H_j} \right)^3 = \frac{6}{(S_G - 1)(1 - \theta)} \left( \frac{\eta^2 U_o^2}{gH_j} \right) \left( \frac{1}{2} + \frac{D_s}{H_j} \right) \]

(27)

Examining Equation (27) for the case of flow separation and Equation (20) for the case of no flow separation reveals that the difference lies in the denominator term that has \( \alpha_\varphi \) and \( \eta \). Both equations can be given in one model equation as

\[ \left( \frac{D_s}{H_j} \right)^3 = \frac{6 C_\phi}{(S_G - 1)(1 - \theta)} \left( \frac{U_o^2}{gH_j} \right) \left( \frac{1}{2} + \frac{D_s}{H_j} \right) \]

(28)

where for the case of no flow separation:

\[ C_\phi = \frac{1}{(1 + \alpha_\varphi)} \]

(29)

and for the case of flow separation,

\[ C_\phi = \frac{\eta^2}{(1 + \alpha_\varphi)} \]

(30)

In addition, \( \eta = 1 \) in case of non-circulating flows while \( \eta < 1 \) in case of circulating flows. Equation (28) is the general mathematical model for prediction of the scour hole’s equilibrium depth whether there is flow separation or not. When \( \alpha_\varphi = 2 \), the ratio of the scour depths of flow separation to that of no flow separation becomes 1.26. This means that when assuming all variables are the same and \( \alpha_\varphi = 2 \), the scour depth in case of flow separation increases by 26% due to increase in turbulence activities. For \( \alpha_\varphi = 3 \) (steeper upstream slope), the percentage increase is 44%. The steeper the
upstream slope of the scour hole, the higher is the flow separation. Flow separation is more likely to occur in flume experiments. This might explain why flume-based scour equations tend to over predict equilibrium scour depths when applied to field cases as will be shown later.

3. Applications of the scour formulas for predictions of scour hole’s depth
The Grand Nile Barrages in Egypt are among the most important hydraulic structures in the Egyptian water resources system as they are spreading across the 1,000 km stretch of the Nile River in Egypt (Figure 2). The Nile Barages serve in regulating the Nile flow discharges and water levels in addition to some barrages producing hydropower. Upstream these barrages, off-take canals withdraw water to irrigate most of the agricultural lands in Egypt. The Egyptian Dams and Barrages system starts at the southern border with the High Aswan Dam (HAD) which was built in 1964. Six kilometers downstream from HAD, the Old Aswan Dam (AD) is located which was built between 1899 and 1902. Distances along the Nile River in Egypt are measured starting from Old AD, i.e. it is considered km zero. The Nile Barages include: Old Isna Barrage at km 166.65, the New Isna Barrage at km 167.85, Naga Hamadi Barrage at km 359.45, Assiut Barrage at km 544.75, Damietta and Rosetta Barrages at km 952.92, Zifta Barrage on Damietta branch at km 1046.7, and Idfina Barrage on Rossetta branch at km 1159.0. Construction has recently been completed for new barrage at Naga Hamadi and has started for a new Assiut low dam. These barrages are very precious hydraulic structures worth millions of US dollars and are therefore very vital to the country’s national economy.

Before the construction of HAD in 1964, just upstream Old Aswan dam, all barrages’ gates during the flood season were completely opened with the barrage acting as a bridge section with constriction scour only expected. In the rising period of the flood, scour occurred while in the falling period, filling of the scoured holes occurred by the flood sediment-laden water. Therefore, it is thought that the current observed scour holes are due to conditions after HAD, i.e. after 1964, because the water is almost clear of sediments. These conditions are resulting from high-velocity jets either from high-head difference on the barrage or high-unit width discharge due to gate operation schemes. Indeed, there might have been scour developed before the events cited herein, but filling of these scour holes by dumping stones and sand sacks had been a common practice to minimize scour effects. Therefore, it could have been assumed that the river bed downstream of the barrages was restored almost to its nearly pre-scour levels.

In the following sections, the existing and newly developed equations are applied to the Grand Nile Barages in Egypt. Details of the hydraulic data for this work are found in Hafez (2004b) and summarized in Table 1. For lack of reported data, it is assumed that $D_{90} = 0.7D_{50}$. In Hafez (2004b), scour predictions were made using a similar equation to Equation (18) while substituting the measured upstream and downstream slopes from the field data. In Hafez (2004b), the constant $1/2$ added to $D_s/H$ in the right-hand side bracket in Equation (18) was assumed erroneously equal to 1.0. It should be noted that field scour data downstream of low-head hydraulic structures are rare. The case of the Shimen Arch Dam, China, is one of these rare cases where a complete data-set exists.

3.1. Local scour prediction downstream of the New Isna Barrage
To replace the historical Old Isna Barrage, the New Isna Barrage in Egypt was built in 1994 at a distance of 168.2 km from Old Aswan Dam. This barrage consists of a powerhouse, flood sluiceway, and navigational lock. The powerhouse has 6 gates that are each 12-m wide and the flood sluiceway has 11 gates that are also each 12-m wide. The floor level elevation is at 66.0 m above the mean sea level (all elevations herein are above the mean sea level).

Scour holes have been formed downstream of the turbine gates and of the flood sluiceway where some measured profiles are shown in Figures 3 and 4. For example, field surveys revealed that the scour depths measured between January 2002 and January 2003 are about 6.5, 8.0, 6.5, 5.0, 4.5, and 3.0 m downstream of the six turbine gates, respectively (Hafez, 2004b). It is noted that at the first three turbine gates, scour is at maximum. This indicates that the first gates were opened more often than the rest. Therefore, conditions of maximum unit discharge prevail at these first three gates with...
Figure 3. Measured scour hole downstream of the New Isna Barrage at turbine gate No. 1, (Hafez, 2004b).

Figure 4. Measured scour hole downstream of the New Isna Barrage at sluiceway No. 10 (Hafez, 2004b).
scour depths of 6.5, 8.0, and 6.5 m. The scour hole with depth 8.0 m at the second gate was symmetrical around the gate axis. Scour is less at the other gates maybe due to partial opening of gates there. Therefore, scour depths ranging from 6.5 to 8.0 m are considered representative of conditions of maximum unit discharge downstream of these turbine gates.

Just right downstream of the 11 flood sluiceway gates, the scour depths are about 3.5, 4.0, 5.0, 6.0, 6.5, 6.0, 5.0, 5.5, 4.8, and 3.0 m, respectively (Hafez, 2004b). Again, it is noted that scour is at its maximum at sluiceway gate numbers 4, 5, and 6 with scour depths of 6.0, 6.5, and 6.0 m, respectively. The same pattern of similarity noticed at the turbine gates appears at the sluiceway gates with maximum value of 6.5 m in the middle, indicating opening of the middle gates more often than the rest. Therefore, scour depths ranging from 6.0 to 6.5 m are considered to be produced by conditions of maximum unit discharge downstream of the sluiceway gates. It was observed that these scour holes are located behind the bed protective cover of the downstream part of the barrage.

The maximum flow allowed to the turbines is 2,000 m$^3$/s which for a total accumulated width of 72 m yields unit width discharge of 27.77 m$^2$/s. This jet unit discharge is assumed to be responsible for the maximum scour depths ranging from 6.5 to 8.0 m observed downstream of turbine gates No. 1, 2, and 3. The maximum observed flow passing the New Isna Barrage is assumed to be as 234 million m$^3$/d = 2,708 m$^3$/s (Hafez, 2004b). With this flow passing only in the flood sluiceway gates along its total width of 132 m (i.e. assuming shutting down the powerhouse), the unit width discharge is 20.5 m$^2$/s. This jet unit discharge is assumed to be responsible for the maximum scour depths ranging from 6.0 to 6.5 m which are observed downstream of flood sluiceway gates No. 4, 5, and 6. The jet depth is assumed to be as 3.0 m for both cases. The ratio of observed upstream and downstream slopes is seen in Table 1 as 2.44 and 3.2 which differs from the assumed value of 2. Dumping of stones which has been a common practice to stop scour progression may be the cause of higher slope ratios. However, for other barrages, the assumed slope ratio of 2 seems to hold.

The predicted scour depths along New Isna Barrage Turbine gates (NEBTG) are shown in Table 2. Equation (8) is not used because of lack of information in the current field data about the sediment gradation $\sigma$, and according to Equation (8), the influence of $\sigma$ cannot be neglected. It is clear from the table that Equation (22) developed in the present study gives computed scour depth of 7.94 m compared to the measured scour depths between 6.5 and 8.0 m and that the rest of the equations are highly overestimating the scour depth. Equation (1) provides close prediction but not as accurate as Equation (22).

The predicted scour depths along New Isna Barrage sluiceway gates (NEBSG) are also shown in Table 2. Again, Equation (22) yields the best prediction of 6.01 m compared to observed scour depths ranging between 6.0 and 6.5 m followed by Equation (1) while the rest of the equations still yield unrealistic scour depths. It is noted that Equation (1) is not sensitive to the varying flow conditions in the two cases discussed herein, while Equation (22) shows sensitivity to the varying flow conditions.

3.2. Local scour prediction downstream of the Old Naga Hamadi Barrage
The Old Naga Hamadi Barrage was built in 1932 with 100 gates, each having a width of 6.0 m. The maximum design head is 4.5 m while the downstream floor level elevation is 58.0 m. Scour holes are severe to the point of initiating the construction of a new barrage. The scour holes (see Figure 5) reached up to 10.0-m depth with upstream slope of 0.20 and downstream slope of 0.10 confirming the assumption previously made that $\alpha_p = 2$. The case that expected to produce this sort of scour is the one for which the outlet jet velocity from the barrage is at its maximum. For a maximum upstream water level of 65.4 m (Hafez, 2004b) and floor level elevation of 58.0 m, the head on the jet is 7.4 m. Note that the head on the jet is not necessarily equal to the head on the barrage. Using Equation (23), the maximum jet velocity becomes 12.05 m/s and for a jet depth of 3.0 m, the resulting maximum unit width discharge becomes 36.15 m$^2$/s. The measured downstream water level was
60.72 m in 4/7/1997 and with floor level of 58.0 m; the downstream water depth was 2.8 m, which is close to the assumed jet depth of 3.0 m.

Table 2 reports the scour depth predictions at the Old Naga Hamadi Barrage (ONHB). It is clear that Equation (22) yields the most realistic value. Equation (22) predicts scour depth of 10.15 m compared to a measured scour depth of 10.0 m. Equation (1) highly under predicts the scour depth as 2.94 m while the rest of the equations highly over predict the scour depth (29.0–118.95 m).

### 3.3. Local scour prediction downstream of Assiut Barrage

Assiut Barrage was built in 1938 at km 544.75 from AD with 111 gates, each gate having a width of 5.0 m. The barrage maximum head difference is 4.2 m, floor level elevation is 43.75 m, and sill level elevation is 43.25 m. Large scour holes (Hafez, 2004b) are formed downstream of the barrage, especially in front of vent No. 58 as seen in Figures 6–8.

Figure 8 reveals two stages of local scour in front of Vent No. 58. The first occurred between January 2002 and January 2003 with a scour depth of about 4.0 m (from level 42.5 to 38.5 m) and the second occurred between January 2003 and December 2003 with a scour depth of about 3.5 m (from level 38.5 to level 35.0 m). The total scour depth between January 2002 and December 2003 is therefore 7.5 m. This large scour hole whose centerline lies downstream of gate No. 58 has dimensions of: length of 110 m, width of 65 m, and depth of 7.5 m, as seen from Figures 7 and 8. The analysis herein is implemented only on the scour hole at gate No. 58 axis with the idea that this scour hole represents a typical fully developed scour hole.

### Table 2. Predicted scour depths downstream of the Nile Barrages, Egypt

| Case   | Equation (1) (m) | Equation (2) (m) | Equation (3) (m) | Equation (4) (m) | Equation (6) (m) | Equation (12) (m) | Present study, Equation (22) | Measured scour |
|--------|------------------|------------------|------------------|------------------|------------------|-------------------|-------------------------------|----------------|
| NEBTG  | 10.81            | 90.03            | 23.43            | 75.55            | 45.84            | 56.83             | 7.94                         | 6.50–8.00 |
| NEBSG  | 10.37            | 72.52            | 17.01            | 75.55            | 33.84            | 40.53             | 6.01                         | 6.00–6.50 |
| ONHB   | 2.94             | 118.95           | 34.30            | 29.00            | 55.35            | 71.25             | 10.15                        | 10.00          |
| ABV58  | 2.75             | 114.05           | 42.08            | 13.75            | 49.59            | 60.69             | 7.80                         | 7.50           |
| RBV23  | 1.99             | 55.01            | 14.78            | 16.00            | 17.21            | 20.89             | 3.68                         | 3.50           |
| DBV24  | 2.87             | 60.05            | 16.69            | 20.45            | 19.43            | 23.28             | 3.68                         | 3.25           |
| ZB     | 2.07             | 57.40            | 21.35            | 9.00             | 17.56            | 21.37             | 3.83                         | 3.50           |
The water depth of the discharge jet is assumed to be 3.0 m (measured downstream water level of 45.0 m—floor elevation level of 42.0 m) while a head difference of 4.2 m was measured in December 2003. The upstream water level was 49.0 m while the downstream water level was 44.8 m, giving also a head of 4.2 m. For an upstream water level of 49.0 m and floor level of 43.75 m, the upstream depth ($H_1$) in Equation (1) becomes 5.25 m. The bed sediment $D_{90}$ downstream of Assiut Barrage was estimated as 0.822 mm. The apron length needed when applying Equation (3) is assumed to be 50.0 m while it is assumed that the initial riverbed level is equal to the apron level ($a = 0.0$ in Equation (1)). From Figure 8, tan $\phi_1$ and tan $\phi_2$ can be calculated as 0.2 and 0.1, respectively, confirming the assumption previously made that $\alpha_\phi = 2$.

It is required to predict the maximum equilibrium scour depth of Assiut Barrage downstream of Vent No. 58. It is expected that conditions which produce this maximum scour are a combination of...
maximum head and maximum unit width discharge which together produce a jet with very high velocity enough to erode the bed and cause significant scour in a relatively short time of almost 2 years.

With head ($H$) equal to 4.2 m, Equation (12) yields a velocity of 9.08 m/s and with depth of 3.0 m, the maximum expected unit width discharge is therefore 27.24 m$^2$/s. Table 2 shows the scour prediction at Assiut Barrage Vent NO. 58 (ABV58) resulting from this unit width discharge of 27.24 m$^2$/s. It is clear that the scour depth prediction equation by Equation (22) gives the nearest value of 7.80 m compared to measured scour depth of 7.5 m. Again, Equation (1) highly under predicts the scour depth as 2.75 m while the rest of the equations highly over predict the scour depth (13.75–114.05 m).

3.4. Local scour prediction downstream of Delta Barrage on Rosetta Branch
Delta Barrage on Rosetta Branch was built in 1939 with 46 gates, each having 8.0-m width. The maximum head on the barrage is 3.8 m and the floor level elevation under the barrage is 11.0 m which is sloping down to a floor level of 9.5 m. A weir exists downstream of the barrage to reduce the head and jet velocity. Shawky (2008) reports field measurement of the scour hole at vent number 23 which looks like a well-defined scour hole with an upstream slope of about 0.15 while the downstream slope is about 0.08, yielding $\alpha_r \approx 1.9$. The measured field scour depth reported by Shawky (2008) was 3.5 m and the length of the scour hole is 63 m.

It is expected that either the maximum head on the barrage or the maximum unit width discharge through the barrage will produce maximum local scour immediately downstream of the barrage. For a head on the barrage of 3.8 m, Equation (23) yields velocity of 8.63 m/s. Assuming (El Kateb, 1982) that the downstream weir will reduce this velocity by 40%, the expected jet velocity becomes 5.18 m/s. Also, the water depth in this vena contracta region is assumed to be 0.6 of the downstream water depth, i.e. $0.6 \times 3.2 = 1.92$ m. With this information, the resulting unit width discharge becomes 10.0 m$^2$/s. Table 2 proves again the success of Equation (22) as the calculated scour depth at Rosetta Barrage Vent 23 (RBV23) was 3.68 m compared to a measured value of 3.5 m. Equation (1) underestimated the scour depth and the rest of the equations yield unrealistically high values.

3.5. Local scour prediction downstream of Delta Barrage on Damietta Branch
Delta Barrage on Damietta Branch was built in 1939 with 34 gates, each having 8.0-m width. The maximum head on the barrage is 3.8 m and the floor level elevation under the barrage is 12.0 m which is sloping down to a floor level of 10.0 m. A weir is constructed downstream of the barrage to reduce
the head and jet velocity. A scour hole (Hafez, 2004b) with depth of 2.0 m exits behind the floor apron downstream of gate No. 18 and another scour hole of depth 3.25 m downstream of gate No. 24.

Following the same lines as in the last section, the resulting jet velocity is 5.18 m/s and with 1.92 m jet depth, the unit depth discharge is 10.0 m$^2$/s. The upstream and downstream slopes of the scour holes are observed as 0.2 and 0.1, respectively, confirming the assumption that $a_p = 2$. Table 2 shows the success of Equation (22) at Damietta Barrage (DBV24) compared to the rest of the equations. Equation (22) yields a scour depth of 3.68 m compared to a measured scour depth of 3.25 m. It is noted that Equation (1) closely predicted the scour depth while the rest of the equations over predicted the scour depth by order of magnitudes (16.69–60.05 m).

### 3.6. Local scour prediction downstream of Zifta Barrage on Damietta Branch

Zifta Barrage (km 1046.7 from AD) on Damietta branch was built in 1903 with 50 gates, each having 5.0-m width. The maximum head on the barrage is 4.0 m and the floor level elevation is 3.5 m. For a head of 4.0 m, the resulting jet velocity from Equation (23) is 8.86 m/s and with 40% reduction due to the weir, the effective jet velocity becomes 5.42 m/s. The jet depth is assumed to be 2.0 (downstream water level of 5.5—floor level 3.5 m). Therefore, the unit width discharge is 10.64 m$^2$/s. For the scour hole along gate No.1 axis, the upstream and downstream slopes are 0.2 and 0.1, respectively, which again confirm the assumption that $a_p = 2$.

Using the above data, the maximum predicted scour depth by Equation (22) is 3.83 m. Observed scour depths along the axes of the barrage gates are equal to or less than 3.5 m. Again, Equation (1) under predicts the scour depth as 2.07 m while the rest of the equations highly over predict the scour depth (9.00–57.40 m).

### 3.7. Scour downstream of the Shimen Arch Dam, China

Lim and Yu (2002) described the case of the Shimen Arch Dam, China. They reported: “The Shimen Arch Dam was built in China along the Bao River in 1973.” The structure has a 20 m apron downstream of the six sluice gates (each 7-m wide and 8-m high). In 1978, the apron protection was extended by 30 m to give a total apron length of 50 m. In the period of 14–25 August 1981, the river was inundated with a one in 300 years flood flow and the water released from the six gates was as high as 4,840 m$^3$/s. Scouring occurred downstream of the sluice gates and the maximum scour depth recorded was about 13.6 m, below the original bed level of the downstream channel. The mean efflux jet velocity at the entrance to the apron floor was estimated to be about 20.8–25.4 m/s, based on the head of water upstream of the sluice. The mean efflux flow depth was estimated to be 4.03 m, based on the flood discharge and the apron width. The downstream water depth was about 53 m. The bed material downstream of the dam consists of layers of cipolin or quartzite. Each layer is less than 10-m thick. The high-speed flow destroyed the rocky layers into loose pieces of rock blocks. Some of these rocks were flushed out by the flow and formed the scour hole during the flood. According to survey on the bed material in the scour hole, it was found that the rock blocks had a volume varying between 24 and 26 m$^3$ and assuming it to be spherical would give the bed material size a value of 3.62 m in diameter.

Lim and Yu (2002) used the above data along with assuming a uniform bed material with $a_p = 1.2$ and $D_{50} = 3.62$ m in their equation (Equation 8), and obtained computed maximum scour depths of 12.9–17.0 m for the velocity range of 20.8–25.4 m/s at the entrance to the apron. Their computed mean scour depth of 14.95 m compares favorably with the measured maximum scour depth of 13.6 m. However, such close matching by them between the computed and measured scour depths would have not been possible unless knowledge exists about bed rock pieces with a diameter of 3.62 m which is very difficult to be predicted prior to any given flood event. For any given high floods, it would have not been possible to make an estimate of the newly formed rock spheres due to very high floods.

In order to apply the present study-developed scour equation which is based on the work transfer theory, care must be given to the type of the rocky bed material at the Shimen Dam. It can be
assumed that the porosity of the rocky bed material is almost zero ($\theta = 0$ for non-porous materials) and that the specific gravity of quartzite is 2.8. These new values of the porosity and specific gravity are inserted into Equation (21) and after casting it in the same form as Equation (22) yields

$$D_s^3 = 0.1133 \frac{q^2}{s} \left( \frac{1}{2} + \frac{D_s}{H_j} \right)$$  \hspace{1cm} (31)

For the lower velocity of $U_o = 20.8 \text{ m/s}$ and jet depth of $4.03 \text{ m}$, the unit flow discharge becomes $83.82 \text{ m}^2/\text{s}$ which is substituted in Equation (31) and solved for $D_s$ to yield a predicted scour depth of $14.97 \text{ m}$. For the higher velocity of $U_o = 25.4 \text{ m/s}$ and jet flow of $4.03 \text{ m}$, the resulting unit flow discharge becomes $102.36 \text{ m}^2/\text{s}$ and after substituting this value in Equation (31), a value of $D_s = 18.09 \text{ m}$ can be computed. It is observed that the predicted scour depth ($14.97$ or $18.09 \text{ m}$) is larger than the measured one ($13.6 \text{ m}$); however, filling of the scour hole during the falling period of the flood is likely to cause the maximum scour depth to be larger than the reported value of $13.6 \text{ m}$ or that the scour did not reach equilibrium conditions. Keeping in mind that only information about the flood hydrodynamics is used in predicting the scour depth of this high flood, the predictions seem to be very impressive.

4. Application of the developed equation for prediction of scour hole’s length

Table 3 shows the results of applying Equation (25) to predict the length of the scour hole in the main flow direction. To use Equation (25), input values for the scour hole’s depth and the slope of its upstream face are needed. Equation (22) is used to provide the scour hole’s depth while the field observed value of the upstream slope of the scour hole is adopted. From topographic survey maps, the measured scour hole’s length can be determined easily; yet, in some cases, its precise value is difficult to be obtained and a range of its value can be settled for.

It is noted that although Equation (25) is very simple, it yields results that are comparable with the observed field values. The predicted values are slightly higher than the measured values which are due to the difference between reality and the theoretical approach used herein. Large differences (Isna and Assiut Barrage cases) can be attributed to the fact that some scour holes have not reached their equilibrium state which is assumed in the theoretical approach herein. The scour length predictions by Chiew and Lim (1996) seem to be of an order of magnitude higher than the measured lengths.

5. Scour downstream of spillway structures due to horizontal jets

In order to test the present approach’s scour equation against laboratory data, detailed data are needed. Due to scale effects, the assumptions made earlier that $\varphi_1 = 2 \tan \varphi_2$ or simply that $\alpha = 2$, $\tan \varphi_1 = \sin \varphi_1$, and $\cos \varphi_1 \approx 1$ (which can be seen from the field data of the scour holes downstream of the Egyptian Barrages in Table 1) cannot be assumed to hold in laboratory data. Therefore, detailed data about the scour hole geometry in terms of its upstream and downstream side slope angles of the scour hole are needed which are rare to find in the case of scour downstream of barrages. Fortunately, such data exist for a nearly similar case which is scour downstream of spillways by Dargahi (2003) as seen in Table 4. Though this is a case of high-head hydraulic structures, the present approach-developed equations still are valid as long as the flow jet downstream of spillway is horizontal with velocity $U_o$ and depth $H_j$.

Dargahi (2003) experiments were conducted in a 22-m-long, 1.5-m-wide, and 0.65-m-deep flume. An overflow spillway of 0.205-m crest height was placed at 16.5 m from the inlet. Two different uniformly graded bed materials were used, one fine sand with $D_{50} = 0.36 \text{ mm}$ and the other medium size gravel with $D_{50} = 4.9 \text{ mm}$. Two series of experiments were carried out, one with a smooth plate at the toe of the spillway and the other with additional roughness on the plate. The flow discharge was varied from 20 to $100 \text{ l/s}$ and the operating head, $h_o$, varied from 38 to 96.2 mm. Experiment S20R in Table 4 means the bed material is sand (S), the flow discharge is $20 \text{ l/s}$, and the protection plate has added roughness ($R$).
Dargahi (2003) experiments show that the scour geometry varied considerably as the bed material was changed from sand to gravel. In gravel tests, the scour cavity became smaller and the slope angles were reduced by 10–20%. A common feature in all tests was that 40% of the final scour depth was reached after about 20 min or 4% of the test duration which confirms the assumption that scour occurs almost simultaneously. It was found for sand tests that the upstream slope was steeper than the downstream slope as was assumed here also in the present method.

As laboratory experiments are characterized by steep slopes of the scour hole, the equation developed for scour with flow separation will be used here. Equation (27) (without assuming $\phi_2$ as small) becomes

$$
\left( \frac{D_s}{H_j} \right)^3 = \frac{6}{(S_G - 1)(1 - \theta)} \left( \frac{1}{\tan \phi_1} + \frac{1}{\tan \phi_2} \right) \sin \phi_2 \frac{U_o^2}{g H_j} \left( \frac{1}{2} + \frac{D_s}{H_j} \right)
$$

(32)

| Location | Predicted scour depth ($D_s$) from Equation (22) (m) | Measured slope of the upstream face of the scour hole, $\tan \phi_1$ | Predicted scour hole-length, $L_s$, Chiew and Lim (1996) Equation (7) (m) | Predicted scour hole-length, $L_s$, present study Equation (25) (m) | Measured scour hole-length, $L_s$ (m) |
|----------|---------------------------------------------------|-----------------|-------------------------------------------------|-------------------------------------------------|-----------------|
| NEBTG    | 7.94                                              | 0.11            | 330                                             | 217                                             | (205–213)       |
| NEBSG    | 6.01                                              | 0.16            | 262                                             | 113                                             | 98              |
| ONHB     | 10.15                                             | 0.2             | 531                                             | 152                                             | 150             |
| ABV58    | 7.80                                              | 0.2             | 350                                             | 117                                             | 110             |
| RBV23    | 3.68                                              | 0.15            | 141                                             | 74                                              | 63              |

Table 3. Predicted length of the scour hole (to nearest meter) downstream of the Nile Barrages, Egypt

| Test | $Q$ (l/s) | $h_s$ (mm) | $\phi_1$ (degrees) | $\phi_2$ (degrees) | Measured $D_s$ (m) | Calculated $D_s$ (m), Equation (32) | Relative error (%) |
|------|-----------|------------|-------------------|-------------------|--------------------|---------------------------------|-------------------|
| S20  | 20        | 38.3       | 27                | 20                | 0.2                | 0.205                           | 2.3               |
| S40  | 40        | 56.4       | 24                | 23                | >0.26              | 0.239                           | −8.8              |
| S60  | 60        | 73         | 19                | 17                | >0.26              | 0.274                           | 5.2               |
| G20  | 20        | 38.5       | 11                | 19                | 0.07               | 0.163                           | 57                |
| G40  | 40        | 57         | 10                | 18                | 0.11               | 0.198                           | 44.5              |
| G60  | 60        | 71         | 13                | 22                | 0.16               | 0.23                           | 30.3              |
| G80  | 80        | 83.2       | 13                | 21                | 0.2                | 0.254                           | 21.1              |
| G100 | 100       | 95.8       | 16                | 22                | >0.26              | 0.288                           | 9.8               |
| S20R | 20        | 38         | 21                | 13                | 0.1                | 0.207                           | 51.7              |
| S60R | 60        | 72.8       | 23                | 17                | >0.26              | 0.286                           | 9.2               |
| G20R | 20        | 37         | 8                 | 7                 | 0.03               | 0.188                           | 84                |
| G60R | 60        | 70.6       | 14                | 18                | 0.13               | 0.246                           | 47.1              |
| G100R| 100       | 96.2       | 17                | 14                | 0.19               | 0.323                           | 41.2              |

Table 4. Predicted scour depths downstream of a Spillway, data of Dargahi (2003)
It is assumed the jet flow depth along the protection plate is 0.8; the operating head, \( h_o \), i.e. \( H_j = 0.8h_o \). Writing the energy equation (neglecting losses) between the midpoint of the operating head and the midpoint in the jet flow depth along the protection plate yields the jet flow velocity, given approximately by

\[
U_o = \sqrt{2g(0.205 + h_o(1-0.8)/2.0)}
\]

With \( U_o \) calculated from Equation (33), \( \phi_1 \) and \( \phi_2 \) known from the measured scour profiles, \( H_j = 0.8h_o \), \( S_G = 2.65 \), and \( \theta = 0.4 \), Equation (33) can be used to calculate the scour depths as seen in Table 4.

It should be noted that the maximum scour depths could not be recorded for tests S4, S60, G100, and S60R because the bed sediment thickness was 0.26 m. The calculated scour depths as seen from Table 4 are in good agreement with the laboratory scour data of Dargahi (2003). When the scour depths are very small for gravel beds, such as at G20 and G20R, the deviations are more which is expected. The comparisons with laboratory data herein clearly demonstrate the effectiveness of the present approach in dealing with both laboratory and field cases and its immunity to scale effects.

6. Scour downstream of grade control structures

Scour downstream of grade control structures is a scour phenomenon closely related to wall jet scour that is investigated herein. Laboratory data, Bormann and Julien (1991), for scour downstream of grade control structures are utilized in further testing of the present approach-developed scour equation. These data have the advantage that a large outdoor flume at Colorado State University, USA, was used which reduces model scale effects. In addition, the upstream slope of the scour hole fluctuates around a value of 0.2 in most runs which is close to the assumption made in this study, i.e. having small upstream scour hole slope. Equation (27), after casting it in terms of the unit width discharge by expressing the unit width discharge of the jet flow as

\[
q = U_oH_j
\]

and substituting \( S_G = 2.65 \), \( \theta = 0.4 \), \( g = 9.81 \text{ m/s}^2 \), and \( \alpha_f = 2 \), is modified by adding the height of the drop structure to the work done by the jet flow which yields

\[
D_s^3 = 0.412 \eta^2 q^2 \left( \frac{1}{2} + \frac{D_p}{b_o} + \frac{D_h}{b_o} \right)
\]

where \( D_s \) is the height of the drop structure. Table 5 shows the number of iterations needed for convergence, the measured input data of: unit discharge, jet flow depth, and drop height, and the predicted scour depths using Equation (34) for three values of \( \eta \) of 1.0, 0.75, and 0.70. It should be noted that the jet flow in grade control structure is not horizontal as assumed in the cases considered in this study but follows the face angle of the structure. The data used here, from Run 1 to 36 the face angle was 45° while for Run 80 to 88 the face angle was \( \approx 18.0° \). The value of \( \eta = 0.7 \) provides the closest agreement to the laboratory data of Bormann and Julien (1991) for grade control structures. It is interesting to note that for a jet with an angle of 45°, its velocity horizontal component is \( U_o \cos 45° \). Therefore, the grade control sloping jet, the data could be converted to horizontal jet by multiplying the velocities (or \( q \)) with 0.7 which is the equivalent to using a value of \( \eta = 0.7 \).

Figure 9 shows the calculated scour depths vs. the measured ones where most data lie above the line of perfect agreement (only 4 points among 37 points are below the line). Figure 10 shows plot of the ratio of the calculated scour depth over the measured one. From Figure 10, it can be deduced that 25 points lie within 0.99–1.5. In other words, 25/37 or nearly 67% of the predicted scour depths have an error less than +50%. Despite the fact that the jet flow angle was inclined in the data of Bormann and Julien (1991) while the analysis here is for horizontal jets, however, very good agreement between the data and the predictions can be assumed.
7. General observations

(1) It should be noted that the mathematical modeling approach developed herein from the principles of the work transfer theory is some sort of a global or general incipient motion concept but in terms of work (or potential energies) rather than in terms of forces or moments. In the classical incipient motion, balance of forces (Dey & Sarkar, 2006; Hopfinger et al., 2004) or moments (Hogg et al., 1997) is applied to a typical sediment bed particle usually $D_{50}$. But, below the original river bed at depths 3.0 m and deeper, the $D_{50}$ value may change from the $D_{50}$ value in the river bed. In the work transfer theory, balance of work is used instead of balance of forces or moments. The whole material in the scour hole is considered as a one big porous particle (mega sediment particle or one unit having a triangular shape). When the jet flow exerts work that is equal to the work needed to move this mega sediment particle out of the scour hole, the equilibrium geometry of the scour hole is attained. This produces a scour equation that is void of the sediment sizes as the sediment unit weight (submerged weight) is more important. This explains theoretically the observation that some scour equations do not include sediment sizes. Simply, what the work transfer theory is stating is that at a certain flow hydrodynamic condition (velocity or depth) and certain sediment properties (sediment-specific gravity and porosity), such conditions produce certain work or energy which can erode the bed material (assuming unlimited bed material along its depth and no armoring) to an extent that the exerted work is exactly the work needed to lift or carry the sediment particles out of the scour hole. From the laws of mechanics, the work done by a group of forces is equal to the work done by the resultant of these forces. This last statement is utilized to deal with the scour hole as one mega particle having resultant weight force acting at the center of mass of the scour hole.

(2) The effect of bed load can be taken in a more direct manner by considering the loss of work or energy due to exerting work in moving the bed load. The work done in moving the bed load inside the scour hole can be given as:

$$ W_{bl} = \rho_s q_b U_b \left( \frac{D_s}{\sin \phi_s} \right) $$

where $W_{bl}$ is the work per unit width exerted by the fluid jet flow in moving the bed load along the sloping length of the scour hole, $q_b$ is the bed load discharge, and $U_b$ is the bed load velocity. This work is subtracted from the work done by the fluid flow which was given in Equation (15) as:

$$ W_{net} = \rho U_o^2 H_o^2 \cos \phi_s \left( \frac{1}{2} + \frac{D_s}{H_j} \right) - W_{bl} $$

Here, $W_{net}$ is the net work done by the attacking jet flow considering the energy lost in moving the bed load. This will result in less work available for removing the bed material out of the scour hole and the scour depth should decrease due to the bed load. It can be assumed that the bed load motion takes the form of dunes with scour hole profile of Figure 1 passing equally through the crests and troughs so that the same area (volume) as given by Equation (16) is still valid. A similar treatment can be done for loss of energy due to suspended load if it is present.

(3) Examining the field data in Tables 1 and 2 reveals the existence of the similarity principal in the cases of scour due to jet flows issuing from gate opening. For example, at the New Isna Barrage turbine gates No. 1 and 3, the same scour depth of 6.5 m for both gates was observed under the same conditions of maximum unit discharge, jet depth, and some bed sediment properties. At the sluiceway, nearly equal scour depths of 6.0–6.5 m appear at gates No. 4, 5, and 6 where hydrodynamic conditions are nearly similar. Even similarity exists between scour at the turbine and sluiceway gates. When the unit discharge was 27.77 m$^3$/s at the turbine gates, scour reached 6.5 m and when the unit discharge was 20.5 m$^3$/s, scour reached 6.0 m for the same jet flow depth of 3.0 m and nearly same sediment properties. A more evident
Table 5. Predicted scour depths for grade control structures data of Bormann and Julien (1991) using Equation (38)

| Run No. | No. of iterations | q (m³/s) | Hj (m) | Dp (m) | Ds (m) Equation 32 \[ \eta = 1.0 \] | Ds (m) Equation 32 \[ \eta = 0.75 \] | Ds (m) Equation 32 \[ \eta = 0.70 \] | Ds (m) measured |
|---------|-------------------|---------|--------|--------|-----------------|-----------------|-----------------|-----------------|
| Runs-1–2–3 | 7                 | 2.25    | 0.94   | 0.15   | 1.735           | 1.35            | 1.271           | 1.02–1.12       |
| Runs-4–5  | 7                 | 2.22    | 0.57   | 0.15   | 2.076           | 1.597           | 1.500           | 1.40–1.46       |
| Runs-6–7–8 | 6                 | 1.72    | 0.88   | 0.15   | 1.403           | 1.095           | 1.033           | 1.01–1.07       |
| Runs-9–10–11–12 | 7 | 1.71 | 0.48 | 0.15 | 1.752 | 1.349 | 1.268 | 1.07–1.28 |
| Run-13    | 6                 | 1.81    | 1.19   | 0.25   | 1.357           | 1.069           | 1.010           | 0.72            |
| Run-14    | 6                 | 1.78    | 0.75   | 0.25   | 1.561           | 1.217           | 1.168           | 0.98            |
| Run-15    | 7                 | 1.78    | 0.46   | 0.25   | 1.886           | 1.457           | 1.370           | 1.30            |
| Run-16    | 4                 | 1.16    | 2.4    | 0.25   | 0.805           | 0.648           | 0.616           | 0.55            |
| Run-17    | 7                 | 2.32    | 0.55   | 0.25   | 2.232           | 1.72            | 1.617           | 1.32            |
| Run-18    | 6                 | 1.93    | 1.17   | 0.05   | 1.383           | 1.082           | 1.021           | 0.66            |
| Run-19    | 7                 | 2.27    | 0.79   | 0.05   | 1.828           | 1.41            | 1.326           | 1.08            |
| Run-20    | 8                 | 2.32    | 0.54   | 0.05   | 2.170           | 1.66            | 1.557           | 1.21            |
| Run-21    | 7                 | 1.94    | 0.52   | 0.05   | 1.865           | 1.428           | 1.341           | 0.97            |
| Run-22    | 7                 | 1.99    | 0.53   | 0.05   | 1.895           | 1.451           | 1.362           | 1.26            |
| Run-23    | 6                 | 2.47    | 1.13   | 0.23   | 1.792           | 1.4             | 1.321           | 0.96            |
| Run-24    | 7                 | 2.32    | 0.58   | 0.23   | 2.176           | 1.678           | 1.578           | 1.06            |
| Run-25    | 7                 | 2.32    | 0.55   | 0.23   | 2.224           | 1.713           | 1.611           | 1.39            |
| Run-26    | 6                 | 1.42    | 1.04   | 0.23   | 1.149           | 0.906           | 0.857           | 0.70            |
| Run-27    | 7                 | 1.46    | 0.43   | 0.23   | 1.614           | 1.248           | 1.175           | 0.89            |
| Run-28    | 7                 | 1.46    | 0.4    | 0.23   | 1.662           | 1.284           | 1.208           | 1.10            |
| Run-29    | 5                 | 0.61    | 0.69   | 0.23   | 0.647           | 0.514           | 0.487           | 0.27            |
| Run-30    | 6                 | 0.58    | 0.34   | 0.23   | 0.784           | 0.615           | 0.581           | 0.29            |
| Run-31    | 6                 | 0.6     | 0.2    | 0.23   | 0.994           | 0.772           | 0.727           | 0.56            |
| Run-32    | 6                 | 0.59    | 0.19   | 0.23   | 1               | 0.776           | 0.731           | 0.62            |
| Run-33    | 5                 | 0.34    | 0.25   | 0.23   | 0.558           | 0.44            | 0.416           | 0.1             |
| Run-34    | 6                 | 0.34    | 0.14   | 0.23   | 0.697           | 0.545           | 0.514           | 0.15            |
| Run-35    | 6                 | 0.33    | 0.12   | 0.25   | 0.73            | 0.57            | 0.537           | 0.39            |
| Run-36    | 6                 | 0.33    | 0.12   | 0.23   | 0.723           | 0.564           | 0.532           | 0.93            |
| Run-80    | 6                 | 2.44    | 1.13   | 0.23   | 1.773           | 1.386           | 1.308           | 0.59            |
| Run-81    | 6                 | 1.47    | 1.1    | 0.23   | 1.163           | 0.918           | 0.868           | 0.30            |
| Run-82    | 7                 | 1.42    | 0.42   | 0.23   | 1.589           | 1.229           | 1.157           | 0.71            |
| Run-83    | 5                 | 0.59    | 0.62   | 0.23   | 0.65            | 0.516           | 0.488           | 0.15            |
| Run-84    | 6                 | 0.61    | 0.24   | 0.23   | 0.936           | 0.729           | 0.687           | 0.57            |
| Run-85    | 6                 | 0.55    | 0.19   | 0.23   | 0.94            | 0.73            | 0.688           | 1.52            |
| Run-86    | 6                 | 0.33    | 0.16   | 0.23   | 0.644           | 0.505           | 0.476           | 0.48            |
| Run-87    | 6                 | 0.32    | 0.13   | 0.23   | 0.682           | 0.533           | 0.502           | 0.56            |
| Run-88    | 6                 | 0.29    | 0.11   | 0.23   | 0.67            | 0.523           | 0.493           | 0.97            |
| Shimen-Dam-L* | 10  | 83.82  | 4.03   | 0    | 20.82           | 15.827          | 14.83           | 13.6            |
| Shimen-Dam-H* | 11  | 102.36 | 4.03   | 0    | 25.22           | 19.138          | 17.92           | 13.6            |

*Equation (37) is used for the Shiemen Arch Dam, China.
case is the similarity at three different locations at Rossetta, Damiatta, and Zifta Barrages. The unit discharge at these three barrages is around 10.0 m$^2$/s, the jet flow depth is about 2.0 m, $D_{90}$ is from 0.772 to 1.0 mm, and the maximum scour depth varies in a narrow range between 3.25 and 3.5 m while the scour hole's shape is the same with $\alpha \phi \approx 2$. In summary, similar hydrodynamic and sediment conditions at different barrage locations produce similar scour pattern (scour depth and scour hole's shape) which confirms the similarity principle.

(4) The theoretical findings in the present study were validated via field data; however, some other studies and laboratory flume data can give more support. For example, Hogg et al. (1997) report that $x_m/x_c = 0.28$ from their model in addition to the experimental results of Rajaratnam (1981) where $x_m$ is the location of the maximum depth of scour and $x_c$ is distance to the crest of the sediment hill. However, plot of the data between $x_m/x_c$ and a flow parameter indicates that all the data points are above the line $x_m/x_c = 0.28$. Now the ratio $x_m/x_c$ can be calculated in the present study as follows. From Figure 1, the distances $x_m$ and $x_c$ are assumed equal to $\frac{u}{\tan \psi_1}$ and $L_{1}$, respectively. Therefore,

$$\frac{x_m}{x_c} = \frac{\frac{u}{\tan \psi_1}}{\frac{D_s}{\tan \psi_1} + \frac{D_s}{\tan \psi_2}} = \frac{1}{\frac{1}{\tan \psi_1} + \frac{1}{\tan \psi_2}} = \frac{1}{1 + \alpha \phi}$$

(37)
It is assumed herein that $x_c$ is nearly equal to the length of the scour hole. For $\alpha_\varphi = 2$, Equation (37) yields $x_m/x_c = 0.33$. The line $x_m/x_c = 0.33$ fits better than the line $x_m/x_c = 0.28$ by Hogg et al. (1997) through the data of Rajaratnam (1981). This agreement confirms not only the similarity in shape of the scour hole between the laboratory data and the theoretical findings here but also the assumption that $\alpha_\varphi = 2$.

(5) The equation for the case of flow separation can also be tested against the barrage field data. Equation (27) for the case of flow separation can be expressed in terms of the unit discharge and jet flow depth in addition to assuming $\alpha_\varphi = 2$, $SG = 2.65$, $\theta = 0.4$, and $g = 9.81 \text{ m/s}^2$ which yields

$$D_s^3 = 0.412 \, \eta^2 \, q^2 \left( \frac{1}{2} + \frac{D_s}{D_o} \right)$$

(38)

Equation (38) is similar in form to Equation (22); however, the coefficient $\eta$ needs to be assumed properly. Table 6 shows the predicted scour depths downstream of the Nile Barrages using different values for $\eta$ such as 1.0, 0.75, and 0.70. It is clear from Table 6 that for $\eta = 1$, the predicted scour depths are relatively higher than with the other two $\eta$ values of 0.75 and 0.70. For these two values, the predicted scour depths are in excellent agreement with the field data.

(6) The case of the Shimen Arch Dam, China, is considered here using a version of Equation (27) (flow separation) in a form similar to Equation (31). In that case, using porosity value $\theta = 0$ in addition to assuming $\alpha_\varphi = 2$, $SG = 2.65$, and $g = 9.81 \text{ m/s}^2$ yields

$$D_s^3 = 0.2266 \, \eta^2 \, q^2 \left( \frac{1}{2} + \frac{D_s}{D_o} \right)$$

(39)

The last two entries in Table 5 show the predicted scour depths using Equation (39) which considers jet flow re-circulation or flow separation. The two values of scour depth prediction (14.83 and 17.92 m), which correspond to the two reported estimated velocity values of 20.8–25.4 m/s, respectively, are acceptable predictions to the measured scour depth of 13.6 m giving the complexity of the problem.

8. Conclusions and recommendations

The developed equations for predicting scour hole’s depth and length due to two-dimensional turbulent wall jets in the present study from the mathematical model that is based on the work transfer theory proved to be very reliable for wall jet scour under field conditions. It yields predictions of the scour hole’s depth and length very close to the observed values and realistic in their order of magnitude when applied to the cases of the Grand Egyptian Barrages contrary to existing scour formulas found in the literature. Close match between the predicted and measured scour depths occurs also for the case of the 300-year flood at the Shimen Arch Dam, China. The success of predicting scour hole’s depth occurred in both cases of non-circulating and re-circulating jet flow. The developed equation for predicating the length of the scour hole showed outstanding performance when compared to field measurements. The close matching of the predicted and measured scour depth and length supports the validity of the work transfer theory and its underlying assumptions such as considering the whole scour hole as mega sediment particle or one unit, the triangular shape of the scour hole section, and that the upstream slope is twice the downstream one.

In addition, scour in case of re-circulating currents in the scour hole can be expressed by the present mathematical model. Comparison of the developed equation against laboratory scour data for scour downstream of a spillway where the data are almost complete clearly demonstrates the effectiveness of the present method. The developed scour equation in this case predicted very well the scour depths measured downstream of grade control structures. The developed equation is based on incipient motion concept in terms of energies rather than in terms of forces or moments, avoids the inclusion of representative bed sediment sizes such as $D_{50}$, but rather includes the bed sediment...
submerged unit weight and porosity as the sediment’s most influencing factors in wall jet scour. In agreement with Melville and Lim (2013), the jet Froude number is found to be the flow hydrodynamic influencing variable while past scour formulas considered the densimetric Froude number. Inclusion of porosity in the scour depth equation enabled successful prediction of scour at a rocky non-porous or non-granular bed material at the Shimen Arch Dam in China.

Based on the above findings, further testing of the analytically developed equation to other cases of low-head hydraulic structures is recommended, especially the ratio \( \alpha \phi \) which is the ratio of the scour hole upstream to downstream slopes. This ratio can be related to the hydrodynamic forces affecting the scour hole formation and also some soil parameters that reflect soil resistivity to erosion. The generalized nature of the work transfer theory might enable developing expressions for scour at abutments (grains or dykes) and scour due to free falling jets at plunging pools, but these will be the subject of future publications.

### Table 6. Predicted scour depths considering flow separation downstream of the Nile Barrages, Egypt

| Case   | \( D_1 \) Equation (38) \( \eta = 1.0 \) (m) | \( D_1 \) Equation (38) \( \eta = 0.75 \) (m) | \( D_1 \) Equation (38) \( \eta = 0.70 \) (m) | \( D_1 \) measured (m) |
|--------|---------------------------------------------|---------------------------------------------|---------------------------------------------|------------------------|
| NEBTG  | 10.97                                       | 8.38                                        | 7.86                                        | 6.50-8.00              |
| NEBSG  | 8.26                                        | 6.34                                        | 5.95                                        | 6.00-6.50              |
| ONHB   | 14.09                                       | 10.73                                       | 10.05                                       | 10.00                  |
| ABV58  | 10.77                                       | 8.23                                        | 7.72                                        | 7.50                   |
| RBV23  | 5.05                                        | 3.88                                        | 3.65                                        | 3.50                   |
| DBV24  | 5.05                                        | 3.88                                        | 3.65                                        | 3.25                   |
| ZB     | 5.27                                        | 4.04                                        | 3.80                                        | 3.50                   |

### Acknowledgments

The author would like to present special thanks and deepest appreciations to the editors and reviewers of this Journal for their valuable comments, suggestions, discussions, and feedback. The author expresses also his deepest appreciation to the staff of the Hydraulic Research Institute (HRI), Egypt, for providing him with the valuable data-sets about the Egyptian Barrages while he was working there in the period 2003–2004. Thanks also go to Mr. Haider A. Chishti, Yanbu Industrial college library services for providing several valuable references. Many Thanks go to Eng. Chandrakant Shitole and Mr. Rojamal Gurumoothri from Royal Commission Yanbu Colleges and Institutes for their help in some graphical work.

### Funding

The author received no direct funding for this research.

### Author details

Youssef I. Hafez
E-mail: mohammedy@rcyci.edu.sa
ORCID ID: http://orcid.org/0000-0002-9503-8348

1 Royal Commission Yanbu Colleges and Institutes, Yanbu University college, Yanbu, Saudi Arabia.

### Citation information

Cite this article as: Scour due to turbulent wall jets downstream of low-/high-head hydraulic structures, Youssef I. Hafez, Cogent Engineering (2016), 3: 1200836.

### Supplementary material

Supplementary material for this article can be accessed here http://dx.doi.org/10.1080/23311916.2016.1200836.

### References

Abedelaziz, S., Bui, M. D., & Rutschmann, P. (2010). Numerical simulation of scour development due to submerged horizontal jet, River Flow. In A. Dittrich, K. Koll, J. Aberle, & P. Geisenhainer. ISBN 978-3-93923-000-7.

Aderibigbe, F., & Rajaratnam, N. (1998). Generalized study of erosion by circular horizontal turbulent jets. Journal of Hydraulic Research, 36, 613–635.

Balachandar, R., & Reddy, P. (Eds.). (2013). Scour caused by wall jets, sediment transport (pp. 177–210). ISBN 980-953-307-557-5.

Bombardelli, F. A., & Gioia, G. (2005). Towards a theoretical model for scour phenomena. In G. Parker & M. Garcia (Eds.), Proceedings of the 4th IAHR Symposium on River, Coastal and Estuarine Morphodynamics, RCEM 2005 (Vol 2, pp. 931–936). Urbana, IL: Taylor & Francis.

Bormann, N. E., & Julien, P. Y. (1991). Scour downstream of grade-control structures. Journal of Hydraulic Engineering, 117, 579–594.

Breusers, H. N. C., & Raudkivi, A. J. (1991). Scouring. Rotterdam: Balkema.

Chiew, Y. M., & Lim, S. Y. (1996). Local scour by a deeply submerged horizontal circular jet. Journal of Hydraulic Engineering, 122, 529–532.

Dargahi, B. (2003). Scour development downstream of a spillway. Journal of Hydraulic Research, 41, 417–426.
Appendix A

There are several ways of proving Equation (15) as follows.

First method

As mentioned before, it is assumed that steady flow conditions exist ($U_j$ and $H_f$ are constants). In addition, it is assumed that the shape of the scour hole does not change with time, i.e. the upstream and downstream slopes of the scour hole are constant ($\phi_1$ and $\phi_2$ are constants). The constancy with respect to time of the scour hole can be seen in the experiments of Rajaratnam (1981) and also the theoretically calculated profiles by Hogg et al. (1997). This leaves the scour depth is the only variable changing with time. Writing Equations (15) and (17) at any given time $t$ yields:

$$W_{in}(t) = \frac{\rho U_j^2 H_f^2 \cos \phi_1}{\sin \phi_1} \left( \frac{1}{2} + \frac{D_s(t)}{H_f} \right)$$  \hspace{1cm} (A1)$$

$$W_{out}(t) = \rho g (S_g - 1) (1 - \theta) \left( \frac{1}{\tan \phi_1} + \frac{1}{\tan \phi_2} \right)$$  \hspace{1cm} (A2)$$

where $W_{in}$ is the work done by the jet flow and $W_{out}$ is the work required to lift the scoured material out of the scour hole, where all the variables are constant with respect to the time, $t$, except the scour depth $D_s(t)$. Now after a very long time at which equilibrium will be established, i.e. at an infinite time, taking the limits of each of Equations (A1) and (A2) as $t \to \infty$ and observing that in this case $D_s(t) \to D_s$ yields Equations (15) and (17) and the rest of the procedure follows.
Second method
Considering the time development of the scour hole, it is assumed that at each time interval (for which significant scour occurs), the scour hole has incremental scour depths of $\Delta D_{s1}$, $\Delta D_{s2}$, $\Delta D_{s3}$, ..., $\Delta D_{sn}$ where $n$ is the number of time increments in order to reach scour equilibrium. Again, it is assumed that the shape of the scour hole remains constant during the scouring process. The work done by the jet flow to reach scour hole equilibrium, $W_{in}$, in this case is expressed as:

$$W_{in} = \frac{\rho U_{1}^{2} H_{j}^{2} \cos \phi_{1}}{\sin \phi_{1}} \left( \frac{1}{2} + \sum_{n} \left( \frac{\Delta D_{s1}}{H_{j}} + \frac{\Delta D_{s2}}{H_{j}} + \frac{\Delta D_{s3}}{H_{j}} + \ldots + \frac{\Delta D_{sn}}{H_{j}} \right) \right)$$  \hspace{1cm} (A3)

Now, as $n \rightarrow \infty$ (or very large), the summation in Equation (A3) tends to $D_{S}/H$ and Equation (A3) becomes identical to Equation (15) and the rest of the procedure follows. The expression inside the summation sign in Equation (A3) is similar to integrating the work term over the duration of scour but in a discrete manner.

Third method
As mentioned before, Kotoulas (1967) found that in case of coarse sand, about 64% of the final scour occurred in the first 20 s, and about 97% of scour depth was attained in 2 h. This leads to assuming in the hypothetical model herein that scour can be considered to occur instantaneously or in a very short time for which Equations (15) and (17) can be established and the rest of the procedure follows. In other words, the analysis is done instantaneously in a very short time in which both Equations (15) and (17) can be written. This leads to considering the whole scour hole as a mega sediment porous particle, i.e. one unit or one big sediment porous particle where the jet flow water flow through it.