The general principles needed to compute the effect of a stationary gravitational field on the quasistationary electromagnetic phenomena in normal conductors and superconductors are formulated from a general relativistic point of view. Generalization of the skin effect, that is the general relativistic modification of the penetration depth (of the time-dependent magnetic field in the conductor) due to its relativistic coupling to the gravitational field is obtained. The effect of the gravitational field on the penetration and coherence depths in superconductors is also studied. As an illustration of the foregoing general results, we discuss their application to superconducting systems in the outer core of neutron stars. The relevance of these effects to electrodynamics of magnetized neutron stars has been shown.

Keywords: Relativity stars; electromagnetic fields; skin effect; penetration depth; coherence length.

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1. Introduction

Since electromagnetic system is a purely relativistic system, its coupling to gravity can be described, in principle, only general-relativistically. In our previous research, the effect of the gravitational field on stationary electromagnetic processes in conductors and superconductors in the presence of constant electric current has been investigated. In fact, the influence of angular momentum of the gravitational source may appear as a galvanogravitomagnetic effect in current carrying conductors,\(^1\) as general-relativistic effect of charge distribution inside conductors in an applied magnetic field,\(^2\) as general-relativistic effects in charged systems.\(^3\) The effect of gravitational field on thermoelectric effects in superconductors and on electromagnetic properties of type II superconductors have been investigated in our papers.\(^4,5\) Here we extend our results to the time-dependent electromagnetic processes.

The paper is organized as follows. First we write Maxwell equations in the Schwarzschild space-time in Section 2. Then in the next part of the paper, we will study the effect of gravity on the quasistationary time-varying electric current and magnetic field penetrating the conductor placed in the gravitational field. Sections 3 is devoted to investigation of the skin effect in the Schwarzschild space-time and in a weak gravitational field when the curvature effects are locally negligible. In particular, new purely general-relativistic effects connected with the gravitational modification of penetration depth of magnetic field will be obtained.

The remarkable quantum phenomena exhibited by a superconductor show that the superconducting state in Ginzburg-Landau theory\(^7\) described by a wavefunction can be spread over the entire superconductor. The existence of macroscopic wavefunction raises the natural question of how it is influenced by the gravitational
field. In the last part of the paper, that is, in Sections 4-7 we shall formulate the general formulae and their solutions needed to determine the influence of stationary gravitational field on the quasistationary electromagnetic phenomena for superconductors, from general-relativistic point of view and study the relevance of general-relativistic effects in superconductors of II type on electrodynamics of magnetized neutron stars. This analysis has been motivated by the mixture commonly found in neutron star models, namely in the outer core region, where superfluid neutrons, superconducting protons and normal electrons are generally thought to coexist (see for example, Ref. 8). However, due to the generality of the present approach, it is equally well applicable to superconducting systems found in more common laboratory contexts in the weak gravitational field of Earth.

In this paper we use a space-like signature \((-, +, +, +)\) and a system of units in which \(G = 1 = c\) (however, for those expressions with a physical application we have written the speed of light explicitly). Greek indices are taken to run from 0 to 3 and Latin indices from 1 to 3; covariant derivatives are denoted with a semi-colon and partial derivatives with a comma.

2. Maxwell Equations in Schwarzschild Space-time

In a coordinate system \((t, r, \theta, \phi)\), the metric for a static spherically-symmetric relativistic star with the total mass \(M\) is

\[
\begin{align*}
\text{ds}^2 &= -N^2 dt^2 + N^{-2} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \\
N^2 &\equiv 1 - 2M/r
\end{align*}
\]

where \(N^2 \equiv 1 - 2M/r\) is the lapse function.

The general form of the first pair of general relativistic Maxwell equations is given by

\[
F_{\alpha\beta,\gamma} + F_{\gamma\alpha,\beta} + F_{\beta\gamma,\alpha} = 0,
\]

where \(F_{\alpha\beta}\) is the electromagnetic field tensor expressing the strict connection between the electric \(E^\alpha\) and magnetic \(B^\alpha\) fields. For an observer with four-velocity \((u^\alpha)_{\text{obs}}\), the covariant components of the electromagnetic tensor are given by

\[
F_{\alpha\beta} \equiv 2(u^\alpha)_{\text{obs}}[A^\beta] + \eta_{\alpha\beta\gamma\delta}(u^\gamma)_{\text{obs}} B^\delta,
\]

where \(A_{[\alpha\beta]} \equiv \frac{1}{2}(A_{\alpha\beta} - A_{\beta\alpha})\) and \(\eta_{\alpha\beta\gamma\delta}\) is the pseudo-tensorial expression for the Levi-Civita symbol \(\epsilon_{\alpha\beta\gamma\delta}\)

\[
\eta_{\alpha\beta\gamma\delta} = -\frac{1}{\sqrt{-g}}\epsilon_{\alpha\beta\gamma\delta}, \quad \eta_{\alpha\beta\gamma\delta} = \sqrt{-g}\epsilon_{\alpha\beta\gamma\delta},
\]

with \(g \equiv \det|g_{\alpha\beta}| = -r^4 \sin^2 \theta\) for the metric (1).

The proper observers have four-velocity components given by

\[
(u^\alpha)_{\text{prop}} \equiv N^{-1} \left(1, 0, 0, 0\right); \quad (u^\alpha)_{\text{prop}} \equiv N \left(-1, 0, 0, 0\right).
\]

In the metric (1) and with the definition (3) referred to the observers (5), the first pair of Maxwell equations (2) take then the form

\[
\begin{align*}
\sin \theta \left(r^2 B^\theta\right)_r &+ \frac{r}{N} \left(\sin \theta B^\theta\right)_\theta + \frac{r}{N} B^\phi = 0, \\
\left(\frac{r}{N} \sin \theta\right) \frac{\partial B^\phi}{\partial \theta} &= E^\phi_{,\phi} - \left(\sin \theta E^\phi\right)_\theta.
\end{align*}
\]

3. Conclusion

This analysis has been motivated by the mixture commonly found in neutron star models, namely in the outer core region, where superfluid neutrons, superconducting protons and normal electrons are generally thought to coexist (see for example, Ref. 8). However, due to the generality of the present approach, it is equally well applicable to superconducting systems found in more common laboratory contexts in the weak gravitational field of Earth.

In this paper we use a space-like signature \((-, +, +, +)\) and a system of units in which \(G = 1 = c\) (however, for those expressions with a physical application we have written the speed of light explicitly). Greek indices are taken to run from 0 to 3 and Latin indices from 1 to 3; covariant derivatives are denoted with a semi-colon and partial derivatives with a comma.
\[
\left( \frac{r}{N} \sin \theta \right) \frac{\partial B^\theta}{\partial t} = \sin \theta \left( rN E^\phi \right)_{,r} - E^\phi_{,r},
\]
\[
\left( \frac{r}{N} \right) \frac{\partial \hat{E}^\phi}{\partial t} = E^r_{,\theta} - \left( rN E^\phi \right)_{,r},
\]
where "hatted" quantities are projected onto a locally orthonormal tetrad \( \{ e^\mu \} = (e^0, e^r, e^\theta, e^\phi) \) carried by a proper observer

\[
e_0^r = \frac{1}{N} (1, 0, 0, 0), \quad e_r^\theta = N (0, 1, 0, 0), \quad e_\theta^\phi = \frac{1}{r} (0, 0, 1, 0), \quad e_\phi^\alpha = \frac{1}{r \sin \theta} (0, 0, 0, 1).
\]

The general form of the second pair of Maxwell equations is given by

\[
F^{\alpha\beta} ;_{\beta} = 4\pi J^\alpha,
\]
where the four-current \( J^\alpha \) is a sum of convection and conduction \( j^\alpha \) currents

\[
J^\alpha = \rho_e u^\alpha + j^\alpha, \quad j^\alpha u_\alpha \equiv 0,
\]
with \( u^\alpha \) being the conductor four-velocity as whole and \( \rho_e \) the proper charge density.

We can now rewrite the second pair of Maxwell equations as\(^{11}\)

\[
\sin \theta \left( r^2 E^r \right)_{,r} + \frac{r}{N} \left( \sin \theta E^\theta \right)_{,\theta} + \frac{r}{N} E^\phi_{,\phi} = \frac{4\pi r^2 \sin \theta}{N} J^t,
\]
\[
\left( \frac{r}{N} \right) \frac{\partial \hat{E}^\phi}{\partial t} = E^r_{,\theta} - \left( rN E^\phi \right)_{,r},
\]
\[
B^r_{,\theta} - \sin \theta \left( rN B^\phi \right)_{,r} = \left( \frac{r \sin \theta}{N} \right) \frac{\partial E^\phi}{\partial t} + 4\pi r \sin \theta J^\phi,
\]
\[
\left( rN B^\phi \right)_{,r} - B^r_{,\phi} = \left( \frac{r}{N} \right) \frac{\partial E^\theta}{\partial t} + 4\pi r J^\phi.
\]

3. Skin Effect in Schwarzschild Gravitational Field

If the conduction current \( j^\alpha \) is carried by the electrons with electrical conductivity \( \sigma \), Ohm’s law can then be written as

\[
j^\alpha = \sigma F_{\alpha\beta} u^\beta.
\]

Equations (14)–(16) can now be rewritten in a more convenient form. Taking four-velocity components of the conductor as (5), we can use Ohm’s law (17) to derive the following explicit components of current \( J^\alpha \) in the proper frame

\[
J^t = \rho_e, \quad J^i = \sigma E^i.
\]

Hereafter we will use the following realistic assumptions. Firstly, we consider \( \sigma \) to be uniform within the conducting media. Secondly, we ignore the contributions coming from the displacement currents. The latter could, in principle, be relevant in the evolution of the electromagnetic fields, but their effects are negligible on timescales that are long as compared with the electromagnetic waves crossing time. In view of this, we
will neglect in (14)–(16) all terms involving time derivatives of the electric field (quasistationarity condition) and use Ohm’s law (18) to rewrite Maxwell equations (14) and (15) as

\[
E^\phi = \frac{c}{4\pi\sigma r} \sin \theta \left[ (\sin \theta B^\phi),_r - B^\phi,_{\phi} \right],
\]

(19)

\[
E^\theta = \frac{c}{4\pi\sigma r} \sin \theta \left[ B^\phi,_{\phi} - \sin \theta \left( rNB^\phi,_{r} \right),_r \right],
\]

(20)

\[
E^r = \frac{c}{4\pi\sigma r} \left[ \left( rNB^\phi,_{\phi} \right),_r - B^\phi,_{,r} \right].
\]

(21)

Using Maxwell equation (13) and Ohm’s law (18), we find that the space charge density \(\rho_e = \rho_e(t, r, \theta, \phi)\) inside the star has a zeroth-order contribution given by

\[
\rho_e = \frac{cN}{4\pi r^2 \sin \theta} \left[ \sin \theta \left( r^2 E^r,_{r} \right),_r + \frac{r}{N} \left( \sin \theta E^\phi,_{\theta} \right),_\phi + \frac{r}{N} E^\phi,_{,r} \right].
\]

(22)

Now with the help of Eqs. (7)–(9), (19)–(21) and Ohm’s laws (18), we obtain the induction equations for the components of time varying magnetic field penetrating the conductor

\[
\frac{\partial B^\phi}{\partial t} = \frac{c^2 N}{4\pi\sigma r^2 \sin \theta} \left\{ \frac{1}{\sin \theta} \left[ B^\phi,_{\phi} - \sin \theta \left( rNB^\phi,_{r} \right),_r \right],_\phi - \left( \sin \theta \left( rNB^\phi,_{r} \right),_r - B^\phi,_{,r} \right),_\theta \right\},
\]

(23)

\[
\frac{\partial B^\theta}{\partial t} = \frac{c^2 N}{4\pi\sigma r \sin \theta} \left\{ \sin \theta \left( N \left[ \left( rNB^\phi,_{r} \right),_r - B^\phi,_{,r} \right] \right),_r - \frac{1}{r \sin \theta} \left[ \left( \sin \theta B^\phi,_{\theta} \right),_\phi - B^\phi,_{,\phi} \right],_\phi \right\},
\]

(24)

\[
\frac{\partial B^r}{\partial t} = \frac{c^2 N}{4\pi\sigma r} \left\{ \frac{1}{r \sin \theta} \left[ \left( \sin \theta B^\phi,_{\theta} \right),_\phi - B^\phi,_{,\phi} \right],_\theta - \left( \frac{N}{\sin \theta} \left[ B^\phi,_{,\phi} - \sin \theta \left( rNB^\phi,_{r} \right),_r \right],_r \right\}.
\]

(25)

Assume a conducting sphere with the conductivity \(\sigma\) occupies the space \(r \leq R\) with empty vacuum space in the region \(r > R\). The surface at \(r = R\) is subjected to dipolar magnetic field varying harmonically with time. For example, the azimuthal component of magnetic field \(B^\phi\) is

\[
B^\phi = B_0^\phi e^{-i\omega t}.
\]

(26)

In order to understand the behavior of the magnetic field inside the spherical conductor we look for a solution of the induction equation (24) in the inner region \(r \leq R\)

\[
-i\omega B^\phi = \frac{c^2 N}{4\pi\sigma r} \left( N \left[ rNB^\phi,_{r} \right],_r \right),
\]

(27)

which can be written as the following differential expression

\[
\frac{\partial^2 B^\phi}{\partial r^2} + \left( \frac{3M}{r^2 N^2} + \frac{2}{r} \right) \frac{\partial B^\phi}{\partial r} + k^2 B^\phi + \frac{M}{r^3 N^2} B^\phi = 0,
\]

(28)

where the parameter \(k^2 = (4\pi\sigma\omega)/(c^2 N^3)\) is introduced.

Analytical solution of this equation can not be found easily, and as we do not need any numerical solution for our purpose we must choose some approximation that do not destroy the structure of possible solution.
Due to this purpose we will see that in linear $M/r$ approximation this equation could be simplified and written as Bessel equation

$$\frac{\partial^2 B^\theta}{\partial r^2} + \frac{2}{r} \frac{\partial B^\theta}{\partial r} + k^2 B^\theta = 0.$$  \hspace{1cm} (29)

The boundary condition for this equation is $B^\theta_0 = B^\theta(R)$. And the solution of this equation is the spherical Bessel function of the first type

$$B^\theta = B^\theta_0 j_0(kr),$$  \hspace{1cm} (30)

where

$$k = \frac{1 + i}{\delta}, \quad \delta = \frac{cN^{3/2}}{\sqrt{2\pi} \sigma \omega}.$$  \hspace{1cm} (31)

We will expand the expression for the magnetic field (30) at the surface of the sphere for the large arguments of $kr \gg 1$. If we take $x = R - r$ as a distance from the surface inwards then the interior magnetic field behaves as

$$B^\theta = B^\theta_0 e^{ikx} = B^\theta_0 e^{ix} + B^\theta_0 e^{-ix},$$  \hspace{1cm} (32)

and the distance $x = \delta$ is called the skin depth of magnetic field penetration (see Fig. 1).

![Fig. 1. The dependence of skin depth in the proton stellar superconductor on the compactness parameter $\frac{2M}{r}$ in the case when conductivity $\sigma \approx 10^{25} s^{-1}$ and the frequency $\omega \approx 10^{3} s^{-1}$.](image)

From Eq. (21) one can find the expression

$$E^\phi = \frac{ckN}{4\pi \sigma} \frac{j_1(kr)}{j_0(kR)} B^\theta_0,$$  \hspace{1cm} (33)

for azimuthal component of the electric field.

Thus the electric field falls off exponentially in $r$, with a spatial oscillation of the same scale, being confined mainly to a depth less than the relativistically modified skin depth.
3.1. Skin effect in weak Earth’s gravitational field

In the weak gravitational field of the Earth the curvature effects are negligible and space-time metric in the Cartesian coordinates is

\[ ds^2 = -c^2 N^2 dt^2 + dx^2 + dy^2 + N^{-2} dz^2 . \]  

(34)

Here \( N \) is equal to

\[ N = 1 + \frac{2g z}{c^2} , \]  

(35)

and \( z \) is the height above some fixed point, when the apparatus at the Earth may be regarded as having an acceleration \( g \) relative to local inertial frame.

The evolution equations for the magnetic field in the metric (34) take the form

\[ \frac{\partial B^\hat{x}}{\partial t} = \frac{c^2}{4 \pi \sigma N} \left\{ N \left[ \left( NB^\hat{y} \right)_z - B^\hat{z}_x \right] \right\} \right\}, \]  

(36)

\[ \frac{\partial B^\hat{y}}{\partial t} = \frac{c^2}{4 \pi \sigma N} \left\{ \left[ B^\hat{y}_x - B^\hat{z}_y \right] - N \left[ B^\hat{y}_z - \left( NB^\hat{y} \right)_z \right] \right\} \right\} \right\}, \]  

(37)

\[ \frac{\partial B^\hat{z}}{\partial t} = \frac{c^2}{4 \pi \sigma N} \left\{ \left[ B^\hat{z}_y - \left( NB^\hat{y} \right)_z \right] - \left[ \left( NB^\hat{x} \right)_z - B^\hat{z}_x \right] \right\} \right\} \right\}. \]  

(38)

Assume a semi-infinite conductor of uniform conductivity \( \sigma \) occupying the space \( z > 0 \). The surface at \( z = 0 \) is subjected to tangential magnetic field \( B^\hat{x} = B^\hat{x}_0 e^{-iz} \) governed by the differential equation

\[ \frac{\partial^2 B^\hat{x}}{\partial z^2} + \frac{6g}{c^4 N} \frac{\partial B^\hat{x}}{\partial z} + \frac{4g^2}{c^4 N^2} B^\hat{x} + k^2 B^\hat{x} = 0 , \]  

(39)

which has a solution

\[ B^\hat{x} = C_1 e^{(-3c^2 g N - \sqrt{5c^4 g^2 N^2 - c^2 k^2 N^2}) z / c^4 N^2} + C_2 e^{(-3c^2 g N + \sqrt{5c^4 g^2 N^2 - c^2 k^2 N^2}) z / c^4 N^2} , \]  

(40)

where \( C_1 \) and \( C_2 \) are the integration constants. Neglecting small terms and using (32) for \( k \) one can get

\[ B^\hat{x} = B^\hat{x}_0 e^{iz/\delta} e^{-z/\delta} . \]  

(41)

4. Effect of Gravity on Coherence Length and Penetration Depth

The coherence length is the measure of the distance within which the properties of the superconductors are not changed appreciably in the presence of a magnetic field. Because we are concerned with the effect of gravity on the coherence length in a superconductor it would be useful first to discuss Ginzburg-Landau equation in a gravitational field\(^7\)

\[ \alpha \psi + \beta |\psi|^2 + \frac{1}{4m} (i \nabla + 2eA)^2 \psi = 0 , \]  

(42)

which could be written as

\[ \alpha \psi + \beta |\psi|^2 + \frac{1}{4m} g^{\mu\nu} (-\nabla_\mu \nabla_\nu + 2ei \nabla_\mu A_\nu + 2ei A_\mu \nabla_\nu + 4e^2 A_\mu A_\nu) \psi = 0 . \]  

(43)
Here $\psi$ is the complex order parameter describing the superconductor, $\alpha$ and $\beta$ are the phenomenological expansion coefficients.

We now rewrite Ginzburg-Landau equation (42) in the Schwarzschild metric (1) in a more useful form

$$\alpha \psi + \beta |\psi|^2 + \frac{1}{4m} \left\{ - \left[ - \frac{1}{N^2} \frac{\partial^2}{\partial r^2} + \frac{1}{r^2 \sin \theta} \left( \sin \theta \frac{\partial}{\partial r} \left[ r^2 N^2 \frac{\partial}{\partial r} \right] + \frac{\partial}{\partial \theta} \left[ \sin \theta \frac{\partial}{\partial \theta} \right] + \frac{1}{\sin \theta} \frac{\partial^2}{\partial \phi^2} \right] \right\} \psi = 0.$$ (44)

Assuming that the order parameter $\psi$ has no variation in coordinates $\theta$ and $\phi$ and all components of vector-potential of electromagnetic field vanish then Eq. (44) reduces to

$$\alpha \psi + \beta |\psi|^2 - \frac{1}{4m} N^2 \left( \frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r} \left[ 1 + \frac{M}{r N^2} \right] \frac{\partial \psi}{\partial r} \right) = 0 ,$$ (45)

which in approximation $M/r \ll 1$ takes the simple form

$$\alpha \psi + \beta |\psi|^2 - \frac{1}{4m} N^2 \left( \frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r} \frac{\partial \psi}{\partial r} \right) = 0 .$$ (46)

Now we introduce the dimensionless wavefunction $\varphi(r)$

$$\varphi(r) = \frac{\psi(r)}{\psi_0} ,$$ (47)

where

$$\psi_0^2(r) = \frac{n_s}{2} = \frac{|\alpha|}{\beta} ,$$ (48)

$$-\varphi + |\varphi|^2 - \xi^2 \left( \frac{\partial^2 \varphi}{\partial r^2} + \frac{2}{r} \frac{\partial \varphi}{\partial r} \right) = 0 .$$ (49)

Here the parameter $\xi^2 = \xi_0^2 N^2 = 4N^2 / m_0$, $\xi_0$ is the Ginzburg-Landau coherence length in the flat Minkowski space-time, $n_{s(\psi)}$ is the density of superconducting electrons. The coherence length is the length scale on which the condensate charge density $2e|\psi|^2$ vanishes, as one approaches the surface of the superconductor from its interior.

Now we distinguish the meaning of $\xi$. The coordinate axis $r$ is perpendicular to the surface of the superconductor $r = R$. Assume that the surface layer is slim and the value of the function $\varphi$ at the surface has small deflection from 1, that is

$$\varphi = 1 - \epsilon(r) ,$$ (50)
and substituting the expression for $\varphi$ into (49) we obtain the Bessel equation
\begin{equation}
\frac{\partial^2 \epsilon}{\partial r^2} + \frac{2}{r} \frac{\partial \epsilon}{\partial r} - 2\xi^{-2} \epsilon = 0
\end{equation}
in the linear approximation in $\epsilon$.

Then taking into account the property $\lim_{r \to 0} \varphi = 1$ one can get $\epsilon(0) = 0$ and find the solution of the Eq. (51) as modified Bessel function of the first type
\begin{equation}
\epsilon = \epsilon(0) \frac{j_0(i\sqrt{2/\xi})}{j_0(i\sqrt{2/R\xi})}.
\end{equation}

It is known that the spherical Bessel function $j_0(r)$ of zeroth order for large arguments $r \gg 1$ is approximated as
\begin{equation}
j_0(r) = \frac{\sin r}{r},
\end{equation}
and due to this approximation near the surface of the sphere expression (52) takes the form
\begin{equation}
\epsilon = \epsilon(0)e^{-\sqrt{2\xi}},
\end{equation}
where $x = R - r$ is the inward distance from the surface of the sphere. It follows that $\xi$ is the length scale of the parameter of order $\varphi$ (See Fig. 2).

![Figure 2](image-url) Exponential growth of the order parameter with the distance $x = R - r$ from the surface of the gravitational object toward its interior; normal line is responsible for Newtonian flat space-time and dashed line is for neutron star with the compactness parameter $2M/r = 0.5$.

In the relativistically generalized London theory, the equation for the superconducting current $j^\alpha_s$ is
\begin{equation}
2j_{(s)[\alpha;\beta]} - 2j_{(s)[\alpha,\beta]}(\ln n_{(s)\beta})_\alpha + c\rho_{(s)}(A_{\alpha\beta} + 2u_{[\alpha}w_{\beta]}) = \frac{c}{4\pi}\lambda^2 L^{-2}F_{\alpha\beta},
\end{equation}
where the notation $\lambda^2_L = (mc^2)/(4\pi n_{(s)}e^2)$ for the London penetration depth is introduced and
\begin{equation}
A_{\alpha\beta} = u_{[\alpha,\beta]} + u_{[\beta}w_{\alpha]}
\end{equation}
is the relativistic rate of rotation, $w_{\alpha} = u_{\alpha,\beta} u^\beta$ is the absolute acceleration of superconductor.

Using the London equation (55) we obtain the expressions for the electric field (we choose $c=1$ for this case too)

$$E^i = 4\pi \lambda^2 [N^{-1} j^i_t - (N j^i),_r - N^{-1} j^i \ln(n),_r + \rho N,_r],$$

$$E^\theta = 4\pi \lambda^2 [N^{-1} j^\theta_t - r^{-1} j^\theta,\theta - N^{-1} j^\theta \ln(n),_r + r^{-1} j^\theta \ln(n),_\phi],$$

$$E^\phi = 4\pi \lambda^2 [N^{-1} j^\phi_t - r^{-1} \sin^{-1} \theta j^\phi,\phi - N^{-1} j^\phi \ln(n),_r + r^{-1} \sin^{-1} \theta j^\phi \ln(n),_\phi].$$

Here the derivatives from the density $n_{\alpha\phi}$ of superconducting pairs are taken to be equal to zero, because the dependence of $n_{\alpha\phi}$ on the coordinates is comparably small. Putting $j^i$ and $j^\phi$ extracted from the second pair of the Maxwell equations (14)–(16) into expressions (57)–(59) in the framework of this approximation and inserting the obtained result into the first pair of Maxwell equations (6)–(9) one obtains the expressions for magnetic field which are the second London equations

$$B^\hat{r} = \frac{\lambda^2}{r \sin \theta} \left\{ \frac{1}{r \sin \theta} \left[ \frac{B^\hat{r}}{r \sin \theta} - \sin \theta \left[ r N B^\hat{\rho} \right],_r \right],_r - \left( \frac{\sin \theta}{r} \left[ r N B^\hat{\rho} \right],_r - B^\hat{\rho},_r \right) \right\},$$

$$B^\hat{\theta} = \frac{\lambda^2}{r \sin \theta} \left\{ \sin \theta \left[ r N B^\hat{\theta} \right],_r - B^\hat{\theta},_r \right\},$$

$$B^\hat{\phi} = \frac{\lambda^2}{r} \left\{ \left( \frac{1}{N r \sin \theta} \left[ \sin \theta B^\hat{\phi} \right],_r - B^\hat{\phi},_r \right) \right\} - \frac{1}{\sin \theta} \left( B^\hat{\phi},_\phi - \sin \theta \left[ r N B^\hat{\phi} \right],_r \right) \right\}.$$

For simplicity of calculations the London equation (61) for $B^\hat{\phi}$ component of the magnetic field

$$B^\hat{\phi} = \frac{\lambda^2}{r} \left( r N B^\hat{\phi} \right),_r = \lambda^2 N^2 \left( -\frac{M^2}{r^4 N^2} B^\hat{\phi} + \frac{2}{r} B^\hat{\phi},_r + \frac{2 M}{r^2 N^2} B^\hat{\phi},_r + B^\hat{\phi},_r \right),$$

can be approximately written as Bessel like equation

$$B^\hat{\phi},_r + \frac{2}{r} B^\hat{\phi},_r - \lambda^2 B^\hat{\phi} = 0,$$

in the linear approximation in the small compactness parameter $M/r$, where parameter $\lambda = \lambda L N$.

The solution of the Eq. (64) is already known and expressed through the Bessel function

$$B^\hat{\phi} = B^\hat{0} \bar{j}_0 \left( \frac{r}{\lambda L N} \right).$$

Using the above mentioned arguments one may get the following expression at the surface of the sphere (see Fig. 3)

$$B^\hat{\phi} = B^\hat{0} e^{-\frac{\phi}{\alpha}}.$$

Our main finding is the dependence of the coherence length and penetration depth on the gravitational field. It is connected with the general relativistic modification of surface electromagnetic energy of the conductors and superconductors due to the effect of gravitational field.
Fig. 3. The exponential decay of magnetic strength inside a superconductor of type II, normal line is for flat space-time, dash line is for a neutron star with the compactness parameter $\frac{2M}{r} = 0.5$. Here $x = R - r$ is the distance from the surface of the star toward its interior.

5. Magnetic Field of Single Vortex

The four-vector of supercurrent density is

$$j_s(\alpha) = -\frac{i e}{m_s} \left\{ \psi^*(\partial_\alpha - 2eA_\alpha)\psi - \psi(\partial_\alpha + 2eA_\alpha)\psi^* \right\}, \quad (67)$$

which could be written through dimensionless function $\varphi = \psi/\psi_0$ as

$$j_s(\alpha) = -i \frac{\Phi_0}{(4\pi\lambda)^2} (\varphi^*\partial_\alpha \varphi - \varphi\partial_\alpha \varphi^*) - \frac{|\varphi|^2}{4\pi\lambda^2} A_\alpha, \quad (68)$$

where

$$\Phi_0 = \frac{hc}{2e} \quad (69)$$

is the quantum of magnetic flux and

$$\lambda^2 = \frac{m_\beta e^2}{8\pi\alpha |\alpha|}. \quad (70)$$

Assume that the wavefunction is $\varphi = |\varphi|e^{i\theta}$. Then superconducting current density (68) takes the form

$$j_s(\alpha) = \frac{|\varphi|^2}{4\pi\lambda^2} \left( \frac{\Phi_0}{2\pi} \partial_\alpha \theta - A_\alpha \right) \approx \frac{1}{4\pi\lambda^2} \left( \frac{\Phi_0}{2\pi} \partial_\alpha \theta - A_\alpha \right). \quad (71)$$

If the parameter of the Ginzburg-Landau theory $\kappa = \lambda/\xi$ is much bigger than 1 then at large distance $r \gg \xi$ we will have $|\varphi|^2 \approx 1$.

In cylindrical coordinates $(t, r, \phi, z)$ the space-time metric around a gravitating source takes the form

$$ds^2 = -N^2 dt^2 + N^{-2} (dr^2 + r^2 d\phi^2 + dz^2). \quad (72)$$
Using this metric we take curl from both sides of Eq. (71) and obtain the differential equations for the components of magnetic field $B^i$:

$$
\frac{1}{r} \left[ (r B^\phi)_r - B^\phi_r \right] = r \left[ B^z_{,zz} - B^z_{,rr} \right] = \frac{r}{\lambda^2 N^2} \left( \frac{\Phi_0}{2\pi} (\nabla \times \nabla \vartheta)^{\phi} - B^\phi \right),
$$

(73)

$$
\frac{1}{r} \left[ B^z_{,\phi} - r B^\phi_{,z} \right] - \frac{1}{r} \left[ (r B^\phi)_r - B^\phi_r \right] = \frac{1}{\lambda^2 N^2} \left( \frac{\Phi_0}{2\pi} (\nabla \times \nabla \vartheta)^{\phi} - \frac{1}{r} B^\phi \right),
$$

(74)

$$
(r \left[ B^i_{,zz} - B^i_{,rr} \right])_{,r} - \frac{1}{r} \left[ B^i_{,\phi} - r B^\phi_{,z} \right] = \frac{r}{\lambda^2 N^2} \left( \frac{\Phi_0}{2\pi} (\nabla \times \nabla \vartheta)^{\phi} - B^i \right).
$$

(75)

According to Stokes Theorem

$$
(\nabla \times \nabla \vartheta)^\mu = 2\pi \delta(x^\alpha - x^\alpha_0) \eta^\mu,
$$

(76)

here $\eta^\mu$ is the unit vector aligned along the vortex.

Assume $B^\phi = B^r = 0$ and $B^z(r) \neq 0$, then Eq. (75) could be written as equation for the $z$ component of magnetic field

$$
B^z - \lambda^2 N^2 \left( B^z_{,rr} + \frac{1}{r} B^z_{,r} \right) = \frac{\Phi_0}{2\pi} (\nabla \times \nabla \vartheta)^{\phi} e^\phi
$$

(77)

where $e^\phi$ is the unit vector along $z$ axis. This is the Bessel equation of complex argument with the boundary condition $B^z(\infty) = 0$. The solution of the Eq. (77) is the Bessel function of the third type, either the McDonald function or Hankel function of complex argument

$$
B^z = \frac{\Phi_0}{2\pi \lambda^2 N^2} K_0(r/\lambda N).
$$

(78)

The asymptotic behaviour of Hankel function of complex argument leads to the following result

$$
B^z = \frac{\Phi_0}{2\pi \lambda^2 N^2} \left( \frac{\pi \lambda N}{2r} \right)^{1/2} e^{-\frac{\pi}{\lambda N}}, \quad \text{for} \quad r \gg \lambda N
$$

(79)

$$
B^z = \frac{\Phi_0}{2\pi \lambda^2 N^2} \ln \left( \frac{\lambda N}{r} \right), \quad \text{for} \quad r \ll \lambda N.
$$

(80)

It follows from (78) and (80) that in the centre of vortex the magnetic field tends to infinity. However, in reality the field can not be infinite and these formulae are not valid near normal core of vortex ($r \sim \xi$). Then the expression (80) can be written as

$$
B(0) = \frac{\Phi_0}{2\pi \lambda^2 N^2} \ln \kappa.
$$

(81)

6. Penetration of Magnetic Field into II Type Superconductor

Assume that space-time admits a time-like Killing vector $\xi^\alpha$

$$
\xi^\alpha \xi_\alpha = -\Lambda,
$$

(82)
where \(-\Lambda = -N^2\) is the norm of time-like Killing vector, \(u^\alpha\) is the four-velocity of superconductor as a whole being parallel to the time-like Killing vector \(\xi^\alpha\)

\[
 u^\alpha u_\alpha = -1 \quad , \quad \xi^\alpha = N u^\alpha .
\]

Then 4-momentum \(P^\alpha\) of the superconducting Cooper pairs is

\[
 P^\alpha = m c u^\alpha .
\]

The four-velocity of superconducting electrons \(u^\alpha_s\) can be decomposed in the form

\[
 u^\alpha_s = u^\alpha + v^\alpha_s \sqrt{1 - v^2_s} ,
\]

where \(v^\alpha_s\) is the relative velocity of the Cooper pairs.

The energy of a superconducting pair is

\[
 W = -P^\alpha \xi_\alpha = m c u^\alpha \xi_\alpha = -m c u^\alpha \sqrt{1 - v^2_s/c^2} = N m c^2 \sqrt{1 - v^2_s/c^2} .
\]

In the case of nonrelativistic motion, the kinetic energy of superconducting pair is

\[
 W = N m c^2 + N m v^2_s .
\]

The density of total kinetic energy of superconducting pair is

\[
 W_{kin} = N m j^2_s / 2 \pi c^2 .
\]

where \(j^\alpha_s = n_s c v^\alpha_s\) is the superconducting current density.

Taking into account Maxwell equations written in vector form

\[
 \nabla \times (N \vec{B}) = \frac{4\pi}{c} \vec{J}_s ,
\]

one can get the expression for kinetic energy (88) in a more useful form

\[
 W_{kin} = N \frac{\lambda^2}{8\pi} (\nabla \times (N \vec{B}))^2 .
\]

According to expressions for the kinetic energy of supercurrent (88) and for energy of magnetic field the free energy of the whole superconductor is equal to

\[
 F_{sB} = F_{s0} + \frac{1}{8\pi} N \int |\vec{B}^2 + \lambda^2 (\nabla \times (N \vec{B}))^2| \sqrt{-g} dV .
\]

According to Leibnitz formula

\[
 (n^\alpha \beta \mu \nu u_\beta A_\mu B_\nu) \alpha = (n^\alpha \beta \mu \nu u_\beta A_{\mu;\alpha}) B_\nu + (n^\alpha \beta \mu \nu u_\beta B_{\nu;\alpha}) A_\mu ,
\]

Expressing vector potential \(A_\mu\) through the magnetic field

\[
 A_\mu = g_{\mu\sigma} n^\sigma \gamma \tau \lambda u_\gamma B_{\tau;\lambda}
\]

and using Stokes Theorem\(^{13}\)

\[
 \int (n^\alpha \beta \mu \nu u_\beta A_\mu B_\nu) \alpha \sqrt{-g} dV = \oint (n^\alpha \beta \mu \nu u_\beta A_\mu B_\nu) n_\alpha dS = 0
\]
one could obtain the following expression for full energy of superconductor
\[ E = \frac{N}{8\pi} \int \vec{B} (\vec{B} + \lambda^2 \nabla \times \nabla \times (N \vec{B})) \sqrt{-g} dV. \] (95)

Taking into account that magnetic field \( \vec{B} \) is governed by Eq. (77) one can get
\[ E = \frac{\Phi_0 N}{8\pi} B(0). \] (96)

Inserting (81) into (96) one can get
\[ E = \left( \frac{\Phi_0}{4\pi\lambda N} \right)^2 N \ln \kappa. \] (97)

For the superconductor embedded in applied external magnetic field the density of Gibbs free energy for the unit length of vortex
\[ G = E - N \frac{\Phi_0 B_0}{4\pi}. \] (98)
takes the minimum value at the equilibrium.

From this formula one can see that for the comparatively weak field \( B_0 \), Gibbs energy \( G > 0 \) and the formation of vortex is impossible. However there exists the magnetic field \( B_{c1} \) from which \( G \) becomes negative and vortex formation becomes energetically favorable. It follows from (98) that
\[ B_{c1} = \frac{4\pi \epsilon}{N \Phi_0} = \frac{\Phi_0}{4\pi\lambda^2 N^2} \ln \kappa. \] (99)

The inner magnetic field should be found as a solution of the interior Maxwell equations. However for the superconductor embedded in an applied magnetic field \( B_{\text{out}} \), the interior magnetic field can be modelled as \( B_0 = B_{\text{out}} N^{-2} \) for simplicity of calculations. Therefore, the free energy for the superconductor is
\[ F_{sB} = F_{s0} + \frac{B_{\text{out}}^2}{8\pi} N = F_{s0} + \frac{B_0^2}{8\pi} N^5. \] (100)

When \( F_{sB} = F_n \), the superconductor is transformed to the normal state and
\[ F_n - F_{s0} = \frac{B_{\text{cm}}^2}{8\pi} N^5. \] (101)

On the other hand according to Ginzburg-Landau theory\(^7\) the free energy of superconductor in the gravitational field in the absence of external magnetic field is
\[ F_{s0} = F_n + N \alpha |\psi|^2 + N \frac{\beta}{2} |\psi|^4. \] (102)

One can find the value of \(|\psi|^2\) in which free energy has a minimum, by solving \( \frac{dF_{s0}}{d|\psi|^2} = 0 \). Simple calculation gives
\[ |\psi|^2 = -\alpha/\beta. \] (103)

Inserting (103) to (102) and equalizing the result to (101) we will have
\[ B_{\text{cm}}^2 = \frac{4\pi \alpha^2}{\beta N^4}. \] (104)
Then using the expressions for $\xi$, $\lambda$ and $\Phi_0$ we obtain for $B_{cm}$

$$B_{cm} = \frac{\sqrt{2}\Phi_0}{4\pi\lambda\xi N^2} .$$

(105)

The second critical magnetic field\(^{17}\) is

$$B_{c2} = \sqrt{2} B_{cm} = \frac{\Phi_0}{2\pi\xi^2 N^2} .$$

(106)

7. Application of General Relativistic Electromagnetic Effects to Neutron Stars Interior

The dependence of the coherence length and penetration depth on the gravitational field may be important for neutron star physics. There are two ways in which external or trapped magnetic flux can penetrate the proton superfluid of neutron stars. This depends upon two important lengths: the proton coherence length $\xi_p$ and the London penetration depth $\lambda_L$. If $\xi_p$ is larger than $\sqrt{2}\lambda_L$, there are small normal regions containing flux interspersed with field-free superconducting regions (type I superconductivity). If instead $\xi_p$ is smaller than $\sqrt{2}\lambda_L$, magnetic flux can penetrate the superconductor without destroying the superconducting state. Realistic estimations show that $\xi_p$ is much less than the London penetration depth $\lambda_L$ for protons in the neutron star interior and hence the proton superconductor is expected to exhibit type II behavior.

However after recent observation of long period precession of neutron stars it was pointed out in the literature, see for example Link,\(^{14}\) that the axis of precession of some neutron stars may not be aligned with the axis of magnetic field. As the rotation of neutron stars causes a lattice of quantized vortices to form in the superfluid neutron state and it is generally believed that the proton superfluid is a type II superconductivity, which means that it supports a stable lattice of magnetic flux tubes in the presence of magnetic field, Link suggested that this two type of vortices might interact quite strongly due to the fact that the axis of the rotation and the axis of magnetic field are not aligned.

Due to this reason it was discussed\(^{18}\) that superconductivity inside neutron stars in fact may be type I because conventional picture of II type superconductivity follows from the standard analysis when only a single proton field is considered. In particular it has been shown the correlation length or coherence length $\xi$ should be replaced by the actual size of proton vortices

$$\bar{\delta} = \xi/\sqrt{\epsilon} .$$

(107)

Since the parameter $\epsilon$ arises from the strong interaction between proton and neutron superfluids, which is about $10^{-2}$, the length (107) is much bigger than the usual proton coherence length. Consequently the Ginzburg-Landau parameter to be bigger than $1/\sqrt{2}$, which in its turn causes that the superconductivity inside neutron stars to be type I.

Naturally the question arises whether the general relativistic effects arising from the strong gravitational field of neutron star can change the type of superconductivity or not. As we have shown here, due to the gravitational effect the coherence length is also modified by factor of $N$ (the value of the parameter $N$ can reach $\approx 0.7$ for the typical neutron stars), but it does not lead to change of the type of superconductivity, as gravitational field reduces the magnitude of the penetration depth by the same factor so then the Ginzburg-Landau parameter will remain unchanged.

It is well known that in superconductors of II type below lower critical field $B_{c1}$ there will be complete expulsion of the field; above an upper critical field $B_{c2}$ superconductivity will be destroyed; in the intermediate
range $B_{c1} < B < B_{c2}$ the superconductor will allow the magnetic field to penetrate, not homogeneously but confined to quantized flux tubes. Each fluxoid carries a quantum of magnetic flux $\Phi_0 = \hbar c/2e = 2 \times 10^{-7}$ Gauss \cdot cm$^2$. Within the core of a fluxoid (of radius $\xi$), the matter is in its “normal” state. The field strength rises to about $B_{c1}$ within this normal core. Around this the matter is in the superconducting state, and the field strength decreases exponentially away from the core with a scale length $\lambda_L$. For protons in the interior of a neutron star, the coherence length is $\xi_p = 0.6 \times 10^{-12} \rho_{p,13}^{1/3} \Delta_p^{-1}$ cm and the London penetration depth is $\lambda_L = 0.9 \times 10^{-11} \rho_{p,13}^{-1/2}$ cm, where $\rho_{p,13}$ is the mass density of protons in units of $10^{13}$ g \cdot cm$^{-3}$ and $\Delta_p$ is the energy gap of the proton superconductor in MeV (Ref. 19).

The vortex filaments consisting of neutron vortices and proton fluxoids inside superconducting region of neutron stars interact each other with an interaction energy which is equivalent to repulsion

$$E_{12} = \left( \frac{\Phi_0}{4\pi \lambda} \right)^2 K_0 \left( \frac{\lambda}{\lambda} \right). \quad (108)$$

Due to the gravitational effect the interaction energy would be less when compared with what could be expected in the flat space-time by a factor

$$N^{-7/2} e^{-\frac{dN}{\xi}} \quad (109)$$

which is tiny and may be neglected during accounting one of the main forces which has been acted on proton fluxoids. Here $d = 5 \times 10^{-10}(B_{12})^{-1}$ is the spacing between proton fluxoids. But there are the most important forces that act on proton fluxoids such as magnus force, buoyancy force, drag force and tension of fluxoids. But there are the most important forces that act on proton fluxoids such as magnus force, buoyancy force, drag force and tension of fluxoids. The collective effect of these forces leads fluxoids to move radially outward. The force

$$f_n = \frac{2\Phi_0 \rho_c R_c \Omega_c (\Omega_c - \Omega_s)}{B_c}, \quad (110)$$

acting on unit length of fluxoid (due to neutron vortices) does not depend on gravitational field and remains unchanged. Here $R_c$ is the core radius, $\Omega_s$ and $\Omega_c$ are the rotation rate of the core superfluid and the rotation rate of the crust respectively. However the other forces such as buoyancy force arising from the existence of the magnetic stress in the core of a fluxoid and drug force that acted by the electron gas to a flux tube moving with the velocity $\mathbf{v}$ depend on the gravitational field and would be modified when compared with the Newtonian ones

$$f_b = \left( \frac{\Phi_0}{4\pi \lambda N} \right)^2 \frac{1}{R_c} \ln \left( \frac{\lambda}{\xi} \right), \quad (111)$$

$$f_d = -\frac{3\pi}{64} n_e e^2 \Phi_0^2 \frac{v_p^2}{c}, \quad (112)$$

where $\tilde{E}_f = NE_f(e)$ is the electron Fermi energy in the stationary gravitational field. In spite of the fact the gravitational field of neutron stars does not change the value of these forces acted on fluxoids enough, the motion of fluxoids inside neutron star might be changed. Considering the equation of motion of fluxoids\(^{21}\) one can eventually see that difference of the rotation rate of the core superfluid from the rotation rate of the crust also depends on the value of the gravitational field.
Since the magnetic field decay from the core of neutron stars depends on interpinning of neutron superfluid and proton superconducting fluid\textsuperscript{21}, then the slightly modified equation of motion leads to the change of the decay timescale, but this modification is very small, and could not be important for considering the lifetime of the magnetic field persisting in neutron stars.

8. Conclusion

Following up our previous research on the electromagnetic properties of superconductors in gravitational field we have considered some general-relativistic effects associated with the influence of stationary gravitational field on quasi-stationary electromagnetic effects in conductors and superconductors.

By solving Maxwell equations for the time-dependent magnetic field penetrating inside the conductor we have shown that the skin depth in the conductors embedded in the gravitational field strongly depends on the gravitational field. However this effect is almost negligible for the laboratory conductors in the weak gravitational field of the Earth.

The penetration depth for the magnetic field and the coherence length in superconductors also depend on the gravitational field and proportional to the lapse function of the gravitational object. However the parameter of Ginzburg-Landau remains unchanged and does not lead to the general relativistic modification of the type of the superconductivity.

Two critical magnetic fields that destroy the superconducting state also depend on the gravitational field: the gravitational field increases the value of the critical magnetic field. Thus it has been shown that the strong gravitational field does effect to the quasi-stationary electromagnetic processes inside superconductors and one must take them into account during studying neutron star physics.

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