Allocation of control rights in the PPP Project: a cooperative game model

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Abstract. Reasonable allocation of control rights is the key to the success of Public-Private Partnership (PPP) projects. PPP are services or ventures which are financed and operated through cooperation between governmental and private sector actors and which involve reasonable control rights sharing between these two partners. After professional firm with capital and technology as a shareholder participating in PPP project firms, the PPP project is diversified in participants and input resources. Meanwhile the allocation of control rights of PPP project tends to be complicated. According to the diversification of participants and input resources of PPP projects, the key participants are divided into professional firms and pure investors. Based on the cost of repurchase of different input resources in markets, the cooperative game relationship between these two parties is analyzed, on the basis of which the allocation model of the cooperative game for control rights is constructed to ensure optimum allocation ration of control rights and verify the share of control rights in proportion to the cost of repurchase.

1. Introduction
Under the traditional project transaction mode, the government, investors, professional firms, intermediaries and other participants are independent, their own responsibility, risk, the source of income is also single. In recent years, the development of PPP projects tends to diversify in participants and input resources, more and more professional firms began to join the PPP project firm as a shareholder, forming “investors-professional firm” role overlap. So at the current stage, PPP project company shareholders are not only pure investors, there will be professional firms and intermediaries too. Therefore, the PPP project company shareholders can be divided into professional firms and pure investors [1].

Different participants determine the different types of inputs to the PPP project, meanwhile the different input resources determine the proportion of control rights. And the allocation of control rights not only to fully mobilize the enthusiasm of the participants injection, but also make full use of the members in different areas of professional technical and management advantages to improve the cooperation efficiency of PPP projects. Therefore, the allocation of control rights should take into account the types of inputs of the participants, aiming at the contribution degree of the input resources and the cost of repurchase to allocate the control rights reasonably.

2. Literature
The allocation of control rights first appeared in the enterprise, used to solve the problems that were arising from separation of ownership and management, Adolf and Means [2] set out the control rights to the actual choice of the directors; Demsetz [3] argued that corporate control is mainly concentrated in the use and disposal of scarce resources, is a group of right bundle which has the nature of exclusive;
Zhou [4] put forward the control right is "a right that dominates company's business operations and decision-making", emphasizing the essential control rights of enterprises are the exclusive rights of using enterprise assets. Based on the above research, Xu et al. [5] believed that the allocation of control rights in PPP project is mainly to solve the project investment decision-making power in the public and private ownership issues; Ye et al. [6] argued that the nature of PPP control rights is based on resource-based corporate control rights and is considered to be the basis for the efficiency of public-private partnerships.

In terms of the allocation of PPP project control rights, Grossman and Hart et al. (GHM) [7, 8] proposed the allocation of control rights to the more important party when studying the allocation of control over the control rights of private goods production in private sectors. Hart et al. (HSV) [9] suggested that the type of partner would affect the allocation of control rights when introducing the public sector into the GHM model. On the basis of HSV, Besley and Ghatak (BG) [10] argued that the type of both parties’ investment products will have an impact on the distribution of control rights and when the two sides invest in public goods, the control rights should be given to the side who have a higher products evaluation. On the basis of the BG model, Zhang et al. [11] proposed to set the control rights proportion as a continuous variable, and from the perspective of reducing the private sector's self-interest and increasing the input of the public, according to establish a mathematical model, the reasonable distribution interval of control rights under different conditions is given. Sun et al. [12] established mathematical model and analyzed the relationship between the allocation of control rights and the incentive between the two parties under the PPP background, and pointed out that under the condition that the initial contract stipulated the income distribution scheme, the optimal control rights allocation scope and the proportion of the distribution of proceeds in the initial contract, the technical resources of both parties, and the two sides’ optimistic about the expected final income.

From the domestic and international research trends, there are more and more factors considered in the allocation of control rights, and the applicability of the study is gradually improving. However, the existing literature still has the following shortcomings: With the diversification of participants and input resources, the different input resources will have an different impact on the ratio of control rights of the participant, and the existing research rarely involved. Therefore, this paper aims at the actual situation of PPP project participation in the diversification of inputs, the key participants is divided into professional firms and pure investors; The paper analyzes the cooperative game relationship between these two parties in view of the repurchase cost of different inputs. On the basis of which the allocation model of the cooperative game for control rights is constructed to ensure the optimal allocation ratio of control rights.

3. Model Assumptions and Construction
Assuming that the achievement of a PPP project requires both capital and technology inputs, pure investors provide part of the funds $i_k$, professional firms provide part of the funds $i_k$, as well as core technology, management and other knowledge $i_k$. Assuming that the PPP project's production function is:

$$y = f(i_k, i_k)$$  \(1\)

Assuming that the price of the PPP project unit needs is $P$. The market price of funds $i_k$ and technology $i_k$ in the input resources is $P_1$ and $P_2$ respectively. Standardizing the price for comparison, setting the unit demand price to $P/P = 1$, the market price of funds to $P_1/P = w_1$, the market price of technology to $P_2/P = w_2$.

Assuming that the cooperative surplus of the PPP project which the pure investor and the professional firm participate is $r$, it is a random variable.

In the process of cooperation, it is assumed that the share of control of the professional firm after the bargaining of both parties is $\beta$, then the share of control of pure investors is $1 - \beta$.

If one party does not choose to cooperate, the other party can only regain the input from the market. At this point, pure investors or professional firms using the spot contract to acquire capital resources (or technology) directly from the market. Assuming that the input resources on the market can not be
bargained during this particular period, that is a buyer’s market, the buyer acquires all "organizational rents", but the buyer must consider the relative purchase cost of funds or technology. Assuming the cost of capital repurchase to \( \xi_k \), the cost of repurchase of technology to \( \xi_i \). That is the professional firm does not cooperate with the pure investor, the pure investor repurchases homogeneous 1 unit input resource \( i_k \) or \( i_i \) from the market required \( \xi_k \) or \( \xi_i \). Obviously, the alternative of input resources (\( i_k \) or \( i_i \)) is more higher, the repurchase cost is more lower. At this point, for pure investors, the total income is:

\[
w_i i_c + r - i_k \xi_k - i_i \xi_i
\]

Thus the benefits of both parties include two parts: the determinative gains determined by the market price and the surplus of the cooperation shared by the control. The benefits of pure investor and professional firm in selecting cooperation and non-cooperation are shown in Table 1.

**Table 1.** Cooperative game payment matrix for pure investor and professional firm

| Professional firm | Cooperation | Non-cooperation |
|-------------------|-------------|-----------------|
| Pure investor     | \(( w_i i_c + (1 - \beta)r \), \( w_i j_a + w_i j_k + \beta r \)) | \(( w_i i_c + r = i_k \xi_k, -i_i \xi_i .0) \) |
| Cooperation       | \(( w_i i_c + r - i_k \xi_k - i_i \xi_i .0) \) | \((0, w_i j_a + w_i j_k + r - i_k \xi_k) \) |
| Non-cooperation   | \((0,0)\) | \((0,0)\) |

Obviously the condition of pure investors choose to cooperate with the professional firm is \( w_i i_c + (1 - \beta)r \geq w_i i_c + r - i_k \xi_k - i_i \xi_i \), otherwise the pure investors will choose not to cooperate, and to reselect the funds and technology resources from the market, they own acquire all the remaining. So as to the maximum limit of cooperation surplus, the surplus investors acquire is:

\[
\beta \leq \frac{i_k \xi_k + i_i \xi_i}{r}
\]

(3)

Similarly, professional firm and pure investor to cooperate with the conditions \( w_i i_c + w_i j_k + \beta r \geq w_i i_c + w_i j_k + r - i_k \xi_k \), otherwise the professional firm will choose not to cooperate, and to reselect the capital resources from the market, they own acquire all the remaining. So that the minimum limit of control that professional firms acquire is:

\[
\beta \geq 1 - \frac{i_k \xi_k}{r}
\]

(4)

Therefore, according to \( 1 - \frac{i_k \xi_k}{r} \leq \beta \leq \frac{i_k \xi_k}{r} + \frac{i_i \xi_i}{r} \), the condition of the pure investor and professional firm is:

\[
[(i_k + i_i) \xi + i_k \xi_k] - r \geq 0
\]

(5)

The formula above defines a cooperative area. As shown in the diagram 1, the slash in the figure is bounded, the right side of the line indicates that the negotiation process of the pure investor and the professional firm on the cooperative surplus distribution, which is a process of cooperative game.
Diagram 1: internal cooperation and market repurchase decision area of PPP project company

In the above formula, \([ (i_{ik} + i_r)\xi_k + i_r\xi_k ]\) can be regarded as the cost of the market when pure investor and professional firm all choose to repurchase input resources, it can be regarded as market transactions. And the cooperative surplus \(r\) can be regarded as value appreciation of the PPP project. The above formula means that the resource owner will choose to project team work when the cost of repurchasing resources through market is higher than the project team cooperation. The above formula actually defines the boundaries of PPP project company. For this reason, it is economically “meaningful” that selecting cooperative game model.

4. Model Analysis and Discussion

4.1 Subjoining bargaining capability
According to the previous discussion, the bargaining capability itself is a function of the relative repurchase cost. And the relative repurchase cost is more lower, the substitutability of the resources in the market is more higher, the bargaining capability is more lower. Therefore, assuming that the funds and technology resources in the PPP project company bargaining capability respectively are:

\[
\alpha_i = \alpha^c_i, \quad \alpha_k = (1 - \alpha^c_k)
\] (6)

Among them, \(\alpha > 0\). it shows that bargaining capability is proportional to the relative repurchase cost.

Assuming that the funds and technology reach a Nash cooperation solution through bargain, by the pay matrix of the pure investors and professional firms, we can get:

\[
\beta = \arg \max \left\{ ([w_i i_k + w_j i_k + r - i_r \xi_k] - (w_i i_k + w_j i_k + \beta r))^n \\
[w_i i_k + r - i_r \xi_k - i_i \xi_k] - [w_i i_k + (1 - \beta) r^n] \right\}
\]

\[
\beta = \arg \max \left\{ (r - \beta r - i_r \xi_k)^n (\beta r - i_i \xi_k - i_i \xi_k)^n \right\}
\]

Calculated:

\[
\beta = \frac{a_r r + a_i i_k \xi_r + (a_i i_k \xi_r - a_j i_j \xi_r)}{(a_i + a_j) r}
\] (7)

Substituting (6) into (7):

\[
\beta = 1 - \alpha^c_i + \frac{\alpha^c_k (i_{ik} + i_r)\xi_k + \alpha^c_k i_i \xi_k - i_i \xi_k}{r}
\] (8)

4.2 Parameter discussion
When \(\xi_k \to 0\), according to the conditions of cooperation \([ (i_{ik} + i_r)\xi_k + i_r\xi_k ] - r \geq 0\), there is \(i_r\xi_k \to r\), by (8) we can get: \(\beta \to 1\). It notes that if the pure investor of PPP project company does not cooperate with the professional firm, and the relative cost of re-selecting the other investors from resources market tends to 0, the professional firm will possess almost all control under the cooperative game framework.

When \(\xi_k \to 0\), according to the conditions of cooperation \([ (i_{ik} + i_r)\xi_k + i_r\xi_k ] - r \geq 0\), there is \((i_{ik} + i_r)\xi_k \to r\), by (8) we can get: \(\beta \to \frac{r - i_r \xi_k}{r}\). It notes that when the PPP project company's
professional firms do not cooperate with pure investors, the relative cost of pure investors re-select other professional firms from the market is tends to 0, in the framework of cooperative games, the original professional firm also has a part control $\beta \to \frac{r - i \xi}{r}$, not completely out of control. Because compared with pure investors, PPP project company's professional firms not only put the technology resources, but also put the capital resources; even if re-purchase the resources of technology, a certain degree of control remain because of the resources of funds.

When $\xi > 0$ and $\xi > 0$, the relative costs of repurchasing the input resource from the market are greater than 0. At this point, the PPP project company control configuration structure is belongs to the sharing type, that is pure investors and professional firms share control, sharing share is determined by the relative repurchase cost. The higher the cost of the relative repurchase in the market, the higher the share of control in the PPP project company.

5. Conclusion
For the situation of professional firm becomes a shareholder in PPP project company, PPP project participant is divided into two categories: professional firm and pure investor. On one hand, the professional firm represents the "investor-professional firm" role overlap participant; On other hand, pure investor includes government, it truly reflects equal participant.

Different category participants input different resources in PPP project, and the different input resources determine the degree of participant’s control right. Based on the repurchase cost of different input resources and the cooperative game relationship between the different category participants, the allocation model of control rights is constructed to ensure optimum allocation ratio of control rights and verify the share of control rights in proportion to the cost of repurchase. It is that the higher the repurchase cost of an input resource in the market, the higher the proportion of control rights for the participant. In a word, the participant with the scarce resources has the majority control rights.

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