Computation of the $E(I)$ characteristics of a superconducting cable using parameters of cable tapes

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Abstract. $E(I)$ characteristics for cable characterization in DC regime plays an important role in dealing with problems that concern superconducting cables. In the contribution a cable model consisting of 16 superconducting tapes connected in parallel is used. Transport properties of each tape were characterized by its $E(I)$ curve with parameters, such as the critical current and the $n$-exponent. The parameters entering the calculation for all tapes were measured on this cable model in which the tapes were localized straight. The tapes had no conducting contact over the whole length. The transport current of the cable was divided into tapes according to their properties and contact resistances between each tape and current termination. It can be described by a set of non-linear equations. The set of nonlinear equations, which entered the computation for the array of the parallel tapes including tapes parameters and contact resistances, were resolved numerically. The contribution shows the way to obtain “global” $E(I)$ characteristics using this calculation. The “global” $E(I)$ characteristics were calculated and compared with characteristics measured for two variants of superconducting cable.

1. Introduction
$E(I)$ relation of superconducting cables carrying a transport current is one of the most important issue for their applications. Particularly for single High Temperature Superconductor (HTS) tapes at currents $I \cdot I_c$, a power-law model, $E=E(I/I_c)^n$ is a good approximation of experimental data ($I_c$-critical current, $E$-electric field, $E_c$-electric field at which the critical current is defined and $n$-value denotes slope). Because the cables have many HTS tapes placed in parallel, the situation for the cables is more complicated. In reality all of the cable tapes are not identical [1]. The parameters characterizing individual HTS cable tapes as critical currents, $n$-exponents and contact resistances between each tape and current terminations are different. Then, current distribution is non-uniform and the voltages on individual tapes are different in this condition [1, 2]. The measurement of the “global” $E(I)$ characteristics of a superconducting cable is possible with the aid of signal measured on the cable current terminations [3]. In our theoretical approach the current distribution and the voltage signals on different tapes are calculated. Subsequently using given cable tapes parameters the “global” $E(I)$ characteristics is evaluated. For comparison of the calculations with measurements we have a simple short model of HTS cable. This single-layer cable model consists of 16 Bi-2223/Ag tapes with critical current of ~30A [1]. Here, we show the way of computation of the “global” $E(I)$ characteristics of a superconducting cable and we compare it with measurement. As a second way to obtain the “global” $E(I)$ characteristics the arithmetic mean of the electric fields was performed and mutually compared.
2. Procedure of computation
First, we calculate how the electrical voltages on individual HTS tapes will depend on the current with the aid of measured parameters. These parameters are linear and non-linear elements entering the computation. Electrical scheme is shown in figure 1 a). The linear elements are contact resistances of individual parallel paths
\[ R_i = R_{ai} + R_{bi} \]
and non-linear elements are superconducting tapes with parameters \( I_{0i} \) and \( n_i \). In order to insert their current-voltage relation, one could use the following empirical two-parameter description for tapes used in our model cable [1].

\[
E_i(I_i) = E_0 \times \begin{cases} 
\left( \frac{I_i}{I_{0i}} \right)^{n_i} & \text{for } I_i \leq I_{0i} \\
1 + 1.3(n_i - 1)^2 \left( \frac{I_i}{I_{0i}} - 1 \right)^2 & \text{for } I_i > I_{0i}
\end{cases}
\]  

(1)

\( I_{0i} \) is the current corresponding to the electric field \( E_0 = 10^4 \) V/m and \( n_i \) is characterizing the steepness of the \( E_i(I_i) \) curve. Solved parallel combination includes \( N \) branches where \( N \) is the number of tapes in the cable containing one tape in each branch. The voltage \( V_{tot} \) is identical for every parallel path and then current flowing in the \( i \)-th tape is given by the equation

\[ R_i I_i + IE_i(I_i) = V_{tot} \]  

(2)

where \( l \) is the tape length. To obtain the total current, Eq.(2) must be solved for every \( i = 1,2,...,N \) and the cable current is

\[ I = I_{cable} = \sum_{i=1}^{N} I_i. \]  

(3)

With help of this procedure we calculate the \( E(I_{cable}) \) characteristics for every cable tapes [1,2]. Next we calculate resistance \( R \) according to simplified electrical scheme of the model cable shown in figure 1 b). There in our model the resistance \( R \) is given by the equation

\[ R = \left( \sum_{i=1}^{N} \frac{1}{R_i} \right)^{-1}. \]  

(4)

The voltage in the resistive part, \( V_R = R \cdot I_{cable} \), thus, the voltage in the superconducting part is \( V = V_{tot} - V_R \) and \( E(I_{cable}) = V/I_{cable} \).
The “global” $E(I_{\text{cable}})$ curve can be obtained also by the arithmetic mean of the electric field calculated for individual HTS tapes of the cable

$$E(I_{\text{cable}}) = \frac{1}{N} \sum_{i=1}^{N} E_i(I_{\text{cable}}).$$

(5)

This $E(I_{\text{cable}})$ curves and curves obtained by previous calculations are graphically compared in the next section.

3. Results and discussion

Measured and calculated curves $E_i(I_{\text{cable}})$ of individual cable tapes for two variants of HTS cables are shown in figure 2 and figure 3. These cables have identical geometries and different parameters of cable tapes and more different parameters of contact resistances. The contact resistance of tape T9 in figure 3 is purposely higher than another. The “global” $E(I)$ characteristics was obtained using two ways of the calculation as mentioned above and two ways of the measurement. Measurement was
carried out by measurement on current terminations [3] and next by arithmetic mean of electric field measured on individual tapes. Every measurement was carried out at fixed value of the cable current, which was set before each measurement. Resulting relations are compared in figures 4 and 5. For both types of HTS cables, resulting characteristics are in very good correspondence, although tape T9 in second cable was worse contacted. The parameters of $E(I)$ characteristics of the first cable are $I_0 = 495\text{A}$, $n = 22$ and of second type of the cable are $I_0 = 473\text{A}$, $n = 20$. It is obvious, that bad contact on one HTS tape can worsen current transport properties of the whole HTS cable.

4. Conclusion
The “global” $E(I)$ characteristics of the superconducting cable were calculated using mathematical model with tapes’ parameters and contact resistances and by arithmetic means of electric field of individual HTS tapes. These relations were compared with results obtained from measurements. All resulting characteristics are in very good correspondence for both variants of HTS cables. The spread in the $E(I)$ characteristics of individual cable tapes can be considerable and it is very complicated to describe the whole cable with individual tape $E(I)$ curves. From these reasons and also from the practical point of view, it is more convenient to characterize the cable with the “global” $E(I)$ characteristics describing the whole cable properties. The advantage is that it can be easily measured and calculated using cable tape parameters for given geometry of HTS cable.

Acknowledgement
The authors would like to acknowledge the financial support from the Centre of Excellence SAV CENG and APVT-20-012902 project.

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