Joint tests of cosmology and modified gravity in light of GW170817

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In this Letter we constrain for the first time both cosmology and modified gravity theories jointly, by combining the GW and electromagnetic observations of GW170817. We provide joint posterior distributions for the Hubble constant $H_0$, and two physical effects typical of modified gravity: the gravitational wave (GW) friction, encoded by the parameter $\alpha_M$, and several GW dispersion relations. Among the results of this analysis, we can improve by 15% the bound of the graviton mass with respect to measurement using the same event, but fixing $H_0$. We obtain a value of $m_g^2 = 2.08_{-1.4}^{+1.3} \cdot 10^{-44} \text{eV}^2/\text{c}^4$ at 99.7% confidence level (CL), when marginalising over the Hubble constant and GW friction term $\alpha_M$. We find poor constraints on $\alpha_M$, but demonstrate that for all the GW dispersions relations considered, including massive gravity, the GW must be emitted $\sim 1.7\text{s}$ before the Gamma-ray burst (GRB). Furthermore, at the GW merger peak frequency, we show that the fractional difference between the GW group velocity and $c$ is $\lesssim 10^{-17}$.

Introduction— The observation of gravitational waves (GWs) from the binary neutron star merger (BNS) GW170817 [1], together with its associated Gamma-ray Burst (GRB) [2] and hosting galaxy NGC4993 [3], have opened the possibility to exploit GWs for cosmological studies [4][14], and also to probe general relativity (GR) on cosmological scales. Indeed, modifications to GR could provide a solution [15] to different open problems in cosmology, such as the nature of dark energy [16] and the tensions between different Hubble constant measurements [17][18]. Hence it is crucial to test GR on cosmological scales.

In many modified gravity models, the evolution of the cosmological background is unchanged but deviations from GR appear in the perturbation equations, including the tensor perturbations on which we focus here [19][20]. Amongst these theories, ones with extra dimensions [21] or a running Planck mass [22] add an effective friction term to the GW propagation equation. Theories which, for instance, break Lorentz invariance [23] or have a massive graviton [24], lead to a modified propagation speed for GWs, together with a possible dispersion relation. Furthermore, these two effects are generally not mutually exclusive and may both be present [25]. In practice, the presence of a GW friction will modify the GW luminosity distance (with respect to the Standard cosmological one) [22][25][29], while the GW dispersion relation might result in a modified GW phase evolution [28][30][40] and observation delay with an associated GRB.

Indeed, in [31] we showed that, in order to obtain an unbiased measurement of the GR deviation parameters (giving the modified friction term and dispersion relation), and in particular when combining results from several events, the Hubble constant $H_0$ must not be fixed, but treated as a free parameter. Moreover, we showed that by combining GW phase studies and timing between GW-GRB, it is not only possible to measure GW dispersion relations but also to provide constraints on the GW-GRB emission delay at the source.

Here, and to our knowledge for the first time, we combine data from the NGC4993 galaxy redshift, GRB-GW delay and GW parameter estimation, to jointly constrain $H_0$ and the GR deviation parameters. We use the statistical method developed in our companion paper [41] together with GW and EM observations from GW170817 to measure $H_0$, the GW friction term $\alpha_M$, different dispersion relations parametrized by $\tilde{\alpha}_j$ and also the GW-GRB emission delay. Using the results from this phenomenological approach, we also provide constraints on the graviton mass and on the time between GW and GRB emission.

Theoretical background— We consider the modified GW propagation equation given by [27][28][42][43]

\begin{equation}
\frac{h''}{a} + 2\left[1 + \alpha_M(\eta)\right] \frac{a'}{a} h' + k^2 c_T^2(\eta, f_d/a) h = 0,
\end{equation}

where $h$ is the GW amplitude (omitting the polarisation indexes; we assume no extra polarisations here), $a$ is the scale factor of $\Lambda$CDM cosmology, and the derivatives are with respect to the conformal time $\eta$. $f_d$ is the detected GW frequency, $\alpha_M$ is the parameter connected to the GW friction and $c_T$ is the phase velocity of GWs in modified gravity [41]

\begin{equation}
c_T^2(\eta, f_d/a) = c^2 \left[1 + \tilde{\alpha}_j(\eta) \left(\frac{f_d}{a}\right)^j\right],
\end{equation}

where $c$ is the speed of light, and the parameters $\tilde{\alpha}_j$ encode the modified GW dispersion relations. For massive gravity for instance, $j = -2$.

Following the results from GW170817 [2], we assume $|\tilde{\alpha}_j(\eta)(f_d/a)^j| \ll 1$, and work to first order in this quantity. Thus, the GW group velocity differs from $c$ by [41]

\begin{equation}
\frac{v_g(\eta, f_d/a)}{c} - 1 \approx -\frac{1}{2} \tilde{\alpha}_j(\eta) \left(\frac{f_d}{a}\right)^j.
\end{equation}
Here, we focus on the event GW170817 at low redshift, and thus approximate \( \alpha_M (\eta) \approx \alpha_M \) and \( \tilde{\alpha}_j (\eta) \approx \tilde{\alpha}_j \).

Eqs. (1)-(3) can be used to define the three central observables of our analysis, see [41]. The first, the GW luminosity distance, depends only on \( \alpha_M \) and \( H_0 \). In terms of the standard luminosity distance \( d_{\text{EM}} (z) \), where \( z \) is the cosmological redshift, it is given by

\[
d_{\text{GW}} (z) = d_{\text{EM}} (z) \exp \left[ \int_0^z \frac{\alpha_M}{1 + z} dz \right].
\]

The second observable is the GW-GRB observation delay

\[
\Delta t_d = (1 + z_s) \tau + \frac{f_{R,d}}{2} \tau_j.
\]

The first term on the right hand side is the standard redshift due to the expansion of the Universe, with \( z_s \) the redshift of the source, and \( \tau \) the GW-GRB emission time delay at the source. The second term is due to modifications of gravity: \( f_{R,d} \) is the GW reference frequency for computing the GRB delay (for GW170817 it is the merger frequency \( f_2 \)), and

\[
\tau_j = \int_0^{z_s} d\zeta \tilde{\alpha}_j \frac{(1 + \zeta')^j}{H_0 \sqrt{\Omega_{m,0} (1 + \zeta')^3 + \Omega_\Lambda}},
\]

we fix \( \Omega_{m,0} = 0.308 \), \( \Omega_\Lambda = 1 - \Omega_{m,0} \) [17]. Note that for low redshift event, we are insensitive to changes in these parameters. The third observable is the fractional GW phase shift [41], encoded in the fractional GW phase deviations to a given Post-Newtonian term \( \beta_{\text{PN}} \):

\[
\delta \psi_{3j+8} = \pi \frac{\tau_j}{\beta_{\text{PN}} (j + 1)}, \quad \text{for } j \neq -1 \quad (7)
\]

\[
\delta \psi_{3j+8} = \pi \frac{\tau_j}{\beta_{\text{PN}} |3j|}, \quad \text{for } j = -1. \quad (8)
\]

The PN parameters are functions of the binary masses and spins and are given up to the 7th order in [41].

**Statistical analysis**— We use the statistical method developed in [41]. We calculate the joint posterior on \( H_0 \), \( \alpha_M \), \( \tilde{\alpha}_j \) and \( \tau \), given the observations of the GW event from LVC data \( x_{\text{GW}} \), GRB data \( x_{\text{GRB}} \) and redshift \( z \) and combining the measurements provided by these 3 datasets denoted as \( \bar{x} \), namely

\[
p (H_0, \alpha_M, \tilde{\alpha}_j, \tau | \bar{x}) = \frac{p (\alpha_M) p (H_0) p (\tilde{\alpha}_j) p (\tau)}{\beta (H_0, \alpha_M)} \int dz d f_{R,d} \times \frac{p (d_{\text{GW}}, \delta \psi | x_{\text{GW}}) p (z | x_{\text{GRB}}) p (\Delta t_d | x_{\text{GRB}})}{\pi (d_{\text{GW}}, \delta \psi) / \pi (z) / \pi (\Delta t_d)}
\]

(for simplicity, we drop the PN index \( 3j + 8 \) on \( \delta \psi \), see Eq. (7)-(8)). Here \( p (d_{\text{GW}}, \delta \psi | x_{\text{GW}}) \) and \( p (\Delta t_d | x_{\text{GRB}}) \) are the posterior distributions of the three observables \( d_{\text{GW}}, \delta \psi, \Delta t_d \) from the LVC GW data \( x_{\text{GW}} \) and Integral/Swift data \( x_{\text{GRB}} \), while \( p (z | x_{\text{GRB}}) \) is the redshift posterior inferred from Surveys data \( x_s \). The functions \( \pi (\cdot) \) are the priors used by the independent measurements to generate the posterior distributions. For LVC data, posteriors are generated with a uniform prior on the PN deviations and a \( d_{\text{GW}} \) prior on the GW luminosity distance. For NGC4993 and the GW-GRB time delay, we assume that the posterior values are originally generated with a uniform prior. For the merger frequency \( f_{R,d} \), we assume a uniform prior distribution between 2200 Hz and 3500 Hz, which are the minimum and maximum values found with different GW waveform models [45]. As in the analysis in [2], for the GW-GRB emission delay, we use a uniform prior \( p (\tau) \) between \([-10, 0]\) s (we allow the GRB emitted up to 10 s after the GW at the source). For \( \tilde{\alpha}_j \) we use a uniform prior in \([\tilde{\alpha}_{j, \text{min}}, \tilde{\alpha}_{j, \text{max}}]\), where these two boundaries are computed from the condition \( [\tilde{\alpha}_j | f_{R,d, \text{min}}] < 10^{-15} \) with \( f_{R,d, \text{min}} = 2200 \) Hz. This condition is set to satisfy the model independent constraints on the GW speed in [2] (also computed allowing the GRB to be emitted 10 seconds after the GW). For the Hubble constant we use a flat in log prior between [40, 120] km Mpc\(^{-1}\)s\(^{-1}\) and for the parameter \( \alpha_M \) a uniform prior between \([-10, 10]\). The lower limit of the \( \alpha_M \) prior has been chosen such that \( d_{\text{GW}} \) is a monotonically increasing function up to redshift \( z = 0.2 \), see [41]. The prior \( p (z | H_0) \) for the cosmological redshift given the Hubble constant value is uniform in comoving volume, which for low redshifts scales as \( \propto z^2 / H_0^3 \). Finally, the denominator \( \beta (H_0, \alpha_M) \) in Eq. (9) ensure the normalization and encodes the selection effects [41, 40].

When assuming no GW dispersion relation (\( \tilde{\alpha}_j = 0 \)), we compute \( p (d_{\text{GW}} | x_{\text{GW}}) \) using the “high-spin” posterior for \( d_{\text{GW}} \) provided by the LVC [45] and generated by fixing the sky position to that of NGC4993 and using the GW waveform generator PhenomPNRT [47]. When we consider \( \tilde{\alpha}_j \neq 0 \), we use the joint posterior samples on the GW luminosity distance and phase shift \( p (d_{\text{GW}}, \delta \psi | x_{\text{GW}}) \) from [40]. These are also generated using the PhenomPNRT. For computing \( p (\Delta t_d | x_{\text{GRB}}) \) we assume a Gaussian posterior with mean \(-1.74 \) s (the GW arrives earlier than the GRB) and standard deviation 0.05 s [2]. For the posterior on NGC4993 redshift, we assume a bivariate gaussian distribution centered at the observed redshift value of \( \hat{z}_\text{obs} = 0.011 \) and peculiar motion redshift \( \hat{z}_\text{pec} = 0.001 \cdot 10^{-4} \) with standard deviations of \( 2 \cdot 10^{-4} \) and \( 5 \cdot 10^{-4} \) respectively [3].

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1 Assuming tight constraints on the GW phase velocity, see [2] [41]

2 The observables are functions of \( \alpha_M, \tilde{\alpha}_j, H_0, z \), see Eqs. (1)-(3).

3 https://dcc.ligo.org/LIGO-P1800061/public

4 https://dcc.ligo.org/LIGO-P1800069-v8/public
TABLE I. Median and 1σ confidence intervals for \( H_0, \alpha_j, \alpha_M \) (3rd, 4th and 5th columns) for Scenarios III-IV. First and Second column: \( v_g \) scaling and correspondent PN for the GW dispersion relations. Correlations between variables are reported in the 6th, 7th, 8th columns.

| \( v_g \) | PN | \( H_0 \) [km Mpc\(^{-1}\)s\(^{-1}\)] | \( \alpha_j \) [eV\(^{-1}\)c\(^2\)] | \( \alpha_M \) | \( C_{H_0-\alpha_M} \) | \( C_{\alpha_0-\alpha_M} \) | \( C_{\alpha_j-\alpha_M} \) |
|-------|-----|-----------------|-----------------|-----|-----------------|-----------------|-----------------|
|       | III-IV |       | III | IV |       | III | IV | IV | IV | IV | IV | IV | IV |
| \( f_d^{-3/4} \) | 0 | 75.1  | 83.7  | 0.8 | 15.1  | -8.5  | 10.0  | -8.3  | 10.0  | 7.9  | 10.4  | 0.11  | 0.02  | 0.62  | -0.18 |
| \( f_d^{-7/4} \) | 1 | 75.1  | 83.7  | 0.8 | 15.1  | 6.1  | 7.4  | 4.9  | 7.4  | 7.9  | 10.4  | 0.09  | 0.13  | 0.67  | 0.08 |
| \( f_d^{-1} \) | 2 | 75.1  | 83.7  | 0.8 | 15.1  | 3.5  | 4.6  | 10.4  | 0.09  | 0.13  | 0.67  | 0.13 |
| \( f_d^{-5/4} \) | 3 | 75.1  | 83.7  | 0.8 | 15.1  | -0.9  | 10.4  | 10.4  | 0.05  | 0.09  | 0.69  | 0.08 |
| \( f_d^{-3/4} \) | 4 | 75.1  | 83.7  | 0.8 | 15.1  | -1.9  | 10.4  | 10.4  | 0.17  | -0.14  | 0.65  | 0.08 |
| \( f_d^{-7/4} \) | 5 | 75.1  | 83.7  | 0.8 | 15.1  | -3.2  | 10.4  | 10.4  | 0.10  | -0.21  | 0.60  | -0.15 |
| \( f_d^{-1} \) | 6 | 75.1  | 83.7  | 0.8 | 15.1  | 3.7  | 4.8  | 10.4  | 0.10  | 0.07  | 0.62  | 0.06 |
| \( f_d^{-5/4} \) | 7 | 75.1  | 83.7  | 0.8 | 15.1  | 2.5  | 3.8  | 10.4  | 0.18  | -0.19  | 0.61  | -0.16 |

**Results** — We discuss measurements of \( H_0, \alpha_M, \alpha_j \) and \( \tau \) in 4 different scenarios. In each of them \( H_0 \) is free, but they differ as follows: scenario I corresponds to GR (\( \alpha_M = \alpha_j = 0 \)); in scenario II only GW friction is present (hence \( \alpha_j = 0 \)); in scenario III only the GW dispersion is present (hence \( \alpha_M = 0 \)); and scenario IV we infer all the parameters. Figures for marginal and joint posterior distributions are provided in the support material.

For scenario I we obtain a value for the Hubble constant of \( H_0 = 74^{+14}_{-14} \) km Mpc\(^{-1}\)s\(^{-1}\), which is consistent with the value of \( 153^{+25}_{-25} \) [1]. Moreover, we obtain a GW-GRB emission delay of \( \tau = -1.72^{+0.05}_{-0.05} s \), i.e. the GW-GRB observed delay is due to an initial one processed by the cosmological expansion.

Scenario II was already analysed in [22], though using a different parametrization of the parameter \( \alpha_M \). We obtain \( H_0 = 85^{+14}_{-14} \) km Mpc\(^{-1}\)s\(^{-1}\) and \( \alpha_M = 14^{+15}_{-16} \) and \( \tau = -1.72^{+0.05}_{-0.05} s \). These values are consistent with GR, but there is a slight preference for higher \( H_0 \) and \( \alpha_M \) given by the strong degeneracy between these two parameters in Eq. 1. There is a preference for positive \( \alpha_M \), since for these values, the GW appears further and the selection function \( \beta \) is small, see the Support material for more details. Note that these larger values of \( \alpha_M \) will be selected by combining more events at redshifts, since for these events, higher values of \( \alpha_M \) will result in exponentially large \( d_{GW} \) which cannot be observed [11].

In scenario III, we focus on several GW dispersion relations separately, namely those corresponding to GW PN orders for which the posterior samples exist. We find that the results on the \( H_0 \) determination are mostly independent of the GW dispersion relation, see Tab. I. In fact, we always obtain values compatible with scenario I. This means that \( H_0 \) is mostly constrained through the GW luminosity distance and not the GW-GRB observation delay or GW phase. Tab. I also reports the values we obtain for \( \alpha_j \). Note that we consider both positive and negative values of \( \alpha_j \) in the posterior distributions, and thus include both sub- and super-luminal GW propagation. Indeed, here we take a purely phenomenological approach in order to fully exploit the information encoded in data. It is important to observe that these combined observations also allow us to put constraints on the GW-GRB observation delay for several GW dispersion relations considered in this Letter (see Fig. 1).

![Fig. 1. Posterior distributions for \( \Delta t_{MG}^d \) (GW-GRB observation delay) and \( v_g/c = 1 \), both evaluated at \( f_{R,\delta} \). Red/orange: scenario III. Blue/orange: scenario IV.](image-url)

As it can be seen, the contribution to the observation delay due to modification of gravity \( (\Delta t_{MG}^d = f_{R,\delta}T_j/2 \) in Eq. 5) is always negligible if compared to the current uncertainty on the observed time delay (0.05 s). As a consequence, for all the dispersion relations, the emission delay posterior converges to a value of \( \tau = -1.72^{+0.05}_{-0.05} \) s, and hence is consistent with the measured GW-GRB observation delay rescaled by a source redshift of \( \sim 0.01 \). It follows that the GW phase, i.e. the deviation from the waveform PN parameters in Eqs. (7-8), is
setting a very tight constraint on the GW speed $v_g(f_{R,a})$ with respect to the one set in [2] by the GW-GRB observation delay. Fig. 1 shows the posteriors for scenarios III-IV obtained for the GW group velocity discrepancy from c.

Finally, we consider scenario IV. By looking at the results for $H_0, \alpha_M$ and $\beta_j$ in Tab. 1, we can see that these are very similar to scenarios II-III. However, the error budget for the dispersion relations are $\sim 20\%$ worse than the previous cases. The marginalization on $\alpha_M$ is contributing to the error budget of $\beta_j$. In other words, for this kind of measurement, the GW friction and the GW dispersion relation are correlated with each other. Indeed, correlations among the parameters that we want to infer play a crucial role for obtaining a bias-free measurement [41]. This means that one should not fix the values of $H_0$ or $\alpha_M$ or $\beta_j$ when trying to infer the remaining parameters.

In Tab. 1 we report the correlations observed between the pairs $H_0, \alpha_M$ and $\beta_j$. We define the correlation as $C = \text{cov}(X, Y)/\sqrt{\text{var}(X)\text{var}(Y)}$, where “cov” and “var” are the covariance and variance operators. In general for the pair $(H_0, \alpha_M)$ we observe a strong correlation. Indeed, we can obtain the same value of the posterior by increasing $\alpha_M$ and increasing $H_0$ or viceversa. This is consistent with the strong degeneracy of these two parameters in Eq. (1) for low redshift events. Concerning the pair $(H_0, \beta_j)$, for almost all the dispersion relations, we observe weak correlation. This is mostly due to the fact that $H_0$ is being constrained from the GW luminosity distance together with $\alpha_M$. Regarding the $(\alpha_M, \beta_j)$ correlation, this is not as significant as that of $(H_0, \alpha_M)$ but it is not completely negligible. Indeed, when combining several events like GW170817 including the increase of the detector sensitivity, these correlations will become more and more important for the determination of $H_0, \alpha_M, \beta_j$.

**Implications** — Here we comment our results for different theories which modify gravity in late-time universe. Our inference on $\alpha_M$ can be converted into constraints on those theories which modify the GW luminosity distance as in Eq. (1). Complementary constraints on $\alpha_M$ are obtained from CMB, e.g. [29, 50] assuming early time modifications of gravity. For instance, for models with extra parameters the parametrization $h \propto d_G^\gamma$ with $\gamma = (D - 2)/2$, $D = 4.08^{+0.05}_{-0.09}$ (1σ CL). For scalar-tensor theories with running Planck mass and no GW dispersion, $d_G/\alpha_{EM} = M^{\text{Pl, eff}}(\text{today})/M^{\text{Pl, eff}}(\text{source}) = 1.14^{+0.20}_{-0.17}$, see also [22]. For non-local RR models [20] with an effective Newton's constant we find $d_G^2/\alpha_{EM}^2 = G(\text{source})/G(\text{today}) = 1.32^{+0.49}_{-0.35}$.

Regarding the GW dispersion relation, the factor $\hat{\alpha}_j$ can be linked to Lorentz invariance breaking [22], or massive gravitons, or scalar/vector fields coupled to the metric [20, 53, 54]. Many of these theories also have a non-trivial $\alpha_M$.

![Fig. 2. Posterior distributions for $H_0, \alpha_M$ and $m_g$](#)

![Fig. 2. Posterior distributions for $H_0, \alpha_M$ and $m_g$](#)

For example, consider massive gravity (corresponding to $j = -2$), allowing also a non-zero $\alpha_M$. The square of the graviton mass, $m_g^2$, is given by $\alpha_{-2}$ divided by the square of the Planck constant. Fig. 2 shows the posteriors on $H_0, \alpha_M$ and $m_g^2$. In all the scenarios, the posteriors are compatible with GR at 1σ CL. In the most general scenario IV, we obtain $m_g^2 = 2.08_{-0.25}^{+0.30} \cdot 10^{-44} \text{eV}^2/c^4$ at 3σ CL. For comparison, the best indirect bounds for $m_g$ are provided from the solar system or weak lensing maps ($0 < m_g^2 < 10^{-64} \text{eV}^2/c^4$) or from binary pulsars ($0 < m_g^2 < 10^{-54} \text{eV}^2/c^4$) [55]. This constraint is tighter than the previous one using GW170817 ($m_g^2 < 9.0 \cdot 10^{-43} \text{eV}^2/c^4$) [40] and obtained by fixing $H_0$ and no information on redshift. The reason being due to the PN parameters posterior samples we are using. In fact, in [40] a dedicated pipeline adding a GW dispersion relation on the entire waveform is used, while the posterior samples that we are using are generated with only the inspiral part (that for GW170817 gives the majority of the SNR). In addition to the improvement related to the PN parameters posteriors, this work allows to improve the bound on $m_g^2$ by 15%. This can be computed running our analysis with a fixed $H_0 = 69.3 \text{km Mpc}^{-1}\text{s}^{-1}$ and providing the NGC4993 redshift information in one case and not in the other. In the former, we obtain $m_g^2 = 1.67^{+0.27}_{-0.10} \cdot 10^{-44} \text{eV}^2/c^4$, while in the latter $m_g^2 = 1.76^{+0.23}_{-0.14} \cdot 10^{-44} \text{eV}^2/c^4$.

**Conclusions** — In this Letter we constrain for the first time both cosmology and modified gravity theories.
conjointly, by combining the GW and electromagnetic observations of GW170817, using the method presented in \[11\]. Our starting point was the modified propagation Eq. \[1\] with frequency dependent GW speed in Eq. \[3\]. Given those assumptions, we measured the Hubble constant, GW friction and dispersion parameters conjointly for the first time. For $H_0$ and $\alpha_M$ we obtain results compatible with previous works \[22\]: adding $\alpha_j$ over and above $(\alpha_M, H_0)$ does not significantly modify the posteriors for either $H_0$ and $\alpha_M$. The reason is that while these variables are correlated with $\alpha_j$, the error budget on $H_0$ is too large for this effect to be observable \[11\]. We have shown that for the single event considered here, the error on $\alpha_j$ is increased by $\sim 20\%$ if, rather than fixing $\alpha_M$, we marginalize over it. We stress that when combining multiple events from a population of BNS sources, the joint correlations between the Hubble constant, GW friction and dispersion relation should be considered so as to avoid biases, hence the results presented in this Letter can be used for future population analyses.

We have also shown for all the GW dispersion relations considered, the GW-GRB observation delay introduced by gravity modifications is negligible relative to the (redshifted) emission delay at the source. As a consequence, combining GW phase and timing of the GRB, one can accurately time the GW emission at the source, concluding that the GRB is emitted exactly 1 second after the GW. This result can also be interpreted as a tighter constraint on the speed of GW at the merger frequency. Indeed, we find in the worst case (namely for \( j = -1/3 \)) \( |v_\mu(f_{R,d})/c - 1| \lesssim 10^{-17} \) while for massive gravity, where \( j = -2 \), \( |v_\mu(f_{R,d})/c - 1| \lesssim 5 \cdot 10^{-20} \). We stress that, though the GW phase is already setting a tight constraint on the GW dispersion relation, as remarked above, a timing of the GRB of the order of millisecond would improve the measurement of the dispersion relations corresponding to the PN parameters 4, 5l, 6 and 7 as we show in Fig. \[1\].

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\[\begin{align*}
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