An asymptotic safety scenario for gauged chiral Higgs-Yukawa models

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in collaboration with
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1. Introduction

2. Gauged chiral Higgs-Yukawa models
   - Questions? (Slide 9)

3. RG flow of the model

4. Fixed-point structure
   - Questions? (Slide 18)

5. Flow from the UV to the electroweak scale

6. Conclusions
   - Questions? (Slide 22)
Perturbation theory:
• Higgs suffers from *triviality problem* → Higgs sector not a fundamental QFT
• Running of Higgs requires *unnaturally fine-tuned* initial conditions to separate EW scale from Planck/GUT scale

Possible solutions:
• new d.o.f.
• new symmetries
• new quantization rules
• *Really cool nonperturbative QFT!* → *Asymptotic Safety*

Landau pole of the Higgs self-interaction

FRG flow equation:
\[ \partial_t \Gamma_k [\Phi] = \frac{1}{2} \text{STr}\{[\Gamma_k^{(2)} [\Phi] + R_k]^{-1} (\partial_t R_k) \} \]

Introduction

[FRG flow equation: \[ \partial_t \Gamma_k [\Phi] = \frac{1}{2} \text{STr}\{[\Gamma_k^{(2)} [\Phi] + R_k]^{-1} (\partial_t R_k) \} \] [C. Wetterich, 1993]

[Standard model of particle physics]

[Today: no gravity]

[FRG flow equation: \[ \partial_t \Gamma_k [\Phi] = \frac{1}{2} \text{STr}\{[\Gamma_k^{(2)} [\Phi] + R_k]^{-1} (\partial_t R_k) \} \] [C. Wetterich, 1993]

[Asymptotic Safety]

[S. Weinberg, 1976]
Introduction

Model:

- **Scalar Higgs-sector** chirally coupled to **top-bottom fermion sector** and **left-handed SU(N_L) gauge group**

\[ S_{cl} = \int d^dx \left[ \frac{1}{4} F_{\mu\nu}^i F^{i\mu\nu}_{\mu\nu} + (D_\mu \phi)^\dagger (D_\mu \phi) + \bar{m}^2 \phi^a \phi^a + \frac{\lambda}{2} (\phi^a \phi^a)^2 \right. \]

\[ \left. + i \left( \bar{\psi}_L^a D^{ab}_\mu \psi_L^b + \bar{\psi}_R^a \bar{\psi}_R^b \right) + \bar{h} \psi_R^a \phi^a \phi^a - \bar{h} \psi_L^a \phi^a \psi_R^a \right] \]

- Reduced model with the same structural deficits as Higgs sector of the standard model
- Perturbation theory of this model → **triviality & hierarchy problem**
- Model is motivated from simpler Higgs-Yukawa systems with UV fixed points for unrealistic parameters

[H. Gies & MMS, 2010]
[H. Gies, S. Rechenberger, MMS, 2010]
Nonperturbative FRG:

- Includes perturbative physics
- Features \textit{nonperturbative threshold phenomena}
  - decoupling of massive modes
  - \textit{dynamically generated masses} can be proportional to coupling $\lambda$

\[ \frac{1}{(k^2 + 2\lambda \rho_{\text{min}})^n} \]

typical loop contribution:

- no naive perturbative expansion even in weak-coupling regime

[J. Berges, N. Tetradis, C. Wetterich, 2002]

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**Higgs potential**

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**SSB**

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**Higgs potential**
Gauged chiral Higgs-Yukawa model

\[ S_{cl} = \int d^d x \left[ \frac{1}{4} F_{\mu \nu} F^{i \mu \nu} + (D^\mu \phi)^\dagger (D_\mu \phi) + \bar{m}^2 \phi^a \phi^a + \frac{\bar{\lambda}}{2} (\phi^a \phi^a)^2 \right. \\
\left. + i \left( \bar{\psi}_L D^{ab} \psi_L^b + \bar{\psi}_R \partial \psi_R \right) + \bar{h} \bar{\psi}_R \phi^a \psi_L - \bar{h} \bar{\psi}_L^a \phi^a \psi_R \right] \]

- Boson mass \( \bar{m} \), scalar self-interaction \( \bar{\lambda} \), Yukawa coupling \( \bar{h} \)
- Complex scalar field (2\( N_L \) real scalar fields): \( \phi^a = (\phi_1^a + i \phi_2^a) / \sqrt{2} \), with invariant: \( \rho := \phi^a \phi^a \)
- Covariant derivatives \( D_\mu \) in fundamental representation of gauge group \( D_{\nu}^{ab} = \partial_\nu - i \bar{g} W_{\nu}^i (T^i)^{ab} \)
- Indices \( a, b, \ldots = 1, \ldots, N_L \)
- Gauge coupling \( \bar{g} \), Yang-Mills vector potential \( W_{\nu}^i \)
- Gauge group generators: \( [T^i, T^j] = if^{ijk} T^k \)
- Non-abelian field strength: \( F_{\mu \nu} = \partial_{\mu} W_{\nu}^i - \partial_{\nu} W_{\mu}^i + \bar{g} f^{ijkl} W_{\mu}^j W_{\nu}^l \) with adjoint indices \( i, j, k, \ldots \)
Gauged chiral Higgs-Yukawa model

\[ S_{cl} = \int d^d x \left[ \frac{1}{4} F^{i\mu\nu} F_{i\mu\nu} + (D^\mu \phi)^\dagger (D_\mu \phi) + \bar{m}^2 \phi_a^\dagger \phi^a + \frac{\lambda}{2} (\phi^a \phi^a)^2 \right. \\
\left. + i \left( \bar{\psi}_L \gamma^\mu D_\mu \psi_L + \bar{\psi}_R \gamma^\mu D_\mu \psi_R \right) + \bar{h} \psi_R \phi^\dagger \phi \phi^a \psi_L - \bar{h} \phi^a \phi^a \phi^a \psi_R \right] \]

- Fermion field (N_g generations*):
  - Left-handed N_L-plet → top-bottom doublet for SU(N_L =2)
  - One right-handed fermion (top-quark component)
  - Right-handed bottom-type components not considered
  - Only top can become massive upon SB

*global Witten anomaly for odd N_g with N_L=2  [E. Witten, 1982]
Flow equation & truncation

- **FRG flow equation:** \( \partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{STr}\{[\Gamma_k^{(2)}[\Phi] + R_k]^{-1}(\partial_t R_k)\} \).

- **Truncation:**

\[
\Gamma_k = \int d^d x \left[ U(\rho) + Z_\phi (D^\mu \phi)^\dagger (D_\mu \phi) \right] \\
+ \bar{\psi}_R \phi^a \psi_L^a - \bar{\psi}_L \phi^a \psi_R^a \\
+ i (Z_L \bar{\psi}_L^a D^{ab} \psi_L^b + Z_R \bar{\psi}_R \partial \psi_R) \\
+ \frac{Z_W}{4} F_{\mu\nu}^i F^{i\mu\nu} + \frac{Z_\phi}{2\alpha} G_i G^i - \bar{c}^i \mathcal{M}^{ij} c^j \right].
\]

- **Left-handed fermions**
- **Right-handed fermion**
- **Higgs effective potential**
- **Higgs**
- **Higgs-Yukawa coupling**
- **Gauge fields**
- **Gauge-fixing term**
- **Ghosts**

\( R_\alpha \) - gauge with gauge fixing parameter \( \alpha \), flow of \( \bar{g} \) with background field formalism.

[L. F. Abbott, 1981]
[W. Dittrich & M. Reuter, 1986]
**Summary:** theory space is parametrized by

- wave function renormalizations of matter fields $Z_\phi, Z_L, Z_R$
- wave function renormalization of gauge fields $Z_W$ or gauge coupling $\bar{g}$
- Higgs-Yukawa interaction $\bar{h}$, the vev $\bar{v}$ and the Higgs effective potential $U$
Dimensionless renormalized parameters:

- Yukawa/gauge couplings: \( h^2 = \frac{k^d\bar{h}^2}{Z\phi Z_L Z_R} \), \( g^2 = \frac{\bar{g}^2}{Z_W k^{d-4}} \).

- Effective potential: \( u(\tilde{\rho}) = k^{-d}U(Z^{-1}\phi k^{d-2}\tilde{\rho}) \), \( \tilde{\rho} = \frac{Z\phi \rho}{k^{d-2}} \).

- Minimum of the potential: \( \kappa = \frac{Z\phi \bar{u}^2}{2k^{d-2}} = \tilde{\rho}_{\min} \).

- Expansion of effective potential: \( u = \frac{\lambda_2}{2!} (\tilde{\rho} - \kappa)^2 + \frac{\lambda_3}{3!} (\tilde{\rho} - \kappa)^3 + \cdots \).

- Anomalous dimensions: \( \eta_\phi = -\partial_t \log Z\phi \), \( \eta_W = -\partial_t \log Z_W \), \( \eta_L = -\partial_t \log Z_L \), \( \eta_R = -\partial_t \log Z_R \).
**Flow equations and fixed points**

**Fixed point and critical exponents:**

- Flow of couplings and \( \beta \)-functions:
  \[ \partial_t g_i = \beta g_i (g_1, g_2, \ldots) \]
- Fixed point:
  \[ \beta_i (g_1^*, g_2^*, \ldots) = 0, \forall i, \]
- Linearized flow and stability matrix:
  \[ \partial_t g_i = B_{ij} (g_j - g_j^*) + \ldots, \quad B_{ij} = \frac{\partial \beta g_i}{\partial g_j} \bigg|_{g=g^*}, \]

**This model gives flow equations for:**

- Effective potential \( u \rightarrow \) self-interactions \( \lambda_i \) and minimum \( \kappa \)
- Yukawa coupling \( h \)
- anomalous dimensions (can be solved as function of \( u, h, \lambda_i, \kappa \))
- gauge coupling \( g \) is kept fixed for the moment!

Set of flow equations:

\[
\begin{align*}
\partial_t \kappa &= \beta_{\kappa} (\kappa, \lambda_2, h^2, g^2), \\
\partial_t \lambda_2 &= \beta_{\lambda} (\kappa, \lambda_2, h^2, g^2), \\
\partial_t h^2 &= \beta_{h} (\kappa, \lambda_2, h^2, g^2),
\end{align*}
\]
Choose finite gauge coupling $g$ (no FP for flow of $g$)

For a given $g$ matter system shows NGFP in SSB regime

Strong dependence of FP on value of $g$

Position of the minimum $\kappa$ diverges as $g \to 0$

\[ \partial_t \kappa = \beta_\kappa(\kappa, \lambda_2, h^2, g^2) = 0, \]
\[ \partial_t \lambda_2 = \beta_\lambda(\kappa, \lambda_2, h^2, g^2) = 0, \]
\[ \partial_t h^2 = \beta_h(\kappa, \lambda_2, h^2, g^2) = 0, \]
\[ \partial_t g^2 \neq 0 \]
NGFP at finite fixed gauge coupling $SU(N_L=2)$

- Position of the minimum $\kappa$ diverges as $g \to 0$
- Asymptotically free couplings as $g \to 0$
- Combinations of couplings approach finite FP as $g \to 0$
  
  $\Rightarrow$ dimensionless mass parameters approach finite FP

\[
\begin{align*}
\partial_t \kappa &= \beta_{\kappa}(\kappa, \lambda_2, h^2, g^2) = 0, \\
\partial_t \lambda_2 &= \beta_{\lambda}(\kappa, \lambda_2, h^2, g^2) = 0, \\
\partial_t h^2 &= \beta_{h}(\kappa, \lambda_2, h^2, g^2) = 0, \\
\partial_t g^2 &\neq 0
\end{align*}
\]

- **FP value of** $(g^2 \kappa/2)$
- **FP value of** $(h^2 \kappa)$
- **FP value of** $(2\lambda \kappa/g^2)$

**“gauge boson mass”**

**“top mass”**

**“Higgs/g^2”**
Is this the Gaussian FP \((g \to 0)\)? No, it’s not!

- True GFP of present model has massless gauge bosons and massless chiral fermions
- Mass parameters arise from interplay of interaction terms in the flow equation in the weak-coupling limit \(\to\) genuine interaction effect
- Critical exponents do not agree with canonical dimensions at GFP (see below!)

\[
g \to 0
\]

- Minimum \(\kappa \to \infty\)
- Higgs mass \(\to 0\)
- Gauge boson mass \(\to\) finite
- Top mass \(\to\) finite

- Scalar potential approaches flatness in the UV with nonvanishing minimum!
For further investigation → mass parametrization:

- dimensionless renormalized mass parameters:
  \[ \mu_W^2 = \frac{1}{2} g^2 \kappa, \]
  \[ m_W^2 = \mu_W^2 k^2, \]

- dimensionful renormalized masses:
  \[ \mu_H^2 = 2 \lambda_2 \kappa, \]
  \[ m_H^2 = \mu_H^2 k^2, \]
  \[ \mu_t^2 = \kappa h^2, \]
  \[ m_t^2 = \mu_t^2 k^2. \]

Flow equations in mass parametrization:

\[ \partial_t \mu_H^2 = 2 (\partial_t \kappa) \lambda_2 + 2 \kappa (\partial_t \lambda_2), \]
\[ \partial_t \mu_t^2 = (\partial_t \kappa) h^2 + \kappa (\partial_t h^2), \]
\[ \partial_t \mu_W^2 = \frac{1}{2} (\partial_t \kappa) g^2 + \frac{1}{2} \kappa (\partial_t g^2), \]
\[ \partial_t g^2 = g^2 \eta_W. \]
**Fixed point in mass parametrization SU(N_L=2)**

- NGFP in mass parametrization in the limit $g \to 0$ (now also $\partial_t g^2 = 0$):

$$
\partial_t g^2 = 0, \quad \partial_t \mu_H = 0,
$$

$$
\chi = \frac{\mu_H^2}{g^2}, \quad \partial_t \chi^2 = \frac{1}{g^2} \partial_t \mu_H^2 - \frac{\mu_H^2}{g^4} \partial_t g^2, \quad \chi^* = -\frac{1}{16\pi^2} \left( \frac{\mu_t^2 (1 + 3 \mu_t^2)}{(1 + \mu_t^2)^3 \mu_W^2} - \frac{9 (1 + 3 \mu_W^2)}{4 (1 + \mu_W^2)^3} \right).
$$

**Mass parametrization uncovers line of NGFPs:**

- Includes NGFP from standard parametrization: $\mathcal{A}: (\mu_t^2, \mu_W^2, \chi^*) \approx (0.38, 0.21, 0.0037)$
Critical exponents $\textbf{SU}(N_L=2)$

Critical exponents on the line of fixed points as a function of the top mass parameter:

- Recover GFP for vanishing top mass parameter with canonical dimensions
- Complex pair of critical exponents for NGFP
- Special NGFP

$\textbf{A}: \left( \mu_t^2, \mu_W^2, \chi^* \right) \simeq (0.38, 0.21, 0.0037)$

$\textbf{A}: \quad \theta_{1/2} = 1 \pm 0.36i, \theta_3 = \theta_4 = 0$

$\textbf{B}: \left( \mu_t^2, \mu_W^2, \chi^* \right) \simeq (0.35, 0.19, 0.0037)$

$\textbf{B}: \quad \theta_{1/2} = 1, \theta_{3,4} = 0$
Summary: We found a line of UV stable fixed points

• 3 physical parameters + 1 FP choice on the line of FPs

• FPs can serve to define UV complete QFT of a gauged Higgs-Yukawa model

• Specifying an RG trajectory yields fully predictive long-range theory
Flow from the UV to the electroweak scale $SU(N_L=2)$

- Similar flows for gauge boson mass and Higgs mass
- Gauge coupling exhibits perturbative asymptotically free running in the UV
- Asymptotically safe trajectories:
  - IR exhibits standard Higgs phase as in perturbative scenario
  - UV controlled by FP at which continuum limit can be taken
**Flow from the UV to the electroweak scale SU(N_L=2)**

- Observation: Typical flows feature Higgs mass of about two orders of magnitude smaller than top and gauge boson mass.

- Only particularly tuned trajectories *Higgs mass to top mass ratio* approach realistic value ~125/175.

**Mass ratios for different initial conditions in the UV**

For 4 different UV choices of gauge coupling (colors)

- "Higgs/top"
- "gauge boson/top"

Towards realistic values!
Flow from the UV to the electroweak scale $SU(N_L=2)$

- Realistic values for “Higgs/top” can be achieved via **walking regime on intermediate scales**

![UV flow](image1)

- **Walking regime** on intermediate scales between **deep UV FP regime** and **IR freeze-out regime**
- Walking regime = “quasi-fixed-point regime” which extends over wide range of scales
- $\beta$-functions small but non-vanishing
  - Remnant of line of FPs at finite gauge coupling

![“Higgs/top”](image2)
Conclusions

- **Line of weak-coupling FPs from threshold effects** in RG flow of gauged chiral Higgs-Yukawa models
- Weak-coupling FPs well-controlled → No strong couplings, anomalous dimensions vanish
- NGFPs define UV complete asymptotically safe QFTs including an elementary scalar
- Critical exponents: 2 relevant + 1 marginally relevant direction + 1 exactly marginal direction
  - linear instead of quadratically running renormalization constants
- Typically small Higgs masses → Realistic masses by particularly tuned flow with walking regime
- Methods to deal with threshold phenomena are also available within perturbation theory
  - Reproduce our results e.g. with a mass dependent RG scheme

Thank you for your attention!

Link to paper: [http://arxiv.org/abs/1306.6508](http://arxiv.org/abs/1306.6508)

More questions
Backup slide 1: Truncation

- **FRG flow equation:**
  \[ \partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{STr}\{[\Gamma_k^{(2)}[\Phi] + R_k]^{-1}(\partial_t R_k)\} . \]

- **Truncation:**
  \[
  \Gamma_k = \int d^d x \left[ U(\rho) + Z_{D^\mu} (D^\mu \phi) \right. \\
  \left. + \bar{\psi}_L \phi^a \psi^a_L - \bar{\psi}_L \phi^a \psi_R \\
  \left. + \frac{1}{2} \bar{\psi}_L \phi^a \psi^a_R \right] .
  \]

**Higgs effective potential**

**Higgs-Yukawa coupling**

**left-handed fermions**

**right-handed fermion**

**gauge fields**

**gauge-fixing term**

**ghosts**

\[ R_\alpha \text{ - gauge with gauge fixing parameter } \alpha, \text{ flow of } \bar{g} \text{ with background field method:} \]

- **Gauge fixing condition:**
  \[ G^i(W) = \partial_\mu W^i_\mu + i\alpha \bar{v} \bar{g} (T^i_\hat{n}_\alpha \Delta \phi^a_1 + iT^i_\hat{n}_\alpha \Delta \phi^a_2) = 0 \]

- **Fadeev-Popov operator:**
  \[ M^{ij} = -\partial^2 \delta^{ij} - \bar{g} f^{ilj} \partial_\mu W^{l\mu} + \sqrt{2} \alpha \bar{v} \bar{g}^2 T^i_\hat{n}_\alpha T^j_\hat{a}_b \Delta \phi^b , \]
Masses in SB regime:

- **Gauge boson mass matrix:**
  \[
  \tilde{m}_{W}^{2} \, ij = \frac{1}{2} Z_{\phi} g^{2} \bar{v}^{2} \{ T^{i}, T^{j} \} \hat{n} \hat{n}.
  \]

- **Basis in adjoint color space:**
  \[
  \tilde{m}_{W}^{2} \, ij = \tilde{m}_{W,i}^{2} \delta^{ij} \quad \text{(no sum over } i \text{).}
  \]

- **Scalar mass matrix:**
  \[
  \tilde{m}_{\phi}^{2} \, ab = \bar{v}^{2} U^{\prime \prime} \left( \frac{\bar{v}^{2}}{2} \right) \hat{n}^{a} \hat{n}^{\dagger b}.
  \]

- **“top mass:”**
  \[
  \tilde{m}_{t} = \frac{\bar{h} \bar{v}}{\sqrt{2}}.
  \]