Implementation of conformal covariance by
diffeomorphism symmetry

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Abstract

Every locally normal representation of a local chiral conformal
quantum theory is covariant with respect to global conformal trans-
formations, if this theory is diffeomorphism covariant in its vacuum
representation.

The unitary, strongly continuous representation implementing con-
formal symmetry is constructed; it consists of operators which are in-
ner in a global sense for the representation of the quantum theory.
The construction is independent of positivity of energy and applies to
all locally normal representations irrespective of their statistical di-
mensions (index).

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1 Introduction

The conformal group in 1+1 dimensions is an infinite dimensional diffeomor-
phism group. Many interesting models exhibit this symmetry and typically
these models factorise into their chiral parts, each of which depends on one
light-cone coordinate only. In this short letter we prove that all locally nor-
mal representations of chiral conformal nets which exhibit diffeomorphism
symmetry in their vacuum representation admit an implementation of global
conformal transformations.

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This automatic implementability is interesting since for a class of conformal models with less symmetry the existence of non-covariant, locally normal representations has been established by Guido, Longo, Wiesbrock \cite{GLW98}. Moreover, our method applies in a very general setting: neither the representation theory of the diffeomorphism group, the index of the respective locally normal representation or even positivity of energy concern our approach at all.

Diffeomorphism covariance of a chiral conformal net $\mathcal{A}$ (see e.g. \cite{GL96} for general properties) means that there is a strongly continuous map $U_0$ from the group of orientation preserving diffeomorphisms of the circle, $\text{Diff}_+(S^1)$, into the unitaries on $\mathcal{H}_0$, the representation space of the vacuum representation of $\mathcal{A}$, implementing a geometric automorphic action $\alpha$ of $\text{Diff}_+(S^1)$:

$$U_0(\phi)\mathcal{A}(I)U_0(\phi)^* \equiv \alpha_\phi(\mathcal{A}(I)) = \mathcal{A}(\phi(I)), \ I \in S^1, \ \phi \in \text{Diff}_+(S^1).$$

The localisation regions are proper, i.e. open and non-dense, intervals $I$, denoted $I \subseteq S^1$, whose causal complements are their open complements, $I'$, in $S^1$. The subgroup $\text{Diff}_I(S^1)$ of diffeomorphisms localised in $I$ consists, by definition, of elements $\phi \in \text{Diff}_+(S^1)$ which act trivially on $I'$. For $\phi \in \text{Diff}_I(S^1)$ the adjoint action of $U_0(\phi)$ is to implement the trivial automorphism of $\mathcal{A}(I')$ and hence it is, by Haag duality of $\mathcal{A}$, a local observable, i.e. $U_0(\phi) \in \mathcal{A}(I)$.

$U_0$ defines a ray representation, as the cocycles $U_0(\phi_1)U_0(\phi_2)U_0(\phi_1\phi_2)^*$ commute with $\mathcal{A}$ and $\mathcal{A}$ is irreducible. With results of Carpi \cite{Car03} this shows that $U_0$ corresponds to a definite value of the central charge of the Virasoro algebra. We require $U_0(id) = \mathbb{1}$ and $\alpha \upharpoonright \text{PSL}(2, \mathbb{R})$ to be identical to the global conformal covariance of $\mathcal{A}$. In models having a stress-energy tensor, the restricted representation $U_0 \upharpoonright \text{PSL}(2, \mathbb{R})$ is in fact a representation of $\text{PSL}(2, \mathbb{R})$. The further analysis does not require the answer to the cohomological question whether this may be achieved always by a proper choice of phases for $U_0$.

We deal with a locally normal representation $\pi$ of $\mathcal{A}$, i.e. a family of normal representations $\pi_I$ of the local algebras $\mathcal{A}(I)$ by bounded operators on a Hilbert space $\mathcal{H}_\pi$, which is required to be consistent with isotony: $I \subset J \Rightarrow \pi_J \upharpoonright \mathcal{A}(I) = \pi_I$; given this condition is fulfilled, we say that the local representations $\pi_I$ are compatible.

By local normality, the maps $\pi \circ U_0 \upharpoonright \text{Diff}_+(S^1)$ define unitary, strongly continuous projective representations of the respective local diffeomorphism
subgroup with cocycles which are phases, since the local algebras are factors. We will use the presence of these representations in order to construct an implementing ray representation of the subgroup of global conformal transformations (Moebius group $\text{PSL}(2,\mathbb{R})$). We begin with a clarification on the relation of general diffeomorphisms to the representation $\pi$ induced by $U_0$.

We introduce the universal $C^*$-algebra $A_{\text{uni}}$ generated by the local algebras of $A$. The properties of $A_{\text{uni}}$ are summarised in

**Proposition 1** [FRS92, GL92] There is a unique $C^*$-algebra $A_{\text{uni}}$ such that

1. For all $I \subset S^1$ there exist injective, compatible embeddings $\iota_I : A(I) \rightarrow A_{\text{uni}}$ and $A_{\text{uni}}$ is generated by its subalgebras $\iota_I(A(I))$.

2. For any compatible family of representations $\{\pi_I\}$ there exists a unique representation $\pi$ of $A_{\text{uni}}$ by bounded operators on $H_\pi$ such that $\pi \circ \iota_I = \pi_I$.

Moreover, every representation $\pi$ of $A_{\text{uni}}$ restricts to a representation of $A$. The vacuum representation $\pi_0$ of $A_{\text{uni}}$ corresponds to the identity (defining) representation of $A$ on $H_0$: $\pi_0 \upharpoonright A(I) = \text{id} \upharpoonright A(I)$.

The action $\alpha$ of $\text{Diff}_+(S^1)$ on the net $A$ can be extended to an action by automorphisms of $A_{\text{uni}}$ through $\alpha \phi \circ \iota_I := \iota_I(U_0(\phi)) \circ \iota_I$. Our next goal is to establish that this action is inner. For this purpose we prove

**Proposition 2** Let $\phi \in \text{Diff}(S^1)$, then for each $J \subset S^1$ we have $\alpha \phi \upharpoonright \iota_J(A(J)) = \text{Ad}_{\iota_I(U_0(\phi))} \upharpoonright \iota_J(A(J))$. Each $\alpha \phi$, $\phi \in \text{Diff}_+(S^1)$, possesses an implementation by unitary elements of $A_{\text{uni}}$.

**Proof:** If there is a proper interval $\hat{J} \supset I \cup J$, the statement is obvious. If $\overline{I \cup J} = S^1$, we choose a covering $\{I_i\}_{i=1,2,3}$ of $I$ such that $I_i \subset S^1$, $I_3 \subset J$ and $I_1, I_2 \cup J$ both are contained in some proper interval. By Lemma 6 we find a factorisation $\phi = \prod_{i=1}^3 \phi_i$, $\phi_i \in \text{Diff}_+(S^1)$, if $\phi$ is contained in the neighbourhood $U_\varepsilon \subset \text{Diff}_+(S^1)$ defined in the Lemma, for $\varepsilon$ sufficiently small.

The two operators $\iota_I(U_0(\phi))$ and $\prod_{i=1}^3 \iota_{I_i}(U_0(\phi_i))$ coincide up to a scalar multiple of $1 = \iota_I(1_{A(I)})$. Thus we have:

$$\text{Ad}_{\iota_I(U_0(\phi))} \upharpoonright \iota_J(A(J)) = \prod_{i=1}^3 \text{Ad}_{\iota_{I_i}(U_0(\phi_i))} \upharpoonright \iota_J(A(J)) .$$
This proves the statement for $\phi \in U \epsilon$.

Now let $\phi$ be an arbitrary diffeomorphism localised in $I$. We will factorise $\phi$ into a product of diffeomorphisms such that the above applies. Let $\varphi$ denote the periodic diffeomorphism of $\mathbb{R}$ which corresponds to $\phi$ (cf proof of lemma 6). Define $\varphi_\lambda(x) = x + \lambda(\varphi(x) - x)$, $x \in \mathbb{R}$, and denote the corresponding element in $\text{Diff}_I(S^1)$ by $\phi_\lambda$.

With the covering $\{I_i\}_{i=1,2,3}$ above there is a $\delta > 0$ such that for all $\lambda \in [0, 1]$ the diffeomorphism $\phi_\lambda \circ \phi^{-1} \in U$. Then we can represent $\phi$ as a product $\phi = \prod_{k=0}^{n-1}(\phi_{k+1/n} \circ \phi^{-1}_{k/n})$.

for $n$ large enough, where each factor and hence $\phi$ satisfies the Proposition.

Since $\text{Diff}_+(S^1)$ is a simple group (theorem of Epstein, Herman, Thurston, cf [Mil83]) each $\phi \in \text{Diff}_+(S^1)$ has a presentation by a finite product of localised diffeomorphisms. One may take this presentation and the results proved so far in order to obtain the desired implementation, which completes the proof.

□

Remark: The implementation of a diffeomorphism $\phi$ by an element of $\mathcal{A}_{uni}$ is not unique, in general: two implementers of $\alpha_\phi$ may differ by an element from the centre of $\mathcal{A}_{uni}$.

In a representation $\pi$ of $\mathcal{A}_{uni}$ the implementers form a projective, unitary representation of $\text{Diff}_+(S^1)$ with cocycle in the centre of $\pi(\mathcal{A}_{uni})$ and which implements the automorphic action $\alpha$; if $\pi$ is a factorial representation, the cocycles are automatically phases. One would like to derive a genuine ray representation of $\text{Diff}_+(S^1)$ with cocycle which is given by phases for any general representation $\pi$, but we only know a way to do this for the subgroup $\text{PSL}(2, \mathbb{R})$ of global conformal transformations.

To this end we analyse the cocycle of the implementation of $\alpha$ and therefore we look for a definite choice of the implementing unitaries for diffeomorphisms close to the identity: For elements $\phi$ of a suitable neighbourhood $U_\epsilon$ of the identity the results of lemma 6 (appendix) allow us to choose a fixed covering $\{I_i\}_{i \in \mathbb{Z}_m}$ of $S^1$ by proper intervals and a fixed set of localisation maps $\Xi_i : U_\epsilon \to \text{Diff}_I(S^1)$ such that $u_\pi(\phi) := \prod_{i=1}^n \pi \circ U_0 \circ \Xi_i(\phi)$ defines a strongly continuous, unitary and unital map. The adjoint action of $u_\pi$ induces an implementation of $\alpha$ in $\pi$, which we will use in the following section.
2 Obtaining the implementation

We will now restrict our attention to the subgroup of global conformal transformations, $\text{PSL}(2, \mathbb{R})$, and construct a unitary, strongly continuous representation of its universal covering group $\text{PSL}(2, \mathbb{R})^\sim$ from $u_\pi |U_\varepsilon \cap \text{PSL}(2, \mathbb{R})$ as defined at the end of the previous section. This representation will implement the automorphic action $\alpha$ of $\text{PSL}(2, \mathbb{R})$ on $\mathcal{A}$ in the representation $\pi$ and will be inner in the global sense, i.e. it will be contained in the von Neumann algebra of global observables, $\pi(\mathcal{A}) := \bigvee_{t \in S^1} \pi_I(\mathcal{A}(I))$.

Let us begin with a closer look at the group $\text{PSL}(2, \mathbb{R})$ itself. We use the symbol $T$ for the one-parameter group of translations, $S$ for the special conformal transformations, $D$ for the scale transformations (dilatations) and $R$ for rotations. We choose parameters for the rotations such that the rotation group $R$ is naturally isomorphic to $\mathbb{R}/2\pi\mathbb{Z}$.

We can write every $g \in \text{PSL}(2, \mathbb{R})$ in the form $g = T(p) D(\tau) R(t)$, where each term depends continuously on $g$ (Iwasawa decomposition, [GF93], appendix I). In fact, any $g \in \text{PSL}(2, \mathbb{R})$ may be written as a product of four translations and four special conformal transformations, each single of them depending continuously on $g$, if one uses the identities:

$$D(\tau) = S(-e^{\tau} - 1) T(1) S(e^{\tau} - 1) T(-e^{-\tau}) , \quad (1)$$
$$R(2t) = S((-1 + \cos t)(\sin t)^{-1}) T(\sin t) S((-1 + \cos t)(\sin t)^{-1}) \quad (2)$$

According to lemma 6 (appendix), there are continuous, identity preserving localisation maps $\Xi_j$, $j = 1, \ldots, m$, which map a neighbourhood of the identity, $U_\varepsilon \subset \text{Diff}_+(S^1)$, into groups of localised diffeomorphisms such that we have $\prod_{j=1}^m \Xi_j(\phi) = \phi$, $\phi \in U_\varepsilon$. If we specialise to translations, this means that there is an open interval $I_\varepsilon$ containing 0 for which the mapping $t \mapsto \prod_j u_\pi(\Xi_j(T(t)))$ is unital and strongly continuous. We extend this map to all of $\mathbb{R}$ through a choice of a $\tau \in I_\varepsilon$, $\tau > 0$, defining $n_t \in \mathbb{Z}$ by its properties $t = n_t \tau + (t - n_t \tau)$, $t - n_t \tau \in [0, \tau[$, and setting

$$\pi^A(T(t)) := \left( \prod_j u_\pi(\Xi_j(T(\tau))) \right)^{n_t} \prod_j u_\pi(\Xi_j(T(t - n_t \tau))) \quad .$$

One can easily check that this is indeed a strongly continuous map into the unitaries of $B(\mathcal{H}_\pi)$ by recognising that the mappings involved are continuous and unital ($\pi(1) = 1$, $\Xi_i(id) = id$).
This procedure applies to the special conformal transformations as well, and we may use the result, the Iwasawa decomposition and (1), (2) to define for each $g \in PSL(2, \mathbb{R})$:

$$
\pi^A(g) := \prod_{i=1}^{4} T^{\pi(A)}(t^i_g) S^{\pi(A)}(n^i_g), \quad g \in PSL(2, \mathbb{R}).
$$

We have $\pi^A(id) = 1$. The following Lemma asserts that the $\pi^A(g)$ define an inner-implementing representation up to a cocycle in the centre of $\pi(A)$. To this end we define operators sensitive to the violation of the group multiplication law:

$$
\zeta^A(g,h) := \pi^A(g)\pi^A(h)\pi^A(gh)^*, \quad g,h \in PSL(2, \mathbb{R}).
$$

Lemma 3 $\pi^A : g \mapsto \pi^A(g)$ defines a strongly continuous mapping with unitary values in $\pi(A)$. The adjoint action of $\pi^A(g)$, $g \in PSL(2, \mathbb{R})$, on $\pi(A)$ implements the automorphism $\alpha_g$. $\zeta^A : (g,h) \mapsto \zeta^A(g,h)$ defines a strongly continuous 2-cocycle with unitary values in $\pi(A)' \cap \pi(A)$.

Proof: Unitarity is obvious. Strong continuity follows since we multiply continuous functions. The implementing property of the $\pi^A(g)$ follows immediately by the decomposition $g = \prod_{i=1}^{4} T(t^i_g) S(s^i_g)$, the subsequent decomposition of these into products of localised diffeomorphisms, the definition of $\pi^A(g)$ and the implementation property of the (generalised) ray representation $u_\pi$ of $\text{Diff}_+(S^1)$. At this point all properties of $\zeta^A$ follow immediately from its definition.

We write the abelian von Neumann algebra generated by the cocycle operators $\zeta^A(g,h)$ as follows: $Z^A \equiv \{ \zeta^A(g,h), \zeta^A(g,h)^* | g,h \in PSL(2, \mathbb{R}) \}''$. Obviously $Z^A$ is contained in the centre of $\pi(A)$. Now we are prepared to realise the construction itself:

Lemma 4 For every $\tilde{g} \in PSL(2, \mathbb{R})^\sim$ there exists a unitary operator $\zeta^A(\tilde{g}) \in Z^A$ such that

$$
U_{\pi}(\tilde{g}) := \zeta^A(\tilde{g})\pi^A(p(\tilde{g}))
$$

defines a unitary, strongly continuous representation, whose adjoint action implements the automorphic action $\alpha \circ p$ on $\pi(A)$; $p$ denotes the covering projection from $PSL(2, \mathbb{R})^\sim$ onto $PSL(2, \mathbb{R})$.

Proof: As $Z^A \subset \pi(A)' \cap \pi(A)'$ we may apply the direct integral decomposition (cf e.g. [KR86], chapter 14). This yields a decomposition of $\mathcal{H}_\pi$ as a
The direct integral of Hilbert spaces $\mathcal{H}_x$ and it implies: the action of $z^A(g, h)$ on $\mathcal{H}_x$, denoted by $z^A(g, h)(x)$, is a multiple of the identity $1_x$ and thereby defines for almost every $x$ a continuous 2-cocycle $\omega(f, g)_x \in \mathbb{S}^1 \subset \mathbb{C}$. The action of the operators $\pi^A(g)$ on $\mathcal{H}_x$, denoted by $\pi^A(g)(x)$, defines for almost every $x$ a unitary, strongly continuous, projective representation of $\text{PSL}(2, \mathbb{R})$, cf [Moo76].

For Lie groups with a simple Lie algebra the lifting criterion is valid [Sim68]. This ensures for almost every $x$ the existence of continuous phases $\omega(\tilde{g})(x), \tilde{g} \in \text{PSL}(2, \mathbb{R})^\sim$, such that $\omega(\tilde{g})(x)\pi^A(p(\tilde{g}))(x)$ defines a representation of $\text{PSL}(2, \mathbb{R})^\sim$. Integrating $\omega(\tilde{g})(x)$ over all $x$ yields a unitary $z^A(\tilde{g}) \in \mathcal{Z}^A$, depending strongly continuously on $\tilde{g}$. Integrating the $\omega(\tilde{g})(x)\pi^A(p(\tilde{g}))(x)$ yields a unitary, strongly continuous representation $U_\pi$ satisfying equation (4). $U_\pi(\tilde{g})$ is an element of $\pi(A)$ for every $\tilde{g}$ and implements $\alpha_{p(\tilde{g})}$ by its adjoint action due to Lemma 3.

□

The outcome of the construction presented above proves our main result; its uniqueness statement is a simple consequence of the fact that $\text{PSL}(2, \mathbb{R})^\sim$ is a perfect group (cf [Kos02], prop. 2):

**Theorem 5** Let $A$ be a chiral conformal, diffeomorphism covariant theory. Then any locally normal representation $\pi$ of $A$ is covariant with respect to the automorphic action of $\text{PSL}(2, \mathbb{R})$. The implementing representation may be chosen to be the unique globally $\pi(A)$-inner, implementing representation $U_\pi$ of $\text{PSL}(2, \mathbb{R})^\sim$.

If there are diffeomorphism covariant theories which possess locally normal representations violating positivity of energy, the construction of the inner-implementing representation given here applies even in cases in which the Borchers-Sugawara construction [Kos02] cannot be used, as the latter depends on the existence of implementations of translations and special conformal transformations which have positive energy. For representations with finite statistical dimension the spectrum condition is always fulfilled because of the theorem we have just derived and results of [BCL98]. For infinite index representations, of which examples are known (cf [Car02]) there exists a criterion for strongly additive theories, which was given in [BCL98], too. In presence of the spectrum condition both constructions agree by uniqueness.

Yngvason [Yng94] discussed conformally covariant derivatives of the $U(1)$-current as interesting examples of chiral conformal theories. It is straight-
forward to see that the first conformally covariant derivative had to exhibit a
diffeomorphism symmetry if it contained a stress-energy tensor (details in [Kös03b]). Guido, Longo, Wiesbrock studied locally normal representations of this model [GLW98] and found representations which manifestly do not admit an implementation of global conformal symmetry. As stated in [GLW98] this excludes the presence of a stress-energy tensor and diffeomorphism symmetry for this model as a consequence of theorem 5. Presence of a stress-energy tensor may be excluded directly as shown in [Kös03a].

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Appendix

The following technical lemma is crucial for the construction in section 2.

Lemma 6 Let \( U_\varepsilon \subset \text{Diff}_+^+(S^1) \), \( \varepsilon > 0 \) denote the neighbourhood of the identity
\[
\{ \phi \in \text{Diff}_+^+(S^1) : \sup_{z \in S^1} |\phi(z) - z| < \varepsilon \inf_{z \in S^1} |\phi(z)'| \}\tag{5}
\]
Let \( \{I_i\}_{i \in \mathbb{Z}} \) be a finite covering of the circle by proper intervals. Then for \( \varepsilon \) sufficiently small there exist continuous localising maps \( \Xi_i : U_\varepsilon \rightarrow \text{Diff}_{I_i}(S^1) \) with the following features:
\[
\phi = \prod_{i=1}^m \Xi_i(\phi), \quad \Xi_i(id) = id.
\]

Proof: We look at the equivalent formulation in terms of periodic diffeomorphisms of the real axis: \( \varphi \in C^\infty(\mathbb{R}), \varphi'(x) > 0, \varphi(x + 2\pi) = \varphi(x) + 2\pi \). The analogue of \( \varphi \) in \( \text{Diff}_+^+(S^1) \) is denoted by \( \hat{\varphi} \). The preimage of an interval \( I \subset S^1 \) under the covering projection \( p \) will be called \( \hat{I} \). We choose a smooth partition \( \mu \) of unity on \( S^1 \) satisfying \( 1 \geq \mu_i \geq 0, \text{supp}(\mu_i) \subset I_i \). On the covering space we define \( \lambda_i(x) := \mu_i(p(x)) \).
Defining \( \psi_k(\varphi)(x) := x + \sum_{i=1}^k \lambda_i(x)(\varphi(x) - x) \), \( k = 0, \ldots, m \), we have:

\[
\Psi_k[\varphi'](x) = (1 - \sum_{i=1}^k \lambda_i(x)) + \sum_{i=1}^k \lambda_i(x)\varphi'(x) + \sum_{i=1}^k \lambda'_i(x)(\varphi(x) - x)
\]

\[
\geq \inf_{\xi \in \mathbb{R}} (\min\{1, \varphi'(x)\}) - \sup_{\xi \in \mathbb{R}} (\sum_{i=1}^m |\lambda'_i(\xi)|) \cdot \sup_{\xi \in \mathbb{R}} |\varphi(\xi) - \xi|
\]

(6)

For a periodic diffeomorphism \( \varphi \) we have \( \inf \varphi'(x) \leq 1 \). With \( \varepsilon > 1 \), (6), imply

\[
\varepsilon^{-1} > \sup_{\xi \in \mathbb{R}} \sum_{k=1}^m |\lambda'_k(\xi)|,
\]

hence \( \Psi_k[\varphi] \) is a periodic diffeomorphism.

Moreover \( \Psi_k[\varphi] \) satisfies the estimate \( |\Psi_k[\varphi](x) - x| \leq |\varphi(x) - x| < \varepsilon \). We can now choose \( \varepsilon \) small enough, such that \( |\chi(x) - x| < \varepsilon \), for all \( x \), \( \chi \in \text{Diff}_+(S^1) \), implies \( \chi(\text{supp} \lambda_k) \subset I_k \). Then \( \Psi_{k-1}[\varphi]^{-1}(I'_k) \subset (\text{supp} \lambda_k)' \). Moreover, \( \Psi_{k-1}[\varphi] \) and \( \Psi_k[\varphi] \) coincide on \( (\text{supp} \lambda_k)' \), hence \( \Xi_k(\varphi) := \Psi_k[\varphi] \circ \Psi_{k-1}[\varphi]^{-1} \) defines a periodic diffeomorphism, whose counterpart in \( \text{Diff}_+(S^1) \), namely \( \tilde{\Xi}_k(\varphi) \), is localised in \( I_k \). We define the localising maps on \( U_\varepsilon \) as follows: \( \Xi_k(\varphi) := \tilde{\Xi}_k(\varphi) \).

Continuous dependence of \( \Xi_k(\varphi) \) on \( \varphi \) is obvious, \( \Psi_k[id] = id \) yields \( \Xi_k(id) = id \). Finally, with \( \Psi_m[\varphi] = \varphi \) and \( \Psi_0[\varphi] = id \):

\[
\prod_{k=1}^m \Xi_k(\varphi) = \Psi_m[\varphi] \circ \Psi_{m-1}[\varphi]^{-1} \circ \Psi_{m-1}[\varphi] \circ \ldots \circ \Psi_0[\varphi]^{-1} = \varphi \circ id
\]

□

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