A SEARCH FOR MILLIMETER EMISSION FROM GAMMA-RAY BURSTS
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ABSTRACT
We have used the two-year Differential Microwave Radiometer data from the Cosmic Background Explorer (COBE) satellite to systematically search for millimetric (31–90 GHz) emission from the gamma-ray bursts (GRBs) in the Burst and Transient Source Experiment (BATSE) GRB 3B catalog. The large beam size of the COBE instrument (7° FWHM) allows for an efficient search of the large GRB positional error boxes, although it also means that fluxes from (point-source) GRB objects will be somewhat diluted. A likelihood analysis has been used to look for a change in the level of millimetric emission from the locations of 81 GRB events during the first two years (1990 and 1991) of the COBE mission. The likelihood analysis determined that we did not find any significant millimetric signal before or after the occurrence of the GRB. We found 95% confidence level upper limits of 175, 192, and 645 Jy or, in terms of fluxes, of 9.6, 16.3, and 54.8 × 10⁻¹³ erg cm⁻² s⁻¹, respectively at 31, 53, and 90 GHz. We also looked separately at three different classes of GRBs, including the top 10 (in peak flux), “short-burst,” and “long-burst” subsets and found a similar upper limits for each subset. While these limits may be somewhat higher than one would like, we estimate that using this technique with future missions could push these limits down to ~1 mJy.

Subject headings: gamma rays: bursts — methods: observational — radio continuum: general

1. INTRODUCTION
The first detection of gamma-ray bursts (GRBs) were made by the Vela nuclear monitoring satellites in the 1960s (Klebesadel, Strong, & Olson 1982). After the launch of the Compton Observatory (CGRO) with the Burst and Transient Source Experiment (BATSE), the number of detected GRBs has greatly increased. Despite this added knowledge, we still do not know what the source of GRBs are. One type of information that could help us unravel the puzzle would be observations at other wavelengths, so we can identify a population of counterparts for separate study. Despite many different counterpart searches over different portions of the radio spectrum, no GRB counterpart population has been identified (Dessenne et al. 1996; Frail et al. 1994; Koranyi et al. 1995; Hjellming & Ewald 1981; Schaefer et al. 1989; for a review see McNamara, Harrison, & Williams 1995). These works concentrated on studying the delayed emission of single GRB events. Two works (Inzani et al. 1982; Baird et al. 1975) have been published so far about an extended sky search for radio-counterparts similar to the work presented in this paper.

Two problems arise when searching for counterparts: (1) the error boxes on GRB locations from BATSE are typically a few degrees, which requires a large amount of searching time with usual astronomical telescopes; and (2) the spectrum, intensity, and duration of the counterpart signals are all very model dependent, so it is hard to optimize counterpart search experiments. We note that the extreme isotropy of the GRB population (Briggs et al. 1996) suggests that the GRBs are cosmological sources. In some cosmological models, in which the GRBs are powered by cosmic fireballs, the GRBs will manifest delayed emission at energies much lower than that of gamma rays. However, the timescale for when the lower frequency emission peaks can be anywhere from hours to months after the GRB, depending on the geometry and other properties of the burst source (see, e.g., Paczynski & Rhoads 1993; Meszaros & Rees 1993, 1997; Katz 1994).

Here we focus on a search for delayed emission from or precursors of GRBs in the millimetric region of the spectrum. This is a more thorough and complete analysis than was presented earlier in Schaefer et al. (1995). Millimetric observations were made fortuitously by the COBE (Cosmic Background Explorer) satellite with the Differential Microwave Radiometer (DMR) experiment. COBE observes the full sky every 6 months at three frequencies (31, 53, and 90 GHz) with six radiometer pairs. (For a more complete description of the COBE mission, see, e.g., Boggess et al. 1992.) For models of GRBs with delayed millimetric emission, the best constraints for the whole GRB population can be found only from the COBE database.

Unfortunately for this study, COBE was not designed for studying point sources. The radiometer horns have a FWHM of 7° to purposely dilute the effect of point sources. The radiometers have rms noises of ~23–59 mK s¹/² in antenna temperature. One mK corresponds to a point-source emission of ~1800 (ν/53 GHz)² Jy. In 1 s of observations, COBE has the following sensitivities: 26.4, 28.8, and 98.6 KJy for, respectively, the 31, 53, and 90 GHz channels. These sensitivities are of the same order of magnitude of the two similar high-sky coverage searches (Baird et al. (1975); Inzani et al. 1982). Given these noise levels, the best use of COBE data is for studying delayed or anticipated emission on long observing times where the noise can be reduced by integration to a useful level. The COBE data has also been used to put upper limits on prompt emission that would be coincident with the GRB event (see Bonnet et al. 1995). Another way to beat the radiometer noise is to average over a large ensemble of GRBs. We have done our analysis on the data from the first two years of publicly available COBE data. Only 207 GRBs were recorded by GRO during this period. This number can be increased by a factor of ~4 by using the remaining two years of COBE data. We plan to analyze the remaining two years’ worth of data in the future.

The study presented in this paper produced upper limits...
that are not very stringent when compared with some theoretical scenario such as, for example, that of Paczynski & Rhoads (1993). The same authors, however, encourage pushing the search for radio counterpart to higher frequencies. According to their model, the peak flux expected is proportional to $v^{0.8}$. This paper presents a search for counterparts at the three highest frequencies ever used. Other observers rarely worked in the GHz range. Another important issue concerns the fact that the GRBs may be seen as if they were only for those GRBs with statistical error radii of less than $5^\circ$. The average position error. Including these bursts with larger error radii of the statistical error circles range from a low of $0^\circ.28$ to a high of $27^\circ$. There are two problems with very large error radii. First, any GRB signal will be severely diluted and so will not contribute much useful information. Second, larger error circles will include many radio sources (galaxies, AGNs, etc.), so the variance of the background-subtracted data does not really decrease with increasing error circle size as argued above. We have decided to carry out our analysis only for those GRBs with statistical error radii of less than $3.5^\circ$, which is the half-width at half-maximum of the COBE beam size. At this angle, the signal is diluted by a factor of 2 when compared to that from a GRB with zero statistical position error. Including these bursts with larger error angles leads to no significant improvement on our limits on the GRB emission, so we do not consider them here any further. The GRB locations and error angles are taken from the BATSE 3B Catalog (Meegan et al. 1996).

2. DATA SELECTION

2.1. Gamma-Ray Burst Selection Criteria

We have had to decide the amount of sky that is included in our “observation” of the gamma-ray burst location. We needed to balance two conflicting trends. The accuracy of our sky measurement, determined by $N$ observations, goes as $N^{-1/2}$, so we want to observe a large area to increase accuracy. On the other hand, if we observe too large a region of sky, we will dilute any signal with observations of blank sky. Intuitively, one might expect that the observations will be optimized if we collect observations in an solid angle area of radius $R$ given by $\pi R^2 = 2\sigma_g^2 + \sigma_e^2$, where $\sigma_g^2$ is the error angle of the GRB location and $\sigma_e = 3^\circ$ is the Gaussian width of the COBE DMR beams. We have used the BATSE team’s recommended error angle, which is determined by the quadrature sum of the statistical error angle and a $1.6^\circ$ systematic error angle. A more formal argument verifying that the choice of radius $R$ is indeed optimal is given in the Appendix.

Having chosen the proper radius of coverage around the GRB central position, we must then decide what range of gamma-ray burst error boxes to include in our analysis. The radii of the statistical error circles range from a low of $0.28$ to a high of $27^\circ$. There are two problems with very large error circles. First, averaging over a large area will mean that any GRB signal will be severely diluted and so will not contribute much useful information. Second, larger error circles will include many radio sources (galaxies, AGNs, etc.), so the variance of the background-subtracted data does not really decrease with increasing error circle size as argued above. We have decided to carry out our analysis only for those GRBs with statistical error radii of less than $3.5^\circ$, which is the half-width at half-maximum of the COBE beam size. At this angle, the signal is diluted by a factor of 2 when compared to that from a GRB with zero statistical position error. Including these bursts with larger error angles leads to no significant improvement on our limits on the GRB emission, so we do not consider them here any further. The GRB locations and error angles are taken from the BATSE 3B Catalog (Meegan et al. 1996).

2.2. Selection of COBE Data

The COBE team has provided the DMR observations as a set of time-ordered data. This set contains the temperature differences observed by each pair of radiometer horns integrated for each half-second of the flight. The average posi-

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1 Two tables from the 3B Catalog have been revised and are included in the on-line version of the catalog at HEASARC.
Fig. 1.—Twenty-four hr average antenna temperatures measured by the COBE satellite. Here we show the observations by averaging both 53 GHz receiver pairs at the location of the GRB 3B 910609, for which we have good coverage over the whole period (1990–1991). (a) Raw antenna temperatures. The solid line is the background signal predicted using the average of the 53A and 53B skymaps. (b) Residual antenna temperatures, which are still largely dominated by Gaussian receiver noise. Large deviations from zero occur when only a few observations were made in that 24 hr period and so are very noisy.

We want to use these residuals to look for delayed emission or precursors from the GRB source. Unfortunately, we must have some model in mind to test the data for goodness of fit. The model cannot be very complicated for testing against the limited sample here because most of the GRB positions are only observed part of the year and the time interval between the GRB and the end of the COBE data is different for each burst. When the full data set is analyzed, we can test more sophisticated models, but for now we model the millimetric GRB signal as a simple increase in emission coincident with the GRB event that remains at a constant level thereafter.

4. RESULTS

We have found the differences of the average signals coming from the GRB location before and after the GRB event for all six DMR receivers (e.g., 31A, 31B, 53A, etc.). We then make a \( \sigma_{\text{channel}}^{-2} \) weighted average of the A and B channels for a final result at each frequency. This is especially important for the 31 GHz channel, as the noise in the 31B GHz channel increased by a factor of more than 2 on 14 October 1991. Since this increase happens in the time after most of the gamma-ray bursts, weighting the averages by \( \sigma_{\text{channel}} \) ensures that the excess noise will not lead to false detections at 31 GHz. We also find the weighted 1 \( \sigma \) error for the differences. We then calculate the likelihood that the ensemble of differences in temperature contain a significant GRB signal.

We have looked at the results of the entire set and three interesting subsets and present the results in the next four subsections.

4.1. All Gamma-Ray Bursts with Error Angle Radii \( \leq 3.5\)°

We look at the complete set of GRB bursts in the BATSE 3B catalog and select all of those with statistical error angle radii less than 3.5°. This leaves us with 81 GRBs that have temperature differences before and after the event.

The likelihood functions are shown in Figure 2a, and the 95% confidence upper limits are shown in Table 1. These limits cover the ensemble of GRB events. We note that including all 81 GRBs leads to a bump in the likelihood of the 31 GHz frequency, which might be construed to be a slight detection. After looking at the temperature differences, we find that this "detection" is caused largely by three GRBs (910712, 910718, and 910807 in the BATSE 3B catalog). Two of these, 910712 and 910807, have somewhat incomplete coverage, which leads to larger variability in both the GRB averages and the background level derived from the skymap. The coverage after the GRB event is particularly spotty in 910807. The daily temperature averages for these GRBs at 31 GHz are shown in Figure 3. It can be seen in Figure 3 that GRB 910718 has more complete coverage than the other two odd events, but we find that the temperature is larger before the event than after the event. Unless this GRB has a higher amount of emission before the GRB event than after, there is no GRB signal here.
Certainly no clear emission features are evident in these data. If these were real detections, we might expect that they would also be seen in at least the 53 GHz and possibly the 90 GHz channels. However, all of these higher frequency temperature differences for these “odd” GRBs are consistent with zero at the 2σ level, so we do not believe that they are real detections. We also calculate what the 31 GHz likelihood (in Fig. 2a) and limit are if we ignore these “odd” GRBs. This limit is denoted by “w/o 3.” The limits obtained from the 53 and 90 GHz channels are little changed, as these GRBs were already consistent with zero in these channels. Lastly, we note that there is another “odd” GRB at 53 GHz (see the temperature history in Fig. 3d). Although the 53 GHz likelihood for all GRBs is completely consistent with no detection, GRB 910626 shows up as a negative detection particularly in the top 10 and short-burst subsets. If we eliminate this burst from the 53 GHz likelihood, the upper limit on the whole set at 53 GHz drops to 192 Jy.

We also note that if all of the channels had the same detector noise temperature, we would expect that the limits would scale as \( v^2 \) when we convert from temperature to flux. However, since the noise varies by almost a factor of 3, we see that the limits from the 53 GHz (the quietest channel) observations are nearly the same as those from the 31 GHz (the noisiest channel) observations.

4.2. Bursts Longer Than and Shorter Than 2 s

We now divide the GRB population into two subsets based on the GRB duration. It has been shown that there are two seemingly different populations (Kouveliotou et al. 1993) that are identified when their \( T_{90} \) parameter, i.e., the length of time in which 90% of the flux is observed, is longer or shorter than 2 s. The short bursts have harder spectra. Within the context of a cosmological GRB interpretation, it
is expected that the short bursts are nearby and the long bursts are more distant (see, e.g., Briggs et al. 1995). Separating these classes might reveal some feature that is diluted when the populations are combined.

Most of the GRBs with error angles ≤ 3.5 are long bursts. This is a consequence of the fact that it is easier for BATSE to get better location information when the burst occurs over a longer time. There are 72 long bursts and nine short bursts in our data set. The likelihood functions for the long bursts are shown in Figure 2b. Since most of the bursts in our selected data are long, it is not surprising that the upper limits from the long bursts are very similar to those for the entire set. In the total set, we found three GRBs, all long bursts, that caused a seeming false detection. The limits and likelihood are calculated excluding those GRBs, just as described above, and are shown in Figure 2b and Table 1.

There are only nine short bursts with error angles ≤ 3.5 in this GRB sample. Since these bursts are usually interpreted as being closer, it is hoped that better model limits may be obtained. The likelihood functions for this set are

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**TABLE 1**

**Upper Limits (95% confidence level) on Delayed or Anticipated Millimetric Emission (in Jy) for Each Subset**

| Channel (GHz) | All* | Long* | Short* | Top 10* |
|--------------|------|-------|--------|--------|
| 31 ......... | 247  | 331   | 562    | 382    |
| 53 ......... | 217  | 202   | 990    | 755    |
| 90 ......... | 645  | 675   | 2205   | 1172   |

Improved Limits after Removing Selected Bursts

| Channel (GHz) | All* | Long* | Short* | Top 10* |
|--------------|------|-------|--------|--------|
| 31 ......... | 175  | 185   | 712    | 375    |
| 53 ......... | 192  | 690   | 555    |        |
| 90 ......... |      | 2700  | 1225   |        |

* When the selected bursts are included, error angles (statistical error circle radii averaged over the subset) are 1.7, 1.6, 2.2, and 0.71 for the data sets containing all bursts, short bursts, long bursts, and the top 10 bursts, respectively.

* Limits without including the three GRBs that imply a detection at 31 GHz (see text).

* Limits without 910626.

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**Fig. 3.**—Averages over 24 hr vs. time of the GRBs that are responsible for the weak “detections” in our likelihood functions. Panels (a–d) are the temperature histories for bursts 910712, 910718, 910807, and 910626, respectively.
shown in Figure 2b. In this case, we see that a weak detection occurs in the 53 GHz channel. This bump seems to be entirely caused by 910626, whose temperature history is shown in Figure 3b. We can see that the observations are extremely sparse for this burst. As with the three “odd” GRBs found in the long-burst sample, this event does not show a significant temperature difference in the 31 and 90 GHz data, so we doubt that this is a real detection. By throwing out this GRB, we improve the 95% confidence limit at 53 GHz by about 30%. However, because of the small number of GRBs in this sample, throwing away one significantly affects the limits in the other channels. In Table 1, we show how the limits weaken in the other channels when we throw away 910626. We have shown only the difference in the 53 GHz likelihood function before and after disposing of 910626 in Figure 2c.

4.3. The Top 10 Gamma-Ray Bursts

Finally, we select the top 10 GRBs in peak flux seen by BATSE during the first two years of COBE data. One might hope that since these GRBs had the strongest gamma-ray emission, they might also show the strongest radio effect. Since these were very strong bursts, enough photons were observed to fix accurate positions so the error radii of these bursts are rather small. As with the other sets, we show the likelihood functions in Figure 2d and the 95% confidence upper limits in Table 1. As in the short-burst set, the 53 GHz channel shows a weak detection. Once again, this seems to be caused by the burst 910626, which is common to both subsets. If we again remove 910626 from the sample, the 53 GHz likelihood function drops dramatically and the 53 GHz upper limit decreases by 30% (see Table 1). However, in this case, dropping one of the 10 GRBs from the sample does not affect the upper limits for the 31 and 90 GHz channels. The 90 GHz limit increases by an amount consistent with (10/9)^1/2, but the 31 GHz limit is nearly unchanged.

5. DISCUSSION AND FUTURE PROSPECTS

As we find no significant detections of GRBs, the upper limits presented in Table 1 are our main results. We can look in detail at the results from the subsets to see if they are consistent with simply observing noise. The main difference between the subsets is the number of GRBs used. The scaling with 1/n^1/2 is seen in the data; however, there is certainly more going on there. This is most noticeable when comparing the limits derived in the short burst and top 10 subsets, as the limits are much larger in the short-burst set.

Part of the reason for this can be discovered when we look at the average GRB error angles in the different subsets (Table 1). The average error angle for the whole set is 1.7; the angles for the short bursts are significantly larger than the average, and the angles for the top 10 bursts are significantly smaller than average. This means there will be different amounts of point-source dilution of the signal in the different subsets. Using the proper beam dilution factors when comparing the results accounts for most of the rest of the difference. For example, the difference between the limits from the 53 GHz channel between the short and top 10 subsets is explained very well by the difference in GRB number and dilution factors. This is reassuring, as 53 GHz is the quietest channel. Interestingly, the limits are more consistent if we consider only those in which the “odd” GRBs are excluded. Lastly, the 90 GHz channel limits for the short bursts are somewhat larger than the scaling relations would predict.

Limits on GRB emission from the total population are roughly what we would expect if the detectors were seeing nothing but detector noise. We take this as an indication that method of “observing” GRBs is working quite well. Since GRO was launched in April 1991, the overlap between COBE and GRO is only 8 months during the first two years of COBE data. Therefore, including the final two years of COBE data will quadruple the number of GRBs observed, so the sensitivity would increase by a factor of 2.

A much larger increase in sensitivity can be achieved if the currently planned satellites INTEGRAL (a gamma-ray observatory satellite) and MAP and/or COBRAS/SAMBA (microwave anisotropy observing satellites) are successfully deployed. We estimate that combining observations from INTEGRAL with either MAP or COBRAS/SAMBA will increase the sensitivity in the 20–90 GHz region by 3 orders of magnitude or more. The improvements come largely from three factors: (1) the detector noise has decreased by about two orders of magnitude; (2) INTEGRAL will provide error boxes on the order of arcminutes, so less blank sky will need to be observed; and (3) the smaller beam sizes add another order of magnitude in sensitivity to point sources. (Note that although the solid angle is over 2 orders of magnitude smaller than COBE’s, one order of magnitude is lost because the satellites will spend less time observing the GRB location.) We point out that COBRAS/SAMBA also has a cryogenic bolometer detector at higher frequencies that should yield considerably better results. The best limit should be obtained from the 143 GHz channel of COBRAS, which has the lowest detector noise. We estimate that if there is no signal from GRBs, this channel will yield an upper limit of about 1 mJy.

6. CONCLUSIONS

We have analyzed the set of GRBs observed by BATSE on GRO for the presence of delayed emission or precursors in the 31–90 GHz region using the data from COBE. We find limits for the amount of possible millimetric emission from this set using a crude model (DC change in emission level) of the millimetric emission. Our main results are the likelihood functions in Figure 3 and the 95% confidence upper limits on GRB emission in Table 1. For the entire set of 81 GRBs with useful positional error boxes (error radii < 3.5”), the 95% confidence upper limits are 175, 192, and 645 Jy at 31, 53, and 90 GHz, respectively.

We have also selected several subsets for study. We have looked at the long- and short-burst (greater than or less than 2 s in duration) subsets, as it has been speculated that they are at different distances in cosmological models. We also examine the top 10 GRBs (rated by BATSE peak flux counts) under the reasoning that if there is any effect, it is likely that the effect will be greatest in the strongest GRBs. We find that these subsets do not show any significant detections and that the limits scale properly from the entire population limits.

These limits are the best on the delayed/precursors flux in the millimetric region of the spectrum. While the particular limits from this study perhaps do not put much pressure on theoretical models, we have demonstrated the viability of using this technique. We expect that using this method with the future INTEGRAL in combination with MAP and/or COBRAS/SAMBA satellites will improve the limits.
by many orders of magnitude, which will be much more interesting theoretically. Lastly we emphasize that with these satellites we have complete sky coverage, so the entire population of GRBs can be observed. Seeing the entire population allows comparisons of different GRB subsets, which is useful for diagnosing the properties of the GRBs. We believe this technique therefore has a promising future.

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APPENDIX

SKY-AVERAGING DILUTION EFFECTS

We show here that there is an optimal angular distance over which one should integrate to get the best limit on GRB emission. We first want to consider the signal seen by a point-source GRB at location \( r_s \), when the COBE beam center is pointing at a position \( s \). The temperature \( T(s) \) seen by the COBE DMR radiometer is then

\[
T(s) = C \exp \left( -\frac{|s - r_s|^2}{2\sigma_g^2} \right),
\]

where \( C \) is the flux of the GRB source converted into antenna temperature and \( \sigma_g = 3\degree \) is the COBE Gaussian beam size.

The GRB location \( r_s \) is not known accurately; instead we give a Gaussian probability of finding it at a location \( r \) within an error angle of \( \sigma_g \) of the central location,

\[
\Pr(r) = \frac{1}{2\pi\sigma_g^2} \exp \left( -\frac{|r - r_s|^2}{2\sigma_g^2} \right),
\]

where \( \sigma_g \) is the quadrature sum of the statistical error (\( \sigma_{\text{stat}} \)) and the systematic error (\( \sigma_{\text{sys}} = 1\degree \)), as recommended by the BATSE team (Meegan et al. 1996). If we define the origin so that \( r_s = 0 \), the expected temperature seen by pointing the radiometer at location \( s \) is

\[
\langle T(s) \rangle = C \int d^2r 2\pi\sigma_g^2 \exp \left( -\frac{|r|^2}{2\sigma_g^2} \right) \exp \left( -\frac{|s - r|^2}{2\sigma_g^2} \right),
\]

\[
= \frac{C}{\sigma_g^2/\sigma_s^2 + 1} \exp \left( -\frac{|s|^2}{2(\sigma_s^2 + \sigma_g^2)} \right),
\]

(A3)

In order to get a measurement, we average the observations made in a circle or radius \( R \) around the GRB central location. The sampling of the sky in this circle is roughly uniform, so the result of our analysis yields an unweighted average of the expected temperature \( \langle T \rangle \) in this circle:

\[
\langle T \rangle = \frac{C}{\sigma_g^2/\sigma_s^2 + 1} \int \frac{d^2s}{\pi R^2} \exp \left( -\frac{|s|^2}{2(\sigma_s^2 + \sigma_g^2)} \right) = \frac{C}{\sigma_g^2/\sigma_s^2 + 1} \frac{1}{x^2} \frac{1}{1 - \exp (-x^2)},
\]

(A4)

where \( x^2 = R^2/[2(\sigma_s^2 + \sigma_g^2)] \). We now can calculate the amount of signal dilution to expect when we make our average of GRB “observations.” If we were pointed directly at a GRB location that had no position error, we would get an antenna temperature of \( C \). The ratio of the term on the RHS of equation (A4) to \( C \) gives us a dilution factor \( D \):

\[
D = \frac{1}{\sigma_g^2/\sigma_s^2 + 1} \frac{1}{x^2} \frac{1}{1 - \exp (-x^2)},
\]

(A5)

which describes the diminishment of any GRB signal in our averaging process. The resulting difference of averaged antenna temperatures before and after the GRB event must be divided by this factor for each GRB. We can also use equation (A4) to define an optimal angle over which to average our temperature observations. As we increase the radius of our observing circle \( R \), the instrumental noise in the average of GRB observations goes down as the inverse square root of the number of observations \( N^{-1/2} \) or as \( R^{-1} \). On the other hand, increasing \( R \) also implies that we will be diluting the signal. The dilution factor in equation (A5) is roughly constant when \( R^2 \lesssim \sigma_s^2 + \sigma_g^2 \) and decreases as \( R^{-2} \) at larger \( R \). The signal-to-noise ratio of our measurement will then be determined by the ratio of the dilution factor to the noise. At small \( R, S/N \propto R \), and at large \( R, S/N \propto R^{-1} \). The maximum ratio occurs then when \( R = 1.02[2(\sigma_s^2 + \sigma_g^2)]^{1/2} \). This is the angular size of the circles we used for making our GRB averages.

Here we also can see explicitly the dependence of the dilution factor on the error angle of the GRB. If we double the maximum size of the error angle, we allow for our GRB to be included, say 7° instead of 3.5°; for the optimal value of \( x \), the signal will be diminished by a factor of \( \sim 1/4 \). Increasing the area searched also allows for more noise sources to be included in the average, and this can be seen to a small degree in the comparison of the long-burst and short-burst subsets.
REFERENCES

Baird, G. A., et al. 1975, ApJ, 196, L11
Boggess, N., et al. 1992, ApJ, 397, 420
Bontekoe, T., Winkler, C., Stacy, J. G., & Jackson, P. D. 1995, Ap&SS, 231, 285
Briggs, M., et al. 1996, ApJ, 459, 40
Briggs, M. 1995, Ap&SS, 231, 3
Dessenne, C. A., et al. 1996, MNRAS, 281, 977
Frail, D. A., et al. 1994, ApJ, 437, L43
Hjellming, R. M., & Ewald, S. P. 1981, ApJ, 246, L137
Inzani, P., Sironi, G., Mandolesi, N., & Morigi, G. 1982, in AIP Conf. Proc. 77, Gamma-Ray Transients and Related Astrophysical Phenomena, ed. R. E. Lingenfelter, H. S. Hudson, & D. M. Worrall (New York: AIP), 79
Katz, J. L. 1994, ApJ, 432, L107
Klebesadel, R. W., Strong, I. B., & Olson, R. A. 1973, ApJ, 182, L85
Kogut, A., et al. 1996, COBE Preprint 96-10
Koranyi, D. M., Green, D. A., Warner, P. J., Waldram, E. M., & Palmer, D. M. 1995, MNRAS, 276, L13
Kouveliotou, C., Meegan, C. A., Fishman, G. J., Bhat, N. P., Briggs, M. S., Koschut, T. M., Paciesas, W. S., & Pendleton, G. N. 1993, ApJ, 413, L101
McNamara, B. J., Harrison, T. E., & Williams, C. L. 1995, ApJ, 452, L25
Meegan, C. A., et al. 1996, ApJS, 106, 65
Meszaros, P., & Rees, M. J. 1993, ApJ, 418, L59
———. 1997, ApJ, 476, 232
Paczynski, B., & Rhoads, J. E. 1993, ApJ, 418, L5
Schaefer, S., et al. 1989, ApJ, 340, 455
Schaefer, R. K., Ali, S., Limon, M., & Piccirillo, L. 1995, ApSS, 231, 331