Modeling and feedforward controller design in 2-dimensional shaking table systems

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Abstract. This paper presents the modeling and basic controller design approach for multi-axis shaking table systems. A target shaking table system in this paper is composed of 4 actuators to move a table in horizontal plane, where coordinated action of each actuator is required to achieve the accurate table motion. In this paper, the simulation model of shaking table mechanism is constructed on the computer aided by multibody dynamics software. Based on the model, feedforward compensators for each actuator are designed to improve the tracking performance.

1. Introduction

Shaking table systems are widely applied for civil engineering materials/structures, automotive/railway industry, seismic engineering field, etc, to analyze the vibration responses and seismic performance for structures [1][2]. The main purpose of shaking table is to evaluate the accurate seismic capacity of structures (specimen) by reproducing the exact earthquake acceleration waveforms on the table. Since the evaluated specimen on the table is shaken by the electro-hydraulic actuators, the reproducibility of target acceleration waveform depends on the control performance of the actuators. However, the disturbances for the actuator control system such as nonlinearity in the actuator, reaction force from the specimen, and overturning moment due to the table and specimen degrade the control performance. In order to improve the control performance, therefore, practical controller design approaches have been proposed to compensate for the disturbances, e.g. offline iterative compensation [3], reaction force feedback compensation [4], model-based feedforward compensation [5][6], adaptive compensation [7][8]. These control approaches mainly focus on the improvement of disturbance suppression capability in the actuator control system.

Not only controller design but also development of shaking mechanism is important elemental technology to improve the shaking performance. Hydrostatic bearing as joint mechanism between actuator and table has been developed to achieve the smooth motion under the low friction and high stiffness [9][10]. In addition, since slide mechanism in the hydrostatic bearing can absorb the mutual interference due to the actuator motion in different shaking direction, each actuator can be independently controlled. On the other hand, link mechanism is widely used in the multi-degree of freedom shaking table systems. By adopting the link mechanism, flexible location of actuator greatly expands the possibility of shaking mechanism design. Space-saving and cost reduction for the installation are realizable by taking advantage of the design
flexibility. However, since the synchronization of each actuator motion is required to achieve the accurate table motion, control algorithm for each actuator becomes complicated.

This paper presents a modeling and controller design approach in 2-dimensional shaking table systems with link mechanisms, where a horizontal shaken laboratory prototype is used as a target system. Since the target shaking table system with link mechanism prototype sets up each actuator to shake the horizontal direction on the side of table, the coordinated control of each actuator is required to achieve the desired table motion. In the target shaking table system, therefore, the references for each actuator are generated on the basis of the target table motion and geometric arrangement. From the controller design point of view, zero phase error tracking algorithm [11] as a feedforward compensator is applied to completely synchronize the motion of all actuators. In order to verify the effectiveness of the table motion accuracy, the target shaking table mechanism is modeled by using a multi-body dynamics (MBD) software (RecurDyn by FunctionBay, Inc.) [12], where a precise simulator including control system and mechanism is constructed by integrating the MBD software and a control design software (MATLAB/Simulink by MathWorks, Inc.) [13]. The effectiveness of the motion of the target shaking table system has been verified by numerical simulations and experiments using a laboratory prototype. Although acceleration reproducibility is actually important for the shaking table system, the effectiveness is evaluated by displacement reference as the initial verifications of modeling and control system design in this paper.

2. Configuration of Target Shaking Table Mechanism

2.1. Configuration of shaking table mechanism

Figure 1 shows a general configuration of shaking table system with link mechanisms. The system is composed of a table and multi-actuator, where the table motion of X, Y, and Z axes is controlled by each actuator. In the mechanism of (a), the table is generally connected with the end of each actuator through the link mechanism, while shaking mechanism of (b) is located horizontal actuators on the side of the table and connect the end of piston with the arm part of table through the ball joint. Since the force generated by the actuator is indirectly given to the table though the arm part, all actuators should be synchronized to reproduce the accurate table motion. From the figure, shaking mechanism of (b) can reduce the installation area. In this way, system design according to the purpose is attained by adopting the link mechanism, while the control system appropriate for the mechanism should be designed to ensure the desired table motion. In this paper, the configuration of (b) is used as a target system.

![Figure 1. Configuration of shaking table mechanism.](image-url)
2.2. Configuration of laboratory prototype

Figure 2 shows an overview of a laboratory prototype with the configuration of figure 1 (b), while figure 3 shows the system configuration. The mechanism is composed of a shaking table coupled on two X-axis actuators and two Y-axis actuators through the ball joints, while opposite side of actuator is coupled on the frame (reaction wall) through the universal joints to prevent the rotation of actuators. Maximum stroke of each actuator is ±25 mm. The table can be horizontally shaken on the basis of the target acceleration or displacement waveform by each actuator. Since the table slides across the coated plate with low friction through the oilless slide plate, the table motion has three-degree-of-freedom on horizontal plane. The table motion can be detected by a 3-axis acceleration sensor attached on the table to evaluate the motion performance, while piston displacement and pressure in the cylinder are detected by a displacement and two pressure sensors to use as feedback signals. The control algorithm is implemented in a controller board with sampling period of 0.5 ms. A servo valve controls the flow in a cylinder based on the control signal calculated by the controller through a power amplifier. The piston excites the table through the joint based on the differential pressure in the cylinder. The sensor signals are fed back to the controller through the AD converters.

![Overview of shaking table mechanism](image1)

![Overview of actuator](image2)

Figure 2. Overview of horizontal shaken laboratory prototype.

![System configuration of prototype](image3)

![Configuration of actuator control system](image4)

Figure 3. System configuration of shaking table.
3. Modeling of proposed shaking table system

3.1. Configuration of simulator

Figure 4 shows a block diagram of simulator that is integrated by the MBD software and MATLAB/Simulink. The shaking table mechanism including table and joint can be modeled by MBD software, while each actuator control system including controller and mathematical plant model is simulated by MATLAB/Simulink. In the MBD software part, the physical quantities, e.g. piston and table displacements, can be calculated by numerical simulation on the basis of the excitation force for each actuator, while excitation forces are calculated by MATLAB/Simulink on the basis of the references and feedback signals.

![Figure 4. Block diagram of simulator integrated MBD software and MATLAB/Simulink.](image)

3.2. Modeling of actuator

The followings are the mathematical expressions for the actuator.

3.2.1. servo valve

The servo valve can be approximated by a constant gain within the control bandwidth as:

\[ q_m = K_{sv}u, \]  
\( (1) \)

where \( q_m \): output flow rate of servo valve, \( u \): control input, and \( K_{sv} \): gain of power amplifier and servo valve.

3.2.2. motion of piston

By assuming the motion of piston near the center of piston, a mathematical model of the cylinder is given as:

\[ q_m = A_a \frac{dy_d}{dt} + K_a \frac{dp_m}{dt} + C_l p_m, \]  
\( (2) \)

where \( y_d \): displacement of piston, \( p_m \): differential pressure in cylinder, \( A_a \): piston area, \( K_a \): internal stiffness of cylinder, and \( C_l \): leakage coefficient in cylinder. Driving force \( f \) generated by the cylinder is given by:

\[ f = A_a p_m. \]  
\( (3) \)
The mechanical motion can be expressed by the following kinetic equations:

\[ M_a y_a = f - d, \]  
\[ y_a = \frac{d^2 y_d}{dt^2}, \]

where \( M_a \): mass of piston, \( y_a \): piston acceleration, \( y_d \): piston displacement and \( d \): friction and uncertainties.

Based on the mathematical expressions, a block diagram of the actuator can be given as figure 5. The model parameters are identified on the basis of the results of frequency response measurement. Figure 6 shows plant characteristics of piston displacement \( y_d \) for control input \( u \) in X1-actuator as an example, where solid lines indicate measurement result and broken lines indicate model characteristic. The models of each actuator are implemented in the MATLAB/Simulink.

3.3. Modeling of shaking mechanism
Shaking table mechanism shown in figure 2 (a) including table and joint mechanisms is modeled by the MBD software. Solid model is constructed by using a CAD software (SolidWorks, Dassault Systemes SolidWorks Corp.) on the basis of the drawings of figure 2 (a), and the model is imported to MBD software. In the MBD software, conditions of constraint are set as shown in figure 7. Rotational or spherical constraints are set to joint parts, while translation constraints are set to piston parts. Table motion is constrained on the horizontal plane. Driving forces \( f \) calculated by MATLAB/Simulink are given to piston parts, while the piston displacement and velocity are transferred to MATLAB/Simulink.

4. Reference Generation and Controller Design
4.1. Reference generation for each actuator
In order to control the accurate table position, position references for each actuator should be generated on the basis of table motion. Figure 8 shows coordinate definition on horizontal plane, where \( C = (0, 0) \): table center position (initial position), \( P_i = (x_{pi}, y_{pi}, 1) \) \( (i = 1 \sim 4) \): initial position of ball joint between table and piston, \( Q_i = (x_{qi}, y_{qi}, 1) \) \( (i = 1 \sim 4) \): initial position of
universal joint between end of actuator and reaction wall, $L$: length between $P_i$ and $Q_i$. Target position $P'_i$ derived from target table position $C' = (x_t; y_t; \theta_t)$ is given as follows:

$$P'_i = \Theta C_t P'^T_i,$$  \hspace{5cm} (6)

where rotation matrix $\Theta$ and translation matrix $C_t$ are given as

$$\Theta = \begin{bmatrix}
\cos \theta_t & -\sin \theta_t & 0 \\
\sin \theta_t & \cos \theta_t & 0 \\
0 & 0 & 1
\end{bmatrix},$$

$$C_t = \begin{bmatrix}
1 & 0 & x_t \\
0 & 1 & y_t \\
0 & 0 & 1
\end{bmatrix}.$$

Based on (6), displacement references for the actuator $r^{ref}_i$ is generated as

$$r^{ref}_i = L'_i - L_i,$$  \hspace{5cm} (7)

where $L'_i$ is length between $P'_i$ and $Q_i$.

4.2. Feedback controller design for each actuator

Feedback controller is designed for each actuator to ensure the system stability. Figure 9 shows the plant characteristics of piston displacement $y_d$ for control input $u$ in the simulator, where solid lines indicate actuator characteristic shown in figure 6 and broken lines indicate plant characteristic which added table mass to piston mass $M_a$. From this figure, a resonant vibration mode exists due to load mass and internal stiffness of cylinder $K_a$ that is determined by fluid compressibility and volume of cylinder [14]. In order to reduce the gain peak, differential pressure signal $p_m$ as a differential value of the velocity $\frac{dy_d}{dt}$ should be fed back on the basis of the actuator
block diagram shown in figure 5. Damping for the vibration poles can be arbitrarily controlled by tuning the pressure feedback gain, where bandpass filter are practically applied for the feedback signal to eliminate offset and sensor noise.

On the other hands, PI (Proportional-Integral) compensator as a basic displacement feedback controller is designed to ensure the system stability and servo performance. Figure 10 shows closed-loop characteristics of piston displacement $y_d$ for the reference to closed-loop system $y_m$. From the figure, gain peak of resonant frequency can be suppressed, while synchronizing to the references including different frequency components for each actuator is difficult due to the phase delay of the closed-loop system.

**4.3. Feedforward controller design**

In order to achieve the desired table motion, the motion of all actuators should be completely synchronized in target frequency range. However, since feedback control system for each actuator
includes phase delay, the synchronization error is caused by the target displacement reference with different frequency components. In order to compensate for the phase delay in each actuator control system, zero phase error tracking controller (ZPETC) [11] as the feedforward compensator is applied to improve the servo performance. The closed-loop transfer function in discrete time domain is expressed as

\[ \frac{y_d(k)}{y_m(k)} = G_c(z^{-1}) = \frac{z^{-d}B_a(z^{-1})B_u(z^{-1})}{A_c(z^{-1})} \] (8)

where \( k \) is a sample number, \( z^{-d} \) is a \( d \)-step delay operator, \( A_c(z^{-1}) \) is a polynomial including poles of closed-loop system, \( B_a(z^{-1}) \) includes stable zeros of closed-loop system, and \( B_u(z^{-1}) \) includes unstable zeros of closed-loop system. The ZPETC is

\[ y_m(k) = \frac{A_c(z^{-1})B_a(z)}{B_a(z^{-1})[B_u(z^{-1})]^2} r_{ref}^{ref}(k + d) \] (9)

where \( r_{ref}^{ref}(k + d) \) is a \( d \)-step advanced reference calculated by reference generator, and \( B_a(z) \) is obtained by replacing \( z^{-1} \) in \( B_a(z^{-1}) \) by \( z \). From (8) and (9), the overall transfer function from the desired output \( y_m \) to the actuator output \( y_d \) is given as

\[ \frac{y_d(k)}{r_{ref}^{ref}(k)} = \frac{B_u(z^{-1})B_c(z)}{[B_c(z^{-1})]^2}. \] (10)

By applying the ZPETC, the frequency response gain remains close to 1 in a low frequency range.

Figure 11 shows the designed control system with ZPETC and reference generator for the target shaking table system. Displacement references \( r_{ref} \) for each actuator are calculated by reference generator on the basis of desired table reference. The feedback compensator \( C_{FB} \) is PI controller and pressure feedback, while \( C_{zptc} \) is ZPETC to improve the servo performance.

5. Validity verifications of designed control system
Effectiveness of the motion performance of shaking table has been verified by numerical simulations using the constructed simulator and experiments using the prototype.

5.1. Numerical simulation results
In the simulation, displacement references of sinusoidal waveforms with 5 Hz for X and Y axes are applied as target table motion, and the stroke is set as \( \pm 15 \) mm. Figure 12 (a) shows piston displacement in each actuator, figure 12 (b) shows table displacement, and figure 12 (c) shows table motion on horizontal plane. In the figure, black solid lines indicate the references, blue broken lines indicate the results of feedback compensation without ZPETC, red dotted lines indicate the results of feedback compensation with ZPETC. On the other hand, figure 13 shows circle motion of table. From these simulation results, the tracking performance can be improved by applying the ZPETC.

5.2. Experimental results
Figure 14 shows experimental results, where (a) indicates the experimental waveforms for diagonal line motion of the table shown in figure 12 and (b) indicates the experimental waveforms for circle motion of the table shown in figure 13. In the figure, black solid lines indicate the references, blue broken lines indicate the results of feedback compensation without ZPETC, red dotted lines indicate the results of feedback compensation with ZPETC. Table displacement
(a) Piston displacement

(b) Table displacement

(c) Lissajous of table motion

Figure 12. Simulation results for diagonal motion of table.

responses are derived by integrating the acceleration signals detected by the acceleration sensor on the table. From these results, the tracking performance can be improved by applying the ZPETC. On the other hand, amplitude errors of table motion still remain due to modeling error between ZPETC and actual closed-loop system. Therefore, adaptive algorithm should be implemented to achieve the accurate table motion in the future work.

6. Conclusions
This paper presented a modeling and controller design approach in 2-dimensional shaking table systems with link mechanisms. The target shaking table mechanism as a horizontal shaken laboratory prototype was modeled, where a precise simulator including control system and mechanism was constructed by integrating the MBD software and the MATLAB/Simulink. In the target shaking table system, displacement references for each actuator were generated on the basis of the desired table motion because the movement of table requires the coordinated control of each actuator due to the unique actuator arrangement. In order to improve the servo performance of each actuator, ZPETC as the feedforward compensator was applied to compensate for the phase delay of closed-loop system for each actuator. The effectiveness of the motion of the target shaking table system was verified by numerical simulations and experiments using a laboratory prototype.
Figure 13. Simulation results for circle motion of table.

Figure 14. Experimental results.

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