Neutron Properties in the Medium

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We demonstrate that for small values of momentum transfer, $Q^2$, the in-medium change of the $G_E/G_M$ form factor ratio for a bound neutron is dominated by the change in the electric charge radius and predict in a model independent manner that the in-medium ratio will increase relative to the free result. This effect will act to increase the predicted cross-section for the neutron recoil polarization transfer process $^4\text{He} (\vec{e}, e' \vec{n}) ~ ^3\text{He}$. This is in contrast to medium modification effects on the proton $G_E/G_M$ from factor ratio, which act to decrease the predicted cross-section for the $^4\text{He} (\vec{e}, e' p) ~ ^3\text{H}$ reaction. Experiments to measure the in-medium neutron form factors via neutron knockout reactions are currently feasible in the range $0.1 < Q^2 < 1 \text{GeV}^2$.

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The discovery by the European Muon Collaboration (EMC) that the structure function of a nucleus, in the valence quark region, is reduced relative to the free nucleon occurred more than twenty years ago [1]. The immediate consequence then implies that the nucleon’s size may also increase when the nucleon is in free space. The uncertainty principle then implies that the nucleon’s size may also increase [2]. This medium modification of nucleon structure should have consequences for nuclear reactions that are sensitive to the properties of a single nucleon. However unambiguous evidence for such modifications have not yet been observed.

Searches for medium modifications have been performed using the $(e, e')$ reaction [3]. The polarization transfer reaction $(\vec{e}, e' \vec{p})$ on a proton target measures quantities proportional to the ratio of the proton’s electric and magnetic form factors [4]. When such measurements are performed on a nuclear target, e.g. the reaction $^4\text{He} (\vec{e}, e' \vec{p}) ~ ^3\text{H}$, the polarization transfer observables are sensitive to the $G_E/G_M$ form factor ratio of a proton embedded in the nuclear environment. Several such $^4\text{He}$ experiments have been performed [5]. The data can be described well by including the effects of medium-modified form factors [6, 7, 8, 9] (in which the ratio is reduced by the influence of the medium) or by including effects from strong charge-exchange final state interactions (FSI) [10]. However, the effects of the strong FSI may not be consistent with measurements of the induced polarization [4]. It is therefore important to find an alternative method to sort out the influence of medium modifications and FSI.

The purpose of this letter is to suggest that important progress can be achieved by measuring neutron recoil polarization in the $^4\text{He} (\vec{e}, e' \vec{n}) ~ ^3\text{He}$ reaction. Recent advances in experimental techniques make such considerations very timely.

Before analyzing the polarization transfer neutron knockout reaction on $^4\text{He}$, it is worthwhile to consider the validity of the general proposition that the structure of a single nucleon is modified by its presence in the nuclear medium. The root cause of any such modification is the interaction between nucleons, so one needs to consider whether the entire concept of single nucleon modification makes sense. Our assertion is that if the kinematics of a given experiment select single nucleon properties, such as in quasi-elastic scattering, it does make sense to consider how a single nucleon is modified. Thus the influence of long-range effects, such as single pion-exchange, occur as multi-nucleon operators and are not considered medium modifications effects of a single nucleon, that we wish to isolate using quasi-elastic scattering. Within the quasi-elastic region it may be possible to characterize these medium modifications by the virtuality of the bound nucleon [11].

We begin the analysis by considering the situation for small values of $Q^2$, where $Q^2$ is the negative of the square of the virtual photon’s four-momentum. In this region the Sachs electric and magnetic form factors [12] for the free proton can be expressed in the form

\[ G_{Ep}(Q^2) \simeq 1 - \frac{1}{6} Q^2 \hat{R}_{Ep}, \]

\[ \frac{1}{\mu_p} G_{Mp}(Q^2) \simeq 1 - \frac{1}{6} Q^2 \hat{R}_{Mp}^2, \]

where $\mu_p$ is the proton magnetic moment and the effective electric and magnetic radii [13] are labeled by $\hat{R}_{Ep}$ and $\hat{R}_{Mp}$, respectively. The effective radii are defined via the Sachs form factors and in a departure from the notation of Ref. [13] will be labeled with a caret, because a superscript # is reserved to denote in-medium quantities. Keeping only the leading $Q^2$ dependence, the proton electric to magnetic form factor ratio can be expressed as

\[ \mathcal{R}_p \equiv \frac{G_{Ep}(Q^2)}{G_{Mp}(Q^2)} \simeq \frac{1}{\mu_p} \left[ 1 - \frac{1}{6} Q^2 (\hat{R}_{Ep}^2 - \hat{R}_{Mp}^2) \right]. \]

For a proton bound in a nucleus we may define an analogous ratio which we label $\mathcal{R}_p$. The influence of the
medium may change any of the three quantities $\mu_p$, $\hat{R}_{Ep}$ and $\hat{R}_{Mp}$. Extensive studies of the EMC effect seem to imply that the nucleon expands in-medium. Therefore, since $\hat{R}_{Ep}^2 \approx \hat{R}_{Mp}^2$ in free space, and if we assume the in-medium changes are similar for the electric and magnetic radii, the influence of the term proportional to $Q^2$ in Eq. (3) would be essentially negligible. However, one may expect that the value of $\mu_p$ in the medium will increase, along with the increasing magnetic radius. In this scenario the super-ratio $\hat{R}_p^2/\hat{R}_p$ would be less than one and largely independent of $Q^2$.

This expectation is borne out by specific model calculations [6, 8, 9] and, more importantly, by the experimental data in Refs. [5]. The basic idea behind the models is that confined quarks in a nucleon – which is treated as a MIT bag in Ref. [6] or as a solution of the relativistic Faddeev equation in Refs. [8, 9] – are influenced by the quarks of neighboring nucleons through the exchange of a scalar meson, which provides the necessary attraction to bind the nucleus. The results of Ref. [9] for the proton super-ratio in nuclear matter are given in Fig. 1. A contrasting model is that of Smith & Miller [14], where the quarks are confined in a chiral soliton which is identified as the nucleon. In-medium the confined quarks are also influenced by the exchange of scalar objects between quarks of neighboring nucleons. In this model the magnetic properties are dominated by the sea, which is resistant to the influence of the medium. Thus $\mu_p$ and $\hat{R}_{Mp}$ remain largely unchanged whereas $\hat{R}_{Ep}$ increases. Once again the super-ratio $\hat{R}_p^2/\hat{R}_p$ is less than unity, however in this model it is expected to vary linearly with $Q^2$. As noted earlier, in the region where data exist all three models are in satisfactory agreement with experiment, as are the effects of including FSI.

There are two lessons from this. Firstly, very different models predict the super-ratio to be less than one for the proton, but for very different reasons. Thus there is a need for another experimental way to determine which, if any, of the relevant parameters are changed in the medium. Secondly, we need more precise data and an increase in the $Q^2$ range of the $(e, e')$ experiments.

One way to help resolve the different mechanisms responsible for the medium modification of nucleons and to also determine the influence of FSI is to consider the neutron in the medium. The analogous expression to Eq. (3) for the neutron, valid at small $Q^2$, is

$$\hat{R}_n \equiv \frac{G_{En}(Q^2)}{G_{Mn}(Q^2)} \simeq \frac{\mu_n}{6} \frac{Q^2}{\hat{R}_{En}^2},$$

where the effective magnetic radius does not appear, since it is the coefficient of a $Q^4$ term. We immediately see that, in contrast with the proton, the medium modifications are generally expected to depend on possible changes in both the electric radius and magnetic moment. This implies that the behavior of the super-ratio $\hat{R}_n^2/\hat{R}_n$ at small $Q^2$ is determined by a competition between the expected increases in both these quantities. The electric radius is more important in Eq. (4) because it enters quadratically. Thus one may expect, in contrast with the proton, that the neutron super-ratio will be larger than one.

It is worthwhile to consider specific models as examples of the previous general statements. In the quark-diquark Nambu–Jona-Lasinio (N贾) model of Refs. [8, 9], both $\hat{R}_{En}$ and $\hat{R}_{Mn}$ increase in-medium, however there is only a small in-medium change in the neutron magnetic moment. Therefore at low $Q^2$ one finds that the super-ratio is dominated by the change in $\hat{R}_{En}$ and therefore increases. This is shown in Fig. 1 where the results of Ref. [9] are illustrated. In the model of Smith & Miller [14] the value of $\mu_n$ and $\hat{R}_{Mn}$ are largely unchanged in the medium, however $\hat{R}_{En}$ increases. Therefore both models predict that the super-ratio goes up for the neutron and down for the proton.

We can also consider placing the relativistic light front constituent quark model of Ref. [15] in the medium. This model for the nucleon is characterized in free space by a confinement scale $1/\alpha$ and a quark mass $m_q$. One might imagine that the medium changes each of these quantities. Numerically the change in $m_q$ is more important for the magnetic moments, and we find that in the medium $\delta \kappa_p / \kappa_p \approx -\delta m_q / m_q$. Thus in this model the percentage change in the neutron and proton anomalous magnetic moments is the same. The nucleon charge and magnetic radii are proportional to $1/\alpha$, therefore the percentage in-medium change of the radii behaves like $\sim \delta \alpha / \alpha$. For the proton, the super-ratio is therefore dominated by the change in the magnetic moment, see Eq. (3), so that the prediction of this model is that the proton super-ratio is less than unity. For the neutron the change in the radius enters quadratically, see Eq. (4), so that once again one expects an increase in the neutron super-ratio. We estimate the size of these in-medium effects by using Eqs. (3)
and (4), along with the appropriate expressions in Ref. [15]. Assuming medium effects increase the radii by 10% and reduce the quark mass by 20% we obtain the results illustrated in Fig [1].

It is possible to generalize our arguments so that they are applicable beyond the $Q^2$ domain where Eqs. (3) and (4) hold, namely $\frac{1}{6} Q^2 \tilde{R}_{E_p, M_p}^2 \ll 1$. The current $Q^2$ range where the neutron polarization transfer reaction is experimentally feasible is probably between 0.1 and 1 GeV$^2$. In this region the proton electromagnetic form factors fall monotonically, so that one may characterize the size of a system by thinking of the width of the electromagnetic form factors $G_{E_p, M_p}(Q^2)$ as a measure of the inverse of the square of a generalized radius. The expectations about the influence of the medium on the generalized radii would be essentially the same as for the radii of Eqs. (1) and (2).

The concept of generalized radii for the neutron is potentially even more interesting, because $G_{E_n}$ rises from zero at $Q^2 = 0$ to a peak at about $Q^2 = 0.4$ GeV$^2$ and then falls monotonically, whereas $G_{M_n}(Q^2)$ simply falls monotonically. One may then characterize the square of a generalized radius in terms of the value of $Q^2$ for which $G_{E_n}$ peaks. This generalized radius can be expected to increase in the medium.

It is intriguing that each of the three models described earlier find that $R^*/R$ is greater than unity for neutrons and less than unity for protons. We shall now try to understand this from a more formal perspective, for both magnetic moments and radii. Consider the expression for the anomalous magnetic moment $\kappa$ derived in Ref. [13], namely

$$\kappa = \langle X | \sum_q e_q \int d^2 b \ b_y \ q_{+}(0, b)q_{+}(0, b) | X \rangle,$$  

(5)

where $q_{+}(x^-, b)$ is a quark-field operator of charge $q$ and $b$ is the impact parameter. The subscript $+$ indicates a light-cone good component of the quark field, defined by $q_+ = \gamma^0 \gamma^+ q$, and therefore the operator $q_{+}(0, b)q_{+}(0, b)$ is a number operator for valence quarks with impact parameter $b$. Explicitly the state $|X\rangle$ has the form

$$|X\rangle = \frac{1}{\sqrt{2}} \left[ |X, +\rangle + |X, -\rangle \right],$$  

(6)

where the first term in Eq. (6) represents a transversely localized state of definite $p^+$ momentum and positive light-cone helicity, whereas the second state has negative light-cone helicity. The state $|X\rangle$ may be interpreted as that of a transversely polarized target \[16, 17\], up to relativistic corrections caused by the transverse localization of the wave packet \[18\].

Define the contribution of the $u$-quarks to the proton matrix element as $2u$, where

$$2u = \langle X | \int d^2 b \ b_y \ q_{u+}^{\dagger}(0, b)q_{u+}(0, b) | X \rangle,$$  

(7)

and the contribution of the $d$-quarks as

$$d = \langle X | \int d^2 b \ b_y \ q_{d+}^{\dagger}(0, b)q_{d+}(0, b) | X \rangle.$$  

(8)

With this definition, and neglecting the contribution from heavy quark flavours, the proton anomalous magnetic moment can be expressed as

$$\kappa_p = \frac{4}{3} u - \frac{1}{3} d.$$  

(9)

Then assuming charge symmetry \[19\], so that the $u$- and $d$-quark contributions in the proton equal the $d$- and $u$-quark contributions in the neutron, we obtain

$$\kappa_n = -\frac{2}{3} u + \frac{2}{3} d.$$  

(10)

In the medium the nucleon matrix elements are modified. Thus $u$ and $d$ are shifted from their free values by $\delta u$ and $\delta d$ respectively. We see no general, model-independent way to relate these two quantities, even in the case of symmetric nuclear matter (with $N = Z$) where the external forces on the confined quarks are flavor independent. This is because of the necessary interplay between the quark orbital angular momentum and spin. Thus the changes in the anomalous magnetic moments are simply

$$\delta \kappa_p = \frac{4}{3} \delta u - \frac{1}{3} \delta d,$$  

(11)

$$\delta \kappa_n = -\frac{2}{3} \delta u + \frac{2}{3} \delta d.$$  

(12)

To determine each of these quantities requires a measurement of both the proton and neutron magnetic moment in the medium. An important point that is worth highlighting is that the change in the proton magnetic moment does not simply equal $\delta \kappa_p$. If the mass of the nucleon changes in the medium there is also a contribution from the Dirac form factor. If the proton magnetic moment is expressed in nuclear magnetons, its change in-medium is given by $\delta \kappa_p$ plus the term $M/M^* - 1$, where $M$ is the free nucleon mass and $M^*$ is the in-medium mass shifted by the influence of the nuclear binding potentials.

Using the relation that the transverse charge density is the two-dimensional Fourier transform of $F_1$ \[16, 20, 21\], one may analyze the nucleon radii in a similar fashion to the anomalous magnetic moments. The $u$-quark sector contribution to the $F_1$ electric charge radius squared is given by

$$2R_{1u}^2 = \langle X, + | \int d^2 b \ \frac{3}{2} b^2 q_{u+}^{\dagger}(0, b)q_{u+}(0, b) | X, + \rangle,$$  

(13)

and the contribution from the $d$-quarks is

$$R_{1d}^2 = \langle X, + | \int d^2 b \ \frac{3}{2} b^2 q_{d+}^{\dagger}(0, b)q_{d+}(0, b) | X, + \rangle.$$  

(14)
The factor $3/2$ accounts for the two-dimensional integration. Recalling that $G_E = F_1 - \frac{Q^2}{4M^2} F_2$, the effective charge radii related to $G_E$ are given by

\begin{align}
\hat{R}_{Ep}^2 &= \frac{4}{3} R_{1u}^2 - \frac{1}{3} R_{1d}^2 + \frac{3}{2 M^2} \kappa_p, \\
\hat{R}_{En}^2 &= -\frac{2}{3} R_{1u}^2 + \frac{2}{3} R_{1d}^2 + \frac{3}{2 M^2} \kappa_n. 
\end{align}

(15)

(16)

For the neutron (but not the proton) the Foldy term $24$, because the anomalous magnetic moments in the Foldy term $24$, is by far the dominant contribution to the charge radius. In-medium this will almost certainly remain true. Therefore for small values of $Q^2$ the leading term of the neutron super-ratio is given by

\[ \frac{R_n^*}{R_n} \simeq \left( \frac{M}{M^*} \right)^2, \]

(17)

because the anomalous magnetic moments in the Foldy terms cancel the neutron magnetic moments. Binding effects imply that $M^* < M$ and therefore we have obtained on general grounds that at small $Q^2$ the super-ratio should be greater than one for the neutron.

This general prediction is worthy of an experimental test, and recent technical developments make this an ideal time to plan such an experiment. Using recoil polarization, high precision, low $Q^2$ measurements of the free proton $25$ and neutron $23$ form factors have already been performed. With a straightforward extension of these experiments, using a similar experimental setup, at for example, the Thomas Jefferson National Accelerator Facility (JLab), it would be possible to perform low $Q^2$ measurements of the reactions $p(e, e'p)$, $d(e, e'p)n$, $d(e, e'n)p$, $^4$He$(e, e'n)^3$H and $^4$He$(e, e'p)^3$He. This would allow a direct test of the predictions made in this letter. Because of the large cross-section for these reactions at low $Q^2$, and the availability of a high current polarization and duty factor electron beam at JLab, these experiments would achieve excellent statistical precision within a relatively short time period. Such experiments could also probe the $Q^2$ dependence of the form factor super-ratios. An experimental proposal to this effect is being developed by the authors for the JLab facility.

Understanding how a nucleon is modified when in the nuclear environment remains a central challenge for the nuclear physics community. In this letter we present a unique model independent result pertaining to the structure of a bound nucleon, which is expressed in Eq. (17), and states that the neutron super-ratio is greater than one at small $Q^2$. We therefore conclude that the measurement of $(e, e'n)$ processes on nuclear targets can provide important additional and complimentary information to that already obtained using the $(e, e'p)$ reaction. These measurements would provide an independent test of any model seeking to explain the EMC effect and offer the hope of providing its long-sought universally accepted explanation.

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