Neutrino Propulsion for Interstellar Spacecraft

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Abstract

An exotic spacecraft propulsion technology is described which exploits parity violation in weak interactions. Anisotropic neutrino emission from a polarized assembly of weakly interacting particles converts rest mass directly to spacecraft impulse.

1 Introduction

The photon rocket has long been a familiar example of special relativistic kinematics [1]. The speed of light as the ultimate rocket exhaust velocity also surfaces in discussions of interstellar travel [2]. But few regard the photon rocket as even remotely practical. This Note outlines a method of spacecraft propulsion which resembles the photon rocket closely, and which appears, in principle, capable of reduction to practice.

Parity nonconservation in the weak interaction can produce thrust by anisotropic neutrino emission from a polarized assembly of weakly unstable particles or nuclei. A conceptual propulsion system, in which decay or nuclear capture of muons from the annihilation of bulk antimatter generates thrust, illustrates the principles involved.

This scheme may be considered a variant on propulsion methods based upon reaction from radioactive decay products, which have appeared in the literature at intervals since their proposal by Goddard and Tsiolkovsky early in this century [3, 4]. It is distinct from these in that the decay particles are
collisionless and electrically neutral, that the anisotropy of parity-violating weak decays resolves the otherwise intractable problem of collimation, and that the small cross section for neutrino interactions with other matter permits thrust generation to take place as a volume, instead of as a surface, effect.

2 Physical Principles

2.1 Parity nonconservation and thrust generation

Consider a nucleus of spin $J$ polarized along $z$ which captures a light particle, accompanied by the emission of a neutrino with energy $Q$, momentum $q = Q/c$. The capture imparts to the daughter nucleus momentum $q$ directed opposite to the neutrino, along with a small recoil energy. As the only preferred axis is the parent nuclear spin $J$, the rate may be written as a Legendre polynomial expansion in $\cos(\theta) = q \cdot J / qJ$:

$$I(\theta) = \sum_l a_l P_l(\cos(\theta))$$  \hspace{1cm} (1)

Because $J$ is an axial vector, while $q$ is a polar vector, their inner product is a pseudoscalar, which is odd under parity. In spherical coordinates, the effect of the parity operator is to transform $\theta$ to $\pi - \theta$. Terms which do not conserve parity thus appear as odd harmonics in eqn. 1. As a result, the average impulse per capture along $z$

$$\frac{\int_{4\pi} d\Omega q_z I(\theta)}{\int_{4\pi} d\Omega I(\theta)} = \frac{1}{3} \frac{Q a_1}{c a_0} \equiv \frac{\eta}{c}$$  \hspace{1cm} (2)

does not vanish. Only a nonvanishing $a_1$ coefficient allows neutrinos to transport a net momentum flux. One concludes that an interaction which conserves parity cannot transport a net flux of momentum from a polarized assembly of unstable particles, that a decay whose angular intensity contains a linear term in $\cos(\theta)$ will violate parity, and that only such an intensity pattern can produce thrust.

2.2 Microscopic basis of thrust generation

The weak processes considered here involve muons produced by charged pion decay following proton-antiproton annihilation. Muons decay by emission
of electron and muon neutrinos in a three-body process. The spectrum of free positive muon decay at rest \(\bar{\nu}_e\) gives \(q_z = Q/10\) for the average momentum carried away by neutrinos, or \(\eta_+ = 1/10\). Negative muons also undergo nuclear capture, subsequently emitting neutrons in over 95% of captures. Collimated neutrino emission results from muon capture in which either the muon or a nonzero spin target nucleus is polarized.

The conversion efficiency \(\eta_+\) for negative muon capture by a polarized nucleus is calculated following [6], incorporating polarization effects after [7]. It will be assumed that capture is dominated by \(L = 1\) and \(L = 2\) multipoles, that recoil corrections \(O(v/c)\) can be neglected and, initially, that the nucleus is 100% polarized and the muon unpolarized. Nuclear matrix elements for individual transitions are computed with a modified closure approximation in the pure shell model [8]. (The net effect of interference terms is estimated to be \(O(\text{recoil})\) for medium-mass nuclei.)

In this manner one may estimate \(\eta_+\) for transitions to a specified final angular momentum and parity. Its magnitude typically lies between 0.25 and 0.3 at the upper end, but can take on values near zero.

Detailed calculations of muon capture [10] find the capture strength concentrated in a small number of final spin/parity states. \(L = 1\) transitions to states of high spin appear favored, with 65-75% of the capture going to these states. It appears reasonable to assume target nuclei exist with net efficiency \(\eta_+\) in the range 0.15-0.25, within recoil corrections \(O(0.04)\).

Hyperfine coupling between the target nucleus and an unpolarized \(1S\) muon will polarize the muon spin slightly at the expense of the target nucleus spin [11]. For high nuclear spins, the polarization loss is small. Muon polarization alters the relative importance of transverse and longitudinal multipoles [12] so as to increase \(\eta_+\) somewhat, by perhaps 25-33% of its magnitude for unpolarized muons in favorable cases.

An estimate for \(\eta_+\) of ca. 0.2 is taken in the following.

## 3 Propulsion System Concept

The propulsion system operates, in outline, as follows: annihilation yields, after free decay of prompt annihilation products, about 3.16 \(\pi^\pm\) and 1.85 \(\pi^0\)'s [13]. It is assumed that neutral annihilation products are discarded so as to contribute negligibly to waste heat production. Muons from pion decay are decelerated to low energies. Positive muons are stopped and (rapidly) polar-
ized, then decay. A polarized nuclear target stops and captures the negative muons, resulting in a neutron flux. The neutrons are moderated and captured in an isotopic mix of nuclei which regenerates the initial target isotope. Both the absorbers which decelerate the muons and the thrust-generating interaction regions are coupled to a high thermal power, low specific mass waste heat rejection system.

3.1 Conversion efficiency

A simple estimate of pion energetics following annihilation results from the pion multiplicities on the assumption that pions share the available mass-energy equally. The resulting mean pion kinetic energy is 236 MeV. Pions are assumed to decay in flight. Muon neutrinos from charged pion decay carry off about 90 MeV on average. It is assumed that charge separation of muons may be achieved with gradient drift, and that they are brought to low kinetic energy by suitable absorbers. Because of the spread in annihilation product kinetic energy, this process requires a degree of momentum sorting.

The fraction of proton pair mass that appears as muon rest mass is

$$\alpha = \frac{n_{\pi}m_{\mu}}{2m_p}$$

(in an obvious notation), divided equally between charge states. Combining the conversion efficiency for free positive muon decay with that for negative muon capture, adjusted for losses from residual nuclear excitation, leads to an overall, ideal efficiency estimate

$$\eta = \frac{\alpha}{2} [\eta_+ + 0.85 \cdot \eta_-] \cong 0.025.$$  

3.2 Muon interactions

Stopped positive muons form muonium with unpaired electrons in most substances. It is assumed that dynamical polarization by muonium hyperfine coupling in a strong magnetic field polarizes muons rapidly compared to their lifetime, whereupon their free decay imparts impulse. The thermal energy deposited in the ultimate target medium by a stopping muon should not, therefore, exceed the hyperfine splitting.

Negative muons are made to stop in a highly polarized target of a suitable nucleus such as $^{51}V$ or $^{55}Mn$, whose most probable muon capture daughters
are all stable. In most targets negative muons rapidly decelerate to rest, whereupon they form muonic atoms. In medium-mass nuclei \((Z > 20\) or so), they overwhelmingly undergo nuclear capture from the muonic \(1S\) state instead of free decay [14].

The target may be gaseous [15]. Negative muons can deposit significant heat in the capture target without compromising nuclear polarization, and thus only require coarse momentum sorting. In fact, muon capture probably must occur in a confined plasma, because hyperfine coupling between \(K\) and \(L\)-shell atomic electrons in the target and the \(1S\) muon-also subject to hyperfine coupling with the target nucleus-poses a risk of catastrophic loss of target nuclear polarization. A thermal energy of 0.5 keV, or just less than \(5 \times 10^6\) K, suffices for most medium-mass nuclei [14].

3.3 Neutron issues

The daughter nucleus deexcites mainly by neutron emission. It is proposed to recover the target nucleus for subsequent reuse by operating a catalytic neutron capture chain on daughter nuclei.

Recovery begins with moderation of the neutrons to thermal energies in a low-neutron absorption moderator such as graphite or \(D_2^{16}O\). A jacket composed of lighter isotopes of the daughter nucleus captures the neutrons, thus running a neutron capture chain terminating in an isobar of the original nucleus that quickly decays back to the parent isotope. Chemical separation of the parent isotope completes the chain.

Proton and alpha channels cause some leakage out of the chain. However, the nuclei are not lost to the system. Daughter nuclei from \((\mu^-, np)\) or \((\mu^-, \alpha)\) reactions lie on the neutron-rich side of stability and will consequently \(\beta\)-decay back to the stability line. An extra neutron capture or two reinserts them into the main chain. Neutrons to supply this loss come from diverting a fraction of the negative muon flux to capture in heavy nuclides whose composition is otherwise immaterial. An estimate based upon the neutron multiplicity per muon capture [14] indicates that a 0.01 penalty to \(\eta\), of recoil order, suffices to redress a 3 % nucleon deficit per capture.

3.4 Waste Heat Rejection

For purposes of discussion it will be assumed that waste heat is rejected by liquid droplet radiators. Nordley [16] estimates that a specific mass of
$10^{-5} \text{kg/W}_t$ at an outlet temperature of 900 K represents an attainable extension of liquid droplet radiator performance. In the present instance, in which no penalty accrues to high operating temperatures, it seems permissible to conjecture that radiators may use as a working fluid a refractory metal at an outlet temperature in the low 2000 K’s. If evaporation losses are low enough, and the nonradiating mass depends weakly upon temperatures, then the specific mass of a liquid drop radiator scales as a function of the working fluid inlet and outlet temperatures only [17]. Estimates for a variety of candidate working fluids indicate specific masses of order $10^{-7} \text{kg/W}_t$ can probably be attained, but extension to greatly lower values appears difficult.

### 3.5 Spacecraft dynamics

Consider a simple rocket propelled by neutrinos. A change in spacecraft rest mass $dM$ yields an impulse $\eta c dM$ and discarded energy $(1 - \eta)c^2 dM$. Let $Nt$ annihilations in a time $t$ impart impulse to the spacecraft at a rate

$$\dot{p} = 2Nm_\mu c\eta, \tag{5}$$

accompanied by local waste heat power

$$\dot{W} \equiv N\delta W \tag{6}$$

within it, where

$$\delta W \approx \left[2m_\mu c^2 - n_{\pi\pm}[m_\mu c^2 + \gamma_0(m_{\pi\pm} - m_\mu)c^2] - \gamma_0 n_{\pi\pm} m_{\pi\pm} c^2 + E_0 \right]. \tag{7}$$

The average Lorentz factor $\gamma_0$ is 2.7. $E_0$ includes ca. 38 MeV from residual nuclear excitation, recoil energy from $\mu^+$ decay, and positron annihilation. Waste heat production per annihilation is about 690 MeV. The propulsion system mass is

$$M = \dot{W}\sigma_W + m_{\text{mod}} + m_{\text{abs}} + m_{\text{plant}}, \tag{8}$$

where $\sigma_W$ is the specific mass for waste heat rejection, and the masses of neutron moderator and catalytic chain absorber (governed by neutron scattering and absorption lengths, respectively) are, along with those of other propulsion plant components, assumed to scale weakly with power. The high waste heat power levels expected in this concept suggest examining the extreme in which the propulsion system fixed mass is dominated by the waste heat
rejection system. In this limit, the final acceleration is given by the ratio of eqns. (5) and (8):

$$a = 2\frac{\eta m_p c}{\delta W \sigma_W} \equiv \epsilon \eta.$$  (9)

With this definition of $\epsilon$ the spacecraft expends mass at the constant rate

$$\frac{\dot{M}}{M_0} = \frac{\epsilon \mu_f}{c}. \quad \text{(10)}$$

Its rapidity $\rho \equiv \tanh^{-1}(v/c)$ obeys the traditional rocket equation [1,2],

$$\rho = -\eta \log(1 - \frac{\epsilon \mu_f t}{c}). \quad \text{(11)}$$

Here $1 - \epsilon \mu_f t/c \equiv \mu(t)$ is the inverse mass ratio; its final value is $M(t_f)/M_0 = \mu_f$. For $\sigma_W = 10^{-7} \text{ kg}/W_t$, $\epsilon = 9.1 \text{cm/s}^2$. The distance traveled in proper time $t$ is

$$s = c \int_{\mu_f}^{1} \beta(\mu(t)) \frac{dt}{d\mu} \, d\mu = \frac{c^2}{\epsilon \mu_f} \int_{1}^{\mu_f} \beta(\mu) \, d\mu; \quad \text{(12)}$$

$$s(\mu_f) = \frac{\eta c^2}{\epsilon} \frac{1}{\mu_f} \left[ \mu_f \log(\mu_f) - \mu_f + 1 \right], \quad \text{(13)}$$

substituting $\rho$ for $v/c \equiv \beta$, since we are concerned with small rapidities $\leq 0.1$.

In addition to the classical rocket, two other concepts are examined:

### 3.5.1 Ram augmentation

In this variant the spacecraft carries its antiprotonic fuel, but obtains protons from the interstellar medium [18]. Four-momentum balance in the rocket rest frame at $t = 0$ gives

$$-\frac{d M}{M} \approx \frac{d \rho}{\exp(-\rho) + 2 \eta - 1}, \quad \text{(14)}$$

which integrates to

$$\log(\mu) = \frac{1}{1 - 2 \eta} \left[ \rho + \log \left( \frac{\exp(-\rho) + 1 - 2 \eta}{2 \eta} \right) \right], \quad \text{(15)}$$

a transcendental equation for $\rho(\mu)$. The expression for ram deceleration is obtained by changing the sign of $\rho$ in eqn. (14).
3.5.2 Pellet stream

The limit of ram augmentation occurs when a stream of fuel pellets is launched in advance of the spacecraft with a tailored profile of pellet speeds and launch times, so that the spacecraft ingests each fuel pellet at zero velocity in its proper frame \[18\]. For the pellet stream,

\[
\log(\mu) = \frac{1}{2\sqrt{\eta - \eta^2}} \left[ \arcsin \left( \frac{1 + (2\eta - 1) \cosh(\rho)}{\cosh(\rho) + (2\eta - 1)} \right) - \frac{\pi}{2} \right]. \tag{16}
\]

Modifications to eqn. (6) for ram-augmented systems are second order in \(v/c\). For the augmented variants, the distance traveled is computed by numerical integration of eqn. (12) but with \(\epsilon\) replaced by \(\epsilon/2\) in order to maintain consistency with the simple rocket.

4 Performance Estimates

Consider a rendezvous mission with a star at a distance of 10 pc, and with a duration held fixed at 1000 years. Table 1 displays mission profiles for hybrids of pure neutrino rocket, ram- and pellet-augmented rocket, and photon sail acceleration at 0.3 gravities \[18\].

There is a critical value of \(\epsilon\) below which it is impossible to travel 10 pc in 1000 years. The ram/ram profile requires \(\epsilon_c = 23.6\). Other profiles in the table have smaller critical values. The photon beam/ram deceleration profile evidently has the greatest latitude in this regard; for \(\eta = 0.025\), \(\epsilon_c = 2.0\). If the efficiency is as low as \(\eta = 0.015\), \(\epsilon_c = 3.2\). The mass ratio for this last example is 2.64.

5 Discussion

The propulsion system concept outlined above delivers semirelativistic rapidity changes for modest mass ratios, but does so at comparatively low efficiency. The inefficiency arises from the small charged pion multiplicity from annihilation at rest. Purely neutrino-powered examples have been described to illustrate the principle most directly, and for comparison with the classical photon rocket. One would doubtless prefer to treat that portion of annihilation energy which cannot be converted into collimated muon neutrinos as something other than waste heat; to generate thrust with it! The
neutrino rocket may furnish a topping cycle for some other, presumably more efficient, method of annihilation-based propulsion.

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Table 1. Sample Rendezvous Profiles

| Method       | photon/ram | photon/rocket | pellet/ram | pellet/rocket | ram/ram |
|--------------|------------|---------------|------------|---------------|---------|
| acc. mass ratio | N/A       | N/A           | 2.35       | 1.84          | 5.13    |
| acc. t(yr)    | 0.11       | 0.10          | 290        | 179           | 339     |
| acc. distance(pc) | $6.1 \times 10^{-4}$ | $4.4 \times 10^{-4}$ | 1.63       | 0.752          | 2.10    |
| max. v/c      | 0.035      | 0.030         | 0.043      | 0.030          | 0.041   |
| coast distance(pc) | 9.06       | 7.52          | 6.91       | 5.30           | 7.39    |
| coast t(yr)   | 849        | 755           | 525        | 557            | 593     |
| dec. mass ratio | 1.70       | 3.28          | 1.86       | 3.44           | 1.82    |
| dec. t(yr)    | 151        | 245           | 185        | 263            | 68      |
| dec. distance(pc) | 0.94       | 3.13          | 1.46       | 3.95           | 0.50    |
| total mass ratio | 1.70       | 3.36          | 4.37       | 6.32           | 9.36    |

$\eta_0 = 0.025$, $\epsilon = 9.0$, except for ram/ram ($\epsilon = 23.6$). Photon sail profiles use an acceleration of 0.3 gravities.