Heal the Privacy: Functional Encryption and Privacy-Preserving Analytics

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Abstract—Secure cloud storage is an issue of paramount importance that both businesses and end-users should take into consideration before moving their data to, potentially, untrusted clouds. Migrating data to the cloud raises multiple privacy issues, as they are completely controlled by a cloud provider. Hence, an untrusted cloud provider can potentially breach users’ privacy and gain access to sensitive information. The problem becomes even more pronounced when the cloud provider is required to store a statistical database and periodically publish analytics. In this work, we first present a detailed example showing that the use of cryptography is not enough to ensure the privacy of individuals. Then, we design a hybrid protocol based on Functional Encryption and Differential Privacy that allows the computations of statistics in a privacy-preserving way.

Index Terms—Cloud Security, Differential Privacy, Functional Encryption

I. INTRODUCTION

Statistics, and data analytics in general, are very important tools for a variety of predictions. From real-time traffic analysis to disease outbreaks discovery, statistics allow societies to predict critical situations and prepare accordingly. However, along with the growth of cloud computing, such prediction services are moving to the cloud, where untrusted third parties may host and control statistical databases. Naturally, this raises several security concerns as the privacy of individuals can often be breached. These concerns become even greater when the analytics in question refer to extra sensitive data, such as medical records. A first response to these problems was presented in [1] with the formalization of differential privacy. Differential Privacy allows sharing information about a dataset while withholding information about the individuals. In a differential private scheme, a curator (data owner) generates a dataset and, upon request of an analyst, releases statistics. FE is an emerging cryptographic technique which allows computations over encrypted data. More precisely, FE schemes provide key generation algorithms that output decryption keys with remarkable capabilities. In contrast to traditional cryptography, each functional decryption key $sk_f$ is associated with a function $f$. Decrypting a ciphertext $\text{Enc}(x)$ using $sk_f$, yields $f(x)$ and thus keeps the $x$ private. More recent works [2] generalized the concept of FE by presenting Multi-Input Functional Encryption (MIFE). In a MIFE scheme, given encryptions $\text{Enc}(x_1), \ldots, \text{Enc}(x_n)$, a user can use $sk_f$ to recover $f(x_1, \ldots, x_n)$. In our work, we combine MIFE with differential privacy to design a scheme that allows the periodical release of statistics in a privacy-preserving way.

Contribution: To the best of our knowledge this is amongst the first works that combine differential privacy with cryptography to ensure the security of datasets, and the first one that does so using FE. More specifically:

C1. By combining FE with differential privacy, we propose a hybrid protocol as solution to the problem of designing encrypted private databases. Our work draws inspiration from both the fields of FE and Symmetric Searchable Encryption [3, 4].

C2. We provide a detailed security analysis of our protocol by demonstrating that it remains secure in the presence of a malicious adversary. Furthermore, we formally prove that our protocol satisfies the notion of differential privacy.

C3. Our solution is considered as efficient since it relies only on symmetric cryptographic primitives.

Organization: The rest of the paper is organised as follows: In section II we present a concrete example that proves that cryptography is not enough to secure statistical databases. In section III we discuss important published works in the fields of functional encryption and differential privacy. Section IV contains all the necessary notations, cryptographic primitives and security notions used throughout the paper, and is followed by section V where we present the detail of our system model. Section VI demonstrates the core contribution of the work as we present a scheme for publishing statistics in a privacy-preserving way. The security of our construction is proved in section VII and finally, section VIII concludes the paper.
II. Motivation and Application Domain

The ultimate goal of this work is to enable authorized users (analysts) to perform statistical analyses over medical datasets in a privacy-preserving way. In order to make this possible, we needed to ensure that our construction would be resistant against both internal (e.g. malicious servers) as well as external (e.g. malicious analysts) attacks.

For our solution, we used structured datasets composed of three different kinds of variables: categorical, ordinal and numerical:

- **Categorical variables** do not have a natural ordering. For example, the medical diagnosis of a patient is a categorical variable.
- **Ordinal variables** are categorical variables for which possible values can be ordered. For example, the condition of a patient, for which we can arbitrarily assume to be mild < severe < critical is considered an ordinal variable.
- **Numerical variables** are expressed using numbers e.g. the age of a patient or systolic and diastolic blood pressures.

To make things clearer, let us consider a scenario in which four patients sought medical care. Following the examination, the hospital stores their medical records to a structured dataset. As a next step, the hospital (who in this case acts as the curator of the dataset) masks all the ordinal and categorical variables in the dataset using a cryptographic hash function, and encrypts the numerical variables with a MIFE scheme. Without loss of generality, we can assume that our dataset looks like the one in Table I where each \( H(\cdot) \) denotes the hash of a variable and each \( c_x \) denotes the ciphertext corresponding to a plaintext \( x \). Finally, the dataset is outsourced to a cloud service provider (CSP), where it will be stored.

For our work, we want to enable analysts to query the CSP in a privacy-preserving manner with queries in the form of “What is the average age of all patients that have been diagnosed with covid19?” or “What is the blood pressure of the patients whose condition is severe?”. In other words, we want to be able to compute a function on the values of the numerical variables that correspond to a specific categorical or ordinal variable.

It should be noted that although cryptography ensures the data confidentiality, it does not ensure the individuals’ privacy. For example, if an analyst were to initially requests the average age for the first three cases in the dataset and subsequently request the average age of all patients, it would become obvious how Scottie’s age influences the average and hence, its value could be deduced. To protect the individuals’ privacy, we rely on the notion of differential privacy. By embedding well-calibrated error in the decryption algorithm, we ensure that the analyst has access to accurate enough results in order to perform any kind of analytics, without breaching the individual’s privacy.

III. Related Work

**Functional Encryption**: While numerous studies with general definitions and generic constructions of FE have been proposed [5]–[10] there is a clear lack of works proposing FE schemes supporting specific functions. To the best of our knowledge, currently the number of supported functionalities is limited to inner products [11]–[13], quadratic polynomials [14] and the \( \ell_1 \) norm of a vector [15]. In this work, we use the symmetric construction for the \( \ell_1 \) norm presented in [15] to design a functionally encrypted private scheme.

**Differential Privacy**: Differential privacy is a notion first formalized in [11], where authors focused on ensuring the individuals’ privacy. More precisely, they proved that by adding well-calibrated noise to the data, the presence or absence of an individual’s information is irrelevant to the output of a database query. Since then, differential privacy has drawn the attention of both researchers [16]–[19] and key industry players such as Google [20] and Uber [21]. Nonetheless, to the best of our knowledge the only work that combines differential privacy with cryptography is the one presented in [22], where authors designed a scheme for private histogram queries. However, the solution presented in [22] relied on homomorphic encryption and hence, queries were restricted to only asking for the value of a counter. In our work, by using FE we allow users to perform any kind of query that is supported by the functionality of the FE scheme.

IV. Background

**Notation**: If \( \mathcal{Y} \) is a set, we use \( y \overset{\$}{\leftarrow} \mathcal{Y} \) if \( y \) is chosen uniformly at random from \( \mathcal{Y} \). The cardinality of a set \( \mathcal{Y} \) is denoted by \( |\mathcal{Y}| \). Vectors are denoted in bold as \( \mathbf{x} = [x_1, \ldots, x_n] \). A probabilistic polynomial time (PPT) adversary \( \text{ADV} \) is a randomized algorithm for which there exists a polynomial \( p(z) \) such that for all input \( z \), the running time of \( \text{ADV}(z) \) is bounded by \( p(|z|) \).

**A. Functional Encryption**

**Definition 1** (Multi-Input Functional Encryption in the Symmetric Key Setting). Let \( \mathcal{F} = \{f_1, \ldots, f_n\} \) be a family of \( n \)-ary functions where each \( f_i \) is defined as follows: \( f_i: \mathbb{Z}^n \rightarrow \mathbb{Z} \). A multi-input functional encryption scheme for \( \mathcal{F} \) consists of the following algorithms:

- Setup(1^\lambda) : Takes as input a security parameter \( \lambda \) and outputs a secret key \( \mathbf{K} = [k_1, \ldots, k_n] \in \mathbb{Z}^n \).
- \( \text{Enc}(\mathbf{K}, i, \mathbf{x}_i) \) : Takes as input \( \mathbf{K} \), an index \( i \in [n] \) and a message \( x_i \in \mathbb{Z} \) and outputs a ciphertext \( ct_i \).
Definition 4 (Laplace distribution). The Laplace distribution centered at 0 and with scale parameter $b$ is given by:

$$\text{Lap}(z) = \frac{1}{2b} e^{-\frac{|z|}{b}}$$

where the mean is 0 and the variance is $2b^2$.

We are now ready to proceed with the definition of the Laplace Mechanism. For the needs of our work, we rely on the one-AD-IND-secure symmetric MIFE scheme for the $\ell_1$ norm, presented in [15]. Informally, one-AD-IND security ensures that given the encryption of two messages $x_1$ and $x_2$, and a functional key $sk_f$ for a function $f$ such that $f(x_1) = f(x_2)$, no PPT adversary should be able to distinguish between them. With the aim of completeness and improved readability, the MIFE scheme for the $\ell_1$ norm is illustrated in Figure 1.

![Fig. 1. one-AD-IND-secure MIFE for the $\ell_1$ norm (MIFE$_{\ell_1}$).](image)

B. Differential Privacy

We proceed by providing the main definitions of $\epsilon$-differential privacy ($\epsilon$-DP) and the main properties of the Laplace mechanism.

Definition 2. Two datasets $DS$ and $DS'$ are neighbouring if:

$$\|DS - DS'\|_1 \leq 1$$

Definition 3 ($\epsilon$-DP). A privacy mechanism $M : \mathbb{N}^{\|DS\|} \rightarrow Im(M)$ is $\epsilon$-DP if $\forall S \subseteq Im(M)$ and $\forall$ neighboring datasets $DS, DS' \in \mathbb{N}^{\|DS\|}$:

$$\text{Pr}[M(\text{DS}) \in S] \leq e^\epsilon \text{Pr}[M(\text{DS'}) \in S]$$

Definition 4 (Laplace distribution). The Laplace distribution centered at 0 and with scale parameter $b$ is given by:

$$\text{Lap}(z) = \frac{1}{2b} e^{-\frac{|z|}{b}}$$

where the mean is 0 and the variance is $2b^2$.

V. ARCHITECTURE

In this section, we introduce the system model by explicitly describing the main entities participating in our protocol along with their capabilities.

We assume the existence of the following four entities:

1. Curator (C): C is responsible for generating an encrypted dataset and outsourcing to the CSP. C also generates a list $L_{MA}$ containing mappings between encryption keys and their unique identifiers. This list is outsourced to MA.

2. Analyst (A): A is an analyst that can perform statistics on the data stored in the CSP.

3. Cloud Service Provider (CSP): We consider a cloud computing environment based on a trusted IaaS provider similar to the one described in [23]. The CSP is responsible for storing an encrypted dataset. Apart from that, upon A’s request the CSP is required to perform a search operation on the encrypted dataset and further communicate with the Master Authority for the generation of secret functional keys.

4. Master Authority (MA): MA is a trusted authority that is responsible for issuing secret functional keys. To do so, MA is required to maintain a list containing mappings between encryption keys and their unique identifiers.

VI. FORMAL CONSTRUCTION

This section presents the core contribution of this work as we formally present Private Searchable Functional Encryption (PSFE). We assume the existence of an IND-CCA2 secure public key cryptosystem and a EUF-CMA secure signature scheme. Finally, we also utilize a first and second preimage resistant hash function $H$. PSFE consists of three algorithms: Gen, Setup and Read such that:

**PSFE.Gen**: Each entity from the described architecture receives a public/private key pair $(pk, sk)$ for an IND-CCA2 secure public cryptosystem, and publishes its public key while keeping the private key secret. Apart from that, all entities generate a signing and a verification key. Below we provide a list of all the generated keys:

- $(pk_C, sk_C), (sig_C, ver_C)$ - public/private, signing/verification and MIFE secret key for the Curator;
- $(pk_A, sk_A), (sig_A, ver_A)$ - public/private and signing/verification key pairs for the Analyst;
- $(pk_{CSP}, sk_{CSP}), (sig_{CSP}, ver_{CSP})$ - public/private and signing/verification key pairs for the cloud service provider;
- $(pk_{MA}, sk_{MA}), (sig_{MA}, ver_{MA})$ - public/private, signing/verification key pairs for the master authority.

**PSFE.Setup**: Represents a three party protocol between C, the CSP and MA. PSFE.Setup is initiated by C who wants to outsource an encrypted dataset (EDS) to the CSP. To encrypt the dataset, C hashes all the categorical and the ordinal entries concatenated with a salt $s$ to prevent dictionary attacks. Apart from that, C also hashes the entries without the salt and stores each pair (salted and unsalted hashed entry) in a list $L_{MA}$.

For the numerical ones, C generates a symmetric key $k$ and
uses it to encrypt the corresponding entry. Apart from that, for each generated \( k \), \( C \) generates a unique index. The keys, along with their indexes are stored in a list \( L_{MA} \). Finally, \( C \) sends \( m_1 = \langle t_1, pk_{CSP}(EDS), \sigma_C(H(t_1 \| EDS)) \rangle \) to the CSP and \( m_2 = \langle t_2, pk_{MA}(L_{MA}, \sigma_{MA}(t_2 \| L_{MA})) \rangle \) to MA. Upon receiving these messages, both the CSP and MA verify their freshness (by looking at the timestamps \( t_1 \) and \( t_2 \)) and the identity of the sender (by verifying the signature). If the verifications are successful, the CSP stores EDS and MA stores both \( L_{MA} \) and \( L_{MA} \). In addition to that, both the CSP and MA send an acknowledgement to \( C \) that they have successfully stored EDS and the two lists via \( m_3 = \langle t_3, \sigma_{CSP}(H(t_3 \| EDS)) \rangle \) and \( m_4 = \langle t_4, \sigma_{MA}(t_4 \| L_{MA}) \| L_{MA} \rangle \) respectively. The encryption of the dataset is presented in detail in algorithm 1 and the flow of PSFE.Setup is illustrated in Figure 2.

Algorithm 1 Dataset Encryption

1: **Input:** A plaintext Dataset DS
2: **Output:** An encrypted Dataset EDS
3: \( K = \{ \} \)
4: \( [r, c] = \) size(DS) \( \triangleright \) Number or rows and columns
5: for \( i = 1 \) to \( r \) do \( \triangleright \) All the cases
6: \( \triangleright \) All the variable
7: \( \triangleright \) All the cases
8: if \( DS(i, j) = \) categorical OR ordinal then
9: \( s_{ij} \leftarrow Z \)
10: \( L_{MA} = L_{MA} \cup H(DS(i, j)) \| H(DS(i, j)) \| s_{ij} \)
11: \( DS(i, j) = H(DS(i, j)) \| s_{ij} \)
12: else
13: Generate \( k_{ij} \in Z \)
14: \( \text{index}_{s_{ij}} = H(k_{ij}) \)
15: \( DS(i, j) = (DS(i, j) + k_{ij}) \| \text{index}_{s_{ij}} \)
16: \( L_{MA} = L_{MA} \cup (k_{ij}) \| \text{index}_{s_{ij}} \)
17: Outsource \( L_{MA} \) and \( L_{MA} \) to MA
18: EDS = DS

PSFE.Read: Represents a tree party protocol between \( A \), the CSP and MA. PSFE.Read is initiated by the analyst \( A \) wishing to perform statistical analysis on the encrypted dataset. To do so, \( A \) first generates a search token \( \tau_s \) as \( \tau_s = \langle H(w_j), H(w_j), f \rangle \), where \( H(w_j) \) refers to a categorical or ordinal value, \( H(w_j) \) refers to a variable, and \( f \) is the description of a function that will be applied to the ciphertexts. Then, \( A \) sends \( m_5 = \langle t_5, \tau_s, \sigma_A(H(t_5 \| \tau_s)) \rangle \)

to the MA. Upon reception, MA verifies the freshness and the signature of \( m_5 \). If the verification is successful, MA retrieves the list \( L_{MA} \), containing the salted hashes, finds which salted values correspond to \( H(w_j) \) and \( H(w_j) \) and sends them to the CSP via \( m_6 = \langle t_6, H(w_j), s_j, H(w_j), s_j, \sigma_{MA}(t_6 \| H(w_j), s_j) \| H(w_j), s_j) \rangle \).

Upon reception, the CSP verifies the freshness and the signature of \( m_6 \). If the verification is successful, the CSP finds the ciphertexts that correspond to \( H(w_j) \) with attribute \( H(w_j) \) and sends the result \( R \) back to \( A \) via \( m_7 = \langle t_7, R, \sigma_{CSP}(H(t_7 \| L_{index} \| f)) \rangle \).

At the same time CSP retrieves the unique index for each ciphertext \( w_j \in R \), and stores them in a list \( L_{index} \) before outsourcing them to MA via \( m_8 = \langle t_8, L_{index}, f, \sigma_{CSP}(H(t_8 \| L_{index} \| f)) \rangle \).

Upon reception of \( m_8 \) (and if the verifications are successful), MA can construct the functional key \( sk_f \) as a linear combination of all the keys \( k_i \) such that \( H(k_i) = L_{index} \). Apart from that, MA samples an error \( e \approx \text{Lap}(1/\epsilon) \) and computes a noisy key \( sk_f = sk_f + e \). Finally, \( sk_f \) is sent back to \( A \) via \( m_9 = \langle t_9, pk_A(sk_f), \sigma_{MA}(H(t_9 \| sk_f)) \rangle \).

Upon receiving \( sk_f \), \( A \) computes the result as follows:

\[
\sum_{i=1}^{n} c_i - sk_f = \sum_{i=1}^{n} (k_i + x_i) - \sum_{i=1}^{n} k_i + e = \sum_{i=1}^{n} x_i + e
\]

PSFE.Read is illustrated in Figure 3.
VII. SECURITY ANALYSIS

In this Section we prove the security of our protocol, and show that the PSFE.Read protocol is \(\epsilon\)-differential private one. Before proceeding to do so, we formally define our threat model.

A. Threat Model

**Threat Model**: Our threat model is similar to the one described in [23], based on the Dolev-Yao adversarial model [24]. We additionally extend it by defining a set of new attacks.

**Attack 1 (Result Substitution Attack)**. Let \(\text{ADV}\) be an adversary that observes the communication between \(A\) and the CSP. \(\text{ADV}\) successfully launches a Result Substitution Attack, if she manages to replace the result list \(R\), sent from the CSP to \(A\), with another one \(R'\).

**Attack 2 (Key Substitution Attack)**. Let \(\text{ADV}\) be an adversary that observes the communication channels between \(A\), the CSP and MA. \(\text{ADV}\) successfully launches a Key Substitution Attack, if \(A\) receives a wrong \(sk'_f\) in a way that is indistinguishable to her.

B. Protocol Security

We will proceed to prove PSFE’s soundness against the attacks defined in Section VII-A.

**Proposition 1** (Result Substitution Attack Soundness). Let \(\text{ADV}\) be an adversary that overhears the communication between \(A\) and the CSP. Then \(\text{ADV}\) cannot successfully launch a Result Substitution Attack.

**Proof**. For \(\text{ADV}\) to successfully launch a Result Substitution Attack, she needs to tamper with the result list \(R\) that is sent from the CSP to \(A\) via \(m_6 = (t_6, R, \sigma_{\text{CSP}}(H(t_6||R)))\). To do so, \(\text{ADV}\) has two choices:

- Replace an old \(m_6\) message
- Replace \(R\) with another result list \(R_{\text{mal}}\)

In the instance where \(\text{ADV}\) overhears the communication between \(A\) and the CSP, we can assume that \(\text{ADV}\) possesses an old \(m_6\) message \(m_{\text{old}} = (t_{\text{old}}, R_{\text{old}}, \sigma_{\text{CSP}}(H(t_{\text{old}}||R_{\text{old}})))\). Thus, when the CSP sends \(m_6\) to \(A\), \(\text{ADV}\) intercepts the communication and replaces \(m_6\) with \(m_{\text{old}}\). Upon receiving \(m_{\text{old}}\), \(A\) verifies the signature, and since \(m_{\text{old}}\) contains a valid CSP’s signature, the verification is successful. However, when \(A\) tries to verify the freshness of the message, she notices that the timestamp is old and thus drops the communication. As a result, the only way for \(\text{ADV}\) to successfully launch the attack, is to use another result list \(R_{\text{mal}}\).

Just like before, when the CSP sends \(m_6\) to \(A\), \(\text{ADV}\) intercepts the communication and replaces \(R\) with \(R_{\text{mal}}\). However, the result list \(R\) is also included in the CSP’s signature. Thus, replacing \(R\) with \(R_{\text{mal}}\) in an indistinguishable way, is equivalent to forging the CSP’s signature. Nonetheless, given the signature scheme’s EUF-CMA security, there is only a negligible probability for this to happen and hence, the attack fails.

\(\Box\)

**Proposition 2** (Key Substitution Attack Soundness). Let \(\text{ADV}\) be an adversary that overhears the communication channels between \(A\), the CSP and MA. Then \(\text{ADV}\) cannot successfully launch a Key Substitution Attack.

**Proof**. Since the encryption keys are elements in \(\mathbb{Z}\), and the secret functional key is a linear combination of the encryption keys, it follows that the functional key lives in \(\mathbb{Z}\) as well. This means that \(A\) is expecting to receive an integer and hence \(\text{ADV}\) could easily replace the real integer number in a way that is indistinguishable for \(A\). As a result, we need to make sure that even the slightest modification in the structure of the messages will have a big impact on what \(A\) receives. For \(\text{ADV}\) to successfully launch a Key Substitution Attack, she needs to replace the key \(sk'_f\) with one of her choice in a way that is indistinguishable for \(A\). To do so, \(\text{ADV}\) can follow the following two approaches:

- Replace the functional key sent from MA to \(A\), as part of \(m_8\), with a key of her choice.
- Force MA to compute a functional key for a function \(g\) such that \(g \neq f\).

Tampering with the \(m_8\) message sent from MA to \(A\) requires \(\text{ADV}\) to either use an old \(m_8\) message or forge the signature of MA. As we saw in the proof for proposition\(^{[1]}\) the use of timestamps and the EUF-CMA security of the signature scheme, ensure that \(\text{ADV}\) can only achieve this with negligible probability. Hence, we conclude that the only way for \(\text{ADV}\) to successfully launch a Key Substitution Attack is to force MA to compute a functional key for a function \(g \neq f\).

Fooling MA into computing a wrong functional key, requires \(\text{ADV}\) to tamper with the \(m_7\) message sent from the CSP to MA. Recall that \((t_7, L_{\text{index}}, f, \sigma_{\text{CSP}}(H(t_7||L_{\text{index}}||f)))\). By observing the structure of \(m_7\), we see that \(\text{ADV}\) can either target \(L_{\text{index}}\), the description of the function \(f\), or both. However, similarly to the proof for proposition\(^{[1]}\) as \(L_{\text{index}}\) and the function \(f\)’s description are also included in the CSP’s signature, tampering with them is equivalent to forging the CSP’s signature, which can only happen with negligible probability due to the signature scheme’s EUF-CMA security. Moreover, the timestamp, ensures that \(\text{ADV}\) cannot replace an older message. We thus prove that in both cases, \(\text{ADV}\) can successfully launch a Key Substitution Attack with negligible probability.

\(\Box\)

C. Differential Privacy

In this section, we prove that the PSFE.Read protocol is \(\epsilon\)-differential private one.

**Theorem 1**. Let \(\mathcal{EDS}, \mathcal{EDS}' \in \mathbb{N}^{[DS]}\) be arbitrary neighbouring datasets, let \(q : \mathbb{N}^{[DS]} \rightarrow \mathbb{R}\) be an arbitrary query and let \(r, r' \in \mathbb{R}\). Moreover, let \(M_L\) be the Laplace Mechanism. Then, the PSFE.Read protocol is \(\epsilon\)-differentially private as per Definition\(^{[3]}\).

\(\Box\)
Proof. Our goal is to prove that issuing a private query $q$ to EDS reveals no more information than what is allowed by the privacy factor $\epsilon$. In our construction, when $A$ uses $sk_f$ to decrypt the result list $R$, she gets a result $r'$. In other words, $q(EDS) = r'$. However, the query contains the secret functional key $sk_f = r + e$, where $e \leftarrow \text{Lap}(\frac{\Delta q}{\epsilon})$, and when the Laplace Mechanism is applied to the query we get:

$$M_L(q, EDS, \epsilon) = r = r' + e \Rightarrow e = r - r' \Rightarrow e = r - q(EDS).$$

However, since $e$ is arbitrarily chosen from the Laplace distribution ($e \leftarrow \text{Lap}(\frac{\Delta q}{\epsilon})$), then, without loss of generality, we can replace $e$ with $\text{Lap}(\frac{\Delta q}{\epsilon})$. Hence, we get:

$$\frac{Pr[M_L(EDS, q, \epsilon) = r]}{Pr[M_L(EDS', q, \epsilon) = r]} = \frac{Pr[\text{Lap}(\frac{\Delta q}{\epsilon}) = r - q(EDS)]}{Pr[\text{Lap}(\frac{\Delta q}{\epsilon}) = r - q(EDB')]}
= \frac{\frac{1}{2\Delta q} \exp\left(-\frac{|r-q(EDS)|}{\Delta q} \epsilon\right)}{\frac{1}{2\Delta q} \exp\left(-\frac{|r-q(EDS')|}{\Delta q} \epsilon\right)}
= \exp\left(-\epsilon |r - q(EDS')| - |r - q(EDS)|\right)
= \exp\left(\epsilon q(EDS') - q(EDS)\right) \leq e^\epsilon \qedhere$$

VIII. CONCLUSIONS

We strongly believe that in the future cloud-based services will rely less on traditional decryption of information and more on computations over encrypted data. With this in mind, we proposed PSFE; a hybrid protocol based on Functional Encryption and differential privacy. Our protocol allows an analyst to periodically query a CSP for the release of statistics, without breaches the individuals’ privacy. We hope that this work will inspire researchers and open new questions in the fascinating field of privacy-preserving computations in untrusted clouds, thus allowing us to create a bridge between the theoretical concepts of FE and real life applications.

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