Rotational stiffness of cold-formed steel roof purlin–sheeting connections

Congxiao Zhao a, Jian Yang b,c,d,e, Feiliang Wang a, Andrew H.C. Chan e

a School of Civil Engineering, University of Birmingham, Edgbaston B15 2TT, UK
b School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, PR China
c State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200240, PR China
d School of Civil Engineering, University of Birmingham, Birmingham, Edgbaston B15 2TT, UK
e School of Science, Information Technology & Engineering, University of Ballarat, 3350, Australia

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Cold-formed steel (CFS) sections are commonly used in modern roof construction. Most purlin members are of thin-walled open cross section. They are usually subjected to roof loading at the top flange in either an upward or a downward direction. The load application points, where the sheeting/purlin connections are located, are often eccentric to the shear centre, and thus inevitably generate a torsional moment that will induce twisting and/or warping deformations in addition to bending deflection. This type of complexity associated with the loading conditions will be exacerbated by the occurrence of single- or mixed-mode buckling (e.g. overall, distortional and local buckling) due to compression flanges tending to move sideways. The connections between purlin and roof sheeting provide a restraining effect on purlin members by preventing such lateral and twisting movements, and thus have a beneficial effect on their load-carrying capacity. In design practice, this effect should be taken into account from a design-efficiency perspective. To this end, a key step is to quantify the rotational restraint stiffness by using an engineering-orientated model. This paper firstly reports a series of torsional restraint tests (F-tests) for both sigma and zed sections. Two loading directions were examined by adjusting the purlin fixing direction. The rotational angles between the connected flange and sheeting were recorded at each loading step, from which the moment–rotation curves were produced and presented for each test case. A linear relationship has been observed for the moment–rotation relationship from all test specimens. Secondly, a hand calculation model for calculating the rotational stiffness at each connection was developed. In that model, the rotation was deemed to be primarily caused by the localised deformation of the roof sheeting and the distortional deformation of the purlin flange. The rotation caused by the separation of connection was found to be negligible. The model was validated by the experimental test results and an example was presented to demonstrate the application of the model proposed. The rotational stiffness calculated by this model can be used to evaluate the input parameters required for numerical modelling of purlin–sheeting interaction.

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1. Introduction

Cold-formed steel (CFS) sections have a wide range of applications in modern construction, such as being used as purlins or as side rails in light-weight buildings [1]. For building structural systems, a purlin is a type of secondary element acting as an intermediate member in the load path to transfer load from the roof sheeting to the primary frame structure. Common types of purlin sections include channel, zed and sigma shapes. In design practice, this group of sections is normally classified as slender because the sections are unlikely to reach their full cross-sectional resistance governed by the yield stress of constituent material [2]. Furthermore, the open and thin-walled cross sections may lead to a high susceptibility to various types of buckling failure, e.g. local, distortional and lateral torsional buckling. Roof sheeting, which is normally attached to purlins using self-drilling screws, can enhance a purlin’s load resistance by supplying it with a certain degree of lateral and rotational restraining effect. Therefore, it is common and economical to treat these two members as an interactive system during the design process [3].

Research into the performance of purlin–sheet systems can date back to the 1960s, including some key research studies mentioned...
below. Lucas et al. [4,5] initially presented a full, and later a simplified, finite element (FE) model to study the interactional behaviour and its effect on the load-carrying capacity of purlin–sheet systems. The full FE model was comprehensive and accounted for both gravity and uplift loading conditions; however, not all modelling information was presented in detail and hence the model is difficult to reproduce. Vieira et al. [6,7] developed an FE model for the purlin–sheeting system, allowing for the material and geometric nonlinearity effects, to investigate the ultimate load and the longitudinal stress in channel shaped purlins. Li et al. [8] has presented an analytical method for predicting the flexural behaviour of zed purlins under uplift load when they are partially restrained by roof sheets. The model adopts the classic asymmetrical beam theory by considering both bending and twisting effects. Research by Sokol [9] focused on the lateral torsional buckling of purlins restrained by sheeting, and developed a semi-analytical method taking into account the effects of anti-sag bars and the moment gradient. All these studies concur that roof sheeting provides both lateral and rotational restraint to purlins. While the lateral restraint is usually considered to be fully effective, the rotational restraint can be variable but plays a vital role in determining the flexural behaviour of purlins [10], e.g. a higher rotational stiffness can lead to a reduced buckling length in the compression zone, a reduced tensile stress in the free flange, and therefore a higher loading resistance [11].

There is a consensus that the effect of rotational restraint of purlin–sheeting systems is associated with a variety of factors such as the shape and thickness of the sheeting, the geometry of the purlin, the number of screws per unit length, and the type of screw and its applied location. Ye et al. [12,13] investigated the effect of the magnitude and location of rotational restraints on buckling.

### Nomenclature

| Symbol | Description |
|--------|-------------|
| \(a\)  | the distance between the screw and the line of contact between purlin and sheeting |
| \(b\)  | the remaining flange width after subtracting \(a\), i.e. \(C - a\) |
| \(b_T\) | the breadth of the single trough of roof sheeting |
| \(C\)  | the flange width |
| \(C_d\) | the rotational stiffness |
| \(D\)  | the bending stiffness of roof sheeting per unit run, \(EJ/T\) |
| \(l_p\) | the second moment of areas of sheeting and purlin |
| \(l_{p_r}\) | the rotation angle of the cantilever sheet at the screw connection |
| \(\theta_p\) | the rotation angle associated with the localised deformation of the sheet at the screw connection |
| \(\theta_k\) | the rotation angle due to the separation between the roof sheet and the purlin flange at the screw connection |
| \(\theta_r\) | the rotation angle due to the purlin flange bending |

### Table 1
Nominal cross section dimensions for sigma sections.

| Section code | Depth (mm) | Flange (mm) | Lips (mm) | Outer-web (mm) | Stiffener (mm) | Thickness (mm) |
|--------------|------------|-------------|-----------|----------------|----------------|----------------|
| \(\Sigma\)20012 | 200        | 62.5        | 20        | 45             | 16             | 1.2            |
| \(\Sigma\)20016 | 200        | 62.5        | 20        | 45             | 16             | 1.6            |
| \(\Sigma\)20025 | 200        | 62.5        | 20        | 45             | 16             | 2.5            |
| \(\Sigma\)24015 | 240        | 62.5        | 20        | 45             | 16             | 1.5            |
| \(\Sigma\)24023 | 240        | 62.5        | 20        | 45             | 16             | 2.3            |
| \(\Sigma\)24030 | 240        | 62.5        | 20        | 45             | 16             | 3.0            |
| \(\Sigma\)30018 | 300        | 75.0        | 20        | 60             | 16             | 1.8            |
| \(\Sigma\)30025 | 300        | 75.0        | 20        | 60             | 16             | 2.5            |
| \(\Sigma\)30030 | 300        | 75.0        | 20        | 60             | 16             | 3.0            |

\(y-y\) is geometric axes of cross section.

### Table 2
Nominal cross section dimensions for zed sections.

| Section code | Depth (mm) | Flange (mm) | Lips (mm) | Thickness (mm) |
|--------------|------------|-------------|-----------|----------------|
| Z14614       | 145        | 62.5        | 20        | 1.4            |
| Z14618       | 145        | 62.5        | 20        | 1.8            |
| Z20618       | 200        | 65          | 20        | 1.8            |
| Z30720       | 300        | 75          | 20        | 2.0            |

\(y-y\) is geometric axes \(u-u\) is principal axes of cross section.
resistance and the buckling mode of purlins. The study from Schafer [14] indicates that an adequate rotational restraint from roof sheeting can partially or even fully eradicate the distortional buckling failure mode. Fiorino et al. [15] carried out an experimental study on the effect of screw connections on the behaviour of the roofing system by considering various sheathing types, loaded edge distances and loading conditions. Katnam et al. [16,17] presented a nonlinear FE model to quantify the rotational stiffness of both single-skin and insulated sandwich sheeting. Vrany [18] adopted a flexibility method to predict rotational stiffness at connection. The numerical model took account of most influencing factors, and hence it became overly complicated and the derivations of some coefficients were unclear.

![Fig. 1. Test set-up for sigma and zed sections.](image1)

![Fig. 2. Dimensions of the roof sheet.](image2)

![Fig. 3. Multiple point loads to simulate the uniformly distributed load (UDL).](image3)
In design practice, the current numerical model adopted in EC3 [19] for determining rotational stiffness was first introduced by Lindner and Gregull [20], who considered the stiffness into three parts: that due to the out-of-plane flexural stiffness of the sheeting, the purlin's resistance to distortion, and the localised deformations at the connection point. The coefficients in the empirical equations were derived from extensive test data, in which assumptions were made to simplify the problem, but they often led to overly conservative outcomes. The \( R \)-factor approach is adopted by the North America Notification [21], which takes account of the effect of rotational and lateral stiffness into a reduction factor. This design method is user-friendly, but the detailed deduction procedure for corresponding \( R \)-factors is unclear.

A closed-form engineering model for rotational stiffness is required for the task of predicting sheeted roof purlin resistance within European design standards (EC3). The semi-analytical model developed by Gao and Moen [22] is to date the latest and most accurate method. The model considers the rotation at connections generated by both localised deformation of a panel and the bending of a restrained purlin flange. The model distinguishes channel and zed sections and provides results of less than 11% deviation in comparison to experimental studies and FE simulations. However, the method has some limitations:

1. The method assumes only one type of contact for zed and channel sections under the uplift loading condition. It is valid for channel sections since the torsional moment caused by the eccentric applied load brings the section into contact with the roof sheet at its flange-lip junction line. However, according to Vrany [10], there are two possible deformation modes for zed sections. Depending on the direction of the eccentric load, the purlin may be in contact with the attached sheet at either the flange-web or the flange-lip junction line. Therefore, this will lead to different behaviour patterns in the resulting rotational stiffness.

2. The method utilises a rather simplified FE model that neglects purlin–sheeting interaction at the screw points; hence to obtain results, engineers need to have access to computer programmes.

| Specimen ID | Measured web depth (mm) | Measured purlin thickness (mm) | Load (N) | Rotation, radian | Vertical deformation (mm) | \( G_c \) (N mm/rad/mm) |
|-------------|-------------------------|-------------------------------|----------|------------------|--------------------------|-------------------------|
| \( \Sigma 20012FD \) 203 | 1.26                     | 141                           | 0.072    | 21.5             | 394                      |
| \( \Sigma 20012FU \) 201 | 1.23                     | 151                           | 0.054    | 19.5             | 502                      |
| \( \Sigma 20016FD \) 200 | 1.58                     | 201                           | 0.070    | 21.5             | 622                      |
| \( \Sigma 20016FU \) 200 | 1.62                     | 241                           | 0.070    | 21.7             | 691                      |
| \( \Sigma 20025FD \) 200 | 2.40                     | 328                           | 0.073    | 21.8             | 895                      |
| \( \Sigma 20025FU \) 198 | 2.42                     | 384                           | 0.075    | 20.8             | 1024                     |
| \( \Sigma 24015FD \) 239 | 1.58                     | 151                           | 0.063    | 23.4             | 593                      |
| \( \Sigma 24015FU \) 241 | 1.58                     | 151                           | 0.052    | 24.5             | 696                      |
| \( \Sigma 24023FD \) 241 | 2.17                     | 301                           | 0.085    | 24.0             | 846                      |
| \( \Sigma 24023FU \) 241 | 2.20                     | 301                           | 0.072    | 23.6             | 1009                     |
| \( \Sigma 24030FD \) 240 | 2.95                     | 385                           | 0.091    | 22.7             | 909                      |
| \( \Sigma 24030FU \) 241 | 2.99                     | 381                           | 0.087    | 22.6             | 1047                     |
| \( \Sigma 30018FD \) 301 | 1.81                     | 201                           | 0.082    | 30.9             | 735                      |
| \( \Sigma 30018FU \) 301 | 1.79                     | 171                           | 0.068    | 29.8             | 753                      |
| \( \Sigma 30025FD \) 300 | 2.37                     | 241                           | 0.079    | 30.8             | 921                      |
| \( \Sigma 30025FU \) 301 | 2.31                     | 301                           | 0.084    | 29.5             | 1078                     |
| \( \Sigma 30030FD \) 301 | 3.04                     | 341                           | 0.105    | 30.8             | 977                      |
| \( \Sigma 30030FU \) 302 | 2.99                     | 381                           | 0.096    | 30.5             | 1191                     |
| \( \Sigma 41614FD \) 147 | 1.56                     | 302                           | 0.086    | 14.2             | 512                      |
| \( \Sigma 41614FU \) 145 | 1.54                     | 331                           | 0.061    | 13.5             | 762                      |
| \( \Sigma 41618FD \) 145 | 1.78                     | 404                           | 0.080    | 14.9             | 730                      |
| \( \Sigma 41618FU \) 147 | 1.78                     | 444                           | 0.078    | 14.9             | 820                      |
| \( \Sigma 20617FD \) 200 | 1.80                     | 261                           | 0.077    | 21.3             | 680                      |
| \( \Sigma 20617FU \) 202 | 1.71                     | 320                           | 0.073    | 19.3             | 876                      |
| \( \Sigma 30720FD \) 299 | 2.08                     | 221                           | 0.084    | 31.2             | 791                      |
| \( \Sigma 30720FU \) 300 | 2.08                     | 261                           | 0.082    | 29.5             | 955                      |

* These recorded loads cause the vertical deflection of the free flange of purlin equal to approximately 1/10 of the purlin web depth.
Validation shows that this model is not suitable for specimens outside the specified range, which suggests that the panel’s local bending stiffness provided in the paper may be calibrated for the tested specimens only.

In this paper, an analytical model is developed for predicting the rotational restraint stiffness of Grade S450 purlins connected to roof sheeting by using self-drilling screws. The model takes account of purlin–sheeting interaction and the effect of loading directions by considering the rotational stiffness being contributed by the localised bending of the roof panel and the bending of the purlin flange panels at the connection point. This analysis is based on Kirchhoff thin plate theory. A series of rotational restraint tests ($F$-test) were conducted and the results were used to validate the analytical model.

2. Experimental study ($F$-test)

2.1. Test specimens

CFS zed and sigma purlin sections provided by Albion Sections Ltd. were used as test specimens. Geometric information is summarised in Tables 1 and 2, respectively. The nominal yield stress of the tested specimen is 450 N/mm$^2$. In this test program, each specimen is assigned with a unique three-part ID indicating the purlin type, cross-sectional dimensions (represent the web depth and the thickness) and purlin directions (“facing down” or “facing up” represents the uplift and gravity loading conditions and are denoted as FD or FU hereafter, respectively). For example, specimen $\Sigma20025FD$ indicates a sigma section with a web depth of 200 mm and cross-sectional thickness of 2.5 mm, which is fixed in a “facing down” manner.

2.2. Test set-up and instrumentation

Torsional restraint tests ($F$-test) adapted on the basis of EC3 guidance [19] were carried out for purlin profiles connected to roof sheets in order to estimate the rotational stiffness under both uplift and downward loading conditions. The test set-up is shown in
A single-skin trapezoidal roof sheet with dimensions of 1070 × 1000 × 0.7 mm was fixed onto a hot-rolled steel channel section with two rows of bolts at the centre of each trough. Timber blocks and steel plates were also used to clamp-fix the base. Such fixing configuration aims at forming a rigidly supported cantilever system. The dimensions of the roof sheet profile are presented in Fig. 2.

A purlin section of 1000 mm length was through-fastened to every sheet trough by self-drilling screws on the central line of the purlin flange. Neoprene washers were used to prevent water leaking at connections. The effect of loading direction was examined by adjusting the purlin fixing direction, i.e. the purlin was facing up under gravity loading so that purlin–sheeting contact took place at the flange-web junction line, and it was facing down under the uplift loading so that the contact took place at the flange lip conjunction line.

A stiff steel plate was bolted to the free flange of the purlin by using several bolts to simulate the load distribution; weights were applied to this steel plate by using a hook (Fig. 3). As a result of such a loading arrangement, the load applied by the weights was always in the vertical direction. Weights were applied in increments until the occurrence of plastic deformation in the roof sheet. The vertical deflections at several locations were recorded at each loading increment by using dial gauges, which were placed at the purlin’s outer webs and free flange (Fig. 4a). The rotational angles near both restrained and free flanges were monitored by using two digital inclinometers, with an accuracy of 0.1°, placed at the outer webs as close to the web-flange junction lines as possible. The lateral displacements of sheet and purlin due to bending were recorded (Fig. 4b).

2.3. F-test results

Tests were continued until the yield failure of roof sheeting occurs. The applied load and deflections were recorded at each loading increment, from which a complete moment against rotation relationship for each screw connection can be established. Table 3 presents a summary for the actual sizes of test specimens, the loads that cause the vertical deflection of the free flange of purlin

![Fig. 9. Cantilever sheet in the test and the calculation model.](image)

![Fig. 10. Comparison of the load-deflection between test and prediction for Σ24015FU.](image)

![Fig. 11. Purlin-sheet interaction at a connection point.](image)
equal to approximately 1/10 of the purlin web depth, the rotation and vertical deformation measured at this load level. The rotational stiffness $C_d$ of each specimen was determined by dividing the moment induced at connection due to that vertical load by the corresponding rotation captured, and is also presented in the same table. This approach of calculating the rotational stiffness is in accordance to the requirement by EC3-1-3 [19].

Typical complete moment–rotation curves recorded for specimen $R_{20012}$ are presented in Fig. 5 for both FU and FD arrangements; similar curves for other specimens are presented in Fig. 15 (later in the paper) in comparison to the theoretical prediction results. A common feature of these curves is that they remain almost linear during the entire loading range. The yield moment ranges from 93 N m for $Z_{14614}$ specimen to 300 N m for $R_{30030}$ specimen. Failed specimens were unloaded and dismounted and the permanent deformation was revealed. It can be seen from Fig. 6 that by the end of loading, localised plastic deformation occurs at each screw connection point, as evidenced by noticeable yield lines formed along the contact line, and two lines linking the screw point to both ends of the contact line. In all tests, the purlin sections have a greater thickness than the roof sheet. Distortional deformation of the purlin can only be clearly observed at the time the plastic yield line is about to be formed. Typical deformation modes of the tested purlin–sheeting system observed during tests are presented in Fig. 7 for both face-down and face-up tests.

3. Analytical model

An analytical model is developed and presented here for predicting the rotational stiffness of the purlin specimen at connection points under the applied loads as seen in the test. The purlin is connected to the roof sheet with self-drilling screws at one flange, but is free to move at the other flange. The total rotation angle captured near the junction line between the connected flange and the adjacent outer web (pivot point line) should comprise the following four components, illustrated in Fig. 8 and summarised as:

1. $\theta_a$, the rotation angle of the cantilever sheet under a row of concentrated moments generated at screw connection points; (2) $\theta_b$, the rotation angle associated with the localised deformation of the sheet under the pulling force in each screw; (3) $\theta_k$, the rotation angle due to the separation of the roof sheet and the purlin flange at the connection point; and (4) $\theta_p$, the rotation angle due to the purlin flange bending.

Typical complete moment–rotation curves recorded for specimen $Z_{20012}$ are presented in Fig. 5 for both FU and FD arrangements; similar curves for other specimens are presented in Fig. 15 (later in the paper) in comparison to the theoretical prediction results. A common feature of these curves is that they remain almost linear during the entire loading range. The yield moment ranges from 93 N m for $Z_{14614}$ specimen to 300 N m for $Z_{30030}$ specimen. Failed specimens were unloaded and dismounted and the permanent deformation was revealed. It can be seen from Fig. 6 that by the end of loading, localised plastic deformation occurs at each screw connection point, as evidenced by noticeable yield lines formed along the contact line, and two lines linking the screw point to both ends of the contact line. In all tests, the purlin sections have a greater thickness than the roof sheet. Distortional deformation of the purlin can only be clearly observed at the time the plastic yield line is about to be formed. Typical deformation modes of the tested purlin–sheeting system observed during tests are presented in Fig. 7 for both face-down and face-up tests.

3.1. Rotation angle of the cantilever sheet, $\theta_a$

During the F-test, the trapezoidal sheet is rigidly fixed at the base and free at the top, behaving as a cantilever one-way plate. The loaded purlin section transfers the load to the sheet in the form of a row of concentrated moments, which produces rotation angle

![Fig. 12. Plate model for predicting rotation due to localised sheet deformation.](image)

![Fig. 13. Comparison of load-localised deflection between test and prediction for specimen $Z_{30018FU}$.](image)
The magnitude of $\theta_c$ (Fig. 9) depends on the geometric configuration and the fixing arrangement of the sheet and the applied moment. Therefore the rotation stiffness associated with this component varies with actual construction details and should be considered separately. In this case, it will be excluded from the overall rotation angle measured from the inclinometer. It is anticipated that the real rotation of the cantilever sheet should be very close to the theoretical calculation based on the cantilever theory. To confirm this, we also included a horizontally placed dial gauge at 200 mm below the screw point to measure the lateral deformation (point A in Fig. 9). The measured deflections at point A were then compared with the calculated ones based on the one-way cantilever plate. The comparison of an example of $\Sigma 24015FU$ is presented in Fig. 10, which shows a reasonably close agreement. The calculation follows $\theta_c = \frac{ML}{EI_c}$ where $L_1$ is the vertical distance between the fixed support and the connection point, and $EI_c$ is the flexural rigidity of the sheet. The reason that the measured behaviour of the roof sheet is stiffer than the calculation model, as shown in Fig. 10, is due to there being a concentrated line load induced by the contact line between the purlin flange and roof sheeting. This load produces a localised deformation in the sheet, which is opposite to the cantilever deflection. The deflection recorded at point A, therefore, will be smaller than the pure cantilever case. However, since the rotation caused by this effect is insignificant compared to other components, it is deemed that the proposed theoretical prediction is acceptable.

### 3.2. Rotation angle caused by the localised deformation of sheet at connection, $\theta_l$

To retain the moment equilibrium, a couple is induced at each purlin–sheeting connection point, with two opposite-and-equally forces $F$ acting at the screw point and the line of contact, respectively. While the reaction force at the line of contact is supported by the webs of the roof sheet, the pulling force from the screw will produce a localised deformation in the sheet in the surrounding area and the rotation angle due to this local deflection is defined as $\theta_l$ (Fig. 11).

The conversion between $F$ and $M$ can be achieved as $F = M/na$, where $F$ is the applied force per unit area, $a$ is the distance between the screw and the line of contact and $n$ is the number of screws. In this case, since the screw is applied at the mid-point of the purlin flange, $a$ equals the half flange width, $C$. This load causes the roof sheet to deform locally around the screw point. Assuming the force at each screw point is identical, the deformation of the sheet at every trough can be treated as a thin rectangular plate subjected to a concentrated load (Fig. 12). By adopting the classic Kirchhoff thin plate theory [23], the governing equation can be expressed as:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 w = \frac{F}{D}$$

where $w$ is the deflection of the plate and $D$ is the bending stiffness, i.e. $EI_c^2 / 12(1 - \nu^2)$. $E$ is the Young’s modulus of the plate material, and $t_1$ is the thickness of the roof sheet.

In line with the deformation pattern, the boundary conditions can be set as follows: two vertical folding lines of the sheet are considered as simply supported ($S$). The pivot point line is considered as the fixed edge ($F$) due to the observation of a very early occurrence of plastic hinge under the applied load. A hypothetical line is chosen at a zero deflection location as the fourth line, also considered as fixed ($F$). Therefore the final boundary conditions along with the dimension symbols are illustrated in Fig. 12, where $b_f$ is the sheet trough width; and $h_T$ is the height of the plate where fixed edges are located; in this case it can be assumed to be equal to the purlin flange width $C$.

A Levy’s solution can be obtained for $w$ (see Appendix A). To facilitate the engineering application, the solution can be simplified into the following form:

$$\theta_l = \frac{w}{a} = \frac{\beta Fh^2}{Et_1^2a} = \frac{\beta Fh^2}{\pi Et_1^2a^2}$$

where $M$ is the moment applied at the connection by the vertical load $P$. The coefficient $\beta$ depends on the $b_f/h_T$ ratio and the location of the screw in relation to the trough panels. For ease of design purposes, common values of $\beta$ are presented in Table 4. To validate this model, the horizontal displacement at points A and B were measured (see Fig. 11). The differential displacements between these two points were calculated and compared with the calculation results. A comparison of specimen $\Sigma 30018FU$ is presented in Fig. 13 with total applied load plotted against horizontal difference between point A and B, which shows a close agreement.

### 3.3. Rotation angle caused by the separation of connection, $\theta_k$

Under the applied load, the purlin profile tends to separate from the sheet due to the resilience of the sealing washer. In our tests, it
has been found during the linear stage that the rotation associated with this effect is relatively small compared to the other components. Therefore this component is ignored in the calculation model.

3.4. Rotation angle caused by the deformation of purlin flange, $\theta_r$

The deformation of the purlin flange and web is calculated with the bending theory for one way slabs. By rotating Fig. 11 by 90°, the flange panel between the pivot line and screw point, as illustrated in Fig. 14, can be assumed as a simply supported one-way plate with a moment applied at one end. Thus the maximum rotation can be expressed in Eqs. (3a) and (3b) depending on the contact:

\[ \theta_r = \frac{Ma}{3EIP} \]  
\[ \theta_r = \frac{Ma}{3EIP} + \frac{Mb}{EI_p} \]

where $a$ is the vertical distance between the screw point and the line of contact, and $EI_p$ is the flexural rigidity of the purlin flange panel. It has been suggested that the contact conditions under differ-

![Graphs showing moment-rotation relationships between test results and analytical predictions.](image)

**Fig. 15.** Moment-rotation relationships between test results and analytical predictions.
different loading directions can be determined for channel and sigma sections, i.e. a flange-web junction line contact for the gravity load and a flange-lip junction line contact for the uplift load. However, for zed sections under a specified loading direction, the purlin flange may be in contact with the sheet in either way, depending on factors such as the screw position, purlin geometry and the loading magnitude [18].

| Specimen name | $C_d$ prediction (N m/rad/m) | $C_d$ test (N m/rad/m) | Test/prediction ratio | Average $C_d$ | FD/FU ratio |
|---------------|-----------------------------|------------------------|-----------------------|---------------|-------------|
| Σ20012 FD     | 403                         | 394                    | 0.98                  | 399           | 0.79        |
| Σ20012 FU     | 508                         | 502                    | 0.99                  | 505           | 0.78        |
| Σ20016 FD     | 607                         | 622                    | 1.02                  | 615           | 0.78        |
| Σ20016 FU     | 772                         | 691                    | 0.89                  | 732           | 0.89        |
| Σ20025 FD     | 930                         | 895                    | 0.96                  | 913           | 0.89        |
| Σ20025 FU     | 1048                        | 1024                   | 0.98                  | 1036          | 0.89        |
| Σ24015 FD     | 588                         | 593                    | 1.01                  | 591           | 0.80        |
| Σ24015 FU     | 731                         | 696                    | 0.95                  | 714           | 0.80        |
| Σ24023 FD     | 860                         | 848                    | 0.99                  | 854           | 0.87        |
| Σ24023 FU     | 987                         | 1009                   | 1.02                  | 998           | 0.88        |
| Σ24030 FD     | 939                         | 909                    | 0.97                  | 924           | 0.86        |
| Σ24030 FU     | 1090                        | 1047                   | 0.96                  | 1069          | 0.88        |
| Σ30018 FD     | 672                         | 735                    | 1.09                  | 704           | 0.83        |
| Σ30018 FU     | 805                         | 753                    | 0.94                  | 779           | 0.88        |
| Σ30025 FD     | 883                         | 921                    | 1.04                  | 902           | 0.88        |
| Σ30025 FU     | 1002                        | 1078                   | 1.08                  | 1040          | 0.88        |
| Σ30030 FD     | 1023                        | 977                    | 0.96                  | 1000          | 0.92        |
| Σ30030 FU     | 1108                        | 1191                   | 1.07                  | 1150          | 0.92        |
| Z14614 FD     | 557                         | 512                    | 0.92                  | 535           | 0.72        |
| Z14614 FU     | 717                         | 762                    | 0.94                  | 740           | 0.81        |
| Z14618 FD     | 723                         | 730                    | 1.01                  | 727           | 0.81        |
| Z14618 FU     | 889                         | 820                    | 0.92                  | 855           | 0.88        |
| Z20617 FD     | 674                         | 680                    | 1.01                  | 677           | 0.78        |
| Z20617 FU     | 868                         | 876                    | 1.01                  | 872           | 0.85        |
| Z30720 FD     | 810                         | 791                    | 0.98                  | 801           | 0.85        |
| Z30720 FU     | 958                         | 955                    | 1.00                  | 957           | 0.85        |

Fig. 16. Proportion of $\theta_l$ and $\theta_p$ in the overall rotation under the FD condition for sigma sections.

Fig. 17. Proportion of $\theta_l$ and $\theta_p$ in overall rotation under the FU condition for sigma sections.

Fig. 18. Proportion of $\theta_l$ and $\theta_p$ in overall rotation under FD condition for Z sections.

Fig. 19. Proportion of $\theta_l$ and $\theta_p$ in overall rotation under FU condition for Z sections.

Based on the abovementioned assumptions and validations, it can be suggested that the terms of $\theta_l$ and $\theta_p$ are considered in our analytical model. The term of $\theta_l$ should be determined based on the actual construction design details and by using the one-way slab theory. The term of $\theta_p$ is deemed to be negligible. The final
rotational stiffness per unit length run at each connection, i.e. \( C_D \), with a unit of N mm/ rad/mm, can be expressed as:

\[
C_D = \frac{M}{\partial_1 + \partial_p} = \frac{M}{\left(\frac{mil^2}{mm^2} + \frac{mol}{mol}\right)} = \frac{E}{\left(\frac{mil^2}{mm^2} + \frac{bT}{Ei}\right)} \tag{4}
\]

when the contact is at the flange-web junction line; and

\[
C_D = \frac{M}{\partial_1 + \partial_p} = \frac{M}{\left(\frac{mil^2}{mm^2} + \frac{mol}{mol} + \frac{4b}{Ei}\right)} \tag{5}
\]

when the contact is at the flange-lip junction line.

4. Results validated by experimental data

The results recorded from the F-tests have been used to validate the analytical model presented above. Typical moment–rotation curves from both test results and analytical predictions are presented in Fig. 15. As in Fig. 5, almost all curves show a notable linear feature during the entire loading range apart from Z30720FD. This suggests that a constant rotation stiffness employed for the purlin–sheeting interaction design is essentially valid. The analytical calculation results for rotational stiffness are compared to the experimental data and summarised in Table 5. It is worth noting that the inclinometers that are used to measure the rotational angles are fixed near the flange-web corner. Although in the test, they have been fixed as close to the corner as possible, it is inevitable that there will be an offset distance \( e \) between the centre and corner. To allow for that, in calculating \( C_D \), the additional angle developed between the corner and inclinometer centre is considered by adding a term of \( e/\partial_p \) to the denominator in both Eqs (4) and (5).

It can be seen that consistently good agreement is achieved between the test and prediction results, which suggests that the analytical model can predict sufficiently accurate results for rotational stiffness. It is noted that the analytical model is a linear model as can be seen from the derivation process. The test/prediction ratios from Table 5 have a standard deviation of 0.05 and a maximum difference of 11%.

The last column in Table 5 shows the ratio of rotational stiffness between FU and FD, and it can be seen that the rotational stiffness in the FU condition is greater than that in the FD condition in all cases regardless of the purlin’s geometry. The FD/FU ratio ranges from 0.72 to 0.92. This trend also agrees with the results based on the EC3 model [19]. The cause of this trend can be easily explained by the additional term in the analytical model, i.e., Eq. (3b). The term that causes the difference is \( b/\partial_p \), which indicates that a purlin with a smaller \( b \) value or a larger thickness is likely to produce a notable difference under FD and FU conditions.

The contribution of \( \theta_1 \) and \( \theta_p \) to the overall rotation for sigma sections is presented in Figs. 16 and 17, for the FD and FU conditions, respectively. In the FD condition, the proportion of \( \theta_1 \) ranges from 35% to 78% for 200 series sections, from 50% to 87% for 240 series sections and from 55% to 86% for 300 series sections. In the FU condition, this proportion has risen to 48–88% for 200 series, 62–92% for 240 series and 69–96% for 300 series. In each series, this proportion will increase with the purlin thickness. This trend agrees with the fact that \( \theta_1 \) is due to the sheet deformation, while \( \theta_p \) is due to the purlin flange panel deformation. The effect of flange width on the proportion of \( \theta_1 \) can also be observed from Figs. 16 and 17. For instance, from section groups \( \Sigma20025/\Sigma30025 \) to \( \Sigma24030/\Sigma30030 \), the flange width rises from 62.5 mm to 75 mm, but the \( \theta_1 \) proportion in the FD condition varies from 79%–74% to 88%/86% and in the FU condition, from 88%/86% to 93%/95%. Therefore the effect of the flange width can be treated as insignificant. Similar patterns are found for the Z sections as shown in Figs. 18 and 19.

5. Parametric study

Studies [17,24] have found that the rotational stiffness of the purlin–sheet system depends on factors such as the shape and thickness of the sheet, the cross-section of the purlin, the number of screws per unit length and the connection details. The analytical model developed in this paper has taken account of these factors, and therefore it can be used for parametric studies. As discussed in the proceeding section, \( \theta_1 \) and \( \theta_p \) are related to the sheet thickness and purlin thickness, respectively. A group of sigma sections

| Table 6 |
| Summary of geometrical details of \( \Sigma24030/FD \) specimen and sheeting. |

| Purlin section | Web depth (mm) | Flange width (mm) | \( \ell_1 \) (mm) | \( \ell_p \) (mm) | \( a \) (mm) | \( b \) (mm) | \( b_1 \) (mm) | \( h_l \) (mm) | \( S \) (mm) | \( L \) (mm) |
|---------------|---------------|-----------------|----------------|----------------|--------|--------|--------|--------|--------|--------|
| \( \Sigma24030 \) | 240           | 62.5            | 0.7            | 2.99           | 31.25  | 31.25  | 130    | 65     | 200    | 1000   |
with a depth 240 mm and under the FD condition have been chosen as an example for parametric analysis. In Fig. 20, \( h \) versus the sheet thickness curves for purlin sections of 3 mm with various flange widths, i.e. 62.5, 65, 75, 100 mm respectively, is presented. Similarly, Fig. 21 presents \( h \) versus the purlin thickness curves for the same range of flange widths. Based on the results, the following observations can be made:

1. \( h \) declines with an increase in sheet thickness. The effect of sheet thickness on \( h \) is more notable for small \( t_s \) values, specifically, less than 0.6 mm. The variation in \( h \) caused by different flange widths \( C \) is inconclusive.

2. The effect of purlin thickness on \( h \) has a similar trend to that of sheet thickness on \( h \). Additionally, when the purlin thickness is up to 1.5 mm, this type of effect is particularly significant. The effect of flange width is only noticeable when the purlin thickness is rather small, e.g. up to 1.5 mm.
6. Application of the developed model in numerical simulation

One of the test specimens, Σ24030FD, is considered as a worked example to demonstrate the application of the above-developed model. For the ease of reference, the geometrical details of both purlin and sheeting are listed in Table 6. The Young’s modulus for both purlin and sheeting is $E = 210$ GPa and the Possion ratio $v = 0.3$. Five screws are used within a length of 1 m, i.e., $n = 5$.

The second moment of area of purlin $I_p = L^3_p/12(1 - v^2) = 1000 \times 2.991/12(1 - 0.3^2) = 2448$ m$^4$; coefficient $b$ for screw at the mid-point of sheeting trough: $b = 0.078$. Therefore, the rotational stiffness is:

$$C_D = \frac{E}{(\beta b T p + b T m) + (0.078 - 0.025^2 + 0.3125 + 0.3125 + 0.3125)} = 1055569 \text{ N mm/rad/m} = 1056 \text{ N m/rad/mm}$$

This result is higher than that listed in Table 5, i.e. 939 N m/rad/mm as it considers the rotation being measured exactly at the flange-web junction line.

The rotational stiffness is often considered in analysing the sheeting-purlin interaction [16–18,25–27]. Due to the complex nature of purlin–sheeting systems, most FEA models choose lateral and/or rotational spring elements as a substitute for the actual interactional effect. Such types of purlin/sheeting interaction are realised through the compression from the contact surface between the purlin flange and sheeting and the tension in the screw connections, under an external loading. It is this tension effect that provides the compression flange an effective restraint in the lateral direction.

The rotational stiffness derived can be applied directly in an FEA model. A numerical model is developed by using ANSYS [28] to illustrate the treatment of purlin/sheeting interaction. The model is for a 6-m simply-supported single span purlin made of Σ24030 subjected to a series of multi-point uplift loads, which has been tested by Yang and Liu [3]. The meshing and the application of boundary conditions are shown in Fig. 22. The effect of sheeting is simulated by applying lateral and rotational spring elements on the compression flange-web junction line. An illustration of spring configurations is provided in Fig. 23. The lateral restraint is set to be fully effective and the rotational spring is assigned with a stiffness based on the above calculation. The meshing density is sufficient for accurate results. H(y) represents the Heaviside function, i.e., $H(y) = 0$ for $y < 0$ and $H(y) = 1$ for $y \geq 0$. The resolved constants can be substituted into Eq. (A1) to calculate the deflection caused by local sheet deformation.

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Appendix A

A Levy’s solution can be obtained by considering the above plate model.

$$w = \sum_{m=1}^{\infty} Y_m \sin \left( \frac{m \pi x}{b_T} \right) \left( \frac{m \pi y}{b_T} \right) + B_m \left( \frac{m \pi y}{b_T} \right) \sinh \left( \frac{m \pi y}{b_T} \right) + C_m \times \sinh \left( \frac{m \pi y}{b_T} \right) + D_m \left( \frac{m \pi y}{b_T} \right) \cosh \left( \frac{m \pi y}{b_T} \right)$$

For a plate with two opposite edges simply supported and the other two fixed (S–F–S–F), the constants can be solved as:

$$A_m = \frac{\left( \frac{m \pi h_T}{b_T} \right)^2 - 4 \beta_1^2 \pi^2 \cosh \left( \frac{m \pi h_T}{b_T} \right)^2 + m^2 \pi^2 h_T^2 + 4 \beta_1^2}{4 \left( \frac{m \pi h_T}{b_T} + 2 \beta_1 \sinh \left( \frac{m \pi h_T}{b_T} \right) \cosh \left( \frac{m \pi h_T}{b_T} \right) \right)^m \pi^2 D}$$

$$B_m = \frac{\left( \frac{m \pi h_T}{b_T} + 2 \beta_1 \sinh \left( \frac{m \pi h_T}{b_T} \right) \cosh \left( \frac{m \pi h_T}{b_T} \right) \right)^m \pi^2 D}{m \pi h_T + 2 \beta_1 \sinh \left( \frac{m \pi h_T}{b_T} \right) \cosh \left( \frac{m \pi h_T}{b_T} \right)}$$

$$C_m = \left( \frac{\left( \frac{m \pi h_T}{b_T} \right) - 2 \beta_1 \sinh \left( \frac{m \pi h_T}{b_T} \right) \cosh \left( \frac{m \pi h_T}{b_T} \right) + m \pi h_T}{m \pi^2 D} \right) \left( \frac{1}{2} - H(y) \right)$$

For engineering applications, $m = 5$ can be deemed to produce sufficiently accurate results. $H(y)$ represents the Heaviside function, i.e., $H(y) = 0$ for $y < 0$ and $H(y) = 1$ for $y \geq 0$. The resolved constants can be substituted into Eq. (A1) to calculate the deflection caused by local sheet deformation.

References

[1] Liu Q, Yang J, Li LY. Pseudo-plastic moment resistance of continuous beams with cold-formed Sigma section sat internal supports: experimental study. Eng Struct 2011;33:947–57.
[2] Rhodes J, Lawsson RM. Design of structures using cold formed steel sections. Ascot: The Steel Construction Institute; 1992.
[3] Yang J, Liu Q. An experimental study into flexural behaviour of sigma purlins attached with roof sheets. Eng Struct 2012;45:481–95.
[4] Lucas RM, AlBermani FGA, Kitipornchai S. Modelling of cold-formed purlin-sheeting systems. 1. Full model. Thin-Wall Struct 1997;27(3):223–43.
[5] Lucas RM, AlBermani FGA, Kitipornchai S. Modelling of cold-formed purlin-sheeting systems. 2. Simplified model. Thin-Wall Struct 1997;27(4):263–86.
