Dark Energy: A Unifying View

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Abstract

Different models of the cosmic substratum which pretend to describe the present stage of accelerated expansion of the Universe like the ΛCDM model or a Chaplygin gas, can be seen as special realizations of a holographic dark energy cosmology if the option of an interaction between pressurless dark matter and dark energy is taken seriously. The corresponding interaction strength parameter plays the role of a cosmological constant. Differences occur at the perturbative level. In particular, the pressure perturbations are intrinsically non-adiabatic.

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I. INTRODUCTION

Since the results of the luminosity distance - redshift observations of supernovae of type Ia suggested an interpretation according to which our Universe entered a stage of accelerated expansion [1], a host of theoretical concepts has been developed to account for this phenomenon (for a review see, e.g., [2]). Within Einstein’s theory a so far unknown ingredient with negative pressure is required, which is called dark energy. By now, many of the dark energy models that were worked out have been tested against observational data of different kind [3, 4, 5]. Different priors and different parametrizations were used to provide limits on the parameters of the models under consideration. Still favored is the ΛCDM model, but it is also clear that the matter is not solved and that there are other contenders. In this situation, lacking a fundamental understanding, one might wish to have a robust phenomenological framework which allows for a unified description of (at least a large part of) the currently favored approaches. In the present essay we demonstrate that holographic dark energy can provide the basis for such a unifying view. We also point out unexpected links to cosmological gas dynamics. Finally, we show that this approach naturally implies the existence of non-adiabatic pressure perturbations.

II. THE EFFECTIVE EQUATION OF STATE

Assume the present cosmic substratum to be described by dark matter and dark energy as the two dynamically relevant components. In the homogeneous and isotropic, spatially flat Universe Einstein’s equations reduce to

\[ 3H^2 = 8\pi G \rho , \quad \frac{\dot{H}}{H^2} = -\frac{3}{2} \left( 1 + \frac{p}{\rho} \right) , \]

where \( \rho = \rho_M + \rho_X \) is the total energy density. Here, \( \rho_M \) and \( \rho_X \) are the energy densities of pressureless dark matter and dark energy, respectively. The pressure of the \( X \) component coincides with the total pressure, \( p = p_X \) and \( H \) is the Hubble expansion rate. Solving the last equation in (1) for \( \frac{p}{\rho} \) results in

\[ \frac{p}{\rho} = \frac{1}{3} (2q - 1) , \]

(2)
where \( q = -1 - \frac{\dot{H}}{H^2} \) is the deceleration parameter. The matter energy density behaves as (see, e.g., [4])

\[
\rho_M = \rho_{M0} \left( \frac{a_0}{a} \right)^3 \frac{f}{f_0}.
\]

(A subscript 0 denotes the value at the present time.) Here we have admitted the possibility that the conventional decay of the matter energy density \( \propto a^{-3} \) is modified by an interaction in the dark sector. Because the total energy has to be conserved, the density \( \rho_X \) of the dark energy component then changes according to

\[
\dot{\rho}_X = -3H \left( 1 + w^{\text{eff}} \right) \rho_X ,
\]

where

\[
w^{\text{eff}} = w + \frac{\dot{f}}{3Hf} r
\]

is the effective equation of state parameter while \( w \) is the corresponding “bare” parameter and \( r \equiv \frac{\rho_M}{\rho_X} \) is the ratio of the energy densities. In case the energy density ratio is constant, we have

\[
r = \text{const} \quad \Leftrightarrow \quad w^{\text{eff}} = -\frac{\dot{f}}{3Hf} \quad \Rightarrow \quad w = (1 + r) w^{\text{eff}}.
\]

Under this condition the total equation of state of the cosmic medium is

\[
\frac{p}{\rho} = w^{\text{eff}}.
\]

It coincides with the effective equations of state of the components. Apparently, this is a very special situation. Therefore it may come as a surprise that on this basis many of the “standard” dark energy models such as the \( \Lambda \)CDM model or the Chaplygin gas can be recovered just by different choices of the interaction. The important aspect here is the following. Via Friedmann’s equation a constant ratio \( r \) implies the dependence \( \rho_X \propto H^2 \).

While this appears to be an almost trivial consequence of the relations used so far, the behavior \( \rho_X \propto H^2 \) itself is anything but trivial. It is exactly this dependence which is found in the context of holographic dark energy models. The central point of the holographic dark energy concept is a field theory based relation between an ultraviolet cutoff and an infrared cutoff [7]. This relation has the attractive feature that, by identifying the infrared cutoff length with the present Hubble scale, the corresponding ultraviolet cutoff energy density turns out to be of the order of the observed value of the cosmological constant parameter. Just this feature is encoded in the dependence \( \rho_X \propto H^2 \). Despite of this
remarkable property the Hubble scale cutoff has fallen out of favor since for \( f = \text{constant} \) it is not consistent with an accelerated expansion of the Universe. This apparent shortcoming can be remedied and, in a sense to be pointed out later on, even made an advantage, if the possibility of an interaction between holographic dark energy and dark matter is not ignored. The relevance of a coupling between both components is easily seen. Combining the relations (2), (6) and (7) we obtain

\[
q = \frac{1}{2} \left( 1 - \frac{\dot{f}}{Hf} \right).
\]  

(8)

The sign of \( q \) crucially depends on the ratio \( \frac{\dot{f}}{Hf} \). For \( \frac{\dot{f}}{f} < H \) we have \( q > 0 \), i.e., decelerated expansion. For \( \frac{\dot{f}}{f} > H \) we have \( q < 0 \) and accelerated expansion. If, in particular, \( f \) is such that the rate \( \frac{\dot{f}}{f} \) changes from \( \frac{\dot{f}}{f} < H \) to \( \frac{\dot{f}}{f} > H \), this corresponds to a transition from decelerated to accelerated expansion under the condition of a constant energy density ratio \( r \) (cf. [8]). This transition is a pure interaction phenomenon.

III. BACKGROUND DYNAMICS

A. The interaction parameter

To advance our discussion, information about the rate \( \frac{\dot{f}}{f} \) is required. Since we know neither the nature of dark matter nor the nature of dark energy, a microphysical interaction model is not available either. However, one may argue that under the conditions of spatial homogeneity and isotropy the only dynamical scale is \( H^{-1} \). For the rate \( \frac{\dot{f}}{f} \) to be cosmologically relevant it should vary at this scale. It seems therefore natural to assume a dependence of the crucial parameter \( \frac{\dot{f}}{3Hf} \) in terms of \( H^{-1} \). We choose

\[
\frac{\dot{f}}{3Hf} = \mu \left( \frac{H}{H_0} \right)^{-n} \Rightarrow \dot{\rho} + 3H \left( 1 - \mu \left( \frac{H}{H_0} \right)^{-n} \right) \rho = 0.
\]  

(9)

The quantity \( \mu \) is an interaction constant. Different interaction rates are characterized by different values of \( n \). A growth of the parameter \( \frac{\dot{f}}{Hf} \) is obtained for \( n > 0 \). In the spatially flat background the ansatz (9) corresponds to an equation of state parameter

\[
\frac{p}{\rho} = -\mu \left( \frac{\rho}{\rho_0} \right)^{-n/2}.
\]  

(10)
At the present time we have $p_0/\rho_0 = -\mu$, i.e., the present equation of state parameter is a direct measure of the interaction parameter $\mu$. Solving the equation for $\rho$ in (9) we find for the background energy density

$$\rho = \rho_0 \left[ \mu + (1 - \mu) \left( \frac{a_0}{a} \right)^{3n/2} \right]^{2/n}.$$ (11)

It has the structure of the energy density of a generalized Chaplygin gas [9]. In the limit $a \ll a_0$ it reproduces a matter dominated universe with $\rho \propto a^{-3}$, while in the opposite limit the energy density is similar to that of a cosmological constant. At first sight this behavior of the energy density might be unexpected since it was derived under the condition of a constant ratio $r$ of the energy densities of both components. However, it is a specific feature of our equation of state parameter, that the dark energy itself behaves as matter at high redshifts ($a \ll a_0$). At high redshifts we have $\dot{f} \ll H$, i.e., the interaction is negligible (for $n > 1$) and we recover a de Sitter universe. It was this property that apparently ruled out a (non-interacting) holographic dark energy model with an infrared cutoff set by the Hubble scale [10, 11]. Here, this unwanted (in the non-interacting model) feature is advantageous since it naturally provides us with an early matter dominated phase during which structure formation can occur.

For $n = 2$ we recover the $\Lambda$CDM model while for $n = 4$ the expression (11) describes the energy density of a “true” Chaplygin gas. The cosmological constant term is determined by the interaction strength parameter $\mu$ of our approach. This is consistent with the circumstance that for the Chaplygin gas the parameter which corresponds to $\mu$ represents the special case of a constant potential term in tachyon field theories [12]. It is also connected with the interaction strength of d-branes [13]. This indicates that there is support from fundamental field theory for an interaction of the type introduced through the ansatz (9).

B. Cosmic force

Another line of understanding the role of the choice (9) emerges if the cosmic medium is studied within a gas dynamical approach. This provides a suggestion for the origin of $\dot{f} \neq 0$ within kinetic theory. In this picture the present phase of accelerated expansion of the Universe is the result of a cosmic force exerted on the particles of the cosmic gas [14]. This force makes the constituents of the cosmic medium move in a non-geodesic manner.
while the macroscopic fluid motion as a whole is geodesic, as required by the cosmological principle. The equation of motion for the gas particles is

$$\frac{Dp^i}{d\tau} = mF^i.$$  \hspace{1cm} (12)

Here, $p^i$ is the 4-momentum of a particle with mass $m$, normalized by $p^ip_i = -m^2$ and $\tau$ is its proper time. The structure of a 4-force, compatible with the requirements of the cosmological principle is

$$mF^i = B \left(-Ep^j + m^2u^j\right),$$  \hspace{1cm} (13)

with $E \equiv -p^iu_j$ being the particle energy as measured by an observer, moving with the macroscopic (geodesic) fluid 4-velocity. The force (13) contains quantities which characterize the same fluid both on the microscopic level (particle momentum, particle energy) and on the macroscopic level (macroscopic 4-velocity). Hence, it describes a self-interaction of the medium. The strength of the force is described by the function $B$. A particle that moves with the geodesic macroscopic 4-velocity is force free,

$$p^i = mu^i \Rightarrow F^i = 0.$$  \hspace{1cm} (14)

Any deviation from this motion corresponds to the action of a non-vanishing force on the particle. The particles are characterized by a one-particle distribution function which is governed by Boltzmann’s equation. Assuming the particles to be non-relativistic, the macroscopic energy balance, obtained from the second moments of the distribution function, is

$$\dot{\rho} + 3H \left(1 - \frac{B}{H}\right)\rho = 0.$$  \hspace{1cm} (15)

The correspondence to (9) is obvious,

$$\frac{B}{H} \Leftrightarrow \mu \left(\frac{H}{H_0}\right)^{-n}.$$  \hspace{1cm} (16)

With this choice of $\frac{B}{H}$ the energy density follows from a gas dynamical approach. In other words, an equation of state parameter $w^{\text{eff}}$ (cf. (6)) can be understood as the result of an effective self-interacting one-particle force that self-consistently acts on the microscopic constituents of the cosmic substratum. The phenomenologically introduced parameter $\mu$ is related to the strength of a force on the gas particles. On the one hand, the relation to the holographic dark energy concept sheds new light on the cosmic force approach, on the other hand the dark energy interaction parameter $\mu$ acquires a counterpart on the level of kinetic theory.
IV. PERTURBATIONS

With $H = \frac{\Theta}{3}$ where $\Theta \equiv u_i^i$ is the fluid expansion scalar, the interaction parameter in (9) is a covariantly defined quantity. Fluctuations of this parameter become part of the perturbation dynamics in a natural way. The quantity $\frac{\dot{\rho}}{\rho}$ which in standard perfect fluids plays the role of an adiabatic sound speed is straightforwardly obtained from (10),

$$\frac{\dot{p}}{\rho} = \left(1 - \frac{n}{2}\right) \frac{\rho}{\dot{\rho}}. \quad (17)$$

However, it is not this quantity that relates the pressure perturbations to the energy density perturbations in our approach. The pressure perturbations in linear order are (cf. (6), (7) and (9))

$$\hat{p} = p \left(\frac{\dot{\rho}}{\rho} - \frac{\dot{H}}{H}\right). \quad (18)$$

Quantities without a hat refer to the homogeneous and isotropic background in the following. The perturbation $\hat{H}$ of $H$ is defined via the expansion scalar $\Theta$ as $\hat{H} = \frac{\hat{\Theta}}{3}$. The pressure perturbations (18) are not simply proportional to the energy density perturbations. This is a consequence of the circumstance that there exists an equation of state $p = p(\rho)$ only in the background (cf. Eq. (10)) but not for deviations from homogeneity and isotropy. The fluctuations of the interaction parameter make the perturbations non-adiabatic. The deviation from adiabatic behavior is most conveniently described by

$$\hat{\rho} - \frac{\dot{\rho}}{\rho} \dot{\rho} = \frac{n}{2} p \left(\frac{\dot{\rho}}{\rho} - 2 \frac{\dot{H}}{H}\right). \quad (19)$$

There are no non-adiabatic contributions only for $n = 0$. The combination $\frac{\dot{\rho}}{\rho} - 2 \frac{\dot{H}}{H}$ on the right hand side of (19) is proportional to the perturbed 3 curvature scalar $R^{(3)}$ of the surfaces orthogonal to $u^i$. In first order we have in the present case

$$\hat{R}^{(3)} = 6 H^2 \left(\frac{\dot{\rho}}{\rho} - 2 \frac{\dot{H}}{H}\right). \quad (20)$$

Thus, the non-adiabatic pressure perturbations of our approach have a direct geometrical meaning. In terms of (gauge invariant) perturbation quantities on comoving hypersurfaces, a combination of the energy and momentum balances of the fluid can be used to eliminate the perturbations of the Hubble parameter. The result is

$$\hat{p} = \frac{p}{\rho} \left[\left(1 + \frac{n}{\gamma}\right) \dot{\rho} + \frac{n}{3 \gamma H} \dot{\rho}\right], \quad \left(\gamma = 1 + \frac{p}{\rho}\right). \quad (21)$$
The remarkable point here is that the pressure perturbations are not just proportional to the energy density perturbations $\hat{\rho}$ as in the adiabatic case. There is an additional dependence on the time derivative $\dot{\hat{\rho}}$ of the energy density perturbations. The relation between $\hat{p}$ and $\dot{\hat{\rho}}$ is no longer simply algebraic, equivalent to a (given) sound speed parameter as a factor relating the two. The relation between them becomes part of the dynamics. In a sense, $\hat{p}$ is no longer a “local” function of $\hat{\rho}$ but it is a function of the derivative $\dot{\hat{\rho}}$ as well: $\hat{p} = \hat{p}(\hat{\rho}, \dot{\hat{\rho}})$. It is only for the background pressure that the familiar dependence $p = p(\rho)$ is retained. Formula (21) is a direct consequence of the structure (9) for the interaction parameter $\dot{f}/f$. While this interaction reproduces known dark energy models in the homogeneous and isotropic background, albeit in a non-standard unifying context, there are differences on the perturbative level which opens the possibility to test the scheme presented here.

V. SUMMARY

Pressureless dark matter in interaction with holographic dark energy with an infrared Hubble scale cutoff is more than just another model to describe an accelerated expansion of the Universe. It sets the stage for a unifying view on a whole class of models, among them the $\Lambda$CDM model and the Chaplygin gas model, which follow as subcases for different interaction rates. The interaction can be interpreted in terms of a 4-force on the constituents of the cosmic gas. The unifying view on the homogeneous and isotropic background is accompanied by a non-adiabatic perturbation dynamics which can be seen as the consequence of a fluctuating interaction rate. The relation between pressure perturbations and energy density perturbations becomes part of the dynamics and is no longer given by a simple sound speed parameter.

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