Mesoscopic fluctuations in diffusive transport of circularly polarized light

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Abstract. We study how the effect of circular polarization memory in a disordered ensemble of resonant Mie particles reveals itself in mesoscopic intensity fluctuations. It is shown that polarization of light enhances the fluctuations. In the vicinity of the first Kerker point, sharp changes in the depolarization rate result in a quasiresonant dependence of the variance of transmission coefficient fluctuations on the wavelength of light.

1. Introduction
Mesoscopic transport in disordered systems is accompanied by long-range correlations in the transmitted flux [1]. For quantum electronic transport, these correlations are responsible for the universal conductance fluctuations in small metal samples. In the optical case (light or microwaves) the long-range correlations are observed in sample specific fluctuations of the speckles formed by multiply scattered waves.

Most of the theoretical results concerned with the problem of correlations between fluctuations of intensity in the speckles were derived within the scalar wave approximation [1] (see, also [2–10]). Such an approach, as applied to diffusive transport of electromagnetic waves, is believed to be valid provided that correlations between the cross-polarized fields are neglected and the polarization of waves is accounted for by doubling the total number of propagating channels (see, e.g., [11,12]).

However, as was shown in Refs. [13,14] for media with large inhomogeneities, the correlations between the cross-polarized waves result in a more complex, as compared with [1], picture of the mesoscopic fluctuations. These correlations are responsible for the polarization state of light and, therefore, until the scattered light is depolarized completely, the contribution from the cross-polarized field correlations to the variance of intensity fluctuations proves to be of the same order as the contribution obtained within the scalar wave theory.

Since the imaginary part of the correlation function of the cross-polarized fields is directly related to the helicity of light, the effect of circularly polarization memory found recently [15] in scattering by resonant Mie particles gives the opportunity to take a new look at mesoscopic phenomena in transport of electromagnetic waves through disordered media. Virtually complete retention of the circular polarization in single scattering results in the depolarization of multiply scattered light to occur deep in the diffusion regime [16]. In such a situation the cross-polarized field correlations should be taken into account in studying the diffusive wave transport of light and the related mesoscopic effects.

Below we analyze the effect of polarization memory on the mesoscopic fluctuations. Our calculations are based on the single-Hikami-box diagram for the intensity correlations. The
Figure 1. Feynman diagrams for the moment $\langle I(z_f, r_\parallel)I(z_f, r_{1\parallel}) \rangle$ (a) and the short-range correlation function (b). Converging solid lines to a point denotes integration over directions of propagation and summation over polarization, $H$ is the Hikami vertex [19].

Spatial spectrum of correlations between the local values of the flux density are calculated beyond the scalar wave approximation and expressed in terms of the Stokes parameters of multiply scattered waves. The inclusion of the cross-polarized field correlations results in the enhancement of intensity fluctuations. Under the first Kerker condition [15,16] the variance of fluctuations can be twice greater compared to that obtained within the scalar theory. As the cross-polarized field correlations decay and the transmitted light is depolarized, the crossover between two regimes of diffusion occurs. In the vicinity of the first Kerker condition the crossover reveals itself as a sharp resonant dependence of the variance of transmission coefficient fluctuations on the wavelength of light.

2. Correlation function and spectrum of intensity fluctuations

Consider a plane electromagnetic wave incident on a sample composed of randomly positioned scattering particles. Interference of multiply scattered waves results in a random distribution of irradiance at the sample boundaries which is known as speckle. The speckle pattern is related directly to the spatial arrangement of scattering particles in the sample and fluctuates from one sample to another.

Sample specific fluctuations in the speckles are strongly correlated. The correlation function between the local values of the flux density (or irradiance) is defined as

$$C(z_f, \rho = r_\parallel - r_{1\parallel}) = \langle I(z_f, r_\parallel)I(z_f, r_{1\parallel}) \rangle - \langle I(z_f, r_\parallel) \rangle\langle I(z_f, r_{1\parallel}) \rangle$$  (1)

where $I(z_f, r_\parallel)$ is the spatial distribution of irradiance over the output boundary of the sample ($z_f = L$ for transmission through the slab of thickness $L$). Brackets $\langle \ldots \rangle$ appearing in Eq.(1) denote averaging over disorder. For brevity, the function $C(z_f, \rho)$ is conventionally referred to as the intensity correlation function.

In addition to the correlation function, it is convenient to introduce the spatial spectrum of intensity fluctuations [7,17]

$$M(z_f, q) = \int d\rho \exp(-i\mathbf{q}\mathbf{\rho})C(z_f, \rho)$$  (2)

The $q$-dependence of the spectrum is related directly to the spatial behavior of the correlation function and is sensitive to the regime of wave propagation in disordered media [7, 17]. The value of the spectrum at $q = 0$ determines the variance of total transmission coefficient $T$.

The intensity correlations are described by the diagram shown in Fig.1(a) (see, e.g., [1]). Based on the results of diagrammatic calculations [13,14] we can express the spectrum of intensity
fluctuations in terms of the Stockes parameters of scattered light,

\[ M(z_f, q) = \left( \frac{2\pi}{k_0} \right)^2 n_0 \int_0^L dz \int d\mathbf{n}' |I_q(z_f|z, \mathbf{n}) - I_q(z_f|z, \mathbf{n}')|^2 S_m(z, \mathbf{n}) Z_{mn}(\mathbf{n}, \mathbf{n}') S_n(z, \mathbf{n}') \]  

(3)

where \( n_0 \) is the number of scattering particles per unit volume, \( k_0 = 2\pi/\lambda \), \( \lambda \) is the wavelength of light, \( Z_{mn}(\mathbf{n}, \mathbf{n}') \) is the phase matrix \([16, 18]\), \( \mathbf{n} \) and \( \mathbf{n}' \) are the directions of wave propagation. Quantities \( S_m \) \((m = 1 \div 4)\) entering into Eq.(3) are the components of the Stokes vector \( \mathbf{S} = (I, Q, U, V) \) which is subject to the vector radiative transfer equation \([18]\). Equation (3) differs from the corresponding result \([7, 17]\) obtained within the scalar wave approximation by the factor \( S_m(z, \mathbf{n}) Z_{mn}(\mathbf{n}, \mathbf{n}') S_n(z, \mathbf{n}') \) which takes into account the polarization state of scattered light.

Formula (3) establishes the interrelation of the mesoscopic intensity fluctuations with the polarization state of the incident light and optical characteristics of the scattering medium, including those that are responsible for depolarization of light.

The spectrum (3) involves all spatial scales of intensity correlations. The correlations at scales of the order of the wavelength \( \lambda \) are described by the diagram with no scattering event in the "outgoing" propagators and the Hikami-vertex (see Fig. 1(b)) \([17]\). The short-range contribution to the correlation function can be presented in the form

\[ C^{(sh)}(z_f, \rho) = \frac{1}{4} \int d\mathbf{n} \int d\mathbf{n}'e^{i\rho(\mathbf{n}-\mathbf{n}')}[\mathbf{n}_m(\mathbf{n} + \mathbf{n}')]^2 (S_m(z_f, \mathbf{n}) D_{mn}(\mathbf{n}, \mathbf{n}') S_n(z_f, \mathbf{n}')) \]  

(4)

where integration is carried out over the forward hemisphere, \( \mathbf{n}_m \) is the inward normal to the output boundary, the matrix \( \mathbf{D} \) is obtained from the matrix \( \mathbf{Z} \) by substitution of the matrix elements \( a_1, a_2, b_1 \) and \( b_2 \) in the form \( a_1(\mathbf{n}'n') = (1/2) \left( 1 + (\mathbf{n}'\mathbf{n}')^2 \right) \), \( a_2(\mathbf{n}'n') = (\mathbf{n}'\mathbf{n}') \), \( b_1(\mathbf{n}'n') = (1/2) \left( (\mathbf{n}'\mathbf{n}')^2 - 1 \right) \), \( b_2 = 0 \).

Further simplifications in Eqs.(3) and (4) are based on the assumptions concerned with the polarization state of light and optical characteristics of the medium. In scattering of circularly polarized light by resonant Mie particles only two components of the Stokes vector, \( I \) and \( V \), survive near the first Kerker point \([16]\), and

\[ S_m(z, \mathbf{n}) Z_{mn}(\mathbf{n}, \mathbf{n}') S_n(z, \mathbf{n}') = \frac{1}{2} (a_1(\mathbf{n}'n') I(z, \mathbf{n})I(z, \mathbf{n}') + a_2(\mathbf{n}'n') V(z, \mathbf{n})V(z, \mathbf{n}')) \]  

(5)

An analogous formula is also valid for the factor \( S_m D_{mn} S_n \) entering into Eq.(4).

3. Short-range correlations in diffusive transport

Propagation of light through thick samples of a weakly absorbing medium \((L \gg l_r)\) and 
\( l_r \ll l_a \), where \( l_r \) is the transport mean free path and \( l_a \) is the absorption mean free path, the angular distribution of the transmitted radiation turns out to be virtually isotropic. In this case the intensity \( I \) can be calculated with the well-known diffusion approximation \([20]\). Such an approximation is also applicable for the fourth Stokes parameter \( V \) near the first Kerker point where the mean free path with respect to depolarizing collisions \( l_{dep} \) is much greater than the transport mean free path \( l_r \) \([16]\).

Within the diffusion approximation, the short-range correlation function (see Eq.(3)) can be presented in the form

\[ C^{(sh)}(L, \rho) = \frac{1}{2} \langle T \rangle^2 (1 + P^2(L)) \frac{J_2^2(k_0\rho)}{(k_0\rho)^2} \]  

(6)
where \( J_1(x) \) is the Bessel function, \( \langle T \rangle \) is the average total transmission coefficient and \( P_C(L) \) is the degree of polarization of transmitted light. In a medium with no absorption

\[
\langle T \rangle = \sqrt{3} \frac{l_{tr}}{L}, \quad P_C(L) = \frac{(L/l_{circ})}{\sinh(L/l_{circ})}.
\]

(7)

Within the diffusion approximation, the depolarization length \( l_{circ} \) is determined by the relation \( l_{circ} = \sqrt{l_{tr} l_{dep}/3} \) [21]. From Eq.(6) it follows that intensity fluctuations at short ranges are positively correlated. The \( \rho \)-dependence of the correlation function is insensitive to the polarization state of light. The spatial scale of correlations proves to be of the order of the wavelength \( \lambda \).

The correlation function (6) is proportional to the square of the transmission coefficient and includes two contributions, from the intensity and the fourth Stokes parameter. The polarization contribution to the function \( C^{(sh)}(\rho) \) depends explicitly on the degree of polarization \( P_C(L) \) and, therefore, vanishes simultaneously with the depolarization of light. Near the first Kerker point, the wavelength dependence of the depolarization length \( l_{circ} \) has a resonance shape [16]. This leads to a similar quasiresonant behavior of the correlation function (6).

Equation (6) differs from the corresponding result of Ref. [22] because of Eq.(6) describes the correlation function decrease in inverse proportion to \( l_{circ} \) and \( l_{tr} \). Near the first Kerker point, where the depolarization length \( l_{circ} \) is determined by the relation \( l_{circ} = \sqrt{l_{tr} l_{dep}/3} \) [21]. For \( l_{circ} = L \), it follows that intensity fluctuations at short ranges exceed noticeably the transport mean free path \( l_{tr} \) [16] the contributions from the Stokes parameters \( I \) and \( V \) to the spectrum can be calculated within the diffusion approximation, to give

\[
M(z_f, q) = \frac{l_{tr}}{4\pi k_0^2} \int_0^L dz \left[ \frac{\partial}{\partial z} I_q(z_f |z|) \right]^2 + q^2 I_q(z_f |z|)^2 \left( I^2(z) + V^2(z) \right)
\]

(8)

where quantities \( I_q(z_f |z|) \), \( I(z) \) and \( V(z) \) are expressed in terms of the integrals of \( I_q(z_f |z, n| \), \( I(z, n) \) and \( V(z, n) \) over directions and subject to to the diffusion equation (see, e.g., [20]). For purely elastic scattering the spectrum of transmitted intensity fluctuations is written as [13,14]

\[
M(L, q) = \frac{9\pi l_{tr}}{k_0^2} \left( F(qL, 0) + F(qL, \frac{L}{l_{circ}}) \right)
\]

(9)

where

\[
F(x, y) = \frac{xy (x \sinh 2y - y \sinh 2x)}{4(x^2 - y^2) \sinh^2 x \sinh^2 y},
\]

(10)

The Fourier transform of Eqs.(9) and (10) leads to the correlation function of the form

\[
C(\rho) = \frac{9\pi l_{tr}}{k_0^2 L^3} \left[ \nu \left( \frac{\rho}{L}, 0 \right) + \nu \left( \frac{\rho}{L}, \frac{L}{l_{circ}} \right) \right]
\]

(11)

In transmission of light through a thick depolarizing slab, \( L > l_{circ} \), the polarization effect reveals itself at all \( \rho \). For relatively small \( \rho, \rho < l_{circ} \), the scalar and polarization contributions to the correlation function decrease in inverse proportion to \( \rho \), but with the different amplitudes. In this case \( \nu (\rho/L, L/l_{circ})/\nu (\rho/L, 0) = (L/l_{circ})^2/\sinh^2 (L/l_{circ}) \), and the polarization term proves to be suppressed as compared to the scalar one (see Fig. 2). For \( l_{circ} < \rho < L \) the polarization contribution \( \nu (\rho/L, L/l_{circ}) \) can be estimated as \( l_{circ}/L \) times smaller than the scalar one \( \nu (\rho/L, 0) \). Just these values of \( \rho \) contribute to the polarization term of the variance of transmission coefficient fluctuations at \( L > l_{circ} \).
Figure 2. Scalar $\nu(\rho/L, 0)$ (solid black line) and polarization $\nu(\rho/L, L/l_{circ})$ (other color lines) contributions to the correlation function of transmitted intensity fluctuations. From upper to lower curves $L/l_{circ} = 0, 1, 3, 7$.

5. Transmission coefficient fluctuations

As follows from definition (2) the value of the spectrum at $q = 0$ determines the variance of the total transmission coefficient [7,17]

$$\langle (\delta T)^2 \rangle = \frac{1}{A} M(L, q = 0),$$

(12)

where $A$ is the area of the slab surface, $T$ is the ratio of the total transmitted flux to the incident one. From Eqs.(9), (10) and (12) it follows that the variance of the transmission coefficient is determined by the expression

$$\frac{\langle (\delta T)^2 \rangle}{\langle T \rangle^2} = \frac{3}{2N} \frac{L}{l_{tr}} \left( \frac{1}{3} + \frac{\sinh 2y - 2y}{4y \sinh^2 y} \right)$$

(13)

where $N = Ak_0^2/(2\pi)$ is the number of propagating modes [12], $y = L/l_{circ}$. Equation (13) describes the crossover in transmission fluctuations which is caused by decay of correlations between the cross-polarized fields and, correspondingly, by depolarization of the incident light. For $L < l_{circ}$ the depolarization of circularly polarized light is of little significance and the variance tends to the value that is twice greater compared to the result $\langle (\delta T)^2 \rangle/\langle T \rangle^2 = L/(2Nl_{tr})$ of the scalar theory [1] which neglects correlations between the cross-polarized fields. As the sample thickness increases, $L > l_{circ}$, the polarization contribution to the relative value of the variance tends to an $L$-independent value (i.e., the polarization contribution is a non-self-averaging quantity):

$$\frac{\langle (\delta T)^2 \rangle}{\langle T \rangle^2} = \frac{1}{2N} \left( \frac{L}{l_{tr}} + \frac{3l_{circ}}{2l_{tr}} \right)$$

(14)

According to Eq.(14), the polarization contribution to the variance of the transmission coefficient is noticeable even for large thickness $L$ where the degree of polarization (see Eq.(7))
Figure 3. Wavelength dependence of the variance of relative transmission fluctuations for an ensemble of silicon spheres near the first Kerker point. From lower to upper curves $L/l_{tr} = 5, 10, 20$.

and, correspondingly, the polarized component of the radiation transmitted through the sample are exponentially small.

Near the first Kerker point, the depolarization length $l_{circ}$ varies sharply with the wavelength [16]. Hence, depending on the wavelength, the sample with given parameters (thickness $L$, radius of particles, their refractive index, etc.) can either retain the initial polarization of the incident light ($L < l_{circ}$) or completely depolarize the light ($L > l_{circ}$). The crossover between these two regimes of propagation manifests itself in the variance of transmission coefficient fluctuations. The wavelength dependence of $\langle (\delta T)^2 \rangle / \langle T \rangle^2$ is illustrated in Fig. 3. As follows from the results of calculations, the fluctuations are enhanced near the Kerker point. If the thickness $L$ is smaller than the length $l_{circ}$, the maximum value of the variance is twice greater than the result of the scalar theory [1]. The peak in the wavelength dependence of the variance is narrowed with increasing $L$, and for thick samples repeats the dependence of the depolarization length on the wavelength $\lambda$ (see Eq.(14)).

6. Effect of absorption

A number of materials with a high refractive index is characterized by noticeable absorption [23] which should be taken into account in studying the depolarization of light and, correspondingly, the intensity fluctuations.

In an absorbing medium, one more spatial scale appears [20]. The length of wave trajectories turns out to be restricted by the absorption mean free path $l_a$. As a result, the intensity of light falls off at depths $z$ of the order of the diffusion length $l_d = \sqrt{l_{tr}l_a/3}$ [20]. The presence of absorption affects also the depolarization length $l_{circ}$. Decay of the degree of circular polarization $P_C = V/I$ is governed by the competition between attenuation of $I$ and $V$, and occurs at scales of the order

$$l_{circ} = l_d \left( \frac{l_{dep}}{l_a} \right) \left( 1 + \sqrt{1 + \frac{l_a}{l_{dep}}} \right) = \begin{cases} \sqrt{l_{tr}l_{dep}/3}, & l_{dep} < l_a, \\ 2l_d (l_{dep}/l_a), & l_{dep} > l_a \end{cases} \tag{15}$$
From Eq. (15) it follows that in the case of relatively high absorption, \( l_a < l_{dep} \), the depolarization length \( l_{circ} \) increases as \( l_{circ} \sim l_{dep} \sqrt{l_{tr}/l_a} \), and exceeds the diffusion length \( l_d \).

As \( L \) increases, the variance of the transmission coefficient tends to

\[
\frac{\langle (\delta T)^2 \rangle}{\langle T \rangle^2} = \frac{1}{2N} \left( \frac{3L}{4l_{tr}} + \frac{3l_d}{4l_{tr}} \frac{2\alpha^2 - 1}{\alpha(\alpha^2 - 1)} \right)
\]

(16)

where \( \alpha = 1 + (l_d/l_{circ}) \). A relative role of the polarization contribution to the variance of transmission fluctuations increases with absorption. If the radiation is depolarized faster than absorbed, \( l_a > l_{dep} \), the polarization contribution to the variance of the transmission coefficient coincides with that entering into Eq.(14). In the opposite case of comparatively high absorption, \( l_a < l_{dep} \), the polarization contribution to the variance increases noticeably. This contribution gains additional factor \( \sqrt{l_{dep}/l_a} \).

7. Conclusions
As has been shown above, the anomalously slow depolarization of circularly polarized light near the first Kerker point [15,16] opens new possibilities for studying optical mesoscopic effects which were previously considered [1] in the scalar approximation. Accounting for the vector nature of light we have obtained the additional contribution to the intensity fluctuations which results from correlations between the cross-polarized fields. Under conditions of circular polarization memory, this contribution survives at great distances because the correlation function of the cross-polarized field involves, as part, the fourth Stokes parameter. We have shown that the main observable effect consists in the enhancement of fluctuations. In the vicinity of the first Kerker point the variance of the transmission coefficient varies resonantly with the wavelength. The peak value of the variance is two times as great as the result of the scalar wave theory [1].

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