Effect of Phonons and Impurities on the Quantum Transport in XXZ Spin-Chains

Amartya Bose

Department of Chemistry, Princeton University, Princeton, New Jersey 08544, USA

Numerical and analytic results have been used to characterize quantum transport in spin chains, showing the existence of both ballistic and diffusive motion. Experiments have shown that heat transfer is surprisingly always diffusive. The scattering from phonons and impurities have been postulated to be the two factors critical in causing the diffusive transport. In this work, we evaluate the transport process by incorporating a bath of phonons and impurities in order to understand the role played by each of the factors. While methods like time-dependent density matrix renormalization group (tDMRG) can be used to simulate isolated spin chains, the coupling with phonons make simulations significantly more challenging. The recently developed multisite tensor network path integral (MS-TNPI) method builds a framework for simulating the dynamics in extended open quantum systems by combining ideas from tDMRG and Feynman-Vernon influence functional. This MS-TNPI is used to characterize dynamics in open, extended quantum systems. Simulations are done with the commonly used sub-Ohmic, Ohmic and super-Ohmic spectral densities describing the phononic bath. We show that while the transport in presence of impurities eventually becomes diffusive, the exact details are dependent on the specifics of the interactions and amount of impurities. In contrast, the presence of a bath makes the transport diffusive irrespective of the parameters characterizing the bath.

I. INTRODUCTION

Non-equilibrium dynamics of quantum systems remain a major focus and important challenge in physics [1–4]. Of special interest are the transport properties of spin chains. This is not only because of possible applications in quantum information processing, information storage, and spintronics, but also because spin chains provide us with a relatively simple model, which can manifest complex aspects of non-equilibrium dynamics. Experimental work done with ultra-cold atoms [5, 6] has demonstrated the possibility of representing these systems as spin-

chains. The two states of a spin represent atoms of different types occupying the given lattice site.

Recent work has studied the transport in quantum systems using a generalization of hydrodynamics [7, 8], successfully predicting ballistic current starting from inhomogenous initial states. However, such theories are unable to predict the dynamics easy-axis regime. It is known that when the systems have parity symmetries \((\mathbb{Z}_2)\), and the initial state is symmetric under the parity operation and under spatial reflection (i.e. \(x \rightarrow -x\)), the transport may lose the ballistic scaling when the observable is odd under the parity [9]. This implies that the “rate” of transport of the conserved quantity across the “boundary” at \(q = 0\) is sub-linear with time. Numerical simulations using time-dependent density matrix renormalization group (tDMRG) [10–13] have been used to explore this regime of quantum transport in XXZ chains [9, 14], demonstrating the superdiffusive dynamics for the isotropic spin chain.

The XXZ-model has proved to be extremely useful, not just for the theoretical study of transport properties [8, 9, 14], but also in describing real systems [15]. Despite the predictions of ballistic heat transport in these systems, Hlubek et al. [15] observed that the transport was diffusive. This was attributed to extrinsic scattering of the spinons off impurities and phonons. It is thus important to understand the contribution of each of these scattering events in bringing about the diffusive transport.

The simulations of spin chains with impurities are relatively simple. One can simulate a statistical ensemble of distribution of impurities in spin chains using tDMRG or the time-dependent variational principle (TDVP) [16–19]. However, the incorporation of coupling to phonons in the dynamics of extended quantum systems at a finite temperature is computationally extremely challenging. The presence of low frequency modes at high equivalent temperatures necessitate the use of large bases for the phononic bath and consequently leads to an exponential growth of computational complexity for wave function based methods. Some interesting work has been done to understand the dynamics of boundary-driven XXZ chain [2, 3].

Open quantum systems are most often simulated by integrating out the phononic bath using path integrals based on the Feynman-Vernon influence functional [20]. It has been shown that it is possible to enhance the performance of influence functional simulations using tensor network [21–24]. Despite such advances, the simultaneous presence of an extended system and the phonons leads to problems that cannot be easily simulated. The existence of local baths introduces a non-Markovian memory, within which the scaling of the computational requirements scale exponentially. For extended systems the base of the scaling is so large that the computations are infeasible even for small memory lengths. We have developed an extension to the framework of tDMRG incorporating influence functionals leading to a method called
multisite tensor network path integral (MS-TNPI) [25]. MS-TNPI is able to simulate the dynamics of open, extended quantum systems accounting for non-Markovian memory. This has been used advantageously to simulate the excitonic dynamics in photosynthetic complexes [26].

The primary objective of this paper is to numerically explore the role played by the scattering from phonons and impurities in the non-equilibrium transport in antiferromagnetic XXZ chain. In Sec. II, the systems explored are described, along with some well-known properties. Standard tDMRG is used for simulating the transport process in presence of impurities. MS-TNPI is used when accounting for the non-Markovian coupling between spins and phonons, and is described in Sec. III. The results of the simulation are presented in Sec. IV. We will show that the presence of phonons causes the dynamics to become diffusive, irrespective of the characteristics of the phononic modes. The presence of impurities on the other hand changes the dynamics in more subtle ways, that are dependent on the particulars of the interactions introduced by the impurities. Therefore, the observation of uniformly diffusive dynamics in Ref. [15] is likely because of interactions with phonons. We end with some concluding remarks and future prospects in Sec. V.

II. SYSTEMS UNDER STUDY

Consider an XXZ spin chain with $n$ spins (for even $n$). The Hamiltonian is given by

$$\hat{H}_0 = \hbar J \sum_{-\frac{n}{2} < q < \frac{n}{2}} \left( \hat{s}^{(1)}_q \hat{s}^{(1)}_{q+1} + \hat{s}^{(2)}_q \hat{s}^{(2)}_{q+1} + \Delta \hat{s}^{(3)}_q \hat{s}^{(3)}_{q+1} \right).$$  \hspace{1cm} (1)

In Eq. (1), $\hat{s}^{(k)}_q$ are the spin-$\frac{1}{2}$ operators for the spin at the spatial location $q$. It is related to the Pauli matrices as $\hat{s}^{(k)}_q = \frac{1}{2} \sigma^{(k)}_q$. The total magnetization $M = \sum_q \hat{s}^{(3)}_q$ is a conserved quantity in the XXZ model. The anisotropy in the system is encoded in $\Delta$. If $\Delta = 0$, the XXZ model reduces to the Frenkel problem, which ubiquitous in the study of exciton transfer. The two eigenstates of $\hat{s}^{(3)}_q$ with eigenvalues of $\pm 1$ are denoted as $|\uparrow, q\rangle$ and $|\downarrow, q\rangle$ respectively, with the $q$ subscript omitted where it does not cause ambiguity. Depending on the sign of $\Delta$, the ground state is either ferromagnetic ($\Delta < 0$) or antiferromagnetic ($\Delta > 0$). Here we consider the antiferromagnetic system with $\Delta > 0$. The excitation spectrum is gapped for $|\Delta| > 1$, and when $|\Delta| < 1$, the system becomes gapless and the correlation functions show power law behavior [14]. The case of $\Delta = 0$ leads to the so-called XX model, also known in the literature as the Frenkel model of exciton transfer.

In this paper, the non-equilibrium transport is studied from the initial state

$$\hat{\rho}(0) = |\uparrow\uparrow \ldots |\uparrow\downarrow \ldots |\downarrow\downarrow \ldots \rangle \langle |\uparrow\uparrow \ldots |\uparrow\downarrow \ldots |\downarrow\downarrow \ldots \|. \hspace{1cm} (2)$$

where the left half spins are in up state ($|\uparrow\rangle$) and the spins in the right half of the chain are in down state ($|\downarrow\rangle$). This fully polarized domain wall is a very special setup that has received attention in the literature [14]. In other explorations an initial state with a more “tilted” domain wall is selected [9, 27]. The exact nature of the non-equilibrium dynamics is sensitive to the exact initial condition. The transport process can be characterized quantitatively through the scaling of the time-dependent spin profile and the magnetization transferred between the two halves given as an integral over the spin current, $j$, at $q = 0$:

$$\Delta m = \int_0^t j(0, t') \, dt' \propto \sum_{q > 0} \left( \hat{s}^{(3)}_q(t) + \frac{1}{2} \right) \propto t^\alpha. \hspace{1cm} (4)$$

A scaling of $\alpha = 1$ implies ballistic motion, and $\alpha = 0.5$ implies diffusive motion. Superdiffusive motion is characterized by $0.5 < \alpha < 1$.

The first and computationally simpler mechanism for the change of the nature of transport that was postulated and is explored here is scattering from impurities. Previous work [28] has shown that addition of even an infinitesimal periodic perturbation can make the thermodynamic limit transport diffusive. Here, we take a different approach to modeling impurities in a spin chain. We assume that an impurity is distinguished from a usual site by the way it interacts with its neighboring spins. Addition of a new type of site introduces two additional types of coupling: the impurity-site and the impurity-impurity couplings. In this work, we take the differences to be in the value of $\Delta$. Different random configurations with certain proportions of impurities are sampled over to obtain the average dynamics. This is explored in Sec. IV A.

Subsequently we explore the effect of scattering from phonons in Sec. IV B. In presence of phonons, the Hamiltonian is modified through interactions with the harmonic bath describing the phonons,

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{spin-phonon}}, \hspace{1cm} (5)$$

where $\hat{H}_0$ is the Hamiltonian of the isolated XXZ spin-chain, Eq. (1), and $\hat{H}_{\text{spin-phonon}}$ is the Hamiltonian corresponding to the dissipative phononic bath interacting with the system at $x$.

$$\hat{H}_{\text{spin-phonon}} = \sum_{-\frac{n}{2} < q < \frac{n}{2}} \sum_j \frac{p_{j,q}^2}{2m_{j,q}} + \frac{1}{2} m_{j,q} \omega_{j,q}^2 \left( x_{j,q} - \frac{c_{j,q} |\uparrow\rangle \langle \uparrow|}{m_{j,q} \omega_{j,q}^2} \right)^2, \hspace{1cm} (6)$$

where $\omega_{j,q}$ and $c_{j,q}$ are the frequency and coupling of the $j$th mode of site $q$. The interaction between the spin chain and the phonons is such that the harmonic oscillators of the phonon get shifted only when the corresponding spin
is in the $|\uparrow\rangle$ state. The bath is usually characterized by a spectral density,

$$J(\omega) = \frac{\pi}{2} \sum_j \frac{c_0^2}{m_j \omega_j} \delta(\omega - \omega_j).$$

One of the most important characteristics of a harmonic bath is the reorganization energy, $\lambda = \frac{2}{\beta} \int_0^\infty d\omega \omega J(\omega)$. For the current study, we use the well-known form with an exponential cutoff:

$$J(\omega) = 2\pi \hbar \xi \frac{\omega^s}{\omega_c} \exp\left(-\frac{\omega}{\omega_c}\right),$$

where $\omega_c$ is the cutoff frequency and $\xi$ is the dimensionless Kondo parameter encoding the strength of spin-bath coupling. The type of the bath is determined by the value of $s$: $s < 1$ defines a sub-Ohmic bath, $s = 1$ defines an Ohmic bath, and $s > 1$ is a super-Ohmic bath. The reorganization energy of the bath is $\lambda = 2\hbar \omega_c \xi \Gamma(s)$.

For the simulations with interaction with phonons, the initial state is taken to be a product state between the reduced density matrix of the spin chain and the thermal density of the isolated bath:

$$\rho(0) = \tilde{\rho}(0) \otimes \frac{Z_{\text{phonon}}}{Z_{\text{phonon}}}.$$  

Here, $U$ is the short-time propagator for the spin chain, $S^+_j$ represents the “forward-backward” state of the spin chain at the $j$th time-point. In the spin chain with many sites, $S^+_j$ is a short-hand for $S^+_{i,j}$, where the first index, $i$, is the index of the spatial location of the site, $q$, and the second index, $j$ gives the time point. The Feynman-Vernon influence functional formalism [20], denoted by $F[\{S^+_j\}]$, is dependent upon the history of the system. The baths are assumed to be site local. Therefore the total influence functional is a product of the individual influence functionals corresponding to each of the sites:

$$F[\{S^+_j\}] = \prod_i \exp\left(-\frac{1}{\beta} \sum_{k=0}^N \Delta s_{i,k} \sum_{k'=0}^k (\eta^{(i)}_{kk'} s^+_{k,k'} - \eta^{(i)*}_{kk'} s^-_{k,k'})\right)$$

where $\Delta s_{i,k} = s^+_{i,k} - s^-_{i,k}$ and $\eta^{(i)}_{kk'}$ are the discretized influence functional coefficients [29, 30]. These $\eta$-coefficients are generally expressed as integrals involving the spectral density.

Here $Z_{\text{phonon}}$ is the partition function for the bath at an inverse temperature of $\beta = 1/k_BT$.

The simulation of transport in presence of phonons at a non-zero temperature is challenging because of the presence of temporally non-local interactions in the form of non-Markovian memory. The recently introduced MS-TNPI method [25] allows us to capture these non-Markovian effects in a numerically exact Feynman-Vernon influence functional-based formalism using tensor networks. This method is described in short in Sec. III.

### III. MULTISITE TENSOR NETWORK PATH INTEGRAL

While tDMRG is well-suited for the simulation of extended quantum systems like spin chains, the presence of dissipative media poses significant computational challenges. The recently developed multisite tensor network path integral (MS-TNPI) [25] extends tDMRG ideas to account for presence of harmonic modes. If the initial state can be expressed as a direct product of the system’s initial reduced density matrix and the bath’s thermal density, then the time-propagated reduced density matrix of the system is given as

$$\langle S^+_N | \tilde{\rho}(N\Delta t) | S^-_N \rangle = \sum_{S^+_N} \sum_{S^+_N-1} \ldots \sum_{S^+_1} \langle S^+_N | \hat{U} | S^+_N-1 \rangle \langle S^+_N-1 | \hat{U} | S^+_N-2 \rangle \ldots \times \langle S^+_1 | \hat{U} | S^+_0 \rangle \langle S^+_0 | \hat{U} | S^-_1 \rangle \ldots \langle S^-_{N-1} | \hat{U} | S^-_N \rangle F[\{S^+_j\}]$$

There are two basic entities required for simulating the path integral: (1) a forward-backward propagator $K(S^+_j, S^+_{j+1}) = \langle S^+_j | \hat{U} | S^+_{j+1} \rangle \langle S^+_j \hat{U}^\dagger | S^+_{j+1} \rangle$, and (2) the influence functional corresponding to each path. For an extended spin chain, the cost of storing the full propagator becomes prohibitive. Thus, following the time-evolving block decimation (TEBD) method for propagation of the wavefunction [13], the “system axis,” $S^+_j$ is also factorized out, yielding a matrix product representation of the forward-backward propagator. In TEBD, this propagator is repeatedly applied to the state to simulate time propagation. This dynamics is Markovian.
FIG. 1. Refactorization of the propagator MPO.

FIG. 2. MS-TNPI network obtained by multiplying the refactorized propagator MPOs.

VISION I. RESULTS

A. Transport in presence of impurities

We start the study by analyzing the transport in an XXZ chain with impurities. These simulations are done with standard TEBD. In Fig. 3, we demonstrate the quantum transport in the isolated XXZ spin chain. As is well-known [14], the transport is ballistic for $0 < \Delta < 1$ ($\alpha = 1$), diffusive for $\Delta > 1$ ($\alpha = 0.5$, not simulated here), and superdiffusive for $\Delta = 1$ ($\alpha \approx \frac{3}{5}$). While $\alpha \approx \frac{3}{5}$ for the fully polarized domain boundary simulated here [9, 14], the exact scaling for the isotropic XXZ model is dependent on the initial condition used. In particular, $\alpha \approx \frac{3}{5}$ at high temperatures [9, 31]. Figure 3 (b) shows striations with certain wavefronts showing ballistic motion. This seems to be qualitatively similar to what was observed in Ref. [27].
Now, we start introducing impurities in the XXZ chain. The impurities considered in this paper can be thought of as replacing the original sites with a different kind of site. Suppose the base pure chain is taken to have an anisotropy of $\Delta = 0$. For every site that has been replaced by an impurity, the interaction with the neighboring sites changes depending on the nature of the neighboring site. For simplicity of simulation, we assume that the value of $J$ remains constant irrespective of whether the interaction is site-site, site-impurity or impurity-impurity. However the anisotropy values are taken to be different. For this preliminary exploration, we assumed

\begin{align}
\Delta_{\text{spin-spin}} &= 0, \\
\Delta_{\text{spin-impurity}} &= 0.5, \\
\Delta_{\text{impurity-impurity}} &= 1.
\end{align}

(Note that in this case, the pure spin chain would show ballistic dynamics, and a spin chain made of 100% impurities should show a super-diffusive dynamics.) To study the effect of impurities, we simulate the dynamics at different proportions of doping. Every site has a certain probability of being an impurity, and the Hamiltonian is consequently defined by the arrangement of these impurities. The overall dynamics is obtained as a statistical average over such arrangements.

First, in Fig. 4 let us consider the dynamics of $\langle \hat{s}_q^{(3)}(t) \rangle$ for the XXZ spin chain with the sites having 25% probability of being an impurity. The amount of magnetization transferred across the initial domain boundary at $q = 0$ is shown as a function of time in Fig. 5 for different percentages of doping. First, notice that the extremely short time dynamics is “super-ballistic,” with $\alpha = 2$. At intermediate time-scales, the dynamics shows a continuous gradation of the effective scaling with the probability of impurity. While the simulations have not been run for long enough durations, the ones with a 75% probability of impurity seems to show a distinct slowing down of dynamics to diffusive transport. This is in line with the observations of long time dissipative dynamics in presence of perturbations that make the system non-integrable [28].

Therefore, the numerical evidence seems to indicate that although the presence of impurities make the dynamics diffusive at long times, there are non-diffusive transients that reflect the exact nature of the impurities. These transients last for longer timespans when the amount of impurity is less. A more thorough exploration of the dynamics, while interesting, is beyond the scope of this work. This study will be a topic of future work.

### B. Transport in presence of phonons

Addition of a dissipative medium (phonons, in this case) to each of the sites “smears” out the dynamics. First, we explore the dynamics of $\langle \hat{s}_q^{(3)}(t) \rangle$ as a function of time in Fig. 6 for $\Delta = 0$. In this case we consider a relatively strongly coupled, cold and fast Ohmic ($s = 1$) bath with $\omega_c = 10J$, $\xi = 1$ and $h\omega_c\beta = 5$. Also shown along with the dynamics as a function of both time and site location, we also report the spin profile various sites at different times. From Fig. 6 (b), it is clear that the spin profile is invariant under a scaling of $q \sim \sqrt{t}$. Thus we have demonstrated that the presence of the Ohmic bath converted the ballistic dynamics of the isolated XXZ chain with $\Delta = 0$ to a diffusive dynamics. Note that the ballistic wavefronts observed in Fig. 3 and Fig. 4 are completely absent in presence of the phonons in Fig. 6. (The
Fig. 4. Dynamics of \( \langle \hat{s}_{q}(t) \rangle \) for an XXZ chain with different probabilities of each site being an impurity with dashed guide for the \( \alpha = 0.5 \) scaling.

Fig. 5. Transfer of magnetization as a function of time for different levels of impurity. Blue dashed line: \( \Delta m \sim t^2 \). Gray dashed line: \( \Delta m \sim t \). Black dashed line: \( \Delta m \sim t^{3/5} \). Magenta dashed line: \( \Delta m \sim t^{1/2} \).

In Fig. 7, we plot the amount of magnetization transferred across \( q = 0 \), \( \Delta m \), in presence of the phonons as a function of time for \( \Delta = 0, 1 \) and \( 2 \). The asymptotic behavior of the system is the same irrespective of the amount of anisotropy. While the initial dynamics is ballistic, the dynamics very quickly becomes diffusive. Notice that the “super-ballistic” transients observed in Fig. 5 has vanished and been replaced by an insignificant duration of ballistic transport rapidly decaying into a uniformly diffusive transport.

Till now we have only considered Ohmic baths. Next consider the effect of different types of bath. Since the biggest change in the nature of the dynamics happens for the XXZ spin chain with \( \Delta = 0 \), we explore this effect for this case. If it is the presence of the bath and not the nature thereof that causes the diffusive transport, the asymptotic behavior of \( \Delta m(t) \) would scale in the same way, irrespective of the value of \( \alpha \). Fig. 8 shows the transfer of magnetization across \( q = 0 \) for a sub-Ohmic \( (\alpha = 0.5) \), Ohmic \( (\alpha = 1) \) and super-Ohmic \( (\alpha = 2) \) baths with \( \omega_c = 10J, \xi = 1 \) and \( \hbar \omega \beta = 5 \). We see that irrespective of whether the bath is Ohmic or sub- or super-Ohmic, the dynamics asymptotically becomes diffusive.

Finally, we investigate the entanglement in the systems for the different cases. Instead of explicitly calculating the entanglement entropy, we report the average bond dimension of the reduced density matrix MPS as an indirect but simpler measure. MS-TNPI simulates the reduced density matrix, consequently, the average bond dimension of the MPS that represents it is going to be larger than that which represents a wavefunction MPS. Therefore, for this comparison, we implemented a density matrix version of TEBD. The average bond dimensions of simulations with different strengths of the phonon bath for the XX system is shown in Fig. 9. The bond dimension is the smallest for the strongest coupled bath. The intuition here is that the entanglement goes out into the bath when it is coupled. The stronger the system-bath coupling, the more efficient the bath is at limiting the growth of entanglement within the system. Figure 10 shows the growth of the bond dimension for various values of \( \Delta \). The bond dimension grows faster at higher anisotropy.

V. CONCLUSIONS

Quantum transport in extended systems is a domain of study that is both incredibly rich both in terms of the physics involved as well as from the perspective of the potential applications. The nature of this dynamics is modified by scattering of the spinons off impurities present in the system and the phonons with which they couple. These phenomena have been postulated to be the cause behind the diffusive heat transport observed by Hlubek et al. [15]. In this paper, we have numerically explored the impact of both mechanisms and attempted
Scattering off impurities is relatively easy to simulate using well established methods like tDMRG and TDVP. Žnidarič [28] has studied the effect of periodic on-site magnetic fields. In this paper, we modeled the impurities as sites with different interactions than the ones in the clean model with $\Delta = 0$. We demonstrated that the impact of the scattering events at intermediate time scales is highly dependent on the amount and the nature of the impurities present. However, at long times, the dynamics in presence of impurities became diffusive. The dynamics was simulated as the average dynamics over arbitrary configurations of impurities. The nature of the transport at intermediate times is bounded by the dynamics in the pure system on one side and the dynamics in a system with 100% impurities on the other hand. The time-scale of the diffusive dynamics setting in is dependent on nature of the impurities. Future work will explore in further detail the various facets of dynamics in presence of such impurities.

The coupling to phonons is significantly more challenging to incorporate because of the presence of non-Markovian memory. Feynman-Vernon influence functional is often used to simulate the dynamics of open quantum systems. We have recently developed the multisite tensor network path integral approach to combine ideas from other tensor network methods like tDMRG and TEBD with influence functional. This allows us to simulate quantum dynamics of open extended systems without invoking perturbation theory or Markovian approximations. MS-TNPI is numerically exact under convergence.
Using MS-TNPI, we analyzed the effect of phonons on the quantum transport. We demonstrated that irrespective of the system and the description of the phononic bath, the thermal transport is always diffusive. This is consistent with the experimental observations in Ref. [15]. We hypothesize that this result will carry over to other structured spectral densities describing the phononic bath. One subtle but notable difference between the dynamics from the system coupled to phonons and the system with impurities is the absence of ballistic wavefronts in presence of the phonons.

The presence of the phonons helps “dissipate” the growing entanglement of the XXZ spin chain. We show that the growth of the average bond dimension of the reduced density matrix corresponding to the extended system is severely restricted by the presence of the baths. The stronger the coupling to the bath, the slower the bond dimension grows.

While the fully polarized domain boundary explored here is a special initial condition in many ways, there have been studies about the dynamics of the XXZ system with a high temperature density matrix [9], and the so-called “tilted” domain boundary [27] initial conditions. These studies explore the rich physics demonstrated by the XXZ system. It would also be very illuminating to study the transport from such nonequilibrium initial conditions in presence of the interaction with phonons.

Given the diffusive nature of transport from the fully polarized boundary initial condition, one may hypothesize that the transport from the other initial conditions in presence of phonons might also be diffusive. We have recently studied the effect of temperature gradient on exciton transport in the Frenkel model [32]. It would be interesting to study the effect of temperature profile on the diffusion observed from these boundary initial conditions.

In this work the phonons were coupled only along the Z-direction. In the future, MS-TNPI will be extended to handle phonon couplings along multiple non-commuting system operators. It will be interesting to explore the effects of different couplings. One wonders if the anisotropy of the couplings could be the main reason behind the uniformly diffusive dynamics observed here.

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Appendix A: Transport dynamics in presence of Ohmic bath

In Sec. IV B, we have shown the dynamics of the XXZ chain with $\Delta = 0$ connected to a phononic bath described by the Ohmic spectral density. Here, in Fig. 11, we show similar graphs for $\Delta = 1$ and $\Delta = 2$. Note that the dynamics looks identical in all these cases, implying that the transport process is completely modulated by the scattering from the phonons.

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FIG. 11. Dynamics of $\langle \hat{s}_q^{(3)}(t) \rangle$ and the invariance of the spin profile at different times for XXZ chains with different values of $\Delta$. Left column shows $\langle \hat{s}_q^{(3)}(t) \rangle$. The black dashed lines guide the eye towards a scaling of $q \sim \sqrt{t}$. Right column shows the spin profiles at different times rescaled by $\alpha = 0.5$. The bath used here is identical to the one in Fig. 6.

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