Asymptotically exact dispersion relations for collective modes in a confined charged Fermi liquid

I. V. Tokatly\textsuperscript{1,*} and O. Pankratov\textsuperscript{1}

\textsuperscript{1}Lerhrstuhl für Theoretische Festkörperphysik, Universität Erlangen-Nürnberg, Staudtstrasse 7/B2, 91058 Erlangen, Germany

(Dated: March 22, 2022)

Using general local conservation laws we derive dispersion relations for edge modes in a slab of electron liquid confined by a symmetric potential. The dispersion relations are exact up to \( \lambda^2 q^2 \), where \( q \) is a wave vector and \( \lambda \) is an effective screening length. For a harmonic external potential the dispersion relations are expressed in terms of the exact static pressure and dynamic shear modulus of a homogeneous liquid with the density taken at the slab core. We also derive a simple expression for the frequency shift of the dipole (Kohn) modes in nearly parabolic quantum dots in a magnetic field.

PACS numbers: 73.21.-b, 71.45.Gm

The examples of importance of a collective motion of particles range from such old problems as energy losses by fast particles in solids to modern studies of a collective response in semiconductor nanostructures and interaction of clusters and molecules with intense laser radiation. However, only a few exact features of collective dynamics in spatially inhomogeneous many-body systems have been found up to now. The most known result is the Generalized Kohn Theorem (GKT) \cite{1} or the Harmonic Potential Theorem in a more general formulation \cite{2}. This theorem has been extensively used for construction of approximations for exchange-correlation (xc) potential in Time Dependent Density-Functional Theory (TDDFT) (see e.g. Ref. 3 and references therein).

In this paper we present another asymptotically exact result which, in some sense, can be viewed as an extension of GKT. We derive the exact dispersion relations for the dipole and monopole plasma modes in a slab of a charged Fermi liquid confined by a symmetric external potential. This result opens a possibility for a direct experimental determination of the shear modulus of an electron liquid. It can also help to control the accuracy of approximations used to describe dynamics of inhomogeneous systems. It has also direct implications for the theory of surface plasmons in simple metals. In addition we derive a few asymptotically exact relations for dipole modes in quantum dots (QD) and wires in a homogeneous magnetic field.

The collective motion of an arbitrary interacting system obeys the exact local conservation laws:

\[ \partial_t n + \partial_{\mu} n v_{\mu} = 0, \]

\[ mn_0 D_{\mu} v_{\mu} + e n (v \times H)_{\mu} + \partial_{\nu} P_{\mu\nu} + n \partial_{\mu} U = 0, \]

where \( D_t = \partial_t + v_{\nu} \partial_{\nu} \), \( n \) is the density of particles, \( v = \frac{1}{n} \) is the velocity, \( U = U_{\text{ext}} + U_{\text{H}} \) is a sum of the external and Hartree potentials, \( H \) is the external static magnetic field and \( P_{\mu\nu} \) is the exact (generally unknown) stress tensor.

Introducing the displacement vector \( u_{\mu} = \partial_t u_{\mu} \), we derive from Eq. (2) the linearized equation of motion

\[ mn_0 \partial_t^2 u_{\mu} + ev_0 (\partial_{\nu} u_{\mu} \times H)_{\mu} + \partial_{\nu} \delta P_{\nu\mu} + n \partial_{\mu} U + \delta n \partial_{\mu} U_0 = 0, \]

where \( \delta n = -\partial_{\mu} n_0 u_{\mu}, \delta P_{\nu\mu} \) and \( \delta U \) are deviations of the density, stress tensor and the potential from their equilibrium values \( n_0, P_{\nu\mu}^{eq} \) and \( U_0 \) respectively. Eigenfrequencies of Eq. (3) define the exact excitation energies of a system.

Let us first consider the dipole modes of a finite system confined in \( D \) dimensions by the potential \( U_{\text{ext}} \) in the presence of a homogeneous magnetic field \( H \). Integration of Eq. (3) leads to the following exact equation for the dipole moment per particle \( d = \frac{1}{N} \int u_{\mu} dx \)

\[ -m \omega^2 d_{\mu} = i \omega e (d \times H)_{\mu} - \frac{1}{N} \int n_0 u_{\nu} \partial_{\mu} \partial_{\nu} U_{\text{ext}} dx. \]

This equation, commonly called the dynamical force sum rule \cite{4}, shows the exact decoupling of the center-of-mass and relative motions for an external potential of the form \( U_{\text{ext}} = \frac{1}{2} m \omega^2 \Omega^2_{\mu\nu} x_{\mu} x_{\nu} \). In this case the dipole (Kohn) modes correspond to the rigid motion with the displacement \( u = d = \text{const} \) being the solution to the Newton equation \cite{2}

\[ -m (\omega^2 \delta_{\mu\nu} - \Omega^2_{\mu\nu}) d_{\nu} = i \omega e (d \times H)_{\mu}. \]

In practice one often deals with the quasiparabolic QD, where the deviation \( \Delta U_{\text{ext}} \) of the confining potential from the parabolic one is small (see Ref. 5 and references therein). The perturbative treatment of Eq. (4) shows that to the first order in \( \Delta U_{\text{ext}} \) the dipole moment still satisfies Eq. (5) but with the renormalized dynamical matrix \( \Omega^2_{\mu\nu} = \Omega^2_{\mu\nu} + \Delta \Omega^2_{\mu\nu} \), where

\[ \Delta \Omega^2_{\mu\nu} = \frac{1}{N} \int n_0 \left( \frac{\partial^2}{\partial x_{\mu} \partial x_{\nu}} \right) \Delta U_{\text{ext}} dx. \]

The corrections to the Kohn-mode frequency has been extensively studied experimentally and theoretically (using...
approximate many-body methods) [5]. The importance of Eq. (6) in this context is that it exactly accounts for the many body effects at the linear in $\Delta U_{\text{ext}}$ level. Therefore it can be helpful for studying correlation phenomena such as Wigner crystallization in a few-electron QD (see e.g. Ref. 6) by means of far-infrared photoabsorption. The transition to the correlated state, which can be tuned by a magnetic field, changes the density distribution $n_0$ and consequently affects the frequency shift according to Eq. (6).

Another useful relation for a dipole mode can be obtained for a system of $N$ charged particles confined by a D-dimensional potential ($D=3,2,1$) with a spherical (dot), cylindrical (wire) or a slab geometry (we consider the case $\mathbf{H} = 0$). We assume that $n_0(r)$ acquires a constant value $\bar{n}$ inside a system and vanishes at infinity. The length $\lambda_0$ of the density variation at the boundary is assumed to be smaller than the system size $a$, which is defined in such a way that the rectangular density distribution $\bar{n}\delta(a-r)$ provides the correct number of particles $N$. Noting that the density fluctuation $\delta n(r)$ for the dipole mode is localized at the length scale $\lambda_0 \ll a$ near the system edges, we obtain from Eq. (4) the following result

$$\omega^2 - \omega_p^2/D = \frac{1}{m\omega_p^2} \int_0^{\infty} \bar{\delta n}(r) \frac{\partial \Delta U_{\text{ext}}}{\partial r} r^{D-1} dr,$$

(7)

where $\bar{\delta n}(r) = \delta n(r) / \int_0^{\infty} \delta n(r) dr$ is the normalized radial part of the density fluctuation and $\Delta U_{\text{ext}} = U_{\text{ext}} - U_{\text{har}}$, where the harmonic potential $U_{\text{har}} = m\omega_p^2 r^2/2D$ corresponds to the same equilibrium density in the internal region ($\omega_p^2 = 4\pi e^2 \bar{n}/m$). It is noteworthy that $\Delta U_{\text{ext}}$ can be arbitrary large in the edge region as the smallness of the right hand side in Eq. (7) is controlled by $\lambda_0/a$.

Equation (7) describes the frequency shift relative to the classical value $\omega_p/\sqrt{D}$. The shift is either positive or negative depending on whether $U_{\text{ext}}$ is “harder” or “softer” than $U_{\text{har}}$. It obviously vanishes for $\Delta U_{\text{ext}} = 0$ in accordance with GKT. Since $\delta n(z)$ has an imaginary part, the collective mode decays due to Landau damping.

For a wire or a slab geometry Eq. (7) gives the frequency at zero wave vector $\mathbf{q}$. Now we consider a general case $\mathbf{q} \neq 0$, which requires a local treatment of the basic Eq. (3).

Consider a slab with a given sheet density $N_s$ confined in z-direction by a symmetric potential $U_{\text{ext}}(z)$. The equilibrium stress tensor can be written as $P_{\mu \nu}^\text{eq} = P_0(z) \delta_{\mu \nu} + \pi_{\mu \nu}^\text{eq}(z)$, where $P_0 = 1/3 \text{Tr} P_{\mu \nu}^\text{eq}$ is the exact pressure and $\pi_{\mu \nu}^\text{eq}$ is a traceless tensor which describes quantum effects. The symmetry requires that $P_{\mu \nu}^\text{eq} = 0$ for $j = x, y$ and $P_{zz}^\text{eq} = P_{\parallel} = P_0 + \pi_0$, $P_{xx}^\text{eq} = P_{yy}^\text{eq} = P_{\parallel} = P_0 - \pi_0/2$.

The equilibrium density $n_0(z)$ satisfies the balance equation (the static version of Eq. (2))

$$\partial_z P_{zz}^\text{eq} + n_0 \partial_z U_0 = 0,$$

(8)

where $U_0 = U_{\text{ext}} + U_{\text{H}}$ and $U_{\text{H}}$ is related to $n_0$ by the Poisson equation

$$\partial_z^2 U_{\text{H}} = -4\pi e^2 n_0.$$

Combining the relation $\delta n = -\partial_\mu n_0 \partial_\mu$ and Eq. (3) and considering the plane wave solution $e^{-i(\omega t - \mathbf{q} \cdot \mathbf{r})}$ we arrive at equation for the density fluctuation $\delta n_{\mathbf{q}, \omega}(z)$ (indexes $\mathbf{q}, \omega$ are suppressed below)

$$m\omega^2 \delta n + \bar{\omega}^2 \delta P_{zz} - q_k q_j \partial_k \partial_j P_{kj} + \partial_z \delta n \partial_z U_0 + \partial_z n_0 \partial_z U - q^2 n_0 \delta U = 0, \quad (10)$$

where $\delta U$ satisfies the equation

$$(\partial_z^2 - q^2) \delta U(z) = -4\pi e^2 \delta n(z).$$

(11)

Up to this point all transformations are exact. From now on we consider the equilibrium density $n_0(z)$ that satisfies the assumption used in the derivation of Eq. (7) [7].

For a slab geometry the width $2a$ is defined as follows

$$\int_{-\infty}^{\infty} n_0(z) dz = 2a \bar{n} = N_s.$$

(12)

Let us choose the origin at the slab “edge” so that the center of the slab is at $z = -a$. Equation (12) implies that the density $n_0(z)$ at $z > -a$ can be represented in the form $n_0(z) = \bar{n} \theta(-z) + \Delta n_0(z)$, where $\int_{-a}^{\infty} \Delta n_0(z) dz = 0$. The function $\Delta n_0(z)$ is localized at the length scale $\lambda_0$.

Our assumption allows to determine the asymptotic form of the stress tensor in the slab interior where $n_0(z) \approx \bar{n}$ and the gradients of the density vanish. Here $\pi_{\mu \nu}^\text{eq} = 0$ and the equilibrium stress tensor reduces to the pressure $P_{\mu \nu}^\text{eq} = \bar{P}_{\mu \nu}$ (all quantities marked with a bar refer to the slab interior). In the same region the local nonequilibrium correction to the stress tensor $\delta P_{\mu \nu}$ is defined by the asymptotic expression

$$\delta P_{\mu \nu} = -K_\omega \delta_{\mu \nu} \partial \mu - \bar{\mu}_\omega \left( \partial_\mu u_\nu + \partial_\nu u_\mu - \frac{2}{3} \delta_{\mu \nu} \partial \mu \right).$$

(13)

Within the limits of Landau Fermi liquid theory the bulk ($K_\omega$) and the shear ($\bar{\mu}_\omega$) moduli are real and can be expressed in terms of Landau parameters [9, 10]. In general $K_\omega$ and $\bar{\mu}_\omega$ have imaginary parts which are related to internal viscosities [9].

Now we focus on two collective modes at the edges of the slab. These are the lowest antisymmetric (dipole) and symmetric (monopole) plasma oscillations. At $q \to 0$ the dipole mode corresponds to the oscillations across the slab (which is the Kohn mode in the case of a parabolic confinement), whereas the monopole mode is a periodic in $q$-direction compression and expansion of the slab (2D plasmon). At $2\mu > 1$ the both modes merge into two surface plasmons.
We derive the dispersion relations for these modes with an accuracy $\lambda^2 q^2$ for arbitrary values of the parameter $aq$, where $\lambda = \max\{\lambda_1, \lambda_2\}$. Let us define an intermediate length scale $a > l > \lambda$ and integrate Eq. (10) over the region $(-l, \infty)$. The result of the integration is

$$\omega^2 = \frac{\omega_p^2}{2} \left( 1 + e^{-2qa} \right) \left[ 1 - q(z\delta n) - \frac{q^2}{2} (z^2 \delta n) \right] - \frac{q^2}{m} \left( \frac{\delta P}{\delta n} \right) - D + O(\lambda^2 q^3), \quad (14)$$

where $\langle \ldots \rangle = \int_{-l}^{\infty} \ldots dz$ and $\delta n(z) = n(z)/\langle \delta n \rangle$. The upper(lower) sign corresponds to the dipole(monopole) mode and the function $D$ is defined as follows

$$D = 2\pi e^2 \left\{ n \theta(-z) z^2 \delta n \right\} + \left\langle \{z - z'\Delta n_0(z) \delta n(z')\} \right\rangle \mp e^{-2qa} \langle z\Delta n_0 \rangle \} \quad (15)$$

Since both $\Delta n_0$ and $\delta n$ vanish as $z \sim \lambda \lesssim l$, all integrals in Eqs. (14) and (15) are independent of the lower limit. Equation (14) is, in fact, an implicit dispersion relation.

To simplify integrals $\langle z\delta n \rangle$ and $\langle z^2 \delta n \rangle$ in Eqs. (14), (15), we multiply Eq. (10) by $z$ and $z^2$ respectively and integrate over $z$ from $-l$ to $\infty$. After calculations we arrive at the following equations

$$\begin{align*}
\langle \omega^2 - \omega_p^2 \rangle (z\delta n) &= \langle \delta n \partial_z \Delta U_{\text{ext}} \rangle \\
- 4\pi e^2 q \left( 1 + e^{-2qa} \right) \langle z\Delta n_0 \rangle + O(\lambda^2 q^2) &= 0, \quad (16) \\
\langle \omega^2/2 - \omega_p^2 \rangle (z^2 \delta n) &= -\langle \delta P / \langle \delta n \rangle \rangle \\
+ \langle z^2 \delta n \delta z \partial_z \Delta U_{\text{ext}} \rangle - D + O(\lambda^2 q) &= 0. \quad (17)
\end{align*}$$

Using Eqs. (14)-(17) and introducing the notation

$$f_{\pm} = 1 \pm e^{-2qa},$$

we obtain the dispersion relations in the final form

$$\omega_{\pm}^2 = \frac{\omega_p^2}{2} f_\pm + \frac{q}{m} f_{\mp} \langle z\delta n \partial_z \Delta U_{\text{ext}} \rangle + \frac{q^2}{2m} (A_\pm + B_\pm) \quad (18)$$

with the following coefficients in the second-order term

$$A_\pm = \left[ \frac{\sqrt{\pi} f_{\mp}}{\omega_p f_\pm} \langle z\delta n \partial_z \Delta U_{\text{ext}} \rangle \right]^2 - \langle z^2 \delta n \partial_z \Delta U_{\text{ext}} \rangle, \quad (19)$$

$$B_\pm = 2\mu_0 / n + \langle \langle \delta P \rangle / \langle \delta n \rangle \rangle + 2\pi^2 \left( 2\langle z\Delta n_0 \rangle f_\pm - \frac{q^2}{2m} \right) \cdot (20)$$

Let us analyze Eqs. (18)-(20) in more detail.

If $U_{\text{ext}}$ differs from the harmonic potential the main contribution to the dispersion comes from the first two terms in the right hand side of Eq. (18). In the limit $2qa \ll 1$ the lower mode becomes a 2D plasmon with the universal dispersion $\omega_\pm^2 = 2\pi e^2 N_a q/m$. On the contrary, the frequency of the higher (dipole) mode

$$\omega_\pm^2 = -q a \omega_p^2 + \langle z\delta n \partial_z \Delta U_{\text{ext}} \rangle / am \quad (21)$$

contains a non-universal shift which is proportional to the inverse width of the slab. At $q = 0$ Eq. (21) recovers the result of Eq. (7) for $D=1$. However a close inspection of Eqs. (14)-(17) shows that, for a dipole mode, they are consistent only for $q > \lambda/a^2$, which seemingly makes the limit $q \to 0$ inaccessible. Yet in the region $1 > qa > \lambda/a$, where Eqs. (14)-(17) are applicable, the dispersion Eq. (21) is a linear function of $q$ which is shifted by a constant as given in Eq. (7). Thus, despite Eqs. (14)-(17) are ill defined at $q \to 0$, the resulting dispersion relations, Eq. (18), are correct up to $q = 0$.

In the opposite limit $2qa \gg 1$ both modes merge into antisymmetric and symmetric combinations of surface plasmons at two slab surfaces with the same frequency $\omega_\pm = \omega_S$.

$$\omega_S^2 = \frac{\omega_p^2}{2} + q \langle \delta n \partial_z \Delta U_{\text{ext}} \rangle / m. \quad (22)$$

In a particular case of the jellium edge $\partial_z \Delta U_{\text{ext}} = -m \omega_p^2 z \theta(z)$ and we recover the result

$$\omega_S^2 = \frac{\omega_p^2}{2} - q \omega_p \int_{0}^{\infty} z \delta n dz, \quad (23)$$

obtained in Ref. 11 using the dynamical force sum rule.

The intriguing feature of Eqs. (18)-(20) is a very special role of the harmonic potential. Both the linear term in Eq. (18) and one of the second order coefficients $A_\pm$, Eq. (19), vanish if $\Delta U_{\text{ext}}(z) = 0$ [12]. This is obviously related to GKT [1]. The existence of the exact GKT mode with $\omega = \omega_p$ at $q = 0$ can be easily observed via direct substitution of the rigid-shift solution: $u_\mu(r,t) = \delta_{\mu z} u_z(t)$, and

$$\delta n(r,t) = -u_z \partial_z n_0, \quad \delta P_{\mu \nu}(r,t) = -u_z \partial_z P_{\mu \nu} \quad (24)$$

into Eqs. (3) and (11) at $q = 0$ with $U_{\text{ext}} = U_{\text{har}}$. Interestingly, the solution Eq. (24) remains asymptotically valid at finite $q$ for both modes under consideration. Substituting Eq. (24) with $u_\mu = u_z(t)e^{iqr}$ into Eqs. (3) and taking into account the Poisson equation for $\delta U$ we obtain the equation

$$\partial_t^2 u_z(t) + 2\pi e^2 N_a \langle 1 + e^{-2qa} \rangle u_z(t) + O(qz) = 0, \quad (25)$$

which shows the validity of the rigid-motion solution in the region $z \ll 1/q$ near the edges of the slab. This observation allows to simplify the remaining coefficient $B_\pm$ [Eq. (20)] in Eq. (18). Using Eq. (24) we find that the integrals of the stress tensor fluctuations (the second term in Eq. (20)) are reduced to the equilibrium pressure per particle in the internal region

$$\langle \delta P \rangle / \langle \delta n \rangle = \langle \delta P \rangle / \langle \delta n \rangle = P_0 / n, \quad (26)$$

whereas the integral $\langle z^2 \delta n \rangle$, which enters the last term in Eq. (20), is transformed to the form

$$2\pi e^2 N_a \langle z^2 \delta n \rangle = 4\pi e^2 \langle z\Delta n_0 \rangle. \quad (27)$$
The right hand side of Eq. (27) is, in fact, the potential of a double layer which is formed by the density distribution $\Delta n_0(z)$. On the other hand, the integration of the balance equation Eq. (8) leads to the following exact relation of the double layer potential to the pressure per particle inside the slab

$$4\pi e^2 \langle z \Delta n_0 \rangle = \bar{P}_0 / \bar{n}. \quad (28)$$

Substituting Eqs. (26)-(28) into Eqs. (20) and (18) we obtain explicit dispersion relations for collective modes in a parabolically confined system

$$\omega_x^2 = \frac{\omega_p^2}{2} f_\pm + \frac{2\bar{P}_0}{m \bar{n}} f_\pm + \frac{2\bar{\mu}_\omega}{m \bar{n}} q^2. \quad (29)$$

In the surface plasmon regime ($2q_0 > 1$, $\omega_x = \omega_S$) this equation simplifies further as

$$\omega_s^2 = \frac{\omega_p^2}{2} + 2q^2 \left( \bar{P}_0 + \bar{\mu}_\omega \right) / m \bar{n} = \omega_p^2 / 2 + v_S^2 q^2. \quad (30)$$

This equation relates the dispersion of plasma modes to the exact pressure $\bar{P}_0$ and shear modulus $\bar{\mu}_\omega$ of a homogeneous electron liquid at a given temperature $T$ and density $\bar{n}$. Equation (29) shows the absence of Landau damping up to $O(\lambda^2 q^2)$. The attenuation is due to the internal viscosity (the imaginary part of $\bar{\mu}_\omega$) which is related to the multi-pair excitations. From exact results Eqs. (29) or (30) we can easily obtain the dispersion in any particular approximation. For example, in Random Phase Approximation (RPA), which assumes Hartree approximation at the static level, we have $\bar{\mu}_\omega = \bar{P}_0$ [10] and, therefore, $v_S^2 = 4\bar{P}_0 / m \bar{n}$ (i.e. $\frac{4}{3} v_F^2$ at $T = 0$). This is times 4/3 of the corresponding coefficient in the RPA bulk plasmon dispersion.

As the exact pressure $\bar{P}_0(\bar{n})$ is presently available from Monte-Carlo calculations, Eq. (29) opens a direct access to the complex dynamic shear modulus $\bar{\mu}_\omega$ in the range $0 < \omega < \omega_p / \sqrt{2}$. To determine $\bar{\mu}_\omega$ one has to measure the dispersion of plasma modes in a wide parabolic quantum wells using e.g. the grating technique [13]. To access the quadratic part of the dispersion, which is governed by the shear stress, one needs thinner grating and, possibly, wider wells than those used in Ref. 13. Such measurements would allow a direct comparison to recent many-body calculations [14].

As any exact result, Eq. (29) should help to control the consistency of approximate methods such as different hydrodynamical approaches [10, 15, 16] or TDDFT schemes with approximate xc kernels $f_{xc}$ [3].

Let us consider hydrodynamics first. Since the dispersion Eq. (29) is controlled by the shear stress, any hydrodynamics with a diagonal stress tensor $\delta P_{\mu \nu} \sim \delta_{\mu \nu}$ [15, 16] does not reproduce the correct dispersion coefficient. In the adiabatic hydrodynamics [15] also the dispersion of bulk plasmon is wrong. This problem is removed in the theory of Ref. 16, which is however valid only for 1D motion, where the tensor structure of $\delta P_{\mu \nu}$ is irrelevant. The generalized hydrodynamics of $\delta P_{\mu \nu}$ gives the exact result since it reproduces the correct structure of $\delta P_{\mu \nu}$.

Similar arguments apply to different approximations for $f_{xc}$ in TDDFT. The kinetic part of $\mu$, which is equal to the pressure of Kohn-Sham particles, is reproduced correctly. The xc part is, however, $f_{xc}$-dependent. For example, the adiabatic approximation gives no additional xc contribution to the shear stress which is completely a non-adiabatic effect. The consistent result can be obtained using the approximate xc kernel of Ref. 8. The correct asymptotic form of the stress tensor Eq. (13) was, in fact, one of the requirements in the derivation of this approximation. The dispersion of surface plasmons is an example of physical situation where this asymptotic requirement leads to an observable effect.

The work of I.T. was supported by the Alexander von Humboldt Foundation and in part by the Russian Federal Program “Integration”.

* On leave from Moscow Institute of Electronic Technology, Zelenograd, 103498, Russia; Electronic address: ilya.tokatly@physik.uni-erlangen.de

[1] L. Brey, N. F. Johnson, and B. I. Halperin, Phys. Rev. B 40, 10647 (1989).
[2] J. F. Dobson, Phys. Rev. Lett B 73, 2244 (1994).
[3] K. Burke and E.K.U. Gross, in Density functionals: Theory and applications (Springer, Berlin, 1998).
[4] R. S. Sorbello, Solid State Commun. 56, 821 (1985).
[5] L. Jacak, P. Hawrylak, and A. Wójcik, Quantum Dots (Springer, Berlin, 1997); O. Astafiev, et.al, Phys. Rev. B 65, 085315 (2002); V. Gudmundsson and R. G. Harding, Phys. Rev. B 43, 12098 (1991); C. A. Ullrich and G. Vignale, Phys. Rev. B 61, 2729 (2000).
[6] C. Yannouleas and U. Landman, Phys. Rev. Lett. 82, 5325 (1999); R. Egger et al., Phys. Rev. Lett. 82, 3320 (1999); S. A. Mikhailov, Phys. Rev. B 65, 115312 (2002).
[7] The upper standing plasma modes in a similar system have been studied in O. Heinonen and W. Kohn, Phys. Rev. B 48, 12240 (1993).
[8] G. Vignale, C. A. Ullrich and S. Conti, Phys. Rev. Lett. 79, 4878 (1997).
[9] S. Conti and G. Vignale, Phys. Rev. B 60, 7966 (1999).
[10] I. V. Tokatly and O. Pankratov, Phys. Rev. B 62, 2759 (2000).
[11] A. Liebsch, Phys. Rev. B 36, 7378 (1987).
[12] The absence of the linear term in the surface plasmon regime was noted in W. L. Schaich, Surface Science, 318, L1157 (1994).
[13] R. F. Pinsukanjana, et. al., Phys. Rev. B 46, 7284 (1992).
[14] R. Nifosi, S. Conti, and M. P. Tosi, Phys. Rev. B 58, 12758 (1998).
[15] F. Bloch, Z. Phys. 81, 363 (1933); E. Zaremba and H. C. Taylor, Phys. Rev. B 49, 8147 (1994).
[16] J. F. Dobson and H. M. Le J. Molecular Structure (Theochem) 501-502 327 (2000); cond-mat/0201267.