\documentclass[12pt]{article}
\usepackage{amsmath}
\begin{document}
\title{U(3)_{C} \times Sp(1)_{L} \times U(1)_{L} \times U(1)_{R}}
\author{Luis Alfredo Anchordoqui}
\date{(Dated: August 2011)}
\maketitle
\section*{Abstract}
We outline the basic setting of the $U(3)_{C} \times Sp(1)_{L} \times U(1)_{L} \times U(1)_{R}$ gauge theory and review the associated phenomenological aspects related to experimental searches for new physics at hadron colliders.
\end{document}
I. GENERAL IDEA

The Standard Model (SM) is a spontaneously broken Yang-Mills theory with gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Matter in the form of quarks and leptons (i.e.
$SU(3)_C$ triplets and singlets, respectively) is arranged in three families ($i = 1, 2, 3$) of left-handed fermion doublets (of $SU(2)_L$) and right-handed fermion singlets. Each family $i$ contains chiral gauge representations of left-handed quarks $Q_i = (3, 2)_{1/6}$ and leptons $L_i = (1, 2)_{-1/2}$ as well as right-handed up and down quarks, $U_i = (3, 1)_{2/3}$ and $D_i = (3, 1)_{-1/3}$, respectively, and the right-handed lepton $E_i = (1, 1)_{-1}$. The hypercharge $Y$ is shown as a subscript of the $SU(3)_C \times SU(2)_L$ gauge representation $(A, B)$. The neutrino is part of the left-handed lepton representation $L_i$ and does not have a right-handed counterpart.

The SM Lagrangian exhibits an accidental global symmetry $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$, where $U(1)_B$ is the baryon number symmetry, and $U(1)_\alpha \ (\alpha = e, \mu, \tau)$ are three lepton flavor symmetries, with total lepton number given by $L = L_e + L_\mu + L_\tau$. It is an accidental symmetry because we do not impose it. It is a consequence of the gauge symmetries and the low energy particle content. It is possible (but not necessary), however, that effective interaction operators induced by the high energy content of the underlying theory may violate sectors of the global symmetry.

The electroweak subgroup $SU_L(2) \times U_Y(1)$ is spontaneously broken to the electromagnetic $U(1)_\text{EM}$ by the Higgs doublet $H = (1, 2)_{1/2}$ which receives a vacuum expectation value $v \neq 0$ in a suitable potential. Three of the four components of the complex Higgs are ‘eaten’ by the $W^\pm$ and $Z$ bosons, which are superpositions of the gauge bosons $W^a_\mu$ of $SU(2)_L$ and $B_\mu$ of $U(1)_Y$,

$$W^\pm_\mu = \frac{1}{\sqrt{2}} W^1_\mu + \frac{i}{\sqrt{2}} W^2_\mu$$

and

$$Z_\mu = \cos \theta_W W^3_\mu - \sin \theta_W B_\mu ,$$

with masses $M^2_W = \pi \alpha v^2 / \sin^2 \theta_W$, $M^2_Z = M^2_W / \cos^2 \theta_W$, and $\alpha \approx 1/128$ at $Q^2 = M^2_W$. The fourth vector field,

$$A_\mu = \sin \theta_W W^3_\mu + \cos \theta_W B_\mu ,$$

persists massless and the remaining Higgs component is left as a $U(1)_\text{EM}$ neutral real scalar. The measured values $M_W \approx 80.4$ GeV and $M_Z \approx 91.2$ GeV fix the weak mixing angle at $\sin^2 \theta_W \approx 0.23$ and the Higgs vacuum expectation value at $\langle H \rangle = v \approx 246$ GeV $[1]$.

Fermion masses arise from Yukawa interactions, which couple the right-handed fermion singlets to the left-handed fermion doublets and the Higgs field,

$$\mathcal{L} = -Y_{d}^{ij} \bar{Q}_{i} H D_{j} - Y_{u}^{ij} \epsilon^{ab} \bar{Q}_{ia} H_{b}^{\dagger} U_{j} - Y_{\ell}^{ij} \bar{L}_{i} H E_{j} + \text{h.c.},$$

where $\epsilon^{ab}$ is the antisymmetric tensor. In the process of spontaneous symmetry breaking these interactions lead to charged fermion masses, $m_{j}^{ij} = Y_{j}^{ij} v / \sqrt{2}$, but leave the neutrinos massless $[2]$. Experimental evidence for neutrino flavor oscillations by the mixing of different mass eigenstates implies that the SM has to be extended $[3]$. The most economic way to get

---

1 One might think that neutrino masses could arise from loop corrections. This, however, cannot be the case, because the only possible neutrino mass term that can be constructed with the SM fields is the bilinear $\bar{L}_i L'^C_j$ which violates the total lepton symmetry by two units ($L'^C_i = C \bar{L}_i^C$). As mentioned above
massive neutrinos would be to introduce the right-handed neutrino states (having no gauge interactions, these sterile states would be essentially undetectable) and obtain a Dirac mass term through a Yukawa coupling.

The SM gauge interactions have been tested with unprecedented accuracy, including some observables beyond even one part in a million [1]. Nevertheless, the saga of the SM is still exhilarating because it leaves all questions of consequence unanswered. The most evident of unanswered questions is why there is a huge disparity between the strength of gravity and of the SM forces. This hierarchy problem suggests that new physics could be at play at the TeV-scale, and is arguably the driving force behind high energy physics for several decades. Much of the motivation for anticipating the existence of such new physics is based on considerations of naturalness. The non-zero vacuum expectation value of the scalar Higgs doublet condensate sets the scale of electroweak interactions. However, due to the quadratic sensitivity of the Higgs mass to quantum corrections from an arbitrarily high mass scale Λ, with no new physics between the energy scale of electroweak unification and the vicinity of the Planck mass, the bare Higgs mass and quantum corrections have to cancel at a level of one part in ∼ 10^{30}. This fine-tuned cancellation seems unnatural, even though it is in principle self-consistent. Thus either the scale of new physics Λ is much smaller than the Planck scale or there exists a mechanism which ensures this cancellation, perhaps arising from a new symmetry principle beyond the SM – minimal supersymmetry (SUSY) is a textbook example [4]. In either case, an extension of the SM appears necessary.

In this talk I will discuss the phenomenology of a newfangled extension of the gauge sector, U(3)_{C} \times Sp(1)_{L} \times U(1)_{L} \times U(1)_{R}, which has the attractive property of elevating the two major global symmetries of the SM (B and L) to local gauge symmetries [5]. The U(1)_{Y} boson Y_{\mu}, which gauges the usual electroweak hypercharge symmetry, is a linear combination of the U(1) of U(3)_{C} gauge boson C_{\mu}, the U(1)_{R} boson B_{\mu}, and a third additional U(1)_{L} field \tilde{B}_{\mu}. The Q_{3}, Q_{1L}, Q_{1R} content of the hypercharge operator is given by,

\[ Q_{Y} = c_{1}Q_{1R} + c_{3}Q_{3} + c_{4}Q_{1L}, \]

with \( c_{1} = 1/2, c_{3} = 1/6, \) and \( c_{4} = -1/2 \) [6]. The corresponding fermion and Higgs doublet quantum numbers are given in Table II. The criteria we adopt here to define the Higgs charges is to make the Yukawa couplings (H^{\dagger}D_{1i}Q_{i}, H^{\dagger}E_{iL}Q_{i}, H^{\dagger}H_{L}D_{1i}, H^{\dagger}N_{Li}L_{i}) invariant under all three U(1)’s. From Table II, \( \bar{U}_{i}Q_{i} \) has the charges (0, 0, -1) and \( \bar{D}_{i}Q_{i} \) has (0, 0, 1); therefore, the Higgs \( H \) has \( Q_{3} = Q_{1L} = 0, Q_{1R} = 1, Q_{Y} = 1/2, \) whereas \( H^{\dagger} \) has opposite charges \( Q_{3} = Q_{1L} = 0, Q_{1R} = -1, Q_{Y} = -1/2. \) The two extra U(1)’s are the baryon and lepton number; they are given by the following combinations:

\[ B = Q_{3}/3 \quad ; \quad L = Q_{1L} \quad ; \quad Q_{Y} = \frac{1}{6}Q_{3} - \frac{1}{2}Q_{1L} + \frac{1}{2}Q_{1R}; \]

total lepton number is a global symmetry of the model and therefore L-violating terms cannot be induced by loop corrections. Furthermore, the U(1)_{B-L} subgroup is non-anomalous, and therefore B – L violating terms cannot be induced even by nonperturbative corrections. It follows that the SM predicts that neutrinos are strictly massless.

2 The fundamental principles of the model are summarized in [6]. Herein though we replace at full length the U(2)_{L} doublets by Sp(1)_{L} doublets. Besides the fact that this reduces the number of extra U(1)’s, one avoids the presence of a problematic Peccei-Quinn symmetry [7], associated in general with the U(1) of U(2)_{L} under which Higgs doublets are charged [8]. A point worth noting at this juncture: the compact symplectic group Sp(1) is equivalent to SU(2); our choice of notation will become clear in Sec. VII.
TABLE I: Quantum numbers of chiral fermions and Higgs doublet.

| Name | Representation | $Q_3$ | $Q_{1L}$ | $Q_{1R}$ | $Q_Y$ |
|------|----------------|-------|---------|---------|-------|
| $Q_i$ | $(3, 2)$       | 1     | 0       | 0       | $\frac{1}{6}$ |
| $\bar{U}_i$ | $(3, 1)$ | $-1$ | 0       | $-1$   | $-\frac{2}{3}$ |
| $D_i$  | $(3, 1)$       | $-1$ | 0       | 1       | $\frac{1}{3}$ |
| $L_i$  | $(1, 2)$       | 0    | 1       | 0       | $-\frac{1}{2}$ |
| $E_i$  | $(1, 1)$       | 0    | $-1$    | 1       | 1     |
| $\bar{N}_i$ | $(1, 1)$ | 0    | $-1$    | $-1$   | 0     |
| $H$    | $(1, 2)$       | 0    | 0       | 1       | $\frac{1}{2}$ |

or equivalently by the inverse relations

$$Q_3 = 3B \quad ; \quad Q_{1L} = L \quad ; \quad Q_{1R} = 2Q_Y - (B - L).$$

Even though $B$ is anomalous, with the addition of three fermion singlets $N_i$ the combination $B-L$ is anomaly free. One can verify by inspection of Table I that these $N_i$ have the quantum numbers of right handed neutrinos, i.e. singlets under hypercharge. Therefore, this is a first interesting prediction of the $U(3)_C \times Sp(1)_L \times U(1)_L \times U(1)_R$ gauge theory: right-handed neutrinos must exist.

Before discussing the favorable phenomenological implications of the model, we detail some desirable properties which apply to generic models with multiple $U(1)$ symmetries.

II. RUNNING OF THE ABELIAN GAUGE COUPLINGS

We begin with the covariant derivative for the $U(1)$ fields in the ‘flavor’ 1, 2, 3, ... basis in which it is assumed that the kinetic energy terms containing $X^i_\mu$ are canonically normalized

$$\mathcal{D}_\mu = \partial_\mu - i \sum_i g'_i Q_i X^i_\mu. \quad (8)$$

The relations between the $U(1)$ couplings $g'_i$ and any non-abelian counterparts are left open for now. We carry out an orthogonal transformation of the fields $X^i_\mu = \sum_j O_{ij} Y^j_\mu$. The covariant derivative becomes

$$\mathcal{D}_\mu = \partial_\mu - i \sum_i \sum_j g'_i Q_i O_{ij} Y^j_\mu$$

$$= \partial_\mu - i \sum_j \bar{g}_j \bar{Q}_j Y^j_\mu, \quad (9)$$

where for each $j$

$$\bar{g}_j \bar{Q}_j = \sum_i g'_i Q_i O_{ij}. \quad (10)$$

Next, suppose we are provided with normalization for the hypercharge (taken as $j = 1$)

$$Q_Y = \sum_i c_i Q_i; \quad (11)$$
hereafter we omit the bars for simplicity. Rewriting (10) for the hypercharge

\[ g_Y Q_Y = \sum_i g'_i Q_i O_{i1} \]  

(12)

and substituting (11) into (12) we obtain

\[ g_Y \sum_i Q_i c_i = \sum_i g'_i O_{i1} Q_i. \]

(13)

One can think about the charges \( Q_{i,p} \) as vectors with the components labeled by particles \( p \). Let us first take the charges to be orthogonal, i.e. \( \sum_p Q_{i,p} Q_{k,p} = 0 \) for \( i \neq k \). Multiplying (13) by \( \sum_p Q_{k,p} \),

\[ \sum_p Q_{k,p} g_Y \sum_i Q_{i,p} c_i = \sum_p Q_{k,p} \sum_i g'_i O_{i1} Q_{i,p}, \]

(14)

we obtain

\[ g_Y c_i = g'_i O_{i1}, \]

(15)

or equivalently

\[ O_{i1} = g_Y c_i g'_i. \]

(16)

Orthogonality of the rotation matrix, \( \sum_i O_{i1}^2 = 1 \), implies

\[ g_Y^2 \sum_i \left( \frac{c_i}{g'_i} \right)^2 = 1. \]

(17)

Then, the condition

\[ P \equiv \frac{1}{g_Y^2} - \sum_i \left( \frac{c_i}{g'_i} \right)^2 = 0 \]

(18)

encodes the orthogonality of the mixing matrix connecting the fields coupled to the flavor charges \( Q_1, Q_2, Q_3, \ldots \) and the fields rotated, so that one of them, \( Y \), couples to the hypercharge \( Q_Y \). Therefore, for orthogonal charges, as the couplings run with energy, the condition \( P = 0 \) needs to stay intact [5].

A very important point is that the couplings that are running are those of the \( U(1) \) fields; hence the \( \beta \) functions receive contributions from fermions and scalars, but not from gauge bosons. As a consequence, if we start with a set of couplings at a high mass scale \( \Lambda \) satisfying \( P = 0 \), this condition will be maintained at one loop as the couplings run down to lower energies (\( Q \)). The one loop correction to the various couplings are

\[ \frac{1}{\alpha_Y(Q)} = \frac{1}{\alpha_Y(\Lambda)} - \frac{b_Y}{2\pi} \ln(Q/\Lambda), \]

(19)

\[ \frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(\Lambda)} - \frac{b_i}{2\pi} \ln(Q/\Lambda), \]

(20)

where

\[ b_Y = \frac{2}{3} \Tr Q_{Y,f}^2 + \frac{1}{3} \Tr Q_{Y,s}^2, \]

(21)
and
\begin{equation}
    b_i = \frac{2}{3} \text{Tr} Q^2_{i,f} + \frac{1}{3} \text{Tr} Q^2_{i,s},
\end{equation}
with \( f \) and \( s \) indicating contribution from fermion and scalar loops, respectively.

Recall that the charges are orthogonal, \( \sum_s Q_{i,s} Q_{k,s} = \sum_f Q_{i,f} Q_{k,f} = 0 \) for \( i \neq k \). Then Eq.(11) implies
\begin{equation}
    \sum_s Q_{Y,s}^2 = \sum_i c_i^2 \sum_s Q_{i,s}^2 \quad \text{and} \quad \sum_f Q_{Y,f}^2 = \sum_i c_i^2 \sum_f Q_{i,f}^2,
\end{equation}

hence
\begin{equation}
    b_Y = \sum_i c_i^2 b_i.
\end{equation}

On the other, the RG-induced change of \( P \) defined in Eq.(18) reads
\begin{equation}
    \Delta P = \Delta \left( \frac{1}{\alpha_Y} \right) - \sum_i c_i^2 \Delta \left( \frac{1}{\alpha_i} \right) = \frac{1}{2\pi} \left( b_Y - \sum_i c_i^2 b_i \right) \ln(Q/\Lambda).
\end{equation}

Thus, \( P = 0 \) stays valid to one loop if the charges are orthogonal [5].

Should the charges not be orthogonal, it is instructive to write Eq. (13) as \( V \cdot Q = 0 \), where
\begin{equation}
    V_i = O_{i1} - \frac{g_Y c_i}{g_i'}.
\end{equation}

Certainly \( V_i = 0 \) still holds as a possible solution. But as the charges do not form a mutually orthogonal basis, one can ask whether other solutions exist. This will be the case if, for non-zero \( V \),
\begin{equation}
    \sum_i V_i Q_i^\alpha = 0
\end{equation}

for each \( \alpha \), where \( Q_i^\alpha \) is the \( U(1) \) charge of the particle \( \alpha \). In the \( U(3) \times U(2) \times U(1) \) gauge group of [8], the right-handed electron is charged only with respect to one of the abelian groups. From (27), this sets one of the \( V \)'s (say \( V_1 \)) equal to zero. For \( \alpha = Q_i, U_i, D_i, L_i, E_i, N_i, H \), there remain at least 4 additional equations satisfied by the remaining components \( V_2 \) and \( V_3 \). The resulting overcompleteness leads to \( V_2 = V_3 = 0 \).

Although in most models the condition \( P = 0 \) holds in spite of the non-orthogonality of the \( Q_i \)'s, the RG equations controlling the running of the couplings lose their simplicity. In particular, since
\begin{equation}
    \text{Tr} Q_Y^2 \neq \sum_i c_i^2 \text{Tr} Q_i^2,
\end{equation}

the RG equations become coupled. In addition, kinetic mixing is generated at one loop level even if it is absent initially [10]. Removal of the mixing term in order to restore canonical gauge kinetic energy requires an additional \( O(3) \) rotation, greatly complicating the analysis.

Here, we are considering models where the underlying symmetry at high energies is \( U(N) \) rather than \( SU(N) \). Following [8] we normalize all \( U(N) \) generators according to
\begin{equation}
    \text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab},
\end{equation}

6
and measure the corresponding $U(1)_N$ charges with respect to the coupling $g_N/\sqrt{2N}$, with $g_N$ the $SU(N)$ coupling constant. Hence, the fundamental representation of $SU(N)$ has $U(1)_N$ charge unity. Another important element of the RG analysis is that the $U(1)$ couplings $(g'_1, g'_2, g'_3)$ run different from the non-abelian $SU(3)$ ($g_3$) and $SU(2)$ ($g_2$). This implies that the previous relation for normalization of abelian and non-abelian coupling constants, $g'_N = g_N/\sqrt{2N}$, holds only at the scale of $U(N)$ unification [5]. The SM chiral fermion charges in Table I are not orthogonal as given ($\text{Tr} Q_{1L} Q_{1R} \neq 0$). Orthogonality can be completed by including a right-handed neutrino.

An obvious question is whether each of the fields on the rotated basis couples to a single charge $\bar{Q}_i$. Let

$$\mathcal{L} = X^T G Q,$$  (30)

be the Lagrangian in the 1, 2, 3, ... basis, with $X^i$ and $Q_i$ vectors and $G$ a diagonal matrix in $N$-dimensional ‘flavor’ space. Now rotate to new orthogonal basis ($\bar{Q}$) for $Q$

$$Q = R\bar{Q};$$  (31)

(30) becomes

$$\mathcal{L} = X^T G R \bar{Q}.$$  (32)

As it stands, each $X^i$ does not couple to a unique charge $\bar{Q}_i$; hence we rotate $X$,

$$X = O\bar{Y},$$  (33)

to obtain

$$\mathcal{L} = \bar{Y}^T O^T G R \bar{Q}.$$  (34)

We wish to see if, for given $O$ and $G$, we can find an $R$ so that

$$O^T G R = \bar{G} \text{ (diagonal)}$$  (35)

This allows each $\bar{Y}^i$ to couple to a unique charge $\bar{Q}_i$ with strength $\bar{g}_i$. To see the problem with this, we rewrite (35) in terms of components

$$(O^T)_{ij} g_j R_{jk} = \bar{g}_i \delta_{ik};$$  (36)

for $i \neq k$, (36) leads to

$$(O^T)_{ij} g_j R_{jk} = 0.$$  (37)

In general, in Eq. (37) there are $N(N-1)$ equations, but only $N(N-1)/2$ independent $O_{ij}$ generators in $SO(N)$; therefore the system is over-determined [11]. Of course, if $G = g I$, the equation becomes

$$O^T R = I,$$  (38)

and so $O = R$.

We illustrate with the case $N = 2$; let

$$R = \begin{pmatrix} C_\varphi & S_\varphi \\ -S_\varphi & C_\varphi \end{pmatrix},$$

$$G = \begin{pmatrix} g'_1 & 0 \\ 0 & g'_3 \end{pmatrix},$$

$$O = \begin{pmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{pmatrix}$$

(39)
then

$$\Omega \mathcal{G} \mathcal{R} = \begin{pmatrix} g_1' C_\theta C_\varphi + g_3' S_\theta S_\varphi & g_1' C_\theta S_\varphi - g_3' S_\theta C_\varphi \\ g_1' S_\delta C_\varphi + g_3' C_\theta S_\varphi & g_1' S_\theta S_\varphi - g_3' C_\theta C_\varphi \end{pmatrix} = \begin{pmatrix} g_1' & 0 \\ 0 & g_3' \end{pmatrix}. \quad (40)$$

From the off diagonal terms we obtain

$$g_1' C_\theta S_\varphi - g_3' S_\theta C_\varphi = 0 \Rightarrow \tan \vartheta = \frac{g_1'}{g_3'} \tan \varphi$$

$$g_1' S_\theta C_\varphi - g_3' C_\theta S_\varphi = 0 \Rightarrow \tan \vartheta = \frac{g_3'}{g_1'} \tan \varphi$$

which implies that $g_1' = g_3' = g$, or equivalently that $\mathcal{G}$ is a multiple of the unit matrix. Next, we consider the diagonal elements using $g_1' = g_2'$ to obtain

$$\cos(\vartheta - \varphi) = 0 \Rightarrow \vartheta = \varphi \quad (41)$$

Note that the matrix $\mathcal{R}$ has one independent variable, and there are two independent homogeneous equations.

Any vector boson $Y'_\mu$, orthogonal to the hypercharge, must grow a mass $M'$ in order to avoid long range forces between baryons other than gravity and Coulomb forces. The anomalous mass growth allows the survival of global baryon number conservation, preventing fast proton decay \cite{12}. It is this that we now turn to study.

III. PREMISES OF THE ANOMALOUS SECTOR

Outside of the Higgs couplings, the relevant parts of the Lagrangian are the gauge couplings generated by the $U(1)$ covariant derivatives acting on the matter fields, and the (mass)$^2$ matrix of the anomalous sector

$$\mathcal{L} = Q^T \mathcal{G} X + \frac{1}{2} X^T \mathcal{M}^2 X, \quad (42)$$

where $X_\mu^i$ are the three $U(1)$ gauge fields in the D-brane basis $(B_\mu, C_\mu, B_\mu)$, $\mathcal{G}$ is a diagonal coupling matrix $(g_1', g_2', g_3')$, and $Q$ are the 3 charge matrices.

Again, perform a rotation $X = \mathcal{O} Y$ and require that one of the $Y$’s (say $Y'_\mu$) couple to hypercharge. We then obtain the constraint on the first column of $\mathcal{O}$ given in \cite{16}. However, there is now an additional constraint: the field $Y'_\mu$ is an eigenstate of $\mathcal{M}^2$ with zero eigenvalue. Under the $\mathcal{O}$ rotation, the mass term becomes

$$\frac{1}{2} X^T \mathcal{M}^2 X = \frac{1}{2} Y^T \mathcal{M}^2 Y, \quad (43)$$

with $\mathcal{M}^2 = \mathcal{O}^T \mathcal{M}^2 \mathcal{O}$. We know that at least $Y'_\mu$ is an eigenstate with eigenvalue zero. We also know that Poincare invariance requires the complete diagonalization of the mass matrix in order to deal with observables. However, further similarity transformations will undo the coupling of the zero eigenstate to hypercharge. There seems no way of eventually fulfilling all these conditions except to require that the same $\mathcal{O}$ which rotates to couple $Y'_\mu$ to hypercharge simultaneously diagonalizes $\mathcal{M}^2$ so that

$$\mathcal{M}^2 = \text{diag}(0, M'^2, M''^2). \quad (44)$$
This implies that the original $M^2$ in the flavor basis is given by

$$M^2 = \mathcal{O} \text{ diag}(0, M^2, M'^2) \mathcal{O}^T,$$

which results in the following baroque matrix:

$$M^2 = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix},$$

where

\begin{align*}
    a &= M'^2(C_\psi S_\theta S_\phi - C_\phi S_\psi)^2 + M''^2(C_\phi C_\psi S_\theta + S_\phi S_\psi)^2, \\
    b &= (M^2 - M'^2)C_\phi C_2 S_\theta S_\phi + C_\phi^2 C_\psi (-M'^2 + M'^2 S_\theta^2) S_\psi + C_\psi (-M''^2 + M''^2 S_\theta^2) S_\phi S_\psi, \\
    c &= C_\theta [M''^2 C_\phi^2 C_\psi S_\theta + M'^2 C_\psi S_\theta S_\phi^2 - (M^2 - M''^2) C_\phi S_\theta S_\psi], \\
    d &= M''^2 (C_\psi S_\theta - C_\phi S_\phi S_\psi)^2 + M'^2 (C_\phi C_\psi + S_\phi S_\phi S_\psi)^2, \\
    e &= C_\theta [(M^2 - M'^2) C_\phi C_\psi S_\theta S_\phi + M'^2 C_\phi^2 S_\theta S_\psi + M''^2 S_\phi S_\psi], \\
    f &= C_\theta^2 (M''^2 C_\phi^2 + M'^2 S_\phi^2). \quad (47)
\end{align*}

We turn now to discuss the phenomenological aspects of anomalous $U(1)$ gauge bosons related to experimental searches for new physics at the Tevatron and at the CERN’s Large Hadron Collider (LHC).

IV. SEARCH FOR NEW GAUGE BOSONS AT HADRON COLLIDERS

Taken at face value, the disparity between CDF \cite{13, 14} and DØ \cite{15} results insinuates a commodious uncertainty as to whether there is an excess of events in the dijet system invariant mass distribution of the associated production of a $W$ boson with 2 jets (hereafter $Wjj$ production). The $M_{jj}$ excess showed up in 4.3 fb$^{-1}$ of integrated luminosity collected with the CDF detector as a broad bump between about 120 and 160 GeV \cite{13}. The CDF Collaboration fitted the excess (hundreds of events in the dijet system) to a Gaussian and estimated its production cross section times the dijet branching ratio to be 4 pb. This is roughly 300 times the SM Higgs rate \( \sigma(pp \rightarrow WH) \times \text{BR}(H \rightarrow b\bar{b}) \). For a search window of 120 – 200 GeV, the excess significance above SM background (including systematics uncertainties) has been reported to be 3.2$\sigma$ \cite{13}. Recently, CDF has included an additional 3 fb$^{-1}$ to their data sample, for a total of 7.3 fb$^{-1}$, and the statistical significance has grown to $\sim 4.8\sigma$ ($\sim 4.1\sigma$ including systematics) \cite{14}. More recently, the DØ Collaboration released an analysis (which closely follows the CDF analysis) of their $Wjj$ data finding “no evidence for anomalous resonant dijet production” \cite{15}. Using an integrated luminosity of 4.3 fb$^{-1}$ they set a 95% CL upper limit of 1.9 pb on a resonant $Wjj$ production cross section.

Although various explanations have been proposed for the CDF anomaly \cite{16}, perhaps the simplest is the introduction of a new leptophobic $Z'$ gauge boson \cite{17}. The suppressed coupling to leptons (or more specifically, to electrons and muons) is required to evade the strong constraints of the Tevatron $Z'$ searches in the dilepton mode \cite{18} and LEP-II measurements of $e^+e^- \rightarrow e^+e^-$ above the $Z$-pole \cite{19}. In complying with the precision demanded of our phenomenological approach it would be sufficient to consider a 1% branching fraction...
to leptons as consistent with the experimental bound. This approximation is within a factor of a few of model independent published experimental bounds. In addition, the mixing of the Z' with the SM Z boson should be extremely small to be compatible with precision measurements at the Z-pole by the LEP experiments \[20\].

All existing dijet-mass searches via direct production at the Tevatron are limited to \(M_{jj} > 200\) GeV \[21\] and therefore cannot constrain the existence of a Z' with \(M_{Z'} \approx 150\) GeV. The strongest constraint on a light leptophobic Z' comes from the dijet search by the UA2 Collaboration, which has placed a 90% CL upper bound on \(\sigma(pp \to Z') \times \text{BR}(Z' \to jj)\) in this energy range \[22\]. A comprehensive model independent analysis incorporating Tevatron and UA2 data to constrain the Z' parameters for predictive purposes at the LHC was recently presented \[23\].\(^3\) As of today the ATLAS and CMS experiments are not sensitive to the Wjj signal \[26\]. However, LHC will eventually weigh in on this issue: if new physics is responsible for the CDF anomaly, an excess in \(\ell jj + E_T\) should become statistically significant in ATLAS and CMS by the end of the year \[26\].

As usual, the \(U(1)\) gauge interactions arise through the covariant derivative

\[
\mathcal{D}_\mu = \partial_\mu - i g_3' C_\mu Q_3 - i g_4' B_\mu Q_{1L} - i g_4 B_\mu Q_{1R},
\]

where \(g_3',\ g_3,\) and \(g_4'\) are the gauge coupling constants. The fields \(Q_3, B_\mu,\) and \(Y'_\mu\) are related to \(Y_\mu, Y'_\mu,\) and \(Y''_\mu\) by the rotation matrix,

\[
\Theta = \begin{pmatrix}
C_\theta C_\psi & -C_\theta S_\psi + S_\theta C_\psi & S_\theta S_\psi + C_\theta S_\psi \\
C_\theta S_\psi & C_\theta C_\psi + S_\theta S_\psi & -S_\theta C_\psi + C_\theta S_\psi \\
-S_\theta & S_\theta & C_\theta
\end{pmatrix},
\]

with Euler angles \(\theta,\ \psi,\) and \(\phi.\) Equation (48) can be rewritten in terms of \(Y_\mu, Y'_\mu,\) and \(Y''_\mu\) as follows

\[
\mathcal{D}_\mu = \partial_\mu - i Y_\mu (-S_\theta g_3' Q_{1R} + S_\theta S_\psi g_4' Q_{1L} + C_\theta C_\psi g_3 Q_3) \\
- i Y'_\mu [C_\theta S_\phi g_1' Q_{1R} + (C_\phi C_\psi + S_\phi S_\psi) g_4' Q_{1L} + (C_\phi S_\theta - S_\phi S_\psi) g_3 Q_3] \\
- i Y''_\mu [C_\theta C_\phi g_1' Q_{1R} + (-C_\psi S_\phi + C_\phi S_\psi) g_4' Q_{1L} + (C_\phi C_\psi S_\theta + S_\phi S_\psi) g_3 Q_3].
\]

Now, by demanding that \(Y_\mu\) has the hypercharge \(Q_Y\) given in Eq. (5) we fix the first column of the rotation matrix \(\Theta\)

\[
\begin{pmatrix}
C_\mu \\
B_\mu \\
Y_\mu
\end{pmatrix} = \begin{pmatrix}
Y_\mu c_3 g_Y / g_3' \\
Y_\mu c_4 g_Y / g_4' \\
Y_\mu c_1 g_Y / g_1'
\end{pmatrix},
\]

and we determine the value of the two associated Euler angles

\[
\theta = -\arcsin[c_1 g_Y / g_1']
\]

and

\[
\psi = \arcsin[c_4 g_Y / (g_4' C_\phi)].
\]

The couplings \(g_3'\) and \(g_4'\) are related through the orthogonality condition (18),

\[
\left( \frac{c_4}{g_4'} \right)^2 = \frac{1}{g_Y^2} - \left( \frac{c_3}{g_3} \right)^2 - \left( \frac{c_1}{g_1'} \right)^2,
\]

\(^3\) Other phenomenological restrictions on Z'-gauge bosons were recently discussed in \[24\].
with $g_3'$ fixed by the relation for $U(N)$ unification: $g_3' = \sqrt{3} g_3$. In what follows, we take 5 TeV as a reference point for running down to 150 GeV the $g_3'$ coupling using (20), that is ignoring mass threshold effects. This yields $g_3' = 0.383$. We have checked that the running of the $g_3'$ coupling does not change significantly by varying the scale of $U(N)$ unification between 3 TeV and 10 TeV.

The phenomenological analysis thus far has been formulated in terms of the mass-diagonal basis set of gauge fields $(Y,Y',Y'')$. As a result of the electroweak phase transition, the coupling of this set with the Higgses will induce mixing, resulting in a new mass-diagonal basis set $(Z,Z',Z'')$. It will suffice to analyze only the $2 \times 2$ system $(Y,Y')$ to see that the effects of this mixing are totally negligible. We consider simplified zeroth and first order (mass)$^2$ matrices

$$(M^2)^{(0)} = \begin{pmatrix} 0 & 0 \\ 0 & M'^2 \end{pmatrix}, \quad (M^2)^{(1)} = \begin{pmatrix} M_Z^2 & \epsilon \\ \epsilon & m^2 \end{pmatrix},$$

where $M'$ is the mass of the $Y'$ gauge field, $M_Z = \sqrt{g_2^2 + g_3'^2}$, $v/2$ is the usual tree level formula for the mass of the $Z$ particle in the electroweak theory (before mixing), $g_2 \simeq 0.651$ is the electroweak coupling constant, $v$ is the vacuum expectation value of the Higgs field, $g_Y \simeq 0.357$, and $\epsilon, m^2$ are of $O(M_Z^2)$.

Standard Rayleigh-Schrödinger perturbation theory then provides the (mass)$^2$ to second order in $M_Z^2$ and wave functions to first order of the mass-diagonal eigenfields $(Z,Z')$ corresponding to $(Y,Y')$

$$M_Z^2 = \bar{M}_Z^2 - \left( \frac{\epsilon}{M'^2} \right), \quad M_{Z'}^2 = M'^2 + m^2 + \left( \frac{\epsilon}{M'^2} \right),$$

and

$$Z = Y - \left( \frac{\epsilon}{M'^2} \right) Y', \quad Z' = Y' + \left( \frac{\epsilon}{M'^2} \right) Y.$$  

(55)

(56)

From Eqs. (55) and (56) the shift in the mass of the $Z$ is given by $\delta M_Z^2 = (\epsilon/M')^2$, so that $\epsilon = M'\sqrt{2}\bar{M}_Z\delta M_Z$. The admixture of $Y$ in the mass-diagonal field $Z'$ is

$$\theta = \frac{\epsilon}{M'^2} = \frac{M_Z}{M'} \sqrt{\frac{2\delta M_Z}{M_{Z}}} \simeq 0.004.$$  

(57)

Since all effects go as $\theta^2 \simeq 1.6 \times 10^{-5}$, all further discussion will be, with negligible error, in terms of $Z'$. By the same token, the admixture of $Y'$ in the eigenfield $Z$ is negligible, so that the discussion henceforth will reflect $Z \simeq Y$ and $\bar{M}_Z^2 \simeq M_{Z'}^2$.

The $f \bar{f} Z'$ Lagrangian is of the form

$$\mathcal{L} = \frac{1}{2} \sqrt{g_Y^2 + g_3'^2} \sum_f \left( \epsilon_{fL} \bar{\psi}_{fL} \gamma^\mu \psi_{fL} + \epsilon_{fR} \bar{\psi}_{fR} \gamma^\mu \psi_{fR} \right) Z'_{\mu},$$

$$= \sum_f \left( (g_{Y'}Q_{Y'})_{fL} \bar{\psi}_{fL} \gamma^\mu \psi_{fL} + (g_{Y'}Q_{Y'})_{fR} \bar{\psi}_{fR} \gamma^\mu \psi_{fR} \right) Z'_{\mu}.$$  

(58)

where each $\psi_{fL(R)}$ is a fermion field with the corresponding $\gamma^\mu$ matrices of the Dirac algebra, and $\epsilon_{fL,R} = v_q \pm a_q$, with $v_q$ and $a_q$ the vector and axial couplings respectively. The
(pre-cut) $Wjj$ production rate at the Tevatron $\sqrt{s} = 1.96$ pb, for arbitrary couplings and $M_{Z'} \approx 150$ GeV, is found to be [23]

$$\sigma(p\bar{p} \rightarrow Z'') \times BR(Z' \rightarrow jj) \simeq \frac{1}{2} \left[ 773 (\epsilon_{u_L}^2 + \epsilon_{d_L}^2) + 138 (\epsilon_{u_R}^2 + \epsilon_{d_R}^2) \right] \times \Gamma(\phi, g'_1)_{Z' \rightarrow q\bar{q}} \text{ pb,}$$

(59)

where $\Gamma(\phi, g'_1)_{Z' \rightarrow q\bar{q}}$ is the hadronic branching fraction. The presence of a $W$ in the process shown in Fig. 1 restricts the contribution of the quarks to be purely left-handed. Since $\epsilon_{u_L} = \epsilon_{d_L}$ and the required branching to quarks is above about 99% (after selection cuts are accounted for) the coupling strength $\epsilon_{d_L}^2$ is fixed by the $Wjj$ production rate. Below, we avoid reference to specific experimental selection cuts and present results for a generous range of possibilities consistent with existing data.

The dijet production rate at the UA2 $\sqrt{s} = 630$ GeV can be parametrized as follows [23]

$$\sigma(p\bar{p} \rightarrow Z') \times BR(Z' \rightarrow jj) \simeq \frac{1}{2} \left[ 773 (\epsilon_{u_L}^2 + \epsilon_{d_L}^2) + 138 (\epsilon_{u_R}^2 + \epsilon_{d_R}^2) \right] \times \Gamma(\phi, g'_1)_{Z' \rightarrow \ell\bar{\ell}} \text{ pb.}$$

(60)

(Our numerical calculation [5] using CTEQ6 [27] agrees within 5% with the result of [23].)

The maximum allowed value of the $\epsilon_{u_R}$ and $\epsilon_{d_R}$ couplings consistent with the UA2 upper limit are shown in Fig. 2. The dijet production rate at UA2 energies is given by

$$\sigma(p\bar{p} \rightarrow Z') \times BR(Z' \rightarrow \ell\bar{\ell}) \simeq \frac{1}{2} \left[ 773 (\epsilon_{u_L}^2 + \epsilon_{u_R}^2) + 138 (\epsilon_{d_L}^2 + \epsilon_{d_R}^2) \right] \times \Gamma(\phi, g'_1)_{Z' \rightarrow \ell\bar{\ell}} \text{ pb,}$$

(61)

where $\Gamma(\phi, g'_1)_{Z' \rightarrow \ell\bar{\ell}}$ is the leptonic branching fraction. From (60) and (58) we obtain the explicit form of the chiral couplings in terms of $\phi$ and $g'_1$

$$\epsilon_{u_L} = \epsilon_{d_L} = \frac{2}{\sqrt{g_Y^2 + g_{Z'}^2}} (C_{\psi} S_{\theta} S_{\psi} - C_{\phi} S_{\psi}) g'_3,$$

$$\epsilon_{u_R} = -\frac{2}{\sqrt{g_Y^2 + g_{Z'}^2}} [C_{\theta} S_{\phi} g'_1 + (C_{\psi} S_{\theta} S_{\psi} - C_{\phi} S_{\psi}) g'_3],$$

$$\epsilon_{d_R} = \frac{2}{\sqrt{g_Y^2 + g_{Z'}^2}} [C_{\theta} S_{\phi} g'_1 - (C_{\psi} S_{\theta} S_{\psi} - C_{\phi} S_{\psi}) g'_3].$$

(62)

The second strong constraint on the model derives from the mixing of the $Z$ and the $Y'$ through their coupling to the Higgs doublet. The last two terms in the covariant derivative

$$D_{\mu} = \partial_{\mu} - i \frac{1}{\sqrt{g_{Z'}^2 + g_Y^2}} Z_{\mu} (g_{Z'}^2 T^3 - g_Y^2 Q_Y) - ig_{Y'} Y_{\mu}' Q_{Y'} - ig_{Y''} Y_{\mu}'' Q_{Y''},$$

(63)

are conveniently written as

$$- i \frac{x_H}{v} M_{Z} Y_{\mu}' - i \frac{y_H}{v} M_{Z} Y_{\mu}''$$

(64)
where
\[
x_H = 1.9\sqrt{g_1^2 - 0.032S_\phi} ,
\]
\[
y_H = 1.9\sqrt{g_1^2 - 0.032C_\phi} ,
\]
and \( T^3 = \sigma^3/2 \). The Higgs field kinetic term \((D_\mu H)^\dagger(D_\mu H)\) together with the anomalous mass terms \((-\frac{1}{2}M'^2Y'^\muY'^\mu - \frac{1}{2}M'^2Y'^\muY'^\mu)\) yield the following mass square matrix\(^4\)
\[
\begin{pmatrix}
    M_Z^2 & M_Z^2x_H & M_Z^2y_H \\
    M_Z^2x_H & M_Z^2x_H^2 + M'^2 & M_Z^2x_Hy_H \\
    M_Z^2y_H & M_Z^2x_Hy_H & M_Z^2y_H^2 + M'^2
\end{pmatrix} .
\]

Next, taking \(M_{Z'} = 150\) GeV we use the two degrees of freedom of the model \((g_1', \phi)\) to demand the shift of the \(Z\) mass to lie within 1 standard deviation of the experimental value and leptophobia. This occurs for \(g_1' = 0.2, \phi = 0.0028\) and \(M_{Z''} = 5\) TeV, corresponding to a suppression \(\Gamma_{Z'' \rightarrow e^+e^-}/\Gamma_{Z'' \rightarrow q\bar{q}} \simeq 1\%\) \([5]\). This also corresponds to \(\theta = -1.103, \psi = -1.227\), and \(g_4' = 0.42\). The \(g_{Y'}Q_{Y'}\) and \(g_{Y''}Q_{Y''}\) couplings to the chiral fields are fixed and given in Table \([1]\). The accompanying values of \(\epsilon_{u_R}\) and \(\epsilon_{d_R}\) are shown in Fig. \([2]\). Now, substituting the above figures into \([51]\) we obtain the projections over \(Y, Y', Y''\)
\[
\begin{align*}
    Y &= 1.8 \times 10^{-1}Q_{1R} + 5.9 \times 10^{-2}Q_3 - 1.8 \times 10^{-1}Q_{1L} \\
    Y' &= 2.5 \times 10^{-4}Q_{1R} + 3.7 \times 10^{-1}Q_3 + 1.4 \times 10^{-1}Q_{1L} \\
    Y'' &= 9.0 \times 10^{-2}Q_{1R} - 1.2 \times 10^{-1}Q_3 + 3.5 \times 10^{-1}Q_{1L} .
\end{align*}
\]

Using Eq. \((7)\) it is straightforward to see that \(Z'\) and \(Z''\) become essentially \(B\) and \(B - L\), respectively.

The \(Z'\) couplings to quarks leads to a large (pre-cut) \(Wjj\) production \((\simeq 6\) pb) at the Tevatron, and at \(\sqrt{s} = 630\) GeV, a direct (pre-cut) \(Z' \rightarrow jj\) production \((\simeq 700\) pb) in the region excluded by UA2 data. However, it is worthwhile to point out that the UA2 Collaboration performed their analysis in the early days of QCD jet studies. Their upper bound depends crucially on the quality of the Monte Carlo and detector simulation which are primitive by today’s standard. They also use events with two exclusive jets, where jets were constructed using an infrared unsafe jet algorithm \([29]\). In view of the considerable uncertainties associated with the UA2 analysis we remain skeptical of drawing negative conclusions. Instead we argue that our model \([3]\) could provide an explanation of the CDF anomaly if acceptance and pseudorapidity cuts reduce the \(Wjj\) production rate by about 35\% - 66\% and the UA2 90\% CL bound is taken as an order-of-magnitude limit \([30]\).

Since the CDF signal is in dispute, it is of interest to study the predictions of the model for a leptophobic \(Z'\) at energies not obtainable at the Tevatron, but within the range of the LHC. The ATLAS Collaboration has searched for narrow resonances in the invariant mass spectrum of dimuon and dielectron final states in event samples corresponding to an integrated luminosity of 1.21 fb\(^{-1}\) and 1.08 fb\(^{-1}\), respectively \([31]\). The spectra are consistent.

\(^4\) We note in passing that two ‘supersymmetric’ Higgses \(H_u \equiv H\) and \(H_d = H^\dagger\), with charges \(Q_3 = Q_{1L} = 0, Q_{1R} = 1, Q_Y = 1/2\) and \(Q_3 = Q_{1L} = 0, Q_{1R} = -1, Q_Y = -1/2\), would also be sufficient to give masses to all the chiral fermions. Here, \((H_u) = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, (H_d) = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, v = \sqrt{v_u^2 + v_d^2}\) and tan \(\beta \equiv v_u/v_d\). It is easily seen that the corresponding mass square matrix is independent of tan \(\beta\) \([3]\).
FIG. 2: The ellipses show the values of $\epsilon_{u_R}$ and $\epsilon_{d_R}$ that saturate the UA2 and DØ limits on direct $Z' \to jj$ and $Wjj$ production, respectively. The solid ellipse is based on the assumption that experimental selection cuts will cause negligible reduction in event rates, the dashed ellipse corresponds to a reduction in event rates by 50%, and the dot-dashed ellipse corresponds to a 66% reduction in DØ event rates and 70% reduction in UA2 event rates. The cross indicates the best eyeball fit that simultaneously ensures small $Z - Z'$ mixing and $\Gamma_{Z' \to e^+e^-}/\Gamma_{Z' \to q\bar{q}} \lesssim 1\%$.

TABLE II: Chiral couplings of $Y'$ and $Y''$ gauge bosons for $\phi = 0.0028$ and $g'_1 = 0.2$.

| Name | $g_{Y'Q_Y}$ | $g_{Y''Q_{Y''}}$ |
|------|-------------|-----------------|
| $Q_i$ | 0.368       | -0.119          |
| $U_i$ | 0.368       | -0.028          |
| $D_i$ | 0.368       | -0.209          |
| $L_i$ | 0.143       | 0.143           |
| $E_i$ | 0.142       | 0.262           |
| $N_i$ | 0.143       | 0.443           |

with SM expectations and thus a lower mass limit of 1.83 TeV on the sequential SM $Z'$ has been set. Therefore, for $M_{Z'} \geq 1$ TeV, we scan the $g'_1 - \phi$ parameter space demanding the shift of the $Z$ mass to lie within 1 standard deviation of the experimental value and small ($\lesssim 1\%$) branching to leptons. We find that for $g'_1 = 0.195$, $\phi = -0.0638$, and $M_{Z''} \geq 2M_{Z'}$, in the sequential SM the $Z'$ has the same couplings to fermions as the $Z$ boson.
TABLE III: Chiral couplings of $Y'$ and $Y''$ gauge bosons for $\phi = -0.0638$ and $g'_1 = 0.195$.

| Name | $g_{Y'Q}$ | $g_{Y''Q}$ |
|------|------------|------------|
| $Q_i$ | 0.370      | -0.112     |
| $U_i$ | 0.365      | -0.033     |
| $D_i$ | 0.375      | -0.190     |
| $L_i$ | 0.154      | 0.154      |
| $E_i$ | 0.159      | 0.338      |
| $N_i$ | 0.149      | 0.495      |

FIG. 3: Comparison of the (pre-cut) total cross section for the production of $pp \to Z' \to jj$ (left) and $pp \to Z' \to \ell\ell$ (right) with the 95% CL upper limits on the production of a gauge boson decaying into two jets (left) and two leptons (right), as reported by the CMS (corrected by acceptance) \[33\] and ATLAS \[31\] collaborations, respectively. We have taken $\phi = -0.0638$, $g'_1 = 0.195$. For isotropic decays (independently of the resonance), the acceptance for the CMS detector has been reported to be $A \approx 0.6$ \[33\]. The predicted $Z'$ production rates are shown for $\sqrt{s} = 7$ TeV and $\sqrt{s} = 14$ TeV.

The ratio $\Gamma_{Z'\to e^+e^-}/\Gamma_{Z'\to q\bar{q}} \lesssim 1\%$ \[3\]. The chiral couplings to the $Z'$ and $Z''$ gauge bosons for these fiducial values are given in Table III. Again, we see that $Z'$ and $Z''$ are essentially $B$ and $B - L$.

The decay width of $Z' \to f\bar{f}$ is given by \[28\]

$$
\Gamma(Z' \to f\bar{f}) = \frac{G_F M_{Z'}^3}{6\pi\sqrt{2}} N_C C(M_{Z'}^2) M_{Z'} \sqrt{1 - 4x} \left[ v_f^2 (1 + 2x) + a_f^2 (1 - 4x) \right],
$$

(69)

where $G_F$ is the Fermi coupling constant, $C(M_{Z'}^2) = 1 + \alpha_s/\pi + 1.409(\alpha_s/\pi)^2 - 12.77(\alpha_s/\pi)^3$, $\alpha_s = \alpha_s(M_{Z'})$ is the strong coupling constant at the scale $M_{Z'}$, $x = m_f^2/M_{Z'}^2$, $v_f$ and $a_f$ are...
the vector and axial couplings, and $N_C = 3$ or 1 if $f$ is a quark or a lepton, respectively. Using the fiducial values of $g'_1$ and $\phi$ fitted in Table III for $M_{Z'} = 1$ TeV, we obtain $\Gamma = 60.9$ TeV. Hence, to compare our predictions (at the parton level) with LHC experimental searches in dilepton and dijets it is sufficient to consider the production cross section in the narrow $Z'$ width approximation,

$$\hat{\sigma}(q\bar{q} \to Z') = K \frac{2\pi}{3} \frac{G_F M_{Z'}^2}{\sqrt{2}} \left[ v_q^2(\phi, g'_1) + a_q^2(\phi, g'_1) \right] \delta \left( \hat{s} - M_{Z'}^2 \right) ,$$

where the $K$-factor represents the enhancement from higher order QCD processes estimated to be $K \approx 1.3$ [32]. After folding $\hat{\sigma}$ with the CTEQ6 parton distribution functions [27], we determine (at the parton level) the resonant production cross section. In Fig. 3 we compare the predicted $\sigma(p\bar{p} \to Z') \times BR(Z' \to \ell\ell)$ (left panel) and $\sigma(p\bar{p} \to Z') \times BR(Z' \to jj)$ (right panel) production rates with 95% CL upper limits recently reported by the ATLAS [31] and CMS [33] collaborations. Selection cuts will probably reduced event rates by factors of 2 to 3. Keeping this in mind, we conclude that the 2012 LHC7 run will probe $3 \text{ TeV} < M_{Z'} < 4 \text{ TeV}$, whereas future runs from LHC14 will provide a generous discovery potential of up to about $M_{Z'} \sim 8 \text{ TeV}$.

We turn now to discuss the string origin and the compelling properties of the $U(3)_C \times Sp(1)_L \times U(1)_L \times U(1)_R$ gauge group.

V. PERTURBATIVE D-BRANE MODELS IN A NUTSHELL

At the time of its formulation and for years thereafter, Superstring Theory was regarded as a unifying framework for Planck-scale quantum gravity and TeV-scale SM physics. Important advances were fueled by the realization of the vital role played by D-branes [34] in connecting string theory to phenomenology. This has permitted the formulation [35] of string theories with compositeness setting in at TeV scales and large extra dimensions, see Appendix A. There are two paramount phenomenological consequences for TeV scale D-brane string physics: the emergence of Regge recurrences at parton collision energies $\sqrt{\hat{s}} \sim \text{string scale} \equiv M_s$; and the presence of one or more additional $U(1)$ gauge symmetries, beyond the $U(1)_Y$ of the SM.

D-brane TeV-scale string compactifications provide a collection of building block rules that can used to build up the SM or something very close to it [36–38]. The details of the D-brane construct depend a lot on whether we use oriented string or unoriented string models. The basic unit of gauge invariance for oriented string models is a $U(1)$ field, so that a stack of $N$ identical D-branes eventually generates a $U(N)$ theory with the associated $U(N)$ gauge group. In the presence of many D-brane types, the gauge group becomes a product form $\prod U(N_i)$, where $N_i$ reflects the number of D-branes in each stack. Gauge bosons (and associated gauginos in a SUSY model) arise from strings terminating on one stack of D-branes, whereas chiral matter fields are obtained from strings stretching between two stacks. Each of the two strings end points carries a fundamental charge with respect to the stack of branes on which it terminates. Matter fields thus posses quantum numbers associated with a bifundamental representation. In orientifold brane configurations, which are necessary for tadpole cancellation, and thus consistency of the theory, open strings become in general non-oriented. For unoriented strings the above rules still apply, but we are allowed many more choices because the branes come in two different types. There are the branes whose
images under the orientifold are different from themselves and their image branes, and also branes who are their own images under the orientifold procedure. Stacks of the first type combine with their mirrors and give rise to $U(N)$ gauge groups, while stacks of the second type give rise to only $SO(N)$ or $Sp(N)$ gauge groups.

The minimal embedding of the SM particle spectrum requires at least three brane stacks [8] leading to three distinct models of the type $U(3)_C \times U(2)_L \times U(1)$ that were classified in [8, 39]. Only one of them (model C of [39]) has baryon number as symmetry that guarantees proton stability (in perturbation theory), and can be used in the framework of TeV strings. Moreover, since $Q_2$ (associated to the $U(1)$ of $U(2)_L$) does not participate in the hypercharge combination, $U(2)_L$ can be replaced by $Sp(1)_L$ leading to a model with one extra $U(1)$, the baryon number, besides hypercharge [40]. Since baryon number is anomalous, the extra abelian gauge field becomes massive by the Green-Schwarz mechanism [41], behaving at low energies as a $Z'$ with a mass in general lower than the string scale by an order of magnitude corresponding to a loop factor [42]. Lepton number is not a symmetry creating a problem with large neutrino masses through the Weinberg dimension-five operator $LLHH$ suppressed only by the TeV string scale.

The SM embedding in four D-brane stacks leads to many more models that have been classified in [9, 43]. In order to make a phenomenologically interesting choice, we focus on models where $U(2)_L$ can be reduce to $Sp(1)$. The minimal SM extension build up out of four stackes of D-branes is $U(3)_C \times Sp(1)_L \times U(1)_L \times U(1)_R$. A schematic representation of the D-brane structure is shown in Fig. 4. The corresponding fermion quantum numbers are given in Table I. Recall that the combination $B - L$ is anomaly free. As mentioned already, anomalous $U(1)$’s become massive necessarily due to the Green-Schwarz anomaly cancellation, but non anomalous $U(1)$’s can also acquire masses due to effective six-dimensional anomalies associated for instance to sectors preserving $N = 2$ SUSY [42]. These two-dimensional ‘bulk’ masses become therefore larger than the localized masses associated to four-dimensional anomalies, in the large volume limit of the two extra dimensions. Specifically for Dp-branes with $(p - 3)$-longitudinal compact dimensions the masses of the anomalous and, respectively, the non-anomalous $U(1)$ gauge bosons have the following generic scale behavior:

$$\begin{align*}
\text{anomalous } U(1)_i : & \quad M_{Z'} = g'_i M_s, \\
\text{non - anomalous } U(1)_i : & \quad M_{Z''} = g'_i M_s^3 V_2.
\end{align*}$$

(71)

Here $g'_i$ is the gauge coupling constant associated to the group $U(1)_i$, given by $g'_i \propto g_s/\sqrt{V_\parallel}$ where $g_s$ is the string coupling and $V_\parallel$ is the internal D-brane world-volume along the $(p - 3)$ compact extra dimensions, up to an order one proportionality constant. Moreover, $V_2$ is the internal two-dimensional volume associated to the effective six-dimensional anomalies giving mass to the non-anomalous $U(1)_i$. E.g. for the case of $D5$-branes, whose common intersection locus is just 4-dimensional Minkowski-space, $V_\parallel = V_2$ denotes the volume of the longitudinal, two-dimensional space along the two internal $D5$-brane directions. Since internal volumes are bigger than one in string units to have effective field theory description, the masses of non-anomalous $U(1)$-gauge bosons are generically larger than the masses of the anomalous gauge bosons. Since we want to identify the light $Z'$ gauge boson with baryon number, which is always anomalous, a hierarchy compared to the second $U(1)$-gauge boson

---

6 In fact, also the hypercharge gauge boson of $U(1)_Y$ can acquire a mass through this mechanism. In order to keep it massless, certain topological constraints on the compact space have to be met.
Particles created by vibrations of relativistic strings populate Regge trajectories relating their spins $J$ and masses $M$, \[ J = J_0 + \alpha' M^2, \] where $\alpha' = M_s^{-2}$ is the Regge slope parameter. Thus, if $M_s$ is of order few TeVs, a whole tower of infinite string excitations will open up at this low mass threshold. Should nature be so cooperative, one would expect to see a few string states produced at the LHC. The leading contributions of Regge recurrences to certain processes at hadron colliders are *universal*. This is because the full-fledged string amplitudes which describe $2 \rightarrow 2$ parton scattering subprocesses involving four gauge bosons as well as those with two gauge bosons and two chiral matter fields are (to leading order in string coupling, but all orders in $\alpha'$) independent of the compactification scheme. Only one assumption will be necessary in order to set up a solid framework: the string coupling must be small for the validity of perturbation theory in the computations of scattering amplitudes. In this case, black hole production and other strong gravity effects occur at energies above the string scale (see Appendix A), therefore at least the few lowest Regge recurrences are available for examination, free from interference with some complex quantum gravitational phenomena. We discuss this next.

**VI. REGGE RECURRENCES**

The most direct way to compute the amplitude for the scattering of four gauge bosons is to consider the case of polarized particles because all non-vanishing contributions can be then generated from a single, maximally helicity violating (MHV), amplitude – the so-called

---

7 In [44] a different (possibly T-dual) scenario with $D7$-branes was investigated. In this case the masses of the anomalous and non-anomalous $U(1)$’s appear to exhibit a dependence on the entire six-dimensional volume, such that the non-anomalous masses become lighter than the anomalous ones.

8 It is important to stress that in SUSY models derived from D-brane compactifications there can be a light $Z'$ even if the string scale is $\mathcal{O}(M_{Pl})$ [43].
partial MHV amplitude [46]. Assume that two vector bosons, with the momenta \( k_1 \) and \( k_2 \), in the \( U(N) \) gauge group states corresponding to the generators \( T^{a_1} \) and \( T^{a_2} \) (here in the fundamental representation), carry negative helicities while the other two, with the momenta \( k_3 \) and \( k_4 \) and gauge group states \( T^{a_3} \) and \( T^{a_4} \), respectively, carry positive helicities. (All momenta are incoming.) Then the partial amplitude for such an MHV configuration is given by

\[
\mathcal{A}(A^-_1, A^-_2, A^+_3, A^+_4) = 4 g^2 \text{Tr} (T^{a_1} T^{a_2} T^{a_3} T^{a_4}) \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} V(k_1, k_2, k_3, k_4) ,
\]

(73)

where \( g \) is the \( U(N) \) coupling constant, \( \langle ij \rangle \) are the standard spinor products written in the notation of Ref. [48], and the Veneziano formfactor,

\[
V(k_1, k_2, k_3, k_4) = V(s, t, u) = \frac{s u}{t M_s^2} B(-s/M_s^2, -u/M_s^2) = \frac{\Gamma(1 - s/M_s^2) \Gamma(1 - u/M_s^2)}{\Gamma(1 + t/M_s^2)}
\]

(74)

is the function of Mandelstam variables, \( s = 2k_1k_2 \), \( t = 2k_1k_3 \), \( u = 2k_1k_4 \); \( s + t + u = 0 \). (For simplicity we drop caret for the parton subprocess.) The physical content of the form factor becomes clear after using the well-known expansion in terms of \( s \)-channel resonances [49]

\[
B(-s/M_s^2, -u/M_s^2) = -\sum_{n=0}^{\infty} \frac{M_s^{2-2n}}{n!} \frac{1}{s - nM_s^2} \left[ \prod_{J=1}^{n} (u + M_s^2 J) \right],
\]

(75)

which exhibits \( s \)-channel poles associated to the propagation of virtual Regge excitations with masses \( \sqrt{n}M_s \). Thus near the \( n \)th level pole \( (s \to nM_s^2) \):

\[
V(s, t, u) \approx \frac{1}{s - nM_s^2} \times \frac{M_s^{2-2n}}{(n-1)!} \prod_{J=0}^{n-1} (u + M_s^2 J) .
\]

(76)

In specific amplitudes, the residues combine with the remaining kinematic factors, reflecting the spin content of particles exchanged in the \( s \)-channel, ranging from \( J = 0 \) to \( J = n + 1 \).\(^9\)

The low-energy expansion reads

\[
V(s, t, u) \approx 1 - \frac{\pi^2} {6} \frac{s u}{M_s^4} - \zeta(3) \frac{s t u}{M_s^6} + \ldots
\]

(77)

Interestingly, because of the proximity of the 8 gluons and the photon on the color stack of D-branes, the gluon fusion into \( \gamma + \) jet couples at tree level [50]. This implies that there is an order \( g^2 \) contribution in string theory, whereas this process is not occurring until order \( g^4 \) (loop level) in field theory. One can write down the total amplitude for this process projecting the gamma ray onto the hypercharge,

\[
\mathcal{M}(gg \to \gamma g) = \cos \theta_W \mathcal{M}(gg \to Yg) = \kappa \cos \theta_W \mathcal{M}(gg \to Cg) .
\]

(78)

\(^9\) There are resonances in all the channels, \( i.e. \) there are single particle poles in the \( t \) and \( u \) channels which would show up as bumps if \( t \) or \( u \) are positive. However, for physical scattering \( t \) and \( u \) are negative, so we don’t see the bumps.
The $C - Y$ mixing coefficient evaluated at the scale for $U(N)$ unification $M_s$ follows from \cite{51} and is given by
\[ \kappa = \frac{c_3 g_Y}{g_3^2} = \frac{g_Y}{\sqrt{6} g_3} \; ; \]
it is quite small, around $\kappa \simeq 0.12$ for couplings evaluated at the $Z$ mass, which is modestly enhanced to $\kappa \simeq 0.14$ as a result of RG running of the couplings up to $\sim 5$ TeV.

Consider the amplitude involving three $SU(N)$ gluons $g_1, g_2, g_3$ and one $U(1)$ gauge boson $\gamma_4$ associated to the same $U(N)$ stack:
\[ T^{a_1} = T^a, \ T^{a_2} = T^b, \ T^{a_3} = T^c, \ T^{a_4} = Q_I, \]
where $I$ is the $N \times N$ identity matrix and $Q$ is the $U(1)$ charge of the fundamental representation. The color factor
\[ \text{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4}) = Q(d^{abc} + \frac{i}{4} f^{abc}) \; , \]
where the totally symmetric symbol $d^{abc}$ is the symmetrized trace while $f^{abc}$ is the totally antisymmetric structure constant \cite{48}.

The full MHV amplitude can be obtained \cite{47} by summing the partial amplitudes \cite{73} with the indices permuted in the following way:
\[ \mathcal{M}(g_1, g_2, g_3, \gamma_4) = 4 g^2 \langle 12 \rangle^4 \sum_\sigma \text{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4}) V(k_{1\sigma}, k_{2\sigma}, k_{3\sigma}, k_{4\sigma}), \]
where the sum runs over all 6 permutations $\sigma$ of $\{1, 2, 3\}$ and $i_\sigma \equiv \sigma(i)$. Note that in the effective field theory of gauge bosons there are no Yang-Mills interactions that could generate this scattering process at the tree level. Indeed, $V = 1$ at the leading order of Eq.(77) and the amplitude vanishes due to the following identity:
\[ \frac{1}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} + \frac{1}{\langle 23 \rangle \langle 31 \rangle \langle 14 \rangle \langle 42 \rangle} + \frac{1}{\langle 31 \rangle \langle 12 \rangle \langle 24 \rangle \langle 43 \rangle} = 0 \; . \]
Similarly, the antisymmetric part of the color factor \cite{81} cancels out in the full amplitude \cite{82}. As a result, one obtains:
\[ \mathcal{M}(g_1^-, g_2^-, g_3^+, \gamma_4^+) = 8 Q d^{abc} g^2 \langle 12 \rangle^4 \left( \frac{\mu(s, t, u)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} + \frac{\mu(s, u, t)}{\langle 12 \rangle \langle 24 \rangle \langle 13 \rangle \langle 34 \rangle} \right), \]
where
\[ \mu(s, t, u) = \Gamma(1 - u/M_s^2) \left( \frac{\Gamma(1 - s/M_s^2)}{\Gamma(1 + t/M_s^2)} - \frac{\Gamma(1 - t/M_s^2)}{\Gamma(1 + s/M_s^2)} \right). \]
All non-vanishing amplitudes can be obtained in a similar way. In particular,
\[ \mathcal{M}(g_1^-, g_2^+, g_3^-, \gamma_4^+) = 8 Q d^{abc} g^2 \langle 13 \rangle^4 \left( \frac{\mu(t, s, u)}{\langle 13 \rangle \langle 24 \rangle \langle 14 \rangle \langle 23 \rangle} + \frac{\mu(t, u, s)}{\langle 13 \rangle \langle 24 \rangle \langle 12 \rangle \langle 34 \rangle} \right), \]
and the remaining ones can be obtained either by appropriate permutations or by complex conjugation.
In order to obtain the cross section for the (unpolarized) partonic subprocess $gg \rightarrow g\gamma$, we take the squared moduli of individual amplitudes, sum over final polarizations and colors, and average over initial polarizations and colors. As an example, the modulus square of the amplitude (82) is:

$$|M(g^-_1, g^-_2, g^+_3, \gamma^+_4)|^2 = 64 Q^2 d^{abc}d^{abc} g^4 \left| \frac{s\mu(s, t, u)}{u} + \frac{s\mu(s, u, t)}{t} \right|^2.$$  \hspace{1cm} (87)

Taking into account all $4(N^2 - 1)^2$ possible initial polarization/color configurations and the formula [51]

$$\sum_{a,b,c} d^{abc}d^{abc} = \frac{(N^2 - 1)(N^2 - 4)}{16N},$$  \hspace{1cm} (88)

we obtain the average squared amplitude [50]

$$|M(gg \rightarrow g\gamma)|^2 = g^4 Q^2 C(N) \left\{ \frac{s\mu(s, t, u)}{u} + \frac{s\mu(s, u, t)}{t} \right\}^2 + (s \leftrightarrow t) + (s \leftrightarrow u),$$  \hspace{1cm} (89)

where

$$C(N) = \frac{2(N^2 - 4)}{N(N^2 - 1)}. \hspace{1cm} (90)$$

Before proceeding, we need to make precise the value of $Q$. If we were considering the process $gg \rightarrow Cg$, then $Q = \sqrt{1/6}$ due to the normalization condition (29). However, for
\( gg \to \gamma g \) there are two additional projections given in [78]: from \( C_\mu \) to the hypercharge boson \( Y_\mu \), yielding a mixing factor \( \kappa \); and from \( Y_\mu \) onto a photon, providing an additional factor \( \cos \theta_W \). This gives

\[
Q = \sqrt{\frac{\pi}{3}} \kappa \cos \theta_W .
\]  

(91)

The two most interesting energy regimes of \( gg \to g\gamma \) scattering are far below the string mass scale \( M_s \) and near the threshold for the production of massive string excitations. At low energies, Eq. (89) becomes

\[
|M(gg \to g\gamma)|^2 \approx g^4Q^2C(N)\frac{\pi^4}{4M_s^8}(s^4 + t^4 + u^4) \quad (s, t, u \ll M_s^2) .
\]  

(92)

The absence of massless poles, at \( s = 0 \) etc., translated into the terms of effective field theory, confirms that there are no exchanges of massless particles contributing to this process. On the other hand, near the string threshold \( s \approx M_s^2 \)

\[
|M(gg \to g\gamma)|^2 \approx 4g^4Q^2C(N)\frac{M_s^8 + t^4 + u^4}{M_s^4(s - M_s^2)^2} \quad (s \approx M_s^2) ;
\]  

(93)

see Appendix B for details.

The general form of (82) for any given four external gauge bosons reads

\[
\mathcal{M}(A_1^+, A_2^*, A_3^+, A_4^+) = 4g^2\langle 12 \rangle^4 \left[ \frac{V_t}{(12)(23)(34)(41)} \text{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4} + T^{a_2}T^{a_1}T^{a_3}T^{a_4}) 
+ \frac{V_u}{(13)(34)(42)(21)} \text{Tr}(T^{a_2}T^{a_1}T^{a_3} + T^{a_1}T^{a_2}T^{a_3}) 
+ \frac{V_s}{(14)(42)(23)(31)} \text{Tr}(T^{a_1}T^{a_2}T^{a_4}T^{a_3} + T^{a_3}T^{a_1}T^{a_4}T^{a_2}) \right] ,
\]  

(94)

where

\[
V_t = V(s, t, u) , \quad V_u = V(t, u, s) , \quad V_s = V(u, s, t) .
\]  

(95)

The modulus square of the four-gluon amplitude, summed over final polarizations and colors, and averaged over all \( 4(N^2 - 1)^2 \) possible initial polarization/color configurations follows from (94) and is given by [52]

\[
|M(gg \to gg)|^2 = g^4 \left( \frac{1}{s^2} + \frac{1}{t^2} + \frac{1}{u^2} \right) \left[ \frac{2N^2}{N^2 - 1} (s^2 V_s^2 + t^2 V_t^2 + u^2 V_u^2) + \frac{4(3 - N^2)}{N^2(N^2 - 1)} \left( s V_s + t V_t + u V_u \right)^2 \right] .
\]  

(96)

The amplitudes involving two gluons and two quarks are also independent of the details of the compactification, such as the configuration of branes, the geometry of the extra dimensions, and whether SUSY is broken or not. This model independence makes it possible to compute all the string corrections to dijet signals at the LHC. The corresponding \( 2 \to 2 \) scattering amplitudes, computed at the leading order in string perturbation theory, are collected in Ref. [52]. The average square amplitudes are given by the following:

\[
|M(gg \to q\bar{q})|^2 = g^4N \frac{t^2 + u^2}{s^2} \left[ \frac{1}{2N} \frac{1}{u t} (t V_t + u V_u)^2 - \frac{N}{N^2 - 1} V_t V_u \right] ,
\]  

(97)
As an illustration, consider the amplitude \( |\mathcal{M}(q\bar{q} \to gg)|^2 = g^4 \frac{t^2 + u^2}{s^2} \left[ \frac{(N^2 - 1)^2}{2N^3} \frac{1}{ut} (t V_t + u V_u)^2 - \frac{N^2 - 1}{N} V_t V_u \right] \), \( (98) \)
and
\[ |\mathcal{M}(gg \to qq)|^2 = g^4 \frac{s^2 + u^2}{t^2} \left[ V_s V_u - \frac{N^2 - 1}{2N^2} \frac{1}{su} (s V_s + u V_u)^2 \right] . \] \( (99) \)

The amplitudes for the four-fermion processes like quark-antiquark scattering are more complicated because the respective form factors describe not only the exchanges of Regge states but also of heavy Kaluza-Klein and winding states with a model-dependent spectrum determined by the geometry of extra dimensions. Fortunately, they are suppressed, for two reasons. First, the QCD \( SU(3) \) color group factors favor gluons over quarks in the initial state. Second, the parton luminosities in proton-proton collisions at the LHC, at the parton center of mass energies above 1 TeV, are significantly lower for quark-antiquark subprocesses than for gluon-gluon and gluon-quark, see Fig. 5. The collisions of valence quarks occur at higher luminosity; however, there are no Regge recurrences appearing in the \( s \)-channel of quark-quark scattering 52.

In the following we isolate the contribution from the first resonant state in Eqs. (96) - (99). For partonic center of mass energies \( \sqrt{s} < M_s \), contributions from the Veneziano functions are strongly suppressed, as \( \sim (\sqrt{s}/M_s)^6 \), over standard model processes; see Eq. (92).

( Corrections to SM processes at \( \sqrt{s} \ll M_s \) are of order \( (\sqrt{s}/M_s)^4 \); see Eq. (77).) In order to factorize amplitudes on the poles due to the lowest massive string states, it is sufficient to consider \( s = M_s^2 \). In this limit, \( V_s \) is regular while
\[
V_t \to \frac{u}{s - M_s^2} , \quad V_u \to \frac{t}{s - M_s^2} . \] \( (100) \)

Thus the \( s \)-channel pole term of the average square amplitude (96) can be rewritten as
\[ |\mathcal{M}(gg \to gg)|^2 = 2 \frac{g^4}{M_s^4} \left( \frac{N^2 - 4 + (12/N^2)}{N^2 - 1} \right) \frac{M_s^8 + t^4 + u^4}{(s - M_s^2)^2} . \] \( (101) \)

Note that the contributions of single poles to the cross section are antisymmetric about the position of the resonance, and vanish in any integration over the resonance. Before proceeding, we pause to present our notation. The first Regge excitations of the gluon \( g \), the color singlet \( C \), and quarks \( q \) will be denoted by \( g^* \), \( C^* \), \( q^* \), respectively. Recall that \( C^*_\mu \) has an anomalous mass in general lower than the string scale by an order of magnitude. If that is the case, and if the mass of the \( C^* \) is composed (approximately) of the anomalous mass of the \( C^*_\mu \) and \( M_s \) added in quadrature, we would expect only a minor error in our results by taking the \( C^* \) to be degenerate with the other resonances. The singularity at \( s = M_s^2 \) needs softening to a Breit-Wigner form, reflecting the finite decay widths of resonances propagating in the \( s \)-channel. Due to averaging over initial polarizations, Eq. (101) contains additive contributions from both spin \( J = 0 \) and spin \( J = 2 \) \( U(3) \) bosonic Regge excitations \( (g^* \) and \( C^*) \), created by the incident gluons in the helicity configurations \((\pm \pm)\) and \((\mp \pm)\).

---

10 As an illustration, consider the amplitude \( a + b/D \) in the vicinity of the pole, where \( a \) and \( b \) are real, \( D = x + i\epsilon \), \( x = s - M_s^2 \), and \( \epsilon = \Gamma M_s \). Then, since \( \text{Re}(1/D) = x/|D|^2 \), the cross section becomes \( \sigma \propto a^2 + b^2/|D|^2 + 2 a b x/|D|^2 \approx a^2 + b^2 \pi \delta(x)/\epsilon + 2 a b \pi x \delta(x)/\epsilon \). Integrating over the width of the resonance, one obtains \( a^2\epsilon + b^2\pi/\epsilon \approx b\pi \), because \( b \propto \epsilon \), \( a \propto g^2 \) and \( \epsilon \propto g^2 \).
respectively. The $M_s^8$ term in Eq. (101) originates from $J = 0$, and the $t^4 + u^4$ piece reflects $J = 2$ activity. Since the resonance widths depend on the spin and on the identity of the intermediate state ($g^*, C^*$) the pole term (101) should be smeared as

$$|\mathcal{M}(gg \to gg)|^2 = 2 \frac{g^4}{M_s^4} \left( \frac{N^2 - 4 + (12/N^2)}{N^2 - 1} \right)^2 \times \left\{ W_{g^* \to gg}^{gg} \left[ \frac{M_s^8}{(s - M_s^2)^2 + (\Gamma_{g^*}^{J=0} M_s^2)^2} + \frac{t^4 + u^4}{(s - M_s^2)^2 + (\Gamma_{g^*}^{J=2} M_s^2)^2} \right] + W_{C^* \to gg}^{gg} \left[ \frac{M_s^8}{(s - M_s^2)^2 + (\Gamma_{C^*}^{J=0} M_s^2)^2} + \frac{t^4 + u^4}{(s - M_s^2)^2 + (\Gamma_{C^*}^{J=2} M_s^2)^2} \right] \right\},$$

(102)

where $\Gamma_{g^*}^{J=0} = 75 (M_s/\text{TeV})$ GeV, $\Gamma_{C^*}^{J=0} = 150 (M_s/\text{TeV})$ GeV, $\Gamma_{g^*}^{J=2} = 45 (M_s/\text{TeV})$ GeV, and $\Gamma_{C^*}^{J=2} = 75 (M_s/\text{TeV})$ GeV are the total decay widths for intermediate states $g^*$ and $C^*$, with angular momentum $J$. The associated weights of these intermediate states are given in terms of the probabilities for the various entrance and exit channels

$$\frac{N^2 - 4 + 12/N^2}{N^2 - 1} = \frac{16}{(N^2 - 1)^2} \left[ \left( \frac{N^2 - 4}{4N} \right)^2 + \left( \frac{N^2 - 1}{2N} \right)^2 \right] \propto \frac{16}{(N^2 - 1)^2} \left[ (N^2 - 1)(\Gamma_{g^* \to gg})^2 + (\Gamma_{C^* \to gg})^2 \right],$$

(103)

yielding

$$W_{g^* \to gg}^{gg} = \frac{8(\Gamma_{g^* \to gg})^2}{8(\Gamma_{g^* \to gg})^2 + (\Gamma_{C^* \to gg})^2} = 0.44,$$

(104)

and

$$W_{C^* \to gg}^{gg} = \frac{(\Gamma_{C^* \to gg})^2}{8(\Gamma_{g^* \to gg})^2 + (\Gamma_{C^* \to gg})^2} = 0.56.$$  

(105)

A similar calculation transforms Eq. (107) near the pole into

$$|\mathcal{M}(gg \to q\bar{q})|^2 = \frac{g^4}{M_s^4} N_f \left( \frac{N^2 - 2}{N(N^2 - 1)} \right) \left[ W_{g^* \to q\bar{q}}^{gq} \frac{ut(u^2 + t^2)}{(s - M_s^2)^2 + (\Gamma_{g^*}^{J=2} M_s^2)^2} + W_{C^* \to q\bar{q}}^{gq} \frac{ut(u^2 + t^2)}{(s - M_s^2)^2 + (\Gamma_{C^*}^{J=2} M_s^2)^2} \right],$$

(106)

where

$$W_{g^* \to q\bar{q}}^{gq} = W_{g^* \to gg}^{gg} = \frac{8\Gamma_{g^* \to gg} \Gamma_{g^* \to q\bar{q}}}{8\Gamma_{g^* \to gg} \Gamma_{g^* \to q\bar{q}} + \Gamma_{C^* \to gg} \Gamma_{C^* \to q\bar{q}}} = 0.71$$

(107)

and

$$W_{C^* \to q\bar{q}}^{gq} = W_{C^* \to gg}^{gg} = \frac{\Gamma_{C^* \to gg} \Gamma_{C^* \to q\bar{q}}}{8\Gamma_{g^* \to gg} \Gamma_{g^* \to q\bar{q}} + \Gamma_{C^* \to gg} \Gamma_{C^* \to q\bar{q}}} = 0.29.$$  

(108)

Near the $s$ pole Eq. (108) becomes

$$|\mathcal{M}(q\bar{q} \to gg)|^2 = \frac{g^4}{M_s^4} \left( \frac{(N^2 - 2)(N^2 - 1)}{N^3} \right) \left[ W_{g^* \to q\bar{q}}^{q\bar{q}} \frac{ut(u^2 + t^2)}{(s - M_s^2)^2 + (\Gamma_{g^*}^{J=2} M_s^2)^2} + W_{C^* \to q\bar{q}}^{q\bar{q}} \frac{ut(u^2 + t^2)}{(s - M_s^2)^2 + (\Gamma_{C^*}^{J=2} M_s^2)^2} \right],$$

(109)

24
whereas Eq. (99) can be rewritten as

\[
|M(qg \rightarrow qg)|^2 = -\frac{g^4}{M_s^2} \left( \frac{N^2 - 1}{2N^2} \right) \left[ \frac{M_s^4 u}{(s - M_s^2)^2 + (\Gamma_{q^*}^{J=1/2} M_s)^2} + \frac{u^3}{(s - M_s^2)^2 + (\Gamma_{q^*}^{J=3/2} M_s)^2} \right].
\] (110)

The total decay widths for the \(q^*\) excitation are: \(\Gamma_{q^*}^{J=1/2} = \Gamma_{q^*}^{J=3/2} = 37 (M_s/\text{TeV}) \text{ GeV}^3\). Superscripts \(J = 2\) are understood to be inserted on all the \(\Gamma\)'s in Eqs. (104), (105), (107), (108). Equation (102) reflects the fact that weights for \(J = 0\) and \(J = 2\) are the same [53].

The \(s\)-channel poles near the second Regge resonance can be approximated by expanding the Veneziano formfactor \(V_t\) around \(s = 2M_s^2\):

\[
V(s, t, u) \approx \frac{u(u + M_s^2)}{M_s^2(s - 2M_s^2)}.
\] (111)

The associated scattering amplitudes and decay widths of the \(n = 2\) string resonances are collected in Ref. [56]. Generally, the width of the Regge excitations will grow at least linearly with energy, whereas the spacing between levels will decrease with energy. This implies an upper limit on the domain of validity for this phenomenological approach [57]. In particular, for a resonance \(R\) of mass \(M\), the total width is given by

\[
\Gamma_{\text{tot}} \sim \frac{g^2}{4\pi} C \frac{M}{4},
\] (112)

where \(C > 1\) because of the growing multiplicity of decay modes [55, 56]. On the other hand, since \(\Delta(M^2) = M_s^2\) the level spacing at mass \(M\) is \(\Delta M \sim M_s^2/(2M)\); thus,

\[
\frac{\Gamma_{\text{tot}}}{\Delta M} \sim \frac{g^2}{8\pi} C \left( \frac{M}{M_s} \right)^2 = \frac{g^2}{8\pi} C n < 1.
\] (113)

For excitation of the resonance \(R\) via \(a + b \rightarrow R\), the assumption \(\Gamma_{\text{tot}}(R) \sim \Gamma(R \rightarrow ab)\) (which underestimates the real width) yields a perturbative regime for \(n \lesssim 40\). This is to be compared with the \(n \sim 10^4\) levels of the string needed for black hole production (see Appendix A).

Given the particular nature of the process we are considering, the production of a TeV-scale resonance and its subsequent \(2\)-body decay, several observables at the LHC are available. Most apparently, one would hope that the resonance would be visible in data binned according to the dijet invariant mass \(M\), after setting cuts on the different jet rapidities, \(|y_1|, |y_2| \leq y_{\text{max}}\) and transverse momenta \(p_T^{1,2} > 50\text{ GeV}\). With the definitions \(Y \equiv \frac{1}{2}(y_1 + y_2)\) and \(y \equiv \frac{1}{2}(y_1 - y_2)\), the cross section per interval of \(M\) for \(pp \rightarrow \text{dijet}\) is given by [58]

\[
\frac{d\sigma}{dM} = M_T \sum_{ijkl} \left[ \int_{-y_{\text{max}}}^{0} dY \ f_i(x_a, M) \ f_j(x_b, M) \int_{-(y_{\text{max}}+Y)}^{y_{\text{max}}+Y} dy \ \frac{d\sigma}{dt} \bigg|_{ij \rightarrow kl} \ \frac{1}{\cosh^2 y} \right. \\
+ \int_{0}^{y_{\text{max}}} dY \ f_i(x_a, M) \ f_j(x_b, M) \int_{-(y_{\text{max}}-Y)}^{y_{\text{max}}-Y} dy \ \frac{d\sigma}{dt} \bigg|_{ij \rightarrow kl} \ \frac{1}{\cosh^2 y} \left. \right].
\] (114)
where \( f(x, M) \)'s are parton distribution functions (we use CTEQ6 [27]), \( \tau = M^2/s, x_a = \sqrt{\tau} e^Y, x_b = \sqrt{\tau} e^{-Y} \), and

\[
|M(ij \rightarrow kl)|^2 = 16\pi s^2 \frac{d\sigma}{dt}|_{ij \rightarrow kl};
\]

we reinstate the caret notation (\( \hat{s}, \hat{t}, \hat{u} \)) to specify partonic subprocesses. The \( Y \) integration range in Eq. (114), \( Y_{\text{max}} = \min\{\ln(1/\sqrt{\tau}), y_{\text{max}}\} \), comes from requiring \( x_a, x_b < 1 \) together with the rapidity cuts \( y_{\text{min}} < |y_1|, |y_2| < y_{\text{max}} \). The kinematics of the scattering also provides the relation \( M = 2p_T \cosh y, \) which when combined with \( p_T = M/2 \sin \theta^* = M/2 \sqrt{1 - \cos^2 \theta^*} \), yields \( \cosh y = (1 - \cos^2 \theta^*)^{-1/2} \), where \( \theta^* \) is the center-of-mass scattering angle. Finally, the Mandelstam invariants occurring in the cross section are given by \( \hat{s} = M^2, \hat{t} = -\frac{1}{2}M^2 e^{-y}/\cosh y, \) and \( \hat{u} = -\frac{1}{2}M^2 e^{+y}/\cosh y \). In what follows we set \( N = 3 \) and \( N_f = 6 \).

The CMS Collaboration has searched for such narrow resonances in their dijet mass spectrum using data from \( pp \) collisions at \( \sqrt{s} = 7 \text{ TeV} \) [59]. After operating for only a few months, with merely 2.9 inverse picobarns of integrated luminosity, the LHC CMS experiment has ruled out \( M_s < 2.5 \text{ TeV} \). The LHC7 has recently delivered an integrated luminosity in excess of 1 fb\(^{-1}\). This extends considerably the search territory for new physics in events containing dijets. The new data exclude string resonances with \( M_s < 4 \text{ TeV} \) [33]. In fact, the LHC has the capacity of discovering strongly interacting resonances via dijet final states in practically all range up to \( \frac{1}{2}\sqrt{s}_{\text{LHC}} \). We discuss this next.

Standard bump-hunting methods, such as calculating cumulative cross sections

\[
\sigma(M_0) = \int_{M_0}^{\infty} \frac{d\sigma}{dM} \, dM
\]

and searching for regions with significant deviations from the QCD background, may allow to find an interval of \( M \) suspected of containing a bump. With the establishment of such a region, one may calculate a signal-to-noise ratio, with the signal rate estimated in the invariant mass window \([M_s - 2\Gamma, M_s + 2\Gamma]\). To accommodate the minimal acceptance cuts on dijets from LHC experiments [60], an additional kinematic cut, \(|y_{\text{max}}| < 1.0\), has been included in the calculation. The noise is defined as the square root of the number of QCD background events in the same dijet mass interval for the same integrated luminosity. Our significant results are encapsulated in Fig. 6, where we show the signal-to-noise ratio as a function of the string scale. It is remarkable that within one to two years of data collection with LHC14, \textit{string scales as large as 6.8 TeV are open to discovery at the } \geq 5\sigma \text{ level}. Once more, we stress that these results contain no unknown parameters. They depend only on the D-brane construct for the SM, and \textit{are independent of compactification details}.\footnote{The only remnant of the compactification is the relation between the Yang-Mills coupling and the string coupling. We take this relation to reduce to field theoretical results in the case where they exist, \textit{e.g.} \( gg \rightarrow gg \). Then, because of the require correspondence with field theory, the phenomenological results are independent of the compactification of the transverse space. However, a different phenomenology would result as a consequence of warping one [61] or more parallel dimensions [62].}
FIG. 6: Signal-to-noise ratio of $pp \rightarrow \text{dijet}$ and $pp \rightarrow \gamma + \text{jet}$ for $\sqrt{s} = 14$ TeV, $\mathcal{L} = 100$ fb$^{-1}$, and $\kappa^2 \approx 0.02$. The approximate equality of the background due to misidentified $\pi^0$'s and the QCD background, across a range of large $p_T^\gamma$, is maintained as an approximate equality over a range of $\gamma$-jet invariant masses with the rapidity cuts imposed ($|y_j^{\text{max}}| < 1.0$ and $|y_{\gamma}^{\text{max}}| < 2.4$). The 95% CL lower limits on the string scale recently reported by the CMS Collaboration are also shown.

Origin for new physics manifest as a resonant structure in LHC data. The Breit-Wigner form for gluon fusion into $\gamma + \text{jet}$ follows from (93) and is given by

$$|\mathcal{M}(gg \rightarrow g\gamma)|^2 \simeq \frac{5g_s^4Q^2}{3M_s^4} \left[ \frac{M_s^8}{(\hat{s} - M_s^2)^2 + (\Gamma_{g^*}^{J=0} M_s)^2} + \frac{\hat{u}^4}{(\hat{s} - M_s^2)^2 + (\Gamma_{g^*}^{J=2} M_s)^2} \right].$$

(117)

From Fig. 5 we see that the dominant $s$-channel pole term of the average square amplitude contributing to $pp \rightarrow \gamma + \text{jet}$ is

$$|\mathcal{M}(gg \rightarrow q\gamma)|^2 = -\frac{g_s^4Q^2}{3M_s^2} \left[ \frac{M_s^4 \hat{u}}{(\hat{s} - M_s^2)^2 + (\Gamma_{q^*}^{J=0} M_s)^2} + \frac{\hat{u}^3}{(\hat{s} - M_s^2)^2 + (\Gamma_{q^*}^{J=2} M_s)^2} \right].$$

(118)

We duplicate the calculation of the signal-to-noise ratio substituting in (114) $d\sigma/dt_{ij \rightarrow kl}$ by $d\sigma/dt_{ij \rightarrow k\gamma}$. For photons, we set $y_{\text{max}} < 2.4$. To minimize misidentification with a high-$p_T$ $\pi^0$, isolation cuts must be imposed on the photon, and to trigger on the desired channel, the hadronic jet must be identified. A detailed study of the CMS potential for isolation of prompt-$\gamma$'s has been carried out, using GEANT4 simulations of $\gamma + \text{jet}$ events generated with Pythia. This analysis (which also includes $\gamma$'s produced in the decays of $\eta$, $K^0_s$, $\omega^0$, $\phi$, and $J/\psi$)
FIG. 7: For a luminosity of 10 fb$^{-1}$, the expected value (dashed line) and statistical error (shaded region) of the dijet ratio of QCD in the CMS detector is compared with LO QCD (dot-dashed line) and LO QCD plus lowest massive string excitation at a scale $M_s = 5$ TeV. From Ref. [54].

and bremsstrahlung photons emerging from high-$p_T$ jets) suggests

$$\beta = \frac{\text{background due to misidentified } \pi^0 \text{ after isolation cuts}}{\text{QCD background from direct photon production}} + 1 \simeq 2 . \quad (119)$$

Therefore, in our numerical calculation we assume the noise is increased by a factor of $\sqrt{\beta}$, over the direct photon QCD contribution. The signal used to obtain the results displayed in Fig. 6 includes the parton subprocesses $gg \rightarrow g\gamma$ (which does not exist at tree level in QCD), $qg \rightarrow q\gamma$, $\bar{q}g \rightarrow \bar{q}\gamma$, and $q\bar{q} \rightarrow g\gamma$. All except the first have been calculated in QCD and constitute the SM background. For string scales as high as 5.0 TeV, observations of resonant structures in $pp \rightarrow \gamma + \text{jet}$ can provide interesting corroboration for stringy physics at the TeV-scale.

Events with a single jet plus missing energy ($E_T$) with balancing transverse momenta (so-called “monojets”) are incisive probes of new physics. As in the SM, the source of this topology is $ij \rightarrow kZ$ followed by $Z \rightarrow \nu\bar{\nu}$. Both in the SM and string theory the cross section for this process is of order $g^4$. Virtual KK graviton emission ($ij \rightarrow kG$) involves emission of closed strings, resulting in an additional suppression of order $g^2$ compared to $Z$ emission. A careful discussion of this suppression is given in [66]. Ignoring the $Z$-mass (i.e. keeping only transverse $Z$’s), the Regge contribution to $pp \rightarrow Z + \text{jet}$ is suppressed relative to the $pp \rightarrow \gamma + \text{jet}$ by a factor of $\tan^2 \theta_W = 0.29$ [67].

We now turn to the analysis of the dijet angular distributions. QCD parton-parton cross sections are dominated by $t$-channel exchanges that produce dijet angular distributions which peak at small center of mass scattering angles. In contrast, non–standard contact
interactions or excitations of resonances result in a more isotropic distribution. In terms of rapidity variables for standard transverse momentum cuts, dijets resulting from QCD processes will preferentially populate the large rapidity region, while the new processes generate events more uniformly distributed in the entire rapidity region. To analyze the details of the rapidity space the DØ Collaboration \[68\] introduced a new parameter,

\[
R = \frac{d\sigma/dM|(|y_1|,|y_2|<0.5)}{d\sigma/dM|(|y_1|,|y_2|<1.0)},
\]

the ratio of the number of events, in a given dijet mass bin, for both rapidities \(|y_1|,|y_2| < 0.5\) and both rapidities \(0.5 < |y_1|,|y_2| < 1.0\). In Fig. 7 we compare the results from a full CMS detector simulation of the ratio \(R\), with predictions from LO QCD and model-independent contributions to the \(q^*, g^*\) and \(C^*\) excitations.\[12\] The synthetic population was generated with Pythia, passed through the full CMS detector simulation and reconstructed with the ORCA reconstruction package \[70\]. For an integrated luminosity of 10 fb\(^{-1}\) the LO QCD contributions with \(\alpha_s = g_s^2/4\pi = 0.1\) (corresponding to running scale \(\Lambda \approx M_s\)) are within statistical fluctuations of the full CMS detector simulation. (Note that the string scale is an optimal choice of the running scale which should normally minimize the role of higher loop corrections.) Since one of the purposes of utilizing NLO calculations is to fix the choice of the running coupling, we take this agreement as rationale to omit loops in QCD and in string theory. It is clear from Fig. 7 that incorporating NLO calculation of the background and the signal would not significantly change the large deviation of the string contribution from the QCD background. String scales \(\sim 5\) TeV can be probed with 10 fb\(^{-1}\) of LHC14 data collection. Because of background reduction by optimized rapidity cuts, the \(R\) parameter can (in principle) extend the LHC discovery reach of Regge excitations.

In closing, we note that for \(e^+e^-\) colliders string theory predicts the precise value, equal to 1/3, of the relative weight of spin 2 and spin 1 contributions \[71\]. This yields a dimuon angular distribution with a pronounced forward-backward asymmetry, which could help distinguishing between low mass strings and other beyond SM scenarios.

VII. CONCLUSIONS

We have considered a low-mass string compactification in which the SM gauge multiplets originate in open strings ending on D-branes, with gauge bosons due to strings attached to stacks of D-branes and chiral matter due to strings stretching between intersecting D-branes. For the non-abelian \(SU(3)\) group, the D-brane construct requires the existence of an additional \(U(1)\) gauge boson coupled to baryon number. In this framework, \(U(1)\) and \(SU(3)\) appear as subgroups of \(U(3)\). In addition, our minimal model contains three other stacks of D-branes to accommodate the electroweak \(Sp(1)\) left and \(U(1)\) fields attached to the lepton D-brane and to the right D-brane. One linear combination of the three \(U(1)\) gauge bosons is identified as the the hypercharge \(Y\) field, coupled to the anomaly free hypercharge current. The two remaining linear combinations \((Y', Y'')\) of the three \(U(1)'s\), which can be naturally associated with \(B\) and \(B - L\), grow masses. After electroweak breaking, mixing with the

\[12\] An illustration of the use of this parameter in a heuristic model where SM amplitudes are modified by a Veneziano formfactor has been presented \[69\].
third component of isospin results in the three observable gauge bosons, where with small mixing $Z'\simeq Y', \ Z''\simeq Y''$.

In our phenomenological discussion about the possible discovery of massive $Z'$-gauge bosons, we have taken as a benchmark scenario the dijet plus $W$ signal, recently observed by the CDF Collaboration at Tevatron. For a fixed $M_{Z'}\simeq 150$ GeV, the model is quite constrained. Fine tuning its free parameters is just sufficient to simultaneously ensure: a small $Z-Z'$ mixing in accord with the stringent LEP data on the $Z$ mass; very small (less than 1%) branching ratio into leptons; and a large hierarchy between $Z''$ and $Z'$ masses.

If the CDF anomaly does not survive additional scrutiny (as indicated by the more recent DØ results), the analysis presented here can be directly applied to the higher energy realm, with a view toward identifying the precise makeup of the various abelian sectors, and pursuing with strong confidence a signal at LHC for Regge excitations of the string.

In D-brane constructions, the full-fledged string amplitudes supplying the dominant contributions to $pp$ scattering cross sections are completely independent of the details of compactification. We have made use of the amplitudes evaluated near the first resonant pole to report on the discovery potential at the LHC for the first Regge excitations of the quark and gluon. The precise predictions for the branching fraction of two different topologies (dijet and $\gamma+\text{jet}$) can be used as a powerful discriminator of low mass string excitations from other beyond SM scenarios. We have long imagined strings to be minuscule objects which could only be experimentally observed in the far-distant future. It is conceivable that this future has already arrived.

Acknowledgments

I’m thankful to Ignatios Antoniadis, Vic Feng, Haim Goldberg, Xing Huang, Dieter Lüst, Satoshi Nawata, Stephan Stieberger, and Tom Taylor for fruitful and enjoyable collaborations. L.A.A. is partially supported by the U.S. National Science Foundation (NSF) under Grant PHY-0757598 and CAREER Award PHY-1053663. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the NSF.

Appendix A: TeV-scale strings and large extra dimensions

For an illustration, consider type II string theory compactified on a six-dimensional torus $T^6$, which includes a $D_p$-brane wrapped around $p-3$ dimensions of $T^6$ with the remaining dimensions along our familiar (uncompactified) three spatial dimensions. We denote the radii of the internal longitudinal directions (of the $D_p$-brane) by $R_i^\parallel$, $i = 1, \ldots, p-3$ and the radii of the transverse directions by $R_j^\perp$, $j = 1, \ldots, 9-p$, see Fig. 8.

The 4-dimensional Planck mass $M_{\text{Pl}} \sim 10^{19}$ GeV, which is related to the string mass scale $M_s$ by

$$M_{\text{Pl}}^2 = \frac{8}{g_s^2} \frac{M_s^8}{(2\pi)^6} V_6,$$  \hfill (A1)

determines the strength of the gravitational interactions. Here,

$$V_6 = (2\pi)^6 \prod_{i=1}^{p-3} R_i^\parallel \prod_{j=1}^{9-p} R_j^\perp.$$  \hfill (A2)
is the volume of $T^6$ and $g_s$ is the string coupling. It follows that the string scale can be chosen hierarchically smaller than the Planck mass at the expense of introducing $n = 9 - p$ large transverse dimensions felt only by gravity, while keeping the string coupling small. E.g. for a string mass scale $M_s \approx O(1 \, \text{TeV})$ the volume of the internal space needs to be as large as $V_6 M_s^6 \approx O(10^{32})$. On the other hand, the strength of coupling of the gauge theory living on the D-brane world volume is not enhanced as long as $R_{\parallel i} \sim M_s^{-1}$ remain small,

$$\frac{1}{g^2} = \frac{1}{2\pi g_s} M_s^{p-3} \prod_{i=1}^{p-3} R_{\parallel i}.$$

The weakness of the effective 4 dimensional gravity compared to gauge interactions (ratio of $\langle H \rangle / M_{Pl}$) is then attributed to the largeness of the transverse space radii $R_{\parallel i} \sim 10^{32} l_s$ compared to the string length $l_s = M_s^{-1}$. A distinct property of these D-brane models is that gravity becomes effectively $(4 + n)$-dimensional with a strength comparable to those of gauge interactions at the string scale. Equation (A1) can be understood as a consequence of the $(4 + n)$-dimensional Gauss law for gravity, with

$$M_s^{2+n} \sim \frac{1}{g_s^2} M_s^{2+n}$$

the effective scale of gravity in $4+n$ dimensions. Taking $M_s \sim 1 \, \text{TeV}$, one finds a size for the extra dimensions $R^\perp \approx 10^{30/n-19} \, \text{m}$. This relation immediately suggests that $n = 1$ is ruled out, because $R^\perp \sim 10^{11} \, \text{m}$ and the gravitational interaction would thus be modified at the scale of our solar system. However, already for $n = 2$ one obtains $R^\perp \sim 1 \, \text{mm}$. This is just the scale where our present day experimental knowledge about gravity ends.

It is important to note that the mass scale $M_{\text{BH}} \sim M_s / g_s^2$, which corresponds to the onset of black hole production, cannot be probed at the LHC. To be specific, we
choose $g_s = 0.1$ and then we obtain $M_{BH} \sim 100 M_s$. It is also noteworthy that TeV-scale string D-brane compactifications naturally and unavoidably give rise to ‘the incredible bulk’ which characterizes the recently proposed dynamical dark matter framework [75].

**Appendix B: Pole residues of the Veneziano form factor**

Consider the product of Gamma functions

$$\Gamma(n) \Gamma(1 - n) = \frac{\pi}{\sin(n\pi)}.$$  \hfill (B1)

In the limit $1 - n = \epsilon \ll 1$, $\sin(n\pi) = \sin(\pi - \pi\epsilon) = \sin(\pi) - \pi\epsilon \cos(\pi) = \pi\epsilon$, and so

$$\Gamma(1 - \epsilon) \Gamma(\epsilon) = \frac{\pi}{\pi\epsilon} = \frac{1}{\epsilon},$$  \hfill (B2)

which in turn leads to

$$\lim_{n \to 1} \Gamma(1 - n) = \frac{1}{1 - n}.$$  \hfill (B3)

Therefore, in the limit of $s/M_s^2 \to 1$,

$$\mu(s, t, u) \to \frac{\Gamma(1 - u/M_s^2)}{\Gamma(1 + t/M_s^2)} \frac{1}{1 - s/M_s^2} = \frac{\Gamma(1 - t/M_s^2)}{\Gamma(1 + u/M_s^2)} \frac{1}{1 - s/M_s^2} = \frac{1 + t/M_s^2}{1 + s/M_s^2}$$  \hfill (B4)

and

$$\mu(s, u, t) \to \frac{\Gamma(1 - t/M_s^2)}{\Gamma(1 + u/M_s^2)} \frac{1}{1 - s/M_s^2} = \frac{\Gamma(1 - u/M_s^2)}{\Gamma(1 + t/M_s^2)} \frac{1}{1 - s/M_s^2} = \frac{1 + u/M_s^2}{1 + s/M_s^2};$$  \hfill (B5)

recall we are working on the physical region (where $t$ and $u$ are negatives) and so the second term in $\mu(s, t, u)$ or $\mu(s, u, t)$ does not develop Regge poles. We can now expand the string squared amplitude,

$$|M(gg \to \gamma g)|^2 \propto \left[\frac{s}{u} \mu(s, t, u) + \frac{s}{t} \mu(s, u, t)\right]^2 + \left[\frac{t}{u} \mu(s, t, u) + \frac{t}{s} \mu(t, u, s)\right]^2$$

$$+ \left[\frac{u}{s} \mu(u, t, s) + \frac{u}{t} \mu(u, s, t)\right]^2,$$  \hfill (B6)

near the pole yielding

$$|M(gg \to \gamma g)|^2 \propto \left|\frac{M_s^2}{u} \frac{\Gamma(1 - u/M_s^2)}{\Gamma(1 + t/M_s^2)} + \frac{M_s^2}{t} \frac{\Gamma(1 - t/M_s^2)}{\Gamma(1 + u/M_s^2)}\right|^2 \frac{1}{(s - M_s^2)^2}$$

$$+ \left|\frac{t}{u} \frac{\Gamma(1 - u/M_s^2)}{\Gamma(1 + t/M_s^2)} + \frac{t}{M_s^2} \frac{\Gamma(1 - t/M_s^2)}{\Gamma(1 + u/M_s^2) - \frac{\Gamma(1 - u/M_s^2)}{\Gamma(1 + t/M_s^2) - \frac{\Gamma(1 - u/M_s^2)}{\Gamma(1 + t/M_s^2)} - \frac{\Gamma(1 - t/M_s^2)}{\Gamma(1 + u/M_s^2)}\right|^2 \frac{1}{(s - M_s^2)^2}$$

$$+ \left|\frac{u}{M_s^2} \frac{\Gamma(1 - u/M_s^2)}{\Gamma(1 + t/M_s^2)} - \frac{u}{t} \frac{\Gamma(1 - t/M_s^2)}{\Gamma(1 + u/M_s^2)}\right|^2 \frac{1}{(s - M_s^2)^2}.$$
Equivalently,
\[
|M(gg \to \gamma g)|^2 \propto \left\{ \frac{M_s^2}{u} A + \frac{M_s^2}{t} B + \left| -\frac{t}{u} A + \frac{t}{M_s^2} (B - A) \right|^2 
+ \left| \frac{u}{M_s^2} (A - B) - \frac{u}{t} B \right|^2 \right\} \frac{1}{(s - M_s^2)^2},
\]
(B7)
where
\[
A = \frac{\Gamma(1 - u/M_s^2)}{\Gamma(1 + t/M_s^2)} = 1 + t/M_s^2 = -u/M_s^2
\]
and
\[
B = \frac{\Gamma(1 - t/M_s^2)}{\Gamma(1 + u/M_s^2)} = 1 + u/M_s^2 = -t/M_s^2
\]
are obtained from Eqs. (B4) and (B5). Then, Eq. (B7) becomes
\[
|M(gg \to \gamma g)|^2 \propto \left\{ 4M_s^4 + \left| t + \frac{t}{M_s^2} (-t + u) \right|^2 + \left| u + \frac{u}{M_s^2} (-u + t) \right|^2 \right\} \frac{1}{(s - M_s^2)^2}
\]
\[
\propto \left[ 4M_s^4 + \frac{4t^4 + 4u^4}{M_s^4} \right] \frac{1}{(s - M_s^2)^2},
\]
(B10)
where we have used the Mandelstam relation: \( u = -M_s^2 - t \).

[1] K. Nakamura et al. [Particle Data Group], J. Phys. G 37, 075021 (2010).
[2] For a thorough introduction to the SM, see for example, F. Halzen and A. D. Martin, “Quarks and Leptons: An Introductory Course in Modern Particle Physics,” (John Wiley & Sons, New York, 1984)
[3] M. C. Gonzalez-Garcia and M. Maltoni, Phys. Rept. 460, 1 (2008) [arXiv:0704.1800 [hep-ph]].
[4] J. Wess and J. Bagger, “Supersymmetry and Supergravity,” 2nd edition (Princeton University Press, Princeton, NJ, 1992).
[5] L. A. Anchordoqui, I. Antoniadis, H. Goldberg, X. Huang, D. Lust and T. R. Taylor, arXiv:1107.4309 [hep-ph].
[6] L. E. Ibanez, F. Marchesano and R. Rabadan, JHEP 0111, 002 (2001) [arXiv:hep-th/0105155];
D. Cremades, L. E. Ibanez and F. Marchesano, JHEP 0307, 038 (2003) [arXiv:hep-th/0302105]; F. G. Marchesano Buznego, arXiv:hep-th/0307252.
[7] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977); S. Weinberg, Phys. Rev. Lett. 40, 223 (1978); F. Wilczek, Phys. Rev. Lett. 40, 279 (1978).
[8] I. Antoniadis, E. Kiritsis and T. N. Tomaras, Phys. Lett. B 486, 186 (2000) [arXiv:hep-ph/0004214].
[9] P. Anastasopoulos, T. P. T. Dijkstra, E. Kiritsis and A. N. Schellekens, Nucl. Phys. B 759, 83 (2006) [arXiv:hep-th/0605226].
[10] F. del Aguila, G. D. Coughlan and M. Quiros, Nucl. Phys. B 307, 633 (1988) [Erratum-ibid. B 312, 751 (1989)].
[11] H. Goldberg, unpublished.
[12] D. M. Ghilencea, L. E. Ibanez, N. Irges and F. Quevedo, JHEP **0208** (2002) 016 [arXiv:hep-ph/0205083].

[13] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. **106**, 171801 (2011) [arXiv:1104.0699 [hep-ex]]; V. Cavaliere, FERMILAB-THESIS-2010-51.

[14] G. Punzi, talk given at the 23th Recontres de Blois, Blois France, May, 2011. For the latest update of CDF analysis see A. Annovi, P. Catastini, V. Cavaliere, and L. Ristori, [http://www-cdf.fnal.gov/physics/ewk/2011/wjj/7_3.html](http://www-cdf.fnal.gov/physics/ewk/2011/wjj/7_3.html).

[15] V. M. Abazov et al. [DØ Collaboration], Phys. Rev. Lett. **107**, 011804 (2011) [arXiv:1106.1921 [hep-ex]].

[16] E. J. Eichten, K. Lane and A. Martin, Phys. Rev. Lett. **106**, 251803 (2011) [arXiv:1104.0976 [hep-ph]]; C. Kilic and S. Thomas, [arXiv:1104.1002 [hep-ph]]; A. E. Nelson, T. Okui and T. S. Roy, [arXiv:1104.2030 [hep-ph]]; B. A. Dobrescu and G. Z. Krnjaic, [arXiv:1104.2893 [hep-ph]]; S. Jung, A. Pierce and J. D. Wells, [arXiv:1104.3139 [hep-ph]]; M. Buckley, P. Fileviez Perez, D. Hooper and E. Neil, Phys. Lett. B **702**, 256 (2011) [arXiv:1104.3145 [hep-ph]]; T. Plehn and M. Takeuchi, J. Phys. G **38**, 095006 (2011) [arXiv:1104.4087 [hep-ph]]; Q. H. Cao, M. Carena, S. Gori, A. Menon, P. Schwaller, C. E. M. Wagner and L. T. M. Wang, JHEP **1108**, 002 (2011) [arXiv:1104.4776 [hep-ph]]; J. Fan, D. Krohn, P. Langacker and I. Yavin, [arXiv:1106.1682 [hep-ph]]; J. L. Evans, B. Feldstein, W. Klemm, H. Murayama and T. T. Yanagida, [arXiv:1106.1734 [hep-ph]]; R. Harnik, G. D. Kribs and A. Martin, [arXiv:1106.2569 [hep-ph]]; J. F. Gunion, [arXiv:1106.3308 [hep-ph]].

[17] M. R. Buckley, D. Hooper, J. Kopp and E. Neil, Phys. Rev. D **83**, 115013 (2011) [arXiv:1103.6035 [hep-ph]]; F. Yu, Phys. Rev. D **83**, 094028 (2011) [arXiv:1104.0243 [hep-ph]]; K. Cheung and J. Song, Phys. Rev. Lett. **106**, 211803 (2011) [arXiv:1104.1375 [hep-ph]]; L. A. Anchordoqui, H. Goldberg, X. Huang, D. Lust and T. R. Taylor, Phys. Lett. B **701**, 224 (2011) [arXiv:1104.2302 [hep-ph]]; P. Ko, Y. Omura and C. Yu, [arXiv:1104.4066 [hep-ph]]; P. J. Fox, J. Liu, D. Tucker-Smith and N. Weiner, [arXiv:1104.4127 [hep-ph]]; Z. Liu, P. Nath and G. Peim, Phys. Lett. B **701**, 601 (2011) [arXiv:1105.4371 [hep-ph]]; M. R. Buckley, D. Hooper and J. L. Rosner, [arXiv:1106.3583 [hep-ph]]; A. E. Faraggi and V. M. Mehta, [arXiv:1106.5422 [hep-ph]]; K. Cheung and J. Song, [arXiv:1106.6141 [hep-ph]]. See also Ref. [5].

[18] D. E. Acosta et al. [CDF Collaboration], Phys. Rev. Lett. **95**, 131801 (2005) [arXiv:hep-ex/0506034]; T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. **99**, 171802 (2007) [arXiv:0707.2524]; T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. **102**, 091805 (2009) [arXiv:0811.0053].

[19] See e.g. R. Barate et al. [ALEPH Collaboration], Eur. Phys. J. C **12**, 183 (2000) [arXiv:hep-ex/9904011]; P. Abreu et al. [DELPHI Collaboration], Phys. Lett. B **485**, 45 (2000) [arXiv:hep-ex/0103025]; J. Abdallah et al. [DELPHI Collaboration], Eur. Phys. J. C **45**, 589 (2006) [arXiv:hep-ex/0512012].

[20] Y. Umeda, G. C. Cho and K. Hagiwara, Phys. Rev. D **58**, 115008 (1998) [arXiv:hep-ph/9805447].

[21] F. Abe et al. [CDF Collaboration], Phys. Rev. D **48**, 998 (1993). F. Abe et al. [CDF Collaboration], Phys. Rev. Lett. **74**, 3538 (1995) [arXiv:hep-ex/9501001]; F. Abe et al. [CDF Collaboration], Phys. Rev. D **55**, 5263 (1997) [arXiv:hep-ex/9702004]; B. Abbott et al. [DØ Collaboration], Phys. Rev. Lett. **82**, 2457 (1999) [arXiv:hep-ex/9807014]; T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D **79**, 112002 (2009) [arXiv:0812.4036].

[22] J. Alitti et al. [UA2 Collaboration], Z. Phys. C **49**, 17 (1991); J. Alitti et al. [UA2 Collaboration], Nucl. Phys. B **400**, 3 (1993).
[23] J. L. Hewett and T. G. Rizzo, arXiv:1106.0294 [hep-ph].

[24] M. Williams, C. P. Burgess, A. Maharana and F. Quevedo, arXiv:1103.4556 [hep-ph].

[25] E. Eichten, K. Lane and A. Martin, arXiv:1107.4075 [hep-ph]; K. Harigaya, R. Sato and S. Shirai, arXiv:1107.5265 [hep-ph].

[26] M. R. Buckley, D. Hooper, J. Kopp, A. Martin and E. T. Neil, arXiv:1107.5799 [hep-ph].

[27] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. Nadolsky and W. K. Tung, JHEP 0207, 012 (2002) arXiv:hep-ph/0201195.

[28] V. D. Barger, K. M. Cheung and P. Langacker, Phys. Lett. B 381, 226 (1996)

[29] Z. Kunszt and D. E. Soper, Phys. Rev. D 46, 192 (1992).

[30] Similar arguments have been previously advocated by Nelson, Okui, and Roy in Ref. [16] and by Liu, Nath, and Peim in Ref. [17].

[31] G. Aad et al. [ATLAS Collaboration], arXiv:1108.1582 [hep-ex].

[32] V. D. Barger and R. J. N. Phillips, “Collider Physics,” (Addison-Wesley, 1987).

[33] S. Chatrchyan et al. [CMS Collaboration], arXiv:1107.4771 [hep-ex].

[34] J. Polchinski, Phys. Rev. Lett. 75, 4724 (1995) arXiv:hep-th/9510017; J. Polchinski, arXiv:hep-th/9611050.

[35] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, Phys. Lett. B 436, 257 (1998)

[36] R. Blumenhagen, B. Kors, D. Lüst, T. Ott, Nucl. Phys. B 616, 3 (2001) hep-th/0107138; M. Cvetic, G. Shiu and A. M. Uranga, Phys. Rev. Lett. 87, 201801 (2001) arXiv:hep-th/0107143; M. Cvetic, G. Shiu and A. M. Uranga, Nucl. Phys. B 615, 3 (2001) arXiv:hep-th/0107166; I. Antoniadis, E. Kiritsis and T. Tomaras, Fortsch. Phys. 49, 573 (2001) [arXiv:hep-th/0111269]. See also Ref. [d].

[37] E. Kiritsis and P. Anastasopoulos, JHEP 0205, 054 (2002) arXiv:hep-ph/0201295; G. Honecker, T. Ott, Phys. Rev. D 70, 126010 (2004) hep-th/0404055; F. Gmeiner, R. Blumenhagen, G. Honecker, D. Lust and T. Weigand, JHEP 0601, 004 (2006) arXiv:hep-th/0510170; F. Gmeiner, G. Honecker, JHEP 0807, 052 (2008) [arXiv:0806.3039 [hep-th]].

[38] For reviews see e.g. E. Kiritsis, Phys. Rept. 421, 105 (2005) [Erratum-ibid. 429, 121 (2006)] [Fortsch. Phys. 52, 200 (2004)] arXiv:hep-th/0310001; R. Blumenhagen, M. Cvetic, P. Langacker and G. Shiu, Ann. Rev. Nucl. Part. Sci. 55, 71 (2005) arXiv:hep-th/0502005. R. Blumenhagen, B. Kors, D. Lüst, S. Stieberger, Phys. Rept. 445, 1 (2007) [hep-th/0610327].

[39] I. Antoniadis and S. Dimopoulos, Nucl. Phys. B 715 (2005) 120 [arXiv:hep-th/0411032].

[40] D. Berenstein and D. Sinitsyn, Nucl. Phys. B 660, 595 (2000) [arXiv:hep-th/9908065].
J. Halverson and R. R. Richter, JHEP 0912, 063 (2009) [arXiv:0905.3379 [hep-th]].

S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

S. Stieberger and T. R. Taylor, Phys. Rev. Lett. 97, 216101 (2006) [arXiv:hep-th/0607184].

S. Stieberger and T. R. Taylor, Phys. Rev. D 74, 126007 (2006) [arXiv:hep-th/0609175].

M. L. Mangano and S. J. Parke, Phys. Rept. 200, 301 (1991) [arXiv:hep-ph/9601359].

L. J. Dixon, Nuovo Cim. A 57, 190 (1968).

S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

S. Stieberger and T. R. Taylor, Phys. Rev. Lett. 97, 211603 (2006) [arXiv:hep-th/0607184].

S. Stieberger and T. R. Taylor, Phys. Rev. D 74, 126007 (2006) [arXiv:hep-th/0609175].

M. L. Mangano and S. J. Parke, Phys. Rept. 200, 301 (1991) [arXiv:hep-ph/9601359].

L. J. Dixon, Nuovo Cim. A 57, 190 (1968).

S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

S. Stieberger and T. R. Taylor, Phys. Rev. Lett. 97, 211603 (2006) [arXiv:hep-th/0607184].

S. Stieberger and T. R. Taylor, Phys. Rev. D 74, 126007 (2006) [arXiv:hep-th/0609175].

M. L. Mangano and S. J. Parke, Phys. Rept. 200, 301 (1991) [arXiv:hep-ph/9601359].

G. Veneziano, Nuovo Cim. A 57, 190 (1968).

S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

S. Stieberger and T. R. Taylor, Phys. Rev. Lett. 97, 211603 (2006) [arXiv:hep-th/0607184].

S. Stieberger and T. R. Taylor, Phys. Rev. D 74, 126007 (2006) [arXiv:hep-th/0609175].

M. L. Mangano and S. J. Parke, Phys. Rept. 200, 301 (1991) [arXiv:hep-ph/9601359].

L. J. Dixon, Nuovo Cim. A 57, 190 (1968).

S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

S. Stieberger and T. R. Taylor, Phys. Rev. Lett. 97, 211603 (2006) [arXiv:hep-th/0607184].

S. Stieberger and T. R. Taylor, Phys. Rev. D 74, 126007 (2006) [arXiv:hep-th/0609175].

M. L. Mangano and S. J. Parke, Phys. Rept. 200, 301 (1991) [arXiv:hep-ph/9601359].

G. Veneziano, Nuovo Cim. A 57, 190 (1968).

S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

S. Stieberger and T. R. Taylor, Phys. Rev. Lett. 97, 211603 (2006) [arXiv:hep-th/0607184].

S. Stieberger and T. R. Taylor, Phys. Rev. D 74, 126007 (2006) [arXiv:hep-th/0609175].

M. L. Mangano and S. J. Parke, Phys. Rept. 200, 301 (1991) [arXiv:hep-ph/9601359].

L. J. Dixon, Nuovo Cim. A 57, 190 (1968).

S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

S. Stieberger and T. R. Taylor, Phys. Rev. Lett. 97, 211603 (2006) [arXiv:hep-th/0607184].

S. Stieberger and T. R. Taylor, Phys. Rev. D 74, 126007 (2006) [arXiv:hep-th/0609175].

M. L. Mangano and S. J. Parke, Phys. Rept. 200, 301 (1991) [arXiv:hep-ph/9601359].

L. J. Dixon, Nuovo Cim. A 57, 190 (1968).

S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

S. Stieberger and T. R. Taylor, Phys. Rev. Lett. 97, 211603 (2006) [arXiv:hep-th/0607184].

S. Stieberger and T. R. Taylor, Phys. Rev. D 74, 126007 (2006) [arXiv:hep-th/0609175].

M. L. Mangano and S. J. Parke, Phys. Rept. 200, 301 (1991) [arXiv:hep-ph/9601359].

L. J. Dixon, Nuovo Cim. A 57, 190 (1968).

S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

S. Stieberger and T. R. Taylor, Phys. Rev. Lett. 97, 211603 (2006) [arXiv:hep-th/0607184].

S. Stieberger and T. R. Taylor, Phys. Rev. D 74, 126007 (2006) [arXiv:hep-th/0609175].

M. L. Mangano and S. J. Parke, Phys. Rept. 200, 301 (1991) [arXiv:hep-ph/9601359].

L. J. Dixon, Nuovo Cim. A 57, 190 (1968).

S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

S. Stieberger and T. R. Taylor, Phys. Rev. Lett. 97, 211603 (2006) [arXiv:hep-th/0607184].

S. Stieberger and T. R. Taylor, Phys. Rev. D 74, 126007 (2006) [arXiv:hep-th/0609175].

M. L. Mangano and S. J. Parke, Phys. Rept. 200, 301 (1991) [arXiv:hep-ph/9601359].

L. J. Dixon, Nuovo Cim. A 57, 190 (1968).

S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

S. Stieberger and T. R. Taylor, Phys. Rev. Lett. 97, 211603 (2006) [arXiv:hep-th/0607184].

S. Stieberger and T. R. Taylor, Phys. Rev. D 74, 126007 (2006) [arXiv:hep-th/0609175].

M. L. Mangano and S. J. Parke, Phys. Rept. 200, 301 (1991) [arXiv:hep-ph/9601359].

L. J. Dixon, Nuovo Cim. A 57, 190 (1968).
106006 (2011) \texttt{arXiv:1012.3466\[hep-ph\]].

[72] I. Antoniadis, Lect. Notes Phys. 720, 293 (2007) \texttt{arXiv:hep-ph/0512182}.

[73] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429, 263 (1998) \texttt{arXiv:hep-ph/9803315}.

[74] C. D. Hoyle, U. Schmidt, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, D. J. Kapner and H. E. Swanson, Phys. Rev. Lett. 86, 1418 (2001) \texttt{arXiv:hep-ph/0011014}; C. D. Hoyle, D. J. Kapner, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, U. Schmidt and H. E. Swanson, Phys. Rev. D 70, 042004 (2004) \texttt{arXiv:hep-ph/0405262}; D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle and H. E. Swanson, Phys. Rev. Lett. 98, 021101 (2007) \texttt{arXiv:hep-ph/0611184}.

[75] K. R. Dienes and B. Thomas, \texttt{arXiv:1106.4546\[hep-ph\]]; K. R. Dienes and B. Thomas, \texttt{arXiv:1107.0721\[hep-ph\]}.