CP Violation in Hyperon Decays from SUSY
with Hermitian Yukawa and $A$ Matrices

Chuan-Hung Chen

Institute of Physics, Academia Sinica, Taipei,
Taiwan 115, Republic of China

Abstract

We show that a large CP asymmetry in hyperon decays can be naturally realized in the framework of SUSY models. The possibility is implemented by the hermiticities of Yukawa and $A$ matrices. And also, the observed values of $\epsilon$ and $\epsilon'$ are explained.
The origin of CP violation (CPV) is one of mysterious phenomena in particle physics since its discovery in the neutral kaon decays in 1964 [1]. Recently, another CP asymmetry, sin 2β, in the decay of $B \to J/\Psi K_s$ is observed by BARBAR [2] and BELLE [3]. Nevertheless, our understanding of CPV is still exiguous. In the standard model (SM), the unique source of CPV is from the Cabibbo-Kobayashi-Maskawa (CKM) matrix [4] induced from the three-generation quark mixings and described by the three angles $\alpha, \beta$ and $\gamma$ or $\phi_1, \phi_2$ and $\phi_3$. Even though the SM prediction on the indirect CP violating parameter $\epsilon$ in the kaon system can be fitted well with current experimental data, due to the large uncertainties from hadronic matrix elements, so far it is unclear whether the result in the SM is consistent with the observed value of the direct CP violating parameter $\epsilon'$ measured by KTeV [5] and NA31 [6]. Furthermore, the problem of baryogenesis is insolvable in the SM. In addition, the predicted CP asymmetry of $O(10^{-5})$ in hyperon decays of $\Xi \to \Lambda \pi \to p \pi \pi$ is about one order of the magnitude smaller than that of $O(10^{-4})$ proposed by the experiment E871 at Fermilab [7]. Hence, it is inevitable to look for new physics which gives observable CPV effects.

Supersymmetric theories not only supply an elegant mechanism for the breaking of the electroweak symmetry and a solution to the hierarchy problem, but provide many new weak CP violating phases. These CP phases usually arise from the trilinear and bilinear supersymmetry (SUSY) soft breaking $A$ and $B$ terms, the $\mu$ parameter for the scalar mixing and gaugino masses, respectively. Unfortunately, it has been shown that with the universal assumption on soft breaking parameters, these phases are severely bounded by electric dipole moments (EDMs) [8] so that the contributions to $\epsilon$ and $\epsilon'$ are far below the experimental values. In the literature, some strategies to escape the constraints of EDMs have been suggested. They are mainly (a) by setting the squark masses of the first two generations to be as heavy as few TeV [9] but allowing the third one being light; (b) by including all possible contributions to EDMs such that somewhat cancellations occur in some allowed parameter space [10, 11]; and (c) with the non-universal soft $A$ terms instead of universal ones. In particular, those models with non-universal parameters have been demonstrated that they can be realized in some string-inspired models [12, 13, 14, 15].

Among the models with non-degenerate soft trilinear terms, for satisfying the bounds of EDMs, the phases in the diagonal elements of the $A$ matrix should be set to be small in any basis artificially although the remaining large phases and the light sfermions are still allowed to explain the observed values of $\epsilon$ and $\epsilon'$. To overcome this problem, it is proposed in Ref. [16] to use hermitian Yukawa and $A$ matrices. The construction of a hermitian Yukawa matrix can be implemented based on some symmetries such as the global (gauged) horizontal $SU(3)_H$ symmetry [17] and left-right symmetry [18]. Although the hermiticity will be broken by renormalization group (RG) effects, it is shown [16] that their contributions to EDMs are two orders of the magnitude below the present experimental limit. Moreover, due to the hermitian property, a special relation is obtained as

$$\left(\delta^d_{12}\right)_{LR} \simeq \left(\delta^{d*}_{12}\right)_{RL}$$

where $\left(\delta^d_{12}\right)_{LR} \equiv (V^{d\dagger} A^d v_d V^d)_{12}/m_{\tilde{q}}^2, A^{d\dagger} \simeq A^d, v_d$ is the vacuum expectation value (VEV) of the Higgs filed $\Phi_d$ for supplying the masses of down-type quarks $A^{d\dagger}$ is the mixing matrix for diagonalizing the mass matrix of down-type quarks and $m_{\tilde{q}}$ is the average mass of squark in super-CKM basis. We note that the mixing matrix for mass eigenstates of left-handed down-type quarks is the same as that for the right-handed one. In this paper, we will show the implication of Eq. [16] on the CP asymmetry of hyperon decays.
The interactions describing $|\Delta S| = 1$ nonleptonic decays of $\Xi$ and $\Lambda$ are the same as those for $K \to \pi\pi$ processes. Therefore, those CP violating effects contributing to hyperon decays will also contribute to $\epsilon'$. As a consequence, the CP observable for hyperon decays is limited to be $O(10^{-5})$ [[21, 22]] level by the bound of $\epsilon'$. One way to avoid the constraint is that the couplings contributing to parity conserving parts of hyperon decays are enhanced but suppressed for parity violating ones.

To understand the CPV in hyperon decays, we start by writing the decay amplitude as

$$Amp(B_i \to B_f\pi) = S + P\hat{\sigma} \cdot \hat{q}$$

where $B_{i,f}$ are the initial and final baryons, $S$ and $P$ denote the parity violating and conserving amplitudes, respectively, and $\hat{q}$ is the momentum direction of outgoing baryon $B_f$. We note that Eq. (2) has to be multiplied by a factor of $G_F m_\pi^2$, with $m_\pi$ being the pion mass, for getting correct decay rate. For simplicity, the amplitudes $S$ and $P$ can be parametrized as

$$S = \sum_i S_i e^{i(\delta^S_i + \theta^S_i)}$$

$$P = \sum_i P_i e^{i(\delta^P_i + \theta^P_i)}$$

where we have separated the strong phases $\delta_i$ generated by final state interactions and the weak CP violating phases $\theta_i$ from decay amplitudes such that $S_i$ and $P_i$ amplitudes are real, with $i$ representing all possible final isospin states. The decay distribution of proton for the chain decays $\Xi \to \Lambda\pi \to p\pi\pi$ with unpolarized $\Xi$ is then given by

$$4\pi \frac{dP}{d\Omega} = 1 + \alpha_{\Xi}\alpha_{\Lambda}\hat{p}_\Xi \cdot \hat{p}$$

and

$$\alpha_H \equiv 2Re(S^*_H P_H)/(|S_H|^2 + |P_H|^2)$$

where $\alpha_{\Xi(\Lambda)}$ is the polarization of $\Lambda(p)$ for $\Xi \to \Lambda\pi$ ($\Lambda \to p\pi$). According to Eq. (4), the direct CP violating observable $A$ can be defined as:

$$A = \frac{\alpha_{\Xi}\alpha_{\Lambda} + \bar{\alpha}_{\Xi}\bar{\alpha}_{\Lambda}}{\alpha_{\Xi}\alpha_{\Lambda} - \bar{\alpha}_{\Xi}\bar{\alpha}_{\Lambda}},$$

$$A \approx A_{\Xi} + A_{\Lambda}$$

with

$$A_H = \frac{\alpha_H + \bar{\alpha}_H}{\alpha_H - \bar{\alpha}_H} \quad (H = \Xi, \Lambda)$$

where $\bar{\alpha}_H$ is the corresponding quantity for the antihyperon $H$. Although the $|\Delta S| = 1$ hyperon decays include two isospin channels $\Delta I = 1/2$ and $\Delta I = 3/2$, the contribution of $\Delta I = 3/2$ amplitude can be neglected so that the asymmetries for $\Xi \to \Lambda\pi$ and $\Lambda \to P\pi$, from Eq. (5), can be obtained as [[23, 24, 25]]

$$A_{\Xi} \approx -\tan(\delta^P_1 - \delta^S_1)\sin(\theta^P_1 - \theta^S_1),$$

$$A_{\Lambda} \approx -\tan(\delta^P_2 - \delta^S_2)\sin(\theta^P_2 - \theta^S_2).$$

It is known that the strong phases for $\Lambda \to p\pi$ decay are $\delta^S_2 = 6.0^o$ and $\delta^P_1 = -1.1^o$. However, for the $\Xi$ decay, we take $\delta^S_2 = 0.2^o$ and $\delta^P_2 = -1.7^o$ calculated by using the
chiral perturbation theory \cite{24}. The result of \( \delta_S^2 \) recently is confirmed in the framework of a relativistic chiral unitary approach \cite{25}. Due to the small values of \( \delta_S^2 \) and \( \delta_P^2 \), consequently, the CP asymmetry of \( \Xi \) is smaller than that of \( \Lambda \) by one order of the magnitude. Hence, in our following analysis, we only concentrate on \( A_{\Lambda} \). To estimate the weak CP violating phases \( \theta_1^P \) and \( \theta_1^S \), we adopt the following approximation \cite{21}

\[
\theta_1^P \approx \frac{\text{Im}[\mathcal{M}(\Lambda \rightarrow p\pi)]}{\text{Re}[\mathcal{M}(\Lambda \rightarrow p\pi)]} \quad (7)
\]

by assuming that the CP violating contributions are much less than the CP conserving ones. Here, \( \mathcal{M}(\Lambda \rightarrow p\pi) \) express the transition matrix elements of relevant effective operators, and their real parts can be obtained from the experimental measurements, with the parity violating and conserving amplitudes of \( l = 0 \) and \( l = 1 \), respectively. In sum, CP violating phases for \( \Lambda \rightarrow p\pi \) decay can be written as

\[
\theta_1^P \approx -\frac{\text{Im}[\mathcal{M}(\Lambda \rightarrow p\pi)]}{9.98 G_F m_\pi^2} \quad (9)
\]

\[
\theta_1^S \approx \frac{\text{Im}[\mathcal{M}(\Lambda \rightarrow p\pi)]}{1.47 G_F m_\pi^2} \quad (8)
\]

where we have used \( S_1 = 1.47 \) and \( P_1 = 0.6 \).

As stated early, the interactions for \( \Lambda \rightarrow p\pi \) are the same as those for \( K \rightarrow \pi\pi \). According to the analysis of Refs. \cite{26, 27}, in SUSY models the main effects for \( \epsilon' \) are from the gluino penguin contributions. The associated effective Lagrangian is given by

\[
\mathcal{L}_{\text{eff}} = C_8(\mu)O_8 + \bar{C}_8(\mu)\bar{O}_8 \quad (10)
\]

where the effective Wilson coefficient \( C_8(\mu) \) and the operator \( O_8 \) are expressed by

\[
O_8 = \frac{g_s m_s}{8\pi^2} d_i \sigma_{\mu\nu} t^a P_R s G^\mu\nu_a, \quad C_8(\mu) = \frac{\alpha_s \pi m_\tilde{g}}{m_\tilde{q}} (\delta_{12})_{LR} \left(-\frac{1}{3}M_1 + 3M_2\right), \quad (11)
\]

respectively, with

\[
M_1(x) = \frac{1 + 4x - 5x^2 + 4x \ln(x) + 2x^2 \ln(x)}{2(1-x)^4},
\]

\[
M_2(x) = \frac{x^2 5 - 4x - x^2 + 2 \ln(x) + 4x \ln(x)}{2(1-x)^4}.
\]

\( \bar{O}_8 \) and \( \bar{C}_8(\mu) \) in Eq. \((11)\) can be obtained from \( O_8 \) and \( C_8(\mu) \) easily by changing the role of chirality therein each other, \( (\delta_{12})_{LR(RL)} \) denotes the mixing effect between left (right)- and right (left)-handed squarks, \( m_\tilde{g} \) is the gluino mass, \( Tr(t^a t^b) = \delta^{ab} / 2 \) and \( x = m_\tilde{q}^2 / m_\tilde{g}^2 \). From Eq. \((11)\), we see that this interaction is no further suppression from the light quark mass.

In terms of Eq. \((10)\), we know that \( \epsilon' \) will be related to \( \text{Im}\left[(\delta_{12}^d)_{LR} - (\delta_{12}^d)_{RL}\right] \) in which minus is from the different chirality. In general, it is not necessary that \( (\delta_{12}^d)_{LR} \) is the same as \( (\delta_{12}^d)_{RL} \). Therefore, it is often concluded that SUSY models can agree with the measured value of \( \epsilon' \). As for the hyperon CPV, from Eqs. \((8)\) and \((9)\), we get
\[ \theta_1^s \propto \text{Im} \left[ (\delta_{12}^d)_{LR} - (\delta_{12}^d)_{RL} \right] \text{ and } \theta_1^\prime \propto -\text{Im} \left[ (\delta_{12}^d)_{LR} + (\delta_{12}^d)_{RL} \right]. \] If we assume that \( \theta_1^s \) is the dominant one, due to the constraint of \( \epsilon' \), the CPV of \( O(10^{-5}) \) in Eq. (1) can be obtained. But, if we set \( \theta_1^\prime \) to be the dominant one, \( \epsilon' \) from the same mechanism will be suppressed. It is interesting to ask whether \( \epsilon'/\epsilon \sim 2 \times 10^{-3} \) and \( A_\Lambda \sim 10^{-4} \) can both be reached in the framework of SUSY.

From the above analysis, we know that it is hopeless if the mechanism for \( \epsilon' \) and for \( A_\Lambda \) is the same. However, the possibility can be realized if Yukawa and non-universal \( A \) matrices are hermitian. As mentioned before, such a kind of SUSY model implies \( (\delta_{12}^d)_{LR} \approx (\delta_{12}^d)_{RL} \). That is, \( \epsilon' \) is suppressed in this gluino penguin contribution but \( \theta_1^\prime \) is enhanced. To emphasize that both large \( \epsilon' \) and \( A_\Lambda \) can be obtained in the SUSY model, for simplicity, we adopt the CP violating phase arises from the Yukawa matrix and other SUSY parameters are real \( [16] \).

Although this SUSY model does not introduce the new weak CP phase \( s \), it still provides an abundant particle spectrum and flavor physics that can be tested in the current and future experiments. As a result, \( W \)-boson and charginos will contribute to \( \epsilon' \). Similar to the SM, by combining these effects altogether, \( \epsilon' \) can be in agreement with the experimental value even though it is sensitive to the uncertainties of hadronic matrix elements. It is shown that even using only chargino box-diagram contributions \( [28] \), with more generic assumptions on SUSY models, the result is also possibly consistent with the data.

As known, \( \epsilon \) from gluino box diagrams can give a bound on \( \text{Im}(\delta_{12}^d)_{LR} \) through \( \text{Im}(\delta_{12}^d)_{LR}^2 \) \( [20] \). In order to constrain \( \text{Im}(\delta_{12}^d)_{LR} \) directly, we consider the contributions of long-distance effects to \( \epsilon \), in which the transition matrix element for \( \langle \bar{K}^0 | H_{\text{eff}} | K^0 \rangle \) comes from the \( \pi, \eta \) and \( \eta' \) poles. According to Ref. \( [29] \), the result is shown as

\[ \epsilon_{LD} \approx \frac{\omega}{40\sqrt{2}(m_K^2 - m_\pi^2)m_K \Delta m_K} < K^0 | \mathcal{L}_{\text{even}} | \pi^0 > < \pi^0 | \mathcal{L}_{\text{odd}} | \bar{K}^0 > \] (12)

where \( \Delta m_K \) is the mass difference of \( K_L \) and \( \bar{K}_S \), \( \omega \) stands for the contributions from different poles and its accessible range is \( 1 < |\omega| < 4 \), \( \mathcal{L}_{\text{even(odd)}} \) denotes the CP-even (odd) interaction and the explicit expression of \( \mathcal{L}_{\text{odd}} \) is

\[ \mathcal{L}_{\text{odd}} = \text{Im} (f_{PC}) \bar{d} i \sigma_{\mu \nu} t^a s G^{\mu \nu} \]

with

\[ f_{PC} = \frac{g_s \alpha_s \eta_g}{16\pi m_3^{1/2}} \left[ (\delta_{12}^d)_{LR} + (\delta_{12}^d)_{RL} \right] \left(-\frac{1}{3}M_1(x) + 3M_2(x)\right) \] (13)

\[ \eta_g = \left(\frac{\alpha_s(\mu_\Lambda)}{\alpha_s(m_\bar{c})}\right)^{-14/27} \left(\frac{\alpha_s(m_c)}{\alpha_s(m_t)}\right)^{-14/25} \left(\frac{\alpha_s(m_t)}{\alpha_s(m_\bar{t})}\right)^{-14/23} \left(\frac{\alpha_s(m_\bar{t})}{\alpha_s(m_\bar{b})}\right)^{-14/21} \]

where \( \eta_g \) is the QCD effects \( [32] \). For the CP-even part, we can use the experimental value \( < K^0 | \mathcal{L}_{\text{even}} | \pi^0 > \approx 2.58 \times 10^{-7} \) GeV. However, for the CP-odd part, according to the MIT bag model \( [30] \), we have \( < \pi^0 | \mathcal{L}_{\text{odd}} | \bar{K}^0 > \approx \text{Im}(f_{PC}) A_{K\pi} \) and \( A_{K\pi} = 0.4 \) GeV for \( \alpha_s \approx 1 \). Hence, the long-distance effects on \( \epsilon \) is

\[ \epsilon_{LD} \approx 4.8 \times 10^6 \omega \text{Im} f_{PC}. \]

Requiring the value of \( \epsilon_{LD} \) being less than \( 2.28 \times 10^{-3} \), the upper bound can be given as \( \text{Im} f_{PC} \leq (4.7/\omega) \times 10^{-10} \).
Due to \((\delta_{12}^d)_{LR} \simeq (\delta_{12}^d)_{RL}\) in our case, the weak phase \(\theta_1^S\) is negligible. By using Eq. (3) and the matrix element calculated by the MIT bag model \([20,31]\), the CP violating phase \(\theta_1^P\) can be given as
\[
\theta_1^P \approx -4.8 \times 10^6 \text{Im}(f_{PC})B_p
\]
where \(B_p\) represents the uncertainty in estimating the matrix elements of hyperon decays and the allowed range is \(0.35 < B_p < 2.6\) \([31]\). In terms of Eq. (3) and the bound of \(\text{Im}f_{PC}\), we obtain
\[
|A_\Lambda| \leq 2.93 \times 10^{-4} \frac{B_p}{|\omega|}.
\]
Although the result is sensitive to the theoretical uncertainty, by taking a proper value, the CP asymmetry \(A_\Lambda\) can reach \(O(10^{-4})\) easily.

In summary, it has an enormous progress in SUSY models since a nonzero value of \(\epsilon'\) is confirmed by the KTeV experiment. Although these models can lead to the observed values of \(\epsilon\) and \(\epsilon'\) well, with the same mechanism and without a further fine tuning, the predicted CP asymmetry in hyperon decays is below the expected value proposed by the E871 experiment. Hence, to obtain large values for both \(\epsilon'\) and \(A_\Lambda\), the Feynman diagrams for each of them should be different. We show that the observed value of \(\epsilon'\) and \(A_\Lambda = O(10^{-4})\) can be reached in the framework of SUSY models naturally if Yukawa and soft breaking A terms are hermitian. In addition, once the CP asymmetry of \(O(10^{-4})\) is measured in hyperon decays, it also gives a strong evidence to support the existence of SUSY.

Acknowledgments
I would like to thank D. Chang, X.G. He, C.Q. Geng and H.N. Li for their useful discussions. This work was supported by the National Science Council of the Republic of China under Contract NSC-90-2112-M-001-069.
References

[1] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turly, Phys. Rev. Lett. 13, 138 (1964).

[2] BABAR Collaboration, B. Aubert et. al., [hep-ex/0107013].

[3] BELLE Collaboration, A. Abashian et. al., Phys. Rev. Lett. 86, 2509 (2001).

[4] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[5] KTeV Collaboration, A. Alavi-Harati et. al., Phys. Rev. Lett. 83, 22 (1999).

[6] NA31 Collaboration, G.D. Barr et. al., Phys. Lett. B317, 233 (1993).

[7] C. White et. al., Nucl. Phys. Proc. Suppl. B71, 451 (1999).

[8] R. Garisto and J.D. Wells, Phys. Rev. D55, 1611 (1997); R. Garisto, ibid, D49 4820 (1994) and the references therein.

[9] K.S. Babu et.al., Phys. Rev. D59, 016004 (1999).

[10] T. Ibrahim and P. Nath, Phys. Rev. D57, 478 (1998).

[11] M. Brhlik, G.J. Good and G.L Kane, Phys. Rev. D59, 115004 (1999).

[12] S.A. Abel and J.M. Frére, Phys. Rev. D55, 1623 (1997).

[13] S. Khalil, T. Kobayashi and A. Masiero, Phys. Rev. D60, 075003 (1999); S. Khalil and T. Kobayashi, Phys. Lett. B460, 341 (1999).

[14] S. Khalil, T. Kobayashi and O. Vives, Nucl. Phys. B580, 275 (2000).

[15] M. Brhlik et. al., Phys. Rev. Lett. 84, 3041 (2000); A. Masiero and O. Vives, Phys. Rev. Lett. 86, 26 (2001).

[16] S. Abel, D. Bailin, S. Khalil and O. Lebedev, Phys. Lett. B504, 241 (2001).

[17] A. Masiero and T. Yanagida, hep-ph/9812228.

[18] K.S. Babu et. al., Phys. Rev. D61, 091701 (2000).

[19] R.E. Marshak, Riazuddin and C.P. Ryan, Theory of weak interactions in particle physics (Wiley, New York, 1969).

[20] J.F. Donoghue and S. Pakavasa, Phys. Rev. Lett. 55, 162 (1985); J.F. Donoghue et. al., Phys. Rev. D34, 833 (1986).

[21] X.G. He and G. Valencia, Phys. Rev. D52, 5257 (1995).

[22] D. Chang and C.H. Chen, Chin. J. Phys. 34, 831 (1996).

[23] L. Roper et al., Phys. Rev. 138, 190 (1965).
[24] M. Lu, M. Savage and M. Wise, Phys. Lett. B337, 133 (1994); A. Datta and S. Pakvasa, Phys. Lett. B344, 430 (1995).

[25] Ulf-G. Meißner and J.A. Oller, Phys. Rev. D64, 014006 (2001).

[26] F. Gabbiani et. al., Nucl. Phys. B477, 321 (1996).

[27] A. Masiero and H. Murayama, Phys. Rev. Lett. 83, 907 (1999).

[28] S. Khalil and O. Lebedev, Phys. Lett. B515, 387 (2001).

[29] J. F. Donoghue and B. Holstein, Phys. Rev. D32, 1152 (1985); H.Y. Cheng, Int. J. Mod. Phys. A7, 1059 (1992).

[30] J.F. Donoghue et. al., Phys. Rev. D23, 1213 (1981).

[31] X.G. He et. al., Phys. Rev. D61, 071701 (2000).

[32] D. Chang et. al., Phys. Rev. Lett. 74, 3927 (1995).