Noise radar range doppler imaging via 2D generalized smoothed-ID

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This paper presents an algorithm to enhance the range-Doppler imaging performance for noise radar. Traditional matched filtering based method suffers from high range and Doppler sidelobes, which makes the weak targets overwhelmed by the sidelobes of strong targets or clutters in the range-Doppler map. Sparse recovery based methods have been widely used to suppress such sidelobes, but most of them assume a repetitive transmit waveform, which cannot be applied to noise radar applications. In this letter, the range-Doppler imaging problem for noise radar is formulated as a sparse recovery model and solved by a 2D generalized smoothed-ID algorithm. The proposed method can deal with the random waveforms varying among different pulses in noise radar. The robustness of the proposed method in strong noise and clutters is validated by simulation results.

Introduction: The Range Doppler imaging aims to estimate the range and Doppler information of targets to perform target localization and motion estimation, and has been a hot topic in radar signal processing [1–3]. In range Doppler imaging applications, due to the use of random transmit waveforms, the noise radar has several advantages over the classical radar [4], such as a good electronic counter-countermeasure (ECCM) capability and the absence of range and Doppler ambiguity [5]. Noise radar has been applied in traffic control, surveillance, and security applications [6]. However, the noise radar suffers from a more serious masking effect than traditional chirp signal based radars, when the matched filters are applied for range-Doppler domain coherent integration. The weak targets become undetectable since they are easily overwhelmed by the high sidelobes of strong targets [7]. The dynamic range is seriously reduced by the masking effect, which obstructs the application of noise radar to the target imaging.

One effective way to deal with the masking effect is to design waveforms with better auto-correlation function (ACF). A non-linear frequency modulated (NLFM) signal is proposed in [8] with phase improvement algorithm to suppress the peak sidelobe level of ACF. Another way is to design the digital receiver with low sidelobes. Sparse recovery based methods have been applied in 2D (i.e. range-Doppler domain) imaging. The sparsity of targets in the range-Doppler map is induced to the imaging problem as a prior knowledge, so that the target information can be recovered more accurately with reduced sidelobes [1]. However, the application of this kind of methods requires the radar to transmit repetitive waveforms, which is not suitable for noise radars whose transmit waveforms show random behaviours. Therefore, in this letter, we establish a new range-Doppler imaging model based on sparse recovery, which can deal with the random waveforms varying among different pulses in noise radar. Then, a corresponding solver is proposed based on a modification of the smoothed-ID algorithm [9]. The proposed method can effectively suppress the sidelobe floors of strong targets and alleviate the masking effect in noise radar, and is robust when the signal-to-noise ratio (SNR) and the signal-to-clutter ratio (SCR) is low.

Signal model: In this section, we establish the signal model of the baseband digital echoes of the noise radar. During one coherent processing interval (CPI), the transmit waveforms are varying among different pulse repetition intervals (PRI), and the waveform during the nth PRI is denoted as \( s_{n,PRI}(n) \), where \( t_f \) denotes the fast-time sampling interval. Suppose \( K \) targets are in the illuminated scene, \( s_{n,k}(n) \) and \( f_{k,m} \) are the \( k \)th target's scattering coefficient, time delay and Doppler frequency respectively. Then, the baseband digital echoes of the noise radar can be expressed as:

\[
y_{n,PRI}(n) = \sum_{k=1}^{K} s_{n,k}(n) e^{j2\pi f_{k,m} n}.
\]

where \( y_{n,PRI}(n) \) denotes the echoes corresponding to the \( n \)th PRI, which can be represented in a matrix form \( Y \in \mathbb{C}^{N \times M} \) (\( M \) is the number of PRIs in one CPI, \( N \) is the number of fast samples during one PRI), \( t_f \) denotes the slow-time sampling interval (i.e. pulse repetition time (PRT)).

Limitations of traditional methods: To recover the range Doppler map from the echoes \( y_{n,PRI}(n) \), traditional matched filtering method can be applied to perform pulse compression for each PRI and then perform coherent integration among different PRIs, but it would suffer from high range and Doppler sidelobes. To eliminate the sidelobes, a sequential sparse model like [10] can be applied here to formulate the following:

\[
\begin{align*}
\text{for } m = 1, \ldots, M: & \min_{X_{n,m}} \|X_{n,m}\|_0, s.t. \|Y(m) - A_n X_{n,m}\|_2^2 < \epsilon \\
\text{for } n = 1, \ldots, N: & \min_{Z(n,m)} \|Z(n,m)\|_0, s.t. \|X(n,:) - D^T Z(n,:)\|_2^2 < \epsilon
\end{align*}
\]

where \( Y(m) \) and \( X_{n,m} \) is the \( m \)th column of the echo matrix \( Y \) (i.e. the echo during the \( m \)th PRI). \( X \in \mathbb{C}^{L_r \times M} \) denotes the range profile matrix, where \( L_r \) denotes the number of range bins. \( A_n \in \mathbb{C}^{N \times L_r} \) is the range-Doppler domain dictionary, whose atoms are constructed as the delayed replica of the \( m \)th transmit pulse. \( Z \in \mathbb{C}^{L_r \times L_d} \) is the range-Doppler image to be recovered (\( L_d \) denotes the number of Doppler bins). \( D \in \mathbb{C}^{N \times M} \) denotes the Doppler domain dictionary. In this sequential model, the range profile matrix \( X \) is constructed first, followed by the recovery of the range-Doppler map \( Z \). Such method would yield satisfying results when the clutters and noise are weak and the illuminated area shows obvious sparse features in range domain, but would suffer performance degradation in the circumstances of strong clutter and noise.

Proposed 2D generalized smoothed-ID (2D-GS-ID) method: To avoid the limitations of the sequential sparse recovery model, we propose an optimization model to jointly consider the sparsity of targets in range-Doppler domain. Considering that the \( l_0 \) norm of a matrix is a discontinuous indicator function, while minimizing such function is highly sensitive to noise, we use a continuous smoothed-\( l_0 \) cost function \( \lim_{\alpha \to 0} f_{\alpha}(Z) = \frac{1}{2} \|X - DZ\|_2^2 \) to approximate the discontinuous \( l_0 \) norm (\( Z_{l,0} \) denotes the \( l \)th column of the matrix \( Z \)). The proposed model can be written as:

\[
\min_{Z} \lim_{\alpha \to 0} f_{\alpha}(Z) = \frac{1}{2} \|X - DZ\|_2^2
\]

s.t. \( \|Y(m) - A_n X_{n,m}\|_2^2 < \epsilon \) for \( m = 1, \ldots, M \)

For small values of \( \alpha \), the minimization of \( f_{\alpha}(Z) \) approaches the \( l_0 \) norm but has a lot of local miniums, while for a larger \( \alpha \), \( f_{\alpha}(Z) \) becomes smoother but has less capability to yield sparse results. In order to avoid the solver trapped into the local minima, the optimization problem (3) is solved with iterations with a gradually decreasing value of \( \alpha \).

For each \( \alpha \), the \( f_{\alpha}(Z) \) is minimized with the constraints of Equation (3) by a gradient descend step to promote sparsity and a projection step to promote accuracy.

In the gradient descend step, we find a matrix \( \hat{Z} \) that makes \( f_{\alpha}(Z) \) smaller by

\[
\hat{Z} = Z - \mu \nabla f_{\alpha}(Z)
\]

where \( \mu \) is the step size and \( \nabla f_{\alpha}(Z) \) denotes the gradient of \( f_{\alpha}(Z) \).

\[
\nabla f_{\alpha}(Z) = Z_{l,0} / \alpha^2 \cdot \exp(\frac{-|Z_{l,0}|^2}{2\alpha^2})
\]

In the projection step, the sparsity optimized matrix \( \hat{Z} \) should be projected into the feasible set to ensure recovery accuracy. This step can be performed by a range profile projection and a 2D range-Doppler map projection. The range profile projection can be considered as an optimization problem:

\[
\hat{X}(n,:) = \min_{X(n,:)} \|Y(n) - A_n X_{n,m}\|_2^2 + \|X(n,:) - \hat{X}(n,:)\|_2^2
\]
where $\hat{X} = \hat{Z}D$ is the range profile corresponding to the optimized $\hat{Z}$ in the gradient descend step. $\lambda$ is the regularization parameter, which ensures that the recovery accuracy is at an acceptable level and the new solution is close to the solution obtained by the steepest descend step. Imposing such regularization term is beneficial to enhancing the algorithm’s robustness to noise [10]. The optimization problem (6) has an analytical solution:

$$\hat{X}(\cdot, m) = (\lambda A_m^H A_m + I_{L_r})^{-1}[\hat{X}(\cdot, m) + \lambda A_m^H Y(\cdot, m)]$$

(7)

where $(\cdot)^{-1}$ denotes the matrix inversion operator. Followed by the range profile projection, the 2D range-Doppler map projection can be obtained by solving:

$$\hat{Z} = \min_Z |\hat{X} - \hat{Z}D|_F = \min_Z |\hat{X}D + \hat{Z}D|_F = (DD^H)^{-1}$$

(8)

The whole steps of the proposed algorithm are shown in Algorithm 1. By initially setting a large enough $\alpha$ and gradually decreasing it during iterations, the algorithm can move quickly toward an initial solution near the actual minimizer (with large $\alpha$), and gradually reach the actual minimum sparse solution (with small $\alpha$), thus avoiding the local minimum and ensures an accurate sparse targets recovery in range-Doppler domain.

**Complexity analysis:** In this section, we analyse the computational complexity of the proposed method. As demonstrated in Algorithm 1, the main complexity lies in the update of $\hat{X}$ and $\hat{Z}$. Updating $\hat{X}$ requires the calculation of $(\lambda A_m^H A_m + I_{L_r})^{-1} [\hat{X}(\cdot, m) + \lambda A_m^H Y(\cdot, m)]$ and the multiplication of $O(L_r^2 M + L_r^2 M + L_r^2 M + L_r^2 M)$. The complexity to update $\hat{Z}$ is $O(L_r^2 M + L_r L_r + L_r L_r)$. Therefore, the total complexity of the proposed method is approximately $O(L_r^2 M + \frac{L_r^2 M + L_r^2 M + L_r^2 MN + L_r^2 MN + L_r^2 MN}{M})$ at each iteration step.

To reduce the complexity and accelerate the proposed method, some useful techniques are suggested. For example, the $(DD^H)^{-1}$ term in $\hat{Z} = \hat{X}D + \hat{Z}D$ can be computed off-line in advance and then be cached in the iterations, since the Doppler dictionary $D$ is known a priori and remain unchanged. The $(\lambda A_m^H A_m + I_{L_r})^{-1}$ term can also be computed off-line if there is a waveform library and the transmit waveforms are known in advance. With the cache technique, the complexity can be reduced to $O(L_r^2 M + L_r MN + L_r L_r + L_r L_r)$. Moreover, the $\hat{X}$ updating process can be accelerated with a parallel implementation, by computing $\hat{X}(\cdot, m)$ for all the $m$ simultaneously.

**Simulation results:** In this section, we demonstrate the range-Doppler imaging performance of the proposed method and its robustness to noise and clutters. The performance is compared with three other methods (i.e. the matched filtering (MF), the sequential sparse recovery method by OMP [11] and SL0 [10]). The parameters of the noise radar in the simulation are shown in Table 1. Eight targets are set in the illuminated area of the noise radar. The range bin, Doppler bin and amplitude of the eight targets are $[40th, 5th, 0 dB]$, $[180th, 11th, 0 dB]$, $[60th, 8th, -5 dB]$, $[80th, 8th, -10 dB]$, $[100th, 8th, -15 dB]$, $[120th, 8th, -20 dB]$, $[140th, 8th, -25 dB]$ and $[160th, 8th, -30 dB]$, respectively. The echoes of the eight targets received by the noise radar are simulated without noise and clutters and then processed by the proposed 2D-GSL0 method and the other three methods. The obtained range-Doppler images are shown in the first row of Figure 1. Then, in order to show the performance of these methods in the environment of noise and clutter, the Gaussian noise is added to the simulated echoes with the SNR $= -10$ dB (PS). In this paper, the SNR/SCR is defined as the ratio of the signal power of the weakest target to the noise/clutter in the echoes). The clutter is randomly generated with a Gaussian Distribution at all the range bins, while the velocity of the clutter is spread within $-0.5$ to $0.5$ m/s to simulate the echoes of stationary objects and slowly-swaying vegetation. The SCR is set as $-60$ dB. The range-Doppler images with the influence of noise and clutters are shown in the second row of Figure 1. As we can see from Figure 1, the MF method yields the poorest range-Doppler image, with much higher sidelobe floors in both cases. The OMP and SL0 have good performances in the noise-less and clutter-less case, but suffer performance degradations when strong clutter and noise exist. The proposed 2D-GSL0 method outperform other methods. The sidelobe/noise floors are low both in range and Doppler domain, and all the targets (even the weakest one) can be accurately recovered. The elapsed time of the proposed 2D-GSL0 method is 0.45 s. With the cache technique, the time can be reduced to 0.21 s. As a reference, the SL0 method and OMP method take 0.31 and 0.28 s, respectively. To further analyse the methods’ robustness to noise and clutters, we calculate the root mean square errors (RMSEs) of the recovered range-Doppler map and the ground truth with varying SNRs and SCRs. The RMSEs are obtained by Monte Carlo simulations with 100 independent runs. Figure 2(a) shows the RMSEs as the SCRs vary from $-60$ to $10$ dB, when the SNR is fixed as $-10$ dB. Figure 2(b) shows the RMSEs as the SNRs vary from $-20$ to $30$ dB, when the SCR is fixed as $-60$ dB. The SNR is defined in the raw data domain before the proposed algorithm is performed. The SNR scope from $-20$ to $30$ dB is reasonable. In fact, for a specific radar system, the SNR mainly depends on the radar cross section (RCS) and the range of the target. For a target with a small RCS and a long range, the SNR is low. For example, according to the parameters in Table 1, for the target of an unmanned rotorcraft with the typical RCS $= 0.01$ m², when the target is far away from the radar with the range $= 4.5$ km, according to the radar equation, the SNR is low (SNR $\approx -20$ dB). However, for the target of a large vehicle with the

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**Algorithm 1 2D Generalized Smoothed-$l_1$ Algorithm**

**Table 1. The parameters of the noise radar in the simulation**

| Bandwidth       | 5 MHz     | Sampling rate          | 10 MHz |
|-----------------|-----------|------------------------|--------|
| PRF             | 30 μs     | Pulse duration          | 10 μs  |
| Carrier frequency| 5.5 GHz  | Antenna Gain            | 30 dB  |
| Peak transmit power | 100 W   | Noise figure of the receiver | 3 dB   |

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**Fig 1** Range-Doppler imaging results of the noise radar by the MF, OMP, SL0 and the proposed 2D-GSL0 method (1st row: Noise-less and clutter-less case, 2nd row: SNR = -10 dB and SCR = -60 dB)
Fig 2 SCR-RMSE curve and SNR-RMSE curve. (a) SCR-RMSE curve (SNR is fixed as -10dB), (b) SNR-RMSE curve (SCR is fixed as −60dB)

typical RCS = 10 m². When the target is close to the radar with the range = 1.6 km, the SNR is very high (approximately 30 dB).

As shown in Figure 2(a,b), the proposed method outperforms other methods with lower RMSEs for different SNRs and SCRs, which demonstrates the 2D-GSL0’s robustness to the varying noise and clutters.

Conclusion: The 2D-GSL0 algorithm is proposed in this paper to enhance the range-Doppler imaging performance for noise radar, where the transmit waveforms are varying from pulse to pulse. Simulation results show its superior target information recovery performance and effectiveness in low SNR and SCR cases. The performance enhancement over other methods is owing to the imposition of the range-Doppler domain sparsity into the optimization model and a precisely designed SL0 based solver to solve it.

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