Exact late time Hawking radiation and the information loss problem
for evaporating near-extremal black holes

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In this paper we investigate the effects of gravitational backreaction for the late time Hawking radiation of evaporating near-extremal black holes. This problem can be studied within the framework of an effective one-loop solvable model on $\text{AdS}_2$. We find that the Hawking flux goes down exponentially and it is proportional to a parameter which depends on details of the collapsing matter. This result seems to suggest that the information of the initial state is not lost and that the boundary of $\text{AdS}_2$ acts, at least at late times, as a sort of stretched horizon in the Reissner-Nordström spacetime.

PACS number(s): 04.70.Dy, 04.62.+v

The discovery of black hole radiation $[1]$ has led to a long standing debate concerning the suggestion $[2]$ that the evaporation process implies a loss of quantum coherence. This conclusion seems inevitable if one assumes the propagation of quantum fields on a fixed classical background. However, backreaction effects could change this picture. ’t Hooft $[3]$ suggested that for an asymptotic observer the interaction between the infalling matter and the outgoing radiation could preserve the information of the initial quantum state of the collapsing matter through non-local effects. Within this alternative viewpoint it has also been proposed a principle of complementarity $[3–6]$, which states that the simultaneous measurements made by an external observer and those made by an infalling observer crossing the horizon are forbidden.

In this letter we shall analyze the evaporation process of a Reissner-Nordström black hole near extremality in a way which is loosely connected with the principle of complementarity. According to it we cannot have a detailed description of the physics near the horizon and, simultaneously, far away from the black hole. We shall restrict the Einstein-Maxwell theory in a region very close to the horizon. If the physical configurations to be considered preserve the spherical symmetry and are close to extremality the resulting effective theory turns out to be equivalent to a solvable two-dimensional model. The effective model remains solvable also at the one-loop quantum level and it has been studied in $[7,8]$, where we found it natural to describe the evaporation of the black hole from the point of view of an infalling observer very close to the horizon. In this paper we shall improve our analysis and consider the same process as it is seen by an asymptotic observer at late retarded times (this is the part of future null infinity which can still be described by our 2d model). The solution we will get is very different in form from the original one (they indeed describe two very different regions of the spacetime), however we will crucially impose that they naturally match at one point, i.e. at the end-point of the evaporation where the solution becomes extremal. In contrast with the standard picture, the Hawking flux goes down at late times and it is not proportional to the total mass of the collapsing matter. Instead, we find that it is proportional to a parameter which admits an infinite series expansion in Planck constant and depends on all the higher order momenta of the classical stress-tensor of the incoming matter. At leading order this parameter is the total mass. All this seems to suggest that the information of the initial state will be then released out to future null infinity during the evaporation process.

We start our analysis presenting the two-dimensional effective theory that describes the near-horizon region of the Einstein-Maxwell theory around extremality (the mass of the extremal hole is $m_0\sim l_\text{p}^{-1}$, where $l_\text{p} = G^{-1/2}$ is Newton’s constant). We refer to $[7,8]$ (and references therein) for the details and for a full description of the methods used in this work . The classical action is given by the Jackiw-Teitelboim model

$$I = \int d^2x \frac{1}{2} \left[ \left( g^{ab} \partial_a \Phi \right)^2 + \frac{4}{l_\text{p}^2} \Phi^2 \right] \sim \frac{1}{2} \frac{l_\text{p}^2}{l_\text{p}^2} \Phi^2 ;$$

where the two-dimensional fields $g^{ab}$ and $\Phi$ appearing in $[1]$ are related to the four-dimensional metric by the expression

$$ds^2_{(4)} = \frac{21}{2} l_0 ds^2_{(2)} + \left( \kappa_0^2 + 4l_\text{p}^2 \Phi^2 \right) d\varphi^2 ;$$

and $l_0 = \lambda l_\text{p}$ is the extremal radius. The field $\Phi$ represents a spherically symmetric scalar field which propagates freely in the region very close to the horizon.
To properly account for backreaction effects we have to consider the corresponding one-loop effective theory. Therefore we have to correct (11) by adding the Polyakov-Liouville term [10]

\[
Z = e^{2\kappa X^g} R^{-\frac{1}{2}} \int P_{\phi} g^{1/2} f_1 f_2 \left( N \sim \frac{Z}{96} e^{2\kappa X^g} g^{-2} \right)^{1/2} \left( \frac{N \sim \frac{Z}{12}}{1} \right) \]

where we have considered the presence of \( N \) scalar fields to have a well-defined theory in the large \( N \) limit. Note that the Polyakov-Liouville action has a cosmological constant term which has been fixed (\( \frac{2}{3} = 1 \)) to ensure that the extremal configuration remains a solution of the quantum theory. In conformal gauge we have the equations of motion derived from (3)

\[
2 \Theta \Theta + 2 \epsilon^2 \cdot \Theta \epsilon = 0; \quad (4)
\]

\[
\Theta \Theta + \epsilon^2 \cdot \Theta \epsilon = 0; \quad (5)
\]

\[
\Theta \Theta + f_1 = 0; \quad (6)
\]

\[
2 \Theta \Theta + 4 \Theta = T \quad (7)
\]

where the chiral functions \( \Theta(x) \), coming from the non-locality of the Polyakov-Liouville action, are related with the boundary conditions of the theory associated with the corresponding observers. The equation (7) is the Liouville equation with a negative cosmological constant. It has a unique solution up to conformal coordinate transformations. It is very convenient to choose the following form of the metric

\[
d s^2 = \frac{2 \kappa^2 g^3 \epsilon^2 \Theta \epsilon}{\Theta \epsilon} \bigg( \frac{N \sim \frac{Z}{12}}{1} \bigg)^2 ; \quad (8)
\]

which, in turn, is a way to fix the conformal coordinates \( x^\tau \) up to Möbius transformations. In these coordinates only the \( \Theta \) terms survive in the quantum part of the constraints (1), i.e. the semiclassical stress tensor is just

\[
\Pi_{\epsilon} = \frac{N \sim \frac{Z}{12}}{1} \Theta ; \quad (9)
\]

and the relevant information of the solutions is therefore encoded in the field \( \epsilon \). The crucial point is then to choose the suitable functions \( \Theta(x) \).

If we want to give a description of the evaporation process for an infalling observer very close to the horizon, the natural boundary conditions are (10) and (11)

\[
\tau(x) = \frac{1}{2} f \epsilon v; \quad (10)
\]

\[
\tau(x) = 0; \quad (11)
\]

which correspond, see eq. (7), to a negative influx of radiation crossing the apparent horizon and no outgoing flux (this is indeed what one gets in the full four dimensional picture in fixed background considering the limit close to the horizon). Alternatively, one can provide a description of the evaporation process from the point of view of an outside observer valid at late times. In this region it is perfectly legitimate to choose the boundary conditions

\[
\tau(x) = 0; \quad (12)
\]

\[
\tau(x) = \frac{1}{2} f \epsilon v; \quad (13)
\]

This gives a positive influx of radiation and vanishing incoming flux. In eqs. (10) and (13) \( u \) and \( v \) are to be identified with the ingoing and outgoing Eddington-Finkelstein coordinates associated with the dynamical Reissner-Nordström metric. Note that we cannot impose simultaneously (10), (12), otherwise the only solution is the classical one. Therefore (10), (11) and (12), (13) are, in a sense, complementary.

It is worth to remark that the relations \( x^+ = x^- (u; \epsilon^-) \), \( x^- = x^- (u; \epsilon^-) \) cannot be given a priori and can only be determined once we solve the equations. At extremality \( x^+ = v \) and \( x^- = u \), up to Möbius transformations, we have \( \tau_\epsilon = 0 = \tau_u \). The conditions (10), (11) and (12), (13) imply the following form of the ingoing and outgoing quantum fluxes

\[
\Pi_{\epsilon}^i = \frac{N \sim \frac{Z}{12}}{1} f \epsilon^2 v; \quad (14)
\]

\[
\Pi_{\epsilon}^o = \frac{N \sim \frac{Z}{12}}{1} f \epsilon v; \quad (15)
\]

In the presence of collapsing matter and neglecting the backreaction the ingoing flux (14) vanishes for an outside observer and the outgoing flux gives the standard Hawking radiation

\[
\Pi_{\epsilon}^o = \frac{N \sim \frac{Z}{12}}{1} f \epsilon v^2 m ; \quad (16)
\]

where \( m \) is the total mass of the collapsing matter.

From the point of view of the infalling observer the solution is the following (18)

\[
\tau = \frac{F(x^+)}{x} + \frac{1}{2} F^0 (x^+) ; \quad (17)
\]

where the function \( F(x^+) \) satisfies the differential equation (here we include a general incoming matter configuration)

\[
F_{t^-}^0 = \frac{N \sim \frac{Z}{12}}{1} f \epsilon^2 v; \quad (18)
\]

which serves to relate the \( x^+ \) and \( v \) coordinates

\[
\frac{d v}{d x^+} = \frac{v \epsilon^2}{F} ; \quad (19)
\]

The metric can also be given in the ingoing Vaidya-type gauge
where \(\phi = 1^-\) and \(m(\nu)\) is the deviation of the mass from extremality. The evaporating mass function satisfies the differential equation

\[
\begin{align*}
\Theta_{\nu m}(\nu) &= \frac{N}{24} \ln(\nu) + T_{\nu m}(\nu) \quad (21)
\end{align*}
\]

The negative incoming quantum flux is given by

\[
T_{\nu m}(\nu) = \frac{N}{24} \ln(\nu) \quad (22)
\]

If the incoming classical matter is turned off at some advanced time \(\nu_f\) then the evaporating solution approaches asymptotically the extremal configuration (up to exponentially small corrections) \(24\). The most delicate point in finding a solution to the above differential equation is the choice of the correct boundary conditions. They come from the requirement that the two descriptions match at the end-point \(\nu = \nu_f\); the evaporating \(T_{\nu m}(\nu)\) exponents then the corrections to eq. \(23\) will of course be different in the two cases and therefore this

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\]
Similarly to (20), the solution can now be expressed in the outgoing Vaidya-type form

$$ds^2 = \frac{2x^2}{x^2 - r^2} \left[ (u) \, du^2 + 2 du \, dv \right]$$

(32)

It is important to point out the fact that $m_\infty (v_\infty)$ depends on the details of the collapsing matter through all the higher-order moments of the classical stress tensor. We observe that for $\sim 0 \, m_\infty (v_\infty)$ is the total classical mass of the collapsing matter and (30) recovers the constant thermal value of a static near-extremal black hole (16). So when backreaction effects are neglected we loose the information of the initial state.

We wish to stress that eq. (30) is the first exact calculation of the Hawking radiation flux for RN black holes at late times which takes into account consistently backreaction effects to all orders in $\sim$. Our result is highly nontrivial because two different expansions in $\sim$ are implicit in (30), one being associated to the exponential $e^{-\pi \gamma_{uv} (u \, v_\infty)}$ and the other inside $m_\infty (v_\infty)$, see (31). While the first expansion is of no surprise because it is nothing but the application of Stefan’s law to this particular situation, the second, consequence of our (natural) choice of boundary conditions for the differential equation (26), is completely unexpected on physical grounds. Actually this suggests, contrary to the earlier predictions based on calculations made in a fixed classical background, that the outgoing radiation may contain all the information about the initial state already at the one-loop semiclassical level (i.e. without recourse to a full quantization of the theory which is still lacking). Obviously our results alone are not enough to prove this conjecture because just from the parameter $m_\infty (v_\infty)$ one cannot reconstruct the whole function $T_{\gamma_{uv}} (v)$ (this is the limitation of our model which, as we have already remarked, can give the exact Hawking flux in the RN spacetime only at late times). Given our present achievements, however, it is our belief that the exact $m_\infty (v_\infty)$ for all $u$ will prove to be an extremely interesting quantity. Unfortunately the semiclassical version of spherically reduced Einstein-Maxwell theory (without the near-horizon approximation considered in this paper) is not solvable, so such a calculation is a much harder challenge (recently, the full backreaction problem for an evaporating RN black hole has been addressed numerically in [11]). Nevertheless we now know that this quantity has to be subjected to the ‘boundary condition’ that for late times $u \sim + 1$ it has to reduce to eq. (30). Therefore the results obtained here cannot be confined to the domain of validity of the particular 2d model considered, but represent the motivation and the starting point for a full four dimensional calculation to be performed in the physical world (we are currently working in this direction).

What is striking is that such an input comes from the simple 2d model (3) and the boundary conditions (27) and (28).

It is also interesting to remark that the radiation measured by an infalling observer and that of an external one at late times $(u \sim + 1)$ are the same under the interchange of $v$ with $u$ (which means a reflection by the curve $x^+ = x^-$). The curve $x^+ = x^-$, which is nothing but the AdS$_2$ boundary, then acts as a sort of stretched horizon $\sim 3$ in the Reissner-Nordström spacetime at least at late times when $v \sim + 1$, $u \sim + 1$. Since the affine distance (as measured along null rays) between the horizon and the boundary is finite, we think that such a surface can be exactly located in the physical spacetime using null rays. There also are indications that the degrees of freedom relevant to account for the Bekenstein-Hawking entropy can be located at the AdS$_2$ boundary $\sim 3$. This also reinforces the idea that it could have a physical meaning in the Reissner-Nordström spacetime.

This research has been partially supported by the CICYT and DGICYT, Spain. D. J. Navarro acknowledges the Ministerio de Educación y Cultura for a FPI fellowship. A.F. thanks R. Balbinot for useful discussions. J. N-S. thanks the Department of Physics of Bologna University for hospitality during the late stages of this work.

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