Fast and slow light in zig-zag microring resonator chains

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We analyze fast and slow light transmission in a zig-zag microring resonator chain. This novel device permits the operation in both regimes. In the superluminal case, a new ubiquitous light transmission effect is found whereby the input optical pulse is reproduced in an almost simultaneous manner at the various system outputs. When the input carrier is tuned to a different frequency, the system permits to slow down the propagating optical signal. Between these two extreme cases, the relative delay can be tuned within a broad range.

The ability to control the transmission delay of optical signals is a major technological target pursued for the next generation of high-performance photonic communications networks [1]. Different physical mechanisms can provide the required control [2]. The use of microring resonator chains [3, 4, 5] is particularly appealing since the large structures in a small-size optical chip provide the required control [2]. The use of microring resonators networks [1] is based on two types of structures [7]: Side Coupled Integrated Spaced Sequence of Resonators (SCISSOR) and Coupled Resonator Optical Waveguides (CROW). We analyze a novel type of structure consisting of a Zig-Zag Ring Resonator (ZZRR) chain. This is shown to provide a group delay which can be controlled by tuning the structure using previous reported techniques [3, 8] in a broad range of values. In the two extremes, we find fast and slow light. A novel type of fast light propagation in which the input signal is placed at several outputs in a nearly simultaneous manner. Potential applications of the new effects and device are briefly discussed.

Figure 1 displays the topology of the physical system under study. Propagation is assumed dispersionless for each waveguide section. The total ring length is $L = 2\pi R$ and we use for reference in the analysis of propagation phenomena the length of one half-ring $L/2$ for which $\tau_0 = L/(2v)$ is the transmission delay, where $v$ is the (phase and group) velocity and $g$ is the half-ring amplification $(g > 1)$ or attenuation $(g < 1)$ factor for active and passive structures, respectively. All the coupling constants are assumed to have the same value of $t$ and $r = \sqrt{1 - T^2}$. A physical length $d$ is assigned to the coupling sections which, in turn, is responsible for the zig-zag geometry of the structure.

The transfer function, as a function of a normalized frequency $\Omega = \tau_0\omega$, is defined by the relation $E_{o,k}(\Omega) = H_k(\Omega)E_i(\Omega)$ and reads

$$H_k(\Omega) = (-1)^k \exp\left(-j(2k + 2)\frac{\Omega d}{L}\right) \times \frac{r - rg^2 \exp(-2j\Omega)}{1 - g^2r^2 \exp(-2j\Omega)} \left[ \frac{t^2g \exp(-j\Omega)}{1 - g^2r^2 \exp(-2j\Omega)} \right]^k,$$

(1)

for $k = 1 \ldots N - 1$, and

$$H_N(\Omega) = (-1)^N \exp\left(-j(2N + 1)\frac{\Omega d}{L}\right) \times \left[ \frac{t^2g \exp(-j\Omega)}{1 - g^2r^2 \exp(-2j\Omega)} \right]^N.$$  

(2)

The relative simplicity of the transfer functions [1] and [2] arises from the zig-zag topology, which brings about the unique properties of this structure. If an additional ring is placed loading the last output, Eq. (1) holds also for the $k = N$ case if multiplied by $r^{-1}$. From Eqs. (1) and (2), we find that, except for the first exponential term corresponding to the delay in coupling sections, the response is periodic in $\Omega$ with $2\pi d$ period. In the following, we will restrict the analysis to the first period of the system response. Also, the system is stable provided that the condition

$$gr < 1$$  

(3)

is fulfilled.

The total transmission group delay, calculated as $\tau_g = \ldots$
\[ \frac{d \arg H(\omega)}{d\omega} \text{ is} \]

\[ \frac{\tau_{g,k}}{\tau_0} = 2(k + 2) \frac{d}{L} - \frac{2g^2(\cos(2\Omega) - g^2)}{1 - 2g^2 \cos(2\Omega) + g^4} + k + (k + 1) \frac{2g^2r^2(\cos(2\Omega) - g^2r^2)}{1 - 2g^2r^2 \cos(2\Omega) + g^4r^4} \]

\[ (4) \]

for \( k = 1 \ldots N - 1 \) and

\[ \frac{\tau_{g,N}}{\tau_0} = 2(N + 1) \frac{d}{L} + N \left( 1 + \frac{2g^2r^2(\cos(2\Omega) - g^2r^2)}{1 - 2g^2r^2 \cos(2\Omega) + g^4r^4} \right). \]

\[ (5) \]

If we tune the system to \( \Omega = \pi/2 \), the group delay is

\[ \frac{\tau_{g,k}}{\tau_0} = 2(k + 2) \frac{d}{L} + \frac{2g^2}{1 + g^2} + k - (k + 1) \frac{2g^2r^2}{1 + g^2r^2} \]

\[ (6) \]

for \( k = 1 \ldots N - 1 \) and

\[ \frac{\tau_{g,N}}{\tau_0} = 2(N + 1) \frac{d}{L} + N \left( 1 - \frac{2g^2r^2}{1 + g^2r^2} \right). \]

\[ (7) \]

If we take the limit \( gr \to 1 \) and \( d/L \to 0 \), the total group delay vanishes \( \tau_{g,N} = 0 \) and

\[ \frac{\Delta \tau_g}{\tau_0} = 2 \frac{d}{L} = 1 - \frac{2g^2r^2}{1 + g^2r^2}. \]

\[ (9) \]

which is a monotonically decreasing function of the product \( gr \) in the range \( 0 < gr < 1 \) and vanishes at \( gr = 1 \) if the term \( 2d/L \) is neglected. For instance, for \( gr = 0.9 \) \( \Delta \tau \approx 0.10\tau_0 \). For \( gr = 0.8 \) \( \Delta \tau \approx 0.22\tau_0 \) 8.

Figures 2 and 3 display the amplitude transfer function and the transmission group delay, respectively, for a structure with \( rg = 0.9 \). In these and all the subsequent results presented, we assume a small value of \( d/L = 0.005 \), \( N = 5 \) and solid, long dashed, short dashed and dotted lines correspond to the 1st, 2nd, 3rd and 4th output, respectively, while the distinct 5th output is identified with small circles. The same value of \( rg = 0.9 \) is considered in all the three cases of figures 2 and 3 with \( g = 9 \) (passive, lossy) in (a), \( g = 1 \) (passive, lossless) in (b) and \( g = 1.8 \) (active) in (c). In the passive lossless case, \( g = 1 \), the amplitude of the \( k \)-th output vanishes.

\[ k = 1, \ldots, N - 1, \text{for the resonances at } \Omega = m\pi \text{ (m integer), while it is maximum for } N\text{-th output. For } g \neq 1, \text{ the output amplitude of all the channels is maximum at the resonances. For the active case (a) a very flat group delay response over a broad bandwidth is found. The amplitude responses in cases (a) and (b) show extreme variations in the amplification or attenuation factors for the different outputs.} \]

If the condition \( g(1 - r^2) = 1 + r^2g^2 \) is satisfied, the system response for all the outputs \( k = 1 \ldots N - 1 \) at \( \Omega = \pm\pi/2 \) takes the same value \( H(k) = r(1 + g^2)(1 + r^2g^2)^{-1} \) and \( |H_N| = 1 \). Figure 4 shows the values of \( r \) and \( g \) satisfying this relation in the stability range \( 0 < rg < 1 \). It is required that the waveguides show amplification, \( g > 1 \), and

\[ r = \sqrt{\frac{g - 1}{g(g + 1)}}. \]

\[ (10) \]

Figure 5 displays the amplitude transfer functions and group delays for the various outputs of a \( N = 5 \) structure and \( rg = 0.9 \) when the condition given in Equation (10) is satisfied.

We now turn to the slow light transmission in the system. If we keep the system parameters fixed, but tune the input to one of the system resonances, \( \Omega = 0 \). The
FIG. 3: Transmission group delay for \( N = 5 \) and \( rg = 0.9 \).
(a) \( r = 0.1 \), (b) \( r = 0.9 \) and (c) \( r = 0.5 \).

FIG. 4: Values of \( r \) and \( g \) in the range \( 0 < rg < 1 \) which produce equal amplitude outputs at \( \Omega = \pm \pi/2 \).

net group delay is

\[
\frac{\tau_{g,k}}{\tau_0} = 2(k + 2) \frac{d}{L} + h + (k + 1) \frac{2g^2r^2}{1 - g^2r^2},
\]

(11)
k = 1 \ldots N - 1, \text{ where } h = 1 \text{ for } g = 1 \text{ and } h = 2g^2(g^2 - 1)^{-1} \text{ otherwise. For the N-th output,}

\[
\frac{\tau_{g,N}}{\tau_0} = 2(N + 1) \frac{d}{L} + N + N \frac{2g^2r^2}{1 - g^2r^2},
\]

(12)

with \( \tau_{g,k} \to \infty \) for \( k = 1 \ldots N \) when \( gr \to 1 \), providing a regime for the slowing down of the optical signal transmission. Again, perfect stopping of the light places the operation point critically at the border of the instability region and smaller values of the product \( gr \) should be considered as more practical conditions. The relative delay between two adjacent outputs in this case is

\[
\frac{\Delta \tau_g}{\tau_0} = \frac{2d}{L} + 1 + \frac{2g^2r^2}{1 - g^2r^2}.
\]

(13)

If we neglect the \( 2d/L \) term, this relative delay is \( \Delta \tau \simeq 9.5\tau_0 \) for \( gr = 0.9 \) and \( \Delta \tau \simeq 19.5\tau_0 \) for \( gr = 0.95 \).

Light propagation in a ZZRR chain has been analyzed. The system exhibits both slow and fast light transmission. In the fast case, a novel type of superluminal propagation, ubiquitous transmission, has been identified and studied. The novel effects addressed in this Letter could find also new applications in optical integrated circuits where ubiquitous signal transmission can be used for in-chip synchronization. Using previously described tuning mechanisms \cite{5,8} one can obtain a relative delay between outputs which can be controlled in a very broad margin. This can be used, for instance, to obtain a tunable system for pulse repetition rate multiplication \cite{9} by combining various outputs.

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\[1\] R. Boyd, D.J.Gauthier, and A. Gaeta, Opt. Photon. News 17, 18 (2006).

\[2\] E. Parra and J. Lowell, Opt. Photon. News 18, 40 (2007).
[3] J. Heebner and R. Boyd, J. Mod. Opt. 49, 2629 (2002).
[4] F. Fraile-Pelaez and P. Chamorro-Posada, Opt. Express 15, 3177 (2007).
[5] F. Liu, Q. Li, Z. Zhang, M. Qiu, and Y. Su, IEEE J. Select. Top. Quantum Electron. 14, 706 (2008).
[6] F. Xia, L. Sekaric, and Y. Vlasov, Nat. Photon. 1, 65 (2007).
[7] J. Scheuer, G. Paloczi, J. Poon, and A. Yariv, Opt. Photon. News 16, 36 (2005).
[8] S. Blair and K. Zheng, Opt. Express 16, 11162 (2006).
[9] M. Preciado and M. Muriel, Opt. Express 16, 11162 (2008).