Irreversible decay of nonlocal entanglement via a "reservoir" of a single degree of freedom

A. Vaglica and G. Vetri
Dipartimento di Scienze Fisiche ed Astronomiche and CNISM, Università di Palermo via Archirafi 36, 90123 Palermo, Italy
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Recently, it has been realized that nonlocal disentanglement may take a finite time as opposite to the asymptotic decay of local coherences. We find in this paper that a sudden irreversible death of entanglement takes place in a two atom optical Stern-Gerlach model. In particular, the one degree non dissipative environment here considered suddenly destroys the initial entanglement of any Bell’s states $|\phi^{\pm}\rangle$ superposition.

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I. INTRODUCTION

Superposition principle is a basic feature of Quantum Mechanics (QM). When applied to composite systems superposition leads to nonlocal coherences and quantum correlations (entanglement) which have been extensively studied in the last years, and are even expected of central interest for physics to come [1].

As it is known, quantum coherences and entanglement are potential resources in various quantum information processes, even if they suffer any sort of environmental action which represents a serious obstacle towards these applications. On the other hand, it is just this fragility with respect to the environment, that is, the propensity to disperse information throughout inaccessible degrees of freedom that explicates the quantum-classical transition 2.

An interesting point raised in [3, 4, 5, 6, 7] concerns the possible finite time disentanglement for bipartite systems, as opposite to the usual local decoherence asymptotic time. An experimental evidence of this peculiar trait of entanglement has been reported recently by Almeida et al. [8]. This issue has been analyzed [4] in a simple and realistic model where two initially entangled two-level atoms separately interact with the multimode vacuum noise of two distinct cavities. The authors find out that the nonlocal decoherence may take place suddenly or at least as fast as the sum of the normal single atom decay rates. This sudden death of entanglement has been analyzed also for two Jaynes-Cummings (JC) atoms [9]. Also in this case the dynamics of the entanglement between the atomic internal variables shows different peculiarities for different initial states, with possible sudden decays that are however followed by periodic revivals, due to recovery of information by the system, being the cavities lossless.

In this paper we wish to investigate if the separate non dissipative interaction of each two-level atom with one only degree of freedom but of continuous variables, may play the role of the interaction with a reservoir leading to an irreversible disentanglement of the bipartite system.

As well known, the optical Stern-Gerlach (OSG) model [10] gives the opportunity of modifying the JC model by including the interaction between the internal and the external atomic dynamics via the e.m. mode of the cavity field, so actually providing the coupling of each qubit to one only degree of freedom. It has already been shown that this non dissipative coupling leads to a decoherence in the dynamics of a single atom [11, 12, 13] and also affects the entanglement in the internal dynamics of two atoms that successively interact with the same cavity [14]. As will shown in what follows, the same interaction gives rise to a disentanglement of the bipartite system which exhibits strong analogies with the disentanglement generated by the vacuum noise [4, 8].

II. OPTICAL STERN-GERLACH MODEL FOR TWO ATOMS

Let us consider two identical isolated two-level atoms, $A$ and $B$, respectively crossing two distinct ideal cavities, $a$ and $b$ (in a direction orthogonal to the cavity axis), as in the OSG model. These two uncoupled subsystems are described by the total Hamiltonian

$$\hat{H} = \hat{H}_A + \hat{H}_B$$

where, in resonant conditions and in the rotating wave approximation for both atoms,

$$\hat{H}_A = \frac{p_A^2}{2m} + \hbar \omega \left( \hat{a}^\dagger \hat{a} + S_z^A + \frac{1}{2} \right) + \hbar \epsilon \sin(kx_A) \left( \hat{a}^\dagger \hat{S}_x^A + \hat{S}_x^A \hat{a} \right)$$

$$\hat{H}_B = \frac{p_B^2}{2m} + \hbar \omega \left( \hat{b}^\dagger \hat{b} + S_z^B + \frac{1}{2} \right) + \hbar \epsilon \sin(kx_B) \left( \hat{b}^\dagger \hat{S}_x^B + \hat{S}_x^B \hat{b} \right).$$

(2a)
To simplify the notation we will sometimes use the indices $A$ and $B$ to indicate the two subsystems $Aa$ and $Bb$, respectively. The cavity fields are, as usually, described by the bosonic operators $\hat{a}$, $\hat{a}^\dagger$ and $\hat{b}$, $\hat{b}^\dagger$, and the two cavity frequencies $\omega = ck$, as well as the coupling constants $\varepsilon$ for the two subsystems, are assumed to be the same. The $1/2$ spin operators $\tilde{S}_z^A$, $\tilde{S}_z^B$ account for the internal dynamics of $j$-th atom, while the conjugate variables $\tilde{x}_j$ and $\tilde{p}_j$ describe the external motion of the same atom along the axis direction of the corresponding cavity ($j = A, B$).

When the initial uncertainties $\Delta x_j$ of the atomic wave packets are sufficiently small with respect to the wavelength $\lambda = 2\pi / k$ of both modes $a$ and $b$, the Hamiltonian assumes the form (see, for example, Ref. [15] for one atom case)

$$\hat{H} = \frac{\hat{p}_A^2}{2m} + \frac{\hat{p}_B^2}{2m} + \hbar \omega (\hat{N}_A + \hat{N}_B) + \hbar \Omega^A \hat{\mu}_x^A + \hbar \Omega^B \hat{\mu}_x^B$$

(3)

where

$$\hat{\mu}_x^A = \frac{\hat{a}^\dagger \hat{S}_x^A + \hat{S}_x^A \hat{a}}{2 \sqrt{N_A}}$$

and

$$\hat{N}_A = \hat{a}^\dagger \hat{a} + \hat{S}_z^A + \frac{1}{2}$$

(4)

are constant of motion. Similarly for the subsystem $B$. $\hat{\Omega}_j = 2\varepsilon k \sqrt{N_j} \tilde{x}_j$ is the (operatorial) Rabi frequency depending on the atomic position (with respect to a nodal point of the related cavity mode function) and on the excitation number of the $j$-th subsystem. Using previous results [15] for the one atom case, it is easy to show that the evolution operator may be written as

$$\hat{U}(t, 0) = e^{-i \hat{H} t / \hbar} \exp \left\{ -\frac{2it}{\hbar} m\hat{a}^\dagger N_A \hat{\mu}_x^A \hat{x}_A \right\} \times \exp \left\{ \frac{-it}{2m\hbar} \hat{p}_A^2 \right\} \exp \left\{ \frac{-it}{2m\hbar} \hat{a}^\dagger \hat{\mu}_x^A \hat{p}_A \right\} \times \exp \left\{ -\frac{2it}{\hbar} m\hat{a} N_B \hat{\mu}_x^B \hat{x}_B \right\} \exp \left\{ \frac{-it}{2m\hbar} \hat{p}_B^2 \right\}$$

$$\times \exp \left\{ \frac{it^2}{\hbar} \hat{a}^\dagger \hat{b} \hat{\mu}_x^B \hat{p}_B \right\} \exp \exp \left\{ \frac{-i\vartheta_0(t)(N_A + N_B)}{2} \right\}$$

(5)

where

$$\hat{\mu}_x^A = a_0 \sqrt{N_j}$$

and it has been assumed that each atom enters the own cavity at the same time.

**III. REDUCED DENSITY OPERATOR FOR THE QUBITS**

In this section we will analyze for the two different configurations considered in Ref.[3], the time evolution of the initial entanglement between the two qubits under the effects of the interaction of each qubit with its own cavity and, consequently, of the coupling with its own translational motion. We will see that, differently from the usual JC model, in this case the elements of the reduced density matrices that describe the atomic internal dynamics show damped oscillations as a consequence of the OSG effect which causes a splitting of the atomic packets. As we will see later, this damping affects the entanglement decay whose rate is actually related to the phase space distance of the scattered packets. The section ends with a brief account of the results for the one atom OSG model, which are useful to explicate relations and differences between the decay rates of nonlocal entanglement and local coherences.

**A. Two atoms**

Let us first suppose that the configuration of the entire system at time $t = 0$ may be written as

$$|\psi(0)\rangle = |\varphi_A(0)\rangle |\varphi_B(0)\rangle |\chi(0)\rangle$$

(7)

where $|\varphi_A\rangle$, $|\varphi_B\rangle$ describe the translational dynamics along the cavity axes of atoms $A$ and $B$, while

$$|\chi(0)\rangle = |\cos \gamma|+\rangle + \sin \gamma |+\rangle |00\rangle, \quad (0 \leq \gamma \leq \frac{\pi}{2})$$

(8)

shows that the two qubits are initially in a superposition of the Bell’s states usually denoted $|\psi^\pm\rangle$, and the two cavities are in the vacuum state. The ket $|+\rangle$ indicates that the qubit $A$ is in the upper state and the qubit $B$ is in the lower state, and so on. Using the dressed states

$$|\chi^\pm\rangle = \frac{1}{\sqrt{2}} (|+\rangle_{0j} \pm |\gamma\rangle_{0j})$$

(9)

state [5] assumes the form

$$|\chi(0)\rangle = \frac{1}{\sqrt{2}} \{ \cos \gamma \left[ |\chi_A^\pm\rangle + |\chi_A^\pm\rangle \right] |0\rangle_B \right.$$  
$$+ \sin \gamma \left[ |\chi_B^\pm\rangle + |\chi_B^\pm\rangle \right] |0\rangle_A \}$$

(10)

Applying the evolution operator [5] to the initial state [7] and using Eq. [10], one obtains the following expression for the state of the entire system at time $t$

$$|\psi(t)\rangle = \frac{\cos \gamma\vartheta_0(t)}{\sqrt{2}} e^{-i\vartheta_0(t)} |\varphi_B(t)\rangle \left( |\phi_A^+(t)\rangle |\chi_A^+\rangle + |\varphi_A^-(t)\rangle |\chi_A^-\rangle \right)$$

$$+ |\phi_B^-(t)\rangle |\chi_B^\pm\rangle |0\rangle_B$$

$$+ \frac{\sin \gamma e^{-i\vartheta_0(t)}}{\sqrt{2}} |\varphi_B(t)\rangle \left( |\phi_B^+(t)\rangle |\chi_B^+\rangle + |\varphi_B^-(t)\rangle |\chi_B^-\rangle \right)$$

$$+ |\phi_B^-(t)\rangle |\chi_B^\pm\rangle |0\rangle_A$$

(11)
where we have set
\[ |\phi_j^+(t)\rangle = \exp \left\{ \pm \frac{it}{\hbar} m_0 \hat{x}_j \right\} \exp \left\{ - \frac{it}{2m\hbar^2} \right\} \]
\times \exp \left\{ \pm \frac{it^2}{2\hbar} \hat{p}_j \right\} |\varphi_j(0)\rangle \] (12)
\[ |\varphi_j(t)\rangle = \exp \left\{ -i \frac{\hat{p}_j^2 t}{2m\hbar} \right\} |\varphi_j(0)\rangle \] (13)
and the following relations have been used
\[ \hat{N}_j |\chi_j^\pm \rangle = |\chi_j^\pm \rangle, \quad \hat{N}_j |0\rangle_j = 0, \]
\[ \hat{\mu}_z^2 |\chi_j^\pm \rangle = \pm \frac{1}{2} |\chi_j^\pm \rangle, \quad \hat{\mu}_z^4 |0\rangle_j = 0. \] (14)
If the initial spatial distributions of the two atoms are gaussian functions of minimum uncertainty
\[ \varphi_j(x_j,0) = \left( \frac{1}{\sqrt{2\pi}\Delta x_0} \right)^{1/2} \exp \left\{ - \frac{(x_j-x_j,0)^2}{4\Delta x_0^2} \right\} \] (15)
centered in \( x_j,0 \) and with the same spread \( \Delta x_0 \), the \( x \)-representation of (12) gives
\[ \phi_j^+(x_j,t) = \left[ \frac{\Delta x_0}{\sqrt{2\pi\beta(t)}} \right]^{1/2} \exp \left\{ \pm \frac{it}{\hbar} m_0 x_j \right\} \]
\times \exp \left\{ - \frac{(x_j-x_j^+(t))^2}{4\beta(t)} \right\} \] (16)
where
\[ x_j^\pm(t) = x_j,0 \mp a_0 t^2/2, \quad \beta(t) = \Delta x_0^2 + iht/(2m). \] (17)
As Eq. (13) shows, \( |\varphi_j(t)\rangle \) describes the translational free motion of the \( j \)-th atom, while the \( x \)-representation of \( |\phi_j^+(t)\rangle \) given by Eq. (16) accounts for the well known splitting of the incoming wave packet in the OSG effect. Since we are interested in the entanglement of the two qubits, we will identify the internal atomic degrees as the system of interest, while the cavities and the external dynamics will serve as environment. As a consequence, the density operator \(|\psi(t)\rangle \langle \psi(t)\rangle\) must be traced on the cavity fields and on the atomic translation variables. Taking into account the normalization of kets (12) and (13), and using relation (9) we obtain
\[ \rho^{AB}(t) = Tr_{\text{transl.field}}(|\psi(t)\rangle \langle \psi(t)|) \]
\[ = a_1 |++\rangle\langle ++| + a_2 |+-\rangle\langle -+| + a_3 |+\rangle\langle +| + a_4 |--\rangle\langle --| \]
\[ + a_5 |-\rangle\langle -| + a_6 |--\rangle\langle --| \] (18)
where
\[ a_1 = \frac{1}{2} \cos^2 \gamma \left( 1 + Re \langle \phi_A(t)|\phi_A(t)\rangle \right) \]
\[ a_2 = \frac{1}{2} \sin^2 \gamma \left( 1 + Re \langle \phi_B(t)|\phi_B(t)\rangle \right) \]
\[ a_3 = \frac{1}{4} \cos \gamma \sin \gamma \left( \langle \phi_A(t)|\phi_A(t)\rangle + \langle \phi_A(t)|\phi_A(t)\rangle \right) \]
\times \left( \langle \phi_B(t)|\phi_B(t)\rangle + \langle \phi_B(t)|\phi_B(t)\rangle \right) \]
\[ a_4 = \frac{1}{2} - \frac{1}{2} \cos^2 \gamma Re \langle \phi_A(t)|\phi_A(t)\rangle \]
\[ - \frac{1}{2} \sin^2 \gamma Re \langle \phi_B(t)|\phi_B(t)\rangle. \] (19)
By using the Eq.s (13) and (10) it is possible to show that the following scalar products hold \[ \langle \phi_j^-(t)|\phi_j^+(t)\rangle = \exp \left\{ -i\Omega_j t \right\} e^{-\Gamma(t)} \] (20a)
\[ \langle \varphi_j(t)|\phi_j^\pm(t)\rangle = \exp \left\{ i\frac{m_0 \tilde{\alpha}^2 t^3}{4\hbar} + i\frac{\Omega_j a_0 t^4}{2} \right\} e^{-\Gamma(t)/4} \] (20b)
where
\[ \Gamma(t) = \frac{\delta x(t)^2}{8\Delta x_0^2} + \frac{\delta p(t)^2}{8\Delta p_0^2}. \] (21)
\( \Delta x_0 \) is the initial uncertainty of the momentum distributions for both atoms, and
\[ \delta x(t) = -a_0 t^2, \quad \delta p(t) = -2m_0 a_0 t, \quad \Omega_{j,0} = 2m_0 a_0 x_j,0/h. \] (22)
In the following we will assume that \( x_{A,0} = x_{B,0} \equiv x_0 \), which implies
\[ \Omega_{A,0} = \Omega_{B,0} \equiv \Omega_0 = \frac{2m_0}{\hbar} a_0 x_0. \] (23)
By inserting this equation and Eq.s (20) in Eq. (19) we get
\[ a_1 = \frac{1}{2} \cos^2 \gamma \left( 1 + \cos(\Omega_0 t)e^{-\Gamma(t)} \right) \] (24a)
\[ a_2 = \frac{1}{2} \sin^2 \gamma \left( 1 + \cos(\Omega_0 t)e^{-\Gamma(t)} \right) \] (24b)
\[ a_3 = \sin \gamma \cos \gamma \cos^2(\Omega_0 t/2)e^{-\Gamma(t)/2} \] (24c)
\[ a_4 = \frac{1}{2} \left( 1 - \cos(\Omega_0 t)e^{-\Gamma(t)} \right). \] (24d)
It is useful to emphasize that \( \Gamma(t) \) is proportional to the square of the adimensional distance in phase space between the average positions of scattered packets in the OSG effect, as defined in Ref. [14] for the one atom case. The same \( \Gamma(t) \) can be also interpreted as the \textit{visibility} of the Rabi oscillations [12] in the ambit of the complementarity relation as introduced by Englert [16].

Considering the ordered basis \(|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\), the reduced density operator (18) takes on the form
\[ \rho^{AB}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\
0 & a_1 & a_3 & 0 \\
a_3 & a_2 & 0 & 0 \\
0 & 0 & 0 & a_4 \end{pmatrix} \] (25)
where \( (a_1 + a_2 + a_4) = 1 \). \( a_3 \) describes the coherence between the two atoms which assumes its maximum value at \( t = 0 \) and, contrarily to what happens in the JC case, goes irreversibly to zero because of the coupling of each qubit with its own external dynamics.

Now we consider the case in which the two qubits are initially in a superposition of the Bell’s states denoted \(|\phi^\pm\rangle\), and the two cavities are in the vacuum state,

\[
|\chi(0)\rangle = [\cos \gamma |++\rangle + \sin \gamma |--\rangle] |00\rangle. \tag{26}
\]

With respect to the more convenient ordered basis \(|++\rangle, |+-\rangle, |--\rangle, |+–\rangle\), the reduced density matrix of the two qubits assumes the form

\[
\rho^{AB}(t) = \begin{pmatrix}
    b_3 & 0 & 0 & 0 \\
    0 & b_2 & b_1 & 0 \\
    0 & b_1^* & b_3 & 0 \\
    0 & 0 & 0 & b_3
\end{pmatrix} \tag{27}
\]

where

\[
b_1 = \frac{1}{4} \sin \gamma \cos \gamma e^{-i\omega_0(t)} \\
\times \left\{ \langle \varphi_A(t)|\phi_A^\dagger(t) \rangle + \langle \varphi_A(t)|\phi_A^\dagger(t) \rangle \right\} \\
\times \left\{ \langle \phi_B(t)|\varphi_B(t) \rangle + \langle \phi_B^\dagger(t)|\varphi_B(t) \rangle \right\}
\]

\[
b_2 = \frac{\cos^2 \gamma}{4} [1 + \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle + \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle] \\
+ \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle \text{Re} |\phi_B(t)|\phi_B^\dagger(t) \rangle \\
\]

\[
b_3 = \frac{\cos^2 \gamma}{4} [1 - \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle + \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle] \\
- \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle \text{Re} |\phi_B(t)|\phi_B^\dagger(t) \rangle \\
\]

\[
b_4 = \frac{\cos^2 \gamma}{4} [1 - \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle + \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle] \\
- \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle \text{Re} |\phi_B(t)|\phi_B^\dagger(t) \rangle \\
\]

\[
b_5 = \frac{\cos^2 \gamma}{4} [1 - \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle + \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle] \\
+ \text{Re} \langle \phi_A(t)|\phi_A^\dagger(t) \rangle \text{Re} |\phi_B(t)|\phi_B^\dagger(t) \rangle + \sin^2 \gamma. \tag{28}
\]

Using the results \[a\] we easily get

\[
b_1 = \sin \gamma \cos \gamma \cos^2 (\Omega_0 t/2) e^{-\Gamma t/2} \\
\times \exp \left\{ -i \left[ \omega t - ma_0^2 t^3 / (3\hbar) \right] \right\} \tag{29a}
\]

\[
b_2 = \frac{1}{4} \cos^2 \gamma \left\{ 1 + \cos (\Omega_0 t) e^{-\Gamma t} \right\}^2 \tag{29b}
\]

\[
b_3 = \frac{1}{4} \cos^2 \gamma \left\{ 1 - \cos (\Omega_0 t) e^{-2\Gamma t} \right\} \tag{29c}
\]

\[
b_5 = \sin^2 \gamma + \frac{1}{4} \cos^2 \gamma \left\{ 1 - \cos (\Omega_0 t) e^{-\Gamma t} \right\}^2 \tag{29d}
\]

where we have used \( b_4 = b_3 \) because of the condition \[a\].

**B. One atom**

To analyze analogies and differences with the disentanglement produced by different environments, it is useful to report in short on the dynamics of a single two-level atom in the OSG model. We assume that the initial state of whole system is

\[
|\psi(0)\rangle = |\varphi(0)\rangle |\chi(0)\rangle \tag{30}
\]

where

\[
|\chi(0)\rangle = [\cos \gamma |+\rangle + \sin \gamma |--\rangle] |0\rangle \tag{31}
\]

and \([\varphi]\) describes the atomic translational dynamics along the cavity axis. Using some results of Ref. \[13\] (see sec. IV) one obtains the time evolution of \[30\],

\[
|\psi(t)\rangle = \sin \gamma |\varphi(t)\rangle |0\rangle \\
+ \frac{\cos \gamma}{\sqrt{2}} e^{-i\omega_0 t} \left[ |\phi^+\rangle \langle \chi^+ \rangle + |\phi^−\rangle \langle \chi^− \rangle \right]. \tag{32}
\]

Removing the environment by tracing on the field and on the atomic translation variables, one gets the reduced one qubit density matrix

\[
\rho(t) = \begin{pmatrix} q_1 & q_2 \\
q_2 & q_3 \end{pmatrix} \tag{33}
\]

where

\[
q_1 = \frac{1}{2} \cos^2 \gamma \left[ 1 + \text{Re} \langle \phi^−(t)|\phi^+\rangle \right] \tag{34a}
\]

\[
q_2 = \frac{1}{2} e^{-i\omega_0 t} \cos \gamma \sin \gamma \left[ \langle \varphi(t)|\phi^+\rangle + \langle \varphi(t)|\phi^−\rangle \right] \tag{34b}
\]

\[
q_3 = \frac{1}{2} \cos^2 \gamma \left[ 1 - \text{Re} \langle \phi^−(t)|\phi^+\rangle \right] + \sin^2 \gamma. \tag{34c}
\]

Both the irreversible evolution of the populations \([34a] \) and \([34c] \) and the decoherence \([34b] \) are to be ascribed to the increasing distance in the phase space between the scattered atomic packets. However, while the population decay depends on the square distance \(\Gamma(t)\) between the packets moving in opposite directions (through the scalar product \(\langle \phi^\pm | \phi^\pm \rangle \)), the coherence experiences a four time slower decay since it depends on the square distance \(\Gamma(t)/4\) between one of the moving packet and the standing packet related to the ground state (scalar product \(\langle \varphi | \phi^\pm \rangle \)). It is moreover to stress that, because of the non dissipative character of the decay, the qubit upper population does not go to zero, as in the spontaneous emission, but tends to half of the initial value.

**IV. ENTANGLEMENT EVOLUTION FOR THE TWO QUBITS**

**A. Concurrence**

As it is generally convenient for these bipartite mixed states we will analyze the time evolution of the entanglement between the internal variables of the two atoms,
that the concurrence of the state (25) is

\[ C(\rho^{AB}) = \max\{0, |a_3| + \sqrt{a_1 a_2} - |a_3| - \sqrt{a_1 a_2}| \} = 2a_3. \]  

(35)

The entanglement with the non relevant systems causes a degradation of the nonlocal correlation between the qubits. According to previous results (see, for example [4, 6]), and as shown in Fig.1 state (23) gives rise to asymptotic disentanglement with the same rate of the decoherence. Because of the particular dynamics of the OSG model, in this case the disentanglement rate is one half time the population decay rate. Finally, we note that, for \( \Gamma(t) = 0 \), i.e. when translation dynamics of the center of mass of the two atoms is not taken into account, the concurrence (35) reduces to that derived in [4] for the two JC atoms case.

More interesting results are expected for the initial state (26), for which the concurrence is

\[ C(\rho^{AB}) = \max\{0, d(t)\} \]  

(36)

where

\[ d(t) = |b_1| + \sqrt{b_2 b_5} - |b_1| - \sqrt{b_2 b_5} - 2b_3 \]

\[ = 2(|b_1| - b_3). \]  

(37)

The behavior of concurrence (36) is shown in Fig. 2. Also in this case, the coherence \( b_1 \) between the two atoms goes to zero as \( a_3 \) in the previous case does. However, the concurrence experiences in this case a sudden death because of the probabilities \( b_3 \) of finding the atoms in the two states \( |+\rangle \) and \(|-\rangle \) which are not correlated to the others and which tend to \( \cos^2(\gamma)/4 \) for long time. Figure (2) makes evident the strong dependence of the entanglement life-time on the parameter of the initial two qubit state (26), similarly to the theoretical [3] and experimental [8] results. However, in this two last papers it is outlined that only some initial entanglements may undergo to sudden death. On the contrary, due to the particular environmental action of the OSG model, there is no threshold in our case. In fact, the function \( d(t) \) of Eq. (37) is negative if \( b_3/|b_1| > 1 \) and it turns out that

\[ b_3/|b_1| \geq \frac{\cot \gamma}{2} \sinh [\Gamma(t)/2]. \]  

(38)

From this equation one can realize that for any value of \( \gamma \) there exists a finite time for which the concurrence goes to negative values and the two qubits become disentangled. In this respect, our model behaves similarly to the continuous variable two-atom model of Ref. [3].

**B. Separability**

We will now inquire on the separability of the reduced density matrices (26) and (27) adopting the Peres-Horodecky test [18, 19]. As it is known, for two qubits the non-negativity of the eigenvalues of the partial transposed matrix works as a necessary and sufficient condition for the separability. Consider first the partial transposed of (26),

\[ \sigma^{AB}(t) = \begin{pmatrix} 0 & 0 & a_3^* \\ 0 & a_1 & 0 \\ a_3 & 0 & a_4 \end{pmatrix} \]  

(39)

whose elements are given by Eqs (24). We easily find that three eigenvalues of (39),

\[ \lambda_1 = a_1, \quad \lambda_2 = a_2, \quad \lambda_3 = \frac{a_4^2}{2} + \frac{1}{2} [a_1^2 + 4a_3^2]^{1/2}, \]  

(40)

are non-negative, while the forth eigenvalue

\[ \lambda_4 = \frac{a_4^2}{2} - \frac{1}{2} [a_1^2 + 4a_3^2]^{1/2} \]  

(41)
assumes only non-positive values, as figure 4 shows.

We wish to point out that for the initial state (8) the two qubits display EPR correlations [20] for all times, but for isolated points (see Fig. (1)). Accordingly, one can see from the analytical expression of $\lambda_4$ (more accurately than from Fig. (4)) that the bipartite system is not separable but for isolated points, and tends to the separability for long times.

Finally, for the density matrix (27) relative to the initial state (26) the partial transposition gives

$$\sigma^{AB}(t) = \begin{pmatrix}
    b_3 & 0 & 0 & b_1^* \\
    0 & b_2 & 0 & 0 \\
    0 & 0 & b_5 & 0 \\
    b_1 & 0 & 0 & b_3
\end{pmatrix}$$

(42)

whose elements are given in this case by Eq.s (29). As in the previous case, three eigenvalues

$$\nu_1 = b_2, \quad \nu_2 = b_5, \quad \nu_3 = b_3 + |b_1|$$

are non-negative, while a forth eigenvalue

$$\nu_4 = b_3 - |b_1|$$

(43)

(44)

takes on negative values for some time intervals (see figure 3), in exact correspondence with the existence of a concurrence (see figure 2).

V. CONCLUSIONS

In this paper it has been analytically derived the time evolution of the initial entanglement of two atoms which separately interact with two distinct cavities. We have used the optical Stern-Gerlach model to include the external translational dynamics of each atom. The presence of entanglement has been ascertained through the concurrence and the separability and we have found, as in Ref. [4], that the two atom internal dynamics follows different patterns and becomes irreversibly separable in different times, depending on the different initial configurations. As found in other contests [4, 6, 8] which use different environments and where the disentanglement is due essentially to spontaneous emission, also in the OSG model the entanglement may undergo to an irreversible sudden death. In addition, the particular non dissipative environment here considered suddenly destroys the initial entanglement of Bell's states $|\phi^\pm\rangle$ superposition (Eq. 20) for any initial value of $\gamma$. This can in part be ascribed to the peculiar dynamics of the qubits in the OSG model in which the population $b_3$ (matrix (27)) or the non correlates states $|+-\rangle$ and $|--\rangle$ does not go to zero, but tend to $\cos^2(\gamma)/4$ for long time. We can consequently conclude that nonlocal coherences manifest an amazing fragility even towards a non dissipative environment made of a single degree of freedom.

* Electronic address: vaglica@fisica.unipa.it

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