Estimation of Broadband Time-Interleaved ADC's Impairments and Performance Using Only Single-Tone Measurements

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ABSTRACT This paper presents a method to estimate the internal impairment parameters of broadband time-interleaved (TI) ADCs using only single-tone measurement data acquired at two different frequencies. The estimated parameters, which include the signal-independent noise power, clock jitter, nonlinearity coefficients, and all sub-ADC mismatch parameters, are further used to predict the error power breakdown and the overall SNDR of the ADC for a multi-tone signal. In this way, the underlying sources of a TI ADC's performance limitations can be separately evaluated with standard and low-cost single-tone measurements. Accurate estimation of the ADC's impairment parameters is helpful for iteratively improving the ADC's performance during design, characterizing prototypes, and potentially simplifying the production test. The proposed method is verified with comprehensive simulations and hardware measurement on a 3.6-GSPS, 12-bit, 2-channel TI ADC whose SNDR for a multi-tone signal is estimated at about 1 dB of its actual value sweeping its internal parameters over a wide range of values.

INDEX TERMS ADC test, bandwidth, broadband, gain, mismatch, multi-tone, nonlinearity, offset, parameter estimation, timing, time-interleaved ADC.

I. INTRODUCTION

The demands on high-resolution broadband analog-to-digital converters (ADCs) continue to increase for emerging communication systems, such as 5G base stations, software-defined radios, and full-bandwidth capture (FBC) cable video receivers [1], [2]. A typical case is an FBC receiver that digitizes the signal spectrum from 50 to 1000 MHz using a single ADC [3], [4]. Although the conversion rate demand of ADCs continues to increase, it remains difficult for an individual ADC circuit to achieve the sampling rate, resolution, and power efficiency simultaneously required for broadband applications. Hence, time-interleaving (TI) is widely used at sampling rates beyond 1GS/s. Much such reported broadband ADCs are realized with multiple SAR [5], [6], pipelined [7] - [9], or pipelined-SAR [10] sub-ADCs operating in TI architectures. Since the performance of TI ADCs is often limited by the various mismatches between sub-ADCs, many comprehensive studies have been conducted to estimate and correct these mismatch errors [11]-[17]. Generally, the dynamic performance of an ADC is evaluated by observing its output spectrum for a single-tone sinusoidal input, which quantifies all of the ADC's impairments into well-known metrics, such as signal-to-noise-and-distortion ratio (SNDR), effective number of bits (ENOB), and total harmonic distortion (THD). This standard method is easy to perform and accurately evaluate the ADC's performance for narrow-band signals. However, it is not straightforward to assess ADC's internal impairments and performance for processing broadband signals [18]. Samples of a broadband signal have an approximately Gaussian distribution. Therefore, the signal-dependent error components in the ADC’s output spectrum for a broadband signal are different from a full-scale sinusoidal signal. The
metrics for evaluating single-tone sinusoidal signals may lead to overdesign or under-design of an ADC for broadband signals [18]. A direct way to evaluate the ADC’s broadband performance is to perform a multi-tone measurement [19]. However, conducting a multi-tone test is much more difficult than a standard single-tone test because it is challenging to realize high-quality multi-tone generators for practical tests. Moreover, a multi-tone test does not explicitly quantify the underlying sources of performance degradation in an ADC. For example, different sources of mismatch and nonlinearity give rise to error/distortion tones (some of which may overlap with signal tones), and both sampling jitter and thermal noise result in an elevated broadband noise floor.

In the estimation and correction of mismatch errors for TI ADCs, the estimation accuracy is critical. Inaccurate identification of the impairments’ parameters will degrade the ADC performance even with the best correction algorithms. Furthermore, two main approaches are known for mismatch calibration: background and foreground calibration. Although background calibration does not require a training input signal or a priori information about the mismatches, it requires a longer convergence time. In addition, background approaches operate continuously, depleting the available power consumption budget, and can occupy a large area. Moreover, back-ground strategies can converge to undesired values if its assumptions about the input statistics are inaccurate. Thus, background calibration is often inappropriate for broadband ADC applications. For fast and accurate convergence of mismatch calibration, foreground methods are preferred. Although it relies on the application of an input training signal with known characteristics, foreground calibration can run during normal ADC operation in particular applications such as wireless communication and radars.

### A. LITERATURE REVIEW

We here specifically review foreground calibration methods for TI ADCs, which is the focus of this work. A single sinusoidal test signal is used in [20] to estimate the offset, timing skew and bandwidth mismatches of an 8-channel TI ADC. In [21] the linear frequency modulated signal (LFM) is employed to estimate the gain, offset, and phase mismatches of a 2-channel TI ADC. The technique used in [22] proposed a frequency-dependent mismatch detection method using a non-return-to-zero (NRZ) multitone signal and a notch filter to extract the mismatch information. With a signal generated from the ADC clock, [23] proposed a method to measure mismatch spurs in the spectrum, including the gain, offset and time mismatches. The estimation of nonlinearity impairments in an 8-channel TIADC is presented in [24] including the estimation of gain and time skew mismatches. In [25] the timing skew mismatch is estimated in a 16-channel TI ADC using only a few resistors, a capacitor, and a clock inverter. A pilot tone is generated in [26] and applied to a 12-channel TI ADC input to estimate its timing mismatches.

An estimation method based on the Least Squares (LS) technique is proposed in [27] to estimate the offset, gain and timing mismatches of an 8-channel TI ADC. Two types of bandwidth mismatch, namely sampler and buffer bandwidth mismatches, are estimated in [28] for a 32-channel TI ADC utilizing least-squares (LS) minimization by injecting two single-tone signals at two different frequencies. The influence of nonlinearity mismatches is analyzed in [29] on a 4-channel TI ADC by using a polynomial model, while the effect of gain and offset errors on the performance of a 2-channel TI ADC is quantified in [30].

While the nonlinearity is included in [24] and [29], they validated the proposed analysis only by simulation. In addition, none of the summarized works [20-30] introduced

| References | Offset | Gain | Timing | BW | Nonlinearity | Clock jitter | Training signal | # of channels | Validation |
|------------|--------|------|--------|----|--------------|--------------|-----------------|---------------|------------|
| [20]       | yes    | no   | yes    | yes| no           | no           | Single-tone     | 8             | Measurement |
| [21]       | yes    | yes  | yes    | no | no           | no           | LFM pulse       | 2             | Measurement |
| [22]       | no     | yes  | no     | yes| no           | no           | NRZ pattern     | 2             | Measurement |
| [23]       | yes    | yes  | yes    | no | no           | no           | Multi-tone      | 16            | Simulation  |
| [24]       | no     | yes  | yes    | no | yes          | no           | Single-tone     | 8             | Simulation  |
| [25]       | no     | yes  | no     | no | no           | no           | Single-tone     | 16            | Measurement |
| [26]       | no     | no   | yes    | no | no           | no           | Pilot tone      | 12            | Measurement |
| [27]       | yes    | yes  | yes    | no | no           | no           | Single-tone     | 8             | Simulation  |
| [28]       | no     | no   | no     | yes| no           | no           | 2x Single-tone  | 32            | Simulation  |
| [29]       | yes    | yes  | yes    | no | no           | no           | Single-tone     | 4             | Simulation  |
| [30]       | yes    | yes  | no     | no | no           | no           | Single-tone     | 2             | Measurement |
| This work  | yes    | yes  | yes    | yes| yes          | yes          | 2x Single-tone  | 2             | Measurement |

TABLE I COMPARISON WITH PRIOR WORK ON MISMATCH ANALYSIS AND PROPOSED WORK

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the clock jitter in their mismatch analysis. We summarize the recent works on mismatch analysis of TI ADCs in Table I and compare them with the work in this paper where we consider most of the internal impairments including offset, gain, timing, non-linearity, bandwidth, and clock jitter mismatches using single-tone measurement at two different frequencies.

B. Contributions of the paper

There is much prior art on the estimation of offset, gain, and timing mismatches in TI ADCs, whereas nonlinearity and bandwidth mismatches have been investigated separately in recent years. However, the prior art has not considered gain, timing, offset, bandwidth, and nonlinearity mismatches and clock jitter simultaneously in TI ADCs. This paper presents a method to estimate the internal impairments in TI ADCs using only single-tone measurement data acquired at two different frequencies. The parameters that can be estimated accurately include the offset, nonlinearity, gain, and overall timing mismatches, as well as the signal-independent noise power, clock jitter, and average nonlinearity coefficients. The estimated parameters are further used to determine the error power breakdown and the overall S/NDR of the ADC for an arbitrary multi-tone signal. Therefore, the underlying sources of a TI ADC's performance limitations can be separately evaluated with standard and low-cost single-tone measurements. The proposed method is verified using comprehensive simulations and practical measurements performed on a 3.6-GSps 12-bit 2-channel TI ADC.

The remainder of this paper is organized as follows. A general TI ADC model with various impairment parameters is introduced in Section II. Section III describes the proposed method for estimating the ADC’s internal impairment parameters, while the expressions to estimate the ADC’s performance for multi-tone signals are derived in Section IV. Section V shows the verification with simulation and hardware measurements. Finally, conclusions are drawn in Section VI.

II. IMPAIRMENT MODELING IN TI ADCS

An \(M\)-channel TI ADC consists of \(M\) sub-ADCs, each of which samples the input signal with the same sampling rate of \(1/(MT_S)\) and a time shift, \(T_s\), between adjacent sub-ADCs. The outputs may be multiplexed, resulting in an overall sampling rate of \(1/(T_S)\). The performance of the TI ADC is not only influenced by the impairments in each sub-ADC but also by various mismatches between the sub-ADCs. The model of an \(M\)-channel TI ADC including several impairments is given in Fig. 1, in which the polynomial function \(T_m(x)\) relates the input and output of the \(m\)-th sub-ADC:

\[
T_m(x) = a_{0,m} + a_{1,m}x + a_{2,m}x^2 + a_{3,m}x^3 + \ldots \quad (1)
\]

where \(a_{0,m}\) is the DC offset, \(a_{1,m}\) is the linear gain, and \(a_{2,m}\) and \(a_{3,m}\) represent the 2\(^{nd}\) and 3\(^{rd}\) order nonlinearity coefficients, respectively. In this paper, nonlinear terms higher than the third order are omitted to simplify the analysis. Normally, \(a_{1,m}\) has a nominal value of 1, and is therefore written:

\[
a_{1,m} = 1 + \epsilon_{m} \quad (2)
\]

where \(\epsilon_{m}\) is each sub-ADC's gain error.

Although the input network for a real ADC can be very complex, it is still acceptable to assume a first-order input structure composed of a sampling capacitor and the on-resistance of a sampling switch for most cases, resulting in a lowpass transfer function for each sub-ADC:

\[
P_m(j\Omega) = \frac{1}{1 + j\Omega/\Omega_{p,m}} = \frac{1}{1 + j\Omega/((1 + \beta_m)\Omega_p)} \quad (3)
\]

where \(\Omega_{p,m} = 1/(R_{S,m}C_{S,m})\) is the 3-dB bandwidth of the \(m\)-th sub-ADC, \(\Omega_p\) is the nominal bandwidth, and \(\beta_m\) is the deviation of the \(m\)-th sub-ADC bandwidth from its nominal value.

As seen from Fig. 1, the ideal sampling instants of the \(m\)-th sub-ADC are \((kM + m)T_S\), where \(k\) is an integer. The parameter \(\Delta T_{1,m}\) represents a fixed time shift from the ideal sampling instants and the difference between these time shifts is termed timing skew.

In summary, there are 5 mismatches in the model: offset, gain, nonlinearity, bandwidth and timing, which in conjunction cause spurious tones in the TI ADC's output spectrum.

Moreover, in broadband ADCs, random clock jitter is another serious problem which causes sampling errors that
depend on the slope of the input signal. Since it is usually a random variable with a Gaussian distribution, the sampling error due to clock jitter will cause signal-dependent noise. Because the clock signals for all sub-ADCs usually come from the same clock source, their clock jitter is here assumed to have the same distribution. Therefore, the sample error due to clock jitter, \( x_{CJ}(t) \) is an additional error source contributing to the overall output, as shown in Fig. 1. The quantization error, \( x_{QN}(t) \), is modeled as yet another additive error source.

In order to derive expressions for the error power caused by each individual impairment, the frequency response of an arbitrary M-channel TI ADC with a linear ADC transfer characteristic is analyzed first. Then the expressions to calculate the error power in a 2-channel TI ADC with nonlinear transfer characteristic are derived.

A. Frequency Response of M-Channel TI ADC

Assuming that the transfer characteristic of each sub-ADC is linear, i.e. \( T_m(x) = a_i x \), and neglecting (for the time being) quantization noise and jitter, the model of the TI ADC in Fig. 1 can be simplified to the one shown in Fig. 2. The fixed clock timing error \( \Delta T_{x,m} \) is translated into a phase shift in the frequency response of the \( m \)-th ADC, \( G_m(\Omega) \):

\[
G_m(\Omega) = P_m(\frac{\Omega}{2\pi}) e^{j(k\Omega T_m)}
\]

(4)

Therefore, the sampled time-domain expression of \( y(t) \) in Fig. 2 can be written as

\[
y(t) = \sum_{m=0}^{M-1} \left[ a_{m} g_m(t) \right] * x(t) \sum_{k=-\infty}^{\infty} \delta(t - mt_{m} - kMT_{x,m})
\]

(5)

where \( g_m(t) \) is the impulse response of \( G_m(\Omega) \), the symbol * denotes convolution, and \( \delta(t) \) is the Dirac Delta function. The frequency-domain equivalent of (5) is

\[
Y(\Omega) = \frac{1}{MT_{x,m}} \sum_{m=0}^{M-1} \left[ a_{m} G_m[\Omega - \frac{k\Omega_{m}}{M}] \right] \cdot \chi(j\Omega - \frac{k\Omega_{m}}{M}) \exp(-j\frac{\pi m}{M})
\]

(6)

where \( \Omega_{S} = (1/T_{S}) \) is the sampling frequency and

\[
F_{m,t}(\Omega) = \frac{1}{M} \sum_{m=0}^{M-1} \left[ a_{m} G_m(\Omega) \exp(-j\frac{\pi m}{M}) \right]
\]

(7)

From (6) and (7), the input spectrum, \( \chi(\Omega) \), is repeated periodically centered at integer multiples of \( \Omega_{S}/M \). Each repeated \( \chi(\Omega) \) is filtered by a different frequency response \( F_{M,k} \), which is a weighted summation of all sub-ADC's frequency responses. Notice that there is a pattern in the summation, repeating at intervals of \( M \) in the index \( k \). For values of \( k \text{ mod } M = 0 \), the frequency responses of all sub-ADCs add up. For values of \( k \text{ mod } M \neq 0 \), the sub-ADC frequency responses cancel each other with a given pattern.

Ideally, if all sub-ADCs have the same \( a_{1,m} G_m(\Omega) \) (without mismatch), only the spectrum at values of \( k \text{ mod } M = 0 \) remains, while other replicas are canceled out completely. Any mismatch in the sub-ADC functions will cause non-zero replicas which deteriorate the SNDR and SFDR of the ADC.

B. Frequency Response of M-Channel TI ADC

Although only expressions for a 2-channel TI ADCs will be derived, similar expressions can be derived for TI ADCs with 4 or more sub-ADCs based on the fundamental spectrum replicating mechanism given by (6) and (7).

As indicated by (6) and (7), there are only two different frequency responses, \( F_{M,0}(\Omega) \), for a 2-channel TI ADC. Therefore, for a real input signal \( x(t) = V_{in}\cos(\Omega_{in}) \) with \( \Omega_{in} = 2\pi f_{in} \), the mismatches of the sub-ADCs will result in a replica of the fundamental tone (hereafter referred to as RFund) at \( (\Omega_{S}/2 - \Omega_{in}) \) in addition to the fundamental tone at \( \Omega_{in} \) (considering the spectrum in the first Nyquist zone).

The two summations in (7) have oppositely signed terms leading to different frequency responses for RFund \((F_{2,0})\) and its replica \((F_{2,1})\), respectively:

\[
F_{2,0} = A_{fund,0} + A_{fund,1} = \frac{a_{1,0} G_0(\Omega_{in}) + a_{1,1} G_1(\Omega_{in})}{2}
\]

(8)

\[
F_{2,1} = A_{fund,0} - A_{fund,1} = \frac{a_{1,1} G_0(\Omega_{in}) - a_{1,1} G_1(\Omega_{in})}{2}
\]

(9)

where \( A_{fund,0} \) and \( A_{fund,1} \) are the frequency responses of the first and second sub-ADCs, respectively, evaluated at the fundamental frequency.

For an input power \( P_{in} \), the replica tone's power is therefore

\[
P_{RFund} = \frac{F_{2,1}^2}{F_{2,0}} P_{in}
\]

(10)

To obtain the dependence of \( P_{RFund} \) on the mismatch parameters, the \( G_m(\Omega) \) in (4) is rewritten

\[
G_m(\Omega) = |P_m(\Omega)| \exp[j\varphi_m(\Omega) + j\Omega \Delta T_{x,m}]
\]

(11)

where \( |P_m(\Omega)| \) and \( \varphi_m(\Omega) \) are the magnitude and phase of \( P_m(\Omega) \) respectively:

\[
|P_0(\Omega)| = \frac{1}{\sqrt{1 + \Omega^2/(1 + \beta_m) \Omega_{x,m}^2}} \approx \frac{1}{\sqrt{1 + \Omega_{x,m}^2/(\Omega_{x,m}^2 - \beta_m)}}
\]

(12)

\[
\varphi_m(\Omega) = -\arctan \left( \frac{\Omega}{(1 + \beta_m) \Omega_{x,m}} \right) \approx -\arctan \left( \frac{\Omega_{x,m}}{\Omega_{x,m} - \beta_m} \right)
\]

(13)

Substituting (2), (11), (12), and (13) into (8), (9) and assuming that \( \varepsilon_m, \beta_m, \text{ and } \Delta T_{x,m} \) are relatively small, \(|F_{2,1}|\) and \(|F_{2,0}|\) can be derived approximately as

\[
|F_{2,0}| \approx \frac{1}{\sqrt{1 + (\Omega_{x,m}^2 - \beta_m)^2}}
\]

(14)
where $\Delta G$ and $\Delta \Phi$ are the gain and combined phase mismatches, respectively:

$$\Delta G \approx \varepsilon_0 - \varepsilon_i$$  \hspace{1cm} (16)

$$\Delta \Phi = \left( (\Delta \varepsilon_{10} - \Delta \varepsilon_{11}) + (\beta_0 - \beta_1) / \Omega_p \right) \cdot \Omega$$  \hspace{1cm} (17)

In the derivations, the contribution from the bandwidth mismatch to the overall gain mismatch is neglected, because it is much smaller than $|\varepsilon_1 - \varepsilon_0|$ in normal cases. However, the bandwidth mismatch gives a significant contribution to the overall phase mismatch, i.e., the small bandwidth mismatch is converted to an extra fixed timing mismatch, as given by (17). Substituting (14) to (17) into (10), the replica tone’s power is rewritten as

$$P_{\text{RFund}} = \frac{1}{4} \left[ (\Delta G)^2 + (\Delta \Phi)^2 \right] P_{\text{in}}$$  \hspace{1cm} (18)

### C. Frequency Response of M-Channel TI ADC

The analysis from the previous section is only suitable for the case where the sub-ADC’s response is linear. When a nonlinear input-output relationship as in (1) is considered, the model of each sub-channel is changed into that shown in Fig. 3. As the system is nonlinear, it can’t be represented by a LTI transfer function. We next proceed with the analysis for a single-tone input.

For $x(t) = V_0 \cos(\Omega_0 t)$, the non-zero $a_{0,m}$, $a_{2,m}$, and $a_{3,m}$ will generate a DC tone, a 2nd-order harmonic tone (H2) at $2\Omega_0$, and a 3rd-order harmonic tone (H3) at $3\Omega_0$, within each sub-ADC. As $a_{2,m}$ also contributes to the DC offset, the overall DC offset of the $m$-th sub-ADC is written as

$$V_{\text{DC,m}} = a_{0,m} + a_{2,m} V_0^2 / 2$$  \hspace{1cm} (19)

Also, considering the contribution of $a_{3,m}$ to the output amplitude at the fundamental frequency, the frequency response for the fundamental tone of the $m$-th sub-ADC changes to

$$A_{\text{Fund,m}} = (a_{0,m} + (3/4)a_{3,m} V_0^2) G_m(j\Omega_0)$$  \hspace{1cm} (20)

Accordingly, the frequency responses for the H2 and H3 tones can be written as

$$A_{\text{H2,m}} = (1/2)a_{2,m} (G_m(j\Omega_0))^2 V_0$$  \hspace{1cm} (21)

$$A_{\text{H3,m}} = (1/4)a_{3,m} (G_m(j\Omega_0))^3 V_0$$  \hspace{1cm} (22)

As implied by the spectrum replicating mechanism from (6), the mismatches in a 2-channel TI ADC will also introduce replica tones of the content DC, H2, and H3, which are located at $\Omega_0/2$, $(\Omega_0/2 - 2\Omega_0)$ and $(\Omega_0/2 - 3\Omega_0)$ and termed as RDC, RH2 and RH3, respectively. As also implied by (7), the DC, fundamental, H2 and H3 tones in the overall output spectrum are the sum of the corresponding components of the 2 sub-ADCs, while RDC, RFund, RH2 and RH3 are obtained as the difference of the corresponding components. Therefore,

$$V_{\text{DC}} = (V_{\text{DC,0}} + V_{\text{DC,1}}) / 2$$  \hspace{1cm} (23)

$$A_{\text{Fund}} = (A_{\text{Fund,0}} + A_{\text{Fund,1}}) / 2$$  \hspace{1cm} (24)

With reasonable approximations, the power of the spurious tones can be derived from (23) and (24):

$$P_{\text{DC}} = P_{\text{RDC}}^2 \approx \left( (a_{0,m} + a_{2,m}) + (a_{2,m} + a_{3,m}) V_0^2 / 2 \right)^2 / 4$$  \hspace{1cm} (25)

$$P_{\text{RDC}} = P_{\text{RFund}}^2 \approx \left( (a_{0,m} - a_{2,m}) + (a_{2,m} - a_{3,m}) V_0^2 / 2 \right)^2 / 4$$  \hspace{1cm} (26)

$$P_{\text{RFund}} = \left( \frac{A_{\text{RFund}}}{A_{\text{Fund}}} \right)^2 P_{\text{in}} = \frac{\left( \Delta G + \frac{3}{4} (a_{0,m} - a_{1,m}) \Delta V \right)^2 + (\Delta \Phi)^2}{4} P_{\text{in}}$$  \hspace{1cm} (27)

$$P_{\text{H2}} = \left( \frac{A_{\text{H2}}}{A_{\text{Fund}}} \right)^2 P_{\text{in}} = \frac{\left( a_{0,m} + a_{1,m} \right)^2 V_0^2}{4} P_{\text{in}}$$  \hspace{1cm} (28)

$$P_{\text{RH2}} = \left( \frac{A_{\text{RH2}}}{A_{\text{Fund}}} \right)^2 P_{\text{in}} = \frac{\left( a_{0,m} - a_{1,m} \right)^2 V_0^2}{4} P_{\text{in}}$$  \hspace{1cm} (29)

$$P_{\text{H3}} = \left( \frac{A_{\text{H3}}}{A_{\text{Fund}}} \right)^2 P_{\text{in}} = \frac{\left( a_{1,m} + a_{3,m} \right)^2 V_0^2}{4} P_{\text{in}}$$  \hspace{1cm} (30)

$$P_{\text{RH3}} = \left( \frac{A_{\text{RH3}}}{A_{\text{Fund}}} \right)^2 P_{\text{in}} = \frac{\left( a_{1,m} - a_{3,m} \right)^2 V_0^2}{4} P_{\text{in}}$$  \hspace{1cm} (31)

Besides the error power of the spurious tones indicated by (25) to (31), noise is also a major concern in ADCs. The signal-independent quantization noise power for an $N$-bit
ADC is

\[ P_{QN} = \frac{1}{12} \left( V_{\text{LSB}} \right)^2 = \frac{1}{12} \left( \frac{V_{\text{FS}}}{2^N} \right)^2 \]  

(32)

where \( V_{\text{LSB}} \) is the voltage for one least-significant-bit (LSB), \( N \) is the resolution of the ADC, and \( V_{\text{FS}} \) represents the full-scale input voltage. Another important noise source in broadband ADCs is the noise caused by clock jitter. Assuming that the absolute jitter of the clock, \( \tau \), is a zero-mean Gaussian random variable with a standard deviation of \( \sigma_j \), the resulting noise power for an input signal \( x(t) = V_{\text{in}} \cos(\Omega_{\text{in}} t) \) is [31]

\[ P_{\text{CKJ}} \approx \sigma_j^2 \Omega_m^2 \cdot V_m^2 / 2 = \sigma_j^2 \Omega_m^2 P_m \]  

(33)

which increases with both the input amplitude and frequency.

Among the errors given by (25) to (33), \( P_{\text{RFund}}, P_{H2}, P_{H3}, P_{\text{RHD}}, P_{\text{RHD}}, \) and \( P_{\text{CKJ}} \) are multiplicative errors that scale with the input signal’s power while \( P_{\text{DC}}, P_{\text{DC}} \) and \( P_{\text{QN}} \) are additive errors. Although \( P_{\text{DC}} \) and \( P_{\text{DC}} \) have signal-dependent components due to nonlinearity, the signal-independent components dominate for small input amplitude.

In order to verify the previous derivations, a 12-bit, 1-GSps, 2-channel TI ADC is modeled in Matlab, with the parameters provided in Table II. Fig. 4 plots the simulated spectra of the ADC with a full-scale sinusoid input signal at frequencies \( \Omega_m = 0.05 \Omega_s \) and \( \Omega_m = 0.45 \Omega_s \). As shown, the noise floor for \( \Omega_m = 0.05 \Omega_s \) is much lower than the spurious tones. For \( \Omega_m = 0.45 \Omega_s \), the noise floor increases remarkably because the jitter induced noise power increases with frequency. The power of RFund also increases with frequency since the phase error is proportional to \( \Omega_m \), as shown by (17).

Using equations (26) to (33), the SNDR of the two-channel TI ADC can be obtained as the ratio of the signal power over the sum of all the error power excluding \( P_{\text{DC}} \). Fig. 5 plots the calculated SNDR for different input frequencies with comparison to the simulated values, showing that the calculated SNDR values match very well with the simulated values.

Fig. 6 further compares the calculated and simulated power of each error source as a function of \( \Omega_m \). All calculated values match well with the simulated values (with an estimation error lower than 1dB) except for the \( P_{\text{RHD}} \) values at high input frequencies, because the noise floor is higher than \( P_{\text{RHD}} \) at high input frequencies. As can be seen, RFund dominates the error power contribution and increases remarkably with \( \Omega_m \) as expected. The power of other error sources decreases slightly with the input frequency because the input signal is slightly attenuated by the low-pass sampling network.

Equations (25) to (33) have been verified through extensive simulations with a wide range of ADC impairment parameters.

### III. IMPAIRMENT PARAMETER ESTIMATION USING TWO SINGLE-TONE MEASUREMENTS

Using equations (25) to (33), the SNDR and error power from various error sources can be calculated directly for a single-tone input if the values of the impairment parameters are known. The ADC’s performance for an arbitrary input signal can also be estimated using the impairment parameters. However, ADC performance is often evaluated simply in terms of its SNDR, SFDR and THD, often measured with a single-tone input, which gives very little information about the internal impairments that are limiting performance.

Since most error sources, including the offset, nonlinearity and time-interleaved channel mismatches, generate spurious tones at known locations in the ADC’s spectrum for a single-tone input, it is possible to calculate the underlying impairment parameters from the output spectra.
measured with a single-tone input. Because in some cases more than one impairment affects the same spurious tones, the spectrum data for a single measurement is not enough for accurate estimation of the error parameters. Fortunately, those impairments impact the spurious tones’ power differently at different frequencies. Therefore, we propose a method to estimate the impairment parameters by performing only two single-tone measurements at different frequencies. In principle, it is also possible to apply the two sinusoid signals in one measurement and extract the relevant parameters from a single ADC output record, but it can be easier and less expensive to generate only a single tone at a time in many laboratory measurement environments.

The two input frequencies should be selected in the ADC’s input bandwidth and high enough to excite noticeable errors due to timing mismatch and clock jitter, while their spacing should be large enough to distinguish between frequency dependent terms and purely level-dependent ones. In addition, the input frequencies should be in prime bins with respect to the clock frequency, which can be realized by synchronized signal generators in a real measurement environment.

The full-scale voltage, $V_{FS}$, is assumed to be known in the method. In addition, the contribution of the 3rd-order nonlinearity mismatch to overall gain mismatch in (27) is neglected along with the contribution of the 2nd-order nonlinearity mismatch to the overall offset mismatch in (26). In parallel, the input amplitude should be reduced to ensure this approximation is accurate.

In the following sections, the frequencies for the 2 single tone measurements are defined as $f_a$ and $f_b$, while the corresponding digital output codes are termed as $D_a$ and $D_b$, respectively.

### A. Estimation of Input Amplitude

In a lab environment, although the input amplitude can often be read out directly from the signal source for test, the path from the signal source to the ADC will introduce an unknown amount of attenuation at the input of the ADC. In that case, it is more accurate to extract the signals’ amplitudes, $V_a$ and $V_b$, from the time domain output codes:

$$V_{ab} = V_{FS} \cdot \sqrt{\frac{2}{N_{FFT}}} \left( D_{ab} \frac{1}{N_{FFT}} \sum_{a,b} D_{ab} \right)^2$$

(34)

where $N_{FFT}$ is the number of time-domain samples acquired for calculation of the Fast Fourier Transformation (FFT).

### B. Estimation of Input Amplitude

Ideally, the noise floor of the ADC’s spectrum is determined only by the quantization noise ($P_{QN}$) and the clock jitter induced noise ($P_{CKJ}$). However, various noise and interference sources may exist in practical ADCs, resulting in higher noise floor and fixed spurs. In order to include all possible noise sources in the calculation, we define $P_{ST}$ as the sum of all signal-independent noise powers. Therefore, the SNR excluding the power of DC, H2, H3, RFund, RH2, RH3 and RDC tones is defined as

$$SNR_{ab} = \frac{P_{in,ab}}{P_{ST} + \sigma_j^2 (2\pi f_{ab})^2 \cdot V_{ab}^2 / 2}$$

(35)

The values of $SNR_a$ and $SNR_b$ can be obtained directly from the two FFT results, and can be used together with $V_{ab}$ from (34) to solve for the unknowns $P_{ST}$ and $\sigma_j^2$:

$$\begin{bmatrix} P_{ST} \\ \sigma_j^2 \end{bmatrix} = \begin{bmatrix} SNR_a \\ SNR_b \end{bmatrix} \cdot \begin{bmatrix} (2\pi f_a)^2 \cdot V_a^2 / 2 \\ (2\pi f_b)^2 \cdot V_b^2 / 2 \end{bmatrix} \begin{bmatrix} V_a^2 / 2 \\ V_b^2 / 2 \end{bmatrix}$$

(36)

### C. Estimation of Nonlinearity Parameters

From (28) to (31), nonlinearity parameters can be solved directly with the normalized power of the harmonic tones and their replica tones from either of the two FFT results:

$$|a_{2,avg}| = \frac{|a_{2,0} + a_{2,1}|}{2} = 2 \sqrt{\frac{P_{H2,ab}}{P_{in,ab}}} \frac{1}{V_{ab}}$$

(37)

$$|\Delta a_2| = |a_{2,0} - a_{2,1}| = 4 \sqrt{\frac{P_{RH2,ab}}{P_{in,ab}}} \frac{1}{V_{ab}}$$

(38)

$$|a_{3,avg}| = \frac{|a_{3,0} + a_{3,1}|}{2} = 4 \sqrt{\frac{P_{H3,ab}}{P_{in,ab}}} \frac{1}{V_{ab}}$$

(39)
\[
|\Delta a_3| = |a_{3,0} - a_{3,1}| = 8 \sqrt{\frac{P_{R\Omega a,b}}{P_{m,a,b}}} \frac{1}{V_{a,b}}
\]  

(40)

D. Estimation of DC Offset and Offset Mismatch

As stated above, the contribution of the 2nd-order nonlinearity mismatch to the overall offset mismatch in (26) is generally negligible. The offset mismatch and average offset can then be calculated as

\[
|\Delta a_0| = |a_{0,0} - a_{0,1}| = 2V_{a,b} \sqrt{P_{R\Omega DC,a,b}}
\]  

(41)

\[
|a_{0,avg}| = \frac{|a_{0,0} + a_{0,1}|}{2} = V_{a,b} \sqrt{P_{DC,a,b}}
\]  

(42)

Averaging the results obtained from (37) to (42) using two separate sets of FFT data for \(f_a\) and \(f_b\) can improve the estimation accuracy.

E. Estimation of Gain and Overall Timing Mismatch

As explained above, the contribution of the third-order nonlinearity mismatch to overall gain mismatch in (27) is also generally negligible. First, overall phase mismatch is written in terms of TI ADC’s timing mismatch,

\[
\Delta \Phi = \Delta T_{tot} \cdot \Omega_m = \Delta T_{tot} \cdot 2\pi f_m
\]  

(43)

where \(\Delta T_{tot}\) is the equivalent overall timing mismatch:

\[
\Delta T_{tot} = (\Delta T_{t,0} - \Delta T_{t,1}) + (\beta_0 - \beta_1) / \Omega_p
\]  

(44)

According to (18), the normalized power of the RFund tone for the two input frequencies can be rewritten as

\[
\frac{P_{RFund,a,b}}{P_{m,a,b}} \approx \left[ (\Delta G)^2 + (\Delta T_{tot})^2 \left( 2\pi f_{a,b} \right)^2 \right] / 4
\]  

(45)

From (45), solutions for \(\Delta G\) and \(\Delta T_{tot}\) can be expressed in matrix form:

\[
\begin{bmatrix}
(\Delta G)^2 \\
(\Delta T_{tot})^2
\end{bmatrix} =
\begin{bmatrix}
1 / 4 & (2\pi f_{a,b})^2 / 4 \\
1 / 4 & (2\pi f_{a,b})^2 / 4
\end{bmatrix}
\begin{bmatrix}
P_{RFund,a,b} / P_{m,a} \\
P_{RFund,b} / P_{m,b}
\end{bmatrix}
\]  

(46)

In summary, with the measured ADC output spectra for \(f_a\) and \(f_b\), the internal impairment parameters for a 2-channel TI ADC can be estimated using expressions (36) - (42) and (46).

IV. PERFORMANCE ESTIMATION FOR MULTI-TONE SIGNAL

In broadband communication receivers, the ADC input spectrum is distributed across a wide range of frequencies. For example, broadband cable TV divides the spectrum from 48 MHz to 1 GHz into channels each with a bandwidth of 6 MHz. Multi-tone signals are often used to evaluate broadband ADCs for such applications, both in simulation and measurement [13], [15]-[19]. In this section, we calculate the error power contributed by each of the

ADC impairments we have identified to the overall SNDR of the ADC for multi-tone signals. This obviates the need for additional simulations or complex input signal generation in the lab, and identifies which specific impairments (jitter, nonlinearity, sub-ADC mismatch, etc.) limit ADC performance with broadband inputs.

A. Statistical Feature and Back-Off of Multi-Tone Signal

A multi-tone signal is obtained by combining several sinusoids with equal amplitude at frequencies that are equally spaced across a frequency band. If the total power of the multi-tone signal is the same as that of a full-scale single-tone signal, the amplitude of each tone is scaled by a factor of \(1/ \sqrt{N_{ch}}\) from the full-scale value, where \(N_{ch}\) is the number of signal channels.

However, the peaks of this combined signal will often exceed the ADC full-scale range [18]. Therefore, further back-off is required. The multi-tone signal is therefore written in term of the back-off coefficient, \(k_{bo}\)

\[
V_{in,mt} (t) = k_{bo} \cdot \frac{1}{\sqrt{N_{ch}}} \sum_{n=1}^{N_S} \frac{V_{FS}}{2} \cos(2\pi f_a t + \phi_n)
\]  

(47)

where \(\phi_n\) is a random phase for the \(n\)-th tone. Assuming that the variance of the \(n\)-th tone is \(D_n\) and all tones are independent, the variance of the combined multi-tone signal is

\[
P_{in,mt} = \sigma_n^2 = \sum_{n=1}^{N_{ch}} D_n = \frac{1}{2} \left( k_{bo} \frac{V_{FS}}{2} \right)^2
\]  

(48)

When \(N_{ch}\) is large enough, samples of the combined signal are nearly Gaussian-distributed random variables with a standard deviation of \(\sigma_n\) that is proportional to \(k_{bo}\). Fig. 7(a) plots the histogram of a 128-tone signal spanning from DC to 0.5 GHz with no back-off (\(k_{bo} = 0\ dB\)) which shows a
Gaussian-like distribution. As expected, a lot of samples exceed the full-scale range of the ADC which results in severe clipping error. Fig. 7(b) plots the histogram of the same signal with -12-dB back-off, showing that the clipping of the ADC is mitigated.

Although applying a larger back-off to the signal will decrease the signal power and also the SNDR of the ADC, it desensitizes the ADC to multiplicative error sources significantly [18]. Fig. 8 plots the weighted power of the clipping error and 3rd-order nonlinear error when an ideal Gaussian signal with \( \sigma_n \) defined by (48) is applied to the ADC model. As can be seen, when \( k_{bo} \) decreases from -11 dB to -12 dB, the clipping error almost disappears, and the nonlinear error also decreases remarkably.

To eliminate the clipping completely, \( k_{bo} \) in (47) should be lower than \( 1/\sqrt{N_{ch}} \). However, this causes the signal power to be too small, resulting in a very poor SNR. The optimum \( k_{bo} \) is the value for which the sum of the clipping and nonlinear error powers are just sufficient to noticeably deteriorate the SNDR. Fig. 9 plots the simulated SNDR as a function of \( k_{bo} \) for 12-bit and 10-bit ADCs when converting the 128-tone signal given by (47), in which only the clipping error and 3rd-order nonlinearity (\( |\alpha_3| = 0.005 \)) are included in the ADC model. As can be seen, when \( k_{bo} \) is small, increasing \( k_{bo} \) increases the SNDR one dB per dB until clipping dominates and the SNDR drops abruptly. Due to the plots’ asymmetry, a nominal \( k_{bo} \) may be chosen slightly below the peak SNDR point on Fig. 9 to accommodate small time variations in input amplitude. Suitable nominal values for \( k_{bo} \) in 12-bit and 10-bit ADCs are about -12 dB and -10 dB, respectively.

**B. Noise Power**

For the multi-tone signal, the signal-independent noise power, \( P_{ST} \), is the same as that of a single tone signal. However, the noise power resulting from clock jitter needs to be re-derived. For the clock jitter, \( r \), with a standard deviation of \( \sigma_r \), the jitter induced error voltage in the \( i \)-th sample can be expressed as

\[
\begin{align*}
  x_{\text{CKJ}}(i) &\approx r(i) \cdot V_{\text{in,m}}(i \cdot T_s) 
\end{align*}
\]

Assuming \( r \) is independent of the signal, the power of \( x_{\text{CKJ}} \) can be expressed as the product of the variances for \( r \) and \( V_{\text{in,m}} \). According to (47), the variance of \( V_{\text{in,m}} \) can be derived as

\[
D(V_{\text{in,m}}') = \frac{1}{2} \left( k_{bo} \frac{V_{FS}}{2} \right)^2 \cdot \frac{1}{N_{ch}} \sum_{n=1}^{N_{ch}} (2\pi f_n)^2
\]

We define \( \Omega_{rms} \) as the root-mean-square angular frequency of the signal tones

\[
\Omega_{rms} = 2\pi f_{rms} = \sqrt{\frac{1}{N_{ch}} \sum_{n=1}^{N_{ch}} (2\pi f_n)^2}
\]

Then, according to (48) - (51), the power of \( x_{\text{CKJ}} \) is obtained as

\[
P_{\text{CKJ,mt}} \approx D(r) \cdot D(V_{\text{in,m}}') = \sigma_r^2 \sigma_n^2 \Omega_{rms}^2
\]

Comparing (52) with (33), for a -12-dB backoff, it can be calculated approximately that \( P_{\text{CKJ,mt}} \) is about -17 dB lower than the \( P_{\text{CKJ}} \) for a full-scale sinusoid input at \( f_s/2 \).

**C. Error Power from Frequency Response Mismatch**

As shown in Section II, for a single-tone signal, mismatch in the frequency response of sub-ADCs in a 2-channel TI-ADC including their gain, timing and bandwidth mismatches generates a replica tone with power of \( P_{\text{Rfund}} \) given by (27). For a multi-tone input signal containing \( N_{ch} \) independent tones, it can be expected that \( N_{ch} \) independent replica tones will be generated. Because of the back-off in the multi-tone signal, the contribution of the 3rd-order nonlinearity mismatch to the gain mismatch can be neglected. Using equations (27), (43), (44) and (47), the resulted overall error power of the replica tones is

\[
P_{\text{Rfund,mt}} \approx \sum_{n=1}^{N_{ch}} \left[ \Delta G^2 + \Delta T_{\text{tot}}^2 (2\pi f_n)^2 \right] P_{\text{in,n}}
\]

where \( P_{\text{in,n}} \) is the power of the \( n \)-th tone in the multi-tone signal. As all the tones have almost the same power \( P_{\text{in,mt}}/N_{ch} \), (53) can be rearranged as

\[
P_{\text{Rfund,mt}} \approx \left( \Delta G^2 + \Delta T_{\text{tot}}^2 \Omega_{rms}^2 \right) \cdot P_{\text{in,mt}}
\]
where $P_{\text{in,mt}}$ is the overall power of the multi-tone signal.

**D. Error Power from DC Offset and Offset Mismatch**

When the multi-tone signal is applied to a 2-channel TI ADC, the DC tone and its replica (RDC) still exist in the output spectrum. Because of the back-off of the multi-tone signal, the contribution of the second-order nonlinearity to $P_{\text{DC}}$ and $P_{\text{RDC}}$ can be neglected. Therefore, the error power of the DC offset and offset mismatch for multi-tone signal can be rewritten as

$$P_{\text{DC,mt}} = \left(\alpha_{0,0} + \alpha_{0,1}\right)^2 / 4$$

(55)

$$P_{\text{RDC,mt}} = \left(\alpha_{0,0} - \alpha_{0,1}\right)^2 / 4$$

(56)

**E. Nonlinear Error Power**

As was the case for a single-tone input, the main 3rd-order nonlinearity error for a multi-tone signal in a 2-channel TI ADC is determined by $a_{3,\text{avg}} = (a_{3,0} + a_{3,1})/2$ which also has a replica determined by $\Delta a_{3} = (a_{3,0} - a_{3,1})$. As indicated by Fig. 8, the back-off of the multi-tone signal will reduce both the main nonlinear error and its replica remarkably. Additionally, since $|\Delta a|_3$ is usually much smaller than $a_{3,\text{avg}}$, the replica power of the 3rd-order nonlinear error is usually even lower and masked by the noise floor. Therefore, only the main 3rd-order nonlinear error needs to be calculated. Similarly, only the main second-order nonlinear error determined by $a_{2,\text{avg}} = (a_{2,0} + a_{2,1})/2$ needs to be calculated for the multi-tone signal.

Frequency analysis of the nonlinear distortion for a multi-tone signal is much more complex than that of a single-tone signal because of the intermodulation problem. Taking only the 3rd-order nonlinearity as an example, $2N_{\text{ch}}(N_{\text{ch}}-1)$ tones will be generated if the signal consists of $N_{\text{ch}}$ tones. Normally, the Volterra series can be used to derive the nonlinear error for an arbitrary signal [32]. However, since the multi-tone signal has an approximately Gaussian distribution, a simpler method based on the probability distribution function is used to calculate the nonlinear error power.

The squared 3rd-order nonlinear error, $(a_{2,\text{avg}}x^3)^2$, is weighted by the probability density function of the input signal, producing the weighted nonlinear error power shown in Fig. 8. The averaged error power is obtained by integrating the weighted nonlinear error along with the input signal range, resulting in,

$$P_{\text{NL,mt}} = \frac{1}{\sigma_x^2} \int_{-\infty}^{\infty} (a_{3,0}x^3)^2 \exp\left(-\frac{x^2}{2\sigma_x^2}\right)dx$$

(57)

With the help of numerical tools, $P_{\text{NL,mt}}$ can be solved as

$$P_{\text{NL,mt}} \approx 15a_{2,\text{avg}}^2 \sigma_x^4 = 15a_{2,\text{avg}}^2 P_{\text{in,mt}}^3$$

(58)

**F. SNDR for Multi-Tone Signal**

The SNDR of the ADC for a multi-tone signal can then be calculated as

$$\text{SNDR}_{\text{mt}} = \frac{P_{\text{in,mt}}}{P_{\text{ST}} + P_{\text{CKJ,mt}} + P_{\text{Rfund,mt}} + P_{\text{RDC,mt}} + P_{\text{NL3,mt}} + P_{\text{NL2,mt}}}$$

(60)

Similar to the single tone case, the power of the DC tone is excluded in (60) for calculation of the multi-tone SNDR.

**V. VERIFICATION VIA SIMULATION AND HARDWARE MEASUREMENT**

**A. Simulation Based Verification**

The proposed methods are verified first through simulation using the 12-bit, 1-GS/s, 2-channel TI ADC model built in MATLAB. The verification procedure is as follows:

1. Simulate the ADC twice with single-tone sinusoidal signals at two different frequencies ($f_b$ and $f_d$);
2. Estimate the impairment parameters from FFTs of the ADC outputs at $f_b$ and $f_d$ using the expressions obtained in Section III. (These may be compared with the actual, known, impairment parameters);
3. Estimate the SNDR for a multi-tone signal with the estimated impairment parameters using the expressions in section IV;
4. Simulate the ADC for the same multi-tone signal and compare the resulting simulated SNDR to the estimated value in step 3.

The process is repeated while sweeping impairment parameters over suitable ranges and repeating steps from 1) to 4).

The magnitudes of the single-tone signals are approximately -4 dBFS and 8192-point FFTs are used for spectral analysis. The two input frequencies are selected as $f_b = 0.05f_b$ and $f_d = 5f_b$ to provide a large enough spacing between the two frequencies.

A 128-tone signal with an optimum back-off of -12 dB is adopted for multi-tone characterization. To prevent the replica tones, harmonic tones and intermodulation tones from overflooded the signal tones, the frequencies of the signal tones are made to deviate from their ideal equally spaced positions slightly with a pseudo random pattern, as shown by the simulated spectrum in Fig. 10.

In all the following simulations, the impairment parameters except for the ones explicitly varied are assigned the values in Table II.

Fig. 11(a) plots the estimated clock jitter's standard deviation, $\sigma_j$, and estimated static noise power, $P_{\text{ST}}$, while
sweeping the actual value of $\sigma_j$. Two cases are included in this simulation to show that the proposed method can separate the jitter induced noise from other static noises accurately. In the first case, only quantization noise is included in the model. With the parameters given in Table II, the quantization noise power is expected to be $1.98 \times 10^{-8}$ V$^2$. In the second case, sampling noise with a uniform distribution (-0.25 to 0.25 $V_{\text{LSB}}$) is included. Therefore, the expected total static noise power for the second case is about $2.48 \times 10^{-8}$ V$^2$. Seen from Fig. 11(a), the estimated $\sigma_j$ and $P_{ST}$ are quite accurate for both cases. The estimated values for the other unaltered parameters are also very close to their actual values. With the extracted parameters, the multi-tone SNDR can then be calculated using the expressions obtained in Section IV. Fig. 11(b) compares the estimated multi-tone SNDR with the simulated values for different $\sigma_j$, showing that the SNDR decreases with the increase of $\sigma_j$. The maximum estimation error of less than 1.6 dB occurs at $\sigma_j = 0.1$ ps for both cases, and the error decreases to less than 1 dB for $\sigma_j > 1$ ps.

Fig. 12(a) plots the estimated $|\Delta a_{3,av}|$ and $|\Delta a_2|$ when sweeping $a_{3,1}$ and the estimated $|\Delta a_{2,av}|$ and $|\Delta a_2|$ when sweeping $a_{2,1}$. These simulations are conducted by sweeping $a_{3,1}$ and $a_{2,1}$ independently. The values of $a_{3,2} + a_{2,2}$ are fixed to -0.005 V$^{-2}$ and -0.005 V$^{-1}$, respectively. As shown, the estimated $|\Delta a_{3,av}|$, $|\Delta a_2|$, $|\Delta a_{2,av}|$, and $|\Delta a_2|$ are very close to their actual values. Fig. 12(b) compares the estimated multi-tone SNDR with the simulated values, showing that the maximum estimation error is less than 2.1 dB. As expected, the nonlinearity parameters have little influence on the multi-tone SNDR due to the back-off of the signal.

Fig. 13 shows the estimated offset mismatch, $|\Delta a_0|$, when sweeping $a_{0,1}$. The value $a_{0,2}$ is fixed to 0.9 $V_{\text{LSB}}$. A deviation of about 0.2 $V_{\text{LSB}}$ can be observed between the
estimated $|\Delta a_0|$ and its actual value, likely caused by the influence of the 2nd-order nonlinearity mismatch. Also shown in Fig. 13, the maximum error between the estimated and simulated multi-tone SNDR for different $a_{0,1}$ is about 1.3 dB.

The estimated gain mismatch, $|\Delta G|$, and the total timing mismatch $|\Delta T_{tot}|$, when sweeping $\Delta G$ are given in Fig. 14(a), in which the actual value of $|\Delta T_{tot}|$ is given by (44). As can be seen, the estimated $\Delta G$ is very close to its actual value even for a relatively large 3rd-order nonlinearity mismatch. The estimated $|\Delta T_{tot}|$ shows a deviation of less than 0.3 ps when the actual value of $\Delta G$ changes within a large range of 2.4%. Fig. 14(b) compares the estimated multi-tone SNDR with the simulated values, showing that the maximum estimation error is about 1.8 dB. As can be seen, the gain mismatch has a large influence on the multi-tone SNDR.

Fig. 15(a) plots the estimated $|\Delta T_{tot}|$ and $|\Delta G|$ when sweeping $\Delta T$ in the presence of different bandwidth mismatch, $\Delta \beta$. The estimated $|\Delta T_{tot}|$ is very close to its actual value even for a relatively large $\Delta \beta$ of 0.006. As expected, the estimated $\Delta G$ almost remains unchanged when $\Delta T$ changes by 10 ps (1% of the sampling period). Fig. 15(b) compares the estimated multi-tone SNDR for different $\Delta T$. When $\Delta \beta$ is small (0.001), the estimation is quite accurate. Simulation shows that the maximum estimation error of the SNDR is about 1.7 dB for a $\Delta \beta$ of 0.003, and increases to about 4 dB for $\Delta \beta$ of 0.006 even though the estimation of the $|\Delta T_{tot}|$ is accurate. The estimation error increases with $\Delta \beta$ because the approximation in (17) shows increasing error with the increase of $\Delta \beta$. Fortunately, proper circuit and layout design usually ensures a $|\Delta \beta|$ less than 0.003. Therefore, the approximation that the bandwidth mismatch contributes to an extra fixed timing mismatch is acceptable.

B. Verification with Hardware Measurement

The proposed estimation method is also verified using a wideband 12-bit TI ADC, the ADC12D1800RF [33], which has two main time-interleaved ADCs with an aggregate sampling rate of 3.6 GHz, an analog bandwidth up to 2.7 GHz and a default $V_{SS}$ of 0.8 V. The ADC has an on-chip auto-calibration engine to calibrate the channel mismatch errors.

In order to generate a high-quality multi-tone signal, a 12-GSOPS, 16-bit DAC, the AD9164 [34], is adopted as the signal generator. The measurement setup is shown in Fig. 16. A low pass filter (LPF) with 1.65-GHz bandwidth is used to filter the image signal of the DAC. A balun is used to convert the single-ended DAC output to a differential input for the ADC.

The verification procedure for hardware measurement is similar to that used for simulation, except that the internal impairment parameters are now unknown for the ADC under test. To estimate the impairment parameters of the ADC, we again use two single-tone measurements at $f_{b}=5f_{s}$ and $f_{b}$ with -4 dBFS amplitude. Several combinations for $f_{s}$ and $f_{b}$ are measured after auto-calibrating the ADC and the obtained codes are processed by an 8192-point FFT with...
Hanning window. The obtained SNDR are given in Fig. 17, which are almost the same as those given in [32]. Based on the FFT data for these combinations of single-tone measurements, the ADC impairment parameters are estimated, as shown in Table III. As can be seen, except for the case of $f_a = 50$ MHz, every estimated parameter shows small variation for all other combinations of the single-tone frequencies. The relatively large deviation in the estimated parameters for $f_a = 50$ MHz is likely because the error power due to timing mismatch and clock jitter is very low and masked by the noise floor, which influences the estimation accuracy. The results prove that the proposed method is almost independent of the frequencies used for the two single-tone measurements. In all the single-tone FFT data, the tones resulting from the nonlinearity mismatches, $\Delta a_2$ and $\Delta a_3$, are very low and masked by the noise floor. Therefore, they are not given in Table III.

### Table III: Estimated ADC Parameters with Different Combinations of Single-Tone Measurements

| $f_a$ (MHz) | $\sigma_t$ (ps) | $P_{ST}$ (V^2) | $|a_{3,avg}|$ (V^2) | $|a_{2,avg}|$ (V^2) | $|\Delta a_2|$ (%$|V_{1,50}|$) | $|\Delta a_3|$ (ps) | $|\Delta a_4|$ (ps) |
|-------------|-----------------|----------------|-----------------|----------------|-----------------|----------------|----------------|
| 50          | 0.38            | 1.97×10^{-7}   | 0.022           | 0.005          | 1.08            | 0.02           | 1.07           |
| 130         | 0.29            | 2.22×10^{-7}   | 0.018           | 0.006          | 0.88            | 0.12           | 0.28           |
| 180         | 0.27            | 2.24×10^{-7}   | 0.015           | 0.007          | 0.86            | 0.13           | 0.46           |
| 230         | 0.29            | 2.33×10^{-7}   | 0.012           | 0.006          | 0.67            | 0.14           | 0.51           |
| 280         | 0.23            | 2.36×10^{-7}   | 0.015           | 0.008          | 0.91            | 0.16           | 0.36           |
| 330         | 0.17            | 2.78×10^{-7}   | 0.019           | 0.009          | 0.67            | 0.16           | 0.46           |

The multi-tone performance of the ADC is measured by applying a 32-tone signal spanning from 80 MHz to 1.65 GHz with -12-dB backoff generated by the DAC. The frequencies of the signal tones again deviate slightly from equally-spaced positions. Fig. 18 plots the measured spectrum for the multi-tone signal after auto-calibration of the ADC, showing an SNDR of 44.4 dB. Moreover, whereas the multi-tone measurements do not reveal the underlying impairment limiting SNDR, the single-tone measurement and parameter extraction procedure described here reveal that the top-2 dominant impairments in this ADC for multi-tone inputs are the signal-independent noise and timing mismatch, which contribute to $P_{ST}$ and $P_{Rf,in,mt}$ in (60), respectively.

Using the estimated parameters given in Table III, the estimated multi-tone SNDR for different combinations of single-tone frequencies is plotted in Fig. 19, which has a small variation from 42.9 to 44.6 dB. Compared with the measured value 44.4 dB, the maximum estimation error is only 1.5 dB.

Besides the on-chip auto calibration engine, the ADC also allows for manual adjustment of the time-interleaved timing and gain mismatch. More measurements are performed by sweeping these values to verify the proposed estimation method over a wide parameter range. The frequencies of the two single-tone measurements used for parameter estimation are 230 MHz and 1150 MHz.

Fig. 20(a) plots the estimated $|\Delta T_{in,mt}|$ and $|\Delta G|$ when adjusting the timing mismatch register code from 0 to the maximum value of 127. The estimated maximum $|\Delta T_{in,mt}|$ is 4.8 ps for a register code of 0. The estimated $|\Delta G|$ shows a small variation of about 0.3% for all the timing mismatch register codes. Fig. 20(b) plots the measured SNDR for the two-single tone signals and the multi-tone signal, as well as
scale voltage. Fig. 21(a) plots the estimated $|\Delta G|$ and $|\Delta T_{skew}|$ when the gain mismatch is adjusted from -4% to 4% manually, showing that the estimation of $|\Delta G|$ is quite accurate. The estimated $|\Delta T_{skew}|$ shows a variation of about 0.6 ps in the whole gain mismatch adjustment range. Fig. 21(b) compares the measured multi-tone SNDR with the estimated values, showing that the estimation error is about 1 dB.

Above verification based on both simulation and hardware measurement proves that the proposed methods provide an accurate estimation of the impairment parameters of broadband TI ADCs. With the estimated internal parameters, the multi-tone performance of the ADC can also be estimated accurately without the challenge of performing broadband multi-tone measurements.

VI. CONCLUSION

A method to estimate the internal impairment parameters of a broadband 2-channel TI ADC using only single-tone measurement data at 2 different frequencies is presented. The estimated parameters are further used to estimate the performance of the ADC with a multi-tone input. In this way, the ADC's performance for broadband signals can be evaluated with standard and low-cost single-tone measurements. In addition, accurate estimation of the ADC's impairment parameters is useful for designers to improve the ADC's performance in design iterations. The proposed method is verified with comprehensive simulations and hardware measurements. The method can also be applied to TI ADCs with 4 or more sub-ADCs with necessary extensions.

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