Calculation of Transient Temperature Field in an Inhomogeneous Rock Mass with a Spherical Cavity Filled with a Heated Liquid

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Abstract. The problem under consideration is determination of temperature fields in a rock mass with a spherical cavity that is partly filled with a heated liquid. At that, the tendency to self-heating of the substance stored in the cavity leads to appearance of high-temperature fields, that in turn results in development of significant thermal stresses. Herewith, real physical and mechanical properties of the rock within the rock mass must be taken into consideration. So, the rock mass material is viewed through the lens of its inhomogeneity, which to a considerable extent is determined by temperature fields for any instant of hot fluid storage. Herewith, the temperature process is transient, but taking into consideration slow redistribution of the thermal field during a long period of time, the problem of its calculation can be considered as a quasi-steady-state problem. Temperature field around the cavity is calculated using an explicit difference scheme.

1. Introduction

The existing methods for creating underground cavities, their use as storages for heated liquid products – all these issues require calculating stress and strain state of the rock mass surrounding such cavity, taking into account its real physical and mechanical characteristics. In our calculations, the rock mass material was assumed as two-dimensionally inhomogeneous.

Figure 1. The calculation scheme of the rock mass
The computational model (figure 1) was chosen as follows: a sphere with radius $r=b$ (usually $b=10a$ is chosen) was cut from the rock massive and at its center a spherical cavity with radius $r=a$ is located that is partly filled with a heated liquid.

In the process of cooling-down of the liquid medium, the rock material warms up and its mechanical properties change. This alteration is characterized by reduction of the rock’s elastic modulus according to the exponential relationship $E_0 e^{-\delta T(r, \theta)}$.

At the initial instance before the cavity is filled with the liquid medium, the soil is assumed to be inhomogeneous. Such inhomogeneity is predetermined by the method of cavity formation. In case the cavity is a result of an underground explosion, local alteration of elastic modulus of the soil takes place, it is described by the known power-law relationship [1].

Thus, the complete dependence of the modulus including the mentioned factors can be presented as follows [2, 3]:

$$E(r, \theta, t) = E_0 e^{-\delta T(r, \theta, t)} \left[ 1 + (k_1 - 1) \left( \frac{a}{r} \right)^m \right].$$

(1)

This equation takes into account the effects of both the temperature field $T(r, \theta, t)$ and the rock fracturing on physical and mechanical properties of the rock mass. Here, $\delta$, $k_1$, $m$ are empirical constants, $a$ is the radius of the spherical cavity, $E_0$ is the elastic modulus of the rock mass at infinity, $t$ is time.

2. Calculation algorithm

It is obvious that the temperature field at any instant of time $t$ is to be calculated to determine the main mechanical characteristic of the rock within the rock mass – its elastic modulus (1).

The temperature field around the cavity at any time instant $t$ is determined by the formula obtained in the papers [4, 5]:

$$T(r, t) = \frac{Q_0}{\delta \pi \sqrt{\pi cyar} \sqrt{\alpha_1} t} e^{-\frac{(r-a)^2}{4 \alpha_1 t}}.$$  

(2)

In case the cavity is partly filled with a hot liquid with the initial temperature $T_0$, the transient heat flow into the rock mass leads to various non-uniform temperature fields for different instants of heating time, which are described by the transient heat conductivity equation [6]:

$$\frac{\partial T(r, \theta, t)}{\partial t} = a_1 \left( \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r^2 \tan \theta} \frac{\partial T}{\partial \theta} \right).$$

(3)

where $a_1$ is a thermal diffusivity coefficient.

As time passes, the zone of the rock mass close to the cavity becomes heated up with subsequent propagation of heat deep into the rock mass and cool-down of the cavity surface. This process is non-steady-state, but taking into consideration slow redistribution of heat field during quite a long subsequent time, the problem can be considered as a quasi-steady-state one.

Temperatures of the wetted part of the cavity surface and of the liquid medium $T_i(t)$ for any instant of time are equal, while the temperature at the non-wetted part of the surface will be determined by heat flow in the meridional direction and will be calculated by the method described below.

For approximate solution of equation (3) for the boundary and initial conditions that were discussed in detail in [7], we will use the finite difference method in the form of explicit difference scheme [8].
Figure 2. Finite-difference grid for modelling a temperature field in a rock mass

Figure 2 shows the finite-difference grid in the polar coordinate system. In the meridional direction, we divide the computational domain into $m$ sectors and in the radial direction – into $n$ annular parts. Let us denote sequential number of circle as $i$, and sequential number of radius as $j$. Then the length of the arc of the $i$-th circle is $d_i = (\pi r_i)/m$, where $r_i = a + (i - 1)h$ and $h = (b - a)/n$ is radial step.

When using the explicit difference scheme, the following set of algebraic equations follows from the differential equation (3):

$$
\frac{T_{ij}^{k+1} - T_{ij}^k}{\tau} = a_1 \left( \frac{T_{i-1,j}^k - 2T_{ij}^k + T_{i+1,j}^k}{h^2} + \frac{1}{r_i} \frac{T_{i+1,j}^k - T_{i-1,j}^k}{h} + \frac{T_{i,j-1}^k - 2T_{ij}^k + T_{i,j+1}^k}{2d_i} \right) + \frac{ctg \theta_i (T_{i,j+1}^k - T_{i,j-1}^k)}{r_i},
$$

(4)

where $\tau$ is value of the time increment chosen from the condition of robustness of the computational algorithm; $k$ is the time layer number; $\theta_j = \pi (j - 1)/m$; $j=1,2,\ldots+n+1$; $i=1,2,\ldots,n+1$.

From the symmetry condition $\frac{\partial T(r, \theta, t)}{\partial \theta} = 0$ (when $\theta = 0$; $\theta = \pi$), we have:

$$
T_{i,0}^k = T_{i,2}^k; \quad T_{i,m+2}^k = T_{i,m}^k,
$$

(5)

where $i=1,2,\ldots,n+1$.

When using finite-difference relations to approximate both parts of the boundary condition at the surface of the cavity, with the quite justified assumption that thermal diffusivity coefficients of the liquid medium and of the rock are equal and $T_i(t_k) = T_i^k$, where $t_k = k \tau$, we get the algebraic equations:

$$
\frac{T_{i,j}^{k+1} - T_{i,j}^k}{\tau} = d_i a_1 \frac{T_{2,j}^k - T_{0,j}^k}{2h},
$$

(6)
where \( j = \bar{j}, \bar{j} + 1, \ldots m + 1 \) \( (\theta_j \geq \bar{\theta}) \).

On the non-wetted part of the spherical cavity surface and at the outer boundary of the rock mass we have the equalities:

\[
T_{0,j}^k = T_{2,j}^k, \quad T_{n+1,j}^k = T_{\infty}^k. \tag{7}
\]

where \( j = 1, 2, \ldots m + 1 \).

The conditions for the initial time instant \( t = 0 \) will be written as:

\[
T_{1,j}^0 = T_0, \tag{8}
\]

where \( j = \bar{j}, \bar{j} + 1, \ldots m + 2; \quad T_{i,j} = T_{\infty} \) for all the remaining \( i \) and \( j \).

The system of algebraic equations (4) through (8) allows computing temperatures for the subsequent instant of time \( t = t_k + \tau \) in the nodes of the finite-difference grid, if temperatures for the previous time instant \( t_k \) are known.

3. Results

Results of temperature field calculations are presented in figures 3 to 5.

Figure 3 shows temperature variations along the radius \( r \) at the filling fraction of the cavity \( \eta = 1 \) after 1 year and 100 days of cool-down of the liquid medium in the cavity. The solid line in this figure corresponds to the accurate solution by formula (2). The dashed line shows temperatures obtained by the finite difference method. The deviation of the numerical solution from the accurate one does not exceed 10%. In the numerical solution, the values \( \tau = 112 \) hours, \( n = 50, m = 24 \) were assumed. Figure 4 shows temperature field around the spherical cavity after a half-year of cool-down of the cavity at the cavity filling fraction \( \eta = 0.5 \).
Figure 4. Temperature field around the spherical cavity after half a year of liquid cool-down at $\eta=0.5$

The relationships $T(r)$ for $\theta = 90^\circ$ and $T(\theta)$ at $r=a$ can be considered as characteristic curves of the temperature field. As one can see, during cooling-down of the liquid in the cavity, a part of the underground rock mass heats up.

Figure 5. Temperature at the contour of the cavity at various cavity filling fractions with heated liquid after $t=0.5$ year

Figure 5 shows variation of temperature at the contour of the cavity for time $t=0.5$ year and for various values of factor $\eta$. As one can see the filling fraction exerts significant influence on the temperature distribution in the rock mass.

The temperature field shown in figure 3 was afterward used to study the effects of the rock inhomogeneity on its stress and strain state [9,10].

4. Conclusions
Results of this study show the necessity to take into account the real material properties of the underground rock mass as well as their changes under the influence of different physical processes when designing the underground cavities.

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