Study of the effect of feedback on dynamic stability of hydraulic

V V Syrkin, Y F Galuza, I N Kvasov and M A Fedorova
Omsk State Technical University, 11 Mira ave., Omsk, 644050, Russia

Abstract. The possibility of using hydraulic feedbacks in hydraulic automation systems to improve the dynamic processes in transient conditions (acceleration or deceleration of actuators), which provide for regulators with elastic shut-off and control elements. As parameters of feedbacks pressure in the hydraulic system, speed of movement of Executive bodies and their derivatives are provided.

Keywords: hydraulic actuator, the feedback pressure, velocity, the derivative of the pressure derivative with respect to speed, dynamic process.

1. Introduction

Designs and studies of hydraulic devices for various technological purposes are widely presented in articles and monographs of Russian [1, 3] and foreign authors [2]. To improve the dynamic processes in hydraulic systems in transient conditions (acceleration, deceleration) effective is the use of feedbacks on the parameters of the system (pressure, velocity and their derivatives). In this case, the dynamic processes become aperiodic, which favorably affects the quality of the Executive movements of the hydraulic drive. As a possible regulator regulating these feedbacks, regulators with elastic shut-off and regulating elements can be used [4].

2. Problem statement

In a simple hydraulic drive (Figure 1) the pressure in the cavity 4 of the hydraulic motor is maintained constant by means of a pressure regulator 2 and regulators 3 (p_4=p_k). In transient modes p_4≠p_k, so as feedback effect on regulator 3 (Figure 2) leads to the appearance of pressure increments Δp, which in the first approximation can be taken proportional to the corresponding parameter by which the hydraulic feedback is introduced.

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![Figure 1. Diagram of a hydraulic drive](image-url)
3. Theory

Taking into account these conditions, as well as neglecting the wave processes in the hydraulic drive lines and friction forces, the following equations of motion of the Executive body with a mass \( m \) can be written:

\[
\begin{align*}
(V_0 - \Delta V)F_2 &= (f_0 - \Delta f)\mu \frac{2p_2}{\rho} + K_2w_2 \frac{dp_2}{dt}; \\
F_1p_1 - F_2p_2 &= P_T + m \frac{dV}{dt}; \\
p_i &= p_k + \varphi_1 \Delta V + \varphi_2 \Delta p + \varphi_3 \Delta p, \\
\end{align*}
\]

where \( V_0 \) – the speed of the steady motion of the Executive body of the hydraulic actuator; \( f_0, p_1 \) and \( p_2 \) are the corresponding in this rate of area of the working window of the throttle and the pressure in the left and right cavities of the hydraulic motor; \( F_1 \) is the effective area of the piston in the left cavity; \( \Delta f, \Delta V \) and \( \Delta p \) – increment of the relevant parameters of a hydraulic actuator and its mode of motion; \( P_T \) – technological force, developing a working body of the hydraulic actuator; \( F_2, W_2, \) and \( K_2 \) the effective area of the piston, the volume ratio and give the right cavity of the hydraulic motor; \( \mu \) is the discharge coefficient of the throttle; \( \rho \) – density of working fluid; \( m \) is the mass of the working body of the hydraulic actuator; \( \varphi_1, \varphi_2, \varphi_3 \Delta p \), the feedback coefficient for the pressure \( P \), the velocity \( V \) and their derivatives, and \( p_k \) is a constant pressure in the left cavity of the hydraulic motor at the steady state motion.

With a step change in the throttle area \( \Delta f \) and taking into account that

\[
(f_0 - \Delta f)\mu \frac{2}{\rho} (p_2 + \Delta p) \approx f_0\mu \frac{2}{\rho} p_2 - \frac{\Delta f}{f_0} \frac{2}{\rho} p_2 + f_0\mu \frac{2}{\rho} p_2 - \frac{\Delta f}{f_0} \frac{2}{\rho} p_2,
\]

get the equation of motion of the hydraulic drive with hydraulic feedbacks:

\[
A_2 \frac{d^2 \Delta V}{dt^2} + A_1 \frac{d\Delta V}{dt} + A_0 \Delta V = c, \tag{2}
\]
\[
A_i = \frac{\phi_{p_i} V_0 [\phi_{p_{i2}} V_0 F_1 (1 - \Delta \bar{f}) + 2 K_s W_i p_2 (K - \phi_{p_{i2}})]}{p_2 (1 - \frac{\phi_{p_{i2}}}{K})(K - \phi_{p_{i2}})(1 - \Delta \bar{f}) V_0 F_2} - \frac{\phi_{p_{i2}}}{K - \phi_{p_{i2}}} \frac{2 p_2 (K - \phi_{p_{i2}}) + \phi_{p_{i2}} V_0 (1 - \Delta \bar{f})}{p_2 \frac{1}{K} (K - \phi_{p_{i2}})(1 - \Delta \bar{f})} + \frac{mv_0 (K - \phi_{p_{i2}}) + V_0 (f_{p_{i2}} m + \phi V^2 F_2)}{p_2 F_2 \frac{1}{K} (K - \phi_{p_{i2}})};
\]

(3)

\[
A_2 = \frac{p_{i2} F_2 (1 - \Delta \bar{f}) + 2 K_s W_i p_2 (K - \phi_{p_{i2}})}{p_2 (1 - \Delta \bar{f})(1 - \frac{\phi_{p_{i2}}}{K})}.
\]

(4)

\[
A_i = \frac{\phi_{p_i} V_0 (1 - \Delta \bar{f}) + 2 K p_2 (1 - \frac{\phi_{p_{i2}}}{K})}{p_2 (1 - \Delta \bar{f})(1 - \frac{\phi_{p_{i2}}}{K})}.
\]

(5)

\[
c = P_{\bar{f}_{1}} K + \frac{K}{1 - \Delta \bar{f}} (1 + \Delta \bar{f}) - \frac{K [p_{k_2} F_2 - \phi_{p_{i2}} (p_{2F_2} + P_{\bar{f}_{1}})]}{p_2 F_2 (K - \phi_{p_{i2}})}.
\]

(6)

Where \( \Delta \bar{f} = \frac{\Delta f}{f_0} \); \( K = \frac{F_2}{F_1} \); \( \Delta \bar{V} = \frac{\Delta V}{V_0} \); \( P_{\bar{f}_{1}} = \frac{P_{\bar{f}}}{p_{2F_2}} \).

From equations (3) – (6) in each of the feedback, it is possible to obtain the equations of motion separately for each of them, and in various combinations.

In particular, when \( \phi_{p_{i2}} = 0, \phi_{p_{i}} = 0, \phi_{p_{i}} = 0 \) and get

\[
\frac{K_s W_i m}{F_2^2} (1 + \frac{\phi_{p_{i2}}}{1 - \phi_{p_{i2}}}) \Delta \bar{V} + \frac{mv_0}{2 F_2 p_2} (1 + \frac{\phi_{p_{i2}}}{K - \phi_{p_{i2}}}) \Delta \bar{V} + \Delta \bar{V} = P_{\bar{f}_{1}} K + \frac{K}{1 - \Delta \bar{f}} (1 + \Delta \bar{f}) - \frac{K [p_{k_2} F_2 - \phi_{p_{i2}} (p_{2F_2} + P_{\bar{f}_{1}})]}{p_2 F_2 (K - \phi_{p_{i2}})}.
\]

(7)

When \( \phi_{p_{i2}} = 0, \phi_{p_{i}} = 0, \phi_{p_{i}} = 0 \); and get:

\[
\frac{K_s W_i m}{F_2^2} \Delta \bar{V} + \frac{mv_0}{2 F_2 p_2} (1 - \Delta \bar{f}) \left[ 1 - \frac{2 \phi_{p_{i2}} p_{2F_2}}{km V_0 (1 - \Delta \bar{f})} \right] \Delta \bar{V} = 1.
\]

(8)
When \( \phi_{p_2} = 0, \phi_r = 0, \phi_{r'} = 0 \), equation (2) takes the form
\[
\frac{K_w m}{F_2^2} \Delta \varphi'' + \frac{m V_0 K(1 - \Delta \tilde{\varphi})}{2 p_2 F_2} \frac{\Delta \varphi'}{1 + \frac{2 W_2 K_p p_2 \phi_r}{Km V_0 (1 - \Delta \tilde{\varphi})}} \Delta \varphi = \Delta \tilde{\varphi}.
\] (9)

When \( \phi_{p_2} = 0, \phi_r = 0, \phi_{r'} = 0 \); and get:
\[
\frac{K_w m}{F_2^2} (1 + \frac{\phi_{r'} F_2}{Km}) \Delta \varphi'' + \frac{m V_0 (1 - \Delta \tilde{\varphi})}{2 F_2 p_2} (1 - \frac{\phi_{r'} F_2}{Km}) \Delta \varphi' + \Delta \varphi = \Delta \tilde{\varphi}.
\] (10)

Periodical transients in the hydraulic actuator will be provided:
\[
\frac{A_1^2}{4 A K V} > 1.
\] (11)

When using \( p_2 \) feedback from expression (7) and (11), the aperiodicity condition will take the form:
\[
\frac{m V_0}{16 p_2 K_w} (1 + \frac{\phi_{p_2}}{K - \phi_{p_2}}))1.
\] (12)

It follows from the expression (12) that the positive pressure feedback \( P_2 \) reduces the oscillation of the transient process, which becomes aperiodic: Using the derivative feedback, we obtain the aperiodic transition process in the hydraulic drive from the expression (8) and (11):
\[
\frac{m V_0^2 (1 - \Delta \tilde{\varphi})^2}{16 p_2^2 K_w} (1 - \frac{2 \phi_{p_2} F_2}{Km V_0 (1 - \Delta \tilde{\varphi})})1.
\] (13)

It follows from the condition (13) that negative feedback should be used to obtain a stable dynamic process.

From expression (9) and (11) we obtain:
\[
\frac{m V_0^2 (1 - \Delta \tilde{\varphi})^2}{16 p_2^2 K_w} \frac{V_0}{\frac{2 K_p (1 - \Delta \tilde{\varphi})}{2 p_2 (1 - \Delta \tilde{\varphi})} + 1} \] (14)

With the \( \phi_{r'} = 0 \) quality of the transition process is improved. From conditions (10) and (11) follows the condition of aperiodicity of process at; \( \phi_r \neq 0; \phi_{r'} = 0; \phi_{p_2} = 0; \phi_{r'} = 0 \) we obtain:
\[
\frac{m V_0^2 (1 - \Delta \tilde{\varphi})^2}{16 K_w p_2^2} (1 - \frac{\phi_{p_2} F_2}{Km})1.
\] (15)

With the simultaneous action of several feedbacks, the transition process is determined by the ratio of the coefficients.

In a particular case, when braking (acceleration) of the Executive body of the hydraulic drive, as a rule, it is necessary that its acceleration is constant.

From equation (2) it follows that the acceleration \( \frac{d \Delta \varphi}{dt} = \text{const} \), if \( A_2 \to 0 \) and \( A_0 \to 0 \). In this case
\[
\frac{d \Delta \varphi}{dt} \approx \frac{A_0}{c}.
\] (16)
According to equations (7) – (10) can be identified that $KW_2 \neq A_0 \rightarrow 0$, if you use feedback $\Delta p_2$ or $\frac{d\Delta V}{dt}$. For the condition $A_0 \rightarrow 0$ it is necessary to use the feedback on $\Delta V$.

When $\phi_y \neq 0$; $\phi_y \rightarrow 0$, $\phi_y \rightarrow 0$ equation of motion of the hydraulic drive is as follows:

$$\frac{K_2W_3}{F_2^2}(1+\frac{\phi_y}{K-\phi_y})\Delta F + \frac{mV_0(1-\Delta f)}{2F_2^2} \left[1 + \frac{2K_2W_3\frac{p_V}{2} - \phi_y}{mV_0(1-\Delta f)(1-\phi_y)} \right] \Delta F = \frac{K_2[F_2^2 - \phi_y(p_2F_2 + \rho F)]}{p_2F(K-\phi_y)}$$

(17)

For $A_2 \rightarrow 0$ it is necessary that $1 + \frac{\phi_y}{K-\phi_y} = 0$, then $K \rightarrow 0$ the ratio $\frac{F_2}{F_1}$ will be minimal.

For $A_2 \rightarrow 0$ it is necessary that $1 + \frac{\phi_y}{2p_2(K-\phi_y)} = 0$, from where follows that $\frac{\phi_y}{K-\phi_y} = \frac{2p_2}{V_0(1-\Delta f)}$.

Since the ratio $K = \frac{F_2}{F_1}$ is limited by the minimum value, the coefficient $A_2 \rightarrow 0$ and acceleration during acceleration and deceleration has restrictions on maintaining a constant value.

The average acceleration can be determined by the expression (16) taking into account the coefficients of the equation (17)

$$\frac{dV}{dt} = \frac{mV_0(1-\Delta f)}{2F_2^2} \left[1 + \frac{2K_2W_3\frac{p_V}{2} - \phi_y}{mV_0(1-\Delta f)(1-\phi_y)} \right] \frac{\phi_y}{K-\phi_y}.$$  (18)

4. Summary

An experimental study of a prototype pressure regulator [4] with feedback of $\Delta p_2$, $\Delta p'_2$ and $\Delta V$ confirmed the theoretical calculations.

The drive of the rotational motion with the liquid was inhibited by instantaneous overlapping of the drain line in the implementation of feedback on $\frac{dp_2}{dt}$ and on $p_2(t)$. The deviation of the acceleration value from the set value did not exceed ±6%.

5. References

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