Particle Physics in Intense Electromagnetic Fields *

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Abstract

The quantum field theory in the presence of classical background electromagnetic fields is reviewed. We give a pedagogical introduction to the Feynman-Furry method of describing non-perturbative interactions with very strong electromagnetic fields. A particular emphasis is given to the case of the plane-wave electromagnetic field for which the charged particles’ wave functions and propagators are presented. Some general features of quantum processes proceeding in the intense electromagnetic background are argued. We also discuss the possibilities of searching new physics through the investigations of quantum phenomena induced by the strong electromagnetic environment.

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1 Introduction

Quantum field theory in the presence of an intense electromagnetic background has a long fruitful history. The interest promoting investigations in this area is based on several reasons. The first one being purely theoretical is associated with the self-consistency problem of the relativistic quantum theory at very high energies. It is well known that in quantum electrodynamics (QED) perturbative vacuum probably is not a true ground state of the theory because of the fictitious pole in photon propagator in the ultraviolet region: $|k^2| \sim m_e^2 \exp(3\pi/\alpha)$ \[1\]. This challenge has inspired many studies dealing with QED of intense fields in which the convergence of perturbation series in fine-structure constant $\alpha = e^2/4\pi$ has been analyzed (see e.g. \[2\], \[3\], \[4\] and references therein). The matter is that ultraviolet behaviour of the theory and the structure of divergencies determine in a uniform way the properties of Green functions in very intense electromagnetic fields and at large momentum transfer. In other words the physics of extremely strong background fields is intimately connected with the one at small distances \[5\],\[6\]. This can be illustrated by the effect of vacuum polarization in QED which modifies the Lagrange density of Maxwell electrodynamics. For weak fields one loop corrections have been calculated long ago by Heisenberg and Euler \[7\].

$$\mathcal{L} = -\mathcal{F} + \frac{2\alpha^2}{45m_e^4}(4\mathcal{F}^2 + 7\mathcal{G}^2), \quad (1)$$

where

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(B^2 - E^2), \quad \mathcal{G} = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = EB. \quad (2)$$

However the above expression is valid only if the field strength is relatively small ($\mathcal{F}, \mathcal{G} \ll m_e^4/e^2$). The domain of extremely strong fields ($\mathcal{F} \gg m_e^4/e^2; \mathcal{G} \ll \mathcal{F}$) is described by another formula \[8\] which in logarithmic approximation can be written as \[9\]

$$\mathcal{L} = -\mathcal{F} + \frac{\alpha}{6\pi}F \ln\left(\frac{2e^2\mathcal{F}}{m_e^4}\right) \quad (3)$$

We see that asymptotically the effective Lagrangian (3) vanishes as far as the field strength $B = \sqrt{2\mathcal{F}}$ tends to the critical value $\sim m_e^2/e \exp(3\pi/\alpha)$. So, the Moscow zero-charge problem \[1\] reveals itself as a nought of effective Lagrangian if we impose the following interrelation:

$$|k^2| \sim eB = e\sqrt{2\mathcal{F}} \quad (4)$$

Two loop calculations \[10\] have confirmed that the analogy (4) is retained in all orders of perturbation. From this viewpoint the physics of intense electromagnetic fields is a powerful theoretical tool for searching the stable non-perturbative vacuum state in which the artifacts mentioned above can be removed.

Let us also note, that the problem of vacuum instability and phase transitions in a strong background field has received additional interest in the context of non-Abelian gauge theories \[11\]. These investigations rely essentially on the philosophy and technique elaborated in the framework of QED with an intense background.

The second motivation for studying particles interactions with very strong electromagnetic fields originates from the quantum theory of synchrotron radiation \[12\]. The process of bremsstrahlung by electrons in a homogeneous magnetic field is has long been known. Classical electrodynamics provides a satisfactory description of the radiation power and spectrum being
observed in synchrotrons (for a recent review see [13]). At the same time particular phenomena require the apparatus of QED with an intense magnetic field to be applied for an adequate theoretical explanation. Systematic investigations of quantum processes in the presence of strong magnetic fields have been initiated in works of Sokolov and Ternov [14] and were continued by their collaborators [13]. These investigations resulted in discovering many interesting theoretical predictions some of which have been confirmed experimentally. Here we can highlight the famous effect of self-polarization of electrons in storage rings due to the synchrotron radiation.

The third reason stimulating interest in physics with intense electromagnetic background is caused by recent CERN experiments involving SPS accelerator in which the particle interactions with single crystals were explored [16], [17]. Notice that, because of regular and systematic structure, electromagnetic interactions in single crystals are substantially enhanced in comparison with the amorphous medium. This gives a unique possibility of creating a strong electromagnetic background for relativistic particles penetrating single crystal near axial or planar directions. Thus we have an opportunity for searching new physics in such unusual conditions because quantum phenomena in the presence of the intense electromagnetic field differ considerably from those occurring in a vacuum. Due to non-linear and non-perturbative influence of the external field one can observe absolutely new processes which as a rule are forbidden under normal conditions. The electromagnetic field open new channels of reactions taking away the bans for transitions between definite quantum states. The most wonderful feature is the possibility for very light particles to decay into heavy species capturing a lacking amount of energy from the intense electromagnetic environment. It would be pertinent to mention here the process of electron-positron pairs production by solitary photons incident along crystal axes. The data of experiments CERN WA-81, CERN NA-046 [17] concerning with this reaction $\gamma \rightarrow e^+e^-$ are in a satisfactory agreement with predictions of QED with the strong electromagnetic field [18] (see also [19]).

Thus, we see that particle physics in the presence of non-perturbative background fields is an impetuously developing branch of physical sciences having many interesting applications. Unfortunately it is impossible to itemize all exploitations of this theoretical technique because they are very numerous. Here I can only mention some other important employments such as the problem of vacuum in quantum chromodynamics [20], astrophysics of neutron stars [21], the cosmology of the early Universe [22] and etc.

Lastly, let us say some words about the method itself. The philosophy of calculations in strong electromagnetic fields originates from the Feynman non-perturbative approach in QED [23] which was elaborated in details by Furry [24]. This technique has become widely known as the “Furry picture”. Classical examples of QED with external fields were given in works of Schwinger [8], [25]. Further developments of the Feynman-Furry method were achieved in the research of Fradkin and Gitman with their collaborators who have generalized and modified the above approach for a wide class of electromagnetic fields with unstable vacuum (see [26] and references therein). It is worth noting that the procedure of second quantization in the intense background field is a separate, uneasy task having common features with the quantum field theory in a curved space-time [27]. A particular difficulty is connected with appropriate treatment of the classical solutions of relativistic wave equations that do not admit one particle interpretation. For instance, in a case of a constant uniform electric field exact solutions of the Dirac equation have a form which cannot be splitted into positive and negative frequency modes corresponding to particle and antiparticle wave functions. The later circumstance is regarded as the instability of the physical vacuum with electric fields in relation with the processes of pairs...
production. However, as was shown by Schwinger [8] the probability of pairs creation from the vacuum becomes feasible only in very strong electric fields being comparable with the critical value $E_{cr} = m_e^2/e = 1.32 \cdot 10^{18} V/m$.

Inasmuch as present values of the field strength being available in experiments are much less than $E_{cr}$ it is advantageous to employ an effective approximation which was proposed in works of Ritus and Nikishov [28]. The main idea comes from a detailed analysis of probabilities of quantum phenomena in an arbitrary constant homogeneous electromagnetic field. Suppose that the initial quantum state is characterized by the only one particle with momentum $p^\mu$. Then, in general the probability of quantum transition to some final state $P_{fi}$ depends on the background field strength $F_{\mu\nu}$ through the following Lorentz-invariant dimensionless parameters:

$$a = \frac{e^2 G}{m_e^4} = \frac{e^2}{m_e^4}EB, \quad b = \frac{e^2 F}{2m_e^4}(B^2 - E^2).$$

The domain of electromagnetic fields that currently can be handled in a laboratory satisfies the conditions: $a \ll 1, b \ll 1$ (i.e. $E \ll E_{cr}, B \ll B_{cr} = m_e^2/e = 4.41 \cdot 10^9 T$). Now, if the initial particle is relativistic ($p_0 \gg m_e$) then $\chi^2 \gg a, b$ and one can expand the probability $P_{fi}$ into a series of these small quantities:

$$P_{fi}(\chi, a, b) = P(\chi, 0, 0) + a \frac{\partial P}{\partial a}(\chi, 0, 0) + b \frac{\partial P}{\partial b}(\chi, 0, 0) + \ldots$$

The first term in eq.(7) represents the probability of quantum transition in a so-called crossed field, which is a combination of the orthogonal electric and magnetic fields having the same magnitude ($F = G = 0$). So, the formula describing a reaction in the crossed field provides a good approximation for that in an arbitrary constant electromagnetic field if one can omit in eq.(7) the corrections proportional to small parameters $a, b$. The merits of the crossed field approximation lie in the most simple form of charged-particle wave functions and propagators which greatly facilitate the scheme of calculations. All what said above may be summarized as the following prescription: if you want to investigate a process in a constant homogeneous electromagnetic field, then the most straightforward way is to calculate the corresponding probability assuming that $F = G = 0$ [2]. The obtained result depending on the sole background field parameter $\chi$ [2] will simulate the exact one as the first term in the expansion (7). Note, that the crossed field is a particular case of the plane-wave electromagnetic field fitting the choice: $A_\mu(\varphi) = a_\mu \varphi; \quad F_{\mu\nu} = k_\mu a_\nu - k_\nu a_\mu$ (see below).

The purpose of this paper is to give a pedagogical introduction to the particle physics in the presence of an intense electromagnetic background. I briefly reproduce the main milestones of the Feynman-Furry method allowing for non-perturbative interactions with very strong electromagnetic fields. Taking into account the spectrum of the known electrically charged particles, I consider the cases of scalar, spinor and vector quantum fields separately and obtain various forms for wave functions and propagators that are suitable for practical calculations (sections 3,4,5). Then I discuss some general features that are typical for quantum phenomena in the background electromagnetic field (section 6). And finally, I analyze the possibilities of searching new physics in the framework of Minimal Standard Model through the investigation of quantum effects induced by the strong electromagnetic background.
2 Scalar Particles in an Intense Electromagnetic Background

Let us consider the Lagrangian of scalar particles with charge $e > 0$ interacting with a strong electromagnetic field.

$$\mathcal{L} = (\partial_\mu \phi^+)(\partial^\mu \phi^-) - m^2 \phi^+ \phi^- + ieA^\mu(\phi^+ \partial_\mu \phi^- - \phi \partial_\mu \phi^+) + e^2 \phi^+ \phi^-(A^\mu A_\mu)$$

(8)

The electromagnetic potential $A_\mu(x)$ is regarded here as a given function of space-time coordinates which corresponds to the classical field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We wish to describe the impact of the external electromagnetic field in a non-perturbative manner employing exact solutions of the equation of motion which follows from the Lagrangian (8).

$$\left[(\partial_\mu \pm ieA_\mu)^2 + m^2\right] \phi^\pm(x) = 0$$

(9)

The greatest embarrassment of this approach is that of solving analytically equation (9) and to obtain non-perturbative wave functions for an arbitrary predetermined configuration of the background field. At present exact solutions are known for the case of a constant uniform electromagnetic field, for the Coulomb field potential and for the so-called Redmond’s configuration (see e.g. [29]). Besides there is an explicit expression for the scalar particle wave function in the presence of a plane electromagnetic wave, which is characterized by some unspecified potential $A_\mu(\varphi)$ depending on space-time coordinates $x^\mu$ only through the phase $\varphi = k_\mu x^\mu = \omega t - kx$. This is the case we intend to discuss in more details.

The starting point of our analysis is to invoke the WKB method for the equation (9) which as it comes out will give us the exact result in the first approximation. Now, substituting $\phi^+(x) = e^{iS(x)}$ and imposing the Lorentz gauge for the vector potential, we get

$$\left(\partial_\mu S + eA_\mu\right)^2 - m^2 = i\partial_\mu \partial^\mu S.$$

(10)

In the semiclassical limit being considered the right-hand side of eq. (10) can be neglected and it gives us the Hamilton-Yacobi equation for the classical action of a relativistic particle moving in a plane-wave electromagnetic field. The classical action as a function of coordinates $x^\mu$ and the vector of initial momentum $p^\mu$ is well known and it can be written in the following form:

$$S(x,p) = -p_\mu x^\mu - \int_0^{kx} \left(\frac{e p^\mu A^\mu(\varphi)}{p^\nu k^\nu} - \frac{e^2 A_\mu(\varphi)A^\mu(\varphi)}{2p_\nu k^\nu}\right) d\varphi + S_0.$$

(11)

Here it is implied that the wave vector $k^\mu$ obeys the isotropic and transversal conditions

$$k^2 = 0, \quad k^\mu A_\mu = 0.$$

(12)

In order to obtain quantum corrections one must insert formula (11) into eq.(10) and take into account the right-hand side of this expression. However it is easy to see that classical action (11) makes both sides of eq. (10) vanish if the momentum vector $p^\mu$ is taken on the mass shell: $p_0 = \sqrt{p^2 + m^2}$. So, we have found an exact classical solution of eq.(3) which should be treated as the scalar particle wave function in a background electromagnetic wave field. For practical use the wave function must be normalized in a proper way. Assuming that the configuration volume space is equal to $V$, we finally get

$$\phi^\pm(x,p) = \frac{1}{\sqrt{2p_0 V}} \exp \left[-ipx - i \int_0^{kx} \left(\pm \frac{epA}{pk} - \frac{e^2 A^2}{2pk}\right) d\varphi\right].$$

(13)
The next step is to realize the second quantization procedure. As usually, we decompose the scalar field into a series of the obtained-above solutions involving the Fock creation and annihilation operators.

\[
\phi^+(x) = \sum_p \left[ a_p \phi^+(x, p) + b^*_p \phi^{*-}(x, p) \right]
\]

\[
\phi^-(x) = \sum_p \left[ a^*_p \phi^{**}(x, p) + b_p \phi^-(x, p) \right]
\]

These operators must satisfy a standard set of commutation relations

\[
[a_p; a^*_p] = \delta_{pp'}, \quad [b_p; b^*_p] = \delta_{pp'},
\]

\[
[a_p; a^*_p'] = [b_p; b^*_p'] = [a_p; b^*_p'] = 0,
\]

which enable us to reproduce the ordinary scheme of quantum field theory. All further reasoning almost exactly repeats the ones of the conventional approach. The negative frequency modes of eq.(9) are regarded as the scalar antiparticles with wave functions \( \phi^-(x, p) \) and the opposite charge \(-e < 0\). The physical vacuum is defined as a ground quantum state with no particles present: \( a_p |0_OID = b_p |0_OID = 0 \), and the Feynman propagator is determined by the formula

\[
D(x, x') = i < 0|T\phi^+(x)\phi^{-}(x')|0 >
\]

So, the essence of the Furry picture of interaction with a background electromagnetic field is to replace the ordinary de Broglie’s wave functions by some exact solutions which incorporate adequately all non-perturbative effects. As a calculation prescription, we present explicitly the modification of the scalar particle propagator (17) in a plane-wave field:

\[
D(x, x') = \int \frac{d^4p}{(2\pi)^4} \frac{1}{m^2 - p^2 - i\epsilon} \exp \left[ -ip(x-x') - i \int_{kx}^{kx'} \left( \frac{epA}{pk} - \frac{e^2A^2}{2pk} \right) d\varphi \right].
\]

Notice that the propagator (18) is a Green function of the Klein-Gordon equation (3) i.e. it obeys inhomogeneous equation with the Dirac \( \delta \)-function in the right-hand side:

\[
[(\partial_\mu + ieA_\mu)^2 + m^2]D(x, x') = \delta^4(x - x')
\]

Another useful representation for the scalar propagator can be obtained from eq.(13) through the Fock-Schwinger proper time formalism [8], [30]. A comprehensive expounding of this method can be found in many text books [31]. For the sake of completeness let us introduce here the final expression for the propagator (17) in such an approach which is equivalent to eq.(18):

\[
D(x, x') = \frac{\Phi(x, x')}{(4\pi)^2} \int_0^\infty \frac{ds}{s^2} \exp \left[ -ism^2 - i \left( \frac{x-x'}{4s} \right)^2 + \frac{ie^2s}{(kx - kx')} \int_{kx}^{kx'} A_\mu(\varphi)A^\mu(\varphi) d\varphi - \frac{ie^2s}{(kx - kx')^2} \int_{kx'}^{kx} d\varphi_1 \int_{kx'}^{kx} d\varphi_2 A_\mu(\varphi_1)A^\mu(\varphi_2) \right],
\]
where the gauge factor \( \Phi(x, x') \) is determined by the integral along the straight line from the point \( x_\mu \) to the point \( x'_\mu \):

\[
\Phi(x, x') = \exp \left[ -ie \int_{x'}^{x} A_\mu(z) dz^\mu \right] = \exp \left[ -ie \frac{(x - x')^\mu}{(kx - kx')} \int_{kx'}^{kx} A_\mu(\varphi) d\varphi \right].
\] (21)

### 3 Charged Fermions in an Intense Electromagnetic Field

The results of previous section can be easily generalized for the spin 1/2 particles being governed by QED-like Lagrangian

\[
\mathcal{L} = \frac{i}{2} \bar{\psi} \gamma^\mu (\partial_\mu \psi) - \frac{i}{2} (\partial_\mu \bar{\psi}) \gamma^\mu \psi - m(\bar{\psi} \psi) - eA_\mu (\bar{\psi} \gamma^\mu \psi),
\] (22)

which gives the Dirac equation for the four component bispinor \( \psi \):

\[
\left( i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m \right) \psi(x) = 0
\] (23)

Exact solutions in a plane electromagnetic wave have been obtained by Volkov [32] and they are deduced from the Klein-Gordon equation (9) by a standard substitution (see also [33]):

\[
\psi(x) = \left( i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu + m \right) \phi(x).
\] (24)

The squared Dirac equation

\[
\left[ (\partial_\mu + ieA_\mu)^2 + m^2 + \frac{e^2}{2} \sigma^{\mu\nu} F_{\mu\nu} \right] \phi(x) = 0
\] (25)
differs from that for scalar particles only in the last term arising because of \( \gamma \)-matrices commutator

\[
\sigma^{\mu\nu} = \frac{i}{2} (\gamma^\nu \gamma^\mu - \gamma^\mu \gamma^\nu).
\] (26)

Note that in a plane electromagnetic wave all powers of this term exceeding the first are equal to zero due to eq.(12). So the desired solution of equation (25) is a slightly modified form of expression (13).

\[
\phi(x) = \exp \left[ -ipx - i \int_{0}^{kx} \left( \frac{eA}{pk} - \frac{e^2 A^2}{2pk} \right) d\varphi \right] \left[ 1 + \frac{e}{2pk} \hat{k} \hat{A}(kx) \right] v,
\] (27)

where

\[
\hat{k} = k_\mu \gamma^\mu, \quad \hat{A} = \gamma^\mu A_\mu,
\]

and \( v \) is an arbitrary constant bispinor. Now, returning to eq.(24), we derive the sought for non-perturbative wave function of a charged fermion in an intense electromagnetic background:

\[
\Psi^\pm(x, p) = \exp \left[ -ipx - i \int_{0}^{kx} \left( \pm \frac{eA}{pk} - \frac{e^2 A^2}{2pk} \right) d\varphi \right] \left[ 1 \pm \frac{e}{2pk} \hat{k} \hat{A}(kx) \right] \frac{u^\sigma(p)}{\sqrt{2p_0V}}
\] (28)
Polarization effects are described here by an ordinary Dirac bispinor
\[ u_\sigma(p) = \sqrt{2p_0 V} (\gamma^\mu p_\mu + m) v \]
which is subject to the following restrictions:
\[ (\gamma^\mu p_\mu - m)u_\sigma(p) = 0 \quad \bar{u}_\sigma(p)u_\sigma(p) = 2m \quad \bar{u}_\sigma(p)\gamma^\mu u_\sigma(p) = 2p^\mu \quad (29) \]
The second quantization is carried out by analogy with eq. (14),(15)
\[ \psi(x) = \sum_{p\sigma} \left[ a_{p\sigma} \Psi^+(x,p) + b^\dagger_{p\sigma} \Psi^-(x,p) \right] \quad (30) \]
\[ \overline{\psi}(x) = \sum_{p\sigma} \left[ a^\dagger_{p\sigma} \overline{\Psi}^+(x,p) + b_{p\sigma} \overline{\Psi}^-(x,p) \right] \quad (31) \]
and involves also the antifermion wave functions being obtained through the $C$-matrix of charge conjugation:
\[ \Psi^-c(x,p) = C\overline{\Psi}^T(x,p) = -\gamma^0 C\Psi^*(x,p) \]
\[ \overline{\Psi}^-c(x,p) = -\overline{\Psi}^T(x,p)C^\dagger \quad (32) \]
A specific form of this matrix is inessential provided that it satisfies the following properties:
\[ C^\dagger = C^{-1} \quad C^T = -C \quad C^{-1}\gamma^\mu C = -\gamma^\mu T \quad (33) \]
For example we can choose $C = -i\gamma^0\gamma^2$ in Weyl and Dirac $\gamma$-matrices representations.
Taking into account the spin-statistics relation, we must impose the anticommutation rules for the Fock operators $a_{p\sigma}, b_{p\sigma}$ which have no difference with the conventional ones:
\[ \{a_{p\sigma}; a^\dagger_{p'\sigma'}\} = \delta_{pp'}\delta_{\sigma\sigma'} \quad \{b_{p\sigma}; b^\dagger_{p'\sigma'}\} = \delta_{pp'}\delta_{\sigma\sigma'} \]
\[ \{a_{p\sigma}; a_{p'\sigma'}\} = \{b_{p\sigma}; b_{p'\sigma'}\} = \{a_{p\sigma}; b^\dagger_{p'\sigma'}\} = 0 \quad (34) \]
The corresponding Feynman propagator is defined as usual
\[ S(x,x') = -i < 0 | T\psi(x)\overline{\psi}(x') | 0 > \quad (35) \]
and has the most simple form in the basis of exact solutions (28):
\[ S(x,x') = \int \frac{d^4p}{(2\pi)^4} \left[ 1 + \frac{ek}{2pk} \hat{A}(kx) \right] \frac{\gamma^\mu p_\mu + m}{p^2 - m^2 + i0} \left[ 1 - \frac{ek}{2pk} \hat{A}(kx') \right] \cdot \exp \left[ -ip(x-x') - i \int_{kx}^{kx'} \left( \frac{epA}{pk} - \frac{e^2A^2}{2pk} \right) d\varphi \right]. \quad (36) \]
It is often more convenient to use the Fock-Schwinger representation which enables us to rewrite the last formula as follows.
\[ S(x,x') = -\Phi(x,x') \int_0^\infty \frac{ds}{s^2} \exp \left[ -ism^2 - i\frac{(x-x')^2}{4s} + i\frac{e^2s}{(kx-kx')} \int_{kx'}^{kx} A_\mu(\varphi) A^\mu(\varphi) d\varphi \right] \]
The corresponding part of the Lagrangian of the Glashow-Weinberg-Salam model has the form field describing charged massive particles with spin 0 and 1. The longitudinal mode of the vector electromagnetic field.

One can easily check that fermion propagator is a Green function of the Dirac equation in the presence of the intense electromagnetic field.

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\[
\frac{ie^2}{(kx - kx')^2} \int_{kx'} \frac{dx}{kx'} \left\{ m + \frac{(\dot{x} - \dot{x}')}{2s} + mesk \hat{A}(kx) - \hat{A}(kx') \right\} \frac{e}{kx - kx'} \int_{kx} \hat{A}(\varphi)d\varphi -
\]

\[
\frac{e}{kx - kx'} \left[ \hat{A}(kx)(\dot{x} - \dot{x}') - (\dot{x} - \dot{x}') \hat{A}(kx') \right] + \frac{e}{kx - kx'} \int_{kx} \hat{A}(\varphi)d\varphi -
\]

\[
- \frac{e}{kx - kx'} \int_{kx'} A_\mu(\varphi)d\varphi - \frac{e^2}{kx - kx'} \hat{A}(kx) \hat{A}(kx') + \frac{e^2}{kx - kx'} \int_{kx} A_\mu(\varphi)A_\mu(\varphi)d\varphi +
\]

\[
+ \frac{e^2}{kx - kx'} \int_{kx'} \left[ \hat{A}(kx) \hat{A}(\varphi) + \hat{A}(\varphi) \hat{A}(kx') \right] d\varphi - \frac{2e^2}{kx - kx'} \int_{kx} \frac{d\varphi}{kx'} \int_{kx'} d\varphi_2 A_\mu(\varphi_1)A_\mu(\varphi_2) \right\}(37)
\]

One can easily check that fermion propagator is a Green function of the Dirac equation in the presence of the intense electromagnetic field.

\[
[i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m]S(x, x') = \delta^4(x - x').
\]

\section{W-bosons in a Background Electromagnetic Field}

Let us now concentrate on electromagnetic interactions of weak intermediate vector bosons $W^\pm$. The corresponding part of the Lagrangian of the Glashow-Weinberg-Salam model has the form (see e.g. [34])

\[
\mathcal{L} = -\frac{1}{2}(\partial_\mu W_\mu^+ - \partial_\nu W_\nu^+)(\partial^\mu W_-^\nu - \partial^\nu W_-^\mu) + m_W^2 W_\mu^+ W_-^\mu - ieF^{\mu\nu} W_\mu^+ W_\nu^- -
\]

\[
-ie(\partial_\mu W_\mu^+ - \partial_\nu W_\nu^+)W_-^\nu A_\nu + ie(\partial_\mu W_\mu^- - \partial_\nu W_\nu^-)W_-^\mu A_\nu - e^2(A_\mu A_\mu)(W_\nu^+ W_-^\nu) +
\]

\[
+ e^2(A_\mu W_\mu^+)(A_\nu W_\nu^-) - \frac{1}{\xi}(\partial^\mu A_\mu^+ + ieA_\mu W_\mu^+)(\partial^\nu W_\nu^- - ieA_\nu W_\nu^-)
\]

The last summand in (39) arises from a non-linear gauge fixing term

\[
\mathcal{L}_{gf} = -\frac{1}{16}\xi(\partial^\mu A_\mu^1 - gA^1_\mu A^2_\mu + \xi m_\varphi \varphi_1)^2 - \frac{1}{2\xi}(\partial^\mu A^2_\mu + gA^1_\mu A^3_\mu + \xi m_\varphi \varphi_2)^2 -
\]

\[
- \frac{1}{2\xi}(\partial^\mu A^3_\mu + \xi m_\varphi \varphi_3)^2 - \frac{1}{2}(\partial^\mu A^0_\mu - \xi m_\varphi \theta_\varphi)^2
\]

(40)

which generalizes the linear $R_\xi$-gauge and is more suitable for calculations in a background electromagnetic field [33]. $A^\mu_\nu$ and $A^\mu_\mu$ denote here the $SU(2)$ and $U(1)$ gauge fields respectively whereas $\varphi_\mu$ designates the Goldstone bosons. The Lagrangian (39) brings the equation of motion

\[
\left( \partial_\nu \pm ieA_\nu \right)^2 + m_W^2 W_\mu^\pm + 1/\xi - 1)(\partial^\mu \pm ieA^\mu)(\partial_\nu W_\nu^\pm \pm ieA_\nu W_\nu^\pm) + 2ieF^{\mu\nu} W_\nu^\pm = 0
\]

(41)

describing charged massive particles with spin 0 and 1. The longitudinal mode of the vector field $W^\pm$

\[
B^\pm(x) = -\frac{1}{\xi m_W}(\partial^\mu W_\mu^\pm \pm ieA_\mu W_\mu^\pm)
\]

(42)
corresponds to the unphysical scalar with mass $\xi m_W$ and it obeys the Klein-Gordon equation

$$\left[(\partial_\nu \pm ieA_\nu)^2 + \xi m_W^2\right]B^\pm(x) = 0. \quad (43)$$

The wave functions of $W$-bosons come from the transversal mode being associated with vector quantum states.

$$V_\mu^\pm(x) = W_\mu^\pm(x) + \frac{1}{\xi m_W} (\partial_\mu \pm ieA_\mu) (\partial^\nu W_\nu^\pm \pm ieA^\nu W_\nu^\pm) \quad (44)$$

The equation for the transversal mode $V_\mu^\pm$ is derived from eq.(41) and it can be written in the following way.

$$\left[(\partial_\nu \pm ieA_\nu)^2 + m_W^2\right]V_\mu^\pm(x) \pm 2ieF^{\mu\nu}V_\nu^\pm(x) = 0 \quad (45)$$

Besides, due to eq.(43) the field $V_\mu^\pm$ satisfies additional condition

$$(\partial^\mu \pm ieA^\mu) V_\mu^\pm(x) = 0 \quad (46)$$

which means that only 3 components of the field $V_\mu^\pm$ are independent (exactly as it should be for spin-1 particles). Thus, we have obtained a Lorentz-covariant decomposition of the field $W_\mu^\pm$ into the scalar and vector parts:

$$W_\mu^\pm(x) = V_\mu^\pm(x) + \frac{1}{m_W}(\partial_\mu \pm ieA_\mu) B^\pm(x) \quad (47)$$

Notice that all physically sensible information is contained in the vector field $V_\mu^\pm$, whereas the scalar field $B^\pm$ plays an auxiliary role and its contribution to the S-matrix is canceled by that of the Goldstone bosons $\varphi^\pm = \frac{1}{\sqrt{2}}(\varphi_2 \pm i\varphi_1)$.

To carry out the scheme of canonical quantization, one needs an explicit form of the $W$-boson wave functions in an intense electromagnetic background. This requires obtaining exact solutions of eq.(45) for a given configuration of the external field. If we restrict ourselves to the case of a plane-wave electromagnetic field with potential $A_\mu(\varphi)$ then eq.(43) can be easily integrated and we get:

$$W_\mu^\pm(x, p) = \exp\left[-ipx - i\int_0^{kx} \left(\pm \frac{epA}{pk} - \frac{e^2A^2}{2pk} \right) d\varphi\right]. \quad (48)$$

where $v_{\mu}^{\pm\nu}(p, \sigma)$ is a complex polarization vector which is subject to the following constraints:

$$v_{\mu}^{\pm\mu} = 0, \quad v_{\mu}^{\pm\rho}v_{\nu}^{\pm\rho} = -1, \quad \sum_{\sigma} v_{\mu}^{\pm\nu}(p, \sigma)v_{\nu}^{\pm\sigma}(p, \sigma) = -g_{\mu\nu} + p_{\mu}p_{\nu}/m_W^2. \quad (49)$$

Now the task is to represent the field $W_\mu^\pm$ as the sum of one-particle quantum excitations. Using expression (47) we expand $W_\mu^\pm$ into series of classical solutions (48), (13)

$$W_\mu^\pm(x) = \sum_{p\sigma} \left[a_{p\sigma}W_\mu^+(x, p) + b_{p\sigma}W_\mu^-\sigma(x, p)\right] + \frac{(\partial_\mu + ieA_\mu)}{m_W} \sum_{p} \left[a_{p0}\phi_+(x, p) + b_{p0}^{\dagger}\phi_-\sigma(x, p)\right], \quad (50)$$
\( W^\pm_\mu (x) = \sum_{p\sigma} [a^\dagger_{p\sigma} W^{\pm \ast}_\mu (x, p) + b_{p\sigma} W^-_\mu (x, p)] + \frac{\left( \partial_\mu - ieA_\mu \right)}{m_W} \sum_p [a^\dagger_p \phi^{\ast \ast}(x, p) + b_p \phi^-(x, p)], \)

which incorporate the Fock operators of two kinds. The observables \( a_{p\sigma}, b_{p\sigma} \) correspond to the vector quantum states of \( W^- \)-bosons and satisfy the usual commutation relations (with a polarization subscript \( \sigma = 1, 2, 3 \))

\[
[a_{p\sigma}; a^\dagger_{p'\sigma'}] = \delta_{pp'} \delta_{\sigma\sigma'} \quad [b_{p\sigma}; b^\dagger_{p'\sigma'}] = \delta_{pp'} \delta_{\sigma\sigma'} \\
[a_{p\sigma}; a_{p'\sigma'}] = [b_{p\sigma}; b_{p'\sigma'}] = [a_{p\sigma}; b_{p'\sigma'}] = [a_{p\sigma}; b^\dagger_{p'\sigma'}] = 0
\]

(52)

At the same time the operators \( a_{p0}, b_{p0} \) connected with the scalar part (12) must obey different rules of quantization:

\[
[a_{p0}; a^\dagger_{p'0}] = -\delta_{pp'} \quad [b_{p0}; b^\dagger_{p'0}] = -\delta_{pp'} \\
[a_{p0}; a_{p'0}] = [b_{p0}; b_{p'0}] = [a_{p0}; b_{p'0}] = [a_{p0}; b^\dagger_{p'0}] = 0
\]

(53)

The odd minus in eq.(53) is a price that must be paid for the Lorentz-covariance. It immediately causes the appearance of indefinite metrics in Hilbert space \( \mathcal{H} \) which is typical for Gupta-Bleuler quantization. However, since longitudinal mode \( B^\pm \) makes no physical sense, one can eliminate it by considering the physical subspace \( \mathcal{H}_{phys} \subset \mathcal{H} \) without these scalar excitations. Similar arguments can be found in many textbooks on quantum field theory [31].

Loop calculations in a background electromagnetic field involve Feynman propagator being defined by the formula:

\[
D_{\mu\nu}(x, x') = -i <0|TW^+_{\mu}(x)W^-_{\nu}(x')|0>,
\]

(54)

and which is a Green function of the equation (11)

\[
((\partial_\alpha + ieA_\alpha)^2 + m_W^2)g^{\mu\nu} + \frac{1}{2}(\partial^\mu + ieA^\mu)(\partial^\nu + ieA^\nu) + 2ieF^{\mu\nu} \right] D_{\nu\lambda}(x, x') = \delta^\mu_\nu \delta^4(x - x')
\]

(55)

Inserting expansions (50) and (51) into eq.(54) and employing the commutation rules (52), (53), one can obtain an explicit covariant form of the \( W^- \)-boson propagator:

\[
D_{\mu\nu}(x, x') = \int \frac{d^4p}{(2\pi)^4} \frac{W_{\mu\alpha}(x, p)W^\ast_{\nu\beta}(x'; p)}{m_W^2 - p^2 - i0} \left[ g^{\alpha\beta} + \frac{p^\alpha p^\beta}{p^2 - \xi m_W^2 + i0} \right].
\]

(56)

This representation is based on the eigen functions of the differential operator corresponding to eq.(45)

\[
W_{\mu\alpha}(x, p) = \exp \left[ -ipx - i \int_0^{kx} \frac{cpA}{pk} - \frac{e^2 A^2}{2pk} d\varphi \right].
\]

\[
\left\{ g_{\mu\alpha} + \frac{e}{pk} \left( k_\mu A_\alpha (kx) - k_\alpha A_\mu (kx) \right) - \frac{e^2}{2(pk)^2} A^2 (kx) k_\mu k_\alpha \right\}
\]

(57)

Let us note that classical solution (15) is simply a particular form of the eigen function (57) taken on the mass shell and contracted through the last index with the ordinary polarization.
vector. For an arbitrary value of gauge fixing parameter $\xi$ a proper time representation of the propagator (54) is rather awkward. However, practically one can use a diagonal gauge ($\xi = 1$) in which expression (56) can be rewritten in the most elegant way:

$$D_{\mu\nu}(x,x') = \frac{\Phi(x,x')}{(4\pi)^2} \int_0^{\infty} ds \frac{1}{s^2} \exp \left[ -ism^2 - \frac{i(x - x')^2}{4s} + \frac{ie^2 s}{(kx - kx')} \int_{kx'}^{kx} A_{\mu}(\varphi)A^{\mu}(\varphi) d\varphi \right] - \frac{ie^2 s}{(kx - kx')^2} \int_{kx'}^{kx} d\varphi_1 \int_{kx'}^{kx} d\varphi_2 A_{\mu}(\varphi_1)A^{\mu}(\varphi_2) \left\{ g_{\mu\nu} + \frac{2es}{(kx - kx')} \left[ k_\mu \left( A_{\nu}(kx) - A_{\nu}(kx') \right) - k_\nu \left( A_{\mu}(kx) - A_{\mu}(kx') \right) \right] - \frac{2e^2 s^2}{(kx - kx')^2} \left( A_\lambda(x) - A_\lambda(x') \right)^2 k_\mu k_\nu \right\} \quad (58)$$

5 General Features of Quantum Processes in Strong Electromagnetic Fields

Many distinctive properties of particle physics with intense electromagnetic background can be traced in the reaction of $W$-boson and neutrino production via the decay of a massive charged lepton $\ell$. It is well known that the process $\ell^- \rightarrow W^- \nu_\ell$ should be forbidden in vacuum if the lepton mass is small in comparison with the ones of final particles. However, in the presence of electromagnetic fields the restriction $m_\ell > m_W + m_\nu$ can be removed and there appears a possibility to observe an effect analogous to quantum transmission of the potential barrier through tunneling. The matrix element of the reaction $\ell^- \rightarrow W^- \nu_\ell$ can be calculated employing the wave functions of the charged lepton $\ell$ (28) and the one of $W$-boson (18):

$$S_{fi} = \frac{ig}{2\sqrt{2}} \int d^4 x W_{\mu}^* (x,p') \gamma^\mu (1 + \gamma^5) \Psi_{\ell'}(x,p) \quad (59)$$

In order to obtain the probability of the decay $\ell^- \rightarrow W^- \nu_\ell$ per unit of time one must integrate $|S_{fi}|^2$ over the phase volume of final particles and sum up their polarizations. In the case of a constant crossed field ($F = G = 0$) the final result can be written as (68):

$$dP \frac{du}{du}(\ell^- \rightarrow W^- \nu_\ell) = -\frac{G_\mu m_W^4}{4\pi^2 \sqrt{2} p_0} \left\{ \frac{1}{2} - \frac{m_\ell^2 + m_\nu^2}{2m_W^2} - \frac{m_\tau^2 - m_\tau^2}{2m_W^2} \right\} \Phi_1(z) + \frac{2m_\rho^2 (\chi^2 u)^{1/3}}{m_W^2 (1 - u)^{1/3}} \left[ 1 + u^2 + (1 - u)^2 \frac{(m_\ell^2 + m_\nu^2)}{2m_W^2} \right] \Phi'(z) \quad (60)$$

This formula represents the differential probability of the process with respect to the variable

$$u = \frac{p''_{\mu} k^\mu}{p_{\lambda} k^\lambda} = \frac{(p''_{\mu} F_{\mu\alpha} F_{\alpha\beta} p^\beta)}{(p_{\mu} F_{\mu\alpha} F_{\alpha\beta} p^\beta)}, \quad (61)$$

where $p''_{\mu}, p_{\mu}$ are momenta of neutrino and $W$-boson, respectively, while $p_{\mu}$ stands for the one of initial lepton. Note that in a crossed electromagnetic field $F_{\mu\lambda}$ the wave vector $k_\mu$ must be substituted by the following expression:

$$k^\mu = \frac{e^2 (pk)}{m_\ell^2 \chi^2} F_{\mu\alpha} F_{\alpha\beta} p^\beta \sim \left( p_0 E^2 - p (E \times B); \quad E (Ep) + B (Bp) - p (BB) + p_0 (E \times B) \right) \quad (62)$$
This interrelation meaning that relativistic particles perceive the electromagnetic background as an incident plane-wave have been noticed long ago by Williams and Weizsäcker [37] who proposed the famous equivalent-photon approximation (see also [38]). In the case of a constant homogeneous electromagnetic field being considered the kinematics of the reaction $\ell^- \rightarrow W^- \nu_\ell$ resembles the one with real photons and is determined by the following constraint of 4-momentum conservation:

$$p_\mu + \frac{e^2 M^2}{2 m_\ell c^2} F^\mu_\alpha F^\alpha_\beta p_\beta = p'_\mu + p''_\mu,$$

(63)

where $M^2$ represents the energy deficit obstructing the decay $\ell^- \rightarrow W^- \nu_\ell$ in a vacuum without fields.

$$M^2 = (p' + p'')^2 - p^2 = m_W^2 + m_\nu^2 - m_\ell^2 + 2 (p' p'') \geq (m_W + m_\nu)^2 - m_\ell^2$$

(64)

Now we see that for light lepton ($m_\ell < m_W + m_\nu$) $M^2$ is always positive and the reaction could proceed only due to the energy-momentum borrowed from the background field.

$$q^\mu = \frac{M^2}{2(pFp)} F^{\mu\alpha} F_{\alpha\beta} p^\beta \quad q^2 = \frac{M^4}{4m_\ell^2} \left( \frac{a}{\chi^2} \right)^2 - \frac{2b}{\chi^2} \approx 0$$

(65)

If we suppose the lepton $\ell$ to be a hypothetical heavy particle of the fourth generation ($m_\ell > m_W + m_\nu$) then for some values of neutrino and $W$-boson momenta $M^2$ can become negative. This reflects another possibility when the background field takes some amount of energy from the initial lepton and absorbs it. Thus electromagnetic environment reveals itself as an additional virtual particle with momentum $q^\mu$ (65) that induces the decay $\ell^- \rightarrow W^- \nu_\ell$ regardless whether it is forbidden or not when the external field is absent.

All said above is described by the formula (60) which involves special functions

$$\Phi(z) = \int_0^\infty dt \cos(zt + t^2/3), \quad \Phi_1(z) = \int_0^\infty dt \Phi(t), \quad \Phi'(z) = \frac{d\Phi(z)}{dz}$$

(66)

which are known in mathematics as the functions of Airy (see the Appendix). The argument $z$ is proportional to the quantity $M^2$ (64) and in terms of spectral variable $u$ (61) it can be written as follows:

$$z = \frac{m_W^2 u + m_\nu^2 (1 - u) - m_\ell^2 u (1 - u)}{m_\ell^2 [\chi u^2 (1 - u)]^{2/3}}$$

(67)

Note that due to the constraint (63) the momentum projection on the wave vector (62) is conserved

$$p_\mu F^{\mu\alpha} F_{\alpha\beta} p^\beta = (p'_\mu + p''_\mu) F^{\mu\alpha} F_{\alpha\beta} p^\beta$$

(68)

that fixes the interval of variations for $u$: $0 \leq u \leq 1$. Now if the reaction $\ell^- \rightarrow W^- \nu_\ell$ is forbidden in vacuum, then $z > 0$ for all $u \in [0; 1]$. In the opposite case: $m_\ell > m_W + m_\nu$ there is a domain of negative values

$$z < 0 \text{ for } u_1 < u < u_2,$$

(69)

which exactly coincide with the phase volume of the process at hand provided that external field is omitted.
The sign and magnitude of the argument $z$ exert primary control over the probability of any quantum process in the background electromagnetic field. This is caused by a particular behaviour of the Airy functions that is examined carefully in the Appendix. Let us only mention that for large positive values of $z$ these functions decrease exponentially as $\exp\left(-\frac{2}{3}z^{3/2}\right)$. So the probabilities of reactions being forbidden under normal conditions must contain a factor of exponential suppression. This exponent can be treated as the well-known semiclassical probability of quantum transmission below the potential energy barrier. The index of the power increasing proportionally with $M^2$ characterizes how much energy is borrowed from the background field. The aforesaid arguments can be confirmed by the asymptotic estimate of the total probability of the decay $\ell^- \rightarrow W^+\nu_\ell$ in the domain of weak electromagnetic fields. Integrating eq.(60) through the use of the saddle-point method one obtains.

$$P(\ell^- \rightarrow W^+\nu_\ell) = \frac{2G_Fm_Wm_\ell^3\chi}{9\pi\sqrt{6}p_0}\exp\left(-\frac{\sqrt{3}m_W^3}{\chi m_\ell^3}\right), \quad \text{for} \quad \chi \ll \left(\frac{m_W}{m_\ell}\right)^3 \tag{70}$$

Here it is implied that masses of particles involved in the reaction satisfy real phenomenological conditions ($m_\ell < m_\ell < m_W$). We see that all leptons being known up to now can produce $W$-bosons only if the field strength $E$ and lepton energy $p_0$ take the values that are comparable with the estimate:

$$\chi = \left(\frac{p_0}{m_\ell}\right)\left(\frac{E}{E_{cr}}\right) \sim \sqrt{3}\left(\frac{m_W}{m_\ell}\right)^3 \approx 6.7 \cdot 10^{15} \tag{71}$$

In other words this exotic process could occur perhaps only in the Early Universe when the energies $p_0 \sim 10^{12} GeV$ were accessible. This ultra-relativistic domain is described by the formula which is derived from eq.(60) by means of the expansions (90), (100) (see the Appendix).

$$P(\ell^- \rightarrow W^+\nu_\ell) = \frac{3G_Fm_Wm_\ell^3\chi}{2\pi\sqrt{6}p_0}, \quad \text{for} \quad \chi \gg \left(\frac{m_W}{m_\ell}\right)^3 \tag{72}$$

The above results can be summed up as the following statement. If the reaction proceeding by the background electromagnetic field is forbidden in vacuum, then for weak fields its probability is exponentially suppressed by the factor which has the lacking energy borrowed from the background field as the power index. This suppression becomes unimportant only in the domain of extremely strong fields and ultra-relativistic energies that practically are inaccessible in experiments, at least now.

The situation changes dramatically for the processes that could take place when the background field is absent. For example, if we consider the decay of a hypothetical heavy lepton of the fourth generation ($m_\ell > m_W + m_\nu$), then from eq.(60) it follows.

$$P(\ell^- \rightarrow W^+\nu_\ell) = \frac{G_F\sqrt{2}}{16\pi p_0}\sqrt{1 - \left(\frac{m_W + m_\nu}{m_\ell}\right)^2}\left[1 - \left(\frac{m_W - m_\nu}{m_\ell}\right)^2\right] \times$$

$$\left[(m_\ell^2 - m_\tau^2)^2 + m_W^2(m_\ell^2 + m_\tau^2) - 2m_W^4\right] + O(\chi^2) \tag{73}$$

This expression exactly reproduces the probability of the reaction $\ell^- \rightarrow W^+\nu_\ell$ which can be calculated through the standard quantum field theory technique. Note, that it arises from the formal substitution $\Phi_1(z) \rightarrow \pi\theta(-z)$ ($\theta(x)$ is the Heaviside step function) giving the first term in the asymptotic expansion of eq.(60) when $\chi \rightarrow 0$. Thus we see that the probability of the permitted in vacuum process acquires in the background field corrections proportional to $\chi^2$ (the omitted term $O(\chi^2)$ in eq.(73)). However these corrections which become essential only in the domain (71) can be neglected if $\chi \ll (m_W/m_\ell)^3$. 

13
6 New Processes in Background Electromagnetic Fields

Now I would like to discuss the possibilities for searching new physics in background electromagnetic fields. We have already noticed that the relativistic particle penetrating through the electromagnetic environment feels the background field as a beam of real photons which can supply it with substantial amount of energy. If the field strength is sufficiently high then new channels of reactions can be opened and the particle decays into more heavy species. Let us consider such a reaction when a light charged particle \( A^\pm \) produces two others, one of which is charged too \( B^\pm \) while the other is neutral \( C^0 \). We assume that the process \( A^\pm \to B^\pm C^0 \) is forbidden in a vacuum \( (m_A < m_B + m_C) \) and wish to estimate the field-energy domains where it could occur. As we have explained in the previous section, in general the probability of the decay \( A^\pm \to B^\pm C^0 \) having a form similar to eq.(60) can be represented through the Airy functions which depend on the following argument:

\[
Z_{\pm} = \frac{m_B^2 u + m_C^2 (1 - u) - m_A^2 u (1 - u)}{m_e^2 \chi u^2 (1 - u)^{2/3}}
\]  

For weak electromagnetic fields that can be handled in experiments the above probability must contain the exponential factor which is connected with the minimal positive value of the argument \( Z_{\pm} \):

\[
P(A^\pm \to B^\pm C^0) \sim \exp\left(-\frac{2}{3} Z_{\pm \min}^{3/2}\right) \equiv \exp(-\gamma)
\]  

Using eq.(74) one can explicitly calculate the power index \( \gamma \) for the arbitrary relationship among the masses of all particles involved in the reaction:

\[
\gamma = \frac{\sqrt{6}}{16 \chi} \left[ 128 \lambda_1^2 \lambda_2 + 80 \lambda_1 (\lambda_1 + \lambda_2 - \lambda_0)^2 + \frac{(\lambda_1 + \lambda_2 - \lambda_0)^4}{\lambda_2} \left( 1 + \frac{32 \lambda_1 \lambda_2}{(\lambda_1 + \lambda_2 - \lambda_0)^2} \right)^{3/2} - 1 \right]^{1/2},
\]  

where

\[
\lambda_0 = \left( \frac{m_A}{m_e} \right)^2, \quad \lambda_1 = \left( \frac{m_B}{m_e} \right)^2, \quad \lambda_2 = \left( \frac{m_C}{m_e} \right)^2
\]  

This quantity governing the range of exponential suppression makes a crucial impact on the probability and fixes a certain threshold of the reaction induced by the electromagnetic background. In the domain of the very small values of parameter \( \chi \) the probability of the decay \( A^\pm \to B^\pm C^0 \) becomes negligible on the account of extremely large magnitude of the power index \( (\gamma \gg 1) \). So the reaction could proceed with a feasible rate only if \( \gamma \sim 1 \). I will refer to this condition as to the threshold of the reaction designating the corresponding value of \( \chi \) by a zero subscript \( (\chi_0) \).

Let us analyze the consequences of the general expression (76). Suppose that neutral particle being produced in the decay \( A^\pm \to B^\pm C^0 \) is very light in comparison with the others. Then the power index \( \gamma \) can be approximated as follows.

\[
\gamma = \frac{\sqrt{3}}{\chi} \left( \frac{m_B}{m_e} \right)^3 \left( 1 - \frac{m_A^2}{m_B^2} \right), \quad \text{for} \quad m_C \ll m_A, m_B.
\]  

A particular case of this formula have been employed in the previous section for the process of \( W \)-boson and neutrino production. The threshold of this reaction \( \ell^- \to W^- \nu_\ell \) coinciding with the estimate (71) can be calculated without resort to the explicit form of the probability (70).
The only thing that we need is the power index (78) being connected with the threshold value of $\chi$ by a simple equation $\gamma = 1$. This reasoning can be extended to some other reactions being of practical interest. Substituting the values of electron, muon and pion masses in eq.(78) we obtain characteristic domains of the following processes induced by the external electromagnetic background.

$$
e^- \to \mu^- \nu_\mu \bar{\nu}_\mu, \quad e^+ \to \mu^+ \bar{\nu}_e \nu_\mu, \quad \chi_0 = 1.53 \cdot 10^7$$

$$
e^- \to \pi^- \nu_e, \quad e^+ \to \pi^+ \bar{\nu}_e, \quad \chi_0 = 3.53 \cdot 10^7$$

$$\mu^- \to \pi^- \nu_\mu, \quad \mu^+ \to \pi^+ \bar{\nu}_\mu, \quad \chi_0 = 1.51 \cdot 10^7$$

(79)

On the other hand, if we assume that the mass of the charged particle emerged in the decay $A^\pm \to B^\pm C^0$ is the smallest parameter, then eq.(76) can be reduced to

$$\gamma = \frac{\sqrt{3} m_B}{\chi m_e} \left( \frac{m_C}{m_e} \right)^2 \left( 1 - \frac{m_A^2}{m_C^2} \right), \quad \text{for} \quad m_B \ll m_A, m_C. \quad (80)$$

Now it is evident that the factor of exponential suppression grows up from the non-zero mass of the charged particle being produced, because the power index $\gamma \ (80)$ vanishes in the limit $m_B \to 0$. By virtue of the fact that there are no charged particles with masses less than the one of electron, it is reasonable to look for the reactions with electrons and positrons in final states. For example, the processes of the inverse beta decay being forbidden in vacuum have the threshold that lies far below the ones of the lepton reactions (79). (This is also caused by a small mass difference between proton and neutron.)

$$p^+ \to ne^+ \nu_e, \quad \tilde{p}^- \to \tilde{n}e^- \bar{\nu}_e, \quad \chi_0 = 1.61 \cdot 10^4 \quad (81)$$

Another interesting possibility arises in connection with the process of neutral particle emission in the intense electromagnetic field $A^\pm \to A^\pm C^0$. The power index $\gamma$ that determines the rate of this reaction can be deduced from eq.(76) by a formal replacement $m_B = m_A$.

$$\gamma = \frac{\sqrt{6}}{16 \chi} \left( \frac{m_C}{m_e} \right)^3 \left[ 1 + 32 \frac{m_A^2}{m_C^2} \right]^{3/2} - 1 + 32 \left( \frac{m_A}{m_C} \right)^2 + 128 \left( \frac{m_A}{m_C} \right)^4 \right]^{1/2}. \quad (82)$$

Now provided that the neutral particle is much heavier than its fore-runner we can rewrite eq.(82) in the form.

$$\gamma = \frac{\sqrt{3} m_A}{\chi m_e} \left( \frac{m_C}{m_e} \right)^2, \quad \text{for} \quad m_A \ll m_C. \quad (83)$$

Here it is relevant to quote the reactions of pion radiation as an example governed by the last formula.

$$e^- \to e^- \pi^0, \quad e^+ \to e^+ \pi^0, \quad \chi_0 = 1.21 \cdot 10^5. \quad (84)$$

In the opposite case corresponding to the relatively small mass of the emitted particle the factor of exponential suppression appears with the following index.

$$\gamma = \frac{\sqrt{3} m_C}{\chi m_e} \left( \frac{m_A}{m_e} \right)^2, \quad \text{for} \quad m_A \gg m_C. \quad (85)$$

So the process of pion radiation from protons could occur only if the field strength and proton energy exceed significantly the ones of the reactions (84).

$$p^+ \to p^+ \pi^0, \quad \tilde{p}^- \to \tilde{p}^- \pi^0, \quad \chi_0 = 8.90 \cdot 10^8. \quad (86)$$
Next we intend to discuss the possibilities of charged particles production via the field induced decay $A^0 \rightarrow B^+C^-$. The argument of the Airy functions through which the probability of this reaction can be expressed has a form that slightly differs from eq. (74).

$$Z_0 = \frac{m_B^2 u + m_C^2 (1-u) - m_A^2 u (1-u)}{m_e^2 [\chi u (1-u)]^{2/3}}$$

(87)

Following the reasoning given above it is straightforward to calculate the exponential factor which governs the probability rate in the domain of weak electromagnetic fields.

$$P(A^0 \rightarrow B^+C^-) \sim \exp\left(-\frac{2}{3}Z_0^{3/2}\right) \equiv \exp(-\gamma)$$

(88)

At this stage, in order to obtain precise estimates of the threshold of the process $A^0 \rightarrow B^+C^-$ we need additional assumptions concerning with the masses of particles being implicated. If the initial neutral particle $A^0$ has a very small mass as compared to the charged ones ($m_A \ll m_B, m_C$) then the power index of eq. (88) can be written as follows:

$$\gamma = \frac{\sqrt{3}}{6\chi}\left[(\lambda_1^2 + 14\lambda_1\lambda_2 + \lambda_2^2)^{3/2} - \lambda_1^3 + 33\lambda_1^2\lambda_2 + 33\lambda_1\lambda_2^2 - \lambda_2^3\right]^{1/2}$$

(89)

The last equation describes, for example, the reactions being caused by neutrinos moving throughout the intense electromagnetic field. This is especially the case when one considers the process which is cross symmetric to the muon decay.

$$\nu_\mu \rightarrow e^+\mu^-\nu_e, \quad \bar{\nu}_\mu \rightarrow e^-\mu^+\bar{\nu}_e \quad \chi_0 = 7.41 \cdot 10^4$$

(90)

Confronting the conditions appropriate to the above reaction with the threshold of another cross symmetric channel adduced in eq. (79) we see that reactions initiated by neutrinos require much less values of energy and background field strength than the ones with electrons. From this viewpoint it is more promising to look for new physics in external electromagnetic fields through the investigations of the processes induced by neutral particles. This conclusion acquires further confirmation from the asymptotic estimate of the power index (88) in the case when one of charged particles produced is extremely light.

$$\gamma = \frac{\sqrt{3}m_C(m_B^2 - m_A^2)}{\chi m_e^3}, \quad \text{for} \quad m_C \ll m_A, m_B.$$ 

(91)

Related conditions are characteristic for the following pion decays, which have the lowest thresholds among the all afore-mentioned reactions.

$$\pi^0 \rightarrow \pi^+e^-\bar{\nu}_e, \quad \pi^0 \rightarrow \pi^-e^+\nu_e \quad \chi_0 = 8.38 \cdot 10^3,$$

(92)

It should also be noted that the power index (91) exactly coincides with the estimate (80) if the interchange $m_B \leftrightarrow m_C$ is made. This fact explains once more the origin of the exponential suppression as a semiclassical probability of the field-induced quantum transmission of the electron from the Dirac sea to the upper continuum. The height of the potential barrier corresponding to the results (80), (91) can be easily calculated if one reproduces the reasoning which is relevant to the famous Klein paradox in quantum mechanics [33].
Finally, I would like to make some remarks about the processes of particle-antiparticle pairs production in the intense electromagnetic fields. The probability of the forbidden in a vacuum decay \(A^0 \rightarrow B^+B^-\) is suppressed in the electromagnetic background by the factor which can be obtained from eq.(87), (88) under the assumption \(m_C = m_B\):

\[
P(A^0 \rightarrow B^+B^-) \sim \exp\left[-\frac{8m_B^3}{3\chi m_e^3}\left(1 - \frac{m_A^2}{4m_B^2}\right)^{3/2}\right]
\]

This formula can be applied for the threshold evaluations related to the following reactions provoked by the external field.

\[
\begin{align*}
\gamma \rightarrow e^+e^- & , \quad \nu_\mu \rightarrow \nu_\mu e^+e^- & \chi_0 = 2.67 \\
\gamma \rightarrow \mu^+\mu^- & , \quad \nu_\mu \rightarrow \nu_\mu \mu^+\mu^- & \chi_0 = 2.36 \cdot 10^7 .
\end{align*}
\]

We see that electron-positron pairs can be easily produced at the energies \(p_0 > 25\ GeV\) if the background field strength is of the order \(E \sim 10^{10} V/m\). Similar conditions have been created in CERN experiments \(\text{[40]}\) for the photons incident on the crystal of Germanium with a small angle to the axial direction. Neutrino fluxes of the like energies could give rise to the same effect caused by the coherent interactions with a regular arrangement of atoms in the single crystal. However due to the very small values of the neutrino couplings the process \(\nu_\mu \rightarrow \nu_\mu e^+e^-\) has an additional suppression by the factor \((m_e/m_W)^4 \sim 10^{-21}\) \(\text{[41]}\) which impedes its immediate experimental observation.

7 Conclusion

In this paper I have presented some methodological ideas providing the basis for practical calculations of quantum processes in very strong electromagnetic fields. The primary motivation of these studies is to look for new physics that could emerge due to the intense electromagnetic background. There is a hope that by combining modern achievements in the laser technique and in electromagnet construction with traditional methods of elementary-particle physics, one could obtain results that are inaccessible in other investigations.

Tracing some general features of quantum processes induced by strong electromagnetic fields I have estimated the energy-field domains that are needed in order to observe a number of reactions which are forbidden under normal conditions. At relatively small values of the field strength there remains the only one parameter \(\chi\) \(\text{[6]}\) which has a crucial influence on the rate of the reaction and governs how the energy threshold moves with the increase of the field intensity. I drew a figure for illustrative purposes (see Fig.1) to display the characteristic domains relevant to the processes mentioned in the previous section. We see that present experimental situation is confined by the bound \(\chi \leq 10\). This conclusion comes from the estimates of electric fields extending over macroscopic distances along strings of atoms in single crystals \(\text{[19]}\). For example, the electric field along a \(< 111 >\) axis in a crystal of Wolfram amounts to \(E \approx 5 \cdot 10^{13} V/m\) \(\text{[19]}\). For crystals of higher atomic number, the electric fields can be still larger than the value given above. However it is insufficient to reach the domain \(\chi \sim 10^2 - 10^5\) as long as the energies of accelerated particles do not exceed the TeV scale. So we can expect that with the particle machines of the new generation there could appear a possibility for experimental observation of some reactions discussed in this paper.
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Appendix

We have employed the Airy functions being connected with the ones described in mathematical handbooks in the following manner.

\[ \Phi(z) = \pi Ai(z), \quad \Phi_1(z) = \pi^2 [Ai(z)Gi'(z) - Ai'(z)Gi(z)], \quad \Phi'(z) = \pi Ai'(z) \]  

(96)

In accordance with standard mathematical notations these functions are defined through the integral representations

\[ Ai(z) = \frac{1}{\pi} \int_0^\infty dt \cos(zt + t^3/3), \quad Gi(z) = \frac{1}{\pi} \int_0^\infty dt \sin(zt + t^3/3) \]  

(97)

and they obey the following differential equations

\[ Ai''(z) - zAi(z) = 0, \quad Gi''(z) - zGi(z) = -\frac{1}{\pi}. \]  

(98)

The Airy functions that are the entire functions of the complex variable \( z \) can be expanded in the Laurent series that converge everywhere (\( |z| < \infty \))

\[ Ai(z) = \frac{3^{-2/3}}{\pi} \sum_{n=0}^{\infty} \Gamma \left( \frac{n+1}{3} \right) \sin \left( \frac{\pi}{3} - \frac{2\pi n}{3} \right) \left( \frac{3^{1/3}z^n}{n!} \right), \]  

(99)

\[ Gi(z) = \frac{3^{-2/3}}{\pi} \sum_{n=0}^{\infty} \Gamma \left( \frac{n+1}{3} \right) \cos \left( \frac{\pi}{3} - \frac{2\pi n}{3} \right) \left( \frac{3^{1/3}z^n}{n!} \right). \]  

(100)

Many properties of the Airy functions can be deduced from their interrelation with the cylindrical (Bessel and McDonald) functions of the fractional index.

\[ Ai(z) = \frac{1}{\pi} \sqrt{\frac{z}{3}} K_{1/3} \left( \frac{2}{3} z^{3/2} \right), \quad Ai(-z) = \sqrt{\frac{z}{3}} \left[ J_{1/3} \left( \frac{2}{3} z^{3/2} \right) + J_{-1/3} \left( \frac{2}{3} z^{3/2} \right) \right] \]  

(101)

\[ Ai'(z) = -\frac{z}{\pi \sqrt{3}} K_{2/3} \left( \frac{2}{3} z^{3/2} \right), \quad Ai'(-z) = \frac{z}{3} \left[ J_{2/3} \left( \frac{2}{3} z^{3/2} \right) - J_{-2/3} \left( \frac{2}{3} z^{3/2} \right) \right] \]  

(102)

For large values of its arguments (\( |z| \to \infty, |\arg z| < \pi \)) there is a possibility to approximate those by the following asymptotic estimates:

\[ Ai(z) = \frac{z^{-1/4}}{2\pi} \exp \left( -\frac{2}{3} z^{3/2} \right) \sum_{n=0}^{\infty} \frac{(-1)^n \Gamma(3n+1/2)}{(2n)!} \left( 9z^{3/2} \right)^{-n} \]  

(103)

\[ Gi(z) = \frac{1}{\pi z} \sum_{n=0}^{\infty} \frac{(3n)!}{n!} \left( 3z^3 \right)^{-n} \]  

(104)
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Figure 1: The energy-field domains plotted for different values of the background field parameter $\chi$ (6). The lines correspond to the following choices: (1) $\chi = 1$, (2) $\chi = 10$, (3) $\chi = 100$, (4) $\chi = 10^3$, (5) $\chi = 10^4$, (6) $\chi = 10^5$. 