SO(2N) (0, 2) SCFT and M Theory on AdS$_7 \times$ RP$^4$

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Abstract

We study M theory on AdS$_7 \times$ RP$^4$ corresponding to 6 dimensional SO(2N) (0, 2) superconformal field theory on a circle which becomes 5 dimensional super Yang-Mills theory at low energies. For SU(N) (0, 2) theory, a wrapped D4 brane on S$^4$ which is connected to a D4 brane on the boundary of AdS$_7$ by N fundamental strings can be interpreted as baryon vertex. For SO(2N) (0, 2) theory, by using the property of homology of RP$^4$, we classify various wrapping branes. Then we consider particles, strings, twobranes, domain walls and the baryon vertex in Type IIA string theory.

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I. INTRODUCTION

In [1] the large $N$ limit of superconformal field theories (SCFT) was described by taking the supergravity limit on anti-de Sitter (AdS) space. One can obtain the scaling dimensions of operators of SCFT from the masses of particles in string/M theory [2]. In particular, $\mathcal{N} = 4$ super $SU(N)$ Yang-Mills theory in 4 dimensions is described by Type IIB string theory on $AdS_5 \times S^5$. This can be done by considering the brane configuration in the string theory and taking a low energy limit which decouples the field theory from gravity and simultaneously considering the near horizon geometry of the corresponding supergravity solution. There are $\mathcal{N} = 2, 1, 0$ superconformal theories in 4 dimensions which have corresponding supergravity description on orbifolds of $AdS_5 \times S^5$ [3]. This proposed duality was tested by studying the Kaluza-Klein (KK) states of supergravity theory and by comparing them with the chiral primary operators of the SCFT on the boundary [4] and used to calculate the energy of quark-antiquark pair [5].

The field theory/M theory duality provides a supergravity description on $AdS_4$ or $AdS_7$ for some superconformal theories in 3 and 6 dimensions, respectively. The maximally supersymmetric theories have been studied in [6,7] and the lower supersymmetric case was also realized on the worldvolume of M theory at orbifold singularities [8]. The KK spectrum description on the twisted states of $AdS_5$ orbifolds was discussed in [9].

Recently, it was observed that the gauge group is replaced by $SO(N)/Sp(N)$ [10] by taking appropriate orientifold operations for the string theory on $AdS_5 \times \mathbb{RP}^5$ (See also [6,11]). By analyzing the discrete torsion for Neveu Schwarz $B$ field and RR $B$ field, many features of gauge theory were described by various wrapping branes.

A system of $N$ D4 branes in the decoupling limit can be well described by considering $N$ M5 branes wrapped on the eleventh dimensional circle in M theory by taking the limit of eleven dimensional Planck length being zero and keeping the radius of circle fixed [12]. See also relevant papers [13,14]. Then $(0,2)$ six dimensional CFT on a circle becomes five dimensional super Yang-Mills theory at low energy. The UV region is described by the $(0,2)$
SCFT on a circle which is dual for large $N$ to M theory on a background $AdS_7 \times S^4$. In the IR region, the theory is described by five dimensional super Yang-Mills theory having a dual supergravity description by the Type IIA D4 brane solution.

In this paper, we generalize the work of [10] to the case of $AdS_7 \times \mathbb{RP}^4$ where the eleventh dimensional circle is one of $AdS_7$ coordinates. So we are dealing with $(0,2)$ six dimensional SCFT on a circle rather than uncompactified full M theory. In section II, for $SU(N)$ $(0,2)$ theory, a wrapped D4 brane on $S^4$ can be interpreted as baryon vertex. There exist $N$ fundamental strings connecting a D4 brane on the boundary of $AdS_7$ with the D4 brane on $S^4$. By putting $N$ M5 branes in the $R^5/Z_2$ orbifold singularity, where the $Z_2$ acts by a reflection of the 5 directions transverse to the M5 branes, and also by changing the sign of the 3-form field $C_3$, we will obtain the large $N$ limit of the $SO(2N)$ $(0,2)$ SCFT and $\mathbb{RP}^4$ orientifold after removing the $R^5/Z_2$ orbifold singularity. Then using the property of homology of $\mathbb{RP}^4$ we classify various wrapping branes and discuss their topological restrictions. In section III, we consider particles, strings, twobranes, domain walls and the baryon vertex in Type IIA string theory. Finally, in section IV, we will come to the summary of this paper and comment on the future direction.

II. $SO(2N)$ $(0,2)$ SCFT AND BRANES ON $\mathbb{RP}^4$

A. The Baryon Vertex in $SU(N)$

Let us consider M theory on $R^5 \times R^5 \times S^1$. For first $R^5$, we can take $(x^0, x^1, x^2, x^3, x^6)$ directions and for second $R^5$, we take $(x^4, x^5, x^7, x^8, x^9)$ transverse to M5 branes. The eleventh coordinate $x^{10}$ is compactified on a circle $S^1$ and is a periodic coordinate of period $2\pi$. For small radius of $S^1$, we can regard M theory as Type IIA string theory which can be described in the context of $AdS_7 \times S^4$ where the D4 branes are M5 branes wrapping on $S^1$. The radial function $\rho = \sqrt{(x^4)^2 + (x^5)^2 + (x^7)^2 + (x^8)^2 + (x^9)^2}$ of second $R^5$ will be one of the $AdS_7$ coordinates, the other six being the ones in $R^5 \times S^1$. The $AdS_7 \times S^4$
compactification has $N$ units of four-form flux on $S^4$ as follows.

$$\int_{S^4} \frac{G_4}{2\pi} = N, \quad (\text{II.1})$$

where $G_4$ is four-form field. When a D4 brane is wrapped on $S^4$, on the worldvolume of the D4 brane, there exists a $U(1)$ gauge field which can couple $G_4$ by

$$\int_{S^4 \times R} a \wedge \frac{G_4}{2\pi}, \quad (\text{II.2})$$

where $R$ is a timelike curve in $AdS_7$. From these two relations (II.1) and (II.2), $G_4$ field has the $N$ units of a charge. There should be $-N$ units of a charge somewhere in order to satisfy the vanishing of a charge in a closed universe. This can be done by putting oriented $N$ fundamental strings ending on the D4 brane. So we can interpret the wrapped D4 brane as baryon vertex or antibaryon vertex. The energy of baryon was calculated as a function of its size in [16] along the similar picture.

In $SU(N)$ gauge theory, the quantity of gauge invariant combination of $N$ quarks should be completely antisymmetric. This antisymmetry of baryon vertex corresponding to the behavior of fundamental string connecting D4 brane to the boundary of $AdS_7$ space can be understood with the following boundary conditions. The time zero section of $AdS_7 \times S^4$ is a copy of $R^5 \times S^1 \times S^4$. We can consider a static D4 brane which has the worldvolume of $S^4 \times R$ where $R$ is a point of $S^4$ and $\tilde{S}^4$ is a large four sphere near infinity in $R^5 \times S^1$. Then our $N$ strings connect between D4 brane on $\tilde{S}^4 \times R$ with D4 brane on $Q \times S^4$ where $Q$ is a point in a time zero slice of $AdS_7$. The former D4 brane and the latter one are linked in

$^1$There are another arguments supporting this baryon picture that seven dimensional Chern-Simons term on $AdS_7$ produces a M2 brane with $N$ units of charge and, wrapping on the eleventh circle $S^1$, gives $N$ open strings joining together in the $AdS_7$ space [13] and, in [14], the baryon vertex was constructed in full M theory by the M5 brane wrapped on $S^4$ balanced by $N$ M theory membranes ending on it. Our picture corresponds to the compactification along the eleventh circle of theirs.
the $\mathbb{R}^5 \times S^1 \times S^4$, but they do not intersect each other. They have linking number $\pm 1$. The string stretching between linked D branes has eight mixed Dirichlet-Neumann boundary conditions. Then the string zero point energy is 0 in the Ramond sector as always, and the zero point energy in NS sector is positive, 1/2. Thus true ground state is a Ramond state, whose Fock space generates Clifford algebra. Thus the strings stretching between the boundary (or a D4 brane) and the D4 brane are certainly fermionic strings.

B. The $\mathbb{RP}^4$ Orientifold

It was observed \cite{17} that the large $N$ limit of $SO(2N)$ $(0,2)$ SCFT in 6 dimensions corresponds to the low energy theories of $N$ M5 branes coinciding at $\mathbb{R}^5/\mathbb{Z}_2$ orientifold singularity \cite{17}. Let us consider M theory on $\mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \times S^1$. The $\mathbb{Z}_2$ acts by sign change on all five coordinates in second $\mathbb{R}^5$ as follows: $(x^4, x^5, x^7, x^8, x^9) \to (-x^4, -x^5, -x^7, -x^8, -x^9)$. The angular directions in $\mathbb{R}^5/\mathbb{Z}_2$ are identified with $\mathbb{RP}^4$. There are $N$ parallel M5 branes which are sitting at an orientifold four-plane (O4 plane) which is located at $x^4 = x^5 = x^7 = x^8 = x^9 = 0$. For small radius of $S^1$, we can regard this as Type IIA orientifold on $\mathbb{R}^5/\mathbb{Z}_2$ which can be described in the context of $AdS_7 \times S^4$. The orientifolding operation replaces four sphere $S^4$ around the origin in $\mathbb{R}^5$ with $\mathbb{RP}^4 = S^4/\mathbb{Z}_2$. We will describe how $SO(2N)$ $(0,2)$ SCFT in 6 dimensions on a circle can be interpreted as M theory on $AdS_7 \times \mathbb{RP}^4$ where the eleventh dimensional circle is in $AdS_7$ space. This is our main goal in this paper.

Let us study the property of $AdS_7 \times \mathbb{RP}^4$ orientifold. Let $x$ be the generator of $H^1(\mathbb{RP}^4, \mathbb{Z}_2)$ which is isomorphic to $\mathbb{Z}_2$. \[\Sigma be a string worldsheet and $w_1(\Sigma) \in H^1(\Sigma, \mathbb{Z}_2)$ be the obstruction to its orientability. Then we only consider the map $\Phi : \Sigma \to AdS_7 \times \mathbb{RP}^4$ such that $\Phi^*(x) = w_1(\Sigma)$. Since $\mathbb{Z}_2$ action on $S^4$ is free (no orientifold fixed points), there is no open string sector. An unorientable closed string worldsheet $\Sigma = \mathbb{RP}^2$ can be identified with the quotient of the two sphere $S^2$ by the overall sign change. The map $\Phi : \mathbb{RP}^2 \to \mathbb{RP}^4$

\[\text{Notice that } H_p(\mathbb{RP}^n, \mathbb{Z}_2) = \mathbb{Z}_2, 0 \leq p \leq n \text{ and } H^p(\mathbb{RP}^n, \mathbb{Z}_2) = \mathbb{Z}_2, 0 \leq p \leq n.\]
satisfying the constraints $\Phi^*(x) = w_1(\mathbb{RP}^2)$ is the embedding $(x_1, x_2, x_3) \to (x_1, x_2, x_3, 0, 0)$.

In M theory, M2 brane and M5 brane are electrically and magnetically charged objects with respect to the three-form potential $C_3$. The Dirac quantization condition for these objects takes the form of flux quantization condition [18] for the four-form field strength $G_4 = dC_3$:

$$2 \int_D \frac{G_4}{2\pi} \equiv \int_D w_4 \mod 2,$$

where $D$ is a four-cycle, a copy of $\mathbb{RP}^4$, and $w_4$ is the fourth Stieffel-Whitney class of the eleven-dimensional space-time. This has a direct consequence in M theory on $\mathbb{R}^5/\mathbb{Z}_2$ orbifold, which implies that the $\mathbb{Z}_2$ fixed plane itself carries the M5 brane charge $-1$. Moreover, one cannot put odd number of M5 branes on the top of it.

It was shown [10] that there exist four kinds of models in Type IIB string theory classified by the discrete torsion of the two two-form fields, NS $B$ field $B_{NS}$ and RR $B$ field $B_{RR}$. The results are summarized as follows:

i) $SO(2N)$ gauge group : $(\theta_{NS}, \theta_{RR}) = (0, 0)$,

ii) $SO(2N + 1)$ gauge group : $(\theta_{NS}, \theta_{RR}) = (0, 1/2)$,

iii) $Sp(2N)$ gauge group : $(\theta_{NS}, \theta_{RR}) = (1/2, 0)$,

iv) $Sp(2N)$ gauge group : $(\theta_{NS}, \theta_{RR}) = (1/2, 1/2)$,  

where $(\theta_{NS}, \theta_{RR})$ are two types of discrete torsion for $B_{NS}$ and $B_{RR}$ respectively.

It is easy to see that the Type IIA brane configurations and its M theory realization considered in [19] can be obtained from the type IIB configurations by Witten [10] by taking T-duality along the $x^6$ direction. The M theory realization of the type IIA O4 planes comes from the two kinds of orientifold, $\mathbb{R}^5 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2$ and $\mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \times S^1$ where $\mathbb{Z}_2$ action on the eleventh circle $S^1$ is $x^{10} \to x^{10} + \pi$. In the first case, since the $\mathbb{Z}_2$ action is free, there is no orientifold fixed point which so generates no M5 brane charge, while in the second case, the orientifold has a singular fixed plane and so carries M5 brane charge $-1$ originated by the singularity. In particular, we should have a single D4 brane stuck on the $O4^-$ plane to
get \( SO(2N+1) \) gauge group. In order to obtain the O4 planes with 0 or +1 D4 brane charge, we should stick one or two M5 branes on the orientifold in the way consistent with the flux quantization condition (II.3). Then we can identify the consistent M theory realization of four types O4 planes obtained by Hori [19]:

\[
i) \quad SO(2N) \text{ with } O4^- : \quad \text{M theory on } \mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \times S^1 \\
ii) \quad SO(2N+1) \text{ with } O4^0 : \quad \text{M theory on } \mathbb{R}^5 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2 \\
iii) \quad Sp(2N) \text{ with } O4^+: \quad \text{M theory on } \mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \times S^1 \\
iv) \quad Sp(2N) \text{ with } \tilde{O}4^+: \quad \text{M theory on } \mathbb{R}^5 \times (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2 . \quad (\text{II.5})
\]

There exist a pair of M5 branes at the \( \mathbb{Z}_2 \) fixed plane for case \( iii \)) and a single M5 brane is stuck on the \( \mathbb{Z}_2 \) invariant cylinder for case \( iv \)).

We will demonstrate that the M theory realization of the four kinds of model in Type IIA string theory can be obtained from the topology of \( B_{NS} \) and the RR \( U(1) \) Wilson line, by using that the relation of the \( B_{NS} \) field and the three-form potential \( C_3 \) (See (II.6)) and by considering the complex line bundle associated with the RR one-form \( A_{RR} \) defined on the circle \( S^1 \). Since the \( SO(N)/Sp(N) \) gauge theories for large \( N \) can be distinguished by the sign of the string \( \mathbb{RP}^2 \) diagram, this can be classified by the topology of the \( B_{NS} \) field.

When we consider M theory on \( (\mathbb{R}^5 \times S^1)/\mathbb{Z}_2 \) or \( \mathbb{R}^5/\mathbb{Z}_2 \times S^1 \), the \( \mathbb{Z}_2 \) involution possibly induces a non-trivial holonomy (RR \( U(1) \) Wilson line) for the former case which corresponds to Type IIA string theory on \( \mathbb{R}^5/\mathbb{Z}_2 \), this can be classified by the topology of the \( A_{RR} \) field.

Since the \( \mathbb{Z}_2 \) action flips the sign of \( G_4 = dC_3 \) and preserves the orientation of the circle \( S^1 \), the \( \mathbb{Z}_2 \) action in ten dimensions flips the sign of the \( H_{NS} = dB_{NS} \) according to the relation

\[
\int_{S^1} \frac{G_4}{2\pi} = \frac{H_{NS}}{2\pi} . \quad (\text{II.6})
\]

This means that a cohomology class \( [H_{NS}] \) takes values in a twisted integer coefficient \( \tilde{\mathbb{Z}} \) where the twisting is determined by an orientation bundle. Since the RR \( U(1) \) gauge field \( A_{RR} \) represents a connection of the \( U(1) \) bundle over \( \mathbb{R}^5/\mathbb{Z}_2 \) where \( U(1) \) group is identified with the circle \( S^1 \), the holonomy of this complex line bundle can be measured by the field.
strength $F_{RR} = dA_{RR}$. Then the relevant cohomology groups measuring the topological types of the fields $B_{NS}$ and $A_{RR}$ are given by respectively

$$H^3(\mathbb{RP}^4, \tilde{\mathbb{Z}}) \approx \mathbb{Z}_2, \quad H^2(\mathbb{RP}^4, \mathbb{Z}) \approx \mathbb{Z}_2. \quad (\text{II.7})$$

If we denote the values of the cohomologies $H^3(\mathbb{RP}^4, \tilde{\mathbb{Z}})$ and $H^2(\mathbb{RP}^4, \mathbb{Z})$ as $(\alpha, \beta)$ respectively, we get the topological classification of the four models:

$O_4^- : (\alpha, \beta) = (0, 0), \quad O_4^0 : (\alpha, \beta) = (0, 1),
O_4^+ : (\alpha, \beta) = (1, 0), \quad \tilde{O}_4^+ : (\alpha, \beta) = (1, 1). \quad (\text{II.8})$

Then how can we obtain full uncompactified M theory on $AdS_7 \times \mathbb{RP}^4$ from Type IIA string theory, which means we take the limit of eleventh circle radius being infinite? There is a big difference between the case $i)$ and remaining $ii), iii) \text{ and } iv)$ in (II.5). The $O_4^-$ plane remains unchanged under the T-S-T transformation where T-duality is taken along the $x^9$ direction [20]. The $O_4^0$ and $\tilde{O}_4^+$ do not produce orientifold plane under T-S-T chain. The $O_4^+$ becomes $O_4^0$ with doubly wrapped D4 brane under the T-S-T transformation which implies under the Montonen-Olive duality they are exchanged each other. We have seen there are two classes of Type IIA string theory corresponding to our M theory configuration which has AdS/CFT correspondence: $i)$ and $iii)$. The difference between these is whether two M5 brane are stuck on the $\mathbb{Z}_2$ fixed plane or not. This M5 brane pair may be separated from the $\mathbb{Z}_2$ fixed plane without cost of energy, if the radius of circle becomes very large (uncompactified M theory). Then there will be no difference between $i)$ and $iii) \text{ as long as }$ the AdS/CFT correspondence is valid and thus the full uncompactified M theory has only one type of orientifold 5-plane as speculated in [20].

3 Note that there is an isomorphism pairing $p$-th homology group $H_p(M, R)$ and $(n - p)$-th cohomology group $H^{n-p}(M, R)$ for all $p$ and modulus $R$ iff $M$ is an orientable compact manifold, which is known as Poincaré duality: $H_p(M, \mathbb{Z}) \approx H^{n-p}(M, \mathbb{Z})$ and $H_p(M, \tilde{\mathbb{Z}}) \approx H^{n-p}(M, \tilde{\mathbb{Z}})$. If $H_n(M, \mathbb{Z}) = \mathbb{Z}$, the $n$-dimensional manifold $M$ is orientable. For an odd dimensional projective space $\mathbb{RP}^n$, the Poincaré duality works since it is orientable.
C. Various Wrapped Branes

Now we consider the possibilities of brane wrapping on $\mathbb{RP}^4$ in the Type IIA string theory. We first recall that fundamental strings, D4 branes and D6 branes in Type IIA string theory are M2 branes, M5 branes and KK monopoles wrapped around the circle $S^1$ in M theory respectively. But our dimension counting of branes in $AdS_7$ is in viewpoint of Type IIA string theory, we hope it does not cause any confusion.

The wrappings of string and NS5 brane are classified by the twisted homology $H_i(\mathbb{RP}^4, \tilde{\mathbb{Z}})$ for wrapped on an $i$-cycle in $\mathbb{RP}^4$ along the line of [10]:
(i) unwrapped string, giving a onebrane in $AdS_7$.

The wrapping modes that would give zero branes are not possible since $H_1(\mathbb{RP}^4, \tilde{\mathbb{Z}}) = 0$.
(ii) unwrapped NS5 brane remains a fivebrane in $AdS_7$,
(iii) wrapped on a two-cycle, to give a threebrane in $AdS_7$, classified by $H_2(\mathbb{RP}^4, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$,
(iv) wrapped on a four-cycle, to give a onebrane in $AdS_7$, classified by $H_4(\mathbb{RP}^4, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$.

The wrappings of D2 and D4 brane are also classified by the twisted homology $H_i(\mathbb{RP}^4, \tilde{\mathbb{Z}})$ for wrapped on an $i$-cycle in $\mathbb{RP}^4$ since the twobrane charge is odd under the orientifolding operation (it comes from the fact that, in M theory, $\mathbb{Z}_2$ action flips the sign of three-form $C_3$) and the fourbrane is dual to the twobrane:
(v) unwrapped D2 brane, giving a twobrane in $AdS_7$,
(vi) wrapped on a two-cycle, to give a zerobrane in $AdS_7$, classified by $H_2(\mathbb{RP}^4, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$,
(vii) unwrapped D4 brane remains a fourbrane in $AdS_7$,
(viii) wrapped on a two-cycle, to give a twobrane in $AdS_7$, classified by $H_2(\mathbb{RP}^4, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$,
(ix) wrapped on a four-cycle, to give a zerobrane in $AdS_7$, classified by $H_4(\mathbb{RP}^4, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$.

The $k$ units of KK momentum mode around the eleventh circle $S^1$ can be identified as $k$ D0 branes which is charged BPS particles. In the five dimensional gauge theory context, they appear as charge $k$ instantons on the D4 brane worldvolume [21]. We cannot consider wrapping modes on $\mathbb{RP}^4$ of D0 branes, since the eleventh circle $S^1$ is extended in $AdS_7$, not $\mathbb{RP}^4$. Similarly, we cannot consider a completely unwrapped sixbrane on $\mathbb{RP}^4$, since
we consider the D6 brane as the KK monopole in M theory which is magnetically charged under the gauge field $A_\mu = G_{\mu 10}$. The wrappings of D6 brane are classified by the ordinary (untwisted) homology $H_i(\mathbb{RP}^4, \mathbb{Z})$ for wrapped on an $i$-cycle in $\mathbb{RP}^4$ since the sixbrane is dual to the zerobrane:

$(x)$ wrapped on a one-cycle, to give a fivebrane in $AdS_7$, classified by $H_1(\mathbb{RP}^4, \mathbb{Z}) = \mathbb{Z}_2$,

$(xi)$ wrapped on a three-cycle, to give a threebrane in $AdS_7$, classified by $H_3(\mathbb{RP}^4, \mathbb{Z}) = \mathbb{Z}_2$.

According to the similar argument done in Type IIB description [10], one can derive a topological restriction on the brane wrappings on $\mathbb{RP}^4$ just described. In particular, the topological restriction coming from the holonomy of the connection $A_{RR}$ on the complex line bundle is trivial since the two cases, $O4^-$ and $O4^+$, we are interested in M theory context have the trivial RR $U(1)$ Wilson line. Nevertheless, we will show there are interesting possibilities that, in intersection of NS5 branes or D6 branes with the O4 plane, the type of the orientifold plane is changed. Therefore it is sufficient only to consider the discrete torsion $\theta_{NS}$ of the field $B_{NS}$. We will show that the description of brane wrappings on $\mathbb{RP}^4$ is consistent with the topological restriction coming from the RR discrete torsion $\theta_{RR}$, which should vanish in our case.

In the case $(iii)$, there is no restriction on wrapping of NS5 branes on $\mathbb{RP}^2 \subset \mathbb{RP}^4$, to make a threebrane in $AdS_7$, since $H^2(\mathbb{RP}^2, \mathbb{Z}) = \mathbb{Z}$. In the case $(iv)$, the NS5 brane can be wrapped on $\mathbb{RP}^4$, to make a string in $AdS_7$, only if $\theta_{RR} = 0$, since $H^2(\mathbb{RP}^4, \mathbb{Z}) = 0$. Similarly, in the case $(vi)$ and $(viii)$, there is no restriction on wrapping of D2 and D4 branes on $\mathbb{RP}^2 \subset \mathbb{RP}^4$, to make a particle and a twobrane in $AdS_7$ respectively, since $H^2(\mathbb{RP}^2, \mathbb{Z}) = \mathbb{Z}$. In the case $(ix)$, the D4 brane can be wrapped on $\mathbb{RP}^4$, to make a particle in $AdS_7$, only if $\theta_{NS} = 0$, since $H^2(\mathbb{RP}^4, \mathbb{Z}) = 0$. Similarly, in the case $(xii)$, the D6 brane can be wrapped on $\mathbb{RP}^3 \subset \mathbb{RP}^4$, to make a threebrane in $AdS_7$, only if $\theta_{NS} = 0$, since $H^2(\mathbb{RP}^3, \mathbb{Z}) = 0$. In the case $(x)$, there is no restriction on wrapping of D6 branes on $\mathbb{RP}^1$, to make a fivebrane in $AdS_7$, since in this case $\mathbb{RP}^2$ cannot be deformed in the D6 brane.

In the cases $(iii)$ and $(iv)$, the system of an O4 plane intersecting with a single NS5 brane is possible. We should remember that the NS5 brane has a unit magnetic charge under the
NS two-form field $B_{NS}$, but does not carry electric or magnetic charge for the RR one-form $A_{RR}$. Thus, in crossing such a fivebrane, the value of $\theta_{NS}$ jumps by one unit, this means that the type of the orientifold is changed as shown in [22]. The possible configurations are $O4^- - NS5 - O4^+$ and $O4^0 - NS5 - \tilde{O}4^+$ by charge conservation. The similar argument can be applied in the system of an O4 plane intersecting with a single D6 brane. This comes from the fact that the D6 brane is a magnetic monopole for the RR $U(1)$ gauge field $A_{RR}$. The case $(x\bar{i})$ exactly corresponds to the same configuration considered in [19], where it was shown that only $O4^- - D6 - O4^0$ and $O4^+ - D6 - \tilde{O}4^+$ configurations are the allowed patterns of dividing an O4 plane by a single D6 brane. This O4-D6 system can be understood by considering a Taub-NUT space realizing this configuration and $\mathbb{Z}_2$ orbifolding of M theory on this space. Since the case $(x)$ corresponds to the situation the O4 plane is completely embedded in the D6 worldvolume which fills out the full $AdS_7$ space, there is no difference of magnetic RR $U(1)$ charge around the O4 plane. Therefore, this case does not cause any change in the type of O4 planes.

III. GAUGE THEORY AND BRANES ON RP$^4$

A. The Pfaffian Particles

Let us consider particles coming from wrapped D2 brane on $\text{RP}^2 \subset \text{RP}^4$ [14] classified by $H_2(\text{RP}^4, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$. We are considering the case of $\nu i$ in section IIC. The number of particles is conserved modulo two and a pair of such particles can annihilate. Since a D2 brane can wrap on $\text{RP}^2$ without any topological restriction for $\theta_{NS}$, we should find a corresponding operator for each gauge group $SO(2k)$ and $Sp(k)$, where $k$ is the number of four-form flux quanta on $\text{RP}^4$:

$$\int_{\text{RP}^4} \frac{G_4}{2\pi} = k,$$

(III.1)

where it is understood that on the double cover $\mathbb{S}^4$, the number of quanta is $N = 2k$. We expect along the similar line suggested in [10] that the D2 brane wrapped on
\( \mathbb{RP}^2 \) should be identified with the Pfaffian particle of \( SO(2k) \) theory where \( \text{Pf}(\Phi) = \epsilon^{a_1 a_2 \cdots a_{2k}} \Phi_{a_1 a_2} \cdots \Phi_{a_{2k-1} a_{2k}} \) is an irreducible gauge invariant polynomials of order \( k \) and \( \Phi_{ab}, a, b = 1, 2, \cdots, 2k \) is an antisymmetric second rank tensor in the adjoint representation of \( SO(2k) \).

In order to identify this with Pfaffians, we will determine the quantum numbers of the low lying states on \( \text{AdS}_7 \) and SCFT sides under R symmetry group of the theory. The manifold \( \mathbb{RP}^4 \) has a symmetry group \( G_0 = SO(5)/\mathbb{Z}_2 \). A subspace \( \mathbb{RP}^2 \) is invariant under \( H_0 = SO(3) \times SO(2)/\mathbb{Z}_2 \). The space of this embedding is the homogeneous space \( G_0/H_0 \) that is the same as \( G/H \) where \( G = SO(5) \) and \( H = SO(3) \times SO(2) \). The quantum states are not ordinary functions on \( G/H \) but sections of a line bundle of degree \( k \). A section of a line bundle on \( G/H \) is a function on the \( G \) manifold. When we identify \( G = SO(5) \) as the group of \( 5 \times 5 \) orthogonal matrices \( g^i_j \) where \( i, j = 1, 2, \cdots, 5 \), one can expand functions on the \( G \) manifold as polynomials in the matrix elements of \( G \). The matrix element of \( g^i_j \) transforms as \( (5, 5) \) under \( SO(5) \times SO(5) \) and as \( (5, 3)_0 \oplus (5, 1)_1 \oplus (5, 1)^*_1 \) under \( SO(5) \times SO(3) \times SO(2) \). We denote \( SO(2) \) charge in the subscript. The polynomials of degree \( k \) in the \( (5, 1)_1 \) transform in the traceless symmetric product of \( k \) copies of \( 5 \). These are holomorphic sections of the line bundle. On the other hand, in the boundary SCFT, the chiral operators from the scalar fields can be constructed. The scalars are transforming in the \( 5 \) of the \( SO(5) \) global symmetry and in the adjoint representation of group \( SO(2k) \). The Pfaffian of the scalars, \( \text{Pf}(\Phi) = \epsilon^{a_1 a_2 \cdots a_{2k}} \Phi_{a_1 a_2} \cdots \Phi_{a_{2k-1} a_{2k}} \) where the \( i \)'s are \( SO(5) \) indices, transforms in the \( k \)-th symmetric product of the \( 5 \).

What is an appropriate operator of an \( Sp(k) \) gauge theory corresponding to the D2 brane wrapped on an \( \mathbb{RP}^2 \) subspace of \( \mathbb{RP}^4 \). This operator will be quite different from the Pfaffian operator in \( SO(2k) \) theory since there is no such operator in \( Sp(k) \) theory. We speculate that the candidate may be given by the holomorphic gauge invariant polynomial

\[ \Phi_{ab}, a, b = 1, 2, \cdots, 2k \]

\[ \text{Pf}(\Phi) = \epsilon^{a_1 a_2 \cdots a_{2k}} \Phi_{a_1 a_2} \cdots \Phi_{a_{2k-1} a_{2k}} \]

\[ \Phi_{ab}, a, b = 1, 2, \cdots, 2k \]

\[ \text{Pf}(\Phi) = \epsilon^{a_1 a_2 \cdots a_{2k}} \Phi_{a_1 a_2} \cdots \Phi_{a_{2k-1} a_{2k}} \] is an irreducible gauge invariant polynomials of order \( k \) and \( \Phi_{ab}, a, b = 1, 2, \cdots, 2k \) is an antisymmetric second rank tensor in the adjoint representation of \( SO(2k) \).
in an adjoint field $\Phi_{ab} = \Phi_{ba}$, $a, b = 1, \cdots, 2k$, that is, $U_k(\Phi) = \frac{1}{k} \text{Tr}\Phi^k$, where trace is taken with the invariant second rank antisymmetric tensor $\gamma^{ab}$. Note that the operator $U_k(\Phi)$ vanishes for odd $k$. The quantum numbers of the state can be determined by the same method as $SO(2k)$ gauge theory. But, we do not find any obvious reason for the modulo two conservation of the number of such quanta.

**B. Strings and Twobranes**

Let us consider “solitonic” strings in $AdS_7$ coming from wrapped NS5 brane on $\mathbb{RP}^4$ which is the case of $iv$). According to the topological restriction, $\theta_{RR} = 0$, the solitonic string is possible for both orthogonal and symplectic group and can induce the sign flip of an orientifold plane, intersecting with the plane. The solitonic strings can annihilate in pairs due to $H_4(\mathbb{RP}^4, \tilde{Z}) = \mathbb{Z}_2$. Let the NS5 brane worldvolume coordinates be $(x^0, x^4, x^5, x^7, x^8, x^9)$ directions and be specified by $x^1 = x^2 = x^3 = x^6 = 0$. Then the solitonic string worldsheet in $AdS_7$ is parametrized by the radial function $\rho$ of $\mathbb{RP}^4$ and $x^0$ and is at $x^1 = x^2 = x^3 = x^6 = 0$. The tension of this string is proportional to the NS5 brane tension, of order $1/\lambda^2$ and stretched to infinity in the radial direction of $AdS_7$. It is not obvious that the solitonic string can carry an external spinor charge as the fat string in $AdS_5$ [10] and it is not clear how to interpret this in the boundary SCFT.

Let us consider D4 brane whose worldvolume is specified by $x^1 = x^2 = x^3 = x^8 = x^9 = 0$, with arbitrary values of $x^0$ and of $x^4, x^5, x^6, x^7$. This is the case of $viii$). From the $AdS_7 \times \mathbb{RP}^4$ point of view such a D4 brane is wrapped on an $\mathbb{RP}^2 \subset \mathbb{RP}^4$ and looks like twobrane in $AdS_7$. The $\mathbb{RP}^2$ is the subspace of $\mathbb{RP}^4$ with $x^8 = x^9 = 0$. The twobrane worldvolume in $AdS_7$ is parametrized by radial function $\rho$ of $\mathbb{R}^5/\mathbb{Z}_2$, $x^6$ and $x^0$ and is at $x^1 = x^2 = x^3 = x^8 = x^9 = 0$. The 4-4 strings connecting the D4 brane in $AdS_7$ ($(x^0, x^1, x^2, x^3, x^6)$ directions) to D4 brane in $(x^0, x^4, x^5, x^6, x^7)$ directions are fermionic strings. Since D4 brane wrapped on $\mathbb{RP}^2 \subset \mathbb{RP}^4$

under an superpotential by an adjoint matter.
and D4 brane in $AdS_7$ meet at $x^1 = x^2 = \cdots = x^5 = x^7 = x^8 = x^9 = 0$, the ground state of the 4-4 string has zero energy. The ground states of these strings give $N$ fermionic zero modes in the spinor representation of $SO(N)$. The D4 brane regarded as a twobrane in $AdS_7$ has an end point at $\rho = 0$. This endpoint lies on the boundary of $AdS_7$ at which there are external spinor charges. At large distances, this twobrane may be used to define “Wilson surface” observable with external spinor charge of the low energy $(0,2)$ SCFT.

C. Domain Walls

Let us consider the objects in $AdS_7 \times S^4$ and $AdS_7 \times \mathbb{RP}^4$ that look like fivebranes in the seven noncompact dimensions of $AdS_7$. Since the $AdS_7$ has six spatial dimensions, the fivebrane could potentially behave as a domain wall, with the string theory vacuum “jumping” as one crosses the fivebrane. In $AdS_7 \times S^4$ and $AdS_7 \times \mathbb{RP}^4$, the only such object is the Type IIA NS5 or D4 branes, which come from the M5 brane. Contrary to Type IIB theory, we cannot obtain the domain wall made by wrapping a brane on $\mathbb{RP}^i \subset \mathbb{RP}^4$ as shown in section IIC.

Since the fivebrane is the magnetic source of the four-form field $G_4$ in M theory, the integrated four-form flux over $S^4$ or $\mathbb{RP}^4$ jumps by one unit when one crosses the fivebrane. This means that the gauge group of the boundary conformal field theory can change, for example, from $SU(N)$ on one side to $SU(N \pm 1)$ on the other side for $AdS_7 \times S^4$. While, for $AdS_7 \times \mathbb{RP}^4$, the change of the gauge group depends on which brane one considers. If one is crossing the D4 brane, it changes from $SO(N)$ to $SO(N \pm 2)$ or from $Sp(N/2)$ to $Sp(N/2 \pm 1)$ since the D4 brane charge changes by two units on double cover. While, if one is crossing the NS5 brane along the eleventh circle $S^1$, there is a jump in discrete torsion $\theta_{NS}$ measured by $H^3(\mathbb{RP}^4, \mathbb{Z}) = \mathbb{Z}_2$ since the fivebrane is a magnetic source of the $B_{NS}$-field and thus a transition between the $SO(N)$ and the $Sp(N)$ gauge group in boundary CFT.
D. The Baryon Vertex in $SO(N)/Sp(N)$

The baryon vertex in $SU(N)$ was obtained by wrapping a D4 brane over $S^4$. By analogy, one expects that the baryon vertex in $SO(N)$ or $Sp(N)$ will consist of a D4 brane wrapped on $RP^4$. If we are considering $SO(2k)$ gauge theory, there are $k$ units of four-form flux on $RP^4$ when the D4 brane wraps once on $RP^4$. But there is no gauge invariant combination, in $SO(2k)$ gauge theory, of $k$ external quarks to obtain a “baryon vertex”. The baryon vertex of $SO(2k)$ gauge theory should couple $2k$ external quarks, not $k$ of them.

Let $\Phi$ be the map of D4 brane worldvolume $X$ to $AdS_7 \times RP^4$. We must impose the condition that the $B_{NS}$ fields should be topologically trivial when pulled back to $X$ as implied in deriving the topological restrictions on the brane wrapping in section II. If one chooses the D4 brane topology as $RP^4$, the condition cannot be satisfied because of $H^3(RP^4, \tilde{Z}) = \mathbb{Z}_2$. Instead, if we choose the D4 brane topology as $S^4$, the topological triviality of the field $B_{NS}$ is obeyed since $H^3(S^4, \tilde{Z}) = 0$. Then the map $\Phi : S^4 \to RP^4$ gives the degree two map, in other words, $S^4$ wraps twice around $RP^4$. Thus we can obtain the correct baryon vertex coupling $2k$ quarks.

In $Sp(k)$ gauge theory, a baryon vertex can decay to $k$ mesons. Thus, one may expect no topological stability for the $AdS_7$ baryon vertex. In previous section, we showed that there is a topological restriction on the D4 brane wrapping on $RP^4$, which is only possible in $SO(N)$ gauge theory. But, we should consider the possibility on an existence of nontrivial torsion class of the $B_{NS}$ fields due to the topology of the D4 brane worldvolume $X$, which is denoted as $W \in H^3(X, \tilde{Z})$. Then this means that the correct global restriction is not that $i^*([H_{NS}]) = 0$ but rather that

$$i^*([H_{NS}]) = W,$$  \hspace{1cm} (III.2)

where $i$ is the inclusion of $X$ in spacetime and $[H_{NS}]$ the characteristic class of the $B_{NS}$ field we have seen in (II.6). A possible $W$ can be determined by using the “connecting homomorphism” in an exact sequence of cohomology groups from the second Stieffel-Whitney
class $w_2(X) \in H^2(X, \mathbb{Z}_2)$. If the baryon vertex decays via compact five-manifold $X$, we will find that $W = 0$ since $w_2(X) = 0$. Consequently, we cannot obtain the baryon vertex in the $O4^+$ type under consideration due to the global restriction $i^*([H_{NS}]) = 0$, which means that $\theta_{NS} = 0$.

In $SO(2k)$ gauge theory, super Yang-Mills theory with 16 supercharges actually has $O(2k)$ symmetry, not just $SO(2k)$. The generator $\tau$ of the quotient $O(2k)/SO(2k) = \mathbb{Z}_2$ behaves as a global symmetry. Since the baryon is odd under $\tau$, it cannot decay to mesons which is even under $\tau$. Since $H_4(\mathbb{RP}^4, \mathbb{Z}_2) = \mathbb{Z}_2$, two baryons can annihilate into $2k$ mesons. This also comes from the fact that, in $O(N)$, a product of two epsilon symbols can be rewritten as a sum of products of $N$ Kronecker deltas. Another possibility is that a baryon vertex constructed from a wrapped D4 brane is transformed to a state containing a wrapped D2 brane, a “Pfaffian” state in section IIIA, plus strings making pairwise connections between external charges since both are odd under $\tau$. We will show that the latter is not the case.

Consider a configuration of a D2 brane ending on a D4 brane. The end of the D2 brane on a D4 brane worldvolume is a magnetic source for the $U(1)$ gauge field $a$ that propagates on the D4 brane. Let $X$ be the D4 brane worldvolume, $E$ the worldvolume of a D2 brane whose boundary is on $X$, $D$ the boundary of $E$. Then, the Poincaré dual of $D$ is a class $[D] \in H^3(X, \mathbb{Z})$. Since $D$ acts as a magnetic source for $a$, the equation which restricts a wrapped D4 brane on $\mathbb{RP}^4$ modifies in the presence of a D2 brane as follows

$$i^*([H_{NS}]) = [D].$$  \hspace{1cm} (III.3)

Thus the configuration of the wrapped D4 brane with a D2 brane ending on it is only possible in the $Sp(k)$ gauge theory since in this case $\theta_{NS} \neq 0$.

Recall that we need $2k$ elementary strings ending on the wrapped D4 brane in order to cancel $2k$ units of the $U(1)$ charge on the D4 brane and even number of D4 branes, near to infinity, in order to provide external quarks with even flavors which are necessary for Witten anomaly \[24\] in 5 dimension, $\pi_5(Sp(k)) = \mathbb{Z}_2$. In $Sp(k)$ gauge theory, when we have taken only two flavors for simplicity, the baryon itself can decay to $k$ mesons.
\[ B = \frac{1}{k!} M_{rs}^k \]  

(III.4)

where \( r, s = 1, 2 \) are flavor indices and \( M_{rs} = \frac{1}{2} \gamma_{ab} \psi^a_r \psi^b_s \) is a meson formed by the invariant second rank antisymmetric tensor \( \gamma^{ab} \). Thus it is expected that the natural decay channel of the baryon vertex constructed from the above configuration will be a state containing \( k \) mesons where \( 2k \) charges on the boundary are connected pairwise by elementary strings and the wrapped D2 brane on \( \mathbb{RP}^2 \subset \mathbb{RP}^4 \):

\[ \frac{1}{k!} M_{rs}^k \oplus \frac{1}{k!} \text{Tr} \Phi^{i_1} \cdots \Phi^{i_k}, \]  

(III.5)

where \( i \)'s are indices of \( R \)-symmetry group.

**IV. DISCUSSION**

To summarize, for \( SU(N) (0, 2) \) theory, we interpreted the baryon vertex as a wrapped D4 brane in \( S^4 \) connected by \( N \) fundamental strings ending on a D4 brane on \( AdS_7 \) boundary. When we go \( SO(2N) (0, 2) \) theory, \( \mathbb{R}^5/\mathbb{Z}_2 \) orbifold singularity was crucial to understand M theory realization of four types of O4 plane. We constructed the possible brane wrappings on \( \mathbb{RP}^4 \) in Type IIA string theory and determined their topological restrictions in each case. According to this classification, it was possible to interpret various wrapping branes on \( \mathbb{RP}^4 \) in terms of particles, strings, twobranes (in \( AdS_7 \)), domain walls and the baryon vertex where the topological properties on \( \mathbb{RP}^4 \) are heavily used.

When we go further compactification of (0, 2) theory on a circle which would be interpreted as the compactified time parameter (in Euclidean space), so temperature, then the low energy theory will be four dimensional \( \mathcal{N} = 4 \) super Yang-Mills theory. If we instead insist on the boundary condition breaking the supersymmetry by taking the fermions to be antiperiodic in going around the circle, the low energy theory will be the pure Yang-Mills theory without supersymmetry. Witten proposed [25] that this \( QCD_4 \) theory has the dual description by the AdS Schwarzschild soution. According to the conjecture, the compactified six dimensional (0, 2) theory considered in this paper can be reduced to the \( QCD_4 \)
theory by introducing a supersymmetry breaking circle. Thus the various spectrums of the 5 dimensional super Yang-Mills theory should be related to 4 dimensional QCD spectrums. It will be then interesting to study $QCD_4$ along this line.

Recently, Aharony and Witten [26] showed that, if $SU(N) (0,2)$ theory is compactified on two torus $\mathbf{T}^2$ or any Riemann surface, the theory has a $\mathbb{Z}_N$ global symmetry and this provides a test of the AdS/CFT correspondence for finite $N$. It is also interesting to study whether our $SO(2N) (0,2)$ theory compactified on $\mathbf{S}^1 \times \mathbf{S}^1$ should have also such topological symmetry, in which case the $SO(2N)$ gauge group has a $\mathbb{Z}_2$ center.

It is natural to ask what happens for the other well known M theory on $\mathbb{R}^3 \times (\mathbb{R}^7 \times \mathbf{S}^1) / \mathbb{Z}_2$, where $\mathbf{S}^1$ is the eleventh dimensional circle. In this case, M2 branes are put at one of orbifold singularities, i.e. the origin of $\mathbb{R}^7$ and $\mathbf{S}^1$ coordinates. In Type IIA viewpoint, the above configuration will appear as D2 branes on the $\mathbb{R}^7 / \mathbb{Z}_2$ orientifold singularity, which produces no orientifold 2-plane when the size of $\mathbf{S}^1$ goes to infinity. Then we expect to have the 3 dimensional $SO(2N)$ maximally supersymmetric SYM theory, which flows in the IR to the $\mathcal{N} = 8$ SCFT corresponding to the M2 branes on the orbifold point $\mathbb{R}^8 / \mathbb{Z}_2$ [6]. Along the same strategy as the case $AdS_7 \times \mathbb{RP}^4$, one can also analyze various brane configurations in Type IIA string theory. It would be interesting to elaborate this further in the future.

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