The white dwarf luminosity function has proved to be an excellent tool for studying some properties of the Galactic disk, such as its age and the past history of the local star formation rate. The existence of an observational luminosity function for halo white dwarfs could provide valuable information about its age and the time that the star formation rate lasted, and it could also constrain the shape of the allowed initial mass functions. However, the main problem is the scarce number of white dwarfs already identified as halo stars. In this Letter, we show how an artificial intelligence algorithm can be used successfully to classify the population of spectroscopically identified white dwarfs, thus allowing us to identify several potential halo white dwarfs and to improve the significance of its luminosity function.

Subject headings: Galaxy: stellar content — stars: luminosity function, mass function — white dwarfs

1. INTRODUCTION

From theoretical (Mochkovitch et al. 1990; Tamanaha et al. 1990) and observational points of view, halo white dwarfs have received continuous interest for almost a decade. From this last observational point of view, Liebert, Dahn, & Monet (1989) studied a high proper-motion sample and derived from it the first—and up to now the only—halo white dwarf luminosity function. Later, Flynn, Gould, & Bahcall (1996) and Méndez et al. (1996) studied the white dwarf content of the Hubble Deep Field, after the suggestion from the MACHO team that most of the dark matter in the galactic halo could be in the form of white dwarfs (Alcock et al. 1997), and came up with negative results.

The observational white dwarf luminosity function of the halo was obtained from a sample of white dwarfs with known parallaxes, large proper motions ($2.5$ yr$^{-1}$ $\geq \mu \geq 0.8$ yr$^{-1}$), using a limiting magnitude of $m_V = 19$ mag, and assuming that only white dwarfs with tangential velocities in excess of 250 km s$^{-1}$ were halo members (Liebert et al. 1989). Consequently, only five white dwarfs contribute to the luminosity function, and thus the statistics is very poor. Besides, in the sample of Liebert et al. (1989), there are no bright halo white dwarfs. The absence of bright halo white dwarfs in this sample could be due to an observational bias, but this issue, which bears important consequences regarding the initial mass function (IMF) of halo stars, remains to be studied, since the biased IMFs recently proposed by Adams & Laughlin (1996) and by Chabrier, Ségretain, & Méra (1996) predict very few bright halo white dwarfs if the halo age is large enough (Isern et al. 1998).

In this Letter, we address the issue of whether or not there exist other halo white dwarfs in the existing catalogs and how to identify them. For this purpose, we use a neural network technique. In the end, this will allow us to present a preliminary luminosity function of halo white dwarfs and to compare it with the theoretical predictions.

2. METHOD AND RESULTS

With the advent of large astronomical databases, the need for efficient techniques to improve the automatic classification strategies has lead to a considerable amount of new development in the field. Among these techniques, the most promising ones are based on artificial intelligence algorithms. Neural networks have been used successfully in several fields such as pattern recognition, financial analysis, biology (see Kohonen 1990 for an excellent review), and astronomy. For instance, Bazell & Peng (1998) used these techniques to discriminate stars from galaxies automatically. Naim et al. (1995) used them to classify galaxies according to their morphology, Serra-Ricart et al. (1996) found the fraction of binaries in stars clusters, and Hernández-Pajares & Floris (1994) used such techniques to classify populations in the Hipparcos Input Catalogue.

The common characteristic of all the existing neural network classification techniques is the existence of a learning process very much in the same manner as human experts manually classify. Generally speaking, there are two different approaches: the supervised and the unsupervised learning methods. The main advantage of the unsupervised methods is that they require a minimum manipulation of the input data, and thus the results are supposedly more reliable. Their leading exponent is the so-called Kohonen self-organizing map (SOM). A thorough description of this technique is beyond the scope of this Letter. Therefore, we refer the reader to the specific literature (Kohonen 1997). However, for our purpose, it is convenient to summarize its basic principle and its properties. The basic principle is to map a multi-dimensional input space ($\mathbf{S}$) into a bi-dimensional space ($\mathbf{\Lambda}$). Similar objects in $\mathbf{S}$ (groups) are mapped in nodes in $\mathbf{\Lambda}$. The most noticeable property of this procedure is the reduction of the dimensionality of the input space, thus allowing, at the same time, the identification of groups in the input data and the automatic classification of individual objects. Besides, neighbor groups in $\mathbf{\Lambda}$ have similar properties.

The catalog of McCook & Sion (1987) is a compilation of the observational data of 1279 white dwarfs. In order to classify the stellar populations that are presumably present in this catalog, a set of variables describing their properties should be adopted. It should be noted that the larger the set of variables adopted, the smaller the number of objects that will have determinations for all the variables. Conversely, if the number of variables in the set is small, we could be disregarding valuable
information. We have adopted a minimal set in order to be able to analyze the largest possible number of objects in the catalog. The variables adopted in this study are the absolute visual magnitude $M_V$, the proper motion $\mu$, the galactic coordinates $(l, b)$, the parallax $\pi$, and a color index $B - V$. This reduces considerably the number of objects with all the determinations but allows a secure classification. We have found it very convenient to use the reduced proper motion, defined as $H = M_V - 5 \log \pi + 5 \log \mu$, instead of $\mu$ itself because the resulting groups are easier to visualize.

The statistical classification of an observational database usually ends up with the detection of groups in the input space that require an “a posteriori” analysis. Since we are interested in detecting different stellar populations simultaneously with the clustering process, we mix into the input data a synthetic population of tracer stars that will allow us to label the groups detected by the classification algorithm as a halo, disk, or intermediate population. The results of the classification procedure are not sensitive to the fine details of these synthetic populations, except for the IMF. These synthetic tracer stars have been produced using a Monte Carlo (MC) simulator. The description of the MC simulation of the disk population can be found in García-Berro & Torres (1997) or García-Berro et al. (1998). A comprehensive discussion of the results of the MC simulation of the halo population will be published elsewhere. However, for the sake of completeness, a brief summary of the inputs is given here. We have adopted a standard, Salpeter-like IMF (Salpeter 1961). The halo was supposed to be formed 14 Gyr ago in an intense burst of star formation of 1 Gyr of duration. The stars are randomly distributed in a sphere of radius 200 pc centered in the Sun according to a density profile given by the expression $\rho(r) \propto (a^2 + R_\odot^2) l(a^2 + r^2)$, where $r$ is the Galactocentric radius, $a \approx 5$ kpc, and $R_\odot = 8.5$ kpc. The velocities of the tracer stars where randomly drawn according to normal distributions for both the radial and the tangential components, with velocity dispersions as given in Marković & Sommer-Larsen (1997); the adopted rotation velocity $V_c$ is 220 km s$^{-1}$. The remaining inputs were the same as those adopted in García-Berro et al. (1998). In order to reproduce accurately the properties of the real catalog, both MC simulations were required to meet an additional set of criteria: $\delta \geq 0.1$, $8.5 \leq M_v \leq 16.5$, $\mu \leq 4.1$ yr$^{-1}$, and $0.006 \leq \pi \leq 0.376$, which are derived from the subset of 232 white dwarfs that have all the determinations. An added value of the above-described procedure of mixing tracer and real stars is that in this way, we can check the accuracy of the classification algorithm and the quality of the MC simulations.

The simulated samples mimic fairly well the observational sample, as can be seen in Figure 1, where the results of the MC simulations for the disk and the halo are compared with the observational sample in the reduced proper-motion color diagram. As can be seen in this diagram, the two simulated samples are visualized easily. Similar diagrams can be produced for each pair of variables, and the results of the MC simulations compare equally well with the real data.

We have run the public domain neural network software

![Fig. 1.—Reduced proper-motion color diagram for the MC simulations of the disk (filled circles) and the halo (solid squares) and of the observational subsample (open triangles).](image)
Groups labeled as halo can be found for each of the halo groups in the sample of Liebert et al. (1989), which is classified in the group $(0, 2)$, an intermediate population. All this evidence points in the same direction: the faintest halo candidate found by Liebert et al. (1988) is located in the group $(0, 0)$; all these objects were already identified as halo members by Liebert et al. (1989) and were used to build their halo white dwarf luminosity function. The only object of the classification scheme can be obtained by checking how many of the synthetic stars are misclassified. This results in the following confusion matrix:

$$
C = \begin{pmatrix}
0.98 & 0.03 \\
0.02 & 0.97
\end{pmatrix},
$$

where the matrix element $C_{11}$ indicates the percentage of disk tracers classified in disk groups, $C_{12}$ is the percentage of disk tracers misclassified in halo groups, and so on. This matrix is very close to unity, and thus the classification seems to be secure. More confidence in this classification comes from the fact that the vast majority of old-disk white dwarfs in the sample of Liebert, Dahm, & Monet (1988) are in the groups $(0, 2)$ and $(2, 1)$, which are labeled as part of the intermediate population. Moreover, LHS 56, LHS 147, and LHS 291 belong to the group $(1, 0)$, which clearly is a halo group, and LHS 2984 belongs to the group $(0, 0)$; all these objects were already identified as halo members by Liebert et al. (1989) and were used to build their halo white dwarf luminosity function. The only object of the sample of Liebert et al. (1989) that is misclassified is LHS 282, which is classified in the group $(0, 2)$, an intermediate population. All this evidence points in the same direction: the classification is correct. The percentages of halo tracers in the groups labeled as halo can be found for each of the halo groups in Figure 2. Since all of them are larger than 80%, all of these groups can be securely labeled as halo. However, for the sake of reliability, we have identified as halo candidates only those white dwarfs belonging to groups that do not have a disk neighbor, namely, $(0, 0), (0, 1), (1, 0)$, and $(2, 0)$. One interesting property of these white dwarfs is that all of them have $M_V \leq 14$, and only four have proper motions in excess of $170$ yr$^{-1}$, being the average $(\mu_\alpha \sin b) = 0.37$ yr$^{-1}$. However, most of them have $\pi \leq 0.03$ and are clustered around $\pi \sim 0.01$, leading to tangential velocities in excess of $200$ km s$^{-1}$ for 11 of our candidates. Only one candidate has a tangential velocity smaller than $100$ km s$^{-1}$. Therefore, the detected population is intrinsically bright and distant. The halo white dwarf candidates detected here can be found in Table 1.

We have used the $1/V_{max}$ method to derive a luminosity function of halo white dwarfs with the candidates found so far. The adopted criteria for deriving such a luminosity function were those of Oswalt et al. (1996). The result is shown in Figure 3 (filled triangles). The error bars have been computed as in Liebert et al. (1988). The number of objects in each luminosity bin is shown on top of the corresponding error bar. The value of $<V/V_{max}> = 0.115$ indicates that the sample is not complete. Thus, this halo white dwarf luminosity function should be considered as preliminary, although it represents a considerable improvement over that of Liebert et al. (1989). According to the previous discussion, the most obvious feature of this luminosity function is the detection of halo candidates for the brightest luminosity bins. The faintest halo candidate found by Liebert et al. (1989), LHS 282, has been classified as part of an intermediate population, and therefore the faintest bin in their luminosity function is absent in ours. Some other small differences for the faintest bins are due to the binning procedure and to the bolometric magnitudes assigned to individual objects.

The SOM Program Package is available at http://www.cis.hut.fi/nnrc/som_pak/, which is prepared by the SOM Programming Team of the Laboratory of Computer and Information Sciences, Helsinki University of Technology, Finland.

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**Fig. 2.**—Self-organizing map of the sample of white dwarfs (see text for details). The group $(0, 0)$ is located in the lower left-hand corner of the diagram, and the group $(4, 4)$ is located in the upper right-hand corner. As a rule of thumb, in the diagram, $H$ increases from right to left, and $M_V$ decreases downward.

**Fig. 3.**—White dwarf luminosity function obtained using the $1/V_{max}$ method with the halo white dwarf candidates found in this work (solid triangles) and the detection limit of Liebert et al. (1988; open triangles). The solid and the dotted lines are theoretical white dwarf luminosity functions obtained assuming a standard IMF and thick H- and He-dominated envelopes, respectively.
jects—we have used the bolometric corrections of Bergeron, Wesemael, & Beuchamp (1995), whereas Liebert et al. (1989) used blackbody bolometric corrections.

For comparison purposes, in Figure 3, we also show two theoretical luminosity functions computed with a standard IMF. The adopted ages were in both cases 14 Gyr, and the durations of the bursts were 1 Gyr. The solid line corresponds to a luminosity function computed with a He-dominated envelope (Wood 1995), whereas the dotted line shows the luminosity function obtained using an He-dominated envelope (Wood & Winget 1989). The method used to compute both luminosity functions is that of Isern et al. (1998), where the rest of the details of the adopted inputs can be found. The detection limit of the very faint white dwarfs (open triangles) of Liebert et al. (1988) is also shown in Figure 3. Both luminosity functions have been normalized to the bin with the smallest error bars, and they present a reasonable agreement with both sets of observational data.

3. CONCLUSIONS

We have shown that an artificial intelligence algorithm is able to classify the catalog of spectroscopically identified white dwarfs and ultimately to detect several potential halo white dwarfs. Some of these white dwarfs were already proposed as halo objects by Liebert et al. (1989). We have found as well that our halo candidates are bright and distant and that most of them have large tangential velocities. Using the $1/V_{\text{max}}$ method, we have computed a preliminary luminosity function of halo white dwarfs. We have found a value of $(V/V_{\text{max}}) = 0.115$, which indicates that our luminosity function is still incomplete. However, this luminosity function is a big improvement over the previous one by Liebert et al. (1989). We have also compared this luminosity function with the theoretical predictions, and we have found a fair agreement with the luminosity functions computed with a standard IMF.

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REFERENCES

Adams, F., & Laughlin, G. 1996, ApJ, 468, 586
Alcock, C., et al. 1997, ApJ, 468, 697
Bazell, D., & Peng, Y. 1998, ApJS, 116, 47
Bergeron, P., Wesemael, F., & Beuchamp, A. 1995, PASP, 107, 1047
Chabrier, G., Ségretain, L., & Méra, D. 1996, ApJ, 468, L21
Flynn, C., Gould, A., & Babcock, J. N. 1996, ApJ, 466, L55
García-Berro, E., & Torres, S. 1997, in White Dwarfs, ed. Isern, M. Hernanz, & E. García-Berro (Dordrecht: Kluwer), 97
García-Berro, E., Torres, S., Isern, J., & Burkert, A. 1998, MNRAS, in press
Hernández-Pajares, M., & Floris, J. 1994, MNRAS, 268, 444
Isern, J., García-Berro, E., Hernanz, M., Mochkovitch, R., & Torres, S. 1998, ApJ, 503, 239
Kohonen, T. 1990, Proc. IEEE, 78(9), 1464
———, 1997, Self-organizing Maps, Springer Ser. Inf. Sci., Vol. 30 (Berlin: Springer)
Liebert, J., Dahn, C. C., & Monet, D. G. 1988, ApJ, 332, 891
———, 1989, in White Dwarfs, ed. G. Wegner (Berlin: Springer), 15
Marković, D., & Sommer-Larsen, J. 1997, MNRAS, 288, 733
McCook, G. P., & Sion, E. M. 1987, ApJS, 65, 603
Méndez, R. A., Minniti, D., De Marchi, G., Baker, A., & Couch, W. J. 1996, MNRAS, 283, 666
Mochkovitch, R., García-Berro, E., Hernanz, M., Isern, J., & Panis, J. F. 1990, A&A, 233, 456
Naim, A., Lahav, O., Sodre, L., & Storrie-Lombardi, M. C. 1995, MNRAS, 275, 567
Osvalt, T. D., Smith, J. A., Wood, M. A., & Hintzen, P. 1996, Nature, 382, 692
Salpeter, E. E. 1961, ApJ, 134, 669
Serra-Ricart, M., Aparicio, A., Garrido, L., & Gaitan, V. 1996, ApJ, 462, 221
Tamanaha, C. M., Silk, J., Wood, M. A., & Winget, D. E. 1990, ApJ, 358, 164
Wood, M. A. 1995, in White Dwarfs, ed. D. Koester & K. Werner (Berlin: Springer), 41
Wood, M. A., & Winget, D. E. 1989, in White Dwarfs, ed. G. Wegner (Berlin: Springer), 282

TABLE 1

HALO WHITE DWARF CANDIDATES

| Name       | Group | $M_\text{p}$ (arcsec yr$^{-1}$) | $\pi$ (arcsec) | $B - V$ | Spectral Type |
|------------|-------|---------------------------------|---------------|---------|---------------|
| LHS 2984*  | (0, 0) | 11.62                           | 0.930         | 0.015   | 0.03          | DA            |
| LHS 3007   | (0, 0) | 13.06                           | 0.636         | 0.028   | 0.29          | DA            |
| G028-027   | (0, 0) | 12.41                           | 0.281         | 0.003   | 0.03          | DQ            |
| G098-018   | (0, 0) | 11.81                           | 0.426         | 0.003   | 0.38          | DA            |
| G138-056   | (0, 0) | 13.34                           | 0.692         | 0.006   | 0.37          | DA            |
| G184-012   | (0, 0) | 13.18                           | 0.427         | 0.017   | 0.26          | DC            |
| LP 640-069 | (0, 0) | 12.75                           | 0.284         | 0.009   | 0.29          | DA            |
| LHS 56*    | (0, 1) | 13.51                           | 3.599         | 0.069   | 0.36          | DA            |
| LHS 147*   | (0, 1) | 13.64                           | 2.474         | 0.016   | 0.40          | DC            |
| LHS 151    | (0, 1) | 13.46                           | 1.142         | 0.053   | 0.33          | DA            |
| LHS 291*   | (0, 1) | 13.39                           | 1.765         | 0.012   | 0.11          | DQ            |
| LHS 529    | (0, 1) | 13.94                           | 1.281         | 0.046   | 0.64          | DA            |
| LHS 1927   | (1, 0) | 11.41                           | 0.661         | 0.009   | 0.11          | DA            |
| G038-004   | (1, 0) | 12.31                           | 0.428         | 0.010   | 0.17          | DA            |
| LHS 3146   | (2, 0) | 11.88                           | 0.579         | 0.024   | 0.17          | DA            |
| G021-015   | (2, 0) | 11.54                           | 0.390         | 0.015   | 0.05          | DA            |
| G035-026   | (2, 0) | 11.12                           | 0.335         | 0.007   | 0.14          | DA            |
| G128-072   | (2, 0) | 12.53                           | 0.457         | 0.025   | 0.21          | DA            |
| G271-106   | (2, 0) | 11.77                           | 0.396         | 0.014   | 0.18          | DA            |
| GR 363     | (2, 0) | 11.39                           | 0.133         | 0.003   | 0.03          | DA            |

* Identified using the neural network algorithm, along with their corresponding groups and properties. The stars already identified in Liebert et al. 1989 are marked with an asterisk.