Analysis of Beams and Frames using Applied Element Method

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Abstract. Applied Element Method (AEM) is an efficient tool for analysing structures numerically. It has a few benefits when conventional methods are concerned. In conventional numerical methods, node-to-node connection is important. Hence, when elements of different sizes are to be connected, transition elements will have to be used. In AEM, rigid elements are connected to each other with the help of springs. Therefore, node-to-node connection is not required. In this paper, some plane stress problems are analysed using AEM. For this, two-dimensional (2D) element is made use of. Continuous beam and 2D frame are analysed using AEM. The results show that AEM is able to analyse beams and frames accurately. Modal analysis of beam clamped on one end and hinged at the other end, gave accurate first natural frequency and mode shape. It was also possible to perform nonlinear analysis of cantilever beam accurately.

1. Introduction
Several numerical methods for the analysis of structures are readily available today. Applied Element Method (AEM) is one of them. AEM is a rigid body method in which springs are introduced between rigid elements to transfer forces. Kimiro Meguro and Hatem Tagel-Din [1-5] are behind the development of the method. Thereafter, the method was used for the analysis of different materials and structures [6-7]. Collapse behaviour of structures also can be predicted using AEM.

AEM is used for the analysis of continuous beam and two-dimensional (2D) frame in this paper. The results were compared with that obtained by the analysis using STAAD. Modal analysis of a beam clamped on one end and hinged at the other end is included in this paper. Nonlinear analysis of cantilever beam with tip load carried out by both force-based and displacement-based approaches is also included.

2. Applied Element Method
Conventionally, AEM has only two elements – 2D element and three-dimensional (3D) element. There are two translational degrees of freedom (DOFs) and one rotational DOF in a 2D element, as shown in figure 1. Two springs – one for transferring normal force and the other to carry shear force make a pair of spring for two-dimensional analysis. In the present study, 2D element was used for the analysis. The connection between the elements in AEM are shown in figure 1. The DOFs are marked at the centre of the elements.

Figure 1. Element connection in 2D AEM.
Every pair of springs has a stiffness matrix, which depends on its position and stiffness as given by equation (1) [8].

$$K = \begin{pmatrix} k_n & 0 & 0 & 0 \\ 0 & k_s & k_{sy} & 0 \\ -k_{sy} & k_{sx} & k_{sy}^2 + k_{sx}^2 & k_{sy} \\ 0 & -k_s & k_{sy} & k_{sy}^2 + k_{sx}^2 \\ k_{sy} & k_{sx} & -k_{sy}^2 + k_{sx}^2 & k_{sy} \\ k_{sy} & k_{sx} & k_{sy} & k_{sy}^2 + k_{sx}^2 \end{pmatrix}$$

(1)

where, $l_s$, $d_s$ and $b_s$ are the length, depth and width of the portion represented by a spring; $(x, y)$ is the coordinate of the location of the spring with respect to the centre of the first element; $E$ and $G$ are the Young’s modulus and modulus of rigidity respectively.

3. Analysis of continuous beam
A two-span continuous beam shown in figure 2 was considered for analysis. The cross-section of both the members were 300 mm × 500 mm. The width of element was 50 mm. 100 springs were used along all the faces. The Young’s modulus and Poisson’s ratio were 25000 N/mm$^2$ and 0.2, respectively.

![Figure 2. Continuous beam considered for analysis.](image)

The stiffness matrix of all the members were determined. The stiffness matrix and force vector were condensed to retain the end DOFs. The stiffness matrix and force vector of both the members were assembled by considering the relevant DOFs alone. Thus, static condensation approach was made use of to carry out the analysis. Figure 3 shows the discretisation of the beam.

![Figure 3. Discretisation of continuous beam.](image)

The deflected shape and reactions obtained using AEM and by the analysis using STAAD are shown in figures 4 and 5, respectively.

![Figure 4. Deflected shape of continuous beam.](image)

![Figure 5. Support reactions by AEM and STAAD (boldface letters indicate results from STAAD).](image)
The percentage difference in the reactions obtained are presented in table 1.

**Table 1. Reactions in continuous beam.**

| Reaction | AEM     | STAAD  | Percentage difference (%) |
|----------|---------|--------|---------------------------|
| $R_A$    | 18.944  | 18.616 | 1.76                      |
| $M_A$    | 20.211  | 20.247 | -0.18                     |
| $R_B$    | 35.304  | 35.675 | -1.04                     |
| $R_C$    | 15.752  | 15.709 | 0.27                      |

* Forces are in kN and moment is in kNm

Figures 4 and 5 show that AEM is able to predict the deformation and reaction of continuous beam accurately. From table 1, it is seen that AEM predicts the reaction with less than 2% difference.

4. **Analysis of 2D frame**

The 2D frame shown in figure 6 was considered for the analysis. The cross-section of all the members were 200 mm $\times$ 400 mm. The width of the element, number of pairs of springs and material properties were same as that of section 3.

![Figure 6. 2D frame.](image)

The discretisation of the frame is shown in figure 7.

![Figure 7. Discretisation of frame.](image)

![Figure 8. Support reactions by AEM and STAAD (boldface letters indicate results from STAAD).](image)
The support reactions and deflected shape of the frame obtained by applied element analysis are shown in figures 8 and 9, respectively. The deflected shape obtained by the analysis using STAAD is also shown in figure 9.

**Figure 9.** Deflected shape by AEM and STAAD (*The displacement is magnified by a factor of 100).

Figures 8 and 9 show that AEM predicts reactions and deformations with reasonable accuracy. The displacement vector and reactions by AEM and by the analysis using STAAD are given in table 2 and 3.

| DOF | AEM  | STAAD | Percentage difference (%) |
|-----|------|-------|--------------------------|
| $u_A$ | 0    | 0     | 0                        |
| $v_A$ | 0    | 0     | 0                        |
| $\theta_A$ | 0  | 0     | 0                        |
| $u_B$ | 0    | 0     | 0                        |
| $v_B$ | 0    | 0     | 0                        |
| $\theta_B$ | -0.002  | -0.002 | 0                        |
| $u_C$ | 6.1054 | 6.2268 | -1.95                    |
| $v_C$ | 0    | 0     | 0                        |
| $\theta_C$ | 0.001  | 0.001 | 0                        |
| $u_D$ | 3.6208 | 3.6854 | -1.75                    |
| $v_D$ | 0.0070 | 0.0076 | -7.89                    |
| $\theta_D$ | -0.001  | -0.001 | 0                        |
| $u_E$ | 3.6091 | 3.6734 | -1.75                    |
| $v_E$ | -0.0644 | -0.0645 | -0.16                   |
| $\theta_E$ | -0.001  | -0.001 | 0                        |
| $u_F$ | 3.6091 | 3.6734 | -1.75                    |
| $v_F$ | -0.0101 | -0.0105 | -3.81                   |
| $\theta_F$ | 0.001  | 0.001 | 0                        |

* The translations are in ‘mm’ and rotations are in ‘rad’. 3 significant digits are considered to find the percentage difference

| Reaction | AEM  | STAAD | Percentage difference (%) |
|----------|------|-------|--------------------------|
| $H_A$    | -24.111 | -23.965 | 0.61                     |
| $V_A$    | 4.652  | 4.998  | -6.92                    |
| $M_A$    | 45.065 | 45.202 | -0.30                    |
| $H_B$    | -5.889 | -6.035 | -2.42                    |
| $V_B$    | 42.937 | 43.037 | -0.23                    |
| $V_C$    | 6.715  | 6.961  | -3.53                    |

* Forces are in kN and moment is in kNm

Table 2 and 3 shows that AEM is capable of analysing 2D frames accurately.

5. Modal analysis of beam
A 2 m long beam with one end clamped and the other end hinged as shown in figure 10 was considered for the modal analysis. The cross-section of the beam was 200 mm × 200 mm. The beam was divided
into 100 elements and they were connected by 100 springs at each face. The density, Young’s modulus and Poisson’s ratio of the material of the beam were 7850 kg/m$^3$, 25000 N/mm$^2$ and 0.2 respectively.

**Figure 10.** Beam considered for modal analysis.

The first natural frequency ($f_1$) of a beam considered is given by equation (2) [9].

$$f_1 = \frac{\lambda_1}{2\pi} \sqrt{\frac{EI}{m}}$$  

(2)

Where, $\lambda_1$, $l$, $E$, $m$ and $I$ are the frequency factor, length of the beam, Young’s modulus, moment of inertia and mass per unit length respectively.

Frequency factor for first mode was 15.418 [9].

The first natural frequency of the beam obtained by AEM and analytically were 61.52 Hz and 63.21 Hz respectively, which are close enough.

The shape of first mode of vibration of the beam is shown in figure 11.

**Figure 11.** First mode shape of the beam.

Figure 11 shows that AEM can predict the mode shape of the beam with reasonable accuracy.

6. **Nonlinear analysis**

A nonlinear analysis done by Moideen and Dewangan [10] using ANSYS was considered for the study. The problem taken was a steel cantilever beam of span 0.5 m and cross-section 70 mm × 70 mm with point load of 60 kN at free end as shown in figure 12. The initial modulus of elasticity, Poisson’s ratio and yield strength were $2.1 \times 10^5$ N/mm$^2$, 0.3 and 250 N/mm$^2$ respectively.

**Figure 12.** Cantilever beam for nonlinear analysis.

The bilinear stress-strain graphs as shown in figures 13(a) and (b) were adopted. These are regarded as case 1 and case 2 respectively.

**Figure 13.** Stress-strain graph for (a) case 1 (b) case 2.
The nonlinear analysis was carried out using AEM by two approaches – force based and displacement based. An increment of 500 N and 0.05 mm were adopted for the force based and displacement based approaches, respectively. The beam was discretised into 100 elements along length and 10 elements along depth. 10 number of springs were provided at all element faces.

**Table 4.** Results of force based incremental method for case 1.

| Load (kN) | Deflection (mm) | Percentage difference (%) | Maximum stress (N/mm$^2$) | Percentage difference (%) |
|-----------|----------------|---------------------------|---------------------------|---------------------------|
|           | AEM  | ANSYS |                  | AEM  | ANSYS |                  |                |
| 1         | 0.10 | 0.10  | 0.88              | 9.08 | 8.31  | -8.48            |
| 5         | 0.50 | 0.50  | 0.88              | 45.40 | 41.55 | -8.48            |
| 10        | 0.99 | 1.01  | 1.89              | 90.80 | 83.10 | -8.48            |
| 20        | 1.98 | 2.02  | 1.89              | 181.60 | 166.18 | -8.49     |
| 30        | 2.98 | 3.03  | 1.83              | 250.48 | 249.27 | -0.48   |
| 40        | 4.29 | 4.60  | 7.13              | 255.67 | 332.36 | 29.99        |
| 50        | 12.46 | 15.52 | 24.57             | 299.41 | 415.45 | 38.75        |
| 60        | 42.36 | 47.32 | 11.70             | 384.15 | 498.54 | 29.78        |

**Table 5.** Results of force based incremental method for case 2.

| Load (kN) | Deflection (mm) | Percentage difference (%) | Maximum stress (N/mm$^2$) | Percentage difference (%) |
|-----------|----------------|---------------------------|---------------------------|---------------------------|
|           | AEM  | ANSYS |                  | AEM  | ANSYS |                  |                |
| 1         | 0.10 | 0.10  | 0.88              | 9.08 | 8.31  | -8.48            |
| 5         | 0.50 | 0.50  | 0.88              | 45.40 | 41.55 | -8.48            |
| 10        | 0.99 | 1.01  | 1.89              | 90.80 | 83.10 | -8.48            |
| 20        | 1.98 | 2.02  | 1.89              | 181.60 | 166.18 | -8.49     |
| 30        | 2.97 | 3.03  | 1.86              | 258.88 | 249.27 | -3.71     |
| 40        | 4.09 | 4.12  | 0.73              | 306.86 | 297.38 | -3.09     |
| 50        | 5.62 | 5.89  | 4.80              | 373.46 | 318.48 | -14.72   |
| 60        | 7.59 | 8.06  | 6.17              | 452.47 | 376.27 | -16.84   |

**Table 6.** Results of displacement based incremental method for case 1.

| Deflection (mm) | Load (kN) | Percentage difference (%) | Maximum stress (N/mm$^2$) | Percentage difference (%) |
|----------------|-----------|---------------------------|---------------------------|---------------------------|
|                | AEM  | ANSYS |                  | AEM  | ANSYS |                  |                |
| 0.10           | 1.01  | 1.00  | -0.82             | 9.16  | 8.31  | -9.23            |
| 0.50           | 5.04  | 5.00  | -0.82             | 45.78 | 41.55 | -9.23            |
| 1.00           | 10.08 | 10.00 | -0.82             | 91.55 | 83.10 | -9.23            |
| 2.00           | 20.17 | 20.00 | -0.82             | 183.11 | 166.18 | -9.24     |
| 3.05           | 30.72 | 30.00 | -2.34             | 250.69 | 249.27 | -0.57     |
| 4.60           | 41.28 | 40.00 | -3.10             | 257.15 | 332.36 | 29.25     |
| 15.50          | 50.84 | 50.00 | -1.64             | 310.94 | 415.45 | 33.61     |
| 47.30          | 60.58 | 60.00 | -0.95             | 393.80 | 498.54 | 26.60     |
The deflection under the load and the maximum stress in beam at various loads, using force based approach, for case 1 and case 2 are given in table 4 and 5 respectively. The load and corresponding maximum stress in the beam at various deflections at free end for Case-1 and Case-2 are given in table 6 and 7 respectively. The analysis has been carried out by displacement-based approach.

| Deflection (mm) | Load (kN) | Percentage difference (%) | Maximum stress (N/mm²) | Percentage difference (%) |
|-----------------|-----------|----------------------------|-------------------------|---------------------------|
|                 | AEM      | ANSYS                      | AEM                     | ANSYS                     |
| 0.10            | 1.01     | 1.00                       | -0.82                   | 9.16                      | 8.31                      | -9.23                     |
| 0.50            | 5.04     | 5.00                       | -0.82                   | 45.78                     | 41.55                     | -9.23                     |
| 1.00            | 10.08    | 10.00                      | -0.82                   | 91.55                     | 83.10                     | -9.23                     |
| 2.00            | 20.17    | 20.00                      | -0.82                   | 183.11                    | 166.18                    | -9.24                     |
| 3.05            | 30.73    | 30.00                      | -2.39                   | 261.98                    | 249.27                    | -4.85                     |
| 4.10            | 40.06    | 40.00                      | -0.15                   | 307.40                    | 297.38                    | -3.26                     |
| 5.90            | 51.51    | 50.00                      | -2.93                   | 385.06                    | 318.48                    | -17.29                    |
| 8.05            | 62.03    | 60.00                      | -3.28                   | 469.69                    | 376.27                    | -19.89                    |

Ta show that the load-deflection values obtained by three approaches – load based incremental method, displacement based incremental method and ANSYS; are similar. Although the maximum stress value obtained from both incremental approaches were similar, they differed from ANSYS results at higher loads.

The load-deflection curve for case 1 and case 2 are shown in figures 14 and 15, respectively.

Figures 14 and 15 show that the load-deflection curve obtained by displacement based incremental method and force based incremental method approximately matched the load-deflection curve obtained from ANSYS.

7. Conclusions
In this paper, a continuous beam and a 2D frame are analysed using AEM. Nonlinear analysis considering material nonlinearity is also done. From the results, it is seen that AEM could determine the deformed shape and reactions of continuous beam and 2D frame accurately. Also, the natural frequency and mode shape of beam are also predicted with reasonable accuracy. The load-deflection curve of cantilever beam with tip load obtained from AEM agrees well with the results of ANSYS.
8. References

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