Bragg diffraction of microcavity polaritons by a surface acoustic wave

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Bragg scattering of polaritons by a coherent acoustic wave is mediated and strongly enhanced by the relevant exciton states resonant with the acoustic and optic fields simultaneously. In this case, in sharp contrast with conventional acousto-optics, the resonantly enhanced Bragg spectra reveal the multiple orders of diffracted light, i.e., a Brillouin band structure of parametrically driven polaritons can be directly visualized. We analyze the above scheme for polaritons in (GaAs) semiconductor microcavities driven by a surface acoustic wave (SAW) and show that for realistic values of the SAW, \( n_{SAW} = 1 \text{ GHz} \) and \( I_{ac} \lesssim 100 \text{ W/cm}^2 \), the main acoustically-induced band gap in the polariton spectrum can be as large as \( \Delta_{MC} \sim 0.6 \text{ meV} \) and the Bragg replicas up to \( n = 3 \) can be observed.

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where $E$ and $P$ are the light field and excitonic polarization, respectively, $J_{\text{ext}}$ is a source of the external optical wave necessary for the Bragg scattering problem, $\hbar \omega_T$ is the energy position of the exciton line, $M_x$ is the in-plane translational mass of QW excitons, $\Omega_x-\gamma$ is the matrix element of the QW exciton – MC photon interaction, and $2 \gamma_x = 1/T_1 = 2/T_2$ is the rate of incoherent scattering of QW excitons. We assume a Cartesian coordinate system with the in-plane $x$-axis along the SAW wavevector $\mathbf{k}$, the $z$-axis along the MC growth direction, and the exciton and light fields linearly polarized along the in-plane $y$-axis. In this case the SAW, which has both transverse and longitudinal displacement components, $u_x$ and $u_z$, is elliptically-polarized in the $x$-$z$ plane (the sagittal plane).

The SAW-induced modulation of the excitonic transition is characterized by the coupling coefficient $m^2 = |J_{\text{ac}}| [u_x]/(|u_x|^2 + |u_z|^2)], m^2 = |J_{\text{ac}}| [u_x]/(|u_x|^2 + |u_z|^2)], m^2 = |J_{\text{ac}}| [u_x]/(|u_x|^2 + |u_z|^2)], m^2 = |J_{\text{ac}}| [u_x]/(|u_x|^2 + |u_z|^2)], m^2 = |J_{\text{ac}}| [u_x]/(|u_x|^2 + |u_z|^2)], m^2 = |J_{\text{ac}}| [u_x]/(|u_x|^2 + |u_z|^2)], m^2 = |J_{\text{ac}}| [u_x]/(|u_x|^2 + |u_z|^2)], m^2 = |J_{\text{ac}}| [u_x]/(|u_x|^2 + |u_z|^2)].$ The phase shift $\delta_{\text{SAW}}$ between $u_x$ and $u_z$ is nearly $\pi/2$, so that both interaction channels interfere constructively. However, because $m^2_{\text{DP}} \gg m^2_{\text{PE}}$ (for $\nu_{\text{SAW}} \lesssim 10 \, \text{GHz}$) and $|u_x|/|u_z| \approx 1.4$, the DP mechanism with $m^2_{\text{DP}} = D_x/(2\hbar^2 \rho \mu_0^3)^{1/2}$ is strongly dominant over the PE one. Here, $\rho$ is the crystal density, $v_x$ is the SAW velocity, and $D_x$ is the exciton – LA-phonon DP. The above hierarchy of the interaction mechanisms is due to the charge neutrality of excitons: In contrast, the electron – SAW interaction is determined by the PE coupling.

The $z$-dependent background dielectric function $\varepsilon_z(z)$ on the left-hand-side (l.h.s.) of Eq. (1) refers to a stack of layers which form the MC structure we analyze, i.e., a GaAs $\lambda$-cavity with an embedded InGaAs QW, symmetrically sandwiched in between two identical AlGaAs/AlAs Bragg reflectors. The Bragg reflectors give rise to the transverse optical confinement and, in the meantime, ensure an optical coupling of the external light wave with the in-plane polariton quasi-eigenstates. At first, however, in order to visualize the quasi-energy spectrum of SAW-driven MC polaritons, we assume 100% optical confinement (i.e., replace the Bragg reflectors by perfect mirrors), no external light source (i.e., $J_{\text{ext}} = 0$ on the r.h.s. of Eq. (1)), and no damping of the exciton states (i.e., $\gamma_x = 0$ on the l.h.s. of Eq. (2)). In this case the polariton spectrum is characterized by the MC Rabi frequency $\Omega_{\text{MC}} = 2(\pi/\varepsilon_b)^{1/2} \Omega_x - \gamma$. In Fig. 1 we plot the quasi-energy spectrum of lower-branch (LB) polaritons in a zero detuning MC with $\Omega_{\text{MC}} = 3.7 \, \text{meV}$, driven by the SAW of $\nu_{\text{SAW}} = 1 \, \text{GHz}$ and $I_{\text{ac}} = 10 \, \text{W/cm}^2$.

The quasi-energy spectrum, which is calculated by the method developed in Ref. [7] for bulk polaritons and cannot be interpreted in terms of the Mathieu functions, show the acoustically-induced band gaps $\Delta_{\text{ac}}^{\text{MC}(n)} \propto I_{\text{ac}}^{n/2}$, due to the $n = 1, 2, \ldots$ – phonon-assisted transitions. Because the energy of SAW phonons is very small, $\hbar \nu_{\text{SAW}} \approx 4.1 \, \text{meV}$ only, the acoustically-induced spectral gaps refer to the in-plane counter-propagating MC polariton states $\{nk/2, \omega_{\text{LB}}^{\text{MC}}(nk/2)\}$ and $\{-nk/2, \omega_{\text{LB}}^{\text{MC}}(nk/2)\}$ (see the nearly horizontal arrows in Fig. 1), where $\omega_{\text{LB}}^{\text{MC}}(p_{||})$ with $p_{||} = \pm nk/2$ is the LB solution of the dispersion equation $c^2 p_{||}^2/\varepsilon_b + \omega_{\text{T}}^2 - \omega^2 = (\Omega_{\text{MC}})^2/\omega_{\text{T}}^2 + \hbar \omega_{\text{T}} p_{||}^2/M_x - \omega^2$. The main acoustically-induced band gap in the LB spectrum is given by $\Delta_{\text{ac}}^{\text{MC}} \equiv \Delta_{\text{ac}}^{\text{MC}(n=1)} = 2|m^2|^2 \varphi_{\text{MC}}(k/2)$, where $\varphi_{\text{MC}}(k/2) = (\Omega_{\text{MC}})^2/\left(\Omega_{\text{MC}}^2 + 4[\omega_{\text{T}} - \omega_{\text{LB}}^{\text{MC}}(k/2)]^2\right)$ is the excitonic component of the polariton states $p_{||} = \pm nk/2$ resonantly coupled via one-SAW-phonon transition. Even for modest SAW intensities the main gap is large, so that $\Delta_{\text{ac}}^{\text{MC}} \gg \hbar \nu_{\text{SAW}}$. For
example, \( I_{ac} = 100 \text{ W/cm}^2 \) yields \( \Delta_{ac}^{MC}/h \simeq 130 \text{ GHz} \). Note, that the conventional acousto-optics deals with the acoustically-induced stop gap \( \Delta_{\gamma_{ac}} \) less than the SAW frequency (usually, \( \Delta_{\gamma_{ac}} \lesssim 100 \text{ MHz} \)) \[14\]. Furthermore, the acoustic band gaps associated with the two- and three-phonon transitions in the MC polariton spectrum are also well developed, so that \( \Delta_{ac}^{MC(n=1)} : \Delta_{ac}^{MC(n=2)} : \Delta_{ac}^{MC(n=3)} = 1 : 0.08 : 0.03 \) for \( I_{ac} = 100 \text{ W/cm}^2 \) (see the insets of Fig. 1). This is in a sharp contrast with the traditional acousto-optic schemes of the Bragg scattering. All the above features of the quasi-energy spectrum of SAW-driven MC polaritons are due to the resonant, exciton-mediated acousto-optic susceptibility.

Due to the periodicity of the acoustic wave, the quasi-energy spectrum can also be interpreted in terms of an extended Brillouin zone, with the band boundaries at \( p_{\parallel} \simeq \pm n k/2 \). The first acoustically-induced Brillouin zone is shown in Fig. 1 by the vertical dashed lines. The energy band boundaries, where the spectral gaps arise and develop with increasing \( I_{ac} \), can be probed in Bragg scattering, by changing the incidence angle \( \alpha \) of the external optical wave. At this point we return to Eqs. (1) and (2) applied to a realistic MC structure with the Bragg reflectors consisting of 34 alternating AlAs and Al\(_{0.13}\)Ga\(_{0.87}\)As \( \lambda/4 \)-layers. The MC polariton quasi-eigenstates can now be excited by the external light field, \( E_{\text{inc}} = E_{\text{inc}}^{(0)} e^{-i\omega t} \) due to the \( J_{\text{ex}} \)-term on the r.h.s. of Eq. (1), and can decay or scatter into the external electromagnetic modes.

In order to calculate the SAW-induced Bragg diffraction of optically excited MC polaritons, we use the Green function technique developed in Ref. \[14\]. Because the in-plane wavevector \( p_{\parallel} \) is conserved in the propagation of the light field through an optically transparent planar structure, one defines the MC photon Green function \( g(z, z'; \omega) \) by the equation:

\[
\frac{d^2}{dz^2} g(z, z'; \omega) + \kappa^2 g(z, z'; \omega) = -4 \pi \delta(z - z'),
\]

where \( \kappa^2 = (\omega/c)^2 \varepsilon_{b}(z) - p_{\parallel}^2 \). The function \( g(z, z'; \omega) \), which satisfies the Maxwellian boundary conditions, is evaluated numerically with the use of \( \varepsilon_{b}(z) \) relevant to the MC structure we analyze. As a next step, we substitute in Eqs. (1) and (2) a Fourier expansion of the \( E \) and \( P \) fields in terms of the in-plane quasi-wavevector \( p_{\parallel} + \ell k \) and quasi-frequency \( \omega + 2 \pi \ell v_{\text{SAW}} \), and evaluate the SAW-mediated acousto-optic susceptibility matrix \( \chi_{\ell, \ell'} \): \( P_{\ell} = \sum_{\ell'} \chi_{\ell, \ell'} E_{\ell'} \) \((\ell = 0, \pm 1, \ldots)\), i.e., \( \ell = \pm n \) corresponds to the \( n \)-phonon transition). The acoustically-induced polarization harmonics \( P_{\ell} \) give rise to the Bragg signal:

\[
E_{\ell} = E_{\ell}^{(0)} + q_{\ell}^2 \int dz' g(z, z'; \omega) \sum_{\ell'} \chi_{\ell, \ell'} E_{\ell'}(z'),
\]

where \( q_{\ell}^2 = (\omega + 2 \pi \ell v_{\text{SAW}})^2 / c^2 \). \( E_{\ell} = E_{\ell}(z) \) is the signal light field, and \( E_{\ell}^{(0)} = E_{\ell}^{(0)}(z) \) is the incoming light field induced by \( E_{\text{inc}} \). The field \( E_{\ell}^{(0)}(z) \) is calculated by using the photon Green function defined by Eq. (3). The integration on the r.h.s. of Eq. (4) is over the QW thickness, so that one can approximate \( E_{\ell}(z') \) by \( E_{\ell}(z' = 0) \).

The l.h.s. amplitude is evaluated from Eq. (1) by putting \( z = 0 \). Finally, the outgoing, diffracted field \( E_{\ell} \) is calculated from the completely defined r.h.s. of Eq. (4). The Bragg angle \( \Theta_{B} \) corresponds to the effective SAW-induced diffraction of MC polaritons with \( p_{\parallel} = \pm k/2 \) and is given by \( \sin \Theta_{B} = |k/(2k_{\text{opt}})| \left( 1 - \left[(\varepsilon_{b} k)/(\varepsilon_{b} \rho_{\text{RT}}^{\omega})\right][((ck)/\Omega_{x}^{\gamma_{MC}})^2] \right) \approx k/2k_{\text{opt}} \), where \( k_{\text{opt}} = ...
\(\omega / c\) is the wavevector of the external optical wave. In Fig. 2a we plot the reflection coefficient \(|r(\omega, I_{ac})|^2\) of the incoming light incident at angle \(\alpha = \Theta_B\) on the SAW-driven MC (\(\nu_{SAW} = 1\) GHz). In this case the r.h.s. boundary at \(p_B = k / 2\) of the acoustically-ionized first Brillouin zone is probed (see Fig. 1). In the vicinity of \(\omega = \omega_{LB}(k / 2)\), with increasing \(I_{ac}\) the reflectivity changes its single-line shape for \(I_{ac} = 0\) (the initial reflection spectrum is shown in Fig. 2a by the bold solid line) to a high contrast double-line shape with the separation \(\Delta_{MC}^{(n=2)}\) between two dips. Furthermore, a similar double-line structure, which refers to the three-phonon-assisted transition, appears and develops with increasing \(I_{ac}\) for the reflectivity at \(\omega \approx \omega_{LB}(3k / 2)\) (see the inset of Fig. 2a). Because the wavevector band \(-k / 2 \leq p_B \leq k / 2\) can also be interpreted in terms of the SAW-induced reduced Brillouin zone, the incident optical wave probes the odd-order SAW-induced energy gaps (the areas marked in Fig. 1 by large solid and dashed circles refer to the \(n = 1\) and \(3\) transitions). The corresponding frequency-downshifted \(-1\) and \(-3\) Bragg replicas give rise to the outgoing optical signal and are plotted in Fig. 2b. The camel-back shape of the replicas follow the band gaps \(\Delta_{MC}^{(n=2)}\) and \(\Delta_{MC}^{(n=3)}\), respectively. In Fig. 3a, \(|r(\omega, I_{ac})|^2\) is shown for \(\alpha = 20^\circ\), so that the r.h.s. boundary at \(p_B = k\) of the second (extended) Brillouin zone is probed. The corresponding \(-2\) Bragg replica is plotted in Fig. 3b. In Figs. 3c-3d we show the energy separation between two spikes in the \(-1\) and \(-2\) replicas. The separation is equal to \(\Delta_{MC}^{(n=2)} \approx I_{ac}^{1/2}\) and \(\Delta_{MC}^{(n=3)} \approx I_{ac}\), respectively.

Bragg diffraction, due to the nonresonant \(\chi^{(2)}_{\gamma-\text{ac}}\), has been observed for an optical wave guided by SAW-driven semiconductor layers \([12, 18, 17]\). The one-phonon diffraction replica \(-1\), calculated for the GaAs-based MC with the use of \(\chi^{(2)}_{\gamma-\text{ac}}\), is plotted in the l.h.s. inset of Fig. 2b. In this case the acoustically-induced main stop gap in the MC photon spectrum is very small and completely screened by the radiative damping, so that it cannot be seen in the Bragg signal as a camel-back structure. Note that the SAW intensities \(I_{ac}\) we discuss are much less than those used to acoustically ionize, by the piezoelectric effect, the excitons in GaAs structures \([18, 19]\).

For the analyzed scattering of MC polaritons by the SAW, the Klein-Cook parameter \(Q = Q(\omega)\), which distinguishes the Raman-Nath (transmission-type) regime from the Bragg (reflection-type) mode, is given by \(Q = v_{pol}(h / \gamma_{pol})(k^2 / k_{opt})\). Here, \(v_{pol}(\omega)\) and \(\gamma_{pol}(\omega)\) are the MC polariton velocity and damping, respectively. For the case analyzed in Figs. 1-3, \(Q \approx 6.5 \gg 1\), i.e., our theory does deal with the Bragg diffraction regime.

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**FIG. 3.** The Bragg spectra of SAW-driven MC polaritons for \(n = 2\) transition. (a) The reflection coefficient \(|r(\omega, I_{ac})|^2\) for \(\alpha = 16.8^\circ\) \((p_B \approx k)\) and \(\omega \approx \omega_{LB}(k)\), see Fig. 1), \(\nu_{SAW} = 1\) GHz and \(I_{ac} = 40\) W/cm\(^2\). (b) The frequency-down-converted \(-2\) Bragg replica. The solid, dashed and dotted lines refer to the excitonic damping \(\hbar \gamma_{ac} = \hbar / T_2 = 5, 10,\) and \(30\) meV, respectively. (c)-(d) The SAW-induced spike separation, i.e., the band gaps \(\Delta_{MC}^{(n=1)}\) and \(\Delta_{MC}^{(n=2)}\), against \(I_{ac}\).

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