Optomechanical analog of two-color electromagnetically-induced transparency: Photon transmission through an optomechanical device with a two-level system

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Some optomechanical systems can be transparent to a probe field when a strong driving field is applied. These systems can provide an optomechanical analogue of electromagnetically-induced transparency (EIT). We study the transmission of a probe field through a hybrid optomechanical system consisting of a cavity and a mechanical resonator with a two-level system (qubit). The qubit might be an intrinsic defect inside the mechanical resonator, a superconducting artificial atom, or another two-level system. The mechanical resonator is coupled to the cavity field via radiation pressure and to the qubit via the Jaynes-Cummings interaction. We find that the dressed two-level system and mechanical phonon can form two sets of three-level systems. Thus, there are two transparency windows in the discussed system. We interpret this effect as an optomechanical analog of two-color EIT (or double-EIT). We demonstrate how to switch between one and two EIT windows by changing the transition frequency of the qubit. We show that the absorption and dispersion of the system are mainly affected by the qubit-phonon coupling strength and the transition frequency of the qubit.

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I. INTRODUCTION

Micro- and nano-scale mechanical resonators [1, 2] provide a platform to explore the transition from the world of quantum physics to the classical world. Such transition can be demonstrated by coupling mechanical resonators to other quantum objects [3], including superconducting qubit circuits [4–12], transmission line resonators [13–17], optical cavities [18–21], nitrogen-vacancy (NV) centers [22–24], and two-level defects [25–26]. For example, the quantization of mechanical oscillations can be demonstrated by phonon blockade [10], which can be measured by a cavity field [27]. Experiments [28–31] showed that mechanical resonators can operate in the quantum regime. This makes it possible to couple different degrees of freedom in hybrid quantum devices [3] by using mechanical resonators as quantum transducers [32, 33], switches, or data buses [34]. Optomechanical systems can be formed [18–21] when mechanical resonators are coupled to electromagnetic fields through radiation pressure. In most recent experiments, the electromagnetic fields of the optomechanical coupling are from microwave to optical wavelengths.

It has been shown [35–40] that mechanical resonators can be used to convert optical quantum states to microwave ones via optomechanical interactions between a mechanical resonator and a single-mode field of both optical and microwave wavelengths. Hybrid electro-optomechanical systems can exhibit controllable strong Kerr nonlinearities even in the weak-coupling regime [41]. This Kerr nonlinearity can enable, in particular, the appearance of the photon blockade or the generation of nonclassical states of microwave radiation (e.g., Schrödinger cat [42] and kitten [43] states). Also when a weak coherent probe field is applied to a cavity with an optomechanical system, the mechanical resonator can act as a switch to control the probe photon transmission such that photons can pass through the cavity one by one [44–47] or two by two [48, 49] in the limit of the strong single-photon optomechanical coupling [50–53]. This photon-induced photon blockade can be used to engineer nonclassical phonon states [54, 55] of macroscopic mechanical resonators in low frequencies. Moreover, optomechanical systems can also become transparent to a weak probe field when a strong driving field is applied to the cavity field. This is an optomechanical analogue of electromagnetically-induced transparency (EIT) [56–59].

Both a cavity field and a mechanical resonator can be coupled to other systems, thus optomechanical systems can also be important ingredients in quantum networks. We found [60] that when a two-level atomic ensemble is coupled to the cavity field of an optomechanical system, then it can be used to enhance the photon-phonon coupling through radiation pressure. We also showed [61] that EIT in a three-level atomic ensemble, interacting with a cavity field of an optomechanical system, can be significantly changed by an oscillating mirror. Optomechanical systems can also be formed by a cavity field and trapped atoms [62, 63]. It has been shown that an optomechanical analogue of EIT can be controlled by a tunable superconducting qubit [64] or a two-level atom [65, 66] coupled to a cavity. Mechanical resonators of optomechanical systems can also be coupled to their intrinsic two-level defects [67] or artificial two-level systems, e.g., two-level superconducting qubits [28]. It is known that the optomechanical EIT [56–58] can occur in bare optomechanical systems. Moreover, EIT and related Autler-Townes splitting phenomenological analysis can be applied to the quantum states of these systems.
ena have also been studied in superconducting artificial atomic systems (e.g., Refs. [68–70] and the many references therein). Therefore, it is an interesting question how a two-level system coupled to a mechanical resonator affects the photon transmission through optomechanical systems.

Here we study a hybrid system, consisting of an optomechanical device and a two-level system, or a two-level defect, which is coupled to a mechanical resonator described by the Jaynes-Cummings Hamiltonian. Our numerical calculations are mainly focused on resonant interactions between the low-frequency mechanical resonator and the two-level system. The latter might be an intrinsic defect inside the mechanical resonator, a superconducting artificial atom, or another two-level system. For simplicity, hereafter we just use a qubit or a two-level system to denote those kinds of systems.

A main result of our work is the observation of the optomechanical analog of two-color EIT and the demonstration how this EIT can be switched to the standard single-color EIT. We are not aware of any other works on this effect in optomechanical systems. Nevertheless, two-color EIT (or double EIT) has already been discussed in some other systems [71], e.g., in an ensemble of two-level atoms coupled to a probe light or, equivalently, a system of two-mode polaritons coupled to one transition of the Λ-type three-level atoms [72]. This effect is also closely related to the EIT in a double-Λ system. Applications of two-color EIT include nonlinear wave-mixing, cross-phase modulation, optical switching, wavelength conversion, etc.

The paper is organized as follows: In Sec. II we describe a theoretical model and the equations of motion for the system operators. In Sec. III we obtain steady-state solutions of the system operators and further study the stability of the system. In Sec. IV the light transmission in this hybrid system is studied through the input-output theory. In particular, the optomechanical analogues of EIT are discussed here. We finally summarize our results in Sec. V.

II. THEORETICAL MODEL

A. Hamiltonian

As schematically shown in Fig. 1 we study a hybrid device in which a qubit is coupled to the mechanical resonator of an optomechanical device [67]. Figure 1 can also describe a system, in which a single-mode cavity field is coupled to a mechanical resonator which interacts with a two-level system as in Refs. [5, 28]. We assume that the coupling between the mechanical resonator and the two-level system is described by the Jaynes-Cummings Hamiltonian. However, the interaction between the mechanical resonator and the cavity field is described by the radiation-pressure Hamiltonian. There is no direct interaction between the two-level system and the cavity field. Thus, the Hamiltonian of the whole system can be written as

\[
H_0 = \hbar \omega_a a^\dagger a + \hbar \omega_b b^\dagger b + \frac{\hbar}{2} \omega_q \sigma_z - \hbar \chi a^\dagger a (b^\dagger + b) + \hbar g (b^\dagger \sigma_- + \sigma_+ b),
\]

where \(a \ (a^\dagger)\) is the annihilation (creation) operator of the single-mode cavity field with frequency \(\omega_a\); \(b \ (b^\dagger)\) is the annihilation (creation) operator of the mechanical mode with frequency \(\omega_b\). The Pauli operator \(\sigma_z\) is used to describe the two-level system the transition frequency \(\omega_q\), while \(\sigma_-\) and \(\sigma_+\) are the ladder operators of the two-level system. The parameter \(\chi\) is the coupling strength between the mechanical resonator and the cavity field, while the parameter \(g\) is the coupling strength between the mechanical resonator and the two-level system.

To demonstrate the relation between the cavity field and the two-level system, let us apply a unitary transform \(U = \exp[-\chi a^\dagger a (b^\dagger + b) / \omega_b]\) to the Hamiltonian in Eq. (1). In this case, we have an effective Hamiltonian \(H'_0 = U H_0 U^\dagger\) with

\[
H'_0 = \hbar \left( \omega_a - \frac{\chi^2}{\omega_b} + \frac{g \chi}{\omega_b} \sigma_x \right) a^\dagger a - \hbar \frac{\chi^2}{\omega_b} a^\dagger a a a + \frac{\hbar}{2} \omega_q \sigma_z + \hbar \omega_b b^\dagger b + \hbar g (b^\dagger \sigma_- + \sigma_+ b),
\]

which shows that both the two-level system and the mechanical resonator can induce a nonlinearity in the cavity field.

Let us now assume that a strong driving field and a weak probe field, with frequencies \(\omega_d\) and \(\omega_p\), respectively, are applied to the cavity. Then the Hamiltonian of the driven hybrid

![FIG. 1: (Color online) Schematic diagram of a hybrid optomechanical system consisting of a cavity (with a photonic mode \(a\)), where one of the mirrors is oscillating corresponding to a quantum mechanical resonator (with a phononic mode \(b\)). The oscillating mirror has a qubit or two-level defect denoted by two lines with the ground \(|g\rangle\) and excited \(|e\rangle\) states inside the dashed circle. The mechanical resonator is coupled both to the cavity (via radiation pressure) and to the qubit (via the Jaynes-Cummings interaction). However, for simplicity, we assume that there is no direct coupling between the cavity and qubit.](image)
Here noise operators are zero, that is, cavity field parameter \( \gamma \) fields, respectively. In the rotating reference frame with frequency \( \omega_d \), the Hamiltonian in Eq. (3) becomes

\[
H_r = H_0 - i\omega_d a^\dagger a + i\hbar (\Omega a^\dagger - \Omega^* a) \\
+ i\hbar \left( \epsilon e^{-i\Delta t} a^\dagger - \epsilon^* e^{i\Delta t} a \right),
\]

with the detuning \( \Delta = \omega_p - \omega_d \) between the probe field with frequency \( \omega_p \) and the strong driving field with frequency \( \omega_d \).

### B. Heisenberg-Langevin equations

Introducing the dissipation and fluctuation terms, and also using the Markov approximation, the Heisenberg-Langevin equations of motion can be written as

\[
\dot{a} = -(\gamma_\alpha + i\Delta_a) a + \Omega + \varepsilon \exp(-i\Delta t) + i\chi a (b^\dagger + b) \\
+ \sqrt{2\gamma_a} \alpha_{in}(t),
\]

\[
\dot{b} = -(\gamma_b + i\epsilon\omega_q) b + i\chi a b - i\gamma b^\dagger b_{in}(t),
\]

\[
\dot{\sigma}_- = - \left( \frac{\gamma}{2} + i\epsilon\omega_q \right) \sigma_- + i\gamma b\sigma_z + \sqrt{\gamma_q} \Gamma_-(t),
\]

\[
\dot{\sigma}_z = - \gamma_q (\sigma_z + 1) - 2i\gamma (b\sigma_- + b^\dagger \sigma_+),
\]

Here \( \gamma_\alpha, \gamma_b, \) and \( \gamma_q \) are the decay rates of the cavity field, mechanical mode, and two-level system, respectively. The parameter \( \Delta_a = \omega_a - \omega_d \) describes the detuning between the cavity field \( a \) with frequency \( \omega_a \) and the strong driving field with frequency \( \omega_d \). The operators \( \alpha_{in}(t), b_{in}(t), \Gamma_-(t), \) and \( \Gamma_z(t) \) denote environmental noises corresponding to the operators \( a, b, \sigma_- \), and \( \sigma_z \). We assume that the mean values of the above noise operators are zero, that is,

\[
\langle \alpha_{in}(t) \rangle = \langle b_{in}(t) \rangle = \langle \Gamma_-(t) \rangle = \langle \Gamma_z(t) \rangle = 0.
\]

However, their correlation functions satisfy the relations

\[
\langle a_{in}^\dagger(t') a_{in}(t) \rangle = n_a \delta(t' - t),
\]

\[
\langle b_{in}^\dagger(t') b_{in}(t) \rangle = n_b \delta(t' - t),
\]

\[
\langle \Gamma_+(t') \Gamma_-(t) \rangle = n_r \delta(t' - t),
\]

\[
\langle \Gamma_+(t') \Gamma_z(t) \rangle = -2n_r \langle \sigma_- \rangle \delta(t' - t),
\]

\[
\langle \Gamma_-(t') \Gamma_z(t) \rangle = 2(n_r + 1) \langle \sigma_- \rangle \delta(t' - t),
\]

\[
\langle \Gamma_z(t') \Gamma_z(t) \rangle = 2 \left( 2n_r + 1 + \langle \sigma_z \rangle \right) \delta(t' - t).
\]

Here \( n_a \) and \( n_b \) are the mean photon and phonon numbers in thermal equilibrium, respectively; \( n_r \) is the mean number of reservoir oscillators (photons or phonons), in thermal equilibrium, interacting with the two-level system.

### III. STEADY STATES AND STABILITY ANALYSIS

#### A. Steady states and linear response to probe field

To analyze the response of the system, in a steady state, to the weak probe field, we now take the mean values corresponding to Eqs. (5)–(9). In this case, we have the following equations

\[
\langle \dot{a} \rangle = -(\gamma_\alpha + i\Delta_a) \langle a \rangle + i\chi \langle a \rangle \left( \langle b^\dagger \rangle + \langle b \rangle \right) + \Omega + \varepsilon \exp(-i\Delta t),
\]

\[
\langle \dot{b} \rangle = -(\gamma_b + i\epsilon\omega_q) \langle b \rangle + i\chi \langle a \rangle \langle b \rangle - i\gamma \langle b^\dagger \rangle \langle b \rangle,
\]

\[
\langle \dot{\sigma}_- \rangle = - \left( \frac{\gamma}{2} + i\epsilon\omega_q \right) \langle \sigma_- \rangle + i\gamma \langle b \rangle \langle \sigma_z \rangle,
\]

\[
\langle \dot{\sigma}_z \rangle = - \gamma_q (\langle \sigma_z \rangle + 1) - 2i\gamma (\langle b \rangle \langle \sigma_- \rangle + \langle b^\dagger \rangle \langle \sigma_+ \rangle).
\]

Here we note that the mean-field approximation, i.e., \( \langle a^\dagger a \rangle = \langle a^\dagger \rangle \langle a \rangle \), was used in the derivation of these equations.

It is very unlikely to obtain exact analytical solutions of the nonlinear Eqs. (16)–(19), because the steady-state response contains an infinite number of components of different frequencies of the nonlinear systems. Instead, we find a steady-state solution, which is exact for the driving field in the parameter \( \Omega \) and correct to first order in the parameter \( \varepsilon \) of the probe field. That is, we assume that the solutions of Eqs. (16)–(19) have the following forms [23]:

\[
\langle a \rangle = A_0 + A_+ \exp(i\Delta t) + A_- \exp(-i\Delta t),
\]

\[
\langle b \rangle = B_0 + B_+ \exp(i\Delta t) + B_- \exp(-i\Delta t),
\]

\[
\langle \sigma_- \rangle = L_0 + L_+ \exp(i\Delta t) + L_- \exp(-i\Delta t),
\]

\[
\langle \sigma_z \rangle = Z_0 + Z_+ \exp(i\Delta t) + Z_- \exp(-i\Delta t).
\]
Here $A_0$, $B_0$, $L_0$, and $Z_0$ correspond to the solutions of $a$, $b$, $\sigma_-$, and $\sigma_z$, respectively, when only the driving field is applied. The parameters $A_\pm$, $B_\pm$, $L_\pm$, and $Z_\pm$ are of the order of $\varepsilon$ of the probe field. These can be obtained by substituting Eqs. (20)–(23) into Eqs. (16)–(19) and comparing the coefficients of the same order. For example, we substitute the expressions $\langle b \rangle$, $\langle \sigma_- \rangle$, and $\langle \sigma_z \rangle$, given by Eqs. (21)–(23), into Eq. (19), then $Z_0$, $Z_+$, and $Z_-$ can be expressed in terms of $L_0$, $L_+$, $L_-$, $B_0$, $B_+$, and $B_-$ as follows

$$Z_0 = \frac{2g}{\gamma_q} (B_0^* L_0 - B_0 L_0^*) - 1,$$

$$Z_+ = -\lambda_1 (B_0 L_+ - B_0^* L_+ - B_0^* L_0 - L_0 B_+^*),$$

$$Z_- = \lambda_1^* (B_0 L_+^* + L_0^* B_- - B_0^* L_- - L_0 B_-^*),$$

with $\lambda_1 = \frac{\sqrt{2}}{\Delta - i\gamma_q}$. Since $\sigma_z$ is a Hermitian operator, the conditions $Z_0^* = Z_0$, $Z_+^* = Z_-$, and $Z_-^* = Z_+$ have been used when Eqs. (24)–(26) were derived. Similarly, we obtain

$$L_0 = \frac{2gB_0Z_0}{\omega_q - i\gamma_q}. \quad (27)$$

Since the two-level system is coupled only to the mechanical mode, then the steady-state value $L_0$ is directly related only to the steady-state value of the mechanical mode and indirectly related to those of the cavity field. We find that $L_+$ and $L_-$ can be expressed with $B_+$ and $B_-$ as

$$L_+ = \lambda_2 B_+ + \lambda_3 B_-^*, \quad (28)$$

$$L_- = \lambda_4 B_- + \lambda_5 B_+^*. \quad (29)$$

Explicit formulas for these and other parameters $\lambda_i$ ($i = 2, 3, ..., 10$) are given in the Appendix.

We substitute $L_0$, given by Eq. (27), into Eq. (24), and then obtain the solution

$$Z_0 = -\frac{\gamma^2_q + 4\omega_q^2}{\gamma^2_q + 4\omega_q^2 + 8\gamma^2 B_0^2}. \quad (30)$$

It is easy to find that the value of $Z_0$ ranges from $-1$ to $0$. If $Z_0 = -1$, then the two-level system is in its ground state. Thus, it is obvious that if the coupling strength $g$ is much smaller than the transition frequency $\omega_q$ of the two-level system, and the phonon number is not very large, then the two-level system is almost in its ground state. We also obtain

$$B_0 = \frac{\chi |A_0|^2 - gL_0}{\omega_b - i\gamma_b}, \quad (31)$$

$$B_+ = \lambda_6 (A_0^* A_+ + A_0 A_+^*),$$

$$B_- = \lambda_7 (A_0^* A_- + A_0 A_-^*), \quad (33)$$

where $B_0$ is the steady-state value of the mechanical mode. Because of the coupling of the mechanical mode to the two-level system and the cavity field, $B_0$ depends both on the steady-state values $L_0$ of the two-level system and on the steady-state value $A_0$ of the cavity field.

We now calculate the coefficients of the steady-state value of the cavity field by substituting the expansions of $\langle a \rangle$, given by Eq. (20), and $\langle b \rangle$, given by Eq. (21), into Eq. (16). Up to first order in the parameter $\varepsilon$, we obtain

$$A_0 = \frac{\Omega}{\gamma_a + i\Delta_a - i\chi (B_0 + B_0^*)^\dagger}, \quad (34)$$

which represents the steady-state value of the cavity field assuming a strong driving field. With the help of Eqs. (32)–(34), we also obtain

$$A_- = \frac{i\chi (\lambda_6^* + \lambda_5) A_0^2 \varepsilon^*}{\gamma^2_a - \gamma^2_0}, \quad (36)$$

which describes the four-wave mixing for the driving field and the weak probe field.

### B. Stability

To analyze the stability of the system, we now present the driving field strength $|\Omega|$ (i.e., the Rabi frequency of the driving field) as a function of the steady value of $|A_0|$ as follows

$$|\Omega| = |A_0| \sqrt{\gamma^2_a + \left[\Delta_a - \frac{2\chi^2 \varepsilon^2 |A_0|^2 (\gamma^2_a + 4\omega^2_q)}{\varepsilon_1^2 + \varepsilon_2^2}\right]^2} \quad (37)$$

with the parameters $\varepsilon_1$ and $\varepsilon_2$ given by

$$\varepsilon_1 = \gamma_b (\gamma^2_a + 4\omega^2_q) - 2\gamma_0 g^2 Z_0,$$

$$\varepsilon_2 = \omega_b (\gamma^2_a + 4\omega^2_q) + 4\omega_q g^2 Z_0. \quad (39)$$

We can conclude from Eq. (30) that if the coupling strength $g \ll \omega_q$ and the phonon number are small then the two-level system has a high possibility to remain in its ground state. In this case the value of $Z_0$ is very close to $-1$ and can be considered constant, then we can see from Eq. (37) that $|A_0|$ can have three real solutions under certain conditions.

In Fig. 2 the steady-state photon number $|A_0|^2$ of the cavity field, corresponding to the steady-state component in Eq. (20), is plotted as a function of the driving field strength $|\Omega|$. This figure shows the bistable behavior of the cavity field of the hybrid optomechanical system. This result is very similar to that of driven optomechanical systems [74]. If we change the coupling strength between the phonon and the two-level system, the steady value and bistable behavior change.

The relation between the phonon mode $B_0$ and the Rabi frequency $\Omega$ can also be calculated as

$$|\Omega|^2 = \left|\frac{\gamma^2_a + (\Delta_a - 2\chi \text{Re} B_0^d) \varepsilon_4 + i\varepsilon_5}{i\chi \varepsilon_3} B_0\right|^2 \quad (40)$$
with the parameters given by

\begin{align}
\varepsilon_3 &= \gamma_a^2 + 4\omega_q^2 \\
\varepsilon_4 &= \gamma_6\varepsilon_3 - 2\gamma_ag^2Z_0 \\
\varepsilon_5 &= \omega_b\varepsilon_3 + 4\omega_dg^2Z_0
\end{align}

In Fig. 3, the steady-state phonon number \(|B_0|^2\) of the mechanical mode, corresponding to the steady-state component in Eq. (21), is plotted as a function of the Rabi frequency \(|\Omega|/(2\pi)|\) of the driving field. We find that the phonon bistability can also occur for the mechanical mode in some parameter regimes, and the two-level system has a little effect on the bistability. From Eqs. (37) and (40), if the variation of \(Z_0\) cannot be ignored, we find that both \(|A_0|\) and \(|B_0|\) can have at most five real solutions, so both the photon and phonon modes can show multistability under certain conditions. According to our numerical calculations shown in Figs. 2 and 3, if the coupling strength \(g\) is much smaller than \(\omega_b\) (or \(\omega_q\)), the photon and phonon modes only exhibit the bistable behavior, no multistability. We note that multistability can occur when the coupling between the two-level system and the phonon mode become very strong or ultrastrong. This coupling might not be easy to produce using natural qubits. However they might become possible using an artificial two-level system, e.g., when the mechanical mode is coupled to a superconducting qubit instead of an intrinsic natural two-level defect.

**IV. ELECTROMAGNETICALLY-INDUCED TRANSPARENCY**

We now study the transmission of a weak-probe field through an optomechanical system which is coupled to a two-level system. Using the input-output theory \cite{75}

\begin{equation}
\langle a_{\text{out}} \rangle + \Omega \sqrt{2\gamma_a} e^{-i\Delta t} + \frac{\varepsilon}{\sqrt{2\gamma_a}} e^{-i\Delta t} = \sqrt{2\gamma_a} \langle a \rangle, \tag{44}
\end{equation}

the output of the cavity field can be obtained as

\begin{equation}
\langle a_{\text{out}} \rangle = A_d + A_s \varepsilon e^{-i\Delta t} + A_{as} \varepsilon e^{i\Delta t}, \tag{45}
\end{equation}

with coefficients

\begin{align}
A_d &= \sqrt{2\gamma_a}A_0 - \frac{\Omega}{\sqrt{2\gamma_a}}, \\
A_s &= \sqrt{2\gamma_a} A_- - \frac{1}{\sqrt{2\gamma_a}}, \tag{46}
\end{align}

\begin{equation}
A_{as} = \frac{\sqrt{2\gamma_a}}{\varepsilon} A_. \tag{47}
\end{equation}

Here \(A_0\) and \(A_s\) are given by Eqs. (34)–(36); \(A_d\) is the output responding to the driving (or control) field with frequency \(\omega_d\); \(A_s\) is the output corresponding to the probe field with frequency \(\omega_p\) (Stokes frequency), and \(A_{as}\) represents the four-wave mixing frequency \(2\omega_d - \omega_p\) (anti-Stokes frequency). We redefine the output field at the frequency \(\omega_p\) of the probe field as \(\varepsilon_T = 2\gamma_a A_- / \varepsilon\) with the real and imaginary parts \(\mu_p = \gamma_a (A_- + A_-^*) / \varepsilon\) and \(\nu_p = \gamma_a (A_- - A_-^*) / (i\varepsilon)\). It is clear that \(\mu_p\) and \(\nu_p\) describe absorption and dispersion to the probe field.

For convenience, let us assume that \(\varepsilon = \sqrt{2\gamma_a}P_s / \hbar\omega_p\) is real. Here \(P_s\) is defined as the input power of the probe field. Then the output power at the Stokes frequency relative to the input power \(P_s\) is

\begin{equation}
G_s = \frac{\hbar \omega_p |\varepsilon A_s|^2}{P_s} = |\sqrt{2\gamma_a} A_s|^2. \tag{49}
\end{equation}

The output power at the anti-Stokes frequency \(2\omega_d - \omega_p\) is \cite{76}:

\begin{equation}
G_{as} = \frac{\hbar (2\omega_d - \omega_p) |\varepsilon A_{as}|^2}{P_s} = |\sqrt{2\gamma_a} A_{as}|^2. \tag{50}
\end{equation}

In the resolved sideband limit \(\omega_b \gg \gamma_a\), it is known that the transmission spectrum exhibits an EIT analogue in optomechanical systems. These phenomena can be mapped to the \(\Lambda\)-type three-level diagram of atomic systems. However, when the mechanical resonator of the optomechanical system is coupled to a two-level system, the transmission of the probe field becomes complicated. This is because the Jaynes-Cummings coupling between the two-level system and the mechanical resonator can lead to dressed states, which have a more complicated energy structure.

In order to better understand the physical meaning of these results, let us use the single-photon and single-phonon excitations as an example to illustrate the nature of photon transmission in this hybrid system. The energy-level diagram for the EIT analogue in optomechanical systems can be understood as in Ref. \cite{57}; the \(\Lambda\)-type three-level systems formed by three states \(|0_a, 0_b\), \(|0_a, 1_b\), and \(|1_a, 0_b\). Here the subscripts \(a\) and
The driving field resonant with the frequency of the mechanical
transmission if the detuning between the cavity field and the
photon and phonon states, respectively. Here $|a\pm\rangle = (|1_b, g\rangle \pm |0_b, e\rangle) / \sqrt{2}$ correspond to the dressed states between the single-phonon state and the
two-level system for $\omega_b = \omega_q$. $b$ denote the photon and phonon states, respectively. How-
ever, when a two-level system is coupled to the mechanical
resonator, the state $|0_a, 1_b\rangle$ is split into two states $|0_a, 1_b+\rangle$
and $|0_a, 1_b-\rangle$. Here $|1_b\pm\rangle$ denote the dressed states [77]
formed by the single-phonon state and the two-level system,
e.g., $|1_b\pm\rangle = (|1_b, g\rangle \pm |0_b, e\rangle) / \sqrt{2}$ for $\omega_b = \omega_q$. This splitting
of the single-phonon state significantly affects the photon
transmission if the detuning between the cavity field and the
driving field resonant with the frequency of the mechanical
resonator, i.e., $\Delta_a = \omega_a$. Clearly this splitting leads to two
transparency windows, which coincide well with the numerical
calculation shown in Fig. 5 and described below. The
case of multi-phonon excitations is very similar to the single-
phonon excitation, but the splitting width of the transparency
windows becomes wider. Moreover, the nonlinear coupling
between the cavity field and mechanical resonator makes the
transmission spectrum more complicated when the excitation
numbers of the photon and phonon are increased.

Figures 5(a) and 5(b) show, respectively, the absorption and
dispersion of the probe field for different values of the coupl-
ing strength $g$ between the mechanical resonator and the
two-level system. These figures show a familiar transparency
window of the optomechanical system, which can occur when
there is no coupling of the two-level system to the mechanical
resonator (as shown by the blue solid curves in Fig. 5). How-
ever, two transparency windows can occur when the two-level
system is coupled to the mechanical resonator (as shown, e.g.,
by the red dashed curves in Fig. 5). The splitting of these two
transparency windows is equal to the splitting width $2g$ that
results from the Jaynes-Cummings coupling between the two-
level system and the mechanical mode. In Fig. 5 the Stokes
and anti-Stokes power spectra are plotted as a function of the frequency
of the probe field. These spectra also show that the two-level system changes the splitting width of the output spectra at the Stokes and anti-Stokes frequencies.

In addition to the coupling strength $g$ between the two-level
system and the mechanical resonator, the transition frequency
$\omega_q$ of two-level system also affects the transmission of the
probe field, which will even more clearly show the main result
of our work, which is the finding of the optomechanical anal-
log of two-color EIT and the demonstration of its switching to
the standard single-color EIT.

In Fig. 7 the absorption spectra of the probe field are plot-
ted as a function of the detuning $\Delta$ between the probe and
driving fields. Different panels of Fig. 7 show the spectra for
different values of the transition frequency of the two-level
system in comparison to the mechanical-mode frequency. We
observe in Fig. 7(a) that there are two nearly symmetric trans-
parency windows [shown also by the red dashed curve in

\[
\begin{align*}
|0_a, 1_b\rangle & = |\psi\rangle, \\
|0_a, 1_b+\rangle & = (|1_b, g\rangle + |0_b, e\rangle) / \sqrt{2}, \\
|0_a, 1_b-\rangle & = (|1_b, g\rangle - |0_b, e\rangle) / \sqrt{2}.
\end{align*}
\]

Fig. 4: (Color online) Schematic diagram for the interaction between
the hybrid system with the driving (control) and probe fields with
a single-particle excitation. The driving field is applied to make a
single-phonon transition from the phonon vacuum state $|0_b\rangle$ to the
single-phonon state $|1_b\rangle$. The probe field is used to measure the trans-
mission when the population of the mechanical mode is not changed.
The $\Lambda$-type three energy levels in the hybrid optomechanical system
occur in the case $\Delta_a = \omega_a$. Here $|1_b\pm\rangle = (|1_b, g\rangle \pm |0_b, e\rangle) / \sqrt{2}$
correspond to the dressed states between the single-phonon state and
the two-level system for $\omega_b = \omega_q$. The splitting of the single-phonon state significantly affects the photon
transmission if the detuning between the cavity field and the
driving field resonant with the frequency of the mechanical
resonator makes the transmission spectrum more complicated when the excitation
numbers of the photon and phonon are increased.

\[
\begin{align*}
|0_a, 1_b\rangle & = |\psi\rangle, \\
|0_a, 1_b+\rangle & = (|1_b, g\rangle + |0_b, e\rangle) / \sqrt{2}, \\
|0_a, 1_b-\rangle & = (|1_b, g\rangle - |0_b, e\rangle) / \sqrt{2}.
\end{align*}
\]
V. CONCLUSIONS

In summary, we have studied the transmission of a probe field through an optomechanical system, consisting of a cavity and a mechanical resonator with a two-level system, for simplicity referred to as a qubit. The qubit might be an intrinsic defect inside the mechanical resonator, a superconducting artificial atom, or another two-level system. We assume that the mechanical resonator is coupled to the qubit via the Jaynes-Cummings interaction and to the cavity field via radiation pressure.

We find that the transmission of the probe field exhibits two transparent windows when the two-level system is resonantly coupled to the mechanical resonator. This is because the interaction between the mechanical resonator and the two-level system might result in two sets of coupling configurations be-
FIG. 8: (Color online) Same as in Fig. 7 but for the dispersion spectra \( \nu_p \) of the probe field.

between the controlling field and the mechanical resonator. We consider this effect as an optomechanical analog of two-color EIT (or double EIT), in contrast to the standard optomechanical single-color (or single-window) EIT exhibiting clear differences in the probe-field spectra. Our examples include: the absorption, dispersion, Stokes, and anti-Stokes spectra. We demonstrated how to switch between one and two EIT windows by changing the transition frequency of the qubit to be in or out of the resonance with the frequency of the mechanical mode. These features might be used to probe the low-frequency two-level fluctuations inside solid-state systems by using a low-frequency mechanical resonator. We note that the control of the transition frequency of the qubit could be realized easier with an artificial two-level system (e.g., a superconducting qubit) rather than with a natural two-level defect.

In addition to optical switching, applications of the optomechanical two-color EIT can include: the generation of nonclassical states of microwave radiation and/or mechanical resonator, nonlinear wave-mixing, cross-phase modulation, wavelength conversion or photon blockade [78], in analogy to such applications of the standard optomechanical single-color EIT.

VI. ACKNOWLEDGEMENT

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FIG. 9: (Color online) The Stokes \( G_s \) and anti-Stokes \( G_{as} \) spectra of the output of the probe field versus the detuning \( \Delta = \omega_p - \omega_d \). In each figure, three curves correspond to different transition frequencies of the qubit: (i) \( \omega_q/(2\pi) = 100 \) MHz (black solid curve) being in resonance with the mechanical-mode frequency \( \omega_b \), as shown in Figs. 7(a) and 8(a); (ii) \( \omega_q/(2\pi) = 80 \) MHz (red dashed curve) corresponding to the case of the red-detuned \( \omega_q \) shown in Figs. 7(b) and 8(b); and (iii) \( \omega_q/(2\pi) = 120 \) MHz (blue dash-dotted curve) corresponding to the case of the blue-detuned \( \omega_q \) shown in Figs. 7(c) and 8(c). All the parameters are the same as in the respective Figs. 7 and 8.
Appendix: Calculation of $A_+$ and $A_-$

From the discussions in Sec. III, one can find the expressions of $Z_0$, $Z_+$, $Z_-$, $L_0$, $L_+$, and $L_-$ in Eqs. (24)–(29) up to first order in the parameter $\varepsilon$ of the probe field by equating the coefficients of the same order. Then the corresponding coefficients are found to be

$$
\lambda_2 = \frac{1}{D_3} \left[ ig Z_0 D_1 - g \lambda_1 B_0 Z_0^* \right] \left(i D_1 - g \lambda_1 |B_0|^2 \right), \quad (A.1)
$$

$$
\lambda_3 = \frac{g_2 I_0}{D_3} \left(g \lambda_1 |B_0|^2 Z_0^* + i g B_0 Z_0 + D_1 L_0 \right), \quad (A.2)
$$

$$
\lambda_4 = \frac{1}{D_5} \left[ ig Z_0 D_2 + g \lambda_1^* B_0 Z_0^* \right] \left(i D_2 + g \lambda_1^* |B_0|^2 \right), \quad (A.3)
$$

$$
\lambda_5 = \frac{g_2 I_0}{D_5} \left(g \lambda_1^* |B_0|^2 Z_0 - i g B_0 Z_0 - D_2 L_0 \right), \quad (A.4)
$$

where the parameters $D_1$, $D_2$, and $D_3$ are given by

$$
D_1 = \frac{\gamma_q}{2} - i \omega_q - ig \lambda_1 |B_0|^2 + i \Delta, \quad (A.5)
$$

$$
D_2 = \frac{\gamma_q}{2} - i \omega_q + ig \lambda_1^* |B_0|^2 - i \Delta, \quad (A.6)
$$

$$
D_3 = \left( \frac{\gamma_q}{2} + i \Delta \right)^2 - 2 ig \lambda_1 |B_0|^2 \left( \frac{\gamma_q}{2} + i \Delta \right) + \omega_q^2. \quad (A.7)
$$

By substituting the expressions of $\langle b \rangle$, $\langle a \rangle$, and $\langle \sigma \rangle$ into the equation of motion for the average value of the operator $b$, we obtain the expressions of $B_0$, $B_+$, and $B_-$ in Eqs. (31)–(33). Here the coefficients $\lambda_6$ and $\lambda_7$ are found as

$$
\lambda_6 = \frac{-g x \lambda_3 + i \chi D_4}{D_4 D_5 - g^2 \lambda_3^* \lambda_5}, \quad (A.8)
$$

$$
\lambda_7 = \frac{-g x \lambda_3 + i \chi D_5^*}{D_4 D_5 - g^2 \lambda_3^* \lambda_5}, \quad (A.9)
$$

with

$$
D_4 = \gamma_b - i (\omega_b - \Delta + g \lambda_5^*), \quad (A.10)
$$

$$
D_5 = \gamma_b + i (\omega_b + \Delta + g \lambda_2). \quad (A.11)
$$

Using similar steps as above, we obtain formulas for $A_0$, $A_\sigma$, and $A_+$ for the average value of the cavity field, up to first order in the parameter $\varepsilon$ of the probe field as given, respectively, by Eqs. (43)–(45) with the parameters

$$
\lambda_8 = \gamma_a + i \left[ \Delta_a - \Delta - \chi B_0 - \chi (\lambda_5^* + \lambda_7) \right] |A_0|^2, \quad (A.12)
$$

$$
\lambda_9 = \gamma_a - i \left[ \Delta_a + \Delta - \chi B_0 - \chi (\lambda_5^* + \lambda_7) \right] |A_0|^2, \quad (A.13)
$$

$$
\lambda_{10} = \chi^2 (\lambda_5^* + \lambda_7)^2 |A_0|^4. \quad (A.14)
$$

It is clear that $A_0$ represents the steady-state value of the cavity field when the probe field is not applied to the cavity. However, $A_\sigma$ describes the linear response of the system to the probe field, and $A_+$ describes the four-wave mixing of the probe and driving fields.
