Determination of geometric parameters for rational load-bearing unit of bridge cylindrical column pier

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Abstract. This paper deals with a bridge pier energywise uniform strength cylindrical steel-concrete load-bearing unit having a mesh case, which is made of an expanded metal mesh, as the most rational and the least resource-demanding one, using the nonwaste technology. The rational parameter selection method, which guarantees cross section uniform strength for the suggested load-bearing unit, is provided. The concrete core of the given element is a hollow cylindrical element that varies in height and whose centre of gravity coincides with an external loading point for any horizontal cross section. In addition to uniform strength, the hollow enables you to control the centre of gravity of the given complicated compound cross section of a cylindrical steel-concrete load-bearing unit. The calculation method to determine the main geometric parameters of the uniform strength load-bearing unit of a bridge cylindrical column pier having the lateral cross section area that varies in height is provided. The geometric parameters are the centre of gravity and its coordinates, central inertia moments and the radius of inertia. To exclude an impact of random eccentricity on a complicated compound cross section of a pier uniform strength steel-concrete load-bearing unit, the method to design a cross section core is provided.

1. Introduction
The constructive design of bridge intermediate column piers has some common decisions [1], as a rule, reinforced concrete ones. However, we have lately witnessed a trend to use various steel-concrete load-bearing units [2]. At the same time, the category of steel-concrete units includes, first of all, tube-concrete ones as well as various systems having confinement reinforcement – lateral reinforcement such as radial rings, hoops, spiral windings etc.

The advantage of a tube-concrete structure is a reduction in concrete content by 1.5-2 times. As a result, structural weight decreases by 1.8-3 and working hours go down by 2 times as rebar placement and casing teardown are not necessary. However, we can witness increased steel content (reinforcement of tube-concrete structures is 3-20% depending on the thickness and diameter of a steel tube), a limited bond between the casing and concrete because a steel tube has smooth surface, increased susceptibility of a solid steel casing to mechanical damage and corrosion as there is no concrete layer to protect it.

An alternative to a tube-concrete structure is the cylindrical steel-concrete load-bearing unit of a bridge pier having a mesh case (figure 1, a) [3, 4].
Figure 1. The cylindrical steel-concrete load-bearing unit of a bridge pier having a mesh case

This casing is a steel plate of the grade St3PS having a mesh with equally distributed cells (fig. 2).

Figure 2. Main geometric characteristics of an expanded metal mesh

This plate is made using nonwaste technology using a special press [3, 4]. This treatment enables us to decrease the weight of a mesh in comparison with a solid plate of the same dimensions, saving required strength parameters (table 1).

| Table 1. Steel consumption per 1 m² of casing of steel-concrete load bearing unit. |
|----------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Solid casing                     | Mesh casing (expanded metal mesh) |
| Thickness of plate-workpiece, mm | Weight, kg/m²   | Plate designation | Thickness of plate-workpiece, mm | Overall thickness, mm | Weight, kg/m² | Steel saving coefficient, % | Weight reduction of casing, % |
| 4                               | 31,4            | PVL 406         | 4                             | 12,7              | 15,7            | 52,0            | 34,0 |
| 5                               | 39,25           | PVL 506         | 5                             | 13,0              | 16,4            | 58,22           | 39,58 |
|                                 |                 | PVL 508         | 5                             | 16,8              | 20,9            | 46,75           | 31,79 |
|                                 |                 | PVL 510         | 5                             | 20,5              | 24,7            | 37,07           | 25,20 |
| 6                               | 47,1            | PVL 606         | 6                             | 13,4              | 17,3            | 63,27           | 43,02 |
|                                 |                 | PVL 608         | 6                             | 17,1              | 21,9            | 53,50           | 36,38 |
|                                 |                 | PVL 610         | 6                             | 20,8              | 26,0            | 44,80           | 30,46 |
A steel-concrete structure with a mesh case can be rated as intermediate between reinforced concrete and tube-concrete structures in terms of strength properties, integrating their best characteristics. In particular, the mesh case, which has volumetric rhombic cells, performs a dispersed reinforcement function in addition to its main function [5, 6]. There is also an increased bond between a plate and a concrete core, using no special anchors, and improved resistance to corrosion and fire as the structure has a concrete cover. In turn, there is a possibility to strengthen the structure owing to various sizes of a plate cell (figure 2) and different angles of generatrix inclination [6, 7].

Taking into consideration that an energywise uniform strength load-bearing unit is the most rational and the least resource demanding one [7, 8], this paper presents the method to select the rational parameters of the given steel-concrete load-bearing unit of a bridge pier in order to guarantee the uniform strength of cross sections in height.

2. The method to determine the main geometric parameters of the complicated compound cross section of the uniform strength steel-concrete load-bearing unit of bridge piers

The concrete core is a cylindrical hollow element whose centre of gravity coincides with an external loading point in any horizontal cross section (figure 3). We believe that the most optimal element has stress that is equal to a concrete strength limit in compression $f_{ck}$ in all points.

To design the given uniform strength bridge pier, we use the following assumptions:
- a lateral compound cross section area changes in height at the expense of an exponential cylindrical hollow (figure 3, b) and it has the maximum shallow diameter $d$ in the top of a load-bearing element (figure 3, a);
- the pier is rigid, i.e. solid, and it is necessary to take into account its weight during computing. A cross section area is determined, according to the formula (1), depending on the distance $x$:

$$A(x) \cdot f_{ck} = F + Q(x),$$  

(1)
where \( A(x) \) is a variable, a cross section area; \( F \) is a total external loading; \( Q(x) \) is structure weight at a distance of \( x \).

Using transformations and taking into account an increase in a cross section area depending on the extension of a bolt by \( dx \) and an increase in a current total loading, we have the equation to determine a pier cross section area of an element in height:

\[
A(x) = A_0 e^{\frac{\gamma x}{f_{ek}}} = \frac{F}{f_{ek}} e^{\frac{\gamma x}{f_{ek}}},
\]

Having had the cross section area, we have the equation to determine the diameter \( d(x) \) of a cylindrical shallow that is also variable in height (figure 3, b):

\[
d(x) = D \sqrt{1 - \frac{4F}{\pi f_{ek} D^2} e^{\frac{\gamma x}{f_{ek}}}}.
\]

We can introduce a dimensionless coefficient that is equal to the ratio of a shallow diameter to a steel-concrete load-bearing element diameter:

\[
\alpha = \alpha(x) = \frac{d(x)}{D},
\]

then

\[
\alpha(x) = \sqrt{1 - \frac{4F}{\pi f_{ek} D^2} e^{\frac{\gamma x}{f_{ek}}}},
\]

where \( D \) is a concrete core diameter; \( d(x) \) is the diameter of a cylindrical shallow; \( f_{ek} \) is concrete core strength in compression; \( \gamma \) is concrete consistency.

To solve the task given, the characteristics of a complicated compound cross section are introduced:

- the coordinates of the gravity centre of a complicated compound cross section are \( z^*, y^* \);
- the dimensionless coordinates of a cylindrical shallow centre:\( \eta_z = \frac{z}{D}, \eta_y = \frac{y}{D} \).
where \( z_0, y_0 \) are the coordinates of the gravity centre of a shallow cylindrical element in relation to the gravity centre of the outside circumference of a cylindrical steel-concrete load-bearing element and its concrete core excluding its hollow.

Then, the coordinates of the gravity centre of a compound cross section are determined:

\[
\begin{align*}
\bar{z} &= \frac{\sum S_{y_i} A_i}{\sum A_i} = \frac{D\alpha^2 \eta_y}{(1 - \alpha^2)}, \\
\bar{y} &= \frac{\sum S_{z_i} A_i}{\sum A_i} = \frac{D\alpha^2 \eta_y}{(1 - \alpha^2)},
\end{align*}
\]

(6)

where

\[
\begin{align*}
\sum S_{y_i} A_i &= \frac{\pi d^2 D\eta_y}{4}, \\
\sum S_{z_i} A_i &= \frac{\pi d^2 D\eta_y}{4}, \\
\sum A_i &= \frac{\pi D^2}{4}(1 - \alpha^2).
\end{align*}
\]

Excluding an impact of eccentricity on a uniform strength steel-concrete element, we believe that an external loading is applied to the gravity centre of a compound cross section, i.e.:

\[
\begin{align*}
e_{z_i} &= \bar{z} = \frac{D\alpha^2 \eta_y}{(1 - \alpha^2)}, \\
e_{y_i} &= \bar{y} = \frac{D\alpha^2 \eta_y}{(1 - \alpha^2)},
\end{align*}
\]

where \( e_{z_i}, e_{y_i} \) are the eccentricities to apply an external loading in relation to a concrete core centre.

We can have the following equations:

\[
\begin{align*}
\eta_z &= \frac{e_{z_i}(1 - \alpha^2)}{D\alpha^2}, \\
\eta_y &= \frac{e_{y_i}(1 - \alpha^2)}{D\alpha^2},
\end{align*}
\]

(7)

and we can introduce the dimensionless coordinates of an external loading point \( \xi_z, \xi_y \) as follows:

\[
\begin{align*}
\xi_z &= \frac{e_{z_i}}{D}, \\
\xi_y &= \frac{e_{y_i}}{D}.
\end{align*}
\]

Then, the coordinates of a cylindrical shallow centre as well as the coordinates of the gravity centre of a complicated compound cross section depend on the ratio of the diameter of a steel-concrete load-bearing element and shallow:
To exclude an impact of a random eccentricity on the complicated compound cross section of a uniform strength steel-concrete pier load-bearing element, we can design a cross section core (figure 4), using the formulae [9]:

\[ \eta_z = \frac{\xi_z(1 - \alpha^2(x))}{\alpha^2(x)}, \]
\[ \eta_y = \frac{\xi_y(1 - \alpha^2(x))}{\alpha^2(x)}. \]  

To exclude an impact of a random eccentricity on the complicated compound cross section of a uniform strength steel-concrete pier load-bearing element, we can design a cross section core (figure 4), using the formulae [9]:

\[ a_u = -\frac{i_u^2}{v_i}, \quad a_u = -\frac{i_v^2}{u_i}, \]

where \( i_u, i_v \) are inertia radii in relation to the main central axes; are the coordinates of the intersections of a neutral line with main axes of a cross section. \( u_i, v_i \)

Figure 4. Determination of the geometric characteristics of the complicated compound cross section of the steel-concrete load-bearing element of a bridge pier

Squared inertia radii are calculated as follows:
\[ i_x = \frac{I_x}{A}, \quad i_y = \frac{I_y}{A}, \]

where \( A = \frac{\pi D^2}{4} (1 - \alpha^2) \);

\[
I_x = \frac{I_z + I_y}{2} + \frac{I_z - I_y}{2} \cos 2\beta - I_{xy} \sin 2\beta ,
\]

\[
I_y = \frac{I_z + I_y}{2} - \frac{I_z - I_y}{2} \cos 2\beta + I_{xy} \sin 2\beta .
\]

The following data are also introduced:

\[
\frac{\alpha^2}{1 - \alpha^2} = \omega, \quad \frac{(1 - \omega)^2}{\omega^2} = \psi .
\]

Then

\[
\eta_x = -\frac{\xi_x (1 - \alpha^2)}{\alpha^2} = -\frac{\xi_x}{\omega}, \quad \eta_z = \frac{\xi_z (1 - \alpha^2)}{\alpha^2} = -\frac{\xi_z}{\omega} .
\]

As a result of the transformations, we have the formulae to determine the inertia moments of a complicated compound cross section in relation to the central axes:

\[
I_z = \frac{\pi D^4}{64} \left[ (1 - \alpha^4) + 16 \xi_x^2 (1 - \psi) \right] ,
\]

\[
I_y = \frac{\pi D^4}{64} \left[ (1 - \alpha^4) + 16 \xi_y^2 (1 - \psi) \right] ,
\]

\[
I_{xy} = \frac{\pi D^4}{4} \xi_x \xi_y \left[ 1 - \alpha^2 \psi \right] .
\]

Then, taking into account all the transformations, the formulae to determine main central inertia moments are as follows:

\[
I_z = \frac{\pi D^4}{64} \left[ (1 - \alpha^4) + 8 (1 - \psi) \left( \xi_x^2 + \xi_z^2 \right) \pm \left( \xi_x^2 - \xi_z^2 \right) \cos 2\beta \right] m ,
\]

where

\[
tg 2\beta_0 = -\frac{2I_{xy}}{I_z - I_y} = -\frac{2 (1 - \alpha^2 \psi)}{(1 - \psi)} \frac{\xi_x \xi_y}{(\xi_y^2 - \xi_z^2)} ,
\]

\[
2\beta_0 = \arctg \left[ -\frac{2 (1 - \alpha^2 \psi)}{\xi_x \xi_y} \left( \frac{\xi_y^2 - \xi_z^2}{(1 - \psi)} \right) \right] .
\]
\[
\sin 2\beta = \sin \left[ \arctan \left( -2 \frac{1 - \alpha^2 \psi}{1 - \psi} \frac{\xi_y \xi_x}{\xi_y^2 - \xi_x^2} \right) \right],
\]
\[
\cos 2\beta = \cos \left[ \arctan \left( -2 \frac{1 - \alpha^2 \psi}{1 - \psi} \frac{\xi_y \xi_x}{\xi_y^2 - \xi_x^2} \right) \right].
\]

The squared inertia radius of a complicated compound cross section and the coordinates of the intersections of neutral lines with the main axes are as follows:

\[
i^2_z = \frac{I_z}{A} = \frac{D^2}{16 (1 - \alpha^2)} \left[ (1 - \alpha^4) + 8(1 - \psi) \left( \xi_x^2 + \xi_y^2 \right) \pm (\xi_x^2 - \xi_y^2) \cos 2\beta \right] m^2;
\]
\[
u = \pm D \cdot \cos \beta,
\]
\[
\nu_y = D - \xi_y \sin \beta,
\]
\[
\nu_y = D(1 + \xi_y \sin \beta).
\]

In order to test the given method, we can specify the parameters of the lateral cross section of a cylindrical component: \( D = 500 \text{mm} \); the hollow diameter is \( 0.1D \) and \( 0.25D \).

The calculation results show the impact of a hollow diameter and the value of the eccentricity of external effort application on component geometric parameters (table 2).

**Table 2.** The main results of the calculation of bridge pier cylindrical uniform strength load-bearing unit geometric parameters

| \( \alpha \) | \( z_0, \text{mm} \) | \( y_0, \text{mm} \) | \( z^*, \text{mm} \) | \( y^*, \text{mm} \) | \( \xi_x \) | \( \xi_y \) | \( I_z, \text{mm}^4 \) | \( I_y, \text{mm}^4 \) | \( I_{zy}, \text{mm}^4 \) |
|---|---|---|---|---|---|---|---|---|---|
| \( D = 500 \text{mm}; \ d = 50 \text{mm}; \ \psi = 9801; \ \omega = 0.01 \) | | | | | | | | | |
| 0.1 | 0 | 0 | 0 | 0 | 0 | 0 | 3.06 \times 10^9 | 3.06 \times 10^9 | 0 |
| 0.25 | 25 | 25 | 0.25 | 0.25 | 5 \times 10^{-4} | 5 \times 10^{-4} | 2.95 \times 10^9 | 2.95 \times 10^9 | -7.44 \times 10^4 |
| 0.50 | 50 | 50 | 0.51 | 0.51 | 1.02 \times 10^{-3} | 1.02 \times 10^{-3} | 2.57 \times 10^9 | 2.57 \times 10^9 | -3.10 \times 10^5 |
| 0.75 | 75 | 75 | 0.76 | 0.76 | 1.52 \times 10^{-3} | 1.52 \times 10^{-3} | 1.94 \times 10^9 | 1.94 \times 10^9 | -6.88 \times 10^5 |
| 1.00 | 100 | 100 | 1.01 | 1.01 | 2.02 \times 10^{-3} | 2.02 \times 10^{-3} | 1.10 \times 10^9 | 1.10 \times 10^9 | -1.21 \times 10^6 |
| 1.25 | 125 | 125 | 1.26 | 1.26 | 2.52 \times 10^{-3} | 2.52 \times 10^{-3} | 1.28 \times 10^7 | 1.28 \times 10^7 | -1.89 \times 10^6 |
| \( D = 500 \text{mm}; \ d = 125 \text{mm}; \ \psi = 195,79; \ \omega = 0.25 \) | | | | | | | | | |
| 0.25 | 0 | 0 | 0 | 0 | 0 | 0 | 3.07 \times 10^7 | 3.07 \times 10^7 | 0 |
| 0.50 | 25 | 25 | 1.46 | 1.46 | 2.93 \times 10^{-3} | 2.93 \times 10^{-3} | 2.97 \times 10^9 | 2.97 \times 10^9 | -296.00 |
| 0.75 | 50 | 50 | 3.33 | 3.33 | 6.67 \times 10^{-3} | 6.67 \times 10^{-3} | 2.63 \times 10^9 | 2.63 \times 10^9 | -1.53 \times 10^6 |
| 1.00 | 75 | 75 | 5.00 | 5.00 | 0.01 | 0.01 | 2.10 \times 10^9 | 2.10 \times 10^9 | -3.45 \times 10^6 |
| 1.25 | 100 | 100 | 6.67 | 6.67 | 1.33 \times 10^{-2} | 1.33 \times 10^{-2} | 1.36 \times 10^9 | 1.36 \times 10^9 | -6.10 \times 10^6 |
| 1.50 | 125 | 125 | 8.33 | 8.33 | 1.67 \times 10^{-2} | 1.67 \times 10^{-2} | 3.89 \times 10^8 | 3.89 \times 10^8 | -9.61 \times 10^6 |

To verify the suggested method, which determines the geometric parameters of a cylindrical uniform strength load-bearing unit, taking into consideration the shortage of the results of analogous calculations, we can compare it with a well-known method [10] without the dislocation of a hollow relative to the gravity centre of a bridge pier cylindrical component (\( z_0 = y_0 = 0 \)). So,
if $\alpha = 0.1$, $I_z = I_y = 3.06 \cdot 10^9 \text{mm}^4$;
if $\alpha = 0.25$, $I_z = I_y = 3.07 \cdot 10^9 \text{mm}^4$.

The results of calculation and comparison testify that the suggested method is positive and it can be used for design and research.

3. Conclusions
The suggested constructive design of the steel-concrete load-bearing unit of a bridge pier uses the nonwaste technology and enables you to reduce steel content by 65% in relation to the tube-concrete element of the same size and casing thickness. At the same time, the bond between the casing and concrete core increases owing to a volumetric mesh structure. The resistance to corrosion and fire increases owing to a concrete cover.

The cylindrical steel-concrete load-bearing element rational parameters, which guarantee the uniform strength of cross sections in height owing to a variable hollow in a concrete core, are determined. In addition to uniform strength, the hollow enables you to control the gravity centre of a complicated compound cross section of a cylindrical steel-concrete load-bearing element.

The calculation method, which allows to determine the main geometric characteristics of the complicated compound cross section of the uniform strength load-bearing element of a cylindrical bridge column pier, is proposed.

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