Spin-orbit fields in asymmetric (001) quantum wells

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We measure simultaneously the in-plane electron g-factor and spin relaxation rate in a series of undoped inversion-asymmetric (001)-oriented GaAs/AlGaAs quantum wells by spin-quantum beat spectroscopy. In combination the two quantities reveal the absolute values of both the Rashba and the Dresselhaus coefficients and prove that the Rashba coefficient can be negligibly small despite huge conduction band potential gradients which break the inversion symmetry. The negligible Rashba coefficient is a consequence of the ‘isomorphism’ of conduction and valence band potentials in quantum systems where the asymmetry is solely produced by alloy variations.

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Symmetry is a thread which runs through all of physics and symmetry reduction discloses basic physical principles. In this letter, we employ crystallographically engineered symmetry reduction to study the intricate effects of spin-orbit interaction on the electron spin in semiconductor nanostructures. Symmetry reduction is an especially powerful tool in semiconductor physics because the variety of crystallographic directions combined with bandgap engineering allow enormous freedom.

The interplay between structure, symmetry and electron spin in semiconductors directly affects the spin relaxation rate $\Gamma_s$ and the effective electron Landé-factor $g$. Early studies of $\Gamma_s$ and $g$ were focused on bulk zincblende material where both entities are isotropic $[1]$. Subsequently, the reduction in symmetry from $T_d$ to $D_{2h}$ symmetry in symmetrical (001)-oriented quantum wells (QWs) was shown to give rise to anisotropy between the in-plane (x,y) and the out of plane (z) directions $[2, 3]$. Further reduction in symmetry to $C_{2v}$ is achieved in (001) quantum wells by removing the mirror symmetry of the quantum well potential and allows an in-plane, two-fold symmetric anisotropy of both $\Gamma_s$ $[4]$ and $g$ $[5]$.

Fundamentally, $\Gamma_s$ and $g$ are both determined by spin-orbit interaction but the basic mechanisms for their anisotropies are quite different. Theoretically the in-plane anisotropy of $g$ is proportional to the asymmetry of the electron wavefunction in the growth direction with proportionality constant given by the Dresselhaus or bulk inversion asymmetry (BIA) spin-splitting coefficient $\gamma$. In contrast, $\Gamma_s$ is in many cases dominated by the Dyakonov-Perel spin-relaxation mechanism and the related in-plane anisotropy depends on the ratio $\alpha/\beta$ of the Rashba structural inversion asymmetry (SIA) to the BIA spin splitting $[4]$. The SIA component is determined in a rather subtle way by the asymmetry of the structure along the growth direction $[3, 5]$.

In this letter, we determine the absolute value of both the Rashba and Dresselhaus coefficients for a series of quantum well structures by simultaneously measuring the in-plane anisotropy of $\Gamma_s$ and $g$ by spin quantum beat spectroscopy $[6]$. The specially designed undoped (001) quantum well samples, with reduced $C_{2v}$ symmetry but without external electric fields, illustrate clearly the different origins of the two anisotropies as they possess a strong anisotropy of $g$ and nearly negligible anisotropy of $\Gamma_s$.

Anisotropies of $\Gamma_s$ and $g$ have been measured previously in symmetrically grown quantum wells in an external electric field $[10, 11]$ but the decisive simultaneous evaluation of Dresselhaus and Rashba components has not been carried out so far. Hanle experiments in undoped asymmetric quantum wells without an applied electric field have revealed a strong in-plane anisotropy of the Hanle depolarization curve $[12]$ but such measurements are unable to distinguish between anisotropy of $\Gamma_s$ and $g$ $[13]$. Recently, Ganichev and co-workers introduced a seminal technique that uses the angular distribution of the spin-galvanic effect and therewith measured the ratio of the Rashba and Dresselhaus coefficients in doped quantum wells $[14, 15]$. Salis and co-workers developed a technique that in principle yields the absolute values of the coefficients in doped structures by optically monitoring the angular dependence of the electrons spin precession $[16]$. However as the calculation of electric fields in these samples is complicated the values of the coefficients can be overestimated $[17]$.

We first summarise the theoretical mechanisms for $g$ and $\Gamma_s$ anisotropy $[1, 6]$. For $g$ a small magnetic field in z-direction $B_z$ deflects the rapid zero-point motion of an electron quantized in z-direction and yields a change of momentum in y-direction. This additional momentum $\delta p_y$ changes the effective Rashba Hamiltonian $\Omega_R(p)$ and Dresselhaus Hamiltonian $\Omega_D(p)$ precession vectors which read for (001) quantum wells in zinc-blend crystals

\begin{equation}
\Omega_R(p) = \alpha/\hbar^2 \begin{pmatrix} p_y & -px \\ px & 0 \end{pmatrix}, \quad \Omega_D(p) = \beta/\hbar^2 \begin{pmatrix} -p_x & p_y \\ p_y & 0 \end{pmatrix}
\end{equation}
where $\alpha$ and $\beta$ are coefficients and $p_{x,y,z}$ are the components of the electron momentum. Inspection of Eq. 1 shows that the Rashba term converts $\delta p_y$ into an additional magnetic field which is parallel to the external magnetic field $B_z$ and thereby alters the diagonal component of the $g$-tensor ($g_{xx} = g_{yy}$). By contrast, the Dresselhaus term $\Omega_D$ converts $\delta p_y$ to an additional magnetic field in $y$-direction, i.e. perpendicular to $B_z$ and thereby generates an off-diagonal component $g_{xy}$. A rigorous theoretical treatment yields [4]

$$g_{xy} = g_{yz} = (2\gamma e/\hbar^2 \mu_B) \left( \langle p_z^2 \rangle \langle z \rangle - \langle p_z^2 \rangle \langle z \rangle \right), \quad (2)$$

where $\mu_B$ is the Bohr magneton and $\langle \rangle$ represents an expectation value for the electron wavefunction. The two terms in Eq. 2 cancel and $g_{xy}$ vanishes if the electron wavefunction is symmetric. The anisotropy of the $g$-tensor is thus proportional to the Dresselhaus coefficient $\gamma$ and determined by asymmetry of the electron wavefunction which may be induced by asymmetry of the confining (conduction band) potential for the electrons. The effective $g$-factor for magnetic field oriented at angle $\phi$ to the (110) axis in the quantum well plane is given by

$$g(\phi) = -\sqrt{g_{xx}^2 + g_{yy}^2 + 2g_{xy}g_{xyz}\sin(2\phi)}. \quad (3)$$

For the spin relaxation which is dominated by the Dyakonov-Perel mechanism the rate in the quantum well plane $\Gamma^{xy}_s(\phi)$ is proportional to $\langle \Omega^2 \rangle$ where $\Omega = \Omega_D + \Omega_R$, the sum of Dresselhaus and Rashba components. It will be anisotropic as a result of interference of the components and is given by [4]

$$\Gamma^{xy}_s(\phi) = \frac{C}{2}(\alpha^2 + \beta^2 + 2\alpha\beta\sin(2\phi)),$$

where $C$ is a constant which depends on the in-plane electron momentum relaxation time which is not well known in general. Thus, the spin relaxation rate anisotropy gives the ratio $\alpha/\beta$, where $\beta \ll \gamma/\hbar$. 

Experimentally, we measure the electron spin relaxation rate along the growth direction ($z$) for a magnetic field applied in the quantum well plane. The magnetic field causes rapid Larmor precession of the electron spins about the magnetic field and the measured relaxation rate is given by the average of $\Gamma^z_s = C(\alpha^2 + \beta^2)$ and $\Gamma^{xy}_s(\phi)$ [11]:

$$\Gamma_s(\phi) = \frac{1}{2}(\Gamma^z_s + \Gamma^{xy}_s(\phi)) = D \left[ 1 + \left( \frac{\alpha}{\beta} \right)^2 + \frac{2\alpha}{3\beta} \sin(2\phi) \right], \quad (4)$$

where $D = 3C\beta^2/4$. Therefore measurement of both anisotropies yields simultaneously the absolute values of $\alpha$ and $\beta$. It is interesting to note that spin relaxation rate anisotropy has the same form as the $g$-factor but with $\beta$ replacing $g_{xx}$ and $\alpha$ replacing $g_{xy}$.

![Diagram](image.png)

**FIG. 1:** (Color online) Conduction band potential profile and numerical calculated electron wavefunction for the $n = 1$ states for a) sample A and b) sample B. c) The measured spin quantum beats at 125 K for sample A for 3 T in-plane magnetic field clearly showing the different electron g-factors for $B \parallel [110]$ and $[110]$ and similar spin relaxation times.

The samples are four molecular beam epitaxy grown, (001)-oriented GaAs/AlGaAs multiple quantum wells with varying asymmetry. Sample A comprises 5 repeats of a 12 nm Al$_{0.4}$Ga$_{0.6}$As barrier, an 8 nm GaAs quantum well followed by a 30 nm alloy layer where the aluminium concentration is varied linearly from 0.04 to 0.4. Samples B-D are equivalent structures but the one sided potential gradient is in the quantum well and has been grown as digital alloy with conduction band gradients equivalent to an electric field of 100 kV/cm for sample B, 50 kV/cm for sample C, and 25 kV/cm for sample D. Figure 1 shows the calculated $n=1$ electron states for samples A and B obtained by numerical solution of the Schrödinger equation. The calculated confinement energies for electrons in samples A to D are 34 meV, 91 meV, 61 meV and 37 meV, respectively.

The samples are mounted on a rotation stage in a liquid helium flow cryostat in a superconducting magnet with the magnetic field oriented in Voigt geometry. The rotation axis corresponds to the growth axis of the sample and is parallel to the direction of excitation. Spin oriented electrons are optically created by circularly polarized picosecond pulses from a mode-locked Ti:Sapphire laser with a repetition rate of 80 MHz, a laser wavelength of 740 nm and a pulse intensity yielding excitation density $\approx 2 \times 10^{10}$ cm$^{-2}$. After excitation the carrier momentum distribution rapidly thermalizes by emission of phonons and scattering with other carriers and the holes lose their spin orientation within the momentum relaxation time.
due to strong valence band mixing and \( k \) dependent spin splitting. The polarized luminescence is spectrally and temporally resolved by a spectrometer and a synchronscan streak camera with two-dimensional readout which provides a resolution of 0.5 nm and 8 ps, respectively. The degree of circular polarizations of the PL, which is proportional to the electron spin polarization, is measured using a switchable liquid crystal retarder and a polarizer.

Figure 2(c) depicts the time evolution of the degree of circular polarization for sample A at 3 T and 125 K for an in-plane magnetic field \( B \) along \([110]\) and \([110]\) directions. The observed oscillations are electron spin quantum beats the frequency being \( \omega_L = g_\mu_B h^{-1} B \) and so a direct measure of \( g \) for the particular magnetic field direction \([110]\). Measurements of beats in \(< S_z >\) in this way do not yield the sign of \( g \) but a comparison with previous measurements on symmetric QWs identifies that \( g \) is negative for samples A, C, and D and positive for sample B \([10, 18]\). The two clearly distinct oscillation frequencies in Fig. 2(c) directly demonstrate the in-plane \( g \) anisotropy whereas the nearly identical decay of the two polarisation transients indicate that \( \Gamma_y \) is very nearly isotropic.

Figure 2 shows in more detail the dependence of \( g_{xx} \) and \( g_{xy} \) on the direction of magnetic field in samples A and B. The black (red online) solid curves in Fig. 2 depict fits of the anisotropy of \( g \) using Eq. 3 which directly yield both \( g_{xx} \) and \( g_{xy} \). The diagonal components of the \( g \)-tensor \( g_{xx} = g_{yy} \) have been previously investigated in symmetrical quantum wells where the dependence on well width, i.e., confinement energy and barrier penetration, is well described by \( k \)-\( p \) theory \([18, 20]\). The solid squares in figure 3(a) show \( g_{xx} \) for all four samples confirming a similar strong dependence of \( g_{xx} \) on confinement energy for asymmetric QWs. The open squares in Fig. 3(a) show \( g_{xy} \) and these values yield by Eq. 2 the dependence of the Dresselhaus spin splitting constant \( \gamma \) on confinement energy (solid dots in Fig. 3(b)). The excellent agreement with data from Ref. 19 illustrates clearly that \( g_{xy} \) provides an accurate measure of \( \gamma \) in asymmetric \((001)\) quantum wells. The remaining deviations of \( \gamma \) from the trend probably result from differences between the actual and the nominal sample structures which lead to uncertainties in the calculation of the wavefunction asymmetry. The distinct decrease of \( \gamma \) with confinement energy is expected from \( k \)-\( p \) theory and has similar origin to the change of \( g_{xx} \) with confinement energy in Fig. 3(a).

Next, we study in detail the anisotropy of the spin relaxation rate. The open circles in Fig. 2(a) and (b) depict \( \Gamma_y(\phi) \) for sample A and B respectively and the grey solid curves are fits according to Eq. 4. Additional temperature and density dependent measurements confirm that the Dyakonov-Perel spin relaxation mechanism dominates \( \Gamma_y \). The measurements clearly show that there
is almost no in-plane anisotropy of $\Gamma_z$ and therefore $\alpha$ is close to zero even though the potential gradients in both samples are large ($> 90 \text{kV/cm}$).

Figure 4 compares $\alpha$ in our samples (solid circles) with previous experiments in external and internal (Hartree) electric fields (open circles) [21, 22]. The comparison of the measurements clearly show that the Rashba spin splitting in AlGaAs heterostructures is large even for a modest external (or internal) electric field but negligibly small in the case of asymmetries produced by alloy variation. Although allowed to be non-zero by the $C_{3v}$ symmetry of the samples, the values of $\alpha$ which are required to fit the present data are zero within experimental uncertainties; they show both positive and negative values with no clear trend as a function of potential gradient and the fitted value of $\alpha/\beta$ is in all cases less than 0.1. The measurements push down by an order of magnitude the previous upper limit of Rashba spin splitting observed in samples with asymmetry from alloy variation [2]. The small values of $\alpha$ are a direct consequence of the ‘isomorphic’ band edges, that is the conduction and valence band potentials are related by a constant factor. This is due to the fact, that the expectation value of the effective electric field always vanishes in the conduction band due to Ehrenfest’s theorem [3] and in ‘isomorphic’ structures as illustrated in the right hand panel of figure 4, will also vanish in the valence band and it is the latter which determines the spin splitting.

In conclusion, we have determined simultaneously the absolute values for the Dresselhaus and the Rashba spin-orbit interaction in undoped low-symmetry (001) quantum wells. All samples show a distinctive anisotropy of the electron g-factor but essentially isotropic spin relaxation rates. This difference highlights the different origins of the two phenomena; the first is a measure of the conduction electron wavefunction asymmetry and the latter a measure of the expectation value of the valence band potential on conduction bands states. Although, a one sided-gradient of the conduction and/or the valence band leads in general to a finite Rashba spin-orbit interaction, the experiment proves that isomorphism of valence and conduction band in GaAs/AlGaAs quantum wells prescribes a sizeable, gradient-induced Rashba spin-orbit splitting.

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