THE EFFECT OF STELLAR EVOLUTION ON POPULATION II CONTACT BINARIES IN THE PERIOD-COLOR RELATION. I. EQUAL-MASS, MARGINAL CONTACT SYSTEMS

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ABSTRACT

Field W Ursae Majoris binaries observe a well-known period-color relation such that systems containing more massive stars are bluer and have longer orbital periods than systems with lower mass components. However, it has been known for a decade that metal-poor W Ursae Majoris binaries are too blue, have too short an orbital period, or both. Correcting the observed color for the reduced line blanketing in the atmosphere of a Population II star accounts for only part of the observed discrepancy. As others have suggested and Rucinski recently showed, the smaller radii of Population II stars and the correspondingly shorter orbital periods are responsible for the remainder. In this paper I investigate the effect of evolution on the location in the period-color plane. This paper addresses the restricted case of equal-mass components and critical contact with their inner Roche lobes but should be applicable to the more general cases to the extent that the relative sizes of stellar components are preserved with metallicity changes. The calculated metallicity-age dependent period-color relations substantially agree with Rucinski & Duerbeck's recent empirically derived corrections to the period-color relation over most of the investigated range of periods. However, our predictions deviate to a greater degree as stellar age increases, since their parameterization does not include the effect of evolution.

Key words: binaries: close — binaries: eclipsing — globular clusters: individual (NGC 6397) —
stars: variables: other

On-line material: color figures

1. INTRODUCTION

W Ursae Majoris overcontact binaries (hereafter, contact binaries) are composed of two stars that are sufficiently close that they both overfill their Roche equipotential surfaces. They are observed in the field (e.g., Rucinski 1997; Hendry & Mochnacki 2000), in open clusters (Rucinski & Kaluzny 1994), and in globular clusters (Mateo et al. 1990; Kaluzny & Krzeminski 1993; Yan & Mateo 1994; Rubenstein & Bailyn 1996; Kaluzny 1997; Mazur, Krzeminski, & Kaluzny 1999; Mirabal & Mateo 1999; Thompson et al. 1999). In these studies, contact binaries typically represent roughly 0.1% of the sample size. W Ursae Majoris binaries are of great interest to the study of stellar populations because they can be used as a test of distance (Rucinski & Duerbeck 1997, hereafter RD97; Rubenstein & Bailyn 1996; Edmonds et al. 1996; Yan & Mateo 1994), because even a tiny fraction of binaries can qualitatively alter the dynamical evolution of dense stellar systems (Heggie & Aarseth 1992), and because they are the most frequently observed type of common-envelope objects.

The internal structure of contact binaries is complicated because of such factors as the tidal deformation of the components, heat flow and mass transfer between the stars, possible thermal relaxation oscillations, chromospheric activity, etc. (cf. Rahnen & Vilhu 1977; Webink 1979; Rahnen 1983; Sarna 1991; Jianmin 1993). However, many of their properties can be derived from spectroscopic observations or via light-curve modeling (see comprehensive text in Kallrath & Milone 1999), especially when the two are used together (Hilditch, King, & McFarlane 1987). In particular, the inclination angle (i), mass ratio (q = M2/M1), and fill-out ratio (F) can be described by study of the Fourier components of the light curve (Rucinski 1993). Light-curve fitting of the Wilson-Devinney (1971, 1973) type (including recent improvements by Kallrath et al. 1998) allows us to derive several additional stellar properties, such as the surface temperature of the common-envelope and starspot sizes, locations, and temperatures, as well as i, q, and F (see Kallrath & Milone 1999 and references therein).

As a result of these robust analyses, we have learned much about the underlying physics of contact systems. For example, we understand the origin of the period-color relation (PCR; Eggen 1967), the observation that W Ursae Majoris systems that are bluer have longer orbital periods. Briefly, a binary containing a massive star will be bluer than a binary composed of lower mass stars. Since massive stars are physically larger than low-mass stars, binaries containing a higher mass component must have a greater orbital separation to accommodate the greater stellar volume within its Roche lobe. This larger separation results in a longer period for these bluer systems.

However, the discovery of W Ursae Majoris binaries in globular clusters (GCs) made it apparent that the PCR depends on additional factors. Yan & Mateo (1994) demonstrated that the reduced line blanketing of metal-poor stellar atmospheres accounted for some of the difference in the GC W Ursae Majoris binaries' positions in the PCR compared with those of field binaries. Rucinski (1995) recognized that metallicity variations will change the stellar radii and therefore the resulting orbital period of W Ursae Majoris systems. He parameterized this effect by including a metallicity-dependent term and found that Δ(B−V) ∝ 0.08 × [Fe/H] for metallicity in the range −0.5 < [Fe/H] < +0.5, while for [Fe/H] < −2, Δ(B−V) ~ −0.15. RD97 have since propagated that parameterization to the PCR (their eqs. [1]−[3]), though they note that the significant scatter in their Figure 1 arises from variations in stellar age
and the resulting structural changes. Recently, Rucinski (2000) used the metallicity-radius relation of Castellani, Degl’Innocenti, & Marconi (1999) to determine the period changes produced by much lower than solar composition of W Ursae Majoris stars. He found that the period decrease was consistent with the deviation of the Population II W Ursae Majoris binaries from the Population I binaries in the period-color plane.

In § 2 of this work, I will discuss the relationship between the period and the color of W Ursae Majoris systems, taking into account evolution of the components, as well as structural changes arising from variations in the chemical composition of systems. I will discuss the results derived for systems composed of equal-mass components that critically fill their inner critical surfaces. Two examples of applications of these metallicity-age dependent period-color relations are given in § 3. The more general case of nonequal-mass components and arbitrary fill-out ratios will be discussed in a forthcoming paper, as will the consequences of rapid rotation.

2. DETERMINING THE METALLICITY-AGE DEPENDENT PERIOD-COLOR RELATION FOR NONSOLAR COMPOSITION BINARIES

Rather than try to determine what combination of "corrections" would translate the position of the Population II W Ursae Majoris binaries onto the Population I period-color relation (PCR), it is more enlightening to determine what the location of the PCR should be for a range of ages and metallicities. The envelopes of low-metallicity stars have lower opacities than those of higher metallicity stars. As noted previously (Yan & Mateo 1994; Rucinski 1995), this reduced opacity results in a higher effective temperature, $T_{\text{eff}}$, and a bluer color. However, the lower opacity also results in a smaller stellar radius for a given mass and evolutionary state. Therefore, the components of a low-metallicity contact binary must be closer together than their Population I counterparts to maintain a contact configuration. The resulting orbital period is shorter. The combination of these factors shifts the expected location of W Ursae Majoris systems in the period-color plane. Merely correcting for the reduced line blanketing of metal-poor stellar atmospheres does not adequately model this shift (cf. Yan & Mateo 1994; Rubenstein & Bailyn 1996).

However, the PCR for a given metallicity can be calculated using the Roche lobe geometry constraint and stellar models that provide the radii of the stars of different masses and ages. Specifically, the volume of the stars and the volumes of their respective Roche lobes must be equal for the stars to be in critical contact (both stars touching the inner Roche potential). For the case of marginal contact, a unique period can be calculated for a given pair of masses and radii by using the formula

$$P_{\text{orb}} = 0.1375 \sqrt{\frac{q - 1}{1 + q} \frac{4 \pi R_1^3}{3 M_1}}$$

where $q = M_2/M_1$ and $M_1$ and $R_1$ are the mass and radius of a component of the binary, respectively (Eggleton 1983). The dimensionless radius

$$r_L \sim \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}$$

is Eggleton’s equation (2); $r_L$ is the stellar radius divided by the separation of the centers of mass of the stars. By using models that span a range of masses, the locus of a PCR can be calculated. One PCR is derived for each set of age and composition.

To study the variation of the PCR over a range of age and chemical composition, I have used stellar models from the Yale Isochrones (Chaboyer et al. 1995, hereafter YI95) to determine the stellar radii for stars. In this work I consider stars below the main-sequence turnoff at solar composition, as well as those with $[\text{Fe/H}] = -0.7, -1.0, -1.5, -2.0,$ and $-3.0$ dex. I calculated the orbital period expected for cases in which two equal-mass stars are both in contact with their inner critical potential. This period and the $B-V$ color of the stars (from YI95) then define the PCR for stars having the same age and composition. The curves in the four panels of Figure 1 show the PCR for these compositions at 2, 5, 10, and 15 Gyr.

How reasonable are the equal-mass component and marginal contact assumptions? Most W Ursae Majoris binaries are not in critical contact and have mass ratios significantly less than 1. Certainly the variation of system properties is important. However, these two restrictions serve as a useful starting point. In any event, many W Ursae Majoris binaries are, in fact, fairly close to critical contact; in Table 1 of Rucinski & Duerbeck (RD97), 15 out of 33 systems have $f \leq 0.15$, i.e., are nearly in critical contact. To determine how reasonable the case of critical contact is, let us consider the variation in expected properties between an idealized system in marginal contact and a second system with an overcontact of 15% (i.e., $f = 0.15$). The 11% variation in stellar volume between these two cases corresponds to a difference in radial separation of less than 4% (see the procedure detailed in Mochnacki 1984, 1985). The orbital period difference is then less than 6%; this factor is negligible compared with the effects of both metallicity and age. Furthermore, the important comparison is between systems of various age-metallicity pairs for a given degree of contact, so only the differential change in orbital period at different age-composition values is relevant and this is an even smaller effect. While the bulk of W Ursae Majoris binaries have mass ratios significantly different from unity, there are exceptions. Table 1 of RD97 indicates that three out of 33 systems have $q \geq 0.75$. The fractional volume difference of two primaries having $q = 1.0$ and 0.75 is only 6%; of course the period difference will be larger. As before, the procedure delineated above depends on the differential period change between systems having different system parameters. The first-order structural changes are calculated in this work. The next paper will correct for these second-order effects.

Do solar-composition W Ursae Majoris binaries fit these calculated PCRs? The asterisks in Figure 1 mark local W Ursae Majoris binaries from Mochnacki (1981) and Rucinski (1983), and the plus signs mark the Baade’s window systems of Rucinski (1998). These calibration systems are plotted in the 2 and 5 Gyr panels only, as it is believed that the mean age of the local W Ursae Majoris binaries is approximately a gigayear (Duerbeck 1984). Clearly this application of YI95 models to equal-mass component, contact-configuration binaries adequately fits the calibration data for systems having approximately solar composition.

What can we learn about metal-poor W Ursae Majoris stars? As expected, models of metal-poor W Ursae Majoris binaries occupy the region further to the blue, with a shorter
Fig. 1.—Metallicity-age dependent period-color relation (MAD-PCR) for contact binaries. These results refer to equal-mass components that are in critical contact with their inner Roche lobes. In each panel, corresponding to stars of 2, 5, 10, and 15 Gyr, the separate lines refer to MAD-PCRs for solar composition, as well as [Fe/H] = −0.7, −1.0, −1.5, −2.0 (and −3.0 at 10 and 15 Gyr). The three circles on each curve correspond to the period-color location for a binary composed of two 0.5, 0.6, or 0.7 $M_\odot$ stars (left to right). Note that in the upper panels all three of these sets of models show the expected shortening of the orbital period at lower metallicity. The reduced orbital period is due to the smaller opacity and resulting smaller radii of the stars, which leads to a smaller orbital separation. Beyond 10 Gyr, the 0.7 $M_\odot$ models show the effect of the faster evolution of low-metallicity stars. The resulting increase in stellar radius more than compensates for the reduced opacity for Population II stars and leads to a period increase. In the upper panels the asterisks and plus signs correspond to local W Ursae Majoris binaries (Mochnacki 1981; Rucinski 1983) and Baade’s window W Ursae Majoris binaries (Rucinski 1998), respectively. The triangle in the 15 Gyr panel refers to V1 in NGC 6397 from Rubenstein & Bailyn (1996); it falls on the [Fe/H] = −2.0, 15 Gyr MAD-PCR. These quantities are roughly the same as the composition and age of NGC 6397, [Fe/H] = −1.9 (Zinn & West 1984; Djorgovski 1993) and 18 Gyr (Chaboyer, Demarque, & Sarajedini 1996), respectively. The errors arising from observational uncertainties are smaller than the triangle. [See the electronic edition of the Journal for color versions of Figs. 1–2.]

We would like to be able to compare the observationally based results of RD97 with those expected from these models. Unfortunately, RD97 and the preceding papers by Rucinski (1994, 1995, 1997) do not include age as a parameter since they are working with calibration objects of approximately known distance but unknown age, rather than with stellar models. Likewise, Rucinski (2000) does not discuss the age of components as it has impact on the PCR. To see what the RD97 relations might look like with various age stars, the YI95 models are combined with the RD97 relationships. The luminosity and stellar volume for a

period than the more metal-rich calibration W Ursae Majoris stars. The circles in each panel correspond to the location in the period-color plane for specific models (from left to right, 0.5, 0.6, and 0.7 $M_\odot$). These show that at younger ages low-metallicity W Ursae Majoris binaries have shorter orbital periods for all masses. Indeed, even for ages of 15 Gyr, metal-poor binaries with masses ≤0.6 $M_\odot$ have shorter orbital periods than more metal-rich systems. However, for ages typical of globular clusters, evolution of those stars at the turnoff leads to slightly longer orbital periods for critical-contact, metal-poor systems.
given mass, age, and metallicity define a particular model. Using that model, an inferred orbital period is calculated as above, and the expected $B-V$ color is determined from equation (1) of RD97. Figure 2 shows this comparison of the metallicity-age dependent (MAD) PCRs and the observationally based metallicity-color parameterization of RD97.

The solar-composition models yield nearly identical results for stars more massive than 0.4 $M_\odot$; the deviations at smaller mass are presumably due to our limited understanding of stellar opacities for low-temperature stars. In each panel, the heavy lines show the relevant MAD-PCR, while the thin lines show the results of RD97. The solid lines represent 2 Gyr in the left panels and 5 Gyr in the right, and the dashed lines represent 10 Gyr in the left panels and 15 Gyr in the right. The deviations between the MAD-PCR and RD97 lines are not significant since the reported 1σ uncertainty of RD97 in magnitude is $\pm 0.25$, implying a color uncertainty of 0.35 mag. It is therefore apparent that the structural changes that arise from the reduced opacity of Population II stars accounts for a significant fraction of the PCR differences for Population I and II stars. Indeed, the shortened orbital period combined with the reduced line blanketing seem to explain the location of Population II contact binaries, at least given the current limited range of binary component properties.

3. APPLICATIONS

With many hundreds of field W Ursae Majoris binaries and more than 50 systems in metal-poor globular clusters (Rucinski 2000 and references therein), we are strongly motivated to test the MAD-CPRs better against real data. As noted above, the bulk of W Ursae Majoris binaries have mass ratios significantly different from 1; for example, Table 1 of RD97 indicates that three out of 33 systems have $q \geq 0.75$. At present I will demonstrate the results of comparing two systems with the $B-V$ MAD-PCRs calculated above, and a more exhaustive comparison of known W Ursae Majoris systems with this technique will be made after the restrictions of the current model are relaxed. The planned work will also include $V-I$ MAD-PCRs since many contact systems are studied in those filters.

In the 15 Gyr panel of Figure 1 the triangle shows the position of V1 in NGC 6397 (Rubenstein & Bailyn 1996). V1 falls on the MAD-PCR for $[\text{Fe/H}] = -2$, consistent with the composition of this globular cluster (Zinn & West

![Figure 2](https://example.com/figure2.png)

**Fig. 2.—**MAD-PCR for five different compositions, shown in ten panels (*top to bottom*: solar to $[\text{Fe/H}] = -2.0$). The left panels show the calculations for 2 and 10 Gyr equal-mass components in critical contact with their inner Roche lobes. The right panels show those for 5 and 15 Gyr components. These calculations agree (to within 1σ) with the parameterized relation for $[\text{Fe/H}]$, $B-V$, and orbital period of RD97. The location of V1 in NGC 6397 (Rubenstein & Bailyn 1996) is consistent with the known properties of its host globular cluster.
The observational errors are smaller than the symbol. The agreement between the metallicity derived from this work and the cluster's known composition and estimated age (≈18 Gyr; Chaboyer, Demarque, & Sarajedini 1996) is encouraging even though Rubenstein & Bailyn's best estimate of V1’s mass ratio is ≈0.75, rather than this work's implicit mass ratio of 1.0.

Another indication of the role that future MAD-PCRs might fill follows. Covarrubias, Duerbeck, & Maza (2001) found a W Ursae Majoris binary roughly 1.4 kpc above the Galactic plane but were uncertain whether it is a Population I or II object. This spectroscopically confirmed binary has an orbital period of 0.35 days and \((B-V)_0 = 0.37\). Inspection of Figures 1 and 2 suggest that the object is likely a Population II binary with \([\text{Fe/H}] \leq -1.0\) dex; however, \([\text{Fe/H}] \approx -0.7\) for an age of only 2 Gyr cannot be ruled out on this basis.

4. CONCLUSION

Metallicity-age dependent period-color relations have been calculated to understand why Population II contact binaries are observed to be too blue or have too short an orbital period compared with their Population I analogs. This work confirms that the structural changes that arise from the reduced opacity of metal-poor stars lead directly to shorter orbital periods. The MAD-PCRs calculated agree with the empirically derived corrections (RD97) to the period-color relation to within 1 \(\sigma\) (about ±0.35 mag in \(B-V\)) down to at least \(P_{\text{orb}} \leq 0.2\) days. However, their parameterization does not include the effect of evolution. Therefore, the predictions deviate to a greater degree as stellar age increases.

Figures 1 and 2 show the MAD-PCRs for a range of ages and compositions. The position of the W Ursae Majoris binary V1 from the metal-poor globular cluster NGC 6397 (Rubenstein & Bailyn 1996) is shown in the 15 Gyr panel of Figure 1 and the \([\text{Fe/H}] = -2.0\) panels of Figure 2. This object falls almost precisely on the MAD-PCR appropriate to this cluster. The success in recovering the age and metallicity of V1 by looking for a best-fit MAD-PCR suggests that this technique may provide an additional way to derive approximate cluster parameters. However, mass transfer may alter the surface abundance of He and metals in the envelope of the accreting star. Therefore, any age-metallicity determinations based on this technique can be applied only to the cluster as a whole for those W Ursae Majoris binaries that have a mass ratio close to unity to ensure that little mass transfer contamination has occurred. Since most W Ursae Majoris binaries do not have mass ratios close to unity (RD97), only a small fraction of contact binaries can be used to obtain age-metallicity values for their host clusters. In most instances, this technique would be used to determine the properties of contact systems for which an age and metallicity were not known (see §3).

The planned follow-up work includes extending the range of mass ratios and degree of contact to test this technique more accurately against systems with known \(q\) and \(F\).

Additional stellar models having a range of helium abundance will also be calculated to determine whether this technique is sensitive to the helium enhancement that likely occurs in the late stages of mass transfer, though structural changes may make this determination difficult. In any event, contact systems with mass ratios close to unity might be able to provide insight into the relative rates of chemical enrichment of helium and iron among open clusters with supersolar abundances. Finally, models of rapidly rotating stars will be used to account for changes in stellar radius and effective temperature.

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REFERENCES

Castellani, V., degl'Innocenti, S., & Marconi, M. 1999, A&A, 349, 834
Chaboyer, B., Demarque, P., Guenther, D. B., Pinsonneault, M. H., & Pinsonneault, L. L. 1995, in The Formation of the Milky Way, ed. E. J. Alfaro & G. Tenorio-Tagle (Cambridge: Cambridge Univ. Press) (Y195)
Chaboyer, B., Demarque, P., & Sarajedini, A. 1996, ApJ, 459, 558
Covarrubias, R., Duerbeck, H. W., & Maza, J. 2001, in preparation
Djorgovski, S. G. 1993, in ASP Conf. Ser. 50, Structure and Dynamics of Globular Clusters, ed. S. G. Djorgovski & G. Meylan (San Francisco: ASP), 373
Duerbeck, H. W. 1984, Ap&SS, 99, 363
Edmonds, P. D., Gilliland, R. L., Guhathakurta, P., Petro, L. D., Saha, A., & Shara, M. M. 1996, ApJ, 468, 241
Eggen, O. J. 1967, MmRAoS, 70, 111
Eggleton, P. P. 1983, ApJ, 268, 368
Heggie, D. C., & Aarseth, S. J. 1975, MNRAS, 257, 513
Hendry, P. D., & Mochnacki, S. W. 2000, ApJ, 531, 467
Hilditch, R. W., King, D. J., & McFarlane, T. M. 1988, MNRAS, 231, 341
Jianmin, W. 1993, ApSS, 209, 273
Kalirai, J., & Milone, E. F. 1999, Eclipsing Binary Stars: Modeling and Analysis, ed. J. Kalirai & E. F. Milone (New York: Springer)
Kalirai, J., Milone, E. F., Terrell, D., & Young, A. T. 1998, ApJ, 508, 308
Kaluzny, J. 1997, A&A, 323, 1
Kaluzny, J., & Krzeminski, W. 1993, MNRAS, 264, 785
Mateo, M., Harris, H. C., Nemec, J., & Olszewski, E. W. 1990, AJ, 100, 469
Mazur, B., Krzeminski, W., & Kaluzny, J. 1999, Acta Astron., 49, 551
Mirabal, N., & Mateo, M. 1999, BAAS, 194, 110508
Mochnacki, S. W. 1981, ApJ, 245, 650
———., 1984, ApJS, 55, 551
———., 1985, ApJS, 59, 445
Rahnen, T. 1983, A&A, 220, 235
Rahnen, T., & Vilhu, Q. 1977, A&A, 56, 99
Rubenstein, E. P., & Bailyn, C. D. 1996, AJ, 111, 260
Rucinski, S. M. 1983, A&A, 127, 84
———., 1993, PASP, 105, 1433
Rucinski, S. M. 1994, PASP, 106, 462
———., 1995, PASP, 107, 648
———., 1997, AJ, 113, 407
———., 1998, AJ, 116, 2998
———., 2000, AJ, 120, 319
Rucinski, S. M., & Duerbeck, H. W. 1997, PASP, 109, 1340 (RD97)
Rucinski, S., & Kaluzny, J. 1994, Mem. Soc. Astron. Italiana, 65, 113
Sarna, M. J. 1991, A&A, 241, 416
Thompson, I. B., Kaluzny, J., Pych, W., & Krzeminski, W. 1999, AJ, 118, 462
Webbink, R. F. 1979, ApJ, 227, 178
Wilson, R. E., & Devinney, E. J. 1971, ApJ, 166, 605
Yan, L., & Mateo, M. 1994, AJ, 108, 1810
Zinn, R., & West, M. J. 1984, ApJS, 55, 45