ARE THE NEARBY AND PHOTOMETRIC STELLAR LUMINOSITY FUNCTIONS DIFFERENT?

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Abstract

The stellar luminosity function derived from the sample of stars within 5.2–20 pc is the nearby luminosity function. The luminosity function obtained from deep low spatial resolution surveys to distances of typically 100–200 pc is the photometric luminosity function. We obtain and tabulate a best estimate of the parent distribution from which Malmquist corrected photometric luminosity functions are sampled. This is presently the most accurate and precise available estimate of the true distribution of stars with absolute magnitude for $M_V \leq 16.5$, as derived from low-resolution surveys. Using rigorous statistical analysis we find that the hypothesis that the nearby and photometric luminosity functions estimate the same parent luminosity function can be discarded with 99 per cent confidence or better for $M_V > 13, M_{bol} > 10$.

Subject headings: stars: low-mass – stars: luminosity function
1 INTRODUCTION

The difference between the faint end of the stellar luminosity function estimated from the stellar sample in the immediate neighbourhood of the Sun and the luminosity function obtained from photographic surveys which reach to distances of 100–200 pc has been discussed by Dahn, Liebert & Harrington (1986). They point out that the nearby luminosity function contains more than 2 sigma more stars in the $M_V = 16 − 17$ double bin than the photometric luminosity function estimated by Reid & Gilmore (1982). Since then there has been a substantial debate on this question (see e.g. Stobie, Ishida & Peacock 1989; Henry & McCarthy 1990; Kroupa, Tout & Gilmore 1991; Reid 1991; Kroupa, Tout & Gilmore 1993) fuelled by the discrepant conclusions made by researchers hampered by small number statistics.

Tinney (1994), however, concludes that “the debate in recent years over the ‘difference’ between the photometrically- and trigonometrically-selected LFs has been somewhat of a mare’s nest”. He bases this conclusion on the extensive photographic survey reported in Tinney, Reid & Mould (1993) which estimates the photometric luminosity function to an unprecedented precision because of the very large sample size, being based on 3538 stars in the magnitude range $8.5 \leq M_{bol} \leq 15.0$. Tinney (1993, 1994) and Reid (1994) find that on the bolometric magnitude scale there is no difference between the photometric luminosity function and the nearby luminosity function at faint magnitudes.

It is because there appear in the recent literature contradictory opinions and views as to the significance of, reality of and reason for the difference between the nearby and photometric luminosity function that we here cast a critical eye on the observational data obtained by a number of researchers in order to (i) establish the significance of the difference between the nearby and photometric luminosity function, and (ii) to verify the results obtained by Tinney (1993) and to study the accuracy of his estimate of the photometric stellar luminosity function. We are able to do this by a statistical analysis of various observational estimates of the photometric luminosity function which were not accessible to Dahn et al. (1986).

We proceed as follows: We first show that the four independent observational estimates of the Malmquist uncorrected photometric luminosity function by Reid & Gilmore (1982), Gilmore, Reid & Hewett (1985), Stobie et al. (1989) and Kirkpatrick et al. (1994) can be used to estimate the parent Malmquist uncorrected photometric luminosity function. We then apply Malmquist corrections and derive a best estimate of the true distribution of stars with absolute $V$-band magnitude as obtained from photographic, or generally deep-sky, surveys. In Section 2 we introduce and tabulate the primary observational data we are concerned with, and in Section 3 we estimate the parent distribution. In Section 4 we test the hypothesis that the nearby luminosity function estimates the same parent distribution as the best estimate of the true photometric luminosity function, and in Section 5 we show that the results carry over into bolometric magnitudes and we compare our best estimate for the true photometric luminosity function with the estimate used by Tinney (1993). Section 6 contains our conclusions.

The possible reasons for the difference are discussed in Kroupa (1995a).

2 THE OBSERVATIONAL ESTIMATES

To estimate the stellar luminosity function from photographic surveys requires identifying all low-mass (i.e. red) stars on the survey photographic plates and estimating stellar number densities using photometric parallax. This is a formidable task because the significant contamination by galaxies, giant stars in the Galaxy and white dwarfs needs to be eliminated. Modern automatic plate scanning machines (e.g. COSMOS and APM, UK) enable this task to be performed. Although only a narrow cone is available the sampling volume in which star counts are complete is large owing to the large sampling distance (100−200 pc).

Photographic star counts need to be corrected for Malmquist bias (see Stobie et al. 1989 for a thorough discussion) which arises because 1) mean absolute magnitudes of stars of a particular colour in a magnitude limited sample are brighter than the mean absolute magnitude in a volume limited sample, and 2) the stellar number density in a magnitude limited sample is larger than in a volume limited sample. Both effects arise because i) photometry has errors, and stars of a given colour or mass have a range of luminosities since they have different metallicities, ages, and may be unresolved binary systems (this scatter in luminosities is referred to as cosmic scatter), and ii) the number of stars increases nonlinearly with increasing distance.
Thus, intrinsically bright stars are overrepresented in a magnitude limited sample. We return to Malmquist corrections in Section 3.3. We refer to the Malmquist uncorrected photometric luminosity function as the ‘raw’ or ‘observed’ photometric luminosity function, $\Psi_{\text{phot}}$. The photometric luminosity function corrected for Malmquist bias is $\Psi_{\text{phot}}$.

Reid & Gilmore (1982) published the first estimate of the faint photometric stellar luminosity function, $\Psi_{\text{RG}}(M_V)$, obtained from a survey of photographic plates in the direction of the South Galactic Pole ($l = 0, b = -90^\circ$). They use the V- and I-band photometric parallax estimation and count 85 stars in the magnitude interval $8.5 \leq M_V \leq 16.5$. We focus our attention on two further photographic surveys using the same photometric bands (Reid (1991) comments on the much larger Malmquist corrections necessary when using the steeper $M_I, R - I$ relation). These are the surveys by Gilmore et al. (1985) in the direction $l = 37^\circ, b = -51^\circ$, $\Psi_{\text{GRH}}(M_V)$, based on 64 stars in the magnitude interval $8.5 \leq M_V \leq 16.5$ and by Stobie et al. (1989) towards the North Galactic Pole ($l = 0, b = +90^\circ$), $\Psi_{\text{SIP}}(M_V)$, based on 178 stars in the magnitude interval $7.5 \leq M_V \leq 16.5$. We also use the recent V-, R- and I-band CCD Transit Instrument survey of faint stars by Kirkpatrick et al. (1994). Their estimate of the photometric luminosity function, $\Psi_{\text{Kirk}}(M_V)$, is based on photometric parallax estimation in the V- and I-bands and has 121 stars in the magnitude interval $10.5 \leq M_V \leq 15.5$. The estimates of the photometric luminosity function by Hawkins & Bessell (1988) and Leggett & Hawkins (1988) agree with the other surveys (see Stobie et al. 1989 and Kirkpatrick et al. 1994, respectively) but we do not use these because their estimates are obtained using photometric parallax in the R- and I-bands (see Section 3.3).

Stobie et al. (1989) show that the Malmquist corrections applied to the luminosity function prior to their study were incomplete [because only point 1) above was corrected for, i.e. Reid & Gilmore (1982) and Gilmore et al. (1985) do not correct for the apparent increase in stellar number density], and that the peak in the photometric luminosity function at $M_V \approx 12$ is much more enhanced after correction for the full Malmquist bias. In particular, raw photometric luminosity functions overestimate stellar number densities at $M_V > 12$. The contribution of unresolved binaries and metallicity to cosmic scatter varies with magnitude which would require an even more elaborate treatment of Malmquist corrections than performed by Stobie et al. who assumed a constant Gaussian cosmic scatter. Kirkpatrick et al. (1994) account for a varying uncertainty in absolute magnitudes as a function of absolute magnitude, but only consider measurement errors in their photometry, $\sigma_{\text{phot}} \approx 0.1$ mag for $10.25 < M_V < 14.00$. This severely underestimates Malmquist bias because it is proportional to $\sigma_{\text{tot}}^2$ (Stobie et al. 1989). The total scatter in absolute magnitudes owing to the dispersion in metalicities, unresolved binary systems, ages and photometry errors is $\sigma_{\text{tot}} \approx 0.5$ mag (Kroupa et al. 1993). Summarising, we find that of all four estimates of the photometric luminosity function only Stobie et al. (1989) have correctly applied Malmquist corrections. To obtain an improved estimate of the parent distribution from which Malmquist corrected photometric luminosity functions are sampled we use the readily available raw luminosity functions to first improve our estimate of the Malmquist uncorrected parent distribution which we correct for Malmquist bias in Section 3.3.

The sample of stars in the neighbourhood of the sun allows estimation of the stellar luminosity function using trigonometric parallax measurements. For $M_V < 11.5$ the complete sampling distance is 10–20 pc (Wielen, Jahreiss & Krüger 1983), but for fainter magnitudes one must resort to the stellar sample within a distance of about 5 pc to obtain probably complete number density estimates. For $M_V \geq 11.5$ we base the nearby luminosity function, $\Psi_{\text{near}}$, on the stellar sample northward of declination $-20^\circ$ and within the 5.2 pc sampling distance (Dahn et al. 1986) extended in Kroupa et al. (1993) to include the discovery by Henry & McCarthy (1990) that one of the ‘stars’ in the 5.2 pc sample is a binary system. The sampling volumes for $\Psi_{\text{near}}$ are $V = 33510.3$ pc$^3$ ($M_V < 7.5$), $V = 25146.9$ pc$^3$ ($7.5 \leq M_V < 9.5$), $V = 3143.4$ pc$^3$ ($9.5 \leq M_V < 11.5$) and $V = 395.2$ pc$^3$ ($11.5 \leq M_V$).

Owing to the small sampling distance the nearby luminosity function is less well constrained for $M_V > 12$ than the photometric luminosity function. We would need an increase of the nearby sample by at least an order of magnitude to significantly improve our estimate of the nearby luminosity function. The parallax limited nearby star count data suffer under a bias similar to the Malmquist bias because of the finite error in parallax measurements (Lutz & Kelker 1973) and because the metallicity and age dispersion smears out the shape of the nearby luminosity function (compare figs. 1 and 20 in Kroupa et al. 1993). Corrections similar to the Malmquist corrections have not been performed on the nearby sample to date. The results
of this paper are, however, not affected by neglecting to correct for these effects because the corrections are smaller than the statistical uncertainties per magnitude bin which are very large for \( M_V > 12 \).

In Table 1 we tabulate the four estimates of the photometric luminosity function not corrected for Malmquist bias by the respective authors. Column 1 lists the absolute visual magnitude of the bin centre. Columns 2 and 3 tabulate the raw estimate of the photometric luminosity function by Reid & Gilmore (1982), \( \Psi_{RG}^{*} \) and the number of stars per magnitude bin, \( n_b \), respectively. We have corrected \( \log_{10}(\Psi_{RG}^{*}) \) for the disk density gradient of +0.11 as suggested by Reid & Gilmore (1982), but have ignored their +0.08 mag Malmquist correction. Similarly, Columns 4 and 5 contain \( \Psi_{GRH}^{*} \), Columns 6 and 7 tabulate \( \Psi_{SIP}^{*} \), and Columns 8 and 9 list \( \Psi_{Kirk}^{*} \). Columns 10 and 11 contain our best estimate of the raw photometric luminosity function obtained in Section 3.2. \( \overline{\Psi}_{\text{phot}}^{*} \). In Table 2 we tabulate our best estimate of the Malmquist corrected photometric luminosity function, \( \overline{\Psi}_{\text{phot}} \) (Columns 2 and 3, derived in Section 3.3) and Columns 4 and 5 list the nearby luminosity function, \( \Psi_{\text{near}} \). We note that in all cases the statistical uncertainty in \( \Psi \) is \( \sigma = \Psi / n_b^{1/2} \). The luminosity functions are accurate to three significant figures only, but we list four to reduce rounding errors in our statistical tests. The four observational estimates of the raw photometric and of the nearby luminosity function are compared in Fig. 1.

It is quite possible that further faint stars will be found within the 5.2 pc sampling distance. However, we do not expect a significant revision of the stellar number density at \( M_V < 16 \). Recent trigonometric parallax measurements suggest that the distances to GL 445 and GJ 1116AB are 5.22 and 5.25 pc, respectively (T. Henry, private communication). These stars are in the 5.2 pc sample of Kroupa et al. (1993), from which the nearby luminosity function in Table 2 is estimated. However, changing the nearby luminosity function to account for the nearby parallax measurements is not warranted at this stage because: (i) The trigonometric parallax measurement uncertainties imply an uncertainty in distance of about 0.1 pc at a distance of 5.2 pc so that both stars cannot be proven to lie outside the 5.2 pc distance limit. (ii) A new sampling volume can be defined: declination northward of \(-20^\circ\) as used by Kroupa et al. (1993), but with a distance limit of 5.23 pc. This change in sampling volume would retain the stars in the sample of Kroupa et al. (1993) but would change the normalisation of the nearby luminosity function in Table 2 by 1.7 per cent which is insignificant. A new star, GL 866C (\( M_V = 15.8 \)) has been added to the 5.2 pc sample (T. Henry, private communication). This would increase the number density near \( M_V = 16 \) increasing the difference between the nearby and photometric luminosity functions. By not counting this star we thus underestimate the difference between the nearby and photometric luminosity functions.

3 THE BEST ESTIMATE PHOTOMETRIC LUMINOSITY FUNCTION

In this section we show that the four estimates of the observed photometric luminosity function listed in Table 1 are consistent with each other and we combine these to our best estimate of the Malmquist uncorrected photometric luminosity function, \( \overline{\Psi}_{\text{phot}}^{*} \). We do not make use of the photometric luminosity function presented by Tinney (1993) in this section. We need to obtain an independent estimate of the photometric luminosity function because in Section 5 it is our aim to test the accuracy of the photometric luminosity function used by Tinney (1993).

3.1 Tests

Consider the hypothesis \( H_1 \) that the four photometric luminosity functions \( \Psi_{RG}^{*}, \Psi_{GRH}^{*}, \Psi_{SIP}^{*}, \Psi_{Kirk}^{*} \) (Table 1) estimate the same underlying parent distribution \( \Psi_{\text{phot}}^{*} \). We test this hypothesis with the chi-squared test and compute the reduced chi-squared

\[
\chi^2_{\nu} = \frac{1}{\nu} \sum_{i=1}^{I} \frac{(\Psi_{1,i}^{*} - \Psi_{2,i}^{*})^2}{\sigma_{1,i}^2 + \sigma_{2,i}^2},
\]

where \( \Psi_{1,i}^{*} \) and \( \Psi_{2,i}^{*} \) are the luminosity function estimates under consideration in magnitude bin \( i \) and \( \sigma_{1,i}^2 \) and \( \sigma_{2,i}^2 \) are their respective uncertainties; \( I \) is the number of magnitude bins, and in our present test the number of degrees of freedom is \( \nu = I \) (no parameters are estimated from the samples).
The significance probability \( p_{\chi^2} = P(\chi^2 \geq \chi^2_0) \) of obtaining a chi-squared value as large as or larger than \( \chi^2_0 \) is the measure of the level of significance of the result. We take as the significance probability at which we reject the hypothesis is \( \alpha_r = 0.01 \), which corresponds to obtaining \( \chi^2 \geq \chi^2_0 \) once in a hundred samples. If \( p_{\chi^2} < \alpha_r \) then we are safe to discard \( H_1 \).

We test the hypothesis on both sides of the maximum in the stellar luminosity function, as well as over the whole range, and tabulate \( \chi^2 \) and \( p_{\chi^2} \) in Table 3. Since \( p_{\chi^2} > \alpha_r \) for all tests in Table 3 we do not reject \( H_1 \).

The chi-squared test is valid if the underlying population distribution is approximately normal. In order to verify our conclusion we apply the non-parametric Wilcoxon signed-rank test, which we discuss in greater detail in Section 4.2. Applying this test on all pairs \( (\Psi^i, \Psi^j) \), where \( k, l = \text{RG,GRH,SIP,} \) Kirk, we obtain significance probabilities \( p_T = P(T^+ \geq x) > \alpha_r \) in all cases, where \( T^+ \) is the sum of the ranks. Details are tabulated in Table 4. The number of nonzero differences is \( n' \). On the basis of the Wilcoxon signed-rank test we cannot reject \( H_1 \).

However, the inability to tell the luminosity functions apart is not equivalent to showing they are alike. The average velocity dispersion of the stellar population in the Galactic disk is about 50 km sec\(^{-1}\). Over a length scale of 400 pc the stellar population will thus mix within about 8 Myrs. Since the Sun is located within a few tens pc of the Galactic midplane we thus have no astrophysical reason to expect significantly different low-mass stellar populations along different lines of sight.

We conclude that it is safe to assume that \( \Psi^\text{RG}, \Psi^\text{GRH}, \Psi^\text{SIP}, \Psi^\text{Kirk} \) can be used to compute an improved estimate of the Malmquist uncorrected photometric luminosity function.

### 3.2 The best estimate of the parent raw photometric luminosity function

We compute the weighted average observed photometric luminosity function

\[
\overline{\Psi}_{\text{phot}} = \frac{\sum_i w_i \Psi^i_{\text{phot}}}{\sum_i w_i}, \tag{2a}
\]

\[
\overline{\sigma} = \left( \sum_i w_i \right)^{-\frac{1}{2}}, \tag{2b}
\]

where \( i = \text{RG,GRH,SIP,Kirk} \) and \( w_i = 1/\sigma^2_i \). It is tabulated in Columns 10 and 11 (where we tabulate \( n_b \), which is the sum of all stars in each magnitude bin, rather than \( \overline{\sigma} \)) in Table 1 and plotted in Fig. 1.

### 3.3 Malmquist corrections

In Section 2 we have outlined the necessity of correcting the observed photometric luminosity function for Malmquist bias. The Malmquist correction is a function of \( M_V \) and can be written as

\[
\Delta \Psi = \Psi^*_{\text{phot}} - \overline{\Psi}_{\text{phot}} = \sigma^2_{\text{tot}} F, \tag{3}
\]

where \( F = \frac{1}{2} \left[ (0.6 \ln10)^2 \Psi_{\text{phot}} - (1.2 \ln10)^2 \right] + \frac{d^2 \Psi_{\text{phot}}}{dM^2} \] (equation 17 in Stobie et al. 1989, cf also with equation 11 in Kirkpatrick et al. 1994). The four estimates of \( \Psi^*_{\text{phot}} \) we use were obtained using the same photometric bands, and can be assumed to stem from the same parent distribution which we estimate to be \( \overline{\Psi}_{\text{phot}} \) in Section 3.2. We assume that the true parent distribution, \( \Psi_{\text{phot}} \), can be approximated by the same Gaussian model as used by Stobie et al. (1989) to evaluate \( F \). We will verify this below by (i) showing in Fig. 3 that our best estimate Malmquist corrected photometric luminosity function, \( \overline{\Psi}_{\text{phot}} \), is indistinguishable from the Malmquist corrected photometric luminosity function estimated by Stobie et al. (1989), and (ii) by deriving the (incorrect) Malmquist corrections applied by Kirkpatrick et al. (1994), \( \Delta \Psi^\text{Kirk}(M_V) \), from our Malmquist corrections, \( \Delta \Psi(M_V) \).

Our assumption implies \( \Delta \Psi = \Delta \Psi_{\text{SIP}} \), which are the Malmquist corrections computed by Stobie et al. (1989). These we plot in the upper panel of Fig. 2, where we demonstrate that (i) the uncertainties, \( \overline{\sigma} \), in our best estimate observed photometric luminosity function are significantly smaller than the Malmquist corrections, \( \Delta \Psi \), for \( 11.5 < M_V < 14 \), (ii) \( \sigma^2_{\text{Kirk}} \approx \sigma^2_{\text{SIP}} \) because both samples have approximately the same
number of stars per magnitude bin, (iii) the Malmquist corrections applied by Kirkpatrick et al. (1994) significantly underestimate the Malmquist bias, and (iv) $\Delta \Psi_{Kirk}^{tot} \approx (\sigma_{b})^2 \Delta \Psi_{SIP}$ is a good estimate of $\Delta \Psi_{Kirk}$, where $\sigma_{M}$ is the uncertainty in $M_V$ used by Kirkpatrick et al. (1994) to compute $\Delta \Psi_{Kirk}$ and is tabulated in their table 7. The correct cosmic scatter is $\sigma_{tot} \approx 0.51$ mag (Stobie et al. 1989, Kroupa et al. 1993).

Point (iv) verifies that our assumption of a universal $F$ for photometric luminosity functions obtained in the V- and I-bands is reasonable because Kirkpatrick et al. (1994) estimate $F$ using $\Psi_{Kirk}$ instead of an assumed model $\Psi_{phot}$ for the parent Malmquist corrected photometric luminosity function.

For comparison with $\Delta \Psi(M_V)$ we plot in the lower panel of Fig. 2 $\Delta \Psi_{TRM}(M_{bol})$, which are the first order Malmquist corrections estimated for the photometric luminosity function in bolometric magnitudes by Tinney et al. (1993). Clearly $\Delta \Psi_{TRM} >> \Delta \Psi$. We also observe that the uncertainties, $\sigma_{TRM}^2$, in the Tinney et al. (1993) survey are significantly better than $\sigma_{SIP}^2$ and $\sigma_{Kirk}^2$. That is, the precision of the estimate of the photometric luminosity function is improved significantly by the survey of Tinney et al. (1993).

We obtain our best estimate, $\overline{\Psi}_{phot}$, of the Malmquist corrected photometric luminosity function in the V-band by evaluating $\overline{\Psi}_{phot} = \overline{\Psi}_{phot} - \Delta \Psi_{phot}$, with $\overline{\sigma} = \overline{\Psi}_{phot}/n_b$, where $n_b$ are the number of stars in each magnitude bin obtained by adding the number of stars in each survey used to estimate the photometric luminosity function (Table 1). Our best estimate for the true distribution of stars with $M_V$, as obtained from deep surveys, is tabulated in Table 2.

4 ARE THEY DIFFERENT?

In this section we compare the nearby luminosity function, $\Psi_{near}$, with the best estimate Malmquist corrected photometric luminosity function, $\overline{\Psi}_{phot}$.

We note from Table 2 and Fig. 1 that the data in the nearby sample are sparse at $M_V > 13$. In this situation it is common practice to group the data into larger intervals to reduce loss of information (see e.g. Bhattacharyya & Johnson 1977, p.18). We combine the $M_V = 14, 15$ and $M_V = 16, 17$ bins to obtain $\Psi_{near,sm} = (13.9 \pm 5.9) \times 10^{-3} \text{pc}^{-3} \text{mag}^{-1}$ in the first two bins, and $\Psi_{near,sm} = (11.4 \pm 5.4) \times 10^{-3} \text{pc}^{-3} \text{mag}^{-1}$ in the last two bins and plot these in Fig. 3.

From the figure it is apparent that the nearby luminosity function differs at faint magnitudes from $\overline{\Psi}_{phot}(M_V)$, but that both agree approximately at bright magnitudes. In this section we assess the significance of the difference between the nearby and photometric luminosity functions evident to the eye-ball in Fig.3.

Consider the hypothesis $H_2$ that $\Psi_{near}(M_V)$ and $\overline{\Psi}_{phot}(M_V)$ estimate the same underlying stellar luminosity function. If the difference suggested in Fig. 3 between the two luminosity functions is a chance fluctuation (Reid 1987, 1991) then $H_2$ cannot be rejected at the level of significance $\alpha_e$ (Section 3.1).

To ensure statistical rigour three tests are applied: the chi-squared test, the Wilcoxon signed-rank test and estimation of the probability of observing the number of stars as are in the different samples.

4.1 The chi-squared test

The large Poisson uncertainties in each magnitude bin in the nearby luminosity function preclude comparing the shapes of the two luminosity functions in detail. We therefore combine the $M_V = 9 - 12$ ($i = 1 - 4$) and the $M_V = 13 - 16$ ($i = 1 - 4$) bins and compute for $\Psi_{near}$ ($j = 1$) and $\overline{\Psi}_{phot}$ ($j = 2$)

$$\Psi_{tot,j} = \sum_{i=1}^{4} \Psi_{i,j}, \quad (4a)$$

$$\sigma_{tot,j} = \left( \sum_{i=1}^{4} \sigma_{i,j}^2 \right)^{\frac{1}{2}}, \quad (4b)$$

and
In Table 5 we tabulate $\chi_{\text{near}}^2$ for the bright and faint ends, and the significance probability $p_{\chi^2} = P(\chi^2 \geq \chi_{\text{near}}^2)$.

As evident from the entries in Table 5 the chi-squared test implies that the hypothesis, $H_2$, that both the nearby and the photometric luminosity functions estimate the same distribution of stars with $9 \leq M_V \leq 12$ cannot be rejected. However, the hypothesis can be rejected with high confidence ($p_{\chi^2} < \alpha_r$) for stars with $13 \leq M_V \leq 16$.

4.2 The Wilcoxon signed-rank test

As mentioned in Section 3.1 the chi-squared test can be applied safely if the underlying population distribution is approximately normal. Because we seek to make a rigorous comparison between the photometric and nearby luminosity functions we also apply a distribution-free, or nonparametric test. The Wilcoxon signed-rank test (see e.g. Bhattacharyya & Johnson 1977, p.519) is designed to allow rigorous comparison between two data sets which can consist of a small ($n' \leq 15$) number of data points. This test requires no assumption about the shape of the population distribution, and allows for the magnitude of the difference between the data in two samples. In our case the data points in one sample are $(\Psi, M_V)$.

We compute the difference $\Psi_{\text{near}} - \bar{\Psi}_{\text{phot}}$ in each magnitude bin, $M_V = 9, 10, ..., 16$, and assign a rank to each of the differences which are ordered according to increasing absolute value. The sum of the ranks of the positive differences is $x$, and the sample size is $n' = 8$. The significance probability of obtaining a signed-rank statistic $T^+$ as large as or larger than $x$ is $p_{T^+} = P(T^+ \geq x)$ which is evaluated from tables. The result is listed in Table 6.

We conclude that hypothesis $H_2$ can be rejected safely because $p_{T^+} < \alpha_r$.

4.3 The Gauß test

In Sections 4.1 and 4.2 we have seen that we have strong confidence that $\Psi_{\text{near}}$ does not stem from the same parent distribution as $\bar{\Psi}_{\text{phot}}$. Nevertheless, we apply another test here which is based on estimating the probability of observing $n$ stars in the photographic (i.e. low spatial resolution) survey if $N$ stars have been counted in the nearby sample.

In Table 7 we list the details. Column 1 contains the $M_V$ bins on which the number of stars is based. We have combined the magnitude bins in which the stellar number density is derived from the same sampling volume $V$ in the nearby luminosity function. Column 2 lists the volume $V$ of the nearby sample, and the number of stars $N$ observed in this volume is listed in Column 3. The number of stars expected to be counted in $V$ in the photographic survey is $n = V \sum_{M_V} \Psi_{\text{phot}}$ and is listed in Column 4. Column 5 contains $z = (N - n)/N^{1/2}$, and Column 6 lists the probability $p_G$ that $n$ or fewer stars are observed given that the population consists of $N$ stars, and is evaluated from the standard normal distribution.

From Table 7 we observe that the number of stars in the photographic survey expected to be seen in the same sampling volume as is available for the nearby sample is consistent with the number of stars seen in the nearby sample for $M_V = 10 - 11$. However, the photographic survey yields far too few stars with $M_V = 13 - 16$. The probability of observing $n = 5$ stars given that we expect $N = 20$ stars is $p_G << \alpha_r$.

We can reject hypothesis $H_2$ with high confidence.

5 THE BOLOMETRIC LUMINOSITY FUNCTIONS

The recent precise luminosity function, $\Psi^*_{\text{TRM}}$, constructed from a large-scale R- and I-band photographic survey by Tinney et al. (1993) is tabulated in table 4 of Tinney (1993) in bolometric magnitudes. Comparing this bolometric luminosity function with the nearby luminosity function he concludes that there is no difference between the two distributions on the bolometric magnitude scale.

In view of our discordant finding in Section 4 from Tinney’s, we now transform our best estimate of the photometric luminosity function to the bolometric magnitude scale. We use equation 2 of Reid (1991)
to transform our $M_V$ magnitudes to $M_K$, and the bolometric correction given by Tinney (1993) in his fig. 5 and obtain
\[ M_V = -5.07 + 1.73 M_{\text{Bol}}, \] (6a)

and
\[ M_K = -1.87 + 0.91 M_{\text{Bol}}. \] (6b)

This is for the present purpose of comparing our photometric luminosity function with Tinney's an adequate approximation in the range $5 < M_K < 11$, i.e. $8.1 < M_V < 17.7$, as can be verified by consulting Reid (1991) and Tinney (1993).

The luminosity function in bolometric magnitudes is given by $\Psi(M_{\text{Bol}}) = \Psi(M_V) dM_V/dM_{\text{Bol}}$. Our best estimate of the Malmquist corrected photometric luminosity function is tabulated in bolometric magnitudes in Table 8, which also includes the nearby luminosity function. Both are obtained from the data in Table 2. Column 1 contains the bolometric magnitude, and Columns 2 and 3 list the best estimate photometric luminosity function in bolometric magnitudes. Columns 4 and 5 contain the bolometric nearby luminosity function. The last four bins have been smoothed as described in Section 4.

In Fig. 4 we compare our best estimate Malmquist corrected photometric luminosity function in bolometric magnitudes with that used by Tinney (1993). It is immediately apparent that Tinney's luminosity function has approximately a factor of two more stars in it than our best-estimate Malmquist corrected photometric luminosity function. This demonstrates that Malmquist bias significantly affects the bolometric luminosity function used by Tinney (1993).

It is because of this that Tinney interprets his fig. 4a to mean that there is no difference between the nearby and his luminosity function. In his fig. 4a Tinney (1993) compares $\Psi_{\text{TRM}}$ with the nearby luminosity function, $\Psi_{\text{WJK}}$, from Wielen et al. (1983) (transformed to $M_{\text{bol}}$ by Reid 1987) and finds they are indistinguishable at $M_{\text{bol}} > 9.50$. However, in his fig. 4a there is a significant excess of bright stars in his luminosity function w.r.t. the nearby sample. The number density of stars he derives at $M_{\text{bol}} = 9.0$ is $\Psi_{\text{Tin}} = (27.0 \pm 1.4) \times 10^{-3} \text{pc}^{-3} \text{mag}^{-1}$. The nearby luminosity function has at $M_{\text{bol}} = 8.96$, $\Psi_{\text{WJK}} = (16.4 \pm 2.9) \times 10^{-3} \text{pc}^{-3} \text{mag}^{-1}$. The difference corresponds to $\chi^2 = 10.8$ which, if both samples are assumed to estimate the same stellar population, has a probability of occurrence of 0.001. Incompleteness of the nearby sample because of “serious completeness problems” for distances equal to or larger than 10 pc (Tinney 1993) cannot be the reason for the apparent deficit of the nearby stellar number density at $M_{\text{bol}} \approx 9, M_V \approx 11$, as fig. 1 of Jahreiss (1994) demonstrates. We note here that the luminosity function constructed by Henry & McCarthy (1990) from the star-count data within 5.2 pc appears in fig. 4a of Tinney (1993) with a significantly smaller number density at $M_{\text{bol}} > 9$, $\Psi_{\text{HM}} \approx 0.007 \text{pc}^{-3} \text{mag}^{-1}$, than the nearby luminosity function, which is based on the same data and has $\Psi_{\text{WJK}} \approx 0.022 \text{pc}^{-3} \text{mag}^{-1}$. This is due to an incorrect scaling of $\Psi_{\text{HM}}$ in his fig. 4a (Tinney, private communication).

In Fig. 5 we plot the first order Malmquist corrected Tinney luminosity function (Tinney et al. 1993), and find good agreement with our best estimate photometric luminosity function at $M_{\text{bol}} > 9.5$. However, at brighter magnitudes significant disagreement persists. Because the Malmquist corrections have been applied to first order only we can at present only take the corrected Tinney luminosity function shown in Fig. 5 to be suggestive rather than conclusive.

In Fig. 5 we also plot the nearby luminosity function, $\Psi_{\text{near}}$, in bolometric magnitudes (Table 8) which we compare to the nearby luminosity function, $\Psi_{\text{WJK}}$, tabulated by Reid (1987). Our transformation of $\Psi_{\text{near}}$ to $M_{\text{bol}}$ agrees with that of Reid although he used different bolometric corrections. We note here that in Fig. 5 we have corrected the erroneous entry in table 1 of Reid (1987) at $M_V = 13, M_{\text{bol}} = 10.17$. Reid lists $\Psi_{\text{near}} = 22.1 \times 10^{-3} \text{pc}^{-3} \text{mag}^{-1}$, which has been adopted by Tinney (1993) in his fig. 4a. The correct value is $\Psi_{\text{near}} = 30.4 \times 10^{-3} \text{pc}^{-3} \text{mag}^{-1}$ which follows from $\Psi_{\text{near}}(M_{\text{bol}}) = \Psi_{\text{near}}(M_V) dM_V/dM_{\text{bol}}$ and Reid’s equation 5a.

We observe from Fig. 5 that (i) our best-estimate photometric luminosity function and the nearby luminosity function agree at $M_{\text{bol}} < 9.5$, (ii) the best-estimate photometric luminosity function and the nearby luminosity function are significantly different at $M_{\text{bol}} > 10$, and (iii) the first order Malmquist corrected photometric luminosity function estimated by Tinney et al. (1993) agrees with our best estimate, $\Psi_{\text{phol}}(M_{\text{bol}})$
for $M_{\text{bol}} > 9.8$. Point (ii) is, naturally, the same difference as already apparent in the photometric V-band (Section 4).

6 CONCLUSIONS

The photographic surveys using V- and I-band photometry by Reid & Gilmore (1982), Gilmore et al. (1985), Stobie et al. (1989) and the CCD survey by Kirkpatrick et al. (1994) yield photometric luminosity functions which can be assumed to estimate the same underlying apparent distribution of stars with luminosity. We estimate the true parent distribution by evaluating a weighted average Malmquist corrected photometric luminosity function, $\Psi_{\text{phot}}(M_V)$, which we tabulate in Table 2 in the V-band and in Table 8 in bolometric magnitudes.

We compare our best estimate of the true Malmquist corrected photometric luminosity function, $\Psi_{\text{phot}}(M_V)$, with the nearby luminosity function using three different tests and conclude with a confidence probability of at least 0.99 that the two observational approaches estimate different distributions. We find substantial evidence that the photometric luminosity function contains significantly fewer stars at faint magnitudes ($M_V > 13$) than the nearby luminosity function. In other words, the hypothesis that the best estimate Malmquist corrected photometric luminosity function is the correct single star luminosity function and that the nearby luminosity function is a random fluctuation of this luminosity function (Reid 1987, 1991) can be rejected safely. Rather, the nature of the stellar population must be different at $M_V > 13$. Varying the Galactic disk scale height does not affect these conclusions provided the vertical Galactic disk scale height $h > 250$ pc approximately.

The difference between the nearby and the photometric luminosity functions persists in bolometric magnitudes. Tinney’s (1993) conclusion that they do not differ is based on a comparison of the nearby luminosity function with his photometric luminosity function which is not corrected for Malmquist bias and which therefore does not describe the distribution of stars with luminosity accurately.

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Table 1. Estimates of the stellar luminosity function (×10^{-3} pc^{-3} mag^{-1})

| M_V   | Ψ^*_{RG} n_b | Ψ^*_{GRH} n_b | Ψ^*_{SIP} n_b | Ψ^*_{Kirk} n_b | Ψ_{phot} n_b |
|-------|---------------|---------------|---------------|----------------|--------------|
| 8     | ...           | ...           | ...           | ...            | 4.571 6      |
| 9     | 2.754 3       | 2.239 2       | 4.074 15      | ...            | 3.336 20     |
| 10    | 6.918 10      | 4.677 6       | 6.026 16      | ...            | 5.827 32     |
| 11    | 9.772 14      | 5.754 8       | 9.333 33      | 9.669 27       | 8.663 82     |
| 12    | 19.95 29      | 11.48 16      | 14.13 51      | 16.42 46       | 14.91 142    |
| 13    | 9.120 13      | 15.49 22      | 14.13 51      | 16.42 46       | 14.91 142    |
| 14    | 6.918 10      | 4.677 6       | 6.026 16      | 2.512 7        | 3.732 39     |
| 15    | 3.467 5       | 2.344 3       | 2.570 3       | 2.188 6        | 2.484 17     |
| 16    | 1.622 1       | 1.585 1       | 2.692 1       | ...            | 1.768 3      |
| 17    | ...           | ...           | ...           | ...            | ...          |

Table 2. The best estimate Malmquist corrected photometric luminosity function and the nearby luminosity function (×10^{-3} pc^{-3} mag^{-1})

| M_V   | Ψ_{phot} n_b | Ψ_{near} n_b |
|-------|--------------|--------------|
| 5     | ...          | 3.223 108    |
| 6     | ...          | 3.611 121    |
| 7     | ...          | 3.044 102    |
| 8     | 3.802 6      | 4.802 6      |
| 9     | 2.573 20     | 4.745 119    |
| 10    | 5.296 32     | ...          |
| 11    | 9.102 82     | 10.18 32     |
| 12    | 12.01 142    | 17.71 7      |
| 13    | 1.956 17     | 22.77 9      |
| 14    | 1.214 3      | ...          |
| 15    | ...          | ...          |
| 16    | ...          | ...          |
| 17    | ...          | ...          |

Table 3. Chi-squared and significance probability.

| M_V   | ν | Ψ^*_{RG} Ψ_{GRH} | Ψ^*_{RG} Ψ_{SIP} | Ψ^*_{RG} Ψ_{Kirk} | Ψ^*_{GRH} Ψ_{SIP} | Ψ^*_{GRH} Ψ_{Kirk} | Ψ^*_{SIP} Ψ_{Kirk} |
|-------|---|------------------|------------------|------------------|------------------|------------------|------------------|
| 9–12  | 4 | 1.35 0.25        | 4.63 0.57        | 2.32 0.75        | 4.09 0.45        | 2.87 0.15        | 0.28 0.78        |
| 13–16 | 4 | 0.85 0.48        | 0.39 0.72        | 1.69 0.22        | 0.59 0.68        | 0.48 0.70        | 0.84 0.48        |
| 9–16  | 4 | 1.10 0.35        | 0.51 0.85        | 1.14 0.35        | 0.76 0.65        | 1.04 0.40        | 0.62 0.69        |

Table 4. Wilcoxon signed-rank test

| Ψ^*_{RG} Ψ_{GRH} | Ψ^*_{RG} Ψ_{SIP} | Ψ^*_{RG} Ψ_{Kirk} | Ψ^*_{GRH} Ψ_{SIP} | Ψ^*_{GRH} Ψ_{Kirk} | Ψ^*_{SIP} Ψ_{Kirk} |
|------------------|------------------|------------------|------------------|------------------|------------------|
| n' x p_{T+}      | n' x p_{T+}      | n' x p_{T+}      | n' x p_{T+}      | n' x p_{T+}      | n' x p_{T+}      |
| 8 28 0.098        | 22 > 0.13        | 11 > 0.16        | 8 26 > 0.13      | 5 5 > 0.16       | 5 9 > 0.16       |

Table 5. Chi-squared test (Ψ_{near}, Ψ_{phot})

| M_V   | Ψ_{near}^2 | Ψ_{phot}^2 |
|-------|-------------|-------------|
| 9–12  | 2.23        | 0.15        |
| 13–16 | 11.31       | < 0.001     |
Table 6. Wilcoxon signed-rank test ($\Psi_{\text{near}}, \Psi_{\text{phot}}$)

| $n'$ | $x$ | $p_{T+}$ |
|------|-----|---------|
| 8    | 35  | 0.008   |

Table 7. The number of stars $n$ expected in the photographic surveys

| $M_V$ | $V[\text{pc}^3]$ | $N$ | $n$ | $z$ | $p_G$ |
|-------|------------------|-----|-----|-----|-------|
| 10–11 | 3143.4           | 55  | 45  | 1.31| 0.19  |
| 13–16 | 395.2            |  20 |  5  | 3.38| 0.0008|}

Table 8. The photographic and nearby bolometric luminosity functions. ($\times 10^{-3} \text{ pc}^{-3} \text{ mag}^{-1}$)

| $M_{\text{Bol}}$ | $\Psi_{\text{phot}}$ | $\Psi_{\text{near}}$ | $\sigma$ |
|------------------|----------------------|----------------------|-----------|
| 8.12             | 4.46                 | 8.22                 | 0.75      |
| 8.70             | 9.17                 | 12.66                | 2.64      |
| 9.27             | 15.77                | 17.63                | 3.12      |
| 9.85             | 20.80                | 30.68                | 11.60     |
| 10.43            | 11.69                | 21.91                | 9.80      |
| 11.00            | 4.19                 | 24.11                | 10.28     |
| 11.58            | 3.39                 | 24.11                | 10.28     |
| 12.16            | 2.10                 | 19.73                | 9.30      |
| 12.73            | ...                  | 19.73                | 9.30      |
REFERENCES

Bhattacharyya, G. K., Johnson, R. A., 1977, Statistical Concepts and Methods, John Wiley & Sons Press, New York
Dahn, C. C., Liebert, J., Harrington, R. S., 1986, AJ 91, 621
Gilmore, G., Reid, N., Hewett, P., 1985, MNRAS 213, 257
Hawkins, M. R. S., Bessell, M. S., 1988, MNRAS 234, 177
Haywood, M., 1994, A&A 282, 444
Henry, T. J., McCarthy, D. W., 1990, ApJ 350, 334
Jahreiss, H., 1994, Ap&SS 217, 63
Kirkpatrick, J. D., McGraw, J. T., Hess, T. R., Liebert, J., McCarthy, D. W., 1994, ApJS 94, 749
Kroupa, P., 1995a, Unification of the Nearby and Photometric Stellar Luminosity Functions, ApJ, in press
Kroupa, P., Tout, C. A., Gilmore, G., 1991, MNRAS 251, 293
Kroupa, P., Tout, C. A., Gilmore, G., 1993, MNRAS 262, 545
Leggett, S. K., Hawkins, M. R. S., 1988, MNRAS 234, 1065
Lutz, T. E., Kelker, D. H., 1973, PASP 85, 573
Reid, N., 1987, MNRAS 225, 873
Reid, N., 1991, AJ 102, 1428
Reid, N., 1994, Ap&SS 217, 57
Reid, N., Gilmore, G., 1982, MNRAS 201, 73
Stobie, R. S., Ishida, K., Peacock, J. A., 1989, MNRAS 238, 709
Tinney, C. G., 1993, ApJ 414, 279
Tinney, C. G., 1994, The Luminosity and Mass Functions at the Bottom of the Main Sequence, In: MacGillivray, H. T., Thomson, E. B., Lasker, B. M., et al., (eds.), Astronomy from Wide-Field Imaging, Kluwer, Dordrecht, p.411
Tinney, C. G., Reid, N., Mould, J. R., 1993, ApJ 414, 254
Wielen, R., Jahreiss, H., Krüger, R., 1983. The Nearby Stars and Stellar Luminosity Function, In: Davis Philip, A. G., Upgren, A. R. (eds.), IAU Colloq. No. 76, New York, Davis Press, p.163
Figure 1. The observational data (Section 2). The photometric luminosity functions not corrected for Malmquist bias are shown as the long-dashed curve (Reid & Gilmore 1982, $\Psi_{RG}^*$), as the dot-short-dashed curve (Gilmore et al. 1985, $\Psi_{GRH}^*$), as the short-dashed curve (Stobie et al. 1989, $\Psi_{SIP}^*$) and as the dot-long-dashed curve (Kirkpatrick et al. 1994, $\Psi_{Kirk}^*$). The open circles are the weighted average, $\Psi_{phot}^*$, of these four luminosity functions, and the solid circles are our best estimate of the Malmquist corrected photometric luminosity function, $\Psi_{phot}$, derived in Section 3.3. The nearby luminosity function, $\Psi_{near}$, is shown as the open squares.

Figure 2. The Malmquist corrections and Poisson uncertainties (Section 3.3). **Upper panel:** We compare the Malmquist correction computed by Stobie et al. (1989), $\Delta \Psi_{SIP}$, and the Poisson uncertainties in their raw photometric luminosity function, $\sigma_{SIP}^*$, with these same quantities computed by Kirkpatrick et al. (1994). The Poisson uncertainties of our weighted average raw photometric luminosity function, $\sigma^*$, are improved substantially over the uncertainties in the individual estimates of the photometric luminosity function (Fig. 1). **Lower panel:** The first order Malmquist corrections estimated by Tinney et al. (1993) and their Poisson uncertainties.

Figure 3. The luminosity function of stars in the photometric V-band (Section 4). The solid histogram is the nearby luminosity function $\Psi_{near}(M_V)$. We have grouped the data into larger bins at $M_V \geq 13.5$. The filled circles are our best-estimate of the Malmquist corrected photometric luminosity function, $\Psi_{phot}(M_V)$. $\Psi_{near}$ and $\Psi_{phot}$ are tabulated in Table 2. The short dashed curve is the Malmquist corrected photometric luminosity function estimated by Stobie et al. (1989).

Figure 4. Our best-estimate photometric luminosity function in bolometric magnitudes, $\Psi_{phot}(M_{Bol})$ (filled circles, Table 8), is compared with the luminosity function used by Tinney (1993), $\Psi_{TRM}(M_{Bol})$ (open triangles) (Section 5).

Figure 5. The solid histogram is the nearby bolometric luminosity function, and the filled circles are our best-estimate Malmquist corrected photometric luminosity function (Section 5). The luminosity function used by Tinney (1993) and plotted in Fig. 4 has been corrected here to first order for Malmquist bias (Tinney et al. 1993). It is shown as open triangles. The open stars are the bolometric luminosity function derived from the nearby star sample by Reid (1987), after correcting his erroneous value at $M_{Bol} = 10.17$. 

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