A Method for Handling Multi-class Imbalanced Data by Geometry based Information Sampling and Class Prioritized Synthetic Data Generation (GICaPS)

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Abstract—This paper looks into the problem of handling imbalanced data in a multi-label classification problem. The problem is solved by proposing two novel methods that primarily exploit the geometric relationship between the feature vectors. The first one is an undersampling algorithm that uses angle between feature vectors to select more informative samples while rejecting the less informative ones. A suitable criterion is proposed to define the informativeness of a given sample. The second one is an oversampling algorithm that uses a generative algorithm to create new synthetic data that respects all class boundaries. This is achieved by finding no man’s land based on Euclidean distance between the feature vectors. The efficacy of the proposed methods is analyzed by solving a generic multi-class recognition problem based on mixture of Gaussians. The superiority of the proposed algorithms is established through comparison with other state-of-the-art methods, including SMOTE and ADASYN, over ten different publicly available datasets exhibiting high-to-extreme data imbalance. These two methods are combined into a single data processing framework and is labeled as “GICaPS” to highlight the role of geometry-based information (GI) sampling and Class-Prioritized Synthesis (CaPS) in dealing with multi-class data imbalance problem, thereby making a novel contribution in this field.

Index Terms—Imbalanced data, SMOTE, ADASYN, Gaussian Mixture Model, Oversampling, Undersampling, GICaPS

I. INTRODUCTION

MAJORITY of existing classification techniques, including deep learning approaches are typically designed to perform well when the distribution of data among classes is balanced. Many of these methods perform poorly on real-world datasets that are inherently class-imbalanced\textsuperscript{[1]} [2] [3] \cite{1} with the majority class(es) forming the bulk of the dataset while a disproportionately smaller share coming from the minority class(es). Spam filtering, network intrusion detection, cancer diagnosis, detecting fraudulent transaction are some of the applications that generate imbalanced datasets. Classifiers trained on such imbalanced datasets are biased towards the majority class making them unreliable for use in several cases where the minority class is of critical interest. For instance, it is very important to detect a fraudulent transaction even when its occurrence is rare compared to the overall number of genuine transactions made over a given period. Same applies to the case of medical diagnosis where a single case of false negative (e.g., failing to detect a malignant tumor) can lead to serious consequences even when the overall classifier accuracy is more than 99%. Many of these applications exhibit high-to-extreme class imbalance where the majority-to-minority class ratio could be more than 10,000:1 thereby, posing serious learning challenges for classifier design\textsuperscript{[5]} [1].

Most of the existing methods for dealing with imbalanced data can be broadly classified into three categories: data-level methods, algorithm-level methods and hybrid approaches\textsuperscript{[2]} [5] [1]. Data-level methods focus on improving the dataset by using methods such as over- or undersampling, feature selection etc. Some of the popular data-sampling approaches include Random Over-Sampling (ROS), Random Under-Sampling (RUS), SMOTE (Synthetic Minority Over-Sampling Technique) and its variants\textsuperscript{[6]} [7] [8] and, ADASYN\textsuperscript{[9]}. On the other hand, algorithm-level methods try to learn the imbalance data distribution from the classes in the datasets. These methods can be further sub-grouped into cost-sensitive methods and ensemble methods. The former works by assigning varying cost or weight to different instances or classifiers in the event of misclassification while the later combines the output of multiple classifiers built on the dichotomies created from the original dataset to improve the classification performance. Bagging and Boosting\textsuperscript{[10]} \cite{10} are two common types of ensemble methods. The hybrid methods combine the advantage of data-level and algorithm-level methods and usually employ more than one machine learning algorithms to improve the classification accuracy\textsuperscript{[12]} [13]\cite{13} [14] [15] [16]. Many of these algorithms have been developed to solve the binary-class data imbalance problem which has been studied more extensively compared to the multi-class imbalance problem\textsuperscript{[5]}. A multi-class data imbalance problem is usually solved by using some decomposition strategy to reduce it to a set of binary-class problems which can be solved by using one of the above techniques\textsuperscript{[17]} [5] [1].

In this paper, we restrict our discussion to sampling-based methods that form a major part of the data-level methods for imbalanced data learning. The data-sampling methods could be further divided into sub-groups, namely, over-sampling and under-sampling methods. While the former aims at adding or replicating instances of the minority class, the later focusses on removing instances of the majority class in a given dataset to reduce to overall data imbalance. The replication or removal of data is either done randomly (e.g. ROS \cite{18} [19] \cite{15}) or through an intelligent algorithm (e.g. SMOTE\textsuperscript{[6]}, ADASYN\textsuperscript{[9]}). The data-sampling algorithms are designed to address problems such as overfitting (during oversampling), loss of valuable information (during undersampling) or the existence of disjuncts (imbalance within a class). The problem becomes

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more challenging when the dataset exhibits considerable class overlap [2] [20]. Overlapping classes have low degree of separability between the classes and the data points on the boundaries may belong to any of the overlapping classes. The class overlapping becomes a more serious issue in the presence of sensor noise or outliers [21]. Various methods have been proposed to address the class overlapping issue within the imbalanced data classification problem. These methods span across the categories mentioned above with varying degree of success. For instance, the authors in [22] suggest a one-vs-one decomposition strategy to alleviate the presence of overlapping without modifying existing algorithms for modifying the dataset. Similarly, the authors in [20] propose a hybrid approach that combines fuzzy SVM with a k-nearest algorithm to deal with data imbalance and class overlapping simultaneously. Several variants of SMOTE [23] have been proposed to address the class separability issue in imbalanced learning problems. In another work [24], authors use manifold distance instead of Euclidean distance to improve the class separability of the samples. In spite of these efforts, the imbalanced learning is still considered as a challenging problem particularly when there exists significant class overlapping.

In this paper, we aim to address the problem of class separability for a multi-class imbalanced learning problem. This is achieved by using a combination of oversampling and undersampling method that use geometric information sampling and class prioritized synthesis that not only improves the separability between classes but also increases the diversity of samples within each class by selectively removing redundant samples. The proposed method is, hence, termed GI-CaPS which is an acronym for Geometric Information-based Sampling and Class-Prioritized Synthesis. The undersampling approach uses distance in the polar (angular) coordinates to remove data points from a majority class while ensuring that valuable information is not lost by constraining the removal of samples only from other orthants (n-dimensional orthogonal half-spaces). Similarly, the oversampling algorithm creates synthetic data samples in a minority class that respects class boundaries. This is ensured by avoiding data synthesis in the regions with class boundaries and addresses the class overlapping problem by avoiding data synthesis in the regions with high class interference, otherwise known as no man’s land. Mathematical formulation for identifying no-man’s land is provided and to the best of our knowledge, such a concept has not been used before in this context.

- The efficacy of the above approaches is established through rigorous analysis on ten different publicly available datasets exhibiting very high class imbalance and is shown to outperform many of the existing state-of-the-art methods in this field.

The rest of the paper is organized as follows. An overview of related work is provided in the next section. The datasets used in this paper are discussed in Section III. The proposed undersampling and oversampling approaches are explained in Section IV. The working of the proposed methods is demonstrated on a simulated 3-dimensional data using a Gaussian Mixture Classifier and is described in Section V. The efficacy of the proposed method is further established by providing comparison with other state-of-the-art methods on ten different datasets that exhibiting high level of imbalance. This along with other analyses are discussed in Section VI. Finally, the conclusion and future scope of this work is discussion in Section VII.

II. RELATED WORK

Handling of imbalance data is a long-lasting problem and lots of work have been done in this area since last few decades [25]. Based on the literature, the approaches of solving this problem can be broadly categorized into three different groups: data-level, cost-sensitive and ensemble learning approaches. Data-level approaches try to balance skewed distribution by using various resampling approaches. Again, it can be categorized into two sub-groups: undersampling and oversampling. Cost-sensitive learning [26], [27] prioritizes accurate classification of minority class samples over majority classes. Target of this approach is to generate a classification model with minimum cost and this is achieved by establishing a cost matrix in which the elements of the matrix indicate the penalty strength for the instances that are misclassified [25]. Ensemble learning approaches [28], [29], [30] combine multiple base classifiers in order to achieve promising results on imbalanced datasets. Various hybrid models are also there in the literature that combines data-level techniques with ensemble learning, resulting into improved performance in imbalanced data classification [8], [31]. Our approach of handling imbalanced data falls under the category of data-level approaches. Rest of this section will thus concentrate on data-level based undersampling and oversampling approaches.

A. Previous works on undersampling

A majority class in an imbalanced dataset often contains a lot of redundant or less informative data which increases unnecessary computation cost and also mis-leads the classifier in predicting accuracy. However, unlike, oversampling approaches, not much works have been done in the direction of undersampling. One of the earliest undersampling technique
used to alleviate the problem of class imbalance in the dataset is Random Under Sampling (RUS) [32]. However, one major drawback of this approach is that it may potentially discard useful information while sampling the data. In real-life datasets, the distribution of data can be such that, it may contain data densely populated in some region and can have very sparse distribution in some other region. As RUS has almost equal probability of picking up a sample from anywhere within the distribution, the densely populated regions will remain dense even after undersampling, and the regions containing very sparse data distribution may lose very informative data. Some research works are also performed in the direction of minimizing the effect of information loss occurs after RUS. EasyEnsemble [33] and BalanceCascade [34] are two of such approaches. Another undersampling approach, commonly known as Edited Nearest Neighbour (ENN), was adopted from the study of Wilson [35]. ENN mainly focuses on instances near the decision boundary and selectively removes majority class instances by considering its k nearest neighbours that belong to the other class. Few extensions of this work include Neighbourhood Cleaning Rule (NCL) [36], [37], [38]. Another data cleaning strategy was introduced by Kubat and Matwin that uses Tomek links [39] to remove only borderline majority samples. All these aforesaid methods mainly focuses on the removal of borderline majority class data and overlooks the desire of removing unnecessary less informative and redundant data which may lie within the majority class. An attempt to remove the redundant data from the distribution of majority class set is presented in an approach commonly known as cluster centroid undersampling [40]. Majority class is undersampled by forming clusters and the sampled data are chosen as cluster centroids. The number of clusters is set by the level of undersampling. Few extensions of this work include [41], [42]. Zhao et al. [41] applied an unsupervised learning algorithm that transforms the classification problem into several classification sub-problems. K-medoids based undersampling approach is applied in [42] and only the cluster centers are considered as sampled data. Again, all these approaches can’t ensure retaining of most informative data as they replaces the real-samples with the cluster centers. In contrast, we propose an undersampling technique based on the angular information among the feature vectors of majority class, that ensures retaining of more informative data and removal of less informative or redundant data from all the regions within the distribution set.

B. Previous works on oversampling

Although data balancing can be best handled by the implication of both oversampling and undersampling approaches, however, researchers have been more frequently applying oversampling approaches to solve this problem. Lots of works have been done in data driven oversampling techniques [43], [44], [6], [7], [9], [45], [46]. Random Over-Sampling (ROS) with replacement [44] is the fundamental concept of oversampling techniques. This method selects a set of \(E\) sampled minority class data from the minority set \(S_{min}\) and then replicates those selected data into the set to make a balanced dataset. As the replacement process of ROS is completely random, this approach does not specify a clear borderline between any two classes. Moreover, simple replication of existing data into the original minority class/es can cause the problem of overfitting [6]. Among the existing data-driven oversampling approaches, Sampling with Data Generation (SMOTE) [6] is still considered as the state of the art in the literature due to its simplicity and easy implementability. However, it is associated with various shortcomings. One important drawback of SMOTE is that, it does not consider possible interferences of data from other classes while generating synthetic examples, thereby increases the chances of multi-class overlapping and also introduces additional noises. Another major issue with the SMOTE is that, it has no control over the number of new data to be generated (it merely replicates same number of data, originally present in a minority class). Therefore, the dataset remains imbalanced even after applying SMOTE. Over the period of time various improvements have been done on SMOTE [23]. Some of those, include Borderline-SMOTE [7], Adaptive Synthetic Sampling Approach for Imbalanced Learning (ADASYN) [9], Ranked minority oversampling in boosting (Ramoboost) [45]. ADASYN is developed based on the idea of Borderline-SMOTE. Unlike SMOTE, Borderline-SMOTE only creates synthetic samples for the data points which are near to the border, while taking into consideration that no synthetic data should be generated for "Noise" instances. ADASYN adapts the concept of Borderline-SMOTE and creates different number of synthetic data based on the data distribution. Unlike, SMOTE and Borderline-SMOTE, ADASYN algorithm can decide the number of synthetic examples that need to be generated for each minority examples by the number of its majority nearest neighbor, i.e., the more the majority nearest neighbor, the more synthetic examples will be created. One important drawback with this approach is that, the synthetic data is generated only near to the boundary. Secondly, it does not consider the possibility of interference of other minority or majority class data while generating synthetic data. Third, both SMOTE and ADASYN do not consider within class imbalance while generating new data. i.e, the data-intensive minority regions may still remain dense, while the data-sparse minority regions may remain sparse [47]. In contrast, the proposed approach of oversampling mainly focuses on the inter-class borderline data over-lapping issues and effectively localizes all the possible data over-lapping regions between any two classes and wisely avoids those regions while synthetically generating new data within the targeted minority class. In addition to that, the proposed oversampling techniques reduces within class imbalance by using K-means clustering before synthetic data generation. Some research works on oversampling techniques that attempted to reduce within class imbalance are Cluster-SMOTE [48], MWMOTE (majority weighted minority oversampling technique) [49], DBSMOTE (density-based synthetic minority oversampling technique) [50], CURE-SMOTE [51], and K-means SMOTE [47]. However, all these methods do not consider interference of other classes boundaries while interpolating data within the selected minority class.
III. IMBALANCED DATA DISTRIBUTION IN DIFFERENT DATASETS

The proposed technique has the capability of handling highly imbalanced datasets with multiple classes. Multi-class datasets used in our experiments are, Abalone, Glass, Wine, Shuttle and Pain with classes 23, 5, 3, 7 and 15 respectively. Remaining datasets contain binary class data. Table I summarizes the details of ten different datasets used in our experiments.

To demonstrate the amount of variations in data distribution among different classes, we present an example in the Table II. The Table II shows pain intensity (class) in the first row and corresponding sample size is given in the second row. Such highly skewed distributions of data among classes motivated us to apply both undersampling and oversampling technique to generate a balanced dataset. It is to be noted that, for synthetic generation of new data, an initial data distribution within a class must have at least two data. For instance, the class labeled as 14, in the Pain database [52] has only one data. We have thus, removed that label in our experiment. Following section presents detailed description of the proposed undersampling and oversampling technique.

IV. PROPOSED IMBALANCED DATA HANDLING TECHNIQUE- GICAPS

The earlier section presented a few instances of extremely imbalanced datasets where the majority classes dominate more than 90 percent of the training data. Therefore, an attempt to balancing the dataset by merely generating synthetic data in the minority classes would make the dataset huge and thereby severely impact the computation during training. We observe that in extremely imbalanced datasets, the majority classes contain many repeated/ less informative occurrences which are in fact redundant to training the classifier. Hence, GICaps is the combination of two algorithms: GICaps-undersampling and GICAPS-oversampling.

A. GICaps undersampling approach

The intention behind applying an undersampling approach to a majority class is to get rid of the data samples those have very less impact on the training of a network. This not only helps balancing the training data, also results in significant decrease in computational burden during network training. It is important to note that the similarity between two feature vectors is directly proportional to their dot product. Hence, by measuring the angle between two feature vectors, one can determine the dissimilarity or distinctiveness between them, which is in fact directly proportional to the angle between the vectors. The main focus of undersampling technique is to remove feature vectors from the majority class while maintaining the intra-class data diversity (as much as possible). However, maintaining diversity is difficult in random undersampling or euclidean distance based approach. Hence, we embrace a sorting technique in the angular space, where we ensure that each feature vector is distinct than the others. The proposed undersampling algorithm works using two principles:

1. discard a feature vector if a similar feature vector already exists in the class
2. reduce the density of data samples in densely populated regions of the class while ensuring the left over features vectors are away from each other in the angular space by certain distance

The undersampling technique has the following attributes:

1) The overlapped and less informative feature vectors are removed.
2) The feature vectors are picked up based on angular information between the vectors in the entire majority class.
3) The approach makes sure that a uniform intra-class data density is maintained within the new dataset of the majority class.
4) The approach is unique in a sense that it also considers the intra-plane information of the vectors while performing undersampling operation. To the best of our knowledge this has not been introduced anywhere else in the literature.

1) GICaps undersampling technique: This section gives a detailed explanation of the proposed undersampling approach. A block diagram presenting different steps involved in the proposed undersampling approach is shown in Figure 1. Given a majority class, the entire class is divided into $K$ clusters with cluster centers $c^k \in \{1, 2, ..., K\}$. We use K-medoids to find the clusters in the majority class. The reason for using K-medoids is that it selects the cluster centers as an original data point while minimizing $L_1$ norm over the entire data which makes it more robust than K-means to outliers. The choice of the number of clusters in the majority class is also important especially when the majority class has a heterogeneous data distribution. The optimal number of clusters are chosen using elbow rule [53], however one can use more efficient techniques as in [54] to determine the optimal number of clusters. The undersampling operation is performed in each of the $k$ clusters of the class. First the angle between the cluster center $c^k$ and all other feature vectors ($x_i$, $i = 1, 2, ...$) is calculated and also the angle between every two features are calculated. Then the algorithm checks for the overlapped or similar feature vector and densely populated regions. The rejection criteria for a data sample are stated in the following.

Rejection criteria 1: The first rejection criteria is set such that the feature vectors are separated at least by the minimum
threshold angle in the angular space. Hence, we start by arranging the angles in ascending order, which places $c^k$ at the top of the stack. Then look for the first feature vector $x_i$ from the remaining data in the stack which has angular displacement with the center that exceeds certain threshold. If the angle between a feature vector $x_i$ and the cluster center $c^k$ is greater than the threshold $\delta \sigma$, then keep the data else remove the feature vector. Keep removing the feature vectors until the initial threshold criteria is satisfied.

Rejection criteria 2: Once the first feature vector is found, look for the second feature vector $x_j$, check if the angular displacement between the first selected feature vector $x_i$ and the feature vector $x_j$ w.r.t center $c^k$ meets the threshold limit, i.e., if the condition $|\theta_{c_i} - \theta_{c_j}| > \delta \sigma_k$ is true, keep the vector or else check if the feature vectors lies in the same hyperoctant or orthant of the center $c^k$ and $x_i$. If all three vectors $c^k$, $x_i$ and $x_j$ are in the same plane then reject $x_j$. Figure 2a illustrates an example of the case of comparing two feature vectors $x_i$ and $x_j$. It also possible that the angular distance between two feature vectors with respect to $c^k$ is small, but if they are not lying in the same orthant then removing the feature vector may lead to loss of necessary information.

The method of checking if two feature vectors lie on the same orthant is given as follows:

1) Find the difference between the cluster center $c^k$ and the two feature vectors $x_i$ and $x_j$.
2) Set each element of the difference vector to either 1 (if it is positive) or 0 (if it is negative).
3) Convert the binary string to decimal equivalent number.

### Algorithm 1 Algorithm for under-sampling data

1: procedure
2: Run K-medoids and divide the majority class in K clusters with associated cluster centers $c_k$.
3: Get the angular spread of each cluster and compute the approx number of data to be kept in each cluster.
4: Remove all scaled / repeated data points from each cluster.
5: Compute the minimum threshold angle $\alpha_k$ between data points by which they are separated from each other.
6: Compute the error vectors by taking the difference between $c_k$ and the other data points in that cluster.
7: Obtain the Orthant of the error vectors.
8: Sort the data points based on their angular distance from $c_k$
9: for For each data point in each cluster do
10: if NOT $|\theta(t+1) - \theta(t)| > \alpha_k$ AND $\theta(t+1) > \theta(t)$ then
11: Discard the data point.
12: end if
13: end for
14: end procedure

The hyper parameter $N_D$ can be kept as high as the total number of data in the majority class; i.e. the majority class is not under-sampled. In that case, the minority classes need to be over-sampled up to the level of the majority class.

### TABLE I: Imbalance nature of different datasets (highest number of data in majority class and least number of data minority class are given in the columns Majority instances and Minority instances respectively.) Among the given datasets, Abalone and Pain have highly imbalanced data with more number of classes ( 23 and 15 respectively).

| Dataset            | # Classes | # Feature | # Data | Minority instances | Majority instances |
|--------------------|-----------|-----------|--------|--------------------|--------------------|
| Abalone            | 23        | 8         | 4177   | 2                  | 689                |
| Spambase           | 2         | 58        | 4601   | 1813               | 2788               |
| Glass              | 5         | 10        | 214    | 9                  | 163                |
| Ionosphere         | 2         | 34        | 351    | 126                | 225                |
| Sonar              | 2         | 60        | 208    | 97                 | 111                |
| Wine               | 3         | 13        | 178    | 48                 | 71                 |
| Pima Indian diabetes | 2      | 8         | 768    | 268                | 500                |
| Shuttle            | 7         | 9         | 43500  | 6                  | 34108              |
| fertility          | 2         | 9         | 100    | 12                 | 88                 |
| Pain               | 15        | 22        | 48198  | 5                  | 39835              |

### TABLE II: Data distribution of pain db. (The terms are: I-Intensity, DS-Data size)

| DS | $I_0$ | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ |
|----|-------|-------|-------|-------|-------|-------|-------|-------|
| DS | 39835 | 2908  | 2349  | 1409  | 802   | 242   | 270   | 53    |
| DS | 79    | 32    | 67    | 76    | 48    | 22    | 1     | 5     |

Fig. 2: (a) A visualization of undersampled data when the proposed undersampling technique is applied to a simulated 3D dataset. The square empty points are original data and the triangular filled points are undersampled data. (b) A pictorial illustration of the proposed undersampling approach when data lies in the same plane.

4) The decimal equivalent number provides the plane information of the feature vector w.r.t the center $c^k$. 

![Diagram](image-url)
This may (or may not) result to a little increase in prediction accuracy but with huge computational cost if the classes are highly imbalanced. Moreover, the minority classes will be over crowded with less-informative data. On the other hand, if we select $N_D$ as small as the number of data in the next largest minority class, we may loose crucial information during under-sampling of the majority classes, which will degrade the prediction accuracy of the algorithm. Hence, the choice of hyper parameter $N_D$ is a trade off between class recognition accuracy and the training time of the algorithm. It also depends on the imbalanceness of the classes. The best practice is to choose an intermediate value for $N_D$, which is lower than the number of data in the majority class but higher than the number of data in the next largest minority class and minority classes are over-sampled to that level. Here, we use cross validation to select the value for $N_D$.

The proposed undersampling algorithm is tested on a 3 dimensional simulated data belong to a single class. The purpose of testing on the 3D dataset is to have a visual realization of the performance. Figure 2b shows the performance of the proposed undersampling approach when applied to the simulated dataset. The squares are original data and the triangular points are undersampled data. For the dataset of size 2000, we choose to select only 600 data using the proposed undersampling approach. It can be observed that, the undersampled data still covers the entire class and maintains uniformity. It removes the unnecessary data from the dataset, yet keeps all the data which are sparsely distributed. Histogram plots of angular distances for two randomly chosen clusters are shown in Figures 3a and 3b. It is observed that the undersampled data in each of the clustered regions are almost uniformly scattered. The redundant or the less informative features are represented either by a single feature vector or by very few of them. It is observed from the histogram plots that the over crowded regions of the original class are made compact by selecting few feature samples. These selected features are the candidates from the crowded regions, which carry important characteristics of the entire class. It is also observed from the plots that no undersampled data is present in certain bins. Such cases can only happen when the data is either lying in the same orthant of the center and another feature vector which has already been selected previously that represents the data under consideration.

B. GICaPS oversampling approach

The oversampling algorithm presented here creates synthetic data samples for minority classes. The algorithm first checks the boundaries of the existing classes and then generates new data samples in the feasible regions. The proposed oversampling approach has the following features:

1) The proposed algorithm does not violate class boundaries while generating new synthetic samples. The new data is generated in such a manner that it would not create confusion for the training module while learning the class distribution. In contrast, the existing algorithms such as SMOTE do not care about the inter class interference while interpolating new data. Other well established approaches, such as Borderline-SMOTE and ADASYN generate synthetic data only in the borderline of the minority classes.

2) The proposed methodology generates new data samples between two data samples considering the possible interference due to the data from neighborhood classes. (Please see Fig. 4).

3) The number of data to be generated in the minority class can be defined by the user.

The key idea of the proposed oversampling technique is that, while interpolating synthetic data between two points $x_m$ and $x_v$, both belong to class $i$, we must consider the interference of other data points belonging to class $j$ with $j \neq i$. This is because, the synthetic data may fall into other class’s ambit if the criteria are not set properly. Such interference would essentially create confusion for the classifier to recognize class identity of a data point in that region. Considering the fact, we calculate the regions where such interference may occur. The regions are denoted as no man’s land. Any data interpolated in that region would be an illegal interpolation. New synthetic data is generated avoiding the no man’s land.

The concept of no man’s land can be better understood by analyzing Figure 4 that shows three different scenarios that one may come across while generating new data points between two existing data points $a$ and $b$ within a given region, say, $V$. The sub-figure (a) shows the first case where existing data points belonging to a class $Q$ may lie only on one side of the line joining data points $a$ and $b$. The sub-figure (b) shows the case where existing data points belonging to a class $Q$ may lie on either side of the line joining data points $a$ and $b$. The sub-figure (c) shows the case where data points belonging to different classes $Q$ and $R$ may lie on either side of the line $\overline{ab}$. Any other case can be analyzed by combining these three cases. In each of these three cases, the no-man’s land represents the region that should be avoided while generating data point between $a$ and $b$. Generating data points in these (no-man’s land) regions will disturb the classifier boundaries between the classes. The exact shape of no-man’s land will depend on the distribution of data points within and between the classes. However, for a two-dimensional dataset, it can be safely represented by rectangles as shown in Figure 4. The height $h$ of this rectangle could be minimum perpendicular distance of the points of class $Q$ or $R$ from the line $\overline{ab}$. The length $l$ of this rectangle includes all the points where the
Data needs to be interpolated. The classes and the class samples need to be generated. Then the data points of class in the neighborhood of the point \( x \) section. It is assumed that the dataset has total approach for calculating no man’s land. Considering all these above criteria we propose a mathematical approach for calculating no man’s land presented below in this section. It is assumed that the dataset has total \( C \) classes \( Q \) and \( R \) lie on either side of the line joining data points \( m \) and \( n \) does not belong the class \( Q \) or \( R \).

Figure 5 shows two neighboring data points \( x_m \) and \( x_v \) belong to the class \( i \) and the filled data points belong to other classes \( j \). A region \( V_m \) contains the points \( x_m \) and \( x_v \) (between which synthetic data need to be generated) and the neighboring points of other classes \( j \).

lines between the points (of class \( Q \)) intersect with the line \( ab \). These points of intersection may also be obtained by drawing projections from the points of class \( Q \) as well, the former providing a more conservative estimate of the parameter \( l \). Considering all these above criteria we propose a mathematical approach for calculating no man’s land. Let’s say, there is a data point \( t_1 \in Q_j \) and a data point \( t_2 \in R_j \) which are to be checked for any interference while interpolating the data along the vector \( ab \) connecting the data points \( x_m \) and \( x_v \). The steps are as follows:

The vector \( aO \) is given as

\[
aO = p_{t1} + s_1 O
\]  

The triangles with points \( t_1, O, s_1 \) and \( t_2, O, s_2 \) forms two similar triangles. Thus, it can be written that the vectors \( p_{t1} \) and \( p_{t2} \) are the projections of the vectors \( at_1 \) and \( at_2 \) respectively on the line \( ab \). Therefore, following relations can
be drawn.

\[ \frac{d_2}{d_1} = \frac{p_2}{p_1} \]  
\[ \frac{d_1 + d_2}{d_1} = \frac{p_2 + p_1}{p_1} \]  
\[ p_1 = \frac{p_2 + p_1}{d_2 + d_1} \times d_1 \]

where,

\[ p_1 + p_2 = \| p_{t2} - p_{t1} \| \]  
\[ d_1 = \| a \| - \| p_{t1} \| \]  
\[ d_2 = \| a \| - \| p_{t2} \| \]

\[ s_1 O = (p_{t2} - p_{t1}) \times \frac{d_1}{d_1 + d_2} \]

The equation 1 can now be written as:

\[ aO = p_{t1} + (p_{t2} - p_{t1}) \times \frac{d_1}{d_1 + d_2} \]

The projections \( p_{t1} \) and \( p_{t2} \) can be calculated as

\[ p_{t1} = \frac{(ab)(ab)^T (at_1)}{(ab)^T (ab)} \]

\[ p_{t2} = \frac{(ab)(ab)^T (at_2)}{(ab)^T (ab)} \]

Let us also assume that the line connecting \( t_1 \) and \( t_2 \) intersects \( ab \) at point \( O \). Till now, it was assumed that the crossing point \( O \) is in between \( x_m \) and \( x_v \). However, there might be cases where \( O \) might fall outside \( ab \). Hence, the next step is to check if the point \( O \) lies on or near to the vector \( ab \). To check if the line joining the vectors \( t_1 \) and \( t_2 \) crosses the intersecting point \( O \) following verification is done.

\[ \| aO \| < \| ab \| \]

and

\[ \| ab - aO \| < \| ab \| \]

If the above conditions are satisfied, then the crossing distance needs to be calculated related to point \( t_1 \) and \( t_2 \). The crossing distance is defined as the shortest distance of the line joining \( t_1 \) and \( t_2 \) from point \( O \). Let us define

\[ O t_1 = aO - at_1 \]

\[ at_{12} = at_2 - at_1 \]

The crossing distance is thus calculated as:

\[ C_{dist} = \| O t_1 \| \times \sin(\theta_0) \]

where, \( \theta_0 \) is the angle between the vectors \( O t_1 \) and \( at_{12} \). The no man’s land is estimated based on the above calculation. A threshold is set to find the region of no man’s land. If \( C_{dist} \) is less than a threshold value, then no data is interpolated within the region. The region thus comes under no man’s land. The intersecting points and the crossing distances are calculated for all the points within the regions \( R_x \) and \( Q_x \). And the same process is repeated for all the \( j \in \{1, \ldots, C\} \) classes. Thus, no man’s land between \( x_m \) and \( x_v \) is identified by calculating all the intersections on \( ab \) for \( C \) classes in the neighborhood. First, the intersections \( O_1, O_2, \ldots, O_K \) (\( K \) is the total number of data samples from the other classes that cause interference) are identified using the procedure explained above. The closest and farthest \( O_k \) to \( x_m \) define the range of no man’s land. Once the no man’s land is identified, the number of new points need to be generated for each data \( x_m \) in the class \( i \) is calculated next. The free space between the vector \( x_m \) and \( x_v \) is given as:

\[ S_{x_m x_v} = \rho\{(ab) - \| O_{max} - O_{min} \|\} \]

where, \( O_{max} \) and \( O_{min} \) are the longest and shortest vector from \( x_m \) to \( O_k \) (farthest and closest intersection). The effect of \( \rho \) is to increase the range of the no man’s land, such that the region does not start and end strictly at closest and farthest \( O_k \) respectively. Total number of data to be interpolated within the \( V_m \) region is given as:

\[ N_{V_m} = \frac{H_i \sum_{x_m} S_{x_m x_v}}{\sum_{x_m} \sum_{x_v} S_{x_m x_v}} \]

and the number of data to be interpolated in between two data points \( (x_m, x_v) \) is given as:

\[ N_{x_v} = \frac{N_{V_m} S_{x_m x_v}}{\sum_{x_m} \sum_{x_v} S_{x_m x_v}} \]

The method of interpolating \( N_{x_v} \) number of data within the free regions of the line \( ab \) for the data vector \( x_m \) is given by:

\[ x_{m_{new}} = x_m + \gamma \frac{x_v - x_m}{N_{x_v}} + r_m \]

where \( \gamma = 1, 2, 3, \ldots, N_{x_m} \) and \( r_m \) is a small random noise.

C. Illustration with a simulated dataset

A two class simulated dataset is created to visualize and compare the performance of the GICaPS oversampling approach with existing well established techniques, such as SMOTE and ADASYN. The results of SMOTE and ADASYN are shown in Figure 10 and Figure 11 respectively and the performance of the GICaPS oversampling approach is already shown in Figure 7. The black triangular points are the original data of minority class and red circles are the synthetically generated data. The green triangular points are the original data of
Algorithm 2: Algorithm for Oversampling data

1: procedure (Given $C$ classes in a dataset and class $c_i$ is under consideration for interpolation. Let us assume that data has to be interpolated between two points $x_{m}$ and $x_{v}$.)
2: for class $c_i$ to $C$ do
3: for each data $x_m$ in the class $c_i$ do
4: Identify region $V_m$ that includes the data points $x_m$ and $x_v$ from class $c_i$ and neighboring points from all other classes $c_j$ where $j = 1, \ldots, C$ and $j \neq i$.
5: for each vector $x_m$ in the region $V_m$ do
6: Find two regions $R_j$ and $Q_j$ near to the line joining the points $x_m$ and $x_v$ representing possible boundary of class $j$.
7: To avoid interference while interpolation select legitimate places for data interpolation between two points.
8: Find No man’s land using the method explained in Section IV-B.
9: for each data point in the region $R_j$ and $Q_j$ do
10: Calculate the intersecting points and crossing distance using the technique described in Section IV-B.
11: repeat the above step for all class $j = 1, \ldots, C$ and $j \neq i$.
12: end for
13: After No man’s lands are identified, calculate the total number of points to be between for each data points $x_m$ within the region $V_m$.
14: Calculate the empty region $S^{x_m}$ between $x_m$ and $x_v$.
15: Find $S^{x_m}$ for all the points in the region $V_m$.
16: end for
17: Calculate the space vector for each data point $x_m$ in the class $c_i$.
18: Calculate the total number of data to be interpolated in the neighborhood of the vector $x_m$.
19: end for
20: end for
21: end procedure

Fig. 8: Oversampling results of proposed approach for a synthetically generated 3 dimensional dataset having two different classes. Class 2 view.

Fig. 9: Oversampling results of proposed approach for a synthetically generated 3 dimensional dataset having two different classes. Overall view.

Fig. 10: Oversampling results of a synthetically generated two class dataset using SMOTE.

Fig. 11: Oversampling results of a synthetically generated two class dataset using ADASYN.

majority class. It can be observed that, the distribution of the synthetically generated data is not uniform in both SMOTE and ADASYN. SMOTE interpolates data between the two existing points without even considering the interference of the majority class. Also, as SMOTE randomly decides which of the K nearest neighbor is to be selected for interpolation, it may happen that more relevant point gets missed and too many data gets interpolated between two closely placed data points. Figure 11 shows the results of synthetic data generation for the minority class data. The observation clearly shows that the new data is generated only near to the border. Even most of the synthetically generated data falls on the territory of the majority class, which will definitely mislead the classifier. The observation gives a clear illustration that the performance of ADASYN is even worse than SMOTE. In contrast to both, the proposed approach takes every neighboring point into consideration, and equal spacing is maintained between all the points while interpolating data. Moreover, unlike SMOTE and ADASYN, the GICaPS decides how many points are to be interpolated between any neighboring points. It also decides whether to interpolate data between two points or not, based on possibility of conflict with other class boundaries. A visualization of the GICaPS oversampling approach is shown in Figure 7.

Table III shows an ablation study performed to proof the efficacy of the GICaPS-Oversampling approach. Two randomly chosen class data are used for this purpose. The chosen minority class is oversampled using GICAPS-oversampling approach, SMOTE and ADASYN. Support vector machine is used to perform this experiment. Minimum distance between support vectors of two classes is calculated for each of the approaches and the same is presented in the Table.
III. We have shown five different datasets for this ablation study. The statistical analysis clearly shows that the proposed approach maintains maximum margin between two classes after oversampling is performed. It is to be noted that, angular information can also be used for the proposed oversampling approach. The main intention of using the angular information in the undersampling approach to remove redundant data and to avoid removal of more informative data which may not be taken care when the Euclidean distance considered as the rejection criteria. As, oversampling involves synthetic data generation, we opted to use Euclidean distance for the sake of lesser computational complexity. However, one can use angular information that will avoid generation of redundant data.

V. RECOGNITION USING MIXTURE OF GAUSSIANS

Gaussian distribution has a wide range of applicability in realistic distributions. The performance and applicability of the estimating model is further enhanced when multiple Gaussians are used in place of one to model the data distribution. In this work, the distribution of the training data is captured using Gaussian mixture model which is a linear combination of finite number of Gaussians. The recognition problem is solved as a regression problem. The class identity is predicted by the regressive model, created by the mixture of Gaussians. We select regression over the standard GMM classification because of two reasons: i) The execution time is faster when the class dimension is high and ii) we want to show that a regressive model can also perform well with the dataset created using the proposed data balancing technique.

A. Recognition model using mixture of Gaussians

The regressive model for the recognition problem is given in the following:

\[ y = f(x) \]  

(20)

where, \( x \in \mathbb{R}^D \) is the feature vector and \( y \in \mathbb{R} \) is the class labels and can take values \( y_i = I_i, \quad i = 1, 2...C \), where \( C \) is the total number of class labels. Let's assume that the random feature vector \( x \) can be matched with a class variable \( y \) and the joint probability density can be modeled using the mixture of Gaussians [55]. The probability distribution of the random variable \( \xi = [x; y] \) fits into the GMM and is given by

\[ p(\xi) = \sum_{k=1}^{K} \pi_k N(\xi; \mu_k, \Sigma_k) \]

(21)

\[ = \sum_{k=1}^{K} \pi_k \frac{1}{\sqrt{(2\pi)^D|\Sigma_k|}} e^{\frac{1}{2}(\xi-\mu_k)^T \Sigma_k^{-1} (\xi-\mu_k)} \]

(22)

where, \( \pi \) is the class prior or prior probability and \( N(\mu_k, \Sigma_k) \) is the \( k^{th} \) Gaussian distribution with \( \mu_k \) being the mean and \( \Sigma_k \) is the co-variance of the distribution and is given by:

\[ \mu_k = \begin{bmatrix} \mu_k^x \\ \mu_k^y \end{bmatrix} \quad \text{and} \quad \Sigma_k = \begin{bmatrix} \Sigma_k^{xx} & \Sigma_k^{xy} \\ \Sigma_k^{yx} & \Sigma_k^{yy} \end{bmatrix}. \]

(23)

The posterior \( p(y|x) \) for a given feature vector \( x \) and component \( k \) can be found using Gaussian mixture regression. The posterior mean estimate \( \hat{y} \) can be found as

\[ \hat{y} = \sum_{k=1}^{K} h(k) \left[ \mu_k^x + \Sigma_k^{xy} (\Sigma_k^{xx})^{-1} (x - \mu_k^x) \right] \]

(24)

where, \( h(k) = \frac{p(y|x, k)p(k)}{\sum_{k'=1}^{K} p(y|x, k')p(k')} \). The class variable \( y \) is given by \( y = g(\hat{y}) \), where, \( g: \mathbb{R} \to \mathbb{R} \quad \forall x \in \mathbb{R}^D \) maps \( \hat{y} \) to its nearest class value. The parameters of the Gaussian distributions are estimated using Expectation maximization (EM), since the maximum likelihood does not work here as there is no closed form solution for GMM. The EM algorithm can be found in [55].

VI. EXPERIMENTAL RESULTS AND DISCUSSIONS

The proposed algorithm has been tested on ten popular imbalanced datasets. The datasets are chosen in such a manner that it contains numeric attributes and no missing data. Unlike other existing imbalanced data handling techniques, this work includes multi-class datasets with number of classes as high as 23 in case of abalone dataset and 15 in case of UNBC-McMaster Shoulder Pain Expression Archive database [52]. Classes containing only one instance have been removed from the datasets as the proposed data handling technique in its current state cannot generate new data with only one sample. In case of pain dataset [52], geometric features vector \( x \in \mathbb{R}^{22} \) is extracted from all the images present in the database. Initially the face is detected using Viola Jones’ face detection
The proposed undersampling and oversampling approaches is during training of the classification model. Source code of generated. This is done to reduce huge computational cost for all the minority classes equivalent number of data are randomly picked and Table V respectively. In case of ADASYN, we have ADASYN and GICaPS are presented in Table IV, Table VI. The distribution of data among classes after applying SMOTE, GICaPS, the data distribution among classes after applying ADASYN. It has been observed that, unlike ADASYN and SMOTE respectively. Whereas, the recognition accuracy of the proposed data handling approach outperforms the recognition accuracy of both ADASYN and SMOTE. The proposed data balancing approach gives an average recognition accuracy of 98.81% which is a significant improvement over SMOTE and ADASYN. The performance of GICaPS is also compared with some of the recent state-of-the-art techniques, such as SIMO [59], WSIMO [59], SWIM [46] and MOCAS (NN) [60]. The statistical comparisons are presented in the Table VII.

VII. CONCLUSIONS

Data imbalance poses serious challenges for classifier performance particularly in cases where minority class is of importance. This problem is addressed in this paper by proposing a data processing framework called GICaPS that uses geometric information-based sampling and class-prioritized synthesis for undersampling and oversampling data in majority and minority classes respectively. The proposed undersampling algorithm uses an angular constraint to remove redundant information in a majority class while ensuring that the valuable information is not lost. This is ensured by restricting the removal of data points only from other orthants. On the other hand, the proposed oversampling method populates the minority class by generating data that respects class boundaries. This is achieved by avoiding data generation in the no-man’s land between the classes. Mathematical expressions are derived for these constraints and concepts, thereby providing a theoretical basis for these algorithms. Pseudocodes for these algorithms are provided for easy implementation. The superiority of the proposed data sampling algorithms is established through rigorous performance comparison analysis with the current state-of-the-art methods on 10 different real-world datasets exhibiting high data imbalance. The future scope of this work would involve extending these concepts to hybrid algorithms to further improve the classification performance on imbalance datasets.

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### TABLE IV: Data distribution of pain db after applying SMOTE. (The terms are: I-Intensity, DS-Data size)

| I   | I_0  | I_1  | I_2  | I_3  | I_4  | I_5  | I_6  | I_7  |
|-----|------|------|------|------|------|------|------|------|
| DS  | 39835| 11632| 9396 | 5636 | 3208 | 968  | 1080 | 212  |

### TABLE V: Data distribution of pain db after applying GICaPS (The terms are: I-Intensity, DS-Data size)

| I   | I_0  | I_1  | I_2  | I_3  | I_4  | I_5  | I_6  | I_7  |
|-----|------|------|------|------|------|------|------|------|
| DS  | 13502| 13502| 13502| 13502| 13502| 13502| 13502| 13502|

The balanced datasets are trained using the GMR model. The recognition performances of all the datasets, in terms of overall accuracy, precision, recall, F-measure and G-Mean are presented in a tabular form as given in the Table VII. The observation shows that, performance of GICaPS is significantly better than ADASYN and SMOTE in almost all the datasets. For instance, in case of pain database, average recognition accuracies of 13.197% and 90.24% are achieved for ADASYN and SMOTE respectively. The recognition accuracy of the proposed data handling approach outperforms the recognition accuracy of both ADASYN and SMOTE. The proposed data balancing approach gives an average recognition accuracy of 98.81% which is a significant improvement over SMOTE and ADASYN. The performance of GICaPS is also compared with some of the recent state-of-the-art techniques, such as SIMO [59], WSIMO [59], SWIM [46] and MOCAS (NN) [60]. The statistical comparisons are presented in the Table VII.

The training and testing are done using 10 fold cross validation technique. To validate the performance of the proposed data balancing technique, we compare the results with well established data handling techniques, such as SMOTE and ADASYN. It has been observed that, unlike ADASYN and GICaPS, the data distribution among classes after applying SMOTE remains skewed in most of the highly imbalanced datasets. Such an instance can be shown using the pain dataset. The distribution of data among classes after applying SMOTE, ADASYN and GICaPS are presented in Table IV, Table VI and Table V respectively. In case of ADASYN, we have randomly picked 10000 data from the majority class and for all the minority classes equivalent number of data are generated. This is done to reduce huge computational cost during training of the classification model. Source code of the proposed undersampling and oversampling approaches is available online [58].
| Dataset          | Evaluation matrices and comparison with different datasets                                                                 |
|------------------|--------------------------------------------------------------------------------------------------------------------------|
| Abalone          | Methods                                                                                                                  |
|                  | OOA | Precision | Recall | F-measure | G-Mean |
| SMOTE            | 96.27 | 96.22 | 96.46 | 86.65 |
| ADASYN           | 91.51 | 43.3 | 77.30 | 45.32 | 39.23 |
| Cost sensitive   | -    | -     | -     | -         |
| SIMO [59]        | -    | -     | -     | -         |
| WSIMO [59]       | -    | -     | -     | -         |
| MOCAS (NN) [60]  | 91.4 | 82.8 | 86.2 | 92.3 |
| Spambase         | GICaPS-O | 96.95 | 92.38 | 92.38 | 92.38 |
|                  | SMOTE | 89.95 | 93.09 | 90.14 | 91.46 | 91.60 |
|                  | ADASYN | 87.35 | 88.81 | 88.81 | 88.81 |
|                  | SWIM [46] | - | - | - | 68.5 |
| Glass            | GICaPS-O | 96.38 | 97.02 | 96.36 | 96.50 | 96.7 |
|                  | SMOTE | 96.27 | 97.08 | 96.22 | 96.46 | 86.65 |
|                  | ADASYN | 91.51 | 43.3 | 77.30 | 45.32 | 39.23 |
|                  | Cost sensitive | - | - | - | - |
|                  | SIMO [59] | - | - | - | - |
|                  | WSIMO [59] | - | - | - | - |
|                  | MOCAS (NN) [60] | 91.4 | 82.8 | 86.2 | 92.3 |
| Ionosphere       | GICaPS-O | 88.50 | 90.38 | 88.49 | 89.00 | 89.43 |
|                  | SMOTE | 84.65 | 91.45 | 84.63 | 87.06 | 87.97 |
|                  | ADASYN | 81.60 | 84.43 | 81.61 | 80.15 | 83.01 |
|                  | Cost sensitive | - | - | - | - |
|                  | SIMO [59] | - | - | - | - |
|                  | WSIMO [59] | - | - | - | - |
|                  | MOCAS (NN) [60] | 91.4 | 82.8 | 86.2 | 92.3 |
| Sonar            | GICaPS-O | 85.55 | 90.39 | 90.48 | 90.28 | 90.43 |
|                  | SMOTE | 80.70 | 87.39 | 82.21 | 84.77 | 84.70 |
|                  | ADASYN | 12.8 | 11.23 | 12.79 | 11.76 | 11.98 |
| Wine             | GICaPS-O | 89.67 | 90.40 | 89.66 | 89.84 | 90.03 |
|                  | SMOTE | 85.60 | 90.06 | 83.60 | 87.22 | 89.01 |
|                  | ADASYN | 20.98 | 18.21 | 20.99 | 19.09 | 19.55 |
|                  | SWIM [46] | - | - | - | 73.0 |
| Pima Indian diabetes | GICaPS-O | 84.15 | 83.77 | 84.17 | 83.88 | 83.97 |
|                  | SMOTE | 76.40 | 81.16 | 76.53 | 78.15 | 78.81 |
|                  | ADASYN | 75.45 | 75.58 | 75.74 | 75.65 | 75.66 |
|                  | SWIM [46] | - | - | - | 50.9 |
|                  | MOCAS (NN) [60] | - | 73.4 | 60.1 | 65.9 | 73.4 |
| Shuttle          | GICaPS | 99.39 | 99.30 | 99.27 | 99.33 | 99.23 |
|                  | SMOTE | 96.59 | 82.35 | 96.60 | 86.29 | 89.19 |
|                  | ADASYN | 56.27 | 55.84 | 56.25 | 55.42 | 56.04 |
| Fertility        | GICaPS-O | 96.60 | 96.56 | 96.56 | 96.56 | 96.56 |
|                  | SMOTE | 76.5 | 81.26 | 78.33 | 78.65 | 79.78 |
|                  | ADASYN | 81.75 | 82.02 | 83.84 | 80.89 | 82.92 |
| Pain             | GICaPS | 98.80 | 98.79 | 92.61 | 98.79 | 95.65 |
|                  | SMOTE | 90.24 | 90.55 | 90.22 | 90.33 | 90.38 |
|                  | ADASYN | 12.7 | 14.73 | 12.19 | 17.45 | 12.94 |

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