Multifractal Detrended Cross-correlation Analysis of Gold and WTI Crude Oil Price Time Series

Burugupalli S
Center for Integrated Studies, University of Hyderabad, Gachibowli, Hyderabad-500 046, India

Abstract
Several papers have documented the cross-correlations across wide range of markets using different methods and the results are being used for the improvrisation of market efficiency. In this paper, we have investigated the cross-correlations between Gold market and WTI Crude oil market using MF-X-DFA. Quantitatively, we have employed the recently developed MF-X-DFA method for the data of Gold and WTI Crude oil markets and we have confirmed the existence of cross-correlation between Gold and Crude oil markets. We have also found that these cross-correlations are strongly multifractal in the short term and weakly multifractal in long term. Moreover, their behavior for small fluctuations is persistent and those of large fluctuations are anti-persistent in the short term. Cross-correlations of very small fluctuations and large fluctuations are persistent but whereas for small fluctuations are anti-persistent in the long term. Furthermore, based on multifractal spectrum, we have produced substantial evidences to determine cross-correlation behaviors exhibit multifractal features. Our results have significant implications to market efficiency.

Keywords: Non-stationary time series; Fractals; Hurst exponent; Multifractal detrended Cross-correlation Analysis

Introduction
The multifractality in real world non stationary time series data are described better from scaling exponents. The observed signals of the physical quantities characterizing any complex system like social, financial, ecological, biological or technological are composed of large number of interacting assorted parameters linked each other nonlinearly and they exhibit long-range correlations. It is of crucial importance and significance to quantify such long-range correlations to have a deep understanding of the dynamics of the underlying complex systems. The theory of fractals proposed by Mandelbrot [1] in contrast to the efficient market hypothesis leads to study of complex system behavior through different method development and approaches. Hurst [2,3] proposed rescaled range analysis (R/S), the most popular analysis method for fractals. To overcome R/S analysis method sensitivity to short-term auto-correlation and non-stationary nature causing bias error; detrended fluctuation analysis (DFA) method was proposed [4] which applied long-range power-law correlation. DFA and its multifractal generalization MF-DFA methods [5] are widely used to describe the fractal properties. Many researchers studied multifractal properties on the different non stationary natural and financial time series data like biological data [6], exchange rate market [7], stock market [8,9] gold market [8,10], crude oil market [9-12]. Two major sources of multifractality which can be found in various time series: one is nonlinear temporal correlation for small and large fluctuations; while the other is fat-tailed probability distribution of increments [8,13,14].

Also multifractality in the daily returns of emerging European stock markets is studied using the Empirical Mode Decomposition (EMD) based Multifractal Detrended Fluctuation Analysis [15]. A comparison study of different fluctuation methods for analyzing non-stationary time series were studied for efficacy of the methods like Fluctuation Analysis (FA); Detrended Fluctuation Analysis (DFA) and Detrended Moving Average (DMA). It was reported that [16] Centred Detrending Moving Average (CDMA) and DFA are the Methods of Choice in determining the Hurst index of time series. CDMA has the best performance while DFA is only slightly worse in some situations. Also CDMA and DFA are less sensitive to the scaling range compared to other methods like FA.

Cross-correlation found to be exists in many simultaneously recorded time series in various real world commodities or financial market data. Therefore, in recent years, many researchers attempted to quantify cross-correlations between non stationary data from complex systems. Detrended Cross-Correlation Analysis (DCCA) was proposed [17] to investigate power-law cross-correlations between two simultaneously recorded time series in the presence of non-stationarity. This new method was applied to different time series for long-range power-law cross-correlations [18]. Subsequently, multifractal generalization of the method was proposed [19] describing the new method as multifractal detrended cross-correlation analysis (MF-DCCA) by combining MFDFA and DCCA approaches. The MFDCCA method was applied [13,20-26] to many different time series for cross correlation analysis.

Efficiency of Brent and WTI crude oil market was analyzed by means of R/S analysis [27]. Dynamics of crude oil prices was studied thru stochastic multi-model approach [28]. Market agent perspective on fractal features in crude oil time series was studied [29,30]. From the analysis it was evident that the crude oil market is a persistent process with long-range memory effects with multifractal structures. The MFDFA analysis on gold price revealed that multifractality is mainly due to the temporal correlation [30]. But there exist a cross correlation between the gold and Crude oil time series to study market behavior...
Methodology
Assume that there are two series x(i) and y(i) where i = 1, 2, . . . , N. Now the MFD CCA method is as follows.

Step 1. Construct the profile

\[ (i) = \sum (x(t) - \bar{x}) \]
\[ Y(i) = \sum (y(t) - \bar{y}) \]

Where \( \bar{x} \) and \( \bar{y} \) are the average of the two time series x(i) and y(i).

Step 2. The profiles x(i) and y(i) are divided into \( N_s = \lfloor N/s \rfloor \) non-overlapping windows of equal length s. Since the length N is not always a multiple of the considered time scale s hence in order not to discard the section of series, the same procedure is repeated starting from the reverse end of each profile. Thus, \( 2N_s \) non-overlapping windows are obtained together.

Step 3. The local trends xν(i) and yν(i) for each segment ν (where, ν=1, 2, 3, . . . , 2Ns) are evaluated by least squares fits of the data, then the detrended covariance is determined by

\[ F_s (i, v) = \sum_{i=1}^{s} X(i) - X^\nu(i) \cdot Y(i) - Y^\nu(i) \]

Here for each segment v, v=1,2, . . . , Ns hence,

\[ F_s (i, v) = \sum_{i=1}^{s} X(N-(v-N_s)s+i) - X^\nu(i) \cdot Y(N-(v-N_s)s+i) - Y^\nu(i) \]

Step 4. Hence the qth order fluctuation function would be,

\[ F_q (s, v) = \left[ \frac{1}{2Ns} \left( \sum_{i=1}^{2Ns} F^2 (i, v) \right) \right]^{1/q} \]

If \( q \neq 0 \) then the qth order fluctuation function would be,

\[ F_q (s) = \exp \left[ \frac{1}{4N^r} \sum_{i=1}^{rs} \ln \left( \sum_{i=1}^{2Ns} F^2 (i, v) \right) \right] \]

When q=2 the MFD CCA is as like CCA.

Step 5. Analyze the scaling behavior of the fluctuations by observing logarithmic plots between \( F_q (s) \) and s for each values of q. If the two series are long-range cross-correlated then the \( F_q (s) \) will increase for values of s and we can obtain a power-law expression as

\[ F_q (s) \sim s^{H_q (q)} \]

Hence taking log on both sides, we could represent this as

\[ \log (F_q (s)) = H_q (q) \log (s) + \log (\lambda) \]

Here the scaling exponent \( H_q (q) \) is known as the generalized cross-correlation Hurst exponent, describing the power-low relationship.
Results and Discussion

To test the presence of cross-correlation quantitatively we need MF-X-FA method which can estimate the cross-correlation exponent. We show the log-log plots of fluctuation function $F_q(s)$ versus time scale $s$ for Gold and WTI Crude oil markets. In Figure 2 we can find that only one line cannot fit the log-log plots of $F_q(s)$ versus time scale $s$ well. Hence we define the "crossover" $s^*$, as the turning point when the linear trend of the curves underwent a fundamental change. The short-term behavior of financial market is easily influenced by the market external factors such as the major events while the long-term behavior is determined by the internal factors. With the time evolving, the short-term shocks gradually decay for the effects of long-term supply and demand mechanism in the markets. The scaling exponents for $s<s^*$ can reflect the short range correlation also imply the correlated behaviors in the short-term. Then, the scaling exponents for $s>s^*$ imply the correlated behaviors in the long-term (Table 1). Thus, we can say that the "crossover" can reflect the lasting period of the effects of the factors which determine the market short-term behavior. We found the "crossover" at about $\ln(s^*)=4.8$ (i.e. 130 days). We provide the slopes of each line, just the scaling exponents for $s<s^*$ and $s>s^*$ in the Tables 2 and 3 respectively.

For $q=2$, the scaling exponent is the Hurst exponent calculated from the method of DFA. When $q=2$, the scaling exponent for Gold and WTI Crude oil markets is markets is 0.49334 for $s<s^*$ indicating that Gold and WTI Crude oil markets are weakly cross-correlated in the short-term. For $s>s^*$, the scaling exponent for Gold and WTI Crude oil markets is 0.50259, larger than that for $s<s^*$ indicating that they are moderately cross-correlated in the long-term.
For other values of $q$, the scaling exponents indicate the cross-correlated behaviors between the kinds of fluctuations related to $q$ showing multifractality behavior. We provide the scaling exponents with $q$ varying from $-10$ to $10$ for $H_x(q)$, $H_y(q)$, and the average scaling exponents $[H_x(q)+H_y(q)]/2$ in the short term and long term in Figures 3 and 4 respectively.

From Figure 3 for $s<s^*$, we can find that $H_x(q)$-Gold market scaling exponents are decreasing from over 0.63 to 0.28 indicating it is exhibiting high multifractal features in the long term. Similarly $H_y(q)$-WTI Crude oil market scaling exponents increase from 0.5 to 0.6 indicating it is moderately multifractal. While the cross-correlated scaling exponents $H_{xy}(q)$ are not varying considerably indicating cross-correlated behavior is weakly multifractal in the long term. Moreover for $q>5$ the scaling exponents are greater than 0.5 indicating that cross-correlated behaviors of large fluctuations are persistent (positive) in the long term. Congruently for $q>1$ the scaling exponents are greater than 0.5 indicating that cross-correlated behaviors of large fluctuations are also persistent (positive) in the long term.

For $q<0$, the scaling exponents $H_y(q)$ are always less than the average scaling exponents $[H_x(q)+H_y(q)]/2$ and for $q>0$, the scaling exponents $H_x(q)$ are greater than the average scaling exponents $[H_x(q)+H_y(q)]/2$ in the short term.

For $q<1$, the scaling exponents $H_{xy}(q)$ are always less than the average scaling exponents $[H_x(q)+H_y(q)]/2$ and for $q>1$, the scaling exponents $H_{xy}(q)$ are greater than the average scaling exponents $[H_x(q)+H_y(q)]/2$ in the long term.

**Conclusion**

Historically gold retains its value during times of crisis and is used as a hedge against inflation, deflation or currency devaluation. Gold also most popularly used as an investment. Crude Oil is a vital source of energy for the world hence higher crude oil prices drive fuel inflation as crude oil demand is inelastic. Diversity of participants like producers, government, extreme socio-political events and speculators drive crude oil market price.

The results demonstrate the overall significance of the cross-correlation based on the analysis of multifractality. We found that the global Hurst coefficient varies with the $q$ and there is multifractality evidenced through the multifractal spectrum also. We get the cross-correlation exponent 0.48. We found that there exists a power-law cross-correlation between the Gold and Crude Oil time series and the multifractal features are significant. The analysis throws light on the structure of crude oil and Gold market as well as its link to macroeconomic conditions and socio-political extreme events.

In this paper, we used multifractal detrended cross-correlation analysis to investigate the cross-correlation properties between Gold and WTI Crude Oil. We find that the cross-correlations display the characteristic of multifractality in the short term. Moreover, the cross-correlations of small fluctuations are persistent, and those of large fluctuations are anti-persistent in the short term, while the cross-correlations of all kinds of fluctuations are persistent in the long term.

Finally, when $q<0$, the cross-correlation exponents are smaller than their average exponents, but larger than the average exponents when $q>0$.

From our analysis we found Gold price has long term correlation with crude oil price. That is the reason why gold is sold off during economic weakness as it’s also used as a kind of proxy currency. Currencies tend to lose their purchasing power over years due to inflation and as time passes it can’t buy the same amount of oil which could have bought years ago while gold could. The main idea behind the gold-crude oil cross correlation is the one which suggests that prices of crude oil partly account for inflation. Increases in oil price increases prices of gasoline which is derived from oil which drives transport of crude oil demand. That is the reason why gold is sold off during economic weakness. Economic weakness as it’s also used as a kind of proxy currency. Currencies tend to lose their purchasing power over years due to inflation and as time passes it can’t buy the same amount of oil which could have bought years ago while gold could. The main idea behind the gold-crude oil cross correlation is the one which suggests that prices of crude oil partly account for inflation. Increases in oil price increases prices of gasoline which is derived from oil which drives transport of crude oil demand. That is the reason why gold is sold off during economic weakness, deflation or currency devaluation. Gold also most popularly used as an investment. Crude Oil is a vital source of energy for the world hence higher crude oil prices drive fuel inflation as crude oil demand is inelastic. Diversity of participants like producers, government, extreme socio-political events and speculators drive crude oil market price.

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goods costly hence the Good prices rises. As a final result inflation hence gold price tend to appreciate with inflation rising. So, an increase in the price of crude oil can eventually translate into higher gold price.

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