Cranked Relativistic Mean Field Description of Superdeformed Rotational Bands

A.V. Afanasjev\textsuperscript{1,2}, G.A. Lalazissis\textsuperscript{1,3} and P. Ring\textsuperscript{1}

\textsuperscript{1} Physik Department der Technischen Universität München
D-85747, Garching, Germany
\textsuperscript{2} Nuclear Research Center, Latvian Academy of Sciences
LV-2169, Salaspils, Miera str. 31, Latvia
\textsuperscript{3} Department of Theoretical Physics, Aristotle University of Thessaloniki,
GR-54006, Thessaloniki, Greece

Abstract The cranked relativistic mean field theory is applied for a detailed investigation of eight superdeformed rotational bands observed in \( ^{151} \text{Tb} \). It is shown that this theory is able to reproduce reasonably well not only the dynamic moments of inertia \( J^{(2)} \) of the observed bands but also the alignment properties of the single-particle orbitals.

In the relativistic mean field (RMF) theory the nucleus is described as a system of point-like nucleons, Dirac spinors, coupled to the meson and Coulomb fields. The nucleons interact via the exchange of several mesons, namely a scalar \( \sigma \)-meson, which provides a strong intermediate range attraction, the isoscalar-vector \( \omega \)-meson responsible for a very strong repulsion at short distances and the isovector-vector \( \rho \)-meson which takes care of the symmetry energy. This theory with only seven parameters fitted to the properties of several spherical nuclei provides an economic and accurate way to describe many properties of finite nuclei throughout the periodic table \cite{1}.

The RMF theory formulated in the rotating frame - cranked RMF theory \cite{2,3} (further CRMF) - has been recently applied for a systematic investigation of superdeformed (SD) rotational bands observed in the \( A \sim 140 - 150 \) mass region \cite{4}. It was shown that this theory provides a rather good agreement with the available experimental data on the dynamic moments of inertia \( J^{(2)} \). It reproduces the trend of the changes of the charge quadrupole moments \( Q_0 \). Moreover, the classification of the SD bands in terms of the number of filled high-\( N \) intruder orbitals, originally suggested within the cranked Nilsson (further CN) model \cite{5}, is supported by the CRMF theory.

As the linking transitions from superdeformed states have not been identified in this mass region, the relative properties of different SD bands play an important role in our understanding of their structure. One way to identify the single-particle orbital by which two SD bands differ is to compare the difference in their dynamic moments of inertia \( J^{(2)} \) observed in experiment with the ones obtained in calculations. However, the deficiency of this approach is that different (especially, non-intruder) orbitals have rather similar contributions to the total \( J^{(2)} \). This prevents a unique definition of the underlying configuration in terms of non-intruder orbitals due to the uncertainty related to the single-particle energies in the SD minimum.

An alternative way to analyse the contributions coming from specific orbitals and thus to identify the configuration is the effective alignment approach suggested by I. Ragnarsson \cite{6}. The effective alignment of two bands is defined as the difference between their spins at constant rotational frequency \( \Omega_x \): \( i_{\text{eff}}^{B,A}(\Omega_x) = I_B(\Omega_x) - I_A(\Omega_x) \). The notation
simply reflects the fact that the spins of the bands under consideration are not experimentally determined. In comparison with the method based on $J^{(2)}$, this approach allows also to study the absolute values of the relative alignment of two SD bands. This provides additional information when theory and experiment are compared. Therefore, the application of this approach should be considered as a necessary test which shows whether theory is able to describe the alignment properties of the single-particle orbitals in the SD minimum.

A systematic investigation of the alignment properties of the single-particle orbitals in the SD minimum of the $A \sim 140 - 150$ mass region has been performed within the CRMF theory and the results will be presented in a forthcoming article [7]. In the present article, the results of the calculations for eight superdeformed bands observed in $^{151}$Tb [8] are reported using the two methods mentioned above. The choice of this nucleus is motivated by several reasons. First, the number of observed SD bands is the largest among all nuclei in this mass region. Second, the SD bands observed in this nucleus have been studied in various non-relativistic theoretical models such as the CN model [8, 10] and the cranked Hartree-Fock approach with Skyrme forces [11, 12, 13]. Thus, a comparison of our calculations with the ones obtained with these models can be made. One should keep in mind that the effective alignment approach has been used mainly within the CN model so far. Third, the pairing correlations play a minor role at high rotational frequencies ($\Omega_x > 0.5$ MeV) for nuclei close to $^{152}$Dy. As a result, the pairing correlations are neglected in the calculations. In this article calculations have been performed using the non-linear parameter set NL1 [14]. Details of the calculations will be given in [7].

In this paper, we restrict ourselves to theoretical configurations discussed in the literature [10, 11, 12, 13]. The results of the calculations and the configuration assignment are presented in Fig. 1 and Table 1. In the upper part of Fig. 1, the effective alignments, $i_{\text{eff}}$ (in units $\hbar$), extracted from the experimental data (unlinked large symbols) and from the assigned calculated configurations (linked small symbols of the same type) are shown. The experimental effective alignment between SD bands A and B is indicated as “A/B”. The band A in the lighter nucleus is taken as a reference so the effective alignment measures the effect of the additional particle. The experimental data are taken from [8, 9, 15, 16]. The compared configurations differ by the occupation of the orbital indicated in the figure. In the lower part of Fig. 1, the experimental dynamic moments of inertia $J^{(2)}$ of observed bands are shown along with the calculated ones for assigned configurations. The same notation is used as in the upper part.

The interpretation of the bands 1-4 as based on proton holes in the doubly magic $^{152}$Dy core discussed in [10, 11, 12, 13] is consistent with present calculations, see Fig. 1. However, the CRMF calculations suggest that the spins of the $^{151}$Tb(1) band relative to the $^{152}$Dy(1) band should be increased by $2\hbar$ compared with the results of the CN calculations [10]. Fig. 1 shows that the effective alignment of the $\pi[651]3/2(r = +i)$ orbital is described much better in the CRMF theory than in the CN model, see Fig. 3 in [10].

The effective alignment of the $^{152}$Dy(1) band relative to the bands 5-8 in $^{151}$Tb is strongly increasing as a function of rotational frequency with total gain in $i_{\text{eff}}$ of $\approx 3\hbar$. Considering that in the vicinity of the $N = 86$ and $Z = 66$ SD shell gaps only the $\nu[770]1/2(r = -i)$ orbital has such properties, this indicates that the $^{151}$Tb(5-8) bands are most likely based on neutron excitations from the $\nu[770]1/2(r = -i)$ orbital into...
Effective alignment \( \ell_{\text{eff}} \)

\[ ^{151}\text{Tb}(1) / ^{152}\text{Dy}(1) \]
\[ \pi[651]3/2(r=+) \]

\[ ^{151}\text{Tb}(2) / ^{152}\text{Dy}(1) \]
\[ \pi[301]1/2(r=+) \]

\[ ^{151}\text{Tb}(3) / ^{152}\text{Dy}(1) \]
\[ \pi[651]3/2(r=-) \]

\[ ^{151}\text{Tb}(4) / ^{152}\text{Dy}(1) \]
\[ \pi[301]1/2(r=-) \]

\[ ^{151}\text{Tb}(1) / ^{151}\text{Tb}(5,6) \]

\[ ^{150}\text{Tb}(1) / ^{151}\text{Tb}(7,8) \]

Rotational frequency \( \Omega \)

Dynamic moment of inertia \( J^{(2)} \) (MeV)

Fig. 1. See text for details.
Table 1: Configuration assignment for the bands 1-8 observed in $^{151}$Tb. Their detailed structure is given relative to the doubly magic $^{152}$Dy core. The energies of intraband $\gamma$-transitions $E_0^0$ populating the lowest observed SD states are listed in column 3. The difference in assigned spins $\Delta I_0$ of the lowest states of observed SD bands in $^{151}$Tb and lowest state of yrast SD band in $^{152}$Dy is shown in column 4. The difference $\Delta Q_0 = Q_0(^{151}\text{Tb(N)}) - Q_0(^{152}\text{Dy(1)})$ in charge quadrupole moments $Q_0$ calculated at $\Omega_x = 0.5$ MeV is given in column 5.

| Band | Configuration | $E_0^0$(keV) | $\Delta I_0(h)$ | $\Delta Q_0$ (eb) |
|------|---------------|-------------|----------------|-----------------|
| 1    | $\pi[651]3/2(r = +i)^{-1}$ | 726.5    | +6.5          | -0.99          |
| 2    | $\pi[301]1/2(r = +i)^{-1}$ | 602.1    | +0.5          | +0.20          |
| 3    | $\pi[651]3/2(r = -i)^{-1}$ | 681.5    | +3.5          | -0.90          |
| 4    | $\pi[301]1/2(r = -i)^{-1}$ | 768.6    | +7.5          | +0.24          |
| 5    | $\pi[651]3/2(r = +i)^{-1}$ | 709.3    | +6.5          | -1.80          |
| 6    | $\nu[770]1/2(r = -i)^{-1}\nu[402]5/2(r = -i)^1$ | 739      | +7.5          | -1.80          |
| 7    | $\pi[651]3/2(r = +i)^{-1}$ | 758      | +9.5          | -1.51          |
| 8    | $\nu[770]1/2(r = -i)^{-1}\nu[521]3/2(r = +i)^1$ | 785      | +10.5         | -1.51          |

the orbitals above the $N = 86$ SD shell gap. This simple consideration shows that the interpretation of band 7 discussed in [13] as based on neutron excitation across the $Z = 66$ SD shell gap is rather unrealistic.

It was suggested in [8] that bands 5 and 6 are most likely based on neutron excitations from $\nu[770]1/2(r = -i)$ to $\nu[402]5/2(r = \pm i)$. Similarly to the CN model, see Fig. 3 in [8], the results of our calculations are in good agreement with experiment which can be considered as an additional confirmation of this interpretation. The consistent interpretation of bands 7 and 8 is more difficult. It was suggested in [8] that these bands are connected with neutron excitations from $\nu[770]1/2(r = -i)$ to $\nu[521]3/2(r = \pm i)$. Although the calculated values of $J^{(2)}$ for configurations built on such excitations are reasonably close to the experimental ones, see Fig. 1, an important discrepancy between experiment and calculations exists for the effective alignments in the $^{150}\text{Tb(1)}/^{151}\text{Tb(7,8)}$ pairs. A strong argument against such an interpretation comes from the large signature splitting between the two signatures of the $\nu[521]3/2$ orbital. This splitting is rather similar both in the CRMF theory and in the CN model and it is in contradiction with the experimental data. One should note that the next orbital above the $N = 86$ SD shell gap, namely, $\nu[514]9/2$ is signature degenerated and configurations based on neutron excitations from $\nu[770]1/2(r = -i)$ to $\nu[514]9/2(r = \pm i)$ have nearly constant effective alignment (which is essentially close to zero) relative to the configuration of the $^{150}\text{Tb(1)}$ band.

In conclusion, cranked relativistic mean field theory has been applied for an investi-
igation of eight SD bands observed in $^{151}$Tb. The calculated dynamic moments of inertia and effective alignments are in reasonable agreement with experiment for bands 1-6. The consistent interpretation of bands 7 and 8 in a pure single-particle picture is more difficult. This might be connected with the influence of the residual interactions neglected in the present calculations.

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