Extraction of the static magnetic form factor and the structure function of the neutron from inclusive scattering data on light nuclei

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Abstract

We show that quasi-elastic inclusive electron scattering data on light nuclei for medium $Q^2$ furnish information on $G^n_M(Q^2)$, whereas the deep-inelastic region for large $Q^2$, provides the Structure Function $F^n_2(x, Q^2)$. Common to the two extractions is the possibility to de-convolute medium effects, which is most accurately done for light targets. Results are independent of the target.

Introduction. Most neutron observables can only indirectly be extracted from experiments on a nuclear medium, in which the $n$ is embedded. We discuss below the neutron static magnetic form factor and its Structure Function (SF).

Consider the reduced cross section for inclusive scattering of unpolarized electron of energy $E$ from non-oriented targets $A$ over an angle $Q^2$

$$\frac{A^{-1}d^2\sigma_{eA}(E;\theta, \nu)/d\Omega d\nu}{\sigma_M(E;\theta, \nu)} = \left[\frac{2xM}{Q^2}F^A_2(x, Q^2) + \frac{2}{M}\tan^2(\theta/2)F^A_1(x, Q^2)\right]$$

$F^A_k(x, Q^2)$ are two nuclear structure functions (SF), functions of $Q^2 = q^2 - \nu^2$ ($\nu, q$ are the energy-momentum transfer) and the Bjorken variable $x = Q^2/2M\nu$, with range $0 \leq x \leq A$. 


(\(M\) is the nucleon mass). Of crucial importance is a relation between the SF of nuclei and of nucleons. For instance (for \(Z = N\))

\[
F^A_k(x, Q^2) = \int_x^A \frac{dz}{z^{2-k}} \left[ F^{P\text{N,A}}_k \left( \frac{x}{z}, Q^2 \right) + F^{n\text{A}}_k \left( \frac{x}{z}, Q^2 \right) \right] / 2
\]  

(2)

The two SF are related by \(f^{P\text{N,A}}\), the SF of a nucleus, composed of point-nucleons. A standard calculation of \(F^A_k\) thus requires data on \(F^p_k\), an assumed form for \(F^n_k\) and in addition, a computed, unphysical \(f^{P\text{N,A}}\).

We separate \(F^N_k\) in NE (\(\Gamma^* + N \rightarrow N\)) and NI parts \((\gamma^* + N \rightarrow \text{hadrons, partons})\), leading to the corresponding components \(F^{A,\text{NE}}_k\) \([2]\) (\(\eta = Q^2 / 4M^2\))

\[
F^{A,\text{NE}}_1(x) = \frac{f^{P\text{N,A}}(x)}{4} G^2_d [(\alpha_p \mu_p)^2 + (\alpha_n \mu_n)^2]
\]

(3a)

\[
F^{A,\text{NE}}_2(x) = \frac{x f^{P\text{N,A}}(x) G^2_d}{2(1 + \eta)} \left[ (\alpha_p \gamma)^2 + \left( \frac{\mu_n \eta}{1 + 5.6\eta} \right)^2 + \eta [(\alpha_p \mu_p)^2 + (\alpha_n \mu_n)^2] \right],
\]

(3b)

where reference to \(Q^2\) has been dropped. Instead of the actual static electromagnetic form factors \(G^{N,M}_M(E,Q^2)\), we use in Eq. (3) their deviations from the standard dipole form \([3,4,5]\).

\[
\alpha_N \equiv \frac{G^N_M / \mu_N G_d}{G^{p}_M} \quad ; N = p, n
\]

(4a)

\[
\gamma = 1 + \theta(Q^2 - 0.3) \approx [1 - 0.14(Q^2 - 0.3)] ; Q^2 \lesssim 5.5
\]

(4c)

For \(G^m_E\) we use the Galster parametrization \([6]\). Nuclear NI components completely dominate cross sections on the inelastic side \(x \lesssim 1\) of the QEP, while for \(x \gtrsim 1\) NE>NI. Those regions will be treated separately.

Quasi-elastic region \(x \lesssim 1\): \(G^{n}_M\). Consider first the \(x, Q^2\) dependence of \(F^{A,\text{NE}}_k(x, Q^2)\). The latter is primarily due to the form factors in Eqs. (3), which decrease with growing \(Q^2\). The \(x\)-dependence resides in \(f^{P\text{N,A}}(x, Q^2)\), which sharply decreases with growing \(|1 - x|\) away from the QEP at \(x \approx 1\). From the above one concludes that \(\ln[\sigma^{A,\text{NE}}/A]\) grows with increasing \(\nu\) (decreasing \(x\) for fixed \(Q^2\)), while in general for \(A \geq 12\) there is a mere break in the slope in the QE region \(|1 - x| \ll 1\) for \(A \geq 12\) (Fig. 1a) \([7]\).
The unusual structure of the lightest nuclei, causes $f^{PN,A}(x, Q^2)$ to be narrow and sharply peaked. With no interference of NI, the above change in slope may develop into a QE peak, as observed for D [8] and $^4$He [4] (Fig. 1b). For the same targets one can compute with great precision ground states [10] and non-diagonal target density matrices in the expression for $f^{PN,A}$ [11,12].

Under the above circumstances one tends to ascribe the total cross sections on the elastic side $x \gtrsim 1$ to NE. With $G_{E,M}^p$ known and small $G_{E}^n$, this enables the extraction of $G_{M}^n$ from NE. Tests for the above allocations are: i) Around $x \lesssim 1$, $\sigma^A/\sigma_M \propto f(x, Q^2)$, i.e. of a bell shape in $1 - x$. ii) $G_{M}^n(Q^2)$ should be independent of the value of the individual $x$ from which the one extracts $G_{M}^n$. iii) Idem for the chosen target.

Our analysis comprises older D data, where separation into transverse and longitudinal SF, with the former $R_T \propto [G_{M}^p]^2 + [G_{M}^n]^2$ [13]. Although direct and simple, it requires high-quality data in order to allow an accurate Rosenbluth separation and to obtain a precise $G_{M}^n$. Table I summarizes all our findings for $\alpha_n(Q^2)$ while Fig. 2 shows all $\alpha_n(Q^2)$, extracted thus far. Our values follow the trend of previously measured values and adds points for intermittent $Q^2$. Hardly any target dependence has been detected.

The deep-inelastic region, $x \ll 1$: extraction of $F_{Q}^n(x, Q^2)$. That region is dominated by NI. We focus on $F_{Q}^n(x, Q^2)$, commonly estimated from the ‘primitive’ ansatz $F_{Q}^n \approx 2F_{2}^D - F_{2}^p$, which is only reliably for $x \lesssim 0.3$. Instead of a vehicle to compute $F_{k}^A$, we now consider Eq. (2) in the inverse sense: Can one, with data on $\sigma^A$, Eq. (1), known $F_{2}^p$ and computed $f^{PN,A}$ extract $F_{2}^n(x, Q^2)$?

Virtually all previous methods addressed a D target (e.g. [14]). We outline and apply a method [19], which with sufficient kinematics available [8], is applicable to all targets (see Refs. [15,16] for treatments of isobar pairs). Again a test is an outcome, independent of $A$. As to $F_{2}^A$, in order to separate it from $F_{1}^A$, one needs in addition to cross sections, an assumption on $R^{-1}(x, Q^2) + 1 \propto 2xF_{1}^A(x, Q^2)/F_{2}^A(x, Q^2)$. Alternatively, one may for every data point determine a relative deviation of theory and data, and ascribe it in equal measure to the two SF. The procedure produces quasi-data for $F_{2}^{A,\text{qd}}$. 

3
All modern data thus far \footnote{4,5} appear to yield $F_A^2$ in disjoint $x, Q^2$ regions, whereas the inversion of Eq. (2) requires data over a large $x$-range for the same $Q^2$. Even with careful binning and/or interpolation, we could only construct a single set for $Q^2 \approx 3.5$ GeV$^2$, $x \gtrsim 0.55$, which $x$-range misses a crucial part of the DI region. Fortunately, one can use the fact, that, independent on $Q^2$, $F_p^2(x, Q^2) \approx 0.32$ for $x \approx 0.16$. Eq. (2) then proves the same for $F_A^2(x, Q^2)$, permitting extrapolation into the vital DI region.

We have used several inversion methods, all based on a parametrization

$$F_n^2(x, Q^2) = C(x, Q^2; d_k) F_p^2(x, Q^2)$$

with mildly constrained parameters. First we take $C(0) = 1$, ensuring a finite outcome for the Gottfried sumrule $S_G(Q^2) = \int_0^1 \frac{dx}{x} [F_p^2(x, Q^2) - F_n^2(x, Q^2)]$. Next we exploit the above 'primitive' ansatz for, say, $x = 0.2$. For the simplest choice $k_{\text{max}} = 2$ only one parameter is left, e.g. $d_0 = C(1)$. It moreover proved useful to parametrize $F_p^2$ as follows

$$F_p^2(x, Q^2) = x^{-a^2} \sum_{m \geq 1} c_m (1 - x)^m; x \geq 0.02$$

$$= 0.42 \quad \text{; } x \leq 0.02$$

In the region $0.02 \lesssim x \lesssim 0.9$, the above practically coincides with the standard parametrization \footnote{17}. Fig. 3 shows our results for $C, F_n^2$ for fixed $Q^2 = 3.5$ GeV$^2$ and given $F_p^2$. The band in $C$ reflects results from several inversion methods and from different targets D,C,Fe. The value of $C$ at the elastic point $x = 1$ has been the subject of several estimates with results, marked by small horizontal lines. All those, as well as our $C$, assumed smooth, i.e. resonance-averaged behaviour of $F_N^2$ (cf. lower part of Fig. 3).

The above is an undesired feature of averaging: the lowest inelastic threshold of $F_N^2(x, Q^2)$, occurs at a mass $M + m_\pi$, or equivalently, at $x_{\text{thr}}(Q^2) = [1 + 2M m_\pi / Q^2]^{-1}$. In particular $x_{\text{thr}}(3.5) \approx 0.93$, which is marked in Fig. 3 by a vertical line. For $x_{\text{th}} < x < 1, F_N^2(x, Q^2)$ is strictly 0. In particular the mention prediction of $C$ out to
the elastic border, merely reflects the different approach to 0 of the $p,n$ SF. As a consequence $C(x \to 1)$ is due to purely NE parts of $F_2^N$, and equals (cf. Eq. (3b))

$$\lim_{x \to 1} C(x, Q^2) = \left[ \frac{\mu_n \alpha_n(Q^2)}{\mu_p \alpha_p(Q^2)} \right] \left[ 1 + \frac{4M^2}{Q^2} \left( \frac{\gamma(Q^2)}{\mu_p} \right)^2 \right]^{-1}, \quad (7)$$

From Eqs. (4), (7) one then computes

$$C(x = 1, 3.5) \approx 0.61, \quad (8)$$

surprisingly close to the extracted value as the ratio of the two $F_2^N$, which tend to 0 in a different way for $x \to 1$. More extensive reports can be found in Refs. [18,19].

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Figure captions

Fig. 1a,b. Partial data and predictions for inclusive cross sections ($E = 4.045$ GeV, $\theta = 15^\circ, 23^\circ, 30^\circ$) on D,Fe.

Fig. 2. $\alpha_n = G_{M}^{n}/\mu_{n}G_{d}$ as function of $Q^2$. Shown are some previous representative results. Filled squares, diamonds, triangles and stars are our results.

Fig. 3. The ratio $C(x, 3.5) = F_{2}^{n}(x, 3.5)/F_{2}^{p}(x, 3.5)$ for $Q = 3.5$ GeV$^2$ from data on D, C, Fe. The drawn line corresponds to $C(1) = 0.54$ and the band represents the spread from averages over different targets and methods. The numbers on the right abscissa are standard quark model and QCD predictions for $C(1)$ with 0.61, the $NE$ limit (7).
TABLES

TABLE I. Extraction of $\alpha_n(Q^2)$ from QE inclusive scattering data on D, $^4$He. Columns give target, beam energy $E$, scattering angle $\theta$, ranges of Bjorken $x$ and $Q^2$, range of SF of target composed of point-nucleons and (between brackets) its maximal value. The last column gives $\alpha_n(Q^2)$ with deviations from average over the considered $x$-intervals.

| target | $E$ (in GeV) | $\theta$ | $x$ (in GeV) | $Q^2$ (in GeV$^2$) | $f^{P.N.A}(x, Q^2)$ | $\alpha_n(Q^2)$ |
|--------|--------------|----------|-------------|-------------------|---------------------|----------------|
| $^4$He | 2.02         | 20°      | 1.125-0.848 | 0.444-0.430       | 0.97-1.49 (1.49)    | 0.988±0.055     |
|        | 3.595        | 16°      | 1.125-0.930 | 0.887-0.864       | 1.16-1.90 (1.90)   | 0.967±0.028     |
|        | 3.595        | 20°      | 1.095-0.925 | 1.295-1.250       | 1.44-2.16 (2.16)   | 0.988±0.018     |
| D     | 4.045        | 15°      | 1.131-0.953 | 0.988-0.972       | 1.31-3.65 (4.30)   | 1.039±0.020     |
|        | 4.045        | 23°      | 1.079-0.978 | 1.976-1.929       | 2.44-5.18 (5.18)   | 1.062±0.009     |
| $D$    | 5.507        | 15°      | 1.063-0.978 | 1.769-1.741       | 2.89-5.04 (5.31)   | 1.047±0.019     |
|        | 2.407        | 41.1°    | 1.081-0.957 | 1.803-1.721       | 2.37-4.89 ((5.32)  | 1.048±0.007     |
|        | 1.511        | 90.0°    | 1.059-0.977 | 1.812-1.728       | 3.21-4.79 (5.26)   | 1.057±0.009     |
| $R_{D,NE}$ | 3.809 | 20°     | 1.141-0.962 | $<Q^2>=1.75$     | 1.79-3.38 (5.31)   | 1.004±0.014     |
| $D$    | 5.507        | 19.0°    | 1.104-1.000 | 2.561-2.501       | 1.69-5.65 (5.98)   | 1.030±0.016     |
|        | 2.837        | 45.0°    | 1.101-0.991 | 2.613-2.500       | 1.69-5.91 (5.94)   | 1.031±0.018     |
|        | 1.968        | 90.0°    | 1.064-0.984 | 2.608-2.474       | 3.06-5.71 (5.90)   | 1.078±0.027     |
| $R_{D,NE}$ | 5.016 | 20°     | 1.068-0.940 | $<Q^2>=2.50$     | 2.92-4.16 (5.94)   | 0.986±0.014     |
| $R_{T}$ | 5.016        | 20°      | 1.051-0.958 | $<Q^2>=3.25$     | 3.50-6.15 (6.43)   | 0.940±0.013     |
| $R_{T}$ | 5.016        | 20°      | 1.079-1.038 | $<Q^2>=4.00$     | 3.80-6.20 (6.50)   | 0.830±0.016     |
\( \frac{1}{A} \frac{d\sigma}{d\Omega d\nu} (\mu b/(\text{sr GeV})) \)

(a) Fe

(b) D

\( \nu(\text{GeV}) \)

\( 15^\circ \)

\( 23^\circ \)

\( 30^\circ \)
\[ \alpha_n(Q^2) \]

\[ Q^2 (\text{GeV}^2) \]
