Generalized thermoelastic interactions in a hollow cylinder with temperature-dependent material properties

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Abstract
In the present work, the generalized thermoelastic interactions in a hollow cylinder with one relaxation time are considered. The modulus of elasticity are taking as function of temperature. Due to the nonlinearity of the governing equations, finite element method is adopted to solve such problem. The exact solution in the case of temperature-independent is discussed explicitly. Numerical results for the temperature distribution, displacement and radial and hoop stresses represented graphically. The accuracy of the finite element method validated by comparing between the finite element and exact solutions for temperature-independent.

Key words: Nonlinear thermoelasticity, Lord-Shulman’s theory, Finite element method, Hollow cylinder, Temperature-dependent, Exact solution

1. Introduction

In fact, the change of body temperature has an effect on the strain/stress fields and conversely, i.e. mechanical action, and corresponding strain produce a temperature field. The numerical value of thermal conductivity varies with temperature, especially if a region of change of temperature is large. So, the thermal conductivity, and may be the heat capacity, should be considered temperature-dependent in most of the practical engineering problems. Serious attention paid to the generalized thermoelasticity theories in solving thermoelastic problems in place of the classical uncoupled/coupled theory of thermoelasticity. The theory of couple thermoelasticity was extended by Lord and Shulman (LS) (Lord and Shulman 1967) and (Green and Lindsay 1972) by including the thermal relaxation time in constitutive relations. The anisotropic case was later developed by (Dhaliwal and Sherief 1980).

Based on these generalized theories, a large number of efforts have been devoted to investigating generalized dynamic problems. Several experimental studies by Kaminski (Kaminski 1990) and (Tzou 1995). Subsequently, several investigations (Abbas 2014e, Abbas 2014f, Abbas 2014b, Abbas 2014a, Abbas 2014d, Abbas 2014c, Abbas 2014g, Abbas and Kunnar 2014, Abbas and Zenkour 2014b, Abbas and Zenkour 2014a, Zenkour and Abbas 2014c, Zenkour and Abbas 2014b, Zenkour and Abbas 2014a, Abbas 2015, Abbas and Zenkour 2015) are carried out based on different generalized theories of thermoelasticity. In most of the problems, the material properties of the medium are taken to be constant. Modern structure elements are often subjected to temperature changes of such magnitude that their material properties may no longer be regarded as having constant values even in approximate sense. The thermal and mechanical properties of materials vary with temperature, so that the temperature dependence of material properties must be taken into consideration in the thermal stress analysis of these elements. (Suhara 1918) solved the thermoelastic problems of hollow
circular cylinder of which only the shear modulus was temperature-dependent. Since his study, many investigators studied
temperature-dependent. (Ezzat, El-Karamany et al. 2004) investigated problem in generalized thermoelasticity with the
modulus of elasticity dependent with temperature. (Youssef 2005a, Youssef 2005b, Youssef and Al-Harby 2007) has
many contributions for temperature-dependent properties of materials.

The aim of the present paper is to investigate a problem of a hollow cylinder, whose material properties like modulus
of elasticity and thermal conductivity vary with temperature, in the context of generalized thermoelasticity theory with
one relaxation parameter. An application of a hollow cylinder is investigated where the inner surface is traction free and
subjected to a decaying-with-time thermal field, while the outer surface is traction free and thermally isolated. Results
are displayed graphically and compared with the results obtained for temperature-independent material properties.

2. Governing equations

In the context of the LS-theory, we consider an isotropic elastic medium with temperature dependent material
properties. The basic equations and constitutive relations can be presented in a unified form as (Lord and Shulman 1967)

\[ \rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) u_{j,j,i} + \mu u_{i,j,j} - \gamma T_j. \]  

(1)

The equation of heat conduction

\[ (kT_j)_j = \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left( \rho c_v T + \gamma T_0 u_{i,j,j} \right). \]  

(2)

The constitutive equations are given by

\[ \tau_{ij} = \lambda u_{i,j} \delta_{ij} + \mu \left( u_{i,j} + u_{j,i} \right) - \gamma \left( T - T_0 \right) \delta_{ij}. \]  

(3)

where \( \rho \) is the mass density; \( T \) the temperature change of a material particle; \( T_0 \) the reference uniform temperature of
the body; \( u_i \) the displacement vector components; \( e_{ij} \) the strain tensor; \( \tau_{ij} \) the stress tensor; \( c_v \) the specific heat at
constant strain; \( \gamma \) the thermal elastic coupling tensor in which \( \gamma = (3\lambda + 2\mu)\alpha c_v \); \( k \) the thermal conductivity; \( \lambda, \mu \)
are elastic parameters; \( \tau_o \) is a relaxation time. For temperature dependent material, we will suppose that

\[ \lambda = \lambda_o f(T), \quad \mu = \mu_o f(T), \quad k = k_o f(T), \quad \rho = \rho_o f(T), \quad \gamma = \gamma_o f(T), \]

where \( \lambda_o, \mu_o, k_o, \rho_o, \) and \( \gamma_o \) are considered constants, \( f(T) \) is a continuous and non-dimensional function in the
domain \( 0 \leq T - T_0 < \infty \). In case of temperature-independent material properties \( f(T) = 1 \) and

\[ \lambda = \lambda_o, \quad \mu = \mu_o, \quad k = k_o, \quad \rho = \rho_o, \quad \gamma = \gamma_o. \]  

Therefore from equations (1)-(3) we have the system of nonlinear partial differential equations:

\[ \rho \frac{\partial^2 u}{\partial t^2} f(T) = \left[ (\lambda_o + \mu_o) u_{j,j} + \mu u_{i,j} - \gamma_o T_j \right] f(T) + \left[ (\lambda_o + \mu_o) u_{j,j} + \mu u_{i,j} - \gamma_o T_j \right] f(T), \]

(4)

\[ (k_o f(T)T_j)_j = \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left( \rho_o c_v f(T) + \gamma_o f(T) T_0 \right) u_{i,j,j}. \]  

(5)

\[ \tau_{ij} = \left[ \lambda_o u_{i,j} \delta_{ij} + \mu_o \left( u_{i,j} + u_{j,i} \right) - \gamma_o \left( T - T_0 \right) \delta_{ij} \right] f(T). \]  

(6)
3. Problem formulation

We consider an isotropic hollow cylinder with internal radius $a$ and external radius $b$. The inner surface is traction free and subjected to a decaying-with-time thermal field, while the outer surface also is traction free but thermally isolated.

We introduce the cylindrical polar coordinates $(r, \theta, z)$ with the $z$-axis lying along the axis of the cylinder. Due to symmetry, the functions considered depending on the radial distance $r$ and the time $t$ where $a \leq r \leq b$. The displacement vector has the components

$$u_r = u(r, t), \quad u_\theta(r, t) = 0, \quad u_z(r, t) = 0.$$  \hspace{1cm} (7)

Rishin et al. (Rishin, Lyashenko et al. 1973) investigated the relationship between modulus of elasticity of several sprayed coatings and temperature, and they found the modulus of elasticity decreases monotonically with the increasing of temperature. For simplicity and without loss of generality, we assume:

$$f(T) = 1 - \alpha(T - T_0).$$ \hspace{1cm} (8)

where $\alpha$ is an empirical material constant. Then, the equations (4-8) yield the following non-linear equations:

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}(\tau_r - \tau_{\theta\theta}) = \rho_o \left(1 - \alpha(T - T_0)\right) \frac{\partial^2 u}{\partial t^2},$$ \hspace{1cm} (9)

$$k_o \frac{\partial}{\partial r} \left( r \left(1 - \alpha(T - T_0)\right) \frac{\partial T}{\partial r} \right) = \left[ \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right] \left(1 - \alpha(T - T_0)\right) \left( \rho_o c_e T + \gamma_o T_0 \left( \frac{\partial u}{\partial r} + \frac{u}{r} \right) \right),$$ \hspace{1cm} (10)

with

$$\tau_r = \left(1 - \alpha(T - T_0)\right) \left[ \lambda_o + 2\mu_o \right] \frac{\partial u}{\partial r} + \lambda_o \frac{u}{r} - \gamma_o (T - T_0),$$ \hspace{1cm} (11)

$$\tau_{\theta\theta} = \left(1 - \alpha(T - T_0)\right) \left[ \lambda_o + 2\mu_o \right] \frac{\partial u}{\partial r} + \left( \lambda_o + 2\mu_o \right) \frac{u}{r} - \gamma_o (T - T_0).$$ \hspace{1cm} (12)

For our convenience, the following non-dimensional variables and notations are used:

$$\left((r', u') = c_1 \chi(r, u), \quad (t', \tau'_{rr}, \tau'_{\theta\theta}) = \frac{1}{\lambda_o + 2\mu_o} \left( \tau_{rr}, \tau_{\theta\theta} \right), \right)$$

$$T' = \frac{T - T_0}{T_0}, \quad c_1 = \sqrt{\frac{\lambda_o + 2\mu_o}{\rho}}, \quad \chi = \frac{\rho_o c_e}{k_o}.$$ \hspace{1cm} (13)

In terms of the non-dimensional quantities defined in equation (13), the above governing equations reduce to (dropping the dashed for convenience)

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \frac{\partial T}{\partial r} - \frac{\beta}{1 - \beta T} \frac{\partial T}{\partial r} \left( \frac{\partial u}{\partial r} + \frac{u}{r} - \frac{\xi_2}{r} T \right) = \frac{\partial^2 u}{\partial t^2},$$ \hspace{1cm} (14)
\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r (1 - \beta T) \frac{\partial T}{\partial r} \right) \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left[ (1 - \beta T) \left( T + \varepsilon \left( \frac{\partial u}{\partial r} + \frac{u}{r} \right) \right) \right].
\]

(15)

\[
\tau_n = (1 - \beta T) \left( \frac{\partial u}{\partial r} + \xi_1 \frac{u}{r} - \xi_2 T \right),
\]

(16)

\[
\tau_{\theta\theta} = (1 - \beta T) \left( \xi_1 \frac{\partial u}{\partial r} + \frac{u}{r} - \xi_2 T \right),
\]

(17)

where \( \beta = \alpha T_0, \xi_1 = \frac{\lambda_o}{\rho_o c_1^2}, \xi_2 = \frac{T_o \gamma_o}{\rho_o c_1^2}, \varepsilon = \frac{\gamma_o}{\rho_o c_2}. \)

The non-dimensional forms of the boundary condition are:

\[
\tau_n (a, t) = 0, \quad T (a, t) = e^{-\omega t}, \quad \tau_n (b, t) = 0, \quad T (b, t) = 0,
\]

(18)

where \( a \) and \( b \) are inner and outer radii of the hollow cylinder and \( \Omega \) is an exponent of the decayed heat flux. The solution of the physical variables can be decomposed in the following form:

\[
(u, T, \tau_n, \tau_{\theta\theta})(r, t) = (u, T, \tau_n, \tau_{\theta\theta})(r) e^{\omega t},
\]

(19)

where \( \omega \) is the angular frequency. Hence, we obtain the following system of nonlinear ordinary differential equations:

\[
\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} - \xi_2^2 \frac{dT}{dr} - \frac{\beta e^{\omega t}}{1 - \beta e^{\omega t}} \frac{dT}{dr} \left( \frac{du}{dr} + \xi_1 \frac{u}{r} - \xi_2 T \right) - \omega^2 u = 0,
\]

(20)

\[
\frac{1}{r} \frac{d}{dr} \left( r (1 - \beta T e^{\omega t}) \right) \frac{dT}{dr} - \left( \omega + \tau_n \omega^2 \right) \left( 1 - 2 \beta T e^{\omega t} \right) \left( T + \varepsilon \left( \frac{du}{dr} + \frac{u}{r} \right) \right) = 0,
\]

(21)

\[
\tau_n = (1 - \beta T e^{\omega t}) \left( \frac{du}{dr} + \xi_1 \frac{u}{r} - \xi_2 T \right) e^{\omega t},
\]

(22)

\[
\tau_{\theta\theta} = (1 - \beta T e^{\omega t}) \left( \xi_1 \frac{du}{dr} + \frac{u}{r} - \xi_2 T \right) e^{\omega t},
\]

(23)

with the boundary conditions

\[
\tau_n (a) = 0, \quad T (a) = e^{-(\omega + \Omega) t}, \quad \tau_n (b) = 0, \quad T (b) = 0,
\]

(24)

4. Numerical scheme

A finite element scheme is used here to get the solutions of nonlinear equations (20-21). The Finite element method is a powerful technique originally developed for numerical solution of complex problems in structural mechanics, and it remains the method of choice for complex systems. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations. For the finite element method one can refer to Abbas and his colleagues (Abbas and Youssef 2013, Abbas and Zenkour 2013, Abbas 2014c, Abbas and Kumar 2014). The finite element equations of a generalized thermoelasticity problem can be readily obtained by following standard procedure. In the finite element method, the displacement component \( u \) and temperature \( T \) are related to the corresponding nodal values by
\[ u = \sum_{i=1}^{m} N_i u_i(t), \quad T = \sum_{i=1}^{m} N_i T_i(t), \]  

(25)

Where \( m \) denotes the number of nodes per element, and \( N \) the shape functions. In the framework of standard Galerkin procedure, the weighting functions and the shape functions coincide. Thus,

\[ \delta u = \sum_{i=1}^{m} N_i \delta u_i, \quad \delta T = \sum_{i=1}^{m} N_i \delta T_i, \]  

(26)

Thus, the finite element equations corresponding to (20) and (21) can be obtained as

\[ \int_{r} \delta u \left( \frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} - \xi^2 \frac{dT}{dr} - \beta e^{\omega t} \frac{dT}{dr} \left( \frac{du}{dr} + \xi^2 u - \xi^2 T \right) - \omega^2 u \right) dr = 0, \]

(27)

\[ \int_{r} \delta T \left[ \frac{1}{r} \frac{dT}{dr} \left( r \left( 1 - \beta T e^{\omega t} \right) \frac{dT}{dr} \right) \right] - (\omega + \tau_0 \omega^2) \left( 1 - 2 \beta T e^{\omega t} \right) \left( T + \epsilon \left( \frac{du}{dr} + \frac{u}{r} \right) \right) dr = 0. \]

(28)

5. Particular case (Exact solution)

By taking In case of temperature-independent material properties \( f(T) = 1 \) and \( \lambda = \lambda_o, \mu = \mu_o, k = k_o, \rho = \rho_o, \) and \( \gamma = \gamma_o \), we obtain the case of temperature-independent material properties as

\[ \frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} - \omega^2 u - \xi^2 \frac{dT}{dr} = 0, \]  

(29)

\[ \frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} - (\omega + \tau_0 \omega^2) \left( T + \epsilon \left( \frac{du}{dr} + \frac{u}{r} \right) \right) = 0, \]  

(30)

Eliminating \( T \) from the equations (23) and (24) we get

\[ \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - p^2_i \right) \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - p^2_i \right) u = 0, \]  

(31)

where \( p^2_1 \) and \( p^2_2 \) are the roots of the following algebraic equation:

\[ p^4 - \left( \omega^2 + (1 + \xi^2) \left( \omega + \tau_0 \omega^2 \right) \right) p^2 + \omega^2 \left( \omega + \tau_0 \omega^2 \right) = 0. \]  

(32)

The general solution of equation (32) can be written in the form:

\[ u(r,t) = \left( A_1 J_1(p_1 r) + A_2 J_1(p_2 r) + A_3 Y_1(p_1 r) + A_4 Y_1(p_2 r) \right) e^{\omega t}, \]  

(33)

where \( A_1, A_2, A_3 \) and \( A_4 \) are parameters depending on \( \omega \) to be determined from the boundary conditions, \( J_1 \) and \( Y_1 \) are the Bessel function of the first and second kind of order one. Using equation (33) into equations (29) and (30), the expression for temperature can be written in the form

\[ T(r,t) = \left( B_1 J_0(p_1 r) + B_2 J_0(p_2 r) + B_3 Y_0(p_1 r) + B_4 Y_0(p_2 r) \right) e^{\omega t}, \]  

(34)

where \( (B_1, B_2) = \sigma_1 (A_1, A_2), (B_3, B_4) = \sigma_2 (A_3, A_4), \sigma_1 = \frac{p^2_1 + \omega^2}{\xi^2 p_i}, \quad i = 1, 2. \)

From the boundary conditions (24), we can obtain
\[
\begin{pmatrix}
A_1 \\ A_2 \\ A_3 \\ A_4
\end{pmatrix}
= \begin{pmatrix}
L_{11} & L_{12} & L_{13} & L_{14} \\
L_{21} & L_{22} & L_{23} & L_{24} \\
L_{31} & L_{32} & L_{33} & L_{34} \\
L_{41} & L_{42} & L_{43} & L_{44}
\end{pmatrix}^{-1}
\begin{pmatrix}
e^{-(\alpha + \Omega)t} \\
0 \\
0 \\
0
\end{pmatrix},
\]

where
\[
L_{11} = \alpha_1 J_0 (p_1 a), \quad L_{12} = \alpha_2 J_0 (p_2 a), \quad L_{13} = \alpha_3 Y_0 (p_1 a), \quad L_{14} = \alpha_4 Y_0 (p_2 a),
\]
\[
L_{21} = \alpha_1 J_0 (p_1 b), \quad L_{22} = \alpha_2 J_0 (p_2 b), \quad L_{23} = \alpha_3 Y_0 (p_1 b), \quad L_{24} = \alpha_4 Y_0 (p_2 b),
\]
\[
L_{31} = \frac{\xi_1 - 1}{a} J_1 (p_1 a) + (p_1 - \xi_2 \alpha_1) J_0 (p_1 a), \quad L_{32} = \frac{\xi_1 - 1}{a} J_1 (p_2 a) + (p_2 - \xi_2 \alpha_2) J_0 (p_2 a),
\]
\[
L_{33} = \frac{\xi_1 - 1}{a} Y_1 (p_1 a) + (p_1 - \xi_2 \alpha_1) Y_0 (p_1 a), \quad L_{34} = \frac{\xi_1 - 1}{a} Y_1 (p_2 a) + (p_2 - \xi_2 \alpha_2) Y_0 (p_2 a),
\]
\[
L_{41} = \frac{\xi_1 - 1}{b} J_1 (p_1 b) + (p_1 - \xi_2 \alpha_1) J_0 (p_1 b), \quad L_{42} = \frac{\xi_1 - 1}{b} J_1 (p_2 b) + (p_2 - \xi_2 \alpha_2) J_0 (p_2 b),
\]
\[
L_{43} = \frac{\xi_1 - 1}{b} Y_1 (p_1 b) + (p_1 - \xi_2 \alpha_1) Y_0 (p_1 b), \quad L_{44} = \frac{\xi_1 - 1}{b} Y_1 (p_2 b) + (p_2 - \xi_2 \alpha_2) Y_0 (p_2 b),
\]

Numerical results for the exact solutions of (33) and (34) are given in tables 1 and 2.

![Fig. 1 Variation of temperature](image1)

![Fig. 2 Variation of displacement](image2)
6. Numerical Results and Discussion

Now, we consider a numerical example to illustrate the problem, the copper material was chosen for purposes of numerical evaluations. The physical data which given as (Youssef 2005b)
\[
\lambda_o = 7.76 \times 10^{10} \text{ (kg) (m)}^{-1} \text{ (s)}^{-2}, \quad \mu_o = 3.86 \times 10^{10} \text{ (kg) (m)}^{-1} \text{ (s)}^{-2}, \quad T_0 = 293 \text{ (K)} , \\
k_o = 3.68 \times 10^2 \text{ (kg) (m)}^{-1} \text{ (s)}^{-3}, \quad c_e = 3.831 \times 10^2 \text{ (m)}^2 \text{ (K)}^{-1} \text{ (s)}^{-2}, \quad \Omega = 1, \\
\rho_o = 8.954 \times 10^3 \text{ (kg) (m)}^{-3}, \quad \alpha_i = 17.8 \times 10^{-6} \text{ (K)}^{-1}, \quad \tau_o = 0.02, \quad \omega = 5, \ t = 0.3.
\]

The effect of temperature-dependency is given in Figures 1 to 4, it is to be noted that \( \beta = 0 \) is used for temperature-independent while \( \beta = 1 \) is used for temperature-dependent. All variable quantities are very sensitive to the temperature-dependent of material properties. Figures 1, 2, 3, and 4 exhibit the variation of the temperature, displacement and radial and hoop stresses with radial distance \( r \). It is obvious from figure 1 that the temperature decreases with the increase of the radial distance and then closed to zero. Figure 2 shows that the displacement increases from the negative value to a positive value. In the positive values, the displacement has a peak value that depends on \( \beta \). It is obvious from figure 3, which gives the radial stress variation with the radial distance. Its magnitude increases from zero to a maximum value after that decreases rapidly as \( r \) increases and is zero at \( r = b \). This is in agreement with the theoretical result that the disturbance is zero beyond the thermal wave front. In which we observed that, the radial stress is zero at \( r = a \) and \( r = b \), which satisfies the boundary conditions of the problem. Figure 4 give the variation of hoop stress versus radial distance. From the numerical results, it can be seen the physical quantities are sensitive to the temperature dependence of material properties. To validate the finite element method (FEM) solutions, one can discuss the following comparison in case temperature-independent ( \( \beta = 0 \) ) which have its exact solutions.

![Fig. 3 Variation of radial stress](image-url)
7. Conclusions

A general finite element model is developed to analyze a generalized coupled thermoelastic problem of a hollow cylinder with temperature-dependent material properties. This kind of coupled problem, involving nonlinear partial differential equations and is difficult to approach by analytical techniques. The finite element method is more efficient and accurate. The temperature-dependent material properties act to reduce the magnitudes of the considered variables, which may be significant in some practical applications, can easily be taken under consideration and accurately assessed.

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