Optimization of oil-mixture ”hot” pumping in main oil pipelines

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Abstract. The optimal conditions for high pour point and high-viscosity oil transportation through the main oil pipeline sections are determined. The optimization of ”hot” pumping is investigated by determining the energy-saving operating conditions of pumps and heating furnaces. The objective function of optimality for the pipeline section with several stations is determined by a minimum costs of energy consumed by pumps and heating furnaces. The problem solving algorithm is constructed by a new dynamic programming approach. The problem is divided into many overlapping subtasks with finding the optimal substructure. The object of each subtask is the cost function of pumps and heating furnaces at the stations.

1. Introduction
The method of ”hot” pumping with associated heating remains a reliable way of pumping of high pour point oil and high-viscosity oil [1]. Objective functions and optimization problems of ”hot” pumping are considered in scientific works [2-4]. Unlike the well-known works, the optimization of ”hot” pumping is investigated by determining the optimal operating conditions for pumping units and heating furnaces.

2. Statement of the problem
The method of oil-mixture ”hot” pumping at the section of main oil pipeline is made by the operation of pumping units and heating furnaces. The objective function of optimality is defined in the form of a minimum of the sum of costs of oil-mixture pumping and heating:

$$\sum_{i=1}^{n} \left( z_{el}^{i} \sum_{j=1}^{m_{pum}^{i}} c_{pum}^{ij} N_{PU}^{ij} (k_{ij}) + z_{fl}^{i} \sum_{j=1}^{m_{fur}^{i}} g_{fur}^{ij} Q_{fl}^{ij} \right) \rightarrow min$$

where $n$ – the number of stations, $m_{pum}^{i}$/$m_{fur}^{i}$ – the quantity of pumps/furnaces at $i$-th station, $z_{el}^{i}/z_{fl}^{i}$ – the electricity cost (tenge/(kW·h))/fuel (tenge/kg) at $i$-th station; $c_{pum}^{ij}/g_{fur}^{ij}$ – the integer variable that has value 1 if pump/furnace is in operation, and 0 otherwise; $N_{PU}^{ij}$ – the power consumption of $j$-th pumping unit at $i$-th station (kW); $k_{ij}$ – the ratio of the rotor speed to nominal speed for this pump; $Q_{fl}^{ij}$ – the speed of fuel delivery to $j$-th furnace of $i$-th station (kg/h).

The criterion (1) is considered together with the safety conditions of pumping:
1) Set point chart satisfaction - pressure limit at the inlet/outlet of the pump station (PS);
2) Safety condition of heating furnaces operating;
3) Avoidance of oil temperature decrease below the critical (pour point) in the pipeline.

The safety condition of the heating furnaces requires the following restriction:

\[ Q_{\text{min},ij}^p \leq Q_{ij}^p \leq Q_{\text{max},ij}^p \]  

(2)

The pressure drop for pumping through group of pumps \( \Delta P^{gr} \) is determined by the formula:

\[ \Delta P^{gr}(Q,k) = \begin{cases} 0, & c_{\text{oper}} = 0 \\ \rho g H \left( \frac{Q}{c_{\text{oper}}}, k \right), & c_{\text{oper}} > 0 \end{cases} \]  

(3)

where \( c_{\text{oper}} \) is the number of operating pumps in the group, \( \rho \) is the density of oil being pumped, \( H \left( \frac{Q}{c_{\text{oper}}}, k \right) \) is the dependence of pressure on the flow rate of any pump in the group.

With a fixed oil flow rate \( Q \) the pressure and power consumption functions generated by pumps or a group of pumps, in addition to the parameter \( k \), will have an additional argument on the temperature at the inlet to the pump (we will denote \( k_i \)).

The pressure coefficients and efficiency characteristics of pumps depend on the viscosity of the oil being pumped, which in turn depends on its temperature. Therefore, for the pressure drop and power consumption, the dependencies are valid:

\[ \Delta P_{ij}^{gr} = \Delta P^{gr}(k_{ij}, T_{ij}^{\text{Pump}}), N_{ij}^{\text{Pump}} = N_{ij}^{\text{Pump}}(k_{ij}, T_{ij}^{\text{Pump}}). \]

The pressure drop \( \Delta P_{ij}^{gr} \) satisfies the pressure balance equation [5]:

\[ P_{\text{init}} + \sum_{i=1}^{n} \sum_{j=1}^{m_i^{gr}} \Delta P_{ij}^{gr}(T_i^{\text{Pump}}) = \sum_{i=1}^{n} \left( \Delta P_{i}^{\text{OHS}} + \Delta P_{i}^{\text{PR}} + \Delta P_{i}^{\text{sec}} \right) + \Delta P_{\text{bp}} + P_{\text{fin}} \]

(4)

where \( P_{\text{init}} \) is the initial pressure; \( m_i^{gr} \) is the quantity of pumping groups on PS; \( \Delta P_{ij}^{gr} \) is the increase in pressure created by the \( j \)-th group of pumps at the \( i \)-th PS; \( \Delta P_{i}^{\text{PR}} \) is the pressure loss after pressure regulator (PR); \( \Delta P_{i}^{\text{sec}} \) is the pressure loss in pipeline with taking into account the difference in hydrostatic pressure at the \( i \)-th and \( (i+1) \)-th PS at the oil-mixture flow rate \( Q \); \( \Delta P_{\text{bp}} \) is the backpressure value at the input of terminal station; \( P_{\text{fin}} \) is the pressure required for oil-mixture pumping to final tank, \( \Delta P_{i}^{\text{OHS}} \) is the pressure loss at oil heating stations (OHS).

In the case of "hot" pumping, the pressure loss varies depending on the oil temperature at the outlet of the station. Therefore, for the safety of pumping the pressure parameters \( \text{max.} \Delta P_{i}^{\text{sec}} \) are temperature functions:

\[ \Delta P_{i}^{\text{sec}} = \Delta P_{i}^{\text{sec}}(T_i + T_i^{\text{OHS}}), \text{max.} \Delta P_{i}^{\text{sec}} = \text{max.} \Delta P_{i}^{\text{sec}}(T_i + T_i^{\text{OHS}}) \]

where \( T_i \) is the oil temperature at the inlet to the \( i \)-th station, \( T_i^{\text{OHS}} \) is the amount of heating at the oil heating station (OHS), \( T_i + T_i^{\text{OHS}} \) is the outlet temperature from the station.

The temperature of the next station can be calculated at a known flow rate \( Q \), using the value \( (T_i + T_i^{\text{OHS}}) \). The value \( T_i \) is equal to the oil temperature at the outlet of the tanks of the initial station. The pressure limits for safe pumping at the inlet \( P_{\text{init}}^{\text{Pump}} \) and at the outlet from the PS to Pressure Regulator (PR) \( P_{\text{out1}}^{\text{Pump}} \) for the \( k \)-th station satisfies the condition:

\[ P_{\text{init}}^{\text{Pump}} = P_{\text{init}} + \sum_{i=1}^{k-1} \sum_{j=1}^{m_i^{gr}} \Delta P_{ij}^{gr} - \sum_{i=1}^{k-1} \left( \Delta P_{i}^{\text{OHS}} + \Delta P_{i}^{\text{PR}} + \Delta P_{i}^{\text{sec}} \right) \geq P_{\text{init}}^{\text{Pump}} \]

(5)

\[ P_{\text{out1}}^{\text{Pump}} = P_{\text{init}} + \sum_{j=1}^{m_k^{gr}} \Delta P_{kj}^{gr} - \Delta P_{k}^{\text{OHS}} \leq P_{\text{out1 max}} \]

(6)
As is known, the heating furnaces are located in different ways relative to pumping unit. If the furnaces are located after the pump, then \( T_{\text{pump}}^{\text{in}} = T_k \). If the furnaces are installed before the pump, then \( T_{\text{in pump}}^{\text{in}} = T_k + \Delta T_{\text{OHS}}^k \) and the conditions of non-cavitation operating for each \( l \)-th group of pumps of the \( k \)-th station are supplemented with a new term:

\[
P_{kl}^{\text{in gr}} = P_{kl}^{\text{in}} - \Delta P_{\text{OHS}}^k + \sum_{j=1}^{l-1} \Delta P_{kj}^{gr} \geq P_{kl}^{\text{minin gr}}
\]

(7)

For the flow rate \( Q_{\text{fur}}^i \) of the heated oil through each \( i \)-th furnace there is a dependence of type:

\[
Q_{\text{fur}}^i = Q_{\text{fur}}^i \left( \Delta P_{\text{OHS}}, g_{\text{fur}} \right)
\]

(8)

where \( \Delta P_{\text{OHS}} \) – the value of pressure drop in bypass pipe, \( g_{\text{fur}} = (g_{1\text{fur}}, g_{2\text{fur}}, \ldots, g_{m\text{fur}}) \) – the vector, signifying the operation of each furnace. The dependence \( Q_{\text{fur}}^i \left( \Delta P_{\text{OHS}}, g_{\text{fur}} \right) \) can be determined taking into account the furnace construction at OHS.

For each heating furnace there is also a dependence of the value of its heating \( \Delta T_{\text{fur}} \) on the oil flow through the furnace \( Q_{\text{fur}} \) and fuel supply rate \( Q_{\text{fl}} \) to this furnace:

\[
\Delta T_{\text{fur}} = C_{\text{fur}} Q_{\text{fur}} Q_{\text{fl}}
\]

(9)

where the constant \( C_{\text{fur}} \) depends on the furnace construction, its condition, composition of fuel, time of the year. The value of \( C_{\text{fur}} \) is proportional to the efficiency coefficient of heating furnace.

Naturally, there is a certain restriction on the temperature of the oil being pumped, to which it can be safely heated in a furnace, so that it does not decompose into separate fractions or evaporate. Let’s call this value \( T_{\text{oil}}^{\text{max}} \). In order to take this into account in the task model, the following restriction is introduced for each \( j \)-th furnace at the \( i \)-th station:

\[
T_i + \Delta T_{ij}^\text{fur} = T_i + C_{ij}^\text{fur} Q_{ij}^\text{fur} Q_{ij}^\text{fl} \leq T_{\text{oil}}^{\text{max}}
\]

(10)

Since heated streams with the flow rates \( Q_{ij}^\text{fur} \) from each furnace are then mixed with a non-heated bypass stream with a flow rate \( \left( Q - \sum Q_{ij}^\text{fur} \right) \), the total heating at the outlet from the OHS with a flow rate \( Q \) is determined by the known mixing formula for flows with a different weighting coefficients:

\[
\Delta T_{\text{OHS}} = \sum_{i=1}^{m\text{fur}} g_{i\text{fur}} Q_{ij}^\text{fur} \Delta T_{ij}^\text{fur}
\]

(11)

Using the equalities (8), (9), (11) we write the dependence of the heating value at OHS on the values \( \Delta P_{\text{OHS}}, g_{\text{fur}} \) and \( Q_{\text{fl}}^i \):

\[
\Delta T_{\text{OHS}} = \frac{1}{Q} \sum_{i=1}^{m\text{fur}} g_{i\text{fur}} C_{ij}^\text{fur} Q_{ij}^\text{fl} Q_{i\text{fur}}^\left( \Delta P_{\text{OHS}}, g_{\text{fur}} \right)^2
\]

(12)

For each type of the oil being pumped the lower limit of the cooling temperature is set at the section (lets denote \( T_{\text{oil}}^{\text{min}} \)). Usually its value is related to the pour point temperature. Therefore, there is a need to enter a restriction:

\[
T(x) > T_{\text{oil}}^{\text{min}}
\]
where $T(x)$ – the value of the oil temperature at the section. For assign of the optimum mode with heating, the objective function and pressure limitations (5) - (7) are formulated, as well as temperature restrictions (2), (10), (12). The main parameters of the task are the following variables:

1) $c_{ij}^{pum}$ – optimal combination of pumps;
2) $g_{ij}^{fur}$ – optimal combination of heating furnaces;
3) $Q_{ij}^{fl}$ – necessary fuel supply to the furnace;
4) $\Delta P_i^{OHS}$ – necessary pressure drop at OHS.

3. Algorithm of solution

Using the solution of the problem for n pumps, it is possible to effectively find solutions for n+1 pumps. A network of the operation states of pumping units is constructed. Each node of the network contains data on the number of pumps used and their parameters, pressure reduction in the PR. The nodes of the network are connected, based on the pressure characteristics of the pumps and the rotational speeds of their rotors. The transition of the subtask solution to the solution of the general problem is found and the correctness of the approach is proved.

The object of each subtask is the function of the dependence of the supplied power consumption $S(P,T)$ on the created differential pressure of the pump. Naturally, $P \geq 0$. The optimal pressures are associated with optimal pumping temperatures. In the search for a solution, instead of a continuous function $S(P,T)$, its discrete variant is used. The pressure value is represented discretely with a sufficiently small step $\varepsilon_P = 0.01$ bar, the temperature value is $\varepsilon_T = 0.05$°C.

The solution of the problem is stored in a discrete array $Info(P,T)$, which contains a list of necessary pumps for each value of $P$.

The cost function $S(P,T)$ and the array of solutions $Info(P,T)$ for the pump have the form:

$$S(P) = \left\{ \begin{array}{ll}
+\infty, & P \neq P_{pum} \\
z N^{PU}(Q), & P = P_{pum}
\end{array} \right.
\quad
Info(P) = \left\{ \begin{array}{ll}
0, & P \neq P_{pum} \\
(pump's \ number), & P = P_{pum}
\end{array} \right.$$  (13)

where $z$ – the electricity cost at the station (tenge/(kWh)), $N^{PU}$ – the power consumption of the pumping unit (kW), $Q$ – the flow rate, which passes through the pump.

The pressure drop $P_{pum}$, created by pump is determines as:

$$P_{pum} = \lfloor \rho g H(Q,T) \rfloor$$

where the operator $\lfloor \rfloor$ means rounding to the nearest rational number with step $\varepsilon_P$.

The initial cost function is defined as follows:

$$S^{st,1}(P,T) = \left\{ \begin{array}{ll}
0, & P = P_{init} \text{ and } T = T_{init} \\
+\infty, & \text{otherwise}
\end{array} \right.$$  (14)

We denote by the union of two functions $S^A$ and $S^B$ the function $S(P)$, which has a value for each $P$:

$$S(P,T) = S^A(P,T) \cup S^B(P,T) = \min(S^A(P,T), S^B(P,T))$$

Similarly, we denote by the union of two arrays $Info^A$ and $Info^B$ the array $Info(P,T)$, which has a value for each $P$:

$$Info(P,T) = Info^A(P,T) \cup Info^B(P,T) = \left\{ \begin{array}{ll}
Info^A(P,T), & S^A(P,T) \leq S^B(P,T) \\
Info^B(P,T), & S^A(P,T) > S^B(P,T)
\end{array} \right.$$
We denote by the "overlaying" of function $S^B$ to function $S^A$ the function $S(P,T)$, which has a value for each $P$:

$$S(P,T) = S^A(P,T) \leftarrow (S^B) = \min \left( S^A(P,T), S^A(P-P^*,T) + S^B(P^*,T) \right)$$

where the value of variable $P^*$ for a specific value $P$ determines as:

$$P^* = \arg\min_{P* \in [0,P]} \left( S^A(P-P^*,T) + S^B(P^*,T) \right)$$

Similarly, we denote by "overlaying" of the array $Info^B$ to the array $Info^A$ the array $Info(P,T)$, which has a value for each $P$:

$$Info(P,T) = Info^A(P,T) \leftarrow Info^B(P,T) = \left\{ \begin{array}{ll}
Info^A(P,T), & S^A(P,T) \leq S^A(P-P^*,T) + S^B(P^*,T) \\
Info^A(P-P^*,T) + Info^B(P^*,T), & \text{otherwise}
\end{array} \right.$$

In the heating mode the cost functions $S^i_{fur,v}$ and the solution arrays $Info^i_{fur,v}$ for each $i$-th furnace must be considered, the arguments of which are: the first argument $P$ indicates the pressure drop at OHS, the second argument $T$ is the oil temperature at the inlet to the furnace, an additional third argument is $\Delta T$ - the increase in the temperature of the total flow rate $Q$ with help of this furnace. The cost function of the furnace has an additional index $v$ - the number of selection of operating furnaces at the OHS from the set $m^{fur}$. Naturally, $v \leq 2^{m^{fur}} - 1$.

The cost function for the operating furnace and its array of solutions are calculated as following:

$$S^i_{fur,v}(P,T,\Delta T) = \begin{cases}
\gamma^i_{fur,v}(P,T,\Delta T) \\
+\infty,
\end{cases} \quad Q^i_{fur,v}(P,T,\Delta T) = \left\{ \begin{array}{ll}
\frac{\Delta T^i_{fur}(P,T)}{C^i_{fur}(P,g^i_{fur}(v))}, & T + \Delta T^i_{fur} \leq T^i_{oil} \\
\frac{Q^i_{fur}(P,g^i_{fur}(v))}{Q^i_{fur}(P,g^i_{fur}(v))}, & \text{otherwise}
\end{array} \right.$$

$$\Delta T^i_{fur}(P,\Delta T) = \frac{Q^i_{fur}(P,\Delta T)}{Q^i_{fur}(P,g^i_{fur}(v))} \Delta T; \quad (15)$$

$$Info^i_{fur,v}(P,T,\Delta T) = \begin{cases}
\left( \begin{array}{c}
\text{"furnace's number"} + Q^i_{fur,v} \\
\text{"the furnace doesn't operate"}
\end{array} \right), & Q^i_{fur,v}(P,T,\Delta T) \in \left[ Q^i_{min,v}, Q^i_{max,v} \right]
\end{cases}$$

where $Q^i_{fur,v}$ – the necessary fuel consumption for the $i$-th furnace for the selection $v$, $\Delta T^i_{fur}$ – the flow heating temperature through this furnace.

The vector $g^i_{fur}(v)$ signifies the operating combination of heating furnaces for the selection $v$. In the obtained solution, the restrictions on the fuel consumption of the furnace (2) and the maximum heating in the furnace (10) are automatically taken into account, since in the determination of $S^i_{fur,v}(P,T,\Delta T)$ the operation "overlaying" is performed on the values of $Q^i_{fur,v}$ and $\Delta T^i_{fur}$.

After calculating of all $S^i_{fur,v}$ and $Info^i_{fur,v}$, it is necessary to define the cost function $S^OHS$ and an array of solutions $Info^OHS$ of the total furnace operating at OHS for each selection $v$. The values $S^OHS$ and $Info^OHS$ are determined by the "overlaying" of the functions of all operating heating furnaces:

$$S^OHS(P,T,\Delta T) = S^1_{fur,v} \leftarrow S^2_{fur,v} \leftarrow \ldots \leftarrow S^g_{fur,v,\text{oper}(v)}$$

$$Info^OHS(P,T,\Delta T) = Info^1_{fur,v} \leftarrow Info^2_{fur,v} \leftarrow \ldots \leftarrow Info^g_{fur,v,\text{oper}(v)} \quad (16)$$
where \( g_{\text{oper}}(v) \) the quantity of operating furnaces for the selection \( v \).

The operation overlaying for the cost functions of furnaces \( A \) and \( B \) will have the following form (similarly for the array of solutions):

\[
S_{\text{fur}}^{A}(P,T,\Delta T) \leftarrow (S_{\text{fur}}^{B}) = \min_{\Delta T^* > 0} \left( S_{\text{fur}}^{A}(P,T,\Delta T) + (S_{\text{fur}}^{B}(P,T,\Delta T - \Delta T^*)) \right)
\]  

For any triple of the values \((P,T,\Delta T)\) the optimal operating of OHS means the optimal selection of operating furnaces from \( m_{\text{fur}} \) furnaces. Therefore, the optimal cost function for OHS is the "union" of all possible cost functions from the \((2^{m_{\text{fur}}}-1)\) selections (similarly for the array of solutions):

\[
S_{\text{OHS}}(P,T,\Delta T) = \bigcup_{v=1}^{2^{m_{\text{fur}}}-1} S_{\text{v}}(P,T,\Delta T)
\]

In the case of non-operating OHS (i.e. when none of the OHS is operating) the calculated cost function and the array of solutions need to be adjusted:

\[
S_{\text{OHS}}(0,T,0) = 0, \text{ Info}_{\text{OHS}}(0,T,0) = "the furnaces don’t operate"
\]

Since, unlike a group of pumps, when the oil passes through the OHS, the flow temperature may increase and the pressure may drop, the "overlaying" of the heating cost function \( S_{\text{OHS}} \) to the cost function of the entire station looks as follows (similar to the array of solutions):

\[
S(P,T) = S_{\text{sec}}(P,T) \leftarrow (S_{\text{OHS}}) = \min_{P_*, \Delta T^* > 0} \left( S_{\text{sec}}(P + P^*, T - \Delta T^*) + S_{\text{OHS}}(P^*, T - \Delta T*, \Delta T^*) \right)
\]

Naturally, if the heating furnaces are located after the pumps, then the cost function at the inlet to the station and its array of solutions the cost function \( S_{\text{OHS}} \) first must be overlaid, and only then the cost functions of the pump groups. Otherwise, vice versa. If there are no heating furnaces at any station, there is no need to calculate the function \( S_{\text{OHS}} \) for it and carry out an "overlay" on the cost function of the station.

The pressure drop and the temperature drop at the section depend on the initial temperature. To properly "cut" the cost functions and arrays of solutions at the outlet from the station and determine the cost functions and an array of solutions at the inlet of the next station, calculate the discrete functions \( \Delta P_{i\text{sec}}(T), \Delta T_{i\text{sec}}(T) \) and \( \max_{\text{sec}} \Delta P_{i\text{sec}}(T), \max_{\text{sec}} \Delta T_{i\text{sec}}(T) \) for each \( i\)-th section with step \( \varepsilon_T \).

Moreover, the function \( \max_{\text{sec}} \Delta T_{i\text{sec}}(T) \) determines the maximum value of the temperature drop between the start point and a point on the \( i\)-th section.

Therefore, functions \( P_{\text{min},i}(T), P_{\text{max},i}(T) \) can be determined for the minimum and maximum permissible pressure at the outlet from station:

\[
P_{\text{out}2,i} \geq P_{\text{out},i} = \max(P_{i+1\text{min}} + \Delta P_i, \max_{\text{sec}} \Delta P_i)
\]

\[
P_{\text{out}1,i} \leq P_{\text{out},i} = \min(P_{\text{out}2,i}, P_{\text{pipe},\text{max}})
\]

Wherein, the condition (12) of the minimum necessary temperature at the outlet from the station takes the form:

\[
T_{\text{out},i}(T) = T_{\text{out}} + \max_{\text{sec}} \Delta T_{i\text{sec}}(T)
\]
Cutting of cost function at the outlet from station \( S_{\text{out}}^{\text{st}}(P,T) \) is determined by using the functions \( P_{\text{out}}^{\text{min}}(T), P_{\text{out}}^{\text{max}}(T), T_{\text{out}}^{\text{min}}(T) \) and is denoted by the function \( \text{CUT}^{\text{out, st}}(P,T) \), which has a value for each \( P \) and \( T \):

\[
\text{CUT}^{\text{out, st}}(P,T) = \text{CUT}(S_{\text{out}}^{\text{st}}(P,T), P_{\text{out}}^{\text{min}}, P_{\text{out}}^{\text{max}}, T_{\text{out}}^{\text{min}}) = \begin{cases} +\infty, & P \notin [P_{\text{out}}^{\text{min}}, P_{\text{out}}^{\text{max}}] \\ +\infty, & T < T_{\text{out}}^{\text{min}}(T) \\ S_{\text{out}}^{\text{st}}(P,T), & \text{otherwise} \end{cases}
\]

The array of solutions is "cut" in a similar way. In the cost function and the array of solutions in the outlet from the OHS will be supplemented with pressure drop on the furnace and in case of its "cutting", the changed pressure conditions (5) and (6) are automatically taken into account.

The cost function at the inlet of the next station \( S_{\text{in}}^{\text{next st}}(P,T) \) is determined by the "shift" operation using the cost function at the outlet from the station \( S_{\text{out}}^{\text{st}}(P,T) \) through \( \Delta P^{\text{sec}}(T) \) and \( \Delta T^{\text{sec}}(T) \). This function has the meaning for each \( P \) and \( T \):

\[
S_{\text{in}}^{\text{next st}}(P,T) = \text{SHIFT}(S_{\text{out}}^{\text{st}}(P,T), \Delta P^{\text{sec}}, \Delta T^{\text{sec}}) = \\
= \min_{T'} \left( S_{\text{out}}^{\text{st}}(P + \Delta P^{\text{sec}}(T'), T + \Delta T^{\text{sec}}(T')) \right)
\]

The "shift" operation for the array of solutions is carried out in a similar way.

Thus, all operations for finding the optimal mode of the "hot" pumping method are defined above. These operations are carried out for all stations except the last in the order of their location on the pipeline section:

\[
k = 1
\]

1) For \( i = 1 \) till \( m^{gr,k} \) calculate \( S_{i}^{gr,k} \);

2) If \( m^{gr,k} > 0 \), then calculate \( S_{k}^{OHS} \) using the (15) – (18)

3) \( S_{\text{out}}^{\text{st,k}}(P,T) = S_{\text{in}}^{\text{st,k}}(P,T) \leftarrow (S_{k}^{OHS}) \leftarrow (S_{1}^{gr,k}) \leftarrow \ldots \leftarrow (S_{m^{gr,k}}^{gr,k}) \)

or \( S_{\text{out}}^{\text{k}}(P,T) = S_{\text{in}}^{\text{k}}(P,T) \leftarrow (S_{1}^{gr,k}) \leftarrow \ldots \leftarrow (S_{m^{gr,k}}^{gr,k}) \leftarrow (S_{k}^{OHS}); \)

(19)

4) Calculation of the functions \( \Delta P_{k}^{\text{sec}}(T), \Delta T_{k}^{\text{sec}}(T), \max_{k} \Delta P_{k}^{\text{sec}}(T), \max_{k} \Delta T_{k}^{\text{sec}}(T); \)

5) \( S_{\text{out}}^{\text{k}}(P,T) = \text{CUT}(S_{\text{out}}^{\text{k}}(P,T), P_{\text{out}}^{\text{min}}, P_{\text{out}}^{\text{max}}, T_{\text{out}}^{\text{min}}); \)

6) \( S_{\text{out}}^{\text{k+1}}(P,T) = \text{SHIFT}(S_{\text{out}}^{\text{k}}(P,T), \Delta P_{k}^{\text{sec}}, \Delta T_{k}^{\text{sec}}); \)

7) \( k = k + 1 \). If \( k \neq n + 1 \), then turn to point 1, otherwise loop termination.

After the conducted loop (19) with the initial condition (14) the optimal final pressure and temperature are calculated as:

\[
(P^{\text{ans}}, T^{\text{ans}}) = \arg\min_{T \geq T^{\text{fin}}, P \geq P^{\text{fin}}} \left( S_{\text{in}}^{\text{st,n+1}}(P,T) \right)
\]

The minimum sum of costs for productivity \( Q \) is the value of the function \( S_{\text{in}}^{\text{st,n+1}}(P^{\text{ans}}, T^{\text{ans}}) \). The optimal combination of operating pumps and heating furnaces, as well as their operating modes and the necessary pressure drops ate the OHS will be stored in the cell of the array \( \text{Inf}_{\text{in}}^{\text{st,n+1}}(P^{\text{ans}}, T^{\text{ans}}) \).
References
[1] Abramzon L S 1979 Optimum parameters of hot pipelines operation Oil industry 2 53-54
[2] Agapkin V M, Krivoshein B L and Yufin V A 1981 Heat and hydrodynamic calculations of pipelines for oil and oil products (Moscow: Nedra) p 256
[3] Yablonsky V S, Novoselov V F, Galeev V B and Zakirov G Z 1965 Design, operation and repair of oil product pipelines (Moscow: Nedra) p 410
[4] Zhabasbayev U K, Makhmotoev E S, Ramazanova G I, Bekibaev T T and Rziev S A 2015 Calculation of the optimum pumping temperature for oil transportation Science & Technologies: Oil and Oil Products Pipeline Transportation 20 61-66
[5] Tugunov P I, Novoselov V F, Korshak A A and Shammasov A M 2002 Typical calculations for the design and operation of gas and oil pipelines (Ufa: DesignPoligraphService) p 658