Nanomechanical topological insulators with an auxiliary orbital degree of freedom

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Discrete degrees of freedom, such as spin and orbital, provide a tool to manipulate electrons, photons and phonons. Topological insulators have stimulated intense interests in condensed-matter physics, optics, acoustics and mechanics, usually with a focus on the spin degree of freedom. However, the orbital degree of freedom constitutes another fundamental attribute in crystals, but has seldom been investigated in topological insulators. Here, we demonstrate topological insulators with an auxiliary orbital degree of freedom on a nanomechanical platform. We realize an adiabatic transition between distinct topological edge states, which constitutes a crucial functionality for integrated circuits accommodating distinct topological edge channels. Beyond the one-dimensional edge states, we further construct zero-dimensional Dirac-vortex states using the orbital degree of freedom. These nanomechanical Dirac-vortex states exhibit strong second-order and third-order nonlinearities. Our results introduce the orbital degree of freedom as an alternative means to manipulate the topological phase transition on an integrated platform.

Topological insulators are a new phase of matter that behave as an insulator in their interior but support backscattering-immune conduction of electrons on their surface. Originating from condensed-matter physics, similar concepts have entered the realms of optics, acoustics and mechanics, and have overturned some of the traditional views on wave propagation and manipulation. The advantages of high controllability and flexibility of these platforms have provided abundant opportunities to go beyond what is possible in natural materials. For instance, exploration of additional degrees of freedom (DOFs) provides a powerful approach for investigating novel topological insulators in synthetic dimensions. Topological insulators can be realized by resorting to a spin or valley-pseudospin DOF. In addition, the orbital constitutes another fundamental attribute of the nanomechanical wave that describes the spatial distribution of wavefunctions inside the unit cell of crystals. By harnessing the orbital DOF, one may realize novel functionalities and exotic topological states beyond backscattering-immune transport along edges, which are important for topological integrated circuits with enhanced functionality and scalability. For example, adiabatic transition between different topological edge channels is a crucial functionality for on-chip integration of distinct topological phases including the well-known quantum spin Hall (QSH) and quantum valley Hall (QVH) insulators. Although an additional spatial or temporal DOF was adopted for realizing adiabatic topological transition, it requires either a three-dimensional or a tuneable two-dimensional (2D) system, which is difficult to apply to on-chip 2D topological circuits with standard thin-film nanofabrication technologies. Another example is the zero-dimensional (0D) topological bound states, which are robust against local perturbations and may serve as high-quality resonators in topological integrated photonic and phononic circuits. However, the orbitals of electrons in condensed matter have limited options due to the constraint of materials and lack of controllability over the Coulomb force of the atomic nucleus. In the photonic and phononic domains, manipulation of orbitals is also challenging because implementation of topological insulators requires careful consideration of the crystalline symmetry and thus imposes strict limitations on the orbitals.

Orbital states of nanomechanical topological insulators

Here, we proposed and experimentally demonstrated topological insulators with an auxiliary orbital DOF based on a 2D nanomechanical crystal (Fig. 1a). We fabricated the devices on a silicon-nitride-on-insulator wafer. We first patterned small holes (600 nm in diameter) in the silicon nitride layer, and then performed wet etching to remove the underlying oxide and release parts of the silicon nitride layer from the substrate. The positions of the small holes were engineered such that the released parts of the silicon nitride layer formed a hexagonal lattice. The basis vectors of this lattice are a1 and a2, with lattice constant a = |a1| = |a2| = 21 μm (Fig. 1a). Each unit cell contains six nanomechanical membranes, which can be classified into two groups due to the C3 rotational symmetry (Fig. 1b). The geometries of these two groups of membranes are governed by the positions of the small holes (r1, r2) = (r1 − δ1, δ0), (r2 − δ1, −δ0) and (r1, r2) = (r1 + δ1, δ0), (r2 + δ1, −δ0) with r0 = 5.5 μm. Non-zero values of δ1 and δ0 break the C3 inversion symmetry and Tp translational symmetry along the vector P = (2a1 + a2)/3 (Fig. 1b), respectively. The QSH insulators demonstrated previously are mainly based on breaking the Tp translational symmetry, which limits the orbitals of phonons to conventional p and d types. Here we explored a more generic type of topological insulator defined in a 2D parameter space of (δ1, δ0) (Fig. 1c) in which both the C3 inversion symmetry and the Tp translational symmetry are broken.

The 2D parameter space in Fig. 1c can be defined with the polar coordinates (δ0, θ) according to (δ1, δ0) = (δ0 cosθ, sinθ). By solving the Kirchhoff–Love plate equation at the Γ point of the first Brillouin zone, we found that there always exist two pairs of degenerate eigenstates |ψn,±(θ)⟩ (n = 1, 2) satisfying the relation |ψn,L(θ)⟩ = K |ψn,R(θ)⟩, where K is the complex conjugate operator. These states are protected by a type of pseudo-time-reversal symmetry T(θ) and can be decomposed as

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where the orbital states \(|u_n(\theta)\rangle\) and spin states \(|s_{n,\pm}\rangle\) describe the spatial amplitude distribution and chirality of the eigenstates \(|\psi_n^{(\theta)}\rangle\) \((n=1, 2)\), respectively (Supplementary Section 2). Fixing \(\delta_3 = 300\) nm, we calculated the modal profiles of \(|\psi_n^{(\theta)}\rangle\) \((n=1, 2)\) at the \(\Gamma\) point and found that their orbital states \(|u_n(\theta)\rangle\) \((n=1, 2)\) are uniquely determined by \(\theta\) (Fig. 1c). The orbitals of the upper-band states \(|u_n(\theta)\rangle\) (lower-band states \(|u_n(\theta)\rangle\)) are the same as those of the lower-band states \(|u_n(\theta+\pi)\rangle\) (upper-band states \(|u_n(\theta+\pi)\rangle\)), indicating that the energy bands are always inverted for \(\theta\) and \(\theta+\pi\).

With the states \(|\psi_{1,\pm}^{(\theta)}\rangle, |\psi_{2,\pm}^{(\theta)}\rangle, |\psi_{1,-}^{(\theta)}\rangle, |\psi_{2,-}^{(\theta)}\rangle\) as the basis, we theoretically obtained the effective bulk Hamiltonian of the system

\[
H(\mathbf{k}) = v_F \left( \sigma_x k_x + \sigma_y k_y - \Delta/2 \cdot \sigma_z \right) - \Delta/2 \cdot \sigma_z, \quad \text{where} \quad \sigma_x, \sigma_y, \sigma_z \text{ and } \tau \text{ are the Pauli matrices, } \mathbf{k} = (k_x, k_y) \text{ is the wavevector, } \Delta = \frac{\omega_{n,\pm}^{(\theta)} - \omega_{1,\pm}^{(\theta)}}{2}, \text{ and } v_F \text{ is the bulk bandgap and } v_D \text{ is the effective Fermi velocity near the } \Gamma \text{ point. We further calculated the spin Chern number for crystals with parameters } (\delta_3 \neq 0, \theta) \text{ and } (\delta_3 = 0, \theta + \pi), \text{ and confirmed that they belong to distinct topological phases (Supplementary Section 2). Interestingly, the eigenfrequencies } \omega_{n,\pm}^{(\theta)} \text{ \((n=1, 2)\) at the } \Gamma \text{ point exhibit a doubly degenerate cone-like dispersion relation near the origin of the 2D parameter space (Fig. 1d), indicating that variation of } \theta \text{ has negligible influence on the degeneracy of } \omega_{n,\pm}^{(\theta)} \text{ \((n=1, 2)\). In other words, the proposed nanomechanical crystals support topological phases with an auxiliary orbital DOF } |u_n(\theta)\rangle \text{ \((n=1, 2)\) determined by the parameter } \theta. \text{ The QVH topological insulators demonstrated previously}^{21-25} \text{ are the special cases with fixed orbitals } |u_n(\theta=0)\rangle \text{ or } |u_n(\theta=\pi)\rangle.\]

One-dimensional topological edge states and adiabatic phase transition

Due to the bulk-edge correspondence, there naturally exists a pair of topological edge states at the domain wall \((y=0)\) between nanomechanical crystals with parameters \((\delta_3 \neq 0, \theta=\theta_0)\) for the region \(y>0\) and \((\delta_3 = 0, \theta=\theta_0 + \pi)\) for the region \(y<0\) (Fig. 2a). The wavefunctions of the topological edge states are determined by the spin and orbital states (Supplementary Section 3):

\[
|\psi_{\text{edge},\pm}^{(\theta_0)}\rangle = |u_1(\theta_0)\rangle \cdot |s_{1,\pm}\rangle \pm j |u_2(\theta_0)\rangle \cdot |s_{2,\pm}\rangle \cdot e^{-j(\Delta/2)v_D}|j|,\]

where \(j\) is the imaginary unit. The corresponding dispersion relations of the edge states are \(\epsilon_{\pm}(k_x) = \mp v_D k_x\). As the bulk bandgap \(\Delta\) is determined by \(\delta_3\) but not \(\theta_0\), the edge states \(|\psi_{\text{edge},\pm}^{(\theta_0)}\rangle\) are confined to the topological domain wall for arbitrary values of \(\theta_0\).

Without loss of generality, we investigated the topological edge states at the domain wall between two nanomechanical crystals with parameters \((\delta_3 = 300\) nm, \(\theta=\theta_0 = \pi/4\)) and \((\delta_3 = 300\) nm, \(\theta=\theta_0 + \pi = 5\pi/4\)), as shown in Fig. 2a. The simulated energy band diagram in Fig. 2b confirms the presence of two topological edge states (purple dots) with an almost linear dispersion relation inside the bulk bandgap from 1.57 to 1.47 MHz. The measured energy band diagram in Fig. 2c agrees well with the simulated results. Note that the structures near the interfacial domain wall slightly break the \(T(\theta_0)\) symmetry, and consequently the two counterpropagating edge states are slightly coupled to each other to form two standing-wave states at the \(\Gamma\) point. The simulated and measured modal profiles of these two standing-wave edge states are shown in Fig. 2d,e and 2f,g, respectively, which confirm that the edge states are highly localized at the topological domain wall. In addition to \(\theta_0 = \pi/4\), the energy band diagrams
of different topological edge states with other $\theta_1$ values were also experimentally demonstrated (Supplementary Fig. 9).

Next, we characterized the propagation properties of the topological edge state along a bent domain wall (Fig. 2h). The nanomechanical crystals in Fig. 2h adopted the same parameters as Fig. 2a. We measured the amplitude distribution of the elastic waves at the frequency of 47.57 MHz (Fig. 2i). Figure 2j shows the intensity spectra of elastic waves measured at selected spots near (points A-C in Fig. 2h) and away from (point D in Fig. 2h) the topological domain wall. The intensity measured at points A–C is orders of magnitude higher than that at point D in the bulk bandgap frequency range of 48.08–51.57 MHz. We measured the bulk bandgap along the $x$ direction and confirmed that it remains nearly constant (Supplementary Fig. 10). Figure 3c shows the measured spatial distribution of elastic waves at the frequency of 50.70 MHz, which confirms that varying $\theta_0$ does not influence the localization function $e^{-\frac{\pi}{2\Delta v_L}|\psi|}$ of the topological edge states. We also observed the backscattering-immune propagating elastic waves in the temporal domain (Supplementary Video 2). Figure 3d shows the intensity spectra of elastic waves measured at selected spots (points A–C in Fig. 3b). In the bulk bandgap frequency range of 48.08–51.57 MHz, the amplitude at points A and B is orders of magnitude higher than that at point C. This suggests that the topological edge state $\psi_{\text{edge}, \pm}^{(\theta_0=\pi/2)}$ could be transformed smoothly into another type of topological edge state $\psi_{\text{edge}, \pm}^{(\theta_0=-\pi/2)}$ by harnessing the auxiliary orbital DOF.

We also experimentally demonstrated that the adiabatic topological phase transition cannot be realized with fixed orbitals. Without loss of generality, we take the topological edge states with fixed orbitals $\psi_n(\theta=\pm\pi/2)$ $(n=1, 2)$ as an example. As shown in Fig. 3e, the only way to realize the topological phase transition is to change the value of $\delta_1 = \delta_1\sin\theta_0$ from positive to negative in region I and
from negative to positive in region II. No matter how smooth this process is, it unavoidably passes an abrupt phase transition point with $\delta_t = 0$ (that is, the origin of the 2D parameter space in Fig. 3e), where the bulk bandgap $\Delta$ becomes zero and the states $|\psi_{\text{edge}, \pm}(\theta_0)\rangle$ are no longer localized near the domain wall but evolve into bulk states gradually. We measured the bulk bandgap along the $x$ direction and confirmed that it closes gradually and then reopens with an increasing $x$ (Supplementary Fig. 10). Figure 3g shows the measured spatial distribution of elastic waves at the frequency of 50.70 MHz.

We also observed the propagating elastic waves in the temporal domain (Supplementary Video 3), which show that the topological edge states evolve into bulk states and experience large backscattering during this process. Figure 3h shows the intensity spectra of elastic waves measured at points A–C in Fig. 3f, which confirm that the topological edge states $|\psi_{\text{edge}, \pm}(\theta_0 = \pi/2)\rangle$ cannot be transformed smoothly into $|\psi_{\text{edge}, \pm}(\theta_0 = -\pi/2)\rangle$ when the orbital DOF is frozen.

**The 0D Dirac-vortex states**

Besides one-dimensional edge states, the auxiliary orbital DOF can also be used to construct 0D Dirac-vortex states, which...
Fig. 4 | Nanomechanical Dirac-vortex states with second-order and third-order nonlinearities. a, Optical microscope image of the device for demonstrating topological Dirac-vortex state. Scale bar, 100 pm. b, Zoomed-in optical microscope image of the nanomechanical crystal. The white hexagon indicates the unit cell at the centre of the vortex. The nanomechanical crystal has parameters $\delta_\phi(r) = \delta_{max} \tanh(R/R_0)$ and $\theta(r) = \varphi$, both of which are a function of the position $r = R(\cos\varphi, \sin\varphi)$. Scale bar, 30 pm. c, Simulated modal profile of the Dirac-vortex state. $W(r)$ denotes the out-of-plane mechanical displacement. Min., minimum; Max., maximum. d, Intensity spectra of elastic waves measured at the white star in b. The grey shaded region indicates the bulk bandgap. $Q$ denotes the quality factor of the mechanical modes. e, f, Measured spatial distributions of elastic waves at the frequencies of $\omega/2\pi = 48.58$ MHz (e) and $\omega/2\pi = 48.66$ MHz (f). g, Measured intensity spectra of elastic waves actuated by an additional pump voltage $V_{\text{pump}} \cos(\Omega t)$ with voltage amplitude $V_{\text{pump}} = 7$ V and frequency $\Omega/2\pi$ sweeping from 77 to 89 kHz, showing the second-order nonlinearity of the structure. h, Measured intensity spectra of elastic waves actuated at various power levels, showing intensity-dependent transmissions caused by the third-order nonlinearity. The observed discontinuities under large drive power are attributed to switching between multiple stable states.

are the nanomechanical analogue of the well-known Majorana bound states in superconductor electronic systems$^{28}$. Such states have been demonstrated in Kekulé-distorted bulk mechanical and acoustic systems$^{29,30}$, and more recently in a Kekulé-distorted nanophotonic system$^{31}$. Here, we proposed a different theoretical scheme to realize the mechanical Majorana states. We designed and fabricated a device as shown in Fig. 4a, with details of the structure near the centre shown in Fig. 4b. The nanomechanical crystals have spatially varying parameters $\delta_\phi(r) = \delta_{max} \tanh(R/R_0)$ and $\theta(r) = \varphi$, with the position vector $r = R(\cos\varphi, \sin\varphi)$. The parameters adopted by the device in Fig. 4 are $R_0 = a/10$ and $\delta_{max} = 300$ nm. Here, the parameter $R_0$ controls the size of the Dirac-vortex mode. The nanomechanical crystals exhibit a double-Dirac-cone-like dispersion relation (Fig. 1d) and are topologically nontrivial in the 2D parameter space. The spatially varying functions $\delta_\phi(r) = \delta_{max} \tanh(R/R_0)$ and $\theta(r) = \varphi$ project this topological nontriviality from the 2D parameter space into the spatial domain, which leads to the emergence of 0D Dirac-vortex states. We define a new set of bulk states $|\psi_{\Theta/D,K}\rangle$ at the $\Gamma$ point:

$$|\psi_{\Theta/D,K}\rangle = |\alpha_{\Theta/D}\rangle \cdot |s_{\pm}\rangle,$$

where $|\alpha_{\Theta/D}\rangle$ denotes circularly polarized orbital states (Supplementary Section 4). By adopting these states as the basis, the nanomechanical crystal can be described by a Dirac Hamiltonian $H(k) = v_0 \cdot \left( \sigma_z k_x - \sigma_y k_y \right) \cdot \tau_z - \Delta/2 \cdot \left( \tau_x \cos \theta + \tau_y \sin \theta \right)$, where $\tau_x$ and $\tau_y$ are the Pauli matrices. The first two terms indicate the double-Dirac-cone dispersion relation in the momentum space and the last two terms are the mass terms that lead to the bulk bandgap. The mass terms can form a complex number $\Delta e^{i\theta}/2$. If $\theta$ is set to be proportional to $\varphi = \text{arg}(r)$, this Hamiltonian can support a 0D defect state similar to the Majorana bound state in condensed-matter physics$^{28}$. 

580 NATURE NANOTECHNOLOGY | VOL 16 | MAY 2021 | 576–583 | www.nature.com/naturenanotechnology
Fig. 5 | Nanomechanical Dirac-vortex states based on generalized scheme to manipulate the orbital states. a, Concept of generalized scheme to realize the Dirac-vortex states. The parameters \( \delta_d, \delta_a, \delta_b \) are implicit functions of \( \phi \). Specifically, \( \delta_d(r) = \delta_{a,b} \text{tanh}(R/R_0) \) with \( R_0 = 3a \) and \( \delta_{a,b} = 300 \text{nm} \). \( \theta(r) \) and \( \phi(r) \) are implicit functions of \( \theta = \arg(r) \), so that \( \phi(\delta_d, \delta_a, \delta_b) \) form a great circle of the Poincaré sphere. b, Spatial distributions of parameters \( (\delta_d, \delta_a, \delta_b) \) for devices defined on different great circles in a. The yellow hexagons indicate the central unit cell of the vortex. The arrows show the direction and magnitude of the vectors defined by \( (\delta_d, \delta_a, \delta_b) \) for each unit cell. c, Intensity spectra of elastic waves measured from devices with parameters shown in b. The Dirac-vortex state always exists near the frequency of 48 MHz (marked by the black arrows). d, e, Measured (d) and simulated (e) modal profiles of the Dirac-vortex states.

The entire structure (Fig. 4b) is mirror-symmetric with respect to the axis of \( \phi = \pi/2 \), so it supports the Dirac-vortex state with the same type of symmetry as shown in the simulated profile in Fig. 4c. Figure 4d shows the intensity spectra of elastic waves measured at the white star in Fig. 4b. There exist two resonances in the bulk bandgap region at frequencies \( \omega_1/2\pi = 48.58 \text{ MHz} \) and \( \omega_2/2\pi = 48.66 \text{ MHz} \) (Fig. 4d). The elastic-wave amplitude distributions measured at these two resonances (Fig. 4e,f) show that they are a hybridization of the Dirac-vortex state and a defect mode induced by the electrode. This can be modelled by a Hamiltonian \( H_{\text{linear}} = \hbar \left[ \omega_{d} a_{d}^{\dagger} a_{d} + \omega_{a} a_{a}^{\dagger} a_{a} - \kappa (a_{M}^{\dagger} a_{d} + \text{h.c.}) \right] \), where \( \hbar \) is the reduced Planck’s constant, \( \omega_{d} \) and \( \omega_{a} \) are the frequencies of the Majorana-like mode and electrode-induced defect mode, respectively, \( \kappa \) is the rate of coupling between the two modes, and \( \text{h.c.} \) denotes the Hermitian conjugate (Supplementary Section 6).

With the tight modal confinement unique on the nanomechanical platform, we also observed strong second-order and third-order nonlinearities in our devices, which are interesting topics of recent theoretical studies\(^{32,33}\). To demonstrate the second-order nonlinearity, we applied an additional pump voltage \( V_{\text{pump}} \cos(2\Omega t) \), which periodically modulated the original frequency \( (\omega_{d}) \) of the electrode-induced defect mode at the pump frequency \( \Omega \). The Hamiltonian of the system including the second-order nonlinearity becomes \( H_{\text{linear}} + H^{(2)} \), with \( H^{(2)} = \hbar \beta^{(2)} V_{\text{pump}} \cos(\Omega t) a_{d}^{\dagger} a_{d}^{\dagger} a_{d} a_{d} \). When \( \Omega \) is near \( \omega_{d} - \omega_{a} \), the nonlinear term \( H^{(2)} \) introduces an effective interaction between the two hybrid modes and modifies the original transmission spectra to exhibit anticrossing phenomena (Fig. 4g). The calibrated second-order nonlinear coefficient...
was found to be $\beta_3 \approx 1.38$ Hz mV$^{-1}$ (Supplementary Section 6). With the second-order nonlinearity, it is possible to realize braking of the Dirac-vortex states by applying spatially resolved electrical tuning gates, where the nontrivial braking phases arise from the adiabatic variation of the phase $\theta(r)$ texture as the vortices are wound around one another$\textsuperscript{44}$. The third-order Kerr nonlinearity was also demonstrated in power-dependent resonant frequency shifts for both the Majorana-like mode and electrode-induced defect mode. Increasing the drive voltage $V_{ac}$ leads to modified transmission spectra with shark-fin-shaped resonant peaks and multiple intensity-dependent stable states$\textsuperscript{35}$ (Fig. 4h). The Hamiltonian including the third-order nonlinearity is $H_{\text{linear}} + H^{(3)}$, with $H^{(3)} = h \left( \beta_3 a_i^\dagger a_m a_i^\dagger a_m + \beta_5^3 a_i^\dagger a_m a_i^\dagger a_e \right)$. The calibrated third-order nonlinear coefficients were found to be $\beta_3 \approx 1.1 \times 10^{-3}$ Hz pm$^{-2}$ and $\beta_5^3 \approx 3.8 \times 10^{-3}$ Hz pm$^{-2}$ (Supplementary Section 6).

**Generalized scheme to manipulate the orbital states**

It is worth elucidating the connections and differences between our scheme and the Kekulé scheme$\textsuperscript{29,30}$. In the Kekulé scheme, a parameter $\phi$ plays a similar role to $\theta$ in our scheme. Varying $\phi$ can also manipulate the modal profiles of the bulk states. We theoretically discovered that both schemes are based on manipulating the orbital polarization $|\mu_c(\theta, \phi)\rangle$ of the bulk states $|\phi_{\mu_c, \nu_c, \delta_c} \rangle \equiv |\mu_c(\theta, \phi)\rangle \otimes |\nu_c, \delta_c \rangle$ ($n = 1, 2$) at the $\Gamma$ point (Supplementary Section 5). The difference is that the two schemes deal with the orbital states on orthogonal great circles of the Poincaré sphere. Our scheme can include certain orbital states that are closely related to the QVH phases, which cannot be realized by using the conventional Kekulé scheme. In this regard, we conducted an experiment of the adiabatic transition between the QSH and QVH phases to demonstrate the unique feature of our scheme (Supplementary Fig. 11 and Supplementary Video 4).

By combining these two schemes, one can realize arbitrary orbital states of nanomechanical topological insulators on the Poincaré sphere. The nanomechanical crystal structure supporting this generalized scheme is determined by the three parameters $(\delta_1, \delta_2, \delta_3)$ and $\delta_1, \delta_2, \delta_3$ are implicit functions of $\delta_1, \delta_2, \delta_3$ and $\delta_1, \delta_2$ are determined by $\delta_1, \delta_2$. The intensity spectra of elastic waves measured from these devices show that there always exists a Dirac-vortex state near the frequency of 48 MHz (Fig. 5c). The measured modal profiles of these resonant modes (Fig. 5d) agree well with the simulated results (Fig. 5e), confirming that the Dirac-vortex states can exhibit various modal profiles with different orbital states as the basis. Note that Case 3 in Fig. 5 corresponds to the Kekulé scheme. With the generalized scheme, the Dirac-vortex state with a winding number of 1 (Case 1 in Fig. 5) can be transformed continuously into that with a winding number of $-1$ (Case 5 in Fig. 5) by rotating the great circle adiabatically in the three-dimensional parameter space. This unique way of manipulating the orbital polarizations of the Dirac-vortex states is unavailable by using the Kekulé scheme only.

**Conclusions**

In conclusion, we have theoretically proposed and experimentally demonstrated topological insulators with an auxiliary orbital DOF. Exploiting this additional DOF, we have realized an on-chip adiabatic topological transition between distinct edge states, which overcomes a major challenge in the further development of topological integrated circuits with enhanced functionality and scalability. We have also realized nanomechanical Dirac-vortex states with strong nonlinearities, which may enable experimental investigation of reconfigurable topological nanomechanical crystals$\textsuperscript{34}$ and exotic phenomena including squeezed states$\textsuperscript{35}$ and solitons in topological nanomechanical metamaterials. Although our experiment was conducted on a nanomechanical platform, the concept of exploiting the orbital DOF can readily be extended and applied to 2D topological circuits in other areas, such as photonics and acoustics.

**Online content**

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41565-021-00868-6.

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**Methods**

**Simulation.** We used commercial software (COMSOL Multiphysics) to solve the 2D Kirchhoff–Love plate elastic equation numerically with a finite-element method. The elastic bulk and edge states in the nanomechanical topological insulators were calculated with the Floquet boundary conditions. In the simulation model, the silicon nitride layer has a density of 3,100 kg m\(^{-3}\), thickness of 140 nm, Young's modulus of 250 GPa, Poisson's ratio of 0.23 and in-plane isotropic residual stress of 1,150 MPa. The etched holes in the silicon nitride layer are ignored because they are too small to affect the simulation results.

**Fabrication.** We fabricated the devices on a silicon-nitride-on-insulator wafer (150 nm silicon nitride layer on 150 nm buried oxide) with processes that are complementary metal–oxide–semiconductor (CMOS) compatible (Supplementary Fig. 1). The handle substrate of the wafer is heavily doped silicon with a low resistivity, which serves as the electrical ground for the devices. First, we defined the patterns of the small holes and the windows for fabricating the ground electrodes in resist ZEP520A by electron-beam lithography, and transferred the patterns to the silicon nitride layer by plasma dry etching. Then, we conducted a short wet-etching process in a buffered oxide etchant to transfer the patterns to the underlying buried oxide layer. Next, we conducted the second electron-beam lithography in resist polymethyl methacrylate followed by metal deposition (chromium, 5 nm; gold, 60 nm) and lift-off processes to fabricate the electrodes. Then, we conducted a long wet-etching process, which further etched the buried oxide in proximity of the patterns defined in the first electron-beam lithography, creating undercuts with an undercut depth of \( \sim 2.9 \mu \text{m} \). This process released parts of the silicon nitride layer from the substrate to form the suspended nanomechanical membranes. Finally, we dried the devices in a critical point dryer for preventing stiction.

**Measurement.** Supplementary Fig. 2 shows the experimental set-up for measuring the frequency-domain response of the devices. We placed the devices inside a vacuum chamber with a pressure of \( 3.75 \times 10^{-3} \) torr. An a.c. voltage \( (V_{\text{ac}}) \) was added on a bias d.c. voltage \( (V_{\text{dc}}) \) and then applied across the signal and ground electrodes (S and G in Supplementary Fig. 1) to actuate the mechanical motions of the nanomechanical membranes. These mechanical motions were optically detected by a homebuilt Michelson interferometer, and then were electrically read out by a photodetector. We used a vector network analyser to generate the a.c. voltage and analyse the response signals collected from the photodetector. By sweeping the frequency of \( V_{\text{ac}} \) and collecting the response signals at the same time, the vector network analyser could obtain the frequency-domain response of the devices. The spectra in Figs. 2j and 3d,h were obtained with a time-gating technique so that only the forward-propagating elastic waves were collected for characterization and analysis (Supplementary Section 7.8). Supplementary Fig. 3 shows the experimental set-up for measuring the temporal-domain response of the devices. In this case, we used a signal generator to generate a sequence of pulse-modulated radio-frequency electrical signals to actuate the mechanical motions, and we used an oscilloscope to record the temporal-domain response signals collected by the photodetector. During the measurement, we used a high-resolution closed-loop micropositioner to control the position of the detecting optical beam in the Michelson interferometer for precise mapping of the spatial behaviours of the elastic waves.

**Data availability**

The data that support the findings of this study are available in the Zenodo repository with the identifier https://doi.org/10.5281/zenodo.4412039. Source data are provided with this paper.

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**Author contributions**

J.M. developed the theoretical concept. X.X. and J.M. performed the device fabrication and characterization. All authors worked together to write the manuscript. X.S. supervised the project.

**Competing interests**

The authors declare no competing interests.

**Additional information**

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