Pros and Cons of GAN Evaluation Measures

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Abstract

Generative models, in particular generative adversarial networks (GANs), have received a lot of attention recently. A number of GAN variants have been proposed and have been utilized in many applications. Despite large strides in terms of theoretical progress, evaluating and comparing GANs remains a daunting task. While several measures have been introduced, as of yet, there is no consensus as to which measure best captures strengths and limitations of models and should be used for fair model comparison. As in other areas of computer vision and machine learning, it is critical to settle on one or few good measures to steer the progress in this field. In this paper, I review and critically discuss more than 19 quantitative and 4 qualitative measures for evaluating generative models with a particular emphasis on GAN-derived models.

1 Introduction

Generative models are a fundamental component of a variety of important machine learning and computer vision algorithms. They are increasingly used to estimate the underlying distribution of a dataset and artificially generate various kinds of data including high-quality images, videos, and audio. They can be utilized for purposes such as unsupervised learning, semi-supervised learning, density estimation, compression, denoising, inpainting, texture synthesis, feature learning, and style transfer. A recent class of generative models known as Generative adversarial networks (GANs) by Goodfellow et al. [31] has attracted a lot of attention. A sizable volume of follow-up papers have been published since introduction of GANs in 2014. There has been substantial progress in terms of theory and applications and a large number of GAN variants have been introduced. However, relatively little effort has been made in evaluating these models and grounded ways to quantitatively and qualitatively assess these models are missing.

Generative models can be classified into two broad categories of explicit and implicit approaches. The former class assumes access to the model likelihood function, whereas the latter uses a sampling mechanism to generate data. Examples of explicit models are variational auto-encoders (VAEs) [43, 42] and PixelCNN [83]. Examples of implicit generative models are GANs. Explicit models are typically trained by maximizing the likelihood or its lower bound. GANs aim to approximate a data distribution \( P \), using a parameterized model distribution \( Q \). They achieve this by jointly optimizing two adversarial networks: a generator and a discriminator. The generator \( G \) is trained to synthesize from a noise vector an image that is close to true data distribution. The discriminator \( D \) is optimized to accurately distinguish between the synthesized images coming from the generator and the real images from the data distribution.

Several evaluation measures have surfaced with the emergence of new models. Some of them attempt to quantitatively evaluate models while some others emphasize on qualitative ways such as user studies or analyzing internals of models. Both of these approaches have strengths and limitations. For example, one may think that fooling a person in distinguishing generated images from real ones can be an ultimate test. Such a measure, however, may favor models that concentrate on limited sections of the data (i.e., overfitting
or memorization; low diversity; mode dropping). Quantitative measures, while being less subjective may not directly correspond to how humans perceive and judge generated images. These, along with other issues such as the variety of probability criteria and the lack of a perceptually meaningful image similarity metric, have made evaluating generative methods notoriously difficult [78]. In spite of no agreement regarding the best GAN evolution measure, few works have already started to benchmark GANs (e.g., [57]). While such studies are indeed helpful, further research is needed to understand the GAN evaluation metrics and assess their strengths and limitations (e.g., [78, 27, 3, 11, 92, 1]).

My main goal here is to critically review available GAN measures and help the researchers objectively assess them. At the end, I will offer some suggestions for designing more efficient measures for fair GAN comparison.

2 GAN Evaluating Measures

Here, I explain the GAN evaluation measures while discussing their pros and cons. They will be organized in two categories: quantitative and qualitative. Table 1 shows the list of measures. Notice that while I tried to include as many measures as possible, it is possible that I have missed some.

2.1 Quantitative Measures

A schematic layout for sample based GAN evaluation measures is shown in Fig. 1. Some measures discussed in the following are “model agnostic” in that the generator is used as a black box to sample images and they do not require a density estimation from the model. On the contrary, some other measures such as average log-likelihood demand estimating a probability distribution from samples.

1. **Average Log-likelihood.** Kernel density estimation (KDE, or Parzen window estimation) is a well-established method for estimating the density function of a distribution from samples. For a probability kernel $K$ (most often an isotropic Gaussian) and i.i.d samples $\{x_1, \cdots, x_n\}$, a density function at $x$ is defined as $p(x) \approx \frac{1}{z} \sum_{i=1}^{n} K(x - x_i)$, where $z$ is a normalizing constant. This allows the use of classical metrics such as KLD and JSD (Jensen Shannon divergence). However, despite its widespread use, its suitability for estimating the density of GANs has been questioned by Theis et al. [78].

Log-likelihood (or equivalently Kullback-Leibler divergence) has been the de-facto standard for training and evaluating generative models [80]. It measures the likelihood of the true data under the generated distribution on $N$ samples from the data, i.e., $L = \frac{1}{N} \sum \log P_{model}(x_i)$. Since estimating likelihood in higher dimensions is not feasible, generated samples can be used to infer something about a model’s log-likelihood. The intuition is that a model with maximum likelihood (zero KL divergence) will produce perfect samples.

The Parzen window approach to density estimation works by taking a finite set of samples generated by a model and then using those as the centroids of a Gaussian mixture. The constructed Parzen
| Measure                                                                 | Description                                                                                                                                                                                                 |
|-----------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Average Log-likelihood [31, 78]                                       | • Log likelihood of explaining realworld held out data using a density estimated from generated data (e.g., using KDE or Parzen window estimation). \( L = \frac{1}{n} \sum \log P_{\text{model}}(x_i) \) |
| Coverage Metric [80]                                                  | • The probability mass of the true data “covered” by the model distribution \( C := P_{\text{data}}(dP_{\text{model}} > t) \) with \( t \) such that \( P_{\text{model}}(dP_{\text{model}} > t) = 0.95 \). |
| Inception Score (IS) [69]                                             | • KL D between conditional and marginal label distributions over generated data. \( \exp(E_k[KL(p(y|x) \| p(y))]) \)                                                                                                    |
| Modified Inception Score (m-IS) [33]                                  | • Encourages diversity within images sampled from a particular category. \( \exp(E_k[E_x[KL(P(y|x) \| P(y|x_j))]]) \)                                                               |
| Mode Score (MS) [10]                                                  | • Similar to IS but also takes into account the prior distribution of the labels over real data \( \exp(E_k[KL(p(y|\hat{x}) \| p(y))]) \)                                                                 |
| AM Score [104]                                                       | • Similar to MS but with reverse order of terms in KL. \( E_x[\sum_k KL(p(y^\text{trans}) \| p(y|x)) - KL(p(y^\text{trans}) \| p(y))]) \)                                         |
| Fréchet Inception Distance (FID) [34]                                | • Wasserstein-2 distance between multi-variate Gaussians fitted to data embedded into a feature space \( FID(r, g) = \|\mu_r - \mu_g\|^2 + Tr(\Sigma_r + \Sigma_g - 2\Sigma_r\Sigma_g^{\frac{1}{2}}) \) |
| Maximum Mean Discrepancy (MMD) [32]                                  | • The critic (e.g., an NN) is trained to produce high values at real samples and low values at generated samples \( W(x_{\text{real}} \mid y_y) = \frac{1}{N} \sum_i f(x_{\text{real}}[i]) - \frac{1}{N} \sum_i f(x_{\text{gen}}[i]) \) |
| Birthday Paradox Test [3]                                             | • Measures the dissimilarity between two probability distributions \( P_1 \) and \( P_n \) using samples drawn independently from each distribution. \( M_n(P_1, P_n) = E_{P_1}[k(x, x')] - 2E_{P_1}[k(x, y)] + E_{P_n}[k(y, y')] \) |
| Classifier Two Sample Test (C2ST) [47]                               | • Measures the support size of a discrete (continuous) distribution by counting the duplicates (near duplicates)                                                                                           |
| Classification Performance [64, 39]                                   | • An indirect technique for evaluating the quality of unsupervised representations (e.g., feature extraction; FCN score)                                                                                      |
| Image Retrieval Performance [86]                                     | • Measures the distributions of distances to the nearest neighbors of some query images (i.e., diversity)                                                                                                  |
| Generative Adversarial Metric (GAM) [38]                             | • Compares two GANs by having them engage in a battle against each other by swapping discriminators or generators. \( p(x|y = 1; M_1)/p(x|y = 1; M_2) = p(y = 1|x; D_1)p(x; G_2)/p(y = 1|x; D_2)p(x; G_1) \) |
| Adversarial Accuracy and Divergence [95]                             | • Adversarial Accuracy. Computes the classification accuracies achieved by the two classifiers, one trained on real data and another on generated data, on a labeled validation set to approximate \( P_y(y|x) \) and \( P_r(y|x) \). Adversarial Divergence: Computes \( KL(P_y(y|x), P_r(y|x)) \) |
| Reconstruction Error [93]                                            | • Measures the reconstruction error (e.g., L2 norm) between a test image and its closest generated image by optimizing for \( z \) (i.e., \( \min_{z} ||G(z) - x^{(test)}||^2 \) ) |
| Image Quality Measures [89, 66, 40]                                   | • Evaluates the quality of generated images using measures such as SSIM, PSNR, and sharpness difference                                                                                                 |
| Low-level Image Statistics [98, 41]                                  | • Evaluates how similar low-level statistics of generated images are to those of natural scenes in terms of mean power spectrum, distribution of random filter responses, contrast distribution, etc. |
| Mode Drop and Collapse [75, 50]                                      | • Over datasets with known modes (e.g., a GMM or a labeled dataset), modes are computed by measuring the distances of generated data to mode centers |
windows mixture is then used to compute a log-likelihood score on a set of test examples. Wu et al. [92] proposed to use annealed importance sampling (AIS) [60] to estimate log-likelihoods using a Gaussian observation model with a fixed variance. The key drawback of this approach is the assumption of the Gaussian observation model which may not work quite well in high-dimensional spaces. They found that AIS is two orders of magnitude more accurate than KDE, and is accurate enough for comparing generative models.

While likelihood is very intuitive, it suffers from several drawbacks [78]:

(a) For a large number of samples Parzen window estimates falls short in approximating a model’s true log-likelihood when the data dimensionality is high. Even for the fairly low dimensional space of $6 \times 6$ image patches, it requires a very large number of samples to come close to the true log-likelihood of a model. See Fig. 8(c).

(b) Theis et al. showed that the likelihood is generally uninformative about the quality of samples and vice versa. In other words, log-likelihood and sample quality are somewhat unrelated. A model can have poor log-likelihood and produce great samples, or have great log-likelihood and produce poor samples. An example in the former case is a mixture of Gaussian distributions where the means are training images (i.e., akin to a look-up table). Such a model will generate great samples but will still have very poor log-likelihood. An example of the latter is a mixture model combined from a good model, with a very low weight $\alpha$, and a bad model with a high weight $1 - \alpha$. Such a model has a large average log-likelihood but generates very poor samples.

(c) Parzen window estimates of the likelihood produce rankings different from other measures.

Due to the above issues, it becomes difficult to answer basic questions, such as whether the GANs are simply memorizing training examples, or whether they are missing important modes of the data distribution. For further discussions on other drawbacks of average likelihood measures consult [37].

2. **Coverage Metric.** Tolstikhin et al. [80] proposed to use the probability mass of the real data “covered” by the model distribution $P_{\text{model}}$ as a metric. They compute $C := P_{\text{data}}(dP_{\text{model}} > t)$ with $t$ such that $P_{\text{model}}(dP_{\text{model}} > t) = 0.95$. A kernel density estimation method was used to approximate the density of $P_{\text{model}}$. They claim that this metric is more interpretable than the likelihood, making it easier to assess the difference in performance of the algorithms.

3. **Inception Score (IS).** Proposed by Salimans et al. [69], it is perhaps the most widely adopted score for GAN evaluation (e.g., in [23]). It uses a pre-trained neural network (the Inception Net [77] trained on the ImageNet [15]) to capture the desirable properties of generated samples: *highly classifiable* and *diverse* with respect to class labels. It measures the average KL divergence between the conditional label distribution $p(y|x)$ of samples (expected to have low entropy for easily classifiable samples; better sample quality) and the marginal distribution $p(y)$ obtained from all the samples (expected to have high entropy if all classes are equally represented in the set of samples; high diversity). It favors low entropy of $p(y|x)$ but a large entropy of $p(y)$.

\[
\exp \left( \mathbb{E}_x [\text{KL}(p(y \mid x) \parallel p(y))] \right) = \exp \left( H(y) - \mathbb{E}_x [H(y|x)] \right),
\]

where $p(y \mid x)$ is the conditional label distribution for image $x$ estimated using a pretrained Inception model [77], and $p(y)$ is the marginal distribution: $p(y) \approx \frac{1}{N} \sum_{n=1}^{N} p(y \mid x_{n} = G(z_{n}))$. $H(x)$ represents entropy of variable $x$.

The Inception score shows a reasonable correlation with the quality and diversity of generated images [69]. IS over real images can serve as the upper bound. Despite these appealing properties, IS has several limitations:

(a) First, similar to log-likelihood, it favors a “memory GAN” that stores all training samples, thus is unable to detect overfitting (i.e., can be fooled by generating centers of data modes [96]). This is aggravated by the fact that it does not make use of a holdout validation set.
Figure 2: Sensitivity of the inception score to image resolution. Top: Training data and synthesized images from the zebra class resized to a lower spatial resolution and subsequently artificially resized to the original resolution (128 × 128 for the red and black lines; 64 × 64 for the blue line). Bottom Left: IS score across varying spatial resolutions for training data and image samples from 64 × 64 and 128 × 128 models. Error bars show standard deviation across 10 subsets of images. Dashed lines highlight the accuracy at the output spatial resolution of the model. Bottom Right: Comparison of accuracy scores at 128 × 128 and 32 × 32 spatial resolutions. Each point represents an ImageNet class. 84.4% of the classes are below the diagonal. The green dot corresponds to the zebra class. Figure from [62].

(b) Second, it fails to detect whether a model has been trapped into one bad mode (i.e., is agnostic to mode collapse). Zhou et al. [102], however, shows results on the contrary.

(c) Third, since IS uses Inception model that has been trained on ImageNet with many object classes, it may favor models that generate good objects rather realistic images.

(d) Fourth, IS only considers $P_g$ and ignores $P_r$. Manipulations such as mixing in natural images from an entirely different distribution could deceive this score. As a result, it may favor models that simply learn sharp and diversified images, instead of $P_r$ [27].

(e) Fifth, it is an asymmetric measure.

(f) Finally, it is affected by image resolution. See Fig. 2.

Zhou et al. [102] provide an interesting analysis of the Inception score. They experimentally measured the two components of the IS score, entropy terms in Eq. 1, during training and showed that $H(y|x)$ behaves as expected (i.e., decreasing) while $H(y)$ does not. See Fig. 3 (top row). They found that CIFAR-10 data are not evenly distributed over the classes under the Inception model trained on ImageNet. See Fig. 3(d). Using the Inception model trained over ImageNet or CIFAR-10, results in two different values for $H(y)$. Also, the value of $H(y|x)$ varies for each specific sample in the training data (i.e., some images are deemed less real than others). Further, a mode-collapsed generator usually
Figure 3: Top: Training curves of Inception Score and its decomposed terms. a) IS during training, b) First term in rhs of Eq. 1, \( H(y) \), goes down with training which is supposed to go up, c) The second term, \( H(y|x) \) decreases in training, as expected. Bottom: Statistics of the CIFAR-10 training images. d) \( p(y) \) over ImageNet classes, e) \( H(y|x) \) distribution with ImageNet classifier of each class, and f) \( H(y|x) \) distribution with CIFAR-10 classifier of each class. Figure compiled from [102].

gets a low Inception score (See Fig. 5 in [102]), which is a good sign. Theoretically, in an extreme case when all the generated samples are collapsed into a single point (thus \( p(y) = p(y|x) \)), then the minimal Inception score of 1.0 will be achieved. Please refer to [4] for further analysis on the inception score.

4. **Modified Inception Score (m-IS).** Inception score assigns a higher value to models with a low entropy class conditional distribution over all generated data \( p(y|x) \). However, it is desirable to have diversity within samples in a particular category. To characterize this diversity, Gurumurthy et al. [33] suggested to use a cross-entropy style score 
\[
- p(y|x) \log(p(y|x_j))
\]
where \( x_j \)s are samples from the same class as \( x_i \) based on the inception model’s output. Incorporating this term into the original inception-score results in:
\[
\exp\left(\mathbb{E}_{x} \left[\mathbb{E}_{x_i} \left[\mathbb{KL}(P(y|x_i)||P(y|x_j))\right]\right]\right).
\]
which is calculated on a per-class basis and is then averaged over all classes. Essentially, m-IS can be viewed as a proxy for measuring both intra-class sample diversity as well as sample quality.

5. **Mode Score.** Introduced in [10], this score addresses an important drawback of the Inception score which is ignoring the the prior distribution of the ground truth labels:
\[
\exp \left(\mathbb{E}_{x} \left[\mathbb{KL} (p(y \mid x) \parallel p(y^{\text{train}}))\right] - \mathbb{KL} (p(y) \parallel p(y^{\text{train}}))\right)
\]
where \( p(y^{\text{train}}) \) is the empirical distribution of labels computed from training data. Mode score adequately reflects the variety and visual quality of generated images [10]. It has been, however, proved that Inception and MODE scores are in fact equivalent. See [104] for a proof.

6. **AM Score.** Zhou et al. [104] argue that entropy terms in Inception and Mode scores are not suitable when the data is not evenly distributed over classes. They swap the order of \( y^{\text{train}} \) and \( p(y|x) \) in the
two KL divergence terms in the inception score which leads to a more sensible metric (arguing that $y^{\text{train}}$ should be the reference term):

$$
\mathbb{E}_{x} \left[ \text{KL} (p(y^{\text{train}}) \mid \mid p(y|x)) \right] - \text{KL} (p(y^{\text{train}}) \mid \mid p(y)) = \mathbb{E}_{x} [H(y^{\text{train}},y|x)] - H(y^{\text{train}}) - H(y^{\text{train}},y) + H(y^{\text{train}}) = \mathbb{E}_{x} [H(y^{\text{train}},y|x)] - H(y^{\text{train}},y) \triangleq \text{AM score.}
$$

(4)

The AM score consists of two cross-entropy terms. The first one is maximized when each sample is far away from the training data overall class distribution. The second term is maximized when the generated samples’ average distribution is the same as training data. The overall class distribution indicated by the training data, i.e., $y^{\text{train}}$, has thus been taken into account, which is important when training data is not evenly distributed.

It has been shown that the Inception score with $p(y|x)$ being the Inception model trained with ImageNet, correlates with human evaluation on CIFAR10. CIFAR10 data, however, is not evenly distributed over the ImageNet Inception model. The entropy term on average distribution of the Inception score may thus not work well (See Fig. 3). With a pre-trained CIFAR10 classifier, the AM score can well capture the statistics of the average distribution. Thus, $p(y|x)$ should be a pre-trained classifier on a given dataset.

7. Fréchet Inception Distance (FID). Introduced by Heusel et al. [34], FID embeds a set of generated samples into a feature space given by a specific layer of Inception Net (or any CNN). Viewing the embedding layer as a continuous multivariate Gaussian, the mean and covariance are estimated for both the generated data and the real data. The Fréchet distance between these two Gaussians (a.k.a Wasserstein-2 distance) is then used to quantify the quality of generated samples, i.e.,

$$
\text{FID}(r,g) = \| \mu_r - \mu_g \|^2 + Tr(\Sigma_r + \Sigma_g - 2(\Sigma_r \Sigma_g)^{\frac{1}{2}})
$$

(5)

where $(\mu_r, \Sigma_r)$ and $(\mu_g, \Sigma_g)$ are the mean and covariance of the real data and model distributions, respectively. FID performs well in terms of discriminability, robustness and computational efficiency. It appears to be a good metric, even though it only takes into consideration the first two order moments of the distributions. However, it assumes that features are of Gaussian distribution which is often not guaranteed. It has been shown that FID is consistent with human judgements and is more robust to noise than IS [34] (e.g., negative correlation between the FID and visual quality of generated samples). Unlike IS however, it is able to detect intra-class mode dropping, i.e., a model that generates only one image per class can score a high IS but will have a bad FID. Also, unlike IS, the FID worsens as various types of artifacts are added to images (See Fig. 4). IS and AM scores measure the diversity and quality of generated samples, while FID measures the distance between the generated and real distributions. An empirical analysis of FID can be found in [57].

8. The Wasserstein Critic. The Wasserstein critic [2] provides an approximation of the Wasserstein distance between the real data distribution $P_r$ and the generator distribution $P_g$:

$$
W(P_r, P_g) \propto \max_{f} \mathbb{E}_{x \sim P_r} [f(x)] - \mathbb{E}_{x \sim P_g} [f(x)]
$$

(6)

where $f : \mathbb{R}^D \rightarrow \mathbb{R}$ is a Lipschitz continuous function. In practice, the critic $f$ is a neural network with clipped weights to have bounded derivatives. It is trained to produce high values at real samples and low values at generated samples (i.e., is an approximation):

$$
\hat{W}(x_{\text{valid}}, x_g) = \frac{1}{N} \sum_{i=1}^{N} \hat{f}(x_{\text{valid}[i]}) - \frac{1}{N} \sum_{i=1}^{N} \hat{f}(x_g[i])
$$

(7)
where \( x_{\text{valid}} \) is a batch of samples from a validation set, \( x_g \) is a batch of generated samples, and \( \hat{f} \) is the independent critic.

This metric measures both overfitting and mode collapse. If the generator memorizes the training set, the critic trained on validation data can distinguish between samples and data. If mode collapse occurs, the critic will have an easy job in distinguishing between data and samples. Further, it does not saturate when the two distributions do not overlap. The magnitude of the distance indicates how easy it is for the critic to distinguish between samples and data.

The Wasserstein distance works well when the base distance is computed in a suitable feature space. A key limitation of this distance is its high sample and time complexity. These make MMD less appealing as a practical evaluation metric, compared to other ones (See [3]).

9. Maximum Mean Discrepancy (MMD). This metric measures the dissimilarity between two probability distributions \( P_r \) and \( P_g \) using samples drawn independently from each [25]. A lower MMD hence means that \( P_g \) is closer to \( P_r \).

The kernel MMD [32] measures (square) MMD between \( P_r \) and \( P_g \) for some fixed characteristic kernel function \( k \) (e.g., Gaussian kernel \( k(x, x') = \exp(\|x - x'\|^2) \)) as follows:

\[
M_k(P_r, P_g) = \mathbb{E}_{P_r}[k(x, x')] - 2\mathbb{E}_{P_r,P_g}[k(x, y)] + \mathbb{E}_{P_g}[k(y, y')].
\] (8)

In practice, finite samples from distributions are used to estimate MMD distance. Given \( X = \{x_1, \cdots, x_n\} \sim P_r \) and \( Y = \{y_1, \cdots, y_n\} \sim P_g \), one estimator of \( M_k(P_r, P_g) \) is:

\[
\hat{M}_k(X, Y) = \frac{1}{\binom{n}{2}} \sum_{i \neq i'} k(x_i, x_{i'}) - \frac{2}{\binom{n}{2}} \sum_{i \neq j} k(x_i, y_j) + \frac{1}{\binom{n}{2}} \sum_{j \neq j'} k(y_j, y_{j'}).
\] (9)

Because of the sampling variance, \( \hat{M}(X, Y) \) may not be zero even when \( P_r = P_g \). Li et al. [48] propose a remedy to address this. Kernel MMD works surprisingly well when it operates in the feature space of

\[1\text{Distinguishing two distributions by finite samples is known as Two-Sample Test in statistics.}\]
a pre-trained CNN. It is able to distinguish generated images from real images, and both its sample complexity and computational complexity are low [27].

Kernel MMD has also been used for training GANs. For example, the Generative Moment Matching Network (GMMN) [49, 22, 48] replaces the discriminator in GAN with a two-sample test based on kernel MMD. See also [7] for more analyses on MMD and its use in GAN training.

10. **Birthday Paradox Test.** This test approximates the support size$^2$ of a discrete distribution. Arora and Zhang [3] proposed to use the birthday paradox$^3$ test to evaluate GANs as follows:

(a) Pick a sample of size $s$ from the generated distribution
(b) Use an automated measure of image similarity to flag the $k$ (e.g., $k = 20$) most similar pairs in the sample
(c) Visually inspect the flagged pairs and check for duplicates
(d) Repeat.

The suggested plan is to manually check for duplicates in a sample of size $S$. If a duplicate exists, then the estimated support size is $S^2$. It is not possible to find exact duplicates as the distribution of generated images is continuous. Instead, a distance measure can be used to find near-duplicates (e.g., using L2 norm). Following this procedure, Arora and Zhang [3] found that with probability $> 50\%$, a batch of about 400 samples generated from the CelebA dataset [53] contains at least one pair of duplicates for both DCGAN and MIX+DCGAN. The birthday theorem assumes uniform sampling. Arora and Zhang [3], however, claim that the birthday paradox holds even if data are distributed in a highly nonuniform way. This test can be used to detect mode collapse in GANs.

11. **Classifier Two-sample Tests (C2ST).** The goal of two-sample tests is to assess whether two samples are drawn from the same distribution [47]. The generator is evaluated on a held out test set. This set is split into a test-train and test-test subsets. The test-train set is used to train a fresh discriminator, while the test-test set is used to evaluate the ability of the trained discriminator to distinguish between generated and real images. Afterwards, the final score is computed as the performance of this new discriminator on the test-test set and the freshly generated images. More formally, assume we have access to two samples $S_P := \{x_1, \ldots, x_n\} \sim P^n(X)$ and $S_Q := \{y_1, \ldots, y_m\} \sim Q^m(Y)$ where $x_i, y_j \in \mathcal{X}$, for all $i = 1, \ldots, n$ and $j = 1, \ldots, m$, and $m = n$. To test whether the null hypothesis $H_0 : P = Q$ is true, these five steps need to be completed:

(a) Construct the following dataset $$\mathcal{D} = \{(x_i, 0)\}_{i=1}^n \cup \{(y_i, 1)\}_{i=1}^n =: \{(z_i, l_i)\}_{i=1}^{2n}.$$ (b) Randomly shuffle $\mathcal{D}$, and split it into two disjoint training and testing subsets $\mathcal{D}_{tr}$ and $\mathcal{D}_{te}$, where $\mathcal{D} = \mathcal{D}_{tr} \cup \mathcal{D}_{te}$ and $n_{te} := |\mathcal{D}_{te}|$.
(c) Train a binary classifier $f : \mathcal{X} \rightarrow [0, 1]$ on $\mathcal{D}_{tr}$. In the following, assume that $f(z_i)$ is an estimate of the conditional probability distribution $p(l_i = 1|z_i)$.
(d) Calculate the classification accuracy on $\mathcal{D}_{te}$:

$$\hat{i} = \frac{1}{n_{te}} \sum_{(z_i, l_i) \in \mathcal{D}_{te}} \mathbb{I}\left[\mathbb{I}\left(f(z_i) > \frac{1}{2}\right) = l_i\right]$$

(10)

as the C2ST statistic, where $\mathbb{I}$ is the indicator function. The intuition here is that if $P = Q$, the test accuracy Eq. 10 should remain near chance-level. In contrast, if binary classifier performs better than chance then it implies that $P \neq Q$.

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$^2$The support of a real-valued function $f$ is the subset of the domain containing those elements which are not mapped to zero.

$^3$The Birthday theorem states that with probability at least 50\%, a uniform sample (with replacement) of size $S$ from a set of $N$ elements will have a duplicate given $S > \sqrt{N}$.
(e) To accept or reject the null hypothesis, compute a p-value using the null distribution of the C2ST.

In principle, any binary classifier can be adopted for computing C2ST. Huang et al. [27] introduce a variation of this metric known as the 1-Nearest Neighbor classifier. The advantage of using 1-NN over other classifiers is that it requires no special training and little hyperparameter tuning. Given two sets of real \( S_r \) and generated \( S_g \) samples with the same size (i.e., \(|S_r| = |S_g|\)), one can compute the leave-one-out (LOO) accuracy of a 1-NN classifier trained on \( S_r \) and \( S_g \) with positive labels for \( S_r \) and negative labels for \( S_g \). The LOO accuracy can vary from 0% to 100%. If the GAN memorizes samples in \( S_r \) and re-generate them exactly, i.e., \( S_g = S_r \), then the accuracy would be 0%. This is because every sample from \( S_r \) would have its nearest neighbor from \( S_g \) with zero distance (and vice versa). If it generates samples that are widely different than real images (and thus completely separable), then the performance would be 100%. Notice that chance level here is 50% which happens when a label is randomly assigned to an image. Lopez-Paz and Oquab [55] offer a revisit of classifier two-sample tests in [55].

12. **Classification Performance.** One common indirect technique for evaluating the quality of unsupervised representation learning algorithms is to apply them as feature extractors on labeled datasets and evaluate the performance of linear models fitted on top of the learned features. For example, to evaluate the quality of the representations learned by DCGANs, Radford et al. [64] trained their model on ImageNet dataset and then used the discriminator’s convolutional features from all layers to train a regularized linear L2-SVM to classify CIFAR-10 images. They achieved 82.8% accuracy on par with or better than several baselines trained directly on CIFAR-10 data.

A similar strategy has also been followed in evaluating conditional GANs (e.g., the ones proposed for style transfer). For example, an off-the-shelf classifier is utilized by Zhang et al. [100] to assess the realism of synthesized images. They fed their fake colorized images to a VGG network that was trained on real color photos. If the classifier performs well, this indicates that the colorizations are accurate enough to be informative about object class. They call this “semantic interpretability”. Similarly, Isola et al. [39] proposed the “FCN score” to measure the quality of the generated images conditioned on an input segmentation map. They fed the generated images to the fully-convolutional semantic segmentation network (FCN) [54] and then measured the error between the output segmentation map and the ground truth segmentation mask.

These measures are useful for evaluating generative models based on the notion that a better generative model should result in better representations for surrogate tasks (e.g., supervised classification). This, however, does not necessarily imply that generated images have high diversity.

13. **Image Retrieval Performance.** Wang et al. [86] proposed an image retrieval measure to evaluate GANs. The main idea is to investigate images in the dataset that are badly modeled by a network. Images from a held-out test set as well as generated images are represented using a discriminatively trained CNN [44]. The nearest neighbors of generated images in the test dataset are then retrieved. To evaluate the quality of the retrieval results, they proposed two measures:

(a) Measure 1: Consider \( d_{i,j}^k \) to be the distance of the \( j^{th} \) nearest image generated by method \( k \) to test image \( i \), and \( d_i^k = \{d_{i,1}^k, \ldots, d_{i,n}^k\} \) the set of \( j^{th} \)-nearest distances to all \( n \) test images (\( j \) is often set to 1). The Wilcoxon signed-rank test is then used to test the hypothesis that the median of the difference between two nearest distance distributions by two generators is zero, in which case they are equally good (i.e., the median of the distribution \( d_i^k - d_i^m \)). If they are not equal, the test can be used to assess which method is statistically better.

(b) Measure 2: Consider \( d_i^j \) to be the distribution of the \( j^{th} \) nearest distance of the train images to the test dataset. Since train and test sets are drawn from the same dataset, the distribution \( d_i^j \) can be considered the optimal distribution that a generator could attain (assuming it generates an equal number of images present in the train set). To model the difference with this ideal distribution,
the relative increase in mean nearest neighbor distance is computed as:

\[ \hat{d}_j^k = \bar{d}_k^j - \bar{d}_t^j, \quad \bar{d}_k^j = \frac{1}{N} \sum_{i=1}^N d_{i,j}^k, \quad \bar{d}_t^j = \frac{1}{N} \sum_{i=1}^N d_{i,j}^t \]

where \( N \) is the size of the test dataset. As an example, \( \hat{d}_1 = 0 \) for a model means that the average distance to the nearest neighbor of a query image is 10% higher than for data drawn from the real distribution.

14. **Generative Adversarial Metric (GAM).** Im et al. [38] proposed to compare two GANs by having them engage in a battle against each other by swapping discriminators or generators across the two models (See Fig. 5). GAM measures the relative performance of two GANs by measuring the likelihood ratio of the two models. Consider two GANs with their respective trained partners, \( M_1 = (D_1, G_1) \) and \( M_2 = (D_2, G_2) \), where \( G_1 \) and \( G_2 \) are the generators, and \( D_1 \) and \( D_2 \) are the discriminators. The hypothesis \( \mathcal{H}_1 \) is that \( M_1 \) is better than \( M_2 \) if \( G_1 \) fools \( D_2 \) more than \( G_2 \) fools \( D_1 \), and vice versa for the hypothesis \( \mathcal{H}_0 \). The likelihood-ratio is defined as:

\[
\frac{p(x|y = 1; M'_1)}{p(x|y = 1; M'_2)} = \frac{p(y = 1|x; D_1)p(x; G_2)}{p(y = 1|x; D_2)p(x; G_1)}
\]

where \( M'_1 \) and \( M'_2 \) are the swapped pairs \((D_1, G_2)\) and \((D_2, G_1)\), and \( p(x|y = 1, M) \) is the likelihood of \( x \) generated from the data distribution \( p(x) \) by model \( M \). Then, one can measure which generator fools the opponent’s discriminator more, \( \frac{D_1(S_2)}{D_2(S_1)} \) where \( S_1 \sim G_1 \) and \( S_2 \sim G_2 \). To do so, Im et al. proposed a sample ratio test to declare a winner or a tie.

A variation of GAM known as generative multi-adversarial metric (GMAM), that is amenable to training with multiple discriminators, has been proposed in [21].

GAM has three main caveats. First, it does not give an absolute value as it offers a relative comparison between two models. Second, it has a constraint where the two discriminators must have an approximately similar performance on a calibration dataset, which can be difficult to satisfy in practice. Third, it is expensive to compute because it has to be computed for all pairs of models (i.e., pairwise comparisons between independently trained GAN).

15. **Adversarial Accuracy and Adversarial Divergence.** Yang et al. [95] proposed two metrics based on the intuition that a sufficient, but unnecessary, condition for closeness of generated data distribution \( P_g(x) \) and the real data distribution \( P_r(x) \) is closeness of \( P_g(x|y) \) and \( P_r(x|y) \), i.e., distributions of generated data and real data conditioned on all possible variables of interest \( y \), e.g., category labels. One way to obtain the variable of interest \( y \) is by asking human participants to annotate the images (sampled from \( P_g(x) \) and \( P_r(x) \)).
Since it is not feasible to directly compare $P_g(x|y)$ and $P_r(x|y)$, they proposed to compare $P_g(y|x)$ and $P_r(y|x)$ instead (following the Bayes rule) which is a much easier task. Two classifiers are then trained from human annotations to approximate $P_g(y|x)$ and $P_r(y|x)$ for different categories. These classifiers are used to compute the following evaluation metrics:

(a) **Adversarial Accuracy**: Computes the classification accuracies achieved by the two classifiers on a validation set (i.e., another set of real images). If $P_g(x)$ is close to $P_r(x)$, then similar accuracies are expected.

(b) **Adversarial Divergence**: Computes the KL divergence between $P_g(y|x)$ and $P_r(y|x)$. The lower the adversarial divergence, the closer the two distributions. The lower bound for this metric is exactly zero, which means $P_g(y|x) = P_r(y|x)$ for all samples in the validation set.

One drawback of these measures is that a lot of human effort is needed to label the real and generated samples. To mitigate this, Yang et al. [95] first trained one generator per category using a labeled training set and then generated samples from all categories.

16. **Reconstruction Error**. For many generative models, the reconstruction error on the training set is often explicitly optimized (e.g., Variational Autoencoders [45]). It is therefore natural to evaluate generative models using a reconstruction error metric (e.g., L2 norm) measured on a test set. In the case of GANs, given a generator $G$ and a set of test samples $X = \{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$ the reconstruction error of $G$ on $X$ is defined as:

$$
L_{rec}(G, X) = \frac{1}{m} \sum_{i=1}^{m} \min_{z} ||G(z) - x^{(i)}||^2
$$

Since it is not possible to directly infer the optimal $z$ from $x$, Xiang and Li [93] used the following alternative method. Starting from an all-zero vector, they performed gradient descent on the latent code to find the one that minimizes the L2 norm between the sample generated from the code and the target one. Since the code is optimized instead of being computed from a feed-forward network, the evaluation process is time-consuming. Thus, they avoided performing this evaluation at every training iteration when monitoring the training process, and only used a reduced number of samples and gradient descent steps. Only for the final trained model, they performed an extensive evaluation on a larger test set, with a larger number of steps.

17. **Image Quality Measures (SSIM, PSNR and Sharpness Difference)**. Some researchers have proposed to use measures from the image quality assessment literature for training and evaluating GANs. They are explained next.

(a) The single-scale SSIM metric [89] compares corresponding pixels and their neighborhoods in two images, denoted by $x$ and $y$, using three quantities—luminance ($I$), contrast ($C$), and structure ($S$):

$$
I(x, y) = \frac{2\mu_x\mu_y + C_1}{\mu_x^2 + \mu_y^2 + C_1} \quad C(x, y) = \frac{2\sigma_x\sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2} \quad S(x, y) = \frac{\sigma_{xy} + C_3}{\sigma_x\sigma_y + C_3}
$$

The variables $\mu_x$, $\mu_y$, $\sigma_x$, and $\sigma_y$ denote mean and standard deviations of pixel intensity in a local image patch centered at either $x$ or $y$ (typically a square neighborhood of 5 pixels). The variable $\sigma_{xy}$ denotes the sample correlation coefficient between corresponding pixels in the patches centered at $x$ and $y$. The constants $C_1$, $C_2$, and $C_3$ are small values added for numerical stability. The three quantities are combined to form the SSIM score:

$$
SSIM(x, y) = I(x, y)^a C(x, y)^b S(x, y)^c
$$

12
SSIM assumes a fixed image sampling density and viewing distance. A variant of SSIM operates at multiple scales. The input images $x$ and $y$ are iteratively downsampled by a factor of 2 with a low-pass filter, with scale $j$ denoting the original images downsampled by a factor of $2^{-j}$.

The contrast $C(x, y)$ and structure $S(x, y)$ components are applied to all scales. The luminance component is applied only to the coarsest scale, denoted $M$. Further, contrast and structure components can be weighted at each scale. The final measure is:

$$\text{MS-SSIM}(x, y) = I_M(x, y)^{\alpha_M} \prod_{j=1}^{M} C_j(x, y)^{\beta_j} S_j(x, y)^{\gamma_j}$$

MS-SSIM ranges between 0 (low similarity) and 1 (high similarity). Snell et al. [66] defined a loss function for training GANs which is the sum of structural-similarity scores over all image pixels,

$$\mathcal{L}(X, Y) = -\sum_i \text{MS-SSIM}(X_i, Y_i),$$

where $X$ and $Y$ are the original and reconstructed images, and $i$ is an index over image pixels. This loss function has a simple analytical derivative [90] which allows performing gradient descent. See Fig. 11 for more details.

(b) PSNR measures the peak signal-to-noise ratio between two monochrome images $I$ and $K$ to assess the quality of a generated image compared to its corresponding real image (e.g., for evaluating conditional GANs [65]). The higher the PSNR (in db), the better quality of the generated image. It is computed as:

$$\text{PSNR}(I, K) = 10 \log_{10} \left( \frac{M A X_I^2}{\text{MSE}_{I,K}} \right)$$

where

$$\text{MSE}_{I,K} = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} (I(m,n) - K(m,n))^2$$

and, $MAX_I$ is the maximum possible pixel value of the image (e.g., 255 for an 8 bit representation). This score can be used when a reference image is available for example in training conditional GANs using paired data (e.g., [39, 65]).

(c) Sharpness Difference (SD) measures the loss of sharpness during image generation. It is compute as:

$$\text{SD}(I, K) = 10 \log_{10} \left( \frac{M A X_I^2}{\text{GRADS}_{I,K}} \right)$$

where

$$\text{GRADS}_{I,K} = \frac{1}{N} \sum_i \sum_j \|(\nabla_i I + \nabla_j I) - (\nabla_i K + \nabla_j K)\|,$$

and

$$\nabla_i I = |I_{i,j} - I_{i,-1,j}|, \quad \nabla_j I = |I_{i,j} - I_{i,j-1}|.$$

Juefei-Xu et al. [40] used the SSIM and PSNR measures to evaluate GANs in image completion tasks. The advantage here is that having 1-vs-1 comparison between the ground-truth and the completed image allows very straightforward visual examination of the GAN quality. It also allows head-to-head comparison between various GANs. In addition to the above mentioned image quality measures, some other metrics such as Universal Quality Index (UQI) [87] and Visual Information Fidelity (VIF) [71] have also been adopted for assessing the quality of synthesized images. It has been reported that MS-SSIM finds large-scale mode collapses reliably but fails to diagnose smaller effects such as loss of variation in colors or textures. Its drawback is that it does not directly assess image quality in terms of similarity to the training set [62].
18. **Low-level Image Statistics.** Natural scenes have certain characteristics (e.g., [28, 72, 81, 68]). It has been shown that statistics of natural images remain the same when the images are scaled (i.e., scale invariance) [76, 106]. The average power spectrum magnitude $A$ over natural images has the form $A(f) = 1/f^{\alpha}$, $\alpha \approx 2$ [17, 13, 9, 24]. Another important property of natural image statistics is the non-Gaussianity [76, 106, 85]. This means that marginal distribution of almost any zero mean linear filter response on virtually any dataset of images is sharply peaked at zero, with heavy tails and high kurtosis (greater than 3) [46]. Recent studies have shown that the contrast statistics of the majority of natural images follows a Weibull distribution [30].

Zeng et al. [98] proposed to evaluate generative models in terms of low-level statistics of their generated images with respect to natural scenes. They considered four statistics including 1) the mean power spectrum, 2) the number of connected components in a given image area, 3) the distribution of random filter responses, and 4) the contrast distribution. Their results show that although generated images by DCGAN [64], WGAN [2], and VAE [43] resemble natural scenes in terms of low level statistics, there are still significant differences. For example, generated images do not have scale invariant mean power spectrum magnitude, which indicates existence of extra structures in these images caused by deconvolution operations.

Karras et al. [41] investigated the multi scale statistical similarities between distributions of local image patches drawn from the Laplacian pyramid [8] representations of generated and real images. They used the Wasserstein distance to compare the distributions of patches. The multi-scale pyramid allows a detailed comparison of statistics. The distance between the patch sets extracted from the lowest resolution indicates similarity in large-scale image structures, while the finest-level patches encode information about pixel-level attributes such as sharpness of edges and noise.

19. **Evaluating Mode Drop and Mode Collapse.** Mode collapse, a.k.a the Helvetica scenario, is the phenomenon when the generator learns to map several different input $z$ vectors to the same output (possibly due to low model capacity or inadequate optimization [3]). It causes lack of diversity in the generated samples as the generator assigns low probability mass to significant subsets of the data distribution’s support. Mode drop occurs when some hard-to-represent modes of $P_r$ are simply “ignored” by $P_g$. This is different than mode collapse where several modes of $P_r$ are “averaged” by $P_g$ into a single mode, possibly located at a midpoint. An ideal GAN evaluation metric should be sensitive to these two phenomena.

Detecting mode collapse in GANs trained on large scale image datasets is very challenging. However, it can be accurately measured on synthetic datasets where the true distribution and its modes are known (e.g., Gaussian mixtures). Srivastava et al. [75] proposed a metric to quantify mode collapse behavior as follows:

(a) First, some points are sampled from the generator. A sample is counted as high quality, if it is within a certain distance of its nearest mode center (e.g., $3\sigma$ over a 2D dataset, or $10\sigma$ over a 1200D dataset).

(b) Then, the number of modes captured is the number of mixture components whose mean is nearest to at least one high quality sample. Accordingly, a mode is considered lost if there is no sample in the generated test data within a certain standard deviations from the center of that mode. This is illustrated in Fig. 6.

The reverse KL divergence over the modes has been used in [50] to measure the quality of mode collapse as follows. Each generated sample is assigned to its closest mode. This induces an empirical, discrete distribution with an alphabet size equal to the number of observed modes in the generated samples. A similar induced discrete distribution is computed from the real data samples. The reverse KL divergence between the induced distribution from generated samples and the induced distribution from the real samples is used as a metric.
The shortcoming of the described metrics is that they only work for datasets with known modes (e.g., synthetic or labeled datasets). Overall, it is hard to quantitatively measure model collapse and mode drop since they are poorly understood. Active research is ongoing in this direction.

2.2 Qualitative Measures

Visual examination of samples is one of the common and most intuitive ways to evaluate GANs. However, evaluating the quality of generated images with human vision is time-consuming, difficult to reproduce, and does not fully reflect the capacity of models. Further, an evaluation based on samples could be biased towards models that overfit and therefore a poor indicator of a good density model in a log-likelihood sense [78] (e.g., fails to tell whether a model drops modes). In fact, the mode dropping problem generally helps visual sample quality as the model can choose to focus on only few common modes. This is because common modes usually correspond to typical samples.

In what follows, I discuss four ways that have been followed in the literature to qualitatively inspect the quality of generated images.

1. Nearest Neighbors. To detect overfitting, traditionally some samples are shown next to their nearest neighbors in the training set (e.g., Fig. 7). There are, however, two concerns regarding this way of evaluation:

   (a) Nearest neighbors are typically determined based on the Euclidean distance which is very sensitive to minor perceptual perturbations. This is a well known phenomenon in the psychophysics literature (See Wang and Bovik [88]). It is trivial to generate samples that are visually almost identical to a training image, but have large Euclidean distances with it [78]. See Fig. 8 for some examples.

   (b) A model that stores transformed training images can trivially pass the nearest-neighbor overfitting test [78]. This problem can be alleviated by choosing nearest neighbors based on perceptual metrics, and by showing more than one nearest neighbor.

2. Rapid Scene Categorization. These measures are inspired by prior studies who have shown that humans are capable of reporting certain characteristics of scenes in a short glance (e.g., scene category, visual layout [63, 70]). To obtain a quantitative measure of quality of samples, Denton et al. [16] asked...
Figure 7: Generated samples nearest to real images from CIFAR-10. In each of the two panels, the first column shows real images, followed by the nearest image generated by DCGAN [64], ALI [20], Unrolled GAN [59], and VEEGAN [75], respectively. Figure from [75].

subjects to distinguish their generated samples from real images. The subjects were presented with the user interface shown in Fig. 9(right) and were asked to click the appropriate button to indicate if they believed the image was real or generated. They varied the viewing time from 50ms to 2000ms (11 durations). Fig. 9(left) shows the results over samples generated by three GAN models. They concluded that their model was better than the original GAN [31] since it did better in fooling the subjects (lower bound here is 0% and upper bound is 100%). See also Fig. 10 for another example of fake vs. real experiment but without time constraints (conducted by Salimans et al. [69]).

This “Turing-like” test is very intuitive and seems inevitable to ultimately answer the question of whether generative models are as good as the nature in generating images. However, there are several concerns in conducting such a test in practice (especially when dealing with models that are far from perfect; See Fig. 9(left)). Aside from experimental conditions which are hard to control in crowd-sourced platforms (e.g., presentation time, screen size, subject’s distance to the screen, subjects’ motivations, age, mood, feedback, etc) and high cost, these tests fall short in evaluating models in terms of diversity of generated samples and may be biased towards models that overfit to training data.

3. Rating and Preference Judgment. These types of experiments ask subjects to rate models in terms of the fidelity of their generated images. For example, Snell et al., [73] studied whether observers prefer reconstructions produced by perceptually-optimized networks or by the pixelwise-loss optimized networks. Participants were shown image triplets with the original (reference) image in the center and the SSIM- and MSE-optimized reconstructions on either side with the locations counterbalanced. Participants were instructed to select which of the two reconstructed images they preferred (See Fig. 11). Similar approaches have been followed in [36, 99, 94, 96, 100, 82, 18, 52, 56]. Often the first few trials in these experiments are spared for practice.

4. Investigating and Visualizing the Internals of Networks. One way of evaluating generative
models is to understand how they learn, what they learn and explore their internal dynamics. While this is a broad topic and many papers fall under it, here I bring few examples to give the reader some insights.

(a) **Space continuity.** The goal here it to understand the landscape of a model’s learned latent space. For example, given two random seed vectors $z_1$ and $z_2$ that generated two realistic images, we can check the images produced using seeds lying on the line joining $z_1$ and $z_2$. If such “interpolated” images are reasonable and visually appealing, then this may be taken as a sign that a model can produce novel images rather than simply memorizing them (e.g., [6]; See Fig. 12).

Radford et al. [64] investigated their trained generators and discriminators in a variety of ways. They proposed that walking on the learned manifold can tell us about signs of memorization (if there are sharp transitions) and about the way in which the space is hierarchically collapsed. If walking in this latent space results in semantic changes to the image generations (such as objects being added and removed), one can reason that the model has learned relevant and interesting representations. They also showed interesting results of performing vector arithmetic on the $z$ vectors of sets of exemplar samples for visual concepts (e.g., smiling woman - neutral woman + neutral man = smiling man; using $z$’s averaged over several samples).

Some other tests can check the existence of semantically meaningful directions in the latent space, meaning that varying the seed along those directions leads to predictable changes (e.g., changes in facial hair, or pose). Some works (e.g., [12, 35, 58, 51]) assess the quality of internal representations by checking whether they satisfy certain properties, such as being “disentangled”. A measure of disentanglement proposed in [35] checks whether the latent space captures the true factors of variation in a simulated dataset where parameters are known by construction (e.g., using a graphics engine). Some other examples include [18, 61]. White [91] suggests that replacing linear interpolation with spherical linear interpolation prevents diverging from a model’s prior distribution and produces sharper samples. Vedantam et al. [84] studied “visually grounded semantic imagination” and proposed several ways to evaluate their models in terms of the quality of the learned semantic latent space.

(b) **Visualizing the discriminator features.** Motivated by previous studies on investigating the representations and features learned by convolutional neural networks trained for scene classification (e.g., [97, 5, 103]), some works have attempted to visualize the internal parts of generators and
discriminators in GANs. For example, Radford et al. [64] showed that DCGAN trained on a large image dataset can also learn a hierarchy of interesting features. Using guided backpropagation [74] they showed that the features learned by the discriminator fire on typical parts of a bedroom, such as beds and windows (See Fig. 5 in [64]).

3 Discussion and Conclusion

In addition to measures discussed above, there exist some other non-trivial or task-specific ways to evaluate GANs. Vedantam et al. [84] proposed a model for visually grounded imagination to create images of novel semantic concepts. To evaluate the quality of the generated images, they proposed three measures including a) correctness: fraction of attributes for each generated image that match those specified in the concept’s description, b) coverage: diversity of values for the unspecified or missing attributes, measured as the difference between the empirical distributions of attribute values in generated set and the true distribution, and c) compositionality: correctness of generated images in response to test concepts that differ in at least one attribute from the training concepts. To measure diversity of generated samples, Zhu et al. [105] randomly sampled from their model and computed the average pair-wise distance in a deep feature space using cosine distance and compared it with the same measure calculated from ground truth real images. This is akin to the image retrieval performance described above. Im et al. [14] proposed to evaluate GANs by exploring the divergence and distance metrics that were used during training GANs. They showed that rankings produced by four metrics including 1) Jensen-Shannon Divergence, 2) Constrained Pearson $\chi^2$, 3) Maximum Mean Discrepancy, and 4) Wasserstein Distance, are consistent and robust across metrics. A new measure known as the Normalized Relative Discriminative Score has also been introduced recently by Zhang et al. [101].

What is the best GAN evaluation metric? There is no trivial answer to this question and there seems not to be a unique one, although some measures seem more plausible than others. Detailed analyses by Theis et al. [78] showed that average likelihood is not a good measure. Parzen windows estimation of likelihood favors trivial models and is irrelevant to visual fidelity of samples. Further, it fails to approximate the true likelihood in high dimensional spaces or to rank models. Quality measures such as nearest neighbor visualizations or rapid categorization tasks may favor models that overfit. Overall, it seems that the main
challenge is to have a metric that evaluates both diversity and visual fidelity simultaneously. The former implies that all modes are covered while the latter implies that the generated samples should have high likelihood. Perhaps due to these challenges, Theis et al. [78] argued against evaluating models for task-independent image generation and proposed to evaluate GANs with respect to specific applications. For different applications then, different measures might be more appropriate. For example, the likelihood is good for measuring compression methods [79] while psychophysics and user ratings are fit for evaluating image reconstruction and synthesis methods [45, 29]. Some measures are suitable for evaluating generic GANs (e.g., input is only a noise vector) while some others are suitable for evaluation of conditional GANs (e.g., FCN score) where correspondences are available (e.g., generating an image corresponding to a segmentation map).

Despite having different formulations, several scores are based on similar concepts. C2ST, adversarial accuracy, and classification performance employ classifiers to determine how separable generated images are from real images (on a validation dataset). FID, Wasserstein and MMD measure the distance between two distributions. Inception score and its variants including m-IS, Mode and AM scores use conditional and marginal distributions over generated data or real data to evaluate diversity and fidelity of samples. Average log likelihood and coverage metric estimate the probability distributions. Reconstruction error and some quality measures determine how dissimilar generated images are from their corresponding (or closest) images in the train set. Some measures use individual samples (e.g., IS) while others need pairs of samples (e.g.,
Figure 12: Top: Interpolations on $z_r$ between real images at 128 $\times$ 128 resolution (from BEGAN [6]). These images were not part of the training data. The first and last columns contain the real images to be represented and interpolated. The images immediately next to them are their corresponding approximations while the images in between are the results of linear interpolation in $z_r$. Middle: Latent space interpolations for three ImageNet classes. Left-most and right-columns show three pairs of image samples - each pair from a distinct class. Intermediate columns highlight linear interpolations in the latent space between these three pairs of images (From [62]). Bottom: Class-independent information contains global structure about the synthesized image. Each column is a distinct bird class while each row corresponds to a fixed latent code $z$ (From [62]).
Figure 13: Measurement of wall-clock time for computing various metrics as a function of the number of samples. As it shows, all metrics are practical to compute for a sample of size 2000, but Wasserstein distance does not scale to large sample sizes. Figure from [27].

MMD). One important concern regarding many measures is that they are sensitive to the choice of the feature space (e.g., different CNNs) as well as the distance type (e.g., L2 vs. L1).

Huang et al. [27] argue that a practical GAN evaluation metric should be computed using a reasonable number of samples and within an affordable computation cost. This is particularly important during monitoring the training process of models. They proposed the following ways to assess evaluation measures:

1. **Sample efficiency** regards the number of samples needed for a metric to discriminate a set of generated samples $S_g$ from a set of real samples $S_r'$. To do this, a reference set $S_r$ is uniformly sampled from the real training data (but disjoint with $S_r'$). All three sets have the same size (i.e., $|S_r| = |S_r'| = |S_g| = n$). An ideal metric $\rho$ is expected to correctly score $\hat{\rho}(S_r,S_r')$ lower than $\hat{\rho}(S_r, S_g)$ with a relatively small $n$. In other words, the number of samples $n$ needed for a metric to distinguish $S_r'$ and $S_g$ can be viewed as its sample complexity.

2. **Computational efficiency.** Fast computation of the empirical metric is of practical concern as it helps researchers monitor the training process and diagnose problems early on (e.g., for early stopping). This can be measured in terms of seconds per number of evaluated samples. See Fig. 13.

What are desirable properties of a good GAN evaluation measure? From what was discussed above, an efficient measure should:

1. be able to distinguish generated samples from real images,
2. be able to identify mode drop and mode collapse, and detect overfitting,
3. favor models that generate high fidelity samples,
4. favor models that generate diverse samples,
5. favor models with disentangled latent spaces,
6. have well defined bounds (lower, upper, and chance),
7. undermine trivial models such as the memory GAN,
8. be sensitive to image distortions and transformations (See Fig. 14),
9. agree with human perceptual judgments and human rankings of models, and
10. have low sample and computational complexity.

These criteria can serve as meta measures to evaluate and compare the GAN evaluations measures. In this work, I provided a critical review of the strengths and limitations of 19 quantitative and 4 qualitative measures that have been introduced so far for evaluating GANs. Seeking appropriate metrics for this purpose continues to be an important open problem, not only for fair model comparison but also for understanding, improving, and developing models. Lack of a universal powerful metric can hinder the progress. In a recent benchmark study, Lucic et al. [57] found no empirical evidence in favor of GAN models who claimed superiority over the original GAN. In this regard, borrowing from other fields such as natural scene statistics and cognitive vision can be rewarding. For example, understanding how humans perceive symmetry [19, 26] or image clutter [67] in generated images versus natural scenes can give clues regarding the plausibility of the generated images.

Ultimately, I suggest the following directions for future research in this area: 1) creating a code repository of evaluation measures, 2) conducting detailed comparative empirical and analytical studies of available metrics, and 3) benchmarking models using several metrics instead of only one or few.

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