Exploring the effect of Lorentz invariance violation in NOνA experiment

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Abstract

Neutrinos are the particles, remain invulnerable to any interactions except weak interaction. Hence, they can propagate large distances without any deviation. This characteristic property can thus provide an ideal platform to investigate Planck suppressed physics through their long distance propagation. In this work, we would like to investigate CPT violation through Lorentz invariance violation (LIV) in the long baseline accelerator based neutrino experiment NOνA. Considering the simplest four-dimensional Lorentz violating parameters, we show their effects on the neutrino oscillation parameters at probability level, on mass hierarchy and CP violation sensitivity of NOνA experiment. Also we calculate the bounds on different Lorentz violating parameters.
I. INTRODUCTION

Neutrinos are considered to be the most interesting particles in nature, possess many unique and interesting features in contrast to the other Standard Model (SM) fermions. The effort of many dedicated neutrino oscillation experiments \[1–7\] over the last two decades, provide us a splendid understanding about the main features of these tiny and elusive particles. Indeed we now know that neutrinos are massive albeit extremely light, and change their flavour as they propagate. This intriguing characteristic, known as neutrino oscillation, bestows the first experimental evidence of physics beyond the standard model. Without the loss of generality, it is believed that the SM is a low-energy effective theory, emanating from a fundamental unified picture of gravity and quantum physics at the Planck scale. To unravel the essence of Plank scale physics through experimental signatures are therefore of great interest, but are extremely challenging to identify. Lorentz symmetry violation constitutes one such signal, which basically associated with tiny deviation from relativity. In recent times, the search for Lorentz violating and related CPT violating signals have been explored over a wide range of systems and at remarkable sensitivities \[8–19\]. A phenomenological consequence of CPT invariance is that a particle and its anti-particle will have exactly the same mass and lifetime and if any difference observed either in their mass or lifetime, would be a clear hint for CPT violation. There exists stringent experimental bounds on Lorentz and CPT violating parameters from kaon and the lepton sectors. For the kaon system, the observed mass difference provides the upper limit on CPT violation as \[|m_{K^0} - m_{K^0}|/m_K < 6 \times 10^{-18}\] \[20\], which is quite stringent. However, parametrizing in terms of \(m_K^2\) rather that \(m_K\), as kaon is a boson and the natural mass parameter appears in the Lagrangian is the squared mass, the kaon constraint turns out to be \(|m_{K^0}^2 - m_{K^0}^2| < 0.25\text{ eV}^2\). This limit is comparable to the bounds obtained from neutrino sector, though relatively weak, hence, the neutrino system can be regarded as a better probe to search for CPT violation. In addition, neutrinos are fundamental particles, unlike the kaons and are therefore, expected to be more suitable for exploring CPT violation. For example, the current neutrino oscillation data provides the most stringent bound: \(|\Delta m^2_{21} - \Delta m^2_{21}| < 5.9 \times 10^{-5}\text{ eV}^2\) and \(|\Delta m^2_{31} - \Delta m^2_{31}| < 1.1 \times 10^{-3}\text{ eV}^2\) \[21\]. Recently, MINOS experiment \[22\] has also provided the bound on the atmospheric mass splitting for the neutrino and antineutrino modes as \(|\Delta m^2_{31} - \Delta m^2_{31}| < 0.8 \times 10^{-3}\text{ eV}^2\) at 3σ C.L.. These probable hints of new physics,
would affect the oscillation phenomena for neutrinos and antineutrinos as well as have other phenomenological consequences, such as neutrino-antineutrino oscillation, baryogenesis etc.

It is well known that the local relativistic quantum field theories are based on three main ingredients: CPT invariance, locality and hermiticity. The CPT violation is intimately related to Lorentz violation, as possible CPT violation can arise from Lorentz violation, non-locality, non-commutative geometry etc. So if CPT violation exists in nature and is related to quantum gravity, which is supposedly non-local and expected to be highly suppressed, long-baseline experiments have the capability to probe such effects. Here, we present a brief illustration about, how the violation of Lorentz symmetry can affect the neutrino propagation. In general, Lorentz symmetry breaking and quantum gravity are interrelated, which requires the existence of a universal length scale for all frames. However, such universal scale is in conflict with general relativity, as length contraction is one of the consequences of Lorentz transformation. Such contradiction can be avoided by the modification of Lorentz transformations (or in other words modifying dispersion relations). It has been shown explicitly in Ref. [24], how the oscillation probability gets affected by the modified dispersion relation, however, for the sake of completeness we will present a brief discussion about it.

The modified energy-momentum relation for the neutrinos can be expressed as

$$E_i^2 = p_i^2 + \frac{1}{2} m_i^2 \left( 1 + e^{2A_i E/m_i^2} \right),$$

(1)

where $m_i$, $E_i$ and $p_i$ are the mass, energy and momentum of the $i$-th neutrino, and $A_i$ is a dimensionful and Lorentz breaking parameter. Assuming that all the neutrinos have the same energy ($E$), the transition probability for two neutrino case is given as

$$P(\nu_\alpha \rightarrow \nu_\beta) = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta p L}{2} \right),$$

(2)

where $\theta$ represents the mixing angle and

$$\Delta p \approx \frac{\Delta m^2}{2E} + \frac{1}{2} (A_\alpha - A_\beta).$$

(3)

Hence, the neutrino oscillation experiments might provide the opportunity to investigate this type of physics. However, if the Lorentz violating parameters are generation independent, then the above oscillation probability reduces to the standard one and the Lorentz violation will show up only in the higher order terms. The possible effect Lorentz violation in neutrino oscillation phenomena has been intensely investigated in recent years [24–27].
In this paper, we are interested to study the phenomenological consequences introduced in the neutrino sector due to the presence of Lorentz violating corrections. Specifically, we focus in our analysis, how the oscillation probabilities of neutrino flavour transitions get modified due to such new contributions in NOνA experiment. We also investigate the implications of LIV effects on the determination of mass ordering as well as the CP violation discovery potential.

The outline of the remainder of the paper is as follows. In section II, we present the theoretical framework for incorporating LIV effects and their implications on neutrino oscillation physics. The simulation details used in this analysis are briefly discussed in section III. The impact of LIV parameters on the $\nu_\mu \rightarrow \nu_e$ oscillation probability is presented in Section IV. Section V contains the discussion on the bounds on LIV parameters, which can be extracted from NOνA experiment. The discussion on how the discovery potential for CP violation and the mass hierarchy sensitivity get affected due to the presence of LIV parameters, the correlation between LIV parameters and $\delta_{CP}$ as well as $\theta_{23}$ are illustrated in Section VI. Finally we present our Conclusion in section VII.

II. THEORETICAL FRAMEWORK

The Lorentz invariance violation effect can be introduced as a small perturbation to the standard physics descriptions of neutrino oscillations. Thus, the effective Lagrangian that describes Lorentz violating neutrinos and anti-neutrinos [8] is given as

$$\mathcal{L} = \frac{1}{2} \bar{\Psi}_A (i \gamma^\mu \partial_\mu \delta_{AB} - M_{AB} + \hat{Q}_{AB}) \Psi_B + \text{h.c.} ,$$

where $\Psi_A$ is a $2N$ dimensional spinor containing the spinor field $\psi_\alpha$ with $\alpha$ ranges from $N$ spinor flavours and their charge conjugates $\psi_\alpha^C = C\bar{\psi}_\alpha^T$, expressed as $\Psi_A = (\psi_\alpha, \psi_\alpha^C)^T$ and the Lorentz violating operator is characterized by $\hat{Q}$. Restricting ourselves to only a renormalizable theory (incorporating terms with mass dimension $\leq 4$), one can symbolically write the Lagrangian density as [28]

$$\mathcal{L}_{\text{LIV}} = -\frac{1}{2} \left[ p_{\alpha\beta}^\mu \bar{\psi}^\gamma_\alpha \gamma_\mu \psi_\beta + q_{\alpha\beta}^\mu \bar{\psi}^\gamma_\alpha \gamma_\mu \psi_\beta - ir_{\alpha\beta}^{\mu\nu} \bar{\psi}^\gamma_\alpha \gamma_\mu \partial_\nu \psi_\beta - i s_{\alpha\beta}^{\mu\nu} \bar{\psi}^\gamma_\alpha \gamma_\mu \partial_\nu \psi_\beta \right] + \text{h.c.} .$$

Since, only left-handed neutrinos are present in the SM, the observable effects which can be explored in the neutrino oscillation experiments can be parametrized as

$$(a_L)_{\alpha\beta} = (p + q)_{\alpha\beta}^\mu , \quad (c_L)_{\alpha\beta}^{\mu\nu} = (r + s)_{\alpha\beta}^{\mu\nu} .$$
These parameters are hermitian matrices in the flavour space and can affect the standard vacuum Hamiltonian. The parameter \((a_L)_{\alpha\beta}^\mu\) is related to CPT violating neutrinos and \((c_L)_{\alpha\beta}^{\mu\nu}\) is associated with CPT-even, Lorentz violating neutrinos. Considering only isotropic terms of Lorentz violation parameters, i.e., \(\mu = \nu = 0\), the Hamiltonian for neutrinos, including Lorentz violating contributions becomes

\[
H = H_{\text{vac}} + H_{\text{mat}} + H_{\text{LIV}},
\]

where \(H_{\text{vac}}\) and \(H_{\text{mat}}\) correspond to the Hamiltonians in vacuum and in the presence of matter effects and \(H_{\text{LIV}}\) refers to the LIV Hamiltonian, which can be expressed as

\[
H_{\text{LIV}} = \begin{pmatrix}
    a_{ee} & a_{e\mu} & a_{e\tau} \\
    a_{e\mu}^* & a_{\mu\mu} & a_{\mu\tau} \\
    a_{e\tau}^* & a_{\mu\tau}^* & a_{\tau\tau}^*
\end{pmatrix} - \frac{4}{3} E \begin{pmatrix}
    c_{ee} & c_{e\mu} & c_{e\tau} \\
    c_{e\mu}^* & c_{\mu\mu} & c_{\mu\tau} \\
    c_{e\tau}^* & c_{\mu\tau}^* & c_{\tau\tau}^*
\end{pmatrix}.
\]

(8)

Since the mass dimensions of \(a_{\alpha\beta}\) and \(c_{\alpha\beta}\) LIV parameters are different, the effect of \(a_{\alpha\beta}\) is proportional to the baseline \(L\), whereas \(c_{\alpha\beta}\) is proportional to \(LE\) and in this work we focus only on the impact of \(a_{\alpha\beta}\) parameters on the physics potential of NO\(\nu\)A experiment.

It should be noted that, the Hamiltonian in the presence of LIV \([7]\), is analogous to that in the presence of NSI in propagation, which is expressed as \([29]\)

\[
H = H_{\text{vac}} + H_{\text{mat}} + H_{\text{NSI}},
\]

with

\[
H_{\text{NSI}} = V_{CC} \begin{pmatrix}
    \epsilon_{ee}^m & \epsilon_{e\mu}^m & \epsilon_{e\tau}^m \\
    \epsilon_{e\mu}^m & \epsilon_{\mu\mu}^m & \epsilon_{\mu\tau}^m \\
    \epsilon_{e\tau}^m & \epsilon_{\mu\tau}^m & \epsilon_{\tau\tau}^m
\end{pmatrix},
\]

(10)

where \(V_{CC} = \sqrt{2}G_F N_e\), \(N_e\) represents the electron number density and \(\epsilon_{\alpha\beta}^m\) characterizes the relative strength between the matter effect due to NSI and the standard scenario. Thus, one obtains a correlation between the NSI and CPT violating scenarios through

\[
a_{\alpha\beta} = \sqrt{2}G_F N_e \epsilon_{\alpha\beta}^m \equiv V_{CC} \epsilon_{\alpha\beta}^m.
\]

(11)

The off-diagonal elements of the LIV Hamiltonian \((a_{e\mu}, a_{e\tau}\) and \(a_{\mu\tau}\)) are the lepton flavor violating LIV parameters, which can affect the neutrino flavour transition, are our subject of interest. These parameters are expected to be highly suppressed and the current limits on their values (in GeV), which are constrained by Super-Kamikande Collaboration \([30]\), as

\[
|a_{e\mu}| < 2.5 \times 10^{-23}, \quad |a_{e\tau}| < 5 \times 10^{-23}, \quad |a_{\mu\tau}| < 8.3 \times 10^{-24}.
\]

(12)
III. SIMULATION DETAILS

In this section, we briefly describe the experimental features of NOνA experiment. NOνA is a currently running long-baseline accelerator experiment, with two totally active scintillation detectors, Near Detector (ND) and Far Detector (FD). ND is placed at around 1 km and FD is at a distance of 810 km away from source and both the detectors are off-axial by 14.6 mrad in nature, which provides a large flux of neutrinos at an energy of 2 GeV, the energy at which oscillation from $\nu_\mu$ to $\nu_e$ is expected to be at a maximum. It uses very high intensity $\nu_\mu$ beam, coming from NuMI beam of Fermilab, with beam power 0.7 MW and 120 GeV proton energy corresponding to $6 \times 10^{20}$ POT per year. This $\nu_\mu$ beam is detected by the ND of mass 280 ton at Fermilab and the oscillated neutrino beam is observed by 14 kton far detector located near Ash River. We assume 45% (100%) signal efficiencies for both electron (muon) neutrino and anti-neutrino signals. The background efficiencies for mis-identified muons (anti-muons) at the detector are 0.83% (0.22%). The neutral current background efficiency for muon neutrino (antineutrino) is 2% (3%). The background contribution coming from the existence of electron neutrino (antineutrino) in the beam, so called intrinsic beam contamination is about 26% (18%). Apart from these, we assume that 5% uncertainty on signal normalization and 10% on background normalization. The auxiliary files and experimental specification of NOνA experiment that we use for the analysis is taken from [31]. We use GLoBES software package along with snu plugin [32, 33] to simulate the experiment. The implementation of LIV in neutrino oscillation scenario has been done by modifying the snu.c according to the Lorentz violating Hamiltonian [7]. We use the values of standard three flavor oscillation parameters as shown in the Table I and consider one LIV parameter at a time while setting all other parameters to zero unless otherwise mentioned. The values of the LIV parameters considered in our analysis are:

| $a_{e\mu}$ | $|a_{\mu\tau}|$ | $|a_{e\tau}|$ | $= 2 \times 10^{-23}$ GeV and $|a_{ee}| = |a_{\mu\mu}| = |a_{\tau\tau}| = 1 \times 10^{-22}$ GeV. |

IV. EFFECT OF LIV PARAMETERS ON $\nu_\mu \rightarrow \nu_e$ AND $\nu_\mu \rightarrow \nu_\mu$ OSCILLATION CHANNELS

In this section, we discuss the effect of LIV parameters $a_{\alpha\beta} = |a_{\alpha\beta}| e^{i\phi_{\alpha\beta}}$, on $\nu_\mu \rightarrow \nu_e$ oscillation channel, as the long-baseline experiments are mainly looking at this oscillation
| Parameter | True values | $3\sigma$ Range |
|-----------|-------------|-----------------|
| $\sin^2 \theta_{12}$ | 0.310 | NA |
| $\sin^2 \theta_{13}$ | 0.02240 | NA |
| $\sin^2 \theta_{23}$ | 0.5 | 0.4 → 0.6 |
| $\delta_{CP}$ | $-\pi/2$ | $[-\pi, \pi]$ |
| $\Delta m^2_{12}$ | $7.39 \times 10^{-5}$eV$^2$ | NA |
| $\Delta m^2_{31}$ | $2.5 \times 10^{-3}$eV$^2$ | $(2.36 \rightarrow 2.64) \times 10^{-3}$eV$^2$ |

TABLE I: The values of oscillation parameters that we consider in our analysis \[34\].

channel. Neglecting higher order terms, the oscillation probability for $\nu_\mu \rightarrow \nu_e$ channel in the presence of LIV for NH can be expressed, which is analogous to the NSI case \[35, 36\], as

$$
P_{\mu e}^{LIV} \approx x^2 f^2 + 2 x y f g \cos(\Delta + \delta_{CP}) + y^2 g^2 + 4 r_A a_{e\mu} \left\{ x f \left[ f s^2_{23} \cos(\phi_{e\mu} + \delta_{CP}) \right. \right.$$

$$+ g c^2_{12} \cos(\Delta + \phi_{e\mu}) \left. + g f^2 s^2_{23} \cos(\Delta - \phi_{e\mu}) \right\} + 4 r_A s_{23} c_{23} \left\{ x f \left[ f \cos(\phi_{e\tau} + \delta_{CP}) - g \cos(\Delta + \phi_{e\tau}) \right] \right.$$

$$- y g \left[ g \cos(\Delta - \phi_{e\tau}) \right] + 4 r_A^2 g^2 c^2_{23} c_{23} a_{e\mu} - s_{23} a_{e\tau}^2 \right\} + 4 r_A f^2 s^2_{23} s_{23} a_{e\mu} + c_{23} a_{e\tau}^2 + 8 r_A f g s_{23} c_{23} \left\{ c_{23} \cos(\phi_{e\mu} - \phi_{e\tau}) \right.$$

$$+ 2 c_{23} a_{e\mu} a_{e\tau} \cos(\phi_{e\mu} - \phi_{e\tau}) - a_{e\mu} a_{e\tau} \cos(\Delta - \phi_{e\mu} + \phi_{e\tau}) \right\} ,$$

where

$$x = 2 s_{13} s_{23} , \quad y = 2 r s_{12} c_{12} c_{23} , \quad r = |\Delta m^2_{21}/\Delta m^2_{31}| , \quad \Delta = \frac{\Delta m^2_{31} L}{4 E}$$

$$f = \frac{\sin(\Delta - r_A(V_{CC} + a_{ee}))}{1 - r_A(V_{CC} + a_{ee})} , \quad g = \frac{\sin(\Delta r_A(V_{CC} + a_{ee}))}{r_A(V_{CC} + a_{ee})} , \quad r_A = \frac{2 E}{\Delta m^2_{31}} ,$$

and $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$. The antineutrino probability $P_{\bar{\mu} \bar{e}}^{LIV}$ can be obtained from (13) by replacing $V_{CC} \rightarrow -V_{CC}$, $\delta_{CP} \rightarrow -\delta_{CP}$ and $\phi_{\alpha\beta} \rightarrow -\phi_{\alpha\beta}$. Similar expression for inverse hierarchy can be obtained by substituting $\Delta m^2_{31} \rightarrow -\Delta m^2_{31}$, i.e., $\Delta \rightarrow -\Delta$ and $r_A \rightarrow -r_A$.

One can notice from Eqn. (13) that only the LIV parameters $a_{ee}$, $a_{e\mu}$ and $a_{e\tau}$ contribute to appearance probability expression at leading order and the rest of the parameters appear only on sub-leading terms.
The expression for disappearance probability $\nu_\mu \rightarrow \nu_\mu$ up to $O(r, s, a_{2, \alpha \beta})$ is

$$P_{\nu_\mu}^{LIV} \simeq 1 - \sin^2 2\theta_{23} \sin^2 \Delta$$

$$- |a_{\mu\tau}| \cos \phi_{\mu\tau} \sin 2\theta_{23} \left[ (2r_A \Delta) \sin^2 \theta_{23} \sin 2\Delta + 4 \cos^2 2\theta_{23} r_A \sin^2 \Delta \right]$$

$$+ (|a_{\mu\mu}| - |a_{\tau\tau}|) \sin^2 2\theta_{23} \cos 2\theta_{23} \left[ (r_A \Delta) \sin 2\Delta - 2r_A \sin^2 \Delta \right]. \quad (15)$$

It is important to observe from the survival probability expression (15) that, LIV parameters involved in $\nu_\mu \rightarrow \nu_e$ transitions do not take part in $\nu_\mu \rightarrow \nu_\mu$ channel. This probability depends only on the new parameters $a_{\mu\mu}, a_{\mu\tau}$ and $a_{\tau\tau}$.

The effect of LIV parameters on $\nu_\mu \rightarrow \nu_e$ channel is displayed in Fig. 1. The left panel of the figure shows how the oscillation probability gets modified in presence of LIV, whereas the absolute difference of standard case from Lorentz violating case is shown in the right panel of the figure. In each plot, the black line corresponds to oscillation probability in the standard three flavor oscillation paradigm and red (blue) dotted line corresponds to the oscillation probability in presence of LIV parameters with positive (negative) value. From Fig. 1, it is clear that $a_{e\mu}$ and $a_{e\tau}$ parameters have significant impact on the oscillation probability. It should be noted from the figure that positive and negative values for LIV parameter $a_{e\tau}$ shift the probabilities in opposite direction, while that of $a_{e\mu}$ create a distortion on the probability. Further, it should also be inferred from the left panel of the figure that the positive and negative values of LIV parameters affect the oscillation probabilities differently. However, the result is qualitatively independent of the actual sign of LIV parameters, hence, one can take the $|a_{\alpha\beta}|$ for sensitivity study of the experiment in presence of LIV parameters.

In Fig. 2, the effect of LIV parameters $a_{\mu\mu}, a_{\mu\tau},$ and $a_{\tau\tau}$ on $\nu_\mu$ disappearance probability is displayed. Analogous to the previous case, here also the effects of the parameters are noticeable, the parameter $a_{\mu\tau}$ significantly modify the probability, whereas the changes due to $a_{\mu\mu}$ and $a_{\tau\tau}$ are negligibly small. In all cases, the sign the LIV parameters are responsible for the decrease or increase of the oscillation probability values.

V. BOUNDS ON THE LIV PARAMETERS

In this section, we analyse the potential of the NO$\nu$A experiment to constrain the LIV parameters. From Eqns.13 and 15, it can be seen that the LIV parameters $a_{e\mu}$ and $a_{e\tau}$ play major role in appearance channel, whereas $a_{\mu\tau}$ influences the disappearance channel. In order
FIG. 1: The $\nu_e$ appearance oscillation probabilities as a function of neutrino energy in presence of Lorentz violating parameters $a_{e\mu}$ and $a_{e\tau}$ in the left panel. Whereas the difference in the oscillation probability with and without LIV is in the right panel.

to see their sensitivities at probability level, we define two quantities, $\Delta P_{\mu e} = \frac{|P_{LIV}^{\mu e} - P_{SM}^{\mu e}|}{P_{SM}^{\mu e}}$ and $\Delta P_{\mu\mu} = \frac{|P_{LIV}^{\mu\mu} - P_{SM}^{\mu\mu}|}{P_{SM}^{\mu\mu}}$, and evaluate their values for various LIV parameters, which are shown in $|a_{\alpha\beta}| - \phi_{\alpha\beta}$ plane in Fig. 3. From the left panel of the figure, one can see that the observable $\Delta P_{\mu e}$ has maximum value for $\phi_{e\mu} \approx 45^\circ$, if $a_{e\mu}$ is positive, whereas for negative value of
\[ P(\nu_\mu \rightarrow \nu_\mu) \]

FIG. 2: The \( \nu_\mu \) disappearance probability as a function of neutrino energy in presence of \( a_{\mu\mu}, a_{\mu\tau}, \) and \( a_{\tau\tau} \) LIV parameters.

\( a_{e\mu}, \Delta P_{\mu e} \) is maximum for \( \phi_{e\mu} \approx -135^\circ \). This nature of \( \Delta P_{\mu e} \) can be easily understood from Eqn.13, as the appearance probability depends on sine and cosine functions of \( \phi_{e\mu} \). However, the nature of \( \Delta P_{\mu e} \) for \( e\tau \) sector is quite different from that of \( e\mu \) sector, even though the appearance probability depends upon sine and cosine functions of \( \phi_{e\tau} \). This is due to the opposite sign on \( a_{e\mu} \) and \( a_{e\tau} \) dependent terms in oscillation probability. As the
LIV parameter $a_{\mu\tau}$ mainly appears on disappearance channel, we calculate $\Delta P_{\mu\mu}$ which has cosine dependence on $\phi_{\mu\tau}$ and display it in the right panel of the figure.

Next, we analyse the potential of NO\textnu\textalpha\, experiment to constrain the various LIV parameters, which are shown in Fig. 4. In order to obtain these values, we compare the true
event spectra which are generated in the standard three flavor oscillation paradigm with the test event spectra which are simulated by including LIV parameters and we show the marginalized sensitivities as a function of allowed values of amplitude of LIV parameters, i.e., $|a_{\alpha\beta}|$. The values of $\chi^2$ are evaluated using the standard rules as described in GLoBES and the details are presented in the Appendix. From the figure, we can see that the bounds on each LIV parameters (in GeV) at 2$\sigma$ C.L. are:

$$
|a_{e\mu}| < 4.6 \times 10^{-23}, \quad |a_{e\tau}| < 1.71 \times 10^{-22}, \quad |a_{\mu\tau}| < 9.35 \times 10^{-23},
$$

$$
-5.9 \times 10^{-22} < a_{ee} < 3.8 \times 10^{-22}, \quad -1.1 \times 10^{-22} < a_{\mu\mu} < 1.2 \times 10^{-22},
$$

$$
-1.2 \times 10^{-22} < a_{\tau\tau} < 9.6 \times 10^{-23},
$$

which are slightly weaker than the bounds obtained from Super-Kamiokande Collaboration.

VI. EFFECT OF LIV ON VARIOUS SENSITIVITIES OF NO$\nu$A

In this section, we discuss the effect of LIV on the sensitivities of NO$\nu$A experiment to determine neutrino mass ordering and CP-violation. In addition to this, we also present the correlations between the LIV parameters and the standard oscillation parameters $\theta_{23}$ and $\delta_{CP}$.

A. CP violation discovery potential

It is well known that the determination of the CP violating phase $\delta_{CP}$ is one of the most challenging issues in neutrino physics today. CP violation in the leptonic sector may provide the key ingredient to explain the observed baryon asymmetry of the Universe through leptogenesis. In this section, we discuss how the CP violation sensitivity of NO$\nu$A experiment gets affected due to impact of LIV parameters. Fig. 5 shows the significance with which CP violation, i.e. $\delta_{CP} \neq 0, \pm \pi$ can be determined for different true values of $\delta_{CP}$. The expression for the test statistics $\chi^2$, which quantifies the CP violation sensitivity is provided in the Appendix. We consider here the true hierarchy as normal, true parameters as given in Table II and vary the true value for $\delta_{CP}$ in the allowed range $[-\pi, \pi]$. Also the possibility of exclusion of CP conserving phases has been shown taking the test spectrum $\delta_{CP}$ value
as 0, ±π. This exclusion sensitivity is obtained by calculating the minimum $\chi^2_{\text{min}}$ after doing marginalization over both hierarchies, NH and IH in their 3σ ranges for $\Delta m^2_{31}$ and for $\sin^2 \theta_{23}$ in its 3σ range. The CPV sensitivity for standard case and in presence of diagonal LIV parameters is shown in the top left panel of Fig. 5. The black line depicts the standard case, and for diagonal elements $a_{ee}$, $a_{\mu\mu}$ and $a_{\tau\tau}$ the corresponding plots are displayed by blue, green and red respectively. Further, we show the sensitivity in presence of non-diagonal LIV parameters in $e\mu$, $e\tau$, and $\mu\tau$ sectors respectively in the top right, bottom left, and bottom right panels of the same figure. As the extra phases of the non-diagonal parameters can affect the CPV sensitivity, we calculate the value of $\chi^2_{\text{min}}$ for a particular value of $\delta_{CP}$ by varying the phase $\phi_{\alpha\beta}$ in its allowed range $[-\pi, \pi]$, which results in a band structure. It can be seen from figure that LIV can significantly affect the CPV discovery potential of the NOνA experiment. All the three non-diagonal LIV parameters have significant impact on CPV sensitivity. It can be seen from the figure that CPV sensitivity spans on both sides of standard case in presence of non-diagonal LIV parameters. Although there is a
FIG. 6: The oscillation probability for NOνA experiment in presence of non-diagonal LIV parameters $a_{\alpha\beta}$.

possibility that the sensitivity can be deteriorated in presence of LIV for some particular true value of the phase of the non-diagonal parameter ($\phi_{\alpha\beta}$), for most of the case the CP violation sensitivity is significantly get enhanced. Moreover, one can expect some sensitivity where there is less or no such significance for $\delta_{CP}$ regions in standard case. Further, the parameters $a_{e\mu}$ and $a_{e\tau}$ have significantly large effect on the sensitivity compared to $a_{\mu\tau}$. Similar observation can also be found by considering inverted hierarchy.

B. MH Sensitivity

Mass hierarchy determination is one of the main objectives of the long baseline experiments. It is determined by considering true hierarchy as NH (IH) and comparing it with the test hierarchy, assumed to be opposite to the true case, i.e., IH (NH). Fig. 6 shows the effect of LIV parameter on MH sensitivity at oscillation probability level. We obtain the bands by varying the $\delta_{CP}$ within its allowed range $[-\pi, \pi]$ and considering the other parameters as given in the Table III The red (green) band in the figure is for NH (IH) case with standard matter effect. There are some overlapped region between the two bands for some values of $\delta_{CP}$, where determination of neutrino mass ordering is difficult. From the figure, it can be seen that the parameter $a_{e\mu}$ has significant effect on the appearance probability energy spectrum compared to other two parameters. The two bands NH and IH shifted to higher values of probability and have more overlapped regions in presence of $a_{e\mu}$. Whereas the effects of $a_{e\tau}$ and $a_{\mu\tau}$ are negligible.
We calculate the $\chi^2$ by comparing true event and test event spectra which are generated for the oscillation parameters in the Table I for each true value of $\delta_{CP}$. In order to get the minimum deviation or $\chi^2_{\text{min}}$, we do marginalization over $\delta_{CP}$, $\theta_{23}$ and $\Delta m^2_{31}$ in their allowed regions. In Fig. 7 we show the mass hierarchy sensitivity of NO$\nu$A experiment for standard paradigm and in presence of diagonal LIV parameter case. The left (right) panel of the figure corresponds to the sensitivity for mass hierarchy as true hierarchy NH (IH). It can be seen from the figure that for standard matter effect case (black line), the test hierarchy can be ruled out in upper half plane (UHP) ($0 < \delta_{CP} < \pi$) and lower half plane (LHP) ($-\pi < \delta_{CP} < 0$) for true NH and IH respectively above $2\sigma$ C.L.. The other half plane is unfavourable for mass hierarchy determination. The parameter $a_{ee}$ is found to give significant enhancement from the standard case compared to $a_{\mu\mu}$.

It should also be emphasized that mass hierarchy can be measured precisely above $3\sigma$ C.L. for most of the $\delta_{CP}$ region in presence of $a_{ee}$ for true value in both NH and IH.

![Mass hierarchy sensitivity as function of $\delta_{CP}$ for NO$\nu$A experiment.](image)

**FIG. 7:** Mass hierarchy sensitivity as function of $\delta_{CP}$ for NO$\nu$A experiment. Left (right) panel is for NH (IH) as true value. Black line represents the standard matter effect case without any LIV parameter. Red and blue dotted lines represent the sensitivity in the presence of diagonal parameters $a_{ee}$, and $a_{\mu\mu}$ respectively.

The MH sensitivity in presence non-diagonal Lorentz violating parameters $a_{\alpha\beta}$ is shown in Fig. 8. As the non-diagonal LIV parameters introduce new phases, we do marginalization over new phases in their allowed range, i.e., $[-\pi, \pi]$ while obtaining the MH sensitivity. In all the three cases, the MH sensitivity expands around the MH sensitivity in the standard three
flavor framework. From the figure, it can be seen that the non-diagonal LIV parameters significantly affect the sensitivity which crucially depends on the value of new phase. Similar analysis can be studied considering IH as the true hierarchy.

FIG. 8: Mass hierarchy sensitivity as a function of $\delta_{CP}$ for NO$\nu$A experiment in presence of $a_{\alpha\beta}$. Black line represents the standard matter effect case without any LIV parameter. Left, middle and right panels represent the sensitivity in presence of non-diagonal parameters $a_{e\mu}$, $a_{e\tau}$ and $a_{\mu\tau}$ respectively.

C. Correlations between LIV parameters with $\delta_{CP}$ and $\theta_{23}$

In this section, we show the correlation between the LIV parameters and the standard oscillation parameters $\theta_{23}$ and $\delta_{CP}$ in $a_{\alpha\beta} - \theta_{23}$ and $a_{\alpha\beta} - \delta_{CP}$ planes. Fig. 9 [10] shows the correlation for $a_{ee}$, $a_{\mu\mu}$, $a_{\tau\tau}$, $|a_{e\mu}|$, $|a_{e\tau}|$, $|a_{\mu\tau}|$ and $\theta_{23}$ ($\delta_{CP}$), at 1$\sigma$, 2$\sigma$, 3$\sigma$ C.L. in two dimensional plane. In both figures upper (lower) panel is for $a_{ee}$, $a_{\mu\mu}$ and $a_{\tau\tau}$ ($a_{e\mu}$, $a_{e\tau}$, $a_{\mu\tau}$). In order to obtain these correlations, we set the true value of LIV parameters to zero and the standard oscillation parameters as given in Table Further, we do marginalization over $\sin^{2}\theta_{23}$, $\delta_{CP}$, and $\Delta m_{31}^{2}$ for both hierarchies. In the case of non-diagonal LIV parameters, $a_{e\mu}$, $a_{e\tau}$, $a_{\mu\tau}$, we also do marginalization over the additional phase $\phi_{\alpha\beta}$. From the plots it can be noticed that precise determination of $\theta_{23}$ will provide useful information about the possible interplay of LIV physics.
FIG. 9: Correlation between LIV parameters and $\theta_{23}$ in $a_{\alpha\beta} - \sin^2 \theta_{23}$ plane at $1\sigma$, $2\sigma$ and $3\sigma$ C.L. for NO$\nu$A experiment.

VII. SUMMARY AND CONCLUSION

It is well known that, neutrino oscillation physics has entered a precision era, and the currently running accelerator based long-baseline experiment NO$\nu$A is expected to shed light on the current unknown parameters in the standard oscillation framework, such as the mass hierarchy as well as the leptonic CP phase $\delta_{CP}$. However, the possible interplay of potential new physics scenarios can hinder the clean determination of these parameters. Lorentz invariance is one of the fundamental properties of space time in the standard version of relativity. Nevertheless, the possibility of small violation of this fundamental symmetry has been explored in various extensions of the Standard Model in recent times and a variety of possible experiments for the search of such signals have been proposed over the years. In this context, the study of neutrino properties can also provide a suitable testing ground to look for the effects of LIV parameters as neutrino phenomenology is extremely rich and spans over a very wide range of energies.

In this work, we have explored the phenomenological consequences introduced in the
neutrino oscillation physics by the presence of Lorentz-Invariance violation on the sensitivity studies of NOνA experiment. We mainly focused on how the oscillation probabilities, which govern the neutrino flavor transitions, get modified in presence of different LIV parameters. In particular, we have considered the impact of the LIV parameters $a_{e\mu}$, $a_{e\tau}$, $a_{\tau\mu}$, $a_{\mu\mu}$ and $a_{\tau\tau}$. We found that the parameters $a_{e\mu}$, $a_{e\tau}$ and $a_{ee}$ significantly affect the $\nu_\mu$ appearance probability $P_{\mu e}$, while the effect of $a_{\tau\mu}$, $a_{\mu\mu}$, $a_{\tau\tau}$ on the disappearance probability $P_{\mu\mu}$ is minimal. We also found that $a_{e\mu}$ creates a distortion on the appearance probability. We analysed the sensitivity of NOνA experiment in constraining the LIV parameters $a_{\alpha\beta}$, and found that the bounds from NOνA experiment is slightly weaker (2σ level) compared to the values extracted from Super-Kamiokande experiment. We further investigated the impact of LIV parameters on the determination of mass hierarchy and CP violation discovery potential and found that the presence of LIV parameters significantly affect these sensitivities. In fact, the mass hierarchy sensitivity and CPV sensitivity are enhanced or deteriorated significantly in presence of LIV parameters as these sensitivities crucially depend on the new CP-violating phase of these parameters. We also obtained the correlation plots between $\sin^2 \theta_{23}$ and $|a_{\alpha\beta}|$.
as well as between $\delta_{CP}$ and $|a_{\alpha\beta}|$. From these confidence regions, it can be ascertained that it is possible to obtain the bounds on the LIV parameters once $\sin^2 \theta_{23}$ is precisely determined.

In conclusion, we found that NO$\nu$A has the potential to explore the new physics associated with Lorentz invariance violation and can provide constraints on these parameters.

**Appendix: Details of $\chi^2$ analysis**

In our analysis, we have performed the $\chi^2$ analysis by comparing true (predicted) event spectra $N_{i}^{\text{true}}$ with test (alternate hypothesis) event spectra $N_{i}^{\text{test}}$, and its general form is given by

$$
\chi^2_{\text{stat}}(\vec{p}_{\text{true}}, \vec{p}_{\text{test}}) = \sum_{i \in \text{bins}} 2\left[ N_{i}^{\text{test}} - N_{i}^{\text{true}} - N_{i}^{\text{true}} \ln \left( \frac{N_{i}^{\text{test}}}{N_{i}^{\text{true}}} \right) \right],
$$

(17)

where $\vec{p}$ is the array of standard neutrino oscillation parameters. However, for numerical calculation of $\chi^2$, we also include the systematic errors using pull method. This is usually done with the help of nuisance systematic parameters as discussed in the GLoBES manual.

Suppose $\vec{q}$ is the oscillation parameter in presence of Lorentz invariance violating parameters. Then the sensitivity of LIV parameter $a_{\alpha\beta}$ can be evaluated as

$$
\chi^2(a_{\alpha\beta}^{\text{test}}) = \chi^2_{\text{SO}} - \chi^2_{\text{LIV}},
$$

(18)

where $\chi^2_{\text{SO}} = \chi^2(\vec{p}_{\text{true}}, \vec{p}_{\text{test}})$, $\chi^2_{\text{LIV}} = \chi^2(\vec{p}_{\text{true}}, \vec{q}_{\text{test}})$. Further, the sensitivities of current unknowns in neutrino oscillation is given by

- **CPV sensitivity:**

$$
\chi^2_{\text{CPV}}(\delta_{CP}^{\text{true}}) = \chi^2(\delta_{CP}^{\text{true}}, \delta_{CP}^{\text{test}} = 0), \chi^2(\delta_{CP}^{\text{true}}, \delta_{CP}^{\text{test}} = \pi)
$$

(19)

- **MH sensitivity:**

$$
\chi^2_{\text{MH}} = \chi^2_{\text{NH}} - \chi^2_{\text{IH}} \quad \text{(for true normal ordering)}
$$

(20)

$$
\chi^2_{\text{MH}} = \chi^2_{\text{IH}} - \chi^2_{\text{NH}} \quad \text{(for true inverted ordering)}
$$

(21)

Further, we obtain minimum $\chi^2_{\text{min}}$ by doing marginalization over all oscillation parameter spaces.
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