A new design to match the vehicle toe-in and camber considering the cornering property of the tire

Daogao WEI*, Yingjie ZHU*, Wei SHI**, Yu WANG***, Bingzhan ZHANG* and Andong YIN*

*School of Mechanical and Automotive Engineering
Hefei University of Technology, Hefei 23009, Anhui, China
E-mail: weidaogao@hfut.edu.cn
**BAIC Group Off-road Vehicle Co., Ltd, Beijing, 100000, China
***Institute of Industrial Science, The University of Tokyo, 4-6-1 Komaba, Meguro-ku, Tokyo, 153-8505, Japan

Received: 28 August 2019; Accepted: 2 September 2019

Abstract
Toe-in and camber on the front and rear wheels are two important parameters which affect tire abrasion of automobiles, driving and braking stability, front wheel shimmy and so on. The reasonable matching calculation between Toe-in and camber has always been a problem in the design of vehicle four-wheel alignment parameters. The cornering property is one of the key factors influencing the matching accuracy between toe-in and camber and it is a difficulty to establish the matching formula of the two. In order to obtain a more accurate matching relationship, a new formula of matching toe-in and camber based on the existing research results assuming that the tires are rigid, which considers the cornering characteristics of tires, is established to decrease the tire abrasion, front wheel shimmy and increase the driving and braking stability of automobiles. The new formula is verified by testing the front wheel sideslip and tire abrasion of a truck model. The test results show that the matching formula adjusted toe-in and camber is reasonable. From previous studies and the principle of balance between lateral force of camber and cornering force of toe-in, a new calculation method to match toe-in with camber reasonably is proposed.

Keywords : Toe-in, Camber, Sideslip, Cornering property, Tire abrasion

1. Introduction

Vehicle four-wheel alignment parameters are important in the dynamic design of the chassis, which can directly influence the handling and stability, shimmy of front wheel and tire abrasion of the vehicle, but are still difficult to design. Moreover, the design has an indirect influence on flexibility and fuel. Camber and toe-in angles are small on solid axles. However, on independent wheel suspension, camber and toe-in angles can play an important role in cornering and straight running. We use the toe-in angle, which functions together with the camber angle to overcome the negative influence caused by the wheel camber, to guarantee that the front wheels are in state of pure rolling and straight running (see Fig. 1).

![Fig. 1 Schematic of the cornering force of toe-in $F_{YA}$ and the thrust force of the camber $F_{Y}$](image-url)
The design method of matching toe-in and camber has been investigated for a long time by many scholars. Assuming that the tire is a rigid body and that the lateral force caused by the toe-in counteracts the force caused by the camber (Wang et al., 1995), the following equation can be derived:

\[ T = \frac{l \cdot d}{2r} \gamma \]

Where \( T \) is the toe-in (mm), \( d \) is the diameter of the rim (mm), \( l \) is length of tire footprint (mm), \( \gamma \) is the camber angle of the front wheel and \( r \) is the radius of rolling tire (mm).

In fact, the wheelbase must be considered while the front wheels are rolling around as a result of the toe-in angle; however, the former does not consider such situation. Consequently, the formula cannot provide a precise matching of the toe-in \( T \) and camber \( \gamma \). Regarding the tire as a rigid body, another relationship between toe-in and camber angles is derived (Kong, 1986), as follows:

\[ T = \frac{2d \cdot \phi \cdot L}{\phi \cdot r + 4\gamma \cdot L} \gamma \]

Where \( L \) is the wheelbase (mm) and \( \phi \) is the centre angle corresponding to the footprint of the tire considering only the camber angle (rad).

Different types of vehicles have different values of the angle \( \phi \). For Jiefang Truck CA10B, \( \phi = 1.5^\circ \), the toe-in is 3 mm, which has been adjusted from the toe-in of 8 mm to 12 mm that has been used for a long time. As a result, the front tire abrasion and shimmy of the front wheel are decreased. However, determining the value of angle \( \phi \) for different types of vehicles is difficult. Moreover, regarding the tire as a rigid body without its special property is incorrect. These two omissions make the formula less precise.

In the study (Cao et al., 1998), a bus is taken as a sample. On the assumption that the tire is rigid, the author developed a formula to describe the relationship between toe-in and camber, as follows:

\[ T = \frac{4d \cdot r \cdot \sin^2(l \cdot \sin \gamma / 2r)}{l \cdot \sin \gamma} \]

However, the formula fails to consider the influence of the wheelbase when the front wheel is rolling. As a result, the formula outputs a larger toe-in.

In the study (Wei et al., 2003), the tire is also assumed to be rigid. Given the construct parameters of vehicles, the equations of the two parameters are expressed as follows:

\[ T = \frac{2d \cdot L \cdot l}{r(4L + l)} \gamma \]

The equation does not consider the influence of the cornering property of the tire. As a result, using the formula in the design of the optimum match between toe-in and camber is inaccurate.

The above research works fail to comprehensively consider the vehicle structural parameters and the elastic characteristics of the tire when deriving the matching formula between toe-in and camber, which affects the calculation accuracy in the design of the toe-in and camber.

In view of the shortcomings of the above research works, in the study (Ma et al., 2012), the rolling radius is considered as a function of the radial stiffness of the tire \( (K_z) \) and the vertical load of the front axle \( (G_1) \). Based on the mechanism of front wheel slideslip, considering the properties of tires and the vehicle structural parameters, the reasonable matching relationship between the toe-in and camber is derived and it is verified by the sideslip tests of prototype car.

\[ T = \frac{24K_z \cdot L \cdot l \cdot d \cdot \gamma}{(8L - l) / 3D \cdot K_z - G_1} \gamma \]
Where $D$ is the nominal diameter of the tire of the front wheel (mm).

Based on this research, in the study (Zhang et al., 2018), the reasonable matching formula for toe-in and camber of the double-front-axle steering automobile are derived. And according to the dynamic model established in the Adams/Car, the matching values of toe-in and camber are simulated and verified.

In the study (Qian et al., 2014), based on the sideslip mechanism of front wheels and the theory of complex carcass deformation, and then with consideration of the tire characteristics and vehicle structural parameters, the matching formula between camber and toe-in is derived, as follows:

$$
T = \left\{ \begin{array}{ll}
4\left( \frac{d}{2R} \right)^2 + a^2 - \frac{d}{2R} \cdot \frac{1}{3R} \left( 1 - \frac{k_\gamma}{k_\gamma} \right) \cdot \gamma \cdot \left( 1 - \frac{a \cdot k_\gamma}{3N_\theta} \right) \cdot 2d \\
3a \cdot \frac{k_\gamma}{k_\gamma} + 2a^2 \cdot \frac{1}{L} \cdot \left( 1 - \frac{a \cdot k_\gamma}{3N_\theta} \right)
\end{array} \right.
$$

Where $a$ is half length of tire footprint (mm), $k_\gamma$ is the tire camber stiffness (N/rad), $k_{c\gamma}$ is the carcass camber stiffness (N/rad), $k_\beta$ is the cornering stiffness (N/rad), $k_{c0}$ is the carcass lateral translation stiffness (N/mm), $N_\theta$ is the carcass torsional stiffness (N/rad).

The formula considers the influence of the complex carcass deformation under the action of toe-in and camber on the trajectory of the front wheel. However, the matching formula needs to calculate many tire characteristic parameters such as $k_\gamma$, $k_{c\gamma}$, $k_\beta$, $k_{c0}$, $N_\theta$ through a large number of tests and formula, which greatly increases the cost and workload of vehicle development. Besides, the carcass bending deformation function for solving the tire cornering stiffness is difficult to obtain, and thus the accuracy of the matching result obtained by the formula is difficult to ensure, which makes it have greater limitations in engineering applications.

Therefore, the study on the basis of ensuring the convenience and accuracy of the calculation, considers the effect of the cornering force under the action of the toe-in and camber on the matching relation between the toe-in and camber has important academic and engineering application value. In this paper, considering the influence of the cornering force generated by the action of camber on the the matching relation between toe-in and camber and the vehicle structural parameters, a new design method of matching toe-in and camber is proposed. The new formula is verified by testing the front wheel sideslip and tire abrasion of a 1026 truck model. The formula provides a more accurate and convenient theoretical calculation method for the reasonable selection of toe-in and camber in the process of vehicle design, and also provides boundary conditions for the optimization of four wheel alignment parameters.

2. Calculating the formulation of the match of the toe-in and the camber

![Fig. 2 Schematic of the left front wheel turning and forcing with only the camber](image)

When a vehicle is running with the effect of camber angle $\gamma$, the wheels tend to roll around Point $O$, where their axes intersect the ground (see Fig. 2). If the wheel not constrained, it will deviate from the front to the left. In fact, the wheels can only move forward because of the constraint of the transaxle. Therefore, lateral force $F_\gamma$ must exist at the centre of the wheels to align the wheels back to their original direction and make the vehicle travel in a straight line. Meanwhile, lateral counterforce $F_{\gamma y}$ is called the camber thrust force in the reverse direction of $F_\gamma$ at the tire contact.
print. Neglecting other factors, such as lateral wind, we take lateral force and lateral counterforce as approximately equal when a vehicle travels in a straight line at a low speed (Yu, 1996, Zhuang, 1996). If the slip angle is no more than $4.0^\circ$ to $5.0^\circ$, then the slip angle $\alpha$ and the cornering force $F_y$ is assumed to be linear.

\[
\begin{align*}
F_{yy} &= k_y \cdot \gamma \\
F_y &= k_y \cdot \alpha \\
F_{yy} &= F_y
\end{align*}
\]  

(1)

Where $k_y$ is the camber stiffness (N/rad), $k_y$ is the cornering stiffness of the front wheel (N/rad), $\alpha$ is slip angle (rad) and $\gamma$ is the camber angle (rad).

As shown in Fig. 3, distance $h$ is derived when the wheels roll with the sideslip. According to the HSRI model (Zhuang, 1996) (Dugoff et al., 1969):

\[
h = \frac{2}{3} l \cdot \tan \alpha
\]

Radial tires are widely used in modern vehicles, and the slip angle is generally less than 1.0° when the vehicle is in low speed. Thus, the previous equation can be rewritten as follows:

\[
h = \frac{2}{3} l \cdot \alpha
\]  

(2)

2.1 The analysis of front wheel movement with only the camber considering the slip angle

In Fig. 2, when the camber of the front wheel equals $\gamma$, the extension line of the axis intersects the ground at Point $O$. Without the transaxle and other restraints, the front wheel moves circular as a cone $AOA_1$ whose centre is Point $O$ with the generatrix $|OA| = R_i$ as radius.

\[
R_i = \frac{r}{\sin \gamma}
\]

Where $R_i$ is the turning radius of the wheel when pure rolling (mm).

The camber angle is designed to be generally smaller than 1.0°, along with the improvements of road construction and the acceleration of vehicle speed. Thus, the equation can be written as follows:

\[
R_i \approx \frac{r}{\gamma}
\]
Figure 3 also shows the offset distance $h$ at the centre of the tire when the sideslip angle is considered. Thus, the previous equation can be rewritten as follows:

$$R_1 \pm \frac{r}{\gamma} h$$  \hspace{1cm} (3)$$

Where $R_1$ is the turning radius of the wheel when pure rolling with cornering property being considered (mm).

2.2 The analysis of front wheel movement with only the toe-in considering the slip angle

As shown in Fig. 4, when neglecting the influence of the sideslip angle of the rear wheels due to the existence of the toe-in and the cornering of the front wheel, we can take the deflection of the left front wheel as a circular motion whose centre is Point $O_1$ with the length of $|OO_0|=R_2$ as the radius. With the cornering effects, an offset distance $h$ occurs at the centre of the tire along the direction of $\overrightarrow{OO_1}$. Accordingly, we can concluded that:

$$R_2 = \frac{L}{\sin(\Lambda + \alpha)} + h$$

Given that the toe-in and sideslip angles are generally less than 1.0°, the previous equation can be rewritten as follows:

$$R_2 \pm \frac{L}{\Lambda + \alpha} + h$$  \hspace{1cm} (4)$$

Where $\Lambda$ is the toe-in of the front wheel (rad).

Figure 5 shows that, from the vertical view of the front axle of the vehicle, we can conclude that the geometric relationship of the toe-in angle (rad) and toe-in (mm):

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Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol.14, No.1 (2020)
\[ \sin \Lambda = \frac{T}{2d} \]
\[ \Lambda = \frac{T}{2d} \]  

(5)

2.3 The analysis of front wheel movement under the influence of the toe-in and camber considering the slip angle

As shown in Fig. 6, under the influence of the cornering force, the centre of the tire foot print where in ground contact deviates from point \( A \) to point \( A_1 \). \( V \) stands for the direction of forward movement of the vehicle. Under the influence of the camber, the front wheel will do conic movement with Point \( O_2 \) as the conic node if the sideslip angle is considered. Under the influence of the toe-in, the front wheel does conic movement with Point \( O_1 \) as the conic node if the sideslip angle is considered. In Fig. 6, the line \( \overline{AG} \) is the common tangent of the arc \( \overline{AC} \) and arc \( \overline{AD} \), on the assumption that the camber, toe-in and sideslip angle of the front wheel are invariant in the process of tire rolling when the vehicle is moving in a straight line. When the core of the front wheel begins to move from point \( A \), along the direction of \( \overline{AX} \) right ahead movement, the front wheel will move along the direction of \( \overline{AD} \) if considering the influence of toe-in and sideslip. Assuming the instant of \( \Delta t \), the rolling distance is the length of arc \( \overline{AD} \). However, when the camber and sideslip of the front wheel are considered, the front wheel will travel in the direction of \( \overline{AC} \). Thus, at the same time, the rolling distance is the length of the arc \( \overline{AC} \).

Actually, during \( \Delta t \), instead of reaching Point \( C \) or \( D \), the front wheel sideslips to Point \( E \) or \( F \) from Point \( C \) or \( D \), respectively. From the analysis of the motion shown in Fig. 6, the included angle formed by \( \overline{AX} \) and \( \overline{AM} \), is determined to be equal to the toe angle \( \Lambda \) and the included angle formed by \( \overline{AM} \) and \( \overline{AG} \), is determined to be equal to the sideslip angle \( \alpha \). Assuming that the toe-in and camber can match reasonably, the sideslip value would be minimum. Thus, a certain geometric relationship can be expressed as follows:

\[ |CE| = |DF| \]  

(6)

Given that the vector \( \overline{AG} \) is the common tangent of circles with centres \( O_2 \) and \( O_3 \), we determine that:

\[ \angle CAG = \frac{1}{2} \angle CO_2 A = \frac{\phi}{2}, \quad \angle DAG = \frac{1}{2} \angle AO_2 D = \frac{\theta}{2}. \]
For the right triangle $A_iCE_i$, \( \angle CA_i E_i = \frac{\phi}{2} - \Lambda - \alpha \) and

\[
|CE_i| = |A_iC| \cdot \sin \left( \frac{\phi}{2} - \Lambda - \alpha \right)
\]  

(7)

For the right triangle $A_iDF_i$, \( \angle DA_i F_i = \frac{\theta}{2} + \Lambda + \alpha \) and

\[
|DF_i| = |A_iD| \cdot \sin \left( \frac{\theta}{2} + \Lambda + \alpha \right)
\]  

(8)

From the lateral force, we derive \( |A_iA| = h \), as indicated in Figs. 3 and 6. From Fig. 6:

\[
|EE_i| = |FF_i| = h \cdot \cos \Lambda
\]  

(9)

In fact, the toe-in is generally less than 1.0°, thus, \( \cos \Lambda = 1 \) and Eq. (9) can be written as follows:

\[
|EE_i| = |FF_i| = h
\]  

(10)

From Fig. 6, we derive the following expression:

\[
|CE| = |CE_i| + |E_iE|
\]  

\[
|DF| = |DF_i| - |F_iF|
\]  

(11)

Substituting Eqs. (7), (8) and (10) into Eq. (11) derives the following expression:

\[
|CE| = |A_iC| \cdot \sin \left( \frac{\phi}{2} - \Lambda - \alpha \right) + h
\]  

\[
|DF| = |A_iD| \cdot \sin \left( \frac{\theta}{2} + \Lambda + \alpha \right) - h
\]  

(12)

If the \( R_1 \) and \( R_2 \) are sufficiently large and the toe-in and camber can match reasonably, then we derive the following expression:

\[
\hat{A_i}C = |A_iC|
\]  

\[
\hat{A_i}D = |A_iD|
\]  

(13)

At the same moment during the process, the following equation must be guaranteed:

\[
\hat{A_i}C = \hat{A_i}D
\]  

(14)

Substituting Eq. (13) into Eq. (14) derives the following expression:

\[
|A_iC| = |A_iD|
\]  

(15)

In addition, considering the sideslip of the front wheel (\( \overrightarrow{CE} \) and \( \overrightarrow{DF} \) in Fig. 6, the direction of the sideslip), we can conclude that Point C or D is where the tire comes in contact with the ground because it is the only condition where
sideslip could ever occur. Then, we can define the length of arcs $\hat{AD}$ and $\hat{AC}$ as $0 \leq \hat{AD} \leq \hat{AC} \leq l$, which is approximately smaller than that of tire footprint $l$ (Zhunag, 1996) (Dugoff et al., 1969) (Mitschke and Wallentowitz, 2004) (Gim and Nikravesh, 1990) (Gim and Nikravesh, 1991).

The maximum length of arc $\hat{AD}$ and $\hat{AC}$ is the length of the tire footprint; that is:

$$\hat{AC} = \hat{AD} = l \quad (16)$$

From Eqs. (14), (15) and (16), we can obtain the following expression:

$$|\hat{AC}| = |\hat{AD}| = l \quad (17)$$

From Eqs (6), (12) and (17), we derive the following expression:

$$l \cdot \sin\left(\frac{\phi}{2} - \Lambda - \alpha\right) + h = l \cdot \sin\left(\frac{\theta}{2} + \Lambda + \alpha\right) - h \quad (18)$$

As shown in Fig. 6, arcs $\hat{AD}$ and $\hat{AC}$ can also be expressed as follows:

$$\hat{AC} = R_1 \phi, \hat{AD} = R_2 \theta \quad (19)$$

Substituting Eq. (19) into Eq. (16) derives the following expression:

$$R_1 \phi = R_2 \theta = l \quad (20)$$

Substituting $R_1 = \frac{r - h}{\gamma}, R_2 = \frac{l}{\Lambda + \alpha} + h$ and $h = \frac{2}{3} l \cdot \alpha$ into Eq. (20) derives the following expression:

$$\frac{r - \frac{2}{3} \gamma \cdot \alpha \cdot l}{\gamma} \phi = \frac{l + \frac{2}{3} (\Lambda + \alpha) \cdot \alpha \cdot l}{\Lambda + \alpha} \theta \quad (21)$$

Solving Eq. (20) for $\phi$ derives the following expression:

$$\phi = \frac{l}{R_1} = \frac{l \cdot \gamma}{r - \frac{2}{3} \gamma \cdot \alpha \cdot l} \quad (22)$$

$$\theta = \frac{l}{R_2} = \frac{l \cdot (\Lambda + \alpha)}{L + \frac{2}{3} (\Lambda + \alpha) \cdot \alpha \cdot l} \quad (23)$$

From Eqs. (22) and (23), we determine that $\gamma < l^\prime, l < r < L$ and that $\phi$ and $\theta$ are generally less than $l^\prime$. In practice, $\sin\left(\frac{\phi}{2} - \Lambda - \alpha\right)$ and $\sin\left(\frac{\theta}{2} + \Lambda + \alpha\right)$ can be expressed approximately as follows:

$$\sin\left(\frac{\phi}{2} - \Lambda - \alpha\right) = \frac{\phi}{2} - \Lambda - \alpha$$

$$\sin\left(\frac{\theta}{2} + \Lambda + \alpha\right) = \frac{\theta}{2} + \Lambda + \alpha \quad (24)$$
Substituting Eq. (24) into Eq. (18) derives the following expression:

$$\theta = \phi - 4\Lambda - \frac{4}{3} \alpha$$  \hspace{1cm} (25)

Substituting Eq. (22) into Eq. (25) derives the following expression:

$$\theta = \frac{l}{r - \frac{2}{3}\gamma \cdot \alpha \cdot l} \gamma - 4\Lambda - \frac{4}{3} \alpha$$  \hspace{1cm} (26)

Substituting Eq. (26) into Eq. (21) derives the following expression:

$$l \cdot \Lambda + l \cdot \alpha = [L + \frac{2}{3}(\Lambda + \alpha) \cdot \alpha \cdot l] \left(\frac{l \cdot \gamma}{r - \frac{2}{3}\gamma \cdot \alpha \cdot l} - 4\Lambda - \frac{4}{3} \alpha\right)$$  \hspace{1cm} (27)

Given that \(\Lambda\) and \(\alpha\) are small, quadratic terms, such as \(\Lambda \cdot \alpha\), \(\Lambda^2\) and \(\alpha^2\), can be neglected:

$$\Lambda = \frac{L \cdot l}{(4L + l)(r - \frac{2}{3}\gamma \cdot \alpha \cdot l)} + \left[\frac{8L}{3(4L + l)} - 1\right] \cdot \alpha$$  \hspace{1cm} (28)

Substituting Eq. (28) into Eq. (5) derives the following expression:

$$T = \frac{2d \cdot L \cdot l}{(4L + l)(r - \frac{2}{3}\gamma \cdot \alpha \cdot l)} \gamma + 2d \cdot \left[\frac{8L}{3(4L + l)} - 1\right] \cdot \alpha$$  \hspace{1cm} (29)

Where \(l\) is the length of the tire footprint and refers to a parameter in the aforementioned equation influenced by multiply factors. When we calculate the toe-in of the sample vehicle, the semiempirical approach can be used (Komandi, 1976), as follows:

$$l = 1.7 \sqrt{(D - \Delta) \Delta}$$  \hspace{1cm} (30)

In the formula, \(D\) is the tire diameter (mm) and \(\Delta\) the radical deformation of the steering tire under the influence of the vehicle vertical load of the front axle. According to the following equation:

$$\Delta = \frac{C \cdot K (0.5G_i)^{0.35}}{b^{0.37} \cdot D_1^{0.45} \cdot p^{0.6}}$$  \hspace{1cm} (31)

Where \(C\) is a coefficient; \(K\) is another coefficient, \(K = 0.0015b + 0.42\); \(G_i\) is the load of the front axle (N); \(b\) is the width of tire section (mm); and \(p\) is the air pressure in tire (MPa).

3. Example calculation

We use the 1026 pick-up truck model as an example to verify the validity of Eq. (29) in matching the toe-in and camber. The parameters needed are listed in Table 1.
Table 1. Parameters of the 1026 truck model needed to calculate the toe-in

| d (mm) | L (mm) | r (mm) | γ (°) | l (mm) | D (mm) |
|--------|--------|--------|--------|--------|--------|
| 400    | 3025   | 330    | 1.0    | 212.7  | 703    |

| C     | K     | G / N  | b (mm) | p / MPa | k_r / k_α / rad |
|-------|-------|--------|--------|---------|-----------------|
| 11.2  | 0.75  | 7500   | 216    | 0.250   | 0.06            |

(1) The calculation of the length of the tire footprint: Substituting the data in Table 1 into Eq. (31) derives \( \Delta = 23.01 \text{ mm} \). Substituting the aforementioned result into Eq. (30) derives \( l = 212.7 \text{ mm} \).

(2) The calculation of toe-in (\( \gamma = 1.0° \)): From Eq. (1) and Table 1, when \( \gamma = 1.0° \) and \( \alpha = 0.06° \), substituting the value of \( l \) and \( \alpha \) and the data in Table 1 into Eq. (29) derives \( T = 1.89 \text{ mm} \).

These theoretical results show that, when the camber angle \( \gamma \) is equal to \( 1.0° \), the corresponding value of the toe-in is \( 1.89 \text{ mm} \), guaranteeing the value of the sideslip minimum theoretically.

4. Experiment

The new design method of matching toe-in and camber proposed in this study is verified by testing the front wheel sideslip and tire abrasion of the 1026 truck model.

4.1 The sideslip test

The sideslip test is performed on the check line of an automobile factory (Wei, et al., 2006). The test instruments and equipment are listed in Table 2. The operating procedures can be summarized as follows:

(1) A 1026 truck model is lifted to check the hanging position of the double wishbone independent suspension, the wheel alignment and the assemblage of steering rods.

(2) The vehicle is set down. Sandbags (65 kg) are placed on the driver position and co-pilot chair. The toe-in of the front wheels is symmetrically adjusted to ensure that the wheels are detected at the double-board sideslip platform.

(3) The vehicle is driven straightaway at a constant velocity that not more than 5 km/h across the double-board sideslip platform when the signal lamp lights.

(4) The toe-in value is \(-3.0\text{mm}, 1.0\text{mm}, 3.0\text{mm}, 5.0\text{mm}, 7.0\text{mm}, 9.0\text{mm} \) and \(11.0\text{mm}\), respectively.

(5) The sideslip value of front wheel is measured by the computer. The test results can be shown from Table 3.

The operational results are shown in Table 3. From the test results, the mathematical relation of the sideslip and toe-in is regressed by the method of orthogonal polynomial regression (Xiang, et al., 1989). The regressing equation is written as follows:
\[ S = 12.45 - 4.06T + 0.095T^2 \]  

(32)

Where \( S \) is the sideslip value \((m/km)\) and \( T \) is the toe-in value \((mm)\).

From Eq. (32), the toe-in value corresponding to the minimum sideslip value (i.e. \( S = 0, \gamma = 1.0^\circ \)) can be obtained as \( T = 1.86 \text{ mm} \). This result is close to what we obtained from the theoretical calculation \((T = 1.89 \text{ mm})\) when \( \gamma \) is equal to \( 1.0^\circ \). This finding indicates that the matching adjusted toe-in and camber is reasonable.

Table 2. Main instrument and equipment for the test

| Instruments            | Model  | Precision       |
|------------------------|--------|-----------------|
| Sideslip platform      | CH-1.5 | ± 0.2m/km       |
| Aligning instrument    | JBC-V3D| ±0.01’          |
| Air pressure           | TG-3   | 15kp            |
| Electric steelyard     | ZCX-10 | III grade       |
| Sandbag                | 65kg   | ±0.2kg          |

Table 3. Testing result of the sideslip of a 1026 pick-up truck model (\( \gamma=1^\circ \))

| Toe-in \( T(mm) \) | -3 | 1 | 3 | 5 | 7 | 9 | 11 |
|--------------------|----|---|---|---|---|---|----|
| Sideslip Value \( S(m/km) \) | 7.2 | 2.4 | -1.9 | -2.1 | -2.6 | -4.3 | -5.1 |

4.2 The tire wear test

Two independent road abrasion tests are performed based on the original match of the toe-in and camber and the match of toe-in adjusted by Eq. (29) and camber to validate the new design method of matching toe-in and camber that has been proposed with the cornering characteristics of tires being considered.

(1) The road abrasion test based on the original match of the toe-in and camber. The 3,000 km road abrasion test is performed. The results are shown in Table 4.

| Wheel location          | Tread depth of the test points | Average depth |
|-------------------------|-------------------------------|---------------|
|                         | 4.6  | 5.4  | 4.5  | 3.6  | 3.1  | 1.3 | 3.8 |
| The left front wheel    | 4.7  | 5.3  | 4.5  | 3.5  | 3.2  | 1.6 |    |
|                         | 4.6  | 5.2  | 4.5  | 3.7  | 3.0  | 1.4 |    |
|                         | 4.5  | 5.4  | 4.5  | 3.6  | 3.0  | 1.7 |    |
| The right front wheel   | 3.9  | 5.8  | 5.2  | 4.5  | 4.7  | 3.6 | 4.6 |
|                         | 4.4  | 5.9  | 5.0  | 4.5  | 3.5  | 3.4 |    |
|                         | 4.9  | 4.4  | 5.7  | 4.6  | 4.6  | 3.4 |    |
|                         | 4.3  | 5.5  | 5.5  | 4.5  | 4.6  | 3.7 |    |
(2) The road wear test based on the adjusted Eq. (29)

We can adjust the toe-in and camber of the vehicle according to Eq. (29). That is, \( T = 1.89 \text{ mm} \) when \( \gamma = 1^\circ \). The 3,000 km road abrasion test is performed. The results are shown in Table 5.

Table 5. The value of tire wear of the vehicle with adjusted toe-in (mm)

| Wheel location     | Tread depth of the test points | Average depth |
|--------------------|-------------------------------|---------------|
| The left front wheel | 9.2  8.8  9.2  9.0  9.3  9.1 | 9.1           |
|                    | 9.2  8.9  9.0  9.3  9.4  8.9 |               |
|                    | 9.4  8.7  8.8  9.2  9.0  9.2 |               |
|                    | 9.1  9.3  9.1  9.1  9.3  9.2 |               |
| The right front wheel | 9.2  9.1  9.3  8.9  8.8  8.9 | 9.0           |
|                    | 8.7  8.9  9.1  9.2  9.0  9.0 |               |
|                    | 8.9  8.8  8.8  9.0  9.2  9.1 |               |
|                    | 9.1  9.0  8.9  9.1  9.2  9.0 |               |

Note: The average tread depth of the new tire is 9.8 mm.

By comparing the results, we determine that the value of tire abrasion decreases when the toe-in and camber of the vehicle are adjusted according to Eq. (29). For the left front wheel, the value decreases from 6.0 mm to 0.7 mm. For the right front wheel, the value decreases from 5.2 mm to 0.8 mm.

5. Conclusion

The matching of the toe-in and camber is one of the important parameters that influence dynamic properties. The precise calculation is difficult to obtain because of the complexity of the tire cornering property. In this study, a new design method of matching toe-in and camber is proposed considering the cornering characteristics of tires and the structure parameters of vehicles. In addition, the new method can ensure the stability of the vehicle and keep the tire abrasion at the minimum. The new formula is verified by testing the front wheel sideslip and tire abrasion of the 1026 truck model. The following conclusions can be drawn:

1. As the tire cornering properties are nonlinear, the formula is established on the assumption of a cornering angle \( \alpha \) under a certain condition.

2. The formula is more accurate because it considers the influence of the length of the tire footprint, the tire cornering angle and the wheelbase. In addition, the formula has been proved by testing the front wheel sideslip and tire abrasion of the 1026 truck model.

3. The formula also provides a theoretical calculation method to obtain the optimum match of the toe-in and camber, the boundary conditions to optimize the wheel alignment parameters and the method to judge abnormal abrasion of the tire.

Acknowledgment

This research was supported by the National Science Foundation of China (grant number 51375130 and 51875154).
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