Radiative corrections to QCD SR for meson distribution amplitudes up to $O(\alpha_s^2\beta_0)$

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Abstract. We obtain QCD radiative corrections to distribution amplitudes of $\pi$ and $\rho$ mesons within QCD sum rules. To this end, we calculate correlators of two different composite vertices at order $O(\alpha_s^2\beta_0)$, where $\beta_0$ is the first coefficient in the expansion of QCD $\beta$-function.

1. Introduction

An important problem in the description of hard processes is to calculate distribution amplitudes (DA) $\varphi_{\text{f}}(x)$ of light mesons $M = \pi, \rho$, etc. with the help of QCD sum rules (SR). These distributions of partons by the fraction $xp$ of an hadron momentum $p$ appear naturally as a consequence of “factorization theorems” for hard exclusive processes [1, 2]. In the factorization approach, DAs accumulate information about long-distance dynamics of partons in hadrons. For helicity-zero vector mesons $M_{\parallel} = \rho_{\parallel}, \rho'_{\parallel}, \ldots$ and for axial mesons $M_A = \pi, a_1^\parallel, \ldots$, the twist-2 DAs are defined by projecting currents $V(z) = d(z)\hat{\gamma}_5u(0)$ and $A(z) = d(z)\hat{\gamma}_5u(0)$ on a meson state $|M\rangle$:

$$
(0|\hat{d}(z)(-i\gamma_5)u(0)|M_{[(A)\parallel]}(p))\big|_{z^2=0} = f_{M_{[(A)\parallel]}(zp)}\int_0^1 dx\, e^{ixzp} \varphi_{M_{[(A)\parallel]}(x)}.
$$

The corresponding definition for a helicity-one state, $M_{\perp} = \pm \rho_\perp$, is as follows:

$$
(0|\hat{d}(z)\sigma_{\mu\nu}u(0)|M_{\perp}^{\pm}(p))\big|_{z^2=0} = if_{M_{\perp}^{\pm}}\left(\varepsilon_{\mu}^{(\lambda)}(p)p_{\nu} - \varepsilon_{\nu}^{(\lambda)}(p)p_{\mu}\right)\int_0^1 dx\, e^{ixzp} \varphi_{M_{\perp}^{\pm}(x)}.
$$

These DAs and their moments can be extracted from a correlator $I(\zeta) = V(z), A(z)$, or $T^\mu(z) = d(z)\sigma_{\mu\alpha}z^\alpha u(0)$ [3–6]—for example, for the local current $T^{\mu(n)}(y) = \hat{d}(y)\sigma_{\mu\alpha}z^\alpha (z\nabla)^n u(y)$, we have

$$
i \int dy\, e^{ipy} \langle 0|T^{\mu(n)}(y)T^{\mu(0)}_{(n)}(0)|0\rangle = -2i^n (zp)^{n+2} I_{(n0)}(p^2).
$$

The key element to calculate the inverse Mellin image of $I_{(n0)}$, $M_{(n\to z)} I_{(n0)} = I(x;p^2)$, at order $O(\alpha_s^2\beta_0)$ was obtained in [7] and will be considered in the next section. The borelized correlator $B_{(M)} I(x;p^2)$ directly determines the pQCD content $\Delta \varphi_M$ of the corresponding DA $\varphi_M$ within QCD SRs. Further, we discuss the impact and main features of these radiative corrections.
2. Two-loop correlators with composite vertices

In order to calculate the required contributions of the order \(O(\alpha_s^2\beta_0)\), we need to evaluate the two-loop diagram of kite topology with one of its external vertices being composite:

\[
\frac{x}{p} \rightarrow \frac{p}{p} = \int \frac{d^Dk_1 \, d^Dk_2}{k_1^2 k_2^2 (k_1 - p)^2 (k_2 - p)^2 [(k_1 - k_2)^2]_n} \frac{(-1)^{n+1} \pi D}{(-p^2)^{n+4-D}} G(n; x; D),
\]

where \(z\) is a lightlike vector and the tick on a line denotes the Dirac \(\delta\)-function that accompanies the composite vertex in the integrand. Even more general kite correlator of two composite vertices (as well as the Mellin moments of it) was considered in [7] with the propagators raised to arbitrary powers. The \(\alpha\) representation of such two-loop correlator can be evaluated directly as a hypergeometric integral. In the general case, the result of integration is expressed in terms of a hypergeometric series in two variables—the Kampé de Fériet functions. A chain of reductions to simpler functions can be found for some special cases. In particular, the correlator (4) amounts to a generalized hypergeometric function:

\[
G(n; x; D) = -\frac{\Gamma^2(-\hat{n})\Gamma(1+\hat{n}-\lambda)\Gamma(\lambda)}{\Gamma(n)\Gamma(1-\hat{n})\Gamma(\lambda-\hat{n})} (x\bar{x})^{\lambda-1} \times \left\{ \frac{\Gamma(n)\Gamma^2((-\hat{n})\Gamma(1-\hat{n})\Gamma(\lambda-\hat{n})}{\Gamma^2(\lambda)\Gamma(2\hat{n})\Gamma(1-2\hat{n})\Gamma(-\hat{n})} + \hat{S} \left[ x^{-\hat{n}} 3F_2 \left( \frac{1}{1-\hat{n}}, \frac{-\hat{n}}{1-\hat{n}}, \frac{-\hat{n}}{\lambda-\hat{n}} \middle| x \right) \right] \right\},
\]

where \(\lambda = D/2 - 1\), \(\hat{n} = n - \lambda\), \(\hat{S}T(x) = T(x) + T(\bar{x})\), and \(\bar{x} = 1 - x\). The integral of \(G(n; x; D)\) over \(x\) coincides with the well-known results in [8–10] after some transformations of \(3F_2(1)\). The Laurent expansions of Eq. (5) near even and odd \(D\) can be obtained with the help of the standard algorithms and results (see [11] and references therein).

3. \(\langle AA \rangle\) and \(\langle VV \rangle\) correlators. Distribution amplitudes of \(\pi\) and \(\rho\)

In what follows, we use the notations \(a_s = \alpha_s(\mu^2)/(4\pi)\), \(\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f\) for the first coefficient of \(\beta\)-function, and \(M^2\) is a parameter of the Borel transform \(\hat{B}_{(M^2)}\) applied to correlators \(\Pi(p^2)\) in the framework of QCD SR (e.g., see [12]). To take into account NLO corrections and a part of N^2LO corrections that are proportional to \(\beta_0\), we have to deal with the following diagrams:

\[i\Delta\varphi^{(1)}_M(M^2; x) = \hat{B}_{(M^2)} + + + \ldots\]

\[i\Delta\varphi^{(2)}_M(M^2; x) = -\frac{3\beta_0}{2n_f} \hat{B}_{(M^2)} + + + \ldots\]

3.1. \(\langle V(A) V(A) \rangle\) correlator at orders up to \(O(\alpha_s^2\beta_0)\)

The contribution of order \(O(a_s)\) was obtained first in [12] and leads to a visible correction that is especially significant near the endpoints. Here, we recalculate it in arbitrary covariant gauge:

\[
\Delta\varphi^{(0+1)}_{M_{\parallel\parallel}}(M^2; x) = \hat{B}_{(M^2)} \Pi_{LO+NLO}(A^2) = \frac{N_c}{2\pi^2} x\bar{x} \left\{ 1 + a_s C_F \left[ 5 - \frac{\pi^2}{3} + \ln^2 \left( \frac{\bar{x}}{x} \right) \right] \right\}.
\]
At $\beta_0 N^2$LO, we obtain the following expression involving dependence on $L_B = \ln \left( \frac{M^2}{\mu^2} e^{-\gamma_E} \right)$:

$$
\Delta \varphi_{M(\perp)}^{(2)} (M^2; x) = \frac{N_c}{4\pi^2} a_s^2 C_F \beta_0 S \left\{ -x\bar{x} \left[ 10 \text{Li}_3(x) - 2 \ln x \text{Li}_2(x) + \frac{5}{6} \ln^2 \left( \frac{x}{\bar{x}} \right) \right] - \frac{1}{3} \ln^3 x + \frac{5\pi^2}{18} - \frac{2\pi^2}{3} \ln x - \frac{7}{6} \right\} - 2x \left[ \text{Li}_2(x) - \frac{\pi^2}{6} - \frac{3}{4} \ln^2 x + \left( \frac{31}{12} - L_B \right) \ln x \right],
$$

(7)

3.2. Perturbative content of twist-2 DA for $\pi$ and $\rho_1$ mesons

Important characteristics of $\Delta \varphi_M$ of DA are the norm $\langle x^0 \rangle_M$ and inverse moment $\langle x^{-1} \rangle_M$:

$$
\langle x^n \rangle_M = \int_0^1 dx x^n \Delta \varphi_M(x), \quad \langle x^0 \rangle_M = \frac{N_c}{12\pi^2} \left[ 1 + a_s C_F 3 + a_s^2 \beta_0 C_F 3 \left( \frac{11}{2} - 4\zeta_3 - L_B \right) \right],
$$

(8)

$$
\langle x^{-1} \rangle_M = \frac{N_c}{4\pi^2} \left[ 1 + a_s C_F 5 + a_s^2 \beta_0 C_F \left( \frac{7}{18} - \frac{5}{3} \zeta_3 + \frac{31}{108} \pi^2 - \frac{\pi^2}{9} L_B \right) \right].
$$

(9)

The $\langle x^0 \rangle_M$ in (8) coincides with the corresponding part of the Adler $D$-function, as expected. The impact of $O(\alpha_s^2 \beta_0)$ contribution to $\Delta \varphi_M$ looks especially significant for intermediate values of $x$, see Fig. 1 (left panel), while in the vicinity of endpoints it is less important, which is reflected by a minor contribution to $\langle x^{-1} \rangle_M(\perp)$ in (9).

**Figure 1.** Comparison of NLO (——) and $\beta_0 N^2$LO (– – –) contributions to DAs: (left panel) pseudoscalar or longitudinally polarized vector mesons, Eqs. (6) and (7); (right panel) transversally polarized vector meson, Eqs. (10) and (11). All curves are for the case of $L_B = \ln(M^2/\mu^2) - \gamma_E = 0$ and $a_s(\mu^2 = 1 \text{GeV}^2) \approx 0.494$.

4. $\langle T \bar{T} \rangle$ correlator and perturbative part $\Delta \varphi_{M \perp}$ of DA for transversal $\rho$ meson

The NLO contribution was derived first in [3] and recalculated by us together with its non-logarithmic part (not shown here):

$$
\Delta \varphi_{M \perp}^{(0+1)} (M^2; x) = \frac{N_c}{2\pi^2} x\bar{x} \left\{ 1 + a_s C_F \left[ 6 - \frac{\pi^2}{3} + \ln^2 \left( \frac{x}{\bar{x}} \right) + \ln(x\bar{x}) + 2L_B \right] \right\}
$$

(10)

The contribution of the NLO corrections is as moderate as in Eq. (6). The $\beta_0 N^2$LO terms read

$$
\Delta \varphi_{M \perp}^{(2)} (M^2; x) = \frac{N_c}{12\pi^2} a_s^2 \beta_0 C_F \left\{ \frac{\pi^2}{6} - L_B^2 \right\} + x \left[ 6 \left( 2 - \bar{x} \right) \ln(x) + 19\bar{x} \right] L_B

+ x\bar{x} \left[ -30\text{Li}_3(x) + 6\text{Li}_2(x) \ln(x) + \ln^2(x) + \ln^2(x) \left[ 2 - 3 \ln(x) \right] + \left( 2\pi^2 + 19 \right) \ln(x)

- 5 \ln(x) \ln(\bar{x}) - \frac{5\pi^2}{6} - \frac{193}{12} \right\} - x \left[ 12\text{Li}_2(x) - 2\pi^2 + 16 \ln(x) - 9 \ln^2(x) \right],
$$

(11)
In comparison with the LO and NLO terms, the contribution of $\Delta\varphi^{(2)}_{M \perp}$ is mainly of an opposite sign and comparable in magnitude with the NLO in the middle region of $x$, see Fig. 1 (right panel).

$$
\langle x^0 \rangle_{M \perp} = \frac{N_c}{12\pi^2} \left[ 1 + a_s C_F \left( \frac{7}{3} + 2 L_B \right) + a_s^2 \beta_0 C_F \left( \frac{\pi^2}{6} - 12 \zeta_3 + \frac{383}{36} + 2 L_B - L_B^2 \right) \right], \quad (12)
$$

$$
\langle x^{-1} \rangle_{M \perp} = \frac{N_c}{4\pi^2} \left[ 1 + a_s C_F 2 (2 + L_B) + a_s^2 \beta_0 C_F \left( 2 \zeta_3 + \frac{19\pi^2}{18} - \frac{493}{36} + \frac{25 - 2\pi^2}{3} L_B - L_B^2 \right) \right], \quad (13)
$$

The $O(a_s^2 \beta_0)$ contribution to $\langle x^{-1} \rangle_{M \perp}$ in (13) is numerically tiny. The norm $\langle x^0 \rangle_M$ in Eq. (12) is in agreement with the result in [13] obtained for a correlator of the corresponding local currents $T^{\mu}_{(0)}$. The magnitude of $O(a_s^2 \beta_0)$ contribution in the norm (12) is moderate.

5. Conclusions

We briefly analyze the perturbative corrections $\Delta\varphi^{(2)}_{M \perp}$ to DAs of leading twist for pion and (longitudinally or transversely) polarized light vector mesons at order $O(a_s^2 \beta_0)$. To this end, we calculate vector-vector (axial-axial), $\langle V(A) V(A) \rangle$, and tensor-tensor $\langle TT \rangle$ correlators with the corresponding composite currents $V(A)$ and $T$ up to the order $O(a_s^2 \beta_0)$. The impact of these $\beta_0 N^2$ LO corrections is moderate, while the sign of the correction in the transverse case $\Delta\varphi^{(2)}_{M \perp}$ is opposite to the one of LO and NLO terms.

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