Contribution of single pion electroproduction to the generalized Gerasimov-Drell-Hearn sum rule for the deuteron

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The generalized Gerasimov-Drell-Hearn (GDH) sum rule is evaluated for the contribution of single pion production on the deuteron by explicit integration up to an energy of 1.5 GeV for both coherent and incoherent production. As elementary $\gamma^*N \rightarrow \pi N$ amplitude the MAID2003 model has been used. For incoherent production final state interaction is included in the final $NN$ and $\pi N$ subsystems. The resulting contribution to the generalized transverse GDH sum rule is considerably smaller than the negative contribution from the disintegration channel $d(e, e'\pi^0)$ which dominates the sum rule at low $Q^2$.

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I. INTRODUCTION

Because of difficulties in the microscopic description of strongly interacting particles within quantum chromodynamics, particular significance is attached to relations between observables obtained directly from general principles of quantum field theory, such as Lorentz invariance, unitarity, causality etc. These relations, connecting different physical aspects of a quantum system, are primarily of strong theoretical significance. Furthermore, they may be considered as a tool allowing one to check the quality of different microscopic models, in which the mentioned principles should be taken into account properly. One of such relations is the Gerasimov-Drell-Hearn (GDH) sum rule connecting the anomalous magnetic moment of a particle to the energy weighted integral over the beam-target spin asymmetry of its total photoabsorption cross section $I_{\pi}^{GDH}$. This sum rule has become of special interest within the last 10-15 years because of the substantial progress in the development of polarized beams and targets. Therefore, considerable success was achieved in measuring the spin asymmetry for the proton as well as for composite light nuclear systems over a large energy range, and even data for particular photoabsorption channels were reported. Besides the GDH sum rule for real photons, the generalized GDH sum rule for electron scattering has come into focus in recent years.

Although the existing analyses allow one to conclude that the GDH sum rule at $Q^2 = 0$ for nucleons can not be violated significantly, there are still points needed to be understood. The first one is the so called neutron puzzle. Whereas the validity of the GDH sum rule for the proton seems to be confirmed both theoretically and experimentally (at least within the experimental errors), the situation of the neutron is much less clear. Indeed, an evaluation of $I_{n}^{GDH}$ in [2] (see also calculations of Ref. [6] and [7]) points to a systematic deviation of about 20 % between the theoretical evaluation of the integral $I_{n}^{GDH}$ and its sum rule value. This fact is quite surprising, especially in view of the much better agreement for the proton within the same model.

The second point concerns a strong modification of the nucleon spin structure as one goes from low and moderate $Q^2$-regions, where the generalized GDH sum rule for the proton $I_{p}^{GDH}(Q^2)$ is predicted to be large and positive, to the deep inelastic scattering region, where the value of the integral becomes negative and scales as $I_{p}^{GDH}(Q^2) \sim -0.14/Q^2$ [8]. There are general indications that it is the resonance region which is mainly responsible for this sharp change of the GDH integral [8]. In this connection it is interesting to see whether the same strong dependence will be exhibited also by the neutron. Thus, the mentioned issues require an investigation of the generalized GDH sum rule for the neutron, so that, in view of the absence of free neutron targets, lightest nuclei, primarily deuteron and $^3$He, become of great relevance. With respect to nuclear targets, it is important to take into account disturbing effects from binding and final state interaction (FSI).

The GDH sum rule for the deuteron at $Q^2 = 0$ has been evaluated in [6] for all important channels, including photodisintegration, single and double pion photoproduction as well as eta photoproduction. These channels were found to nearly saturate the sum rule. The contribution of higher multiple meson final states, being quite important for the unpolarized cross section, is not expected to be very significant in the sum rule. This is primarily because of a slowly increasing phase space available for reactions with more then three particles in the final state. As a result, multiple pion production starts to come into play at rather high energies $\omega^{ab} > 1$ GeV so that the corresponding contribution to the GDH integral is suppressed by the weight $1/\omega^{ab}$. The second and probably more important reason...
is that with increasing multiplicity of the final reaction channels, the spin dependence of its amplitude is expected to become less and less pronounced, so that different spin configurations tend to appear with equal probability and thus are largely canceled in the sum rule.

With respect to the generalized sum rule of the deuteron, the contribution of the electrodisintegration channel \(d(e,e'\pi^0)np\) has been evaluated in Ref. [11]. The calculation was based on a conventional nonrelativistic framework using a realistic \(NN\)-potential and including contributions from meson exchange currents, isobar configurations and leading order relativistic terms. By integrating up to a maximal internal excitation energy of the final \(np\)-system \(E_{np} = 1\ \text{GeV}\), good convergence was achieved. The prominent feature of the electrodisintegration channel to the generalized GDH sum rule as function of the squared four-momentum \(Q^2\) is a pronounced deep negative minimum, \(I^\gamma_{d\rightarrow np}(Q^2) \approx -9.5\ \text{mb}\), at low \(Q^2 \approx 0.006\ (\text{GeV}/c)^2\) (see Fig. [10]) which is almost exclusively driven by the nucleon isovector anomalous magnetic moment contribution to the magnetic dipole transition to the \(1S_0\) scattering state. Above \(Q^2 = 0.8\ (\text{GeV}/c)^2\) the integral \(I_{d\rightarrow np}^\gamma(Q^2)\) approaches zero rapidly.

Apart from the aspects related to the neutron, the generalized GDH sum rule for the deuteron is interesting by itself. Indeed, as has been shown in in Ref. [4], an almost vanishing value for the GDH integral at \(Q^2 = 0\) according to the sum rule value \(I_{d}^\gamma = 0.65\ \text{mb}\) as dictated by the very small anomalous magnetic moment of the deuteron is provided by an almost exact cancelation between nucleon degrees of freedom (photodisintegration channel) and subnucleon degrees of freedom (mainly pion as manifest in single and double pion photoproduction). In view of the large negative contribution from the photodisintegration channel of about \(-380\ \text{mb}\), the mentioned cancelation has to eliminate the three leading decimals. This feature requires a consistent treatment of nucleon, pion and other subnucleon degrees of freedom.

In view of the above mentioned dramatic change with increasing \(Q^2\) of the contribution \(I_{d\rightarrow np}^\gamma(Q^2)\) from electrodisintegration to the generalized sum rule, going through a deep minimum, the natural question arises, whether this large negative contribution will be canceled again by the contribution of meson electroproduction similar to the case of real photons. Clearly, to that end the spin asymmetry of electroproduction on the deuteron must strongly increase above the pion production threshold. On the one hand, the strength of magnetic transitions, dominating pion production at lower energies increases with \(Q^2\), at least as long as a suppression of the long range mechanisms (resonance excitation) does not start to come into play in the region of high momentum transfers. On the other hand, with increasing \(Q^2\) the nuclear structure becomes more effective in leading to a decrease of the reaction rate via the nuclear form factor, which must be particularly pronounced in the case of a deuteron, having a rather extended structure. Thus the question about the behavior of \(I_{d}^\gamma(Q^2)\) at \(Q^2 > 0\) is by no means trivial and must be studied within a sufficiently refined model. The latter has to take into account correctly nuclear effects, such as Fermi motion, off-mass shell corrections and final state interactions.

The aim of the present paper is an evaluation of the leading contribution above pion production threshold to the generalized GDH integral \(I_{d}^\gamma(Q^2)\), namely incoherent and coherent single pion electroproduction, \(d(e,e'\pi)NN\) and \(d(e,e'\pi^0)d\). These reactions were considered in detail in Ref. [11]. Special attention had been given to polarization observables and to the role of \(NN\) and \(\pi N\) interactions in the final state. The calculation requires proper treatment of the elementary electroproduction reaction \(N(e,e'\pi)N\). The physical picture underlying the electroproduction in the region of low and medium energies is usually presented in terms of transitions from the nucleon to \(N^*\) and \(\Delta\) resonances. These have nonperturbative character and, therefore, need a phenomenological model for their description. In Ref. [11] the MAID2003 analysis [12] was used, which describes electroproduction of pions via excitation of \(S\)-channel resonances with nonresonant contributions from the nucleon poles as well as meson exchange in the \(t\)-channel. Utilizing MAID2003 developed up to a total c.m. energy \(W = 2\ \text{GeV}\), the calculation on the deuteron in [11] could be extended up to a photon lab energy \(\omega_{lab} = 1.5\ \text{GeV}\), thus covering the major part of the resonance region.

As for \(NN\) and \(\pi N\) FSI effects, it turned out that their influence is similar to that noted previously for the photoproduction processes in Ref. [6]. Namely, the most significant role is played by the \(np\) interaction in the \(^3S_1\) state in the neutral channel \(d(e,e'\pi^0)np\), resulting in a visible reduction of the corresponding impulse approximation (IA). In the charged channels, \(NN\) FSI leads to much smaller effects, about 1-2 \% in the first resonance region. The origin of such a different role of FSI in the charged and neutral channels was found in a spurious contribution of the coherent process to the incoherent process in IA. This spurious contribution is admixed unavoidably to the incoherent one if the \(np\) FSI is neglected, because of the nonorthogonality between the plane wave of the impulse approximation and the bound state wave function. After elimination of this spurious contribution by a projection technique, the remaining FSI effect is comparable in size to that seen in the \(\pi^+nn\) and \(\pi^-pp\) channels. The \(\pi N\) rescattering is insignificant over the whole energy region \(\omega_{lab} \leq 1.5\ \text{GeV}\). It is also worth noting that although in the unpolarized cross section FSI may safely be neglected at least not very close to the threshold, their role may appear to be important in polarization observables, especially in the target polarization, as was shown in Ref. [11].

In the next two Sections [III] and [IV] we give relevant formulas for the generalized GDH sum rule for the deuteron. In Sect. [V] the contribution to the sum rule from single pion electroproduction is presented. In Sect. [V] we summarize the results and present some conclusions.
II. THE GENERALIZED GDH SUM RULE

The generalized GDH sum rule is determined by the vector beam-target asymmetry \( A_{10}^V \) of the inclusive electron deuteron scattering cross section for longitudinally polarized electrons. For electroproduction the latter has the form \([11]\) (correcting some misprints)

\[
\frac{d^3 \sigma}{d E d \Omega_{\gamma'}} = \frac{\alpha_{\text{qcd}} k_{\gamma'} e}{Q^4} \left[ \rho_L F_{10}^{00} + \rho_T F_{10}^{00} + P_1^d \left( \rho_L' F_{11}^{11} \cos \phi_d + \rho_T L F_{11}^{11} \sin \phi_d \right) \right]
\]

\[
+ P_2^d \left( \rho_L F_{20}^{10} + \rho_T F_{20}^{10} \right) \alpha_{\text{qcd}} \approx \frac{k_{\gamma'}}{k_e} \left[ \rho_L F_{10}^{00} + \rho_T F_{10}^{00} + P_1^d \left( \rho_L' F_{11}^{11} \cos \phi_d + \rho_T L F_{11}^{11} \sin \phi_d \right) \right]
\]

\[
+ P_2^d \left( \rho_L F_{20}^{10} + \rho_T F_{20}^{10} \right) \alpha_{\text{qcd}} \approx \frac{k_{\gamma'}}{k_e} \left[ \rho_L F_{10}^{00} + \rho_T F_{10}^{00} + P_1^d \left( \rho_L' F_{11}^{11} \cos \phi_d + \rho_T L F_{11}^{11} \sin \phi_d \right) \right]
\]

\[
+ P_2^d \left( \rho_L F_{20}^{10} + \rho_T F_{20}^{10} \right) \alpha_{\text{qcd}} \approx \frac{k_{\gamma'}}{k_e} \left[ \rho_L F_{10}^{00} + \rho_T F_{10}^{00} + P_1^d \left( \rho_L' F_{11}^{11} \cos \phi_d + \rho_T L F_{11}^{11} \sin \phi_d \right) \right]
\]

(1)

with \( \alpha_{\text{qcd}} \) as electromagnetic fine structure constant and where incoming and scattered electron energies and momenta are denoted by \( E_\gamma, k_e \) and \( E_{\gamma'}, k_{\gamma'}, \) respectively. Furthermore, \( \rho_\alpha \) with \( \alpha \in \{L, T, LT, TT\} \) denote the virtual photon polarization parameters, \( P_1^d \) and \( P_2^d \) the vector and tensor deuteron polarization parameters, respectively, \( (\theta_d, \phi_d) \) the spherical angles of the deuteron orientation axis, and \( h \) the degree of longitudinal electron polarization. The kinematics is displayed in Fig. 1.

![FIG. 1: Kinematics of single pion electroproduction on the deuteron in the \( \gamma^* d \) c.m. system.](image)

The various form factors \( F_\alpha^{(\gamma)^{M}} \) are defined in \([11]\) and the reader is referred to this work for more details. They are functions of the energy transfer \( \omega \) in the \( \gamma^* d \) c.m.-system and the squared four momentum transfer \( Q^2 = -q^2 \). At the photon point, the transverse form factors are related to the corresponding asymmetries of the total photoproduction cross section as listed in eq. (83) of \([13]\)

\[
F_\alpha^{00} = \frac{\omega W}{\pi^2 E_d \sigma^0}, \quad F_\alpha^{10} = \frac{\omega W}{\pi^2 E_d \sigma^0}, \quad F_\alpha^{20} = \frac{\omega W}{\pi^2 E_d \sigma^0}, \quad F_\alpha^{22} = \frac{\omega W}{\pi^2 E_d \sigma^0},
\]

(2)

where \( \sigma^0 \) denotes the unpolarized inclusive photoproduction cross section, \( T_\alpha \) the tensor target asymmetry, \( T_\alpha^c \) the beam-target vector asymmetry for circularly polarized photons and \( T_\alpha^l \) the beam-target tensor asymmetry for linearly polarized photons. (N.B.: In the corresponding relations listed in eq. (60) of \([11]\) a factor \( \sigma^0 \) is missing on the right hand sides of \( F_\alpha^{20}, F_\alpha^{10}, \) and \( F_\alpha^{22} \).)

The various beam, target and beam-target asymmetries of inclusive scattering are defined by the following general form of the inclusive cross section

\[
\sigma_{\alpha,\pi}(h, P_1^d, P_2^d, \theta_d, \phi_d) = \sigma_{\alpha,\pi}^0 \left( 1 + P_1^d A_{\pi}^V (\theta_d, \phi_d) + P_2^d A_{\pi}^T (\theta_d, \phi_d) \right)
\]

\[
+ h [A_{\pi}^V (\theta_d, \phi_d) + P_1^d A_{\pi}^V (\theta_d, \phi_d) + P_2^d A_{\pi}^T (\theta_d, \phi_d)]
\]

(3)

where

\[
\sigma_{\alpha,\pi}^0 = \frac{\alpha_{\text{qcd}} k_{\gamma'}}{Q^4} \left( \rho_L F_{10}^{00} + \rho_T F_{10}^{00} \right)
\]

(4)
denotes the unpolarized inclusive scattering cross section. By proper choices of the polarization parameters \( h, P_1^d \), and \( P_2^d \), one can separate the various asymmetries. For example, the beam-target vector asymmetry is obtained by combining four different settings of the polarization parameters according to

\[
A_{cd}^V(\theta_d, \phi_d) = \frac{1}{4 h P_1^d \sigma^0_c} \left[ (\sigma_e(h, P_1^d, P_2^d, \theta_d, \phi_d) - \sigma_e(-h, P_1^d, P_2^d, \theta_d, \phi_d) \\
- \sigma_e(h, -P_1^d, P_2^d, \theta_d, \phi_d) + \sigma_e(-h, -P_1^d, P_2^d, \theta_d, \phi_d) \right].
\]  

(5)

Comparing (3) with (1), one can express the asymmetries in terms of form factors and kinematic quantities. For the vector asymmetry \( A_{cd}^V \) one obtains

\[
A_{cd}^V(\theta_d, \phi_d) = \frac{\alpha_{gcd}}{Q^4} \frac{k_{c'}}{k_e} \left( \rho' T^1_{10} \cos \theta_d + \rho' L T^2_{10} \cos \phi_d a^1_{10}(\theta_d) \right),
\]

(6)
yielding for \((\theta_d, \phi_d) = (0, 0)\), i.e. deuteron orientation axis parallel to \( \vec{q} \),

\[
A_{cd}^V(0, 0) = \frac{\alpha_{gcd}}{Q^4} \frac{k_{c'}}{k_e} \rho' T^1_{10}.
\]

(7)

The expression \( 2 \sigma^0_c A_{cd}^V(0, 0) \) describes exactly the total electroproduction cross section asymmetry for completely polarized electrons \((h = 1)\) and complete deuteron spin aligned parallel and antiparallel to the momentum transfer, which means \( P_1^d = \pm \sqrt{3/2} \), respectively, and \( P_2^d = 1/\sqrt{2} \). Because defining

\[
\sigma^{P/A}_{c,\pi} = \sigma_{c,\pi}(1, \pm \sqrt{3/2}, 1, \pm \sqrt{2}, 0, 0)
\]

\[
= \frac{\alpha_{gcd}}{Q^4} \frac{k_{c'}}{k_e} \left[ \rho_L(F_{L0}^{00} + \frac{1}{\sqrt{2}} F_{L2}^{00}) + \rho_T(F_{T0}^{00} + \frac{1}{\sqrt{2}} F_{T2}^{00}) \right] + \frac{\sqrt{3}}{\sqrt{2}} \rho' T^1_{10},
\]

(8)
one finds for the cross section asymmetry

\[
\sqrt{\frac{2}{3}} (\sigma^{P}_{c,\pi} - \sigma^{A}_{c,\pi}) = 2 \frac{\alpha_{gcd}}{Q^4} \frac{k_{c'}}{k_e} \rho' T^1_{10}.
\]

(9)

In view of the spin asymmetry for real photons

\[
\sigma^P_{\omega} - \sigma^A_{\omega} = 2 \sigma_0 T^c_{10}(\omega) = 2 \frac{\pi^2}{W q} E_d F_{T_{10}}^0(W, Q^2 = 0),
\]

(10)

using the expressions in Eq. (2), we introduce for transverse virtual photons the parallel and antiparallel spin aligned cross sections

\[
\sigma^{P/A}_{T,\gamma}(\omega, Q^2) = \frac{\pi^2}{W q} \left[ F_{T0}^{00}(\omega, Q^2) + \frac{1}{\sqrt{2}} F_{T2}^{00}(\omega, Q^2) \pm \frac{\sqrt{3}}{\sqrt{2}} \rho' T F_{T_{10}}^1(\omega, Q^2) \right],
\]

(11)

where \( \rho' T / \rho T \) describes the degree of circularly polarized virtual photons. Accordingly, we introduce as spin asymmetry for transverse virtual photons

\[
\Sigma_{\gamma T}(\omega, Q^2) = \sqrt{\frac{2}{3}} \frac{\rho T}{\rho' T} (\sigma^{P}_{T,\gamma}(\omega, Q^2) - \sigma^{A}_{T,\gamma}(\omega, Q^2)) = 2 \frac{\pi^2}{W q} F_{T_{10}}^1(W, Q^2),
\]

(12)

which coincides at the photon point with (111). Correspondingly, we take as extension of the GDH integral from real to virtual photons the definition

\[
I^{GDH}_d(Q^2) = \int_{\omega_{th}}^{\infty} \frac{d\omega}{\omega_{th}} \Sigma_{\gamma T}(\omega, Q^2) = \frac{2 \pi^2}{W q} \int_{\omega_{th}}^{\infty} \frac{d\omega}{\omega_{th}} E_d F_{T_{10}}^1(W, Q^2) g(\omega, Q^2),
\]

(13)
where $W$ denotes the invariant mass which is a function of $\omega$ and $Q^2$

\[ W = \omega + \sqrt{M_d^2 + q^2} = \sqrt{(2\omega_{\text{lab}} + M_d)M_d - Q^2} \]

(14)

with $M_d$ as deuteron mass. The threshold invariant mass is given by $W_{\text{th}} = 2M + m_\pi$ with $M$ and $m_\pi$ as nucleon and pion masses, respectively, and thus the threshold lab energy by $\omega_{\text{lab}} = (W_{\text{th}}^2 + Q^2 - M_d^2)/2M_d$. Furthermore, $E_d$ and $(\omega, q)$ denote the deuteron energy and the virtual photon four-momentum in the $\gamma^*d$ c.m. system, respectively. The factor $g(\omega_{\text{lab}}, Q^2)$ in (13) takes into account the fact, that the generalization of the GDH integral is to a certain extent arbitrary. The only restriction for this factor is the condition that at the photon point $Q^2 = 0$ one has

\[ g(\omega_{\text{lab}}, 0) = 1, \]

(15)

and that

\[ \lim_{\omega_{\text{lab}} \to \infty} g(\omega_{\text{lab}}, Q^2) = 1 \]

remains finite. As simplest extension we choose here $g(\omega_{\text{lab}}, Q^2) \equiv 1$. For the explicit integration over a finite range of $\omega_{\text{lab}}$ we introduce the finite GDH integral by

\[ I_{d}^{\text{GDH}}(Q^2, \omega_{\text{max}}) = 2\pi^2 \int_{\omega_{\text{lab}}^{\text{max}}}^{\omega_{\text{lab}}^{\text{max}}} d\omega \frac{E_d}{W} \frac{q}{q} F_T^{10}(W, Q^2). \]

(17)

Transforming (13) into an integral over $W$, using

\[ \omega_{\text{lab}} = \frac{1}{2M_d} (W^2 + Q^2 - M_d^2), \]

(18)

one obtains

\[ I_{d}^{\text{GDH}}(Q^2) = 4\pi^2 \int_{W_{\text{th}}}^{\infty} dW E_d(W, Q^2) \frac{F_T^{10}(W, Q^2)}{(W^2 + Q^2 - M_d^2)}, \]

(19)

where now $E_d$ and $q$, the three-momentum in the $\gamma^*d$ c.m.-system, have to be considered as functions of $W$ and $Q^2$, i.e.

\[ E_d(W, Q^2) = \sqrt{M_d^2 + q^2(W, Q^2)}, \]

(20)

\[ q(W, Q^2) = \frac{1}{2W} \sqrt{((W - M_d)^2 + Q^2)((W + M_d)^2 + Q^2)}. \]

(21)

\[ \text{III. THE REACTION AMPLITUDE} \]

In the present work the contribution of single pion production to the generalized GDH sum rule of (19) is evaluated by explicit integration up to a maximal lab virtual photon energy $\omega_{\text{max}} = 1.5$ GeV. The evaluation is based on the formalism developed in Ref. [11] which we will briefly review. The general form of the $T$-matrix is given by

\[ T_{\text{sm}, \mu m_d}(W, Q^2, p_\pi, \Omega_\pi, \Omega_p) = \langle (-\bar{p}) sm_s, \vec{p}_\pi | J_{\gamma \pi, \mu}(\vec{q}) | 1m_d \rangle = \sqrt{2\pi} \sum_L i^L \langle (-\bar{p}) sm_s, \vec{p}_\pi | O_\mu^L | 1m_d \rangle, \]

(22)

where $s$ and $m_s$ denote the total spin and its projection on the relative momentum $\vec{p}$ of the outgoing two nucleons, and $m_d$ correspondingly the deuteron spin projection on the $z$-axis as quantization axis. Furthermore, $\mu \in \{0, \pm 1\}$ enumerates the spherical current components with the provision that $J_{\gamma \pi, 0}$ is identified with the charge density. We use throughout the notation $\hat{L} = \sqrt{2L + 1}$. The kinematic quantities and the geometry is explained in Fig. [1]. The symbol $O_{\mu M}$ denotes charge ($O_{\mu M}^L$) and transverse multipoles ($E_{\mu M}^L$ and $M_{\mu M}^L$) according to

\[ O_{\mu M}^L = \delta_{\mu 0} C_{\mu M} + \delta_{\mu |1|} (E_{\mu M}^L + \mu M_{\mu M}^L). \]

(23)
Using a partial wave decomposition of the final states, one can separate the $\phi_\pi$-dependence

$$T_{sm,\mu m_d}(W, Q^2, p_\pi, \Omega_\pi, \Omega_p) = e^{i(\mu + m_d - m_\pi)\phi_\pi} t_{sm,\mu m_d}(W, Q^2, p_\pi, \theta_\pi, \theta_p, \phi_{\pi\pi}),$$

(24)

where the small $t$-matrix depends besides $W$, $Q^2$ and the pion momentum $p_\pi$ only on $\theta_\pi$, $\theta_p$, and the relative azimuthal angle $\phi_{\pi\pi} = \phi_p - \phi_\pi$. For further details see Ref. [11].

According to (13) the form factor $F_T^{10}$ is the relevant quantity which determines the generalized GDH sum rule. It is expressed in terms of the small $t$-matrix elements via integration over the final phase space

$$F_T^{10} = \frac{1}{\sqrt{6}} \int dp_\pi d\Omega_\pi d\Omega_p c(W, Q^2, p_\pi, \Omega_\pi, \Omega_p, \phi_{\pi\pi}) \sum_{sm_\pi} \left| t_{sm_\pi,11} \right| - \left| t_{sm_\pi,1-1} \right|,$$

(25)

where

$$c(W, Q^2, p_\pi, \theta_\pi, \theta_p, \phi_{\pi\pi}) = \frac{M^2 p_\pi^2 p_\pi^2}{8(2\pi)^4 E_\pi(E_{12p} + \frac{1}{2}p_\pi(E_1 - E_2) \cos \theta_{\pi\pi})}.$$

(26)
FIG. 3: Transverse spin asymmetry $\Sigma_{\gamma^* d \rightarrow \pi^+ n n}$ as function of $\omega_{\text{lab}}$ (left panel) and finite GDH integral $I_{\text{GDH}}^{\gamma^* d \rightarrow \pi^+ n n}$ as function of $\omega_{\text{max}}^{\text{lab}}$ (right panel) of $\pi^+$ electroproduction on the deuteron $d(e, e'\pi^+)nn$ for various constant squared four-momentum transfers $Q^2$.

denotes a kinematic phase space factor in which all kinematic quantities refer to the $\gamma^* d$ c.m. system ($E_\pi$ and $p_\pi$ pion energy and momentum, $E_1$ and $E_2$ nucleon energies, and $p$ relative nucleon momentum, see Fig. 1).

As already noted, we used MAID2003 to calculate the elementary pion electroproduction amplitude, which is sandwiched between the $NN$ initial and final state wave functions. The usual calculational method, taking into account the interaction between the final particles, follows the scheme

$$T = T^{IA} + T^{NN} + T^{\pi N},$$

where $T^{IA}$ corresponds to the pure spectator model, whereas the other two terms include $NN$ and $\pi N$ rescatterings treated up to the first order in the corresponding two-body $t$-matrices $t_{NN}$ and $t_{\pi N}$.

For the deuteron wave function as well as for the final two nucleon state we used the separable representation of the Paris potential from Ref. [14]. This model fits the $NN$-phases up to a lab kinetic energy of 330 MeV. Although in the final $NN$ subsystem also energies above this value appear in the calculation, the contribution of such events is insignificant as has been shown in [13], so that the implementation of the model [14] is justified. For the $\pi N$ scattering matrix we also used the separable representation from Ref. [14].
FIG. 4: Transverse spin asymmetry $\Sigma^{d \rightarrow \pi^0 np}$ as function of $\omega_{lab}$ (left panel) and finite GDH integral $I_{GDH}^{d \rightarrow \pi^0 np}$ as function of $\omega_{\text{lab max}}$ (right panel) of incoherent $\pi^0$ electroproduction on the deuteron $d(e,e'\pi^0)np$ for various constant squared four-momentum transfers $Q^2$.

IV. RESULTS FOR SPIN ASYMMETRY AND FINITE GDH INTEGRAL

In the left panels of Figs. 2 through 5 we present our results for the spin asymmetry $\Sigma_{\gamma^*d \rightarrow \pi^0 np}$ as function of the photon energy $\omega_{\text{lab}}$ for different values of the squared four-momentum transfer $Q^2$. The right panels of these figures exhibit the finite GDH-integral as function of the upper integration limit $\omega_{\text{lab max}}$ in order to check the convergence of $I_{GDH}^{\gamma^*d \rightarrow \pi^0 np}(Q^2, \omega_{\text{lab max}})$ within the restricted energy domain of the present work. The corresponding results on the asymmetry and the finite GDH-integral for the sum of all channels are exhibited in Fig. 6. Furthermore, we show in Fig. 7 a comparison of the transverse spin asymmetries of single pion production on nucleon and deuteron for $Q^2 = 0.01, 0.1$ and $1.0 \ (\text{GeV/c})^2$.

Before turning to the discussion we note that the impulse approximation $T^{IA}$ (first term in eq. (27)) provides quite an adequate description of pion photoproduction in the incoherent channels, whereas two-body mechanisms are of little importance, at least not very close to the threshold. At the same time, it is reasonable to expect that the amplitude $T^{IA}$ has only a weak sensitivity to the details of the deuteron wave function as long as $Q^2$ is not too large. Therefore, the total spin asymmetry in the incoherent channel should obey approximately the simple relation

$$\sigma_{T,\gamma^*}^{P} - \sigma_{T,\gamma^*}^{A} \approx (\sigma_{3/2}^{P} + \sigma_{3/2}^{n}) - (\sigma_{1/2}^{P} + \sigma_{1/2}^{n}),$$

(28)
which is in line with the assumption that the nucleon spins are aligned along the deuteron spin except for a small pollution by the presence of the D-state. The relation (28) rests furthermore on the assumption that Fermi motion, FSI and other two-body effects can be disregarded and furthermore interference effects between proton and neutron amplitudes can be neglected. However, already at the photon point as well as for low $Q^2$ values one notes significant deviations (see Ref. [6] and Fig. 7) for which the Fermi motion is mainly responsible. Moreover, in the low energy region and at higher values of $Q^2$ one may expect that short range mechanisms, where two-body effects are expected to become important, play a more and more significant role. Therefore, in these regions larger deviations from (28) may occur.

From Figs. 2 through 5 one readily notes, that in the region of low $Q^2$ the general energy dependence of the asymmetry becomes similar to that of photoproduction [6]. Just above the threshold, due to the large wave length of the virtual photon the main mechanism of the reaction is described in terms of dipole transitions, i.e. $E1$ and $M1$. At low energies the $E1$ amplitude leads to $s$-wave pion production primarily via the Kroll-Ruderman term with a small $d$-wave correction. As a consequence, $\pi^+$ and $\pi^-$ production (Figs. 2 and 3) is mainly governed by a strong $\sigma_{T,\gamma}$ contribution dominating over $\sigma_{P,\gamma}$, up to energies of about $\omega_{lab} = 200$ MeV. In this region the values of $\Sigma_{\gamma}$ for the charged channels $\pi^- pp$ and $\pi^+ nn$ are comparable in magnitude. The small difference is due to the different dipole
in agreement with the results shown in Figs. 2 and 3.

In the neutral channels (Figs. 4 and 5), due to the small pion mass resulting in a vanishingly small dipole moment of π0N, the E1 transition is strongly suppressed below the first resonance and the corresponding spin asymmetries Σγ∗d→π0np and Σγ∗d→π0d are comparable with zero. This property is seen at all values of Q2.

In the first resonance region the spin structure is mainly governed by the incoherent sum of M1 and E1 transitions originating from the P33(1232) electroexcitation and the Born terms, respectively. In the neutral channel π0np the electric transitions remain insignificant up to an energy ωlab = 700 MeV, where the D13(1520) resonance is exited via absorption of an E1 photon. An additional very small contribution comes from the electric E1 component of the convection current due to the Fermi motion of the bound nucleons. As a result, in the whole energy region up to ωlab = 500 MeV the electroproduction of π0 proceeds almost exclusively via the M1 transition to the P33(1232)
resonance, which is especially well seen in this channel. This mechanism is slightly enhanced by the nucleon pole terms in the s- and u-channels, coming from the magnetic coupling of the photon to the nucleon.

Starting from $\omega^{lab} = 600$ MeV, the behavior of $\Sigma_{\gamma^*}$ is mainly determined by the localization of the dipole $E1$ strength in the region of the $D_{13}(1520)$ and the quadrupole $E2$ transition via excitation of $F_{15}(1680)$. As a result, in $\pi^0 nn$ and $\pi^0 np$ the curves exhibit two peaks at $\omega^{lab} = 780$ MeV and 1 GeV. The latter is almost invisible in the $\pi^- pp$ channel due to a relatively weak coupling of the $F_{15}(1680)$ to $\gamma n$. Above the second resonance region the spin asymmetry remains small in all channels and demonstrates quite a smooth behavior.

The $Q^2$ evolution of $\Sigma_{\gamma^*}$ is partially determined by the $Q^2$ dependence of the spin asymmetry of the elementary nucleon reactions, which in our case is given by the MAID2003 parametrization used in the present work. In particular, nucleon Born terms and vector meson exchange are parametrized with a standard dipole form factor. It is worth noting, that above the $P_{33}(1232)$ resonance the strong background in the charged channels is to a large extent canceled in the spin asymmetry $\Sigma_{\gamma^*}$.

As for the resonance sector, the corresponding experimental information on the $Q^2$ dependence is still quite scarce, even for the transverse helicity components $A_\lambda$ with $\lambda = 1/2, 3/2$. In the region of low $Q^2$ one may hope that the largest contribution to the sum rule is still provided by the low lying nucleon resonances, in particular by the $P_{33}(1232)$ resonance whose internal spatial structure is quite well understood. This is however not the case for higher values of $Q^2$ where higher resonances start to come into play. Therefore the MAID2003 analyses still leaves room for some variation of $\Sigma_{\gamma^*}(\omega^{lab}, Q^2)$.

The $Q^2$ dependence of the $N \rightarrow P_{33}(1232)$ transition was studied in a wide range and is shown to have an almost constant ratio of the electric to the magnetic components. As a result, the contribution of $E2$ remains vanishingly
small and the relation

$$\frac{\sigma_{T,\gamma^*}^{P(M1)}}{\sigma_{T,\gamma^*}^{A(M1)}} \approx \frac{\sigma_{1/2}^{3/2}}{\sigma_{1/2}} \approx 3.$$  \hspace{1cm} \text{(30)}$$

holds over a wide region of $Q^2$.

Quite remarkable is the rapid decrease of $\Sigma_{\gamma^*}(Q^2)$ in the second and the third resonance region. This is especially well seen in the $\pi^+nn$ channel where, starting from about $Q^2 = 0.3$ (GeV/c)$^2$, the asymmetry becomes predominantly negative. This increasing relative contribution of the antiparallel component $\sigma_{T,\gamma^*}^A$, especially seen in $\pi^+$ photoproduction was also noted in Ref. [5]. The helicity amplitudes $A_{\lambda}$ ($\lambda = 1/2, 3/2$) of MAID give for the transitions $N \rightarrow D_{13}(1520)$ and $N \rightarrow F_{15}(1680)$ respectively

$$\left(\frac{A_{p(D_{13})}^{P(D_{13})}}{A_{p(D_{13})}^{A(D_{13})}}\right)^2 \approx 31 \left(48 \pm 36\right), \quad \left(\frac{A_{p(F_{15})}^{P(F_{15})}}{A_{p(F_{15})}^{A(F_{15})}}\right)^2 \approx 29 \left(79 \pm 64\right),$$  \hspace{1cm} \text{(31)}$$

where the corresponding PDG values from [16] are given in parentheses. As a result, at low $Q^2$ we see a strong dominance of the parallel component $\sigma_{T,\gamma^*}^P$. As is discussed in [5], with increasing virtuality of the photon, the amplitude $A_{3/2}$ drops faster than $A_{1/2}$, so that the resulting helicity asymmetry eventually runs through a zero. In the $\pi^+nn$ channel this effect even forces the integral $I_{GDH}^{\gamma^*d \rightarrow \pi^+nn}(Q^2)$ to become negative at about $Q^2 = 0.3$ (GeV/c)$^2$ (upper left panel in Fig. 8). Another reason for the predominance of $\sigma_{T,\gamma^*}^A$ at higher $Q^2$ should be a rather smooth $Q^2$ dependence of the $S_{11}(1535)$, so that this resonance, contributing exclusively to the antiparallel component, remains excited at rather large values of momentum transfer.

Finally, in the region $Q^2 > 0.5$ (GeV/c)$^2$, in the absence of the main contribution from the first and partially the second resonance region, higher resonance states start to dominate the GDH integral. As a result, its value becomes...
FIG. 9: Total finite GDH integral of single pion electroproduction on the deuteron \(d(e, e'\pi)\) (solid curve) and on neutron plus proton (dashed curve) integrated up to \(\omega_{\text{lab}}^{\text{max}} = 1.5\) GeV as function of squared four-momentum transfer \(Q^2\).

FIG. 10: Total GDH integral of single pion electroproduction on the deuteron \(d(e, e'\pi)\) (solid curve), electrodisintegration \(d(e, e'np)\) (dotted curve), and on neutron plus proton (dashed curve) as function of squared four-momentum transfer \(Q^2\). The curves representing pion production (solid and dashed curves) are magnified by a factor 10.
sensitive to the behavior of the cross section at higher energies. An important consequence of this fact is, as already noted above, that the integral \( I_{CDH}^{\omega_{\pi N}}(Q^2) \) has not reached convergence for these higher \( Q^2 \)-values within the energy region of our study. Therefore, one needs in future studies to extend the elementary multipole analysis at least up to lab energies above 2 GeV. For the neutral channels in comparison to \( \pi^\pm \) the convergence appears to be better for the low \( Q^2 \)-values whereas again one notes at higher \( Q^2 \) that convergence has not been reached.

The resulting values of the GDH integral as function of \( Q^2 \) in different channels are presented in Fig. 12. As is noted in [17], the maximum in \( I_{\gamma d-\pi NN}^{GDH}(Q^2) \) for \( \pi^+ nn \) and \( \pi^- pp \) seen at about 0.05 (GeV/c)\(^2\) is simply due to a different \( Q^2 \)-dependence of the Kroll-Ruderman and the \( P_{33}(1232) \) terms. Namely, the \( P_{33}(1232) \) contribution dominating the parallel component \( \sigma_{P,\gamma}^{2} \), at lower \( Q^2 \) changes quite slowly up to a value where \( \sqrt{Q^2} \) becomes comparable to the \( \rho \) meson mass (see the experimental results compiled in [18]). Above this point, the \( P_{33}(1232) \) form factor visibly suppresses the \( P_{33}(1232) \) resonance peak. On the contrary, the \( \sigma_{A,\gamma}^{3} \), component, dominated by the Kroll-Ruderman term, behaves like the inverse momentum of the incident virtual photon, and thus rapidly vanishes. This interplay leads to the slow increase of \( I_{\gamma d-\pi NN}^{GDH}(Q^2) \) up to about \( Q^2 = 0.05 \) (GeV/c)\(^2\) with a subsequent fall-off forced by the strong \( Q^2 \) dependence of the \( P_{33}(1232) \) form factor.

Coherent pion photoproduction on the deuteron in the first resonance region is almost totally determined by the spin independent part of the \( M1 \) transition to the \( P_{33}(1232) \) resonance. As a result, the cross section is strongly dominated by the \( \sigma_{P,\gamma}^{2} \) component. The resonance \( P_{33}(1232) \) is especially well seen in this channel up to quite high \( Q^2 \) values, and the GDH integral [3] is saturated at already rather low \( \omega_{\pi N} \). At the same time, the extended structure of the deuteron results in quite a rapid fall-off of \( I_{\gamma d-\pi NN}^{GDH}(Q^2) \) with increasing \( Q^2 \) via the deuteron form factor.

Finally, we show in Fig. 11 a comparison of the GDH integral of single pion production with the one of electrodisintegration. One readily notes for the disintegration channel the pronounced deep negative minimum near \( Q^2 = 0.006 \) (GeV/c)\(^2\), which we had mentioned in the introduction, whereas the pion production channel exhibits below \( Q^2 = 0.1 \) (GeV/c)\(^2\) a nearly constant contribution of, however, much smaller size, and then a rapid fall-off at higher \( Q^2 \) values. Thus the disintegration channel remains the dominant feature for the generalized GDH sum rule.

V. SUMMARY AND CONCLUSIONS

The beam-target spin asymmetry of single pion electroproduction on the deuteron for transverse virtual photons and the associated generalized Gerasimov-Drell-Hearn integral have been evaluated by explicit integration up to an energy \( \omega_{\pi N} = 1.5 \) GeV and for squared momentum transfers between 0.001 and 1.2 (GeV/c)\(^2\). Whereas below pion production threshold the main contribution (negative) to the transverse spin asymmetry from electrodisintegration comes from the strong \( M1 \) transition to the resonant \( ^1S_0 \) scattering state near the disintegration threshold (so-called antibound state), mainly driven by the large nucleon anomalous magnetic moment with additional meson exchange current and relativistic contributions [10], above the threshold single nucleon mechanisms, primarily single pion electroproduction start to dominate the spin asymmetry.

According to our results, the contribution of pion production to \( I_{\gamma d-\pi NN}^{GDH}(Q^2) \) coming from the energy region below \( \omega_{\pi N} = 1.5 \) GeV is to a large extent saturated by resonance electroexcitations. The calculation based on the MAID2003 model for the elementary pion production amplitude shows that at low \( Q^2 \) the dominant contribution comes from the \( P_{33}(1232) \) resonance. As the virtuality of the photon increases, the role of higher resonances tends to be more and more important. This effect is in addition amplified by the damping of the \( P_{33}(1232) \) at higher \( Q^2 \) as well as by a rapid increase of the relative contribution of the antiparallel component \( \sigma_{A,\gamma}^{3} \) above the first resonance. As a result, we find a strong \( Q^2 \) dependence of the integral \( I_{\gamma d-\pi NN}^{GDH}(Q^2) \) so that at \( Q^2 = 1 \) (GeV/c)\(^2\) it comprises only about 5 \( \mu b \).

In general, our calculation shows that for \( Q^2 \leq 0.5 \) (GeV/c)\(^2\) the major contribution of single pion electroproduction to \( I_{\gamma d}^{GDH}(Q^2) \) is contained in the region \( \omega_{\pi N} \leq 1.5 \) GeV. For higher values of \( Q^2 \) the finite GDH integral does not exhibit good convergence in the considered energy range pointing to the need for an extension of the present analysis beyond the energy region considered in this work. Moreover, very likely one would also need to consider two pion production.

As for the question about the behavior of the total sum rule \( I_{\gamma d}^{GDH}(Q^2) \) comprising disintegration and pion production at finite \( Q^2 \), we see that with increasing \( Q^2 \) the single pion photoproduction does not compensate the rapid change of the negative contribution of the nucleonic channel \( I_{\gamma d-\pi NN}^{GDH}(Q^2) \), so that the resulting total integral \( I_{\gamma d}^{GDH}(Q^2) \) exhibits almost the same strong \( Q^2 \) dependence as the disintegration channel alone. It is very unlikely that multiple meson production, which is so far not included in this study, will be able to visibly change the value of \( I_{\gamma d}^{GDH}(Q^2) \). Therefore, the strong dominance of the disintegration channel, leading to a large negative value of \( I_{\gamma d}^{GDH}(Q^2) \) below
about $Q^2 = 1 \text{ (GeV/c)}^2$ seems to remain a special feature of the generalized GDH sum rule of the deuteron.

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