A method for the asteroseismic analysis of \( \beta \) Cephei stars is presented and applied to the star \( \nu \) Eridani. The method is based on the analysis of rotational splittings and their asymmetries using differentially rotating asteroseismic models. Models with masses around 7.13 \( M_\odot \), and ages around 14.9 Myr, were found to fit better in 10 of the 14 observed frequencies, which were identified as the fundamental radial mode and the three \( \ell = 1 \) triplets \( g_1, p_1, \) and \( p_2 \). The splittings and asymmetries found for these modes recover those provided in the literature, except for \( p_2 \). For this last mode, all its non-axysymmetric components are predicted by the models. Moreover, opposite signs of the observed and predicted splitting asymmetries are found. If identification is confirmed, this can be a very interesting source of information about the internal rotation profile, in particular in the outer regions of the star. In general, the seismic models that include a description for shellular rotation yield slightly better results as compared with those given by uniformly rotating models. Furthermore, we show that asymmetries are quite dependent on the overshooting of the convective core, which makes the present technique suitable for testing the theories describing the angular momentum redistribution and chemical mixing due to rotationally induced turbulence.

Key words: stars: evolution – stars: individual (\( \nu \) Eridani) – stars: interiors – stars: oscillations (including pulsations) – stars: rotation – stars: variables: other

1. INTRODUCTION

The \( \beta \) Cephei star \( \nu \) Eridani (HD 29248) is nowadays one of the most in-depth studied stars. Classified as a B2III star, it presents a relatively simple internal structure characterized by a large convective core. The \( \kappa \) mechanism located in the metal opacity bump situated around 2 \( \times 10^5 \) K (Dziembowski & Pamiatnykh 1993; Gautschy & Saio 1993) drives its oscillations. In addition, the pulsation periods of \( \beta \) Cephei stars, varying in the range of 3–8 hr, make them suitable for detecting and analyzing oscillation frequencies. Several photometric and spectroscopic multisite observational campaigns for \( \nu \) Eridani have been set up since 2002 with subsequent frequency analysis from spectroscopy (Aerts et al. 2004) and from photometry (Handler et al. 2004; Jerzykiewicz et al. 2005). In these latter works, two independent low-frequency, high-order \( g \) modes were detected. As a consequence, \( \nu \) Eridani can also be classified as a SPB star. In terms of light curves, these multisite observations constitute the largest time series ever collected for a \( \beta \) Cephei star. Nowadays, \( \nu \) Eridani presents the richest oscillation spectrum (14 independent frequencies) of the \( \beta \) Cephei-type class. Such a privileged scenario for asteroseismology has led several authors to analyze its oscillation spectrum and perform seismic models of the star.

Several attempts have been carried out to provide a plausible seismic model which explains the observed frequencies of \( \nu \) Eridani. In Ausseloos et al. (2004, hereafter ASTA04), a massive exploration of standard and non-standard stellar models was undertaken in order to fit the oscillation data. The authors showed that an increase in the relative number fraction of iron throughout the whole star, or a large decrease in the initial hydrogen abundance, made the stellar models satisfy all the observational constraints; in particular, the modes around the fundamental radial mode are predicted to be unstable.

Pamyatnykh et al. (2004) performed a seismic analysis of the oscillation spectrum of \( \nu \) Eridani taking the excitation of modes into account. In that work, only three frequencies were fitted, failing to reproduce mode excitation in the broad observed frequency range of the \( (\ell = 1, p_2) \) modes, associated with the highest frequency peak in the spectrum. Nevertheless, they also inferred some properties of the internal rotation rate using the rotational splittings of two dipole \( (\ell = 1) \) modes identified as \( g_1 \) and \( p_1 \). In particular, their results suggest that the mean rotation rate in the core and the \( \mu \)-gradient zone is about 3 times higher than in the envelope for their two standard models fitting the three aforementioned frequencies.

Recently, Dziembowski & Pamyatnykh (2008) have analyzed the impact of considering uncertainties in the opacity and element distributions on the interpretation of \( \nu \) Eridani’s oscillation spectrum. No satisfactory explanation of the low-frequency modes was found. Moreover, the authors concluded that some enhancement of the opacity in the driving zone is required.

The rotational splitting asymmetries of \( \nu \) Eridani have also been studied under different hypotheses. In particular, Dziembowski & Jerzykiewicz (2003) suggested that the asymmetry of the \( \ell = 1 \) triplet (around 5.64 days \(^{-1}\)), as measured by van Hoof (1961), could be explained by two principal effects: the quadratic effects of rotation and a strong magnetic dipole field of the order of 5–10 kG. Such a magnetic field was searched for by Schnerr et al. (2006) using spectropolarimetry with no success.

Motivated by these results, the present work aims at performing a complete modeling of \( \nu \) Eridani, taking into account the effect of rotation up to second order, with the special feature of considering the presence of a radial differential rotation in the seismic modeling. To do so, a method based on the analysis of rotational splittings and their asymmetries is discussed.
The paper is organized as follows. Section 2 describes the modeling procedure and provides details of both the evolutionary models and the oscillation spectra computation. In Section 3, the different sources for constraining the stellar parameters are compared, which includes a stability analysis. Then, Sections 4 and 5 explain the method presented here and discuss its application to the particular case of \( \nu \) Eridani. Finally, conclusions and final remarks are written in Section 6.

2. SEISMIC MODELING

The seismic modeling described below consists in the computation of evolutionary models and their corresponding adiabatic and non-adiabatic oscillation spectra. This is described in the following sections.

2.1. Equilibrium Models

To theoretically characterize \( \nu \) Eridani, we build equilibrium models that are representative of the star with the evolutionary code CESAM (Morel 1997). In particular, models taking into account first-order effects of rotation are constructed. Such models are the so-called pseudo-rotating models, whose spherically averaged contribution of the centrifugal acceleration is included by means of an effective gravity \( g_{\text{eff}} = g - A_{\text{eff}}(r) \), where \( g \) is the local gravity, \( r \) is the radius, and \( A_{\text{eff}}(r) = 2/3 r \Omega^2(r) \) is the centrifugal acceleration of matter elements. This spherically averaged component of the centrifugal acceleration does not change the order of the hydrostatic equilibrium equations (Kippenhahn & Weigert 1990). The non-spherical components of the centrifugal acceleration (which are not included in the equilibrium models) are included in the adiabatic oscillation computations (see the following section) by means of a linear perturbation analysis according to Soufi et al. (1998; see also Suárez et al. 2008). It is possible to evaluate the impact of a differential rotation using two simple hypotheses when prescribing the rotation profile (Suárez et al. 2006): (1) instantaneous transport of angular momentum in the whole star (global conservation) which thus yields a uniform rotation or (2) local conservation of the angular momentum (shellular rotation), except in the convective core whose rotation is assumed to be rigid. In both cases, no mass loss is considered at any evolutionary stage, that is, the total angular momentum is assumed to be conserved. These hypotheses represent extreme cases, so the reality is presumably somewhere in between. In fact, similar rotation profiles have been found when analyzing the evolution of giant stars including rotationally induced mixing of chemical elements and transport of angular momentum (Maeder & Meynet 2004).

Input physics have been adequately chosen for main sequence B stars. Particularly for the mass range treated in this work (7–13 \( M_\odot \)), the CEFF equation of state (Christensen-Dalsgaard & Daempen 1992) is used, in which the Coulombian correction to the classical EFF (Eggleton et al. 1973) has been included. The opacity tables are taken from the OPAL package (Iglesias & Rogers 1996). For the metal mixture, the abundances given by Grevesse & Noels (1996) are used.

A weak electronic screening is assumed, which is valid in the evolutionary stages considered in this work (see Clayton 1968 for more details). For adiabatic stellar models, the Eddington’s \( T(r) \) law (gray approximation) is considered for the atmosphere reconstruction. For non-adiabatic stellar models, the atmospheres are reconstructed using the Kurucz equilibrium atmospheric models (Kurucz 1993) from a specific Rosseland optical depth until the last edge (around \( \tau = 10^{-3} \)) of the star is reached.

Convection is treated with the mixing-length theory (Böhm-Vitense 1958), which is parameterized with \( \alpha_{\text{MLT}} = 1/H_p \), where \( l \) is the mean path length of the convective elements and \( H_p \) is the pressure scale height. In addition, we use the overshoot parameter, defined as \( d_{\text{ov}} \) (\( l_{\text{ov}} \) being the penetration length of the convective elements).

The grid of equilibrium models has been constructed with steps of 0.002, 0.01, and 0.05 dex in the metallicity \( Z \), overshoot parameter \( d_{\text{ov}} \), and mass \( M \), respectively.

2.2. Oscillations

Adiabatic eigenfrequencies of selected pseudo-rotating models described in the previous section have been computed with the oscillation code FILOU (see Tran Minh & Léon 1995; Suárez 2002). In this code, oscillation frequencies are obtained by means of a perturbative method taking into account up to second-order effects of rotation. In the case of near-degenerate frequencies, i.e., when two or more frequencies are close to each other (\( \omega_{\text{nlm}} \sim \omega_{\text{nl'm}} \)), the corrections for near degeneracy are included. As detailed in Suárez et al. (2006), the oscillations computation also takes into account the presence of a radial differential rotation profile under the form

\[
\Omega(r) = \tilde{\Omega}(1 + \eta_0(r)),
\]

where \( \tilde{\Omega} \) represents the angular rotational velocity at the surface and \( \eta_0(r) \) is a radial function. This rotation profile is equivalent to the shellular rotation profile obtained with the pseudo-rotating models described in the previous section.

Concerning the instability computations, non-adiabatic theoretical observables are obtained using the GRACO code (Moya et al. 2004). This code solves the stellar pulsation equation in a non-adiabatic, non-rotating frame by dividing the star in two parts: (1) the interior, by means of the non-adiabatic equations described in Unno et al. (1989), and (2) the atmosphere, taking into account the interaction with the pulsation as prescribed by Dupret et al. (2002).

2.3. Comparison of Numerical Seismic Packages

As is widely known, in stellar modeling, and in particular in the field of stellar seismology, the use of different codes with different numerical techniques can be crucial for the correct interpretation of seismic data (see Moya et al. 2008). ASTA04 compared two seismic packages: the LIEGE PACKAGE, which comprises the evolutionary code CLES (Scuflaire et al. 2007) plus the oscillation code LOSC (Boury et al. 1975), and the WARSAW–NEW JERSEY PACKAGE (by Dziembowski & Jerzykiewicz 2003), which comprises the WARSAW–NEW JERSEY code and its corresponding oscillation code. Following ASTA04, these codes are compared (through a mass–metallicity relation) with our seismic modeling package, the GRANADA PACKAGE, which is composed of the evolutionary code CESAM (Morel 1997) and the oscillation codes GRACO (Moya et al. 2007) and FILOU (Suárez et al. 2008), described in Sections 2.1 and 2.2, respectively. Hereafter, these three packages are called LP, WP, and GP, respectively.

Similar to what was done in ASTA04, the mass \( M \), metallicity \( Z \), overshoot parameter \( d_{\text{ov}} \), and initial hydrogen content \( X_i \) are then varied to fit the two observed frequencies, \( f_1 \) and \( f_2 \) (Table 1) by using non-rotating models. This procedure yields a mass–metallicity relation for each \( d_{\text{ov}} \) value which is compatible with the similar study described in ASTA04 (see Figure 1). It is found that, under similar conditions, i.e. for a given metallicity,
α_{MLT} and d_{ov}, GP predicts higher mass values, around 0.25 \ M_\odot. These differences increase slightly when d_{ov} increases. Such a different behavior can be explained by the different treatments of the overshooting implemented in the evolutionary codes. Indeed, for the overshooting description, the LÎŒGE evolutionary code only takes the density variations (in the overshooted region) into account, which slightly affects the temperature gradient, whereas CESAM considers an additional restriction by imposing that the real temperature gradient must be equal to the adiabatic one, i.e. \nabla = \nabla_{ad}. This would imply that either transport of heat is purely radiative, or it is efficiently transported outward from the stellar core through convective movements (as a result of the overshooting). This constitutes an interesting challenge for asteroseismology because the oscillation modes are sensitive to the physical description of the \mu-gradient zone.

3. CONSTRAINING STELLAR PARAMETERS

The modeling of any star entails the constraint of its stellar parameters. The choice of the set of free parameters to be fixed generally depends on the observational material available. In the case of \nu Eridani, the following space of free parameters was chosen:

\[ \mathcal{P} = \mathcal{P}(M, t, d_{ov}, Z, \Omega), \]

where \( M \) is the stellar mass, \( t \) is the age, \( d_{ov} \) is the overshooting parameter, \( Z \) is the metallicity, and \( \Omega \) is the angular rotational velocity. From photometric and spectroscopic observations, we search for an estimate of the star location in the Hertzsprung–Russell (H–R) diagram. This allows us to constrain the metallicity and to have an estimate of the evolutionary stage of the star. Stability analysis significantly reduces the region of the H–R diagram in which representative models can be searched.

Then, the fitting of four of the observed frequencies permits us to better constrain the mass, metallicity, evolutionary stage (age), and overshooting parameter of the star. The small observed \( \nu \text{sin} i \) of \nu Eridani allows the use of non-rotating models for this exercise. However, further model constraining makes it necessary to take the stellar rotation into account. In particular, seismic models including shellular rotation profiles are used (see Section 2). Finally, analysis of the rotationally split modes and their asymmetries allows us to make a refined search for representative models of the star.

3.1. Locating \nu Eridani in the H–R Diagram

In order to locate the star in the H–R diagram we followed the works by Morel et al. (2006) and De Ridder et al. (2004), based on high-precision spectroscopy. The error box shown in Figure 2 takes into account the results reported in both papers.

On the other hand, the location of stars in the H–R diagram depends on both the rotational velocity of the star and the inclination angle with respect to the observer, \( i \). A technique to estimate and correct for the effect of fast rotation on the determination of fundamental parameters for pulsating stars is described in Michel et al. (1999). This technique was then refined by Pérez Hernández et al. (1999), who applied it to \( \delta \) Scuti stars in clusters. A first consequence that can be extracted from those works is that rotation increases the size of the uncertainty box of stars in the H–R diagram (when obtained...
predicts and better fits the unstable observed frequencies \( f \) velocity of \( \nu \) inclination of the star). For \( \nu \) Eridani, a projected rotational velocity of \( v \sin i \sim 16 \) km s\(^{-1}\) has been observed. This value has been derived by Morel et al. (2006) from high-resolution spectroscopy, taking into account the broadening due to oscillations. In principle, it can be considered as a slow rotator so that the additional uncertainties coming from the effect of rotation can be considered within the already large photometric uncertainties.

3.2. Instability Predictions

Once the star is located in the H–R diagram, models which predict the observed frequencies to be unstable (considering the constraints on physical parameters given above) are searched for.

To do so, no specific ad hoc modification of the iron mixture throughout the star is proposed. The search for the best models is then enhanced by using the \( M–Z \) relations presented in Section 2.3. We recall that lines in Figure 1 represent the models which fit simultaneously both \( f_1 \) and \( f_1 \). These models constitute a first guess to the best solution. Finally, our best solution is given by those models that also fit \( f_6 \) and \( f_6 \). As is the case with ASTA04, when using standard solar mixture for the metallicity, an initial hydrogen content of \( X_1 \sim 0.50 \) is required in order to predict the unstable observed frequencies. In Figure 2, the different evolutionary tracks shown have been computed using \( X_1 = 0.50 \) in the zero-age main sequence (ZAMS). From our grid of models (see Section 2.1), the best model (filled circle in Figure 2) was selected to be the one which predicts (and better fits) the unstable observed frequencies \( f_1, f_3, f_6, \) and \( f_6 \). This best non-rotating model is characterized by a mass of 7.13 \( M_\odot \), a solar metallic mixture (with an initial hydrogen content of \( X_1 = 0.50 \)), and by the physical parameters \( \alpha_{\text{MLT}} = 1 \), \( d_{\nu} = 0.28 \) (NR1 in Table 2). Note that rotation has not been taken into account in the instability predictions described above. The lack of theories describing the effect of rotation on mode stability makes it difficult to estimate this effect for \( \nu \) Eridani. Nevertheless, as stated by Pamyatnykh (1975), mode stability depends predominantly on the effective temperature of the models (see Suárez et al. 2007 for an interesting discussion), whose variations due to rotation are expected to be small (due to the low rotational velocity of the star).

The results of the stability analysis obtained for that model are depicted in Figure 3 in terms of growth rate as a function of the oscillation frequency. As expected, in the frequency region around the fundamental radial mode, the results obtained for the different spherical degrees \( \ell \) are quite similar, which implies that this parameter cannot be discriminated.

4. PROCEDURE

As discussed in Sections 2.1 and 2.2, the small rotational velocity of the star makes it plausible to initially adopt the parameters of the non-rotating models which were found to be representative of the star. In particular, in order to work with models located in the H–R error box which predict the unstable observed frequencies, a value of \( X_1 \sim 0.5 \) for the initial hydrogen fraction is kept (see the previous section). Indeed, such a value is rather unrealistic. This can be solved either by considering an ad hoc iron enhancement in the driving zone (as done by Pamyatnykh et al. 2004), or by modifying (uniformly throughout the stellar interior) the relative number fraction of iron (as done by ASTA04). However, the modeling techniques used here do not allow us to make such modifications. Instead, following ASTA04, we simulate the metallicity change by modifying the initial hydrogen fraction. This yields, similarly to ASTA04 (see Table 1 in that paper), models around 7 \( M_\odot \) and \( Z = 0.018–0.019 \). Similarly to previous works, standard models, i.e. those with \( X_1 \sim 0.7 \), within the H–R error box present masses higher than 8 \( M_\odot \), \( Z \sim 0.015 \), for similar surface rotational velocities. Such variations with respect to the non-standard models have no significant impact either on the rotation profile or on the splitting asymmetries.

A systematic search for representative models within the error box was then performed by locally varying the mass, initial rotational velocity, and age of the models. The mass was varied around 7.10 \( M_\odot \), in particular from 7 to 7.20 \( M_\odot \), in steps of 0.01 \( M_\odot \). Rotation was considered under two main assumptions: uniform rotation and differential rotation (see Section 2 for more details). For both assumptions, the rotational velocities considered ranged from 5 to 20 km s\(^{-1}\) at the stellar surface. Then, the modeling was refined by analyzing the rotational splittings and their asymmetries. Rotational splittings are defined as

\[
S = \frac{1}{2}(v_{+1} - v_{-1})
\]

and asymmetries of these splittings are defined as

\[
A = v_{-1} + v_{+1} - 2v_0,
\]

where sub-indices \( \pm 1 \) represent the value of the azimuthal order \( m \) for a given \( \ell \). The information provided by the asymmetries can be completed by the semisplittings, which corresponds with \( \Delta^+ = v_{+1} - v_0 \), and \( \Delta^- = v_0 - v_{-1} \). Both quantities are related such that Equation (4) transforms into

\[
A = \Delta^+ - \Delta^-.
\]

Moreover, this analysis also permits the extraction of information about the internal rotation profile of the star since, as we discuss in the following sections, \( S \) and \( A \) are both sensitive to changes in the rotation profile.
Models with a mean density of about $\bar{\rho} = 0.064 \text{ g cm}^{-3}$ were found to better fit the observed frequencies. The models within $\pm 0.05 \ M_\odot$ of the mass value calibrated using non-rotating models yield similar results, but for slightly different ages. Due to the extremely small rotational velocity, it is plausible to assume the same $\alpha_{\text{MLT}}$ and $d_\nu$ parameters which were calibrated by the non-rotating models. Therefore, only the mass, age, and initial rotational velocity of the models were varied.

Models were then selected to fit at least the observed frequencies $f_1$ (identified as the fundamental radial mode) and the triplet $(f_3, f_4, f_2)$, identified as $g_1$. With these criteria, a model with a mass of $7.13 \ M_\odot$, a rotational velocity (in the surface) of about $7 \text{ km s}^{-1}$, and an age of about $14.9 \text{ Myr}$ (which corresponds with $\log T_{\text{eff}} = 4.365$) was found to better fit the observed frequencies. As expected, when rotation is taken into account, the stellar parameters of the models are similar to those of the non-rotating best model (NR1). For each type of rotation, i.e. uniform and shellular rotation, the models better matching the observed frequencies, splittings, and asymmetries are UR1 and SR1, whose characteristics are summarised in Table 2 (their corresponding list of frequencies is reported in Table 3), and whose internal rotation profiles are depicted in Figure 4. Figure 5 shows the evolution of the theoretical growth rates $\eta$ as a function of the oscillation frequency. Each curve represents the growth rate obtained for oscillation modes with different degrees ($\ell$ ranging from 0 to 3). The lowest frequencies correspond to the SPB-type pulsation frequencies. Only the frequencies around the fundamental radial mode are predicted to be unstable (positive growth rate), which are those kept for the present investigation.

5. RESULTS AND DISCUSSION

Models with $\pm 0.05 \ M_\odot$ of the mass value calibrated using non-rotating models yield similar results, but for slightly different ages. Due to the extremely small rotational velocity, it is plausible to assume the same $\alpha_{\text{MLT}}$ and $d_\nu$ parameters which were calibrated by the non-rotating models. Therefore, only the mass, age, and initial rotational velocity of the models were varied.

Models were then selected to fit at least the observed frequencies $f_1$ (identified as the fundamental radial mode) and the triplet $(f_3, f_4, f_2)$, identified as $g_1$. With these criteria, a model with a mass of $7.13 \ M_\odot$, a rotational velocity (in the surface) of about $7 \text{ km s}^{-1}$, and an age of about $14.9 \text{ Myr}$ (which corresponds with $\log T_{\text{eff}} = 4.365$) was found to better fit the observed frequencies. As expected, when rotation is taken into account, the stellar parameters of the models are similar to those of the non-rotating best model (NR1). For each type of rotation, i.e. uniform and shellular rotation, the models better matching the observed frequencies, splittings, and asymmetries are UR1 and SR1, whose characteristics are summarised in Table 2 (their corresponding list of frequencies is reported in Table 3), and whose internal rotation profiles are depicted in Figure 4. Figure 5 shows the evolution of the theoretical growth rates $\eta$ as a function of the oscillation frequency. Each curve represents the growth rate obtained for oscillation modes with different degrees ($\ell$ ranging from 0 to 3). The lowest frequencies correspond to the SPB-type pulsation frequencies. Only the frequencies around the fundamental radial mode are predicted to be unstable (positive growth rate), which are those kept for the present investigation.

### Table 2

| Model | Z   | $M/M_\odot$ | $R/R_\odot$ | $\log T_{\text{eff}}$ | $\log(L/L_\odot)$ | $d_\nu$ | $\omega_\nu$ | $\omega_\Omega$ | Age (Myr) | $X_c$ | $r_c/R$ |
|-------|-----|-------------|-------------|------------------------|-------------------|--------|-------------|---------------|-----------|-------|--------|
| NR1   | 0.019 | 7.13        | 5.704       | 4.364                  | 3.778             | 0.3923 | 0.28        | 0             | 0         | 14.82 | 0.139  |
| SR1   | 0.019 | 7.13        | 5.714       | 4.364                  | 3.777             | 0.3923 | 0.28        | 0.258         | 0.681     | 14.90 | 0.138  |
| SR2   | 0.019 | 7.13        | 5.727       | 4.362                  | 3.775             | 0.3918 | 0.24        | 0.301         | 0.778     | 14.60 | 0.129  |
| UR1   | 0.019 | 7.13        | 5.511       | 4.368                  | 3.808             | 0.3909 | 0.28        | 0.377         | 0.377     | 15.80 | 0.159  |

Notes. The different columns are, from left to right: the model identification, the metallicity $Z$, the mass $M$, the radius $R$, the logarithm of the effective temperature $T_{\text{eff}}$ (in K), the logarithm of the gravity $\log g$ (in cgs), the logarithm of the luminosity (cgs), the overshooting parameter (in the H scale), the rotational frequency of the surface $\omega_\nu$ (in $\mu$Hz), the rotational frequency of the core $\omega_\Omega$ (in $\mu$Hz), the age (in Myr), the hydrogen abundance in the core $X_c$, and the radius of the convective core $r_c$.

5.1. Analysis of Rotational Splittings and Asymmetries

In Figure 6, the predicted semisplittings (left column) and asymmetries (right column), are compared with the corresponding observed values given in Table 4. For all panels in
values given by the best models SR1 and UR1 (see Table 2).

Notes. The model A does not take rotation into account, hence only the frequencies of the \( m = 0 \) components are reported.

Figure 6, the shaded vertical regions indicate the range of \( m \) frequencies of the

Table 3

| Mode   | \( \nu_{\text{NR},0} \) | \( \nu_{\text{SR1},0} \) | \( \nu_{\text{SR2},0} \) | \( \nu_{\text{UR},0} \) |
|--------|----------------|----------------|----------------|----------------|
| F0     | 5.75818 | 5.74936 | 5.72332 | 5.77681 |
| \( \beta_1 \), -1 | 5.63099 | 5.63402 | 5.67979 | 5.64512 |
| \( g_1 \), 0 | 5.61397 | 5.65885 | 5.62652 |
| \( p_1, -1 \) | 6.26192 | 6.31067 | 6.27800 |
| \( p_1, 0 \) | 6.24442 | 6.26865 | 6.25494 |
| \( p_1, 1 \) | 6.21785 | 6.26212 | 6.23102 |
| \( p_2, -1 \) | 7.87975 | 7.87980 | 7.94832 |
| \( p_2, 0 \) | 7.88829 | 7.87398 | 7.85263 | 7.91898 |
| \( p_2, 1 \) | 7.84899 | 7.82361 | 7.88731 |

Table 4

| Mode   | \( \Delta^+ \) | \( \Delta^- \) | \( \Delta^\theta \) |
|--------|----------------|----------------|----------------|
| \( g_1 \) | 0.016630 | -0.01723 | -0.000508 |
| \( p_1 \) | -0.0203 | -0.0012 |
| \( p_2 \) | 0.0161 | -0.0156 | 0.0005 |
| \( p_3 \) | -0.01 \times 10^{-4} | 0.02 \times 10^{-4} |

Notes. Quantities are given in days\(^{-1}\). Uncertainties, \( \delta \), are calculated from data in Jerzykiewicz et al. (2005).

Figure 5. Evolution of the theoretical oscillation frequencies corresponding, from top to bottom, to the \( \ell = 1 \) triplets \( p_2, p_1 \), and \( g_1 \), for the selected 7.13 \( M_\odot \) model. Horizontal lines represent the observed frequencies, bottom to top, \( f_4, f_6 \), and \( f_8 \), identified as the \( m = 0 \) components of the triplets \( p_2, p_1 \), and \( g_1 \), respectively.

\( \log T_{\text{eff}} = 4.35 \). In general, the evolution of the semisplittings for both types of rotation is found to be quite different. This is somehow expected since g modes are very sensitive to variations of the rotational velocity of the core. In fact, it was shown by Suárez et al. (2006) that a shellular rotation profile modifies significantly the radial displacement eigenfunctions, especially for g and mixed modes. This implies that the presence of shellular rotation may affect both the rotational splitting itself and its asymmetry. The general definition of the first-order rotational splitting kernel can be written as

\[
\mathcal{K} = \left( \frac{2 y_{01} z_0 + z_0^2}{y_{01}^2 + \Lambda z_0^2 - 2 y_{01} z_0} \right) \rho_0 r^4 \int_0^R \left( y_{01}^2 + \Lambda z_0^2 \right) \rho_0 r^4 \, dr
\]

(6)

where \( \Lambda = \ell (\ell + 1) \), and y and z represent the vertical and horizontal displacement normalized eigenfunctions, respectively. The second term within the square brackets accounts for the presence of shellular rotation through \( v_\theta \), defined in Equation (1). When a uniform rotation is considered, this term becomes null. In Figure 7 such kernels are depicted for the \( g_1, p_1 \), and \( p_2 \) triplets. Notice that, when considering a shellular rotation profile, a bump in the energy distribution near the \( \mu \)-gradient zone (see Figure 4) comes up, for \( g_1 \), at the expense of the energy of the outer layers, which could explain the
Figure 6. Evolution of the theoretical semisplittings (left column) and asymmetries (right column) as a function of the effective temperature (logarithmic scale) for the selected $7.13 \, M_\odot$ model with a rotational velocity of 7 km s$^{-1}$, approximately. The observed values are depicted with horizontal lines, and the corresponding observational uncertainties are represented by horizontal shaded bands. Each panel row shows the results, from top to bottom, for the $\ell = 1$ triplets $g_1$, $p_1$, and $p_2$, respectively. In the left panels, solid lines represent the positive semisplittings $\Delta^+$, and dashed lines the negative ones $\Delta^-$. For the asymmetries, the results obtained from the UR and SR models are represented by dashed and solid lines, respectively. The shaded vertical region indicates the range of effective temperatures defined by the models SR1 and UR1 (more details in Section 5.1).

different behavior of the semisplittings predicted by the UR and SR models.

Concerning the asymmetries, $A_{g_1}$ are predicted to diminish for decreasing effective temperature (Figure 6, top right panel). In other words, asymmetries decrease while the star evolves. For UR models, such a decrease is more rapid than for the SR models, and the asymmetries fit the observed value at $\log T_{\text{eff}} \sim 4.357$, which represents a difference of $\sim 400$ K with respect to the models that fit the observed frequencies (shaded area). On the other hand, the asymmetries predicted by the SR models never exactly fit the observed value. In the shaded region, the deviation of the asymmetry, defined as $\epsilon_A = A_{g_1} - A_{g_1}^o$, is found to be $6 \times 10^{-3}$ days$^{-1}$ for the SR models whereas $\epsilon_A = 10^{-4}$ days$^{-1}$ for the UR models. Such deviations are respectively two and three orders of magnitude larger than the observed asymmetry uncertainty.

As for the $g_1$ semisplittings, better results are found for cooler models, especially for UR models around $\log T_{\text{eff}} = 4.357$, and for SR models near $\log T_{\text{eff}} = 4.35$. In any case, for low-order g and p modes, asymmetries are sensitive to variations of the rotation profile near the core. Indeed, as shown by Suárez et al. (2006), the analytical form of $A$ (using a perturbative theory) can be written as the second-order term

$$A = -\frac{6 m^2 \Omega^2}{4 \Lambda - 3 \omega_0 \mathcal{J}_c},$$

(7)

where $\Omega$ is the rotational velocity at the stellar surface and $\omega_0$ is the unperturbed oscillation frequency. The $\mathcal{J}_c$ integral contains
First-order rotational splitting kernel, from top to bottom, for the $g_1$, $p_1$, and $p_2$ modes for the selected uniformly rotating models (UR, dashed line) and differentially rotating models (SR). For the latter models, two $d_0$ values are considered: 0.28 (continuous line) and 0.24 (dot-dashed line). Note that the position of the zeros and maxima for the UR models is slightly shifted with respect to those of the SR models. This can be explained by small differences in the radii of both models, which, as expected, principally affects the $g_1$ and $p_1$ split modes.

...a complex combination of structure and oscillation terms which are modified by the rotation profile and its derivatives. Analysis of this term is definitely necessary to construct simplified kernels for $A$ (work in progress). Such kernels will help us to better understand the behavior of the asymmetries and especially their sensitivity to variations of the internal rotation profile.

5.1.2. The $p_1$ Triplet

For $p_1$, the predicted curves of $|\Delta^+|$ remain lower than those of $|\Delta^-|$ in the whole range of effective temperature studied, which is compatible with the observations (Figure 6, middle left panel). Note that, in the shaded region, the SR models predict absolute values for the semisplittings closer to the observed ones ($\varepsilon_{\Delta^+} \sim 10^{-4}$ days$^{-1}$) than those predicted by the UR models, for which $\varepsilon_{\Delta^-} = 4 \times 10^{-3}$ days$^{-1}$. However, this situation is reversed for the splitting asymmetries (Figure 6, middle right panel). In particular, around log $T_{\text{eff}} = 4.365$, the asymmetries predicted by the UR models ($\varepsilon_A = 6 \times 10^{-4}$ days$^{-1}$) are slightly closer to the observed values than those predicted by the SR models ($\varepsilon_A = 2 \times 10^{-4}$ days$^{-1}$). Even so, such differences are of the order of magnitude of the observational uncertainty of $A_{p_1}$, which makes it difficult to discriminate between both types of rotation. Contrary to the $g_1$ results, cooler models do not fit the observations better.

5.1.3. The $p_2$ Triplet

In the case of $p_2$, the predicted semisplittings are larger than the observed value in the whole range of effective temperatures, except for log $T_{\text{eff}} \sim 4.35$, for which SR predictions are almost coincident with the observations (Figure 6, bottom left panel). In the shaded region $\varepsilon_{\Delta^-} = 6 \times 10^{-3}$ days$^{-1}$ for the SR models and $\varepsilon_{\Delta^+} = 1.6 \times 10^{-2}$ days$^{-1}$ for the UR models. Similarly to the $g_1$ case, the best results are given by the SR models for effective temperatures around the cooler limit of the photometric uncertainty box (log $T_{\text{eff}} = 4.35$). Furthermore, it is worth noting that, contrary to the $p_1$ and $g_1$ cases, the observed semisplitting $|\Delta^-|$ is smaller than $|\Delta^+|$, whereas both SR and UR models predict $|\Delta^-| > |\Delta^+|$ (Figure 6, bottom left panel). This results in a positive observed asymmetry, whereas both UR and SR models predict negative values (Figure 6, bottom right panel). In the vicinity of log $T_{\text{eff}} = 4.365$ (shaded region), the difference between the observed and predicted asymmetries is about $\varepsilon_A = 10^{-3}$ days$^{-1}$ ($\varepsilon_A = 1.5 \times 10^{-3}$ days$^{-1}$ for the SR models and $\varepsilon_A = 2.8 \times 10^{-3}$ days$^{-1}$ for the UR models), which represents a difference of one order of magnitude with respect to the observed value. Such an apparently marginal contradiction between predictions and observations could be a consequence either of (1) an incorrect mode identification, that is, the observed frequencies concerned do not belong to the rotationally split mode, or (2) the use of a wrong description for the rotation profile, particularly in the outer shells of the star (note the small influence of the selected rotation profile near the $\mu$-gradient zone, Figure 7). A priori, none of these possibilities can be discarded. In order to solve this problem, improvements on both the observations and modeling are required. From the observational side, an improvement of accuracy with which the concerned observed frequencies are
determined might help to confirm the observed asymmetry and its sign. Moreover, the detection of additional frequencies (e.g., with the help of space missions) may provide new insight into the current mode identification.

From the theoretical side, the second possibility given above is related to angular momentum redistribution, which plays an important role. In particular, the balance between rotationally induced turbulence and meridional circulation generates mixing of chemicals and redistribution of angular momentum (Zahn 1992), which affects the rotation profile and the evolution of the star. The present technique is, therefore, especially suitable for testing that theory by providing estimates for the coefficients of turbulence using only asteroseismic observables. This can be illustrated by artificially modifying the physical conditions beyond the convective core, which can be done by varying the overshooting parameter.

As expected, such variations principally affect the low-order $g_1$ and $p_1$ (see Figure 7 for a comparison between the rotational kernels of UR, SR ($d_{ov} = 0.28$), and SR ($d_{ov} = 0.24$) models). In Figure 8, the results for the asymmetries given in Figure 6 (right column), which were computed with an overshooting parameter of $d_{ov} = 0.28$, are compared with those obtained from SR models computed with $d_{ov} = 0.24$. The best model found for this overshooting parameter value is SR2 (see Tables 2 and 3). In particular, a variation in $d_{ov}$ of 0.04 results in differences in the asymmetry of the order of those found between UR and SR ($d_{ov} = 0.28$) models, i.e. a few $10^{-4}$ days$^{-1}$ for $g_1$ and $p_1$, and about $10^{-4}$ days$^{-1}$, which represents almost one order of magnitude smaller than the difference between the results yielded by the UR and SR ($d_{ov} = 0.28$) models. Moreover, in the case of $p_1$, the SR ($d_{ov} = 0.24$) models fit the observed asymmetry in the limits of the observational uncertainties.

Furthermore, recent theoretical studies (L. Andrade et al. 2008, in preparation) seem to indicate that even rough variations of the rotation profile may modify the asymmetries of the split modes. This modification can be as important as to change the sign of the asymmetries. According to this, the frequencies around $p_2$ cannot be discarded as belonging to a rotationally split mode. However, if confirmed, the presence of such different asymmetries in the oscillation spectrum is very interesting, since they may significantly constrain the models providing information about the structure and rotation profile in the zones where they have more amplitude.

6. CONCLUSIONS

An asteroseismic analysis of the $\beta$ Cephei star $\nu$ Eridani is presented, with focus on the study of the internal rotation profile. To this aim, a new method is presented based on the analysis of rotational splittings and their asymmetries. Some of the most updated asteroseismic modeling techniques are used, in particular an analysis of mode stability and improved descriptions for rotation effects. Regarding the latter, the so-called pseudo-rotating models are used, which consider radial differential rotation profiles (shellular rotation) in both the evolutionary models and adiabatic oscillations computations. This represents an important qualitative step with respect to previous theoretical works.

The present work is divided into two parts. In the first part, a comparison of the different numerical packages (Liège, Warsaw–New Jersey, and Granada packages) was performed (the latter has been used in the present work). This comparison was performed using a mass–metallicity relation of the models fitting two of the observed frequencies. In that case, important
differences in mass, up to 0.25 $M_\odot$ between Liège models and ours, were found. Interestingly, differences between ASTA04’s models and ours can be explained by the different treatments of the overshooting implemented in the evolutionary codes. ASTA04’s models only take into account density variations in the overshooted region, which slightly affects the temperature gradient. On the other hand, our models were constructed considering an additional restriction by imposing that the real temperature gradient must equal the adiabatic gradient. Physically, this implies that either the heat is transported efficiently outward from the stellar core through convective movements due to the overshooting (the latter case), or purely by radiation (ASTA04’s treatment). This constitutes a very interesting challenge for asteroseismology, since the oscillation modes (low-order g and p modes) are sensitive to the physical description of the $\mu$-gradient zone.

Then, the previous exercise is extended to four frequencies with mode excitation included. In that case, models were also constrained to match the fundamental radial mode and three $m = 0$ splitting components. Non-standard models built with a significant decrease of the initial hydrogen abundance, $X_i = 0.50$, were necessary to match and excite the modes. This provided models similar, but not identical, to those found by ASTA04 using a solar iron relative abundance, and the same initial hydrogen abundance. The remaining differences between ASTA04’s models and ours indicate that, for this level of the model accuracy, the present modeling still depends on the core overshooting. This parameter may change the physical conditions beyond the convective core. In that region, other physical processes take place, such as the mixing of chemical elements due to rotation. In particular, the balance between rotationally induced turbulence and meridional circulation generates mixing of chemicals and redistribution of angular momentum (Zahn 1992). This affects the rotation profile and the evolution of the star.

Secondly, the method here presented studies the asymmetries of the split modes in order to refine the modeling and, more importantly, provide information about the rotation profile of the star. To do so, pseudo-rotating models were built using the physical parameters provided in the first part of the work. Models with masses around 7.13 $M_\odot$, and ages around 14.9 Myr, were found to better fit 10 of the 14 observed frequencies, which were identified as the fundamental radial mode and the three $\ell = 1$ triplets $g_1$, $p_1$, and $p_2$. For these modes, a comparison between the observed and predicted splittings and their asymmetries was performed. Two types of rotation profiles were considered: uniform rotation and shellular rotation profiles. Differences between predictions and observations were found to be of the order of $5 \times 10^{-3}$ and $10^{-4}$ days$^{-1}$, for the rotational splittings and their asymmetries, respectively. For this last mode, none of the selected models reproduce either the splittings, generally larger than the observed ones, or the asymmetries, whose predictions have the opposite sign than the observed values. Although this result might indicate that the frequencies around the $p_2$, $m = 0$ mode do not belong to this rotationally split mode, other possibilities cannot be discarded. In fact, a wrong physical description of the rotation profile may be responsible for such a peculiar result, particularly in the outer regions of the star. This result is very important because, up to now, none of the physical phenomena in which rotation plays a role predict variations of the rotation profile in that region. Furthermore, it is shown that asymmetries are quite dependent on the overshooting of the convective core. Therefore, the method presented here is suitable for testing the theories describing the angular momentum redistribution and chemical mixing due to rotationally induced turbulence.

In general, the seismic models which include a description for shellular rotation yield slightly better results as compared with those given by uniformly rotating models. Even so, further improvements are necessary in order to better constrain the modeling of $\nu$ Eridani. In particular, efforts should be focused in searching for a better description of the rotation profile, for which a detailed study of asymmetries is required. This may be enhancing by analyzing other $\beta$ Cephei stars with larger rotational velocities, so that splittings and asymmetries are not of the same order as the frequency uncertainties.

J.C.S. acknowledges support by the “Instituto de Astrofísica de Andalucía” by an I3P contract financed by the European Social Fund and from the Spanish “Plan Nacional del Espacio” under project ESP2007-65480-C02-01. P.J.A. acknowledges financial support from a “Ramon y Cajal” contract of the Spanish Ministry of Education and Science. C.R.L. acknowledges financial support from an “Ángeles Alvarino” contract of the “Xunta de Galicia,” local government.

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5 See the explanation given above when matching only two frequencies.
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