Tensor-Optimized Few-Body Model for s-Shell Nuclei

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Received: 20 September 2011 / Accepted: 24 December 2011 / Published online: 27 April 2012
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Abstract We propose a tensor-optimized few-body model (TOFM) in the few-body framework with bare nucleon–nucleon interaction. The physical concept of TOFM comes from the tensor-optimized shell-model (TOSM), which is applicable for the study of medium and heavy nuclei. The TOSM wave function describes the deuteron-like tensor correlation and provides a good reproduction of the binding energy with the bare nucleon–nucleon interaction. Using the spirit of the TOSM approximation, we show the performance of TOFM for s-shell nuclei. It is found that the TOFM can account for the contribution of the tensor interaction very well and almost reproduces the total energy and various energy components as compared with rigorous few-body calculations. The relative correlation function is also provided to improve the performance of TOSM for the study of heavy nuclei.

1 Introduction

It is important to describe nuclear structure by using bare nucleon–nucleon (NN) interaction [1, 2]. The difficulty of solving nuclear ground states by using the NN interaction is the presence of the strong short range repulsion and the medium range tensor interaction caused by the pion exchange. These features of the NN interaction have to be handled in any theoretical frameworks for a quantitative account of nuclear states. In recent years, very important methods were proposed for the description of finite nuclei. The tensor optimized shell model (TOSM) is a method to treat the strong tensor interaction in the 2p-2h model space of the shell model basis [3, 4]. The strong tensor interaction acting on two nucleons in a spin-saturated shell model state ought to excite them into two particle states. Hence, it is a minimum requirement to take 2p-2h excitations in order to treat the tensor interaction, which provides a large attraction to form nucleus.

A theoretical study was then performed by Myo et al. [3] to use a bare NN interaction in TOSM for 4He and the numerical results were compared with those of few-body methods. They used the central correlation

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part of the unitary correlation operator method (UCOM) [5] in order to treat the short range repulsion in the NN interaction. The comparison was quantitatively very successful. Hence, the TOSM can be used for the discussion of nuclear structure of medium and heavy nuclei. However the tensor matrix element comes out to be about 80 % of that of few-body methods, and the binding energy was about 3 MeV smaller than that one. Then, it is important to clarify the origin of the difference between the TOSM and the rigorous calculations and what should be done in order to improve the TOSM description of nuclei.

On the other hand, we have a powerful few-body technique to describe light nuclei by using the relative coordinates. It is much easier to take into account the tensor interaction and the short range repulsion. However one of the reasons of the difficulty is a large variational model space in which the whole energy is minimized. Hence, it may be a good idea to introduce the spirit of the TOSM approximation in the few-body framework. This idea corresponds to take the S-wave configuration as the basis and add a single S-wave in a relative coordinate and do not take double and/or triple D-wave configurations. The purpose of the present study is then to develop a tensor-optimized few-body model using the spirit of TOSM and compare with results of rigorous calculations in a few-body framework and to obtain a good approximate method to study larger mass nuclei.

2 Tensor Optimized Few-Body Model

We would like to state first the formulation of TOFM briefly. On the basis of the TOSM approximation, we only have to take all the states connected by one operation of the tensor interaction and take enough Gaussian functions to express both the tensor and short range correlations. We should end up introducing the following tensor-optimized wave function for \( A = 2, 3 \) and 4 body systems by taking the essential features of rigorous calculations in the few-body method. Hence, we write a few-body wave function as a linear combination of \( S \) and \( D \) wave components,

\[
|\Psi\rangle = |\Psi_S\rangle + |\Psi_D\rangle.
\]

Here, \( |\Psi_D\rangle \) should contain the \( Y_2 \)-function only once. Particularly the most essential D-wave state is to introduce a \( Y_2 \)-function in the Jacobi coordinate \( x_1 = r_1 - r_2 \) and perform all the necessary permutations so that all the particle pairs profit the use of the strong tensor interaction. The S-wave and D-wave components for s-shell nuclei with \( A \leq 4 \) are written as

\[
|\Psi_{S,D}\rangle = \sum_{i=1}^{N_{S,D}} C_{S,D}^{B} |\chi_{LM}(B_{S,D}, \{x_i\})\rangle (\chi_s(\{\sigma_i\}) \chi_t(\{\tau_i\})).
\]

\( \chi_s(\{\sigma_i\}) \) represents the spin(iso-spin) wave function. On the other hand, the spatial wave function is written as [6]

\[
\psi_{LM}(B_{S,D}(x_i)) = \exp(-\frac{1}{2} \tilde{B}_{S,D} x)^T Y_{LM}(\tilde{u}x)
\]

Here, \( x \) represents the relative coordinate vector \( x = (x_1, x_2, \ldots) \) and \( \tilde{B}_{S,D} x \) means the short-hand notation of \( \sum_{ij}^{N_p} B_{S,D}^{ij} x_i \cdot x_j \) in the correlated gaussian basis.

In addition, we introduce the representation of the global vector \( \tilde{u}x \) for the basis wave functions in the same way as the Niigata group [6]. The global vector \( \tilde{u}x \) is defined as the linear combination of the vector \( x \) as \( \tilde{u}x = \sum_{i}^{N_p} u_i x_i \). We obtain the wave function and energy in a few-body framework by taking minimization of the total energy. The ranges of the spatial wave functions \( B_{S,D} \) and their amplitudes \( C_{S,D}^{B} \) are chosen variationally. We work out numerical calculations using the stochastic variational method (SVM) of the Niigata group [6], where the gaussian ranges \( B_{S,D} \) are generated randomly [7].

3 Numerical Results with TOFM

We show numerical results in TOFM and compare with the rigorous few-body calculation for \( A = 2, 3 \) and 4 systems and the TOSM calculation of Myo et al. for \(^4\text{He} \) [3] by using the AV8’ potential. We show in Table 1 the total energy and various energy components as the kinetic energy, the central interaction energy, the tensor
Table 1 Various energy components in unit of MeV with the AV8' interaction

| Nucleus      | Energy | Kinetic | Central | Tensor | LS    |
|--------------|--------|---------|---------|--------|-------|
| Deuteron     | −2.23  | 19.95   | −4.49   | −16.64 | −1.03 |
| $^3$H(TOFM)  | −7.54  | 46.67   | −21.98  | −30.47 | −1.95 |
| SVM[6]       | −7.76  | 47.57   | −22.49  | −30.84 | −2.00 |
| $^4$He(TOFM) | −24.08 | 95.53   | −54.61  | −60.95 | −4.05 |
| SVM[3]       | −22.30 | 90.50   | −55.71  | −54.55 | −2.53 |
| SVM[1]       | −25.92 | 102.35  | −55.23  | −68.32 | −4.71 |

interaction energy and the spin-orbit interaction energy for the deuteron, $^3$H and $^4$He. We do not include the Coulomb energy in this comparison. The present result is compared with the rigorous SVM calculation by Suzuki et al. [6]. As for $^4$He, we also compare the results with TOSM. We would like to discuss the comparison of TOFM with SVM for $^3$H and $^4$He. The TOFM results are compared almost perfectly with the SVM results for $^3$H. For $^4$He, the binding energy is close to the total energy of SVM. The central interaction energy and the spin-orbit energy are quite close to those of SVM. On the other hand, the tensor component and the kinetic energy are slightly underestimated due to the small-model space of TOFM. Of course the full values are obtained by adding all the finite angular momentum components in the variational wave function [6,8]. As for the comparison with the TOSM calculation for $^4$He, the present calculation of the energy value is better than the TOSM result [3]. Large differences are found in the matrix elements of the kinetic energy and the tensor interaction. This difference should mean that the TOSM calculation can be improved by taking a more general UCOM correlation function.

4 Correlation Function

It is very important to see correlation functions of the S-wave light nuclei in order to see the behavior of wave functions at short distance. It is also very important to compare with the correlation function of UCOM, the function form of which is extracted from the short range behavior of wave functions obtained with an effective interaction as the MT-V interaction [5]. The MT-V interaction has a short range repulsion without the tensor interaction [9]. We define the correlation function as

$$C_{S,D}(r) = \frac{1}{4\pi r^2} \langle \Psi_{S,D} | \delta(|r_1 - r_2| - r) | \Psi_{S,D} \rangle$$  \hspace{1cm} (4)$$

Then, we would like to show the S-and D-wave correlation functions $C_S(r)$ and $C_D(r)$ as functions of the relative distance $r$ for $A = 2, 3$ and 4 nuclei in Fig. 1. As for S-wave, we see a dip structure below 1 fm reflecting the presence of the strong short range repulsion. The D-wave components are found significant and similar among the three nuclei. Since all the correlation functions among the three nuclei look similar, we normalize the correlation functions to those of $^4$He and show them in Fig. 2. We see essentially the same short range behaviors below 1 fm for these three nuclei for both the S-wave and D-wave components. We have to obtain correlation functions as those found here even for heavy nuclei in order to optimize the role of the tensor interaction and the short range repulsion.
We would like to discuss here the improvement of the TOSM results where the short range correlation function is treated by UCOM. We calculate $C_{S}(r)$ in the few-body framework with the MT-V interaction [9] for $^{4}$He and compare the result with $C_{S}(r)$ of the AV8$'$ interaction in Fig. 3. We see quite a large difference between the two correlation functions particularly at short distance ($r < 1$ fm). The short range behaviors of the correlation function depend largely on the properties of interactions. It would be very interesting therefore to modify the correlation function in UCOM in the study of heavy nuclei in the TOSM description in order to improve the TOSM results.

5 Conclusion

We have formulated a tensor-optimized few-body model (TOFM) in the spirit of TOSM. The TOFM approximation makes the variational space much smaller that the one of the rigorous calculation. We have calculated $A=2$, $3$ and $4$ body systems in TOFM and compared with the rigorous calculations. As for $A=3$, we can get the good energy reproduction of the same results as the full model space calculation. As for $A=4$, we again obtain good reproduction of the rigorous results, but TOFM slightly underestimates the tensor interaction with single $Y_2$ component.

We have calculated correlation functions $C(r)$ in the short distance for $A=2$, $3$ and $4$ systems. The short range behaviors of the correlation functions for both the S-wave and D-wave components are very similar among the three nuclei. We have compared S-wave correlation function $C_{S}(r)$ of the MT-V interaction with AV8$'$ interaction. The large difference between these S-wave correlation functions means that the short range correlation function used in UCOM should be studied further for the calculation in finite nuclei in the TOSM framework. The present study is very encouraging to extend our study for nuclei with $A \geq 5$ in TOFM and to describe medium and heavy nuclei using the TOSM approximation with better description of the short range correlation.
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