Modeling a broadband detector for an experiment that measures the speed of gravity over short distances

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Abstract: In order to investigate the behavior of gravitational signals while travelling through a medium an experiment was designed, aimed at measuring the speed of these signals over short distances. The experiment contains 2 sapphire devices that behave as a detector, which are suspended in vacuum and cooled down to 4.2 K. The amplitude of the detecting device is measured by an ultralow, phase-noise microwave signal that uses resonance in the whispering gallery modes. Since sapphire has a quite high mechanical Q, the detection band is expected to be small, thus reducing the detection sensitivity. A new shape for the detecting device is presented in this work, yielding a detection band of several hundred Hertz. With the aid of a Finite Element Program the normal mode frequencies of the detector are determined.

1. Introduction
The Brazilian research group GRAVITON is dedicated to the study of gravity, with gravitational waves consisting of its main area of investigation. As neutron stars are important candidate sources of gravitational waves, the group devotes part of its goals to study pulsars[1]. The announcement of the first direct detection of gravitational waves in history happened in 2016 after a series of experimental runs planned in 2010 [2,3,4]. Gravitational waves had then a very strong indirect evidence of its existence through the observation of binary pulsar systems like PSR B1913+16 (also known as PSR J1915+1606, PSR 1913+16, and the Hulse-Taylor binary), whose orbital period is decreasing with time, a phenomenon explained theoretically with high precision assuming the emission of gravitational waves [5]. The first attempts to directly detect gravitational waves date from the early 1960's [6], using resonant-mass gravitational wave detectors [7,8,9,10].

GRAVITON’s efforts toward the direct detection of gravitational waves (GW) involve the SCHENBERG detector, whose antenna consists of a solid sphere with 0.65m in diameter, made of a Cu6%Al alloy. Six sensors are attached to the sphere’s surface, arranged in a semi-dodecahedron
distribution, and each of them amplifies the motion of the sphere’s surface which it is connected to. This mechanically amplified motion excites a membrane in a resonant cavity where microwaves are stored. As these waves leave the cavity they generate a signal that contains information on the GW’s amplitude. The direction of propagation of GW can also be determined from the joint analysis of the output signals of the six transducers [11,12,13].

Some of the investigations carried by the GRAVITON group are presented in these references [14-36]. The SCHENBERG detector schematics is displayed in Figure 1.

![Figure 1: The resonant-mass gravitational wave detector SCHENBERG (schematics by Xavier P. M. Gratens).](image)

The knowledge acquired in the field of GW detection gave our group expertise to design an experiment to measure the speed of gravity over short distances. The determination of this speed is essential for the proper design of any GW detector, and in general it is estimated that it coincides with the speed of light. In order to measure this speed a quadrupolar distribution of masses must rotate at a very high speed in a very stable motion, demanding an engine operating at a very high and very stable rotational speed as well.

2. A model for artificial generation of gravitational signals

Frajuca and Ruiz [37] proposed the experiment shown in Figure 2, where two bodies with masses $M$ rotate around each other within a radius $r$. The rotation axis is at a distance $a$ from the detector. This project became the basis for the development of a prototype and the theoretical model was applied to the calculation of forces between emitter and detector. The greatest challenge of this experiment is to find the best motorization and control.

The initial experiment had an elementary design, with a basic control system that used the vibration of sapphire bars[38]. However, the detector bandwidth was not satisfactory, a problem that is solved here using a new design for the detecting device.
3. Details of the experiment

In this work we show that a new broadband detector could improve the sensitivity of the experiment described in [39], which consists of three devices that involve sapphire bars. These devices would be separated 5.0m from each other (see Fig. 3), suspended as described in [38]. The ones at the extremities would emit gravitational signals due to vibrations generated by PZT systems, and those signals are expected to excite the central device, which would detect them using ultra-low-noise microwaves (Fig. 4). The experiment would operate at 4.2 K, subjected to high vacuum.

Sapphire was chosen due to its physical characteristics and properties: mass density of 3.98 g.cm\(^{-3}\), sound speed of 9.4 km.s\(^{-1}\) and mechanical quality factor of 3×10\(^9\).
Figure 3: Planned design of the project with PZT in phase signal. Distance of 5 m between emitters (in the extremities) and detector (in the middle).

Figure 4: Diagram of the gravitational wave detector using the low noise oscillator connected to the detecting sapphire bar. From the authors.

The determination of the gravitational tidal force applied to the central device (detector) is done using the model shown in Figure 5 and is presented in Ref. [39]. Also in this reference the viability of the experiment is proven, and the change proposed below improves those results.
4. The detector’s modes
The detector has three resonant, detecting modes that were found with the aid of a Finite Element Modeling (FEM) program. In Fig. 6 we show the new experiment mounting. Figures 7, 8 and 9 display the detector’s vibrational modes, which occur at the frequencies of 4722.0 Hz, 5958.7 Hz and 7169.6 Hz. As can be seen this bandwidth is of the order of 2500 Hz, what increases the bandwidth by a big factor.
5. Amplitude, quantum limit, equipment sensitivity and thermal noise

In the calculations of the next subsections we use the following parameters [39]:

- Oscillator phase noise: -165 dBc/Hz at 1 kHz offset;
- \( M_{\text{eff}} = 1 \text{kg} \);
- Distance between the masses: 5 m;
- \( Q = 10^9 \);
- \( G = 6.67 \times 10^{-11} \text{m}^3\text{kg}^{-1}\text{s}^{-2} \);
- \( a = 10^{-4} \) (Vibration amplitude of the bars);
- \( b = 0.2 \text{ m} \) (Equivalent size of the bars);
- \( x = 5.0 \text{ m} \) (Distance between detector and emitter);
- Frequency bandwidth (BW): 2500 Hz;
- \( \hbar = 6.626 \times 10^{-34} \text{J} \cdot \text{s} \);
- \( f = 10^4 \text{ Hz} \) (instead of 5958 Hz, because it gives a better distance for the experiment);
- \( K = \text{Boltzmann constant} \).

5.1 Signal amplitude

Following the work done in [39]:

\[
\Delta \delta = \frac{OGM_{\text{eff}}2ab^2}{w^2x^2} = 3 \times 10^{-38} \text{ m}
\]  

\[\text{(1)}\]

5.2 Quantum Limit

This corresponds to the minimum limit, because the smallest number of phonons is 1. Therefore,

\[E = \hbar \omega.\]  

\[\text{(2)}\]
\[ \hbar w = \frac{1}{2} A^2 \omega^2 m \implies \hbar w = \frac{A^2 \omega^2 M_{\text{eff}}}{2} \]

\[ \Delta b_{\text{QL}} = \sqrt{\frac{\Delta b}{M_{\text{eff}}}} \]

\[ \Delta b_{\text{QL}} = A = \sqrt{\frac{M_{\text{eff}}^2}{\omega^2}} = \frac{2 \times 10^{-14}}{1.2 \times 10^9} = 3.6 \times 10^{-19} \]  

(3)

(4)

5.3 Equipment Sensitivity Limit [39]

\[ S_s(f) = \left( \frac{df}{\omega} \right)^{-2} S_q(f) f^2 \]  

(5)

\[ S_s = \sqrt{10^{-32}} = 10^{-16} \frac{m}{\sqrt{\text{Hz}}} \]  

(6)

Using BW (bandwidth) = 2500 Hz:

\[ \Delta b_{\text{ES}} = 2 \times 10^{-18} m \]  

(7)

5.4 Thermal Noise Limit

\[ \Delta b_{\text{th}} = \sqrt{\frac{kT}{2 M_{\text{eff}} \omega Q(BW)}} \]  

(8)

\[ \Delta b_{\text{th}} = 1.3 \times 10^{-20} m \]  

(9)

6. Conclusion

The knowledge of the variation of the equivalent size of the bars (\( \Delta b \)) is equal to \( 3 \times 10^{-18} m \), limited by:

- Quantum limit: \( \Delta b_{\text{QL}} = 3.6 \times 10^{-19} m \);  
- Equipment sensitivity limit: \( \Delta b_{\text{ES}} = 2 \times 10^{-18} m \);  
- Thermal noise limit: \( \Delta b_{\text{th}} = 1.3 \times 10^{-20} m \).  

The experiment with this new shape for the detector increases the sensitivity of the experiment and allows it to work in a smaller frequency which makes the experiment feasible. Moving to lower frequencies will improve \( \Delta b_{\text{ES}} \) and \( \Delta b \) keeping the others' limits in a good range. It will depend on the size the detector can be manufactured, but 1 kHz seems to be a good choice.

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References

[1] Magalhaes N S, Miranda T A, Frajuca C 2012 The Astrophysical Journal 755, 54
[2] The Gravitational Waves International Committee Roadmap (GWIC). A global pan. June 2010. Glasgow: University of Glasgow - Department of Physics and Astronomy - Kelvin Building (G12 8QQ), 117p.
[3] Taylor J H, Hulse R A, Fowler L A, Gullahorn GE, Rankin J M 1976 Astrophysical Journal 206 L53
[4] Weber J 1960 Physical Review 117 306
[5] Thorne K S 1987 “300 years of gravitation”. Cambridge: Cambridge University Press: 1987, p.330.
[6] Blair D G “The detection of Gravitational Waves,” 1991 Cambridge: Cambridge University Press
[7] Richard J P 1984 Physical Review Letters 167 165
[8] Aguiar O D et al. 2006 Journal Class. Quantum Grav. 23, 239
[9] Frajuca C et al. 2004 Class. Quantum Grav. 21 1107
[10] Velloso W F, Aguiar OD and Magalhaes NS Proc. First International Workshop for an Omnidirectional Gravitational Radiation Observatory 1997 Singapore:World Scientific
[11] Magalhaes N S et al. 1997 Astrophysical Journal 475, 462
[12] Magalhaes N S et al. 1995 MNRAS 274, 670
[13] Magalhaes N S et al 1997 Gen. Relat. Grav. 29 1511
[14] Aguiar O D et. al. 2005 Class. Quantum Grav. 22, 209
[15] Frajuca et al 2002 Class. Quantum Grav. 19 1961
[16] Ribeiro K L et al. 2004 Class. Quantum Grav. 21, 1225
[17] Aguiar O D et al. 2012 Journal of Physics: Conference Series 363, 012003
[18] Van Albada et al 2000 Review of Scientific Instruments 71 1345
[19] Frajuca, Bortoli F S, Magalhaes N S 2005 Brazilian Journal of Physics 35 1201
[20] Frajuca, Bortoli F S, Magalhaes N S 2006 Journal of Physics: Conference Series 32 319
[21] Aguiar O D et al. 2004 Class. Quantum Grav. 21 459
[22] Bortoli F S et al. 2010 Journal of Physics: Conference Series 228 012011.
[23] Andrade L A et al. 2004 Class. Quantum Grav. 21, 1215
[24] Aguiar O D et al 2002 Brazilian Journal of Physics 32 866
[25] Bortoli F S et al. 2019 Brazilian Journal of Physic 49 133
[26] Frajuca, Bortoli F S, Magalhaes N S, Horiguti A M 2008 Journal of Physics: Conference Series 122 012029
[27] Magalhaes N S, Miranda T A, Frajuca C 2012 Astrophysical Journal 755, 54[26] Frajuca C, Bortoli F S 2006 Journal of Physics: Conference Series 32 315
[28] Bortoli F S et al. 2016 Brazilian Journal of Physic 46 308
[29] Frajuca C et al. 2018 Journal of the Brazilian Society of Mechanical Sciences and Engineering 40 319
[30] Bortoli F S et al. 2020 Brazilian Journal of Physic 50 541
[31] Sepulveda J et al 2012 25th Symposium on Integrated Circuits and Systems Design
[32] Sepulveda J et al 2013 IEEE 4th Latin American Symposium on Circuits and Systems 6519016
[33] Aguiar O D et al 2002 The Ninth Marcel Grossmann Meeting: On Recent Developments in Theoretical and Experimental General Relativity, Gravitation and Relativistic Field Theories 1891
[34] Sepulveda J et al 2013 15th Annual Conference on Genetic and Evolutionary Computation 167
[35] Sepulveda J et al 2013 26th Symposium on Integrated Circuits and Systems Design 6644851
[36] Souza RC et al. 2008 Materials Research 11 89
[37] Ruiz W 2014 2015 Experimento para medir a velocidade da interacão gravitacional utilizando um motor de relutância variável
[38] Ramalho W C S 2016 Transdutor de Safira para medicação da velocidade da interacão gravitacional, Master Thesis IFSP
[39] Frajuca C, Bortoli F S 2019 Journal of Physics: Conference Series 1391 012029