Theoretical Status of $B \to X_s \gamma$ Decays

Matthias Neubert
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract:
We review the theoretical understanding of the branching ratio and photon-energy spectrum in $B \to X_s \gamma$ decays at next-to-leading order in QCD, including consistently the effects of Fermi motion. For the Standard Model, we obtain $B(B \to X_s \gamma) = (3.29 \pm 0.33) \times 10^{-4}$ for the total branching ratio, and $B(B \to X_s \gamma) = (2.85^{+0.34}_{-0.40}) \times 10^{-4}$ if a cut $E_{\gamma}^{\text{lab}} > 2.1 \text{ GeV}$ is applied on the photon energy, as done in the recent CLEO analysis. A precise measurement of the photon spectrum would help reducing the theoretical uncertainty and yield important information on the momentum distribution of $b$ quarks inside $B$ mesons.

To appear in the Proceedings of the
XXIXth International Conference on High Energy Physics
Vancouver, B.C., Canada, 23–29 July 1998
THEORETICAL STATUS OF $B \rightarrow X_s\gamma$ DECAYS

M. NEUBERT

Theory Division, CERN, CH-1211 Geneva 23, Switzerland
E-mail: Matthias.Neubert@cern.ch

We review the theoretical understanding of the branching ratio and photon-energy spectrum in $B \rightarrow X_s\gamma$ decays at next-to-leading order in QCD, including consistently the effects of Fermi motion. For the Standard Model, we obtain $B(B \rightarrow X_s\gamma) = (3.29 \pm 0.33) \times 10^{-4}$ for the total branching ratio, and $B(B \rightarrow X_s\gamma) = (2.85 \pm 0.40) \times 10^{-4}$ if a cut $E_{\gamma} > 2.1$ GeV is applied on the photon energy, as done in the recent CLEO analysis. A precise measurement of the photon spectrum would help reducing the theoretical uncertainty and yield important information on the momentum distribution of $b$ quarks inside $B$ mesons.

1 Introduction

About three years ago, the CLEO Collaboration reported the first measurement of the inclusive branching ratio for the radiative decays $B \rightarrow X_s\gamma$, yielding $B(B \rightarrow X_s\gamma) = (3.32 \pm 0.57 \pm 0.35) \times 10^{-4}$. At this Conference, this value has been updated to $B(B \rightarrow X_s\gamma) = (3.15 \pm 0.35 \pm 0.26) \times 10^{-4}$, where the first error is statistical, the second systematic, and the third accounts for model dependence. The ALEPH Collaboration has reported a measurement of the corresponding branching ratio for $b$ hadrons produced at the $Z$ resonance, yielding $B(H_b \rightarrow X_s\gamma) = (3.11 \pm 0.80 \pm 0.72) \times 10^{-4}$. Theoretically, the two numbers are expected to differ by at most a few percent. Taking the weighted average gives

$$B(B \rightarrow X_s\gamma) = (3.14 \pm 0.48) \times 10^{-4}.$$  \hspace{1cm} (1)

Being rare processes mediated by loop diagrams, radiative decays of $B$ mesons are potentially sensitive probes of New Physics beyond the Standard Model, provided a reliable calculation of their branching ratio can be performed. The theoretical framework for such a calculation is set by the heavy-quark expansion, which predicts that to leading order in $1/m_b$ inclusive decay rates agree with the parton-model rates for the underlying decays of the $b$ quark. The leading nonperturbative corrections have been studied in detail and are well understood. The prediction for the $B \rightarrow X_s\gamma$ branching ratio suffers, however, from large perturbative uncertainties if only leading-order expressions for the Wilson coefficients in the effective weak Hamiltonian are employed. Therefore, it was an important achievement when the full next-to-leading order calculation of the total $B \rightarrow X_s\gamma$ branching ratio in the Standard Model was completed, combining consistently results for the matching conditions, matrix elements, and anomalous dimensions. The leading QED and electroweak radiative corrections were included, too. As a result, the theoretical uncertainty was reduced to a level of about 10%, which is slightly less than the current experimental error. During the last year, the next-to-leading order analysis was extended to the cases of two-Higgs-doublet models and supersymmetry, so that accurate theoretical predictions are now at hand also for the most popular extensions of the Standard Model.

The fact that only the high-energy part of the photon spectrum in $B \rightarrow X_s\gamma$ decays is accessible experimentally introduces a significant additional theoretical uncertainty. For instance, in the new CLEO analysis reported at this Conference a lower cut on the photon energy of 2.1 GeV in the laboratory is imposed, which eliminates about three quarters of phase space in this variable. After reviewing the theoretical status of calculations of the total $B \rightarrow X_s\gamma$ branching ratio, we thus discuss to what extent the effects of a photon-energy cutoff can be controlled theoretically, pointing out the importance of measurements of the photon spectrum for reducing the theoretical uncertainty in the extraction of the total branching ratio.

2 $B \rightarrow X_s\gamma$ branching ratio

The starting point in the analysis of $B \rightarrow X_s\gamma$ decays is the effective weak Hamiltonian

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i(\mu_b) O_i(\mu_b).$$  \hspace{1cm} (2)

The operators relevant to our discussion are

$$O_2 = \bar{s}_L \gamma_\mu c_L \bar{e}_L \gamma^\mu b_L,$$

$$O_7 = \frac{e m_b}{16\pi^2} \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R,$$

$$O_8 = \frac{g_s m_b}{16\pi^2} \bar{s}_L \sigma_{\mu\nu} G^{\mu\nu}_a t_ab_R.$$  \hspace{1cm} (3)

To an excellent approximation, the contributions of other operators can be neglected. The renormalization scale $\mu_b$ in (3) is chosen of order $m_b$, so that all large logarithms reside in the Wilson coefficients $C_i(\mu_b)$. For inclusive decays, the relevant hadronic matrix elements of the local operators $O_i$ can be calculated using the heavy-quark expansion. The complete theoretical prediction for the
\[ B \rightarrow X_s \gamma \] decay rate at next-to-leading order in \( \alpha_s \) has been presented for the first time by Chetyrkin et al. It depends on a parameter \( \delta \) defined by the condition that the photon energy be above a threshold given by \( E_\gamma > (1-\delta)E_{\gamma}^{\text{max}} \). The prediction for the \( B \rightarrow X_s \gamma \) branching ratio is usually obtained by normalizing the result for the corresponding decay rate to that for the semileptonic rate, thereby eliminating a strong dependence on the \( b \)-quark mass. We define
\[
R_{th}(\delta) = \frac{\Gamma(B \rightarrow X_s \gamma)|_{E_{\gamma} > (1-\delta)E_{\gamma}^{\text{max}}}}{\Gamma(B \rightarrow X_c e \bar{\nu})} = \frac{6\alpha}{\pi f(z)} \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 K_{\text{NLO}}(\delta),
\]
where \( f(z) \approx 0.542 - 2.23(\sqrt{z} - 0.29) \) is a phase-space factor depending on the quark-mass ratio \( z = (m_c/m_b)^2 \). The fine-structure constant \( \alpha \) is renormalized at \( q^2 = 0 \), as is appropriate for real-photon emission. The quantity \( K_{\text{NLO}}(\delta) \) contains the next-to-leading order corrections. In terms of the theoretically calculable ratio \( R_{th}(\delta) \), the \( B \rightarrow X_s \gamma \) branching ratio is given by \( B(B \rightarrow X_s \gamma) = 0.105N_{\text{SL}} R_{th}(\delta) \), where \( N_{\text{SL}} = B(B \rightarrow X_c e \bar{\nu})/10.5\% \) is a normalization factor to be determined from experiment. To good approximation \( N_{\text{SL}} = 1 \). The current experimental situation of measurements of the semileptonic branching ratio of \( B \) mesons and their theoretical interpretation are reviewed in Refs. 22,23.

In the calculation of the quantity \( K_{\text{NLO}}(\delta) \) we consistently work to first order in the small parameters \( \alpha_s, 1/m_b^2 \) and \( \alpha/\alpha_s \), the latter ratio being related to the leading-logarithmic QED corrections. The structure of the result is
\[
K_{\text{NLO}}(\delta) = \sum_{i,j=2,7,8}^{s,t} k_{ij}(\delta, \mu_b) \text{Re}[C_i(\mu_b) C_j^*(\mu_b)], \tag{5}
\]
where the Wilson coefficients \( C_i(\mu_b) \) are expanded as
\[
C_i^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} C_i^{(1)}(\mu_b) + \frac{\alpha}{\alpha_s(\mu_b)} C_i^{(\text{em})}(\mu_b) + \ldots \tag{6}
\]
The coefficients \( C_i^{(k)}(\mu_b) \) are complicated functions of the ratio \( \eta = \alpha_s(m_W)/\alpha_s(\mu_b) \), which also depend on the values \( C_i(m_W) \) of the Wilson coefficients at the weak scale. In the Standard Model, these initial conditions are functions of the mass ratio \( x_t = (m_t/m_W)^2 \). Whereas the leading-order coefficients \( C_i^{(0)}(\mu_b) \) are known since a long time, the next-to-leading terms in \( \eta \), which must be kept for the coefficient \( C_7(\mu_b) \), have been calculated only recently. The expression for \( C_7^{(1)}(\mu_b) \) can be found in eq. (21) of Ref. 16, and the result for \( C_7^{(\text{em})}(\mu_b) \) is given in eq. (11) of Ref. 18.

Explicit expressions for the functions \( k_{ij}(\delta, \mu_b) \) in (4) can be found, e.g., in Ref. 18, where we have corrected some mistakes in the formulae for real-gluon emission used by previous authors. (The corrected expressions are also given in the Erratum to Ref. 16.) Bound-state corrections enter the formulae for the coefficients \( k_{ij} \) at order \( 1/m_b^2 \) and are proportional to the hadronic parameter \( \lambda_2 = \frac{1}{4}(m_b^2 - m_s^2) \approx 0.12 \text{ GeV}^2 \). Most of them characterize the spin-dependent interactions of the \( b \) quark inside the \( B \) meson. However, a peculiar feature of inclusive radiative decays is the appearance of a correction proportional to \( 1/m_c^2 \) in the coefficient \( k_{27} \), which represents a long-distance contribution arising from \((c\bar{c}) \) intermediate states.

### 2.1 Definition of the total branching ratio

The theoretical prediction for the \( B \rightarrow X_s \gamma \) branching ratio diverges in the limit \( \delta \to 1 \) because of a logarithmic soft-photon divergence of the \( b \to sg\gamma \) subprocess, which would be canceled by an infrared divergence of the \( O(\alpha) \) corrections to the process \( b \to sg \). We have argued that a reasonable definition of the “total” branching ratio is to use an extrapolation to \( \delta = 1 \) starting from the region \( \delta \sim 0.5-0.8 \), where the theoretical result exhibits a weak, almost linear dependence on the cutoff. The extrapolated value so defined agrees, to a good approximation, with the result obtained by taking \( \delta = 0.9 \), and hence we define the total branching ratio using this particular value of the cutoff.

The theoretical result is sensitive to the values of various input parameters. For the (one-loop) quark pole masses we take \( m_c/m_b = 0.29 \pm 0.02, m_b = (4.80 \pm 0.15) \text{ GeV}, \) and \( m_t = (175 \pm 6) \text{ GeV}. \) The corresponding uncertainties in the branching ratio are, respectively, \( \pm 5.9\%, \pm 1.0\%, \) and \( \pm 1.6\%. \) We use the two-loop expression for the running coupling \( \alpha_s(\mu) \) with the initial value \( \alpha_s(m_Z) = 0.118 \pm 0.003, \) which induces an uncertainty of \( \pm 2.7\%. \) For the ratio of the CKM parameters in (4) we take the value \( |V_{ts}^* V_{tb}|/|V_{cb}| = 0.976 \pm 0.010 \) obtained from a global analysis of the unitarity triangle. This gives an uncertainty of \( \pm 2.1\%. \) Finally, we include an uncertainty of \( \pm 2.0\% \) to account for next-to-leading electroweak radiative corrections. The theoretical uncertainty arising from the variation of the renormalization scale will be addressed below. We find an uncertainty of \( \pm 6.3\% \) from the variation of the scale \( \mu_b \), and of \( \pm 2.2\% \) from the variation of the scale \( \mu_t \) entering the expression for the semileptonic decay rate in the denominator in (4). Adding the different errors in quadrature gives a total uncertainty of \( \pm 9\% \) (adding them linearly would lead to the more conservative estimate of \( \pm 24\% \)). For the total \( B \rightarrow X_s \gamma \) branching ratio in the Standard Model
we obtain
\[ B(B \to X_s\gamma) = (3.29 \pm 0.33) \times 10^{-4} N_{\text{SL}}, \] (7)
in good agreement with the experimental value in [1].

### 2.2 Sensitivity to New Physics

Possible New Physics contributions would enter the theoretical prediction for the $B \to X_s\gamma$ branching ratio through non-standard values of the Wilson coefficients of the dipole operators $O_7$ and $O_8$ at the weak scale $m_W$. To explore the sensitivity to such effects, we normalize these coefficients to their values in the Standard Model and introduce the ratios $\xi_i = C_i(m_W)/C_i^{\text{SM}}(m_W)$ with $i = 7, 8$. In the presence of New Physics, the parameters $\xi_7$ and $\xi_8$ may take (even complex) values different from 1. Similarly, New Physics may induce dipole operators with opposite chirality to that of the Standard Model, i.e. operators with right-handed light-quark fields. If we denote by $O^R$ the Wilson coefficients of these new operators, expression (3) can be modified to include their contributions by simply replacing $C_iC_j^* \to C_iC_j^* + C_i^{R}C_j^{R*}$ everywhere. We thus define two additional parameters $\xi_i^R = C_i^R(m_W)/C_i^{\text{SM}}(m_W)$ with $i = 7, 8$, which vanish in the Standard Model. Since the dipole operators only contribute to rare flavour-changing neutral current processes, there are at present rather weak constraints on the values of these parameters [1]. On the other hand, we assume that the coefficient $C_2$ of the current–current operator $O_2$ takes its standard value, and that there is no similar operator containing right-handed quark fields. With these definitions, the $B \to X_s\gamma$ branching ratio can be decomposed as

\[ \frac{1}{N_{\text{SL}}} B(B \to X_s\gamma)|_{E_\gamma > (1-\delta) E_{\text{max}}} = B_{27}(\delta) + B_{77}(\delta) (|\xi_7|^2 + |\xi_8|^2) + B_{28}(\delta) (|\xi_8|^2 + |\xi_8^R|^2) \]
\[ + B_{78}(\delta) \text{Re}(\xi_7\xi_8^*) + B_{78}^{R}(\delta) \text{Re}(\xi_7^R\xi_8^*)] . \] (8)

In Table 1, the values of the components $B_{ij}(\delta)$ obtained with $\delta = 0.9$ are shown for different choices of the renormalization scale. Assuming $N_{\text{SL}} = 1$, the Standard Model result for the total branching ratio is given by $B_{\text{SM}} = \sum_{ij} B_{ij}$. The most important contributions are $B_{27}$ and $B_{22}$ followed by $B_{77}$. The smallness of the remaining terms shows that there is little sensitivity to the coefficient $C_6(m_W)$ of the chromo-magnetic dipole operator. Once the parameters $\xi_i$ and $\xi_i^R$ are calculated in a given New Physics scenario, the result for the $B \to X_s\gamma$ branching ratio can be derived using the numbers shown in the table. For the remainder of this talk, however, we assume the validity of the Standard Model.

### 2.3 Perturbative uncertainties

The components $B_{ij}(\delta)$ in (8) are formally independent of the renormalization scale $\mu_b$. Their residual scale dependence results only from the truncation of perturbation theory at next-to-leading order and is conventionally taken as an estimate of higher-order corrections. Typically, the different components vary by amounts of order 10–20% as $\mu_b$ varies between $m_b/2$ and $2m_b$. The good stability is a result of the cancelation of the scale dependence between Wilson coefficients and matrix elements achieved by a next-to-leading order calculation.

Previous authors [4, 5, 7] who estimated the $\mu_b$ dependence of the total $B \to X_s\gamma$ branching ratio in the Standard Model found a more striking improvement over the leading-order result, namely a variation of only $\pm 0.1\%$ as compared with $\pm 2\%$ at leading order. However, the apparent excellent stability observed at next-to-leading order is largely due to an accidental cancelation between different contributions to the branching ratio. A look at Table 1 shows that the residual scale dependence of the individual contributions $B_{ij}$ is much larger than that of their sum, which determines the total branching ratio in the Standard Model. Note, in particular, the almost perfect cancelation of the scale dependence between $B_{27}$ and $B_{22}$, which is accidental since the magnitude of $B_{27}$ depends on the top-quark mass, whereas $B_{22}$ is independent of $m_t$. In such a situation, the apparent weak scale dependence of the sum of all contributions is not a good measure of higher-order corrections. Indeed, higher-order corrections must stabilize the different components $B_{ij}$ individually, not only their sum. The variation of the individual components as a function of $\mu_b$ thus provides a more conservative estimate of the truncation error than does the variation of the total branching ratio. For each component, we estimate the truncation error by taking one half of the maximum variation obtained by varying $\mu_b$ between $m_b/2$ and $2m_b$. The truncation error of the sum is then obtained by adding the individual errors in quadrature. In this way, we find a total truncation error of $\pm 6.3\%$, which is more than a factor of 2 larger than the estimates obtained by previous authors. An even larger truncation error could be justified given that the choice of the range of variation of $\mu_b$ is ad hoc.
and that the scale dependence of the various components is not symmetric around the point $\mu_b = m_b$.

3 Partially integrated branching ratio and photon-energy spectrum

Whereas the explicit power corrections to the functions $k_{ij}$ are small, an important nonperturbative effect not included so far is the motion of the $b$ quark inside the $B$ meson caused by its soft interactions with the light constituents. It leads to a modification of the photon-energy spectrum, which must be taken into account if a realistic cutoff is imposed. This so-called “Fermi motion” can be included in the heavy-quark expansion by resumming an infinite set of leading-twist contributions into a shape function $F(k_+)$, which governs the light-cone momentum distribution of the heavy quark inside the $B$ meson. The physical decay distributions are obtained from a convolution of parton-model spectra with this function. In the process, phase-space boundaries defined by parton kinematics are transformed into the proper physical boundaries defined by hadron kinematics.

The shape function is a universal characteristic of the $B$ meson governing the inclusive decay spectra in processes with massless partons in the final state, such as $B \to X\gamma$ and $B \to X_e \ell \nu$. However, this function does not describe in an accurate way the distributions in decays into massive partons such as $B \to X_c \ell \nu$. Unfortunately, therefore, the shape function cannot be determined using the lepton spectrum in semileptonic decays, for which high-precision data exist. On the other hand, there is some useful theoretical information on the moments $A_n = \langle k_n^+ \rangle$ of the function $F(k_+)$, which are related to the forward matrix elements of local operators.

In particular, $A_1 = 0$ vanishes by the equations of motion (this condition defines the heavy-quark mass), and $A_2 = \frac{1}{2} k_+^2$ is related to the kinetic energy of the heavy quark inside the $B$ meson. For our purposes it is sufficient to adopt the simple form $F(k_+) = N (1-x)^a e^{(1+a)x}$ with $x = k_+ / \Lambda \leq 1$, where $\Lambda = m_B - m_b$. This ansatz is such that $A_1 = 0$ by construction. The parameter $a$ can be related to the second moment, yielding $\mu^2_\pi = 3 \Lambda^2 / (1+a)$. Thus, the $b$-quark mass (or $\Lambda$) and the quantity $\mu^2_\pi$ (or $a$) are the two parameters of our function. We take $m_b = 4.8$ GeV and $\mu^2_\pi = 0.3$ GeV$^2$ as reference values, in which case $a \approx 1.29$.

Let us denote by $B_{ij}^p(\delta_p)$ the various components in calculated in the parton model, where the cutoff $\delta_p$ is defined by the condition that $E_\gamma \geq \frac{1}{2} (1 - \delta_p) m_B$. Then the corresponding physical quantities $B_{ij}(\delta)$ with $\delta$ defined such that $E_\gamma \geq \frac{1}{2} (1 - \delta) m_B$ are given by

$$B_{ij}(\delta) = \int_{m_B(1-\delta) - m_b}^{m_B - m_b} dk_+ F(k_+) B_{ij}^p \left(1 - \frac{m_B(1-\delta)}{m_b + k_+}\right).$$

This relation is such that $B_{ij}(1) = B_{ij}^p(1)$, implying that the total branching ratio is not affected by Fermi motion. The effect is, however, important for realistic values of the cutoff $\delta$.

As an illustration of the sensitivity of our results to the parameters of the shape function, the upper plots in Figure show the predictions for the partially integrated $B \to X_\gamma \gamma$ branching ratio as a function of the energy cutoff $E_\gamma^\text{min} = \frac{1}{2} (1 - \delta) m_B$. In the left-hand plot we vary $m_b$ keeping the ratio $\mu^2_\pi / \Lambda^2$ fixed. The gray line shows the result obtained using the same parameters as for the solid line but a different functional form given by $F(k_+) = N (1-x)^a e^{-b(1-x)^2}$. For comparison, we show the data point $B(B \to X_\gamma \gamma) = (2.04 \pm 0.47) \times 10^{-4}$ obtained in the original CLEO analysis with a cutoff at 2.2 GeV. In the right-hand plot, we keep $m_b = 4.8$ GeV fixed and compare the parton-model result (gray dotted curve) with the results corrected for Fermi motion using different values for the parameter $\mu^2_\pi$. This plot illustrates how Fermi motion fills the gap between the parton-model endpoint at $m_b/2$ and the physical endpoint at $m_B/2$. (The true endpoint is actually located at $(m_B^2 - (m_K + m_\pi)^2)/2m_B \approx 2.60$ GeV, i.e. slightly below $m_B/2 \approx 2.64$ GeV.) Comparing the two plots, it is evident that the uncertainty due to the value of the
b-quark mass is the dominant one. Variations of the parameter $\mu_Z^2$ have a much smaller effect on the partially integrated branching ratio, and also the sensitivity to the functional form adopted for the shape function turns out to be small. This behaviour is a consequence of global quark–hadron duality, which ensures that even partially integrated quantities are rather insensitive to bound-state effects. The strong remaining dependence on the b-quark mass is simply due to the transformation by Fermi motion of phase-space boundaries from parton to hadron kinematics.

Taking the three curves in the left-hand upper plot for a representative range of parameters and applying a small correction for the boost from the B rest frame to the laboratory as appropriate for the CLEO analysis, we show in Table 2 the predictions for the partially integrated branching ratio for three different values of the cutoff on the photon energy. The first error refers to the dependence on the various input parameters discussed previously, and the second one accounts for the uncertainty associated with the description of Fermi motion. In general, this second error can be reduced in two ways. The first possibility is to lower the cutoff on the photon energy. A first step in this direction has already been taken in the new CLEO analysis reported at this Conference, in which the cutoff has been lowered from 2.2 to 2.1 GeV. If a value as low as 2 GeV could be achieved, the theoretical predictions would become rather insensitive to the parameters of the shape function. To what extent this will be possible in future experiments will depend on their capability to reject the background of photons from other decays. The second possibility is that future high-precision measurements of the photon spectrum will make it possible to adjust the parameters of the shape function from a fit to the data. For the purpose of illustration, the photon spectra corresponding to the various parameter sets are shown in the lower plots in Figure 2.

Such a determination of the shape-function parameters from $B \rightarrow X_s \gamma$ decays would not only help to reduce the theoretical uncertainty in the determination of the total branching ratio, but would also enable us to predict the lepton spectrum in $B \rightarrow X_s \ell \nu$ in a model-independent way. This may help to reduce the theoretical uncertainty in the value of $|V_{ub}|$. A detailed analysis of the photon spectrum will therefore be an important aspect in future analyses of inclusive radiative $B$ decays.

### Table 2: Partially integrated $B \rightarrow X_s \gamma$ branching ratio for different values of the cutoff on the photon energy

| $E_{\gamma, \text{min}}^{\text{lab}}$ (GeV) | $B(B \rightarrow X_s \gamma)$ [10^{-4} N_{3L}] |
|----------------------------------------|-----------------------------------------------|
| 2.2 GeV                                | 2.56 ± 0.26 ±0.31 |
| 2.1 GeV                                | 2.85 ± 0.29 ±0.18 |
| 2.0 GeV                                | 3.01 ± 0.30 ±0.09 |

4 Conclusions

The inclusive radiative decays $B \rightarrow X_s \gamma$ play a key role in testing the Standard Model and probing the structure of possible New Physics. A reliable theoretical calculation of their branching ratio can be performed using the operator product expansion for inclusive decays of heavy hadrons combined with the twist expansion for the description of decay distributions near phase-space boundaries. To leading order in $1/m_b$ the decay rate agrees with the parton-model rate for the underlying quark decay $b \rightarrow X_s \gamma$. With the completion of the next-to-leading order calculation of the Wilson coefficients and matrix elements of the operators in the effective weak Hamiltonian, the perturbative uncertainties in the calculation of this process have been reduced to a level of about 10%. Bound-state corrections to the total decay rate are suppressed by powers of $1/m_b$ and can be controlled in a systematic way.

A more important effect is the Fermi motion of the heavy quark inside the meson, which is responsible for the characteristic shape of the photon-energy spectrum in $B \rightarrow X_s \gamma$ decays. It leads to the main theoretical uncertainty in the calculation of the branching ratio if a restriction to the high-energy part of the photon spectrum is imposed. Fermi motion is naturally incorporated in the heavy-quark expansion by resumming an infinite set of leading-twist operators into a non-perturbative shape function. The main theoretical uncertainty in this description lies in the value of the b-quark mass. Other features associated with the detailed functional form of the shape function play a minor role. The value of $m_b$ and other shape-function parameters could, in principle, be extracted from a precise measurement of the photon-energy spectrum, but also gross features of this spectrum such as the average photon energy would provide valuable information. For completeness, we note that besides the photon-energy spectrum also the invariant hadronic mass distribution in radiative $B$ decays can be studied. Investigating the pattern of individual hadron resonances contributing to the spectrum, one can motivate a simple description of the hadronic mass spectrum with only a single parameter, the $B \rightarrow K^{*+} \gamma$ branching ratio, to be determined from experiment.

Possible New Physics contributions would enter the theoretical predictions for the $B \rightarrow X_s \gamma$ branching ratio and photon spectrum through the values of parameters $\xi$ and $\xi_R$, which are defined in terms of values of Wilson coefficients at the scale $m_W$. This formalism allows one to account for non-standard contributions to the magnetic and chromo-magnetic dipole operators, as well
as operators with right-handed light-quark fields. Quite generally, New Physics would not affect the shape of the photon spectrum but could change the total branching ratio by a considerable amount. This implies that the analysis of the photon-energy and hadronic mass spectra, which is crucial for the experimental determination of the total branching ratio, can be performed without assuming the correctness of the Standard Model. On the other hand, the total branching ratio will provide a powerful constraint on the structure of New Physics beyond the Standard Model as experimental data become more precise.

Acknowledgments

The work reported here has been done in a most pleasant collaboration with Alex Kagan, which is gratefully acknowledged.

1. M.S. Alam et al. (CLEO Collaboration), Phys. Rev. Lett. 74, 2885 (1995).
2. T. Skwarnicki (CLEO Collaboration), Talk no. 702 presented at this Conference.
3. R. Barate et al. (ALEPH Collaboration), Phys. Lett. B 429, 169 (1998).
4. J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B 247, 399 (1990).
5. I.I. Bigi, N.G. Uraltsev and A.I. Vainshtein, Phys. Lett. B 293, 430 (1992) [E: 297, 477 (1993)]; I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. Lett. 71, 496 (1993); B. Blok, L. Koyrakh, M.A. Shifman and A.I. Vainshtein, Phys. Rev. D 49, 3356 (1994) [E: 50, 3572 (1994)].
6. A.V. Manohar and M.B. Wise, Phys. Rev. D 49, 3367 (1994).
7. A.F. Falk, M. Luke and M.J. Savage, Phys. Rev. D 49, 3392 and 4623 (1994).
8. G. Buchalla, G. Isidori and S.J. Rey, Nucl. Phys. B 511, 594 (1998).
9. M. Ciuchini et al., Phys. Lett. B 316, 127 (1999).
10. A.J. Buras, M. Misiak, M. M"unz and S. Pokorski, Nucl. Phys. B 242, 374 (1994).
11. K. Adel and Y.P. Yao, Phys. Rev. D 49, 4945 (1994).
12. C. Greub and T. Hurth, Phys. Rev. D 56, 2934 (1997).
13. A.J. Buras, A. Kwiatkowski and N. Pott, Phys. Lett. B 414, 157 (1997); Nucl. Phys. B 517, 353 (1998).
14. A. Ali and C. Greub, Phys. Lett. B 361, 146 (1995); see also the extended version in Preprint DESY 95-117 [hep-ph/9506373].
15. C. Greub, T. Hurth and D. Wyler, Phys. Lett. B 380, 385 (1996); Phys. Rev. D 54, 3350 (1996).
16. K. Chetyrkin, M. Misiak and M. M"unz, Phys. Lett. B 400, 206 (1997) [E: 425, 414 (1998)].
17. A. Czarnecki and W.J. Marciano, Phys. Rev. Lett. 81, 277 (1998).
18. A.L. Kagan and M. Neubert, Preprint CERN-TH/98-99 hep-ph/9805303, to appear in Eur. Phys. J. C.
19. M. Ciuchini, G. Degrassi, P. Gambino and G.F. Giudice, Nucl. Phys. B 527, 21 (1998).
20. F.M. Borzumati and C. Greub, Preprint ZU-TH-31-97 hep-ph/9802391.
21. M. Ciuchini, G. Degrassi, P. Gambino and G.F. Giudice, Preprint CERN-TH/98-177 hep-ph/9806308.
22. P. Drell, Preprint CLNS-97-1521 hep-ex/9711020, to appear in the Proceedings of the 18th International Symposium on Lepton–Photon Interactions, Hamburg, Germany, July 1997.
23. M. Neubert, Preprint CERN-TH/98-2 hep-ph/9801268, to appear in the Proceedings of the International Europhysics Conference on High Energy Physics, Jerusalem, Israel, August 1997.
24. A.F. Falk and M. Neubert, Phys. Rev. D 47, 2965 (1993).
25. M.B. Voloshin, Phys. Lett. B 397, 275 (1997).
26. A. Khodjamirian, R. R"uckl, G. Stoll and D. Wyler, Phys. Lett. B 402, 167 (1997).
27. Z. Ligeti, L. Randall and M.B. Wise, Phys. Lett. B 402, 178 (1997).
28. A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei, Phys. Rev. D 56, 3151 (1997).
29. G. Buchalla, G. Isidori and S.J. Rey, Nucl. Phys. B 511, 594 (1998).
30. A.J. Buras, Proceedings of the 28th International Conference on High-Energy Physics, Warsaw, Poland, July 1996, edited by Z. Ajduk and A.K. Wroblewski (World Scientific, Singapore, 1997), pp. 243.
31. For a review, see: J. Hewett, Talk no. 1617 at this Conference.
32. A.L. Kagan and M. Neubert, Preprint CERN-TH/98-1 hep-ph/9803368, to appear in Phys. Rev. D.
33. M. Neubert, Phys. Rev. D 49, 3392 and 4623 (1994).
34. I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Int. J. Mod. Phys. A 9, 2467 (1994); R.D. Dikeman, M. Shifman and N.G. Uraltsev, Int. J. Mod. Phys. A 11, 571 (1996).
35. T. Mannel and M. Neubert, Phys. Rev. D 50, 2037 (1994).