Multi-channel Sensing And Resource Allocation in Energy Constrained Cognitive Radio Networks

Kedar Kulkarni, Adrish Banerjee

Department of Electrical Engineering, Indian Institute of Technology, Kanpur, 208016, India

Abstract

We consider a cognitive radio network in a multi-channel licensed environment. Secondary user transmits in a channel if the channel is sensed to be vacant. This results in a tradeoff between sensing time and transmission time. When secondary users are energy constrained, energy available for transmission is less if more energy is used in sensing. This gives rise to an energy tradeoff. For multiple primary channels, secondary users must decide appropriate sensing time and transmission power in each channel to maximize average aggregate-bit throughput in each frame duration while ensuring quality-of-service of primary users. Considering time and energy as limited resources, we formulate this problem as a resource allocation problem. Initially a single secondary user scenario is considered and solution is obtained using decomposition and alternating optimization techniques. Later we extend the analysis for the case of multiple secondary users. Simulation results are presented to study effect of channel occupancy, fading and energy availability on performance of proposed method.

Keywords: Cognitive radio, energy constrained networks, resource allocation, sensing-throughput tradeoff

1. Introduction

Cognitive radio (CR) facilitates efficient spectrum use of current licensed spectrum that is highly underutilized and is considered as a potential solution to the
problem of spectrum scarcity [1, 2]. In CR networks, secondary users (SU) opportunistically access spectrum allocated to licensed or primary users (PU) in such a way that quality of service (QoS) requirements of PUs are satisfied. For this purpose, SUs periodically sense the spectrum for presence of PUs. While many spectrum sensing techniques exist, energy detection method is widely used due to its low complexity and easy implementation [3–5] and is optimal when form of signal to be detected is unknown [6]. SU transmits in a channel only if the channel is sensed to be vacant. This method of spectrum access is widely known as interweave-mode [7]. Due to channel fading and noise, spectrum sensing may result in missed detection or false alarm. Longer sensing periods lead to better sensing performance, but at the cost of reduced transmission time as a node cannot transmit and sense simultaneously. This sensing-throughput tradeoff necessitates selection of optimal sensing time to maximize SU throughput while sufficiently protecting PU [8].

Sensing throughput tradeoff where SU determines optimal sensing time has been studied under various PU QoS constraints such as fixed target detection probability [8, 9], collision probability constraint [10] and PU outage constraint [11]. Kaushik et al. [12] studied effect of estimation time on the tradeoff considering PU signal of unknown power. When multiple SUs are present, better sensing performance can be achieved in less time using cooperative sensing. In [9] and [11], authors optimized sensing time in cooperative sensing framework assuming availability of a single PU band. Pei et al. [13] considered multiple PUs multiplexed using orthogonal frequency division multiple access (OFDMA) and a SU equipped with wideband antenna, which enabled simultaneous sensing of all PU channels. The authors determined optimal sensing time to achieve given target detection probability and proposed power allocation method to maximize SU throughput. Using the same model, Sharkasi et al. [14] studied sensing throughput tradeoff under PU outage constraint. In practice, maximum bandwidth that can be scanned by SU is limited by its radio-frequency (RF) frontend and analog-to-digital converter (ADC) sampling unit. For a SU device having narrowband antenna or low sampling rate, simultaneous sensing of all bands in a wideband spectrum is not possible, prompting SU to optimally select sensing and transmission time in each PU band. In this work, we aim to address this multi-channel sensing-throughput tradeoff.

In energy harvesting (EH) wireless networks, users are often energy constrained [15, 16]. In this case, in addition to tradeoff arising from sensing and transmission time, tradeoff in energy becomes critical. In energy constrained CR networks, as sensing time increases, more energy is used in sensing, leav-
ing less energy available for transmission. Considering EH-CR network, Park et al. [17] determined optimal sensing threshold for SU under energy causality and collision probability constraints. In [18], authors found optimal sensing time to minimize average energy cost under constraints on SU transmission rate. Yin et al. [19] divided SU frame duration in three parts for— harvesting, sensing and transmission— and proposed optimal time division to maximize SU throughput under a fixed target detection probability. A fractional programming framework was proposed in [20] to find optimal sensing time and power allocation to maximize energy efficiency of SU. In [21–23], authors proposed energy efficient dynamic control policies using Markov decision process (MDP) approach where SU can choose to stay silent, carry out sensing or transmit based on its belief about PU occupancy. MDP based techniques have high computational complexity and require knowledge of transition probabilities between different PU occupancy states. In practice, information of state transition probabilities is not readily available due to sparse spectrum activity over long term. Existing spectrum availability studies only document duty cycle of a channel which is the probability of a channel being occupied by a PU [24–27]. Also, the works mentioned above considered CR systems with a single PU channel with fixed target detection probability as PU’s QoS criteria. Availability of multiple PU channels poses a challenge as SU has to allocate available time and energy appropriately in sensing and transmission tasks in each channel.

In multi-channel environment, if channel conditions are such that the channel does not yield good throughput, SUs should not transmit in the channel. Hence SUs should not be required to sense it. Further, choice of channels for sensing and transmission can be made based on occupancy probability of the channel. To maximize average SU throughput, SU must appropriately allocate limited time and energy for tasks of sensing and transmission in each channel. In this paper, we address the problem of finding optimal sensing time and power allocation in a multi-channel PU environment where channels have to be sensed sequentially such that expected bit-throughput of SU is maximized in a given duration. We consider two main constraints— average rate constraint of PU to maintain QoS of PU and total energy constraint that results from limited energy availability. Our contribution in this work is as follows.

- We first consider a single SU case and formulate the joint sensing time-energy-throughput tradeoff problem to maximize aggregate average bit-throughput of SU. The optimization problem is a non-convex one. We decompose the problem in subproblems with separable objectives. We propose sensing and
resource allocation (SRA) method which iteratively solves the subproblems and finds optimal sensing time, transmission time and transmission energy for each channel.

- We then extend SRA method for multiple SU scenario to maximize sum-throughput of SU network.

- We present numerical results to study performance of proposed approach under various channel and energy availability conditions. We also compare SRA with heuristics based best channel selection (BCS) and proportional energy-time allocation (PETA) methods.

Rest of the paper is organized as follows. In Section 2 system model is presented and optimization problem is formulated. In Section 3 we propose SRA and find solution to the optimization problem. In Section 4 we propose SRA for the multiple SU case. Simulation results are presented in Section 5. We conclude in Section 6.

2. System model and problem formulation

Initially we consider a cognitive radio system with one SU that opportunistically accesses PU spectrum of $M$ non-overlapping narrowband channels of equal bandwidth. The model for multiple SU scenario is explained later in Section 4. PU and SU follow time slotted synchronous communication with frame duration $T$ [28]. PU is active in $i$th channel with occupancy probability $\pi_{1,i}$, $i = 1, 2, \ldots, M$. Thus, probability of $i$th channel being vacant is $\pi_{0,i} = 1 - \pi_{1,i}$. SU has a-priori knowledge of channel occupancy probabilities which can be obtained by observing the spectrum for long duration or from existing spectrum database [24, 27, 29]. All channels between different source-destination pairs are independent Rayleigh block fading, that is, channel gains remain constant in one frame and vary independently from frame to frame. Instantaneous channel gain of SU source to SU destination link on $i$th channel is denoted as $g_i$, $i = 1, 2, \ldots, M$. 
\[ \pi_{0,i} \quad \text{Probability of } i\text{th PU channel being vacant} \]
\[ g_i \quad \text{Channel power gain of SU-SU link on } i\text{th channel} \]
\[ \sigma^2_N \quad \text{AWGN noise power} \]
\[ \tau_{s,i} \quad \text{Sensing time allocated to } i\text{th channel} \]
\[ \tau_{t,i} \quad \text{Transmission time allocated to } i\text{th channel} \]
\[ p_{t,i} \quad \text{SU transmit power in } i\text{th channel} \]
\[ p_s \quad \text{Sensing power} \]
\[ f_s \quad \text{Sampling frequency} \]
\[ T \quad \text{Frame time} \]

Table 1: Key notation used in this paper

Noise at SU receiver is additive white Gaussian (AWGN) with variance \( \sigma^2_N \). PU transmit power in each channel is \( p_{PU} \). In Table 1, we list key notation used in this paper.

2.1. Sensing and spectrum access

SU is equipped with a single narrowband antenna that limits the sensing capability to one channel at a time. At the beginning of each frame, SU performs spectrum sensing using energy detection with sampling frequency \( f_s \). Sensing takes place at a constant power \( p_s \) \([18]\). SU senses \( i\)th PU channel for time \( \tau_{s,i} \).

Assuming PU signal to be complex valued phase-shift keying (PSK) signal, we write detection probability \( P_{d,i} \) and false alarm probability \( P_{f,i} \) as \([8]\)

\[
P_{d,i} = Q \left( \left( \frac{\epsilon_i}{\sigma^2_N} - \gamma_i - 1 \right) \sqrt{\frac{f_s \tau_{s,i}}{2\gamma_i + 1}} \right), \tag{1}
\]
\[
P_{f,i} = Q \left( \left( \frac{\epsilon_i}{\sigma^2_N} - 1 \right) \sqrt{\frac{f_s \tau_{s,i}}{\gamma_i}} \right), \tag{2}
\]

where \( \epsilon_i \) is the detection threshold and \( \gamma_i \) is the average PU signal-to-noise ratio (SNR) received at SU source over \( i\)th channel. For a target detection probability \( P_{d,i} \), we can write false alarm probability as \([8]\)

\[
P_{f,i} = Q \left( \sqrt{\frac{2\gamma_i}{\gamma_i + 1}} Q^{-1} (P_{d,i}) + \gamma_i \sqrt{f_s \tau_{s,i}} \right). \tag{3}
\]

Note that depending on channel occupancy probabilities, channel conditions and available energy, SU may not sense a PU channel, which results in \( \tau_{s,i} = 0 \). After the sensing phase is over, for the remaining frame duration, SU transmits
in vacant PU channels (interweave mode) with appropriate power so as maximize bit-throughput. For $i$th channel, time spent in transmission and transmit power are denoted as $\tau_{t,i}$ and $p_{t,i}$ respectively. The frame structure of SU is shown in Fig. [1]

2.2. System constraints

2.2.1. Primary rate constraint

Quality of service (QoS) criterion of $i$th PU demands that the PU should be able to transmit $\bar{B}_{p,i}$ bits on average in each frame duration. If SU correctly senses a channel as active, there is no interference with the PU transmission. In this case, average number of bits transmitted by $i$th PU is given by $T \mathbb{E}[R_{p,i}]$ where $\mathbb{E}[R_{p,i}]$ is the average transmission rate that depends on PU source-PU destination link. In case of missed detection, SU transmits and interferes with $i$th PU for time $\tau_{t,i}$. There is no interference to the PU transmission for time $(T - \tau_{t,i})$. We consider a strong interference channel between SU and PU. Thus, transmission rate achieved under interference is negligible. Then average number of bits transmitted in a frame by $i$th PU is

$$B_{p,i} \cong P_{d,i} T \mathbb{E}[R_{p,i}] + (1 - P_{d,i}) (T - \tau_{t,i}) \mathbb{E}[R_{p,i}]. \quad (4)$$

Let $\tau_{p,i} = \bar{B}_{p,i}/\mathbb{E}[R_{p,i}]$ where $\tau_{p,i} \in [0, T]$. Higher value of $\tau_{p,i}$ indicates that required average bit-throughput $\bar{B}_{p,i}$ is higher. Then the QoS constraint $B_{p,i} \geq \bar{B}_{p,i}$ can be written as

$$P_{d,i} \geq \bar{P}_{d,i} = \max \left[ 0, 1 - \frac{T - \tau_{p,i}}{\tau_{t,i}} \right]. \quad (5)$$

Thus, to transmit in $i$th channel, detection probability should be greater than detection probability threshold $\bar{P}_{d,i}$ which depends on transmission time $\tau_{t,i}$. As $\tau_{t,i}$ increases, required detection probability increases. To achieve increasing $P_{d,i}$, sensing time $\tau_{s,i}$ increases, leaving less time available for transmission. This results in the sensing-throughput tradeoff. Optimal sensing time is such that constraint in (5) is satisfied with equality [8].

2.2.2. Energy constraint

SU is energy constrained i.e. in each frame, SU has limited energy to spend in sensing and transmission. This may happen when SU is not powered by conventional sources and harvests energy from surroundings. SU employs a greedy policy where it uses all the available energy in one frame for sensing and transmission subject to maximum power constraint. Suppose energy $e_{tot}$ is available
at SU in each frame. Then total energy spent in sensing and transmission cannot exceed \( e_{\text{tot}} \), that is,

\[
\sum_{i=1}^{M} p_s \tau_{s,i} + \sum_{i=1}^{M} p_t \tau_{t,i} \leq e_{\text{tot}}.
\]  

(6)

2.2.3. Peak transmit power constraint

As response of power amplifier is non-linear at high values of transmit power, there is a limit \( p_{\text{max}} \) on allowed maximum transmit power. Thus, we have

\[
p_{t,i} \leq p_{\text{max}}, \ i = 1, 2, \ldots, M.
\]  

(7)

2.2.4. Total time constraint

In a frame, total time spent in sensing and transmission cannot exceed frame duration. Thus, we have

\[
\sum_{i=1}^{M} \tau_{s,i} + \sum_{i=1}^{M} \tau_{t,i} \leq T.
\]  

(8)

2.3. Problem formulation

If \( i \)th channel is vacant, instantaneous rate achieved by SU on the channel is

\[
\log_2 \left( 1 + \frac{g_i p_{t,i} \sigma^2}{N} \right) \text{ bits/s assuming normalized bandwidth.}
\]

In case of missed detection, PU interferes with SU transmission. Our interest is in maximizing throughput achieved in transmission over a vacant band. Thus, we consider transmission rate achieved under interference as negligible. This is especially true when interference channel between PU and SU is strong. Then average bit-throughput of SU over \( i \)th channel, which is defined as average number of bits transmitted over \( i \)th channel in a frame by SU, is written as

\[
B_{s,i} = \pi_{0,i} \left( 1 - P_{f,i} (\hat{P}_{d,i}, \tau_{s,i}) \right) \tau_{t,i} \log_2 \left( 1 + \frac{g_i p_{t,i} \sigma^2}{N} \right).
\]  

(9)

In this paper, our objective is to maximize average aggregate bit-throughput of SU in a frame duration, given by \( B_s = \sum_{i=1}^{M} B_{s,i} \) under aforementioned constraints in Section 2.2. Thus, we can write the maximization problem as

\[
\max_{\tau_{s,i}, \tau_{t,i}, p_{t,i}} \sum_{i=1}^{M} \pi_{0,i} \left( 1 - P_{f,i} (\tau_{t,i}, \tau_{s,i}) \right) \tau_{t,i} \log_2 \left( 1 + \frac{g_i p_{t,i} \sigma^2}{N} \right)
\]

s. t. (5), (6), (7), (8),

\( \tau_{s,i}, \tau_{t,i}, p_{t,i} \geq 0, \ i = 1, 2, \ldots, M, \)

(10)
where $\tau_s = [\tau_{s,1}, \ldots, \tau_{s,M}]^T$, $\tau_t = [\tau_{t,1}, \ldots, \tau_{t,M}]^T$ and $p_t = [p_{t,1}, p_{t,2}, \ldots, p_{t,M}]^T$. The optimization problem in (10) is non-convex due to non-convex nature of $P_{f,i}(\tau_{t,i}, \tau_{s,i})$. Also, the energy constraint given by (6) is non-convex due to product terms of optimization variables $p_{t,i}$ and $\tau_{t,i}$. In following section, we reformulate the problem so that all constraints are affine and the objective function is separable.

3. Sensing and resource allocation (SRA): Single user scenario

To make constraint (6) affine, we reconstitute problem (10) as energy and time allocation problem. Suppose SU uses energy $e_{t,i}$ to transmit in $i$th channel. Then energy constraint in (6) can be written as

$$\sum_{i=1}^{M} e_{t,i} + p_s \sum_{i=1}^{M} \tau_{s,i} \leq e_{tot}. \quad (11)$$

Transmit power in $i$th channel is $p_{t,i} = e_{t,i}/\tau_{t,i}$. Thus, we write peak power constraint in (7) as

$$e_{t,i} \leq p_{max} \tau_{t,i}, \quad i = 1, 2, \ldots, M. \quad (12)$$

Suppose SU allocates time $t_s = \alpha T$, $\alpha \in [0, 1]$ for sensing and time $t_t = (1 - \alpha) T$ for transmission. Then we can write time constraint (8) as two separate constraints given by

$$\sum_{i=1}^{M} \tau_{s,i} \leq \alpha T, \quad (13)$$

$$\sum_{i=1}^{M} \tau_{t,i} \leq (1 - \alpha) T. \quad (14)$$

Let $B_{s,i} = f_{1,i}(\tau_{t,i}, \tau_{s,i}) \cdot f_{2,i}(\tau_{t,i}, e_{t,i})$ where

$$f_{1,i}(\tau_{t,i}, \tau_{s,i}) = 1 - P_{f,i}(\tau_{t,i}, \tau_{s,i}), \quad (15)$$

$$f_{2,i}(\tau_{t,i}, e_{t,i}) = \pi_{0,i} \tau_{t,i} \log_2 \left(1 + \frac{g_i e_{t,i}}{\sigma_N^2 \tau_{t,i}}\right). \quad (16)$$
Now we can reformulate optimization problem in (10) as follows:

\[
\max_{\alpha, \{\tau_s, \tau_t, e_t\}} \sum_{i=1}^{M} f_1(\tau_{t,i}, \tau_{s,i}) f_2(\tau_{t,i}, e_{t,i})
\]

\[(17a)\]

s. t. \[
\sum_{i=1}^{M} e_{t,i} + p_s \sum_{i=1}^{M} \tau_{s,i} \leq e_{\text{tot}},
\]
\[
e_{t,i} \leq p_{\text{max}} \tau_{t,i}, \quad i = 1, 2, \ldots, M,
\]
\[
\sum_{i=1}^{M} \tau_{s,i} \leq \alpha T,
\]
\[
\sum_{i=1}^{M} \tau_{t,i} \leq (1 - \alpha) T,
\]
\[
e_{t,i}, \tau_{s,i}, \tau_{t,i} \geq 0, \quad i = 1, 2, \ldots, M,
\]
\[
\alpha \in [0, 1],
\]
\[(17g)\]

where \(\tau_s = [\tau_{s,1}, \ldots, \tau_{s,M}]^T\), \(\tau_t = [\tau_{t,1}, \ldots, \tau_{t,M}]^T\) and \(e_t = [e_{t,1}, \ldots, e_{t,M}]^T\).

In the problem above, all constraints are affine. Objective in (17a) is concave in optimization variable \(e_{t,i}\). But the problem is still non-convex in \(\tau_{s,i}\) and \(\tau_{t,i}\).

To solve (17a), we first fix \(\alpha\) and decompose the problem into three subproblems as follows.

**Subproblem P1**

We first fix \((\tau_t, e_t)\) and find optimal sensing time \(\tau_s\) subject to constraints (11), (17d) and (27f). Objective in (17a) is monotonically increasing with \(\tau_{s,i}, \quad i = 1, 2, \ldots, M\). For fixed \((\tau_t, e_t)\), we can write problem of finding optimal \(\tau_s\) as

\[
\max \sum_{i=1}^{M} B_{s,i}(\tau_{s,i})
\]

\[(18)\]

s. t. \[
\sum_{i=1}^{M} \tau_{s,i} \leq \min \left[ \alpha T, \frac{e_{\text{tot}} - \sum_{i=1}^{M} e_{t,i}}{p_s} \right],
\]

Problem in (18) can be modelled as a general non-linear knapsack problem (NKP). We use greedy algorithm [30, 31] to solve it with complexity \(O(M \log (f_s T))\).
We define $u_s = 1/f_s$ as the smallest time unit that can be allocated to the sensing time of a channel. Incidentally $u_s$ is also the time between successive samples. We initialize sensing time as $\tau_{s,i} = 0$, $i = 1, 2, \ldots, M$. Increase in bit-throughput due to addition of one sensing time unit can be viewed as reward of the action. Thus, reward of adding a unit to $i$th channel in $k$th iteration is given by

$$ r^{(k)}_{s,i} = B_{s,i} \left( \tau^{(k)}_{s,i} + u_s \right) - B_{s,i} \left( \tau^{(k)}_{s,i} \right). \quad (19) $$

In each iteration, one time unit is added to the channel $\hat{i}$ where $\hat{i}$ is the channel that gives maximum reward, i.e. $\hat{i} = \arg \max_i \{ r^{(k)}_{s,i} \}$. Thus, in each iteration, sensing time is updated as

$$ \tau^{(k+1)}_{s,i} = \begin{cases} \tau^{(k)}_{s,i} + u_s & \text{for } i = \hat{i}^{(k)} \\ \tau^{(k)}_{s,i} & \text{for } i \neq \hat{i}^{(k)} \end{cases}. \quad (20) $$

The process continues until $\sum_{i=1}^{M} \tau^{(k)}_{s,i} \leq \min \left[ \alpha T, \frac{e_{tot} - \sum_{i=1}^{M} e_{t,i}}{p_s} \right]$.

**Subproblem P2**

Keeping optimal $(\tau_s, e_t)$ in P1 fixed, we now optimize $\tau_t$ subject to constraints (17e) and (17f). The problem of optimizing $\tau_{t,i}$ is

$$ \max \sum_{i=1}^{M} B_{s,i} (\tau_{t,i}) \quad (21) $$

s. t. $\sum_{i=1}^{M} \tau_{t,i} \leq (1 - \alpha) T$.

On similar lines of subproblem P1, we can find optimal $\tau_{t,i}$ by greedy method for NKP using $u_t = 1/f_s$ as the smallest time unit. From (15) and (16), we see that $f_{1,i}$ is a monotonically decreasing function of $\tau_{t,i}$ while $f_{2,i}$ is a monotonically increasing function of $\tau_{t,i}$. Thus, in a region where $B_{s,i}(\tau_{t,i})$ is monotonically decreasing with $\tau_{t,i}$, addition of a unit $u_t$ to transmission time of $i$th channel results in negative reward value. When reward values for all channel are negative, any further increase in transmission time $\tau_{t,i}$ results in decreasing bit-throughput. Thus, the greedy algorithm stops when all rewards become negative or when constraint (17e) is violated. Using reward $r^{(k)}_{t,i} = B_{s,i} \left( \tau^{(k)}_{t,i} + u_t \right) - B_{s,i} \left( \tau^{(k)}_{t,i} \right)$, transmission
time is updated in each iteration as

\[
t(t+1)_{i,i} = \begin{cases} t(k)_{i,i} + u_i & \text{for } i = \hat{i}(k) \\ t(k)_{i,i} & \text{for } i \neq \hat{i}(k) \end{cases},
\]

where \( \hat{i}(k) = \arg \max_i \{ r(k)_{i,i} \} \). The process continues until \( \sum_{i=1}^M t(i) \leq (1 - \alpha) T \) and \( \max \{ r(k)_{i,i} \} \geq 0 \).

**Subproblem P3**

Keeping optimal \((\tau_s, \tau_t)\) in P1 and P2 fixed, we now optimize over \(e_t\) subject to constraints (17b), (17c) and (17f). Let \( e_{th} = e_{tot} - p_s \sum_{i=1}^M (t_i - \tau_{t,i}) \). Since the problem (17a) is convex in \( e_{t,i} \), we solve it using Lagrangian method. The Lagrangian for P3 is

\[
L(e_t, \lambda, \mu) = \sum_{i=1}^M f_{1,i} \cdot f_{2,i}(e_{t,i}) - \lambda \left( \sum_{i=1}^M e_{t,i} - e_{th} \right) - \sum_{i=1}^M \mu_i \left( e_{t,i} - p_{max} \tau_{t,i} \right),
\]

where \( \lambda \) and \( \mu = [\mu_1, \ldots, \mu_M]^T \) denote the dual variables associated with constraints (17b) and (17c). The dual problem of P3 is given by

\[
\min_{\lambda, \mu} \max_{e_t} L.
\]

For fixed \((\lambda, \mu)\), we find optimal primal variable by differentiating \( L \) with respect to \( e_{t,i} \) and equating it to zero as

\[
e_{t,i} = \left[ \frac{\pi_{0,i} \tau_{t,i} f_{1,i}}{\ln(2) (\lambda + \mu_i)} - \frac{\sigma^2 N \tau_{t,i}}{g_i} \right]^+, \tag{23}
\]

where \([\cdot]^+ = \max[\cdot, 0]\). Since the dual function of \( L \) has unique maximizers, we use gradient descent method to find \((\lambda, \mu)\) as

\[
\lambda^{(k+1)} = \lambda^{(k)} + \epsilon_{\lambda} \left( \sum_{i=1}^M e_{t,i} - e_{th} \right), \tag{24}
\]
\[ \mu^{(k+1)}_i = \mu^{(k)}_i + \epsilon_\mu (e_{t,i} - p_{max}), \]  

(25)

where \( \epsilon_\lambda \) and \( \epsilon_\mu \) are step sizes. Iteration index is denoted by \( k \). The process of calculating \( e_t \) and updating \( (\lambda, \mu) \) is repeated until convergence. In this way the subproblem P3 is solved with complexity \( \mathcal{O}(M^2) \).

All subproblems aim to maximize objective in (17a). For fixed \( \alpha \), we repeat the three step process of solving P1, P2 and P3. This process of finding \((\tau_s, \tau_t, e_t)\) that maximize SU throughput is Block Coordinate Minimization (BCM) method which converges to stationary solution for non-convex problems as proven in [32]. Convergence is achieved as long as initialization of \((\tau_t, e_t)\) in subproblem P1 is done to satisfy constraints (17c), (17e) and \( \sum_{i=1}^{M} e_{t,i} \leq e_{tot} - p_s \alpha T \). The BCM method runs over all values of \( \alpha \in [0, 1] \) and value of \( \alpha \) that corresponds to the maximum SU bit-throughput is chosen.

4. Sensing and resource allocation (SRA): Multi-user scenario

In this section, we propose sensing and resource allocation for the case where multiple SUs are present in the system. We consider a secondary network of \( N \) SUs governed by a central base station (BS) employing cooperative sensing. BS acts as the fusion centre for sensing data of individual SUs. Alternatively, in absence of BS, one of the SUs can act as the controller. We assume that SUs always have data to transmit and all SUs transmit to a common destination. To avoid inter-SU interference, BS employs time division multiple access (TDMA). We assume that BS has knowledge of channels gains on all SU source to SU destination links and PU source to SU source links, denoted as \( g_{ij} \) and \( h_{ij} \), \( i \in \{1, \ldots, M\} \), \( j \in \{1, \ldots, N\} \) respectively. Assumption of perfect channel knowledge gives us the upper bound on throughput performance and serves as a baseline for the case with imperfect or limited channel knowledge. Prior to sensing and transmission, BS determines optimal sensing time, transmission time allocation and transmission energy allocation for each channel and communicates it to the SUs over a low bandwidth control channel as done in [33].

Time allocated for sensing and transmission in \( i \)th PU channel is \( \tau_{s,i} \) and \( \tau_{t,i} \) respectively. Sensing data is reported to BS over a low bandwidth control channel. BS performs data fusion and takes a decision on presence of PU in a given band. In this case, false-alarm probability in sensing \( i \)th PU channel is written as [8]

\[ P_{f,i}' = Q \left( \sqrt{2\hat{\gamma}_i} + 1 Q^{-1} (\hat{P}_{d,i}) + \hat{\gamma}_i \sqrt{\tau_{s,i}} \right), \]  

(26)
where $\bar{\gamma}_i = p_{PU} \sum_{j=1}^{N} h_{ij}$ and $\bar{P}_{d,i}$ is the target detection probability given in (5).

If $i$th PU is sensed to be absent, each SU transmits its own data to SU destination. Due to TDMA, transmission time of each SU in $i$th channel is $\tau_{t,i}$. Energy used by $j$th SU to transmit in $i$th channel is $e_{t,ij}$. Energy available at $j$th SU is denoted by $e_j$. A SU participates in the joint-sensing and transmission process only if it has minimum required energy to sense a channel for whole frame duration, that is $e_j \geq p_s T$, $j = 1, \ldots, N$.

Our objective is to find optimal access time, transmission time and energy allocation to maximize sum-throughput of SU system. Let $f'_{1,i} = 1 - P'_{f,i}$. Then the optimization problem is

$$\max_{\alpha, \{\tau_s, \tau_t, e_t\}} \sum_{i=1}^{M} f'_{1,i}(\tau_{t,i}, \tau_{s,i}) \pi_0, i \sum_{j=1}^{N} \frac{\tau_{t,i}}{N} \log_2 \left(1 + \frac{g_{ij} N e_{t,ij}}{\sigma^2 N \tau_{t,i}}\right)$$

s. t. \hspace{1cm} \sum_{i=1}^{M} e_{t,ij} + p_s \sum_{i=1}^{M} \tau_{s,i} \leq e_j, \hspace{1cm} j = 1, 2, \ldots, N, \hspace{1cm} (27b)

$$e_{t,ij} \leq p_{max} \frac{\tau_{t,i}}{N}, \hspace{1cm} j = 1, 2, \ldots, N,$$

$$\sum_{i=1}^{M} \tau_{s,i} \leq \alpha T, \hspace{1cm} (27d)$$

$$\sum_{i=1}^{M} \tau_{t,i} \leq (1 - \alpha) T, \hspace{1cm} (27e)$$

$$e_{t,ij}, \hspace{0.5cm} t_i, \hspace{0.5cm} \tau_{t,i} \geq 0, \hspace{0.5cm} i = 1, \ldots, M, \hspace{0.5cm} j = 1, \ldots, N,$$

$$\alpha \in [0, 1], \hspace{1cm} (27f)$$

where $\tau_s = [\tau_{s,1}, \ldots, \tau_{s,M}]^T$, $\tau_t = [\tau_{t,1}, \ldots, \tau_{t,M}]^T$ and $e_t = [e_{t,ij}]_{M \times N}$. Objective function (27a) is concave in $e_t$, but non-convex in $\tau_s$ and $\tau_t$. Total energy used by a SU in sensing and transmission cannot exceed energy available at the SU. This gives rise to a per-user energy constraint in (27b). Constraint in (27c) is the peak power constraint for each user. Time constraints (27d) and (27e) remain unchanged from single-user scenario. We see that all constraints are affine.

Along the lines of Section 3 we can decompose the optimization problem in three subproblems for fixed value of $\alpha$. Subproblem P1 solves sensing time allocation for fixed $(\tau_t, e_t)$ using greedy algorithm for NKP under constraint
\[
\sum_{i=1}^{M} \tau_{s,i} \leq \min \left[ \alpha T, \tau_{th,1}, \tau_{th,2}, \ldots, \tau_{th,N} \right], \quad \text{where} \quad \tau_{th,j} \quad \text{is given by}
\]
\[
\tau_{th,j} = e^{-j - \sum_{i=1}^{M} e^{t_{ij}}}, \quad j = 1, 2, \ldots, N.
\]

In subproblem P2, to find optimal \( \tau_{t} \), let \( f'_{2,i} = \pi_{0,i} \sum_{j=1}^{N} \tau_{t,i} N \log_2 \left( 1 + \frac{g_{ij} N e^{t_{ij}}}{\sigma^2_N} \right) \).

We see that \( f'_{1,i} \) is monotonically decreasing function of \( \tau_{t,i} \) and \( f'_{2,i} \) is a monotonically increasing function of \( \tau_{t,i} \). Thus, for fixed value of \( (\tau_{s}, e_{t}) \), optimal \( \tau_{t} \) can be found by greedy algorithm with modified stopping criteria as done in subproblem P2 in Section 3.

In subproblem P3, we keep optimal \( (\tau_{s}, \tau_{t}) \) fixed and optimize over \( e_{t} \) under constraints (27b), (27c) and (27f). P3 is a convex-programming problem that is solved by Lagrangian method using steps similar to those used in Section 3. We omit the steps here for brevity and write closed form expression for \( e_{t,ij} \) as
\[
e_{t,ij} = \left[ \frac{\pi_{0,i} \tau_{t,i} f'_{1,i}}{\ln(2) (\lambda_{j} + \mu_{ij}) N} - \frac{\sigma^2_N \tau_{t,i}}{g_{ij} N} \right]^{+}, \quad (28)
\]

where \( \lambda_{j} \) and \( \mu_{ij} \) are Lagrange’s multipliers that are chosen to satisfy per-user energy constraint \( \sum_{i=1}^{M} e_{t,ij} \leq e_j - p_s \sum_{i=1}^{M} \tau_{s,i} \) and peak power constraint \( e_{t,ij} \leq p_{\max} \frac{\tau_{t,i}}{N} \) respectively. Subproblems P1, P2 and P3 are executed recursively until all variables converge. The process of finding optimal \( (\tau_{s}, \tau_{t}, e_{t}) \) is repeated over all values of \( \alpha \in [0, 1] \) and \( \alpha \) that maximizes SU bit-throughput is chosen.

5. Simulation results and discussion

In this section, we first study performance of proposed Sensing and Resource Allocation (SRA) method under different channel conditions and energy availability scenarios for single-SU case. We compare the performance with Best Channel Selection (BCS) and Proportional Energy and Time Allocation (PETA) methods. In BCS, SU chooses the best channel for sensing and transmission, based on a heuristic that depends on channel gains, PU occupancy and QoS constraint. The heuristic \( \mathcal{H}_i \) for \( i \)th channel is defined as
\[
\mathcal{H}_i = \frac{\pi_{0,i} g_i}{\tau_{p,i}}.
\]

Value of \( \mathcal{H}_i \) is high for a channel with low occupancy probability, low value of \( \tau_{p} \) and good SU-SU channel. In each frame, SU chooses the best channel \( \hat{i} \) that has
highest value of $H_i$, i.e. $\hat{i} = \arg \max_{i \in \{1, \ldots, M\}} H_i$. In PETA, total available time and energy is divided between channels such that time and energy for each channel is proportional to the channel heuristic. Let $T_i$ and $e_{\text{tot},i}$ be the time allocated to $i$th channel. Then we have $T_i = kH_iT$ and $e_{\text{tot},i} = kH_i e_{\text{tot}}$ where normalization factor $k$ is calculated as $k = 1/\sum_{i=1}^{M} H_i$. For each channel, optimal sensing and transmission time $\tau_{s,i}, \tau_{t,i} \leq T_i$ as well as transmission energy $e_{t,i} \leq e_{\text{tot},i}$ is found using SRA. We also compare the performance with single channel transmission scheme (SRA-SC). In SRA-SC, expected throughput is calculated for a single channel at a time. Only the channel that gives maximum expected throughput is selected for transmission. Further, as a baseline for performance comparison, we use optimal sensing under target detection probability $P_d = 0.95$ proposed in [8] combined with best channel selection.

For simulation, we consider $M = 10$. Values of frame time and sampling frequency are $T = 100$ ms and $f_s = 1$ MHz. As all channels are Rayleigh faded, SU-SU channel gains $g_i, i = 1, \ldots, M$ are exponentially distributed. We take the average channel gain $\sigma^2_g = -10$ dB unless mentioned otherwise. Channel occupancy probabilities are $\pi_1 = [0.7, 0.7, 0.7, 0.7, 0.7, 0.5, 0.5, 0.5, 0.5, 0.5]^T$. PU QoS threshold for each PU channel is $\tau_{p,i} = 0.9, i = 1, \ldots, M$. Maximum power threshold is $p_{\text{max}} = 1$ W and power required for sensing is $p_s = 0.1$ W [17]. Noise power is $\sigma^2_N = 0.1$ W. Average received PU SNR is $\gamma_i = \gamma_p = -10$ dB, $i = 1, \ldots, M$, unless mentioned otherwise.

5.1. Effect of energy availability

Fig. 2 plots simulation and analytical results for bit-throughput achieved in proposed SRA method against available energy for different values of received PU power. As available energy $e_{\text{tot}}$ increases, more energy can be used in transmission and average bit-throughput $B_s$ increases. At high value of $e_{\text{tot}}$, peak power constraint in (17c) becomes dominant and limits transmission energy in each channel. Even though energy is available, more energy cannot be used in transmitting. Thus, $B_s$ becomes constant at high value of $e_{\text{tot}}$.

When received PU power is high, sensing time required to achieve target detection probability is low. Also, false alarm probability is low. This leaves more time for transmission in a channel. Thus, throughput achieved is higher. If received PU power is low, more sensing time is required to detect a PU correctly. Thus, time available for transmission decreases, resulting in less bit-throughput.

Fig. 3 shows that bit-throughput in SRA is better than BCS, PETA and SRA-SC. In BCS, best channel chosen by SU may have desirable properties like low
Figure 2: Bit-throughput $B_s$ versus available energy $e_{tot}$ for different values of average received PU SNR $\gamma_p$ for $N = 1$.

Figure 3: Comparison of different allocation methods in terms of bit-throughput $B_s$ for $N = 1$ and $\gamma_P = -10$ dB.
occupancy and loose QoS constraint but may also have low channel gain. By choosing a single channel to transmit, diversity provided by multiple channels is sacrificed in BCS, hence resulting in less throughput. PETA fixes maximum energy and time allocated to a channel based on channel heuristics and only optimizes over individual channels, which is suboptimal compared to SRA. Fig. 3 also plots throughput for optimal sensing in [8] using best channel selection. We see that SRA performs better than fixed $P_d$ based method.

5.2. Effect of primary occupancy and QoS constraint

Fig. 4 plots bit-throughput $B_s$ against occupancy probability $\pi_{1,1}$ for different values of QoS constraint $\tau_{p,1}$ for $e_{tot} = 10$ mJ, $M = 4$ and $N = 1$.

PU occupancy and QoS constraints also affect probability of SU accessing a channel as shown in Fig. 5. Channel access probability is defined as probability of SU transmitting in the channel. It is represented in Fig. 5 in grayscale color tone where darker shade indicates lower access probability. As $\pi_{1,1}$ and $\tau_{p,1}$ increase, SU does not transmit in the channel unless SU-SU channel gain is high.
Figure 5: Channel access probability versus PU occupancy probability $\pi_{1,1}$ and QoS constraint $\tau_{p,1}$ for $T = 10$ ms, $f_s = 0.1$ MHz, $M = 4$ and (a) $\epsilon_{tot} = 10^{-4}$ J (b) $\epsilon_{tot} = 10^{-2}$ J.

Thus, channel access probability decreases. When available energy $\epsilon_{tot}$ is high, maximum power constraint limits the energy that can be transmitted in a channel. Thus, some energy is distributed in other channels even though their occupancy probability may be higher and QoS constraints may be tighter. Thus, overall access probability is higher than low energy availability case. Access probability decreases sharply only at high values of $\pi_{1,i}$ and $\tau_{p,i}$ as seen in Fig. 5(b).

5.3. Sum-throughput in multi-user scenario

Heuristic based methods are suboptimal and SRA clearly outperforms them as seen in Fig. 3. Also, for multi-user scenario, allocating resources for each SU-PU channel pair based on heuristics is a separate optimization problem in itself. Hence, we omit the comparison of SRA with heuristic based methods. In Fig. 6, we plot sum-throughput of a SU network against number of SUs $N$ for different energy availability scenarios. Energy available at each SU is denoted by $\epsilon_{tot}$. Initially, as number of SUs increase, cooperative sensing lowers required sensing time to achieve target detection probability leaving more time for transmission. This results in increasing sum-throughput. But increasing number of SUs decrease transmission time allotted to each SU, given by $\tau_{t,i}^N$, $i = 1, \ldots, M$. Due to peak power constraint $\epsilon_{t,ij} \leq p_{max} \frac{\tau_{t,i}^N}{N}$, transmission energy decreases with decreasing value of $\frac{\tau_{t,i}^N}{N}$. Thus, with increasing $N$, transmission time as well as transmission energy of each SU decreases, resulting in decreasing throughput.

6. Conclusion

We considered a CR system with multiple PU channels where simultaneous sensing of all channels is not possible. Also, SU has limited energy for sensing and
transmission. The problem of maximizing SU bit-throughput while maintaining QoS of PUs was formulated as a resource allocation problem with time and energy as available resources. We proposed sensing and resource allocation (SRA) method that solves the problem by decomposing it in non-linear knapsack subproblem and convex optimization subproblem. We then extended the framework for multiple SU scenario where SUs can achieve benefits of cooperative sensing. Simulation results show that throughput increases as more energy is available. It was observed that with more number of SUs, throughput performance benefits from cooperative sensing. But as SUs further increase, throughput decreases as less time is available for transmission of each SU.

References

[1] M. A. McHenry, “NSF spectrum occupancy measurements project summary.” Shared Spectrum Company, 2005.

[2] S. Haykin, “Cognitive radio: brain-empowered wireless communications,” IEEE J. Sel. Areas Commun., vol. 23, pp. 201–220, Feb 2005.

[3] T. Yucek and H. Arslan, “A survey of spectrum sensing algorithms for cognitive radio applications,” IEEE Commun. Surveys Tuts., vol. 11, pp. 116–130, First 2009.
[4] H. Sun, A. Nallanathan, C.-X. Wang, and Y. Chen, “Wideband spectrum sensing for cognitive radio networks: a survey,” IEEE Wireless Commun., vol. 20, pp. 74–81, April 2013.

[5] F. Digham, M.-S. Alouini, and M. K. Simon, “On the energy detection of unknown signals over fading channels,” IEEE Trans. Commun., vol. 55, pp. 21–24, Jan 2007.

[6] H. Urkowitz, “Energy detection of unknown deterministic signals,” Proc. IEEE, vol. 55, pp. 523–531, April 1967.

[7] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, “Breaking spectrum gridlock with cognitive radios: An information theoretic perspective,” Proc. IEEE, vol. 97, pp. 894–914, May 2009.

[8] Y.-C. Liang, Y. Zeng, E. Peh, and A. T. Hoang, “Sensing-throughput trade-off for cognitive radio networks,” IEEE Trans. Wireless Commun., vol. 7, pp. 1326–1337, April 2008.

[9] E. Peh, Y.-C. Liang, Y. L. Guan, and Y. Zeng, “Optimization of cooperative sensing in cognitive radio networks: A sensing-throughput tradeoff view,” IEEE Trans. Veh. Technol., vol. 58, pp. 5294–5299, Nov 2009.

[10] S. Zarrin and T. J. Lim, “Throughput-sensing tradeoff of cognitive radio networks based on quickest sensing,” in IEEE Int. Conf. Commun. (ICC 2011), pp. 1–5, June 2011.

[11] M. Cardenas-Juarez and M. Ghogho, “Spectrum sensing and throughput trade-off in cognitive radio under outage constraints over Nakagami fading,” IEEE Commun. Lett., vol. 15, pp. 1110–1113, October 2011.

[12] A. Kaushik, S. Sharma, S. Chatzinotas, B. Ottersten, and F. Jondral, “Sensing-throughput tradeoff for cognitive radio systems with unknown received power,” in Cognitive Radio Oriented Wireless Networks (M. Weichold, M. Hamdi, M. Z. Shakir, M. Abdallah, G. K. Karagiannidis, and M. Ismail, eds.), vol. 156 of Lecture Notes of the Institute for Computer Sciences, Social Informatics and Telecommunications Engineering, pp. 308–320, Springer International Publishing, 2015.

[13] Y. Pei, Y.-C. Liang, K. Teh, and K. H. Li, “Sensing-throughput tradeoff for cognitive radio networks: A multiple-channel scenario,” in Proc. IEEE 20th
Int. Symp. Personal, Indoor and Mobile Radio Commun. (PIMRC 2009), pp. 1257–1261, Sept 2009.

[14] Y. Sharkasi, M. Ghogho, and D. McLernon, “Sensing-throughput tradeoff for OFDM-based cognitive radio under outage constraints,” in Proc. Int. Symp. Wireless Commun. Syst. (ISWCS 2012), pp. 66–70, Aug 2012.

[15] J. Paradiso and T. Starner, “Energy scavenging for mobile and wireless electronics,” IEEE Pervasive Comput., vol. 4, pp. 18–27, Jan 2005.

[16] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han, “Wireless networks with RF energy harvesting: A contemporary survey,” CoRR, vol. abs/1406.6470, 2014.

[17] S. Park, H. Kim, and D. Hong, “Cognitive radio networks with energy harvesting,” IEEE Trans. Wireless Commun., vol. 12, pp. 1386–1397, March 2013.

[18] S. Wang, Y. Wang, J. Coon, and A. Doufexi, “Energy-efficient spectrum sensing and access for cognitive radio networks,” IEEE Trans. Veh. Technol., vol. 61, pp. 906–912, Feb 2012.

[19] S. Yin, E. Zhang, L. Yin, and S. Li, “Saving-sensing-throughput tradeoff in cognitive radio systems with wireless energy harvesting,” in Proc. IEEE Global Commun. Conf. (GLOBECOM 2013), pp. 1032–1037, Dec 2013.

[20] X. Wu, J.-L. Xu, M. Chen, and J. Wang, “Optimal energy-efficient sensing and power allocation in cognitive radio networks,” 2014.

[21] A. T. Hoang, Y.-C. Liang, D. Tung Chong Wong, R. Zhang, and Y. Zeng, “Opportunistic spectrum access for energy-constrained cognitive radios,” in Proc. IEEE Veh. Technol. Conf. (VTC Spring 2008), pp. 1559–1563, May 2008.

[22] A. Sultan, “Sensing and transmit energy optimization for an energy harvesting cognitive radio,” IEEE Wireless Commun. Lett., vol. 1, pp. 500–503, October 2012.

[23] S. Park, S. Lee, B. Kim, D. Hong, and J. Lee, “Energy-efficient opportunistic spectrum access in cognitive radio networks with energy harvesting,” in Proceedings of the 4th International Conference on Cognitive Radio and
Advanced Spectrum Management, CogART ’11, (New York, NY, USA), pp. 62:1–62:5, ACM, 2011.

[24] A. Chakraborty and S. R. Das, “Measurement-augmented spectrum databases for white space spectrum,” in Proceedings of the 10th ACM International on Conference on Emerging Networking Experiments and Technologies, CoNEXT ’14, (New York, NY, USA), pp. 67–74, ACM, 2014.

[25] D. Das and S. Das, “A survey on spectrum occupancy measurement for cognitive radio,” Wireless Personal Communications, vol. 85, no. 4, pp. 2581–2598, 2015.

[26] J. J. Lehtomaki, R. Vuohoniemi, and K. Umebayashi, “On the measurement of duty cycle and channel occupancy rate,” IEEE J. Sel. Areas Commun., vol. 31, pp. 2555–2565, November 2013.

[27] J. Xue, Z. Feng, and P. Zhang, “Spectrum occupancy measurements and analysis in Beijing,” {IERI} Procedia, vol. 4, pp. 295 – 302, 2013. 2013 International Conference on Electronic Engineering and Computer Science (EECS 2013).

[28] Q. Zhao, L. Tong, A. Swami, and Y. Chen, “Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework,” IEEE J. Sel. Areas Commun., vol. 25, pp. 589–600, April 2007.

[29] G. Naik, S. Singhal, A. Kumar, and A. Karandikar, “Quantitative assessment of TV white space in India,” in Proc. National Conf. Commun. (NCC 2014), pp. 1–6, Feb 2014.

[30] D. S. Hochbaum, “A nonlinear knapsack problem,” Operations Research Letters, pp. 103–110, 1995.

[31] Y. Zhang and C. Leung, “Subcarrier, bit and power allocation for multiuser OFDM-based multi-cell cognitive radio systems,” in Proc. IEEE Veh. Technol. Conf. (VTC 2008-Fall), pp. 1–5, Sept 2008.

[32] M. Razaviyayn, M. Hong, and Z.-Q. Luo, “A unified convergence analysis of block successive minimization methods for nonsmooth optimization,” SIAM Journal on Optimization, vol. 23, no. 2, pp. 1126–1153, 2013.
[33] T. Li, N. B. Mandayam, and A. Reznik, “A framework for distributed resource allocation and admission control in a cognitive digital home,” IEEE Trans. Wireless Commun., vol. 12, pp. 984–995, March 2013.

Kedar Kulkarni received his B.Tech. degree in Electronics and Telecommunication from College of Engineering Pune, India, in 2010. He is presently pursuing Ph.D. degree in Department of Electrical Engineering at Indian Institute of Technology (IIT) Kanpur, India. His research interests include resource allocation in cognitive radio networks, cooperative networks, and energy harvesting wireless communications.

Adrish Banerjee received his B.Tech. degree in Electronics and Electrical Communication Engineering from Indian Institute of Technology (IIT) Kharagpur, India, in 1996 and M.S. and Ph.D. degrees in Electrical Engineering from University of Notre Dame, Indiana in 1998 and 2003, respectively. He is currently an Associate Professor in the Department of Electrical Engineering at IIT Kanpur, India. His research interests include physical layer aspects of wireless communications, particularly error control coding, cognitive radio, and green communications.
This figure "author1.jpg" is available in "jpg" format from:

http://arxiv.org/ps/1701.04901v1
This figure "author2.jpg" is available in "jpg" format from:

http://arxiv.org/ps/1701.04901v1