Short-range nucleon correlations and neutrino emission by neutron stars

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Abstract. We argue that significant probability of protons with momenta above their Fermi surface leads for proton concentrations $p/n \geq 1/8$ to the enhancement of termally excited direct and modified URCA processes within a cold neutron star, and to a nonzero probability of direct URCA processes for small proton concentrations ($p/n \leq 1/8$). We evaluate high momentum tails of neutron, proton and electrons distributions within a neutron star. We expect also significantly faster neutrino cooling of hyperon stars.

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INTRODUCTION

A normal neutron star is bound by gravitational interactions. Global characteristics of neutron stars follow from the equations for the hydrostaticequilibrium in the general relativity, see [1]. A neutron star can be divided into several layers: the crust, the outer and the inner cores. The outer core extends up to the densities $\rho \sim (2-3)\rho_0$, where $\rho_0 \approx 0.16\text{nucleon}/\text{fm}^3$ is the nuclear matter density. The inner core extends to the center of the neutron star where densities can be significantly larger $\sim 5-10\rho_0$ and may contain muons, hyperons, and exotic matter. Due to inverse $\beta$ decay, the nuclear matter dissolves into a uniform liquid composed of neutrons at the density $\sim 1/2\rho_0$, with

$$x = N_p/N_n \sim 5 \div 10\%,$$

admixture of protons and equal admixture of electrons and tiny admixture of muons, see Ref. [2, 3]. In the inner core the value of proton fraction is probably larger: $\sim 10 \div 13\%$ [4]. The most efficient neutrino cooling reactions are due to direct URCA processes involving neutron $\beta$ decay:

$$n \to p + e + \bar{\nu}_e,$$

and $\beta$ capture in

$$e + p \to n + \nu_e.$$

Thus it is worth to analyse how internucleon interactions influence termally excited direct URCA processes within cold neutron stars. Standart cooling scenario assumes that direct URCA processes can occur in the inner core only [5].

In the ideal gas approximation the zero temperature neutron star is described as the system of degenerate neutron, proton and electron gases with the ratio of proton and neutron densities, $x \ll 0.1$. For any positive neutron density the Pauli blocking in the
electron and proton sectors guarantees stability of a neutron star to the neutron $\beta$-decay cf. [6, 7]. The number densities of protons and electrons are equal to ensure electrical neutrality of the star, so $k_F(e) = k_F(p)$. The neutron Fermi momentum is significantly larger than the proton Fermi momentum because of the larger number of the neutrons:

$$x^{1/3}k_F(n) = k_F(p). \quad (4)$$

The internucleon interaction produces nucleons with momenta above Fermi surfaces, cf. Eqs. (5, 6). To guarantee conservation of the electric and the baryon charges nucleon occupation numbers below the corresponding Fermi surfaces - $f_i(k, T = 0)$ should be smaller than unity especially for protons. The nonrelativistic Schrödinger equation with realistic nucleon-nucleon interactions gives occupation numbers for protons with zero momenta $\approx 70\%$ for the nuclear matter density. Even a larger depletion of occupation numbers is found for protons with momenta near the Fermi surface [8].

The Landau Fermi liquid approach [9] where momentum distribution of quasiparticles coincides with the Fermi distribution for the ideal gas of fermions is effective starting approximation for describing strongly interacting liquid. It has been explained by A.B.Migdal that nucleon distribution at zero temperature should exhibit the Migdal jump at $k = k_F$ which justifies applicability of the Fermi step distribution at zero temperature. The value of the Migdal jump is equal to the renormalization factor $Z \leq 1$ of the single-particle Green’s function in the nuclear matter. The condition $Z < 1$ follows from the probability conservation [10] and implies that occupation numbers for nucleons with momenta $k < k_F$ are below one. In the limit of small proton concentration Fermi surface nearly disappears since proton neighborhood is predominantly strongly interacting neutron medium. So the height of the Migdal jump for the proton distribution should decrease $\propto x$ for $x \to 0$. (Decrease of the Migdal jump due to a large probability of SRC has been discussed a long time ago for the liquid $^3$He [11].) Thus for a highly asymmetric mixture of protons and neutrons the interaction tends to extend proton momenta well beyond $k_F(p)$.

We show that for the temperatures $T \ll 1\text{MeV}$ the presence of the high momentum proton tail leads to a different value and temperature dependence of URCA processes for $x \geq 1/8$, cf. Eq. 16 as compared to that in Refs. [12, 13, 14] where the Fermi momentum distribution for quasiparticles was used. As the consequence of the presence of the high momentum proton tail the neutrino luminosity due to direct URCA processes differs from zero even for $x < 1/8$ i.e. in the region forbidden in the ideal gas approximation for quasiparticles by the Pauli blocking and the momentum conservation.

The electron gas within neutron star is ultrarelativistic. So the Coulomb parameter $e^2/v \ll 1$. Here $e$ is the electric charge of electron and $v = p/E \approx c$ is its velocity. Thus approximation of the free electron gas is justified. The Coulomb interaction between protons with momenta $k \geq k_F(p)$ and electrons produces electrons with momenta above the electron Fermi surface, although with a tiny probability cf. Eq. [7]. So the occupation probability for electrons: $f_e(k_e \leq k_F(e), T = 0)$ is slightly less than one.

Thus the interaction produces holes in all Fermi seas removing the absolute Pauli blocking for the direct neutron, muon, hyperon $\beta$-decays. We show however that the account of the Pauli blocking in the electron sector ensures stability of a neutron to the direct $\beta$ decay in the outer core of a neutron star. Condition of stability may be violated
in the inner core where however use of nucleon degrees of freedom is questionable.

If hyperon stars exist (for the review of this subject and references see [13]), neutrino luminosity due to direct $\beta$- decay may appear significantly larger than for a neutron star.

**THE ROLE OF THE INTERACTION**

High momentum nucleon component of the wave function of a neutron star follows directly from the Schrödinger equation in the limit $k \gg k_F$ where $k_F$ is Fermi momentum. The derivation of the formulae is similar to that in [16, 17].

At the leading order in $(k_F^2/k^2)$ the occupation numbers for protons and neutrons with momenta above Fermi surface are:

\[ f_n(k, T = 0) \approx (\rho_n)^2 \left( \frac{V_{nn}(k)}{k^2/m_N} \right)^2 + 2x \left( \frac{V_{pn}(k)}{k^2/m_N} \right)^2, \]  

(5)

and

\[ f_p(k, T = 0) \approx (\rho_n)^2 \left( x^2 \frac{V_{pp}(k)}{k^2/m_N} \right)^2 + 2x \left( \frac{V_{pn}(k)}{k^2/m_N} \right)^2. \]  

(6)

Here $\rho_i$ is the density of constituent $i$. The factor $V_{NN}(k)$ describes the high momentum tail of the potential of the $NN$ interaction. The factor 2 in the above formulae accounts for the number of spin states. In the first term, this factor is cancelled due to the identity of nucleons within the pair. In the derivation of the formulae for the probability of SRCs we used the approximation of nucleon density uniform in coordinate space to describe the uncorrelated part of the wave function. Thus, the value of the high momentum tail depends strongly on the nucleon density in the core of a neutron star. Since $k_F(p)$ is significantly smaller than $k_F(n)$, the probability to find a proton with $k \geq k_F(p)$ for a neutron density close to the nuclear density should be significantly larger than in nuclei where $x \approx 1$. Note also that the analysis of the recent data on SRCs in the symmetric nuclear matter found a significant $\sim 20\%$ probability of nucleons above the Fermi surface in nuclei which is predominantly due to $I = 0$ SRCs [18].

The Coulomb interaction between protons from SRCs and electrons produces electrons with momenta above the electron Fermi surface. Such electrons are ultrarelativistic so Feynman diagrams approach should be used to evaluate the high momentum electron component rather than the nonrelativistic Schrödinger equation. We find for the high momentum electron component approximate expression:

\[ f_e(k_e \geq k_F(e), T = 0) \approx \frac{1}{2} \int (d^3 k_p/(2\pi))^3 f_p(k_p) \theta(k_p - k_F(p)) \rho_e \cdot \]  

(7)

\[ \cdot (1 - f_p(k_p, T = 0)) \left( \frac{k_e + \frac{3}{4} k_F(e)}{\sqrt{k_e} \cdot \sqrt[3]{\frac{3}{4} k_F(e)}} \right) \left( \frac{V_{Coulomb}(k)}{k_e - k^2/2m_N - \frac{3}{4} k_F(e)} \right)^2. \]

The factor $1 - f_p(k_p, T = 0)$ is the number of proton holes which prevent Pauli blocking for the proton after interaction with the electron. Effectively, Eq.(7) gives the probability
for triple (e-p-n) short range correlations. This equation can be simplified for applications by using average quantities:

\[
fe(k_e \geq k_F(e), T = 0) \approx (1/2) P_{pn} \langle H \rangle.
\]

Here \( P_{pn} \) is the probability of pair nucleon correlation and \( \langle H \rangle \approx P_{pn} \).

The factor

\[
1/2 \left( \frac{\sqrt{k^2 + m_e^2} + \sqrt{<k_e^2 + m_e^2>}}{(k^2 + m_e^2)^{1/4} <k_e^2 + m_e^2>^{1/4}} \right)
\]

follows from the Lorentz transformation of the electron e.m. current, conveniently calculable from the Feynman diagrams. Here \( <k_e^2> \) is the average value of the square of electron momentum within the electron Fermi sea.

**IMPACT OF SRC ON THE DIRECT AND MODIFIED URCA PROCESSES AT SMALL TEMPERATURES**

In the Landau Fermi liquid approach at finite temperature, \( T \) the direct URCA process Eqs.2 and 3 is allowed by the energy-momentum conservation law if the proton concentration exceeds \( x = 1/8 \) [14]. The restriction on the proton concentration follows from the necessity to guarantee the momentum triangle:

\[
k_F(p) + k_F(e) \geq k_F(n),
\]

in the absorption of electrons by the protons.

If proton concentration is below threshold or direct URCA process is suppressed due to nucleon superfluidity neutrino cooling proceeds through the less rapid modified URCA processes:

\[
n + (n, p) \rightarrow p + (n, p) + e + \bar{\nu}_e,
\]

and

\[
e + p + (n, p) \rightarrow n + (n, p) + \nu_e,
\]

in which additional nucleon enables momentum conservation.

The neutrino luminosity resulting from the direct and modified URCA processes, \( \varepsilon_{URCA} \), was evaluated in Ref. [14] for \( x \geq 1/8 \) where the Fermi distribution:

\[
fi_{bare}(k, T) = \frac{1}{1 + \exp \frac{E_i - \mu_i}{kT}},
\]

describes the Pauli blocking factors \( 1 - fe(k, T) \) and \( 1 - fp(k, T) \) in the final state. After integration over the phase volume of the decay products it was found:

\[
\varepsilon_{URCA} = c(kT)^6 \theta(k_F(e) + k_F(p) - k_F(n)).
\]
Here \( c(x \geq 0.1) \) has been calculated in terms of the square of the electroweak coupling constant relevant for low energy weak interactions and the phase volume factors.

In the case of realistic NN interactions significant fraction of protons has momenta above the proton Fermi momentum. So Eq\([9]\) is satisfied for the proton large momentum tail even for \( x \) smaller than 0.1. For the sake of illustrative estimate we substitute in the probability of neutron \( \beta^- \)-decay the Pauli blocking factor \((1 - f_{p,\text{bare}}(k, T))\), by the actual distribution of protons within the core of a neutron star. We account for the probability of additional neutron from (p,n) correlation by the additional factor \( P_{pn} \).

To simplify the discussion we will ignore here tiny probability for electron holes at zero temperature and parametrize neutrino luminocity as

\[
\epsilon_{\text{URCA}} = c(kT)^6 R, \tag{14}
\]

where \( R \) accounts for the role of SRC in neutrino luminosity at small temperatures. We find

\[
R \approx \kappa_{pn}^2 \left[ \int [(1 - f_p(k_p, T)) \theta(k_F(p) - k(p)) + f(k_p, T) \theta(k_p - k_F(p)) \theta(k_F(e) + k(p) - k_F(n)) d^3k_p/(2\pi)^3 \right] \cdot \left[ \int (1 - f_{p,\text{bare}}(k_p, T)) \theta(k_F(e) + k_F(p) - k_F(n)) d^3k_p/(2\pi)^3 \right]^{-1}. \tag{15}
\]

Here \( f_p(k_p, T) \) is the occupation number of protons accounting the interaction and \( f_{p,\text{bare}}(T, k) \) is the Fermi distribution function over proton momenta at nonzero temperature. The factor \( \kappa_{pn} \) is the overlapping integral between a component of the wave function of the neutron star containing pair nucleon correlation and the mean field wave function of the star. For the numerical estimate we use approximation: \( \kappa_{pn}^2 = P_{pn} \). the first term in the numerator of the above formulae and put \( T = 0 \) in the second term. Using for the estimate \( V_{NN}(k) \propto 1/k^2 \) for \( k \gg k_F \) and Eq.(6) to evaluate large \( k \) behavior of \( f_p \propto (1/k)^8 \) we obtain:

\[
R \approx \frac{(P_{pn}^2/5)\rho_n}{(m_NkT)^{3/2}}, \tag{16}
\]

where \( P_{pn} \) is the the probability for a proton to have momentum \( k \geq k_F(p) \). For the illustration, we numerically evaluate the enhancement factor \( R \) for neutron density close to \( \rho_0 \), \( x = \rho_p/\rho_n = 0.1 \), and \( P_{pn} = 0.1 \). So,

\[
R \approx 0.16P_{pn}(\text{MeV}/kT)^{3/2}. \tag{17}
\]

The enhancement is significant for \( kT \ll 1\text{MeV} \). Remember that after one year a neutron star cools to the temperatures \( T \leq 0.01 \text{ MeV} \).

Neutrino luminocity due to direct URCA processes decreases with decrease of \( x \) but differ from zero even for the popular option: \( x \leq 0.1 \). So investigation of the neutrino luminocity of the neutron stars may help to narrow down the range of the allowed values of \( x \).
\( \beta \) STABILITY OF NEUTRON WITHIN THE OUTER CORE OF ZERO TEMPERATURE NEUTRON STAR

Normal neutron star is bound by gravity. Gravity does not forbid decays of constituents of the star if energy and momentum are conserved in the decay (the equivalence principle).

Constraints due to the energy-momentum conservation law and the Pauli blocking in the electron sector work in the opposite directions. Indeed, the maximal momentum of an electron from \( \beta \)-decay of a neutron with momentum \( k_n \) is \( \approx 1.19 MeV/(1 - k_n/m_p) \). Hence, an electron produced in the neutron \( \beta \) decay may fill the electron hole with momentum \( k \approx 1MeV/c \) only. The dominant process which may lead to the formation of electron holes is the elastic interaction of an energetic proton with electrons within the free electron gas. Energy-momentum conservation is fulfilled in the case of nonrelativistic nucleons if electron in the Fermi sea kicked out by proton has minimal energy in the range:

\[
E_{\text{hole}}(k) = ((p - k_f + k)^2 - p^2)/2m_N + k_f. \tag{18}
\]

Here \( p \) is the proton momentum and \( k_f \) is the electron momentum in the final state. Scattered electron has energy \( k_f \geq E_F(e) \), so it is legitimate to neglect by the electron mass. Hence, the minimal energy of the hole (when electron and proton momenta are antiparallel in the initial state) is

\[
E_{\text{hole}} \sim (1/2 \div 1/3)E_F(e), \tag{19}
\]

for the proton momenta around \( p = 0.4 \div 0.5GeV/c \) typical for SRC and decreases with increase of \( p \). Evident mismatch between energies of produced electron holes and electrons in the neutron decay guarantees that an electron from \( \beta \)-decay of a neutron can not fill an electron hole.

In the case of ultrarelativistic nucleon gas (inner core of a star?) energy-momentum conservation does not restrict energies of electron holes produced in (e-p) interaction:

\[
E_{\text{hole}}(k) = \sqrt{(m_N^2 + (p + k - k_f)^2) - \sqrt{(m_N^2 + p^2) + (m_e^2 + k_f^2)}} \tag{20}
\]

In the limit \( p/m_n \to \infty \) we obtain expression for minimal energy of hole:

\[
E_{\text{hole}}(k) = \sqrt{(m_e^2 + k_f^2)} - k_f + k \approx k + m_e^2/k_f. \tag{21}
\]

However in this regime use of nucleon degrees of freedom would be questionable. We will not discuss further in this paper interesting question on the possible \( \beta \) instability of neutron within the inner core of star.

Direct \( \beta \) decay of muon produces electrons with momenta up to \( m_\mu/2 \) which are not far from the electron Fermi momentum. So evaluation of Pauli blocking for muon, hyperon \( \beta \)-decays requires model building.
CONCLUSIONS

The reduction of the difference between neutron and proton momentum distributions influences collective modes. The most significant effect would be the tendency to suppress the superfluidity of protons (superconductivity) due to the deformation of the proton Fermi surface because of an increase of the fraction of protons having momenta above the Fermi surface. Existence of SRC will not strongly influence the possible superfluidity of neutrons. Note that superfluidity of neutrons will further suppress neutron $\beta$ decay due to formation of neutron Cooper pairs near the Fermi surface.

Electrons and neutrinos in the $\beta$ decays of hyperons, muons, are vastly more energetic than in neutron decay. Hence, if hyperon or muon stars exist, they should decay significantly more rapidly than the neutron stars and produce larger neutrino flux.

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REFERENCES

1. R. G. Tolman, Proc. Nat. Acad. Sci. US 20 169 (1934); J. R. Oppenheimer, G. M. Volkov, Phys. Rev. 55 374 (1939).
2. G. Baym, in Proceedings of "Quark Confinement and the Hadron Spectrum 7", Ponta Delgada, Azores, Portugal, (2006).
3. H. Heiselberg and V. Pandharipande, Ann. Rev. Nucl. Part. Sci. 50, 481 (2000) [arXiv:astro-ph/0003276].
4. D. G. Yakovlev and C. J. Pethic, Annu. Rev. Astron. Astrophys. 42 169 (2004).
5. J. M. Lattimer and M. Prakash. Science 304, 536 (2004) [arXiv:astro-ph/0405262].
6. S. Weinberg. "Gravitation and Cosmology" John Wiley and Sons (1972).
7. S. L. Shapiro and S. A. Teukolsky. "Black Holes, White Dwarfs and Neutron Stars. The physics of compact objects" John Wiley and Sons (1983).
8. T. Frick, H. Muther, A. Rios, A. Polls and A. Ramos, Phys. Rev. C71 014313 (2005).
9. L. D. Landau, E. M. Lifshitz, "Statistical physics", Pergamon Press, 1980.
10. A. B. Migdal, JETP 32 399 (1957), "Theory of finite Fermi systems and applications to atomic nuclei". Interscience Publishers, Inc. London, England, (1967) Fig.3
11. A. M. Dygaev, The theory of quantum non-degenerate liquids, Hartwood Academic Publisher, 1990.
12. J. N. Bahcall and R. A. Wolf, Phys. Rev. 140 B1445 (1965).
13. B. L. Friman and O. V. Maxwell, Astrophys. J. 232 541(1979).
14. J. M. Lattimer, C. J. Pethick, M. Prakash, P. Haensel, Phys. Rev. Lett. 61 2701 (1991).
15. The Hans Bethe Centennial Volume. Phys. Rep. 442 (2007).
16. L. L. Frankfurt and M. I. Strikman, Phys. Rept. 76 215 (1981).
17. C. Ciofi degli Atti, S. Simula, L. L. Frankfurt and M. I. Strikman, Phys. Rev. C 44 7 (1991).
18. E. Pisetzky, M. Sargsyan, L. Frankfurt, M. Strikman, J. W. Watson, Phys.Rev.Lett. 97 162504 (2006); R. Subedi et al. R. Subedi et al., Science 320, 1476 (2008).