MSSM Higgs Physics: Theoretical Developments*

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Abstract

The corrections to the MSSM Higgs boson masses and couplings performed in this millennium are briefly reviewed. For the lightest MSSM Higgs boson mass, $m_h$, we list the current status of the intrinsic uncertainties (due to unknown higher-order corrections) and the parametric uncertainties (due to the imperfect experimental knowledge of the input parameters). The need for high-precision calculations in the MSSM Higgs boson sector is exemplified in a realistic example.

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1 Introduction

Disentangling the mechanism that controls electroweak symmetry breaking is one of the main tasks of the current and next generation of colliders. The prime candidates are the Higgs mechanism within the Standard Model (SM) or within the Minimal Supersymmetric Standard Model (MSSM)\(^1\). Contrary to the SM, two Higgs doublets are required in the MSSM, resulting in five physical Higgs bosons: the light and heavy CP-even $h$ and $H$, the CP-odd $A$, and the charged Higgs bosons $H^{\pm}$. The Higgs sector of the MSSM can be expressed at lowest order in terms of $M_Z$, $M_A$, and $\tan \beta = v_2/v_1$, the ratio of the two vacuum expectation values. The MSSM Higgs boson sector receives large corrections from SUSY-breaking effects in the Yukawa sector of the theory. The leading one-loop correction to the lightest MSSM Higgs boson mass, $m_h$, is proportional to $m_t^4$. For instance, the leading logarithmic one-loop term (for vanishing mixing between the scalar tops) reads\(^1\)

$$\Delta m_h^2 = \frac{3G_F m_t^4}{\sqrt{2} \pi^2 \sin^2 \beta} \ln \left( \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right).$$

Here $m_{\tilde{t}_1,2}$ denote the masses of the scalar top quarks. Corrections of this kind have dramatic effects on the predicted value of $m_h$ and many other observables in the MSSM Higgs sector. The one-loop corrections, which are known completely, can shift $m_h$ by 50–100\%. Since this shift is related to effects from a part of the theory that does not enter at tree level, corrections even of this size do not invalidate the perturbative treatment.

The large corrections to $m_h$ have to be compared with the anticipated accuracy at future colliders. At the LHC $m_h$ can possibly be measured down

\(^1\)We assume all parameters in the MSSM to be real.
to $\delta m_h^{\exp,\text{LHC}} \approx 200$ MeV. A future $e^+e^-$ linear collider (LC) can even go down to $\delta m_h^{\exp,\text{LC}} \approx 50$ MeV.

The effect of these large corrections is two-fold. On the one hand, reliable investigations (comparisons of experimental results with theoretical predictions) require multi-loop higher-order corrections. On the other hand, the strong dependence of $m_h$ on the whole SUSY spectrum, especially on the scalar top sector, makes $m_h$ an extremely powerful precision observable. $m_h$ can be used to obtain information on otherwise unknown parameters. This allows in particular to obtain indirect information on the mixing in the scalar top sector, which is very important for fits of the SUSY Lagrangian to (prospective) experimental data.\(^2\)

2 Recent Calculations In The MSSM Higgs Sector

Concerning the two-loop effects, their computation is quite advanced, see\(^3\) and references therein. They include the strong corrections at $\mathcal{O}(\alpha_t\alpha_s)$, and Yukawa corrections, $\mathcal{O}(\alpha_s^2)$, to the dominant one-loop $\mathcal{O}(\alpha_t)$ term, as well as the strong corrections from the bottom/sbottom sector at $\mathcal{O}(\alpha_b\alpha_s)$ in the limit of $\tan\beta \to \infty$.\(^2\) For the $b/\tilde{b}$ sector corrections also an all-order resummation of the $\tan\beta$-enhanced terms, $\mathcal{O}(\alpha_b(\alpha_s\tan\beta)^n)$, is known\(^5\). Most recently the $\mathcal{O}(\alpha_t\alpha_b)$ and $\mathcal{O}(\alpha_b^2)$ corrections have been derived\(^6\). Finally a “full” two-loop effective potential calculation (including even the momentum dependence for the leading pieces) has been published\(^8\). However, the latter results have been obtained using a certain renormalization in which all quantities, including SM gauge boson masses and couplings, are $\overline{\text{DR}}$ parameters. This makes them not applicable in the Feynman-diagrammatic (FD) approach using the on-shell renormalization scheme, see Sect. 5.

3 Intrinsic Uncertainties

The current intrinsic error (due to unknown higher-order corrections) consists of four different pieces:

− missing momentum-independent two-loop corrections: By varying the renormalization scale at the one-loop level, these two-loop uncertainties can be estimated to be $\pm 1.5$ GeV.\(^9\)

\(^2\)For the result for arbitrary $\tan\beta$ see\(^4\).
\(^3\)Leading corrections in the MSSM with non-minimal flavor violation have recently been obtained in\(^7\).
\(^4\)We do not consider here the “full” two-loop effective potential calculation presented in\(^8\) for the reasons outlined above.
− missing momentum-dependent two-loop corrections: since at the one-loop level the momentum corrections are below the level of 2 GeV, it can be estimated that they stay below ±0.5 GeV\textsuperscript{3,10}.

− missing 3/4-loop corrections from the $t/\bar{t}$ sector: by applying three different methods (changing the renormalization scheme at the two-loop level; direct evaluation of the leading terms in a simplified approximation; numerical iterative solution of the renormalization group equations) these corrections have been estimated to be at about ±1.5 GeV (see\textsuperscript{3} and references therein).

− missing 3/4-loop corrections from the $b/\bar{b}$ sector: the corrections from the $b/\bar{b}$ sector can be large if both, $\mu$ and $\tan \beta$ are sufficiently large. For $\mu > 0$ it can be shown\textsuperscript{4} that the QCD two-loop corrections give already an extremely precise result, provided that the resummation of $(\alpha_s \tan \beta)^n$ terms\textsuperscript{5} is taken into account. On the other hand, for $\mu < 0$ the 3-loop corrections can be up to ±3 GeV\textsuperscript{4}. Since the results for $a_\mu$ favor a positive $\mu$ we do not consider this possibility here.

The current intrinsic error can thus be estimated to be\textsuperscript{3}
\[
\delta m_h^{\text{intr, today}} \approx 3 \text{ GeV},
\]  
(2)
depending in detail on the investigated point in the MSSM parameter space.

If the full two-loop calculation (in an FD suitable renormalization) as well as the leading 3-loop (and possibly the very leading 4-loop) corrections are available, the intrinsic error could be reduced to less than about ±0.5 GeV.

This seems to be possible within the next 5–10 years.

### 4 Parametric Uncertainties

The currently induced error by $M_W$ and $m_t$ are already almost negligible, and will be irrelevant with the future precision of these input parameters\textsuperscript{11}. On the other hand, $m_t$ and $\alpha_s$ play a non-negligible role. Currently we have\textsuperscript{11} (for a recent reevaluation leading to similar results, see\textsuperscript{10})
\[
\delta m_t^{\text{exp, today}} \approx 4.3 \text{ GeV} \quad \Rightarrow \quad \delta m_h^{\text{para, } m_t} \approx 4 \text{ GeV} \quad (3)
\]
\[
\delta \alpha_s^{\text{exp, today}} \approx 0.002 \quad \Rightarrow \quad \delta m_h^{\text{para, } \alpha_s} \approx 0.3 \text{ GeV} \quad (4)
\]

From the LC one can hope to achieve in the future
\[
\delta m_t^{\text{exp, future}} \approx 0.1 \text{ GeV} \quad \Rightarrow \quad \delta m_h^{\text{para, } m_t} \approx 0.1 \text{ GeV} \quad (5)
\]
\[
\delta \alpha_s^{\text{exp, future}} \lesssim 0.001 \quad \Rightarrow \quad \delta m_h^{\text{para, } \alpha_s} \approx 0.1 \text{ GeV} \quad (6)
\]

The error induced by the experimental uncertainties of the SUSY parameters is very difficult to estimate. It will depend substantially on the values of
the parameters realized in nature. For most of the MSSM parameter space no analysis of the anticipated experimental errors is available yet.

5 The Need For Precision

We present one example emphasizing the need for a drastic improvement in the intrinsic error of $m_h$ (assuming that the experimental and parametric error will be well enough under control, see above). The evaluation of $m_h$ is based on the code FeynHiggs\textsuperscript{12,13,14}. We consider the SPS 1b benchmark scenario\textsuperscript{15}, where it will be very challenging to measure the trilinear Higgs-stop coupling, $A_t$. In Fig. 1 we show the dependence of $m_h$ on $A_t$ in this scenario\textsuperscript{11}. The band reflects the effects of parametric uncertainties of the MSSM parameters (including prospective experimental errors, see text). The light shaded (green) area corresponds to $\delta m_t^{\exp} = 2$ GeV, the dark shaded (blue) area to $\delta m_t^{\exp} = 0.1$ GeV (see text). The possible experimental measurement of $m_h$ is shown (including the error). Two further bands are shown, demonstrating the effect of an intrinsic error of 3 GeV (today) and 0.5 GeV (future). From

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{The dependence of $m_h$ within the SPS 1b scenario is shown as a function of $A_t$. The band reflects the effects of parametric uncertainties of the MSSM parameters (including prospective experimental errors, see text). The light shaded (green) area corresponds to $\delta m_t^{\exp} = 2$ GeV, the dark shaded (blue) area to $\delta m_t^{\exp} = 0.1$ GeV (see text). The possible experimental measurement of $m_h$ is shown (including the error). Two further bands are shown, demonstrating the effect of an intrinsic error of 3 GeV (today) and 0.5 GeV (future).}
\end{figure}
the intersection of the experimental $m_H$ determination, including the intrinsic error, with the SPS 1b band allows to determine $A_t$ indirectly (up to a sign ambiguity, see for a determination of its sign). Besides the need for a precise $m_t$ measurement, it becomes obvious that with the current intrinsic $m_H$ uncertainty hardly any bound could be set. If, on the other hand, a reduction of the intrinsic error down to $\sim 0.5$ GeV could be performed, determination of $A_t$ better than $\sim 10\%$ seems feasible.

References

1. J. Ellis, G. Ridolfi, F. Zwirner, Phys. Lett. B257 (1991) 83; Y. Okada, M. Yamaguchi, T. Yanagida, Prog. Theor. Phys. 85 (1991) 1; H. Haber, R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815.
2. R. Lafaye, T. Plehn, D. Zerwas, hep-ph/0404282; P. Bechtle, K. Desch, P. Wienemann in LHC/LC Study Group Report, eds. G. Weiglein et al., see: www.ippp.dur.ac.uk/~georg/lhclc.
3. G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. Weiglein, Eur. Phys. J. C28 (2003) 133.
4. S. Heinemeyer, W. Hollik, H. Rzehak, G. Weiglein, CERN–PH–TH/2004–126, in preparation.
5. M. Carena, D. Garcia, U. Nierste, C. Wagner, Nucl. Phys. B577 (2000) 577; see also: H. Eberl, K. Hidaka, S. Kraml, W. Majerotto, Y. Yamada, Phys. Rev. D 62 (2000) 055006.
6. G. Degrassi, A. Dedes, P. Slavich, Nucl. Phys. B672 (2003) 144.
7. S. Heinemeyer, W. Hollik, F. Merz, S. Peñaranda, hep-ph/0403228.
8. S. Martin, Phys. Rev. D66 (2002) 096001; D67 (2003) 095012; D68 075002 (2003); hep-ph/0312092; hep-ph/0405022.
9. M. Frank, S. Heinemeyer, W. Hollik, G. Weiglein, hep-ph/0202166.
10. B. Allanach, A. Djouadi, J. Kneur, W. Porod, P. Slavich, hep-ph/0406166.
11. S. Heinemeyer, S. Kraml, W. Porod, G. Weiglein JHEP0309 (2003) 075.
12. M. Frank, S. Heinemeyer, W. Hollik and G. Weiglein, hep-ph/0212037; S. Heinemeyer, hep-ph/0407244.
13. S. Heinemeyer, W. Hollik, G. Weiglein, Eur. Phys. J. C9 (1999) 343.
14. S. Heinemeyer, W. Hollik, G. Weiglein, Comp. Phys. Comm. 124 (2000) 76; T. Hahn, S. Heinemeyer, W. Hollik, G. Weiglein, in preparation. The code can be obtained at www.feynhiggs.de.
15. B. Allanach et al., Eur. Phys. J. C25 (2002) 113.
16. K. Desch, E. Gross, S. Heinemeyer, G. Weiglein, L. Zivkovic, hep-ph/0406322.