Influence of magnetic non-uniformity existing in a rigid rotor supported by a superconducting magnetic bearing on its whirling

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Abstract. Superconducting magnetic bearings (SMBs) have a significant feature over conventional bearings in terms of supporting a shaft without physical contact while attaining its stability without control. In their large-scale rotary applications, magnetization distribution of a rotor in the circumferential direction can be non-uniform and it would be better to know influence of such circumferential magnetic non-uniformity existing in a rotor on its dynamics, especially on its behaviors in the vicinity of the critical speed. In this study, further developing our previous research, we improved our analytical model so that we can adjust several different degrees of magnetic non-uniformity by arranging multiple magnetization vectors and investigated its influence. First, we simulated dynamical behavior of the system by numerical calculations and their results show that, with increasing the degree of magnetic non-uniformity, the whirling amplitude of the system, together with the difference of the amplitudes in the orthogonal directions in the whirling plane, get larger. Further, the rotational frequency at which the whirling amplitude takes its peak gets lower, which is caused by nonlinearity of the electromagnetic force. We carried out experiments and verified our numerical predictions.

1. Introduction
A superconducting magnetic bearing (SMB) has various merits such as low damping, supporting high rotational speed, high energy efficiency and utilization under special environment due to maintenance-free property, because they are able to support a rotor without physical contact. However, it shows complicated nonlinear vibration in the vicinity of the critical speed due to nonlinearity of electromagnetic force [1]. Nevertheless, it is often required to spin at high rotational speed over the critical speed. For these reasons, we need to evaluate behaviors near the critical speed of a system using a SMB. Magnetization distribution in the circumferential direction can be non-uniform, because SMBs, if applied to large systems including flywheel energy storage systems, will support a rotor with a single large magnet or multiple magnets connected with each other [2]. For mechanical design of such a rotary system, it would be better to know influence of such magnetic non-uniformity on rotordynamics, because it is conceivable that this magnetic non-uniformity can produce periodic change of electromagnetic force supporting a rotor, which may cause instability of rotor whirling. We previously investigated vibrations of the elastic shaft with and without circumferentially non-uniform magnetization distribution supported by a SMB and theoretically pointed out that anisotropy of its horizontal stiffness and periodic change of that stiffness leading to parametric
excitation newly arose from magnetic non-uniformity [3]. We also found that the equations of motion of a rotor with magnetic non-uniformity supported by a SMB are equivalent to those of an asymmetrical shaft supported by a mechanical bearing [4]. We further numerically confirmed that an unstable frequency region, predicted in the linear theory, does not exist because of nonlinearity and that the amplitudes in the horizontal directions, where there is anisotropy of stiffness, take their peaks simultaneously, which is also different from linear prediction. In this study, we investigate influence of magnetic non-uniformity existing in a rigid rotor supported by a SMB on its whirling in more detail by numerical calculations and experiments. First, we improve our analytical model and perform numerical calculations with several different degrees of magnetic non-uniformity of a rotor. Further, by carrying out corresponding experiments, we verify our numerical predictions.

2. Theory
As shown in Fig. 1, our analytical model consists of a rigid shaft and a magnet attached at the bottom of the rotor shaft and a superconducting bulk fixed under the bottom end of the shaft. In this study, as shown in Fig. 2, we assume magnetic non-uniformity of the magnet in the circumferential direction, so that the magnetic flux density due to the magnet changes sinusoidally in a circumferential direction, as given by $B = B_0 + \Delta B \cos \theta$. Here $\theta$ is a circumferential angle around the central axis normal to the $x - y$ plane. The nondimensional equation of motion with and without magnetic non-uniformity are shown in (1), (2). The electromagnetic force due to the superconducting bulk is evaluated by the advanced mirror image method [5], and is expanded into Taylor series around the equilibrium position up to the 3rd order terms. We assume one permanent magnet as an individual magnetic dipole to simplify evaluation of the electromagnetic force on the basis of the advanced mirror image method. To satisfy above assumption of magnetic non-uniformity, we arrange additional magnetization vectors $m_1$ on both sides of a main central magnetization vector $m_0$, as shown in Fig. 3, and adjust the degree of magnetic non-uniformity by changing the value of $m_1$.

![Figure 1. Analytical model.](image1.png)

![Figure 2. Assumption of magnetic non-uniformity in the circumferential direction.](image2.png)

![Figure 3. Arrangement of additional magnetization vectors.](image3.png)

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\begin{align*}
\ddot{x} + 2\gamma \dot{x} + \omega^2 x &= \epsilon \nu^2 \cos \nu t + \Delta_{xyy} y^2 + \Delta_{xxx} x^3 \\
\ddot{y} + 2\gamma \dot{y} + \nu^2 y &= \epsilon \nu^2 \sin \nu t + \Delta_{yxx} x^2 y + \Delta_{yyy} y^3
\end{align*}
\]

(1)

\[
\begin{align*}
\ddot{x} + 2\gamma \dot{x} + p_x x &= \epsilon \nu^2 \cos \nu t + \Delta_{11} x \cos 2\nu t + \Delta_{12} y \sin 2\nu t + \Delta_{xxy} y^2 + \Delta_{xxx} x^3 \\
\ddot{y} + 2\gamma \dot{y} + p_y y &= \epsilon \nu^2 \sin \nu t + \Delta_{21} x \sin 2\nu t - \Delta_{22} y \cos 2\nu t + \Delta_{yxx} x^2 y + \Delta_{yyy} y^3
\end{align*}
\]

(2)

Here $\gamma$ is the nondimensional damping coefficient and $\epsilon$ is the nondimensional eccentricity. $p_x$ and $p_y$ are the nondimensional stiffnesses in the $x$ and $y$ directions, respectively, $\nu$ is the nondimensional excitation frequency. $\Delta_{11}$, $\Delta_{12}$, $\Delta_{21}$ and $\Delta_{22}$ are nondimensional coefficients of parametric excitation terms and $\Delta_{xxy}$, $\Delta_{xxx}$, $\Delta_{yxx}$ and $\Delta_{yyy}$ are nondimensional coefficients of nonlinear terms. In our previous study [3], we discussed the force terms in the equations of motion and theoretically pointed out following significant characteristics inherent to SMB
systems appearing under the condition of magnetic non-uniformity: anisotropy of its horizontal stiffnesses and periodic change of that stiffness at integral multiplication of rotational frequency which may lead to parametric excitation. We theoretically predicted that there might be some difference between the resonant frequencies in two orthogonal horizontal directions at which respectful horizontal stiffnesses differ, and that there may be also some regions of the rotational frequency at which the whirl amplitude can be unstable, and that the system could show the soft-spring nonlinearity so that the resonant peak might bend to lower frequencies in the frequency response curve and thus a phenomenon of jump in vibration to higher amplitudes may happen near the resonance. However, from our previous numerical calculations [3], we also confirmed that an unstable region does not exist and the amplitudes in the $x$ and $y$ directions take their peaks simultaneously.

3. Numerical calculation

Equations (1), (2) were solved numerically by the Runge-Kutta method. To adjust the degree of non-uniformity of magnetization distribution in the circumferential direction of a rotor, we changed the ratio of $m_1$ to $m_0$ to be 0, 1/36, 1/9 and 1/4. The degree of magnetic non-uniformity gets larger as the ratio of $m_1$ to $m_0$ increases. Fig. 4 (a)–(d) show calculated frequency response curves indicating relations between the rotational frequency and the whirling amplitude. The whirling amplitude and the rotational frequency are respectively nondimensionalized by the distance $H$ between the superconducting bulk and the permanent magnet and the mean-square value of eigenfrequencies in the $x$ and $y$ directions. According to Fig. 4 (a)–(d), the resonant amplitudes in the $x$ and $y$ directions get larger as the degree of magnetic non-uniformity increases. Difference of amplitudes in the $x$ and $y$ directions similarly get larger. Moreover, the rotational frequency at which the whirling amplitudes in the $x$ and $y$ directions take their peaks gets lower.

4. Experiment

We carried out experiments to verify our numerical predictions above. Our experimental model is shown in Fig. 5. As shown in Fig. 6, to adjust degrees of non-uniformity of magnetization distribution in the circumferential direction of a rotor, we additionally located two smaller permanent magnets with symmetry against a main central permanent magnet in the rotor and changed the radii of the additional magnets $r_1$ to be 0 mm, 1.5 mm, 2 mm and 3 mm. The radius of the main central magnet was 6 mm in all cases. Magnetic non-uniformity increases as the
radius of $r_1$. By measuring displacements of the rotor, we obtained their time histories with their FFT spectra. Fig. 7 (a)−(d) show frequency response curves, nondimensionalized as in Fig. 4. Here $H = 6.6$ mm and the mean-square value of eigenfrequencies are 7.69 Hz in (a), 7.32 Hz in (b), 7.33 Hz in (c), 8.09 Hz in (d). As the degree of magnetic non-uniformity increases, the resonant amplitudes in the $x$ and $y$ directions and their difference get larger. Furthermore, the rotational frequency at which the whirling amplitudes in the $x$ and $y$ directions take their peaks gets lower. Our experimental results were in good qualitative agreement with our numerical results in most cases. One of possible reasons for quantitative differences may be dipole approximation of permanent magnets.

**Figure 5.** Schematic diagram of our experimental setup.

**Figure 6.** Arrangement of the additional magnets.

**Figure 7.** Experimentally obtained frequency response curves.

5. Conclusion

We numerically and experimentally investigated dynamical behaviors of a rotor with circumferentially non-uniform magnetization distribution supported by a SMB. Obtained results of numerical calculations with increasing the degree of magnetic non-uniformity showed that the whirling amplitude of the system gets larger and that difference of the amplitudes in the $x$ and $y$ directions also gets larger. Further, the rotational frequency at which the amplitudes in the $x$ and $y$ directions take their peaks gets lower. These numerical predictions were verified by experiments.

References

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