Astrophysics, cosmology, and fundamental physics with compact binary coalescence and the Einstein Telescope

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Abstract. The second-generation interferometric gravitational wave detectors, currently under construction are expected to make their first detections within this decade. This will firmly establish gravitational wave physics as an empirical science, and will open up a new era in astrophysics, cosmology, and fundamental physics. Already with the first detections, we will be able to, among other things, establish the nature of short-hard gamma ray bursts, definitively confirm the existence of black holes, measure the Hubble constant in a completely independent way, and for the first time gain access to the genuinely strong-field dynamics of gravity. Hence, it is time to consider the longer-term future of this new field. The Einstein Telescope (ET) is a concrete conceptual proposal for a third-generation gravitational wave observatory, which will be \( \sim 10 \) times more sensitive in strain than the second-generation detectors. This will give access to sources at cosmological distances, with a correspondingly higher detection rate. We have given an overview of the science case for ET, with a focus on what can be learned from signals emitted by coalescing compact binaries. Third-generation observatories will allow us to map the coalescence rate out to redshifts \( z \sim 3 \), determine the mass functions of neutron stars and black holes, and perform precision measurements of the neutron star equation of state. ET will enable us to study the large-scale structure and evolution of the Universe without recourse to a cosmic distance ladder. Finally, we have discussed how it will allow for high-precision measurements of strong-field, dynamical gravity.

1. Introduction
A network of second-generation, kilometre scale interferometric gravitational wave detectors is currently under construction: Advanced LIGO in the US [1], Advanced Virgo in Europe [2], and KAGRA in Japan [3]; a further large interferometer might be built in India [4]. In Germany, GEO-HF is operational [5]. The first direct detection of gravitational waves (GWs) is expected before the end of the decade. Potential sources include pulsars, and indeed, the (now decommissioned) first-generation detectors already put interesting bounds on the non-axisymmetry of the Crab pulsar [6]. Next, there is the possibility of observing a stochastic gravitational wave background; here too, the initial observatories are already able to put relevant constraints [7]. Among the most promising sources are coalescing compact binaries composed of neutron stars and/or black holes [8]. Observing these will reveal the nature of short-hard gamma ray bursts (GRBs) [9], establish once and for all the existence of black holes or uncover the presence of alternative compact objects such as Boson stars [10], and enable us to independently study the...
expansion of the Universe by using coalescing binaries as ‘standard sirens’ [11]. They may also
give us empirical access to the neutron star equation of state (EOS) [12, 13, 14, 15, 16, 17, 18].
Last but not the least, they will allow us to probe the genuinely strong-field dynamics of
spacetime [19, 20, 21, 22, 23, 24, 25]. (For a recent review, see [26].) Thus, an exciting field
of observational gravitational physics is about to open up, with implications for astrophysics,
cosmology, and fundamental physics.

It is appropriate to start considering the longer-term future of this new field. In 2008-2011,
a conceptual design study was conducted for a third-generation gravitational wave observatory
called Einstein Telescope (ET) [27]. ET is envisaged to be a large (∼10 km), underground,
partially cryogenic facility housing several interferometers arranged in a triangular configuration.
It will be about a factor of ten more sensitive in strain than the second-generation detectors.
While the second-generation observatories may see up to hundreds of sources per year [8], ET
might have as many as a million detections. A first study of the considerable data analysis
challenges associated with such a high detection rate was presented in [28]. In this paper, we
have investigated in general terms what kind of novel science can be expected from ET. In doing
so, we have not attempted to be exhaustive and focus on compact binary coalescence. (For a
more complete overview, see the recent paper [29].)

This paper is structured as follows: In Section 2, we have discussed the detailed re-
construction of the compact binary coalescence rate as a function of redshift, which (a network
of) third-generation detectors will allow for the accuracy with which the mass function of neutron
stars and black holes can be determined, and the possibility of measuring the EOS of neutron
stars. In Section 3, we have turned to cosmology, in particular the potential of coalescing
binaries as the so-called standard sirens. Finally, in Section 4, we have described how high-
precision tests of the strong field dynamics of general relativity could be performed with ET. A
summary is given in Section 5.

Throughout this paper, we have used units such that \(G = c = 1\), unless stated otherwise.
We have denoted binary neutron stars by BNS, neutron star-black hole binaries by NSBH, and
binary black holes by BBH.

2. Astrophysics

2.1. Reconstruction of the coalescence rate

ET will be able to see coalescing binaries out to cosmological distances. The coalescence rate
as a function of redshift is expected to rise with increasing redshift \(z\), reaching a peak around
\(z \sim 1.5\), after which it must decrease again and eventually reach zero at some large redshift
value where no binaries had yet formed. The coalescence rate per redshift bin is given by [30]:

\[
\frac{dR}{dz}(z) = \frac{\dot{\rho}_c(z)}{dV/dz},
\]

where \(dV/dz\) is the derivative of comoving volume with respect to redshift, and

\[
\dot{\rho}_c(z) \propto \int_{t_d}^{\infty} \frac{\dot{\rho}_s(z_f, t_d)}{1 + z_f} P(t_d) \, dt_d,
\]

with \(\dot{\rho}_s\), the star formation rate, \(z\) the redshift at which merger takes place, \(z_f\) the redshift at
which the binary is formed, and \(P(t_d)\) a probability distribution of the delay time \(t_d\) between
formation and merger. The factor \((1 + z_f)^{-1}\) within the integral takes into account the difference
between intrinsic and observed rates. The advanced detectors, which will see mergers out to
\(z \sim 0.3\), will be able to measure the coalescence rate density at \(z = 0\), \(\dot{\rho}_c(0)\); even knowing this
one number would already be valuable in understanding the fate of extremely massive stars. At
larger redshifts, there is significant disagreement between various predictions for the shape of
Figure 1. Four different models for the coalescence rate as a function of redshift \[32, 33, 34, 35\]. Assuming each of these in turn to be the underlying rate, one can simulate to what extent ET will be able to map it. The results are given by the ‘data points’ with \(1 - \sigma\) error bars.

the function \(dR/dz(z)\), as illustrated in Figure 1, where coalescence rates are normalized such that \(\dot{\rho}_c(0)\) is the same for all of them.

In order to arrive at a crude idea of how well a network of ETs, e.g., with locations at the two LIGO sites and the Virgo site, would be able to map the coalescence rate, one can proceed as follows: With such a network, one would be able to partially dis-entangle the luminosity distance \(D_L\) from the angles appearing in the GW amplitude (see Eq. (3) below). Fisher matrix estimates indicate that distance could be estimated with \(1 - \sigma\) relative error \(\sigma_0 \sim 1/\rho\), with \(\rho\) being the network SNR, if detector noise were the only source of uncertainty. However, distance measurements will also be affected by weak lensing. Modelling the associated relative distance error as \(\sigma_w = 0.05 z\) \[31\], one can take the combined uncertainty to be \(\Delta D_L/D_L = (\sigma_0^2 + \sigma_w^2)^{1/2}\).

Next, consider a large number of simulated ‘catalogues’ of \(\mathcal{O}(10^5)\) sources each, distributed in redshift according to one of four different underlying true rate distributions \(dR/dz(z)\) taken from recent literature \[32, 33, 34, 35\]. Within each catalogue, take the ‘measured’ distance for each source to be Gaussian distributed, centred around the true distance and with spread \(\Delta D_L/D_L\).

A measured distance \(\hat{D}_L\) can be converted into a measured redshift \(\hat{z}\) using a cosmological model, e.g., a ΛCDM cosmology with parameter values from the recent WMAP results \[36\], and by counting the number of sources in each redshift bin, one arrives at a ‘measured’ coalescence rate as a function of redshift, for a given catalogue. Starting from \(z \sim 0.7\), this rate needs to be corrected for loss of GW detection efficiency due to inconvenient positioning and/or orientation of the source. Finally, in each redshift bin, one can compute the average and variance of the measured \(d\hat{R}/dz(z)\) over all the simulated catalogues. The results are shown as the ‘data points’ and \(1 - \sigma\) error bars in Figure 1. For redshifts up to \(z \sim 1.5\), the uncertainty on the rate is \(\sim 1\%\). In particular, one would be able to tell the difference between the four theoretical distributions considered here.

2.2. Re-construction of the neutron star and black hole mass functions

Current measurements of neutron star masses pertain to a small sample of neutron stars within our own galaxy (see e.g., \[37\]). Accordingly, the true distribution of neutron star masses, and in particular, the mass region where the transition to black holes occurs, remains unknown. The
possible time evolution of these mass functions over cosmological timescales is also something we currently have no access to. In Figure 2, we have shown Fisher matrix estimates of how accurately the mass of a canonical $(1.4 \, M_\odot)$ neutron star in a BNS or NSBH system can be determined from the inspiral signal only, as a function of the companion’s mass, and for $z = 1, 2, 3$. (The results suggest that in the near-equal mass case, it will be difficult to estimate the mass; however, this is due to a singularity of the Fisher matrix itself.) As we have seen, star formation models suggest that the rate at which coalescences occur will peak at $z \approx 1 - 3$, a range where ET can measure masses with high accuracy. Thus, ET will be able to make a detailed map of the neutron star and black hole mass functions during the most interesting part of star formation history.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** The percentage uncertainty in measuring the mass of a canonical $(1.4 \, M_\odot)$ neutron star in a BNS or NSBH system, as a function of the companion’s mass, for sources at $z = 1, 2, 3$. Comparing with Figure 1, we have seen that ET would be able to measure component masses with $0.5 - 3\%$ uncertainty within the redshift interval where most of the inspirals take place.

### 2.3. Determining the equation of state of neutron stars

The later part of BNS or NSBH inspiral, as well as the merger, will bear imprints of the neutron star EOS. To the linear order, the tidal field $\mathcal{E}_{ij}$ of the companion object will cause a spherically symmetric neutron star to acquire a quadrupole moment $Q_{ij} = -\lambda \mathcal{E}_{ij}$. The tidal deformability parameter $\lambda$ will be a function of the star’s mass $m$. The neutron star’s deformation affects the orbital motion and enters the inspiral waveform’s phase starting from 5PN order (in the usual notation), where it contributes $\propto \lambda / M^5 (v/c)^5$, with $M$, the total mass of the binary and $v$, the orbital velocity. Since, $\lambda \propto R^5$, with $R$, the neutron star radius, and $R/M = O(10)$ for a neutron star, the tidal contribution to the phase will be non-negligible despite occurring at such high order in $v/c$. The function $\lambda(m)$ is set by the neutron star EOS.

In [12], a variety of EOS were considered, from very stiff to very soft, for different neutron star compositions (npe$\mu$ only, $\pi$/hyperon/quark matter, and strange quark matter). At $m = 1.4 \, M_\odot$, the resulting values for $\lambda(m)$ vary over an order of magnitude. Fisher matrix results indicate that in the gravitational wave frequency range 10 Hz - 450 Hz, with the advanced detector network, and a single high-SNR source, we will only have access to the stiffest EOS leading to the largest values of the deformability $\lambda$. (For a more recent assessment which also includes the plunge, see [16].) On the other hand, ET will have access to the entire range considered in [12],
In [13], merger and post-merger BNS waveforms were numerically computed for a range of EOS with varying stiffness. For very soft EOS, the merger is followed by the prompt formation of a black hole. By contrast, for a very stiff EOS, an unstable bar is formed, which spins down over a timescale of milliseconds before also collapsing to a black hole. With second-generation detectors, in the post-merger, it is again the stiffest EOS that might cause a detectable signal because of the unstable bar. Thus, already with the upcoming advanced detector network, there is good hope of gaining information about the EOS of neutron stars in this way. However, also in the post-merger phase, ET will enable far more accurate measurements due to better sensitivity in the kHz region, where the bar mode signal, if present, will have the most power.

In [14, 15], NSBH signals were considered, in which case larger induced neutron star quadrupole moments are to be expected. It was found that, for mass ratios of 2 or 3, advanced detectors would be able to extract \( \lambda/M^5 \) with 10 – 40% accuracy for a single source at a distance of 100 Mpc, and in particular, they would be able to distinguish between a neutron star and a black hole of the same mass. ET would be able to do better by an order of magnitude.

Finally, as shown in [17], combining information from multiple BNS inspirals observed with an advanced detector network will allow us to at least tell the difference between a soft, moderate, and stiff EOS, and constrain the EOS to 10% accuracy with as few as \( \mathcal{O}(10) \) sources near the SNR threshold.

In summary, the prospects are good for empirically studying the EOS of neutron stars already with advanced detectors, but detailed measurements may have to wait until the ET era.

3. Cosmology

3.1. Compact binary coalescences as standard sirens

As noted by Schutz as early as 1986 [11], compact binary coalescences can be used as ‘standard sirens’, a designation based on ‘standard candles’, but referring to GW rather than electromagnetic sources. The observed strain amplitude of inspiral signals goes like:

\[ A(t) \propto \frac{1}{D_L} \mathcal{M}_{\text{obs}}^{5/3} F(\theta, \phi, \iota, \psi) F^{2/3}(t). \]  

(3)

Here, \( \mathcal{M}_{\text{obs}} \) is the observed chirp mass (which is related to the physical chirp mass \( \mathcal{M}_{\text{phys}} \) by \( \mathcal{M}_{\text{obs}} = (1 + z) \mathcal{M}_{\text{phys}} \)), and \( F \) is a known function of sky position and orientation. The chirp mass can be measured very accurately from the phase. Thus, if some information is available on sky position and orientation, one can infer the luminosity distance \( D_L \) directly from the gravitational wave signal, with no need for calibration through a cosmic distance ladder, unlike the currently used standard candles. This opens up the possibility of completely independent cosmological measurements.

Assuming that at sufficiently large scales, the Universe can be modelled as a Friedman-Lemaître-Robertson-Walker (FLRW) spacetime, its evolution is governed by the Hubble constant \( H_0 \), the density of matter \( \Omega_M \) relative to the critical density, the density of dark energy \( \Omega_{\text{DE}} \), the contribution from spatial curvature \( \Omega_k \), and the dark energy EOS parameter \( w \). The latter is defined as \( w = p_{\text{DE}}/\rho_{\text{DE}} \), with \( \rho_{\text{DE}} \) and \( p_{\text{DE}} \) are the density and pressure of dark energy respectively; note that we have allowed for the possibility that \( w \) is time dependent. Given a number of sources for which the luminosity distance \( D_L \) can be measured, as well as the redshift \( z \), the cosmological parameters \( \Omega = (H_0, \Omega_M, \Omega_{\text{DE}}, \Omega_k, w) \) can be inferred by fitting the distance-redshift relationship \( D_L(\Omega; z) \).

This means that information about the redshifts of sources will also be required. In recent years, a variety of methods, as given below, have been developed for utilizing binary inspiral signals as standard sirens. The first three will already allow for an independent measurement of \( H_0 \) with an advanced detector network.
(i) Some fraction of inspirals will have electromagnetic counterparts, such as GRBs, which may allow for the identification of the host galaxy, and hence the redshift [38].

(ii) Assuming knowledge of the distribution of physical neutron star masses $m_{\text{phys}}$, the observed masses $m_{\text{obs}} = (1 + z) m_{\text{phys}}$ in BNS sources lead to a redshift estimate without the need for electromagnetic counterparts [39].

(iii) Approximate knowledge of sky position by using multiple detector sites together with the distance uncertainty leads to a 3-dimensional volume within which the inspiral will have occurred. Using a galaxy catalogue, each event can be associated with a list of possible redshifts, which for moderate distances can be translated into a list of possible values of $H_0$. A second inspiral event will lead to a different list of $H_0$ values, which will only be partially consistent with the first. As more events are observed, the true value of $H_0$ will quickly emerge. This method allows the use of all inspiral events, including BBH [40].

(iv) If the EOS of neutron stars has already been determined (e.g., using some subset of the observed sources), one can extract the redshift directly from the GW waveform through the effect of tidal deformations on the orbital motion, which involves the intrinsic masses [41].

In [31] and [42], detailed studies were made of how accurately the cosmological parameters $\Omega$ could be measured with the use of electromagnetic counterparts, i.e., using the method (i) above. With $O(1000)$ such events over the course of 5 – 10 years, $\Omega_M$ and $\Omega_{\text{DE}}$ could be measured with an accuracy comparable to what is possible with CMB measurements, but in a completely independent way. One can also consider the possible time dependence of the EOS parameter $w$, taking the CMB measurements of $\Omega_M$, $\Omega_{\text{DE}}$, and $\Omega_k$ as priors. It is convenient to approximate $w \approx w_0 + (1 - a) w_a$, with $w_0$ the present-day value, and $a$, the scale factor. In Figure 3, we have compared accuracies in measuring $w_0$ and $w_a$, on the one hand using standard sirens seen by ET, and on the other hand, considering the SNAP Type Ia supernova survey, which may be available on the same timescale as ET [43]. The measurement quality is comparable in the two cases, but we stress once again that standard sirens allow for an independent measurement; in particular, there will be no need for a cosmic distance ladder. More recent work by Taylor and Gair [44] based on method (ii), which allows the use of all BNS inspiral events seen in gravitational waves, has demonstrated that a network of ETs would, in fact, be able to beat the predicted accuracy of the future supernova surveys.

4. Probing the strong-field dynamics of spacetime

Until the 1970s, all tests of general relativity (GR) took the form of observing test particles (photons, planets modelled as test masses,...) moving in a weak, stationary gravitational field [45]. This changed with the discovery of tight binary pulsars [46, 47, 48], whose orbital parameters were found to be changing in accordance with the emission of gravitational radiation as predicted by GR. Even so, the most interesting aspect of classical GR, namely the strong-field dynamics of the gravitational field itself, remains out of reach. In the foreseeable future, only the direct detection of gravitational radiation, in particular from coalescing compact binaries, will enable significant progress in this direction.

The authors of [49, 50, 20] have first pointed out the possibility of a very broad test of the dynamics of GR, based on a consistency requirement on the post-Newtonian coefficients appearing in the phase of an inspiral waveform. In the stationary phase approximation, the phase as a function of frequency, takes the form [51, 52]:

$$\Psi(f) = 2\pi f t_c - \phi_c + \sum_{j=0}^{7} \left[ \psi_j + \psi_j^{(l)} \ln f \right] f^{(j-5)/3},$$

(4)

where $t_c$ and $\phi_c$ are respectively the time and phase at coalescence. Now, according to general relativity, in the non-spinning case, all of the coefficients $\psi_j$ and $\psi_j^{(l)}$ only depend on the two
component masses \((m_1, m_2)\), and GR makes a specific prediction for this dependence. Li et al [23, 24] subsequently developed a test of GR in the context of Bayesian inference, which computes an odds ratio based on the data \(d\), and whatever background information \(I\) one may hold:

\[
O_{\text{GR}}^{\text{modGR}} = \frac{P(\mathcal{H}_{\text{modGR}}|d, I)}{P(\mathcal{H}_{\text{GR}}|d, I)}.
\]  

(5)

Here, \(\mathcal{H}_{\text{GR}}\) is the hypothesis that GR is correct, while \(\mathcal{H}_{\text{modGR}}\) is the hypothesis that one or more of the \(\psi_j, \psi_j^{(l)}\) do not have the functional dependence on component masses as well as spins that GR predicts, without specifying which ones. Computing the posterior probability \(P(\mathcal{H}_{\text{modGR}}|d, I)\) amounts to testing a large number of independent sub-hypotheses, in each of which a particular subset of the phase coefficients are allowed to vary freely, while the others have the dependences on masses and spins as in GR. This has been dubbed the TIGER method. The framework is well suited to a situation, where almost all sources have low SNR, as will be the case with the advanced detectors. It also trivially allows for combining information from multiple sources: For each source, the same ‘yes/no’ question is asked, whose correct answer is the same for all sources, so that evidence for or against GR has a tendency to build up.

As an example, one can consider a possible violation of GR in the coefficient \(\psi_3\), which is of particular interest, because 1.5 PN is the lowest post-Newtonian order at which so-called ‘tail effects’ become apparent, which roughly speaking result from the interaction of gravitational waves with the rest of spacetime [53, 54]. As shown in [23], with \(O(10)\) BNS detections, the Advanced LIGO and Virgo network will be able to pick up a 10% deviation in this coefficient with essentially zero false alarm probability, and constraining it will be possible at the \(\sim 1\%\) level. More generally, terms in Eq. (4) with \(j > 0\) are only accessible with direct gravitational wave detection.

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1 This is an example of an essentially field-theoretic effect, rather than just pertaining to the behaviour of test masses in a given gravitational field. Such effects cannot be probed with the currently observed binary neutron stars, due to their low orbital velocity \((v/c \sim 10^{-3})\) and orbital compactness \((GM/c^2 R \sim 10^{-6})\) compared with a binary on the verge of merger (which has \(v/c \sim 0.4\) and \(GM/c^2 R \sim 0.2\) respectively). More generally, terms in Eq. (4) with \(j > 0\) are only accessible with direct gravitational wave detection.
level. In the case of ET, the Fisher matrix results of [20] suggest that already with a single source, we will be able to measure $\psi_3$ with an accuracy better than 0.1% using the inspiral signal only. Presumably, these numbers will be even better for BBH when dynamical spins as well as merger and ringdown are taken into account.

It should be emphasized that a much wider class of GR violations can be picked up by using the TIGER method, not just simple shifts in post-Newtonian coefficients. (See [23, 24] for some striking examples.) A likely condition for a generic violation to be detectable in this way is that, for fixed coalescence phase, it induces a shift in the waveform of half a cycle around the frequency where the detector is the most sensitive; both for the advanced detectors and for ET this is $\sim 150$ Hz. Finally, we have noted that although TIGER will be very efficient in finding a deviation from GR, it will not necessarily allow for a determination of its precise nature. For the latter, one may want to use one of the general parameterized post-Einsteinian (ppE) waveform families of [19, 21, 25].

As shown in [55], observing the ringdown signal with ET will allow for a powerful test of the No Hair Theorem. As the black hole resulting from merger evolves towards a quiescent state, it is well modelled as a perturbed Kerr black hole. The Einstein’s equations dictate that the frequencies and damping times of the quasi-normal modes only depend on the mass $M$ and spin $J$ of the underlying Kerr geometry. For a 500 $M_\odot$ black hole at 6 Gpc, a departure as small as 8% in the frequency of the dominant mode could be detected. Kamaretsos et al went a step further by relating ringdown mode amplitudes to the progenitor masses and spins [56, 57]. It would be of great interest to cast these ideas into the language of TIGER, in which case smaller deviations can presumably be found even for a single source, and information from multiple sources can be combined.

5. Summary

The advanced interferometric gravitational wave detectors that are currently being built have the potential to settle important scientific questions: What is the local coalescence rate, and what does this imply for our understanding of the evolution of massive stars? What is the nature of short-hard gamma ray bursts? Do black holes really exist, or is there some alternative kind of very compact object? What does the EOS of neutron stars approximately look like? Are estimates of the present-day expansion of the Universe correct? Is general relativity really the correct theory of gravity in the strong-field, dynamical regime?

Einstein Telescope will take us to much larger detection rates and out to cosmological distances, where new questions can be addressed. What is the mass distribution of neutron stars and black holes, and how does it evolve in time? What is the true EOS of neutron stars (and does this, too, have time dependence)? In cosmology, we will arrive at a comprehensive, completely independent determination of all the basic parameters governing the Universe. Is the EOS of dark energy time dependent? Last but not the least, ET will probe the strong-field dynamics of spacetime in an unprecedented way.

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