Dipole low-order g-mode instability of metal-poor low-mass main-sequence stars due to the $\varepsilon$ mechanism

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ABSTRACT
We analysed the vibrational stability of metal-poor low-mass main-sequence stars due to the $\varepsilon$ mechanism. Since outer convection zones of metal-poor stars are limited only to the very outer layers, uncertainty in the treatment of convection does not affect the result significantly. We found that the dipole $g_1$ and $g_2$ modes definitely become unstable due to the $\varepsilon$ mechanism for $Z \lesssim 6 \times 10^{-4}$. Besides that, we found that as the metallicity decreases the mass range of $\varepsilon$-mechanism instability extends toward higher mass.

Key words: stars: abundances – stars: low-mass – stars: oscillations – stars: Population II.

1 INTRODUCTION

The structure and evolution of stars born in metal-poor environments or at times when heavy elements were significantly deficient in the Universe are considerably different from those of stars born much later with higher heavy-element abundances. This is mainly because opacities and nuclear reactions are highly dependent on metallicity. Opacity decreases with metallicity because of the lack of absorption by heavy elements; hence it makes the luminosity of a star higher. To maintain energy equilibrium in this situation, stars with low metallicity need to be compact compared with metal-rich stars. As a result, the main sequence of metal-poor stars on the Hertzsprung–Russell (HR) diagram moves toward the bluer and higher luminosity side. With this, the convective envelope of stars with $M \simeq 1 M_\odot$ becomes thinner and limited to the very outer layer close to the surface (Sonoi & Shibahashi 2011, hereafter Paper I).

The decrease in metallicity also makes CNO-cycle energy generation less efficient. In the Population I case, the pp chain is dominant for $M \lesssim 1.2 M_\odot$, while the CNO cycle is influential for more massive stars. As the metallicity and CNO abundance decrease, however, the CNO cycle becomes replaced by the pp chain for more massive stars. In the Population III case, the nuclear energy source is only the pp chain at the zero-age main sequence (ZAMS) stage for stars with $M \lesssim 13 M_\odot$ (Sonoi & Shibahashi 2012, hereafter Paper I).

In the case of pp-chain burning, we should note the vibrational instability of low-degree low-order g modes due to the $\varepsilon$ mechanism. Such instability and the resultant material mixing were once proposed as a possible solution to the solar neutrino problem (Dilke & Gough 1972), and the detailed numerical calculations of linear stability analyses demonstrated that such instability is likely to occur at a certain early evolutionary stage of the Sun and solar-like stars (Christensen-Dalsgaard, Dilke & Gough 1974; Boury et al. 1975; Shibahashi, Osaki & Unno 1975; Noels et al. 1976). The presence of a convective envelope, which occupies the outer 20–30 per cent of the stellar radius, however, has made it hard to reach a definite conclusion on vibrational stability because of uncertainty in the treatment of the convective envelope.

However, the situation is different in metal-poor stars. As described above, with decreased metallicity the convective envelopes of solar-like stars become thin enough that uncertainty in the treatment of convection does not significantly affect the results of vibrational stability analyses.

Indeed, the present authors have shown that metal-free Population III stars are vibrationally unstable against dipole low-order g modes due to the $\varepsilon$ mechanism of the pp chain (Paper I and II). To extend our analyses to stars with $Z \neq 0$ but still having only a very thin convective envelope, we examined the stability of stars with low metallicity through non-adiabatic analysis and determined the upper-limit metallicity for which this instability appears without uncertainty.

2 EVOLUTIONARY EQUILIBRIUM MODELS

We adopted the same code as in Paper II, MESA (Paxton et al. 2011), to calculate stellar evolution. Evolutionary models were constructed by calculating from the pre-main-sequence Hayashi phase with initial hydrogen abundance $X_0 = 0.75$ and metallicities $Z = 1, 2, 4, 6, 8 \times 10^{-4}$ and $1 \times 10^{-3}$. The mixing-length parameter was set to be $\alpha_{\text{MLT}} = 1.0$.

Fig. 1 shows the top and bottom boundaries of an outer convection zone for ZAMS models with different metallicity. As described in Section 1, with decreasing metallicity the outer convection zones for $M \simeq 1 M_\odot$ stars become thinner, limited to outer layers above $r/R \simeq 0.99$, and as thin as or thinner than those of Population I stars with $1.6–2.0 M_\odot$, which correspond to the blue edge of the $\delta$ Scuti...
The thick dashed line means the red edge of the region in which such models are located.

Figure 1. Top and bottom boundaries of an outer convection zone for ZAMS models with $Z = 4 \times 10^{-4}, 1 \times 10^{-3}$ and $2 \times 10^{-2}$. The initial hydrogen abundance is set to be $X_0 = 0.75$. The abscissa is the central hydrogen abundance $X_c$, which is an indicator of stellar evolution. The range is set to be from $X_c = 0.75$–0.20.

3 LINEAR FULLY NON-ADIABATIC STABILITY ANALYSIS

We performed a linear fully non-adiabatic analysis for the above equilibrium models using a Henyey-type code developed in Paper II. By following Unno et al. (1989), we linearized the equations of continuity, motion, energy conservation and radiative diffusion and Poisson’s equation, while any perturbation terms are expressed in terms of a combination of a spatial function and a time-varying function. The latter is expressed by $\exp(\sigma t)$, where $\sigma$ denotes the eigenfrequency. The former, the spatial part, is decomposed into a spherical harmonic function, which is a function of the colatitude and the azimuthal angle, and a radial function. Equations governing the radial functions lead to a set of sixth-order differential equation, of which coefficients are complex, including terms of frequency $\sigma$. Together with proper boundary conditions, this set of equations forms a complex eigenvalue problem with an eigenvalue $\sigma$. The real part of $\sigma$, $\sigma_R$, represents the oscillation frequency and the imaginary part of $\sigma$, $\sigma_I$, gives the growth rate or the damping rate, depending on its sign. We adopted the ‘frozen-in convection’ approximation, i.e. simply neglected convective flux perturbation. This approximation is acceptable to this study, since the outer convection zones of the chosen models are very thin and the result of the stability analysis may not depend significantly on our treatment of convection.

For a stability analysis relevant to the $\varepsilon$ mechanism, we should adopt temperature and density dependences of nuclear reactions through the perturbation $\varepsilon_T \equiv (\partial \ln \varepsilon/\partial \ln T)_\rho$ and $\varepsilon_\rho \equiv (\partial \ln \varepsilon/\partial \ln \rho)_T$, which are different from those for the evolutionary time-scale. For example, for the pp-I branch in the pp chain the temperature dependence of energy generation all along the branch in equilibrium is governed by the slowest reaction, $^1$He($^1$H,$e^+$)$^2$He, and $\varepsilon_T \approx 4$ at $\log T = 7$. However, we should adopt the effective temperature dependence of the nuclear reaction through the perturbation, which is mainly governed by $^3$He($^3$He,$e^+$)$^4$He and $\varepsilon_T \approx 11$ (Dilke & Gough 1972; Boury & Noels 1973; Unno 1975; Unno et al. 1989). We evaluate such effective dependences of the pp chain and the CN cycle separately through perturbation and then average them with their contribution to the total nuclear energy generation to obtain net values. More details are presented in Paper II.
4 RESULTS

4.1 Variation of stability with stellar evolution and stellar mass dependence

In this section, we discuss the variation of stability with stellar evolution and the stellar mass dependence of stability, while showing results in the $Z = 1 \times 10^{-4}$ case. Fig. 4 shows the variation of growth rates of the dipole ($l = 1$) $g_1$ and $g_2$ modes for stars with $Z = 1 \times 10^{-4}$. A positive value of $-\sigma / (2\pi)$ means instability. Except for lower mass stars, the $g$ modes are stable during the ZAMS stage, become unstable in the middle stage of core hydrogen burning and eventually become stable again. As well as the Population III case introduced in Papers I and II, the $g_1$ and $g_2$ modes become most unstable or least stable at $X_e = 0.5-0.6$ and $X_e = 0.6-0.7$, respectively. This delicate change of stability is caused by the variation in amplitude distribution with stellar evolution.

In the case of pp-chain burning, at the stellar centre, $^3$He is consumed by $^3$He($^1$H,$^2$H)$^4$He or by $^3$He($^1$H,$\gamma$)Be immediately after it is generated. However, since the $^3$He reactions are highly sensitive to temperature it does not occur efficiently in the outer part of the nuclear-burning core. As a consequence, $^3$He accumulates in an off-centred shell. Hence the most favourable situation for vibrational instability is that the temperature perturbation has a large amplitude in such an off-centred $^3$He shell.

Table 1 and Fig. 5 show the properties of the $g_1$ mode for a 1.2-$\text{M}_\odot$ star with $Z = 1 \times 10^{-4}$ at different evolutionary stages. The core of a metal-poor low-mass star is convective at the ZAMS stage even with pp-chain burning, because of the high central temperature, and gravity waves are evanescent there. Note that the convective core will barely affect vibrational stability, since the convective timescale there ($\sim$ yr) is much longer than the oscillation period ($\sim$ h).

In such a situation, it is plausible that the convective flux does not react to pulsation (e.g. Goldreich & Nicholson 1977; Pesnell 1987; Guzik et al. 2000). As the convective core shrinks with stellar evolution, gravity waves start to propagate in the deep interior. The $g$ mode then starts to have a large amplitude around the $^3$He shell and is unstable due to the $\kappa$ mechanism of the $^3$He$-$He reaction. For higher mass stars, on the other hand, the pp-II and III branches and the CNO cycle contribute to nuclear energy generation due to the high temperature. Collisional reactions belonging to them have a high temperature dependence comparable with that of the $^3$He$-$He reaction and the $g$ mode becomes stable again.

We found that the above instability appears for stars, for example, with $M \leq 2.3 \text{ M}_\odot$ in the $Z = 1 \times 10^{-4}$ case. More massive stars, on the other hand, keep a substantial size of the convective core because of the dominant contribution of the CNO cycle rather than the pp chain. In this situation, gravity waves cannot propagate in the nuclear-burning core enough for vibrational instability to occur. Instead, just outside the nuclear-burning core the amplitude of the $g$ mode is relatively large and strong damping is induced, as shown in Fig. 6. The $g$ modes are then not destabilized in such stars.

For less massive stars, for example ones with $M \leq 1.1 \text{ M}_\odot$ in the $g_1$-mode case, the growth rate strongly increases in the late stage of core hydrogen burning (Fig. 4). Since such stars are located inside the classical instability strip, the $g$ mode is excited by the $\kappa$ mechanism of helium ionization. As the $g$ mode starts to have a p-mode-like behaviour in the envelope at later stages, the $\kappa$ mechanism begins to work efficiently and leads to stronger instability.

Figure 4. Variation of growth rates of the dipole ($l = 1$) $g_1$ (top) and $g_2$ modes (bottom) for different mass stars with $Z = 1 \times 10^{-4}$. The abscissa is the same as in Fig. 3.

Figure 5. Properties of models and the dipole $g_1$ mode at different evolutionary stages for a 1.2-$\text{M}_\odot$ star. (a) Work integral normalized with the total oscillation energy in a whole star. (b) Radiative luminosity perturbation normalized with the value at the photosphere. (c) Squared temperature perturbation normalized with the peak value in the deep interior, marked with an open circle. (d) Effective temperature dependence of nuclear reactions, $\nu_T$. (e) $^3$He mass fraction.

Table 1. Properties of the $g_1$ mode shown in Fig. 5.

| $X_e$ | Age (yr) | Period (h) | Growth time-scale (yr) |
|------|---------|-----------|------------------------|
| 0.75 | $1.1 \times 10^7$ | 1.78 | $-1.9 \times 10^7$ |
| 0.55 | $8.0 \times 10^6$ | 1.33 | $2.6 \times 10^7$ |
| 0.41 | $1.2 \times 10^6$ | 0.997 | $-1.6 \times 10^7$ |
Although the $g_2$ mode is also excited by the $\epsilon$ mechanism, its instability appears mainly for lower mass stars. Since damping just outside the nuclear-burning core is stronger compared with the $g_1$ mode, the $g_2$-mode is not destabilized for higher mass stars (Fig. 6). For lower mass stars, on the other hand, such damping is weaker and the $g_2$ mode is excited mainly by the $\epsilon$ mechanism.

The $g_2$-mode instability also appears in the late stage of $M \simeq 1.6 M_\odot$ stars with $Z = 1 \times 10^{-4}$. In this case, although the convective core size is not substantial, the contribution of the CNO cycle starts to exceed that of the pp chain and hence the $\mu$ gradient around the nuclear-burning core becomes steeper. In this situation, gravity waves are trapped in the steep $\mu$-gradient zone and the $\epsilon$ mechanism works efficiently.

4.2 Metallicity dependence

Fig. 7 shows the boundaries of instability regions on the $T_{\text{eff}}$–$g$ plane for the $g_1$ and $g_2$ modes. They are cut off on the lower temperature side, where the convective envelope extends below $r/R = 0.99$. In the $Z = 1 \times 10^{-3}$ case, instability appears for stars with $M \lesssim 1.2 M_\odot$ and is induced mainly by the $\kappa$ mechanism. As the metallicity decreases, the instability region, particularly for the $g_1$ mode, extends toward more massive stars.

One of the reasons for this is that metal-poorer stars have lower density contrast between the inner and outer regions than metal-rich stars. Fig. 8 shows the ratio of the central density to the average density in a whole star, which represents the density contrast, for $Z = 1 \times 10^{-3}$ and $1 \times 10^{-4}$. The ratio for metal-poorer stars is lower than that for metal-rich stars having the same mass, since metal-poorer stars are more compact and thus have higher average density, while the central density is not substantially different for different metallicities. Due to this, the $g$-mode amplitude is relatively larger in the deep interior and the $\epsilon$ mechanism can work efficiently for metal-poorer stars.

Another reason is that as the metallicity decreases the CNO cycle becomes replaced by the pp chain for more massive stars. As discussed in Section 4.1, dominance of the pp-chain is favourable for $\epsilon$-mechanism instability, while CNO-cycle burning maintains a substantial size of convective core during stellar evolution.

Fig. 9 shows regions corresponding to instability with more than 50 per cent contribution of the $\epsilon$ mechanism to the total excitation energy. Such instability appears with $Z \lesssim 6 \times 10^{-4}$ and with $\log T_{\text{eff}} \gtrsim 3.90$ in the $g_1$-mode case. In this temperature range, corresponding to the bluer region outside the classical instability strip, the $\kappa$ mechanism of helium ionization does not work efficiently and the $\epsilon$ mechanism is responsible for the instability. In the $g_2$-mode
case, on the other hand, such instability appears inside the classical instability strip, while not on the bluer side except for the late stage of \( M \approx 1.6 \, M_\odot \) stars with \( Z = 1 \times 10^{-4} \), for which \( \varepsilon \)-mechanism instability is induced because of the steep \( \mu \) gradient as described in Section 4.1. As shown in Fig. 6, the \( g_2 \) mode is strongly excited by the \( \varepsilon \) mechanism in the lower mass star case, while strong damping just outside the nuclear-burning core avoids instability in the higher mass star case.

5 DISCUSSION

Due to \( \varepsilon \)-mechanism instability, the stars corresponding to the unstable models might exhibit pulsations with period \( \sim 1 \, h \). The \( g \) modes have relatively substantial amplitude at the surface, as shown in Fig. 5, although a non-linear analysis is necessary to obtain the absolute value of the amplitude.

Dziembowski (1982, 1983) estimated the dipole \( g_1 \)-mode amplitude in the solar photosphere to be about \( 20 \, \text{cm} \, \text{s}^{-1} \) by adopting the three-mode coupling theory. In particular, Dziembowski (1983) considered the special cases of parametric resonance, in which a parent linearly unstable mode is coupled with two linearly stable daughter modes. The oscillation energy of the parent mode is converted into that of the two daughter modes at a rate proportional to the product of the amplitudes of the three modes. The most effective case for low-degree low-order parent \( g \) modes appears to be coupling with a pair of similar high-degree daughter \( g \) modes to a state in which the amplitudes of all three modes are steady.

The value of \( 20 \, \text{cm} \, \text{s}^{-1} \) is too small for \( g \)-mode oscillations to be detectable for stars other than the Sun. On the other hand, slowly pulsating \( B \) (SPB) stars and \( \gamma \) Doradus stars exhibit \( g \)-mode oscillations with much higher amplitude. Appourchaux et al. (2010) suggested that such stars have only shallow convection zones, and therefore daughter \( g \) modes propagate much higher in the envelope and dissipate much more strongly, thus being limited themselves to much lower amplitudes and thereby being less able to extract energy from their parent. Such a situation might also be valid for \( g \) modes in the metal-poor low-mass main-sequence stars analysed in this study, although the linear growth rate is much smaller than in the SPB and \( \gamma \) Doradus cases. It is worth searching for stellar pulsations induced by the \( \varepsilon \) mechanism, since such pulsations have not been detected yet and would be a great tool with which to examine the first derivative of the nuclear energy generation rate.

Metal-poor stars have attracted attention as clues to chemical evolution of the Universe. Thanks to advances on the observational side, many metal-poor stars have been detected to date (e.g. Suda et al. 2008). Although most metal-poor stars are very far from the Sun, faint metal-poor pulsating stars have been detected in addition to Cepheids and RR Lyr stars in other galaxies, e.g. 20 \( \delta \) Scutum stars at \( V \sim 23 \) in the Carina dwarf spheroidal galaxy at \( \sim 100 \, \text{kpc} \) (Mateo, Hurley-Keller & Nemec 1998). Recently, a high number of faint variables were found in the Large Magellanic Cloud through the Optical Gravitational Lensing Experiment (OGLE) project (Poleski et al. 2010).

Most metal-poor stars are thought to be considerably old or to have an equivalent age to the Universe. The \( \varepsilon \)-mechanism instability reported in this paper, however, appears in the early stages of the main sequence. In some galaxies such as dwarf irregulars, on the other hand, chemical evolution might be delayed and metal-poor star-formation regions may still remain now. In such galaxies, young metal-poor stars may exist and are expected to be candidates for \( \varepsilon \)-mechanism pulsators.

We also state the possibility that growth of amplitude due to vibrational instability might induce material mixing and have a significant influence on the later evolution of a star, as was expected concerning the solar neutrino problem by Dilke & Gough (1972). In particular, if mixing occurred in the core, the surrounding cooler and hydrogen-rich matter would be incorporated into the core. This phenomenon might induce rejuvenation of the star and prolong the lifetime. To obtain a solution to this problem, we have to pursue its non-linear evolution.

6 CONCLUSION

We performed a linear non-adiabatic stability analysis of metal-poor low-mass main-sequence stars. We restricted our analysis to evolutionary models having negligibly thin convective envelopes. We found that the dipole \( g_1 \) and \( g_2 \) modes definitely become unstable due to the \( \varepsilon \) mechanism for \( Z \lesssim 6 \times 10^{-4} \). The mass range of this instability extends toward higher mass with decreasing metallicity.

One of the reasons for this is that for metal-poorer stars the density contrast between the inner and outer regions is lower and the \( g \)-mode amplitude is relatively larger in the nuclear-burning core. Another reason is that as the metallicity decreases CNO-cycle burning becomes replaced by pp-chain burning, which is favourable for \( \varepsilon \)-mechanism instability. As a result of this instability, stellar pulsation or material mixing is expected to occur. To confirm these, we...
require highly precise observations of faint metal-poor stars and a non-linear analysis of the oscillations.

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REFERENCES

Appourchaux T. et al., 2010, A&AR, 18, 197
Boury A., Noels A., 1973, A&A, 24, 255
Boury A., Gabriel A., Noels A., Scuflaire R., Ledoux P., 1975, A&A, 41, 279
Christensen-Dalsgaard J., Dilke F. W. W., Gough D. O., 1974, MNRAS, 169, 429
Dilke F. W. W., Gough D. O., 1972, Nat, 240, 262
Dziembowski W., 1982, Acta Astron., 32, 147
Dziembowski W., 1983, Solar Phys., 82, 259

Goldreich P., Nicholson P., 1977, Icarus, 30, 301
Guzik J. A., Kaye A. B., Bradley P. A., Cox A. N., Neuforge C., 2000, ApJ, 542, L57
Mateo M., Hurley-Keller D., Nemec J., 1998, AJ, 115, 1856
Noels A., Boury A., Gabriel M., Scuflaire R., 1976, A&A, 49, 103
Pamyatnykh A. A., 2000, in Breger M., Montgomery M. H., eds, ASP Conference Series Vol. 210, Delta Scuti and Related Stars. Astron. Soc. Pac., San Francisco, p. 215
Paxton B., Bildsten L., Dotter A., Herwig F., Lesaffre P., Timmes F., 2011, ApJS, 192, 3
Pesnell W. D., 1987, ApJ, 314, 598
Poleski R. et al., 2010, Acta Astron., 60, 1
Shibahashi H., Osaki Y., Unno W., 1975, PASJ, 27, 401
Stellingwerf R. F., 1979, ApJ, 227, 935
Sonoi T., Shibahashi H., 2011, PASJ, 63, 95 (Paper I)
Sonoi T., Shibahashi H., 2012, PASJ, 64, 2 (Paper II)
Suda T. et al., 2008, PASJ, 60, 1159
Tsvetkov Ts. G., 1982, Ap&SS, 89, 435
Unno W., 1975, PASJ, 27, 81
Unno W., Osaki Y., Ando H., Saio H., Shibahashi H., 1989, Nonradial oscillations of stars, 2nd edn. University of Tokyo Press, Tokyo

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