Dynamics of elliptic breathers in saturable nonlinear media with linear anisotropy

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Abstract
We have introduced a class of dynamic elliptic breathers in saturable nonlinear media with linear anisotropy. Two kinds of evolution behavior for the dynamic breathers, rotations and molecule-like librations, are both predicted by the variational approach, and confirmed in numerical simulations. The dynamic elliptic breathers can rotate even though they have no initial orbital angular momentum (OAM). As the media are linear anisotropic, OAM is no longer conserved, and hence the angular velocity is not constant but a periodic function of the propagation distance. When the linear anisotropy is large enough, the dynamic elliptic breathers librate like molecules. The dynamic elliptic breathers are present in media with not only saturable nonlinearity but also nonlocal nonlinearity; indeed, they are universal in nonlinear media with linear anisotropy.

Keywords: dynamic elliptic breathers, rotation mode, molecule-like libration mode, variational approach

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1. Introduction
The self-trapping and self-focusing of optical beams in nonlinear media of both linear isotropy (isotropic diffraction) and nonlinear isotropy have been studied extensively for over three decades [1–3]. In isotropic media, the circular symmetry is conserved, and therefore the fundamental solitons have circular shapes. Introducing nonlinear anisotropy into a medium, self-trapping beams with ellipse-shaped spots can be obtained. Coherent elliptic strongly nonlocal solitons were observed experimentally in lead glass [4], where nonlinear anisotropy was achieved using rectangular boundaries in the transverse direction. Elliptical discrete solitons can form in an optically induced two-dimensional photonic lattice, where nonlinear anisotropy stems from the enhanced photorefractive anisotropy and nonlocality under a nonconventional bias condition [5]. It was reported recently that the anisotropic nonlocal nonlinearity of the diffusive (thermal) type can stabilize the dipole-mode solitons, which were completely unstable in the isotropic medium [6].

The linear anisotropy is also important in many soliton phenomena. In [7], the self-focusing of the beam propagating in any direction in uniaxial crystals was discussed. There exists an elliptic self-trapping beam for the extraordinary light in uniaxial crystals. Stationary elliptic quadratic solitons [8] and elliptic nonlocal solitons [9] were found, respectively, in biaxial crystals and nematic liquid crystals with large birefringence. The generation of multiple solitons [10, 11] were reported theoretically and experimentally to be the consequence of linear anisotropy. Linear isotropy can also elicit elliptic optical beams without any initial orbital angular momentum (OAM) during linear propagation [12], and the rotation angle will monotonically approach the value determined by the media and the input parameters of the beam. It was predicted very recently that the spiraling elliptic solitons with the OAM can exist in isotropic saturable nonlinear media [13] and in isotropic nonlocal nonlinear media [14], where the OAM can bring in the effective linear anisotropy. The nonlinear propagation of elliptic optical beams in saturable nonlinear medium with linear anisotropy is analyzed in this paper. In such media, a class of dynamic elliptic breathers is introduced that can rotate even without an initial OAM. Angular velocity is no longer constant but a periodic function of the propagation...
distance. Libration of the dynamic elliptic breathers as in molecules is produced when the linear anisotropy of the media is large enough.

2. The variational solution of the dynamic elliptic breathers

The propagation of optical beams in saturable nonlinear media with linear anisotropy can be modeled using the saturable nonlinear Schrödinger equation (NLSE) [2, 7]

\[
\frac{\partial A}{\partial z} + \frac{1}{2k} \left( \frac{\partial^2 A}{\partial \xi^2} + \frac{\partial^2 A}{\partial \eta^2} \right) + \frac{\omega \beta}{n_0} \frac{I}{1 + I / I_t} A = 0, \tag{1}
\]

where \( A(\xi, \eta, \zeta) \) is the amplitude for the paraxial beam, \( I = |A|^2 \) the beam intensity, \( I_t \) the saturation intensity, \( n_0 \) the nonlinear index coefficient, \( \zeta \) the longitudinal coordinate, \( \xi \) and \( \eta \) are the transverse coordinates, \( k \) is the wavenumber in the media without nonlinearity, \( n_0 \) the linear refractive index of the media, and \( \alpha_1 \) and \( \alpha_2 \) are the diffraction coefficients along \( \xi \) and \( \eta \) directions, respectively. The larger these coefficients are, the stronger the optical beam diffracts in those directions. If \( \alpha_1 = \alpha_2 \), a circularly shaped optical beam will diffract equally along any direction [14]. Nevertheless here we consider the case of linear anisotropy, i.e., \( \alpha_1 \neq \alpha_2 \).

Introducing the scaled dimensionless variables \( x = \xi / w_0 \), \( y = \eta / w_0 \), \( z = \zeta / L_d \), \( \phi = \left( \frac{\omega \beta I_t}{n_0} \right)^{1/2} A \), where \( L_d = 2k w_0^2 \) is the Rayleigh distance, the equation (1) becomes the form

\[
\frac{i \partial \phi}{\partial z} + \alpha_1^2 \frac{\partial^2 \phi}{\partial x^2} + \alpha_2^2 \frac{\partial^2 \phi}{\partial y^2} + \frac{\lambda \beta^2}{1 + \lambda \beta^2} \phi = 0, \tag{2}
\]

where \( \gamma \) is the dimensionless saturation intensity; \( \gamma = 1 \) is assumed throughout. Through the variable transformation \( X = x / \alpha_1, Y = y / \alpha_2, Z = z \), the NLSE (2) becomes

\[
\frac{i \partial \phi}{\partial Z} + \frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} + \frac{\lambda \beta^2}{1 + \lambda \beta^2} \phi = 0, \tag{3}
\]

where the optical beam is changed to \( \phi(X, Y, Z) \). The Lagrangian of (3) can be expressed as [13]

\[
L = i \bar{I} \int \left( \frac{\partial^2 \phi}{\partial Z^2} - q \partial^2 \phi / \partial Z \right) dX dY - H, \quad \text{where} \quad H \quad \text{is the Hamiltonian of the system}
\]

\[
H = \int \left( \frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} \right) - \lambda \beta^2 \phi + \ln \left( 1 + \phi^2 \right) dX dY. \tag{4}
\]

We introduce a trial function [13]

\[
\phi = \frac{P}{\sqrt{\sigma b(\zeta) c(Z)}} G \left[ \frac{\xi}{b(\zeta)} \right] G \left[ \frac{\eta}{c(Z)} \right] \exp (i \phi), \tag{5}
\]

where the Gaussian envelope is \( G(t) = \exp \left( -i t^2 / 2 \right) \), the phase \( \phi = \xi \cos(\beta Z) + \Theta(Z) \), \( \xi = X \cos(\beta Z) + Y \sin(\beta Z) \), \( \eta = -X \sin(\beta Z) + Y \cos(\beta Z) \), and \( P = \int |\phi|^2 dX dY \) is the power. From (5), we can obtain the OAM, \( M = \text{Im} \int \phi^*(X \partial \phi / \partial x - Y \partial \phi / \partial y) dX dY = \frac{1}{2} P (b^2 - c^2) \theta \). Inserting the trial solution (5) into the Lagrangian, \( L \) can be analytically determined. Using the variational approach [15], we obtain \( P = 0, H = 0, M = 0, \beta = 2(b^2 + c^2) \theta / (b^2 - c^2) \), and \( b = 4b_b, c = 4c_q \), where the primes indicate derivatives with respect to the variable \( Z \). Inserting relational expressions above into the Hamiltonian (4), we obtain

\[
H = \frac{P}{8} \left( b^2 + c^2 \right) + \Pi, \tag{6}
\]

where \( \Pi = \frac{P}{8} \left[ \frac{1}{b^2} + \frac{1}{c^2} + 4 \sigma \phi^2 / (b^2 - c^2)^2 \right] - z b c L_i \left( - \frac{P}{8c} \right) - P \) is the potential of the system with \( \sigma = M / P \). Here, \( L_i(\beta, t) \) is the dilogarithm function, defined by \( L_i(\beta, t) = \int_0^1 \frac{du}{u} \ln (1 - u / q) / q \). Solitons correspond to the extrema of the potential \( H(\beta, c) \). Hence, letting \( d\Pi / db = d\Pi / dc = 0 \), we obtain

\[
\frac{4}{b^3} + \frac{16 \sigma \phi^2 \left( b^2 + 3c^2 \right)}{\left( b^2 - c^2 \right)^3} + 4 \pi c \frac{F(b, c)}{P} = 0, \tag{7}
\]

\[
\frac{4}{c^3} - \frac{16 \sigma \phi^2 \left( 3b^2 + c^2 \right)}{\left( b^2 - c^2 \right)^3} + 4 \pi b \frac{F(b, c)}{P} = 0, \tag{8}
\]

where \( F(b, c) = \ln \left( 1 + \frac{P}{8c} \right) + L_i \left( - \frac{P}{8c} \right) \). If \( b \) and \( c \) are given, the critical power and the critical OAM can be obtained from (7) and (8), with which the optical beam can propagate keeping its elliptic profile changeless and rotating stably in the XYZ-coordinate frame. The rotational angular velocity is

\[
\omega = \beta^2 \left( \frac{b^2 + c^2}{(b^2 - c^2)^2} \right). \tag{9}
\]

We can then obtain \( \beta(Z) = \omega Z + \beta_0 \), where \( \beta_0 \) is the initial inclination of the elliptic optical beam at \( Z = 0 \) in the XYZ-coordinate frame. Here, we give an example [13], when \( b = 4.26 \) and \( c = 2.13 \), the critical power and the critical OAM can be obtained as \( P = 127.32 \pi \) and \( \sigma = 0.35 \), respectively. The rotational angular velocity can also be determined as \( \omega = 0.17 \). The optical beam expressed in the XYZ-coordinate
frame (the laboratory frame) is of the form
\[
\varphi = \frac{P}{\sqrt{\pi bc}} \exp \left[ -\left( \frac{x \cos \beta + y \sin \beta}{a_1} \right)^2 \right.
\left. - \left( \frac{y \cos \beta - x \sin \beta}{a_2} \right)^2 \right]
\times \exp \left[ i\Theta \left( \frac{x \cos \beta + y \sin \beta}{a_1} \right) + i\theta \right].
\] (10)

As an example, the evolution of the optical beam in the \(xyz\)-coordinate system is shown in figure 1, where the input beam is expressed as (10) with \(\alpha_1 = 1.01\) and \(\alpha_2 = 1.32\). The optical beam rotates during propagation, whereas its shape changes periodically. To confirm the validity of the approximate analytical solution, we compare the two half widths obtained from variational solution, \(w_b = \alpha_1 b^{-2} \cos^2 \varphi_0 z + c^{-2} \sin^2 \varphi_0 z)^{-1/2}\) and \(w_c = \alpha_2 (c^{-2} \cos^2 \varphi_0 z + b^{-2} \sin^2 \varphi_0 z)^{-1/2}\), with those from the numerical simulation of (2); we find excellent agreement (figure 2). The method of numerical simulation used here is the split-step Fourier method [16] using (10) as the input beam at \(z = 0\).

3. Rotation mode

We now focus on rotation of the dynamic elliptic breathers in media with linear anisotropy. To this end, we should transform the expression for the elliptic optical beam (10), to its standard elliptic form by a coordinate rotation through angle \(\vartheta\). We obtain
\[
\tan (2\vartheta) = \frac{r_{\varphi}}{r_{\theta}} = \frac{\gamma_{\varphi}}{\gamma_{\theta}}
\]
where \(\gamma_{\varphi} = (b^{-2} \cos^2 \beta + c^{-2} \sin^2 \beta) / a_1^2\), \(\gamma_{\theta} = (b^{-2} \sin^2 \beta + c^{-2} \cos^2 \beta) / a_2^2\), and \(\gamma_{xy} = 2 \sin \beta \cos \beta (b^{-2} - c^{-2}) / (a_1 a_2)\). One of the semi-axes of the standard elliptic optical spot is
\[
w_b = \frac{1}{\sqrt{\gamma_{xx} \cos^2 \vartheta + \gamma_{yy} \sin^2 \vartheta + \gamma_{xy} \sin \vartheta \cos \vartheta}}
\] (11)
the other being
\[
w_c = \frac{1}{\sqrt{\gamma_{xx} \sin^2 \vartheta + \gamma_{yy} \cos^2 \vartheta - \gamma_{xy} \sin \vartheta \cos \vartheta}}.
\] (12)

From (11) and (12), the two semi-axes of the elliptic optical beam are found in general to vary with propagation distance \(z\) for \(\alpha_1 \neq \alpha_2\) and \(b \neq c\). No elliptic solitons with rotation mode and molecule-like libation mode exist in the model (2); only dynamic elliptic breathers exist, the semi-axes of which vary with \(z\) periodically. From (11) and (12) for media with linear isotropy, i.e., \(\alpha_1 = \alpha_2\), we obtain \(w_b = \alpha_1 c\) and \(w_c = \alpha_1 b\), which appears in [13] for elliptic solitons rotating with constant angular velocity forming in saturable nonlinear media with linear isotropy. For \(b = c\), we obtain \(\Theta (z) = 0\), that is, the two semi-axes of the elliptic solitons lie on the \(x\) and \(y\) axes at all times. There is no rotation or molecule-like libation. In [7], an elliptic self-trapping beam for the extraordinary light was reported to exist in an uniaxial crystal and the semi-axes of the elliptic optical beam were aligned in the principal plane of the uniaxial crystal.
The angular velocity of the optical beam in the \(xyz\)-coordinate frame is

\[
\omega = \frac{d\theta}{dz} = \frac{f_1(b, c, \omega, \alpha_1, \alpha_2, z)}{f_2(b, c, \omega, \alpha_1, \alpha_2, z)},
\]

where

\[
f_1(b, c, \omega, \alpha_1, \alpha_2, z) = a_1 a_2 (b^2 c^2 - \omega^2 (a_1^2 + a_2^2)(b^2 - c^2)
+ (a_1^2 - a_2^2)(b^2 + c^2) \cos 2(\omega z + \beta_0)) \text{ and}
+ 2(a_1^2 b^2 - a_2^2 c^2)^2 \sin^2(\omega z + \beta_0) + [a_1^2 a_2^2 (b^4 + c^4)
+ (a_1^4 - 4a_1^2 a_2^2 + a_2^4)b^2 c^2] \sin^2(\omega z + \beta_0). \]

The angular velocity \(\omega\) is a periodic function of \(z\) but not a constant, (see figure 3(a)). We can prove that \(f_1 \geq 0\) in (13). If \(f_1\) is always positive or negative at any propagation distance \(z\), the dynamic elliptic breathers will rotate anticlockwise or clockwise. Mathematically, positive \(f_1\) at any \(z\) only requires that the minimum of \(f_1\) is positive, i.e., \(\min(f_1) \geq 0\). Similarly, negative \(f_1\) at any \(z\) only requires that the maximum of \(f_1\) is negative, i.e., \(\max(f_1) \leq 0\). From this we obtain criteria for the rotation of the dynamic elliptic breathers, \(\min(\rho, 1/\rho) \leq \rho_M \leq \max(\rho, 1/\rho)\) with \(\rho_M \neq 1\), where \(\rho = b/c\), and \(\rho_M = \alpha_1/\alpha_2\) parameterizes the linear anisotropy of the media. The larger the difference between \(\rho_M\) and 1, the larger is the linear anisotropy of the media. We find that \(f_1\) is always positive when \(\min(\rho, 1/\rho) \leq \rho_M \leq \max(\rho, 1/\rho)\); that is, the dynamic elliptic breather will rotate anticlockwise when the linear anisotropy of the media is small enough. Indeed, we have used the positivity of \(\omega\) here, which ensures \(f_1\) is always positive. Of course, we can also use negative \(\omega\), then the dynamic elliptic breather will rotate clockwise when \(\min(\rho, 1/\rho) \leq \rho_M \leq \max(\rho, 1/\rho)\). From (9), the sign of \(\omega\) is found to depend on the sign of \(\sigma\). Nevertheless, the sign of \(\sigma\) has no effect in (7) and (8). Taking parameters \(b = 4.26, c = 2.13, \alpha_1 = 1.0\) and \(\alpha_2 = 1.3\) as an example, we find parameter \(\rho_M\) satisfy \(1/\rho < \rho_M < 1\), and predict that the

![Figure 3. Variation of rotational angular velocity with propagation distance. The parameters are the same as those in figures 1 and 2 for (a), and the parameters for (b) are \(b = 4.26, c = 2.13, P = 127.32, \theta = 0.052, \alpha_1 = 2.5, \alpha_2 = 1.0, \beta_0 = 73^\circ\).](image)

![Figure 4. Evolution of an elliptic breather over a single period. The parameters are the same as those in figure 1 except for \(a_1 = 1, a_2 = 2\) so that \(\rho_M = 1/\rho\).](image)
dynamic elliptic breathers will rotate anticlockwise, as con-

firmed in figures 1 and 3(a).

The OAM of the optical beam in the \(xyz\)-coordinate
frame is obtained as
\[
m = \text{Im} \int \int \phi^* \left( \frac{\partial \phi}{\partial y} - y \frac{\partial \phi}{\partial x} \right) dx dy = \frac{\rho_\theta f_1(b, c, \omega, \rho_\alpha, \rho_\beta, \zeta)}{4 \omega_\alpha \alpha (b^2 - c^2)}.
\]
If we set \(f_1(z = 0) = 0\), the dynamic elliptic
breathers will have no initial OAM. Meanwhile, by setting
\[f_1(z) = 0\] or \(\max(f_1) = 0\), the dynamic elliptic
breathers without an initial OAM will rotate anticlockwise or
clockwise. We then have dynamic elliptic breathers without initial
OAM rotating anticlockwise if \(\rho = 1\) and \(\beta = \pi/2\). The rotations
of such breathers is different from the rotation resulting from
those with initial OAM in isotropic media [4, 14].

From the expression for dynamic elliptic breathers (10), we obtain period
\(T = \pi/\omega\), where ‘period’ refers to as the
propagation distance \((z)\) at which the dynamic breather
completes one rotation or one libration. The period for the
dynamic elliptic breather in figure 4 is about 18.4, where
\(\omega = 0.17\), but we find that its shape after a period is not the
same as that at input \(z = 0\). This is because the exact solutions
of NLSE (2) are not Gaussian functions; the variational
approach introduces deviations if we take a Gaussian trial
solution such as (5). The deviations will be expected to dis-
appear if we use the numerical iterative solution of NLSE (2)
as the input beam at \(z = 0\). Nonetheless, we can correctly
predict the behaviors of the rotation and libration of the
dynamic elliptic breathers using the variational approach.

4. Molecule-like libration mode

If we increase the linear anisotropy of the media so that
\(\rho_M \geq \max(\rho, 1/\rho)\) or \(\rho_M \leq \min(\rho, 1/\rho)\), the angular velo-
city \(\omega\) will change its sign during the propagation of the
dynamic elliptic breathers. That is, the direction of the rota-
tion of the dynamic elliptic breathers will change, and the
molecule-like libration will appear (figures 3(b) and 5). The
breather rotates clockwise from \(z = 0\) to \(z = 3\), then changes its
direction and rotates anticlockwise until \(z = 17\). The critical
angle at which the rotation direction changes is found to be
\[
\tan(2\theta_c) = \frac{1 - \rho^2}{\sqrt{\rho^2 - 1 - \rho^4 + \rho^2 \rho_M^2}}.
\]
If \(\rho = 2\) and \(\rho_M = 2.5\), the critical angle is \(-25^\circ\), which
agrees well with that from a numerical simulation (figure 5(d)).

Indeed, the propagation of the optical beam in nonlinear
media with linear anisotropy can be modeled by the following
generalized NLSE:
\[
i \frac{\partial \varphi}{\partial z} + \alpha_1^2 \frac{\partial^2 \varphi}{\partial x^2} + \alpha_2^2 \frac{\partial^2 \varphi}{\partial y^2} + \Delta n \varphi = 0, \quad (15)
\]

where the nonlinear index $\Delta n$ can have various functional forms, for example, saturable nonlinearity
\[
\Delta n = \varphi \gamma |\varphi|^2 + |\varphi|^4
\]
and nonlocal Kerr nonlinearity
\[
\Delta n = -\int R(\mathbf{x}, \mathbf{y}) \varphi(\mathbf{x}, \mathbf{y})^2 \text{d}x \text{d}y.
\]
The OAM is not conserved for the generalized NLSE (15) when $\alpha_1 \neq \alpha_2$. Therefore, the rotation mode of the dynamic elliptic breathers for small anisotropy and the molecule-like libration mode for large anisotropy all can be supported by the generalized NLSE (15). For example, we can simply explore the molecule-like libration mode of the dynamic elliptic breathers in the nonlinear media with the nonlocal
\[
\Delta n = -\int R(\mathbf{x}, \mathbf{y}) \varphi(\mathbf{x}, \mathbf{y})^2 \text{d}x \text{d}y,
\]
where the response function is
\[
R = 1/(2\omega_0^2) \exp \left[ -(x^2 + y^2)/(2\omega_0^2) \right].
\]
We take the optical beam (10) as an input beam, where the parameters, $w_m = 15$, $b = 2$, $c = 1$, $P = 1.25 \times 10^5$, and $\Theta = 0.374$, are the same as for figure 1(a) of [14]. However, instead of $\alpha_1 = \alpha_2 = \frac{\sqrt{2}}{2}$ in [14], here we use the diffraction coefficients $\alpha_1 = \frac{5\sqrt{2}}{8}$ and $\alpha_2 = \frac{\sqrt{2}}{4}$ to make sure that the parameter of the linear anisotropy, $\rho_M = 2.5$, and $\rho = b/c = 2$ are the same as those in figure 5 of this paper. The evolution of the dynamic elliptic breather is shown in figure 6, where librations also happen. The dynamic elliptic breather rotates anticlockwise from $z = 0$ to $z = 6$, then changes the rotational direction and rotates clockwise until $z = 13$, then changes the rotational direction again, and rotates anticlockwise until $z = 19$, then changes the rotational direction again, and rotates clockwise until $z = 25.6$. The shape of the dynamic elliptic breather at $z = 25.6$ after one period is almost the same as that of the input beam at $z = 0$. The larger the degree of nonlocality, the less is the difference in shape between the beam after one period and the input beam. The details of the rotation and molecule-like libration of the dynamic elliptic breathers in nonlocal nonlinear media with linear anisotropy will be reported elsewhere.

5. Conclusion

We have introduced a class of dynamic elliptic breathers in saturable nonlinear media with linear anisotropy. The dynamic elliptic breathers have light spots of elliptical shape, and have two different kinds of modes of evolution, that is, the rotational mode and the molecular-like libration mode. The dynamic elliptic breathers can rotate even though they
have no initial OAM. Because of the anisotropy of the media, the angular velocity is not constant but a periodic function of the propagation distance. When the anisotropy of the media is large enough, the dynamic elliptic breathers librate like molecules. The dynamic elliptic breathers are universal in nonlinear media with linear anisotropy, such as saturable nonlinear media and nonlocal nonlinear media. The predictions of rotation and libration for dynamic elliptic breathers using the variational approach were confirmed in numerical simulations.

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