Self-similarity in jet events following from p-p collisions at LHC

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Abstract

Using a Tsallis nonextensive approach, we simultaneously analyze recent data obtained by the LHC ATLAS experiment on distributions of transverse momenta of jets, $p_T^\text{jet}$, together with distributions of transverse momenta of particles produced within these jets (defined relative to the jet’s axis), $p_T^\text{rel}$, and their multiplicity distributions, $P(N)$. The respective nonextensivity parameters for distributions of jets, $q_{\text{jet}}$, for distributions of particles in jets, $q_{\text{rel}}$, and the global nonextensivity parameter obtained from $P(N)$, $q_N$, were then compared with nonextensivity parameters $q$ obtained from minimum bias $pp$ collisions at energies corresponding to the energies of these jets. The values of the corresponding nonextensivity parameters were found to be similar, strongly indicating the existence of a common mechanism behind all these processes. We tentatively identify this as a self-similarity property known to be present there and resulting in Tsallis type distributions. If confirmed, this would considerably strengthen the nonextensive Tsallis approach.

Keywords: p−p collisions, jets, nonextensivity, self-similarity

1. Introduction

For some time now it is known that transverse momentum spectra of different kinds measured in multiparticle production processes, which change character from exponential at small values of $p_T$ to power-like at large $p_T$, can be described by a simple two-parameter formula,

$$h(p_T) = C \left(1 + \frac{p_T}{nT}\right)^{-n}.$$  \hspace{1cm} (1)

This was first proposed in \cite{Hagedorn} as the simplest formula extrapolating the large $p_T$ power behavior expected from parton collisions to exponential behavior observed for $p_T \to 0$. At present it is known as the QCD-based Hagedorn formula \cite{Hagedorn} and was used in many fits to recent data. However, in many branches of physics Eq. (1), with $n$ replaced by $n = 1/(1 - q)$, is more widely known as the Tsallis formula \cite{Tsallis}. In this case, $q$ is known as a nonextensivity parameter. In this form, Eq. (1) is usually supposed to represent a nonextensive generalization of the Boltzmann-Gibbs exponential distribution, $\exp(-p_T/T)$, used in a statistical description of multiparticle production processes, with $q$ being a new parameter, in addition to previous “temperature” $T$. Such an approach is known as nonextensive statistics \cite{Tsallis} in which the parameter $q$ summarily describes all features causing a departure from the usual Boltzmann-Gibbs statistics (in particular it can be shown that it is directly related to the possible intrinsic, nonstatistical fluctuations of the temperature $T$ \cite{Temperatures, Non-extensive}). However, the Tsallis distribution also emerges from a number of other more dynamical mechanisms, for example see \cite{Tsallis} for more details and references. In all possible scenarios leading to Eq. (1), the "temperature", or, in general, scale parameter $T$, is given by the mean value of the transverse momentum, $\langle p_T \rangle = 2nT/(n - 3)$, and we do not discuss here its possible dependence on energy and the nonextensivity parameter. For large values

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of transverse momenta, \( p_T \gg nT \), Eq. (1) becomes scale free (independent of \( T \)) distribution. The Tsallis distribution was successfully used for a description of all kinds of multiparticle production processes in a wide range of incident energy (from few GeV up to few TeV) and in a broad range of transverse momenta (see, for example, reviews \([5, 6]\)) In particular, it turned out that it also successfully describes transverse momenta of charged particles measured by LHC experiments, the flux of which changes by over 15 orders of magnitude \([7, 8]\).

The Tsallis distribution was recently used in an analysis of the distribution of the longitudinal component of momenta of particles within jets produced in pp collisions \([9]\) which, from this point of view, is similar to what was found in \( e^+e^- \) collisions \([10]\). Recent ATLAS data \([12, 13]\) allow us to extend such an analysis to transverse characteristics of jets and charged particles within them. This is because they provide both the distributions of transverse momenta of jets produced at LHC energies, \( p_T^{\text{jet}} \), and distributions of transverse momenta of particles produced within these jets (defined relative to the jets), \( p_T^{\text{rel}} \). One can then retrieve and discuss the respective nonextensivity parameters of jets, \( q_{\text{jet}} \), and particles produced within them, \( q_{\text{rel}} \). In addition, because \([12]\) at the same time also provides multiplicity distributions within jets, \( P(N) \), it is possible to confront both nonextensivities with that obtained from an analysis of \( P(N) \), \( q_N \). This is the subject of the present work.

2. Transverse momentum distributions of jets and particles within jets

In what follows we shall concentrate on ATLAS data \([12]\). They were taken at energy 7 TeV and in rapidity window \(|y| < 1.9\) measured jets observed in very narrow jet cones defined by \( R = \sqrt{\Delta \eta^2 + \Delta \phi^2} \) (where \( \Delta \phi \) and \( \Delta \eta \) are, respectively, the azimuthal angle and the pseudorapidity of the hadrons relative to that of the jet, \( \eta = -\ln \tan \theta \), with \( \theta \) being the polar angle), namely \( R = 0.6 \). Distributions of transverse momenta, \( p_T^{\text{jet}} \), of jets of charged particles were observed,

\[
f(p_T^{\text{jet}}) = \frac{1}{N_{\text{jet}}} \frac{dN_{\text{jet}}}{dp_T^{\text{jet}}}
\]

and also distributions of transverse momenta

\[
p_T^{\text{rel}} = \frac{|\vec{p} \times \vec{p}_T^{\text{rel}}|}{|\vec{p}_T^{\text{jet}}|}
\]

of all \( N \) particles (only charged) in the jet,

\[
f(p_T^{\text{rel}}) = \frac{1}{N} \frac{dN}{dp_T^{\text{rel}}}.
\]

In addition, \([12]\) also provides multiplicity distributions of particles produced within observed jets, \( P(N) \).

It should be stressed that the pure power law distribution, \( f(p_T) \sim p_T^\gamma \), is not experimentally observed for jets. The observed slope parameter \( \gamma \) depends on \( p_T \), \( \gamma = \gamma(p_T) \). However, a Tsallis distribution \([11]\) emerges if one accounts for this dependence and assumes it in the following two parameter \((n \text{ and } T)\) form,

\[
\gamma(p_T) = n \frac{\ln(nT + p_T) + (n-1)\ln(nT) + \ln(n-1)}{\ln(p_T)}.
\]

In this case, the transverse momentum distribution for jets can be fitted by a Tsallis formula \([11]\) with \( n = 7 \) and \( T = 0.45 \text{ GeV} \), cf. Fig. 1.

Data on distributions of transverse momenta \( p_T^{\text{jet}} \) of particles produced within the jet are presented in two papers. In \([12]\), data for \( p_T^{\text{jet}} \leq 40 \text{ GeV} \) and in \([13]\) for \( p_T^{\text{jet}} > 40 \text{ GeV} \). All can be fitted by a Tsallis formula \([11]\) and results are shown in Fig. 2 and Table 1 for the first set, and in Fig. 3 and Table 2 for the second one. It must be stressed at this point that the uncertainty is large in getting the precise values of parameters \( T \) and \( n \) from fits because both variables are correlated. One also has to remember that data from \([13]\) presented in Fig. 3 differ from those from \([12]\) and presented in Fig. 2. Namely, they were collected for \( |\eta| < 1.2 \) and \( p_T^{\text{track}} > 0.5 \text{ GeV} \) (to be compared with \( |\eta| < 1.9 \) and \( p_T^{\text{track}} > 0.3 \text{ GeV} \) in the former case). This fact influences multiplicity in jets (which is smaller in the
From our experience with applications of Tsallis statistics to multiparticle production processes, we know\cite{14} that multiplicity distribution of particles energy spectra of later case), which, in turn, influences the value of the parameter $T$.

Notice the negative values of the parameter $n$ (or, correspondingly, $q < 1$ values of the nonextensivity parameter) for small values of the $p_T^{jet}$, i.e., for small values of the energy of such jets seen in Fig. 2. This fact is connected with the limitation of the available phase space in this case. Actually, maximal values for the ratios $p_T^{rel}/p_T^{jet}$ for data in Fig. 2 are in the range $0.09 - 0.15$ and in Fig. 3 in the range $0.006 - 0.09$. The nonextensivity parameter drops below unity for distributions with $p_T^{rel}/p_T^{jet} > 0.12$.

3. Multiplicity distributions within jets

From our experience with applications of Tsallis statistics to multiparticle production processes, we know\cite{14} that multiplicity distribution of particles energy spectra of which follow Tsallis distribution has Negative Binomial form (NBD)\cite{3}.

$$P(N) = \frac{\Gamma(N + k)}{\Gamma(N + 1) \Gamma(k)} \left( \frac{\langle N \rangle}{k} \right)^N (1 + \frac{\langle N \rangle}{k})^{-k-N}, \quad (6)$$

with

$$\frac{1}{k} = \frac{\text{Var}(N)}{\langle N \rangle^2} - \frac{1}{\langle N \rangle} = q_N - 1. \quad (7)$$

Whereas for NBD $q > 1$ and parameter $k$ in (7) is positive, for the $q < 1$ case $k$ becomes negative ($k \rightarrow -k$) and NBD becomes a binomial distribution (BD),

$$P(N) = \frac{\Gamma(k + 1)}{\Gamma(N + 1) \Gamma(k - N + 1)} \left( \frac{\langle N \rangle}{k} \right)^N (1 - \frac{\langle N \rangle}{k})^{k-N}, \quad (8)$$

and

$$\frac{1}{k} = 1 - q_N. \quad (9)$$

Cf., also\cite{15} where similar results were obtained from apparently different point of views. In fact there is a parameter equivalent to $q$ and a resulting distribution can be written in Tsallis form.
For both the NBD and BD we expect the following to hold:

\[
\frac{(N + 1)P(N + 1)}{P(N)} = a + bN
\]  

(10)

with

\[
a = \frac{\langle N \rangle}{k + \langle N \rangle}, \quad b = \frac{a}{k} \quad \text{for NBD,} \\
a = \langle N \rangle, \quad b = 0 \quad \text{for Poisson,} \\
a = \frac{\langle N \rangle k}{k - \langle N \rangle}, \quad b = \frac{a}{k} \quad \text{for BD},
\]

(11) (12) (13)

From data on multiplicity distributions, \( P(N) \), measured in jets \([12]\) (for \( p_T^{\text{jet}} \leq 40 \text{ GeV} \) only) one can check the behavior of Eq. (10). As can be seen from in Fig. 3 this relation is linear, i.e., the corresponding \( P(N) \) are indeed of NBD or BD type (the deviation from linearity occurs only for \( N = 1 \), for which one encounters experimental difficulties and which, in fact, can be omitted from our analysis). From parameters \( a \) and \( b \) obtained this way we can deduce, using Eqs. (11) - (13), values of \( \langle N \rangle \), \( \text{Var}(N) \) and \( k \) or \( \kappa \) (i.e., values of the corresponding nonextensivity parameter \( q_N \)) which are presented in Table 3. Notice that their values correspond closely to those obtained from the distributions of \( p_T \) in jets presented in Table 1.

### 4. Self-similarity property of the multiparticle production processes

The values of nonextensivity parameters obtained from an analysis of multiplicity distributions and distributions of \( p_T \) of jets and in jets can now be compared with the respective nonextensivity parameters obtained in measurements of \( p_T \) distributions in other experiments on minimum bias \( pp \) collisions in which the range of \( p_T \) and multiplicities were similar and energies of which were similar to energies of the jets investigated. The corresponding results for the dependence of the resulting nonextensive parameters \( q \) as a function of the measured mean multiplicity \( \langle N \rangle \) are presented in Fig. 5. The approximate similarity of these results is clearly visible. \footnote{A word of comment on Fig. 5 is in order here. So far we were estimating the parameter \( q \) from distributions of \( p_T \) or \( N \) and discussing its energy dependence, \( q(s) \), as obtained from different experiments.}

| \( p_T^{\text{jet}} \) [GeV] | \( \langle N \rangle \) | \text{Var}(N) | \( q_N - 1 \) |
|------------------------|-----------------|--------------|----------------|
| 4 - 6                  | 4.41            | 2.31         | -0.11          |
| 6 - 10                 | 5.72            | 3.83         | -0.058         |
| 10 - 15                | 7.11            | 6.61         | -0.0098        |
| 15 - 24                | 7.56            | 11.2         | 0.063          |
| 24 - 40                | 7.80            | 18.1         | 0.097          |

### Table 2: Fit parameters for Fig. 3; \( q = 1 + 1/n \).

| \( p_T^{\text{jet}} \) [GeV] | \( T \) [GeV] | \( n \) | \( q \) |
|------------------------|--------------|--------|-------|
| 25 - 40                | 0.25         | 70     | 1.014 |
| 40 - 60                | 0.25         | 25     | 1.040 |
| 60 - 80                | 0.25         | 18     | 1.056 |
| 80 - 110               | 0.25         | 15     | 1.067 |
| 110 - 160              | 0.25         | 12     | 1.083 |
| 160 - 210              | 0.25         | 10     | 1.100 |
| 210 - 260              | 0.25         | 9      | 1.111 |
| 260 - 310              | 0.25         | 9      | 1.111 |
| 310 - 400              | 0.25         | 9      | 1.111 |
| 400 - 500              | 0.25         | 7.5    | 1.133 |

### Table 3: \( P(N) \) characteristics for jets with different \( p_T^{\text{jet}} \).
The results presented here can be summarized in the following way: (i) A Tsallis distribution successfully describes inclusive $p_T$ distributions in a wide range of transverse momenta for all energies measured so far [5, 6, 7]. (ii) This is also true for the distribution of transverse momenta of jets as shown in Fig. 1. The nonextensivity parameter in this case, $q = 1.14$, is comparable to $q = 1.15$ describing inclusive distributions of transverse momenta of particles at the same energy 7 TeV [7]. (iii) The Tsallis distribution also describes transverse momentum distributions of particles in jets. The values of $q$ obtained in this case are roughly the same as those obtained from an analysis of multiplicity distributions in these jets. It should be noted that, as seen in Fig. 5, values of the nonextensivity parameter $q$ for particles in jets correspond rather closely to values of $q$ obtained from the inclusive distributions measured in $pp$ collisions (for the corresponding energies available for production) in the similar ranges of transverse momenta.

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18. Here we would like to compare distributions of particles in $p + p$ collisions to those in jets, for which, unfortunately, we do not know the corresponding energy $\sqrt{s}$. On the other hand, we know $\langle N \rangle$ both for $p + p$ collisions and for particles produced in jets, so it is reasonable instead to show $q$ as a function of $\langle N \rangle$.

5 Results discussed here could be regarded as related to the phenomenon of geometrical scaling for $p_T$ distributions discussed recently (cf. [24] and references therein), apparently being a consequence of gluon saturation at some scale $Q_s$. It turns out that scaled distributions can be described by a Tsallis formula [25] with the saturation scale being hidden in the parameter $T$ (not $q$); in fact to get scaling one has to allow for $T$ being dependent on $p_T$. One should, however, be aware of the fact that in the energy domain discussed here scaling seems to be violated [26].
a statistical equilibrium of an undetermined number of all kinds of fireballs, each of which in turn is considered to be a fireball. In fact this was used as a justification in the first proposed generalization of the Hagedorn model, considered as a statistical model, to $q$-statistics, cf., [22]. In the pure dynamical QCD approach to hadronization, one encounters the same idea, as, for example, that presented in [23]. In it partons fragment into final state hadrons through multiple sub-jet production. As a result one has a self-similar behavior of cascade of jets to sub-jets to sub-sub-jets... to final state hadrons.

5. Summary

Using the Tsallis nonextensive approach, we have analysed recent data found by the LHC ATLAS experiment [12, 13] on transverse momentum distributions of jets, particles within jets and their multiplicity distributions. The values of the respective nonextensivity parameters obtained this way, when compared with the corresponding values obtained from the inclusive distributions measured in $pp$ collisions for the corresponding energies available for production and in similar ranges of transverse momenta, were found to be similar. This can be considered as strong evidence of the existence of some common mechanism behind all these processes which we tentatively identify with a self-similarity property and cascade type processes based on multiplicative noise [20]. They are known to lead to a Tsallis distribution (with $n - 2 = \langle \eta \rangle/\text{Var}(\eta)$ given by fluctuations of multiplicative noise $\eta$ [20]) of the same type as those describing statistical or thermodynamical systems (with $q - 1 = \text{Var}(T)/\langle T \rangle$ given by fluctuations of temperature $T$ [4, 5, 6]).

It is worth reminding at this point that both Tsallis distribution and the Negative Binomial Distribution can be regarded as a consequence of using a gamma distribution for clusters formed before fragmentation. Whereas the former arises from the fluctuations of temperature in a Boltzman-Gibbs distribution, the latter arises from the fluctuations of mean multiplicity in a Poissonian distribution. The common feature is that in both cases fluctuations are given by a gamma distribution which is stable under the size distribution, i.e., exhibits self-similarity and scaling behavior (actually, NBD is also a self-similar distribution [27]). This indicates once more that self-similarity encountered in processes under consideration is the physical ground of the observed similarities discusses here. Results presented here could possibly open discussion about the validity of thermal models [28].

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