A Quantum Optomechanical Transistor Based on a Cavity-Optomechanical System

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Abstract

We theoretically propose a scheme to realize an all-optically controlled quantum optomechanical transistor based on a cavity-optomechanical system, where the cavity photons interfere with the input signal photons while the pump field controls the transmission spectrum of the signal laser. Theoretical analysis shows such a quantum optomechanical transistor can be switched on or off by turning on or off the pump laser, which corresponds to amplification or attenuation of the signal laser, respectively. The results further demonstrate that the output signal gain is enhanced abruptly with increasing the input pump power. The scheme proposed here will pave the way towards many important applications such as all-optical logic circuits and quantum repeaters.

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I. INTRODUCTION

In the pursuit of improved platforms for global information exchange and telecommunications, electrons can no longer serve as a signal carriers due to the heat effect, restricted transmission rate, energy loss, and the coherence between them [1]. In this case, the continued increase in computing and communications highlights the need for new devices that reduce the effects of this electronic bottleneck by operating entirely in the optical domain [2]. Photons are the best choice to substitute electrons as a signal carrier, which have robustness against decoherence and have a high transmission efficiency. One of the critical devices based on photons is the all-optical transistor-a device where a small optical ‘gate’ field is used to control the propagation of another optical ‘signal’ field via a specific process [3, 4] such as electromagnetically induced transparency (EIT) [5, 6], optical Kerr effect [7], and cascaded second-order nonlinearity [8]. Recently, Hwang et al have demonstrated that a single dye molecule can be operated as an optical transistor and coherently attenuate or amplify a tightly focused laser beam [10].

Several materials and systems have been used to develop and demonstrate optical transistor including photonic crystal [11, 12], atomic gas [13], among others [14, 15]. However, these systems tend to be experimentally complex and most do not lend themselves to miniaturization. To this end, there has been a notable effort to create all-optical transistor using conveniently controlled manufacturing and miniaturization system. With this backdrop, in the present letter, we theoretically propose an all-optically controlled quantum optical transistor with an optomechanical system consisting of a Fabry-Perot cavity and a mechanical resonator, which we refer it as quantum optomechanical transistor. Recently, Kippenberg and his research group [16] have experimentally demonstrated that optomechanically induced transparency (OMIT) is equivalent to EIT in a cavity optomechanical system operated in the resolved sideband regime. Attracted by unique features of the optomechanical system, such as on-chip integration, large bandwidth of operation and long delay times, many theoretical and experimental researchers are focusing on its optical effects including the slow light effect and fast light effect [17, 18], normal splitting [19] and other effects [20, 21]. Here we demonstrate that such an optomechanical system can indeed be served as a quantum optical transistor, where the cavity photons interfere with the input signal photons while the pump field controls the transmission of the signal laser. This quantum optomechanical...
transistor has the obvious merits that it can be switched on or off just by turning the pump beam on or off, which corresponds to amplification or attenuation of transmitted signal laser, respectively. The scheme proposed here will pave the way towards many important applications such as quantum communication and quantum repeaters.

II. THEORY

In what follows, we consider the canonical situation in which a driven high-finesse optical cavity is coupled by momentum transfer of the cavity photons to a micro-mechanical resonator. The physical realization is shown in Fig.1(a), which a toroidal nanocavity is coupled to the mechanical radial breathing mode of the structure via radiation pressure. This toroidal nanocavity system can be modeled as a typical optomechanical system consisting of a Fabry-Perot cavity and a mechanical resonator as shown in Fig.1 (b) [16]. The mechanical mode can be treated as phonon mode. The cavity is driven by a strong pump field and a weak signal field with frequencies of \( \omega_p \) and \( \omega_s \), respectively. In the adiabatic limit, the input laser drives only one cavity mode \( \omega_c \), and the cavity free spectrum range \( L/2c \) ( \( c \) is the speed of light in the vacuum and \( L \) is the effective cavity length [22, 23]) is much larger than the frequency of mechanical mode(\( \omega_m \)). Hence, we can ignore the scattering of photons to other cavity modes.

In a rotating frame at a pump field frequency \( \omega_p \), the Hamiltonian of the system can be written as [24, 25]

\[
H = \hbar \Delta_p b^+ b + \frac{1}{2} \hbar \omega_m (P^2 + Q^2) + \hbar G_0 b^+ b Q + i \hbar E_p (b^+ - b) + i \hbar E_s (b^+ e^{-i\delta t} - b e^{i\delta t}),
\]

where \( \Delta_p = \omega_c - \omega_p \) is the detuning between cavity frequency and pump frequency, \( \delta = \omega_s - \omega_p \) is the frequency detuning of the signal and pump field. \( b \) and \( b^+ \) denote the annihilation and creation operators with commutation relation \([b, b^+] = 1\). \( Q \) and \( P \) with commutation relation \([Q, P] = i\) are dimensionless position and momentum operators of mechanical mode, respectively. The parameter \( G_0 = (\omega_c/L\sqrt{\hbar/m\omega_m}) \) is the coupling rate between the cavity and the oscillator \((m \) is the effective mass of the mechanical mode [26]). \( E_p \) and \( E_s \) are slowly varying envelope of the pump field and signal field, respectively, which are related to the laser power \( \mathcal{P} \) by \(|E_p| = \sqrt{2\mathcal{P}_p\kappa/\hbar\omega_c} \) and \(|E_s| = \sqrt{2\mathcal{P}_s\kappa/\hbar\omega_s} \) (\( \kappa \) is the cavity amplitude decay rate).
The temporal evolution of the lowering operator $b$ and the dimensionless position operator $Q$ is determined by the Heisenberg equation of motion $i\hbar(dO/dt) = [O, H]$ by adding the damping terms, thus the semiclassical equations are given by

\[
\frac{d}{dt}\langle b \rangle = -(i\Delta_p + \kappa)\langle b \rangle + iG_0\langle b \rangle \langle Q + E_p + E_s e^{-i\delta t} \rangle, \tag{2}
\]

\[
\frac{d^2}{dt^2}\langle Q \rangle + \gamma_m \frac{d}{dt}\langle Q \rangle + \omega_m^2\langle Q = \omega_m G_0\langle b^+ \rangle \langle b \rangle \rangle, \tag{3}
\]

where $\gamma_m$ is the damping rate of mechanical mode. In order to solve these equations, we make the ansatz [27]:

\[
b(t) = b_0 + b_+ e^{-i\delta t} + b_- e^{i\delta t}, \quad Q(t) = Q_0 + Q_+ e^{-i\delta t} + Q_- e^{i\delta t}. \tag{4}
\]

In the steady state we obtain [18]

\[
b_+ = \frac{E_s}{f(\delta)} [(\kappa - i\delta - i\Delta_p)\omega_m^2 + iG_0^2w_0(\eta + 1)],
\]

where $f(\delta) = (\kappa - i\delta)^2\omega_m^2 + [\omega_m\Delta_p - G_0^2w_0(\eta + 1)]^2 - G_0^4\eta^2w_0^2$ and $\eta(\delta) = (\omega_m^2)/(\omega_m^2 - i\gamma_m\delta - \delta^2)$. The parameter $w_0(= |b_0|^2)$ is determined by the equation

\[
w_0[\kappa^2 + (\Delta_p - \frac{G_0^2}{\omega_m}w_0)^2] = E_p^2. \tag{5}
\]

Using the standard input-output relation $b_{out}(t) + b_{in}(t) = \sqrt{2}\kappa b(t)$ [28], where $b_{out}(t)$ and $b_{in}(t)$ are the output and input operators, respectively, we obtain $\langle b_{out}(t) \rangle = b_{out0} + b_{out+} e^{-i\delta t} + b_{out-} e^{i\delta t} = \sqrt{2}\kappa(b_0 + b_+ e^{-i\delta t} + b_- e^{i\delta t})$. Finally we can get the relationship of $b_{out+} = \sqrt{2}\kappa b_+$, which corresponds to the linear optical susceptibility [16, 18].

### III. RESULTS AND DISCUSSIONS

For illustration of the numerical results, we choose the realistic optomechanical system [29]. The parameters used in calculation are $(G_0, \omega_m, \kappa, \gamma_m) = (0.9, 10, 2\pi \times 0.215, 2\pi \times 0.14) MHz$. Fig.2 shows the energy levels of cavity photons while dressing with mechanical vibrations (phonon modes).

In the following, we present the physical condition of quantum optomechanical transistor. Fig.1(c) shows the transmission spectrum of the signal field under the strong pump field. The top curve in Fig.1 (c) displays the transmission spectrum when we shelve the pump beam in this transistor but only applying a signal beam ($\Delta_s = \omega_s - \omega_c$). This plot shows that in the absence of the pump beam the system attenuates the weak signal beam totally. This dip arises from the usual cavity absorption resonance. However, as the pump beam
turns on, and fixing the pump beam detuning $\Delta_p = -\omega_m = -10MHz$, the dip becomes a peak immediately (see the bottom curve in Fig.1 (c)). As the pump power increases even further, we can observe more amplification of the signal beam as shown in Fig.3. Fig.3(a) is the transmission spectra of the signal beam as a function of signal-cavity detuning, which indicates that when the input pump power increases, the signal transmission increases rapidly. This result agrees well with the recent experiment in circuit micro-cavity electromechanics [30]. This pump beam, just like a switch, dramatically controls the transmission spectrum of the signal beam. These plots in Fig.3(a) demonstrate that an optomechanical system can indeed act as a quantum optical transistor. This amplification behavior is caused by quantum interference between the dressed states while applying two optical fields. Fig.3(b) shows the origin of this three-photon resonance physical processing. Here the cavity photons makes a transition from the energy level $|n_p, n_m\rangle$ to the dressed level $|n_p + 1, n_m + 1\rangle$ by the simultaneous absorption of two pump photons and emission of a signal photon at $\Delta_s = 0$, as indicated by the region of amplification of the signal beam in Fig.3(a). In this case, optomechanics based optical transistor is an all-optical process, which is very different from atom and molecule optical transistor. Because of the all-optical physical situation, this optomechanical transistor can act as an optical repeater, which can provide enough energy for the transmitted signal laser while the light loses its energy during the transmission and makes the signal light amplified. For more specific description, we further investigate the transistor characteristic curve by plotting the amplification of the signal beam as a function of the input pump power as shown in Fig.3 (c). In this case, the gain of the signal beam can be enhanced abruptly by increasing the input pump power, which is the typical behavior of the optical transistor. Therefore this cavity optomechanical system can be referred as a quantum optomechanical transistor.

IV. CONCLUSIONS

In summary, we have theoretically implemented an all-optically controlled quantum optomechanical transistor with a toroidal nanocavity-optomechanical system. Theoretical analysis has shown that the transmission of the weak signal beam can be controlled conveniently by the pump field power and the gain of the output signal laser is enhanced abruptly by increasing the input pump power. The scheme proposed here will provide an important
clue towards the realization of all-optical devices and network communications. Finally, we hope that our proposed scheme in the present work will be testified by experiments in the near future.

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Figure Captions

Fig.1 Schematic diagram of a toroidal nanocavity-optomechanical system. (a) The input signal laser comes from the “in” port of optical fiber while the pump field exists in the nanocavity previously. Otherwise, the output signal laser can be detected at the “out” port of optical fiber. The nanocavity-resonator is coupled to the pump and signal fields using an optical fiber. (b) The typical cavity-optomechanical system with a fixed mirror and a movable mirror. (c) Attenuation (pump off) and amplification (pump on) of the signal beam in the case $\Delta_p = -10MHz$, $G_0 = 0.9MHz$, $\omega_m = 10MHz$, $\kappa = 2\pi \times 215KHz$, $\gamma_m = 2\pi \times 140KHz$. $P_p$ is the pump laser power.

Fig.2 The energy levels of nanocavity photons while dressing with the mechanical modes (phonon modes) (a) The initial energy levels of the cavity photons and mechanical vibration of nanocavity. $G_0$ is the coupling between the cavity photon and mechanical vibration of nanocavity. The vibration modes of nanocavity are treated as phonon modes. (b) The split energy levels of the cavity photons when dressing the mechanical vibration mode. $|n_p\rangle$ and $|n_m\rangle$ denote the number states of the photon and phonon, respectively.

Fig.3 (a) The transmission of the signal beam with the same parameters as Fig.1(c). The transmission of the signal laser increases as the power of pump beam increases. (b) The energy levels and transitions during the signal amplification. (c) The characteristic curve of quantum optomechanical transistor by plotting the output signal gain and the input pump power.
FIG. 1:
FIG. 2:
FIG. 3: