Angular Distribution of Charming $B \rightarrow VV$ Decays and Time Evolution Effects

Cheng-Wei Chiang*

Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213

Abstract

Angular distributions of a $B$ meson decaying into two vector mesons are discussed with emphasis on time evolution effects on the complete set of amplitude bilinears. Time integrated quantities are suggested to observe substantial CP violation in decays with charm quarks in the final state particles. Relations among the nine observables at $t = 0$ are found to be useful for a consistency check of experimentally extracted quantities. Numerical estimates of the nine observables are made using form factor models and the assumption of the factorization hypothesis. Branching ratio asymmetries for $B^+_u \rightarrow D^{*+}\bar{D}^{*0}$ and $B_d \rightarrow D^{*+}D^{*-}$ can be as large as $-3\%$ and $-4\%$, respectively.

*E-mail: chengwei@andrew.cmu.edu
I. INTRODUCTION

The decays of the $B$ meson into two vector mesons $B \rightarrow V_1 + V_2$, either with charm quarks in the final state particles, such as $B \rightarrow J/\Psi \rho$, or with particles without charm quarks, such as $B \rightarrow \rho K^*$, have been calculated in many models [1–9]. The time evolution effects in neutral $B$ meson decays are also discussed in [2,7]. In this work, we would like to extend the general discussion on time evolving observables and to emphasize the charming decays in numerical analysis.

One major advantage of analyzing $B \rightarrow VV$ decays is that the interference of CP-even and CP-odd final states appear in the angular distributions. These interference terms provide good opportunities to observe CP or T violating effects. Since it is possible to measure all nine observables in certain decays [11], the physically interesting quantities such as $\beta$ and $\eta$ can be determined from experiments given sufficient statistics. In addition, relations among the nine observables provide a consistency check for the amplitude bilinears obtained experimentally.

The decay amplitude involves the hadronic matrix element of a $B$ meson decaying to two vector mesons through a weak current, which at present can not be calculated from first principles. Thus, our numerical evaluation of these observables at $t = 0$ is based upon the assumptions of factorization and no final state interactions and form factor models, where the updated Wilson coefficients [11] are used.

This paper is organized as follows: In Section II, we review the observables in the angular distributions of $B \rightarrow VV$ decays. In Section III, we derive the time-dependent formulas for the observables and list the complete results in the Appendix. Section IV considers the situation of no time evolution or at $t = 0$. The case with no strong phases is discussed in Section V. Results of single weak amplitude decays are presented in Section VI, wherein CP asymmetries are also extensively discussed. In Section VII, we present the numerical estimation of the nine observables. We summarize this paper in Section VIII.
II. OBSERVBABLES AND ANGULAR DISTRIBUTIONS IN $B \to VV$ DECAYS

To extract the $CP$-odd and $CP$-even or $T$-odd and $T$-even components more easily, the angular distribution is often written in the transversity basis. Let us define the amplitude of $B \to V_1 V_2$ in the rest frame of $V_1$. According to their polarization combinations, the amplitude can be decomposed into 

$$A(B \to V_1 V_2) = A_0 \epsilon_{V_1}^* \epsilon_{V_2}^* - \frac{A_\parallel}{\sqrt{2}} \epsilon_{V_1}^T \cdot \epsilon_{V_2}^T - i \frac{A_\perp}{\sqrt{2}} \epsilon_{V_1}^* \times \epsilon_{V_2}^* \cdot \hat{p},$$

(1)

and similarly for $\bar{B} \to \bar{V}_1 \bar{V}_2$. In Eq. (1), $\epsilon_{V_1}$ and $\epsilon_{V_2}$ are the unit polarization vectors of $V_1$ and $V_2$, respectively. $\hat{p}$ is the unit vector along the direction of motion of $V_2$ in the rest frame of $V_1$, $\epsilon_{V_1}^* \equiv \epsilon_{V_1} \cdot \hat{p}$ and $\epsilon_{V_1}^T = \epsilon_{V_1}^* - \epsilon_{V_1}^L \hat{p}$. It is easy to see that $A_\perp$ is odd under the parity transformation because of the appearance of $\epsilon_{V_1}^* \times \epsilon_{V_2}^* \cdot \hat{p}$, whereas $A_0$ and $A_\parallel$ are even.

The nine observables in the squared amplitude $A^* A$ are [10]

$$K_1(t) = |A_0(t)|^2, \quad K_4(t) = Re \left[ A_0^*(t) A_\parallel(t) \right], \quad L_4(t) = Im \left[ A_0^*(t) A_\parallel(t) \right],$$

$$K_2(t) = |A_\parallel(t)|^2, \quad K_5(t) = Im \left[ A_0^*(t) A_\perp(t) \right], \quad L_5(t) = Re \left[ A_0^*(t) A_\perp(t) \right],$$

$$K_3(t) = |A_\perp(t)|^2, \quad K_6(t) = Im \left[ A_0^*(t) A_\perp(t) \right], \quad L_6(t) = Re \left[ A_0^*(t) A_\perp(t) \right].$$

(2)

So we have

$$A^*(t) A(t) = K_1(t) X_1(\Omega) + K_2(t) X_2(\Omega) + K_3(t) X_3(\Omega) + K_4(t) X_4(\Omega) + L_5(t) X_5(\Omega) + L_6(t) X_6(\Omega) + L_4(t) Y_4(\Omega) + K_5(t) Y_5(\Omega) + K_6(t) Y_6(\Omega),$$

where the quantities $X_i(\Omega)$ and $Y_i(\Omega)$ represent polarizations or polarization correlations of the final vector mesons and $\Omega$ stands for the angles of the outgoing particles.

In general, the angular distribution of the decay in the transversity basis can be written as

$$\frac{d^3 \Gamma(t)}{d \cos \theta_1 d \cos \theta_2 d \phi} = \sum_i K_i(t) f_i(\theta_1, \theta_2, \phi),$$

(3)
where $K_i$'s are the amplitude bilinears that contain the dynamics and generally evolve with time, and $f_i(\theta_1, \theta_2, \phi)$ are the corresponding angular distribution functions.

One can classify the decays into three types of processes according to the properties of the final product particles as follows:

_Type I:_ For the case in which the decays of $V_1$ and $V_2$ are both into two pseudoscalar mesons, one can immediately translate the tensor correlations into angular distributions \[10\]. The normalized angular distribution of the decays $B \to V_1(\to P_1 P'_1) V_2(\to P_2 P'_2)$, where $P^{(i)}_1$ and $P^{(i)}_2$ denote pseudoscalar mesons, is:

$$
\frac{1}{\Gamma_0} \frac{d^3\Gamma(t)}{d \cos \theta_1 d \cos \theta_2 d \phi} = \frac{9}{8\pi} \left\{ \frac{K_1(t)}{\Gamma_0} \cos^2 \theta_1 \cos^2 \theta_2 + \frac{K_2(t)}{2\Gamma_0} \sin^2 \theta_1 \sin^2 \theta_2 \cos^2 \phi \\
+ \frac{K_3(t)}{2\Gamma_0} \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \phi + \frac{K_4(t)}{2\sqrt{2}\Gamma_0} \sin 2\theta_1 \sin 2\theta_2 \cos \phi \\
- \frac{K_5(t)}{2\sqrt{2}\Gamma_0} \sin 2\theta_1 \sin 2\theta_2 \sin \phi - \frac{K_6(t)}{2\Gamma_0} \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi \right\} (4)
$$

Here $\theta_1$ ($\theta_2$) is the angle between the $P_1$ ($P_2$) three-momentum vector in the $V_1$($V_2$) rest frame and the $V_1$($V_2$) three-momentum vector defined in the $B$ rest frame, and $\phi$ is the angle between the normals to the planes defined by $P_1 P'_1$ and $P_2 P'_2$, in the $B$ rest frame. Examples of such decays are $B^+ \to D^*(\to D^0\pi^0)\rho^+(\to \pi^+\pi^0)$, $B_d \to D^{*-}(\to \bar{D}^0\pi^-)\rho^+(\to \pi^+\pi^0)$, and $B_d \to D^{*-}(\to \bar{D}^0\pi^-)D^{*+}(\to D^0\pi^+)$. 

_Type II:_ For the case of the decay $B \to V_1(\to P_1 P'_1) V_2(\to l^+ l^-)$, suppose we observe that $l^-$ is a right-handed particle and comes out in the direction $\vec{k}_2 = (\sin \theta_2 \cos \phi, \sin \theta_2 \sin \phi, \cos \theta_2)$ and the momentum of $P_1$, $\vec{k}_1 = (\sin \theta_1, 0, \cos \theta_1)$ with angles defined in the same fashion as in the previous type, we have instead the differential angular distribution \[10\]:

$$
\frac{1}{\Gamma_0} \frac{d^3\Gamma}{d \cos \theta_1 d \cos \theta_2 d \phi} = \frac{9}{16\pi \Gamma_0} \left\{ K_1 \cos^2 \theta_1 \sin^2 \theta_2 + \frac{K_2}{2} \left( \sin^2 \theta_1 \cos^2 \theta_2 \cos^2 \phi + \sin^2 \theta_1 \sin^2 \phi \right) \\
+ \frac{K_3}{2} \left( \sin^2 \theta_1 \cos^2 \theta_2 \sin^2 \phi + \sin^2 \theta_1 \cos^2 \phi \right) + \frac{K_4}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos \phi \\
- \frac{K_5}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin \phi - \frac{K_6}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi \\
+ \frac{L_4}{\sqrt{2}} \sin 2\theta_1 \sin \theta_2 \sin \phi - \frac{L_5}{\sqrt{2}} \sin 2\theta_1 \sin \theta_2 \cos \phi + \frac{L_6}{2} \sin^2 \theta_1 \cos \theta_2 \right\} (5)
$$
To obtain the result for the other possible final state with a left-handed outgoing $l^-$, one only needs to flip the signs of $L_4$, $L_5$, and $L_6$. The muon polarization is equal to the sum of the terms $L_4$, $L_5$, $L_6$ divided by the sum of the other 6 terms. For the case of $L_6$ it is seen that the polarization does not vanish after integrating over $\theta_1$ and $\phi$ and so the observation can be made without observing the $V_1$ decay. Such decay modes are $B^+_u \to J/\Psi(\to l^+l^-)K^{*+}(\to \pi^0 K^+$), $B^+_u \to J/\Psi(\to l^+l^-)\rho^+(\to \pi^+\pi^0)$, $B_d \to J/\Psi(\to l^+l^-)K^*(\to \pi K)$, $B_d \to J/\Psi(\to l^+l^-)\rho^0(\to \pi\pi)$, $B_d \to J/\Psi(\to l^+l^-)\omega(\to \pi^+\pi^-\pi^0)$, $B_s \to J/\Psi(\to l^+l^-)K^*(\to \pi K)$, and $B_d \to J/\Psi(\to l^+l^-)\phi(\to K^+ K^-)$. Although $\omega$ decays into three pions, they are still correlated so that one can pick the normal direction to the decay plane formed by the three pions in the $\omega$ rest frame to define the direction $\theta_2$ and $\phi$.

Although the $B \to V(\to PP)V(\to P\gamma)$ modes have a different decay pattern from that of $B \to V(\to PP)V(\to l^+l^-)$, they share the same differential angular distribution, with the direction of $l^-$ in the latter case replaced by that of $\gamma$. For instance, for the decay with a right-handed circularly polarized photon in the final state, the angular distribution is the same as Eq. (5). Such examples are $B^+_u \to D^{*+}(\to D^{+\gamma})\bar{D}^{*0}(\to \bar{D}^{0}\pi)$, $B_d \to D^{*+}(\to D^{+\gamma})D^{*-}(\to \bar{D}^{0}\pi)$, $B_s \to D^{*+}(\to D^{-\gamma})D^{*+}(\to D^{0}\pi)$. If one does not measure the polarization of the product particles, the angular distribution would be the one by doubling Eq. (5) and eliminating the $L_{4,5,6}$ terms.

Type III: Next we consider the decay $B \to V(\to P\gamma)V(\to P\gamma)$. Since it is experimentally impractical to measure the polarizations of both photons in the final state, we just give here the differential angular distribution with no polarization measured:

$$
\frac{1}{\Gamma_0} \frac{d^3\Gamma}{d \cos \theta_1 d \cos \theta_2 d \phi} = \frac{9}{8\pi\Gamma_0} \Bigg\{ K_1 \sin^2 \theta_1 \sin^2 \theta_2 \\
+ \frac{K_2}{2} (\cos^2 \theta_1 \cos^2 \theta_2 \cos^2 \phi + \cos^2 \theta_1 \sin^2 \phi + \cos^2 \theta_2 \sin^2 \phi + \cos^2 \phi) \\
+ \frac{K_3}{2} (\cos^2 \theta_1 \cos^2 \theta_2 \sin^2 \phi + \cos^2 \theta_1 \cos^2 \phi + \cos^2 \theta_2 \sin^2 \phi + \sin^2 \phi) \\
- \frac{K_4}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos \phi + \frac{K_5}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin \phi \\
+ \frac{K_6}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi \Bigg\} 
$$

(6)
III. TIME EVOLUTION OF THE AMPLITUDE BILINEARS

The time evolution of an arbitrary neutral $B$ meson state $a|B^0(t)⟩ + b|\bar{B}^0(t)⟩$ is governed by the Schrödinger equation

$$i \frac{d}{dt} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \mathcal{H} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}. \quad (7)$$

If we write the mass eigenstates, $|B_{L,H}\rangle$, with eigenvalues $m_{L,H} - \frac{i}{2} \Gamma_{L,H}$ in terms of $|B^0\rangle$ and $|\bar{B}^0\rangle$ as

$$|B_{L,H}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle, \quad (8)$$

then the time evolutions of $B^0$ and $\bar{B}^0$ are

$$|B^0(t)⟩ = g_+(t)|B^0(0)⟩ + \frac{q}{p} g_-(t)|\bar{B}^0(t)⟩,$$

$$|\bar{B}^0(t)⟩ = \frac{p}{q} g_-(t)|B^0(0)⟩ + g_+(t)|\bar{B}^0(t)⟩, \quad (9)$$

where

$$g_{\pm}(t) = \frac{1}{2} \left( e^{-im_{L,H} t} e^{-\frac{1}{2} \Gamma_{L,H} t} \pm e^{-im_{L,H} t} e^{-\frac{1}{2} \Gamma_{H,H} t} \right). \quad (10)$$

Suppose $|f_{\eta}\rangle$ is a state with definite CP property, namely, $CP|f_{\eta}\rangle = \eta_i|f_{\eta}\rangle$ for $i = 1, 2, 3$ and $\eta = 0, ||, \perp$, respectively. The CP eigenvalues $\eta_1 = \eta_2 = +1$ and $\eta_3 = -1$. Suppose we write the decay matrix element of $B^0$ decaying into the final states $f_\eta$ at time $t = 0$ as

$$A_\eta(0) \equiv \langle f_\eta | B^0(0) \rangle = Y_{CKM}^T e^{i\theta_\eta} (T_\eta + P_\eta e^{i\phi_w} e^{i\delta_\eta}). \quad (11)$$

$Y_{CKM}^T$ is the overall CKM factors appearing in the amplitudes. $\theta_\eta$ are the factored strong phases of $A_\eta$, but only the relative phases are essential. Conventionally, we take $\theta_{\perp} = 0$. $T_\eta$ and $P_\eta$ are the absolute values of two types of amplitudes that differ by a relative weak phase $\phi_w$ and a relative strong phase $\delta_\eta$. We will refer to them by “tree” and “penguin” amplitudes, respectively. Similarly, for the CP conjugate mode we have

$$\bar{A}_\eta(0) \equiv \langle f_{\eta}^* | \bar{B}^0(0) \rangle = Y_{CKM}^T \ast e^{i\theta_{\eta}} (T_\eta + P_\eta e^{-i\phi_w} e^{i\delta_{\eta}}). \quad (12)$$
Here we may assume that $|f_\eta\rangle$ and $|\bar{f}_\eta\rangle$ are the same state that both $|B^0\rangle$ and $|\bar{B}^0\rangle$ can decay into (e.g., $B^0, \bar{B}^0 \to J/\Psi \phi, D^{*+} D^{*-}$). They can also be conjugate states so that only $|B^0\rangle$ (or $|B^+\rangle$) can decay into $|f_\eta\rangle$ and only $|\bar{B}^0\rangle$ (or $|B^-\rangle$) to $|\bar{f}_\eta\rangle$. According to the time evolution, the decay amplitude at time $t$ would be

$$A_\eta(t) = \langle f_\eta | B^0(t) \rangle = A_\eta(0) \left[ g_+ (t) + \eta \lambda_\eta g_- (t) \right],$$  

where

$$\lambda_\eta = \frac{q Y_{CKM}^T * T_\eta + P_\eta e^{-i\phi_w} e^{i\delta_\eta}}{p Y_{CKM}^T T_\eta + P_\eta e^{i\phi_w} e^{i\delta_\eta}}.$$  

It is convenient to define a phase $\phi$ by

$$e^{i\phi} \equiv \frac{q Y_{CKM}^T *}{p Y_{CKM}^T},$$  

and

$$R_\eta \equiv Re \left[ \frac{T_\eta + P_\eta e^{-i\phi_w} e^{i\delta_\eta}}{T_\eta + P_\eta e^{i\phi_w} e^{i\delta_\eta}} \right], \quad I_\eta \equiv Im \left[ \frac{T_\eta + P_\eta e^{-i\phi_w} e^{i\delta_\eta}}{T_\eta + P_\eta e^{i\phi_w} e^{i\delta_\eta}} \right].$$  

Note that $R_\eta^2 + I_\eta^2 = 1$ if and only if $\delta_\eta, \phi_w = 0$ (mod $\pi$). If either (i) no nontrivial relative weak phase (0 or $\pi$), (ii) negligible tree contributions ($T_\eta \simeq 0$), or (iii) negligible penguin contributions ($P_\eta \simeq 0$) happens, then $R_\eta = 1$ and $I_\eta = 0$, apart from a possible overall phase.

With the above definitions, one can get, for example, the time evolving $|A_\eta(t)|^2$ as follows:

$$|A_\eta(t)|^2 = |A_\eta(0)|^2 e^{-\Gamma t} \left\{ \frac{1 + R_\eta^2 + I_\eta^2}{2} \cos \left( \frac{\Delta \Gamma t}{2} \right) + \frac{1 - R_\eta^2 - I_\eta^2}{2} \cos (\Delta m t) \right. \right.$$

$$
+ \eta_i \left[ (R_\eta \cos \phi - I_\eta \sin \phi) \sinh \left( \frac{\Delta \Gamma t}{2} \right) - (R_\eta \sin \phi + I_\eta \cos \phi) \sin (\Delta m t) \right],  
\left. \right\},  

$$|\bar{A}_\eta(t)|^2 = |A_\eta(0)|^2 e^{-\Gamma t} \left\{ \frac{1 + R_\eta^2 + I_\eta^2}{2} \cos \left( \frac{\Delta \Gamma t}{2} \right) - \frac{1 - R_\eta^2 - I_\eta^2}{2} \cos (\Delta m t) \right. \right.$$

$$
+ \eta_i \left[ (R_\eta \cos \phi - I_\eta \sin \phi) \sinh \left( \frac{\Delta \Gamma t}{2} \right) + (R_\eta \sin \phi + I_\eta \cos \phi) \sin (\Delta m t) \right].  
\left. \right\}.$$
A complete list of all the observable amplitude bilinears and their CP conjugates is given in the Appendix.

Before we proceed the discussion, let’s define the CP asymmetry parameters,

\[ \zeta_i(t) \equiv K_i(t) - \bar{K}_i(t) \quad \text{for} \quad i = 1, 2, 3, \ldots, 6 \]

and

\[ \xi_i(t) \equiv L_i(t) - \bar{L}_i(t) \quad \text{for} \quad i = 4, 5, 6. \]

These nine parameters measure the changes of the amplitude bilinears under the CP transformation. For instance, from Eqs. (17) and (18), we obtain

\[
\zeta_1(t) = K_1(0)e^{-\Gamma t} \left[ (1 - R_0^2 - I_0^2) \cos(\Delta m t) - 2(R_0 \sin \phi + I_0 \cos \phi) \sin(\Delta m t) \right].
\]

This relation along with others for \( K_2,3(t) \) provide information on \( \phi \) given \( \Delta m \) and \( \Gamma \) extracted from other experiments and theoretical estimates of \( K_{1,2,3}(0), R_0, I_0 \), \( \parallel \), \( \perp \).

### IV. CASE I: NO TIME EVOLUTION

If we take \( t = 0 \) in Eqs. (A1)-(A5) and (A6)-(A10), we get the bilinear formulas for neutral \( B \) meson decays at time \( t = 0 \), or the charged \( B \) meson decays. The relations between the conjugate amplitude bilinears and amplitude bilinears are

\[
\bar{K}_i = \left( R_\eta^2 + I_\eta^2 \right) K_i, \quad \text{for} \quad i = 1, 2, 3,
\]

\[
\bar{K}_4 = \left( R_\parallel R_0 + I_\parallel I_0 \right) K_4 - \left( I_\parallel R_0 - R_\parallel I_0 \right) L_4,
\]

\[
\bar{K}_{5,6} = \left( R_\perp R_{0,\parallel} + I_\perp I_{0,\parallel} \right) K_{5,6} + \left( I_\perp R_{0,\parallel} - R_\perp I_{0,\parallel} \right) L_{5,6},
\]

\[
\bar{L}_4 = \left( R_\parallel R_0 + I_\parallel I_0 \right) L_4 + \left( I_\parallel R_0 - R_\parallel I_0 \right) K_4,
\]

\[
\bar{L}_{5,6} = \left( R_\perp R_{0,\parallel} + I_\perp I_{0,\parallel} \right) L_{5,6} - \left( I_\perp R_{0,\parallel} - R_\perp I_{0,\parallel} \right) K_{5,6}.
\]

As discussed in the paragraph after Eq. (16), if none of the relative strong and weak phases are trivial, \( i.e., \) 0 or \( \pi \), CP asymmetry exists in the above bilinears. However, if there are no strong phases (including all the factored strong phases and relative phases) but the relative weak phase is nontrivial, then one can simplify the above equations to get

\[ \bar{K}_{1,2,3,4} = K_{1,2,3,4}, \quad L_{5,6} = L_{5,6}, \quad \bar{K}_{5,6} = -K_{5,6}, \quad \text{and} \quad L_4 = -L_4. \]

This effect is purely due to
that fact that there is a relative weak phase and \(Im[A^*_0A_\parallel], Im[A^*_0A_\perp],\) and \(Im[A^*_\parallel A_\perp]\) are CP odd quantities [10].

The observation of CP asymmetries in any of the bilinears indicates that nontrivial strong and weak phases are involved in the decay. Therefore, if the relative weak phase within the Standard Model is trivial, that is, effectively only one weak amplitude dominates, then no CP asymmetry will be observed among all the bilinears.

The formulae presented in this section can be applied to \(B^+_u \rightarrow D^{*+} \bar{D}^*\), \(B^+_u \rightarrow J/\Psi \rho^+\), \(B^+_u \rightarrow D^* s \bar{D}^*\), and \(B^+_u \rightarrow J/\Psi K^{*+}\). One can only measure \(K_{1-6}\) in the first decay mode because it is a Type I decay. There is no nontrivial weak phases in the latter two decays. Therefore, one should not expect to observe CP asymmetries in the observables; but \(K_{5,6}\) and \(L_4\) may be nonzero, and provide evidence for strong phases due to final state interactions. The observation of CP asymmetries in such modes indicates new CP violating source from physics beyond the Standard Model.

V. CASE II: NO STRONG PHASES

If there is no strong phases involved in the decays, then \(R^2_{\eta} + I^2_{\eta} = 1\). One can write

\[
R_{\eta} = \cos 2\alpha_{\eta}, \quad I_{\eta} = -\sin 2\alpha_{\eta},
\]

where \(\alpha_{\eta}\) is the phase of \(T_{\eta} + P_{\eta} e^{i\phi}\). With Eq. (21) and the definition of the phase \(\phi\) in Eq. (15), one can get the nine CP asymmetry parameters

\[
\zeta_i(t) = 2\eta_{k} K_i(0) e^{-\Gamma_t t} \sin(2\alpha_{\eta} - \phi) \sin(\Delta m t), \text{ for } i = 1, 2, 3,
\]

\[
\zeta_4(t) = K_4(0) e^{-\Gamma_t t} \left[ \sin(2\alpha_0 - \phi) + \sin(2\alpha_0 - \phi) \right] \sin(\Delta m t)
\]

\[
- L_4(0) e^{-\Gamma_t t} \left[ \cos(2\alpha_0 - \phi) - \cos(2\alpha_0 - \phi) \right] \sin(\Delta m t),
\]

\[
\zeta_5(t) = K_5(0) e^{-\Gamma_t t} \left\{ \cosh \left( \frac{\Delta \Gamma t}{2} \right) + \cos(\Delta m t) 
\right. 
\]

\[
+ \left[ \cos(2\alpha_0 - \phi) - \cos(2\alpha_0 - \phi) \right] \sinh(\frac{\Delta \Gamma t}{2}) 
\]

\[
- \cos(2\alpha_0 - 2\alpha_0) \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) - \cos(\Delta m t) \right] \right\}
\]

\[ +L_5(0)e^{-\Gamma t}\left\{ \left[ \sin(2\alpha_0 - \phi) + \sin(2\alpha_\perp - \phi) \right] \sinh \left( \frac{\Delta \Gamma t}{2} \right) \\
- \sin(2\alpha_0 - 2\alpha_\perp) \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) - \cos(\Delta m t) \right] \right\}, \tag{24} \]

\[ \xi_4(t) = L_4(0)e^{-\Gamma t}\left\{ \cosh \left( \frac{\Delta \Gamma t}{2} \right) + \cos(\Delta m t) \\
+ \left[ \cos(2\alpha_0 - \phi) + \cos(2\alpha_\parallel - \phi) \right] \sinh \left( \frac{\Delta \Gamma t}{2} \right) \\
+ \cos(2\alpha_0 - 2\alpha_\parallel) \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) - \cos(\Delta m t) \right] \right\} \\
-K_4(0)e^{-\Gamma t}\left\{ \left[ \sin(2\alpha_\parallel - \phi) - \sin(2\alpha_0 - \phi) \right] \sinh \left( \frac{\Delta \Gamma t}{2} \right) \\
- \sin(2\alpha_\parallel - 2\alpha_0) \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) - \cos(\Delta m t) \right] \right\}, \tag{25} \]

\[ \xi_5(t) = L_5(0)e^{-\Gamma t} \left[ \sin(2\alpha_\perp - \phi) + \sin(2\alpha_0 - \phi) \right] \sin(\Delta m t) \\
+ K_5(0)e^{-\Gamma t} \left[ \cos(2\alpha_\perp - \phi) - \cos(2\alpha_0 - \phi) \right] \sin(\Delta m t). \tag{26} \]

The formulas for \( \zeta_6(t) \) and \( \xi_6(6) \) can be obtained by replacing “0” in Eq. (24) and (26) by “\( \parallel \)” . Thus, in principle, one may extract information about the phase combinations \( 2\alpha_{0,\parallel,\perp} - \phi \). One should notice that \( \xi_4 \) and \( \zeta_{5,6} \) can be nonzero at \( t = 0 \), whereas the others are identically zero. Although the assumption of no strong phases is unlikely to be true in charming decays, it may be applied to charmless decays such as \( B \to \rho \rho \).

**VI. CASE III: NO RELATIVE WEAK PHASE**

If there is no relative weak phase in each transversity amplitude, namely, \( \phi_w = 0 \), then one gets \( R_\eta = 1 \) and \( I_\eta = 0 \). This case is equivalent to the cases where only one type of amplitude dominates the decay process. For completeness, we list time evolutions of the nine observables in Tables I and II [1].

So the nine CP asymmetry parameters are

\[ ^1\text{While we agree with [2] in } K_{1-6} \text{ and } \bar{K}_{1-4}, \text{ our } \bar{K}_{5,6} \text{ differ from theirs by an overall minus sign.} \]
| Bilinear | Time evolution |
|----------|----------------|
| $K_i(t)$ | $K_i(0)e^{-\Gamma t}\left\{\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \eta_i \left[\cos \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right) - \sin \phi \sin (\Delta mt)\right]\right\}$ for $i = 1, 2, 3$, |
| $K_4(t)$ | $K_4(0)e^{-\Gamma t}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \cos \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right) - \sin \phi \sin (\Delta mt)\right]$ |
| $L_4(t)$ | $L_4(0)e^{-\Gamma t}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \cos \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right) - \sin \phi \sin (\Delta mt)\right]$ |
| $K_{5,6}(t)$ | $K_{5,6}(0)e^{-\Gamma t} \cos (\Delta mt) - L_{5,6}(0)e^{-\Gamma t}\left[\sin \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right) + \cos \phi \sin (\Delta mt)\right]$ |
| $L_{5,6}(t)$ | $L_{5,6}(0)e^{-\Gamma t} \cos (\Delta mt) + K_{5,6}(0)e^{-\Gamma t}\left[\sin \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right) + \cos \phi \sin (\Delta mt)\right]$ |

**TABLE I.** Time evolutions of observables in the decay of an initially pure $B_q$ meson into a self-conjugate state of two vector mesons.

| Bilinear | Time evolution |
|----------|----------------|
| $\bar{K}_i(t)$ | $\bar{K}_i(0)e^{-\Gamma t}\left\{\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \eta_i \left[\cos \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right) + \sin \phi \sin (\Delta mt)\right]\right\}$ for $i = 1, 2, 3$ |
| $\bar{K}_4(t)$ | $\bar{K}_4(0)e^{-\Gamma t}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \cos \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right) + \sin \phi \sin (\Delta mt)\right]$ |
| $\bar{L}_4(t)$ | $\bar{L}_4(0)e^{-\Gamma t}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \cos \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right) + \sin \phi \sin (\Delta mt)\right]$ |
| $\bar{K}_{5,6}(t)$ | $\bar{K}_{5,6}(0)e^{-\Gamma t} \cos (\Delta mt) - \bar{L}_{5,6}(0)e^{-\Gamma t}\left[-\sin \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right) + \cos \phi \sin (\Delta mt)\right]$ |
| $\bar{L}_{5,6}(t)$ | $\bar{L}_{5,6}(0)e^{-\Gamma t} \cos (\Delta mt) + \bar{K}_{5,6}(0)e^{-\Gamma t}\left[-\sin \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right) + \cos \phi \sin (\Delta mt)\right]$ |

**TABLE II.** Time evolutions of observables in the decay of an initially pure $\bar{B}_q$ meson into a self-conjugate state of two vector mesons.

\[
\zeta_i(t) = -2\eta_i K_i(0)e^{-\Gamma t} \sin \phi \sin (\Delta mt), \quad \text{for} \quad i = 1, 2, 3, \\
\zeta_4(t) = -2 K_4(0)e^{-\Gamma t} \sin \phi \sin (\Delta mt), \\
\zeta_{5,6}(t) = -2 L_{5,6}(0)e^{-\Gamma t} \sin \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right), \\
\xi_4(t) = -2 L_4(0)e^{-\Gamma t} \sin \phi \sin (\Delta mt), \\
\xi_{5,6}(t) = 2 K_{5,6}(0)e^{-\Gamma t} \sin \phi \sinh\left(\frac{\Delta\Gamma t}{2}\right).
\]  (27)

These equations hold even if there are nontrivial strong phases.

If we fix the overall strong phases of the transversity amplitudes by the following con-
vention: $A_{\perp}(0) = |A_{\perp}(0)|$, $A_{0}(0) = |A_{0}(0)|e^{-i\delta_{0}}$, and $A_{\parallel}(0) = |A_{\parallel}(0)|e^{-i\delta_{\parallel}}$, then $K_{4,5,6}(0)$ and $L_{4,5,6}(0)$ can be rewritten as

$$K_{4}(0) = \sqrt{K_{1}(0)K_{2}(0)} \cos(\delta_{0} - \delta_{\parallel}), \quad L_{4}(0) = \sqrt{K_{1}(0)K_{2}(0)} \sin(\delta_{0} - \delta_{\parallel}),$$

$$K_{5}(0) = \sqrt{K_{1}(0)K_{3}(0)} \cos(\delta_{0}), \quad L_{5}(0) = \sqrt{K_{1}(0)K_{3}(0)} \sin(\delta_{0}),$$

$$K_{6}(0) = \sqrt{K_{2}(0)K_{3}(0)} \cos(\delta_{\parallel}), \quad L_{6}(0) = \sqrt{K_{2}(0)K_{3}(0)} \sin(\delta_{\parallel}).$$

(28)

One can readily reach four relations among them:

$$K_{1}(0)K_{2}(0) = K_{4}(0)^{2} + L_{4}(0)^{2}, \quad K_{2}(0)K_{3}(0) = K_{6}(0)^{2} + L_{6}(0)^{2},$$

$$K_{3}(0)K_{1}(0) = K_{5}(0)^{2} + L_{5}(0)^{2}, \quad \frac{L_{4}(0)}{K_{4}(0)} = \frac{L_{5}(0)K_{6}(0) - K_{5}(0)L_{6}(0)}{K_{5}(0)K_{6}(0) + L_{5}(0)L_{6}(0)}.$$  \hspace{1cm} (29)

All experimentally measured nine amplitude bilinears should obey the above consistency relations. If the strong phases $\delta_{0}$ and $\delta_{\parallel}$ are nontrivial, one could possibly get sizeable $L_{4,5,6}$ that can be observed experimentally. We can then obtain information on the strong phases $\delta_{0}$, $\delta_{\parallel}$, the mass difference $\Delta m$, the decay width difference $\Delta \Gamma$, and $\sin \phi$ from Eqs. (27).

Since some of them share the same time evolution pattern, they also provide a consistency check for the experimental results.

At $t = 0$, there is no $CP$ asymmetry at all. So for charged $B$ decays where one weak amplitude dominates in the Standard Model, one should get the same amplitude bilinears for the particle and its conjugate modes. However, for neutral $B$ decays, the asymmetries develop as time goes on due to the mixing effect. In particular, $\zeta_{1-4}(t)$ and $\xi_{4}(t)$ have a sinusoidal time dependence, while $\zeta_{5,6}$ and $\xi_{5,6}(t)$ decay exponentially at $B_{L}$’s decay rate, $\Gamma_{L}$, in the large $t$ limit.

It is, nevertheless, interesting to look at the time integrated quantities for sizeable $CP$ or $T$ violating effects. The particle total decay rate, after time integration, is

$$\int_{0}^{\infty} dt \left[ K_{1}(t) + K_{2}(t) + K_{3}(t) \right] = \frac{1}{\Gamma} \left\{ \frac{4}{4 - y^2} \left[ K_{1}(0) + K_{2}(0) + K_{3}(0) \right] \right\}$$

Here we ignore the common weak factor that will be cancelled in all amplitude bilinears.
\[
\left(\cos\phi \frac{2y}{4-y^2} - \sin\phi \frac{x}{1+x^2}\right) [K_1(0) + K_2(0) - K_3(0)].
\]

(30)

Similarly, the time integrated anti-particle total decay rate can be obtained by simply reversing the sign of \( \phi \) in Eq. (30). One can obtain \( \sin\phi \) from the asymmetry between the time integrated total rates of conjugate modes and \( \cos\phi \) from the untagged analysis. This then eliminates the discrete ambiguity in the angle \( \phi \).

By integrating Eq. (27) from \( t = 0 \) to \( t = \infty \), we find the asymmetries to be

\[
\begin{align*}
\int_0^\infty dt \zeta_i(t) &= -2\eta_i K_i(0) \frac{1}{\Gamma 1 + x^2} \sin\phi, \text{ for } i = 1, 2, 3, \\
\int_0^\infty dt \zeta_4(t) &= -2K_4(0) \frac{1}{\Gamma 1 + x^2} \sin\phi, \\
\int_0^\infty dt \zeta_5(t) &= -2L_5(0) \frac{1}{\Gamma 1 + y^2} \sin\phi, \\
\int_0^\infty dt \zeta_6(t) &= -2L_6(0) \frac{1}{\Gamma 1 + y^2} \sin\phi,
\end{align*}
\]

(31)

In the above equations, \( x \equiv \Delta m/\Gamma \) and \( y \equiv \Delta\Gamma/\Gamma \). For \( B_d \), \( x = 0.73 \) and \( y \) is negligibly small; for \( B_s \), \( x > 14.0 \) (CL = 95\%) and \( y < 0.67 \) (CL = 95\%) \[12\]. From these relations, one can also directly extract \( \sin\phi \) given the information about the amplitude bilinears at initial time.

In principle, one can extract information about \( \phi \) and strong phases \( \delta_0, \delta_\parallel \) either from the time-dependent CP asymmetries \( \zeta \)'s and \( \xi \)'s or from the integrated asymmetries once the bilinears are determined experimentally or from models. \( \sin 2\beta \) has been measured from the mixing-induced CP asymmetry of \( B_d \to J/\Psi K_S \) \[13\]. For \( B_d \to J/\Psi K^*(\to \pi K_S) \), the \( B_d - \bar{B}_d \) and \( K - \bar{K} \) mixings and the CKM factor in the weak decay amplitude also give \( \phi = -2\beta \) \[14,2\]. Therefore, this offers an alternative way of measuring \( \sin 2\beta \) through the angular distribution analysis of tagged \( B_d \) decays. In addition, the \( \cos 2\beta \) dependence in \( K_{5,6} \) and \( L_{5,6} \) helps resolving the discrete ambiguity of the CKM angle \( \beta \) \[2\].

For \( B_s \to D_s^+ D_s^- \) and \( B_s \to J/\Psi \phi, \phi = 2\lambda^2\eta = \mathcal{O}(0.03) \) to an extremely good approximation, where \( \lambda = 0.22 \) is the Cabibbo angle and \( \eta \) is one of the Wolfenstein parameters
In this case, the CP asymmetries $\zeta_{1,2,3}(t)$ can be used to provide an unambiguous determination of the sign of $\phi$, and therefore the sign of $\eta$.

The decays that one may apply the results in this section to include: $B_s \to D_s^{*+} D_s^{*-}$, $B_d \to J/\Psi K^*(\to \pi K_S)$, $B_d \to J/\Psi \phi$. The first mode is a Type III decay, whereas the latter two are Type II decays.

Notice that $\sin \phi$ appears in all CP asymmetries, where $\phi$ is the phase of mixing and the single CKM factor involved in the decay amplitude. Since the amplitude has only one CKM factor, no CP violation effects would be found in the nine observables at $t = 0$. Yet the mixing will produce differences between the particle and anti-particle decay modes as time goes on. So any observation of the CP asymmetries in such modes indicates CP violation due to mixing and decay.

VII. NUMERICAL CALCULATION

In this section, we apply the factorization hypothesis [16–21] to the calculation of hadronic decay amplitudes. In general, factorization is expected to hold more strongly for color-allowed processes, such as $B_q \to D_s^{*+} \bar{D}_q^*$ with $q \in s, d, u$, though it is doubtful in color-suppressed modes, such as $B_q \to J/\Psi V$ with $(q, V) \in (s, \phi), (d, K^{*0}), (u, K^{*+})$ [22–25].

Throughout the calculations, we ignore the strong phases $\theta_\eta$ in Eq. (11) for simplicity but keep the strong phases in the Wilson coefficients [26]. These strong phases may be extracted from experimental data as mentioned in the previous section.

In our calculations, the Wolfenstein parameters are $(\rho, \eta) = (0.18, 0.37)$. The decay constants used are $F_{D_s} = 230 MeV$, $F_{D_s^*} = 275 MeV$, $F_{J/\Psi} = 394 MeV$, $F_{K^{*0}} = 221 MeV$, and $F_{\phi} = 237 MeV$. Extracting dominant Wilson coefficients, $a_1$ and $a_2$, from experimental decay rates has been performed in [11]. We apply their results to $B^\pm$ and $B_d$ decays with $b \to s$ quark level transitions, i.e., $B_u \to D_s^{*+} \bar{D}_q^*$, $B_d \to D_s^{*+} D^{*-}$, $B_u \to J/\Psi K^{*+}$, and $B_d \to J/\Psi K^{*0}$. We then extend the results to other decays according to the final state configurations, color-allowed ($B_q \to D_s^{*+} \bar{D}_q^*$ and $B_q \to D^{*+} D^*$) or color-suppressed ($B_q \to$
\( J/\Psi V \), charged or neutral. Assuming heavy quark symmetry, we use the \( B \to D^* \) decay form factors for the \( B \to D_s^* \) transitions.

The bilinears \( K_i \) and \( L_i \) in the following tables are normalized by dividing with \( \Gamma_0 \equiv K_1 + K_2 + K_3 \). The branching ratio asymmetry is defined by

\[
a_{CP} \equiv \frac{A - \bar{A}}{A + \bar{A}},
\]

where \( A \) and \( \bar{A} \) are the branching ratios for the particle and antiparticle decays, respectively. In the following tables, we list all nine amplitude bilinears for each mode even if \( L_{4,5,6} \) may not be able to be observed from the angular distributions of some of them (Type I decays).

In Tables III and IV, we take the modified BSW (or BSW II) model \([17,18]\) for the form factors in the evaluation of hadronic matrix elements. In Tables V and VI, the Neubert-Stech (NS) model \([27]\) is used. The relativistic light-front (LF) model \([28,29]\) is applied to the calculations in Tables VII and VIII.

We see that in general (1) there are no CP asymmetries for the nine normalized observables at the initial time (yet the CP asymmetries do exist for the unnormalized observables); (2) the branching ratio CP asymmetry is larger in decays involving the \( b \to d \) quark level processes because of the relative phase between the CKM factors of two weak amplitudes; and (3) \( L_4, K_5, \) and \( K_6 \) that involve the imaginary parts of the amplitude bilinears are essentially zero because the tree amplitudes dominates over the penguin contributions in these decays and we ignore possible final state interaction phases.

Although the branching ratios and nine observables may vary as one uses different form factor models, the asymmetries are roughly the same. It is found that \( b \to d \) type decays have larger asymmetries than \( b \to s \) type ones, as one would expect. From the models we analyze, the asymmetries for \( B_u^+ \to D^{*+}D^{*0} \) and \( B_d \to D^{*+}D^{*-} \) range from \(-3.29\%\) to \(-3.53\%\) and from \(-4.14\%\) to \(-4.46\%\), respectively. Unlike the charmless decays where \( A_0 \) is the dominant component in the transversity amplitudes, both \( A_0 \) and \( A_\parallel \) are about the same size. The parity odd component \( A_\perp \) is still small in \( B \to D^*D^* \) type transitions, but larger in \( B \to J/\Psi V \) decays.
VIII. SUMMARY

The decays of a $B$ meson into two vector mesons, which subsequently decay into two lighter particles via CP conserving currents, have a specific pattern in the differential angular distributions. The coefficient of each angular function in the distribution is a bilinear of amplitudes with certain CP properties. The knowledge of these amplitude bilinears can help us observe and understand CP violating effects. In particular, the time evolution of these bilinears in the cases of neutral $B$ meson decays further reveals the information such as the mass and decay width differences and CKM parameters. In situations where we can measure the polarization of the final product particles, all the nine combinations of amplitude bilinears are observable.

Under certain special circumstances, one can find simple relations among the nine observables. Therefore, experimental determination of them is valuable in testing our theoretical assumptions in the calculations, such as the factorization hypothesis and form factor models. The results are particularly simplified when there is only one weak amplitude dominating in the decay process. In such cases, one can test the Standard Model from the CP asymmetries at $t = 0$. The time development of CP asymmetries provides a window for observing CP violations due to mixing effects.

We provide numerical estimates of the observables in 12 sets of charming $B \to VV$ decays using three different form factor models. We find that the results do not depend strongly on the models used. In particular, we find bigger branching ratio asymmetries in $b \to d$ type decays, and those for $B_u^+ \to D^{*+}\bar{D}^{*0}$ and $B_d \to D^{*+}D^{*-}$ are as large as $-3\%$ and $-4\%$, respectively.

ACKNOWLEDGMENT

This research work is supported by the Department of Energy under Grant No. DE-FG02-91ER40682. The author is grateful to F. Gilman, L. Wolfenstein for useful discussion and to A. Leibovich for his comments. He also would like to thank the hospitality of Academia Sinica in Taiwan where part of this work is done.
APPENDIX: TIME-DEPENDENT AMPLITUDE BILINEARS

The time evolutions of the amplitude bilinears are as follows:

\[
|A_\eta(t)|^2 = |A_\eta(0)|^2 e^{-\Gamma t} \left\{ \frac{1 + R_0^2 + I_0^2}{2} \cosh \left( \frac{\Delta \Gamma t}{2} \right) + \frac{1 - R_0^2 - I_0^2}{2} \cos (\Delta m t) \right. \\
+ \left. \eta_i \left[ \left( R_\eta \cos \phi - I_\eta \sin \phi \right) \sinh \left( \frac{\Delta \Gamma t}{2} \right) - \left( R_\eta \sin \phi + I_\eta \cos \phi \right) \sin (\Delta m t) \right] \right\},
\]

where \( i = 1, 2, 3 \) for \( \eta = 0, \parallel, \perp, \) respectively, and \( \eta_{1,2} = -\eta_3 = 1; \)

\[
Re \left[ A_0^* (t) A_\parallel (t) \right] = Re \left[ A_0^*(0) A_\parallel (0) \right] e^{-\Gamma t} \times \\
\left\{ \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) + \cos (\Delta m t) \right] \\
+ \left( R_\parallel \cos \phi - I_\parallel \sin \phi \right) \sinh \left( \frac{\Delta \Gamma t}{2} \right) - \left( R_\parallel \sin \phi + I_\parallel \cos \phi \right) \sin (\Delta m t) \\
+ \left( R_0 \cos \phi - I_0 \sin \phi \right) \sinh \left( \frac{\Delta \Gamma t}{2} \right) - \left( R_0 \sin \phi + I_0 \cos \phi \right) \sin (\Delta m t) \\
+ \left( R_\parallel R_0 + I_\parallel I_0 \right) \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) - \cos (\Delta m t) \right] \right\};
\]

\[
Im \left[ A_0^* (t) A_\parallel (t) \right] = Im \left[ A_0^*(0) A_\parallel (0) \right] e^{-\Gamma t} \times \\
\left\{ \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) + \cos (\Delta m t) \right] \\
+ \left( R_\parallel \cos \phi - I_\parallel \sin \phi \right) \sinh \left( \frac{\Delta \Gamma t}{2} \right) - \left( R_\parallel \sin \phi + I_\parallel \cos \phi \right) \sin (\Delta m t) \\
+ \left( R_0 \cos \phi - I_0 \sin \phi \right) \sinh \left( \frac{\Delta \Gamma t}{2} \right) - \left( R_0 \sin \phi + I_0 \cos \phi \right) \sin (\Delta m t) \\
+ \left( R_\parallel R_0 + I_\parallel I_0 \right) \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) - \cos (\Delta m t) \right] \right\} + \\
Re \left[ A_0^*(0) A_\parallel (0) \right] e^{-\Gamma t} \times \\
\left\{ \left( R_\parallel \sin \phi + I_\parallel \cos \phi \right) \sinh \left( \frac{\Delta \Gamma t}{2} \right) + \left( R_\parallel \cos \phi - I_\parallel \sin \phi \right) \sin (\Delta m t) \right\}.
\]
\[-(R_0 \sin \phi + I_0 \cos \phi) \sinh \left(\frac{\Delta \Gamma t}{2}\right) - (R_0 \cos \phi - I_0 \sin \phi) \sin (\Delta m t)\]
\[+(I_{\parallel} R_0 - R_{\parallel} I_0) \left[ \cosh \left(\frac{\Delta \Gamma t}{2}\right) - \cos (\Delta m t) \right] \}\); (A3)

\[Re[A_0^*(t)A_\perp(t)] = Re[A_0^*(0)A_\perp(0)] e^{-\Gamma t} \times \left\{ \cosh \left(\frac{\Delta \Gamma t}{2}\right) + \cos (\Delta m t) \right\} \]
\[-(R_{\perp} \cos \phi - I_{\perp} \sin \phi) \sinh \left(\frac{\Delta \Gamma t}{2}\right) + (R_{\perp} \sin \phi + I_{\perp} \cos \phi) \sin (\Delta m t)\]
\[+(R_0 \cos \phi - I_0 \sin \phi) \sinh \left(\frac{\Delta \Gamma t}{2}\right) - (R_0 \sin \phi + I_0 \cos \phi) \sin (\Delta m t)\]
\[-(R_{\perp} R_0 + I_{\perp} I_0) \left[ \cosh \left(\frac{\Delta \Gamma t}{2}\right) - \cos (\Delta m t) \right] \}\]
\[+Im[A_0^*(0)A_\perp(0)] \frac{e^{-\Gamma t}}{2} \times \left\{ \cosh \left(\frac{\Delta \Gamma t}{2}\right) + \cos (\Delta m t) \right\} \]
\[-(R_{\perp} \sin \phi + I_{\perp} \cos \phi) \sinh \left(\frac{\Delta \Gamma t}{2}\right) + (R_{\perp} \cos \phi - I_{\perp} \sin \phi) \sin (\Delta m t)\]
\[+(R_0 \sin \phi - I_0 \cos \phi) \sinh \left(\frac{\Delta \Gamma t}{2}\right) + (R_0 \cos \phi - I_0 \sin \phi) \sin (\Delta m t)\]
\[+(I_{\perp} R_0 - R_{\perp} I_0) \left[ \cosh \left(\frac{\Delta \Gamma t}{2}\right) - \cos (\Delta m t) \right] \}\]; (A4)

\[Im[A_0^*(t)A_\perp(t)] = Im[A_0^*(0)A_\perp(0)] e^{-\Gamma t} \times \left\{ \cosh \left(\frac{\Delta \Gamma t}{2}\right) + \cos (\Delta m t) \right\} \]
\[-(R_{\perp} \cos \phi - I_{\perp} \sin \phi) \sinh \left(\frac{\Delta \Gamma t}{2}\right) + (R_{\perp} \sin \phi + I_{\perp} \cos \phi) \sin (\Delta m t)\]
\[+(R_0 \cos \phi - I_0 \sin \phi) \sinh \left(\frac{\Delta \Gamma t}{2}\right) - (R_0 \sin \phi + I_0 \cos \phi) \sin (\Delta m t)\]
\[-(R_{\perp} R_0 + I_{\perp} I_0) \left[ \cosh \left(\frac{\Delta \Gamma t}{2}\right) - \cos (\Delta m t) \right] \}\]
\[-Re[A_0^*(0)A_\perp(0)] \frac{e^{-\Gamma t}}{2} \times \left\{ \cosh \left(\frac{\Delta \Gamma t}{2}\right) + \cos (\Delta m t) \right\} \]
\[-(R_{\perp} \sin \phi + I_{\perp} \cos \phi) \sinh \left(\frac{\Delta \Gamma t}{2}\right) + (R_{\perp} \cos \phi - I_{\perp} \sin \phi) \sin (\Delta m t)\]
\[+(R_0 \sin \phi + I_0 \cos \phi) \sinh \left(\frac{\Delta \Gamma t}{2}\right) + (R_0 \cos \phi - I_0 \sin \phi) \sin (\Delta m t)\]
\[+(I_{\perp} R_0 - R_{\perp} I_0) \left[ \cosh \left(\frac{\Delta \Gamma t}{2}\right) - \cos (\Delta m t) \right] \}\]. (A5)
Similar formulas for $Re \left[ A^*_\parallel(0)A_\perp(0) \right]$ and $Im \left[ A^*_\parallel(0)A_\perp(0) \right]$ can be obtained from Eq. (A4) and (A7) by replacing “0” with “∥”, respectively.

The time evolution formulas for the CP conjugate amplitude bilinears are:

$$|\tilde{A}_\parallel(t)|^2 = |A_\parallel(0)|^2 e^{-\Gamma t} \left\{ \frac{1+R_\parallel^2+I_\parallel^2}{2} \cosh \left( \frac{\Delta \Gamma t}{2} \right) - \frac{1-R_\parallel^2-I_\parallel^2}{2} \cos (\Delta m t) \right\} + \eta_i \left[ (R_\parallel \cos \phi - I_\parallel \sin \phi) \sinh \left( \frac{\Delta \Gamma t}{2} \right) + (R_\parallel \sin \phi + I_\parallel \cos \phi) \sin (\Delta m t) \right] \right\}; \quad (A6)$$

$$Re \left[ \tilde{A}^*_\parallel(t)\tilde{A}_\parallel(t) \right] = \left[ Re \left[ A^*_\parallel(0)A_\parallel(0) \right] \left( R_\parallel R_0 + I_\parallel I_0 \right) - Im \left[ A^*_\parallel(0)A_\parallel(0) \right] \left( I_\parallel R_0 - R_\parallel I_0 \right) \right] e^{-\Gamma t} \left\{ \frac{1}{R_\parallel^2 + I_\parallel^2} \right\} \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) + \cos (\Delta m t) \right]$$

$$+ \frac{1}{R_\parallel^2 + I_\parallel^2} \left[ (R_\parallel \cos \phi - I_\parallel \sin \phi) \sinh \left( \frac{\Delta \Gamma t}{2} \right) + (R_\parallel \sin \phi + I_\parallel \cos \phi) \sin (\Delta m t) \right]$$

$$- \frac{1}{R_\parallel^2 + I_\parallel^2} \left[ -\left( R_\parallel \sin \phi + I_\parallel \cos \phi \right) \sinh \left( \frac{\Delta \Gamma t}{2} \right) - \left( R_\parallel \cos \phi - I_\parallel \sin \phi \right) \sin (\Delta m t) \right] \right\}; \quad (A7)$$

$$Im \left[ \tilde{A}^*_\parallel(t)\tilde{A}_\parallel(t) \right] = \left[ Re \left[ A^*_\parallel(0)A_\parallel(0) \right] \left( I_\parallel R_0 - R_\parallel I_0 \right) + Im \left[ A^*_\parallel(0)A_\parallel(0) \right] \left( R_\parallel R_0 + I_\parallel I_0 \right) \right] e^{-\Gamma t} \left\{ \frac{1}{R_\parallel^2 + I_\parallel^2} \right\} \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) + \cos (\Delta m t) \right]$$

$$+ \frac{1}{R_\parallel^2 + I_\parallel^2} \left[ (R_\parallel \cos \phi - I_\parallel \sin \phi) \sinh \left( \frac{\Delta \Gamma t}{2} \right) + (R_\parallel \sin \phi + I_\parallel \cos \phi) \sin (\Delta m t) \right]$$

$$+ \frac{1}{R_\parallel^2 + I_\parallel^2} \left[ (R_\parallel \cos \phi - I_\parallel \sin \phi) \sinh \left( \frac{\Delta \Gamma t}{2} \right) + (R_\parallel \sin \phi + I_\parallel \cos \phi) \sin (\Delta m t) \right]$$

$$+ \frac{1}{R_\parallel^2 + I_\parallel^2} \left[ (R_\parallel \cos \phi - I_\parallel \sin \phi) \sinh \left( \frac{\Delta \Gamma t}{2} \right) + (R_\parallel \sin \phi + I_\parallel \cos \phi) \sin (\Delta m t) \right] \right\}; \quad (A7)$$
\[
\left\{ \frac{1}{R^2_0 + I^2_0} \left[ (R_0 \sin \phi + I_0 \cos \phi) \sinh\left( \frac{\Delta \Gamma}{2} t \right) - (R_0 \cos \phi - I_0 \sin \phi) \sin \left( \Delta m t \right) \right] \right. \\
- \frac{1}{R^2_0 + I^2_0} \left[ (R_0 \sin \phi + I_0 \cos \phi) \sinh\left( \frac{\Delta \Gamma}{2} t \right) - (R_0 \cos \phi - I_0 \sin \phi) \sin \left( \Delta m t \right) \right] \\
+ \frac{1}{(R^2_0 + I^2_0)(R^2_0 + I^2_0)} \left[ \cosh\left( \frac{\Delta \Gamma}{2} t \right) - \cos \left( \Delta m t \right) \right] \} \right); \quad (A8)
\]

\[
\text{Re} \left[ A^*_0(t) \bar{A}_1(t) \right] = \left[ \text{Re} \left[ A^*_0(0) A_\perp(0) \right] \left( R_\perp R_0 + I_\perp I_0 \right) - \text{Im} \left[ A^*_0(0) A_\perp(0) \right] \left( I_\perp R_0 - R_\perp I_0 \right) \right] \frac{e^{-\Gamma t}}{2} \times \\
\left\{ \cosh\left( \frac{\Delta \Gamma}{2} t \right) + \cos \left( \Delta m t \right) \right. \\
- \frac{1}{R^2_0 + I^2_0} \left[ (R_\perp \cos \phi - I_\perp \sin \phi) \sinh\left( \frac{\Delta \Gamma}{2} t \right) + (R_\perp \sin \phi + I_\perp \cos \phi) \sin \left( \Delta m t \right) \right] \\
+ \frac{1}{R^2_0 + I^2_0} \left[ (R_0 \sin \phi + I_0 \cos \phi) \sinh\left( \frac{\Delta \Gamma}{2} t \right) - (R_0 \cos \phi - I_0 \sin \phi) \sin \left( \Delta m t \right) \right] \\
- \frac{1}{(R^2_0 + I^2_0)(R^2_0 + I^2_0)} \left[ \cosh\left( \frac{\Delta \Gamma}{2} t \right) - \cos \left( \Delta m t \right) \right] \} \}; \quad (A9)
\]

\[
\text{Im} \left[ A^*_0(t) \bar{A}_1(t) \right] = \left[ \text{Re} \left[ A^*_0(0) A_\perp(0) \right] \left( R_\perp R_0 - I_\perp I_0 \right) + \text{Im} \left[ A^*_0(0) A_\perp(0) \right] \left( I_\perp R_0 + R_\perp I_0 \right) \right] \frac{e^{-\Gamma t}}{2} \times \\
\left\{ \cosh\left( \frac{\Delta \Gamma}{2} t \right) + \cos \left( \Delta m t \right) \right. \\
- \frac{1}{R^2_0 + I^2_0} \left[ (R_\perp \cos \phi - I_\perp \sin \phi) \sinh\left( \frac{\Delta \Gamma}{2} t \right) + (R_\perp \sin \phi + I_\perp \cos \phi) \sin \left( \Delta m t \right) \right] \\
+ \frac{1}{R^2_0 + I^2_0} \left[ (R_0 \sin \phi + I_0 \cos \phi) \sinh\left( \frac{\Delta \Gamma}{2} t \right) - (R_0 \cos \phi - I_0 \sin \phi) \sin \left( \Delta m t \right) \right] \\
- \frac{1}{(R^2_0 + I^2_0)(R^2_0 + I^2_0)} \left[ \cosh\left( \frac{\Delta \Gamma}{2} t \right) - \cos \left( \Delta m t \right) \right] \} \]}
\[-\frac{1}{(R^2_\perp + I^2_\perp)(R^2_0 + I^2_0)}(I_\perp R_0 - R_\perp I_0) \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) - \cos (\Delta m t) \right] \right\}. \quad (A10)\]

Similar formulas for \(Re \left[ \bar{A}^*_\parallel(0) \bar{A}_\perp(0) \right] \) and \(Im \left[ \bar{A}^*_\parallel(0) \bar{A}_\perp(0) \right] \) can be obtained from Eq. \((A9)\) and \((A10)\) by replacing “0” with “\(\parallel\)”, respectively.
REFERENCES

[1] A.S. Dighe, I. Dunietz, H.J. Lipkin, and J.L. Rosner, Phys. Lett. B369, 144 (1996).
[2] A.S. Dighe, I. Dunietz, and R. Fleischer, Eur. Phys. J. C6, 647 (1999).
[3] G. Kramer and W.F. Palmer, Phys. Rev. D45, 193 (1992).
[4] G. Kramer and W.F. Palmer, Phys. Lett. B279, 181 (1992).
[5] G. Kramer, W.F. Palmer, and H. Simma, Nucl. Phys. B428, 77 (1994).
[6] G. Kramer, T. Mannel, and W.F. Palmer, Z. Phys. C55, 497 (1992).
[7] R. Fleischer, Phys. Rev. D 60, 073008 (1999).
[8] D. London, N. Sinha, and R. Sinha, Phys. Rev. D 60, 074020 (1999).
[9] C.W. Chiang and B. Tseng, hep-ph9905338.
[10] C.W. Chiang and L. Wolfenstein, hep-ph/9911338 (to appear in Phys. Rev. D) (1999).
[11] H.Y. Cheng and K.C. Yang, Phys. Rev. D 59, 092004 (1999).
[12] Particle Data Group, Eur. Phys. J. C3 (1998).
[13] F. Abe et. al., CDF Collaboration, Phys. Rev. Lett. 81, 5513 (1998).
[14] Y. Nir, Lectures given at 20th Annual SLAC Summer Institute on Particle Physics: The Third Family and the Physics of Flavor, Stanford, CA, 13-24 Jul 1992.
[15] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).
[16] D. Fakirov and B. Stech, Nucl. Phys. B133, 315 (1978).
[17] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C29, 637 (1985).
[18] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C34, 103 (1987).
[19] J. Körner and G. Goldstein, Phys. Lett. 89B, 105 (1979).
[20] J.D. Bjorken, Nucl. Phys. Proc. Suppl. B11, 325 (1989).
[21] T.E. Browder, K. Honscheid, and D. Pedrini, Annu. Rev. Nucl. Part. Sci. 46, 395 (1996).
[22] R. Aleksan, A. Le Yaouanc, L. Oliver, O. Pène and Y.C. Raynal, Phys. Lett. B316, 567 (1993).
[23] J. Bijnens and F. Hoogeveen, Phys. Lett. B283, 434 (1992).
[24] J.L. Rosner, Phys. Rev. D 42, 3732 (1990).
[25] T. Mannel, W. Roberts, and Z. Ryzak, Phys. Rev. D 44, 18 (1991).
[26] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. 43, 242 (1979).
[27] M. Neubert and B. Stech, in Heavy Flavours, edited by A.J. Buras and M. Lindner, 2nd ed. (World Scientific, Singapore, 1998), p. 294, hep-ph/9705292.
[28] W. Jaus, Phys. Rev. D 41, 3394 (1990).
[29] H.Y. Cheng, C.Y. Cheung, and C.W. Hwang, Phys. Rev. D 55, 1559 (1997).
TABLE III. Charming $B \to VV$ decays involving the $b \to d$ underlying quark processes. BSW II form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, $-3.29\%$, $-0.38\%$, $-4.14\%$, $-0.38\%$, $-0.97\%$, and $-0.38\%$, respectively.
| Processes | $\text{Br}(\times 10^{-3})$ | $K_1$ | $K_2$ | $K_3$ | $K_4$ | $K_5$ | $K_6$ | $L_4$ | $L_5$ | $L_6$ |
|-----------|----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $B_u^+ \to D_s^{*+} D^{*0}$ | 34.6 | 0.491 | 0.438 | 0.071 | $-0.464$ | 0 | 0 | 0 | $-0.177$ | 0.187 |
| $B_u^- \to D_s^{*+} \bar{D}^{*0}$ | 34.5 | 0.491 | 0.438 | 0.071 | $-0.464$ | 0 | 0 | 0 | $-0.177$ | 0.187 |
| $B_u^+ \to J/\Psi K^{*+}$ | 1.988 | 0.358 | 0.397 | 0.245 | $-0.377$ | 0 | 0 | 0 | $-0.312$ | 0.296 |
| $B_u^- \to J/\Psi K^{*-}$ | 1.987 | 0.358 | 0.397 | 0.245 | $-0.377$ | 0 | 0 | 0 | $-0.312$ | 0.296 |
| $B_d^0 \to D_s^{*+} D^{*-}$ | 22.6 | 0.491 | 0.438 | 0.071 | $-0.464$ | 0 | 0 | 0 | $-0.177$ | 0.187 |
| $\bar{B}_d^0 \to D_s^{*+} \bar{D}^{*-}$ | 22.5 | 0.491 | 0.438 | 0.071 | $-0.464$ | 0 | 0 | 0 | $-0.177$ | 0.187 |
| $B_d^0 \to J/\Psi K^{*0}$ | 1.988 | 0.358 | 0.397 | 0.245 | $-0.377$ | 0 | 0 | 0 | $-0.312$ | 0.296 |
| $\bar{B}_d^0 \to J/\Psi \bar{K}^{*0}$ | 1.987 | 0.358 | 0.397 | 0.245 | $-0.377$ | 0 | 0 | 0 | $-0.312$ | 0.296 |
| $B_s^0 \to D_s^{*+} D_s^{*-}$ | 31.28 | 0.491 | 0.442 | 0.068 | $-0.466$ | 0 | 0 | 0 | $-0.173$ | 0.182 |
| $\bar{B}_s^0 \to D_s^{*+} \bar{D}_s^{*-}$ | 31.25 | 0.491 | 0.442 | 0.068 | $-0.466$ | 0 | 0 | 0 | $-0.173$ | 0.182 |
| $B_s^0 \to J/\Psi \phi$ | 2.049 | 0.353 | 0.413 | 0.235 | $-0.382$ | 0 | 0 | 0 | $-0.311$ | 0.288 |
| $\bar{B}_s^0 \to J/\Psi \phi$ | 2.049 | 0.353 | 0.413 | 0.235 | $-0.382$ | 0 | 0 | 0 | $-0.311$ | 0.288 |

TABLE IV. Charming $B \to VV$ decays involving the $b \to s$ underlying quark processes. BSW II form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, 0.18%, 0.02%, 0.23%, 0.02%, 0.05%, and 0, respectively.
| Processes | $Br$  | $K_1$  | $K_2$  | $K_3$  | $K_4$  | $K_5$  | $K_6$  | $L_4$  | $L_5$  | $L_6$  |
|-----------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $B^+_u \rightarrow D^{++} \bar{D}^{*0}$ | 1.24  | 0.554  | 0.396  | 0.050  | $-0.469$ | 0  | 0  | 0 | $-0.140$ | 0.166 |
| $B^-_u \rightarrow D^{*-} \bar{D}^{*0}$ | 1.32  | 0.554  | 0.396  | 0.050  | $-0.469$ | 0  | 0  | 0 | $-0.140$ | 0.166 |
| $B^+_u \rightarrow J/\Psi \rho^+$ | 0.0957 | 0.493  | 0.367  | 0.141  | $-0.425$ | 0  | 0  | 0 | $-0.227$ | 0.263 |
| $B^-_u \rightarrow J/\Psi \rho^-$ | 0.0963 | 0.493  | 0.367  | 0.141  | $-0.425$ | 0  | 0  | 0 | $-0.227$ | 0.263 |
| $B^0_d \rightarrow D^{*-} \bar{D}^{*+}$ | 0.81  | 0.554  | 0.396  | 0.050  | $-0.469$ | 0  | 0  | 0 | $-0.140$ | 0.166 |
| $B^-_d \rightarrow D^{*-} \bar{D}^{*+}$ | 0.88  | 0.554  | 0.396  | 0.050  | $-0.469$ | 0  | 0  | 0 | $-0.140$ | 0.166 |
| $B^0_d \rightarrow J/\Psi \rho^0$ | 0.0957 | 0.493  | 0.367  | 0.141  | $-0.425$ | 0  | 0  | 0 | $-0.227$ | 0.263 |
| $B^-_d \rightarrow J/\Psi \rho^0$ | 0.0963 | 0.493  | 0.367  | 0.141  | $-0.425$ | 0  | 0  | 0 | $-0.227$ | 0.263 |
| $B^0_s \rightarrow D^{*-} \bar{D}^{*+}$ | 1.09  | 0.552  | 0.401  | 0.047  | $-0.470$ | 0  | 0  | 0 | $-0.137$ | 0.161 |
| $B^-_s \rightarrow D^{*-} \bar{D}^{*+}$ | 1.11  | 0.552  | 0.401  | 0.047  | $-0.470$ | 0  | 0  | 0 | $-0.137$ | 0.161 |
| $B^0_s \rightarrow J/\Psi K^{*0}$ | 0.131 | 0.489  | 0.383  | 0.128  | $-0.433$ | 0  | 0  | 0 | $-0.222$ | 0.250 |
| $B^-_s \rightarrow J/\Psi K^{*0}$ | 0.132 | 0.489  | 0.383  | 0.128  | $-0.433$ | 0  | 0  | 0 | $-0.222$ | 0.250 |

**TABLE V.** Charming $B \rightarrow VV$ decays involving the $b \rightarrow d$ underlying quark processes. The NS model form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, $-3.29\%$, $-0.30\%$, $-4.14\%$, $-0.30\%$, $-0.97\%$, and $-0.30\%$, respectively.
| Processes        | Br  | $K_1$ | $K_2$ | $K_3$ | $K_4$ | $K_5$ | $K_6$ | $L_4$ | $L_5$ | $L_6$ |
|------------------|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $B_u^+ \to D_s^{*+} D^{*0}$ | 36.09 | 0.531 | 0.420 | 0.049 | −0.472 | 0  | 0  | 0  | −0.144 | 0.162 |
| $B_u^- \to D_s^{*-} D^{*0}$ | 35.96 | 0.531 | 0.420 | 0.049 | −0.472 | 0  | 0  | 0  | −0.144 | 0.162 |
| $B_u^+ \to J/\Psi K^{*+}$ | 2.411 | 0.484 | 0.392 | 0.124 | −0.436 | 0  | 0  | 0  | −0.220 | 0.245 |
| $B_u^- \to J/\Psi K^{*-}$ | 2.410 | 0.484 | 0.392 | 0.124 | −0.436 | 0  | 0  | 0  | −0.220 | 0.245 |
| $B_d^0 \to D_s^{*+} D^{*-}$ | 23.59 | 0.531 | 0.420 | 0.049 | −0.472 | 0  | 0  | 0  | −0.144 | 0.162 |
| $B_d^0 \to D_s^{*-} D^{*+}$ | 23.48 | 0.531 | 0.420 | 0.049 | −0.472 | 0  | 0  | 0  | −0.144 | 0.162 |
| $B_d^+ \to J/\Psi K^{*0}$ | 2.4114 | 0.484 | 0.392 | 0.124 | −0.436 | 0  | 0  | 0  | −0.220 | 0.245 |
| $B_d^- \to J/\Psi K^{*0}$ | 2.4107 | 0.484 | 0.392 | 0.124 | −0.436 | 0  | 0  | 0  | −0.220 | 0.245 |
| $B_s^0 \to D_s^{*+} D_s^{*-}$ | 32.54 | 0.529 | 0.425 | 0.047 | −0.474 | 0  | 0  | 0  | −0.141 | 0.157 |
| $B_s^0 \to D_s^{*0} D_s^{*-}$ | 32.51 | 0.529 | 0.425 | 0.047 | −0.474 | 0  | 0  | 0  | −0.141 | 0.157 |
| $B_s^0 \to J/\Psi \phi$ | 3.166 | 0.478 | 0.408 | 0.114 | −0.441 | 0  | 0  | 0  | −0.216 | 0.234 |
| $B_s^0 \to J/\Psi \phi$ | 3.165 | 0.478 | 0.408 | 0.114 | −0.441 | 0  | 0  | 0  | −0.216 | 0.234 |

TABLE VI. Charming $B \to VV$ decays involving the $b \to s$ underlying quark processes. The NS model form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, 0.18%, 0.02%, 0.23%, 0.02%, 0.05%, and 0.02%, respectively.
\begin{table}[h]
\centering
\begin{tabular}{l|cccccccc}
\hline
 Processes & $B^+_u \to D^{*+} \bar{D}^{*0}$ & $B^-_u \to D^{*-} \bar{D}^{*0}$ & $B^+_u \to J/\Psi\rho^{+}$ & $B^-_u \to J/\Psi\rho^{-}$ & $B^0_d \to D^{*+} D^{*-}$ & $\bar{B}^0_d \to D^{*+} D^{*-}$ & $B^0_d \to J/\Psi\rho^0$ & $\bar{B}^0_d \to J/\Psi\rho^0$ \\
\hline
 & 1.23 & 0.557 & 0.376 & 0.066 & -0.458 & 0 & 0 & 0 & -0.158 & 0.192 \\
 & 1.32 & 0.557 & 0.376 & 0.066 & -0.458 & 0 & 0 & 0 & -0.158 & 0.192 \\
 & 0.084 & 0.592 & 0.334 & 0.074 & -0.445 & 0 & 0 & 0 & -0.157 & 0.209 \\
 & 0.085 & 0.592 & 0.334 & 0.074 & -0.445 & 0 & 0 & 0 & -0.157 & 0.209 \\
 & 0.81 & 0.558 & 0.376 & 0.066 & -0.458 & 0 & 0 & 0 & -0.158 & 0.192 \\
 & 0.88 & 0.558 & 0.376 & 0.066 & -0.458 & 0 & 0 & 0 & -0.158 & 0.192 \\
 & 0.084 & 0.592 & 0.334 & 0.074 & -0.445 & 0 & 0 & 0 & -0.157 & 0.209 \\
 & 0.085 & 0.592 & 0.334 & 0.074 & -0.445 & 0 & 0 & 0 & -0.157 & 0.209 \\
 & 1.10 & 0.555 & 0.382 & 0.063 & -0.460 & 0 & 0 & 0 & -0.155 & 0.187 \\
 & 1.19 & 0.555 & 0.382 & 0.063 & -0.460 & 0 & 0 & 0 & -0.155 & 0.187 \\
 & 0.131 & 0.536 & 0.371 & 0.093 & -0.446 & 0 & 0 & 0 & -0.186 & 0.224 \\
 & 0.132 & 0.536 & 0.371 & 0.093 & -0.446 & 0 & 0 & 0 & -0.186 & 0.224 \\
\hline
\end{tabular}
\caption{Charming $B \to VV$ decays involving the $b \to d$ underlying quark processes. The LF model form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, $-3.53\%$, $-0.29\%$, $-4.46\%$, $-0.29\%$, $-1.03\%$, and $-0.29\%$, respectively.}
\end{table}
| Processes                     | Br  | $K_1$ | $K_2$ | $K_3$ | $K_4$ | $K_5$ | $K_6$ | $L_4$ | $L_5$ | $L_6$ |
|------------------------------|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $B_u^+ \to D_s^{*+}D^{*0}$   | 35.42 | 0.533 | 0.401 | 0.067 | -0.462 | 0     | 0     | 0     | -0.163 | 0.188 |
| $B_u^- \to D_s^{*-}D^{*0}$   | 35.29 | 0.533 | 0.401 | 0.067 | -0.462 | 0     | 0     | 0     | -0.163 | 0.188 |
| $B_u^+ \to J/\Psi K^{*+}$    | 2.402 | 0.529 | 0.381 | 0.090 | -0.449 | 0     | 0     | 0     | -0.185 | 0.218 |
| $B_u^- \to J/\Psi K^{*-}$    | 2.401 | 0.529 | 0.381 | 0.090 | -0.449 | 0     | 0     | 0     | -0.185 | 0.218 |
| $B_d^0 \to D_s^{*+}D^{*-}$   | 23.06 | 0.533 | 0.401 | 0.067 | -0.462 | 0     | 0     | 0     | -0.163 | 0.188 |
| $B_d^0 \to D_s^{*-}D^{*+}$   | 22.94 | 0.533 | 0.401 | 0.067 | -0.462 | 0     | 0     | 0     | -0.163 | 0.188 |
| $B_d^0 \to J/\Psi K^{*0}$    | 2.4025 | 0.529 | 0.381 | 0.090 | -0.449 | 0     | 0     | 0     | -0.185 | 0.218 |
| $B_d^0 \to J/\Psi \bar{K}^{*0}$ | 2.4017 | 0.529 | 0.381 | 0.090 | -0.449 | 0     | 0     | 0     | -0.185 | 0.218 |
| $B_s^0 \to D_s^{*+}D_s^{*-}$ | 32.30 | 0.530 | 0.407 | 0.063 | -0.464 | 0     | 0     | 0     | -0.160 | 0.183 |
| $B_s^0 \to D_s^{*-}D_s^{*+}$ | 32.27 | 0.530 | 0.407 | 0.063 | -0.464 | 0     | 0     | 0     | -0.160 | 0.183 |
| $B_s^0 \to J/\Psi \phi$     | 2.330 | 0.496 | 0.376 | 0.128 | -0.432 | 0     | 0     | 0     | -0.220 | 0.252 |
| $B_s^0 \to J/\psi \phi$     | 2.329 | 0.496 | 0.376 | 0.128 | -0.432 | 0     | 0     | 0     | -0.220 | 0.252 |

**TABLE VIII.** Charming $B \to VV$ decays involving the $b \to s$ underlying quark processes. The LF model form factors are used in this table. The branching ratio asymmetries of the paired modes are, from top to bottom, 0.19%, 0.02%, 0.25%, 0.02%, 0.05%, and 0.02%, respectively.