Understanding Impedance Ratio Criteria for Converter-Based AC Power System

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Abstract—Nyquist criterion-based impedance ratio criteria (IRCs) have been widely applied for inspecting the risk of small-signal instability among converter-based AC power systems. Aided by a comparative study on voltage source converter, including the single-input single-output (SISO) and multiple input multiple output (MIMO) analyses in both the dq and the sequence domain, two aspects are emphasized in this paper: 1) the sufficiency of SISO analysis when the mapping function (MF) is observable to potentially unstable modes, and 2) the inconvenience of IRCs with an unintended right-half plane pole emergence of MF due to the source-load partition. The strictness of analyses is proved by a systematical deduction of explicit analytical impedance models using the state space. Moreover, a novel criterion that relies on the logarithmic derivative of MFs is proposed, which can identify the system modes directly, serve as an alternative to IRCs, and be extended to other transfer function-based stability analyses.

Index Terms—Impedance model, state space, stability criterion, source-load partition, AC power systems, mapping function

I. INTRODUCTION

IMPEDEANCE ratio criteria (IRCs) for converter-based AC power systems, which were first deduced in the dq [1] and then extended to the sequence domain [2], is proved to be a powerful tool for analyzing the small-signal stability that can protect the privacy of vendors. A relevant standard [3] has been formed to guide the construction of wind farms in China, which confirms the practicability of IRCs in the real world.

Port impedance/admittance models (IMs/AMs) of both load and source subsystems at a single converter-grid interface are required before applying the two-dimensional (2-d) IRC and forms a multiple input multiple output (MIMO) analysis. In general, the frequency responses instead of the detailed expressions are available for the 2-d transfer function matrices with no zero-sequence differential-mode path in the three-phase AC system. Fig.1 shows that a couple of port IM/AM can export to two decoupled one-dimensional (1-d) loop IMs, and two 1-d IRCs can be derived after partitioning loop IMs [4] to achieve the single input single output (SISO) analyses. Suppose that the converter operates in a current source mode, that is, no right half plane (RHP) pole exists in the 2-d AM, the marginal stability condition of various IRCs should be the same. Such an equivalence is guaranteed by the linear frame transformation [4] and the uniformity of mapping function (MF) using Schur complement (MF maps the Nyquist contour in s plane to the Nyquist plots in ω plane according to Cauchy’s argument principle). However, the lack of authentic zeros and poles (Zs and Ps, mainly the Ps) for MFs may cause difficulties in the practical graphical stability assessments.

The scope of this paper is to conduct a comparative study, reveal several overlooked details of IRC, and inspire future stability analysis. The order of targeted grid-connected system is quite low, thus IMs/AMs can be deduced from the state space to obtain the exact Zs and Ps. Nyquist plots are checked with the Zs and Ps of MFs to clarify the advantage of 1-d IRC, and it is found that the stable operation of AC subsystems does not guarantee no RHP pole in the loop IM. Hence, it is recommended to apply the proposed stability criterion based on the logarithmic derivative of loop IM with only frequency responses used, which is easy to implement and worthy of extending to other transfer function-based stability analyses [5].

![Fig. 1. A generalized representation of the closed-loop system.](image)

II. IM/AM DERIVATION USING STATE SPACE

A. System Overview

Fig. 2 shows the typical grid-following control loops of a three-phase symmetrical two-level voltage source converter (VSC), which act as a current source in parallel with an output impedance in the frequency domain, i.e., the load subsystem [2]. The d-axis outer loop regulates the DC voltage while the reference of q-axis outer loop is set to 0. The use of VSC average model in the simulation avoids the complicated effects of PWM on stability analyses. Table I lists the key parameters for comparative studies.

![Fig. 2. Control loops of VSC. Superscript * means reference.](image)

| TABLE I | KEY PARAMETERS OF SYSTEM |
|---|---|

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B. Modeling Process

It is well-recognized that the modes of the complete system can be identified through the eigenvalue analysis, and the negative damping mode induces the oscillation. By computing the system equilibrium, regarding the small-signal of the system, the eigenvalue analysis is employed, and the eigenvalues of each element of port AM, which is a necessary and sufficient condition for the independent stability of the subsystems.

\[ \Delta x_i = A_i x_i + B_{ii} u_i, \quad \Delta u_i = [C_{ii} (sI - A_{ii})^{-1} B_{ii} + D_{ii}] y_{ii}^{-1} \]

where \( A, B, C, \) and \( D \) are Jacobian matrices; \( x, y, \) and \( u \) are state, output, and input vectors formed by the 8 variables in Fig. 2, including 3 of energy storage device and 5 of integrator output, and the variables are distinguished by whether appear in the dq pairs (e.g. \([s_a, s_b]\)). The prefix \( \Delta \) and the subscript \( dq \) represent the small signals and the system dq axis [7]; \( Y \) and \( Z \) represent AMs and YMs, while \( s \) represents the imaginary unit.

Realizing there is a combination of rotation and complex transformation from a \( dq \) domain real vector to a sequence domain complex vector (with the subscript \( dq \) and \( pn \) respectively) [7]; (2) is transformed to (3):

\[ \Delta x_p = A_p x_p + B_{pp} u_p, \quad \Delta u_p = [C_{pp} (sI - A_{pp})^{-1} B_{pp} + D_{pp}] y_{pp}^{-1} \]

In (3), the item \( j \omega_0 I \) reflects the rotation, where \( j \) is the imaginary unit, \( \omega_0 \) is the rated angular frequency, and \( I \) is the utility matrix. \( T \) represents the complex transformation, whose order is determined by the vector of subscript, where the submatrix 0.5×\([1, 1; -1, 1]\) cooperates with the \( dq \) pairs [7].

Compared with IM/AM derived by frequency domain linearization and intermediate variables elimination, the distinct advantage of the proposed modeling process is that it can ensure the number of AM poles equals that of state variables, which helps observe the real case of zero-pole cancellation and avoids the dimension explosion of IM/AM.

Applying the 1-d IRC requires a reconstruction of source-load IM from the 2-d port IM obtained by (1) and (3) [4]:

\[ Z_{\text{L/S}}^{\text{1/d}} = Z_{\text{L/S}}^{\text{1/2}} - \frac{Z_{\text{L/S}}^{\text{1/2}} (Z_{\text{L/S}}^{\text{1/2}} + Z_{\text{L/S}}^{\text{1/2}})}{Z_{\text{L/S}}^{\text{1/2}} + Z_{\text{L/S}}^{\text{1/2}}}, \quad Z_{\text{L/S}}^{\text{1/2}} = \frac{Z_{\text{L/S}}^{\text{1/2}} (Z_{\text{L/S}}^{\text{1/2}} + Z_{\text{L/S}}^{\text{1/2}})}{Z_{\text{L/S}}^{\text{1/2}} + Z_{\text{L/S}}^{\text{1/2}}} \]

where the subscript \( L/S \) represents the load/source subsystem, the superscript of a two-digit number indicates the coordinate of element in the 2-d port IM, while the superscript of a one-digit number indicates the order of obtained equivalent 1-d port IM in (4). It can be verified that the sum of \( Z_{\text{L/S}}^{\text{1/2}} \) and \( Z_{\text{L/S}}^{\text{1/2}} \) is equal to the 1-d loop IM deduced from (1) and (3), hence (4) is also a source-load partition of the 1-d loop IM regardless of modeling in the \( dq \) and the sequence domain.

III. Stability Analysis Using IRCs

A. Simulation Results

For both cases in Table I, VSC is ensured to be in stable operation when powered by the ideal voltage source (the phase and amplitude of the voltage source are determined by the power flow with various grid inductors \( L_g \)), and IRCs are allowed to use from the conventional perspective [2]. Taking the condition of \( L_g = \max(L_g) \) as an example, by calculating the eigenvalues of \( A_{\text{diq}} \), Table II shows that no RHP pole exists in each element of port AM, which is a necessary and sufficient condition for the independent stable subsystems of operations.

| Case 1  | \(-1.32\pm j4.53\) | \(-0.40\pm j0.75\) | \(-0.09\pm j0.49\) | \(-8.91\) | \(-0.35\) |
|--------|--------------------|--------------------|--------------------|---------|---------|
| Case 2  | \(-7.67\pm j1.60\) | \(-1.13\pm j6.31\) | \(-0.26\pm j3.24\) | \(-0.62\pm j0.86\) |

The time-domain simulation is then performed in PSCAD to serve as a standard for theoretical analyses. Fig. 3 shows the waveform of DC voltage \( u_{\text{dc}} \) in case 1. The system becomes unstable if the grid \( L_g \) steps from 0.9 mH to 1 mH, and a similar phenomenon happens when \( L_g \) steps from 2.2 mH to 2.3 mH in case 2 (the waveform is neglected for brevity). Such phenomena conform to a general cognition that the system tends to be unstable due to the interaction between the phase-locked loop and weak grid, and can be analyzed using IRCs at the AC converter-grid interface since the MFs discussed below are observable to the unstable mode [5].

Fig. 3. Simulation waveforms of case 1.

B. Study on Case 1

Since the case-by-case simulation is time-consuming, a high-fidelity theoretical analysis is of practical requirements, and it is significant to deepen the understanding of mainstream IRCs. Nyquist plots in Fig. 4 and 5 are obtained by first calculating the matrix division of frequency response of source to load subsystems, and then calculate the eigenvalues of the matrix quotients. The legends of \( d, q, p, \) and \( n \) below represent the trajectories of each domain in 1-d IRCs with explicit physical meaning, while the legends of eig 1&2 represent the trajectories of eigenvalues in 2-d IRCs.
Fig. 4. Nyquist plots of case 1. (a) 1-d (sequence, p). (b) 2-d (sequence, eig 1). (c) 1-d (dq, d&q). (d) 2-d (dq, eig 1&2).

TABLE III

| Zs-Ps of MFs (×10^2, Case 1) |
|-----------------------------|
| Z   | ds | Zq   | 0.0018±j0.62 | -0.61±j4.07 | -0.27±j0.48 | -0.35 | -2.26 |
|     |     |      | -1.02±j4.72 | -0.38±j0.93 | -0.10±j0.49 | -0.35 | -2.19 |
|     |     |      | -0.94±j3.77 | -0.30±j0.44 | 0.10±j0.58  | -0.35 | -8.91 |
| Lg= | 1mH | P    | 0.0018±j3.76 | -0.61±j7.21 | -0.27±j3.62 | -0.35±j3.14 | -2.26±j3.14 |
|     |     | n    | 0.0018±j2.52 | -0.61±j0.93 | -0.27±j2.67 | -0.35 | -2.19 |
| p   |     |      | -0.52±j3.12 | -0.55±j7.18 | -1.72±j1.02 | -0.41±j3.76 |
|     |     |      | -0.068±j3.63 | -0.35±j3.13 | -0.28±j2.67 | 0.024±j2.48 |
| q   |     |      | -0.52±j3.17 | -0.55±j0.90 | -1.72±j7.31 | -0.41±j2.52 |
|     |     |      | -0.068±j2.65 | -0.35±j3.15 | -0.28±j3.61 | 0.024±j3.80 |
| Lg= | 0.9mH | P    | 0.015±j0.62 | -0.66±j4.10 | -0.27±j0.49 | -0.35 | -2.46 |
|     |     | n    | -1.05±j4.73 | -0.40±j0.91 | -0.10±j0.49 | -0.35 | -2.41 |
| p   |     |      | -0.95±j3.81 | -0.30±j0.45 | 0.002±j0.58 | -0.35 | -8.92 |
| q   |     |      | -0.015±j3.76 | -0.66±j0.96 | -0.27±j3.63 | -0.35±j3.14 | -2.46±j3.14 |
|     |     |      | -0.015±j2.52 | -0.66±j7.24 | -0.27±j2.65 | -0.35 | -2.46 |
| Lg= | 0.8mH | P    | -5.54±j3.10 | -0.58±j7.22 | -1.69±j1.03 | -0.41±j3.77 |
|     |     | n    | -0.068±j3.63 | -0.35±j3.13 | -0.27±j2.66 | 0.002±j2.48 |
| p   |     |      | -5.54±j3.19 | -0.58±j0.94 | -1.69±j7.32 | -0.41±j2.51 |
| q   |     |      | -0.068±j2.65 | -0.35±j3.15 | -0.27±j3.62 | 0.002±j3.80 |
| Lg= | 0.7mH | P    | 0.033±j0.62 | -0.71±j4.13 | -0.27±j0.51 | -0.35 | -2.70 |
|     |     | n    | -1.05±j4.73 | -0.40±j0.91 | -0.10±j0.49 | -0.35 | -2.41 |
| p   |     |      | -0.95±j3.81 | -0.30±j0.45 | 0.002±j0.58 | -0.35 | -8.92 |
| q   |     |      | -0.033±j3.76 | -0.71±j7.26 | -0.27±j2.64 | -0.35±j3.14 | -2.70±j3.14 |
|     |     |      | -0.033±j2.53 | -0.71±j0.98 | -0.27±j3.65 | -0.35 | -8.92 |
| Lg= | 0.6mH | P    | -5.60±j3.10 | -0.62±j7.26 | -1.65±j1.05 | -0.41±j3.78 |
|     |     | n    | -0.071±j3.63 | -0.35±j3.13 | -0.26±j2.64 | -0.022±j2.48 |
| p   |     |      | -5.60±j3.20 | -0.58±j0.98 | -1.69±j7.33 | -0.41±j2.50 |
| q   |     |      | -0.071±j2.65 | -0.35±j3.15 | -0.26±j3.65 | -0.022±j3.80 |

Fig. 4 illustrates the Nyquist plots of case 1. To improve the persuasiveness of graphical stability assessments, the Zs and Ps of the decoupled type-a MFs, i.e., MFs, are calculated and shown in Table III, and MFs is defined as:

\[
MF(s) = 1 + Zs^2 / Zq^2 = 1 + [n_s / d_q] / [n_q / d_s]
\]

\[
= [d_q(n_s + d_s(n_q))] / [d_s(n_q)], \text{ for } d \text{ or } p, 2 = q \text{ or } n.
\]

The numerator of MFs is the closed-loop characteristic polynomial of the system, whose roots are equal to the system eigenvalues. Please note that MFs is preferred for 1-d IM, since establishing the Smith-Macmillan form of transfer functions matrix is complicated, and due to the dimension explosion, even if using the MATLAB function minreal, it is unlikely to obtain the accurate zeros whose number should be equal to that state variables. Because of the nonzero non-diagonal elements of the source subsystem (i.e., the grid) port IM, minreal is unavoidable when calculating the Zs and Ps of the dq domain in Table III, and determining the tolerance of Z-P cancellation is a matter of trial and error. Besides, it is intuitive from Table III that the conjugate symmetries exist for two MFs mutually about jΩs in the sequence domain while separately about 0 in the dq domain, and an offset jΩs exists for the zeros between MFs in two domains. Therefore, when the frequency range is selected as –100 to 100 Hz for the sequence domain while –50 to 150 Hz for the dq domain, the consistency of stability assessments in the two domains and the feasibility of omitting a trajectory of the sequence domain in Fig. 4 and 5 are ensured.

With the bold RHP Zs and Ps in Table III and the arrows to show the directions of (–1, 0j) encirclement, the particularity of case 1 is analyzed as below:

1) For the sequence domain analysis, Fig. 4 (a) of 1-d IRC shows that there is one clockwise encirclement when \(L_g = 1\) mH but one counterclockwise encirclement when \(L_g = 0.9\) mH, both of (–1, 0j). One can conclude that the system is unstable when \(L_g = 1\) mH but cannot conclude that the system is stable when \(L_g = 0.9\) mH except by inspecting the RHP poles of MFs (or the open loop gain MF_c–1, the results are the same), and an RHP pole emerges for the 1-d IM when \(L_g = 1\) mH even if both subsystems can stable operate separately. Furthermore, if one avoids the 1-d IRC but focuses on the 2-d IRC, it is more confusing that the feature of trajectory does not change when \(L_g\) decreases from 1 mH to 0.9 mH in Fig. 4 (b). Here the instability when \(L_g = 1\) mH cannot even be ascertained since the trajectory encircles (–1, 0j) counterclockwise and the poles are unavailable.

2) For the dq domain analysis, Fig. 4 (c) of 1-d IRC shows that when \(L_g = 1\) mH, there are two clockwise encirclements of (–1, 0j) for the d-trajectory instead of the q-trajectory. One may infer that the d-axis control leads to the instability, but it is not rigorous since q-MFs owns two Zs and two Ps, and the no encirclement can also judge the system unstable, which is unusual for using IRC and further indicates the significance of acquiring RHP poles of MFs. Combining 1), the consistency of stability assessment using 1-d IRCs in two domains is proved by only a frequency shift of Zs in Table III. The rule of rest trajectories in Fig. 4 (c) and 4 (d) satisfies the general cognition of IRCs, which is related to a much simpler Z-P distribution in Table III.

3) The preference of selecting a certain intersection point between the trajectory and unit circle as the unstable mode
does not work in case 1, especially for the sequence domain analysis in Fig.4(a), since none of the frequency of intersection points corresponds to the simulated (50±9.9) Hz, and the phase margin derived by various IRCs are not consistent obviously. Such a risk on the system mode identification error should be realized in the frequency domain analyses, since IRCs can only judge whether or not the system is stable with no more accurate quantitative information offered theoretically.

C. Study on Case 2

To realize that the findings of case 1 are not general, case 2 is well-studied with a similar process to case 1. On the whole, it is intuitive for each type of IRC to judge the stability through Fig. 5 with the simpler Z-P distribution in Table IV, except the 2-d IRC in Fig. 5 (a) since the counterclockwise encirclement of (−1, 0j) can be observed, but the feature of trajectory changes when \( L_e \) decreases from 2.3mH to 2.2mH, which differs from Fig. 4 (b), and case 2 is similar with the usage of IRCs in [4] but cannot cover the special case 1. That is to say, only if the stable closed-loop system with no RHP pole can ensure the output of intersection points corresponds to the simulated (50±9.9) Hz, and the phase margin derived by various IRCs is intuitive for each type of IRC to judge the stability through quantitative information offered theoretically.

Fig. 5. Nyquist plots of case 2. (a) 2-d (sequence, eig 1). (b) 2-d (dq, eig 1&2). (c) 1-d (sequence, p). (d) 1-d (dq, d&q).

D. Summary

From the comparative study, even if 1-d IRCs of both domains correctly judge the stability, the unintended RHP poles of MF due to the source-load partition result in inconvenience for using IRCs, because the analyses will rely on accurate Z-P calculation and is not practical in application, which promotes a novel criterion using the frequency response of MF.

IV. LOOP IM-BASED STABILITY CRITERION

A. Basic Principle

The loop IM and be selected as another MF and defined as the type-b, i.e., MFb, since MFb and MFs hold the same closed-loop characteristic polynomial:

\[
\text{MF}_s(s) = Z_{L}^2 + Z_{L}^2 + n(s) / d(s) + n(s) / d(s)
\]

\[
= \{d(s) n(s) + d(s) n(s) / d(s) d(s) / n(s) \}/l = d \_p, p = q \_n.
\]

Rewriting MFb in a factored Z-P form gives:

\[
\text{MF}_b(s) = \prod_{l=1}^{i} a(s - Z) / \prod_{l=1}^{i} a(s - P), z = j\omega.
\]

(7)

where \( a \) is the flat gain. Since the essential scope of stability analysis is to identify a positive real part of \( Z \_b \), the basic unit of MFb, a first-order polynomial \( g(\omega) = a(\omega - \omega_0), \lambda = a \_p \_e \omega_0 \), is focused. Recalling that the criterion in [6] regards the rest of MFb as a whole, requires complex square root operations and irrational approximations, the logarithmic derivative acts on \( g_2 \) is to eliminate the effect of \( \omega_2 \) and equalize each basic unit:

\[
D_1(g_2) = d \log(g_1) / d\omega = d(g_1) / (g_1 d\omega)
\]

\[
= j/(\omega - \omega_0) = j[-\omega_0 + j(\omega - \omega_0)].
\]

(8)

The real-imaginary (Re-Im) separation projects a complex output of \( D_1(g_2) \) to two real outputs for quantitative analysis:

\[
\{\text{Re}[D_1(g_2) = (\omega - \omega_0) / (\omega - \omega_0)^2 + \omega_0^2]
\]

\[
\{\text{Im}[D_1(g_2) = \omega_0 / (\omega - \omega_0)^2 + \omega_0^2]
\]

\[
\Rightarrow \{\text{Re}[D_1(g_2)]_\omega = 0, \text{Im}[D_1(g_2)]_\omega = -1 / \omega_0.
\]

(9)

Eq. (9) shows that at \( \omega = \omega_0 \), a zero-crossing point exists for \( \text{Re}[D_1(g_2)] \). To better understand \( D_1(g_2) \) at \( \omega = \omega_0 \), the first and second derivatives of \( \{\text{Re}[D_1(g_2)] \) and \( \{\text{Im}[D_1(g_2)] \) are deduced:

\[
d^1\{\text{Re}[D_1(g_2)] \} / d\omega = -(\omega - \omega_0)^2 + \omega_0^2
\]

\[
d^2\{\text{Re}[D_1(g_2)] \} / d\omega^2 = -2(\omega - \omega_0) \lambda \omega_0 /
\]

\[
\Rightarrow d^2\{\text{Re}[D_1(g_2)] \} / d\omega^2 = -2(\omega - \omega_0)^2 \lambda \omega_0^2
\]

\[
\Rightarrow d^2\{\text{Im}[D_1(g_2)] \} / d\omega^2 = -8(\omega - \omega_0) \lambda \omega_0^2 + 2\lambda^2
\]

\[
\Rightarrow d^2\{\text{Im}[D_1(g_2)] \} / d\omega^2 = 2(\omega - \omega_0)^2 \lambda \omega_0^2 + 2\lambda^2
\]

(10)

Therefore, when \( a \_p \_e > 0 \), a negative minimum of \( \text{Im}[D_1(g_2)] \) and a positive slope of \( \text{Re}[D_1(g_2)] \) coexist at \( \omega = \omega_0 \), and a dual relationship satisfies for \( \text{Re}[D_1(1/g_2)] \) and \( \text{Im}[D_1(1/g_2)] \), where \( (1/g_2) \) can be deemed as a basic unit that contains a pole in MFb. The frequency selectivity induced by the inverse square term of \( \omega \) in (9) suppresses the interactions between each Z-P, thus the weak negative damping mode can be identified quite exactly using (9) and (10). By substituting MFb into \( D_1(l) \), the complete
process of applying the novel stability criterion is:

1) Obtaining the frequency responses of loop IMs, by either adopting the frequency sweep in analytical expressions or employing the data-driven method for black-box VSCs [5].

2) Calculating the logarithmic derivative of frequency responses based on the first line in (8), cooperating with the difference method (using a small step such as 0.01Hz).

3) The system is determined as unstable when a minimum of \( \text{Im}[D_L(g_z)] \) and a positive slope of \( \text{Re}[D_L(g_z)] \) coexist at the same frequency (\( \omega_0 = \omega_z \)), and \( \alpha_Z \) is identified using the output of \( \text{Im}[D_L(g_Z(\omega_z))] \) and (9).

The proposed criterion draws lessons from Section III that a 1-d MF is adequate to judge the stability. It avoids the influence of P’s on the stability analysis since it merely focuses on the parameter identification of Zs of MFs, and thus there is no need to partition MF to use 1-d IRC [4]. One more superiority of the proposed criterion is that the loop IM of MFs is not sensitive to the initial phase of ideal voltage source, and the basic idea can be extended to any other transfer-function based stability analysis as long a proper MF is selected.

B. Validations

The stability assessments of case 1 are illustrated in Fig. 6. It can be observed that the magnitude of each MF owns a pair of nadirs that are symmetrical about the central frequency (0 Hz for the sequence domain, 50 Hz for the \( dq \) domain, and the peaks/nadirs at \( \pm 50 \) Hz in Fig. 6 (b) and 6 (c) due to the calculation error of difference method are neglected). Consequently, it is more credible that using various MFs can achieve the same stability analysis result compared with the inconsistent feature of trajectories using IRCs in Fig. 4 (a) and 4 (c), which is an advantage of the proposed criterion.

![Fig. 6. Impedance characteristics with Re-Im parts of \( D_L(MF_b) \).](image)

(a) \( p. \) (b) \( d. \) (c) \( q. \)

The logarithmic derivatives and Re-Im separations are then performed on each MFs to check the stability. It is found that the aforementioned nadirs lead to the zero-crossing points of \( \text{Re}[D_L(MF_b)] \) with definite positive slopes when \( L_g \) varies, and a Z of each MF exists over the certain frequency range. Minimums when \( L_g=1 \)mH and maximums when \( L_g=0.9 \& 0.8 \)mH both for \( \text{Im}[D_L(MF_b)] \) are observed, and the extremes are very close in both domains, thus the system is judged as unstable only when \( L_g=1.8 \)mH no matter which MF is selected. In addition, \( \alpha_Z \) is estimated as 0.1821, –1.4749, and –3.2555 when dividing each extreme value by 2\( \pi \) (the x-axis represents \( f \) instead of \( \omega \) in Fig. 6) in the sequence domain, and the results match the theoretical calculation in Table III and the simulated divergence rate of \( u_d \) in Fig. 3. In a word, the mode is estimated accurately enough to judge the system unstable using the proposed criterion.

V. Conclusion

To deepen the understanding of a typical transfer function-based small-signal stability assessment, the well-known IRCs are comparatively studied using two cases in both \( dq \) and the sequence domain for converter-based AC systems. It is confirmed that the SISO analysis is successful as long as a proper MF is selected, which is proved by detailed Z-P calculations, and thus the MIMO analyses can be fully avoided. The source-load partition introduces uncertainties to the open-loop Ps even if the subsystems can operate stably when powered by ideal sources, and to avoid such influences on the stability assessment, the system modes implied by the closed-loop characteristic polynomial are especially focused. A novel loop IM-based criterion is proposed aided by the logarithmic derivative of MF frequency responses, which can identify the unstable mode successfully. Such ideas can be extended to the stability analysis of the multi-terminal AC/DC, the unbalanced AC, and the harmonically distorted AC systems in the future.

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