Theoretical description of power transfer in a magneto-acoustic resonator

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Abstract

We derive an analytical model for the power flows in a magnetoacoustic resonator. The resonator consists of a piezoelectric-magnetostrictive bilayer system that can function as an electromagnetic transducer. The derived model captures the dynamic magnetic influence onto the elastodynamics via an effective frequency dependent stiffness constant. This allows to calculate both the transducer magnetic and elastic power loss as well as its efficiency in function of the frequency while also considering the resonance conditions. The model is applied onto an example system consisting of piezoelectric ScAlN and magnetostrictive CoFeB. In addition, the influence of the magnetic material parameters onto the power and efficiency are determined by comparing CoFeB with Nickel and Terfenol-D magnetostrictive layers.

Keywords: Magnetoelasticity, magnetostriction, magnetoelastic resonator, piezoelectric-magnetostrictive composite
I. INTRODUCTION

In recent years, there has been a strong interest in magnetoelectric transducers that efficiently convert electrical signals into the magnetic domain, and vice versa. There are several types of magnetoelectric transducers such as inductive antennas\[1,2\], transducers based on spin-orbit torques\[3,4\] and spin transport torque\[5\]. However, all these mentioned transducers are current-based and therefore inevitably have associated Joule heating which reduces their efficiency when scaling down the device.

Another type of magnetoelectric transducer utilizes the combination of the piezoelectric and magnetostrictive effect. These transducers are based on voltages instead of currents, and therefore, are expected to have lower losses and thus higher efficiencies at nanoscale dimensions. Inside these voltage-based transducers, elastodynamics is generated via the piezoelectric effect which then couples to the magnetic domain via the magnetoelastic effect. Therefore, much research is spent on how the magnetization dynamics dynamically couples with the elastodynamics.

There are several approaches and device concepts that generate magnetization dynamics via dynamic strain. A first and common type for the generation of magnetization dynamics via strain consists of a magnetostrictive element on top of a piezoelectric substrate\[8\]. In this way, the substrate can be stretched or compressed by applying a voltage and the introduced strain then affects the magnetization. Similarly, other methods utilize a piezoelectric/magnetostrictive bilayer to generate magnetization dynamics via an applied voltage. In this case, the voltage induces strain in the full structure which consequently modifies the magnetic anisotropy in the magnetic layer\[9,10\]. However, the two previous approaches do not include elastodynamic resonance behavior that can exist inside the devices. Instead, they consider the strain amplitude as independent of the frequency and therefore, the resonance effect cannot be exploited.

In other works, piezoelectric-magnetostrictive magnetoelectric transducers that support elastodynamic resonances are studied. These devices consist of high overtone bulk acoustic resonators (HBARs) with an additional magnetostrictive layer in which the magnetization dynamics is generated or laminates consisting of piezoelectric and magnetostrictive elements\[13,21\]. Despite these works accurately describe the magnetoelastic coupling near resonance, they do not have simple expressions for the power flow and transducer efficiency or
do not include piezoelectric coupling\textsuperscript{21}.

In this work, we present a theory that describes the magnetoelastic coupling in a piezoelectric/magnetostrictive thin bilayer bulk acoustic resonator near resonance. Simple expressions for both the elastic power loss as well as the power transfer to the magnetic domain are derived and discussed. These power flows are then utilized to describe the transducer efficiency. Finally, a case study of a transducer is presented and the influence of the material and dimensional parameters are comprehensively discussed.

II. GEOMETRY AND CONFIGURATION

The structure consists of a piezoelectric/magnetostrictive bilayer of thickness $d + t$, as shown in Fig. 1. The lateral dimensions are assumed to be much larger than the thickness which allows to approximate the system as a 1D system with only spatial variation in the thickness direction. There are free elastic boundary conditions at the top and bottom of the structure, similar to an idealistic FBAR device.

The resonator can be excited by applying an electric field over the piezoelectric element which results in strain generation. This strain travels through the device and, consequently, also excites magnetization dynamics in the magnetostrictive medium.

There are several different configurations possible depending on the magnetization and electric field orientations. Some configurations give only weak second order coupling between the elastic and magnetic domain whereas others have strong first order coupling. In this paper, we study the case where the electric field is applied lateral along the x-direction and parallel with the static magnetization, as shown in Fig. 1. This configuration allows for shear elastic wave confinement along the structure thickness and results in strong first order magnetoelastic coupling\textsuperscript{12,22}.

III. BOUNDARY VALUE PROBLEM

In this section, first, the differential equations describing the dynamics inside the piezoelectric and magnetostrictive medium are derived. Thereafter, the boundary conditions are stated and the overall solution of the boundary value problem is presented. Then, this solution is utilised to calculate the magnetic and elastic power absorption as well as the
FIG. 1: Structure of the magneto-acoustic resonator

transducer efficiency.

A. Differential equation in piezoelectric

The constitutive relations of the piezoelectric medium are given by

\[
\sigma_{ij} = c^E_{ijkl} S_{kl} - e_{kij} E_k \quad (1)
\]

\[
D_i = e_{ikl} S_{kl} + \varepsilon^S_{ij} E_k \quad (2)
\]

with \( \sigma_{ij} \) the stress tensor components, \( c^E_{ijkl} \) the components of the stiffness tensor under constant electric field, \( S_{ij} \) the strain tensor components, \( e_{kij} \) the components of the piezoelectric constant tensor with units \([\text{C/m}^2]\), \( E_i \) the electric field components, \( D_i \) the electric displacement components and \( \varepsilon^S_{ij} \) the components of the permittivity tensor under constant strain.

For a laterally oriented electric field and piezoelectrics with tetragonal or hexagonal crystal symmetries, the piezoelectrically induced displacement is parallel with the electric field direction and thus also along the x-direction \cite{23}. Moreover, assuming spatially uniform variables in the lateral directions results in only the shear strain, \( S_{xz} = S_{zx} = \frac{1}{2} \frac{\partial u_x}{\partial z} \), to be different from zero because there is only variation in the z-direction. In this case, according to Eq. 1 there is also only one stress component different from zero given by

\[
\sigma_{xz} = \sigma_{zx} = c^E_{55} (2 S_{xz}) - e_{15} E_x
\]

with \( c^E_{55} \) the shear stiffness constant under constant electric field, \( e_{15} \) the piezoelectric coupling constant, and \( E_x \) the applied electric field. In the further part, the indices will be
omitted and replaced by the superscript $^p$ for the variables in the piezoelectric and $^m$ for the variables in the magnetostrictive medium.

The elastodynamic equation for the piezoelectric layer is given by

$$\rho^p \frac{\partial^2 u^p}{\partial t^2} = f^p_{el} \quad (4)$$

with $\rho^p$ the piezoelectric medium density, $u^p = u_x$ the displacement and $f^p_{el}$ the elastic body force given by

$$f^p_{el} = \frac{\partial}{\partial z} \sigma^p = \frac{\partial}{\partial z} \left[ c^p (2S^p) - cE \right] = c^p \frac{\partial^2 u^p}{\partial z^2}. \quad (5)$$

Substitution of this body force into the equation of motion results into the wave equation

$$\frac{\partial^2 u^p}{\partial t^2} = v^p \frac{\partial^2 u^p}{\partial z^2} \quad (6)$$

with $v^p = \sqrt{\frac{\rho^p}{\rho^p}}$ the phase velocity and dispersion relation $\omega = v^p k$.

Note that losses can be included by considering a complex stiffness constant $c^p_i = c^p_r + ic^p_i$ with $i = \sqrt{-1}$. The real part of the stiffness determines the wave’s wavelength and dispersion relation whereas the complex part results in wave damping. The complex part is typically some orders of magnitude lower than the real part.$^{23}$

The most common loss at GHz frequencies in piezoelectric media is viscoelastic damping for which the complex stiffness can be written as$^{23,25}$

$$c^p_i = \omega \eta \quad (7)$$

with $\omega$ the frequency and $\eta$ the materials viscosity constant. However, in a resonator, a more generic loss factor, that can be experimentally obtained, is often utilized, i.e. the $Q$-factor. This $Q$-factor can be related to the complex stiffness constant via$^{23}$

$$c^p_i = \frac{c^p_r}{Q}. \quad (8)$$

When considering losses, besides the stiffness, also the wavenumber becomes a complex quantity given by

$$k^p_i = \omega \sqrt{\frac{\rho^p}{c^p_r}} = \omega \sqrt{\frac{\rho^p}{c^p_r + ic^p_i}}. \quad (9)$$

The root of a complex number is given by

$$\sqrt{z} = \sqrt{r \frac{z + r}{|z + r|}} \quad (10)$$
with \( r \) the modulus of the complex number. Considering this identity and assuming that \( c_i^p \ll c_r^p \), the effective complex wavenumber can be rewritten as

\[
k_i^p \approx k_i^p - ik_i^p
\]

with

\[
k_i^p = \frac{c_i^p}{2c_r^p} k_r = \frac{k_r}{2Q}.
\]

**B. Differential equation in magnetostrictive**

In the magnetostrictive layer, there is an additional magnetoelastic body force \( f_{mel} \) besides the elastic body force \( f_{el}^m \). Hence, the elastodynamic equation of motion inside the magnetostrictive layer becomes

\[
\rho^m \frac{\partial^2 u^m}{\partial t^2} = f_{el}^m + f_{mel}.
\]

The elastic body force is given by

\[
f_{el}^m = \frac{\partial}{\partial z} \sigma^m = \frac{\partial}{\partial z} \left[ c_0^m (25m^m) \right] = c_0^m \frac{\partial^2 u^m}{\partial z^2}
\]

and the magnetoelastic body force is given by (see appendix A)

\[
f_{mel} = 2 \frac{B_1}{M_s^2} \begin{bmatrix} M_x \frac{\partial M_x}{\partial x} \\ M_y \frac{\partial M_y}{\partial y} \\ M_z \frac{\partial M_z}{\partial z} \end{bmatrix} + \frac{B_2}{M_s^2} \begin{bmatrix} M_x \left( \frac{\partial M_y}{\partial y} + \frac{\partial M_z}{\partial z} \right) + M_y \frac{\partial M_x}{\partial y} + M_z \frac{\partial M_y}{\partial z} \\ M_y \left( \frac{\partial M_x}{\partial x} + \frac{\partial M_z}{\partial z} \right) + M_x \frac{\partial M_y}{\partial x} + M_z \frac{\partial M_y}{\partial z} \\ M_z \left( \frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} \right) + m_x \frac{\partial M_z}{\partial x} + M_y \frac{\partial M_z}{\partial y} \end{bmatrix}.
\]

Considering only variation in the thickness direction, i.e. \( \frac{\partial M}{\partial z} \neq 0 \), gives

\[
f_{mel} \approx 2 \frac{B_1}{M_s^2} \begin{bmatrix} 0 \\ 0 \\ M_z \frac{\partial M_z}{\partial z} \end{bmatrix} + \frac{B_2}{M_s^2} \begin{bmatrix} M_x \frac{\partial M_x}{\partial z} + M_z \frac{\partial M_z}{\partial z} \\ M_y \frac{\partial M_y}{\partial z} + M_z \frac{\partial M_z}{\partial z} \\ 0 \end{bmatrix}.
\]

Further considering the linear magnetic regime, i.e. \( M_y, M_z \ll M_s \) and \( M_x \approx M_s \), allows to neglect the second order terms in the magnetization and results in

\[
f_{mel} \approx B \frac{\partial M_z}{\partial z} \hat{e}_x
\]
with \( B = B_2 \).

Via the linearised LLG equation and assuming the magnetostrictive layer much thinner than the acoustic wavelength, the magnetization in the magnetostrictive layer can be written in function of the shear strain (see appendix B)

\[
M_z = \frac{2B \gamma \omega_y}{\omega^2 - \omega_y \omega_z} S^m = \frac{B \gamma \omega_y}{\omega^2 - \omega_y \omega_z} \frac{\partial u^m}{\partial z}.
\]

with \( \omega_y = \omega_0 + i \omega \alpha \) and \( \omega_z = \omega_0 + \omega_M + i \omega \alpha \). Substitution of this into the magnetoelastic body force gives

\[
f_{\text{mel}} = \frac{B^2 \gamma \omega_y}{M_s (\omega^2 - \omega_y \omega_z)} \frac{\partial^2 u}{\partial z^2}.
\]

Substitution of the body forces, Eqs. 14 and 20, in the equation of motion Eq. 13 gives again the regular wave equation

\[
\frac{\partial^2 u^m}{\partial t^2} = v^m_t \frac{\partial^2 u^m}{\partial z^2}
\]

with the phase velocity \( v^m_t = \sqrt{\frac{c^m_t}{\rho^m}} \) and an effective stiffness

\[
c^m_t = c^m_0 + \frac{B^2 \gamma \omega_y}{M_s (\omega^2 - \omega_y \omega_z)} = c^m_0 + c^m_B.
\]

The normal stiffness constant \( c^m_0 \) is frequency independent whereas the magnetic contribution \( c^m_B \) is complex and frequency dependent. Moreover, by inspecting Eq. 22 it is seen that \( c^m_B \) has the same shape as the dynamic magnetic susceptibility. Hence, the magnetic contribution to the effective stiffness allows to treat the system as a regular lossy elastic medium with a complex and frequency dependent stiffness and, by this, takes into account the magnetic influence on the elastodynamic behavior.

To understand the magnetic power absorption, the effective stiffness is rewritten in a real and imaginary part \( c^m_t = c_r^m + ic_i^m \). Assuming weak magnetic damping, i.e. \( \alpha \ll 1 \), the real and imaginary part of the effective stiffness respectively become (see appendix B)

\[
c_r^m = c_0^m + \frac{B^2 \gamma \omega_0 (\omega^2 - \omega_r^2)}{M_s (\omega^2 - \omega_r^2)^2 + \omega^2 \alpha^2 (2\omega_0 + \omega_M)^2}
\]

and

\[
c_i^m = \frac{B^2 \gamma}{M_s (\omega^2 - \omega_r^2)^2 + \omega^2 \alpha^2 (2\omega_0 + \omega_M)^2} \omega \alpha (\omega_0^2 + \omega^2).
\]

Considering regular ferromagnetic material parameters, the magnetic contribution to the stiffness is much smaller than \( c_0^m \). Therefore, the real part of the magnetic contribution can be neglected and allows to approximate the real part by \( c_r^m \approx c_0^m \). Similarly, the modulus of
the effective stiffness can be approximated by $|c^m| = \sqrt{(c^m)^2 + (c^m)^2} \approx c^m$. Furthermore, similar to the complex part of the dynamic magnetic susceptibility, also the complex part of the stiffness $c^m$ reaches its maximum at the magnetic resonance frequency. This maximum is given by

$$c^m(\omega_m) = \frac{B^2 \gamma \omega_0}{M_\omega \alpha (2\omega_0 + \omega_M)}$$  \hspace{1cm} (25)

with $\omega_m = \sqrt{\omega_0(\omega_0 + \omega_M)}$ the magnetic resonance frequency.

Similar to the lossy piezoelectric medium, the complex stiffness of the magnetostrictive medium also results in a complex wavenumber given by

$$k^m = k^m_r - ik^m_i$$  \hspace{1cm} (26)

with

$$k^m_r = \omega \sqrt{\frac{\rho^m}{c^m}}$$  \hspace{1cm} (27)

and

$$k^m_i = \frac{c^m_i}{2c^m} k^m_r.$$  \hspace{1cm} (28)

C. Solution of boundary value problem

As shown in the previous sections, the dynamics is described by the wave equation for both the piezoelectric and magnetostrictive layer. Hence, a wave ansatz can be proposed for both layers. However, note that both layers have a different phase velocity and thus also a different wavenumber for a given frequency. Inside the piezoelectric, the displacement and strain ansatz respectively become

$$u^p(z) = A_1e^{ik^p_z} + A_2e^{-ik^p_z}$$ \hspace{1cm} (29)

$$S^p(z) = \frac{ik^p_z}{2} \left(A_1e^{ik^p_z} - A_2e^{-ik^p_z}\right).$$  \hspace{1cm} (30)

According to Eq. 3, the total stress in the piezoelectric layer then becomes

$$\sigma^p(z) = c^p_i(2S^p(z)) - eE$$ \hspace{1cm} (31)

$$= ic^p_i k^p_z \left(A_1e^{ik^p_z} - A_2e^{-ik^p_z}\right) - eE.$$ \hspace{1cm} (32)

Inside the magnetostrictive layer, the displacement and strain are given by

$$u^m(z) = A_3e^{ik^m_z} + A_4e^{-ik^m_z}$$ \hspace{1cm} (33)

$$S^m(z) = \frac{ik^m_z}{2} \left(A_3e^{ik^m_z} - A_4e^{-ik^m_z}\right).$$  \hspace{1cm} (34)
Similarly to the total body force, the total stress in the magnetostrictive layer has both an elastic and magnetoelastic contribution. The magnetoelastic stress tensor is derived in appendix A. Considering both contributions, the total stress becomes

\[ \sigma^m(z) = c_0^m(2S^m(z)) + \frac{B}{M_s} M_z \]

\[ = 2c_0^m S^m(z) + \frac{2B^2\gamma_\omega y}{M_s(\omega^2 - \omega_0^2\omega^2)} S^m(z) \]

\[ = i c^m k_t^m (A_3 e^{ik_t^m z} - A_4 e^{-ik_t^m z}) . \]  

The boundary conditions impose zero stress at the top surface, continuity of displacement and stress at the interface between piezoelectric and magnetostrictive and zero stress at the bottom surface, i.e.

\[ \sigma^p(z = d) = 0 , \]

\[ \sigma^p(z = 0) = \sigma^m(z = 0) , \]

\[ u^p(z = 0) = u^m(z = 0) , \]

\[ \sigma^m(z = -t) = 0 . \]

These four conditions allow to find the four constants \( A_1, A_2, A_3 \) and \( A_4 \). Subsequently inserting these constants into the Eqs. \(30\) and \(34\) gives expressions for the strain inside the device. In the piezoelectric layer, the strain becomes

\[ S^p(z) = eE \frac{c^m k^m \sin(k^m t) \cos(k^m z) + 2c^p k^p \sin \left( \frac{dk^p}{2} \right) \cos(k^m t) \cos \left( \frac{1}{2}k^p (d - 2z) \right)}{2c^m c^p k^m \cos(dk^p) \sin(k^m t) + 2(k^p)^2 k^p \sin(dk^p) \cos(k^m t)} = eE S^p_k , \]

whereas for the magnetostrictive layer, the strain is given by

\[ S^m(z) = eE \frac{k^m \sin^2 \left( \frac{dk^p}{2} \right) \sin(k^m(t + z))}{c^m k_t^m \cos(dk^p) \sin(k_t^m t) + c^p k_t^p \sin(dk^p) \cos(k_t^m t)} . \]

This last expression can be further simplified by considering the magnetic layer much thinner than the acoustic wavelength, i.e. \( k_t^m (t + z) \ll 1 \), and results in

\[ S^m(z) \approx eE(t + z) \frac{(k_t^m)^2 \sin^2 \left( \frac{dk^p}{2} \right)}{c^m k_t^m \cos(dk^p) \sin(k_t^m t) + c^p k_t^p \sin(dk^p) \cos(k_t^m t)} = eE(t + z) S^m_k . \]

Note that the strain in both layers is a complex number as the stiffness and wavenumber are both complex quantities.
IV. MAGNETIC POWER TRANSFER

The strain in the magnetostrictive layer results in a dynamic magnetoelastic field which generates magnetization dynamics. Hence, a specific amount of the total input energy is converted to the magnetic domain. This power transfer density to the magnetic system, originating from the magnetoelastic field, is given by

\[ p(z) = \frac{i\omega}{2} \mu_0 M_{\text{dyn}} \cdot H_{\text{mel}}^*. \]  

Here, \( i\omega \) originates from the power as time derivative, the factor \( 1/2 \) originates from taking the RMS value of the waves and the complex conjugate is utilised because we are working with complex waves. Considering the magnetization and magnetoelastic field from Eqs. 19 and 43 results in

\[ p(z) = \frac{i\omega\mu_0}{2} M_s H_{\text{mel},z} \]  

\[ = \frac{i\omega\mu_0}{2} \left( \frac{2B\gamma\omega y S_m}{\omega^2 - \omega y\omega_z} \right) \left( \frac{-2B(S_m^*)}{\mu_0 M_s} \right) \]  

\[ = -i \frac{2\omega B^2\gamma\omega y}{M_s(\omega^2 - \omega y\omega_z)} |S_m(z)|^2 \]  

\[ = -i 2\omega c_m^m |S_m(z)|^2. \]

This power density can be rewritten in a real and complex part, i.e. \( p(z) = p_r(z) + ip_i(z) \), in a similar way as was done for the effective magnetostrictive stiffness. The power absorption corresponds to the real part of the power whereas the reactive power oscillation corresponds to the imaginary part of the power. The total power absorption is obtained by integration of the real part over the magnetic volume. The power per unit area then becomes

\[ P_m = \int_{-t}^{0} p_r(z)dz = 2\omega c_m^m \int_{-t}^{0} |S_m(z)|^2 dz. \]

Substitution of Eq. 44 and performing the integration gives the total absorbed magnetic power

\[ P_m(\omega) = 2\omega c_m^m e^2 E^2 f^3 \frac{1}{3} |S_k^m(\omega)|^2. \]

Note that this expression has a strong frequency dependence because of the resonance behavior of both the magnetic and elastic system. The frequency behavior of the magnetic
system is captured in the complex part of the stiffness constant \( c_i^m(\omega) \) whereas the frequency behavior of the elastic system is captured in the strain magnitude factor \( S_k^m(\omega) \). At the magnetic resonance frequency, the power absorption per unit area is given by

\[
P_m(\omega_r^m) = \frac{2}{3} E^2 t^3 \frac{e^2 B^2 \gamma \omega_0}{M_s \alpha (2\omega_0 + \omega_M)} |S_k^m(\omega_r^m)|^2.
\]

(52)

This equation shows several interesting aspects of the magnetic power absorption in a piezoelectric-magnetostrictive bilayer. First of all, the magnetic power absorption increases quadratically with the applied electric field, \( P_m \propto E^2 \). Hence, further increasing the field results in much stronger magnetization dynamics until the non-linear regime is met. Besides the external field, also the piezoelectric constant \( e \) quadratically relates to the power, \( P_m \propto e^2 \). For the magnetic material parameters, \( B \), \( M_s \) and \( \alpha \), it is much harder to predict their influence as these parameters also indirectly influence the strain magnitude.

In section \([VII]\) the power is calculated for a specific use case and special attention is spent to the influence of the material parameters on the power absorption.

From a geometrical perspective, the power absorption strongly depends on both the magnetic and piezoelectric layer thickness. Similarly as for the magnetic material parameters, these thicknesses indirectly influence the strain magnitude and therefore no simple relation can be predicted. In section \([VII]\) this will be studied in more detail.

The maximum power transfer is obtained when the elastic resonance frequency matches the magnetic resonance frequency, \( c_i^m(\omega) \) and \( S_k^m(\omega) \) peak at the same frequency. However, it is difficult to tune the elastic resonance frequency without changing the structure dimensions. Fortunately, the magnetic resonance frequency can be easily tuned by modifying the external magnetic field \( H_0 \). In the following, an approximate expression for the external field is derived to achieve mutual resonance of both the elastic and magnetic system. By this, the frequency that gives the maximal power transfer is found.

Without losses, the strain denominator \( D(\omega) \) becomes zero at the resonance frequencies, giving infinite strain amplitude at resonance. However, when losses are considered, the denominator becomes a complex number. At resonance, the real part of the denominator becomes zero and the strain amplitude is still limited because of the finite complex part of the denominator. Hence, to find the resonance frequencies, the real part of the denominator has to be set equal to zero and solved for the frequency, \( P_m \propto E^2 \). Nevertheless, this expression is still rather cumbersome to solve and further simplifications
can be made. When the imaginary part of the complex stiffness and wavenumber is small as compared to the real part, its influence onto the resonance frequency can be neglected. Therefore, it is allowed to neglect the imaginary components of the stiffness and wavenumber, i.e. $c_i^m \approx 0$ and $c_i^p \approx 0$. As a result, the resonance condition becomes

$$c_r^m k_r^m \cos(k_r^m d) \sin(k_r^m t) + c_r^p k_r^p \sin(k_r^m d) \cos(k_r^m t) = 0.$$  \hspace{1cm} (53)

For a thin magnetic layer, i.e. $t \ll d$, the solutions of this equation, i.e. the resonance frequencies, can be approximated by

$$f_{r,el} \approx \frac{nv_r^p}{2(d + t)} \hspace{1cm} (54)$$

with $n$ the mode number. This corresponds to the resonance frequency of a single piezoelectric layer of thickness $d + t$.

To obtain the maximal power transfer, the overlap of the magnetic and elastic resonance is required. This can be obtained by tuning the magnetic resonance frequency via the external magnetic field. The condition for mutual resonance is

$$f_{el} = f_{m} \Rightarrow \frac{nv_r^p}{2(d + t)} = \frac{\sqrt{\omega_0(\omega_0 + \omega_M)}}{2\pi} \hspace{1cm} (55)$$

which results in the quadratic equation

$$\omega_0^2 + \omega_M \omega_0 - \left(\frac{n\pi v_r^p}{d + t}\right)^2 = 0.$$  \hspace{1cm} (56)

The (real) solution of this equation is given by

$$H_0 = \frac{-\omega_M + \sqrt{\omega_M^2 + 4 \left(\frac{n\pi v_r^p}{d + t}\right)^2}}{2\gamma_0}.$$  \hspace{1cm} (57)

Hence, this field needs to be applied to obtain mutual resonance of the elastic and magnetic system in a magnetoelastic resonator.

V. ELASTIC POWER LOSS

In the previous section, the magnetic power induced by the dynamic displacement was derived. Here, the viscoelastic power loss inside the resonator is determined. The generic expression for the elastic power density is given by

$$p_{el} = \frac{i\omega}{2} \sum_{i,j,k,l=1}^{3} S_{ij} (c_{ijkl} S_{kl})^*. \hspace{1cm} (58)$$
In the geometry discussed in this article, only the shear strain \( S_{xz} = S_{zx} \) is present and thus only this component contains power. For systems with tetragonal or hexagonal crystal symmetries, the power density contained in the shear strain becomes

\[
p_{el}(z) = i\omega \left( c_t^p |S^p(z)|^2 + c_0^m |S^m(z)|^2 \right) .
\]

(59)

Similarly as for the magnetic power, the power per unit area is obtained by integration over the structure thickness and gives

\[
P_{el, t} = i\omega \left( c_t^p \int_0^d |S^p(z)|^2 dz + c_0^m \int_{-t}^0 |S^m(z)|^2 dz \right) .
\]

(60)

\[
\approx i\omega c_t^p \int_0^d |S^p(z)|^2 dz
\]

(61)

\[
= i\omega (c_t^p - ic_t^m) e^2 E^2 \int_0^d |S^p_k(z)|^2 dz .
\]

(62)

Here, the approximation is valid because the magnetostrictive layer is much thinner than the piezoelectric layer. The real part of this expression represents the elastic power loss due to viscoelastic effects and is given by

\[
P_{el} = \omega E^2 c_t^p e^2 \frac{Q}{d} \int_0^d |S^p_k(z)|^2 dz .
\]

(63)

Similar as for the magnetic power absorption, also here a quadratic dependence on the applied electric field and piezoelectric constant is found, \( i.e. P_{el} \propto e^2 E^2 \). All the other parameters also indirectly influence the strain magnitude and thus no direct influence on the power can be predicted. Hence, numerical calculations need to be done to better understand the influence of the material parameters and resonator thickness.

VI. EFFICIENCY

The total input power consists of three main parts: the reactive power, elastic power loss and magnetic power transfer. The reactive part corresponds to the power that resonantly oscillates inside the transducer. This power does not get lost and needs only to be supplied in the transient regime of the resonator. The elastic and magnetic power losses are more
important to describe the transducer efficiency. The magnetic power loss is the desired power in the resonator. This is the part of the transducer input power that is transferred to the magnetic domain and then eventually gets dissipated in this domain. This power transfer is only present in the magnetostrictive layer. On the other hand, the elastic power loss represent the parasitics of the transducer which has to be minimized. The elastic losses are mainly present in the piezoelectric medium as this volume is much larger than the magnetostrictive volume.

The goal of the transducer is to transfer as much as possible of the active power to the magnetic domain and as less as possible dissipation. By comparing these power flows, the efficiency of the magneto-acoustic resonator can be determined

\[ \eta = \frac{P_m}{P_m + P_{el}}. \]  

(64)

This efficiency factor indicates the portion of the active power that is converted to the magnetic domain. The higher the efficiency, the better the transducer. From Eqs. 51 and 63, it is already seen that the efficiency is independent on the resonator area, piezoelectric constant \( e \) and appleid electric field \( E \). Hence, this leads to several interesting conclusions. First of all, this shows the scaling behavior that indicates that the transducer efficiency is unaffected by reducing the transducer area. Furthermore, also a reduction of the applied electric field or voltage does not harness the transducer efficiency. And surprisingly, also the piezoelectric constant does not affect the transducer efficiency. This indicates that the viscoelastic losses and Q-factor are more important to consider than the piezoelectric coupling constant when determining the piezoelectric material.

VII. USE CASE

To illustrate the derived model of power transfer in the magneto-acoustic resonator, we evaluate the equations for a piezoelectric-magnetostrictive bilayer. The piezoelectric material is ScAlN with the following material parameters obtained from literature: piezoelectric coupling constant \( e = 0.3 \text{ C/m}^2 \), mass density \( \rho_p = 3.5 \text{ g/cm}^3 \) and stiffness constant \( c_{55}^p = c^p = 100 \text{ GPa} \). The piezoelectric layer thickness is \( d = 200 \text{ nm} \). The \( Q \)-factor of the single piezoelectric layer is assumed to be \( Q = 10^3 \) and is used to determine the imaginary part of the piezoelectric stiffness via \( c_i^p = c_r^p/Q \).\textsuperscript{29} For the magnetostrictive layer, CoFeB is used
as main material to explain the different coupling regimes. In addition, also Nickel and Terfenol-D are analysed and compared to CoFeB to identify the influence of the material parameters. These material parameters are obtained from literature and summarised in Table I.

The frequency dependence of the CoFeB influence on the elastodynamics is seen from its effective stiffness $c_m$, which is plotted in Fig. 2. The blue dotted line corresponds to the normal CoFeB stiffness without magnetoelastic coupling, i.e. $c_m^0 = 70$ GPa, whereas the solid blue and red line respectively indicate the real and imaginary part of the effective stiffness when magnetoelastic coupling is considered. This figure shows that the magnetization dynamics perturbs the real part of the CoFeB stiffness and creates an additional imaginary component. This imaginary component accounts for the losses in the magnetic system and is about an order of magnitude lower than the real part. The frequency dependence and shape of the effective stiffness is the same as the shape of the dynamic magnetic susceptibility as the effective stiffness is linearly proportional with the susceptibility. As a result, the frequency band over which the magnetic effect is present strongly depends on the resonance width of the magnetic system which depends on the magnetic damping parameter $\alpha$. Note that this magnetic resonance width, and thus also the width of $c_i^m$, can differ from the elastic resonance width which is determined by the Q-factor. In conclusion, the magnetic perturbation to the stiffness is located in a narrow frequency band around the magnetic resonance frequency which corresponds to the peak of the imaginary part of the effective stiffness $c_i^m$. Note that this resonance frequency can be tuned by the external magnetic field $H_0$ to match the elastic resonance frequency of the structure.

The effective stiffness solely depends on material parameters and is not modified by
FIG. 2: Effective stiffness of CoFeB in function of the frequency. The blue dotted line is the normal CoFeB stiffness and the solid blue and red line respectively correspond to the real and imaginary part of the effective stiffness. The external applied field is 111.75 GHz which result in a magnetic resonance frequency at 10.96 GHz.

changing the structure dimensions. Nevertheless, the dimensional parameters influence the strain amplitude and volume integration which result in modified power transfers and transducer efficiencies. To illustrate this dimensional influence, the magnetic and elastic power transfer per area and squared electric field, i.e. $P_m/E^2$ and $P_el/E^2$, are plotted in function of CoFeB layer thickness in Fig.3. Here, the green and red line respectively correspond to the magnetic and elastic power absorption in the resonator. Note that for every thickness, the external magnetic field is modified such that the magnetic resonance frequency matches the elastic resonance frequency, resulting in maximal power absorption.

The elastic power is seen to gradually decrease whereas the magnetic power shows a maximum in function of the magnetic layer thickness. To explain this behavior, it is necessary to understand how the strain amplitude varies near resonance as the strain plays an important factor in both the magnetic as well as the elastic power.

At resonance, the strain amplitude reaches a peak value because the real part of the strain denominator becomes zero, i.e. $D_r(\omega_r) = 0$. This means that the strain amplitude at resonance is determined by the imaginary part of the strain denominator which reflects the losses in the system, i.e. $S(\omega_r) \propto 1/D_i$. In the magnetoelastic resonator, there are both elastic and magnetic losses that contribute to the imaginary part of the strain denominator.
When magnetic losses are low, the imaginary part of the strain denominator as well as the strain amplitude are determined by the elastic losses. This means that the strain amplitude is mainly determined by the properties of the piezoelectric layer and not by the magnetic material properties. By contrast, for high magnetic losses, the strain denominator and amplitude are determined by the magnetic layer properties.

Now the behavior of the power curves in Fig. 3 can be explained. For thin magnetic layer thicknesses, the magnetic power loss is low in comparison with the elastic power loss due to a small magnetic volume. This means that the magnetic contribution to the strain denominator can be neglected and thus results in large strain amplitudes that gives large elastic power absorption. For \( t = 0 \), the elastic power reaches the maximum as there is no magnetic material and the fundamental elastic mode in the piezoelectric is not disturbed by an additional layer. When increasing the magnetic layer thickness, the magnetic power absorption also increases mainly because the magnetic volume is increasing. When the magnetic power reaches a similar magnitude as the elastic power loss, then the magnetic contribution to the strain denominator cannot be neglected anymore which results in further decreasing strain amplitude and thus also decreasing elastic power. At some point, this decreasing strain amplitude also affects the magnetic power and counteracts the effect of increasing magnetic volume which results in a peak in the magnetic power in function of the layer thickness. Hence, both the magnetic as well as the elastic power are affected by changing the magnetic layer thickness because both depend quadratically on the strain amplitude inside the device. However, for the magnetic power, this decreasing strain counteracts the effect of increasing magnetic volume. The combination of the two effects leads to a maximum at around 20 nm CoFeB thickness. For \( t < 20 \) nm, the effect of increasing magnetic volume is more important whereas for \( t > 20 \) nm the effect of decreasing strain amplitude is dominant.

Figure 4 (a), (b) and (c) respectively show the magnetic power per unit area and squared electric field, elastic power per unit area and squared electric field and power transfer efficiency for CoFeB, Nickel and Terfenol-D. For Nickel and Terfenol, the same trends as for CoFeB can be seen, i.e. peak in magnetic power and consistent decreasing elastic power for increasing thicknesses. However, the maximum in magnetic power is shifted to lower thickness values. This means that the elastic and magnetic power loss are of similar magnitude for less magnetic volume for Nickel and Terfenol as compared to CoFeB. This difference between the materials can be understood from the complex part of the effective stiffness \( c_{im} \).
that reflects how much power can be absorbed for a given strain amplitude. When calculating the effective stiffness, it is seen that Terfenol-D and Nickel have a higher value than CoFeB because of their higher magnetoelastic coupling constant $B$. As a result, Terfenol-D and Nickel absorb more power for a given strain amplitude and thus require less magnetic volume to have equal elastic and magnetic power absorption.

From this, the effective stiffness is used to derive a Figure of Merit (FoM) for the magnetoelastic resonator. According to Eq. (24) the effective stiffness depends on the magnetic material parameters and the frequency. However, the resonance frequency is roughly the same for different magnetostrictive materials as the very thin magnetic layer does not have a big influence on the elastic resonance frequency. As a result, the frequency factor in $c^m_i$ can be omitted when comparing different magnetostrictive materials and the figure of merit (FoM) can be defined as

$$FoM = \frac{B^2 H_0}{M_s \alpha (2H_0 + M_s)}.$$  \hfill (65)

Materials with a higher FoM absorb more magnetic power with respect to elastic power and reach higher efficiencies for a specific thickness. To illustrate this, the external applied field, resonance frequency, power absorption, efficiency and FoM are calculated for a 20 nm thick CoFeB, Ni and Terfenol layer. The results are summarised in table [II]. From this table, it is seen that the resonance frequency for the three materials is roughly the same and the transducer efficiency correlates positively with the FoM. The highest efficiency and FoM is
CoFeB | Ni | Ter
--- | --- | ---
$B_0$ (mT) | 111.75 | 218.97 | 145.70
$f_{res}$ (GHz) | 10.96 | 10.84 | 10.83
$P_m/E^2$ ($\mu$W/V$^2$) | 1.94 | 0.53 | 0.442
$P_{el}/E^2$ ($\mu$W/V$^2$) | 0.57 | 0.01 | 0.0065
Efficiency (%) | 77 | 98 | 99
FoM ($MN^2/(Am^3)$) | 1.4 | 27 | 69

TABLE II: External applied field $B_0$, resonance frequency $f_{res}$, magnetic and elastic power absorption, efficiency and FoM for CoFeB, Nickel and Terfenol-D respectively. Piezoelectric and magnetostrictive layer thickness’s are 200 nm and 20 nm respectively.

obtained for Terfenol because of its high magnetoelastic coupling constant. Note that the $FoM$ gives insight into the efficiency but does not explain the absolute power absorption. To better understand the absolute power absorption, further investigation of the influence of the material parameters is required.

The density $\rho^m$, stiffness $c^m$ and saturation magnetization $M_s$ only differ a small factor between the different materials and thus lead to a minor influence in the power absorption difference between different materials. On the other hand, the magnetoelastic coupling constant $B$ and Gilbert damping constant $\alpha$ can differ more than an order of amplitude between the different materials and thus are expected to have a significant influence on the power differences. To better understand their effect, the magnetoelastic coupling constant and magnetic damping are artificially varied several magnitudes and their influence on the power absorption is determined. In all the following simulations, the magnetic layer thickness is kept constant at 20 nm.

In Fig. 5, the magnetic and elastic power absorption for a material with CoFeB parameters and varying magnetoelastic coupling constant is shown. From this figure, different regimes can be identified indicated by the different background colors. In the blue regime, for low $B$ values, there is a quadratic increase in the magnetic power and a quasi-constant value of the elastic power. In this regime, the elastic losses are dominant and thus determine the strain denominator at resonance. This means that the strain amplitude is determined by the piezoelectric properties and is independent of the magnetic system. Hence, increasing $B$
FIG. 4: Magnetic power, elastic power and efficiency in function of the magnetostrictive layer thickness for CoFeB (CFB), Nickel (Ni) and Terfenol-D (Ter).

does not affect the strain amplitude but increases $c_i^m$ and thus results in constant elastic and more magnetic power absorption, i.e. $P_m \propto c_i^m \propto B^2$. When further increasing the coupling constant, the magnetic and elastic power reach similar magnitudes, indicated by the yellow region in Fig. 5. As a result, the magnetic contribution to the imaginary part of the strain denominator cannot be neglected anymore and results in decreasing strain amplitudes for increasing $B$ values. This decreasing strain results in decreasing elastic power and slows down the increasing magnetic power until a maximum is reached. After this maximum, the decreasing strain has a stronger effect than the increasing magnetoelastic coupling, resulting in decreasing magnetic power.

At even more elevated values of the magnetoelastic coupling constant, in the purple regime, both the magnetic and elastic power reach a minimum and then start to increase again. This change is explained by considering the effective stiffness which consists of the normal CoFeB stiffness $c_0^m$ and a magnetic contribution $c_B^m$ as shown in Fig. 2. For normal CoFeB $B$ values around 7 MJ/m$^3$, this magnetic contribution is weak as compared to the normal CoFeB stiffness, i.e. $c_i^m/c_0^m \approx 0.03$. By contrast, for high $B$ values, the effective stiffness is strongly modified by the magnetic contribution, resulting in completely different elastodynamics in the magnetic layer. As a result, there is a strong mismatch between the magnetostrictive and piezoelectric layer which decouples the dynamics in the two layers from each other. This decoupling allows the increase in strain in the decoupled piezoelectric layer whereas the strain in the magnetostrictive layer slowly keeps decreasing. As a result, the strain discontinuity at the interface between the two layers grows with increasing $B$ values in this regime. In addition to the decoupling between the layers, there is also a splitting in res-
onance frequency for high $B$ values. This splitting originates from the magnetic contribution to the real part of the effective stiffness $c_r^{\text{eff}}$ that cannot be neglected anymore. As seen from Fig. 2, the real part has two antisymmetric peaks around the resonance frequency. These two peaks result in two shifted zeros in the real part of the strain denominator. The zeros in $D_r$ generate the resonance peaks and thus result in a splitting of the resonance frequency. This splitting is illustrated in Fig. 6 (a), (b) and (c) where respectively the strain amplitude, magnetic power and elastic power are mapped in function of the frequency and magnetoelastic coupling constant. At $B \approx 10 \text{ MJ/m}^3$, the frequency splitting becomes visible and results in two branches. Note that the real CoFeB magnetoelastic coupling constant is $B \approx 7 \text{ MJ/m}^3$ for which the splitting is not yet visible and which corresponds to the yellow regime in Fig. 5.

![Graph](image)

**FIG. 5**: Magnetic and elastic power for a material with CoFeB parameters and varying magnetoelastic coupling constant $B$. Three regimes are identified that are color encoded.

In Fig. 7 the influence of $B$ for CoFeB is compared with the other magnetostrictive materials. Here, Fig. 7 (a), (b) and (c) respectively show the magnetic power, elastic power and efficiency for materials with CoFeB, Ni and Terfenol parameters and for changing magnetoelastic coupling constants. From these plots, it is seen that the three materials show the same trends, i.e. a regime with dominant elastic losses, regime with similar elastic and magnetic losses and regime with dominant magnetic losses which results in decoupling of the two layers. In Fig. Figure 7 (c), the efficiency shows a strong increase with increasing $B$.
FIG. 6: Strain, magnetic power and elastic power in function of the frequency and magnetoelastic coupling constant for a material with CoFeB parameters. A clear resonance frequency is identified and splitting of this frequency is seen for elevated frequencies until a maximum is reached for the three materials. This maximum is located at the elastic power minimum as from that point, the two layers get decoupled and the elastic power starts to increase again. Considering the real magnetoelastic coupling constants indicated in table I, it is seen that the Terfenol efficiency is the highest followed by Nickel and then CoFeB efficiency. However, the absolute magnetic power transfer is the highest for CoFeB because this material is located in the yellow regime on Fig. 5 where the elastic and magnetic power are of similar order and thus result in high absolute strain and power values.

FIG. 7: Magnetic power, elastic power and efficiency in function of the magnetoelastic coupling constant for materials with CoFeB (CFB), Nickel (Ni) and Terfenol-D (Ter) parameters.

In Fig. 8 the magnetic and elastic power for CoFeB is calculated and plotted in function of the magnetic Gilbert damping constant $\alpha$. From the equations, the damping constant is seen to have to opposite effect as the magnetoelastic coupling constant $B$ because the
imaginary part of the stiffness constant is inversely proportional with the damping, \( i.e. \ P_m \propto c_i^m \propto B^2/\alpha \). This is also confirmed by comparing Figs. 8 and 5 from which a horizontal flip of the curves can be identified. The three different regimes are again color encoded and correspond to the same regimes as discussed above. This means that the low magnetic damping regime corresponds to the high \( B \) regime where the two layers are decoupled from each other and a frequency splitting was observed. Further increasing the damping results in the yellow regime where the magnetic power loss is of similar magnitude as the elastic power loss. At high damping values, the effective stiffness is low and result is low magnetic power loss. As a result, the elastic power loss becomes dominant resulting constant strain amplitude. This constant strain gives saturating elastic power and decreasing magnetic power with increasing Gilbert damping.

Besides the power amplitude, the magnetic damping also influences the resonance width of the system. In the blue regime where the elastic losses are dominant, the strain denominator and thus also the resonance width is determined by the piezoelectric layer properties, \( i.e. \) the Q-factor. However, in the yellow and blue regime, the magnetic contribution cannot be neglected anymore and the resonance width converges the the resonance width of the magnetic system, determined by the magnetic damping constant.

![Graph](image-url)

FIG. 8: Magnetic and elastic power for a material with CoFeB parameters and varying Gilbert damping constant \( \alpha \). Three regimes are identified that are color encoded.

Figure 9 (a), (b) and (c) respectively show the magnetic power, elastic power and efficiency for materials with CoFeB, Ni and Terfenol parameters and for changing Gilbert
damping constant. From these graphs, it is seen that the three materials show the same trends. Similarly as for the magnetoelastic coupling constant, also here the efficiency shows a maximum when the elastic power reaches the minimum.

![Graphs showing magnetic power, elastic power, and efficiency as functions of the Gilbert damping constant for materials with CoFeB (CFB), Nickel (Ni) and Terfenol-D (Ter) parameters.](image)

**FIG. 9:** Magnetic power, elastic power and efficiency in function of the Gilbert damping constant for materials with CoFeB (CFB), Nickel (Ni) and Terfenol-D (Ter) parameters.

In summary, three different regimes can be identified by comparing the elastic and magnetic power loss in the resonator. The actual working regime depends on the magnetic material properties. Besides these magnetic material parameters, another parameter that can vary quite broadly is the Q-factor of the piezoelectric element. The influence of this factor is visualised in Fig. 10. Figure 10 (a) shows the magnetic and elastic power per area and per squared electric field in function of the Q-factor for a 20 nm CoFeB magnetostrictive layer. For low Q values, the magnetic power strongly increases with increasing Q values whereas for high Q values the magnetic power saturates. On the other hand, the elastic power first increases and then decreases with increasing Q-factor. This behavior can be explained by considering the strain amplitude inside the device.

For low Q values, the elastic losses in the piezoelectric dominate. This means that the strain denominator and amplitude are determined by the piezoelectric properties such as the Q-factor itself. Hence, increasing the Q-factor results in decreasing imaginary part of the strain denominator and consequently increases the strain amplitude. This increasing strain quadratically increases the magnetic power as seen from Eq. 51, i.e. \( P_m \propto S^2 \). On the other hand, the elastic power also quadratically depends on the strain but also on the imaginary component of the stiffness which is inversely proportional to the Q-factor, i.e. \( P_{el} \propto c_i^p S^2 \propto S^2/Q \). As a result, the elastic power increases less strongly as compared...
to the magnetic power. When further increasing the Q-values, the magnetic and elastic power start to have similar intensities. Consequently, the strain amplitude is not solely determined by the Q-factor, and the magnetic contribution to the strain denominator cannot be neglected anymore. For even further elevated Q-values, the magnetic loss dominates the strain denominator and the strain amplitude solely depends on the magnetic layer properties and thus becomes independent of the Q-factor. As a result, in this regime, further increasing the Q value does not influence the strain anymore and thus the magnetic power is saturating. On the other hand, the elastic power still depends on the Q-factor via the complex stiffness and thus keeps decreasing for increasing Q values. This behavior explains the peak in the elastic power curve.

Figure 10 (b), (c) and (d) respectively show the magnetic power, elastic power and efficiency for CoFeB, Ni and Terfenol in function of changing piezoelectric Q-factor. From these graphs, it is seen that the three materials show the same trends. The efficiency strongly increases for increasing Q-values as expected from the increasing magnetic power.

VIII. CONCLUSION

In this work, the formal theory of power flow in an acoustic resonator consisting of a piezoelectric-magnetostrictive bilayer is presented. The differential equations describing the dynamics for both layers are derived and solved for free boundary conditions. The general solution is presented and utilised to derive expressions for both the magnetic and elastic power loss inside the device. In addition, also the efficiency in function of the frequency is determined. The magnetic power is shown to be strongly dependent on the complex part of the effective stiffness that captures the influence of the dynamic magnetization onto the elastodynamics. Besides this stiffness, both the magnetic and elastic power are shown to strongly depend on the strain inside the device.

In the second part of the work, an example bilayer structure is used to illustrate the influence of dimensional and material parameters onto the transducer efficiency. From this, different regimes where found depending on the ratio between the magnetic and elastic power loss. For dominant elastic losses, it was shown that the piezoelectric layer parameters determine the transducer efficiency. On the other hand, in the case where the magnetic losses where dominant, the magnetic material parameters strongly determine the transducer
FIG. 10: Magnetic power, elastic power for a 200/20 nm ScAlN/CoFeB bilayer with varying Q-factor. The magnetic power, elastic power and efficiency is compared for between different magnetostrictive materials, i.e. CoFeB (CFB), Nickel (Ni) and Terfenol-D (Ter).

efficiency. In addition also a figure of merit was defined that describes the transducer efficiency based on the effective magnetostrictive stiffness constant.

Appendix A: Magnetoelastic stress and body force

The magnetoelastic energy for a material with cubic (or higher) crystal symmetry is given by

\[ E_{\text{mel}} = \int_V \left[ \frac{B_1}{M_s^2} \sum_i M_i^2 S_{ii} + B_2 \sum_{i \neq j} M_i M_j S_{ij} \right] dV. \]  (A1)
Here, $S_{ij}$ are the strain components and $B_{1,2}$ are the magnetoelastic coupling constants. The magnetoelastic stress tensor then becomes

$$\sigma_{\text{mel},ij} = \frac{\delta E_{\text{mel}}}{\delta S_{ij}}.$$  \hspace{1cm} (A2)

Considering only the shear strain $S_{xz} = S_{zx}$ to be different from zero, as is the case in the configuration in this work, results in only shear stress $\sigma_{xz} = \sigma_{zx}$ to be different from zero. This stress component is then given by

$$\sigma_{\text{mel},xz} = \frac{B_2}{M_s^2} M_x M_z.$$ \hspace{1cm} (A3)

The body force is found by taking the derivative of the stress tensor and becomes

$$f_{\text{mel},i} = \frac{\partial}{\partial x_j} \frac{\delta E_{\text{tot}}}{\delta S_{ij}}.$$ \hspace{1cm} (A4)

**Appendix B: Magnetization in function of strain**

In this appendix, the amplitude of the magnetization dynamics in function of the strain amplitude is determined. The action of the strain onto the magnetization is captured by the magnetoelastic field given by

$$H_{\text{mel}} = -\frac{1}{\mu_0} \frac{\delta E_{\text{mel}}}{\delta M}$$ \hspace{1cm} (B1)

$$= -\frac{2}{\mu_0 M_s^2} \begin{bmatrix} B_1 S_{xx} M_x + B_2 (S_{xy} M_y + S_{zx} M_z) \\ B_1 S_{yy} M_y + B_2 (S_{xy} M_x + S_{yz} M_z) \\ B_1 S_{zz} M_z + B_2 (S_{zx} M_x + S_{yz} M_y) \end{bmatrix}.$$ \hspace{1cm} (B2)

with $\mathbf{m} = \mathbf{M}/M_s$ the normalised magnetization. For the considered geometry and configuration, only the shear strain $S_{xz} = S_{zx}$ is present which results in

$$H_{\text{mel}} = -\frac{2B_2 S_{zx}^m}{\mu_0 M_s^2} \begin{bmatrix} M_z \\ 0 \\ M_x \end{bmatrix}.$$ \hspace{1cm} (B3)

Note that in the further part of the text, the subscripts are removed to improve readability giving $B_2 = B$ and $S_{zx}^m = S^m$.

The other contributions to the effective magnetic field include the exchange field, demagnetization field and the external applied field $H_0$. However, when considering the magnetic
layer much smaller than the acoustic wavelength, i.e. \( k_m t \ll 1 \), then there is only very weakly varying strain over the magnetic layer thickness. This can be seen as a quasi-uniform magnetic excitation over the thickness and thus also results in very weak magnetization variation along the thickness direction. This allows to neglect the exchange interaction and approximate the demagnetization field by

\[
H_d = \begin{bmatrix} 0 \\ 0 \\ -M_z \end{bmatrix}. \quad (B4)
\]

Summing the external, demagnetization and magnetoelastic fields together results in the effective field

\[
H_{\text{eff}} = \begin{bmatrix} H_0 - \frac{2BS_m}{\mu_0 M_s} M_z \\ 0 \\ -M_z - \frac{2BS_m}{\mu_0 M_s} M_x \end{bmatrix}. \quad (B5)
\]

Inserting the effective field into the LLG equation and then linearising the equation gives

\[
i\omega \begin{bmatrix} M_y \\ M_z \end{bmatrix} = \begin{bmatrix} -(\omega_0 + \omega_M)M_z - 2BS_m\gamma \\ \omega_0 M_y \end{bmatrix} + i\omega_0 \begin{bmatrix} -M_z \\ M_y \end{bmatrix}. \quad (B6)
\]

Rewriting the equations allows to find the relation between the magnetization components and the strain

\[
M_y = -i\frac{2B\gamma\omega}{\omega_y\omega_z - \omega^2} S^m(z = 0)
\]

\[
= -i\frac{2B\gamma\omega}{\omega^2 - \omega^2 + i\omega(2\omega_0 + \omega_M)} S^m 
\]

\[
M_z = -\frac{2B\gamma\omega_y}{\omega_y\omega_z - \omega^2} S^m 
\]

\[
= -\frac{2B\gamma\omega_y}{\omega^2 - \omega^2 + i\omega(2\omega_0 + \omega_M)} S^m 
\]

with, \( \omega_y = \omega_0 + i\omega_0 \) and \( \omega_z = \omega_0 + \omega_M + i\omega_0 \). Assuming that \( \omega \ll 1 \), we can write

\[
\omega_y\omega_z \approx \omega_0(\omega_0 + \omega_M) + i\omega_0(2\omega_0 + \omega_M) = \omega_r^2 + i\omega_0(2\omega_0 + \omega_M) \quad (B11)
\]

with \( \omega_r^2 = \omega_0(\omega_0 + \omega_M) \) the magnetic resonance frequency.

Note that, from this it can be seen that the dynamic magnetization has the same weak thickness variation as the strain \( S^m(z) \).
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