1. Introduction

The Wolf–Rayet phenomenon is a stellar phenomenon. This means that its length scale is essentially that of the star, i.e. some $10^9$ cm. A typical nebular size is $10^{14}$ cm or higher, which immediately illustrates that it may not be easy to make the connection between the star and the nebulae and observations confirm this. In this paper I will try to discuss some ways in which the nebular properties could be changed by the presence of a [WR] central star.

The star’s *actio in distans* is through two means

1. Photons

   The ionizing photons from the star heat and ionize the gas. This can have non–trivial consequences for the shaping of the PN, as shown by Marten & Schönberner (1991), Mellema (1994), and Mellema (1995). However, although the spectral energy distribution of a [WR] star is quite different from that of a ‘normal’ central star of a PN (CSPN), I do not expect this difference to cause any big differences in the hydrodynamic structure of the PN. The detailed ionization structure can be quite different, as illustrated by the photo–ionization modelling of Crowther et al. (1999) for the nebulae M1–67, but the ways in which the onset of ionization modifies the density of the circumstellar matter through large or small scale ionization fronts will probably not be very different.

2. Stellar wind

   The stellar wind differs in two aspects from a normal CSPN wind

   a) **Mass loss rate.** The stellar wind from a [WR] star is generally a factor 10–100 more massive than from a normal CSPN, see
for instance Leuenhagen et al. (1998). This means that the wind luminosity ($\frac{1}{2}Mv^2$) and its momentum ($Mv$) will be a factor 10–100 higher. This should have an effect on the formation of the PN.

**b) Abundances.** The wind from a typical [WC] star consists for approximately 0% of H, 50% of He, 40% of C and 10% of O, abundances radically different from the usual cosmic or solar abundances. As a wind with these abundances starts interacting with the environment its radiative cooling behaviour will be very different from the standard case. Also this will have an effect on the shaping of the PN.

Apart from what we observe now the mass loss history of the star is also important for understanding the shaping. Did the star become a [WR] directly after the AGB or is it the result of a late or very late He-shell flash? In the first case, was the mass loss during the AGB different for the [WR] stars or did the event which turned the star into a [WR] star influence its mass loss? Which role does binarity play in [WR] systems, and are there accretion disks?

These are all questions to which the answer is difficult to give since even for normal PNe they are not always well known. The standard model to explain the formation of PNe is the Interacting Stellar Winds [ISW] model or sometimes called Generalized ISW [GISW] model. In this model the nebulae is formed from the interaction between the slow AGB wind and the faster post–AGB wind. The usual assumption (confirmed by observations) is that the AGB wind has an aspherical density distribution (disk or torus), which leads to the formation of an aspherical PNe. The ISW model goes back to the late 70-ies (Kwok et al. 1978).

In this paper I will discuss how the formation of a PN can be different for [WR] central stars, taking the ISW model as the basis. However, recently the ISW model has received some criticism which may be good to list here, even though we do not know how relevant these problems are for the PNe around [WR] stars.

− Although aspherically distributed circumstellar matter is commonly observed in PNe, it is not around AGB stars. The puzzle then is how and when did the mass loss turn from more or less spherical to aspherical. The ISW model does not address this point, it just assumes the asphericity. A complete model should include a mechanism for producing aspherical mass loss. Most observational evidence indicates that the transition to aspherical mass loss happens right at the end of the AGB.

− Jets and other collimated outflow phenomena are observed in some PNe. Although the ISW model can produce some degree of collimation,
it seems unlikely at this point that the observed jets can be explained by the ISW, especially since they often show point-symmetry.

- Point–symmetry is not only shown by jets and collimated outflow phenomena, but even by whole nebulae. This is indicative of changes in direction of the symmetry axis of the system. Again, this seems difficult to establish in the case of the ISW model, since this would require a warped density distribution in the AGB wind.

- Young PNe, and many pre-PNe show aspherical morphologies, often with very peculiar shapes (see for example Sahai & Trauger 1998; Ueta et al. 1999). Since the post–AGB wind only becomes energetic for high effective temperatures, it is unclear how the ISW model can produce the observed pre-PNe and young PNe. Also, some of the shapes of the young PNe seem to be in conflict with the ISW model; we see objects with several symmetry axes, wave function–like morphologies, etc.

To deal with these difficulties some alternatives and/or additions to the ISW model have been proposed. The role of companions (stellar or substellar) for establishing aspherical mass loss has become generally accepted, albeit more for lack of alternatives than solid observational proof. See Soker (1998) and references therein for ways in which gravitational interaction with a companion can lead to aspherical mass loss. To explain jets and jet-like phenomena accretion disks have been proposed, possibly around a companion object. Sahai & Trauger (1998) even suggested that a short jet phase is responsible for moulding the initially spherical AGB wind into a torus like density distribution. A third alternative is the existence of strong magnetic fields in the post–AGB wind which through hoop stresses can lead to the formation of aspherical PNe (Chevalier & Luo 1995). These ‘MHD models’ can possibly also explain jets and point-symmetry (Garcia–Segura et al. 1999).

Thus, the caveat is that the ISW model may not be the whole story, but since much remains unclear, I will in this paper still concentrate on the ISW model.

2. Observations

The observations of PNe around [WC] stars are reviewed by Górny in this volume. The summary is that there are few to no differences. The only differences to be noted are

- The average expansion velocity of the ionized material is observed to be somewhat higher than in the case of ‘normal’ PNe (Górny & Stasińska 1995).

- The line shapes in WR-PNe seem to require a higher value for the turbulent velocity component than in normal PNe (Geśicki & Acker 1996).
As we will see in the following sections both these effects can be understood from the differences between normal fast winds and those from [WR] stars.

The most puzzling aspect of the observations is the lack of correlation between [WR] stars and certain types of PN morphologies. All types of morphology (elliptical, bipolar, attached shells, etc.) are found around [WR] stars in approximately the same fractions as in normal PNe. Somehow one would expect the morphology of a PN to be dependent on the mass loss history of the star. For a [WR] star this must have been different since the entire H envelope was lost. Reversely, it has been shown that bipolar PNe are associated with more massive progenitors, and one would expect that the mass of the star is important to determine whether it becomes a [WR] star or not. Still there seems to be the normal fraction of bipolar PNe around [WR] stars.

The conclusion thus seems to be that the process which determines the shapes of PNe, i.e. the start of aspherical mass loss, is unrelated to the process which turns the star in a [WR]–type star, or at least is not influenced much by the way in which the star loses its envelope.

This is a rather amazing conclusion. If one for example considers a common envelope scenario, in which the entire envelope is ejected, one would think that the case in which all of the H–envelope is removed is more extreme than the one in which only part of the H-envelope is removed, which would lead to different morphologies of the subsequent PN. But the observations show that this is not the case.

This behaviour can be used as a test for proposed mechanisms to introduce asphericity in the AGB/post-AGB system. For example one might argue that mixing will be stronger in the case of faster rotation, and therefore some relation between nebulae around H-poor central stars and more extreme morphologies should be present. Since there is no such relation, rotation is a less likely mechanism to introduce asphericity.

Although useful in principle, the application of this test is complicated by the fact that the real mechanism for producing H-poor central stars is not fully understood. All the proposed mechanisms centre around thermal pulses (either a final pulse on the AGB or a late pulse in the post-AGB phase, see the contributions of Blöcker and Herwig in this volume), which play a marginal role in most of the proposed mechanisms for aspherical mass loss.

Also, the set of [WR]–PNe has not been studied in much detail. A more thorough investigation of image and kinematic data of individual nebulae may still reveal some differences, gone unnoticed when using catalogue data. A more detailed study of a sample of [WR]–PNe should be done before the above test becomes really hard, but such a study would be worth it.
3. Massive stellar winds

The typical fast wind of a normal CSPN has a mass loss rate in the range $10^{-9} - 10^{-7} \text{ M}_\odot \text{ yr}^{-1}$, whereas the for the [WR] stars the reported rates are $10^{-7} - 10^{-5} \text{ M}_\odot \text{ yr}^{-1}$, roughly a factor 100 higher. The wind velocities do not differ much, which is not surprising since for radiatively driven winds the terminal wind velocity is of order the escape velocity from the surface of the star.

The result of a more massive stellar wind is that the momentum ($\dot{M}v$) and energy ($\frac{1}{2} \dot{M}v^2$) input is a 100 times larger. The main effect of this is an increase in the expansion velocity of the nebula.

Nebulae formed by a stellar wind come in two types, depending on how well the stellar wind cools when it is shocked (see e.g. Lamers & Cassinelli 1999, Ch. 12.3). If this cooling is efficient enough to radiate away all the energy injected by the stellar wind, the wind–driven bubble is said to be ‘momentum–driven’. The structure of the bubble is as shown in Fig. 1a, the stellar wind fills the entire volume of the bubble and there is a thin cooling zone separating the stellar wind from the material swept up from the environment. In this case it is the ram pressure of the stellar wind which sweeps up a bubble.

If the cooling of the stellar wind is inefficient, a volume of hot, shocked fast wind material forms and can fill most of the volume of the bubble, with an inner shock lying relatively close to the star, see Fig. 1b. This type of bubble is said to be ‘energy–driven’, it is the thermal pressure of the
volume of hot shocked fast wind material which pushes the nebula into the environment.

There are of course intermediate cases, but in general the division between the two holds.

Kahn (1983) showed that for an energy–driven bubble the expansion velocity can be approximated by

\[ v_{\text{exp}} = \lambda v_{\text{slow}} \]  

where \( \lambda \) is given by the solution of the cubic equation

\[ \lambda(\lambda - 1)^2 = \frac{2}{3} \frac{M_{\text{fast}} v_{\text{fast}}^2}{M_{\text{slow}} v_{\text{slow}}^2}, \]  

which depends on the ratio of the two wind luminosities \( \frac{1}{3} M v^2 \). An alternative solution was presented by Koo & McKee (1992). They derive

\[ v_{\text{exp}} = \left( 1 + \left( \frac{2 \pi}{3} \frac{\Gamma_{\text{rad}} \xi}{M_{\text{fast}} v_{\text{fast}}^2} \right)^{1/3} \right) v_{\text{slow}}, \]

in which \( \Gamma_{\text{rad}} \) is the fraction of the energy injected by the stellar wind (in the case of no radiative losses equal to 1), and \( \xi \) a numerical constant of order unity (whose value is not given by the authors). These two solutions are equivalent.

For the momentum–driven case Kahn & Breitschwerdt (1990) found that

\[ v_{\text{exp}} = \left( 1 + \left( \frac{M_{\text{fast}} v_{\text{fast}}}{M_{\text{slow}} v_{\text{slow}}^2} \right)^{1/2} \right) v_{\text{slow}}, \]

which depends on the ratio of the two wind momentum rates \( \dot{M} v \).

Fig. 2 shows a plot of the expansion velocity for both cases as a function of \( \dot{M}_{\text{fast}} \) for fixed \( M_{\text{slow}}, v_{\text{slow}}, \) and \( v_{\text{fast}} \). One sees that the expansion velocity increases as a function of fast wind mass loss rate, but that the difference becomes large only for very high mass loss rates. For the chosen parameters the effect is stronger for the energy–driven case.

Figure 2 should not be overinterpreted. The spread in mass loss rates and velocities in a sample of PNe will make the correlation less clear, especially since aspherical nebulae will have different expansion velocities in different directions. Also, Fig. 2 neglects any evolution of the winds; we know that mass loss rates and wind velocities are changing throughout the post–AGB phase but in calculating the expansion velocities the fast wind properties are assumed to be constant (the actual assumptions are that
Figure 2. Expansion velocity of a spherical wind bubble for different values of the fast wind mass loss rate.

\((\dot{M}v)_{\text{fast}}\) is constant in time for the momentum–driven case and \((\dot{M}v^2)_{\text{fast}}\) for the energy–driven case. Still the conclusion is that we should not be surprised to find an on average higher expansion velocity for the nebulae around [WR] stars.

4. Radiative Cooling

As was outlined above the efficiency of the radiative cooling in the shocked fast wind determines the character of the wind–driven bubble. Since the cooling processes are mostly two body interactions, the cooling rate is proportional to the square of the particle density, \(n^2\). The dependence on the temperature is much stronger. For a gas of cosmic abundances, the cooling is very strong between temperatures of \(10^4\) to \(10^6\) K (see e.g. Dalgarno & McCray 1972). Since the immediate post–shock temperature for a strong shock is given by

\[
T_s = \frac{3}{16} \frac{\mu m_H}{k} v^2
\]  

this means that shocks with \(v\) between 30 and 400 km s\(^{-1}\) are likely to be radiative. The velocity \(v\) here is the pre–shock velocity in the frame in which the shock is stationary.

As a star evolves through the post–AGB phase its wind velocity will go up and mass loss rate down. This means that one expects the bubble to be initially momentum–driven and later make a transition to energy–
Kahn & Breitschwerdt (1990) analytically calculated the conditions for which this transition happens and found that for a wide range of parameters the bubble makes the transition from momentum to energy–driven when \( v_{\text{fast}} \sim 150 \text{ km s}^{-1} \). The reason for this is the strong dependency of the post–shock cooling time on the wind velocity

\[
t_{\text{cool}} = 0.255 \frac{(1 + \sqrt{\alpha})^2}{\alpha q} \frac{v_{\text{slow}}}{M_{\text{slow}}} v_{\text{fast}}^5 t^2,
\]

(6)

in which

\[
\alpha = \frac{\dot{M}_{\text{fast}} v_{\text{fast}}}{\dot{M}_{\text{slow}} v_{\text{slow}}},
\]

(7)

and \( q \) is a constant from the assumed analytical cooling function which has a value of \( 4 \times 10^{32} \text{ cm}^6 \text{ g}^{-1} \text{ s}^{-4} \) for normal cosmic abundances.

Mellema (1994) and Dwarkadas & Balick (1998) numerically calculated the evolution of wind–driven bubbles with evolving fast winds and confirmed this behaviour. Also when using detailed radiative cooling or a somewhat less accurate cooling curve, the bubble makes the transition from momentum–driven to energy–driven at a fast wind velocity of approximately 150 km s\(^{-1}\).

Dwarkadas & Balick (1998) investigated the momentum–driven phase somewhat more closely and found that during this phase the bubble is sensitive to the Nonlinear Thin Shell Instability (NTSI). The effects of this instability survive even beyond the momentum–driven phase as they found that bubbles which evolved through a momentum–driven phase possess a much more disturbed interior.

Another property of bubbles in the momentum–driven phase is that they are much more affected by asphericities in the fast wind. The reason for this is that the ram pressure of the fast wind is the driving force, so if the wind pushes harder in one direction than in another, the shape of the bubble will reflect this. During the momentum–driven phase it is possible to form an aspherical PNe using an aspherical fast wind and a spherical slow wind.

On the other hand, in the energy–driven phase, it is the thermal pressure of the hot shocked fast wind which drives the bubble and as thermal pressure is locally uniform, and any large scale pressure variations in the hot bubble are quickly smoothed out, possible asphericities in the fast wind will during the energy–driven phase only influence the shape of the bubble in a watered down manner, if at all.

The ‘canonical’ value of 150 km s\(^{-1}\) for the transition from one phase to the other depends on

- Density of the fast wind, and hence its mass loss rate (since cooling goes with \( n^2 \))
Details of the cooling process expressed by the value of $q$, which is partly determined by the abundances.

Since the winds from [WR] stars have both high densities and peculiar abundances one can expect the value of the transition velocity to change. To find out how much requires a detailed calculation taking into account the cooling rates of all the different ions, which as far as I know has never been attempted. An approach similar to that described in Raga et al. (1997) should work well for this kind of problem.

A simple estimate can be made using Eq. (6). Considering the ionic cooling curves from Cox & Tucker (1969) one can estimate that an increase of the abundance of C by a factor 500-1000 (and factors 6 and 50 for He and O respectively) will mean an increase in the cooling efficiency by a similar factor. Raising $q$ by $10^3$ and $\alpha$ by $10^2$ in Eq. (6) means that the critical velocity goes up with a factor 10 or so. In other words up to wind velocities of 1000 km s$^{-1}$ or even higher the bubbles around [WR] stars would be momentum–driven. This includes all late–type [WC] stars.

There are claims from bubbles around Pop. I WR stars that this is in fact the case (Chu 1982), but for the [WR] nebulae this question has as yet not been considered. The consequences would be more chaotic nebulae due to the Nonlinear Thin Shell Instability, and more extreme morphologies due to aspherical mass loss during the post-AGB phase.

For the first there is little indication in the observed morphologies, although more careful analysis may show otherwise. There is the reported need for a higher turbulent velocity needed to explain the line shapes from PN around [WR] stars (Geşicki & Acker 1996), which may be due to this effect.

As pointed out above, there is no evidence for the second effect. This absence of any correlation between [WR] stars and nebular morphology then implies that mass loss in the post–AGB phase does not deviate much from spherical.

5. Born–again Planetary Nebulae

One model for the formation of [WR] stars is the occurrence of a late to very late He–shell flash (Blöcker, this volume). This model seems at the moment to be able to explain the observed abundances the best. However, the fact that there appear to be no differences between the nebulae around [WR] and normal CSPN causes problems for this scenario.

The reason is that the formation of PN empties the region around the star. The fast wind blows a bubble which is almost as empty as interstellar space. Models show that a typical density inside a PN bubble is 1 to 10 cm$^{-3}$. If one assumes that a very late He–shell flash happens after
10^4 years and that the PN expands with a velocity of 30 km s^{-1}, one obtains a PN radius of \sim 10^{16} cm at the time of the flash, which means a total mass of about 10^{-8} M_\odot yr^{-1} interior to this first PN, insufficient to sweep up a second PN, which would require between 0.01 and 1 M_\odot.

One can think of three possible solutions to this problem. Firstly, ‘reuse’ the old PN. The implication of this would be that the PNe around born–again post–AGB stars are ‘old’, at least older than the apparent age of the star. In the case of A30, A78, and Sakurai’s object this certainly seems to be the case, but for the majority of [WR] stars this is not true, their PNe are very similar to those seen around stars which supposedly evolved straight off the AGB, implying that they have not suffered a late to very late He-shell flash.

Perhaps it would be possible to accrete part of the old PN back to the star. The scenario would be that when the fast wind stops, the pressure inside the nebula starts dropping and material from the swept–up nebula starts diffusing back in. There is little observational evidence for a process like this, but it is true that the Helix nebula is actually not as empty as one expects. The ‘hot bubble’ is filled with cometary knots and a more diffuse high ionization gas of a density of about 100 cm^{-3} (see e.g. Maeburn et al. 1998). The evolution of ‘old’ PNe has not been studied well and requires some more attention. Still it is doubtful that a new PN made out of the material of a diffused old PN would look the same as a ‘normal’ PN.

Thirdly, the star could lose 0.01 to 1 M_\odot with a slow velocity, this way mimicking AGB mass loss. Also this seems unlikely, since the stars do not have that much mass to lose.

In all, the (very) late He–shell model seems to be irreconcilable with the fact that the PNe around [WR] look so ‘normal’.

6. Conclusions

The [WR] phenomenon shows us again that it is most useful to consider a star and its circumstellar environment together. The observed properties of the nebula around these stars can help us in understanding the nature of the [WR] phenomenon, and at the same time properties of [WR] stars can help us understand the formation of PNe. To sum up the conclusions reached in this paper:

1. The higher average expansion velocity of PNe around [WR] stars can be understood as being due to the higher mass loss rates from the star.
2. The typical [WC] abundances are expected to lead to a longer lasting momentum–driven phase in the formation of the nebulae. This phase may last until the wind reaches velocities of 1000 km s^{-1}.
3. A longer lasting momentum–driven phase should lead to the nebulae becoming more affected by instabilities. This may be the explanation for the fact that a higher turbulent velocity is needed to explain the line shapes of PNe around [WR] stars.

4. A longer lasting momentum–driven phase allows asphericities in the stellar wind to have a larger effect on the shape of the PN. Since the observed nebulae show no sign of this, it implies that the post–AGB wind is mostly spherical.

5. Producing a second PNe in the case of a born–again PN scenario is difficult to impossible. Consequently, born–again PNe should show a discrepancy between the age of the PN and that of the star. Since this is not observed for most [WR] stars, the born–again scenario seems not to apply.

6. The lack of correlation between PN morphology and the [WR] phenomenon shows that the ultimate mechanism to produce aspherical PNe is unrelated to the process which produces the [WR] star.

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