Pauli Paramagnetism of Neutron Star Matter and the Upper Limit for Neutron Star Magnetic Fields

Soma Mandal\textsuperscript{a)} and Somenath Chakrabarty\textsuperscript{a),b)}

\textsuperscript{a}) Department of Physics, University of Kalyani, Kalyani 741 235, India and \textsuperscript{b}) Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune 411 007, India

(October 31, 2018)

Abstract

A relativistic version of Pauli paramagnetism for $n - p - e$ system inside a strongly magnetized neutron star has been developed. An analytical expressions for the saturation value of magnetic field strength for each of these constituents at which they are completely polarized have been obtained. From the fully polarized configuration of electronic component, an upper limit for neutron star magnetic field is predicted. It has been concluded that indeed, magnetars, as strongly magnetized young neutron stars can not exist if the constituents are electron, proton and neutron in $\beta$-equilibrium. An alternative model has been proposed.

1. INTRODUCTION

The study of the effect of strong quantizing magnetic field of neutron stars on dense nuclear matter has gotten a new dimension after the discovery of a few magnetars. These

\textsuperscript{*}E-Mail: soma@klyuniv.ernet.in

\textsuperscript{†}E-Mail: somenath@klyuniv.ernet.in
exotic objects are believed to be strongly magnetized young neutron stars of surface magnetic field \( \approx 10^{15} \text{G} \). The use of scalar virial theorem shows that the magnetic field strength at the core region may go up to \( 10^{18} \text{G} \). It is therefore very much advisable to study the effect of such strong magnetic field on various physical properties of dense neutron star matter as well as on various physical processes taking place inside neutron stars. An extensive studies have already been done on the equation of state of dense neutron star matter in presence of strong magnetic field. Such studies are based on quantum mechanical effect of strong magnetic field. The effect of strong quantizing magnetic field on the gross properties, e.g., the mass, radius, moment of inertia etc., of neutron stars, which are strongly dependent on the equation of state of matter have also been obtained. In the second kind of studies, how the weak processes (reactions and decays) are affected by the quantum mechanical effect of strong magnetic field have been obtained. As a consequence the \( \beta \)-equilibrium condition also depends on the strength of magnetic field. Since the cooling of neutron stars is dominated by the emission of neutrinos produced by weak processes inside the stars, these studies also give an idea of the effect of strong magnetic field on thermal evolution of neutron stars. Not only that, the presence of strong magnetic field can change significantly, both qualitatively and quantitatively the transport coefficients (viscosity, thermal conductivity, electrical conductivity etc.) of dense neutron star matter. The magnetic field can change the tensorial character of transport coefficients of neutron star matter in presence of strong magnetic fields. Such qualitative change in transport coefficients can cause some significant changes in thermal evolution of neutron star matter and the evolution of its magnetic field. There are another kind of studies; the effect of strong quantizing magnetic field on quark-hadron phase transition. It was shown explicitly that a first order quark-hadron phase transition is absolutely forbidden if the strength of magnetic field exceeds \( 10^{15} \text{G} \). However, a metal insulator type (color insulator to color metal) second order phase transition is possible unless the field strength
exceeds $10^{20}$ G. It has also been shown, that even if there is a first order quark-hadron phase transition for magnetic field strength $< 10^{15}$ G at the core region of a neutron star, an investigation of chemical evolution of quark matter, with various initial conditions leads to the system in $\beta$-equilibrium, revealed that the system becomes energetically unstable in chemical equilibrium \[18\]. In some completely different type of studies, the stability and some of the gross properties of deformed stellar objects are analyzed with general relativity \[19,20,21,22,23\]. The presence of strong magnetic field destroys the spherical symmetry of neutron star. Then it is possible for a deformed and rotating neutron star to emit gravity waves, which in principle may be detected. Very recently we have critically studied the ferromagnetism of neutron star matter which could be one of the sources of residual magnetism of old neutron stars/sources of magnetic field of millisecond pulsars. In these studies, we have shown that a spontaneous Ferromagnetic transition in absence of external magnetic field is not possible in neutron star matter in $\beta$-equilibrium. However in the case of neutrino trapped neutron star matter (Proto-neutron star matter), the possibility of such a transition can not be ruled out, provided the neutrinos carry some non-zero mass \[24\]. We have also analyzed the problem with the occupancy of zeroth Landau levels by electrons/protons, which occur in presence of ultra-strong magnetic fields \[25\]. It has been argued in this critical analysis that in presence of a strong quantizing magnetic field the existence of neutron star matter in $\beta$-equilibrium is questionable. Which further opens up a vital question on the possibility of magnetars as young and strongly magnetized neutron stars.

In this paper we shall present a relativistic version of Pauli paramagnetism of neutron star matter in $\beta$-equilibrium. We shall extend the work of Shul’man \[26\] to the relativistic region (i.e., in the high density regime of neutron star matter) and study the paramagnetism of relativistic nuclear matter. The aim of this paper is to show that the fully polarized configuration of electronic components puts a restriction on the upper limit of neutron star magnetic field. We have also discussed the alternative picture of neutron star structure, if
the magnetic field exceeds that limit. The paper is organized in the following manner. In section 2, we shall develop the formalism of relativistic version of Pauli paramagnetism, in section 3 we study the Pauli paramagnetism of neutron star matter and in the last section, we shall conclude and discuss the importance of this work.

2. FORMALISM

We consider a Fermion gas of density \( n \) in presence of a strong magnetic field \( B \) at zero temperature \( (T = 0) \). Then the number density is given by [26]

\[
n = \frac{[2\mu B(2\mu B + 2m)]^{3/2}(2x^{3/2} - 1)}{6\pi^2} \tag{1}
\]

where \( \mu \) and \( m \) are respectively the magnetic dipole moment and mass of the particle and

\[
x = \frac{\epsilon^2(B) - m^2}{2\mu B(2\mu B + 2m)} \tag{2}
\]

is a dimensionless quantity, known as the energy variable and

\[
\epsilon(B) = [m^2 + x[R_{3/2}(x)]^{-2/3}(\epsilon_0^2 - m^2)]^{1/2} \tag{3}
\]

is the single particle energy, \( \epsilon_0 \) is the value of \( \epsilon(B) \) for \( B = 0 \) and

\[
R_{3/2}(x) = \frac{1}{2}(2x^{3/2} - 1) \tag{4}
\]

Then it is a matter of simple algebraic manipulation to obtain the number densities with spin parallel and anti-parallel to the direction of magnetic field \( B \), and are given by

\[
n^\uparrow(B) = \frac{n}{2}x^{3/2}[R_{3/2}(x)]^{-1}
\]

\[
n^\downarrow(B) = n^\uparrow(B)(1 - x^{-3/2}) \tag{5}
\]

It is obvious from these two expressions that for \( x \to \infty \), for which \( B \to 0 \) (no field situation), \( n^\uparrow = n^\downarrow = n/2 \), which indicates that both up and down spin states are equi-probable. On
the other hand for $x \to 1$ we have $n^\uparrow = n$ and $n^\downarrow = 0$, which are the saturation densities (fully polarized scenario). In the case of spin saturation, the single particle energy is given by

$$
\epsilon(B) = \epsilon^{(s)}(B) = [2^{2/3} \epsilon_0^2 - (2^{2/3} - 1)m^2]^{1/2}
$$

(6)

Since for $x < 1$, the value of $n^\downarrow$ becomes negative, the lowest value of $x$ is 1 and the corresponding saturated value for magnetic field strength is given by

$$
B_s = \frac{1}{2\mu^2}[(6\pi^2n)^{2/3} + m^2]^{1/2} - m
$$

(7)

This is also the maximum value of magnetic field strength which the neutron star matter can sustain. In fig.(1) we have plotted the values of $n^\uparrow(B)$ and $n^\downarrow(B)$ (in terms of $n$) for a Fermi system against $x$. The upper curve is for $n^\uparrow(B)$ and lower one corresponds to $n^\downarrow(B)$. It is obvious from the figure that $x=1$ corresponds to fully polarized scenario, whereas $x \to \infty$ corresponds to completely unpolarized picture.

### 3. Pauli Paramagnetism of Nuclear Matter

We next consider a $n-p-e$ system inside a neutron star in presence of a strong magnetic field at $T = 0$ (since the chemical potential $\mu_i\ast$ for the $i$th species is $>>$ the temperature of the system $T$). Now in presence of a strong magnetic field, the chemical potential for the $i$th component is given by

$$
\mu_i^\ast = \epsilon_i(B) - \mu_iB
$$

(8)

where $i = n, p, \text{or } e$. The charge neutrality condition gives $n_p = n_e$ where $n_i$ is the number density of $i$th species. Finally, the baryon number density is given by

$$
n_B = n_n + n_p
$$

(9)
which we treat as a constant parameter. It has been noticed from some critical investigations that the $\beta$-equilibrium condition can not be achieved in a polarized neutron star matter ($n - p - e$ system in presence of a strong magnetic field [25]). We have assumed some arbitrary iso-spin symmetry, defined by the parameter

$$\beta = \frac{n_n - n_p}{n_n}$$

(10)

with $0 \leq \beta \leq 1$ (since the matter with excess protons is highly unstable, we have not considered negative values of $\beta$). Then combining eqns.(9) and (10) we have

$$n_n = \frac{n_B}{2 - \beta} \quad \text{and} \quad n_p = n_n(1 - \beta)$$

(11)

Hence after a little algebraic manipulation, we have from eq.(2), the dimensionless energy variable for the species $i$

$$x_i = \left[0.5 + \left\{ \frac{2\mu_i B(2\mu_i B + m_i)}{(3\pi^2 n_i)^{2/3}} \right\}^{-2/3} \right]^{3/2}$$

(12)

It clearly shows that the dimensionless energy variables for the constituents are functions of magnetic field strength $B$, density of matter $n_B$ and the iso-spin symmetry parameter $\beta$. We have taken $\mu_p = 2.79\mu_N$, $\mu_n = -1.91\mu_N$ and $\mu_e = 1.001\mu_B$, where $\mu_N = 3.15 \times 10^{-14}\text{MeV}/T$ and $\mu_B = 5.79 \times 10^{-11}\text{MeV}/T$ are respectively the nuclear magneton and Bohr magneton. In figs.(2), (3) and (4) we have shown that the variations of $x_i$ ($i = n, p, e$) with $B$ for $n_B = 2n_0$ and $\beta = 0, 0.5, 1.0$ (to avoid some numerical problem we have taken $x = 0.99$ instead of 1.0). All these figures show that within the range of magnetic field strength relevance for magnetized neutron stars, neutrons and protons never become fully polarized ($x_n, x_p >> 1$) whereas the value of $x_e$ is very close to unity which means that the electrons are in very close to fully polarized configuration. In figs.(5), (6) and (7) we have plotted the same quantities for $n_B = 6n_0$. In these sets of figure also very close to fully polarized states for electrons is possible whereas neutrons and protons remain in the states with almost unpolarized configuration. However both the neutron matter and the proton matter can be made fully polarized by increasing the strength of magnetic field.
4. CONCLUSIONS

We have noticed that although the electronic component becomes fully polarized for $B \sim 10^{16}\text{G}$, the nuclear matter can only become fully polarized if and only if the magnetic field strength exceeds $10^{20}\text{G}$. We have also observed that for such strong magnetic fields make $n_e^\uparrow$ negative, which is completely unphysical. Therefore, the fully polarized configuration of electronic component restricts the upper limit of a neutron star magnetic field to $\approx 10^{16}\text{G}$. However the inclusion of pions ($\pi^-$) or kaons ($K^-$) instead of electrons does not impose any restriction on the upper limit. At the same time the inclusion of these two components instead on electrons allow the $n - p$ system to become fully polarized even in $\beta$-equilibrium condition. We can therefore conclude that the magnetars, if they exist at all and are assumed to be strongly magnetized young neutron stars, the constituents are possibly $n - p - \pi^-$ or $n - p - K^-$ instead of $n - p - e$. Otherwise the existence of such objects is impossible. The incorporation of interaction in nuclear matter sector does not change the conclusion. The observed maximum polarization in the electronic sector whereas almost unpolarized configuration of nuclear matter regime is because of several orders of magnitude difference in the magnitudes of magnetic dipole moments. In the case of electron it is $\sim$ Bohr magneton, whereas in the case of nucleons it is $\sim$ nuclear magneton. This is independent of the type of interaction in the neutron star matter.
FIG. 1. The variation of $n^+$ and $n^-$ (in terms of $n$) with $x$.

FIG. 2. Variation of dimensionless energy variables $x_i$'s with $B$, expressed in terms of $B_c^{(e)}$, for $n_B = 2n_0$ and $\beta = 0$. 
FIG. 3. Variation of dimensionless energy variables $x_i$'s with $B$, expressed in terms of $B_{e}^{(e)}$, for $n_B = 2n_0$ and $\beta = 0.5$.

FIG. 4. Variation of dimensionless energy variables $x_i$'s with $B$, expressed in terms of $B_{e}^{(e)}$, for $n_B = 2n_0$ and $\beta = 0.99$. 
FIG. 5. Variation of dimensionless energy variables $x_i$'s with $B$, expressed in terms of $B_c^{(e)}$, for $n_B = 6n_0$ and $\beta = 0$.

FIG. 6. Variation of dimensionless energy variables $x_i$'s with $B$, expressed in terms of $B_c^{(e)}$, for $n_B = 6n_0$ and $\beta = 0.5$. 
FIG. 7. Variation of dimensionless energy variables $x_i$'s with $B$, expressed in terms of $B_{\xi}^{(e)}$, for $n_B = 6n_0$ and $\beta = 0.99$. 
REFERENCES

[1] R.C. Duncan and C. Thompson, Astrophys. J. Lett. 392, L9 (1992); C. Thompson and R.C. Duncan, Astrophys. J. 408, 194 (1993); C. Thompson and R.C. Duncan, MNRAS 275, 255 (1995); C. Thompson and R.C. Duncan, Astrophys. J. 473, 322 (1996).

[2] P.M. Woods et al., Astrophys. J. Lett. 519, L139. (1999)

[3] C. Kouveliotou, et al., Nature 391, 235 (1999); K. Hurley, et al., Astrophys. Jour. 442, L111 (1999).

[4] S. Mereghetti and L. Stella, Astrophys. Jour. 442, L17 (1999).

[5] J. van Paradijs, R.E. Taam and E.P.J. van den Heuvel, Astron. Astrophys. 299, L41 (1995); S. Mereghetti, astro-ph/9911125; see also A. Reisenegger, astro-ph/0100301.

[6] S. Chandrasekhar and E. Fermi, Astrophys. Jour. 118, 116 (1953); D. Lai and S.L. Shapiro, Astrophys. Jour. 383, 745 (1991).

[7] S. Chakrabarty, D. Bandopadhyay and S. Pal, Phys. Rev. Lett. 78, 2898 (1997); D. Bandopadhyaya, S. Chakrabarty and S. Pal, Phys. Rev. Lett. 79, 2176 (1997); D. Bandopadhyaya, S. Chakrabarty, Pranyick Dey and S. Pal, Phys. Rev. D58, 121301 (1998)

[8] C.Y. Cardall, M. Prakash and J.M. Lattimer, astro-ph/0011148

[9] Y.F. Yuan and J.L. Zhang, Astrophys. J. 525, 920 (1999).

[10] A. Broderick, M. Prakash and J.M. Lattimer, Astrophys. Jour., 537, 351 (2000).

[11] S. Chakrabarty and P.K. Sahu, Phys. Rev. D53, 4687 (1996); K. Konno, gr-qc/0105013.

[12] C.Y. Cardall, M. Prakash and J.M. Lattimer, astro-ph/0011148 and references therein; E. Roulet, astro-ph/9711206; L.B. Leinson and A. Pérez, astro-ph/9711216; D.G. Yakovlev and A.D. Kaminkar, The Equation of States in Astrophysics, eds. G. Chabrier and E.
Schatzman P.214, Cambridge Univ. Press, 1994.

[13] V.G. Bezchastrov and P. haensel, astro-ph/9608090.

[14] D.G. Yakovlev and D.A. Shalybkov, Sov. Astron. Lett. 16, 86 (1990).

[15] D.G. Yakovlev and D.A. Shalybkov, Astrophys. & Space Sc. 176, 171 (1991); ibid 176, 191 (1991).

[16] Sutapa Ghosh, Sanchayita Ghosh, Kanupriya Goswami, Somenath Chakrabarty, Ashok Goyal, Int. Jour. Mod. Phys. D 2002 (in press).

[17] S. Chakrabarty, Phys. Rev. D51, 4591 (1995); S. Chakrabarty, Phys. Rev. D54, 1306 (1996).

[18] T. Ghosh and S. Chakrabarty, Phys. Rev. D63, 043006-1 (2001); T. Ghosh and S. Chakrabarty, Int. Jour. Mod. Phys. D10, 89 (2001).

[19] A. Melatos, Astrophys. Jour. 519, L77 (1999); A. Melatos, MNRAS 313, 217 (2000).

[20] S. Bonazzola et al, Astron. & Atrophysics. 278, 421 (1993).

[21] M. Bocquet et al, Astron. & Atrophysics. 301, 757 (1995); Shin Yoshida, astro-ph/0207118.

[22] B. Bertotti and A.M. Anile, Astron. & Atrophysics. 28, 429 (1973); C. Cutler and D.I. Jones, gr-qc/0008021; K. Konno, T. Obata and Y. Kojima, gr-qc/9910038; A.P. Martinez et al, hep-ph/0011399; M. Chaichian et al, Phys. Rev. Lett. 84, 5261 (2000); Guangjun Mao, Akira Iwamoto and Zhuxia Li, astro-ph/0109221.

[23] A. Melatos, Astrophys. Jour. 519, L77 (1999); A. Melatos, MNRAS 313, 217 (2000); R. González Felipe et al, astro-ph/0207150 and references therein.

[24] S. Ghosh, S. Mandal and S. Chakrabarty, astro-ph/0205445.
[25] S. Ghosh, S. Mandal and S. Chakrabarty. astro-ph/0207492

[26] G.A. Shul'man, Sov. Phys. Astron. 35, 50 (1991).