EFFECTS OF LEAKAGE NEUTRAL PARTICLES ON SHOCKS

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Received 2012 February 21; accepted 2012 September 3; published 2012 October 4

ABSTRACT

In this paper, we investigate effects of neutral particles on shocks propagating into the partially ionized medium. We find that for $120 \text{ km s}^{-1} < u_{\text{sh}} < 3000 \text{ km s}^{-1}$ ($u_{\text{sh}}$ is the shock velocity), about 10% of upstream neutral particles leak into the upstream region from the downstream region. Moreover, we investigate how the leakage neutral particles affect the upstream structure of the shock and particle accelerations. Using four-fluid approximations (upstream ions, upstream neutral particles, leakage neutral particles, and pickup ions), we provide analytical solutions of the precursor structure due to leakage neutral particles. It is shown that the upstream flow is decelerated in the precursor region and the shock compression ratio becomes smaller than without leakage neutral particles, but the total compression ratio does not change. Even if leakage of neutral particles is small (a few percent of total upstream particles), this smaller compression ratio of the shock can explain steep gamma-ray spectra from young supernova remnants. Furthermore, leakage neutral particles could amplify the magnetic field and heat the upstream region.

Key words: acceleration of particles – cosmic rays – ISM: supernova remnants – plasmas – shock waves

Online-only material: color figures

1. INTRODUCTION

Supernova remnants (SNRs) are thought to be the origin of Galactic cosmic rays (CRs). The most popular SNR acceleration mechanism is diffusive shock acceleration (DSA; Axford et al. 1977; Krymsky 1977; Bell 1978; Blandford & Ostriker 1978). In fact, Fermi and AGILE observed GeV gamma rays due to CRs from SNRs (e.g., Abdo et al. 2009, 2010; Tavani et al. 2010; Giuliani et al. 2011; Giordano et al. 2012). However, gamma-ray spectra from SNRs are steeper than that expected from the standard DSA theory. The steep spectra can be interpreted as effects of energy-dependent escape (Ptuskin & Zirakashvili 2005; Ohira et al. 2010; Caprioli et al. 2010) and diffusion (e.g., Ohira et al. 2011) for middle-aged SNRs ($\sim 10^4$ yr old). For young SNRs ($\lesssim 10^3$ yr old), some ideas to explain the steep spectra have been proposed (Kirk et al. 1996; Zirakashvili & Ptuskin 2009; Ohira et al. 2009; Ohira & Takahara 2010; Bell et al. 2012), but it is still an open issue.

On the other hand, some authors considered effects of neutral particles (such as hydrogen atoms) on particle accelerations and shock structures. SNR shocks propagating into a partially ionized medium have been observed as Balmer-dominated shocks (Chevalier & Raymond 1978). Moreover, X-ray synchrotron radiation has been observed from the Balmer-dominated shocks (Cassam-Chenaï et al. 2008; Helder et al. 2009). A neutral fraction of the interstellar medium around SNRs is often found to be order of unity (Ghavamian et al. 2000, 2002). Neutral particles reduce growth rates of CR streaming instabilities, which are indispensable for DSA (Drury et al. 1996; Reville et al. 2007). In contrast, ions produced from the neutral particles trigger other plasma instabilities and are important for the injection into particle accelerations (Ohira et al. 2009; Ohira & Takahara 2010). A recent review of Balmer-dominated shocks can be found in Heng (2010).

Interactions between neutral particles and ions have been directly observed in the solar wind. There are two peculiar particles, energetic neutral atoms (McComas et al. 2009) and pickup ions (Gloeckler et al. 1993) in the solar wind. Although their origin has not been completely understood, an attractive idea has been proposed. Neutral particles penetrate into the inner solar system from the surrounding local interstellar medium. The neutral particles have a drift velocity comparable to the solar wind velocity in the rest frame of the solar wind. After they are ionized by charge exchange and photoionization, they gyrate around magnetic field lines of the solar wind and their mean velocity becomes the solar wind velocity in the observer frame, so that they become pickup ions with a large velocity dispersion in the solar wind. After passing over the termination shock, some pickup ions become neutral atoms by charge exchange. The neutral atoms can propagate toward the sun and are observed as energetic neutral atoms.

Applying these pictures to SNR shocks propagating into a partially ionized medium, we expect leakage of neutral particles from the downstream region to the upstream region. Raymond et al. (2008) discussed formation and neutralization of pickup ions in the downstream region of SNR shocks. Therefore, we can expect leakage neutral particles not only from pickup ions produced in the upstream region but also from pickup ions produced in the downstream region. Upstream pickup ions originate from leakage neutral particles, so that downstream pickup ions should be the dominant source of leakage neutral particles.

The leakage neutral particles become pickup ions by collisional ionization or charge exchange in the upstream region. If leakage is significant, the upstream flow is decelerated and heated by the pickup ions. As a result, a precursor is produced by leakage neutral particles. Very recently, Blasi et al. (2012) showed that neutral particles really leak into the upstream region from the downstream region by solving the Vlasov equation of neutral particles. They showed the formation of the precursor due to the leakage neutral particles and provided the precise velocity distribution of neutral particles by assuming that only the ion distribution is a Maxwellian.

In this paper, we investigate the precursor structure by a different approach, which is a four-fluid approximation. We consider upstream ions, upstream neutral particles, leakage
neutral particles, and pickup ions, respectively. Then, we obtain analytical solutions of the precursor structure. Our results in this paper are qualitatively similar to those of Blasi et al. (2012).

We first estimate the number density and the velocity of leakage neutral particles in Section 2. We then provide some length scales for collisional ionization and charge exchange in Section 3, provide four-fluid models to describe the precursor structure in Section 4, and provide two approximate solutions in Sections 4.1 and 4.2. Section 5 is devoted to the discussion.

2. DISTRIBUTION FUNCTION OF LEAKAGE NEUTRAL PARTICLES

In this section, we estimate the distribution function of leakage neutral particles in the shock rest frame. Leakage neutral particles originate from hot neutral particles produced in the downstream region. We here consider charge exchange, collisional ionization, and Coulomb collision as interactions in the downstream region. Although ionization by electrons could be important, it depends on the electron temperature, which has not been understood yet (Ohira & Takahara 2007, 2008; Rakowski et al. 2008). We here do not take into account ionization by electrons because it is sub-dominant compared to that by protons as long as the relative velocity is larger than about 2000 km s\(^{-1}\) (e.g., Heng & McCray 2007), which is a typical shock velocity of young SNRs. Cross sections of charge exchange and collisional ionization depend on a relative velocity, \(u_{\text{rel}}\) (e.g., Heng & McCray 2007). Collisional ionization of hydrogen atoms by protons is dominant for \(u_{\text{rel}} \gtrsim 3000\) km s\(^{-1}\), and its cross section is typically \(\sigma_i \sim 10^{-16}\) cm\(^2\). On the other hand, charge exchange is dominant for \(u_{\text{rel}} \lesssim 3000\) km s\(^{-1}\), and its cross section between protons and hydrogen atoms is about \(\sigma_{\text{ce}} \sim 10^{-16}\) cm\(^2\) for \(u_{\text{rel}} \sim 3000\) km s\(^{-1}\) and \(\sigma_{\text{ce}} \sim 10^{-15}\) cm\(^2\) for \(u_{\text{rel}} \lesssim 1000\) km s\(^{-1}\). The ionization cross section of hydrogen atoms by hydrogen atoms, \(\sigma_{\text{IHH}} (H + H \rightarrow p + e^- + H)\), is quite similar to that by protons (Barnett et al. 1990), so that we let \(\sigma_{\text{IHH}} = \sigma_i\). All relative velocities in the downstream region are typically the shock velocity, \(u_{\text{sh}}\), which is about 3000 km s\(^{-1}\) for young SNRs, so that both collisional ionization and charge exchange are important processes for neutral particles.

A neutral fraction of the interstellar medium around SNRs is often found to be order of unity (Ghavamian et al. 2000, 2002). The typical ISM density is 1 cm\(^{-3}\). Therefore, we use \(n_n = n_{\text{ion}} = 0.5\) cm\(^{-3}\) as fiducial values in this paper, where \(n_n\) and \(n_{\text{ion}}\) are number densities of upstream neutral particles and ions, respectively.

In the shock rest frame, upstream ions are decelerated and heated at the shock, while upstream neutral particles are not decelerated because the shock dissipation is due to electromagnetic interactions (Chevalier & Raymond 1978). After passing over the shock front, the upstream neutral particles are mainly ionized by hot ions in the downstream region. Then, the upstream neutral particles become pickup ions in the downstream region (Raymond et al. 2008). Therefore, the ionization timescale of upstream cold neutral particles in the downstream region, \(t_{\text{i,cold}}\), is given by

\[
i_{\text{i,cold}} = \frac{1}{(\sigma_i + \sigma_{\text{ce}}) n_{\text{ion},\text{hot}} u_{\text{rel}}},
\]

where \(n_{\text{ion},\text{hot}} = r n_{\text{ion}}\) is the number density of hot ions in the downstream region and \(r\) is the shock compression ratio. Hot neutral particles are produced from hot ions by charge exchange in the ionization length scale of penetrating neutral particles, \(L_{\text{i,down}} = u_{\text{sh}} t_{\text{i,cold}}\), where \(u_{\text{sh}}\) is the shock velocity. The charge-exchange timescale of hot ions, \(t_{\text{ce}}\), is given by

\[
t_{\text{ce}} = \frac{1}{\sigma_{\text{ce}} n_n u_{\text{rel}}}.
\]

The crossing timescale of hot neutral particles that move toward the shock with a negative velocity of \(v_x\) is given by

\[
t_{\text{cross,n}}(v_x) = \frac{L_{\text{i,down}}}{|v_x|} = t_{\text{i,cold}} \frac{u_{\text{sh}}}{|v_x|},
\]

where \(v_x\) is a velocity in the direction of the shock normal and \(v_x < 0 (x = 0\) and \(x = -\infty\) are the positions of the shock and the far upstream region, respectively\). Hot neutral particles are mainly ionized by hot ions, so that the ionization timescale of hot neutral particles, \(t_{\text{i,hot}}\), is given by

\[
t_{\text{i,hot}} = \frac{1}{(\sigma_i + \sigma_{\text{ce}}) n_{\text{ion},\text{hot}} u_{\text{rel}}}.
\]

Therefore, in the shock rest frame, the steady-state distribution function of hot neutral particles with \(v_x < 0\) at the shock, that is, the distribution function of leakage neutral particles at the shock, is approximately given by

\[
f_{\text{leak,sh}}(v) = \frac{f_{\text{i,hot}}(v)}{t_{\text{ce}}} \times \min(t_{\text{cross,n}}(v_x), t_{\text{i,hot}}),
\]

where \(f_{\text{i,hot}}(v)\) is the distribution function of downstream hot ions in the shock rest frame.

Although the velocity distribution of hot ions in the downstream region, \(f_{\text{i,hot}}(v)\), has not been understood for SNR shocks, especially in the partially ionized medium, we here assume that the dominant source of leakage neutral particles is pickup ions produced in the downstream region because they should have large velocity dispersion and high density compared to those of other components. The velocity distribution of pickup ions produced in the downstream region becomes approximately an isotropic shell distribution in the downstream rest frame (Raymond et al. 2008). The isotropic velocity in the downstream rest frame, \(v_{\text{d,r}}\), is the velocity of upstream neutral particles in the downstream rest frame, \((1 - r^{-1})u_{\text{sh}}\), where \(r\) is the shock compression ratio. Therefore, the ionization timescale of hot neutral particles is smaller than the crossing timescale of hot neutral particles in the ionization region \(t_{\text{i,hot}} < t_{\text{cross,n}}(v_x)\) because \(v_x \geq -(1 - 2r^{-1})u_{\text{sh}}\) for pickup ions produced in the downstream region. In this case, leakage neutral particles with the negative velocity, \(v_x\), are produced in the production region, \(0 \leq x \leq t_{\text{i,hot}}|v_x|\). Then, the crossing timescale of downstream pickup ions in the production region is given by

\[
t_{\text{cross,PUI}}(v_x) = \frac{|v_x|}{u_{\text{sh}}} t_{\text{i,hot}}.
\]

Furthermore, from Equation (5), the distribution function of leakage neutral particles in the shock rest frame, \(f_{\text{leak,sh}}(v)\), is expressed by

\[
f_{\text{leak,sh}}(v) = \frac{t_{\text{i,hot}} n_{\text{PUI,down}}}{t_{\text{ce}} 4\pi v_d^2} \delta\left(v_d - (1 - r^{-1})u_{\text{sh}}\right),
\]

where \(v_d = (v_x - u_{\text{sh}} r^{-1})^2 + v_y^2 + v_z^2)^{1/2}\) is a particle speed in the downstream rest frame and \(v_y, v_z, \delta[\ldots]\), and \(n_{\text{PUI,down}}\) are
velocities perpendicular to the shock normal, the delta function, and the number density of downstream pickup ions contributing to the leakage neutral particles, respectively.

Downstream pickup ions are eliminated by charge exchange and Coulomb collision. The relaxation timescale of pickup ions by Coulomb collision is given by (Spitzer 1962)

\[ t_{\text{Coulomb}} = \frac{m_p^2 U_{\text{rel}}}{8\pi n_{\text{hot},e} e^4 \ln \Lambda}, \tag{8} \]

where \( e \) and \( \ln \Lambda \) are the elementary charge and the Coulomb logarithm, respectively. Then, the number density of pickup ions contributing to the leakage neutral particles, \( n_{\text{PUI,down}} \), is approximately given by

\[ n_{\text{PUI,down}} = n_n \times \min(t_{\text{cross,PUI}}(v_x), t_{ce}, t_{\text{Coulomb}}). \tag{9} \]

The crossing timescale of downstream pickup ions, \( t_{\text{cross,PUI}}(v_x) \), is always smaller than the charge-exchange timescale of hot ions, \( t_{ce} \), as long as \( n_{\text{ion}}/n_n \geq 0.5 \). We here consider the case of \( t_{\text{cross,PUI}} < t_{ce} \) because the shock dissipation has not been completely understood yet for the low ionization fraction. The maximum value of \( t_{\text{cross,PUI}}(v_x) \) is about \( t_{i,\text{hot}} \). Therefore, we consider two cases: \( t_{i,\text{hot}} < t_{\text{Coulomb}} \) and \( t_{i,\text{hot}} > t_{\text{Coulomb}} \). From the condition \( t_{i,\text{hot}} = t_{\text{Coulomb}} \), we obtain the critical shock velocity, \( u_{sh,c} \), as

\[ u_{sh,c} = 120 \text{ km s}^{-1} \left( \frac{\ln \Lambda}{40} \right)^{\frac{3}{4}} \left( \frac{\sigma_i + \sigma_{ce}}{10^{-15} \text{ cm}^2} \right)^{-\frac{1}{4}}, \tag{10} \]

where we assume \( U_{\text{rel}} \sim u_{sh} \).

For \( t_{i,\text{hot}} < t_{\text{Coulomb}} (u_{sh} > u_{sh,c}) \), relaxation by Coulomb collision is negligible in the production region of leakage neutral particles, \( 0 \leq x \leq t_{i,\text{hot}}|v_x| \). Then, from Equations (2), (4), (7), and (9), the distribution function of leakage neutral particles in the shock rest frame, \( f_{\text{leak},\sh}(v) \), is expressed by

\[ f_{\text{leak},\sh}(v) = r n_n \frac{n_n}{n_{\text{ion},e}} \frac{\sigma_{ce}}{\sigma_i + \sigma_{ce}} u_{sh} \frac{1}{4\pi v_d^2} \delta[v_d - (1 - r^{-1})u_{sh}], \tag{11} \]

where \( v_d = (v_x - u_{sh} f^{-1})^2 + v_y^2 + v_z^2)^{1/2} \). Then, the number density of leakage neutral particles at the shock, \( n_{\text{leak},\sh} \), is given by

\[ n_{\text{leak},\sh} = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_0^1 dv_z f_{\text{leak},\sh}(v), \tag{12} \]

where \( F(r) = 3(1 - 2r^{-1})^2/(1 - r^{-1}) (F = 1 \text{ for } r = 4) \). Hence, for \( 120 \text{ km s}^{-1} < u_{sh} \lesssim 3000 \text{ km s}^{-1} \), about 10% of upstream neutral particles leak into the upstream region from the downstream region. For \( u_{sh} > 3000 \text{ km s}^{-1} \), the cross section of ionization is larger than that of charge exchange (\( \sigma_i > \sigma_{ce} \)), so that leakage of neutral particles becomes smaller. Moreover, in the shock rest frame, the mean velocity of leakage neutral particles at the shock, \( u_{\text{leak},\sh} \), is given by

\[ u_{\text{leak},\sh} = \frac{1}{n_{\text{leak},\sh}} \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_0^1 dv_z f_{\text{leak},\sh}(v) v_x = -\frac{1}{3} u_{sh} G(r). \tag{13} \]

where \( G(r) = 2(1 - 2r^{-1}) (G = 1 \text{ for } r = 4) \). Hence, the mean velocity of leakage neutral particles in the shock rest frame is about one-third of the shock velocity for \( u_{sh} > u_{sh,c} \). For the smaller compression ratio, \( n_{\text{leak},\sh} \) and \( |u_{\text{leak,sh}}| \) become smaller and there is no leakage for \( r \lesssim 2 \).

For \( t_{i,\text{hot}} > t_{\text{Coulomb}} (u_{sh} < u_{sh,c}) \), relaxation by Coulomb collision is not negligible. For leakage neutral particles satisfying \( t_{\text{Coulomb}} < t_{\text{cross,PUI}}(v_x) (v_x < (1 - r^{-1})u_{sh}) \), from Equations (2), (7), (8), and (9), the distribution function of leakage neutral particles at the shock, \( f_{\text{leak,sh}}(v) \), is expressed by

\[ f_{\text{leak,sh}}(v) = \frac{1}{r} n_n \frac{n_n}{n_{\text{ion},e}} \frac{\sigma_{ce}}{\sigma_i + \sigma_{ce}} u_{sh} \frac{1}{4\pi v_d^2} \delta[v_d - (1 - r^{-1})u_{sh}], \tag{14} \]

where we assume \( U_{\text{rel}} \sim u_{sh} \). Note that the shock velocity dependence is very strong. For leakage neutral particles with a smaller speed of \( |v_x| \) satisfying \( t_{\text{Coulomb}} > t_{\text{cross,PUI}}(v_x) \), \( |v_x| \leq (1 - r^{-1})u_{sh} \), the distribution function, \( f_{\text{leak,sh}}(v) \), is given by Equation (11) because relaxation by Coulomb collision is negligible.

### 3. Relevant Length Scales

In this section, we briefly summarize relevant length scales in the precursor due to leakage neutral particles. We here consider charge exchange, collisional ionization, and Coulomb collision as interactions in the upstream region. Ionization of upstream neutral particles by electrons could be important compared to ionization by leakage neutral particles and pickup ions produced in the upstream region because the number density of electrons is larger than that of leakage neutral particles and pickup ions. However, the electron temperature in the precursor region has not been completely understood and ionization by electrons depends on the electron temperature. Therefore, we here do not take into account ionization by electrons for simplicity.

In the shock rest frame, leakage neutral particles have the mean velocity in the direction of the shock normal of \( u_{\text{leak,sh}} \approx -u_{sh}/3 \) as shown in Equation (13). Then, the relative velocity between upstream particles and leakage neutral particles becomes \( u_{\text{rel,leak}} \approx 4u_{sh}/3 \). The shock velocity is typically \( u_{sh} \approx 3000 \text{ km s}^{-1} \) for young SNRs, so that both collisional ionization with upstream particles and charge exchange with upstream ions are important processes for leakage neutral particles. Therefore, the precursor length scale in the shock rest frame, \( L_{\text{pre}} \), is given by

\[ L_{\text{pre}} = \frac{|u_{\text{leak,sh}}|}{u_{\text{rel,leak}}} \left( \begin{array}{c} \sigma_i + \sigma_{ce} \\ 10^{-15} \text{ cm}^2 \end{array} \right) \left( \begin{array}{c} n_{\text{ion}} + n_n \\ 1 \text{ cm}^{-3} \end{array} \right)^{-1} \times \left( \begin{array}{c} \sigma_i + \sigma_{ce} \\ 10^{-15} \text{ cm}^2 \end{array} \right)^{-1} \left( \begin{array}{c} n_{\text{ion}} + n_n \\ 1 \text{ cm}^{-3} \end{array} \right)^{-1}, \tag{15} \]
where \( n_{\text{ion}} \) and \( n_n \) are the number densities of the upstream ions and upstream neutral particles, respectively.

Leakage neutral particles are ionized in the above region. Then, they become pickup ions and are advected into the shock. The relaxation length scale of pickup ions due to Coulomb collision in the shock rest frame, \( L_{\text{Coulomb}} \), is given by

\[
L_{\text{Coulomb}} = \frac{m_i^2 \mu_{\text{rel, leak}} u_{\text{sh}}}{8 \pi n_{\text{ion}} e^4 \ln \Lambda} \\
= 2 \times 10^{21} \text{ cm} \left( \frac{u_{\text{sh}}/\mu_{\text{rel, leak}}}{0.75} \right)^{-3} \left( \frac{n_{\text{ion}}}{0.5 \text{ cm}^{-3}} \right)^{-1} \\
\times \left( \frac{\ln \Lambda}{40} \right)^{-1} \left( \frac{u_{\text{sh}}}{3000 \text{ km s}^{-1}} \right)^4. \tag{16}
\]

The relaxation length scale, \( L_{\text{Coulomb}} \), is larger than the precursor length scale, \( L_{\text{pre}} \), as long as \( u_{\text{sh}} \gg 100 \text{ km s}^{-1} \). Therefore, Coulomb collision between pickup ions and upstream ions is negligible, that is, pickup ions do not relax to upstream ions in the precursor region as long as \( u_{\text{sh}} > 100 \text{ km s}^{-1} \).

Another loss process of pickup ions in the precursor region is charge exchange between pickup ions and neutral particles. The charge-exchange length scale of pickup ions in the shock rest frame, \( L_{\text{ce, PUI}} \), is given by

\[
L_{\text{ce, PUI}} = \frac{u_{\text{sh}}}{\mu_{\text{rel, leak}} \sigma_{\text{ce}} (n_n + n_{\text{leak, sh}})} \\
= 1.5 \times 10^{16} \text{ cm} \left( \frac{u_{\text{sh}}/\mu_{\text{rel, leak}}}{0.75} \right) \\
\times \left( \frac{\sigma_{\text{ce}}}{10^{-16} \text{ cm}^2} \right)^{-1} \left( \frac{n_n + n_{\text{leak, sh}}}{0.5 \text{ cm}^{-3}} \right)^{-1}. \tag{17}
\]

The precursor length scale, \( L_{\text{pre}} \), is always smaller than the charge-exchange length scale of pickup ions, \( L_{\text{ce, PUI}} \), as long as the leakage velocity, \( |u_{\text{leak, sh}}| \), is smaller than the shock velocity, \( u_{\text{sh}} \). Therefore, we can neglect charge exchange of pickup ions for \( |u_{\text{leak, sh}}| < u_{\text{sh}} \).

Upstream neutral particles are ionized by collision and charge exchange with leakage neutral particles and pickup ions. The ionization length scale of upstream neutral particles in the shock rest frame, \( L_{i, \text{up}} \), is given by

\[
L_{i, \text{up}} = \frac{u_{\text{sh}}}{\mu_{\text{rel, leak}} \sigma_i (n_{\text{leak, sh}} + n_{\text{PUI}}) + \sigma_{\text{ce}} n_{\text{PUI}}} \\
\approx 1.5 \times 10^{17} \text{ cm} \left( \frac{u_{\text{sh}}/\mu_{\text{rel, leak}}}{0.75} \right) \\
\times \left( \frac{\sigma_i + \sigma_{\text{ce}}}{10^{-16} \text{ cm}^2} \right)^{-1} \left( \frac{n_{\text{leak, sh}} + n_{\text{PUI}}}{0.05 \text{ cm}^{-3}} \right)^{-1}. \tag{18}
\]

where \( n_{\text{PUI}} \) is the number density of pickup ions and we assume \( n_{\text{leak, sh}} \sim 0.1 n_n \sim 0.05 \text{ cm}^{-3} \) as shown in Equation (12). Therefore, the ionization of upstream neutral particles is negligible in the precursor because \( L_{i, \text{up}} > L_{\text{pre}} \).

Upstream neutral particles interact not only with leakage neutral particles but also with upstream ions. The relative velocity between upstream neutral particles and upstream ions, \( u_{\text{rel, up}} \), would become larger than their thermal velocity but smaller than the shock velocity (\( \sim 3000 \text{ km s}^{-1} \)) for high Mach number shocks. Therefore, charge exchange is dominant and the charge-exchange length scale of upstream neutral particles with upstream ions in the shock rest frame, \( L_{\text{ce, up}} \), is given by

\[
L_{\text{ce, up}} = \frac{u_{\text{sh}}}{\mu_{\text{rel, up}} \sigma_{\text{ce}} n_{\text{ion}}} \\
= 2 \times 10^{16} \text{ cm} \left( \frac{u_{\text{sh}}/\mu_{\text{rel, up}}}{10} \right) \\
\times \left( \frac{\sigma_{\text{ce}}}{10^{-15} \text{ cm}^2} \right)^{-1} \left( \frac{n_{\text{ion}}}{0.5 \text{ cm}^{-3}} \right)^{-1}. \tag{19}
\]

where we assume that upstream ions are decelerated to about 90% of the shock velocity, \( 0.9 u_{\text{sh}} \) (see Section 4). If leakage is small, the upstream plasma flow does not change significantly, so that \( L_{\text{ce, up}} > L_{\text{pre}} \). Then, upstream neutral particles rarely interact with upstream ions in the precursor, that is, the flow velocity of upstream neutral particles does not change. We consider this decoupling case in Section 4.1. If leakage is large or the shock velocity is larger than 3000 km s\(^{-1} \), the upstream plasma flow is significantly decelerated, \( u_{\text{rel, up}} \sim u_{\text{sh}} \), or the precursor length scale becomes large because the ionization cross section becomes small, so that \( L_{\text{ce, up}} < L_{\text{pre}} \). Then, upstream neutral particles interact many times with upstream ions in the precursor. As a result, the velocity distribution of upstream neutral particles quickly becomes that of upstream ions at each point in the precursor, that is, the flow velocity (mean velocity) of upstream neutral particles becomes that of upstream ions. However, it should be noted that the relative velocity between upstream ions and upstream neutral particles is not approximately zero because of their thermal velocity. We consider this tight-coupling case in Section 4.2.

According to the DSA theory, accelerated particles diffuse into the upstream region. The diffusion length scale is given by

\[
L_{\text{diff}} = \frac{\eta_e c E}{3 \varepsilon B u_{\text{sh}}} \\
= 10^{15} \text{ cm} \left( \frac{\eta_e}{1} \right) \left( \frac{u_{\text{sh}}}{3000 \text{ km s}^{-1}} \right)^{-1} \\
\times \left( \frac{B}{100 \mu G} \right)^{-1} \left( \frac{E}{1 \text{ TeV}} \right), \tag{20}
\]

where \( \eta_e, B, \) and \( E \) are the gyrofactor, the magnetic field, and the energy of accelerated particles, respectively. Therefore, particles are accelerated in the precursor due to leakage neutral particles, that is, leakage of neutral particles is important for the particle acceleration up to \( 1-10 \text{ TeV} \). The diffusion length scale of pickup ions is much smaller than the precursor scale, \( L_{\text{pre}} \). Hence, we can neglect diffusion of pickup ions.

4. FOUR-FLUID MODEL IN A NEUTRAL PARTICLE PRECURSOR

In this section, we calculate the steady-state precursor structure due to leakage neutral particles. There are cold ions and neutral particles in the upstream region. In addition, we consider leakage neutral particles and pickup ions originating from leakage neutral particles. We here adopt a four-fluid model to describe the precursor. Our treatment is similar to that of Heng et al. (2007). Continuity equations of the steady state are given by

\[
\frac{d}{dx}(n_n \mu_n) = -Q_{\text{ion}}, \tag{21}
\]

\[
\frac{d}{dx}(n_{\text{leak}} u_{\text{leak}}) = -Q_{\text{PUI}}, \tag{22}
\]
\[
\frac{d}{dx}(n_{\text{ion}}u_{\text{ion}}) = Q_{\text{ion}},
\]
(23)

\[
\frac{d}{dx}(n_{\text{PUI}}u_{\text{PUI}}) = Q_{\text{PUI}},
\]
(24)

where the subscripts “n,” “leak,” “ion,” and “PUI” denote upstream neutral particles, leakage neutral particles, upstream ions, and pickup ions, respectively, and \( x \) is the coordinate of the direction along the shock normal (\( x = 0 \) and \( x = -\infty \) are the positions of the shock and the far upstream region, respectively). We assume that the fluid velocity of pickup ions is the same as that of upstream ions because of electromagnetic interactions, \( u_{\text{PUI}} = u_{\text{ion}} \). \( Q_{\text{ion}} \) and \( Q_{\text{PUI}} \) are source terms and given by

\[
Q_{\text{ion}} = (n_{\text{leak}} + n_{\text{PUI}})u_n\sigma_{\text{ce}}\sigma_{\text{rel, leak}},
\]

\[
Q_{\text{PUI}} = (n_n + n_{\text{PUI}} + n_{\text{ion}})u_{\text{leak}}\sigma_{\text{ce}}\sigma_{\text{rel, leak}},
\]

where we approximate all relative velocities as a constant (see Equation (31)). The first two terms of \( Q_{\text{ion}} \) are due to collisional ionization of upstream neutral particles with leakage neutral particles and pickup ions, respectively. The last term of \( Q_{\text{ion}} \) is due to charge exchange of upstream ions with leakage neutral particles. The first three terms of \( Q_{\text{PUI}} \) are due to collisional ionization of leakage neutral particles with upstream neutral particles, pickup ions, and upstream ions, respectively. The last term of \( Q_{\text{PUI}} \) is due to charge exchange of leakage neutral particles with upstream ions. For the decoupling case (\( L_{\text{ce, up}} > L_{\text{pre}} \)), we can neglect charge exchange and collisional ionization between upstream ions and upstream neutral particles because their interaction length scales are larger than the precursor length scale. For the tight-coupling case (\( L_{\text{ce, up}} < L_{\text{pre}} \)), we approximately treat upstream ions and neutral particles as a single fluid at each point in the precursor instead for solving charge-exchange processes between upstream ions and upstream neutral particles. Once we assume the tight coupling between upstream ions and upstream neutral particles, charge exchange between upstream ions and neutral particles approximately does not change anything and collisional ionization between upstream ions and neutral particles does not change dynamics of the single fluid. Therefore, we do not take into account charge exchange and collisional ionization between upstream ions and neutral particles for both cases. Moreover, we can neglect charge exchange of pickup ions as discussed in Equation (17).

In the next subsections, we solve Equations (21)–(24) by using momentum and energy conservations. Boundary conditions are as follows:

\[
n_n(-\infty) = n_{\text{n,0}}, \quad u_n(-\infty) = u_0,
\]

\[
n_{\text{ion}}(-\infty) = n_{\text{ion,0}}, \quad u_{\text{ion}}(-\infty) = u_0,
\]

\[
n_{\text{PUI}}(-\infty) = 0,
\]

\[
n_{\text{leak}}(0) = n_{\text{leak,sh}}, \quad u_{\text{leak}}(0) = u_{\text{leak,sh}},
\]

where the subscripts “0” and “sh” represent quantities at the far upstream region and at the shock, respectively. Note that \( u_{\text{leak,sh}} \) is negative. All quantities are normalized by \( u_0 \) and \( n_{\text{ion,0}} + n_{\text{n,0}} \) in Sections 4.1 and 4.2. Although we estimated \( n_{\text{leak,sh}} \approx 0.1n_{\text{n,0}} \) and \( u_{\text{leak,sh}} \approx -u_0/3 \) in Section 3, there are a few uncertainties, especially for the velocity distribution of downstream hot ions. Hence, we treat \( n_{\text{n,0}}/(n_{\text{ion,0}} + n_{\text{n,0}}) \), \( n_{\text{leak,sh}}/(n_{\text{ion,0}} + n_{\text{n,0}}) \), and \( u_{\text{leak,sh}}/u_0 \) as free parameters in Sections 4.1 and 4.2.

We treat leakage neutral particles as a cold fluid in this paper, that is, we neglect the velocity dispersion of leakage neutral particles. Then, the relative velocity, \( u_{\text{rel, leak}} \), is approximately given by

\[
u_{\text{rel, leak}} = u_0 - u_{\text{leak,sh}}.
\]

(31)

4.1. Decoupling Approximation (\( L_{\text{ce, up}} > L_{\text{pre}} \))

In this subsection, we solve equations of the four-fluid system using the decoupling approximation corresponding to small leakage of neutral particles. We here assume that fluid velocities of upstream neutral particles and leakage neutral particles do not change from that at the far upstream region, \( u_0 \), and that at the shock, \( u_{\text{leak,sh}} \), respectively. Then, continuity equations are given by

\[
u_0 \frac{d}{dx}n_n = -Q_{\text{ion}}, \quad \frac{d}{dx}n_{\text{leak}} = -Q_{\text{PUI}},
\]

(32)

(33)

\[
u_0 \frac{d}{dx}(n_{\text{ion}}u_{\text{ion}}) = Q_{\text{ion}}, \quad \frac{d}{dx}(n_{\text{PUI}}u_{\text{PUI}}) = Q_{\text{PUI}}.
\]

(34)

(35)

Momentum and energy conservation laws of upstream ions and pickup ions are approximately given by

\[
u_0 \frac{d}{dx}\left\{m(n_{\text{ion}} + n_{\text{PUI}})u^2_{\text{ion}} + P_{\text{ion}} + P_{\text{PUI}}\right\} = m(u_0 Q_{\text{ion}} + u_{\text{leak,sh}} Q_{\text{PUI}}),
\]

(36)

\[
u_0 \frac{d}{dx}\left\{\frac{1}{2}m(n_{\text{ion}} + n_{\text{PUI}})u^3_{\text{ion}} + \frac{\gamma}{\gamma - 1}u_{\text{ion}}(P_{\text{ion}} + P_{\text{PUI}})\right\} = \frac{1}{2}m(u_0^2 Q_{\text{ion}} + u_{\text{leak,sh}}^2 Q_{\text{PUI}}),
\]

(37)

where \( m, P_{\text{ion}}, P_{\text{PUI}}, \) and \( \gamma \) are the particle mass, the pressure of upstream ions, the pressure of pickup ions, and the adiabatic index, respectively. In the first term of the right-hand side of Equations (36) and (37), we approximate the fluid velocity of upstream ions as that at the far upstream region, \( u_0 \). This approximation is valid as long as deceleration of upstream ions is small.

To make the expression simple, hereafter velocities, densities, pressures, and spatial coordinate are normalized by \( u_0, (n_{\text{ion,0}} + n_{\text{n,0}}), m(n_{\text{ion,0}} + n_{\text{n,0}})u^2_0, \) and \( (n_{\text{ion,0}} + n_{\text{n,0}})^{-1}\sigma_i^{-1} \), respectively. Normalized quantities are denoted with a bar. From Equations (32)–(37), one can make four conserved quantities. Hence, Equations (32)–(37) reduce to the following two equations:

\[
u_0 \frac{d}{dx}\bar{n}_n = -\bar{u}_{\text{rel, leak}}\bar{n}_{\text{leak}} \left[\bar{n}_n - \frac{1}{\bar{u}_n}\right] \times \left\{(1 - \bar{n}_n)\left(\frac{\sigma_{\text{ce}}}{\sigma_i}\right) + \bar{n}_n\bar{u}_{\text{leak,sh}}\right\},
\]

(38)

\[
u_0 \frac{d}{dx}\bar{n}_{\text{leak}} = -\bar{u}_{\text{rel, leak}}\bar{n}_{\text{leak}} \left[\bar{n}_n + \frac{1}{\bar{u}_n}\right] \times \left\{(1 - \bar{n}_n)\left(1 + \frac{\sigma_{\text{ce}}}{\sigma_i}\right) - \bar{n}_n\bar{u}_{\text{leak,sh}}\right\},
\]

(39)
where \( \bar{u}_{\text{ion}} \) can be expressed by

\[
\bar{u}_{\text{ion}} = \frac{B + \sqrt{B^2 - 4AC}}{2A},
\]

(40)

where \( A = (\gamma + 1)(1 - \bar{n}_n - \bar{n}_{\text{leak}} \bar{u}_{\text{leak,sh}}) \),

\[ B = 2\gamma \left( 1 + \frac{\bar{n}_{\text{ion,0}}}{\gamma M_{\text{ion,0}}^2} - \bar{n}_n - \bar{n}_{\text{leak}} \bar{u}_{\text{leak,sh}}^2 \right), \]

\[ C = (\gamma - 1) \left( 1 + \frac{2\bar{n}_{\text{ion,0}}}{(\gamma - 1) M_{\text{ion,0}}^2} - \bar{n}_n - \bar{n}_{\text{leak}} \bar{u}_{\text{leak,sh}}^2 \right). \]

Moreover, the evolution of the Mach number of upstream ions and pickup ions, \( M_{\text{ion}}(\bar{x}) \), can be expressed by

\[
M_{\text{ion}} = \frac{\bar{u}_{\text{ion}}}{\sqrt{\frac{\bar{n}_n + \bar{n}_{\text{PUI}}}{\bar{P}_n + \bar{P}_{\text{PUI}}}}} = \left( 1 + \frac{\bar{n}_{\text{ion,0}}}{\gamma M_{\text{ion,0}}^2} - \bar{n}_n - \bar{n}_{\text{leak}} \bar{u}_{\text{leak,sh}}^2 \right)^{-\frac{1}{2}} \left( \frac{1}{1 - \bar{n}_n \bar{u}_{\text{leak,sh}}^2 / \bar{u}_{\text{ion}}} - 1 \right)^{\frac{1}{2}}. \]

(44)

Figure 1 shows numerical solutions to Equations (38)–(44), where \( M_{\text{ion,0}} = 100, \gamma = 5/3, \bar{n}_{n,0} = 0.5, \bar{n}_{\text{leak,sh}} = 0.05, \bar{u}_{\text{leak,sh}} = -1/3, \) and \( \sigma_{\text{ce}}/\sigma_1 = 1. \) For the same input parameters, analytical approximations of Equation (39) give about 0.1% accuracy. There is no solution with \( M_{\text{ion}} < 1. \) The flow velocity of upstream ions and pickup ions, \( \bar{u}_{\text{ion}} \), is slightly decelerated by small leakage of neutral particles. However, the Mach number becomes small significantly because the pressure of pickup ions is large. These features are qualitatively the same as results of Blasi et al. (2012). The number density of upstream neutral particles, \( \bar{n}_n \), is almost constant. As already mentioned in Section 3, ionization of upstream neutral particles is negligible in the precursor region for \( \bar{n}_{\text{leak,sh}} \ll 1. \) Therefore, the number density of upstream neutral particles, \( \bar{n}_n \), can be regarded as a constant. Then, one can obtain analytical solutions at the shock.

Because we have all quantities at the shock, we can calculate the shock jump condition. Collisionless shocks are formed only by the plasma because the dissipation length of the plasma is much smaller than the interaction length of neutral particles (Chevalier & Raymond 1978; Chevalier et al. 1980). It should be noted that we have to take into account sink terms due to leakage of neutral particles when we derive the Rankin–Hugoniot relations at the shock. Even though leakage neutral particles are not ions, the origin is hot ions in the downstream region. By using the Rankin–Hugoniot relations between the far upstream region and the downstream region, one can easily obtain the shock jump condition. This is because there is no net sink between the far upstream region and the downstream region. Therefore, the total compression ratio between the far upstream region and the downstream region is given by

\[
r_{\text{tot}} = \frac{\gamma + 1}{\gamma - 1 + 2M_0^2}. \]

(45)

where

\[
M_0 = \left( \frac{(\bar{n}_{\text{ion,0}} + \bar{n}_{n,0})/(\bar{P}_{\text{ion,0}} + \bar{P}_{n,0})}{} \right)^{1/2} = M_{\text{ion,0}} \]

is the Mach number of all particles at the far upstream region. The partial compression ratio between the far upstream region and the front of the shock \( (\bar{x} = -\epsilon) \) is \( \bar{u}_{\ion}^{-1} \), so that the compression ratio of the shock is given by

\[
r_{\text{sh}} = \frac{\gamma + 1}{\gamma - 1 + 2M_0^2} \bar{u}_{\ion}(\bar{x} = 0). \]

(46)

One can obtain the analytical approximation of the shock compression ratio, \( r_{\text{sh}} \), from Equations (40) and (46) because of \( \bar{n}_n \approx \bar{n}_{n,0}. \) Figure 2 shows the analytical results of the fluid velocity of upstream ions at the shock, \( \bar{u}_{\ion}(\bar{x} = 0) \), and the spectral index of accelerated particles, \( s = (r_{\text{sh}}/2 + 1)/r_{\text{sh}} - 1. \) Even though leakage is small (\( \bar{n}_{\text{leak,sh}} \ll 0.1 \)), the spectral index becomes larger than 2. As already mentioned by Blasi et al. (2012), this can explain the observed gamma-ray spectra slightly steeper than the simplest prediction of DSA. Effects of leakage neutral particles become significant for the low ionization fraction and the large leakage flux. Note that the decoupling approximation may not be valid for \( \bar{n}_{\text{leak,sh}} \gtrsim 0.1 \) because the relative velocity between upstream neutral particles and upstream ions becomes large (see Section 3).

### 4.2. Tight-coupling Approximation \( L_{\text{ce,up}} < L_{\text{pre}} \)

In this subsection, we solve equations of the four-fluid system using the tight-coupling approximation corresponding to large
leakage of neutral particles. We here assume that the fluid velocity of leakage neutral particles does not change from that at the shock, $u_{\text{leak,sh}}$. In addition, we assume that the fluid velocity of upstream neutral particles is the same as that of upstream ions because of the tight coupling, $u_n = u_{\text{ion}}$. Then, continuity equations are given by

$$\frac{d}{dx}(n u_{\text{ion}}) = -Q_{\text{ion}},$$

$$\frac{d}{dx}(n_{\text{leak}}) = -Q_{\text{PU1}},$$

$$\frac{d}{dx}(n_{\text{leak}}u_{\text{ion}}) = Q_{\text{ion}},$$

$$\frac{d}{dx}(n_{\text{PU1}}u_{\text{ion}}) = Q_{\text{PU1}}.$$  

Momentum and energy conservation laws of upstream neutral particles, upstream ions, and pickup ions are given by

$$\frac{d}{dx}\left[m (n + n_{\text{ion}} + n_{\text{PU1}}) u_{\text{ion}}^2 + P_n + P_{\text{ion}} + P_{\text{PU1}}\right] = m u_{\text{leak,sh}} Q_{\text{PU1}}.$$  

$$= \frac{1}{2} m u_{\text{leak,sh}}^2 Q_{\text{PU1}},$$  

where $P_n$ is the pressure of upstream neutral particles.

To make the expression simple, hereafter velocities, densities, pressures, and spatial coordinate are normalized by $u_0, (n_{\text{ion},0} + n_{n,0}), m(n_{\text{ion},0} + n_{n,0})u_0,_{\text{leak,sh}},$ and $(n_{\text{ion},0} + n_{n,0})^{-1}\sigma_i^{-1}$, respectively. Normalized quantities are denoted with a bar. From Equations (47)–(52), one can make four conserved quantities. Hence, Equations (47)–(52) reduce to the following two equations:

$$\frac{d}{dx} \left( \tilde{n} u_{\text{ion}} \right) = - \tilde{u}_{\text{rel,leak}} \tilde{n}_{\text{leak}} \left[ \tilde{n}_n \left( 1 + \frac{\sigma_{\text{ce}}}{\sigma_i} \right) - \tilde{n}_{\text{leak}} u_{\text{leak,sh}} \right] - \frac{1}{u_{\text{ion}}} \left\{ \tilde{n} n_{\text{leak,sh}} + \left( \frac{\sigma_{\text{ce}}}{\sigma_i} \right) \right\},$$

(53)

where $\tilde{u}_{\text{ion}}$ can be expressed by

$$\tilde{u}_{\text{ion}} = \frac{B + \sqrt{B^2 - 4AC}}{2A},$$

$$A = (\gamma + 1)(1 - \tilde{n}_{\text{leak}} \tilde{u}_{\text{leak,sh}}),$$

$$B = 2\gamma \left( 1 + \frac{1}{\gamma M_0^2} - \tilde{n}_{\text{leak}} \tilde{u}_{\text{leak,sh}}^2 \right),$$

$$C = (\gamma - 1) \left( 1 + \frac{2}{\gamma - 1} M_0^2 \right).$$

From Equations (53)–(55), one can obtain solutions of $\tilde{n}_n(\bar{x}), \tilde{n}_{\text{leak}}(\bar{x}),$ and $\tilde{u}_{\text{ion}}(\bar{x})$ by numerical computations. By assuming that the brackets terms of Equations (53) and (54) are constant, one can obtain analytical approximations. Using the solutions of $\tilde{n}_n, \tilde{n}_{\text{leak}}$ and $\tilde{u}_{\text{ion}},$ the other quantities, $\tilde{n}_{\text{ion}}, \tilde{n}_{\text{PU1}},$ and $\tilde{P}_n + \tilde{P}_{\text{ion}} + \tilde{P}_{\text{PU1}},$ can be expressed by

$$\tilde{n}_{\text{ion}} = \frac{1}{u_{\text{ion}}} \tilde{n}_n,$$

$$\tilde{n}_{\text{PU1}} = - \tilde{u}_{\text{leak,sh}} \tilde{n}_{\text{leak}},$$

$$\tilde{P}_n + \tilde{P}_{\text{ion}} + \tilde{P}_{\text{PU1}} = 1 + \frac{1}{\gamma M_0^2} - \tilde{n}_{\text{leak}} \tilde{u}_{\text{leak,sh}}^2 - (1 - \tilde{n}_{\text{leak}} \tilde{u}_{\text{leak,sh}}) \tilde{u}_{\text{ion}}.$$  

Moreover, the evolution of the Mach number, $M(\bar{x}),$ can be expressed by

$$M = \sqrt{\frac{\bar{n}_n + \bar{n}_{\text{ion}} + \bar{n}_{\text{PU1}}}{\bar{P}_n + \bar{P}_{\text{ion}} + \bar{P}_{\text{PU1}}}} = \gamma^{-\frac{1}{2}} \left\{ 1 + \frac{1}{\gamma M_0^2} - \tilde{n}_{\text{leak}} \tilde{u}_{\text{leak,sh}}^2 \right\} \frac{1}{\left( 1 - \tilde{n}_{\text{leak}} \tilde{u}_{\text{leak,sh}} \right) \tilde{u}_{\text{ion}}}.$$  

Figure 3 shows numerical solutions to Equations (53)–(59), where $M_0 = 100, \gamma = 5/3, \bar{n}_{n,0} = 0.5, \bar{n}_{\text{leak,sh}} = 0.3,$ and $\bar{n}_{\text{leak,sh}} = -0.5.$ For the same input parameters, analytical approximations of Equations (53) and (54) give an accuracy of about 5%. There is no solution with $M < 1$. Unlike the decoupling approximation, the flow velocity of upstream neutral particles, upstream ions, and pickup ions, $\tilde{u}_{\text{ion}},$ is significantly decelerated by large leakage of neutral particles. The number densities of upstream neutral particles, $\tilde{n}_n,$ and upstream ions, $\tilde{n}_{\text{ion}},$ become large because of compression. Moreover, the increase rate of the number density of upstream ions is slightly larger than that of upstream neutral particles because some of upstream neutral particles are ionized in the precursor region. Unlike the decoupling approximation, the flow velocity of upstream ions and the Mach number, Equations (55) and (59), do not explicitly depend on the number density of upstream neutral particles. Therefore, one can obtain analytical solutions at the shock. From Equation (46), one can obtain the analytical solution of the shock compression ratio, $r_{sh}.$ Figure 4 shows
the analytical solutions of the fluid velocity of upstream ions at the shock, $\tilde{u}_{\text{ion}}(\tilde{x} = 0)$, and the spectral index of accelerated particles, $s = (r_c + 2)/(r_{sh} - 1)$. For $n_{\text{leak}, \text{sh}} \sim 0.1$, the spectral index becomes larger than 2. As already mentioned by Blasi et al. (2012), this can explain the observed gamma-ray spectra slightly steeper than the simplest prediction of DSA. Note that the tight-coupling approximation is not valid for $n_{\text{leak}, \text{sh}} \ll 0.1$ because the fluid velocity of upstream ions does not significantly change (see Section 3).

5. DISCUSSION

We here discuss other important effects of leakage neutral particles. When leakage neutral particles are ionized, their velocity distribution in the upstream rest frame is initially a beam-like or ring-like distribution. These velocity distributions excite electromagnetic fields and amplify the magnetic field (Wu & Davidson 1972; Lee & Ip 1987; Raymond et al. 2008; Ohira et al. 2009; Ohira & Takahara 2010). This is a promising mechanism to explain some observations concerning the strong magnetic field (Vink & Laming 2003; Berezhko et al. 2003; Bamba et al. 2005; Uchiyama et al. 2007). Moreover, the electromagnetic instabilities could heat the upstream region. $H_\alpha$ observed from the upstream region (Lee et al. 2010) can be interpreted as the result of leakage neutral particles.

In Sections 4.1 and 4.2, we assumed that the adiabatic index of pickup ions is 5/3. However, there is no guarantee that the behavior of pickup ions is the same as the standard gas because of the collisionless system. Especially, the behavior of pickup ions at the shock is important for the shock jump condition and particle accelerations. Even though the Mach number at the shock is not so small, the compression ratio could be smaller than 4 because the adiabatic index of pickup ions could be larger than 5/3 (Fahr & Chalov 2008; Wu et al. 2009). If pickup ions drain the large fraction of the shock kinetic energy, upstream ions are not heated up to $T = 3mu_{\text{sh}}^2/16$. This can explain the recent observation of $H_\alpha$ (Helder et al. 2009), which showed that the temperature derived from the line width of $H_\alpha$ is much smaller than derived from the proper motion. Furthermore, pickup ions produced in the precursor region could be preferentially accelerated by DSA because their velocity is larger than that of upstream ions (Ohira & Takahara 2010).

These issues are crucial not only for particle accelerations and shock dissipation but also for the amount of leakage neutral particles from the downstream region to the upstream region. Therefore, we treated values concerning the leakage neutral particles as free parameters in Sections 4.1 and 4.2.

In this paper, we did not specify neutral particles. Helium atoms have a smaller cross section than that of hydrogen atoms. We expect large length scale of the precursor region compared with that of hydrogen atoms. However, the ionization fraction of helium depends on time because helium atoms are ionized by radiation from the downstream region (Ghavamian et al. 2000). Therefore, the precursor length scale and the injection of helium ions into DSA could depend on an SNR age. The CR injection history of helium ions is important to understand the CR helium spectrum observed at the Earth (Drury 2011; Ohira & Ioka 2011).

6. SUMMARY

In this paper, we have investigated effects of leakage neutral particles on shocks. We have found that if the dominant source of leakage neutral particles is pickup ions produced in the downstream region, about 10% of upstream neutral particles leak into the upstream region (Equation (12)) and the mean leakage velocity is about one-third of the shock velocity (Equation (13)). Moreover, we have calculated the precursor structure due to the leakage neutral particles by using four-fluid approximations (upstream neutral particles, upstream ions, leakage neutral particles, and pickup ions). We have found analytical solutions of the precursor structure by using the decoupling approximation or the tight-coupling approximation, where the decoupling means that upstream neutral particles do not interact with upstream ions and the tight coupling means the opposite case. We have found that even when leakage is small, the shock compression ratio becomes significantly small. This can explain the observed gamma-ray spectra slightly steeper than the simplest prediction of DSA. In addition, leakage neutral particles could heat the precursor region, and pickup ions produced in the precursor region are important for the injection into DSA.
We thank T. Terasawa and S. Matsukiyo for useful comments about physics of the solar wind. We also thank the anonymous referee for valuable comments. This work is supported in part by grant-in-aid from the Ministry of Education, Culture, Sports, Science, and Technology (MEXT) of Japan, No. 24·8344.

REFERENCES

Abdo, A. A., Ackermann, M., Ajello, M., et al. 2009, ApJ, 706, L1
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2010, ApJ, 710, L92
Axford, W. I., Leer, E., & Skadron, G. 1977, Proc. 15th Int. Cosmic Ray Conf., 11 (Plovdiv: Bulgarian Academy of Sciences), 132
Bamba, A., Yamazaki, R., Yoshida, T., Terasawa, T., & Koyama, K. 2005, ApJ, 621, 793
Barnett, C. F., Hunter, H. T., Kirkpatrick, M. L., et al. 1990, NASA STI/Recon Technical Report N, 91, 13238
Blandford, R. D., & Ostriker, J. P. 1978, ApJ, 221, L29
Bell, A. R. 1978, MNRAS, 182, 147
Bell, A. R., Schure, K. M., & Reville, B. 2012, MNRAS, 418, 1208
Berezhko, E. G., Ksenofontov, L. T., & Völk, H. J. 2003, A&A, 412, L11
Blasi, P., Moriño, G., Bandiera, R., Amato, E., & Caprioli, D. 2012, ApJ, 755, 121
Caprioli, D., Amato, E., & Blasi, P. 2010, Astropart. Phys., 33, 160
Cassam-Chenaï, G., Hughes, J. P., Reynoso, E. M., Badenes, C., & Moffett, D. 2008, ApJ, 680, 1180
Chevalier, R. A., Kirshner, R. P., & Raymond, J. C. 1980, ApJ, 235, 186
Chevalier, R. A., & Raymond, J. C. 1978, ApJ, 225, L27
Drury, L. O’C. 2011, MNRAS, 415, 1807
Drury, L. O’C., Duffy, P., & Kirk, J. G. 1996, A&A, 309, 1002
Fahrb, H. J., & Chalov, S. V. 2008, A&A, 409, L35
Ghavamian, P., Raymond, J., Hartigan, P., & Blair, W. P. 2000, ApJ, 535, 266
Ghavamian, P., Winkler, P. F., Raymond, J. C., & Long, K. S. 2002, ApJ, 572, 888
Giordano, F., Naumann-Godo, M., Ballet, J., et al. 2012, ApJ, 744, L2
Giuliani, A., Cardillo, M., Tavani, M., et al. 2011, ApJ, 742, L30
Gloeckler, G., Geiss, J., Balsiger, H., et al. 1993, Science, 261, 70
Helder, E. A., Vink, J., Bassa, C. G., et al. 2009, Science, 325, 719
Heng, K. 2010, PASA, 27, 23
Heng, K., & McCray, R. 2007, ApJ, 654, 923
Heng, K., van Adelsberg, M., McCray, R., & Raymond, J. C. 2007, ApJ, 668, 275
Kirk, J. G., Duffy, P., & Gallant, Y. A. 1996, A&A, 314, 1010
Krymsky, G. F. 1977, Dokl. Akad. Nauk SSSR, 234, 1306
Lee, J.-I., Raymond, J. C., Park, S., et al. 2010, ApJ, 715, L146
Lee, M., & Ip, W.-H. 1987, J. Geophys. Res., 92, 11041
McComas, D. J., Allegrini, F., Bochsler, P., et al. 2009, Science, 326, 959
Ohira, Y., & Ioka, K. 2011, ApJ, 729, L13
Ohira, Y., Murase, K., & Yamazaki, R. 2010, A&A, 513, A17
Ohira, Y., Murase, K., & Yamazaki, R. 2011, MNRAS, 410, 1577
Ohira, Y., & Takahara, F. 2007, ApJ, 661, 171
Ohira, Y., & Takahara, F. 2008, ApJ, 688, 320
Ohira, Y., & Takahara, F. 2010, ApJ, 721, L43
Ohira, Y., Terasawa, T., & Takahara, F. 2009, ApJ, 703, L59
Ptuskin, V. S., & Zirakashvili, V. N. 2005, A&A, 429, 755
Rakowski, C. E., Laming, J. M., & Ghavamian, P. 2008, ApJ, 684, 348
Raymond, J. C., Isenberg., P. A., & Laming, J. M. 2008, ApJ, 682, 408
Reville, B., Kirk., J. G., Duffy, P., & O’Sullivan 2007, A&A, 475, 435
Spitzer, L. 1962, Physics of Fully Ionized Gases (2nd ed.; New York: Wiley)
Tavani, M., Giuliani, A., Chen, A. W., et al. 2010, ApJ, 710, L151
Uchiyama, Y., Aharonian, F. A., Tanaka, T., Takahashi, T., & Maeda, Y. 2007, Nature, 449, 576
Vink, J., & Laming, J. M. 2003, ApJ, 584, 758
Wu, P., Winske, D., Gary, S. P., Schwadron, N. A., & Lee, M. A. 2009, J. Geophys. Res., 114, A08103
Wu, S. C., & Davidson, R. C. 1972, J. Geophys. Res., 77, 5399
Zirakashvili, V. N., & Ptuskin, V. S. 2009, in AIP Conf. Proc. 1085, High Energy Gamma-Ray Astronomy, ed. F. A. Aharonian, W. Hofmann, & F. M. Rieger (Melville, NY: AIP), 336