Nuclear composition of magnetized gamma-ray burst jets

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Abstract

We investigate the fraction of metal nuclei in the relativistic jets of gamma-ray bursts associated with core-collapse supernovae. We simulate the fallback in jet-induced explosions with two-dimensional relativistic hydrodynamics calculations and the jet acceleration with steady, radial, relativistic magnetohydrodynamics calculations, and derive the detailed nuclear composition of the jet by post-processing calculation. We found that if the temperature at the jet launch site is above $4.7 \times 10^9$ K, quasi-statistical equilibrium is established and heavy nuclei are dissociated into light particles such as $^4$He during the acceleration of the jets. The criterion for the survival of metal nuclei is written in terms of the isotropic jet luminosity as $L_{j}^{iso} \lesssim 3.9 \times 10^{50} (R_i/10^7 \text{ cm})^2 (1 + \sigma_i)$ erg s$^{-1}$, where $R_i$ and $\sigma_i$ are the initial radius of the jets and the initial magnetization parameter, respectively. If the jet is initially dominated by radiation field (i.e., $\sigma_i \ll 1$) and the isotropic luminosity is relatively high ($L_{j}^{iso} \gtrsim 4 \times 10^{52}$ erg s$^{-1}$), the metal nuclei cannot survive in the jet. On the other hand, if the jet is mainly accelerated by magnetic field (i.e., $\sigma_i \gg 1$), metal nuclei initially contained in the jet can survive without serious dissociation even in the case of a high-luminosity jet. If the jet contains metal nuclei, the dominant nuclei are $^{28}$Si, $^{16}$O, and $^{32}$S and the mean mass number can be $\langle A \rangle \sim 25$.

Key words: gamma-ray burst: general — magnetohydrodynamics (MHD) — nuclear reactions, nucleosynthesis, abundances

1 Introduction

Gamma-ray bursts (GRBs) are one of the most energetic phenomena in the Universe. They radiate enormous energies of the order of $10^{51}$ erg mainly in the form of gamma-rays with short duration, typically of $\sim 10$ s. Although the detailed radiation mechanism of gamma-rays is under debate, the radiation is thought to originate from ultra-relativistic jets with $\Gamma \gtrsim 100$, where $\Gamma$ is the Lorentz factor of the jets (e.g., Mészáros 2006). Furthermore, it is known that GRBs with durations longer than 2 s, called long GRBs, are associated with highly energetic type Ic supernovae (e.g., Iwamoto et al. 1998). This implies that the relativistic jets are launched from deep inside massive progenitor stars after the onset of core-collapse.

The nuclear composition of GRB jets has been studied by several authors. Pruet, Guiles, and Fuller (2002), Lemoine (2002), and Beloborodov (2003) investigated within the framework of the standard fireball model.
(e.g., Mészáros 2006), assuming that a GRB jet initially consists of free nucleons. The nucleons can recombine into deuterium and/or $\alpha$ particles as the jet expands and cools but metal nuclei heavier than carbon cannot be produced. Thus, they conclude that the GRB jet consists only of light nuclei.

However, there is a caveat that the initial composition of the jet is not necessarily dominated by the free nucleons because the mechanism to launch a well-collimated relativistic jet is not specified. The proposed mechanisms include neutrino annihilation (e.g., Woosley 1993; MacFadyen & Woosley 1999) and magnetic field (e.g., Blandford & Znajek 1977; Brown et al. 2000; McKinney 2006). In fact, some observations of GRBs suggest that their relativistic jets were initially dominated by the magnetic field energy flux (e.g., Zhang & Pe’er 2009; Guiriec et al. 2011). When the relativistic jets are launched by the magnetic field, the jets could include heavy nuclei such as $^{56}$Ni (Fujimoto et al. 2008), and thus the nuclear composition of the jets may be different from the one that is derived within the framework of standard fireball model.

Horiuchi et al. (2012) investigated the survival of metal nuclei in relativistic jets. They analytically estimated the conditions for metal nuclei to survive photodisintegration and spallation at the base of the jets, in the accelerating jets, and at the emission region of a GRB. They argued that the nuclei can avoid destruction for a range of jet parameters. For example, the metal nuclei can survive at the base of the jet if the radius of the central engine is greater than $\sim 10^8$ cm or if the radiation luminosity is less than $\sim 10^{58}$ erg s$^{-1}$. They also investigated the possibility of entrainment of metal nuclei from stellar material during the jet propagation in the progenitor star and suggested that the entrainment is possible depending on the model parameters. However, they neither calculate detailed nuclear reactions nor presented final nuclear compositions.

In this paper, we calculate nonequilibrium nuclear reactions during the fallback and jet acceleration, based on the thermal histories derived by the relativistic hydrodynamics calculation and the steady, radial, relativistic magnetohydrodynamics calculation. Then we present the criterion for the metal nuclei to survive without serious dissociation. For the initial composition, we assume that the GRB jet is initially made from falling matter during a relativistic jet-induced explosion. We adopt Wolf–Rayet stars proposed to be progenitors of GRB-Supernovae (GRB-SN) and circumstantially treat the jet acceleration by thermal and/or magnetic pressure gradient and nucleosynthesis in relativistic jets.

This paper is organized as follows. In section 2, we describe the method and the model of the GRB jet. In section 3, we present the fallback in jet-induced explosions, the hydrodynamic properties of the accelerating jet, and the final nuclear composition of the jet. The conclusion is presented in section 4. Finally, the discussions are presented in section 5.

2 Method and model
2.1 Method
We calculate the nuclear composition of magnetized GRB jets with the following two steps. (1) We follow fallback in a relativistic jet-induced explosion with a two-dimensional relativistic hydrodynamics calculation in order to derive the initial composition of jets. (2) We follow the acceleration of magnetized GRB jets after the launch of jets with a steady, radial, relativistic magnetohydrodynamics calculation, assuming an interaction between the jet and stellar mantle does not influence the acceleration of the GRB outflow.

According to thermodynamical histories during the fallback and acceleration, nuclear reactions are calculated as a post-process (e.g., Hix & Thielemann 1996, 1999). The nuclear reaction network includes 281 isotopes up to $^{79}$Br. Here, we adopt an equation of state for relativistic particles, $p = e_{\text{int}}/3$, which means $\gamma_a = 4/3$, where $\gamma_a$ is the adiabatic index, and assume that the internal energy is dominated by contributions from photons and $e^\pm$ pairs.

The temperature $T$ is derived by following equation (e.g., Freiburghaus et al. 1999):

$$e_{\text{int}} = aT^4 \left( 1 + \frac{7}{4} \frac{T_9^4}{T_9^2 + 5.3} \right),$$  

(1)

where $a = 7.57 \times 10^{-15}$ erg cm$^{-3}$ K$^{-4}$ is the radiation constant and $T_9 = T/(10^9$ K). We note that the nonrelativistic gas pressure is negligible compared with radiation pressure because of the high entropy.

2.1.1 Fallback in relativistic jet-induced explosions
In jet-induced explosions, a considerable fallback takes place along the equatorial plane (e.g., Maeda & Nomoto 2003), and thus an interaction between the jets and cocoon and the stellar mantle determines which mass elements fall back to a central remnant. Therefore, a numerical simulation is required to correctly treat the fallback in explosions with relativistic jets. Hence, we calculate relativistic jet-induced explosions of C+O stars by the use of a two-dimensional relativistic Eulerian hydrodynamic code with Newtonian self-gravity (Tominaga et al. 2007; Tominaga 2009).

Since the GRB jets consist of the falling matter, the initial composition of the outflow is set to be an integration of
falling matter. However, as the freefall time of the matter at the inner boundary is short (< 0.1 s), materials that fall through the inner boundary well before the initiation of the jet injection are likely to have been accreted to the central remnant before the jet injection and are not likely to be re-ejected as the relativistic jets. Therefore, in this paper, only the matter falling after the initiation of the jet injection is assumed to be reejected and integrated as the initial composition of the outflow.

As the material falls, the temperature increases and nucleosynthesis may take place. Nucleosynthesis during the infall from the presupernova location to \( r = R_i \) (initial radius of the outflow) is calculated with a thermodynamical history taking into account heating due to the infall and cooling due to the Urca process (Bisnovatyi-Kogan 2002).

2.1.2 Steady relativistic magnetized outflow

We treat a GRB jet as a steady, radial, magnetized outflow with efficient magnetic dissipation. We assume that magnetic fields in the outflow are dominated by a toroidal component and that the field efficiently dissipates via magnetic reconnection as the outflow expands. Such efficient dissipation of magnetic fields creates a strong magnetic pressure gradient which enables a direct conversion of magnetic field energy into kinetic energy (Drenkhahn 2002; Drenkhahn & Spruit 2002). We neglect gravitational force because the gravitational energy is small compared with the radiation energy or magnetic field energy in the models considered here.

The mass, momentum, and energy conservation equations for the steady, radial outflow with a toroidal magnetic field are described as follows (e.g., Lyutikov & Blandford 2003):

\[
\frac{1}{\tau^2} \frac{\partial}{\partial r} (r^2 \rho \Gamma v) = 0, \tag{2}
\]

\[
\frac{1}{\tau^2} \frac{\partial}{\partial r} \left\{ r^2 \left[ (w + b^2) \Gamma^2 v^2 + \frac{b^2}{2} \right] \right\} + \frac{\partial p}{\partial r} = 0, \tag{3}
\]

\[
\frac{1}{\tau^2} \frac{\partial}{\partial r} \left[ r^2 (w + b^2) \Gamma^2 v \right] = 0, \tag{4}
\]

where \( r, \rho, \Gamma, v, p, w, \) and \( b \) are the distance from the center, proper mass density, outflow Lorentz factor, outflow velocity, pressure, enthalpy, and toroidal component of the magnetic four-vector, respectively. The enthalpy is defined by \( w = \rho + e_{\text{int}} + p \) where \( e_{\text{int}} \) is the internal energy. Here we adopt the unit system in which the speed of light is unity.

We employ the following evolution equation for the magnetic field which includes the dissipation of a non-axisymmetric magnetic field produced by an inclined rotator (Drenkhahn 2002):

\[
\frac{\partial}{\partial r} (r b \Gamma v) = -\frac{r \Gamma b}{\tau_{\text{dis}}}, \tag{5}
\]

where \( \tau_{\text{dis}} \) is the dissipation time scale derived as follows:

\[
\tau_{\text{dis}} = \frac{2 \pi \Gamma^2}{\epsilon \Omega} \sqrt{1 + u_A^2}, \tag{6}
\]

where \( u_A = b/\sqrt{w} \) is the Alfvén four-velocity, \( \Omega \) is an angular frequency of the central object, and \( \epsilon \) is a dimensionless factor.

From equations (2)–(6), the evolution of the Lorentz factor can be written as

\[
\frac{\partial \Gamma}{\partial r} = \frac{\gamma_s v^2 \Gamma^3}{(\Gamma^2 - \gamma_s^2)} \left[ \frac{2p}{r} + \frac{(2 - \gamma_s)b^2}{\gamma_s v \tau_{\text{dis}}} \right], \tag{7}
\]

where \( \Gamma_i \) is a Lorentz factor corresponding to the phase velocity of a fast magnetosonic wave and can be expressed as (e.g., Lyutikov & Blandford 2003)

\[
\Gamma_i = \frac{\gamma_s p + b^2}{w + b^2}. \tag{8}
\]

Equation (7) indicates that the flow is accelerated only when \( \Gamma > \Gamma_i \) and the acceleration of the flow is infinity when \( \Gamma = \Gamma_i \). These behaviors of the flow are due to the absence of the gravity in the above formulation. Thus, although the magnetic field possesses a part of the total energy, we assume that the flow is accelerated as if there is no magnetic field and the magnetization parameter defined by \( \sigma = b^2/w \) is constant until \( \Gamma \) reaches \( 2 \Gamma_i \). Then, the flow is accelerated with equation (7) at \( \Gamma < \Gamma_i \).

2.2 Model

We parametrize the GRB outflow with six parameters: isotropic energy deposition rate \( L_{\text{iso}} \), initial radius of the outflow \( R_i \), initial Lorentz factor \( \Gamma_i \), maximum Lorentz factor \( \Gamma_{\text{max}} \), angular frequency of central object (including dimensionless factor) \( \epsilon \Omega \), and initial magnetization parameter \( \sigma_i = b_i^2/w_i \), where \( b_i \) and \( w_i \) are the initial toroidal component of the magnetic four-vector and the initial enthalpy, respectively.

In this paper, we investigate the dependence of the final nuclear composition on \( L_{\text{iso}}, R_i, \) and \( \sigma_i \) for the C+O star models with metallicity \( Z = 0 \) and 0.02. We adopt progenitor stars constructed from C+O cores of 40 \( M_\odot \) stars with \( Z = 0 \) and \( Z = 0.02 \) (Umeda & Nomoto 2005) by attaching C+O envelopes in hydrostatic and thermal equilibriums.
The envelope connects to the core structures continuously and smoothly in the first order differentials (e.g., Saio et al. 1988) and extends down to a density $\rho = 10^{-10}$ g cm$^{-3}$. The density at the O layer is one order of magnitude lower in the model with $Z = 0.02$ than in the model with $Z = 0$. The O layer can be divided into two layers according to the abundances of C and Mg: O+Mg and O+C layers. The boundaries between the two layers for the models with $Z = 0$ and $Z = 0.02$ are $3 \times 10^5$ cm and $8 \times 10^8$ cm, respectively.

The parameter ranges of the outflow are as follows. (1) We adopt $L_{iso}^{52} = L_{iso}/(10^{52}$ erg s$^{-1}) = (4, 10)$ because the GRB isotropic luminosity is normally distributed in the range of $L_{iso}^{51-54}$ erg s$^{-1}$ (Ghirlanda et al. 2010). (2) $R_i$ corresponds to the size of the central engine that is constrained by the time variability of the GRB prompt emission. Since an interval of the time variability $\Delta t$ is of the order of $10^{-3}$ s, $R_i$ is presumed to be smaller than $c\Delta t \sim 10^8$ cm. Therefore, we vary $R_i$ for a range of $R_{i,7} = R_i/(10^7$ cm) = 1–10. (3) $\sigma_i$ is determined by the mechanism to launch relativistic jets. We construct the models with $\sigma_i = 0$, which correspond to the standard fireball model, and with a range of $\sigma_i = 0.1–10$, which correspond to magnetized jet models. We note that the strength of magnetic field realizing adopted $\sigma_i \sim 10^{12-15}$ gauss at $R_i$. (4)

For the other parameters, we fix $\Gamma_{max} = 100$ referring to a requirement to avoid the compactness problem (Piran 2004), and expeditely set $\epsilon \Omega = 10^2$ s$^{-1}$, assuming $\epsilon = 0.1$ and $\Omega = 10^8$ s$^{-1}$. We note that $\epsilon \Omega$ is rather uncertain. The initial Lorentz factor of the outflow is fixed at $\Gamma_1 = 1.23$, which corresponds to the sound velocity of ultra-relativistic fluids $c_s = 1/\sqrt{3}$. We name the model with $(L_{iso}^{52}, Z) = (4, 0)$ Model A, that with $(L_{iso}^{52}, Z) = (10, 0)$ Model B, $(L_{iso}^{52}, Z) = (4, 0.02)$ Model C, and $(L_{iso}^{52}, Z) = (10, 0.02)$ Model D. The model parameters are summarized in Table 1.

In the two-dimensional relativistic hydrodynamics calculation, we set parameters of the relativistic jets corresponding to the parameters of GRB outflow. The relativistic jets are injected from the inner boundary at an enclosed mass 1.4$M_\odot$ corresponding to a radius $R_{in} = 900$ km. The initial half angle of jet is set to $15^\circ$ (Zhang et al. 2003), and thus the energy deposition rates of jets $L_j$ are set to $L_j,51 = L_j/10^{51}$ erg s$^{-1} = 1.5$ and 3.4, which give $L_{iso}^{52} = 4$ and 10, respectively. We employ the Lorentz factor of the jet at the inner boundary as $\Gamma = 2.5$ by reference to subsection 3.2 and the energy density of the jet as to accelerate the jet to $\Gamma = 100$. 

### Table 1. Summary of the model parameters.

| Model | $L_{iso}^{52}$ | $L_j$ | $Z$ | $\Gamma_{max}$ | $\Gamma_1$ | $\epsilon \Omega$ |
|-------|---------------|-------|-----|----------------|-----------|-----------------|
| A     | 4             | 1.5   | 0   | 100            | 1.23      | 10^3            |
| B     | 10            | 3.4   | 0   | 100            | 1.23      | 10^3            |
| C     | 4             | 1.5   | 0.02| 100            | 1.23      | 10^3            |
| D     | 10            | 3.4   | 0.02| 100            | 1.23      | 10^3            |

3 Results

#### 3.1 Initial composition

Figures 1a and 1b show the falling regions of the models. The matter at the intersection between the circumference of the falling region and the jet axis is located at the inner boundary when the jet injection initiates. In the models with $Z = 0$ (Models A and B), the jet injection initiates at an early time for the model with $L_{iso}^{52} = 10$ (Model B), while it has to wait until the large portion of the material in the O layer falls in the model with $L_{iso}^{52} = 4$ (Model A) because the ram pressure of the jets cannot overcome that of the falling matter in the O layer (see Maeda & Tominaga 2009). As a result, the falling region of the model with $L_{iso}^{52} = 4$ is more extended than that of the model with $L_{iso}^{52} = 10$. On the other hand, in the models with $Z = 0.02$ (Models C and D), due to the low-density O layer the jet injections in both of the models with $L_{iso}^{52} = 4$ and 10 initiate at similar epochs and thus their falling regions are similar. The initial composition of the GRB outflow is derived from the integration of matter at the filled region because the matter at the shaded region is likely to be accreted to the central remnant before the initiation of the jet injection.

The composition of a mass element falling to $r = R_i$ depends on the maximum temperature, which is higher for smaller $R_i$ and for the matter that is initially located at the inner layers. The mean mass numbers $(A)$ of the initial composition of the outflow are shown as a function of $R_i$ in figure 2. Model B has small $(A) = 1–6$ for $R_{i,7} = 1–10$ because the jets consist of the matter that is initially located in the inner layer, while Model A can have $(A) \geq 15$ for $R_{i,7} \geq 5.5$ because the heavy nuclei can survive in the outer matter that falls to the outer $R_i$. The most abundant nucleus in the models with $(A) \geq 15$ is $^{28}$Si, due to the O burning during the fallback. The turnover at $R_{i,7} \sim 8$ stems from the fact that the maximum temperature of the mass elements in the outer layer is not high enough to ignite $^{16}$O.

On the other hand, the jet injection in the models with $Z = 0.02$ (Models C and D) is initiated at an early time and the outflow contains the matter that is initially located in the O+Mg layer. The falling regions are similar in both models and thus the resultant $(A)$ values of the models are similar. In the models with $Z = 0.02$, the presupernova temperature...
Fig. 1. Initial locations of the falling mass elements (filled and shaded regions), for (a) Z = 0 models (Models A and B) and (b) Z = 0.02 models (Models C and D). The shaded and filled regions represent the mass elements that fell before and after the initiation of the jet injection, respectively. We assume that the outflow consists of the materials initially present at the filled regions. The color of the regions represent the models with $L_{\text{iso}}^{10^{-52}}$, $R_i$, $\sigma_i$ = 4 (green) and $L_{\text{iso}}^{10^{-52}}$, $R_i$, $\sigma_i$ = 10 (red) and the background circles represent the boundaries between the inner O+Mg layer with $X(^{24}\text{Mg}) > X(^{12}\text{C})$ and the outer O+C layer with $X(^{24}\text{Mg}) \leq X(^{12}\text{C})$. (Color online)

of the infalling matter is lower than that in the models with $Z = 0$, due to the low-density O layer. Therefore, $(A) \geq 15$ is realized at $R_i \geq 7.3$ and the most abundant nucleus is $^{28}\text{Si}$ in these models.

3.2 Outflow dynamics

Hydrodynamical properties of the outflow for the models with $(L_{\text{iso}}^{10^{-52}}, R_i, \sigma_i) = (4, 6, 0)$ and $(4, 6, 3)$ are shown in figures 3a–3d.

Figures 3a and 3b show the evolutions of the Lorentz factor as functions of $r$ and fluid proper time $t_p$, which is calculated by an integration $t_p = \int_{R_i}^{r_i} 1/(\upsilon \Gamma) dr$, respectively. In the model with $\sigma_i = 0$, the Lorentz factor evolves linearly with radius until it reaches $\Gamma_{\text{max}}$, as expected from the standard fireball model. The acceleration takes place at $t_p \sim 10^{-5} - 10^{-2}$ s. On the other hand, in the models with $\sigma_i = 3$, the outflow is accelerated initially by thermal pressure, like the fireball model, and later by magnetic field with the time scale of $\tau_{\text{dis}}$. The Lorentz factor evolves more slowly than the model with $\sigma_i = 0$.

Figures 3c and 3d show the evolutions of the temperature as functions of $r$ and $t_p$, respectively. In the both models with $\sigma_i = 0$ and 3, the temperature decreases exponentially with...
3.3 Nuclear composition

Figures 4a and 4b show the time evolution of nuclear composition of the outflow for models with \((L_{i,52}, Z, R, \sigma) = (4, 0, 6, 0)\) and \((4, 0, 6, 3)\), respectively. Nuclear reaction ceases at \(t_p \sim 10^{-2}\) s in both models (a) and (b), because the temperature after the epoch falls below \(10^8\) K (figure 3d). The epoch corresponds to \(r \sim 10^{10}\) cm for the model with \((L_{i,52}, Z, R, \sigma) = (4, 0, 6, 0)\) and \(r \sim 4 \times 10^9\) cm for the model with \((L_{i,52}, Z, R, \sigma) = (4, 0, 6, 3)\).

In the model with \((L_{i,52}, Z, R, \sigma) = (4, 0, 6, 0)\), which corresponds to the standard fireball model, heavy and intermediate-mass nuclei are almost dissociated until \(t_p \sim 10^{-5}\) s, and \(^4\)He and \(^d\) (deuterium), \(^p\), and \(^n\), remain in the outflow after the acceleration (figure 4a). This demonstrates that the standard fireball model with \((L_{i,52}, Z, R, \sigma) = (4, 0, 6, 0)\) destroys heavy nuclei even if they are initially contained in the outflow.

On the other hand, in the model with \((L_{i,52}, Z, R, \sigma) = (4, 0, 6, 3)\), whereas some nuclei such as \(^{56}\)Ni are dissociated at \(t_p < 10^{-5}\) s and resynthesized at \(t_p > 10^{-3}\) s, other nuclei such as \(^{28}\)Si survive without dissociation and remain abundant after acceleration (figure 4b). This illustrates that the metal nuclei can survive in the magnetized jet.

Figures 5a–5d show mean mass number of final nuclear composition as functions of \(R_i\) and \(\sigma_i\) for Models A, B, C, and D (see table 1).

When the condition,

\[
T_0 > 3.6 \left( \frac{0.04}{t_p} \right)^{1/3},
\]

is satisfied, quasi-statistical equilibrium (QSE) is attained (e.g., Woosley et al. 1973). The white solid line in figures 5a–5d represents the criterion to establish QSE. Therefore, in the models with lower \(\sigma_i\) and smaller \(R_i\) than the white solid line, the composition can be described by QSE and the metal nuclei are almost destroyed due to the high entropy. This criterion is well fitted with a contour of initial temperature \(T_{i,0} = T_i/10^9\) K = 4.7 (green dotted line in figure 5). On the other hand, as long as QSE is not attained, the models with larger \(R_i\) yield final compositions with heavier mean mass numbers, except for the model B, which initially does not have heavy nuclei (figure 2).

Table 2 shows the ten most abundant nuclei and their mass fractions for Models A, B, C, and D with \((R_i, \sigma_i) = (10, 10)\). In Models A, C, and D, \(^{28}\)Si is most abundant and its mass fraction reaches \(\sim 40\%\). The mass fractions of \(^{16}\)O and \(^{32}\)S are also high in Models A, C, and D. In particular, the mass fraction of \(^{16}\)O reaches \(\sim 30\%\) in the model A. On the other hand, in the model B, the most abundant nucleus is \(^4\)He, the mass fraction of which is about \(\sim 52\%\), and the mass fractions of individual metal nuclei are less than 10%.

So far, we fix the isotropic jet luminosity at \(L_{i,52} = 4\) and 10 referring to canonical GRB isotropic luminosity. However, there are GRBs belonging to a less energetic class called low-luminosity GRBs (LLGRBs) with \(L_{i,52} \lesssim 10^{49}\) erg s\(^{-1}\) (e.g., Liang et al. 2007). From equation (4) and the equation of state, the isotropic luminosity \(L_{i,52}\) of a GRB jet is related to the initial temperature through \(e_{int}\) with equation (1) as \(L_{i,52} \simeq 16\pi R_i^2 e_{int} \Gamma_t^3 v_i (1 + \sigma_i)/3\). Applying the condition for \(T_i\) to avoid QSE establishment (i.e., \(T_{i,0} < 4.7\)), the condition in terms of the isotropic luminosity is obtained as follows:

\[
L_{i,52} \lesssim 3.9 \times 10^{50} R_i^2 (1 + \sigma_i) \text{ erg s}^{-1}.
\]
Fig. 5. Mean mass numbers of the final nuclear composition in the outflow as functions of $R_i$ and $\sigma_i$ for (a) Model A, (b) Model B, (c) Model C, and (d) Model D. The model parameters are summarized in table 1. The white solid lines and green dotted lines represent the criterion for the QSE establishment and the contour of $T_{i,9} = 4.7$, respectively.

Table 2. Ten most abundant nuclei and their mass fractions for Models A, B, C, and D with $(R_i, \sigma_i) = (10, 10)$.

|   | Model A   | Model B   | Model C   | Model D   |
|---|-----------|-----------|-----------|-----------|
| 1 | $^{28}\text{Si}$ | $^{4}\text{He}$ | $^{28}\text{Si}$ | $^{28}\text{Si}$ |
|   | $3.7182E-01$ | $5.1509E-01$ | $3.9266E-01$ | $3.8889E-01$ |
| 2 | $^{16}\text{O}$ | $^{58}\text{Ni}$ | $^{16}\text{O}$ | $^{32}\text{S}$ |
|   | $2.9626E-01$ | $9.0864E-02$ | $2.0150E-01$ | $1.7979E-01$ |
| 3 | $^{32}\text{S}$ | $^{32}\text{S}$ | $^{32}\text{S}$ | $^{32}\text{S}$ |
|   | $2.0439E-01$ | $7.2190E-02$ | $1.7598E-01$ | $1.6323E-01$ |
| 4 | $^{36}\text{Ar}$ | $^{28}\text{Si}$ | $^{54}\text{Fe}$ | $^{54}\text{Fe}$ |
|   | $5.3067E-02$ | $5.8950E-02$ | $6.9657E-02$ | $8.7961E-02$ |
| 5 | $^{40}\text{Ca}$ | $^{54}\text{Fe}$ | $^{36}\text{Ar}$ | $^{36}\text{Ar}$ |
|   | $4.1669E-02$ | $5.7403E-02$ | $3.9364E-02$ | $4.0427E-02$ |
| 6 | $^{54}\text{Fe}$ | $^{55}\text{Co}$ | $^{40}\text{Ca}$ | $^{40}\text{Ca}$ |
|   | $1.0189E-02$ | $3.3212E-02$ | $2.2114E-02$ | $2.4969E-02$ |
| 7 | $^{24}\text{Mg}$ | $^{36}\text{Ar}$ | $^{55}\text{Co}$ | $^{55}\text{Co}$ |
|   | $7.1779E-03$ | $2.4232E-02$ | $1.6240E-02$ | $1.9738E-02$ |
| 8 | $^{56}\text{Ni}$ | $^{57}\text{Ni}$ | $^{24}\text{Mg}$ | $^{56}\text{Ni}$ |
|   | $2.8607E-03$ | $1.9790E-02$ | $1.0641E-02$ | $1.4527E-02$ |
| 9 | $^{55}\text{Co}$ | $^{16}\text{O}$ | $^{58}\text{Ni}$ | $^{58}\text{Ni}$ |
|   | $2.1998E-03$ | $1.8796E-02$ | $8.9718E-03$ | $1.3446E-02$ |
| 10 | $^{58}\text{Ni}$ | $^{56}\text{Ni}$ | $^{56}\text{Ni}$ | $^{56}\text{Ni}$ |
|    | $1.2994E-03$ | $1.4433E-02$ | $8.8717E-03$ | $1.0203E-02$ |

According to this condition, the metal nuclei can be involved in the jets of the LLGRBs even within the framework of the standard fireball model (i.e., $\sigma_i = 0$).

Figure 6 shows the cooling radius $R_{cool}$ above which the condition to establish QSE is no longer satisfied; $R_{cool} \sim 10^{7-8}$ cm depending on $R_i$ and $\sigma_i$. Since QSE is not attained at $r > R_{cool}$, the metal nuclei, which have not been dissociated at $r \sim R_{cool}$, can survive after the acceleration. Furthermore, if the metal nuclei mix from the stellar mantle at $r > R_{cool}$, as discussed in Horiuchi et al. (2012), they are involved in the outflow without serious dissociation.

4 Conclusion

In this paper, we investigated the nuclear composition of GRB jets assuming that the jets initially possess metal nuclei. We calculated fallback in a relativistic jet-induced explosion with a two-dimensional relativistic hydrodynamics calculation and derived the initial composition of the jet. Then, we calculated the acceleration of magnetized GRB jets and detailed nuclear reactions in the jets with the initial compositions.

We found that the composition of the falling matter after the jet injection depends on the radius to which the matter falls off and thus the initial composition of the jet depends on the size of the central engine of the jet. If the size of the
central engine is larger than \( R_{i,7} \geq 5 \), the matter contains metal nuclei in abundance except for the model with \( Z = 0 \) and \( L_{\gamma,52} = 10 \) (Model B). Model B involves only a small fraction of metal nuclei with the mean mass number of \( \langle A \rangle \leq 6 \) even for \( R_{i,7} \geq 5 \), because the jets consist of mass elements with high temperatures in the presupernova star.

We conclude that the metal nuclei can survive in the jet if QSE is not established. The metal nuclei are dissociated mostly to \(^4\)He once QSE is established. This is due to the high entropy of the jet being accelerated to \( \Gamma = 100 \), in which \( \alpha \)-rich freezeout takes place. Therefore, the final nuclear composition of the jet is dominated by \(^4\)He for the QSE-established models. The criterion for QSE establishment is well fitted with the contour of \( T_{i,9} = 4.7 \). The criterion leads to the condition of the isotropic jet luminosity \( L_{\gamma,iso}^{j} \) as \( L_{\gamma,iso}^{j} \lesssim 3.9 \times 10^{50} \frac{R_{i,7}}{1 + \sigma_i} \) erg s\(^{-1} \), and this is consistent with the results of Horiuchi et al. (2012) since the beaming correction for the jet reduces \( L_{\gamma,jiso}^{i} \) by the order of \( \sim 2 \).

The most popular model for the acceleration of a GRB jet is the fireball model, in which thermal pressure accelerates the jet from sub-relativistic to ultra-relativistic (Goodman 1986; Paczynski 1986; Meszaros 2006). In such a standard fireball model, \( R_{i,7} \geq 4 \) is required for the metal nuclei to survive in the jet with \( L_{\gamma,iso}^{j} \geq 1 \). If the jet has a luminosity of \( L_{\gamma,iso}^{j} \lesssim 4 \), the size of the central engine must be larger than \( 10^5 \) cm for the survival of metal nuclei. However, this violates a constraint on the size of the central engine from the time variability of the flux, i.e., \( R_{i,7} \lesssim 10 \). Therefore, at least, metal nuclei initially contained in the GRB jet with \( L_{\gamma,iso}^{j} \lesssim 4 \) should be destroyed and the final nuclear composition is dominated by light nuclei and free nucleons within the framework of the standard fireball model.

On the other hand, the magnetized jet has been proposed to explain spectra of some GRBs. For example, Zhang and Pe’er (2009) suggests that GRB 080916C involves a magnetized jet because a thermal component, which should appear in the spectrum if the jet energy is initially dominated by thermal energy, was not detected. Also, Guiriec et al. (2011) reports that GRB 100724B exhibits a typical nonthermal spectrum, called a band spectrum, with a significant thermal component and suggests that a highly magnetized jet can explain the feature, which is quite challenging for the standard fireball model. In such a magnetized jet, the energy is initially possessed by the magnetic field, and thus the condition for QSE is avoidable in the jet, even with high luminosities like \( L_{\gamma,iso}^{j} \lesssim 10 \), while satisfying \( R_{i,7} \lesssim 10 \), although \( R_{i,7} \gtrsim 5 \) is required for metal nuclei to initially exist in the jet in our model.

5 Discussion

In this paper, we constrain the size of the central engine with the time variability of the gamma-ray emission since it could reflect the time variability of the central engine, for example, in the internal shock model. However, the time variability of the gamma-ray emission may stem from other mechanisms. For example, an order of \( \sim 1 \) s variability can arise from the interaction between the jet and the progenitor star (Morsony et al. 2010), or the time variability may be related to the emission mechanism itself, e.g., turbulent motion at the gamma-ray emitting region (e.g., Zhang & Yan 2011). In these cases, the size of the central engine is not necessarily constrained, at least, by the time variability, and the larger \( R_{i,7} (> 10) \) is possible. If the size of the central engine is as large as \( R_{i,7} = 50 \), the criterion for the survival of the metal nuclei is \( L_{\gamma,iso}^{j} \lesssim 10^{53} (1 + \sigma_i) \) erg s\(^{-1} \), indicating that a large fraction of GRBs can possess metal nuclei in the jet even with \( \sigma_i = 0 \).

We treated the GRB jet as a steady, radial, magnetized outflow for simplicity. However, in reality, the jet should propagate through the progenitor and interaction between them is expected. Such an interaction could affect the dynamics of the jet. For example, it is suggested that the initial confinement by the collapsing stellar material has an important role on the collimation of magnetically dominated jets from magnetars (Uzdensky & MacFadyen 2007). The cocoon, which is a shocked hot gas surrounding the jet, also has a role in collimating the jet (e.g., Morsony et al. 2007; Mizuta & Aloy 2009; Bromberg et al. 2011). If the jet is collimated, the evolution of the jet cross-section \( \Sigma(r) \) may be expressed as \( \Sigma(r) \propto r^2 \) with \( \xi < 2 \). In this case, the temperature, and also the density, will decrease more slowly than in the radial flow, and thus the criterion, equation (10), will possibly tighten because the fluid will tend to keep a high temperature at the initial stage.
Metzger, Giannios, and Horiuchi (2011b) investigated the nucleosynthesis from free nucleons in the magnetically dominated jet in the context of the protomagnetar model for the central engine of GRBs (Metzger et al. 2011a). The main difference between their work and this paper is the parameter range of the entropy per baryon. They estimated the entropy per baryon in the jet with the analytic expression derived in Qian and Woosley (1996) for the neutrino-driven winds. They suggested that, since the entropy per baryon in the neutrino driven wind is sufficiently low, the heavy nuclei beyond the iron peak, i.e., $A \gtrsim 56$ where $A$ is the mass number, can be synthesized in the jet. On the other hand, we do not consider such a sufficiently low-entropy environment for synthesis of metal nuclei from free nucleons and we focus only on the survival of the metal nuclei in this paper. Future studies on the central engine of GRBs will reveal whether such a low-entropy environment is realized or not.

It is known that GRBs are one of the candidates for the origin of ultra-high energy cosmic rays (UHECRs) (e.g., Waxman 1995; Vietri 1995) and LLGRBs are also possible candidates for the origin of UHECRs (e.g., Murase et al. 2006; Gupta & Zhang 2007). The acceleration of metal nuclei up to ultra-high energies is also possible in both usual high-luminosity GRBs and LLGRBs (Wang et al. 2008; Murase et al. 2008). Abraham et al. (2010) reported that the observed UHECRs are dominated by heavy nuclei at high energies, i.e., $E \gtrsim 10^{19}$ eV (see, however, Abbassi et al. 2010). Hooper and Taylor (2010) suggested that the result of Abraham et al. (2010) can be quantitatively reproduced only if UHECR composition at the accelerating site mainly consists of intermediate-mass nuclei, such as nitrogen, together with a considerable fraction of heavy nuclei such as iron. Therefore, GRBs with magnetized jets or LLGRBs have the potential to be the origin of UHECRs in terms of, at least, the nuclear composition, because intermediate-mass nuclei, e.g., $^{28}\text{Si}$, are abundant in the jet. However, in order to explore whether the model considered in this paper can quantitatively explain the UHECR observations, the acceleration process, the escape from the source, and the propagation of the UHECR nuclei from the source to the Earth also have to be considered. Such calculations are beyond the scope of this paper and will be studied elsewhere.

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