Optical trapping of transversal motion for an optically levitated mirror

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Optomechanical systems are suitable systems to elucidate quantum phenomena at the macroscopic scale. The systems should be well isolated from the environment to avoid classical noises, which conceal quantum signals. Optical levitation is a promising way to isolate optomechanical systems from the environment. In order to realize optical levitation, all degrees of freedom need to be trapped. So far, longitudinal trapping and rotational trapping with optical radiation pressure have been well studied and validated with various experiments. On the other hand, less attention has been paid to transversal trapping. Here, we experimentally confirmed transversal trapping of a mirror of a Fabry-Pérot cavity for the first time by using a torsional pendulum. By this demonstration, we proved experimentally that optical levitation is realizable with only two Fabry-Pérot cavities that are aligned vertically. This work paves the way toward optical levitation and realizing a macroscopic quantum system.

I. INTRODUCTION

Quantum mechanics is an established theory in modern physics. However, no experiment has shown quantum phenomena of a massive object, and it is not fully understood what makes classical systems classical. Thus, there are active discussions about whether quantum mechanics breaks at some mass scale or not. One possible scenario is that quantum mechanics is valid over all scales, but macroscopic systems tend to couple strongly to the environment. This interaction between the macroscopic systems and the environment causes thermal decoherence. As a result of this thermal decoherence, the systems lose their quantumness. Other scenarios are that quantum mechanics have to be modified or include gravitational effects. For example, the Continuous Spontaneous Localization model [1] is a new theory that modifies quantum mechanics by adding a collapse mechanism. Gravitational decoherence is also one of the decoherence models [2,3].

These discussions lead to a demand for the realization of a macroscopic quantum system to test such proposed models. In addition to the discussion about the decoherence mechanism, the effect of the gravity matters in a macroscopic quantum system. Thus, a macroscopic quantum system can be a platform to experimentally elucidate quantum gravity and ultimately lead to the unification of the theories of gravity and quantum mechanics [4,9].

Recent progress in experimental techniques gives us a good chance to realize a macroscopic quantum system. In particular, optomechanical systems have been attracting growing attention as a promising candidate for macroscopic quantum systems. In an optomechanical system, a mechanical oscillator couples to an optical field, and we can measure the displacement of the oscillator with laser beams. The sensitivity is so high that it can reach the quantum regime.

So far, below nanogram scales, several systems are reported to reach quantum regime [10,12] in the sense that they hit the standard quantum limit. Though there are also experimental efforts with even heavier mechanical oscillators in the range of micrograms to kilograms, no experiments have yet succeeded in preparing their systems into reaching quantum regimes [13-20]. As an oscillator gets massive, thermal decoherence due to the mechanical structure to support the oscillator prevents the system from reaching quantum regime. In other words, thermal noise conceals the quantum signal in the massive oscillators.

Among efforts on getting high mechanical quality factor to lower the thermal noise, optical levitation is an alternative way to eliminate thermal noise from mechan-
tical support. By supporting a mirror just by the radiation pressure of laser lights, mechanical structures that introduces thermal noise can be avoided.

A widely used method to levitate an object is to employ optical tweezers \[21\] \[26\]. Optical tweezers use optical gradient force. Thus, levitated objects have to be inside a highly focused laser beam. Therefore, the size of the levitated objects with optical tweezers are so far limited to the nanogram-scale. An alternative way to levitate more massive objects is to levitate a highly reflective mirror by the radiation pressure of laser light \[27\] \[29\].

For any type of optical levitation, stable trapping of all degrees of freedom of the mirror is necessary. Here, the direction in which we measure the displacement of the oscillator is called the longitudinal direction, and the transversal direction is defined to be transverse to the longitudinal direction, as shown in Fig. 1. In previous researches, longitudinal and rotational trapping with optical radiation pressure were well studied and tested with experiments. For longitudinal trapping, the optical spring effect is often utilized \[30\] \[31\]. Radiation pressure in a Fabry-Perot cavity behaves like a spring at a detuned point. As for rotational degrees of freedom, it is known that the rotational motion of a suspended Fabry-Perot cavity is unstable due to the radiation pressure inside the cavity. This is called Sildes-Sigg instability \[32\]. We can avoid Sildes-Sigg instability by using a triangular cavity, and several experiments have already utilized triangular cavities to stabilize their suspended mirrors \[14\] \[15\] \[33\]. As an alternative way, Sildes-Sigg instability can be mitigated by angular feedback control with optical radiation pressure \[34\] \[35\].

On the other hand, transversal trapping for a disk mirror has not been fully examined experimentally. Although some configurations for stable optical levitation were proposed \[27\] \[29\], they have not been experimentally realized. In the configuration proposed in Ref. \[27\], two horizontal laser beams are introduced as optical tweezers to trap the levitated mirror in the horizontal direction. In Ref. \[28\], they proposed to build three Fabry-Perot cavities in a tripod form below the levitated mirror. Since the Fabry-Perot cavities are slightly inclined from the vertical axis, the optical springs can trap the horizontal motion of the levitated mirror. In Ref. \[29\], the levitated mirror is sandwiched by two Fabry-Perot cavities aligned vertically. The horizontal motion of the levitated mirror is trapped by making use of the geometry of the Fabry-Perot cavities.

In this paper, we show our first experimental demonstration of the optical transversal trapping of a mirror. Our system utilizes a torsional pendulum as a force sensor to measure the optical trapping force acting on a mirror. Here, we consider the sandwich configuration \[29\] because the sandwich configuration employs a transversal restoring force due to the optical radiation pressure in the Fabry-Perot cavities aligned vertically as explained in more detail in the next section. We conducted measurements to evaluate the horizontal restoring force on the mirror in a Fabry-Perot cavity to show that the Fabry-Perot cavity gives significant positive restoring force.

### II. SANDWICH CONFIGURATION

The sandwich configuration is a levitation configuration proposed in Ref. \[29\]. Two Fabry-Perot cavities are built above and below a levitated mirror as shown in Fig. 2. The lower cavity produces radiation pressure that support the levitated mirror, while the upper cavity is introduced to stabilize the levitated mirror.

To realize levitation, the levitated mirror must be stable in all degrees of freedom, which are \((x, y, z)\) for position and \((α, β, γ)\) for rotation, where \(α\), \(β\) and \(γ\) are the rotation angle around \(x\), \(y\) and \(z\) axes, respectively. Here, we set the origin of the coordinates to the center of the curvature of the levitated mirror, with the \(z\)-axis being vertical. Since the sandwich configuration is symmetric in the \(z\)-axis, it is enough to consider the motion in \((x, z, β)\) for the stability.

For stable levitation, all restoring force in each degrees of freedom must be positive. Let \((δF_x, δF_z, δN_β)\) represent the restoring forces and the restoring torque corresponding to the small displacement \((δx, δz, δβ)\). The linear response matrix, \(K\), is given in the diagonalized form by

\[
K = \begin{pmatrix}
K_L^\text{hor} + K_U^\text{hor} & 0 & 0 \\
0 & K_L^\text{opt} + K_U^\text{opt} & 0 \\
0 & 0 & mgR
\end{pmatrix}, \tag{1}
\]

where \((δF_x, δF_z, δN_β) = K(δx, δz, δβ)\). Therefore, we can consider the stability of each degrees of freedom separately.
In the vertical direction, $z$, the levitated mirror can be trapped by double optical springs \cite{30}. $K_L^{\text{opt}}$ and $K_U^{\text{opt}}$ in Eq. (1) represent spring constants due to the optical spring of the lower and upper cavities, respectively.

As for the $\beta$ rotational direction, the levitated mirror is trapped with gravitational potential like an ordinary suspended pendulum when the mirror is convex downwards as shown in Fig. 3. Accordingly, the spring constant can be expressed as $mgR$ where $m$ is the mass of the levitated mirror, $g$ is the gravitational acceleration and $R$ is the radius of curvature of the mirror.

The stability in the horizontal direction, $x$, is nontrivial as it is due to the geometrical configuration. When the levitated mirror moves slightly in the horizontal direction, the optical paths in the cavities incline. The inclined radiation pressure of the cavities provides restoring force in the horizontal direction. In the Fourier domain, the spring constant for horizontal direction, $K_j^{\text{hor}}$, is given by

$$K_j^{\text{hor}} = k_j^{\text{hor}} + i\omega_j^{\text{hor}}$$

$$= \pm \frac{1}{a_j} \frac{2P_j}{c} \left[ 1 - i\omega_j \frac{\pi l_j}{F_j c (1 - G_j)} \right],$$

(2)

where $a_j$ is the distance between the center of curvatures, $P_j$ is the intracavity power, $F_j$ is the cavity finesse, $l_j$ is the cavity length and $c$ is the speed of light. $J = \text{U, L}$ indicates the upper or lower cavity. $G_j$ is defined as $G_j = (1 - l_j/R_j)(1 - l_j/R_l)$. The sign is positive for the upper cavity and negative for the lower cavity. We focus on $k_j^{\text{hor}}$ because the damping term, $\gamma_j^{\text{hor}}$, is negligible with realistic parameters aimed at reaching the standard quantum limit \cite{29}. The intracavity power in the lower cavity must be larger than that in the upper cavity in order to support the levitated mirror. Nevertheless, the total restoring force can be positive if we adopt an enough small $a_U$. This horizontal trapping scheme is unique to the sandwich configuration and had not been demonstrated experimentally.

### III. EXPERIMENTAL METHOD

Our purpose of the experiment is to demonstrate the trapping of a mirror in the horizontal direction with the sandwich configuration. The trapping is realized if the sandwich configuration gives a positive restoring force to the levitated mirror. Therefore, we need to evaluate the restoring force in horizontal direction precisely.

In order to detect restoring force due to the sandwich configuration, we utilized a torsional pendulum as a force sensor as shown in Fig. 3. A mirror is attached to the edge of the torsional pendulum; we treat this mirror as the levitated mirror in the sandwich configuration, and build a cavity with this mirror. Restoring force on the mirror is reflected in the restoring torque of the torsional pendulum. We note that the rotational motion of the pendulum and the horizontal motion of the mirror are identical for the mirror if the displacement of the motion is small compared to the length of the pendulum bar. Since a torsional pendulum generally has a small restoring force, it is sensitive to an extra restoring force, which comes from the optical trapping force in the sandwich configuration in our case. In addition, we can omit the lower cavity of the sandwich configuration in this experiment because the restoring force in the horizontal direction is given by the upper cavity and the torsional pendulum supports the mirror against gravitational force instead of the lower cavity.

The extra applied restoring force on the pendulum due to the sandwich configuration is reflected on the shift in the resonant frequency of the pendulum. The spring constant from the extra force can be expressed as

$$k_{\text{ext}} = \frac{2\pi l^2}{L^2} (f_0^2 - f_0^2),$$

(3)

where $I$ is the moment of inertia of the pendulum, $f_0$ is the original resonant frequency of the pendulum, $f_{\text{eff}}$ is the resonant frequency of the pendulum when the extra force is applied and $L$ is the distance between the suspension point of the pendulum bar and the beam spot position on the mirror. We derived the restoring force due to the sandwich configuration by measuring the resonant frequency shift according to Eq. (3).

The resonant frequency of the pendulum is determined by measuring the open-loop transfer function of the pendulum control. The torsional motion of the pendulum is monitored with an optical lever. The optical lever beam at the wavelength of 850 nm was injected to the center of the pendulum, where an aluminum mirror is attached, and the position of the reflected beam is read out with a position sensitive detector. The torsional mode of the pendulum is feedback controlled by applying differential signals to coil-magnet actuators attached on both arms of the pendulum. With the feedback control, the horizontal motion of the mirror is stabilized to 1.3 $\mu$m in the root mean square value, which is enough to form a cavity.

The pendulum is designed to have a small restoring force to be a sensitive force sensor. The moment of inertia of the pendulum is $I = 7.2 \times 10^{-6}$ kg m$^2$ and the mass of the pendulum is 8.8 g. A small moment of inertia is better because the susceptibility of the pendulum will be larger. Therefore, we used a quarter-inch mirror on the edge of the pendulum to reduce the moment of inertia and used a thin aluminum pole for the arm. The length of the pendulum bar is $2L = 17$ cm. The suspension wire length is 105 mm and the radius of the wire is 20 $\mu$m. We have to reduce the restoring force in the pendulum itself to detect the tiny extra restoring force on the pendulum, so we chose an ultra thin tungsten wire. Thanks to its tensile strength, this thin wire can suspend the pendulum while keeping the restoring force small enough.

The cavity is illuminated by a laser beam at the wavelength of 1550 nm. The maximum output power of the laser source is 2 W. The radii of curvature of the pendulum mirror and the input mirror are both 75 mm. The
distance between the center of curvatures was measured to be $a_U = 8.9 \pm 0.8$ mm from transverse mode spacing measurements. The finesse of the cavity was measured to be $880 \pm 90$. The cavity length is stabilized to the resonance using a piezoelectric actuator attached on the input mirror. We used the Pound-Drever-Hall method [36] to obtain the error signal for the cavity length control. We injected different input power to the cavity to see the power dependence of the trapping force.

The setup needs to be isolated from environmental disturbances to keep the cavity on the resonance stable and to measure the transfer function precisely. The setup is in vacuum at the level of 1 Pa to prevent air turbulence. The pendulum is a double pendulum; the intermediate mass is suspended with three wires and has strong eddy-current damping introduced by surrounding neodymium magnets to minimize translational fluctuation.

IV. RESULT AND DISCUSSION

We measured the transfer function of the pendulum, which is shown in Fig. 4. Around the resonant frequency, the phase of the transfer function flips, so it can be determined. The result shows that the resonant frequency of the pendulum itself was $32.2 \pm 1.1$ mHz. The resonant frequency shifted to $43.5 \pm 0.4$ mHz when the cavity circulated light with a power of $29.7 \pm 8.0$ W. The shift of the resonant frequency indicates an additional positive restoring force on the pendulum, and we can estimate

FIG. 3. (a) Experimental setup. The cavity is built on the edge of the torsional pendulum. The horizontal rotation of the torsional pendulum is read out with an optical lever. The torsional pendulum and the cavity are in the vacuum chamber, and the laser beam is introduced through an optical fiber. EOM: electro-optic modulator, PZT: piezoelectric transducer. (b) The torsional pendulum. The quarter-inch mirrors are put on the edges of the bar. The aluminum mirror attached on the center of the bar is used for the optical lever. The bar is suspended with a 105 mm wire. The pendulum is a double pendulum to isolate the system from seismic motion, and the intermediate mass is suspended with 76.6 mm wires. Two neodymium magnets are attached on the both arms 35 mm apart from the center of the bar for coil-magnet actuators.

FIG. 4. Measured transfer functions of the torsional pendulum. The points represent each measured values and the lines are fitted lines. The blue points and lines represent the data when the laser is off. The red points and lines represent the data when the laser is on, while the intracavity power is $29.7 \pm 8.0$ W. The fitted parameters are the resonant frequency, $Q$ value of the resonance, and overall gain factor. The resonant frequencies and $Q$ values are fitted with the phase data of the measured transfer function. The overall gain factors are fitted with the gain data of the measured transfer function.
FIG. 5. Spring constant from the extra restoring force derived from the sandwich configuration. Firstly, we measured the restoring force of the torsional pendulum itself. The measurement point with zero intracavity power corresponds to this measurement. Then, we injected a laser beam with various power. The results when the laser is on are represented by the other five measurement points. The shaded region represents the estimated values that are calculated with the cavity parameters based on Eq. (2).

the amplitude of the swinging is identical for all measurements, so the fluctuation of the intracavity power is in proportion to the intracavity power in principal in our case.

We repeated measuring the transfer function three times for each intracavity power. We obtained the estimated value of the resonant frequency for every measured transfer functions. The statistical uncertainty of the resonant frequency was estimated and it was the dominant source of uncertainty of the spring constant estimation.

V. CONCLUSION

We proved the stability of the sandwich configuration in the horizontal direction. In order to measure the restoring force on the mirror within the sandwich configuration, we utilized a torsional pendulum as a force sensor. By observing the resonant frequency shift of the torsional pendulum, we evaluated the extra restoring force that comes from the sandwich configuration. The measured restoring force was positive and the dependence on the intracavity power was consistent with the theoretical prediction.

Our work gives the first experimental validation to the transversal trapping for an optically levitated mirror of a Fabry-Pérot cavity. Consequently, we showed all degrees of freedom can be trapped stably by optical levitation. Thus, optical levitation in the sandwich configuration will be able to actualize. Our research is a crucial step towards realizing and elucidating macroscopic quantum systems. We have also established a method to measure the horizontal restoring force that acts on a levitated mirror, and this method is applicable to other levitation configurations for evaluating their stability.

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