Microscopic Theory of the Spin Hall Magnetoresistance

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(Dated: June 1, 2020)

We consider a microscopic theory for the spin Hall magnetoresistance (SMR). We generally formulate a spin conductance at an interface between a normal metal and a magnetic insulator in terms of spin susceptibilities. We reveal that SMR is composed of two contributions: the static and the dynamic parts. The static part, which is almost independent of the temperature, originates from spin flip caused by an interfacial exchange coupling. On the other hand, the dynamic part, which is induced by the creation or annihilation of magnons, has an opposite sign from the static part. By the spin-wave approximation, we predict that the latter results in a nontrivial sign change of the SMR signal at a finite temperature. We also derive the Onsager relation between a spin conductance and a thermal spin-current noise.

I. INTRODUCTION

Magnetoresistance is one of the fundamental phenomena in the research field of spintronics. Giant magnetoresistance and tunneling magnetoresistance are now essential ingredients in spintronics technology for sensors, memories, and data storage. Recently, a novel type of magnetoresistance called spin Hall magnetoresistance (SMR) has been observed in a bilayer system composed of a normal metal (NM) and a ferromagnetic insulator (FI)²⁻¹⁸. The SMR is explained by a combination of charge-spin conversions in the NM and a loss of the spins at the NM/FI interface²⁻¹⁰. When an in-plane current is applied to the NM layer with large spin-orbit interaction, a spin accumulation is caused near the NM/FI interface by the spin Hall effect (SHE). An amount of spin accumulation is affected by the orientation of the FI magnetization because it changes a rate of spin loss at the interface. A backward spin current due to spin diffusion is converted into the charge current again by the inverse spin Hall effect (ISHE), and induces longitudinal magnetoresistance dependent on the orientation of the FI magnetization. The strength of SMR is of the order of $\theta_{SH}^2$, where $\theta_{SH}$ is the spin Hall angle.

Recently, the SMR has been reported for the bilayer system composed of a NM and an antiferromagnetic insulator (AFI)¹⁹⁻²⁵, where the orientation of the Néel vector of the AFI is fixed by a strong external field or by the orientation of the magnetization of the FI using the NM/AFI/FI trilayer structure. It is remarkable that its sign of the SMR is opposite to the one in the NM/FI bilayer system with respect to the external magnetic field. This sign change of SMR can be explained if the Néel vector of the AFI is fixed as perpendicular to the external magnetic field or to the magnetization of the FI via an exchange bias. We note that a similar sign change of the SMR has been observed in a non-collinear-ferrimagnet/NM bilayer system²⁷,²⁸.

The SMR can be described theoretically by combining the spin-diffusion theory with the boundary condition at the interface in terms of the spin-mixing conductance.²⁻¹⁰

In this theory, however, the spin-mixing conductance at the interface is a phenomenological parameter to be determined in experiments, and therefore, its temperature dependence cannot be predicted. Furthermore, the magnetization-orientation dependence of the spin-mixing conductance is assumed phenomenologically by its definition. This semiclassical description of the SMR seems to be insufficient for studying quantum features of magnetic insulators such as the effect of thermally excited magnons. Recently, a microscopic theory of the SMR has been proposed based on a local mean-field approach,²⁹⁻³⁰, which is justified at high temperatures. However, a general microscopic theory applicable to a wide parameter region is still lacking.

In addition, the thermal noise of the pure spin current at the NM/FI interface has been measured using the ISHE.²⁵. Although the Onsager relation between the thermal noise and the spin-mixing conductance at the interface has been discussed qualitatively,²⁵, it has not been derived from a microscopic theory so far. Explicit derivation of the Onsager relation provides an important basis for a theory of nonequilibrium spin-current noise, which generally includes important information on spin transport,²⁵ as suggested from studies of the electronic current noise.³⁹⁻⁴₀.

In this paper, we construct a microscopic theory of the SMR based on the method of the tunnel Hamiltonian. We derive a general formula of a spin conductance at the interface for both NM/FI and NM/AFI bilayer systems. By applying the spin-wave approximation, we discuss the temperature dependence of the SMR well below the magnetic transition temperature. We also formulate the spin-current noise in the
same framework, and derive the Onsager relation, i.e., the relation between the thermal spin-current noise and the spin conductance.

This paper is organized as follows. In Sec. II, we theoretically describe the SMR by combining the spin diffusion theory in the NM and the spin conductance at the interface. In Sec. III, we give the microscopic Hamiltonian for the NM/FI (or NM/AFI) bilayer system. In Sec. IV, we formulate the spin conductance at the interface using the method of the tunnel Hamiltonian. In Sec. V and Sec. VI we discuss the temperature dependence of the spin conductance using the spin-wave approximation. In Sec. VII we briefly discuss the relevance of the present study to the SMR experiments. In Sec. VIII we formulate the spin current noise, and derive the Onsager relation explicitly. Finally, we summarize our results in Sec. IX. Details of the derivation are given in two appendices.

II. THEORETICAL DESCRIPTION OF SMR

We theoretically describe the SMR by improving the spin diffusion theory given in Ref. 10. Let us first consider a bilayer system composed of NM and FI layers. When we apply an electric field to the NM layer in the $+x$ direction, a spin current $J_{s}^{\mathrm{SH}} = \theta_{\mathrm{SH}} \sigma E_{x}$ is driven in the $-y$ direction by the spin Hall effect, where $\theta_{\mathrm{SH}}$ is a spin Hall angle, $\sigma$ is an electric conductivity, and $E_{x}$ is a $x$ component of the electric field. This spin current induces spin accumulation at an interface between the NM and FI layers as shown in Fig. 1(a) and (b). In the figure, the direction of the spins accumulated at the interface is denoted with $\sigma$. In a steady state, the spin current $J_{s}^{\mathrm{SH}}$ is balanced with a backflow spin current

$$j_{s}^{\mathrm{B}} = -\langle \sigma / 2e \rangle \partial_{y} \mu_{s}(y).$$

(1)

Here, $\mu_{s}(y)$ is a spin chemical potential defined as

$$\mu_{s}(y) = \mu_{\uparrow}(y) - \mu_{\downarrow}(y).$$

(2)

By a continuity equation taking account of spin relaxation, we can show that $\mu_{s}(y)$ obeys a differential equation

$$\frac{d^{2} \mu_{s}}{dy^{2}} = \frac{\mu_{s}}{\lambda^{2}},$$

(3)

where $\lambda$ is a spin diffusion length. The spin chemical potential $\mu_{s}(y)$ is obtained as a function of $y$ by solving this equation under the boundary conditions

$$j_{s}^{\mathrm{B}}(y) - J_{s0}^{\mathrm{SH}} = \begin{cases} j_{s}^{\uparrow}, & (y = 0), \\ 0, & (y = d_{N}), \end{cases}$$

(4)

where $d_{N}$ is a thickness of the NM layer, and $j_{s}^{\uparrow}$ is a spin absorption rate at a NM/FI interface that depends on the direction of the magnetization of the FI.

![FIG. 1. (Color online) Schematics of a NM/FI bilayer system for the SMR measurement. When an external charge current is applied to a NM in the $x$-direction, a spin current $I_{s}$ with a $z$ component flows in the $y$-direction due to the spin Hall effect. The spin current induces spin accumulation $\sigma$ at the NM/FI interface with a magnetization $M$. (a) A parallel configuration of $\sigma \parallel M$. (b) A perpendicular configuration of $\sigma \perp M$. (c) A chemical potential difference $\delta \mu_{s} = \mu_{\uparrow} - \mu_{\downarrow}$ defined for a quasi-equilibrium steady state is shown as a function of the position $y / \lambda$, where $\lambda$ is a spin diffusion length, and the thickness of the NM is set as $d_{N} = 6 \lambda$. The NM/FI interface is located at $y = 0$.](https://example.com/fig1.png)

In this paper, we employ another definition of a spin current at the interface. We define $I_{s}$ as a decay rate of the spin angular momenta at the NM/FI interface. This can be related with $j_{s}^{\uparrow}$ as follows:

$$j_{s}^{\uparrow} = \frac{e}{\hbar / 2 S} I_{s},$$

(5)

where $S$ is a cross section area of the NM/FI interface. We define a dimensionless spin conductance as

$$G_{s} = \lim_{\mu_{s}(0) \to 0} \frac{I_{s}}{\mu_{s}(0)},$$

(6)

where $\mu_{s}(0)$ is a chemical potential difference at the NM/FI interface. By solving a self-consistent equation for $\mu_{s}(0)$ in combination of the differential equation (4), we obtain

$$\mu_{s}(0) = \frac{\mu_{s0}}{1 + g_{s} \coth(d_{N} / \lambda)}$$

(7)

where $\mu_{s0}$ is a chemical potential difference in the absence of the NM/FI interface, and $g_{s}$ is a dimensionless factor.
defined as
\[ g_s = \frac{4e^2}{\hbar} \sigma S/\lambda, \]
that indicates amplitude of the absorption rate at the interface.

In Ref. 10, the magnetization-orientation dependence of \( G_s \) was discussed in terms of the spin mixing conductance. In this discussion, the spin absorption rate at the interface, \( g_s \), is largest (smallest) when the magnetization \( \mathbf{M} \) is perpendicular (parallel) to the accumulated spin \( \sigma \) (see Fig. 1(a) and (b)). Then, the spatial profile of \( \mu_s(y) \) changes depending on the direction of \( \mathbf{M} \) (see Fig. 1(c)). Here, we stress that in this paper, no assumption is made on the magnetization-orientation dependence of \( G_s \). As shown later, the magnetization-orientation dependence of the spin absorption rate which is implicitly assumed in the discussion based on the mixing conductance is not sufficient to discuss the temperature dependence of the SMR signal.

The backflow current \( j_B^B \) induces a charge current in the \( x \) direction due to the inverse spin Hall effect. A longitudinal magnetoresistance is then calculated as \( \Delta \rho / \rho = g_s \frac{\tan \hbar(d_N/2\lambda)}{1 + g_s \coth(d_N/\lambda)}. \)

Thus, the SMR is related to the spin conductance \( G_s \) via the factor \( g_s \). In the subsequent sections, we calculate \( G_s \) as a function of the angle between \( \mathbf{M} \) and \( \sigma \).

The theoretical description of the SMR for a NM/AFI bilayer is almost the same as the NM/FI bilayer. The SMR can be discussed by calculating \( G_s \) as a function of an angle between the Néel vector and \( \sigma \).

### III. MODEL

In this section, we introduce a model for the NM/FI and NM/AFI bilayers. After we give the Hamiltonian for bulk systems of NM (Sec. IIIA), FI (Sec. IIIB), and AFI (Sec. IIIC), we describe the model of the interfacial exchange coupling in Sec. IIID.

#### A. Normal Metal

The Hamiltonian for a bulk NM is given as
\[ H_{NM} = \sum_{\sigma} \xi_k c_{k\sigma}^\dagger c_{k\sigma}, \]
where \( \xi_k = c_k - \mu \) is an energy dispersion measured from a chemical potential, and \( \sigma = \uparrow, \downarrow \) is a \( z \) component of an electron spin. We assume that the spin accumulation at an interface induced by SHE is described by quasi-thermal equilibrium states with spin-dependent chemical potential shifts, \( \pm \mu_s/2 \), where \( \mu_s \) is recognized as its value at the interface, \( \mu_s(0) \), given in the previous section.

The density matrix for this quasi-thermal equilibrium state is given as \( \rho = e^{-\beta H_{NM}} / Z \), where
\[ \mathcal{H}_{NM} = \sum_{k\sigma} (\xi_k - \sigma \mu_s/2) c_k^\dagger c_{k\sigma}, \]
where \( \beta \) is an inverse temperature, and \( Z = \text{Tr}(e^{-\beta \mathcal{H}}) \) is the ground partition function.

#### B. Ferromagnetic Insulator (FI)

For the Hamiltonian of a bulk FI, we consider the Heisenberg model given as
\[ H_{FI} = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \hbar \gamma \hbar_{dc} \sum_i S_i^z, \]
where \( \mathbf{S}_j \) is a localized spin, \( J < 0 \) is a ferromagnetic exchange coupling, \( \langle i,j \rangle \) indicates a pair of nearest-neighbor sites, \( \gamma \) is the gyromagnetic ratio, and \( \hbar_{dc} \) is a static magnetic field. Here, we have introduced a new coordinate \((x', y', \z')\), and assume that the net magnetization is aligned in the +z' direction in this new coordinate by a magnetic field (see Fig. 2(a)):
\[ \langle \mathbf{S}_j \rangle_0 = (\langle S_j^x \rangle_0, \langle S_j^y \rangle_0, \langle S_j^z \rangle_0) = (0, 0, \tilde{S}_0), \]
where \( \langle \cdot \cdot \cdot \rangle_0 \) indicates a thermal average in the bulk FI, and \( \tilde{S}_0 \) is an amplitude of the magnetization per site, which depends on the temperature.

#### C. Antiferromagnetic Insulator

For the Hamiltonian of a bulk AFI, we consider the Heisenberg model on a lattice composed of two sublattices, A and B:
\[ H_{AFI} = J \sum_{\langle i,j \rangle} \mathbf{S}_{A,i} \cdot \mathbf{S}_{B,j}, \]
where $S_{I,J}$ denotes a localized spin on the sublattice $\nu$ ($= A, B$), the antiferromagnetic exchange is denoted with $J > 0$, and $\langle i,j \rangle$ indicates the nearest-neighbor pairs between two sublattices. We assume that the magnetization on the sublattice $A(B)$ is aligned in the $+z'(-z')$ direction. (see Fig. 2(b)):

$$\langle S_{A,j} \rangle = \langle (S_{A,j}'^z, S_{A,j}'^y, S_{A,j}'^x) \rangle = (0, 0, \tilde{S}_0),$$  \hspace{1cm} (15)

$$\langle S_{B,j} \rangle = \langle (S_{B,j}'^z, S_{B,j}'^y, S_{B,j}'^x) \rangle = (0, 0, -\tilde{S}_0),$$  \hspace{1cm} (16)

where $\tilde{S}_0$ is an amplitude of the staggered magnetization per site, which depends on the temperature.

D. Exchange coupling at an interface

The interfacial exchange coupling between the FI (or AFI) and NM is described by the Hamiltonian

$$\begin{align*}
H_{\text{ex}} &= \sum_{\nu} \sum_{\langle i,j \rangle} \left[ T_{ij}^\nu S_{\nu,i}^+ S_{\nu,j}^- + (T_{ij}^\nu)^* S_{\nu,i}^- S_{\nu,j}^+ \right],
\end{align*}$$  \hspace{1cm} (17)

where $S_{\nu,i}^\pm = S_{\nu,i}^y \pm S_{\nu,i}^y$ are creation and annihilation operators of the spin in the laboratory coordinate, $T_{ij}^\nu$ is an exchange coupling between a pair of interfacial sites, $\langle i,j \rangle$, and $\nu$ is the sublattice of the localized spins. The sublattice is unique ($\nu = A$) for the case of FI, whereas there are two sublattices ($\nu = A,B$) for the case of AFI.

To proceed in calculation, we need to rewrite the Hamiltonian (17) in the spin operators in the magnetization-fixed coordinate (x', y', z'). The conversion formula of the spin operators from the magnetization-fixed coordinate (x', y', z') to the laboratory coordinate (x, y, z) is given as

$$S_{\nu,j}^x = \cos \theta S_{\nu,j}'^x - \sin \theta S_{\nu,j}'^y,$$  \hspace{1cm} (18)

$$S_{\nu,j}^y = S_{\nu,j}'^y,$$  \hspace{1cm} (19)

$$S_{\nu,j}^z = \sin \theta S_{\nu,j}'^x + \cos \theta S_{\nu,j}'^y,$$  \hspace{1cm} (20)

where $\theta$ is an angle of the magnetization (see Fig. 2). By this coordinate transformation, we obtain

$$S_{\nu,j}'^+ = \cos^2(\theta/2)S_{\nu,j}^+ - \sin^2(\theta/2)S_{\nu,j}^- - \sin \theta S_{\nu,j}'^x,$$  \hspace{1cm} (21)

$$S_{\nu,j}'^- = \cos^2(\theta/2)S_{\nu,j}^- - \sin^2(\theta/2)S_{\nu,j}^+ + \sin \theta S_{\nu,j}'^x,$$  \hspace{1cm} (22)

where $S_{\nu,j}^\pm = S_{\nu,j}'^\pm \pm S_{\nu,j}^y$. Then, the interface exchange interaction can be divided into three parts as

$$H_{\text{ex}} = \sum_{a=1}^{3} H_{\text{ex}}^{(a)},$$  \hspace{1cm} (23)

$$H_{\text{ex}}^{(a)} = g_a(\theta) \sum_{\langle i,j \rangle} \left[ T_{ij}^\nu S_{\nu,i}^{(a)} S_{\nu,j}^x + (T_{ij}^\nu)^* (S_{\nu,i}^{(a)})^x S_{\nu,j}^x \right],$$  \hspace{1cm} (24)

where $S^{(a)}$ and $g_a(\theta)$ ($a = 1, 2, 3$) are defined as

$$S_{I,J}^{(1)} = S_{I,J}', \hspace{1cm} g_1(\theta) = -\sin \theta,$$  \hspace{1cm} (25)

$$S_{I,J}^{(2)} = S_{I,J}', \hspace{1cm} g_2(\theta) = \cos^2(\theta/2),$$  \hspace{1cm} (26)

$$S_{I,J}^{(3)} = S_{I,J}', \hspace{1cm} g_3(\theta) = -\sin^2(\theta/2).$$  \hspace{1cm} (27)

IV. SPIN CURRENT

Next, we calculate the spin current using the second-order perturbation with respect to the exchange coupling at the interface. We derive a general formula for the spin current and the spin conductance. Our formula is expressed in terms of the spin susceptibilities in the NM and FI (or AFI) layers, and general in a sense that the formula does not depend on a specific model.

A. Definition

We define the spin current as the absorption rate of the z component of the spin angular momenta in the NM side at the interface:

$$\dot{I}_s = -\hbar \partial_z s^z_{\text{tot}} = i[s^z_{\text{tot}}, [H]],$$  \hspace{1cm} (28)

$$s^z_{\text{tot}} = \frac{1}{2} \sum_{ij} \left( c_{ij \uparrow}^\dagger c_{ij \downarrow} - c_{ij \downarrow}^\dagger c_{ij \uparrow} \right).$$  \hspace{1cm} (29)

The spin current is calculated in the form

$$\dot{I}_s = \sum_{a=1}^{3} \dot{I}_s^{(a)},$$  \hspace{1cm} (30)

$$\dot{I}_s^{(a)} = -ig_a(\theta) \sum_{\langle i,j \rangle} \langle T_{ij}^\nu S_{\nu,i}^{(a)} S_{\nu,j}^z - \text{h.c.} \rangle.$$  \hspace{1cm} (31)

Here, we should note that the $z$ component of the total spin is not conserved at the interface as the magnetization of the FI (or AFI) is not necessarily aligned in the $z$ direction.

B. Second-order perturbation

We calculate the spin current within the second-order perturbation with respect to the interfacial exchange coupling. For simplicity, we assume that the correlation between the exchange coupling for different pairs vanishes after impurity averaging:

$$\langle T_{ij}^\nu (T_{ij'}^\nu)^* \rangle_{\text{imp}} = |T_{ij}^\nu|^2 \delta_{i,i'} \delta_{j,j'} \delta_{\nu,\nu'}.$$  \hspace{1cm} (32)
Then, the spin current is calculated as \(38, 41\)
\[
\langle \dot{I}_s^{(a)} \rangle = \hbar^2 g_a(\theta)^2 \sum_{\nu, \nu'} A_{\nu} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left(-\frac{N_N}{N_N'}/N_N\right) \sum_{k, q} \chi^<(q, \omega)G_{\nu \nu'}^{R,(a)}(k, \omega) + \chi^A(q, \omega)G_{\nu \nu'}^{c,(a)}(k, \omega),
\]
(33)
where \(A_{\nu} = 2\sum_{(i, j)} |T_{ij}^{(2)}|^2 / \hbar, N_N\) is a number of sites in NM, and \(N_N'\) is a number of unit cells in FI (or AFI). Hereafter, we set \(A = A_A\) for the case of the FI, and assume that the exchange coupling is equivalent for the two sublattices as \(A = A_A = A_B\) for the case of the AFI. The advanced and lesser spin susceptibilities of NM, \(\chi^A(q, \omega)\) and \(\chi^<(q, \omega)\), are defined by the Fourier transformation of the following functions:
\[
\chi^A(q, t) = \frac{i}{N_N\hbar} \theta(-t)\langle [s^+_{q}(t), s^-_{q}(0)]\rangle_0,
\]
(34)
\[
\chi^<(q, t) = \frac{i}{N_N\hbar} \langle [s^-_{q}(0), s^+_{q}(t)]\rangle_0,
\]
(35)
where \(\langle \cdots \rangle_0\) indicates an average for the unperturbed Hamiltonian, \(s^\pm_q\) is the spin operator of conduction electrons, and \(s^\pm_q(t) = e^{iH_{NM}/\hbar} s^\pm_q e^{-iH_{NM}/\hbar}\). For quasithermal equilibrium states, we can prove the dissipation-fluctuation theorem
\[
\chi^<(q, \omega) = -2i f(\hbar \omega + \mu_s) \text{Im} \chi^A(q, \omega),
\]
(36)
by utilizing the Lehmann representation, where \(f(E) = (e^{\beta E} + 1)^{-1}\) is the Fermi distribution function. The retarded and lesser spin correlation functions of the FI (or the AFI), \(G^{R,(a)}_{\nu \nu'}(k, \omega)\) and \(G^{c,(a)}_{\nu \nu'}(k, \omega)\), are defined by the Fourier transformation of the following functions:
\[
G^{R,(a)}_{\nu \nu'}(k, t) = \frac{i}{\hbar} \theta(t)\langle [S^{(a)}_{\nu,k}(t), (S^{(a)}_{\nu,k}(0))]\rangle_0,
\]
(37)
\[
G^{c,(a)}_{\nu \nu'}(k, t) = -\frac{i}{\hbar} \langle [S^{(a)}_{\nu,k}(0)], S^{(a)}_{\nu,k}(t)\rangle_0,
\]
(38)
where \(S^{(a)}_{\nu,k}(a = 1, 2, 3)\) are the spin operators defined by spatial Fourier transformation of Eqs. (35)–(37). To continue calculation, we introduce a fluctuating part of the spin correlation function as \(\delta S^{(a)}_{\nu,k}(t) = S^{(a)}_{\nu,k}(t) - \langle S^{(a)}_{\nu,k}\rangle_0\). From Eqs. (35) and (38), we obtain
\[
G^{R,(a)}_{\nu \nu'}(k, \omega) = \delta G^{R,(a)}_{\nu \nu'}(k, \omega),
\]
(39)
\[
G^{c,(a)}_{\nu \nu'}(k, \omega) = \frac{2\pi i N_N S_0^2}{\hbar} \delta_{\nu,1} \delta_{k,0} \delta(\omega)
+ \delta G^{c,(a)}_{\nu \nu'}(k, \omega),
\]
(40)
where \(\delta G^{R,(a)}_{\nu \nu'}(k, \omega)\) and \(\delta G^{c,(a)}_{\nu \nu'}(k, \omega)\) are correlation functions defined by replacing \(S^{(a)}_{\nu,k}(t)\) with \(\delta S^{(a)}_{\nu,k}(t)\) in Eqs. (37) and (38). By utilizing the Lehmann representation, we can prove the dissipation-fluctuation theorem
\[
\delta G^{c,(a)}_{\nu \nu'}(k, \omega) = 2i f(\hbar \omega) \text{Im} \delta G^{R,(a)}_{\nu \nu'}(k, \omega).
\]
(41)
Combining Eqs. (36), (39)–(41), the spin current is calculated as
\[
\langle \dot{I}_s \rangle = I_{s,1} + I_{s,2},
\]
(42)
\[
I_{s,1} = \hbar A \sin^2 \theta S_0^2 N_N \text{Im} \chi^{R}_{\text{loc}}(0),
\]
(43)
\[
I_{s,2} = \frac{3}{\hbar} \sum_{\nu} 2g_a(\theta)^2 A \sum_{\nu} \int \frac{d(\hbar \omega)}{2\pi} \text{Im} \chi^{R}_{\text{loc}}(\omega)
\times (\text{Im} \delta G^{R,(a)}_{\nu \nu', \text{loc}}(\omega))[f(\hbar \omega) - f(\hbar \omega + \mu_s)],
\]
(44)
where \(N_N\) is a number of the sublattices (\(N_N = 1\) for FI and \(N_N = 2\) for AFI), and the local spin correlation functions, \(\chi^{R}_{\text{loc}}(\omega)\) and \(G^{R,(a)}_{\nu \nu', \text{loc}}(\omega)\), are defined as
\[
\chi^{R}_{\text{loc}}(\omega) = \frac{1}{N_N} \sum_{q} \chi^{R}(q, \omega),
\]
(45)
\[
G^{R,(a)}_{\nu \nu', \text{loc}}(\omega) = \frac{1}{N_N} \sum_{k} G^{R,(a)}_{\nu \nu'}(k, \omega).
\]
(46)

Finally, let us summarize the physical meaning of the spin current formula given by Eqs. (42)–(44). We stress that the present formula is written in terms of spin susceptibilities for bulk materials, and therefore is applicable to general systems such as a NM with strong electron-electron interaction. The spin current is composed of two parts. The first part, \(I_{s,1}\), describe a static part, which is induced by spin flipping due to a static effective transverse magnetic field via the interfacial exchange coupling. Actually, the static part \(I_{s,1}\) is almost independent of the temperature well below the magnetic transition temperature, and takes a maximum when the accumulated spin at the interface in the side of the NM is perpendicular to the magnetization of the FI (or the Néel vector of the AFI), i.e., \(\theta = \pi/2\). This feature of \(I_{s,1}\) coincides with the theory based on the spin mixing conductance in Refs. [4] and [14]. There exists, however, an additional contribution \(I_{s,2}\), which is induced by creation or annihilation of magnons. This part can be regarded as a dynamical part. In the subsequent sections, we will show that this dynamical part depends on the temperature, and that at sufficiently low temperatures \(I_{s,2}\) has a maximum at \(\theta = 0\), i.e., has angle dependence opposite to the static part.

C. Spin conductance

From Eqs. (42)–(44), the (dimensionless) spin conductance at the interface defined in Eq. (6) is calculated as
\[
G_s = G_{s,1} + G_{s,2},
\]
(47)
\[
G_{s,1} = G_0 \sin^2 \theta, \quad \theta = \pi/2\n\]
(48)
\[
G_{s,2} = 2 G_0 g_a(\theta)^2 \frac{1}{N_N} \sum_{\nu} \int \frac{dE}{2\pi} E \times \left( -\frac{1}{S_0^2} \text{Im} \delta G^{R,(a)}_{\nu \nu', \text{loc}}(E/\hbar) \right) \left[ -\frac{df}{dE} \right],
\]
(49)
where \( G_0 = \hbar A S_0^2 N_c \pi N(0)^2 \) and \( N(0) \) is a density of states in the NM at the Fermi energy. We note that the spin chemical potential \( \mu_s(0) \) is now recognized as \( \mu_s \) in the model of the NM (see Eq. (31)).

V. SMR IN FI/NM BILAYERS

In this section, we calculate the spin conductance by employing the spin-wave approximation for the ferromagnetic Heisenberg model. Hereafter, we assume that the amplitude of spins for the ground state, \( S_0 = \tilde{S}(T = 0) \), is sufficiently large, and that the temperature is much lower than the magnetic transition temperature.

By applying the Holstein-Primakoff transformation to the spin operators in the magnetization-fixed coordinate, the Hamiltonian of the FI is approximately written as

\[
H_{FI} = \sum_k \hbar \omega_k b_k^\dagger b_k, \tag{50}
\]

\[
\hbar \omega_k \approx D k^2 + E_0, \tag{51}
\]

where \( b_k (b_k^\dagger) \) is a magnon annihilation (creation) operator, \( E_0 = h \gamma h_{dc} \) is the Zeeman energy, and \( D = J S_0 a^2 \).

In the spin-wave approximation, the local spin susceptibility is calculated as \( \text{Im} \delta G^{R,(1)}_{\text{loc}}(E/\hbar) = 0 \), and

\[
\text{Im} \delta G^{R,(2)}_{\text{loc}}(E/\hbar) = -\text{Im} \delta G^{R,(3)}_{\text{loc}}(-E/\hbar) = -2\pi S_0 D_F(E), \tag{52}
\]

where \( D_F(E) \) is the density of states for magnon excitation per unit cell:

\[
D_F(E) = \frac{1}{N_F} \sum_k \delta(E - \hbar \omega_k). \tag{53}
\]

Although the magnetization \( \tilde{S}_0 \) depends weakly on the temperature within the spin-wave approximation, we neglect it for simplicity (\( \tilde{S}_0 \approx S_0 \)). Then, the spin conductance takes a form

\[
G_s = G_{s,0} + \Delta G_s \sin^2 \theta, \tag{54}
\]

where \( G_{s,0} \) is a part independent of \( \theta \), and the second term describes the angle dependence, i.e., the SMR. The amplitude of the SMR is calculated as

\[
\Delta G_s = G_0 (1 - g_F(T)), \tag{55}
\]

\[
g_F(T) = \frac{1}{S_0} \int_0^\infty dE E D_F(E) \left[ -\frac{dF}{dE} \right]. \tag{56}
\]

We note that the first term \( G_0 \) in Eq. (55) comes from the static part \( G_{s,1} \), while the second term \( -G_0 g_F(T) \) from the dynamic part \( G_{s,2} \). The factor \( g_F(T) \) is small under the condition \( S_0 \gg 1 \), for which the spin-wave approximation works well. If we neglect \( g_F(T) \), we recover usual positive SMR behavior, \( \Delta G_s = G_0 \sin^2 \theta \). We should note that the factor \( g_F(T) \) weakens the positive SMR. When the Zeeman energy is neglected \( (E_0 \approx 0) \), the temperature dependence of the SMR signal is obtained at sufficiently low temperatures as

\[
\frac{\Delta G_s}{\Delta G_s (T = 0)} \approx 1 - \frac{5.2}{S_0} \left( \frac{k_B T}{E_c} \right)^{3/2}, \tag{57}
\]

where \( E_c = D k_c^2 \) is the cutoff energy, which is of order of the transition temperature (for details, see Appendix B). If \( g_F(T) \) exceeds 1, the sign of the SMR changes. The temperature of the sign change, \( T_r \), is roughly estimated as

\[
k_B T_r \sim \left( \frac{S_0}{5.2} \right)^{2/3} E_c. \tag{58}
\]

We note that for \( S_0 \gg 1 \), \( T_r \) becomes of order of \( E_c \) for which the spin-wave approximation is not justified. This estimate indicates that the sign change of SMR occurs if \( S_0 \) is not large.

VI. SMR IN AFI/NM BILAYERS

In the spin-wave approximation, the Hamiltonian for the FI is obtained in the leading order of \( 1/S_0 \) as

\[
H_{AFI} = \sum_k \hbar \omega_k (\alpha_k^\dagger \alpha_k + \beta_k^\dagger \beta_k), \tag{59}
\]

where \( \alpha_k \) and \( \beta_k \) are annihilation operators for magnons, \( \omega_k = v_m |k| \) is the dispersion relation, and \( v_m = z J S_0 a/(\sqrt{3} \hbar) \) is the velocity of magnons (see Appendix A). Here, we have approximated the magnon dispersion as a linear dispersion in the long wavelength limit \((|k| \rightarrow 0)\). The local spin susceptibility is calculated as \( \text{Im} \delta G^{R,(1)}_{\text{loc}}(E/\hbar) = 0 \) and

\[
\sum_{\nu=A,B} \text{Im} \delta G^{R,(2)}_{\nu,\text{loc}}(E/\hbar) = -\sum_{\nu=A,B} \text{Im} \delta G^{R,(3)}_{\nu,\text{loc}}(-E/\hbar) = -2\pi S_0 F(E) (D_{AF}(E) - D_{AF}(-E)), \tag{60}
\]

where \( D_{AF}(E) \) and \( F(E) \) are the density of states for magnon excitation and the form factor, respectively (see Appendix A):

\[
D_{AF}(E) = \frac{1}{N_F} \sum_k \delta(E - \hbar \omega_k), \tag{61}
\]

\[
F(E) = \frac{\sqrt{3} h v_m}{|E/a|}. \tag{62}
\]

Using these results, the spin conductance is calculated in the form of Eq. (57). The amplitude of the SMR is then given as

\[
\Delta G_s = G_0 (1 - g_{AF}(T)), \tag{63}
\]

\[
g_{AF}(T) = \frac{1}{S_0} \int_0^\infty dE E F(E) D_{AF}(E) \left[ -\frac{dF}{dE} \right]. \tag{64}
\]
For the case of the AFI, the temperature dependence of the SMR signal is given as

$$\frac{\Delta G_s}{\Delta G_s(T = 0)} \simeq 1 - \frac{4A}{S_0} \left( \frac{k_B T}{E_c} \right)^2,$$

(65)

where $E_c = \hbar v_m k_c$ is the cutoff energy (see Appendix B). The temperature of the sign change is roughly estimated as

$$k_B T_c \sim \left( \frac{S_0}{4A} \right)^{1/2} E_c.$$

(66)

This estimate indicates that the sign change of SMR may occur if $S_0$ is not large.

VII. COMPARISON WITH EXPERIMENTS

Let us first consider the SMR for the FI. SMR experiments for FI have been performed mainly for Pt/YIG bilayer systems. In the present theory, the temperature of the sign change of SMR signal estimated for Pt/YIG exceeds the magnetic transition temperature using Eq. (65) and $S_0 = 10$. This indicates that the correction by the factor of $g_F(T)$ is small, and no sign change occurs. The detailed temperature dependence of SMR in Pt/YIG has been measured in Refs. 14 and 15, where the observed temperature dependence has been interpreted by temperature dependence of the spin diffusion length. At present, effect of magnon excitation in the Pt/YIG bilayer system has not been observed possibly because of its smallness.

The SMR have been measured also for AFI/NM bilayer systems in several experiments. In Fig. 3, we show the measured temperature dependence of the SMR in the Pt/NiO/YIG bilayer systems taken from Ref. 19. In this experiment, an estimate of the factor $g_F \text{coth}(d_N/\lambda)$ is much smaller than 1 using $\lambda = 1.5$ nm, $d = 4$ nm, $\sigma^{-1} = 860$ $\Omega \cdot$ nm, and $G_s/(\hbar/2e^2) \sim 3 \times 10^{12}$ $\Omega^{-1} \cdot$ m$^{-2}$. Therefore, the SMR is proportional to the spin conductance (see Eq. (65)). As seen from Fig. 3, the sign of the SMR changes at 80, 140 and 180 K for NiO thickness of 2.0, 2.2, and 2.7 nm. We also show the fitted curve using quadratic temperature dependence given in Eq. (66) in the figure. This fitting indicates that the quadratic temperature dependence well explains the experimental data at low temperatures. If we employ $S_0 = 0.94$ and $E_c = 1500$ K from the magnon dispersion measured by the neutron experiment, the temperature of this sign change is estimated for bulk NiO as $T_r = 690$ K (see also Appendix B). This estimated temperature for the sign change is much larger than the experimental observation shown in Fig. 3. We, however, should note that the Néel temperature of NiO ($T_N = 525$ K) is suppressed for thin layers, indicating reduction of the magnon velocity. We also note that the spin-wave approximation holds well only at low temperatures compared with the Néel temperature.

In this paper, we have discussed the SMR at low temperatures using the spin-wave approximation. Because the spin current formula derived in this paper is general, the SMR can be evaluated for arbitrary temperatures by employing a numerical method such as the Monte Carlo method. Detailed numerical analysis beyond the spin-wave approximation as well as consideration of the roughness of the interface is left as a future problem.

VIII. ONSAGER RELATION

In this section, we formulate a noise in thermal equilibrium ($\delta \mu_s = 0$), and derive the Onsager relation, which relates the thermal noise with the spin conductance.

We define the noise power of the spin current as

$$S = \int_{-\infty}^{\infty} dt \langle \dot{I}_S(t) \dot{I}_S(0) \rangle + \langle \dot{I}_S(0) \dot{I}_S(t) \rangle. \quad (67)$$

In second-order perturbation with respect to the exchange coupling at an interface, we can replace the average with that for an unperturbed system as $\langle \cdots \rangle \simeq \langle \cdots \rangle_0$. By a similar procedure as the spin current, the
noise power is calculated as
\[
S = S_1 + S_2,
\]
\[
S_1 = 2\hbar^2 A S_{loc}^2 N_\nu \sin^2 \theta \\
\times \lim_{\omega \to 0} (-i) [\chi_{loc}^<(\omega) + \chi_{loc}^>(\omega)],
\]
\[
S_2 = \sum_{a=1}^{3} 2\hbar^2 A_{a}(\theta)^2 \sum_{\nu} \int_{-\infty}^{\infty} \frac{d(\hbar \omega)}{2\pi} \\
\times \left[ \chi_{loc}^<(\omega) \delta G_{\nu\nu_{\nu}, loc}^{R,(a)}(\omega) + \chi_{\nu}^{\nu} (\omega) \delta G_{\nu\nu_{\nu}, loc}^{R,(a)}(\omega) \right].
\]  
(70)

Here, \(\chi_{loc}^>(\omega)\) and \(G_{\nu\nu_{\nu}, loc}^{R,(a)}(\omega)\) are the greater components of the local spin susceptibilities defined as
\[
\chi_{loc}^>(\omega) = \frac{i}{N^2 \hbar} \sum_{q} \int d\omega e^{i\omega t} \langle s_+^q(t) s_-^q(0) \rangle,
\]
\[
\delta G_{\nu\nu_{\nu}, loc}^{R,(a)}(\omega) = -\frac{i}{N^2 \hbar} \sum_{k} \int d\omega e^{i\omega t} \\
\times \langle \delta S_{\nu k}^{(a)}(t) \delta S_{\nu_{\nu k}}^{(a)}(0) \rangle^t_0,
\]  
(72)

Using the dissipation-fluctuation relations
\[
\chi_{loc}^>(\omega) = 2i(1 + f(\hbar \omega + \delta \mu_R)) \text{Im} \chi_{loc}^R(\omega),
\]
\[
\delta G_{\nu\nu_{\nu}, loc}^{R,(a)}(\omega) = 2i(1 + f(\hbar \omega)) \text{Im} \delta G_{\nu\nu_{\nu}, loc}^{R,(a)}(\omega),
\]  
(74)

the thermal spin-current noise is calculated as
\[
S_1 = 2\hbar k_B T G_0 \sin^2 \theta,
\]
\[
S_2 = \sum_{a=1}^{3} 8\hbar^2 A_{a}(\theta)^2 \sum_{\nu} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \text{Im} \chi^R(q, \omega) \\
\times \text{Im} G_{\nu\nu_{\nu}, loc}^{R,(a)}(k, \omega) f(\hbar \omega)(1 + f(\hbar \omega)).
\]  
(76)

Using the identity
\[
f(\hbar \omega)(1 + f(\hbar \omega)) = k_B T \left( \frac{df}{dE} \right),
\]  
(77)

and comparing these results with Eqs. (77)-(79), we can prove the Onsager relation
\[
S = 4\hbar k_B T G_s.
\]  
(78)

We stress that the present proof is general, and this relation holds at arbitrary temperatures for a wide class of the Hamiltonian for NM and FI(or AFI).

\[\text{IX. SUMMARY}\]

We constructed a microscopic theory for spin magnetoresistance observed in bilayer systems composed of a normal metal and a ferromagnetic (or antiferromagnetic) insulator. We formulated the spin current at an interface in terms of spin susceptibilities, and clarified that it is composed of static and dynamic parts. The static part of the spin current originates from spin flip due to an effective magnetic field induced by an interfacial exchange coupling. This part is almost independent of the temperature, and takes a maximum when the magnetization (or the Néel vector) is perpendicular to accumulated spins in a normal metal consistent with intuitive discussion in previous experimental works. On the other hand, the dynamic part, which is induced by creation or annihilation of magnons, depends on the temperature, and has opposite magnetization dependence, i.e., takes a maximum when the magnetization (or the Néel vector) is parallel to accumulated spins in a normal metal. The dynamic part becomes larger when the amplitude of the localized spin, \(S_0\), is smaller. This indicates that the sign of the SMR changes at a specific temperature if \(S_0\) is sufficiently small. We discussed that the measured temperature dependence of the SMR in the Pt/NiO/YIG trilayer system is consistent with our results. Finally, we have proved the Onsager relation between a spin conductance and a thermal spin-current noise by a microscopic calculation.

Our general formalism, which is applicable to various systems for arbitrary temperatures, will be an important ingredient for describing spin magnetoresistance. Theoretical analysis beyond the spin-wave approximation as well as that for effect of interfacial randomness is left for a future problem.

\[\text{ACKNOWLEDGMENTS}\]

T.K. is financially supported by JSPS KAKENHI Grant Numbers JP20K03831. M.M. is financially supported by the Priority Program of Chinese Academy of Sciences, Grant No. XDB28000000 and KAKENHI (No. 20H01863) from MEXT, Japan.

Appendix A: Spin-Wave Approximation for the AFI

In this appendix, we briefly summarize the spin-wave approximation for the AFI. Throughout this appendix, the amplitude of spins for the ground state, \(S_0 = \bar{S}(T = 0)\), is much larger than 1, and the temperature is much lower than the magnetic transition temperature.

By standard procedure based on the Holstein-Primakoff transformation, the Hamiltonian of the AFI is approximately written as
\[
H_{\text{AFI}} \simeq J z S_0 \sum_k \left( a_k^\dagger b_k + b_k^\dagger a_k \right) \left( \frac{1}{\zeta_k} \frac{1}{\zeta_k} \right),
\]  
(A1)

where \(a_k\) and \(b_k\) are annihilation operators of spins on A and B sublattices, respectively, and \(z\) is the number of the nearest neighbor sites. For the cubic lattice (\(z = 6\),...
\[ \eta_k \text{ is calculated as } \]
\[ \zeta_k = \frac{1}{3} (\cos k_x a + \cos k_y a + \cos k_z a), \quad (A2) \]

To diagonalize the Hamiltonian, we introduce the Bogoliubov transformation:
\[ a_k = u_k \alpha_k - v_k \beta_k^\dagger, \quad (A3) \]
\[ b_k = v_k \alpha_k - u_k \beta_k^\dagger. \quad (A4) \]

The exchange relation of the bosonic operators leads to the constraint \( u_k^2 - v_k^2 = 1 \). By straightforward calculation, we obtain the diagonalized Hamiltonian \[ \text{[70]} \] in the main text using the solutions
\[ u_k^2 = v_k^2 + 1 = \frac{1}{2} \left( \frac{1}{\sqrt{1 - \zeta_k^2}} + 1 \right), \quad (A5) \]

In the long-wavelength limit \( (k \rightarrow 0) \), the dispersion relation becomes \( \omega_k = v_m |k| \), where \( v_m = J z S_0 a / (\sqrt{3} \hbar) \) is a velocity of magnons. Using this diagonalized Hamiltonian, the local spin susceptibilities are written as Eq. \[ \text{[60]} \] with the density of state of magnons given by Eq. \[ \text{[62]} \]. We note that the spin susceptibilities include the factor \( u_k^2 + v_k^2 \), which is rewritten in the limit of \( k \rightarrow 0 \) by the form factor
\[ F(E) = \frac{1}{\sqrt{1 - \zeta(E)^2}} = \frac{J z S_0}{E}, \quad (A6) \]
where \( \zeta(E) = \zeta_k |\omega_k = E| \).

### Appendix B: Cutoff Energy

For estimate of the temperature at which the sign of the SMR changes, we approximate the magnon dispersion as that for the continuum limit \( (k \rightarrow 0) \). We introduce the cutoff wavenumber, \( k_c \), as \( N_F = V (2\pi)^{-3} \int_{k_c}^{\infty} dk 4\pi k^2 \), leading to \( k_c a = (6\pi^2)^{1/3} \gamma \equiv \alpha \), where \( \alpha \) is a lattice constant. The density of states of magnons is calculated for the FI using its cumulative function defined as \[ D_F(E) = ((E - E_0) / E_c)^{3/2} \], where \( E_c = DK^2 \). When we neglect the Zeeman energy \( E_0 \), we obtain
\[ D_F(E) = \frac{dD_F(E)}{dE} = \frac{3}{2E_c} \left( \frac{E}{E_c} \right)^{1/2}. \quad (B1) \]

For the AFI, the cumulative function is given as \[ D_{AF}(E) = (E / E_c)^2 \], where \( E_c = h\nu_m k_c \). We obtain the density of state of magnons as
\[ D_{AF}(E) = \frac{dD_{AF}(E)}{dE} = \frac{3}{E_c} \left( \frac{E}{E_c} \right)^2. \quad (B2) \]

In estimate of the SMR for the Pt/NiO/YIG bilayer system, we have used the parameters obtained from the neutron scattering experiment\[ \text{[10]} \]. The antiferromagnetic structure of NiO is described by four sublattices, each of which is a simple cubic lattice with strongest antiferromagnetic coupling, \( J \approx 230 \) K, for nearest neighboring sites, leading to \( E_c = 1500 \) K.

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