The two-degree-of-freedom parallel control for inverse response plus time delay

Guangzhi Xu\textsuperscript{a}, Tong Wu\textsuperscript{b}, Jinggang Zhang\textsuperscript{c} and Guijie Yue\textsuperscript{a}

\textsuperscript{a}Innovation Center, China Academy of Electronics and Information Technology, Beijing, China; \textsuperscript{b}School of Cyberspace Security, Beijing University of Posts and Telecommunications, Beijing, China; \textsuperscript{c}School of Electronic Information Engineering, Taiyuan University of Science and Technology, Taiyuan, China

ABSTRACT
A two-degree-of-freedom (2-DOF) control method with parallel control structure (PCS) is proposed for inverse response process plus time delay. The parallel control structure decouples the set-point tracking response from disturbance rejection response. Set point tracking control uses the method of internal model control (IMC) based on PCS. This method not only eliminates the non minimum phase parts in the denominator of the characteristic equation, but also has only one adjustable parameter which sets controller parameter using dynamic performance index, which makes the system have a good tracking performance. Disturbance rejection controller uses the principle of internal model controller, and the maximum sensitivity function is also applied to set the controller parameter. It avoids the blindness of parameter selection and improves the robustness and disturbance rejection performance of the system. Theoretical analysis and simulation results show that the proposed method presents good set-point tracking performance, disturbance rejection performance and robustness.

ARTICLE HISTORY
Received 27 February 2019
Accepted 26 August 2019

KEYWORDS
Inverse response; two-degree-of-freedom control; Smith predictive control; parallel control structure

1. Introduction
Inverse response process exists in the chemical process, such as level of drum boiler in a distillation column and the exit temperature of a tubular exothermic reactor (Seborg, Mellichamp, Edgar, & Doyle, 2010). The main feature of inverse response process is that the initial step response direction to an input is opposite direction of the final steady-state value (Guangzhi & Jinggang, 2015). This phenomenon is due to the transfer function of the inverse response process existed an odd number of right-half-plane (RHP) zeroes (Ghousiya Begum, Seshagiri Rao, & Radhakrishnan, 2017; Nasution, Jeng, & Huang, 2011). And the inverse response process in the industrial control process is often accompanied with the time delays which prevent control signal transmitting timely and cannot effectively control the controlled object. So the general method to control the inverse response process plus time delay has much great difficulty (Geng, Hao, Liu, & Zhong, 2019).

In order to control the inverse response process plus time delay more effectively, many scholars have researched this problem and put forward a variety of the designs of controllers. Currently, there are two types design methods of the controller: the first is proportional–integral (PI) or proportional–integral–derivative (PID) controllers, which can’t tune control parameters exactly and the set-point tracking performance and the disturbance rejection performance cannot be obtained simultaneously in the same time (Chen, Zhang, & Zhu, 2006; Chien, Chung, Chen, & Chuang, 2003; Luyben, 2000). The second category is based on the model of inverse response compensation, in which most of the inverse response compensator is designed based on Smith predictor. However, Smith predictor alone cannot make the system good disturbance rejection response and robustness (Álcantara, Pedrett, Vilanova, & Zhang, 2009; Jeng & Lin, 2011; Zhang, Xu, & Sun, 2000). The parallel structure, proposed firstly in (Ajmeri & Ali, 2015) and (Karunagaran & Wenjian, 2011), is used for inverse response process plus time-delay in this paper. Literatures (Ajmeri & Ali, 2015; Karunagaran & Wenjian, 2011) illustrate that the parallel structure is good at controlling of the integrating process with good performance. However, it does not present an effective control method for the inverse response process plus time delay. In this paper, a two-degree-of-freedom control method with parallel control structure is proposed for inverse response process plus time-delay. The set-point tracking performance and disturbance rejection performance of system are affected by means of designing the set-point tracking controller and disturbance rejection controller separately, in which two adjustable parameters are directly related.
system performance. The system has different set-point tracking performance, disturbance rejection performance and robustness if different parameter values of the two controls were selected.

2. The basic characteristics of the inverse response process

In the field of chemical industry control, the most typical inverse response process is constituted by two first order processes with opposite effects, shown in Figure 1, where: \( K_1, K_2, T_1 \) and \( T_2 \) are positive numbers. \( K_1, K_2 \) are the proportional gain of two first order processes; and \( T_1, T_2 \) are the time constants; the open loop transfer function of the inverse response process can be obtained from Figure 1 as follows:

\[
G_p(s) = \frac{K_1}{T_1s + 1} - \frac{K_2}{T_2s + 1} = \frac{(K_1T_2 - K_2T_1)s + (K_1 - K_2)}{(T_1s + 1)(T_2s + 1)} = \frac{-as + 1}{T_1s + 1}(T_2s + 1)
\]  

(1)

Where \( K_p = K_1 - K_2 > 0, a = (K_2T_1 - K_1T_2)/(K_1 - K_2). \)

If \( a > 0 \), then the transfer function exists a right-half plane zero.

When the step signal is applied to the input of system process, the process 2 plays a leading role owning to the relatively fast response rate at the initial stage of the output process. The output value of the process 1 becomes gradually larger than that of the process 2 as time goes on. So the curve direction of the total response starting will change to the opposite direction. The steady-state value of process 1 is larger than the steady-state value of the process 2 in the final.

In addition, due to the actual process of inverse response generally contains delay time, without loss of generality, the inverse response process plus time delay can be expressed as:

\[
G_p(s) = \frac{K_p(-as + 1)}{(T_1s + 1)(T_2s + 1)}e^{-\tau s} \]  

(2)

3. Parallel control structure

The parallel control structure used in this paper is shown in Figure 2, where signals \( r, d \) and \( y \) denote the set-point, disturbance and process output, respectively. \( G_{c1}(s) \) is the set-point tracking controller and \( G_{c2}(s) \) is the disturbance rejection controller. \( G_p(s) \) is the actual process and \( G_m(s) \) is the model process respectively. The form in Figure 2 can change to Figure 3 if the RHP zero and time delay term (non-minimum phase portion) existed in the model \( G_m(s) \) are separated from the closed-loop of \( G_{c1}(s), G_m(s) \). According to the structure of Figure 3, the denominator of the set-point tracking response transfer function does not contain non-minimum phase portion.

If the model is accurate, the closed-loop transfer function of the set-point tracking response shown in Figure 3 is given by Equation (3).

\[
G_{yr}(s) = \frac{G_{c1}(s)G_{m0}(s)G_{m1}(s)}{1 + G_{c1}(s)G_{m0}(s)}
\]  

(3)

The closed-loop transfer function of the disturbance rejection response is given by Equation (4)

\[
G_{yd}(s) = \frac{G_p(s)}{1 + G_{c2}(s)G_p(s)}
\]  

(4)

According to the Equations (3) and (4), the set-point tracking performance of the system is controlled by controller.
G\(c_1(s)\) and the disturbance rejection performance of system is controlled by controller G\(c_2(s)\). Therefore, the set-point tracking performance and the disturbance rejection performance of system are decoupled in the parallel control structure.

4. Design method of set-point tracking controller G\(c_1(s)\)

In case of nominal condition and no disturbance, \(d = 0\), the set-point tracking response in Figure 3 can be equivalent to the response in Figure 4, according to Equation (3).

Note that, in Figure 4, the objective of the proposed control system is to get the non-minimum phase dynamics out of the feedback loop. As a result, the design of controller G\(c_1(s)\) only faces the minimum phase part, and some of the control limitations imposed by right half-phase zero and time delay disappear. According the above analysis, the controller G\(c_1(s)\) can be designed by considering only the minimum phase portion of the process, which makes the controller design easier. In Figure 4, the closed-loop transfer function of dashed box is given by

\[
H(s) = \frac{G_{c1}(s)G_{m0}(s)}{1 + G_{c1}(s)G_{m0}(s)} \tag{5}
\]

In this study, the IMC method is used for designing the controller G\(c_1\). According to the theory of IMC, the controller G\(c_1\) can be designed as

\[
G_{c1}(s) = \frac{Q_1(s)}{1 - Q_1(s)G_{m0}(s)} \tag{6}
\]

Where

\[
G_{m0}(s) = \frac{K_p}{(T_1s + 1)(T_2s + 1)} \tag{7}
\]

\[
Q_1(s) = f_1(s)G_{m0}^{-1}(s) \tag{8}
\]

Where G\(m_0\) is the minimum phase part of the controlled object model, f\(_1\)(s) is a low-pass filter, and the filter is introduced for physical realizability of the IMC controller and for taking into account the modelling errors.

The simplest filter has the following form

\[
f_1(s) = \frac{1}{\lambda s + 1} \tag{9}
\]

The filter time constant \(\lambda\) constitutes the tuning parameter of the controller. By substituting the formulas (7), (8) and (9) into Equation (6), the IMC-PID controller G\(c_1\) can be obtained as:

\[
G_{c1}(s) = \frac{(T_1s + 1)(T_2s + 1)}{K_p\lambda s} \tag{10}
\]

4.1. Tuning parameter \(\lambda\) based on dynamic performance index

The unit step response of system can be deduced from Figure 3 as follows

\[
y(t) = \begin{cases} 
0 & (t \leq \tau) \\
1 - \left(\frac{\lambda + a}{\lambda}\right) e^{-\frac{t - \tau}{\lambda}} & (t > \tau) 
\end{cases} \tag{11}
\]

From Equation (11) it is found that the response does not have overshoot when step signal is input the system. Based on the formula (11) the parameter \(\lambda\) can be tuned with the way of taking advantage of the rise time \(t_r\). The relationship between the rise time and the response speed is that the shorter the rise time, the faster the response.

From the Equation (11) it follows that

\[
1 - \left(\frac{\lambda + a}{\lambda}\right) e^{-\frac{t - \tau}{\lambda}} \geq 0.9 \tag{12}
\]

\[
\left(1 + \frac{a}{\lambda}\right) e^{-\frac{t - \tau}{\lambda}} \leq 0.1 \tag{13}
\]

If \(\lambda \leq a\), \((1 + a/\lambda)\) locates between 2 and infinity, then the formula (13) has no solution; If \(\lambda \geq a\), \((1 + a/\lambda)\) locates between 1 and 2, the solution of formula (13) is given in formula (14):

\[
e^{-\frac{t - \tau}{\lambda}} \leq 0.05 \tag{14}
\]

Because of \(e^{-2.994} = 0.05\), it yields:

\[
\frac{t_r - \tau}{\lambda} \geq 2.994 \tag{15}
\]

and

\[
a \leq \lambda \leq \frac{t_r - \tau}{2.994} \tag{16}
\]

The relationship between parameters \(\lambda\) and \(t_r\) can be obtained from the formula (16). Based on the relationship, the parameter \(\lambda\) can be chosen to make a good set-point tracking performance.
5. Design method of disturbance rejection controller $G_{c2}(s)$

According to Equation (4), it is found that the system closed-loop transfer function of the disturbance rejection response is the same as the closed-loop transfer function of one degree of freedom conventional control structure. Therefore, the following design method can be used for the design of controller $G_{c2}(s)$ in this paper:

In order to change the control problems of inverse response process plus time delay to the second order process with time delay, the portion $(-as + 1)$ in Equation (2) is approximated in the way of $(-as + 1) \approx e^{-\alpha s}$. Therefore, the process can be derived as

$$G_p(s) = \frac{K_p}{(T_1 s + 1)(T_2 s + 1)} e^{-(\alpha + \tau)s}$$  \hspace{1cm} (17)

The IMC can be used for the design of the controller $G_{c2}(s)$ as well, because the IMC controller has only one parameter simplified tuning method. The filter can be chosen based on the theory of IMC as follows

$$f_2(s) = \frac{1}{\mu s + 1}$$  \hspace{1cm} (18)

The controller $G_{c2}(s)$ with the form of PID can be deduced as

$$G_{c2} = \frac{T_1 T_2 s^2 + (T_1 + T_2) s + 1}{K_p (\mu + a + \tau) s}$$  \hspace{1cm} (19)

5.1. Tuning parameter $\mu$ based on maximum sensitivity ($M_S$)

The closed-loop control system robustness is measured with $M_S$, which indicates the sensitivity of the closed-loop transfer function to parameters with the process model changing. If the maximum sensitivity value of the system is smaller, then the control system more strong robustness for mismatched model condition. However, with smaller value of maximum sensitivity, the response speed of system slows down. The value of $M_S$ should be 1.1–2.0. when $M_S$ is generally taken 1.6, the response of system has a good robustness and a fast response speed as well (Alfaro & Vilanova, 2012). The maximum sensitivity is defined as follows:

$$M_S = \max_{0<\omega<\infty} \left| \frac{1}{1 + G_{c2}(j\omega) G_p(j\omega)} \right|$$  \hspace{1cm} (20)

By substituting the formulas (13) and (17) into Equation (20), it shows that maximum sensitivity function is only relevant to the parameters $a$, $\tau$ and $\mu$. Parameter $a$ and $\tau$ come from the process model. The parameter $\mu$ is the tuning parameter of controller, which is directly related to closed-loop control system robustness. Figure 5 describes the three-dimensional relationship among parameters $a$, $\tau$ and $\mu$ with the maximum sensitivity being 1.6. Using surface fitting toolbox of MATLAB, the available equation, containing $a$, $\tau$ and $\mu$, is given as follows:

$$\mu = 0.6096 - 1.16\tau + 1.639a$$  \hspace{1cm} (21)

From the above formulation, the parameter $\mu$ can be worked out.

6. Illustrative example

A second-order process with inverse response and time delay is considered as following formulation (Jeng & Lin, 2012).

$$G(s) = \frac{2(-3s + 1)}{(2s + 1)(s + 1)} e^{-0.5s}$$  \hspace{1cm} (22)

This example is used to compare the proposed design method with the methods of Chen et al. (2006), Jeng and Lin (2012) and Chien et al. (2003). Considering the fairness of comparison, the tuning parameters $\lambda$ and $\mu$ in the proposed method are selected such that the respective ISE values in the accurate model condition are similar to those of the methods of Chen et al. (2006), Jeng and Lin (2011) and Chien et al. (2003). Sequently, the respective ISE values of various methods are compared in mismatched model condition. The ISE to a step set-point change of these systems can be derived analytically using the following equation

$$\text{ISE} = \int_0^\infty |e^2(t)|dt = \int_0^\infty |r(t) - y(t)|^2dt$$  \hspace{1cm} (23)

In the closed-loop control system, a unit step set-point change is applied at $t = 0$. A step disturbance is added at $t = 40$ with amplitude $-0.5$ in the input process. Figure 6 shows the closed-loop responses of the nominal system with the same ISE value, while the parameters of controller $G_{c1}(s)$ are tuning to be $T_r = 15s$, $\lambda = 3.97$.
and the parameters of controller $G_{c2}(s)$ are tuning to be $\mu = 4.94$. The parameters of the compared methods are that $K_C = 0.156$, $\tau_I = 2$, $\tau_D = 1$ in the reference Chien et al. (2003) and $\lambda = 0.688$, $\tau_F = 0.079$ in Chen et al. (2006), and $\lambda = 4.396$ in Jeng and Lin (2011).

Next, considering the case of model mismatch, the robust performance is checked by assuming a perturbation of $\pm20\%$ in process parameters in the worst direction, where $T_1$, $T_2$ decrease $20\%$ and $a$, $K_p$, $\tau$ increase $20\%$. Figure 7 illustrates the responses of this perturbed system, which indicates that the achieved response of system with the proposed method is fast and less oscillatory. Table 1 lists the values of ISE performance index with those methods.

The results in Figure 7 and Table 1 clearly show that the value of the ISE performance index of the proposed method in this paper is smaller than those of the other three methods. The proposed method is also more robust than other methods when all systems have similar performance in the nominal condition. It can be concluded that the proposed method provides a smaller overshoot and a less oscillatory response, and maintain superior system robustness.

7. Conclusions

In this paper, the PID controllers of two-degree-of-freedom control method with parallel control structure are designed by IMC approach for inverse response process plus time delay. The advantage of PCS is that set-point tracking performance and disturbance rejection performance of system are decoupled, so the disturbance rejection response will not affect the tracking response. With a single easily tuning control parameter the set-point tracking controller ($G_{c1}$) and the disturbance rejection controller ($G_{c2}$) can manipulate three important attributes of the control loop (set-point tracking performance, disturbance rejection performance and robustness). The parameter of control $G_{c1}$ is tuned by the index of dynamic performance, which makes the close-loop system have faster set-point tracking response. The parameter of control $G_{c2}$ is tuned by the maximum sensitivity, which makes the close-loop system have disturbance rejection response and robustness. Simulation results show that the proposed method makes the system get a good set-point tracking performance, the disturbance rejection performance and robustness. The control method is easily implemented that can be used for engineering practice easily.

Acknowledgment

Firstly, I am deeply grateful to my honourable supervisor, who have checked through my thesis with patience and given me instructive suggestions, and she also played an important role in indicating a bright road in my future writing.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Ajmeri, M., & Ali, A. (2015). Direct synthesis based tuning of the parallel control structure for integrating processes. *International Journal of Systems Science*, 46(13), 2461–2473.

Alcántara, S., Pedrett, C., Vilanova, R., & Zhang, W. D. (2009). Analytical $H_\infty$ design for a Smith-type inverse-response compensator. *American control conference, 2009. ACC’09* (pp. 1604–1609). IEEE.

Alfaro, V. M., & Vilanova, R. (2012). Performance analysis of model reference robust tuned 2DoF PI controllers for over damped processes. 2012 20th Mediterranean conference on control & automation (MED) (pp. 872–877). IEEE.

Chen, P., Zhang, W., & Zhu, L. (2006). Design and tuning method of PID controller for a class of inverse response processes. *American control conference, 2006. IEEE*, 6.
Chien, I.-L., Chung, Y.-C., Chen, B.-S., & Chuang, C.-Y. (2003). Simple PID controller tuning method for processes with inverse response plus dead time or large overshoot response plus dead time. *Industrial & Engineering Chemistry Research, 42*(20), 4461–4477.

Geng, X., Hao, S., Liu, T., & Zhong, C. (2019). Generalized predictor based active disturbance rejection control for non-minimum phase systems. *ISA Transactions, 87*, 34–45.

Ghousiya Begum, K., Seshagiri Rao, A., & Radhakrishnan, T. K. (2017). Enhanced IMC based PID controller design for non-minimum phase (NMP) integrating processes with time delays. *ISA Transactions, 68*, 223–234.

Guangzhi, X., & Jinggang, Z. (2015). Control of two-degree-of-freedom smith predictor for inverse response processes with time delay. *Journal of Applied Sciences, 33*(4), 449–458.

Jeng, J.-C., & Lin, S.-W. (2012). Robust proportional-integral-derivative controller design for stable/integrating processes with inverse response and time delay. *Industrial & Engineering Chemistry Research, 51*(6), 2652–2665.

Jeng, J. C., & Lin, S. W. (2011). PID controller tuning based on Smith-type compensator for second-order processes with inverse response and time delay. *2011 8th Asian control conference (ASCC)* (pp. 1147–1152). IEEE.

Karunagaran, G., & Wenjian, C. (2011). The parallel control structure for transparent online tuning. *Journal of Process Control, 21*(7), 1072–1079.

Luyben, W. L. (2000). Tuning proportional—integral controllers for processes with both inverse response and deadtime. *Industrial & Engineering Chemistry Research, 39*(4), 973–976.

Nasution, A. A., Jeng, J.-C., & Huang, H.-P. (2011). A simple non-minimum phase zero's predictor for open-loop unstable processes. *Asia-Pacific Journal of Chemical Engineering, 6*(3), 441–451.

Seborg, D. E., Mellichamp, D. A., Edgar, T. F., & Doyle, F. J. (2010). *Process dynamics and control*. â€œHoboken, NJ: John Wiley & Sons.

Zhang, W., Xu, X., & Sun, Y. (2000). Quantitative performance design for inverse-response processes. *Industrial & Engineering Chemistry Research, 39*(6), 2056–2061.