Symmetry Subgroup Defense Against Adversarial Attacks

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Abstract

Adversarial attacks and defenses disregard the lack of invariance of convolutional neural networks (CNNs), that is, the inability of CNNs to classify samples and their symmetric transformations the same. The lack of invariance of CNNs with respect to symmetry transformations is detrimental when classifying transformed original samples but not necessarily detrimental when classifying transformed adversarial samples. For original images, the lack of invariance means that symmetrically transformed original samples are classified differently from their correct labels. However, for adversarial images, the lack of invariance means that symmetrically transformed adversarial images are classified differently from their incorrect adversarial labels. Might the CNN lack of invariance revert symmetrically transformed adversarial samples to the correct classification? This paper answers this question affirmatively for a threat model that ranges from zero-knowledge adversaries to perfect-knowledge adversaries. We base our defense against perfect-knowledge adversaries on devising a Klein four symmetry subgroup that incorporates an additional artificial symmetry of pixel intensity inversion. The closure property of the subgroup not only provides a framework for the accuracy evaluation but also confines the transformations that an adaptive, perfect-knowledge adversary can apply. We find that by using only symmetry defense, no adversarial samples, and by changing nothing in the model architecture and parameters, we can defend against white-box PGD adversarial attacks, surpassing the PGD adversarial training defense by up to \(\sim 50\%\) even against a perfect-knowledge adversary for ImageNet. The proposed defense also maintains and surpasses the classification accuracy for non-adversarial samples.

1. Introduction

Convolutional neural networks (CNNs) have achieved state-of-the-art status in computer vision [22, 26] by incorporating data symmetries. CNNs have the translation invariance built-in with their convolutional layers, which are equivariant with respect to translations due to kernel sliding [19, 29]. Other layers such as pooling in CNNs also support equivariance. In addition, CNNs learn invariance with respect to other transformations with data augmentation. For example, random cropping, image flips, color shifting / whitening are used for training on the ImageNet dataset [26].

CNNs lack invariance in the classification of samples that have been symmetrically transformed, even only slightly [4, 17], despite being engineered to incorporate translation, horizontal flipping and rotational symmetries. CNNs have been found to only provide approximate translation invariance [4, 5, 17, 24]. In addition, CNNs have been shown to be unable to learn invariances with respect to symmetries such as rotation and horizontal flipping with data augmentation [4, 5, 17].

CNNs also lack adversarial robustness [21, 43], which remains unsolved. Attacks such as the Projected Gradient Descent (PGD) attack [32] and the Carlini-Wagner attack [12] cause CNNs to misclassify adversarial samples. One of the first and still undefeated defenses against adversarial attacks is adversarial training (AT) [27, 32, 43]. AT uses adversarial samples in training, which requires advance knowledge of the attack [32] that might not be available.

Although the lack of adversarial robustness and the lack of invariance with respect to symmetric transformations are both inherent to CNNs, they have generally been studied separately with only one exception. To the best of our knowledge, only Engstrom et al. [17] have considered the lack of adversarial robustness and the lack of invariance together to study their interplay by applying rotation/translation transformations before adversarial perturbation, not after adversarial perturbation as we propose.

Considered together, CNN lack of adversarial robustness and CNN lack of invariance lead to the lack of equivariance of both CNN classifier boundary finding and finding adversarial samples, with respect to symmetry transformations. First, the lack of invariance of CNN classification with respect to symmetry transformations [4, 5, 17, 24] means that
CNN classifier boundary finding lacks equivariance with respect to symmetry transformations. Taking pixel inversion as an example, Figure 1 shows that inverting all dataset images and finding the classifier boundary do not commute with each other and would result in different class boundaries because otherwise CNN classification would not lack invariance with respect to symmetry transformations. However, CNN classification has been shown to lack invariance with respect to many symmetry transformations even after data augmentation [4, 5, 17]. Second, the lack of equivariance of CNN classifier boundary finding with respect to symmetry transformations leads to the lack of equivariance of finding adversarial samples with respect to symmetry transformations. On a conceptual level, all adversarial attacks [12, 21, 32, 35, 43] aim to change an original sample with a small perturbation in order to obtain an adversarial sample that is on the other side of the classifier boundary and misclassifies as a result. It stands to reason that different class boundaries in symmetric settings would cause adversarial attacks to find non-equivariant adversarial samples. In summary, the lack of invariance of CNN classification with respect to symmetry transformations leads to different classifier boundaries in symmetric settings, which further leads to different adversarial samples in these symmetric settings. The question is whether we would be able to recover the correct classification of adversarial samples after symmetry transformations.

In this paper, we propose a defense that uses only symmetry to recover the correct classification of adversarial samples. The proposed defense has these main contributions:

- We consider a threat model that ranges from zero-knowledge to perfect-knowledge adversaries based on the threat model suggested for adversarial defenses in [8], discussed in Section 3.
- We propose a novel symmetry defense that exceeds the accuracy of the robust PGD adversarial training by up to 50 percentage points for ImageNet without using adversarial samples and without changing the model architecture or its parameters. This is shown in Table 1 and Table 2 against a zero-knowledge adversary, and in Table 3 against a perfect-knowledge adversary.
- We devise a symmetry subgroup of image transformations that provides a framework for how the accuracy is evaluated and also confines an adaptive perfect-knowledge adversary to the subgroup transformations by the closure property of the subgroup. The defined subgroup is in Section 5.2.1.
- We incorporate a new artificial symmetry of pixel intensity inversion in the devised symmetry subgroup, which shows that the proposed method can be extended even to datasets without inherent symmetries.
- The proposed defense maintains and even exceeds the default accuracy for non-adversarial samples, as shown in Table 1 and Table 2 against a zero-knowledge adversary, and in Table 3 against a perfect-knowledge adversary.

2. Related Work

Symmetry. A transformation that leaves an object invariant is called a symmetry of that object. Some examples of symmetry transformations from images are rotation, horizontal flipping, inversion [34]. Symmetries can be discrete or continuous. Rotations where we allow any angle are examples of continuous symmetries. On the other hand, rotations where we allow only certain angles are examples of discrete symmetries.

Following are some definitions from group theory.

Binary Operation. According to [16], a binary operation * on a set G is a function *: G × G → G. Instead of writing the binary operation * on a, b ∈ G as a function *(a, b), we can write it as a * b.
**Closure.** Suppose that $*$ is a binary operation on the set $G$ and $H$ is a subset of $G$. If $*$ is a binary operation on $H$, that is, $\forall a, b \in H$, $a * b \in H$, then $H$ is said to be closed under the $*$ binary operation [16].

**Group.** According to [16], a group is an ordered pair $(G, \ast)$ where $G$ is a set and $\ast$ is a binary operation on $G$ that satisfies these axioms:

- **Associativity.** $\ast$ is associative: $\forall a, b, c \in G$, $(a \ast b) \ast c = a \ast (b \ast c)$.
- **Identity.** There exists an identity element $e \in G$, such that $a \ast e = e \ast a = a$, for $\forall a \in G$.
- **Inverse.** Every element in $G$ has an inverse: $\forall a \in G$, there exists $a^{-1} \in G$ such that $a \ast a^{-1} = a^{-1} \ast a = e$.

**Subgroup.** According to [16], a subset $H$ of $G$ is a subgroup of $G$ if $H$ is nonempty and $H$ is closed under products and inverses (that is, $x, y \in H$ implies that $x^{-1} \in H$ and $x \ast y \in H$). A subgroup $H$ of group $G$ is written as $H \subseteq G$. Informally, the subgroup of a group $G$ is a subset of $G$, which is itself a group with respect to the binary operation defined in $G$.

The **Subgroup Criterion.** A subset $H$ of a group $G$ is a subgroup if and only if $H \neq \emptyset$ and $\forall x, y \in H$, $x \ast y^{-1} \in H$ [16].

The **Finite Subgroup Criterion.** If $H$ is finite, then it suffices to check that $H$ is nonempty and closed under $\ast$ [16].

### 2.1. Equivariance and Invariance in CNNs.

**Equivariance.** A function $f$ is equivariant with respect to a class of transformations $\mathcal{T}$ if $\forall T \in \mathcal{T}$ of input $x$, a corresponding transformation $T'$ of the function output $f(x)$ can be found, such that $f(Tx) = T'f(x)$, according to [40].

**Invariance.** A function $f$ is invariant with respect to a class of transformations $\mathcal{T}$ if $f(Tx) = f(x)$, $\forall T \in \mathcal{T}$, according to [40]. Invariance is a special case of equivariance where $T'$ is the identity transformation. Invariant functions can be constructed by simply discarding information, thus limiting the functions and discarding relevant information [18].

CNNs learn better features by using domain knowledge that utilizes symmetries. Stacking equivariant mappings followed by an invariant map is a common blueprint used in machine learning [6] to ensure that the resulting mapping is invariant with regard to transformations [23]. For this purpose, CNNs use stacked convolution and pooling layers [20] to achieve translation equivariance. In addition, CNN training data is augmented with samples transformed by other symmetries so that CNNs can learn these symmetries.

**CNN Translation Equivariance.** CNNs have image translation symmetry encoded in to their architecture [15]. The convolutional layers of deep classifiers [26, 28] are engineered to be equivariant to translation symmetries in the image domain. CNNs achieve translation equivariance with the computation of feature maps over the translation symmetry group [19, 42]. Thus, CNNs are equivariant to translation symmetries that characterize image classification, where the position of the object in an image should not impact its classification. The equivariance of CNNs with respect to translations is built-in in CNNs by having alternating layers of local feature extraction and pooling [6, 15, 20]. The pooling layers of CNNs positioned after convolutional layers enable local invariance to translation [15]. By summing or maxing nearby input features, the output of the pooling operation does not change when the position of features changes within the pooling region. By typically stacking convolutional and pooling layers, CNNs comply with the common blueprint of stacking equivariant mappings followed by an invariant map that is common in machine learning [6, 23]. Cohen and Welling [13] show that convolutional layers, pooling, arbitrary pointwise nonlinearities, batch normalization and residual blocks are equivariant.

**CNN Translation Non-Equivariance.** Although equivariance to translation is built-in in CNNs, there are studies that suggest that translation equivariance in CNNs is not complete [4, 5, 17, 24, 45]. Engstrom et al. [17] find that CNNs are not invariant to even small translations or rotations. Azulay and Weiss [4] contend that the convolutional architecture is not fully translation-invariant because it ignores the sampling theorem. According to [4], it is the aliasing effects caused by the subsampling of the convolutional stride that make the output not invariant. Zhang [45] shows that CNNs are not translation invariant because of the max pooling, average pooling and strides that they use, for which Zhang [45] proposes an anti-aliasing solution. Kayhan and Gemert [24] also contend that CNNs are not translation-invariant because they learn filters that respond to particular absolute locations through image boundary effects. Boucachourt et al. [5] claim that the translation invariance built into CNNs is approximate, furthermore claiming that in ImageNet models, translation invariance is primarily learned from the data and enhanced by data augmentation.

**Equivariance Through Data Augmentation.** Unlike translation, other symmetries are not built-in into CNN architecture but are left to the CNN to learn using data augmentation. Symmetries such as rotations, horizontal flipping, scaling, shearing, perturbations to brightness, contrast and color jittering, and cropping are learnt by CNNs during training when the training data is augmented with original images that have been transformed with the symmetries that the CNN is supposed to learn. In the case of the ImageNet dataset, data augmentation can consist of a random crop, a random horizontal flip, as well as color jitter and color transforms of original images [1]. Previous work [30] has
found that an AlexNet CNN model learns representations that are equivariant to flipping, scaling and rotation symmetries.

**Non-Equivariance Through Data Augmentation.** Although data augmentation dominates as the method for CNNs to learn invariances with respect to non-translation symmetry transformations [22], studies have shown that data augmentation is only marginally effective [4, 5, 17]. It has been shown that learning symmetries from data augmentation is not as effective as incorporating the symmetries in the model architecture [13, 25]. More recent studies [4, 5, 17] show that CNNs are unable to learn invariances with data augmentation. Engstrom et al. [17] find that data augmentation with rotation and translation only marginally improved invariance in for CNN classifiers of CIFAR10 and ImageNet datasets. Azulay and Weiss [4] find that data augmentation only enables CNNs to be invariant to symmetric transformations for images that are similar to typical images from the dataset, obtained in the same way as the training images. Bouchacourt et al. [5] claim that non-translation invariance is learned from the data independently from data augmentation, finding that the main source of invariance is the training data and data augmentation only increases the invariances that have been learnt.

**Building More Equivariant CNNs.** There have been several approaches to build equivariant CNNs that address the limitations of convolutional layers and data augmentation in terms of equivariance. Schmidt and Roth [40] learn image priors that are rotation-invariant and build rotation-equivariant and invariant descriptors of features. Invariant scattering CNNs [7,41] construct representations with translation and rotation invariance using a cascade of wavelet decompositions and transforms. Anselmi et al. [2] theorize a hierarchical architecture with basic moduli with invariance properties. The deep symmetry networks [19], or symnets, generalize CNNs by forming feature maps over symmetry groups. Group equivariant convolutional neural networks [13, 14] use a new type of layer with G-convolutions that exploits discrete symmetry groups generated by translations, reflections and rotations. Dieleman et al. [15] introduce four operations that can be combined to provide partial equivariance with respect to rotations. Dieleman et al. [15] generate rotated and flipped images that are all used in training, which allows parameter sharing between the representations of different orientations. Zhou et al. [46] address CNN lack of equivariance with respect to rotations with active rotating filters (ARFs) that rotate during convolution and result in feature maps with encoded location and orientation. Marcos et al. [33] propose rotation equivariant vector field networks that encode rotation equivariance, invariance and covariance by applying filters at many orientations and recording the orientation with the highest score. Finzi et al. [18] construct convolutional layers that are equivariant with respect to transformations any specific Lie group with a surjective exponential map. Romero and Cordonnier [38] formulate self-attention with equivariance to arbitrary symmetry groups by defining positional encodings that are invariant to the group transformations.

**Datasets in CNNs with Better Equivariance.** Although there are architectures that handle symmetry better than traditional CNNs, they have not been shown to work for the ImageNet dataset but only for simple or synthetic datasets. Schmidt and Roth [40] conduct experiments on MNIST and an aerial car detecting dataset of 30 images. Invariant scattering CNNs [7,41] reports on the MNIST image dataset [7] and texture datasets [31]. Anselmi et al. [2] show their results for a small face dataset. Deep symmetric networks [19] uses MNIST and the synthetic NORB dataset. Group equivariant convolutional networks [13] reports on CIFAR10 and MNIST. Dieleman et al. [15] use the Plankton dataset of 30,336 images, the Galaxies dataset 61,578 images, and the Massachusetts buildings dataset of 151 images. Zhou et al. [46] show results with their ARFs with rotated MNIST and CIFAR10. Marcos et al. [33] show their results with rotated MNIST, a small dataset of 15 Google Map images of cars, and a 20K, 64 × 64 dataset of cathedral images. Finzi et al. [18] apply their method on the rotated MNIST dataset, and the QM9 regression dataset of 134K small inorganic molecules. Romero and Cordonnier [38] show classification results in rotated MNIST, CIFAR10 and a dataset of tumorous/non-tumorous images.

**Summary.** Relevant to the proposed defense, we derive the following key points from the related work:

- Adversarial attacks are still an open problem as one of the main defenses necessitates knowing the attack in advance.
- CNNs do not achieve full equivariance with respect to symmetries such as translation, horizontal flipping and rotation despite being designed and trained with data augmentation to incorporate these symmetries.
- Studies that better incorporate symmetries into CNNs have not been successful for bigger datasets such as ImageNet.

### 2.2. Adversarial Attacks

Szegedy et al. [43] defined the problem of generating adversarial samples as starting from original samples and applying small perturbation that results in misclassification. Formally, Szegedy et al. [43] defined generating adversarial samples as a minimization that is based on the classifier gradient while minimizing the perturbation. For targeted attacks that target a specific label for the misclassification, the minimization is:
minimize \( c \cdot \|\delta\| + \text{loss}_f(x + \delta, l) \) \hspace{1cm} (1)

such that \( x + \delta \in [0, 1]^d \).

where \( f \) is the classifier function, \( \text{loss}_f \) is the classifier function loss, and \( l \) is an adversarial label, \( c \) is a constant, \( \|\delta\| \) is the \( L_p \) norm of perturbation. By using this Minimization 1, attacks start out with original samples in order to generate adversarial samples by using the classifier gradient to generate a small perturbation \([12, 32, 35]\).

PGD Attack. The PGD attack \([27]\) is one of the strongest current adversarial attacks. PGD is an iterative attack with a parameter that defines the magnitude of the perturbation of each step. PGD starts from an initial sample point \( x_0 \) and then iteratively finds the perturbation of each step and projects the perturbation on an \( L_p \)-ball.

2.3. Adversarial Defenses

Adversarial Training. Adversarial Training (AT) \([27, 32, 35]\) is one of the first and few defenses that have not been defeated. AT works by training classifiers with adversarial samples that are labeled correctly. The robust PGD AT defense \([32]\) is formulated as a robust optimization problem and is considered to be one of the most successful adversarial defenses \([31]\). Despite AT being the best current defense, AT necessitates knowing the attack in advance and generating adversarial samples during training. This is unrealistic for real attacks and increases training time due to adversarial sample computation.

Other Failed Defenses. Many other defenses have been shown to fail against an adaptive adversary. Defensive distillation has been shown to be not robust to adversarial examples \([9]\), many adversarial detection defenses have been bypassed \([10, 11]\), obfuscated gradient defenses \([3]\) and other defenses \([44]\) have been circumvented.

2.4. Lack of Adversarial Robustness and Lack of Invariance Together

To the best of our knowledge, only one prior work \([17]\) examines the lack of invariance and the lack of adversarial robustness together by looking at the interplay between rotations/translations and \( L_\infty \) adversarial perturbation by applying first the rotation/translation and then the adversarial perturbation. Whereas Engstrom et al. \([17]\) largely find rotation/translation to be orthogonal to adversarial perturbation, they apply symmetries before the adversarial perturbation. In contrast, we examine symmetry transformations of adversarial samples, applying the symmetry transformations after the adversarial perturbation.

3. Threat Model

We base our threat on the threat model recommendations in \([8]\) for adversarial defenses. The threat model consists of three separate scenarios. In all scenarios, the adversary is assumed to know the model and its parameters and to be able to conduct white-box attacks.

- Zero-Knowledge Adversary. This is the weakest threat model where the adversary is unaware of the symmetry defense.
- Perfect-Knowledge Adversary. The adversary is aware that the classifier is using symmetry against adversarial attacks and attempts to evade.
- Limited-Knowledge Adversary. Based on \([8]\), this evaluation only needs to be done if the zero-knowledge attack fails and the perfect-knowledge attack succeeds.

4. Experimental Setting

We conduct experiments to evaluate the proposed symmetry defense against the adversaries identified in the threat model based on \([8]\) in Section 3: zero knowledge adversary, perfect-knowledge adversary, and limited-knowledge adversary.

Overall, we use the same experimental setting as the robust PGD AT implementation \([1]\) for the ImageNet dataset. Based on \([1]\), the evaluation is done on logits, non-softmax output.

We use the ImageNet \([39]\) dataset to evaluate the proposed defense. ImageNet \([39]\) is a 1000-class dataset of over 1.2M training images, and 50K testing images.

We use the ResNet50 \([22]\) architecture for training with the ImageNet \([39]\) using the same architecture, parameters, and training as used for default training in \([1]\). The ResNet50 architecture model is trained with the SGD optimizer with a momentum of 0.9, a learning rate decaying by a factor of 10 every 50 epochs, and a batch size of 256. The classifier takes as input images with \([0, 1]\) pixel value ranges that get normalized with channel mean and standard deviation values.

Data Augmentation. We use the same data augmentation as \([1]\) in all the models we train: random resized crops, random horizontal flips.

Image Preprocessing. Being based on \([1]\), our implementation uses the following image preprocessing: Fancy PCA and color jitter. We have examined both these transformations in the context of equivariance as defined earlier. The reason we have examined them is because if the mapping in each transformation is not equivariant with relation to the symmetries, then we can end up with different representations depending on which is applied first, the transformation mapping or the symmetry. If the representations are
not symmetric, then the adversarial samples generated from them might be even more different than they would otherwise be in different settings. Therefore, in order to show that the proposed defense does not necessitate this advantage, we examine the transformations. Further, we perform experiments both with and without non-equivariant transformations to show that the proposed method succeeds in both cases.

**Fancy PCA.** The Fancy Principal Component Analysis (PCA) is a form of data augmentation that changes the intensities of RGB channels in the training images. This method performs PCA analysis on the ImageNet [39] images. Then, the method adds to every training image multitudes of the principal components it has found. The magnitudes are proportional to the eigenvalues as well as to a random variable drawn from a Gaussian distribution with a mean of 0 and standard deviation valued at 0.05 for the Fancy PCA pre-processing of ImageNet images in the implementation of [1] that we follow.

**Equivariance of Fancy PCA with Respect to Intensity Inversion.** We conduct experiments to verify whether the mapping of the Fancy PCA transformation is equivariant with respect to intensity inversion because we find the PCA analysis of inverted images that they have the same principal components as original images. Based on this experiment, we conclude that the Fancy PCA transformation is equivariant with respect to intensity inversion. Therefore, we keep this transformation in the pre-processing of ImageNet images in all models.

**Color Jitter.** The ColorJitter transform used in the PGD AT defense implementation [1] randomly changes the brightness, contrast and saturation of images.

**Inequivariance of Color Jitter with Respect to Intensity Inversion.** We have examined the equivariance of this transformation with respect to image inversion and found that color jitter is not equivariant with respect to pixel intensity inversion. This means that classifier boundaries of two models with the same architecture trained one with original images and the other with inverted images would differ more than they would differ when color jitter is not used. Therefore, the color jitter data augmentation has the potential of increasing the inequivalence. In order to show that the proposed method does not necessitate color jitter, we conduct experiments both with and without color jitter. Whenever color jitter is used, it is used in both training and testing. Whenever color jitter is not used, it is not used in both training and testing.

**Models.** Here, we clarify the models we have used in our experiments and how they were trained. All are ImageNet ResNet50 models trained according to [1]. The following models differ from each other in what symmetric transformations get applied to the original samples before training. Apart from the pretrained model, all other models are trained based on the implementation in [1].

- **M-Pretrained.** The Pytorch ResNet50 pretrained model.
- **M-Orig.** A model trained on original images without jitter in preprocessing.
- **M-Orig-Jitter.** A model trained on original images with jitter in preprocessing.
- **M-Invert.** A model trained on inverted images without jitter in preprocessing.
- **M-Invert-Jitter.** A model trained on inverted images with jitter in preprocessing.
- **M-Both.** A model trained on original and inverted images without jitter in preprocessing.
- **M-Both-Jitter.** A model trained on original and inverted images with jitter in preprocessing.

**PGD Attack Settings.** To evaluate the proposed defense, we evaluate the classification accuracy against $L_2$ and $L_\infty$ PGD [27] attacks parametrized according to [1] for ImageNet. In $L_2$ PGD attacks, we use $\epsilon$ values of 0.5, 1.0, 2.0, 3.0 based on [1]. In $L_\infty$ PGD attacks, we use $\epsilon$ values of 4/255, 8/255, 16/255 based on [1]. All PGD [27] attacks have 100 steps, with a step perturbation value defined as the ratio of $2.5 \times \epsilon$ over the number of steps, according to [1]. All PGD attacks are targeted, with the target label chosen uniformly at random among the labels other than the ground truth label.

**Tools.** The proposed defense method was written in Python 3.9.7 using PyTorch [37]. Adversarial samples were generated with IBM Adversarial Robustness 360 Toolbox (ART) toolbox 1.8.1 [36].

5. The Proposed Method

The proposed method is inspired by the lack of invariance of CNNs with respect to the classification of symmetry transformations of samples. As discussed in Section 2.1, CNNs are not completely equivariant with respect to symmetry transformations such as translation, rotation and flipping despite CNN architecture and the data augmentation techniques. Attempts to build symmetries better into CNNs have not been shown to succeed with the ImageNet dataset, as discussed in Section 2.1.

CNNs’ lack of invariance with respect to the classification of symmetry transformations of samples means different things for original samples and adversarial samples. For original samples, the lack of invariance means that CNNs would classify a transformed original sample differently from the correct classification, which is undesirable. For adversarial samples, the lack of invariance means that CNNs
would classify a transformed adversarial sample differently from the adversarial sample classification, which has the potential to be the correct classification.

The proposed method examines whether we can use symmetry transformations of adversarial samples to recover the correct classification of the adversarial samples. Following the threat model in Section 3, we examine first the proposed defense of CNNs against zero-knowledge adversaries in Section 5.1. Next, we examine the proposed defense of CNNs against perfect-knowledge adversaries in Section 5.2. Next, we examine the proposed defense of CNNs against limited-knowledge adversaries in Section 5.3.

Two Types of Preprocessing. We consider two types of preprocessing: with color jitter and without color jitter. For each case, the training and testing are consistent. When color jitter is used, it is used in the preprocessing of both training and testing samples. When color jitter is not used, it is not used in the processing of training or testing samples.

5.1. The Defense Against A Zero-Knowledge Adversary

Here, we assume to have a zero-knowledge adversary that is not aware of the symmetry defense but has otherwise access to the model and its parameters and is able to conduct white-box attacks. The attacks are $L_2$ and $L_\infty$ PGD attacks. For the defense against zero-knowledge adversaries, we do not examine adaptive attacks because the adversaries are not aware of the defense.

Two Symmetries. First, we consider the horizontal flipping symmetry which is a symmetry that is present in the ImageNet dataset and for which data augmentation is used for CNNs to learn it. We utilize here the findings that CNN do not learn symmetries well with data augmentation. Second, we consider the intensity inversion symmetry where a pixel value $a \in [0, 1]$ changes to the value $1 - a$. The purpose of using this symmetry is to show that the proposed defense method can be extended to the classification of datasets without inherent symmetries.

5.1.1 Defense Using Horizontal Flipping Symmetry Against Zero-Knowledge Adversary

For these experiments, we generate the adversarial samples from original samples using the M-Orig (no jitter) or the M-Pretrained (with jitter) model, applying jitter only when using the M-Pretrained model that has been trained with jitter images. Next, we horizontally flip the adversarial samples and test the classification of the flipped adversarial samples with the same model that was used to generate them - M-Orig or M-Pretrained respectively. Figure 2 outlines the flip symmetry defense against a zero-knowledge adversary and Table 1 shows the results.

5.1.2 Defense Using Artificial Intensity Inversion Symmetry Against Zero-Knowledge Adversary

Since the intensity inversion symmetry is not present by default in the image domain, we train two models with inverted samples instead of original samples - M-Invert and M-Invert-Jitter. The M-Invert model is trained without jitter preprocessing of samples, the M-Invert-Jitter model is trained with jitter preprocessing of samples. For these experiments, we generate the adversarial samples from original samples using the M-Orig or the M-Orig-Jitter model, applying jitter only when using the M-Orig-Jitter model that has been trained with images with jitter. Then, we invert the intensity of the pixels of the adversarial image and classify the inverted adversarial image with the M-Invert or the M-Invert-Jitter model respectively. Figure 3 outlines the invert symmetry defense against a zero-knowledge adversary and Table 2 shows the results.

5.2. The Defense Against A Perfect-Knowledge Adversary

Here, we assume that the white-box adversary is aware of the proposed symmetry defense.

To counter a perfect-knowledge adversary, a defense would need to use more than one symmetry transformation to succeed. A defense that uses only one symmetry would not know whether the perfect-knowledge adversary has applied any symmetry transformations to the adversarial sample. For example, aware that the defense in Section 5.1.2 flips (or inverts) samples before classifying them, the perfect-knowledge adversary can flip (or invert) the adversarial samples after generating them. Followed by the defense’s transformation described in Section 5.1.2, this would result in the adversarial sample having no symmetry transformation because flipping (or inverting) image samples twice is the same as the identity transformation.

To counter a perfect-knowledge adversary, a defense would need the additional symmetry transformations to be discrete, which rules out using rotations or translations. If non-discrete symmetries were used, then the perfect-knowledge adversary would have infinite number of choices which would not be easy for the defense to verify exhaustively.

We decide to use two symmetries together: the horizontal flip symmetry of image domain, and intensity inverting which is an artificial symmetry that inverts the pixels values from $a \in [0, 1]$ to the value $1 - a$. We use the flip symmetry because CNN do not learn the flip symmetry despite data augmentation, as discussed in Section 2.1. We use the additional, artificial, discrete symmetry of pixel value inversion because we expect CNNs to not be able to learn this symmetry either based on how they learn the translation, rotation and flipping symmetries in the image domain, as discussed in Section 2.1.
We impose a symmetry subgroup that is comprised of these transformations: identity, flipping, inverting, flipping and inverting. We impose the subgroup by training the CNN with samples on which a subgroup transformation has been applied: original samples (identity transformation), inverted original samples (inverting transformation), and using data augmentation with horizontal flipping on the original training dataset (flipping transformation) and on the inverted training dataset (flipping and inverting transformation). The imposed symmetry subgroup is discussed more in detail in Section 5.2.1.

The testing of the CNN model is also based on the defined symmetry subgroup in two aspects that are shown in Figure 4. First, the adversary can construct adversarial samples from any of the four possible states an image sample can be in according to the imposed group shown in Figure 5: original, flipped, inverted, flipped and inverted. Therefore, we test all four possibilities. Second, after the adversarial sample has been created by the adversary, we obtain an adversarial sample created by the adversary, for which we do not know the transformations, if any, that were applied after the adversarial image generation. Therefore, we apply all four subgroup symmetry transformations to the adversarial image and decide on the classification label when the classifier classifies at least two of the four symmetric images the same. The results are shown in Table 3.
Table 2. Adversarial accuracy of defense that uses invert symmetry against zero-knowledge adversary.

| Perturbation | PGD ε-test | PGD Defense | No Defense | Proposed Defense | No Jitter | Proposed Defense | With Jitter | Proposed Defense |
|--------------|------------|-------------|------------|------------------|----------|------------------|------------|------------------|
| −            | ε = 0.0    | 47.91%*     | 77.00%     | 76.95%           | 76.13%   | 76.88%           |            |                  |
| L_2          | ε = 0.5    | 54.24%      | 34.46%     | 75.92%           | 36.66%   | 4.17%            | 75.08%     |                  |
|              | ε = 1.0    | 50.67%      | 4.38%      | 75.03%           | 0.17%    | 74.06%           | 73.88%     |                  |
|              | ε = 2.0    | 43.04%      | 0.22%      | 74.08%           | 0.02%    | 74.06%           | 73.88%     |                  |
|              | ε = 3.0    | 35.16%      | 0.03%      | 73.69%           | 0.02%    | 74.06%           | 73.88%     |                  |
| L_∞          | ε = 4/255  | 33.58%      | 0.01%      | 73.47%           | 0.00%    | 73.87%           | 73.67%     |                  |
|              | ε = 8/255  | 19.63%      | 0.00%      | 71.33%           | 0.00%    | 71.39%           | 71.39%     |                  |
|              | ε = 16/255 | 5.00%       | 0.00%      | 65.77%           | 0.00%    | 65.80%           | 65.80%     |                  |

Against a zero-knowledge adversary, the proposed defense is shown to surpasses the accuracies of the robust PGD AT defense by up to 60 percentage points using the intensity inversion symmetry.

*The accuracy results of the robust PGD AT defense for non-adversarial samples vary depending on the perturbation ε used in training. The results shown here are obtained from [1] on which we base the implementation of the proposed defense. More detailed results in Table 5.

5.2.1 Definition of the discrete group of image transformations with regard to which the ImageNet classifier is invariant

**Definition of the subgroup of transformations.** We define the discrete set of operations $H = \{e, a, b, c\}$, where $e$ denotes the identity element, $a$ denotes horizontal flipping, $b$ denotes intensity inversion, $c$ is the composition of $a$ and $b$ that denotes flipping and inverting. The operation $\ast$ is simply that one transformation follows another. We build the Cayley table in Table 4 to show that the subgroup is closed since compositions of the elements also belong to the subgroup.

**Theorem.** $H = \{e, a, b, c\}$ is a subgroup of the group of symmetry transformations of images.

**Proof.** Based on the finite group criterion in Section 2, a subset of a group should need only to be nonempty and closed under operation $\ast$ [16]. $H$ is nonempty because it has four elements and is a subset of the symmetry transformations of images. Based on the definition of closure in Section 2, for $H$ to be closed under the $\ast$ operation, we need to show that $\forall a, b \in H$, we get that $a \ast b^{-1} \in H$. Table 4 shows that $\forall a, b \in H$, we get that $a \ast b^{-1} \in H$. Table 4 also shows that each element is its own inverse element because $\forall b \in H$, we get $b \ast b = c$, which means that $b = b^{-1}$. From $\forall a, b \in H, a \ast b \in H$ and $\forall b \in H, b = b^{-1}$, we derive that $\forall a, b \in H, a \ast b^{-1} \in H$.

The subgroup $H = \{e, a, b, c\}$ that we have defined is known as the **Klein four-group**, which is a four-element group where each element is its own inverse and where
Figure 4. Here, we show how the proposed defense classifies adversarial samples from a perfect-knowledge adversary and original samples. We show that both the adversary and the defense use the symmetry subgroup. The adversary can generate the adversarial sample after applying to the original sample any of the transformations of the subgroup. After generating the adversarial sample, there is no point for the adversary to transform the adversarial sample because the defense applies all the subgroup transformations to the adversarial sample. The defense applies all four transformations of the subgroup to the adversarial sample and assigns a class when the classification labels of two of the transformed samples agree.

composing any two of the non-identity elements results in the third non-identity element. Another way to define the Klein four-group $H$ is: $H = \{a, b| a^2 = b^2 = (a*b)^2 = e\}$, which agrees with the transformations of the subgroup used in the proposed defense.

5.2.2 Adaptive Attack To The Proposed Defense Against Perfect-Knowledge Adversaries

Being aware of the defense, a perfect-knowledge adversary will adapt its attack and the defense needs to take this into account [8].

The perfect-knowledge adversary is limited in what it can do because the proposed defense uses the inherent CNN lack of invariance and makes no change in the the model architecture or image pre-processing. When the proposed defense uses or not color jittering, testing is done with the same jittering pre-processing applied during training.

By training with original and inverse samples and flipped samples from data augmentation, the proposed defense
Table 3. Accuracies in the threat model of a perfect-knowledge adversary as compared to PGD accuracies.

| Norm | PGD** | Proposed Defense | Proposed Defense | Proposed Defense | Proposed Defense |
|------|-------|------------------|------------------|------------------|------------------|
|      | Identity transf. | Flipping transf. | Inverting transf. | Flipping and Inverting |
|      | No | Proposed | No | Proposed | No | Proposed | No | Proposed |
| - $\epsilon = 0.0$ | 47.91%* to 76.13% | 76.37% | 77.75% | 76.22% | 77.75% | 76.28% | 77.85% | 76.15% | 77.85% |
| $L_2$ | 54.42% | 34.23% | 73.15% | 34.13% | 73.30% | 34.06% | 72.94% | 34.00% | 73.04% |
| $L_\infty$ | 50.67% | 3.91% | 70.44% | 3.89% | 70.70% | 4.02% | 70.34% | 4.07% | 70.29% |
| $\epsilon = 4/255$ | 43.04% | 0.16% | 68.52% | 0.18% | 68.65% | 0.17% | 68.46% | 0.19% | 68.35% |
| $\epsilon = 8/255$ | 35.16% | 0.02% | 67.67% | 0.03% | 67.77% | 0.02% | 67.71% | 0.02% | 67.64% |

We observe that the proposed defense surpasses the accuracies of the robust PGD AT defense for adversarial samples regardless of the symmetry subgroup transformation that the adversary chooses to apply to original samples before generating adversarial samples. Unlike the robust PGD AT defense, the performance of which deteriorates on non-adversarial samples for bigger perturbations, the performance of the proposed defense even improves compared to the default 76.13%.

*The accuracy results of the robust PGD AT defense for non-adversarial samples vary depending on the perturbation $\epsilon$ used in training. The results shown here are obtained from [1] on which we base the implementation of the proposed defense. More detailed results in Table 5.

**Robust PGD AT defense [1] results duplicated because Robust PGD AT defense does not examine both no-jitter and with-jitter cases image preprocessing, but only does with-jitter preprocessing.

Table 4. The Cayley table for the $H$ subgroup shows the composition of the elements of the $H$ subgroup, where the elements of the subgroup are $a$, $b$, $c$ and the identity element $e$. From the Cayley table, we can see that the subgroup is closed because compositions of the elements belong to the subgroup.

| * | a | b | c |
|---|---|---|---|
| identity | - | e | a | b | c |
| flip | - | a | e | b | c |
| invert | - | b | c | e | a |
| flip and invert | - | c | b | a | e |

against a perfect-knowledge adversary gives the CNN model the opportunity to learn all the subgroup symmetries so that the adversary can succeed better. The defense performance results show that the defense succeeds nevertheless.

What the perfect-knowledge adversary can do is utilize the subgroup symmetries and try to apply subgroup transformations before or after the generation of adversarial samples. Both have been taken into account and do not succeed because of the closure of the defined subgroup. To account for sample transformations by the perfect-knowledge adversary before the generation of adversarial samples, we take all possible cases into account and show results for each case in Table 3. To account for sample transformations after the generation of adversarial samples, the proposed defense applies all subgroup transformations to adversarial samples to obtain four samples out of classification of which the defense decides the classification. This means that there is no point for the perfect-knowledge adversary to apply any subgroup transformations after the generation of the adversarial sample because that has no effect on the defense obtaining all four possible samples due to the subgroup closure. Therefore, the perfect-knowledge adversary cannot utilize the defined symmetry subgroup to its advantage.
5.3. The Defense Against a Limited-Knowledge Adversary

According to [8], we would only need to do this evaluation if the zero-knowledge attack fails and the perfect-knowledge attack succeeds. Based on the results, this evaluation is not needed.

5.4. Summary of Results

The proposed symmetry defense exceeds robust adversarial training defense. We have shown that the proposed defense exceeds the robust adversarial training defense without using adversarial samples even when the attacker is aware of the defense.

6. Computing Resources

Here, we analyse the computing resources of the proposed defense and the adversary relative to the computing resources of default classification. We discuss the computing resources in each scenario of the threat model defined in Section 3.

Zero-knowledge adversary. Against a zero-knowledge adversary, the training and testing resources are exactly the same as the training and testing resources of a default classifier. The adversary also uses the same resources in generating the adversarial sample.

Perfect-knowledge adversary. During training, the proposed defense trains with original dataset samples and inverted samples as well maintaining the same batch size. Therefore, the defense uses twice the time used in training the default classifier, and twice the computation. Space requirements remain the same because the batch size is the same as in the default classifier. During testing, the defense applies the four subgroup transformations to the sample, causing space, time and inference computation requirements to remain the same $O(4n) = O(n)$, $O(1)$ computation overhead compared to default classifier. The perfect-knowledge adversary chooses which subgroup transformation to apply to the original image, leading to overhead of $O(1)$.

Limited-knowledge adversary. We do not examine this scenario and its resource usage based on [8] where it is stated that this evaluation only needs to be done if the zero-knowledge attack fails and the perfect-knowledge attack succeeds, which is not the case for the proposed defense.

7. Discussion and Conclusion

The proposed defense succeeds in defending CNN classifiers by using only symmetry against adversaries ranging from zero-knowledge to perfect-knowledge. The defense exceeds by far the accuracies of the current best defense without even necessitating using adversarial samples.

8. Appendix

In Table 5, we detail the performance of the robust PGD AT defense with original non-adversarial samples from results in [1].

Table 5. Non-adversarial accuracy of robust PDG AT defense.

| $\epsilon$-test | Original Samples | $\epsilon$-train | $L_2$ Norm | $L_\infty$ Norm |
|-----------------|------------------|-----------------|-----------|----------------|
| $\epsilon = 0.0$ | 76.13\%          | 57.90\%         | 62.42\%  | 47.91\%        |
| $\epsilon = 3.0$ |                  |                 |           |                |
| $\epsilon = 8/255$ |                 |                 |           |                |

Robust PGD AT accuracies with non-adversarial samples, as obtained from [1], decrease as the perturbation increases.

dition, the defense also maintains, and even exceeds, default performance on non-adversarial samples.

The proposed defense is the first symmetry defense. Current adversarial attacks have no answer to a symmetry subgroup defense because lack of invariance is an inherent property of current CNNs. Against a symmetry defense that changes nothing in the model architecture and nothing in image preprocessing (succeeding with and without color jitter), an adaptive perfect-knowledge adversary can only use the symmetry subgroup. However, the closure of the defined finite subgroup limits what the attack can do and does not succeed.

The proposed defense employs an additional artificial symmetry not present in the dataset, which extends the applicability of the defense to datasets without inherent symmetries.

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