Singly charged higgses at linear collider

K. Huitu\textsuperscript{a,1}, J. Laitinen\textsuperscript{a,2}, J. Maalampi\textsuperscript{b,3} and N. Romanenko\textsuperscript{b,4}

\textsuperscript{a}Helsinki Institute of Physics and \textsuperscript{b}Department of Physics

P.O.Box 9, FIN-00014 University of Helsinki, Finland

We consider the production of singly charged Higgs bosons in the Higgs triplet and two Higgs doublet models. We evaluate the cross sections for the pair production and the single production of charged higgses at linear collider. The decay modes of $H^+$ and the Standard Model backgrounds are considered. We analyze the possibilities to differentiate between triplet and two Higgs doublet models.

Key words: Singly charged Higgs boson; triplet Higgs model; two Higgs doublet model

PACS: 12.60.Fr

\textsuperscript{1} katri.huitu@helsinki.fi
\textsuperscript{2} jouni.laitinen@helsinki.fi
\textsuperscript{3} jukka.maalampi@helsinki.fi
\textsuperscript{4} nikolai.romanenko@helsinki.fi
1 Introduction

The observation of a singly charged Higgs boson in future collider experiments would be a definitive evidence of physics beyond the Standard Model (SM). The scalar sector of the SM consists of a single isodoublet $\phi = (\phi^+, \phi^0)$, and there is only one physical scalar field in the theory, a neutral Higgs. The charged scalar degrees of freedom are eaten by the weak boson $W^\pm$ via the Higgs mechanism, and they do not show up in the spectrum of physical particles. Nevertheless, a common feature of almost all imaginable extensions of the SM is the existence of at least one physical charged Higgs field.

In this paper we shall study the production of a singly charged Higgs scalar in $e^+e^-$ collisions in a future linear collider. We shall concentrate on two particular models of different types that both predict one charged Higgs scalar, a model with two $SU(2)_L$ Higgs doublets (2HDM) and a model with an $SU(2)_L$ triplet (HTM).

The best known example of the 2HDM is the minimal supersymmetric extension of the Standard Model (MSSM), which requires the existence of two isodoublets of Higgs fields to give masses separately to up and down-type fermions and to cancel anomalies [1]. We will use it as our framework when analysing the two-doublet model and implement the mass relations it predicts in calculating the branching ratios of charged Higgs decays. We will assume, however, that the supersymmetric partners of SM particles are too heavy to affect the processes we will consider.

The triplet model we will consider is a simple extension of the SM, with a Higgs sector including, in addition to the ordinary isodoublet, a complex $\Delta = (\delta^{++}, \delta^+, \delta^0)$ [2] isotriplet with hypercharge $Y = 2$. This kind of a model is particularly interesting for it can create mass to neutrinos via spontaneous symmetry breaking due to Yukawa coupling of the triplet with leptons, without requiring the existence of sterile right-handed neutrinos. (A strong evidence of non-vanishing neutrino masses was recently obtained in the Super-K measurements of the atmospheric neutrino fluxes [3].) The vacuum expectation value of the $\delta^0$ field breaks the global lepton number symmetry and could in principle lead to the existence of a majoron, as in the case of the so-called triplet majoron model [4]. The triplet majoron is, however, ruled out experimentally by the LEP measurements of the invisible width of the $Z$ boson [5]. We will
consider the version of the HTM where the lepton number is explicitly broken and no majoron appears [2]. This model serves as an example of the class of models which do not contain a majoron.

Isotriplet Higgs scalars arise naturally in the left-right symmetric model [6]. Although this model differs from the triplet model we will consider due to the extended gauge symmetry, the phenomenology of the singly charged Higgs particle is quite similar in both models. One feature common to all triplet models is that the charged triplet Higgs does not couple to quarks, in contrast with the charged higgses in isodoublets. The physical charged higgses are, however, mixtures of isotriplet and isodoublet higgses, the magnitude of the mixing depending on the ratio of the vacuum expectation values of the corresponding neutral Higgs fields, so that they do couple to quarks but with a much smaller strength than the ordinary higgses.

The object of our study are the phenomenological features that would distinguish between purely isodoublet and predominantly isotriplet singly charged higgses in $e^+e^-$ collisions. The possibility of the their detection has been previously studied in [2,7], where the effects of the tree-level $H^\pm W^\mp Z$ vertex, present in triplet models but absent in doublet models, were considered. This vertex is suppressed in the model with one complex $Y = 2$ triplet [2] considered in this paper, but may be sizeable in a model which has in addition a real $Y = 0$ triplet [8]. In the previous studies the possible couplings of triplet charged Higgs to leptons were not considered (the model in [8] with real and complex triplets does not have tree-level couplings of charged Higgs with fermions). We study the effects of the non-vanishing triplet Yukawa couplings, which are essential for the neutrino mass generation.

We have organized this paper as follows. In Section 2 we briefly describe 2HDM and HTM models. In Section 3 we investigate the decay of the singly charged higgses in both models. The production processes, as well as the SM background, are studied in Section 4. Section 5 is devoted to the summary and conclusions.
2 The models

In this chapter we will define and summarize several basic features of the two above mentioned models. We will also discuss the existing experimental bounds on the parameters of these models.

The two Higgs doublet model (2HDM). The conventional framework to study the singly charged higgses has been the Standard Model with an extended scalar sector consisting of two Higgs isodoublets. Usually a discrete symmetry between the doublet Higgs fields \( \phi_1 \) and \( \phi_2 \) has been invoked in order to avoid the flavour changing neutral currents [1]. Depending on the symmetry, the resulting models have been called Model I and Model II [1]. The Higgs sector in Model II includes that of the Minimal Supersymmetric Standard Model (MSSM). In our calculations of the 2HDM, we will use the Model II.

The two Higgs doublets

\[
\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^0 \\ \phi_2^- \end{pmatrix},
\]

have hypercharges \( Y = 1 \) and \( -1 \), respectively. The VEV of \( \phi_1 \) gives masses to down-type quarks and leptons and \( \phi_2 \) gives masses to up-type quarks. A combination of the charged components of the doublets, \( \phi_1^+ \) and \( \phi_2^+ \), corresponds to the Goldstone modes \( G^\pm \) needed to have \( W^\pm \) massive. The other combination orthogonal to this forms a physical charged Higgs \( H^\pm \). Its couplings to leptons are given by

\[
\frac{m_L g}{2\sqrt{2}M_W} \tan \beta \overline{\nu_l}(1 + \gamma_5) l H^+, \tag{2}
\]

where \( \tan \beta \) is defined as

\[
\tan \beta = \frac{\langle \phi_2^0 \rangle}{\langle \phi_1^0 \rangle} = \frac{v_2}{v_1}. \tag{3}
\]

Correspondingly, the couplings of the charged Higgs with quarks in the 2HDM are given by

\[
L = \frac{g}{2\sqrt{2}M_W} (V_{ij} m_{U_i} \cot \beta \overline{u_i}(1 - \gamma_5) d_j + V_{ij} m_{D_j} \tan \beta \overline{u_i}(1 + \gamma_5) d_j) H^+. \tag{4}
\]
where \( V_{ij} \) are the CKM matrix elements.

The Higgs triplet model (HTM). The Higgs content of the HTM is more complicated than that of the 2HDM. The model contains at least one Higgs triplet with weak hypercharge \( Y = 2 \) which has lepton number violating couplings to leptons but does not couple to quarks. In addition to the triplet, the model should contain an ordinary SM doublet to create fermion masses. The minimal Higgs multiplet content of the triplet model is thus [2]

\[
\Delta = \begin{pmatrix} \delta^+ / \sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+ / \sqrt{2} \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \tag{5}
\]

Here the neutral components of the triplet and doublet multiplets can get VEV’s, denoted as \( \langle \delta^0 \rangle = w / \sqrt{2} \) and \( \langle \phi^0 \rangle = v / \sqrt{2} \).

In the 2HDM the electroweak \( \rho \)-parameter is equal to unity at the tree-level while the loop-level contributions slightly increase this value, but within the experimental limits. In contrast to this, in the triplet model the \( \rho \)-parameter is less than unity already at the tree-level due to the isotriplet contributions to the masses of electroweak bosons. This forces the VEV \( w \) of the triplet Higgs to be small compared with the VEV \( v \) of the doublet. We will assume \( w \lesssim 15 \) GeV. \(^5\) Our results are, however, not very sensitive to the actual value of \( w \).

The physical charged Higgs will be a mixture of the triplet and doublet charged components,

\[
\begin{pmatrix} H^+ \\ G^+ \end{pmatrix} = \begin{pmatrix} -\sin \theta_H & \cos \theta_H \\ \cos \theta_H & \sin \theta_H \end{pmatrix} \begin{pmatrix} \phi^+ \\ \delta^+ \end{pmatrix}, \tag{6}
\]

where

\[
\sin \theta_H = \frac{\sqrt{2} w}{\sqrt{v^2 + 2w^2}}, \quad \cos \theta_H = \frac{v}{\sqrt{v^2 + 2w^2}}. \tag{7}
\]

\(^5\) It corresponds to the 3-\( \sigma \) of the modern LEP bounds [5]. Besides this we would like to note that in [9] the value of the \( \rho \)-parameter was discussed in a model with an additional \( Y = 0 \) triplet. It was found that at one-loop level the \( \rho \)-parameters in the SM and the triplet model are experimentally indistinguishable.
The gauge interactions of the triplet Higgs are governed by the term

\[ L_{\delta}^{\text{kin}} = \text{Tr} \left\{ (D_\mu \Delta)^\dagger (D^\mu \Delta) \right\} \]  

with

\[ D_\mu \Delta = \partial_\mu \Delta + ig' B_\mu + ig \frac{2}{3} W_\mu^a [\tau^a, \Delta]. \]  

A characteristic feature of the models involving Higgs triplets is the existence of the \( WZH \)-coupling at tree-level. This is different from the SM or the 2HDM, where \( WZH \)-coupling does not exist at tree-level because of the isospin invariance [10]. The vertex can appear in these models at loop-level, but it has been found to be too small to be observed in the case of MSSM [11]. Nevertheless, in a general 2HDM, the vertex can be enhanced [12]. For large \( \tan \beta \) and large mass differences between \( H^+ \) and \( A \) the branching ratio \( H^\pm \to W^\pm Z \) can be \( O(10^{-2}) \). In this work we consider the parameter region which is consistent with the MSSM. Obviously, the coupling \( \gamma W^+ H^- \) is absent in both models at tree-level because of the electromagnetic gauge invariance.

The lepton-triplet Yukawa couplings are given by

\[ L = -ih_{ll'} \Psi_{lL}^T C \tau_2 \Delta \Psi_{l' L} + h.c., \]  

where \( C \) is the charge conjugation matrix and \( \Psi_{lL} \) denotes the left-handed lepton doublet with flavour \( l \). After symmetry breaking, the interaction (10) yields lepton number-violating Majorana masses \( m_{\nu_l} = h_{ll \nu} w \) for neutrinos. The experimental upper bounds for the neutrino masses are [13] \( m_{\nu_e} < 2.3 \) eV, \( m_{\nu_\mu} < 170 \) keV and \( m_{\nu_\tau} < 18.2 \) MeV. The values of \( h_{ll} \) and \( w \) which we will use in our calculations are consistent with these limits.

For the first and second generation there exist experimental constraints on the Yukawa couplings \( h_{ll'} \) [14]. Stringent constraints come from the non-observation of the decays \( \mu \to eee \) and \( \mu \to e\gamma \):

\[ h_{e\mu} h_{ee} < 3.2 \times 10^{-11} \text{GeV}^{-2} M_{\delta^+}^2, \]
\[ h_{e\mu} h_{\mu\mu} < 2 \times 10^{-10} \text{GeV}^{-2} M_{\delta^+}^2. \]  

From Bhabha scattering follows an upper limit for \( h_{ee} \):

\[ h_{ee}^2 < 9.7 \times 10^{-6} \text{GeV}^{-2} M_{\delta^+}^2. \]  

6
and from \((g - 2)_{\mu}\) an upper limit for \(h_{\mu\mu}\):

\[
h_{\mu\mu}^2 \lesssim 2.5 \times 10^{-5} \text{ GeV}^{-2} M_{\delta^{++}}^2. \tag{13}
\]

From muonium-antimuonium transition one finds the following bound:

\[
h_{\mu\mu} \lesssim 5.8 \times 10^{-5} \text{ GeV}^{-2} M_{\delta^{++}}^2. \tag{14}
\]

For the third generation Yukawa couplings \(h_{\tau\tau}, h_{\tau e}\) and \(h_{\tau\mu}\) there are no limits so far.

We will assume that the Yukawa couplings \(h_{ll'}\) are diagonal. One deduces from the above constraints that all values of the diagonal coupling constants below the perturbation limit of \(h_{ll} \lesssim 1\) are allowed if the mass of the doubly charged Higgs exceeds 300 GeV.

The interactions of \(H^+\) with the leptons are given by

\[
\frac{1}{2} \bar{\nu_l} \left[ h_{ll} \cos \theta_H (1 - \gamma_5) - \sin \theta_H \frac{\sqrt{2} m_l}{v} (1 + \gamma_5) \right] lH^+ + h.c. \tag{15}
\]

The latter term, which originates from the Yukawa couplings of the isodoublet, is suppressed by both the small value of \(\sin \theta_H\) and the ratio \(m_l/v\). It will be neglected in the following.

The triplet Higgs \(\delta^+\) has no couplings to quarks due to conservation of weak hypercharge. Nevertheless, the physical charged Higgs \(H^+\) does have couplings to quarks because of the doublet-triplet mixing (6), but they are suppressed by the small mixing angle \(\sin \theta_H\).

The classical scalar potential of the model (see Eq. (2) of ref. [2]) contains among other terms a dimension-3 coupling:

\[
\phi^T \tau_2 \Delta^\dagger \phi + h.c., \tag{16}
\]

which breaks lepton number explicitly. As discussed in the introduction, this term is necessary to avoid the existence of majorons. The Higgs potential implies the following relations of various scalar masses:
Fig. 1. Branching ratios of $H^+$ in the HTM, with $h_{\tau\tau} = 1, 0.1$ and 0.

\[
M_{\delta^{++}}^2 = M_{H^+}^2 - \frac{\lambda_5}{2} v^2 \\
\frac{1}{2} M_{H_{im}^0}^2 = M_{H^+}^2 + \frac{\lambda_5}{2} v^2
\]

(17)

(18)

where $H_{im}^0$ is the physical combination of $\text{Im}(\delta^0)$ and $\text{Im}(\phi^0)$ and $\lambda_5$ is a coupling constant appearing in the scalar potential. One can conclude from these relations that the doubly charged and singly charged higgses are quite degenerate in mass, except when the coupling constant $\lambda_5$ happens to be large.

3 Decay modes of $H^\pm$

Signatures of charged higgses produced are determined by their decay modes. Depending on the unknown parameters, like scalar masses, $\tan\beta$ and the triplet Higgs Yukawa couplings $h_{ll'}$, the decay modes and their branching ratios may be quite different in the 2HDM and HTM. In both models there exist
the following two-body decay channels:

\[ H^+ \rightarrow l\nu_l, \, u_i\bar{d}_i, \, W^+H^0, \, W^+Z^0. \]  

(19)

The branching ratios in the HTM are presented in Fig.1 for two values of the third generation Majorana Yukawa coupling \((h_{\tau\tau} = 1 \text{ and } 0.1)\). We remind that this Yukawa coupling is not restricted by the present experiments. We have assumed here that the couplings \(h_{ee}\) and \(h_{\mu\mu}\) are small compared with \(h_{\tau\tau}\). Below the boson pair threshold the dominant decay channel of the charged Higgs is \(H^+ \rightarrow \tau^+\nu_\tau\). The other important channel \(H^+ \rightarrow c\bar{s}\) is suppressed by the factor \(\sin\theta_H\) as quarks do not couple to the isotriplet component of \(H^+\) but only with the isodoublet component. If the Yukawa coupling \(h_{\tau\tau} \lesssim 10^{-3}\), the mode \(H^+ \rightarrow c\bar{s}\) would have a comparable or larger branching ratio than the mode \(H^+ \rightarrow \tau^+\nu_\tau\). For the values \(h_{\tau\tau} = 0.1 - 1\) the \(c\bar{s}\)-channel is negligible. The same is true if one has \(h_{ee}\) or \(h_{\mu\mu} = 0.1 - 1\), which are consistent with the bounds (12), (13) and (14) when \(m_{H^\pm} \gtrsim 300\) GeV.

When the mass of \(H^+\) is above the \(H^0W\) threshold (where \(H^0\) is either the pseudoscalar neutral Higgs or the real part of the neutral triplet \(\delta^0\)), the decay mode \(H^+ \rightarrow W^+H^0\) quickly takes over the \(\tau\nu_\tau\)-channel and becomes the dominant channel. The \(W\) boson produced decays into a pair of quarks or a lepton and a neutrino, and the neutral Higgs, if it is \(\delta^0_{re}\), decays most of the time invisibly into neutrinos. The signal of this decay mode will therefore be a charged lepton plus missing energy, just like in the direct leptonic decay \(H^+ \rightarrow l^+\nu_l\), or a pair of quarks with invariant mass \(M_W\) plus missing energy. There exists also the decay channel to \(W^+\phi^0_{re}\), where \(\phi^0_{re}\) is the real part of the neutral doublet Higgs (this is the one which resembles the SM Higgs), but it is less important because the relevant vertex is suppressed by the factor \(\sin\theta_H\). (It is ignored in the Fig.1.) The interactions \(H^+W^-Z^0\) and \(H^+q\bar{q}\) are also suppressed by the doublet-triplet mixing angle \(\sin\theta_H\), and the corresponding branching ratios are therefore too small to be visible in Fig. 1.

We depict in Fig. 1 the branching ratios also when all the Majorana Yukawa couplings \(h_{ll'}\) are set to zero. This changes the decay pattern dramatically for small values of \(M_{H^+}\). In this case the mode \(\tau\nu\) is absent and the channel to \(c\bar{s}\) is the dominant one. For a heavier \(H^+\) also the decays to \(W^+Z^0\) and \(t\bar{b}\) can have significant branching ratios depending on the value of \(M_{\delta^0}\). The chan-
nel $H^+ \rightarrow \delta_0^c W^+$ will also in this case be the dominant one, whenever it is kinematically allowed. The branching ratio of the relatively rare decay $W^+ H^0$ is not depicted in Fig. 1 for this case of vanishing Majorana Yukawa couplings.

In the case of vanishing Majorana Yukawa couplings, the total width of $H^\pm$ is below the $W \delta_0^c$ threshold proportional to $\sin^2 \theta_H$. Consequently, if the doublet-triplet mixing is small enough, $H^+$ would leave the detector before it decays. This kind of semi-stable charged particle would give a clear signature.

The branching ratios for the case of 2HDM, in the framework of the MSSM, have been extensively studied e.g. in ref. [15] It is found that for a low $M_{H^+}$, the decays to $c \bar{s}$ or $\tau \nu_{\tau}$ are the most important ones. Also the decays $H^+ \rightarrow A^0 W^+, h^0 W^+$, with virtual Higgs bosons, may have sizeable branching ratios in the case of a light $H^+$ and a small $\tan \beta$. The channel $H^+ \rightarrow t \bar{t}$ is always the dominant channel when $M_{H^+}$ exceeds the appropriate threshold. Actually this channel, with a virtual $t$, may have a substantial branching ratio already below the threshold, in particular in the case of small $\tan \beta$. The subdominant mode is $H^+ \rightarrow \tau \nu_\tau$, but its branching ratio is much smaller, in particular for small values of $\tan \beta$, and the $H^+$ decays almost exclusively to the $t \bar{b}$ pair. In the presence of SUSY, new decay modes may open, providing that there exist light enough SUSY particles. For example, the two doublet case is realized in a SUSY model, e.g. in the MSSM, the decays of $H^+$ into lightest neutralinos and charginos might be kinematically allowed for large values of $M_{H^+}$. This would drastically suppress the branching ratio of the $H^+ \rightarrow t \bar{b}$ channel [16].

To summarise, in the large mass range and if at least one of the Yukawa couplings is not extremely small, a charged Higgs of the HTM and that of the 2HDM can be easily distinguished from each other through their decay characteristics, the final state being $l^+ +$ missing energy in the former case and $t \bar{t}$ in the latter case. In the low mass range the situation is less clear, the dominant channel in both cases being to $\tau \nu_\tau$, if we assume that $h_{\tau \tau}$ is the largest Majorana Yukawa coupling. The absence of both the $\tau \nu_\tau$ and $c \bar{s}$ decay modes would be a favourable indication towards the HTM and would imply large $h_{ee}$ or $h_{\mu \mu}$ couplings. In the two doublet case there exists a sizeable $c \bar{s}$ component, which the HTM in general misses if at least one Majorana Yukawa
coupling is of the order 0.1-1.

4 Production of $H^\pm$

Our main emphasis in the following will be placed on the single production of the charged Higgs, but let us start with a comment concerning the pair production $e^+e^- \rightarrow H^+H^-$. 

In the 2HDM the pair production occurs through a $\gamma$ and $Z^0$ exchange in s-channel. The unpolarized cross section is given e.g. in ref. [17]. In the HTM there is an additional amplitude, the t-channel neutrino exchange. In Fig. 2 we present the differential cross sections of the process in the HTM for the parameter values $M_H = 300$ GeV and $h_{ee} = 0, 0.1$ and 1. The t-channel neutrino diagram increases the pair production cross section by several orders of magnitude if the Yukawa coupling $h_{ee}$ has a value close to 1. It also changes the angular distribution from the $\sin^2 \theta$ form obtained in the 2HDM to a forward peaked one. As follows from the structure of the coupling (15), the $t$-channel diagram can be turned off if the electron beam has right-handed polarization. This provides a method to test the existence and the magnitude of the neutrino exchange amplitude.

Let us proceed to the single production of $H^\pm$. Of course, this production channel opens below the pair production threshold and could therefore be the first place to discover a charged Higgs particle. A two-body associate production occurs through the channel

$$e^+e^- \rightarrow H^+W^-.$$ 

(20)

This process is mediated by the t-channel neutrino exchange and s-channel $Z$-exchange. In the triplet model the cross section for this process is, however, very small. This is so because the neutrino-exchange diagram is proportional to the electron neutrino mass, which is constrained to be less than a few eV’s. The cross section remains very small for $\sqrt{s} = 0.5 - 2$ TeV. The $Z$-exchange diagram, in turn, has the $\sin \theta_H$-mixing angle suppression from the $WZH$-vertex.

On the other hand, if the triplet Yukawa coupling is nonvanishing, one may
Fig. 2. Angular distribution for \( \sqrt{s} \), \( h_{ee} = 1 \) (solid) 0.1 (dashed) 0 (dotted) and energy dependence of the cross section \( h_{ee} = 1 \) (solid) 0.1 (dot-dashed) in HTM, \( m_{H^+} = 300 \text{ GeV} \).

We expect larger cross sections for the leptonic three-body reactions

\[
e^+ e^- \rightarrow l^- \nu_l H^+, \; l = e, \mu, \tau. \tag{21}
\]

We have evaluated the cross sections for the processes (21) in the HTM and 2HDM. The relevant Feynman rules for both models were generated with the
Fig. 3. Feynman diagrams for the process $e^+e^- \rightarrow e^-\nu_e H^+$ in the Higgs triplet model.

LanHEP package [18] (the Feynman rules for the 2HDM can also be found in [1]). The symbolic and numerical calculations of the cross sections were carried out using the CompHEP package [19].

In the 2HDM, the value of the cross section depends on two unknown independent parameters, $M_{H^\pm}$ and $\tan \beta$. In the HTM, the unknown model parameters are the three Majorana Yukawa couplings $h_{ll}, l = e, \mu, \tau$, the VEV $w$ of the neutral triplet scalar and the masses of the Higgs bosons. For definiteness, the masses of all the neutral and the doubly charged scalars are taken to be 300 GeV.

Feynman diagrams for the reaction $e^+e^- \rightarrow e^-\nu_e H^+$ in the HTM are presented in Fig. 3. The cross section of this process is plotted in Fig. 4 as a function of centre-of-mass energy $\sqrt{s}$ for the triplet Yukawa couplings $h_{ll} = 0, 0.1, 1$ $(l = e, \mu, \tau)$ and $M_{H^\pm} = 300$ GeV and 500 GeV. As can be seen, for large Yukawa couplings $h_{ll}$ cross sections at a few picobarn-level are possible. In the limit where all the triplet Yukawa couplings vanish only two Feynman amplitudes survive, which both contain a $WZH$ vertex and are thus suppressed by the
Fig. 4. The cross section for the $e^+e^- \rightarrow e^-\nu_e H^+$ as a function of center of mass energy for (a) $M_{H^+} = 300$ GeV and (b) $M_{H^+} = 500$ GeV in the HTM. The triplet Yukawa couplings are $h_{ll} = 0$ (dotted), $h_{ll} = 0.1$ (dashed) and $h_{ll} = 1$ (solid), $l = e, \mu, \tau$.

Therefore the corresponding cross sections, plotted by dotted lines in Fig. 4, are relatively small.

The cross section for the process $e^+e^- \rightarrow e^-\nu_e H^+$ is in the 2HDM always very small. The reason is that the $H^+e^-\nu_e$ Yukawa coupling is in this model suppressed by the factor $m_e/M_W$, as can be seen from the expression (2). Clearly, this process allows for separation between the two models: if one measures a large cross sections for this reaction, it could be a signal of the existence of triplet Higgs representations with a large Yukawa coupling $h_{ee}$.

Of course, it is quite naturally to expect the hierarchy in Yukawa couplings $h_{ee} < h_{\mu\mu} < h_{\tau\tau}$. In this case, when $h_{ee}$ is too small to observe the above mentioned reaction, the study of $\mu^+\mu^- \rightarrow \mu^-\nu\mu H^+$ may be quite useful. For
Fig. 5. Feynman diagrams for the process $e^+e^- \rightarrow \tau^-\nu_\tau H^+$ in the triplet model.

In this reaction the results depicted in Fig. 4 remain unchanged if $h_{ee} = 0$.

Feynman diagrams of the process (21) with $l = \tau$ are shown in the Fig. 5 in the HTM. All the amplitudes are proportional to the triplet Yukawa couplings except the one with a virtual $Z$ boson decaying via the $WZH$ coupling. The total cross sections of this process are plotted in Fig. 6 (a)-(d) as a function of centre-of-mass energy $\sqrt{s}$. In Fig. 6 (a) and (b) all the Yukawa couplings are taken to be equal whereas in Fig. 6 (c) and (d) only the third generation coupling $h_{\tau\tau}$ differs from zero. In the latter case the last diagram of Fig. 5 does not contribute, since it is proportional to the first generation triplet Yukawa coupling $h_{ee}$. The solid and dashed lines in these plots demonstrate the effect of the triplet Yukawa couplings. For simplicity, the triplet VEV $w$ is set to zero in evaluating these curves. The dotted lines show the cross section in the case when all the triplet Yukawa couplings are set to zero and the triplet VEV has a nonzero value $w = 15$ GeV. In this case there is only one diagram which contributes to the cross section, the one involving an $s$-channel $Z$ and a $WZH$ vertex. The $\sin \theta_H$ suppression of the vertex makes the cross section negligible.

Comparing the Fig. 6 (a),(c) and 6 (b),(d) one can see the large effect of the $t$-channel neutrino exchange diagram that involves the coupling $h_{ee}$. If this coupling is $h_{ee} \simeq 1$, the cross section is about three orders of magnitude larger than the cross section for $h_{ee} = 0$.

For the case of nonvanishing Yukawa couplings, the cross section can be in the region $\sqrt{s} > 2M_{H^\pm}$ well approximated with the real $H^\pm$ pair production cross section multiplied by the branching ratio of the decay $H^+ \rightarrow \tau\nu$. In the region $\sqrt{s} < 2M_{H^\pm}$ the $H^\pm$-pair production is not kinematically allowed. The single $H^\pm$ production may in this case still be allowed. This energy range is therefore of particular interest. As seen from the Fig. 6 (a) the cross section
Fig. 6. The cross section for the $e^+e^- \rightarrow \tau^- \nu_\tau H^+$ as a function of center of mass energy for (a), (c) $M_{H^+} = 300$ GeV and (b), (d) $M_{H^+} = 500$ GeV in HTM. In (a) and (b) the triplet Yukawa couplings are taken to be equal for all three generations: $h_\ell \ell = 1$ (solid), $h_\ell \ell = 0.1$ (dashed) and $h_\ell \ell = 0$ (dotted). In (c) and (d) only the third generation triplet Yukawa coupling is nonzero: $h_\tau \tau = 1$ (solid), $h_\tau \tau = 0.1$ (dashed) and $h_\tau \tau = 0$ (dotted).

In this region may reach a few hundred femtobarn level for favourable model parameter values.
Fig. 7. The cross section for the $e^+e^- \rightarrow \tau^- \nu_{\tau}H^+$ as a function of center of mass energy for $M_{H^+} = 300$ GeV (a) and $M_{H^+} = 500$ GeV (b) in 2HDM. The solid line corresponds to $\tan \beta = 50$ and dashed line corresponds to $\tan \beta = 3$.

The cross sections for the reaction (21) with $l = \tau$ in the 2HDM are depicted in the Fig. 7 for $\tan \beta = 3$ and 50. The cross section is larger for larger $\tan \beta$ and is at most a few femtobarns for the choice $\tan \beta = 50$. For $m_{H^+} = 300$ GeV the threshold effects of the $H^+H^-$ pair production can be be seen around $\sqrt{s} = 600$ GeV. The cross sections for the production of muon and muon neutrino can be obtained from the $\tau\nu_{\tau}$ cross section simply by multiplying by $(m_\mu/m_\tau)^2$. 

17
The cross section for the $\mu^+ e^- \rightarrow \mu^+ \nu_e H^-$ as a function of center of mass energy for (a) $M_{H^+} = 300$ GeV and (b) $M_{H^+} = 500$ GeV in HTM. The triplet Yukawa couplings are $h_{ll} = 1$ (solid), $h_{ll} = 0.1$ (dashed) and $h_{ll} = 0$ (dotted).

We have also considered the production of a single $H^\pm$ in the reaction

$$\mu^+ e^- \rightarrow \mu^+ \nu_e H^-$$

in the $e^- \mu^+$ collider. For this initial state, the pair production of real $H^\pm$'s is not possible. The cross sections for this process in the HTM are depicted in the Fig. 8 (a) and (b). The cross sections are at most a few hundred femtobarns for large Yukawa couplings.
Fig. 9. Angular distributions of the final state $\tau$'s for the $e^+ e^- \rightarrow \tau^- \nu \tau$ in the triplet Higgs model for (a) $M_{H^+} = 300$ GeV and (b) $M_{H^+} = 500$ GeV. Solid lines correspond to $h_{ll} = 1$ and dashed lines to $h_{ll} = 0.1$.

It is in principle possible to make distinction between the HTM and 2HDM by looking at the angular distribution of the outgoing lepton. In the 2HDM, the distribution is symmetric and resembles the $\sin^2 \theta$-form of the pair production differential cross section, whereas in the HTM the peak of the distribution is shifted forward. The shift is significant only if the first generation Majorana Yukawa coupling is large, since in this case there are large t-channel neutrino exchange diagrams which contribute to the process altering the form of the distribution. This is illustrated in the Fig. 9, which shows the angular distributions of the charged lepton in the HTM. Clearly a deviation from the $\sin^2 \theta$ distribution, after the SM background has been removed, would be a signal of triplet higgses. The magnitude of this effect, as discussed, depends on the model parameters.

Let us now discuss the SM background to the process (21). The main background comes from the $W$-pair production $e^+ e^- \rightarrow W^- W^+$ with subsequent decays $W^\pm \rightarrow \tau^\pm \nu$. The cross section for the W-pair production is about 7.5 pb at $\sqrt{s} = 500$ GeV. Multiplying by $\text{Br}(W \rightarrow \tau \nu)^2 \approx 0.013$ gives the value
98 fb for the \(e^+e^- \rightarrow \tau^-\tau^+\nu\nu\) cross section.

In this SM background process the final state leptons are preferentially emitted in the direction of the beam axis, whereas in the signal process (21) the angular distribution of the leptons vanishes in the beam directions. Thus, this background can be substantially reduced by requiring the scattered leptons to be away from the direction of the beam.

There is also background from the pair production of Z’s with one Z decaying into a pair of charged leptons and the other into a neutrino pair. This background can be reduced by requiring the invariant mass of the charged lepton pair to be different from \(M_Z\).

5 Conclusions

In conclusion, we have investigated the production of singly charged Higgs boson \(H^\pm\) at future linear collider in the framework of two models, the triplet Higgs model (HTM) and two-Higgs-doublet model (2HDM). The aim has been to find out signatures that would allow one to make distinction between the models.

The single production of \(H^\pm\) in the process \(e^+e^- \rightarrow l^-\nu H^+\) can be useful in exploring the scalar sector if the charged Higgs is so heavy that it cannot be pair produced. In the HTM, the cross sections depend on the unknown Majorana Yukawa couplings and the mass \(M_{H^\pm}\) of the charged Higgs. In the supersymmetry-motivated 2HDM, the free parameters are \(M_{H^\pm}\) and \(\tan \beta\). The process \(e^-e^- \rightarrow e^-\nu_e H^+\) is the most promising one for separating the two models. In the HTM, the cross section for this reaction can be of the order of 1 pb for \(100 < M_{H^\pm} < 300\) GeV. In the case when \(h_{ee} = 0\) the same results may be obtained for the reaction \(\mu^+\mu^- \rightarrow \mu^-\nu_\mu H^+\) at muonic collider. The reaction \(e^-e^- \rightarrow \tau^-\nu_\tau H^+\) is also well observable and provides the separation between HTM and 2HDM through angular distribution of final \(\tau\)’s and decay modes of \(H^+\).

In 2HDM, on the other hand, the couplings of fermions with the higgses are always proportional to the ratio \((m_f/M_W)\). This mass hierarchy between Yukawa couplings doesn’t have to hold in the HTM and its nonexistence can be used
to differentiate between the two models, since light fermions can have large couplings with the charged Higgs boson. In our calculations we have reduced the number of independent parameters in the 2HDM by assuming the MSSM supersymmetric relations among the Higgs potential parameters. The only free parameters in our calculations are then the charged Higgs mass $m_{H^\pm}$ and $\tan \beta$.

The signatures of the processes at the detector are determined by the decays of the charged Higgs. In the 2HDM, the dominant decay channel for the charged Higgs is $H^+ \to b\bar{t}$ above the threshold. In the HTM this channel is suppressed by the ratio of triplet and doublet vevs. The dominant decay channels in the HTM are likely to be $H^+ \to l\nu$ or, when kinematically allowed, $H^+ \to H^0 W^+$. Below the $b\bar{t}$ threshold the dominant decay mode in the 2HDM is $H^+ \to c\bar{s}$, while in the HTM the dominant channel is $H^+ \to l^+\nu_l$ provided the appropriate Yukawa coupling $h_{ll} \geq 0.1$.

Acknowledgments

One of us (N.R.) is grateful to the Theoretical Physics Division at the Department of Physics of Helsinki University for warm hospitality. It is also a great pleasure to thank Alexander Pukhov for helpful instructions for using the CompHEP package. This work has been supported also by the Academy of Finland under the contract 40677 and project number 163394, and by RFFI grant 98-02-18137.

References

[1] For a review, see J.F. Gunion, H.E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter’s Guide* (Addison-Wesley, 1990).

[2] R. Godbole, B. Mukhopadhyaya, and M. Nowakowski, Phys. Lett. **B352** (1995) 388; D.K. Ghosh, R.M. Godbole, B. Mukhopadhyaya, Phys. Rev. **D55** (1997) 3150.
[3] Y. Fukuda et al., Phys. Rev. Lett. **81** (1998)1562, ibid **82** (1999) 2644.

[4] G.B. Gelmini and M. Roncadelli, Phys. Lett. **B99** (1981) 411.

[5] The LEP Collaborations, Phys. Lett. **276B** (1992) 247.

[6] J.C. Pati and A. Salam, Phys. Rev. **D10** (1974) 275; R.N. Mohapatra and J.C. Pati, Phys. Rev. **D11** (1975) 566, 2558; G. Senjanovic and R.N. Mohapatra, Phys. Rev. **D12** (1975) 1502; R.N. Mohapatra and R.E. Marshak, Phys. Lett. **91B** (1980) 222.

[7] K. Cheung, R.J. Phillips and A. Pilaftsis, Phys. Rev. **D51** (1995) 4731.

[8] H. Georgi and M. Machacek, Nucl. Phys. **262** (1985) 463; S. Chanowitz, M. Golden, Phys. Lett. **B165** (1985) 105.

[9] T. Blank, W. Hollik, Nucl.Phys. **B514** (1998) 113.

[10] J.A. Grifols, A. Mendes, Phys.Rev. **D22** (1980) 1725.

[11] A. Mendez, A. Pomarol, Nucl.Phys. **B349** (1991) 369; M. Capdequi Peyranère, H. E. Haber, P. Irulegui, Phys. Rev. **D44** (1991) 191.

[12] S. Kanemura, hep-ph/9710237.

[13] C. K. Jung, in Proceedings of the EPS HEP '99 Conference, July 15-21, 1999, Tampere, Finland, eds. K. Huitu, H. Kurki-Suonio, J. Maalampi (IOP publishing 2000).

[14] M.L. Swartz, Phys.Rev. **D40** (1989) 1521; M. Lusignoli, S. Petrarca, Phys.Lett. **B226** (1989) 397; R. Mohapatra, Phys.Rev. **D46** (1992) 2990.

[15] A. Djouadi et al., hep-ph/0002258 (2000).

[16] F. Borzumati and A. Djouadi, hep-ph/9806301.

[17] S. Komamiya, Phys. Rev. **D38** (1988) 2158
[18] A. Semenov, hep-ph/9609488; A. Semenov, Nucl. Inst & Meth. \textbf{A393} (1997) 293.

[19] P. A. Baikov et al., Physical Results by means of CompHEP, in Proc. of X Workshop on High Energy Physics and Quantum Field Theory (QFTHEP-95), eds. B. Levchenko, V. Savrin, Moscow, 1996, p. 101, hep-ph/9701412; E. E. Boos, M. N. Dubinin, V. A. Ilyin, A. E. Pukhov, V. I. Savrin, hep-ph/9503280.