Geometric Phases of Electrons Due to the Spin-rotation Coupling in Rotating C\textsubscript{60} Molecules

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The rapidly rotational motion of C\textsubscript{60} molecules will provide us with an ingenious way to test Mashhoon’s spin-rotation coupling. Geometric phases arising from the time-dependent spin-rotation coupling of electrons in the rotating C\textsubscript{60} molecule is considered in the present Letter. It is shown that geometric phases of electrons in C\textsubscript{60} molecules may be measured through the photoelectron spectroscopy of C\textsubscript{60}. A physically interesting fact that the information about rotation and precession of C\textsubscript{60} molecules in the orientational ordered (or disordered) phase may be read off from the photoelectron spectroscopy of C\textsubscript{60} is also demonstrated.

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The information on the rotational dynamics of C\textsubscript{60} molecule in condensed phases have been obtained from nuclear-magnetic resonance (NMR) studies \cite{1}. Both NMR spectroscopy and quasielastic neutron scattering experiments \cite{2} indicated the rapid rotation of C\textsubscript{60} molecules whose rotational correlation time may be picoseconds in the orientationally disordered phase. Historically, many researches were in connection with the molecular-dynamics of C\textsubscript{60} rotation \cite{3}. Heiney \textit{et al.} found from x-ray diffraction and calorimetric measurements that solid C\textsubscript{60} exhibits a phase transition near 249 K from a simple cubic structure at low temperatures to a face-centered-cubic structure at high temperatures \cite{3–5}. These studies suggest that the phase above the transition temperature is characterized by free rotation or rotational diffusion and that the phase below the transition is characterized by jump rotational diffusion between symmetry-equivalent orientations. The correlation time for both phases (orientationally ordered phase below 249 K and orientationally disordered phase above 249 K) were measured to fit with an Arrhenius law, which leads to the fact that in the low-temperature phase, the correlation time \( \tau \) increases by a factor of \( \sim 40 \). For example, \( \tau \) varies from 0.44 to 0.29 ns as the temperature varies between 241 and 254 K \cite{4}. In order to investigate the dynamics of C\textsubscript{60} rotation, Johnson \textit{et al.} performed detailed measurements of the reorientational correlation time for solid C\textsubscript{60} over the temperature range 240 to 331 K and again showed that the correlation time satisfies the Arrhenius behavior \cite{4}. Cheng \textit{et al.} presented molecular-dynamics investigation of orientational freezing in pure solid C\textsubscript{60} based on a pairwise-additive atom-atom intermolecular potential \cite{3}. Kiefl \textit{et al.} reported the study of the molecular dynamics and electronic structure of \( \mu^+\text{-C}_{60} \) radical in a highly crystalline sample of pure C\textsubscript{60} and showed a signal at room temperature which is a \( \mu^+\text{-C}_{60} \) radical in crystalline sample C\textsubscript{60} undergoing a quasi-free rotation \cite{5}. Thus it follows that the molecular-dynamics of C\textsubscript{60} rotation is of great importance, since it is related close to the molecular thermal motion, phase transition and crystal structure of solid C\textsubscript{60}.

In this Letter we propose a new approach to the investigation of molecular-dynamics of C\textsubscript{60}. We suggest that both the spin-rotation coupling of electrons and the consequent geometric phases can provide us with an insight into the rotational dynamics, intermolecular interaction and thermal motion of C\textsubscript{60} molecules.

Basically speaking, the spin-rotation coupling considered here is one of the gravitational effects since the nature of the inertial force (e.g., the Coriolis force) is the gravitational force according to the principle of equivalence. Mashhoon showed that a particle with an intrinsic spin possesses a gravitomagnetic moment that can be coupled to the gravitomagnetic fields \cite{6}. By using the coordinate transformation of gravitomagnetic vector potentials in Kerr metric from the fixed frame of reference to the rotating frame \cite{7}, it is readily verified that the rotating frequency, \( \tilde{\omega} \), of a rotating frame relative to the inertial frame of reference can be regarded as a gravitomagnetic field \cite{7}. The Hamiltonian of spin-rotation coupling, \( H^{\text{spin-rot}} = \tilde{\omega} \cdot \vec{S} \), can be obtained \cite{6–9} by making use of the Dirac equation in the curved spacetime \cite{10}, where \( \vec{S} \) denotes the spin operator of a spin-\( \frac{1}{2} \) particle.

Fleming \textit{et al.} have considered the electron-spin relaxation of C\textsubscript{60} induced by the coupling of the electron spin to the molecular-rotational-angular momentum \cite{11}. Since in some certain cases the rotating frequency (e.g., \( 10^{11} \) rad/s in orientationally disordered phases \cite{4}) of C\textsubscript{60} molecule can be compared to the electron Larmor frequency in a...
magnetic field arising from molecule rotation, it is necessary to consider the gravitational analogue of Fleming’s case. Moreover, in some cases such as in the low-temperature phase, the precession of C_{60} molecules around cones is faster than the molecular rotation (see, for example, in the following evaluation) and therefore it is also essential to deal with the precessional motion of C_{60} molecules and the consequent geometric phases of electrons in molecules.

First we consider the geometric phase of electrons arising from the time-dependent spin-rotation coupling in the rotating C_{60} molecule which undergoes a precessional motion. Note that the gravitational Aharonov-Bohm effect (also termed Aharonov-Carni effect [12]) due to the Lorentz magnetic force (i.e., the Coriolis force) can be ignored in our following consideration, since the Berry’s phase, $2m\vec{ω} \cdot \vec{A}$ with $\vec{A}$ being the area vector surrounded by a closed path along which the electron moves, appearing in the gravitational Aharonov-Bohm effect is negligibly small compared with the geometric phase due to the interaction of electron spin with the rotating frame (spin-rotation coupling). In what follows the precession of angular momentum of rotating C_{60} molecules is taken into account. Since the interaction between C_{60} molecules in solid C_{60} is the Van der Waals potential, the coupling is relatively weak and therefore the magnitude of $|\vec{ω}|$ does not easily alter, except that it gives rise to the variations of the direction of $\vec{ω}$. By the aid of the above assumptions (e.g., $\vec{d} \cdot \vec{ω}^2 = 0$), the phenomenological equation for the rotational motion of the C_{60} molecule weakly coupled to its neighbors is derived as $\frac{\partial}{\partial t} L = \vec{B} \times \vec{L}$ with the effective gravitomagnetic field strength $\vec{B}$ (which leads to the precessional motion of C_{60} molecules) being $\vec{ω} \times \vec{ω}$, where dot denotes the derivative of $\vec{ω}$ with respect time $t$; the angular momentum $\vec{L}$ of the C_{60} molecule is defined to be $\vec{L} = \vec{I} \vec{ω}$ with the moment of inertia, $I$, being approximately $\frac{2}{3}ma^2$. Here $m$ and $a$ (0.355 Å) denote the mass and radius of C_{60} molecule, respectively. Note that here the rotational angular velocity $\vec{ω}$ of C_{60} molecules acted upon by intermolecular interactions is time-dependent due to the molecular thermal motion and thermal fluctuations in solid C_{60}. So, the Hamiltonian describing the coupling of spin of electrons (such as those in the delocalized $\pi$ bond which has the conjugation effect) to the time-dependent $\vec{ω}(t)$ is given $H^{s\rightarrow r}(t) = \vec{ω}(t) \cdot \vec{S}$ and the time-dependent Schrödinger equation governing this electron spin-rotation coupling is

$$H^{s\rightarrow r}(t) |\psi_\sigma(t)\rangle = i\hbar \frac{\partial}{\partial t} |\psi_\sigma(t)\rangle.$$

(1)

For convenience, we set $\vec{ω}(t) = \omega_0 [\sin(\theta(t)) \cos(\varphi(t)), \sin(\theta(t)) \sin(\varphi(t)), \cos(\theta(t))]$ where $\omega_0$ is time-independent and $\theta(t)$ and $\varphi(t)$ stand for the angle displacements in the spherical polar coordinate system. In order to exactly solve the time-dependent Schrödinger equation (1), in what follows use is made of the Lewis-Riesenfeld invariant theory [13] and the invariant-related unitary transformation formulation [14,15]. So, according to the Lewis-Riesenfeld theory [13] an invariant $I(t)$ that satisfies the Liouville-Von Neumann equation $\frac{\partial}{\partial t} I(t) + \frac{i}{\hbar} [I(t), H^{s\rightarrow r}(t)] = 0$ should be constructed in terms of the spin operator $S_\pm$ and $S_3$ with $S_\pm = S_1 \pm iS_2$. Thus the invariant $I(t)$ is of the form $I(t) = \frac{1}{2} \sin(\lambda(t)) \exp [-i\gamma(t)] S_+ + \frac{1}{2} \sin(\lambda(t)) \exp [i\gamma(t)] S_- + \cos(\lambda(t)) S_3$, where the time-dependent parameters, $\lambda(t)$ and $\gamma(t)$, are determined by the following two auxiliary equations

$$\dot{\lambda} = \omega_0 \sin(\theta(t) \cos(\varphi(t) - \gamma)), \quad \dot{\gamma} = \omega_0 [\cos(\theta(t) \sin(\varphi(t)) \cos(\varphi(t) - \gamma)]$$

(2)

with dot denoting the time derivative of $\lambda(t)$ and $\gamma(t)$. Note that insertion of expressions for $I(t)$ and $H^{s\rightarrow r}(t)$ into the Liouville-Von Neumann equation yields Eq.(2). In accordance with the Liouville-Von Neumann equation, the invariant $I(t)$ possesses time-independent eigenvalues, which enables us to obtain the exact solutions of the time-dependent Schrödinger equation (1), for it is readily verified that the particular solutions of (1) is different from the eigenstates of the invariant $I(t)$ only by a time-dependent $c$-number factor [13]. If once the eigenvalue equation of $I(t)$ is exactly solved, then the solutions of the time-dependent Schrödinger equation (1) is easily obtained. By utilizing a unitary transformation operator [14,16,17] $V(t) = \exp [\beta S_+ - \beta^* S_-]$ with $\beta(t) = -\frac{\lambda(t)}{\gamma(t)} \exp [i\gamma(t)]$, the time-dependent invariant $I(t)$ can be transformed into a time-independent one, i.e., $I_V(t) \equiv V(t)^\dagger I(t) V(t) = S_3$, where the time-independent eigenvalue of $S_3$ is $\sigma = \pm \frac{1}{2} \hbar$ for the electron.

Since $\frac{\partial}{\partial t} I_V = 0$, the Liouville-Von Neumann equation under this unitary transformation may be rewritten $[I_V, H_V^{s\rightarrow r}(t)] = 0$. It follows that $H_V^{s\rightarrow r}(t)$ depends only on the third component $S_3$ of the spin operator of the electron. Further calculation yields

$$H_V^{s\rightarrow r}(t) \equiv V(t)^\dagger [H^{s\rightarrow r}(t) - i\hbar \frac{\partial}{\partial t} V(t)] = \omega_0 [\cos(\lambda \cos(\theta(t) + \sin(\lambda \sin(\theta(t))) S_3 + \gamma(t) (1 - \cos(\lambda))) S_3].$$

(3)

This, therefore, means that the particular solution $|\psi_\sigma(t)\rangle_V$ of the time-dependent Schrödinger equation $H_V^{s\rightarrow r}(t) |\psi_\sigma(t)\rangle_V = i\hbar \frac{\partial}{\partial t} |\psi_\sigma(t)\rangle_V$ with $|\psi_\sigma(t)\rangle_V = V(t) |\psi_\sigma(t)\rangle$ is different from the eigenstate of $I_V$ only by a time-dependent $c$-number factor $\exp \left\{ \frac{i}{\hbar} \left[ \phi_\sigma^{(d)}(t) + \phi_\sigma^{(g)}(t) \right] \right\}$, where the nonadiabatic noncyclic dynamical phase is
\[ \phi^{(d)}_{\sigma}(t) = \sigma \int_{0}^{t} \omega_{0} \{ \cos \lambda(t') \cos \theta(t') + \sin \lambda(t') \sin \theta(t') \cos [\gamma(t') - \varphi(t')] \} \, dt' \]  

and the nonadiabatic noncyclic geometric phase is

\[ \phi^{(g)}(t) = \sigma \int_{0}^{t} \{ \dot{\gamma}(t') [1 - \cos \lambda(t')] \} \, dt' . \]  

If the adiabatic cyclic evolution process is taken into consideration, then it is verified that the present calculation is self-consistent. This may be illustrated as follows: in the adiabatic cyclic case, where the parameters \( \lambda = \theta = \text{const}., \) \( \gamma = \varphi, \) and the precessional frequency \( \dot{\gamma} = \Omega \) is very small, the nonadiabatic geometric phase (5) in one cycle \( (T = \frac{2\pi}{\dot{\gamma}}) \) is reduced to the expression for Berry’s adiabatic cyclic topological phase \( \phi^{(g)}(T) = 2\pi \sigma (1 - \cos \theta) \) \[18\], where \( 2\pi (1 - \cos \theta) \) is a solid angle over the parameter space of the C\(_{60}\) rotational frequency \( \omega(t) \). This fact implies that geometric phases possess the topological and global properties of time evolution in time-dependent quantum systems \[16,18\].

If the eigenstate of \( I_{V} \) corresponding to the eigenvalue \( \sigma \) is \( |\sigma \rangle \), then the eigenstate of the invariant \( I(t) = V(t) |\sigma \rangle \).

Hence, in accordance with the Lewis-Riesenfeld invariant theory, the particular solution corresponding to \( \sigma \) of Eq.(1) reads

\[ |\psi_{\sigma}(t)\rangle = \exp \left( \frac{1}{i\hbar} \left[ \phi^{(d)}(t) + \phi^{(g)}(t) \right] \right) V(t) |\sigma \rangle . \]  

The influence of spin-rotation coupling and geometric phases discussed above on the photoelectron spectroscopy in the C\(_{60}\) molecule deserves considerations. The total Hamiltonian of an electron in the C\(_{60}\) molecule is altered by the external perturbation (e.g., the radiation fields) can be written as \( H = H_{0} + H^{\text{ext}}(t) + H^{'}(t) \), where \( H_{0} \) represents the Hamiltonian of the electron in the C\(_{60}\) molecule when no spin-rotation coupling and external perturbation exist, and \( H^{'}(t) \) describes the external perturbation acting on the electrons. Since \( H_{0} \) is assumed to be time-independent, the eigenvalue equation of \( H_{0} \) may be written \( H_{0} \exp \left[ \frac{i}{\hbar} \epsilon_{n,\sigma} t \right] |\phi_{n,\sigma} \rangle = \epsilon_{n,\sigma} |\phi_{n,\sigma} \rangle \). The initial state \( |\psi_{n,\sigma}(t = 0)\rangle \) of Eq.(1) may be taken to be \( |\phi_{n,\sigma} \rangle \). Thus, the particular solution of the time-dependent equation \( [H_{0} + H^{\text{ext}}(t)] |\Phi_{n,\sigma}(t)\rangle = i\hbar \frac{d}{dt} |\Phi_{n,\sigma}(t)\rangle \) is given \( |\Phi_{n,\sigma}(t)\rangle = \exp \left\{ \frac{i}{\hbar} \left[ \phi_{n,\sigma}(t) + \epsilon_{n,\sigma} t \right] \right\} V(t) |\phi_{n,\sigma} \rangle \) with \( \phi_{n,\sigma}(t) = \phi^{(d)}(t) + \phi^{(g)}(t) \). Note, however, that here the magnetic interactions such as spin-orbit and spin-spin couplings in \( H_{0} \) have been combined into \( H^{\text{ext}}(t) \) (i.e., this leads to the commutation relation \([H_{0}, H^{\text{ext}}(t)] = 0\) in order that the exact particular solution can be obtained conveniently via the above unitary transformation method. The solution of the time-dependent Schrödinger equation \([H_{0} + H^{\text{ext}}(t) + H^{'}(t)] |\Psi(t)\rangle = i\hbar \frac{d}{dt} |\Psi(t)\rangle \) associated with the total Hamiltonian is assumed to be \( |\Phi(t)\rangle = \sum_{n,\sigma} a_{n,\sigma}(t) |\Phi_{n,\sigma}(t)\rangle \). Substitution of this expression into this time-dependent Schrödinger equation yields \( i\hbar \frac{d}{dt} a_{m,\sigma}(t) = \sum_{n,\sigma} a_{n,\sigma}(t) \langle \Phi_{m,\sigma}(t) | H^{'}(t) |\Phi_{n,\sigma}(t)\rangle \). Further calculation shows that

\[ \langle \Phi_{m,\sigma}(t) | H^{'}(t) |\Phi_{n,\sigma}(t)\rangle = \exp \left[ \frac{1}{i\hbar} \phi_{\text{tot}}(m\sigma', n\sigma; t) H_{m,\sigma', n\sigma}^{'}(t) \right] . \]  

where \( H_{m,\sigma', n\sigma}^{'}(t) = \langle \Phi_{m,\sigma'} | V^\dagger(t) H'(t) V(t) |\Phi_{n,\sigma} \rangle \) and \( \phi_{\text{tot}}(m\sigma', n\sigma; t) = [\phi_{\sigma}(t) + \epsilon_{n,\sigma} t] - [\phi_{\sigma}(t) + \epsilon_{m,\sigma} t] \).

If the initial state is \( |\Phi_{k,\sigma}\rangle \), i.e., \( a_{k,\sigma}(t = 0) = 1 \), then \( a_{m,\sigma}(t) = \frac{1}{i\hbar} \int_{0}^{t} H_{m,\sigma', k\sigma}^{'}(t') \exp \left[ \frac{1}{i\hbar} \phi_{\text{tot}}(m\sigma', k\sigma; t') \right] \, dt' \), and the transition probability from the state \( |\Phi_{k,\sigma}\rangle \) to \( |\Phi_{m,\sigma'}(t)\rangle \) is \( W_{k\sigma \rightarrow m\sigma'} = a_{m,\sigma'}^{*}(t) a_{m,\sigma'}(t) \).

Note that the exact solutions of the auxiliary equations (2) are often of complicated form. As an illustrative example, here we consider a physically interesting solution to Eq.(2), which stands for a typical case where the precessional frequency \( \dot{\phi} \) is constant (denoted by \( \dot{\Omega} \)) and the nutational frequency \( \dot{\theta} \) vanishing. The explicit expression for this simple solution is \( \gamma(t) = \varphi(t) = \dot{\Omega} t, \) \( \dot{\lambda} = \dot{\theta} = 0 \), where the time-independent \( \Omega = \frac{\omega_{0} \sin(\lambda - \theta)}{\sin \lambda} \). Here the C\(_{60}\) molecule precesses at an angular velocity \( \Omega \) about the z-axis (\( \omega \) deviates from z-axis by a constant angle \( \theta \)). Thus it follows that in this case the nonadiabatic noncyclic dynamical phase and geometric phase of the electron are proportional to time \( t \), i.e., \( \phi^{(g)}(t) = [\omega_{0} \sigma \cos(\lambda - \theta)] t \) and \( \phi^{(d)}(t) = \Omega [1 - \cos \lambda] t \), respectively, and the total phase difference \( \phi_{\text{tot}}(m\sigma', k\sigma; t) \) in transition matrix element \( \langle \Phi_{m,\sigma'}(t') | H'(t) |\Phi_{k,\sigma}(t)\rangle \) is therefore of the form

\[ \phi_{\text{tot}}(m\sigma', k\sigma; t) = \{ (\sigma - \sigma') [\omega_{0} \cos(\lambda - \theta) + \Omega (1 - \cos \lambda)] + (\epsilon_{k,\sigma} - \epsilon_{m,\sigma'}) \} t . \]  

It follows from the expression (8) that the photon energy absorbed or emitted in the transition process from \( |\Phi_{k,\sigma}(t)\rangle \) to \( |\Phi_{m,\sigma'}(t)\rangle \) is shifted by \( (\sigma - \sigma') [\omega_{0} \cos(\lambda - \theta) + \Omega (1 - \cos \lambda)] \) due to the electron spin-rotation coupling and the consequent geometric-phase effect. Apparently, the energy shifted by the dynamical phase and geometric
photoelectron spectroscopy, which enables physicists to investigate the rotational dynamics and phase transition of the spin-rotation geometric phases of electrons in C$_{60}$ itself appears to possess rich physical significance and should therefore be considered in more detail.

reasonably believed that, from the point of view of equivalence principle in General Relativity, this geometric phase

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interaction energy of C$_{60}$ interaction between the spinning particle and the time-dependent gravitomagnetic fields. In the present Letter, the time-dependent gravitomagnetic field strength is just the rotating frequency \( \vec{\omega} \) and the effective gravitomagnetic field is \( \vec{B} = \vec{\omega} \times \vec{L} \). As is assumed above, the angular velocity of C$_{60}$ molecule is \( \vec{\omega}(t) = \omega_0 (\sin \theta \cos \Omega t, \sin \theta \sin \Omega t, \cos \theta) \), and the effective gravitomagnetic field is therefore \( \vec{B} = \Omega \sin \theta (-\cos \theta \cos \Omega t, -\cos \theta \sin \Omega t, \sin \theta) \) that is apparently perpendicular to the angular momentum \( \vec{L} \). Thus the magnitude of intermolecular torque acting on the rotating C$_{60}$ molecule is \( |\vec{M}| = |\vec{B} \times \vec{L}| = \omega_0 \Omega I \sin \theta \) with \( I \approx 1.0 \times 10^{-43} \text{ Kg m}^2 \) [4] being the moment of inertia of C$_{60}$ molecule.

The order of magnitude of \( |\vec{M}| \) may be approximately equal to (or less than) the Van der Waals potential energy (0.001 \( \sim \) 0.1 eV). Since it follows that in the high-temperature phase (orientationally disordered phase), \( \omega_0 \) may be \( 10^{11} \text{ rad/s} \), the precessional frequency \( \Omega \) is therefore compared to \( \omega_0 \), i.e., \( \Omega \approx \omega_0 \) ranges from \( 10^{10} \) to \( 10^{12} \text{ rad/s} \). However, in the low-temperature phase (orientationally ordered phase), \( \omega_0 \) decreases (e.g., \( \omega_0 \approx 10^9 \text{ rad/s} \) and in turn the precessional frequency \( \Omega \) increases by a factor of \( \sim 100 \). Thus, the precessional frequency of C$_{60}$ molecule is much greater than the rotating frequency \( \omega_0 \) (which means in this case the dynamical phases due to spin-rotation coupling can be ignored) and the frequency shift in the transition matrix element due to geometric phases can be compared to the typical energy of an electron in solid. It follows that the effects resulting from the spin-rotation coupling and geometric phases in C$_{60}$ molecules deserve further investigations both theoretically and experimentally.

It is apparently seen that investigation of the spin-rotation geometric phases of electrons in C$_{60}$ molecules is of physical interest:

(i) it enables us to study the relation between the photoelectron spectroscopy and the rotational motion of C$_{60}$ molecules. Moreover, since geometric phases of electrons in one C$_{60}$ molecule depend on other C$_{60}$ molecules (via the effective gravitomagnetic field strength \( \vec{\omega} \times \vec{L} \) and hence the intermolecular torque \( \vec{M} \) and thus imply the information about the thermal motion and rotational dynamics of C$_{60}$, it is helpful to analyze the condensed phases (and hence the phase-transition behavior) of solid C$_{60}$ [19].

(ii) for the present, Mashhoon’s spin-rotation coupling can be tested only in microwave experiments [9], since this coupling is relatively weak due to the smallness of the rotational frequency of various rotating frames on the Earth. Fortunately, here the rotational motion of C$_{60}$ molecule can provide us with an ingenious way to test this weakly gravitational (gravitomagnetic) effect. Since the rotational angular velocity, \( \omega_0 \), of rotating C$_{60}$ molecules is much greater than that of any rotating bodies on the Earth, the C$_{60}$ molecule is an ideal noninertial frame of reference for the electrons in the C$_{60}$ molecule, where the effects resulting from the electron spin-rotation coupling may be easily observed experimentally.

(iii) as is claimed previously, in addition to the Aharonov-Carmi geometric phase due to the Coriolis force (gravitomagnetic Lorentz force) [12], there exists another geometric phase associated with gravitational fields arising from the interaction between the spinning particle and the time-dependent gravitomagnetic fields. In the present Letter, the time-dependent gravitomagnetic field strength is just the rotating frequency \( \vec{\omega}(t) \) of C$_{60}$ molecules. It may be reasonably believed that, from the point of view of equivalence principle in General Relativity, this geometric phase itself appears to possess rich physical significance and should therefore be considered in more detail.

To summarize, we study the C$_{60}$ molecule precession and geometric phases of electrons due to the time-dependent spin-rotation coupling in C$_{60}$ molecules. Since in the orientationally ordered phase the precessional frequency of C$_{60}$ is fairly great, the effects of geometric phases will be apparent and even may therefore be read off from the photoelectron spectroscopy, which enables physicists to investigate the rotational dynamics and phase transition of C$_{60}$. Additionally, the present work will make possible a test of Mashhoon’s spin-rotation coupling by measuring the spin-rotation geometric phases of electrons in C$_{60}$ molecules with rapid rotation. We hope all these physical phenomena and effects would be investigated experimentally in the near future.

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