Confronting particle emission scenarios with strangeness data

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Abstract

We show that a hadron gas model with continuous particle emission instead of freeze out may solve some of the problems (high values of the freeze out density and specific net charge) that one encounters in the latter case when studying strange particle ratios such as those by WA85. This underlines the necessity to understand better particle emission in hydrodynamics to be able to analyze data. It also re-opens the possibility of a quark-hadron transition occurring with phase equilibrium instead of explosively.

1. Introduction

An enhancement of strangeness production in relativistic nuclear collisions (compared to e.g. proton-proton collisions at the same energy) is a possible signature of the much sought-after quark-gluon plasma. It is therefore particularly interesting that current data at AGS (Alternating Gradient Synchrotron) and SPS (Super Proton Synchrotron) energies do show an increase in strangeness production (see e.g. \cite{2}). At SPS energies, this increase seems to imply that something new is happening: in microscopical models, one has to postulate some previously unseen reaction mechanism (color rope formation in the RQMD...
code, multiquark clusters in the Venus code, etc) while hydrodynamical models have their own problems (be it those with a rapidly hadronizing plasma or those with an equilibrated hadronic phase, preceded or not by a plasma phase). In this paper, we examine the shortcomings of the latter class of hydrodynamical models and suggest that they might be due to a too rough description of particle emission.

To be more precise, let us assume that a hadronic fireball (region filled with a hadron gas, or HG, in local thermal and chemical equilibrium) is formed in heavy ion collisions at SPS energies and that particles are emitted from it at freeze out (i.e. when they stop interacting due to matter dilution). One then runs into (at least) three kinds of problems when discussing strange particle ratios.

First, the temperature \(T_{f.out} \sim 200\) MeV and baryonic potential \(\mu_{b,f.out} \sim \text{few 100 MeV}\) needed at freeze out to reproduce WA85 and NA35 strangeness data actually correspond to high particle densities: this is inconsistent with the very notion of freeze out.

Second, to reproduce strange particle ratios, it turns out that the strange quark potential \(\mu_s\) must be small and the strangeness saturation factor \(\gamma_s\) of order 1 (this quantity, with value usually between 0 and 1, measures how far from chemical equilibrium the strange particles are, 1 corresponds to full chemical equilibrium of the strange particles). Both facts

\(^1\)The main problem for the former class of hydrodynamical models is the difficulty to yield enough entropy after hadronization.

\(^2\)While WA85 and NA35 data for strange particle ratios are comparable and lead to high T’s and \(\mu_b\’s\), NA36 data are different and lead to lower T’s (Phys. Lett. B327 (1994) 433) for similar targets but a somewhat different kinematic window. However the rapidity distribution for \(\Lambda\)’s (E.G. Judd et al. Nucl. Phys. A590 (1995) 291c) as well as \(\bar{\Lambda}\)’s and \(K^0_s\’s\) (J.Eschke et al. Heavy Ion Phys.4 (1996) 105) for NA36 are quite below that of NA35; NA44 midrapidity data for \(K^\pm\) agree with that of NA35.
are expected in a quark-gluon plasma hadronizing suddenly, not normally in a hadronic 
fireball [13,14].

Third, using the values at freeze out of the temperature, baryonic potential and satura-
tion factor extracted to reproduce WA85 strange particle ratios, one can predict the value 
of another quantity, the specific net charge (ratio of the net charge multiplicity to the total 
charge multiplicity). This quantity has been measured not by WA85, but in experimental 
conditions similar to that of WA85 by EMU05 [15]. It turns out that the predicted value is 
too high, (while it might be smaller if a quark-gluon plasma fireball had been formed) [16,3].

In what follows, we study how problems 1 and 3 are related to the mechanism for particle 
emission normally used, freeze out, and suggest that the use of continuous emission instead 
of freeze out, might shed some light on these questions. (We also re-discuss problem 2.) 
This underlines the necessity to understand better particle emission in hydrodynamics and 
re-opens perspectives (see conclusion) for scenarios of the quark-hadron transition.

2. Fluid behavior and particle spectra

First let us see in more details what are the two particle emission mechanisms just 
mentioned. In the usual freeze-out scenario, hadrons are kept in thermal equilibrium until 
some decoupling criterion has become satisfied (then they free-stream toward the detectors). 
For example, in the papers mentioned above where experimental strange particle ratios are 
reproduced, the freeze out criterion is that a certain temperature and baryonic potential have 
been reached. The formula for the emitted particle spectra used normally is the Cooper-
Frye formula [17]. In the particular case of a gas expanding longitudinally only in a boost 
invariant way, freezing out at some fixed temperature and chemical potential, the Cooper-
Frye formula reads

\[
\frac{dN}{dp_{\perp}dp_{\perp}} = \frac{gR^2}{2\pi^2} \tau_{f.out} m_\perp \sum_{n=1}^{\infty} (\mp)^{n+1} \exp \left( n\mu_{f.out}/T_{f.out} \right) K_1(nm_\perp/T_{f.out}). \tag{1}
\]

(The plus sign corresponds to bosons and minus, to fermions.) It depends only on the 
conditions at freeze out: \( T_{f.out} \) and \( \mu_{f.out} = \mu_{B_{f.out}}B + \mu_{S_{f.out}}S \), with B and S the baryon
number and strangeness of the hadron species considered, and \( \mu_{S\text{ f.out}}(\mu_{b\text{ f.out}}, T_{f\text{.out}}) \) obtained by imposing strangeness neutrality. So the experimental spectra of particles teach us in that case only what the conditions were at freeze out.

In the continuous emission scenario developed in [18,19], the basic idea is the following. Due to the finite dimensions and lifetime of the fluid, a particle at space-time point \( x \), has some chance \( P \) to have already made its last collision. In that case, it will fly freely towards the detector, carrying with it memory of what the conditions were in the fluid at \( x \). Therefore the spectrum of emitted particles contains an integral over all space and time, counting particles where they last interacted. In other words, the experimental spectra will give us in principle information about the whole fluid history, not just the freeze out conditions. For the case of a fluid expanding longitudinally only in a boost invariant way with continuous particle emission, the formula that parallels (1) is

\[
\frac{dN}{dyp_\perp dp_\perp} \sim \frac{2g}{(2\pi)^2} \int_{P=0.5} d\phi d\eta m_\perp \cosh \eta \tau_F \rho d\rho + p_\perp \cos \phi \rho F \tau d\tau \exp((m_\perp \cosh \eta - \mu)/T) \pm 1
\]

where \( \tau_F(\rho, \phi, \eta; v_\perp) \) (resp. \( \rho_F(\tau, \phi, \eta; v_\perp) \)) is the time (resp. radius) where the probability to escape without collision \( P = 0.5 \) is reached. \( P \) is given by a Glauber formula, \( \exp[-\int \sigma v_{rel} m(\tau')d\tau'] \) and depends in particular on location and direction of motion. We are using both (1) and (2) in the following. Clearly, in (2), various \( T \) and \( \mu = \mu_b B + \mu_S S \) appear (again \( \mu_S(\mu_b, T) \) is obtained from strangeness neutrality), reflecting the whole fluid history, not just \( T_{f\text{.out}} \) and \( \mu_{b\text{.out}} \).

So to predict particle spectra, in the case of continuous emission, we need to know the fluid history. To get it, we fix some initial conditions \( T(\tau_0, \rho) = T_0 \) and \( \mu_b(\tau_0, \rho) = \mu_{b0} \) and solve the equations of conservation of momentum-energy and baryon number for a mixture of free and interacting particles, using the equation of state of a resonance gas (including the 207 known lowest mass particles) and imposing strangeness neutrality. As a result we get \( T(\tau, \rho), \mu_b(\tau, \rho) \) and we can use these as input in the formula for the particle spectra (2). The procedure is similar to that of a massless pion gas [18,19] but is numerically more involved.
An important result [18,19] for the following is that for heavy particles, the spectrum (2) is dominated by the initial conditions, precisely a formula similar to (1) with freeze out quantities replaced by initial conditions could be used as an approximation, (particularly at high $p_\perp$); for light particles the whole fluid history matters. To understand this fact, one can consider equation 2 and compare particles emitted at $T(\tau, \rho)=200$ and 100 MeV. For particles with mass of 1 GeV, the exponential term gives a thermal suppression above one hundred between these two temperatures. The multiplicative factors in front of the exponential are in principle larger at the lower temperature but do not compensate for such a big decrease. This is why heavy particles are abundantly emitted at high temperatures. On the other side for pions, the thermal suppression is only a factor of 2. This is why light particles are emitted significantly in a larger interval of temperatures.

Note that since heavy particle and high $p_\perp$ particle spectra are sensitive mostly to the initial values of $T$ and $\mu_b$, the exact fluid expansion does not matter very much for them; in particular the assumption of boost invariance should play no part in the forthcoming analysis of strange high $p_\perp$ particle ratios\textsuperscript{3}. It would be however interesting to include continuous emission in, e.g. a hydrodynamical code, to obtain the fluid evolution and study pion data and low $p_\perp$ data.

3. Particle ratios

Once the spectra have been obtained, they can be integrated to get particle numbers, taking into account eventual experimental cutoffs and correcting for resonance decays. Since we had to specify the initial conditions to solve the conservation equations and use this solution as input into (2), the particle numbers depend on $T_0, \mu_b_0$. In contrast, for the freeze

\textsuperscript{3}Note also that the data considered below are in a small rapidity window, near midrapidty. Were it not for this fact, boost invariance should not be assumed, because the rapidity distributions do not have this symmetry.
out case, particle numbers depend on the conditions at freeze out, $T_{f,\text{out}}$ and $\mu_{b,\text{out}}$.

We look for regions in the $T_0, \mu_{b,0}$ space which permit to reproduce the latest WA85 experimental data on strange baryons \cite{11} for $2.3 < y < 2.8$ and $1.0 < p_\perp < 3.0$ GeV: $\bar{\Lambda}/\Lambda = 0.20 \pm 0.01$, $\Xi^-/\Xi = 0.41 \pm 0.05$ and $\Xi^-/\Lambda = 0.09 \pm 0.01$ ($\Xi^-/\bar{\Lambda} = 0.20 \pm 0.03$ follows). In fact there is no set of initial conditions which permits to reproduce all the above ratios. A similar situation occurs with freeze out, as noted in \cite{20}. (Note that with the older experimental data \cite{21}, a region in the $T_{f,\text{out}} - \mu_{b,\text{out}}$ space permitting to reproduce all the above ratios existed \cite{6,8–10}.)

In the comparison of our model with WA85 data we have assumed however complete chemical equilibrium for strangeness production. As already mentioned in the introduction, this is not expected for a HG. In order to account for incomplete strangeness equilibration, we introduce the additional strangeness saturation parameter $\gamma_s$ by making the substitution $\exp \mu_S S \rightarrow \gamma_s |S| \exp \mu_S S$ in the (Boltzmann) distribution functions \cite{22}. In our case, a priori, $\gamma_s$ depends on the space-time location $x$, however since as already mentioned, the initial conditions dominate in the shape and normalization of the spectra of heavy particles (particularly at high $m_\perp$), we take

$$\frac{dN}{dp_\perp dp_\perp} \sim \gamma_s |S| (\tau_0) \frac{dN_{\text{eq}}}{dp_\perp dp_\perp}$$

with $dN_{\text{eq}}/dp_\perp dp_\perp$ given by (2). In figure 1a, we see that for $\gamma_s (\tau_0) = 0.58$, there exists an overlap region in the $T_0, \mu_{b,0}$ plane where all the above ratios are reproduced. For the freeze out case, a similar situation occurs as noted in \cite{20}, namely there exists an overlap region for $\gamma_s = 0.7$.

In the freeze out case, the values of the parameters in the overlap region correspond to high particle densities and so it is hard to understand how particles have ceased to interact: this is the problem 1 mentioned in the introduction. In the continuous emission case, $T_0$ and $\mu_{b,0}$ in the overlap region lead to high initial densities, but this is of course quite reasonable since these are values when the HG started its hydrodynamical expansion. Note also that the value we used for $<\sigma v_{rel}>$ comes from using a Breit-Wigner formula for the $\Lambda - \pi$ cross
section and computing $<\sigma v_{rel}>_{p^+_T\geq 1.0}$ at various temperatures.

The aim of picture 1a is to allow an easy comparison with freeze out results such as [20], however it is not physically complete: so far we have neglected hadronic volume corrections. For freeze out, this correction cancels between numerator and denominator in baryon ratios so can be ignored [10] but for continuous emission, since we are considering the whole fluid history to get particle numbers (and then their ratio), it must be included. There are various ways to do this. (Some of the) thermodynamical quantities for pointless particles can be multiplied by a factor (determined from pointlike quantities), for example $1/(1 + V n_0)$, where $V$ is a typical hadronic volume and 0 indicates pointlike quantities. However there is no consensus on whether this correction involves a net baryonic density $n_0$ [23,24,10,25] or the total particle density $n_0 = \sum_j n^0_j$ [26,27] or a compromise [8]. Here we modify all particle densities as follows $n_i = n^0_i/(1 + V \sum_j n^0_j)$ as in [26,27]. Taking into account hadronic volumes, we get the overlap region shown in figure 1b, which is shifted towards smaller $T$’s and $\mu_b$’s but not very different from that of figure 1a. Given that simulations of QCD on a lattice indicate a quark-hadron transition for temperatures around 200 MeV, it seems more reasonable to consider initial conditions $T_0 \sim 190$ MeV and $\mu_{b0} \sim 180$ MeV, i.e. the bottom part of the overlap region. The precise location of the overlap region (and exact value of $\gamma_s$) is sensitive to changes in the equation of state -as we have just seen- as well as in the cross section or cutoff $P = 0.5$ in eq. 2. (As a cross check, we have also remade our calculations using a Walecka-type equation of state [28] and found similar results to figure 4.

In these simulations, the transition temperature depends strongly on the number of flavors $N_f$ (being $> 200$ MeV for $N_f = 0$ and $< 200$ MeV for $N_f > 1$) and somehow on the method of extraction. Since the created plasma might remain gluon dominated with quarks away from equilibrium, up to the transition (this is already predicted to be the case for RHIC and LHC energies), it is not clear what value of the transition temperature should be used but one should probably not take results from lattice QCD as a rigid value in our context.
1b.) Therefore problem 1 (whether the overlap region is physically reasonable) is taken care of.

To be complete, we also examined the more recent ratios obtained by WA85 \[29\] (at midrapidity):
\[
\frac{\Omega^-}{\Omega^-_{m_\perp \geq 2.3 GeV}} = 0.57 \pm 0.41, \quad \frac{(\Omega^- + \Xi^-)}{(\Xi^- + \Xi^-)_{m_\perp \geq 2.3 GeV}} = 1.7 \pm 0.9,
\]
\[
\frac{K_0^0}{\Lambda_{p_\perp > 1.0 GeV}} = 1.43 \pm 0.10, \quad \frac{K_0^0}{\bar{\Lambda}_{p_\perp > 1.0 GeV}} = 6.45 \pm 0.61 \quad \text{and} \quad \frac{K^+}{K^-_{p_\perp > 0.9 GeV}} = 1.67 \pm 0.15.
\]

We looked for a region in the \(T_0, \mu_{b0}\) plane where \(\Omega^-/\Omega^-_{m_\perp \geq 2.3 GeV}\) is reproduced: due to the large experimental error bars, this does not bring new restrictions to fig. 1b. We also calculated our value for \((\Omega^- + \Xi^-)/{(\Xi^- + \Xi^-)_{m_\perp \geq 2.3 GeV}}\) in the overlapping region and found \(\sim 0.7\), in marginal agreement with the above experimental values. The three ratios involving kaons depend on more than just initial conditions (kaons are intermediate in mass between pions and lambdas so part of the fluid thermal history must be reflected in their spectra), in particular \(\gamma_s(x) \sim \gamma_s(\tau_0) \sim \text{cst}\) may not be a good approximation and we are still working on this. The above experimental ratios concern SW collisions, data with SS are not so extensive yet but not very different \[30\] so a similar overlapping region can be found.

The apparent temperature extracted from the experimental \(p_\perp\) spectra for \(\Lambda, \bar{\Lambda}, \Xi^-\) and \(\Xi^-\) is \(\sim 230\) MeV \[11\]. Given that we extracted from ratios of these particles, temperatures \(T_0 \geq 200\) MeV, we conclude that heavy particles exhibit little transverse flow, which is compatible with the fact that they are emitted early during the hydrodynamical expansion\[5\].

### 4. Specific net charge

We now turn to
\[
D_q = (N^+ - N^-)/(N^+ + N^-)
\]  \(\text{(4)}\)

\[^5\]Note that WA85 data concern high \(p_\perp\) strange particles. But in \[18,19\], we showed that (a simpler version of) our model with continuous emission and no transverse flow, reproduces the shape of the NA35 S+S \(p_\perp\) spectra for strange particles, which extend to low \(p_\perp\) (but are not restricted to midrapidity).
using the continuous emission scenario. As mentioned in the introduction, for HG models with freeze out the predicted $D_q$ is too high, when using values of the freeze out parameters that fit strangeness data, e.g. $T_{f,\text{out}} \sim 200$ MeV, $\mu_{b,\text{out}} \sim 200$ MeV and $\gamma_s \sim 0.7$. For continuous emission, due to thermal suppression, particles *heavier than the pion* are approximately emitted at $T_0 \sim 200$ MeV, $\mu_{b,0} \sim 200$ MeV and $\gamma_s \sim 0.49$ (fig. 1b), so $D_q$ so far is similar to that of freeze out. However there is an additional source of particles that enters the denominator of (4), namely pions are emitted at $T_0$ *and then on* (since they are not thermally suppressed). So we expect to get a lower value for $D_q$ in the continuous emission case than in the freeze out case. (We recall that pions are the dominant contribution in $N^+ + N^-$.)

This would go into the direction of solving problem 3, it is still under investigation.

### 5. Conclusion

Our present description is simplified, for example we do not include the transverse expansion of the fluid, use similar interaction cross sections for all types of particles, etc. In addition, we need to look systematically at strangeness data from other collaborations as well as other types of data such as Bose-Einstein correlations. Nevertheless, we have seen that the continuous emission scenario with a HG may shed light on problems 1 and 3 (discussed in the introduction) that a freeze out model with a HG encounters. Namely, in the overlap region of the parameters needed to reproduce WA85 data, the density of particles is high and this is consistent with the emission mechanism, since it is the initial density of the thermalized fluid. We also expect $D_q$ to be smaller for continuous emission than freeze out. But (problem 2) the value of the strangeness saturation parameter may be high for a HG, particularly at the beginning of its hydrodynamical expansion. However what we really need to get fig. 1b, is that $\Xi^-/\Lambda = \gamma_{\Xi^-}/\gamma_{\Lambda,\Lambda|\text{eq.}}$ and $\Xi^-/\Xi^- = \gamma_{\Xi^-}/\gamma_{\Xi^-,\Xi^-|\text{eq.}}$ with $\gamma_{\Xi^-}/\gamma_{\Lambda} = 0.49$. We expect indeed that multistrange $\Xi^-$ and $\Xi^-$ are more far off chemical equilibrium than singlestrange $\Lambda$ and $\Xi^-\Lambda$ so that $\gamma_{\Xi^-}/\gamma_{\Lambda} < 1$. The result $\gamma_{\Xi^-} = 0.49$ arises if one makes the *additional hypothesis* that quarks are independent degrees of freedom inside
the hadrons so that one has factorizations of the type \( \gamma_A \exp \mu_A/T = \gamma_s \exp 2\mu_q/T \exp \mu_s/T \) and \( \gamma_\Xi \exp \mu_\Xi/T = \gamma_s^2 \exp \mu_q/T \exp 2\mu_s/T \). Therefore problem 2 may not be so serious.

The fact that we may cure some of the problems of the HG scenario does not mean that no quark-gluon plasma has been created before the HG, in fact it may open new possibilities for scenarios of the quark-hadron transition (e.g. a quark-hadron plasma evolving into an equilibrated HG with continuous emission); in particular it may not be necessary to assume an explosive transition [5] or a deflagration/detonation scenario [31–33].

But our main conclusion is that the emission mechanism may modify profoundly our interpretation of data (for example, does the slope in transverse mass spectrum tell something about freeze out or initial conditions?). In turn this modifies our discussion of what potential problems (such as 1 and 3) are emerging. Therefore we believe it is necessary to devote more work to get a realistic description of particle emission in hydrodynamics, [18,19] being a first step in that direction. We remind that the idea that particles are emitted continuously and not on a sharp freeze out surface is supported by microscopical simulations at AGS energies [34] and SPS energies [35].

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**Note added**

After completing this paper, we learned that G.D. Yen, M.I. Gorenstein, W. Greiner and S.N. Yang suggested [36] another solution to problems 1 and 3 above, in term of the excluded volume approach of [20], for the preliminary Au+Au (AGS) and Pb+Pb (SPS) data.
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FIGURE CAPTIONS

FIG. 1 Overlap region in the $T_0 - \mu_{b0}$ plane for WA85 data, with $<\sigma_{v_{rel}}>=1\, fm^2$ a) without b) with hadronic volume corrections.
Fig. 1.a  $\gamma_s = 0.58$
