A SURVEY OF DUE-DATE RELATED SINGLE-MACHINE WITH TWO-AGENT SCHEDULING PROBLEM

HONGWEI LI, YUVRAJ GAJPAL* AND C. R. BECTOR

Department of Supply Chain Management
I.H. Asper School of Business, University of Manitoba
Winnipeg, Manitoba, R3T 5V4, Canada

(Communicated by Ada Che)

ABSTRACT. In this paper, we consider due-date related single machine scheduling problems, where two agents compete for utilizing a common processing resource (i.e. a single machine). In this paper, we provide a detailed and a systemic literature review of the two-agent scheduling problem dealing with models with a given due date. We consider the following four main cases: 1) the earliness and tardiness, 2) the maximum lateness, 3) the number of tardy jobs and 4) the late work criteria. To do so, we classify due-date related, two-agent scheduling problems into two categories on the basis of the objective function setting, (i.e. the feasibility model and the minimality model). The feasibility model minimizes the objective function of one agent subject to an upper bound on the objective for the other agent. The minimality model assigns certain weights for two agents and as a result minimizes their weighted objectives. In the present paper, we list the computational complexities and proposed algorithms for the due-date related, two-agent scheduling problem, investigated in the literature since 2003.

1. Introduction. Scheduling problems for optimizing resource allocation have been studied for many years. Nowadays, in both manufacturing and service industries, scheduling is playing a critical role, since the scheduling related decisions have a significant impact on improving the productivity and the efficiency of production systems. In a two-agent scheduling problem, two agents compete for limited processing resources. Each agent has a set of jobs to be processed using a common processing resource. Furthermore, each agent may have different objective functions at a time (for instance, the minimization of weighted tardiness and earliness, the minimization of total completion time, and the minimization of the number of tardy jobs). The primary purpose of setting these objective functions is to assess the performance of the schedules from multiple perspectives.

Baker and Cole Smith (2003) and Allesandro Agnetis, Mirchandani, Pacciarelli, and Pacifici (2004) introduced the two-agent scheduling problem, in which a single machine is being used by two agents. The two-agent scheduling problem faces the dilemma of minimizing two conflicting objectives associated with two agents. The objectives of two agents are in conflict because the completion time of the jobs for

2010 Mathematics Subject Classification. Primary: 90B35; Secondary: 68M20.
Key words and phrases. Two-agent scheduling, due date, survey (literature review), the earliness and tardiness, the maximum lateness, the number of tardy jobs.

* Corresponding author: Yuvraj Gajpal.
two agents are mutually independent. To address the conflicting nature of objective functions, Suresh and Chaudhuri (1996) suggested two measures. The first measure is to minimize the objective function of one agent subject to an upper bound on the objective function for the other agent. The second measure is to assign some weights to each objective function and to minimize the weighted objective functions. The two measures suggested by Suresh and Chaudhuri (1996) set the two research directions in two-agent scheduling problem. The work of Baker and Cole Smith (2003) suggested a set of two-agent scheduling problems with the first research direction and the work of Allesandro Agnetis et al. (2004) considers a set of two-agent scheduling problems with the second research direction. T. E. Cheng, Ng, and Yuan (2008) named these two types of single machine with two-agent scheduling problems as “feasibility model” and “minimality model”, respectively.

Furthermore, during the last decades, there has been an increase in interest in single-machine with two-agent scheduling problems. The application of single-machine with two-agent scheduling problems has been studied in literature after the work of Baker and Cole Smith (2003) and Allesandro Agnetis et al. (2004). We provide several applications of the two-agent scheduling problem:

- Brewer and Plott (1996) explored the allocation of rights in using railroad tracks. In this problem, Sweden central rail administration allocated access to the railroad tracks to private firms. They studied this project for the purpose of improving the efficiency of the railroad network operation.
- Soomer and Franx (2008) studied an airline landing operation problem. They presented a method to schedule aircraft landings. Minimizing airlines operating costs was considered as the objective in the study. The critical decision variable of this problem is the landing time which provides a safe and efficient aircraft landing schedule.
- Peha (1995) explored the application of competing scheduling problem in integrated-service packet-switched networks for minimizing the weighted number of tardy jobs and the weighted completion time. The integrated network is responsible for carrying various data types, for instance, voice, video and image. In this type of communication network, the data is divided into small data packages. These data packages are transmitted by the network.
- Pandey, Wu, Guru, and Buyya (2010) Mezmaz et al. (2011) and Li, Gajpal, and Bector (2018) studied the cloud resource scheduling problem. The scheduling algorithm plays a critical role in optimum utilization of cloud infrastructure to deliver the economical cloud service in a cut throat competition environment.
- Lun, Lai, Ng, Wong, and Cheng (2011) and Zhang, Ng, Tang, Cheng, and Lun (2011) studied the due-date-based multi-agent scheduling problem in the shipping industry. In their study, the port was considered as a common processing machine, the container ships at the port were considered as the jobs to be processed. They assumed that the ships belonged to different shipping companies, and these shipping companies were considered as different agents.
- Gerstl and Mosheiov (2014) considered the due-date-based two-agent scheduling problem under the Just-In-Time (JIT) manufacturing environment.

Compared with previous literature, this paper considers a new classification method. The scheduling problems reviewed in this paper are classified into two categories based on the scheduling problem settings:
a) Feasibility Model: Minimize the objective function of one agent subject to an upper bound on the objective function for the other agent.

b) Minimality Model: Assign weight to the objective function of each agent and minimize the weighted objectives.

We review the papers which were published after the work of Baker and Cole Smith (2003). The due-date-related objective functions considered in this survey paper include the maximum lateness, the earliness and lateness, the number of tardy jobs, and the late work criteria.

This paper conducts a literature survey of the due-date involved single-machine with two-agent scheduling problems. To our knowledge, no survey paper has been published on single-machine with two-agent scheduling problems; therefore, in this paper, we attempt to do so. In the present survey paper, the due-date parameter is considered as a deterministic variable, instead of a decision variable. In other words, the due date is taken as input to the scheduling problem. Due-date related single-machine with two-agent scheduling problem literature has two research directions. First research direction considers the due date parameter as a decision variable (Mor & Mosheiov, 2017; Wang, Yin, Cheng, Cheng, & Wu, 2016; Yin, Wang, Wang, & Cheng, 2017). The second research direction considers the due date parameter as a given variable. Nevertheless, only a limited number of papers have considered the due-date assignment related two-agent scheduling problems. Therefore, due-date assignment related two-agent scheduling papers are not reviewed in this paper.

2. Notation and classification. This section provides the notation, terminology and the classification of the due date related, single-machine, two-agent scheduling problems.

\[ n_A, n_B \]
\[ n \]
\[ p_j \]
\[ d_j \]
\[ r_j \]
\[ \sigma \]
\[ f(\sigma) \]
\[ Q \]
\[ C_j \]
\[ C_{max} \]
\[ E_j/L_j \]
\[ L_{max} \]
\[ T_j \]
\[ V_j \]
\[ \sum C_j/\sum w_jC_j \]
\[ \sum E_j/\sum w_jE_j \]
\[ \sum T_j/\sum w_jT_j \]
\[ \sum U_j/\sum w_jU_j \]

The notation \( \alpha | \beta | \gamma \) describes the scheduling problems reviewed in the present paper. The parameter \( \alpha \) describes the shop environment, \( \beta \) describes the information of processing characteristics and \( \gamma \) describes the objective to be minimized. For instance, consider a two-agent single-machine scheduling problem to minimize the maximum lateness of one agent, subject to an upper bound \( Q \) on the total weighted completion time of the other. This problem is denoted by
In Figure 1 below, we provide the relationships among different objective functions, in which the notation used is already explained.

As already said, this paper considers a set of two-agent scheduling problems, in which a set of given jobs for each agent are processed by a common machine. Each agent has an objective function, which depends on the completion time of jobs. All papers reviewed in this survey are classified into two categories, on the basis of the way the objective function of two agents is minimized.

3. Feasibility model. The goal of this type of two-agent scheduling problem is to minimize the objective of one agent, subject to an upper bound on the objective of the other agent. Most of the two-agent-related scheduling problems follow this direction to deal with conflicting objective functions.

3.1. Maximum lateness. The maximum lateness $L_{\text{max}}$ is utilized to measure the worst violation of the due dates (Pinedo, 2015). It is defined as $L_{\text{max}} = \max_{j=1,2,...,n} \{L_j = C_j - d_j\}$, where as already stated $d_j$ is the due date of job $j$. In a single-machine with two-agent scheduling problem, maximum lateness can be minimized by arranging the jobs in Earliest Due Date first (EDD) rule.

T. E. Cheng et al. (2008) and Alessandro Agnetis, de Pascale, and Pacciarelli (2009) proposed and studied a two-agent scheduling problem to minimize the weighted total completion time of the first agent subject to an upper bound on the maximum lateness of the other agent (i.e. $1 \mid \sum w_j C_j^B Q \mid L_{\text{max}}^A$). T. E. Cheng et al. (2008) proved that this problem is strongly NP-hard. Alessandro Agnetis et al. (2009) proposed that the A-jobs and B-jobs are scheduled in Weighted Shortest Processing Time first (WSPT) rule and Earliest Due Date first (EDD) rule in an optimal schedule, respectively. Alessandro Agnetis et al. (2009) relaxed the constraints on B-jobs to get a Lagrangian problem: $L(\delta^B) =$
min_{\sigma \in S} \sum_{i=1}^{n_A} \delta_i^A p_i^A C_i^A(\sigma) + \sum_{j=1}^{n_B} \frac{1}{\delta_j^B} p_j^B (C_j^B(\sigma) - Q_j) \}. The relaxed problem was integrated with a Branch-and-Bound algorithm to solve the problem.

A number of papers took the release time into consideration. The release time \( r_j \) is the time when job \( j \) arrives at the system. It is also referred to as the earliest time at which the job \( j \) can be processed. Yin, Wu, Cheng, and Wu (2012) studied and proved the problem \( 1 | L_{\text{max}}^B \leq Q, r_{ij} | \sum C_i^A \) to be unary NP-hard. T. E. Cheng, Chung, Liao, and Lee (2013) and Yin, Wu, Cheng, Wu, and Wu (2015) added the weight parameter into their consideration. These two papers studied a two-agent scheduling problem to minimize the total weighted completion time of one agent, subject to an upper bound on the maximum lateness of the other agent. This problem is denoted by \( 1 | L_{\text{max}}^B \leq Q, r_{ij} | \sum w_i^A C_i^A \). They proved that this problem is strongly NP-hard. The jobs in an optimal schedule must follow two theorems. The first theorem states that the job \( i \) needs to be scheduled before job \( j \), if \( r_i + p_i \leq r_j \). The second theorem states that \( C_i^A \leq r_{A1} \) for the jobs of the first agent and \( C_j^B - d_j^B \leq Q \) for the jobs of the second agent. With the help of these conditions, a Branch-and-Bound algorithm was developed to find the optimal solution. T. E. Cheng et al. (2013) adopted a depth-first search to execute the branching procedure which only requires little storage space. As per the result of computational experiments, the proposed Branch-and-Bound algorithm was capable to find optimal solutions for up to 24 jobs within a reasonable time. Furthermore, T. E. Cheng et al. (2013) proposed a simulated annealing (SA) algorithm and Yin et al. (2015) proposed a Marriage in Honey-Bees Optimization (MBO) algorithm for finding near-optimal solutions. As per the result of computational experiments, both SA and MBO algorithms performed quite well regarding the solution quality and stability.

Yin et al. (2012) and Lee, Chung, and Hu (2012) studied a two-agent scheduling problem to minimize the total tardiness of the first agent subject to an upper bound on the maximum lateness of the second agent. This problem was proved to be unary NP-hard. Yin et al. (2012) developed a Branch-and-Bound algorithm and a Mixed Integer Programming model (MIP) to find an optimal solution. The proposed Branch-and-Bound algorithm, Yin et al. (2012) adopted the method of Baptiste, Carlier, and Jouglet (2004) for the propose of searching the lower bound. They also developed an approximation algorithm and Honey-Bees Optimization algorithm (MBO) to find the near-optimal solution. The initialization step of MBO algorithm schedules all B-jobs in EDD order, followed by arranging the A-jobs in non-decreasing order of their sum of processing time and release time. It was observed that MBO algorithm performed well in the both aspects of solution quality and efficiency. Furthermore, the result generated by MBO algorithm was considered as the initial incumbent solution in the Branch-and-Bound algorithm. However, the proposed Branch-and-Bound algorithm could not solve the problem instances for more than 14 jobs. Lee et al. (2012) proposed three Genetic Algorithms (GA) to solve the problem. The numerical experiment showed that the average percentage deviation was less than 0.2%, which revealed a very good performance of the proposed combined genetic algorithm.

Wang et al. (2015) studied five maximum lateness related two-agent scheduling problems. The four problems are: \( 1 | L_{\text{max}}^B \leq Q, L_{\text{max}}^A, (\text{Problem 1}), 1 | L_{\text{max}}^B \leq Q, \sum T_i^A, (\text{Problem 2}), 1 | L_{\text{max}}^B \leq Q, \sum w_i^A T_i^A, (\text{Problem 3}), 1 | L_{\text{max}}^B \leq Q, \sum U_i^A, (\text{Problem 4}) \) and \( 1 | L_{\text{max}}^B \leq Q, \sum w_i^A U_i^A, (\text{Problem 5}) \), where the maximum lateness of jobs in agent B is kept within a pre-specified limit. The optimal schedule
of these problems can be found if the B-jobs are scheduled in the SPT – EDD order. Additionally, there exists an optimal solution, if A-jobs are scheduled in SPT – EDD order for Problem 1 and Problem 2; if A-jobs are scheduled in SPT order for Problem 3, Problem 4 and Problem 5. Regarding the computational complexities of these problems, this paper proved that the Problem 1, Problem 2 and Problem 4 can be solved in \( O(n_A \log n_A + n_B \log n_B) \) time. Problem 5 was proved to be NP-hard and it can be solved in time \( O(n_A^2 \min\{d^A_{\text{max}}, \sum_{i=1}^{n_A} w^A_i\} + n_B \log n_B) \).

Yin, Cheng, Cheng, Wu, and Wu (2013) investigated a two-agent scheduling problem with release dates and due dates to minimize the number of tardy jobs of the first agent subject to an upper bound on the maximum lateness of the second agent (i.e. \( 1 \mid \sum L_{\text{max}}^B \leq Q, r_{ij} \mid \sum U_i^A \)). This problem was proved to be strongly NP-hard in the paper. They studied two polynomial solvable cases for the problem. The first case considered assigning a common release time (i.e. time zero) to all jobs of the first agent (i.e. \( 1 \mid \sum L_{\text{max}}^B \leq Q, r^A_i = 0, r^B_j \mid \sum U_i^A \)). Also, it can be solved in \( O(n_A \log n_A + n_B \log n_B) \) time. The second special case considered assigning a common processing time to all jobs of both agents (i.e. \( p^B_{i,j} = p \), where \( X \in \{A, B\} \)), yielding a problem that can be solved in \( O((n_A + n_B)^2) \) time. This paper provided a Branch-and-Bound algorithm and a simulated annealing (SA) algorithm for finding an optimal solution and near-optimal solutions. Apart from these two algorithms, this paper also proposed an algorithm for calculating the lower bound.

Wang, Kang, Shiu, Wu, and Hsu (2017) studied the problem \( 1 \mid L_{\text{max}}^B \leq Q \mid \sum V_i^A \). They proved that this problem is NP-hard. To solve this problem optimally, a branch-and-bound algorithm and two pseudo-polynomial time dynamic programming algorithms were developed in the paper. The proposed dynamic-programming-based algorithms can solve the problem in time \( O(n_A n_B \min\{\sum_{i=1}^{n_A} p_i^A + \sum_{j=1}^{n_B} p_j^B, \max\{\sum_{i=1}^{n_A} (p_i^A + d_i^A - 1), Q - d^B_{\text{max}}\}\}) \) and time \( O(n_A n_B \sum_{i=1}^{n_A} p_i^A) \), receptively. A tabu search algorithm also was designed to solve this problem. The optimal solution for this problem (no pre-emption allowed) follows four properties: (1) all A-jobs and B-jobs are processed consecutively without idle time and the first job starts at time 0; (2) all early and partially early A-jobs and all B-jobs are scheduled before all tardy A-jobs; (3) the early and partially early A-jobs are scheduled in EDD order; (4) the B-jobs are scheduled in non-decreasing order of \( Q - d^B_{\text{max}} \).

Yuan, Ng, and Cheng (2015) studied a two-agent scheduling problem to minimize the maximum lateness of the first agent subject to an upper bound on the maximum objective value of the second agent. This problem can be denoted by \( 1 \mid \max\{f^B(C^B_j)\} \leq Q, r_{ij}, \text{pmtn} \mid L_{\text{max}}^A \). They used the pmtn-LS (i.e. pre-emptive list schedule) method proposed by Horn (1974) to address the problem. Yuan et al. (2015) proved that the problem can be solved optimally by using the algorithm pmtn-LS in \( O(n_A n \log n) \) time.

A summary of maximum lateness related two-agent scheduling problem is provided in Table 1.

3.2. Earliness, lateness and tardiness. In a Flexible Manufacturing System (FMS) and in a Just-in-Time (JIT) production system, all jobs are required to be finished at the time of due date. If a job is finished before its due date, it will incur a storage costs. On the other hand, if a job is finished after its due date, it will incur a penalty cost. The lateness of a job is defined as \( L_j = C_j - d_j \). Positive results represent that the job is completed late, while negative results means that the job is completed early. The earliness of a job is denoted by \( E_j = \max(0, d_j - C_j) \), and the tardiness of a job is denoted by \( T_j = \max(C_j - d_j, 0) = \max(L_j, 0) \). Some
papers also considered to minimize the maximum job tardiness, which is defined as $T_{max} = \max_j \{T_j\}$. S.-R. Cheng (2014) proposed and studied a set of earliness-based two-agent scheduling problems. This author considered different objective functions, such as total weighted earliness, total weighted earliness and tardiness, and maximum earliness. Cheng proposed an algorithm to solve the problem $1 \mid f_{max}^B \leq Q \mid \max_j f_j^X(E_j^X)$, where $X \in \{A, B\}$ in $O(n_A + n_B)^2$ time. Table 2 summarizes the objective functions studied in the paper and the corresponding complexity, where $AD$ denotes identical weights and reversely agreeable due dates, and $AW$ denotes identical due dates for all jobs and reversely agreeable weights. Here, $X \in \{A, B\}$.

Gerstl and Mosheiov (2013) addressed a problem in which all jobs have an identical processing time and a common due date. This problem is denoted by

**Table 1. Summary of reviewed maximum lateness scheduling problem.**

| Problem | Complexity | Reference | Approach/Result |
|---------|------------|-----------|-----------------|
| 1. $1 \mid f_{max}^B \leq Q \mid \sum w_i C_i^A$ | strongly NP-hard | T. E. Cheng et al. (2008); Alessandro Agnetis et al. (2009) | Branch-and-Bound method; in an optimal schedule, $A$-jobs are scheduled in WSPT order, and $B$-jobs are scheduled in EDD order; $B$-jobs are scheduled consecutively. |
| 2. $1 \mid f_{max}^B \leq Q, r_j^B \mid \sum C_i^A$ | unary NP-hard | Yin et al. (2012) | $A$-jobs are scheduled in SPT order, and $B$-jobs are scheduled in EDD order. |
| 3. $1 \mid f_{max}^B \leq Q, r_j^B \mid \sum w_i^B C_i^A$ | strongly NP-hard | T. E. Cheng et al. (2013); Yin et al. (2015) | Branch-and-Bound method; Simulated Annealing algorithm; Marriage in Honey-bees Optimization algorithm; in an optimal schedule, (1) the job $i$ needs to be scheduled before job $j$, if $r_i + p_i \leq r_j$ for any jobs in both agents; (2) $C_i^B \leq C_j^B$ for the jobs of the first agent and $C_i^A = C_j^A$ for the jobs of the second agent. |
| 4. $1 \mid f_{max}^B \leq Q, r_j^B \mid \sum T_i^A$ | unary NP-hard | Lee et al. (2012); Baptiste et al. (2004) | Branch-and-Bound algorithm; Mixed Integer Programming model; Marriage in Honey-Bees Optimization algorithm; in the initialization step, $B$-jobs are scheduled in EDD order. $A$-jobs are scheduled in non-decreasing order of their sums of processing time and release time. |
| 5. $1 \mid f_{max}^B \leq Q \mid \max_j f_j^P (P1)$, $1 \mid f_{max}^B \leq Q \mid \sum T_i^A (P2)$, $1 \mid f_{max}^B \leq Q \mid \sum w_i T_i^A (P3)$, $1 \mid f_{max}^B \leq Q \mid \sum C_i^A (P4)$, $1 \mid f_{max}^B \leq Q \mid \sum w_i C_i^A (P5)$ | NP-hard | Wang et al. (2015) | There exists an optimal solution. $A$-jobs are scheduled in SPT-EDD order for $P1$ and $P2$; $A$-jobs are scheduled in SPT order for $P3$, $P4$ and $P5$; $B$-jobs are scheduled in SPT-EDD order for all five problems. |
| 6. $1 \mid \sum T_i^B \leq Q, r_j^B \mid \sum U_i^A$ | strongly NP-hard | Yin, Cheng, et al. (2013) | Branch-and-Bound method; Simulated Annealing algorithm. |
| 7. $1 \mid f_{max}^B \leq Q \mid \sum U_i^A$ | NP-hard | Wang et al. (2017) | Branch-and-Bound method; Tabu-Search Heuristic; in an optimal schedule, (1) all $A$-jobs and $B$-jobs are processed consecutively without idle time and the first job starts at time 0; (2) all early and partially early $A$-jobs and all $B$-jobs are scheduled before all tardy $A$-jobs; (3) the early and partially early $A$-jobs are scheduled in EDD order; (4) the $B$-jobs are scheduled in non-decreasing order of $Q - C_i^B$. |
| 8. $\max \{f_j^B(C_j^B) \leq Q, r_j^B, p_{min} \mid L_{max} \}$ | $O(n_A + n_B \log n)$ | Yuan et al. (2015) | Algorithm Ptns-LS (optimal algorithm); EDD rule. |
Table 2. Summary of problems and their computational complexity studied by S.-R. Cheng (2014)

| Problem | Complexity |
|---------|------------|
| 1 \( | f_{\text{max}}^B \leq Q \) \( | \max \sum \bar{w}_i X_{\bar{E}_i}^X \) | \( O((n_A + n_B)^2) \) |
| 1 \( | f_{\text{max}}^B \leq Q \) \( | \sum w_j B_j X_j + T_j X_j \) | strongly NP-hard |
| 1 \( | f_{\text{max}}^B \leq Q \) \( | \sum w_j A_j X_j \) | strongly NP-hard |
| 1 \( | f_{\text{max}}^B \leq Q \) \( | \sum w_j B_j X_j \) | strongly NP-hard |
| 1 \( | f_{\text{max}}^B \leq Q \) \( | \sum w_j A_j X_j + w_j B_j T_j X_j \) | strongly NP-hard |

1 \( | \max(w_j B_j E_j^B + w_j B_j T_j B_j) \leq Q, p_A^i = p_B^i = p, d_A^i = d_B^i = d | \sum(w_i A_i E_i^A + w_i A_i T_i^A) \), and can be solved within time \( O(n^4) \). The optimal schedule should follow, (1) the first scheduled job starts at time zero, or (2) at least one B-job has cost value of the upper bound, or (3) a A-job is completed exactly at the due date. A solution procedure was introduced in their paper for solving each of the three above situations of the problem. The first and the third situations were reduced to a single linear assignment problem (LAP) and the second situation was reduced to a series of LAPs.

Yin, Wu, Wu, Hsu, and Wu (2013) studied a hybrid two-agent scheduling problem with maximum earliness and weighted earliness objectives. They denoted the problem by 1 \( | E_{\text{max}}^B \leq Q | \sum w_i A_i E_i^A + \sum w_i B_j E_j^B \). It is worthwhile to point out that the objective of the problem considered was the combined objective for both agents, instead of the objective for a single agent. They proved that the problem is strongly NP-hard. Authors developed a mathematical programming formulation for finding optimal solution. An appropriate, powerful integer linear programming software, such as CPLEX, can be used to solve the problem optimally. Furthermore, a Branch-and-Bound method was proposed for finding an optimal solution. Based on the computational result, the branch-and-bound method was able to solve up to 28 jobs in a reasonable time. For solving a large-size problem instance, a serious of simulated annealing (SA) algorithms were proposed in the paper. Kirkpatrick, Gelatt, and Vecchi (1983)’s approach was adopted by the proposed SA algorithm in order to avoid being trapped in a local minimum.

Mor and Mosheiov (2010) addressed two-maximum-earliness-based two-agent scheduling problems. These problems are denoted by 1 \( | E_{\text{max}}^B \leq Q | E_{\text{max}}^A \) and 1 \( | E_{\text{max}}^B \leq Q | \sum w_i A_i E_i^A \). They proved that the problems are binary NP-hard and strongly NP-hard, respectively. They also proposed that, for the first problem, an optimal solution exists, if

1. the first job starts at time \( D - (P_A^i + P_B^i) \), where \( D \) represents the common due date, \( P_A^i \) and \( P_B^i \) represents the total processing time of A-jobs and B-jobs, respectively,
2. B-jobs are scheduled in a non-decreasing order of \( d_j^B - p_j^B \) (i.e. MST order), and
3. A-jobs are scheduled in MST order.
A polynomial time algorithm with complexity $O(n_A \log n_B + n_B \log n_B)$ was introduced for solving the first problem. An efficient heuristic was developed for the second problem. In a special case, a common due date and identical weights were considered in the first agent. However, there is no exact algorithm has been proposed in the paper.

Yazdani and Jolai (2013) studied a two-agent scheduling problem with periodic unavailability. The objective is to minimize the total completion time of the first agent subject to an upper bound on the maximum lateness of the second agent. In this problem, the machine is not continuously available for processing the jobs from two agents. It is denoted by $1 \mid T_{\max}^B \leq Q, nr - a \mid \sum C_i^A$, where $nr$ represents non-resemble case and $a$ represents availability constraints. They proved that the problem is strongly NP-hard. The paper proposed a Genetic Algorithm (GA) with modified crossover method. Compared with the uniform crossover method, the modified crossover method had a better performance. Unfortunately, there was no algorithm to find the optimal solution. The performance of the proposed modified GA algorithm was evaluated using the lower bound value. A summary of earliness, lateness and tardiness related two-agent scheduling problem is provided in Table 3.

### Table 3. Summary of reviewed earliness, lateness and tardiness related two-agent scheduling problem.

| Problem                                                                 | Complexity                           | Reference              | Approach/Result                                                  |
|------------------------------------------------------------------------|--------------------------------------|------------------------|------------------------------------------------------------------|
| $1.1 \mid \max(w_i^A \cdot \beta_i^B + w_i^B \cdot \beta_i^A) \leq$  | $Q, p^I = p^J = 0, d^I = d^J = d \mid \sum(w_i^A \cdot \beta_i^B + w_i^B \cdot \beta_i^A)$ | Gers etl and (2013)    | Reduce the problem to a single linear assignment problem (LAP). The global optimum is determined by the minimal-cost LAP. |
|                                                                       |                                      | Mosheiov               |                                                                  |
|                                                                       |                                      |                        |                                                                  |
| $2.1 | R_{\max}^B \leq Q \mid \sum w_i^A \cdot \beta_i^A + w_i^B \cdot \beta_i^B$ | strongly hard                     | Yin, Wu, et al (2013) | Mathematical Programming Formulation method; Branch-and-Bound method; Simulated Annealing algorithm. |
|                                                                       |                                      |                        |                                                                  |
| $3.1 \mid \beta_{\max}^B \leq Q \mid \beta_{\max}^A$               | binary NP-hard                      | Mor and (2010)         | A HDuristic. There exists an optimal solution, if (1) the first job starts at time $0 \mid (p^B + p^A) \cdot B - (p^B + p^A) \cdot A$, where $B$ represents the common due date, $p^A$ and $p^B$ represent the total processing time of $A$-jobs and $B$-jobs, respectively, (2) $B$-jobs are scheduled in a non-decreasing order of $d_i^B - p_i^B$ (i.e. MST order), and (3) $A$-jobs are scheduled in MST order. |
|                                                                       |                                      | Mosheiov               |                                                                  |
|                                                                       |                                      |                        |                                                                  |
| $4.1 \mid \beta_{\max}^B \leq Q \mid \sum w_i^A \cdot \beta_i^A$   | strongly hard                      | Mor and (2010)         | A WSPT and MST based HDuristic.                                  |
|                                                                       |                                      | Mosheiov               |                                                                  |
|                                                                       |                                      |                        |                                                                  |
| $5.1 \mid \gamma_{\max}^B \leq Q, nr - a \mid \sum C_i^A$          | strongly hard                      | Yazdani and Jolai (2013)| Genetic Algorithm.                                              |

3.3. **Number of tardy jobs.** The number of tardy jobs refers to the unit penalty of the job. It is defined as

$$U_j = \begin{cases} 1, & \text{if } C_j > d_j \\ 0, & \text{otherwise} \end{cases}$$

Moore (1968) introduced an exact algorithm for solving the problem $1 || \sum U_j$ optimally in $O(n \log n)$ time. In an optimal schedule, the first set of jobs finishes on
time, while the second set of jobs finishes after the due date. Karp (1972) proved the problem $1\parallel \sum w_jU_j$ is NP-hard.

Allesandro Agnetis et al. (2004) investigated three types of two-agent scheduling problems with the objectives of minimizing the number of tardy jobs. These three problems were denoted by $1\mid \sum U_j^B \leq Q \mid \sum U_i^A$ (Problem 1); $1\mid \sum U_j^B \leq Q \mid \sum C_i^A$ (Problem 2); and $1\mid \sum w_i^A C_i^A$ (Problem 3). In the paper, they proved and summarized the computational complexity for these problems. The reference theorems were also given by authors. They proposed that Problem 1 can be optimally solved by using the dynamic programming in $O(n^3)$. In addition, in an optimal solution, (1) all early jobs are scheduled consecutively in EDD order at the beginning of the schedule, and (2) all tardy jobs are scheduled consecutively at the end of the schedule. The computational complexity of Problem 2 and Problem 3 are open and binary NP-hard, respectively. Ng, Cheng, and Yuan (2006) proposed and proved a lemma for Problem 2. The lemma states that in the optimal solution, all A-jobs are ordered in the Shortest Processing Time first (SPT) rule, and all B-jobs are scheduled in the Earliest Due Date first (EDD) rule. Furthermore, they developed a pseudo polynomial time algorithm for the problem to search an optimal solution. Ng et al. (2006) also proved that the problem $1\mid \sum U_j^B = 0 \mid \sum w_i^A C_i^A$ is strongly NP-hard.

Lee, Chung, and Huang (2013) studied a single-machine two-agent scheduling problem to minimize a linear combination of the maximum lateness and total completion time of one agent, subject to zero tardy jobs for the other agent. This problem is denoted by $1\mid \sum U_j^B = 0 \mid \theta \sum C_i^A + (1 - \theta)L_{max}^A$ where $0 < \theta < 1$. A Branch-and-Bound algorithm and a simulated annealing (SA) algorithm were proposed by Lee et al. (2013) to solve the problem. The Earliest Due Date first (EDD) rule and the Shortest Processing Time first (SPT) rule-based SA algorithms were designed to search near-optimal solutions. Three neighborhood generation methods were considered in the SA algorithm: the pairwise interchange (PI), the extraction and forward-shift reinsertion (EFSR), and the extraction and backward-shifted reinsertion (EBSR). The SA algorithm was able to achieve a solution within 0.5% of the optimal solution.

Wu (2013) studied a two-agent scheduling problem, denoted by $1\mid \sum U_j^B = 0 \mid \sum T_i^A$, with the objective of total tardiness and number of tardy jobs. Since the total tardiness scheduling problem without multi-agent was proved to be NP-hard, this problem is NP-hard as well. A branch-and-bound method was proposed for searching an optimal solution, and a Genetic algorithm (GA) was proposed for finding a near-optimal solution.

Reisi-Nafchi and Moslehi (2015) extended the study of Wu (2013). They studied a two-agent scheduling problem with a weighted number of tardy jobs and weighted lateness objectives. This problem was denoted by $1\mid \sum w_j^B U_j^B \leq Q \mid \sum w_i^A L_i^A$. This paper proved that there exists an optimal solution, if (1) A-jobs are scheduled in WSPT order; (2) in which the agent B tardy accepted orders are sequenced arbitrarily at the end of sequence; (3) in which the global arrangement of the agent B non-tardy accepted orders follows the EDD order and they are replaced at the last
possible positions before their due time. They proved that the problem is NP-hard and proposed an integer linear programming model to find an optimal solution. A hybrid meta-heuristic algorithm combining a genetic algorithm and linear programming method was developed for the purpose of solving large size problem instances. Although the proposed hybrid meta-heuristic is based on the Genetic Algorithm, it is more powerful and more efficient as the numerical experiment.

The problem $1 \mid \sum U^B_j = 0, r, p = \alpha + \beta t \mid \sum w^A_i C^A_i$ was extended and studied by Lee, Wang, Shiau, and Wu (2010), where a linear deterioration assumption was taken into consideration. They assumed that a jobs normal processing time is $\alpha_j$ and that all jobs have a common deterioration rate $\beta$. The actual processing time for a job is determined by formula $\alpha_j + \beta t$, where $t$ denotes the jobs starting time. In such a case, an optimal schedule exists such that job $i$ is the preceding job of job $j$ when $r_i + p_i \leq r_j$. They developed a Branch-and-Bound algorithm and three effective heuristics for searching the lower bound and approximate solutions, respectively.

A summary of the number of tardy jobs related to the two-agent scheduling problems reviewed is provided in Table 4.

3.4. Late work criteria. The late work criteria first proposed by Blazewicz (1984). Van Wassenhove and Potts (1991) studied a total late work minimization problem. Potts and Van Wassenhove (1992a) proposed a branch-and-bound method in order to solve the problem. Hariri, Potts, and Van Wassenhove (1995) introduced a dynamic programming approach to solve the binary NP-hard problem $1 \mid \sum w_j V_j$ in $O(n^2 \sum p_j)$ time. The late work criteria is a parameter used to measure the amount of processing performed after the due date (Potts & Van Wassenhove, 1992b). It combines the characteristics of job tardiness and the number of tardy jobs (Sterna, 2011). The late work criteria (i.e. $V_j$) is defined as:

$$V_j = \begin{cases} 
0, & C_j \leq d_j \\
C_j - d_j, & d_j < C_j < d_j + p \\
p_j, & d_j + p \leq C_j 
\end{cases}$$

The late-work-criteria-related optimization problems are considered in many control systems (Blazewicz, Drozdzowski, Formanowicz, Kubiak, & Schmidt, 2000; Potts & Van Wassenhove, 1992b), such as, the ERP system and the MRP system. Sterna (2011) provided a systemic review of late-work-criteria-related scheduling problems. The paper investigated the literature published between 1984 and 2009. This section presents one two-agent-based late work related scheduling paper, which were not reviewed in Sterna (2011)s survey.

Xingong and Yong (2017) studied the problem $1 \mid f^B_{\text{max}} \leq Q, \sum w^A_i V^{A}_i$. They proved that this is NP-hard, and it can be solved in $O(n^2 Q(\sum p^A_i + \sum p^B_j) + n^B \log n^B)$ time. Two special cases of the problem were considered, the first case considers assigning all A-jobs with a common due date (i.e. $1 \mid f^B_{\text{max}} \leq Q, d^A_i = d^A, \sum w^A_i V^{A}_i$), and the second case considers assigning all A-jobs a common processing time (i.e. $1 \mid f^B_{\text{max}} \leq Q, p^A_i = p^A, \sum w^A_i V^{A}_i$). In the first case, the problem can be solved in $O(n^A \log n^A + n^B \log n^B)$ time. If the pre-emptive jobs are allowed in this case, there exists an optimal solution that all B-jobs are scheduled in the reserved interval it is associated with. Furthermore, the pre-emption constraint does not impact the optimal solution. In other words, the total weighted late works
are same, no matter if the pre-emption constraint is considered or not. The second special case can be solved in $O(n_A^3 + n_B \log n_B)$ time.

Another late work criteria related paper was published by Wang et al. (2017). They studied the problem $1 \mid L_{\text{max}}^B \leq Q \mid \sum V_i^{A}$, proved its computational complexity and proposed two exact solution method based on branch-and-bound dynamic programming algorithms to solve the problem. A detailed review was presented in Section 3.1.

A summary of the late-work criteria related to two-agent scheduling problem is provided in Table 5.

### 4. Minimality model

The goal of this type of two-agent scheduling problem is to assign some weight to the objective function of each agent and minimize the weighted objectives. In other words, the overall objective is a weighted sum of performance measures (Baker & Cole Smith, 2003).
Table 5. Summary of reviewed late work criteria related two-agent scheduling problems.

| Problem | Complexity | Reference | Approach/Result |
|---------|------------|-----------|-----------------|
| 1. $f_{\text{max}}^B \leq Q \mid \sum \omega_i^A V_i^A$ | $O(n_A^2 Q \left( \sum_{j=1}^{n_A} p_j^A + \sum_{j=1}^{n_B} p_j^B \right) + n_B \log n_B)$ | Xiong and Yong (2017) | In an optimal schedule, all B-jobs are scheduled in the reserved interval it associated with. The pre-emption constraint does not impact the optimal solution |
| 2. $f_{\text{max}}^B \leq Q, d_i^A = \sum \omega_i^A V_i^A$ | $O(n_A \log n_A + n_B \log n_B)$ | | |
| 3. $f_{\text{max}}^B \leq Q, p_i^A = p_i^A$ | $O(n_A^3 + n_B \log n_B)$ | | |
| 4. $l_{\text{max}}^B \leq Q \mid \sum V_i^A$ | NP-hard | Wang et al. (2017) | Branch-and-Bound method; Tabu-Search Heuristic; In an optimal schedule, (1) all A-jobs and B-jobs are processed consecutively without idle time and the first job starts at time 0; (2) all early and partially early A-jobs and all B-jobs are scheduled before all tardy A-jobs; (3) the early and partially early A-jobs are scheduled in EDD order; (4) the B-jobs are scheduled in non-decreasing order of $Q - d_i^B$ |

4.1. **Maximum lateness.** With respect to the single-machine with two-agent scheduling problem, Baker and Cole Smith (2003) introduced different problems with multiple objective functions, such as maximum lateness, the makespan, and the weighted completion time etc. This paper is the first study on the two-agent scheduling problem. In the paper, all problems followed the second research direction, where the overall objective of both agents is minimized by assigning weights to the objective function of each agent. The computational complexity of problem $1 || (\sum \omega_i^A C_i^A, L_{\text{max}}^B)$ has been proved by Baker and Cole Smith (2003) to be NP-complete. Soltani, Jolai, and Zandieh (2010) proposed a Genetic Algorithm (GA) and a Hybrid Kangaroo Simulated Annealing (HKSA) algorithm to solve the problem. The proposed GA algorithm applied three disparate crossover methods (i.e. uniform crossover, one-point crossover and two-point crossover) and three disparate mutations operations (i.e. pair-wise interchange mutation, inverse mutation and shift mutation) for the propose of generating good solutions. The problem $1 || (\sum C_i^A, L_{\text{max}}^B)$ was also studied by Baker and Cole Smith (2003). This problem can be solved optimally when the A-jobs are ordered in SPT rule and processed consecutively, and the B-jobs are ordered in EDD rule.
A summary of studied scheduling problems along with computational complexity, approach/result is provided in Table 6.

Table 6. Summary of two-agent scheduling problems, computational complexity and approach/result studied by Baker and Smith (2003).

| Problem | Complexity | Approach/Result |
|---------|------------|-----------------|
| 1. $1 || (L_{max}^A, L_{max}^B)$ | NP-hard | Processing the jobs of two agents in EDD rule; Using dynamic programming to find an optimal solution. |
| 2. $1 || (C_{max}^A, L_{max}^B)$ | $O(n_2 \log n_2)$ | Processing the jobs of the first agent consecutively; Processing the jobs of the other agent in EDD rule; Polynomial time algorithm. |
| 3. $1 || \sum w_i^A c_i^A, L_{max}^B$ | NP-complete | Processing the jobs of the first agent may not in WSPT rule; Processing the jobs of the other agent in EDD rule; For a special case with equal weights, using SPT and EDD rule. It can be solved by using the dynamic programming. |
| 4. $1 || (C_{max}^A, L_{max}^B, \sum w_i^C c_i^C)$ | NP-hard | Processing the jobs of the first agent consecutively; Combining the contributions of the objectives of the first and third agent; |

A two-agent problem with release date and pre-emption was studied by Yuan et al. (2015) to minimize the maximum lateness of jobs from both agents simultaneously. The problem can be denoted by $1 | r_{ij}, pmtu | (L_{max}^A, L_{max}^B)$. This is a Pareto Optimization problem and the aim of this problem is to search all the Pareto points. The Earliest Due Date first (EDD) rule was employed in order to search the Pareto optimal points. Furthermore, authors proved that this problem can be solved by the Algorithm Pareto-List-Revising in $O(n_A n_B n \log n)$ time.

4.2. Earliness, lateness and tardiness. T. E. Cheng, Liu, Lee, and Ji (2014) addressed a weighed combination of two-agent scheduling problems. The objective was to minimize the weighted sum of the total completion time of one agent and the total tardiness of the other agent. This problem is denoted by $1 || \theta \sum C_i^A + (1 - \theta) \sum T_j^B$, where $0 \leq \theta \leq 1$. A branch-and-bound method was developed for finding the optimal solution. In the paper, T. E. Cheng et al. (2014) proved the properties for optimal solutions as (1) if jobs $i$ and $j$ are belonging the agent A, and the processing time of job $i$ is less than job $j$, then job $i$ should be scheduled before job $j$; (2) if jobs $i$ and $j$ are belonging to the agent B, the processing time of job $i$ is less than job $j$, and the due time of job $i$ is less than job $j$, then job $i$ should be scheduled before job $j$. In the branch-and-bound method, the solution of the genetic algorithms was implemented in the initialization step as an initial incumbent solution. The proposed branch-and-bound method can solve up to 40 jobs in reasonable time. In addition, many solution methods were proposed for the purpose of solving the problem. These solution methods include a SPT-based heuristic, an EDD-based heuristic, a simulated annealing (SA) algorithm and a genetic algorithm (GA). Compared with the proposed SA algorithm, the GA algorithms have a better performance, since these significantly improves the quality of solution while consuming less CPU time.
Wu, Yin, Wu, Wu, and Hsu (2014) studied a two-agent scheduling problem with increasing linear deterioration of job processing time. The objective is to minimize the sum of maximum weighted tardiness of one agent and total weighted tardiness of the other agent. They proved that this was NP-hard. For solving this problem, they developed a branch-and-bound method, an ant colony optimization algorithm, and a simulated annealing algorithm. In this paper, the actual processing of job $j$ was defined as $p_j(t) = p_j(a + bt)$, where $a$ and $b$ were positive constants, and $t$ was the start processing time of job.

A summary of the earliness, lateness and tardiness related to two-agent scheduling problem (Minimality Model) is provided in Table 7.

### Table 7. Summary of reviewed late work criteria related two-agent scheduling problems (Minimality Model).

| Problem | Complexity | Reference | Approach/Result |
|---------|------------|-----------|-----------------|
| $1||\theta \sum C_i^A + (1-\theta) \sum W_j^B$ | NP-hard | T. E. Cheng et al. (2014) | Branch-and-Bound method; Genetic Algorithm; Simulated Annealing algorithm; In an optimal solution, (1) if jobs $i$ and $j$ are belongling the agent $A$, $p_i < p_j$, then job $i$ should be scheduled before job $j$; (2) if jobs $i$ and $j$ are belongling the agent $B$, $p_i < p_j$ and $d_i < d_j$, then job $i$ should be scheduled before job $j$. |
| $2||\sum w_i^A T_i^A + \sum w_j^B T_j^B$ | NP-hard | Wu et al. (2014) | Branch-and-Bound method; Ant Colony Optimization algorithm; Simulated Annealing algorithm. |

5. **Future studies.** The literature survey reveals that the due-date related two-agent scheduling problems are important for both researchers and mangers in industries. Obviously, the research in this area has not been carried out to its completeness as many scheduling models still remain untouched, such as minimizing number of tardy jobs and optimizing late work criteria. The production/logistics managers are facing the difficult situations for delivering the product or freight on time to avoid penalties. Therefore, more research needs to be done on the due-date related two-agent scheduling problems.

In literature, many due-date related scheduling models are still not touched. Only two papers have considered the late work criteria based two-agent scheduling problem. Furthermore, algorithms or mathematical models are needed to be proposed for solving existing problems. For instance, although Xingong and Yong (2017) studied the complexities and optimal algorithm properties of three late work criteria based scheduling models, no algorithms were proposed to solve those three problems. A similar situation happened in the study of minimizing number of tardy jobs, the problems $1|\sum U_j^B \leq Q| \sum w_i^A C_i^A$, $1|\sum U_j^B = 0| \sum w_i^A C_i^A$ and $1|\sum U_j^B \leq Q| \sum T_i^A$ have not been solved yet. Therefore, more efforts are needed to be made in order to propose exact algorithms and to solve the problems optimally.

Regarding the scheduling problems which have been solved, there is also necessary to proposed more efficient meta-heuristics. As we all know, majority of two-agent scheduling problems are NP-hard, which means that the exact algorithms have a limited capability to solve large size problems. Hence, more efficient meta-heuristics (such as Genetic Algorithms, Simulated Annealing algorithms, Marriage
in Honey-Bees Optimization algorithms and Ant Colony Optimization algorithms) are required for the purpose of solving large size problems.

Using optimization software for solving scheduling problems is a new trend. As more and more scheduling problems are being formulated in linear integer mathematical models, optimization software such as CPLEX can be used to solve those problems in a more efficient approach.

It is evident from the literature review, the practical application of the problem is missing on most of the papers. Researchers should focus on solving practical problems using the models proposed in the literature. The two-agent scheduling problem can be used in many situations, such as cloud computing, telecommunications and linehaul truck scheduling.

Furthermore, the two-agent scheduling problems with various production environment settings that need to be considered and studied in the future studies, including computational complexity study, algorithm study and review study. A number of publications on two-agent scheduling in parallel machine setting or in flowshop setting are worth to be discussed, such as Yin, Cheng, Wang, and Wu (2017), Yin, Cheng, Cheng, Wang, and Wu (2016), Elvikis and Tkindt (2014) and Elvikis, Hamacher, and Tkindt (2011). The classification approach proposed in this paper can be used in future survey paper to classify two agent scheduling problem in other machine setting environment.

6. Conclusions. In this paper, we review/survey the work done by other researchers on two-agent scheduling with due dates that have appeared since 2003. This survey classifies the literature on two-categories-based objective function settings: feasibility model and minimality model. The feasibility model is further divided in four sub-categories: (1) maximum lateness model, (2) earliness, lateness and tardiness model, (3) number of tardy jobs model, and (4) late work criteria model. The minimality model is further divided in two sub categories: (1) maximum lateness model, and (2) earliness, lateness and tardiness model. Clearly, the number of tardy job and late work criteria related minimality model is missing from literature.

The common proposed algorithms are Branch-and-Bound algorithm, Genetic Algorithm (GA), Simulated Annealing (SA) algorithm and Marriage in Honey-Bees Optimization algorithm (MBO). Apart from these algorithms, Ant Colony Optimization (ACO) algorithm, Dynamic Programming (DP) method and Tabu search were also considered in some papers. Moreover, Reisi-Nafchi and Moslehi (2015) proposed an efficient hybrid meta-heuristic to solve a two agent scheduling problem with the objective of a weighted number of tardy jobs and weighted lateness objectives.

A vast majority of surveyed papers studied both the Maximum Lateness and Earliness, Lateness and Tardiness objectives. Most of these problems have been solved by proposed optimal algorithms and meta-heuristics. In this paper, we have not only reviewed the proposed algorithms but have also summarized the theorems and properties in the optimal schedules. This review paper provides a comprehensive picture for future studies. Although not many papers considered the objective functions of the number of tardy jobs and late work criteria, these two problem categories have great potential, the late work criteria related (i.e. $V_j$) problems in particular. As shown in Table 4, the problem $1 | \sum U_j^B \leq Q | \sum C_i^A$ is still open. Meanwhile, for such problems, as $1 | \sum U_j^B \leq Q | \sum w^A_i C_i^A$ and $1 | \sum U_j^B \leq Q | \sum T_i^A$, in future studies, researchers need to propose exact algorithm(s) to solve these optimally. The
late work criteria objective is a new orientation in the two-agent scheduling study, and so far, only two papers have considered this type of problem. The late-work-criteria-related objective functions are used to evaluate a work schedules quality according to the duration of late parts of jobs. Late work criteria related scheduling problems are able to meet all applications where penalty is considered (Sterna, 2011). More importantly, by minimizing the late work parameter, both customer and manufacturer can get an advantage in minimizing late parts of orders and reducing financial loss, respectively. Applications of late work criteria are easily found in multiple fields, such as control systems, manufacturing systems, agriculture and the land cultivation process optimization.

Acknowledgments. We are thankful to the reviewers for their valuable comments. This research is supported by NSERC discovery Grant 318689.

REFERENCES

[1] A. Agnetis, G. de Pascale and D. Pacciarelli, A Lagrangian approach to single-machine scheduling problems with two competing agents, Journal of Scheduling, 12 (2009), 401–415.
[2] A. Agnetis, P. B. Mirchandani, D. Pacciarelli and A. Pacifici, Scheduling problems with two competing agents, Operations Research, 52 (2004), 229–242.
[3] K. R. Baker and J. Cole Smith, A multiple-criterion model for machine scheduling, Journal of Scheduling, 6 (2003), 7–16.
[4] P. Baptiste, J. Carlier and A. Jouglet, A branch-and-bound procedure to minimize total tardiness on one machine with arbitrary release dates, European Journal of Operational Research, 158 (2004), 595–608.
[5] J. Blazewicz, Scheduling preemptible tasks on parallel processors with information k s. (1984).
[6] J. Blazewicz, M. Drozdowski, P. Formanowicz, W. Kubiak and G. Schmidt, Scheduling preemptable tasks on parallel processors with limited availability, Parallel Computing, 26 (2000), 1195–1211.
[7] P. J. Brewer and C. R. Plott, A binary conflict ascending price (BICAP) mechanism for the decentralized allocation of the right to use railroad tracks, International Journal of Industrial Organization, 14 (1996), 857–886.
[8] S.-R. Cheng, Some new problems on two-agent scheduling to minimize the earliness costs, International Journal of Production Economics, 156 (2014), 24–30.
[9] T. E. Cheng, Y.-H. Chung, S.-C. Liao and W.-C. Lee, Two-agent single-machine scheduling with release times to minimize the total weighted completion time, Computers & Operations Research, 40 (2013), 353–361.
[10] T. E. Cheng, C.-Y. Liu, W.-C. Lee and M. Ji, Two-agent single-machine scheduling to minimize the weighted sum of the agents’ objective functions, Computers & Industrial Engineering, 78 (2014), 66–73.
[11] T. E. Cheng, C. Ng and J. Yuan, Multi-agent scheduling on a single machine with max-form criteria, European Journal of Operational Research, 188 (2008), 603–609.
[12] D. Elvikis, H. W. Hamacher and V. T’kindt, Scheduling two agents on uniform parallel machines with makespan and cost functions, Journal of Scheduling, 14 (2011), 471–481.
[13] D. Elvikis and V. T’kindt, Two-agent scheduling on uniform parallel machines with min-max criteria, Annals of Operations Research, 213 (2014), 79–94.
[14] E. Gerstl and G. Mosheiov, Scheduling problems with two competing agents to minimized weighted earliness-tardiness, Computers & Operations Research, 40 (2013), 109–116.
[15] E. Gerstl and G. Mosheiov, Single machine just-in-time scheduling problems with two competing agents, Naval Research Logistics (NRL), 61 (2014), 1–16.
[16] A. M. Hariri, C. N. Potts and L. N. Van Wassenhove, Single machine scheduling to minimize total weighted late work, ORSA Journal on Computing, 7 (1995), 232–242.
[17] W. Horn, Some simple scheduling algorithms, Naval Research Logistics (NRL), 21 (1974), 177–185.
[18] R. M. Karp, Reducibility among combinatorial problems, Complexity of Computer Computations(Proc. Sympos., IBM Thomas J. Watson Res. Center, Yorktown Heights, N. Y., 1972), Springer,(1972), 85–103.
[19] S. Kirkpatrick, C. D. Gelatt and M. P. Vecchi, Optimization by simulated annealing, *Science, 220* (1983), 671–680.

[20] W.-C. Lee, Y.-H. Chung and M.-C. Hu, Genetic algorithms for a two-agent single-machine problem with release time, *Applied Soft Computing, 12* (2012), 3580–3589.

[21] W.-C. Lee, Y.-H. Chung and Z.-R. Huang, A single-machine bi-criterion scheduling problem with two agents, *Applied Mathematics and Computation, 219* (2013), 10831–10841.

[22] W.-C. Lee, W.-J. Wang, Y.-R. Shiau and C.-C. Wu, A single-machine scheduling problem with two-agent and deteriorating jobs, *Applied Mathematical Modelling, 34* (2010), 3098–3107.

[23] J. Y.-T. Leung, M. Pinedo and G. Wan, Competitive two-agent scheduling and its applications, *Operations Research, 58* (2010), 458–469.

[24] H. Li, Y. Gajpal and C. Bector, Single machine scheduling with two-agent for total weighted completion time objectives, *Applied Soft Computing, 70* (2018), 147–156.

[25] Y. Lun, K. Lai, C. Ng, C. Wong and T. Cheng, Editorial: Research in Shipping and Transport Logistics, *International Journal of Shipping and Transport Logistics, 2011*.

[26] M. Mezmaz, N. Melab, Y. Kessaci, Y. C. Lee, E.-G. Talbi, A. Y. Zomaya and D. Tuyttens, A parallel bi-objective hybrid metaheuristic for energy-aware scheduling for cloud computing systems, *Journal of Parallel and Distributed Computing, 71* (2011), 1497–1508.

[27] J. M. Moore, An n job, one machine sequencing algorithm for minimizing the number of late jobs, *Management Science, 15* (1968), 102–109.

[28] B. Mor and G. Mosheiov, Scheduling problems with two competing agents to minimize minmax and minsum earliness measures, *European Journal of Operational Research, 206* (2010), 540–546.

[29] B. Mor and G. Mosheiov, A two-agent single machine scheduling problem with due-window assignment and a common flow-allowance, *Journal of Combinatorial Optimization, 33* (2017), 1454–1468.

[30] C. Ng, T. C. Cheng and J. Yuan, A note on the complexity of the problem of two-agent scheduling on a single machine, *Journal of Combinatorial Optimization, 12* (2006), 387–394.

[31] S. Pandey, L. Wu, S. M. Guru and R. Buyya, A particle swarm optimization-based heuristic for scheduling workflow applications in cloud computing environments, Paper presented at the Advanced information networking and applications (AINA), 2010 24th IEEE international conference on, 2010.

[32] J. M. Peha, Heterogeneous-criteria scheduling: minimizing weighted number of tardy jobs and weighted completion time objectives, *Computers & Operations Research, 22* (1995), 1089–1100.

[33] M. Pinedo, *Scheduling*, Theory, algorithms, and systems. Fourth edition. Springer, New York, 2012.

[34] C. N. Potts and L. N. Van Wassenhove, Approximation algorithms for scheduling a single machine to minimize total late work, *Operations Research Letters, 11* (1992), 261–266.

[35] C. N. Potts and L. N. Van Wassenhove, Single machine scheduling to minimize total late work, *Operations Research, 40* (1992), 586–595.

[36] M. Reisi-Nafchi and G. Moslehi, A hybrid genetic and linear programming algorithm for two-agent order acceptance and scheduling problem, *Applied Soft Computing, 33* (2015), 37–47.

[37] R. Soltani, F. Jolai and M. Zandieh, Two robust meta-heuristics for scheduling multiple job classes on a single machine with multiple criteria, *Expert Systems with Applications, 37* (2010), 5951–5959.

[38] M. Soomer and G. J. Franx, Scheduling aircraft landings using airlines’ preferences, *European Journal of Operational Research, 190* (2008), 277–291.

[39] M. Sterna, A survey of scheduling problems with late work criteria, *Omega, 39* (2011), 120–129.

[40] V. Suresh and D. Chaudhuri, Bicriteria scheduling problem for unrelated parallel machines, *Computers & industrial engineering, 30* (1996), 77–82.

[41] L. N. Van Wassenhove and C. N. Potts, Single machine scheduling to minimize total late work, *Oper. Res., 40* (1992), 586–595.

[42] D.-J. Wang, C.-C. Kang, Y.-R. Shiau, C.-C. Wu and P.-H. Hsu, A two-agent single-machine scheduling problem with late work criteria, *Soft Computing, 21* (2017), 2015–2033.

[43] D.-J. Wang, Y. Yin, S.-R. Cheng, T. Cheng and C.-C. Wu, Due date assignment and scheduling on a single machine with two competing agents, *International Journal of Production Research, 54* (2016), 1152–1169.
[44] D.-J. Wang, Y. Yin, J. Xu, W.-H. Wu, S.-R. Cheng and C.-C. Wu, Some due date determination scheduling problems with two agents on a single machine, *International Journal of Production Economics*, 168 (2015), 81–90.
[45] W.-H. Wu, An exact and meta-heuristic approach for two-agent single-machine scheduling problem, *Journal of Marine Science and Technology*, 21 (2013), 215–221.
[46] W.-H. Wu, Y. Yin, W.-H. Wu, C.-C. Wu and P.-H. Hsu, A time-dependent scheduling problem to minimize the sum of the total weighted tardiness among two agents, *Journal of Industrial & Management Optimization*, 10 (2014), 591–611.
[47] Z. Xingong and W. Yong, Two-agent scheduling problems on a single-machine to minimize the total weighted late work, *Journal of Combinatorial Optimization*, 33 (2017), 945–955.
[48] M. Yazdani and F. Jolai, A Genetic Algorithm with Modified Crossover Operator for a Two-Agent Scheduling Problem, *Shiraz Journal of System Management*, 1 (2013), 1–13.
[49] Y. Yin, S.-R. Cheng, T. Cheng, D.-J. Wang and C.-C. Wu, Just-in-time scheduling with two competing agents on unrelated parallel machines, *Omega*, 63 (2016), 41–47.
[50] Y. Yin, S.-R. Cheng, T. Cheng, W.-H. Wu and C.-C. Wu, Two-agent single-machine scheduling with release times and deadlines, *International Journal of Shipping and Transport Logistics*, 5 (2013), 75–94.
[51] Y. Yin, T. Cheng, D.-J. Wang and C.-C. Wu, Two-agent flowshop scheduling to maximize the weighted number of just-in-time jobs, *Journal of Scheduling*, 20 (2017), 313–335.
[52] Y. Yin, W. Wang, D. Wang and T. Cheng, Multi-agent single-machine scheduling and unrestricted due date assignment with a fixed machine unavailability interval, *Computers & Industrial Engineering*, 111 (2017), 202–215.
[53] Y. Yin, C.-C. Wu, W.-H. Wu, C.-J. Hsu and W.-H. Wu, A branch-and-bound procedure for a single-machine earliness scheduling problem with two agents, *Applied Soft Computing*, 13 (2013), 1042–1054.
[54] Y. Yin, W.-H. Wu, S.-R. Cheng and C.-C. Wu, An investigation on a two-agent single-machine scheduling problem with unequal release dates, *Computers & Operations Research*, 39 (2012), 3062–3073.
[55] Y. Yin, W.-H. Wu, T. Cheng, C.-C. Wu and W.-H. Wu, A honey-bees optimization algorithm for a two-agent single-machine scheduling problem with ready times, *Applied Mathematical Modelling*, 39 (2015), 2587–2601.
[56] J. Yuan, C. Ng and T. E. Cheng, Two-agent single-machine scheduling with release dates and preemption to minimize the maximum lateness, *Journal of Scheduling*, 18 (2015), 147–153.
[57] F. Zhang, C. Ng, G. Tang, T. Cheng and Y. Lun, Inverse scheduling: Applications in shipping, *International Journal of Shipping and Transport Logistics*, 3 (2011), 312–322.

Received November 2017; 1st revision July 2018; 2nd revision September 2018.

E-mail address: lih34526@myumanitoba.ca
E-mail address: Yuvraj.Gajpal@umanitoba.ca
E-mail address: Chhajju.Bector@umanitoba.ca