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Investigation of Size Effect on the Pull-in Instability of Beam-type NEMS under van der Waals Attraction

Y. Tadi Benia*, M.R. Abadyanb, Aminreza Noghrehabadi

aFaculty of Engineering, University of Shahrekord, Shahrekord, Iran
bMechanical Engineering Group, Ramsar branch, Islamic Azad university, Ramsar, Iran
cDepartment of Mechanical Engineerin, Shahid Chamran University, Ahvaz, Iran

Abstract

When the dimensions decrease to nano-scale, many essential phenomena appear which are not important at macro scales. In this paper, two effects of these sub-micron phenomena are demonstrated and considered for simulation of pull-in instability of beam-type nano-actuator. The first phenomenon that becomes important at nano-distances is the presence of dispersion forces such as van der Waals attraction. The second effect that appears at nano scale is the size dependency of material characteristics. The obtained results agree well with numerical solution and other models in the literature.

Keywords: NEMS; MAD; Modified couple stress theory; Size effect; van der Waals force

1. Introduction

Beam-type NEMS are increasingly used in many applications such as electrical, medical, optical and microscopy devices [1,2]. Consider a beam-type actuator constructed from two conductive electrodes which one is fixed and the other is movable. Applying voltage difference between these electrodes causes the movable electrode to deflect towards the fixed electrode (ground electrode), because of the electrostatic forces. At a critical voltage, which is known as pull-in voltage, the electrode becomes unstable and pulls-in onto the ground electrode. The pull-in behavior of micro-electromechanical systems (MEMS) has been studied for over two decades without considering nano-scale effects [3-5]. With

* Corresponding author. Tel.: +98-381-4424401; fax: +98-381-4424438.
E-mail address: tadi@eng.sku.ac.ir.
decrease in dimensions to nano-scale, many essential phenomena appear which are not important at macro scales. In this paper, two effects of these sub-micron phenomena are demonstrated and considered for simulation of pull-in instability of beam-type nano-actuator.

The first phenomenon that becomes important at nano-distances is the presence of dispersion forces such as van der Waals attraction. This attraction can significantly influence the NEMS performance when the initial gap between the components of nanoactuator is typically below several ten nanometers [2]. In this case, the attraction between two surfaces is proportional to the inverse cube of the separation and is affected by material properties [6]. Rotkin [7] obtained an analytical relation to express the effect of van der Waals force on the pull-in gap and voltage of a nano-actuator. Ramezani et al. [8] applied distributed parameter model to study the pull-in instability of nano-cantilevers.

The second effect that appears at nano scale is the size dependency of material characteristics. The classical continuum mechanics theory is not able to explain the size-dependent behavior of materials and structures at sub-micron distances. To overcome this problem, non-classical continuum theories such as couple stress [9] are developed considering the size effects. Some of the important experimental works in this area were made by Fleck et al. [10], Stolken and Evans [11], Chong and Lam [12] and McFarland and Colton [13]. These experimental results demonstrate that the size dependency is an inherent property of conductive metals when the characteristic size of the structures is comparable to the internal material length scale. For beam-type nanostructures, the characteristic size is usually the beam thickness [14] and is in the order of the metal length scale parameter. Therefore, the size dependent deformation must be considered in modeling the instability of nano-structures [15]. Recently, new modified couple stress theory has been proposed by Yang et al. [16]. In this theory, two classical material constants in couple stress theory are reduced to only one single additional internal material length scale parameter. In this view, this non-classical elasticity theory has been applied to investigate Euler micro-beams by many researchers such as Kong et al. [14,17].

To the knowledge of the authors, none of the two mentioned phenomena have contributed together in any of the pull-in models proposed by previous researchers. In this study, modified couple stress theory is introduced to demonstrate the effects of van der Waals attraction and size dependency together on the pull-in behavior of beam-type NEMS for the first time. The Euler beam model is applied as a time-saving continuum approach to obtain constitutive governing equations [18-20]. In order to solve the constitutive equation of nano-structures, modified Adomian decomposition (MAD) is utilized. The MAD results are compared with the numerical data as well as other results reported in the literature.

2. Preliminaries

The modified couple stress theory was presented by Yang et al. [16], in which the strain energy density is written as

\[ \bar{w} = \frac{1}{2} (\sigma : \varepsilon + m : \chi) \]  

(1)

where the stress tensor \( \sigma \), strain tensor \( \varepsilon \), deviatoric part of the couple stress tensor \( m \) and symmetric curvature tensor \( \chi \) are defined by the following:

\[
\sigma = \lambda \text{tr}(\varepsilon) I + 2\mu \varepsilon, \quad \varepsilon = \frac{1}{2} \left( (\nabla u) + (\nabla u)^T \right) \\
m = 2l^2 \mu \chi, \quad \chi = \frac{1}{2} \left( (\nabla \theta) + (\nabla \theta)^T \right)
\]

(2)

where \( \lambda, \mu \) and \( l \) are Lame constant, shear modulus and the material length scale parameter, respectively.
For small deformation in elastic range which is considered here by neglecting the Poisson’s effect to facilitate the formulation of a simple beam theory, it is yield [14]

\[ \varepsilon_{xx} = -z \frac{\partial^3 w}{\partial X^2} \frac{E}{X^2} \]
\[ \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{yy} = \varepsilon_{yy} = \varepsilon_{zz} = 0 \]
\[ \sigma_{xx} = -Ez \frac{\partial^3 w}{\partial X^2} \frac{E}{X^2} \]
\[ \sigma_{yy} = \sigma_{zz} = \sigma_{yy} = \sigma_{yy} = \sigma_{zz} = \sigma_{yy} = 0 \]

where \( E \) is Young’s modulus. Same as above one conclude that [14]

\[ \chi_{yy} = -\frac{1}{2} \frac{\partial^3 w}{\partial X^2} \frac{X}{g} \]
\[ m_{yy} = -\mu l^2 \frac{\partial^3 w}{\partial X^2} \frac{X}{g} \]

3. Governing equation

Figure 1 show clamp-clamp supported (CC) NEMS actuators. The actuators are modelled by a beam of length \( L \) with a uniform rectangular cross section of width \( B \) and thickness \( H \) which are suspended over a conductive substrate. Note that considering axial tractions and forces along the beam is out of the scope of this work and these effects will be considered in further publications.

In order to develop the governing equation of the beams, we apply the minimum energy principle which implies equilibrium when the free energy reaches a minimum value. Substituting Eqs. (4-5) into Eq. (1), integrating and considering the work done by external forces, the energy of the system can be obtained, then Hamilton principle can be applied to determine the governing equilibrium equation as:

\[ (EI + \mu A l^2) \frac{d^4 w}{dx^4} = \frac{E_0 B_v^2}{2(g-w)^2} \left[ 1 + 0.65 \left( \frac{g-w}{B} \right) \right] + \frac{\bar{A} B}{6 \pi (g-w)^3}, \quad w(0) = w(L) = \frac{dw(0)}{dx} = \frac{dw(L)}{dx} = 0 \]

where \( I \) is the second moment of cross-sectional area around \( Z \)-axis, \( A \) is the cross-sectional area of the beam, in Eq. (6) first and second right hand terms are the electrostatic force and van der Waals forces per unit length of the beam, respectively. \( E_0 = 8.854 \times 10^{-12} \) C^2 N^{-1} m^{-2} is the permittivity of vacuum, \( V \) is the applied voltage, \( g \) is the initial gap between the movable and the ground electrode and \( \bar{A} \) is the Hamaker constant. In this equation, the constitutive material of the nano-actuator is assumed linear elastic and only the static deflection of the nano-beam is considered. The electrostatic force enhanced with first order fringing correction [21]. In this study, only the nanoactuators that are wider than the separation (\( g/w \leq 1 \)) are considered [22]. One can use the substitutions \( \tilde{w} = w/g \) and \( x = X/L \) to transform Eq. (6) into the following dimensionless equation:

\[ \frac{d^4 \tilde{w}}{dx^4} = \frac{\alpha}{(1 - \tilde{w}(x))^3 (1 + \delta)} + \frac{\beta}{(1 - \tilde{w}(x))^2 (1 + \delta)} + \frac{\gamma \beta}{(1 - \tilde{w}(x))(1 + \delta)} \]
In above equations, the non-dimensional parameters, $\alpha$, $\beta$, $\gamma$ and size effect parameter $\delta$ are defined as

$$\alpha = \frac{ABL^4}{6\pi \gamma^2EI}, \quad \beta = \frac{\varepsilon BV L^2}{2g^2 E I}, \quad \gamma = 0.65 \frac{g}{B}, \quad \delta = \frac{\mu A L^2}{EI}$$ (8)

Relations (7-8) present the governing equation of beam-type nanostructures. In order to study pull-in behavior of nanostructures, Eq. (7) solved numerically using MAPLE commercial software. Furthermore, MAD is applied to the boundary value problem and the analytical results are compared with those of numerical solution in the following section (for detail of MAD see [20]). Geometry and constitutive material of the beams are identified in Table 1.

Table 1. parameters and material properties of the nano-beam

| Material Properties | Geometrical Dimensions |
|---------------------|------------------------|
| $E$ (GPa)           | $v$                    | $L$ (µm) | $W$ (µm) | $H$ (µm) | $g$ (µm) |
| 77                  | 0.33                   | 300      | 0.5      | 1        | 2.5      |

The comparison between pull-in voltages obtained by MAD and those of the literature is presented in Table 2. It reveals that the difference between MAD and numerical value is within the range of those of other methods presented in the literature.

Table 2. pull-in voltage($V$) comparison for cantilever beam, vdW force is neglected

| Ref. [3] | Ref. [23] | Ref. [24] | Ref. [8] | Numerical | MAD |
|----------|-----------|-----------|----------|-----------|-----|
| 1.23     | 1.20      | 1.21      | 1.29     | 1.24      | 1.27|

4. Results and discussion

In order to investigate the pull-in behavior of the NEMS thoroughly, one case including clamped-clamped is investigated. Figure 2 show the centerline deflection of clamped-clamped. Increase of voltage from zero to $\beta_{PI}$, increases $\dot{w}$ from its initial value to the pull-in deflection. While the maximum deflection of clamped-clamped occurs at $x=0.5$. Effect of van der Waals force on the pull-in behavior of clamped-clamped is illustrated in figure 3. This figure presents the obtained results for various $\alpha$ values and depict that the intermolecular force decreases the pull-in deflection and voltage of the nano-actuators.

![Fig. 2. Deflections of nano-beam ($\delta=0$, $\alpha=25$)](image)
Figures 2 and 3 also reveal that the beam has an initial deflection due to the presence of intermolecular force even without applying voltage ($\beta = 0$).

![Deflection graph](image1)

Fig. 3. Deflection as a function of $\beta$ for different $\alpha$ values in nano-beam with $g/W=1$

Figure 4 show the strong size dependency of the pull-in voltage for clamped-clamped. As seen from this figure, the size effect highly influences $\beta_{PI}$ of nanostructures. By increasing $\delta$ from 0 to 0.5, $\beta_{PI}$ increases more than 1.8 in the case of clamped-clamped structures. On the other hand, the pull-in deflection is less sensitive to the size effect in comparison with the pull-in voltage.

![Deflection graph](image2)

Fig. 4. Deflection of nano-beam ($\delta=0,0.25,0.5$)

5. Conclusions

In this article, modified couple stress theory in conjunction with MAD solving method has been introduced to investigate the effect of van der Waals attraction and size dependency on the nonlinear pull-in behavior of supported NEMS. Results reveal that van der Waals force decreases the pull-in voltage and increase deflection of NEMS in submicron scales. On the other hand, size effect can highly increase the pull-in parameters of nano-beams. The pull-in deflection of NEMS is more sensitive to the size effect compared to the pull-in voltage. The proposed MAD solutions avoid time-consuming numerical iterations and make parametric studies possible.

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