Multi attribute outranking approach for supplier selection

Nitesh Kumar¹, Tarun Soota¹², Neetesh Gupta¹ and Sunil Kumar Rajput¹
1. Department of Mechanical Engineering, Bundelkhand Institute Of Engineering & Technology, Jhansi
2. Corresponding Author
Email-niteshkumar1127@gmail.com, tarunsoota@rediffmail.com

Abstract
Multi attribute outranking approach has been widely used in supply chain management. Compromise ranking and Outranking method has been used extensively for solving the multi criteria decision problems. Elimination and Choice Translating Reality (Electre) and Preference Ranking Organization Method for Enrichment Evaluations (Promethe) are the most widely used outranking methods to solve multi-criteria decision problems. The above methods may be used to solve problems involving both conflicting qualitative and quantitative criteria. This work focuses on the application of improved outranking based method electre, for solving a real problem of supplier selection. A case study is done for selection of supplier for the spherical roller bearing for finding the rank of the alternatives. The need of this case study is to select the best possible supplier out of the available alternative suppliers.

1. Introduction
One major task of the purchasing department is supplier selection which includes the acquisition of required materials, services and equipment for all type of business organization. The increasing importance of supplier selection decision is forcing the organization to rethink their purchasing and evaluation strategies because a successful purchasing decision directly depends on selecting the best supplier to fulfill the strategies goals apart from the operational requirements of the organization. One of the important areas in purchasing research that has significant practical implication is supplier evaluation and selection. The decision of selecting the best supplier from a wide supplier base is an unstructured, complicated and time consuming task. The decision-making process involves evaluation of different alternative based on various criteria. [Chatterjee et al. 2011]. A general decision making process can be divided into the following steps : Define the problem, Determine requirements, Establish goals, Identify alternatives, Define criteria, Select a decision making tool, Evaluate alternatives against criteria and Validate solutions against problem statement.

Roy (1991) introduced the concept of Elimination and Choice Expressing Reality (ELECTRE) which evolved as ELECTRE I, ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE IS and ELECTRE TRI (ELECTRE Tree). This method consists of two sets of parameters: importance coefficient and the veto thresholds. Effect of Normalization Norms in Flexible manufacturing system selection using multi criteria decision making tool Promethe has been investigated by Chatterjee et al 2014.
Various research works have been done in the past on Multi criteria decision making, using outranking methods like Electre, Promethee, etc. In Electre method a pair wise comparison of alternatives in each attribute is done in order to determine partial binary relations denoting the strength of preference of one alternative over the other. Roodhooft et al. (1996) proposed an activity based costing approach for supplier selection and evaluation which allow computation of total cost caused by a supplier in an organization’s production process, thereby increasing the objectivity in the selection process. Savadogo et al. (2006) applied Electre (Elimination and Choice Expressing the Reality) method which would provide a more precise selection of material for a particular application while producing a material selection decision matrix and criteria sensitivity analysis. Almeida et al. (2007) proposed a multi-criteria decision model for outsourcing supplier selection using contribution from utility theory associated with Electre method. Chatterjee et al. (2011) attempted to solve the supplier selection problem using two most potential multi-criterion decision making approach and compare the relative performance for a giving organization environment. Electre and Vikor method are used to rank the alternative supplier for whom several requirement are consider simultaneously. Chatterjee et al. (2014) study on the application of a very popular Multi-Criteria Decision-Making (MCDM) tool, i.e. Elimination and Et Choice Translating Reality (ELECTRE) for solving an automated inspection device selection problem in a discrete manufacturing environment. Kumar et al. (2015) compared the different multi criteria decision making methods (MCDM) such as TOPSIS & VIKOR for the selection of alternative industrial welding machine. Both the method are aggregating function that represent closeness to the ideal solution.

2. ELECTRE METHOD

The ELECTRE (Elimination and Choice Translating Reality) method was first introduced in 1966 to overcome some deficiencies of popularly used MCDM tools to deal with ordinal attributes without the need for transforming them into cardinal values. ELECTRE is a well known MCDM method that has a history of successful real world applications for its robust ranking technique. It has been applied in various types of decision-making situations. The basic concept of the ELECTRE method is to deal with “outranking relations” by using pairwise comparisons among alternatives under each one of the criteria separately. In the ELECTRE approach, the concordance and dis-concordance indexes are used for outranking the supplier in choosing one alternative over the other alternative. In the first stage the opinion of experts and the study of outranking literature is used to recognize variables and effective criteria in supplier selection. The dominating criteria are extracted, which will be used in the evaluation. Thereafter,
the lists of qualified suppliers are identified. The decision criteria identified are then approved by
decision-making team, who assigns the weights on them. Finally the calculated weights of the criteria are
approved by decision making team and the ranks are determined, using ELECTRE method. Based on the
average ranking, then that alternative is selected which has best average rank. (Chatterjee et al 2014)

The basic concept of the ELECTRE method is to deal with "outranking relations" by using pairwise
comparisons among alternatives under each one of the criteria separately.

- Concordance-C (i, j): for any two alternative i and j, this is a weighted measure of the number of criteria
  for which i is preferred over alternative j.
- Discordance-D (I, j): It handles the set of criteria for which i is not preferred over j and gives a measure of
  the degree of “discomfort or discontent” as a result of preferring i to j.

The ELECTRE II method is basically devoted to the ranking problems and the obtained results are in
the form of a total ranking preorder among the non-dominated alternatives. The procedural steps as
involved in ELECTRE II method are presented below

- In the first step determine the objective and to identify the attribute value for each alternative.
- Then develop the initial decision matrix, X

\[
X = [X_{ij}]_{m \times n} = \begin{bmatrix}
X_{11} & X_{12} & \cdots & X_{1n} \\
X_{21} & X_{22} & \cdots & X_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
X_{m1} & X_{m2} & \cdots & X_{mn}
\end{bmatrix}
\tag{1}
\]

where \(x_{ij}\) is the performance value of \(i^{th}\) alternative on \(j^{th}\) criterion, \(m\) is the number of alternatives
compared and \(n\) is the number of criteria.

- Then using the above matrix to develop the normalized decision matrix using the equation (2) The
  purpose of normalization is to calculation of weight of different attribute and obtain dimensionless values
  of different criteria to make them comparable with each other. Several normalization techniques have been
  proposed by the past researchers to transform the different units into dimensionless values. In ELECTRE
  based methods, vector normalization is generally adopted in which each element of the quantified
decision-making matrix is divided by its own Euclidian norm. The norm represents the square root of the
addition of element value squares, according to each criterion.[10]

\[
G = [g_{ij}]_{mn} = X_{ij}/[\sum_{i=1}^{m} X_{ij}^2]^{1/2} \quad 1 \leq i \leq m, \quad 1 \leq j \leq n
\tag{2}
\]
Where $x_{ij} = \begin{cases} \frac{1}{x_{ij}} & \text{for beneficial attribute} \\ x_{ij} & \text{for non-beneficial attribute} \end{cases}$

- Depending upon the relative importance of different attributes obtain weight for each attribute which can be calculated given as

$$w_j = v_j / \sum_{i=1}^{m} v_j \text{ and } \sum_{j=1}^{m} w_j = 1$$

(3)

Where $v_j$ is the variance of each attribute which can be calculated as

$$V_j = \frac{1}{n} \sum_{i=1}^{n} (g_{ij} - g_{ij\text{mean}})^2$$

(4)

- Determine the weighted normalization decision matrix $Y$

$$Y = [y_{ij}]_{m \times n} = w_j \times g_{ij} \quad (i=1,2,\ldots,m; j=1,2,\ldots,n)$$

(5)

Where $g_{ij}$ is the normalized performance value of $i^{th}$ alternative on $j^{th}$ criterion and $w_j$ is the weight of $j^{th}$ criterion.

- Determine the concordance index $c(i,j)$ for every pair of the alternatives $A_i$ and $A_j$

$$c(i,j) = \sum_{k=1}^{n} w_k \text{ for } i,j = 1,2,\ldots,m \text{ and } i \neq j$$

(6)

where $g_k(i)$ and $g_k(j)$ are the normalized measures of performance of the $i^{th}$ and $j^{th}$ alternative respectively with respect to the $j^{th}$ criterion in the decision matrix. Thus, for an ordered pair of alternatives $(A_i, A_j)$, the concordance index $c(i,j)$ is the sum of all the weights for those criteria where the performance score of $A_i$ is at least as that of $A_j$. Clearly, the concordance index lies between 0 and 1.

- The discordance matrix $D$ expresses the degree that a certain alternative $A_i$ is worse than a competing alternative $A_j$. The elements $d_{ij}$ of the discordance matrix are defined as follows:

$$d(i,j) = \begin{cases} 0 & \text{if } y_k(A_i) \geq y_k(A_j) \text{ for all } k \\ \frac{\max(y_k(i)-y_k(j))}{\max(y_k(j)-y_k(i))} & \text{otherwise} \end{cases}$$

(7)

- Compute pure concordance and pure discordance indices as follow.

Pure Concordance index

$$C_j = \sum_{k=1}^{n} c(j,k) - \sum_{j=1}^{m} c(k,j) \quad (j \neq k)$$

(8)

Pure Disconcordance index

$$D_j = \sum_{k=1}^{n} d(j,k) - \sum_{j=1}^{m} d(k,j) \quad (j \neq k)$$

(9)

Once two indices are estimated, two ranking are obtained on the basis of these two indices and an average ranking is determined from these two ranking. Based on the average ranking, then that alternative is selected which has best average rank.
3. Case Study
In order to show the application of ELECTRE method, the supplier selection problem for the Spherical Roller Bearing is considered. The Spherical Roller Bearing (Bearing no-22326) considered has a Bore diameter, Outside diameter, width and radius of 130mm, 280mm, 93mm and 4mm respectively. The data used for supplier selection for spherical roller bearing shown in Table 6.1 has been taken from government workshop located at Jhansi. For this spherical roller bearing there are four alternative suppliers with different criteria. The suppliers are NBC, TIMKEN, FAG and SKF indicated by S1, S2, S3, S4 as per academic interest only. The assumptions for supplier selection are that the delivery time, performance, service and price are similar in nature. Hence the criteria considered for selection of supplier are dynamic load (D), static load (S), mass (M) and reference speed (R). Among these four criteria, D, S, R are beneficial attribute where higher value is desirable and M is non-beneficial attribute requiring smaller value.

### Table 1. Quantitative data for Supplier selection

| Suppliers | Basic Dynamic Loading Rating(KN) | Basic Static Load Rating(KN) | Mass (Kg) | Reference Speed (rpm) |
|-----------|----------------------------------|----------------------------|-----------|-----------------------|
| S1        | 1043                             | 1343.6                     | 26.8      | 1800                  |
| S2        | 1270                             | 952                        | 28.2      | 1900                  |
| S3        | 1250                             | 1370                       | 28        | 2400                  |
| S4        | 1176                             | 1320                       | 29        | 2400                  |

The normalized decision matrix for calculating weight criteria shown in Table 2 calculated by using equation (2) according to beneficial and non beneficial criteria

\[
G = \left[ g_{ij} \right]_{m \times n} = \frac{X_{ij}}{\sum_{i=1}^{m} x_{ij}^2}^{1/2} \\
1 \leq i \leq m \quad 1 \leq j \leq n
\]

(2)

Where \( x_{ij} = \begin{cases} x_{ij} & \text{for beneficial attribute} \\ \frac{1}{x_{ij}} & \text{for non – beneficial attribute} \end{cases} \)

For Supplier S1 beneficial attribute D, S & R

\[
X_{ij} = 1043^2 + 1270^2 + 1250^2 + 1176^2 = 5646225 \quad g_{11} = 1043/[5646225]^{1/2} = 0.4389
\]

Similarly for \( X_{ij} = 6330864.96 \), \( g_{21} = 0.5339 \)
For $X_{ij}^2 = 1837000$ , \[ g_{41} = 0.4199 \]

For Supplier S1 non-beneficial attribute M
\[ 1/ X_{ij}^2 = 5.1144 \times 10^{-3} \]
\[ g_{31} = 0.5224 \]

Table 2. Normalized decision matrix for selection of bearing

| Supplier | D  | S  | M  | R  |
|----------|----|----|----|----|
| S1       | 0.4389 | 0.5339 | 0.5224 | 0.4199 |
| S2       | 0.5344 | 0.3783 | 0.4957 | 0.4433 |
| S3       | 0.5260 | 0.5444 | 0.5001 | 0.5599 |
| S4       | 0.4949 | 0.5246 | 0.4817 | 0.5599 |

The variance of different attribute shown in Table 3 calculated by using equation (4)
\[ V_j = (1/n) \left[ \sum_{i=1}^{n} (g_{ij} - g_{j\text{mean}})^2 \right] \] (4)

For Dynamic Loading
\[ V_1 = [(0.4389-0.4985)^2 + (0.5344-0.4985)^2 + (0.5260-0.4985)^2 + (0.4949-0.4985)^2]/4 = 0.0014 \]

Table 3. Variance of different attribute

| Variance | D  | S  | M  | R  |
|----------|----|----|----|----|
| $V_j$    | 0.0014 | 0.0046 | 0.0002 | 0.0042 |

The weight of different attribute shown in Table 4 calculated by using equation (3)
\[ w_j = V_j \sum_{i=1}^{m} v_j \sum_{i=1}^{m} w_j = 1 \] (3)

\[ w_1 = 0.0014 / 0.0104 = 0.14 \]

Similarly $w_2 = 0.44$, $w_3 = 0.02$, $w_4 = 0.40$

Table 4. Weight of different attributes

| Weight | D  | S  | M  | R  |
|--------|----|----|----|----|
| $W_j$  | 0.14 | 0.44 | 0.02 | 0.40 |

The supplier selection problem is solved by using ELECTRE- II
The normalized decision matrix of ELECTRE Method shown in Table 5 calculated by using equation 2 according to beneficial and non beneficial criteria

| Suppliers | D    | S    | M    | R    |
|-----------|------|------|------|------|
| S1        | 0.4389 | 0.5339 | 0.5224 | 0.4199 |
| S2        | 0.5344 | 0.3783 | 0.4957 | 0.4433 |
| S3        | 0.5260 | 0.5444 | 0.5001 | 0.5599 |
| S4        | 0.4949 | 0.5246 | 0.4817 | 0.5599 |

Weighted normalized matrix shown in Table 7 calculated by using equation (5)

\[
Y = [y_{ij}]_{m \times n} = w_j \times g_{ij} \quad (i=1,2, \ldots m; j=1,2, \ldots n)
\]

Weight criteria for Supplier S1
\[
y_{11} = 0.14 \times 0.4389 = 0.0614
\]
Similarly \( y_{12} = 0.2349, y_{13} = 0.0104, y_{14} = 0.1679 \)

| Supplier | D    | S    | M    | R    |
|-----------|------|------|------|------|
| S1        | 0.0614 | 0.2349 | 0.0104 | 0.1679 |
| S2        | 0.0748 | 0.1664 | 0.0099 | 0.1773 |
| S3        | 0.0736 | 0.2395 | 0.0100 | 0.2239 |
| S4        | 0.0692 | 0.2308 | 0.0096 | 0.2239 |

Concordance matrix shown in Table 8 calculated by using equation (6) While calculating the concordance index, if there are ties between the alternatives, they would receive one half of the criteria weights

\[
c(i,j) = \sum w_k \quad i,j=1,2, \ldots m, i \neq j
\]

\[
g_k(i) \geq g_k(j)
\]
comparing normalized matrix of S1 with S2 then \( g_{12} > g_{21} \) & \( g_{13} > g_{23} \)
the corresponding weight is 0.44 and 0.02
\[ c(1,2) = 0.44 + 0.02 = 0.46 \]

Comparing normalized matrix of S1 with S3 then \( g_{13} > g_{33} \)
the corresponding weight is 0.02
\[ c(1,3) = 0.02 \]

Comparing normalized matrix of S1 with S4 then \( g_{12} > g_{42} \) & \( g_{13} > g_{43} \)
the corresponding weight is 0.44 & 0.02
\[ c(1,4) = 0.44 + 0.02 = 0.46 \]

### Table 8. Concordance matrix

| Supplier | S1   | S2   | S3   | S4   |
|----------|------|------|------|------|
| S1       | -    | 0.46 | 0.02 | 0.46 |
| S2       | 0.54 | -    | 0.14 | 0.16 |
| S3       | 0.98 | 0.86 | -    | 0.80 |
| S4       | 0.54 | 0.84 | 0.20 | -    |

Discordance matrix shown in Table 9 calculated by using equation (7)

\[
d(i,j) = 0 \text{ if } y_k(A_i) \geq y_k(A_j) \text{ for all } k
\]

\[
d(i,j) = \frac{(\max(y_k(A_i) - y_k(A_j)))}{(\max(y_k(A_i) - y_k(A_j)))} \quad (7)
\]

\[ d(1,2) = \frac{0.0748 - 0.0614}{|0.1664 - 0.2349|} = 0.1956 \]

Similarly \( d(1,3) = 1, d(1,4) = 1 \)

### Table 9. Dis-concordance matrix

| Supplier | S1   | S2   | S3   | S4   |
|----------|------|------|------|------|
| S1       | -    | 0.1956 | 1     | 1     |
| S2       | 1    | -    | 1     | 1     |
| S3       | 0.0071 | 0.0164 | -    | 0     |
| S4       | 0.0737 | 0.0869 | 1    | -     |

Pure Concordance index and Pure Disconcordance index shown in Table 10 calculated by using equation (8) and (9)

\[
C_j = \sum_{k=1}^{n} c(j,k) \cdot \sum_{i=1}^{n} c(k,i)
\]  

(8)
Pure dis-concordance index

\[ D_j = \sum_{k=1}^{n} d(j, k) \sum_{i=1}^{n} d(k, j) \]

(9)

\[ C_1 = (0.46+0.02+0.46)-(0.54+0.98+0.54) = -1.12 \]

\[ D_1 = (0.1895+1+1)-(1+0.0071+0.0737) = 1.1087 \]

4. RESULT

| Supplier | Pure Concordance index | Initial Ranking | Pure Disconcordance index | Initial ranking | Average ranking | Final rank |
|----------|------------------------|-----------------|---------------------------|-----------------|-----------------|------------|
| S1       | -1.12                  | 3               | 1.1087                    | 4               | 3.5             | 3          |
| S2       | -1.32                  | 4               | 2.7104                    | 3               | 3.5             | 4          |
| S3       | 2.28                   | 1               | -2.9765                   | 1               | 1               | 1          |
| S4       | 0.16                   | 2               | -0.8394                   | 2               | 2               | 2          |

The pure concordance and pure discordance indices for the supplier are computed using Equations (8) and (9) respectively, as exhibited in Table 10. From this table, the ranking of the supplier is observed as S3>S4>S1>S2. S3 is the best supplier, S4 emerges out as the second best choice and S2 is the worst device.

5. Conclusion

The present study explores the use of Electre methods in solving a supplier selection problem and the results obtained can be valuable to the decision maker in framing the supplier selection strategies. MCDM methodology applied for selection of supplier for spherical roller bearing. With the help of normalized
decision matrix we estimate criteria weight so that human judgment can be avoided by assigning weights to different attributes. The results show that supplier 3 is best in term of ranking index.

6. References

[1] Almeida, A.T., 2007, Multi-criteria decision model for outsourcing contract selection based on utility function and Electre method, *Computer & Operation Research, 34*, 3569-3574.

[2] Chatterjee P., Mukherjee P., & Chakraborty S., 2011, Supplier selection using compromise ranking and outranking methods, *Journal of Industrial Engineering International, 7*, 61-73.

[3] Chatterjee P., Mondal S., & Chakraborty S., 2014, A Comparative Study of Preference Dominance-based Approaches for Selection of Industrial Robots, *Advances in Production Engineering & Management, 9*, 5-20.

[4] Chatterjee, P. and Chakraborty, S., 2014. Investigated the effect of Normalization Norms in Flexible manufacturing system selection using Multi criteria decision making methods, *Journal of Engineering Science and Technology review, 7(3)*, 141-150.

[5] Govindan K., and Jepsen M.B., 2014, Electre: A comprehensive literature review on methodologies and applications, *Eur. K. Oper. Res.*, 250, 1-29.

[6] Kumar M., Vrat P., & Shankar R., 2006, A fuzzy programming approach for vendor selection problem in a supply chain. *International Journal of production Economics, 101*, 273-285.

[7] Kumar J., Soota T., & Sagar G., 2015, Compare the different multi criteria decision making method (MCDM) for the selection of alternative industrial welding machine, *Journal of scientific research, 2*, 125-130.

[8] Roy, B., 1991, The outranking approach and the foundations of Electre Methods, Theory and Decision, 31, 49-73.

[9] Rao R.V., 2010, A subjective and objective integrated multiple attribute decision making method for material selection, *Materials and Design, 31*, 4738-4747.

[10] Rao, R.V., 2007, Decision making in the manufacturing environment: Using graph theory and fuzzy multiple attribute decision making methods, *Springer Science & Business Media*. 