Double scatter of vector electromagnetic waves from rough metal and dielectric surfaces using the Kirchhoff approximation

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Abstract. This paper presents calculations of a formulation of the 3D Kirchhoff approximation which allows calculation of the scattering of vector waves from 2D rough surfaces containing infinite slopes. Results are presented for scattering from surfaces with rectangular surface structures. This type of surface has applications, for example, in remote sensing and in testing or imaging of printed circuits. Some preliminary calculations for rectangular-shaped grooves in a plane are presented for the 2D surface method.

1. Introduction

Surfaces with infinite surface slopes are interesting in many areas of research [1-6]. This type of surfaces involves rectangular structures and some examples are man-made structures (vehicles or buildings) in remote sensing or pattern surfaces used for growing nanostructures. However, the study of the scattering of light from this type of surface has proved to be more complicated than the study of scattering from surfaces with smaller slopes. There have been efforts to calculate scattering from 1D surfaces with infinite slopes using modal methods and integral equation methods which are difficult to generalize to 2D surfaces or geometrical optics methods (ray-tracing or specular point theory) which are limited in their range of application. In this paper we present the application of the Kirchhoff approximation to calculate the scattering of light from 2D rough surfaces with infinite slopes. In particular, in this paper, results are presented for the case of a metallic surface material, although any surface material can be considered in the numerical calculations.

The Kirchhoff approximation has been one of the most popular methods for calculating the scattering of light from rough surfaces in recent years [4,5]. Multiple scattering formulations based on the Kirchhoff approximation have been developed to calculate the enhanced backscatter effect in randomly rough surfaces with Gaussian statistics and higher surface slopes. However, higher slopes also produce instabilities in the Kirchhoff approximation due to the presence in the Kirchhoff integral of a term which is proportional to the slope. This means, for example, that for an infinite-sloped surface, such as a surface with rectangular shaped grooves, the traditional, or Beckman, formulation of the multiple scatter Kirchhoff approximation cannot be used.

Recently a reformulation of the Kirchhoff method was presented which permits the multiple scatter calculation of infinite-slope surfaces [4,5]. The theory of the method applied to general materials (dielectrics and metals) is resumed in section 2 for the single-scatter contribution and in section 3 for...
the double scatter contribution. In section 4 preliminary results of the numerical method are presented to verify the method.

2. Theory

Vector wave scattering from 2D rough surface can be calculated using the Stratton-Chu equation [6]

\[ E_{sc}(r) = \frac{i \exp(ikr)}{4\pi r} \int \left[ (n \times E) \times k_{sc} - k \eta (n \times H) \right] \exp(-ik_{sc} \cdot r) dS(r_s) \]  

(1)

where \( E \) and \( H \) are the electric and magnetic fields, respectively, on the scattering surface, the subscript \( sc \) refers to the scattered field, \( r \) is the position of the detector, \( n \) is the unit vector normal to the surface at the point \( r_s \), \( k \) is the wave vector and its magnitude, \( k \) is the scatter unit vector normal to the surface area around the point \( r_s \), and \( \eta \) is the permittivity of the medium through which the incident and scattered waves propagate, and \( \epsilon \) the permittivity of this medium.

It can be shown that the Kirchhoff approximation can be written [6]

\[ n \times E = E_{inc} \left[ (1-R_v)(n \cdot k_{inc})(k_{inc} \times e) + (1-R_v)(n \cdot k_{inc})(e \cdot t)(t \times k_{inc}) + (1+R_v)(e \cdot t)(n \times t) \right] \]  

(2)

\[ \eta n \times H = E_{inc} \left[ (1+R_v)(n \cdot e)k_{inc} - (1-R_H)(n \cdot k_{inc})e + (R_v + R_H)(e \cdot t)(k_{inc} \cdot n) \right] \]  

(3)

where \( e \) is the direction of the incident electric field and \( t \) is the vector which is perpendicular to the local plane of incidence at each point of the surface, the subscript \( inc \) refers to the incident field, \( R \) is the Fresnel reflection coefficient for a plane surface, and the subscripts \( V \) and \( H \) are the vertical and horizontal polarizations respectively. The Fresnel reflection coefficient depends on the material properties of the surface and the local incident angle which varies from point to point on the surface.

The combination of equations (1), (2) and (3) is the single scatter (light which interacts only once with the surface) contribution to the scattered field. To correctly account for the scattering contributions from different parts of the surface, the shadowing must be taken into account. Usually, shadowing is included in the calculation explicitly by introducing functions inside the integral of equation (1) in the form of geometric (ray-trace) shadow functions to take into account the parts of the surface which are illuminated and those which are visible from the detector.

Figure 1. The geometry of a segment of a 2D surface

The combination of equations (1), (2) and (3) involves a contribution of the form \( n dS \) in each of the terms of the integral, and this term is the one which governs whether the method can be used for surfaces with infinite slopes or not.

Figure 1 shows the geometry of a single surface segment which is the result of the discretization of the surface. As is usual for the Kirchhoff approximation, each segment is assumed flat so that the approximation of using the Fresnel reflection coefficient is valid. In this figure \( x \) and \( y \) are the unit
vectors along the x and y sides of the segment, \( dx \), \( dy \), \( dh_x \) and \( dh_y \) are the components of these vectors, \( dS_x \) and \( dS_y \) are the lengths of the x and y sides of the segment, \( \beta \) is the angle between the segment and the x-axis, \( \alpha \) is the angle between the segment and the y-axis and \( \theta \) is the angle between the vectors x and y. The normal to the surface can be written, in terms of the parameters shown in figure 1 as

\[
\mathbf{n} = \frac{1}{\sin \theta} \left( \sin \beta \cos \alpha, -\cos \beta \sin \alpha, \cos \beta \cos \alpha \right)
\]

(4)

and \( dS \) can be written in the form

\[
dS = \left| \mathbf{dS}_x \times \mathbf{dS}_y \right| = dS_x dS_y \sin \theta \frac{\sin \theta}{\cos \alpha \cos \beta} \, dx \, dy
\]

(5)

so that

\[
\mathbf{n} \, dS = \left( -\tan \beta, -\tan \alpha, 1 \right) dx \, dy = \left( -m_x, -m_y, 1 \right) dx \, dy
\]

(6)

where \( m_x \) and \( m_y \) are the gradients of the surface segment in the x and y directions, respectively.

This is the traditional formulation of the Kirchhoff calculation. It can be seen that if, for example, we have a vertical wall in the surface parallel to the y-axis, then \( m_x = \infty \) and equation (6), and hence the diffraction integral of equation (1) will be undetermined. Also the sampling of the vertical wall will be very poor, because all points on the wall have the same value of x, i.e. \( dx = 0 \) for the wall.

Now, to resolve this problem, the terms in the components of this vector can be written in terms of their increments in distance [7,8]:

\[
\mathbf{n} = \frac{1}{\sin \theta} \left( \frac{dy}{dS_x} dS_x, \frac{dx}{dS_y} dS_y, \frac{dh_x}{dS_x} dS_x, \frac{dh_y}{dS_y} dS_y \right)
\]

(7)

giving

\[
\mathbf{n} \, dS = \left( -dy \, dh_x, -dx \, dh_y, dx \, dy \right)
\]

(8)

It can now be seen that substitution of equation (8) in equations (1), (2) and (3) will give three integrals and the surface slope will not appear in any of these terms. The integrations are over the combinations of variables \( dy \, dh_x \), \( dx \, dh_y \) and \( dx \, dy \). If, for example, a surface segment has \( dx = 0 \), i.e. a vertical wall parallel to the y-axis, the first integral of variables \( dy \, dh_x \) will contribute to the total scattered field giving the correct contribution for this part of the surface. Thus, the combination of equations (1),(2), (3) and (8) means that the vector wave scattering Kirchhoff approximation can be applied to 2D surfaces with infinite slopes.

The double scatter contribution can be calculated by using the same equations, but calculating first the light scattered from one point on the surface to another point on the surface and then using this scattered field as the incident field to calculate the field scattered from the second point to the detector. Here, an extra shadow function must be considered to take into account the visibility of the second surface point from the first point.

3. Results

As a first test, the single scatter contribution for a flat surface has been calculated. The values of the reflection coefficients as a function of wavelength for silver, gold, copper and aluminium are shown in Figure 2. The values of the refractive indices were taken from reference [9]. It can be seen that the reflection coefficients have the expected values [10].
Figure 2. Calculated reflection coefficients for a flat surface

Figure 3 shows the geometry of the rough surfaces used to test the method for 3D surfaces. The surface of size $L$ by $L$ contains a groove of depth $d$ and width $a$. The groove runs perpendicular to the plane of incidence defined by the incident light, of wavelength $\lambda$, which illuminates the surface at an angle $\theta_{\text{inc}}$. The polarization of the light is divided into horizontal polarization, $H$, which is the component parallel to the plane of the surface; and vertical polarization, $V$, which is perpendicular to $H$.

Figure 3. the geometry used for the calculations
Figure 4. Single scattered light from a surface with $\lambda = 0.633 \mu m, L = 10 \lambda, a = 2 \lambda, d = 1 \lambda$ and $\theta_{\text{inc}} = 20^\circ$.

Figure 5. as figure 4 but for the double scattered contribution.
The silver surface used for testing had the parameters: $L = 10\lambda$, $a = 2\lambda$, $d = 1\lambda$ and $\theta_{inc} = 20^\circ$. Both single and double scattered contributions have been calculated for the test surfaces and the coherent sum of these contributions gives the total scattered intensity. Figures 4, 5 and 6 show the scattered light patterns for a wavelength of 0.633\,\mu m. It can be seen that the single scatter contributions have a maximum in the specular direction and the double scattered contributions have a maximum in the retro dispersion direction, as expected.

The total scattered energy for HH and HV polarizations is shown in figure 7 as a function of wavelength. It can be seen that both contributions follow the same behavior as the Fresnel coefficient for silver.
4. Conclusions
A formulation of the Kirchhoff approximation which allows calculation for surfaces with infinite slopes and general (metallic and dielectric) materials has been presented. The 3D formulation has been presented along with some preliminary results showing the validity of the method. In the future the results of calculations with this method will be compared to experimental results.

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