W_4 Toda Example as Hidden Liouville CFT^{1,2}

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Abstract—We construct correlators in the W_4 Toda 2d conformal field theory for a particular class of representations and demonstrate a relation to a W_2 (Virasoro) theory with different central charge. The relevance of the classical limits of the constructed 3-point functions and braiding matrices to problems in 4d conformal theories is discussed.

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1. INTRODUCTION

There are few explicit results for the basic structures of the 2d conformal field theories (CFT) based on higher rank algebras, such as the WZW models, or their quantum Drinfeld–Sokolov reductions, the Toda CFT. The main motivation of this work is to extend the examples in the literature [1, 2] (see also [3–5]), mostly for the W_3 case, for a particular Toda CFT. The latter is selected by the possible applications to the 4d models with superconformal symmetry in the context of the AdS/CFT correspondence.

The talk is based mostly on [6]. In Section 2 we exploit the Coulomb gas construction technique of [1, 2] to compute the 3-point functions for a class of representations of the W_4 algebra and then we discuss their light charge classical limit comparing with computations in the supergravity approximation to the string theory. The 3-point constants are used in Section 3 to solve the crossing symmetry equation for the braiding matrices of the 4-point conformal blocks in which one of the fields is labelled by the highest weight of the 6-dimensional fundamental s(2,2|4) representation. In Section 4 the local 4-point function is shown to admit an explicit integral representation. Surprisingly, the latter is identified with a 4-point Liouville correlator with one degenerate field. The vertex operators are described by representations of the Virasoro algebra with modified central charge, which differs from the central charge of the Virasoro subalgebra of the W_4 algebra. This relation, announced in [6], is demonstrated here for the fusing matrices of the two theories. In the last Section 5 we discuss the relevance of the heavy charge classical limit for a braiding identity and its solutions for the study of the quasiclassics of sigma models in the context of the AdS/CFT correspondence.

2. 3-POINT FUNCTIONS

We consider the W_4 CFT with central charge parameterized by real b

\[ c_T = 3(1 + 2Q^2) = 3\left(41 + 20\left(b^2 + \frac{1}{b^2}\right)\right), \]

where

\[ Q = \frac{1}{b}. \]

The scalar primary fields \( V_\beta(z, \overline{z}) \) are labelled by a generic s(4) weight in the free field (Coulomb gas) representation, see [1, 7] for details. Equivalent representations carry charges related by the action of the Weyl reflection group

\[ \omega = Q \rho + w(\beta - Q \rho) \] (2.2)

and the operators are related by a reflection amplitude

\[ V_\beta(z, \overline{z}) = R_w(\beta)\overline{V}_{-\beta}(z, \overline{z}). \]

The action (2.2) preserves the three quantum numbers characterising the W_4 representations, in particular, the conformal dimension

\[ \Delta(\beta) = \frac{1}{2} (\beta, 2Q \beta - \beta). \] (2.3)

Here \( \rho = \sum_{i=1}^{3} \omega_i \) is the Weyl vector and \( \omega_i, i = 1, 2, 3 \) are the three s(4) fundamental weights (\( \omega_i, \alpha_j \)) = \( \delta_{ij} \) for \( \alpha_j \) denoting the s(4) simple roots.
We start with computing the 3-point OPE constant for a particular set of symmetric $\beta_\alpha = \beta_\alpha^*$ highest weights
\[
(\beta_\alpha, \alpha_1) = 0 = (\beta_\alpha, \alpha_3), \text{ for } \alpha = 1, 2, \quad (2.4a)
\]
\[
(\beta_\alpha, \alpha_2) = (\beta_\alpha, \alpha_3). \quad (2.4b)
\]
The weights (2.4a) correspond to scalars in the context of the 4d conformal group representations, with 4d conformal dimension $\Delta = (\beta, \omega_3)/b$.

To compute the 3-point constant we exploit the technique of [1] based on the Baseilhac – Fateev (BF) integral relation [8]. It allows to derive a recurrence relation for the corresponding Coulomb gas integrals in which the three charges are restricted by a charge neutrality condition. The solution is then analytically continued similarly to the Liouville case with the result
\[
C(\beta_1, \beta_2, \beta_3) = (b^{2(1-b)} \pi \mu)^{(2bQ-\beta_1, \beta_2, \beta_3)}/b \prod_{\alpha=\alpha_1, \alpha_3} \Gamma_{\alpha}((\beta_3 - \rho Q, \alpha) + Q) \times \left( \prod_{\alpha=\alpha_1, \alpha_3} \Gamma_{\alpha}(\beta_3 - \rho Q, \alpha) \right)
\]
\[
\times \prod_{\alpha=1,2} \prod_{\alpha=\alpha_1, \alpha_3} \left( \prod_{\alpha=\alpha_1, \alpha_3} \Gamma_{\alpha}(\beta_{123} - 2\beta_\alpha, \omega_2 - \omega_1 + Q) \times \Gamma_{\alpha}(\beta_{123} - 2\beta_\alpha, \omega_2 - \omega_1) - 2Q \right)
\]
\[
\times \prod_{\alpha=1,2} \left( \prod_{\alpha=\alpha_1, \alpha_3} \Gamma_{\alpha}(\beta_{123} - 2\beta_\alpha, \omega_2 - \omega_1) - Q \right) Y_b((\beta_{123} - 2\beta_\alpha, \omega_2 - \omega_1) - 3Q). \quad (2.5)
\]

Here $\beta_{123} = \sum_{\alpha=1,2} \beta_\alpha^* \beta_3^* = \beta_{123} - 2\beta_3$. Recall that $Y_b(x)$ is an entire function with zeros at $x = -nb - mb/b$ and $x = Q + nb + mb/b$, $n, m \in Z_{20}$, satisfying the functional relations
\[
Y_b(x + b) = e^{(\beta_3 - 2\beta_\alpha)} Y_b(x), \quad \epsilon = \pm 1, \quad \epsilon(x) = \Gamma(x)/\Gamma(1-x). \quad (2.6)
\]

We introduce notation for the Coulomb gas OPE constant reproduced as a double residue
\[
ce(\beta_1, \beta_2, 2pQ - \beta_3) := \text{res}_{\beta_1^*, \alpha_2^*}(\beta_1^*, \omega_2 - 2\omega_0)/(b^{2(1-b)} \pi \mu)^{(2bQ-\beta_1, \beta_2, \beta_3)} \times C(\beta_1^*, \beta_2^*, 2pQ - \beta_3^*) \quad (2.7)
\]
where $s_1$ and $s_2 - 2s_1$ are nonnegative integers. In particular if $\beta_1^* = -b\omega_2$, corresponding to the highest weight $\lambda_2 = \omega_3$, the 6-dim $sl(4)$ representation, the formula (2.7) reproduces the structure constants of the fusion of the fundamental field $V_{-\omega_0, b}$ and $V_{\beta}$ corresponding to the shifts with three of the six points of the weight diagram $\Gamma_{\omega_3}$.

In particular $c_{\beta}(\beta) = 1$. The remaining three OPE constants, computed in general in [1] vanish, in agreement with the vanishing of the inherited from the $s(4)$ tensor product decomposition multiplicities for the particular partially degenerate weight (2.4a).

Consider the special case when all three weights in (2.5) are of the type (2.4a). It is instructive to display the asymptotics of (2.5) in the classical limit $b \to 0$ with “light” charges, i.e., $(\beta_\alpha, \alpha_2) = \sigma_\alpha$ remain finite. More precisely, consider the modified 3-point constant $\bar{C}(\beta_1, \beta_2, \beta_3)$ in which all $Q$-factors are replaced by $Q \to b$. This is achieved by factorizing a finite number of $\gamma(z + 1/\beta)$ – factors extracted by applying the functional relation (2.6). Then the light charge classical limit reads
\[
\bar{C}(\sigma_1, \beta_2, \sigma_1, \beta_2, \sigma_3) \sim \Gamma\left(\frac{\sigma_1 \sigma_2 \sigma_3}{2} - 2\right) \prod_{\alpha=1}^{\infty} \Gamma\left(\frac{\sigma_1 \sigma_2 \sigma_3}{2} - \sigma_\alpha\right) \times \Gamma\left(\frac{\sigma_1 \sigma_2 \sigma_3}{2} - 3\right) \prod_{\alpha=1}^{\infty} \Gamma\left(\frac{\sigma_1 \sigma_2 \sigma_3}{2} - \sigma_\alpha\right) . \quad (2.9)
\]

This expression reproduces, up to field normalisation, the 4d scalar 3-point correlators defined as an integral over $AdS_5$ of three bulk-boundary kernels representing the vertex operators in the supergravity approximation to string theory [9]. This result suggests that the modified 3-point constant $\bar{C}(\beta_1, \beta_2, \beta_3)$ describes the WZW counterpart of the Toda 3-point constant; see [10] for a discussion of the quantum Drinfeld–Sokolov reduction on the level of correlators.

Analogous to (2.9) classical limit arises from the “matter” counterpart in the dual central charge region with the parameter $b \to ib$. It reproduces the integral over $S^4$, see [6] for more details and a further discussion of a BPS type relation for the charges in the two dual regions, leading to a trivial 3-point constant of the corresponding 4d superconformal correlator.

3. 4-POINT FUNCTION AND FUSING MATRIX

Our next step is to consider the local 4-point function $\langle V_{\beta, \omega Q} \bar{V}_{\beta, \omega Q}, \bar{V}_{\beta, \omega Q}, \bar{V}_{\beta, \omega Q} \rangle$ of primary spinless operators $V_{\beta}(z, \overline{z})$ one of which is labelled by a fundamental
highest weight, in our case \( f = -b\omega_2 \). All the other three weights \( \beta_i, a = 1, 2, 3 \) are supposed to be in the class \((2.4a)\) labelling doubly reducible Verma modules with at most two singular vectors since \((\beta_a, \omega_2)\) are assumed to take generic values.

The standard approach to the study of the 4-point functions with degenerate fields is to exploit the differential equations for the chiral blocks arising from the factorization of all singular vectors (and possibly some descendent states) along with the restrictions imposed by the Ward identities of the \( W_3 \) algebra. In higher rank CFT this is a difficult technical task, see [11] for an early discussion of the peculiarities of the Toda correlators. On the other hand we expect that in the particular example of several reducible representations the set of restrictions will be sufficient to determine the chiral blocks and furthermore to reduce the fusion channels to the ones determined by the OPE coefficients of the primary fields with the fully degenerate field \( V_{-\omega_2} \).

In the next section we shall confirm this expectation by deriving an explicit alternative integral representation for the 4-point function.

The 4-point function admits different equivalent diagonal decompositions in conformal blocks. They are related by linear transformations, realizing the action of the elements of braiding group with generators \( e_i, i = 1, 2, 3 \) on the plane (Riemann sphere) with 4 holes; \( e_i \) is exchanging the chiral vertex operators at the \( i \)-th and \( i + 1 \)-th points. In particular the generator \( e_2 \) (for the above order of the corresponding chiral vertex operators) is represented by non-trivial braiding matrix \( B \) proportional to the fusing matrix \( F \)

\[
B_{\beta_i - h_b \beta_j - h_b} \begin{bmatrix} \beta_1 & \beta_2 \end{bmatrix} = e^{i\pi\epsilon(\Delta(\beta_i) + \Delta(f) - \Delta(\beta_j) - \Delta(\beta_2) - \Delta(\beta_3) - \Delta(h_b))} F_{\beta_i - h_b \beta_j - h_b} \begin{bmatrix} \beta_2 & f \end{bmatrix} \begin{bmatrix} \beta_3 & \beta_1 \end{bmatrix}. \tag{3.1}
\]

The generators \( e_i \) and \( e_s \), which exchange the operators in the first two, respectively last two, fixed points, reduce to diagonal matrices.

The crossing symmetry leads to a set of equations relating the 3-point constants and the fusing matrix elements

\[
\sum_{h_b \in \Gamma_{\omega_2}} c_h(\beta_1) C(\beta_2 - h_b \beta_2, \beta_3) F_{h_b} = \delta_{h_b, \omega_2} = \delta_{h_b, -\omega_2}. \tag{3.2}
\]

As discussed above we are left with summation over 3 of the 6 weights in the weight diagram \( \Gamma_{\omega_2} \), as given in (2.8). A shorthand notation for the matrix \( F_{h_b, \omega_2} = F_{h_b} \) in the last equality in (3.2) is used.

The inverse matrix \( F^{-1} \) is given by

\[
(F^{-1})_{h_b, \omega_2} = F_{h_b, \omega_2}(\beta_1 - h_b \omega_2, \beta_2, \beta_3). \tag{3.3}
\]

According to (3.2) it can be identified with the matrix formed by the ratio of constants times \( F \), i.e.,

\[
c_h(\beta_1) C(\beta_2 - h_b \beta_2, \beta_3) F_{h_b, \omega_2}(\beta_1, \beta_2, \beta_3) = F_{h_b, \omega_2}(\beta_1, \beta_2, \beta_3). \tag{3.4}
\]

Given the 3-point constants (2.5) and the OPE coefficients (2.8) one can solve the equations (3.3), (3.4) for the \( 3 \times 3 \) fusing matrix \( F \). We shall cast the solution of [6] in a compact form introducing first some notation. Define for weights of type \((\beta_a, \omega_2), a = 1, 2, 3 \) and \( \epsilon, \epsilon' = \pm 1 \)

\[
\Gamma(1 + b(2p Q - \beta_j, \alpha_2)) \Gamma(h(\beta_2 \omega_2 + \epsilon \beta_2, 2(1 - \epsilon)Q\omega_2 + \epsilon' \beta_2, \beta_3)). \tag{3.5}
\]

Then with \( s, s' = \pm 1 \), referring to the shifts \( \beta_2 \pm s\omega_2 b, \beta_2 \pm s'\omega_2 b \), and \( s, s' = 0 \), referring to \( \beta_2 - \bar{h}_2 b, \beta_2 \), with \( \bar{h} \) defined in (2.8) we have

\[
F_{s, s'} \begin{bmatrix} \beta_2 - b\omega_2 & \beta_3 \\ \beta_1 \end{bmatrix} = \sum_{p = 1, 2} \frac{G_{p, x}(\beta_1, \omega_2, b, 2s'Q\omega_2)}{G_{p, x}(\beta_1 + 2sQ\omega_2, \omega_2 b, \beta_1)} \times G_{-p, x}(\beta_1 + 2sQ\omega_2, \omega_2 b, \beta_1). \tag{3.6}
\]

Here \( \mu = s \) if \( s = \pm 1 \), and if \( s = 0 \) one can choose one of the two values.
The matrix (3.6) coincides with the $3 \times 3$ fusing matrix in the Liouville theory with one of the charges given by the degenerate h.w. $\gamma = -\tilde{b}$, where the parameter parameterizing the central charge $c_V = 13 + 6 \left( \tilde{b}^2 + \frac{1}{\tilde{b}^2} \right)$ is $\tilde{b}^2 = \frac{-Q}{b}$ (i.e., $c_V < 1$ for real $b$), namely:

$$F_{x,s'} \left[ \begin{array}{c} \beta_2 \\ \beta_3 \\ \beta_1 \end{array} \right] = F_{y_1 + x, y_2 + x, \tilde{b}} \left[ \begin{array}{c} \gamma_2 \\ \gamma_3 \\ \gamma_1 \end{array} \right],$$

(3.7)

under the identification of $W_2$ and $W_4$ highest weights

$$2\gamma_a \tilde{b} = 1 + Qb - (\beta_a, \alpha_2)b$$

(3.8)

$$= 2(1 + (b \alpha_2 - \beta_a, \omega_0)b), \ a = 1, 2, 3.$$

The reason for this coincidence will be cleared in the next Section.

### 4. INTEGRAL REPRESENTATION FOR THE 4-POINT FUNCTION

The BF [8] integral formula which allows to solve recursively some multiple Coulomb integrals representing 3-point functions can be also exploited in order to give meaning of the so far formal 4-point correlator of Section 3. Namely one can interpret the latter as the analytic continuation of a 4-point Coulomb correlator with the four charges constrained by a charge neutrality condition—the latter is transformed, achieving an alternative double integral representation of Coulomb type, so that the whole dependence on the finite number of screening charges is located in the overall constant, which admits an analytic continuation in the standard way. The computation is analogous to the one for Liouville correlators in [12], but unlike that case the type of the integral representation is not preserved. We obtain ($w = w_{12321}$)

$$\langle \langle \langle V_{-\rho, b}(x) V_{\gamma_1}(0) V_{\gamma_2}(1) V_{\gamma_3}(\rho) \rangle \rangle \rangle = R_{\alpha}(\beta_a) \langle \langle \langle V_{-\rho, b}(x) V_{\gamma_1}(0) V_{\gamma_2}(1) V_{\gamma_3}(\rho) \rangle \rangle \rangle = \Omega((\beta_a)) \times \left[ \frac{2(4\rho - \beta_{1234}, \alpha_2)}{2(\beta_{1234}, \alpha_0) - 3Q} \right] \times \prod_{a=1,2,3} Y_{\delta}((\beta_{1234} - 2\beta_a, \omega_0) + b) Y_{\delta}((\beta_{1234} - 2\beta_a, \omega_0) - Q + b)$$

(4.4)

The integral in (4.1) is equivalent [12] up to a constant and an overall $x$ factor to the Liouville 4-point function $\langle \langle \langle V_{-\rho, b}(x) V_{\gamma_1}(0) V_{\gamma_2}(1) V_{\gamma_3}(\rho) \rangle \rangle \rangle^L$ with three arbitrary weights $\gamma_a$ and one degenerate $-\tilde{b}$, in our notation

$$\gamma_a = \gamma_a + \gamma_3 + \tilde{b}, \ a = 1, 2, \ \gamma_3 = \frac{1}{b} \gamma_2 - \gamma_3,$$

(4.5)

The relation between the highest weights of the vertex operators in the $W_4$ and the $W_2$ CFT takes the universal form (3.8) if we furthermore change notation $\beta_3 \rightarrow w_{2132} \ast \beta_1 = 4Q\omega_2 - \beta_3$ in (4.1), which leaves invariant the expression (3.8) for the $F$ matrix elements.

### 5. BRAIDING IDENTITY

One checks that in all the products

$$M_{\alpha\beta}(\beta_1, \beta_2, \beta_3) := F_{\alpha\beta}(\beta_1, \beta_2, \beta_3) F_{\beta_1\beta_2}(\beta_2, \beta_2, \beta_3)$$

(5.1)

the Gamma functions combine according to $\Gamma(z) \Gamma(1 - z) = \pi \sin \pi z$, so that (5.1) turn into ratios of trigonometric functions. These values are actually solutions of an equation which arises from the braiding identity

$$\Omega \Omega \Omega := (e^{2i}) (e^{-2i}) (e^{-1}) \times (e^{2i}) (e^{-2i}) (e^{-1})$$

(5.2)

The identity (5.2) expresses the triviality up to phase of the composition of monodromies around the three ver-
tex coordinates: this is a property of the braid group on the sphere with 4 holes. The first and the last factors in the matrix relation (5.2) are diagonal. The eigenvalues $e^{2\pi i \rho(\beta,b)}$ of the monodromy matrix $\Omega$ are computed from the difference of Toda dimensions, cf. (3.1)

$$p(\beta,h) = \Delta(\beta - hb) - \Delta(\beta) = \Delta(-bb) = 2bQ + b(\beta - \rho Q,h).$$

(5.3)

Multiplying both sides of (5.2) with $\Omega^{-1}$ and taking the trace one gets an equation for the products (5.1)

$$\sum_{h,h} e^{2\pi i \rho(\beta,\beta')} M_{h,h}(\beta,\beta') e^{2\pi i \rho(\beta,h)} = e^{-4\pi i \Delta(f)} \sum_{h} e^{-2\pi i \rho(\beta,h)}.$$

(5.4)

In our case the summation over the weight diagram $\Gamma_{\rho_0}$ is restricted to the three points in (2.8). The equation is checked to hold true for the explicit data presented in Section 3.

- In the limit $b \to 0$ the r.h.s of the matrix relation (5.2) becomes an identity for any of the fundamental weights $f = -\alpha_0 b$; in our case $4\Delta(-\alpha_0 b) \to 0$ mod 2.

Taking this limit for three "heavy" charges $\beta_a = \eta_a/b$ with finite $\eta_a, a = 1, 2, 3$, one has $p(\eta_a/b,h) \to (\eta_a,h)$ mod 2.

The resulting simplified identities are precisely of the type encountered [13, 14] in the construction of the quasiclassical 3-point functions of sigma models (for the example of $AdS_3 \times S^3$) in the framework of the AdS/CFT correspondence. To make a connection with these considerations one identifies the parameter $b^2$ with the inverse of the 't Hooft coupling $b^2 = 1/\sqrt{\kappa}$. In the sigma model case the generators of the braid group depend on the spectral parameter, i.e., $\eta = \eta(x)$, and $F = F(x)$. However the $F(x)$ products in (5.1) solving the equation (5.4) in the quasiclassical limit are identical as (trigonometric) functions of $\eta_a(x)$ to the ones found in the CFT. Given these products, in the next step [13, 14] find integral representations for the nontrivial individual $F(x)$ matrix elements with measure depending on the spectral curve. We see that Toda CFT (and more generally, the WZW models, related to them via the quantum Drinfeld–Sokolov reduction) provide important data on the string side of the constructions of the AdS/CFT correspondence.

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