Model-independent comparisons of pulsar timings to scalar–tensor gravity

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Abstract
Observations of pulsar timing provide strong constraints on scalar–tensor theories of gravity, but these constraints are traditionally quoted as limits on the microscopic parameters (like the Brans–Dicke coupling, for example) that govern the strength of scalar–matter couplings at the particle level in particular models. For binary pulsars whose all five post-Keplerian parameters have been measured, we present fits to timing data directly in terms of the phenomenological couplings (masses, scalar charges, moment of inertia sensitivities and so on) of the stars involved, rather than to the more microscopic parameters of a specific model. For instance, for the double pulsar PSR J0737-3039A/B, we find at the 68\% confidence level that the masses are bounded by $1.28 < m_A/m_\odot < 1.34$ and $1.19 < m_B/m_\odot < 1.25$, while the scalar charge-to-mass ratios satisfy $|a_A| < 0.21$, $|a_B| < 0.21$ and $|a_B - a_A| < 0.002$, independent of the details of the scalar–tensor model involved, and of assumptions about the stellar equations of state. Whenever it is possible to do so, we urge observers to express their results in this more model-independent way, which potentially can then be used by theorists to constrain a great variety of specific models by computing the fit quantities as functions of the microscopic parameters in any particular model. For the Brans–Dicke and quasi-Brans–Dicke models, the constraints coming from the double pulsar obtained in this manner are consistent with, and slightly weaker than, those quoted in the literature.

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(Some figures may appear in colour only in the online journal)
1. Introduction

Although general relativity (GR) has many applications in astrophysics and cosmology, incomplete knowledge about the gravitating bodies involved prevents using most of these as tests of the theory itself for the vast majority of systems outside of the solar system. Binary pulsars provide the rare exception to this rule, due to the great precision with which timing measurements allow the properties of their orbits to be inferred [1–5]. These orbital properties are parameterized by the inferred values of the Keplerian parameters (such as semi-major axis $a$ and eccentricity $e$) that define the characteristics of the Newtonian orbit of the pulsar and its partner, together with its orientation in space. They are also parameterized by a suite of post-Keplerian (PK) parameters that describe observable slow, secular orbital changes over time, as well as relativistic effects in the timing of radio signals emitted by the pulsar. In the literature, results of pulsar timing observations are reported by presenting the inferred values of the Keplerian and post-Keplerian parameters. Theorists may then use these values in order to test GR and alternative theories, rather than directly working with the enormous data set of pulse arrival times.

1.1. The classic constraints

GR predicts the values for these PK parameters in terms of the underlying Keplerian parameters and the ADM [6] masses $m_{A,B}$ of the two orbiting bodies. To test these predictions, bands are drawn in the $m_A - m_B$ plane of the form

$$\xi^\text{th}(m_A, m_B) = \xi^\text{obs} \pm \Delta,$$

where $\xi^\text{th}$ denotes the theoretically predicted value of a PK parameter, $\xi^\text{obs}$ denotes its observed value and $\Delta$ denotes the observational error. The validity of GR requires all such bands to have a non-empty intersection, providing a substantive test provided at least three PK parameters can be measured. The area of mutual overlap then gives the GR-inferred masses of the stars, to within some tolerance.

Thus far, GR has passed these stringent tests, using a number of binary pulsars. For two of these—PSR B1534+12 [7–10] and the double pulsar, PSR J0737-3039A/B [11–19]—the tests are particularly redundant since it is possible to infer observational values for five independent PK parameters. It is also possible to infer the spin–orbit precession frequency [10, 19], which can be considered as a sixth PK parameter. Moreover, for the double pulsar, PSR J0737-3039A/B, both stars are pulsars and a measurement of the ratio of the semi-major axes yields another constraint on the masses. The masses are constrained quite accurately [13]

$$m_A = 1.3381(7)m_\odot \quad \text{and} \quad m_B = 1.2489(7)m_\odot \quad \text{(GR)}.$$  

The success of GR in describing pulsar orbits also constrains alternative theories of gravity; requiring their predictions to agree with GR to within present errors. Prominent among these alternatives are scalar–tensor theories [20, 21], for which long-range gravitational forces are mediated by both the metric, $g_{\mu\nu}$, and a very light scalar, $\phi$, described by the action

$$S = -\int \frac{d^4x}{c}\sqrt{-g}\left[\frac{1}{2\kappa^2}g^{\mu\nu}(R_{\mu\nu} + \partial_\mu\phi\partial_\nu\phi)\right] + S_m.$$  

3 In order for these masses to be well defined, the bodies must be widely separated, which is the case for the binary pulsars that have been precisely timed.
Here $\kappa^2 = 8\pi G/c^4 = \hbar/(M_p^2 c^3)$ denotes the gravitational coupling, and $g$ is the determinant of $g_{\mu\nu}$, while $S_m = S_m[g_{\mu\nu}, \phi, \Psi_i]$ denotes the matter action, and controls how $\phi$ and $g_{\mu\nu}$ couple to the various other `matter’ fields, $\Psi_i$, which we take in what follows to have the form

$$S_m = S_m[A^2(\phi)g_{\mu\nu}, \Psi_i].$$  \hfill (1.4)

This form of coupling has several motivations. First, it is favoured by strong observational constraints [22, 23] on violations of the weak equivalence principle, which are evaded by actions of the form of equation (1.4). Second, it is also the kind of theory that actually describes the low-energy limit of certain types of extra-dimensional theories [24].

In scalar–tensor theories with matter action (1.4), the gravitational constant which is measured in a Cavendish experiment is given by [21]

$$\tilde{G} = GA^2(\phi_{\infty})(1 + a^2(\phi_{\infty})), $$ \hfill (1.5)

where $\phi_{\infty}$ is the value of the scalar field asymptotically far away from the experimental apparatus carrying out the measurement, and

$$a(\phi) := \frac{d \ln A}{d \phi} $$ \hfill (1.6)

is the effective coupling of scalars to matter. For isolated or widely separated bodies, the total ADM [6] mass energy computed using the Einstein-frame metric $g_{\mu\nu}$ is conventionally denoted by $m$, and that computed using the so-called Jordan-frame metric $\tilde{g}_{\mu\nu} \equiv A^2(\phi)g_{\mu\nu}$ is denoted by $\tilde{m}$. It may be shown that $\tilde{m} = m/A(\phi_{\infty})$ [21], where $\phi_{\infty}$ is the value of the scalar field asymptotically far away from the body. In this paper, units will be used in which $A(\phi_{\infty}) = 1$, so that the Jordan-frame and Einstein-frame masses coincide.

The predictions of theories with action (1.3)–(1.4) have been compared in detail with binary pulsar data [25–27], with the coupling function assumed to have the particular quasi-Brans–Dicke form,

$$A(\phi) = \exp \left[ a_s \phi + \frac{b_s}{2} \phi^2 \right], $$ \hfill (1.7)

for which the effective coupling of scalars to matter (1.6) turns out to have the strength

$$a(\phi) = a_s + b_s \phi. $$ \hfill (1.8)

This form is motivated by the idea that the field $\phi$ does not vary appreciably in any particular system of interest, and so $a(\phi)$ is approximately constant.

Comparison with the solar system and pulsar data is found to constrain the microscopic couplings for this theory, $a_s$ and $b_s$, to be consistent with zero, with $a_s$ strongly constrained from the solar system data and $b_s$ restricted by pulsars. The constraints on these microscopic couplings are usually presented as an exclusion plot in the $b_s$–$a_s$ plane, and in particular it was found that [25–27]

$$b_s \gtrsim -4.5, \quad |a_s + b_s \phi_{\infty}^{SS}| < 3.4 \times 10^{-3}, $$ \hfill (1.9)

where $\phi_{\infty}^{SS}$ is the value of the scalar field asymptotically far away from the solar system, which is conventionally taken to be zero.

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4 The quantity $G$ denotes the Einstein-frame ‘bare’ gravitational constant. Its relation to the ‘physical’ gravitational constant measured in a Cavendish experiment is given in equation (1.5).

5 The Brans–Dicke theory corresponds to the specific choice $b_s = 0$. Our notation follows that of [31], and differs in minor ways from that used elsewhere in the literature [21], as described in detail in table 1.
Table 1. The first three (microscopic) quantities enter into the action and are defined in equations (1.3)–(1.8). The last four quantities are phenomenological parameters that depend on the internal structure of a star. They are defined in section 2.2.

| Our notation | Notation of [21] | Meaning |
|--------------|------------------|---------|
| $a_i$        | $\alpha$         | Microscopic scalar–matter coupling constant |
| $b_i$        | $\beta$          | Microscopic scalar–matter coupling constant |
| $a(\phi)$    | $\alpha(\phi)$  | Microscopic scalar–matter coupling function |
| $a_{A,B}$    | $\alpha_{A,B}$   | Effective scalar–matter coupling of a star |
| $b_{A,B}$    | $\beta_{A,B}$    | Effective scalar–matter coupling of a star |
| $k_{A,B}$    | $\kappa_{A,B}$   | Sensitivity of a star’s moment of inertia to scalar field |
| $Q_{A,B}$    | $\omega_{A,B}$   | Scalar charge of a star |

1.2. A more model-independent approach

There are two related drawbacks to traditional comparisons between scalar–tensor theory and observations. First, because limits like equation (1.9) are quoted directly for the microscopic couplings, $a_i$ and $b_i$, a completely new analysis is required for each new assumed functional form for the coupling function $A(\phi)$. Second, as discussed below (and remarked by the original authors), these bounds are subject to uncertainties that are hard to quantify, due to limits of our understanding of the nuclear equation of state that applies within the pulsars.

One approach towards robustness that has been taken in the literature is to generalize the form taken for the phenomenological Lagrangian describing the two-body interactions of the gravitating objects, restricting attention to theories of gravity whose predictions for the orbital dynamics may be derived from a boost-invariant Lagrangian (at least to the first post-Newtonian (1PN) order). Will [23] and Damour and Taylor [2] have shown that such a Lagrangian may be characterized by a set of body-dependent phenomenological parameters, of which there are five—$m_A$, $m_B$, $G$, $\epsilon$, $\xi$—in the most general Lagrangian [23, 2]:

$$L = \frac{V^2}{2} + \frac{GM}{R} + \frac{1}{8c^2}(1 - 3v)V^4 + \frac{GM}{2Rc^2} \left( (\epsilon + v)V^2 + v(\vec{N} \cdot \vec{V})^2 - \frac{GM}{R} \right),$$

(1.10)

where $\vec{R}$ and $\vec{V}$ are the relative position and velocity vectors, $\vec{N} = \vec{R}/R$, and $G, m_A, m_B, \epsilon$ and $\xi$ are phenomenological parameters and $M = m_A + m_B$, $v = m_A m_B / M^2$. Wex and Kramer have used the observed values of the PK parameters for the double pulsar to constrain these Will–Damour–Taylor (WDT) parameters [11].

However, a drawback of this approach is that it does not describe radiation effects, and in particular the PK parameter $P_b$ (which describes the shortening of the orbital period due to emission of gravitational radiation). This is because $P_b$ cannot be expressed in terms of the WDT parameters. This limitation exists because radiative effects come in at higher order in powers of $V/c$ (at 1.5PN order for dipole emission and 2.5PN order for quadrupole emission), whereas the WDT parameters only describe 1PN effects. Extending the phenomenological parameterization to higher PN orders introduces too many new parameters for the data to usefully constrain, however. Thus, in order to use all PK parameters, including $P_b$, to obtain interesting constraints on alternative theories of gravity, it is necessary to further restrict the class of theories that one considers.

In this paper we take a complementary approach to confronting pulsar observations with scalar–tensor models, based on the observation that the scalar–tensor predictions for the PK parameters depend only on a relatively small number of macroscopic quantities that characterize how the two stars couple to the scalar field. There turn out to be seven of these (defined in detail in section 2.2): $m_{A,B}$, $a_{A,B}$, $b_{A,B}$ and $k_A$. These generalize the two masses—$m_A$ and $m_B$—that suffice to make predictions within GR, to include four new quantities—$a_{A,B}$ and
$b_{A,B}$—that characterize the strength with which the scalar field couples to the pulsar and its orbital partner, plus one variable, $k_A$, related to the pulsar’s moment of inertia.

The key point is that model-dependent complications, such as the detailed form of $A(\phi)$ and knowledge of the nuclear equation of state, enter only into the predictions for the quantities $m_{A,B}$, $a_{A,B}$, $b_{A,B}$ and $k_A$ as functions of the microscopic couplings in a particular scalar–tensor theory. But these complications do not enter at all into the formulae that express how $m_{A,B}$ through $k_A$ determine the observed PK parameters.

This makes it useful to phrase the confrontation between theory and experiment in two steps: first use the observations to constrain the quantities $m_{A,B}$ through $k_A$ once and for all in a model-independent way; then compute these quantities within specific scalar–tensor models as functions of the underlying model parameters that define the function $A(\phi)$.

It is the goal of this paper to perform the first—and model-independent—one of these steps. At first sight, this might seem to be impossible to do, since it appears to involve constraining more quantities than the five observable PK parameters\(^6\). However, it turns out that the present data nonetheless allow useful constraints to be achieved, for two reasons. First, constraints are possible because the dependence of the PK parameters on two of the parameters, $b_{A,B}$, is very weak whenever $a_{A,B}$ is small. Thus, the number of free variables is effectively reduced from seven down to five\(^7\).

Second, one combination of Keplerian parameters—the ratio of projected semi-major axes of the two orbits, $R \equiv x_B/x_A$ (see section 2.1 for details of notation)—can sometimes also be measured. For instance for the double pulsar, observations give $R = 1.0714(11)$. This measurement is useful because the theoretical prediction for this quantity in all Lorentz-invariant theories of gravity is $R_{th} = m_A/m_B + \mathcal{O}(1/c^4)$ and so is completely independent of the scalar couplings $a_{A,B}$ and $k_A$ [26] (see also section 2.1). When this ratio is measurable, the number of observational constraints to be satisfied rises from five to six.

In the end we find that interesting model-independent constraints on $m_{A,B}$ and $a_{A,B}$ are possible. The great virtue of these bounds is that they directly constrain the stellar parameters on which the PK parameters depend, independent of any assumptions about the function $A(\phi)$ and the nuclear equation of state.

Once the observational constraints on these quantities are known, they can be used to constrain the microscopic parameters for any choice of $A(\phi)$ or equation of state as a separate step. When we do so for the quadratic model $A(\phi) = \exp(a_\phi \phi + b_\phi \phi^2/2)$, we find agreement with earlier work\(^8\), which already rules out a large part of the most interesting region (that of spontaneous scalarization [25–28]).

It might come as a surprise to the reader that this second step might depend on the details of the neutron equation of state, since it is an important feature of GR that predictions for the PK parameters depend only on the two masses, $m_{A,B}$, of the stars, and on none of their other properties. This happy property is called the ‘principle of effacement’ of internal structure [29], and it is this feature that allows a precise prediction of all PK parameters in GR using only the masses, without a need for detailed knowledge about the star’s structure.

In scalar–tensor gravity (as in most other alternative theories of gravity), the principle of effacement does not hold. The prediction for the PK parameters actually depends in principle on all seven quantities which characterize the internal gravitational fields: $m_{A,B}$, $a_{A,B}$, $b_{A,B}$, $k_A$. For now, it is important to note that these are all a priori independent. Once a particular equation of state is specified, then the equations of stellar structure can be solved, and on a

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\(^6\) Since the theoretical prediction for the spin–orbit precession frequency has not yet been calculated in scalar–tensor gravity, we do not include it in the present analysis.

\(^7\) This is explained in detail in section 3.2.

\(^8\) A detailed comparison with earlier work is given at the end of section 3.2.
given branch of stellar configurations, the quantities \( a, b \) and \( k \) for each star can be expressed in terms of its mass \( m \) and the underlying parameters—such as \( A \) and \( B \)—that define the scalar–tensor model. This is the approach taken in [25, 26]. The resulting bounds are then subject to uncertainties in the equation of state, which are hard to quantify.

In the end, the phenomenological analysis to which we are led in this paper is similar in spirit to that carried out by Wex and Kramer [11]. The difference is that we specialize to a more restricted class of theories of gravity, for which the energy loss due to emission of gravitational radiation has been calculated and takes a relatively simple form. Consequently, unlike Wex and Kramer, we are able to make use of all observed PK parameters, including \( \dot{P}_b \), when obtaining constraints.

In the literature, results of pulsar timing measurements are presented graphically in the \( m_A - m_B \) plane (as described in the beginning of section 1.1) for the purpose of testing GR. Whenever it is possible to do so, we urge observers to additionally present bounds on the quantities \( m_{A,B}, a_{A,B}, b_{A,B}, k_A \) within scalar–tensor gravity, allowing one to robustly infer some of the physical properties of the objects involved—like the masses \( m_{A,B} \)—within scalar–tensor gravity, allowing one to robustly infer some of the physical properties of the objects involved—like the masses \( m_{A,B} \)—within scalar–tensor gravity, allowing one to robustly infer some of the physical properties of the objects involved—like the masses \( m_{A,B} \). Moreover, these bounds potentially can be used by theorists to constrain a great variety of specific models.

2. Formalism

In this section we collect expressions for the Keplerian and post-Keplerian parameters in scalar–tensor theories, following the results of [2–5] and references therein. We do so both to establish notation and to provide context for the bounds obtained in the next section. Experts and readers in a hurry should feel free to skip this part completely.

2.1. Keplerian and post-Keplerian parameters

First, a reminder of how Keplerian orbits are described, followed by the PK parameters that describe slow secular changes to the Keplerian parameters.

2.1.1. Orbital description. The non-relativistic gravitational two-body problem famously predicts bound orbits to be ellipses. More specifically, in the centre of mass frame, the relative position vector, \( \vec{r} = \vec{r}_A - \vec{r}_B \), sweeps out the trajectory of an ellipse whose shape is described by two parameters: the semi-major axis \( a \), and the eccentricity \( e \). The positions \( \vec{r}_A \) and \( \vec{r}_B \) also separately trace out ellipses, with semi-major axes satisfying \( a_A/a = m_B/M \) and \( a_B/a = m_A/M \), where \( M = m_A + m_B \) is the system’s total mass.

The time taken to traverse this orbit is given by the orbital period \( P_b \), and is related to the semi-major axis by Newton’s modification of Kepler’s third law:

\[
P_b = \frac{2\pi a^{3/2}}{GM}.
\]

For timing measurements, a reference time \( T_0 \) is also needed to specify the time of passage through periastron.

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9 In scalar–tensor theories, there may be multiple branches of stellar configurations [28, 31].

10 Of course, scalar–tensor theories are not the only interesting alternatives to GR, and it would be worthwhile to obtain such bounds in other classes of alternative theories.

11 In order for the method to be applicable, all five PK parameters must have been measured.

12 \( \tilde{G} \) is the effective gravitational coupling constant between the two bodies. In scalar–tensor gravity with units chosen so that \( A(\phi_\infty) = 1 \), where \( \phi_\infty \) is the value of the scalar field asymptotically far away from the binary system, we have \( \tilde{G} = G(1 + a_\phi a_\phi) = \tilde{G}(1 + a_\phi a_\phi)/(1 + a_\phi a_\phi) \), where \( a_\phi \) is shorthand notation for \( a(\phi_\infty) \), and the quantities \( G \), \( \tilde{G} \) and \( a(\phi) \) have been defined in equations (1.3), (1.5) and (1.6), respectively.
The orientation of the orbital ellipse with respect to a reference triad is specified by three angles. In celestial mechanics, these angles are conventionally taken to be the longitude of ascending node, \( \eta \), the orbital inclination, \( i \), of the orbit relative to the plane of the sky; and the argument of periastron, \( \omega \).

Standard techniques allow the positions and momenta of the two bodies to be inferred from the variables \((a, e, \eta, i, \omega, T_0)\) by means of the action-angle formalism \([30]\).

Deviations from the Newtonian two-body problem cause the shape and orientation of the Keplerian orbits to change with time. In practice, this change is slow enough to be regarded as a small secular evolution in each of the Keplerian parameters. Two of these have been accurately measured for several binary pulsars: the decrease in the orbital period due to emission of gravitational radiation \( \dot{P}_b \); and the precession of the orbital periastron \( \dot{\omega} \).

2.1.2. Pulsar timing. Pulsars emit electromagnetic signals at regular intervals that are measured on the Earth by radio telescopes. The problem of relating the time of emission \( T_e \) at the pulsar to the time of arrival \( \tau_a \) on the Earth is usually split into two parts. The first part involves relating \( T_e \) to the time of arrival \( t_a \) at the solar system barycentre, neglecting the time delay due to the solar gravitational field. The second part involves relating \( t_a \) to \( \tau_a \), which depends only on the motion of Earth with respect to the solar system barycentre (and is not considered here).

The solution to the first problem is the pulsar timing formula, and is conventionally written as

\[
Dt_a = T_e + \Delta_R + \Delta_E + \Delta_S + \mathcal{O}(1/c^4). \tag{2.1}
\]

This type of formula was first considered by Blandford and Teukolsky \([5]\), and subsequently improved upon by Damour and Deruelle \([4]\). Its terms have the following origin.

- The Doppler factor \( D \) describes the time dilation caused by the motion of the solar system relative to the binary pulsar.
- The term \( \Delta_R \) is called the Römer time delay. It is \( \mathcal{O}(1/c) \), and is due to the dependence of the light path on the position of the pulsar. The variable that sets the timescale of the Römer delay is the projection of the semi-major axis along the line of sight, measured in units of time. It is conventionally denoted as \( x_{A,B} \equiv a_{A,B} \sin i/c \), and is called the light crossing time.
- The term \( \Delta_E \) is called the Einstein time delay. It is \( \mathcal{O}(1/c^2) \), and is due to the time dilation caused by the motion of the pulsar A and the gravitational field of the companion B. The variable that sets the timescale of the Einstein delay is called \( \gamma \), and is given by equation (2.7).
- The term \( \Delta_S \) is called the Shapiro time delay. It is \( \mathcal{O}(1/c^3) \), and is caused by the effect of curved space on the propagation of light. It can be thought of as the first relativistic correction to \( \Delta_R \). The variable that sets the timescale of the Shapiro delay is called \( r \) (the range of the Shapiro delay) and is given by equation (2.8). It is proportional to \( G_{\infty B} \), the gravitational coupling between the companion B and a photon, whereas all the other PK parameters depend on \( G_{AB} \), the gravitational coupling between the two orbiting bodies. It is intuitively clear that the importance of the Shapiro effect depends strongly on the inclination angle of the orbit. The variable which parameterizes this dependence is called \( s \) (the shape of the Shapiro delay) and is given by equation (2.9).
2.2. PK parameters in scalar–tensor gravity

When solving the problem of stellar structure in scalar–tensor gravity, it is necessary to specify the boundary condition for the scalar field asymptotically far away from the star \( \phi_{\infty} \). All of the properties of the star, such as its mass, therefore depend implicitly on \( \phi_{\infty} \). (Because these properties can be multiple-valued functions of \( \phi_{\infty} \), for some purposes it can be useful instead to follow the dependence on the value of the scalar field at the star’s centre \[31\].)

Although the asymptotic field is simple to specify for isolated stars, it is a more complicated concept for an orbiting binary system. For isolated stars, the scalar field sourced by the star becomes small far from its position, leading to the generic weak-field large-distance form

\[
\Phi_{A,B}(t, \vec{r}) = \Phi_{\infty} + \frac{GQ_{A,B}}{|\vec{r} - \vec{r}_{A,B}(t)|c^2} + \cdots, \tag{2.2}
\]

where \( Q_{A,B} \) defines the scalar ‘charges’ of the two stars, and \( \vec{r}_{A,B}(t) \) are their trajectories\[13\]. We here make the choice that the scalar field approaches \( \Phi_{\infty} \) asymptotically far away from both stars of the binary system. It is convenient to choose units of length so that \( \Phi_{\infty} = 0 \).

Now consider a binary system of stars A and B, and assume that the separation between the two stars is large enough to justify the near-Newtonian weak-field limit. In this regime the fields sourced by the two stars can be superposed so the total scalar field is \( \Phi = \Phi_A + \Phi_B \).

Consider now the region much closer to one of the two stars, and let \( \phi_A \) and \( \phi_B \) denote the scalar fields of each star in this ‘internal’ regime. The boundary conditions for this ‘internal’ field therefore are (approximately)

\[
(\phi_A)_{\infty}(t) \simeq \Phi_B(t, \vec{r}_A(t)), \quad (\phi_B)_{\infty}(t) \simeq \Phi_A(t, \vec{r}_B(t)). \tag{2.3}
\]

These expressions show that \( (\phi_{A,B})_{\infty} \) are small whenever \( GM/rc^2 \sim (v/c)^2 \ll 1 \), where \( r \) is the separation between the two stars, and \( v \) is their orbital velocity.

Now, the couplings that are relevant for computing PK quantities govern how the mass of star A depends on the boundary condition \( (\phi_A)_{\infty} \). For instance, expanding \( m_A \) in a power series about \( (\phi_A)_{\infty} = 0 \) defines the coefficients \( a_A \) and \( b_A \):

\[
m_A[(\phi_A)_{\infty}] = m_A \cdot \left[ 1 + a_A \cdot (\phi_A)_{\infty} + \frac{1}{2} (b_A + (a_A)^2) (\phi_A)_{\infty}^2 + \cdots \right], \tag{2.4}
\]

and an expansion of \( m_B \) about \( (\phi_B)_{\infty} = 0 \) similarly defines \( a_B \) and \( b_B \). It is a general property of scalar–tensor systems that couplings defined in this way agree with those defined from \( A(\phi) \) using equation (1.6) in the limit of weakly coupled non-relativistic systems \[21\].

Pulsars rotate and the frequency of pulsation is given by the rotational frequency \( \Omega = J/I \), where \( J \) is the ‘spin’ angular momentum of the pulsar, and \( I \) is its moment of inertia. It can be found by solving the equations of stellar structure, and just like the mass, it depends on the boundary conditions for the scalar field:

\[
I_A[(\phi_A)_{\infty}] = I_A \cdot \left[ 1 - k_A \cdot (\phi_A)_{\infty} + \cdots \right]. \tag{2.5}
\]

The predictions for the PK parameters in scalar–tensor gravity turn out to be determined by the Keplerian parameters, together with the coefficients \( m_{A,B}, a_{A,B}, b_{A,B} \) and \( k_A \) \[26\]. Explicitly, we find

\[
\dot{\omega} = \frac{n}{1 - e^2} \left( \frac{G_{AB}Mn}{c^3} \right)^{2/3} \left( 3 - a_Aa_B \right)^{1/2} \frac{X_Bb_Ba_A^2}{2(1 + a_Aa_B)^2}, \tag{2.6}
\]

\[
\gamma = \frac{eX_B}{n(1 + a_Aa_B)} \left( \frac{G_{AB}Mn}{c^3} \right)^{2/3} (X_B(1 + a_Aa_B) + 1 + k_Aa_B), \tag{2.7}
\]

\[13\] Knowledge of the stellar equation of state allows \( Q_{A,B} \) to be computed as functions of the corresponding mass \( m_{A,B} \) and the asymptotic value of the scalar field.
\[ r = G_{\infty B} m_B / c^3, \]  
\[ s = \frac{n_A}{X_B} \left( \frac{G_{\infty B} M_n}{c^3} \right)^{-1/3}, \]

where \( G_{AB} = G(1 + a_A a_B) \) is the total (graviton plus scalar) weak-field coupling between the two bodies, and \( G_{\text{S}} = G(1 + \tilde{a}_n a_B) \) is the coupling between the companion \( B \) and a photon. The quantity \( d_{\text{SC}} \) is given by the value of the scalar–matter coupling \( a(\phi) \) (defined in equation (1.6)) asymptotically far away from the binary pulsar. As before, \( M = m_A + m_B \) is the total mass, while \( n = 2 \pi / P_b \) is the orbital angular frequency and \( X_{A,B} = m_{A,B} / M \).

The expression for the decay of the orbital period is similarly given by
\[
\dot{P}_b = \dot{P}^\text{mon}_b + \dot{P}^\text{dip}_b + \dot{P}^\text{quad}_b + \dot{P}^\text{kin}_b + \dot{P}^\text{gal}_b, \tag{2.10}
\]
where the different contributions are
\[
\dot{P}^\text{mon}_b = - \frac{3 \pi X_A X_B}{1 + a_A a_B} \left( \frac{G_{\infty B} M_n}{c^3} \right)^{5/3} \frac{e^2 (1 + e^2/4)}{(1 - e^2)^{7/2}} \times \left[ \frac{5}{3} (a_A + a_B) - \frac{2}{3} (a_A X_A + a_B X_B) + \frac{b_A a_B + b_B a_A}{1 + a_A a_B} \right]^2, \tag{2.11}
\]
\[
\dot{P}^\text{dip}_b = - \frac{2 \pi X_A X_B}{1 + a_A a_B} \left( \frac{G_{\infty B} M_n}{c^3} \right)^{5/3} \frac{(1 + e^2/2)}{(1 - e^2)^{5/2}} (a_A - a_B)^2 \left[ 1 + \mathcal{O} \left( \frac{1}{e^2} \right) \right] + \mathcal{O} \left( \frac{a_A - a_B}{e^2} \right), \tag{2.12}
\]
\[
\dot{P}^\text{quad}_b = - \frac{32 \pi X_A X_B}{5(1 + a_A a_B)} \left( \frac{G_{\infty B} M_n}{c^3} \right)^{5/3} \frac{(1 + 73e^2/24 + 37e^4/96)}{(1 - e^2)^{7/2}} (6 + [a_A X_B + a_B X_A]_c^2), \tag{2.13}
\]
where ‘mon’, ‘dip’ and ‘quad’ denote monopole, dipole and quadrupole radiation, respectively. In the limit of GR—i.e. when \( a_{A,B} \to 0 \) and \( b_{A,B} \to 0 \)—the monopole and dipole contributions vanish. Note that the monopole term is of order \( 1/c^2 \), because the total scalar charge of the binary system is constant in time.

The last two terms in equation (2.10) arise due to the relative motion between the binary pulsar and the solar system [32]. The kinetic contribution to \( \dot{P}_b \), also called the Shklovskii effect [33], is given by
\[
\dot{P}^\text{kin}_b = \dot{v}^2 / cd, \tag{2.14}
\]
where \( \dot{v} \) is the velocity of the binary pulsar relative to the solar system, \( T \) denotes the component transverse to the line of sight and \( d \) is the distance between the binary pulsar and the solar system.

The galactic contribution to \( \dot{P}_b \) is given by
\[
\dot{P}^\text{gal}_b = a g / c, \tag{2.15}
\]
where \( \ddot{a} \) is the acceleration of the binary pulsar relative to the solar system, and \( R \) denotes the component along the line of sight. We have [2–5]
\[
\ddot{a} = \left[ \frac{1 + \tilde{a}_{\text{gal}}}{1 + (\tilde{a}_{\text{gal}})^2} \right] \tilde{g}_{\text{psr}} - \tilde{g}_{\text{ss}}, \tag{2.16}
\]
where \( \tilde{g}_{\text{psr}} \) is the acceleration of the binary pulsar relative to the galactic centre, as predicted by a Newtonian galactic model, and \( \tilde{g}_{\text{ss}} \) is the corresponding quantity for the solar system.
quantity $a_{\text{gal}}^{\infty}$ is the value of the scalar–matter coupling function $a(\phi)$ (defined in equation (1.6)) asymptotically far away from the galaxy, and

$$ a_{\text{psr}} = X_A a_A + X_B a_B $$

is the charge-to-mass ratio of the binary pulsar system as a whole.

A simple galactic model may be used to relate $a_{\text{gal}}^{\infty}$ to $a_{\text{psr}}^{\infty}$, and so in principle the PK parameters depend on $a_{\text{psr}}^{\infty}$ (and the galactic model) in addition to the seven quantities $m_A, b_A, a_A, b_B, a_B$ and $k_A$. However, it turns out in practice that the contributions of $a_{\text{psr}}^{\infty}$ are much too small to be measurable, so we henceforth set $a_{\text{psr}}^{\infty} = a_{\text{gal}}^{\infty} = 0$. This ensures that GR is recovered asymptotically far away from the binary pulsar, and asymptotically far away from the galaxy.

Note that equations (2.6)–(2.10) are invariant under

$$ a_A \rightarrow -a_A, \quad a_B \rightarrow -a_B, \quad a_{\text{psr}}^{\infty} \rightarrow -a_{\text{psr}}^{\infty}, \quad a_{\text{gal}}^{\infty} \rightarrow -a_{\text{gal}}^{\infty} \quad \text{and} \quad k_A \rightarrow -k_A, $$

which corresponds to switching the sign of the scalar field.

3. Constraints

3.1. Statistics

In this section, we use the method of least squares [34] to compare the observed values of the PK parameters to the predictions of scalar–tensor gravity, thereby obtaining constraints on the phenomenological stellar parameters $\{m_A, b_A, a_A, b_B, a_B, k_A\}$. For brevity, we will denote these phenomenological stellar parameters by $\{\Gamma_i\}_{i=1}^7$.

For a given pulsar of interest, let $\{\xi_i\}_{i=1}^N$ run over as many of the quantities $\dot{P}_b, \dot{\omega}, \gamma, r, s$ and $R$ as are measured. Assume that the measurement process for $\xi_i$ can be described by a normal (Gaussian) distribution with standard deviation $\Delta_i$, and assume that correlations between the different $\xi_i$ can be neglected.

If we know a priori that the pulsar timing is correctly described by the phenomenological stellar parameters $\{\Gamma_i\}$ within scalar–tensor gravity, then the probability of measuring any given set of values of $\{\xi_i\}$ is given by

$$ P(\xi_1, \ldots, \xi_N | \Gamma_1, \ldots, \Gamma_7) = \frac{e^{-\chi^2/2}}{(2\pi)^{N/2} \Delta_1 \cdots \Delta_N}, $$

where

$$ \chi^2 = \sum_{i=1}^N \left[ \frac{\xi_i - \xi_i^{\text{th}}(\Gamma_1, \ldots, \Gamma_7)}{\Delta_i} \right]^2, $$

(3.2)

where $\xi_i^{\text{th}}$ are the theoretically predicted values in scalar–tensor gravity. By means of Bayes’ theorem, this probabilistic statement may be turned around: if we know a priori that a given set of values of $\{\xi_i\}$ have been observed, then the probability that the pulsar is described by the phenomenological parameters $\{\Gamma_i\}$ in scalar–tensor gravity is given by

$$ P(\Gamma_1, \ldots, \Gamma_7 | \xi_1, \ldots, \xi_N) = \frac{P(\Gamma_1, \ldots, \Gamma_7) P(\xi_1, \ldots, \xi_N | \Gamma_1, \ldots, \Gamma_7)}{P(\xi_1, \ldots, \xi_N)}. $$

(3.3)

Assume that we have no prior information about $\{\xi_i\}$ or $\{\Gamma_i\}$. Then, combining equations (3.1) and (3.3) yields

$$ P(\Gamma_1, \ldots, \Gamma_7 | \xi_1, \ldots, \xi_N) \sim e^{-\chi^2/2}. $$

(3.4)
Table 2. Keplerian parameters for the double pulsar J0737-3039. Data taken from [13].

| Symbol | Meaning | Value |
|--------|---------|-------|
| $x_A$  | Light crossing time of A | 1.415 032(1) s |
| $x_B$  | Light crossing time of B | 1.5161(16) s |
| $e$    | Eccentricity | 0.087 7775(9) |
| $P_b$  | Orbital Period | 0.102 251 562 48(5) days |
| $\omega$ | Argument of periastron of A | 87.0331(8) |
| $T_0$  | Time at periastron of A | 53 155.907 4280(2) MJD |

In principle, equation (3.4) may be calculated numerically, and integrated over the seven-dimensional parameter space to find the constraints of interest. For instance, the mean value and variance of $\Gamma_i$ are given by

$$\bar{\Gamma}_i = \frac{\int d^7 \Gamma \Gamma_i e^{-\chi^2/2}}{\int d^7 \Gamma e^{-\chi^2/2}}$$

and

$$\sigma_{\Gamma_i}^2 = \frac{\int d^7 \Gamma (\Gamma_i - \bar{\Gamma}_i)^2 e^{-\chi^2/2}}{\int d^7 \Gamma e^{-\chi^2/2}}$$

respectively. In practice, it is very difficult to calculate (3.5) and (3.6) directly. Therefore, we resort to approximation methods. Assume that $\chi^2$ has a global minimum at $\left\{ \Gamma^*_i \right\}$, and approximate it near this minimum by a quadratic form:

$$\chi^2(\Gamma_1, \ldots, \Gamma_7) = \chi^2_{\text{min}} + \sum_{i,j=1}^{7} (\Gamma_i - \Gamma^*_i)C_{ij}(\Gamma_j - \Gamma^*_j).$$

Substituting equation (3.7) into equations (3.4)–(3.6) shows that $\Gamma_i$ are normally distributed, with means

$$\bar{\Gamma}_i = \Gamma^*_i,$$

and variances

$$\sigma_{\Gamma_i}^2 = \frac{1}{C_{ii}}.$$

The off-diagonal components of $C$ are related to the correlation coefficients between the different $\left\{ \Gamma_i \right\}$.

3.2. Implementation

We have implemented the statistical analysis described in the previous section for two pulsars, for which at least five PK parameters have been measured. For the double pulsar PSR J0737-3039, all of the quantities $P_b$, $\omega$, $\gamma$, $r$, $s$, $R$ have been measured (see [13] for the latest values), and so $N = 6$. For the binary pulsar PSR B1534+12, by contrast, $R$ is not measured (see [7] for the latest values) and so $N = 5$.

The double pulsar PSR J0737-3039A/B consists of two pulsars, A and B, that are bound in a relativistic orbit. Pulsar A, which has a period of 23 ms, was discovered by Burgay et al with the Parkes radio telescope in 2003 [18]. Subsequent data analysis by Lyne et al has revealed that companion B is also a pulsar, with a much slower period of 2.8 s [17]. The Keplerian parameters of this binary system are summarized in table 2, and the PK parameters are summarized in table 3. The PK timing parameters $\gamma$, $r$ and $s$ all pertain to pulsar A. The light crossing time $x$ has been measured for both pulsars. For reviews of this system and
other relevant literature, see [12–16, 19]. For the binary pulsar PSR B1534+12 discovered by Wolszczan in 1991 [9], the Keplerian parameters are summarized in table 4, and the post-Keplerian parameters are summarized in table 5. For other relevant literature on this system, see [7, 8, 10].

For these two pulsars, we have calculated (3.2) numerically on a grid in the seven-dimensional parameter space, and found that $\chi^2$ has a global minimum near $a_A = a_B = 0$. This is not surprising, because these pulsars are very well described by GR. We have also found that the minimum value of $\chi^2$ is very close to zero—$\chi^2_{\text{min}} \sim 10^{-3}$ for the double pulsar, and $\chi^2_{\text{min}} \sim 10^{-1}$ for 1534+12. This is also not surprising, because we have more parameters than data points.

Since it is hard to visualize the seven-dimensional parameter space, we will define the following functions in order to present the results of our numerical calculations:

$$\chi^2_{m}(m_A, m_B) \equiv \min_{a_A, a_B, b_A, b_B, k_A} \chi^2(m_A, m_B, a_A, a_B, b_A, b_B, k_A),$$

(3.10)

$$\chi^2_{a}(a_A, a_B) \equiv \min_{m_A, m_B, b_A, b_B, k_A} \chi^2(m_A, m_B, a_A, a_B, b_A, b_B, k_A).$$

(3.11)

In principle, there are $\binom{7}{2} = 21$ possible functions of this form. However, in practice (3.10) and (3.11) are the only ones which lead to interesting constraints. In particular, it is not possible to
constrain $b_A$ and $b_B$, because these two quantities enter into the PK parameters only through the combinations $a_A b_B$, $a_A^2 b_B$, $a_B b_A$ and $a_B^2 b_A$ (see equations (2.6) and (2.11)). Consequently, for pulsars whose timing data are consistent with GR, arbitrarily large values of $b_A$, $b_B$ are permissible, provided that $a_A$, $a_B$ are sufficiently small. It is this weak dependence of the PK parameters on $b_A$, $b_B$ in the vicinity of $a_A$, $a_B = 0$ which enables interesting constraints to be obtained in the first place, by effectively reducing the number of free variables from seven down to five.

If we use the quadratic form approximation (3.7), then $\chi^2_a$ and $\chi^2_m$ are also quadratic forms, whose coefficients can be explicitly calculated in terms of the components of $C$ [37]. Moreover, it can be shown that [37]

$$\min_{a_B} \chi^2_a (a_A^* + \delta, a_B) = \chi^2_{a_B} + \delta^2 / \sigma_{a_A}^2, \quad (3.12)$$

$$\min_{a_A} \chi^2_a (a_A, a_B^* + \delta) = \chi^2_{a_B} + \delta^2 / \sigma_{a_B}^2, \quad (3.13)$$

and similar relations hold for $\chi^2_m$. Thus, the standard deviations may be found from contour plots of $\chi^2_a$ and $\chi^2_m$.

For the double pulsar PSR J0737-3039A/B, contours of $\chi^2_a$ are plotted in the left panel of figure 1. This plot shows that $a_A$ is very close to $a_B$, with a precision that is more easily seen in the right panel of the same figure. The robustness of this constraint can be simply understood from equation (2.12)—if $|a_B - a_A|$ gets too large, then this equation predicts too much dipole radiation, inconsistent with observations. Changing variables from $(a_A, a_B)$ to $(a_A, a_B - a_A)$ in equations (3.12) and (3.13), we infer that

$$\sigma_{a_A} = 0.21, \quad \sigma_{a_B - a_A} = 0.002, \quad (3.14)$$

while the mean values of both $a_A$ and $a_B - a_A$ are zero. Also, note that the plots in figure 1 are both symmetric under $a_A, a_B \rightarrow -a_A, a_B$, as expected from the symmetry (2.18).

This same analysis may be carried out for the pulsar PSR B1534+12. The contour plots of $\chi^2_m$ are shown in figure 2. We find that

$$\sigma_{a_A} = 0.44, \quad \sigma_{a_B - a_A} = 0.004. \quad (3.15)$$

For the double pulsar PSR J0737-3039A/B, contours of $\chi^2_m$ are plotted in the left panel of figure 3. The masses lie very close to the line $m_A = R m_B$, in good agreement with the general
Figure 2. Contour plot of $\chi^2$ for PSR B1534+12 in the $a_A - a_B$ plane (left panel), and with $a_A - a_B$ shown versus $a_A$ (right panel). The value of $\chi^2$ is minimized over $m_A, m_B, b_A, b_B, k_A$.

Figure 3. Contour plot of $\chi^2$ for the double pulsar J0737-3039 in the $m_A-m_B$ plane (left panel). The right panel plots the same information using a variable that emphasizes the accuracy of the test of the prediction $R = m_A/m_B$. The value of $\chi^2$ is minimized over $a_A, a_B, b_A, b_B, k_A$. Units on both axes are in solar masses.

prediction. The deviation from this line is shown in more detail in the right panel of the same figure, by plotting $x_B m_B - x_A m_A$ versus $m_A$. The minimum value of $\chi^2_m$ is close to the mass values inferred in GR—$m_A = 1.3381(7)m_\odot$ and $m_B = 1.2489(7)m_\odot$—on the top right of the line in the left panel of figure 3, and on the centre right of the right panel of the same figure.

Note that the behaviour of $\chi^2_m$ near its minimum value is very asymmetric. This asymmetry is caused by the contribution of equation (2.9) to $\chi^2$. At the minimum value of $\chi^2_m$, we have $s \sim 1$. The value of $\chi^2_m$ increases very rapidly as we enter the region $s > 1$, whereas $\chi^2_m$ increases much more slowly as we enter the region $s < 1$. Theoretically, $s$ is the sine of the orbital inclination angle, i.e. $s = \sin i$ and so $s \leq 1$. Strictly speaking, this constraint should have been imposed by a prior probability in equation (3.3). However, we see that in practice, for the double pulsar, this constraint is automatically enforced by the rapid growth of $\chi^2_m$.

This strong asymmetry also shows that the quadratic form approximation to $\chi^2$, equation (3.7), is not a good one, and that strictly speaking, the use of equations (3.12)
Figure 4. Contour plot of $\chi^2$ for PSR B1534+12. The value of $\chi^2$ is minimized over $a_A, a_B, b_A, b_B, k_A$. Units on both axes are in solar masses.

and (3.13) is not justified. However, we will still use the $\chi^2_m = \chi^2_{\min} + 1$ contour to estimate the allowed range for the masses at the 68% confidence level. We find

$$1.28 \leq m_A/m_\odot \leq 1.34, \quad 1.19 \leq m_B/m_\odot \leq 1.25.$$  

(3.16)

Now for the pulsar PSR B1534+12, contours of $\chi^2_m$ are plotted in figure 4. The minimum value is at $m_A \sim 1.23m_\odot$ and $m_B \sim 1.29m_\odot$. The masses are not as highly correlated as in the double pulsar case, because the ratio $R$ has not been measured. Also, the asymmetry of $\chi^2_m$ is not as pronounced as in the double pulsar case, because the PK parameter $s$ has been measured much less accurately than for the double pulsar. Looking at the contour $\chi^2_m = \chi^2_{\min} + 1$, we find that at the 68% confidence level,

$$0.97 \leq m_A/m_\odot \leq 1.28, \quad 1.15 \leq m_B/m_\odot \leq 1.31.$$  

(3.17)

In GR, if the bands (1.1) defined by the measured values$^{14}$ of all five PK parameters for PSR B1534+12 are plotted in the $m_A$–$m_B$ plane, it is found that they do not intersect, and thus, GR is ruled out at the one-sigma level [7]. However, if the orbital period decay measurement $\dot{P}_b$ is thrown out, and only the four PK parameters $\dot{\omega}$, $\gamma$, $r$, and $s$ are used, then the bands do intersect, and it is found that the GR-inferred mass range is given by $m_A = 1.3332(10)m_\odot$ and $m_B = 1.3452(10)m_\odot$ [7, 8], which lies in the top-right corner of figure 4, and is outside of the range (3.17). The justification for throwing out the $\dot{P}_b$ measurement [7] is that the measurement of the distance to the pulsar$^{15}$ derived from the Taylor and Cordes model [35] is unreliable [36]. In fact, an improved estimate for the distance to PSR B1534+12 has been obtained by assuming the correctness of GR, and requiring all five bands (1.1) in the $m_A$–$m_B$ plane to intersect [7]. Since the subject of this paper involves alternative theories of gravity, we do not use a distance measurement that was derived by assuming the correctness of GR. Rather, we use the distance measurement derived from the Taylor and Cordes [35] model, and warn the reader that the results (3.15) and (3.17) obtained therefrom should be taken with a grain of salt.

Having obtained the model-independent constraints (3.14), (3.16) for the double pulsar, and (3.15), (3.17) for PSR B1534+12, we now illustrate how these results may be used to obtain bounds on the microscopic scalar–matter coupling parameters in a particular model. For simplicity, we consider the quasi-Brans–Dicke model with $A(\phi) = \exp(a_5\phi + b_5\phi^2/2)$, and use the same relativistic polytrope models for neutron stars as in [28, 31], which are called EOS II and EOS A. For each pair of values $(a_i, b_i)$, we find all stable stellar configurations,
Figure 5. Theory-space exclusion plot for the quasi-Brans–Dicke model. The regions above and to the left of the curves are ruled out. If units are chosen such that $\phi = 0$ asymptotically far away from the binary pulsar, then $a_{\text{psr}} = a_s$.

and check if our bounds on the masses and scalar couplings may be satisfied. The results of this calculation are presented as a theory-space exclusion plot in figure 5.

Strictly speaking, the bounds (3.14)–(3.17) are only valid in the limit when the scalar–matter coupling vanishes asymptotically far away from the binary pulsar, i.e. $a_{\text{psr}}^{\text{psr}} = 0$, and thus figure 5 is only accurate close to the horizontal axis. Since the Cassini bound in the solar system together with a simple galactic model implies that $a_{\text{psr}}^{\text{psr}} \lesssim 10^{-2}$, we do not explore the dependence of our bounds on $a_{\text{psr}}^{\text{psr}}$ in great detail\(^{16}\).

Figure 5 is to be compared with figure 11 of [27], which was obtained by calculating the PK parameters as functions of the masses $m_{A,B}$ of the two stars, and checking if the bands in the $m_A–m_B$ plane defined by the measured values of the PK parameters have a non-empty intersection. In the latter figure, it is shown graphically that all binary pulsar timing measurements place the constraint $b_s \gtrsim -5$, in agreement with figure 5. If one specializes to the Brans–Dicke theory, i.e. $b_s = 0$, then figure 11 of [27] shows that timing of the double pulsar leads to the constraint $a_s \lesssim 0.04$, whereas timing of PSR B1534+12 leads to the constraint $a_s \lesssim 0.08$. In comparison, our bound on $a_s$ coming from the double pulsar is weaker by a factor of 4 to 5, and for PSR B1534+12, our bound is weaker by at least the same factor\(^{17}\).

It is not surprising that our bounds on $a_s$ are weaker than those obtained by directly working with the PK parameters, because information about correlations is lost when intermediate variables are introduced. This is the price paid for the simplicity of working directly with the masses and scalar couplings of the individual stars, rather than the PK parameters, and the curves defined by them in the $m_A–m_B$ plane.

4. Conclusions

In summary, we have shown that the existing data for two pulsars are constraining enough to place model-independent bounds directly on the stellar parameters that control the size of PK effects in scalar–tensor models. The virtue of these bounds is that they do not depend on

\(^{16}\) We have run our numerical code for the double pulsar with $a_{\text{psr}}^{\text{psr}} = 0.1$, and found that the bounds on the scalar couplings (3.14) change to $\sigma_{a_A} = 0.27$ and $\sigma_{a_{A,B}} = 0.002$, while the bounds on the masses (3.16) change to $1.26 \leq m_A/m_\odot \leq 1.35$, and $1.18 \leq m_B/m_\odot \leq 1.26$.

\(^{17}\) Figure 5 has been cut off at $a_s \sim 0.2$, because the plotted curves are only accurate for small $a_s$, as explained in the preceding paragraph.
the particular form for the function $A(\phi)$ that defines which model is of interest, or on the details of the stellar equations of state. In particular we find that existing data impose strong and model-independent constraints on the relative size of the two scalar charges and masses.

These bounds can also be used to constrain particular models by computing in these models the masses and couplings as functions of the microscopic parameters. It is at this point that dependence on things like the stellar equation of state enters.

We applied these methods in particular to the double pulsar J0737-3039A/B, and found $1.28 \leq m_A/m_\odot \leq 1.34$, $1.19 \leq m_B/m_\odot \leq 1.25$, $|a_{A,B}| < 0.21$, and $|a_B - a_A| < 0.002$, with 68% confidence, for all choices of scalar–matter coupling function, and for all nuclear equations of state.

A similar analysis of the pulsar PSR B1534+12 yields $0.97 \leq m_A/m_\odot \leq 1.28$, $1.15 \leq m_B/m_\odot \leq 1.31$, $|a_{A,B}| < 0.44$ and $|a_B - a_A| < 0.004$.

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