Chiral symmetry breaking, the superspace gap equation and the no-renormalization theorem

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Abstract

Solutions of the superspace Schwinger-Dyson equations, describing mass generation and chiral symmetry breaking in supersymmetric gauge theory, need not be constrained to vanish by no-renormalization theorems, nor by special choices of gauge parameter. Thus symmetry breaking vacuum structures remain possible (as in non-supersymmetric gauge theory), inviting comparison with predictions of an alternative approach based on holomorphy and the Wilsonian effective action.

The gap equation formalism has frequently been employed to describe the vacuum structure of strongly interacting gauge theories. Chiral symmetry breaking provides an especially interesting application. The resulting conclusions can now be compared with those of independent approaches based on holomorphy and the Wilsonian effective action [1], in the special case of supersymmetric gauge theories. Some previous supersymmetric gap equation analyses [2, 3] employ the component formalism, with supersymmetry not manifest, complicating their interpretation. In this letter, we will study the gap equation in a superspace formulation.

A previous treatment of this topic [4] concluded that chiral symmetry does not break unless the Lagrangian explicitly breaks supersymmetry. We will follow a similar program, but arrive at different conclusions, arguing that the vanishing of gap equation coefficients in special gauges does not prohibit chiral symmetry breaking. We shall separately emphasize that in general the supersymmetric no-renormalization theorem for the 1PI effective potential does not apply to a significant
class of theories, and is less restrictive than sometimes assumed elsewhere in the literature.

The simplest gauge theory, Abelian supersymmetric QED, suffices to illustrate the main features of the gap equation analysis. Non-abelian generalizations, whatever their behaviors in alternative analyses, introduce no essential complications in the one-loop truncated gap equation. We thus begin with the SQED action

\[ \int d^4x d^4\theta \left\{ \frac{1}{8} V D \bar{D} D D V - \frac{1}{\xi} V D \bar{D} D V \right\} \]

\[ + \int d^4x d^4\theta \left\{ \Phi_+^\dagger e^{2eV} \Phi_+ + \Phi_-^\dagger e^{-2eV} \Phi_- \right\} \]

\[ + \int d^4x d^2\theta \ m \Phi_+ \Phi_- + \int d^4x d^2\theta \ m \Phi_+^\dagger \Phi_-^\dagger \]

using standard conventions for supersymmetric operator normalizations and identities. The chiral field \( \Phi_+ \) contains a left handed fermion and its scalar partner, while \( \Phi_- \) contains the left-handed anti-fermion and another scalar partner. If the bare mass \( m \) vanishes, the theory has a chiral symmetry, separate phase rotations on \( \Phi_+ \) and \( \Phi_- \) (or \( SU(N) \times SU(N) \) rotations, in the \( N \) flavor case).

The gauge invariant condensate \( \langle \int d^2\theta \ \Phi_+(z) \Phi_-(z) \rangle \) varies under chiral transformations, and provides a suitable order parameter for chiral symmetry breaking. Since this is the zero-distance limit of (the fermion part of) the matter field chiral-chiral propagator, we can shift our attention to the matter field propagators (quantum corrected beyond finite order in perturbation theory). We write the full GRS chiral-chiral propagator (for Euclidean momenta) as

\[ i \frac{-1}{p_E^2 \ Z^2(p_E^2) + \Sigma^2(p_E^2)} \ \frac{\Sigma^2(p_E^2) \ D^2}{-4 p_E^2} \ \delta^4(\theta_1 - \theta_2) \] (2)

and the chiral-antichiral propagator as

\[ i \frac{-1}{p_E^2 \ Z^2(p_E^2) + \Sigma^2(p_E^2)} \ Z^2(p_E^2) \ \delta^4(\theta_1 - \theta_2) \] (3)

Coupled Schwinger-Dyson equations relate the inverse of these full propagators to the inverse of the corresponding bare propagators, together with the fully resummed one loop corrections shown in Fig. 1. Our superspace gap equations will be similar to those of Clark and Love, up to some sign differences and our retention of the order \( e^2 \) seagull graph contributing to wave function.
renormalization. We will similarly use vertex functions related to a single wave function factor \( [4, 6] \), in accordance with the Ward identity, rather than attempting to solve the infinite set of coupled higher-point equations.\(^1\)

\[
\begin{align*}
\Phi_+ & \quad \Phi_- \\
\Phi^+_i & \quad \Phi^-_i
\end{align*}
\]

Figure 1: Contributions to mass and wave function gap equations for the matter superfield. Solid lines with blobs are matter superfield propagators, solutions of the gap equation through which the indicated momentum flows. Wavy lines are gauge superfield propagators, and black vertex blobs are corrected vertices.

To obtain the gap equations, we must evaluate the diagrams. After Wick rotation, the mass renormalization diagram (Fig. 1a) has the value

\[
-4 \xi^2 E(p) \xi^2 - 2 \xi (p \cdot q) \xi^2 \int d^4 \theta \Phi^*_m(p, \theta, \bar{\theta}) \Phi_n(-p, \theta, \bar{\theta}).
\]

For the chiral superfield wave function renormalization, there are two graphs (Fig. 1b) at lowest order in \( e^2 \). The gauge emission/absorption diagram is

\[
\begin{align*}
\Phi_+ & \quad \Phi_- \\
\Phi^+_i & \quad \Phi^-_i
\end{align*}
\]

and the seagull gauge loop is

\[
\begin{align*}
\Phi_+ & \quad \Phi_- \\
\Phi^+_i & \quad \Phi^-_i
\end{align*}
\]

Here \( \Gamma_3 \) and \( \Gamma_4 \) are the fully corrected three and four point vertices. An approximate form relying on the Ward identities sets them to \( Z(M^2) \), where \( M^2 \) is the maximum of their squared (Euclidean)
arguments. The gap equations are then
\[
\Sigma(p_E^2) = m + p_E^2 4e^2 (1 - \xi) \int \frac{d^4 q_E}{(2\pi)^4} \frac{Z(M^2) \Sigma(q_E^2)}{q_E^2 Z^2(q_E^2)} \frac{1}{(p_E - q_E)^4} \quad (7)
\]
and
\[
Z(q_E^2) = 1 + 4e^2 \int \frac{d^4 q_E}{(2\pi)^4} \frac{Z(M^2) Z(q_E^2)}{q_E^2 Z^2(q_E^2)} \frac{1}{(p_E - q_E)^4} \left[ p_E \cdot q_E - \frac{\xi}{2} (p_E^2 + q_E^2) \right] \\
- 2e^2 (1 - \xi) \int \frac{d^4 q_E}{(2\pi)^4} \frac{1}{q_E^4}. \quad (8)
\]
We will not explicitly renormalize the ultraviolet log in Eq. (8); this could be straightforwardly accomplished by changing the leading term of unity to a scheme-dependent constant \( Z_\phi (\mu) \) corresponding to fields \( \Phi_\pm \) renormalized at some scale \( \mu \).

The mass gap equation (7) involves a nonvanishing integral, but there is no conflict with the supersymmetric no-renormalization theorem. The latter is sometimes misconstrued in the literature as prohibiting all corrections to the one-particle-irreducible (as opposed to Wilsonian) perturbative effective superpotential. Stated precisely, however, it merely restricts the possible form of such corrections [10]. (Power counting still indicates the absence of ultraviolet divergences [10, 11].) Substituting \( \Sigma = m \) and \( Z = 1 \) in the integral of Eq. (7) reproduces the one loop contribution to the one-particle-irreducible self energy. This correction is not however strictly a superpotential contribution: the factor of external momentum squared corresponds to a derivative operator, although not even that was required by the no-renormalization theorem. For instance, the massless Wess-Zumino model, with or without a Yang-Mills sector, has perturbative mass corrections without prefactors of external momenta [10]. Massless particles are necessary for such non-derivative superpotential contributions, but not sufficient (as shown by Eq. (7) induced by the massless SQED gauge multiplet).

The gauge parameter dependence of this result offers another possible argument for a vanishing dynamical mass. Clark and Love [11] noted that the gap equation integral is multiplied by zero in Feynman gauge (\( \xi = 1 \)), leaving only the trivial solution. They concluded that SQED in the gap equation formalism does not break chiral symmetry, unless the Lagrangian contains explicit supersymmetry breaking. We should note, however, that the integral evaluated for gap equation
solutions could cancel the vanishing prefactor. This in fact happens in non-supersymmetric gauge theory, and also in supersymmetric gauge theory in component formalism. The linearized gap equation (valid in parameter ranges where the dynamical mass is small) has solutions independent of $\xi$, including the limit of the prefactor approaching zero. Carrying out the integral in superspace is more complicated, due to difficulties in regulating infrared divergences (arising, as Clark and Love discussed, from propagating a supersymmetry-gauge artifact—the lowest component of the vector superfield). The favored infrared regulator prescription gives finite answers only for gauge invariant Green functions; however, the propagators of interest here are of course gauge dependent. Demonstrating the existence of explicit solutions will require further analysis.

We conclude that gauge independence arguments, and the no-renormalization theorem, do not prevent superfield gap equations from yielding chiral symmetry breaking, even without explicit supersymmetry breaking in the Lagrangian. This is reassuring for the corresponding calculations in component notation, which cannot impose manifest supersymmetry on the off-shell Green’s functions in the gap equation. Since gauge dependent infrared contributions are however harder to handle in superspace, the two formalisms play a complementary role in comparisons of gap equation techniques with holomorphic/Wilsonian ones.

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