On the Meissner Effect in the Relativistic Anyon Superconductors

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Abstract

The relativistic model with two types of planar fermions interacting with the Chern-Simons and Maxwell fields is applied to the study of anyon superconductor. It is demonstrated, that the Meissner effect can be realized in the case of the simultaneous presence of the fermions with a different magnetic moment interactions. Under the certain conditions there occurs an extra plateau at the magnetization curve. In the order under consideration the spectrum of the electromagnetic field excitations contains the long-range interaction and one massive "photon" state.
1. Introduction

The zero-temperature Meissner effect presented in the 2+1 dimensional anyon matter, provoked a considerable efforts in order to promote the Chern-Simons gauge theory as a hypothetical candidate for the high $T_c$ superconductivity.

The most important points in that development are existence of the massless (Goldstone) pole in the current correlators [1], cancelation of bare and induced C-S terms [3], and detailed calculations of effective action and thermodynamical potential for the fermions ineracting with C-S and Maxwell fields [2, 4, 5, 6].

Among the others it was shown, that the Meissner effect is partial, i.e. magnetic field starts to penetrate into the sample at any non-zero temperature [5]. The second result concerns the effective mixing of Maxwell and C-S fields and occurrence of two distinct massive excitations [4].

In the present paper we try to give some complementary insights into those intriguing questions.

It is demonstrated, that the Meissner effect exists only if the matter consists of the two types of fermions with opposite signs of magnetic moment interaction. Such a system can be naturally realized considering the planar relativistic fermions. Note, that the different versions of the relativistic anyon superconductivity have been considered in [3, 4, 5].

Studying the gauge field propagators, we found out, that results cited in [4], can be understood as a consequenses of momentum expansion of the certain structure functions. Rising the order of terms taken into account, it can be shown, that there exists the single massive excitation.

The paper is organized in the following way. Section 2 containes the description of model and some basic definitions.

In Section 3 we present the thermodynamical potential and relevant physical quantities.

In Section 4 we show, that the Meissner effect can take place if the system contains two types of fermions. Further we calculate the dependence of critical magnetic field on the temperature and establish the existence of additional plateaux at the magnetization curves.

In Section 5 we derive the electromagnetic field propagators and analyze the spectrum.

2. The Model

Consider the system of Dirac fermions in 2+1 dimensions interacting with Maxwell electromagnetic field and Chern-Simons gauge field.
The total lagrangian for this system is a sum of matter and gauge field lagrangians

\[ \mathcal{L} = -\frac{\delta}{4\mu_0} F_{\mu\nu} F^{\mu\nu} - \frac{c e^2 \nu_0}{\hbar} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + e c n_e A_0 + \]

\[ + \bar{\psi} \{ i \hbar c \gamma^\mu D_\mu - \sigma mc^2 \} \psi, \]

\[ D_\mu = \partial_\mu + i(e/\hbar)(A_\mu + a_\mu), \quad m > 0, \quad \sigma = \pm 1, \]

where \( \delta \sim 10^{-9} m \) is the interplanar distance in the multilayered system and its presence is justified by the use of the three dimensional Maxwell field and electric charge. The Chern-Simons field is in fact two-dimensional, but will be measured in three dimensional units. \( \psi \) is the 2-component fermion field and \( e n_e \) is the planar density of the background neutralizing charges. In 2+1 dimensions the sign of the mass term is essential and describes the helicity of the fermion.

In what follows, we study the static properties of this relativistic system. Analysis will be performed in the self consistent field approximation, developed in [5]. It means, that the gauge fields must satisfy the equations

\[ -\frac{\delta}{\mu_0} \Delta A_0 + e c n_e = \langle J^0(r) \rangle, \]

\[ -\frac{\delta}{\mu_0} \varepsilon^{kn} \partial_n B = \langle J^k(r) \rangle, \]

\[ \frac{c e^2 \nu_0}{\hbar} b = \langle J^0(r) \rangle, \]

\[ \frac{c e^2 \nu_0}{\hbar} \varepsilon^{kn} \partial_n a_0 = \langle J^k(r) \rangle, \]

where the thermal averages are defined in terms of the grand canonical ensamble

\[ \langle \cdots \rangle \text{Tr} e^{-(H_e - \mu mc^2 N)/k_B T} = \text{Tr} \left\{ \cdots e^{-(H_e - \mu mc^2 N)/k_B T} \right\}. \]

Here \( k_B, T \) and \( \mu \) are the Boltzmann constant, temperature and the dimensionless chemical potential respectively, while \( N \) denotes the particle number operator

\[ N = \int \psi^\dagger(r) \psi(r) dr. \]

The fermion hamiltonian is given by

\[ H_e = \int \psi^\dagger(r) \{ i \hbar c \gamma^0 \gamma_k D_k(r) + ec A_0(r) + e c a_0(r) + \sigma mc^2 \gamma^0 \} \psi(r) dr. \]
Introduce the thermodynamic potential
\[ \Omega_e(T, \mu, A, a) = -k_B T \ln \text{Tr} \exp \left\{ \frac{-H_e(A, a) - \mu mc^2 N}{k_B T} \right\}, \]
in terms of which the current averages are given by
\[ \langle J^\mu(r) \rangle = \frac{\delta \Omega_e}{\delta A_\mu(r)} = \frac{\delta \Omega_e}{\delta a_\mu(r)}. \] (5)

The gauge fields are presented as a sums of background and fluctuating parts
\[ A_\mu = A^b_\mu + A^f_\mu, \quad a_\mu = a^b_\mu + a^f_\mu. \]
Backgrounds correspond to the uniform Maxwell and Chern-Simons magnetic fields, i.e. \( A^b_0 = a^b_0 = 0, B^b = \text{const}, b^b = \text{const}. \)
Separate the fermionic hamiltonian into the dimensionless free and interacting parts
\[ H_e = mc^2(H_0 + H_{int}), \]
\[ H_0 = \int \psi^\dagger(r) \{ i \ell_0 \gamma^0 \gamma^k D^b_k(r) + \sigma \gamma^0 \} \psi(r) \text{d}r, \] (6)
\[ D^b_k(r) = \partial_k + i(e/\hbar)(A^b_k + a^b_k), \]
\[ H_{int} = \frac{1}{mc^2} \int J^\mu(r) \{ A^f_\mu(r) + a^f_\mu(r) \} \text{d}r, \]
where \( \ell_0 = \hbar/mc \) is the Compton wave length for the fermions.
Applying the perturbation theory formalism, we get for the thermodynamic potential
\[ \Omega_e = \Omega_0 - k_B T \ln \left\langle \text{Tr} \exp \left\{ - \int \beta H_{int}(\tau) \text{d}\tau \right\} \right\rangle_0, \quad \beta = \frac{mc^2}{k_B T}. \] (7)
\( \Omega_0 \) is the thermodynamic potential for the system in the uniform magnetic background
\[ \Omega_0(T, \mu, A^b, a^b) = -k_B T \ln \text{Tr} \exp \left\{ -\beta (H_0(A^b, a^b) - \mu N) \right\} \] (8)
and \( \langle \cdots \rangle_0 \) is defined as
\[ \langle \cdots \rangle_0 \text{Tr} \left\{ e^{-\beta(H_0-\mu N)} \right\} = \text{Tr} \left\{ \cdots e^{-\beta(H_0-\mu N)} \right\}. \]
In (7) $H_{\text{int}}(\tau)$ is the interaction hamiltonian in the Matsubara representation

$$H_{\text{int}}(\tau) = \frac{1}{mc^2} \int \bar{\psi}(\tau, \mathbf{r}) \gamma^\mu \psi(\tau, \mathbf{r}) \{ A^\dagger_\mu(\mathbf{r}) + a^\dagger_\mu(\mathbf{r}) \} d\mathbf{r}$$

and $T$ denotes $\tau$-ordering. Matsubara fields are given by

$$\psi(\tau, \mathbf{r}) = e^{i \mu \tau} \sum_{np} \{ a_{np} u_{np}(\mathbf{r}) e^{-\omega_n \tau} + b_{np}^\dagger v_{np}(\mathbf{r}) e^{\omega_n \tau} \}, \quad (9)$$

$$\bar{\psi}(\tau, \mathbf{r}) = e^{-i \mu \tau} \sum_{np} \{ a_{np}^\dagger \bar{u}_{np}(\mathbf{r}) e^{\omega_n \tau} + b_{np} \bar{v}_{np}(\mathbf{r}) e^{-\omega_n \tau} \}. \quad (10)$$

Remark, that except the lowest energy eigenvalues the solutions of Dirac equation are always paired. For the lowest energy there is asymmetry, i.e. there is no $v_0$ mode for $\sigma \varepsilon = +1$ and no $u_0$ mode for $\sigma \varepsilon = -1$, where $\varepsilon \equiv \text{sgn}(eB^b + eb^b)$.

3. Thermodynamic potential

In this item we shall find the analytical expression for the thermodynamic potential in the second order approximation with respect to the gauge field fluctuations. All the operator expressions such as hamiltonian, currents, etc. are assumed to be normal ordered.

Substituting $H_0$ into (8) and taking into account the degeneracy of the Landau levels we get

$$\frac{\Omega_0(A^b, a^b)}{\text{Area}} = \frac{k_B T}{2\pi \ell^2} \sum_n \ln(1 - \rho_n^+) + \frac{k_B T}{2\pi \ell^2} \sum_n \ln(1 - \rho_n^-). \quad (11)$$

where $\rho_n^+$ and $\rho_n^-$ are the Fermi distribution functions for the particles and antiparticles respectively

$$\rho_n^\pm = \{ 1 + \exp[\beta(\omega_n \mp \mu)] \}^{-1}, \quad \omega_n = \sqrt{1 + 2hn},$$

$$h = \frac{\ell_0^2}{\ell^2}, \quad \frac{1}{\ell^2} = \frac{1}{\hbar} |eB^b + eb^b| \quad (12)$$

and $\ell$ is the magnetic length.

Thermodynamic potential in the second order approximation is given by the functional

$$\Omega_\varepsilon(A, a) = \Omega_0(A^b, a^b) + e c \int \Pi^\mu(\mathbf{r}) \{ A^\dagger_\mu(\mathbf{r}) + a^\dagger_\mu(\mathbf{r}) \} d\mathbf{r} +$$
\[ + \frac{e^2}{2m} \int \Pi^\mu(\mathbf{r}_1, \mathbf{r}_2) \{A^f_\mu(\mathbf{r}_1) + a^f_\mu(\mathbf{r}_1)\} \{A^f_\nu(\mathbf{r}_2) + a^f_\nu(\mathbf{r}_2)\} d\mathbf{r}_1 d\mathbf{r}_2, \]

where \( \Pi^\mu(\mathbf{r}) \) and \( \Pi^\mu(\mathbf{r}_1, \mathbf{r}_2) \) read as

\[ \Pi^\mu(\mathbf{r}) = \frac{1}{2\pi\ell^2} \sum_n \rho_n^+ \bar{u}_n \gamma^\mu u_n - \frac{1}{2\pi\ell^2} \sum_n \rho_n^- \bar{v}_n \gamma^\mu v_n = \text{const}, \]

\[ \Pi^\mu(\mathbf{r}_1, \mathbf{r}_2) = \int \frac{dk}{8\pi^3} e^{i k (\mathbf{r}_1 - \mathbf{r}_2)/\ell} \Pi^\mu(\mathbf{k}). \]

Polarization operator possesses the following tensor form

\[ \Pi^{00}(\mathbf{k}) = \frac{2\pi}{h} \Pi_E(\mathbf{k}^2/2), \]

\[ \Pi^{0k}(\mathbf{k}) = \frac{2\pi}{\sqrt{h}} i \varepsilon^{k,l} \Pi_{CS}(\mathbf{k}^2/2), \]

\[ \Pi^{kl}(\mathbf{k}) = 2\pi \varepsilon^{km} \varepsilon^{ln} k^m k^n \Pi_M(\mathbf{k}^2/2), \]

where \( \Pi_E, \Pi_{CS} \) and \( \Pi_M \) are the structure functions, presented in the Appendix. Explicit calculations show that

\[ \Pi_0 = \frac{1}{2\pi\ell^2} \sum_n \rho_n^+ - \frac{1}{2\pi\ell^2} \sum_n \rho_n^-, \quad \Pi_k = 0 \] (13)

and the final expression for the thermodynamic potential is given by

\[ \Omega_e(A, a) = \Omega_0(A^b, a^b) + ec \Pi_0 \int \{A^f_0(\mathbf{r}) + a^f_0(\mathbf{r})\} d\mathbf{r} + \]

\[ + \frac{e^2}{2m} \frac{m^2 c^2}{h^2} \int \{A^f_0(\mathbf{r}) + a^f_0(\mathbf{r})\} \hat{\Pi}_E \{A^f_0(\mathbf{r}) + a^f_0(\mathbf{r})\} d\mathbf{r} + \]

\[ + \frac{e^2}{2m} \frac{2mc}{h^2} \int \{A^f_0(\mathbf{r}) + a^f_0(\mathbf{r})\} \hat{\Pi}_{CS} \{B^f(\mathbf{r}) + b^f(\mathbf{r})\} d\mathbf{r} + \]

\[ + \frac{e^2}{2m} \int \{B^f(\mathbf{r}) + b^f(\mathbf{r})\} \hat{\Pi}_M \{B^f(\mathbf{r}) + b^f(\mathbf{r})\} d\mathbf{r}, \]

where \( \hat{\Pi} \) stands for the differential operator \( \Pi \left(-\ell^2 \Delta/2\right) \) (\( \Delta \) is the Laplace operator). Using (5), the current averages can be expressed as a linear functions of gauge field fluctuations

\[ \langle J^0(\mathbf{r}) \rangle = ec \Pi_0 + \frac{e^2 mc^2}{h^2} \hat{\Pi}_E \left(A^f_0 + a^f_0\right) + \frac{e^2 c}{h} \hat{\Pi}_{CS} \left(B^f + b^f\right), \] (14)
\[ \langle J^k(r) \rangle = -\varepsilon^n \partial_n \left\{ \frac{\varepsilon^2_c}{\hbar} \hat{\Pi}_{\text{CS}} \left( A^f_0 + a_0^f \right) + \frac{\varepsilon^2_m}{m} \hat{\Pi}_{\text{M}} \left( B^f + b^f \right) \right\}. \] (15)

4. Uniform magnetic field and the Meissner effect

As a starting point, consider the system in the zeroth order approximation, i.e. \( A^f_\mu = a^f_\mu = 0 \). Equations of motion (1) and (3) are reduced to

\[ n_e = \Pi_0, \] (16)

\[ b^b = \frac{\pi \hbar}{e \nu_0} n_e, \] (17)

where \( \Pi_0 \) is given by (13).

Note, that \( n_e \) is the free fermion density in the sample and (16) serves to define the chemical potential \( \mu = \mu(T, B^b, b^b, n_e) \).

Equation (17) indicates that the Chern-Simons magnetic background is created by the free fermion density. For the typical value \( n_e = 10^{18} \text{m}^{-2} \) we get

\[ |b^b| = \frac{2 \cdot 10^7}{|\nu_0|} \text{Gauss}. \] (18)

We shall study the compound system, consisting of two sorts of fermions with equal gauge couplings \( e_1 = e_2 = e \) and the different helicities \( \sigma_1 = -\sigma_2 \), corresponding to the different signs of the magnetic moment interaction, which on its turn can be associated with spin up and spin down fermions. In that case the r.h.s. of the equation (16) gets the contributions, defined by (13) from both sorts of particles. Remark that, since the different sorts have the equal charges, corresponding magnetic lengths are also equal. Moreover, as it follows from (18), realistic values of \( B^b \) are small compared with those of \( b^b \) and can be neglected:

\[ \varepsilon = \text{sgn}(eB^b + eb^b) = \text{sgn}(eb^b). \]

In other words, \( \varepsilon \) can be considered as independent of \( B^b \), and without loss of generality we can set \( \sigma_1 \varepsilon_1 = -\sigma_2 \varepsilon_2 = 1 \). Taking into account the contributions from both types, equation of motion (16) can be rewritten in the following form

\[ n_e = \frac{\hbar}{2\pi \ell_0^2} (\nu_1 + \nu_2), \] (19)

where \( \nu_1 \) and \( \nu_2 \) are the corresponding filling fractions

\[ \nu_1 \equiv \sum_{n=0} \rho^+(\mu_1) - \sum_{n=1} \rho^-(\mu_1), \] (20)
\[ \nu_2 \equiv \sum_{n=1}^{\infty} \rho^+_n(\mu_2) - \sum_{n=0}^{\infty} \rho^-_n(\mu_2), \quad (21) \]

and \( \mu_{1,2} \) are the chemical potentials for the types 1 and 2 respectively. In (20) and (21) we took into the consideration the spectral asymmetry of the one-particle Hamiltonian, which is reflected in the absence of \( n = 0 \) modes in certain terms.

Introduce the partial contributions to the particle density

\[ n_e^{(1,2)} = \frac{h}{2\pi \ell_0^2} \nu_{1,2}. \quad (22) \]

By means of (19) we can express \( h \) in terms of the average filling fraction

\[ h = \frac{\pi n_e \ell_0^2}{\nu}, \quad \nu = \frac{\nu_1 + \nu_2}{2}. \quad (23) \]

Using (22), (23), we express the partial filling fractions in terms of the average one

\[ \nu_{1,2} = \frac{2n_e^{(1,2)}}{n_e^{(1)} + n_e^{(2)}} \nu. \quad (24) \]

Substituting \( h, \nu_{1,2} \) from (23) and (22) into (20) and (21) we see that the chemical potentials depend on temperature and the average filling fraction \( \nu \).

The same time, expression for the magnetic length (12) together with equations (17) and (23) yields

\[ \frac{1}{\nu} = \frac{\varepsilon}{\nu_0} + \frac{\varepsilon e B^b}{\pi n_e h}, \quad (25) \]

which reflects the one to one correspondence between \( \nu \) and \( B^b \). Consequently, any quantity depending on \( B^b \) can be also viewed as a function of \( \nu \), and vice versa.

The value of the background magnetic field is determined by the external magnetic field \( B_{\text{ext}} \) and the magnetization \( M \)

\[ B^b = B_{\text{ext}} + M(B^b), \quad M(B^b) = -\frac{\mu_0}{\delta} \frac{dF(B^b)}{dB^b}, \quad (26) \]

where \( F = F_1 + F_2 \) is the Helmholtz free energy density of the composite system. The separate contributions are

\[ F_{1,2}(B^b) = \frac{1}{Area} \{ \Omega_0(\mu_{1,2}) + \mu_{1,2} mc^2 \langle N_{1,2} \rangle \}, \]

where \( \langle N_{1,2} \rangle \) are the thermal averages of the fermion numbers determined by

\[ \langle N_1 \rangle_{\text{Area}} = \frac{h}{2\pi \ell_0^2} \sum_{n=0}^{\infty} \rho^+_n(\mu_1) - \frac{h}{2\pi \ell_0^2} \sum_{n=1}^{\infty} \rho^-_n(\mu_1) = n_e^{(1)}, \]

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\[ \frac{\langle N_2 \rangle}{\text{Area}} = \frac{\hbar}{2\pi\ell_0^2} \sum_{n=1}^{\infty} \rho_n^+(\mu_2) - \frac{\hbar}{2\pi\ell_0^2} \sum_{n=0}^{\infty} \rho_n^-(\mu_2) = n_e^{(2)}. \]

Here the quantities \( h, \mu_1 \) and \( \mu_2 \) are the functions of \( \nu \). In order to exhibit the global behaviour of \( F(\nu) \) we have used the numerical methods. Consider the case of the equal concentrations \( n_e^{(1)} = n_e^{(2)} = n_e/2 \). Corresponding results are plotted in figure 1. As one can see the free energy density of the composite system has a local minima at the integer values of the average filling fraction, however the separate contributions of each type of particles exhibit no minima.

The cusp like structure of the free energy is the manifestation of the Meissner effect, when the system tries to expell the magnetic field from inside the sample [5].

As we see, in contrast with the previous calculations, the Meissner effect does not exist in the single fermion system, but only in the composite one, where the diversity in the magnetic moment interaction plays the decisive role.

Magnetization is expressed with the help of the free energy. The later contains the additive contributions from the one-particle state energies. One particle hamiltonian considered in [5], takes into account the magnetic moment interaction only with Maxwell magnetic field. Further simplification is achieved by taking

\[ E_n = \frac{\hbar^2}{m\ell^2} \left( n + \frac{1}{2} \right), \]  

assuming the vanishing magnetic moment interaction.

In our approximation the magnetic interaction term contains the contribution from both Maxwell and Chern-Simons magnetic fields. The nonrelativistic limit of one-particle energy spectrum is given by

\[ E_n(\sigma \varepsilon) = \frac{\hbar^2}{m\ell^2} \left( n + \frac{1 - \sigma \varepsilon}{2} \right). \]

Remark, that (27) is in fact the half sum of \( E_n(+) \) and \( E_n(-) \). Separate use of \( E_n(+) \) or \( E_n(-) \) does not lead to the free energy with localized minima and only their simultaneous contribution has a cusp like form.

Equations for the chemical potentials cannot be solved in the global form. However, one can find the analytic form of \( \mu_{1,2}(\nu) \) nearby the integer values of \( \nu \), where the free energy achieves the local minima. This enables to analyze the system in more details near these minima, where the Meissner effect just takes the place. Below we present these calculations for the asymmetric concentrations i.e. when \( n_e^{(1)} \neq n_e^{(2)} \).

Here we shall deal with the minimum corresponding to \( \nu = 1 \) and for concretness set \( e > 0 \) and \( \varepsilon = 1 \). In that case we have \( \nu_0 = 1 \) and

\[ h = \pi n_e \ell_0^2 (1 + \alpha), \quad \alpha = \frac{eB}{\pi n_e \hbar} \]  

(28)
Note that for the characteristic values of the internal magnetic field \((B < 200 \text{Gauss})\) we have \(\alpha < 10^{-5}\), \((1 + \alpha)^{-1} = 1 - \alpha\) and present (24) as

\[
\nu_1 = (1 + \bar{\alpha})(1 - \alpha) \equiv 1 - \alpha_1,
\]

\[
\nu_2 = (1 - \bar{\alpha})(1 - \alpha) \equiv 1 - \alpha_2.
\]

\[
\bar{\alpha} = \frac{n_e^{(1)} - n_e^{(2)}}{n_e^{(1)} + n_e^{(2)}}.
\]

Here \(\bar{\alpha}\) measures the asymmetry in the concentrations of the different types of fermions.

Further simplifications are due to the fact that in the considered range of temperatures we have \(\beta > 3 \cdot 10^7\), and consequently for \(\mu_{1,2} > 0\)

\[
\rho_{n}^{-}(\mu_{1,2}) = \frac{1}{1 + e^{\beta(\omega_{n} + \mu_{1,2})}} < e^{-10^7},
\]

meaning that the main contributions to (20) and (21) come from \(\rho_{n}^{+}(\mu_{1,2})\), forcing \(\nu_{1,2}\) to be positive. With this assumption we take \(\mu_{1} > 0\) and present it as

\[
\mu_{1} = 1 + \frac{1 - w_{1}}{2} h,
\]

where \(w_{1}\) is to be found. Due to (28) one has \(h \sim 5 \cdot 10^{-7}\). Consequently, for the Landau levels with \(2hn << 1\) we can use \(\omega_{n} = 1 + hn\) and write down

\[
\beta(\omega_{n} - \mu_{1}) = \beta h \left( n - \frac{1}{2} + \frac{w_{1}}{2} \right).
\]

Characteristic values of \(\beta\) and \(h\) are such that \(\beta h > 15\), allowing to neglect the contributions coming from the higher Landau levels. This permits to write (20) as \(\nu_{1} = \rho_{n}^{+}(\mu_{1}) + \rho_{n}^{-}(\mu_{1})\) or in the equivalent form

\[
1 - \alpha_1 = \frac{1}{1 + e^{-\eta e^{\eta w_{1}}}} + \frac{1}{1 + e^{\eta e^{\eta w_{1}}}},
\]

where \(\eta \equiv \pi n_e \ell_{0}^{2} \beta (1 + \alpha)/2\). From this equation we easily solve \(w_{1}\) as

\[
e^{\pm \eta w_{1}} = \frac{1}{1 + \alpha_1} \left( \sqrt{1 + \alpha_1^2 \sin^2 \eta} \pm \alpha_1 \sin \eta \right), \quad \alpha_1 = \alpha - \bar{\alpha} + \alpha \bar{\alpha}.
\]

The same time, we take

\[
\mu_{2} = 1 + \frac{3 - w_{2}}{2} h
\]

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and performing the same manipulations obtain
\[ e^{\pm \eta w_2} = \frac{1}{1 \mp \alpha_2} \left( \sqrt{1 + \alpha_2^2 \delta^2 \eta} \pm \alpha_2 \chi \eta \right), \quad \alpha_2 = \alpha + \bar{\alpha} - \alpha \bar{\alpha}. \] (30)

So, the leading contributions to the different physical quantities come from the following Fermi distribution functions
\[ \rho_0^+(\mu_1) = \frac{1}{1 + e^{-\eta w_1}}, \quad \rho_1^+(\mu_1) = \frac{1}{1 + e^{\eta w_1}}, \] (31)
\[ \rho_0^+(\mu_2) = \frac{1}{1 + e^{-\eta w_2}}, \quad \rho_1^+(\mu_2) = \frac{1}{1 + e^{\eta w_2}}, \] (32)
where \( e^{\eta w_1} \) and \( e^{\eta w_2} \) are defined by (29) and (30).

In this approximation the system magnetization is
\[ M = M_1 + M_2 \]
where
\[ M_1(B^b) = -\frac{e \mu_0 mc^2}{2\pi \hbar \delta} \sum_{n=1,2} \left\{ \frac{1}{\beta} \ln [1 - \rho_n^+(\mu_1)] + \frac{\hbar n}{\omega_n} \rho_n^+(\mu_1) \right\} \approx \frac{\mu_0 e \xi}{4m \delta} \left\{ f(\alpha - \bar{\alpha}) + 1 \right\}, \] (33)
\[ M_2(B^b) = -\frac{e \mu_0 mc^2}{2\pi \hbar \delta} \sum_{n=1,2} \left\{ \frac{1}{\beta} \ln [1 - \rho_n^+(\mu_2)] + \frac{\hbar n}{\omega_n} \rho_n^+(\mu_2) \right\} \approx \frac{\mu_0 e \xi}{4m \delta} \left\{ f(\alpha + \bar{\alpha}) - 1 \right\}, \] (34)
\[ f(z) = \frac{1}{\xi} \ln \frac{\sqrt{z^2 + 4e^{-\xi} - z}}{\sqrt{z^2 + 4e^{-\xi} + z}}, \quad \xi = \pi n_e \ell_0^2 \beta. \] (35)

These quantities define the magnetic and thermal properties of the system.

In the case of the symmetric concentrations (\( \bar{\alpha} = 0 \)) one gets
\[ M(T, B^b) = \frac{\mu_0 e \xi}{2m \delta} \frac{1}{\xi} \ln \frac{\sqrt{\alpha^2 + 4e^{-\xi} - \alpha}}{\sqrt{\alpha^2 + 4e^{-\xi} + \alpha}}, \quad \alpha = \frac{eB^b}{\pi n_e \hbar}. \]

Behaviour of \( M \) as a function of \( B^b \) is depicted in figure 2 for the different values of temperature. Presented curves are the same as given in earlier work [5].

On figure 3 we present the dependence of \( B^b \) on \( B^{ext} \) which is set by (26). As one can see, when the external magnetic field is applied, the internal one is in general
non-zero. In other words, the Meissner effect is not complete. This observation was first made in \[5\]. The same time the magnitude of \(B^b\) is quite small until \(B^{\text{ext}}\) reaches some critical value \(B^{\text{cr}}(T)\), which depends on the temperature. Above this value external magnetic field begins the notable penetration in the sample.

The critical value of the external magnetic field is evidently related to the small interval at \(B^b\) axis, where the magnetization curve drastically changes its direction, i.e. where the curve passes the point of maximal curvature (PMC). The lower is temperature, more narrow is the interval and it is easier to establish the corresponding critical magnetic field. In order to find the approximated value of \(B^{\text{cr}}(T)\) we can use the behaviour of the derivative

\[
\frac{\partial M}{\partial B^b} = -\frac{\mu_0 e^2 k_B T}{\pi^2 h^2 n_e \delta} \left( \alpha^2 + 4e^{-\pi n_e h^2 / mk_B T} \right)^{-1/2}.
\]

Until the curve \(M(B^b)\) reaches the PMC, its slope can be considered to be constant and therefore can be determined by its value at the origin, where it is of the order of \(10^5\) for \(T = 50^\circ\)K and \(10^{50}\) for \(T = 10^\circ\)K. On the other hand, in the low temperature regime the curve becomes practically horizontal after passing the PMC. Obviously, somewhere in the vicinity of PMC one has \(\partial M/\partial B^b = -1\) and using this relation as the definition of the location of PMC, one gets the corresponding value of the internal magnetic field to be

\[
B_0(T) = \left\{ \left( \frac{\mu_0 e k_B T}{\pi h \delta} \right)^2 - \left( \frac{2\pi n_e h}{e} \right)^2 e^{-\pi n_e h^2 / mk_B T} \right\}^{1/2}.
\]

Substituting \(B_0(T)\) into \(B^{\text{cr}} = B_0 - M(B_0)\) and keeping the leading terms we get

\[
B^{\text{cr}}(T) = \frac{\mu_0 e n_e h}{2m \delta} + \frac{\mu_0 e k_B T}{\pi h \delta} - \ln \frac{\mu_0 e^2 k_B T}{2\pi^2 h^2 n_e \delta}.
\]

Corresponding curve is presented at figure 4. We observe that the value of the critical magnetic field decreases for higher temperatures and completely vanishes near \(T = 90^\circ\)K, which seems to be the critical temperature for the system. However, the area of the maximal curvature becomes smeared for \(T = 90^\circ\)K, and the critical magnetic field is ill defined, causing the critical temperature to be also ill defined. On the contrary, the PMC is well located for \(T = 0^\circ\)K and the corresponding value of the critical magnetic field is given by

\[
B^{\text{cr}}(0) = \frac{\mu_0 e n_e h}{2m \delta}.
\]

Situation with the asymmetric concentrations is presented at figures 5 and 6. In the interval, which includes \(B^b = 0\), magnetization practically vanishes. The width of the plateaux, is equal to \(2\alpha (\pi n_e h / e)\) and does not depend on temperature. This kind
of behaviour of the magnetization can be explained by the fact, that the magnetization curves of each kind of fermions have the standard form set by (35), but shifted by $\bar{\alpha}$ relatively to each other according to (33) and (34).

The genuine value of the asymmetry parameter $\bar{\alpha}$ must be determined by the equilibrium conditions imposed on the chemical potentials.

5. Gauge field propagators and mass spectrum

Equations of motion for the gauge fluctuations can be introduced as the extremals of the effective lagrangian

$$L_{\text{eff}} = \frac{\delta}{2\mu_0} (\partial_k A_0^f)^2 - \frac{e^2 m c^2}{2\hbar^2} (A_0^f + a_0^f) \hat{\Pi}_E^{\text{tot}} (A_0^f + a_0^f) -$$

$$- \frac{\delta}{2\mu_0} B^f B^f - \frac{e^2}{2m} (B^f + b^f) \hat{\Pi}_M^{\text{tot}} (B^f + b^f) +$$

$$+ \frac{e^2 \nu_0 c}{\pi \hbar} a_0^f b^f - \frac{e^2 c}{\hbar} (A_0^f + a_0^f) \hat{\Pi}_CS^{\text{tot}} (B^f + b^f),$$

where the script "tot" means that the corresponding quantities include the contributions from both types of fermions.

In order to study the gauge field propagators we introduce the gauge fixing term

$$L_\alpha = \frac{1}{2\alpha} (\partial_n A_n^f)^2 + \frac{1}{2\alpha} (\partial_n a_n^f)^2$$

and present the total lagrangian as

$$L_{\text{eff}} + L_\alpha = \frac{1}{2} A^T D A, \quad A^T = \left( ec A_0^f, e A_n^f, eca_0^f, ea_n^f \right).$$

Gauge field propagators are defined by the inverse of the matrix $D$

$$DG = GD = 1.$$
and can be presented in the following form

\[
G = \begin{pmatrix}
G^{00}_0 & G^{0m} & G^{00}_1 & G^{0m} \\
G^{00}_1 & G^{mm} & G^{00}_4 & G^{nm} \\
G^{00}_3 & G^{0m} & G^{00}_2 & G^{nm} \\
G^{00}_3 & G^{mm} & G^{00}_2 & G^{mm}
\end{pmatrix}
\]

Here the tensor blocks are given by

\[
G^{0n} = -G^{n0} = \frac{i \varepsilon^{nm} k^m}{k^2} G^{CS},
\]

\[
G^{nm} = \frac{\varepsilon^{ni} \varepsilon^{mj} k^i k^j}{k^4} G^M + \frac{k^n k^m}{k^4} G^\parallel,
\]

\[
G^{00}_1 = \frac{\delta}{D \mu_0 c^2} \left\{ \frac{\mu_0 e^2}{m \delta} \Pi^\text{tot}_M + \left( 1 - \frac{\pi}{\nu_0} \Pi^\text{tot}_\text{CS} \right)^2 - \frac{\pi^2}{\nu_0^2} \Pi^\text{tot}_E \Pi^\text{tot}_M \right\},
\]

\[
G^{CS}_1 = \frac{\pi}{D \nu_0} \left\{ \Pi^\text{tot}_\text{CS} \Pi^\text{tot}_\text{CS} - \Pi^\text{tot}_E \Pi^\text{tot}_M - \frac{\nu_0}{\pi} \Pi^\text{tot}_\text{CS} \right\},
\]

\[
G^{M}_1 = \frac{m}{D} \Pi^\text{tot}_E - \frac{\delta k^2}{D \mu_0 c^2} \left\{ \left( 1 - \frac{\pi}{\nu_0} \Pi^\text{tot}_\text{CS} \right)^2 - \frac{\pi^2}{\nu_0^2} \Pi^\text{tot}_E \Pi^\text{tot}_M \right\},
\]

\[
D = \Pi^\text{tot}_\text{CS} \Pi^\text{tot}_\text{CS} - \Pi^\text{tot}_E \Pi^\text{tot}_M - \frac{m \delta}{\mu_0 e^2} \Pi^\text{tot}_E + \frac{\delta^2 k^2}{\mu_0 e^2 c^2} \left\{ \frac{\mu_0 e^2}{m \delta} \Pi^\text{tot}_M + \left( 1 - \frac{\pi}{\nu_0} \Pi^\text{tot}_\text{CS} \right)^2 - \frac{\pi^2}{\nu_0^2} \Pi^\text{tot}_E \Pi^\text{tot}_M \right\},
\]

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and \((\nu_0/\pi)^2\mathcal{D}\) is the determinant of \(D\). In the Coulomb gauge \((\alpha = 0)\) we obtain

\[
G_{10}^{00}(k) = \frac{g_0^E + g_2^E k^2 + g_4^E k^4 + \cdots}{d_0 + d_2 k^2 + d_4 k^4 + \cdots},
\]

\[
G_{10}^{0n}(k) = i \varepsilon^{nm} k^m \left( \frac{g_{0}^{CS}}{k^2} + \frac{g_{2}^{CS} k^2 + g_{4}^{CS} k^4 + \cdots}{d_0 + d_2 k^2 + d_4 k^4 + \cdots} \right),
\]

\[
G_{1n}^{nm}(k) = \left( \delta^{nm} - \frac{k^n k^m}{k^2} \right) \left( \frac{g_{0}^{M} + g_{2}^{M} k^2 + g_{4}^{M} k^4 + \cdots}{d_0 + d_2 k^2 + d_4 k^4 + \cdots} \right).
\]

The constants \(g\)'s are the coefficients of the momentum expansion of the structure functions in the nominators of (36), (37) and (38), while \(d\)'s are set by \(\mathcal{D} = \sum d_{2n} k^{2n}\).

The ratios

\[
\frac{g_0 + g_2 k^2 + g_4 k^4 + \cdots}{d_0 + d_2 k^2 + d_4 k^4 + \cdots}
\]

represent the short range interaction.

The leading contributions to the first two coefficients of the determinant expansion are given by

\[
d_0 = \Pi_{CS}^{\text{tot}}(0) \Pi_{CS}^{\text{tot}}(0) - \frac{m\delta}{\mu_0 e^2} \Pi_{E}^{\text{tot}}(0),
\]

\[
d_2 = - \frac{m\delta}{\mu_0 e^2} \frac{\ell^2}{2 \hbar^2} \frac{d \Pi_{E}^{\text{tot}}(x)}{dx} \bigg|_{x=0}.
\]

Using (A.1) – (A.5), one can show that \(d_0\) and \(d_2\) are the nonvanishing positive numbers. Consequently keeping the lowest order terms one can write

\[
\frac{g_0 + g_2 k^2 + g_4 k^4 + \cdots}{d_0 + d_2 k^2 + d_4 k^4 + \cdots} \rightarrow \frac{g_0}{d_0 + d_2 k^2} = \frac{g_0}{d_2} = \frac{1}{k^2 + \mathcal{M}^2},
\]

where \(\mathcal{M}^2 = d_0/d_2 > 0\) gives rise to the short range interaction. Using the zero temperature values of the structure functions (see Appendix), one can estimate the corresponding value of the mass parameter

\[
\mathcal{M}^2 = \frac{\hbar^2 \mu_0 e^2 n_e}{m \delta} = \frac{\hbar^2}{\lambda_L^2},
\]

(39)

where \(\lambda_L\) is the London penetration depth. Note, that \(\mathcal{M}\) does not depend on \(N\).

So, in the lowest order approximation \((\mathcal{D} = d_0 + d_2 k^2)\) the spectrum of the short range interaction consists of one single mass \(\mathcal{M}\). According to [4], the low temperature
mass spectrum in $k^4$ approximation consists of two masses $M_+ << M_-$ where $M_+$ exactly coincides with $M$ presented above.

Consider this point more carefully. Taking the case of $\nu = 1 (N = 0)$ we present the zero temperature expression for $D$ up to the $k^6$ terms

$$\pi^2 D(x) = 1 + 2\alpha x - \frac{3}{2} \alpha \left(1 - \frac{8}{3} \gamma\right) x^2 + \frac{11}{18} \alpha \left(1 - \frac{81}{11} \gamma\right) x^3,$$

(40)

$$x = \frac{k^2}{2\pi n_e h^2}, \quad \alpha = \pi n_e \lambda^2 L, \quad \gamma = \pi^2 n_e \lambda^2 \ell_0^2.$$

The analysis of [4] is based on the following $k^4$ approximation

$$\pi^2 \Delta(x) = 1 + 2\alpha x - \frac{3}{2} \alpha \left(1 - \frac{8}{3} \gamma\right) x^2$$

and the solution to the equation $\Delta(x) = 0$ leads to the conclusion about the existence of the second mass.

In our case the $k^4$ approximation leads to the following factorization

$$D \sim (k^2 + M^2) (k^2 - a^2),$$

where the second factor is of a tachionic type. Situation can be clarified if the higher order terms are taken into account. For the typical values of the parameters we have $\gamma \sim 0.0416$ and one can easily verify that the equation

$$\frac{6\pi^2}{\alpha} D'(x) = 11 \left(1 - \frac{81}{11} \gamma\right) x^2 - 18 \left(1 - \frac{8}{3} \gamma\right)x + 12 = 0$$

has no real solution, i.e. $D(x)$ exhibits no extrema. This fact together with $D'(x) > 0$ implies that $D'(x) > 0$ for all $x$. Moreover, we have $D(0) = 1$ which means that defining equation $D(x) = 0$ has a single real solution $x = x_0 < 0$. It is not difficult to guess that $x_0$ tends to $-M^2$ when we go to the lower approximation $\pi^2 D = 1 + 2\alpha x$. In other words, $k^6$ order corrections do not affect the lower order result: the short range interaction spectrum derived from (40) consists of a single mass defined by (39). The same time it is obvious, that the presented assertion depends on the approximation used and can be changed in higher orders.

Constant parameters, corresponding to the long range components are given by

$$g^{CS} = \frac{\pi}{\nu_0 d_0} \left\{ \Pi_{CS}^{\text{tot}}(0) \Pi_{E}^{\text{tot}}(0) \Pi_{M}^{\text{tot}}(0) - \frac{\nu_0}{\pi} \Pi_{CS}^{\text{tot}}(0) \right\},$$

$$g^{M} = \frac{m}{d_0} \Pi_{E}^{\text{tot}}(0),$$
and using (A.6) – (A.8) one can show that they vanish at $T = 0$ for any $\nu_0 = N + 1$.

So, at finite temperatures the Coulomb forces carry the finite range character, while the charge-current and the current-current interactions include the short as well as the long range parts. When the temperature is decreased, integer number of the Landau levels are kept to be completely filled, and we see that the strength of the long range interaction decreases until vanishes at $T = 0^\circ K$, where the Maxwell field completely acquires the finite range character.

Appendix

Structure functions for a one type fermion system can be presented in the following analytical form

\[ \Pi_E(x) = -\frac{\hbar}{2\pi} I(x) - \frac{1}{4\pi} e^{-x} S_3(x, x) + \frac{\hbar}{2\pi} e^{-x} \frac{\partial}{\partial x} x \frac{\partial}{\partial x} S_0(x, y) \bigg|_{y=x}, \tag{A.1} \]

\[ \Pi_{CS}(x) = \frac{1}{4\pi} e^{-x} \left[ \frac{\partial}{\partial x} S_1(x, y) - \frac{\partial}{\partial x} S_2(x, y) + S_2(x, y) \right] \bigg|_{y=x}, \tag{A.2} \]

\[ \Pi_M(x) = -\frac{1}{2\pi} e^{-x} \frac{\partial^2}{\partial x \partial y} S_0(x, y) \bigg|_{y=x}, \tag{A.3} \]

where $I(x)$ and $S_a(x, y)$ are given by

\[ I(x) = e^{-x} \sum_{n=0}^{\infty} \sum_{\alpha=1}^{\infty} \frac{n!}{(n+\alpha)!} \alpha^2 \Theta_0(n, \alpha) x^{\alpha-1} \overline{L_n^\alpha(x)} \overline{L_n^\alpha(x)}, \tag{A.4} \]

\[ S_a(x, y) = \sum_{n=0}^{\infty} \Theta_a(n) \overline{L_n(x)} \overline{L_n(y)} + \]

\[ + \sum_{n=0}^{\infty} \sum_{\alpha=1}^{\infty} \frac{n!}{(n+\alpha)!} \Theta_a(n, \alpha) (x^{\alpha} + y^{\alpha}) \overline{L_n^\alpha(x)} \overline{L_n^\alpha(y)}. \tag{A.5} \]

Here $\overline{L_n^\alpha(x)}$ are the generalized Laguerre polynomials, while $\Theta_a$ are the temperature dependent constants

\[ \alpha \Theta_a(n, \alpha) = \theta_a(n) - \theta_a(n + \alpha), \quad \Theta_a(n) = \lim_{\alpha \to 0} \Theta_a(n, \alpha), \]

\[ \theta_0(n) = \frac{\rho_n}{\omega_n}, \quad \theta_1(n) = \sigma \frac{\rho_n}{\omega_n} + \varepsilon \bar{\rho}_n, \quad \theta_2(n) = \sigma \frac{\rho_{n+1}}{\omega_{n+1}} - \varepsilon \bar{\rho}_{n+1}, \]

where $\rho_n$, $\omega_n$, $\sigma$, $\varepsilon$ and $\bar{\rho}_n$ are determined by the specific system.
\[ \theta_3(n) = \frac{1}{2} \left( \frac{\omega_n + \frac{1}{\omega_n}}{\omega_n} \right) \rho_n + \frac{1}{2} \left( \frac{\omega_{n+1} + \frac{1}{\omega_{n+1}}}{\omega_{n+1}} \right) \rho_{n+1} + \sigma \varepsilon (\rho_n - \bar{\rho}_{n+1}), \]

\[ \rho_n \equiv \rho_n^+ (\mu) + \rho_n^- (\mu), \quad \bar{\rho}_n \equiv \rho_n^+ (\mu) - \rho_n^- (\mu). \]

Structure functions which include the contributions from both sorts of fermions are denoted as \( \Pi_{E}^{\text{tot}}(x) \), \( \Pi_{CS}^{\text{tot}}(x) \) and \( \Pi_{M}^{\text{tot}}(x) \). For our purposes we consider the case of \( T = 0 \) and \( \bar{\alpha} = 0 \). If the applied magnetic field \( B^{\text{ext}} \) does not exceed the critical one, then at \( T = 0 \) it is completely expelled from the sample \( (B^b = 0) \), and according to (24) and (25) we have \( \nu_1 = \nu_2 = \nu = \nu_0 = N + 1 \) where \( N \) is any nonnegative integer. Consequently, only the nonvanishing terms in (20) and (21) in the zero temperature limit \( (\beta = \infty) \) are

\[ \rho_0^+ (\mu_1) = \cdots = \rho_{N-1}^+ (\mu_1) = \rho_N^+ (\mu_1) = 1, \]

\[ \rho_1^+ (\mu_2) = \cdots = \rho_N^+ (\mu_2) = \rho_{N+1}^+ (\mu_2) = 1, \]

while all others, including the antiparticle contributions vanish exactly. These values of the Fermi distribution functions lead to the following zero temperature expressions

\[ \Pi_{E}^{\text{tot}}(x) = \frac{1}{2\pi} \sum_{n=0}^{N} \left\{ -4x + 3x^2(2n + 1) - \frac{x^3}{9} (30n^2 + 30n + 11) \right\}, \quad (A.6) \]

\[ \Pi_{CS}^{\text{tot}}(x) = \frac{1}{\pi} \sum_{n=0}^{N} \left\{ 1 - \frac{3x}{2} (2n + 1) + \frac{x^2}{12} (30n^2 + 30n + 11) \right\} + \]

\[ + \frac{x^3}{72\pi} \sum_{n=0}^{N} (70n^3 + 105n^2 + 85n + 25), \quad (A.7) \]

\[ \Pi_{M}^{\text{tot}}(x) = \frac{1}{\pi} \sum_{n=0}^{N} \left\{ 2n + 1 - \frac{3x}{2} (2n^2 + 2n + 1) \right\} + \]

\[ + \frac{x^2}{12\pi} \sum_{n=0}^{N} (30n^3 + 30n^2 + 32n + 11) + \]

\[ + \frac{x^3}{72\pi} \sum_{n=0}^{N} (35n^4 + 70n^3 + 120n^2 + 85n + 25), \quad (A.8) \]

where the structure functions are expanded up to \( x^3 \) i.e. up to \( k^6 \) terms. Note, that the closed expressions for the nonrelativistic structure functions are presented in [9].
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Figure 1. Free energies (conventional units) versus $\nu$ for $\bar{\alpha} = 0$ and $T = 50^\circ K$.

Figure 2. $M$ [Gauss] versus $B^b$ [Gauss] for $\bar{\alpha} = 0$. 
Figure 3. $B^b$ [Gauss] versus $B^{\text{ext}}$ [Gauss] for $\bar{\alpha} = 0$.

Figure 4. Critical magnetic field [Gauss] versus temperature [$^\circ$K] for $\bar{\alpha} = 0$. 
Figure 5. $M \ [\text{Gauss}]$ versus $B^b \ [\text{Gauss}]$ for $(\pi n_e h / e) \cdot \bar{\alpha} = 30 \text{ Gauss}$ and $T = 0^\circ K$.

Figure 6. $M \ [\text{Gauss}]$ versus $B^b \ [\text{Gauss}]$ for $(\pi n_e h / e) \cdot \bar{\alpha} = 30 \text{ Gauss}$ and $T = 40^\circ K$. 