Quantum speed limit time for the damped Jaynes-Cummings and Ohmic-like dephasing models in Schwarzschild spacetime

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Quantum theory sets limit on the minimal evolution time between initial and target states. This minimal evolution time can be used to specify the maximal speed of the dynamics of open and closed quantum systems. Quantum speed limit is one of the interesting topic in the theory of open quantum systems. If the quantum speed limit time decreases then the dynamics is faster than longer quantum speed limit time. In this work we consider the quantum speed limit time in Schwarzschild spacetime for two various model consist of damped Jaynes-Cummings and Ohmic-like dephasing. At first, we will show how quantum coherence is affected by Hawking radiation. Given the dependence of quantum speed limit time on quantum coherence and the dependence of quantum coherence on relative distance of quantum system to event horizon $R_0$, we will show the quantum speed limit is decreased by increasing $R_0$ for damped Jaynes-Cummings model. Conversely, we will show that the quantum speed limit is increased by increasing $R_0$ for Ohmic-like dephasing model.

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I. INTRODUCTION

Quantum theory sets a bound on the speed of the evolution of quantum systems. The minimum time of the evolution of a quantum system from an initial state at time $τ$ to a state at time $τ + τ_D$ is known as quantum speed limit (QSL) time $τ_{QSL}$, where $τ_D$ is driving time. QSL time can be interpreted as a generalization of the time-energy uncertainty principle. This limit, determines the maximum speed of a quantum evolution. It is used for many topics in quantum information theory such as, quantum communication [1], exploration of accurate bounds in quantum metrology [2], computational bounds of physical systems [3] and quantum optimal control algorithms [4].

For closed quantum systems, which is isolated from its surroundings, the dynamics is unitary and different bounds on QSL time have been obtained based on Bures angle and relative purity as the distance measure between initial state and target state [5–11]. The most famous of bounds on QSL time for closed quantum systems are the Mandelstam-Tamm (MT) bound [10] and Margolus-Levitin (ML) bound [11]. The inescapable interaction between the quantum system and its environment has made the study of open quantum systems an important topic in quantum information theory [12–14]. Therefore, it is interesting to evaluate the QSL time for open quantum systems. Based on various distance measure, different bounds on the QSL time have been proposed for open quantum systems [15–23]. In general, one can say that there are two types of QSL time, Mandelstam-Tamm bound and Margolus-Levitin bound. In Refs. [7, 24], the authors have provided the generalization of the (MT) and (ML) bounds to nonorthogonal states and to driven systems. Deffner et al. formulated the unified bound of QSLT including both (MT) and (ML) types for non-Markovian dynamics [19]. In recent years, QSL time has been studied from different perspectives such as the effect of the decoherence on QSL time [25–28], the role of the initial state on QSL time [29] and the applications of the QSL time in quantum phase transition[30].

It is worth noting that QSL time $τ_{QSL}$ can be interpreted as the potential capacity for further evolution acceleration. If $τ_{QSL} = τ_D$ then the evolution is now in the situation with the highest speed, thus the evolution has not the potential capacity for further acceleration. However, when $τ_{QSL} < τ_D$ the potential capacity for further acceleration will be greater. On the basis of this fact, achieving a shorter quantum speed limit time $τ_{QSL}$ is one of the most important issues in the study of the evolution of quantum systems. Another important point to be noted here is that, when the coupling strength between the system and environment is weak $τ_{QSL}$ tends to the actual driving time $τ_D$. On the contrary, in the strong coupling between the system and environment, $τ_{QSL}$ can be reduce below the actual driving time $τ_D$ [19]. In the other words, strong coupling can increase the speed of the quantum evolution while the weak system-environment couplings can not increase the speed of the quantum evolution. In this paper, we use relative purity as the distance measure to derive a quantum speed limit time for open system dynamics. we use this to determine the quantum speed limit time because it is applicable to both mixed and pure initial states[22].

In Ref. [22], Zhang et al. have shown that quantum speed limit depend on quantum coherence. They have represented that in the damped Jaynes-Cummings model QSL time decreases by increasing the quantum coherence while in Ohmic-like dephasing model QSL time increase by increasing quantum coherence. It should be noted that the quantum coherence can also be affected by Hawking radiation in curved spacetime [31, 32]. It has been known that the quantum coherence of quantum system would degrade when quantum get close to the event horizon. Given this fact, it can be said that Hawking radiation can changed the QSL time. It is interesting to investigate how Hawking radiation would affect on the quantum speed limit time. To demonstrate this, we use the simplest black hole: Schwarzschild black hole with Dirac field states.
Here, we consider the setting in which quantum system freely falls into the Schwarzschild black hole and then hovers near the event horizon, finally quantum system interacts with surroundings in the damped Jaynes-Cummings or Ohmics like dephasing models. This work is organized as follows. In Sec. II, QSL time have been obtained based on relative purity as a distant measure. In Sec. III, we review the Dirac fields in the Schwarzschild spacetime. In Sec. IV, the quantum speed limit time of a single qubit system is investigated in the damped Jaynes-Cummings and Ohmics like dephasing models respectively. The conclusion of this work is given in Sec. V.

II. QUANTUM SPEED LIMIT TIME BASED ON RELATIVE PURITY

In this section we consider the evolution of the open quantum system. The state of the system at time \( t \) is described by density matrix \( \rho_t \). The evolution of such a quantum system can be described by the time-dependent nonunitary equations of the form \( \dot{\rho}_t = \mathcal{L}_t(\rho_t) \), where \( \mathcal{L}_t \) is the positive generator [14]. Here we want to calculate the minimum time which is necessary to evolve from the state at time \( \tau \) to target state at time \( \tau + \tau_D \), where \( \tau_D \) is the driving time of the open quantum system. In order to obtain this minimal time one should choose an appropriate distance measure to define the quantum speed limit time. In Ref. [22], Zhang et al. have introduced the QSL time based on relative purity. The advantage of their QSL time is that, it is applicable to both mixed and pure initial states. One can define the relative purity \( f(\tau) \) between initial state \( \rho_I \) and target state \( \rho_T \), as [33]

\[
f(\tau + \tau_D) = \frac{\text{tr}(\rho_I \rho_T e^{\tau D})}{\text{tr}(\rho_T^2)}. \tag{1}
\]

Using the method outlined in Ref. [22], one can derive the (ML) bound of quantum speed limit time for nonunitary dynamics as

\[
\tau \geq \frac{|f(\tau + \tau_D) - 1| \text{tr}(\rho_T^2)}{\sum_{i=1}^{n} \sigma_i \rho_i}, \tag{2}
\]

where \( \sigma_i \) and \( \rho_i \) are the singular values of \( \mathcal{L}_t(\rho_t) \) and \( \rho_I \), respectively and \( \mathcal{A} = \frac{1}{\tau_D} \int_{t}^{t+\tau_D} \text{Adt} \). In a similar way, (MT) bound of QSL time for nonunitary evolution can be obtain as

\[
\tau \geq \frac{|f(\tau + \tau_D) - 1| \text{tr}(\rho_T^2)}{\sum_{i=1}^{n} \sigma_i^2}. \tag{3}
\]

From Eqs. (2) and (3), unified bound can be formulated as follows

\[
\tau_{QSL} = \max \left\{ \frac{1}{\sum_{i=1}^{n} \sigma_i \rho_i}, \frac{1}{\sum_{i=1}^{n} \sigma_i^2} \right\} \times |f(\tau + \tau_D) - 1| \text{tr}(\rho_T^2). \tag{4}
\]

Zhang et al. have shown that this QSL time is depend on the coherence of initial state \( \rho_I \) [22].

III. DIRAC FIELDS IN THE SCHWARZSCHILD SPACETIME

In this section, we discuss over important exclusivity of Dirac fields in the Schwarzschild spacetime to express the QSL time for non-unitary open quantum dynamic in the background of a black hole. Let’s consider a Schwarzschild spacetime which is given by the metric in the form below

\[
ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{5}
\]

where \( M \) and \( r \) are the mass and radius of the black hole, respectively. \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \), is the line element in the unit sphere. It is worth noting that, near the event horizon \( r_h \), this metric has similar structure with Rindler horizon in flat spacetime [34].

Let’s consider the massless Dirac equation in the curved spacetime as [35]

\[
\frac{\gamma_0}{\sqrt{1 - \frac{2M}{r}}} \frac{\partial \psi}{\partial t} + \gamma_1 \sqrt{1 - \frac{2M}{r}} \left( \frac{\partial}{\partial r} + \frac{1}{r} + \frac{M}{2(r - 2M)} \right) \psi + \gamma_2 \left( \frac{\partial}{\partial \theta} + \frac{1}{2} \cot \theta \right) \psi + \frac{\gamma_3}{r \sin \theta} \frac{\partial \psi}{\partial \phi} = 0, \tag{6}
\]

where \( \gamma_i \)’s \( (i = 0, 1, 2, 3) \) are the 4 by 4 Dirac matrices. Solving massless Dirac equation (10) near the event horizon \( r_h \) leads to positive (fermions) frequency outgoing solutions outside and inside regions of the event horizon as [36, 37]

\[
\psi_k = \begin{cases} 
\xi e^{-i\omega u} & r > r_h \\
\xi e^{i\omega u} & r < r_h 
\end{cases}, \tag{7}
\]

where \( u = t - r^* \) and \( r^* = r + 2M \ln \left| \frac{2M - r}{2M} \right| \) is the is the tortoise coordinate. The goal is to determine the vacuum structure for different observer. For this, Let’s introduce the light-like Kruskal coordinates as [38]

\[
U = -\frac{1}{k} \exp [-k(t - r^*)], \quad V = -\frac{1}{k} \exp [k(t + r^*)], \tag{8}
\]

where \( k = 1/4M \) is the surface gravity. With regard to light-like Kruskal coordinates, the Schwarzschild metric has the following form [39, 40]

\[
ds^2 = -\frac{1}{2kr} e^{-2kr} dU dV + r^2 d\Omega^2. \tag{9}
\]

There exist three regions with different physical time-like vectors. In first region, for time-like vector in the form \( \partial_t \propto (\partial_U + \partial_V) \), specific parameter \( t \) is associated with the proper time of a free-falling observer (Alice) near the horizon. This time-like vector is similar to the Minkowskian time-like Killing vector and the structure of the Hartle–Hawking vacuum \( |0_H\rangle \) is similar to the structure of the Minkowski vacuum \( |0_M\rangle \). In second region, \( \partial_t \propto (\partial_U + \partial_V) \) is the time-like Killing vector, which is related to an observer
(Bob) with proper acceleration \( a = k/√{1 - 2M/r_0} \), where \( r_0 \) is the position of the observer with respect to event horizon, it is assumed that this distance is small enough. \( \partial_t \) is similar to Rindler vacuum in flat space. Note that Rindler approximation is only logical when \( R_0 - 1 \ll 1 \), where \( R_0 = r_0/r_h = r_0/2M \) [38]. For this time-like Killing vector, the vacuum structure is known as the Boulware vacuum \( |0\rangle_i \). However, \( -\partial_t \) is another Killing vector. \( -\partial_t \) enables us to introduce another Boulware vacuum which is known as Anti-Boulware vacuum \( |0\rangle_{II} \). The Hartle–Hawking vacuum structure is made from a variety of frequency modes as \( |0\rangle_H = \otimes |0\rangle_{\omega_i} \), in a similar way for first excitation \( |1\rangle_H = \otimes |1\rangle_{\omega_i} \). One can define the Hartle-Hawking vacuum \( |0_{\omega_i}\rangle \) and its first excitation \( |1_{\omega_i}\rangle \) as [38, 41]

\[
|0_{\omega_i}\rangle_H = \frac{1}{\sqrt{1 + e^{-\Omega}}} |0_{\omega_i}\rangle_I |0_{\omega_i}\rangle_{II} + \frac{1}{\sqrt{1 + e^{-\Omega}}} |1_{\omega_i}\rangle_I |1_{\omega_i}\rangle_{II}, \tag{10}
\]

\[
|1_{\omega_i}\rangle_H = |1_{\omega_i}\rangle_I |0_{\omega_i}\rangle_{II}, \tag{11}
\]

where \( \Omega = \frac{\omega_i}{T_H} = \frac{2\pi}{\omega} \) is the mode frequency measured by Bob and \( T_H = k/2\pi \) is the Hawking temperature. This formulation enables us to check the QSL time as a function of distance of Bob to the event horizon and Hawking temperature \( T_H \).

IV. QUANTUM SPEED LIMIT TIME OF THE DYNAMICS IN THE SCHWARZSCHILD SPACETIME

In this section we investigate the effect of Hawking radiation on QSL time \( \tau_{QSL} \). First, let’s assume that the quantum system \( B \) is in possession of Bob in single-qubit state \( \rho^B_0 = \frac{1}{2}(I + \sum_{i=1}^3 r_i \sigma_i) \), where \( I \) is the identity operator of the qubit, \( \sigma_i \)'s \( (i = 1, 2, 3) \) are the Pauli operators, and \( r_i \)'s are the components of Bloch vector. Bob falls toward the black hole and then locates at a fixed distance \( r_0 \) outside the event horizon.

Let us assume that Bob has a detector which only detects mode with frequency \( \omega \). Thus, the states associated with mode \( \omega \) must be specified in Boulware basis. From the viewpoint of Bob, under the single-mode approximation, Hartle-Hawking vacuum and its excitation in Boulware basis is rewritten as

\[
|0_{\omega_i}\rangle_H = j_-|0_{\omega_i}\rangle_I |0_{\omega_i}\rangle_{II} + j_+ |1_{\omega_i}\rangle_I |1_{\omega_i}\rangle_{II}, \tag{12}
\]

\[
|1_{\omega_i}\rangle_H = |1_{\omega_i}\rangle_I |0_{\omega_i}\rangle_{II},
\]

where \( j_+ = [1 + \exp(\Omega \sqrt{1 - \frac{1}{r_0^2}})]^{1/2} \) and \( j_- = [1 + \exp(-\Omega \sqrt{1 - \frac{1}{r_0^2}})]^{-1/2} \). The subscripts \( I \) and \( II \) show the states associated to the Rindler region \( I \) and region \( II \) respectively. In particular, after transforming Bob’s states according to Eq. (12) and by tracing over the degrees in region \( II \), one can obtain the density matrix for single-qubit system under the effect of Hawking radiation as

\[
\rho^B_{I0} = \frac{1}{2} (I + \sum_{i=1}^3 r_i \sigma_i), \tag{13}
\]

where \( r_1 = j_1 r_1, r_2 = j_- r_2 \) and \( r_3 = \frac{1}{2} (j_2^2 - j_-^2 - 1)(1 + r_0^2 + 1) \). When Bob falls toward the black hole and locates at a fixed distance \( r_0 \) outside the event horizon then one can find the coherence of the Bob’s system as \( C(\rho^B_0) = r_0^2 + r_0^2 \). Due to the mathematical form of the coefficient \( j_- \) one can see the quantum coherence decreases when the distance of Bob from event horizon, i.e. \( R_0 \) decrease. In Fig. 1 the quantum coherence is plotted as a function of \( R_0 \) for the state with initial coherence \( C(\rho^B_0) = 1 \). As can be seen from Fig. 1, the quantum coherence decreases strongly when Bob falls toward black hole. Also, one can see the quantum coherence decreases strongly when the frequency of the mode increases.

In our scenario the quantum system \( B \) is located at a fixed distance \( r_0 \) outside the event horizon and it is in a direct non-interaction with the environment. We consider two types of decoherence model to investigate the effect of Hawking radiation on QSL time : damped Jaynes-Cummings and Ohmic-like dephasing models.

A. Damped Jaynes-Cummings

Let us consider the exactly solvable damped Jaynes-Cummings model for a two-level system which is coupled to a leaky single mode cavity [42, 43]. The environment is assumed to be initially in a vacuum state. The non-unitary positive generator of the dynamics of the system is given by

\[
\mathcal{L}_\epsilon(\rho_t) = \gamma_t (\sigma_- \rho_t \sigma_+ - \frac{1}{2} \sigma_+ \sigma_- - \frac{1}{2} \rho_t \sigma_+ \sigma_-), \tag{14}
\]

where \( \sigma_{\pm} = \sigma_1 \pm i\sigma_2 \) are the Pauli operators and \( \gamma_t \) is the time-dependent decay rate. If there exist one excitation in the
compound atom-cavity system then the environment can be described by an effective Lorentzian spectral density as

\[ J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda}{(\omega_0 - \omega)^2 + \lambda^2}, \]

where \( \omega_0 \) is the frequency of the single-qubit system, \( \lambda \) denotes the spectral width and \( \gamma_0 \) represents the coupling strength.

The time-dependent decay rate can be written as

\[ \gamma_t = \frac{2\gamma_0 \lambda \sinh(dt/2)}{d \cosh(dt/2) + \lambda \sinh(dt/2)}, \]

where \( d = \sqrt{\lambda^2 - 2\gamma_0 \lambda} \). When the coupling is weak i.e. \( \lambda > 2\gamma_0 \), the dynamics is Markovian and information flow to the environment in irreversible manner. In contrast, when we have the strong coupling between system and environment i.e. \( \lambda < 2\gamma_0 \) the dynamics is non-Markovian and information flow back to the system from environment. One can find the reduced density operator \( \rho_B^{\tau} \) of the system which is in Bob’s possession near the event horizon as

\[ \rho_B^{\tau} = \left( \frac{2 - (1 - \hat{r}_3)p_t}{(\hat{r}_1 + i\hat{r}_2)\sqrt{p_t}} \right), \]

where \( p_t = \exp(-\int_0^t \gamma_t dt') \), we also introduced \( \hat{r}_i \)'s at the beginning of this section.

Considering Eq. (4), we can obtain QSL time for damped Jaynes-Cummings in Schwarzschild spacetime. Here after, we choose an initial state with maximum quantum coherence i.e. \( C(\rho_0^B) = 1 \). In Fig.2 the quantum speed limit time is plotted as a function of the initial time parameter \( \tau \) in the weak coupling regime \( \gamma_0 = 0.1\lambda \). As can be seen QSL time is decreased by increasing the distance \( R_0 \) between Bob and the event horizon. Fig. 3 represents the quantum speed limit time as a function of the initial time parameter \( \tau \) in strong coupling regime. Fig. 3 shows in strong coupling the quantum speed limit time is shorter than weak coupling. As can be seen, QSL time is decreased by increasing \( R_0 \).

In order to show the effects of coupling strenght on QSL time, the QSL time is plotted as a function of \( \gamma_0 \) in Fig. (4) for the constant driving time \( \tau_D = 1 \). As can be seen the QSL time decreases in strong coupling. It also shows that the QSL time decrease when \( R_0 \) increases. Directly, in order to show the effect of the frequency of the mode \( \Omega \) and \( R_0 \) on QSL time we plot the QSL time as a function of \( R_0 \) in strong and weak coupling limit for different value of \( \Omega \) in Fig. 5. As can be seen for both of strong and weak coupling the QSL time is increased by decreasing \( R_0 \). Also, this figure shows that the QSL time is decreased by increasing \( \Omega \). As a result, by comparing the Figs. (2,3,4,5) with Fig.(1) one can say as the quantum coherence is decreased by decreasing \( R_0 \), the Quantum speed limit time is increased by decreasing \( R_0 \). This is what we expected because Zhang et al. have shown the QSL time is decreased by increasing quantum coherence for damped Jaynes-Cummings model.

\[ \mathcal{L}_t(\rho_t) = \gamma_t (\sigma_3 \rho_t \sigma_3 - \rho_t), \]

where the time-dependent decay rate is written as

\[ \gamma_t = \int_0^\infty d\omega J(\omega) \coth\left( \frac{\hbar \omega}{2K_B T} \right) \frac{1 - \cos \omega t}{\omega^2}, \]

where \( T \) is temperature and \( K_B \) is Boltzmann constant. Here, we consider the Ohmic-like density spectral as

\[ J(\omega) = \frac{\omega^s}{\omega_c^{s-1}} \exp\left( -\frac{\omega}{\omega_c} \right), \]
where $\omega_c$ is the cutoff frequency and $\eta$ is a dimensionless coupling constant. The type of the environment is characterized by $S$. For $s \leq 1$, $s = 1$ and $s \geq 1$ we have sub-Ohmic, Ohmic and super Ohmic environment respectively. In the limit of zero temperature $T = 0$, for $t > 0$ and $s \geq 0$, the dephasing rate can be written as

$$\gamma_t = \eta \left[ 1 - \frac{\cos((s - 1) \arctan(\omega_c t)) \Gamma(s - 1)}{1 + \omega_c^2 t^2} \right], \quad (21)$$

where $\Gamma(\cdot)$ is the Euler Gamma function. Note that, in special case when $s \to 1$, the dephasing rate is obtained as $\gamma_t(s = 1) = \eta \ln(1 + \omega_c^2 t^2)$. In this decoherence model we find the reduced density operator $\rho^B_{ti}$ as

$$\rho^B_{ti} = \left( \begin{array}{cc} (1 + \dot{r}_3) & (\dot{r}_1 - i \dot{r}_2) q_t \\ (\dot{r}_1 + i \dot{r}_2) q_t & (1 - \dot{r}_3) \end{array} \right), \quad (22)$$

where $q_t = \exp[-\gamma_t]$.

Considering Eq. (4), we can obtain QSL time for damped Jaynes-Cummings in Schwarzschild spacetime. We consider the maximally coherent state as an initial state.

In Fig. (6) the quantum speed limit time is plotted as a function of the initial time parameter $\tau$, when Ohmicity parameter is $s = 2$ and the evolution is Markovian. As can be seen QSL time is increased by increasing the distance $R_0$, between Bob and the event horizon. It is due to the fact that the quantum speed limit time for Ohmic-like dephasing model is depend on quantum coherence in linear manner, i.e. the quantum speed limit time is increased(decreased) by increasing(decreasing) quantum coherence [22]. Fig. (7) represents the quantum speed limit time as a function of the initial time parameter $\tau$, when Ohmicity parameter is $s = 2.5$ and the evolution is Non-Markovian. Comparing Fig. (6) with Fig. (7), shows for non-Markovian evolution the quantum speed limit time is shorter than Markovian evolution. Also one can see in both Markovian and bob-Markovian evolution the quantum speed limit is increased by increasing $R_0$. 
In order to show the effects of Ohmicity parameter on QSL time, the QSL time is plotted as a function of $s$ in Fig. (8) for the constant driving time $\tau_D = 1$. As can be seen the QSL time decreases for super-Ohmic environment, when the evolution is non-Markovian. It also shows that the QSL time increases when $R_0$ increases. Directly, in order to show the effect of the frequency of the mode $\Omega$ and $R_0$ on QSL time we plot the QSL time as a function of $R_0$ in Markovian $s = 2$, and non-Markovian $s = 3$ evolution for different value of $\Omega$ in Fig. (9). As can be seen for both of Markovian and non-Markovian evolution the QSL time is decreased by decreasing $R_0$. Also, this figure shows that the QSL time is increased by increasing $\Omega$. It is due to the fact that, as can be seen from Fig.(1), the quantum coherence is increased by increasing $\Omega$. Consequently, with comparison Figs. (6,7,8,9) and Fig.(1), It can be concluded that QSL time has a linear dependence with quantum coherence for Ohmic-like dephasing model. As quantum coherence increases with increasing $R_0$ and $\Omega$. Due to the linear dependence of QSL time on coherence, QSL time is also increased by increasing $R_0$ and $\Omega$.

V. CONCLUSION

In this work we studied the effect of Hawking radiation on quantum speed limit time for two types of decoherence model. The results obtained in this article are in satisfactory agreement to those obtained previously by Zhang et al. for damped Jaynes-Cummings and Ohmic-like dephasing models [22]. We have shown that for damped Jaynes-Cummings model the quantum speed limit time has the inverse relation with quantum coherence in Schwarzschild spacetime. The quantum coherence is decreased by decreasing the distance between of quantum state with the event horizon, thus the quantum speed limit for damped Jaynes-Cummings model in Schwarzschild spacetime will be increase when $R_0$ decreases. The situation of QSL time in Schwarzschild spacetime for the Ohmic-like dephasing model is completely different. In this model the QSL time is depend on quantum coherence linearly[22]. We have shown that in Schwarzschild spacetime for Ohmic-like dephasing model, the QSL time is increased by increasing $R_0$ and $\Omega$.

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