Multiobjective Tuning and Performance Assessment of PID Using Teaching–Learning-Based Optimization

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ABSTRACT: There have been many studies on the optimal tuning and control performance assessment (CPA) of the PID controller. In the optimal tuning, the trade-off between the setpoint tracking and the disturbance rejection performance is a challenge. Minimum output variance (MOV) is very widely used as a benchmark for CPA of PID, but it is difficult to be observed due to the non-convex optimization problem. In this paper, a new multiobjective function, considering both the OV in the CPA problem and integral of absolute error, is proposed to tune PID for this trade-off. The CPA-related non-convex problem and tuning-related multiobjective problem are solved by teaching–learning-based optimization, which guarantees a tighter lower bound for MOV due to the excellent capability of local optima avoidance and has higher computational efficiency due to the low complexity. The numerical examples of CPA problems show that the algorithm can generate better MOV than existing methods with less calculation time. The relationship between the weight of the multiobjective function and the performance, including setpoint tracking, stochastic response coefficients of the closed-loop transfer function as polynomials in unknown controller parameters and used sums of squares programming to solve the related optimization problem, which can guarantee a lower bound on the solution. Some researchers employed global optimization methods to solve this non-convex problem to guarantee a lower bound. Nevertheless, because few of them analyze the calculation time, they are inappropriate to be applied in online CPA. Fu et al. transformed the non-convex problem into a convex problem, which was solved by a low-complexity algorithm called iterative convex programming to promote the online application. To further reduce the calculation time, Shahni et al. proposed a fast method by using a fixed length of impulse response coefficients to remove the iteration. This method with a weighting parameter can get a tighter lower bound, but the calculation time is longer. As stochastic optimization methods show great performance in solving global optimization problems, Pillay and Govender proposed a hybrid algorithm combining Nelder-Mead simplex with the Particle Swarm algorithm to solve this non-convex problem, but the results are not so competitive.

1. INTRODUCTION

In the last few decades, researchers have been working on the autonomous operation of industrial process control systems. PID control is the most widely used control method in industries, and its performance maintenance has received much attention from both academia and industry. Most of the process control systems suffer from their performance deteriorating during long time operation under the influence of malfunction and the environment, such as equipment faults from the sensor and actuator, controller tuning problem, and changes of disturbance characteristic. Therefore, there are many techniques to solve these problems, such as control performance assessment (CPA), fault diagnosis, and controller retuning. CPA aims to provide a benchmark for PID control systems to indicate the room for improvement, which plays a vital role in the autonomous operation of control systems. Once the model of the system and the characteristic of the environment are changed, a retuning process is needed to maintain the system performance.

Minimal output variance (MOV) is very widely used as a benchmark for the CPA of PID control systems. However, it is not easy to be observed due to the non-convexity of the relevant optimization problem. Therefore, many approaches have been proposed to solve this problem. Some approaches adopted local optimization methods based on the gradient, but these methods can only provide an upper bound on MOV of PID because they do not ensure global optimality. Karivala reformulated the computation of MOV so as to ensure a lower bound. Sendjaja and Karivala represented the impulse response coefficients of the closed-loop transfer function as polynomials in unknown controller parameters and used sums of squares programming to solve the related optimization problem, which can guarantee a lower bound on the solution. Some researchers employed global optimization methods to solve this non-convex problem to guarantee a lower bound. Nevertheless, because few of them analyze the calculation time, they are inappropriate to be applied in online CPA. Fu et al. transformed the non-convex problem into a convex problem, which was solved by a low-complexity algorithm called iterative convex programming to promote the online application. To further reduce the calculation time, Shahni et al. proposed a fast method by using a fixed length of impulse response coefficients to remove the iteration. This method with a weighting parameter can get a tighter lower bound, but the calculation time is longer. As stochastic optimization methods show great performance in solving global optimization problems, Pillay and Govender proposed a hybrid algorithm combining Nelder-Mead simplex with the Particle Swarm algorithm to solve this non-convex problem, but the results are not so competitive.
Many methods have been proposed to tune the PID controller, some for setpoint tracking performance, some for disturbance rejection performance, and some for the trade-off between these two performances. For the setpoint tracking performance, the most presented in the literature are optimal tuning methods based on objective functions. Among them, integral of absolute error (IAE) and integral of the squared error are the most commonly used criteria, but they lead to the contradiction between settling time and overshoot. Therefore, integral of time multiplied by absolute error and integral of squared time multiplied by squared error are proposed to overcome this problem. However, these object functions cannot optimize all criteria, such as overshoot, rise time, settling time, and steady error, and some multiobjective functions have been proposed recently to solve this problem. Sahib and Ahmed proposed a new multiobjective function considering the four performance criteria simultaneously, and a decision making process is designed to select the best optimum from the Pareto optimal set. Skogestad proposed a novel multiobjective function taking into account the mean of time-weighted absolute error, settling time, overshoot, and steady error. Gaing proposed a new time domain performance criterion, the minimization of which corresponds to parameters with good step response.

For the disturbance rejection performance, Skogestad aimed at deriving the maximum limit on the controller to tune PID for acceptable disturbance rejection. Krohling and Rey proposed to describe the disturbance rejection as $H_{\infty}$-norm, and then it is used as a constraint for the controller tuning with optimal disturbance rejection. Alagöz et al. proposed a reference to the disturbance ratio (RDR) to investigate the disturbance rejection performance of the PID controller. Subsequently, they proposed a graphical methodology to improve the disturbance rejection capability. Shamsuzzoha and Lee designed optimum IMC filter structures for the PID controller and then tuned it with the IMC–PID method to improve the disturbance rejection capability. Leva et al. tuned the PID controller to minimize the magnitude of the nominal disturbance-to-output frequency response to obtain excellent disturbance rejection performance, and, in the meantime, to avoid the peak and plateau. Vrančić et al. proposed the disturbance rejection magnitude optimum method based on the magnitude optimum (MO) method to enhance the disturbance rejection capability of the PI controller and the PID controller, and a two-degrees-of-freedom (2DOF) structure with a setpoint filter is used to retain the tracking performance.

The trade-off between the two performances is a challenge to tune the PID controller. The multiloop model reference control structure is efficient in improving the disturbance rejection without severely deteriorating the tracking performance. Tepljakov et al. proposed a multiloop MRAC-FOPID control structure to improve the disturbance rejection and tracking performance. A 2DOF control structure with a setpoint filter is also a common method to achieve the trade-off. Some studies addressed the trade-off for a one-degree-of-freedom PID controller. Alcántara et al. proposed a $\gamma$-tuning procedure based on the model matching approach. Arrieta et al. proposed performance degradation indexes and autotuning methods. Recently, the multiobjective optimization strategy is also used to solve this trade-off. Ozbey et al. presented a multiobjective tuning method to use the mean squared control error (MSCE) to measure the tracking performance and use RDR to measure the disturbance rejection. Tufenkci et al. presented a multiobjective controller design strategy considering the optimal placement of the minimum angle system pole and a predefined RDR design specification.

The OV of a control loop in the CPA problem is a direct measurement of the stochastic disturbance rejection capability, and the IAE criterion of the PID control loop mainly studies the setpoint tracking performance. In this paper, a new multiobjective function considering both OV and IAE is proposed as a performance criterion to tune the PID controller. The proper weight can achieve the trade-off between the disturbance rejection and setpoint tracking. With the multistage tuning strategy presented in the literature, the weight of the function can compromise another performance to improve the disturbance performance in the steady stage and improve the setpoint tracking performance in the initial stage. So far, few methods have the ability to find MOV of PID with accurate estimation and high efficiency simultaneously. To solve this problem, this paper employs a stochastic optimization method named teaching–learning-based optimization (TLBO) that can balance these two aspects due to the superiorities of local optimality and low complexity compared with the existing methods to solve the CPA problem. Meanwhile, the algorithm is easy to implement and does not need any specific parameters.

Furthermore, the optimal tuning and the CPA problem are extended to the PI/P cascade control case, which is a practical control strategy in process control systems. Several simulation examples from the literature are utilized to test the performance of the algorithm in solving the CPA problem of PID. The results verify that it can obtain better MOV and runtime than existing methods on most problems. The proposed tuning method is applied to two temperature control systems, and the simulation results reveal the relationship between the weight and the performance including setpoint tracking, stochastic disturbance rejection, and step disturbance rejection of the single-loop and cascade control.

This paper is organized as follows: Section 2 introduces the TLBO algorithm. Section 3 described the achievable performance and tuning method of the PID control. In Section 4, the results of the simulation examples are presented. Finally, Section 5 shows the conclusion.

## 2. TEACHING–LEARNING-BASED OPTIMIZATION

The TLBO algorithm, proposed by Rao et al. in 2011, has been a powerful meta-heuristic optimization algorithm in solving engineering problems due to the two-phase strategy, that is, teacher phase and learner phase. This strategy imitates the process that learners improve their knowledge through teaching and learning behaviors. The algorithm involves two populations named learners and teachers, and the learner with the best fitness value at every iteration is chosen as the teacher. In the teacher phase, learners learn knowledge from the teacher to approach the global optimum, which guarantees the exploitation capability of the algorithm. In the learner phase, the learners learn knowledge from each other to get more chances to find the global optimum, which make the algorithm have excellent exploration capability.

### 2.1. Teacher Phase

In this phase, the teacher tries to improve the mean of all learners at any iteration $G$. Supposing that the number of learners is $N$ (for any learner $i, i = 1, 2, \ldots, N$) and the dimension of a learner is $D$ (for any dimension $j, j = 1, 2, \ldots, D$). The learners can be updated by the following law
It is further assumed that there is no setpoint change, that is, \( y_s(t) = 0 \). The output of the system can be described as follows

\[
y(t) = -y(t)G_c(q^{-1})G_d(q^{-1}) + a(t)G_d(q^{-1}) \tag{4}
\]

When the structure of the controller is restricted to PID described as follows

\[
G_d(q^{-1}) = \frac{k_i + k_d q^{-1} + k_d q^{-2}}{1 - q^{-1}} \tag{5}
\]

where \( k_i = k_p + k_d \), \( k_d = -(k_p + 2 k_d) \), and \( k_i = k_p, k_d, k_d \) and \( k_d \) represent proportional, integral, and derivative gains, respectively. This discrete controller consists of an incremental PID controller and an integrator. Due to the fact that the controller parameters are solved in the discrete-time-domain but most industrial controllers are obtained in the continuous-time domain, an appropriate sampling time satisfying the sampling theorem is crucial when the process model is discretized. In addition, if only a single shock \( a(0) \) is introduced to the system, according to the convolution theorem, the calculation of output sequence \( \tilde{y} = [y(0), y(1), ..., y(n)]^T \) is expressed as follows

\[
y = -k_i \tilde{y}_m - k_d F \tilde{y}_m - k_d F^2 \tilde{y}_m + na(0) \tag{6}
\]

where \( n = [g(0), g(1), ..., g(n)]^T \) is the impulse response of the disturbance model, \( F \) is the forward shift matrix, and \( I_m \) is the matrix consisting of the impulse response \( \tilde{g} = [g(1), g(2), ..., g(n)]^T \) of the process model, that is

\[
F = \begin{bmatrix}
0 & 0 \\
1 & \ddots \\
\vdots & \ddots & \ddots \\
0 & 1 & 0 \\
\end{bmatrix}_{(n+1) \times (n+1)}
\]

\[
I_m = \begin{bmatrix}
g(0) & 0 & 0 & \cdots & 0 \\
g(1) & 0 & 0 & \cdots & 0 \\
g(2) & g(1) & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
g(n) & g(n-1) & g(n-2) & \cdots & 0
\end{bmatrix}
\]

The output sequence can be set as

\[
y = (1 + k_1 I_m + k_2 F I_m + k_d F^2 I_m)^{-1} a(0) = \varphi a(0) \tag{7}
\]

where \( \varphi = [\varphi(1), \varphi(2), ..., \varphi(n)]^T \) is the impulse response of the closed-loop model and the OV can be calculated as

\[
\sigma_y^2 = \varphi^T \varphi \sigma_a^2 \tag{8}
\]

where \( \sigma_y^2 \) is the variance of disturbance. Therefore, the CPA problem of PID control is described as follows

\[
\gamma = \frac{\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4}{4}
\]

where \( \gamma_1, \gamma_2, \gamma_3, \gamma_4 \) are the achievable gains, respectively.
The impulse response with finite length \( p \) (i.e., \( \phi = [\phi(0), \phi(1), \ldots, \phi(p)]^T \)) is utilized to approximate the OV, and the non-convex problem can be redescribed as follows

\[
J^*_t = \min f_t(k_1, k_2, k_3 ) = \min_{k_1, k_2, k_3} \sigma^2 = \min_{k_1, k_2, k_3} \phi^T \phi \sigma^2
\]

The impulse response with finite length \( p \) (i.e., \( \phi_p = [\phi(1), \phi(2), \ldots, \phi(p)]^T \)) is utilized to approximate the OV, and the non-convex problem can be redescribed as follows

\[
J^*_t = \min f_t(k_1, k_2, k_3 ) \approx \min_{k_1, k_2, k_3} \phi_p^T \phi_p \sigma^2
\]

### 3.2. Achievable Performance of PI/P Cascade Control.

Figure 2 shows a cascade control system with the outer loop model \( G_1(q^{-1}) \) and inner loop model \( G_2(q^{-1}) \). \( a_1(t) \) and \( a_2(t) \) are disturbances in the outer and in the inner loops, respectively. The disturbance models are \( G_{d1}(q^{-1}) \) and \( G_{d2}(q^{-1}) \). When the setpoint \( y_{sp}(t) = 0 \), and the structures of the primary controller \( G_{1} \) and the secondary controller \( G_{2} \) are restricted to PI and P, respectively, that is

\[
G_1 = \frac{k_1 + k_1q^{-1}}{1 - q^{-1}}, \quad G_2 = k_2
\]

The outputs of the outer loop \( y_1(t) \) and the inner loop \( y_2(t) \) are

\[
y_1(t) = y_2(t)G_1(q^{-1}) + a_1(t)G_{d1}(q^{-1})
\]

\[
y_2(t) = k_2 \left[ -y_1(t) \frac{k_1 + k_1q^{-1}}{1 - q^{-1}} - y_2(t) \right] G_2(q^{-1}) + a_2(t)G_{d2}(q^{-1})
\]

When the system is only influenced by the initial shocks of the disturbances \( a_1(0) \) and \( a_2(0) \), the output vectors are formulated as follows

\[
y_{1}(0) = I_m \tilde{y}_{1} + n_1 a_{1}(0)
\]

\[
y_{2}(0) = -k_1k_2S_2\tilde{y}_{1} - k_1k_0FS_2\tilde{y}_{1} - k_1k_0S_2\tilde{y}_{2} + \bar{n}_{2} a_{2}(0)
\]

where

\[
\tilde{y}_{1} = [y_{1}(0), y_{1}(1), \ldots, y_{1}(n)]^T \quad \text{(i = 1, 2)}
\]

\[
\tilde{y}_{2} = [g_{1}(0), g_{1}(1), \ldots, g_{d}(n)]^T \quad \text{(i = 1, 2)}
\]

\[
S_2 = [s_{2}(1), s_{2}(2), \ldots, s_{2}(n)]^T
\]

The output vector can be expressed in the following form

\[
\tilde{y}_{1} = \phi a_{1}(0) + \phi a_{2}(0)
\]
Table 1. Benchmark Problems of PID Performance Assessment

| example | $G$ | $G_D$ |
|---------|-----|-------|
| 1       | $0.2q^{-5}$ | $\frac{1}{(1 - q^{-1})(1 + 0.4q^{-1})}$ |
|         | $1 - 0.8q^{-1}$ | |
|         | $0.08919q^{-12}$ | $0.08919$ |
|         | $1 - 0.8669q^{-7}$ | $1 - 0.8669q^{-7}$ |
| 2       | $0.5108q^{-28}$ | $0.5108$ |
|         | $1 - 0.9604q^{-2}$ | $1 - 0.9604q^{-2}$ |
| 3       | $q^{-6}$ | $(1 - 0.5q^{-7})(1 - 0.6q^{-7})(1 + 0.7q^{-7})$ |
|         | $1 - 0.8q^{-2}$ | $(1 - 0.5q^{-7})(1 - 0.6q^{-7})(1 + 0.7q^{-7})$ |
| 4       | $q^{-6}$ | $(1 - 0.5q^{-7})(1 - 0.6q^{-7})(1 + 0.7q^{-7})$ |
|         | $1 - 0.8q^{-2}$ | $(1 - 0.5q^{-7})(1 - 0.6q^{-7})(1 + 0.7q^{-7})$ |
| 5       | $q^{-6}$ | $(1 - 0.5q^{-7})(1 - 0.6q^{-7})(1 + 0.7q^{-7})$ |
|         | $1 - 0.8q^{-2}$ | $(1 - 0.5q^{-7})(1 - 0.6q^{-7})(1 + 0.7q^{-7})$ |
| 6       | $0.1q^{-5}$ | $0.1q^{-6}$ |
|         | $1 - 0.8q^{-2}$ | $0.1q^{-6}$ |
| 7       | $0.1q^{-5}$ | $0.1q^{-6}$ |
|         | $1 - 0.8q^{-2}$ | $0.1q^{-6}$ |
| 8       | $0.1q^{-5}$ | $0.1q^{-6}$ |
|         | $1 - 0.8q^{-2}$ | $0.1q^{-6}$ |
| 9       | $0.1q^{-5}$ | $0.1q^{-6}$ |
|         | $1 - 0.8q^{-2}$ | $0.1q^{-6}$ |
| 10      | $0.1q^{-5}$ | $0.1q^{-6}$ |
|         | $1 - 0.8q^{-2}$ | $0.1q^{-6}$ |

$q_1 = (1 + W)^{-1} y_1$
$q_2 = (1 + W)^{-1}[I_{in} + (I + k_d I_{in})^{-1}] y_2$
$W = k_d k_{Iin} (I + k_d I_{in})^{-1} S_2 + k_{Iin} k_d (I + k_d I_{in})^{-1} F S_2$

The variance of the output is

$$\sigma^2 = q_1^T \sigma_1^2 + q_2^T \sigma_2^2 + 2q_1^T \sigma_3 \sigma_2$$

The CPA problem of the PI/P cascade control can be described as follows

$$J_2 = \min_{k_1, k_2, k_d} \sigma_2^2 = \min_{k_1, k_2, k_d} \left( q_1^T \sigma_3 \sigma_2 + q_2^T \sigma_3 \sigma_2 + 2q_1^T \sigma_3 \sigma_2 \right)$$

3.3. PID Tuning Based on a New Multiobjective Function. A single-objective function cannot optimize all performance criteria of a control system at the same time. For example, the most commonly used objective function IAE can optimize the setpoint tracking but cannot optimize the disturbance rejection at the same time because they conflict with each other. Multiobjective optimization can solve optimization problems with two or more conflicting object functions. Unlike single-objective optimization with only one “best solution”, it always has a set of alternative optima. These solutions are called Pareto optimal set, and a decision-making process is needed to select an appropriate compromise solution from the set.

The disturbance rejection is one of the most important performances of a control system. Many measurement criteria have been proposed for this performance in literature to tune the PID controller, such as RDR, the magnitude of the nominal disturbance-to-output frequency response.32 OV of a control loop in the CPA problem is also a direct measurement of the disturbance rejection performance and when the disturbance is a zero mean white noise, the result calculated by eq 8 is consistent with the magnitude of the nominal disturbance-to-output frequency response.32 However, few studies have designed objective functions taking into account it. Here, a new multiobjective function considering both OV and IAE is designed to achieve the trade-off between the disturbance rejection and setpoint tracking performance, which is described as follows

$$J_3 = \min_{k_{PID}} \left( k_{PID} \min_{k_{sp}} \int_0^\infty e(t) dt + \rho \sigma_3^2 \right)$$

where $k_{PID}$ is the PID parameter, $e(t) = y_{sp}(t) - y(t)$ is the error between the setpoint and the output, $\sigma_3$ is the OV calculated by eq 8, and $\rho$ is a weight. It should be noted that the calculation of IAE = $\int_0^\infty |e(t)| dt$ is in the case that the system is influenced by the setpoint but not the disturbance, and the calculation of OV is just the opposite case.

By adjusting the weight $\rho$ in a proper range, this function can balance the two performances. With the increase of $\rho$, the objective function places more weight on OV and the controller has better disturbance rejection performance; otherwise, the controller will have better setpoint tracking performance. Owing to the fact that IAE is always much larger than OV, a relatively large weight is necessary. Otherwise, too small a weight cannot attain a better performance of disturbance rejection.

The setpoint tracking performance mainly concerns the initial stage of step response, but disturbance rejection concerns the steady stage. Therefore, combining this tuning method with the multistage PID tuning strategy can resolve the contradiction between the disturbance rejection and the setpoint tracking...
In order to generate stable results considering the computation number of learners is \( N \), the search space of PID parameters can be set as \([-50, 50]\), and there are two termination criteria. The first one is that iteration number \( G \) is bigger than \( G_{\text{min}} \) and the second one is that the difference between the fitness value of the current teacher \( f(P_{\text{teacher}}^G) \) and the teacher in the \( G_{\text{min}} \) iteration before the current iteration \( f(P_{\text{teacher}}^{G_{\text{min}-1}}) \) is less than \( DV \). If the first criterion is not met, the second criterion is not checked. With larger \( N \) and \( G_{\text{min}} \) and smaller \( DV \), the optimal value can be found more easily. However, considering the runtime, appropriate values of these parameters should be selected. The teacher (controller parameter values) and its fitness value are returned at the end of the algorithm.

3.4. Steps of Algorithm. TLBO is a stochastic algorithm that solves the optimization problem by comparing the objective function values (fitness values) of the points selected according to some random rules. The objective functions (10) and (17) are used when the CPA problems are solved by the algorithm, and the objective function (18) is used when the tuning problem is solved. The steps of the algorithm are presented in Figure 3. In order to generate stable results considering the computation efficiency, some parameters of the algorithm can be adjusted: the number of learners is \( N \), the search space of PID parameters can be set as \([-50, 50]\), and there are two termination criteria. The first one is that iteration number \( G \) is bigger than \( G_{\text{min}} \) and the second one is that the difference between the fitness value of the current teacher \( f(P_{\text{teacher}}^G) \) and the teacher in the \( G_{\text{min}} \) iteration before the current iteration \( f(P_{\text{teacher}}^{G_{\text{min}-1}}) \) is less than \( DV \). If the first criterion is not met, the second criterion is not checked. With larger \( N \) and \( G_{\text{min}} \) and smaller \( DV \), the optimal value can be found more easily. However, considering the runtime, appropriate values of these parameters should be selected. The teacher (controller parameter values) and its fitness value are returned at the end of the algorithm.

4. RESULTS AND DISCUSSION

4.1. CPA of Single-Loop Case. In this section, 10 benchmark problems (as shown in Table 1) adopted from the literature are used to verify the excellent performance of the algorithm in solving the non-convex problem. To reduce the

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**Table 2. Best Known MOV (BKMOV) and Results of the Algorithm**

| example | MV    | BKMOV | mean    | std     | worst   | time (s) |
|---------|-------|-------|---------|---------|---------|----------|
| 1       | 2.9427| 3.0728| 3.0728  | 3.36 × 10^{-10} | 3.0728  | 0.3106   |
| 2       | 0.0306| 0.0310| 0.0310  | 2.15 × 10^{-11} | 0.0310  | 0.7524   |
| 3       | 3.0112| 3.0238| 3.0232  | 5.16 × 10^{-10} | 3.0232  | 3.6852   |
| 4       | 3.4004| 3.4065| 3.4064  | 4.94 × 10^{-7}  | 3.4064  | 0.3624   |
| 5       | 11.9528| 13.8076| 13.8068 | 5.18 × 10^{-7}  | 13.8068 | 0.3800   |
| 6       | 58.3406| 87.7377| 87.7069 | 7.88 × 10^{-10} | 87.7069 | 0.4128   |
| 7       | 0.2978| 0.4246| 0.4246  | 5.36 × 10^{-8}  | 0.4246  | 0.2691   |
| 8       | 3.0000| 3.2032| 3.2032  | 3.40 × 10^{-9}  | 3.2032  | 0.1900   |
| 9       | 0.3144| 0.4268| 0.4267  | 2.50 × 10^{-10} | 0.4267  | 0.3395   |
| 10      | 0.0023| 0.0024| 0.0024  | 2.41 × 10^{-10} | 0.0024  | 0.1436   |

The bolder ones mean the best results.

**Table 3. Mean and Std of PID parameters ([k_1, k_2, k_3])**

| example | mean                  | std                  |
|---------|-----------------------|----------------------|
| 1       | [2.8408, -4.4059, 1.7486] | [1.51 × 10^{-3}, 9.22 × 10^{-4}, 4.53 × 10^{-5}] |
| 2       | [1.8236, -3.3531, 1.5299] | [1.31 × 10^{-3}, 6.64 × 10^{-4}, 3.12 × 10^{-4}] |
| 3       | [0.4989, -0.9663, 0.4674] | [1.17 × 10^{-3}, 3.71 × 10^{-4}, 2.03 × 10^{-4}] |
| 4       | [0.1354, -0.2523, 0.1170] | [8.00 × 10^{-4}, 1.47 × 10^{-4}, 7.19 × 10^{-5}] |
| 5       | [0.7241, -1.2058, 0.5178] | [1.25 × 10^{-3}, 3.34 × 10^{-4}, 1.82 × 10^{-4}] |
| 6       | [0.8327, -1.4003, 0.6094] | [5.00 × 10^{-4}, 7.67 × 10^{-5}, 4.33 × 10^{-5}] |
| 7       | [8.0941, -13.1891, 5.5927] | [7.27 × 10^{-4}, 4.69 × 10^{-5}, 2.55 × 10^{-5}] |
| 8       | [6.5338, -9.2379, 3.3583] | [3.74 × 10^{-4}, 1.79 × 10^{-4}, 1.16 × 10^{-4}] |
| 9       | [8.2318, -13.7793, 5.9701] | [1.00 × 10^{-4}, 2.51 × 10^{-5}, 1.45 × 10^{-5}] |
| 10      | [6.1676, -8.5741, 3.0332] | [5.73 × 10^{-4}, 1.35 × 10^{-4}, 7.63 × 10^{-5}] |
error of approximation to obtain an accurate MOV, the length of the impulse response is selected as \( p = 8d \), where \( d \) is the time delay of the process model. The appropriate values of the adjusted parameters are \( N = 20 \), \( G_{\text{min}} = 22 \), and \( DV = 10^{-7} \). All experiments were performed 30 times independently to test the stability of the algorithm and were run on Matlab R2017a on Intel(R) Core(TM) i5-4460 CPU @ 3.20 GHz with 12 GB RAM*.

The results of the algorithm and the best known results of refs 17, 19, and 20 are shown in Table 2, where “MV” is the minimum variance benchmark, “BKMOV” is the best known results, and “Mean”, “Std”, “Worst”, and “Time” are the mean, the standard derivation, the worst, and the mean calculation time of 30 runs, respectively. It shows that the algorithm has better MOV on problems 3, 4, 5, 6, and 9 and has the same MOV on other problems. Particularly, the calculation time of the algorithm is less than 1 s on most problems. The mean and standard derivation of 30 runs of the MOV-related PID parameters are shown in Table 3. It reveals that the algorithm can solve the non-convex problem with accurate estimation, high efficiency, and good stability.

4.2. Tuning of Single-Loop Case. The tuning method based on the multiobjective optimization for the single-loop PID is applied to a high-precision air temperature control system, which provides an environment with high-temperature stability for precision instruments such as laser interferometers and lithography tools. As shown in Figure 4, the temperature control system aims to supply air with high-temperature stability to the temperature chamber, that is, to maintain the temperature of the point “T1” measured by a thermistor. The temperature is controlled by a pipe heater adjusted by a power regulator receiving 4–20 mA current signal.

The input of the single-loop temperature control system is the current (mA), and the output is the temperature (°C) of the point “T1”. A step test is implemented to identify a first-order plus dead time model as follows

\[
G(s) = \frac{y(s)}{u(s)} = \frac{0.39}{90.28s + 1} e^{-31.98s}
\]

This model is discretized with 10 s as the sampling time, which shows satisfying performance in practice. The discrete form is

\[
G(q^{-1}) = \frac{0.0413q^{-4}}{1 - 0.8952q^{-1}}
\]

The disturbance model for the process is simulated as

\[
h(q^{-1}) = \frac{0.2}{1 - 0.8951q^{-1}}
\]

with the variance of the noise being \( \sigma_a^2 = 10^{-5} \). Therefore, the model of the process is

\[
y = \frac{1}{1 - q^{-1}} \left[ \frac{0.0413q^{-4}}{1 - 0.8952q^{-1}}(1 - q^{-1})u + \frac{0.2}{1 - 0.8952q^{-1}}d \right]
\]

The tuning method is verified by comparing the results of four weights. The appropriate values of the adjusted parameters are \( N = 20 \), \( G_{\text{min}} = 50 \), and \( DV = 10^{-12} \). The OV values are presented in Table 4, and the response curves under the step change of the set point are shown in the left of Figure 5. It reveals that adjusting the weight can improve the stability of temperature control but leads to a large overshoot in the initial stage. To solve this problem, a relatively small weight can be used in the initial stage. The right of Figure 5 shows the step disturbance rejection performance of the four weights, which indicates that the weight with better stochastic disturbance rejection performance has better step disturbance rejection performance. It can be concluded that a large weight produces a high gain controller, which exhibits excellent disturbance rejection capability.

4.3. Tuning of PI/P Cascade Control. The temperature control system of immersion liquid in an immersion lithography tool (as shown in Figure 6) adopted from the literature is tested to investigate the tuning of the PI/P cascade control based on the multiobjective function. The controlled variable is the temperature of immersion liquid “T2”, and the manipulated variable is the flow rate of process cooling water controlled by a valve. Because the pipe between “T3” and “T4” is long, a cascade control is used to improve the disturbance rejection, and the sensor of the inner loop is “T3”. The models of the outer loop and inner loop of this system are described as follows

\[
G_1(s) = \frac{1.0092}{1 + 138.06s} e^{-35.75s},
\]

\[
G_2(s) = \frac{-1.3361}{1 + 11.834s} e^{-11.63s}.
\]

The discrete models with the sampling time of 6 s are

\[
G_1(q^{-1}) = \frac{0.04292}{1 - 0.9575q^{-1}} q^{-7},
\]

\[
G_2(q^{-1}) = \frac{-0.5314}{1 - 0.6023q^{-1}} q^{-3}.
\]

The disturbance models are simulated as

\[
G_{dl}(q^{-1}) = \frac{1}{1 - 0.9575q^{-1}},
\]

\[
G_{d2}(q^{-1}) = \frac{1}{1 - 0.6023q^{-1}}
\]

and the variances of the disturbances are set as \( \sigma_{a1}^2 = 0.0005 \), \( \sigma_{a2}^2 = 0.005 \).

The test results of four weights are shown in Table 5 and Figure 7. The appropriate values of the adjusted parameters are \( N = 40 \), \( G_{\text{min}} = 50 \), and \( DV = 10^{-12} \). It indicates that a larger weight relates to a smaller OV value, but the settling time is longer. To solve this conflict, a relatively smaller weight can be used to stabilize the system quickly, and then a larger weight is utilized to improve the disturbance rejection to attain a better performance of temperature control. From the right of Figure 7 showing the step disturbance rejection performance of the four weights, the weight with the best stochastic disturbance rejection performance does not show the best step disturbance rejection performance, which is different from the single-loop case.
5. CONCLUSIONS

This paper proposes a multiobjective function considering both OV and IAE for PID tuning to make a trade-off between the setpoint tracking performance and the disturbance rejection performance. The TLBO algorithm is employed to solve the multiobjective optimization problem and the CPA-related non-convex problem. Furthermore, the tuning method and CPA are extended to the PI/P cascade control. The algorithm is tested on 10 numerical CPA examples adopted from the literature. The results show that the algorithm obtains better MOV and calculation time than the existing methods. The tuning method is applied to a single-loop air temperature control system and a cascade immersion liquid temperature control. The results of the single-loop reveal that a large weight of the multiobjective function can generate a high gain controller, which exhibits excellent stochastic and step disturbance rejection capabilities, but the setpoint tracking performance deteriorates. The results of the cascade control indicate that a larger weight means better stochastic disturbance rejection and poorer setpoint tracking performance.

Table 5. OV of PI/P Cascade Control

| ρ (×10^6) | [k_p, k_i, k_d] | σ_y (×10^{-4}) |
|-----------|----------------|-----------------|
| 0         | [2.7638, −2.6554, −0.8436] | 0.9551          |
| 1         | [3.0563, −2.9922, −0.9631] | 5.3566          |
| 10        | [2.8715, −2.8482, −1.0054] | 4.9421          |
| 100       | [2.9088, −2.8420, −0.9538] | 4.8117          |

5. CONCLUSIONS

This paper proposes a multiobjective function considering both OV and IAE for PID tuning to make a trade-off between the setpoint tracking performance and the disturbance rejection performance. The TLBO algorithm is employed to solve the multiobjective optimization problem and the CPA-related non-convex problem. Furthermore, the tuning method and CPA are extended to the PI/P cascade control. The algorithm is tested on 10 numerical CPA examples adopted from the literature. The results show that the algorithm obtains better MOV and calculation time than the existing methods. The tuning method is applied to a single-loop air temperature control system and a cascade immersion liquid temperature control. The results of the single-loop reveal that a large weight of the multiobjective function can generate a high gain controller, which exhibits excellent stochastic and step disturbance rejection capabilities, but the setpoint tracking performance deteriorates. The results of the cascade control indicate that a larger weight means better stochastic disturbance rejection and poorer setpoint tracking performance.
performance, but the step disturbance rejection capability is weaker, which is a little different from the single-loop case. By combining the proper adjustment of the weight of the proposed tuning method with the multistage PID tuning strategy, a trade-off between the disturbance rejection and setpoint tracking can be realized in practice.

**ASSOCIATED CONTENT**

Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acsomega.1c04428.

Code of programs and simulation schemes: https://github.com/HUSTGDong/ACS-Omega-revision1.git (ZIP)

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Notes

The authors decline any competing financial interest.

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