Focus on out-of-equilibrium dynamics in strongly interacting one-dimensional systems

A J Daley¹,², M Rigol³ and D S Weiss³

¹ Department of Physics and SUPA, University of Strathclyde, Glasgow G4 0NG, Scotland, UK
² Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA
³ Department of Physics, The Pennsylvania State University, University Park, PA 16802, USA

E-mail: andrew.daley@strath.ac.uk

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Abstract

In the past few years, there have been significant advances in understanding out-of-equilibrium dynamics in strongly interacting many-particle quantum systems. This is the case for 1D dynamics, where experimental advances—both with ultracold atomic gases and with solid state systems—have been accompanied by advances in theoretical methods, both analytical and numerical. This ‘focus on’ collection brings together 17 new papers, which together give a representative overview of the recent advances.

Keywords: many-body dynamics, out-of-equilibrium quantum systems, 1D systems, quantum quench, thermalisation

1. Introduction

Until recently, the out-of-equilibrium dynamics of many-body quantum systems was a relatively unexplored field. However, advances in experiments with strongly interacting systems in both cold quantum gases and solid state physics have begun to make such dynamics accessible in a highly controllable way [1, 2]. These advances have been accompanied by
important developments in numerical methods and analytical tools that allow theoretical insight into these highly complex systems, especially in one dimension (1D) [3].

There has been much recent progress in experiments with ultracold atomic [4] and molecular [5] gases. These systems are microscopically well understood, can be modeled by simple many-body Hamiltonians, and allow for a high degree of experimental control over microscopic parameters via external fields. This includes direct control over the strength of the interaction between particles (e.g., via Feshbach resonances [6]), and over the effective dimensionality of the system via optical dipole traps formed with laser beams, or via magnetic traps [4]. This has made the realization of systems where particles are confined to move in one dimension—and transverse degrees of freedom are ‘frozen out’—routine in experiments. Moreover, these systems are well isolated from their environment and coherent dynamics occur on larger length scales and slower timescales than in typical solid state systems. Systems can be repeatedly prepared in the same state and then measured after progressive evolution times. Far-from-equilibrium dynamics can be tracked in experiments for a wide range of microscopic parameters, allowing many interesting phenomena to be studied [1–3].

Understanding non-equilibrium dynamics in the resulting 1D systems has fundamental implications, especially in explaining the mechanisms behind thermalization in many-body systems [1, 2, 7–9], and in characterizing phenomena such as quantum transport after quantum quenches [10–14]. With regard to thermalization, several remarkable experimental observations have already been made. It was demonstrated that, in the range from intermediate to strong coupling, 1D Bose gases thermalize very slowly, if at all [15]. Instead they evolve to what has more recently been called a ‘prethermalized’ state, which is something of a misnomer for integrable systems, which never thermalize [16]. Whether it is similarly a misnomer for nearly integrable systems, like 1D Bose gases that can be made in a lab, is an open question. The details of prethermalized states can sometimes be understood using a generalized Gibbs ensemble (GGE), in which the entropy is maximized under the constraints imposed by a full set of (quasi-)conserved quantities in the model [9, 16, 18]. Prethermalization has also been observed in weakly coupled 1D Bose gases [19], using a different, complementary set of measurement tools. These experiments and GGE theory have motivated an extensive theoretical exploration of the dynamics of low-dimensional quantum systems at integrable points or close to them [1–3]. In parallel, there have been many experimental studies of quench dynamics in lattice systems, including recent observations of proliferating quasi-particles after a quench in a bosonic Mott insulator [20, 21], transport dynamics as an initial trap is opened [22–24], and studies of quenches and adiabatic processes within effective spin models made with tilted optical lattices [25–28].

The study of dynamics in 1D has been strongly supported by developments in theoretical methods, including computational techniques and quantum field theory, as well as by new Bethe–Ansatz solutions of specific integrable problems [3, 29, 30]. New insights have also resulted from connections to the quantum information community [31, 32]. In particular, there have been significant advances in numerical techniques for studying many-body dynamics in 1D systems. Those include the development of the time-dependent density matrix renormalization group (t-DMRG) and related matrix product state techniques [33–39], as well as matrix product operator methods [39–41], which have enabled the treatment of systems with long-range interactions [42, 43]. Furthermore, one can also use numerical linked cluster expansions (NLCEs) to calculate observables after relaxation following a quench [44]. NLCEs provide results in the thermodynamic limit and can be used in arbitrary dimensions.
Non-equilibrium phenomena are also important in the ongoing development of quantum simulators with ultracold quantum gases [45]. The understanding and control that is available with these systems can be used to engineer microscopic Hamiltonians, and then explore the corresponding many-body physics—essentially using the experiment as a special-purpose computer for determining ground states and out-of-equilibrium dynamics. It is important to verify that experimental quantum simulation with such systems is reliable; since the goal is to ultimately work in regimes that are not accessible by computation, at least without a quantum computer, theoretical checks will necessarily be incomplete [46]. 1D systems are useful in this regard, since it has proven easier to develop computational tools in 1D that can handle correlated quantum systems. Theory/experiment cross-checks can be taken farther in 1D. Even so, exact dynamics in 1D can be solved numerically for only a limited time after a quantum quench, since the rapid growth of entanglement produces regimes where matrix-product-state based methods become exponentially costly [14, 47–52].

In further developing techniques to engineer many-body states in quantum simulators, control over non-equilibrium dynamics is also of key importance. For example, adiabatic preparation schemes using time-dependent control over system parameters provide a promising route toward engineering a range of complex many-body states [53–57]. It is also necessary to theoretically understand dissipative processes and their associated non-equilibrium dynamics, both to account for them in the comparison of theory to experiment and to develop ways to suppress them in experiments, as well as to generate new ways to cool quantum gases [58–60].

Within this broad field, the goal of this ‘focus on’ collection is to sample the state-of-the-art and emerging ideas related to non-equilibrium dynamics in one-dimensional many-body systems. The issue contains 17 contributions from leading groups in the field, ranging from new developments in numerical methods and analytical understanding of these systems to recent advances in experiments. Here, we summarize the topics and some of the main findings of the contributions to this issue.

### 2. Unique equilibrium and out-of-equilibrium properties of 1D systems

In equilibrium, 1D systems are known to exhibit bizarre properties [3]. For example, a 1D bosonic gas with contact interactions becomes more strongly correlated as the density of particles is reduced. In general, such strange behavior in 1D results from strong quantum fluctuations, which in 1D are generally stronger than in higher dimensions, the fact that (quasi-) particles can have statistics other than bosonic and fermionic, and the existence of bosonic Hamiltonians that can be mapped onto fermionic ones and vice versa. In addition, there are 1D models of correlated particles (such as bosons with contact interactions) and spins (such as the Heisenberg model) that have nontrivial sets of conserved quantities that make them integrable. In classical mechanics, integrable models contain as many constraints as degrees of freedom and, consequently, lack a property that is ubiquitous in nature, namely, ergodicity. Because of this, 1D systems can provide unique possibilities for charge and energy transport, as well as for quantum information processing and storage.

Motivated by those possibilities, there are several papers in this issue that deal with transport in 1D systems. Chancellor and Haas [61] explore how to use the $J_1$-$J_2$ Heisenberg spin chain to transport qubit states adiabatically. Temme, Wolf and Verstraete [62] look at the interplay between stochastic exclusion processes and coherent transport. Their study is based on
a master equation, which allows for a classical stochastic hopping process as well as coherent tunneling. Pletyukov and Gritsev [63] study transport in the context of a chiral 1D channel. They derive a general formalism to describe the propagation of multi-particle wave packets in such a multimode chiral channel coupled to an ensemble of emitters at arbitrary positions along the channel.

One-dimensional systems also provide an excellent context for studying the behavior of few interacting particles, or gases of complex pairs. Counterintuitive results on the stability of repulsively bound atom pairs in an optical lattice were first measured by Winker et al [64] and the stability of such bound doublons was later investigated by Strohmaier et al [65]. In this ‘focus on’ collection, Santos and Dykman [66] study the stability of doublon pairs in a Bose–Hubbard system and also of neighboring excited spins in an anisotropic Heisenberg spin-1/2 chain. They investigate in detail the role of quantum interference in suppressing the decay of such bound pairs due to scattering by collisions with other pairs or through defects in the system. Also on this theme, Kolovsky, Link and Wimberger [67] investigate the tunneling of energetically bound bosons through a barrier in an optical lattice, discussing the atomic cotunneling both within and beyond the regime where the pair can be understood as a point particle. 1D systems can also be a testbed for interesting polaron problems, in which a small number of impurity atoms interact with a larger reservoir of a different species. In this collection, Massel et al [68] investigate single trapped impurity atoms in a bath of fermionic or bosonic atoms in an optical lattice.

3. Quenches and thermalization in far-from-equilibrium 1D systems

Given their properties in equilibrium or near equilibrium it is no surprise that, in recent years in which there has been increasing interest in understanding many-body quantum systems far from equilibrium [1, 2], 1D systems have been intensively studied. Experimentally, 1D dynamics can emerge in multiple settings like, for example, in non-equilibrium edge-channel spectroscopy of systems in the integer quantum Hall regime [69]. Motivated by those experiments, in this collection Karzig et al [70] study how three-body collisions lead to energy relaxation of particles injected into quantum Hall edges.

The suppression of thermalization that has been observed in 1D gases [15] makes them a natural arena in which to study prethermalization. This was recently done with split 1D Bose gases on atom chips [19], and in this collection Adu Smith et al [71] elaborate on the experimental evidence for prethermalization in that system. The observable of choice in those experiments is the full quantum mechanical probability distribution function of the matter wave interference. This observable is also studied theoretically by the authors within the Tomonaga–Luttinger model, and they obtain excellent agreement between their theoretical and experimental results.

The Tomonaga–Luttinger model is an integrable model that provides an effective low-energy description of 1D gapless systems [3]. Soon after 1D bosonic systems with hard-core interactions were shown not to thermalize after a quantum quench [16], similar results were found for the Tomonaga–Luttinger model [72]. In a quantum quench, the system is initially taken to be in a stationary state of a given Hamiltonian (usually in the ground state) and then a (some) parameter(s) of the Hamiltonian is(are) suddenly changed so that the time evolution is driven by the new Hamiltonian. Reporting in this collection, Rentrop et al [73] study quenches
in the Tomonaga–Luttinger model with momentum-dependent two-particle potentials. They find that there are nonthermal properties of the fermionic momentum distribution function at long times after the quench that are universal. Furthermore, they verified that the one-particle Green’s function at long times can be computed using a GGE.

As alluded to above, the model that describes one-dimensional bosons with contact interactions, the Lieb–Liniger model [74], is also integrable. Quenches in which the interaction in the Lieb–Liniger is turned off are studied by Mossel and Caux in [75]. Mossel and Caux compute the time averages of observables after the quench and show that they are well described by the GGE. For small numbers of particles, for which the fluctuations of the particle number in the GGE are large while the systems after the quench have a fixed particle number, the authors find that the GGE does not provide accurate estimates of the time average of observables. A generalized canonical ensemble (similar to the GGE but with a fixed number of particles [76]), on the other hand, does provide a very accurate description of the time average of the observables.

While quenches in integrable systems are not expected to lead to thermalization in general, it is not precluded that specific experimentally relevant initial states can produce nearly thermal correlations at long times after a quench within an integrable model. As a matter of fact, specific kinds of initial states have been shown to exhibit this behavior, though, admittedly, the effective temperatures after the quench appear to be very large or infinite for this to occur [76–78]. In [79], Chung et al show that specific types of bipartite quantum entanglement and a gap in the spectrum of the initial Hamiltonian (the case in [76, 77]) lead to a GGE that can be arbitrarily close to the Gibbs ensemble. They also show that the effective temperature in the latter can be related to the entanglement spectrum. Hence, experimental measurements of the effective temperature could be used to characterize entanglement in (nearly) integrable systems.

Thermalization in isolated generic (nonintegrable) systems after a quench has attracted major attention in the field of nonequilibrium quantum dynamics. Early studies suggested that even in the absence of integrability thermalization does not occur [80, 81]. However, later studies found indications that thermalization does occur away from integrability [7] and that it can be understood to be a result of eigenstate thermalization [7, 82–84]. Systematic studies since then have looked at how the transition between integrability and nonintegrability occurs and how it affects the outcome of the relaxation dynamics in isolated quantum systems [8, 9, 18, 85–90]. The transition between integrability and nonintegrability, and its relation to thermalization, has been related to a localization–delocalization transition in Fock space [90] or in quasi-particle space [91]. In this collection, Canovi et al [92] study the latter scenario in detail, and relate it to the behavior of observables and their full probability distribution function (also studied in the experiments in [19, 71]) as one breaks integrability.

4. Developments in numerical methods

As was noted in the introduction, there have been very significant developments in recent years regarding numerical approaches for treating out-of-equilibrium dynamics in 1D systems. The advent of methods based on matrix product states, which led to t-DMRG and related techniques [33–39] have opened new ways in which to access and understand non-equilibrium dynamics. The matrix product state ansatz on which they are based is efficient provided that the many-body state is in some sense weakly entangled in space. In practice, these methods work well
over long timescales provided that the system has a 1D character (e.g., local or rapidly decaying interactions in a 1D geometry), and that the dynamics are not too far from equilibrium. However, trying to compute a general evolution where the initial state is not an eigenstate or close to an eigenstate tends to lead to a rapid growth of entanglement, as is typical in a quantum quench [11, 48, 49, 93, 94]. This results in a situation where the method only faithfully represents the dynamics over relatively short timescales, and afterward becomes exponentially costly in time [47–51].

In this context, advances that allow for an extension of the timescales over which these methods can be applied, or optimized algorithms for particular classes of problems, can lead to a substantial extension of the physics that can be captured and understood by these methods. In this collection, two articles present important steps in this direction. Müller-Hermes, Cirac and Bañuls [95] analyze a new folding algorithm that can be used to compute the dynamics of infinite spin chains, and demonstrate the advantages over previous algorithms in various regimes. This analysis extends to time evolutions, and also the computation of ground and thermal states. Wall and Carr [96] present two new algorithms operating on matrix product states, and based on the philosophy of matrix product operators methods [39–41]. These algorithms are designed to find excited states of generic 1D Hamiltonians, or to improve computation of the time evolution for a generic time-dependent 1D Hamiltonian.

5. Weakly interacting systems and c-field methods

The study of non-equilibrium dynamics in cold atom systems spans the wide range from weakly-interacting systems, which are amenable to mean-field descriptions, to strongly-interacting systems, where newer theoretical tools are needed. The possibility in experiments with cold atoms to tune between these regimes by varying the lattice geometry and/or the use of Feshbach resonances leads to a vast variety of phenomena, and there seems to be no part of the strong to weak coupling space, from 1D to 3D, that is not amenable to experimental and theoretical study with the broad range of tools available.

On the more weakly interacting side, there have been many theoretical developments in recent years, especially originating from classical field methods [97]. Within this broad space, there are three articles in this collection that represent more weakly interacting physics. Two of these deal with out-of-equilibrium physics in two-component Bose–Einstein condensates: Vidanović van Druten and Haque [98] study spin instabilities and phase separation dynamics in such systems, while Sabbatini, Zurek and Davis [99] study the formation of defects through a phase transition engineered in such a binary Bose–Einstein condensate. Schmidt et al [100] investigate single-particle momentum spectra as solitary waves evolve in a 1D gas, and open new perspectives by relating these dynamics to critical phenomena far from thermal equilibrium.

6. Summary and outlook

As briefly summarized in this editorial, this ‘focus on’ collection gives many examples of the current state-of-the-art in research of non-equilibrium dynamics in 1D systems. This is a very exciting time, promising new understanding of open fundamental physics questions such as thermalization, as well as the possibility to explore many novel phenomena.
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