Estimating the viscoelastic properties of local hot-mix asphalt

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Abstract. The mechanistic empirical method for designing flexible pavement requires determination of the strain that results from applying wheel load. To achieve this, the viscoelastic properties of asphalt concrete is required to be characterised numerically in order to permit simulating such asphalt concrete in finite element modelling for layered systems of flexible pavement. Accordingly, the estimation of the viscoelastic properties of local hot mix asphalt (HMA) was the target of this study. This was achieved using the interconversion process, with the master curve of shear modulus estimated from the master curves of dynamic modulus and phase angle developed within the Hirsch model by substituting the binder dynamic and volumetric properties of local Hot-Mix Asphalt (HMA) into the model. The master curve of shear modulus was constructed with aid of the interconversion process from the creep compliance resulting from Indirect Tensile Tests (ITT) under different testing conditions. The calculated master curve of shear modulus was then used to validate the estimated results. The validation revealed that the estimation method offered good ability to determine the properties required. Numeric values, known as the Prony series, used to characterise asphaltic pavement layers numerically, were established by fitting the normalised master curve of shear modulus. The research results showed that this estimation method is an appropriate way to determine the viscoelastic properties of HMA as opposed to using costly and complex laboratory tests.

Key words
Hot Mix Asphalt, viscoelastic, Hirsch model, dynamic modulus, phase angle, storage modulus, Prony series, relaxation modulus, shear modulus, creep compliance.

1. Introduction
Much interest has arisen in the study of the stresses and strains on, and related distresses developed in, flexible pavement due to applied wheel loads [1, 2], and recent development in hardware and software has boosted the use of numerical approaches to simulating flexible pavement layers subjected to wheel loads [3]. During the last few decades, the use of finite element analysis has thus helped researchers in understanding flexible pavement behaviour by allowing simulation of layer responses when subjected to various types of applied loads [4].

The complexity and difficulty of simulating the multiple different variables related to flexible pavement design has, however, led to the continuation of the use of empirical design methods, which depend on statistical models to correlate pavement thickness and the number of passes of standard axle load a pavement section can handle during its service life. These design methods have succeeded in predicting pavement capacity generally, but lack high levels of accuracy [5, 6 and 7]. The search for more accuracy has led to a shift towards mechanistic-empirical methods. This approach seeks links between the strain developed in flexible sections and the number of standard wheel load passes required to make the pavement section reach its failure limit [8 and 9].
Simulating flexible pavement layers numerically requires an understanding of the behaviours of hot mix asphalt (HMA) subjected to wheel loads under different conditions. The presence of asphalt makes this simulation more difficult, as it is a viscoelastic material affected by both temperature and loading duration. Accordingly, it has been deemed important to focus on determining the numeric values known as the Prony series to fit the normalised master curve of the shear modulus using certain formulae to characterise the viscoelastic properties of HMA tested under different load frequencies and testing temperatures [10]. Throughout this research, the normalised master curve of shear modulus was estimated for local HMA by using the Hirsch model [11] and an interconversion process [12]. This was validated by comparing the results with those resulting from altering the relaxation modulus calculated from creep compliance determined during Indirect Tensile Test (ITT) [13] carried out on the cylindrical specimens of local HMA at different deformation rates and testing temperatures.

The results of this research suggest that it is useful to use HMA volumetric properties with binder dynamic characteristics to estimate the viscoelastic properties of HMA rather than using costly and time consuming laboratory tests.

2. Background

Many studies have been carried out to characterise the viscoelastic properties of HMA [14], which reflects the need to simulate flexible pavement and analyse its response under applied wheel loads. The viscoelastic properties of HMA are used to express the time-temperature dependency of the stress-strain relations represented by different types of modulus within the time or frequency domains at different testing temperatures [15].

2.1 Dynamic or complex modulus and creep compliance

Dynamic modulus is the ratio of peak stress to the corresponding peak strain, lagged by phase angle (Ø), [16], which results from the applied sinusoidal wave of the repeated load on the sample of HMA as shown in Figure (1, A). The rheological nature of the asphaltic material affects the response of the HMA subjected to the repeated load. There are two main parts to the response to the applied load: (1) the in-phase elastic (storage) component, which represents the storage energy used to recover the original shape after the applied load is removed; and (2) the out-of-phase viscous (loss) component, which represents the energy dissipated as heat. These components are represented in the literature as horizontal (storage moduli) and vertical (loss moduli) projections of the dynamic (complex) modulus vector, as shown in Figure (1, B). The ratio of the loss modulus to the storage modulus is the tangent value of the phase angle, which represents the damping value (resistance to recovering the original shape) and explains the response and elastic and plastic strain of the HMA sample subjected to repeated loads, as shown in Figure (1,C).
Figure 1: (A) Stress strain waves resulting from the repeated load test, (B) Complex modulus components, and (C) HMA sample response subjected to repeated load [16].

The Creep compliance is the ratio of strain to the corresponding axial stress developed due to the application of constant stress, as in the creep test, or constant strain, as in the ITT test, to HMA samples [17].

2.2 Inter conversion process
This process is used to determine the relaxation modulus from either the creep compliance or dynamic modulus.

2.2.1 Relaxation modulus from creep compliance
The creep compliance can be converted to the relaxation modulus using a variety of formulas. One of the simplest ways is to use equations 1-3 presented below [10].

\[ \int_0^t E(t - \tau) D(\tau) d\tau = t \quad \text{for } t > 0 \]  
\[ E(t)D(t) = \frac{\sin \pi n}{\pi n} \]  
\[ n = \left| \frac{d \log D(t)}{d \log t} \right| \]

where
E(t) = Relaxation modulus and

\[ \int_0^t E(t - \tau) D(\tau) d\tau = t \quad \text{for } t > 0 \]  
\[ E(t)D(t) = \frac{\sin \pi n}{\pi n} \]  
\[ n = \left| \frac{d \log D(t)}{d \log t} \right| \]
$$D(t) = \text{creep compliance}$$

2.2.2 Relaxation modulus from dynamic modulus

The storage modulus (horizontal component of the dynamic modulus) master curve is fitted with numeric values known as Prony parameters that are used to create the relaxation modulus master curve as explained below:

2.2.3 Prony parameters

These parameters are numerical values used with formula 4 to fit the master curves of storage modulus. These parameters are used to fit the storage modulus $E'(\omega)$ master curve as a function of angular frequency ($\omega$). The form of this formula is thus [12]

$$E'(\omega) = Ee + \sum_{i=1}^{n} \frac{\omega^2 \rho_i^2 E_i}{\omega^2 \rho_i^2 + 1} \quad (4)$$

where

$Ee$ = equilibrium modulus,

$E_i$ = relaxation strengths, and

$\rho_i$ = relaxation times.

2.3 Shear modulus from relaxation modulus

The relaxation modulus can be converted to the shear modulus using equation 5, which correlates the relaxation modulus and the shear modulus using Poisson’s ratio [10].

$$G(t) = \frac{E(t)}{2(1+\mu)} \quad (5)$$

where

$G(t)$ = Shear modulus,

$E(t)$ = Relaxation modulus, and

$\mu$ = Poisson’s ratio.

2.4 Prony series to fit the master curve of shear modulus

This series is used with the formula presented in equation 6 to fit the master curve of normalised shear modulus $NG(t)$ as a function of time ($t$) [18].

$$NG(t) = 1 - \sum_{i=1}^{n} Gi(1 - e^{-\frac{t}{\tau_i}}) \quad (6)$$

where

$Gi$ = material constants and

$\tau_i$ = retardation times

3. Methodology

The methodology adopted in this paper (shown in Figure 2) can be summarised as follows:

1. Estimate the shear modulus master curve from the dynamic modulus,
2. Calculate the shear modulus master curve from the creep compliance,
3. Validate and normalise the estimated shear modulus master curve based on the calculated one,
4. Find the viscoelastic properties of HMA represented by the Prony series that fits the normalised master curve of shear modulus.
Estimate the dynamic modulus $IE^*I$ and phase angle ($\phi$) using Hirsch model with an input of local HMA volumetric properties and binder dynamic characteristics resulting from Cox-Mirtz rule.

Carry out the ITT test to the specimens made with local HMA requirements at:
- Different testing temperature
- Different deformation rates

Establish the Prony series
Fitted the normalized shear modulus with established Prony series

Validate the estimated shear modulus by comparing its normalized master curve with that of calculated one

Figure 2: Methodology flow chart.
4. Estimated Shear modulus master curve

With aid of the interconversion process, the master curve of shear modulus was estimated from the estimated master curve of dynamic modulus as presented below.

4.1 Dynamic modulus master curve

Many models have been developed to estimate the dynamic modulus for HMA based on binder and mix volumetric properties; of these, the Witczak and Hirsch models are the most commonly used. The Witczak model and its modifications were adopted by NCHRP report 1-40D as a mechanistic empirical method, as reported by [19], while the Hirsch model is built on a limited data range compared to the Witczak model [19] but can be used to estimate the phase angle in addition to the HMA dynamic modulus required to estimate the storage modulus. The Hirsch model (equations 7 to 11) [20] requires the shear modulus and phase angle for the asphalt binder as input; thus, the Cox-Mertz model (equations 12 to 15) [20] was used to determine these binder properties.

\[
\delta_b = 90 - 0.1785 \times \log\left(\eta_{fs,T}\right)^{2.3814} \times \left(f_s\right)^{0.3507 - 0.0782VTS} \\
\left| G'_b \right| = 1.469 \times 10^{-9} \times \log\log\left(\eta_{fs,T}\right)^{12.0056} \times \left(\sin\delta_b\right)^{0.6806}
\]

\[
\left| E' \right|_m = P_c \left[ 4,200,000 \left(1 - \frac{VMA}{100}\right) + \left| G'_b \right| \left(\frac{VFA+VMA}{10,000}\right) \right] + \frac{(1-P_c)}{4,200,000 + 3\left| G'_b \right| \left(VFA\right)}
\]

\[
\phi = -2\left(\log P_c\right)^2 - 55 \log P_c
\]

\[
P_c = \left(\frac{20+3 \left| G'_b \right| \left(VFA\right)/\left(VMA\right)}{650+3 \left| G'_b \right| \left(VFA\right)/\left(VMA\right)}\right)^{0.58}
\]

\[
\log\log \eta_{fs,T} = A' + VTS' \log T_R
\]

\[
A' = 0.9699 \times f_s^{-0.0527} \times A
\]

\[
VTS' = 0.9668 \times f_s^{-0.0575} \times VTS
\]

\[
f_s = f_c/2\pi
\]

where

\[\eta = \text{Viscosity (cP)}\]

\[f = \text{Frequency (Hertz)}\]

\[f_c = \text{Frequency under axial compression}\]

\[f_s = \text{Dynamic shear frequency}\]

\[\left| E' \right|_m = \text{Dynamic modulus of HMA (psi)}\]

\[\phi = \text{Phase angle of HMA (degree)}\]

\[P_c = \text{Aggregate volume contact}\]

\[A = \text{Intercept of temperature susceptibility relationship}\]

\[VTS = \text{Slope of temperature susceptibility relationship}\]

\[TR = \text{Temperature in Rankine}\]

4.2 Sigmoidal formula

As reported by [17], the AASHTO guide 2002 presented the mathematical forms to be used to construct the master curve of HMA moduli resulting from tests at different testing temperatures and applied load durations. Equations 16 to 18 [12] show the forms of this formula
\[
\log\left(\frac{|E^*|}{\gamma f} \right) = \delta + \frac{\alpha}{1 + e^{\beta + \gamma \log(f_r)}} 
\]

(16)

\[
\phi(f_r) = \xi_1 + \xi_2 \log f_r - \xi_3 \pi \frac{\alpha y}{2} e^{\beta + \gamma \log(f_r)^2}
\]

(17)

\[
\log [aT] = a T_i^2 + b T_i + c
\]

(18)

where

\[ \delta, \alpha, \beta, \gamma, \xi_{1,2,3}, a, b, c = \text{regression constants and} \]
\[ aT, T_i = \text{shift factor, testing temperature at } i \text{ degree centigrade.} \]

4.3 Local HMA Properties

The term local refers to the materials and mix properties used in the middle of Iraq, especially in Baghdad city. Tables 1 to 3 are devoted to presenting these HMA material properties.

**Table 1**: Physical properties of aggregates

| Property                                | Designation | Aggregate |
|-----------------------------------------|-------------|-----------|
| Bulk specific gravity                   |             |           |
|                                        |             | Course    |
|                                        |             | Fine      |
| Apparent specific gravity               | C127-04     | 2.610     |
|                                        | C128-01     | 2.690     |
| Percentage of water absorption          |             | 0.464     |
|                                        |             | 0.715     |
| Percentage of wear (Los Angeles abrasion)| 35-45 Max   | C131-01   |
|                                        |             | 22.00     |
| Soundness loss by sodium sulphate solution,% | 10-20 Max   | C88-05    |
|                                        |             | 3.55      |
| Fractured pieces, %                     | 95 Min.     | -----     |
|                                        |             | 96        |
| Sand equivalent, %                      | 45 Min.     | D2419-02  |
|                                        |             | -----     |
|                                        |             | 53        |

**Table 2**: Aggregate gradation

| Sieve size (inch) | 1/2 | 3/8 | No.4 | No.8 | No.50 | No.200 |
|-------------------|-----|-----|------|------|-------|--------|
| Local limits [27] (SCRB, 2003) | 90-100 | 76-90 | 44-74 | 28-58 | 5-21 | 4-10   |
| % Passing         | 95  | 83  | 59   | 43   | 13   | 7      |

**Table 3**: Filler properties.

| Property               | Result |
|------------------------|--------|
| Specific Gravity       | 2.720  |
| Passing Sieve No.200 (0.075 mm) | 95%    |
The local asphalt binder was classified by [28] based on testing of a sample taken from Durah refinery with penetration grade of 40 to 50; the performance grade of this binder was 64 to 16, with the other properties as shown in Table 4 [28].

Table 5 shows the volumetric properties of HMA used in surface courses as required by local standards R9/5 [27]. These properties were used as input for the Hirsch model, with the upper value of air voids used to match the limitations and requirements of the model.

Table 4: Local asphalt cement properties [28].

| Test                                      | Result                      |
|-------------------------------------------|-----------------------------|
| Rotational Viscosity Pa.sec @135°C         | 0.516                       |
| G*/sin δ, kPa @64°C                       | 2.37 (2.17)*                |
| G*/sin δ, kPa @70°C                       | 0.958 (1.06)                |
| True Grade 69.83°C                        | 1                           |
| Penetration @25°C                         | 45                          |
| Softening Point                           | 48°C                        |
| Rotational Thin Film Oven (RTFO) G*/sin δ, kPa @64°C | 4.05 (4.04)                |
| G*/sin δ, kPa @70°C                       | 1.887 (1.72)                |
| True Grade 69.13°C                        | 2.2                         |
| Loss (%) 0.73                             | <1                          |
| Penetration @25°C                         | 29                          |
| Softening Point                           | 52°C                        |
| Pressure Aging Vassal (PAV) Phase angle δ | @25°C 53°(63°)              |
| m-value @25°C                             | 0.525 (0.638)               |
| m-value @28°C                             | 0.578 (0.692)               |
| G*/sin δ, kPa @25°C                       | 7320 (22453)                |
| G*/sin δ, kPa @28°C                       | 4700 (12334)                |
| Creep Stiffness, mPa @-16°C               | 182 (200)                   |
| Creep Stiffness, mPa @-22°C               | 426 (490)                   |
| True Grade -18.9°C                        | 300                         |
| Slope m-value @-16°C                      | 0.399 (0.477)               |
| Slope m-value @-22°C                      | 0.289 (0.269)               |
| True Grade -21.4°C                        | 0.3                         |
| Penetration @25°C                         | 20                          |
| Softening Point                           | 57°C                        |

Measured properties (estimated properties using BANDS 2 Software)’

Table 5: Volumetric properties of local HMA for surface courses according to local standards SCRB R9/5 [27] as used in the Hirsch model.

| Mix volumetric property                  | Value          |
|------------------------------------------|----------------|
| % Air voids (%AV)                       | 3.0 – 5.0 %    |
| % Voids Filled with Asphalt (%VFA)       | More than 70 % |
| % Voids in Mineral Aggregates (%VMA)     | More than 14 % |
4.4 Estimating the dynamic modulus master curve

For PG 64-16, [19], the values for A and VTS to be substituted in the Cox-Martin rule, which were 11.375 and -3.822 respectively, were used in addition to other required inputs of the Hirsch model to estimate the dynamic modulus. Consequently, the master curve was created with the aid of the sigmoidal formula presented in Figure 3, at testing temperatures of 5, 15, 25, and 38 °C, and applied frequencies of 0.1, 0.5, 1, 1.25, 2.5, and 5 Hz.

Figure 3: Estimated dynamic and phase angle master curve.

4.5 Estimated Relaxation modulus master curve

By using equation 4, the master curve of relaxation modulus ($E(t)$) was created, as shown in figure 4, using the Prony parameters ($E_i, P_i$) to fit the master curve of storage modulus (the horizontal component of dynamic modulus).

Figure 4: The master curve of relaxation modulus.
4.6 Shear modulus master curve

The master curve of shear modulus was created by substituting the relaxation modulus in equation 5 and assuming a value of 0.35 for Poisson’s ratio. The master curve of estimated shear modulus was normalised, then validated by comparing it with the calculated one in order to identify the Prony series required to characterize the viscoelastic properties of local HMA, as shown in figure 7.

5. Calculated shear modulus

5.1 Experimental work

Samples of 6” (150mm) diameter gyratory compacted local HMA of wearing layer specification with ½” maximum nominal size were prepared as required by [29], using the N\text{design} of 100 gyrations of 600 kPa (87 psi) to reach the upper limit of air voids, as required by [27]. Specimens of 2” (50mm) thickness were tested using the Indirect Tensile Test (ITT) based on ASSHTO T322-07 and British standards BS EN 12697-23-2003. The ITT was carried out as shown in figure 5 with different deformation rates (0.01, 0.03, and 0.05 in/min; 0.25, 0.75, 1.25 mm/min) for testing temperatures of 30, 20, and 10 °C respectively. A fully closed chamber with a double glazed door was used to maintain the testing temperature during the test. A pair of linear variable differential transducers (LVDTs) with a length of 3” (75mm) was used to measure the deformation at the core of each tested specimen. Three samples were tested per specimen, and the average value of the results was adopted, with outliers omitted. The results of the tests are shown in Figure 6 in terms of the relationship between the horizontal deformation at the core of the sample and reaction load measured by a load cell of 75kN capacity as recorded by cameras facing the load cell and LVDT displays.

![Figure 5: ITT test with different deformation rates and testing temperatures.](image-url)
Figure 6: ITT results for different deformation rates and testing temperatures.

5.2 Creep compliance master curve

As [13] used the linear viscoelastic solution to produce the following formulas 19 and 20 for calculating the Poisson’s ratio and the creep compliance, a Poisson’s assumed value of 0.35 was determined by using these formulas, and the creep compliance was calculated for the elastic part (early stage of the test); the master curve of creep compliance was also developed as shown in Figure 7.

\[ \mu = -\frac{\alpha_1 U(t) + V(t)}{\alpha_2 U(t) + \alpha_3 V(t)} \]  \hspace{1cm} (19)

\[ D(t) = -\frac{d}{p} \left[ \beta_1 U(t) + \beta_2 V(t) \right] \]  \hspace{1cm} (20)

where

\( \mu \) = Poisson’s ratio,

\( D(t) \) = Creep compliance 1/GPa,

\( U(t) \) = horizontal displacement in m,

\( V(t) \) = vertical displacement in m,

\( d \) = specimen thickness in m,

\( p \) = applied load in N, and

\( \alpha_1 = 4.59, \alpha_2 = 1.33, \alpha_3 = 3.311, \beta_1 = 0.415, \) and \( \beta_2 = 1.034 \) (\( D = 150 \text{ mm} \) and gauge length = 76.2 mm).

Figure 7: Creep compliance master curve.
5.3 Shear modulus from Creep compliance

The relaxation modulus was calculated from the creep compliance using equations 1 to 3, and the master curve was created as shown in figure 8; this was converted to shear modulus by using equation 5 and an assumed value of 0.35 for the Poisson’s ratio. The created master curve of shear modulus was normalised to validate the estimated one, as shown in Figure 8.

![Figure 8: Relaxation modulus master curve.](image)

6. Normalised Shear modulus and Prony series

As presented earlier, the estimated and calculated master curve of shear modulus were normalised and drawn on the same graph using both normal and logarithmic scales to validate the estimated master curve, as shown in Figure 9. A Prony series as presented in Table 6 was established to fit the normalised master curve by using the formula in equation 6, as presented in Figure 10. To ensure better fitting with the established Prony series, the normalised master curve of the shear modulus was drawn with both normal and logarithmic scales versus log-reduced time. Hence, Figure 10 shows how the established Prony series is fitted with the master curve of the shear modulus.

![Figure 9: Normalised shear modulus master curve.](image)
Table 6: Prony series established to fit the master curve of the normalised shear modulus.

| $i$ | $G_i$  | $\tau_i$ |
|-----|--------|----------|
| 1   | 1.06E-01 | 5.00E-07 |
| 2   | 1.33E-01 | 5.00E-06 |
| 3   | 1.72E-01 | 5.00E-05 |
| 4   | 2.06E-01 | 5.00E-04 |
| 5   | 1.68E-01 | 5.00E-03 |
| 6   | 1.14E-01 | 5.00E-02 |
| 7   | 5.76E-02 | 7.00E-01 |
| 8   | 2.27E-02 | 1.00E+01 |
| 9   | 1.51E-02 | 5.50E+02 |

Figure 10: Fitting the normalised master curve of the shear modulus with the established Prony series.

7. Discussion
The use of the Hirsch model and the interconversion process offer good capability to estimate the relaxation modulus that can be used to calculate the shear modulus after selecting a suitable value for the Poisson’s ratio. Due to the Hirsch model’s limitations with regard to testing temperatures ranging from 5 °C to 38 °C, the sigmoidal function was used to increase the range of estimated moduli and create a master curve with an extended domain. The Prony series that fits the normalised shear modulus curve was established after validating the shear modulus, and the validation made with the master curve was calculated from the creep compliance results for tested specimens made to local HMA requirements using the principles of the ITT with multiple deformation rates and testing temperatures. There was a slight difference between the estimated and calculated shear modulus, which may be due to the differences in volumetric properties between the specimens tested to generate the data for the Hirsch model and the specimens made with local properties for the purposes of this research. The latter samples were prepared to meet the limit of air voids required by local standards for HMA used in surface courses, while the specimens that were tested to drive the Hirsch model were prepared with air voids in the range of 5 to 11. The effects of the materials’ properties and testing conditions must also be considered. The Hirsch model needs to enlarge its data limitations to cover a wider range of testing conditions.
temperatures, mix properties, and binder grades to meet the properties of HMA made to the requirements of local standards. The outcomes of this research could nevertheless be very useful for evaluate and designing flexible pavement models that simulate the asphaltic layers in finite elements models to determine pavement response under different applied wheel loads and different environmental conditions. This study should be followed by more comprehensive studies to evaluate local HMA and to determine viscoelastic properties according to local standards and conditions.

8. Conclusion
The main points of this research are summarised as follows:

- The use of the Hirsch model and the interconversion process produces a good estimation of the shear modulus compared with the results of creep compliance and the corresponding calculated shear modulus obtained from the multi deformation rates of the ITT at different testing temperatures.
- It is useful to use sigmoidal formulas and shift factors to enlarge the domain of the master curve created with limited testing temperatures and load frequencies.
- The obtained sigmoidal functions and shape formula parameters can be used to classify the HMA based on dynamic or monotonic testing with different applied frequencies, deformation rates, or applied stresses at different testing temperatures.
- The methodology used in this paper can be extended to evaluate HMA or modified HMA behaviours within finite element models to study flexible pavement response under applied wheel loads by representing the required viscoelastic properties with a Prony series to model different flexible pavement asphaltic layers.

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