Nonlinear Distortion Correction of Camera Based on Straight Lines

Zhang Zhengyu, Huang Xuhui, Ying Jiang and Zhou Run

ABSTRACT

Nonlinear distortion correction of camera is used to promote the photogrammetric accuracy. According to the principle which a straight line in three-dimensional space projecting on the CCD plane of camera remains a straight line, a nonlinear distortion correction method based on straight lines is proposed. Experimental results have demonstrated that the nonlinear distortion coefficients are gradually converged to the correct result, with the increase of uniform distribution of the selected straight lines on the CCD. This method does not require the high-cost calibration plate, as well as the camera external parameters, only using the straight lines in the photos, and the time-consuming process of coupling optimization involving distortion coefficients and camera external parameters is avoided. Therefore it has value.

KEYWORDS

machine vision; photogrammetry; camera calibration; distortion correction

INSTRUCTIONS

The camera optical system is not an ideal pin-pole model, there existing problems like the lens distortion, distort the central of photography and collinear relationship between the measuring point, its imaging point and center of camera lens etc. The camera calibration can be used to get the distortion coefficients, and it is divided into the three kinds[1-13].

The first is based on collinear equation[1-3]. It needs high precision calibration object whose surface is printed many points whose accuracy 3D coordination is known to stablish the constraint relationship following collinear equation to solve the image distortion coefficients and external parameters of camera, but as the width of a calibration plate more than 400mm, its manufacturing and storage maintenance costs increase significantly. This kind of method includes Tsai's two-step calibration[7], the iterative method proposed by Weng[8] and Martin[9]'s double plane calibration[13].

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The second is based on coplanar equation\(^4\). It needs no calibration object, only by the coplanar relationship of homonymous points on different view images to establish the constraint relationship following coplanar equation, so it is applied widely. But it needs to solve nonlinear coplanar equations, and the results is therefore sensitive to initial values and noise, and lack of robustness\(^4,5,10,11,14\).

The third is called active vision\(^1,7\), which needs an active visual platform. The movement of camera can be accurately controlled on the platform, this method can obtain linear solution, but it must be equipped with a precise control of the camera motion platform.

Distortion correction method based on straight lines nowadays comes to a focus in the field of imaging measurement, which using the geometrical invariance of the straight lines’ projection \(^{15}\) without solving external parameters of camera and to obtaining distortion coefficients directly, therefore the time-consuming process of coupling optimization involving distortion coefficients and camera external parameters is avoided.

As a result, a nonlinear distortion correction method based on the straight lines for camera is presented in this paper. Following the principle that the projection of a 3D straight line on camera CCD is a 2D straight line or a point, the nonlinear equations containing distortion coefficients are built and the numerical calculation method are studied.

### SOLUTION OF NONLINEAR DISTORTION BASED ON STRAIGHT LINES

#### Preparing the New File With the Correct Template

Radial distortion\(^{13}\) causes the image point to move along the radial direction. The point on the CCD is farther to the center, its distortion is larger. A mathematical model\(^{6,7,13}\) of radial distortion without more than two order parts can be described as

\[
\begin{align*}
\delta x_r &= x[k_1(x^2 + y^2) + k_2(x^2 + y^2)^2] + \delta x_r \\
\delta y_r &= y[k_1(x^2 + y^2) + k_2(x^2 + y^2)^2] + \delta y_r
\end{align*}
\]  

(1)

where the \((x, y)\) is the image coordinates of the point on CCD, \(k_1\) and \(k_2\) are radial distortion coefficients. The eccentric distortion is composed of the radial deformation component and the tangential deformation component, and its mathematical model\(^{6,7,13}\) without more than two order parts is,

\[
\begin{align*}
\delta x_e &= p_1(x^2 + y^2) + 2p_2xy + \delta x_e \\
\delta y_e &= 2p_1xy + p_2(x^2 + 3y^2) + \delta y_e
\end{align*}
\]  

(2)

where \(p_1\) and \(p_2\) are eccentric distortion coefficients. The third sort of distortion is caused by manufacturing errors of optical lens and CCD sensor array, and its mathematical model\(^{6,7,13}\) without more than two order parts is,

\[
\begin{align*}
\delta x_p &= s_1(x^2 + y^2) + \delta x_p \\
\delta y_p &= s_2(x^2 + y^2) + \delta y_p
\end{align*}
\]  

(3)

where \(s_1\) and \(s_2\) are distortion coefficients of thin prism.

Given a given point \((x_n, y_n)\) on CCD, its correction point \((x_{u,n}, y_{u,n})\) can be calculated by
For the three different points $P_1(x_{u1}, y_{u1}, -f)$, $P_2(x_{u2}, y_{u2}, -f)$ and $P_3(x_{u3}, y_{u3}, -f)$ on a projection line of a 3D straight line, $P_1P_2 \times P_1P_3$ is

\[
\begin{vmatrix}
i & j & k \\
x_{u2} - x_{u1} & y_{u2} - y_{u1} & 0 \\
x_{u3} - x_{u1} & y_{u3} - y_{u1} & 0 \\
\end{vmatrix} = 0
\]  

The equation (5) can be described by 6 distortion coefficients substituting $(x_{u,n}, y_{u,n})$, $n=1,2,3$ using equation (1), equation (2), equation (3) and equation (4), that is

\[
P_1P_2 \times P_1P_3 = f_i(k_i, k_2, p_1, p_2, s_1, s_2) = 0, i = 0,1,..m - 1, m \geq n
\]  

where $m$ is sum of selected straight lines used to solve distortion coefficients. The generalized inverse method of least square solution is employed to compute $X = (k_i, k_2, p_1, p_2, s_1, s_2)$, and its Jacobian matrix is

\[
J = \begin{bmatrix}
\frac{\partial f_0}{\partial k_0} & \frac{\partial f_0}{\partial k_1} & \frac{\partial f_0}{\partial s_1} & \frac{\partial f_0}{\partial s_2} & \frac{\partial f_0}{\partial p_1} & \frac{\partial f_0}{\partial p_2} \\
\frac{\partial f_1}{\partial k_0} & \frac{\partial f_1}{\partial k_1} & \frac{\partial f_1}{\partial s_1} & \frac{\partial f_1}{\partial s_2} & \frac{\partial f_1}{\partial p_1} & \frac{\partial f_1}{\partial p_2} \\
\frac{\partial f_2}{\partial k_0} & \frac{\partial f_2}{\partial k_1} & \frac{\partial f_2}{\partial s_1} & \frac{\partial f_2}{\partial s_2} & \frac{\partial f_2}{\partial p_1} & \frac{\partial f_2}{\partial p_2} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial f_{m-1}}{\partial k_0} & \frac{\partial f_{m-1}}{\partial k_1} & \frac{\partial f_{m-1}}{\partial s_1} & \frac{\partial f_{m-1}}{\partial s_2} & \frac{\partial f_{m-1}}{\partial p_1} & \frac{\partial f_{m-1}}{\partial p_2} \\
\end{bmatrix}
\]  

where, Its iterative formula is

\[
X^{(k+1)} = X^{(k)} - \alpha_k Z^{(k)}
\]  

$Z^{(k)}$ is determined by $J(k) Z^{(k)} = F^{(k)}$, $J^{(k)}$ is Jacobian matrix of $X^{(k)}$,

\[
F^{(k)} = (f_0^{(k)}, f_1^{(k)}, \ldots, f_{m-1}^{(k)})^T
\]  

where $f_i^{(k)} = f_i[k_i, k_2, p_1, p_2, s_1, s_2], i = 0,1,\ldots m-1$, $\alpha_k$ is the point making

\[
\sum_{i=0}^{m-1} f_i^{(k+1)}
\]

get its minimum. The initial value of $X$ is zero, and then the 6 distortion coefficients are obtained by iteration.

EXPERIMENTS

The Nikon D80SLR (3872pixel×3872pixel) with Nikon 24 mm lens is used to take photos on which many straight lines are, and then the 6 nonlinear distortion coefficients are solved by the method presented in this paper.
1) Canny edge detection is used to obtain image points of a given straight line \( L_m \) \((m = 1, 2, \ldots, M)\), and the image points of \( L_m \) are fit to get straight line \( L_{fit_m} \) \((m = 1, 2, \ldots, M)\). Figure 1 (a) shows a photo containing 7 straight lines, on which the extracted straight lines whose color is blue shown as figure 1 (a), and the red one is the selected line whose enlarged view is shown as figure 1 (b).

2) To solve \( k_1, k_2, p_1, p_2, s_1 \) and \( s_2 \), and the corrected straight line \( L_{cor_m} \) of \( L_m \) is get by fit the corrected image points of \( L_m \) followed equation (4).

3) To observe the affections of sum of selected straight lines on the result of distortion correction and the distortion coefficients.

Table 1. Selected lines sum and the solved distortion coefficients.

| sum of lines | \( k_1 \)  | \( k_2 \)  | \( p_1 \)  | \( p_2 \)  | \( s_1 \)  | \( s_2 \)  |
|--------------|------------|------------|------------|------------|------------|------------|
| 6            | 2.01\times10^{-4} | -4.29\times10^{-7} | 4.23\times10^{-5} | 1.70\times10^{-6} | -1.22\times10^{-4} | -1.27\times10^{-4} |
| 7            | 1.20\times10^{-4} | -2.46\times10^{-7} | 6.45\times10^{-5} | 1.15\times10^{-4} | 1.70\times10^{-4} | -5.38\times10^{-4} |
| 8            | 5.58\times10^{-4} | -1.20\times10{-6} | -2.98\times10^{-4} | -4.45\times10^{-4} | -7.20\times10^{-4} | 2.17\times10^{-4} |
| 10           | 6.82\times10^{-5} | 3.01\times10^{-7}  | -9.25\times10^{-5} | 9.40\times10^{-6} | -5.31\times10^{-4} | 1.72\times10^{-4} |
| 11           | 6.26\times10^{-4} | -1.54\times10^{-7} | -2.02\times10^{-4} | -8.10\times10^{-4} | -1.97\times10^{-4} | 5.84\times10^{-4} |
| 12           | 1.73\times10^{-4} | -1.84\times10^{-7} | -8.92\times10^{-5} | -1.33\times10^{-5} | -7.87\times10^{-5} | 7.15\times10^{-5} |
| 13           | 1.62\times10^{-4} | -7.02\times10^{-7} | -2.43\times10^{-5} | -5.19\times10^{-5} | -1.11\times10^{-5} | -2.42\times10^{-5} |
| 14           | 1.65\times10^{-4} | -3.89\times10^{-7} | 3.99\times10^{-5} | -5.90\times10^{-5} | 2.52\times10^{-5} | -1.53\times10^{-5} |
| 15           | 1.40\times10^{-4} | -3.86\times10^{-7} | 6.55\times10^{-5} | -2.42\times10^{-5} | 5.05\times10^{-5} | -4.70\times10^{-5} |
| 16           | 1.10\times10^{-4} | -5.45\times10^{-7} | -1.25\times10^{-7} | 4.71\times10^{-7} | -4.91\times10^{-7} | 4.62\times10^{-5} |
| 17           | 1.18\times10^{-4} | -1.29\times10^{-7} | -7.15\times10^{-5} | -1.70\times10^{-7} | -8.51\times10^{-7} | 3.88\times10^{-5} |
| 18           | 1.15\times10^{-4} | -1.36\times10^{-7} | -4.01\times10^{-7} | -5.90\times10^{-8} | -4.92\times10^{-7} | -1.25\times10^{-5} |
| 19           | 1.13\times10^{-4} | -5.58\times10^{-8} | 1.67\times10^{-7} | -7.12\times10^{-7} | -5.10\times10^{-7} | -1.28\times10^{-7} |
| 20           | 1.23\times10^{-4} | -1.27\times10^{-8} | -6.76\times10^{-7} | 9.38\times10^{-7} | -6.55\times10^{-7} | -1.01\times10^{-5} |
| 21           | 1.43\times10^{-4} | -9.44\times10^{-9} | -3.88\times10^{-7} | 2.62\times10^{-7} | -4.34\times10^{-7} | -1.42\times10^{-5} |
| 22           | 1.09\times10^{-4} | -1.20\times10^{-7} | -4.03\times10^{-7} | 3.26\times10^{-7} | -3.20\times10^{-7} | -1.17\times10^{-5} |

The 16 groups of distortion coefficients are calculated shown as table 1. With the increase of straight lines used to solve the distortion coefficients, the variation trend of the 6 distortion coefficients is shown in Figure 2. When the sum of straight lines is arrange from 6 to 12, the distortion coefficients are not stable, the reasons are: 1) in the first 6 groups of experiments, the distribution of selected lines on the CCD is not uniform enough, therefore the distortion coefficients solved by the least squares method is only the solution in the mathematical sense, which cannot represent the lens distortion coefficients. With the increase of the selected lines, the distribution of the
Distortion straight lines on CCD becomes gradually uniform, thus the solved distortion coefficients are consistent with the lens distortion coefficients, so the final solution tends to be stable.

The straightness of $L_{cor_m}$ is used to evaluate the accuracy of distortion coefficients and the correction results. The standard deviations of distances from the image points to $L_{cor_m}$ (or $L_{fit_m}$) represent the straightness of $L_{cor_m}$ (or $L_{fit_m}$). The smaller the straightness is, the better correction results are. The calculated straightness for each experiment and its straightness tendency chart shows in figure 3, in which the straightness of $L_{cor_m}$ are obviously reduced, which shows the effect of distortion correction presented by this paper is obvious. The maximum distances from the image points to $L_{cor_m}$ (or $L_{fit_m}$) for each experiment are computed, and figure 4 is the tendency chart between the maximum distance and the sum of selected lines, where we can see the maximum distance is 1.485 pixel without correction, which reflects the maximum distortion value of these experiments. After correction, the maximum distance is reduced to 0.418 pixel, which shows the effect of distortion correction in this paper is remarkable.

This method is also used to get distortion coefficients of Daheng MER-200-20UC with the 25 mm fixed focus lens. The relationships between sum of selected lines and the distortion coefficients are shown in figure 5. The distortions with and without correction are shown in figure 6.
CONCLUSIONS

A nonlinear distortion correction method based on straight lines is established. Experimental results have shown that this method does not need high precision calibration board (or moving platform), expensive manufacturing and maintenance costs, and the time-consuming process of coupling optimization involving distortion coefficients and camera external parameters is avoided, only using the straight lines on the images to obtain the nonlinear distortion coefficients and then achieve image distortion correction, so it has certain practical value.

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