Quantum Lax Pair From Yang-Baxter Equations

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Abstract

We show explicitly how to construct the quantum Lax pair for systems with open boundary conditions. We demonstrate the method by applying it to the Heisenberg XXZ model with the general integrable boundary terms.

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Ever since the introduction of the quantum Lax pair into statistical mechanics by McCoy and Wu [1] and of the notion of a one-parameter family of commuting transfer matrices by Baxter [2], a great deal of effort has been expended in the search for spin chains with Hamiltonian $\mathcal{H}$ and two-dimensional statistical models with transfer matrix $\tau$ which have the integrability property.

The quantum inverse scattering method places the theory of completely integrable quantum system and solvable statistical models in a unified framework [3]. The basis to apply it to a completely integrable system is to associate an operator version of an auxiliary linear problem [4]:

$$\Psi_{n+1} = L_n \Psi_n, \quad \Psi_n = A_n \Psi_n, \quad (1)$$

where $L_n$ and $A_n$ are matrix operators depending on the spectral parameter $u$, and a dot signifies a time derivative. The consistency condition for equations (1) with $u=0$ yields the Lax pair equation:

$$\dot{L}_n = A_{n+1}L_n - L_nA_n. \quad (2)$$

All the solved integrable models appear to imply that a model is completely integrable if we can find a Lax pair $\{L_n, A_n\}$ such that the Lax equation (2) is equivalent to the equation of motion of the model [4].

Starting with a local Lax operator $L_n$ (a matrix acting in the auxiliary space $V$ with elements acting in the quantum space $h_n$, at site $n$), for most quantum integrable system the product direct of two Lax operators $L_n$, with different spectral parameters satisfy a similarity relation

$$\mathcal{R}(u-v)L_n(u) \otimes L_n(v) = L_n(v) \otimes L_n(u)\mathcal{R}(u-v) \quad (3)$$

with $\mathcal{R}$ a $c$-number matrix acting in $V \otimes V$. The above relation is referred to as the local Yang-Baxter relation and can be represented graphically by figure 1.
In terms of the operators $L_n$, the monodromy matrix for a chain with length $N$ is expressed as

$$T(u) = L_N(u)L_{N-1}(u) \cdots L_2(u)L_1(u)$$

and if $L'_n$s with different $n$ commute, we further have

$$\mathcal{R}(u - v)T(u) \otimes T(v) = T(v) \otimes T(u)\mathcal{R}(u - v)$$

called global Yang-Baxter relation.

We assume the auxiliary space $V$ and the quantum space $\mathcal{h}$ are the same and regard the elements of $L$ and $\mathcal{R}$ as those of the $R$-matrix ($\mathcal{R} = \mathcal{P}R$ with $\mathcal{P}$ the permutation operator which interchanges spaces of $V_n$ with $V_{n-1}$), in terms of which the local Yang-Baxter relation (3) has the form

$$R_{12}(u - v)R_{13}(u)R_{23}(v) = R_{23}(v)R_{13}(u)R_{12}(u - v),$$

where $R_{ij}$ is a matrix in $V \times V \times V$, which acts non-trivially in the spaces $V_i$ and $V_j$ only.

In two-dimensional statistical mechanics, the elements of $R$ may be considered as vertices of a vertex model. Baxter first noticed the importance of the relation (3) in that context and regarded it as the solvability condition of the vertex model [2]: we introduce the transfer matrix $\tau(u)$ as a trace of the monodromy matrix $\tau(u) = \text{tr} T(u)$. The relation (3) indicates that there exists a family of commuting transfer matrices and that $u$-expansion of $\tau(u)$ gives a set of conserved quantities which are involutive.

The integrability conditions for a system with open boundary condition are formulated in order that both the Yang-Baxter equation and the boundary Yang-Baxter equations (or reflection equations) are satisfied [3]. To a quantum system on a finite interval with independent boundary conditions at each end, we have to introduce reflection matrices $K^{\pm}(u)$ to describe such boundary conditions. For a $PT$-invariant $R$-matrix, the fundamental reflection-factorization relations obeyed by $K^{-}(u)$ and $K^{+}(u)$ are [3]:

$$R_{12}(u - v)K^{-}_1(u)R_{21}(u + v)K^{-}_2(v) = K^{-}_2(v)R_{12}(u + v)K^{-}_1(u)R_{21}(u - v)$$

which is represented graphically by figure 2.
For the left boundary we have a similar equation:

\[ R_{12}(-u + v)K_1^+(u)t_1 M_1^{-1}R_{21}(-u - v - 2\rho)M_1 K_2^+(v)t_2 = K_2^+(v)t_2 M_1 R_{12}(-u - v - 2\rho)M_1^{-1}K_1^+(u)t_1 R_{21}(-u + v) \]  

(8)

Here \( t_i \) denotes the transposition in the \( i^{th} \) space; \( K_1^\pm = K^\pm \otimes 1, K_2^\pm = 1 \otimes K^\pm \), etc. \( \rho \) is a crossing parameter and \( M \) is a crossing matrix, both being specific to a given \( R \)-matrix. These relations correspond to the constraint of factorized scattering in the presence of a wall.

There is an automorphism \([5, 6]\) between the \( K^- \) and \( K^+ \) namely, given a solution \( K^-(u) \), then \( K^-(u - \rho)tM \) is a solution \( K^+(u) \).

Given an \( R \)-matrix and \( K \)-matrices satisfying (7) and (8), the corresponding open chain transfer matrix \( t(u) \) is given by

\[ t(u) = \text{tr} \left( K^+(u)T(u)K^-(u)T^{-1}(-u) \right) \]  

(9)

which constitutes a one-parameter commutative family.

Hamiltonians with boundary terms are obtained from the first derivative of the transfer matrix (9) \([\text{II}]\):

\[ \mathcal{H} = \sum_{n=2}^{N} H_{n,n-1} + \frac{1}{2} \frac{K_1^-'(0)}{K_1^-(0)} + \frac{\text{tr}_a[K_a^+(0)H_{Na}]}{\text{tr}K^+(0)} \]  

(10)

where the local sum is given by

\[ \sum_{n=2}^{N} H_{n,n-1} = \alpha \frac{d}{du} \ln \tau(u) \bigg|_{u=0} + \text{const } I \]  

(11)

The index \( a \) denotes the auxiliary space and \( \alpha \) is a constant.
The corresponding Lax pair formulation may be obtained by direct consideration of the equations of motion which can be written in the Lax form considering the following operator version of an auxiliary linear problem

\[ \Psi_{n+1} = L_n \Psi_n \quad n = 1, 2, ..., N \quad \text{and} \quad \Psi_n = A_n \Psi_n \quad n = 2, 3, ..., N \]  

(12)

and

\[ \Psi_1 = Q_1 \Psi_1 \quad \text{and} \quad \Psi_{N+1} = Q_N \Psi_{N+1}. \]  

(13)

Note that \( Q_1(u) \) and \( Q_N(u) \) are responsible for the boundary terms in the equations of motion of the system. The consistency conditions for these equations are the following Lax equations:

\[ \dot{L}_n = A_{n+1} L_n - L_n A_n \quad \text{for} \quad n = 2, ..., N - 1. \]  

(14)

and

\[ \dot{L}_1 = A_2 L_1 - L_1 Q_1 \quad \text{and} \quad \dot{L}_N = Q_N L_N - L_N A_N \]  

(15)

These equations specify the evolution only of \( L_n \). It means that \( A_n \), \( Q_1 \) and \( Q_N \) have to be determined in terms of \( L_n \) and \( H_{n,n-1} \).

In this note we will show how to find the Lax pair from the boundary Yang-Baxter equations. In this way we are including the general integrable boundary terms in a previous work of Zhang [7] where the periodic case was considered. We also apply the method to the open \( \text{XXZ} \) model and compare the result with previous calculations.

To do this we consider an application of the local Yang-Baxter relation, where the vertices are being commuted horizontally rather than vertically:

\[ R_{n,n-1}(\epsilon) L_n(u + \epsilon) L_{n-1}(u) = L_n(u) L_{n-1}(u + \epsilon) R_{n,n-1}(\epsilon). \]  

(16)

Here the spectral parameters of the local \( L \)-operators differ only by an infinitesimal amount \( \epsilon \). This relation can be represented graphically by figure 3.

![figure 3](image)

This is a form of the star-triangle equation used in [7, 8, 9]. This formulation follows also from the non-local quantum Lax pair [10, 11].
Using the normalization $\mathcal{R}(0) = 1$ we have the following $\epsilon$-expansions

$$\mathcal{R}(\epsilon) \sim 1 + \epsilon(\alpha^{-1}H + \beta I) + o(\epsilon^2),$$

$$L_n(u + \epsilon) \sim L_n(u) + \epsilon L'_n(u) + o(\epsilon^2),$$

where a prime stands for $u$-derivative, with $\alpha$ and $\beta$ some constants and $H_{n,n-1}$ of the local sum in (11) is the $H$ in (17) acting on the quantum spaces at sites $n$ and $n - 1$.

Substituting (17) and (18) into (16) we get as the first non-trivial consequence the following commutation relation

$$\alpha^{-1}[H_{n,n-1}, L_n(u) L_{n-1}(u)] = L_n(u) L'_{n-1}(u) - L'_n(u) L_{n-1}(u)$$

$$n = 2, 3, ..., N$$

We multiply out the above equation to arrive at

$$[H_{n,n-1}, L_n] = \alpha(L_n L'_{n-1} L_{n-1}^{-1} - L'_n) - L_n[H_{n,n-1}, L_n] L_{n-1}^{-1}$$

$$[H_{n+1,n}, L_n] = \alpha(L'_n - L_{n+1}^{-1} L'_{n+1} L_n) - L_{n+1}^{-1}[H_{n+1,n}, L_{n+1}] L_n$$

for $n = 2, 3, ..., N - 1$.

In order to include the boundary terms into (20) and (21) we also shall consider a reflection equation where the vertices are being commuted horizontally:

$$[L_1(u) L_0(u + 2\epsilon) K_1^-(\epsilon)] K_2^-(u + \epsilon) = [K_1^-(\epsilon) L_1(u + 2\epsilon) L_0(u)] K_2^-(u + \epsilon)$$

This equation can be represented graphically by figure 4

![Figure 4](image-url)

Using the the normalization $K^- (0) = 1$ and

$$K^-(\epsilon) \sim 1 + \epsilon K^-(0) + o(\epsilon^2),$$

we get from (22) a new commutation relation

$$\left[\frac{1}{2} K_1^-(0), L_1(u) L_0(u)\right] = L_1(u) L'_0(u) - L'_1(u) L_0(u)$$

(24)
We multiply out the above equation and arrive at
\[ \frac{1}{2} K_1^{-'}(0), L_1 = (L_1 L_0^0 L_0^{-1} - L_1') - L_1 \frac{1}{2} K_1^{-'}(0), L_1 L_0^{-1} \]
(25)
Note that in these equations we have extended the chain in order to include the site \( n = 0 \) at the right boundary and made use of the non-singular property of the \( L \)-operators.

Now we recall the equation (20) to see that the identification
\[ \alpha^{-1} H_{1,0} = \frac{1}{2} K_1^{-'}(0) \]
(26)
allows its continuation for \( n = 1 \).

By similar considerations one can derive the left boundary relations. Now the equation (21) is continued for \( n = N \) with the identification
\[ \alpha^{-1} H_{N+1,N} = \frac{\text{tr} [K_2^+(0) H_{N,n}]}{\text{tr} K^+(0)} \]
(27)
These results tell us that we can keep all considerations made by Zhang for the periodic case\[7\]. The main difference consist in remove the term \( H_{N,N+1} = H_{N,1} \) from the periodic Hamiltonian and adding two boundary terms \( H_{1,0} \) and \( H_{N+1,N} \) determined by the matrices \( K^\pm(u) \).

The equation of motion for \( L_n \) is the Heisenberg equation
\[ \dot{L}_n = i[H, L_n] = i[H_{n+1,n}, L_n] + i[H_{n,n-1}, L_n], \]
(28)
here we have set \( \hbar = 1 \). The second equality is because of the locality of the \( H \) and \( L_n \). Combining (20), (21) and (28), we have
\[ \dot{L}_n = -i L_{n+1}^{-1} \{\alpha L'_{n+1} + [H_{n+1,n}, L_{n+1}]\} L_n + i L_n \{\alpha L'_{n-1} - [H_{n,n-1}, L_{n-1}]\} L^{-1}_{n-1} \]
(29)
By comparing with the Lax equation (14), we can read off the second Lax operator
\[ A_n = -i L_n^{-1} \{\alpha L'_{n} + [H_{n,n-1}, L_{n}]\} \]
(30)
or
\[ A_n = -i \{\alpha L'_{n-1} - [H_{n,n-1}, L_{n-1}]\} L^{-1}_{n-1} \]
(31)
The compatibility of (30) and (31) is guaranteed by the commutation relation (20).

Therefore, the second Lax operator for completely integrable open chains has the following form: In the bulk it is the same for the corresponding periodic chain
\[ A_n = i H_{n,n-1} - i L^{-1}_{n} H_{n,n-1} L_n - i \alpha L^{-1}_{n} L'_{n} \]
\[ n = 2, 3, ..., N, \]
(32)
at the right boundary it is given by

\[ Q_1 = i\alpha \left\{ \frac{1}{2} K_{1}^t(0) - \frac{1}{2} L_1^{-1} K_{1}^t(0) L_1 - L_1^{-1} L_1' \right\} \] (33)

and at the left boundary it is read off from the equation (31):

\[ Q_N = iH_{N+1,N} - iL_N H_{N+1,N} L_N^{-1} - i\alpha' L_N L_N^{-1} \] (34)

where \( H_{N+1,N} \) is given by (27).

Finally, we shall apply this method to a concrete model, the one-dimensional Heisenberg XXZ open chain with Hamiltonian [5, 13]:

\[ H = -N \sum_{k=2}^{N} \left( \sigma^+_k \sigma^-_{k-1} + \sigma^-_k \sigma^+_k + \frac{1}{2} \cos 2\eta \sigma^z_k \sigma^z_{k-1} \right) + \sin 2\eta \left( A_- \sigma^+_1 + B_- \sigma^+_N + C_- \sigma^-_1 + A_+ \sigma^z_N + B_+ \sigma^z_1 + C_+ \sigma^-_N \right) \] (35)

where

\[ A_\mp = 1/2 \cot(\xi_\mp), \quad B_\mp = \frac{b_\mp}{\sin \xi_\mp}, \quad C_\mp = \frac{c_\mp}{\sin \xi_\mp} \] (36)

Here \( \sigma^x, \sigma^y, \sigma^z \) and \( \sigma^\pm = 1/2(\sigma^x \pm i\sigma^y) \) are the usual Pauli spin-1/2 operators. \( \eta \) is a parameter associated with the anisotropy, \( b_\mp, c_\mp \) and \( \xi_\mp \) are some constants describing the boundary effects.

It is not difficult to verify that the equations of motion derived from the Hamiltonian (35) may be cast in the Lax form (14) and (15).

The first Lax operator \( L_n \) is identified with the \( R \)-matrix of the 6-vertex model [14]

\[ L_n = R_{na} = \begin{pmatrix} w_4 + w_3 \sigma^z_n & 2w_1 \sigma^-_n \\ 2w_1 \sigma^+_n & w_4 - w_3 \sigma^z_n \end{pmatrix} \]

\[ = w_4 + w_3 \sigma^x \sigma^z_n + 2w_1 (\sigma^- \sigma^+_n + \sigma^+ \sigma^-_n) \] (37)

where the elements are parametrized by trigonometric functions of \( u \)

\[ w_4 + w_3 = \sin(u + 2\eta), \quad w_4 - w_3 = \sin u, \quad 2w_1 = \sin 2\eta \] (38)

Using the \( \epsilon \)-expansion (17) we have

\[ \alpha = -\sin 2\eta \quad \text{and} \quad \beta = -\frac{1}{2} \cot 2\eta \] (39)

The inverse and the first derivative of \( L_n \) are well-defined:

\[ L_n^{-1} = v_4 + v_3 \sigma^z \sigma^z_n + 2v_1 \left( \sigma^- \sigma^+_n + \sigma^+ \sigma^-_n \right) \]

\[ L'_n = w'_4 + w'_3 \sigma^z \sigma^z_n \] (40)
where
\[ v_4 + v_3 = \frac{1}{\Delta} \sin(u - 2\eta), \quad v_4 - v_3 = \frac{1}{\Delta} \sin u, \quad 2v_1 = -\frac{1}{\Delta} \sin 2\eta \]
\[ \Delta = \sin(u - 2\eta) \sin(u + 2\eta) \]  

(41)

Now, by direct substitution of these data in (32) one can easily find the second Lax operator

\[ A_n = \text{const.} I + id(u)\sigma_n^z \sigma_{n-1}^z + if(u) \left( \sigma_n^- \sigma_{n-1}^+ + \sigma_n^+ \sigma_{n-1}^- \right) \]
\[ + i\sigma^z \left[ g(u) \left( \sigma_n^- \sigma_{n-1}^+ - \sigma_n^+ \sigma_{n-1}^- \right) - d(u) \left( \sigma_n^z + \sigma_{n-1}^- \right) \right] \]
\[ - i\sigma^+ \left[ p(u) \left( \sigma_n^- \sigma_{n-1}^z - \sigma_n^z \sigma_{n-1}^- \right) + q(u) \left( \sigma_n^- + \sigma_{n-1}^z \right) \right] \]
\[ - i\sigma^- \left[ p(u) \left( \sigma_n^z \sigma_{n-1}^+ - \sigma_n^+ \sigma_{n-1}^z \right) + q(u) \left( \sigma_n^+ + \sigma_{n-1}^z \right) \right] \]

(42)

where
\[ d(u) = \frac{1}{4\Delta} \sin 4\eta \sin 2\eta, \quad f(u) = \frac{1}{\Delta} \sin \eta \left( \sin \eta \cos 2u + \sin 3u \right), \]
\[ g(u) = \frac{1}{2\Delta} \sin 2\eta \sin 2u, \quad p(u) = \frac{1}{2\Delta} \sin 4\eta \sin u, \quad q(u) = \frac{1}{2\Delta} \sin 4\eta \sin u \]
\[ \text{const.} = \frac{1}{2\Delta} \sin 2\eta \left( \sin 2u - \sin 2\eta \cos 2\eta \right) \]

(43)

This solution is the Lax operator for the periodic XXZ spin chain which was obtained by Sogo and Wadati [12] in the trigonometric limit of the Lax pair operators for the one-dimensional XYZ Heisenberg spin chain.

Now we have to compute the corresponding boundary operators. By a direct calculation with the operators \( L_1 \) and \( L_1^{-1} \) given by (40) and together with (26), one gets the following results for each term of \( Q_1 \):

\[ iH_{10} = -i\alpha \left( A_- \sigma_1^z + B_- \sigma_1^+ + C_- \sigma_1^- \right) \]

(44)

\[ -i\alpha L_1^{-1} L_1 = -i\alpha \left( v_4 w_4' + v_3 w_3' \right) - i\alpha \left( v_4 w_3' + v_3 w_4' \right) \sigma^z \sigma_1^z \]
\[ - 2i\alpha \left( w_4' - w_3' \right) \left( \sigma^- \sigma_1^+ + \sigma^+ \sigma_1^- \right) \]

(45)

and

\[ -iL_1^{-1} H_{01} L_1 = i\alpha A_- (v_4 w_4 + v_3 w_3 - 2v_1 w_1) \sigma_1^z + i\alpha A_- (v_4 w_3 + v_3 w_4 + 2v_1 w_1) \sigma^z \]
\[ + 2i\alpha A_- [(v_4 - v_3) w_1 - v_1 (w_4 - w_3)] (\sigma^- \sigma_1^+ + \sigma^+ \sigma_1^-) \]
\[ + i\alpha \left( v_4 (v_4 w_4 - v_3 w_3) (B_- \sigma_1^+ + C_- \sigma_1^-) + i\alpha (v_3 w_4 - v_1 w_3) \sigma^z (B_- \sigma_1^+ - C_- \sigma_1^-) \right) \]
\[ + i\alpha \left( w_1 (v_4 + v_3) + v_1 (w_4 + w_3) \right) \left( B_- \sigma^+ + C_- \sigma^- \right) \]
\[ + i\alpha \left[ w_1 (v_4 + v_3) - v_1 (w_4 + w_3) \right] \left( B_- \sigma^+ - C_- \sigma^- \right) \sigma_1^z \]

(46)
We thus find the following Lax operator

\[ Q_1 = \text{const.} \mathbb{1} + \frac{i \sin^2 2\eta}{\Delta \sin \xi_-} \times \]

\[
\begin{pmatrix}
-\frac{1}{2} \sin(2\eta + \xi_-) \sigma^+_1 + \frac{1}{2} \cos \xi_- \sin 2\eta + b_- t(u) \sigma^+_1 + c_- r(u) \sigma^-_1 & \sin(u - \xi_-) \sigma^-_1 \\
+ b_- \sin 2\eta \cos u - c_- \sin u \cos 2\eta \sigma^+_1 & + b_- \sin 2\eta \cos u - c_- \sin u \cos 2\eta \sigma^-_1 \\
- \sin(u + \xi_-) \sigma^+_1 & -\frac{1}{2} \sin(2\eta - \xi_-) \sigma^+_1 - \frac{1}{2} \cos \xi_- \sin 2\eta + b_- r(u) \sigma^+_1 + c_- t(u) \sigma^-_1
\end{pmatrix}
\]

(47)

where

\[ t(u) = \frac{\sin u \cos(u - \eta)}{\cos \eta} - \sin 2\eta \quad \text{and} \quad r(u) = \frac{\sin u \cos(u + \eta)}{\cos \eta} - \sin 2\eta. \]

(48)

The left boundary operator \( Q_N \) is obtained using the equation (34). This results that \( Q_N \) is the transposition of \( Q_1 \), followed by the following substitution:

\[ \xi_- \rightarrow \xi_+, \quad b_- \rightarrow c_+, \quad c_- \rightarrow b_+ \quad \text{and} \quad \sigma_1 \rightarrow \sigma_N \]

(49)

These Lax operators are the trigonometric limit of the Lax pair for the open XYZ spin chain given in ref.[15]. In particular, the diagonal case \( b_\pm = c_\pm = 0 \), was first derived in [14].

To summarize: solving the Yang-Baxter equation together with the reflection equations, we can read off \( L_n, R \) and \( K^\mp \) operators; if \( \mathcal{H} \) has the form (10) with its bulk \( [1] \) is related to \( \mathcal{R} \) by (17), then the corresponding \( A_n \) operator is given by (32). Moreover, for every solution \( K^- (K^+) \) of the reflection equation (7) (8) with \( K^- (0) \neq 0 \) \((\text{Tr} K^+(0) \neq 0)\), we can read off the right (left) boundary term of \( \mathcal{H} \) which has the form (26) (27), then the corresponding \( Q_1 (Q_N) \) operator is given by (33) (34).

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