Adaptive Switched Lattice Reduction-Aided Linear Detection Techniques for MIMO Systems

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Abstract—Lattice reduction (LR) aided multiple-input-multiple-out (MIMO) linear detection can achieve the maximum receive diversity of the maximum likelihood detection (MLD). By employing the most commonly used Lenstra, Lenstra, and L. Lovász (LLL) algorithm, an equivalent channel matrix which is shorter and nearly orthogonal is obtained. And thus the noise enhancement is greatly reduced by employing the LR-aided detection. One problem is that the LLL algorithm can not guarantee to find the optimal basis. The optimal lattice basis can be found by the Korkin and Zolotarev (KZ) reduction. However, the KZ reduction is infeasible in practice due to its high complexity. In this paper, a simple algorithm is proposed based on the complex LLL (CLLL) algorithm to approach the optimal performance while maintaining a reasonable complexity.

I. INTRODUCTION

MIMO systems are fundamental for the next generation wireless networks for their great potential in improving the system capacity and performance. Two key factors that are responsible for the advantages of MIMO system are how to design the optimized signal transmission form at the transmit side and the appropriate signal detection at the receive side. The nonlinear maximum likelihood detection (MLD) can guarantee the best bit error rate (BER) performance. However, the MLD is usually impractical due to its complexity that grows exponentially with the number of constellation points and the number of transmitted streams. The linear detectors such as zero-forcing (ZF) and minimum mean square error (MMSE) receivers have the lower complexity while the simplification is the cost of sacrificing BER performance [1].

Recently, the lattice reduction (LR) aided detection attracted significant research efforts for it can achieve near-optimal performance with very low complexity. The LR algorithm in conjunction with MIMO detection techniques was first considered by Yao and Wornell [2]. From the simulations of [2], the symbol error rate (SER) curves can parallel those of the MLD algorithm by using LR-aided detection schemes. It was proved in [2], [3] that the LR-aided MIMO linear receivers can achieve the maximum diversity as the MLD. Hence, a great deal of interest has been devoted to exploring the application of LR in MIMO systems. The LR-aided detection schemes with respect to the MMSE criterion have been extended by Wuebben et al [5]. In [6], not only the

LR-aided SU-MIMO detection but also the LR-aided SU-MIMO precoding has been investigated. LR-aided MIMO precoding for decentralized receivers was discussed in [7]. The quantitative error-rate analysis of LR aided detection was given in [11].

The optimal lattice basis can be found by Korkin and Zolotarev (KZ) reduction [12]. KZ reduction is an exponential-time lattice reduction algorithm, hence, it is infeasible in practice due to its high complexity [10], [13]. The most commonly used LR reduction is the polynomial-time LLL algorithm which was first proposed by Lenstra, Lenstra, and L. Lovász in [8]. The LLL algorithm guarantees to find a lattice basis within a factor to the optimal one in polynomial time [8], [10]. The essence of the LLL algorithm is try to orthogonalize the columns of the channel matrix and reduce its size as well. The Gram-Schmidt orthogonalization (GSO) procedure and size reduction are the two core components of the LLL algorithm. Through the LLL algorithm, only a real value-based matrix can be processed which may lead to extra unnecessary complexity when the channel has large dimensions. In order to reduce the computational complexity further, the complex LLL (CLLL) algorithm was proposed in [9]. The overall complexity of CLLL algorithm is nearly half of the LLL algorithm without sacrificing any performance.

In this paper, we will employ the CLLL algorithm to implement the LR-aided detection techniques for the MIMO system. As mentioned earlier, the CLLL algorithm can not guarantee to find the optimal lattice basis, hence, there will be a room left for us to improve the performance of CLLL algorithm further and maintain a low complexity at the same time. This is the motivation of this paper, and then the switched scheme is developed and proposed to improve the performance of LR-aided detection techniques.

This paper is organized as follows. In Section II the system model and brief review of the algorithms are given. In Section III the proposed switched LR-aided MIMO detection algorithms are described in detail. Simulation results and conclusions are presented in Section IV and Section V.

Notation: Matrices and vectors are denoted by upper and lowercase boldface letters, and the transpose, inverse, pseudoinverse of a matrix \( B \) by \( B^T \), \( B^{-1} \), \( B^† \), respectively. The \( \mathbb{R} \) and \( \mathbb{C} \) prefixes denote the real and imaginary parts. \( \lfloor x \rfloor \) rounds to a closest integer, while \( \lfloor x \rfloor \) to the closest integer smaller than or equal to \( x \).
II. PRELIMINARIES

A. System Model

In this paper, we assume an uncoded spatial multiplexing (SM) MIMO broadcast channel, where \( N_T \) transmit antennas are employed at the base station (BS) and \( N_R \) receive antennas are equipped at the user terminal (UT). Actually, MIMO techniques can be combined with orthogonal frequency division multiplexing (OFDM) technique. By decomposing the channel into multiple orthogonal sub-channels, the OFDM techniques can successfully transform frequency selective fading channels into flat fading channels. Hence, we assume a flat fading MIMO channel in this paper. The system model is illustrated in Fig.1, and the received signal is given by:

\[
y = Hx + n,
\]

where \( H \in \mathbb{C}^{N_R \times N_T} \) is the complex Gaussian channel matrix with zero mean and unit variance. We assume \( H \) is a full-rank MIMO channel, i.e., \( H \) consists of \( N_T \) (\( N_T \leq N_R \)) linearly independent row vectors. \( x \in \mathbb{C}^{N_T} \) is the transmitted data vector, and \( n \in \mathbb{C}^{N_R} \) is the Gaussian noise with i.i.d. entries of zero mean and variance \( \sigma_n^2 \).

Fig. 1. MIMO System Model

B. Complex Lattice Reduction Algorithm

A complex lattice is a set of points \( L(H) = \{Hx| x_i \in \mathbb{Z} + j \mathbb{Z}\} \), where \( H = \{h_1, h_2, \ldots, h_{N_T}\} \) contains the bases of the lattice \( L(H) \). Actually, any matrix \( \tilde{H} \) can generate the same lattice as \( H \) if and only if \( \tilde{H} = HU \), where \( U \) is a unimodular matrix (\( \text{det}(U) = 1 \)) and all elements of \( U \) are Gaussian integers, i.e. \( u_{i,k} \in \mathbb{Z} + j \mathbb{Z} \).

The aim of the LR algorithm is to find a new basis \( \tilde{H} \) which is shorter and nearly orthogonal compared with the original matrix \( H \). Given the QR decomposition of \( H \), \( H = QR \), where \( Q \) is an orthogonal matrix with unit length (\( Q^*Q = I_{N_T} \)) and the upper-triangular matrix \( R \) is a rotated and reflected representation of \( H \). Thus, each column vector \( h_k \) of \( H \) is given by [5]

\[
h_k = \sum_{l=1}^{k} r_{l,k} q_l,
\]

where \( q_l \) is the columns of \( Q \). If \( |r_{1,k}|, \ldots, |r_{k-1,k}| \) are close to zero, we can say that \( h_k \) is almost orthogonal to the space spanned by \( h_1,\ldots,h_{k-1} \). Similarly the QR decomposition of \( \tilde{H} \) is \( \tilde{H} = \tilde{Q} \tilde{R} \). Then, the basis for \( L(H) \) is CLLL-reduced if both of the following conditions are satisfied:

\[
|\Re(\tilde{r}_{l,k})| \leq \frac{1}{2}|\Re(\tilde{r}_{l,l})|, \quad |\Im(\tilde{r}_{l,k})| \leq \frac{1}{2}|\Im(\tilde{r}_{l,l})|, 1 \leq l < k \leq N_T.
\]

\[
\delta\|\tilde{r}_{k-1,k-1}\|^2 \leq \|\tilde{r}_{k,k}\|^2 + \|\tilde{r}_{k-1,k}\|^2, \quad 2 \leq k \leq N_T
\]

where \( \delta \in (\frac{1}{2}, 1] \) influences the quality of the reduced basis and the computational complexity. We usually choose \( \delta = \frac{3}{4} \) to achieve a trade-off between good performance and complexity [8]. The main steps of the CLLL reduction algorithm are described clearly in [9].

Obviously, \( \tilde{H} \) is not the unique basis for the lattice \( L(H) \). The lattice \( L(H) \) can have infinitely many different bases other than \( \tilde{H} \). For any unimodular matrix \( U \) which satisfy \( \text{det}(U) = 1 \) and \( u_{i,k} \in \mathbb{Z} + j \mathbb{Z} \), there will be a corresponding basis \( \tilde{H} \). Among all the bases, the optimal one can be found by KZ reduction. From [10], an arbitrary matrix is KZ-reduced if and only if its upper-triangular representation is KZ-reduced. For convenient, we study the upper-triangular matrix \( \tilde{R} \) of \( \tilde{H} \),

\[
\tilde{R} = \begin{pmatrix}
{r_{1,1}} & {r_{1,2}} & \cdots & {r_{1,N_T}} \\
0 & {r_{2,2}} & \cdots & {r_{2,N_T}} \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & {r_{N_T,N_T}}
\end{pmatrix}
\]

(5)

the KZ reduction is recursively defined if each of the following three conditions holds:

\[
|r_{k,N_T}| \leq \frac{|r_{N_T,N_T}|}{2}, \quad \text{for } k = 2, \ldots, N_T
\]

\[
\begin{pmatrix}
{r_{1,1}} & {r_{1,2}} & \cdots & {r_{1,N_T-1}} \\
0 & {r_{2,2}} & \cdots & {r_{2,N_T-1}} \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & {r_{N_T-1,N_T-1}}
\end{pmatrix}
\]

is KZ-reduced

Any KZ-reduced matrix is clearly also CLLL-reduced. The optimal basis can be obtained by using KZ reduction, however, finding a KZ reduced basis would be too time-consuming and infeasible in practice. Considering the computational complexity, it is reasonable to employ the low complexity CLLL reduction algorithm to implement the lattice reduction. Consequently, there will be a performance gap between the CLLL and KZ algorithms. Regarding the performance gap, strategies are developed in section III to approach the optimal performance and offer a trade-off between performance and complexity.
C. CLR Aided MIMO Detection

The LR-reduced channel matrix $\tilde{H}$ has better channel quality compared to the original channel matrix $H$. Therefore, if the MIMO receivers were designed based on $\tilde{H}$, a better detector performance can be achieved due to less noise enhancement increased by $\tilde{H}$. The LR aided MIMO detection structure is illustrated in Fig.2.

$$\begin{align*}
\tilde{H} & \xrightarrow{\text{Shift}} \mathbf{U} \\
\mathbf{U} & \xrightarrow{\text{LR Detector}} \mathbf{y} \\
\mathbf{y} & \xrightarrow{\text{ShiftScale}} \mathbf{z} \\
\mathbf{z} & \xrightarrow{\text{U}} \hat{x}
\end{align*}$$

Fig. 2. LR Aided MIMO Detection Structure

In Fig.2, $x$ is the transmit M-QAM signal. The set of M-QAM constellation is given by $\mathbb{S} = \{\pm \frac{1}{\sqrt{M}} a, \pm \frac{1}{\sqrt{2M}} a, \ldots, \pm \frac{\sqrt{M-1}}{\sqrt{2M}} a\}$ with $\sqrt{M}$ representing the modulation index. The parameter $a = \sqrt{6}/M-1$ is used for normalizing the power of the transmit signals to 1. From Fig.2, the received signal is,

$$y = Hx + n = HUU^{-1}x + n = \tilde{H}z + n, \quad (9)$$

where $\tilde{H} = HU$ is the CLLL-reduced channel matrix and $z = U^{-1}x$ is the equivalent transmit signal. Similarly to the conventional MIMO detector, there are several CLR-aided MIMO detection techniques:

1) CLR-aided ZF Detector

ZF detection strategy is the simplest one, obviously, if we want to cancel out the impact of the fading channel at the receiver side, we can set the receiver filter as the pseudoinverse of $\tilde{H}$ [1].

$$\tilde{G}_{ZF} = (\tilde{H}^{H} \tilde{H})^{-1} \tilde{H}^{H}, \quad (10)$$

And then, the received signal is $\tilde{z} = z + \tilde{G}_{ZF}n$.

Since $U$ is a unimodular matrix, the statistical properties of $Un$ are identical to those of $n$. Therefore, the estimation error covariance matrix of ZF detection is,

$$\varphi_{ZF} = E\{(\hat{x} - x)(\hat{x} - x)^{H}\} = \sigma_{n}^{2} \tilde{G}_{ZF} \tilde{G}_{ZF}^{H}, \quad (11)$$

From $\varphi_{ZF}$, it is clear that $\tilde{G}_{ZF}$ increased the noise interference on the signal, and thus the BER performance of the ZF detector will be degraded by $\tilde{G}_{ZF}$. Since $\tilde{G}_{ZF}$ is based on the CLR-reduced channel $\tilde{H}$, it is more likely to be well conditioned compared to $G_{ZF}$ which is based on $H$, hence the effect of noise enhancement will be moderated by $\tilde{G}_{ZF}$.

2) CLR-Aided MMSE Detector

As an improvement to reduce the effects of noise amplification caused by the ZF filter, the MMSE filter taking the noise term into account:

$$G_{MMSE} = (H^{H}H + \sigma_{n}^{2}I_{NT})^{-1}H^{H}, \quad (12)$$

As shown in [14], the MMSE detection is equal to ZF with respect to an extended system model. The extended channel matrix $\tilde{H}$ and the extended received signal $\mathbf{y}$ is given by,

$$\tilde{H} = \begin{bmatrix} H \\ \sigma_{n}I_{NT} \end{bmatrix} \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0}_{NT} \end{bmatrix}, \quad (13)$$

From (14), it is the condition of $\tilde{H}$ that determines the noise amplification not the condition of $\tilde{H}$ in the CLR-aided MMSE detection case. We can compute $\tilde{G}_{MMSE}$ according to (15), however, the BER performance will be discounted for this mismatching. A better performance can be obtained by performing the CLR for the extended channel matrix $\tilde{H}$, i.e. $\tilde{H} = \tilde{H} \mathbf{U}$, and by computing

$$\tilde{G}_{MMSE} = \tilde{H}^{H} \tilde{H} (\tilde{H}^{H} \tilde{H})^{-1} \tilde{H}^{H}, \quad (14)$$

then, the received signal is $\tilde{z} = \tilde{G}_{MMSE} \mathbf{y}$.

3) CLR-Aided SIC Detector

The filters employed in Successive Interference Cancellation (SIC) detection are mainly based on the linear filters which were discussed above. The key idea of SIC detection is layer peeling, that is, the first symbol is decoded first, and then cancelling the decoded symbol in the next layer peeling, repeat this manipulation layer by layer until all the symbols are decoded from the received signal. By layer peeling, the interference caused by the already detected symbols is canceled. The SIC detection can be equivalently implemented by a QR decomposition, $\tilde{H} = QR$, and then computes $\tilde{y} = Q^{H}y$. The detection steps is summarized bellowed,

$$\tilde{z}_{N_{T}} = \begin{bmatrix} \tilde{y}_{N_{T}} \\ r_{N_{T},N_{T}} \end{bmatrix}, \quad (15)$$

$$\tilde{z}_{i} = \begin{bmatrix} \tilde{y}_{i} - \sum_{j=i+1}^{m} r_{i,j} \tilde{z}_{j} \\ r_{i,i} \end{bmatrix}, \quad (16)$$

where $i = N_{T} - 1, \ldots, 1$. And finally, $\tilde{z} = [\tilde{z}_{1}, \ldots, \tilde{z}_{N_{T}}]^{T}$.

After the receive filter, the estimated transmit signal $\tilde{x}$ can be obtained by $\tilde{x} = U \tilde{z}$. However, the elements of $U \tilde{z}$ are far from integers and serious performance loss will be caused if $U \tilde{z}$ was quantized directly. In order to avoid the quantization error caused by rounding as much as possible, proper shifting and scaling work should be done before multiplying the received signal by $U$. The estimation of $z$ is,

$$\tilde{z} = a(\hat{\tilde{z}} + \frac{1}{2}U^{-1}\mathbf{1}_{N_{T}}), \quad (17)$$

where $\hat{\tilde{z}} = [\frac{1}{2}\tilde{z} - \frac{1}{2}U^{-1}\mathbf{1}_{N_{T}}]$. Finally the estimated transmit signal $x$ can be easily obtained by $\hat{x} = U \hat{\tilde{z}}$.

III. PROPOSED SWITCHED LR-AIDED DETECTION

The proposed algorithm is motivated by the fact that the CLLL-reduced channel matrix $\tilde{H}$ is neither the optimal nor the unique basis in the lattice space. Therefore, we can build more LR-reduced candidate matrices and choose the best one during each detection stage. Observing the formulation $\tilde{H} = HU$, it is not difficult to find that there are two ways if we want to get more LR-reduced candidates.
The first way is achieved by constructing different satisfied $U$ matrix. As long as $U$ always meet two requirements, (1) $\det(U) = 1$, (2) all elements of $U$ are Gaussian integers. The physical meaning of the first requirement is that we cannot change the transmit power through the LR transformation. Under the first limitation, we need to construct the different $U$ matrices by the four units 1, -1, j and -j.

The selection criterion called orthogonality defect factor (ODF) is defined as

$$\text{ODF}(H) = \left(\frac{\|h_1\|^2\|h_2\|^2\ldots\|h_{N_T}\|^2}{\det(H^H H)}\right),$$

where $h_i$s are the columns of the basis $H$. Clearly, $\text{ODF}(H) \leq 1$ with equality of an orthogonal basis. Therefore, for candidates $\tilde{H}_1$ and $\tilde{H}_2$, we can say $\tilde{H}_1$ is better reduced than $\tilde{H}_2$ if $\text{ODF}(\tilde{H}_1) \leq \text{ODF}(\tilde{H}_2)$. By using the ODF selection metric, we can choose the best candidate $\tilde{H}_{op}$ from $\{\tilde{H}_1, \tilde{H}_2, \ldots, \tilde{H}_K\}$, and then the received signal can be rewritten as

$$y = \tilde{H}_{op}x + n,$$

consequently, the receive filter is calculated based on $\tilde{H}_{op}$ as well. Since $\tilde{H}_{op}$ has better channel quality, the BER performance will be improved by the proposed algorithm.

The complexity of the proposed algorithm is determined by the total candidate number $K$. The different trade-offs between performance and complexity can be achieved by altering $K$ depending on specific situation. For example, we can set $K = 1$, and compare the orthogonality between the candidate and the original channel matrix. Then, choose the better one to compute the detection filter. Hence, the performance will be always better or equal to the conventional CLR-aided detection.

A graceful trade-off between BER performance and computational complexity offered by the proposed algorithm which is summarized in Table I. The simulation results are given in the next section.

IV. SIMULATION RESULTS

In this section, the linear MIMO detection techniques are given and compared with the proposed algorithms. A system with $N_T = 6$ transmit antennas and $N_R = 6$ receive antennas is considered. We assume an uncorrelated block fading channel, that is, the channel is static during each transmit packet and there is no correlation among the antennas.

The number of trials used to average the curves is 1000 and the packet length is 100 symbols for the simulations. The SNR is defined as $SNR = \frac{N_T \sigma_s^2}{\sigma_n^2}$, and the $E_b/N_0$ is defined as $E_b/N_0 = SNR \frac{N_T}{N_R R_m}$ with $R_m$ is the number of information bits transmitted per channel symbol $x_l$. 

![Fig. 3. Switched LR Aided Detection Structure](image-url)
Table I: Proposed Switched LR-Aided Algorithm

| Function                  | Description                                      |
|---------------------------|--------------------------------------------------|
| \( H_{\text{KL}} = \text{KLR}(H, K) \) |                                                     |
| 1: \( H_{\text{LR}} = \text{CLLL}(H) \)     |                                                     |
| 2: \( T_{\text{obj}} = \text{ODF}(H_{\text{LR}}) \) |                                                     |
| 3: \( P = \text{Rand}_1 \text{Permute}(K) \) |                                                     |
| 4: for \( i = 1 : K \) |                                                     |
| 5: \( H_{\text{LR}}^i = \text{CLLL}(H_{\text{LR}}) \) |                                                     |
| 6: \( T_{\text{ao}}^i = \text{ODF}(H_{\text{LR}}^i) \) |                                                     |
| 7: end                     |                                                     |
| 8: \([\text{Min Idx}] = \text{Min}(T_{\text{ao}}^i) \) |                                                     |
| 9: if \( T_{\text{ao}}^i \text{Idx} < T_{\text{ao}} \) |                                                     |
| 10: \( H_{\text{KL}} = H_{\text{LR}}^i \)     |                                                     |
| 11: else                   |                                                     |
| 12: \( H_{\text{KL}} = H_{\text{LR}} \)     |                                                     |
| 13: end                     |                                                     |
| 14: return \( H_{\text{KL}} \)             |                                                     |

Fig. 4 showed the BER performance of the proposed randomly switched CLR-aided schemes with ZF detection. From Fig. 4, we can find that the ZF detection has the worst BER performance compared to the other techniques since the high noise amplification without LR. The BER performance of CLR-aided ZF detection is better than the conventional ZF detection while worse than the proposed algorithm. In addition, the performance of the proposed scheme gradually improved as the candidates number \( K \) increases. It is clear that the proposed scheme have better performance compared to ZF detection and LR-aided ZF detection. At the BER of \( 10^{-3} \), the proposed scheme with \( K = 10 \) has more than 4.5dB gains compared to the CLR-aided ZF detection. It is worth noting that even with \( K = 1 \), the proposed scheme has almost 2dB gains over the CLR-aided ZF linear detection.

Fig. 5 illustrated the MMSE,MMSE-SIC and their corresponding proposed algorithms respectively. The MMSE based detection algorithms have better BER performance compared to their existing ZF counterparts in Fig.4. At 25dB, the BER of conventional MMSE and CLR-aided extended MMSE detection are all higher than \( 10^{-4} \). While, the BER of the proposed scheme achieved very good performance at 25dB, they are all below \( 10^{-4} \) and even approached \( 10^{-5} \) with \( K = 10 \). Since there is a interference cancellation during each SIC detection layer, the improved performance of the proposed randomly switched CLR-aided MMSE-SIC is quite limited compared with the randomly switched technique with linear detection. However, the proposed algorithm still offer almost 2dB gain with \( K = 10 \) at \( 10^{-4} \) compared to CLR-aided MMSE-SIC.

Fig. 6 illustrated the BER performance with 16-QAM modulation. Obviously, the BER performance of all corresponding algorithms are becoming worse compared to the QPSK modulation scenario due to the smaller signal point distance. Similarly, the proposed schemes offered better performance in the 16-QAM modulation.

These simulation results support the analysis that the LR algorithms based on LLL can not guarantee an optimal basis, however, we can approach the optimal performance by using
the proposed scheme. Actually, if the equivalent channel is strictly orthogonal between each other, the linear detection is identical to MLD [5].

V. CONCLUSION

In this paper, the randomly switched technique was proposed to improve the performance of LR-aided MIMO detection, which gracefully offered a tradeoff between the complexity and performance. Simulation results evidence that our proposed algorithms have substantial performance gains compared to the existing MIMO linear and LR-aided linear detection. As for nonlinear SIC detection, the gain improved by the proposed algorithm is limited. How to reduce the gap between the proposed algorithm based on SIC detection and the ML or SD detection need further study.

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