Leptogenesis from the Asymmetric Texture

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Abstract

We investigate non-resonant thermal leptogenesis in the context of the $SU(5) \times T_{13}$ “asymmetric texture”, where both Dirac and Majorana $CP$ violation arise from a single phase in the tribimaximal seesaw mixing matrix. We show that the baryon asymmetry of the universe can be explained in this model only when flavor effects are considered for right-handed neutrino masses of $O(10^9 - 10^{12})$ GeV. The sign of the baryon asymmetry also determines the sign of the previously predicted Dirac $CP$ phase $|\delta_{CP}| = 1.32\pi$, consistent with the latest global fit $\delta_{CP}^{PDG} = 1.37 \pm 0.17\pi$.

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1. INTRODUCTION

The observable lepton mixing in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix has been measured to contain two large and one small angle, unlike the nearly-identity Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2]. This inspires contemplating the large mixing angles originating from the unknown $\Delta I = 0$ physics, whereas the small reactor angle comes entirely from the “Cabibbo haze” [3] of the $\Delta I = \frac{1}{2}$ sector. This idea is implemented in the $SU(5)$ “asymmetric texture” [4] where the $\Delta I = 0$ seesaw matrix is assumed to be diagonalized by the tribimaximal (TBM) mixing [5] with a $CP$ phase. The asymmetry, introduced minimally in the down-quark and the charged-lepton Yukawa matrices, is essential to explain the reactor angle and it determines the TBM phase up to a sign. This single phase brings all three lepton mixing angles within $3\sigma$ of their Particle Data Group (PDG) value and predicts $CP$ violation in the lepton sector consistent with the current global fits [1, 2].

The asymmetry of the texture singles out $T_{13} \equiv Z_{13} \times Z_3$ [6], an order 39 discrete subgroup of $SU(3)$, as the smallest family symmetry. The electroweak sector of the texture is explained in an $SU(5) \times T_{13}$ model in Ref. [7] and its seesaw sector is explored in Ref. [8]. Guided by minimality in the particle content and simplicity in the vacuum structure of the scalars, this model yields the normal ordering of light neutrino masses such that $m_{\nu_1} = 27.6$, $m_{\nu_2} = 28.9$ and $m_{\nu_3} = 57.8$ meV through the seesaw mechanism involving four right-handed Majorana neutrinos. The sum of these masses almost saturates the Planck bound $\sum_i m_i \leq 120$ meV [9] and will be probed further by near-future experiments [10]. This model also predicts neutrinoless double beta decay [11] with the invariant mass parameter $|m_{\beta\beta}| = 13.02$ or 25.21 meV, within an order of magnitude of the latest upper bound of $61 - 165$ meV measured by the KamLAND-Zen experiment [12] and sensitive to several next-generation experiments [13].

In this paper we expand the analysis of the asymmetric texture to investigate the generation of the baryon asymmetry [14] of the universe through leptogenesis [15]. Baryon asymmetry is defined as the ratio of the net number of baryons to the number of photons: $\eta_B = (N_B - N_{\bar{B}})/N_\gamma$. The abundance of matter over antimatter in the universe implies $\eta_B > 0$, as evidenced by the measurement from Cosmic Microwave Background (CMB) data [16]:

$$\eta_B^{CMB} = (6.12 \pm 0.04) \times 10^{-10}. \quad (1)$$

Sakharov identified three necessary conditions for successful generation of the baryon asymmetry [17]: (i) the existence of baryon number, $B$, violating elementary processes, (ii) violation of $C$ and $CP$, and (iii) a departure from thermal equilibrium. In leptogenesis, lepton asymmetry is generated from the $C$ and $CP$ out-of-equilibrium decays of the Majorana neutrinos into leptons and Higgs bosons. These decays violate the total lepton number $L$, which is partially converted into violation of the baryon number $B$ by $B - L$-preserving sphaleron processes [18], fulfilling Sakharov’s conditions.

We discuss leptogenesis in the so-called “strong washout” regime where only decays and
inverse decays of the Majorana neutrinos describe generation of the asymmetry [19, 20]. We show that the low energy $CP$ phases of the model do not yield any high energy $CP$ asymmetry unless “flavor effects” [21, 23] are considered. The relevant Boltzmann equations are solved numerically for the non-hierarchical mass spectrum of the Majorana neutrinos in three-flavor approximation. Successful leptogenesis occurs for Majorana masses of $O(10^9 – 10^{12})$ GeV and constrains the parameter space of the model.

The signs of the low energy leptonic $CP$ violation and the baryon asymmetry can, in general, be correlated [24]. In the $SU(5) \times T_{13}$ model the baryon asymmetry is generated by the single TBM phase whose sign was unresolved in the previous works [4, 8]. We demonstrate that the final asymmetry is sensitive to this sign. The Dirac $CP$ phase $\delta_{CP}$ predicted in this model is $\pm 1.32\pi$, compared to the latest PDG fit $\delta_{CP}^{PDG} = 1.36 \pm 0.17\pi$ [1]. We find that positive baryon asymmetry occurs for the ‘correct’ sign of $\delta_{CP}$ for a particular mass ordering of the Majorana neutrinos: $M_2 < M_1 < M_3$.

The paper is organized as follows. In section 2, we set up the lepton sector of the $SU(5) \times T_{13}$ model presented in Refs. [4, 7, 8] in a basis relevant for leptogenesis calculation. In section 3, we briefly review thermal leptogenesis in the non-hierarchical mass spectrum of the Majorana neutrinos. In section 4, we discuss the relation between low energy $CP$ phases and high energy $CP$ asymmetry and show that leptogenesis is only viable in this model in the flavored approximation. Section 5 describes our results for the Majorana masses required for leptogenesis. In section 6, we discuss how the sign ambiguity in the TBM phase is resolved from leptogenesis and we conclude in section 7.

2. LEPTON SECTOR OF THE $SU(5) \times T_{13}$ MODEL

The “asymmetric texture” [4] is inspired by the $SU(5)$ Georgi-Jarlskog texture [25] with a $45$ Higgs coupling to the (22) element of the down-quark and the charged-lepton Yukawa matrices $Y^{(-\frac{1}{3})}$ and $Y^{(-1)}$, respectively, and a $5$ Higgs coupling elsewhere:

$$Y^{(-\frac{1}{3})} \sim \text{diag} (\lambda^8, \lambda^4, 1),$$

$$Y^{(-1)} \sim \begin{pmatrix}
  bd\lambda^4 & a\lambda^3 & b\lambda^3 \\
  a\lambda^3 & c\lambda^2 & g\lambda^2 \\
  d\lambda & g\lambda^2 & 1
\end{pmatrix} \quad \text{and} \quad Y^{(-1)} \sim \begin{pmatrix}
  bd\lambda^4 & a\lambda^3 & d\lambda \\
  a\lambda^3 & -3c\lambda^2 & g\lambda^2 \\
  b\lambda^3 & g\lambda^2 & 1
\end{pmatrix}.$$  \hspace{1cm} (2)

The $O(1)$ prefactors $a = c = \frac{1}{3}$, $g = A$, $b = A\sqrt{\rho^2 + \eta^2}$, $d = \frac{2}{3A}$ are determined in terms of the Wolfenstein parameters $A$, $\rho$, and $\eta$. The asymmetry of $O(\lambda)$ lies along the (13) – (31) axis of $Y^{(-\frac{1}{3})}$ and $Y^{(-1)}$. The up-quark Yukawa matrix $Y^{(\frac{1}{3})}$ is assumed to be diagonal. $SU(5)$ dictates $Y^{(-\frac{1}{3})}$ to be transpose of $Y^{(-1)}$ and the factor of $-3$ in the later comes from the vacuum expectation value of the $45$ Higgs. The Yukawa matrices are unitarily diagonalized as $Y^{(q)} = U^{(q)\ D^{(q)}} Y^{(q)\ t}$, where $U^{(-\frac{1}{3})} = U_{CKM}$ and

$$U^{(-1)} = \begin{pmatrix}
  1 - \left(\frac{2}{9A^2} + \frac{1}{18}\right) & \lambda^2 & \frac{2A}{3A} \\
  -\frac{1}{3} & 1 - \frac{\lambda^2}{18} & A\lambda^2 \\
  -\frac{2A}{3A} & (-A - \frac{2}{9A}) & 1 - \frac{2A^2}{3A^2}
\end{pmatrix} + O(\lambda^3).$$  \hspace{1cm} (3)
Together with the complex-TBM seesaw mixing $U_{\text{seesaw}} = \text{diag}(1, 1, e^{i\delta}) \, U_{\text{TBM}}$, where $|\delta| \simeq 78^\circ$, this texture reproduces the GUT-scale mass ratios and the mixing angles of quarks and charged leptons and predicts Dirac and Majorana $CP$ violating phases in the lepton sector.

A straightforward explanation of the asymmetric term in $Y(-\frac{1}{3})$ and $Y(-1)$ requires an $SU(3)$-subgroup family symmetry with at least two different triplets. The smallest discrete group that fits the bill is $T_{13}$ \cite{7}. An $SU(5) \times T_{13}$ model of effective interactions, where the $SU(5)$ matter fields transform as different triplets of $T_{13}$ but the Higgs bosons are family singlets, explains the structure of the texture \cite{7} and the origin of the complex-TBM seesaw mixing \cite{8} through simple vacuum alignment of gauge-singlet family-triplet familons. The generic setup of three Majorana neutrinos appears to be in tension with the oscillation data. A minimal extension of the seesaw sector with a fourth Majorana neutrino resolves this and predicts normal ordering of the light neutrino masses.

The aim of this paper is to further investigate the seesaw sector of the model to see if the low energy $CP$ phases can explain the baryon asymmetry of the universe at high energies through leptogenesis. We assume that both the gauge and the family symmetry are broken down to the Standard Model gauge group before this happens, so that the Majorana neutrinos decay into Standard Model leptons and Higgs. This implies that the mass of the Majorana neutrinos should be lower than $10^{16}$ GeV, the breaking scale of the gauge and family symmetry.

In the following subsections, we will briefly review the seesaw sector of the $SU(5) \times T_{13}$ model and its breaking to the Standard Model gauge group. Then we will set up the relevant parameters in the appropriate basis for discussing leptogenesis in the subsequent sections.

2.1. From $SU(5) \times T_{13}$ to the Standard Model Gauge Group

The seesaw Lagrangian of the $SU(5) \times T_{13}$ model \cite{8} is given by

$$L_{\text{ss}} \supset y_A F \Lambda H_5 + y'_A \bar{N} \bar{\varphi}_A + y_B \bar{N} \bar{\varphi}_B + M_\Lambda \Lambda \Lambda + y'_v \bar{N} \bar{\varphi}_v + m \bar{N}_4 \bar{N}_4.$$  \hspace{1cm} (4)

This Lagrangian has a $Z_{12}$ ‘shaping’ symmetry to prevent unwanted operators. \cite{8} Charged leptons reside in the field $F$ and the Majorana neutrinos in $\bar{N}$ and $\bar{N}_4$. There are three familons $\varphi_A$, $\varphi_B$ and $\varphi_v$, and their vacuum expectation values (VEVs) are given by:

$\langle \varphi_A \rangle_0 = \sqrt{m_{\nu} b_1 b_2 b_3} (-b_2^{-1} e^{i\delta}, b_1^{-1}, b_3^{-1})$, \hspace{1cm} $\langle \varphi_B \rangle_0 = (b_1, b_2, b_3)$, \hspace{1cm} $\langle \varphi_v \rangle_0 = \sqrt{m m'_{\nu}} (2, -1, e^{i\delta})$,

where $b_1, b_2, b_3, m \neq 0$. The vacuum alignments of $\varphi_A$ and $\varphi_B$ are related to each other, as required for the complex-TBM diagonalization of the seesaw matrix. The interactions are mediated by a heavy vector-like messenger $\Lambda$. The transformation properties of the relevant fields are given in Table 1.

\footnote{Ref. \cite{8} also discusses a $Z_{14}$ ‘shaping’ symmetry for a slightly different particle content. We have checked that this case does not yield successful leptogenesis at least for simple vacuum alignments of familons and therefore will not be pursued in the paper.}
Using oscillation data, the parameters $m_\nu$ and $m'_v$ were determined in Ref. [8] as

$$|m_\nu| = 57.8 \text{ meV}, \quad |m'_v| = 5.03 \text{ or } 14.2 \text{ meV}. \quad (5)$$

For our calculation, we will adopt $m_\nu = 57.8 \text{ meV}$ and $m'_v = 5.03 \text{ meV}$. This leaves four undetermined parameters: $b_1, b_2, b_3$ and $m$. In this paper we will discuss how these parameters are constrained when successful leptogenesis occurs and both the signs of the baryon asymmetry and the low energy $\mathcal{C}P$ violation are consistent with current data.

Integrating out the heavy messenger $\Lambda$ from the Lagrangian in Eq. (4) gives the dimension-5 operators

$$\frac{1}{M_\Lambda} F \tilde{N} H_5 \varphi_A \quad \text{and} \quad \frac{1}{M_\Lambda} F \tilde{N} H_5 \varphi_v.$$  

These operators yield the Dirac Yukawa matrix $Y(0)$ when the familon $\varphi_A$ and $\varphi_v$ develop nonzero VEVs spontaneously breaking the $T_{13} \times Z_{12}$ symmetry [8]:

$$Y(0) \equiv \sqrt{b_1 b_2 b_3 m_\nu} \begin{pmatrix}
0 & b_3^{-1} & 0 & 2 \sqrt{\frac{m m'}{b_1 b_2 b_3 m_\nu}} \\
0 & b_1^{-1} & 0 & -\sqrt{\frac{m m'}{b_1 b_2 b_3 m_\nu}} \\
0 & 0 & -e^{i\delta} b_2^{-1} & e^{i\delta} \sqrt{\frac{m m'}{b_1 b_2 b_3 m_\nu}} \\
0 & 0 & 0 & 0
\end{pmatrix}. \quad (6)$$

The effective operator $F \tilde{N} H_5$ further gives rise to the interaction $\ell_i^\dagger H^* \tilde{N}_i$ when the $SU(5)$ symmetry is broken down to the Standard Model gauge group and generates the decays:

$$\tilde{N}_i \rightarrow \ell_i^\dagger + H^*, \quad i = 1, 2, 3, 4; \quad \alpha = e, \mu, \tau. \quad (7)$$

The $4 \times 4$ Majorana mass matrix gets contribution from the VEV of the familon $\varphi_B$ and can be expressed as [8]

$$\mathcal{M} \equiv \begin{pmatrix}
0 & b_2 & b_3 \\
b_2 & 0 & b_3 \\
b_3 & b_1 & 0 \\
0 & 0 & m
\end{pmatrix}. \quad (8)$$

It is a complex symmetric matrix and its Takagi factorization [26] yields

$$\mathcal{M} = \mathcal{U}_m \mathcal{D}_m \mathcal{U}_m^T. \quad (9)$$

Here $\mathcal{D}_m = \text{diag}(M_1, M_2, M_3, M_4)$ is the diagonal mass matrix with the positive square root of real eigenvalues of $\mathcal{M} \mathcal{M}^\dagger$ and $\mathcal{U}_m$ is the unitary matrix containing the corresponding eigenvectors of $\mathcal{M} \mathcal{M}^\dagger$.\footnote{The matrices in Eqs. (6) and (8) are valid at the grand unified scale ($\sim 10^{16}$ GeV). For simplicity we neglect the effects of their running and assume that they are valid at the Majorana mass scale ($\sim 10^{11}$ GeV) too. See Ref. [27] for more discussion on the effect of running seesaw parameters on leptogenesis.}

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Table 1. Charge assignments of matter, Higgs, messenger and familon fields in the seesaw sector.

|         | $F$ | $\tilde{N}$ | $\tilde{N}_i$ | $H_5$ | $\Lambda$ | $\varphi_A$ | $\varphi_B$ | $\varphi_v$ |
|---------|-----|-------------|---------------|-------|-----------|-------------|-------------|------------|
| $SU(5)$ | $\mathbf{5}$ | $1$ | $1$ | $5$ | $1$ | $1$ | $1$ | $1$ |
| $T_{13}$ | $\mathbf{3}_1$ | $\mathbf{3}_2$ | $1$ | $1$ | $\mathbf{3}_1$ | $\mathbf{3}_2$ | $\mathbf{3}_2$ | $\mathbf{3}_1$ |
| $Z_{12}$ | $\omega$ | $\omega^3$ | $1$ | $\omega^9$ | $\omega^2$ | $\omega^{11}$ | $\omega^6$ | $\omega^2$ |

Here $\omega^{12} = 1$. The $Z_{12}$ ‘shaping’ symmetry is required to prevent unwanted tree-level operators.

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2.2. Rotating to the Weak Basis

In leptogenesis we usually work in the so-called weak basis, where the charged-lepton Yukawa matrix and the right-handed Majorana matrix are diagonal with real, positive entries [28]. After spontaneous breaking of the $SU(5) \times T_{13} \times Z_{12}$ symmetry, the relevant terms in the Lagrangian are

$$L \supset \ell \gamma(-1) \bar{e} H + \ell \gamma(0) \bar{N} H^* + \bar{N}^T M \bar{N},$$

(10)

Redefining the fields $\ell \rightarrow U(-1) \ell$, $\bar{e} \rightarrow V(-1) \bar{e}$, and $\bar{N} \rightarrow U_m^* \bar{N}$, it becomes

$$L \supset \ell \gamma(-1) \bar{e} H + \ell \gamma(-1) \gamma(0) \bar{N} H^* + \bar{N}^T D_m \bar{N},$$

(11)

and we identify the light neutrino Yukawa matrix:

$$Y_\nu = U(-1) \gamma(0) U_m^*.$$  

(12)

$Y_\nu$ serves as a key input for leptogenesis.

3. THERMAL LEPTOGENESIS IN THE NON-HIERARCHICAL MASS SPECTRUM

In this section we will briefly review the formalism of thermal leptogenesis relevant for our discussion later. Majorana neutrinos are produced in the early universe from Yukawa interactions of leptons and Higgs bosons in a thermal bath right below the very high reheating temperature $T_{RH} \lesssim 10^{15}$ GeV [29]. Any pre-existing asymmetry is washed out and the Majorana neutrinos are in thermal equilibrium. As the temperature falls below their mass, their overabundance above the equilibrium density prompts decays into leptons (with a decay width $\Gamma_{i\alpha}$) or into antileptons (with a decay width $\Gamma_{i\alpha}^\prime$). These $L$, $C$, and $CP$ processes go out of equilibrium as the decay rate becomes smaller than the expansion rate of the universe. At $10^2 \ll T$ (GeV) $\ll 10^{12}$, sphaleron processes, which violate both $B$ and $L$ but conserve $B-L$, are in equilibrium and convert part of the generated lepton asymmetry to the baryon asymmetry [18].

Leptogenesis is a battle between decays and inverse decays of the Majorana neutrinos. The minimal scenario involves a hierarchical mass spectrum, where the asymmetry generated by the decay of the heavier Majorana neutrinos is washed out as the temperature comes down to the scale of the lightest mass and the final baryon asymmetry is generated entirely from its decay. Such scenarios appear, for example, in $SO(10)$-inspired models [30], where the Majorana masses follow the hierarchy of the up-quark masses with a suppression of $O(\lambda^4)$ between families. For a non-hierarchical mass spectrum, however, one must consider the decay of all Majorana neutrinos, since the asymmetry generated by the decay of the heavier ones are not completely washed out [31].
All flavors of the leptons in the decay product can be considered identical as long as the lightest Majorana neutrino mass is above $10^{12}$ GeV, a scenario known as “unflavored leptogenesis”. Flavor plays an important role for smaller mass scales and can enhance the final asymmetry significantly [22, 23].

In the following, we will discuss both of these cases and express the relevant equations in terms of the seesaw parameters $Y_\nu$ and $M_i$.

3.1. “Flavored” leptogenesis

The evolution of number density of the Majorana neutrino $N_i$ is kinematically described by the following Boltzmann equation [31, 32]:

$$\frac{dN_i}{dz} = -(D_i + S_i)(N_i - N_{eq}^i),$$

(13)

where $z \equiv M_{min}/T$ and $M_{min} \equiv \min(M_i)$.

Introducing the notation $x_i \equiv M_i^2/M_{min}^2$ and $z_i \equiv z\sqrt{x_i}$, the equilibrium number density can be expressed in terms of the modified Bessel functions of the second kind [19]:

$$N_{eq}^i(z_i) = \frac{1}{2}z_i^2K_2(z_i),$$

(14)

so that $N_{eq}^i(z_i \ll 1) = 1$. The decay factor $D_i$ is given by [19, 33]

$$D_i \equiv \frac{\Gamma_{D,i}}{H(z_i)} \equiv K_i x_i \frac{K_1(z_i)}{K_2(z_i)},$$

(15)

where $\Gamma_{D,i} \equiv \Gamma_i + \bar{\Gamma}_i$ is the total decay rate and $H(z_i)$ is the Hubble expansion rate. The decay parameter $K_i$ is given by [32, 34]:

$$K_i \equiv \frac{\tilde{\Gamma}_{D,i}}{H(z_i = 1)} = \frac{(Y_\nu^i)^T Y_\nu}_ii M_i m_*,$$

(16)

where $\tilde{\Gamma}_{D,i} \equiv \Gamma_{D,i}(z_i = \infty)$ and $m_* \simeq 1.07$ meV is the effective neutrino mass [35]. The flavor-dependent decay parameter is defined as

$$K_{i\alpha} \equiv K_i P_{i\alpha},$$

(17)

where $P_{i\alpha} = \frac{|(Y_\nu)_i\alpha|^2}{\sum_\gamma |(Y_\nu)_i\gamma|^2}$

is the branching ratio for $N_i$ decaying into $\ell_\alpha$ [23]. If we limit our discussion to the scenario when $K_i \gg 1$, i.e., the so-called “strong washout” region, the dynamics can be explained well by considering only decays and inverse decays [19] and the $\Delta L = 1$ scattering term $S_i$ can be neglected.

The evolution of the $B - L$ asymmetry is split into individual equations for each flavor $\alpha = e, \mu, \tau$ [21]:

$$\frac{dN_{\Delta\alpha}}{dz} = -\sum_i \varepsilon_{i\alpha} D_i (N_{N_i} - N_{eq}^{N_i}) - N_{\Delta\alpha} \sum_i P_{i\alpha} W_i.$$

(18)
The flavor-dependent \( CP \)-asymmetry parameter \( \varepsilon_{i\alpha} \) represents the \( B-L \) asymmetry produced by the decay of each \( N_i \) into \( \ell_\alpha \) and Higgs. It is perturbatively calculated from the interference of the tree-level with the one-loop and the self-energy diagrams when \( |M_j - M_i|/M_i \gg \max[(Y^\dagger \nu Y_\nu)^{ij}]/(16\pi^2v^2) \) [28, 36]:

\[
\varepsilon_{i\alpha} \equiv \frac{\Gamma_{i\alpha} - \bar{\Gamma}_{i\alpha}}{\Gamma_i + \bar{\Gamma}_i} = \frac{1}{8\pi v^2} \sum_{j \neq i} \text{Im} \left[ \frac{(Y^\dagger \nu Y_\nu)^{ij}}{(Y^\dagger \nu Y_\nu)^{ii}} \right] \xi \left( \frac{x_j}{x_i} \right)
+ \frac{1}{8\pi v^2} \sum_{j \neq i} \text{Im} \left[ \frac{(Y^\dagger \nu Y_\nu)^{ij}}{(Y^\dagger \nu Y_\nu)^{ii}} \right] \zeta \left( \frac{x_j}{x_i} \right).
\]

(19)

Here \( v = 174 \) GeV is the Higgs VEV and the loop factors are given by

\[
\xi(x) = \sqrt{x} \left( 1 + x \right) \log \left( \frac{1 + x}{x} \right) + \frac{1}{x - 1} - 1, \quad \text{and} \quad \zeta(x) = \frac{1}{1 - x},
\]

(20)

which blow up if there is a mass degeneracy \( x_i = x_j \). Although exact degeneracies cannot generate \( CP \) asymmetry, nearly degenerate masses can significantly enhance the \( CP \) asymmetry leading to a scenario known as “resonant leptogenesis” [37].

The washout term \( W_i \) represents the washout of the generated asymmetry for each Majorana neutrino. Subtracting the resonant contribution from \( \Delta L = 2 \) processes \( (\ell_\alpha + H^* \leftrightarrow \bar{\ell}_\alpha + H) \) to the inverse decays, it is given by [31]

\[
W_i \equiv W_{ID}^i(z) = \frac{1}{4} K_i \sqrt{x_i} K_1(z_i) z_i^3.
\]

(21)

Solving the system of equations (13) and (18) yields the flavor-dependent asymmetry \( N_{\Delta\alpha} \). The final value of the \( B-L \) asymmetry is the summation of all flavor contributions:

\[
N_{fB-L} = \sum_{\alpha} N_{f\Delta\alpha}^f, \quad \text{and} \quad \eta_B = \frac{N_{fB-L}}{N_{\gamma}^{rec}} \approx 0.96 \times 10^{-2} N_{fB-L},
\]

where the sphaleron conversion coefficient is \( a_{sph} = 28/79 \) [18] and the baryon-to-photon number ratio at recombination is \( N_{\gamma}^{rec} \approx 37 \) [31]. Successful leptogenesis requires \( \eta_B \) to match the measured value in Eq. (1).

3.2. “Unflavored” leptogenesis

All flavor-dependent parameters are summed over the flavor index \( \alpha \) in “unflavored” leptogenesis. This yields the flavor-independent \( CP \)-asymmetry parameter [31, 32]:

\[
\varepsilon_i \equiv \frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i} = \frac{1}{8\pi v^2} \sum_{j \neq i} \text{Im} \left[ \frac{((Y^\dagger \nu Y_\nu)^{ij})^2}{(Y^\dagger \nu Y_\nu)^{ii}} \right] \xi \left( \frac{x_j}{x_i} \right).
\]

(23)

Eq. (13) still represents the evolution of number densities of Majorana neutrinos in the strong washout region. The flavor-independent \( B-L \) asymmetry is described by the
following Boltzmann equation [31, 32]:
\[
\frac{dN_{B-L}}{dz} = - \sum_i \varepsilon_i D_i (N_{N_i} - N_{N_i}^{eq}) - N_{B-L} \sum_i W_i^{ID}.
\] (24)

Eqs. (13) and (24) can be solved as coupled first order differential equations and their solution yields \(N_{B-L}\) in the unflavored case.

4. RELATING LOW ENERGY CP VIOLATION TO HIGH ENERGY CP ASYMMETRY

The relation between low energy CP violation in the PMNS matrix and high energy CP violation required for leptogenesis has been discussed extensively in literature [38]. In general, the existence of CP phases in the PMNS matrix do not guarantee CP asymmetry in unflavored leptogenesis. However, barring accidental cancellations, observation of low energy CP violation necessarily implies generation of the baryon asymmetry in flavored leptogenesis [39].

In the asymmetric texture, the only source for both Dirac and Majorana CP violation is the TBM phase \(\delta^3\), appearing in the matrix diag(1, 1, \(e^{i\delta}\)) multiplying the real TBM matrix from the left. In this section we will argue that this particular placement of the phase results in vanishing CP asymmetry in the unflavored case.

The seesaw matrix is given by
\[
S \equiv Y^{(0)} \mathcal{M}^{-1} Y^{(0)T} = \left[ D_{m}^{-1/2} U_m^0 Y^{(0)T} \right]^T \left[ D_{m}^{-1/2} U_m^0 Y^{(0)T} \right],
\] (25)
where \(D_{m}^{-1/2} \equiv \text{diag}(M_1^{-1/2}, M_2^{-1/2}, M_3^{-1/2}, M_4^{-1/2})\) is a diagonal matrix with all positive entries. Diagonalization of the seesaw matrix by the complex-TBM mixing implies
\[
S = \text{diag}(1, 1, \(e^{i\delta}\)) \ U_{TBM} D_{\nu} U_{TBM}^{T} \ \text{diag}(1, 1, \(e^{i\delta}\))
\] (26)
where \(D_{\nu} \equiv \text{diag}(m_1^{1/2}, m_2^{1/2}, m_3^{1/2}, m_4^{1/2})\). In general the entries in \(D_{\nu}\) can be either positive or negative. Comparing Eqs. (25) and (26), we find that \(D_{m}^{-1/2} U_m^0 Y^{(0)T}\) has the following form:
\[
D_{m}^{-1/2} U_m^0 Y^{(0)T} \equiv PW \text{diag}(1, 1, \(e^{i\delta}\)),
\] (27)
where \(W\) is a real matrix and \(P\) is a diagonal phase matrix with entries either 1 or \(i\) (so that \(P^T P = \text{diag}(\pm 1, \pm 1, \pm 1, \pm 1)\)).

It is useful to define an orthogonal matrix \(R\) in the Casas-Ibarra parametrization [40] to relate the low energy parameters to the high energy CP asymmetry:
\[
R \equiv D_{m}^{-1/2} U_m^0 Y^{(0)T} \text{diag}(1, 1, \(e^{-i\delta}\)) U_{TBM} D_{\nu}^{-1/2},
\] (28)

To clarify, it is related to but not the same as the Dirac phase \(\delta_{CP}\) in the PMNS matrix.
where $R$ is complex in general. Then, from Eq. (27),

$$R = PU_{TBM}D_{\nu}^{-1/2}. \quad (29)$$

In this parametrization, the neutrino Dirac Yukawa matrix can be written as, cf. Eq. (12):

$$Y_\nu = U_{PMNS}D_1^{1/2}R^TD_1^{1/2}, \quad (30)$$

where $U_{PMNS} = U_{-}^{-1}\text{diag}(1, 1, e^{i\delta})U_{TBM}$, so that

$$Y_\nu^\dagger Y_\nu = P^\dagger(D_1^{1/2}WW^T D_1^{1/2})P. \quad (31)$$

The relation between low energy $CP$ phases and high energy $CP$ asymmetry is evident from Eqs. (30) and (31). $CP$ asymmetry in unflavored leptogenesis depends on $\text{Im}[(Y^\dagger_\nu Y_\nu)^2]/(Y^\dagger_\nu Y_\nu)_{ii}$ for $j \neq i$, cf. Eq. (23). From Eq. (31), the diagonal elements of $Y_\nu^\dagger Y_\nu$ are real and the off-diagonal elements are either real or purely imaginary. Hence the $CP$-asymmetry parameter vanishes and the low energy $CP$ phases do not result in unflavored leptogenesis.

However, from Eq. (19), the $CP$-asymmetry parameter in flavored leptogenesis depends on $\text{Im}[(Y^\dagger_\nu Y_\nu)_{ij}]/(Y^\dagger_\nu Y_\nu)_{ii}$ for $j \neq i$. The $CP$ phases in the PMNS matrix do not vanish in $(Y^\dagger_\nu Y_\nu)_{ij}$, cf. Eq. (30), in general, and the $CP$-asymmetry parameter is nonzero. In the next section we will discuss flavored leptogenesis in more detail.

5. FLAVORED LEPTOGENESIS IN THE $SU(5) \times T_{13}$ MODEL

In this section we will employ the formalism developed so far to calculate the baryon asymmetry in the $SU(5) \times T_{13}$ model through flavored leptogenesis. We will match the calculated asymmetry to the observed value to constrain the undetermined model parameters $b_1, b_2, b_3$ and $m$.

The predictions for the light neutrino masses and neutrinoless double beta decay in this model do not depend on the particular value of $b_1, b_2, b_3$ except that they must be nonzero [8]. However, in the spirit of simplicity in vacuum alignments of the familons in the electroweak sector of the model [7], we are motivated to set $b_1, b_2$ and $b_3$ to be of the same order: $(b_1, b_2, b_3) \equiv b(1, f, 1)$, where $f \neq 1$ is an $O(1)$ prefactor. The Dirac Yukawa matrix becomes:

$$Y^{(0)} = \sqrt{bf_{\nu}} \begin{pmatrix} 0 & 1 & 0 & 2\beta \\ 1 & 0 & 0 & -\beta \\ 0 & -f^{-1}e^{i\delta} & \beta e^{i\delta} \\ \end{pmatrix}, \quad (32)$$

where $\beta \equiv \sqrt{\frac{am_\nu}{f_{\nu}}} \text{ and } a \equiv \frac{m}{b}$.

The Majorana matrix is given by

$$\mathcal{M} = b \begin{pmatrix} 0 & f & 1 & 0 \\ f & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & a \end{pmatrix}. \quad (33)$$
Its Takagi factorization, cf. Eq. (9), yields

$$M_1 = bf, \quad M_2 = \frac{b}{2} \left( \sqrt{f^2 + 8} - f \right), \quad M_3 = \frac{b}{2} \left( \sqrt{f^2 + 8} + f \right), \quad M_4 = ab,$$

(34)

and

$$U_m = \begin{pmatrix}
-\frac{i}{\sqrt{2}} & \frac{i}{2} \sqrt{1 - \frac{f}{\sqrt{f^2 + 8}}} & \frac{1}{2} \sqrt{1 + \frac{f}{\sqrt{f^2 + 8}}} & 0 \\
\frac{i}{\sqrt{2}} & \frac{i}{2} \sqrt{1 - \frac{f}{\sqrt{f^2 + 8}}} & \frac{1}{2} \sqrt{1 + \frac{f}{\sqrt{f^2 + 8}}} & 0 \\
0 & \frac{i}{2} \sqrt{1 + \frac{f}{\sqrt{f^2 + 8}}} & \frac{1}{\sqrt{2}} \sqrt{1 - \frac{f}{\sqrt{f^2 + 8}}} & 0 \\
0 & 0 & 0 & 1 
\end{pmatrix}. \quad (35)$$

For simplicity, we will limit our discussion to non-resonant thermal leptogenesis where the Majorana neutrino masses are required to be away from degeneracy. In Eq. (34), $M_1$ and $M_2$ are degenerate for $f = 1$, which justifies our assumption $f \neq 1$. $f$ lifts the degeneracy and makes leptogenesis viable.

For concreteness, we will set $f = 2$ for the remainder of our discussion whenever a numerical value is required\footnote{We require $f \sim \mathcal{O}(1)$ to avoid hierarchy among components of the VEVs of the familons, inspired from the VEVs of the electroweak familons of the model presented in Ref. [7]. We have verified that the final results relevant for leptogenesis are in the same order of magnitude as long as $f \sim \mathcal{O}(1)$.}. This leaves us with two undetermined parameters $b$ and $a$, and yields the following mass spectrum:

$$\frac{M_1}{b} = 2, \quad \frac{M_2}{b} = \sqrt{3} - 1, \quad \frac{M_3}{b} = \sqrt{3} + 1, \quad \frac{M_4}{b} = a.$$

(36)

Since only $M_4$ depends on $a$, it can be degenerate with $M_1$, $M_2$ and $M_3$ for $a = 2$, $\sqrt{3} - 1 \approx 0.73$ and $\sqrt{3} + 1 \approx 2.73$, respectively, as shown in Figure 1. To avoid resonant enhancement near degeneracies we split the parameter space into four regions: (i) $0.1 \leq a \leq 0.65$, (ii) $0.8 \leq a \leq 1.9$, (iii) $2.1 \leq a \leq 2.65$, and (iv) $a \geq 2.85$. These regions represent particular mass ordering of the Majorana neutrinos. For example, region (ii) corresponds to $M_2 < M_4 < M_1 < M_3$. In this section we will discuss leptogenesis in the regions (ii) and (iii), shown as shaded in Figure 1. We will argue in section 6 that the sign of the generated asymmetry in the other two regions is in tension with the sign of the low energy $CP$ phases.

We assume that there is no asymmetry present in any flavor in the universe before the decay of the Majorana neutrinos occur: $N_{\Delta \alpha}(z = 0) = 0$, and the reheating temperature of inflation is sufficiently higher than the mass of the heaviest Majorana neutrino, so that the asymmetry generated by the heavier ones is not washed out prior to the decay of the lightest one.

The parameters of the Boltzmann equations are expressed in terms of the undetermined model parameters $a$ and $b$. We will treat $a$ as a free parameter and $b$ as the overall
mass scale and rescale all leptogenesis parameters as a function of $a$ only, so that the Boltzmann equations can be numerically solved for the range of $a$ in regions (ii) and (iii).

The neutrino Yukawa matrix $Y_\nu$ is proportional to the factor $\sqrt{b}$, cf. Eqs. (12) and (32). Suppose $b \sim 10^{11}$ GeV, a reasonable scale for the Majorana neutrino masses. This yields $\sqrt{b f m_\nu} \sim \mathcal{O}(1)$ GeV. For convenience we will consider $Y_\nu$ to be determined up to the factor $\sqrt{b f m_\nu}$ so that $Y_\nu/\sqrt{b f m_\nu}$ is a dimensionless matrix.

The decay parameter $K_{i\alpha}$ is given by:

$$K_{i\alpha} = P_{i\alpha} K_i \equiv \frac{|(Y_\nu)_{i\alpha}|^2}{\sum_\gamma |(Y_\nu)_{i\gamma}|^2} \frac{(Y_\nu \bar{Y}_\nu)_{ii}}{M_i m_\star}$$

so that the dependence on $b$ is cancelled out. An explicit calculation yields

$$K_1 = \frac{m_\nu}{m_\star} \approx 57.8, \quad K_2 = \frac{m_\nu}{m_\star} \frac{f (1 - f^2) + (f^2 + 1) \sqrt{f^2 + 8}}{f \sqrt{f^2 + 8} \left(\sqrt{f^2 + 8} - f\right)} \approx 64.5,$$

$$K_4 = \frac{6 m_\nu}{m_\star} \approx 30.2, \quad K_3 = \frac{m_\nu}{m_\star} \frac{f (f^2 - 1) + (f^2 + 1) \sqrt{f^2 + 8}}{f \sqrt{f^2 + 8} \left(\sqrt{f^2 + 8} + f\right)} \approx 35.6,$$

justifying the strong washout approximation.

The $CP$-asymmetry parameter $\varepsilon_{i\alpha}$ can be determined up to $bfm_\nu$:

$$\varepsilon_{i\alpha} = \frac{1}{8 \pi v^2} \sum_{j \neq i} \frac{\text{Im} \left[ (Y_\nu^* \sqrt{bfm_\nu})_{ij} (Y_\nu \sqrt{bfm_\nu})_{i\alpha} (Y_\nu^* \sqrt{bfm_\nu})_{j\alpha} (Y_\nu \sqrt{bfm_\nu})_{ij} \right]}{((Y_\nu \sqrt{bfm_\nu})_{i\alpha} (Y_\nu \sqrt{bfm_\nu})_{i\alpha} (Y_\nu \sqrt{bfm_\nu})_{ij} (Y_\nu \sqrt{bfm_\nu})_{ij})} \xi \left(\frac{x_i^2}{x_f^2}\right)$$

$$+ \frac{1}{8 \pi v^2} \sum_{j \neq i} \frac{\text{Im} \left[ (Y_\nu^* \sqrt{bfm_\nu})_{ij} (Y_\nu \sqrt{bfm_\nu})_{i\alpha} (Y_\nu^* \sqrt{bfm_\nu})_{j\alpha} (Y_\nu \sqrt{bfm_\nu})_{ij} \right]}{((Y_\nu \sqrt{bfm_\nu})_{i\alpha} (Y_\nu \sqrt{bfm_\nu})_{i\alpha} (Y_\nu \sqrt{bfm_\nu})_{ij} (Y_\nu \sqrt{bfm_\nu})_{ij})} \xi \left(\frac{x_i^2}{x_f^2}\right).$$
since \( x_i \equiv M_2^2/M_{\text{min}}^2 \) is independent of \( b \).

The Boltzmann equations can be rewritten in terms of \( a \)-dependent parameters as

\[
\frac{dN_{N_i}}{dz} = -K_i x_i z \frac{K_1(z \sqrt{x_i})}{K_2(z \sqrt{x_i})} (N_{N_i} - N_{N_i}^{eq}), \quad i = 1, 2, 3, 4
\]

(38)

\[
\frac{d}{dz} \left( \frac{N_{\Delta \alpha}}{b f m_\nu} \right) = \sum_i \frac{\varepsilon_i a}{b f m_\nu} \frac{dN_{N_i}}{dz} - \frac{N_{\Delta \alpha}}{b f m_\nu} \sum_i \frac{1}{4} K_i a x_i^2 K_1(z \sqrt{x_i}) z^3.
\]

(39)

We solve the Boltzmann equations (38) numerically to calculate the number densities \( N_{N_i}(z) \), assuming first thermal initial abundance \( N_{N_i}(z = 0) = N_{N_i}^{eq}(z = 0) \), and then dynamical initial abundance \( N_{N_i}(z = 0) = 0 \). The results are shown in Figure 2 for two representative cases: (a) \( a = 1.4 \) for region (ii), and (b) \( a = 2.4 \) for region (iii). In both cases, the number densities at \( z \gg 1 \) are identical for both initial conditions.

The number densities are then fed into the Boltzmann equation (39) to calculate the \( B - L \) asymmetry for the individual flavors \( \alpha \) up to the factor \( b f m_\nu \) for the initial condition \( N_{\Delta \alpha}(z = 0) = 0 \). Their sum yields the total asymmetry, up to \( b f m_\nu \), as shown in Figure 2. The final asymmetry does not depend on the initial conditions.

Replicating the analysis for the regions \( 0.8 \leq a \leq 1.9 \) and \( 2.1 \leq a \leq 2.65 \), we calculate the final \( B - L \) asymmetry up to the factor \( b f m_\nu \). The results are shown in Figures 3a and 3c, respectively.

The final \( B - L \) asymmetry is related to the final baryon asymmetry \( \eta_B \) through Eq. (22). The parameter \( b \) is determined by demanding that \( \eta_B \) equals the central value of the observed baryon asymmetry from the CMB in Eq. (1):

\[
b = \frac{6.12 \times 10^{-10}}{0.96 \times 10^{-2} \times f m_\nu \times \left( |N_{B-L}^f|/(b f m_\nu) \right)}.
\]

(40)
Figure 3. Final value of the $B - L$ asymmetry up to the factor $b f m_\nu$ for $f = 2$ in the regions $0.8 \leq a \leq 1.9$ and $2.1 \leq a \leq 2.65$. The parameter $b$ is fitted to match the calculated baryon asymmetry to the observed value $6.12 \times 10^{-10}$. Majorana neutrino masses are determined with the fitted $b$.

All Majorana neutrino masses can now be calculated as functions of $a$, cf. Eq. (34). Figures 3b and 3d show the mass spectrum in GeV for the two regions. For successful leptogenesis, the Majorana neutrino masses are of $O(10^{-9} - 10^{-12})$ GeV and $O(10^9 - 10^{11})$ GeV, respectively, in these regions.

6. SIGN OF THE $CP$ PHASES AND THE BARYON ASYMMETRY

In the asymmetric texture, the TBM phase is determined as $\delta \simeq \pm 78^\circ$ \[4\] from the requirement to match the observed reactor angle.\[5\] The sign ambiguity in $\delta$ is not resolved from the physics of the electroweak sector. In this section we will explore the possibility of relating this sign to the sign of the baryon asymmetry.

The dependence on $\delta$ comes through the neutrino Yukawa matrix $Y_\nu$, which appears

\[5\] See appendix for a discussion on the robustness of the leptogenesis results when $\delta$ is allowed to vary in the range that still reproduces all three PMNS angles within $3\sigma$ of their PDG value.
in the decay parameter $K_{i\alpha}$ and the $CP$-asymmetry parameter $\varepsilon_{i\alpha}$. From Eqs. (30) and (31), it goes away in $|(Y_\nu)_{i\alpha}|^2$ and $(Y_\nu^* Y_\nu)_{ii}$; hence $K_{i\alpha}$ in Eq. (17) does not depend on $\delta$. This implies that the number density of the Majorana neutrinos in the first Boltzmann equation, Eq. (13), can be determined independently of $\delta$.

To see the $\delta$-dependence of the $CP$-asymmetry parameter $\varepsilon_{i\alpha}$ in Eq. (19), we write $Y_\nu$ as

$$Y_\nu = U^{(-1)*} \text{diag}(1, 1, e^{i\delta}) W^{T} PD_{m}^{1/2}$$

following Eqs. (29) and (30). Then Eqs. (41) and (31) yield

$$\text{Im} \left[ (Y_\nu^*)_{ai}(Y_\nu)_{aj} (Y_\nu^T Y_\nu)_{ij} \right] = P^2_{ii} P^2_{jj} (D_m)_{ai} (D_m)_{aj} \sum_{\beta,\gamma,\kappa} U^{(-1)*}_{\beta\alpha} U^{(-1)*}_{\gamma\alpha} W_{i\beta} W_{j\gamma} W_{i\kappa} W_{j\kappa} \times \text{Im} \left[ (\text{diag}(1, 1, e^{-i\delta}))_{\beta\beta} (\text{diag}(1, 1, e^{i\delta}))_{\gamma\gamma} \right],$$

where $j \neq i$. The imaginary part on the right-hand side is nonzero, and proportional to $\sin \delta$, when either $\beta = 3$ or $\gamma = 3$. Similar arguments apply for $\text{Im} \left[ (Y_\nu^*)_{ai}(Y_\nu)_{aj} (Y_\nu^T Y_\nu)_{ji} \right]$. Therefore the $CP$-asymmetry parameter in Eq. (19) is sensitive to the sign of $\delta$.

Individual flavored $B - L$ asymmetries depend on the $CP$-asymmetry parameters, since the solution to the second Boltzmann equation, Eq. (18), can be written as:

$$N_{\Delta a}(z) = \sum_{i} \varepsilon_{i\alpha} \int_{0}^{z} dz' \frac{dN_{i\alpha}}{dz'} e^{-\sum_{i} \int_{0}^{z} dz'' P_{i\alpha W_{i}^{D}}(z'')}.$$

The integral contains parameters from the first Boltzmann equation and is independent of $\delta$. The final $B - L$ asymmetry is the sum of the flavor-dependent components and the baryon asymmetry is related to the final $B - L$ asymmetry by Eq. (22). Hence the baryon asymmetry is proportional to $\sin \delta$, and demanding that the calculated asymmetry has a positive sign fixes the sign of $\delta$.

For $\delta \simeq -78^\circ$, the Dirac $\mathcal{CP}$ phase and the Jarlskog-Greenberg invariant [41] predicted by the asymmetric texture are [4]:

$$\delta_{\mathcal{CP}} = 1.32\pi, \quad J = -0.028,$$

compared to the latest PDG global fit $\delta_{\mathcal{CP}} = 1.37 \pm 0.17\pi$ at $1\sigma$ [1]. Hence, the sign of low energy $CP$ violation and high energy baryon asymmetry would be consistent with data if the generated asymmetry is positive for negative $\delta$.

Numerically solving the Boltzmann equations for the four non-degenerate regions in the parameter space, as shown in Figure 1, we find that both the sign of the generated baryon asymmetry and the sign of $\delta_{\mathcal{CP}}$ are compatible with data in the regions (ii) and (iii). This distinguishes a particular mass ordering of the Majorana neutrinos: $M_2 < M_4 < M_3$, and implies

$$\frac{1}{2}(\sqrt{f^2 + 8} - f) < a < \frac{1}{2}(\sqrt{f^2 + 8} + f),$$

further constraining $a$. For $f = 2$, this condition translates into $0.73 < a < 2.73$. 
With $\delta \simeq -78^\circ$, the sign of the Majorana invariants [42] is also fixed and the invariants are given by [8]

\[ \mathcal{I}_1 = -0.106, \quad \mathcal{I}_2 = -0.011. \] (46)

Although there are still no strict bound on the Majorana phases from current experiments [43], the prediction for $\delta_{CP}$ in the asymmetric texture is consistent with the current PDG fit. Recently $\delta_{CP} = 0$ has been excluded by the T2K experiment at $3\sigma$ level [44], and upcoming experiments DUNE [45] and Hyper-K [46] are expected to measure $\delta_{CP}$ with $5\sigma$ precision in the next decade.

7. CONCLUDING REMARKS

In this paper we have investigated non-resonant thermal leptogenesis in the context of the asymmetric texture in the $SU(5) \times T_{13}$ model proposed in Refs. [4, 7, 8]. Baryon asymmetry is generated through the decay of four right-handed Majorana neutrinos and is intimately related to the single $\mathcal{CP}$ phase in the TBM seesaw mixing. The sign and magnitude of the asymmetry constrains the parameter space of the model and resolves the sign ambiguity in the TBM phase.

We have shown that low energy $\mathcal{CP}$ violation does not yield any high energy $\mathcal{CP}$ asymmetry in this model when all flavors of the generated leptons are considered equivalent. This happens because the only source of $\mathcal{CP}$ violation, the TBM phase, is introduced in a diagonal matrix and does not enter in the calculation of the $\mathcal{CP}$ asymmetry. However, flavored leptogenesis remains viable and the low energy $\mathcal{CP}$ phase generates non-vanishing $\mathcal{CP}$ asymmetry.

The conventional analysis of thermal leptogenesis assumes a hierarchical mass spectrum of the Majorana neutrinos, where the asymmetry generated by the heavier ones are washed out completely and only the decay of the lightest one yields the baryon asymmetry. Such a generic picture does not apply to the model discussed in this paper as the mass spectrum is non-hierarchical in the parameter space of interest. We have considered the decay of all four Majorana neutrinos in flavored leptogenesis and the resulting Boltzmann equations have been solved numerically. Our calculation of the baryon asymmetry relates the previously undetermined parameters of the model and determines the masses of the Majorana neutrinos to be of $\mathcal{O}(10^9 - 10^{12})$ GeV.

We have illustrated that the unresolved sign of the TBM phase is related to the mass ordering of the Majorana neutrinos. Simultaneously requiring the sign of baryon asymmetry and the sign of the Dirac phase $\delta_{CP}$ to be compatible with data further constrains the model parameters and yields $M_2 < M_4 < M_3$. The discussion in this paper has been limited to thermal leptogenesis in the strong washout regime, where the dynamics are described by simpler Boltzmann equations considering only decays and inverse decays. However, the mass spectrum in the model offers richer phenomenology. The parameter space includes regions of nearly degenerate Majorana neutrinos, where the $\mathcal{CP}$ asymmetry is enhanced resonantly. This can further lower
the required mass scale for reproducing the observed baryon asymmetry, even to the TeV scale. The discussion of resonant leptogenesis in this model remains out of the scope of this paper and will be addressed in a future work.

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Appendix A: Robustness of the results with respect to $\delta$

The only source of low energy $CP$ violation in the asymmetric texture is the TBM phase $|\delta| = 78^\circ$, whose magnitude was determined in Ref. [4] to match the reactor angle to its 2018 PDG central value [17]. In this appendix we investigate if this value of $\delta$ is contained in the range that reproduces all three PMNS angles within 3$\sigma$ of their 2020 PDG central values and if the leptogenesis results derived in section 5 are robust with respect to the variation of $\delta$ within this range.

The dependence of the PMNS angles on $\delta$ is shown in Figure 4. The shaded regions represent the 3$\sigma$ range of the latest PDG fit [1]. For $66^\circ \leq \pm \delta \leq 85^\circ$, all three PMNS angles are within 3$\sigma$ of their PDG central value. The corresponding range for the Dirac $CP$ phase is $1.27\pi \leq \mp \delta_{CP} \leq 1.35\pi$, consistent with the PDG fit $\delta_{CP}^{PDG} = 1.37 \pm 0.17\pi$ [1]. Hence the results of Ref. [4] are compatible with the 2020 PDG data.

We now investigate how the leptogenesis results are impacted if $\delta$ is allowed to vary. The correct signs of the baryon asymmetry and the Dirac $CP$ phase are achieved when $\delta$ is negative and $0.8 \leq a \leq 1.9$ or $2.1 \leq a \leq 2.65$. We show the $B-L$ asymmetry up to the factor $bfm_\nu$ for $-85^\circ \leq \delta \leq -66^\circ$ in Figure 5. The generated asymmetry and therefore, the Majorana neutrino mass spectrum, are in the same order of magnitude compared to the values calculated for $\delta = -78^\circ$ in Figure 3. This shows that the results are robust with respect to the variation in $\delta$.
(a) $-85^\circ \leq \delta \leq -66^\circ$, $0.8 \leq a \leq 1.9$ and $f = 2$  
(b) $-85^\circ \leq \delta \leq -66^\circ$, $2.1 \leq a \leq 2.65$ and $f = 2$

Figure 5. Final $B - L$ asymmetry up to the factor $bfm_{\nu}$. The upper boundary of the shaded regions represents $\delta = -66^\circ$ and the lower boundary represents $\delta = -85^\circ$. 
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