Design of a control with multiple inputs multiple outputs by decoupling

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Abstract. In industry, most processes can be modeled by a system of equations that can be monovariable or multivariable. In our case, we will present a multivariable system more adjusted to real cases and on which a control system is available. This article presents a decoupling control method for two inputs two outputs through a centralized multivariable controller using a decoupling network and a diagonal controller, this technical advantage allows reduction in computing time. It shows the methodology by which a multivariate system can be represented as the composition of different coupled systems in this way that the control of the variables involved in it can be done separately, facilitating the control system, this implies a modification in the outputs to be affected. In the same way, the two-inlet with 2-outlet distillation system was also studied.

1. Introduction
Multivariate systems are prevalent in the process industry and by their very nature involve the need for forms as an automatic control problem. In the present work, a control system with several inputs and outputs is executed, where the distillate compositions are configured with the input variables, the flow in the upper part and the heat flow in the lower part. This exchange of temperatures occurs due to the entropic processes that exist in thermodynamic systems. This last feature makes the plant's control system even more difficult since the temperature gradients are highly non-linear.

The system we are working on is called a boiler, a two-by-two interaction system, similar to these are the so-called mixing systems, widely used when there are multiple inputs and multiple outputs that are coupled, this coupling is generally built in a type of circuit series, this type of plant is common to see in the industry for its multiple applications and for its low-cost regarding reliability [1].

It should be noted that the aforementioned systems have a great difference due to the rapid deterioration of the plant itself, since the variables imply high and rapid variations in temperature and molecular exchanges are more severe, for this reason the behavior of the plant and the control technique to be implemented must be planned with a greater degree of characteristics that implicitly imply the nature of the problem, some of these are: response speed, efficiency, stability, therefore they must be designed controllers that are capable of bringing the system from unstable states to stable states, all
through the state variables that for this purpose will be decoupled in order to guarantee greater computational efficiency and therefore real.

2. Mathematical model
In the following section the mathematical description of the control model necessary for the implementation will be presented. First the block diagram Figure 1 is presented in which the multivariable control system is described, in which it can be visualized the plant to be studied in the frequency domain the control system, the input, output and error signals which are necessary for the description of the system, later the transfer functions which are necessary for describe the system in a domain more suitable to work.

**Figure 1.** Multivariable control system.

Here, we have the transfer function that relates the inputs and outputs of the system (Equation (1) to Equation (7)) [1]. As the first step, the transfer function must be found, which relates the input to the output as seen in the following Equation (1) and Equation (2).

$$
H(s) = \begin{bmatrix}
H_{11}(s) & \cdots & H_{1q}(s) \\
H_{21}(s) & \cdots & H_{2q}(s) \\
\vdots & \ddots & \vdots \\
H_{pq}(s) & \cdots & H_{pq}(s)
\end{bmatrix},
$$

(1)

$$
C(s) = \begin{bmatrix}
C_{11}(s) & \cdots & C_{1p}(s) \\
C_{21}(s) & \cdots & C_{2p}(s) \\
\vdots & \ddots & \vdots \\
C_{q1}(s) & \cdots & C_{qp}(s)
\end{bmatrix},
$$

(2)

Equation (3) shows us the control output which is related to the control function.

$$
U(s) = \begin{bmatrix}
U_1(s) \\
\vdots \\
U_q(s)
\end{bmatrix}.
$$

(3)

Next, the system of equations in the frequency domain is described in which it is more appropriate than the time domain since in it we can analyze the differential equations as algebraic equations, Equation (4) and Equation (5).

$$
U(s) = C(s)E(s),
$$

(4)

$$
Y(s) = H(s)U(s).
$$

(5)

The output of the system is described as a column vector, there the information provided by the plant is stored Equation (6).

$$
Y(s) = \begin{bmatrix}
Y_1(s) \\
\vdots \\
Y_p(s)
\end{bmatrix}.
$$

(6)
Finally, the reference signal which is useful for the controller to take a guide is described through Equation (7).

\[
R(s) = \begin{bmatrix} R_1(s) \\ \vdots \\ R_p(s) \end{bmatrix},
\]

(7)

The variables \( p \) and \( q \) correspond to the number of inputs and outputs respectively. As mentioned in Equation (1) to Equation (7) which are necessary to be able to describe the system in the frequency domain and thus be able to implement an adequate control system which for our case is multivariable since it has two inputs and two outputs. This type of control is more robust in that it presents manipulation of several signals in a plant, this type of investigation is of great importance since they are the ones that are most required in the experience in the industry.

3. Control by decoupling

In the decentralized control, the manipulated or control variables \((u)\) and the controlled variables \((y)\) are ungrouped in different sets. Then, these sets are coupled to produce non-overlapping pairs for which regulators are designed that operate in a completely independent way. The design problem is trivial when the interactions (static or dynamic) between the inputs and outputs of different pairs are weak. It is well known that strong interactions can even cause that it is not possible to achieve stability or good performance with a decentralized control structure. The centralized control is basically a control structure in which all the measured variables can influence the calculation of all the control actions, that is, all the control actions are available to counteract the effect of any disturbance in any of the variables controlled. Model-based control is among the most representative methods of centralized control [2]. These control structures make centralized control using a process model to calculate the control actions.

Next, as shown in Figure 2, a diagram is represented in which the possible variants that are used are described about the control systems, in our case the one that will be used in this article is shown through the black line research, which is a coupled nonlinear multivariable system, to solve the technique based on diagonal decoupling.

![Figure 2. Control system.](image)

Centralized multivariable control techniques seek decoupling between controlled variables and input or reference signals. The decoupling control has two different approaches: using a decoupling network and a diagonal controller or using a purely centralized controller. Figure 3 shows the block diagram of a control system by decoupling with decoupling network. The decoupling or the compensator \( D(s) \), and the diagonal controller \( C'(s) \), it can be observed that said decoupling will allow to have two plants which will be easier to control due to the simplicity that is generated.
Figure 3. Control system by decoupling.

where \( C(s) \) is defined according to Equation (8).

\[
C(s) = D(s)C'(s).
\]

(8)

The diagonal decoupled matrix is also defined by Equation (9).

\[
H'(s) = H(s)D(s).
\]

(9)

The compensating block \( D(s) \), or decoupling network, is designed with the intention of eliminating, or at least reducing, the process interactions, in such a way that the decentralized controller \( C(s) \) manipulates the variables \( V(s) \) instead of the \( U(s) \) variables. With this configuration it is intended that the controller see the new apparent process \( Q(s) \) as a set of \( n \) totally independent processes or with much less interaction, for which the decentralized control \( C(s) \) would be designed. The multivariable controller resulting from this design would be composed of the diagonal control and the decoupling network. Next, we present the analysis and deduction of the equations to determine the parameters of the coupling network and the diagonal controller [3,4].

The network or compensation matrix is defined below, Equation (10).

\[
D(s) = \begin{bmatrix} 1 & D_{12}(s) \\ D_{21}(s) & 1 \end{bmatrix}.
\]

(10)

For the system transfer matrix given by Equation (11).

\[
H(s) = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix}.
\]

(11)

We have that the uncoupled diagonal matrix \( H'(s) \) is Equation (12).

\[
H'(s) = \begin{bmatrix} H'_{11}(s) & 0 \\ 0 & H'_{22}(s) \end{bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \begin{bmatrix} 1 & D_{12}(s) \\ D_{21}(s) & 1 \end{bmatrix}.
\]

(12)

From where the values of said matrix are obtained in terms of the transfer matrix and the compensation matrix, where the mutual coupling that the system of Equation (13) and Equation (14) has outside the main diagonal can be observed and own in Equation (15) and Equation (16) [5].

\[
0 = H_{12}(s) + H_{22}(s)D_{21}(s) \rightarrow D_{21}(s) = -\frac{H_{21}(s)}{H_{12}(s)}.
\]

(13)

\[
0 = H_{11}(s)D_{12}(s) + H_{12}(s) \rightarrow D_{12}(s) = -\frac{H_{12}(s)}{H_{11}(s)}.
\]

(14)

\[
H'_{11} = H_{11}(s) + H_{12}(s)D_{21},
\]

(15)

\[
H'_{22} = H_{21}(s)D_{12}(s) + H_{22}(s).
\]

(16)

In the same way, the system for the matrix of the coupled controller is presented in terms of the compensation matrix and the diagonal controller (Equation (17) to Equation (21)).
\[
C(s) = \begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{bmatrix} = \begin{bmatrix} 1 & D_{12}(s) \\ D_{21}(s) & 1 \end{bmatrix} \begin{bmatrix} C'_1(s) & 0 \\ 0 & C'_2(s) \end{bmatrix},
\]

\[
C_{11}(s) = C'_1(s),
\]
\[
C_{12}(s) = D_{12}(s)C'_2(s),
\]
\[
C_{21}(s) = D_{21}(s)C'_1(s),
\]
\[
C_{22}(s) = C'_2(s).
\]

4. Analysis and results

Next, the analysis will be run in a two-input distillation system with 2 outputs, and we will implement decoupled control in it. The following block diagram (Figure 4) represents a decoupled control system, it can be seen that the outputs are associated with their required transfer function, this technical advantage allows the reduction in computing time and the ease of finding solutions according to systems with multiple inputs and outputs. The idea of decoupling is a methodological proposal that allows us to be more numerically efficient because the input signal does not advance in cascade but now advances in parallel allowing simultaneous work and on simpler systems.

![Figure 4: Control system decoupled TITO.](image)

The functions of the transfer matrix for a distillation system [6] are given by (Table 1).

| \( H_{11} \) | \( H_{12} \) | \( H_{21} \) | \( H_{22} \) |
|----------------|----------------|----------------|----------------|
| \( \frac{-2.2e^{-3}}{1+7s} \) | \( \frac{-2.8e^{-1.8s}}{1+9.5s} \) | \( \frac{1.3e^{-0.35}}{1+7s} \) | \( \frac{4.3e^{-0.355}}{1+9.2s} \) |

The parameters of the compensation network are Equation (22) and Equation (23).

\[
D_{21}(s) = \frac{-H_{21}(s)}{H_{22}(s)} = \frac{1.3e^{-0.35}}{4.3e^{-0.355}} = \frac{-1.3e^{-0.35}(1+9.2s)}{4.3e^{-0.355}(1+7s)}. \tag{22}
\]

\[
D_{12}(s) = \frac{-H_{12}(s)}{H_{11}(s)} = \frac{1.3e^{-0.35}}{-2.2e^{-3}} = \frac{2.8e^{-1.8s}(1+7s)}{2.2e^{-9}(1+9.5s)}. \tag{23}
\]

In Figure 5, the theory built with specific data for the transfer functions and for the input signals, which will be unit steps, is implemented due to their immediate jump in an instant of time, which implies a challenge for the controller.

In Figure 6 you can see the output signals that are obtained from implementing the system of Figure 5 in them you can see how the control signals stabilize in the advance of time.
The controller matrix for the decoupled system is obtained [7,8] through the closed loop shown in Figure 7 for the transfer functions $H_{11}$ and $H_{22}$.

In Figure 7 it can be seen unlike Figure 4 how the decoupled block diagram is described, in which it can be seen how the transfer functions change to be more efficient when finding the outputs of the system. For the diagonal control matrix, the following values have been obtained (Table 2) [11,12].

Unlike Figure 5, the system in Figure 8 is decoupled which allows greater efficiency.
5. Conclusions
A multivariable proportional integral differential (PID) controller design methodology for two input and two output systems has been presented. The design is carried out in two steps: in the first, a decoupling network with integral action is designed, which minimizes the effects of the interaction, and the system has a zero-position error. In a second step, the decoupling network is approximated by a network of delayed PID controllers. In this way, a completely centralized controller matrix is achieved and easily transferable to commercial distributed control systems.

The decoupled control strategy allows to control a process with multiple input and multiple output (MIMO) in a more efficient way, depending on the values of the gains involved. The result is best for variations in the reference signal. For the decoupling strategy, the results for changes in the reference are those predicted by the theory, the inapplicability of this strategy was verified for the case studied, but with a general projection, for the disturbances.

References
[1] Duarte M, Sepúlveda F, Castillo A, Contreras A, Lazcano V, Gimenez P, Castelli L 1999 A comparative experimental study of five multivariable control strategies applied to a grinding plant Power Technology 104 1-28
[2] Garrido J 2012 Diseño de Sistemas de Control Multivariable por Desacoplo con Controladores PID (Spain: Universidad de Córdoba)
[3] Bristol E 1996 On a new measure of interaction for multivariable process control IEEE Transactions on Automatic Control 11(1) 133-134
[4] Cai W J, Ni W, He M J, Ni C Y 2008 Normalized decoupling-A new approach for MIMO process control system design Industrial and Engineering Chemistry Research 47(19) 7347-7356
[5] Quintero C, Oñate J, Jimenez J 2018 Control Automático Aplicado (Colombia: Universidad del Norte)
[6] Bristol E H 1966 On a new measure of interaction for multivariable process control IEEE Transactions on Automatic Control 11(1) 133-134
[7] Calderón Osorio J C Control de Procesos Multivariables Mediante un Sistema de Control Distribuido Modificado (Colombia: Universidad Nacional de Colombia)
[8] Chen J, He Z F, Qi X 2011 A new control method for MIMO first order time delay non-square systems Journal of Process Control 21 538–546
[9] Astrom K, Hagglund T 1995 PID Controllers: Theory, Design and Tuning (North Carolina: Instrument Society of America)
[10] Llata J R, Oria J P, Sarabia E G, Arce J, Robles A 2006 Control Predictivo de Tanques Acoplados (Spain: Universidad de Cantabria)
[11] Zhang W, et al. 2016 Multivariable disturbance observed-based H2 analytical decoupling control design for multivariable systems International Journal of Systems Science 47 179-193
[12] He G, et al. 2015 Decoupling control design for the module suspension control system in maglev Mathematical Problems in Engineering 2015 865650:1-13