Capture reactions into borromean two-proton systems at rp-waiting points

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Abstract. We investigate even-even two-proton borromean systems at prominent intermediate heavy waiting points for the rapid proton capture process. The most likely single-particle levels are used to calculate three-body energy and structure as a function of proton-core resonance energy. We establish a linear dependence between two- and three-body energies with the same slope, but the absolute value slightly dependent on partial wave structure. Using these relations we estimate low-lying excited states in the isotones following the critical waiting points. The capture rate for producing a borromean bound state is described based on a full three-body calculation for temperatures about $0.1 - 10$ GK. In addition, a simple rate expression, depending only on a single resonance state, is found to comply with the three-body calculation for temperatures between 0.1 and 4 GK. The rate calculations are valid for both direct and sequential capture paths. As a result the relevant path of the radiative capture reactions can be determined.

1. Motivation and purpose
The rapid-proton capture (rp) process is the process whereby nuclei in stellar environments capture an increasing number of protons, and become increasingly heavier \cite{1}. Naturally, these nuclei eventually reach the proton dripline where the process is hindered, and further evolution must wait for the much slower $\beta$-decay. This typically occurs in the accretion disk of a close binary system containing a neutron star \cite{2}. These nuclei along the dripline are called waiting points, and the waiting points with the longest lifetime are called the critical waiting points. Another option, instead of waiting for a natural $\beta$-decay, is to skip the gap in the proton dripline by capturing two protons, which would affect the effective lifetime. This effective lifetime of the critical waiting points plays a vital part in determining the overall time scale and relative distributions of elements in the system \cite{3}. The critical waiting points among the relatively heavy nuclei, are $^{64}$Ge, $^{68}$Se, and $^{72}$Kr \cite{2}, where $^{68}$Se is thought to be most important \cite{4}.

These systems are ideal candidates to be studied from a three-body perspective, as adding one proton to the critical waiting point creates an unbound system, but adding two protons would create a bound system, which is the very definition of a borromean system. Another argument in favour of a three-body treatment is that the first excited state in the critical waiting points are 0.9 \cite{5}, 0.9 \cite{6}, and 0.7 \cite{7} MeV for $^{64}$Ge, $^{68}$Se, and $^{72}$Kr respectively, which means they can be considered fairly inert.
This work will focus on studying the proton capture on the critical waiting points from a three-body perspective. More elaborate considerations on the same subject are presented in Ref. [1]. The purpose of this is two-fold; First of all we will examine the relation between two- and three-body energies in the system. Secondly, we will use this in combination with very limited experimental information to estimate the reaction rate of the proton capture process.

2. Theory and method

The few-body formalism applied is the so-called hyperspheric, adiabatic expansion of the three-body Faddeev equations in coordinate space. In the interest of brevity the details of this method will not be explained here, but the interested reader is referred to Refs. [1, 8]. Briefly explained the idea is to introduce one hyperradial, \( r \), and five hyperangular, \( \Omega \), coordinates, and then solve the angular Faddeev equations for given \( r \). This is then used to solve the radial part. The idea is very similar to the idea behind the Born-Oppenheimer approximation.

Using this formalism the only thing that must be chosen is the three two-body interactions in the system. As we are dealing with a core and two protons, there are only two unique two-body interactions. For the proton-proton interaction a well-established phenomenological interaction, presented in Ref. [9], is used. For the core-proton interaction a Woods-Saxon potential with a central and a spin-orbit part is used,

\[
V(r) = V_C(r) + \frac{V_0}{1 + e^{(r-R)/a}} + 1 \cdot s \cdot \frac{d}{dr} \frac{V_0^{ls}}{1 + e^{(r-R_{ls})/a_{ls}}},
\]

where \( R, R_{ls}, a, \) and \( a_{ls} \) are radial shape and diffuseness parameters kept constant at \( R = 7.2, R_{ls} = 6.3, a = 0.65 \) and \( a_{ls} = 0.5 \) all in fm. The only free parameters are then the strengths \( V_0 \) and \( V_0^{ls} \). The intent is to isolate only the partial waves that are most relevant using the free strength parameters. Based on shell model calculations the critical waiting points are known to be situated in the middle of the \( fpg \)-shell. However, recent experiments show that the \( g_{9/2} \) orbital is not important around \( A = 70 \) and \( N \simeq Z \) [10]. The \( f_{5/2} \) and \( p_{3/2} \) orbitals are then assumed to be dominating, which is also indicated by the mirror nuclei [11]. To form a negative parity state, and to see the effect of the electric dipole transition \((E1)\), an \( d_{5/2} \) orbital is at times used instead of \( f_{5/2} \).

3. Two- and three-body energies

Using \( V_0^{ls} \) to push up the undesired spin-orbit partner, and using \( V_0 \) to settle on a specific orbital energy, the relation between two- and three-body energies can be studied. This is seen in Fig. 1 for \( ^{68}\text{Se} + p + p \).

The single-particle energies of \( p \) and \( f \) (or \( p \) and \( d \)) are always identical in Fig. 1. If, for instance, the energy of the \( f \) orbital was increased above the energy of \( p \), the black and the green lines would be shifted up by the same amount. The reason for the energy shift being identical is explained in Ref. [1]. However, at some point they would coincide with the corresponding lines with only \( p \) and they would not move further if the energy of \( f \) was increased even more. In other words, there is a limit to how much the black and the green line can be moved, and the excitation energy is basically unchanged. The red line is limited in the same way, if the energy of \( d \) is increased. But the blue line would move up without limit, if the energy of \( d \) was increased, as both \( p \) and \( d \) is needed to form the negative parity state. As a result the excitation energy for excitations to the \( 1^- \) state is not constant, and can increase without limit, depending on the single-particle energies. This has profound consequences for the reaction rate.

As the borromean region of interest is confined by \( 0 < E_{2h} \) and \( E_{3b} < 0 \), Fig. (1) places significant constraints on the possible energy levels. Recently, it was possible to measure the proton separation energy of \( ^{69}\text{Br} \) \((^{68}\text{Se} + p)\) to be 0.641 MeV, and the lowest state looked to be an \( f_{5/2} \) state [12]. The energy of \( p_{3/2} \) is then limited by the borromean region in Fig. 1.
Maxwell-Boltzmann distribution is what is finally calculated, in a stellar environment, with temperature $T$. The interested reader is referred to Refs. [1, 13]. Seeing that the proton capture takes place with the discretized three-body continuum state. The proton has to tunnel through the long-range Coulomb barrier, while the photon decay width depends on the short range properties, in particular through the overlap of the final three-body waves, as well as for the one-step $\gamma$ three-body transition $p+p+c\rightarrow A+\gamma$. This depends on the photodissociation cross section, which in turn depends on $\langle \Psi_0|\Theta_f|\psi^{(i)}_e \rangle$.

If the resonances are well separated and very narrow, they can be approximated by a Breit-Wigner shape, in which case the integral can be solved analytically. The final, energy average reaction rate would then become

$$\langle R_{ppc}(E) \rangle = 4\pi^3(2\ell+1)\hbar^5 \frac{\Gamma_{eff}(E_R)}{(\mu c \gamma c_{ppc})^{3/2}} \frac{T^3}{E_R} \exp(-E_R/T),$$

where $\ell$ is the electric multipole order, while the effective width, $\Gamma_{eff}$, is given by the proton, $\Gamma_{ppc}$, and the photon, $\Gamma_{\gamma}$, decay width by $\Gamma_{eff}(E) = (\Gamma_{ppc}(E))^{-1} + (\Gamma_{\gamma}(E))^{-1}$, and the total width is $\Gamma = \Gamma_{ppc} + \Gamma_{\gamma}$. The proton decay width depends on the long range properties, as the proton has to tunnel through the long-range Coulomb barrier, while the photon decay width depends on the short range properties in particular through the overlap of the final three-body state with the discretized three-body continuum state.

The two-body energy is chosen to be 0.641 MeV for both $p$ and $f$ (or $d$). The reaction rates are then calculated according to Eqs. (2) and (3). The results are shown in Fig. 2, where the full lines are from the full calculation of Eq. (2), the dashed lines are from the approximation in Eq. (3) using only the lowest resonance state and nothing else, and the dotted line is also from Eq. (3) using the spectrum of resonance states up to 5 MeV. There is only one $1^-$ resonance state. It should be noted how well the approximation fits with the full calculation using only a single resonance. It should also be noted that the dipole transition has been reduced by a factor of $10^4$ to make it fit in the same figure. However, increasing the energy of the $d_{5/2}$ state to just 3 MeV makes the Coulomb barrier unable to support the $1^-$ resonance, and the rate then drops by many orders of magnitude, making the quadrupole transition the dominating transition.

This result is surprisingly comprehensive, as explained in greater detail in Ref. [1]. The only significant uncertainty is the two-body energy of the lowest $p_{3/2}$ state.

4. Reaction rates

As with the three-body formalism, the reaction rate theory will not be explained in detail. The interested reader is referred to Refs. [1, 13]. Seeing that the proton capture takes place in a stellar environment, with temperature $T$, the energy averaged rate over the corresponding Maxwell-Boltzmann distribution is what is finally calculated,

$$\langle R_{ppc}(E) \rangle = \frac{1}{2T^3} \int E^2 R_{ppc}(E) \exp(-E/T) dE,$$

where $R_{ppc}$ is the reaction rate for the one-step $\gamma$ three-body transition $p+p+c\rightarrow A+\gamma$. This depends on the photodissociation cross section, which in turn depends on $\langle \Psi_0|\Theta_f|\psi^{(i)}_e \rangle$.

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5. Deduced reaction mechanism and adjusted reaction rate

Often, when dealing with three-body capture reactions, it is necessary to assume that the process is either a so-called direct or a sequential process. With the current method not only are both
types of reactions included, as well as combinations of the two, it is actually possible to deduce the reaction mechanism from the calculations.

Figure 2. (Color online) The reaction rates for the dipole and quadrupole transition for $^{68}$Se + $p + p \rightarrow ^{70}$Kr + $\gamma$ where the allowed single-particle orbitals are adjusted to an energy of 0.641 MeV. The result of the full calculations from Eq. (2) is given by the solid lines. The dashed lines in the same colour are the result of applying Eq. (3) to the lowest resonance. The dotted line is the sum of contributions for spectrum of resonances below 5 MeV using Eq. (3). The $E1$ rates have been scaled down by $10^4$ to make the figure readable.

Figure 3. (Color online) The square of the angular wavefunction mentioned in Sec. 2, multiplied by the phase factor $\cos^2(\alpha) \sin^2(\alpha)$ and integrated over $\Omega_x$ and $\Omega_y$, as a function of hyperradius $\rho$ and hyperangle $\alpha$ for first angular wave function of the $2^+$ state in the set of Jacobi coordinates with first (x) coordinate between core and proton.

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