Pricing-based edge caching resource allocation in fog radio access networks

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Pricing-based edge caching resource allocation in fog radio access networks

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Abstract: The edge caching resource allocation problem in Fog Radio Access Networks (F-RANs) is investigated. An incentive mechanism is introduced to motivate Content Providers (CPs) to participate in the resource allocation procedure. We formulate the interaction between the cloud server and the CPs as a Stackelberg game, where the cloud server sets nonuniform prices for the Fog Access Points (F-APs) while the CPs lease the F-APs for caching their most popular contents. Then, by exploiting the multiplier penalty function method, we transform the constrained optimization problem of the cloud server into an equivalent non-constrained one, which is further solved by using the simplex search method. Moreover, the existence and uniqueness of the Nash Equilibrium (NE) of the Stackelberg game are analyzed theoretically. Furthermore, we propose a uniform pricing based resource allocation strategy by eliminating the competition among the CPs, and we also theoretically analyze the factors that affect the uniform pricing strategy of the cloud server. We also propose a global optimization-based resource allocation strategy by further eliminating the competition between the cloud server and the CPs. Simulation results are provided for quantifying the proposed strategies by showing their efficiency in pricing and resource allocation.

Key words: fog radio access networks; edge caching; resource allocation; Stackelberg game; nonuniform pricing; Nash equilibrium; competition

1 Introduction

Driven by the dramatic growth of intelligent devices and mobile applications, wireless networks have been suffering unprecedented data traffic pressure in recent years. Numerous repetitive downloads and redundant transmissions occur when vast and various User Equipments (UEs) request the same contents. Advanced network architectures, such as Cloud Radio Access Networks (C-RANs), have been developed to relieve the traffic pressure and improve the quality of service. Centralized cloud caching and computing in C-RANs can provide reliable and stable service for UEs. However, ever-increasing mobile data traffic brings tremendous pressure on C-RANs that have capacity-limited fronthaul links and centralized baseband unit pools, which may cause communication interruptions or traffic congestions especially at peak traffic moments[1]. As a complementary network architecture, Fog Radio Access Network (F-RAN) has been further developed and attracted increasing attention[2-7]. In F-RANs, Fog Access Points (F-APs) equipped with edge caching and computing resources can effectively accommodate data traffic pressure by caching popular contents in their local storages. However, limited edge caching resources restrict performance improvement. Therefore, efficient edge caching resource allocation becomes challenging and indispensable.

Recently, researchers have been paying much attention to the edge caching resource allocation problem
from different aspects\cite{8-11}. In Ref. [8], the authors studied the problem of joint resource allocation and content caching to improve the quality of service and relieve backhaul congestion by utilizing radio and content storage resources. In Ref. [9], the problem of joint caching, channel assignment, and interference management was formulated to maximize the system throughput in coordinated small-cell cellular networks. In Ref. [10], the resource allocation problem was formulated to maximize the energy efficiency under the constraints of transmit power, caching status, and fronthaul capacity. In Ref. [11], the problems of user association, caching strategy, and power allocation were formulated to optimize cross-tier interference. As stated previously, resources are limited, thereby leading to competition among the participants in the resource allocation procedure. However, most of the above works on resource allocation ignore the competition among the participants in the resource allocation procedure.

Game theory has the capability to stress the competition relationship and has been extensively utilized to solve the competitive resource allocation problem\cite{12-18}. In Ref. [12], the problem of multi-user computation offloading for mobile cloud computing was investigated under a dynamic environment. In Ref. [13], a distributed iterative power control algorithm was proposed to handle the power control problem for interference management. In Ref. [14], the two-tier game theoretic framework with static and dynamic pricing models was proposed to allocate spectrum resource in femtocell networks. In Ref. [15], a cooperative bargaining game was formulated in cognitive small-cell networks to solve the joint uplink sub-channel and power allocation problem. In Ref. [16], the distributed power and channel allocation problem was formulated in cognitive femtocell networks by using coalitional game. In Ref. [17], a Stackelberg game was formulated to model the interaction between Small-cell Base Station (SBS) and multiple Content Providers (CPs). In Ref. [18], a pricing-based resource allocation strategy was proposed to minimize the transmission latency and mitigate the redundant transmission in small-cell cellular networks. However, most of the above works on resource allocation do not take the spatial distribution of SBSs into consideration and do not consider the influence of competition elimination on the corresponding resource allocation strategies.

Motivated by the aforementioned discussions, we propose pricing-based edge caching resource allocation strategies in F-RANs. Our main contributions are summarized as follows:

1. We introduce an incentive mechanism to motivate the CPs to participate in the edge caching resource allocation procedure. The optimization problems of the cloud server and the CPs are formulated by considering the spatial distribution of the F-APs. Then, the interaction between them is formulated as a Stackelberg game. The competition among the CPs is further formulated as a non-cooperative sub-game because of resource limitations.

2. We propose the nonuniform pricing based resource allocation strategy to maximize the profits of the cloud server and the CPs. The constrained optimization problem of the cloud server is transformed into an equivalent non-constrained one by exploiting the multiplier penalty function method. Furthermore, the existence and uniqueness of the Nash Equilibrium (NE) are analyzed theoretically.

3. We propose the uniform pricing based resource allocation strategy by eliminating the competition among the CPs, and we also theoretically analyze the factors that affect the uniform pricing strategy of the cloud server. Then, we propose the global optimization-based resource allocation strategy by further eliminating the competition between the cloud server and the CPs.

4. We validate the efficiency of the proposed three resource allocation strategies through simulations. The nonuniform pricing based strategy can efficiently improve the cache hit rate of each CP, whereas the uniform pricing based strategy can reduce the computational complexity. In comparison, the global optimization-based strategy can achieve the largest average cache hit rate of all CPs.

The rest of this paper is organized as follows. In Section 2, the system model is briefly described. In Section 3, the profit functions are modeled and the Stackelberg game is formulated. The nonuniform pricing based resource allocation strategy is presented in Section
4. In Section 5, the uniform pricing-based and global optimization-based edge caching resource allocation strategies are presented, respectively. Simulation results are shown in Section 6, and final conclusions are drawn in Section 7.

2 System model

As illustrated in Fig. 1, we consider the F-RAN including one cloud server, multiple F-APs, and \( N \) CPs, which are denoted by \( \mathcal{N} = \{1, 2, \ldots, n, \ldots, N\} \). CP \( n \) owns \( C_n \) contents, and the contents owned by different CPs have different popularity distributions. UEs make independent requests of the \( c \)-th content with the popularity \( p_{n,c} \) owned by CP \( n \). Generally, the content popularity follows a Zipf distribution\(^{[19]}\). Correspondingly, \( p_{n,c} \) can be expressed as follows:

\[
p_{n,c} = \frac{1/e^{\beta_n}}{\sum_{c=1}^{C_n} 1/e^{\beta_c}}
\]

where \( \beta_n \) denotes the content preference parameter that reflects the steepness of the popularity distribution of the contents owned by CP \( n \) with a positive value. According to Eq. (1), we can readily establish: \( p_{n,1} > p_{n,2} > \cdots > p_{n,c} > \cdots > p_{n,C_n} \).

Assume that F-APs are spatially distributed as a Homogeneous Poisson Point Process (HPPP) with intensity \( \lambda^{[20,21]} \), where \( \lambda \) denotes the number of F-APs per unit area. Let \( \tau_n \) denote the fraction of the F-APs leased by CP \( n \) from the cloud server. Obviously, we can establish \( 0 \leq \tau_n \leq 1 \) and \( \sum_{n=1}^{N} \tau_n \leq 1 \). The leased F-APs for CP \( n \) are selected with equal probability. Each of the leased F-APs for CP \( n \) can cache at most \( Q_n \) contents. Therefore, the distribution of these leased F-APs can be modeled as thinned HPPP with intensity \( \tau_n \lambda \). Assume that each F-AP can serve UEs within the region of radius \( R \). Let \( H_n \) denote the probability that any UE is covered by any F-AP leased by CP \( n \). Then, \( H_n \) can be expressed as follows\(^{[22]}\):

\[
H_n = 1 - \exp\left(-\pi \tau_n \lambda R^2\right)
\]

If the \( c \)-th content has been cached in the leased F-APs of CP \( n \), its cache hit rate can be expressed as follows:

\[
H_{n,c} = p_{n,c} \left[1 - \exp\left(-\pi \tau_n \lambda R^2\right)\right]
\]

3 Problem formulation

In this section, we firstly model the profits of the cloud server and the CPs, respectively. Then, we formulate the edge caching resource allocation problem as a Stackelberg game. To motivate the CPs to participate in the resource allocation procedure, we introduce an incentive mechanism which is regulated by the pricing strategy of the cloud server. In other words, the cloud server can attract the CPs to lease the F-APs by setting proper prices. Moreover, the NE of the Stackelberg game is investigated.

3.1 Profit of the cloud server and CPs

For the cloud server, the profit comes from leasing F-APs to the CPs. Let \( s_n \) denote the nonuniform price set by the cloud server for leasing the F-APs to CP \( n \) and define \( s = [s_1, s_2, \ldots, s_n, \ldots, s_N]^{T} \) and \( \tau = [\tau_1, \tau_2, \ldots, \tau_n, \ldots, \tau_M]^{T} \). Then, the profit function of the cloud server can be modeled as follows:

\[
P_c(s, \tau) = \sum_{n=1}^{N} \tau_n s_n
\]

For each CP, the profit consists of two parts: (1) the gain brought by the cache hit rate and (2) the cost of leasing F-APs from the cloud server. To achieve the maximal cache hit rate, CP \( n \) will cache the most popular contents in its leased F-APs. Therefore, the profit function of CP \( n \) can be modeled as follows:

\[
P_n(s_n, \tau_n) = \sum_{c=1}^{Q_n} H_{n,c} - s_n \tau_n
\]

3.2 Formulation of the Stackelberg game

Generally, the Stackelberg game is a strategic game that consists of a leader and several followers that compete with each other for certain resources\(^{[23]}\). The leader

---

**Fig. 1** Illustration of the F-RAN.
moves firstly, and the followers move subsequently. In this work, there are two kinds of players, the cloud server and the CPs, competing for the edge caching resource. Therefore, we formulate their interaction as a Stackelberg game, where the cloud server controlling the F-APs is the leader and the CPs willing to lease the F-APs are the followers. The CPs strictly compete with each other in a non-cooperative fashion due to resource limitation. Therefore, the sub-game of the proposed Stackelberg game is modeled as a non-cooperative game. Firstly, the cloud server sets the leasing price vector \( \mathbf{s} \) for the CPs. Then, according to the determinate leasing price vector \( \mathbf{s} \), the CPs determine the leasing fraction vector \( \mathbf{r} \).

### 3.2.1 Optimization problem of the cloud server

The objective of the cloud server is to maximize its profit as formulated in Eq. (4). It is obvious that the leasing prices of the F-APs set by the cloud server influence the fraction of F-APs that each CP tends to lease. Correspondingly, the optimization problem of the cloud server can be formulated as follows:

\[
\begin{align*}
\max \quad & P_{c} (\mathbf{s}, \mathbf{r}) \\
\text{s.t.} \quad & s_{n} \geq 0, \quad \forall n \in \mathcal{N}
\end{align*}
\]  

(6)

### 3.2.2 Optimization problem of the CPs

The objective of each CP is to maximize its profit as formulated in Eq. (5). For CP \( n \), once the leasing price of F-APs is determinate, its profit completely depends on the leasing fraction \( \tau_{n} \). If CP \( n \) leases more F-APs from the cloud server, it can cache more contents in its leased F-APs which will increase the cache hit rate and hence bring more gain. However, the cost increases with the leasing fraction \( \tau_{n} \). Correspondingly, the optimization problem of CP \( n \) can be formulated as follows:

\[
\begin{align*}
\max \quad & P_{n} (s_{n}, \tau_{n}) \\
\text{s.t.} \quad & 0 \leq \tau_{n} \leq 1, \quad \forall n \in \mathcal{N}, \\
& \sum_{n=1}^{N} \tau_{n} \leq 1
\end{align*}
\]  

(7)

Generally, the optimization problems in Formulas (6) and (7) formulate a Stackelberg game.

### 3.3 Nash equilibrium

The objective of the Stackelberg game is to find the NE point where neither the cloud server nor the CPs have incentives to deviate\(^{18,24} \). For the proposed Stackelberg game, the NE is defined below.

**Definition 1** Let \( \mathbf{s}^{*} \) and \( \mathbf{r}^{*} \) denote the two solutions of the optimization problems in Formulas (6) and (7), respectively. Define \( \mathbf{s}^{*} \triangleq [s_{1}^{*}, s_{2}^{*}, \ldots, s_{n}^{*}, \ldots, s_{N}^{*}]^{T} \) and \( \mathbf{r}^{*} \triangleq [r_{1}^{*}, r_{2}^{*}, \ldots, r_{n}^{*}, \ldots, r_{N}^{*}]^{T} \). An NE point represents an equilibrium status that normal solutions will approach. Then, \( (\mathbf{s}^{*}, \mathbf{r}^{*}) \) is an NE point for the proposed Stackelberg game if the following conditions are satisfied:

\[
\begin{align*}
&P_{c} (\mathbf{s}^{*}, \mathbf{r}^{*}) \geq P_{c} (\mathbf{s}, \mathbf{r}^{*}) , \\
&P_{n} (s_{n}^{*}, \tau_{n}^{*}) \geq P_{n} (s_{n}, \tau_{n}) , \quad \forall n \in \mathcal{N}
\end{align*}
\]  

(8)

Generally, the NE for a Stackelberg game can be obtained by finding its sub-game perfect NE. For the proposed Stackelberg game, its sub-game, which formulates the competition among the followers, is non-cooperative, and its NE is defined as the operating point at which no player can improve its utility by changing its strategy unilaterally\(^{18} \). Therefore, we can define the NE of the proposed Stackelberg game as shown in Formula (8).

### 4 Nonuniform pricing based edge caching resource allocation strategy

In this section, we will propose a nonuniform pricing based edge caching resource allocation strategy to solve the above formulated optimization problems. Since the proposed Stackelberg game is a two-layer game, we propose to exploit the backward induction method to obtain the NE\(^{25} \). Correspondingly, the optimal solution of the sub-game should be obtained at first, i.e., the leasing fraction of the F-APs for each CP should be obtained firstly by solving the optimization problem in Formula (7). Since there is only one leader in the proposed Stackelberg game, the nonuniform pricing of the cloud server can be obtained sequentially by solving the optimization problem in Formula (6).

#### 4.1 Leasing fractions of the F-APs

It can be readily verified that the objective function in Formula (7) is strictly concave with respect to \( \tau_{n} \). Therefore, with the given leasing price vector \( \mathbf{s} \), the optimal leasing fraction of the F-APs for CP \( n \) can be
After some mathematical manipulations, the profit with a given indicator $s$ will increase its leasing price where $F$ in the following equivalent form:

$$
F = e^{-\pi \lambda R^2} \prod_{n=1}^{N} F_n.
$$

The constrained optimization problem in Formula (12) involves two logarithmic functions and one constraint in the form of continued product, which requires a great computational burden to solve by using the traditional sub-gradient method\cite{26}. In general, the exterior penalty function method can be used to eliminate the constraint in a convex optimization problem\cite{27}. However, the solution obtained by solving the transformed non-constrained optimization problem may not satisfy the given constraint. Furthermore, as the penalty factor approaches to infinity, the corresponding Hessian matrix becomes infinite, which will bring a heavy computation burden. Therefore, we propose to transform the constrained optimization problem in Formula (12) into a non-constrained one by using the multiplier penalty function method, which can eliminate the constraint with a fixed penalty factor\cite{28}.

### 4.2 Nonuniform pricing of the cloud server

Substitute the optimal solution in Eq. (9) into Eq. (4). After some mathematical manipulations, the profit function of the cloud server can be reexpressed as follows:

$$
P_c(s) = \sum_{n=1}^{N} \frac{1}{\pi \lambda R^2} \xi_n (s_n - s_n \ln s_n)$$

where $\xi_n$ denotes the indicator with $\xi_n = 1$ if $s_n \leq F_n$ and $\xi_n = 0$ otherwise. The profit function in Eq. (10) is non-convex with respect to $s_n$ due to $\xi_n$. However, with a given indicator $\xi_n$, it can be readily verified that the above function is convex. At the beginning of the proposed game, all CPs will participate in the resource allocation procedure. Therefore, in the following, we commence that $\xi_n = 1, \forall n \in \mathcal{N}^{[19]}$. To further simplify the optimization problem, we define $\tilde{P}_c(s)$ as follows:

$$
\tilde{P}_c(s) = \sum_{n=1}^{N} (-s_n \ln F_n + s_n \ln s_n)
$$

By considering the constraint $\sum_{n=1}^{N} \tau_n^* \leq 1$, the optimization problem in Formula (6) can be re-expressed in the following equivalent form:

$$\min_{s} \tilde{P}_c(s), \quad \text{s.t.} \quad \prod_{n=1}^{N} s_n \geq F$$

where $F = e^{-\pi \lambda R^2} \prod_{n=1}^{N} F_n$.

4.2.1 Elimination of the constraint

Firstly, variable $y$ is introduced to transform the inequality constraint into the equality one. Then, the optimization problem in Formula (12) can be expressed in the following equivalent form:

$$\min_{s} \tilde{P}_c(s), \quad \text{s.t.} \quad \left( \prod_{n=1}^{N} s_n - F \right) - y^2 = 0$$

Let $\phi(s, w, \sigma, y)$ denote the augmented Lagrangian function of the above optimization problem, where $w$ and $\sigma$ denote the Lagrangian multiplier and the penalty factor, respectively. Then, it can be expressed as follows:

$$
\phi(s, w, \sigma, y) = \sum_{n=1}^{N} (-s_n \ln F_n + s_n \ln s_n) - w \left( \prod_{n=1}^{N} s_n - F - y^2 \right) + \frac{1}{2} \sigma \left( \prod_{n=1}^{N} s_n - F - y^2 \right)^2
$$

By applying the method of completing the square\cite{28}, $y^2$ can be calculated as follows:

$$
y^2 = \left( \frac{N}{\prod_{n=1}^{N} s_n - F - \frac{w}{\sigma}} \right)^+$$

Substitute Eq. (15) into the augmented Lagrangian function in Eq. (14). Then, we have
\[
\phi(s, w, \sigma) = \sum_{n=1}^{N} (-s_n \ln F_n + s_n \ln s_n) + \\
\frac{1}{2\sigma} \left\{ \left( w - \sigma \sum_{n=1}^{N} s_n + \sigma F \right)^2 - w^2 \right\}
\]  
(16)

Correspondingly, the constrained optimization problem in Formula (13) can be transformed into the non-constrained one as follows:

\[
\min \phi(s, w, \sigma)
\]
(17)

where

\[
\phi(s, w, \sigma) = \begin{cases}
g_1(s, w, \sigma), \sum_{n=1}^{N} s_n - F \geq w / \sigma; \\
g_2(s, w, \sigma), \sum_{n=1}^{N} s_n - F < w / \sigma
\end{cases}
\]
(18)

\[
g_1(s, w, \sigma) = \sum_{n=1}^{N} (-s_n \ln F_n + s_n \ln s_n) - w^2 / 2\sigma
\]
(19)

\[
g_2(s, w, \sigma) = \sum_{n=1}^{N} (-s_n \ln F_n + s_n \ln s_n) + \\
\frac{1}{2\sigma} \left( w - \sigma \sum_{n=1}^{N} s_n + \sigma F \right)^2 - w^2 
\]
(20)

Then, we have the following theorem.

**Theorem 1** The non-constrained optimization problem in Formula (17) is equivalent to the constrained one in Formula (12).

**Proof** The optimization problems in Formulas (12) and (13) are obviously equivalent. According to Ref. [29], the optimal solution of the non-constrained optimization problem in Formula (17) is equivalent to the locally optimal solution of the constrained optimization problem in Formula (12). Based on the definition of \( \hat{P}_c(s) \) in Eq. (11), we can readily establish

\[
\frac{\partial^2 \hat{P}_c(s)}{\partial s_n^2} = \frac{1}{s_n} > 0, \forall n \in \mathcal{N}
\]
(21)

\[
\frac{\partial^2 \hat{P}_c(s)}{\partial s_n \partial s_{\bar{n}}} = 0, \forall n, \bar{n} \in \mathcal{N}, n \neq \bar{n}
\]
(22)

Therefore, the Hessian matrix of \( \hat{P}_c(s) \) is positive definite and \( \hat{P}_c(s) \) is strictly convex. Correspondingly, the locally optimal solution of the constrained optimization problem in Formula (12) is its globally optimal solution[28].

### 4.2.2 Nonuniform pricing of the cloud server

To obtain the optimal solution of the optimization problem in Formula (17), the Lagrange multiplier \( w \) needs to be updated iteratively to revise the augmented Lagrangian function[28]. Let \( w(t) \) denote the Lagrange multiplier for the \( t \)-th iteration. According to the multiplier penalty function method[28], the iterative relationship between \( w(t+1) \) and \( w(t) \) can be established as follows:

\[
w(t+1) = \left( w(t) - \sigma \left( \prod_{n=1}^{N} s_n - F \right) \right)^+
\]
(23)

Since the penalty factor \( \sigma \) is fixed in the multiplier penalty function method, \( s \) is the only variable in both \( g_1(s, w, \sigma) \) and \( g_2(s, w, \sigma) \) after the Lagrange multiplier \( w \) is updated in each iteration. For \( g_1(s, w, \sigma) \), it is a convex function with respect to \( s_n \). Consequently, the optimal solution can be obtained by taking the first derivative with respect to \( s_n \). Let the first derivative equal zero. Then, \( s_n \) can be obtained as follows:

\[
s_n = F_n / e, \quad \forall n \in \mathcal{N}
\]
(24)

where \( e \) is the base of a natural logarithm. For \( g_2(s, w, \sigma) \), we propose to use the simplex search method, which can solve the non-constrained optimization problem without derivation and with low computational complexity[28]. The details of the nonuniform pricing procedure of the cloud server are presented in Algorithm 1.

**Algorithm 1 Nonuniform pricing of the cloud server**

1: procedure Nonuniform Pricing
2: Initialization: \( \varepsilon, \Delta = \varepsilon, \sigma, t = 0, w^{(t)} \).
3: while \( \Delta \geq \varepsilon \) do
4: if \( \prod_{n=1}^{N} s_n - F > w^{(t)} / \sigma \), then
5: Update \( s_n \) according to Eq. (24);
6: else
7: Update \( s_n \) via the simplex search method;
8: end if
9: Update \( w^{(t+1)} \) according to Eq. (23);
10: \( \Delta = |w^{(t+1)} - w^{(t)}| \);
11: \( t \leftarrow t + 1 \);
12: end while
13: end procedure
4.3 Existence and uniqueness of the NE

NE offers a predictable and stable outcome for the Stackelberg game. However, there may be several NEs in one game. For the proposed Stackelberg game, we have the following theorem.

**Theorem 2** There exists one and only one NE point for the proposed Stackelberg game.

**Proof** As stated previously, we propose to use the backward induction method to obtain the NE. Therefore, we need to prove that there exists only one NE for the non-cooperative sub-game at first and then verify the only NE of the proposed Stackelberg game.

According to Ref. [30], for the non-cooperative sub-game, the NE exists if the strategy space of each player is a non-empty and closed-bounded set in the Euclidean space. Note that the profit function $P_n(s_n)$ for CP $n$ is continuous and convex with respect to $s_n$. Correspondingly, the strategy space of each CP is a non-empty and closed-bounded convex set in the Euclidean space. Therefore, the existence of the NE in the sub-game can be proved. The Hessian matrix of $P_n(s_n, \tau_n)$ can be easily verified to be negative definite. Correspondingly, the profit function for each CP is strictly concave, and the optimization problem in Formula (7) has a unique optimal solution. Therefore, the uniqueness of the NE for the sub-game can then be proved.

There is only one leader in the proposed Stackelberg game, and the profit function of the leader is convex as stated previously, which means that the optimal solution of the optimization problem in Formula (6) exists uniquely. Consequently, the existence and uniqueness of the NE for the proposed Stackelberg game can be proved. ■

5 Uniform pricing based and global optimization based edge caching resource allocation strategies

In this section, we propose two other edge caching resource allocation strategies. To eliminate the competition among the CPs, we propose a uniform pricing based resource allocation strategy. To further eliminate the interaction between the cloud server and the CPs, we propose a global optimization-based resource allocation strategy.

5.1 Uniform pricing based strategy

For the proposed nonuniform pricing based edge caching resource allocation strategy, the competition among the CPs is formulated as a non-cooperative game due to resource limitations. However, the competition will make the edge caching resources concentrate on a few CPs that own more popular contents due to the unbalanced content popularity distributions among different CPs.

To solve the above issue, we propose the uniform pricing based strategy to eliminate the competition among the CPs by setting the same leasing price for the F-APs, i.e., $s = s_1 = s_2 = \cdots = s_N$. According to Eq. (9), the leasing fraction $\tau_n$ for CP $n$ can be expressed as follows:

$$\tau_n = \left[\ln\left(\frac{F_n}{s}\right) / \pi \lambda R^2\right]^+$$

Substitute Eq. (25) into Eq. (4). Then, the profit function of the cloud server can be reexpressed as follows:

$$P_c(s) = \sum_{n=1}^{N} \ln F_n - N \ln s$$

Taking the constraint $\sum_{n=1}^{N} \tau_n \leq 1$ into account, the optimization problem of the cloud server in Formula (6) can be reexpressed as follows:

$$\max_s P_c(s), \quad \text{s.t. } s \geq \exp\left(\sum_{n=1}^{N} \ln F_n - \pi \lambda R^2\right) / N$$

After some mathematical manipulations, the uniform pricing strategy of the cloud server can be obtained as follows:

$$s = \exp\left(\sum_{n=1}^{N} \ln F_n - \min \{N, \pi \lambda R^2\}\right) / N$$

It can be seen from Eq. (28) that the pricing strategy of the cloud server has a close relationship with $F_n$. As shown previously, there is a positive correlation between $F_n$ and the storage capacity $Q_n$ of the F-APs. Therefore, the storage capacity of the F-APs is one factor that affects the uniform pricing strategy of the cloud server. When the F-APs have larger storage capacity, the cloud server will increase the leasing price. The reason is that
the F-APs with larger storage capacity can cache more contents. Therefore, the CPs can obtain a higher cache hit rate when they lease the same fraction of the F-APs, which will motivate them to lease more F-APs. The competition among the CPs will become intensive and the cloud server will then benefit from increasing the leasing price of the F-APs.

It can also be seen from Eq. (28) that the number of the participant CPs \( N \) is another factor that affects the uniform pricing strategy of the cloud server. Correspondingly, we have the following theorem.

**Theorem 3** If a new CP, which owns more popular contents than any other CP in the current game, participates in the resource allocation procedure, the uniform price set by the cloud server will increase.

**Proof** According to Eq. (28), we consider the following two cases: \( N < \pi \lambda R^2 \) and \( N \geq \pi \lambda R^2 \). In each case, we will calculate the price increment brought by the new CP at first. Then, we will show how this new CP affects the uniform pricing strategy of the cloud server.

1. If \( N < \pi \lambda R^2 \), after some mathematical manipulations, the uniform price can be reexpressed as follows:

   \[
   s_1(N) = \exp \left( \frac{1}{N} \sum_{n=1}^{N} \ln \frac{\sum_{c=1}^{Q_n} p_{n,c}}{\ln (\pi \lambda R^2)} + \ln \frac{\sum_{n=1}^{N} \ln \sum_{c=1}^{Q_n} p_{n,c}}{1} \right)
   \]

   Let \( h_1(N) = \ln [s_1(N)] \). Then, \( h_1(N) \) can be expressed as follows:

   \[
   h_1(N) = \frac{1}{N} \sum_{n=1}^{N} \ln \frac{\sum_{c=1}^{Q_n} p_{n,c}}{\ln (\pi \lambda R^2)} + \ln \frac{\sum_{n=1}^{N} \ln \sum_{c=1}^{Q_n} p_{n,c}}{1} - 1
   \]

   Let \( \Delta_1(N) = h_1(N + 1) - h_1(N) \), which denotes the increment when the number of CPs increases from \( N \) to \( N + 1 \). Then, we have

   \[
   \Delta_1(N) = \frac{N \ln \sum_{c=1}^{Q_n} p_{n+1,c} - \ln \sum_{c=1}^{Q_n} p_{n,c}}{N \ln (N + 1)}
   \]

   \[
   = \frac{N \ln \left( \sum_{c=1}^{Q_n} p_{n+1,c} \right) - \ln \left( \prod_{n=1}^{N} \sum_{c=1}^{Q_n} p_{n,c} \right)}{N \ln (N + 1)}
   \]

   Let \( p(n) = \sum_{c=1}^{Q_n} p_{n,c} \) and \( p_{n,\max} = \max \{ p(n) \}_{n=1}^{N} \).

   Then, we have

   \[
   \Delta_1(N) = \frac{1}{N(N + 1)} \ln \left( \frac{\prod_{n=1}^{N} p(n)}{p_{n,\max}} \right) \geq \frac{1}{N + 1} \ln \left( \frac{\prod_{n=1}^{N} p(n)}{p_{n,\max}} \right)
   \]

   \[
   \text{(32)}
   \]

   It can be seen from Formula (32) that if the new participant CP owns more popular contents than any other CP in the current game, i.e., \( p(N + 1) > p_{n,\max} \), the increment \( \Delta_1(N) \) will be larger than zero. The uniform price set by the cloud server will then increase.

2. If \( N \geq \pi \lambda R^2 \), the uniform price can be re-expressed as follows:

   \[
   s_2(N) = \exp \left[ \frac{1}{N} \sum_{n=1}^{N} \ln \sum_{c=1}^{Q_n} p_{n,c} - \pi \lambda R^2 \right] + \ln (\pi \lambda R^2)
   \]

   Let \( h_2(N) = \ln [s_2(N)] \). Then, \( h_2(N) \) can be expressed as follows:

   \[
   h_2(N) = \frac{1}{N} \sum_{n=1}^{N} \ln \sum_{c=1}^{Q_n} p_{n,c} - \pi \lambda R^2 \] + \ln (\pi \lambda R^2)

   \[
   \text{(33)}
   \]

   Let \( \Delta_2(N) = h_2(N + 1) - h_2(N) \), which denotes the increment when the number of CPs increases from \( N \) to \( N + 1 \). Then, we have

   \[
   \Delta_2(N) = \frac{N \ln \sum_{c=1}^{Q_n} p_{n+1,c} - \ln \sum_{c=1}^{Q_n} p_{n,c} + \pi \lambda R^2}{N(N + 1)}
   \]

   \[
   = \frac{\Delta_1(N) + \frac{1}{N(N + 1)} \pi \lambda R^2}{\Delta_1(N) + \frac{1}{N(N + 1)} \pi \lambda R^2}
   \]

   \[
   \text{(34)}
   \]

   It can be seen from Eq. (35) that the increment \( \Delta_2(N) \) is larger than \( \Delta_1(N) \). Therefore, the uniform price set by the cloud server will increase if the new participant CP owns more popular contents than any other CP in the current game.

**5.2 Global optimization-based strategy**

In this section, we propose a global optimization-based edge caching resource allocation strategy, where the competition of the players is eliminated completely.

Let \( P_e(r) \) denote the global profit, including the profits of the cloud server and all CPs. Then, we have
Let $P_g(\tau) = P_c(\tau) + \sum_{n=1}^{N} P_n(\tau) = \sum_{n=1}^{N} \sum_{c=1}^{Q_n} p_{n,c} \left[1 - \exp\left(-\pi\lambda r_n R^2\right)\right]$ \hspace{1cm} (36)

Correspondingly, the global optimization problem can be formulated as follows:

$$\max_{\tau} P_g(\tau),$$

s.t. $0 \leq \tau_n \leq 1, \forall n \in \mathcal{N},$ \hspace{1cm} (37)

It can be seen from Formula (37) that the global optimization problem is a typical water-filling one. Obviously, the objective function in Formula (37) is convex. The Lagrangian multiplier $\eta$ is introduced as follows:

$$L(\tau, \eta) = \sum_{n=1}^{N} \sum_{c=1}^{Q_n} p_{n,c} \left[1 - \exp\left(-\pi\lambda r_n R^2\right)\right] - \eta \left(\sum_{n=1}^{N} \tau_n - 1\right)$$ \hspace{1cm} (38)

Then, the corresponding optimization problem can be solved by using the sub-gradient method, which is generally used to solve the water-filling problem.\(^{[31]}\)

Let $\eta^{(t)}$ denote the Lagrangian multiplier for the $t$-th iteration, the iterative relationship can be established as follows:

$$\eta^{(t+1)} = \left(\eta^{(t)} + \left(\sum_{n=1}^{N} \tau_n - 1\right)\right)^+$$ \hspace{1cm} (39)

Based on the Karush-Kuhn-Tucker (KKT) condition, $\tau$ can be readily obtained by setting $\partial L(\tau, \eta)/\partial \tau_n = 0.$ Correspondingly, we have

$$\tau_n^{(t+1)} = \left(\ln\left(\frac{\pi \lambda r_n^2 \sum_{c=1}^{Q_n} p_{n,c}}{\eta^{(t+1)}}\right)/\pi \lambda r_n^2\right)^+$$ \hspace{1cm} (40)

The details of the proposed global optimization-based strategy are presented in Algorithm 2.

### 5.3 Comparison of proposed strategies

For the nonuniform pricing based resource allocation strategy, the cloud server gains the largest profit. The reason is that the selfishness of the cloud server and the CPs is considered, and the interaction between them is formulated as a Stackelberg game where the cloud server has the first-mover advantage. For the uniform pricing based resource allocation strategy, it has the lowest computational complexity due to the closed-form solution. However, the profit of the cloud server is reduced due to the elimination of competition among the CPs. For the global optimization-based resource allocation strategy, it achieves the largest average cache hit rate of the CPs. The reason is that the competition among the cloud server and the CPs is eliminated completely, which means that the CPs can cache contents in the storages of the F-APs without paying the cloud server.

### 6 Simulation results

In this section, we evaluate the efficiency of the proposed three edge caching resource allocation strategies, namely, Nonuniform Pricing, Uniform Pricing, and Global Optimization, which are referred to as NUP, UP, and GO, respectively. For comparison, the edge caching resource allocation strategy in Ref. \([18]\) is used as the baseline. The system parameters are set as follows: The number of CPs $N = 4$, the content preference parameters of the CPs $\beta_1 = 1.6, \beta_2 = 1.2, \beta_3 = 0.8$, $\beta_4 = 0.4$, the number of contents owned by the CPs $C = C_1 = C_2 = C_3 = C_4 = 5000$, and the serving radius of the F-APs $R = 500$ m.

In Fig. 2, we show the cache hit rate of the CPs vs. number of iterations for NUP and the baseline. It can be observed that the cache hit rate of NUP is apparently superior to that of the baseline. The reason is that the CPs in the latter strategy cache the contents with equal probability, whereas the CPs in our proposed strategy cache the contents by considering the content popularity.
In Fig. 3, we show the leasing fraction of the F-APs for the CPs vs. the number of iterations for NUP. NUP can converge to a stable state quickly. Moreover, the CP with a larger content preference parameter tends to lease more F-APs. A larger content preference parameter means that the contents owned by the corresponding CP are more popular. Therefore, it can bring a larger cache hit rate and then increase the profit by leasing the F-APs to the CP that owns more popular contents.

In Fig. 4, we show the profit of the cloud server vs. storage capacity of the F-APs $Q$ for NUP and UP. It can be observed that the profit of the cloud server in NUP is always higher. The reason is that the cloud server can benefit from the competition among the CPs. The profit of the cloud server increases with both $Q$ and $N$, which is consistent with our analytical results presented in Section 5.2.
speed.

In Fig. 7, we show the average cache hit rate of the CPs vs. storage capacity of the F-APs for three proposed strategies. It can be observed that the average cache hit rate of the CPs gradually increases with the competition elimination from NUP to GO, which is consistent with the analysis presented in Section 5.3.

7 Conclusion

In this paper, we have proposed three edge caching resource allocation strategies for F-RANs. Firstly, we have formulated a Stackelberg game via the introduced incentive mechanism and proposed the nonuniform pricing based resource allocation strategy to maximize the profits of the cloud server and the CPs by using the multiplier penalty function method. Secondly, we have proposed the uniform pricing based resource allocation strategy by eliminating the competition among the CPs. Thirdly, we have proposed the global optimization-based resource allocation strategy by further eliminating the competition between the cloud server and the CPs. Simulation results have shown that the nonuniform pricing based strategy has a larger cache hit rate compared with the baseline, and has brought the cloud server more profits than the uniform pricing-based strategy. However, the latter strategy has lower computational complexity due to the closed-form solution. Besides, the global optimization-based strategy has achieved the largest average cache hit rate due to the complete elimination of competition among the cloud server and the CPs.

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