Novel arrangement of Routh array for order reduction of \( z \)-domain uncertain system

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**ABSTRACT**

This paper presents a novel arrangement of Routh table array for deriving an approximate model of a higher order \( z \)-domain uncertain system. The demand for this computation is to procure a lower order model which is easy to be exercised in comparison to their original large scale systems. Additionally, the derived model should preserve fewer dynamic characteristic of the comprehensive higher order systems. The mentioned new arrangement is achieved from the arena of different combinations of numerator and denominator polynomials. The combinations are validated by their practice over the conventional example from the literature. This precise blend is then applied to a real-time system for its rational acceptability. The paper also offers a future scope for fellow researchers.

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Discrete-time uncertain systems; bilinear transformation; order reduction; Routh approximation

1. Introduction

Order reduction, an appealing topic of research for control engineers, came into existence in the late 1960s. Its prime concern is to derive an approximate model from the higher order system retaining fewer of the dynamic characteristics. Since then, many reduction methodologies emerged such as aggregation method, balanced truncation method, Pade approximation, continued fraction method, dominant eigenvalue method, Routh approximation (RA) and many others. These methods are available in the literature (Bultheel & Barel, 1986; Feng, Zheng, Francis, & Mantooth, 2008; Genesio & Milanese, 1976; Gugercin & Antoulas, 2004; Samuel, Knockaert, & Dhaene, 2014; Sinha & Lastman, 1990). Among the methodologies, RA is highly accepted for being computationally effortless and significantly retaining system stability. Various available articles on RA (Hwang & Lee, 1997; Narwal & Prasad, 2016; Paraskevopoulos, 1980; Prasad, 2000; Shamash, 1975; Singh, 2005; Singh, Prasad, & Gupta, 2006) prove it.

The rise in the demand of thoroughly analysing the practical system that includes the model dynamics, noise, variation in parameters; disturbances leads to the outcome of uncertain structures. Thus, uncertain or interval systems are systems with constant coefficients, but unknown within a limited range. Since, no competent technique developed for reducing the uncertainty descriptions, investigation moved towards the simplification of the order of uncertain schemes. Algorithm pioneering the approximation of such system is Routh-Pade (Bandyopadhyay, Ismail, & Gorez, 1994), an extension of deterministic systems (Shamash, 1975). It expresses the derivation of a stable reduced model by truncating Routh’s table for denominator coefficients and matching time moments for numerator coefficients. The same author proposes a formulation of \( \gamma - \delta \) parameters for obtaining reduced-order coefficients (Bandyopadhyay, Upadhye, & Ismail, 1997), resulting in increased computational effort with retention of system stability. These methods pose the limitation of not attaining stable reduced model and is highlighted in the work of Hwang and Yang (1999). This restriction is an outcome of the implementation of interval Routh expansion and inversion algorithms, which confine that algorithm cannot guarantee success in generating a full interval Routh array and the approximant may not be stable even if the original uncertain system is robustly stable. The reason for drawback is the irreversibility of interval arithmetic and is considered by Dolgin and Zeheb (2003), where modification of the method in Bandyopadhyay et al. (1994) assures retention of model stability. Few more queries attempt to sort this limitation through numerical examples (Dolgin, 2005; Yang, 2005). Sastry, Raja Rao, and Mallikarjuna Rao (2000) elaborate an algorithm for 0
deriving the denominator and numerator coefficients by computing only $\gamma$ parameters.

With time span, an efficient outcome of digital signals and systems amused the research towards order reduction of discrete-time uncertain systems. Literature available for such systems are few but proficient – remarkably, Padé approximation allowing dominant poles’ retention (Ismail, Bandyopadhyay, & Gorez, 1997) and $\mu$-dependent approach (Zhang, Boukas, & Shi, 2009). Hsu and Wang (2000) developed a higher order integrator approach to approximate discrete-time interval model for the continuous-time interval systems sampled by a ZOH. Dolgin and Zeheb (2004) states the demand of assuming the required order polynomial coefficients at an earlier stage, for computation of error between the polygons in the complex plane (original uncertain system) and the point representing the resulted reduced-order fixed-coefficients systems for deriving the reduced-order model. In recent times, algorithms that expressed their advancement from deterministic systems to uncertain systems are work by Choudhary and Nagar (2013a, 2013b, 2015b, 2016), namely direct truncation, gamma–delta approximation and Routh-Pade approximation. Freshly, the article (Choudhary & Nagar, 2015a) offers the application of existing algorithms to power systems components.

From the available and cited literature, RA techniques advanced from deterministic systems to uncertain structures. This paper addresses a novel approach for order reduction of the discrete-time uncertain system. The prime motive of this method is to offer a satisfactorily reduced model that retain the significant property of model stability. The realm for attaining novelty is the possible combinations (mentioned as cases in the paper) of the numerator and denominator polynomials for computing the Routh table array. The paper is organized in five sections. Section 2 presents the proposed methodology. It also contains the higher and lower order system representation, the proposed algorithm cases, validation and stability check procedures. Section 3 includes an example available from literature in support of the proposal. This segment also validates the confronted combination case over a real-time test system. A brief discussion of the findings and the limitation through the proposed algorithm is in Section 4. Finally, Section 5 ends with an outcome of the new and promising algorithm.

2. Problem methodology

2.1. System representation

Let the higher order discrete-time uncertain system be

$$T_n(z) = \frac{B_n - 1 z^{n-1} + B_{n-2} z^{n-2} + \cdots + B_0}{A_n z^n + A_{n-1} z^{n-1} + \cdots + A_0} = \frac{B_n(z)}{A_n(z)}, \quad (1)$$

where $B_i = [B_i^-, B_i^+]$ and $A_i = [A_i^-, A_i^+] \quad i = 0, 1, 2, \ldots, n-1, n$.

And its reduced approximate model be (2), where $m < n$

$$R_m(z) = \frac{b_{m-1} z^{m-1} + b_{m-2} z^{m-2} + \cdots + b_0}{a_m z^m + a_{m-1} z^{m-1} + \cdots + a_0} = \frac{B_m(z)}{A_m(z)}, \quad (2)$$

where $b_i = [b_i^-, b_i^+]$ and $a_i = [a_i^-, a_i^+] \quad i = 0, 1, 2, \ldots, m-1, m$.

2.2. Proposed algorithm

In the classical control system, RA is the favourite tool to check system stability. Similarly, for deriving a stable reduced model, RA is taken into consideration. Since RA is a continuous-time domain algorithm and the article discusses the discrete-time domain system, an appropriate transformation of latter to its former equivalent is in demand here. This requirement is fulfilled by the use of Bilinear or Tustin transformation $z = (1 + w/1 - w)$ as applied earlier in Hwang and Shin (1982), Hwang and Hsieh (1990) as well as Hwang and Lee (1997), resulting $T_n(z)$ in

$$T_n(w) = \frac{x_n w^n + x_{n-1} w^{n-1} + \cdots + x_0}{y_n w^n + y_{n-1} w^{n-1} + \cdots + y_0} = \frac{X_{n\in\mathbb{k}}(w)}{Y_{n\in\mathbb{k}}(w)}, \quad (3)$$

where $x_i = [x_i^-, x_i^+]$ and $y_i = [y_i^-, y_i^+] \quad i = 0, 1, 2, \ldots, n$.

Henceforth, the novel arrangement along with the various arrangements (Cases) of Routh table array elaborates below. Firstly for a quick review of the usual Routh table array, consider Table 1, with all the entries of interval form below. Firstly for a quick review of the usual Routh table array, consider Table 1, with all the entries of interval form below.

| $a_{0,0}$ | $a_{0,1}$ | $a_{0,2}$ | $a_{0,3}$ | $\cdots$ |
|-----------|-----------|-----------|-----------|-----------|
| $a_{1,0}$ | $a_{1,1}$ | $a_{1,2}$ | $a_{1,3}$ | $\cdots$ |
| $a_{2,0}$ | $a_{2,1}$ | $a_{2,2}$ | $\cdots$ | $\cdots$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $a_{n-1,0}$ | $a_{n-1,1}$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $a_{n,0}$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |

Henceforth, the novel arrangement along with the various arrangements (Cases) of Routh table array elaborates below. Firstly for a quick review of the usual Routh table array, consider Table 1, with all the entries of interval form as $a_{0,0} = [a_{0,0}^-, a_{0,0}^+]$, in general, $a_{ij} = [a_{ij}^-, a_{ij}^+]$ with $i = 0, 1, 2, \ldots$ and $j = 0, 1, 2, \ldots$.

Equation (3) in two different arrangements offers four cases as mentioned below for the entries of first two rows of the respective Routh tables for numerator and denominator polynomials. Once, the above two rows are available for the stated cases, the conventional Routh algorithm computes the entries in the tables from the third row as

$$a_{ij} = a_{i-2,j+1} - \frac{(a_{i-2,0} \times a_{i-1,j+1})}{a_{i-1,0}}, \quad (4)$$

where $a_{ij} = [a_{ij}^-, a_{ij}^+]$ with $i = 2, 3, 4, \ldots, n-1, n$ and $j = 0, 1, 2$. 

| $a_{0,0}$ | $a_{0,1}$ | $a_{0,2}$ | $a_{0,3}$ | $\cdots$ |
|-----------|-----------|-----------|-----------|-----------|
| $a_{1,0}$ | $a_{1,1}$ | $a_{1,2}$ | $a_{1,3}$ | $\cdots$ |
| $a_{2,0}$ | $a_{2,1}$ | $a_{2,2}$ | $\cdots$ | $\cdots$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $a_{n-1,0}$ | $a_{n-1,1}$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $a_{n,0}$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
These defined cases are then compared within themselves through an example in the next section to present the best among them.

The first two cases and their respective entries for first two rows is from Equation (3) as

Case 1
For numerator

1st Row; \( a_{ij} = x_k \) where \( i = 0; j = 0, 1, 2, 3, \ldots \)
\[ k = n, n - 2, n - 4, \ldots \]
2nd Row; \( a_{ij} = x_k \) where \( i = 1; j = 0, 1, 2, 3, \ldots \)
\[ k = n - 1, n - 3, n - 5, \ldots \]

For denominator

1st Row; \( a_{ij} = y_k \) where \( i = 0; j = 0, 1, 2, 3, \ldots \)
\[ k = n, n - 2, n - 4, \ldots \]
2nd Row; \( a_{ij} = y_k \) where \( i = 1; j = 0, 1, 2, 3, \ldots \)
\[ k = n - 1, n - 3, n - 5, \ldots \]

Case 2
For numerator

1st Row; \( a_{ij} = x_k \) where \( i = 0; j = 0, 1, 2, 3, \ldots \)
\[ k = n, n - 2, n - 4, \ldots \]
2nd Row; \( a_{ij} = y_k \) where \( i = 1; j = 0, 1, 2, 3, \ldots \)
\[ k = n - 1, n - 3, n - 5, \ldots \]

For denominator

1st Row; \( a_{ij} = y_k \) where \( i = 0; j = 0, 1, 2, 3, \ldots \)
\[ k = n, n - 2, n - 4, \ldots \]
2nd Row; \( a_{ij} = x_k \) where \( i = 1; j = 0, 1, 2, 3, \ldots \)
\[ k = n - 1, n - 3, n - 5, \ldots \]

For other two cases, reciprocate Equations (3) to (5) as

\[ \hat{T}_n(w) = \frac{x_0w^n + x_1w^{n-1} + \cdots + x_{n-1} + x_n}{y_0w^n + y_1w^{n-1} + \cdots + y_{n-1} + y_n} = \frac{X_{N\Xi K}(w)}{Y_{N\Xi K}(w)}. \tag{5} \]

Case 3
For numerator

1st Row; \( a_{ij} = x_k \) where \( i = 0; j = 0, 1, 2, 3, \ldots \)
\[ k = 0, 2, 4 \ldots \]
2nd Row; \( a_{ij} = x_k \) where \( i = 1; j = 0, 1, 2, 3, \ldots \)
\[ k = 1, 3, 5 \ldots \]

For denominator

1st Row; \( a_{ij} = y_k \) where \( i = 0; j = 0, 1, 2, 3, \ldots \)
\[ k = 0, 2, 4 \ldots \]
2nd Row; \( a_{ij} = y_k \) where \( i = 1; j = 0, 1, 2, 3, \ldots \)
\[ k = 1, 3, 5 \ldots \]

Case 4
For numerator

1st Row; \( a_{ij} = x_k \) where \( i = 0; j = 0, 1, 2, 3, \ldots \)
\[ k = 0, 2, 4 \ldots \]
2nd Row; \( a_{ij} = x_k \) where \( i = 1; j = 0, 1, 2, 3, \ldots \)
\[ k = 1, 3, 5 \ldots \]

As stated above, the conventional Routh algorithm completes the Routh tables. The discussion underneath is to derive the desired order reduced transfer function using the entries in the Routh tables.

Let \( R_m(w) \), where \( m < n \) is the reduced transfer function and is assembled with \((n + 1 - m)th\) and \((n + 2 - m)th\) rows of denominator table, along with \((n + 1)th\) and \((n + 2)th\) rows of numerator table, be represented as

\[ R_m(w) = \frac{c_{(n+1),0}w^{m-1} + c_{(n+2),0}w^{m-2} + \cdots + c_{(n+1),1}w^{m-2} + \cdots}{d_{(n+1-m),0}w^m + d_{(n+1-m),1}w^{m-1} + \cdots} = \frac{X_m(w)}{Y_m(w)}. \tag{6} \]

with the respective coefficients in uncertain form.

An appropriate inverse bilinear transformation from \( R_m(w) \) to the desired \( R_m(z) \) results. The next section discusses the different cases through numerical examples.

### 2.3. Validation tools

Integral Square Error is the convenient tool used here for performance analysis, with a modification towards discrete-time systems. Its adaptation leads to the weighted error sum over a fixed interval of time, which
can be expressed as
\[ J = \sum_{k=0}^{\infty} (y_n(k) - y_m(k))^2, \]  
(7)
where \( y_n(k) \) and \( y_m(k) \) are the step responses of the original \( T_n(z) \) system and reduced \( R_m(z) \) model, respectively.

Ideally, minimum \( J \) guarantees an approximate model of the higher order system. As the paper considers uncertain system, two individual transfer functions (i) with only lower limits and (ii) with only upper limits) account for computing the error. Thus, the calculated \( J \) under two error columns lower and upper limits make the analysis and comparison easy with the recognized techniques.

Step response of the original system and reduced models also authenticate the proposed algorithm.

### 2.4. Stability analysis

Ever since the discovery of uncertainty in the system, the worry of its stability exists. This problem, also a key focus of the paper, is checked by applying Kharitonov theorem (Barmish, 1989) as stated below.

Consider the set of real interval polynomials of degree \( n \) of the form
\[ \lambda(s) = \lambda_0 + \lambda_1 s + \lambda_2 s^2 + \lambda_3 s^3 + \lambda_4 s^4 + \cdots + \lambda_n s^n, \]  
(8)
where the coefficients lie within given ranges \( \lambda_0 \in [x_0,y_0], \lambda_1 \in [x_1,y_1], \ldots, \lambda_n \in [x_n,y_n] \).

Then, every polynomial in the family is Hurwitz if and only if the following four extreme polynomials are Hurwitz:

\[
K^1(s) = x_0 + x_1 s + y_2 s^2 + y_3 s^3 + x_4 s^4 + x_5 s^5 + \cdots \\
K^2(s) = x_0 + y_1 s + x_2 s^2 + x_3 s^3 + x_4 s^4 + y_5 s^5 + \cdots \\
K^3(s) = y_0 + x_1 s + x_2 s^2 + x_3 s^3 + y_4 s^4 + x_5 s^5 + \cdots \\
K^4(s) = y_0 + y_1 s + x_2 s^2 + x_3 s^3 + y_4 s^4 + y_5 s^5 + \cdots.
\]  
(9)

The Kharitonov theorem involves the standard Routh algorithm to check the stability of above polynomials. Thus, an uncertain family of polynomials \( \lambda(s) \) is robustly stable if, and only if, the Kharitonov polynomials are stable. The stated theorem holds true in the discrete-time domain also (Cieslik, 1987; Hollot & Bartlett, 1986).

Graphical interpretation for the stability of derived reduced model is performed by the Bode plot in the frequency domain that includes the original system also.

### 3. Illustrative examples

An example considered from the literature determines the best Case among the above stated agreements. Comparison of error sum performs the assessment between the proposed cases with the existing techniques. The step responses also verify the obtained results. In the course of attaining the prime motive for the proposed algorithm (i.e. model stability), a limitation of no importance is discovered. For illustrating the proficiency of the proposed cases, the limitation is taken into account and then neglected on a later stage. Additionally, the obtained case of the numerator and denominator polynomial is applied to a real-time test system for assuring its possible liability. In both the examples, the sampling time considered is 0.001 s.

**Example 1:** Consider third-order uncertain system available from literature (Choudhary & Nagar, 2013a, 2013b; Ismail et al., 1997) as
\[ T_3(z) = \frac{[1,2]z^2 + [3,4]z + [8,10]}{[6,6]z^3 + [9,9.5]z^2 + [4.9,5]z + [0.8,0.85]}. \]  
(10)
After bilinear transformation, the above system is represented as
\[ T_3(w) = \frac{[-9, -5]w^3 + [17,27]w^2}{[0.55, 1.2]w^3 + [5.9, 6.65]w^2}{+ [19.45, 20.2]w + [20.7, 21.35]}. \]  
(11)
First two rows of the Routh table and the reduced models fetched as per the four cases are observed below. Consider Equation (11) for first two cases

**Case 1:** See Tables 2 and 3.

\[
R_1(z) = \frac{[12,16]z + [12,16]}{[35.8,39.83]z + [2.22,6.25]},
\]  
(12)
\[
R_2(z) = \frac{[-19.77, 0.48]z^2 + [24,32]z + [27.52, 47.77]}{[41.7,46.48]z^2 + [28.1,30.9]z + [8.12,12.9]}.
\]  
(13)

**Case 2:** See Tables 4 and 5.

\[
R_1(z) = \frac{[20.7,21.35]z + [20.7,21.35]}{[30.32,35.95]z + [-7.95, -2.32]},
\]  
(14)
\[
R_2(z) = \frac{[2.27,29.91]z^2 + [41.4,42.7]z + [12.14, 39.78]}{[47.32,62.95]z^2 + [-30,-2]z + [9.05,24.68]}.
\]  
(15)

**Table 2.** Denominator array.

| \(\omega^3\) | \([0.55, 1.2]\) |
|-------------|----------------|
| \(\omega^2\) | \([5.9, 6.65]\) |
| \(\omega\)   | \([20.7, 21.35]\) |

**Table 3.** Numerator array.

| \(\omega^3\) | \([-9, -5]\) |
|-------------|----------------|
| \(\omega^2\) | \([-34, -24]\) |
| \(\omega\)   | \([17,27]\)  |
|             | \([12,16]\)  |
For Case 3 and Case 4, reciprocate $T_3(w)$ to $\hat{T}_3(w)$ and draft the two cases as

$$\hat{T}_3(w) = \frac{[12, 16]w^3 + [-34, -24]w^2}{[20.7, 21.35]w^3 + [19.45, 20.2]w^2 + [5.9, 6.65]w + [0.55, 1.2]}.$$

Case 3: See Tables 6 and 7.

$$R_1(z) = \frac{[-9, -5]z + [-9, -5]}{[5.13, 7.28]z + [3.38, 5.53]}$$

(16)

$$R_2(z) = \frac{[2, 20.23]z^2 + [22, 50.46]z + [16, 34.23]}{[24.58, 27.48]z^2 + [36.5, 39.3]z + [13.92, 16.82]}.$$  

(17)

Case 4: See Tables 8 and 9

$$R_1(z) = \frac{[0.55, 1.2]z + [0.55, 1.2]}{[-11.1, -1.4]z + [2.9, 12.6]},$$

$$R_2(z) = \frac{[16.56, 27.87]z^2 + [32.02, 53.34]z + [14.81, 26.12]}{[-45.1, -25.4]z^2 + [-58, -30]z + [-46.6, -26.9]}.$$  

(18)

(19)

The computed error $J$, through different Cases, is made known in Table 10 offering the support to Case 2 for calculating minimum error when compared to the existing techniques. The next section explains the limitation encountered during the error computation.

Another practice to authenticate the algorithm is step response depicted in Figures 1 and 2 for lower and upper limit models of first order. Figures 3 and 4 show the response for lower and upper limit models of second order. The responses displayed are for the models obtained from Case 2 and the existing techniques. Limitation explained in the next section can be observed from the demonstrated figures.

Deriving the reduced models confer the fair representation of the higher order system to its lower equivalent. Since the prime focus of the paper is to retain the model stability, it is checked through Kharitonov theorem and displayed in the frequency domain (Bode Plot). Figures 5 and 6 show the Bode plot of first-order reduced model with lower and upper limits and the following Figures 7 and 8 present for second-order model with lower and upper limits. In all the figures, the models have an infinite gain margin and a positive phase margin, clearly indicating the derivation of a stable reduced model through Case 2. Thus, the prime motive of the paper is acknowledged.

From the above example, Case 2 is sorted to be the satisfactory arrangement of the Routh tables providing a minimum error, an adequate step response and retaining the model stability. To accept the arrangement’s rational acceptability, Case 2 is applied to the real-time system through the example below.

### Table 10. Error for first- and second-order reduced models through the four cases and other prevailing techniques.

| Methods                             | Error                          |
|-------------------------------------|-------------------------------|
|                                     | First-order | Second-order |
|                                     | Lower limit | Upper limit | Lower limit | Upper limit |
| Proposed Case 1                     | 0.3456      | 0.3271      | 0.2894      | 0.1287      |
| Proposed Case 2                     | 0.3644      | 0.1497      | 0.0211      | 0.0233      |
| Proposed Case 3                     | 9.4259      | 1.8765      | 0.4812      | 1.9492      |
| Proposed Case 4                     | 0.0801      | 96.0295     | 0.7302      | 6.1975      |
| Pade Appr. and dominant pole (Ismail et al., 1997) | 0.1398    | 0.0195      | 0.1810      | 0.0741      |
| Direct truncation (Choudhary & Nagar, 2013a) | 2.1491    | 2.7778      | 0.0278      | 0.0077      |
| Gamma–Delta Appr. (Choudhary & Nagar, 2013b) | 0.0157    | 0.0035      | 0.1292      | 0.0250      |

Note: The Proposed Case 2 present an appropriate acceptance as compared to rest of the algorithms.
Example 2: Consider the real-time digital control system shown in Figure 9, where

\[
D(z) = \frac{1.68z^6 - 0.566z^5 + 0.356z^4 - 0.204z^3}{z^6 + 1.159z^5 + 0.76z^4 + 0.466z^3 + 0.096z^2 - 0.016z + 0.003},
\]

With \( T = (0.5)^{0.5} \) s, and accepting the robustness of the system into count, the overall transfer function is crafted as

\[
T_S(z) = \frac{[1.6484, 1.7156]z^7 + [1.0937, 1.1383]z^6 + [-0.2142, -0.2058]z^5 + [0.1490, 0.1550]z^4 + [-0.5263, -0.5057]z^3 + [-0.2672, -0.2568]z^2 + [0.0431, 0.0449]z + [-0.0061, -0.0059]}{[23.52, 24.48]z^8 + [-1.7156, -1.6484]z^7 + [-1.1383, -1.0937]z^6 + [0.2058, 0.2142]z^5 + [-0.1550, -0.1490]z^4 + [0.5057, 0.5263]z^3 + [0.2672, 0.2672]z^2 + [-0.0449, -0.0431]z + [0.0059, 0.0061]}.
\]
By the proposed algorithm cases, the reduced-order model is obtained as

Case 1:

\[
R_1(z) = \frac{[1.92, 2.07]z + [1.92, 2.07]}{[110.83, 291.57]z + [-247.58, -66.84]} \quad (22)
\]

\[
R_2(z) = \frac{[13.21, 21.14]z^2 + [3.84, 4.14]z + [-17.15, -9.22]}{[-49.49, 739.4]z^2 + [-852.78, 365.74]z + [-407.9, 380.99]} \quad (23)
\]

Case 2:

\[
R_1(z) = \frac{[1.92, 2.07]z + [1.92, 2.07]}{[36.43, 39.85]z + [4.14, 7.63]} \quad (24)
\]

\[
R_2(z) = \frac{[14.94, 182.65]z^2 + [3.84, 4.14]z + [-178.66, -10.95]}{[-1222.74, 886.15]z^2 + [-1649.72, 2563.3]z + [-1254.96, 853.93]} \quad (25)
\]
Case 3:

\[
R_1(z) = \frac{[-0.07, 0.07]z + [-0.07, 0.07]}{[129.23, 279.82]z + [81.24, 231.83]} \]  (26)

\[
R_2(z) = \frac{[-5.69, -4.03]z^2 + [-11.24, -8.2]z + [-5.69, -4.03]}{[-130.46, 762.05]z^2 + [-568.48, 917.58]z + [-491.52, 400.92]} \]  (27)

Case 4:

\[
R_1(z) = \frac{[-0.07, 0.07]z + [-0.07, 0.07]}{[17.24, 22.39]z + [-30.25, -25.6]} \]  (28)

\[
R_2(z) = \frac{[181.98, 211.74]z^2 + [364.1, 423.34]z + [181.98, 211.74]}{[-1724.76, 589.39]z^2 + [-3534.1, 1087.12]z + [-1716.9, 597.25]} \]  (29)

Cumulative error \( J \) for \( R_1(z) \) and \( R_2(z) \) of example 2 is presented in Table 11.
4. Discussion

Table 10 depicts the error for first- and second-order reduced models through the discussed four Cases and other prevailing techniques for Example 1. It offers support towards Case 2. Similarly, computation of minimum error for Example 2, real-time test system also support Case 2 in Table 11. Both the tables satisfactorily state the acceptance of Case 2 for computation of minimum error. Thus, among the four cases elaborated, the arrangement for the desired reduced model is Case 2. Authors would firmly state that the reduction methodology discussed in this paper may be good for one practical system as considered here and not for other. However, the paper presents an
explicit arrangement for computing the stable reduced model.

As stated earlier, the limitation discovered is observed in Table 10. It is the computation of higher error by Case 2 in comparison to other techniques (e.g. Error of 1st order lower limit is more than the other in the same column). Precisely, error obtained by Case 2 is more than the usual method (Ismail et al., 1997). Similarly, in Table 11, the error in second-order upper limit column by Case 2 is more than in Case 1 and Case 3.

Prime motive of the paper to retain the model stability is fulfilled at the stake of the encountered limitation. Thus, the limitation is taken into consideration for Case 2 arrangement of Routh table. Stability of the interval systems and the reduced models is checked with the Kharitonov theorem at the department laboratory using Kharitonov software package. However, Figures 5–8 demonstrate the frequency response of first- and second-order models with lower and upper limits, respectively. The responses in figures are for the models derived from Case 2.

Through the above illustration, it is observed that the tracking of the step responses of the reduced models is not accurate with the original response although it gives an appropriate approximation. The tracking proposes a clear indication towards the future work to a control engineer or a researcher, to design an appropriate controller that tracks the response of the reduced models accurately with the original response.

Table 11. Error for first- and second-order reduced models through the four cases.

| Methods       | First-order |           | Second-order |           |
|---------------|-------------|-----------|--------------|-----------|
|               | Lower limit | Upper limit | Lower limit | Upper limit |
| Proposed Case 1 | 3.1075*10^{-4} | 0.0030 | 17.5841 | 0.0033 |
| Proposed Case 2 | 0.0036 | 0.0033 | 0.0049 | 0.0250 |
| Proposed Case 3 | 0.0050 | 0.0040 | 0.0189 | 0.0064 |
| Proposed Case 4 | 0.0073 | 0.0036 | 0.0402 | 0.2480 |

Note: The Proposed Case 2 present an appropriate acceptance as compared to rest of the algorithms.

5. Conclusion

The property of RA to yield a stable model is the key focus of discussion in the paper and is successfully achieved. The computational simplicity of RA intends this work proposal. The arrangement of Routh table in a varied form confers the methodology. The domain of various cases constructed by the different combinations of numerator and denominator polynomials explores the novelty of the algorithm. An example available from the literature is used to get the blueprint of the arrangement and then tested over a real-time system. Both the examples in the paper offer a significant contribution towards the institution of a superior algorithm over the other prevailing techniques based on the performance measure, step response and stability check. The paper also exploits the limitation encountered and provides a future scope to the fellow researchers.

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References

Bandyopadhyay, B., Ismail, O., & Gorez, R. (1994). Routh-Pade approximation for interval systems. IEEE Transactions on Automatic Control, 39(12), 2454–2456. doi:10.1109/9.362850
Bandyopadhyay, B., Upadhye, A., & Ismail, O. (1997). y-Routh approximation for interval systems. IEEE Transactions on Automatic Control, 42(8), 1127–1130. doi:10.1109/9.618241
Barmish, B. R. (1989). A generalization of Kharitonov’s four-polygonal concept for robust stability problems with linearly dependent coefficient perturbations. IEEE Transactions on Automatic Control, 34(2), 157–165. doi:10.1109/9.210887
Bultheel, A., & Barel, M. V. (1986). Padé techniques for model reduction in linear system theory: A survey. Journal of Computational and Applied Mathematics, 14(3), 401–438. doi:10.1016/0377-0427(86)90076-2
Choudhary, A. K., & Nagar, S. K. (2013a). Direct truncation method for order reduction of discrete interval system. Proceedings of annual IEEE India conference (INDICON), pp. 1–4. Mumbai, India: IEEE. doi:10.1109/INDICON.2013.6726040
Choudhary, A. K., & Nagar, S. K. (2013b). Gamma Delta approximation for reduction of discrete interval system. Proceedings of international conference on advances in recent technologies in electrical and electronics (ARTEE), pp. 91–94. Bangalore, India: Institute of Doctors Engineers and Scientists.
Choudhary, A. K., & Nagar, S. K. (2015a). Application of various order reduction methodologies over power system components. 9th IFAC symposium on control of power and energy systems (CPES-15), Vol. 48, pp. 185–190. IIT Delhi, India. doi:10.1016/j.ifacol.2015.12.375
Choudhary, A. K., & Nagar, S. K. (2015b). Order reduction of uncertain system using the advantages of two varied approximations. Proceedings of the Michael faraday IET international summit (MFIIIS), pp. 49–51. Kolkata, WB, India.

Choudhary, A. K., & Nagar, S. K. (2016). Revisiting approximation techniques to reduce order of interval system. Fourth international conference on advances in control and optimization of dynamical systems (ACODS 2016), pp. 241–246.

Ciesiak, J. (1987). On possibilities of the extension of Kharitonov’s stability test for interval polynomials to the discrete-time case. IEEE Transactions on Automatic Control, 32(3), 237–238. doi:10.1109/TAC.1987.1104585

Dolgin, Y. (2005). Author’s reply to comments on ‘On Routh-Pade model reduction of interval systems’. IEEE Transactions on Automatic Control, 50(2), 274–275. doi:10.1109/TAC.2005.843849

Dolgin, Y., & Zeheb, E. (2003). On Routh-Pade model reduction of interval systems. IEEE Transactions on Automatic Control, 48(9), 1610–1612. doi:10.1109/TAC.2003.816999

Dolgin, Y., & Zeheb, E. (2004). Model reduction of uncertain FIR discrete-time systems. IEEE Transactions on Circuits and Systems II: Express Briefs, 51(8), 406–411. doi:10.1109/TCSII.2004.832766

Feng, Y., Zheng, W., Francis, A. M., & Mantooth, H. A. (2008). Model order reduction by Miller’s theorem and root localisation. IET Computers & Digital Techniques, 2(5), 363–376. doi:10.1049/iet-cdt:20050216

Genesi, R., & Milanese, M. (1976). A note on the derivation and use of reduced-order models. IEEE Transactions on Automatic Control, 21(1), 118–122. doi:10.1109/TAC.1976.1101127

Gugercin, S., & Antoulas, A. C. (2004). A survey of model reduction by balanced truncation and some new results. International Journal of Control, 77(8), 748–766. doi:10.1080/00207170410001713448

Hallot, C., & Bartlett, A. (1986). Some discrete-time counterparts to Kharitonov’s stability criterion for uncertain systems. IEEE Transactions on Automatic Control, 31(4), 355–356. doi:10.1109/TAC.1986.1104268

Hsu, C.-C., & Wang, W.-Y. (2000). Discrete modelling of uncertain continuous systems having an interval structure using higher-order integrators. International Journal of Systems Science, 31(4), 467–477. doi:10.1080/002077200291055

Hwang, C., & Hsieh, C.-S. (1990). Reduced-order modeling of discrete-time systems via bilinear Routh approximation. Journal of Dynamic Systems, Measurement, and Control, 112(2), 292. doi:10.1115/1.2896138

Hwang, C., & Lee, Y. C. (1997). A new family of Routh approximants. Circuits Systems and Signal Processing, 16(1), 1–25. doi:10.1007/BF01183172

Hwang, C., & Shin, Y.-P. (1982). Routh approximations for reducing order of discrete systems. Journal of Dynamic Systems, Measurement, and Control, 104(1), 107–109. doi:10.1115/1.3149620

Hwang, C., & Yang, S. F. (1999). Comments on the computation of interval Routh approximants. IEEE Transactions on Automatic Control, 44(9), 1782–1787. doi:10.1109/9.788553

Ismail, O., Bandypadhyay, B., & Gorez, R. (1997). Discrete interval system reduction using Padé approximation to allow retention of dominant poles. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 44(11), 1075–1078. doi:10.1109/81.641771

Narwal, A., & Prasad, B. R. (2016). A novel order reduction approach for LTI systems using cuckoo search optimization and stability equation. IETE Journal of Research, 62(2), 154–163. doi:10.1080/03772063.2015.1075915

Paraskevopoulos, P. N. (1980). Padé-type order reduction of two-dimensional systems. IEEE Transactions on Circuits and Systems, 27(5), 413–416. doi:10.1109/TCS.1980.1084833

Prasad, R. (2000). Padé type model order reduction for multivariable systems using Routh approximation. Computers & Electrical Engineering, 26(6), 445–459. doi:10.1016/S0045-7906(00)00002-1

Samuel, E. R., Knockaert, L., & Dhagne, T. (2014). Model order reduction of time-delay systems using a Laguerre expansion technique. IEEE Transactions on Circuits and Systems I: Regular Papers, 61(6), 1815–1823. doi:10.1109/TCSI.2013.2295011

Sastry, G. V. K. R., Raja Rao, G., & Mallikarjuna Rao, P. (2000). Large scale interval system modelling using Routh approximants. Electronics Letters, 36(8), 768. doi:10.1049/el:20000571

Shamash, Y. (1975). Model reduction using the Routh stability criterion and the Padé approximation technique. International Journal of Control, 21(3), 475–484. doi:10.1080/0020717050020777508922004

Singh, V. (2005). Obtaining Routh-Padé approximants using the Luus-Jaakola algorithm. IEEE Proceedings – Control Theory and Applications, 152(2), 129–132. doi:10.1049/ip-cta:20041305

Singh, N., Prasad, R., & Gupta, H. O. (2006). Reduction of linear dynamic systems using Routh Hurwitz array and factor division method. IETE Journal of Education, 47(1), 25–29. doi:10.1080/09747338.2006.11415859

Singh, N. K., & Lastman, G. J. (1990). Reduced order models for complex systems – a critical survey. IETE Technical Review, 7(1), 33–40. doi:10.1080/02526460.1990.11438579

Yang, S.-F. (2005). Comments on ‘On Routh-Pade model reduction of interval systems’. IEEE Transactions on Automatic Control, 50(2), 273–274. doi:10.1109/TAC.2004.841885

Zhang, L., Boukas, E.-K., & Shi, P. (2009). μ-dependent model reduction for uncertain discrete-time switched linear systems with average dwell time. International Journal of Control, 82(2), 378–388. doi:10.1080/00207170802126856