Thermodynamics of black holes in the Schrödinger space

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Abstract
A black hole and a black hyperboloid solution in the space with the Schrödinger isometries are presented and their thermodynamics is examined. The on-shell action is obtained by the difference between the extremal and non-extremal ones with the unusual matching of the boundary metrics. This regularization method is first applied to the black brane solution in the space of the Schrödinger symmetry and is shown to correctly reproduce the known thermodynamics. The actions of the black solutions all turn out to be the same as the AdS counterparts. The phase diagram of the black hole system is obtained in the parameter space of the temperature and chemical potential and the diagram contains the Hawking–Page phase transition and instability lines.

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1. Introduction

Pioneered by Son [1], and Balasubramanian and McGreevy [2], developments in the Galilean holography have been recently seen. They are aimed to be the non-relativistic generalizations of the AdS/CFT correspondence [3–5].

The generators of the Galilean algebra correspond to the temporal and spatial translations, rotational transformations, as well as to the Galilean boost and mass operators. One can extend the algebra by including the dilatation operator, and for the special case where the powers of the time coordinate scale twice as much as the spatial coordinates, the full non-relativistic conformal symmetry, that includes the special conformal transformation, can be obtained. The non-relativistic conformal symmetry is also known as the Schrödinger algebra [6, 7]. The authors of [1, 2] have discovered the $d + 3$ dimensional spacetime geometry whose isometry group is the Schrödinger symmetry of $d$ spatial dimensions, and based on the AdS/CFT correspondence, they proposed that the gravitational system of this Schrödinger geometry is
the holographic dual of the non-relativistic conformal field theories at strong couplings. The expectation is that the duality would provide important information about the strongly coupled real world non-relativistic systems (see [1] for the examples of such systems), as it has been the case for the AdS/CFT correspondence.

Among the subsequent developments, the finite temperature generalization of the Galilean holography was discussed in [9–11]. In parallel to the finite temperature AdS/CFT correspondence [12], they have proposed a planar black solution, a black brane, in the Schrödinger space as the holographic dual of the non-relativistic CFT at finite temperature. The authors embed the holographic set up in string theory. References [9, 11] start with the near horizon geometry of D3-branes which is the AdS3 black brane times S5. Then they apply a solution generating procedure known as the Null Melvin Twist [13, 14] to this system, and upon the KK-reduction of S5, they find that the resulting geometry is a black brane solution whose extremal and asymptotic limits reduce to the Schrödinger geometry2. Note that by construction, this procedure yields the planar black solutions. The thermodynamic and transport properties were discussed in the references and Herzog et al and Maldacena et al [9, 10] noted the similarity to the usual Schwarzschild-AdS black branes.

Just as for the asymptotically AdS case, the planar black solution does not exhibit the Hawking–Page phase transition [15]. The phase transition, in the AdS case, is observed for the black holes, as opposed to the black branes, and the phase structure of the black holes tends to be richer than the planar solutions. Moreover, in the context of the AdS/CFT correspondence, the dual field theory is supposed to go through the confinement/deconfinement phase transition, corresponding to the gravitational Hawking–Page phase transition [12]. Therefore, it is natural to search for the black hole solution in the Schrödinger space and this is the topic of this paper.

Our approach to this problem is not to embed the story in string theory but to directly deal with the (d+3)-dimensional action whose solution is supposed to be dual to the d spatial dimensional non-relativistic conformal field theory. (To be concrete, we will concentrate on the case with d = 2.) Such an action has been proposed by Son [1] and by the authors Herzog et al, Maldacena et al and Adams et al [9–11]. We seek for the black hole solution to this action and we indeed find one by mimicking the relationship between the black brane and black hole in AdS.

To examine the thermodynamics of a given black solution, we need to obtain the value of the action and the thermodynamic quantities. Because of the asymptotic structure of the Schrödinger space and the volume integral which must be carried out, the on-shell value of the action diverges and we must somehow regularize this. For this purpose, Herzog et al [9]3 proposed boundary terms, similarly to the counterterm technique of asymptotically AdS systems. This proposal has a few problems. One problem is that the resulting finite action seems to depend on arbitrary coefficients of the boundary terms introduced. In the reference, some minimal number of the boundary terms were considered, however, there are more terms that can contribute to the finite action [e.g., (AμAμ)2]. This ambiguity may be eliminated by requiring the first law to be satisfied for the resulting thermodynamic quantities. However, given the action with the counterterms, it is natural to compute the thermodynamic quantities through the definition proposed by Brown and York [16]. (Herzog et al [9] computed thermodynamic quantities differently, as we will do in section 2.3.) This attempt fails unless one chooses unjustifiably awkward normalization for the time-

2 One notes in [9, 11] that the dynamical exponent ν of the resulting geometry is 1. The dynamical exponent ν determines the relative scaling of the time coordinate and others. For ν = 1, the powers of the scaling for the time is twice that of translationally invariant coordinates and as noted, this is the special case where the Galilean symmetry extends to the Schrödinger.

3 See also the newer version of [11].
like boundary Killing vector field. The most important problem for this paper is that the similar counterterm technique fails miserably for our black hole solution. The counterterms in this case are not in the form to cancel the divergences coming from the bulk action and the Gibbons–Hawking surface term, mainly because of the nonzero scalar field (dilaton) at infinity.

Another way to obtain the thermodynamic quantities was proposed in [10]. The idea was to transform the metric to the asymptotically AdS form by using the symmetry of the system. (The symmetry actually is that of the eight-dimensional supergravity. See the appendix of [10] for details.) Then compute the relevant quantities by utilizing the established method [17–20] for the AdS case. This procedure, however, does not directly apply for the black hole solution, mainly because the horizon of the black hole does not possess the spatial translation invariance. (However, see the discussion in section 4 where we argue that there should be a similar procedure that is applicable for the black hole.)

We therefore propose a new regularization method that is apt for all the black solutions discussed in this paper. This actually is the oldest regularization technique proposed by Gibbons, Hawking and Page [15, 21], namely, the subtraction method, but with an unusual boundary matching. This method proceeds as follows. One needs to evaluate the action (with a Gibbons–Hawking surface term) on the spaces with and without the black hole but due to the volume integral, each action is divergent. To regulate the divergences, one puts the systems in a large box and the metrics on the wall of the box (the first fundamental forms) must be matched. Then one subtracts the regulated action from the other and the wall of the box is removed to infinity. In the Schwarzschild-AdS black hole case the difference is finite and the sign of the difference action determines the thermodynamically preferred solution (that is, it is suitable in searching for the Hawking–Page phase transition). In our geometry, we find a problem for this procedure: the boundary matching cannot be achieved because of the peculiar nature of one of the coordinates. Therefore, in this paper, we propose a kind of ‘partial matching’ of the first fundamental forms where we scale the coordinates on the wall to match the metrics, except for the special coordinate.

In section 2, the procedure is first applied to the known black brane case and checked to reproduce the thermodynamics of [9, 10]. It is noteworthy that Herzog et al [9] and ourselves find the finite action be the same as that of the Schwarzschild-AdS black brane. Next in section 3, the black hole solution is presented and the same procedure is applied to this solution. Remarkably, the resulting action turns out to be the same as that for the Schwarzschild-AdS black hole. The thermodynamic quantities resulting from the action are computed and the phase diagram of the system is obtained. Among other structures in the phase diagram, we find the Hawking–Page phase transition. Section 4 is dedicated to the discussions. The appendix presents the black hyperboloid solution and examines its thermodynamic properties.

2. Black brane

In this section, we mainly review the results obtained in [9, 10]. We, however, propose a new way of computing the finite on-shell action in section 2.2. The same method will be applied to the black hole solution in the following section and this section serves to show the validity of the method.

2.1. The solution

In [9–11], the Schrödinger geometry discovered in [1, 2] was embedded in the framework of string theory. Herzog et al and Adams et al [9, 11] achieve this by applying the Null
Melvin Twist procedure [13, 14] to the D3-brane geometry. Let us start with the non-extremal D3-brane geometry in the near horizon region (Schwarzschild-AdS black brane)

\[
ds^2 = \left( \frac{r}{R} \right)^2 \left( -h \, dt^2 + dy^2 + d\tilde{x}^2 \right) + \left( \frac{R}{r} \right)^2 h^{-1} \, dr^2 + R^2 \, d\Omega_5^2,
\]

\[
\Phi = 0, \\
B = 0,
\]  

where \( R \) is the AdS scale, \( \tilde{x} := (x_1, x_2) \), \( h \) is \( 1 - (r_H/r)^4 \) and \( d\Omega_5^2 \) is the line element of the unit 5-sphere. The location \( r = r_H \) is where the horizon is and note that setting \( r_H = 0 \) reduces the metric to the extremal case. The fields \( \Phi \) and \( B \) are the dilaton and NSNS 2-form, respectively. We have the RR 4-form potential as well but it does not play an important role in our discussion and hence is omitted. For convenience, [9–11] adopt the \( U(1) \) Hopf fibration over \( P^2 \) to write

\[
d\Omega_5^2 = \eta^2 + ds^2_{P^2},
\]

where \( \eta \) is the 1-form in the Hopf fiber direction and \( ds^2_{P^2} \) is the metric on \( P^2 \) (see appendix of [11] for more details). As always, \( \eta^2 \) is understood to be the symmetric tensor product. (When we intend to mean alternating projections, we write wedges explicitly.)

The coordinate \( y \) is arbitrarily singled out from the three translationally invariant spatial directions (the other directions are denoted by \( \tilde{x} \)) and it is the direction that plays a special role in the Null Melvin Twist procedure. The procedure yields the system,

\[
ds^2 = K^{-1} \left( \frac{r}{R} \right)^2 \left[ -(1 + b^2 r^2)h \, dt^2 - 2b^2 r^2 h \, dt \, dy + (1 - b^2 r^2 h) \, dy^2 + K \, d\tilde{x}^2 \right]
\]  

\[
\Phi = -\frac{1}{2} \ln K, \\
B = K^{-1} \left( \frac{r}{R} \right)^2 b(h \, dt + dy) \wedge \eta,
\]

where \( K := 1 - (b - 1) b^2 r^2 \) and the parameter \( b \) has the dimension of inverse length and is related to the twist factor given in the \( \eta \)-direction (see [9, 11, 13, 14]). The KK reduction of the 5-sphere is a consistent truncation [10] and this provides a five-dimensional system. In doing so, following Herzog \textit{et al} [9], we introduce the light-cone coordinates

\[
x^+ := bR(t + y), \quad \text{and} \quad x^- := \frac{1}{2bR} (t - y).
\]

The particular choice of the normalization will be explained shortly. We then have the five-dimensional system

\[
ds^2 = K^{-2/3} \left( \frac{r}{R} \right)^2 \left[ - \left\{ \frac{h - 1}{(2bR)^2} + \left( \frac{r}{R} \right)^2 h \right\} \, dx^+ dx^- + (1 + h) \, dx^+ dx^+ + (bR)^2 (1 - h) \, dx^- dx^- 
\]  

\[
+ K \, d\tilde{x}^2 \right] + K^{1/3} \left( \frac{r}{R} \right)^2 h^{-1} \, dr^2,
\]

\[
\Phi = -\frac{1}{2} \ln K, \\
A = K^{-1} \left( \frac{r}{R} \right)^2 b \left\{ \frac{h + 1}{2bR} \, dx^+ + bR (h - 1) \, dx^- \right\}.
\]
where we changed to the Einstein frame and renamed the KK-reduced 2-form field \( B \) to \( A \) which is a 1-form field. (The reader is warned about the slightly confusing notations here: \( dx^+ = (dx^+)^2 \) and \( dx^- = (dx^-)^2 \).) As suggested by Son [1], \( x^+ \) is reinterpreted as the time coordinate and \( x^- \) is assumed to be a compactified direction to yield the discrete mass spectrum of the system.

The extremal case is given by the value \( h = 1 \) and the non-extremal case approaches this at asymptotically large \( r \). Note that the normalization of the light-cone coordinates in equations (4) is designed to yield the extremal system which is free of the parameter \( b \). This means that the parameter is not physical in the extremal case and this fact makes the normalization of the light-cone coordinates (4) unique, even for the non-extremal case because that should be fixed by the boundary metric\(^4\).

The five-dimensional system (5) can be obtained from the equations of motion that follow from a five-dimensional action. Such an action was originally proposed by Son [1] for the extremal system and [9–11] find the action that also supports the non-extremal system

\[
S_5 = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R - \frac{4}{3} (\partial\Phi)(\partial\Phi) - \frac{1}{4} R^2 e^{-4\Phi/3} F_{\mu\nu} F^{\mu\nu} - 4A_{\mu} A^{\mu} - V/R^2 \right].
\]

(6)

where \( G_5 \) is the five-dimensional Newton constant, \( g \) is the determinant of the five-dimensional metric, \( R \) is the scalar curvature, \( F : = dA \) and the potential \( V \) is defined to be

\[
V : = 4e^{2\Phi/3}(e^{2\Phi} - 4).
\]

(7)

Son’s original action is recovered by setting \( \Phi \)-field to zero and the potential \( V \) is the negative cosmological constant.

We wrap up this subsection by briefly discussing the geometric properties of the metric, especially the causal development of it. Because of the nonzero \( g^{++} \) component, the geometry is stationary but not static. We interpret this as the rotating black brane in the compactified \( x^- \) direction. Let us rewrite the metric in the ADM form

\[
dr^2 = K^{-2/3} \left( \frac{r}{R} \right)^2 \left[ - \frac{1}{(bR)^2(1-h)} + \left( \frac{r}{R} \right)^2 \right] h \, dx^+ \]

\[
+ (bR)^2(1-h) \left\{ dx^- - \frac{1+h}{2(bR)^2(1-h)} \, dx^+ \right\}^2 + K \, d\vec{x}^2 \]

\[
+ K^{1/3} \left( \frac{R}{r} \right)^2 h^{-1} \, dr^2.
\]

(8)

From this form, we can pick up some information about this geometry. First, recall that \( h(r_H) = 0 \), so we clearly see the existence of the ergo region, as typical for a rotating black hole. Second, the angular velocity of the horizon \( \Omega_H \) measured in the units of \( x^- \)-circumference is given by

\[
\Omega_H = \frac{1}{2(bR)^2},
\]

(9)

and the coordinate angular velocity diverges at the boundary where \( h \to 1 \).

In the usual notion of geometrodynamics, \( x^+ \) is supposed to describe the Cauchy development of the space-like 4-surface with the metric

\[
d\bar{x}^2 = K^{-2/3}(br)^3(1-h) \, dx^- + K^{1/3} \left( \frac{r}{R} \right)^2 \, d\bar{x}^2 + K^{1/3} \left( \frac{R}{r} \right)^2 h^{-1} \, dr^2.
\]

(10)

\(^4\) This fact was pointed out to the author by Ofer Aharony and Zohar Komargodski.
This poses a problem for the extremal case with $h = 1$, because the $g_{4-}$ component is degenerate. The surface of constant $x^+$ is not space-like and the 4-surface has zero measure. This fact stems from the light-like nature of the coordinate $x^-$ before we reinterpret $x^+$ as the time coordinate. Therefore, we see that $x^+$ is not an appropriate time function in the five-dimensional spacetime point of view. One can, actually, see that this is not just due to the particular choice of the time coordinate. For example, if we use time $t$ as in (3), we see that the lapse function squared always becomes negative at sufficiently large $r$. The fact is that this spacetime is not globally hyperbolic, just like AdS and pp-wave spacetimes and to the latter, the Schrödinger geometry is conformal. The causal pathology of the geometry (5) was pointed out in [9] and argued to be the evidence of the holographic dual of non-relativistic CFT which should allow the action-at-a-distance. Though we feel that this interesting point should be investigated further, we will not dwell on this issue in this paper and leave it for future consideration.

2.2. The difference action

As noted before, the black solution (5) found in [9–11] approaches the extremal solution near the boundary and therefore naturally interpreted as the finite temperature generalization of the Galilean holography. This is similar to the finite temperature AdS/CFT correspondence [12]. We thus would like to examine the thermodynamic properties of this finite temperature system. We have mentioned in the introduction that the methods of computing the regularized action and thermodynamic quantities adopted in [9, 10] are not directly applicable to non-planar black solutions. We, therefore, use the subtraction method which we describe in detail here. In what follows, ‘the action’ is meant to be the sum of the bulk action and the Gibbons–Hawking surface term.

We first analytically continue $x^+$ to $i x^+$ and put the system into a box by the cutoff $r = r_B$. The cutoff $r_B$ is assumed to be much larger than the scale $R$ but finite. We subtract the action of the extremal solution from the non-extremal one. In doing so, it is instructed in [15] to match the metrics of those geometries at the wall $r = r_B$. In our case, we have a problem with the boundary matching: since the $g_{4-}$ component of the extremal metric (with $h = 1$) in equation (5) is degenerate, there is no way we can match the metrics at the wall. As noted before, the $x^+$-constant box boundary slice has zero measure for the extremal case. Therefore, we propose to match the boundary metric of the extremal geometry to the non-extremal one for the $x^-$-constant three-dimensional slices parametrized by $x^+$. We can achieve this by appropriately scaling those three coordinates. As for the $x^-$ direction, we scale this coordinate by a constant and adjust the constant so that the resulting difference action is finite in the limit $r_B \to \infty$. It turns out that this constant is just unity for all the black solutions discussed in this paper, so we do not include the constant in the computations below.

Concretely, we have the analytically continued and scaled extremal metric and the fields

$$\Phi = 0,$$

$$d\sigma^2 = \left(\frac{r}{R}\right)^2 \left[\left(\frac{r}{R}\right)^2 H^2_B \, dx^2 - 2i H_B \, dx^+ \, dx^- + G^2_B \, dx^2 \right] + \left(\frac{R}{r}\right)^2 \, dr^2, \quad (\text{11})$$

5 The analytic continuation does not yield a Euclidean section, as noted by Herzog et al [9]. This is because though the system is not static, we do not have an appropriate ‘rotation parameter’ whose simultaneous analytic continuation provides a Euclidean section as in Kerr black holes. We take the same stance as Herzog et al: we carry out the analysis with the complex section because the on-shell action and other relevant quantities will all be real.

6 One does not attempt to match the second fundamental forms of the boundary surface, so it is too strong to say that we match the boundary ‘geometry’.
\[ A = i \left( \frac{r}{R} \right)^2 \frac{H_B}{R} \, dx^+, \]

where we have defined the scaling factors

\[ H_B := \left( K(r_B) \right)^{-2/3} \left\{ \frac{h(r_B) - 1}{(2bR)^2} + \left( \frac{r_B}{R} \right)^2 h(r_B) \right\}^{1/2} \left( \frac{r_B}{R} \right)^{-1}, \quad G_B := K(r_B)^{1/6}. \]  

We then compute the action

\[ S_0 = S_{\text{bulk}} + S_{\text{GH}}, \]

where \( S_{\text{bulk}} \) is the action (6) evaluated on solution (11). The second term \( S_{\text{GH}} \) is the Gibbons–Hawking term

\[ S_{\text{GH}} = - \frac{1}{8 \pi G_5} \int d^4 \sqrt{g_B} (Tr K_0), \]

where \( g_B \) is the determinant of the boundary first fundamental form and \( (Tr K_0) \) is the trace of the second fundamental form with respect to the outward pointing unit normal vector. Since the invariants in the action are not affected by the scaling, the factors of \( H_B \) and \( G_B \) come in only from the metric determinants. Similarly, we compute the action \( S = S_{\text{bulk}} + S_{\text{GH}} \) evaluated on the non-extremal solution (5). The difference \( (S - S_0) \) is now finite in the limit \( r_B \to \infty \) and we have

\[ \lim_{r_B \to \infty} (S - S_0) = \frac{V_4}{16 \pi G_5} \frac{r_H^4}{R^3}, \quad \text{with} \quad V_4 := \int d^4 x. \]

This result is in agreement with [9] whose authors have noted that this is the same result as the Schwarzschild-AdS\(_5\) black brane. In the discussion of the black hole solution, we will find that the action is the same as the Schwarzschild-AdS\(_5\) black hole.

### 2.3. Thermodynamics

We now reproduce the thermodynamics of [9, 10] that follows from the regulated action. For this purpose, we need to identify the temperature, chemical potential, energy, charge and entropy of the system. Let us start with the temperature. We identify the temperature in the traditional way, that is, we set \( \beta = \frac{2 \pi}{\kappa} \) where \( \kappa \) is the horizon surface gravity. In the computation of \( \kappa \), the Killing vector field is taken as

\[ \chi = \partial_v + \Omega_H \partial_{x^-}, \]

where the horizon angular velocity \( \Omega_H \) is shown in equation (9) and it is measured in the units of the compact \( x^- \) circumference in the coordinate length. We then have

\[ \beta = \pi b R^3 / r_H. \]

and we set the (coordinate) circumference of \( x^+ \) to this value. One can also obtain the same result by requiring the analytic continuation of the metric (5) to have the smooth geometry in the \( x^- \)-\( r \) slice. (In doing so, one must bring the metric to the ADM form (8), just as in the Kerr black hole case.)

Let us now discuss the chemical potential of the system. As discussed before, our metric (5) is stationary but not static and it describes the black brane rotating in the compactified \( x^- \) direction. We then naturally interpret the angular momentum and velocity as the charge and the conjugate chemical potential of the system, respectively. This offers

\[ \hat{\mu} := \Omega_H = \frac{1}{2(bR)^2}, \]
where the hat on $\mu$ reminds us that the angular velocity is measured in the units of the $x^-$ circumference. This identification, however, is subtle. In the usual rotating black hole systems, a chemical potential is taken to be the difference between the angular velocities at the horizon and the boundary. However, as we saw previously, our horizon angular velocity is $\Omega_1^H$ and the boundary coordinate angular velocity diverges. Hence the usual identification does not work. Despite this subtlety, we will see shortly that definition (18) yields the consistent entropy, suggesting that the identification is correct.

To obtain the energy, the charge (conjugate to the chemical potential just defined) and the entropy, we define the zero-loop saddle point free energy

$$F := -(16\pi G_s V_3^{-1} \lim_{r_\bar{x} \to \infty} (S - S_0) = -\beta \left( r_1^4 / R^5 \right)$$

$$= -\frac{\pi^4 R^3}{4\hat{\beta}^3 \hat{\mu}^2},$$

(19)

where $V_3$ represents the integration over $x^-$ and $\bar{x}$. Note that the product $G_s V_3^{-1}$ is dimensionless, hence so is $F$. From this, we have

$$E = \left( \frac{\partial F}{\partial \hat{\beta}} \right)_{\hat{\mu}} - \hat{\mu} \beta^{-1} \left( \frac{\partial F}{\partial \hat{\mu}} \right)_{\beta} = r_1^4 / R^5$$

$$Q = -\beta^{-1} \left( \frac{\partial F}{\partial \hat{\mu}} \right)_{\beta} = -4b^2 r_1^4 / R^3$$

$$S = \beta \left( \frac{\partial F}{\partial \beta} \right)_{\hat{\mu}} - F = 4\pi br_1^3 / R^2.$$

These results have been derived in [9, 10]. These quantities, by construction, satisfy the first law. One non-trivial result, though, is that the horizon area (the 3-volume of $x^+$-constant and $r = r_H$ slice of the metric (5)) is exactly four times the entropy derived above, apart from the overall factor in the definition of the free energy (19). This makes us confident of the chemical potential identified above.

To examine the local thermodynamic stability of the system, one can compute the Hessian of $\beta (E - \hat{\mu} Q) - S$ with respect to the thermodynamic variables $(r_H, b)$ and evaluate it at the on-shell values of $(\beta, \hat{\mu})$. This gives $32\pi^2 r_1^4 / R^4$, which is always positive. Therefore, the system is thermodynamically stable.

Finally, we note that the free energy (19) is negative, implying that the non-extremal black brane solution is always thermodynamically preferred to the thermal Schrödinger space without the black brane. This means that the system does not possess the Hawking–Page phase transition.

3. Black hole

It is common that a single gravitational action with a negative cosmological constant supports the black solutions whose horizon geometries are $\mathbb{R}^n, S^n$ and $\mathbb{H}^n$. We find that it is also the case for the five-dimensional action (6). In this section, we present the spherical solution and the hyperbolic case is worked out in the appendix. We will see that the spherical case has a richer phase structure than the other solutions such as the Hawking–Page phase transition.

7 Here, the Hessian is the determinant of the Hessian matrix.
3.1. The solution

Consider the Schwarzschild-AdS black hole solution of type IIB supergravity compactified on $S^5$

$$ds^2 = \left(\frac{r}{R}\right)^2 \left(-h dt^2 + R^2 d\Omega_5^2\right) + \left(\frac{R}{r}\right)^2 h^{-1} dr^2 + R^2 d\Omega_2^2,$$

where $h := 1 + \left(\frac{R}{r}\right)^2 - \left(r_0 R/r^2\right)^2$ and $r_0$ is the non-extremality parameter in that the extremal solution is given by $r_0 = 0$. This parameter can be re-expressed in terms of $r_H$ by solving $h(r_H) = 0$. Recall that the only differences between the spherical and planar black solutions are the function $h$ and the metric $d\Omega_2^2$. To further exploit this similarity to the planar case, we write

$$R^2 d\Omega_2^2 = \eta^2 + dX^2,$$

where we have defined

$$\eta := \frac{R}{2} (d\psi + \cos \theta d\phi), \quad \text{and} \quad dX^2 := \left(\frac{R}{2}\right)^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$

This form of the metric has the advantage that the $S^2$ part is not degenerate at any value of $\psi$. (This is the feature of the $U(1)$ fibration of a sphere.) Using this expression, the ten-dimensional metric above can be trivially rewritten as

$$ds^2 = \left(\frac{r}{R}\right)^2 \left[-(1 + b^2 r^2) h dt^2 - 2b^2 r^2 h dt \eta + (1 - b^2 r^2 h) \eta^2 + K dX^2\right] + K^{1/3} \left(\frac{R}{r}\right)^2 h^{-1} dr^2,$$

where $K := 1 - (h - 1)b^2 r^2$. One can check that this set indeed is a solution to the equations of motion that follow from the five-dimensional action (6). Note that this solution reduces to the Schwarzschild-AdS black hole solution by setting $b = 0$, like the KK-reduced solution (3) becomes the Schwarzschild-AdS black brane. Also, just as in the AdS case, this solution’s isometry group is the maximal compact subgroup of the Schrödinger group. Thus, we see that this is the (asymptotically) Schrödinger space which has $S^2$-foliation for the slice of constant $t$ and $\eta$. To identify the Hamiltonian generator and the compactified direction, we need to define the light-cone-like basis and we introduce the following sets of the coframe:

$$\omega^+ = bR(dt + \eta), \quad \omega^- = \frac{1}{2bR} (dt - \eta),$$

$$\omega^\theta = \frac{R}{2} d\theta, \quad \omega^\phi = \frac{R}{2} \sin \theta d\phi, \quad \omega^r = dr,$$
and the frame

\[ e_+ = \frac{1}{2bR} \left( \partial_t + \frac{2}{R} \partial_\phi \right), \quad e_- = bR \left( \partial_t - \frac{2}{R} \partial_\phi \right), \]
\[ e_\theta = \frac{2}{R} \partial_\theta, \quad e_\phi = \frac{2}{R} \left( -\cot \theta \partial_\theta + \frac{1}{\sin \theta} \partial_\phi \right), \quad e_r = \partial_r. \]  

(27)

Note that we have \( \omega^i e_j = \delta^i_j \), as required for a frame and the corresponding coframe and also observe that \( \omega^\pm \) are defined similar to the light-cone coordinates of equations (4). This clearly is a non-coordinate basis and we have the non-vanishing Lie product

\[ [e_\theta, e_\phi] = 2bR^2 e_- - \frac{2}{R} \cot \theta e_\phi. \]  

(28)

Because of this non-vanishing Lie bracket, the usual definitions of geometric quantities based on the coordinate basis must be modified (see MTW [22]). Among other things, the Christoffel symbols \( \Gamma^\alpha_{\mu \nu} \) defined with respect to a coordinate basis are modified and we no longer necessarily have the symmetry \( \Gamma^\alpha_{\mu \nu} = \Gamma^\alpha_{\nu \mu} \) in the non-coordinate basis.

Given the new coframe, we can now write the solution as

\[ ds^2 = K^{-2/3} \left( \frac{r}{R} \right)^2 \left[ \frac{h - 1}{(2bR)^2} + \left( \frac{r}{R} \right)^2 h \right] \omega^+ \omega^+ \]
\[ - (1 + h) \omega^+ \omega^- + (bR)^2 (1 - h) \omega^- \omega^- + K \omega^2 \]  

+ \( K^{1/3} \left( \frac{R}{r} \right)^2 h^{-1} \omega^\phi \omega^\phi, \]  

(29)

where we have defined \( \omega := (\omega^\theta, \omega^\phi) \). Here, one sees that the coframe is chosen so that the solution appears congruous to the black brane solution (5). In parallel to the black brane case, we interpret \( \omega^+ \) as the direction of time and \( \omega^- \) as the compactified direction with some circumference. Though obvious, it is an interesting exercise to check that the equations of motion are satisfied in the new frame and coframe with the modified geometric quantities.

We remark that for this solution to make sense, we must impose the restriction

\[ bR < 1. \]  

(30)

This is because we have \( K = 1 - (bR)^2 + (bRr_0/r)^2 \), and without the restriction, \( K \) can become zero or negative. This is a special condition for the black hole solution and we do not have it for the black brane case.

As in the previous section, the geometry (29) has the ill-defined causal structure. We can bring the metric to the ADM form identical to equation (8) with the obvious replacements. A quick inspection reveals that with the function \( h \) of the black hole solution, the ADM form is highly pathological. For example, the extremal case \( (r_0 = 0) \) with condition (30) has negative definite lapse function squared. Therefore, as in the black brane case, we must give up on the causal development of the hypersurface defined by constant \( \omega^+ \). Currently the consequences of this observation are unclear and it must be investigated in future whether this has something to do with the holographic dual of non-relativistic CFT on \( S^2 \).

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9 Some care is necessary for this. For example, we have \( F_{\mu \nu} := \omega_{\nu,\mu} - A_{\mu,\nu} \) and we do not replace the semi-colons to just commas, because the symmetry of the Christoffel symbols is lost. Another subtle point is that the definition of the Riemann tensor involves the non-trivial derivatives with respect to \( e_\theta \) which is different from \( \partial_\theta \) by the factor of \( 2/R \) and this must be done with care.
3.2. The difference action

Let us now proceed to compute the on-shell difference action following the procedure described in the previous section. Unlike the previous case, the $\omega^-$ direction is in general not degenerate for the extremal ($r_0 = 0$) nor non-extremal ($r_0 \neq 0$) black hole solutions. However, simple scaling of the cobasis cannot achieve the full matching of the extremal $r = r_B$ boundary metric to the non-extremal one. Therefore, we match the metrics only for the $\omega^-$-constant 3-slices parametrized by $\omega^+, \theta, \phi$, just as was done in the previous section.

We analytically continue $\omega^+$ to $i\omega^+$ and set the cutoff at $r = r_B$. We then have the scaled extremal metric and fields

\[ ds^2 = K_0^{-2/3} \left( \frac{r}{R} \right)^2 \left[ -\left\{ \frac{h_0 - 1}{(2bR)^2} + \left( \frac{r}{R} \right)^2 \right\} H_B^2 \omega^{+2} - (1 + h_0) H_B \omega^+ \omega^- \\
+ (bR)^2 (1 - h_0) \omega^- \right] + K_0^{1/3} \left( \frac{R}{r} \right)^2 \frac{1}{h_0} \omega^2, \]

\[ \Phi = -\frac{1}{2} \ln K_0, \]

\[ A = K_0^{-1} \left( \frac{r}{R} \right)^2 b \left\{ \frac{h_0 + \frac{1}{2bR} H_B \omega^+ + bR(h_0 - 1) \omega^-} {2bR} \right\}, \]

where we have defined

\[ H_B := (K(r_B))^{-2/3} \left\{ h(r_B) - 1 \left( \frac{R}{2bR} \right)^2 + \left( \frac{r}{R} \right)^2 h(r_B) \right\}^{-1/2} \]

\[ \times \left( K_0(r_B))^{-2/3} \left\{ h_0(r_B) - 1 \left( \frac{R}{2bR} \right)^2 + \left( \frac{r}{R} \right)^2 h_0(r_B) \right\}^{-1/2}, \]

\[ G_B := \{K(r_B)/K_0(r_B)\}^{1/6}, \]

\[ h_0 := 1 + (R/r)^2, \]

\[ K_0 := 1 - (bR)^2. \]

Next we compute the action

\[ S_0 = S_{\text{bulk}} + S_{\text{GH}}, \]

where $S_{\text{bulk}}$ is action (6) evaluated on solution (31) and the second term $S_{\text{GH}}$ is the Gibbons–Hawking term. Note that due to the scaling in (31), the coframe (26) and frame (27) must be scaled accordingly, including the factor of ‘$i$’ that comes from the analytic continuation. This results in the modified Lie bracket (28) which in turn affects the non-coordinate-based geometric quantities. Using those quantities, one can check that the invariants in the action $S_0$ are independent of the scaling and the scaling factors come in only from the metric determinants, as expected.

Similarly, we compute the action $S = S_{\text{bulk}} + S_{\text{GH}}$ evaluated on the non-extremal solution (29). The difference $(S - S_0)$ is finite in the limit $r_B \to \infty$ and we have

\[ \lim_{r_B \to \infty} (S - S_0) = \frac{V_4}{16 \pi G_5} \left( r_B^2 - R^2 \right)^2 \frac{r_B}{R^5}, \quad \text{with} \quad V_4 := \int \omega^+ \wedge \omega^- \wedge \omega^\theta \wedge \omega^\phi. \]

Rather surprisingly, this action is the same as the Schwarzschild-AdS$_5$ black hole.

3.3. Thermodynamics

In this subsection, we examine the thermodynamics following from the action calculated in equation (34). We proceed in parallel with the previous section.
First, we identify the temperature. We can compute this just as in the previous section, either through the surface gravity (using the same Killing vector field as equation (16) with the replacements $\partial_\pm \rightarrow e_\pm$) or by requiring the smooth geometry. The result is
\[
\beta = 2\pi br_H R^3 / (2r_H^2 + R^2). \tag{35}
\]
Now, it is a little naive to identify this to the ‘circumference’ of $\omega^+$, because it is not a coordinate basis. However observe that at each fixed point on $S^2$, parametrized by $(\theta, \phi)$, $\omega^+$ is a coordinate basis. We thus write $V_4$ of equation (34) as
\[
V_4 = \beta V_3, \tag{36}
\]
where $V_3$ includes the ‘circumference’ of the $\omega^-$ direction.

Second, we identify the chemical potential to the angular velocity of the horizon as before and it turns out that it is the same
\[
\hat{\mu} = \Omega_H / (2\pi b R^2). \tag{37}
\]
To obtain the thermodynamic quantities, we define the zero-loop saddle point free energy
\[
F := -\left(16\pi G_5\right) V_3^{-1} \lim_{r_H \rightarrow \infty} (S - S_0) = -\beta \left(r_H^2 - R^2\right) r_H^2 / R^5 \tag{38}
\]
where $f := (\pi R)^2 (1 + \sqrt{1 - (4\beta^2 \hat{\mu})/(\pi R)^2})$ and this is defined purely for the presentation purpose and we do not mean anything deeper in this function $f$. Then we compute the thermodynamic quantities as in equations (20) and obtain
\[
E = \left(r_H^2 + 2R^2\right) r_H^2 / R^5, \quad Q = -2b^2 r_H^2 / R^3 \left(2r_H^2 + R^2\right), \quad S = 4\pi b r_H^3 / R^2. \tag{39}
\]
Again, by construction, these quantities satisfy the first law. The entropy, thus calculated, is consistent with the quarter-area law which is a non-trivial result and indicates the right identification of the chemical potential.

The local thermodynamic stability can be examined by computing the Hessian of $\beta(E - \hat{\mu}Q) - S$ with respect to the thermodynamic variables $(r_H, b)$ and evaluate it at the on-shell values of $(\beta, \hat{\mu})$. The result is
\[
\text{Hessian} = \frac{64\pi^2 r_H^2 (2r_H^2 - R^2) (r_H^2 + R^2)}{R^4 (2r_H^2 + R^2)^2}. \tag{40}
\]
This implies the stability threshold $r_H = R / \sqrt{2}$ which translates to the following curve in the $\hat{\mu}-T$ parameter space:
\[
\hat{\mu} = \frac{\pi^2}{4} R^2 T^2. \tag{41}
\]
The mechanism of this instability is very similar to the usual Schwarzschild-AdS black hole, namely, it is the merger point of the large and small black holes in the parameter space. Though we have not explicitly discussed the small black hole, it exists. A quick way to see this is to note that there are two solutions to equation (35) with respect to variable $r_H$ and these solutions degenerate at the threshold.

Let us now discuss the Hawking–Page phase transition [15]. It is clear from equation (38) that the free energy changes sign at $r_H = R$. This implies that the thermodynamically preferred solution changes from the thermal space without black hole to the space with black hole, and
where the chemical potential $\hat{\mu}$ is measured in the units of the circumference of the compactified direction. Therefore, this restriction may be alternatively interpreted as the minimal size of the compactification.

The discussions above are summarized in the phase diagram figure 1.

4. Discussions

In this paper, we have discussed the thermodynamics of the black solutions to the action that describes the space with Schrödinger symmetry. The black hole and black hyperboloid solution are newly found solutions and also we have proposed a way to compute the finite actions. For the black hole case, we have found the Hawking–Page phase transition, as well as the instability lines in the phase diagram.

We have shown that the on-shell actions of all solutions discussed in this paper are identical to those of the black solutions in AdS. This seems to imply that there is a symmetry that transforms the asymptotically Schrödinger solutions to the corresponding asymptotically AdS solutions. For the planar case, [10] explicitly show such a symmetry. Therefore, it is
likely that the symmetry transformation of the reference can be generalized to the non-planar solutions discussed in this paper. If this expectation is borne out, the method similar to that proposed in appendix C.2 of [10] can be applied to the solutions other than the black branes and the resulting thermodynamic quantities computed this way would agree with our results in this paper. It is undeniable that the matching procedure proposed in this paper is highly ad hoc. We have checked that our method produces correct results for the black brane case in section 2 and also we have seen that the same method applied for the black hole case in section 3 produces reasonable results. However, it would be more desirable if we could cross-check the results using different regularization methods. We have not yet found such an alternative procedure and further understanding of this is required. The key issue is closely related to the (so far not so clear) physical meaning of the coordinate, because the procedure cannot match the boundary first fundamental forms for this component. Perhaps, this is related to the fact that the boundary CFT is defined on the coordinate that is not including .

We have mentioned in section 2.3 that there is a certain subtlety in identifying the chemical potential, because of the difference of the angular velocities at the horizon and the boundary diverges. However, we have shown that our identification of the chemical potential yields the entropy that is consistent with the quarter-area law. In contrast, it is advocated in [1, 11] that the chemical potential should be identified with the coefficient of the term in the component. This identification yields the entropy that is not consistent with the quarter-area law. However, it is not clear to us what is inappropriate with their identification. Clarification on this issue is desired.

As pointed out at the end of sections 2.1 and 3.1, the spacetimes discussed in this paper are causally pathological. The problem is clear when we bring the metric into the ADM form with respect to the direction which the Galilean holography seems to uniquely select as the time. It is likely that this behavior of the spacetime is closely related to the fact that this is the (conjectured) dual of non-relativistic conformal field theories. We expect that investigation into this issue would deepen our understanding of the holography.

It is known that the R-charged black holes possess a rich phase structure [26, 27]. It is possible to include the R-charge into the Galilean holography by applying the Null Melvin Twist to the spinning D3-brane geometry [28–30] and taking the near horizon limit of the resulting geometry. (The RR potential, in this case, becomes non-trivial and this itself is interesting to investigate.) From this solution, one could obtain the five-dimensional action and its spherical (and hyperbolic) solution. We are then able to observe the interplay between the chemical potential discussed in this paper and that for the R-symmetry. It would be very interesting to see how the phase structure of the R-charged black holes would be extended.

In this paper, we have exclusively worked out the gravity side of the proposed Galilean holography. It is of natural interest to investigate into the CFT side of the corresponding phenomena found in this work. Most importantly, we would like to see in detail how the non-relativistic conformal field theory behaves in accordance with the Hawking–Page phase transition found in this paper. Learning from the AdS/CFT correspondence, it is likely that the phase transition corresponds to the confinement/deconfinement phase transition in the CFT side. It would be intriguing to see how this actually works in the non-relativistic set up.

Meanwhile, we should always keep in mind that this holography is initiated in the hope to understand the real world strongly coupled non-relativistic field theories. The gravity duals being discussed in the literature (including this work) most likely do not have the real world

10 One example of an alternative is the method proposed in [23–25] However, this method relies on the assumption of asymptotic AdS and it is not straightforward to generalize this to spacetime with a different asymptote.
field theory duals. But as in the AdS/CFT correspondence, we expect the Galilean holography to provide important universal properties of non-relativistic field theories. The fact is that the experimental data exists on the field theory side of the proposed duality. It is crucial to see if one could come up with the gravity dual that is consistent with the experimental data.

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Appendix: Black hyperboloid
In this appendix, we present the black hyperboloid solution and examine its thermodynamics. The discussion follows the structure of the main text.

We would like to find the solution to action (6) with $H^2$ horizon. For this purpose, we find the appropriate form of the metric for $H^3$ as

$$ds^2 = \eta^2 + dX^2,$$

with

$$\eta = \frac{R}{2}(d\psi + \cosh \chi \, d\phi), \quad \text{and} \quad dX^2 = \left(\frac{R}{2}\right)^2(d\chi^2 - \sinh^2 \chi \, d\phi^2).$$

We then find the desired solution

$$d\bar{s}^2 = K^{-2/3} \left(\frac{r}{R}\right)^2 \left[ - \left( \frac{h-1}{(2bR)^2} + \left(\frac{r}{R}\right) \right)^2 \omega^+ - (1 + h) \omega^+ \omega^- + (bR)^2 (1 - h) \omega^2 
+ K (\omega^+^2 - \omega^-^2) \right] + K^{1/3} \left(\frac{r}{R}\right)^2 r^{-1} \omega^r^2,$$

$$\Phi = -\frac{1}{2} \ln K,$$

$$A = K^{-1} \left(\frac{r}{R}\right)^2 b \left\{ h + \frac{1}{2bR} \omega^+ + bR(h - 1) \omega^- \right\},$$

$h := 1 - \left(\frac{R}{r}\right)^2 - \left(\frac{r_0 R}{r}\right)^2,$

$K := 1 - (h - 1)b^2 r^2 - 1 + (bR)^2 + \left(\frac{r_0 bR}{r}\right)^2,$

where we have defined the coframe

$$\omega^+ = bR(dt + \eta), \quad \omega^- = \frac{1}{2bR} (dt - \eta),$$

$$\omega^\chi = \frac{R}{2} d\chi, \quad \omega^\phi = \frac{R}{2} \sinh \chi \, d\phi, \quad \omega^r = dr,$$

and the frame

$$e_+ = \frac{1}{2bR} \left( \partial_t + \frac{2}{R} \partial_\phi \right), \quad e_- = bR \left( \partial_t - \frac{2}{R} \partial_\phi \right).$$
\[ e_\chi = \frac{2}{R} \partial_\chi, \quad e_\phi = \frac{2}{R} \left( - \coth \chi \partial_\phi + \frac{1}{\sinh \chi} \partial_\phi \right), \quad e_r = \partial_r. \] (A.5)

With this non-coordinate basis, the non-vanishing Lie bracket is

\[ [e_\chi, e_\phi] = -2b e_+ + \frac{1}{bR^2} e_- - \frac{2}{R} \coth \chi e_\phi. \] (A.6)

Note that the non-extremality parameter \( r_0 \) is expressed as

\[ r_0 = r_H^2 \sqrt{1 - \left( \frac{R}{r_H} \right)^2}, \] (A.7)

so we have the extremality at \( r_H = R \) and the horizon radius must be larger than this value.

One can proceed to compute the difference action just as in the main text and find that

\[ \lim_{r_B \to \infty} (S - S_0) = \frac{V_4}{16\pi G_5} \left( \frac{r_H^2 + R^2}{R^5} \right)^2, \quad \text{with} \quad V_4 := \int \omega^+ \wedge \omega^- \wedge \omega^\chi \wedge \omega^\phi. \] (A.8)

This is the same as the Schwarzschild-AdS\(_5\) black hyperboloid. The temperature can be computed in the usual way and one obtains

\[ \beta = \frac{2\pi b r_H R^3}{(2r_H^2 - R^2)}. \] (A.9)

Since we have the restriction \( r_H > R \), we see that this solution does not have zero temperature limit. The chemical potential is the same as in other cases

\[ \hat{\mu} = \frac{1}{2(b R)^2}. \] (A.10)

To obtain the thermodynamic quantities, we define the zero-loop saddle point free energy

\[ F := -(16\pi G_5) V_5^{-1} \lim_{r_B \to \infty} (S - S_0) = -\beta \left( \frac{r_H^2 + R^2}{R^5} \right)^2 \]
\[ = -\frac{1}{32 R^3 \beta^3 \hat{\mu}^5} f(\beta, \hat{\mu})^2 \left[ f(\beta, \hat{\mu}) + 6b^2 \hat{\mu} \right], \] (A.11)

where \( f := R^2 (1 + \sqrt{1 + (4b^2 \hat{\mu})/(\pi R)^2}) \). Then we compute the thermodynamic quantities as in equations (20) and obtain

\[ E = \left( \frac{r_H^2}{R^5} - 2R^2 \right)^2, \quad Q = -\frac{2b^2 r_H^2}{R^3} \left( 2r_H^2 - R^2 \right), \quad S = \frac{4\pi b r_H^2}{R^2}. \] (A.12)

By construction, these quantities satisfy the first law and the entropy is consistent with the quarter-area law.

The local thermodynamic stability can be examined by computing the Hessian of \( \beta (E - \hat{\mu} Q) - S \) with respect to the thermodynamic variables \((r_H, b)\) and evaluate it at the on-shell values of \((\beta, \hat{\mu})\). The result is

\[ \text{Hessian} = \frac{64\pi^2 r_H^2 \left( 2r_H^2 + R^2 \right) \left( r_H^2 - R^2 \right)}{R^4 \left( 2r_H^2 - R^2 \right)^2}. \] (A.13)

Because of the restriction \( r_H > R \), we see that the system is always locally stable. Note that for the range of the horizon radius \( R < r_H < \sqrt{2}R \), we have \( E < 0 \), but the system is stable.

Finally we remark that the free energy is always negative, implying that the black solution is always preferred and there is no Hawking–Page phase transition.
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