Doorway states for one-nucleon transfer reactions as a test for current approaches to nuclei*

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Abstract

The naturalness concept of the effective field theory is not confirmed.

1 Introduction

The doorway states for one-nucleon transfer reactions are eigenstates of nucleon in the static field of nucleus which is the convolution of the free-space nucleon-nucleon forces with the nucleon density distributions in nucleus [1]. They are model-independent quantities because (a) the free-space $NN$ forces are independent of nuclear medium effects and (b) the nucleon density distributions are deduced from the electron–nucleus [2] and proton–nucleus [3] elastic scattering data. The corresponding eigenvalue problem is that of nucleon in central field which can be solved with any desired accuracy. For these reasons the doorway states can be used as a very trustworthy test for current nuclear models.

Let us discuss some results of Ref.[1] from this point of view.

1. The nuclear relativity within the Walecka [4] model is confirmed to be actually existing phenomenon.

2. The dominant contribution to the isovector part of the static field arises from the many-particle $NN$ forces because the $\rho$ meson (vector–isovector) and $\delta$ meson (scalar–isovector) fields arising from the two-particle forces nearly cancel each other. At the same time the isovector nuclear potential is exclusively of the $\rho$ meson origin within the quantum hadrodynamics [5, 6]. The reason for this wrong QHD result is the neglect of the $\delta$ meson field in spite of the fact that both the $\rho$ and $\delta$ meson exchanges are taken into account in the two-particle $NN$ forces [7, 8, 9].

3. The contributions from the two-particle, three-particle and four-particle forces to the isoscalar part of the static field are found to be $U_2 \approx -80\,\text{MeV}$, $U_3 \approx +96\,\text{MeV}$, $U_4 \approx -104\,\text{MeV}$. This is in conflict with such leading principles of the effective field theory as the

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naive dimensional analysis and the naturalness [10, 11]. Indeed, the values of the forces which are estimated according to the above principles are [12]

\[ V_2 \approx 30 \text{ MeV}, \quad V_3 \approx \frac{V_2^2}{m} \approx 1 \text{ MeV}, \quad V_4 \approx \frac{V_3^2}{m^2} \approx 0.03 \text{ MeV}. \] (1)

(\(m\) is the mass of nucleon) and therefore the expected relation between \(U_2\), \(U_3\) and \(U_4\) is \(|U_2| : |U_3| : |U_4| \approx 1 : 10^{-\frac{1}{2}} : 10^{-3}\). In fact it is \(1 : 1.2 : 1.3\) thus suggesting that there is something wrong with the naturalness. Discussion of this point is continued in the next Section.

2 Nonlinearity as a source of many-particle forces

As discussed in Ref.[13] the isoscalar part of the static field may contain contributions from higher (five-particle, six-particle, etc.) many-particle forces. They could be taken into account by increasing the number of terms in the power series expansion

\[ U_{st}(r) = \sum_{n=2}^{\infty} a_n \rho^n(r) \] (2)

for the static field [1] (\(\rho(r)\) is the nucleon density distribution) thus introducing an indeterminate number of additional adjustable parameters. Instead we use the fact that ultimately the underlying reason for many-particle forces is the nonlinearity of strong interaction. We introduce an auxiliary scalar-isoscalar field \(\phi\) with the Lagrangian density

\[ L = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U(\phi) - \bar{\psi} \psi \phi \] (3)

\[ U(\phi) = \frac{1}{2} \Lambda^2 \phi^2 + \frac{1}{3} \lambda_3 \phi^3 + \frac{1}{4} \lambda_4 \phi^4, \] (4)

thus obeying the following equation:

\[ \Lambda^2 \phi + \lambda_3 \phi^2 + \lambda_4 \phi^3 = -g \rho_s + \Delta \phi, \] (5)

\(\rho_s(r) = \langle A_0 | \bar{\psi}(r) \psi(r) | A_0 \rangle\) is the nuclear scalar density. As discussed in Ref.[1] the many-particle forces of all ranks are taken into account in this way.

The field \(\phi\) contains the ”two-particle” component \(\phi_2\) obeying the equation

\[ \Lambda^2 \phi_2 = -g \rho_s + \Delta \phi_2. \] (6)

This component must be eliminated because the ”two-particle” contribution to the static field is determined by the free-space two-particle forces [8, 9]. So the many-particle contribution to the scalar–isoscalar field is

\[ W(r) = g(\phi(r) - \phi_2(r)). \] (7)

Let us analyse this expression disregarding for a moment the Laplace terms of Eqs. (5) and (6) which are responsible for the finite range of the forces, although these terms are included in
the actual calculations. But as demonstrated in Ref. [13] they are of little importance thus not affecting the results of the below analysis. As demonstrated in Ref. [1] the radial dependence of $W(r)$ has the form which is schematically shown in Fig. 1. As seen from the figure it is negative at $r < r_1$, positive at $r > r_1$ with a maximum $W_m$ in this region, and vanishing at $r = r_1$. Without the Laplace terms

$$W(r) = -\frac{g\lambda_1}{\Lambda^2} \phi^2(r) \left( \frac{\lambda_3}{\lambda_4} + \phi(r) \right).$$

(8)

So

$$\phi(r_1) = -\frac{\lambda_3}{\lambda_4}.$$  

(9)

But as follows from (6) and (7)

$$\phi(r_1) = -\frac{g}{\Lambda^2} \rho_s(r_1)$$  

(10)

and therefore

$$\frac{\lambda_3}{\lambda_4} = \frac{g\rho_1}{\Lambda^2},$$

(11)

where $\rho_1 = \rho_s(r_1)$. Let us introduce the dimensionless quantities $y(r)$ and $y_2(r)$,

$$\phi(r) = -\frac{g}{\Lambda^2} \rho_1 y(r), \quad \phi_2(r) = -\frac{g}{\Lambda^2} \rho_1 y_2(r).$$

(12)

In these units

$$W(r) = -\frac{g^4\rho_1^3}{\Lambda^8} y^2(r)(1 - y(r))\lambda_4.$$  

(13)

The maximum $W_m$ occurs at $y = 2/3$, so

$$\lambda_4 = -\frac{27\Lambda^8 W_m}{4g^4\rho_1^3}, \quad \lambda_3 = -\frac{27\Lambda^6 W_m}{4g^4\rho_1^2}.$$  

(14)

The parameter $\lambda_4$ is negative since $W_m > 0$, see the figure. The parameter $\lambda_3$ is negative too provided the coupling constant $g$ is positive. Actually the sign of $g$ is insignificant since the physical field is $g\phi$ thus being expressed through $g^2$.

The potential energy of the scalar field is also of the form (4) within the relativistic mean-field approach [14, 15], the parameters $\lambda_3$ and $\lambda_4$ being negative too. In this way the sign of the RMF parameters is confirmed. It should be mentioned that the values of the RMF parameters are determined from the experimental data which include the important correlation effects (binding energies, density distributions, low-energy spectra etc.), and therefore they are model-dependent (the model-independent treatment of the correlations does not exist). In contrast, our parameters are determined from the doorway state energies thus being model-independent.

In terms of the $y$ and $y_2$ quantities the contribution to the scalar–isoscalar field from the many-particle forces is

$$W(r) = -\frac{9xW_m}{4}(y(r) - y_2(r)) + \frac{1}{2} \beta \left( \rho_s^-(r) \right)^2$$

(15)

$$x = \frac{4g^2\rho_1}{9\Lambda^2 W_m}, \quad \rho_s^-(r) = \rho_{sn}(r) - \rho_{sp}(r).$$

(16)
The second term in the rhs of (15) arises from the symmetry energy. The quantities \( y(r) \) and \( y_2(r) \) obey the equations

\[
y(r) + \frac{3}{x} y^2(r)(1 - y(r)) = \frac{\rho_s(r)}{\rho_1} + \frac{1}{\Lambda^2} \Delta y(r) \\
y_2(r) = \frac{\rho_s(r)}{\rho_1} + \frac{1}{\Lambda^2} \Delta y_2(r).
\]

(17)  
(18)

The details of calculations will be described in the forthcoming publication. The resulting values of the parameters are found to be

\[
\rho_1 = 0.146 \text{ fm}^{-3}, \quad W_m = 11.393 \text{ MeV}, \quad x = 16.004, \quad \Lambda = 986.64 \text{ MeV}, \quad \beta = 5.583 \text{ fm}^5.
\]

(19)

The NDA prescription \([10, 11]\) for the scalar field potential energy is \([16]\)

\[
U(\phi) = f_\pi^2 \Lambda^2 \sum_{n=2}^{\infty} \frac{\kappa_n}{n!} \left( \frac{\phi}{f_\pi} \right)^n,
\]

where \( f_\pi = 93 \text{ MeV} \). According to the concept of naturalness all the coefficients \( \kappa_n \) must be of order of unity. Comparison between (20) and (4) together with (14) and (16) gives

\[
\kappa_2 = 1, \quad \kappa_3 = \frac{2f_\pi}{\Lambda^2} \lambda_3 = -4\lambda_2 \frac{f_\pi}{x \rho_1} \left( \frac{\rho_1}{x W_m} \right)^{1/2}, \quad \kappa_4 = \frac{6f_\pi^2}{\Lambda^2} \lambda_4 = -8\lambda_2^2 \frac{f_\pi^2}{x^2 \rho_1 W_m}.
\]

(21)

As follows from the values (19) of the parameters \( \kappa_3 = -1.6, \kappa_4 = -20.5 \), the concept of naturalness thus being not confirmed.

As demonstrated by the calculations for the few-nucleon systems the effect of many-particle forces is relatively small \([12]\). This result is confirmed but the underlying physical reason is different from that provided by the effective field theory. According to the latter it is decrease of the strength with increasing rank of the force, see Eq.(1). As follows from above this scenario does not hold: the actual reason is the cancellation of the contributions from many-particle forces of different ranks (the physics is believed to be the same for complex nuclei and few-nucleon systems).
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Figure 1: Many-particle contribution to the static field.