Non-singlet structure functions: Combining the leading logarithms resummation at small -$x$ with DGLAP

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The explicit expressions for the non-singlet DIS structure functions $F_1$ and $g_1$, obtained at small $x$ by resumming the leading logarithmic contributions to all orders, are discussed and compared in detail with the DGLAP evolution for different values of $x$ and $Q^2$. The role played by the DGLAP inputs for the initial parton densities on the small-$x$ behavior of the non-singlet structure functions is discussed. It is shown that the singular factors included into the fits ensure the Regge asymptotics of the non-singlet structure functions and mimic the impact of the total resummation of the leading logarithms found explicitly in our approach. Explicit expressions are presented which implement the NLO DGLAP contributions with our small -$x$ results.

PACS numbers: 12.38.Cy

I. INTRODUCTION

The non-singlet components of the structure function $F_1$ ($\equiv f^{NS}$) and of the spin structure function $g_1$ ($\equiv g_1^{NS}$) have been investigated in great detail in deep inelastic scattering (DIS) experiments. The standard theoretical framework is provided by DGLAP[1]. In this approach, any of $f^{NS}(x, Q^2), g_1^{NS}(x, Q^2)$ can be represented as a convolution of the coefficient functions and the evolved quark distributions calculated in LO[1], NLO[2] and NNLO[3]. Combining these results with appropriate fits for the initial quark distributions, provides a good agreement with the available experimental data (see e.g.[4] and the recent paper [5]).

On the other hand, the DGLAP evolution equations were originally applied in a range of rather large $x$ values, where higher-loop contributions to the coefficient functions and the anomalous dimensions are small. According to the results of Refs [6, 7], such corrections are becoming more and more essential when $x$ is decreasing and DGLAP should not work so well at $x \ll 1$. Nevertheless, it turns out that DGLAP predictions are in a good agreement with available experimental data, leading to the conclusion that the impact of the higher-order corrections is negligibly small for the available values of $x$ and those corrections may be relevant at asymptotically small $x$ only.

In the present paper we use our previous results [6, 7] to show that the impact of the high-order corrections on the $Q^2$ and $x$ -evolutions of the non-singlet structure functions is indeed quite sizable at available values of $x$. We also show that the reason for the success of phenomenological analysis of the data based on DGLAP at $x < 10^{-2}$ is related to the sharp $x$ -dependence assumed for the initial parton densities, which is able to mimic the role of high-order corrections.

The paper is organized as follows: In Sect. 2 we introduce our notations and discuss the difference of our approach with DGLAP. In Sect. 3 the DGLAP and our predictions for the small-$x$ asymptotics are explicitly compared. It allows us to clarify the role played by the singular terms in $x$ in the DGLAP inputs for the initial parton densities. In Sect. 4 we give a detailed numerical comparison of our results with DGLAP, and discuss also the role played by the regular part of the initial parton densities. As our approach deals with small $x$ only, in Sect. 5 we suggest a possible method to combine DGLAP with our approach in order to obtain equally correct expressions for both large and small values of $x$. Finally, Sect. 5 contains our conclusions.
II. COMPARISON BETWEEN DGLAP AND OUR APPROACH

As the DGLAP -expressions for the non-singlet structure functions are well-known, we discuss them briefly only. In this approach, both \( f^{NS}(x, Q^2) \) and \( g_1^{NS}(x, Q^2) (\equiv f^{DGLAP}(x, Q^2)) \) can be represented as a convolution

\[
f^{DGLAP}(x, Q^2) = \int_x^1 \frac{dy}{y} C(x/y) \Delta q(y, Q^2)
\]

of the coefficient functions \( C(x) \) and the evolved quark distributions \( \Delta q(x, Q^2) \). Similarly, \( d\Delta q(x, Q^2)/\ln(Q^2) \) can be expressed through the convolution of the splitting functions and the initial quark densities \( \delta q(x \approx 1, Q^2 \approx \mu^2) \) where \( \mu^2 \) is the starting point of the \( Q^2 \)-evolution. It is convenient to represent \( f^{DGLAP}(x, Q^2) \) in the integral form, using the Mellin transform:

\[
f^{DGLAP}(x, Q^2) = (c_q^2/2) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega \cdot C(\omega) \cdot \delta q(\omega) \cdot \exp \left[ \gamma(\omega) \int_{\mu^2}^{Q^2} \frac{dk^2}{k^2} \alpha_s(k^2) \right]
\]

where \( C(\omega) \) are the non-singlet coefficient functions, \( \gamma(\omega) \) the non-singlet anomalous dimensions and \( \delta q(\omega) \) the Mellin transforms of the initial non-singlet quark densities. In the \( x \)-space, the standard DGLAP inputs for \( \delta q(x) \) for the non-singlet parton densities (see e.g. Refs. [7]) include terms which are singular when \( x \to 0 \), as well as regular ones for all \( x \). For example, the fit \( A \) of Ref. [7] is chosen as follows:

\[
\delta q(x) = N\eta x^{-\alpha}(1-x)^{\beta} (1 + \gamma x^\delta) = N\eta x^{-\alpha} \cdot \phi(x), \quad \phi(x) \equiv (1-x)^{\beta} (1 + \gamma x^\delta),
\]

with \( N, \eta \) being normalization factors, \( \alpha = 0.576, \beta = 2.67, \gamma = 34.36 \) and \( \delta = 0.75 \). As the first term \( N\eta x^{-\alpha} \) in the rhs of Eq. (8) is singular when \( x \to 0 \) whereas the second one, \( \phi(x) \) is regular, we will address them as the singular and regular parts of the fit respectively. Obviously, in the \( \omega \)-space Eq. (8) is a sum of pole contributions:

\[
\delta q(\omega) = N\eta \left[ (\omega - \alpha)^{-1} + \sum_{k=1}^{\infty} m_k \left( (\omega + k - \alpha)^{-1} + \gamma(\omega + k + 1 - \alpha)^{-1} \right) \right],
\]

with \( m_k = \beta(\beta - 1)...(\beta - k + 1)/k! \), so that the first term in Eq. (11) (the leading pole) corresponds to the singular term \( x^{-\alpha} \) of Eq. (8) and the second term, i.e. the sum of the poles, corresponds to the interference between the singular and regular terms. In contrast to the leading pole position \( \omega = \alpha \), all other poles in Eq. (11) occur at negative values, because \( k - \alpha > 0 \). As usual, when \( \omega \) is integer: \( \omega = n = 1, 2, ... \), the integrand of Eq. (2) gives the \( n \)-th moment of \( f \), (\( \equiv \Gamma_n(Q^2) \)). The standard DGLAP -technology of calculating \( f(x, Q^2) \) corresponds to first calculating \( \Gamma_n(Q^2) \) and then reconstructing \( f(x, Q^2) \) from the moments, choosing appropriate forms for \( \Delta q \). Presently \( C(\omega) \) for the non-singlet structure functions are known with two-loop [2] and three-loop [3] accuracy.

In order to make the all-order resummation of the double-logarithmic contributions to \( f^{NS}(x, Q^2) \) and \( g_1^{NS}(x, Q^2) \), i.e. the contributions \( (\alpha_s/\omega^2)^k \) to \( C(\omega) \) and \( \gamma(\omega) \) in Eq. (12), an alternative approach was used in Refs. [7], by introducing and solving infrared evolution equations with fixed \( \alpha_s \). This approach was improved in Refs. [7], where single-logarithmic contributions were also accounted for and the QCD coupling was running in the Feynman graphs contributing to the non-singlet structure functions. In contrast to the DGLAP parametrization \( \alpha_s = \alpha_s(k^2) \), we used in Refs. [7] another parametrization where the argument of \( \alpha_s \) in the quark ladders is given by the time-like virtualities of the intermediate gluons. Arguments in favor of such a parametrization were given in Ref. [8]. In particular, it was shown that it coincides with the DGLAP -parametrization when \( x \to 1 \), but it differs sensibly when \( x \ll 1 \). Instead of Eq. (2), Refs. [7] suggest the following formulae for the non-singlet structure functions \(^1\):

\[
f^{NS}(x, Q^2) = (c_q^2/2) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega C^{(+)NS}(\omega) \cdot \delta q(\omega) \cdot \exp \left( H^{(+)}(\omega)y \right)
\]

\(^1\) Integration over \( \omega \) in Eq. (1) was performed in Refs. [7, 10], however with simplified assumptions for \( \alpha_s \).
\[ g_1^{NS}(x, Q^2) = (e_q^2/2) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i}(1/x)^\omega C^{(-)}_{NS}(\omega) \delta q(\omega) \exp(H_{NS}^{(-)}(\omega)y) , \]  

with \( y = \ln(Q^2/\mu^2) \) so that \( \mu^2 \) is the starting point of the \( Q^2 \) -evolution. The new coefficient functions \( C^{(\pm)}_{NS} \) are expressed in terms of new anomalous dimensions \( H^{(\pm)}_{NS} \):  

\[ C^{(\pm)}_{NS} = \frac{\omega}{\omega - H^{(\pm)}_{NS}(\omega)} \]  

and the new anomalous dimensions \( H^{(\pm)}_{NS}(\omega) \) which account for the resummation of the double- and single-logarithmic contributions are  

\[ H^{(\pm)}_{NS} = (1/2) \left[ \omega - \sqrt{\omega^2 - B^{(\pm)}(\omega)} \right] \]  

where  

\[ B^{(\pm)}(\omega) = (4\pi C_F(1 + \omega/2)A(\omega) + D^{(\pm)}(\omega))/(2\pi^2) . \]  

Finally \( D^{(\pm)}(\omega) \) and \( A(\omega) \) in Eq. (8) are expressed in terms of \( \rho = \ln(1/x) \), \( b = (33 - 2n_f)/12\pi \) and the color factors \( C_F = 4/3, N = 3 \):  

\[ D^{(\pm)}(\omega) = \frac{2C_F}{b^2N} \int_0^\infty d\eta e^{-\omega\eta} \ln \left( \frac{\rho + \eta}{\rho + \eta + 1/\eta} \right) \left[ \frac{\rho + \eta}{(\rho + \eta)^2 + \pi^2} \right] , \]  

\[ A(\omega) = \frac{1}{b} \left[ \frac{\eta}{\eta^2 + \pi^2} - \int_0^\infty d\rho e^{-\omega\rho} \right] \]  

\( A(\omega) \) is the Mellin transform of \( \alpha_s \) with the time-like argument \( k^2 \), \( \alpha_s(k^2) = 1/(b \ln(-k^2/\Lambda^2)) \). The comparison of Eq. (2) to Eqs. (5) (6) clearly shows the difference between DGLAP and our approach:  

First, our approach accounts for the double- and single-logarithmic contributions to all orders in the QCD coupling whereas in the DGLAP -expressions the coefficient functions and the anomalous dimensions are obtained up to fixed-order accuracy (two and three loops).  

Then a second difference is related to the treatment of \( \alpha_s \). In DGLAP, the argument of \( \alpha_s \) in any Feynman graph is \( k^2 \), with \( k \) being the momentum of the ladder quarks, and therefore \( \alpha_s \) is not involved in the Mellin transform. This leads to the \( Q^2 \) dependence of the non-singlet structure functions with the exponent \( \sim \ln \ln Q^2 \). In our approach \( \alpha_s \) depends on time-like virtualities of the ladder gluons and therefore, it is involved in the Mellin transform.  

The inclusion of \( \alpha_s \) in the Mellin transform explicitly shows that the factorization between the longitudinal and transverse spaces used in DGLAP is not valid for small \( x \). However, as shown in Ref. [8], our approach and DGLAP converge when \( x \sim 1 \) where the factorization does hold. The parametrization of \( \alpha_s(k^2) \) in the form of Eq. (11) has recently been used in Refs. [11] for calculating the double- and single-logarithmic corrections to the Bjorken sum rules.  

### III. THE ROLE OF THE SINGULAR FACTOR FOR \( \delta q \) IN DGLAP FITS  

In the first place let us study the small-\( x \) asymptotics of the non-singlet structure functions. When \( x \rightarrow 0 \), one can use the saddle point method in order to estimate the integrals in Eqs. (11) and derive much simpler expressions for the non-singlet structure functions. When the initial parton densities \( \delta q(x) \) are regular in \( x \) (e.g. given by \( \phi(x) \) only in Eq. (2)), they do not contribute to the saddle point position, which is:  

\[ \omega^{(\pm)}_0 = \sqrt{B^{(\pm)}} \left[ 1 + (1 - q^{(\pm)}_0)^2(y/2 + 1/\sqrt{B^{(\pm)}})^2/(2 \ln^2 \xi) \right] , \]  

with  

\[ q^{(\pm)}_0 = \frac{d\sqrt{B^{(\pm)}}/d\omega |_{\omega=\omega^{(\pm)}_0}} \]
and \( y = \ln(Q^2/\mu^2), \xi = Q^2/(x^2\mu^2) \).

It immediately leads to the Regge asymptotics for the non-singlets:

\[
f^{NS} \sim e_0^2 \delta q(\omega_0^{(\pm)})\Pi_{NS}^{(\pm)}e^{\omega_0^{(\pm)}y}/2, \quad g_1^{NS} \sim e_0^2 \delta q(\omega_0^{(\pm)})\Pi_{NS}^{(\pm)}e^{\omega_0^{(\pm)}y}/2,
\]

with

\[
\Pi_{NS}^{(\pm)} = \frac{2(1-q^{(\pm)})\sqrt{B^{(\pm)}}}{\pi^{1/2} \ln^{3/2} \xi} \left( y/2 + 1/\sqrt{B^{(\pm)}} \right)^{1/2}.
\]

For \( Q^2 \ll 150 \text{ GeV}^2, y \ll 2/\sqrt{B^{(\pm)}}, \) so \( \omega_0^{(\pm)} \) do not depend on \( y \). Therefore Eq. (14) can be rewritten as:

\[
f^{NS}(x, Q^2) \sim e_0^2 \delta q(\omega_0^{(\pm)})e^{T^{(\pm)}(\xi)}(\xi), \quad g_1^{NS}(x, Q^2) \sim e_0^2 \delta q(\omega_0^{(\pm)})e^{T^{(\pm)}(\xi)}(\xi)
\]

with

\[
T^{(\pm)}(\xi) = \xi \omega_0^{(\pm)}/2/\ln^{3/2} \xi
\]

and the intercepts \( \omega_0^{(\pm)} = 0.38 \) and \( \omega_0^{(-)} = 0.43 \) (see [7] for details). The factors \( e^{(\pm)} \),

\[
e^{(\pm)} = \left[ 2(1-q)/(\pi \sqrt{B^{(\pm)}}) \right]^{1/2}
\]

do not depend on \( y \). Eq. (16) predicts the asymptotic scaling for the non-singlet structure functions: in the region \( Q^2 \ll 150 \text{ GeV}^2, \) with \( T^{(\pm)} \) depending on one argument \( \xi \) instead of \( x \) and \( Q^2 \) independently. In the opposite case when \( Q^2 \gg 150 \text{ GeV}^2, \) the scaling is valid for \( f^{NS}/y \) and \( g_1^{NS}/y \). The small-\( x \) dependence of Eq. (16) is confirmed by various analyses of the experimental data in Refs. [13], while the \( Q^2 \)-dependence of these formulae has not been checked yet.

On the other hand, under the same hypothesis that \( \delta q(x) \) is regular (e.g. given again by \( \phi(x) \) in Eq. (3)), the small-\( x \) asymptotics of \( f_{DGLAP} \) is well-known (we drop the inessential pre-factor here):

\[
f^{NS}_{DGLAP} \sim \exp \sqrt{\left( (2C_F)/\pi b \right)(\ln(1/x))\left( (\ln(Q^2/\Lambda^2))/((\ln(\mu^2/\Lambda^2)) \right)}
\]

Obviously, this behavior is different and less steep than the Regge asymptotics of Eq. (16). However when the singular part in \( \delta q \) is also included in the DGLAP input of Eq. (3), the DGLAP asymptotics for the non-singlet structure functions is controlled by the leading singularity \( \omega = \alpha = 0.576 \) of the fit in the integrand in Eq. (4) so that the asymptotics of \( f_{DGLAP}(x, Q^2) \) is also of the Regge type:

\[
f^{NS}_{DGLAP} \sim (e_0^2/2)C(\alpha)(1/x)^{\gamma} \left( (\ln(Q^2/\Lambda^2))/((\ln(\mu^2/\Lambda^2)) \right)^{\gamma(\alpha)/b}
\]

with \( b = (33 - 2n_f)/12\pi \). The comparison of Eq. (16) and Eq. (20) shows that both DGLAP and our approach lead to the Regge asymptotic behavior in \( x \) for the non-singlet structure functions. In particular, Eq. (6) makes the DGLAP prediction be more singular in \( x \) than ours. Also they differ as far as the predictions for the \( Q^2 \)-behavior are concerned. However, it is important to stress that our intercepts \( \omega_0^{(\pm)} \) are naturally obtained by the total resummation of the leading logarithmic contributions, without assuming any singular input for \( \delta q \), whereas the DGLAP intercept \( \alpha \) in Eq. (20) is generated by the phenomenological factor \( x^{-0.57} \) of Eq. (3), which makes the structure functions grow when \( x \) decreases, and mimics in fact the total resummation\(^2\). In other words, the role of the higher-loop radiative corrections on the small-\( x \) behavior of the non-singlets is, actually, incorporated into DGLAP phenomenologically, through the initial parton densities fits. It also means that the singular factors can be dropped from such fits when the coefficient functions account for the total resummation of the leading logarithms and therefore regular functions of \( x \) only can be chosen for the initial densities \( \delta q \).

\(^2\) We remind that our estimates for the intercepts \( \omega_0^{(\pm)} \) were confirmed by several analyses\(^[12]\) of the experimental data.
IV. THE ROLE OF THE REGULAR PART IN THE INITIAL PARTON DENSITIES

As discussed in Sect. II, the regular part in the initial quark distribution in the DGLAP approach, e.g. \( \phi(x) = N(1-x)^\alpha(1+\gamma x^\beta) \) in Eq. (21), corresponds to the sum of pole contributions \( \phi(\omega) \):

\[
\phi(\omega) = N \sum_{k=1}^{\infty} m_k \left( (\omega+k)^{-1} + \gamma(\omega+k+1)^{-1} \right).
\] (21)

We have introduced here the normalization factor \( N \). Obviously, \( \phi \to N \) when \( x \to 0 \). An input \( \delta q(x) = N \) means that the shape of the initial quark distribution does not depend on \( x \). In order to study the influence of the initial parton densities in more detail, we compare numerically our results (6) for two situations: (a) a regular fit \( \delta q(x) = \phi(x) \) is used, (b) \( \delta q(x) = N \). We define \( R_1(x) \) as follows:

\[
R_1(x) = g_1^{NS}[\delta q(x) = \phi(x)] / g_1^{NS}[\delta q(x) = N]
\] (22)

at \( Q^2 = 10 \text{ GeV}^2 \). For all numerical studies in the present section we choose the starting point of the \( Q^2 \)-evolution \( \mu = 1.5 \text{ GeV} \) and assume \( \Lambda = 0.15 \text{ GeV} \) and \( n_f = 4 \). The results for \( R_1 \) are plotted in Fig. 1 (curve 1). It shows that one can approximate \( \phi(x) \) by a constant distribution for \( x < 0.05 \) whereas at larger values of \( x \) the \( x \)-dependence of the initial quark distributions is essential.

The comparison of our results (6) with DGLAP when the regular fit \( \phi(x) \) is used is also shown in Fig. 1 (curve 2) for the ratio \( R_2 \) defined as

\[
R_2(x) = g_1^{NS}[\delta q(x) = \phi(x)] / g_1^{NS}[\delta q(x) = \phi(x)]
\] (23)

where we choose again \( Q^2 = 10 \text{ GeV}^2 \). This shows that when the input for \( \delta q \) does not have a singular term, the difference between our results and DGLAP grows fast for \( x < 0.05 \). On the contrary, when the singular term is included into the DGLAP input, the DGLAP results grow faster than ours. This is shown in Fig. 1 (curve 3) for the ratio \( R_3 \):

\[
R_3(x) = g_1^{NS}[\delta q(x) = \phi(x)] / g_1^{NS}[\delta q(x) = x^{-\alpha}\phi(x)]
\] (24)

with \( Q^2 = 10 \text{ GeV}^2 \). A fast growth of \( f_{DGLAP}^{NS}[\delta q(x) = x^{-\alpha}\phi(x)] \) for small \( x \) also contradicts the analyses of experimental data obtained in Refs. [13].

Our results (21, 22) differ slightly from DGLAP also for the \( Q^2 \)-evolution, depending on \( x \). This is shown in Fig. 2 where the ratio \( R(Q^2) \):

\[
R(Q^2) = g_1^{NS}[\delta q(x) = \phi(x)] / g_1^{NS}[\delta q(x) = \phi(x)]
\] (25)

is plotted for \( x = 0.01 \) (curve 1), \( x = 0.001 \) (curve 2) and for \( x = 0.0001 \) (curve 3). Fig. 2 also suggests that the difference between our and DGLAP predictions for the \( Q^2 \)-evolution slowly grows with increasing \( Q^2 \).

V. IMPLEMENTATION OF DGLAP WITH OUR HIGHER-LOOP CONTRIBUTIONS

We have shown that Eqs. (22, 23) account for the resummation of the double- and single logarithmic contributions to the non-singlet anomalous dimensions and the coefficient functions that are leading when \( x \) is small. However, the method we have used does not allow us to account for all other contributions which can be neglected for \( x \) small but become quite important when \( x \) is not far from 1. On the other hand, such contributions are naturally included in DGLAP, where the non-singlet coefficient function \( C_{DGLAP} \) and anomalous dimension \( \gamma_{DGLAP} \) are known with the two-loop accuracy.\(^3\)

\(^3\) See for details Ref. [12] and the review [13].
\[ C_{\text{DGLAP}} = 1 + \frac{\alpha_s(Q^2)}{2\pi}C^{(1)}, \]
\[ \gamma_{\text{DGLAP}} = \frac{\alpha_s(Q^2)}{4\pi}\gamma^{(0)} + \left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2\gamma^{(1)}. \]

Therefore, we can borrow from the DGLAP formulæ the contributions which are missing in Eqs. (5,6), by adding \( C_{\text{DGLAP}} \) and \( \gamma_{\text{DGLAP}} \) to the coefficient functions and anomalous dimensions of Eqs. (28). Of course, both \( C_{\text{DGLAP}} \) and \( \gamma_{\text{DGLAP}} \) contain also terms already included in Eqs. (5,6), so in order to avoid a double counting these terms should be extracted from \( C_{\text{NS}}^{(\pm)}, H_{\text{NS}}^{(\pm)}. \)

In order to do so, let us consider the region of \( x \sim 1 \) where the effective values of \( \omega \) in Eqs. (5,6) are large. In this region we can expand \( H_{\text{NS}}^{(\pm)} \) and \( C_{\text{NS}}^{(\pm)} \) into a series in \( 1/\omega \). Retaining the first two terms in each series and expressing them through \( A \) and \( D^{(\pm)} \), we arrive at
\[ C_{\text{NS}}^{(\pm)} = \tilde{C}_{\text{NS}}^{(\pm)} + O(\alpha_s^2), H_{\text{NS}}^{(\pm)} = \tilde{H}_{\text{NS}}^{(\pm)} + O(\alpha_s^3) \]

with
\[ \tilde{C}_{\text{NS}}^{(\pm)} = 1 + \frac{A(\omega)CF}{2\pi}[1/\omega^2 + 1/2\omega], \]
\[ \tilde{H}_{\text{NS}}^{(\pm)} = \frac{A(\omega)CF}{4\pi}[2/\omega + 1] + \left(\frac{A(\omega)CF}{4\pi}\right)^2(1/\omega)[2/\omega + 1]^2 + D^{(\pm)}(\omega)[1/\omega + 1/2]. \]

Now let us define the new coefficient functions \( \tilde{C}_{\text{NS}}^{(\pm)} \) and new anomalous dimensions \( \tilde{\gamma}_{\text{NS}}^{(\pm)} \) as follows:
\[ \tilde{H}_{\text{NS}}^{(\pm)} = \left[H_{\text{NS}}^{(\pm)} - \tilde{H}_{\text{NS}}^{(\pm)}\right] + \frac{A(\omega)}{4\pi}\tilde{\gamma}^{(0)} + \left(\frac{A(\omega)}{4\pi}\right)^2\tilde{\gamma}^{(1)}, \]
\[ \tilde{\gamma}_{\text{NS}}^{(\pm)} = \left[C_{\text{NS}}^{(\pm)} - \tilde{C}_{\text{NS}}^{(\pm)}\right] + 1 + \frac{A(\omega)}{2\pi}\tilde{\gamma}^{(1)}. \]

These new, "implemented" coefficient functions and anomalous dimensions of Eq. (28) include both the total resummation of the leading contributions and the DGLAP expressions in which \( \alpha_s(Q^2) \) is replaced by \( A(\omega) \). As shown in Ref. [7], this parametrization differs from the DGLAP one, \( \alpha_s = \alpha_s(k_F^2) \), at small \( x \), though both parameterizations coincide when \( x \) is large. The main point is that the factorization of the phase space into transverse and longitudinal spaces used in DGLAP is a good approximation for large \( x \) only. The implemented coefficient functions \( \tilde{C}_{\text{NS}}^{(\pm)} \) and anomalous dimensions \( \tilde{\gamma}_{\text{NS}}^{(\pm)} \) of Eq. (28) can be used either in the standard DGLAP way by letting \( \omega \) be integer, i.e. \( \omega = n, \) with \( n = 1, 2, \ldots \) and thereby obtaining the moments of the non-singlet structure functions, or alternatively by using \( \tilde{C}_{\text{NS}}^{(\pm)}, \tilde{H}_{\text{NS}}^{(\pm)} \) in Eqs. (5,6) where \( \omega \) is complex. In the latter case, \( C_{\text{DGLAP}} \) and \( \gamma_{\text{DGLAP}} \) should first be continued analytically from the integer values \( \omega = n \). We have verified that the new formulæ in Eq. (28) provide a much better agreement with NLO DGLAP in the region of large \( x \).

We notice that our approach is qualitatively close to the one suggested in Ref. [14], however they are not quite identical: indeed they first differ in the treatment of \( \alpha_s \) which is running in our picture in every Feynman graph involved, whereas in Ref. [14] the DL contributions are obtained from Refs. [8] where \( \alpha_s \) was kept fixed at an unknown scale. To make it running, in Ref. [14] the parametrization \( \alpha_s = \alpha_s(Q^2) \) is suggested in the final formulæ, which is of course incorrect at small \( x \) (see Refs. [8] for details). In addition, the single-logarithmic contributions were not accounted in Ref. [14] at all.

VI. DISCUSSION AND CONCLUSIONS

As is well known, the conventional DGLAP approach was originally suggested for the region of rather large values of \( x \), where logarithms of \( Q^2 \) gave the most important contributions. At the same time, DGLAP neglects the total resummation of logs of \( x \). In our approach we have been able to account for those contributions to the non-singlet structure functions at small \( x \). The results are presented in Eqs. (5,6). Similarly to DGLAP, the new anomalous dimensions \( \tilde{\gamma} \) control the \( Q^2 \) -evolution while new coefficient functions \( \tilde{C} \) evolve the initial quark densities from their empirical values \( \delta_q \) at \( x \sim 1 \) to \( x \ll 1 \).

The extrapolation of Eqs. (5,6) into the asymptotic region \( x \to 0 \) leads to the Regge asymptotic formulæ of Eqs. (14,15). The extrapolation of the DGLAP expressions with the standard assumption of Eq. (16) for the initial quark distribution \( \delta_q \) also leads to the Regge behavior Eq. (20). However, our approach provides the Regge behavior
because of the total resummation of the leading logarithms of $x$ whereas the DGLAP Regge asymptotics is generated by the phenomenological factor $x^{-\alpha}$ in the fit of Eq. (3) for $\delta q$. This allows us to conclude that the singular part of the DGLAP input for the initial quark densities mimics the impact of total resummation and can be dropped when the resummation is done, simplifying the structure of the input densities to expressions regular in $x$. It also turns out that the DGLAP small-$x$ asymptotics, using the fit of Eq. (3) for $\delta q$ for the non-singlet, is more singular than our predictions. This contradicts the various analyses of the experimental data of Refs. [15], which agree with our predictions. Our asymptotics also differ from DGLAP as far as the $Q^2$-dependence is concerned: indeed we predict an asymptotic scaling, where the non-singlet structure functions asymptotically depend on one argument $\xi = Q^2/x^2$ instead of two arguments $Q^2$ and $x$, separately.

Giving up the singular factor from the DGLAP fit Eq.(3) and using its regular part $\phi(x)$ only, we have compared numerically our formulae Eqs. (5,6) with the NLO DGLAP results. Such a comparison confirms that in absence of the singular terms the DGLAP small-$x$-dependence and $Q^2$-dependence of the non-singlets are slower than ours. The results have been displayed in Figs. 1, 2.

Finally, in order to obtain an overall approach which is consistent and accurate in both regions of small and large $x$, we have suggested to combine our results and those of DGLAP by an appropriate definition of the anomalous dimensions and the coefficient functions. Indeed, the expressions of Eqs. (28) for the improved anomalous dimensions and coefficient functions account for both NLO DGLAP terms and our total resummation of the most important logs of $x$. Therefore, Eqs. (5,6) can be used both at large and small $x$. Then the new inputs for $\delta q$ in this approach could be much simpler functions of $x$ than in the standard DGLAP approach.

VII. ACKNOWLEDGEMENT

The work is supported partly by grant RSGSS-1124:2003.2

VIII. FIGURE CAPTIONS

Fig. 1: The $x$-dependence for: $R_1$ of Eq. (22) (curve 1), $R_2$ of Eq. (23) (curve 2) and $R_3$ of Eq. (24) (curve 3). All curves correspond to $Q^2 = 10$ GeV$^2$.

Fig. 2: The $Q^2$-dependence for $R(Q^2)$ of Eq. (25). Curves 1,2,3 correspond to $x = 10^{-2}$, $10^{-3}$, $10^{-4}$ respectively.

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FIG. 1:

FIG. 2: