Post-recombination gravity-generated contributions to the cosmic microwave background anisotropies and cosmological parameter estimation.

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Gravitational interaction of cosmic microwave background (CMB) photons with matter perturbations present along the line-of-sight to the surface of last scattering modifies the shape of the CMB anisotropy power spectrum. Here I focus on (linear) integrated Sachs-Wolfe and (non-linear) gravitational lensing effects and discuss the detectability of the resulting distortions and their possible consequences for the CMB-based estimation of cosmological parameters. Specifically, I discuss if any of those effects may allow us to use CMB experiments to put independent constraints on the curvature of the universe and the cosmological constant, i.e. breaking the so-called geometrical degeneracy in CMB parameter estimation discussed by Bond, Efstathiou & Tegmark (1997) and Zaldarriaga, Spergel & Seljak (1997).

I address that issue using the Fisher matrix approach and show that gravitational lensing of the CMB temperature and polarisation patterns might be detectable by the Planck Surveyor satellite, leading to useful independent and precise constraints on the cosmological constant and spatial curvature. The integrated Sachs-Wolfe effect though bound to restrict those parameters only very weakly still may set constraints more stringent than those currently available.

1. INTRODUCTION. In standard inflationary scenarios of structure formation in the Universe the primordial perturbations generated during the inflation phase are amplified during the subsequent stages of the evolution, leading to the abundance of structures extending at the present from the scale well over 100h⁻¹Mpc down to galactic scales. Byproducts of the matter perturbation evolution are the anisotropies in the photon background, which fills the Universe with a present day temperature of $T_0 = 2.726$K and energies in the microwave band. While these anisotropies are expected (and measured) to be very small, they are suitable for linear analysis and therefore provide a convenient tool for exploring the Universe. Furthermore, while they appear to be very sensitive to the evolution of the Universe as well as details of the perturbations present in the Universe CMB investigations offer an opportunity to constrain numerous important yet poorly till now known parameters.

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These include the parameters of the homogeneous FRW background as well as those describing statistical properties of the primordial perturbations.

In any viable model the CMB photons stop interacting non-gravitationally with matter at very high \( z \gtrsim 50 \) redshift (for definiteness hereafter I will assume the standard thermal scenario (Peebles 1968) with the surface of last scattering (SLS) at \( z \sim 1100 \) and neglect the possibility of inhomogeneous reionisation and existence of small-scale effects such as the Sunyaev-Zel’dovich effect). Since then photons travel freely along the geodesics carrying on the image of the SLS.

![Figure 1](image.png)

**FIGURE 1.** The thick solid lines show the locii of the cosmological models indistinguishable for CMB observations if post-recombination contributions to observed anisotropies are neglected. Also, lines of the constant Hubble parameter (spanning the densely shaded area) and of constant age (in Gyr) are depicted with corresponding values as given at the edges of the figure.

The high redshift of the SLS means that at that time the CMB photon distribution is not affected yet by either the cosmological constant or the curvature of the Universe. Hence the models with very different values of those two parameters may produce identically perturbed universes at the time of last scattering. The angular scales at which those 3D structures are observed at the present are determined solely by one parameter: a comoving distance travelled by photons from SLS to the present denoted as \( r_{SLS} \equiv r_{SLS} (\omega_b, \omega_c, \omega_h, \omega_\gamma, \omega_\nu, \omega_K, \omega_\Lambda) \) and usually referred

\[ \omega_i = \Omega_i h^2, \]

where \( \Omega_i \) denotes, as usual, the density of the \( i \)th component in critical units, \( h \) – the Hubble parameter in units of 100km/s/Mpc, and \( i = b, c, h, \gamma, \nu, K, \Lambda \) stands for baryons, cold dark matter, hot dark matter, photons, massless neutrinos, the curvature and the cosmological constant respectively.

\[ \dagger \] I use physical densities \( \omega_i = \Omega_i h^2 \), where \( \Omega_i \) denotes, as usual, the density of the \( i \)th component in critical units, \( h \) – the Hubble parameter in units of 100km/s/Mpc, and \( i = b, c, h, \gamma, \nu, K, \Lambda \) stands for baryons, cold dark matter, hot dark matter, photons, massless neutrinos, the curvature and the cosmological constant respectively.
to as the distance to the SLS. Consequently, any two models with different values of the cosmological constant and the curvature chosen to produce an identical distance to the SLS, and equal values of all the other parameters ($\omega_i$) may produce statistically indistinguishable patterns of observed CMB anisotropies if normalised appropriately (see Stompor & Efstathiou 1998 (hereafter SE98) for a correct normalisation procedure). That property of CMB anisotropies has been nicknamed a geometrical degeneracy.

If the geometrical degeneracy were exact, then it would be a distance to the SLS which could be inferred from precise CMB measurements, rather than values of the curvature and the cosmological constant separately. Those would remain almost unconstrained – a serious setback for those hoping to use CMB for parameters estimation. Fortunately, it appears not to be the case.

2. POST-RECOMBINATION EPOCH. As was noticed early on (e.g. Sachs & Wolfe 1967, Rees & Sciama 1968), though the Universe stays mostly neutral after last scattering (by definition) the photons travelling through space are affected by the gravity of the growing matter inhomogeneities. The most important contribution is that on the largest angular scales where the anisotropy imprinted by the evolving potential wells may dominate the one generated at the time of last scattering. The effect, called the integrated Sachs-Wolfe (ISW) effect, is linear and may be produced either just after recombination (high-$z$ ISW), when radiation density still affects overall expansion or at low redshifts ($z \lesssim 10$) (low-$z$ or late ISW) with the curvature or/and the cosmological constant interfering with the pace of the expansion. As mentioned above, the amplitude of this effect, which is often dominant on the largest angular scales, is expected to decrease rapidly with angular scale, because gains and losses of the energy of photons crossing the potential wells of the perturbations with sizes much smaller than $r_{SLS}$ tend to average out very efficiently giving a negligible net effect.

The remaining effects: the Rees-Sciama effect (or the non-linear ISW effect) and gravitational lensing are of second (and higher) order nature in a perturbative approach and therefore unavoidably small. Both were investigated in the past in considerable detail by many authors (see e.g. Seljak 1996ab, SE98 and references therein) who generally concluded that, of the two, it is lensing that might be of more importance in viable cosmologies. Hereafter I therefore discuss only the linear ISW and lensing effects. It is important to observe that though both effects are generated by gravity, it is only lensing that may distort a polarisation pattern. That property together with the fact that lensing-introduced distortions become larger on smaller angular scales implies that predominantly lensing is of importance for a CMB-based parameter estimation as demonstrated in the following.

Both effects are displayed in Fig. 2 and contrasted against the cosmic variance (full sky coverage assumed). What is presented in the figure are distortions of power spectra of the CMB anisotropies $C_\ell$ defined as a variance of coefficients with a polar number $\ell$ of a multipole decomposition of the two dimensional CMB maps into a series of spherical harmonics. In the realm of Gaussian theories the power spectra provide a complete statistical description of the CMB anisotropies.

The normalisation of the models follows the recent cluster abundance analyses giving a standard deviation of mass on scale of $8h^{-1}\text{Mpc}$ equal to $\sigma_8 \simeq 0.52 \Omega^{-0.6}_0$ (e.g. Eke, Cole & Frenk 1996). To express the magnitude of discussed contributions
I employ here a $\chi^2$ statistics (per degree of freedom) defined as (I neglect here possible cross-correlations between $C_\ell$ coefficients, see Wandelt, Hivon & Górski 1998 for an exact treatment in a case of azimuthally symmetric sky cuts)

$$\chi^2_\alpha = \frac{1}{\ell_{\text{max}} - 1} \sum_{\ell=2}^{\ell_{\text{max}}} \frac{(\tilde{Z}_{\ell,\alpha} - Z_{\ell,\alpha})^2}{\sigma_{\alpha}^2}, \quad \alpha = S \text{ or } BJ.$$ 

Here $Z_{\ell,\alpha}$ is a function of $C_\ell$ solely and is considered to be a Gaussian variable. $\sigma_{\alpha}$ is the corresponding standard deviation. I consider here two possibilities. The first one is a standard high-$\ell$ approximation ($\alpha = S$) with $Z_{\ell}^S \equiv C_\ell$ and $\sigma_S^2 \equiv 2 (2\ell + 1)^{-1} f_{\text{sky}}^{-1} (C_\ell + w^{-1} b_\ell^{-2})$. The second choice ($\alpha = BJ$) follows that of Bond & Jaffe (1998) and is supposed to account for the non-gaussian character of $C_\ell$ and is therefore applicable on large angular scales (i.e. at the low-$\ell$ tail). The appropriate definitions read in this case $Z_{\ell}^{BJ} \equiv \ln (C_\ell + w^{-1} b_\ell^{-2})$ and $\sigma_{BJ}^2 \equiv 2 [f_{\text{sky}} (2\ell + 1)]^{-1}$. Hereafter $b_\ell$ denotes an antenna beam pattern, and $w$ is a squared product of the noise level per pixel and the angular size of the pixels. The tilde over a quantity means that the post-recombination contribution has been taken into account. $\ell_{\text{max}}$ has been always chosen to maximize a corresponding $\chi^2$ value. The assumed specifications of the experimental setup follow those for the combined three best Planck channels. $f_{\text{sky}}$ is assumed to be 0.65. (See SE98 for a detailed description.)
For the particular cosmological model shown in fig. 2, the corresponding $\chi^2$ values are $\chi^2_{BJ} \simeq 13.2$ (ISW) and $\chi^2_{BJ} \simeq 0.6$ (lensing) for the temperature power spectrum and $\chi^2_{BJ} \simeq 1.4$ for the polarisation one (lensing only). It is therefore not only apparent that a strong detection of the ISW contribution is to be expected, but also that the lensing distortions might be detectable especially if a polarisation measurement is performed (cf. Fig. 2c). Note that the major contribution to $\chi^2$ comes from the very low-\ell modes in the ISW distortion case (i.e. $\ell_{\max} = 2$) and the computed value can be improved upon neither by improving on the resolution or noise of an experiment. In the contrary, the major contribution in the lensing case comes from the highest accessible \ell-modes ($\ell_{\max} \simeq 2650$ for a Planck-like observation) and therefore susceptible to further improvements.

For lensing the ($\alpha = S$) choice of variables would give $\chi^2_S \simeq 0.6$ and 1.4 for temperature and polarisation respectively, in good agreement with the previous numbers, showing that the use of these variables in lensing analyses is justified.

3. COSMOLOGICAL PARAMETERS ESTIMATION. While, as shown above, both ISW and lensing-generated distortions may be detectable for a Planck-type experiment, thus breaking the geometrical degeneracy, it is still to be determined to what precision this can be done.

The fashionable way to investigate this problem is through the Fisher matrix analysis (e.g. Tegmark, Taylor & Heavens 1997). Here under the assumption of the nearly Gaussian character of the likelihood as a function of cosmological parameters, the expected uncertainties of the determination of cosmological parameters are estimated using the curvature matrix of the likelihood at its peak.

Involved computations require knowledge of the derivatives of the lensing contribution with respect to various cosmological parameters. In the case of gravitational lensing the smallness of these contributions makes this a serious problem. Here I skip the description of the performed calculations referring the reader to SE98 for a discussion. Derivatives of ISW distortions were estimated using a finite differencing scheme of the computed power spectra of the ISW effect alone.

| $\omega_K$ | $\omega_\Lambda$ | $[\delta \omega_K]_{GL}$ | $[\delta \omega_\Lambda]_{GL}$ | $[\delta \omega_K]_{ISW}$ | $[\delta \omega_\Lambda]_{ISW}$ |
|---|---|---|---|---|---|
| 0.0085 | 0.2515 | 0.1 | 1.2 | 0.8 | 10 |
| 0.175 | 0.085 | 0.3 | 1.0 | 1.8 | 6.5 |

Hereafter, I limit the parameter space to only two dimensions ($\omega_K, \omega_\Lambda$) with all other cosmological parameters fixed. A discussion of changes accompanying an increase in the dimensionality of the parameter space can be found in SE98. Note, that the constraints set by the Fisher matrix analysis do not depend on the choice of $Z_\ell$ variables. A selection of the results is given in Table 1.
4. CONCLUSIONS. The imprint of gravitational lensing on the power spectra of CMB anisotropies is likely to be detected by future high resolution and precision experiments including the instruments on board the Planck satellite. As a result, the non-linear gravitational lensing contribution may have to be taken into account in the analysis of the future experimental data. One of the consequences of that fact, apart from the associated complications for the analysis itself, is rather fortunate. Lensing-generated distortions appear to contribute to breaking of the geometrical degeneracy, improving substantially on the constraints that can be set on the curvature and the cosmological constant separately if only linear CMB anisotropy power spectra are considered. Though it is true in a case of both temperature and polarisation it is polarisation which appears to be much more sensitive to the lensing-generated distortions.

The forecasted errors in the determination of both of those parameters depend on the assumed normalisation of the primordial perturbations. However, keeping in mind the rather low normalisation inferred recently from the cluster abundance (and adopted for the work presented here), they are not expected to be underestimated too significantly. Hence high resolution and sensitivity CMB measurements on their own may provide an independent data set to be crosschecked for consistency with data from other sources in an attempt to verify theoretical assumptions underlying a choice of cosmological model parameter space under considerations.

If lensing distortions are not unambiguously detected then the ISW effect may break the geometrical degeneracy of the linear CMB power spectra. However the inferred errors are almost an order of magnitude worse than those set with the help of lensing. Yet even those may seem attractive from the present-day perspective.

The higher sensitivity of polarisation to the lensing effects makes its measurement not only desirable for the purpose of degeneracy breaking, but also as a test of the presence of nonlinear contributions to the temperature power spectrum.

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