Abstract

We study the heavy-quark contributions to the proton structure function $F_2(x, Q^2)$ at next-to-leading order using compact formulas at small values of Bjorken’s $x$ variable. The formulas provide a good agreement with the modern HERA data for $F_2(x, Q^2)$.

Keywords: Deep inelastic scattering; nucleon structure functions; QCD coupling constant.

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1 Introduction

In the last year the H1 [1, 2] and ZEUS [3, 4] Collaborations at HERA are presented a new data on the charm structure function (SF) $F_2^c$. Moreover, recently H1 and ZEUS have been demonstrated the preliminary combine data of $F_2^c(x, Q^2)$ [5].

In the framework of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) dynamics [6] there are two basic methods to study heavy-flavor physics. One of them [7] is based on the massless evolution of parton distribution functions (PDFs) and the other one is based on the photon-gluon fusion (PGF) process [8]. There are also some interpolating schemes (see Ref. [9] and references cited therein).

In this short paper, we apply compact low-$x$ approximations for the SF $F_2^c(x, Q^2)$ obtained [10] in the PGF framework at the first two orders of perturbation theory to these new HERA experimental data [1, 2, 3, 4, 5]. We show a good agreement between experimental data and the approach which was found without any additional free parameters. All PDF parameters have been fitted earlier [11] from $F_2(x, Q^2)$ HERA experimental data only.

2 Approach

We now present our basic formula for $F_2^c(x, Q^2)$ appropriate for small values of $x$, where only the gluon and quark-singlet contributions matter, while the non-singlet contributions are negligibly small.

\[
F_2^c(x, Q^2) = \sum_{a=g,l} \sum_{r=+,-} C_{2,a}^r(x, Q^2, \mu^2) \otimes x f_a^r(x, \mu^2) \approx \sum_{r=+,-} C_{2,g}^r(x, Q^2, \mu^2) \otimes x f_g^r(x, \mu^2) \tag{1}
\]

The papers [1, 2, 3, 4] contain also the references on the previous data on deep-inelastic (DIS) structure function (SF) $F_2^c$ at small $x$ values.

Here and in the following, we suppress the variables $m_c$ in the argument lists of the structure and coefficient functions for the ease of notation.
where \( l \) generically denotes the light-quark flavors, \( r = \pm \) labels the usual + and − linear combinations of the gluon and quark-singlet contributions, \( C_{2,a}^g(x) \) are the DIS coefficient functions, which can be calculated perturbatively in the parton model of QCD, \( \mu \) is the factorization scale and same time the renormalization one appearing in the strong-coupling constant \( \alpha_s(\mu) \), and the symbol \( \otimes \) denotes convolution according to the usual prescription, \( f(x) \otimes g(x) = \int_0^1 (dy/y) f(y) g(x/y) \). Massive kinematics requires that \( C_{2,a}^g = 0 \) for \( x > b = 1/(1 + 4c) \), where \( c = m_c^2/Q^2 \). We take \( m_c \) to be the solution of \( \overline{m}_c(m_c) = m_c \), where \( \overline{m}_c(\mu) \) is defined in the modified minimal-subtraction (\( \overline{\text{MS}} \)) scheme. The simplification in the r.h.s. of (1) is obtained by neglecting the contributions due to incoming light quarks and antiquarks, which is justified because they vanish at LO and are numerically suppressed at NLO for small values of \( x \).

Exploiting the low-\( x \) asymptotic behavior of \( f_a^r(x, Q^2) \) \[12\] \[13\],

\[
f_a^r(x, \mu^2) \xrightarrow{x \to 0} \frac{1}{x^{1+\delta_r}} \tilde{f}_a^r(x, \mu^2) \quad \text{(hereafter \( a = q, g \))},
\]

where the rise of \( \tilde{f}_a^r(x, \mu^2) \) as \( x \to 0 \) is less than any power of \( x \), Eq. (1) can be rewritten as

\[
F_2^g(x, Q^2) \approx \sum_{r=+,-} M_{2,g}^r (1 + \delta_r, Q^2, \mu^2) x f_a^r(x, \mu^2),
\]

where

\[
M_{2,g}^r(n) = \int_0^{b_i} dx x^{n-2} C_{2,g}^r(x)
\]

is the Mellin transform, which is to be analytically continued from integer values \( n \) to real values \( 1 + \delta_r \). Strictly speaking, the equation (3) is correct for non-singular Mellin moments \( M_{2,g}^r(n) \) at \( n \to 1 \). The generalization of (3) for the moments containing singularity will be done in subsection 4.2.

### 3 Gluon density

As demonstrated in Refs. \[14\] \[11\], HERA data for \( F_2(x, Q^2) \) support the modified Bessel-like behavior of PDFs at small \( x \) values predicted in the framework of the so-called generalized double-asymptotic scaling regime. \[3\] In this approach, DGLAP dynamics \[6\] starting at some initial value \( \mu_0^2 \) with flat \( x \) distributions:

\[
xf_a(x, \mu_0^2) = A_a,
\]

where \( A_a \) are unknown constants to be determined from the data.

In the framework of the generalized double-asymptotic scaling regime, the main ingredients of the results for gluon density include the following. \[4\] It is presented in terms of two components (“+” and “−”)

\[
f_g(x, \mu^2) = f_g^+(x, \mu^2) + f_g^-(x, \mu^2),
\]

which are obtained from the analytic \( \mu^2 \)-dependent expressions of the corresponding (“+” and “−”) PDF moments. Here, \( e_i \) is the fractional electric charge of heavy quark \( i \), \( e = (\sum_{i=1}^4 e_i^2)/f \) is the average charge square and \( f \) is the number of active quark flavors.

The small-\( x \) asymptotic results for the PDF \( f_g^\pm \) are at the next-to-leading order (NLO) \[16\]

\[
x f_g^+(x, \mu^2) = A_g^+ f_0(\sigma)e^{-D_+ s-D_+ p} + O(p),
x f_g^-(x, \mu^2) = A_g^- e^{-D_- s-D_- p} + O(x),
\]

\[3\] It is a generalization of earlier studies \[15\].

\[4\] The results for quark densities may be found in Refs. \[14\] \[11\].
where $I_\nu$ are the modified Bessel functions,
\[ D_\pm = d_{\pm\pm} - \frac{\beta_1}{\beta_0} d_\pm \] (8)
and similar for $\bar{D}_\pm$ and $\overline{D}_\pm$,
\[ A_g^+ = \left(1 - \frac{80f}{81} a_s(Q)\right) A_g + \frac{4}{9} \left(1 + \left(3 + \frac{f}{27}\right) a_s(Q_0) - \frac{80f}{81} a_s(Q)\right) A_g, \quad A_g^- = A_g - A_g^+ \] (9)
and
\[ a_s(\mu) = \frac{a_s(\mu_0)}{4\pi}, \quad \hat{d}_+ = -\frac{12}{\beta_0}, \quad \bar{d}_+ = 1 + \frac{20f}{27\beta_0}, \quad d_- = \frac{16f}{27\beta_0}, \]
\[ \hat{d}_{+-} = \frac{412f}{27\beta_0}, \quad \bar{d}_{+-} = \frac{8}{\beta_0} \left(36\zeta_3 + 33\zeta_2 - \frac{1643}{12} + \frac{2f}{9} \left[68 - 4\zeta_2 - \frac{13f}{243}\right]\right), \]
\[ d_{--} = \frac{16}{9\beta_0} \left(2\zeta_3 - 3\zeta_2 + \frac{13}{4} + f \left[\frac{23}{18} + \frac{13f}{243}\right]\right), \] (10)
with $\zeta_2$ and $\zeta_3$ are Euler functions. $\beta_0$ and $\beta_1$ are first two coefficients of QCD $\beta$-function.

The variables $s$, $p$, $\sigma$, and $\rho$ are given by
\[ s = \ln \frac{a_s(\mu_0)}{a_s(\mu)}, \quad p = a_s(\mu_0) - a_s(\mu), \quad \sigma = 2\sqrt{(\hat{d}_+ s + \hat{D}_+ p) \ln x}, \quad \rho = \frac{\sigma}{2\ln(1/x)}. \] (11)

The transformation to the LO is simple: in Eqs. (7), (9) and (11) we should put $p = a_s(Q) = a_s(Q_0) = 0$ and replace the variable $s$ by its LO approximation: $s^{LO} = \ln(a_s^{LO}(\mu_0)/a_s^{LO}(\mu))$.

### 4 Wilson coefficients

One has $M_{2,g}^+(1) = M_{2,g}^-(1)$ if $M_{2,g}^+(n)$ are devoid of singularities in the limit $\delta_r \to 0$, as we assume for the time being. Such singularities actually occur at NLO, leading to modifications to be discussed below. Defining $M_{2,g}(1) = M_{2,g}^+(1)$ and using $f_g(x, Q^2) = \sum_{r=\pm} f_g^r(x, Q^2)$, Eq. (3) may be simplified to become
\[ F_2^c(x, Q^2) \approx M_{2,g}(1, Q^2, \mu^2) x f_g(x, \mu^2). \] (12)

Through NLO, $M_{2,g}(1, Q^2, \mu^2)$ exhibits the structure
\[ M_{2,g}(1, Q^2, \mu^2) = e_i^2 a_s(\mu) \left\{ M_{2,g}^{(0)}(1, c) + a_s(\mu) \left[M_{2,g}^{(1)}(1, c) + M_{2,g}^{(2)}(1, c) \ln \frac{\mu^2}{m^2}\right]\right\} + O(a_s^3). \] (13)

#### 4.1 LO results

The LO coefficient function of PGF can be obtained from the QED case \[17] by adjusting coupling constants and color factors, and they read \[18, 19\]
\[ C_{2,g}^{(0)}(x, c) = -2x\{[1 - 4x(2 - c)(1 - x)]c - [1 - 2x(1 - 2c) + 2x^2(1 - 6c - 4e^2)]L(\beta)\}, \] (14)
where
\[ \beta(x) = \sqrt{1 - \frac{4cx}{1 - x}}, \quad L(\beta) = \ln \frac{1 + \beta}{1 - \beta}. \] (15)

Performing the Mellin transformation \[10\] in Eq. (4), we find
\[ M_{2,g}^{(0)}(1, c) = \frac{2}{3}[1 + 2(1 - c)J(c)] \] (16)
with
\[ J(c) = -\sqrt{b} \ln t, \quad t = \frac{1 - \sqrt{b}}{1 + \sqrt{b}}, \quad b = \frac{1}{1 + 4c}. \] (17)
4.2 NLO results

The NLO coefficient functions of PGF are rather lengthy and not published in print; they are only available as computer codes [20]. For the purpose of this letter, it is sufficient to work in the high-energy regime, defined by $x \ll 1$, where they assume the compact form [21]

$$C^{(j)}_{2,g}(x, c) = \beta R^{(j)}_{2,g}(1, c),$$  \hspace{1cm} (18)

with

$$R^{(1)}_{2,g}(1, c) = \frac{8}{9} C_A [5+(13-10c)J(c)+6(1-c)I(c)], \hspace{0.5cm} R^{(2)}_{2,g}(1, c) = -4C_A M^{(0)}_{2,g}(1, c),$$  \hspace{1cm} (19)

where $C_A = N$ for the color gauge group SU(N), $J(c)$ is defined by Eq. (17), and

$$I(c) = -\sqrt{b} \left[ \zeta(2) + \frac{1}{2} \ln^2 t - \ln (bc) \ln t + 2 \text{Li}_2(-t) \right],$$  \hspace{1cm} (20)

where $t$ is given in (17) and $\text{Li}_2(x) = -\int_0^x (dy/y) \ln(1 - xy)$ is the dilogarithmic function.

As already mentioned above (see the end of Section 2), the Mellin transforms of $C^{(j)}_{k,g}(x, c)$ exhibit singularities in the limit $\delta_r \to 0$, which lead to modifications in Eqs. (3) and (12). As was shown in Refs. [13, 14, 11], the terms involving $1/\delta_r$ correspond to singularities of the Mellin moments $M^{(j)}_{2,g}(n)$ at $n \to 1$ and depend on the exact form of the subasymptotic low-$x$ behavior encoded in $\tilde{f}_g^*(x, \mu^2)$. The modification is simple:

$$\frac{1}{\delta_r} \to \frac{1}{\delta_r}, \hspace{1cm} \frac{1}{\delta_r} = \frac{1}{f_g^*(\hat{x}, \mu^2)} \int_{\hat{x}}^{1} \frac{dy}{y} \tilde{f}_g^*(y, \mu^2),$$  \hspace{1cm} (21)

where $\hat{x} = x/b$. In the generalized double-asymptotic scaling regime, the + and $-$ components of the gluon PDF exhibit the low-$x$ behavior (7). We thus have [10, 11]

$$\frac{1}{\delta_+} \approx \frac{1}{\rho(\hat{x})} \frac{I_1(\sigma(\hat{x}))}{I_0(\sigma(\hat{x}))}, \hspace{1cm} \frac{1}{\delta_-} \approx \ln \frac{1}{\hat{x}},$$  \hspace{1cm} (22)

where $\sigma$ and $\rho$ are given in (11).

Because the ratio $f_g^*(x, Q^2)/f_g^+(x, Q^2)$ is rather small at the $Q^2$ values considered, Eq. (12) is modified to become

$$F_2^c(x, Q^2) \approx \tilde{M}_{2,g}(1, \mu^2, c) x f_g(x, \mu^2),$$  \hspace{1cm} (23)

where $\tilde{M}_{2,g}(1, \mu^2)$ is obtained from $M_{2,g}(n, \mu^2)$ by taking the limit $n \to 1$ and replacing $1/(n - 1) \to 1/\delta_+$. Consequently, one needs to substitute

$$M^{(j)}_{2,g}(1, c) \to \tilde{M}^{(j)}_{2,g}(1, c) \hspace{1cm} (j = 1, 2)$$  \hspace{1cm} (24)

in the NLO part of Eq. (13). Using the identity

$$\frac{1}{I_0(\sigma(\hat{x}))} \int_{\hat{x}}^{1} \frac{dy}{y} \beta \left( \frac{x}{y} \right) I_0(\sigma(y)) \approx \frac{1}{\delta_+} - \ln (bc) - \frac{J(c)}{b},$$  \hspace{1cm} (25)

we find the Mellin transform (4) of Eq. (18) to be [7]

$$\tilde{M}^{(j)}_{2,g}(1, c) \approx \left[ \frac{1}{\delta_+} - \ln (bc) - \frac{J(c)}{b} \right] R^{(j)}_{2,g}(1, c) \hspace{1cm} (j = 1, 2),$$  \hspace{1cm} (26)

with $R^{(j)}_{2,g}(1, a)$ (j = 1, 2) are given in [26]. The rise of the NLO terms as $x \to 0$ is in agreement with earlier investigations [24].

\footnote{Note, that $\delta_+$ determines the behavior of the slope of gluon density (see [2]) and also mostly the slope of SF $F_2$. The form (22) of $\delta_+$ is in good agreement [22] with the corresponding HERA experimental data.}
5 Results

We are now in a position to explore the phenomenological implications of our results. As for our input parameters, we choose $m_c = 1.25$ GeV in agreement with Particle Data Group \cite{PDG}. While the LO result Eq. (15) is independent of the unphysical mass scale $\mu$, the NLO formula (13) does depend on it, due to an incomplete compensation of the $\mu$ dependence of $a(\mu)$ by the terms proportional to $\ln(\mu^2/Q^2)$, the residual $\mu$ dependence being formally beyond NLO. In order to fix the theoretical uncertainty resulting from this, we put $\mu^2 = Q^2 + 4m_c^2$, which is the standard scale in heavy quark production.

The PDF parameters $\mu_0^2$, $A_q$ and $A_g$ shown in \cite{H1, CEPA, ZEUS} have been fixed in the fits of $F_2$ experimental data. Their values depend on conditions chosen in the fits: the order of perturbation theory and the number $f$ of active quarks.

Below $b$-quark threshold, the scheme with $f = 4$ has been used \cite{H1} in the fits of $F_2$ data. Note, that the $F_2$ structure function contains $F_2^g$ as a part. In the fits, the NLO gluon density and the LO and NLO quark ones contribute to $F_2^g$, as the part of $F_2$. Then, now in PGF scattering the LO coefficient function (14) corresponds in $m \to 0$ limit to the standard NLO Wilson coefficient (together with the product of the LO anomalous dimension $\gamma_{gg}$ and $\ln(m^2/Q^2)$). It is a general situation, i.e. the coefficient function of PGF scattering at some order of perturbation theory corresponds to the standard DIS Wilson coefficient with the one step higher order. The reason is following: the standard DIS analysis starts with handbag diagram of photon-quark scattering and photon-gluon interaction begins at one-loop level.

To analyze $F_2^g$ at the LO of PGF process one should take $x_f(x, Q^2)$ from the fit of $F_2$ at NLO with $f = 4$. In practice, we use the following parameters, $Q_0^2 = 0.523$ GeV$^2$, $A_g = 0.060$ and $A_q = 0.844$.

Correspondingly, to analyze PGF process at NLO one needs to know the gluon density extracted from $F_2$ data at NNLO, which is not yet known \cite{PDG} in generalized double-asymptotic scaling regime. However, as we can see from the modern global fits \cite{H1}, the difference between NLO and NNLO gluon densities is not so large. So, we can safely apply the NLO form (7) of $x_f(x, Q^2)$ to our NLO PGF analysis.

The results for $F_2^g$ are presented in Fig.1. We can see a good agreement between our compact formulas (12), (16) and (26) and the modern experimental data \cite{H1, CEPA, ZEUS} for $F_2^g(x, Q^2)$ structure function. To keep place on Fig.1, we show only the H1 \cite{H1} and the combine preliminary H1-ZEUS \cite{ZEUS} data.

The good agreement between generalized double-asymptotic scaling approach and $F_2$, $F_2^c$ data demonstrates an equal importance of the both parton densities (gluon and sea quark) at low $x$. It is due to the fact that $F_2$ relates mostly to the sea quark distribution, while the $F_2^g$ relates mostly to the gluon one. Dropping sea quarks in analyze leads to the different gluon densities extracted from $F_2$ or from $F_2^g$. (see, for example, \cite{ZEUS}).

6 Conclusions

We presented a compact formulas for the heavy-flavor contributions to the proton structure functions $F_2$ valid through NLO at small values of Bjorken’s $x$ variable. Our results agree with modern experimental data \cite{H1, CEPA, ZEUS} well within errors without a free additional parameters. In the $Q^2$ range probed by the HERA data, our NLO predictions agree quite well with the LO ones. Since we worked in the fixed-flavor-number scheme, our results are bound to break down for $Q^2 \gg 4m_c^2$, which manifests itself by appreciable QCD correction factors and scale dependence. As is well known, this problem is conveniently solved by adopting the variable-flavor-number scheme, which we leave for future work.

\footnote{The difficulty to extend the analysis \cite{H1, CEPA, ZEUS} to NNLO is related to the appearance of the pole $\sim 1/(n-1)^2$ in the three-loop corrections to the anomalous dimension $\gamma_{gg}$ (see \cite{H1}). The pole $\sim 1/(n-1)^2$ violates the Bessel-like solution (7) of DGLAP equation for PDFs at low $x$ values with the flat initial condition (8).}
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