Predicting Soil-Water Characteristic Curves of Clayey Sand Soils Using Area Computation

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Received 26 November 2019; Revised 5 May 2020; Accepted 15 May 2020; Published 3 June 2020

1. Introduction

The engineering properties of partially saturated soils are now easily predicted using mathematical models designed for unsaturated soil mechanics; these models are popular and accepted worldwide [1, 2]. Soil-water characteristic curves (SWCCs) are used to indirectly predict hydraulic conductivity and shear strength parameters through a hydraulic conductivity function and a shear strength function, respectively [3–6].

The concept of these predictions started as early as the 1960s when Brooks and Corey [7] studied the hydraulic properties of porous media and introduced a framework for the behavior of partially saturated soils.

Models describing the state of suction at variable water content have been introduced by many researchers [1, 8–10]. Common reference terms related to the SWCC include the air-entry value (AEV), residual suction (Rs), and residual water content (Wr). The suction measured at the point when the air starts to take up a space within the water soil system is referred to as the air-entry value. When the suction pressure increases, the water content is reduced until a level where all further reductions in water content are small compared to the applied suction. The suction and water content at this point are both considered residual values. Geometric or volumetric water content plotted against soil suction is generally used to establish the SWCC. There are several factors that can influence the SWCC profile. These include and are not limited to particle and pore size distribution, soil chemistry and fabric, soil density, and soil temperature [11–13].

Montanez [14] studied the suction and volume changes for compacted mixtures of sand and bentonite. Agus and Schanz [15] studied the wetting and drying suction characteristic curves of bentonite sand. Pei-yong and Qing [16] covered the SWCC and water retention of bentonite sand mixtures. These three studies confirmed the increase of...
retention capacity with the increase in clay content. Al-Mahbashi et al. [17–19] conducted several works on the soil-water characteristics for selected soils at different conditions. Zhou et al. [13] presented a model to predict the SWCC for different initial density conditions by introducing a new parameter calibrated using experimental results.

Most of the studies considered Fredlund and Xing’s [1] model to fit the laboratory data of moisture and soil suction. The construction of a single SWCC is a time-consuming process and may involve several techniques if the entire suction range is to be studied. As sand bentonite mixtures are frequently used for liners and hydraulic barriers, the need for a quick and yet reliable procedure is of interest to engineers. This work is aimed at utilizing results collected from the experimental work for a wide range of suction and moisture content in order to establish a prediction chart for similar soils with variable clay content.

## 2. Material and Characteristics

### 2.1. Material Used

The highly plastic and highly expansive clay of Al-Qatif which has been proposed by different researchers for use in liner and waste barrier applications was selected for this study. The fine sand material used in many construction activities was obtained from Riyadh, Saudi Arabia. This material is poorly graded in general, with occasional silt components. The coefficient of uniformity and the coefficient of curvature were measured as 1.737 and 1.078, respectively. The specific gravity of the sand used is 2.66. The specific gravity, liquid limit, plastic limit, and plasticity index for the bentonite material used are given in Table 1.

The Eastern Province of Saudi Arabia is known for expansive clay reported in Al-Qatif region. Al-Qatif region is a historic, oasis located along the western coast of the Arabian Gulf in the Eastern Province of Saudi Arabia (26° 56’ 0” N, 50° 1’ 0” E). This clay is highly plastic and classified as CH in accordance with [20]. According to Azam [21], this clay is of high smectite content. All natural clay used in this study were obtained from Al-Qatif using a hydraulic excavator for near-surface material. The depth of samples was between 0.5 m and 3 m. Detailed geotechnical characterization was carried out in King Saud University laboratories. Other materials used included commercial bentonite HY OCMA obtained by a local supplier. The index properties of HY OCMA bentonite are given in Table 2.

### 2.2. Compaction Curves of Mixtures

The SWCCs were investigated for a selected range of clay content. The samples were normally prepared at the maximum dry density and optimum moisture content. The Al-Qatif sand mixtures with clay contents spanning from 5% to 100% pure clay were prepared. Other mixtures including bentonite (HY OCMA) and Al-Qatif clay were also investigated for the purpose of prediction and validation of the prediction model. The compaction tests were carried out in accordance with the standard Proctor compaction method, ASTM D698—method A [22]. For the sand-Al-Qatif clay mixtures, the range of clay content investigated included 0%, 5%, 10%, 15%, 30%, and 60% by dry weight of sand. For the sand-commercial bentonite mixtures, the range of clay content studied included 15%, but the proportion of natural clay (Al-Qatif) and bentonite was varied from one-third to two-thirds. All samples of mixtures were prepared with different water contents ranging from 5% to 36%. Distilled water was added to the mixtures to obtain the desired water contents.

The mixtures were allowed to hydrate for 24 hours, while placed in plastic bags prior to compaction. A 5.5 lb hammer was used to compact the mixtures into a 100 mm mold (inside diameter) with a height of 101.3 mm. Three layers of mixtures were placed and compacted in order to ensure uniform compaction. The process of establishing compaction curves includes plotting the weight of the soil in the mold after compaction and the associated water content. Three typical curves for 15%, 30%, and 60% are presented in Figure 1. The general profile for the maximum dry density and optimum moisture content is presented in Table 3.

The moisture-density relationship of clay-sand mixtures was investigated for variable clay content. Figure 1 presents an example of a dry density-moisture profile as obtained for a clay-sand mixture with 15% clay content. The maximum dry density was reported as 18.0 kN/m³, and the optimum moisture content was found to be 15%. The values of the maximum dry density are presented in Table 3.

When replacing Al-Qatif clay with one-third or two-thirds volume as bentonite, the variation in the maximum dry density and optimum moisture content is small and not significant.

## 3. Experimental Work and Procedures

### 3.1. General

Two main approaches were followed to test Al-Qatif clay-sand mixtures for soil-water characteristics. The first procedure involved axis translation technique, while the second utilized the filter paper technique. These methods depend on the suction level, and the first method is more reliable for suction values between 10 kPa and 1,500 kPa utilizing a pressure plate in accordance with ASTM D6836 [23], while filter paper techniques are used for measuring matric suction greater than 1,500 kPa.

### Table 1: Properties of bentonite.

| Property                  | Value   |
|---------------------------|---------|
| Specific gravity, GS      | 2.6–2.7 |
| Liquid limit, LL (%)      | 480     |
| Plastic limit, PL (%)     | 49.6    |
| Plasticity index, PI (%)  | 430     |

### Table 2: Properties of Al-Qatif clay.

| Property                           | Value   |
|------------------------------------|---------|
| Specific gravity, GS               | 2.71–2.75 |
| Liquid limit, LL (%)               | 137–140  |
| Plastic limit, PL (%)              | 45–60    |
| Plasticity index, PI (%)           | 95–99    |
| Unified soil classification system | CH      |
3.2. Pressure Plate. The initial molding for the clay-sand mixtures was prepared at the maximum dry density (MDD) and optimum moisture content (OMC). The specimens were extracted in two inch diameter rings and then inundated in distilled water for at least 24 hours. Extra time was allowed to ensure saturation. A 7 kPa stress was applied to the specimens during the saturation period.

Upon removal of the specimen from the inundation cell, it was observed that a saturated surface dry condition was achieved. This was accomplished by using a mop or paper towel. High-air-entry ceramic disks of 5 bars to 15 bars are also needed to be saturated in water overnight.

Assuring good contact between saturated soil specimens and the saturated high-air-entry ceramic disk, the whole assembly is placed in the pressure chamber. The chamber is then tightly closed using screw bolts (Figure 2). Outer air pressure source was used to apply suction in increments starting from 10 kPa and increasing to 50 kPa. Further increments included 100, 200, 400, 800, and 1,400 kPa. A flow-out tube leading to a graduated cylinder was attached. The time allowed for each increment was in the range of two to three days until equilibrium was achieved, i.e., when no more water was expelled from the specimen. The device cell is dismantled following equilibration between the filter paper and the disk specimen. The suction corresponding to the moisture content of the sample was obtained from a calibrated chart of the filter paper used in the test (Figure 3). More details about the procedures for calibrating the filter paper are given by Al-Mahbashi [17].

3.4. Fitting Method. Many curve fitting methods may be considered for the soil suction and water content profile. For this particular research, Fredlund and Xing’s [1] method was selected:

$$\theta_w(\Psi) = \theta_i \left[ 1 - \frac{\ln \left( 1 + \left( \Psi/h_t \right) \right)}{\ln \left( 1 + (10^6/h_t) \right)} \right]^m \left[ \frac{1}{\ln \left( \exp \left( 1 + (\Psi/a)^n \right) \right)} \right]^m,$$

where $\Psi$ is the soil suction, $\theta_w$ is the water content, $\theta_i$ is the saturated water content, $h_t$ is the suction related to the inflection point on the curve, $n$ is the soil parameter related to the slope at the inflection point, $m$ is the soil parameter related to the residual water content, and $h_b$ is the suction related to the volumetric residual water content.

Excellent correlation can be obtained with $R^2$ values in excess of 0.95. The selection of fitting approach is dependent on the evaluation of the researcher.

The AEV and residual suction values have been determined using the graphical method proposed by Vanapalli et al. [4] and represent the points of intersection of tangents to the saturation zone, desaturation zone, and residual zone.

3.5. The Area Concept. While observing many SWCCs and the main control points of the curve, major variations in the area under the curve for different mixtures with different clay contents were noted. The order of variation seems to be related to the first part of the curve, starting from very low suction values up to the inflection point. The trend beyond the inflection point generally converges towards a low moisture zone at high suction levels. This encouraged the researchers to investigate this parameter in order to use it in the prediction of the SWCC for a wide range of clay content. This will certainly reduce the time for design and laboratory
Table 3: Compaction data for variable clay content within mixtures.

| Proportion of clay within the mixture (%) | Maximum dry density | Optimum moisture content |
|------------------------------------------|---------------------|--------------------------|
| 5                                        | 17.5                | 13.5                     |
| 10                                       | 17.7                | 15.4                     |
| 15                                       | 18.0                | 16.8                     |
| 30                                       | 16.5                | 20.0                     |
| 60                                       | 14.1                | 25.0                     |
| 100                                      | 12.0                | 32.0                     |

Typical area parameters suggested in this study are shown in Figures 4 and 5 for a sand-bentonite liner and a pure sand material.

The variation in the area below the curve starts well before the AEV and continues up to and beyond the inflection point.

4. Results and Discussion

4.1. SWCC Control Points. The SWCC constructed for pure sand and sand with 5%, 10%, 15%, 30%, and 60% of Al-Qatif clay and other mixtures involving commercial bentonite revealed control points, as presented in Table 4.

4.2. Saturation Water Content. The clay mineralogy is the most important factor that determines the saturation moisture content in the SWCC plot. Expanding minerals play a significant role in attracting and retaining water, and this is clearly reflected by the increase in saturation moisture content with the increase of clay content. SWCCs are widely spaced for low matric suction, and the spacing is reduced by increasing the matric suction. The curves for all sand-clay mixtures are shown in Figures 3 and 4.
mixture profiles converge towards a zone of high matric suction between 10,000 and 100,000 kPa [24].

The saturated water content value is closely related to the porous nature of the mixture and governed by the retention capacity of the clay and proportion of sand. When the clay and fines within the mixture are dominant, a higher degree of saturation is expected. Volume increase and shrinkage are strongly related to the water retention in liners and wastewater barriers made of sand enhanced by clay.

4.3. SWCC Trends for Sand-Clay Mixtures. The SWCC for sand indicated nearly horizontal moisture-suction slope for low suction pressures (0.1 kPa or less). This is also valid for different selected clay mixtures, but the moisture content is variable. The slope of the SWCC changes gradually when the matric suction is increased until a point of inflection is reached. The slope beyond the inflection point indicates a reversed trend and change of direction. Table 4 presents the AEV, Rs, and Wr based on Fredlund and Xing’s [1] fit.

The SWCC of sand-clay mixtures, in general, indicates trend with different control points and slopes. Figures 6-8 present Fredlund and Xing’s fit for sand with varying clay content.

4.4. Areas under the Curve. Measurements of the area under the curve can be performed graphically or by computations. The Simpson rule can be utilized if the area is subdivided to many slices of equal width from initially reported suction up to the inflection point. Integrating the function over the same range is found to give reliable results. In this study, one type of sand material is used with different clay contents. Figure 5 shows the contribution of sand alone to the area under the curve. This may be investigated when different types of sand are used with different clay contents. In this part of research, the influence of sand is ignored or assumed to be identical in all tested samples.

The scale of the two axes and dimensions need to be identical when measuring or comparing the area using this procedure. The scale must be semilogarithmic with moisture content in the vertical axis and the matric suction for the horizontal axis. It is suggested to start measuring areas for very low suction (0.01 kPa).

By integrating equation (1), the area under the curve up to the inflection point was calculated and presented for a sample mixture. The first step is to establish the inflection points in order to identify the limits of the required area. Fredlund and Xing’s [1] equation is simplified and integrated over the range from 0.01 kPa suction up to the suction value at the inflection point shown as 1,905.46 for the selected mixture.

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sand-clay mixtures transforms from a unimodal form to a bimodal form due to two of pores sizes involved: macropores and micropores. When considering this new concept of shape analysis using areas under the curve, a quantitative evaluation is possible. This is expected to be beneficial for geotechnical designers and laboratory engineers due to the amount of time saved. Areas with similar clay mineralogy can establish a single SWCC chart for different clay additives. This can be repeated or reproduced for other sands commonly used in the liner industry.

Figure 10, with experimental data performed in the laboratory, can be used to establish the zones of different Al-Qatif clay contents for any proposed clay-sand liners. SWCC chart, Figure 11, is constructed to help practicing engineers to use the area under the curve concept. The chart was idealized based on repeated tests and selected clay content. The area function is not linear, and statistical fit indicated a polynomial trend of the second order. Therefore, the shift of the SWCC profile shall not be considered linear and needs to be adjusted following the area under the curve concept. When plotting control points, it is worth noting that the position of the AEV and suction need to be determined by finding the moisture content as correlated to the area factor; similarly, the suction pressure can be obtained from correlations between area factor and suction pressure at AEV. For the residual point, it was observed that it occurs at high suction and is not majorly deviating from the straight lines converging towards 100,000 kPa suction pressure. For the purpose of validation of the prediction chart, the points with 30% and 60% Al-Qatif clay are shown in the plot (Figure 12). The SWCC for the pure sand used is indicated in the prediction chart. When the type of sand is altered, a similar prediction chart can be constructed.

When using different types of clay of similar mineralogy in the form of mixtures of local clay and highly plastic bentonite, deviation from the general trend was found to be not serious but depends on the concentration of bentonite used. This can be seen from various plots of the SWCC of 5% bentonite plus 10% Al-Qatif clay and 10% bentonite and 5% Al-Qatif clay.

| Mixture       | Air-entry value (AEV), kPa | Residual water content (WR), % | Saturated water content (WS), % | Residual suction (Rs), kPa | R²   |
|---------------|----------------------------|-------------------------------|-------------------------------|---------------------------|------|
| Pure sand     | 8.6                        | 0.8                           | 11.5                          | 700                       | 0.997|
| 5% Q          | 5.0                        | 1.3                           | 12.0                          | 3800                      | 0.988|
| 10% Q         | 27.0                       | 1.4                           | 13.0                          | 22000                     | 0.957|
| 15% Q         | 42.0                       | 1.4                           | 14.5                          | 45000                     | 0.978|
| 30% Q         | 8.6                        | 15.3                          | 26.0                          | 300                       | 0.992|
| 60% Q         | 11.5                       | 22.0                          | 35.0                          | 700                       | 0.988|
| 100% Q        | 68.0                       | 7.5                           | 71.0                          | 50000                     | 0.991|
| 5% B, 10% Q   | 30.0                       | 1.9                           | 16.0                          | 25000                     | 0.978|
| 10% B, 5% Q   | 98.0                       | 0.9                           | 17.0                          | 13000                     | 0.973|
| 15% B         | 170.0                      | 1.2                           | 22.5                          | 10000                     | 0.988|

Figure 6: SWCC for sand-clay mixtures with clay content of 0, 5, 10, 15, 30, and 60%.

Figure 7: SWCC for Al-Qatif pure clay compared with other selected mixtures.
Figure 12 presents a comparison between measured and predicted data for mixtures including 15% and 30% of Al-Qatif clay. The obtained results fit very well with the proposed area under the curve model. It is worth mentioning that the predicted lines for the SWCC are rather uniform up to mixture containing 20% of Al-Qatif clay, and beyond this level, the variation is not regular. The upward shift increases with increase of clay content above 20%. The practical range of Al-Qatif clay that is to be used in the liner industry is not expected to go beyond 25% content. This procedure can be further utilized to compare other clay based on the area under the curve. This can be in the form of a slight shift of the prediction lines and will be of sufficient accuracy for the preliminary design of liner and prediction of geotechnical properties.

Cai et al. [5] discussed different models used in the prediction of the permeability function. They classified these models into three groups. They are empirical methods, macroscopic models, and statistical models. The theoretical approach conducted by Cai et al. [5] included adding an
additional density parameter to enable a wide range prediction and not for a single initial dry density. The theoretical approach in this study is limited to the SWCC of a particular soil fitted using Fredlund and Xing’s method [1] enhanced with computations of the area under the curve as obtained for variable clay contents. The area under the curve parameter was found to provide a prediction chart with reliable accuracy that can enable successful geotechnical engineers.

5. Conclusion

This study considered the use of the area under the curve concept to predict idealized SWCC trends for sand-clay mixtures used in liners. The call for establishing a liner chart for areas of similar mineralogical nature will be useful for design-practicing engineers and laboratory testing agencies. Considerable time will be saved, which will be most welcomed by designers and environmental specialists.

Prediction charts established can form a library for use by designers and research groups in the field of geotechnical and environmental engineering. The SWCC control points were established experimentally for Al-Qatif clay when used with the common fine sand which is used in the construction industry. When using different clay of similar mineralogy or mixtures including local clay and highly plastic bentonite, a deviation from the general trend may be noticed. This deviation was found not significant, and its level depends on the concentration of bentonite used.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors extend their appreciation to the Deanship of Scientific Research at King Saud University for funding this work through the Research Group Program (RG-1440-073).

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