Study on Voxel Finite Element Analysis of Open-Cell Polyurethane Foam

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Abstract. Establishment of a design method for the macroscopic stress-strain relationship of open-cell foam has been investigated because of the foam’s complicated microstructures. In this study, a material design method for the mechanical behavior of open-cell polyurethane foam using voxel finite element analysis is proposed to consider the microscopic structure and matrix properties. A finite-strain hyperelastic model was applied to a matrix of polyurethane foam. The Mooney-Rivlin model was adopted for the potential energy function of the hyperelastic model. To determine the mechanical properties of the matrix, original, solid specimens of polyurethane were prepared. The material parameters of the matrix were identified based on the tensile loading test results of the solid specimens. Microstructures of the open-cell polyurethane foam were determined using a CT-scanning system. Finite element models constrained by the original microstructural configurations were applied to numerical simulations. Periodic finite element models based on the modified microstructural configurations, the relative density of which was the same as that of the original models, were applied to the simulation. Based on the numerical results, verification of the design performance of the simulation code for open-cell polyurethane foam was performed. The simulation results were reasonably consistent with compression loading test results up to the plateau region.

1. Introduction

Polymer foam materials have been widely used in a variety of applications such as car seats, shoe soles and packing materials. The strong points of polymer foam are that it is light weight, has good impact energy absorption and has manufactural formability [1]. Polymer foam exhibits a nonlinear stress-strain relationship, which is divided into three regions – the linear elastic region, the plateau region and the high-density region. It is known that the mechanical properties of polymer foam depend on its microstructure and matrix [2]. Prediction methods for the mechanical properties of polymer foam have been proposed by numerous researchers. For example, Gibson and Ashby proposed a formula to describe the relationship between relative density and initial stiffness [2]. Relative density represents the ratio of the matrix in the foam. Studies to analyze the elastic properties of open-cell foams using the Kelvin cell foam model have been done [3, 4]. However, while this method can be applied to the initial stiffness, prediction of the stress-strain relationship with greater strain remains difficult.

Especially in the medical field, the voxel FEM has been used to predict the mechanical properties of materials with complex microstructures, such as human bones [5, 6]. The validity of this method has been confirmed in numerous studies. In this study, the applicability of voxel FEM analysis to predict the stress-strain relationship up to the large-strain region of polymer foam material was verified. CT scan data of polyurethane foam was acquired in voxel form. Parts of the CT scan data were applied to voxel FEM analysis. The material parameters of the polyurethane matrix employed in the simulations were identified using tensile loading test results for solid specimens. In addition, a periodic model which represents the microstructures of polymer foam
was also prepared. The simulation results were in near agreement with experimental results up to the plateau region.

2. Tensile loading tests on solid specimens

Preparing a specimen without voids is difficult because numerous voids in polymer foam are generated during the chemical reaction. For this reason, there have been few discussions of the precise mechanical properties of the matrix. In this study, in order to identify the material parameters of the polyurethane matrix, original solid specimens of a polyurethane foam matrix without air voids were prepared. Fig. 1 shows the solid specimens. Tensile loading tests on three solid specimens were conducted using a uniaxial machine (Autograph AG-20kNXplus, Shimadzu Corporation); the strain rate for all tests was set to 0.1%/s. Fig. 2 shows the stress-strain relationships given by the tensile loading tests. The three specimens showed similar stress-strain relationships. The results did not exhibit the complicated mechanical characteristics peculiar to polyurethane foam. Thus, the mechanical properties of the polyurethane matrix which were obtained with the loading tests excluded the influence of the microstructure.

To determine the material parameters of the polyurethane matrix, the incompressible hyperelasticity was applied to the mechanical behavior given by the tensile loading test results. Thus, the second Piola-Kirchhoff stress tensor $\mathbf{S}$ of a hyperelastic material is given by the partial differentiation of the strain energy function $\mathcal{W}$ with respect to the right Cauchy-Green deformation tensor $\mathbf{C}$ as follows:

$$
\mathbf{S} = 2 \frac{\partial \mathcal{W}(\overline{\mathbf{C}})}{\partial \mathbf{C}}
$$

(1)

where $\overline{\mathbf{C}}$ is the modified right Cauchy-Green deformation tensor. The following equation for elastic energy function was applied to represent the stress-strain relationships of the polymer matrix:

$$
\mathcal{W}(\overline{\mathbf{C}}) = \mathcal{W}(\overline{I}_1, \overline{I}_2)
= C_{11} (\overline{I}_1 - 3) + \frac{C_{12}}{2} (\overline{I}_1 - 3)^2 + \frac{C_{13}}{3} (\overline{I}_1 - 3)^3 + \frac{C_{14}}{C_{15}} \exp\{C_{15} (\overline{I}_1 - 3)\}
$$

(2)

where $\overline{I}_1$ is the modified first invariant, $\overline{I}_2$ is the modified second invariant, and $C_{11}, C_{12}, C_{13}, C_{14}$, and $C_{15}$ are material parameters. The material parameters were identified using the least square method with the tensile loading test results for the solid specimens. Table 1 shows the material parameters of the polyurethane matrix approximated using the above method.

| Table 1. Material parameters identified from the result of tensile loading test for solid specimens |
|---|---|---|---|---|
| $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{14}$ | $C_{15}$ |
| 2.37 | -0.0486 | 0.0312 | 3.08 | -6.75 |
In order to determine the mechanical properties of polyurethane foam, compression loading tests were conducted. The relative densities of the compression test specimens were 3.73%, 4.82% and 5.94%, respectively. The specimens were compressed up to a nominal strain of 0.6, and unloaded. The strain rate was set to 0.1 \%/s for all tests. Fig. 3 shows the stress-strain relationships given by the compression loading test. As confirmed in previous research, initial stiffness increases as relative density increases \[6\]. All specimens exhibited linear-elastic regions, plateau regions and high-density regions. These are characteristics peculiar to polymer foam material. Initial stiffness was calculated based on the definition that the linear-elastic region is up to a strain of 0.05. Fig. 4 shows the relationship between initial stiffness and relative density. These results confirmed that this polyurethane foam satisfies the theoretical formula for the relationship between initial stiffness and relative density, which was introduced by Gibson and Ashby \[2\]. The formula is as below:

\[
\frac{E_f}{E_m} = C_1 \left(\frac{\rho_f}{\rho_m}\right)^2
\]

(3)

Where $E_f$ is the initial stiffness of the foam material, $E_m$ is the initial stiffness of the matrix, $C_1$ is the constant of proportionality, $\rho_f$ is the density of the foam material and $\rho_m$ is the density of the matrix.
Voxel FEM analysis based on CT scan data

Voxel FEM analysis with the model based on CT scan data was conducted in order to consider the effect of microscopic structures on the mechanical properties of polymer foam. CT scan data for the polyurethane foam was acquired in voxel form. The polyurethane foam scanned was same as the specimens used for the compression loading tests described in a later section. Rectangular areas were extracted from the CT scan data. The length of one side of each area was 2 mm. Hereinafter this model is referred to as the CT model. Voxel FEM analysis of the CT model was conducted under the following conditions. The top and bottom surfaces were completely constrained in x₁, x₂ and x₃ directions, and the side surfaces were free. Strain was applied to the top surface in increments of 0.015 up to a strain of 0.3 at the 20th increment. The material parameters of the matrix as determined by the tensile loading tests of the solid specimens were applied to this simulation. Fig. 5 is a diagram of the model at strains 0, 0.15 and 0.3. Fig. 6 shows the stress-strain relationships obtained with the simulations. The percentage annotations in the graph indicate the relative density of each model. The simulation results all roughly agree with each other. These results suggest that the sample areas contained representative microstructures of polymer foam. In addition, the results show the bend of stress-strain relationship, which represents the plateau region. Therefore, it is implied that applying the voxel FEM with a CT model can predict the stress-strain relationship of polymer foam up to the plateau region.
5. Voxel FEM analysis of the body-centered cubic model

The number of elements of the CT model was so large that the simulation required a high calculation cost. Therefore, a simplified voxel model reproducing the microstructure of the polymer foam was prepared by creating voids in the cubes. The length of one side of each cube was 30 voxel, and the model was a body-centered cubic lattice, which had cavities in the center and corners. The radius of the center void and the corner void were 19 voxel and 5 voxel, respectively. This simplified model is referred to as the BCC model (the Body-Centered Cubic model) because of its structure. Analysis conditions were the same as those of the voxel FEM analysis of the CT model. Fig. 7 shows diagrams of the model at strains of 0, 0.15, and 0.3. Fig. 8 shows the stress-strain relationship obtained with the simulation. As with the CT model, the initial stiffness and the bend of the stress-strain relationship were reproduced reasonably well, which indicates good agreement with the simulation results of the CT model.
6. Result and discussion

6.1. Verification of the applicability of the voxel FEM to polymer foam material

In order to verify the applicability of the voxel FEM to polymer foam materials, comparison of the experimental results with simulations was required. However, since the relative densities of the CT model specimens were higher than those of the compression loading test specimens, simple comparison was not permitted. Thus, an assumption that the equation (3) was possible to apply up to the plateau region was introduced. If it is possible, the formula holds:

$$\frac{\sigma_1}{\sigma_2} = C_1 \left(\frac{\rho_1}{\rho_2}\right)^2$$  \hspace{1cm} (4)

Where, $\sigma_1$ and $\sigma_2$ are nominal stresses obtained by experiment or simulation, and $\rho_1$ and $\rho_2$ are relative densities of test specimens or models. To verify the applicability of this assumption, two experimental results of which relative densities were 4.82% and 5.94% with modification by equation (4), were compared. Thus, for this study, the following values were used in formula (4): $\sigma_2$ was the nominal-stress experimental data obtained for specimens with a relative density of 4.82%, $\rho_1$ was the estimated result’s relative density of 5.94%, $\rho_2$ was the test specimens’s relative density of 4.82%, solving for $\sigma_1$. This result is referred to as 4.82% modified, and its curve is compared to the 5.94% curve in Fig. 9. The estimated stress-strain relationship shows rough correspondence with experimental results over the entire range of the strain. Therefore, equation (4) was considered to be applied up to the plateau region of experimental results which was conducted in this research.
The results of the voxel FEM analysis of the CT model include randomness affected by the relative density. By applying formula (4) to the results, estimated stress-strain relationships were calculated with the condition that relative density was 5.94%, which is the same as the compression test specimens. Fig. 10 shows a comparison of the estimated stress-strain relationship based on the CT model and the experimental results. The estimated results are consistent with experimental results. Therefore the voxel FEM is reasonably applicable to polymer foam material.

![Stress-strain relationships of compression loading test and result modified with formula (4)](image)

Fig. 9. Stress-strain relationships of compression loading test and result modified with formula (4)

![The results of voxel FEM of CT model modified with formula (4) and experimental result](image)

Fig. 10. The results of voxel FEM of CT model modified with formula (4) and experimental result

6.2. Verification of the applicability of the BCC model

The stress-strain relationship of the BCC model with a relative density of 5.94%, was calculated with equation (4). Fig. 11 shows a comparison of the modified stress-strain relationship of the BCC model and the experimental results. From these results, it can be seen that the bending of the stress-strain relationship of the BCC model is consistent with experiment results. In addition, the results obtained with the CT model also reproduced the bending strain. Therefore, voxel FEM can reproduce the stress-strain relationship of polymer foam up to the plateau region using a model that contains some randomness of structure. The simulated initial stiffness was lower than that of the experimental results due to the boundary conditions. The microscopic boundary conditions of the BCC model in the voxel FEM simulation did not correspond exactly to the microscopic boundary conditions of the real
specimens in the loading tests. The microscopic deformation of the real specimens was satisfied with the equilibrium of stress with surrounding parts. Conversely, the BCC model was not stressed from the side surfaces by the adjacent cells. For this reason, the differences in the initial stiffness between the voxel FEM simulation and the loading tests are thought to be due to the boundary conditions rather than the microstructure.

![Graph](image)

**Fig. 11.** The results of voxel FEM of BCC model modified with formula (4) and experimental result

### 7. Conclusion

The applicability of voxel FEM analysis to polymer foam was investigated. Incompressible hyperelasticity was applied to the voxel FEM code. CT models obtained using a CT-scanning system and BCC models that reproduced the microscopic structure of polymer foam were prepared. The simulated results of the CT model employing material parameters of the matrix were consistent with the experimental results. Voxel FEM using the CT model made it possible to design polymer foam that incorporate its macroscopic stress-strain relationship by considering its microscopic structure and matrix properties. Additionally, the results for the BCC models were consistent with the experimental results in terms of bending strain. It was possible to consider the geometrical effects of microscopic voids on the mechanical characteristics of polymer foam using the proposed voxel methods. Voxel FEM simulation using the results of the CT-scanning system were applicable to the mechanical characteristics of polymer foam up to the plateau region.

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