Absorption losses in periodic arrays of thin metallic wires

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We analyze the transmission and reflection of the electromagnetic wave calculated from transfer matrix simulations of periodic arrangements of thin metallic wires. The effective permittivity and the absorption is determined. Their dependence on the wire thickness and the conductance of the metallic wires is studied. The cutoff frequency or effective plasma frequency is obtained and is compared with analytical predictions. It is shown that the periodic arrangement of wires exhibits a frequency region in which the real part of the permittivity is negative while its imaginary part is very small. This behavior is seen for wires with thickness as small as 17 µm with a lattice constant of 3.33 mm.

Rapidly increasing interest in left-handed metamaterials (LHM) (for a recent review, see Ref. 9) raised some interesting questions about the electromagnetic (EM) properties of composites which contain very thin metallic components. The most simple example of such a composite is a periodic array of thin metallic wires. Pendry et al. predicted that such system behaves as a high pass filter with an effective permittivity

$$\epsilon_{\text{eff}} = 1 - \frac{f_p^2}{f^2 + 2i\gamma f}. \quad (1)$$

In Eq. (1), $f_p$ is the effective plasma frequency, or cutoff frequency, and $\gamma$ is the damping factor. Various theoretical formulas were derived for the dependence of the plasma frequency on the lattice period $a$ and the wire radius $r$. Pendry et al. obtained that

$$f_p^2 = \frac{\epsilon_{\text{light}}^2}{2\pi a^2 \ln(a/r)}. \quad (2)$$

Shalaev and Sarychev obtained that

$$f_p^2 = \frac{\epsilon_{\text{light}}^2}{2\pi a^2 \ln(a/\sqrt{2}r) + \pi/2 - 3}, \quad (3)$$

while Maslovski et al. obtained that

$$f_p^2 = \frac{\epsilon_{\text{light}}^2}{2\pi a^2 \ln(2a^2/4\pi\mu(a - r))}. \quad (4)$$

In Eqs. (2)-(4), $c_{\text{light}}$ is the velocity of light in vacuum.

Periodic arrangements of thin metallic wires are used as a negative-epsilon medium in the left-handed structures. 9,10,11 It is therefore important to understand how the electromagnetic response - not only the effective plasma frequency, but also the factor $\gamma$ - depends on the structural parameters of the wire system. Recently, Ponizhovskaya et al. claimed that for small wire radius, the absorption in the wire system is so large that the transmission losses do not allow any propagation of EM wave in left-handed structure. Very low transmission, measured in the original experiments on the LHM 9,10,11 seemed to agree with their pessimistic conclusion. However, it is not clear why the transmission was so low in the original experiments. Recent experimental measurements 12,13 established that the transmission of LHM could be as good as in the right-handed systems.

Our aim in this Letter is to study numerically how the effective permittivity of the periodic arrangement of metallic wires depends on the wire radius and on the conductance of the wires. We present results for the real ($\epsilon_{\text{eff}}^r$) and imaginary ($\epsilon_{\text{eff}}^i$) part of the effective permittivity of the wire medium, estimate the transmission losses and the plasma frequency and compare our results with the analytical formulas given in Eqs. (1)-(4).

In our numerical simulations we use the transfer matrix method (TMM). Details of the method are given elsewhere. 14 Here we only point out the main advantage of the TMM, namely that it gives directly the transmission $t$, reflection $r$ and absorption $A = 1 - |t|^2 - |r|^2$ of the EM plane wave passing through the system. Contrary to this, in the FDTD method, which was used in Ref. 9, one obtains $t$ and $r$ form the time development of the wave packet, which is much more complicated and probably also less accurate. To be able to obtain $\epsilon_{\text{eff}}$, one also needs the phase of $r$ and $t$, in addition to their amplitudes. This is also easily achieved in the TMM.

From the obtained data for the transmission and reflection, we calculate the effective permittivity of the system. 14 The refraction index is given by the equation

$$\cos(nkL) = \frac{1}{2t} \left[1 - r^2 + t^2\right]. \quad (5)$$

Since we do not expect any magnetic response, we fixed the value of the permeability to be $\mu = 1$. The permittivity is then found as $\epsilon_{\text{eff}} = n^2$.

The way we discrete the space in the TMM might control the accuracy of our results. To test how discretization influences our results, we repeated the numerical simulation for different discretizations. The wire is represented as a rectangular with square cross section $2r \times 2r$, $r$ being the "wire radius". The obtained results for the effective permittivity $\epsilon_{\text{eff}}$ (both real and imaginary part) are almost independent of the discretization procedure.
Figure 1 shows how the effective permittivity depends on the wire radius. We analyzed four different wire arrays with period $a = 3.33$ mm. For all of them the real part of the effective permittivity is negative and could be fitted by Eq. 1. This enables us to obtain easily the plasma frequency. Only for the smallest wire thickness studied ($17 \times 17 \mu m$) one gets a relatively large $\varepsilon''_{\text{eff}}$ for small frequencies. In this region relation 1 is not valid. Nevertheless, for frequencies larger than 5 GHz, $\varepsilon''_{\text{eff}}$ is small and $\varepsilon'_{\text{eff}}$ negative.

The right lower panel of fig. 1 shows data for wires with the cross section of $17 \times 300 \mu m$. These parameters were used in the experiment of Shelby et al. We again see that the formula given by Eq. 1 qualitatively agree with our data. Thus, there is no doubt that this array of wires really produces a medium with negative $\varepsilon'_{\text{eff}}$, which can then be used in the creation of the left handed systems.

Figure 2 compares our data for plasma frequency with the analytical formulas given by Eqs. 2-4. Accepting some uncertainty in the estimation of the plasma frequency from the numerical data, we can conclude that for thin wires our data agree with the theoretical formula of Shalaev and Sarychev. For thicker wires, our results are in agreement with the formula of Maslovski et al.

We also study the role of the conductance of the metallic wires. In the simulations shown in fig. 1, we consider the metallic permittivity to be $\varepsilon_m = (-3 + 588 i) \times 10^{-3}$. We are aware that this value of $\varepsilon_m$ is smaller than the permittivity of realistic metallic wires: for instance for copper $\varepsilon_m \approx 5 \times 10^{-7} i$, as follows from the relation between the permittivity and the conductivity (the conductivity of copper is $\sigma \approx 5.9 \times 10^7 (\Omega m)^{-1}$). Our data in fig. 1 therefore underestimate losses, because transmission losses are smaller for higher values of the metallic permittivity. This is clearly shown in fig. 3 where we present the ratio $\kappa = |\varepsilon''_{\text{eff}}/\varepsilon'_{\text{eff}}|$ vs frequency for two systems which differ only in the value of the imaginary part of the metallic permittivity. Fig. 3 shows also the frequency dependence of the absorption as obtained from the numerical simulations. The absorption also exhibits a maximum in the neighborhood of the plasma frequency.

Fig. 1 also confirms that $\varepsilon''_{\text{eff}}$ increases when the wire radius decreases. For instance, the parameter $\gamma$ is only 0.003 GHz for $r = 100 \mu m$. As we will see below, (fig. 4) $\gamma$ increases up to 1.2 GHz when the wire radius decreases to 15 $\mu m$. Nevertheless, even for very thin wires, losses are much less than what was claimed in Ref. 6. As it is shown in fig 1, also an array of wires with thickness $17 \times 17 \mu m$ creates a negative-$\varepsilon$ medium. As this is in strong contrast with the results of Ref. 6, we decided to study exactly the same system as that of. Results of our simulations are shown in fig. 4. Although such systems are not used in experimental arrangements of the LHM, our results give a comparison between two different numerical treatments. Our data again clearly show that $\varepsilon''_{\text{eff}}$ is negative for $f < f_p$. As it is shown in the inset, the transmission losses are also small.

We believe that the present data are more accurate, than those published previously in, not only because they agree with the theoretical analysis but also because the TMM gives the reflection and its phase straightforward. This is important because the main difference between our results and those of seems to be in the estimation of reflection $R = |r|^2$. When comparing our data for absorption, given in the inset of fig. 4 with those given in fig 2b of Ref. 6, we see that our absorption is much less than that estimated in Ref. 6.

In conclusion, we analyzed numerically the transmission properties of a periodic arrangement of thin metallic wires. From the transmission and reflection data we calculate the effective permittivity and plasma frequency, which agrees qualitatively with theoretical predictions. Both the effective permittivity and the absorption data confirm that the array of thin metallic wires, used in the recent experiments on the left-handed metamaterials, indeed behaves as a negative permittivity medium with low losses.

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FIG. 1: Effective permittivity as a function of frequency for various shapes of the metallic wires. The lattice period in all cases is $a = 3.33$ mm. We used the metallic permittivity $\epsilon_m = (-3 + 588i) \times 10^3$.

FIG. 2: Plasma frequency as a function of the wire radius. The lattice constant is $a = 5$ mm. Solid, dashed and dot-dashed line is the result of Pendry et al. (2), Shalaev and Sarychev (3) and Maslovski et al. (4), respectively. Dot line is a fit of our data to the function $f_p = a_0/\sqrt{\ln(a_1/r)}$ with parameters $a_0 = 20.9$ and $a_1 = 0.84$.

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FIG. 3: Ratio $\kappa = |\varepsilon''/\varepsilon'_{\text{eff}}|$ for a lattice of wires with radius 50\,\mu m. Metallic permittivity is $\varepsilon_m = (-3 + 588 \, i) \times 10^3$ (open circles) and $\varepsilon_m = (-3 + 5880 \, i) \times 10^3$ (full circles). Dashed (solid) line is absorption for the corresponding systems obtained numerically by the TMM. These numerical results confirm that losses are smaller for higher metallic permittivity and that the value of the plasma frequency, estimated approximately from the position of the maximum of $\kappa$, does not depend on the value of the metallic permittivity.

FIG. 4: Effective permittivity for a lattice of thin metallic wires. The wire radius is $r = 15\,\mu m$ and the lattice constant is $a = 5$ mm. The metallic permittivity is $\varepsilon_m = -2000 + 10^6 \, i$. Two different discretizations are used with mesh sizes of 30\,\mu m (open symbols) and 15\,\mu m (full symbols). The solid and dashed lines are fit to Eq. 1 with $f_p = 11.1$ GHz and $\gamma = 1.2$ GHz. The length of the system was up to 60 unit lengths for open symbols and 10 unit lengths for full symbols. Inset shows the numerically calculated absorption as a function of frequency.
