Modeling of Wake Effects in Steady State Mixing Plane Simulations of a High Lift Turbine Cascade with Different Combinations of Wake Passing Frequency and Wake Orientation

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Abstract. Due to operation at low Reynolds numbers, low pressure turbines of aircraft engines mostly show large laminar boundary layers and transitional separation bubbles which considerably change their viscous losses when interacting with impinging wakes. The change of loss depends on several wake parameters, among others on wake passing frequency and wake orientation. In the present work, these parameters are expressed in terms of Strouhal number and flow coefficient and their influence is investigated by means of unsteady Reynolds-averaged Navier-Stokes (RANS) simulations. Different combinations of both wake parameters which are typical of aircraft engine conditions, are prescribed upstream of a high lift turbine cascade, while the Reynolds number and Mach number are kept constant. The solver TRACE by DLR and MTU Aero Engines together with the $\gamma-\text{Re}_{\Theta}$ transition model by Langtry and Menter has been used. Further, the wake profile is representative for upstream turbine profiles and is prescribed by a correlation framework which has been calibrated in previous work. A newly developed quasi-unsteady wake model (QUWM) is applied in order to model the effects of periodically passing wakes in steady state simulations involving mixing plane interfaces. It is shown that the gap between unsteady and steady state simulations is narrowed significantly by the QUWM while still maintaining quick turnaround times that are crucial in industrial flow solver applications.

1. Introduction

While scale-resolving simulations, such as Large-Eddy Simulation (LES), are widely used in recent academical applications of turbulence and transition modeling, the required computational resources and expertise greatly exceed the increase in accuracy so that industrial applications still rely on RANS methods as a robust analysis tool as concluded by Zhiyin [1]. In order not to sacrifice too much of the performance for the higher prediction accuracy, there is still a need to enhance reduced-complexity models and simplified setups for quick turnaround times. In this work, the widely used two equation $\gamma-\text{Re}_{\Theta}$ transition model by Langtry and Menter [2] was applied in order to model the laminar-turbulent boundary layer transition. In another study by
the authors [3], it was concluded that in steady state mixing plane simulations, the model failed to accurately predict the laminar-turbulent boundary layer transition for a high-lift low pressure turbine cascade involving a laminar separation bubble (LSB) in the presence of impinging wakes. In order to further extend the baseline transition model, the authors proposed a quasi-unsteady transition model approach [4] that was validated with a generic stage involving zero pressure gradient flat plates as well as a 1.5 stage low-pressure turbine. In this study, the aim is the application of the model to a broad range of different wake convection characteristics in terms of wake passing frequency and wake convection angle. The importance of wakes for the correct prediction of laminar-turbulent boundary transition in turbomachinery has been studied by many researchers such as Stieger et. al. [5, 6] and Mayle [7, 8] who observed that wakes are able to temporally and instantaneously increase the boundary layer intermittency and interact with laminar separation bubbles.

2. Numerical Method

For all simulations conducted, the parallel CFD solver TRACE of DLR and MTU Aero Engines has been applied, cf. [9] [10] [11] [12]. The turbulence model in use is the two equation $k - \omega$ model by Wilcox (1988 version) [13] together with the stagnation point anomaly fix of Kato and Launder [14]. The laminar-turbulent transition of the boundary layer is modeled by the $\gamma - Re_\Theta$ model by Langtry and Menter [2]. Being widely used for the prediction of the boundary layer transition, it is a local correlation based transition model to evaluate the local flow features in terms of natural, bypass and separation induced transition. Only the wall distance formulation used is not local and computed in a pre-processing step. For further information regarding the $\gamma - Re_\Theta$ transition model, please refer to the work of Langtry [15]. All boundary layers have been resolved with a dimensionless wall distance of $y^+ \leq 1$. In this study, the quasi-unsteady wake model (QUWM) by the authors [4] has been applied. The QUWM is an extension of the $\gamma - Re_\Theta$ model that aims to enhance the prediction accuracy of steady state mixing plane CFD computations regarding wake effects. While passing wakes and the associated effects are inherently unsteady phenomena, the QUWM is an approach to model the time-mean effect of the wake boundary layer interaction. This happens through an extensive analysis of the unsteadiness at the upstream side of the mixing plane interface. The QUWM reconstructs the time-mean flow field in terms of involved effective transition model variables and improves the quantitative prediction of wake effects. The correct prescription of the freestream turbulence quantities in terms of the turbulent kinetic energy $k$ as well as the turbulent length scale $l_t$ is essential for the prediction of the laminar-turbulent boundary layer transition as shown by Bode et. al. [16]. The model utilizes two points of interaction with the used baseline $\gamma - Re_\Theta$ transition model. The first modeling step is the increase of boundary layer intermittency that occurs through the impinging wakes. The second is the scaling of the Reynolds momentum thickness number $Re_\Theta$ correlation in order to correctly predict the start of the laminar-turbulent transition in the boundary layer in MP simulations. The need of scaling the correlation equation is based on the fact that the strongly non-linear empirical correlation of the Reynolds momentum thickness number and the local turbulence intensity $Tu$ are not able to predict the correct value of $Re_\Theta$ downstream of MP interfaces since $Tu$ is only locally available as a time-averaged value in steady state computations. The wake intermittency $\gamma_w$ is modeled using a transport variable according to eqn. (1).

$$\frac{\partial (\rho \gamma_w)}{\partial t} + \frac{\partial (\rho U_j \gamma_w)}{\partial x_j} = P_{\gamma_w} - D_{\gamma_w} + \frac{\partial}{\partial x_j} \left[ \sigma_w (\mu + \mu_t) \frac{\partial \gamma_w}{\partial x_j} \right]$$

(1)

With the production and destruction terms eqn. (2) and eqn. (3) where the quantity $F_{\Theta}$ is a boundary layer blending function of the baseline model that equals 1.0 in the boundary layer.
and 0.0 in the freestream, $S$ is the strain rate and $\Omega$ is the vorticity.

$$\begin{align*}
P_{\gamma\theta} &= F_{\theta\theta} \max(P_{\text{BL}}, P_{\text{sep}}) = f(S, \Omega, F_{\text{BL}}) \\
D_{\gamma\theta} &= F_{\theta\theta} D_{\text{BL}} + (1.0 - F_{\theta\theta}) D_{\text{FS}} = f(S, \Omega, F_{\text{BL}})
\end{align*}$$

(2)

(3)

The interaction of the wake intermittency $\gamma_w$ with the baseline transition model is done with eqn. (4). The effective intermittency $\gamma_{\text{eff}}$ is directly influencing the production and destruction terms of the underlying turbulence model.

$$\gamma_{\text{eff}} = \max(\gamma, \gamma_{\text{sep}}, \gamma_w)$$

(4)

The scaling of the $Re_{\theta}$ correlation that influences the transported $Re_{\theta}$ is done with a scaling factor $s_0$ that is determined at the MP interface through eqn. [5] - [8] with the index $dn$ denoting variables at the upstream side of the MP that are transformed into the downstream reference system.

$$Tu_{dn} = \sqrt{\frac{2}{3} k_{udn}}$$

(5)

$$Re_{\theta, dn} = f(Tu_{dn})$$

(6)

$$Re_{\theta, dn, MP} = f(Tu_{dn})$$

(7)

$$s_0 = \frac{Re_{\theta, dn}}{Re_{\theta, dn, MP}}$$

(8)

$Re_{\theta}$ is calculated at the MP through the correlation equations of the underlying baseline model [2] using eqn. [9] as well as $F(\lambda_{\theta}) = 1.0$.

$$Re_{\theta} = \begin{cases} 
1173.51 - 589.428 Tu + \frac{0.2196}{Tu^2} & Tu \leq 1.3 \\
331.50 \left[ Tu - 0.5658 \right]^{-0.671} F(\lambda_{\theta}) & Tu > 1.3 
\end{cases}$$

(9)

The scaled local correlation value of $Re_{\theta, eff}$ is computed according to eqn. [11] with $F_{\theta w}$ from eqn. [10] being a function that is 1.0 inside of wakes and boundary layers and 0.0 outside and $k_0$ as a turbulence kinetic energy average computed at the interface.

$$F_{\theta w} = \min \left( \max \left( 1.0 e^{-\left(\frac{y}{\delta}\right)^4}, 1.0 - \left( \frac{\gamma - 1/c_{\text{c2}}}{1.0 - 1/c_{\text{c2}}} \right)^2 \right), 1.0 \right)$$

(10)

$$Re_{\theta, eff} = \left( 1.0 - F_{\theta w} \right) s_{row, 0} \sqrt{ \frac{k}{k_0} + F_{\theta w} } Re_{\theta}$$

(11)

3. Design of Simulations

The test case presented in this study makes use of the widely known T106 blade profile in its variant 'C' resulting in a pitch-to-chord ratio of $t/l = 0.95$. The geometry of the profile itself is identical to the originally published T106A case ($t/l = 0.799$), also known as AGARD-case E/CA-6 by Hoheisel [17]. The design of the setup generally follows the procedure already applied in another work by the authors [3], while the computed high lift turbine cascade profile differs. The case aims to give a performance oriented reproduction of a rotor-stator interaction scenario that occurs in multistage applications. For this matter, an inviscid block, the "rotor block", is translated upstream of the cascade that acts as the stator. The cascade is computed at exit conditions of $Ma_2 = 0.59$ and $Re_2 = 150 \times 10^3$. The turbulent freestream inlet conditions are
Figure 1: Illustration of the rotor (not modeled) and the stator cascade T106C, every fourth node shown. Wakes mechanics are akin to a modeled upstream blade wake.

Figure 2: Experimental results by Hoheisel [18] (l.) and Michálek et. al. [19] (r.) at two different homogeneous inflow conditions and corresponding RANS setup solution.

set to a turbulent intensity of $Tu = 1\%$ and a turbulent length scale of $l_t = 0.18 \text{mm}$. In order to make sure the trend in blade performance is captured correctly with respect to a change of inflow conditions, two experiments at related operating conditions by Hoheisel [18] as well as Michálek et. al. [19] were computed and showed a good agreement of the computational setup with the experimental data (see Fig. 2). A synthetic wake that was produced from a correlational framework developed in an earlier study by the authors [20] is mapped onto the inlet boundary in order to model the wakes originating from an upstream rotor (see fig. 1). In order to ensure the wake mechanics are akin to the wakes of actual LPT blades, the wake of a reference computation of the used blade profile was used to calibrate the correlational framework for the wake synthesis. The wake convection mechanics are characterized by two quantities being the Strouhal number $Sr$ (eqn. (12)) as well as the flow coefficient $\phi$ (eqn. (13)) with $u_x$ and $u_y$ describing the axial and circumferential velocities as well as $c_{ax}$ as the axial blade chord length and $t_w$ as the circumferential distance between the impinging wakes. While $Sr$ gives information about the reduced frequency of the passing wakes, $\phi$ is implicitly describing...
Figure 3: Typical values of $Sr$ and $\phi$ for LTP blades found in aircraft engine operating conditions and cascade experiment conditions (l.) as well as wake convection characteristics at the most extreme combinations of $Sr$ and $\phi$ respectively (r.).

the wake convection angle.

$$Sr = \frac{u_y \times c_{ax}}{u_x \times t_w} = \frac{c_{ax}}{t_w \times \phi}$$

(12)

$$\phi = \frac{u_x}{u_y}$$

(13)

For the full design of simulations, please refer to [3]. The range of the characteristic variables is $0.25 \leq Sr \leq 1.9$ and $0.41 \leq \phi \leq 2.0$ respectively with the maximum difference between the actual $Sr$ and the targeted $Sr$ of 5% in terms of $Sr$ as shown in Fig. 3 (l.) since the Strouhal number can only be varied using integer multiples of wakes and modeled stator blades according to eqn. (12). The flow coefficient $\phi$ can be fixed exactly by choosing the appropriate circumferential velocity $u_y$. The variation of either $Sr$ or $\phi$ generally change the wake convection to a large degree as the frequency and convection angle of the passing wakes determine in what way the boundary layer is subjected to increased turbulence levels and wake jet effects. Fig. 3 (r.) illustrates the change in wake characteristics when either $Sr$ or $\phi$ are changed independently showing that an increase in $Sr$ leads to a greater number of impinging wakes while an increase of $\phi$ changes the way the wake convects through the downstream passage. For the simulations, the wake characteristics in terms of velocity defect, width, and turbulence levels are fixed in order to isolate the varied quantities as much as possible. However, since the inlet total quantities are also fixed between the wake profiles, the operating point of the stator blade varies slightly. This procedure works similar to an immersed boundary setup with wake generating bars. As opposed to an immersed boundary setup, however, the average inflow angle is fixed for all cases as this removes the incidence angle as a potential reason for the change in loss behavior. The unsteady simulations are resolved using 256 physical timesteps per period with a duration of ten blade passing periods, starting from a converged steady state solution, in order to sufficiently capture the unsteady effects.
4. Results

For the T106C LPT cascade, a total of 48 combinations of $Sr$ and $\phi$ have been computed and the integral loss parameter $\zeta$ as well as the size of the LSB were evaluated. Figure 4 shows the normalized total pressure loss coefficient $\zeta/\zeta_0$ for the baseline model formulation as given in [2] as well as the losses obtained by the QUWM, compared to the unsteady time-mean results using a direct mapping zonal interface (cf. Yang et. al. [21]) instead of a mixing plane. The most prominent phenomenon in the $Sr - \phi$ overviews of loss coefficients is the area of locally increased losses at $Sr \approx 0.53$, $\phi \geq 0.6$ that is captured in both the unsteady time-mean as well as the QUWM computations. In comparison, the baseline steady state setup is not able to show such a nuanced overview of the loss change matrix as it predicts almost a constant $\zeta/\zeta_0$ for the lower left half of the matrix. Looking at the integral total pressure loss prediction capabilities of both steady state setups, it is noticeable that the QUWM performs better than the baseline model formulation in the given environment using mixing plane interfaces. On the other hand, the QUWM results also differ slightly from the reference unsteady time-mean predictions as the local area of increased total pressure loss is slightly larger and more pronounced quantitatively.

While the baseline model generally underpredicts the total pressure loss change in low $Sr$ and low $\phi$ scenarios and slightly overpredicts them in high $Sr$ and high $\phi$ environments, the overall trend follows the unsteady time-mean results. As a large portion of the losses of the T106C test case is due to separated suction side flow, the predicted time-mean size of the LSB is compared. A good prediction of the LSB dimensions is also crucial in order to assess the reliability of the blade profile at various operating conditions. Figure 5 shows the length $l_{sep}$ of the separation bubble, normalized with the overall length of the suction side boundary layer $l$. The difference between $l$ and the widely used chord length $c$ is that $l$ is the actual integrated length of the suction side of the blade in order to provide a more accurate evaluation. The comparison of the steady state results with the unsteady time-mean reference solution shows a similar trend as the change in total pressure loss. In order to give a better understanding of the normalized result, the relative length of the LSB in homogeneous freestream conditions (no wakes) of $l_{sep}/l = 0.26$ is a quantity of interest. Thus, all combinations of $Sr$ and $\phi$ lead to a decrease in separated boundary layer length. At $Sr \approx 0.53$, $\phi \geq 0.6$, the LSB is comparably large which is only
Figure 5: Size of laminar suction side separation bubble $l_{sep}$ with respect to the suction side boundary layer length $l$ compared for the baseline steady state (left), unsteady time-mean (mid) and QUWM steady state (right) case with wakes.

shown by the unsteady simulation as well as the QUWM results while the baseline steady state computation omits this information. The area of locally increased losses as well as larger relative separated length is particularly interesting since it is captured by both the unsteady time-mean simulation as well as the QUWM. In order to better understand the unsteady interaction of the wakes with the boundary layer, a time-space diagram of the suction side boundary layer shape factor $H_{12}$ for three Strouhal numbers $Sr = [0.25, 0.6, 1.8]$ at a constant $\phi = 1.5$ is shown in Fig. 6 for the same normalized time scale. As $\phi$, i.e. the wake convection, is fixed for all three cases, they only differ in wake frequency.

Figure 6: Time-space diagram of the unsteady computation suction side boundary layer shape factors at three different $Sr$ and a constant $\phi = 1.5$. Black lines denote separated flow. Dashed black lines at normalized boundary layer length $s/l = [0.6, 0.8]$ for orientation.

The time-space diagram of $Sr = 0.25$ shows two impinging wakes in the captured time period.
that interact with the boundary layer leading to a temporarily shorter separation. In between the wakes, $H_{12}$ suggests laminar flow upstream of the separation point at around $s/l \approx 0.6$. However, the separation bubble does not seem to recover to a steady state, but instead keeps changing its position on the suction side of the airfoil. For $Sr = 0.6$, there are 5 wakes per period penetrating the boundary layer of the T106C stator blade. The dynamics of the interaction between wake and boundary layer seem to be different though. While each wake is able to decrease the bubble size, the separation point is not delayed much compared with $Sr = 0.25$ despite having 2.5 times more wakes per time period. Also, the separation bubble is reacting to the passing wakes in a more direct but also more resistant way. The length of the separation is temporarily decreased right when the wake convects through the boundary layer. However, the separation immediately re-establishes itself and returns to a longer average size than for the $Sr = 0.25$ case. In the $Sr = 1.8$ diagram, the large number of wakes (15 per normalized time period) leads to a separation bubble that is almost suppressed. Also, the shape factor upstream of the flow separation suggests that the boundary layer has already partially undergone transition before separating. The general behavior of the separation bubble for $Sr = 0.6$ compared to the other two combinations could be due to the damping properties of the boundary layer with respect to particular wake frequencies. As the length of the separation bubble as well as the losses do not deviate much from the homogeneous inflow computation, the interaction of the separation region with the impinging wakes seems to be only weak. This is also shown for most of the other computations at this specific wake frequency (Fig. [1][5]). These findings are supported by the LES results of the T106A blade by Michelassi et al. [22] that also show a local maximum in mixed out losses for a certain but lower reduced frequency range at a similar flow coefficient. A detailed boundary layer stability analysis in terms of amplified and damped frequencies should be part of further studies.

5. Computation Time
Compared to the steady state baseline computation, the QUWM consumes around 12% more time per iteration while keeping the total number of iterations needed at the same level. The time-resolved simulations consume up to 30 times more cpu hours than the baseline model computations as multiple stator blades have to be modeled while the steady state setups utilize mixing plane interfaces removing the need for a synchronization of rotor and stator pitch.

6. Conclusions
A high lift low pressure turbine cascade has been computed in the presence of impinging wakes using steady and unsteady RANS methods. The wake-related quantities Strouhal number and flow coefficient were altered in a set of 48 combinations changing the wake convection and overall number of upstream wakes per modeled blade. The simulations were conducted in order to compare a newly developed quasi-unsteady wake model to the baseline $\gamma$ - $Re_\Theta$ model formulation in steady state mixing plane simulations with the respective time-resolved unsteady simulation as the reference solution. The integral total pressure loss coefficient as well as the normalized laminar separation bubble length was evaluated for all setups. The unsteady results show an area where the wakes do not alter separation bubble size or loss behavior much. An investigation of the shape factor in a time-space diagram shows that the suction side separation bubble may be more resistant against certain wake frequencies regardless of the flow coefficient. The comparison between the steady state results shows that the QUWM is able to predict both evaluated quantities in a more accurate way than the baseline model in steady state mixing plane simulations while not increasing the computation time to a large degree. As the QUWM captures flow features that are not included in the baseline model results while keeping the computation time in a reasonable range, it is a useful extension for turbomachinery flow involving unsteady rotor-stator interaction phenomena.
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