Axion-Radiation Conversion by Super and Normal Conductors

Aiichi Iwazaki

International Economics and Politics, Nishogakusha University,
6-16 3-bantyo Chiyoda Tokyo 102-8336, Japan.
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We have proposed a method for the detection of dark matter axion. It uses superconductor under strong magnetic field. As is well known, the dark matter axion induces oscillating electric field under magnetic field. The electric field is proportional to the magnetic field and makes charged particles oscillate in conductors. Then, radiations of electromagnetic fields are produced. Radiation flux depends on how large the electric field is induced and how large the number of charged particles is present in the conductors. We show that the electric field in superconductor is essentially identical to the one induced in vacuum. It is proportional to the magnetic field. It is only present in the surface because of Meissner effect. On the other hand, although the magnetic field can penetrate the normal conductor, the oscillating electric field is only present in the surface of the conductor because of the skin effect. The strength of the electric field induced in the surface is equal to the one in vacuum. We obtain the electric field in the superconductor by solving equations of electromagnetic fields coupled with axion and Cooper pair described by Ginzburg-Landau model. The electric field in the normal conductor is obtained by solving equations of electromagnetic fields in the conductor coupled with axion. We compare radiation flux from the cylindrical superconductor with that from the normal conductor with same size. We find that the radiation flux from the superconductor is a hundred times larger than the flux from the normal conductor. We also show that when we use superconducting resonant cavity, we obtain radiation energy generated in the cavity two times of the order of the magnitude larger than that in normal conducting resonant cavity.

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I. INTRODUCTION

Axion is the Nambu-Goldstone boson associated with Peccei-Quinn symmetry. The symmetry is a global U(1) chiral symmetry. It naturally solves the strong CP problem. Because the strong CP violation has not yet been observed, we need to explain why the CP violating term $G_{\mu,\nu}\tilde{G}^{\mu,\nu}$ is absent or extremely small in QCD Lagrangian where $G_{\mu,\nu}$ ( $\tilde{G}^{\mu,\nu}$ ) denotes ( dual ) fields strength of gluons. The axion is a real scalar field which is the phase of a complex scalar field carrying the U(1) charge. It acquires the mass $m_a$ through chiral anomaly. That is, instantons in QCD give rise to the mass of the axion. The axion is called as QCD axion. In the paper we mainly consider the QCD axion. The axion is generated in early universe and becomes one of dominant components of dark matter in the present universe when the Peccei-Quinn symmetry is broken after inflation.

In the present day, the axion is one of the most promising candidates for the dark matter. There proceed many projects for the detection of the axion. There are three types of the projects: Haloscope, Helioscope and others. The dark matter axion in our galaxy is searched with Haloscope, while axion produced in the Sun is searched with Helioscope. Haloscope projects are ADMX, CARRACK, HAYSTAC, ABRACADABRA, ORGAN, etc. Helioscope projects are CAST, SUMICO, etc.. The others are LSW ( Light Shining through a Wall ), VMB ( Vacuum Magnetic Birefringence ) etc. Since axion mass is unknown, the mass range in the search is very wide, $10^{-20}\text{eV} \sim 1\text{keV}$, or more. There are axion-like particles proposed whose masses are not limited, differently to the QCD axion mass. ( The axion-like particles are those which couple with electromagnetic fields just as the QCD axion does. ) But we assume the QCD axion in this paper and expect that the appropriate mass range is of the order of $10^{-6}\text{eV} \sim 10^{-4}\text{eV}$, It comes from cosmological consideration and simulation in lattice gauge theory.

In previous paper we have proposed a method of the detection of radiations generated by dark matter axion. It is to use a superconductor of cylindrical shape. In general, the dark matter axion induces oscillating electric field under strong magnetic field. The oscillating electric field makes Cooper pair oscillate in the superconductor so that the radiations are emitted. Similarly, we expect that radiations are also emitted by electrons in normal metal under the magnetic field. In this paper we examine both cases in detail and show that the radiation flux from the metal is a hundred times smaller than that from the superconductor.

Our proposal is a type of Haloscope, in which the dark matter axion is converted to photon ( radio wave ) under magnetic field. How large amount of radiation is generated depends on materials we use. Here, we consider
superconductor and normal conductor (sometimes we call it simply as metal). In the superconductor there are Cooper pairs with large number density \( \sim 10^{22} \text{cm}^{-3} \), while in the metal there are electrons with the number density almost identical to that of the Cooper pairs. These charged particles emit radiations when they are forced to oscillate by the oscillating electric fields. The strength of the electric fields are proportional to external magnetic field we impose. But, it is different depending on each material.

In this paper we show that the electric fields induced in the superconductor and the normal conductor are essentially identical to the one induced in the vacuum. It is proportional to the magnetic field in the conductors. However, the electric field in the superconductor is only present in the surface because the magnetic field is only present in the surface to a penetration depth owing to the Meissner effect. On the other hand, although the magnetic field penetrates the normal conductor, the oscillating electric field is only present in its surface because of the skin effect. Theses oscillating electric fields generate oscillating electric currents. The oscillating electric currents in both conductors only flow in their surfaces and emit electromagnetic radiations.

To find how strong electric field is generated in the superconductor, we analyze Ginzburg-Landau model for the superconductor. The superconductivity is represented as a condensed state of Cooper pair in the model. We solve equations of electromagnetic fields coupled with axion and Cooper pair described by the model. On the other hand, to find the electric field in the metal, we solve equations of electromagnetic fields coupled with axion in the normal conductor characterized by permeability \( \mu \) and electric conductivity \( \sigma \).

The presence of the oscillating electric fields leads to radiations from the conductors. We show that the radiation flux from the superconductor is two times the order of the magnitude larger than that from the normal conductor. The difference arises from the difference between the penetration depth and the skin depth. (The penetration depth in the superconductor is shorter than the skin depth as long as the frequency of the radiation is approximately less than 100GHz.) The radiation flux from the superconductor is so large that existing radio telescope such as one with parabolic dish antenna of radius \( \sim 10 \text{m} \) can easily observe the radiation, even when the superconductor is put at a hundred meter away from the telescope. Indeed, the radiation flux from the cylindrical superconductor with radius \( \sim 1 \text{cm} \) and length \( \sim 10 \text{cm} \) under magnetic field \( \sim 5 \text{T} \) is of the order of \( \sim 10^{-18} \text{W} \).

We also show that radiation energy generated in superconducting cavity is two times of the order of the magnitude larger than that in normal conducting cavity, e.g. copper. The difference arises from the difference in the depth from the surface in which the oscillating electric current flows.

In the next section, we introduce axion photon coupling and find electric field in vacuum induced by axion under magnetic field. In the section (III), we introduce Ginzburg-Landau model for superconductor, in which Cooper pair is described by the field \( \Phi \). The model represents the coupling with electromagnetic field and the Cooper pair, and describes Meissner effect characterizing the superconductivity. We derive electric field in the superconductor induced by axion under magnetic field. In the section (IV) we solve the equations of electromagnetic fields in normal conductor coupled with axion. We find that electric field in normal conductor induced by axion is only present in the surface of the conductor. In the section (V) we numerically show the radiation fluxes from the superconductor and normal conductor of cylindrical shape. In the section (VI) we estimate radiation energy generated in superconducting resonant cavity. In the final section (VII) summary and discussion follow.

## II. ELECTRIC FIELD IN VACUUM

First we show that the coherent axion induces an electric field in vacuum under a magnetic field. It is well known that the axion \( a(\vec{x}, t) \) couples with both electric \( \vec{E} \) and magnetic fields \( \vec{B} \) in the following,

\[
L_{\alpha EB} = k_0 a(\vec{x}, t) \frac{\alpha a(\vec{x}, t) \vec{E} \cdot \vec{B}}{f_\alpha \pi} = g_{\alpha \gamma \gamma} a(\vec{x}, t) \vec{E} \cdot \vec{B}
\]

with the decay constant \( f_\alpha \) of the axion and the fine structure constant \( \alpha \approx 1/137 \), where the numerical constant \( k_0 \) depends on axion models; typically it is of the order of one. The standard notation \( g_{\alpha \gamma \gamma} \) is such that \( g_{\alpha \gamma \gamma} = k_0 a/f_\alpha \pi \approx 0.14(m_\alpha/\text{GeV})^2 \) for DFSZ model[16] and \( g_{\alpha \gamma \gamma} \approx -0.39(m_\alpha/\text{GeV})^2 \) for KSVZ model[17]. In other words, \( k_0 \approx 0.37 \) for DFSZ and \( k_0 \approx 0.96 \) for KSVZ. The axion decay constant \( f_\alpha \) is related with the axion mass \( m_\alpha \) in the QCD axion: \( m_\alpha f_\alpha \approx 6 \times 10^{-6} \text{eV} \times 10^{12} \text{GeV} \).

We show that the coupling parameter \( k_0 a/f_\alpha \pi \) in the Lagrangian eq[16] is extremely small for the dark matter axion \( a(t) \). Furthermore, the dark matter axion \( a(t) \) can be treated as a classical field because the axions are coherent.

We note that the energy density of the dark matter axion is given by
\[ \rho_a = \frac{1}{2} (a^2 + (\partial a)^2 + m_a^2 a^2) \simeq m_a^2 a^2 \]  

where \( a(t) = a_0 \cos(t \sqrt{m_a^2 + p_a^2}) \simeq a_0 \cos(m_a t) \), because the velocity \( \rho_a/m_a \) of the axion is about \( 10^{-3} \) in our galaxy. The local energy density of dark matter in our galaxy is supposed such as \( \rho_a \simeq 0.3 \text{GeV cm}^{-3} \simeq 2.4 \times 10^{-42} \text{GeV}^4 \). Assuming that the density is equal to that of the dark matter axion, we find extremely small parameter \( a/f_a \simeq \sqrt{\rho_a/(m_a f_a)} \simeq 10^{-19} \). The energy density also gives the large number density of the axions \( \rho_a/m_a \simeq 10^{15} \text{cm}^{-3} (10^{-6} \text{eV/m}_a) \), which causes their coherence. This allows us to treat the axions as the classical axion field \( a(t) \). Anyway, we find that the parameter \( k_a a/(f_a \pi) \) is extremely small.

The interaction term in eq(1) between axion and electromagnetic field slightly modifies Maxwell equations in vacuum,

\[
\begin{align*}
\partial \cdot \vec{E} + g_{a\gamma\gamma} \partial \cdot (a(\vec{x}, t) \vec{B}) &= 0, \\
\vec{E} + g_{a\gamma\gamma} a(\vec{x}, t) \vec{B} &= 0, \\
\vec{B} + \partial \times \vec{E} + \partial_t \vec{B} &= 0.
\end{align*}
\]

From the equations, we approximately obtain the electric field \( \vec{E} \) generated by the axion \( a \) under the static background magnetic field \( \vec{B}(\vec{x}) \),

\[ \vec{E}_a(r, t) = -g_{a\gamma\gamma} a(\vec{x}, t) \vec{B}(\vec{x}), \]

assuming the parameter \( k_a a/(f_a \pi) \) extremely small. This is the electric field in vacuum induced by the dark matter axion \( a(t, \vec{x}) = a_0 \cos(\omega_a t - p_a \cdot \vec{x}) \simeq a_0 \cos(m_a t) \) with \( \omega_a = \sqrt{m_a^2 + p_a^2} \simeq m_a \). We note out that the magnetic field configuration is arbitrary.

### III. ELECTRIC FIELD IN SUPERCONDUCTOR

Now we proceed to examine the electric field induced in superconductor. Especially, we suppose that the superconductor is present at \( x > 0 \) and uniform in \( y \) and \( z \) directions. The magnetic field is parallel to the surface of the superconductor. It is also uniform in \( y \) and \( z \) directions, and points to \( z \) direction. We suppose that the superconductor is described by Ginzburg-Landau model with Cooper pair \( \Phi \),

\[ L_{GL} = |(\partial_t - iq A_0)\Phi|^2 - |(\partial_x + iq \vec{A})\Phi|^2 - h(|\Phi|^2 - v_0^2|^2) \]

with electric charge \( q = 2e \) ( electron charge \( e \) ) and coupling constant \( h \), where \( A_0 \) and \( \vec{A} \) denote gauge fields; \( \vec{E} = -\partial A_0 - \partial_t \vec{A} \) and \( \vec{B} = \partial \times \vec{A} \). We identify a field \( \Phi \) used in nonrelativistic Ginzburg-Landau model such that \( \Psi = \Phi \sqrt{2m_e} \) with mass \( m_e \) of Cooper pair. Because the number density \( n_c \) of the Cooper pair is represented as \( n_c = |\langle \Psi \rangle|^2 \) in the nonrelativistic Ginzburg-Landau model, it is given by \( 2m_e |\langle \Phi \rangle|^2 = 2m_e v_0^2 \) in the relativistic Ginzburg-Landau model. \( m_c = 2m_e \) with electron mass \( m_e \). The superconducting state is represented by the condensed state \( \langle \Phi \rangle = v_0 \) of the Cooper pair.

When we include the effect of the Cooper pair, the modified Maxwell equations are led to the following,

\[ \begin{align*}
\partial \cdot \vec{E} + g_{a\gamma\gamma} \partial \cdot (a(\vec{x}, t) \vec{B}) &= 2q^2 A_0 |\Phi|^2 + i q \Phi^\dagger \partial_\Phi + C.C. = 0, \\
\vec{E} + g_{a\gamma\gamma} a(\vec{x}, t) \vec{B} &+ \partial_t \left( \vec{E} + g_{a\gamma\gamma}(\vec{x}, t) \vec{B} \right) - 2q^2 A_0 |\Phi|^2 + i q |\Phi|^2 \partial_\Phi + C.C. = 0, \\
\vec{B} &= \vec{B}, \quad \partial \times \vec{E} + \partial_t \vec{B} = 0.
\end{align*} \]

In order to see magnetic field configuration in the superconductor, we solve the second equation with external magnetic field \( \vec{B} = (0, 0, B_z(x)) \) with \( B_z(x) = B_0 \) for \( x < 0 \), neglecting the axion field,

\[ -\partial \times \vec{B} - 2q^2 \vec{A}_0 |\Phi|^2 = -\partial \times \vec{B} - 2q^2 A_0 v_0^2 = 0 \quad \Rightarrow \quad B_z(x) = B_0 \exp(-x/\lambda) \quad \text{for} \quad x \geq 0 \]
with $\Phi = \nu_0$ and the penetration depth $\lambda = (\sqrt{2}q\nu_0)^{-1}$. We have taken the value $\Phi = \nu_0$ of Cooper pair in the superconductor. The equations (6) and (8) trivially hold when $A_0 = 0$ and $\Phi_0 = \nu_0$. We find the magnetic field configuration in the superconductor; it penetrates the surface to the depth $\lambda$. That is, it represents the Meissner effect. Then, we expect that the electric field induced by the axion is also present only in the surface. We will derive the electric field $\vec{E}_a$ induced by the axion under the magnetic field $\vec{B}(x) = (0, 0, B_z(x))$. Supposing the parameter $k_a(\alpha/a)\pi$ extremely small and small momentum of axion $\vec{q} = 0$, we derive the equations for $\delta \Phi$ and $\delta \vec{A}$ from Ginzburg-Landau Lagrangian in eq(5) and modified Maxwell eq(7) ($\Phi = \Phi_0 + \delta \Phi$ and $\vec{A} = \vec{A}_0 + \delta \vec{A}$ with $\vec{B} = \vec{\partial} \times \vec{A}_0$),

$$0 = -\vec{\partial}^2 \delta \Phi + (\vec{\partial} + iq\vec{A}_0)^2 \delta \Phi + 2(\vec{\partial} + iq\vec{A}_0) \cdot (-iq\delta \vec{A})\nu_0 - 2hv_0^2\delta \Phi^1 + C.C.$$ (10) 

$$0 = (\vec{\partial}^2 - \vec{\partial}^2)\delta \vec{A} - 2q^2v_0^2\delta \vec{A} + g_{a\gamma\gamma}\partial_0 \partial_0 B - 2q^2\vec{A}_0\nu_0(\delta \Phi + \delta \Phi^1) - iqv_0\vec{\partial} \delta \Phi + C.C.$$ (11)

with the gauge condition $\vec{\partial} \cdot \delta \vec{A} = 0$, where $\delta \Phi$ and $\delta \vec{A}$ is of the order of $k_a(\alpha/a)\pi$. In addition to eq(11) and eq(11), we have the equation (6) for the gauge field $A_0 = \delta A_0$,

$$0 = \vec{\partial} \cdot (\vec{E}_a + g_{a\gamma\gamma}\partial_0 B) + 2q^2\delta A_0v_0^2 + iqv_0\partial_0 \delta \Phi + C.C.$$ (12)

All of the variables $\delta \vec{A}$, $\delta A_0$, and $\delta \Phi$ may oscillate according to the oscillation $\alpha(t) \times \cos(m_\alpha t)$. Because we assume that the fields are uniform in $y$ and $z$ directions, we simplify the equation (11) by taking $\delta \vec{A} = (0, 0, \delta A_z)$ as well as $\vec{A}_0 = (0, A_{0y}, 0)$ and $\delta A_0 = 0$ ($\vec{E}_a = (0, 0, -\partial_0 \delta A_z)$ and $\vec{B} = (0, 0, \partial_z A_{0y})$),

$$0 = (\vec{\partial}^2 - \vec{\partial}_z^2)\delta A_z - 2q^2v_0^2\delta A_z + g_{a\gamma\gamma}\partial_0 B,$$ (13)

It is easy to obtain a solution of eq(10), eq(12) and eq(13),

$$\delta \Phi = 0, \quad \delta A_z = A'_0 \cos(\omega t) \exp(-\frac{X}{\lambda'}) - \frac{g_{a\gamma\gamma}\partial_0 a}{m_a^2}B_z,$$ (14)

with arbitrary amplitude $A'_0$ and frequency $\omega$, where we have $\lambda' \approx \lambda(1 + \omega^2 \lambda^2/2)$, assuming $\omega^2 \ll \lambda^{-2} = 2q^2v_0^2$. Namely, the electric field is

$$\vec{E}_{a,z} = \omega A'_0 \sin(\omega t) \exp(-\frac{X}{\lambda'}) - g_{a\gamma\gamma}a(t)\vec{B}$$ (15)

with $\vec{E}_a = (0, 0, -\partial_0 A_z) = (0, 0, \vec{E}_{a,z})$.

The first term in $\vec{E}_{a,z}$ represent a radiation with frequency $\omega$ entering from outside the superconductor, while the second term represents the electric field $\vec{E}_a = -g_{a\gamma\gamma}a(t)\vec{B}(x)$ induced by the dark matter axion. It is just equal to the one in eq(11) in the vacuum. The radiation entering the superconductor from outside only penetrates the surface of the superconductor to the penetration depth $\lambda' \approx \lambda$ owing to the Meissner effect.

It is instructive to see that we have small suppression factor $(m_a\lambda)^2$ in the electric field $\vec{E}_a$ when the second term $2q^2v_0^2\delta \vec{A} = \lambda^{-2}\delta \vec{A}$ in eq(13) is absent. Namely, $\vec{E}_a = -\lambda(m_a\lambda)^2g_{a\gamma\gamma}a(t)\vec{B}$. (Typically, $\lambda \sim 10^{-6}$ cm and $m_a^{-1} \sim 10$ cm.) The second term represents the effect of the Cooper pair and cancels the derivative $\vec{\partial}_z^2 \delta \vec{A}$ in eq(13) because $B_z \propto \exp(-x/\lambda)$. Thus, we find that the suppression factor is absent in the superconductor; $\vec{E}_a(x,t) = -g_{a\gamma\gamma}a(t)\vec{B}(x)$. The electric field $\vec{E}_a$ is only present in the surface of the superconductor because the magnetic field $\vec{B}$ is only present in the surface. These results were used in the previous paper [14].

Here we make a brief comment that similar electric field $\vec{E}_a = -g_{a\gamma\gamma}a(t)\vec{B}$ is induced around magnetic vortices in type 2 superconductor. As we know, the magnetic field penetrates the type 2 superconductor and forms vortices. They are described as classical solutions in the Ginzburg-Landau model. A solution of the magnetic vortex located at $\rho = 0$ is characterized in the following,

$$\Phi_v = |\Phi_v(\rho)|\exp(i\theta) \quad \text{with} \quad |\Phi_v(\rho = 0)| = 0 \quad \text{and} \quad |\Phi_v(\rho)| - \nu_0 \propto \exp(-\rho/\xi) \quad \text{for} \quad \rho \gg \xi$$ (16)

$$\vec{B}_v = (0, 0, B_z^v(\rho)) \quad \text{with} \quad B_z^v(\rho) \propto \exp(-\rho/\lambda) \quad \text{for} \quad \rho \gg \lambda$$ (17)
with integer \( n \), where the cylindrical coordinate \((\rho, \theta, z)\) is used. The important feature of the vortex is the flux quantization: \( \oint d\rho d\theta B_\theta^2 = 2\pi n/q \). The type 2 superconductor has a property that the penetration depth \( \lambda \) is larger than the coherent length \( \xi \) of the Cooper pair; \( \lambda > \xi \). Thus, we may approximate these fields such as \( B_\theta(\rho) \propto 1/\rho \) for \( \rho > \lambda \). In the region \( \rho > \lambda \), we have a similar equation to eq(13) for \( \delta \Phi = \Phi - \Phi_0 = (0, 0, \delta A_\rho) \) in the cylindrical coordinate by putting \( \delta \Phi = \Phi - \Phi_0 = 0 \) and \( \Phi_0 = (0, A_\rho, 0) \), \( B_\theta^2(\rho) = \partial_\rho A_\rho + A_\theta/\rho \) and \( \delta \Phi = \Phi - \Phi_0 \). Then, we find that the solution of the electric field \( \vec{E}_r = -\vec{\partial}_t \delta \vec{A} \) is given such as

\[
\vec{E}_r(\rho) = -g_{a\gamma\gamma} a \vec{B}_v(\rho).
\]

The solution holds only in the region \(( \rho > \xi )\) outside the vortex core. This is the electric field expected naively when the magnetic field \( B_v(\rho) \) is present. Obviously, it is attached to the vortex.

We would like to mention that the current density \( J_r \) induced by the axion is given such as \( J_r = 2g^2 v_0^2 \delta A_\rho = \lambda^{-2} \delta A_\rho = -\lambda^{-2} \int d\tau E_r(t') = \lambda^{-2} \partial_\tau E_r(t)/m_a^2 \) in the superconductor; \( E_r = \cos(m_a t) \). The current \( J_r \propto \sin(m_a t) \) has no dissipation of its energy; \( \int_0^{2\pi/m_a} dt J_r(t) E_r(t) = 0 \). Namely, the current is a superconducting current even in the presence of the electric field. The fact is valid both for the surface current in the superconductor and the current flowing along the magnetic vortex.

We note that the penetration depth is rewritten such as \( \lambda = \sqrt{1/2q^2 v_0^2} = \sqrt{m_a/q^2 n_c} \) in terms of the number density \( n_c \), where we use the relation \( n_c = 2m_e v_0^2 \) mentioned above. Then, the current density is also rewritten such as \( J_r = -\lambda^{-2} \int d\tau E_r(t') \approx q^2 n_c E_r(t)/m_a n_c \). We will explain later that the formula can be obtained by using the Drude model. The model describes the classical motion of Cooper pair under the electric field \( E_r \).

In this section, we find that the electric field induced in the superconductor is essentially identical to the one induced in the vacuum, although the electric field is confined to the surface of the superconductor. It oscillates with the frequency identical to the one of the axion field \( a(t) \propto \cos(\omega A t) \). Similarly, the current density \( J_r \) also oscillates with the same frequency.

IV. ELECTRIC FIELD IN NORMAL CONDUCTOR

Next, we derive the electric field in the normal conductor (metal) with permeability \( \mu \) and electric conductivity \( \sigma \). The configuration of the metal is the same as the case of the superconductor. That is, the metal is present at \( x > 0 \) and uniform in \( y \) and \( z \) directions. We impose magnetic field \( \vec{B} = (0, 0, B_z) \) parallel to the surface (at \( x = 0 \)) of the metal is continuous at the surface, the magnetic field \( \vec{B}_m \) is obtained such that \( \vec{B}_m = \vec{B}_0/\mu_0 \) where we denote the vacuum permeability \( \mu_0 \) (\( \mu_0 = 1 \) in natural unit). For instance, \( \mu \approx \mu_0 \) for copper or \( \mu \approx 5000 \times \mu_0 \) for iron. We should note that in general, the permeability \( \mu \) depends on the magnetic field \( \vec{B} \) and that the strength \( \vec{B}_m = \vec{B}_0(\mu)/\mu_0 \) saturates as the external field \( \vec{B} \) increases. The maximal magnetic field \( B_{m, \max} \) is at most \( 1T \sim 2T \). Thus, we cannot make \( B_m \) increase unlimitedly. Hereafter, we assume that \( \mu \) is independent of \( B \) and we use natural unit \( \mu_0 = 1 \).

The electromagnetic fields in the metal are described by the modified Maxwell equations including non trivial permeability of the metal,

\[
\vec{\partial} \cdot \vec{B} + g_{a\gamma\gamma} \vec{\partial} \cdot (a(x, t) \vec{B}_m) = 0, \quad \vec{\partial} \times \left( \vec{H}_m - g_{a\gamma\gamma} a(x, t) \vec{E} \right) - \partial_t \left( \vec{D} + g_{a\gamma\gamma} a(x, t) \vec{B}_m \right) = \vec{J}_r, \quad \vec{\partial} \cdot \vec{E}_m = 0, \quad \vec{\partial} \times \vec{E} + \partial_t \vec{B}_m = 0.
\]

where \( \vec{H}_m = \mu^{-1} \vec{B}_m \) and \( \vec{D} = \epsilon \vec{E} \) with permittivity \( \epsilon \). The permittivity \( \epsilon \) is nearly equal to 1 in the metal for radiations with frequency \( \sim 1GHz \). In eq(18) we have included the current \( \vec{J}_r = \sigma \vec{E} \) induced by electric field \( \vec{E} \), but we have neglected external current generating the background magnetic field \( \vec{B} \).

When we neglect axion contribution, we obtain magnetic field \( B_{m, in}^0 = \mu \vec{B} \) uniform inside the metal where \( B \) is external magnetic field imposed. Obviously, there is no electric field inside the metal. When we take into account the axion contribution, the oscillating electric field is induced. Naively we expect that the electric field is given such that \( \vec{E}_{a, in} = -g_{a\gamma\gamma} a(t) \mu \vec{B} / \epsilon \). But, this is not correct as we show below.

Assuming the parameter \( g_{a\gamma\gamma} a(x, t) \) small and noting that the electric field is the order of \( g_{a\gamma\gamma} a(x, t) \), we derive the equations,

\[
\vec{\partial} \cdot \delta \vec{E} = 0, \quad \vec{\partial} \times \delta \vec{B} = \mu \vec{J}_r + \mu_0 \partial_t \delta \vec{E}, \quad \vec{\partial} \cdot \delta \vec{B} = 0, \quad \text{and} \quad \vec{\partial} \times \delta \vec{E} + \partial_t \delta \vec{B} = 0.
\]
with $\delta E = \bar{E} - E_a^{in}$ and $\delta B = B_{in} - \mu \bar{B} = B_{in} - B_{0}^{in}$, where we have used the relation $\bar{\delta} \times E_a^{in} = 0$ because $\bar{\delta} \times \bar{B} = 0$ inside the metal. Here, we should note that $\delta B$ is the order of $g a_{\gamma} \alpha (\bar{x}, t)$. The electric field $E_a^{in}$ is the naive one expected in the metal.

Using the Ohm’s law $J_c = \sigma E$ in eqs(19), we derive the equation for $\bar{E}$,

$$
(\bar{\delta}^2 - \mu \sigma \bar{\delta}^2) \bar{E} = \sigma \bar{\delta} \partial_t \bar{E} - \mu \sigma \bar{\delta}^2 \bar{E}_a^{in}
$$

where we note that $\bar{E}_a^{in} \propto \cos(m_a t)$. Then, we find the solution,

$$
\bar{E} \simeq \bar{E}_0 \exp(-\frac{a}{\delta} \cos(\omega t - \frac{x}{\delta}) + \frac{\epsilon}{\sigma} \partial_t \bar{E}_a^{in}),
$$

with arbitrary field strength $\bar{E}_0$, and frequency $\omega$. The skin depth $\delta$ is given by $\delta = \sqrt{2/\mu \sigma \omega}$. In the derivation, we have neglected the term $\mu \sigma \bar{\delta}^2 \bar{E}$ in the left hand side of eq(20), which is much smaller than the term $\sigma \mu \partial_t \bar{E}$ in the right hand side. Namely, we have used the inequality $\mu \sigma \gg \omega (\sim m_a)$ because the electric conductivity $\sigma \sim 10^4 \text{eV}$ in copper or iron is much larger than the axion mass $m_a = 10^{-6} \text{eV} \sim 10^{-4} \text{eV}$ under consideration.

The first term in the solution $\bar{E}$ represents oscillating electric field with the skin depth $\delta$, while the second term represents the oscillating electric field $\bar{E}_a^{in}$ inside the metal; $\bar{E}_a^{in} = \frac{\epsilon}{\delta} \partial_t \bar{E}_a^{in}$. The first term is only present in the surface of the metal and represents a radiation with frequency $\omega$ entering from outside the metal. The strength $\bar{E}_0$ is determined by the boundary condition at the surface $x = 0$. In our case it is determined by electric field induced outside the metal, i.e. electric field in the vacuum. It is just $\bar{E}_a = -g a_{\gamma} \alpha (t) \bar{B}$. Because of the continuity of the electric field parallel to the surface, the electric field in the surface is given by $\bar{E}_a^{suf} = - g a_{\gamma} \alpha_0 \bar{B} \exp(-\frac{a}{\delta} \cos(m_a t - \frac{x}{\delta})$. (Similar consideration in the superconductor leads to the electric field $\bar{E}_a$ derived in the previous section.)

On the other hand, the electric field $\bar{E}_a^{in}$ present inside the metal is the one induced by the dark matter axion. It is suppressed by the factor $m_a/\sigma$ compared with the naive one $\bar{E}_a^{in}$. This oscillating electric field induces the oscillating current $J_c = \sigma \bar{E}_a^{in} = \epsilon \partial_t \bar{E}_a^{in}$.

In general, electric field is absent inside conductor with finite size because the field is screened by the electric field produced by surface charge. It is induced on the surface of the boundary between the conductor and vacuum. (Even if the electric field is present inside the metal, free electrons move to make the field cancelled by the surface charge.) In our case there is no such surface charge because we consider the conductor extended infinitely in the direction $z$ and $y$. Thus we have non vanishing electric field $\bar{E}_a^{in}$ inside the metal.

When the conductor has finite size, the electric field $\bar{E}_a^{in}$ is absent. But we show that oscillating electric current $\bar{J}_c = \epsilon \partial_t \bar{E}_a^{in}$ is present. We suppose that the shape of the metal is cylindrical and the metal has finite size; the metal has upper and down surfaces. The external magnetic field $\bar{B}$ imposed parallel to the cylinder is perpendicular to the upper and down surfaces. The perpendicular magnetic field is continuous at the surfaces. Then, the electric field $\bar{E}$ just outside the upper or down surfaces induced by the axion is given by $\bar{E} = -g a_{\gamma} \alpha (t) \bar{B}_a^{in} = \bar{E}_a^{in}$, where $\bar{B}_a^{in} = \mu \bar{B}$ is the magnetic field inside the metal. It is also the magnetic field just outside the metal. Differently to the case of the magnetic field, the electric field is absent inside the metal. Thus the electric field perpendicular to the surfaces is discontinuous at the surfaces. Then, there are surface charges density $\sigma_f$ on the upper and down surfaces; $\sigma_f = \pm E_a^{in}$. Because the field $\bar{E}_a^{in}$ oscillates, the charge density also oscillates. It means that an oscillating electric current is produced [13] such as $\bar{J}_c = \partial_t \sigma_f = \partial_t \bar{E}_a^{in}$. This current flows in the surface to the skin depth. It is the physical reason why the oscillating current $\bar{J}_c$ is generated in the metal. (We have set $\epsilon = 1$ in the argument because $\epsilon \simeq 1$ for the electric field with the frequency $m_a/2\pi \sim 1 \text{GHz}$.)

We make a comment that the electric field $\bar{E}_a^{in} = \frac{\epsilon}{\delta} \partial_t \bar{E}_a^{in}$ vanishes in the limit of infinite conductivity, $\sigma \rightarrow \infty$. Namely, there is no electric field in perfect conductor. It means that the above formulation cannot be applied to the superconductor, although the superconductor is perfect conductor. We have the electric field $\bar{E}_a$ in the surface of the superconductor, as we have shown. We need a model like Ginzburg-Landau model for the superconductor to appropriately describe electromagnetic fields in the superconductor.

The current density $J_c = \partial_t E_a = m_a g a_{\gamma} \alpha_0 B \sin(m_a t)$ flows in the surface of the metal, while there is an additional current $\bar{J}_c^{suf}$ flowing in the surface. It is given such that $\bar{J}_c^{suf} = \sigma \bar{E}_a^{suf} = - g a_{\gamma} \alpha_0 \bar{B} \exp(-\frac{a}{\delta} \cos(m_a t - \frac{x}{\delta}))$, because the electric field $\bar{E}_a^{suf}$ is present in the surface. Obviously, the current density $\bar{J}_c^{suf}$ is much larger than $J_c$. Differently to the superconducting current $\bar{J}_c$, the energy of the current $\bar{J}_c^{suf}$ is dissipated; $\int_0^{2\pi/m_a} dt \bar{J}_c^{suf}(t) \bar{E}_a^{suf}(t) \neq 0$. Because the current oscillates, it generates dipole radiation from the metal.
In this section we find that electric field \( \vec{E}^{\text{suf}} = -g_{a\gamma}a_0\vec{B}\exp(-\frac{x}{\lambda})\cos(m_at - \frac{\tau}{\lambda}) \) is induced in the surface of the metal, which produces the surface current \( \vec{J}^{\text{suf}} = \sigma\vec{E}^{\text{suf}} \). The strength of the electric field \( \vec{E}^{\text{suf}} \) is almost identical to that of the electric field \( \vec{E}_a \) in the superconductor; \( \vec{E}^{\text{suf}}(x = 0) = \vec{E}_a \).

V. RADIATION FLUX FROM CYLINDRICAL CONDUCTORS

We proceed to show how large amount of radiation is emitted from the superconductor as well as the normal conductor. The radiation is generated by the oscillating current \( \vec{J}_e \) (\( \vec{J}^{\text{suf}} \)) in the superconductor (normal conductor) induced by the oscillating electric fields \( \vec{E}_a \) (\( \vec{E}^{\text{suf}} \)). Because the oscillation is harmonic, the radiation is dipole radiation. The current is carried by Cooper pair (electrons) in the superconductor (normal conductor).

According to the Drude model, we give a simple argument for the form of the current density \( \vec{J}_e = q^2E_an_e/m_am_e \) \( (\vec{J}^{\text{suf}} = \sigma\vec{E}^{\text{suf}}) \). The Cooper pair in the superconductor oscillates with the frequency \( m_at/2\pi \) according to the equation of motion, \( m_\epsilon\dot{v}_\epsilon = qE_a\cos(m_at) \); \( v_\epsilon \) denotes velocity. We note that the motion of the Cooper pair is not disturbed by impurities in the superconductor, so there is no dissipative term in the equation of motion. On the other hand, the electrons in the metal obey the equation of motion, \( m_\epsilon\dot{v}_\epsilon = eE_a\cos(m_at) \); \( v_\epsilon \) denotes relaxation time; when \( E_a^{\text{suf}} = 0 \), \( v_\epsilon \propto \exp(-t/\tau) \). Obviously, the second term \( -m_\epsilon\tau^{-1}v_\epsilon \) in the equation describes the dissipation of electron energy. From these equations of motion, we derive the velocity such that \( v_\epsilon = q\int dt'E_a(t')/m_e \approx qE_a/m_am_e \) \( (v_\epsilon = eE_a/\tau/m_e \) for the superconductor (metal).

Then, the current density in the superconductor is given by \( J_e = qn_\epsilon v_\epsilon = q^2E_an_e/m_am_e \) with the number density \( n_e \) of the Cooper pair; \( n_e \approx 10^{27}\text{cm}^{-3} \). The formula is identical to the one obtained in the Ginzburg-Landau model. Similarly, the current density \( J_e \) in the metal is given by \( J_e = en_\epsilon v_\epsilon = e^2n_\epsilon E_a/\tau/m_e \approx e\vec{E}^{\text{suf}} \) with the electric conductivity \( \sigma \equiv e^2n_\epsilon/\tau/m_e \). Both of the currents flow only in the surface of the conductors.

We make a comment that the current density \( J_e \) has a phase different by \( \pi/2 \) with the electric field \( E_a \) \( \propto \sin(m_at) \). It leads to the dissipation less current in the superconductor, as we have shown in the previous section. On the other hand, the current \( J_e = en_\epsilon v_\epsilon = e^2n_\epsilon E_a/\tau/m_e \) has the identical phase to that of the electric field \( E_a^{\text{suf}} \) so that the current in the normal conductor is dissipative.

The current oscillates with the frequency \( m_at/2\pi \) and flows in the direction of the magnetic field. The spectrum of the axions has the peak frequency \( m_a/2\pi \) with small bandwidth \( \Delta\omega \approx 10^{-6} \times m_a \), because of the velocity \( v_\epsilon \sim 10^{-3} \) of the dark matter axion in our galaxy. Thus, the oscillating current has the same spectrum as that of the axion.

We have proposed a method for the conversion of the dark matter axions to electromagnetic waves. We use a superconductor of cylindrical shape, on which the magnetic field \( \vec{B} \) parallel to the direction along the length of the superconductor is imposed. We take the direction as \( z \) direction. The magnetic field is expelled from the superconductor. But the field penetrates into the superconductor to the depth \( \lambda = \sqrt{m_e/q^2n_e} \) (London penetration depth). Thus, the oscillating current is present only in the surface to the depth \( \lambda \). In general, the oscillating current in conductors is only present in the surface with the skin depth \( \delta = \sqrt{2/\mu_0\omega} \). But, the skin depth is larger than the penetration depth, only to which the magnetic field is present. Typically, the penetration depth is \( \lambda \sim 10^{-5}\text{cm} \), while the skin depth is \( \delta \sim 10^{-4}\text{cm} \) for copper with the frequency \( \omega = 1\text{GHz} \). Thus, the oscillating current \( J_e \propto \vec{B} \) in the superconductor is only present to the penetration depth.

Now, we estimate the radiation flux emitted by the cylindrical superconductor under the magnetic field \( B \). We suppose that the superconductor has radius \( R = 1\text{cm} \) and length \( l = 10\text{cm} \). Then, the flux of the dipole radiation is given by

\[
S_c = \frac{m_a^2(2\pi R\lambda J_e)^2}{3} = \frac{m_a^2(q^2E_0n_e)^2(2\pi R\lambda)^2}{3m_e^2m_e^2} = \frac{(k_a\alpha B)^2(2\pi R)^2\rho_a}{3\pi^2m_a^2f_a^2\lambda^2}
\]

with \( E_0 \equiv E_0\cos(m_at) \) \( (E_0 = -k_0\alpha\alpha_0B/f_a\pi = -g_\alpha\gamma\alpha_0B) \), where we have used the formulae of the penetration depth \( \lambda = \sqrt{m_e/q^2n_e} \).

When we use the superconductor \( \text{Nb}_3\text{Sn} \), the penetration depth is about \( \lambda = 8 \times 10^{-6} \text{cm} \). Then, we numerically estimate the flux \( S_c \),

\[
S_c \approx 4.4 \times 10^{-18}\text{W}\left(\frac{8 \times 10^{-6}\text{cm}}{\lambda}\right)^2\left(\frac{B}{5\text{T}}\right)^2\left(\frac{R}{1\text{cm}}\right)^2\left(\frac{l}{10\text{cm}}\right)^2\left(\frac{k_a}{1.0}\right)^2\left(\frac{\rho_a}{0.3\text{GeV/cm}^3}\right),
\]
with $k_a \simeq 0.37$ for DFSZ model and $k_a \simeq -0.96$ for KSVZ model, where there is no dependence on the axion mass. The spectrum of the radiation shows a sharp peak at the frequency $m_a/2\pi$ with the bandwidth $\Delta \omega \sim 10^{-6}m_a/2\pi$. It is remarkable that the radiation flux in eq. (23) is four times of the order of magnitude larger than that obtained in resonant cavity [3, 20].

The flux is obtained by integrating a Poynting vector over the sphere with radius $r \gg 2\pi/m_a$ around the superconductor: $S_c = \int S_c(\theta, r)^2 r^2 d\Omega = \int S_c(\theta, r)^2 r^2 \sin \theta d\theta d\phi$, where

$$S_c(\theta, r) = \frac{m_a^2 (2\pi R l J)^2 (\sin \theta)^2}{8\pi r^2},$$

(24)

where we have taken the polar coordinate.

The dipole radiation is emitted mainly toward the direction ($\theta = \pi/2$) perpendicular to the electric current flowing in $z$ direction. Thus, when we measure the radiation emitted in the direction, we receive relatively strong flux density. For example, when we observe the radiation using the radio telescope of parabolic dish antenna with the diameter 32m by putting the cylindrical superconductor 100m away from the telescope (e.g. Yamaguchi 32-m radio telescope of National Astrophysical Observatory of Japan), the observed flux per frequency $P_c$ is given by

$$P_c = \frac{\int S_c(\theta, r)^2 r^2 d\Omega}{\Delta \omega} = \frac{\int^{\pi/2 - \delta_t}_{\pi/2 + \delta_t} S_c(\theta, r)^2 \sin \theta d\theta d\phi}{\Delta \omega} \simeq \frac{3\pi \delta t^2}{8\Delta \omega} \cdot \frac{m_a^2 (2\pi R l J)^2 (\sin \theta)^2}{8\pi r^2} \cdot \frac{40}{\Delta \omega} \simeq 4 \times 10^{-22} W/Hz,$$

(25)

with $\delta_t \simeq 16m/100m = 0.16$ and $\Delta \omega = 10^3 Hz(m_a/(6 \times 10^{-6} eV))$, where the center of the parabolic antenna is set in the direction $\theta = \pi/2$. Thus, we find that the antenna temperature is approximately $T_a \equiv P\eta \simeq 1.5K$ with the unit $kB = 1$, assuming the antenna efficiency $\eta \simeq 0.6$. Therefore, the radiation can be observed with the radio telescope with diameter such as 32m.

We estimate the detection sensitivity. When we observe the radiation over time $t$ with bandwidth $\delta \omega$, the ratio of signal to noise is given by $S/N = \frac{S}{T_{sys}} \sqrt{t/\delta \omega}$ where $S_c = \frac{3\pi}{8} S_c/8 \sim t^{-2} S_c$ denotes the radiation flux received by the telescope 100m away from the superconductor and $T_{sys}$ is the system noise temperature. For instance, $T_{sys} = 40K$ for Yamaguchi 32m radio telescope. Therefore, we find:

$$\frac{S}{N} \sim 40 \times \frac{1 MHz}{\delta \omega} \sqrt{t \frac{g_{15}}{m_a}} \frac{2 \times 10^{-6} \text{cm}}{\lambda} \frac{5 \times 10^{-6} \text{cm}}{R} \frac{7 \pi}{2 \text{cm}} \frac{l}{20 \text{cm}} \frac{0.3 \text{GeV/cm}^2}{\rho_a}$$

(26)

with $T_{sys} = 40K$, where we have taken the physical parameters $B = 7 T$, $R = 2 cm$ and $l = 20 cm$ to have better detection sensitivity. Here, we put $g_{15} \equiv g_{\gamma \gamma}/(10^{-12} \text{GeV}^{-1})$ and $m_a \equiv m_a/(10^{-6} eV)$. Thus, even for DFSZ axion ($\left(\frac{m_a}{m_{\chi}}\right)^2 \simeq 0.1$), we reach the sensitivity $S/N \sim 4$ when we observe the radiation over 1 second with the bandwidth $\delta \omega = 1\text{MHz}$. In the formula we do not use the relation $m_a f_a \simeq 6 \times 10^{-6} eV \times 10^{12} \text{GeV}$ specific to the QCD axion. The formula holds even for axion-like particle.

Obviously, the larger radiation flux can be obtained when we put the superconductor nearer the radio telescope than 100m. Furthermore, even when we use a radio telescope with smaller radius than 32m, large $S/N$ ratio can be achieved if we put the apparatus near the telescope. The radiation flux is determined by the solid angle of the parabolic dish antenna viewed from the superconductor. The larger solid angle leads to larger radiation flux. The merit of our proposal is that we can simultaneously search wide bandwidth of the radio frequency without tuning the shape of the superconductor. In this way, we can observe the radiation from the dark matter axion.

We make a comment on the actual setup for the observation. The strong magnetic field $B$ parallel to the direction along the length of the cylindrical superconductor is produced by coils surrounding it. The coils should have open space for the dipole radiations to escape outside the coils and reach the telescope. In particular, the open space should be present in the $\theta = \pi/2 \pm \delta$ directions (e.g. $\delta \simeq 16cm/100cm = 0.16$) perpendicular to the cylindrical superconductor. That is, the coils are composed of two parts; one covers the upper side of the superconductor and the other one covers the lower side. Then, there is an open space in the coils through which the radiations can escape from the coils. Furthermore, the whole of the apparatus must be cooled. We need to use a glass container for liquid helium so as for the radiation to pass the container and reach radio telescope.

We make an additional comment on radiation from magnetic vortex. We use type 2 superconductor for the apparatus in order for the strong magnetic field not to break the superconductivity. Then, the magnetic vortices are formed inside the superconductor and the electric field $E_a$ is induced around the vortices. But, the radiations from magnetic vortices do not arise. This is owing to the fact that each vortex is surrounded by the superconducting state $\langle \Phi \rangle \neq 0$. The electromagnetic waves do not pass the state.
For comparison, we estimate the radiation flux $S_e$ from the normal conductor with the shape identical to the one in the superconductor.

$$S_e = \frac{m_a^2 (2\pi R \delta J_e)^2}{3} = \frac{m_a^2 (\sigma E_0)^2 (2\pi R \delta)^2}{3} = \frac{4 (k_a \alpha B)^2 (2\pi R)^2 / \rho_a}{3 \pi^3 m_a^2 f_a^2 \delta^2} \approx 2.8 \times 10^{-20} W \left( \frac{2 \times 10^{-4} \text{m}}{\delta} \right)^2 \left( \frac{m_a}{4 \times 10^{-6} \text{eV}} \right)^2 \left( \frac{B}{5 \text{T}} \right)^2 \left( \frac{R}{1 \text{cm}} \right)^2 \left( \frac{\rho_a}{1.0} \right)^2 \left( \frac{0.3 \text{GeV/cm}^2}{\text{c}} \right),$$

(27)

with $E_0 = g_{a\gamma} a_0 a B$ and $\delta = \sqrt{2/m_a \sigma}$, where we put $\mu = 1$ for simplicity.

Differently to the case of the superconductor, the radiation flux depends on the axion mass. This is because the skin depth $\delta$ depends on the frequency of the radiation. The difference between the flux from the superconductor and that from the normal conductor comes from the difference in the depth in which the currents flow. This difference causes the big difference in the flux. That is, the radiation flux $S_e$ emitted from the normal conductor is about a hundred times smaller than $S_e$ from the superconductor because typically $\delta \sim 10^{-4} \text{cm}$ and $\lambda \sim 10^{-5} \text{cm}$.

**VI. SUPERCONDUCTING RESONANT CAVITY**

We would like to briefly show radiation energy generated in superconducting resonant cavity. The cylindrical resonant cavity (tube) has been considered for the detection of the dark matter axion for more than 30 years ago[13]. The cavity is formed of normal conductor, e.g., copper. In this section we consider superconducting resonant cavity instead of the normal conductor. We should note the difference between the superconductor and the normal conductor. The difference is the currents flowing the conductors. That is, the electric currents $J_e = E_a/(m_a \lambda^2)$ flows in the surface in the superconductor and $J_e = 2 E_a/(m_a \delta^2)$ in the superconducting region of the cavity, where we assume $2 \pi/(m_a \delta) \lesssim \omega_a / \Delta \omega \sim 3 \times 10^6$ with $\omega_a = m_a / 2\pi$. $J_0(x)$ denotes a Bessel function of the first kind. Here, the time average is taken: $\frac{2\pi}{\omega_a} \int_{0}^{2\pi} \delta(x) \, dt = \langle Q \rangle$

Then, when we use the superconducting resonant cavity, we have

$$U_e = \frac{16 \pi^2 g_{a\gamma} B^2 V \rho_a}{m_a^2 \lambda^2} \left( \frac{\delta}{\sqrt{2 \lambda}} \right)^2$$

with typical values $\delta \sim 10^{-4} \text{cm}$ and $\lambda \sim 10^{-5} \text{cm}$. Therefore, a hundred times larger amount of the radiation energy is generated when we use the superconducting resonant cavity. Contrary to the radiation energy inside the cavity, the flux absorbed in the superconducting cavity vanishes because the time average of the Poynting vector vanishes at the surface $\rho = R_c$ of the cavity: $\int_{0}^{2\pi} \omega_a \, dt \, \delta E \times \delta B = 0$. This is owing to the fact that the superconducting current is dissipationless. To find the radiation flux absorbed in the cavity we need to take into account the effect of electrons remaining in the superconductor without forming Cooper pairs at nonzero temperature. The electrons absorb the radiation.
VII. SUMMARY AND DISCUSSION

We have shown that electric field induced by axion in superconductor is essentially identical to the one in vacuum. The electric field is proportional to magnetic field in the superconductor. The electric field is only present in the surface of the superconductor because of the Meissner effect; magnetic field is expelled from the superconductor. The result is obtained by analyzing equations of electromagnetic fields coupled with the axion and Cooper pairs. The Cooper pairs are described by a Ginzburg Landau model.

On the other hand, although the magnetic field is present inside normal conductor, the electric field induced by axion is absent in normal conductor. It is only present in the surface of the conductor because of skin effect of the oscillating electric field. The strength of the electric field is almost equal to the one in vacuum. The result is obtained by analyzing equations of electromagnetic fields in the metal coupled with the axion.

These electric fields oscillate with the frequency given by the axion mass so that Cooper pairs (electrons) in the superconductor (normal conductor) are forced to oscillate and emit radiations with the frequency. We have estimated the radiation fluxes from the cylindrical conductors. In particular, the flux from the superconductor is sufficiently large to be observed by existing radio telescopes. For instance, even when the superconductor with radius 1cm and length 10cm is put 100m away from the radio telescope of the parabolic dish antenna with the diameter 32m, the flux received by the telescope is four times of the order of the magnitude larger than that in the resonant cavity recently used in ADMX.

The flux from the superconductor or metal is inversely proportional to the square of the penetration depth $\lambda$ in the superconductor or the skin depth $\delta$ in the normal conductor, respectively. In general, the penetration depth $\sim 10^{-5}$ cm is shorter than the skin depth $\sim 10^{-4}$ cm for frequency $\sim 1$GHz. This difference results in the difference of each flux.

In this paper we have mainly considered the QCD axion whose mass is expected in the range from $10^{-6}$eV $\sim 10^{-4}$eV. This expectation comes from the previous our paper[21], in which we have predicted the axion mass $\sim 7 \times 10^{-6}$eV. The prediction comes from the analysis of the spectrum of fast radio bursts (FRBs)[22, 23]. The FRBs are radio bursts with typical frequency 1GHz, flux $\sim 10^{40}$erg/s and duration $\sim 1$ms. It is still a mysterious phenomena in astrophysics. Our model[24] for the FRBs is that the fast radio bursts arise from the collision between axion star[25] and magnetized dense electron gases such as neutron star or geometrically thin accretion disk around black hole with larger mass than $\sim 10^{3} M_{\odot}$. The axion star is gravitationally bound state of axions, which is more dense than the dark matter axion $\rho_a$ under the consideration. The emission mechanism of radiations from the astrophysical objects is identical to the one discussed in the present paper. That is, the strongly magnetized electron gas emit radiations when they collide the dense axions. For this reason, our main interest in the axion mass is in the range mentioned above.

Obviously, our method for the detection of the dark matter axion can be applicable for much wide mass range beyond the range $10^{-6}$eV $\sim 10^{-4}$eV. We need sensitive receiver for the capture of the radiations from the cylindrical superconductor. The receiver should have surface area with the large solid angle as possible viewed from the superconductor. We also need to fabricate appropriate magnet, and glass container for liquid helium, in order for the radiations to pass through the magnet and the container and to reach the receiver. Then, we can search the wide range of the axion mass with high sensitivity.

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