deSitter brane universe induced by phantom and quantum effects

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ABSTRACT

Five-dimensional braneworld cosmology with deSitter (inflationary) brane universe induced by classical and quantum matter is discussed. It is shown that negative energy phantom field with quantum CFT supports the creation of deSitter universe. On the same time, pure phantom or dust with quantum effects, or Chaplygin gas with quantum effects may naturally lead to the occurrence of Anti-deSitter brane universe but not deSitter one. It is also interesting that unlike to four-dimensional gravity, for phantom with (or without) quantum contribution the standard cosmological energy conditions may be effectively satisfied.

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1. The interpretation of recent astrophysical data [1] indicate to the acceleration of the scale factor of the observable universe. It looks quite possible now that not only the early universe passed the inflationary stage, but also observable universe is (or enters) in deSitter phase. The origin of deSitter universe could be caused by the dark energy. There are recent speculations[2, 3] considering negative energy (phantom) theories which produce the accelerating scale factor. Unfortunately, such phantom theories break the standard cosmological energy conditions. However, the combination of phantom matter with quantum effects may improve the situation with energy conditions (see, for instance, [4]).

From another side, the dark energy naturally appears in the braneworld cosmology (see [5] and references therein). It is sometimes easier to achieve brane FRW-cosmology with accelerating scale factor even without brane matter. Then it is quite interesting to consider braneworld cosmology with phantom brane matter and quantum brane CFT in order to check the possibility for deSitter universe occurence. The present Letter is devoted to the study of above question. It is shown that 4d brane deSitter occurs for combination of brane matter as phantom and quantum CFT or phantom, quantum CFT and dust. Moreover, in such a case NEC and WEC may be effectively satisfied. For pure phantom brane matter (unlike to four-dimensional case[3, 4]) there is no deSitter solution (only Anti-deSitter one) but all energy conditions are satisfied. For combination of matter as Chaplygin gas and quantum CFT there occurs only brane Anti-deSitter universe.

2. Let the 3-brane is embedded into the 5d bulk space as in ref.[6]. Let $g_{\mu\nu}$ be the metric tensor of the bulk space and $n_\mu$ be the unit vector normal to the 3-brane. Then the metric $q_{\mu\nu}$ induced on the brane has the following form:

$$q_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu .$$

The initial gravitational action is

$$S = \int d^5 x \sqrt{-g} \left\{ \frac{1}{\kappa_5^2} R^{(5)} - 2\Lambda + \cdots \right\} + \int \sqrt{-q} (-2\lambda + \text{matter Lagrangian density}) .$$

Here $\Lambda$ is the bulk cosmological constant, $\lambda$ is the tension of the brane. In the following, the 5d quantities are denoted by the suffix $^{(5)}$ and 4d ones by $^{(4)}$. 

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In (2), \( \cdots \) expresses the matter contribution. The bulk Einstein equation is given by

\[
\frac{1}{\kappa_5^2} \left( R^{(5)}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{(5)} \right) = T_{\mu\nu}
\]  

(3)

If one chooses the metric near the brane as:

\[
ds^2 = d\chi^2 + q_{\mu\nu} dx^\mu dx^\nu,
\]

(4)

the energy-momentum tensor \( T_{\mu\nu} \) has the following form:

\[
T_{\mu\nu} = T^{\text{bulk matter}}_{\mu\nu} - \Lambda g_{\mu\nu} + \delta(\chi) (\lambda q_{\mu\nu} + \tau_{\mu\nu})
\]

(5)

Here \( T^{\text{bulk matter}}_{\mu\nu} \) is the energy-momentum tensor of the bulk matter and \( \tau_{\mu\nu} \) expresses the contribution due to brane matter. Without the bulk matter \((T^{\text{bulk matter}}_{\mu\nu} = 0)\), following the procedure in [6, 7], the bulk Einstein equation can be mapped into the equation on the brane:

\[
\frac{1}{\kappa_5^2} \left( R^{(4)}_{\mu\nu} - \frac{1}{2} q_{\mu\nu} R^{(4)} \right) = -\frac{1}{2} \left( \Lambda + \frac{\kappa_5^2 \Lambda^2}{6} \right) q_{\mu\nu} + \frac{\kappa_5^2 \Lambda}{6} \tau_{\mu\nu} + \kappa_5^2 \pi_{\mu\nu} - \frac{1}{\kappa_5^2} E_{\mu\nu}.
\]

(6)

Here \( \pi_{\mu\nu} \) is given by

\[
\pi_{\mu\nu} = -\frac{1}{4} \tau_{\mu\alpha} \tau^\alpha_{\nu} + \frac{1}{12} \tau^2 \tau_{\mu\nu} + \frac{1}{8} q_{\mu\nu} \tau_{\alpha\beta} \tau^\alpha \tau^\beta - \frac{1}{24} q_{\mu\nu} \tau^2.
\]

(7)

On the other hand, \( E_{\mu\nu} \) is defined by the bulk Weyl tensor \( C^{(5)}_{\mu\nu\rho\sigma} \):

\[
E_{\mu\nu} = C^{(5)}_{\alpha\beta\gamma\delta} n^\alpha n^\beta q^\gamma_{\mu} q^\delta_{\nu}.
\]

(8)

Note that one may identify the effective 4d gravitational constant \( \kappa_4 \) and 4d cosmological constant \( \Lambda_4 \) with

\[
\frac{1}{\kappa_4^2} = \frac{6}{\lambda \kappa_5^4}, \quad \Lambda_4 = \frac{\kappa_5^2 \Lambda}{2} \left( \Lambda + \frac{\kappa_5^2 \Lambda^2}{6} \right).
\]

(9)

As one of the matter fields on the brane, we may consider the phantom field \( C \), whose energy-momentum tensor has the following form:

\[
\tau_{C_{\mu\nu}} = \partial_\mu C \partial_\nu C - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \partial_\alpha C \partial_\beta C.
\]

(10)
This scalar field has negative energy density and its solution in 4d deSitter space is [3]

\[ C = at + b , \]  

(11)

where a,b are some arbitrary constants. With above phantom the negative energy density \( \rho_C \) and the negative pressure \( p_C \) is given by

\[ \rho_C = p_C = -\frac{a^2}{2} \]  

(12)

It was argued in [4] that phantom behaves as some effective QFT in deSitter space.

The space is assumed to be 5d AdS space, where

\[ R^{(5)}_{\mu\nu\rho\sigma} = -\frac{1}{l^2} (g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) . \]  

(13)

Here \( \Lambda = -\frac{6}{\kappa_5^2l^2} \). Then \( C^{(5)}_{\alpha\beta\gamma\delta\nu\mu}n^\alpha n^\beta q^\gamma q^\delta q^\nu = 0 \). We also assume the brane is the 4d deSitter space, whose metric is taken in the following form:

\[ ds^2 = -dt^2 + L^2 \cosh^2 \frac{t}{L} d\Omega_3^2. \]  

(14)

Here \( d\Omega_3^2 \) is the metric of the 3d sphere with unit radius. Then \((t,t)\)-component of (6) has the following form:

\[ 0 = -\frac{3}{\kappa_5^2L^2} - \frac{1}{2} \left( -\frac{6}{\kappa_5^2l^2} + \frac{\kappa_5^2\lambda^2}{6} \right) + \frac{\kappa_5^2\lambda}{6} \left( -\frac{a^2}{2} + \rho_m \right) \]

\[ + \kappa_5^2 \left( \frac{a^4}{48} - \frac{a^2 \rho_m}{12} + \frac{\rho_m^2}{12} \right) , \]  

(15)

and \((i,j)\)-component corresponding to the 3d sphere part is:

\[ 0 = \frac{3}{\kappa_5^2L^2} - \frac{1}{2} \left( -\frac{6}{\kappa_5^2l^2} + \frac{\kappa_5^2\lambda^2}{6} \right) + \frac{\kappa_5^2\lambda}{6} \left( -\frac{a^2}{2} + p_m \right) \]

\[ + \kappa_5^2 \left( \frac{a^4}{16} - \frac{a^2 \rho_m}{6} - \frac{a^2 p_m}{12} + \frac{\rho_m^2}{12} + \frac{\rho_m p_m}{6} \right) . \]  

(16)
Here $\rho_m$ and $p_m$ express the (classical or quantum) contribution from the matter on the brane besides the phantom contribution. The bulk cosmological constant $\Lambda$ is

$$\Lambda = -\frac{6}{\kappa_5^2 l^2}.$$  \hspace{1cm} (17)

By combining (15) and (17), one gets

$$0 = \frac{\kappa_5^2 \lambda}{6} (-a^2 + \rho_m + p_m) + \kappa_5^2 \left( \frac{a^4}{12} - \frac{a^2 \rho_m}{4} - \frac{a^2 p_m}{12} + \frac{\rho_m^2}{6} + \frac{\rho_m p_m}{6} \right),$$  \hspace{1cm} (18)

$$0 = -\frac{6}{\kappa_5^2 L^2} + \left( -\frac{6}{\kappa_5^2 l^2} + \frac{\kappa_5^2 \lambda^2}{6} \right) + \frac{\kappa_5^2 \lambda}{6} (\rho_m - p_m) + \kappa_5^2 \left( -\frac{a^4}{24} + \frac{a^2 \rho_m}{12} + \frac{a^2 p_m}{12} - \frac{\rho_m^2}{12} - \frac{\rho_m p_m}{6} \right).$$  \hspace{1cm} (19)

First we consider the case that there are no matter fields besides the phantom $C$ on the brane, that is, $\rho_m = p_m = 0$. Then Eq.(18) gives $a^2 = 0$, or $a^2 = 2\lambda$. For the latter case, Eq.(19) gives $0 = \frac{1}{L^2} + \frac{1}{l^2}$. Then there might be anti-deSitter solution where $L^2 = -l^2 < 0$ but there is no inflationary brane solution (compare with [3]).

As a next step, we may include the quantum effects from the conformal brane matter. As usually, the simplest way to do so is to consider the conformal anomaly:

$$\tau^A = b \left( F^{(4)} + \frac{2}{3} \Box R^{(4)} \right) + b' G^{(4)} + b'' \Box R^{(4)},$$  \hspace{1cm} (20)

where $F^{(4)}$ is the square of 4d Weyl tensor, $G^{(4)}$ is Gauss-Bonnet invariant. In general, with $N$ scalar, $N_{1/2}$ spinor, $N_1$ vector fields, $N_2$ (=$0$ or $1$) gravitons and $N_{\text{HD}}$ higher derivative conformal scalars, $b$, $b'$ and $b''$ are given by

$$b = \frac{N + 6 N_{1/2} + 12 N_1 + 611 N_2 - 8 N_{\text{HD}}}{120(4\pi)^2},$$

$$b' = -\frac{N + 11 N_{1/2} + 62 N_1 + 1411 N_2 - 28 N_{\text{HD}}}{360(4\pi)^2}, \hspace{1cm} b'' = 0.$$  \hspace{1cm} (21)

Note that $b'$ is negative for the usual matter ($N_{\text{HD}} = 0$).
For inflationary brane (14), one gets
\[ \rho_A = -p_A = -\frac{6b'}{L^4}. \] (22)

Then Eqs.(18) and (19) are:
\[ \begin{align*}
0 &= -\frac{k_5^2 a^2 \lambda}{6} + \frac{k_5^2}{24} \left( \frac{a^4}{12} + b' a^2 \right), \quad (23) \\
0 &= -\frac{6}{k_5^2 L^2} + \left( -\frac{6}{k_5^2 L^2} + \frac{k_5^2 \lambda^2}{6} \right) + \frac{2k_5^2 \lambda b'}{L^4} \\
&\quad + \frac{k_5^2}{24} \left( \frac{a^4}{24} + \frac{3b'^2}{L^8} \right). \quad (24)
\end{align*} \]

Especially when there is no phantom field \((a = 0)\) and \(\lambda = \frac{6}{k_5^2}\), Eq.(23) is trivial and Eq.(24) has the following form:
\[ \left( \frac{1}{l} - \frac{k_5^2 b'}{L^4} \right)^2 = \frac{1}{L^2} \left( 1 + \frac{L^2}{l^2} \right) \quad \text{or} \quad \pm \frac{1}{L} \sqrt{1 + \frac{L^2}{l^2} - \frac{1}{l}} = -\frac{k_5^2 b'}{L^4}, \quad (25) \]
which (plus sign) reproduces the result [8](see also, [9, 10]). In other words, we demonstrated that for the particular choice of the boundary terms, our equation describes the quantum creation of deSitter (inflationary) brane which glues two AdS spaces. Such inflationary brane-world scenario is sometimes called Brane New World [10].

When \(a^2 \neq 0\), Eq.(23) gives
\[ a^2 = -\frac{12b'}{L^4} + 2\lambda. \] (26)

Since usually \(b' < 0\), if \(\lambda > 0\), it does not conflict with \(\lambda = \frac{6}{k_5^2}\). Substituting (26) into (24), one gets
\[ 0 = -\frac{6}{k_5^2 L^2} - \frac{6}{k_5^2 L^2} \frac{k_5^2 \lambda b'}{L^4} - \frac{3k_5^2 b'^2}{L^8}. \] (27)

Generally Eq.(27) has non-trivial solution when \(\lambda\) is large enough. If we define a new variable \(y\) by \(y = \frac{1}{L^2}\), Eq.(27) can be rewritten as
\[ 0 = \lambda^2 k_5^2 \left\{ -3b'^2 \left( y + \frac{1}{6b'} \right) + \frac{1}{12} \right\} - \frac{6}{k_5^2 L^2} - \frac{6}{k_5^2} \sqrt{\lambda y}. \] (28)
Then in the limit that $\lambda$ is large, one has a deSitter solution
\[
\frac{1}{L^2} = 0, \quad \text{or} \quad \frac{1}{L^2} = \lambda y = -\frac{\lambda}{3b'} > 0.
\]
Thus, we demonstrated that braneworld which contains phantom and quantum CFT on the brane admits deSitter brane solution gluing two AdS bulks. It is interesting that in pure 4d space the same type of matter also leads to occurrence of inflationary universe [4] in easier way.

As a next case, we consider the situation that the matter on the brane is dust
\[
\rho_d = \frac{\alpha}{L^3}, \quad p_d = 0.
\]

If also the phantom field $C$ presents on the brane, Eqs.(18) and (19) are:
\[
0 = -\frac{\kappa_5^2 a^2 \lambda}{6} \left( -a^2 + \frac{\alpha}{L^2} \right) + \kappa_5^2 \left( \frac{a^4}{12} - \frac{\alpha a^2}{L^3} + \frac{a^2}{L^6} \right),
\]
\[
0 = -\frac{6}{\kappa_5^2 L^2} + \left( -\frac{6}{\kappa_5^2 L^2} + \frac{\kappa_5^2 \lambda^2 \alpha}{6L^3} \right) + \kappa_5^2 \left( -\frac{a^4}{24} + \frac{a^2 \alpha}{12L^3} - \frac{\alpha^2}{12L^6} \right).
\]
Eq.(31) can be solved as
\[
\frac{\alpha}{L^3} = \frac{1}{2} \left\{ -\left( \frac{\lambda}{6} - a^2 \right) \pm \sqrt{\frac{2a^4}{3} + \frac{\lambda a^2}{3} + \frac{\lambda^2}{36}} \right\}.
\]
Then $+$-sign in (33) always gives a non-trivial solution if $\lambda > 0$. If $0 < \lambda < \frac{a^2}{2}$, the $-$-sign in (33) is also a solution.

We may consider the combination of the dust and the quantum effect coming from the conformal anomaly (22), where
\[
\rho_m = \frac{\alpha}{L^3} - \frac{6b'}{L^4}, \quad p_m = \frac{6b'}{L^4}.
\]

Without phantom field ($a = 0$), Eqs.(18) and (19) are:
\[
0 = 0 = \frac{\kappa_5^2 \alpha}{6L^3} \left( \lambda + \frac{\alpha}{L^3} - \frac{6b'}{L^4} \right),
\]
\[
0 = -\frac{6}{\kappa_5^2 L^2} + \left( -\frac{6}{\kappa_5^2 L^2} + \frac{\kappa_5^2 \lambda^2}{6} \right) + \frac{\kappa_5^2 \lambda}{6} \left( \frac{\alpha}{L^3} - \frac{12b'}{L^4} \right) - \frac{\kappa_5^2}{12} \left( \frac{\alpha}{L^3} - \frac{6b'}{L^4} \right) \left( \frac{\alpha}{L^3} + \frac{6b'}{L^4} \right).
\]
Then in order that Eq. (35) has non-trivial solution, one gets

$$\frac{6b'}{L^4} = \lambda + \frac{\alpha}{L^3}.$$  

(37)

By substituting (37) into (36), we obtain

$$\frac{6}{\kappa_5^2 L^2} = -\frac{6}{\kappa_5^2 l^2} - \frac{\lambda^2 \kappa_5^2}{12}.$$  

(38)

Since the right-hand-side of (38) is negative, there is no deSitter solution, where $L^2 > 0$ although there might be an anti-deSitter solution, where $L^2 < 0$. Thus, in such situation brane deSitter space does not occur.

3. There are several standard types of the energy conditions which are usually assumed to be fulfilled in cosmology:

- Null Energy Condition (NEC): $\rho + p \geq 0$.
- Weak Energy Condition (WEC): $\rho \geq 0$ and $\rho + p \geq 0$.
- Strong Energy Condition (SEC): $\rho + 3p \geq 0$ and $\rho + p \geq 0$.
- Dominant Energy Condition (DEC): $\rho \geq 0$ and $\rho \pm p \geq 0$.

They are violated in four-dimensional space for pure phantom field. Nevertheless, when phantom field is considered in combination with quantum CFT some of the energy conditions survive in 4d deSitter universe[4]. Let us check what happens with them in our braneworld.

Eqs. (15) and (16) give the effective energy density $\rho_{\text{eff}}$ and the effective pressure $p_{\text{eff}}$

$$\rho_{\text{eff}} = -\frac{a^2}{2} + \rho_m + 6 \left( \frac{a^4}{48} - \frac{\alpha^2 \rho_m}{12} + \frac{\rho_m^2}{12} \right),$$  

(39)

and $(i,j)$-component corresponding to the 3d sphere part has the following form:

$$p_{\text{eff}} = -\frac{a^2}{2} + p_m + 6 \left( \frac{a^4}{16} - \frac{\alpha^2 \rho_m}{6} - \frac{a^2 \rho_m}{12} + \frac{\rho_m^2}{12} + \frac{\rho_m p_m}{6} \right).$$  

(40)
If there is no non-trivial matter besides the phantom field, one obtains

\[
\rho_{\text{eff}} = p_{\text{eff}} = -\frac{a^2}{2} + \frac{a^4}{4\lambda} = \frac{a^2 (a^2 - 2\lambda)}{4}.
\]

(41)

Then if \(a^2 > 2\lambda\), all the energy conditions are effectively satisfied even for pure phantom field. This is an interesting feature of phantom braneworld, as phantom leads to breaking of all energy conditions in usual 4d deSitter space[3]. On the other hand, without the phantom field

\[
\rho_{\text{eff}} = \rho_m \left(1 + \frac{\rho_m}{2\lambda}\right), \quad p_{\text{eff}} = p_m + \frac{1}{\lambda} \left(\frac{\rho_m^2}{2} + \rho_m p_m\right).
\]

(42)

Then if \(0 > \lambda > -\frac{b^2}{2}, \rho_{\text{eff}} < 0\) even if \(\rho_m = 0\). Therefore WEC and DEC are effectively violated. Furthermore since

\[
\rho_{\text{eff}} + p_{\text{eff}} = (\rho_m + p_m) \left(1 + \frac{\rho_m}{\lambda}\right),
\]

(43)

if \(0 > \lambda > -\rho_m\), which is the condition weaker than (43), all the energy conditions are effectively violated even if \(\rho_m + p_m \geq 0\) and \(\rho > 0\). This is known property of deSitter braneworld. If we consider the contribution from phantom field and the conformal anomaly, Eqs.(39) and (40) are

\[
\rho_{\text{eff}} = \left(-\frac{a^2}{2} - \frac{6b'}{L^4}\right) \left\{ 1 + \frac{1}{2\lambda} \left(\frac{a^2}{2} - \frac{6b'}{L^4}\right) \right\},
\]

\[
p_{\text{eff}} = -\frac{a^2}{2} + \frac{6b'}{L^4} + \frac{6}{\lambda} \left(\frac{a^4}{16} + \frac{b'a^2}{2L^4} - 3b'^2\right).
\]

(44)

Then

\[
\rho_{\text{eff}} + p_{\text{eff}} = a^2 \left\{ -1 + \frac{1}{2\lambda} \left(a^2 + \frac{12b'}{L^4}\right) \right\}
\]

(45)

When \(\lambda > 0\), if \(a^2 < -\frac{12b'}{L^4}\) or \(a^2 > -\frac{12b'}{L^4} + 4\lambda\), we have \(\rho_{\text{eff}} > 0\) and if \(a^2 > -\frac{12b'}{L^4} - 2\lambda\), we have \(\rho_{\text{eff}} + p_{\text{eff}} > 0\). On the other hand, when \(\lambda < 0\), if \(-\frac{12b'}{L^4} > a^2 > -\frac{12b'}{L^4} + 4\lambda\), we have \(\rho_{\text{eff}} > 0\) and if \(a^2 < -\frac{12b'}{L^4} - 2\lambda\), we have \(\rho_{\text{eff}} + p_{\text{eff}} > 0\). Then, for example, NEC is effectively satisfied if

\[
a^2 \geq -\frac{12b'}{L^4} - 2\lambda \quad (\lambda > 0)
\]

\[
a^2 \leq -\frac{12b'}{L^4} - 2\lambda \quad (\lambda < 0)
\]

(46)
and WEC is effectively satisfied if
\[-\frac{12b}{L^2} - 2\lambda \leq a^2 \leq -\frac{12b}{L^2} \text{ or } a^2 \geq -\frac{12b}{L^2} + 4\lambda \quad (\lambda > 0)\]
\[-\frac{12b}{L^2} > a^2 > -\frac{12b}{L^2} + 4\lambda \quad (\lambda < 0). \quad (47)\]

Thus, in the presence of phantom and quantum CFT it is easier to fulfill the standard energy conditions in the braneworld.

4. As a matter, one may consider Chaplygin gas\[11]\], which satisfies
\[p_{Ch} = -\frac{A}{\rho_{Ch}}, \quad (48)\]
where \(A\) is a positive constant. For simplicity, we consider \(a = 0\) case, that is, the case without phantom. Then Eqs.(18) and (19) have the following form:
\[0 = (\rho + \lambda) (\rho^2 - A), \quad (49)\]
\[0 = -\frac{6}{\kappa_5^2 L^2} + \left( -\frac{6}{\kappa_5^2 l^2} + \frac{\kappa_5^2 \lambda^2}{6} \right) + \frac{\kappa_5^2 \lambda}{6} \left( \frac{\rho_{Ch} + A}{\rho_{Ch}} \right) + \kappa_5^2 \left( -\frac{\rho_{Ch}^2}{12} + \frac{A}{6} \right). \quad (50)\]
The solutions of (49) are easily obtained by
\[\rho_{Ch} = \pm \sqrt{A}, \quad \rho_{Ch} = -\lambda. \quad (51)\]
By substituting the former solution \(\rho = \pm \sqrt{A}\) into (50), one obtains
\[\frac{6}{\kappa_5^2 L^2} = \frac{\kappa_5^2}{12} \left( (\sqrt{A} \mp 2\lambda)^2 - 2\lambda^2 - \frac{72}{\kappa_5^2 l^2} \right). \quad (52)\]
Therefore in order that \(L\) is real, we obtain the following condition
\[\sqrt{A} < \pm 2\lambda - \sqrt{2\lambda^2 + \frac{72}{\kappa_5^4 l^2}} \text{ or } \sqrt{A} > \pm 2\lambda + \sqrt{2\lambda^2 + \frac{72}{\kappa_5^4 l^2}}. \quad (53)\]
On the other hand, choosing the latter solution \(\rho = -\lambda\) in (51) and substituting it into (50), one gets
\[\frac{6}{\kappa_5^2 L^2} = -\frac{6}{\kappa_5^2 l^2} - \frac{\kappa_5^2 \lambda^2}{12}. \quad (54)\]
Since the right-hand-side of (54) is negative, there is no real solution for $L$.

We may add the contribution due to the conformal anomaly (22) to (48):

$$\rho_m = \rho_{Ch} - \frac{6b'}{L^4}, \quad p_m = \frac{A}{\rho_{Ch}} + \frac{6b'}{L^4}. \quad (55)$$

Then instead of (49) and (50), one obtains

$$0 = \left( \rho + \lambda - \frac{6b'}{L^4} \right) \left( \rho^2 - A \right), \quad (56)$$

$$0 = -\frac{6}{\kappa^2 L^2} + \left( -\frac{6}{\kappa^2 l^2} + \frac{\kappa^2 \lambda^2}{6} \right) + \frac{\kappa^2 \lambda}{6} \left( \rho_{Ch} + \frac{A}{\rho_{Ch}} - \frac{12b'}{L^4} \right)$$

$$-\frac{\kappa^2}{12} \left( \rho_{Ch} - \frac{6b'}{L^4} \right) \left( \rho_{Ch} - \frac{2A}{\rho_{Ch}} + \frac{6b'}{L^4} \right). \quad (57)$$

The solution of (56) is:

$$\rho_{Ch} = \pm \sqrt{A}, \quad \rho_{Ch} = -\lambda + \frac{6b'}{L^4}. \quad (58)$$

Substituting the former solution $\rho = \pm \sqrt{A}$ into (50), we get

$$\frac{6}{\kappa^2 L^2} = \frac{\kappa^2}{12} \left\{ \left( \sqrt{A} \pm \left( 2\lambda - \frac{6b'}{L^4} \right) \right)^2 - 2\lambda^2 - \frac{72}{\kappa^2 l^2} \right\}. \quad (59)$$

It is difficult to solve (59) with respect to $L^2$ but in order to get real $L$, the following condition should be satisfied:

$$\sqrt{A} < \pm \left( 2\lambda - \frac{6b'}{L^4} \right) - \sqrt{2\lambda^2 + \frac{72}{\kappa^2 l^2}}$$

$$\text{or} \quad \sqrt{A} > \pm \left( 2\lambda - \frac{6b'}{L^4} \right) + \sqrt{2\lambda^2 + \frac{72}{\kappa^2 l^2}}. \quad (60)$$

Substituting the latter solution $\rho = -\lambda + \frac{6b'}{L^4}$ (58) into (57), one gets

$$\frac{6}{\kappa^2 L^2} = -\frac{6}{\kappa^2 l^2} - \frac{\kappa^2 \lambda^2}{12}. \quad (61)$$
Since the right-hand-side of (61) is negative, there is no real solution for $L$. Then there might be an anti-deSitter solution, where $L^2 < 0$ but there is no deSitter solution, where $L^2 > 0$.

To conclude, the combination of Chaplygin gas with quantum CFT (unlike to phantom with quantum CFT) does not support the occurrence of deSitter brane in AdS bulk. On the same time, the combination of brane quantum CFT and phantom may lead to the inducing of 4d deSitter universe. It would be interesting to develop such inflationary braneworld scenario in more detail.

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**References**

[1] S. Perlmutter et al., *Astrophys.J.* **517** (1999) 565; A. Riess et al., *Astron.J.* **116** (1998), 1009.

[2] R.R. Caldwell, *Phys.Lett.* **B545** (2002) 23, astro-ph/9908168; S. Hannestad and E. Mortsell, *Phys.Rev.* **D66** (2002) 063508, astro-ph/0205096; B. Boisseau, G. Esposito-Farese, D. Polarski and A.A. Starobinsky, *Phys.Rev.Lett.* **85** (2000) 2236, gr-qc/0001066; V. Faraoni, *Int.J.Mod.Phys.* **D11** (2002) 471, astro-ph/0110067; V.K. Onemli and R.P. Woodard, *Class.Quant.Grav.* **19** (2002) 4607, gr-qc/0204065; I. Maor, R. Brustein, J. McMahon and P.J. Steinhardt, *Phys.Rev.* **D65** (2002) 123003, astro-ph/0112526; S. M. Carroll, M. Hoffman and M. Trodden, astro-ph/0301273; A. Feinstein and S. Jhingan, hep-th/0304069.

[3] G. Gibbons, hep-th/0302199.

[4] S. Nojiri and S. D. Odintsov, hep-th/0303117.

[5] N.J. Kim, H.W. Lee and Y.S. Myung, *Phys.Lett.* **B504** (2001) 323; V. Sahni, Y. Shtanov, astro-ph/0202346;
P. Singh, R.C. Vishwakarma and N. Dadhich, hep-th/0206193. J. Yokoyama and Y. Himemoto, Phys.Rev. D64 (2001) 083511, hep-ph/0103115; T. Shiromizu, T. Torii and D. Ida, JHEP 0203 (2002) 007, hep-th/0105256; B. Chen and F.-L. Lin, Phys.Rev. D65 (2002) 044007, hep-th/0106054; S.I. Vacaru and D. Gontsa, hep-th/0109114; L. Anchordoqui et al, Phys.Rev. D64 (2001) 084027; J.P. Gregory and A. Padilla, Class.Quant.Grav. 19 (2002) 4071, hep-th/0204218; R. Neves and C. Vaz, Phys.Rev. D66 (2002) 124002, hep-th/0207173. S. Seahra, H.R. Sepangi and J. Ponce de Leon, gr-qc/0303095.

[6] T. Shiromizu, K.-I, Maeda, M. Sasaki, Phys.Rev. D62 (2000) 024012, gr-qc/9910076; Phys.Rev. D62 (2000) 024008, hep-th/9912233.

[7] S. Kanno and J. Soda, Phys.Rev. D66 (2002) 043526, hep-th/0205188; hep-th/0303203.

[8] S. Nojiri S and S.D. Odintsov, Phys.Lett. B484 (2000) 119, hep-th/0004097; Grav.& Cosm. 8 (2002) 73, hep-th/0105160; hep-th/0303011.

[9] S. Nojiri S, S.D. Odintsov and S. Zerbini, Phys.Rev. D62 (2000) 064006, hep-th/0001192.

[10] S.W. Hawking, T. Hertog T and H.S. Reall, Phys. Rev. D62 (2000) 043501, hep-th/0003052.

[11] G. Gibbons, Grav.& Cosm. 8 (2002) 2.