Indirect detection of long-lived particles in a rich dark sector with a dark Higgs boson portal

Krzysztof Jodłowski,\textsuperscript{a} Leszek Roszkowski,\textsuperscript{b,a} Sebastian Trojanowski\textsuperscript{b,a}

\textsuperscript{a}National Centre for Nuclear Research, Pasteura 7, 02-093 Warsaw, Poland
\textsuperscript{b}Astrocent, Nicolaus Copernicus Astronomical Center Polish Academy of Sciences, ul. Rektorska 4, 00-614, Warsaw, Poland
E-mail: krzysztof.jodlowski@ncbj.gov.pl, leszek.roszkowski@ncbj.gov.pl, strojanowski@camk.edu.pl

ABSTRACT: Simplified models of light new physics provide a convenient benchmark for experimental searches for new physics signatures, including dark matter. However, in less simplified – and more realistic – scenarios invoking additional degrees of freedom additional modes of detection may arise. In this study, we examine a model in which the dark sector couples to the Standard Model sector via a light dark Higgs boson portal but it also contains a secluded scalar dark matter candidate with the mass around the TeV scale. In this model involving both light and heavy particles in the dark sector we find some new interesting phenomenological features that allow one to avoid otherwise stringent cosmological bounds and lead to new complementary probes in intensity frontier searches for light long-lived particles, in indirect detection searches for dark matter and cosmic microwave background surveys. We also highlight possible non-local effects present in the indirect detection searches for dark matter that could significantly affect usual detection strategies and allow one to distinguish this model from the usual signatures based on simplified models.
1 Introduction

The nature of the particle content of the dark matter (DM) sector of the Universe remains unknown. The dominant paradigm of the last several decades has been based on the assumption that DM is made up of some weakly interacting massive particles (WIMPs) in the mass range of a few GeV to a few TeV. WIMP DM could naturally be produced in the early Universe via the popular thermal freeze-out mechanism, yielding a correct relic density for WIMP interactions set by a fraction of electroweak interactions of the Standard Model (SM); cf., e.g., Ref [1–3] for a review. However, lack of experimental signal in a wide array of worldwide searches has led one to explore alternative candidates for DM, its production mechanisms, as well as detection methods; see e.g., Refs [5, 6].

For instance, the standard freeze-out mechanism can be generalized to a broad ranges of DM mass and coupling constants [7], see also a discussion in Ref. [8]. While too large values of DM couplings are constrained by unitarity, lowering them down by even several orders of magnitude below the electroweak strength can still render the correct relic density, however, for DM mass much below the GeV scale. In various alternative scenarios some
light mediator particle(s) beyond the Standard Model (BSM) spectrum are present. Their role is to specify the interaction between the SM and some DM particle with the mass in the GeV range \([9, 10]\), or below. Experimental searches for such light DM (LDM) via direct detection (DD) invoke new strategies \([3, 5]\), while further signatures of associated frameworks may be possible with searches for unstable light mediator species; cf. Refs \([11–13]\) for recent reviews.

Such studies are usually performed within a framework of simplified models where the DM particle is assumed to be part of some secluded (dark) sector that interacts with the SM sector via one mediator particle only, for instance a dark photon or a dark Higgs boson; cf. Refs \([14–16]\) for recent reviews. It is, however, understood that in more UV-complete models further experimental probes could be possible and could provide other, complementary ways of their experimental tests, which triggers interest in considering non-minimal scenarios.

In models with a richer dark sector, there are typically more BSM particles with masses spanning wide ranges of values, even up to several orders of magnitude. If such a rich dark sector remains secluded from the SM sector, with only a weak portal communicating between them, the dark couplings of the BSM species are only mildly constrained and can lead to new phenomenological effects. This is especially relevant for cosmological tests and indirect detection (ID) searches of DM that can probe scatterings in the dark sector with the visible signals generated via subsequent decays of BSM species. Notable examples of such effects include, e.g., secluded \([10, 17]\) or Sommerfeld-enhanced \([18]\) DM annihilations into the dark sector particles that modify ID bounds and can be constrained by Cosmic Microwave Background (CMB) radiation measurements.\(^2\)

In this study, we focus on such an interplay between light and heavy dark species in models that can simultaneously be probed by experiments targeting light and long-lived particles (LLPs). As we will show, already in a simple extension of the most minimal scenarios in which we add a heavy DM candidate and much lighter secluded dark species one can find new complementary detection prospects in future CMB surveys and DM ID searches, in addition to standard searches. We will highlight possible unique non-local effects in DM ID that could appear in the models with very long-lived light mediators; cf. Refs \([20–24]\). In this case, the interplay between different channels of DM ID observations could be challenging to interpret in standard WIMP scenarios, while it could be indicative of the existence of a dark sector comprising of both heavy WIMP-like DM particles and much lighter and long-lived species.

We will focus on a popular dark Higgs boson portal, which is often considered as a convenient mediator between the SM and even very complicated BSM sectors \([25–29]\). In particular, such a portal can naturally be related to the hidden valley scenarios predicting the existence of some LLPs \([30]\). Motivated by this, we will assume that the relevant new scalar species – in our case it will be SM singlet mixing with the SM Higgs – has a mass below 1 GeV and suppressed couplings to the SM sector. The remaining non-minimal BSM

\(^2\)Collider probes of light new physics can also be modified in the presence of large dark coupling constants via, e.g., possible secondary production of new BSM species in front of or inside the detectors \([19]\).
content of our model will further contain particles at a wide mass scales between $\sim 1 \text{ MeV}$ and $10 \text{ TeV}$, or so, including a heavy secluded DM candidate that could be targeted in future ID observations.

This paper is organized as follows. In section 2, we introduce the BSM model of our interest. In section 3 we examine the relic abundance of both stable and long-lived dark species present in this scenario. In section 4 we discuss past and future possible bounds on this model. Finally, we present our results in section 5 and present our conclusions in section 6. Technical details of our analysis, including, i.a., involved cross section and decay width formulae, as well as the discussion of the photon spectrum in DM ID are relegated to appendices A and B.

2 Model

The BSM model that we study in this article contains a fairly rich dark sector that is coupled to the visible (SM) sector via the mixing between a light, sub-GeV dark Higgs boson $h_D$ and its heavier SM counterpart $H$. It also includes a dark photon $A'$, a gauge boson of the secluded $U(1)'$ group. The dark Higgs boson is charged under $U(1)'$ and the dark vector obtains its mass via the dark Higgs mechanism. Furthermore, we introduce an additional stable complex scalar field $\eta$ of a similar mass, which also carries a non-zero charge under the $U(1)'$ group. The field $\eta$ will constitute some fraction of the DM in the Universe. Notably, this scenario corresponds to one of the prototype models with LDM at the MeV-GeV scale which is discussed in intensity frontier studies, cf., e.g., Refs [31–33] for recent analyses. In fig. 1, we schematically present the connection between the SM and the dark sectors in the model. The fields charged under the dark gauge group $U(1)'$ are shown in green.

In this study, our aim is to identify possible new features that may arise when this simple scenario is extended to include some additional heavy and light secluded dark species. In particular, we introduce a new, heavy and stable complex scalar field $\chi$ that will play the role of a dominant component of DM. It couples to $\eta$ (a minor DM component) via a heavy spectator real scalar field $\phi$, which plays only an auxiliary role in our model and a direct renormalizable contact operator, $|\chi|^2|\eta|^2$, could also be used instead. Finally, the full dark sector will likely contain some additional light degrees of freedom that are not essential for our analysis. We assume that they are charged under some $U(1)''$ group, which adds to the spectrum of particles a light dark gauge boson $A''$. As we shall see later, its presence will be needed for phenomenological reasons. We further assume that the dark Higgs field $h_D$ is not charged under $U(1)''$.

The Lagrangian of the model can be written as

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DS}} + \mathcal{L}_{\text{portal}}$$

where $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian, $\mathcal{L}_{\text{DS}}$ corresponds to the dark sector, and $\mathcal{L}_{\text{portal}}$ describes interactions between the SM and the dark sector, as well as between the two parts of the dark sector charged under the gauge groups $U(1)'$ and $U(1)''$.

$$\mathcal{L}_{\text{portal}} = -\lambda_{h_D} |\Phi|^2 |\phi|^2 + \frac{\epsilon'}{2} F_{\mu\nu}' F'^{\mu\nu} - \frac{\epsilon''}{2} F_{\mu\nu}'' F''^{\mu\nu} - \frac{\tilde{\epsilon}'}{2} F_{\mu\nu}' (F'^{\mu\nu})''.$$  (2.2)
Figure 1: Schematic illustration of the model. The dark sector (DS) is connected to the Standard Model (SM) through a light dark Higgs boson portal \((h_D)\), which mixes with the Higgs particle. We denote the mixing angle by \(\theta_{hDH}\). The core of the model under study consists of the SM gauge groups extended by a single \(U(1)'\) group. The respective gauge boson is denoted by \(A'\). Both this vector field, as well as the dark Higgs field \(h_D\) and complex scalar dark matter \(\eta\) charged under this group are shown in green. In the study, we focus on the impact on this scenario from its possible extensions towards both heavy and light degrees of freedom. These can contain many fields, while in the figure we have focused on the ones with a direct impact on our discussion, i.e., the heavy complex scalar dark matter \(\chi\) and the auxiliary scalar field \((\phi)\), indicated with the black color, and the light new dark gauge boson \(A''\) of the additional \(U(1)''\) group shown in blue. The \(\eta\) field is a minor DM component, which also couples to the heavier dark sector species, \(\chi\) and \(\phi\). Instead, the interactions between the fields charged under the \(U(1)'\) and \(U(1)''\) groups arise due to kinetic mixing. The presence of additional light and heavy dark sector components allows for avoiding stringent cosmological bounds and leads to non-standard phenomenological effects in DM indirect detection discussed in the draft.

In the following, we assume that both dark vectors have vanishing kinetic mixing with the SM photon, \(\epsilon' = \epsilon'' = 0\). The motivation behind it is that our goal is to primarily focus our study on identifying possible new phenomenological aspects of the model – with respect to usual dark photon signatures – that might appear in the presence of extremely long-lived dark vectors. Importantly, in our model, there is also no perturbative generation of the kinetic mixing terms with the SM photon, as will be discussed later. The vanishing kinetic mixing parameters also result in effective decoupling of the dark vectors from the SM \(Z\) boson, cf., e.g., Ref. [34]. On the other hand, we allow for a non-zero kinetic mixing between the two dark vectors. This can be loop-induced by additional heavy fields charged under both the dark \(U(1)'\) groups, which leads to \(\epsilon' < 1\).

In eq. (2.2) \(\Phi\) denotes the SM Higgs doublet while \(\sigma\) corresponds to the dark Higgs boson singlet. We parameterize both fields in the unitary gauge after the spontaneous symmetry breaking of both the electroweak and the dark gauge symmetries as follows

\[
\Phi = \begin{pmatrix} 0, (v_h + h) / \sqrt{2} \end{pmatrix}^T, \quad \sigma = (v_D + H_D) / \sqrt{2}
\]  

(2.3)

where the SM Higgs vacuum expectation value is \(v_h = 246\) GeV, while the corresponding quantity for the dark Higgs field is denoted by \(v_D\). The portal term \(\lambda_{Hh_D}|\Phi|^2|\sigma|^2\) introduces non-diagonal mixing of \(h\) and \(H_D\); for details see a recent review [35]; which necessitates a change of basis into the mass eigenstate basis. We denote resulting physical eigenvector states by \(H\) and \(h_D\), respectively.

As discussed above, we assume that the lighter DM component \(\eta\) is coupled to both the dark vector \(A'\) and the dark Higgs boson \(h_D\), while the auxiliary heavy scalar field \(\phi\)
provides the connection between both DM components $\chi$ and $\eta$

$$
\mathcal{L}_{\text{DS}} \supset \mu_\chi |\chi|^2 \phi + \mu_\eta |\eta|^2 \phi + (q_H' g_D)^2 A'_\mu h_D |^2 \\
+ i q_H' g_D A'_\mu [\eta^* (\partial^\mu \eta) - (\partial^\mu \eta^*) \eta] + (q_H' g_D)^2 A'_\mu A'^\mu |\eta|^2.
$$

(2.4)

Here, $q_H'$ denotes the charges of particle $i$ under the dark gauge group $U(1)'$. In further discussion, we set $q_H' = q'_H = 1$. As we will discuss below, invoking two components of DM scalar fields allows one to relax otherwise stringent bounds on heavy WIMP-like scalars coupled to the SM sector via light mediator species, cf. Refs [36, 37].

The dark sector Lagrangian also describes the dark Higgs boson potential. It contains usual mass terms, as well as the cubic and quartic interaction terms between the dark Higgs boson and the lighter scalar $\eta$,

$$
\mathcal{L}_{\text{DS}} \supset \mu_D |\sigma|^2 - \frac{1}{2} \lambda_D |\sigma|^4 \\
+ m_\chi^2 |\chi|^2 + m_\eta^2 |\eta|^2 \\
- \lambda_{h_D} h_D^2 |\eta|^2 - (\mu_{h_D} h_D |\eta|^2 + \text{h.c.} ).
$$

(2.5)

As a result of the spontaneous breaking of the dark $U(1)'$ gauge symmetry, the dark bosons $A'$ and $h_D$ obtain their masses as $m_{A'} = g_D v_D$ and $m_{h_D} = \sqrt{\lambda_D} v_D$. Moreover, the dark Higgs boson mixing with the SM Higgs can be parameterized by some angle $\theta_{h_D} \simeq \lambda_{h_D} v_D v_h / m_H^2$, valid whenever $m_{h_D} \ll m_H$, which will be the case in the following.

In the part of the dark sector charged under the group $U(1)''$, a similar dark Higgs mechanism with additional light scalar $H''_D$ could also be present and generate a non-zero mass for the corresponding gauge boson $m_{A''}$. In the following, we assume for simplicity that $m_{A''} \ll m_{A'}$ and that this part of the dark sector remains secluded from the SM, while the presence of additional light dark species, not indicated in the equations below, would render the $A''$ vector unstable. In the following, we will not focus on this part of the dark sector. However, below we will discuss the direct impact of $A''$ on the phenomenology of the species charged under the $U(1)'$ group.

The most relevant couplings for our discussion appear after a dark vector field redefinition, $A' \rightarrow A' - \tilde{\epsilon} \delta A'$. They are induced by moving to the mass basis and removing the non-canonical kinetic mixing term between the two dark field-strength tensors in eq. (2.2). We stress the presence of an additional multiplicative factor $\delta = (m_{A''} / m_{A'})$ that modifies the field shift with respect to the case where one of the vector fields is massless [38, 39].

In the following, we will denote $\tilde{\epsilon} = \epsilon' \delta$ and assume $\tilde{\epsilon} \sim 10^{-6}$ in our benchmark scenarios. This requires $m_{A''} \gtrsim 10^{-3} m_{A'}$ so that the $A''$ mass lies in the MeV – GeV range.

The aforementioned field redefinition leads to

$$
\mathcal{L}_{\text{DS}} \supset \frac{1}{2} m_{A''} A''^2 - i \tilde{\epsilon} g_D A''_\mu [\eta^* (\partial^\mu \eta) - (\partial^\mu \eta^*) \eta].
$$

(2.6)

As can be seen, the lighter DM scalar $\eta$ acquires an $\tilde{\epsilon}$-suppressed coupling to the light dark vector $A''$. We note that a similar coupling could be obtained for $\tilde{\epsilon}' = 0$ by introducing a tiny (milli)charge of the $\eta$ field with respect to the $U(1)''$ group. In this case, the $A''$ boson could be even much lighter.
The dark sector spans at least several orders of magnitude in mass, as shown on the left side of fig. 2. Even for all the dark charges set to unity, the model is still characterized by a set of 12 free parameters. In order to organize our discussion and better highlight interesting phenomenological prospects of this scenario, we assume below the following mass hierarchy in the BSM sector of the model

\[(m_\phi \gg) \; m_\chi > m_\eta > m_{A'} > m_{h_D} > 2m_f \;
\text{and} \; m_{A''} \sim m_{h_D}. \tag{2.7}\]

With this assumption, we find below that the dominant DM component is the heavier scalar field $\chi$ which decouples earlier from thermal plasma than the lighter species $\eta$. In particular, the decoupling of $\chi$ proceeds via annihilations through an exchange of the intermediate heavy scalar, $\chi\chi \to (\phi^*) \to \eta\eta$. In addition, $2 \to 3$ annihilation processes are possible with an outgoing dark vector emitted from the $\eta$ leg, $\chi\chi \to (\phi^*) \to \eta A'$. The latter process will play a crucial role in our discussion of ID observables. On the right side of fig. 2, we show the respective Feynman diagram along with other key processes leading to indirect signals of DM in our model.

As can be seen, the dark photon produced in the $2 \to 3$ process subsequently decays into dark Higgs boson $h_D$ and the dark vector $A' \to h_D A''$, which is kinematically allowed; cf eq. (2.7). This decay takes place via a radiative process with the $\eta$ particle exchange in
the triangle loop. As a result, the decay width is naturally suppressed and $A'$ may have a very large lifetime of astrophysical relevance,

$$c\tau_{A'} \simeq 1 \text{kpc} \left( \frac{1}{g_D} \right)^2 \left( \frac{10^{-6}}{\epsilon} \right)^2 \left( \frac{4 \times 10^{-6}}{\lambda h_D \eta} \right)^2 \left( \frac{m_\eta}{150 \text{GeV}} \right)^4 \left( \frac{10 \text{GeV}}{m_{A'}} \right)^5 . \tag{2.8}$$

The approximate result in eq. (2.8) is derived from a full expression given in eq. (A.2) under the assumption that $m_\eta \gg m_{A'} \gg m_{h_D}$. The surprising dependence of $c\tau_{A'}$ on only two powers of the coupling $g_D$ is due to the fact that, after the spontaneous symmetry breaking in the dark sector, the coupling $g_{\eta h_D}$ between the $\eta$ particle exchanged in the loop and the dark Higgs boson is proportional to the dark vacuum expectation value, $g_{\eta h_D} \propto v_D \propto m_{A'}/g_D$. This partially cancels the dependence on $g_D$ while introducing additional powers of $m_{A'}$ in the expression. We provide a complete list of decay width and annihilation cross sections relevant for our discussion in appendix A.

As can be seen from eq. (2.8), the dark vector can travel galactic-scale distances before decaying and producing visible signals due to subsequent decays of the dark Higgs bosons, $h_D \to f \bar{f}$, where $f \bar{f} = e^+ e^-, \mu^+ \mu^-$, or hadrons obtained from quark pairs $q \bar{q}$. We require $h_D$ to decay before the Big Bang Nucleosynthesis (BBN) in the early Universe, i.e., $\tau_{h_D} \lesssim 0.1 \text{s}$. In fact, the lifetime is often within the reach of the intensity frontier searches for light new physics.

Before we discuss phenomenological aspects of this scenario, it is worthwhile to comment further on the vanishing kinetic mixing terms in eq. (2.2) that are proportional to $\epsilon'$ and $\epsilon''$. We note that, in general, even if there is no such a kinetic mixing between hypercharge and dark-photon field-strength tensors at the tree level, it can be generated at the loop level [41] provided that charge conjugation symmetry is also broken in the dark sector; for details see a recent review [42]. In order to obtain such a symmetry breaking, one can either introduce a new particle (or several new particles) which is charged under both SM and dark gauge groups or else introduce interactions in the dark sector that break this symmetry, in analogy with the SM where weak interactions are responsible for the violation. Otherwise, there can be no perturbative generation of the kinetic mixing term [42].

In the model considered here there is no such a symmetry breaking and, therefore, non-zero kinetic mixing could only appear at the tree level. This can also be seen directly since for each loop diagram that could a priori contribute to kinetic mixing there exists the same diagram but with a reversed particle flow, which corresponds to virtual antiparticles, and the contributions of both diagrams cancel pairwise. In the following, we will therefore assume a negligible kinetic mixing which, in the light of the discussion above, is

---

Note that other loop-induced decays into the two lighter dark vectors, $A' \to A'' A''$, or two scalars, $A' \to h_D (h_D$ or $H$), are excluded by the scalar QED analog of the Furry theorem that affects loop diagrams with an odd number of external vector fields, cf. Ref. [40]. Decays of $A'$ into a pair of the SM Higgs and the light dark Higgs boson would, either way, correspond to only narrow regions of the parameter space of the model studied in section 5. The similar process involving two dark Higgs bosons in the final state is also excluded at a more general level, as it would involve the initial state particle with $J = 1$ and two identical real scalar bosons in the final state that due to Bose statistics must have even $J$. 

---

3
not regenerated perturbatively. This way we will be able to also avoid stringent terrestrial and astrophysical bounds on light dark vector species [14], as well as possible BBN constraints [43].

3 Relic density

We begin the discussion of specific features of our model by examining the relic density of both DM components, the dominant one $\chi$ and a minor one $\eta$ – for which we require $\Omega_\chi h^2 + \Omega_\eta h^2 \approx 0.12$ [44] – as well the abundance of the unstable long-lived dark vector $A'$. To this end, we numerically solve a set of Boltzmann equations that extend the familiar assisted freeze-out mechanism discussed for two-component dark sectors [45] to compute the relic densities of three dark species:

$$
\frac{dY_\chi}{dx} = -\frac{\lambda_\chi}{x^2} \left( Y_\chi^2 - \frac{Y_\eta^2}{(Y_\chi^\text{eq})^2} (Y_\chi^\text{eq})^2 \right),
$$

$$
\frac{dY_\eta}{dx} = -\frac{\lambda_\eta}{x^2} \left( Y_\eta^2 - (Y_\eta^\text{eq})^2 \frac{Y_{A'}^2}{(Y_{A'}^\text{eq})^2} \right) + \frac{\lambda_\chi}{x^2} \left( Y_\chi^2 - \frac{Y_\eta^2}{(Y_\chi^\text{eq})^2} (Y_\chi^\text{eq})^2 \right),
$$

$$
\frac{dY_{A'}}{dx} = \frac{\lambda_\eta}{x^2} \left( Y_\eta^2 - (Y_\eta^\text{eq})^2 \frac{Y_{A'}^2}{(Y_{A'}^\text{eq})^2} \right) - \frac{\lambda_{A'}}{x^2} \left( Y_{A'}^2 - (Y_{A'}^\text{eq})^2 \right),
$$

where $Y_i$ is the respective yield, $i = \chi, \eta, A'$, and $x = m_\eta/T$. We stress again that we assume the lifetime of the dark Higgs boson $h_D$ to be short enough to decay before the era of BBN. A calculation of its abundance is, therefore, not needed in further analysis. The parameters $\lambda_i$ depend on the annihilation cross sections of the processes that play the dominant role in determining the abundances in the dark sector, given mass ordering shown in eq. (2.7):

$$
\lambda_\chi \equiv \frac{s(m_\eta)}{H(m_\eta)} \langle \sigma_{\chi\chi\rightarrow \eta\eta} v \rangle \simeq \frac{1.32 g_{s*}(m_\eta)}{\sqrt{g_*(m_\eta)}} m_\eta m_{\text{Pl}} \langle \sigma_{\chi\chi\rightarrow \eta\eta} v \rangle,
$$

$$
\lambda_\eta \equiv \frac{s(m_\eta)}{H(m_\eta)} \langle \sigma_{\eta\eta\rightarrow A'A'} v \rangle \simeq \frac{1.32 g_{s*}(m_\eta)}{\sqrt{g_*(m_\eta)}} m_\eta m_{\text{Pl}} \langle \sigma_{\eta\eta\rightarrow A'A'} v \rangle,
$$

$$
\lambda_{A'} \equiv \frac{s(m_\eta)}{H(m_\eta)} \langle \sigma_{A'A'\rightarrow h_D h_D} v \rangle \simeq \frac{1.32 g_{s*}(m_\eta)}{\sqrt{g_*(m_\eta)}} m_\eta m_{\text{Pl}} \langle \sigma_{A'A'\rightarrow h_D h_D} v \rangle,
$$

\footnote{We note that, in the absence of a direct coupling between $\chi$ and $A'$, the contribution in the Boltzmann equation for $Y_{A'}$ corresponding to the $\chi - A'$ interactions will appear only at next-to-leading order and we neglect it in the following. Similarly, given the suppressed couplings of the light vector $A''$ to the dark sector species charged under the $U(1)'$ group, its impact on their relic density is also subdominant and we neglect it in the following. As mentioned above, we assume that the $A''$ field itself is unstable and decays into much lighter dark species which is not relevant for the discussion of the $\chi, \eta$, and $A'$ abundances. We solve the Boltzmann equations assuming a partial wave decomposition of the thermally averaged annihilation cross sections for each process, $\langle \sigma v \rangle$. Due to large mass hierarchies among the three species, the times of their thermal freeze-out are well separated which means that is not essential to keep their full thermal dependence of $\langle \sigma v \rangle$.}
where \( m_{\text{Pl}} = 2.44 \times 10^{18} \text{ GeV} \) is the reduced Planck mass, \( s \equiv s(T) \) is the entropy density and \( H \equiv H(T) \) is the Hubble rate. The effective number of degrees of freedom for the entropy and energy densities of the thermalized SM-DS plasma at temperature \( T \) is denoted by \( g_{\ast\ast}(T) \) and \( g_\ast(T) \), respectively. The equilibrium comoving yield of each particle is defined as \( Y_i^{\text{eq}} = n_i^{\text{eq}}/s \), and it explicitly reads:

\[
Y_i^{\text{eq}}(x) = \frac{g_i}{g_{\ast\ast}(x)} \frac{45}{4\pi^2} (r_i x)^2 K_2[r_i x].
\]  

(3.3)

Here, \( r_i = m_i/m_\eta \) and the number of internal degrees of freedom of particle \( i \) is denoted by \( g_i \). The resulting relic density is obtained from \( \Omega_\chi h^2 = (\rho_\chi/\rho_{\text{crit}}) h^2 = (s_0 Y_{\chi,0} m_\chi/\rho_{\text{crit}}) h^2 \), where \( s_0 \) is the present-day entropy density and \( Y_{\chi,0} \) is the final yield of the dark species \( i \) after its freeze-out.

In the case of the dominant heavy DM component \( \chi \), additional contributions to the total relic density may come from \( 2 \rightarrow 3 \) processes \( \chi \bar{\chi} \rightarrow \eta \bar{\eta} A' \). We take this into account when presenting the results below by properly tuning the \( \mu_\chi \) and \( \mu_\eta \) parameters in eq. (2.4), as well as by modifying the heavy scalar mass \( m_\phi \), such that the total pair-annihilation cross section of \( \gamma \chi \) assumes a thermal value. In the case of the relic abundance of \( \eta \), possible annihilation modes are into the \( A' A' \) and \( h_D h_D \) as well as \( A' h_D \) final state. In practice, the annihilation mode into a pair of dark vectors dominates for typical large value of the dark coupling constant \( g_D \gtrsim 0.1 \). The coupling \( g_D \) takes such values in the cosmologically allowed region of the parameter space because it also determines the annihilation rate of dark photons into \( h_D h_D \) pairs, and we require the cross section in the dark sector \( \langle \sigma v \rangle_{A' A' \rightarrow h_D h_D} \) to be up to a few orders of magnitude larger than the thermal value. This suppresses the abundance of \( A' \) and allows one to avoid stringent cosmological bounds, cf. section 4.1. As a result, in the allowed region of the parameter space of the model we typically find

\[
\Omega_{\chi} h^2 \simeq 0.12 \gg \Omega_\eta h^2 \sim \Omega_{A'} h^2.
\]  

(3.4)

In the left panel of fig. 3 we illustrate the evolution of the yield \( m_i Y_i \) for all three dark species as a function of \( x \). We assume \( m_\chi = 1.5 \text{ TeV} \), \( m_\eta = 150 \text{ GeV} \), \( m_{A'} = 20 \text{ GeV} \), and \( m_{h_D} = 250 \text{ MeV} \). We also fix the coupling constants in the Higgs sector of the model as \( \lambda_{h h D} = 10^{-4} \) and \( \lambda_{h D \eta} = 4 \times 10^{-7} \), cf. discussion in section 5 for the justification of such choice. The heavy scalar \( \phi \) mass and the coupling constants \( \mu_\chi \) and \( \mu_\eta \) are chosen such that the annihilation cross section \( \langle \sigma v \rangle_{\chi \chi \rightarrow \eta \eta} \) achieves the thermal value. Due to the assumed mass hierarchy, the heavier DM species freezes out almost independently of the other two, resulting in \( \Omega_\chi h^2 \simeq 0.12 \).

Instead, for sufficiently large \( g_D \) the \( A' \) abundance may become strongly suppressed with respect to the total DM relic density. For the assumed values of the model parameters \( g_D = 5 \), as shown in the figure we obtain \( \Omega_{A'} \lesssim 10^{-3} \Omega_\chi \). With such a small abundance, one can avoid stringent BBN bounds for even very long-lived dark vectors. Importantly, one can see that in this case the predicted abundance of the lighter DM species \( \eta \) is even smaller than for the unstable dark vector \( (m_Y)_\eta < (m_Y)_{A'} \). This is so due to the fact that the dominant annihilation modes for both of these particles depend on \( g_D \) in a way that guarantees \( \langle \sigma v \rangle_{\eta \eta \rightarrow A' A'} > \langle \sigma v \rangle_{A' A' \rightarrow h_D h_D} \), cf. eqs. (A.9) and (A.12) for \( m_\eta \gtrsim m_{A'} \).

---

- 9 -
The yield of both $A'$ and $\eta$ grows with decreasing dark coupling constant $g_D$, e.g., $Y_{A'} \propto 1/(\sigma v)_{A' \to h_D h_D} \propto g_D^{-4}$, cf. eq. (A.12). We illustrate this in the right panel of fig. 3 which shows $(mY)$ as a function of $g_D$. Here, we also schematically present the BBN bounds violated for too low values of the coupling constant that predict both the increased abundance of late-time decaying $A'$ and its large lifetime, $\tau_{A'} \propto 1/g_D^2$, cf. eq. (2.8). We defer to section 4.1 the discussion of a more precise implementation of the BBN bounds in our analysis. CMB data constrain excessively large lifetimes for which $A'$-induced electromagnetic energy injection could happen around recombination. This excludes even very suppressed values of the dark vector yield, even if it survived until this time. As can be seen in the figure, both cosmological constraints become weaker for larger $g_D$. This simultaneously increases the sub-thermal $2 \to 3$ annihilation cross section $\chi \chi \to \eta \eta A'$ that is responsible, as we shall see below, for DM ID signals in our model. In the following, we will focus on regions of the parameter space of the model with $g_D \gtrsim 0.1$.

Correcting for the temperature difference in the SM and dark sector In the scenario of our interest, the dark sector couples to the SM via the light mediator particle $h_D$, while some of the stable or long-lived particles can freeze out late. In this case it is important to take into account the impact of the possible early kinetic decoupling on the abundances of the dark species [46–48]. We model this effect following Ref. [49] and implement the differences in the dark sector temperature $T_{DS}$ and the SM temperature $T$ in the above expressions determining the relic densities.

One has to note that this simple approach relies on a few assumptions which are not
necessarily valid in our case. In particular, we assume that the chemical potential of \( h_D \) vanishes until decoupling due to its interactions with the SM bath. In Ref. [48], however, it was shown that whenever the mass of the mediator is comparable to the mass of the DM particle, this can lead to underestimating the DM abundance by more than even an order of magnitude. On the other hand, for the mass hierarchy \( m_{\text{DM}} \gg m_{\text{med}} \), this effect becomes negligible and we expect it to play a minor role in our discussion, cf. eq. (2.7). We stress that the effect of the early kinetic decoupling affects only fractions of the available parameter space of our model presented in section 5 for the dark Higgs boson mass close to the di-electron threshold, \( m_{h_D} \gtrsim 2m_e \).

4 Current and future constraints from astrophysics, cosmology and colliders

Although most of the dark sector species present in the model considered here remain secluded from the SM, their indirect couplings via a light dark Higgs boson portal \( h_D \) still induce experimental constraints. Below, we briefly summarize the resulting current bounds on our scenario. They will define allowed regions of the parameter space of the model, as will be discussed in section 5. We then discuss future experimental and observational prospects of the considered scenario.

4.1 Current bounds

Accelerator-based searches Light dark Higgs bosons are certainly among the primary targets of intensity frontier searches for sub-GeV new physics as they correspond to one out of only a few available simple renormalizable portals to the dark sector. Based on available data one can put upper bounds on the mixing angle \( \theta \) between \( h_D \) and the SM Higgs boson \( H \). Of particular relevance to our study are constraints obtained from E949 data on rare kaon decays [50] and LHCb searches for rare \( B \) meson decays [51, 52], as well as results of beam-dump searches in the CHARM [53], MicroBooNE [54], and NA62 [55] experiments. We implement them following Refs [11, 56–59].

Astrophysical and cosmological bounds Additional constraints on light dark scalars can be obtained from their possible impact on astrophysical and cosmological observations. In particular, below we employ applicable bounds that arise due to possible modifications of the supernovae cooling rate and the neutrino emission from SN1987A [60], as well as BBN constraints on late-time decaying unstable BSM species [43].

Important bounds are also related to the residue relic abundance of potentially very long-lived dark vector \( A' \). We implement them following Ref. [61] for the BBN constraints on late-time electromagnetic energy injection. We also use the results of Ref. [62] for the CMB bounds based on the combined data from the Planck [44] and COBE/FIRAS [63] satellite missions. In the latter case, we follow Refs [64, 65] and modify the CMB constraints from Ref. [62] by taking into account a finite fraction of the electromagnetic energy transferred to the intergalactic medium, \( f_{\text{eff}} < 1 \), characteristic of different cascade annihilation SM final states present in the model. The final shape of the cosmological bounds derived this way depends on the interplay between the \( A' \) lifetime, cf. eq. (2.8), and its relic abundance obtained by solving the Boltzmann equations discussed in section 3.
The rich dark sector of our model also offers very good discovery prospects in various future searches targeting different BSM species. Below, we first discuss expected constraints for the light dark Higgs boson. These are related to the upcoming intensity frontier searches for LLPs. We then move to complementary ID of DM experiments that will probe the heavier part of the secluded BSM sector of the model. We finally focus on future prospects for detecting signatures of a very long-lived $A'$ in CMB observations.

4.2 Intensity frontier searches for light dark Higgs boson

As discussed above, rare meson decays provide stringent bounds on light, sub-GeV, dark Higgs bosons from various past accelerator-based experiments. Similar searches in the future are expected to further constrain the available parameter space of the model. For the dark Higgs boson mass range of our interest important bounds on light dark Higgs bosons below the di-muon threshold \( m_{h_D} < 2m_{\mu} \) are expected to come from rare kaon decays in the proposed KLEVER experiment \[11, 66\], a possible upgrade of the KOTO detector, referred to as KOTO step-2 \[67\], and the next run of the NA62 experiment \[68\]. Further constraints on both light and somewhat heavier scalars, but below the kaon threshold for the \( h_D \) production \( m_{h_D} < m_K - m_\pi \) can be obtained by Fermilab Short-Baseline Neutrino Experiments. We present expected bounds from the NuMI-ICARUS and BNB-SBND detectors following Ref. \[57\].

For a heavier dark Higgs boson, up to a mass of order several GeV, the most stringent future constraints are expected to come from displaced visible decays of \( h_D \)s produced in beam-dump or collider experiments. We present below future sensitivity reach contours for the proposed SHiP experiment \[69\] at CERN SPS, as well as the LHC searches at Codex-b (300 fb\(^{-1}\)) \[70, 71\], FASER 2 \[72, 73\], and MATHUSLA \[74, 75\]. In addition, we also show expected sensitivity due to the search for dark scalars in rare \( B \) meson decays at Belle-II \[76, 77\].

We note that direct searches for heavier dark species at the LHC will be very challenging to obtain in our model since we assume that they couple to the SM only via the \( h_D \) portal and the corresponding mixing angle with the SM Higgs \( H \) is suppressed. This also leads to typically very low BSM decay rates of \( H \) that will in this case be SM-like with suppressed invisible branching fraction \( B(H \to \text{inv.}) < 0.1\% \).

4.3 Dark matter detection

The parameters of our model can also be probed in DM experiments. This is especially the case for indirect searches. In this case, the corresponding discovery prospects rely on annihilation rates that can depend only on unsuppressed couplings present in the secluded dark sector. In our case, however, further caveats to this scenario appear that lead to distinct phenomenological features, as discussed below. On the other hand, future DD searches remain much less promising. While lighter DM species \( \eta \) couple to the SM via \( h_D \), their suppressed couplings to light quarks and negligible abundance typically yield very low expected signal rates in DD experiments. This is also true for the heavier DM species \( \chi \) for which the scattering rates off nuclei appear only via intermediate \( \eta \) and \( \phi \) fields.
Indirect detection of χ DM ID signatures in our model can arise from annihilations of both DM species, the heavier dominant scalar χ and the lighter one η. The latter process, however, is highly suppressed by the tiny abundance of η. This is not the case for annihilations of the dominant DM component. At the leading order, though, the main annihilation channel is purely into invisible final states χχ → ηη. Instead, the dominant contribution to ID signal rates appears at the next-to-leading order due to the 2 → 3 process χχ → ηηA′ shown in fig. 2. This is especially important for a growing value of the dark coupling constant gD which increases the chance of the final-state A'-strahlung off the η leg, cf. eq. (A.13) and eq. (A.14).

Notably, as we have already argued above, larger values of gD are also preferred by cosmological and collider constraints.

In order to examine possible DM ID signatures of the less-simplified scenario considered here, we first note that it has at least three distinct features that differentiate it from the simplest secluded DM models. Firstly, it relies on the 2 → 3 annihilation process producing a continuous (not monochromatic) spectrum of the meta-stable A' mediator energies. Secondly, it employs multi-step cascade decays contributing to the final photon flux, A' → hDA", followed by hD → 4e, 4µ, hadrons → γ, cf. Ref. [78, 79]. Thirdly, this allows for interesting non-local effects in DM ID present for very long-lived A' [20–24]. In particular, the first two features above result in smearing the spectrum of the final state SM particles which can be further non-trivially changed by non-local effects described in the third feature. As a result, in the model one effectively avoids bounds from the searches for the peaked spectral features in the positron data, cf. Refs [80, 81].

A particularly promising way of probing these scenarios is via searches for a diffuse DM-induced γ-ray flux. The corresponding differential flux of photons coming to a detector from the angular region in the sky ∆Ω is given by

$$\left( \frac{d\Phi}{dE_\gamma} \right)_{\text{standard}} = \frac{1}{8\pi} \left( \frac{\langle \sigma v \rangle}{m_\chi^2} \right) \int_{\Delta \Omega} d\Omega \int_{\ell.o.s.} \rho_\chi^2 ds \left( \frac{dN_\gamma}{dE_\gamma} \right)_\chi ,$$

where on the left hand side we have denoted that this result corresponds to the “standard” regime in which the long-lived vector mediator A' decays after traveling distances much shorter than a kpc. In our analysis below, we employ the Einasto DM profile,

$$\rho_\chi \equiv \rho_{\text{Einasto}}(r) = \rho_s \exp \left( -\frac{2}{\alpha} \left( \frac{r}{r_s} \right)^\alpha - 1 \right) ,$$

where \( \rho_s = 0.079 \text{GeV/cm}^3 \), \( r_s = 20 \text{kpc} \) and \( \alpha = 0.17 \) [82]. In eq. (4.1) the photon spectrum from 2 → 3 annihilations of χ and subsequent three-body and cascade decays of A' is denoted by \( (dN_\gamma/dE_\gamma)_\chi \), cf. appendix B for further discussion. Importantly, the above chain of processes results in a much softer photon spectrum than one could expect based on the DM mass \( m_\chi \) which is set to be above the TeV scale in the discussion below. In particular, a typical A' energy after the initial χ annihilation is of the order \( E_{A'} \sim (0.1 – 0.2)m_\chi \). The peak energy of photons produced as a result of A' decays is then

5The dark vectors could also be produced in cross interactions χη → χηA' which, however, play a subdominant role with respect to the 2 → 3 annihilations of the χ8 due to the suppressed relic density of η.
further shifted towards smaller energies, $E_{\gamma} \lesssim 100$ GeV, although it contains a high-energy tail extending towards larger $E_{\gamma}$.

In section 5, we will present the expected sensitivity reach of the forthcoming DM ID experiment in the TeV mass regime, namely the Cherenkov Telescope Array (CTA) [83], cf. also Refs [84, 85]. Instead of performing full detector simulations for the scenario of our interest, which is beyond the scope of our study, we illustrate the impact of the CTA on the parameter space of the model with some approximate expected bounds. We obtain them by employing the CTA sensitivity plots for the secluded DM regime following Ref. [86] which we, however, modify by taking into account characteristic $A'$ energies obtained after the $2 \to 3$ annihilation process. We expect this simplified procedure to properly encompass the most essential effects in our study. Notably, the applicable CTA bounds for secluded DM only mildly depend on the DM mass in the regime between $100$ GeV $\lesssim m_{\text{DM}} \lesssim$ TeV.

We treat in a similar way past bounds from the Fermi-LAT search for DM-induced signals from dSphs and present them following Ref. [87]. We note that even stronger bounds could be derived based on Fermi-LAT searches towards the GC [88]. These could improve the current constraints by up to a factor of a few in $\langle \sigma v \rangle \sim g_{\text{D}}^2$ for the $2 \to 3$ process, although the impact of these bounds could be ameliorated both by a smeared photon spectrum and a possible non-standard morphology of the signal discussed below.

**Non-local effects in $\gamma$-ray DM ID for $\chi$** The presence of an exceptionally long-lived vector mediator $A'$ has further important consequences for the ID of DM in our model. Signals can be modified by at least two additional non-local effects: enhanced DM-induced signal rates from extensive regions outside the Galactic Center (GC), and decreased signals from individual small regions, e.g., around the dSphs.

In the non-local case, the DM-induced $\gamma$-ray flux is partially driven outside the dense region around the GC by a very long-lived mediator $A'$. As a result, both the energy spectrum and morphology of the signal might be affected. The respective photon flux reads

$$\left( \frac{d\Phi}{dE_{\gamma}} \right)_{\text{non-local}} = \sum_{\text{bins} E_{A'}} \int_{\Delta \Omega} d\Omega \int_{\text{lo.s.}} ds \int_{V_\chi} d^3 \vec{r}_\chi \times \frac{\bar{d}_{A'}}{\rho_\chi^2 |(\vec{r}_\chi - \vec{r}_{GC})|^2} \exp \left( -\frac{|\vec{r}_A' - \vec{r}_\chi|}{d_{A'}} \right) \gamma_{A'} (1 - \beta_{A'} \cos \theta) \frac{f(\theta)}{4\pi} \left( \frac{dN_{\gamma}}{dE_{\gamma}} \right)_{\chi | E_{A'}}$$

where $\bar{d}_{A'} = c \tau_{A' \gamma A' \beta_{A'}}$ is the decay length of $A'$ in the Galactic frame and vectors $\vec{r}_\chi$, $\vec{r}_A'$, and $\vec{r}_{GC}$ correspond, respectively, to the position of the $\chi$ annihilation, the $A'$ decay, and the GC with respect to the detector on Earth. As can be seen, compared with the standard case, eq. (4.1), in the non-local DM ID an additional integration appears over the position of the initial $\chi$ annihilation. This takes into account the fact that the long-lived mediator $A'$ produced in $2 \to 3$ processes at $\vec{r}_\chi$ can travel long-distances before decaying at position $\vec{r}_A'$. In particular, the initial position $\vec{r}_\chi$ can lie outside the region of interest (RoI) in a given DM ID analysis. The mediator decay probability decreases exponentially with the growing distance $|\vec{r}_A' - \vec{r}_\chi|$. Hence, typically only a limited region in the Galaxy around
the RoI contributes to the observed DM-induced photon flux, although this depends on the value of $\bar{d}_{A'}$.

In eq. (4.3), we also employ anisotropy factors that depend on the angle $\theta$ defined as the angle between the $A'$ boost direction and the detector,

$$\cos \theta = \frac{\vec{r}_{A'} \cdot (\vec{r}_X - \vec{r}_{A'})}{|\vec{r}_{A'}| |\vec{r}_X - \vec{r}_{A'}|}, \quad (4.4)$$

The function $f(\theta)$ then reads

$$f(\theta) = \frac{(1 + \tan^2 \theta)^{3/2}}{\tan^2 \theta} \left[ \frac{(\beta/\tilde{\beta}_{h_D}) + \cos \tilde{\theta}}{(\beta/\tilde{\beta}_{h_D}) \cos \theta + 1} \right] \sin^2 \tilde{\theta}, \quad (4.5)$$

in which $\tilde{\theta}$ is the relevant angle in the $A'$ rest frame and we obtain $\cos \tilde{\theta} = \cos \tilde{\theta}_+$ for $\theta \leq \pi/2$ and $\cos \tilde{\theta} = \cos \tilde{\theta}_-$ otherwise, where

$$\cos \tilde{\theta}_+,- = \frac{-\gamma^2 \tan^2 \theta \frac{\beta}{\tilde{\beta}_{h_D}} \pm \sqrt{\gamma^2 \tan^2 \theta \left( 1 - \frac{\beta^2}{\tilde{\beta}_{h_D}^2} \right) + 1}}{\gamma^2 \tan^2 \theta + 1}, \quad (4.6)$$

and $\tilde{\beta}_{h_D} = \sqrt{1 - \left(2m_{h_D}/m_{A'}\right)^2}$. In the simplest case (not relevant for our full model), in which the travelling mediator directly decays into a pair of photons, one would reproduce a known expression for radiative beaming, $f(\theta) = \gamma^2_{A'} (\beta_{A'} \cos \tilde{\theta} + 1)^2$. The anisotropy factors appear due to the boost of the decaying $A'$ in the Galactic frame. They affect the final observed photon flux at Earth, since, in each given region in the Galaxy, decaying mediators preferentially come from the direction of the GC. However, in the non-relativistic and local limit we obtain $f(\theta) \to 1$.

We illustrate the impact of non-local effects on the total integrated flux of $\gamma$-rays in a toy DM model with $m_{DM} = 100$ GeV and a long-lived mediator mass $m_{med} = 10$ GeV in the left panel of fig. 4. Here, we assume for simplicity that the mediator decays directly into a pair of photons. We show with the red lines the ratio between the flux obtained in the long and short mediator lifetime regimes for several different RoIs. In particular, the red solid line corresponds to a large region around the GC characterized by $|b|, |l| < 12^\circ$.\(^6\) While this is a larger region than for typical CTA analyses, we employ it to better highlight the difference between the searches focused on the closed vicinity of the GC and the possible extended Galactic center survey present in the non-local DM ID regime, see, e.g., Refs [83, 85] for further discussion about CTA analyses. The large RoI employed in our study extends to roughly $d_{RoI} \sim R_0 \sin b \simeq 2.3$ kpc distance away from the GC, where $R_0 = 9$ kpc is the distance between the Earth and the GC.

As can be seen in the figure, for $\bar{d}_{med} \lesssim d_{RoI}$ the impact of non-local effects on the observed spectrum is very small and the photon spectrum resembles the one obtained in the short lifetime regime denoted by the horizontal black solid line, $\Phi_{non-local}/\Phi_{stand.} \simeq 1$. In contrast, for very large decay lengths of $A'$ the expected DM-induced photon flux coming

\(^6\)We use the Galactic coordinate system with the Galactic longitude $l$ and latitude $b$.  


from the RoI drops down much below the standard expectations. This is due to efficient escape of mediators away from the GC before they decay. The decrease of the flux is roughly linear in growing $\bar{d}_{\text{med}}$, as can be seen from eq. (4.3) in the limit of $\bar{d}_{\text{med}} \gg |\vec{r}_\text{med} - \vec{r}_\chi| \sim d_{\text{RoI}}$. The relative increase of the DM-induced photon flux for intermediate values of $\bar{d}_{\text{med}} \sim d_{\text{RoI}}$ can be understood as follows [22]. In this case, most of the mediators produced close to the GC decay within the RoI. Only a small fraction of dark photons produced at the GC would generate photons traveling towards the Earth. In the standard case, the signal from other mediators will be lost. In the non-local regime, however, this can be partially overcome by dark vectors traveling away from the GC before decaying. At these remote positions, they can produce photons in the direction of the Earth, which would not be seen had they been produced close to the GC. As a result, the DM ID signal rates from distant positions within the RoI do not only receive contributions from DM annihilations happening locally but also from annihilations occurring close to the GC. This increase is even more pronounced for a modified RoI around the GC in which we exclude the innermost region with the $2\degree$ size. We present this with the dashed red line in the figure. In this case, the photon flux in the non-local regime gains even more from the dark vectors traveling inside the RoI from the innermost region close to the GC.

In addition, we also show in the figure the expected photon fluxes for much smaller RoIs. Here, with a red dotted line we present the results for a small RoI characteristic for the CTA Galactic center survey, in which we have additionally excluded part of the region very close to the GC, i.e., we assume $0.3\degree < |b| < 1\degree$ and $|l| < 1\degree$. Instead, with a red dash-dotted line we present the flux for an even smaller region around the GC with $|b|, |l| < 0.5\degree$. The size of this region encompasses a typical DM halo size for dwarf galaxies in the Fermi-LAT analyses [89]. As can be seen, for both small RoIs, the relative growth of the flux for smaller $\bar{d}_{\text{med}}$ is hard to reconstruct, while for the decay length of order several kpc the flux is already suppressed. We note that if dSphs are modeled as point-like sources in the analysis with the $0.1\degree \times 0.1\degree$ bin size [90], the impact of non-local effects is even stronger. Last but not least, we stress that in the non-local regime the DM-induced photons escaping from small RoIs could also affect the analysis based on local background expectations around each of the dSphs. This could further ameliorate the relevant bounds.

As can be seen, for $\bar{d}_{\text{med}} \sim$ a few kpc, the difference between the impact of non-local effects for DM ID focusing on extensive regions around the GC and for searches targeting small RoIs can reach up to a factor of a few in the predicted photon flux. One can then expect a weakening of the relevant DM constraints derived based on stacked dwarf analyses, while DM-induced signals could be stronger for searches using larger RoIs around the GC. This might open new possibilities in explaining persisting anomalies in DM searches, see, e.g., Ref. [24] for the relevant discussion about the Galactic Center Excess (GCE) and non-local DM ID effects. However, as mentioned above, such effects would also modify the morphology of the DM-induced signals, which could then no longer follow the original DM profile but would appear to be less cuspy. In particular, as shown in Ref. [24], when compared to the standard short-lived regime, DM solutions to the GCE employing non-local effects struggle to improve the global $\chi^2$ fit for this anomaly. In the following, we will then rather focus on heavy DM $\chi$ with the mass above the TeV scale which corresponds
Annihilation of DM to long lived mediators

Figure 4: Left: The ratio of the integrated photon fluxes obtained for increasing decay length \( d \) of the mediator and in the standard regime of prompt decays shown with the horizontal black solid line. The figure has been prepared for the toy model with the DM and mediator masses equal to \( m_{DM} = 100 \text{ GeV} \) and \( m_{med} = 10 \text{ GeV} \), respectively, and assuming that the mediator decays directly into a pair of photons. We integrate the fluxes over the energy range between 0.1 GeV and 100 GeV. The solid (dashed, dotted, dash-dotted) red line corresponds to the DM ID observation region around the GC defined by different longitude and latitude limits, as indicated in the figure. Right: The CTA sensitivity for the secluded WIMP DM scenario in the \((m_{DM}, \langle \sigma v \rangle)\) plane is shown with the black solid line following Ref. [86]. For comparison, we also present with the red dotted line the expected such reach for the toy model with the long-lived mediator with \( \tau_{med} = 10^9 \text{ s} \) and \( m_{med} = 10 \text{ GeV} \) which decays into light quarks. The red dashed line corresponds to a re-scaled sensitivity for a larger RoI, as indicated in the figure (see text for details).

To the next important target in the upcoming DM ID searches.

In the right panel of fig. 4 we illustrate the impact of non-local effects on DM ID searches for the aforementioned toy model. To this end, we compare the expected sensitivity reach of the CTA in secluded WIMP DM scenario presented in Ref. [86] with the relevant reach obtained for a very long-lived mediator with \( \tau_{med} = 10^9 \text{ s} \) and fixed \( m_{med} = 10 \text{ GeV} \). Here, we assume that the mediator decays into light quarks. In the figure larger values of the mass of annihilating DM \( m_{DM} \) implies larger values of the boost factor of the mediator and the corresponding decay length \( \bar{d}_{med} \sim (m_{DM}/1 \text{ TeV}) (10 \text{ GeV}/m_{med}) \times 1 \text{ kpc} \). This results in an effective suppression of the DM-induced signal from a small RoI around the GC for \( m_{DM} \gtrsim \) a few hundred GeV, as indicated with the dotted red line in the figure. We also show there with the red dashed line the expected sensitivity for a larger RoI \(|b|, |l| < 12^\circ\) which, for illustrative purposes, has been re-scaled to match the sensitivity of the small RoI in the limit of low \( m_{DM} \). As can be seen, for the extended RoI, the weakening of the future bounds corresponds to larger DM masses than for small RoI. In this case, we also observe a relative improvement of the bound in the intermediate region of \( m_{DM} \sim \) a few TeV. This is due to the excess DM-induced photon flux for \( \bar{d}_{med} \sim d_{RoI} \), as discussed above.

**DM ID of the lighter scalar DM component \( \eta \)** The subdominant DM component \( \eta \) can also contribute to DM ID signals. The relevant annihilation rate is, however, suppressed by by a very small abundance, typically \( \Omega_\eta \lesssim 10^{-6} \Omega_\chi \). Since the DM ID signal rate depends quadratically on the number density of annihilating species, this is not compensated for by
the increasing annihilation cross section $\sigma_{\eta\bar{\eta}}$ for the process $\eta\bar{\eta} \to A'A'$ and the resulting signal is suppressed with respect to the dominant $\chi$ annihilations by several orders of magnitude. This suppression becomes smaller for the growing mass gap between $\eta$ and the dark vector, $m_\eta \gg m_{A'}$, due to a possible Sommerfeld enhancement (SE) of $\sigma_{\eta\bar{\eta}}$. However, we have verified numerically that this enhancement is not larger than a factor of order $10^4$ in the regions of the parameter space of the model that we explore in section 5, which still renders the $\eta$ contribution to the total DM ID rate subdominant. To implement the Sommerfeld effect, we followed Refs \cite{91–93} and approximated the Yukawa potential generated by the dark photon exchange by the Hulthen potential. Similarly, we do not obtain large SE of $\eta$ annihilations that could be induced by lighter dark vectors $A''$. This is due to much suppressed couplings between $A''$ and $\eta$.

The annihilations of the dominant DM species $\chi\bar{\chi} \to \eta\bar{\eta}(A')$ will also produce a flux of a boosted lighter scalar $\eta$. It can travel through the Galaxy or from distant galaxies, and can scatter off a background $\chi$ or annihilate again with background $\bar{\eta}$. In both cases further unstable dark vectors can be produced either via a next-to-leading order scattering process $\eta\chi \to \eta\chi A'$ or in the annihilation $\eta\bar{\eta} \to A'A'$. The relevant contributions are, however, extremely small. In particular, the probability that $\eta$ produced in the GC will annihilate before leaving the Galaxy can be estimated as

$$P_{\eta,\text{Galaxy}} \sim 1 - \exp \left( -\frac{\sigma_{\text{th}}}{m_\eta} \left( \frac{\sigma_{\eta\bar{\eta}}}{\sigma_{\text{th}}} \frac{\Omega_\eta}{\Omega_\chi} \right) \int_0^{R_{\text{max}}} \rho_{\text{Einasto}}(\ell) \, d\ell \right) \sim 10^{-16}$$

(4.7)

where $\sigma_{\text{th}} \sim \text{a few } \times 10^{-9} \text{ GeV}^{-2}$ is the typical thermal annihilation cross section. We note that the quantity in the square brackets is roughly equal to 1. Here, we assume the Einasto DM profile eq. (4.2) for background $\bar{\eta}$ and set $R_{\text{max}} = 100$ kpc for concreteness, although a precise value of the Galactic DM halo size has a negligible impact on the final result. In the last step, we have estimated the interaction probability for the benchmark scenario presented in the left panel of fig. 3. As can be seen, lighter scalars $\eta$ produced in $\chi$ annihilations close to the GC can easily escape the Galaxy. A typical distance they can travel is set by their mean free path in the background DM in the Universe, $d_\eta \sim (m_\eta/\sigma_{\text{th}}) (1/\rho_{\text{av}}) \sim 10^{17}$ Gpc, where we used $\rho_{\text{av}} = 1.1 \times 10^{-6}$ GeV/cm$^3$ \cite{44}. This is, again, an extremely large distance, which renders any extragalactic contributions to DM ID signal rates negligible in our analysis.

4.4 Future Cosmic Microwave Background surveys

As we have already discussed in section 4.1, CMB observations provide a further complementary way of probing the BSM scenario of our interest. We will implement them below following Ref. \cite{62}. In particular, future surveys are expected to significantly improve such bounds on CMB spectral distortions and to essentially exclude the BSM scenarios predicting the mediator lifetime between $\tau_{A'} \sim 10^5$ s and $10^{12}$ s. The relevant bounds will constrain the relic abundance of unstable very long-lived species to be not larger than a tiny fraction of the total DM relic density, $\Omega_{A'} \lesssim 10^{-6} \Omega_{\text{DM}}$, but, depending on $\tau_{A'}$, even much more stringent limits can be derived of order $10^{-12} \Omega_{\text{DM}}$. Instead, for the CMB anisotropy data, the expected improvement over current bounds is less pronounced but will also result in
more stringent constraints by about a factor of a few in $\Omega_{A'}$. This is relevant for the large lifetime regime, $\tau_{A'} > 10^{12}$ s.

In the following, we will present expected combined bounds from the Planck [44] and the proposed Primordial Inflation Explorer (PIXIE) [94] satellite mission. In addition, we will also employ more stringent constraints expected from the combination of the future data from the Polarized Radiation Imaging and Spectroscopy Mission (PRISM) [95], ground-based CMB Stage-4 (CMB S-4) searches [96] and the space mission LittleBIRD [97]. Similarly to the discussion in section 4.1, for the future CMB surveys we also include finite $f_{\text{eff}} < 1$ factors in our analysis. We stress that, while the CMB data remain complementary to DM ID searches for intermediate mediator lifetimes $\tau_{A'} \gtrsim 10^5$ s, they will provide the best way of probing scenarios with extremely long-lived $A'$s that would predict much-suppressed ID signal rates.

5 Results

The non-minimal content of our model results in 12 free parameters that contain the masses of the dark species and various dark sector couplings, cf. section 2. A thorough investigation of a rich phenomenology of the model would therefore require extensive and sophisticated numerical methods. We therefore limit our discussion to only slices of this multidimensional parameter space. Below, we first justify our choice for the fixed parameters, and later we present the results of our analysis in the most convenient two-dimensional ($m_{A'}, g_D$) plane.

We present our result for three different values of the dark Higgs boson mass: $m_{h_D} = 20$, 250, and 500 MeV. They correspond to distinct dominant $h_D$ decays into $e^+e^-$, $\mu^+\mu^-$, and pion ($\pi\pi$) pairs, respectively, with important consequences for experimental searches. In our case, $h_D$ is the lightest dark sector particle charged under the $U(1)'$ group. We further assume a large hierarchy between the dominant heavy scalar DM $\chi$ and the lighter dark species $\eta$ and $A'$ such that the long-lived dark photons produced in $2 \rightarrow 3$ annihilations $\chi\chi \rightarrow \eta\eta A'$ could become additionally boosted and travel galactic-scale distances with $d_{A'} \gtrsim 1$ kpc. We note, though, that for the scalar $\eta$ that is too light the loop-suppressed $A'$ lifetime would be driven again to smaller values, cf. eq. (2.8), in which case no non-local effects in DM ID would be expected. We therefore set $m_{\chi} = 1$ TeV and $m_{\eta} = 150$ GeV for concreteness. We note that the dominant $\chi$ DM mass around and above the TeV-scale corresponds to the best reach of DM ID searches in CTA, while it goes beyond the previous constraints for lighter DM.

In presenting our results we find it most convenient to vary the dark photon mass $m_{A'}$ and the coupling constant $g_D$. The former assumes a limited mass range, $m_{h_D} < m_{A'} < m_{\eta}$, cf. eq. (2.7), which allows for both the $A' \rightarrow h_D A''$ decays and $\eta \eta \rightarrow A'A'$ annihilation channels to be open. In particular, they drive the $\eta$ abundance to negligible levels. Instead, for $m_{\eta} < m_{A'}$ direct annihilations of $\eta$ into the SM species via the light scalar portal $h_D$ and suppressed annihilations into the lighter vectors $\eta\eta \rightarrow A''A''$ will lead to an overabundance of thermally-produced DM, unless the induced mixing angle between the dark Higgs boson and the SM Higgs boson acquires large values close to the current bounds. Even in this case, however, the large hierarchy $m_{h_D} \ll m_{\eta}$ would generate too large $f_{\text{eff}}$ and already...
excluded – values of the Sommerfeld-enhanced annihilation cross section of $\eta$ around the time of recombination. In the following, we therefore focus on the case with $m_{A'} < m_{\eta}$ and fit the auxiliary parameters in the dark sector, $m_\phi, \mu_A$, and $\mu_B$, such that at each point in the parameter space shown in the figures the heavy scalar DM obtains the correct value of the thermal relic density, $\Omega h^2 \simeq 0.12$, while $\Omega_{\eta} h^2$ is negligible.

The values of the dark coupling constant $g_D$ are a priori only limited by the perturbativity bound, which for simplicity we take to be $g_D < 4\pi$. On the other hand, as we have already discussed in section 3 and will also show below, the astrophysical, cosmological, and collider bounds discussed in section 4.1 constrain too low values of this coupling constant, and we, effectively, obtain $g_D \gtrsim 0.1$. In particular, lower values of $g_D$ would lead to too large abundance of $A'$ and its late-time decays would violate BBN and CMB limits. As we will see in the figures, collider bounds also affect the region of the parameter space with too low values of $g_D$. This is because for fixed $m_{A'}$ and $\lambda_{HhD}$ the mixing angle $\theta$ increases with decreasing $g_D$, cf. discussion in section 2. It, eventually, reaches the level at which it has already been probed in the previous searches for the dark Higgs boson.

Finally, we set the dark coupling constant between the dark Higgs boson and the lighter scalar DM component $\eta$ such that the lifetime of $A'$ can be large and of astrophysical relevance, cf. eq. (2.8). For concreteness, below we present our results for $\lambda_{hD\eta} = 4 \times 10^{-7}, 4 \times 10^{-6},$ and $4 \times 10^{-5}$. We also assume fixed small value of the kinetic mixing parameter between the two dark vectors $\tilde{\epsilon} = 10^{-6}$.

In the left panel of fig. 5 we show the results for $m_{hD} = 500$ MeV and $\lambda_{hD\eta} = 4 \times 10^{-6}$. Already excluded regions of the parameter space are shown in gray and labeled with proper bounds. As can be seen, in this case in the allowed region of the model’s parameter space one prefers the values of the dark coupling constant $g_D$ right in the ballpark for future searches in the proposed Codex-b, FASER 2, MATHUSLA, or SHiP detectors, as well as within the reach of Belle-II. The sensitivity reach of each experiment corresponds to the region below the accordingly marked line in the figure. In particular, MATHUSLA could cover almost the entire available region in the parameter space, except for the upper left corner.

For $m_{hD} = 500$ MeV, the dark Higgs boson decays hadronically and further bounds on the model may arise from past and future ID searches for DM-induced $\gamma$-rays. We mark in gray the upper bounds obtained from null searches for DM signals in dwarf galaxies performed by Fermi-LAT following Refs [87, 89]. Importantly, this bound puts a constraint on too large values of the dark coupling constants $g_D$. For larger $g_D$ the $2 \rightarrow 3$ annihilation cross section of $\chi$ grows which in turn enhances the expected $A'$-induced signal rates. We note that these bounds become much weaker with the increasing dark vector lifetime $\tau_{A'}$. This can be seen for lower values of $m_{A'} \lesssim 10$ GeV. This is due to the diffusion of the DM-induced signal outside dwarf galaxies, as discussed in section 4.3. Boosted and very-long-lived dark vector particles often decay away from their parent dSphs. The relevant region of the parameter space is, however, at least partially constrained by past CMB surveys because very long-lived dark vectors with $\tau_{A'} \gtrsim 10^{12}$ s could decay during the recombination epoch and the subsequent dark ages.

For smaller values of the lifetime of the dark vector additional bounds can be derived
\textbf{Figure 5:} Left: The impact on the parameter space of the model under study of various past bounds (gray-shaded region, see the text for details) and future searches shown in the \((m_{A'}, g_D)\) plane. We fix other parameters of the model to \(m_{\text{D}} = 500\) MeV, \(m_\text{h} = 150\) GeV, and \(m_{\chi} = 1.5\) TeV, as well as \(\lambda_{\text{D,eff}} = 4 \times 10^{-6}\) and \(\bar{\epsilon} = 10^{-6}\). In each point in the figure, the parameters \(m_{\chi}, \mu_\chi,\) and \(\mu_\tau\) are chosen such that the correct value of the heavy DM relic density is obtained, \(\Omega_{\chi} h^2 \simeq 0.12\). The colorful lines indicate the expected sensitivity reach for future searches, as discussed in the text. We also show with a black dash-dotted line the contours of the fixed \(A'\) lifetime of \(\tau_{A'} = 10^{11}\) and \(10^9\) s from left to right. Right: For the same benchmark scenario as in the left panel, the expected CTA sensitivity is shown for the standard case with diminishing decay length of \(A'\) (solid line) and the true scenario that takes into account the non-local effects. For the latter, we present the results for the RoIs characterized by \(|b|, |l| < 1\degree\), \(|b|, |l| < 12\degree\) (dashed line) and \(|b|, |l| < 1\degree\), \(|b|, |l| < 12\degree\) (dotted line). A smaller portion of the parameter space of the model presented in the right panel is indicated with a dotted rectangle in the left panel.

in future DM ID searches at CTA. We show the relevant sensitivity curve with the purple dotted contour which constrains the region of the parameter space above the line. It corresponds to DM-induced photons coming from the region around the GC with \(0.3\degree < |b| < 1\degree\) and \(|l| < 1\degree\). In this case, we also observe the diminishing reach in the limit of increasing \(\tau_{A'}\). Notably, in the large lifetime regime complementary probes of the model will be available thanks to future CMB surveys. These can cover parts of the allowed region below the dotted red and light blue lines corresponding, respectively, to the future Planck and PIXIE data, as well as to the combined data from CMB-S4, LittleBIRD, and PRISM (indicated as CMB-S4 in the figure), cf. section 4.4.

In the right panel of fig. 5 we compare the CTA bounds obtained by taking into account the non-local DM ID regime for large \(d_A'\) with the respective ones with neglecting these effects. The latter sensitivity reach line is denoted as “Standard” in the figure. As can be seen, the presence of non-local effects significantly weakens the impact of CTA on the parameter space of the model. This could be ameliorated in studies focusing on larger RoI, as schematically illustrated in the figure for the RoI characterized by \(|b|, |l| < 12\degree\). While we expect the bounds derived for the larger RoI to be shifted with respect to the
Figure 6: Same as the left panel of fig. 5 but for $m_{h_D} = 20$ MeV, $\lambda_{h_D \eta} = 4 \times 10^{-7}$ (left) and $m_{h_D} = 250$ MeV, $\lambda_{h_D \eta} = 4 \times 10^{-5}$ (right).

ones obtained for the smaller region around the GC, in the plot, for illustration, we have artificially rescaled the reach for $|b|, |l| < 12^\circ$ to match the standard one in the regime of the small $A'$ lifetime and better highlight the relative difference in the non-local effect in both cases. For the larger RoI, the weakening of the bounds is seen only for much smaller values of $m_{A'}$. We also note the presence of a small improvement of the expected CTA sensitivity in the small region of the figure for $m_{A'} \simeq (3 - 4)$ GeV. It is caused by the anisotropy effects discussed in section 4.3.

In fig. 6 we present similar bounds for $m_{h_D} = 20$ and 250 MeV in the left and right panel, respectively. We also fix the value of the coupling constant $\lambda_{h_D \eta} = 4 \times 10^{-7}$ and $4 \times 10^{-5}$ for the lighter and the heavier $h_D$, respectively. As expected from eq. (2.8), a larger $\lambda_{h_D \eta}$ implies a shorter $A'$ lifetime. As a result, for larger $\lambda_{h_D \eta}$ cosmological bounds constrain smaller fractions of the available parameter space and the respective bounds are shifted towards smaller values of the dark vector mass. This can be seen by comparing the size of the currently excluded gray-shaded regions in both panels. In both cases, the expected photon flux from DM ID is too low for the CTA to probe the allowed region in the parameter space of the model. On the other hand, the future CMB and collider searches will both be able to constrain these scenarios.

In the left panel of fig. 6 where $m_{h_D} = 20$ MeV the currently allowed region of the parameter space is constrained by the supernova SN198a and NA62 bounds on the dark Higgs boson, and BBN and CMB constraints on long-lived $A'$. In this case, $h_D$ decays predominantly into electrons and has a large lifetime, beyond the reach of much intensity frontier experiments targeting displaced decays of LLPs. Therefore, it can be searched for in rare kaon decays in the future KLEVER and NA62 detectors which could cover almost the entire allowed region of the parameter space below the dark blue dotted line. On the other hand, the expected future CMB bounds in this case are somewhat weaker.
and can only cover regions below the dotted red and light blue lines corresponding to $m_{A'} \lesssim \text{a few GeV}$.

The heavier dark Higgs boson with $m_{h_D} = 250$ MeV (the right panel of fig. 6) decays dominantly into a muon pair and is, therefore, characterized by a much smaller lifetime, as determined by its Yukawa-like couplings to the SM fermions. Hence, a good fraction of the parameter space can be probed by the future Codex-b, FASER 2, KLEVER, MATHUSLA, and SHiP detectors, as well as by the neutrino experiments NuMI-ICARUS and BNB-SBN. The complementarity between these searches and future CMB surveys will therefore lead to probing a large region in the $(m_{A'},g_D)$ plane across a wide range of the $A'$ lifetime.

6 Conclusions

Light sub-GeV portal models of dark matter have gained much interest in recent years and have sparked both experimental and theoretical activity in the field. Most studies so far have focused on simplified frameworks with only a limited number of new species added to extend the SM. These typically correspond to the most popular interaction operators and are supposed to encompass essential phenomenological aspects of such simplified scenarios. While this approach allows one to do a manageable comparison among many experimental proposals, it is also understood that new effects might be observed in more elaborate, and more realistic, models incorporating a larger number of BSM species with a mass hierarchy that can be quite complex and in particular span several orders of magnitude.

The presence of the light portal particle that thermally connects the SM sector and a heavy DM in the early Universe leaves important observational imprints that can be distinct from those in both more popular heavy WIMP models and from scenarios predicting the existence of only light DM. The biggest effects can be expected in indirect DM searches where a secluded heavy DM can lead to non-negligible interaction rates via displaced decays of the light portal particles, provided that the dark coupling constants remain large. Importantly, the light BSM species can be even very long-lived and additionally constrained by cosmology. This opens up new detection prospects for this kind of LLPs in the lifetime regime that remain highly complementary to intensity frontier searches.

In this study, we have found such effects and exposed them in a model with a rich dark sector and the popular scenario in which the coupling between the SM and the dark sectors arises only due to the mixing between the sub-GeV dark Higgs boson and the SM Higgs boson. The dark sector of the model further contains particles with masses up to 10 TeV or so. In particular, it invokes a heavy scalar DM with the mass above the TeV scale and a potentially very long-lived dark vector mediator $A'$ that is secluded from the SM. This scenario remains beyond the reach of current and near-future DD searches. However, we have shown here that the best way of probing this type of model, besides intensity frontier searches for LLPs, is to employ DM-induced signatures in both future ID and CMB experiments.

The presence of unstable subdominant dark species produced in annihilations of much heavier dominant DM particles can lead to further striking signatures. We have illustrated this in the case of the dark vector $A'$ that can feature a very large loop-suppressed lifetime.
and an astrophysically interesting decay length $\gamma_{\beta c} \sim \text{kpc}$. This can lead to interesting non-local effects in DM ID searches, for instance, enhanced DM signal rates from the GC and simultaneously suppressed corresponding rates expected from the dwarf galaxies. Similarly, the DM-induced $\gamma$-ray flux might in this case be characterized with a distinct morphology which does not have to follow a true DM density distribution. A thorough experimental testing of such scenarios will require going beyond the traditional approach to DM ID. In particular, studying possible DM ID signatures in both small and larger regions of interests around the GC might lead to important differences in expected photon fluxes and in the bounds on the annihilation cross section compared to simplified scenarios studied so far.

In searching for new physics it remains essential to encompass a broad range of possible BSM scenarios and to study all possible complementarities among different types of experiments. An attractive framework base on the popular thermal DM paradigm has lead to numerous studies in the past years. Models of this class that predict the existence of a light sub-GeV portal to heavy secluded DM deserve special attention in such efforts. As we have shown here, their characteristic signatures are absent in more popular simplified scenarios but may be successfully probed in extensive experimental programs in the coming years.

**Acknowledgements** We would like to thank Brian Batell for useful remarks and comments on the manuscript. We would like to thank Luc Darmé, Ifthah Galon, Andrzej Hryczuk, Arvind Rajaraman for useful discussions. In our analysis, we employed python module *vegas* [98], which uses an algorithm developed in Refs [99, 100]. KJ and LR are supported by the National Science Centre, Poland, research grant No. 2015/18/A/ST2/00748. ST and LR are supported by the grant “AstroCeNT: Particle Astrophysics Science and Technology Centre” carried out within the International Research Agendas programme of the Foundation for Polish Science financed by the European Union under the European Regional Development Fund. ST is supported in part by the Polish Ministry of Science and Higher Education through its scholarship for young and outstanding scientists (decision no 1190/E-78/STYP/14/2019).

**A Particle physics formulae**

Below, we provide expressions for all the decay widths and scattering cross sections relevant for our analysis.

**A.1 Decay widths**

**Decay width of $h_D$** The partial decay width of the dark Higgs boson into a pair of leptons, $h_D \rightarrow l^+ l^-$, reads [35]:

$$
\Gamma (h_D \rightarrow l^+ l^-) = \frac{\theta_{H_1}^2 m_l^2 m_{h_D}}{8 \pi v_{SM}^2} \left(1 - \frac{4 m_{l}^2}{m_{h_D}^2} \right)^{3/2},
$$

(A.1)
where \( v_{SM} = 246 \text{ GeV} \) and the mixing angle between the dark and SM Higgs bosons, \( \theta_{HhD} \), is given by \( \theta_{HhD} \approx \lambda_{HhD} v_D / m_H^2 \), valid for \( m_{hD} \ll m_H \).

The total decay width of \( h_D \) can be obtained from, \( \Gamma_{hD} = \Gamma (h_D \to l^+ l^-) / B(h_D \to l^+ l^-) \), where the branching fraction into leptons can be found, e.g., in Ref. [101].

**Decay width of \( A' \)**

\[
\Gamma_{A' \to A'h_D} = \frac{|M_{A' \to A'h_D}|^2 \sqrt{m_{A'}^2 - 4m_{hD}^2}}{16\pi m_{A'}^2},
\]

where \( \mu_{hD} = \lambda_{hD} m_{A'}/g_D \). The amplitude \( M_{A' \to A'h_D} \) is divergent and before renormalization is given by the formula:

\[
M_{A' \to A'h_D} = \epsilon(p)^{\nu}(p_1)^{\mu}(q_{A'h_D}^{2}\mu_{hD})C_{12}(m_{A'}^2, m_{hD}^2, 0, m_\chi, m_\eta, m_\eta) + 4g_{\mu\nu}C_{00}(m_{A'}^2, m_{hD}^2, 0, m_\chi, m_\eta, m_\eta)
\]

where \( p \) is the incoming momentum of \( A' \) and \( r \) is its polarization, while \( p_1 \) is the outgoing momentum of \( A'' \) and \( s \) is its polarization. The loop functions \( C_{00} \) (UV divergent) and \( C_{12} \) (UV finite) were obtained using the package described in Ref. [102]. We follow conventions presented therein, e.g., a constant factor \( 1/(16\pi)^2 \) is factored out from the definition of loop functions and it reappears in eq. (A.2). We add a suitable counterterm to our Lagrangian to render the \( C_{00} \) function finite and we work in the \( \overline{\text{MS}} \) renormalization scheme with dimensional regularization.

In the mass regime of eq. (2.7), the loop functions simplify significantly, and the amplitude squared is given by the following compact expression:

\[
|M_{A' \to A'h_D}|^2 = \frac{g_{A'}^2 g_D^2 \mu_{hD}^2}{72 m_\eta^4} (m_{A'}^2 - m_{hD}^2)^2.
\]

The resulting lifetime can be very large, as shown in the left panel of fig. 7.

**Decay width of \( \phi \)**

While it is not essential for our study, we also provide for completeness the decay width of the heavy scalar \( \phi \), which can decay into both the \( \chi \bar{\chi} \) and \( \eta \bar{\eta} \) pairs:

\[
\Gamma_\phi = \Gamma_{\phi \to \chi \chi} + \Gamma_{\phi \to \eta \eta} = \frac{\mu_\chi^2 \sqrt{m_\phi^2 - 4m_\chi^2}}{16\pi m_\phi^2} + \frac{\mu_\eta^2 \sqrt{m_\phi^2 - 4m_\eta^2}}{16\pi m_\phi^2}.
\]

In the assumed mass scheme, eq. (2.7), and for \( \mu_\chi = \mu_\eta, \mu_\chi/m_\phi \sim 0.1 \), we obtain a very small lifetime of \( \phi \), which then does not have any implications on our discussion of the cosmological bounds

\[
\tau_\phi \sim \frac{8\pi m_\phi}{\mu_\chi^2} \sim 10^{-25} \text{ s} \left( \frac{1 \text{ TeV}}{\mu_\chi} \right)^2 \frac{m_\phi}{10 \text{ TeV}}.
\]
Figure 7: Left: Lifetime of $A'$ due to the decay into a pair of $h_D$'s as a function of $m_\eta$ (particle flowing in the loop). We fix the dark Higgs boson mass, $m_{h_D} = 250$ MeV, the coupling constant $\lambda_{h_D\eta} = 4 \times 10^{-6}$, and the kinetic mixing angle $\tilde{\epsilon} = 10^{-6}$. These values also correspond to the right panel of fig. 6. The black solid (dashed, dotted) lines correspond to fixed dark vector masses, $m_{A'} = 100, 10,$ and $1$ GeV, respectively. Right: Dotted black line denotes the cross section $\langle \sigma v (\chi \bar{\chi} \to \eta \bar{\eta} A') \rangle$ relevant for indirect detection of $\chi$ DM as a function of $m_{A'}$, while the solid black line corresponds to the thermally averaged cross section, which yields $\Omega \chi h^2 \sim 0.1$.

Figure 8: Heavy DM annihilation processes contributing to the $\chi$ relic density.

### A.2 Annihilation cross sections

$\chi \chi \to \eta \bar{\eta}$ The relic density of $\chi$ is driven by annihilations into two lighter species $\eta$, as shown on the left in fig. 8. The relevant cross section is given by

$$
\langle \sigma v \rangle_{\chi \chi \to \eta \bar{\eta}} = \frac{\mu^2_{\chi} \mu^2_{\eta}}{32 \pi m^3_{\chi}} \sqrt{m^2_{\chi} - m^2_{\eta}} \left( \frac{m^2_{\eta} - 4 m^2_{\chi} + \Gamma^2_{\phi} m^2_{\phi}}{m^2_{\phi}} \right) \sim 10^{-9} \text{ GeV}^{-2} \left( \frac{\mu}{m_{\phi}} \right)^4 \left( \frac{30 \text{ GeV}}{m_{\chi}} \right)^2 .
$$

(A.7)

$\eta \eta \to A'A'$ The relic density of $\eta$ is obtained via its annihilations into the light mediators, $A'$ and $h_D$, as shown in figs. 9 and 10. Assuming the mass hierarchy in eq. (2.7), the dominant annihilation mode is into a pair of dark vectors with the corresponding cross
\[ \langle \sigma v \rangle_{\eta\eta \rightarrow A^\prime A^\prime} = \frac{g_D^4 (3m_{A'}^2 - 24m_A^2m_{\eta}^2 + 88m_A^4 - 128m_A^2m_{\eta}^4 + 64m_{\eta}^8)}{16\pi m_{\eta}^4} \] 

\[ + \frac{2g_D^2 \lambda_{h_D\eta} (3m_{A'}^4 - 8m_A^4m_{\eta}^2 + 8m_{\eta}^4)}{m_{A'}^2 (m_{H}^2 - 4m_{\eta}^2)} + \frac{\lambda_{h_D\eta}^2 (3m_{A'}^4 - 4m_A^4m_{\eta}^2 + 4m_{\eta}^4)}{(m_{H}^2 - 4m_{\eta}^2)^2} \] 

In the limit of \( m_{\eta} \gg m_{A'} \gg m_{h_D} \), the above expression can be simplified to:

\[ \langle \sigma v \rangle_{\eta\eta \rightarrow A^\prime A^\prime} \simeq \frac{g_D^4 m_{\eta}^2}{\pi m_{A'}^4}. \]  

(A.9)

For completeness, we also provide the expression for the \( \eta\bar{\eta} \rightarrow h_D h_D \) cross section contributing sub-dominantly to the total annihilation rate in our case.

\[ \eta\bar{\eta} \rightarrow h_D h_D \] The cross section for \( \eta \) annihilations into light dark Higgs bosons reads

\[ \langle \sigma v \rangle_{\eta\eta \rightarrow h_D h_D} = \frac{\lambda_{h_D\eta}^2 \sqrt{m_{\eta}^2 - m_{h_D}^2}}{64\pi g_D^2 m_{\eta}^4 \left( m_{h_D}^4 - 6m_{h_D}^2 m_{\eta}^2 + 8m_{\eta}^4 \right)} \times \] 

\[ (g_D^2 (5m_{h_D}^4 - 18m_{h_D}^2 m_{\eta}^2 + 16m_{\eta}^4)) - 2\lambda_{h_D\eta} m_{A'}^2 (m_{h_D}^2 - 4m_{\eta}^2))^2, \] 

which, in the limit of \( m_{\eta} \gg m_{h_D} \), simplifies to

\[ \langle \sigma v \rangle_{\eta\eta \rightarrow h_D h_D} = \frac{\lambda_{h_D\eta}^2}{16\pi m_{\eta}^2}. \]  

(A.11)

For the values of \( \lambda_{h_D\eta} \) relevant for analysis, this cross section remains much suppressed with respect to eq. (A.9).
The metastable relic density of the dark vector is obtained thanks to its annihilations into $h_D h_D$ pairs, as shown in fig. 11. The relevant expression reads

$$
\langle \sigma v \rangle_{A' A' \rightarrow h_D h_D} = \frac{g_D^4 \sqrt{1 - r (r^2 - 2r - 2)^2}}{3 \pi m_{A'}^2 (8 - 6r + r^2)^2} \sim 0.01 \frac{g_D^4}{m_{A'}^2},
$$

where $r = m_{h_D}^2 / m_{A'}^2 \ll 1$. Note that both eq. (A.12) and eq. (A.8) depend on the dark gauge coupling $g_D$, which, due to stringent BBN bounds on metastable relic density of $A'$, means that $\Omega_\eta \sim \Omega_{A'}$. In fact, eq. (A.8) leads to the cross section proportional to $m_{h_D}^2 / (\pi m_{A'}^4)$, which can be compared to the factor $0.01 / m_{A'}^2$ present in eq. (A.12). Given the mass scheme that we consider in our study, eq. (2.7), this leads to $\Omega_\eta \lesssim \Omega_{A'}$.

### $\chi \chi \rightarrow \eta A'$

For large values of the dark gauge coupling $g_D$, the dark brehmstrahlung process can become important, cf. the right panel of fig. 7. The amplitude of the process depends on, i.a., the momenta of all the particles and the total energy in the CM frame $s = (p_1 + p_2)^2$:

$$
M_{\chi \chi \rightarrow \eta A'} = \frac{g_D \mu_{\chi} \mu_{\eta}}{s - m_\phi^2} \left( \frac{(p_3 + 2p_4)\mu}{(p_3 + p_4)^2 - m_4^2} + \frac{(-p_3 - 2p_5)\mu}{(p_3 + p_5)^2 - m_5^2} \right) \epsilon_\mu(p_3, m_3),
$$

where the incoming $\chi$ particles have momenta $p_1, p_2$, the outgoing $\eta$ particles have momenta $p_4, p_5$ and the outgoing $A'$ has momentum $p_3$. The total cross section is given by [103]:

$$
\sigma_{\chi \chi \rightarrow \eta A'} = \frac{1}{64 \pi^4 \sqrt{\lambda(s, m_1^2, m_2^2)}} \int_{E_3}^{E_3^{\text{max}}} dE_3 \sqrt{E_3^2 - m_3^2} \int_0^\pi d\theta \sin \theta \int_0^\pi d\theta^* \sin \theta^* \int_0^{2\pi} d\varphi^* \frac{1}{8} \sqrt{((m_{45} + m_4)^2 - m_\phi^2)((m_{45} - m_4)^2 - m_\phi^2)} \left| M_{\chi \chi \rightarrow \eta A'} \right|^2,
$$

where we have already performed a trivial integral over the azimuth angle $\phi$, which gives a factor of $2\pi$. The angles written without the asterisk are evaluated in the CM frame of pair of particles with momenta $p_1$ and $p_2$. In turn, quantities with the asterisk (*) are evaluated in the CM frame of pair of particles with momenta $p_3$ and $p_4$. For example, $\theta$ is the scattering angle between the particle with momentum $p_3$ and the collision axis of the particles with momenta $p_1$ and $p_2$ in their CM frame, while $\theta^*$ and $\phi^*$ denote the angles of the particle with momentum $p_3$ in the frame where $\vec{p}_4 + \vec{p}_5 = 0$.

---

We provide a general expression valid for five different masses, expanding and correcting the expressions from [103].
In our case, we set:

\[
m_1 = m_2 = m_\chi,
\]

\[
m_3 = m_{A'}, m_4 = m_5 = m_\eta,
\]

\[
E_{3}^{\text{min}} = m_3,
\]

\[
E_{3}^{\text{max}} = \frac{s + m_3^2 - (m_4 + m_5)^2}{2\sqrt{s}}.
\]  

(A.15)

In order to obtain the quantities with the asterisk, one needs to use the following boost factor

\[
\gamma = \frac{E_{45}}{m_{45}}, \quad \beta = \sqrt{1 - \frac{1}{\gamma^2}}.
\]  

(A.16)

The invariant mass \(m_{45}\) and the energy of the pair \(E_{45}\) are given by

\[
m_{45} = \sqrt{(p_4^0 + p_5^0)^2} = \sqrt{s - 2\sqrt{s}E_3 + m_3^2},
\]

\[
E_{45} = \frac{s + m_{45}^2 - m_3^2}{2\sqrt{s}}.
\]  

(A.17)

We also write down the expressions for all components of \(p_4\) and \(p_5\):

\[
\begin{align*}
p_4^0 &= \frac{m_{45}^2 + m_3^2 - m_4^2}{2m_{45}} \quad \text{and} \quad p_5^0 = \frac{m_{45}^2 + m_3^2 - m_5^2}{2m_{45}} \quad (A.18) \\
p_4^1 &= p_{45}^{\text{CM}} \sin \theta^* \cos \varphi^* \quad \text{and} \quad p_5^1 = -p_{45}^{\text{CM}} \sin \theta^* \cos \varphi^* \\
p_4^2 &= p_{45}^{\text{CM}} \sin \theta^* \sin \varphi^* \quad \text{and} \quad p_5^2 = -p_{45}^{\text{CM}} \sin \theta^* \sin \varphi^* \\
p_4^3 &= p_{45}^{\text{CM}} \cos \theta^* \quad \text{and} \quad p_5^3 = -p_{45}^{\text{CM}} \cos \theta^*
\end{align*}
\]

where

\[
p_{45}^{\text{CM}} = \sqrt{(p_4^0)^2 - m_3^2}. \]  

(A.19)

Finally, we can express \(p_4\) and \(p_5\) in the CM frame as a function of \((E_3, \theta, \theta^*, \varphi^*)\) by applying the following boost and rotation transformations

\[
p_{4,5} = \text{Rot}_y(\theta + \pi) \cdot \text{Boost}_z(\beta) \cdot p_{4,5}^*,
\]  

(A.20)

where the usual boost and rotation matrices are given by

\[
\text{Boost}_z(\beta) = \begin{bmatrix}
\gamma & 0 & 0 \\
0 & 1 & 0 \\
\beta \gamma & 0 & 1
\end{bmatrix}, \quad \text{Rot}_y(\theta) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta & 0 & \sin \theta \\
0 & 0 & 1 & 0 \\
0 & -\sin \theta & 0 & \cos \theta
\end{bmatrix}. \]  

(A.21)

\section*{B DM-induced \(\gamma\)-ray spectrum}

In this appendix, we provide further details about our modelling of the DM-induced photon spectrum in the model under study. For the primary spectra of \(\gamma\) rays we rely on the PPPC data [104], cf. also Ref. [105] for useful discussion. We denote the relevant spectrum obtained in the rest frame of the dark Higgs boson by \((dN_\gamma/dE_{\gamma}^{hD})_{hD}\). In order to obtain
the differential flux of photons produced in the DM-induced cascade annihilation process described in the text for the model under study, shown as a function of the photon energy. In the figure, we show with the black solid line the result for the standard scenario, in which the dark vector mediator decays promptly after being produced in the $2 \rightarrow 3$ annihilation process of $\chi$ DM. Instead, the black dash-dotted, red dotted, and blue dashed lines correspond to the long-lived mediator case with the relevant typical decay length of boosted $A'$s equal to 0.1, 3, and 30 kpc, respectively.

In order to obtain the final $\gamma$-ray spectrum $(dN_\gamma/dE_\gamma)\chi$, we then convolute this with the actual continuous $A'$ spectrum from $2 \rightarrow 3$ annihilations, $\chi\chi \rightarrow \eta\eta A'$, cf. eq. (A.14) for the relevant differential cross section. In order to simplify our analysis, we evaluate this as a discretized weighted average

$$
\left(\frac{dN_\gamma}{dE_\gamma}\right)\chi = \sum_{b} \frac{\langle \sigma v \rangle_{E_\gamma}}{\langle \sigma v \rangle_{E_{\gamma}}} \left(\frac{dN_\gamma}{dE_\gamma}\right)\chi,$$

where $\langle \sigma v \rangle_{E_{\gamma}}$ corresponds to the $2 \rightarrow 3$ annihilation cross section integrated over a limited range of the outgoing $A'$ energies (within the energy bin centered around $E_{A'}$), while $\langle \sigma v \rangle$ is, to a good approximation, the Galactic frame for cold DM $\chi$. 

---

**Figure 12:** The differential flux of photons produced in the DM-induced cascade annihilation process described in the text for the model under study, shown as a function of the photon energy. In the figure, we show with the black solid line the result for the standard scenario, in which the dark vector mediator decays promptly after being produced in the $2 \rightarrow 3$ annihilation process of $\chi$ DM. Instead, the black dash-dotted, red dotted, and blue dashed lines correspond to the long-lived mediator case with the relevant typical decay length of boosted $A'$s equal to 0.1, 3, and 30 kpc, respectively.
is the total such cross section. We used 20 equally spaced bins in $x_{A'} = E_{A'}/E_\chi$ in the logarithmic scale, which have been numerically verified to be sufficient for the results presented in section 5.

Example of such a photon spectrum is shown with the black solid line in fig. 12. We have obtained it for the fixed masses of the dark sector species $m_\chi = 1.5$ TeV, $m_\eta = 150$ GeV, $m_{A'} = 10$ or 3 GeV, and $m_{h_D} = 500$ MeV, as well as for the dark coupling constant set to $g_D = 0.01$. As expected, even though the assumed $\chi$ mass is above TeV, the resulting photon spectrum is much softer and peaked at $E_\gamma \sim$ tens of GeV.

In fig. 12, we further present several such photon spectra corresponding to very long-lived mediators with the decay length of order $\bar{d}_{A'} \sim 0.1$, 3 and 30 kpc. We present the photon fluxes for the $|b|, |l| < 12^\circ$ region around the GC. As discussed in section 4.3, for $\bar{d}_{A'} < d_{\text{RoI}} \simeq 2.3$ kpc the impact of non-local effects on the observed spectrum is very small and the photon spectrum resembles the one obtained in the short lifetime regime. This is indicated with the black dash-dotted line in the figure. Instead, for larger values of the decay length, $\bar{d}_{A'} \sim d_{\text{RoI}}$, we see a relative increase in the photon flux observed from this RoI due to anisotropy effects, as shown with the red dotted line. Finally, for very large decay lengths, $\bar{d}_{A'} \gg d_{\text{RoI}}$, the flux becomes suppressed, which is illustrated with a blue dash-dotted line. The suppression is more pronounced for energetic photons as they originate from more boosted dark vectors. These can more easily escape the RoI before decaying. As a result, the observed photon spectrum is even stronger shifted towards lower energies.

References

[1] G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, M. Pierre et al., The waning of the WIMP? A review of models, searches, and constraints, Eur. Phys. J. C 78 (2018) 203 [1703.07364].
[2] L. Roszkowski, E.M. Sessolo and S. Trojanowski, WIMP dark matter candidates and searches—current status and future prospects, Rept. Prog. Phys. 81 (2018) 066201 [1707.06277].
[3] e.a. Billard, Julien, Direct Detection of Dark Matter – APPEC Committee Report, 2104.07634.
[4] D.J.E. Marsh, Axion Cosmology, Phys. Rept. 643 (2016) 1 [1510.07633].
[5] M. Battaglieri et al., US Cosmic Visions: New Ideas in Dark Matter 2017: Community Report, in U.S. Cosmic Visions: New Ideas in Dark Matter, 7, 2017 [1707.04591].
[6] R. Alves Batista et al., EuCAPT White Paper: Opportunities and Challenges for Theoretical Astroparticle Physics in the Next Decade, 2110.10074.
[7] J.L. Feng and J. Kumar, The WIMPless Miracle: Dark-Matter Particles without Weak-Scale Masses or Weak Interactions, Phys. Rev. Lett. 101 (2008) 231301 [0803.4196].
[8] H. Baer, K.-Y. Choi, J.E. Kim and L. Roszkowski, Dark matter production in the early Universe: beyond the thermal WIMP paradigm, Phys. Rept. 555 (2015) 1 [1407.0017].
[9] C. Boehm and P. Fayet, Scalar dark matter candidates, Nucl. Phys. B 683 (2004) 219 [hep-ph/0305261].
[10] M. Pospelov, A. Ritz and M.B. Voloshin, Secluded WIMP Dark Matter, Phys. Lett. B 662 (2008) 53 [0711.4866].

[11] J. Beasham et al., Physics Beyond Colliders at CERN: Beyond the Standard Model Working Group Report, J. Phys. G 47 (2020) 010501 [1901.09966].

[12] J. Alimena et al., Searching for long-lived particles beyond the Standard Model at the Large Hadron Collider, J. Phys. G 47 (2020) 090501 [1903.04497].

[13] P. Agrawal et al., Feebly-Interacting Particles:FIPs 2020 Workshop Report, 2102.12143.

[14] M. Fabbrichesi, E. Gabrielli and G. Lanfranchi, The Dark Photon, 2005.01515.

[15] A. Filippi and M. De Napoli, Searching in the dark: the hunt for the dark photon, Rev. Phys. 5 (2020) 100042 [2006.04640].

[16] G. Arcadi, A. Djouadi and M. Kado, The Higgs-portal for dark matter: effective field theories versus concrete realizations, Eur. Phys. J. C 81 (2021) 653 [2101.02507].

[17] B. Batell, M. Pospelov and A. Ritz, Probing a Secluded U(1) at B-factories, Phys. Rev. D 79 (2009) 115008 [0903.0363].

[18] T. Bringmann, F. Kahlhoefer, K. Schmidt-Hoberg and P. Walia, Strong constraints on self-interacting dark matter with light mediators, Phys. Rev. Lett. 118 (2017) 141802 [1612.00845].

[19] K. Jodlowksi, F. Kling, L. Roszkowski and S. Trojanowski, Extending the reach of FASER, MATHUSLA, and SHiP towards smaller lifetimes using secondary particle production, Phys. Rev. D 101 (2020) 095020 [1911.11346].

[20] I.Z. Rothstein, T. Schwetz and J. Zupan, Phenomenology of Dark Matter annihilation into a long-lived intermediate state, JCAP 07 (2009) 018 [0903.3116].

[21] D. Kim, J.-C. Park and S. Shin, Dark Matter "Transporting" Mechanism Explaining Positron Excesses, J. High Energ. Phys. 2018 (2018) 93 [1702.02944].

[22] X. Chu, S. Kulkarni and P. Salati, Dark matter indirect signals with long-lived mediators, JCAP 11 (2017) 023 [1706.08543].

[23] S. Gori, S. Profumo and B. Shakya, Wobbly Dark Matter Signals at Cherenkov Telescopes from Long Lived Mediator Decays, Phys. Rev. Lett. 122 (2019) 191103 [1812.08694].

[24] K. Agashe, S.J. Clark, B. Dutta and Y. Tsai, Nonlocal effects from boosted dark matter in indirect detection, Phys. Rev. D 103 (2021) 083006 [2007.04971].

[25] Z. Chacko, H.-S. Goh and R. Harnik, The Twin Higgs: Natural electroweak breaking from mirror symmetry, Phys. Rev. Lett. 96 (2006) 231802 [hep-ph/0506256].

[26] R.M. Schabinger and J.D. Wells, A Minimal spontaneously broken hidden sector and its impact on Higgs boson physics at the large hadron collider, Phys. Rev. D 72 (2005) 093007 [hep-ph/0509209].

[27] B. Patt and F. Wilczek, Higgs-field portal into hidden sectors, hep-ph/0605188.

[28] G.C. Branco, P.M. Ferreira, L. Lavoura, M.N. Rebelo, M. Sher and J.P. Silva, Theory and phenomenology of two-Higgs-doublet models, Phys. Rept. 516 (2012) 1 [1106.0034].

[29] N. Craig, A. Katz, M. Strassler and R. Sundrum, Naturalness in the Dark at the LHC, JHEP 07 (2015) 105 [1501.05310].
[30] M.J. Strassler and K.M. Zurek, *Echoes of a hidden valley at hadron colliders*, Phys. Lett. B 651 (2007) 374 [hep-ph/0604261].

[31] L. Darmé, S. Rao and L. Roszkowski, *Light dark Higgs boson in minimal sub-GeV dark matter scenarios*, JHEP 03 (2018) 084 [1710.08430].

[32] L. Darmé, S. Rao and L. Roszkowski, *Signatures of dark Higgs boson in light fermionic dark matter scenarios*, JHEP 12 (2018) 014 [1807.10314].

[33] M. Duerr, T. Ferber, C. Hearty, F. Kahlhoefer, K. Schmidt-Hoberg and P. Tunney, *Invisible and displaced dark matter signatures at Belle II*, JHEP 02 (2020) 039 [1911.03176].

[34] A. Berlin and F. Kling, *Inelastic Dark Matter at the LHC Lifetime Frontier: ATLAS, CMS, LHCb, CODEX-b, FASER, and MATHUSLA*, Phys. Rev. D 99 (2019) 015021 [1810.01879].

[35] O. Lebedev, *The Higgs portal to cosmology*, Prog. Part. Nucl. Phys. 120 (2021) 103881 [2104.03342].

[36] E. Ma, *Inception of Self-Interacting Dark Matter with Dark Charge Conjugation Symmetry*, Phys. Lett. B 772 (2017) 442 [1704.04666].

[37] M. Duerr, K. Schmidt-Hoberg and S. Wild, *Self-interacting dark matter with a stable vector mediator*, JCAP 09 (2018) 033 [1804.10385].

[38] M. Bauer, P. Foldenauer and J. Jaeckel, *Hunting All the Hidden Photons*, JHEP 07 (2018) 094 [1803.05466].

[39] B. Batell, “Private communication.”

[40] M.E. Peskin and D.V. Schroeder, *An Introduction to quantum field theory*, Addison-Wesley, Reading, USA (1995).

[41] B. Holdom, *Two U(1)’s and Epsilon Charge Shifts*, Phys. Lett. 166B (1986) 196.

[42] T. Gherghetta, J. Kersten, K. Olive and M. Pospelov, *Evaluating the price of tiny kinetic mixing*, Phys. Rev. D 100 (2019) 095001 [1909.00696].

[43] J. Berger, K. Jedamzik and D.G.E. Walker, *Cosmological Constraints on Decoupled Dark Photons and Dark Higgs*, JCAP 11 (2016) 032 [1605.07195].

[44] PLANCK collaboration, *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. 641 (2020) A6 [1807.06209].

[45] G. Belanger and J.-C. Park, *Assisted freeze-out*, JCAP 03 (2012) 038 [1112.4491].

[46] M. Duch and B. Grzadkowski, *Resonance enhancement of dark matter interactions: the case for early kinetic decoupling and velocity dependent resonance width*, JHEP 09 (2017) 159 [1705.10777].

[47] T. Binder, T. Bringmann, M. Gustafsson and A. Hryczuk, *Early kinetic decoupling of dark matter: When the standard way of calculating the thermal relic density fails*, Phys. Rev. D 96 (2017) 115010 [1706.07433].

[48] T. Bringmann, P.F. Depta, M. Hufnagel and K. Schmidt-Hoberg, *Precise dark matter relic abundance in decoupled sectors*, 2007.03696.

[49] J.L. Feng, H. Tu and H.-B. Yu, *Thermal Relics in Hidden Sectors*, J. Cosmol. Astropart. Phys. 2008 (2008) 043 [0808.2318].

[50] E949 collaboration, *New measurement of the $K^+ \to \pi^+ \nu \bar{\nu}$ branching ratio*, Phys. Rev. Lett. 101 (2008) 191802 [0808.2459].

– 33 –
[51] LHCb collaboration, *Search for hidden-sector bosons in $B^0 \to K^{*0} \mu^+ \mu^-$ decays*, *Phys. Rev. Lett.* 115 (2015) 161802 [1508.04094].

[52] LHCb collaboration, *Search for long-lived scalar particles in $B^+ \to K^+ \chi(\mu^+ \mu^-)$ decays*, *Phys. Rev. D* 95 (2017) 071101 [1612.07818].

[53] CHARM collaboration, *Search for Axion Like Particle Production in 400-GeV Proton - Copper Interactions*, *Phys. Lett. B* 157 (1985) 458.

[54] MicroBooNE collaboration, *Search for a Higgs Portal Scalar Decaying to Electron-Positron Pairs in the MicroBooNE Detector*, *Phys. Rev. Lett.* 127 (2021) 151803 [2106.00568].

[55] NA62 collaboration, *Measurement of the very rare $K^+ \to \pi^+ \nu \bar{\nu}$ decay*, *JHEP* 06 (2021) 093 [2103.15389].

[56] M.W. Winkler, *Decay and detection of a light scalar boson mixing with the Higgs boson*, *Phys. Rev. D* 99 (2019) 015018 [1809.01876].

[57] B. Batell, J. Berger and A. Ismail, *Probing the Higgs Portal at the Fermilab Short-Baseline Neutrino Experiments*, *Phys. Rev. D* 100 (2019) 115039 [1909.11670].

[58] B. Batell, J.A. Evans, S. Gori and M. Rai, *Dark Scalars and Heavy Neutral Leptons at DarkQuest*, *JHEP* 05 (2021) 049 [2008.08108].

[59] G. Lanfranchi, M. Pospelov and P. Schuster, *The Search for Feebly-Interacting Particles*, 2011.02157.

[60] J.H. Chang, R. Essig and S.D. McDermott, *Supernova 1987A Constraints on Sub-GeV Dark Sectors, Millicharged Particles, the QCD Axion, and an Axion-like Particle*, *JHEP* 09 (2018) 051 [1803.00993].

[61] V. Poulin, J. Lesgourgues and P.D. Serpico, *Cosmological constraints on exotic injection of electromagnetic energy*, *J. Cosmol. Astropart. Phys.* 03 (2017) [1610.10051].

[62] M. Lucca, N. Schöneberg, D.C. Hooper, J. Lesgourgues and J. Chluba, *The synergy between CMB spectral distortions and anisotropies*, *JCAP* 02 (2020) 026 [1910.04619].

[63] D.J. Fixsen, E.S. Cheng, J.M. Gales, J.C. Mather, R.A. Shafer and E.L. Wright, *The Cosmic Microwave Background spectrum from the full COBE FIRAS data set*, *Astrophys. J.* 473 (1996) 576 [astro-ph/9605054].

[64] T.R. Slatyer, *Indirect dark matter signatures in the cosmic dark ages. I. Generalizing the bound on s-wave dark matter annihilation from Planck results*, *Phys. Rev. D* 93 (2016) 023527 [1506.03811].

[65] T.R. Slatyer, *Indirect Dark Matter Signatures in the Cosmic Dark Ages II. Ionization, Heating and Photon Production from Arbitrary Energy Injections*, *Phys. Rev. D* 93 (2016) 023521 [1506.03812].

[66] KLEVER Project collaboration, *KLEVER: An experiment to measure BR$(K_L \to \pi^0 \nu \bar{\nu})$ at the CERN SPS*, 1901.03099.

[67] T. Nomura, *A future $K_L^0 \to \pi^0 \nu \bar{\nu}$ experiment at J-PARC*, *J. Phys. Conf. Ser.* 1526 (2020) 012027.

[68] K. Bondarenko, A. Boyarsky, T. Bringmann, M. Hufnagel, K. Schmidt-Hoberg and A. Sokolenko, *Direct detection and complementary constraints for sub-GeV dark matter*, *JHEP* 03 (2020) 118 [1909.08632].

– 34 –
[69] S. Alekhin et al., A facility to Search for Hidden Particles at the CERN SPS: the SHiP physics case, Rept. Prog. Phys. 79 (2016) 124201 [1504.04855].

[70] V.V. Gligorov, S. Knapen, M. Papucci and D.J. Robinson, Searching for Long-lived Particles: A Compact Detector for Exotics at LHCb, Phys. Rev. D 97 (2018) 015023 [1708.09395].

[71] G. Aielli et al., Expression of interest for the CODEX-b detector, Eur. Phys. J. C 80 (2020) 1177 [1911.00481].

[72] J.L. Feng, I. Galon, F. Kling and S. Trojanowski, ForwArd Search ExpeRiment at the LHC, Phys. Rev. D 97 (2018) 015023 [1708.09395].

[73] FASER collaboration, FASER’s physics reach for long-lived particles, Phys. Rev. D 99 (2019) 095011 [1811.12522].

[74] J.P. Chou, D. Curtin and H.J. Lubatti, New Detectors to Explore the Lifetime Frontier, Phys. Lett. B 767 (2017) 29 [1606.06298].

[75] D. Curtin et al., Long-Lived Particles at the Energy Frontier: The MATHUSLA Physics Case, Rept. Prog. Phys. 82 (2019) 116201 [1806.07396].

[76] A. Filimonova, R. Schäfer and S. Westhoff, Probing dark sectors with long-lived particles at BELLE II, Phys. Rev. D 101 (2020) 095006 [1911.03490].

[77] A. Kachanovich, U. Nierste and I. Nišandžić, Higgs portal to dark matter and $B \to K^{(*)}$ decays, Eur. Phys. J. C 80 (2020) 669 [2003.01788].

[78] J. Mardon, Y. Nomura, D. Stolarski and J. Thaler, Dark Matter Signals from Cascade Annihilations, JCAP 05 (2009) 016 [0901.2926].

[79] G. Elor, N.L. Rodd and T.R. Slatyer, Multistep cascade annihilations of dark matter and the Galactic Center excess, Phys. Rev. D 91 (2015) 103531 [1503.01773].

[80] L. Bergstrom, T. Bringmann, I. Cholis, D. Hooper and C. Weniger, New Limits on Dark Matter Annihilation from AMS Cosmic Ray Positron Data, Phys. Rev. Lett. 111 (2013) 171101 [1306.3983].

[81] I. John and T. Linden, Cosmic-Ray Positrons Strongly Constrain Leptophilic Dark Matter, 2107.10261.

[82] L. Pieri, J. Lavalle, G. Bertone and E. Branchini, Implications of High-Resolution Simulations on Indirect Dark Matter Searches, Phys. Rev. D 83 (2011) 023518 [0908.0195].

[83] CTA Consortium collaboration, B.S. Acharya et al., Science with the Cherenkov Telescope Array, WSP (11, 2018), 10.1142/10986, [1709.07997].

[84] A. Hryczuk, K. Jodłowski, E. Moulin, L. Rinchiuso, L. Roszkowski, E.M. Sessolo et al., Testing dark matter with Cherenkov light - prospects of H.E.S.S. and CTA for exploring minimal supersymmetry, JHEP 10 (2019) 043 [1905.00315].

[85] CTA collaboration, Sensitivity of the Cherenkov Telescope Array to a dark matter signal from the Galactic centre, JCAP 01 (2021) 057 [2007.16129].

[86] C. Siqueira, G.N. Fortes, F.S. Queiroz and A. Viana, Indirect Searches for Secluded Dark Matter, 2107.04053.

[87] S. Profumo, F.S. Queiroz, J. Silk and C. Siqueira, Searching for Secluded Dark Matter with H.E.S.S., Fermi-LAT, and Planck, JCAP 03 (2018) 010 [1711.03133].
[88] K.N. Abazajian, S. Horiuchi, M. Kaplinghat, R.E. Keeley and O. Macias, Strong constraints on thermal relic dark matter from Fermi-LAT observations of the Galactic Center, Phys. Rev. D 102 (2020) 043012 [2003.10416].

[89] Fermi-LAT collaboration, Searching for Dark Matter Annihilation from Milky Way Dwarf Spheroidal Galaxies with Six Years of Fermi Large Area Telescope Data, Phys. Rev. Lett. 115 (2015) 231301 [1503.02641].

[90] Fermi-LAT, DES collaboration, Searching for Dark Matter Annihilation in Recently Discovered Milky Way Satellites with Fermi-LAT, Astrophys. J. 834 (2017) 110 [1611.03184].

[91] S. Cassel, Sommerfeld factor for arbitrary partial wave processes, J. Phys. G: Nucl. Part. Phys. 37 (2010) 105009 [0903.5307].

[92] R. Iengo, Sommerfeld enhancement: General results from field theory diagrams, JHEP 05 (2009) 024 [0902.0688].

[93] T.R. Slatyer, The Sommerfeld enhancement for dark matter with an excited state, JCAP 1002 (2010) 028 [0910.5713].

[94] A. Kogut et al., The Primordial Inflation Explorer (PIXIE): A Nulling Polarimeter for Cosmic Microwave Background Observations, JCAP 07 (2011) 025 [1105.2044].

[95] PRISM collaboration, PRISM (Polarized Radiation Imaging and Spectroscopy Mission): An Extended White Paper, JCAP 02 (2014) 006 [1310.1554].

[96] CMB-S4 collaboration, CMB-S4 Science Book, First Edition, 1610.02743.

[97] T. Matsumura et al., Mission design of LiteBIRD, J. Low Temp. Phys. 176 (2014) 733 [1311.2847].

[98] P. Lepage, “gplepage/vegas: vegas version 4.0.1.” https://doi.org/10.5281/zenodo.4746454, May, 2021.

[99] G.P. Lepage, A New Algorithm for Adaptive Multidimensional Integration, J. Comput. Phys. 27 (1978) 192.

[100] G.P. Lepage, Adaptive multidimensional integration: VEGAS enhanced, J. Comput. Phys. 439 (2021) 110386 [2009.05112].

[101] F. Bezrukov and D. Gorbunov, Light inflaton Hunter’s Guide, J. High Energ. Phys. 2010 (2010) 10 [0912.0390].

[102] H.H. Patel, Package-X 2.0: A Mathematica package for the analytic calculation of one-loop integrals, Comput. Phys. Commun. 218 (2017) 66 [1612.00009].

[103] J.C. Romao, “Techniques for Calculations in Quantum Field Theory: W-strahlung in e colliders.” https://porthos.tecnico.ulisboa.pt/CTQFT/files/Wstrahlung.pdf.

[104] M. Cirelli, G. Corcella, A. Hektor, G. Hutsi, M. Kadastik, P. Panci et al., PPC 4 DM ID: A Poor Particle Physicist Cookbook for Dark Matter Indirect Detection, JCAP 03 (2011) 051 [1012.4515].

[105] J.-F. Fortin, J. Shelton, S. Thomas and Y. Zhao, Gamma Ray Spectra from Dark Matter Annihilation and Decay, 0908.2258.