On the direct evidence of time-reversal non-invariance
in the $K^0 - \bar{K}^0$ system

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Abstract

The measurements of the CP-violation parameters by the CPLEAR experiment are
reviewed. It is shown that attempts to prove T-violation from the semileptonic asymmetries
are flawed by logical inconsistencies.

1 Introduction

The CPLEAR Collaboration has recently published a series of papers [1, 2, 3, 4] reporting high
precision measurements of the parameters which describe the CP-violation in the $K^0 - \bar{K}^0$
system. In the first paper [1], the measurement of the semileptonic decay-rate asymmetry,

$$A_T(\tau) = \frac{R(\bar{K}_t^0 \to e^+\pi^-\nu_{t=\tau}) - R(\bar{K}_t^0 \to e^-\pi^+\bar{\nu}_{t=\tau})}{R(\bar{K}_t^0 \to e^+\pi^-\nu_{t=\tau}) + R(\bar{K}_t^0 \to e^-\pi^+\bar{\nu}_{t=\tau})}$$  \hspace{1cm} (1)

is presented. Assuming CPT-symmetry in the semileptonic decays and the $\Delta S = \Delta Q$ rule, the
$A_T$ asymmetry is identified to the Kabir asymmetry [5],

$$A_K = \frac{P(K^0 \to K^0) - P(\bar{K}^0 \to \bar{K}^0)}{P(K^0 \to K^0) + P(\bar{K}^0 \to \bar{K}^0)}$$  \hspace{1cm} (2)

and the measured value $A_T = (6.6 \pm 1.64) \times 10^{-3}$ is claimed to be a direct evidence for time-
reversal non-invariance.

The validity of the identification of $A_T$ and $A_K$ has been discussed from diverse points of
view [6, 7, 8], and it has been asserted [6, 7, 9] that its proof was established by a subsequent
measurement [3]. We discuss here this assertion.

2 Semileptonic asymmetries

Let us first define the notations. The $K_S$ and $K_L$ states are written

$$K_S = K_1 + \epsilon_S K_2$$
$$K_L = K_2 + \epsilon_L K_1$$  \hspace{1cm} (3)

in terms of the CP-eigenstates $K_1$ and $K_2$, and the $\epsilon_{S,L}$ are decomposed into

$$\epsilon_S = \epsilon + \delta, \quad \epsilon_L = \epsilon - \delta$$  \hspace{1cm} (4)
The violations of CP and \(\Delta S = \Delta Q\) in the semileptonic decays are described by the parameters \(y, x_+\) and \(x_-\). The constraints from \(\Delta S = \Delta Q\), CPT-, CP- and T-conservation are:

\[
\Delta S = \Delta Q \\
CPT \quad \delta = y = x_+ = 0 \\
CP \quad \text{Re}(y) = \text{Re}(x_-) = \text{Im}(x_+) = 0 \\
T \quad \text{Re}(\epsilon) = \text{Im}(y) = \text{Im}(x_+) = \text{Im}(x_-) = 0
\]  

(5) 
(6) 
(7) 
(8)

Only first order terms in the small parameters, \(\epsilon, \delta, x_+\) and \(y\) are retained in the theoretical expressions.

Writing \(\tilde{R}_+ = \tilde{R}_+(\tau)\) instead of \(R(\tilde{K}_t^0 \rightarrow e^+\pi^-\nu_{e}\tau)\), \(\tilde{N}_+\) the corresponding number of decays and \(\tilde{N}\) the total number of \(\tilde{K}_0^0\),

\[
A_T = \frac{\tilde{R}_+ - \tilde{R}_-}{\tilde{R}_+ + \tilde{R}_-} = \frac{\tilde{N}_+ - \tilde{N}_-}{\tilde{N}_+ + \tilde{N}_-}
\]  

(9)

In order to get a high precision determination of \(\tilde{N}/\tilde{N}\), the CPLEAR Collaboration uses its measurements of the \(\pi^+\pi^-\) decays. Since the decay-ratios must follow the law:

\[
R_{\pi^+\pi^-}^{K_0/K_0^0} = C(1 \mp 2\text{Re}(\epsilon_L))[e^{-\Gamma_S\tau} + |\eta_{+\mp}|^2 e^{-\Gamma_L\tau} \pm 2|\eta_{+\mp}| e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)\tau}\cos(\delta m\tau - \phi_{+\mp})]
\]  

(10)

it is required that the parameter \(\text{Re}(\epsilon_L)\) determined from the \(\pi^+\pi^-\) decays is equal to \(\delta / 2\), where \(\delta / 2\) is the measured charge asymmetry of the \(K_L\). The constraint can be written:

\[
(1 + \delta / 2)\tilde{N}_+ \tilde{N}_{\pi\bar{\pi}} = (1 - \delta / 2)\tilde{N}_- \tilde{N}_{\pi\bar{\pi}}
\]

and the experimental asymmetry is:

\[
A_T^{\exp}(\tau) = \frac{N_{\pi\bar{\pi}} N_+(\tau)(1 + \delta / 2) - N_{\pi\bar{\pi}} N_-(\tau)(1 - \delta / 2)}{N_{\pi\bar{\pi}} N_+(\tau)(1 + \delta / 2) + N_{\pi\bar{\pi}} N_-(\tau)(1 - \delta / 2)} = \frac{N_{\pi\bar{\pi}} \tilde{N}_+(\tau) - N_{\pi\bar{\pi}} \tilde{N}_-(\tau)}{N_{\pi\bar{\pi}} \tilde{N}_+(\tau) + N_{\pi\bar{\pi}} \tilde{N}_-(\tau)} + \delta / 2
\]  

(11)

neglecting second order in the CP-violation.

In a following paper \cite{2}, the CPT-violating parameter \(\text{Re}(\delta)\) is measured by the use of another asymmetry:

\[
A_\delta(\tau) = A_1(\tau) + A_2(\tau)
\]  

(12)

where the \(A_1\) and \(A_2\) asymmetries are defined by:

\[
A_1(\tau) = \frac{\tilde{R}_+(\tau) - \tilde{R}_-(\tau)(1 + 4\text{Re}(\epsilon_L))}{\tilde{R}_+(\tau) + \tilde{R}_-(\tau)(1 + 4\text{Re}(\epsilon_L))}, \quad A_2(\tau) = \frac{\tilde{R}_-(\tau) - \tilde{R}_+(\tau)(1 + 4\text{Re}(\epsilon_L))}{\tilde{R}_-(\tau) + \tilde{R}_+(\tau)(1 + 4\text{Re}(\epsilon_L))}
\]  

(13)

In order to get rid of the normalizations, \(N\) and \(\tilde{N}\), the parameter \(\text{Re}(\epsilon_L)\) is extracted from the measured \(\pi^+\pi^-\) decay rates and the experimental expressions of \(A_1\) and \(A_2\) are:

\[
A_1(\tau) = \frac{N_{\pi\bar{\pi}} \tilde{N}_+(\tau) - N_{\pi\bar{\pi}} \tilde{N}_-(\tau)}{N_{\pi\bar{\pi}} \tilde{N}_+(\tau) + N_{\pi\bar{\pi}} N_-(\tau)}, \quad A_2(\tau) = \frac{N_{\pi\bar{\pi}} \tilde{N}_-(\tau) - N_{\pi\bar{\pi}} \tilde{N}_+(\tau)}{N_{\pi\bar{\pi}} \tilde{N}_-(\tau) + N_{\pi\bar{\pi}} \tilde{N}_+(\tau)}
\]  

(14)

With the above notations, the \(A_T^{\exp}\) asymmetry is:

\[
A_T^{\exp}(\tau) = A_1(\tau) + \delta / 2
\]  

(15)

The theoretical expressions of the asymmetries, without any assumption on CPT or \(\Delta S = \Delta Q\), are \cite{3, 14, 11, 13}:

\[
A_1(\tau) = 2\text{Re}(\epsilon - y - x_-) + 2\text{Re}(\delta) + F_1(\tau; \text{Im}(x_+), \text{Re}(x_-))
\]  

(16)

\[
A_2(\tau) = -2\text{Re}(\epsilon - y - x_-) + 6\text{Re}(\delta) + F_2(\tau; \delta, \text{Im}(x_+), \text{Re}(x_-))
\]  

(17)

\[
\delta / 2 = 2\text{Re}(\epsilon - y - x_-) - 2\text{Re}(\delta)
\]  

(18)
with, for \( \tau \gg \tau_S \), \( F_1 = F_2 = 0 \).

The \( \delta_l \) asymmetry and the time-independent parts of \( A_1 \) and \( A_2 \) carry information on two parameters only, \( \text{Re}(\epsilon - y - x_-) \) and \( \text{Re}(\delta) \). The \( A_{T}^{\text{exp}} \) and \( A_3 \) asymmetries are two of their possible estimators; the optimal ones can be obtained, for instance, by a simultaneous fit of the three asymmetries.

The time-dependent parts bring information on \( \text{Im}(\delta) \), \( \text{Re}(x_-) \) and \( \text{Im}(x_+) \) and some additional information on \( \text{Re}(\delta) \). With the normalization procedures used by CPLEAR (see appendix) there is no independent information on \( \epsilon \) and \( y \) in the decay-ratio asymmetries.

The proof of T-violation (\( \text{Re}(\epsilon) \neq 0 \)) from \( A_{T}^{\text{exp}} \) requires a measurement of \( \text{Re}(y + x_-) \). Such a measurement has been given by CPLEAR [8] but, to be used, it must be independent of \( A_{T}^{\text{exp}} \).

It is easy to check that, on the contrary, the measurement of \( \text{Re}(y + x_-) \) is given by \( A_{T}^{\text{exp}} \).

### 3 Global fit

The determination of the CP-violation parameters in [3] is made through a global fit based on the Bell-Steinberger relation [13]:

\[
\text{Re}(\epsilon) - i \text{Im}(\delta) = \left[ \frac{\Gamma_L + \Gamma_S + 2i(m_L - m_S)}{\Sigma \gamma_{S,L}^\pi \eta_{S,L} + \Sigma \gamma_{S,L}^{3\pi} \eta_{S,L} + 2\gamma_{L}^{\pi \nu}(\text{Re}(\epsilon) - y - i \text{Im}(x_+ + \delta))] \right. \tag{19}
\]

where \( \gamma_{S,L}^\pi \) is the partial width of the \( K_{S,L} \) in the final state \( f \).

The experimental data entered in the fit are the partial widths, the \( \eta \) amplitudes and also the semileptonic asymmetries, \( A_1(\tau) \), \( A_2(\tau) \) and \( \delta_l \). All the information used to measure \( A_{T}^{\text{exp}} \) and \( A_3 \) [1, 2] is also exploited, in an optimal way, in the global fit.

As mentioned before the simultaneous fit of the semileptonic asymmetries gives optimized measurements of \( \text{Re}(\delta) \) and \( \text{Re}(\epsilon - y - x_-) \), from the constant terms, of \( \text{Re}(x_-) \) and \( \text{Im}(x_+) \) from the time-dependence. They can be used to compute the quantity \( \text{Re}(\epsilon - y) - i \text{Im}(x_+) \) appearing on the right-hand side of the Bell-Steinberger relation which gives \( \text{Re}(\epsilon) \) and \( \text{Im}(\delta) \). However, the semileptonic contribution to eq. (19) being tiny, the parameters \( \text{Re}(\epsilon) \) and \( \text{Im}(\delta) \) are, in a very good approximation, independent of the others and determined, as usual, by the hadronic decays [8].

The only reason to include the Bell-Steinberger relation in a global fit is to constrain the fit of the time-dependent part of \( A_2 \) by the knowledge of \( \text{Im}(\delta) \). The errors on the already measured parameters can be slightly improved but there is no spontaneous generation of information on new parameters. Anyway, the value of the correlation coefficient \( C(\text{Re}(y), \text{Re}(x_-)) = -0.997 \) [3] shows that the additional information from the time-dependence is very small.

So the parameter \( \text{Re}(\delta) \) is determined by the \( A_3 \) asymmetry, and the parameter \( \text{Re}(\epsilon - y - x_-) \) by \( A_{T}^{\text{exp}} \). More exactly, they are determined by the optimal combinations of \( \delta_l \) and the time-independent parts of \( A_1 \) and \( A_2 \), constructed by the fit. The measurement of \( \text{Re}(y + x_-) \) is nothing else that the difference of \( \text{Re}(\epsilon - y - x_-) \) measured by the optimized \( A_{T}^{\text{exp}} \) and \( \text{Re}(\epsilon) \) measured by the hadronic decays through the Bell-Steinberger relation. Using it to correct the measurement of \( A_{T}^{\text{exp}} \) for the possible CPT-violation in the decay would just give back the value of \( \text{Re}(\epsilon) \) provided by the Bell-Steinberger relation.

### 4 Conclusion

In the very complete and precise set of measurements [1, 2, 3, 4] published by the CPLEAR Collaboration, the \( A_{T}^{\text{exp}} \) asymmetry serves to measure neither the T-violating parameter \( \text{Re}(\epsilon) \) [8] nor \( \text{Re}(\delta) \) [8] but the CPT-violating parameter \( \text{Re}(y + x_-) \). There is no logical way to use the measured \( A_{T}^{\text{exp}} \) as a direct evidence of time-reversal non-invariance without assuming CPT-conservation in the semileptonic decays. The proof of T-violation still rests on the Bell-Steinberger relation.
5 Appendix

In the following, each time-dependent asymmetry $A(\tau)$ is decomposed into a constant, asymptotic part $\tilde{A}$ and a time-dependent part $F_a$:

$$A(\tau) = \tilde{A} + F_a(\tau), \quad F_a = 0 \quad \text{for} \quad \tau \gg \tau_S$$  \hspace{1cm} (20)

From the four semileptonic decay-ratios, $R_+, R_-, \tilde{R}_+$ and $\tilde{R}_-$, three independent asymmetries can be constructed. One is the CP-conserving $A_{\delta m}$:

$$A_{\delta m} = \left( \frac{R_+ + \tilde{R}_- - R_- - \tilde{R}_+}{R_+ + \tilde{R}_- + R_- + \tilde{R}_+} \right) = F_{\delta m}(\tau; \text{Re}(x_+), \text{Im}(x_-))$$  \hspace{1cm} (21)

Assuming $\text{Im}(x_-) = 0$, it has been used by CPLEAR [1] to measure $|\delta m|$ and $\text{Re}(x_+)$. The two others are CP-violating, for instance:

$$A_T(\tau) = \left( \frac{\tilde{R}_+ - R_-}{R_+ + \tilde{R}_-} \right) = 4\text{Re}(\epsilon) - 2\text{Re}(y + x_-) + F_T(\tau)$$  \hspace{1cm} (22)

$$A_{CP T}(\tau) = \left( \frac{\tilde{R}_- - R_+}{R_- + \tilde{R}_+} \right) = 4\text{Re}(\delta) + 2\text{Re}(y + x_-) + F_{CP T}(\tau)$$  \hspace{1cm} (23)

with $F_T = F_1$ and $F_{CP T} = F_2$. Their asymptotic values give:

$$\tilde{A}_T + \tilde{A}_{CP T} = 4\text{Re}(\epsilon), \quad \tilde{A}_T - \tilde{A}_{CP T} = 2\delta_l$$  \hspace{1cm} (24)

Since the measurement of the $R_{\pi^+\pi^-}$ and $\tilde{R}_{\pi^+\pi^-}$ decay-ratios allows (eq. 10) a determination of $\text{Re}(\epsilon_L) = \text{Re}(\epsilon - \delta)$, the T-violating parameter $\text{Re}(\epsilon)$ can be determined from single-channel decay-ratio measurements.

The determination of $\text{Re}(\epsilon)$ sketched above rests, however, on the assumption that the total numbers of $K^0$ and $\bar{K}^0$ are exactly known. If they are not, and the $\pi^+\pi^-$ decay-channel is used as normalization [2,3], the $A_T$ and $A_{CP T}$ asymmetries are replaced by the previously defined

$$A_1 = A_T - 2\text{Re}(\epsilon_L) \quad \text{and} \quad A_2 = A_{CP T} - 2\text{Re}(\epsilon_L)$$

with

$$\tilde{A}_1 + \tilde{A}_2 = 8\text{Re}(\delta), \quad \tilde{A}_1 - \tilde{A}_2 = 2\delta_l$$  \hspace{1cm} (25)

and the independent information from $R_{\pi^+\pi^-}/\tilde{R}_{\pi^+\pi^-}$ is lost.

The determination of $\text{Re}(\delta)$, which is the greatest achievement of the CPLEAR measurement of semileptonic decays, is still possible but there is no more information in the remaining part of the asymptotic asymmetries than in $\delta_l$.

The connection of $A_T^{exp}$ to a measurement of $\text{Re}(\delta)$ mentioned in 8 can be described in a different way. $A_T^{exp}$ is an estimator of $4\text{Re}(\epsilon - y - x_-)$ constructed from $\text{Re}(\delta)$, given by the semileptonic asymmetries measured by CPLEAR, and from $\delta_l$, given by a combination of CPLEAR asymmetries and the PDG value. As stated before, this estimator is not the best one; for instance, using simply the PDG value [4] of $\delta_l$ and the CPLEAR measurement [2] of $\text{Re}(\delta)$, gives $4\text{Re}(\epsilon - y - x_-) = 2\delta_l + \tilde{A}_\delta/2 = (7.74 \pm 1.36) \times 10^{-3}$.

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