Determination of earth's gravitational acceleration and moment of inertia of rigid body using physical pendulum experiments

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Abstract. To determine the value of earth's gravitational acceleration, physical pendulum experiment is preferred over simple pendulum experiment that was previously more popular. In physical pendulum experiment, the object is a rigid body. The swing period and swing shaft position are measured. From these two variables and using the applicable laws of motion, equations can be determined that involve constants which imply the value of the earth's gravitational acceleration and the moment of inertia of objects pivoting at the center of mass. The independent variable in the equation is a function of the distance of the pendulum shaft to the center of mass based on the parallel-axis theorem, while the dependent variable is a swing period. Difficulty in solving equations is found because of the terms of the equation involving variables in different ranks. In order for the equation to be linear, a dependent variable and an independent variable are combined into new variables. After that, with the linear regression method, the constants of the equation can be determined which can then be determined the value of the earth's gravitational acceleration and the moment of inertia of the object. The results obtained are quite satisfactory with a relatively smaller error value of 5%. Thus this physical pendulum experiment can be used to determine the earth's gravitational acceleration and the moment of inertia of an object simultaneously.

1. Introduction

In everyday practice, a gravitational acceleration can be determined using simple pendulum experiment, free fall motion, or conservation of mechanical energy on a swing. A simple pendulum consists of a certain object with a mass suspended using a lightweight string of a certain length. The shape and size of the object can be ignored, but its mass must be considered. Similarly, the mass of the string, must be ignored. In addition, when the object is swinging, the angle of displacement must be small so that the movements are classified as simple harmonic motion. In the experiment of free fall and the conservation of mechanical energy, conditions for measuring period are automatically required, both when the object starts to fall and when the object reaches a certain distance, as well as the neglect of forces other than the earth’s gravitational force.
Simple Pendulum Experiments are often practiced in schools and in the Basic Physics Laboratory in colleges, so that the concept of simple pendulum is well-known to both students and college students. Meanwhile, the fact that the mass distribution of object is not the same, and cannot be ignored by the mass of the string, causes the concept of "the length of the pendulum " in the formula of the pendulum period to become a complex question [1].

In fact, it is difficult to meet the conditions required for a simple pendulum, especially the string mass and the shape of the object which is very difficult to ignore. Similarly, in experiments of free fall and the conservation of mechanical energy, especially in the neglect of non-gravitational forces, as well as the accuracy of practical free fall period measurements. From the free fall motion experiment, it is obtained an average value of earth’s gravitational acceleration of 9.19 m/s². The value of the earth's gravitational acceleration obtained in the free fall motion experiment is too small when compared to its supposed value. This is caused by a source of error in the form of remanence of the magnetic field which results in a delay when the falling object is made of metal [2].

Determination of the earth’s gravitational acceleration can also be done by using a rubber ball that is reflected by measuring the time of contact and the height of its reflection. The relative error of g obtained is 2% - 3%. In this experiment, problems related to contact time and the effect of elasticity of the rubber ball on temperature arose[3]. Whereas the determination of the moment of inertia of rigid objects has been carried out by Spurr et al. The rigid object used is a tennis rackets [4].

In previous studies, the determination of the earth's gravitational acceleration and the moment of inertia of rigid objects used different tools. However, in this study the determination of the earth's gravitational acceleration and moment of inertia used a set of experimental devices. Thus, the purpose of this study is to determine the acceleration of the earth's gravity and the moment of inertia simultaneously using a physical pendulum experiment which covers two concepts. By using this tool, it is expected that this physical pendulum experiment will become an effective and efficient practicum module.

2. Methods
The method used in this research is descriptive analysis of data collection at the Applied Physics Laboratory of Politeknik Negeri Bandung with the concept of physical pendulum vibrations. The data observed from the concept of vibration using physical pendulum is the period (T) of a homogeneous rod with a mass of 0.465 kg and a length of 0.75 m with variations in the distance of the rotary axis to the center of mass (d) every 0.025 m. The data processing method uses linear regression without the weight of the relationship $dT^2$ to $d^2$.

3. Results and discussion

3.1. Simple harmonic rotation motion
Physical pendulum is a rigid object, with a certain shape and size, which is swung by a pivot point on the object. Swing motion is a simple harmonic rotational motion with a small deviation angle so that it meets $\sin \theta = \tan \theta = \theta$ with the cause of motion is only torque due to the gravity of the object.

Simple harmonic motion is the alternating motion of objects through a certain equilibrium point with a constant motion period. While the rotational motion is the motion that experiences rotation of a shaft. Simple harmonic rotational motion is characterized by the alternating motion with the trajectory being part of a rotation, with a constant period of motion. This rotational motion is caused by the torque $\tau$ which causes the object to accelerate the angle $\alpha$ based on the equation:

$$\sum \tau = I\alpha$$  \hspace{1cm} (1)

In objects that swing with a particular swing shaft on the object, the cause of the motion is torque due to the gravity. When the object is deviated from the angle deviation $\theta$ from the balanced state, torque works for

$$\tau = -mgd \sin \theta$$  \hspace{1cm} (2)
here \( m \) is the mass of the object, \( g \) is the acceleration of gravity, \( d \) is the distance of the swing shaft to the center of mass. From equations (1) and (2), it is obtained

\[
-mgd \sin \theta = I \alpha
\]

The angular acceleration is the second derivative of the angle on time \( d^2\theta/dt^2 = \alpha \). For a simple aligned rotational motion, it applies

\[
\alpha = -\omega^2 \theta.
\]

If the equation (3) is substituted for equation (2), it will be obtained

\[
-Mgd \sin \theta = -I \omega^2 \theta
\]

and for a small angle \( \theta \), \( \sin \theta \approx \theta \) applies, it is obtained

\[
\omega = \sqrt{\frac{mgd}{I}}
\]

\[
T = 2\pi \sqrt{\frac{I}{mgd}}.
\]

\( T \) is the physical pendulum period [5][6][7].

3.2. Physical pendulum experiment

The physical pendulum tried in this research is a homogeneous straight rod, with several holes in a certain position for the placement of the swing axis (pivot point) as shown in Figure 1. In this figure, it is shown that \( G \) is the centre of mass of the object, \( P \) is the swing axis which its position can be moved, \( PG \) is \( d \), and the length of the stem is \( L \).

![Figure 1. Physical pendulum.](image)

If the rod is swung freely with the \( P \) axis which coincides with \( G \), then based on equation 4, the swing period is

\[
T = 2\pi \sqrt{\frac{I}{mgd}}
\]

where \( m \) is the mass of the rod, and \( g \) is the acceleration of gravity. Because the pendulum used is a straight rod along \( L \) which is homogeneous with mass \( m \), with the shaft located at the end of the stem, then

\[
d = \frac{L}{2}
\]

and

\[
I = \frac{1}{12} mL^2
\]

So, the swing period can also be written as

\[
T = 2\pi \sqrt{\frac{I + md^2}{mgd}}
\]

If the swing axis \( P \) does not coincide with the center of mass \( G \) then the rod will have a moment of inertia with the \( P \) axis equals to \( I_P \), which based on parallel-axis theorem (Steiner) equals to

\[
I_P = I + m d^2
\]

where \( I \) represents the moment of inertia of an object with a shaft through the center of mass \( G \).
Equation (10) can be written in the form of:

$$T^2 = \frac{4\pi^2 l}{mg} + \frac{4\pi^2}{g} d$$

(12)

By changing the position of the shaft, which also simultaneously changes the value of $d$, then the swing period $T$ will also change. If $T^2d$ and $d^2$ are used as new variables, the equation is:

$$dT^2 = \frac{4\pi^2 l}{mg} + \frac{4\pi^2}{g} d^2$$

(13)

Assuming $d^2 = x$, and $T^2d = y$, the above equation can be written as a linear equation:

$$Y = A + Bx$$

with

$$A = \frac{4\pi^2 l}{mg}$$

(14)

and

$$B = \frac{4\pi^2}{g}$$

(15)

### 3.3. Data processing results

Below is presented the results of the Physical Pendulum experiment using a straight rod with a mass of 0.465 kg with a length of 0.75 m and rotated on a shaft (P) which is shifted every 0.025 m from the center of mass (point G).

**Table 1.** Data from the processing of physical pendulum experiments.

| Experiment | Distance between the pivot point and the center of mass $d$ [m] | Angle of swing [degree] | Period $T$ [s] | $T^2$ [s$^2$] | $d^2$ [m$^2$] | $dT^2$ [m.s$^2$] |
|------------|---------------------------------------------------------------|-------------------------|----------------|---------------|----------------|------------------|
| 1          | 0.350                                                         | 10                      | 1.360          | 1.850         | 0.1225         | 0.6474           |
| 2          | 0.325                                                         | 10                      | 1.381          | 1.906         | 0.1056         | 0.6195           |
| 3          | 0.300                                                         | 10                      | 1.350          | 1.823         | 0.0900         | 0.5546           |
| 4          | 0.275                                                         | 10                      | 1.333          | 1.777         | 0.0756         | 0.4886           |
| 5          | 0.250                                                         | 10                      | 1.330          | 1.769         | 0.0625         | 0.4422           |
| 6          | 0.200                                                         | 10                      | 1.320          | 1.742         | 0.0400         | 0.3527           |
| 7          | 0.188                                                         | 10                      | 1.332          | 1.775         | 0.0352         | 0.3328           |
| 8          | 0.175                                                         | 10                      | 1.342          | 1.800         | 0.0306         | 0.3150           |
| 9          | 0.150                                                         | 10                      | 1.362          | 1.855         | 0.0225         | 0.2783           |
| 10         | 0.125                                                         | 10                      | 1.415          | 2.002         | 0.0156         | 0.2503           |
| 11         | 0.100                                                         | 10                      | 1.523          | 2.320         | 0.0100         | 0.2320           |
| 12         | 0.075                                                         | 10                      | 1.725          | 2.977         | 0.0056         | 0.2233           |
| 13         | 0.050                                                         | 10                      | 1.990          | 3.960         | 0.0025         | 0.1967           |

From table 1 above, a $dT^2$ graph is made as the y-ordinate and $d^2$ as the x-abscissa as shown in figure 2.
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Figure 2. Graph $dT^2$ to $d^2$ from a rod-shaped physical pendulum experiment.

By using Linear Regression or commonly known as the Least Squares Method, from figure 2, the value of $A = 0.1909$ and $B = 4.0246$ is obtained. The value of $A$ is inserted to equation (14) and the value of $B$ is inserted to equation (15). Thus, the value of the gravitational acceleration $g$, and moment of inertia of rod I, are 9,799 m/s² and 0,0220 kg m². When compared to the actual value of gravitational acceleration based on Lowrie [8] which is 9.728 m/s², as well as the moment of inertia of the straight rod which is theoretically calculated using equation 9, which is equal to 0.0218 kg m² then a relative error of the gravitational acceleration value is obtained by 0,1% and relative error of stem moment of inertia values of 1.2%. Thus, the physical pendulum experiment is significant to be used to determine the value of gravitational acceleration and the moment of inertia. Therefore this experiment is feasible to become a topic in the Basic Physics Practicum at the college of science or engineering.

Physical pendulum experiment can be developed to determine the moment of inertia of objects other than rods. The object is attached to the physical pendulum rod in any position. The combined system of objects is swung freely, so that a period is obtained, and then the combined moment of inertia of the objects can be calculated, and finally the inertia moment of the objects can be determined by considering the axis shift in accordance with the parallel-axis theorem [9]. Spurr et al. have used this principle to determine the inertia of a tennis racket. The oscillation was recorded using a Panasonic HDC-SD9 camcorder with 25 frames per second as much as 50 times of oscillations. The accuracy of inertia moment measurement of the racket is 99% [4]. However, it should be noted that limitation of the angle deviation of the pendulum[10], because if the angle is too large then it needs to be corrected in the form of an approach, at least first-order degrees, using the concept of conservation of mechanical energy [11].

4. Conclusion

Physical pendulum experiment can be used to determine the value of the gravitational acceleration and the moment of inertia of an object simultaneously. The results obtained through physical pendulum experiment can compensate for the correction needed when using simple pendulum, free fall motion, or mechanical energy conservation. The experiment can be done simply to get the swing period with enough data, which can be processed using linear regression to determine the value of the gravitational acceleration and the moment of inertia.

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