The production of jets from quasi real photons
in $e^+ e^-$ collisions

P. Aurenche, J.-Ph. Guillet
Laboratoire de Physique Théorique ENSLAPP∗ – Groupe d’Annecy
LAPP, IN2P3-CNRS, B.P. 110, F-74941 Annecy-le-Vieux Cedex, France

M. Fontannaz
Laboratoire de Physique Théorique et Hautes Energies†
Université de Paris XI, bâtiment 211, F-91405 Orsay Cedex, France

Y. Shimizu, J. Fujimoto
Physics Department, KEK
Tsukuba, Ibaraki 305, Japan

K. Kato
Physics Department, Kogakuin University
Shinjuku, Tokyo 160, Japan

Abstract

We consider the production of jets in photon-photon collisions beyond the
leading logarithm approximation. Theoretical uncertainties as well as uncer-
tainties due to the virtuality of the initial photons are discussed in detail. The
comparison with TOPAZ data is performed and good agreement is found between
experiment and theory. It is expected that future high precision TRISTAN data
will constrain the non-perturbative component of the photon structure function.

ENSLAPP-A-482/94
KEK preprint 94-66
KEK CP-019
July 1994

∗URA 14-36 du CNRS, associée à l’Ecole Normale Supérieure de Lyon, et au Laboratoire
d’Annecy-le-Vieux de Physique des Particules.
† Laboratoire associé au CNRS (URA 63).
1 Introduction

The study of large transverse momentum processes in photon-photon collisions already has a long history since in the late 70’s and early 80’s, several experimental collaborations at DESY and SLAC have collected data on the reaction $\gamma\gamma \rightarrow h X$ [1, 2]. The most recent results concerning this process were published last year [2] and are shown in Fig. 1 where the next-to-leading-logarithmic QCD predictions are also displayed [3]: the situation is somewhat puzzling since ”perfect” agreement between theory and data is obtained at rather low transverse momentum where the theory is not very reliable (because of the importance of the poorly known hadronic or $VDM$ component of the photon) while the theory falls below the data, by a factor 2 to 5, at large $p_T$ where the physics is dominated by the ”QED” process $\gamma\gamma \rightarrow q\bar{q}$. We observe a disagreement in the $p_T$ dependence which may be attributed to the fact that the data follow the scaling behavior $p_T^3 \frac{d\sigma}{dp_T} = \text{constant}$ [2], whereas one would expect this quantity to fall with $p_T$ due to the $q \rightarrow h$ fragmentation process.

In the following we consider another version of large $p_T$ processes, namely $\gamma\gamma \rightarrow \text{jet } X$. We first present the theoretical expressions and discuss some uncertainties inherent to the perturbative approach. Since the data concern the reaction $e^+e^- \rightarrow e^+e^- \text{ jet } X$ via two photon exchange we have to study the validity of the Weizsäcker-Williams [4] approximation and the effect of the photon virtuality. Finally we compare the theory with the very recent TOPAZ anti-tag data [5, 6] and stress the importance of the gluon content of the photon in the kinematical range covered by the data.

2 Theoretical expressions for $\gamma + \gamma \rightarrow \text{jet } + X$

As extensively discussed at this workshop [7], the photon can couple to the hard sub-process either directly or through its quark or gluon content. The cross section for the production of a jet of a given $p_T$ and pseudorapidity $\eta$ can therefore be decomposed as

\[
\frac{d\sigma}{d\vec{p}_T d\eta} = \frac{d\sigma^D}{d\vec{p}_T d\eta} + \frac{d\sigma^{SF}}{d\vec{p}_T d\eta} + \frac{d\sigma^{DF}}{d\vec{p}_T d\eta}
\]  

(1)

where each term is now being specified. Beyond the leading logarithm approximation the ”direct” cross section takes the form

\[
\frac{d\sigma^D}{d\vec{p}_T d\eta}(R) = \frac{d\sigma^{\gamma\gamma\rightarrow\text{jet}}}{d\vec{p}_T d\eta} + \frac{\alpha_s(\mu)}{2\pi} K^D(R; M).
\]  

(2)

with the corresponding diagrammatic decomposition shown in Fig.2. The parameter $R$ specifies the jet cone size [5], while $\mu$ and $M$ are the renormalization
and factorization scales respectively. When one photon couples directly and the other one through its “structure function”, it leads to

\[
\frac{d\sigma_{SF}}{d\vec{p}_T d\eta}(R) = \sum_{i=q,g} \int dx_1 F_{i/\gamma}(x_1, M) 
\alpha_s(\mu) \left( \frac{d\sigma_{i\gamma\rightarrow jet}}{d\vec{p}_T d\eta} + \frac{\alpha_s(\mu)}{2\pi} K_{i\gamma}^{SF}(R; M, \mu) \right) 
\]

\[
+ \sum_{j=q,g} \int dx_2 F_{j/\gamma}(x_2, M) 
\alpha_s(\mu) \left( \frac{d\sigma_{j\gamma\rightarrow jet}}{d\vec{p}_T d\eta} + \frac{\alpha_s(\mu)}{2\pi} K_{j\gamma}^{SF}(R; M, \mu) \right)
\]

(3)

where some diagrams representative of the \(\mathcal{O}(\alpha_s)\) and \(\mathcal{O}(\alpha_s^2)\) terms on the right hand side are shown in Fig. 3 a) and b) respectively. The underlined diagrams in Fig. 2 b) and 3 a) are in fact the same but they contribute to different region of phase space. When the final state quark is not collinear to the initial photon (as in Fig. 2 b)) the exchanged propagator has a large virtuality (shown by the fat line) and the corresponding contribution is associated to the hard subprocess \(K^D\); when the final quark becomes almost collinear to the initial photon (as in Fig. 3 a)) the virtuality of the exchanged propagator is small: the interaction is soft (long range) and the corresponding contribution reflects the properties of the photon fragmenting in a \(q\bar{q}\) pair and is then naturally associated to the photon structure function. Roughly speaking the factorization scale \(M\) separates the hard region from the soft region and changing this arbitrary scale shifts contributions from \(d\sigma^D\) to \(d\sigma_{SF}\) but clearly does not affect the sum \(d\sigma^D + d\sigma_{SF}\). More precisely, the photon structure function satisfies the evolution equation of type [1].

\[
\frac{dF_{i/\gamma}(M)}{d\ln M^2} = P_{i\gamma} + \sum_{j=q,g} P_{ij} \otimes F_{j/\gamma}(M) \tag{4}
\]

where the \(P_{i\gamma}, P_{ij}\) are the relevant Altarelli-Parisi splitting functions. The scale variation associated to the inhomogeneous term, \(P_{i\gamma}\), induces a change in \(d\sigma_{SF}\) which is compensated by a corresponding variation of \(K^D(R, M)\) as described above: this effect is unique to reactions involving photons as external legs. As for the remaining variation associated to the homogeneous term in eq.(4) it is compensated by a variation of \(K_{SF}^e(R, M, \mu)\), as it occurs in purely hadronic reactions.

We turn now to the last component in eq.(1) where both photons interact via their structure functions

\[
\frac{d\sigma_{DF}}{d\vec{p}_T d\eta} = \sum_{i,j=q,g} \int dx_1 dx_2 F_{i/\gamma}(x_1, M) F_{j/\gamma}(x_2, M) \left( \frac{\alpha_s(\mu)}{2\pi} \right)^2
\]
\[
\left( \frac{d\sigma_{ij\rightarrow \text{jet}}}{d\vec{p}_T d\eta} + \frac{\alpha_s(\mu)}{2\pi} K_{ij}^{DF}(R; M, \mu) \right)
\]
and the corresponding diagrams in Fig. 4 a) and b). Similarly to our previous discussion, the higher order diagrams of \( d\sigma^{SF} \) generate the "Born" contribution to \( d\sigma^{DF} \) as well as the higher order contributions \( K^{SF} \) to \( d\sigma^{SF} \) with the consequence that scale dependent terms in \( \sum_i \left( \frac{\alpha_s}{2\pi} \right)^2 F(\gamma(M) \otimes K^{SF} \) compensate the variation in \( M \) of \( d\sigma^{DF} \).

In conclusion, only the sum eq. (1) has a physical meaning. In particular, it is not legitimate to associate \( d\sigma^{SF} \) and \( d\sigma^{DF} \) to experimentally measured "once resolved" and "twice resolved" components. Let us finally remark that the renormalization scale \( \mu \) variation is compensated, as in purely hadronic cross sections, within the "Born" and higher-order corrections in eqs. (3) and (5) separately.

To illustrate quantitatively the variation of the theoretical predictions under changes of \( M \) and \( \mu \) we consider jet production at TRISTAN \( (\sqrt{s_{ee}} = 58 \text{ GeV}, p_T = 5.24 \text{ GeV}/c) \). The photon structure functions of ref. [10] have been used and the proper convolutions have been made to construct, from the \( \gamma\gamma \rightarrow \text{jet} X \) reaction, the \( e^+e^- \) cross section with the relevant experimental cuts of TOPAZ [3] (see below). Fig. 5 a) is obtained when setting arbitrarily \( K^D = K^{SF} = K^{DF} = 0 \) (the so-called leading logarithmic predictions) while Fig. 5 b) takes into account the full expressions eqs. (2)-(5): the gain in stability is remarkable despite the fact that no saddle-point or extremum is found [5]. The same patterns are also observed at \( \sqrt{s_{ee}} = 1 \text{ TeV} \). In the following we always use for definiteness \( M = \mu = p_T \).

### 3 From \( \gamma\gamma \) to \( e^+e^- \) cross sections

The usual procedure is to use the Weizäcker-Williams approach [5, 11] which approximates the \( e^+e^- \) cross section by the convolution

\[
\frac{d\sigma_{ee\rightarrow \text{jet}}}{d\vec{p}_T d\eta} = \int dz_1 dz_2 F_{\gamma/e}(z_1, E) F_{\gamma/e}(z_2, E) \frac{d\sigma_{\gamma\gamma\rightarrow \text{jet}}}{d\vec{p}_T d\eta}
\]

where \( F_{\gamma/e}(z, E) \) is the spectrum of collinear photons emitted by an electron or positron of energy \( E \) (see Fig. 6). In the above approximation one has neglected the dependence of the \( \gamma\gamma \) cross section on the virtuality \( q^2, q'^2 \) of the photons.

Usually, experiments have an anti-tagging condition which restricts the angle between the incoming and outgoing electron \( \theta < \theta_{\text{Max}} \) such that \( q_{\text{Max}}^2 = -E^2(1 - z)\theta_{\text{Max}}^2 \) for sufficiently small angles. The quasi-real photon spectrum becomes

\[
F_{\gamma/e}(z, E, \theta_{\text{Max}}) = \frac{\alpha}{\pi} \left( \frac{1 + (1 - z)^2}{z} \right) \ln \frac{E(1 - z)\theta_{\text{Max}}}{z m_e}
\]
with $m_e$, the electron mass. Neglecting the photon virtualities in $d\sigma^{\gamma\gamma}$ typically introduces an error of \( (s/s) \leq \frac{E^2 \theta^2_{\text{Max}}}{4p_T^2} \)

which is less than 10% in the case of TOPAZ (\( \theta_{\text{Max}} = 3.2^\circ, \ p_T \geq 2.5 \text{ GeV}/c \)).

For large $p_T$ processes another effect may become relevant. A photon of virtuality $q^2$ has a transverse momentum given by $k_T^2 \sim -(1-z)q^2$ which may be as high as $2.5 \text{ GeV}^2$ in the case of TOPAZ. This $k_T$ plays the role of an "intrinsic" transverse momentum in hadronic collisions and it is well-known that, because of "trigger bias" effects, this intrinsic momentum distorts the shape of the jet $p_T$ distribution at not too large $p_T$. Since the Weiszäcker-Williams spectrum (eq.(7)) assumes the photons to be collinear to the electron this effect is neglected in eq.(6). To quantitatively test the reliability of our approximations we consider the process $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ (same kinematics as $e^+e^- \rightarrow e^+e^-q\bar{q}$) and study the distribution $d\sigma / d\vec{p}_T d\eta$ for a single $\mu$ using, on the one hand, the exact matrix element and, on the other hand, eq.(6) with the photon spectrum eq.(7). For the case of TOPAZ we find that the approximate result overestimates the exact one by 8%, independent of $p_T$ up to $p_T = 8.5 \text{ GeV}/c$. Such a correction will be included in our subsequent calculation.

This analysis covers the effects of the photon virtuality in the direct process(eq.(2)) but, unfortunately, it is not complete in the case of the SF and DF processes. Indeed, for these processes there appear new terms, in the photon structure functions, of type $\ln M^2 / |q^2|$ or $q^2/m_\rho^2$ where $m_\rho$ is a typical vector meson mass as we are now going to discuss. Consider the schematic SF process as shown in Fig.7 where the parton $i$ hard scatters with a characteristic mass scale $M$ to produce a jet.

In a schematic notation the cross section for the initial electron to produce a jet can be written

$$d\sigma^{e\rightarrow \text{jet}} = \frac{\alpha}{2\pi} \int_0^1 \frac{dz}{z} \frac{1 + (1-z)^2}{z} \int_{Q_{\text{Min}}^2}^{Q_{\text{Max}}^2} \frac{dQ^2}{Q^2} \int_0^2 dx \ F_{i/\gamma}(\frac{x}{z}, M, Q) \ d\sigma^{i\rightarrow \text{jet}} \ (8)$$

where we have introduced the structure function of a virtual $\gamma$ of mass $q^2 = -Q^2$, with the condition $Q^2 \ll M^2$. It satisfies eq.(4) and similarly to the real photon case the solution to the evolution equation is written as

$$F_{i/\gamma}(z, M, Q) = F_{i/\gamma}^{\text{Pert}}(z, M, Q) + F_{i/\gamma}^{\text{VDM}}(z, M, Q). \ (9)$$

We now make a model for each term on the right hand side. Following the analysis of ref. [12] we assume

$$F_{i/\gamma}^{\text{Pert}}(z, M, Q) \equiv F_{i/\gamma}^{\text{Pert}}(z, M) \sim \ln \frac{M^2}{Q_0^2} \quad \text{for} \quad Q^2 < Q_0^2$$

$$F_{i/\gamma}^{\text{Pert}}(z, M, Q) \sim \ln \frac{M^2}{Q^2} \quad \text{for} \quad Q^2 \geq Q_0^2 \ (10)$$

with $Q_0 = 6 \text{ GeV}/c$. It is also assumed that the structure function of the virtual $\gamma$ vanishes for $z \rightarrow 0$, $z \rightarrow 1$ and $Q^2 \rightarrow 0$.
where the function with two arguments on the right hand side of the equation refers to the real photon. The value of $Q_0^2$ is chosen to be $.5 \, \text{GeV}^2$ as in ref. [10]. Inserting this in eq.(8) we find

$$d\sigma^{e\rightarrow \text{jet}}|_{\text{pert}} \sim \left( \ln \frac{M^2}{Q_0^2} \ln \frac{Q_0^2}{Q_{\min}^2} - \frac{1}{2} \ln^2 \frac{Q_0^2}{Q_{\min}^2} \right) d\sigma^{i\rightarrow \text{jet}}$$  (11)

The first term would be obtained, had we used $F_{i/\gamma}(z, M)$ over the $Q^2$ integration range while the second term is the reduction factor due to the $Q^2$ dependence of $F_{i/\gamma}$. For TOPAZ it never exceeds a negligible 2%. Following an observation of Borzumati and Schuler [12], it is argued in ref.[13] that the gluon structure function should be more suppressed (as $\ln^2 \frac{M^2}{Q^2}$ when $Q^2 \geq Q_0^2$) than the quark structure function and should lead to a somewhat larger reduction factor in that case.

Turning to the $VDM$ component in eq.(9) we make the usual $\rho$-pole dominance ansatz and write

$$F_{i/\gamma}^{VDM}(z, M, Q) = \left( \frac{m^2_\rho}{m^2_\rho + Q^2} \right)^2 F_{i/\gamma}^{VDM}(z, M)$$  (12)

to obtain

$$d\sigma^{e\rightarrow \text{jet}}|_{\text{VDM}} \sim \left( \ln \frac{Q_{\max}^2}{Q_{\min}^2} - \left( \ln \frac{m^2_\rho + Q_{\max}^2}{m^2_\rho} + \frac{Q_{\max}^2}{m^2_\rho + Q_{\max}^2} \right) \right) d\sigma^{i\rightarrow \text{jet}}$$  (13)

where the correction factor is now 10% to 12% in the case of TOPAZ and is taken into account in our numerical estimates.

4 Comparison to TOPAZ data and conclusions

The TOPAZ collaboration has measured the single jet spectrum at $\sqrt{s_{e^+e^-}} = 58 \, \text{GeV}$ under some specific anti-tagging conditions (mainly $\theta < 3.2^\circ$): the error bars include the systematic errors added linearly to the statistical ones [5] (Fig. 8). The next-to-leading QCD predictions, based on the photon structure function of ref.[10], are also shown in the figure: the top curve is obtained using the standard set of structure functions derived from a comparison to the $F_2(x, M)$ deep inelastic data [14] while the bottom curve is obtained by arbitrarily setting the $VDM$ component equal to 0 in eq.(9). Both curves are compatible with the data for $p_T > 4 \, \text{GeV}/c$. The middle curve is the prediction when the $VDM$ component in $F_{i/\gamma}(x, M, Q)$ is divided by 2 a choice still compatible with the deep-inelastic photon data [14]. Little change is observed in the predictions when instead of varying the $VDM$ normalization one varies the shape of the quark and gluon distributions in $F_{i/\gamma}^{VDM}$ [8]. Concerning the role of the higher order corrections, we find that for the scales $M = \mu = p_T$ they increase the lower
order result by 25% at $p_T = 3 \text{ GeV/c}$ and leave it practically unchanged at large $p_T$. The pattern of the higher order corrections is quite different for the different components of eq.(1): while $d\sigma^D$ is decreased by 15%, independently of $p_T$, and $d\sigma^{SF}$ is left practically unchanged, $d\sigma^{DF}$ is increased by 70%.

In conclusion, we find it extremely encouraging that the theory is able to account for both the data on the deep-inelastic photon structure function and jet production in $\gamma\gamma$ collisions. The somewhat too high theoretical predictions at low $p_T$ using our "standard" set of structure functions is attributed to the neglect, in the calculation, of the charm quark mass. An estimate of this effect leads to a reduction of the cross section of about 15% at $p_T = 3 \text{ GeV/c}$ and only 2% at large $p_T$. Taking this into account the agreement of our standard set of predictions with the data is quite good. It is obvious that the new TOPAZ (and AMY) data, with errors reduced by a factor 2, will provide a very powerful tool to constrain the non-perturbative input to the photon structure function. Combining this with future results from LEP on photon deep-inelastic scattering [14] as well as jet photoproduction [10, 17] will lead [18] to a quantitative understanding on the hadronic structure of the photon.

Acknowledgements

We would like to thank CNRS-IN2P3 (France) and Monbusho (Japan) for the support to our collaboration. Three of us (P.A., J.-Ph.G., M.F) are also indebted to the EEC programme "Human Capital and Mobility", Network "Physics at High Energy Colliders", contract CHRX-CT93-0357 (DG 12 COMA) for financial support. They also thank Prof. G. Jarlskog for the organization of a very interesting and enjoyable workshop.

References

[1] TASSO collaboration: R. Brandelik et al., Phys. Lett. B107 (1981) 290; Phys. Lett. B138 (1984) 219.

[2] MARK II collaboration: D. Cords et al., Phys. Lett. B302 (1993) 341.

[3] P. Aurenche, R. Baier, A. Douiri, M. Fontannaz and D. Schiff, Z. Phys. C29 (1985) 423; CERN yellow report CERN 86-02 (1986) 193, J. Ellis and R. Peccei eds; D.J. Miller, ECFA workshop on LEP 200, CERN 87-08 (1987) 202, A. Bohm and W. Hoogland eds.

[4] C.F. Weizsäcker, Z. Phys. 88 (1934) 612; E.J. Williams, Phys. Rev. 45 (1934) 729.

[5] TOPAZ collaboration: H. Hayashii et al., Phys. Lett. B314 (1993) 149.
[6] see also: AMY collaboration, R. Tanaka et al., Phys. Lett. **B325** (1994) 248.

[7] See in particular the talks by M. Fontannaz, A. Vogt and P. Zerwas at this workshop.

[8] P. Aurenche, J.-Ph. Guillet, M. Fontannaz, Y. Shimizu, J. Fujimoto and K. Kato, Prog. Theor. Phys. **92** (1994) 175.

[9] E. Witten, Nucl. Phys. **B120** (1977) 189;
W.A. Bardeen and A.J. Buras, Phys. Rev. **D20** (1979) 166; Phys. Rev. **D21** (1980) 2041 E.

[10] M. Fontannaz, Orsay preprint LPTHE-93-22, talk given at the 21st International meeting on Fundamental Physics, Miraflores de la Sierra, Spain, May 1993;
P. Aurenche, M. Fontannaz and J.Ph. Guillet, LPTHE 93-37, October 1993.

[11] see the talk by P. Kessler at this workshop.

[12] G.A. Schuler, CERN-TH.6427/92, proceedings of the DESY workshop on physics at HERA, Hamburg, Oct. 1991;
F.M. Borzumati and G.A. Schuler, Z. Phys. **C58** (1993) 139.

[13] M. Drees and R.M. Godbole, U. of Wisconsin preprint, MAD/PH/819, March 1994.

[14] PLUTO collaboration: Ch. Berger et al., Nucl. Phys. **B281** (1987) 365;
AMY collaboration: T. Sasaki et al., Phys. Lett. **B252** (1990) 491;
JADE collaboration: W. Bartel et al., Z. Phys. **C24** (1984) 231.

[15] F. Kapusta, talk at this workshop;
D. Miller, *ibid*.

[16] see the talk by M. Erdmann at this workshop.

[17] H1 collaboration: I. Abt et al, Phys. Lett. **B314** 1993 436;
ZEUS collaboration: M. Derrick, Phys. Lett. **B322** (1994) 287.

[18] D. Bödeker, G. Kramer and S.G. Salesh, DESY preprint DESY 94-042;
J.Ph. Guillet, talk at this workshop; preprint in preparation.
This figure "fig1-1.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9409294v1
This figure "fig1-2.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9409294v1
This figure "fig1-3.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9409294v1