Coupling hybrid inflation to moduli

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Abstract. Hybrid inflation can be realized in low energy effective string theory, as described using supergravity. We find that the coupling of moduli to $F$-term hybrid inflation in supergravity leads to a slope and a curvature for the inflaton potential. The $\epsilon$ and $\eta$ parameters receive contributions at tree and one loop level which are not compatible with slow roll inflation. Furthermore the coupling to the moduli sector can even prevent inflation from ending at all. We show that introducing shift symmetries in the inflationary sector and taking the moduli sector to be no-scale removes most of these problems. If the moduli fields are fixed during inflation, as is usually assumed, it appears that viable slow roll inflation can then be obtained with just one fine-tuning of the moduli sector parameters. However, we show this is not a reasonable assumption, and that the small variation of the moduli fields during inflation gives a significant contribution to the effective inflaton potential. This typically implies that $\eta \approx -6$, although it may be possible to obtain smaller values with heavy fine-tuning.

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1. Introduction

The theory of inflation provides an explanation for many observed properties of our visible universe, in particular its homogeneity and isotropy (see e.g. [1] and [2] for reviews). It also gives an explanation for the origin of small density perturbations which eventually gave rise to the structures in our universe. These aspects are well understood. However, from the particle physics point of view, a suitable candidate for the inflaton field has yet to be found [3]. Among the many candidates, hybrid inflation [4] is a particularly well motivated one, especially in supersymmetric models [5]–[10].

There has been a lot of recent work investigating the possibility of realizing inflation within string theory (see e.g. [11]–[13] for reviews). The most natural framework for brane inflation is the brane–antibrane system where an attractive force leads to a potential for the interbrane distance [14]–[19]. Inflation ends with an open string tachyonic instability similar to hybrid inflation. This scenario has been extensively studied and leads to interesting phenomena such as $D$-string formation at the end of inflation. One particular issue in these models comes from the need for moduli stabilization during inflation. If this is not achieved, the potential for the moduli is of runaway type, destroying the existence of slow roll inflation. This problem can be circumvented using the KKLT scenario whereby...
the Kahler moduli are stabilized once the complex moduli have been stabilized and non-perturbative gaugino condensation has occurred on D7 branes \textsuperscript{20}. The coupling to inflation has been studied in KKLMMT where a fine-tuning of the inflaton superpotential has been advocated \textsuperscript{21}. One can also use a shift symmetry to alleviate the $\eta$ problem \textsuperscript{22}–\textsuperscript{26}. Another way of realizing hybrid inflation in string models can be obtained using the D3/D7 system; see, e.g., \textsuperscript{27}–\textsuperscript{31}. In this case the end of inflation happens at a lower scale than the Planck scale, when the charged open string fields between the branes condense. The D3/D7 system can be modelled using an $N = 1$ supersymmetric $D$-term inflation model \textsuperscript{28}. The interbrane distance plays the role of the inflaton and the charged open strings between the branes are the waterfall fields. When the supergravity corrections are neglected, i.e. in global supersymmetry, and assuming that the Kahler moduli have been stabilized, the inflaton direction is flat. Slow roll inflation is driven by the one loop logarithmic corrections as the one loop quadratic divergences are inflaton independent.

In this paper, we are investigating $F$-term hybrid inflation with the inflationary superpotential

$$W_{\text{inf}}(\phi^+, \phi) = \sqrt{2} g \left( \phi^+ \phi^- - x^2 \right) \phi,$$

where $g$ is the $U(1)$ gauge coupling, in the presence of additional moduli fields. It would be very interesting to derive the origin of $x$ from fundamental string theory \textsuperscript{32,33}. We treat the supergravity case going beyond the global supersymmetry analysis. We show that the treatment of Fayet–Iliopoulos $D$-terms (i.e. when $x = 0$) needs extra care in supergravity. To include such a term would require additional fields and the study of their stabilization. For simplicity we therefore do not include inflationary $D$-terms in our models. Inflation is then driven by the presence of the non-vanishing $x$.

We show that supergravity corrections, and supergravity-induced interactions with a moduli sector, induce a tree level slope and mass for the inflaton \textsuperscript{3}. Moreover there are additional contributions from the quadratic divergence part of the one loop corrections, since they are inflaton dependent in supergravity, as opposed to inflaton independent as in global supersymmetry. In general these effects spoil the flatness of the inflationary potential. We show that most of the tree level problems can be removed by including shift symmetries in the inflaton sector, and taking the moduli sector to be no-scale. It then appears that a viable slow roll inflation model can be found with limited fine-tuning of the moduli sector. However we also find that the coupling between the inflaton and the moduli has the effect of inducing a small variation of the moduli during inflation. Despite the small size of this variation, it gives a significant contribution to the inflaton slope and violates slow roll inflation.

The paper is arranged as follows. In section 2, we construct the full potential for our combined model of inflation and moduli stabilization. We calculate tree and one loop level parts of the potential, and describe symmetries which help maintain the flatness of the inflaton potential. In section 3 we determine the role of moduli fields during inflation, and show that their effect is significant. We find the constraints corresponding to the COBE normalization and the WMAP3 spectral index constraint in section 4. Our arguments and analysis apply to a general moduli sector, although as a specific example, we also give explicit results for the KKLT model, which has just one moduli field $T$. In section 5 we study particular choices of racetrack superpotentials for the KKLT scenario, and find that the running of the moduli field during inflation typically leads to a large value of
A small value of \( \eta \) could only be achieved if the moduli superpotential is heavily fine-tuned. Finally in section 6 we consider models where a Minkowski vacuum after inflation is obtained without the need for a lifting term. A model with this property is also interesting as it may give a small gravitino mass, as is required to have a low energy sparticle spectrum. We derive a no-go theorem which states that, for no-scale models with no lifting term, no such stable supersymmetry breaking Minkowski vacua can be found. We conclude in section 7.

2. Combining hybrid inflation with moduli fields

As already discussed, many string theory inflation scenarios can be modelled at low energy with a supergravity description of hybrid inflation. The system uses three fields, the inflaton \( \phi \) measuring the interbrane distance and two charged fields \( \phi^\pm \) (which in the D3/D7 system would represent the open strings between the two types of branes). The fields interact according to the superpotential (1.1) with \( x = 0 \), which has been studied extensively in the literature. In [28] it was shown how the Fayet–Iliopoulos D-term arose from fluxes on the D7 brane, allowing \( D \)-term inflation in this system. Here we will focus almost exclusively on hybrid inflation which is driven only by \( F \)-terms (so \( x \neq 0 \)), although exactly how such terms are embedded in string theory remains to be found. Including inflationary \( D \)-terms in a supergravity theory creates extra complications, as we will discuss in the following subsection.

2.1. Supergravity \( P \)-term inflation

Using (1.1) and taking to \( x \neq 0 \) we obtain

\[
V = |\partial_i W_{\text{inf}}|^2 + V_D = 2g^2 \left( |\phi^+|^2 + |\phi^-|^2 + |\phi^+ - \phi^- - \xi_1|^2 \right) + \frac{g^2}{2} \left( |\phi^+|^2 - |\phi^-|^2 - \xi_1 \right) .
\]

This potential has an \( N = 2 \) origin which was derived in [30]. During inflation, when the waterfall fields \( \phi^\pm = 0 \), the potential is

\[
V = \frac{g^2 \xi_1^2}{2} + 2g^2 x^4 ,
\]

which gives a positive contribution to the inflation energy.

Extending this model to a supergravity theory, and working in units with \( M_{\text{Pl}} = 1 \), we replace the above \( F \)-term by

\[
V_F = e^{K_{\text{inf}}} (K^{ij} D_i W_{\text{inf}} D_j \bar{W}_{\text{inf}} - 3 |W_{\text{inf}}|^2)
\]

where \( D_i W_{\text{inf}} = \partial_i W_{\text{inf}} + K_{i} W_{\text{inf}} \), and \( K_{\text{inf}} \) is the Kahler potential for \( \phi \) and \( \phi^\pm \). The simplest possibility would be \( K_{\text{inf}} = |\phi|^2 + |\phi^-|^2 + |\phi^+|^2 \).

It was originally thought that extending the \( D \)-term in the above model to SUGRA would be straightforward. However it was later discovered that in order to have a Fayet–Iliopoulos term \( \xi_1 \) appearing in a supergravity theory, the superpotential must have charge \( -\xi_1 \) [34]. Thus the extension of (2.1) to SUGRA described in the earlier paper [30] requires reconsideration. To preserve the properties of \( \phi \) and its role as the inflaton, it should be...
uncharged [34]. This is only possible when \( x = 0 \) and the charges of \( \phi^\pm \) are shifted to \( q_\pm = \pm 1 - \rho_\pm \xi_1 \), where \( \rho_+ + \rho_- = 1 \). Now all the contributions to \( W \) from any other sectors of the theory would also have to be charged. This would be for instance the case with the MSSM superpotential. Since they are, \textit{a priori}, unrelated to inflation, this seems unnatural. Hence we have an incompatibility between a non-vanishing \( x \) and a Fayet–Iliopoulos term \( \xi_1 \). In the usual inflationary scenario, inflation is driven by the Fayet–Iliopoulos term.

One possibility is to introduce a Fayet–Iliopoulos term with a gauge invariant superpotential. This can be achieved in supergravity with a \( D \)-term by making the \( U(1) \) pseudo-anomalous. We extend the model to include one modulus \( X \) and matter fields \( A_a \), all of which are charged under the pseudo-anomalous \( U(1) \) gauge group. The relevant part of the Kahler potential for these fields is

\[
K(X, A^a) = - \log \left( X + \bar{X} - \xi_1 V_1 \right) + \sum_i e^{-q_a \lambda_i} |A^a|^2,
\]

(2.4)

where the gauge field, \( V_1 \), has been included (the usual Kahler potential appearing in the scalar potential expression is obtained by putting \( V_1 = 0 \) formally).

The potential arising from the \( D \)-term is then

\[
V_D = \left( \frac{g^2}{X + \bar{X}} \left( |\phi^+|^2 - |\phi^-|^2 - \frac{\xi_1}{X + \bar{X}} + \sum_a q_a |A^a|^2 \right) \right)^2
\]

(2.5)

where we have taken the gauge coupling function to be \( f = X/g^2 \) in order to cancel the \( U(1) \) anomaly. We see that a Fayet–Iliopoulos term can give rise to inflation when \( x = 0 \), assuming that \( X \) and \( A^a \) can be stabilized by a suitable superpotential.

During inflation, the waterfall fields \( \phi^\pm \) vanish. This implies that the \( U(1) \) \( D \)-term and the \( x \) dependent part of the \( F \)-term potential give rise to the inflation energy

\[
V_0 = \frac{g^2}{(X + \bar{X})^3} \left( \xi_1 - (X + \bar{X}) \sum_a q_a |A^a|^2 \right)^2 + \frac{2g^2 x^4}{(X + \bar{X})^3} e^{2 \xi_1 |A^a|^2}.
\]

(2.6)

When \( x = 0 \) and the extra fields are stable, this resembles the usual inflation energy in hybrid inflation.

If on the other hand we just use the \( F \)-term to give inflation, there is no need to introduce the additional fields \( A^a \) and \( X \). The Fayet–Iliopoulos \( D \)-term must then be zero, due to the gauge invariance of \( W \), so \( \xi_1 = 0 \). We will take \( x \neq 0 \) and assume this is the case for the rest of the paper. The string theory origin for \( x \) has yet to be found [33].

2.2. Combined tree level potential

We are interested in the hybrid inflation scenario based on the superpotential above (1.1). We are also taking into account the effects of a separate sector, so the total superpotential is

\[
W(Z^I, \phi, \phi^\pm) = W_{\text{inf}}(\phi, \phi^\pm) + W(Z^I).
\]

(2.7)

The \( Z^I \) are moduli and matter fields, which are usually assumed to be stable during inflation. With one modulus in \( F \)-theory no stabilization was obtained [35]. Here we
consider the most general case in supergravity with more than one moduli field and generic superpotential \( W(Z^I) \). The moduli dependent part of the superpotential \( W(Z^I) \) is then responsible for stabilizing the moduli fields.

The total Kahler potential has the form \( \mathcal{K} = K_{\text{inf}} + K(Z^I) \) with
\[
K_{\text{inf}} = -\frac{1}{2} (\phi - \tilde{\phi})^2 + (\phi^+ - \tilde{\phi}^-)(\tilde{\phi}^+ - \phi^-). \tag{2.8}
\]
We have imposed two shift symmetries in the \( (\phi, \phi^\pm) \) sector to alleviate the \( \eta \) problem of supergravity inflation. This is an extension to the usual \( \phi \) shift symmetry [22]–[26]. In string theory, this corresponds to the translational invariance of the brane system. For the full superpotential (2.7), the \( F \)-term part of the potential can be expanded as
\[
V_F = e^{\mathcal{K}} \left[ \sqrt{2} g \phi^- + (\tilde{\phi}^+ - \phi^-) (W_{\text{inf}} + W) \right]^2 + \sqrt{2} g (\phi^+ - x^2) (W_{\text{inf}} + W)^2 + V_2 |W_{\text{inf}}|^2 + 2 \text{Re} \{ V_1 W_{\text{inf}} \} + V_S, \tag{2.9}
\]
where \( \text{Re}(z) \) denotes the real part of \( z \), and we have defined
\[
V_S = (K^{IJ} D_I W D_J \bar{W} - 3 |W|^2), \quad V_1 = (K^{IJ} D_I W K_J - 3 W), \quad V_2 = (K^{IJ} K_I K_J - 3). \tag{2.10}
\]
There is also a contribution to \( V \) from the \( D \)-term
\[
V_D^{(1)} = \frac{1}{2 \text{Re} [f(Z^I)]} (|\phi^+|^2 - |\phi^-|^2)^2. \tag{2.11}
\]

In this paper we will be particularly interested in Kahler potentials with the no-scale property, such as \( K = -3 \ln(T + \bar{T}) \) in the KKLT scenario. In this case the above expressions reduce to
\[
V_2(Z^I) = 0, \quad V_1(Z^I) = K^{IJ} \partial J W K_J. \tag{2.12}
\]

At the end of inflation, the charged fields \( \phi^\pm \) condense and the inflaton \( \phi \) is zero. The \( U(1)_A \)-term part of the potential is then zero, and \( F \)-term part reduces to \( V_F = V_S(Z^I) \). This part of \( V_F \), which comes from the moduli dependent part of the superpotential \( W(Z^I) \), is then responsible for stabilizing the moduli fields. For \( W \neq 0 \) its minimum is an AdS\(_4\) vacuum where supersymmetry is preserved, i.e. \( F_I = 0 \) for all the fields. Since we want a Minkowski vacuum at the end of inflation, it is necessary to add an additional lifting term, \( V_{\text{lift}}(Z^I) \) to the potential. In the KKLT scenario a non-supersymmetric lifting term is used:
\[
V_{\text{NS}} = \frac{E}{(T + \bar{T})^n}, \tag{2.13}
\]
where \( n = 2, 3 \) depending on the origin of the term, i.e. \( n = 2 \) for anti-D3 branes and \( n = 3 \) for fluxes on D7 branes.

An alternative possibility is that the minimum is raised by an uplifting \( D \)-term [36, 37]. This allows one to lift the AdS\(_4\) vacuum while avoiding the need for a non-supersymmetric potential. In this case the set of fields \( Z^I \) comprises one modulus \( T \) and matter fields \( \chi^I \), all
of which are charged under a second, pseudo-anomalous $U(1)_2$ gauge group. The Kahler potential in the moduli sector can be written as

$$K(Z^I) = -3 \ln \left( T + \bar{T} - \frac{\xi_2}{3} V_2 \right) + \sum_i e^{-\tilde{q}_i} |\chi_i|^2. \quad (2.14)$$

Unlike the KKLT case it is not no-scale, although it is straightforward to write down a no-scale version.

The potential arising from the $D$-term for the additional $U(1)_2$ is then

$$V_D^{(2)} \propto \frac{1}{T + \bar{T}} \left( \sum_i \tilde{q}_i |\chi_i|^2 + \frac{\xi_2}{T + \bar{T}} \right)^2. \quad (2.15)$$

If $\xi_2$ and all $\tilde{q}_i$ are positive, this potential will be non-zero and provide a suitable lifting term. As explained in [38], an AdS$_4$ vacuum in supergravity cannot normally be lifted by $D$-terms. One can circumvent this argument by using non-analytic superpotentials with positive definite $D$-terms, as described in [37].

The total moduli stabilization potential is then

$$V_{\text{stab}}(Z^I) = e^K V_S(Z^I) + V_{\text{lift}}(Z^I), \quad (2.16)$$

where $V_{\text{lift}}$ is given by, for example, (2.13) or (2.15). After inflation, the potential possesses a Minkowski vacuum thanks to the presence of the lifting term.

For the rest of the paper we will concentrate just on the case where there are no additional matter fields $\chi^I$. This implies that the gauge invariance of the superpotential can only be maintained if $T$ is gauge invariant, and we must use the non-supersymmetric lifting term (2.13). This is the minimal setting for studying hybrid inflation coupled to moduli.

2.3. Inflation

The structure of this potential is rather rich due to the possible interplay between the moduli and other fields. During hybrid inflation, the fields $\phi^\pm$ vanish, and the inflaton field is real, so $\bar{\phi} = \phi$. The full (tree level) potential then reads

$$V = V_{\text{stab}}^{(\text{tree})} + V_0^{(\text{tree})} - 2\sqrt{2} g x^2 e^K \text{Re}\{V_1\} \phi + 2g^2 x^4 e^K V_2 \phi^2, \quad (2.17)$$

where we have identified

$$V_{\text{stab}}^{(\text{tree})}(Z^I) = e^K V_S(Z^I) + V_{\text{lift}}(Z^I) \quad (2.18)$$

and

$$V_0^{(\text{tree})}(Z^I) = 2g^2 x^4 e^K. \quad (2.19)$$

In addition to the terms above, there will be contributions from loop corrections, which we will discuss in the next subsection. Note that the inflaton is much smaller than the Planck mass, i.e. $\phi = \bar{\phi}$ is such that $\phi \ll M_{\text{pl}}$. During inflation, the Minkowski vacuum is lifted by $V_0$. We see that in general we will get large, tree level contributions to the slope and mass of the inflaton [3] (from $V_1$ and $V_2$ respectively). In other words the model has $\epsilon$ and $\eta$ problems. This new $\eta$ problem comes from the inclusion of the moduli sector, and is distinct from the usual $\eta$ problem which comes from embedding inflation in
supergravity. In the case of no-scale models $V_2 = 0$, and so this $\eta$ problem can be removed by a symmetry of the Kahler potential $K$. To obtain a small inflaton slope, we will need to fine-tune $V_1$.

The form of the potential discussed so far was based on the hybrid inflationary epoch, in which the fields $\phi^\pm$ vanish. Inflation ends when the mass matrix acquires a tachyonic direction. This usually occurs when one of the two charged fields $\phi^\pm$ (or a combination thereof) has a negative mass, i.e. when $m_2^2 < 0$. Without moduli fields, this instability occurs at $\phi = x$. However, the inclusion of the moduli sector modifies this result. For simplicity we take $V_2 = 0$ in what follows (as is the case for a no-scale $K$). Working out the mass matrix for the $\phi^\pm$ sector, we find

$$\phi_{\text{end}} = \sqrt{x^2 + \frac{V_1^2}{8g^2} - \frac{V_1}{2\sqrt{2}g}},$$

(2.20)

where we have assumed $W$ to be real. If $V_1 = 0$, we recover the standard result $\phi = x$. However, due to the presence of the moduli fields, the value of $\phi_{\text{end}}$ is altered. Note that if we had used a more general $K_{\text{inf}}$, which did not have the shift symmetries, then $\phi_{\text{end}}$ would have received far more corrections. Taking a canonical Kahler potential, and neglecting the contributions of order $V_1$ in the regime $x \ll 1$, the lowest mass is given by

$$m_2^2 = e^K \left[ 2g^2(\phi^2 - x^2) + W^2 + 2\sqrt{2}gW\phi \right].$$

(2.21)

We see that if $W \geq \sqrt{2}gx$, inflation never ends as the mass is always positive. We naturally expect that $W$ will be much larger than the inflation scale $x$ in order to stabilize moduli during inflation. We see that both shift symmetries must be included in $K_{\text{inf}}$ for inflation to be viable.

In the above discussion we have considered how the moduli sector will alter inflation. Of course we must not forget that the inflationary sector can also interfere with the moduli stabilization, since the inflation potential $V_0$ acts as a perturbation of the moduli potential $V_{\text{stab}}$. In the KKLT scenario the minimum of $V_{\text{stab}}$ is separated from $T = \infty$ (which also has $V_{\text{stab}} = 0$) by a potential barrier. Denoting the height of the barrier by $V_{\text{max}}$, we see that the value of the above potential (2.17) must be less than $V_{\text{max}}$ during inflation, or the modulus field $T$ will roll off to infinity, and inflation will not finish in the correct vacuum. The slope terms in the above potential must be small (in order to have slow roll inflation), so we need $e^K g^2 x^4 \lesssim V_{\text{max}}$. This guarantees that the minimum of the moduli potential is only shifted during inflation and not completely destroyed.

Typically $V_{\text{max}} \sim -V_{\lambda_{\text{AdS}}} \sim 3W^2e^K$, where $V_{\lambda_{\text{AdS}}}$ is the energy of the AdS minimum of $V$ (if $V_{\text{lift}}$ were absent). This provides a rough estimate of the barrier height. In this case we need $gx^2 \lesssim W$, which is reasonable if the inflation scale is far below the moduli stabilization scale.

2.4. Loop corrections

The one loop corrections to the effective potential for a general theory, with cut-off $\Lambda$, are [39]

$$V_{\text{loop}} = \frac{1}{32\pi^2} \text{Str} M^2 \Lambda^2 + \frac{1}{64\pi^2} \text{Str} M^4 \log \frac{M^2}{\Lambda^2}. $$

(2.22)
The supertrace is $\text{Str} M^2 = M^2_{(\text{boson})} - M^2_{(\text{fermion})}$. Expressions for the boson and fermion mass matrices are given in the appendix. As long as the masses are below the cut-off scale, the log corrections are smaller than the quadratic divergences. This is guaranteed provided the potential $W(Z_I)$ is lower than the cut-off, i.e., lower than the Planck scale in practice. This is also a phenomenological requirement in order to obtain a hierarchy between the Planck mass and the gravitino mass. We therefore concentrate on the quadratic divergences in what follows.

We see there are corrections to all parts of the inflationary potential (2.17). In particular we find

$$\tilde{V}_0 = c_4 V_0 = 2 g^2 x^4 \left( 1 + \frac{\Lambda^2}{16 \pi^2} [2 + \delta_I^2] \right) e^K. \quad (2.23)$$

In general, the corrections to the other parts of the potential are complicated, and not particularly illuminating. For simplicity we will explicitly consider the case for one modulus field only, i.e. $W = W(T)$. We will use $f(T) \propto T$. In the Minkowski background

$$\text{Str} M^2_{\text{Mink}} = \frac{2n(n+1)E}{3(2T)^n} + \frac{2W'^2}{3T} - \frac{2WW'}{T^2} - \frac{W^2}{2T^3}. \quad (2.24)$$

During inflation we have instead

$$\text{Str} M^2_{\text{Inf}} - \text{Str} M^2_{\text{Mink}} = \frac{g^2 x^4}{2T^3} (3 - 2\phi^2) + \frac{2\sqrt{2}gx^2}{T^3} (TW' + W)\phi. \quad (2.25)$$

We see that the loop corrections are $\phi$ dependent. Note that we have included not only the contributions from $\phi^\pm$, but also those from the moduli sector.

For the KKLT scenario, we find the tree level potential to be (after taking $T$ to be real)

$$V^{(\text{Mink})}_{\text{tree}} = \frac{W'(TW' - 3W)}{6T^2} + \frac{E}{(2T)^n}. \quad (2.26)$$

where $W' = dW/dT$. Adding on the loop corrections (2.24) gives the full potential for the Minkowski background

$$\tilde{V}_{\text{stab}} = \frac{W'^2}{6T} \left[ 1 + \frac{\Lambda^2}{8\pi^2} \right] - \frac{WW'}{2T^2} \left[ 1 + \frac{\Lambda^2}{8\pi^2} \right] - \frac{W^2}{T^3} \frac{\Lambda^2}{32\pi^2} + \frac{E}{(2T)^n} \left[ 1 + \frac{\Lambda^2 n(n+1)}{48\pi^2} \right]. \quad (2.27)$$

Adding the loop corrections will change the value of $T$ at the minimum. Furthermore the minimum will no longer be Minkowski. The parameter $E$ needs to be retuned to fix this.

During inflation, i.e. for $\phi$ non-vanishing, we get

$$V^{(\text{Inf})}_{\text{tree}} = V^{(\text{Mink})}_{\text{tree}} + \frac{\sqrt{2}x^2 g W' \phi}{2T^2} + \frac{x^4 g^2}{4T^3}, \quad (2.28)$$

to which we need to add the loop corrections (2.25).

In this paper, we take the cut off at the Planck scale. Hence, as long as $W'$ and $W$ do not vanish during inflation, the radiative corrections induce a slope for the inflationary potential, which is dependent on $W'$, and which can in principle be a danger to the flatness of the inflationary potential. This effect is only present in supergravity, as in
global supersymmetry the quadratic divergences are $\phi$ independent. For one modulus field $T$, the full potential during inflation now reads
\[ V_{\text{total}}^{(\text{Inf})} = \tilde{V}_{\text{stab}} + \tilde{V}_0 - 2\sqrt{2}g x^2 e^K \text{Re}\{\tilde{V}_1\} \phi + 2g^2 x^4 e^K \tilde{V}_2 \phi^2. \] (2.29)
For $T$ real, the full moduli stabilization potential is given above (2.27). We denote the value of $T$ at the minimum of $\tilde{V}_{\text{stab}}$ by $T_0$. Similarly
\[ \tilde{V}_1 = -2TW\left[1 + \frac{\Lambda^2}{8\pi^2}\right] - \frac{W\Lambda^2}{4\pi^2} \] (2.30)
\[ \tilde{V}_2 = -\frac{\Lambda^2}{4\pi^2}. \] (2.31)
As it is a no-scale model, the only contribution to $\tilde{V}_2$ comes from loop corrections.

3. Moduli fields during $F$-term hybrid inflation

In this section we will discuss the form of the effective inflationary potential, taking into account the moduli stabilization mechanism. It is usually assumed that moduli fields are stabilized before inflation begins. The energy scale of the corresponding potential is much higher than that of inflation, and so the effects of the inflaton will not significantly alter the values of the moduli fields. It therefore seems reasonable to assume the moduli are fixed during inflation. As we will show, this is not the case.

Let us now analyse the moduli stabilization mechanism when the inflation sector is present. We denote the values of the moduli fields at the minimum of $\tilde{V}_{\text{stab}}$ by $Z'_0$, and define $\delta Z^I = Z^I - Z'_0$ to be the deviation of the moduli fields from this value. Expanding the inflationary potential around this minimum we find
\[ V(\phi, Y^a) = \frac{1}{2} \sum_a \lambda_a(Y^a)^2 - 2\sqrt{2}g x^2 \phi \sum_a \text{Re}(e^K \tilde{V}_1)_a Y^a \]
\[ + \tilde{V}_0 - 2\sqrt{2}g x^2 \phi e^K \text{Re}(\tilde{V}_1) + 2g^2 x^4 \phi^2 e^K \tilde{V}_2 + \mathcal{O}(Y^3, Y^2 x^2 \phi, Y g^2 x^4), \] (3.1)
where $Y^a$ are independent combinations of $\delta Z^I$ and $\delta Z^I$ which satisfy $\sum_b \tilde{V}_{\text{stab},ab}(Z'_0) Y^b = \lambda_a(Z'_0) Y^a$. Minimizing this with respect to $Y^a$, we find
\[ Y^a = 2\sqrt{2}g x^2 \phi \frac{\text{Re}(e^K \tilde{V}_1)_a}{\lambda_a} + \mathcal{O}(g^2 x^4). \] (3.2)
So we see that the moduli fields are now stabilized along a valley depending on $\phi$. If $\lambda_a \gg g x^2 \phi$ (which is perfectly reasonable), then $\delta Z^I \ll Z'_0$, and the variation of the moduli fields is tiny, as is usually assumed. The correction to the inflationary potential is tiny too. However the slope of $V$ is also tiny, and so the contribution of the moduli fields can be comparable to the other effects we are considering. Hence it is not reasonable to ignore their variation during inflation after all. Now if we substitute the above expression (3.2) back into the potential (3.1), we obtain the effective inflationary potential
\[ V(\phi) = \tilde{V}_0 + C\phi + \frac{M^2}{2} \phi^2 + \mathcal{O}(g^3 x^6), \] (3.3)
with slope
\[ C = -2\sqrt{2}g x^2 e^K \text{Re}(\tilde{V}_1) \]  
(3.4)

and effective mass
\[ M^2 = 4g^2x^4 \left[ e^K \tilde{V}_2 - 2 \sum_a \frac{(\partial_a \text{Re}(e^K \tilde{V}_1))^2}{\lambda_a} \right]. \]  
(3.5)

We remind the reader that loop corrections are included in these expressions. Hence, we see for a wide range of models (including the KKLT scenario) that \( M^2 < 0 \), and so the inflaton is tachyonic. This tachyonic instability is not necessarily a problem if \( \phi \) is small, or the mass term is subdominant. Since we need the field to roll towards zero during inflation we also require that \( C \) is positive.

It is tempting to use a moduli sector whose superpotential satisfies \( W = \partial_I W = 0 \) at the minimum, since \( \tilde{V}_1 = 0 \) in this case, and so we automatically avoid any large contributions to the slow roll parameter \( \epsilon \). This type of model has other appealing features, such as a small gravitino mass [44]. If we consider just one moduli field \( T \), and take a no-scale \( K \), then we see from (2.31) that loop corrections give a small negative inflaton mass. The effects of the moduli variation will make it even more negative. Even if this mass is small, it will still dominate the behaviour of \( \phi \), and cause it to roll away from zero. Hence we see that a moduli potential with \( W = W' = 0 \) at its minimum (i.e. unbroken supersymmetry) cannot be successfully combined with \( F \)-term hybrid inflation.

4. Slow roll inflation constraints

We have established that during inflation the effective potential has the leading order form
\[ V(\phi) = \tilde{V}_0 + C\phi + \frac{M^2}{2}\phi^2, \]  
(4.1)

with \( C \) positive and \( M^2 \) negative. The potential has the form of an inverted parabola. Since we want inflation to end by the tachyonic instability of the \( \phi^- \)-field, \( \phi \) has to roll towards zero. This means that during inflation \( \phi < \phi_c \equiv -C/M^2 \). Thus, an additional constraint on the model parameters is that \( \phi_c > \phi_{\text{end}} \), as given in (2.20). To get an idea about the allowed parameter ranges for the theory, making only modest assumptions about the moduli sector, we provide some general constraints on potentials of the form (4.1), leaving \( \tilde{V}_0 \), \( C \) and \( M^2 \) as free parameters, but with \( M^2 \) negative. Note that \( \tilde{V}_0 \), \( C \) and \( M^2 \) depend on the underlying theory and are not independent. As shown in the previous section, they depend on the details of the moduli sector as well as the parameters \( g \) and \( x \). An inflationary model has to satisfy certain constraints in order to agree with observations. Firstly, in order to realize a period of slow roll inflation, the two slow roll parameters
\[ \epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \approx \frac{1}{2} \left( \frac{C + M^2 \phi}{\tilde{V}_0} \right)^2 = \frac{C^2}{2\tilde{V}_0^2}(1 - \phi/\phi_c)^2 \]  
(4.2)
and

$$\eta = \frac{V''}{V} \approx \frac{M^2}{V_0}$$  \hspace{1cm} (4.3)$$

have to be small (we have approximated $V \approx \tilde{V}_0$ during inflation).

Also, in order to get the right amplitude for the density perturbations from inflation, the COBE normalization must hold [3]:

$$\frac{V}{\epsilon} \equiv \delta \approx 24\pi^2 (5 \times 10^{-5})^2.$$  \hspace{1cm} (4.4)$$

The current WMAP +2dFGRS constraint on the spectral index is [40]

$$n_s = 0.948^{+0.014}_{-0.018}.$$  \hspace{1cm} (4.5)$$

(a weaker bound has been found in [41,42]). In the case that $\phi \ll \phi_c$, the above two constraints are fulfilled when

$$\frac{2 \tilde{V}_0^3}{C^2} \approx 24\pi^2 (5 \times 10^{-5})^2$$  \hspace{1cm} (4.6)$$

and

$$n_s - 1 = 2\eta - 6\epsilon = \frac{2M^2}{V_0} - \frac{3C^2}{V_0^2}.$$  \hspace{1cm} (4.7)$$

The constant $C$ can be eliminated from the last expression, using (4.6). Since $M^2$ is negative and $n_s \approx 0.95$, this restricts the parameters to be in the region (in Planck units)

$$0 \leq \tilde{V}_0 \lesssim 7 \times 10^{-9},$$  \hspace{1cm} (4.8)$$

$$0 \geq M^2 \gtrsim -6 \times 10^{-11},$$  \hspace{1cm} (4.9)$$

$$0 < C \lesssim 10^{-9},$$  \hspace{1cm} (4.10)$$

implying an upper bound for the inflationary scale of order $2 \times 10^{16}$ GeV. The allowed region in the $\tilde{V}_0-M^2$ plane is shown in figure 1.

The parameters $V_0$, $C$ and $M^2$ depend on the details of the moduli superpotential as well as on $g$ and $x$. In order to generate slow roll inflation, we require that

$$\epsilon \approx \frac{C^2}{2V_0^2} = \left(\frac{\text{Re}(\tilde{V}_1)}{gx^2c_s}\right)^2 \ll 1$$  \hspace{1cm} (4.11)$$

and

$$|\eta| \approx \left| \frac{M^2}{V_0c_s} - 2 \sum_a \frac{\partial_a \text{Re}(e^K \tilde{V}_1)}{e^K \lambda_a} \right| \ll 1.$$  \hspace{1cm} (4.12)$$

Note that if we had not included the contribution from the variation of the moduli fields, we would instead have $|\eta| = (2/c_s)|\tilde{V}_2| \ll 1$. In general, the above contributions to $\eta$ from the moduli sector are likely to be large. We see this type of model may have another $\eta$ problem, which is not removed by the symmetries we used in section 2 to avoid the other two $\eta$ problems.
Figure 1. Constraints on the parameters $M^2$ and $\tilde{V}_0$ coming from the observed amplitude of the spectrum (4.6) and the observed spectral index $n_s$ (4.5). The allowed region is the grey area and the values are given in Planck units. It is assumed that $M^2$ is negative, as predicted in these theories.

Using (4.8), which follows from the COBE and spectral index constraints (4.4) and (4.5), we find that

$$g x^2 e^{K/2} \lesssim 6 \times 10^{-5}. \quad (4.13)$$

This is consistent with $g x^2 \ll 1$, as we have assumed (and is required, for example, for $V_{\text{stab}}$ to retain its minimum during inflation).

Finally, for the maximal allowed value of $\tilde{V}_0^{1/4} = 2 \times 10^{16}$ GeV, the relative contribution of gravitational waves $r \approx 12.4 \epsilon$ is 13%. However, it can be easily shown that the field variation $\Delta \phi$ during inflation, related to the e-fold number by

$$N = \int \frac{V}{V'} d\phi \approx \frac{\tilde{V}_0}{C} \Delta \phi, \quad (4.14)$$

is larger than $M_{\text{Pl}}$ (with $N = 60$), as long as $\tilde{V}_0^{1/4} \gtrsim 7 \times 10^{15}$ GeV. Since field variations below the Planck scale are preferred theoretically, it is very likely that the predicted amount of gravitational waves is much lower.

4.1. Moduli destabilization and the gravitino mass

Here we study the stability of a modulus during inflation. In particular, the previous approximation of assuming that the moduli potential is quadratic when perturbed by the inflaton is only valid if the inflaton does not remove the potential barrier between the inflation minimum and infinity. We have to ensure that the term linear in $\phi$ in the potential is not too big; otherwise this term will completely destabilize the modulus $T$. 
The condition for this to happen is that for at least 60 e-folds of inflation the term linear in \( \phi \) in (2.29) is smaller than the barrier height, \( V_{\text{max}} \). Typically \( V_{\text{max}} \sim 3W^2e^K \) which implies

\[
2\sqrt{2}gx^2e^K\hat{V}_1\Delta \phi < V_{\text{max}} \sim 3W^2e^K. \tag{4.15}
\]

Using (4.14) to eliminate \( \Delta \phi \) from this equation, we get

\[
W \gtrsim \sqrt{NW'}T_0, \tag{4.16}
\]

where we have used \( \hat{V}_1 \approx -2T_0W' \) (ignoring loop corrections), and have dropped numbers of order one. Using the COBE normalization, we get

\[
m_{3/2} \gtrsim \sqrt{\frac{N}{\delta}V_0} \tag{4.17}
\]

where \( m_{3/2} = We^{K/2} \). This is the condition guaranteeing that the modulus is not destabilized during inflation. It states that the gravitino mass is never small. In terms of the soft breaking terms and the sparticle spectrum obtained with a large gravitino mass, this implies that the scalar sparticles have very large masses. This is the type of scalar spectrum advocated in split SUSY models [43]. Of course if one is to observe scalars at the LHC, the scalars cannot be very massive and the gravitino mass must be reduced. This requires a superpotential which gives a small gravitino mass, but which still produces a stabilization potential with a high barrier. The model in section 5.2 has these properties. It is even possible to have \( m_{3/2} = 0 \) and still stabilize the moduli in a Minkowski vacuum, as described in [44]. In this case there is no lifting term \( \hat{V}_{\text{lift}} \), and the vacuum is supersymmetric. We could then obtain a gravitino mass from additional supersymmetry breaking effects. As we will show in section 6, if the vacuum is be Minkowski, the SUSY breaking must involve a lifting term. Otherwise the vacuum will be unstable.

5. KKLT scenario

As has already been noted, the parameters \( \hat{V}_0, C \) and \( M^2 \) are not independent, but depend on details of the moduli sector as well as on \( g \) and \( x \). As a simple example we will discuss the KKLT scenario [20], which has just one modulus field \( T \) and uses a non-supersymmetric lifting term (2.13). During inflation the full potential is then given by (2.27) and (2.29)–(2.31). The stabilization potential’s minimum is at \( T = T_0 \), so \( V_{\text{stab}}'(T_0) = V_{\text{stab}}(T_0) = 0 \). This allows us to eliminate \( E \), and one other parameter.

Following [20], we will use a racetrack superpotential

\[
W(T) = Ae^{-aT} + Be^{-bT}, \tag{5.1}
\]

where \( B, A, a \) and \( b \) are real constants. The above superpotential comes from gaugino condensation. We take \( T \gg 1 \) and \( aT, bT \gg 1 \) to guarantee the validity of the supergravity approximation and the weak gauge coupling limit.
5.1. Simple KKLT superpotential

Let us first focus on the KKLT model obtained with $b = 0$ and $aT_0 \gg 1$. To leading order this gives $W(T_0) \approx -(2/3)AaT_0e^{-aT_0}$ and

$$\tilde{V}_1 \approx 2AaT_0e^{-aT_0} \left[ 1 + \frac{5\Lambda^2}{24\pi^2} \right], \quad \tilde{V}_2 = -\frac{A^2}{4\pi^2},$$

$$\left( e^K \tilde{V}_1 \right)' \approx -\frac{Aa^2}{4T_0}e^{-aT_0} \left[ 1 + \frac{\Lambda^2}{8\pi^2} \right], \quad \lambda \approx \frac{A^2a^4}{3T_0}e^{-2aT_0} \left[ 1 + \frac{\Lambda^2}{8\pi^2} \right],$$

(5.2)

where $\lambda = \tilde{V}_{\text{stab}}''(T_0)$.

During inflation we also require the slow roll parameter $\epsilon \approx \frac{V^2}{|V|^2} < 1$. We see above that $\tilde{V}_1 \sim W$, and so we need $W \lesssim gx^2$. We observe that $\tilde{V}_2$ is small, suggesting $\eta$ is small too. However there are also corrections to $\eta$ from the modulus field variation during inflation (see section 3), which cannot be ignored. These imply

$$\eta \approx 2 \left( \tilde{V}_2 - 2\frac{(\left| e^K \tilde{V}_1 \right|)^2}{e^K \lambda} \right).$$

(5.3)

We see that for the above choice of $F$-term model, the slow roll parameter is found to be negative, independent of the parameters, and large: $\eta \approx -6.03$. This value is not compatible with the slow roll condition and hence not compatible with observational constraints. In general there could also be corrections to $\eta$ from variations of the imaginary part of $T$. However for the examples in this section we find that the potential is always minimized at $\text{Im}(T) = 0$, even during inflation.

Another constraint on the model comes from requiring that the minimum of $\tilde{V}_{\text{stab}}$ does not disappear during inflation. This would be the case if $\tilde{V}_0 > V_{\text{max}}$. For the above example we find that the maximum of the potential is at $T_{\text{max}} \approx T_0 + (1/a) \log(gaT_0)$, with $y = 2[1 + \Lambda^2/(8\pi^2)]/[n + (5n - 6)\Lambda^2/(24\pi^2)]$. The barrier height is then

$$V_{\text{max}} \approx \frac{1}{6T_0} A^2a^2e^{-2aT_0} \left[ 1 + \frac{\Lambda^2}{8\pi^2} \right].$$

(5.4)

We see that $V_{\text{max}} \approx 3W^2e^K$, as we assumed in the previous section.

We need $V_0 \sim gx^4e^K \lesssim V_{\text{max}}$, implying $gx^2 \lesssim W$. However this contradicts $gx^2 > W$, which was required to have small $\epsilon$. Hence the moduli superpotential in this subsection fails on both slow roll parameters.

5.2. Racetrack superpotential

As a second case, consider the racetrack models with $b \approx a$, and take the limit $aT_0, bT_0 \gg 1$. In this case we find $W(T_0) \approx -A(a - b)e^{-aT_0}/a$ and

$$\tilde{V}_1 \approx A(a - b)e^{-aT_0} \frac{3}{a} \left[ 1 + \frac{5\Lambda^2}{24\pi^2} \right], \quad \tilde{V}_2 = -\frac{A^2}{4\pi^2},$$

$$\left( e^K \tilde{V}_1 \right)' \approx -A(a - b)e^{-aT_0} \frac{a}{4T_0} \left[ 1 + \frac{\Lambda^2}{8\pi^2} \right],$$

$$\lambda \approx A^2(a - b)^2e^{-2aT_0} \frac{a^2}{3T_0} \left[ 1 + \frac{\Lambda^2}{8\pi^2} \right].$$

(5.5)
Small \( \epsilon \) requires \( V_1 < gx^2 \) which, as in the previous subsection, implies \( W < gx^2 \). This suggests that it will again be impossible to have small \( \epsilon \) and a stable modulus \( T \).

Using an asymptotic analysis of \( \tilde{V}_{\text{stab}} \), we find that this time the maximum of the potential occurs at \( T_{\text{max}} \approx T_0 + 1/a \), and the height of the barrier is

\[
V_{\text{max}} \approx \frac{1}{6T_0} A^2(a - b)^2 e^{-2aT_0} \left[ 1 + \frac{A^2}{8\pi^2} \right].
\]

(5.6)

In contrast to the previous subsection we find \( V_{\text{max}} \) is not of order \( W^2 e^K \). In this case it is possible to have \( \epsilon \ll 1 \) and \( V_0 \ll V_{\text{max}} \).

Unfortunately when we calculate \( \eta \), we run into the same problem as before. If the modulus field \( T \) were fixed at \( T_0 \), \( \eta \) would be small. However this does not occur, and (5.3) applies instead. We find again \( \eta \approx -6.03 \), ruling out this case too.

### 5.3. Analytic arguments

We will now use approximate analytic arguments to determine if any choices of \( W \) could give viable slow roll inflation. Using the expressions for \( \tilde{V}_{\text{stab}} \) (2.27) and \( \tilde{V}_1 \) (2.30), we obtain

\[
n \tilde{V}_{\text{stab}} + n \tilde{V}_1' = \frac{W'' W}{2T} - \left[ \frac{W''}{6T} + \frac{(2 - n)}{4T^3} W \right] V_1 + \frac{(n - 4)}{24T^3} V_1^2 + O \left( \frac{A^2}{8\pi^2} \right) = 0
\]

(5.7)

at \( T = T_0 \). Now the requirement that \( \epsilon \ll 1 \) implies that \( V_1 \ll gx^2 \), and since \( gx^2 \ll 1 \) we can take \( V_1 \ll 1 \) to find approximate solutions to (5.7). Noting that \( 1/(8\pi^2) \approx 0.01 \), the effect of the loop corrections will also be small, allowing us to neglect them. For small \( V_1 \) (5.7) is only satisfied if either \( W \) or \( W'' \) is very small. For the former we obtain

\[
W = -\frac{V_1}{3} + O \left( V_1^2, \frac{A^2}{8\pi^2} \right).
\]

(5.8)

Using this, the expression for \( \lambda = \tilde{V}_{\text{stab}}''(T_0) \) simplifies to \( \lambda = W''/(3T) \), to leading order in \( V_1 \). From (2.30) we obtain

\[
(e^K V_1)' = -\frac{TW'' - 2W'}{4T^3} + O \left( \frac{A^2}{8\pi^2} \right).
\]

(5.9)

To leading order in \( V_1 \) this is \( -W''/(4T^2) \). Substituting all this into the expression for \( \eta \) (5.3), which includes the effect of varying \( T \), we find

\[
\eta \approx -4 \left( \frac{[e^K \tilde{V}_1]'}{e^K \lambda} \right)^2 \approx -6.
\]

(5.10)

So we retrieve an \( \eta \) problem along this branch of solutions. The models discussed in the previous two subsections fall into this category.

The second approximate solution to (5.7) has

\[
W'' = \frac{(n - 2)}{2T_0^2} V_1 + O \left( V_1^2, \frac{A^2}{8\pi^2} \right).
\]

(5.11)
This again allows us to simplify \( \lambda \), and we find \( \lambda \approx -WW'''/(2T^2) \). Expanding in \( V_1 \), (5.9) implies \((e^K V_1)' \approx -V_1/(8T^4)\). Combining all this, (5.3) implies

\[
\eta \approx -\frac{V_1^2}{T_0^2 W''W}.
\]

(5.12)

Noting that \( \tilde{V}_1 \approx -2TW' \ll 1 \) in order to get successful inflation, we need both \( TW' \) and \( T^2W'' \ll (T^3 W'''W)^{1/2} \) at the minimum \( T = T_0 \). If this fine-tuning is not satisfied then slow roll inflation does not occur. This fine-tuning of the moduli superpotential is in addition to the fine-tuning required for the stabilization potential \( V_{\text{stab}} \) to have a Minkowski minimum. Furthermore, \( W \) must also be such that the maximum of \( V_{\text{stab}} \) is greater than the inflationary scale (so that minimum of the moduli potential does not disappear). This all adds up to a huge amount of fine-tuning, and the two term racetrack potential used above (5.1) does not seem to have enough parameters to satisfy all the conditions simultaneously. This suggests that a heavily fine-tuned, three term \( W \) would be needed to give viable inflation. It is hard to see how such a fine-tuned model could arise naturally.

6. Supersymmetry breaking Minkowski vacuum

In this section we consider Minkowski vacua with a non-vanishing gravitino mass. This is the traditional framework used in particle physics. At the end of inflation, the vacuum is a non-supersymmetric configuration with a vanishing cosmological constant. One would also like to impose \( W \ll 1 \) in order to have a hierarchy between the Planck scale and the sparticle masses. Supersymmetry breaking could be achieved with \( F \)-terms, although as we will show this leads to an unstable vacuum. It is therefore necessary to include a lifting term in the theory, such as those discussed at the end of section 2.2.

Consider a theory with only one modulus field, no lifting term, the following no-scale Kahler potential

\[
K = -3 \ln(T + \bar{T})
\]

and \( W = W(T) \). It is convenient to transform the field according to

\[
T = \frac{1 - iz}{1 + iz}.
\]

(6.2)

In terms of the new variable \( z \), one obtains

\[
K = -3 \ln(1 - |z|^2)
\]

(6.3)

and

\[
W(z) = \left(\frac{1 + iz}{\sqrt{2}}\right)^3 W \left(\frac{1 - iz}{1 + iz}\right).
\]

(6.4)

The Kahler transformation (6.2) preserves \( G = K + \ln |W|^2 \), which is the only relevant combination in supergravity. Using the new variable \( z \) one obtains

\[
V(z) = \frac{1}{3(1 - |z|^2)^2}(|W'|^2 - |3W - zW'|^2),
\]

(6.5)
where primes denote here $d/dz$. Imposing that the potential has an extremum with a vanishing cosmological constant leads to the following relations:

$$\left| 3 \frac{W'}{W'} - z \right| = 1 \quad (6.6)$$

and

$$\frac{W'^2}{WW''} = \frac{2}{3} \quad (6.7)$$

at the extremum when $W \neq 0$ and $W' \neq 0$, i.e. when supersymmetry is broken at the extremum. The extremum equations are also satisfied for $W = W' = 0$. The mass at the extremum involves

$$V_{zz} = \frac{1}{3(1 - |z|^2)^2} (|W''|^2 - |2W' - zW''|^2) \quad (6.8)$$

When $W \neq 0$ and $W' \neq 0$, the extremum equations give

$$V_{zz} = 0 \quad (6.9)$$

at the extremum $z_0$. As $V_{zz}$ is generically non-zero, this implies that the extremum is a saddle point where the direction along $\text{Im}(z - z_0)$ (for $V_{zz}$ chosen real positive) is tachyonic. This is a generic property of no-scale models and implies that the modulus cannot be stabilized with a vanishing potential and broken supersymmetry.

In contrast when supersymmetry is preserved, we find

$$V_{zz} = \frac{|W''|^2}{3(1 - |z|^2)} > 0 \quad (6.10)$$

as long as $W''$ does not vanish, guaranteeing the stability of the model [45].

This argument can be generalized to an arbitrary number of fields in the no-scale case

$$K = -3 \ln \left( T + \bar{T} - \sum_i |\chi_i|^2 \right) \quad (6.11)$$

where the $\chi^i$ are, for example, matter fields. Indeed putting $z^i = (1 + iz)\chi^i$ and collectively $z^I = \{z, z^I\}$, the Kahler superpotential becomes

$$K = -3 \ln(1 - z^I \delta_{ij} \bar{z}^j) \quad (6.12)$$

and the superpotential $W(z, \chi^i)$ is turned into

$$W(z^I) \equiv \left( \frac{1 + iz}{\sqrt{2}} \right)^3 W \left( \frac{1 - iz}{1 + iz}, \frac{z^I}{1 + iz} \right). \quad (6.13)$$

With these redefinitions, the scalar potential reads

$$V = \frac{1}{3(1 - |z|^2)^2} (|W_I|^2 - |3W - W_I z^I|^2) \quad (6.14)$$

The vanishing of the cosmological constant at the extremum implies that

$$|W_I|^2 = |3W - W_I z^I|^2. \quad (6.15)$$
The minimum equation reads
\[ (2W_I - z^J W_{J I}) = \frac{W_{IJ} \delta^{JK} \bar{W}_{K I}}{3W - z^J W_J} \] (6.16)
when supersymmetry is broken. In the supersymmetric case, the minimum equations are \( W = W_I = 0 \).

The stability of the extremum depends on the mass matrix which reads
\[ V_{IJ} = \frac{1}{3(1 - |z|^2)^2} \left[ W_{IJ} \delta^{JK} \bar{W}_{KJ} - (2W_I - z^K W_{KI})(2\bar{W}_J - \bar{z}^K \bar{W}_{KJ}) \right]. \] (6.17)
Using the extremum equation we find that
\[ V_{IJ} = \frac{1}{3|W|^2(1 - |z|^2)^2} \left( |W_I|^2 W_{IK} \delta^{KL} \bar{W}_{LJ} - W_{IL} \delta^{JK} W_K \bar{W}_{JN} \delta^{MN} \right) \] (6.18)
in the supersymmetry breaking case. This mass matrix has nice algebraic properties. When \( W_{IJ} \) has zero eigenvalues, the mass matrix \( V_{IJ} \) vanishes in these directions. In the directions orthogonal to the zero eigenstates of \( W_{IJ} \), all the eigenvalues of \( V_{IJ} \) are positive but one which vanishes along the eigenvector \( f^I = W^{IJ} W_J \) where \( W^{IJ} W_{JK} = \delta^K_I \). Hence the mass matrix \( V_{IJ} \) vanishes along \( f^I \) and the zero eigenstates of \( W_{IJ} \).

In the supersymmetric case, we find
\[ V_{IJ} = \frac{1}{3(1 - |z|^2)^2} \left[ W_{IK} \delta^{JK} \bar{W}_{KJ} - z^K W_{KI} \bar{z}^K \bar{W}_{KJ} \right]. \] (6.19)
Defining a vector space basis with \( e_i^I = z^I / |z|^2 \) where \( |z|^2 = z^I \delta_{IJ} \bar{z}^J \), and orthogonal vectors \( e_i^I, i > 0 \), we can use the sum rule \( \delta^{JK} = \sum_i e_i^K \bar{e}_i^K \) to find that
\[ V_{IJ} = \frac{1}{3(1 - |z|^2)^2} \left[ \left( \frac{1}{|z|^2} - 1 \right) z^K W_{KI} \bar{z}^K \bar{W}_{KJ} - \sum_{i>0} W_{IK} e_i^K \bar{W}_{KJ} e_i^K \right] \] (6.20)
which is a positive definite Hermitian matrix as \( V_{IJ} u^I \bar{u}^J > 0 \) for any \( u^I \) provided the matrix \( W_{IJ} \) does not have zero eigenstates. This corroborates the results in [45].

Hence we have found that the mass matrix \( V_{IJ} \) has zero eigenstates when supersymmetry is broken. Now for a generic supersymmetry breaking model, the holomorphic mass matrix is such that \( V_{IJ} f^I f^J \neq 0 \). Expanding along the direction \( z^I = z_0^I + \sigma f^I + O(\sigma^2) \) where \( z_0^I \) is the minimum leads to a potential
\[ V = \frac{1}{2} V_{IJ} f^I f^J \sigma^2 + (c.c.) + O(\sigma^2). \] (6.21)
Notice that this leads to a tachyonic direction as before, implying that the supersymmetry breaking extremum with zero cosmological constant is not stable. Hence we see that a supersymmetry breaking Minkowski vacuum from a no-scale theory which does not use
a lifting term cannot be stable. Since such a set-up does not even stabilize the moduli fields, it is certainly not suitable to use in a hybrid inflation model.

7. Conclusions

In this paper we considered $F$-term hybrid inflation in supergravity, when moduli fields are present. In the literature it is usually assumed that moduli fields are stabilized before the period of slow roll inflation begins and that they therefore have no impact on the dynamics of the inflaton field. We have shown that this is not necessarily the case. Firstly, the tree level coupling of moduli and inflaton fields in supergravity induces a slope and, in general, a mass term in the potential for the inflaton field. On top of this, loop corrections from both the inflation and moduli sectors of theory contribute to the slope and curvature of the potential. A further problem comes from the evolution of the moduli fields during inflation. Since they are stabilized by a steep potential, they are roughly constant during inflation. However, despite the small size of their variation, we have shown that they still give a significant contribution to the effective inflaton potential. This generally leads to a large and tachyonic mass for the inflaton.

We find that this class of model has up to three different $\eta$ problems. Firstly there is the usual one coming from embedding hybrid inflation in supergravity (which is avoided with the help of shift symmetries). Secondly there are tree level contributions to the inflaton mass from the coupling of the inflaton to the moduli sector (although these are absent for no-scale models). Finally there is the negative contribution to the effective inflaton mass from the rolling of the moduli fields during inflation. It is not obvious that this third $\eta$ problem can be removed with natural additional symmetries. The model also has an $\epsilon$ problem due to the tree level contributions to the inflaton slope from the moduli sector.

We found that in the popular model for moduli stabilization, the KKLT scenario, the induced mass term for the inflaton field is generally too big to give a period of slow roll inflation. It seems that only with a heavily fine-tuned superpotential for the moduli sector may viable slow roll inflation take place.

To conclude, we have shown that the coupling of moduli fields to hybrid inflation can have an important impact on the effective inflationary potential. Neglecting this coupling is not consistent and, as we have seen, generally leads to incorrect conclusions about the inflationary dynamics. These considerations are also relevant to other inflationary scenarios. In each case it should be checked that the moduli fields, even if thought to be stabilized, do not spoil the flatness of the inflationary potential and therefore the dynamics of the inflaton field.

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Appendix: Masses

We have $M^2_{\text{boson}} = 2K^{ij} \partial_i \partial_j V + 6f_R^{-1} \sum_{i=\pm} |\phi|^2$, where $f_R = \text{Re}(f)$. Note that we are assuming the only gauge group in the theory is $U(1)_1$. If there were additional gauge groups, the additional gauge sector would also give contributions to the mass matrices. For the fermions

$$M^2_{\text{fermion}} = 2K^{ij} m_{ij} K^{ki} \bar{m}_{k\ell} + 4K^{ij} m_{i\lambda} \bar{m}_{j\lambda} + 2|m_{\lambda\lambda}|^2 + 4m_{3/2}^2$$  \hspace{1cm} (A.1)

where

$$m_{ij} = e^{K/2} D_i D_j \mathcal{W} = e^{K/2} \left( W_{ij} + K_{ij} \mathcal{W} + K_{ij} \partial_i \mathcal{W} + K_{ij} \partial_j \mathcal{W} - \Gamma_{ij}^k D_k \mathcal{W} \right)$$  \hspace{1cm} (A.2)

with $\Gamma_{ij}^k = K^{k\ell} \partial_i K_{j\ell}$, and

$$m_{i\lambda} = \frac{\sqrt{2q_i} \phi_i}{\sqrt{f_R}}, \quad m_{\lambda\lambda} = \frac{e^{K/2} K^{ji} \bar{J} \partial_i f}{2f_R}, \quad m_{3/2} = e^{K/2} \mathcal{W},$$  \hspace{1cm} (A.3)

where $q_{\pm} = \pm 1$, and $q_i = 0$ otherwise. Define

$$m_{1j}^{(0)} = e^{K/2} D_j D_j \mathcal{W}, \quad m_{1j}^{(1)} = e^{K/2} [K_{1j} + K_{j1} - \Gamma_{1j}^k K_k].$$  \hspace{1cm} (A.4)

The diagonal entries of the boson mass matrix for the Minkowski background are

$$M^2_{\pm \pm} = (2g^2 x^2 + 2|W|^2 + V_S) e^K + \frac{x^2}{f_R},$$

$$M^2_{\phi \phi} = (4g^2 x^2 + 2|W|^2 + V_S) e^K, \quad M^2_{1j} = (e^K V_S + V_{\text{lift}})_{1j},$$

and non-zero elements of the fermion mass matrix are

$$m_{\phi \phi} = m_{\pm \pm} = -m_{3/2}, \quad m_{\phi \pm} = e^{K/2} \sqrt{2g} x, \quad m_{\pm \lambda} = \pm \frac{\sqrt{2x}}{\sqrt{f_R}},$$

$$m_{\lambda \lambda} = \frac{e^{K/2} K^{1j} D_j \bar{W} \partial_i f}{2f_R}, \quad m_{1j} = m_{1j}, \quad m_{3/2} = e^{K/2} W.$$  \hspace{1cm} (A.6)

Hence we obtain

\[ \text{Str} M^2_{\text{Mink}} = 2e^K |W|^2 + 6e^K V_S + 2K_{1j} (e^K V_S + V_{\text{lift}})_{1j} \]
\[- 2|m_{1j}^{(0)} K_{1j}|^2 - \frac{e^K}{2f_R} |\partial_j f K_{1j} D_j W|^2. \]  \hspace{1cm} (A.7)

For the inflationary background, the diagonal entries of the boson mass matrix are

$$M^2_{\phi \phi} = [2(3g^2 x^4 + 4g^2 x^2 \phi^2 - 2\sqrt{2g} x^2 \phi \text{Re}(W) + |W|^2)]$$
\[ + V_S - 2\sqrt{2g} x^2 \phi \text{Re}(V_1) + 2g^2 x^4 (1 + \phi^2) V_2] e^K, \]

$$M^2_{\pm \pm} = [2(g^2 x^4 + g^2 [2x^4 + 2x^2 + 1] \phi^2 - \sqrt{2g} \phi \text{Re}(W)[1 + 2x^2] + |W|^2)$$
\[ + V_S - 2\sqrt{2g} x^2 \phi \text{Re}(V_1) + 2g^2 x^4 \phi^2 V_2] e^K, \]

$$M^2_{1j} = 2g^2 x^4 e^K (K_{1j} + K_{j1}) + (e^K V_S + V_{\text{lift}})_{1j}$$
\[- 2\sqrt{2g} x^2 \phi (e^K \text{Re} V_1)_{1j} + 2g^2 x^4 \phi^2 (e^K V_2)_{1j}, \]  \hspace{1cm} (A.8)
while the fermion mass matrix has
\[ m_{\phi} = -m_{3/2}, \quad m_{\pm} = e^{K/2} (\sqrt{2} g x^2 - W), \]
\[ m_{\phi J} = -e^{K/2} \sqrt{2} g x^2 K_{J}, \quad m_{\lambda \lambda} = e^{K/2} \frac{\partial f}{\partial R} (D_{J} W - \sqrt{2} g x^2 \phi K_{J}), \]
\[ m_{IJ} = m_{IJ}^{(0)} - \sqrt{2} g x^2 \phi m_{IJ}^{(1)}, \quad m_{3/2} = e^{K/2} (W - \sqrt{2} g x^2 \phi). \]

This implies
\[
\text{Str} M_{\text{Inf}}^2 = \text{Str} M_{\text{Mink}}^2 + 4 \left[ 2 + \delta J \right] g^2 x^4 e^K \]
\[ - 4 \sqrt{2} g x^2 \phi \text{Re} \left[ (W + 3 V_1) e^K + K^{IJ} (e^K V_1)_{,IJ} \right. \]
\[ - m_{IJ}^{(0)} K^{IA} \bar{m}_{AB}^{(1)} K^{JB} - \frac{e^K}{4 f_R^2} (K^{IJ} K_{J} \partial I f) (K^{BA} D_B W \partial A f) \]
\[ + 4 g^2 x^4 \phi^2 \left[ 1 + V_2 \right] e^K + K^{IJ} (e^K V_2)_{,IJ} \]
\[ - \left| m_{IK}^{(1)} K^{KJ} \right|^2 - \frac{e^K}{4 f_R^2} \left| K^{IJ} K_{J} \partial I f \right|^2. \]

(A.9)
Coupling hybrid inflation to moduli

[18] Garcia-Bellido J, Rabaneda R and Zamora F, Inflationary scenarios from branes at angles, 2002 J. High Energy Phys. JHEP01(2002)036 [SPIRES] [hep-th/0112147]

[19] Majumdar M and Davis A C, D-brane anti-brane annihilation in an expanding universe, 2003 J. High Energy Phys. JHEP12(2003)012 [SPIRES] [hep-th/0304153]

[20] Kachru S, Kallosh R and Linde A, De Sitter vacua in string theory, 2003 Phys. Rev. D 68 046005 [SPIRES] [hep-th/0301240]

[21] Kachru S, Kallosh R, Linde A, Maldacena J M, McAllister L and Trivedi S P, Towards inflation in string theory, 2003 J. Cosmol. Astropart. Phys. JCAP10(2003)013 [SPIRES] [hep-th/0308055]

[22] Kawasaki M, Yamaguchi M and Yanagida T, Natural chaotic inflation in supergravity, 2000 Phys. Rev. Lett. 85 3572 [SPIRES] [hep-ph/00041243]

[23] Yamaguchi M and Yokoyama J, New inflation in supergravity with a chaotic initial condition, 2001 Phys. Rev. D 63 043506 [SPIRES] [hep-th/0007021]

[24] Yamaguchi M and Yokoyama J, Chaotic hybrid new inflation in supergravity with a running spectral index, 2003 Phys. Rev. D 68 123520 [SPIRES] [hep-th/0307373]

[25] Hsu J P and Kallosh R, Volume stabilization and the origin of the inflaton shift symmetry in string theory, 2004 J. High Energy Phys. JHEP04(2004)042 [SPIRES] [hep-th/0402047]

[26] Brax P and Martin J, Shift symmetry and inflation in supergravity, 2005 Phys. Rev. D 72 023518 [SPIRES] [hep-th/0504168]

[27] Dasgupta K, Herdeiro C, Hirano S and Kallosh R, D3/D7 inflationary model and M-theory, 2002 Phys. Rev. D 65 126002 [SPIRES] [hep-th/0203019]

[28] Dasgupta K, Hsu J P, Kallosh R, Linde A and Zagernann M, D3/D7 brane inflation and semilocal strings, 2004 J. High Energy Phys. JHEP08(2004)030 [SPIRES] [hep-th/0405247]

[29] Hsu J P, Kallosh R and Prokushkin S, On brane inflation with volume stabilization, 2003 J. Cosmol. Astropart. Phys. JCAP12(2003)009 [SPIRES] [hep-th/0311077]

[30] Kallosh R and Linde A, P-term, D-term and F-term inflation, 2003 J. Cosmol. Astropart. Phys. JCAP10(2003)008 [SPIRES] [hep-th/0306058]

[31] Chen P, Dasgupta K, Narayan K, Shmakova M and Zagermann M, Brane inflation, solitons and cosmological solutions: I, 2005 J. High Energy Phys. JHEP09(2005)009 [SPIRES] [hep-th/0501185]

[32] Koyama F, Tachikawa Y and Watari T, Hybrid inflation model from D3–D7 system, 2004 Phys. Rev. D 69 106001 [SPIRES] Koyama F, Tachikawa Y and Watari T, 2004 Phys. Rev. D 70 129907 [SPIRES] [hep-th/0311191]

[33] Dasgupta K, 2006 private communication

[34] Binetruy P, Dvali G, Kallosh R and Van Proeyen A, Fayet–Iliopoulos terms in supergravity and cosmology, 2004 Class. Quantum Grav. 21 3137 [SPIRES] [hep-th/0402046]

[35] Robbins D and Sethi S, A barren landscape, 2005 Phys. Rev. D 71 046008 [SPIRES] [hep-th/0405011]

[36] Burgess C P, Kallosh R and Quevedo F, de Sitter string vacua from supersymmetric D-terms, 2003 J. High Energy Phys. JHEP10(2003)056 [SPIRES] [hep-th/0309187]

[37] Achucarro A, de Carlos B, Casas J A and Doplicher L, de Sitter vacua from uplifting D-terms in effective supergravities from realistic strings, 2006 Preprint hep-th/0601190

[38] Choi K, Falkowski A, Nilles H P and Olechowski M, Soft supersymmetry breaking in KKLT flux compactification, 2005 Nucl. Phys. B 718 113 [SPIRES] [hep-th/0503216]

[39] Ferrara S, Kounnas C and Zwirner F, Mass formulae and natural hierarchy in string effective supergravities, 1994 Nucl. Phys. B 429 589 [SPIRES] Ferrara S, Kounnas C and Zwirner F, 1995 Nucl. Phys. B 433 255 [SPIRES] [hep-th/9405188]

[40] Spergel D N et al, Wilkinson microwave anisotropy probe (WMAP) three year results: implications for cosmology, 2006 Preprint astro-ph/0603449

[41] Kinney W H, Kolb E W, Melchiorri A and Riotto A, Inflation model constraints from the Wilkinson microwave anisotropy probe three-year data, 2006 Preprint astro-ph/0605338

[42] Martin J and Ringeval C, Inflation with WMAP3: confronting the slow roll and exact power spectra to CMB data, 2006 Preprint astro-ph/0605367

[43] Arkani-Hamed N and Dimopoulos S, Supersymmetric unification without low energy supersymmetry and signatures for fine-tuning at the LHC, 2005 J. High Energy Phys. JHEP06(2005)073 [SPIRES] [hep-th/0405159]

[44] Kallosh R and Linde A, Landscape, the scale of SUSY breaking, and inflation, 2004 J. High Energy Phys. JHEP12(2004)004 [SPIRES] [hep-th/0411011]

[45] Blanco-Pillado J J, Kallosh R and Linde A, Supersymmetry and stability of flux vacua, 2005 Preprint hep-th/0511042