Particle Production via Dirac Dipole Moments in the Magnetized and Non-magnetized Exponentially Expanding Universe

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In the present paper, we solve the Dirac equation in the 2+1 dimensional exponentially expanding magnetized by uniform magnetic field and non-magnetized universes, separately. Asymptotic behaviors of the solutions are determined. Using these results we discuss the current of a Dirac particle to discuss the polarization densities and the magnetization density in the context of Gordon decomposition method. In this work we also calculate the total polarization and magnetization, to investigate that the magnetic field how can effect on the particle production. Furthermore, the electric and the magnetic dipole moments calculated, and based on these we have discussed the effects of the dipole moments on the charge distribution of the universe and its conductivity for both the early and the future time epoch in the presence/absence a constant magnetic field and exponentially expanding spacetime.
Introduction

One of the most interesting and important results of the formulation of the relativistic quantum mechanics in curved spacetime is the particle creation event in the expanding universe which was firstly discussed by Parker for the scalar particles and Dirac particles (1–3). So, he computed the number density of the created particles by means of the Bogolibov transformation by using the combining out vacuum states with constructed from the solutions of the relativistic particles wave equations. After these important works of Parker, the solutions of the relativistic particle wave equations have extensively been studied in various 3+1 dimensional spacetime backgrounds (4–13).

The Gordon decomposition of the Dirac currents is another useful tool for discussing the particle creation phenomena (4, 8, 9, 14). In the decomposition method, the Dirac currents constructed from the solutions of the Dirac equations are separated into three parts, the convective, polarization with three components, and magnetization with three components, in the 3+1 dimensional spacetime. This method includes some complexities stemming from the 3+1 dimensional spacetime. Using this method in a 2+1 dimensional curved spacetime, the densities of the particle currents are separated into three parts, as in the 3+1 dimensional spacetime, but polarization density has two components and the magnetization density has only one component (14), and, moreover, as the Dirac spinor can be defined by only two components, the computations in the 2+1 dimensional spacetime become more simple than that of 3+1 dimensional spacetime. Because of the simplicity stemming from the dimensions, the dipole moments that are computed from the polarization and magnetization densities of the Dirac electron under influence in a constant magnetic field are easily computed and their result are, furthermore, compatible with the current experimental results (23). With these motivations, in this study, we solve the Dirac equation in the 2+1 dimensional exponentially expanding magnetized by uni-
form magnetic field and non-magnetized universes, separately, and discuss the particle creation event by means of the Dirac currents written in terms of these solutions. As a result, we observe that the polarization and magnetization parts of the currents are affected differently whether the exponentially expanding universe is magnetized or not and find expressions for the electric and magnetic dipole moments by integrating the polarization and magnetization densities on hypersurface.

The outline of the work is as follows; in Section 2, we, at first, discuss the Dirac equation solutions in the 2+1 dimensional exponentially expanding universe. In Section 3, the Dirac equation is solved in the 2+1 dimensional exponentially expanding universe with a constant magnetic field. In Section 4, we derive the components of Dirac currents for the solutions obtained in the Sections 2 and 3, and also compute the polarization density, the magnetization density, total polarization (electric dipole moment) and the total magnetization (magnetic dipole moment). Finally, last Section, conclusion, includes a discussion about the results of this work.

1 Dirac particle in the 2+1 dimensional exponentially expanding universe

The behavior of the electron in 2+1 dimensional curved space is represented by the covariant form of the Dirac equation (14), which is important application in curved spacetime (15–22)

\[ \{ i \bar{\sigma}^\nu(x) [\partial_\nu - \Gamma_\nu + i e A_\nu] \} \Psi(x) = m \Psi(x), \] (1)

where \( \Psi(x) = (\psi_1, \psi_2) \) is the Dirac spinorial wave function with two components that are positive and negative energy eigenstates, \( m \) is the mass of Dirac particle, \( e \) is the charge of the Dirac particle and \( A_\nu \) are 3-vectors of electromagnetic potential. Using triads, \( e_\nu^{(i)}(x) \), Dirac matrices that dependent on spacetime, \( \bar{\sigma}^\nu(x) \), are written in terms of the constant Dirac matrices, \( \bar{\sigma}^i \);

\[ \bar{\sigma}^\nu(x) = e_\nu^{(i)}(x) \bar{\sigma}^i. \] (2)
So, we choose the constant Dirac matrices, $\sigma^j$, in the flat spacetime as follows:

$$\sigma^j = (\sigma^0, \sigma^1, \sigma^2)$$ \hspace{1cm} (3)

with

$$\sigma^0 = \sigma^3, \quad \sigma^1 = i\sigma^1, \quad \sigma^2 = i\sigma^2,$$ \hspace{1cm} (4)

where $\sigma^1$, $\sigma^2$ and $\sigma^3$ are Pauli matrices. The spin connection, $\Gamma_\mu(x)$, for the diagonal metrics is defined as:

$$\Gamma_\mu(x) = -\frac{1}{4} g_{\tau\alpha} \Gamma^\alpha_{\nu\beta} \left[ \sigma^\tau(x), \sigma^\beta(x) \right],$$ \hspace{1cm} (5)

where $\Gamma^\alpha_{\nu\beta}$ is Christoffel symbol. Also, the metric tensor $g_{\beta\nu}(x)$ is written in terms of triads as follows;

$$g_{\beta\nu}(x) = e^{(i)}_{\beta}(x)e^{(j)}_{\nu}(x)\eta(i)(j),$$ \hspace{1cm} (6)

where $\beta$ and $\nu$ are curved spacetime indices run from 0 to 2, $i$ and $j$ are flat spacetime indices run 0 to 2 and $\eta(i)(j)$ is the signature with (1,-1,-1).

The (2+1) dimensional de Sitter space-time metric can be written as (24);

$$ds^2 = dt^2 - e^{2Ht} \left[ dr^2 + r^2 d\phi^2 \right]$$ \hspace{1cm} (7)

where $H$ is Hubble parameter. From the Eqs.(3)-(7), the spin connections for the metric read

$$\Gamma_0 = 0, \quad \Gamma_1 = -\frac{H}{2} e^{Ht} \frac{\Delta^0}{\Delta^1} \quad \text{and} \quad \Gamma_2 = -\frac{1}{2} \left[ rHe^{Ht} \frac{\Delta^0}{\Delta^1} \right].$$ \hspace{1cm} (8)

Using the Eqs.(7), (2) and (8), then, the Dirac equation in the 2+1 dimensional exponentially expanding universe becomes,

$$\left\{ \sigma^0 (\partial_t + H) + im + e^{-Ht} [\sigma^1 (\partial_r + \frac{1}{2r}) + \sigma^2 \partial_\phi] \right\} \Psi(x) = 0.$$ \hspace{1cm} (9)
Letting the Eqs.(4) and Eqs.(9) and the Dirac spinor, \( \Psi(x) = (\Psi_1, \Psi_2) \), we write the Dirac equation in explicit form as follows:

\[
\begin{align*}
[\partial_t + im + H] \psi_1 + ie^{-Ht}[\partial_r + \frac{1}{2r} - \frac{i}{r} \partial_\phi] \psi_2 &= 0 \\
[\partial_t - im + H] \psi_2 - ie^{-Ht}[\partial_r + \frac{1}{2r} + \frac{i}{r} \partial_\phi] \psi_1 &= 0.
\end{align*}
\] (10)

To find the solutions of Eqs.(10), thanks to the separation of variables method, the wave function components can be defined as

\[
\left( \begin{array}{c} \Psi_1 \\ \Psi_2 \end{array} \right) = e^{ik\phi} \left( \begin{array}{c} T_1(t) R_1(r) \\ T_2(t) R_2(r) \end{array} \right),
\] (11)

By these definitions, the Dirac equation are separated into the following two differential equation systems:

\[
\begin{align*}
\left[ \frac{d}{dr} + \frac{1}{2r} + \frac{k}{r} \right] R_2(r) &= -\lambda R_1(r) \\
\left[ \frac{d}{dr} + \frac{1}{2r} - \frac{k}{r} \right] R_1(r) &= \lambda R_2(r)
\end{align*}
\] (12)

and

\[
\begin{align*}
\left[ \frac{d}{dt} + im + H \right] T_1(t) &= i\lambda e^{-Ht} T_2(t) \\
\left[ \frac{d}{dt} - im + H \right] T_2(t) &= i\lambda e^{-Ht} T_1(t),
\end{align*}
\] (13)

where \( \lambda \) is a separation constant, and we find the solutions of Eqs.(12) in terms of the Bessel and confluent hypergeometric functions as follows:

\[
\begin{align*}
R_1(r) &= AJ_{k-\frac{1}{2}}(\lambda r) = A \frac{e^{-i\lambda r} (\lambda r)^{k-\frac{1}{2}}}{\Gamma \left( k + \frac{1}{2} \right)} \, _1F_1 \left[ k, 2k; i2r\lambda \right] \\
R_2(r) &= BJ_{k+\frac{1}{2}}(\lambda r) = B \frac{e^{-i\lambda r} (\lambda r)^{k+\frac{1}{2}}}{\Gamma \left( k + \frac{3}{2} \right)} \, _1F_1 \left[ k + 1, 2k + 2; i2r\lambda \right].
\end{align*}
\] (14)
On the other hand, to solve Eqs. (13), we must define a new variable such as \( z = \lambda e^{-Ht} \). With the definition \( n = -i \frac{m}{H} \), the solutions of the Eqs. (13) are obtained in terms of Bessel functions or confluent hypergeometric functions;

\[
T_1 (z) = C z^{\frac{3}{2}} e^{-iz \frac{1}{2}} V^{n + \frac{1}{2}} (z) = C z^3 \left( \frac{z}{2} \right)^{n + \frac{1}{2}} \frac{1}{\Gamma (n + \frac{3}{2})} \; \sum_{k=0}^{n} \frac{(-i)^k}{k!} \left( \frac{z}{2} \right)^k J_n (\lambda r) \left( \frac{z}{2} \right) J_{n+1} (\lambda r) \]

\[
T_2 (z) = D z^{\frac{3}{2}} e^{-iz \frac{1}{2}} V^{n - \frac{1}{2}} (z) = D z^3 \left( \frac{z}{2} \right)^{n - \frac{1}{2}} \frac{1}{\Gamma (n + \frac{3}{2})} \; \sum_{k=0}^{n} \frac{(-i)^k}{k!} \left( \frac{z}{2} \right)^k J_n (\lambda r) \left( \frac{z}{2} \right) J_{n-1} (\lambda r) \]

Then, the wave function, \( \Psi(x) \), can be written as

\[
\Psi = N z^{\frac{3}{2}} e^{ik\phi} \left( \frac{J_{n+\frac{1}{2}} (z) J_{k-\frac{1}{2}} (\lambda r)}{J_{n-\frac{1}{2}} (z) J_{k+\frac{1}{2}} (\lambda r)} \right) \]

where \( N \) is normalization constant. To find the normalization constant, we use the Dirac-delta normalization condition (4):

\[
\int \sqrt{|g|} \overline{\Psi} \Psi \sigma d\sigma = \delta (\lambda - \lambda') \]

where \( g \) is determinant of the metric tensor and \( \overline{\Psi} = \Psi^{*} \sigma^0 \). Thus, the normalization constant are computed as

\[
|N|^2 = \frac{H^2}{4 \pi z \lambda J_{n-\frac{1}{2}} (z) J_{n+\frac{1}{2}} (z)},
\]

where we use the following relation (26)

\[
\int_0^\infty r J_{k+\frac{1}{2}} (\lambda r) J_{k+\frac{1}{2}} (\lambda' r) dr = \frac{1}{\sqrt{\lambda' - \lambda}} \delta (\lambda - \lambda').
\]

2 Dirac particle in the 2+1 dimensional exponentially expanding magnetized universe

It is interesting in discussing what the universe would happen if it was under influence in an external constant magnetic field in the beginning time. Therefore, an electromagnetic potential can be chose as \( A_0 = 0 \) and \( \overrightarrow{A} (r, \phi) = \frac{1}{2} B_0 r \hat{\phi} \) for a constant magnetic field in 2+1 dimensional
spacetime. Then, the Dirac equation in the 2+1 dimensional exponential expanding universe with a constant magnetic field becomes

\[
\left\{ \sigma^0 (\partial_t + H) + im + e^{-Ht} \sigma^1 (\partial_r + \frac{1}{2r}) + \sigma^2 \left( \frac{1}{2r} \partial_\phi + \frac{ieB_0}{2} \right) \right\} \Psi(x) = 0
\]

and, thus, the explicit form of the equation is written as

\[
(\partial_t + im + H) \psi_1 + ie^{-Ht} \left( \partial_r + \frac{1}{2r} - \frac{i}{r} \partial_\phi + \frac{eB_0}{2} \right) \psi_2 = 0
\]

\[
(\partial_t - im + H) \psi_2 - ie^{-Ht} \left( \partial_r + \frac{1}{2r} + \frac{i}{r} \partial_\phi - \frac{eB_0}{2} \right) \psi_1 = 0
\]  

(18)

To solve the Eqs(18), we use the same procedure as section before. The solutions of the equations are

\[
\Psi = Nz^\frac{3}{2} e^{ik\phi} 
\left( J_{n+\frac{1}{2}} (z) \rho^{\frac{1}{2}} W_{\kappa,\eta}(\rho) 
J_{n-\frac{1}{2}} (z) [\rho^{\frac{1}{2}} (\rho-2\eta-1)(\sqrt{e^2 B_0^2 - 4\lambda^2} - eB_0) W_{\kappa,\eta}(\rho) - \sqrt{e^2 B_0^2 - 4\lambda^2} W_{\kappa+1,\eta}(\rho)] \right)
\]

(19)

where \( \kappa = -\frac{eB_0}{\sqrt{e^2 B_0^2 - 4\lambda^2}} \), \( \eta = k - \frac{1}{2} \), \( \rho = \sqrt{e^2 B_0^2 - 4\lambda^2}r \), and \( N \) is normalization constant.

As all the contributions for particle creation and dipole moments are taking place from the boundaries, we can write the wave function in the following asymptotic form (25):

\[
\Psi \sim Nz^\frac{3}{2} e^{ik\phi} \rho^{\frac{1}{2}} e^{-\frac{z^2}{2}} \left( \frac{1}{\Gamma(n+\frac{1}{2})} \left( \frac{\pi}{2} \right)^n + \frac{1}{\Gamma(n+\frac{1}{2})} \left( \frac{\pi}{2} \right)^n \rho^{\frac{1}{2}} (\rho-2\eta-1)(\sqrt{e^2 B_0^2 - 4\lambda^2} - eB_0) W_{\kappa,\eta}(\rho) - \sqrt{e^2 B_0^2 - 4\lambda^2} W_{\kappa+1,\eta}(\rho) \right) \right)
\]

(20)

Then, the normalization constant can be obtained from the Eqs.(17) as follows:

\[
|N|^2 \sim \frac{H^2 (\eta + 1/2)^2 (1 - 4n^2)}{\cos (n\pi)} \frac{1}{z^2 \left( (\eta + 1/2)^2 - \kappa^2 \right) \Gamma(2\kappa) + (1 - 4n^2) \Gamma(2\kappa - 2)}
\]

where we use \( \Gamma \left( -n + \frac{3}{2} \right) \Gamma \left( n + \frac{3}{2} \right) = \frac{\pi (1 - 4n^2)}{4 \cos (n\pi)} \) and \( \Gamma \left( -n + \frac{1}{2} \right) \Gamma \left( n + \frac{1}{2} \right) = \frac{\pi}{\cos (n\pi)} \).

3 Dirac currents

The 2+1 dimensional Dirac current is written as

\[
J^\nu = \overline{\Psi} \sigma^\nu (x) \Psi
\]

(21)
where $\overline{\Psi}$ is hermitian conjugate of the Dirac spinor $\Psi$ and equal to $\overline{\Psi} = \Psi^{\dagger} \sigma^0 = \Psi^{\dagger} \sigma^3$ (14). As showed in (14), the Eqs. (21) expressed in explicit form as follow;

$$J^\tau = \frac{1}{2m} (\overline{\Psi} \sigma^\tau \sigma^0 (t, r) \Psi)_{,\nu} - \frac{1}{2m} \overline{\Psi} \left( \frac{i}{2} \sigma^\nu \overleftarrow{\partial} - eA^\nu \right) \Psi - \frac{i}{4m} \overline{\Psi} \left[ \sigma^\nu (t, r), \sigma_\nu (t, r) \right] \Psi$$

$$- \frac{i}{2m} \overline{\Psi} \sigma^\nu \Gamma_v \sigma^\nu (t, r) \Psi - \frac{i}{4m} \overline{\Psi} \left[ \sigma^\nu (t, r), \sigma^\nu (t, r) \right] \Psi \quad (22)$$

The components of the Dirac current in the 2+1-dimensional exponential expanding universe, $J^0$ and $J^k$, are

$$J^0 = \frac{1}{2m} \partial_0 \left[ \overline{\Psi} \sigma^0 (t, r) \Psi \right] - \frac{1}{2m} \overline{\Psi} \left( \frac{i}{2} \overleftarrow{\partial} - qA^0 \right) \Psi - \frac{i}{2mr} \exp (-Ht) \overline{\Psi} \sigma^1 \sigma^0 \Psi \quad (23)$$

and

$$J^k = \frac{1}{2m} \partial_i \left[ \overline{\Psi} \sigma^0 (t, r) \Psi \right] + \frac{1}{2m} \sigma^k \overline{\Psi} \sigma^0 (t, r) \Psi - \frac{1}{2m} \overline{\Psi} \left( \frac{i}{2} \overleftarrow{\partial}^k - qA^k \right) \Psi$$

$$+ i \frac{3H}{2mr} \exp (-Ht) \overline{\Psi} \sigma^0 \sigma^k \Psi + i \overline{\Psi} \sigma^0 \sigma^k \Psi \quad (24)$$

where $k, l = 1, 2, \sigma^0 = i/2 \left[ \sigma^0, \sigma^k (t, r) \right], \sigma^{0l} = i/2 \left[ \sigma^l (t, r), \sigma^k (t, r) \right]$ and $\delta_{k2}$ is Dirac-delta function. Also this components can be rewritten in terms of the convective, the polarization and magnetization parts as follows:

$$J^0 = \partial_k P_k + \rho_{\text{convecitive}} - \frac{i}{2mr} \exp (-Ht) \overline{\Psi} \sigma^1 \sigma^0 \Psi$$

and

$$J^k = \partial_0 P_k + \partial_l M_{lk} + J^k_{\text{convecitive}} + \frac{3H}{2mr} \exp (-Ht) \overline{\Psi} \sigma^0 \sigma^k \Psi + i \overline{\Psi} \sigma^0 \sigma^k \Psi$$

where $P_{0k}$ are polarization densities and $M_{lk}$ is magnetization density, and their explicit forms are given by,

$$P^{0k} = \frac{1}{2m} \overline{\Psi} \sigma^{k0} (t, r) \Psi \quad (25)$$
and
\[ M^{[lk]} = \frac{1}{2m} \overline{\Psi} \sigma^{lk} (t, r) \Psi, \]  
(26)
respectively. From these relations, the total polarizations, \( p^0_l \), and magnetization, \( \mu \), are defined as
\[ p^0_l = \int P^{0k} d\Sigma_{kl}, \]
(27)
and
\[ \mu = \int M^{kl} d\Sigma_{kl}, \]
(28)
where \( d\Sigma_{kl} \) is a hypersurface for \( t = \text{constant} \) and \( d\Sigma_{kl} = \sqrt{|g|} d^2x = e^{2Ht} r dr d\phi \).

Now, we are going to discuss the Dirac currents and the dipole moments expressions for the exponentially expanding universe. So, inserting the Eqs. (16) and its conjugate into the Eqs. (21), we compute the components of the Dirac currents in asymptotic region as follow:

\[ J^0 \approx \frac{H^2}{2\lambda^2 \pi^2} \frac{z^2}{r}, \]
\[ J^1 \approx \frac{H^3 z^2 [(2n + 1)^2 - z^2]}{4\pi^2 \lambda^3 (2n + 1) r} \sin \left( \lambda r - \frac{k\pi}{2} \right), \]
\[ J^2 \approx i \frac{H^3 z^2 [z^2 - (2n + 1)^2]}{4\pi^2 \lambda^3 (2n + 1) r} \sin \left( \lambda r - \frac{k\pi}{2} \right), \]
(29)
Similarly, substituting Eqs. (16) and its conjugate in the Eqs. (25) and Eqs. (26), the components of the polarization densities and the magnetization density are written as follows:

\[ P^1 = A(z) J_{k+\frac{1}{2}} (\lambda r) J_{k-\frac{1}{2}} (\lambda r), \]
\[ P^2 = B(z) \frac{1}{r} J_{k+\frac{1}{2}} (\lambda r) J_{k-\frac{1}{2}} (\lambda r), \]
\[ M_{12} = \frac{\lambda^4 r}{8\pi m^3} \left[ J_{k+\frac{1}{2}} (\lambda r) J_{k+\frac{1}{2}} (\lambda r) - J_{k-\frac{1}{2}} (\lambda r) J_{k-\frac{1}{2}} (\lambda r) \right], \]
(30)
respectively, where we use the following abbreviations:
\[ A(z) = i \frac{H^3 z^3}{8\pi \lambda^2 m} \left[ \frac{J_{n+\frac{1}{2}} (z)}{J_{n-\frac{1}{2}} (z)} - \frac{J_{n-\frac{1}{2}} (z)}{J_{n+\frac{1}{2}} (z)} \right], \]
\[ B(z) = -i A(z). \]

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Giving to the polarization densities and magnetization density depending spacetime coordinates, we can say that particle production event takes place. To calculate the total polarizations and magnetization, we insert the Eqs. (30), the polarization densities and magnetization density, in the Eqs. (27) and Eqs. (28), respectively, and later integrate them on the hypersurface. Then, we obtain the total polarization densities (electric dipole moments) and magnetization density (magnetic dipole moment) as follows:

\[
\begin{align*}
p^1 &= 0, \\
p^2 &= \frac{\pi \lambda B(z)}{H^2}, \\
\mu &= 0,
\end{align*}
\]

where we use the integral representation of Bessel function (25) and \( p^1 \) and \( p^2 \) are total polarizations, i.e. electric dipole moment components, and also \( \mu \) is total magnetization, i.e. magnetic dipole moment. From these results, we see that the particle creation events are affected from only \( p^2 \) total polarization density, electric dipole moment. On the other hand, in the limit \( t \to -\infty \ (z \to \infty) \), \( p^2 \) vanishes by the following way

\[
p^2 \to \frac{zH}{4\lambda m} \left[ \frac{2 \cos^2 \left( \frac{z - \frac{n\pi}{2}}{2} \right) - 1}{\sin \left( 2z - n\pi \right)} \right] \to 0,
\]

that is, there is not particle production in this limit or the universe has a symmetric charge distribution in the beginning time and, of course, the universe has not any dipole moments, but, in the limit \( t \to +\infty \ (z \to 0) \), \( p^2 \) becomes

\[
p^2 \to \frac{e\hbar H}{4\lambda mc^2} \left[ \frac{z^2 - \left( 1 - \frac{2mc^2}{\hbar H} \right)^2}{1 - \frac{2mc^2}{\hbar H}} \right] \to -\frac{e}{2\lambda \delta} \exp \left( -i\delta \right),
\]

where \( \hbar \) is Planck constant, \( c \) is the speed of light and \( \delta = \frac{2mc^2}{\hbar H} \), and, thus, the universe has a permanent complex dipole moment, which it oscillates with Zitterbewegung frequency, \( \frac{2mc^2}{\hbar} \).
To calculate the Dirac current components for the Dirac particle in the 2+1 dimensional exponentially expanding magnetized universe, we insert the Eqs. (20) and its conjugate in the Eqs. (21). Thus, we find that the current components are

\[
J^0 \approx \frac{H^2 k^2}{z^2 (k^2 - \kappa^2) \Gamma (2\kappa) + e^2 B_0^2 (1 - 4n^2) \Gamma (2\kappa - 2)} \rho^{2\kappa - 1} e^{-\rho} \\
\left[ z + \frac{(1 - 4n^2)}{z (k^2 - \kappa^2) \rho^2} [(2k - \rho) (k + \kappa) + 2k\rho]^2 \right]
\]

\[
J^1 \approx i \frac{4nH^3 e B_0 k^2 \kappa}{\pi (k^2 - \kappa^2) \rho^2} z^2 (k^2 - \kappa^2) \Gamma (2\kappa) (2k - \rho) (k + \kappa) + 2k\rho \rho^{2\kappa - 2} e^{-\rho}
\]

\[
J^2 \approx \frac{4H^3 \kappa^3 z^4}{\pi (k^2 - \kappa^2) \rho^2} [(2k - \rho) (k + \kappa) + 2k\rho] \rho^{2\kappa - 3} e^{-\rho}.
\]

Using Eqs. (20) in the Eqs. (25) and in the Eqs. (26), the components of polarization and magnetization can be found as follows:

\[
P^1 \approx \frac{2H^3 e B_0 k^2 \kappa^4}{(k^2 - \kappa^2) \pi m 2z^2 \lambda^2 \kappa^2 \Gamma (2\kappa) + e^2 B_0^2 (1 - 4n^2) \Gamma (2\kappa - 2)} \rho^{2\kappa - 2} e^{-\rho} (2k - \rho) (k + \kappa) + 2k\rho
\]

\[
P^2 \approx i \frac{4nH^3 \kappa^3 z^4}{\pi m (k^2 - \kappa^2) \rho^2} [(2k - \rho) (k + \kappa) + 2k\rho] \rho^{2\kappa - 3} e^{-\rho}
\]

\[
M_{12} \approx \frac{2H^4 e B_0 k^3 \kappa^5 (4n^2 - 1) \kappa}{\pi m (k^2 - \kappa^2) [4z^2 \lambda^2 \kappa^3 \Gamma (2\kappa) + e B_0 (1 - 4n^2) \Gamma (2\kappa - 2)]} \rho^{2\kappa - 2} e^{-\rho}
\]

\[
\left[ \frac{z}{(1 - 4n^2)} + \frac{1}{z^2 (k^2 - \kappa^2) \rho^2} [(2k - \rho) (k + \kappa) + 2k\rho]^2 \right]
\]

To calculate the total polarizations and total magnetization, we insert the Eqs. (33) in Eqs. (27) and Eqs. (28). Then, using the integral representation of Bessel function (25), we find the following dipole moments expressions:

\[
p_1 \approx \frac{He^3 B_0^{\kappa 4} z^2}{\kappa^3 m} \frac{2 \rho^{2k^2 - 2}}{(2\kappa) \rho^{2k^2 - 2} \Gamma (2\kappa) + (1 - 4n^2) \Gamma (2\kappa - 2)}
\]

\[
p_2 \approx \frac{2nHe^3 B_0^{\kappa 5} z^2}{\kappa^4 m} \frac{(2k^2 - 2k^2 + 4k + \kappa - k) \rho^{2k^2 - 2} \Gamma (2\kappa) + (1 - 4n^2) \Gamma (2\kappa - 2)}
\]

\[
\mu \approx \frac{H^2 e^3 B_0^{\kappa 5} z^3}{\kappa^3 m}.
\]

From these expressions, we see that, in finite time intervals, the particle creation influenced by both polarization and magnetization components. On the other hand, in the limit \( t \rightarrow \)
\[ z \to 0 \) the magnetic dipole moment, \( \mu \), goes to zero faster than the electric dipole moment components, \( p^1 \) and \( p^2 \). Therefore, the electric dipole moments in the particle creation events become more dominant than the magnetic dipole moment in finite time intervals if there exists in an external constant magnetic field.

\section*{4 Summary and Conclusion}

We exactly solve the Dirac equation in existence of the exponentially expanding magnetized and non-magnetized universe and, from these solutions, derive some expressions for the Dirac current components and dipole moments. The particle creation in the exponentially expanding universe are only affected by the \( p^2 \) polarization in the finite time interval. However, this component goes to zero in the limit \( t \to -\infty \), i.e. in the beginning of the universe, but, in the limit \( t \to +\infty \), the universe has a permanent complex dipole moment oscillating with Zitterbewegung frequency, \( \frac{2mc^2}{\hbar} \): \( p^2 \simeq -\frac{e}{2\lambda\delta} \exp(-i\delta) \). The complexity of the dipole moment points out the conductivity of the exponentially expanding universe. Also, the universe has the electric and magnetic dipole moments which are dependent on time in existence of an external constant magnetic field with the expansion such that, in the limit \( t \to -\infty \), the dipole moment expressions become infinite, but, in the limit \( t \to \infty \), they go to zero. The dependence on time of the polarization and magnetization show that the particle creation happens. Furthermore, in the limit \( t \to \infty \) (\( z \to 0 \)), the particle creation events are affected only via the polarization because the magnetization, \( m \), goes to zero faster than \( p^1 \) and \( p^2 \). From the point of view, we point out that the exponential expansion of the universe causes a particle creation, a permanent complex electric dipole moment and asymmetric charge distribution, but, in existence of an external constant magnetic field with exponential expansion in time, the universe charge distribution is getting symmetric and thus all the dipole moments become zero as \( t \to \infty \).
Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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