Root locus approach in design of PID controller for cruise control application

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Abstract. The Proportional Integral Derivative (PID) controller is an effective and common feedback control design used in closed loop control systems. One such best consideration of closed loop control system would be cruise control system. This is a system that automatically controls the speed of an electric vehicle despite external disturbances. In this paper, the goal is to design a PID controller using root locus technique for a closed loop cruise control system. By root locus approach, the controller constants and controller design is finalized. Simulation results through MATLAB environment validate the effectiveness of controller design.

1. Introduction
Cruise control system is a controller used to control speed of vehicles automatically as per the driver command, and adapting itself to road condition and external disturbances. It uses a servo mechanism which takes over the throttle of the vehicle to regulate the speed. The main aim of the cruise control system is to provide ease to drivers by automatically setting the speed of vehicles. Nowadays it’s been a general feature that comes with most of the vehicles. It’s the most important feature in all auto pilot vehicles. The cruise controllers in vehicles are in demand of controllers that can automatically adjust the different attributes of cruise control such as speed, velocity and acceleration to maintain the desired speed of the vehicle as per the road conditions.

The first ideology of cruise control came to automobile in the year 1900 by Wilson Pilcher [1]. The same cruise control was also used by James Watt in steam engines. Ralph Teetor in 1948 developed an advanced version of cruise control [2]. After many years, i.e, from 2000 due to the development of highways there was a need to develop cruise control system to assist the drivers with ease of driving [3]. The key issue in Cruise control system is to maintain the cruise speed, with varying environmental disturbances and road condition. To make this possible there a need of controllers such as PID controllers, fuzzy controllers, feedforward controllers etc. The cruise control system is usually built on a radar sensor or Adaptive Cruise Control sensor (ACC) at its core to perpetuate its adaptive nature. The equipment, which is mounted at the front of the vehicle, constantly monitors the road ahead. ACC maintains the driver's speed as long as the road ahead is clear. If a slower vehicle is detected within the system's detecting range, the system gradually slows speed by releasing the accelerator or actively engaging the brake control system. The ACC automatically accelerates to the driver's selected speed if the car ahead accelerates or changes lanes. Thus, a rear-end collisions are avoided and a safe distance is maintained. The sensor in use, is of dimension 45*20*15mm and has a working voltage of 5V,
working current of 15mA, working frequency of 40Hz, maximum range of 4m and measuring angle of 15°.
In the modelling of different variant of car, the complexity is increased when we consider all the car
parameters. In order to focus the control, only on speed control of car with respect to the road and
external disturbances, a balance car model with lesser parameter, without ignoring the real car model
is considered here. After the complete modelling of car or cruise, designed a PID controller by root
locus approach and the stability analysis is also done to validate the model.

2. Modelling of Cruise Control System

In [10], the model of a cruise control is considered and as given below,

\[
\begin{align*}
\text{Figure1. Cruise Car simple model[10]}\\
\end{align*}
\]

we consider a simple vehicle dynamic to model according to above shown diagram. A control force \( u \) acts on a vehicle of mass \( m \), where the force \( u \) is the force generated by external factors. While
modelling the system, it is assumed that, the controlling variable is the force generated by the vehicle,
neglecting changes in the power train, friction between tyre and road, aerodynamic drag, gradient, etc.
The rolling resistance force and aerodynamic force due to wind are assumed to be varying linearly
with output velocity \( v \). we assume that the car is not travelling in steep or valley positions but travels
in a straight road. We model this entire system as a first order mechanical system, such that the sum of
forces in horizontal direction for the figure 1, applying Newton’s second law of motion as

\[
mv + bv = u
\]

since our aim is controlling the speed, the output equation (\( y \)) can be written as

\[
y = v
\]

we assume the vehicle mass as 1000kg and damping coefficient (b) as 50Ns/m in this paper.

2.1 State space model

Considering the kinetic energy of the car, the equation (1), (2) can be dealt as first order system, and is
represented in state space model

\[
\dot{x} = [\dot{v}] = \left[ \begin{array}{c} -b \\ m \end{array} \right] [v] + \left[ \begin{array}{c} 1 \\ m \end{array} \right] [u]
\]

\[
y = [1][v]
\]

2.2 Frequency domain model

Applying Laplace transformation to equation (3),(4) and combining with equation of (1) and (2) we
get,

\[
P(s) = \frac{V(s)}{U(s)} = \frac{1}{ms+b} [\frac{m/s}{N}]
\]

3. Design of controllers

Among the various controllers, proportional controller is best suited for closed loop continuous
systems and hence preferred in this work. This [12] controller, minimizes the overshoots in the control
variable and helps in reaching the desired position in a much faster rate. The improved version of
Proportional controller (P) is the Proportional plus Integral controller (PI). Due to the inclusion of
integral term, this controller achieves much faster response time than Integral only controller (I). The
A combination of above controllers will give a proportional plus integral plus derivative (PID) controller by combining the benefits of all the controller, PID control is the most widely used controller for time domain systems. This controller gives a faster response time and a lower or zero offset. So PID controller is the best form of controller. Considering \( r(s) \) as input, ‘e’ as error between input set speed and actual speed, \( c(s) \) as the PID controller, \( p(s) \) is the cruise car model or plant and \( y(s) \) is the output velocity, a unity feedback PID system can be shown as follows.

![Figure 2. Block diagram of PID controller](image)

The time domain design constrains for the cruise control were taken as setting time <10sec, Rise time<5sec.

3.1 Proportional Control

First, we find the closed loop transfer function with a proportional controller \( C(s) = kp \)

The closed loop transfer function with unity feedback proportional controller is given as

\[
T(s) = \frac{Y(s)}{R(s)} = \frac{P(s)C(s)}{1+P(s)C(s)} = \frac{K_p}{ms+b+K_p}
\]

![Figure 3. Step response for proportional control](image)

According to the above graph the response seems to be unrealistic that is because in real-time the velocity of cruise control cannot change the speed of vehicle from zero to ten meter per seconds in less than 0.5 seconds due to drive train and engine constrains.

3.2 Proportional plus Integral control

The Transfer Function of this closed loop cruise control system with PI controller is given by

\[
T(s) = \frac{Y(s)}{R(s)} = \frac{P(s)C(s)}{1+P(s)C(s)} = \frac{K_p(s+Ki)}{ms^2+(b+K_p)s+Ki}
\]
An introduction of integral controller to the system removes the steady state error of system. For implementations let us consider $K_p$ as 600 and $K_i$ as 1.

$$T(s) = \frac{Y(s)}{R(s)} = \frac{P(s)C(s)}{1+P(s)C(s)} = \frac{K_ds^2 + K_p s + K_i}{(m+K_d) s^2 + (b+K_p) s + K_i}$$

3.3 Proportional plus Integral plus Derivative control

The controller design can be finalized for the PI controller only but the most significant part of derivatives will be missing. So, we go for PID controller design in the due process. Still the PI controller would also serve the purpose of efficient controller.
4. Root locus controller design

Using the root locus method, we find the damping ratio and natural frequency. The following equations are used to find them.

\[ \tau \geq \frac{\ln^2(Mp)}{\pi^2 + \ln^2(Mp)} \]  

(10)

where

- \( \omega_n \) = frequency of natural oscillation
- \( \tau \) = damping factor
- \( T_r \) = rise time in seconds
- \( Mp \) = Peak overshoot in percentage

Let us consider to have the rise time of 5 seconds. For achieving this rise time natural frequency must be greater than 0.36, and damping ratio must be more than 0.6 as per the above equation. With these we, plot a root locus design and obtained the below output.

![Figure 6. Root locus plot of the obtained result](image)

From the plot we infer that the damping ratio of the system is more than 0.6 between the lines and less than 0.6 outside these lines. The semi ellipse indicates a constant natural frequency.

5. Conclusion

A simple mathematical model of cruise control was derived with state space and transfer function mathematical model. A PID controller was designed for the mathematical model. The root locus analysis was done for the same. We have also concluded how a PID controller would serve a better purpose than a PI controller. Autonomous cruise control is the future of motorized vehicles. This work can be extended further with the introduction of more number of sensors like LiDAR camera sensor, odometry sensor, etc. The time response of such a system can be studied for further improvements in the model of autonomous vehicles which can prevent phantom traffic jams. Stop-and-go waves can be dissipated with the help of ACC equipped vehicles. Furthermore, the validity and real-time or near real-time applications of human-in-loop cyber physical systems in intelligent electromechanical systems such as the one described in this paper, can be tested and their functionalities be improved.

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