Theory of magnetotunneling spectroscopy in spin triplet \( p \)-wave superconductors

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We study the influence of a magnetic field \( H \) on the zero-bias conductance peak (ZBCP) due to zero-energy Andreev bound state (ZES) in normal metal / unconventional superconductor. For \( p \)-wave junctions, ZBCP does not split into two by \( H \) even for sufficiently low transparent junctions, where ZBCP clearly splits for \( d \)-wave. This unique property originates from the fact that for \( p \)-wave superconductors, perpendicularly injected quasiparticle form ZES, which contribute most dominantly on the tunneling conductance. In addition, we show that for \( p_x + ip_y \)-wave superconductor junctions, the height of ZBCP is sensitive to \( H \) due to the formation of broken time reversal symmetry state. We propose that tunneling spectroscopy in the presence of magnetic field, i.e., magnetotunneling, is an promising method to determine the pairing symmetry of unconventional superconductors.

**KEYWORDS:** triplet superconductor, tunneling conductance, Doppler shift, chirality

Nowadays, the formation of zero-energy Andreev bound states (ZES) at surfaces\(^1,2\) or interfaces\(^3\) of unconventional superconductors is receiving increasing attention. This state, which is created by injected and reflected quasiparticle’s feeling different signs of the pair potential, can play an important role in determining the pairing symmetry of anisotropic superconductors. The detection of the ZES is reflected as an observation of a zero-bias conductance peak (ZBCP) in tunneling conductance, which is generally considered as a clear signature of anisotropic pairing.\(^4,6-9\) The ZBCP is observed in various superconductors having anisotropic pairing symmetry, such as the cuprates,\(^5,10-13\) Sr\(_2\)RuO\(_4\),\(^14,15\) and UBe\(_{13}\).\(^16\) Possibility of ZBCP has also been theoretically predicted for organic superconductors (TMTSF)\(_2\)X very recently.\(^17-19\) The existence of the ABS has crucial influences on many transport phenomena in unconventional superconductor junctions.\(^3,20-25\)

Since the existence of ZES is a universal phenomena expected for unconventional superconductors having pair potential that changes sign on the Fermi surface, difficulty may arise in determining the pairing symmetry only from conventional tunneling spectroscopy. In order to overcome this difficulty, we require some \textit{in situ} way of probing the symmetry from the tunneling spectroscopy. Here, we will show that a promising way is to use a tunneling spectroscopy in the presence of a magnetic field. For \( d_{z^2} - y^2 \)-wave junction, it has been shown that screening currents shift the ZES spectrum (Doppler shift) and lead to a splitting of ZBCP.\(^26-29\)

By contrast, we show in the present paper that for \( p \)-wave cases, ZBCP does not split into two in the presence of a magnetic field since the most dominant contribution to tunneling conductance is given by perpendicular injection, where the energy shift of the quasiparticles does not occur at all because the component of the Fermi velocity parallel to the interface is zero for a cylindrical Fermi surface. We also show phenomenologically the absence of Doppler shift for more general shapes of the Fermi surface, assuming inversion symmetry of crystal, where the component of the Fermi velocity parallel to the interface always have the same magnitude but different signs between perpendicularly injected and reflected quasiparticle states. Finally, we show that for a \( p_x + ip_y \)-wave superconductor, the magnitude of ZBCP is enhanced or suppressed depending on which way (+z or −z direction) to apply the magnetic field. We propose that this dependence on the magnetic field can be used in detecting the broken time reversal symmetry superconducting state.

We calculate tunneling conductance in normal metal / unconventional superconductor junctions by solving Bogoliubov-de Gennes (BdG) equation using quasiclassical approximation as in our previous theory.\(^3,30\) As regards the triplet pairing cases, we assume that Cooper pairs are formed by electrons having antiparallel spins, but an extension to more general cases including parallel spin pairing or non-unitary cases\(^8,9\) is straightforward. Now, we consider the case where a specularly reflecting surface or interface run along the \( y \)-direction. The insulator located at the interface between normal metal and \( d \)-wave superconductor is expressed using \( U(x) \) as

\[
U(x) = \begin{cases} 
0 & x < -d_i \\
U_0 & -d_i < x < 0 \\
0 & x > 0 
\end{cases}
\]

(1)

where the width and height of the barrier are \( d_i \) and \( U_0 \), respectively. The magnetic field is applied parallel to the \( z \)-axis, so that the vector potential can be chosen as \( A(r) = (0, A_y(x), 0) \). The pair potential depends only on \( x \) since the system is homogeneous along the \( y \)-direction. Since we are now considering the situation where the coherence length of the pair potential \( \xi \) is much smaller than the penetration depth of the magnetic field \( \lambda \), we can ignore the spatial dependence of \( A_y(x) \) in the following calculations. We assume \( A_y(x) = A_y = -H \lambda \), where \( H \) is the applied magnetic field. The normalized
tunneling conductance \( \sigma_T(eV) \) under the bias voltage \( V \) is given by\(^3\) \( ^4\) \( ^30\)

\[
\sigma_T(E) = \frac{\int_{-\pi/2}^{\pi/2} d\theta \sigma_S(\theta, E)}{\int_{-\pi/2}^{\pi/2} d\theta \sigma_N(\theta) \cos \theta}.
\]

The normalized tunneling conductance is given by

\[
\sigma_N(\theta) = \frac{4Z^2}{(1 - Z^2) \sinh^2(\lambda_0 d_i) + 4Z^2 \cosh^2(\lambda_0 d_i)}.
\]

\[
Z = \frac{\lambda_0 \cos \theta}{\sqrt{1 - \lambda_0^2 \cos^2 \theta}}, \quad \lambda_0 = \sqrt{2mU_0/h^2}, \quad \kappa = k_F/\lambda_0,
\]

\[
\sigma_S(\theta, E) = \sigma_N(\theta) \sigma_R(\theta, E) \cos \theta,
\]

\[
\sigma_R(\theta, E) = \frac{1 + \sigma_N(\theta) |\Gamma_+|^2 + |\sigma_N(\theta) - 1| |\Gamma_+|^2 |\Gamma_-|^2}{|1 + \sigma_N(\theta) - 1| |\Gamma_+|^2}.
\]

\[
\Gamma_\pm = \begin{cases} 
\frac{\Delta_0 f(\theta_\pm)}{\sqrt{E^2_\pm - |\Delta_0 f(\theta_\pm)|^2 + E_\pm}}, & E_\pm > 0, \\
\frac{-\Delta_0 f(\theta_\pm)}{\sqrt{E^2_\pm - |\Delta_0 f(\theta_\pm)|^2 - E_\pm}}, & E_\pm < 0,
\end{cases}
\]

\[
E_\pm = E + ev_Fg\lambda = E + \frac{\Delta_0 Hg_0}{H_0},
\]

with the injected and reflected angles being \( \theta_+ = \theta \) and \( \theta_- = \pi - \theta \), respectively, \( H_0 = \frac{\Delta_0}{e\xi_F} = \frac{\phi_0}{\xi_0 \lambda \pi^2} \), \( \xi_0 = h/\xi_F(\pi \Delta_0) \), \( E = eV \), and \( g_\pm = v_Fg_\pm/v_F \), where \( k_F \) and \( v_F \) is the Fermi momentum and the Fermi velocity, respectively, taken to be common in superconductor and normal metal for simplicity. \( \Delta_0 \) is the magnitude of the maximum value of the pair potential and \( E_\pm \) are the Doppler shifted energies of injected and reflected quasiparticle states. In the above, we have assumed a spatially constant pair potential in the superconductor. As shown in our previous paper, if we concentrate on the qualitative feature of \( \sigma_T(E) \) at low voltage, this assumption is justified.\(^3\) \( ^{30}\)

We take \( \lambda_0 d_i = 3 \) and \( \kappa = 0.5 \), where the transmission coefficient perpendicular to the interface is about 0.02. In the following, we will look at \( \sigma_T(eV) \) and \( \sigma_S(\theta, eV) \) for various cases.

We have calculated \( \sigma_T(eV) \) for a \( d_{x^2-y^2} \)-wave junction with \( (110) \) oriented surface and for a \( p_\Sigma \)-wave junction with \( (100) \) oriented surface, where \( f(\theta) \) is given as \( f(\theta) = \sin(2\theta) \) and \( f(\theta) = \cos(\theta) \), respectively, as a prototype of \( d \)-wave and \( p \)-wave superconductor junctions. In \( d_{x^2-y^2} \)-wave junctions, \( \sigma_T(eV) \) has a peak splitting [see curve \( b \) and \( c \) in Fig. 1(a)] in the presence of magnetic field \( H^{29, 30} \) since the transparency of the junction is sufficiently small. On the other hand, in the \( p_\Sigma \)-wave junctions, there is no ZBCP splitting with the increase of the magnitude of \( H \), where only the height of the ZBCP is reduced and the width becomes broad [Fig. 1(b)]. Thus, the response of ZBCP to the magnetic field is quite different from that in the \( d_{x^2-y^2} \)-wave case.

In order to understand this difference, we look into \( \sigma_S(\theta, eV) \), i.e., the angle resolved conductance in the superconducting state. For \( H = 0 \), the height of ZBCP \( \sigma_S(\theta, 0) \) is given as \( \sigma_S(\theta, 0) = 2 \cos \theta \) for \( f(\theta) \neq 0 \) independent of \( \sigma_N(\theta) \) due to the formation of ZES and \( \sigma_S(\theta, 0) = 2 \sigma_N(\theta) \) for \( f(\theta) = 0 \), respectively.\(^4\) In the absence of the magnetic field, \( \sigma_S(\theta, eV) \) always has ZBCP except for special cases, i.e., \( \theta = 0 \) for the \( d_{x^2-y^2} \)-wave junction and \( \theta = \pm \pi/2 \) for both junctions. For the \( d_{x^2-y^2} \)-wave junction, the width of the ZBCP \( W \) takes its maximum value for some oblique injection angle \( \theta_m \), while for \( p_\Sigma \)-wave, both \( W \) and \( \sigma_S(\theta, 0) \) becomes largest at \( \theta = 0 \).

By applying \( H \), the positions of peak shift from zero. In the present case, since positive \( H \) is chosen, quasiparticle energy \( E \) increases (decreases) for \( \theta > 0 \) (\( \theta < 0 \)), so that the peak position of \( \sigma_S(\theta, eV) \) becomes located in the negative (positive) voltage region. However, a remarkable feature is that the ZBCP for perpendicular injection, i.e. \( \sigma_S(0, eV) \), is not changed at all by \( H \). This is because \( v_Fg = 0 \) at \( \theta = 0 \), so that there is no Doppler shift for perpendicular injection. For the \( p_\Sigma \)-wave case, perpendicular injection contributes most dominantly to
the low bias behavior of the integrated normalized tunneling conductance $\sigma_T(eV)$, and consequently the ZBCP is robust against $H$. In the above, $v_F\gamma = 0$ at $\theta = 0$ is a consequence of assuming a cylindrical Fermi surface. In fact, the absence of Doppler shift for perpendicular injection can be shown in more generalized cases with $v_F\gamma(k_x,k_y) \neq 0$. Namely, there is generally an inversion symmetry of the crystal, $\epsilon(k) = \epsilon(-k)$, so the injected and reflected quasiparticle states have different sign of $v_F\gamma(v_F\gamma(k_x,0) = -v_F\gamma(-k_x,0))$ for perpendicular injection. In order to look into this general situation, we consider a phenomenological analogue of $\sigma_R(\theta, E)$ as

$$\rho(E) = \frac{1 + \tilde{\sigma}|\Gamma_+|^2 + |\tilde{\sigma} - 1||\Gamma_+|^2 |\Gamma_-|^2}{1 + |\tilde{\sigma} - 1||\Gamma_+|^2 |\Gamma_-|^2}. \quad (5)$$

Here we take $\tilde{\sigma} = 0.1$ as a typical value for a low transparent barrier, and calculate $\rho(E)$ for two cases, i.e.,

- $v_F\gamma(k_x,0) = -v_F\gamma(-k_x,0)$ i.e., $g_+ = g_-$ and
- $v_F\gamma(k_x,k_y) = v_F\gamma(-k_x,k_y)$ i.e., $g_+ = g_-$. As seen from

Fig. 4(a), there is no Doppler shift of ZBCP with the increase of $H$ for case (i). The absence of the shift is quite different from that of the conventional case (ii) [Fig. 4(b)], where the magnitude of the shift of the peak position is proportional to the magnetic field.²⁹ The absence of Doppler shift originates from the cancellation of the additional phase in the product of $\Gamma_+\Gamma_-$ due to the different sign of $E_+$ and $E_-$ at $E = 0$. The cancellation of the additional phase shift of the wave function is a quite novel feature originating from the difference in the sign of $v_F\gamma$ between reflected and injected quasiparticle. Although the present argument for $v_F\gamma(k_x,0) = -v_F\gamma(-k_x,0) \neq 0$ is phenomenological, recently, we have shown for a tight binding model of a possibly triplet superconductor (TMTSF)$_2$X that zero-energy peak in the surface density of states does not split in the presence of a magnetic field, not only for $p$-wave pairing, but also for a triplet $f$-wave pairing.²⁹

Finally, we now look into the chiral $p$-wave ($p_x+ip_y$-wave) case, where $f(\theta)$ is given as $f(\theta) = \exp(i\theta)$. This is a pairing symmetry possibly realized in Sr$_2$RuO$_4$. As seen in Fig. 5, ZBCP again does not split in the presence of a magnetic field, while its height is enhanced for positive $H$ ($H$ in the $+z$ direction; curves $b$ and $c$), but is reduced for negative $H$ ($H$ in the $-z$ direction; curves $d$ and $e$). Such an asymmetric $H$ dependence of ZBCP around $H = 0$ does not appear in $d_{x^2-y^2}$-wave and $p_y$-wave cases. In order to understand this asymmetric feature in detail, we also plot in Fig. 6 the angle resolved tunneling conductance as in Figs. 2 and 3. As seen in Fig. 6, the peak position of $\sigma_S(\theta, eV)$ has a strong $\theta$ dependence. Namely, the $p_x+ip_y$-wave pairing induces a broken time reversal symmetry state (BTRSS), so that the peak positions for $\theta \neq 0$ is no longer located at $E = 0$ even in the absence of a magnetic field. Now, in the presence of a positive magnetic field, all the peaks for $\theta \neq 0$ are shifted toward $E = 0$, [Fig. 6(b)] while they are shifted away from $E = 0$ for a negative magnetic field [Fig. 6(c)]. This unique feature can be interpreted as follows. In $p_x+ip_y$ state, due to the formation of BTRSS, there is a surface current which flows parallel to the interface and the spontaneous magnetic field even without applied magnetic field. If the direction of the applied magnetic field $H$ is the same (opposite) as that of spontaneous field, the effective magnetic field is enhanced (reduced) for $H < 0$ ($H > 0$). Then the resulting magnitude of ZBCP in total conductance $\sigma_T(eV)$ is reduced (enhanced) for negative (positive) $H$. In summary, we have calculated tunneling conductance in normal metal / unconventional superconductor junctions, where $d_{x^2-y^2}$-wave, $p_x$-wave and $p_x+ip_y$-wave pairings have been chosen as a prototype. We focused on the influence of the applied magnetic field on ZBCP. For $p$-
wave cases, ZBCP does not split into two by magnetic field since the most dominant contribution to tunneling conductance at zero-energy originates from perpendicular injection, where Doppler shift does not occur. The absence of Doppler shift has been shown in general situations, where the component of the Fermi velocity parallel to the interface has the same magnitude but different signs between injected and reflected quasiparticle states. As for $p_x + ip_y$-wave superconductor junctions, we have shown that ZBCP does not split, while its height is sensitive to the direction of the applied magnetic field, which is a consequence of a broken time reversal symmetry. This behavior should be observed in the tunneling experiments of Sr$_2$RuO$_4$, if it is actually a chiral $p$-wave superconductor.

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