Solitons in SO(5) Superconductivity

R. MacKENZIE
Laboratoire René-J.-A.-Lévesque, Université de Montréal, Montréal, Québec H3C 3J7
and
Department of Physics and Astronomy, University of British Columbia, 6224 Agriculture Rd, Vancouver BC V6T 1Z4

J.M. CLINE
Department of Physics, McGill University, 3600 University St., Montréal, Québec H3A 2T8

A model unifying superconductivity and antiferromagnetism using an underlying approximate SO(5) symmetry has injected energy into the field of high-temperature superconductivity. This model might lead to a variety of interesting solitons. In this paper, the idea that superconducting vortices may have antiferromagnetic cores is presented, along with the results of some preliminary numerical work. An outlook for future work, including speculations about other possible exotic solitons, is presented.

1 Introduction

The phase diagrams of a variety of exotic superconductors (high-temperature superconductors, heavy fermion superconductors, organic superconductors) have a very rich structure. Although profound differences in these phase diagrams exist, it is surprising that in all of them two features are common: superconductivity and antiferromagnetism. It is extremely enticing to speculate that these materials, despite the incredible range of underlying structures, may have some common underlying reason for the appearance of these two phases.

This idea was formalized by S.C. Zhang, who observed that both superconductivity and antiferromagnetism involve spontaneous symmetry breaking. Superconductivity is essentially spontaneous breaking of electromagnetism (it is, in fact, the first example of what we now call dynamical symmetry breaking); antiferromagnetism is spontaneous breaking of spin-rotation symmetry. The first symmetry group is U(1), or equivalently SO(2); the second is SO(3). Zhang’s suggestion, borrowing heavily on ideas from particle physics, was that these two symmetries might be unified into a larger symmetry group. He presented a strong case for the group SO(5). His work has given rise to a minor cottage industry of SO(5) phenomenology, not to mention fueling a heated debate (with some of the heavyweights of condensed matter physics appearing
on opposite sides) over the merits and possible fundamental flaws of the idea.

In this work (which, admittedly, uses a rather broad interpretation of the theme of this Institute – “Electroweak Physics”), the ABCs of superconductivity and antiferromagnetism will be briefly reviewed, and Zhang’s unified description of the two will be outlined. The case for exotic solitons will then be discussed. Superconducting vortices with antiferromagnetic cores will be examined in some detail (though much work remains), and other, even more speculative, possibilities will be discussed.

2 The SO(5) Model

2.1 Superconductivity

Superconductivity is a phenomenon which occurs in a huge number of materials, if a sufficiently low temperature is reached. Superconductors display several striking features, among them zero resistance, the Meissner effect, the existence of a gap in the spectrum of low-energy excitations (this is not always the case, but usually it is so), and a transition to a normal state as the temperature increases.

Most of these phenomena can be described by a phenomenological model, known as a Ginzburg-Landau model. The great success of many workers in the late fifties and early sixties, culminating in the work of Bardeen, Cooper and Schrieffer (who first succeeded in an essentially complete description of all “conventional” superconductors), was the derivation of this GL model from an underlying microscopic model.

For our purposes, the GL model of superconductivity is essentially a non-relativistic version of what we in particle physics call the Abelian Higgs model. The distinction between nonrelativistic and relativistic models is irrelevant here, so I will discuss only the relativistic case. The Lagrangian is

$$\mathcal{L} = |D_\mu \phi|^2 - \frac{\lambda}{4} (|\phi|^2 - v^2)^2 - \frac{1}{4} F_{\mu\nu}^2,$$

(1)

where $D_\mu \phi = \partial_\mu \phi - i2eA_\mu \phi$ and where $\phi = c_{p,\uparrow}^\dagger c_{-p,\downarrow}^\dagger$ is the Cooper pair creation operator.

When $\langle \phi \rangle \neq 0$ (as is the case in the above Lagrangian), the O(2) symmetry is spontaneously broken, and it is easy to derive such features as the Meissner effect and a gap in the excitation spectrum from this fact.

2.2 Antiferromagnetism

In elementary discussions of spontaneous symmetry breaking, often the first example given is the ferromagnet. At high temperatures a spin system has spins
oriented randomly. As the temperature is decreased, it can occur that the spins prefer to be aligned in the same direction as their neighbours. This alignment amounts to the selection of an \textit{a priori} random direction in space which is singled out. Rotations of the resulting configuration about this direction do not alter the system (that symmetry is not broken), yet rotations about any other direction do change the system (those symmetries are broken). Thus the ferromagnet breaks spin rotational symmetry from SO(3) (rotations about any direction when the average magnetism in any region is zero) to SO(2) (rotations about the direction in which the spins are aligned).

Somewhat less familiar, but no more complicated (for our purposes, at least), is the case of antiferromagnetism. There, adjacent spins prefer to be anti-aligned at low energies. Once one spin’s direction is chosen, all the others must follow suit (alternating in direction from site to site) in order to minimize the energy. Once again, SO(3) spin rotation symmetry is broken to SO(2).

The order parameter in the case of ferromagnetism is (somewhat loosely) the average value of the spin $\langle \vec{S} \rangle$; in the case of antiferromagnets this averages to zero, but one defines a staggered spin vector $\vec{n} = \langle (-)^n \vec{S} \rangle$, where the extra sign is positive or negative depending whether one is on a site an even or odd number of translations away from some reference site.

2.3 \textit{Combined superconductivity and antiferromagnetism}

We have seen that superconductivity can be described by a complex field $\phi$ (or, equivalently, by a real doublet of fields defined by $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$), and that antiferromagnetism can be described by a real triplet field $\vec{n}$. Zhang put forth the idea that, since these two order parameters seem to be relevant to such a wide variety of systems, perhaps there is an underlying (approximate) symmetry which includes these two as subgroups, rather like the central idea of Grand Unified Theories. This theory would then be described by a five-component vector $\vec{N} = (n_1, n_2, n_3, \phi_1, \phi_2)$; if the dynamics of the system is such that the expectation value of $\vec{N}$ lies in the upper three-dimensional subspace the system is antiferromagnetic, while if it lies in the lower subspace the system is superconducting. If the expectation value of $\vec{N}$ is zero the system is neither superconducting nor antiferromagnetic. (And a fourth possibility, seemingly not realized in nature, is that the system could in principle be in a state which breaks both superconductivity and antiferromagnetism.)

Zhang’s work suggested that the SO(5) symmetry is explicitly broken by small terms, in a similar way to the explicit breaking of chiral symmetry by small quark masses; as a result some of the Goldstone bosons which would arise due to the breaking of SO(5) would, in fact, be pseudo-Goldstone
bosons (low-energy but not quite massless); low-energy excitations seen in high-temperature superconductors were among Zhang’s original motivations for introducing this model.

3 Solitons in the SO(5) Model

Whenever a model exhibits spontaneous symmetry breaking, there is a family of equivalent vacua (which are rotated into one another by the broken symmetry generators). The possibility then arises that topological solitons could exist. General topological arguments can be applied to any case to see if, in fact, solitons are realized.

One of the simplest examples of solitons is superconducting vortices. These appear in the appropriate GL model, or equivalently in the Abelian Higgs model \( \mathbb{I} \), in 2+1 dimensions. The potential is the familiar Mexican-hat potential, with a ring of vacua given by \( |\phi| = v \). For finiteness of energy, \( \phi \) must go to a vacuum at infinity, but there is no need for this to be the same vacuum along different directions. We can construct a configuration such that the phase of \( \phi \) changes by \( 2\pi \) as we go around a circle at infinity; such a configuration cannot be unwound by continuous deformations (without wandering away from the vacuum at infinity, which would cost an infinite energy). Furthermore, if the field configuration is continuous, it is a topological necessity that somewhere there must be a zero of the field. In the simplest, most rotationally symmetric configuration, this zero will be at the origin; we may write

\[
\phi(r, \theta) = v f(r)e^{i\theta},
\]

where the function \( f(r) \) interpolates from zero at the origin to 1 at infinity.

In the SO(5) theory, the order parameter is considerably more complicated, and interesting and exotic possibilities for solitons might arise, as we will now see.

The potential is assumed to be exactly invariant under rotations of the superconducting and antiferromagnetic order parameters, and we may assume that \( V \) has the following form, depending only on the magnitudes of these order parameters, \( \phi = |\phi| \) and \( n = |\vec{n}| \):

\[
V(\phi, n) = -\frac{m_1^2}{2} \phi^2 + \frac{\lambda_1}{4} \phi^4 - \frac{m_2^2}{2} n^2 + \frac{\lambda_2}{4} n^4 + \frac{\lambda_3}{2} \phi^2 n^2 + \text{const.}
\]

Here, we have added a constant so that the minimum of the potential is zero, included even terms up to fourth order, and assumed the quadratic terms are such that symmetry breaking in both sectors is favoured. The potential is
assumed bounded below at all directions at infinity, which will be true if the quartic couplings satisfy $\lambda_{1,2} > 0$ and $\lambda_3 > -\sqrt{\lambda_1 \lambda_2}$.

Suppose furthermore that the parameters of the potential are such that $V$ is as shown in Fig. 1. There are two important features. First, the global minimum is at a nonzero value of $\phi$ with $n = 0$; this corresponds to having a superconducting ground state. Second, if we were to force $\phi$ to be zero, the potential $V(0, n)$ is minimized at a nonzero value of $n$. These features do occur if the parameters obey the following conditions:

$$\frac{\lambda_3}{\lambda_1} > \frac{m_2^2}{m_1^2}, \quad \frac{m_1^4}{\lambda_1} > \frac{m_2^4}{\lambda_2}.$$  (4)

The second feature is no mere mathematical curiosity, since at the core of a superconducting vortex $\phi$ is indeed zero. Thus, if the potential energy had its way, the superconducting vortex core would surely be antiferromagnetic. In fact, the energetics is somewhat more complicated, and there is a range of parameters where the core is antiferromagnetic; outside this range the core is normal.

To see this, we must solve the coupled equations for $\phi$ and $n$. These
The equations come from the energy functional (we assume planar geometry):

\[ E = \int d^2x \left\{ \frac{1}{2} \left( \frac{\nabla \phi}{2} \right)^2 - \frac{1}{2} m_1^2 \phi^2 \right\} + \frac{\lambda_1 |\phi|^4}{4} \]
\[ + \frac{1}{2} \left( \frac{\nabla \vec{n}}{2} \right)^2 - \frac{1}{2} m_2^2 |\vec{n}|^2 \right\} + \frac{\lambda_2 |\phi|^2 |\vec{n}|^2}{4} + \frac{\lambda_3 |\phi|^2 |\vec{n}|^2}{2} \right\} \]  

(5)

The equations of motion are straightforward, and can be rewritten in the following form with a rotationally symmetric ansatz \( \phi(x) = \nu \phi(r)e^{i\theta} \), \( \vec{n}(x) = \vec{n}_0 n(r) \) (where \( \vec{n}_0 \) is a constant unit vector), and with appropriate field and coordinate rescalings:

\[
\frac{d^2\phi}{du^2} + \frac{1}{u} \frac{d\phi}{du} + \left( 1 - \frac{1}{u^2} \right) \phi - \delta n^2 \phi - \phi^3 = 0, 
\]

(6)

\[
\frac{d^2n}{du^2} + \frac{1}{u} \frac{dn}{du} + \beta n - \alpha \phi^2 n - n^3 = 0, 
\]

(7)

where the constants \( \alpha, \beta \) and \( \delta \) are \( \alpha = \lambda_3/\lambda_1 \), \( \beta = m_2^2/m_1^2 \) and \( \delta = \lambda_3/\lambda_1 \).

The equations can be solved subject to four boundary conditions: \( \phi \) must go to zero at the origin and to 1 at infinity, and \( n \) must have zero slope at the origin and must go to zero at infinity. The crucial observation, however, is that there is no need for \( n \) to be zero at the origin, and we find that for some parameters \( n \) goes to a nonzero value, corresponding to an antiferromagnetic core for the vortex.

As an example, Figure 2 displays the functions \( \phi(u) \) and \( n(u) \) (\( u \) being a scaled radial variable), for specific values of the parameters. Since \( n \) is nonzero at \( u = 0 \), the core of the vortex in this case is superconducting. For other values of the parameters (for example, if \( \beta \) is reduced to a sufficiently low value), one finds that \( n(u) = 0 \) for all \( u \), indicating a normal core.

Still to be done is a systematic scan of parameter space to see under what conditions vortices do indeed have antiferromagnetic cores. It is also important to make contact between the parameters of the GL model above and experimental parameters of exotic superconductors.

Finally, it is not difficult to see that other types of solitons could in principle have similar exotic structure. One example would occur in the antiferromagnetic phase of these materials, if the antiferromagnetism is of an “easy-plane” variety. (This means that, rather than being truly isotropic, a plane is favoured for antiferromagnetism, due to crystal asymmetry.) In this case, it is easy to imagine antiferromagnetic vortices of a very similar structure to the superconducting ones discussed above, and depending on the parameters it is possible that such vortices would have superconducting cores.
A second example might actually be observed in certain underdoped high-temperature superconductors which display striping. In these materials, the striping can be understood in terms of the formation of antiferromagnetic domain walls (domain boundaries, really, since the materials are effectively planar), with superconductivity occurring in the domain wall. This might fit in nicely with the SO(5) model, since it is fairly straightforward to construct antiferromagnetic domain boundaries which have a superconducting core.

References
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