Abstract. In this article, the data notion is mathematically conceptualized as typed information based on the two concepts of information and computable functionality. A data type is defined as a pair of a set of distinguishable characters (an alphabet) and a set of operations (surjective, computable functions) that operate on this alphabet as domain and capture the intent of a parameterizable concept.

Two different ways to construct new data types from existing ones are described: restriction and extension. They lead to two different partial orders on types in the sense of subtyping as formulated by Liskov and Wing.

It is argued that the proposed data concept matches the concept of characteristics (Merkmale) of the automation industry.

Keywords: data, information, data type, operation type, type system, type relation, subtyping, computability, characteristics, interoperability

1 Introduction

What are data? Or — what is data? What is the difference between information and data? It might seem strange that in 2017 someone writes an article about the concept of data. But, one of the consequences of the youth of informatics, in contrast to other, more settled disciplines, like, lets say physics, seems to be the heterogeneity of even some of its rather fundamental concepts - like data or information.

Surely, there will not be the one-and-only meaning of the term "data" in our natural language. But it seems to be a worthwhile undertaking to develop a mutually agreed meaning in the specialist language of the informatic people.

The Merrian-Webster Dictionary\footnote{https://www.merriam-webster.com/dictionary/data} says that "data" is used both as a plural noun (like earnings) and as an abstract mass noun (like information). It gives three different definitions of data, all based on the notion of information and two also explicitly referring to their processing:

1. factual information (such as measurements or statistics) used as a basis for reasoning, discussion, or calculation
2. information output by a sensing device or organ that includes both useful and irrelevant or redundant information and must be processed to be meaningful
3. information in numerical form that can be digitally transmitted or processed

However, and perhaps surprisingly, there is good evidence, that a substantial part of the scientific community has a different model in mind when reflecting about data and information. Chaim Zinn [1] documented 130 definitions of data, information, and knowledge from 45 scholars of 16 countries. Interestingly, even for the definition of information, Claude Shannon was mentioned only once. Many scholars seemed to be the opinion that there was somehow a hierarchical ordering in abstraction between the three terms in the sense that knowledge can be defined in terms of information and information can be defined in terms of data. Jennifer Rowley [2] investigates the so called "Knowledge Pyramid" Data - Information - Knowledge - Wisdom as "one of the fundamental, widely recognized and ‘taken-for-granted’ models in the information and knowledge literature" and states that there is "some consequent lack of definitional clarity" with respect to the relation of these concepts in the investigated literature.

Why is that surprising? Because it was the notion of information as developed by Ralph V. L. Hartley [3], Claude Shannon [4], and others that stood at the beginning of the field of informatics. It was their breakthrough idea to introduce a completely new perspective on the physical world around them, a perspective that disregards the quality of the physical states, be it voltage, pressure, current, etc. and take interest only in the values of these quantities as they can be distinguished as values of information. To talk about information transport and processing, we have to agree on the names of these distinguishable values. I name these values "characters" and the character sets "alphabets".

As already Claude Shannon emphasized: to describe communication as the problem of reproducing at one point either exactly or approximately a character selected at another point, requires us to disregard the "semantic aspects of communication" [4]. Following Claude Shannon, it’s information that is transported and not meaning. In this sense, meaning cannot — by definition — be transported, but meaning is locally attributed to information by its processing. We therefore can qualify anything that classifies the processing of information as a semantic technique.

Information processing is the domain of the theory of computable functionality as it was developed by Kurt Gödel, Stephen Kleene [5] and others.

The contribution of this article is a simple data concept. It is based on combining the two concepts information and computable functionality to capture the intent of other, parameterizable concepts, with typing. Additionally we will show the implications of the definition with respect to data type relations.

In short, we can say that data is information which we know how to process in principle. Capitalizing on the tight relation between semantics and processing, talking about data instead of information, obviously then has a strong semantic connotation. Additionally, we draw the relation to the current discussion on characteristics based system description in the automation.
1.1 Preliminaries

Elements and functions are denoted by small letters, sets and relations by large letters, and mathematical structures by large calligraphic letters. The components of a structure may be denoted by the structure’s symbol or, in case of enumerated structures, index as subscript. The subscript is dropped if it is clear to which structure a component belongs.

Characters are distinguishable values with no other property. Sets of characters are named alphabets. Alphabets are assumed to be enumerable. If not stated otherwise, characters can be vectors.

2 Data

The data concept dominates the very influential entity-relationship-model of Peter P.-S. Chen [6] and others, the de-facto standard for data models. There entities (individual, identifiable objects of the real world) are characterized by attributes and relationships. It was extended by generalization and specialization [7].

Intuitively, the data concept is tightly related to typing. Data types in informatics are supposed to carry quite desirable properties. They ought to assign meaning to bits and bytes, they seem to carry the intent of the programmer and last but not least seem to prevent inconsistencies in data processing. Even prominent institutions as the UN have taken serious effort to overcome semantic issues in business communication with the help of a type system [8,9].

Types were introduced to informatics by Alonso Church in 1940 [10] as a means to guarantee well-formedness of formulas of his $\lambda$-calculus of function application published in 1932 [11]. In the typed $\lambda$-calculus, simple types $\sigma$ for simple terms and function types $\sigma \rightarrow \tau$ for $\lambda$-terms are defined. Church did not commit himself to any concrete interpretation, but pointed out that “We purposely refrain from making more definite the nature of the types ..., the formal theory admitting of a variety of interpretations in this regard”.

Meanwhile it is generally agreed that a data type defines a set of (data) values together with a set of operations, having this value set as their domain [12,13,14,15,16]. Although, there had been other opinions viewing types only as sets of values (e.g. [17]) or as equivalence classes of variables (e.g. [18]).

Let us assume, that our informatics perspective has already resulted in an alphabet $V$, a signal (as a mapping from a time domain $T$ onto the value set $V$) can represent. In the following we will express this type concept in the language of mathematics and study some of its consequences.

2.1 Computable functionality with natural numbers

As the denotation of the distinguishable values of the information sets are arbitrary, looking at them as natural numbers, as the pioneers of computability did, is possible.
Be $F_n$ the set of all functions on natural numbers with arity $n$ and there exists a set of initial computable functions (the successor, the constant and the identity function). Then, based on work of Kurt Gödel, Stephen Kleene [5] showed that there are three rules to create all computable functions:

1. **Comp**: Be $g_1, \ldots, g_n \in F_m$ computable and $h \in F_n$ computable, then $f = h(g_1, \ldots, g_n)$ is computable.
2. **PrimRec**: Are $g \in F_n$ and $h \in F_{n+2}$ both computable and $a \in \mathbb{N}^n$, $b \in \mathbb{N}$ then also the function $f \in F_{n+1}$ given by $f(a, 0) = g(a)$ and $f(a, b + 1) = h(a, b, f(a, b))$ is computable.
3. **µ-Rec**: Be $g \in F_{n+1}$ computable and $\forall a \exists b$ such that $g(a, b) = 0$ and the $\mu$-operation $\mu_b[g(a, b) = 0]$ is defined as the smallest $b$ with $g(a, b) = 0$. Then $f(a) = \mu_b[g(a, b) = 0]$ is computable.

### 2.2 Computable functionality with arbitrary alphabets

We can reformulate the three rules for arbitrary alphabets $V, W, X$. Again, we assume a set of atomic functions, in this case for every alphabet:\footnote{We refrained from rephrasing also the enumerating loop parameters as we would then have to introduce a successor relation on these alphabets, which would bring us either way back to the natural numbers.}

1. **Comp**: Be $g_i : V \to W_i$, $(i = 1 \ldots n)$ computable, $W = W_1 \times \cdots \times W_n$ and $h : W \to X$, then $f = h(g_1, \ldots, g_n)$ is computable.
2. **PrimRec**: Are $g : V \to W$ and $h : V \times \mathbb{N} \times W \to W$ both computable and $a \in V$, $b \in \mathbb{N}$, then also the function $f : V \times \mathbb{N} \to W$ given by $f(a, 0) = g(a)$ and $f(a, b + 1) = h(a, b, f(a, b))$ is computable.
3. **µ-Rec**: Be $g : V \times \mathbb{N} \to \mathbb{N}$ computable and $\forall a \exists b \in \mathbb{N}$ such that $g(a, b) = 0$ and the $\mu$-operation $\mu_b[g(a, b) = 0]$ is defined as the smallest $b$ with $g(a, b) = 0$. Then $f(a) = \mu_b[g(a, b) = 0]$ is computable.

### 2.3 Data types

We first define:

**Definition 1.** We call a surjective computable function with a defined alphabet $V$ as domain an operation.

If we want to relate more than one operation to the same alphabet as their domain, all of them must adhere to the same naming conventions we use to denote the characters. Hence, we need a concept that brings together the alphabets and the operations that operate on them. The question is, how should we determine the set of operations operating on the alphabet? Is it that it’s all operations that operate on a given alphabet? Actually no. For example, the datum of a length, a diameter, and an inner diameter relate to the same alphabet, that represent a length. It’s the set of operations that differentiates these entities in a hierarchical way (see below).
Obviously, it is the kind of entity whose parameterization is supposed to be represented by a character, that let us determine whether a given operation is appropriate to process it or not. Or, to be even more precise: We have a certain concept in mind together with a parameterization as a necessary alphabet such that it can be represented by a signal. Then we are able, first, to define the alphabet, and second, to mark all operations (on that alphabet) that we think do capture our intent how to deal with it. All this is achieved by typing in the following sense.

**Definition 2.** A data type $T$ is a pair of two sets $T = (V, F)$, an alphabet $V$ and a set $F$ of operations with $V$ as their domain that together capture the intent of a parameterizable concept $C$. We then say for a character $c \in V$ that it is of type $T$ and call it a datum. We also write $F^V$.

So, if we say that a character is a datum we say three things: first, there is an alphabet which it belongs to. Second, this character can be processed by all of the operations belonging to a given type. And third, that it represents a value of a parameterized concept, that corresponds to the type. Additionally, we can also say that a signal is typed in this sense.

Comparing this definition with the initial Merrian-Webster definition shows that we are pretty close to the colloquial meanings of data.

Obviously, we can now reformulate all conditions in rules 1-3 relating to the required domain of the operations with the set $F^V$ of our type definition.

Referring to our initial statement that the meaning of the characters of an alphabet is provided by their processing, we can say that the set of operations of a type defines the abstract meaning of the characters of the alphabet of a type.

Please note, that a data type in the mentioned sense is a mathematical structure where the set of operations need not be explicitly given. This is in contrast to abstract data types or objects in the object oriented sense, which at first sight seem to be similar combinations of sets of values and operations. But in abstract data types or objects the set of operations is usually comparatively small, explicitly defined and related to a dedicated signal (= internal state). While the set of operations of a type is constructively open in a sense that new operations can be constructed and marked to belong to a given type without changing the semantics of the type. For example, the semantics of th C-data type `double` does not change if we add a new operation that is supposed to process a double variable, which would be the case for an abstract data type or an object. However, there are some authors (e.g. Robert W. Sebesta, [12], p. 248) representing the idea that the set of operations of a type is predefined in the sense of objects.

It is important to understand that this type concept does not presuppose any concept of a programming language or any other formal calculus, which are usually used as a vehicle to introduce types in informatics, for example when Luca Cardelli says, “the fundamental purpose of a type system is to prevent the occurrence of execution errors during the running of a program.” [19]. The main purpose of the presented approach is to introduce a useful, mathematically founded data concept that captures somehow most of the scope of the intuitive
meaning of this concept and, in addition to that, allows the derivation of further useful consequences. One such consequence is surely to use our knowledge about data types in our design of programming languages for the important purpose Luca Cardelli points out.

2.4 Relations of derived types

A key concept of typing is to derive new types from already defined ones. Barbara H. Liskov and Jeannette M. Wings [20] formulated the "Substitutional Principle" of subtyping: Let \( \phi(x) \) be a property of all objects \( x \) of type \( T \). Then \( \phi(y) \) should be true for objects \( y \) of type \( S \) where \( S \) is a subtype of \( T \).

Other authors seem to assume that subtyping means subset relations between the value sets while leaving the set of operations invariant (e.g. David A. Watt and William Findlay in [14], p.191 or Benjamin C. Pierce [16], p.182) while other authors (e.g. John C. Mitchell [15], p. 704) relate subtyping to a subset relations between the set of operations.

We will see that there are at least two Liskov-Wing-subtype relations for data types creating two different partial orders on our type-graph. To proceed, we need the following relation between two sets of operations.

**Definition 3.** Be \( F^V, F^{V'} \) two sets of operations with the domains \( V \supseteq V' \neq \emptyset \). If \( F^{V'} \) contains all restricted operations \( f' = f|_{V'} \) with \( f \in F^V \) in addition to all operations operating only on the alphabet \( V' \), we say that \( F^{V'} \) contains restricted \( F^V \) and write \( F^V \sqsubseteq F^{V'} \).

This relation between \( F^V \) and \( F^{V'} \) is similar to a subset relation, but it does not for example necessarily imply that the \( F^{V'} \) is larger than or equal in size to \( F^V \) (as a subset relation in the finite case would do), because there are more then one operation of \( F^V \) that restrict to one operation of \( F^{V'} \). In case that \( V = V' \) it degenerates to a subset relation.

**Restriction/Expansion** The first Liskov-Wing-subtype property we look at is \( \Phi(x) = \)"Character x is being processable by every operation \( f \in F \) of type \( T' \)." It is useful for restricting an existing type.

**Definition 4.** Be \( T = (V, F) \) a defined data type. We derive a restricted type \( T' = (V', F') \) by requiring \( V \supseteq V' \neq \emptyset \) and \( F \sqsubseteq F' \). We call \( T \) the expanded type and \( T' \) the restricted type.

From the subset relations of the alphabets immediately follows:

**Lemma 1.** Every character \( c \in V' \) of the restricted data type \( T' \) can be processed by every operation \( f \in F \) of the expanded type \( T \).

In other words, every character of type \( T' \) can be treated as if it were of type \( T \). We also say that every character of type \( T' \) can be "safely R-casted\(^3\) (=expanded)" to type \( T \). Clearly, the following subtyping proposition holds.

\(^3\)"R" stands for restriction as the basic subtyping mechanism.
Proposition 1. Be $T'$ a restricted data type of $T$, then $T'$ is an (R-)subtype of $T$ in the Liskov-Wing sense with respect to the property $\Phi(x) = "\text{Character } x \text{ is being processable by every operation } f \in F \text{ of type } T"$.

Example: Be $\text{Char}40$ the type with all character sequences of length 40 as alphabet. It is possible to define the R-subtype $\text{Alphanum}40$, relating to all alphanumeric character sequences of length 40, by restricting the alphabet in relation to the alphabet of $\text{Char}40$.

- $V_{\text{Alphanum}40} \subseteq V_{\text{Char}40}$: Strings of length 40 with alphanumeric characters are just a subset of strings of the same length of all possible characters
- $F_{\text{Alphanum}40} \supseteq F_{\text{Char}40}$: Each operation capable of processing all elements of $V_{\text{Char}40}$ is also able to process all elements of $V_{\text{Alphanum}40}$.

Thus, a character of type $\text{Alphanum}40$ can safely be R-casted (or expanded) to $\text{Char}40$, but not vice versa.

Another example, where the alphabet remains invariant, but the set of operations is enlarged by R-subtyping is a length type that is restricted to a diameter type that is further restricted to an inner diameter type. Every inner diameter can be treated as a length or as a general diameter, that is, it can be processed by all operations of the diameter type or length type. Also every diameter can still be processed by all operations of the length type - but not by all operations of type inner diameter, and so on.

So, the case that we can have different types, referring to the same alphabet, reminds us to the semantic connotation a type carries.

**Truncation/Extension** The second Liskov-Wing-subtype property we look at is $\Phi(x) = "\text{The projection } \pi(x) \text{ of character } x \text{ is being processable by every operation } f \in F \text{ of type } T"$. It is useful for extending an existing type.

**Definition 5.** Be $T = (V, F)$ a data type. We derive an extended type $T' = (V', F')$ by requiring the existence of a projection function $\pi : V \cup V' \rightarrow V$ such that $\pi(V) = V$, $V \supseteq \pi(V') \neq \emptyset$, and $F^V \subseteq F^{\pi(V')}$. We call $T$ the truncated type and $T'$ the extended type.

And again, from the subset relation $\pi(V') \subseteq V$ it follows immediately:

**Lemma 2.** The projection $\pi(c)$ of every character $c \in V'$ of the extended data type $T'$ can be processed by every function $f \in F$ of the truncated data type $T$.

In other words, every projected character of type $T'$ can be treated as if it were of type $T$. We also say that every character of type $T'$ can be ”safely P-casted“ (=truncated) to type $T$. Finally, the following subtyping proposition holds.

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4 A projection function $\pi$ fulfills the equality $\pi = \pi \circ \pi$. Therefore its codomain must be a subset of its domain.

5 ”P“ stands for projection as the basic subtype mechanism.
Proposition 2. Be $T'$ an extended data type of $T$ with the required projection $\pi$. Then $T'$ is a (P-)subtype of $T$ in the Liskov-Wing sense with respect to the property $\Phi(x) =$ "The projection $\pi(x)$ of character $x$ is being processable by every operation $f \in F$ of type $T$".

Example: Be $\text{Alphanum}20$ a type having the alphabet of all alphanumeric characters together with an extra character $\text{unknown}$ in sequences of length 20 as value set. Then we can construct a P-subtype $\text{Char}40$ as an extension by providing a projection $\pi : V_{\text{Char}40} \rightarrow V_{\text{Alphanum}20}$ such that $\pi(c_i) = c_i$ if the $i$-th character is alphanumeric and else $\pi(c_i) = \text{unknown}$.

- $V_{\pi(\text{Char}40)} \subseteq V_{\text{Alphanum}20}$: Each element in the projected set $V_{\pi(\text{Char}40)}$ is also part of the value set $V_{\text{Alphanum}20}$ of the truncated type.
- $F_{\pi(\text{Char}40)} \supseteq F_{\text{Alphanum}20}$: Each operation capable of processing all elements of $\text{Alphanum}20$ is also able to process all elements of the projected set $\pi(\text{Char}40)$ of the extended type $\text{Char}40$.

With the truncation operation being the projection, a character of type $\text{Char}40$ can be safely P-casted (or truncated) to $\text{Alphanum}20$, but not vice versa.

As the example illustrates, extension does not just mean to extend the value set, but also to assure that really all projected values belong to the original alphabet and therefore can be processed by the original operations. Additional dimensions of the extended type can simply be truncated.

2.5 Data type hierarchies

Obviously, we now have two ways to create data type hierarchies: either by starting from some top level type and restrict it more and more, or by starting from some bottom-level type and extend it more and more. However, in both cases the subtypes are the derived types.

Both subtypings define a partial order on the their derived types. As both subtypings can be combined, we get a type graph with two kinds of edges.

As long as extension is restricted to extend the elements of an alphabet without changing its dimensions, we can have circles in our type graph, resulting in safe casting in "opposite" directions.

Example: Be $\mathcal{T}$ a type with $V = \{a\}$. We extend it to the P-subtype $\mathcal{T}^{\text{ext}}$ with $V^{\text{ext}} = V \cup \{b\}$ and $\pi : \{a, b\} \rightarrow \{a\}$ such that $\pi(a) = a$ and $\pi(b) = a$. We can now restrict $\mathcal{T}^{\text{ext}}$ to the R-subtype $\mathcal{T}'$ with $V' = V^{\text{ext}}/\{b\} = \{a\}$. Obviously $\mathcal{T} = \mathcal{T}'$.

We can now cast a character of type $\mathcal{T} = \mathcal{T}'$ safely to $\mathcal{T}^{\text{ext}}$ and back. Any character of the original set $V$ (in our example only $a$) thereby remain invariant, but an eventually chosen character of type $\mathcal{T}^{\text{ext}}$ that is not an element of $V$ is changed by P-casting to some character in $V$ (in our example to $a$).

If we extend a type’s alphabet $V$ by adding some dimensions, then we have no circles anymore, because of the different requirement between the subset relation between the alphabets of type restriction, which requires a nonempty
subset, and the projection relation between the alphabets of type extension, which allows dimension reduction.

2.6 Relation to characteristic based system description

Standardizing the meaning of system properties by stipulating their types is a common technique (e.g. [9,21]). In automation engineering, there have been substantial efforts to standardize the meaning of characteristics (German "Merkmale") to simplify interoperability (see for example eClass, Prolist).

According to Ulrich Epple [22,23], a characteristic is a classifying property of a system whose manifestations can be represented by single values - which is essentially our definition of the alphabets of data types in section 2.3. Hence, each characteristic in this sense can be assigned a type in our sense.

He distinguishes characteristics from state quantities by their dynamics. State quantities change over the considered time scale and thereby parameterize the timewise behavior of systems while characteristics can be viewed as constant and therefore are well-suited to classify systems. We may add that in contrast to a state quantity, a characteristic like "stability" may not be possibly represented explicitly by the system at all. For a classification of system properties in this sense, see [24]. IEC61987 [25] is an example of a characteristic-based catalog standard of classes of systems.

Ulrich Epple [22] gives two examples for hierarchical relations. One for types of the carrier of the characteristics (that is, systems) on different levels of abstraction: a measuring device with the characteristic "measurement range" is more abstract than a flow meter with a "cross section" is more abstract than an inductive flow meter with a "minimum conductivity". This hierarchy fits nicely with the truncation/extension relation of data types. Especially as he demands that the less abstract device must "inherit" all characteristics of the more abstract device. The other hierarchy specializes characteristics: an inner diameter specializes an diameter specializes a length. Our usage of this example further above shows that this hierarchy fits nicely with the restriction/expansion of data types.

In summary, the data concept with its data types and type hierarchies match the proposed structure of system characteristics.

2.7 Operation types

As already Alonso Church pointed out, operations themselves can be processed, and therefore typed. To distinguish ordinary operations from operations that process operations, we call the ordinary ones also first order operations and the other one second order operations. As processing an operation by a second order operation may entail the execution of the first order operation, we have to additionally take into account the domain and codomain of the operations for their typing. That is, we combine only operations to a common type which have identical domains and codomains.
Definition 6. An operation type $\mathcal{T}$ is a 4-tuple $\mathcal{T} = (V, F, D, C)$, where $V$ is the alphabet of operations, $F$ a set of second order operations with $V$ as domain, and $D$ and $C$ are domain and codomain of all operations in $V$.

Please note that the operations of a data type coincide only in their domain. Thus, they usually belong to different operation types.

Restriction/Expansion To define the R-Liskov-Wing subtype of an operation type, we have to think about what kind of additional change to the domain and codomain sets would be compatible with the first Liskov-Wing condition. Clearly, this would be the case for the domain of the subtype to be a superset and the codomain becoming a subset of the original sets. However, a simple subset relation on the operation set $V$ requires that the domain and codomain remains invariant. We therefore define:

We therefore define:

Definition 7. Be $\mathcal{T} = (V, F, D, C)$ a defined operation type. We derive a restricted type $\mathcal{T}' = (V', F', D', C')$ by restricting $V$ such that $V \supseteq V' \neq \emptyset$, $F \subseteq F'$, $D = D'$ and $C = C'$. We call $\mathcal{T}$ the expanded type and $\mathcal{T}'$ the restricted type.

It should be clear that

Proposition 3. Be $\mathcal{T}$ an expanded operation type of $\mathcal{T}'$, then $\mathcal{T}'$ is an R-subtype of $\mathcal{T}$.

An example is the set of compare operations on strings. Be $S$ the set of all strings, then $\text{compare} : S \times S \rightarrow \{<,=,>\}$, where the result indicates some lexical order. There is one compare operation for each possible lexical order. We derive a restricted operation subtype if, for example, we restrict the compare operations to only those that sort numbers before characters. Please note that we do not get an R-subtype if we restrict the domain of the compare operations, for example, to documents of a given grammar, as some second order operation might call the compare operation internally still with some general string instead of the intended document.

Another example would be the type of all linear combinations of sine and cosine operations, which could be restricted to the type of all sine or all cosine operations.

Truncation/Extension To define the P-Liskov-Wing subtype of an operation, we have to understand the effect of a projection on an operation. An operation is a mapping and can therefore be represented by its graph, that is by a set of pairs where the first entry is from the operation’s domain and the second entry is from its codomain. The relevant projection $\pi$ then maps this set of pairs onto the set of pairs from the domain and codomain of the original type’s operations such that the operations of the original type remain invariant.
Definition 8. Be $T = (V, F, D, C)$ an operation type. We derive an extended type $T' = (V', T', D', C')$ by requiring the existence of a projection function $\pi = (\pi_V, \pi_D, \pi_C)$ with $\pi_V : V \cup V' \to V$ such that $\pi(V) = V$, $\pi_D : D \cup D' \to D$ such that $\pi(D) = D$, $\pi_C : C \cup C' \to C$ such that $\pi(C) = C$ and $V \supseteq \pi(V') \neq \emptyset$, and $F^V \subseteq F^{\pi(V')}$. We call $T$ the truncated type and $T'$ the extended type.

And again the respective subtype proposition:

Proposition 4. Be $T$ a truncated operation type of $T'$ with the required projection $\pi$, then $T'$ is a P-subtype of $T$.

Examples of extensions of operations are additional return parameters or the extension of domain or codomain of the original operation type.

3 Conclusion

The presented data model is essentially a type concepts that combines alphabets and sets of operations to capture parameterizable concepts. There is no way to derive some canonical set of operations from an alphabet. Thus, there can be different sets of operations to a given alphabet, representing different concepts. These sets can be related as in a type hierarchy or they can be unrelated.

We interpret the freedom to relate alphabets and sets of operations as the possibility to express our intent of the meaning of characters of the alphabet in an abstract sense. If we say that a certain alphabet should represent for example a temperature, we determine that it can only be processed by operations that are intended to work on values of temperature. We therefore must know beforehand what a temperature is as far as the construction of the operations requires it. By defining a data type we implicitly state that we know how to process information of this kind in general.

With this conception of type semantics, the role and limitations of common type systems to facilitate interoperability become comprehensible. Agreeing on common data types within an interaction implies that every interaction partner now has exactly the information she needs to avoid an unintended mismatch between the structure of the received information and the structural expectations of the operations with respect to their input. How much semantic connotation is provided by a type depends on how specific the concept is, it represents. We see as the practical limitation of this type semantics the fact that the interaction of networking, reactive systems cannot be represented by operations, mapping characters to characters, due to their nondeterministic interactions [26]. A concept that rests completely on the relation between information and its processing by operations cannot completely capture the interactive aspects of these interaction semantics.

It is obvious that our tools to create operations, namely modern imperative programming languages, should contain language elements to describe data types and their relations in the sense of this article. It is therefore quite surprising that virtually no modern programming language that we know of is expressive enough
to represent the complete data type relation model of this article. It would be an endeavor of its own to investigate what aspects of our proposed type model can be found in which programming language. C allows the definition of composed types and also operation types but does not support any type relations. Pure so-called "object oriented language" like JAVA not even allow the declaration of data types. Script languages like ECMAScript often are only very weakly typed. The language ADA is an example of a programming language that actually supports data type restrictions. For example \texttt{subtype Int10 is Integer range 1..10;} defines Int10 as an integer type with a restricted value set of 1 \ldots 10.

Currently we see a dramatic increase in the interest in data-oriented computing, like in the area of big data. We think that it would be worthwhile to develop truly data oriented programming paradigms based on the presented type concept. Due to the much more flexible relation between alphabets and operations in the world of types compared to the world of objects, we would expect a data oriented programming paradigm also to be much more flexible.

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