Six-Degree of Freedom Mathematical Dynamic Model of a Light-Sport Aircraft

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Abstract

The paper considers the applications of six-degree of freedom mathematical model of a light-sport aircraft and analytically reveals aircraft dynamic response both longitudinal and lateral-directional stability. The model composes of both the kinematics equations of motion and the kinetics equations of aerodynamic forces and moments, and it is known as the aerodynamic model equations. Simulation results show responses of the perturbed dynamic system at the trim condition, and indicate the dynamic stability of both short-period pitching oscillation and phugoid in longitudinal axes. The spiral, roll subsidence, and dutch roll modes in lateral-directional axes are dynamically stable as well. These are essential to understand and evaluate the dynamic behavior, stability, safety and other aspects of the designed aircraft through mathematical model before conducting operational flight test.

Keywords: mathematical model, dynamic stability, light-sport aircraft

1. Introduction

A two-seater light amphibious airplane special project supported by Thai Research Fund (henceforth, TRF) aims to build a light amphibious airplane called “NAX-5” in which categorized into a light-sport aircraft (henceforth, LSA). In short, LSA is defined as a small non-pressurized aircraft, fixed-wing, one crew and one passenger, 600 kg maximum take-off weight (650 kg for seaplane), and driven by a fixed pitch propeller connect to a single piston engine. A certificate of airworthiness can be issued to the LSA that comply with ASTM F2245 standard for the safety and performance. One of the compliance method through the certification process is analytically proven in both longitudinal and lateral-directional aircraft stability [1], [2]. Before the aircraft will be demonstrated the aerodynamic stability through a flight test program, an aircraft designer normally prefers to investigate the dynamic response of initial design by proper simulation [5], [6], [7].

A mathematical model of aircraft usually describes the aerodynamic forces and moments in terms of relevant measurable quantities such as control surface deflections, aircraft linear velocity, angular velocities, and the orientation of the aircraft respect to the relative wind [11]. When the mathematical model is parametric, aerodynamic parameters can be quantified as the functional dependence of the aerodynamic forces and moments on measureable quantities. In general, aircraft mathematical model is non-linear, comprised of complex aerodynamic function depends on aircraft flight configuration [9], [10]. To ensure the aircraft safety and performance design, dynamics simulation
through a mathematical model is required before conducting the first flight test so that suitable engineering correction can be made.

This paper presents a six-degree of freedom equation of motion of an aircraft model. With a small perturbation theory, one can analytically linearize the general equation of motion composed of kinematics equations of motion and kinetics equations of forces and moments at trim. Decoupling technique is used to simplify the equation of motion into a set of longitudinal equations and lateral-directional equations [8]. Laplace transform technique is also used to achieve a set of the input-output transfer functions.

### Abbreviations

| Symbol | Description |
|--------|-------------|
| $U, V, W$ | linear velocity vector component about the axes $\alpha x$, $\alpha y$ and $\alpha z$ |
| $X, Y, Z$ | moment vector component about the axes $\alpha x$, $\alpha y$ and $\alpha z$ |
| $p, q, r$ | angular velocity vector component about the axes $\alpha x$, $\alpha y$ and $\alpha z$ |
| $L, M, N$ | moment vector component about the axes $\alpha x$, $\alpha y$ and $\alpha z$ |
| $I_x, I_y, I_z$ | moment of inertia about the axes $\alpha x$, $\alpha y$ and $\alpha z$ |
| $I_{xy}, I_{xz}, I_{yz}$ | product of inertia about the axes $\alpha x$, $\alpha y$ and $\alpha z$ |
| $\dot{u}, \dot{v}, \dot{w}$ | linear disturbance velocities |
| $X_u, X_v, X_w$ and so on | aerodynamic stability derivatives |
| $\xi, \eta, \zeta$ | aileron deflection angle, elevator deflection angle, and rudder deflection angle |
| $\phi, \theta, \psi$ | components of airplane attitude in roll, pitch, and yaw angle |
| $\alpha$ | incidence angle |
| $\beta$ | side slip angle |
| $m$ | mass |
| $\tau$ | thrust |
| $g$ | gravity |
| $V_o$ | velocity of the airplane |

### 2. Light sport aircraft model

The “NAX” project initiated by the Royal Thai Navy research team has been started since 2002, aiming to build a light versatile amphibious airplane that can serve in military and civilian purposes. The initial version of airplane, NAX-3, has been designed from the concept of the EDRA Aeronautic Super Pétrel, an amphibious pusher configuration biplane. The NAX-3 has demonstrated its satisfaction flying qualities in both performance and safety, but still, the total weight of airplane is above 650 kilograms which are the number classified as a light sport airplane category stated by the Civil Aviation Authorities of Thailand (CAAT).

The new design of NAX-5 project has started with the concept of building an amphibious pusher high-wing configuration [12]. The airplane main structure is mostly made from composite, gains benefits in lightweight control and enhances the strength of the structure. With retractable landing gears, the airplane can take-off and landing on a short airfield or water runway. A more powerful 115 Hp, Rotax 914 piston engine with three blades constant pitch propeller is installed as “pusher” high-wing configuration. The airplane basic design performance is shown in Table 1.
Figure 1. A two-seater light amphibious airplane “NAX-5”

| Specifications:                      |
|--------------------------------------|
| Crew:                                |
| Capacity:                            |
| Length:                              |
| Wingspan:                            |
| Wing area:                           |
| Cabin width:                         |
| Empty weight:                        |
| Max fuel capacity:                   |
| Max takeoff weight:                  |

| Performance:                        |
|-------------------------------------|
| Maximum speed:                      |
| Cruise speed:                        |
| Stall speed:                         |
| Endurance:                           |
| Rate of Climb:                       |
| Take off Distance:                   |
| Landing Distance:                    |

Table 1. “NAX-5” specification and performance

The flying motion of the aircraft analyzing at the center of gravity $cg$ coincident with the origin $o$ in an orthogonal axis set $(oxyz)$, in the general body, as shown in Figure 2. Then, the force and moment equation comprised the generalized six-degree of freedom equations of motion. The uniform mass distribution of an airframe can be expressed as the following equations:

\[
m(\dot{U} - W + qV) = X_a + X_o + X_c + X_p + X_d \\
\dot{U} = \dot{p} + q \ddot{r} - \dot{r} + \dot{r}_x (pq + \dot{r}) = L_a + L_o + L_c + L_p + L_d \\
\dot{p} = \dot{r}_x (pq + \dot{r}) + \dot{r}_x (p^2 - r^2) = M_a + M_o + M_c + M_p + M_d \\
\dot{r} = \dot{r}_x (pq + \dot{r}) + \dot{r}_x (p^2 - r^2) = N_a + N_o + N_c + N_p + N_d
\]

Assuming that the disturbing forces and moments on the right side of the equation are the effects resulting from aerodynamic, gravitational, aerodynamic flight control moment, power from engine, and atmospheric disturbances. The equations are in general highly non-linear, and unpractical to achieve their solution by analytical mean. To continue solving the analytical solution, linearization of these equations can be accomplished by restraining the motion of the aircraft to small perturbation at the trim.
condition. If the aircraft at the undisturbed axis (oxyz) subjects a small perturbation of linear disturbance
velocities ($u, v, w$) and angular disturbance velocities ($p, q, r$) about the trim; therefore, in the disturbed
motion, the overall linear velocity vector components of the aircraft at the center of gravity are given by

$$\begin{align*}
U &= U_e + u \\
V &= V_e + v \\
W &= W_e + w \\
\end{align*}$$

(2)

At the flying circumstance in regular trim (none of the roll, sideslip, and yaw angle), there is no effect of an undisturbed atmosphere ($X_d = Y_d = Z_d = L_d = M_d = N_d = 0$). Weight and moment about any
axis are considered ($L_q = M_q = N_q = 0$). The gravitational force components are $X_g = -mg \sin \theta_e - mg \theta \cos \theta_e$, $Y_g = -mg \sin \phi_e + mg \phi \cos \theta_e$, $Z_g = mg \cos \phi_e - mg \theta \sin \phi_e$.

The aerodynamic terms are expressed by Taylor’s expansion and neglected all small terms (i.e.
$X_a = X_{ae} + \frac{\partial X_u}{\partial u} u + \frac{\partial X_v}{\partial v} v + \frac{\partial X_w}{\partial w} \omega + \frac{\partial X_p}{\partial p} p + \frac{\partial X_q}{\partial q} q + \frac{\partial X_r}{\partial r} r$, and $\frac{\partial X}{\partial u} = X_u$ for shorthand notation). Also in the steady trimmed, $X_{ae} = mg \sin \theta_e$, $Y_{ae} = 0$, $Z_{ae} = -mg \cos \phi_e$, $L_{ae} = M_{ae} = N_{ae} = 0$. The aerodynamic control
derivatives (i.e. pitching moment $M = \frac{\partial X}{\partial \phi} + \frac{\partial Y}{\partial \theta} + \frac{\partial Z}{\partial \phi}$) can be used to describe the response of all primary aerodynamic control such as the elevator, ailerons, and rudder. The relationship between the trust from the engine and throttle level angle in power term is also expressed in shorthand notation such as, $Z_p = Z_{e \tau}$.

The contributions of the final equation are also written comparably, therefore, the equations of motion for small perturbations can be shown as

$$
m(\ddot{u} - q\dot{W}_e) = X_u \dot{u} + X_v \dot{v} + X_w \dot{w} + X_p \dot{p} + X_q \dot{q} + X_r \dot{r} - mg \theta \cos \theta_e + X_\theta \xi + X_\phi \eta + X_\psi \zeta + X_{\tau} \\
m(\ddot{v} - p\dot{W}_e + r\dot{U}_e) = Y_u \dot{u} + Y_v \dot{v} + Y_w \dot{w} + Y_p \dot{p} + Y_q \dot{q} + Y_r \dot{r} + mg \phi \cos \theta_e + mg \phi \sin \theta_e + Y_\theta \xi + Y_\phi \eta + Y_\psi \zeta + Y_{\tau} \\
m(\ddot{w} - q\dot{W}_e + r\dot{U}_e) = Z_u \dot{u} + Z_v \dot{v} + Z_w \dot{w} + Z_p \dot{p} + Z_q \dot{q} + Z_r \dot{r} - mg \sin \phi_e + Z_\theta \xi + Z_\phi \eta + Z_\psi \zeta + Z_{\tau} \\
l_{\theta} \ddot{\theta} - l_{\phi} \ddot{\phi} = m \ddot{\phi} - m \ddot{\phi} = M_u \dot{u} + M_v \dot{v} + M_w \dot{w} + M_p \dot{p} + M_q \dot{q} + M_r \dot{r} + M_\theta \xi + M_\phi \eta + M_\psi \zeta + M_{\tau} \\
l_{\phi} \ddot{\phi} - l_{\psi} \ddot{\psi} = m \ddot{\psi} - m \ddot{\psi} = N_u \dot{u} + N_v \dot{v} + N_w \dot{w} + N_p \dot{p} + N_q \dot{q} + N_r \dot{r} + N_\theta \xi + N_\phi \eta + N_\psi \zeta + N_{\tau}$$

(3)

Equation (3) are small perturbations equations of motion describing the transient response of the aircraft at trim. They compose of six simultaneous linear differential equation written conventionally with inputs. Since longitudinal and lateral dynamics are fully coupled in the equations of motion, but the longitudinal and lateral coupling can be negligible when considering small perturbation of motion. As a result, these equations can be simplified by decoupling the longitudinal and lateral motion.

**Decoupled equations of motion**

**Longitudinal equations of motion**

The motion is described by the longitudinal plane of symmetry (oxyz plane), therefore only axial force, normal force, and pitching moment are considered in the equations of motion. By this assumption, the motion variables and their derivative in lateral along with aileron and rudder deflections are all zero, thus

$$\begin{align*}
\dot{X}_e &= \dot{X}_p = \dot{X}_r = \dot{Z}_e = \dot{Z}_p = \dot{Z}_r = \dot{M}_e = \dot{M}_p = \dot{M}_r = 0 \\
\dot{X}_\xi &= \dot{X}_\eta = \dot{Z}_\xi = \dot{Z}_\eta = \dot{M}_\xi = \dot{M}_\eta = 0 \\
\end{align*}$$

Also, with the assumption that the aircraft is in level flight

$$\theta_e = W_e = 0$$

The simplified longitudinal equations of motion are rearranged as

$$\begin{align*}
m \ddot{u} - X_u \dot{u} - X_v \dot{v} - X_w \dot{w} - X_p \dot{p} &+ q \dot{m} \theta = X_\theta \xi + X_{\tau} \\
- Z_u \dot{u} + (m - Z_w) \dot{w} - Z_w \dot{w} - (Z_q + m U_e) \dot{q} = Z_\theta \xi + Z_{\tau} \\
- M_u \dot{u} + M_w \dot{w} - M_p \dot{p} + l_p \dot{q} - M_q \dot{q} = M_\theta \eta + M_{\tau} \\
\end{align*}$$

(4)
Lateral-directional equations of motion

The motion contains yaw, roll, and sideslip; hence, yawing moment, rolling moment, and side force are considered in the equations of motion. The elevator deflection, thrust variation, all motion variables and their derivative in longitudinal do not influence lateral-directional motion and do not take into account, thus

\[
\begin{align*}
Y_u &= Y_w = Y_q = L_u = L_w = L_q = N_u = N_w = N_q = 0 \\
Y_\eta &= Y_\epsilon = L_\eta = L_\epsilon = N_\eta = N_\epsilon = 0
\end{align*}
\]

Also, with the assumption that the aircraft is in level flight

\[\theta_e = W_\theta = 0\]

The lateral-directional equations of motion are simplified and rearranged in the form

\[
\begin{align*}
lv \dot{v} - Y_\eta v - p Y_\eta &= \left( Y_\epsilon - m \dot{u}_e \right) r - mg \phi = Y_\epsilon \xi + Y_\zeta \\
- L_\zeta v + I_x \dot{p} - L_p \dot{p} - I_{zx} r - L_r r &= L_\epsilon \xi + L_\zeta \zeta \\
- N_\zeta v - I_{xz} \phi - N_p \ddot{p} + I_{zr} r - N_r r &= N_\epsilon \xi + N_\zeta \zeta
\end{align*}
\]

(5)

3. The longitudinal and lateral-directional response transfer functions

When small perturbation motion is taken into consideration and thrust remains constant in trim, the Laplace transform with the assumption of zero initial conditions of longitudinal equation (4) can be written in the matrix form

\[
\begin{pmatrix}
\left(Ms - X_\alpha^*\right) & -\left(X_\alpha^* s - X_\alpha^*\right) & -\left(\dot{\alpha}_q\right) s - mg \\
-Z_\alpha^* & -\left(Z_\alpha^* s - Z_\alpha^*\right) & \left(Z_\alpha^* s + m \dot{u}_e\right) \\
-M_\alpha^* & -\left(M_\alpha^* s + M_\alpha^*\right) & \left(I_\alpha s^2 - M_\alpha^* s\right)
\end{pmatrix}
\begin{bmatrix}
\alpha(s) \\
\phi(s) \\
\gamma(s)
\end{bmatrix} =
\begin{bmatrix}
\frac{\dot{\alpha}(s)}{\phi(s)} \\
\frac{\phi(s)}{\gamma(s)} \\
\frac{\gamma(s)}{\gamma(s)}
\end{bmatrix}
\begin{bmatrix}
u(s) \\
w(s) \\
\eta(s)
\end{bmatrix}
\]

(6)

By Cramer’s rule, one can write the longitudinal transfer functions as

\[
\frac{\dot{\alpha}(s)}{\phi(s)} = \frac{N_\alpha^p(\xi) \gamma(s)}{\Delta(s)} = \frac{N_\alpha^p(\xi)}{\Delta(s)} \frac{\gamma(s)}{\gamma(s)}
\]

Similarly, we can rewrite the lateral-directional in the matrix form, as shown in equation (5).

\[
\begin{pmatrix}
\left(Ms - Y_\zeta^*\right) & \left(Y_\zeta^* s + mg\right) & -\left(\dot{\zeta}_q\right) s - mg \phi \\
-L_\zeta & \left(I_x s^2 - L_p s\right) & \left(I_{zx} s^2 + I_r s\right) \\
-N_\zeta & \left(I_{xz} s^2 + N_p s\right) & \left(I_\zeta s^2 - N_r s\right)
\end{pmatrix}
\begin{bmatrix}
\dot{\zeta}(s) \\
\phi(s) \\
\gamma(s)
\end{bmatrix} =
\begin{bmatrix}
\frac{\dot{\zeta}(s)}{\phi(s)} \\
\frac{\phi(s)}{\gamma(s)} \\
\frac{\gamma(s)}{\gamma(s)}
\end{bmatrix}
\begin{bmatrix}
u(s) \\
w(s) \\
\eta(s)
\end{bmatrix}
\]

(7)

By Cramer’s rule, one can write the lateral-directional transfer functions as

\[
\frac{\dot{\zeta}(s)}{\phi(s)} = \frac{N_\zeta^p(\xi) \gamma(s)}{\Delta(s)} = \frac{N_\zeta^p(\xi)}{\Delta(s)} \frac{\gamma(s)}{\gamma(s)}
\]

Incidence and sideslip response transfer functions

For other related variables, one can easily compose incidence angle in the longitudinal decouple equation by the assumption for small perturbation motion incidence as
\[ \alpha \equiv \tan \alpha = \frac{w}{v_e} \]  

(8)

since \( U_e \rightarrow V_o \) implying the perturbation has tendency going to zero, then incidence \( \alpha \) can be written as aircraft vertical velocity over the steady air stream velocity.

Similarly, the sideslip angle (\( \beta \)) can be acquired from the lateral-directional equation by the relationship

\[ \beta \equiv \tan \beta = \frac{v}{v_e} \]  

(9)

**Aerodynamic stability derivative**

All aerodynamic derivative appeared in the linearized equation of motion play significant roles in determining the accuracy of the mathematical models. Analytically, the aerodynamic derivative describing a dynamic characteristic of aircraft can be determined from the relationship of various aerodynamic coefficients and parameters which are linearized about the operating point of interest. However, since the flow conditions across the airframe are complicated, the explanation for the mathematical aerodynamic phenomena must result in compromise. Wind tunnel measurement and flight test measurement are considered to be most reliable methods to gain the correct aerodynamic coefficients and parameters of the specific aircraft model, but a number of equipment and expenses must be taken into account.

In this paper, all longitudinal stability derivative and lateral-directional stability derivative are obtained from NASA CR-1975 Handling Quality Light Aircraft report [3], [4] referring to the NAX-5 design dimension and configurations.

**4. Simulation results**

To simulate and verify aircraft stability dynamic through the equations of motion, the initial input data of the aircraft characteristic is determined and inserted into the simulation program. The flight condition corresponding to level cruising flight is the altitude at 5,000 ft. and aircraft’s speed at Mach 0.2.

By using MATLAB, the transfer functions of both longitudinal and lateral-directional aircraft motions are shown in equations (10) and (11)

**Longitudinal**

\[
\begin{align*}
\eta(s) &= \frac{-1.18(s-10.76)(s^2+3.72s+7.662)}{(s^2-0.0306s+0.0465)(s^2-3.46s+5.1833)} \\
\eta(s) &= \frac{-9.075(s+28.59)(s^2+0.044s+0.085)}{(s^2+0.0306s+0.0465)(s^2-3.46s+5.1833)} \\
\eta(s) &= \frac{-5.364s(s+0.0708)(s+1.82)}{(s^2-0.0306s+0.0465)(s^2-3.46s+5.1833)} \\
\eta(s) &= \frac{5.364s(s+0.0708)(s+1.82)}{(s^2+0.0306s+0.0465)(s^2-3.46s+5.1833)} \\
\eta(s) &= \frac{-1.1881(s+28.59)(s^2+0.044s+0.085)}{(s^2-0.0306s+0.0465)(s^2-3.46s+5.1833)} \\
\eta(s) &= \frac{0.1881(s+0.026)(s+7.299)(s-7.21)}{(s^2-0.0306s+0.0465)(s^2-3.46s+5.1833)} \\
\eta(s) &= \frac{0.1881(s+0.026)(s+7.299)(s-7.21)}{(s^2-0.0306s+0.0465)(s^2-3.46s+5.1833)} \\
\eta(s) &= \frac{0.1881(s+0.026)(s+7.299)(s-7.21)}{(s^2-0.0306s+0.0465)(s^2-3.46s+5.1833)} \\
\eta(s) &= \frac{0.1881(s+0.026)(s+7.299)(s-7.21)}{(s^2-0.0306s+0.0465)(s^2-3.46s+5.1833)}
\end{align*}
\]  

(10)

The phugoid stability mode is determined by the first pair of complex poles with the value of damping ratio \( \zeta_p = 0.0712 \) and undamped natural frequency \( \omega_p = 0.215 \text{ rad/s} \). The short-period pitching oscillation mode is also determined by the second pair of complex poles with the value of damping ratio \( \zeta_p = 0.761 \) and undamped natural frequency \( \omega_p = 2.28 \text{ rad/s} \). The characteristic of these modes indicates the degree of stability of the aircraft.

The dynamic responses of the aircraft from a pulse elevator input (5 degrees pulse, held for 5 seconds and return to zero and another -5 degrees pulse, held for 5 seconds and return to zero) are shown in Figure 4. The results of this study clearly show dynamic stability both short-period pitching oscillation and phugoid modes. It should be noted that each response variable shows a different scale in term of each stability mode.
Lateral-directional

\[
\begin{align*}
\beta(s) &= \frac{1.411(s+0.124)(s+9.22)}{(s+0.061)(s+8.81)(s^2+0.46s+1.91)} \\
\xi(s) &= \frac{15.41(s-0.012)(s^2+0.702s+1.53)}{(s+0.061)(s+8.81)(s^2+0.46s+1.91)} \\
p(s) &= \frac{0.0963(s+0.0445)(s+8.81)(s+18.1)}{(s+0.061)(s+8.81)(s^2+0.46s+1.91)} \\
\zeta(s) &= \frac{0.5963(s-0.011)(s-1.45)(s+7.85)}{(s+0.061)(s+8.81)(s^2+0.46s+1.91)} \\
r(s) &= \frac{-1.719(s+8.823)(s^2+0.055s+0.0938)}{(s+0.061)(s+8.81)(s^2+0.46s+1.91)} \\
\phi(s) &= \frac{0.77561(s-1.77)(s+7.71)}{(s+0.061)(s+8.81)(s^2+0.46s+1.91)} \\
\zeta'(s) &= \frac{0.05061(s-1.77)(s+7.71)}{(s+0.061)(s+8.81)(s^2+0.46s+1.91)} \\
\end{align*}
\]

The spiral mode is determined by the first real of complex poles with the value of the time constant \( T_s = \frac{1}{0.061} \approx 16.39 \text{ sec} \). The roll subsidence mode is determined by the second real pole with the value of the time constant \( T_s = \frac{1}{8.81} \approx 0.1135 \text{ sec} \). The oscillatory dutch roll mode is also determined by the pair of complex poles with the value of damping ratio \( \zeta_d = 0.166 \) and undamped natural frequency \( \omega_d = 1.38 \text{ rad/sec} \).

Since all poles of the system transfer function have negative real parts, the mode characteristics indicate that the aircraft is aerodynamically stable. The dynamic responses from an aileron impulse input and a rudder impulse input of the aircraft are shown in Figures 5 and 6, respectively.

![Figure 3. Impulse input](image-url)
Figure 4. Aircraft dynamic response from elevator command
Figure 5. Aircraft dynamic response from aileron command

Figure 6. Aircraft dynamic response from rudder command
5. Conclusions
This study set out to investigate the mathematical model of a light-sport at the trim condition. The model composed of kinematics equations of motion and kinetics equations of forces and moments. This study has identified the system transfer function in both longitudinal and lateral-directional axes. It also shows a stable aerodynamic response with a suitable degree of stability. The investigation of the assumption of level flight and stable thrust has shown that the longitudinal equations are disturbed with five degrees of elevator deflection in both directions. The results revealed that it does not only make airspeed, vertical speed, pitch rate, pitch angle, incidence angle and flight path angle deviated, but it also turns to steady value in such a period. One of the more significant findings to emerge from this study is that when five degrees of aileron and rudder deflection is made, the roll rate, yaw rate and sideslip angle turn to steady-state values in a short time. However, the roll angle keeps slowly changing because a small degree of stability allows some changes to the aircraft attitude. These findings suggest that in general the adverse roll can be noticed from the graph since a non-minimum phase numerator term appeared in some system transfer function. Overall, this study strengthens the idea that the stability of aircraft dynamic and ensure safety before conducting actual test flight should be encouraged.

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