Numerical simulation of dynamics of a gas bubble in liquid near a rigid wall during its growth and collapse

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Abstract. A numerical technique of calculating the dynamics of a cavitation bubble near a plane rigid wall is presented. The bubble at its collapse can become toroidal. The liquid is assumed inviscid and incompressible, its flow being potential. The bubble surface movement is determined by the Euler method, the normal component of the liquid velocity on the bubble surface is found by the boundary element method. The technique also includes an algorithm for calculating the velocity and pressure fields in the liquid. The convergence of the numerical solution with refining the temporal and spatial steps is demonstrated. The results of the present technique are compared with some known numerical and experimental data by other authors, their satisfactory agreement is found. To illustrate the capabilities of the present technique the process of growth and collapse of a bubble in water near a wall is considered. The liquid pressure contours in the stage of the bubble collapse are given and the radial liquid pressure profiles on the wall and at a small distance from the wall where the liquid pressure is maximum are shown.

1. Introduction

The vapor bubbles near a wall are known to lead to its cavitation damage. In particular, this happens to the blades of marine propulsion propellers, the walls of pipe systems, the blades of hydraulic turbines, etc. It was first believed that the cavitation damage is caused by the pressure pulses resulting from the spherical collapse of bubbles. However, in 1944, Kornfeld and Suvorov [1] suggested that the destructive action of bubbles on a solid is realized by means of jets formed at collapse of cavitation bubbles near a solid wall. This hypothesis was later confirmed by Plesset and Chapman [2] using the finite difference method. After that, the dynamics of bubbles near a wall with the formation of the jets was widely studied, for example, in [3-7]. In most papers devoted to investigating the dynamics of a bubble in an inviscid incompressible fluid the boundary element method was used. In [8] a numerical technique based on the boundary element method was first proposed, which was later applied in [3] for studying the axisymmetric deformations of an empty spheroidal cavity near a rigid wall. Much later a similar approach was utilized in [4] for studying the growth and collapse of a bubble near a wall, in [5, 6] it was adapted to the spatial problems of bubble dynamics, and in [9-13] it was modified to simulate the toroidal phase of bubble dynamics. The toroidal phase is characteristic of the liquid flow circulation around the toroidal bubble.
In this paper, a numerical technique for studying the evolution of a bubble near a wall is presented. The liquid flow rotation in the toroidal phase of bubble dynamics is taken into account by introducing an artificial plane section perpendicular to the axis of symmetry [9]. Unlike the other authors [11, 14], the numerical instability is eliminated by a smoothing cubic spline. The capabilities of the technique are illustrated by computing a problem of growth and collapse of a bubble in water at a small distance from a wall.

2. Main relations and solution method

Axisymmetric growth and collapse of a cavitation bubble in liquid at a distance from a plane motionless rigid wall is considered. The axis of symmetry is orthogonal the wall. The pressure in the bubble varies as

\[ p_b = p_{bo}(V_0/V)^\kappa \]  

where \( p_{bo} \) is the initial pressure in the bubble, \( V_0, V \) are the initial and current volumes of the bubble, respectively, \( \kappa \) is the adiabatic index. The shape of the bubble at its maximum size due to the presence of the wall is non-spherical. Upon collapse a cumulative jet can arise on the bubble surface. This jet hits the opposite side of the bubble surface so that the bubble becomes toroidal.

It is assumed that the liquid is inviscid and incompressible, its flow is potential and is governed by the equations

\[ \Delta \Phi = 0, \]

\[ \Phi_s + \frac{1}{2} (\nabla \Phi)^2 + \frac{p - p_s}{\rho} = 0 \]

where \( p, p_s \) are the liquid pressures at any point and at a great distance from the bubble, respectively, \( \rho \) is the liquid density, \( \Phi \) is the liquid velocity potential, \( \mathbf{v} = \nabla \Phi \) is the liquid velocity, \( t \) is the time.

On the bubble surface, the following dynamic

\[ p^+ = p^- - 2\sigma H \]

and kinematic

\[ \frac{dx_s}{dt} = \mathbf{v}_s \]

boundary conditions are posed. In (4), (5), \( p^+, p^- \) are the liquid and gas pressure on the bubble surface, respectively, \( \sigma \) is the surface tension, \( H \) is the mean curvature at a point of the bubble surface; \( x_s, \mathbf{v}_s = (\nabla \Phi)\big|_s \) are respectively the position-vector and the velocity of a surface point.

On the rigid wall the condition of impermeability

\[ v_z = \frac{\partial \Phi}{\partial z} = 0 \]

is posed where \( z \) is the axial coordinate, \( v_z \) is the axial component of the liquid velocity.

In the process of evolution of a bubble with the formation of the cumulative jet on its surface one can separate out two phases. The first phase corresponds to the formation of the jet. It ends at the moment of impact of the jet \( t_c \) on the opposite side of the bubble surface. The second phase corresponds to the motion of the toroidal bubble resulting from that impact. Accordingly, the numerical technique of the present paper is also divided into two stages, with the first (second) stage corresponding to the first (second) phase. At that, the bubble surface movement and deformation are determined by the Euler method from kinematic (4) and dynamic (5) boundary conditions, the liquid velocity and its potential on the bubble surface are evaluated by the boundary element method [15].

The first stage of the numerical technique used for computing the first phase of the bubble dynamics is described in detail in [7]. Its utilizing without any smoothing procedure can lead to numerical instability that manifests itself in high-frequency distortions of the bubble shape. To suppress such distortions, a smoothing procedure proposed in [7] is adopted in the present technique. That procedure is based on a cubic spline. It is used for smoothing the functions \( \Phi(s), r(s), z(s) \), which respectively define the liquid velocity potential, the radial and axial coordinates of the bubble surface along its contour in its axial section (\( s \) is the contour arc length).
The second phase of the bubble dynamics begins with formation of a toroidal bubble resulting from the jet impact on the opposite side of the bubble surface. In the present numerical technique the transition from singly connected to doubly connected bubble surface in its axial section is performed by the algorithm proposed in [9]. In the doubly connected flow region the velocity potential is not uniquely defined. To resolve this ambiguity, a straight infinitely-thin cut perpendicular to the symmetry axis and connecting the symmetry axis with a point of the torus section is introduced in the area of the jet. Across this cut the velocity potential is assumed discontinuous. The value of this discontinuity is constant along the cut. It is equal to the difference $\Delta \Phi$ of the axial potential values at the points of the jet end and the opposite side of the bubble surface attained at the end of the first phase of the bubble dynamics. The only difference of the second stage of the present numerical technique from the first one is that an additional term containing an integral along the cut appears in the boundary integral equation [9].

In addition to the first and second stages, the present numerical technique includes the construction of the liquid velocity and pressure fields. To this end, the potential $\Phi$ is determined by the boundary integral equation whereas the velocity components are calculated by the finite-differences of the second order of accuracy. The pressure in the liquid is found by equation (3) where $\partial \Phi / \partial t$ is also calculated by the finite-differences.

3. Numerical convergence. Comparison with known numerical and experimental results

The following dimensionless parameters are introduced: $R_0^* = R_0 / R_{\text{max}}$, $\gamma = d_0 / R_{\text{max}}$, $r^* = r / R_{\text{max}}$, $z^* = z / R_{\text{max}}$, $v^* = v / (p_\alpha / \rho_f)^{1/3}$, $p^* = p / p_\alpha$, $t^* = t / [R_{\text{max}}(p_\alpha / \rho_f)^{1/3}]$ where $R_0$ is the initial bubble radius, $R_{\text{max}}$ is the mean radius of the bubble at its maximum size, $d_0$ is the initial distance from the center of the bubble to the wall, $r$ is the radial coordinate, $v = |\mathbf{v}|$ is the liquid velocity value.

To estimate the numerical convergence of the present technique in computing the first phase of the bubble dynamics (until the moment $t_i^*$ at which the jet touches the opposite side of the bubble surface), a problem of initially spherical bubble growth and collapse in water is considered for $R_0^* = 0.159$, $\gamma = 0.7$, $\kappa = 1.33$, $p_{\text{ho}}^* = 37.76$, $v_0^* = 10.37$, $R_{\text{max}} = 6.3 \cdot 10^{-4}$ m, $p_\alpha = 10^7$ Pa.

![Figure 1](image1.png)

**Figure 1.** The contours of the bubble surface at the moment $t_i^*$ for $n = 50$ (curve 1), 100 (curve 2), 200 (curve 3), 400 (curve 4), 800 (curve 5) (a) and their enlargements near the jet end for the same values of $n$ (b).

![Figure 2](image2.png)

**Figure 2.** The contours of the bubble surface in the toroidal phase of the bubble dynamics at the moment $t^* = 0.0635$ for $\gamma = 0.4 \cdot 10^{-4}$ and $n = 100$ (dotted curve), 200 (dashed curve) and 400 (solid curve) (a); their enlargements in the vicinity of the splash for the same values of $n$ (b); the similar...
fragments of the bubble surface contours for \( n = 200 \) and the initial time steps \( \tau_0^* = 2.1 \cdot 10^{-4} \) (curve 1), \( 0.9 \cdot 10^{-4} \) (curve 2), \( 0.4 \cdot 10^{-4} \) (curve 3), \( 0.2 \cdot 10^{-4} \) (curve 4) (c).

Figure 1 shows the contours of the bubble surface in its axial section at the moment \( t^* \), computed for consecutively increasing contour discretization defined by the contour segment number \( n \). One can see numerical converge with increasing \( n \). At that, the contours for \( n > 100 \) are only different in the areas with large curvature (at the end of the jet and in the region most distant from the symmetry axis). Moreover, the difference of the solutions for \( n = 400 \) and 800 is small.

In the toroidal phase of the bubble dynamics following the jet impact, liquid circulation through the hole in the toroidal bubble occurs. This liquid flows onto the wall and radially spreads on it. At the same time, there is an oppositely directed liquid flow from the far field. The collision of these flows can result in an upward splash on the bubble surface (Figure 2) [12, 17, 18]. The curvature of the contour within the splash is greater than outside it, which impairs the numerical convergence. Figure 2 illustrates this feature in the case of increasing \( n \) (at \( t^* = 0.0635 \)). It should be noted that for \( n = 200 \) and 400 the shape of the bubble is almost the same. In Figure 2c, the numerical convergence of the splash is demonstrated by refining the initial time step (in the case of \( n=200 \)). One can see that the solutions corresponding to \( \tau_0^* = 0.4 \cdot 10^{-4} \) and \( \tau_0^* = 0.2 \cdot 10^{-4} \) are graphically coincident. Similar convergence in the other significantly-smoother part of the bubble contour is derived for all the considered values of \( \tau_0^* \).

4. Growth and collapse of a cavitation bubble near a rigid wall

To assess the reliability of the technique of the present work its results are compared with the numerical results of [12] (Figure 3) and with the experimental data of [18] (Figure 4). In the both cases a problem of growth and collapse of an initially spherical bubble in water near a solid wall is considered. In the first case the simulation is carried out for \( R_0^* = 0.159 \), \( \gamma = 1.0 \), \( \kappa = 1.4 \), \( \rho_{\infty} = 100 \), \( p_\infty = 10^3 \) Pa. In the second case it is performed for \( R_0^* = 2.5 \cdot 10^{-4} \), \( \gamma = 2.0 \), \( R_{\text{max}} = 1.45 \cdot 10^{-3} \) m, \( \rho_s = 1000 \) kg/m\(^3\), \( p_c = 10^5 \) Pa where \( R_0 \), \( R_{\text{max}} \), and \( \gamma \) are taken from [18]. It follows from Figures 3 and 4 that in both cases the agreement is quite satisfactory.

To illustrate the capabilities of the present technique a problem of growth and collapse of a cavitation bubble in water near a rigid wall is considered with
\[ R_0 = 0.159, \gamma = 0.7, \, p_{\text{at}} = 37.76, \, v_0 = 10.37, \, \kappa = 1.33, \, p = 10^5 \text{ Pa}, \, \rho = 1000 \text{ kg/m}^3, \, n = 400. \]

**Figure 5.** The shape of the bubble at six time moments (a) and the pressure field in the liquid surrounding the bubble at the moment \( t^* \) (b).

Figure 5 (a) shows the change in the shape of the bubble from its inception (moment 1) until the transformation into a torus (moment 6). The maximum growth is reached at moment 2 \( (t^* = 1.104) \). At moment 6 \( (t^* = 2.138) \) the cumulative jet impacts on the bubble surface side nearest to the wall. One can see that at the moment of maximum bubble size the lower part of the bubble surface is flattened, which predetermines the bubble shape at the moment of the jet impact. The jet velocity at the moment \( t^* \) is equal to 7.22. Figure 5 (b) shows the liquid pressure field near the jet end at the moment \( t^* = 2.138 \). Maximum pressure \( p^* = 3.8 \) is observed at the bottom of the jet.

**Figure 6.** The bubble contours at six time moments at the beginning of the toroidal phase of the bubble dynamics (a), the liquid pressure field near the jet end at \( t^* = 1.6 \cdot 10^{-5} \) (b), the radial pressure profiles on the wall \( z^* = 0 \) (c) and at the level \( z^* = 0.012 \) (d) at four time moments.

Figure 6 (a) shows the change of the bubble contour in the vicinity of the jet end at the beginning of the toroidal phase of the bubble dynamics. Figure 6 (b) presents the liquid pressure field near the jet end at a moment at the beginning of the toroidal phase. The maximum pressure in the liquid \( p^* = 1100 \) is observed on the periphery of the circular area of contact of the jet end with the thin liquid layer be-
tween the bubble and the wall. The maximum pressure on the wall equal to 660 is located at a point of the axis of symmetry. The radial pressure profiles on the wall $z^* = 0$ and at the level $z^* = 0.012$ are shown in figures 6 (c), (d) for a number of time moments. One can see that with time the pressure on the wall as well as at the level $z^* = 0.012$ significantly decreases. The liquid layer between the bubble and the wall weakens the jet action on the wall.

Figure 7. The shapes of the bubble (a), the pressure fields in the liquid surrounding the bubble (b) and the radial pressure profiles on the wall at three time moments in the toroidal phase of the bubble dynamics (c).

Further dynamics of the toroidal bubble is demonstrated in figure 7 where the shapes of the bubble (a), the pressure fields in the liquid surrounding the bubble (b) and the pressure profiles on the wall (c) are shown for three time moments. One can see that as the process of the bubble collapse progresses a splash arises on the bubble surface near the wall. At that, the pressure on the wall decreases everywhere except the neighborhood of the splash where it slightly grows.

5. Conclusion
A numerical technique for investigating the dynamics of a cavitation bubble near a rigid wall in an inviscid and incompressible liquid is presented. The technique is based on the boundary element method and consists of two stages. The first stage is devoted to calculating the bubble dynamics until the impact of the cumulative liquid jet arising on the surface of the bubble at its collapse on the bubble surface side closest to the wall. In the second stage the dynamics of the toroidal bubble resulting from that impact is evaluated. The numerical instability is eliminated by smoothing the functions defining the bubble surface and its liquid velocity potential by a smoothing procedure using a cubic spline. The numerical convergence of the technique with increasing the spatial and temporal discretization is shown. A comparison with numerical and experimental results of other authors shows satisfactory
agreement. The problem of growth and collapse of an initially spherical bubble near a wall is com-puted. The corresponding pressure fields in the liquid surrounding the bubble and the radial pressure profiles on the wall are presented.

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