Time Resolved Phase Space Tomography of an Optomechanical Cavity

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We experimentally study the phase space distribution (PSD) of a mechanical resonator that is simultaneously coupled to two electromagnetic cavities. The first one, operating in the microwave band, is employed for inducing either cooling or self-excited oscillation, whereas the second one, operating in the optical band, is used for displacement detection. A tomography technique is employed for extracting the PSD from the signal reflected by the optical cavity. Measurements of PSD are performed in steady state near the threshold of self-excited oscillation while sweeping the microwave cavity detuning. In addition, we monitor the time evolution of the transitions from an optomechanically cooled state to a state of self excited oscillation. This transition is induced by abruptly switching the microwave driving frequency from the red-detuned region to the blue-detuned one. The experimental results are compared with theoretical predictions that are obtained by solving the Fokker-Planck equation. The feasibility of generating quantum superposition states in the system under study is briefly discussed.

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Optomechanical cavities are currently a subject of intense basic and applied study [1, 10]. These devices can be employed in various sensing [11, 16] and photonic applications [17, 23]. Moreover, optomechanical cavities may allow experimental study of the crossover from classical to quantum mechanics [3, 24, 50] and the observation of macroscopic quantum behavior in mechanical systems [28, 30, 32, 35, 37, 38]. When the finesse of the optical cavity that is employed for constructing the optomechanical cavity is sufficiently high, the coupling to the mechanical resonator that serves as a vibrating mirror is typically dominated by the effect of radiation pressure [7, 54, 61]. On the other hand, bolometric effects can contribute to the optomechanical coupling when optical absorption by the vibrating mirror [62, 63] is significant [11, 64, 74]. In recent years a variety of cavity optomechanical systems have been constructed and studied [3, 5, 7, 10, 28, 36, 44, 49, 52, 61, 75, 81], and phenomena such as mode cooling [49, 76, 81, 83], self-excited oscillation [5, 64, 70, 74, 80, 84, 87] and optically induced transparency [88, 91] have been investigated.

In this work we experimentally study a hybrid system made of a single mechanical resonator and two cavities, one operating in the microwave band and the other in the optical band. We study self-excited oscillation induced by driving the microwave cavity in the blue-detuned region. The technique of state tomography is employed in order to construct the phase space distribution (PSD) of the mechanical resonator [92], whose displacement is detected using the optical cavity. We begin by mapping out the PSD in steady state near the threshold of self-excited oscillation. We then employ time resolved measurements of PSD for both, studying the drifting in time of the phase of self-excited oscillation, and for monitoring the transition from cooling to self-excited oscillation. In the latter case, the frequency of the driving signal that is injected into the microwave cavity is abruptly switched from the red-detuned region to the blue-detuned one, allowing thus the recording of the time evolution from an initial state, in which the mechanical resonator is cooled down, to a final state, in which the system undergoes self-excited oscillation. The possibility of employing such abrupt switching for the creation of quantum superposition states is briefly discussed.

The experimental setup is schematically depicted in Fig. 1. A photo-lithography process is used to pattern a microwave microstrip cavity made of 200 nm thick aluminum on a high resistivity silicon wafer. At the open end of the microstrip, a mechanical resonator in the shape of a 100 × 100 µm² trampoline supported by four beams is fabricated [74]. At the other end, the microwave cavity is weakly coupled to a feedline, which guides both the injected and reflected microwave signals. Details of the fabrication process can be found elsewhere [93].

The sample is mounted inside a closed Cu package, which is internally coated with Nb. Measurements are performed in a dilution refrigerator operating at a temperature of 0.54 K and in vacuum. The injected microwave signal is first attenuated by a 20 dB directional coupler at the same temperature before entering the feedline. The reflected microwave signal is amplified using a cryogenic amplifier at 4 K, and measured in both the frequency domain (using a network analyzer and a spectrum analyzer) and in the time domain (using a power diode connected to an oscilloscope). The fundamental microwave cavity resonance frequency is \( f_c = \omega_c / 2\pi = 2.783 \text{GHz} \), the corresponding linear damping rate is \( \gamma_c = 4.2 \text{MHz} \), the fundamental mechanical resonance frequency is \( f_m = \omega_m / 2\pi = 662.7 \text{kHz} \) and the corresponding mechanical linear damping rate is \( \gamma_m = 2.5 \text{Hz} \).

A single mode optical fiber coated with Nb is placed above the suspended trampoline (see Fig. 1). In the presence of the coated fiber, two optomechanical cavities are formed, one is a superconducting cavity operating in the microwave band and the other is a fiber-based cavity.
operating in the optical band. The coupling between the mechanical resonator and the microwave cavity, originating by the capacitance between the coated fiber and the suspended trampoline \[92\], is dominated by the effect of radiation pressure, whereas bolometric effects are responsible for the coupling between the mechanical resonator and the optical cavity. The fact that both optomechanical cavities share the same mechanical resonator can be exploited for inducing mechanically mediated coupling between microwave and optical photons \[94,102\].

A fiber Bragg grating (FBG) mirror and a focusing lens are made near the fiber tip. To allow optical transmission, the core of the fiber at the tip is exposed by etching the Nb coating using focused ion beam (FIB) \[93\]. A cryogenic piezoelectric 3-axis positioning system having sub-nanometer resolution is employed for manipulating the position of the optical fiber. The reflected signal off the optical cavity, which is measured both by a spectrum analyzer and by an oscilloscope, is employed for displacement detection. A tunable laser operating near the Bragg wavelength \[\lambda_B = 1545.8 \text{ nm}\] of the FBG together with an external attenuator are employed to excite the optical cavity. In order to avoid back-reaction effects originating by bolometric effects \[103\], the input laser power feeding the optical cavity is kept at sufficiently low level (below \[60 \mu \text{W}\]), allowing the utilization of the optical cavity for displacement detection without any significant effect on the dynamics of other parts of the system.

Self-excited oscillation can be induced by injecting a monochromatic pump signal into the feedline of the microwave cavity provided that the angular frequency of the pump signal \[\omega_p\] is blue-detuned with respect to the angular cavity resonance frequency \[\omega_c\], i.e., provided that \[\omega_p > \omega_c\], and provided that the injected power \[P_p\] exceeds a threshold value. Panel (b) of Fig. 1 shows time traces of the off reflected signals of both, the optical and the microwave cavities, in the region where the system exhibits self-excited oscillation induced by a microwave monochromatic pump tone having normalized detuning \[d = (\omega_p - \omega_c) / \omega_m = 0.7\] and power \[P_p = 0.63 \mu \text{W}\]. The off reflected optical signal is measured using a photodetector, and the off reflected microwave signal is measured using a power diode. The relative phase between the two oscillating signals allows the direct measurement of the retardation \[104\] in the response of the microwave cavity to mechanical oscillation (note that the retardation in the response of the optical cavity is negligibly small).

The PSD provides an important insight on the effect of noise on the dynamics of the system under study in both the classical and quantum regimes. Armour and Rodrigues \[105\] have calculated the Wigner quasi-probability PSD for an optomechanical cavity for the case where the coupling between the mechanical resonator and the cavity is dominated by the effect of radiation pressure. They found that in the semiclassical approximation (in which third-order derivative terms in the equation of motion for the Wigner function are neglected) the master equation leads to Langevin equations corresponding to the classical dynamics of the system. Furthermore, the assumption that the dynamics of the mechanical resonator is slow on the time-scale of the cavity dynamics allows employing the technique of adiabatic elimination in order to derive an evolution equation for the complex amplitude \[A\] of the mechanical resonator, which is found...
where both the effective angular resonance frequency detuning $\Omega_{\text{eff}}$ and the effective damping rate $\Gamma_{\text{eff}}$ are real even functions of $A_x \equiv |A|$. To second order in $A$, they are expressed as $\Omega_{\text{eff}} = \omega_0 + \omega_2 A^2$ and $\Gamma_{\text{eff}} = \gamma_0 + \gamma_2 A^2$. The fluctuating term $\langle 106 \rangle \xi (t) = \xi_x (t) + i \xi_y (t)$, where both $\xi_x$ and $\xi_y$ are real, represents white noise and the following is assumed to hold: $\langle \xi_x (t) \xi_x (t') \rangle = 2 \delta (t - t')$ and $\langle \xi_x (t) \xi_y (t') \rangle = 0$, where $\Theta = \gamma m k_B T_{\text{eff}}/4 \hbar \omega_m$. $k_B$ is the Boltzmann’s constant and $T_{\text{eff}}$ is the effective noise temperature.

In the absence of optomechanical coupling $\Omega_{\text{eff}}$ and $\Gamma_{\text{eff}}$ respectively represent the angular resonance frequency detuning (with respect to $\omega_m$) and the damping rate of the isolated mechanical resonator, i.e. $\Omega_{\text{eff}} \approx \omega_0$ and $\Gamma_{\text{eff}} = \gamma_0 = \gamma_m$. However, with a finite optomechanical coupling both $\Omega_{\text{eff}}$ and $\Gamma_{\text{eff}}$ are modified when the cavity is externally driven. Consider the case where a monochromatic tone having normalized amplitude $a_p$ and angular frequency $\omega_p$ is injected into the feedline in order to drive the microwave cavity. For this case the effective linear angular frequency detuning $\omega_0$ and the effective linear damping rate $\gamma_0$ become $\omega_0 = 2G^2 E c \omega_m^3 \Im \Xi_1$ and $\gamma_0 = \gamma_m + 2G^2 E c \omega_m^{-1} \Re \Xi_1 (d, g)$, respectively, where $\Xi_1 (d, g) = [-i(d + l) + g]^{-1} + [-i(d - l) - g]^{-1}$, $g = \gamma c/\omega_m$ is the normalized cavity damping rate, and $E_c$ is the average number of photons in the cavity in steady state, which is related to $a_p$ by $E_c = |a_p|^2 \omega_m^{-2} (d^2 + g^2)^{-1}$ when optomechanical coupling is disregarded $\langle 106 \rangle \langle 107 \rangle$. The optomechanical coupling constant $G$ represents the shift in the effective cavity angular resonance frequency induced by displacing the mechanical resonator by its zero point amplitude.

In the current study we focus on the region close to the threshold of self-excited oscillation, for which the effect of resonance frequency detuning $\Omega_{\text{eff}}$ can be disregarded. For such a case the Langevin equation $\langle 11 \rangle$ for the complex amplitude $A$ can be expressed in a two-dimensional vector form as $\dot{A} = \nabla H - \xi$ where $A = (A_x, A_y)$, $A_x$ and $A_y$ are the real and imaginary parts of $A$, respectively, the noise term is $\xi = (\xi_x, \xi_y)$, and the scalar function $H$ is given by $H = (\gamma_0/2) A_x^2 + (\gamma_2/2) A_y^4 \langle 108 \rangle \langle 109 \rangle$. The corresponding Fokker-Planck equation for the PSD $\mathcal{P}$ can be written as $\langle 106 \rangle$

$$\frac{\partial \mathcal{P}}{\partial t} - \nabla \cdot (\mathcal{P} \nabla H) - \Theta \nabla \cdot (\nabla \mathcal{P}) = 0 \ . \ (2)$$

Consider the case where $\gamma_2 > 0$, for which a supercritical Hopf bifurcation occurs when the linear damping coefficient $\gamma_0$ vanishes. Above threshold, i.e. when $\gamma_0$ becomes negative the amplitude $A_e$ of self-excited oscillations is given by $A_e0 = \sqrt{-\gamma_0/\gamma_2}$. The Fokker-Planck equation $\langle 2 \rangle$ can be used to evaluate the normalized PSD function in steady state $W(A)$, which is found to be given by $W = Z^{-1} e^{-H/\Theta}$, where $Z$ is a normalization constant (partition function) $\langle 106 \rangle \langle 109 \rangle \langle 110 \rangle$. Note that $W$ is independent on the angle $A_0$ of the complex variable $A$.

Panel (a) of Fig. 2 exhibits the measured PSD in steady state as a function of the normalized detuning $d$. The measured PSD seen in panel (a) is extracted using the technique of state tomography. The calculated PSD in panel (b) is obtained from the steady state solution of Eq. (2). The following device parameters have been employed in the calculation: $G = 0.013$ Hz (corresponding to frequency shift per displacement of 55 MHz $\mu$m$^{-1}$) and $\gamma_2 d_m^2 \approx 2.0 \times 10^{-4}$. The rate of nonlinear damping $\gamma_2$ is taken to be independent on the cavity driving parameters since the optomechanical contribution to $\gamma_2$ is found to be negligible small.

FIG. 2: (color online) PSD in steady state as a function of the normalized detuning $d$. The measured PSD seen in panel (a) is extracted using the technique of state tomography. The calculated PSD in panel (b) is obtained from the steady state solution of Eq. (2). The following device parameters have been employed in the calculation: $G = 0.013$ Hz (corresponding to frequency shift per displacement of 55 MHz $\mu$m$^{-1}$) and $\gamma_2 d_m^2 \approx 2.0 \times 10^{-4}$. The rate of nonlinear damping $\gamma_2$ is taken to be independent on the cavity driving parameters since the optomechanical contribution to $\gamma_2$ is found to be negligible small.

The phase of self-excited oscillation in steady state ran-
FIG. 3: (color online) Measured (left) and calculated (right) PSD in steady state self-excited oscillation. The normalized dwell time $\gamma_m t_d$ is 0.12, 1.2, 7.5 and 12 for the panels labeled by the letters a, b, c, and d, respectively.

domly drifts in time due to the effect of external noise. In addition, noise gives rise to amplitude fluctuations around the average value $A_{r0}$. To experimentally study these effects self-excited oscillation is driven by injecting a monochromatic pump tone having normalized detuning $d = 0.7$ and power $P_p = 0.77 \mu W$ into the feedline. The off-reflected signal from the optical cavity is recorded in two time windows separated by a dwell time $t_d$. While the data taken in the first time window is used to determine the initial amplitude and phase of self-excited oscillation, the data taken in the second one is used to extract PSD using tomography. The results are seen in Fig. 3 for 4 different values of the dwell time $t_d$. While the left panels show the measured PSDs, the panels on the right exhibit the calculated PSDs obtained by numerically integrating the Fokker-Planck equation (2). For the longest dwell time the phase of self-excited oscillation becomes nearly random, and consequently the PSD becomes nearly independent on the angle $A_\theta$.

While cavity excitation in the measurements of PSD in steady state is performed using a single microwave synthesizer, the time-resolved experiment, in which the transient from cooling to heating is recorded, is performed using two synthesizers. The first one, having negative normalized detuning $d_1 = -0.475$, serves during the cooling stage, whereas the second one, having the opposite normalized detuning $d_2 = -d_1$, is employed for driving self-excited oscillation. Both synthesizers are connected to an RF switch through two $-26$ dB directional couplers. The attenuated signal from the couplers is connected to the input ports of a microwave switch, while the through signal from the two synthesizers is mixed using an RF mixer. The IF port of the mixer is used for triggering a pulse generator, which, in turn, triggers the RF switch. Both the time delay between the two triggers and the power levels of both synthesizers are carefully tuned in order to achieve a smooth transition from cooling to heating. This is done by monitoring the off-reflected signal shortly after switching and by minimizing signal ringing. A smooth transition is achieved when both amplitude and phase of cavity fixed point before switching coincide with the amplitude and phase corresponding to the cavity fixed point after switching (which is unstable). Note that the relative phase is a periodic function of the time delay with a period given by the inverse frequency difference between the two synthesizers $\pi/d_2 \omega_m = 1 \mu s$. The power level of both synthesizers is set close to 1.0 $\mu W$ (with fine adjustments to eliminate ringing).

Fig. 4 shows the PSD as a function of the normalized elapsing time $\gamma_m t$ since the switching from cooling to heating. Panel (a) exhibits the measured PSD whereas the calculated PSD, which is obtained by numerically integrating the Fokker-Planck equation (2), is seen in panel (b).
Analytical approximation to the time evolution of \( P \) that is generated by the Fokker-Planck equation (2) can be obtained for the case of short delay times \( t > 0 \), for which \( A_r \) is sufficiently small to allow disregarding the effect of nonlinear damping. For this case, \( P \) is found to be a Gaussian distribution given by \( P = \pi^{−1} \delta_H^{−2} \exp \left(−A_r^2/\delta_H^2\right) \), with exponentially growing width \( \delta_H \) given by

\[
\delta_H = \sqrt{\frac{2\Theta}{\gamma_b + \gamma_m} \left(1 + \frac{2\gamma_b}{\gamma_b - \gamma_m} (e^{2(\gamma_b - \gamma_m) t} - 1)\right)},
\]

where \( \gamma_{ba} = 2G^2\omega\omega_0^{-1}\text{Re} \Xi_1(d_1, g) \) is the back-action contribution to the mechanical damping rate.

The process of PSD expansion, which is triggered by switching cavity detuning from cooling to heating, can be employed under appropriate conditions for the generation of a quantum superposition state \( |\psi\rangle = |0\rangle + |1\rangle \). Increasing the value of \( \gamma_{ba} \) by increasing the power injected into the microwave cavity allows both, enhancing cooling efficiency before switching, and accelerating PSD expansion after switching [see Eq. (3)]. This latter is highly desirable since experimental observation of a superposition state is possible only if the time needed to generate the state is shorter than the corresponding decoherence time. On the other hand, while nonlinearity in the response of the microwave cavity has been disregarded in the data analysis presented above, such effects may play an important role at higher power levels \( \|93\|113\|117\). Taking full advantage of nonlinearity for enhancing the efficiency of cooling \( 112 \) and for the suppression of decoherence \( 107 \) may open the way for experimental exploration of non-classicality at a macroscopic scale in such systems.

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