SINGLE IMAGE SUPER-RESOLUTION OF NOISY 3D DENTAL CT IMAGES USING TUCKER DECOMPOSITION

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ABSTRACT

Tensor decomposition has proven to be a strong tool in various 3D image processing tasks such as denoising and super-resolution. In this context, we recently proposed a computationally effective, canonical polyadic decomposition (CPD) based algorithm for single image super-resolution. In this work, we investigated the added value brought by Tucker decomposition. While CPD allows a joint implementation of the denoising and deconvolution steps, with Tucker decomposition the denoising is followed by the deconvolution. This way the ill-posedness of the deconvolution caused by noise is partially mitigated. The results achieved using the two different tensor decomposition techniques were compared, and the robustness against noise was investigated. For validation, dental images were used. The superiority of the proposed method is shown in terms of peak signal-to-ratio, structural similarity index, the root canal segmentation accuracy, and runtime.

Index Terms— 3D single image super-resolution, tensor factorization, Tucker decomposition, dental CT

1. INTRODUCTION

Single image super-resolution (SISR) techniques aim to improve the observed image without further measurements (e.g. radiation), offering a safe and cost-effective reconstruction method. In dentistry, the position and structure of the tooth canal is determined using cone-beam computed tomography (CBCT), and it is outstandingly important in case of routine root canal treatments, where the current success rate of 60-85% [1] could be improved with SISR.

The SISR problem assumes an image degradation model where the high-resolution (HR) image is convolved by a blurring kernel, downsampled, and corrupted by additive noise, resulting in the low-resolution (LR) observed image. The size of 3D images is further complicating the computations. In our previous work [2] an SISR technique (TF-SISR) using canonical polyadic decomposition (CPD) was proposed. In that work promising results were obtained for high SNR. In this paper an SISR method is proposed based on the Tucker tensor-decomposition (TD), already used in multi- and hyperspectral imaging [3]. The proposed algorithm, denoted by TD-SISR, is shown to be more accurate and more robust to noise than the previous TF-SISR method.

First, tensor operations will be presented. Section 3 summarizes TF-SISR and presents the TD-SISR. Section 4 compares the results for simulated and real CBCT data. Section 5 concludes this work and presents possible future directions.

2. TENSOR OPERATIONS

A 3D image can be defined as a third-order tensor $X \in \mathbb{R}^{I \times J \times K}$. Its mode-n fibers are the analogues of columns and rows ($X(:,j,k)$, $X(i,:,k)$ and $X(i,j,:)$). $X$ can be multiplied by a 2D matrix along each of its n dimensions ($P_1 \in \mathbb{R}^{I' \times I}$, $P_2 \in \mathbb{R}^{J' \times J}$, $P_3 \in \mathbb{R}^{K' \times K}$, $I', J', K' \in \mathbb{Z}$ respectively), called the mode-n products ($\odot$) [2]

$$ T = X \otimes P_1 \otimes P_2 \otimes P_3 = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} X(i,j,k) P_1(i,:) \otimes P_2(:,j) \otimes P_3(:,k), $$

where $T \in \mathbb{R}^{I' \times J' \times K'}$, and $\otimes$ denotes the outer product, as $[P_1(:,i) \otimes P_2(:,j) \otimes P_3(:,k)]_{i,m,n} = P_1(l,i) P_2(m,j) P_3(n,k)$.

The tensor unfolding flattens a tensor into a 2D matrix. It can happen along any of the modes; the mode-n fibers are columns of the matrix in lexicographical order.

2.1. Canonical Polyadic Decomposition

The CPD [4] used in our previous work [2] factorizes the tensor $X$ as a sum of $R$ rank-1 tensors (outer product of three 1D arrays, here $U^n(:,r)|n=1,2,3$)

$$ X \approx \sum_{r=1}^{R} U^1(:,r) \otimes U^2(:,r) \otimes U^3(:,r), $$

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where \( \mathcal{U} = \{ U^1 \in \mathbb{R}^{I \times R}, U^2 \in \mathbb{R}^{I \times R}, U^3 \in \mathbb{R}^{K \times R} \} \) is a set of three 2D matrices, known as the CPD of the tensor \( \mathbf{X} \). The minimal number of such rank-1 tensors that can exactly compose the original tensor is called the rank of the tensor, \( R \). Finding the tensor rank is NP-hard, but using it a unique decomposition can be achieved under mild conditions [5]. The shorthand of this decomposition is \( \mathbf{X} = [U^1, U^2, U^3] \).

2.2. Tucker Decomposition

The n-rank of a tensor (in 3D \{1-rank, 2-rank, 3-rank\}) is the number of its independent mode-n fibers. TD (also called higher order SVD, or multi-linear SVD) is the multidimensional generalization of the 2D SVD, written as

\[
\mathbf{X} \approx \Sigma \times_1 V_1 \times_2 V_2 \times_3 V_3,
\]

where \( \Sigma \) is a core tensor, and \( V_1 \in \mathbb{R}^{I \times R_1}, V_2 \in \mathbb{R}^{J \times R_2}, V_3 \in \mathbb{R}^{K \times R_3} \) are the orthonormal bases of the subspaces spanned by the mode-n fibers [4]. If \( R_1, R_2, R_3 \) equal the n-ranks of \( \mathbf{X} \), the decomposition is unique, but for lower numbers and in the presence of noise it becomes inexact. \( \Sigma \) is not diagonal, but shows the level of interaction between the different modes. The shorthand of TD is \( \mathbf{X} = \{ \Sigma; V_1, V_2, V_3 \} \).

3. METHODS

3.1. Problem formulation

The SISR image degradation model considered herein assumes that the HR image \( \mathbf{X} \) is blurred by a separable (usually Gaussian) kernel \( \mathbf{H} \), is downsampled by rate \( r \) \( \{ \downarrow_r \} \), and corrupted by additive white Gaussian noise \( \mathbf{N} \). The output LR image \( \mathbf{Y} \in \mathbb{R}^{I/r \times J/r \times K/r} \) is given by

\[
\mathbf{Y} = \downarrow_r \{ \mathbf{H} \ast \mathbf{X} \} + \mathbf{N}.
\]

In this work the blurring kernel is assumed to be known, and is estimated for the current application from the LR image as explained in [6].

3.2. Previous TF-SISR using CPD

In our previous work [2] the denoising, deconvolution and up-sampling of \( \mathbf{Y} \) were implemented jointly, separated for the 3 dimensions (with \( H_1, H_2, H_3 \) block-circulant matrices with circulant blocks, defined from \( \mathbf{H} \), and \( D_1, D_2, D_3 \), \( r \)-fold downsampling operators)

\[
\begin{align*}
U^1 &= (D_1H_1)^\dagger \mathbf{Y}^{(1)}(D_1H_3U^3 \circ D_2H_2U^2)^T \\
U^2 &= (D_2H_2)^\dagger \mathbf{Y}^{(2)}(D_2H_3U^3 \circ D_1H_1U^1)^T \\
U^3 &= (D_3H_3)^\dagger \mathbf{Y}^{(3)}(D_2H_2U^2 \circ D_1H_1U^1)^T,
\end{align*}
\]

as derived in [2]. These steps are repeated until \( \mathbf{U} \) converges. Denoising is realized by choosing a small enough \( R \). The symbol \( \dagger \) denotes the Moore-Penrose pseudoinverse with Tikhonov regularization.

3.3. Proposed TD-SISR

The idea of the method \(^1\) proposed herein is to denoise the image before deconvolution, in order to stabilize this ill-posed operation, as earlier suggested in the literature [7].

As explained in Section 2.2, the singular values of each mode \( (SV_1, SV_2, SV_3) \) can be calculated from \( \Sigma \). Similarly to the 2D case, by picking the relevant components having a singular value higher than a threshold \( R_n \), a denoised version of \( \mathbf{Y}, \hat{\mathbf{Y}} \) may be achieved [8].

\[
\hat{\mathbf{Y}} = \mathbf{V}_n \times_1 \mathbf{V}_1 \times_2 \mathbf{V}_2 \times_3 \mathbf{V}_3,
\]

where

\[
\mathbf{V}_n = \{ V_n (\cdot ; i) | SV_n(i) \geq R_n \}
\]

Unlike in the 2D case, this truncated approximation might not be optimal in the least squares sense, but gives a reasonable estimate with bounded error [4, 9].

After obtaining the denoised image, the deconvolution is realized using a Tikhonov-regularized deconvolution separated for the three modes.

\[
\hat{\mathbf{X}} = \hat{\mathbf{Y}} \times_1 (D_1H_1)^\dagger \times_2 (D_2H_2)^\dagger \times_3 (D_3H_3)^\dagger. \tag{8}
\]

4. RESULTS

4.1. Datasets and metrics

HR dental images were acquired using a QuantumFX micro-CT system (Perkin Elmer, resolution 10 LP/mm at 50% MTF). In simulation the HR image (lower premolar, 280×268×492 pixels) was blurred (Gaussian kernel, with standard deviations \( \sigma_1 = \sigma_2 = \sigma_3 = 8 \)), downsampled (at \( r = 2 \)) and Gaussian noise was added at different SNR levels. For real data experiments (an upper molar, 324×248×442 pixels) the LR images were CBCT images, acquired with a Carestream 81003D system (resolution 1 LP/mm at 50% MTF). See all sample images in Fig. 1. The Gaussian blurring kernel \( (\sigma_1 = 8.2, \sigma_2 = 7.5, \sigma_3 = 1.3) \) was estimated from the CBCT data [6]. To further explore the robustness of the SISR algorithms, Gaussian noise corresponding to different SNRs was artificially added to the LR data.

The improvement of the LR images was measured through their peak signal-to-noise ratio (PSNR) and structural similarity index (SSI), against the micro-CT HR images (considered ground truth due to their high spatial resolution and SNR.

\(^1\)The Matlab codes associated with the methods are available at www.irit.fr/~Adrian.Basarab/img/Demo_TensorSR.zip
in this ex vivo setting, but are unavailable in vivo because of the excessive radiation dose). The metrics were calculated excluding the background, with values scaled to $(0, 1)$.

The efficiency of the algorithms was also measured on the segmented root canals [10], calculating their volume and Dice coefficient against the segmented HR micro-CT volumes. The algorithms were implemented in Matlab 2019, and for basic tensor operations Tensorlab [11] was used.

### 4.2. Simulation results

For TF-SISR $R = 500$ was chosen, and in both methods $\epsilon = 1$ was set for Tikhonov-regularization following [2]. As it can be seen in Fig. 2, the singular values decay rapidly (mind the logarithmic scale). For TD-SISR $R_1 = R_2 = R_3 = 40$ (SV values under 1) were generally sufficient for all noise levels based on the calculated metrics.

Table 1 shows the quantitative results of the SISR methods. The PSNR is improved for each case and both methods, compared to the simulated LR image. TD-SISR gave better results, except for the extremely noisy, 20 dB case. The SSI gave similar results, with no improvement in the 20 dB case. For illustration purpose, the segmentation was carried out at 25 dB. The improvement is confirmed by the Dice coefficients, showing the superiority of the TD-SISR method.

### 4.3. Real data results

For the real data, TF-SISR was used with the same settings as in simulation, and for TD-SISR $R_1, R_2, R_3 = 50$ was set after plotting the SVs (Fig. 3).

The metrics have shown milder improvements compared to the simulation, but both PSNR and SSI improved in all cases with both methods, and TD-SISR gave superior results. The volume of the segmented 25 dB images also improved regarding the Dice coefficient.

### 5. CONCLUSION

In contrast to the earlier TF-SISR method no iterations are applicable in TD-SISR. In TD-SISR 3 thresholds $R_1, R_2, R_3$ have to be defined for the three modes, while in TF-SISR only
one parameter, $R$ influences the denoising step. However, the singular values of TD-SISR correspond to the importance of the components, while $R$ in TF-SISR bears no such meaning. This makes the setting of TD-SISR parameters easier, and its efficiency is validated by the qualitative and quantitative results. Images of $280 \times 268 \times 492$ and $324 \times 248 \times 442$ pixels were super-resolved under 2 s with standard Matlab implementation. The runtime of TD-SISR is lower because of the lack of iterations, but calculating the SVD for even larger volumes might be a bottleneck. In future work, the connection between the TF-SISR and TD-SISR parameters along with their robustness, and the general inverse problem including thresholding constraint will be investigated.

6. COMPLIANCE WITH ETHICAL STANDARDS

The teeth were donated anonymously for research and had been extracted for reasons unrelated to the current study.

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