MODELING J/ψ PROPERTIES AT FINITE TEMPERATURE

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Abstract

We model J/ψ using an effective lagrangian approach and calculate its spectral function in a gas of light mesons at finite temperature. We explore the hadronic landscape estimating cross sections for elastic and inelastic channels. Effects of form factors are tested in a general, but consistently gauge invariant manner. Relevance to heavy ion experiments is discussed.

1 Introduction

Experimental efforts at Brookhaven’s Relativistic Heavy Ion Collider will soon bring to fruition laboratory studies of nuclear systems heated and compressed to energy densities far exceeding that of the proton. Quantum chromodynamics (QCD), which is believed be the appropriate description in the subatomic domain, predicts that matter produced in this regime could have its subhadronic degrees of freedom liberated from hadronic boundaries and allowed to move about as a plasma of quarks and gluons\cite{1}. Identification of the quark-gluon plasma (QGP) concerns much of the current activities as models and interpretation seem to point toward a common conception regarding the nature of the transition or crossover from ordinary hadronic matter to QGP. However, many issues remain unsettled\cite{2} providing a wealth of opportunities in the vibrant and rapidly advancing research field of ultra-relativistic heavy ion physics.

Possibly the best available tool for studying nuclear systems in such states of excitation are electromagnetic probes of photons and lepton pairs. Each has
its kinematical advantages, but both share the property that once produced in
the system, undeflected flight to detectors ensues. Since virtual photons couple
directly to the neutral component of the vector hadronic current, dileptons
have been raised to a level of premier importance as they allow study of in-
medium properties of the vector mesons. In the low mass sector, effects of
the \( \rho \) meson clearly appear in proton-nucleus reactions whereas in heavy-ion
experiments a significant broadening of the “\( \rho \)” distribution is observed\[3\].
The importance of broadening effects through relatively high scattering rates
were highlighted several years ago in Ref. [4]. Since then sophisticated hadronic
models have been developed which seek to include in a consistent formalism
these and other effects into vector meson spectral functions\[5\]. The emerging
lore favors a \( \rho \) distribution in matter which is broadened essentially beyond
recognition\[6\].

In the higher mass sector, \( J/\psi \) appears as a promising tool for spectroscopy.
Within conventional hadronic scenarios it is expected to contribute a measur-
able muon-pair signal, whereas in QGP scenarios color screening effectively
inhibits \( c\bar{c} \) binding and limits \( J/\psi \) survival probabilities\[7\]. \( J/\psi \) yields are
therefore expected to be severely suppressed when plasma is produced. The
notion has recently gained a great deal of attention as \( J/\psi \) yields from Pb+Pb
reactions at 158 AGeV\[8\] show “anomalous” suppression compared simple ex-
trapolations from lighter projectiles’ results. Models of absorption on hadronic
comovers are able to provide interpretation for the lighter system\[3\], but not
consistently for the lead results. Very intriguing analyses involving QGP sce-
narios have been put forward\[10\] which are able to explain the yields. Mean-
while, purely hadronic approaches have been proposed too, but necessary in-
put of \( J/\psi \) scattering cross sections with light hadrons\[11\] are up to now quite
uncertain\[12\]. We report here on results of effective field theoretical methods
of estimating cross sections for \( J/\psi \) with light hadrons and we use them to
construct a spectral function for \( J/\psi \) in hot hadronic matter.

2 Effective Lagrangian

Hadronic interactions are here modeled with meson exchange, consequently a
symmetry embodying strangeness and charm is needed. We therefore begin
with \( SU(4) \) and introduce pseudoscalar \( (\phi = \varphi_a \lambda_a) \) and vector \( (V^\mu = v^\mu_a \lambda_a) \)
meson matrices, where \( \varphi_a \) and \( v^\mu_a \) are pseudoscalar and vector multiplets and
the \( \lambda_a \) are \( SU(4) \) generators. The large charm quark mass and its symmetry
breaking effects are duly noted, but since we use physical mass eigenstates
and we incorporate empirical constraints on the model where possible, we
expect reasonably reliable results as evidenced by the coupling constants’ near universality [13].

The free meson Lagrangian is written as

\[ \mathcal{L}_0 = \text{Tr}(\partial^\mu \phi^\dagger \partial_\mu \phi) - \text{Tr}\left((\partial_\mu V_\nu^\dagger) (\partial^\mu V^\nu - \partial^\nu V^\mu)\right) + \text{mass terms}. \]  

The pseudoscalar meson mass matrix that leads to properly normalized mass terms is

\[ \phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^+ & \bar{D}^0 \\ \frac{\pi^-}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\eta \sqrt{\frac{2}{3}} + \frac{\eta}{\sqrt{12}} & D_s^- \end{pmatrix}, \]  

while that for the vector multiplet is (suppressing the Lorentz index)

\[ V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \frac{\rho^-}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & -\omega \sqrt{\frac{2}{3}} + \frac{J/\psi}{\sqrt{12}} & D_s^{*-} \end{pmatrix}. \]

We then introduce interactions through a gauge covariant minimal substitution \( \partial_\mu f \rightarrow D_\mu f = \partial_\mu f + [A_\mu, f] \), where \( A_\mu = -ig/2 V_\mu \). As an aside remark, we note that the appearance of the \( g/2 \) instead of a mere \( g \) is nothing but a choice of convention. Since the model is calibrated to data, the choice is irrelevant. We must collect terms up to order \( g^2 \) for a consistently gauge invariant description for the hadronic currents. They are (since \( \phi^\dagger = \phi \) and \( V^\dagger = V \))

\[ \mathcal{L}_{\text{int}} = \frac{ig}{2} \text{Tr}(\phi V^\mu \partial_\mu \phi - \partial^\mu \phi V_\mu \phi) + \frac{1}{2} g^2 \text{Tr}(\phi V^\mu V_\nu \phi - \phi V^\mu \phi V_\nu) \]

\[ + \frac{ig}{2} \text{Tr}(\partial^\mu V^\nu [V_\mu, V_\nu] + [V^\mu, V^\nu] \partial_\mu V_\nu) + g^2 \text{Tr}(V^\mu V^\nu [V_\mu, V_\nu]). \]  

Carrying out the matrix algebra, we arrive at the totality of interaction terms allowed in the symmetry group.

The model is calibrated to observed hadronic decays, tuned to respect vector dominance, or as a last resort, to respect \( SU(4) \) symmetry. Details can be found in Ref. [13].
2.1 The $J/\psi + h$ cross section

Of particular importance are cross sections for a light hadron to knock apart the charmonium leaving $D$, $D^*$ or antiparticles when appropriate for conservation of relevant quantities. We present in Fig. 1 the results for the $\pi^-$, $K^-$, and $\rho$-induced dissociation. Purely elastic $J/\psi + h \to J/\psi + h$ are found to be of order femtobarns for pions, microbarns for rho and nanobarns for kaons, clearly too small for any significance.

![Figure 1: Dissociation cross sections for pions, kaons and rho mesons. Bands of uncertainties (not shown) are estimated to be $\sim 20$--50%.](image)

3 The $J/\psi$ spectral function

We next place $J/\psi$ in a strongly interacting medium and explore possible modifications in spectral properties as compared to the vacuum. From previous studies on vector mesons, one has seen that two-loop effects are much stronger than one-loop\cite{1, 14}. We have checked to see this trend continue for charmonium. We here adopt the approach to neglect one-loop effects and use free masses throughout.

We calculate the expected broadening of the $J/\psi$ distribution from collisions with light pseudoscalar and vector mesons in the hot hadronic matter. The extra width induced in the distribution by a reaction of type $J/\psi 2 \to 34$, where 2, 3, and 4 are arbitrary species is \cite{4, 15}.
\[ \Gamma(\omega, \vec{p}) = \frac{1}{2\omega} \int d\Omega n_2(E_2)(1 + n_3(E_3))(1 + n_4(E_4))|\mathcal{M}(J/\psi 2 \rightarrow 34)|^2, \quad (5) \]

where \( \omega = \sqrt{\vec{p}^2 + m_{J/\psi}^2} \), \( \vec{p} \) being the three-vector of the \( J/\psi \). Note that 3 or 4 can be a \( J/\psi \). In Eq. (5),

\[ d\Omega = d\bar{p}_2 d\bar{p}_3 d\bar{p}_4 (2\pi)^4 \delta(p + p_2 - p_3 - p_4), \quad (6) \]

where we have used shorthand notation

\[ d\bar{p}_i = \frac{d^3p_i}{(2\pi)^3 2E_i}. \quad (7) \]

A direct connection can be drawn between the rate in Eq. (5) and the eventual structure of the spectral function, utilizing at the intermediate stages such field theoretical concepts as the \( J/\psi \) propagator and imaginary part of the self-energy\[15\]. In an on-shell approximation, one has for the full spectral function

\[ A_{J/\psi}(\omega, \vec{p}) = \frac{2m_{J/\psi} \Gamma_{J/\psi}}{(p^2 - m_{J/\psi}^2)^2 + m_{J/\psi}^2 \Gamma_{J/\psi}^2}, \quad (8) \]

where \( \Gamma_{J/\psi} \) contains the vacuum width as well as contributions from elastic and inelastic collisions. Discussion of Eq. (8) in the context of more general formalism will be published elsewhere\[16\].

Now we consider \( J/\psi \) in a finite temperature gas consisting of \( \pi \)s, \( K \)s, \( \rho \)s and \( K^* \)s. The spectral function at 150 MeV is shown below in Fig. 2. Severe broadening of the spectral distribution with an accompanying suppression of the peak is immediately clear already for temperatures on the order of the pion mass. Studies of temperature dependence as well as specific channels’ contributions will also be included in Ref. [16].

4 Effects of form factors

Up to now pointlike hadrons have been considered. Effective lagrangian approaches are not considered complete until finite size effects are incorporated, typically accomplished with vertex form factors. Here we outline the procedure for implementation. Each \( t \)-channel Feynman graph is given a product of monopoles, one for each vertex, having the following structure

\[ h(t) = \left( \frac{\Lambda_1 + t_{max} - m_\alpha^2}{\Lambda_1 + t_{max} - t} \right) \cdot \left( \frac{\Lambda_2 + t_{max} - m_\alpha^2}{\Lambda_2 + t_{max} - t} \right), \quad (9) \]
where $\Lambda_1$ and $\Lambda_2$ are cutoff parameters depending on the species of on-shell particle, and $m_\alpha$ is the mass of the exchanged meson. Two cases will be examined: first we choose a fixed cutoff for all vertices of 2, 3 and 4 GeV, and second we allow each vertex to be given its own $\Lambda_i = m_i$, where $m_i$ is the mass of the on-shell charmed meson entering or leaving the vertex.

Exchange graphs, or $u$-channels, are given similar form-factor structure. Let us call the function $g(u)$. Finally, contact graphs are given general Lorentz structure with several expansion coefficients. The idea is then to determine the coefficients (in terms of Lorentz invariants and $h$ and $g$) which result in a manifestly gauge invariant amplitude for the most general case of functions $h$ and $g$. Due to space limitation, we do not include here the full expression. More complete details will be published elsewhere[16].

As an example of form-factor effects, we show in Fig. 3 the dissociation cross section for one of the pion channels ($K$ and $\rho$ cross sections are similarly affected). We are inclined to point to the results for $\Lambda_\alpha = m_\alpha$ as likely being the most consistent, although means of constraining the form factors are under investigation[16].
5 Conclusions

We have studied the spectral function for $J/\psi$ modified from its vacuum structure due to interactions with a gas of light mesons. Results suggest that the spectral function gets considerably modified due mostly to dissociation-type reactions. An environment much like the one we studied here is known to occur in ultrarelativistic heavy ion collisions. We stress that the approach we have taken is fully consistent in terms of conserving currents and respecting gauge invariance. Some issues will require further attention before we can take the next step of employing a spacetime model to ultimately compare with heavy ion data. Since the $SU(4)$ symmetry is known to be only approximately valid, some coupling strengths are not completely pinned down. Also, the final results will depend fairly sensitively on what we do with the form factors. Work on these and related issues is in progress [16].

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