Lagrangian-based investigation of gaseous jets injected into water by finite-time Lyapunov exponents

Jia-Ning Tang, Ning-Fei Wang

School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, China

Abstract

Gaseous jets injected into water are found in a variety of engineering applications, and the flow is essentially unsteady and turbulent. Additionally, the high water-to-gas density ratio can result in complicated flow structures; hence, it is important to investigate their flow structures to predict the dynamical behaviors effectively. Lagrangian coherent structures (LCS) defined by the ridges of the finite-time Lyapunov exponent (FTLE) is utilized in this study to elucidate the multiphase interactions in gaseous jets injected into water under the framework of the Navier-Stokes flow computations. The highlighted phenomena of the jet transportation can be observed by the LCS method, including expansion, bulge, necking/breaking, and back-attack. Besides, the observation of the LCS reveals that the back-attack stage is caused due to the injected gas has difficulties approaching the downstream region after the necking/breaking. The results indicate that the FTLE field has the potential to identify the structures of multiphase flows, and the LCS can capture the interface/barrier or the vortex/circulation region.

© 2011 Published by Elsevier Ltd. Open access under CC BY-NC-ND license. Selection and/or peer-review under responsibility of ICAE2011.

Keywords: Finite-Time Lyapunov Exponents; Lagrangian Coherent Structures; Multiphase flow; Gaseous jets injected into water

1. Introduction

When a gaseous jet is injected into water through a nozzle, the flow structure and process are essentially unsteady and turbulent due to the high water-to-gas density ratio and complicated shock wave structures [1]. This process can be found in a variety of engineering applications, such as direct-contact condensers, metallurgical processes, and underwater cutting/propulsion. As a result, it is important to investigate the flow structures of the gaseous jets injected into water to predict its dynamical behaviors effectively.

Recently, Haller and Yuan [2] have developed a trajectory-based approach from a Lagrangian...
perspective by considering the fluid as a dynamical system of fluid particles rather than a continuum. It is so-called Lagrangian Coherent Structures (LCS). A clearer introduction of the theory and computation details of the LCS has been established by Shadden et al. [3], enabling this method to become more useful to evaluate the flow structures in a wide range of scope. O’Farrell and Dabiri [4] have proposed a criterion for identifying vortex ring pinch-off in the axisymmetric jet flow based on the LCS. They have found out that the appearance of a new LCS is indicative of the initiation of the vortex pinch-off. Lekien and Leonard [5] have analyzed the high-frequency radar data collected in Monterey Bay by using the LCS, and a much clearer observation of the current structures along the coastlines can be observed. Shadden et al. [6] have measured the quasi-steadily propagating vortex rings generated by a mechanical piston-cylinder apparatus and the vortex ring wake of a free-swimming Aurelia aurita jellyfish [7]. Their researches have shown that the LCS is more effective in capturing the dynamical features of the flow, which can be missed in the traditional Eulerian analyses based on the velocity or vorticity fields. Tang et al. [8] have selected the LCS method to assess the three-dimensional unsteady wind field because this approach is inherently frame independent, and it ensures the physical objectivity in continuum mechanics.

Compared with other methods, the LCS method has several outstanding advantages. Firstly, it requires no velocity derivatives, which could cause additional noise in the study of a turbulent flow. Secondly, it is frame-independent and remains invariant with respect to rotation of the reference frame. Thirdly, it is robust to the anomalies of the velocity field [4]. Finally, it shows the true structures of trajectories and is easy interpretable even for unsteady vector fields. This study utilizes the FTLE and the LCS to get a better understanding of the flow dynamics and underlying physics of the gaseous jet injected into water.

2. Finite-time Lyapunov exponent and Lagrangian coherent structures

For a time-dependent velocity field $v(x, t)$, a trajectory $x(t; t_0, x_0)$ starting at point $x_0$ at time $t_0$ is defined as the solution of:

$$\dot{x}(t; t_0, x_0) = v[x(t; t_0, x_0), t]$$

(1)

with the initial conditions:

$$x(t_0; t_0, x_0) = x_0, \quad \dot{x}(t_0; t_0, x_0) = v_0$$

(2)

The trajectory is dependent on the initial position $x_0$, the initial time $t_0$, and the final time $t$.

Considering two points $x_0$ and $x_0 + \delta x_0$, each of which will generate a trajectory in the space. Use one of the trajectories as a reference, the divergence between the two trajectories can be written as a function of the time and the initial location with the form $\delta x(x_0, t)$. The mean exponential rate of separation of two close trajectories can be computed by using the following formula:

$$\sigma = \lim_{\|\delta x\| \to 0} \frac{1}{t} \ln \left| \frac{\delta x(x_0, t)}{\|\delta x\|} \right|$$

(3)

where $\sigma$ is the Lyapunov exponent. The Lyapunov exponent of a dynamical system is a quantity that characterizes the rate of separation of infinitesimally close trajectories. There are three possibilities of $\sigma$ [9]: $\sigma < 0$, the system attracts to a stable fixed point or a stable periodic orbit; $\sigma = 0$, the system is neutrally stable with a constant separation; $\sigma > 0$, the system is unstable. The neighborhood points will diverge to any arbitrary separation no matter how close they are initially.

In the finite time period $[t_0, t_0 + T_{LE}]$, the solution of the dynamical system given in Eq. (1) shows a flow map which takes points from their initial position $x_0$ at time $t_0$ to their new position, $x(t_0 + T_{LE}; t_0, x_0)$, after a time interval $T_{LE}$. The finite time version of the Cauchy-Green deformation tensor [3], $\Delta$, at the given point $x_0$ is defined as:
\[
\Delta^T_{t_0} (x_0) = \left( \frac{\partial x(t_0 + T_{LE}; t_0, x_0)}{\partial x_0} \right)^T \Delta x(t_0 + T_{LE}; t_0, x_0) \tag{4}
\]

where \((\cdot)^T\) is the transpose of the deformation gradient tensor. The maximum eigenvalue of \(\Delta^T_{t_0} (x_0)\) is defined as \(\lambda_{\text{max}}(\Delta^T_{t_0} (x_0))\). It represents the maximum stretching, and the corresponding eigenvector provides the direction and vector which \(\delta x_0\) will align to. Then the largest finite-time Lyapunov exponent with a finite integration time \(T_{LE}\) is defined as:

\[
\sigma^T_{t_0} (x_0) = \frac{1}{T_{LE}} \ln \left( \frac{\lambda_{\text{max}}(\Delta^T_{t_0} (x_0))}{\lambda_{\text{max}}(\Delta_{t_0} (x_0))} \right) \tag{5}
\]

The Finite-Time Lyapunov Exponent (FTLE) represents the maximum stretching rate for infinitesimal close particles. Ridges in the FTLE field are named as Lagrangian Coherent Structures (LCS) [3]. The theoretical foundation of FTLE has been established in detail by Haller and Yuan [2] and Shadden et al. [3].

3. Results and discussions

The schematic of the gaseous jets injected into water is shown in Fig. 1. The underlying data used to compute the FTLE is obtained from the time-dependent Navier-Stokes flow computations under the framework of the volume of fluid (VOF) model, the details of the computational setups are introduced in Ref. [1]. The process of gaseous jets injected into water consists of four stages [1]: expansion, bulge, necking/breaking, and back-attack. Each stage results in the deformation of the jet, which can be clearly captured by the LCS, as shown in Fig. 2-Fig. 8. The magnitude of the integration time \(T_{LE}\) used to compute the FTLE is \(4 \times 10^{-3}\) second (4000 numerical time steps), and the reference time scale, \(t_\infty\), is \(10^{-3}\) second. The normalized time scale, \(t^* = t / t_\infty\), is used from Fig. 2-Fig. 8.

Fig. 1. The schematic of the jet system

3.1. Expansion

When the gas jets enter the water initially, the pressure of the gas is not high enough to overcome the inertia effect of the water due to the large density ratio \((\rho_w / \rho_g \approx 800)\). Therefore, a “gas bag” enclosed by the surrounding water will form behind the nozzle exit [10] as shown in Fig. 2(a). The corresponding forward-time FTLE field at this moment, \(t^* = 6.8\), is presented in Fig. 2(b), which is normalized by the maximum FTLE value. The LCS can be observed as the ridges of the FTLE field, and the FTLE field itself corresponds well to the gaseous region, indicating we can use the LCS to identify the jet transportation in this case. In order to evaluate the relationship between the structures of LCS and vortex, the contour of the conventional vortex definition criterion, \(Q\), is shown in Fig. 2(c). \(Q\) is defined as \(Q=1/2( |\Omega|^2 - |S|^2)\) [11], where \(\Omega\) is the vorticity tensor, and \(S\) is the rate-of-strain tensor. Comparing Fig. 2(b) with (c), it is clear that the LCS indicates that there is a pair of vortex generating in the “gas bag”. The LCS in this region depicts the boundary of the vortex/circulation region. According to the Lagrangian criterion proposed by O’Farrell and Dabiri [4], the vorticity flux generated near the nozzle solid wall boundary, jet boundary, and shock wave will fuel the growth of the vortexes, enabling them to evolve.
3.2. Bulge

At the bulge stage, since a small bulged bubble appears several times near the nozzle exit [1], [10] as shown in Fig. 3(a), the gas oscillates intensively in the direction perpendicular to the flow and leads to the newly-generated LCS as shown in Fig. 3(b). The vortexes in Fig. 2 have already shed downstream from the original structure as shown in Fig. 3(c), resulting in the detachment of the LCS in Fig. 3(b). Besides, the strength of these shed vortexes become weaker in Fig. 3(c) compared with that in Fig. 2(c). This will weaken the strength of the detached LCS as shown in Fig. 3(b). At this stage, the FTLE field also has the ability to highlight the gas region.

3.3. Necking/breaking

Fig. 4 shows the liquid volume fraction and the FTLE contours of the necking stage ($t^*=23.4$), and Fig. 5 shows those of the breaking stage ($t^*=24$). At the necking stage, the gas jet will be compressed in the direction perpendicular to the jet center line, and so is the LCS as shown in Fig. 4(b). Comparing Fig. 4(a) with (b), we can observe that the LCS is not close to the gas-water interface. Besides, it highlights the main jet passage excluding the backward gas flow, meaning that the interfaces between the forward and backward flows are captured by the LCS as shown in Fig. 4(b). The gas inside the LCS is flowing forward, and the gas outside the LCS is flowing backward, which is illustrated in Fig. 6. This boundary is hard to be distinguished in the Eulerian method. Furthermore, after the gas breaks into two parts, the LCS is highly dense in the gas region attached to the nozzle exit and builds up a barrier obstructing the jet passage as shown in Fig. 5.
In order to get a better understanding for the development of necking/breaking stage, we seed three groups of particles in the flow field at the beginning of the necking stage when \( t^* = 23.4 \) as shown in Fig. 6(a). The blue group is located around the center line near the nozzle exit, the purple group is near the necking position outside the jet central region where the value of the FTLE is approximately zero, and the brown group is outside the interfaces of forward and backward gas flow which is denoted in Fig. 4(b). After a time interval when \( t^* = 24 \), some of the blue particles have already moved to the downstream region, and some are still locked in the region near the nozzle exit and inside the barrier created by the LCS as shown in Fig. 6(b). The purple particles located in the low FTLE region have slight movement to the jet center line. At the same time, the brown particles move upstream toward the necking position. It indicates the mechanism and source to narrow the main flow passage, and also proves that the LCS highlighted in Fig. 5(b) can be regarded as the interface between forward and backward gas flow.

3.4. Back-attack

After necking/breaking, a lot of gas inside the LCS barrier will assemble behind the nozzle exit and impact on the nozzle surface, which is called “back-attack” as shown in Fig. 7(a). Shi et al. [10] and Aoki et al. [12] also have observed the back-attack phenomenon occurs after the jet necking with the experimental method. From Fig. 7(b), it can be observed that the LCS at the back-attack stage is mainly located in the gas region attached to the nozzle, which is similar to the breaking stage. The FTLE field also corresponds to the gas region.
There are many explanations for the underlying physics of the back-attack phenomenon. Shi et al. [10] have suggested that the back-attack can be a shockwave feedback phenomenon, while Tang et al. [1] have described the back-attack is a backward flow generated due to the strong resistance effect after the necking/breaking process. To evaluate the development of the back-attack phenomenon, we refer to the particle tracking and the LCS technique. As shown in Fig. 8(a), the black particles are initially seeded in the flow field at the beginning of the back-attack stage ($t^* = 24$). After a time interval when $t^* = 26.4$, the particles are spread as shown in Fig. 8(b). An interesting phenomenon is that the particles initially located in the high FTLE region will move upstream to match the ridges of the FTLE field orderly. However, the particles with low FTLE value will flow straight toward the downstream region. Besides, the reversing particles keep moving to the nozzle part. After impacting on the nozzle surface, which will be illustrated in Figure 10, the particles will become more chaotic when $t^* = 30$ as shown in Fig. 8(c). It is clear that the back-attack phenomenon is caused by the backward flow at the region where particles has difficulties moving downstream.

Fig. 7. Liquid volume fraction contour and FTLE plot of the gaseous jets injected into water at back-attack process, $t^* = 26.4$

Fig. 8. Locations of the LCS and the Lagrangian tracers at three different times

(a) The beginning of the back-attack stage $t^* = 24$ (b) Back-attack stage $t^* = 26.4$ (c) The end of the back-attack stage $t^* = 30$

4. Conclusions

In this study, the application of the Finite-Time Lyapunov Exponent (FTLE) and Lagrangian Coherent Structures (LCS) for the gaseous jets injected into water is investigated. The main findings are summarized below:

(1) The FTLE field has the potential to identify the phase transportation in this case, indicating it is possible to track velocity field only to obtain the corresponding phase change process experimentally. The LCS can also describe the interfaces or barriers with the specific physical meaning. For example, the LCS
in gaseous jets injected into water can capture the interfaces between the forward and backward flows inside the gas region. It can be used to highlight the main jet core excluding the backward gas flow.

(2) The main flow characteristics in gaseous jet injected into water, including expansion, bulge, necking/breaking, and back-attack, are analyzed by the FTLE. For the expansion process, the gas injects into water and generates a pair of vortex in the gas bag region, which is captured by the LCS clearly. During the bulge process, a series of bulged bubbles cause a separation of the LCS, and so does the vortex shedding. Furthermore, at the necking/breaking stage, the jet and LCS will be compressed in the direction perpendicular to the jet center line. The LCS becomes the barrier to categorize the forward and backward flows, and will result in the back-attack stage later.

(3) Based on the LCS and particle tracking technique, it can be observed that the back-attack phenomenon in gaseous jets injected into water is caused due to the injected gas has difficulties approaching the downstream region after the necking/breaking stage. As a result, the injected gas in this region reverses to the nozzle and impacts on the nozzle surface.

References

[1] Tang J.-N., Tseng, C.-C., Wang, N.-F., Shyy, W.: Flow Structures of Gaseous Jets Injected into Water for Underwater Propulsion. 49th AIAA Aerospace Sciences Meeting, Orlando, Florida, Paper No. 2011-185 (2011)
[2] Haller, G. and Yuan, G.: Lagrangian coherent structures and mixing in two-dimensional turbulence. Physica D 147: 352-370 (2000)
[3] Shadden, S. C., Lekien, F. and Marsden, J. E. Definition and properties of Lagrangian coherent structures from finite-time Lyapunov exponents in two dimensional aperiodic flows. Physica D 212: 271-304 (2005)
[4] O’Farrell, C. and Dabiri, J. O.: A Lagrangian approach to identifying vortex pinch-off. CHAOS 20: 017513 (2010)
[5] Lekien, F., Leonard, N.: Dynamically Consistent Lagrangian Coherent Structures. 8th Experimental Chaos Conference. 132-139 (2004)
[6] Shadden, S. C., Dabiri, J. O. and Marsden, J. E.: Lagrangian analysis of fluid transport in empirical vortex ring flows. Physics of Fluids 18: 047105 (2006)
[7] Franco, E., Pekarek, D. N., Peng, J., Dabiri, J. O.: Geometry of unsteady fluid transport during fluid–structure interactions. Journal of Fluid Mechanics 589: 125-45 (2007)
[8] Tang, W., Mathur, M., Haller, G., Hahn, D. C., Ruggiero, F. H.: Lagrangian Coherent Structures near a Subtropical Jet Stream. Journal of the Atmospheric Sciences 67: 2307-2319 (2010).
[9] Elert, G.: The Chaos Hypertextbook. http://hypertextbook.com/chaos/. (2007)
[10] Shi, H.H., Wang, B.Y., Dai, Z.Q.: Research on the mechanics of underwater supersonic gas jets. Science China 53(3), 527-535 (2010)
[11] Haller, G.: An Objective Definition of a Vortex. Journal of Fluid Mechanics 525, 1-26(2005)
[12] Aoki T, Masuda S, Hatano A.: Characteristics of submerged gas jets and a new type bottom blowing tuyere. In: Injection Phenomena in Extraction and Refining. Wraith A E, ed. Newcastle: Department of Metallurgy and Engineering Materials (University of Newcastle upon Tyne), A1-A36 (1982)