An empirical model of the global distribution of plasmaspheric hiss based on Van Allen Probes EMFISIS measurements

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Key Points:
- The empirical model is a third-order polynomial function of L-shell, magnetic local time (MLT), magnetic latitude (MLAT), and AE.
- The model can perform the hiss amplitude distribution by L/MLT and /MLAT.
- All data are based on Van Allen Probes EMFISIS measurements.

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Abstract: Using wave measurements from the EMFISIS instrument onboard Van Allen Probes, we investigate statistically the spatial distributions of the intensity of plasmaspheric hiss waves. To reproduce these empirical results, we establish a fitting model that is a third-order polynomial function of L-shell, magnetic local time (MLT), magnetic latitude (MLAT), and AE. Quantitative comparisons indicate that the model’s fitting functions can reflect favorably the major empirical features of the global distribution of hiss wave intensity, including substorm dependence and the MLT asymmetry. Our results therefore provide a useful analytic model that can be readily employed in future simulations of global radiation belt electron dynamics under the impact of plasmaspheric hiss waves in geospace.

Keywords: hiss; Van Allen Probes; global model

1. Introduction

Plasmaspheric hiss is a broadband incoherent whistler mode emission with frequencies ranging from ~20 Hz to ~2 kHz that is observed predominantly inside the plasmasphere or high density plumes (Thorne et al., 1973, 1979; Meredith et al., 2004; Li W et al., 2013, 2015a, b; Ni BB et al., 2014; Shi R et al., 2017). It can persist under quiet conditions; its fluctuations are positively correlated with the level of solar activity (Smith et al., 1974; Thorne et al., 1973, 1974). Broadband amplitudes of plasmaspheric hiss range from a few pT to as high as 100 pT (Thorne et al., 1973; Smith et al., 1974; Meredith et al., 2004). Plasmaspheric hiss is mainly observed at a broad range of wave normal angle, e.g., its propagation near the geomagnetic equator is predominantly field-aligned, but oblique at higher latitudes (Santolik et al., 2001; Bortnik et al., 2008).

It has been well recognized that electron scattering by plasmaspheric hiss acts as a dominant contributor to formation of the slot region, which separates the radiation belts into inner (1.2 < L < 2) and outer (3 < L < 7) parts (Lyons et al., 1972; Lyons and Thorne, 1973; Albert, 1994; Abel and Thorne, 1998a, b). Resonant interactions with plasmaspheric hiss result in electron scattering losses with decay time scales varying from less than 1 hour to tens or hundreds of days. Decay times are closely dependent on electron energy, ambient magnetic field, and plasma density, as well as on wave amplitude, spectral intensity, and wave normal angle (WNA) distribution of the hiss waves as a function of spatial location and geomagnetic activity level (Meredith et al., 2007; Ni BB et al., 2014; Yu J et al., 2017). As reported in Summers et al. (2007a, b), the pitch angle diffusion coefficient of resonant electrons in quasi-linear formalisms is proportional to the wave amplitude. Thus, the development of a reliable global model of hiss wave amplitude is essential to facilitate quantification of hiss-induced electron scattering rates and resultant global variations of radiation belt electron distribution.

A number of empirical models of hiss wave global distribution have been constructed in previous studies (Meredith et al., 2004; Kim et al., 2015; Li et al., 2015b). Data from the Combined Release and Radiation Effect Satellite (CRRES) are utilized to investigate the features of plasma waves. Meredith et al. (2004) presented elaborate global maps of hiss wave intensity and described its distribution features. Orlova et al. (2014) produced empirical quadratic fittings of $R_S$ as a function of $K_p$, $L$, and geomagnetic latitude ($\lambda$) for the daytime and nighttime sectors respectively. However,
the CRRES wave instrument has limitations in space and frequency band, in particular it records only electric field data; accordingly, some important wave information such as magnetic field spectral intensity was estimated, based on theoretical assumptions. Characteristics of whistler mode waves have also been analyzed by Agapitov et al. (2013) with Cluster data, which separate chorus from hiss merely by \( f_{\text{ce}} \) (the electron cyclotron frequency), i.e., chorus frequency above 0.1\( f_{\text{ce}} \) and hiss below 0.1\( f_{\text{ce}} \), though their frequencies may actually overlap. Subsequently Agapitov et al. (2014) parameterized the hiss wave activity at \( L < 2 \) based on Akebono spacecraft observations. In addition, combined observations of Cluster (Agapitov et al., 2013) and Polar (Tsurutani et al., 2015) showed that hiss can be widely distributed over the magnetic latitude, extending to \( \lambda > 45^\circ \).

Recently, using data from Van Allen Probes, Spasojevic et al. (2015) and Yu J et al. (2017) constructed one-dimensional fitting models of hiss wave amplitude as a function of \( AE \), \( L \), MLT, and \( Kp \), \( L \), MLT, respectively. Highlighting the dependence of hiss amplitude on single variables, those studies focus on the fluctuations of the fitting curves associated with each variable in their hypothetical formulae and do not reproduce the observed latitudinal variations of hiss wave amplitude. Since the dependence on MLAT cannot be neglected in accurate predictions of hiss amplitude, as previous studies have done, the present investigation considers the combined effects of \( L \), MLT, MLAT, and \( AE \), and intends to establish a more comprehensive empirical model of plasmaspheric hiss amplitude as a function of these four input parameters.

2. Data and Methodology

Van Allen Probes were launched on September 8, 2012 with peri-
gee of \(-1.1 R_E \) (radius of the Earth), apogee of \(-5.8 R_E \), inclination of 10\(^\circ\), and an orbital period of \(-9 \) hours. Equipped with identical electromagnetic detectors, the EMFISIS instruments onboard Van Allen Probes can acquire high quality measurement of whistler-mode waves in the inner magnetosphere (Kletzing et al., 2013). The waveform receiver (WFR) on EMFISIS provides wave power spectral densities ranging from 10 Hz–12 kHz with a temporal resolution of 6 s. The high frequency receiver (HFR) records electric spectral information between 10 and 400 kHz, in which the traces of upper hybrid resonance (UHR) frequency can be used to estimate the ambient plasma density (Mosier et al., 1973) and identify the location of the plasmapause that separates the regions outside and inside the plasmasphere (e.g., He F et al., 2011, 2013, 2016, 2017; Katus et al., 2015; Verbanac et al., 2015; Zhang XX et al., 2017a, b).

According to the wave characteristics of hiss emissions, in this study we identify a hiss event with the ellipticity \( > 0.7 \) (i.e., right-hand polarized) and the frequency band within 10–2000 Hz in the plasmasphere. By doing so, we distinguish the hiss waves from chorus waves outside the plasmasphere and from magnetosonic waves with nearly linear polarization. Subsequently, we integrate the wave spectral intensity in the determined frequency band to calculate the wave amplitude \((B_w)\). In our following analysis we concentrate on the hiss emissions with \( B_w \geq 5 \) pT.

The adopted database consists of observations made during the period from September 8, 2012 to June 30, 2017: L-shell (\( L \)), magnetic local time (MLT), magnetic latitude (MLAT), \( AE^* \) (averaged value of the \( AE \) index in the previous hour), and hiss wave amplitude \((B_w)\) with temporal resolution of 6 s. We then implement two methods to construct our empirical global model of hiss wave amplitude. In each method, we divide the entire database data into two groups, i.e., the training group and the test group. The training group is used as the baseline to obtain the fitting coefficients in the regression, while the test group is regarded as a comparison to examine the model’s performance and verify its reliability.

As tabulated in Table 1, for Method #1, data in first three years are selected as the training group (Database 1) and the remaining datasets are reserved as the test group (Database 2). For Method #2, we divide them in another way: data collected during odd-number days are set as the training group (Database 3), and the others are the test group (Database 4). Furthermore, we divide each Database (1–4) into 12 sub-groups in terms of \( AE^* \) and MLT, to take into account the geomagnetic effect and MLT variation. Hence, we rank the \( AE^* \) values into three geomagnetic levels (quiet, \( AE^* < 100 \) nT; moderate, 100 nT \( \leq AE^* \leq 300 \) nT; active, \( AE^* > 300 \) nT) and classify all the MLTs into four sectors (dayside, 09–15; duskside, 15–21; nightside, 21–03; dawnside, 03–09). Correspondingly, twelve groups of fitted parameters need to be derived from regression of these training groups, and we then examine the feasibility of the resulting fitting functions through self-consistency and verification with the test groups. To check differences between the model and observations, we define the variable, \( R_d \) given as \[ \frac{|w_{\text{obs}} - w_{\text{mod}}|}{w_{\text{obs}}} \] where \( w_{\text{obs}} \) is the observed hiss wave amplitude and \( w_{\text{mod}} \) is the model result, as a criterion to evaluate our empirical model. Clearly, lower values of \( R_d \) represent higher similarity between observations and the model.

Taking \( L \), MLT, MLAT, \( AE^* \), and \( B_w \) as known quantities and \( F(i, j, k, l) \) as unknown, we implement the fits of a third-order polynomial function as follows:

\[ \log_{10}(B_w) = \sum_{i=0}^{3} \sum_{j=0}^{2} \sum_{k=0}^{2} \sum_{l=0}^{2} F(i, j, k, l) A^i B^j C^k D^l, \] (1)

where the parameters \( A, B, C, \) and \( D \) are quantified as \( L/10 \), MLT/24, MLAT/20, and \( AE^*/500 \), respectively. Following previous studies of Spasojevic et al. (2015) and Yu et al., (2017), we set the

| Table 1. Classification of each adopted database. |
|-----------------------------------------------|
| Type                                      | Method #1                              | Method #2                              |
| Training group                           | Every day during the period of 2012/9/8–2015/9/8 (Database 1) | Odd-number days during the period of 2012/9/8–2017/6/30 (Database 3) |
| Test group                               | Every day during the period of 2015/9/9–2017/6/30 (Database 2) | Even-number days during the period of 2012/9/8–2017/6/30 (Database 4) |

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maximum values of $i$, $j$, $k$, and $l$ as 3, 2, 2, and 2, respectively; consequently we have 108 coefficients for the fit to each training group.

3. Analysis Results

Based on the two methods described above, we do the fits for the training groups, compute the model results with the obtained polynomial fitting functions, and perform quantitative comparisons with the statistical observations of the test groups.

First, we follow Method #1 to establish the empirically analytic model of plasmaspheric hiss intensity, the results of which are shown in Figures 1−4. Figures 1 and 2 show the global distributions of time-averaged root-mean-square (RMS) hiss wave amplitudes in the equatorial plane and the meridian plane, at resolution of $0.5L \times 1MLT$ and $1° \times 0.1L$, respectively, obtained using Database 1 (the period from 2012/9/8–2015/9/8), and comparisons between these statistical observations and results of the empirical model.

The first-row subplots in Figure 1 reveal that the hiss wave amplitude strengthens significantly when $AE^*$ increases and that the waves on the dayside are much stronger than those at the nightside. Overall, hiss wave activity shows a strong dependence on $L$-shell, MLT, and geomagnetic level. Correspondingly, the second-row panels illustrate model results of hiss wave amplitude for the same time period as in Figures 1a–1c, obtained using the fitted three-order polynomial function (i.e., Equation (1)). It is evident

![Figure 1](image-url)

**Figure 1.** (a–c) Global $L$–MLT distribution of hiss wave amplitude from Van Allen Probe observations of Database 1; (d–f) self-consistent model results of Database 1; (g–i) relative differences ($R_d$) between them; (j–l) corresponding numbers of samples collected during quiet, moderate, and active times (left to right).
that the fitted function reflects the major features of the observed global dynamics of plasmaspheric hiss. The relative differences ($R_d$) between the observations and model results are shown in Figures 1g−1i. It is clearly seen that during most bins the relative differences are less than 15%, though the differences are larger in some limited regions of $L = 4.5–6$ and MLT ~23–06.

Figure 2 results are similar to those in Figure 1 but as a function of $L$ and MLAT in the indicated four MLT sectors. It can be seen clearly that, besides its $L$-shell and MLT dependence, the hiss wave amplitude also depends on MLAT but in a less significant manner. Figures 2i−2l confirm that during most bins the relative differences between the observational and model results are less than 20%, showing good agreement between model and empirical data.

Figures 3−4 present Database 2 (the test data) results in the same format as Figures 1−2, to verify consistent model performance during periods exclusive to the training group.

While the wave observations are slightly different from those in Database 1, the dominant trends are favorably similar. In order to validate the feasibility of the fitting function, we compare the observations from Database 2 (Figures 3a–3c) with the model results for the same period that are obtained using the empirically analytic function model as a function of $L$, MLT, MLAT, and $AE^*$ derived using Database 1 (Figures 3d−3f). Apparently, large differences between the observational and model results occur especially during geomagnetically moderate and active times, showing that the value of $R_d$ can be well above 1 in association with large deviation of the model results from the observations. In addition, it is distinct in Figure 4 that the model results become much less reliable compared to the observations, especially for the interval of MLT = 03–09. These discrepancies between model results and observations during the time period exclusive to the test group tell that the empirical model obtained using Method #1 has limitations that make it unsuited to the task of obtaining a global hiss wave distribution.

Accordingly, we adopt Method #2 to establish an empirical model of the global distributions of hiss intensity; that is, the data during
the odd-number days are set as the training group (Database 3) and used to acquire the fitting function \( F(i, j, k, l) \), and the data during the even-number days are set as the test group to verify the model’s performance.

First, we use Database 3 for the fits and self-consistency comparisons, the results of which are shown in Figures 5 and 6. Evidently, the fitting model results and the observations exhibit favorable agreement in most \( L \)-MLT bins (Figure 5), justifying the feasibility of the empirical model. While the differences between the observations and the model become relatively larger in the \( L \)-MLAT bins at high \( L \) shell, the majority of \( R_d \) values are below \( \sim 20\% \) (Figure 6), thereby confirming good performance of the model.

Again, to validate the performance of the fitting model obtained using Database 3 as the training group, we use Database 4 to apply the model and compare the model results with the observations, the results of which are shown in Figures 7 and 8.

Compared with the statistical observations (Figures 7a–7c), the model can reproduce well the global distribution of hiss wave amplitude including the MLT asymmetry and geomagnetic activity dependence (Figures 7d–7f). As shown in Figures 7g–7i, the relative differences (\( R_d \)) are < 15% for over 80% of \( L \)-MLT bins, justifying good performance of the fitting model, while the model becomes less feasible at high \( L \)-shells on the nightside, especially for active conditions with \( AE^* > 300 \) nT. In addition, the empirical model can reproduce well the major features of hiss wave amp-...
Global distribution of hiss wave intensity by implementing third-order polynomial function fits to the long-term Van Allen Probes EMFISIS wave data, which is determined by Equation (1) and the polynomial coefficients tabulated in Tables S1–S12 in the Appendix for the four considered MLT sectors (i.e., 03−09, 09−15, 15−21, and 21−03) under three geomagnetic conditions (i.e., quiet-time: $AE^* < 100$ nT; moderate-time: $100$ nT $\leq AE^* \leq 300$ nT; active-time: $AE^* > 300$ nT).

### 4. Concluding Remarks

In the present study, we have used approximately five years of Van Allen Probes EMFISIS wave data to analyze statistically the global distribution of plasmaspheric hiss intensity at $L = 2−6$ under various conditions of geomagnetic activity. During quiet times or times of weak substorm activity ($AE^* < 100$ nT), the hiss wave amplitude distribution is not dependent on MLT. However, when substorm activity intensifies, the hiss wave amplitude distribution is more dependent on MLT and the intense hiss events are concentrated at high $L$ ($L > 5$) on the morning side and the nightside (MLT ~ 00:00−06:00). Furthermore, the observation results (the first row of our Figures 2, 4, 6, and 8) suggest that the hiss amplitude distribution is slightly dependent on MLAT in the data on which the present study is based, which means the distribution is mainly controlled by $L$ and MLT. The hiss wave amplitude is very intense inside the dayside plasmasphere (the first row of Figures 2, 4, 6, and 8).
Inspired by observations that hiss wave amplitudes vary sensitively with spatial location and $AE^*$, we have then performed numerical fits to the statistical observations in terms of a third-order polynomial as a function of $L$, MLT, MLAT, and $AE^*$. By doing so, we have constructed an empirically analytic model of the global distribution of plasmaspheric hiss amplitude. The significance mainly lies in the treatment of fluctuation. The hiss amplitude can be obtained through the model simply by using $AE^*$ and position information. The results of previous studies (Spasojevic et al., 2015, Yu J et al., 2017) show good agreement with hiss amplitude observations when for single variables ($L$ or MLT) are considered. Since the 2D model including $L$/MLT or $L$/MLAT is of greater importance, in this study we present the fitting model to investigate the relationship between $B_w$ and $L$/MLT, and also $B_w$ and $L$/MLAT. Our 2D model can perform the hiss amplitude distribution by $L$/MLT and $L$/MLAT simultaneously. Beyond this, our model can be simplified for 1D usage; the results show good consistency with Spasojevic et al. (2015) and Yu J et al. (2017).

Two different methods have been implemented to separate the training group and the test group, as shown in Table 1. Our analysis indicates that Method #2, which uses the odd-number days as the training group and the even-number days as the test group, is superior in deriving well-performing fitting functions for the global hiss wave model. One possible reason can be that the training and test groups in Method #2 may reflect the phases of a solar cycle more reasonably. Several other factors that may affect the modeling results should also be considered: first, the latitudin-

Table 1: 2012/9/8–2017/6/30 (odd number of days)

| $AE^*$ | # of samples |
|--------|-------------|
| $< 100$ nT | 100 |
| $100 \leq AE^* \leq 300$ nT | 50 |
| $> 300$ nT | 0 |

Figure 5. (a−c) Global $L$-MLT distribution of hiss wave amplitude from observations of Database 3; (d−f) self-consistent model results of Database 3; (g−i) relative differences ($R_d$) between them; (j−l) corresponding numbers of samples collected during quiet, moderate, and active times (left to right).
distribution of the number of samples is dependent on the MLAT, which could be another reason why Method #1 is worse than Method #2; second, Databases 1 and 2 have more differences in the MLAT distribution, which may also be the cause of differences between the observations and model results. These unsolved questions require further investigation in future studies.

In summary, it is well demonstrated, by quantitative comparisons between the statistical observations and model results, that the empirical model in terms of fitted functions can reflect favorably the major features of the global distribution of hiss wave intensity, including the MLT asymmetry, substorm dependence, and latitudinal variations (Xiang Z et al., 2017). Because both the energy spectra and pitch angle distributions of radiation belt electrons are critically affected by hiss wave scattering (e.g., Ni BB et al., 2013, 2014, 2017, 2019; Zhao H et al., 2019; Li LY et al., 2008; Hua M et al., 2019), and because the scattering rates increase proportionally to the square of hiss amplitude for the near-resonance cases, our results therefore provide an empirically useful analytic model to be readily used for numerical quantification of hiss-driven electron diffusion coefficients and global simulations of resultant modulation of radiation belt electron dynamics in response to varying conditions of solar wind and geomagnetic activities.

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Figure 6. (a−d) Latitudinal L-MLAT distribution of hiss wave amplitude from observations in Database 3; (e−h) self-consistent model results of Database 3; (i−l) relative differences ($R_d$) between them; (m−p) corresponding numbers of samples during different MLT sectors (left to right).
Fitting coefficients of global hiss wave amplitudes described by Equation (1), with $A$, $B$, $C$, and $D$ standing for $L/10$, MLT/24, MLAT/20, and $AE^*/500$ respectively in the tables below.

**Supplementary Materials**

Fitting coefficients of global hiss wave amplitudes described by Equation (1), with $A$, $B$, $C$, and $D$ standing for $L/10$, MLT/24, MLAT/20, and $AE^*/500$ respectively in the tables below.

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Database 4: 2012/9/8−2017/6/30 (even number of days)

L = 6

L = 2

L = ...

Observation

Model

$R_d$ (%)

# of samples

Figure 7. (a−c) Global $L$-MLT distribution of hiss wave amplitude from observations of Database 4; (d−f) model results calculated by Method #2 of Database 4; (g−i) relative differences ($R_d$) between them; (j−l) Corresponding numbers of samples during quiet, moderate, and active times (left to right).
Table S1. For the case of $3 \leq \text{MLT} < 9$ when $AE^* < 100$ nT.

|        | $A^3$       | $A^2$       | $A$            | 1              |
|--------|-------------|-------------|----------------|----------------|
| $B^2C^2D^2$ | $-266514.9971$ | $396133.7612$ | $-184520.1083$ | $22417.0169$   |
| $B^2C^2D$   | $54078.0900$   | $-73746.8387$  | $34075.0324$   | $-4159.9805$   |
| $B^2C^2$    | $-5621.4548$   | $6523.7330$     | $-2538.6389$   | $289.3744$     |
| $B^2CD^2$   | $711003.3528$  | $-869701.6097$  | $336135.7473$  | $-36131.3944$  |
| $B^2CD$     | $-162514.4515$ | $198471.9394$   | $-77133.7749$  | $8403.3444$    |
| $B^2C$      | $5460.4956$    | $-7038.6107$    | $2935.1354$    | $-339.9128$    |
| $B^2D^2$    | $12383.3494$   | $36038.8107$    | $-26108.7679$  | $4090.9090$    |
| $B^2D$      | $-5046.8114$   | $-11409.0749$   | $8713.9449$    | $-1313.8818$   |
| $B^2$       | $1167.8761$    | $-144.9262$     | $-286.1891$    | $57.2291$      |
| $BC^2D^2$   | $109462.4900$  | $-189592.8830$  | $95759.0780$   | $-12030.0396$  |

Figure 8. (a−d) Latitudinal $L$-MLAT distribution of hiss wave amplitude from observations in Database 4; (e−h) model results calculated by Method #2 of Database 4; (i−l) relative differences ($R_d$) between them; (m−p) corresponding numbers of samples during different MLT sectors (left to right).
Continued from Table S1

|      |   A³   |   A²   |   A    |   1    |
|------|--------|--------|--------|--------|
| BC²D | −22770.5610 | 35900.9413 | −17996.1522 | 2267.7925 |
| BC²  | 2614.9908   | −3200.3276 | 1298.0470  | −150.7319 |
| BCD² | −399735.7029 | 486461.0926 | −185965.6797 | 19759.4410 |
| BCD  | 89648.4882  | −108883.2325 | 41859.1725  | −4512.8917 |
| BC   | −3072.1548  | 3905.0191   | −1596.3226  | 181.6559   |
| BD²  | 13119.3847  | −34517.3428 | 17091.5814  | −2350.3062 |
| BD   | −3057.1654  | 10660.3297  | −5661.1607  | 768.2433   |
| B    | −133.1410   | −385.0074   | 285.6188    | −41.8407   |
| C²D² | −8925.5333  | 19872.9552  | −11002.8916 | 1400.9854  |
| C²D  | 1961.5531   | −3790.8888  | 2059.6952   | −260.8511  |
| C²   | −260.5572   | 337.9240    | −141.9811   | 16.4166    |
| CD²  | 49165.7159  | −59712.5852 | 22671.5047  | −236.0183  |
| CD   | −10827.3537 | 13107.7268  | −4999.7083  | 531.5406   |
| C    | 374.1982    | −471.3872   | 189.8032    | −260.8511  |
| D²   | −3683.7443  | 5466.8612   | −2190.9823  | 278.3865   |
| D    | 1167.1453   | −1918.8436  | 821.3382    | −101.4842  |
| 1    | −47.9098    | 104.0285    | −49.1017    | 7.6187     |

Table S2. For 3 ≤ MLT < 9 when 100 nT ≤ AE* ≤ 300 nT.

|      |   A³   |   A²   |   A    |   1    |
|------|--------|--------|--------|--------|
| b⁶c⁴D³ | 331653.7991 | −308724.7892 | 85439.2375 | −6834.3308 |
| b⁶c⁴D | −241950.8713 | 225856.0542 | −63900.4468 | 5298.6881  |
| b⁶c² | 42092.5539   | −39622.5307 | 11528.0842 | −989.3248  |
| b⁶CD² | −30330.8124  | 36914.0125  | −16526.4601 | 2262.0703  |
| b⁶CD | −813.0019    | −3918.5268  | 6060.2020  | −1315.9209 |
| b⁶C | 2057.5584    | −824.1222   | −811.5823  | 243.6292   |
| b⁶D² | −137886.6057 | 140268.7276 | −42807.3836 | 3577.8427  |
| b⁶D | 106875.8033  | −110329.2476 | 33724.7500 | −2866.8768 |
| b⁶ | −19767.3055  | 20493.2503  | −6391.3540 | 555.4488   |
| BC²D² | −186784.8467 | 173025.2772 | −47277.7337 | 3707.6257  |
| BC²D | 137389.8139  | −127709.9661 | 35625.4506 | −2887.3134 |
| BC² | −23877.6413  | 22387.1216  | −6417.5656 | 537.2465   |
| BCD² | 17663.7459   | −20102.7491 | 8714.8459  | −1168.0038 |
| BCD | −1797.5484   | 3467.5402   | −3431.5976 | 692.6035   |
| BC | −792.7300    | 316.6578    | 415.8051   | −123.0988  |
| BD³ | 70177.6085   | −71475.3936 | 21189.5131 | −1740.4780 |
| BD | −54224.0265  | 55197.0739  | −16574.8150 | 1386.2145  |
| B | 10003.5159   | −10274.1115 | 3168.2252  | −271.3915  |
| C²D² | 23356.0741   | −21140.9820 | 5549.2345  | −406.8674  |
Continued from Table S2

|   | \(A^2\)     | \(A^2\)     | \(A\)       | 1       |
|---|-------------|-------------|-------------|---------|
| \(C^2D\) | -17446.3873 | 15895.2195  | -4271.5491 | 324.4526 |
| \(C^2\)  | 3086.8747   | -2847.3167  | 790.1641   | -62.5019 |
| \(CD^2\) | -1941.2948  | 2015.7225   | -843.7662  | 110.3938 |
| \(CD\)   | 311.1382    | -340.1256   | 319.5751   | -65.1887 |
| \(C\)    | 74.4323     | -51.4628    | 31.4615    | 11.3554  |
| \(D^2\)  | -7709.1922  | 7698.6539   | -2219.8704 | 175.7890 |
| \(D\)    | 6041.5387   | -6004.5814  | 1750.7038  | -141.2786|
| 1         | -1111.2043  | 1114.1224   | -333.5586  | 29.1836  |

Table S3. For \(3 \leq \text{MLT} < 9\) when \(AE^* > 300\) nT.

|   | \(A^2\)     | \(A^2\)     | \(A\)       | 1       |
|---|-------------|-------------|-------------|---------|
| \(B^2C^2D^2\) | 6132.7613   | -27930.8705 | 11651.5562 | -1113.2363|
| \(B^2C^2D\)  | -42815.7524 | 82673.4313  | -30777.2382| 2865.2209 |
| \(B^2C^2\)   | 37990.7017  | -55785.8343 | 19486.3362 | -1779.3551|
| \(B^2CD^2\)  | -64036.0325 | 51549.2193  | -12378.0540| 847.4860  |
| \(B^2CD\)    | 114298.9448 | -90899.8767 | 21070.5138 | -1340.9157|
| \(B^2C\)     | -45958.2849 | 35709.9717  | -7758.8220 | 413.6833  |
| \(B^2D^2\)   | 16006.5829  | -7613.0548  | 801.5318   | 6.6229    |
| \(B^2D\)     | -28049.8313 | 11604.8521  | -500.9973  | -103.9934 |
| \(B^2\)      | 7399.3240   | 132.7468    | 1469.2754  | 195.2219  |
| \(BC^2D^2\)  | 7989.1234   | 7255.6867   | -4462.3906 | 465.6041  |
| \(BC^2D\)    | 3203.5654   | -30890.6393 | 13397.9060 | -1306.4381|
| \(BC^2\)     | -12601.4592 | 24661.2024  | -9224.4162 | 861.4047  |
| \(BCD^2\)    | 34438.2147  | -26848.3917 | 6264.1346  | -419.9600 |
| \(BCD\)      | 3203.5654   | -30890.6393 | 13397.9060 | -1306.4381|
| \(BC\)       | -12601.4592 | 24661.2024  | -9224.4162 | 861.4047  |
| \(BD^2\)     | -11028.8682 | 5649.5698   | -775.4366  | 22.5878   |
| \(BD\)       | 19404.6519  | -8840.1046  | 820.2465   | 14.5789   |
| \(B\)        | -5664.8571  | 820.0595    | 630.4501   | -92.2497  |
| \(C^2D^2\)   | -3547.9859  | 880.4892    | 148.3611   | -28.8548  |
| \(C^2D\)     | 4192.7312   | 584.0480    | -909.5064  | 107.4561  |
| \(C^2\)      | -350.2315   | -1663.0556  | 808.4976   | -81.9262  |
| \(CD^2\)     | -4156.7722  | 3070.2755   | -688.0119  | 44.8482   |
| \(CD\)       | 6954.1099   | -5030.6860  | 1065.6385  | -62.0595  |
| \(C\)        | -2606.0522  | 1792.1408   | -330.7502  | 12.9542   |
| \(D^2\)      | 1830.1727   | -989.6628   | 157.7775   | -7.3790   |
| \(D\)        | -3206.6072  | 1671.6640   | -226.6916  | 7.6697    |
| 1            | -1146.7147  | -400.4969   | -6.6888    | 7.2731    |
Table S4. For 9 ≤ MLT < 15 when AE* < 100 nT.

|       | A    | A^2   | A     | 1     |
|-------|------|-------|-------|-------|
| b^2c^2d^2 | 629884.9981 | -704561.3756 | 236123.6731 | -24458.1348 |
| b^2c^2d  | -186449.5938 | 214143.2284 | -73522.9196 | 7600.7679  |
| b^2c^2   | 9859.9713   | -12173.5796  | 4461.3441  | -472.1497  |
| b^2cd^2  | 40426.9278  | -74824.5923  | 27475.3091 | -1057.7387 |
| b^2cd    | 26081.4585  | -22956.4169  | 7566.3318  | -1206.4892 |
| b^2c     | -3150.9226  | 3150.7331    | -1054.2248 | 130.7475   |
| b^2d^2   | -425735.9035 | 445958.9123  | -137447.9455 | 12161.9005 |
| b^2d     | 102505.6268 | -109727.0210 | 34662.5765 | -3156.3718 |
| b        | -4963.7051  | 5566.8348    | -1842.3759 | 172.6923   |
| bc^2d^2  | -578928.5758 | 643869.6617  | -215185.3916 | 22565.6035 |
| bc^2d    | 171190.6501 | -197223.3796 | 68048.7862 | -7118.7784 |
| bc^2     | -9095.0724  | 11370.7693   | -4215.4383 | 450.4360   |
| bc^2d^2  | -53180.7345 | 88403.4235   | -32210.0716 | 1505.6013  |
| bcd      | -25102.6522 | 21531.6270   | -6873.9905 | 1130.0184  |
| bc       | 3372.6792   | -3349.1747   | 1098.5430  | -133.1092  |
| bd^2     | 445754.0520 | -469938.1203 | 146202.9813 | -13142.2304 |
| bd       | -105703.5663 | 114052.7380  | -36428.4483 | 3371.1101  |
| b        | 5136.6792   | -5815.8431   | 1949.8871  | -185.6119  |
| c^2d^2   | 128119.2472 | -140783.7539 | 46586.5678 | -4936.3869 |
| c^2d     | -37662.4911 | 43430.8074   | -15013.9783 | 1586.4379  |
| c^2      | 1998.3167   | -2533.6223   | 950.2927   | -102.4939  |
| cd^2     | 16734.6294  | -25521.5664  | 9214.2985  | -499.8170  |
| cd       | 5836.5497   | -4845.1243   | 1480.8104  | -252.5459  |
| c        | -883.7282   | 870.9973     | -279.6956  | 32.9784    |
| d^2      | -114081.9787 | 120791.1982  | -37827.7391 | 3439.3908  |
| d        | 26693.0495  | -28982.9744  | 9339.3403  | -873.6237  |
| 1        | -1282.2221  | 1458.1511    | -491.7567  | 48.6827    |

Table S5. For 9 ≤ MLT < 15 when 100 nT ≤ AE* ≤ 300 nT.

|       | A    | A^2   | A     | 1     |
|-------|------|-------|-------|-------|
| b^2c^2d^2 | -169864.1685 | 147455.0384 | -32886.8818 | 1505.1670 |
| b^2c^2d  | 130383.1144 | -112560.8228 | 24526.2227 | -962.8418 |
| b^2c^2   | -22365.0240 | 19117.4984  | -3981.7294 | 114.3034  |
| b^2cd^2  | -139011.8288 | 145722.3046 | -48273.5030 | 4794.3010 |
| b^2cd    | 115734.3602 | -122340.2537 | 40873.8247 | -4056.8831 |
| b^2c     | -21231.0770 | 22556.7890  | -7555.3091 | 740.8990  |
| b^2d^2   | -37444.2493 | 48027.6866  | -18563.9424 | 1952.8074 |
| b^2d     | 26352.4851  | -34065.8492 | 13311.0788 | -1439.8912 |
| b^2      | -4917.4746  | 6152.4054   | -2368.9573 | 257.3706  |

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Continued from Table S5

\begin{table}[h]
\centering
\begin{tabular}{cccc}
\hline
 & $A^1$ & $A^2$ & $A$ & 1 \\
\hline
$BC^2D^2$ & 193321.4523 & $-174237.8943$ & 42071.8227 & $-2420.7938$ \\
$BC^2D$ & $-148952.9454$ & 133411.3280 & $-31560.3969$ & 1655.2613 \\
$BC^2$ & 25620.7780 & $-22754.5790$ & 5203.0059 & $-233.4869$ \\
$BCD^2$ & 143821.9669 & $-149879.9036$ & 49306.6154 & $-4866.6454$ \\
$BCD$ & $-119807.0605$ & 125903.8889 & $-41793.2646$ & 4125.1234 \\
$BC$ & 22021.6420 & $-23271.2594$ & 7751.7619 & $-756.4568$ \\
$BD^2$ & 35237.0632 & $-45609.4155$ & 17734.8507 & $-1864.9925$ \\
$BD$ & $-24541.1752$ & 32148.4689 & $-12680.9205$ & 1373.0821 \\
$B$ & 4612.8304 & $-5824.8700$ & 2259.7143 & $-245.7838$ \\
$c^2D^2$ & $-53549.9666$ & 49738.0385 & $-12707.6404$ & 829.3473 \\
$c^2D$ & 41574.0809 & $-38362.6236$ & 9630.8017 & $-590.3443$ \\
$c^2$ & $-7201.5844$ & 6603.9642 & $-1618.0443$ & 90.3080 \\
$CD^2$ & $-36699.157$ & 37931.4620 & $-12344.1107$ & 1204.9972 \\
$CD$ & 30593.7282 & $-31889.1734$ & 10479.5269 & $-1023.9428$ \\
c & $-5641.7512$ & 5917.5459 & $-1953.9292$ & 188.9122 \\
$D^2$ & $-7846.7149$ & 10377.5080 & $-4099.8631$ & 433.1066 \\
D & 5360.0358 & $-7230.9882$ & 2914.8737 & $-317.1766$ \\
1 & $-980.3300$ & 1267.2780 & $-501.2689$ & 56.7315 \\
\hline
\end{tabular}
\caption{For 9 ≤ MLT < 15 when $AE^+ > 300$ nT.}
\end{table}

Table S6. For 9 ≤ MLT < 15 when $AE^+ > 300$ nT.

\begin{table}[h]
\centering
\begin{tabular}{cccc}
\hline
 & $A^1$ & $A^2$ & $A$ & 1 \\
\hline
$B^2C^2D^2$ & $-31993.8136$ & 36350.2045 & $-12567.1004$ & 1153.6804 \\
$B^2C^2D$ & 65046.8285 & $-76045.8471$ & 26740.9094 & $-2483.7590$ \\
$B^2C^2$ & $-29375.6610$ & 35116.2003 & $-12477.5683$ & 1147.1708 \\
$B^2CD^2$ & $-150.4137$ & $-931.4761$ & 888.6293 & $-135.8866$ \\
$B^2CD$ & 17481.1852 & $-15612.7851$ & 3539.6859 & $-188.5353$ \\
$B^2C$ & $-15577.0949$ & 14709.2991 & $-3816.5509$ & 263.8378 \\
$B^2D^2$ & 1501.7008 & $-2734.6008$ & 1196.6681 & $-122.1471$ \\
$B^2D$ & $-5178.1268$ & 8533.7132 & $-3533.0662$ & 353.4630 \\
$B^2$ & 3170.0384 & $-5116.3017$ & 2086.0565 & $-209.4092$ \\
$BC^2D^2$ & 32561.2017 & $-37073.5770$ & 12812.5938 & $-118.6440$ \\
$BC^2D$ & $-65390.7075$ & 76840.6779 & $-27080.6938$ & 2522.4960 \\
$BC^2$ & 28769.8107 & $-34819.4777$ & 12474.0677 & $-1155.9478$ \\
$BCD^2$ & $-775.3958$ & 2061.5781 & $-1276.5211$ & 171.4736 \\
$BCD$ & $-15580.6001$ & 13246.3806 & $-2720.2183$ & 111.5433 \\
$BC$ & 14743.2103 & $-13635.5241$ & 3438.1251 & $-227.0693$ \\
$BD^2$ & $-1534.6187$ & 2744.9817 & $-1183.0081$ & 119.6319 \\
$BD$ & 5378.4352 & $-8676.4002$ & 3526.4529 & $-348.9188$ \\
$B$ & $-3102.6496$ & 5018.5670 & $-2032.9596$ & 202.7243 \\
$c^2D^2$ & $-8149.1462$ & 9285.7846 & $-3200.7904$ & 293.8390 \\
\hline
\end{tabular}
\caption{Continued from Table S5}
\end{table}
Continued from Table S6

|      | $A^3$       | $\hat{A}^3$ | $A$      | $1$     |
|------|-------------|-------------|----------|---------|
| $C^2D$ | 16073.3979  | -18981.3168 | 6694.7619| -624.9779|
| $C^2$  | -6818.1265  | 8377.6824   | -3026.2587| 282.4056|
| $CD^2$ | 472.4027    | -830.8122   | 420.3314 | -51.8246|
| $CD$   | 3289.2412   | -2608.3565  | 448.6455 | -6.4834 |
| $C$    | -3397.5641  | 3064.5458   | -743.7383| 45.6803 |
| $D^2$  | 411.2663    | -703.0306   | 293.8160 | -29.3386|
| $D$    | -1452.1749  | 2246.5070   | -885.0933| 86.5118 |
| 1      | 830.0756    | -1301.6374  | 516.5562 | -49.1952|

Table S7. For $15 \leq \text{MLT} < 21$ when $AE^* < 100$ nT.

|      | $A^3$       | $\hat{A}^3$ | $A$      | $1$     |
|------|-------------|-------------|----------|---------|
| $B^2C^2D^2$ | 502956.9265 | -539128.3919 | 162997.4350| -11347.8250|
| $B^2C^2D$ | -113445.6282| 119077.1007 | -34329.3681| 2265.3844|
| $B^2C^2$ | 3276.0336   | -3230.7921  | 770.7469 | -29.5738|
| $B^2CD^2$ | 157210.5677 | -213224.4394| 81353.1972| -9973.3017|
| $B^2CD$ | -74359.6575 | 94332.0713  | -34624.8291| 3840.0784|
| $B^2C$ | 4960.6349   | -6373.8833  | 2359.7361| 255.2794 |
| $B^2D^2$ | -325638.9011| 359494.1791 | -115501.3005| 9258.6430|
| $B^2D$ | 51615.4180  | -56416.3949 | 17634.4180| -1328.2824|
| $B^2$ | -573.9870   | 603.2576    | -145.7251| -1.0749 |
| $BC^2D^2$ | -718212.2574| 771041.5231 | -233097.7411| 16232.2811|
| $BC^2D$ | 159873.2797 | -168175.695 | 48398.1224| -3172.3739|
| $BC^2$ | -4424.4420  | 4377.1854   | -1025.1029| 35.5277 |
| $BCD^2$ | -164889.3781| 237262.1277 | -93580.3798| 12233.4214|
| $BCD$ | 97030.0976  | -124752.9570| 46246.4867| -5224.7885|
| $BC$ | -6799.8377  | 8825.3793   | -3292.1941| 359.7621 |
| $BD^2$ | 494266.5954 | -545293.6129| 175472.2080| -14174.7311|
| $BD$ | -79249.9878 | 86620.8634  | -27190.6987| 2080.3337|
| $B$ | 971.3968    | -1018.7705  | 257.5490 | -3.1859 |
| $C^2D^2$ | 262395.7950 | -282540.0548| 85653.9028| -6033.4447|
| $C^2D$ | -57857.2535 | 61113.1883  | -17628.2573| 1163.2656|
| $C^2$ | 1576.1757   | -1571.5676  | 367.7632 | -12.7950|
| $CD^2$ | 36477.0645  | -59567.5142 | 24948.3281| -3593.2854|
| $CD$ | -30929.8153 | 40537.0913  | -15224.6182| 1756.3501|
| $C$ | 2308.0058   | -3034.7736  | 1142.4299| -126.1022|
| $D^2$ | -189357.3284| 208806.6558 | -67321.9007| 5486.7131|
| $D$ | 31157.3121  | -34050.0090 | 10738.0056| -835.8921|
| 1    | -441.6704   | 456.7027    | -118.3040| 4.9437  |
Table S8. For 15 ≤ MLT < 21 when 100 nT ≤ AE* ≤ 300 nT.

| $b^2c^2d^1$ | $a^2$ | $a$ | 1 |
|-------------|-------|-----|---|
| 93608.8811  | -99129.7690 | 34001.2091 | -4022.7276 |
| -76919.5127 | 83277.6117  | -28706.9010 | 3299.9839  |
| 12047.6256  | -13441.2021 | 4741.1544  | -554.2596  |
| -101043.1854| 108320.6616 | -34647.3669 | 3143.5469  |
| 88592.0627  | -95344.3339 | 30571.9557 | -2754.5159 |
| -15301.5783 | 16315.5162  | -5164.9895 | 448.3235   |
| -90468.5901 | 102938.6861 | -35797.5100 | 3751.9817  |
| 72443.6599  | -81900.6731 | 28201.0568 | -2915.3123 |
| -12884.7832 | 14594.5450  | -5019.4980 | 512.0447   |
| -131111.6890| 138410.6585 | -47580.7641 | 5719.2727  |
| 107716.9153 | -116697.5797| 40429.3929 | -4720.5602 |
| -16725.0565 | 18755.6724  | -6678.4186 | 796.3718   |
| 160921.7848 | -173343.3489| 55671.2223 | -5053.0726 |
| -140460.4094| 151887.3397 | -48922.0289| 4420.9577  |
| 24425.4249  | -26222.1361 | 8360.2948  | -732.0562  |
| 134092.0390 | -153291.3326| 53557.2123 | -5635.3154 |
| -108035.3083| 122651.0370 | -42407.2877| 4399.2731  |
| 19075.0580  | -21693.7179 | 7491.9321  | -768.7761  |
| 45135.6942  | -47371.0058 | 16302.6130 | -1992.7369 |
| -37088.5944 | 40149.0026  | -13979.0482| 1659.5441  |
| 5691.3770   | -6414.5665  | 2309.8626  | -281.7614  |
| -62815.8241 | 67885.5183  | -21856.3421| 1983.0585  |
| 54705.6403  | -59355.6320 | 19176.0729 | -1736.4545 |
| 9583.8709   | 10341.7678  | -3314.4556 | 292.2754   |
| -49409.4172 | 56695.8478  | -19880.8569| 2097.0375  |
| 40032.1588  | -45603.7988 | 15819.2237 | -1644.4106 |
| -6979.0670  | 7962.5121   | -2758.1727 | 286.2224   |

Table S9. For 15 ≤ MLT < 21 when AE* > 300 nT.

| $b^2c^2d^2$ | $a^2$ | $a$ | 1 |
|-------------|-------|-----|---|
| -1829.3591  | -587.4156 | 1164.1513 | -259.9248 |
| -1757.2425  | 4277.4628 | -3162.8914 | 686.3738 |
| -3003.5657  | 3324.1760 | -371.8121 | -184.5379 |
| -12211.0051 | 10877.9200 | -2708.4724 | 203.9189 |
| 34058.6401  | -32021.0297| 8632.3466 | -686.0533 |
| -15231.1783 | 14413.9471 | -3857.5838 | 297.3967 |
| 4236.7924   | -3758.4586 | 941.0589 | -45.7887  |
| -6926.4342  | 6521.6285  | -1678.5517 | 61.2233   |
| 2985.1491   | -3203.8975 | 910.2741 | -37.0109  |
Continued from Table S9

|     | $A^2$   | $A^2$   | $A$     | 1       |
|-----|---------|---------|---------|---------|
| $BCD^2$ | 700.9561 | 2672.3186 | -2260.9915 | 430.0345 |
| $BCD$  | 5060.9111 | -8754.0784 | 5522.8809  | -1092.7459 |
| $BC^2$ | 3854.3675 | -4227.9542 | 245.1139   | 304.1879  |
| $BCD^2$ | 17249.2805 | -15189.3713 | 3705.5840  | -272.1701 |
| $BCD$  | -48633.6842 | 45430.6074 | -12127.1988 | 952.8634 |
| $BC$   | 21845.9720 | -20534.2901 | 5436.1308  | -413.5728 |
| $BD^2$ | -5921.7142 | 5368.5874  | -1374.5254 | 69.1577  |
| $BD$   | 10043.8037 | -9731.4713 | 2574.3572  | -104.3332 |
| $B$    | -4786.1755 | 5257.6537  | -1535.5133 | 73.1924  |
| $C^2D^2$ | 224.8326 | -1408.1570 | 963.8340   | -169.6245 |
| $C^2D$ | -2273.1720 | 3649.4858 | -2214.6980 | 420.1827 |
| $C^2$  | -1467.1437 | 1555.0067  | -47.5418   | -117.9417 |
| $CD^2$ | -6012.4862 | 5221.5247  | -1241.6993 | 88.5275  |
| $CD$   | 17125.1575 | -15871.8048 | 4184.9588  | -324.8277 |
| $C$    | -7718.2414 | 7195.9406  | -1879.5879 | 141.0361 |
| $D^2$  | 2083.5260 | -1928.0878 | 503.1514   | -26.1625 |
| $D$    | -3685.4162 | 3651.8279  | -984.5011  | 43.4187  |
| 1      | 1962.9860 | -2184.9943 | 650.9175   | -33.2332 |

Table S10. For 21 ≤ MLT < 3 when $AE^+ < 100$ nT.

|     | $A^2$   | $A^2$   | $A$     | 1       |
|-----|---------|---------|---------|---------|
| $B^2C^2D^2$ | 78763.3201 | -70953.4215 | 17898.7742 | -1439.3581 |
| $B^2C^2D$  | -21308.5777 | 19385.5327  | -4916.1973 | 371.5218  |
| $B^2C^2$   | 1355.2223 | -1248.0642 | 324.4378  | -24.3469  |
| $B^2CD^2$  | -81512.5192 | 86332.2005 | -26434.5405 | 2334.6949 |
| $B^2CD$    | 20283.1122 | -21191.7675 | 6459.3212  | -575.1379 |
| $B^2C$     | -1352.0203 | 1402.9724  | -429.5032  | 38.8220   |
| $B^2D^2$   | -44623.4589 | 53791.7013 | -20025.8662 | 2291.6462 |
| $B^2D$     | 14313.1686 | -16487.0934 | 5830.2838  | -636.0968 |
| $B^2$      | -1096.0362 | 1209.4196  | -405.8388  | 41.1090   |
| $BC^2D^2$  | -77283.1734 | 68446.0694 | -16839.1821 | 1364.0079 |
| $BC^2D$    | 21090.8216 | -18943.0997 | 4712.5379  | -358.0203 |
| $BC^2$     | -1362.0066 | 1246.7419  | -321.9076  | 24.5684   |
| $BCD^2$    | 91013.8294 | -97245.0751 | 30272.9614 | -2722.3949 |
| $BCD$      | -22484.0807 | 23802.3541 | -7400.8736 | 671.8152  |
| $BC$       | 1478.9923 | -1556.7355 | 485.3099  | -44.5170  |
| $BD^2$     | 46269.0705 | -55171.8689 | 20368.9337 | -2334.2277 |
| $BD$       | -14739.9411 | 16875.8488 | -5941.3381 | 649.9034  |
| $B$        | 1126.6437 | -1242.7771 | 417.5811  | -42.5079  |
| $C^2D^2$   | 3222.2702 | -2493.1073 | 584.5990  | -79.1772  |
Continued from Table S10

|       | $A^3$       | $A^2$       | $A$        | 1        |
|-------|-------------|-------------|------------|----------|
| $C^2D$ | -1104.8296  | 964.6767    | -251.1849  | 27.5374  |
| $C^3$  | 75.8759     | -68.7066    | 18.7087    | -1.9568  |
| $CD^2$ | -10859.7647 | 12081.3927  | -4078.3578 | 401.0778 |
| $CD$   | 2518.3306   | -2831.8652  | 970.0610   | -97.1894 |
| $C$    | -147.7759   | 165.3412    | -56.4296   | 5.6820   |
| $D^3$  | -1896.6714  | 2283.9892   | -893.5652  | 119.4325 |
| $D$    | 677.7034    | -764.1779   | 272.3790   | -33.1051 |
| 1      | -51.6867    | 54.9470     | -17.4602   | 3.2163   |

Table S11. For $21 \leq \text{MLT} < 3$ when $100 \text{ nT} \leq \text{AE}^* \leq 300 \text{ nT}$.  

|       | $A^3$       | $A^2$       | $A$        | 1        |
|-------|-------------|-------------|------------|----------|
| $B^2C^2D^2$ | -57019.3361 | 43046.7962  | -10082.0642 | 734.2537 |
| $B^2C^2D$   | 26850.9381  | -18283.2664 | 3771.4534  | -233.2010|
| $B^2C^2$    | -2572.1777  | 1423.4839   | -204.7980  | 4.6060   |
| $B^2CD^2$   | -28576.4602 | 23138.1742  | -5344.0995 | 331.6583 |
| $B^2CD$     | 23254.2636  | -19765.9681 | 4821.6157  | -324.0250|
| $B^2C$      | -4627.3395  | 4144.7511   | -1084.7928 | 81.1160  |
| $B^2D^2$    | -3724.8879  | 5266.9111   | -1561.2734 | 101.8974 |
| $B^2D$      | 6397.9463   | -1341.9042  | 187.9051   | -3.8626  |
| $B^2$       | -1655.9537  | 1735.3889   | -481.9074  | 33.9444  |
| $BC^2D^2$   | 54049.7630  | -40590.7766 | 9461.8151  | -686.6623|
| $BC^2D$     | -25582.3355 | 17266.6460  | -3531.5461 | 216.7897 |
| $BC^2$      | 2466.9510   | -1341.9042  | 187.9051   | -3.8626  |
| $BCD^2$     | 29740.3530  | -24332.8418 | 5690.6267  | -357.2131|
| $BCD$       | -24353.1457 | 20852.1989  | -5133.2978 | 347.5365 |
| $BC$        | 4808.6367   | -4323.6752  | 1136.2007  | -85.0516 |
| $BD^2$      | 4602.6476   | -6209.6000  | 1885.5551  | -136.1870|
| $BD$        | -6905.4185  | 7690.0970   | -2191.3714 | 159.2823 |
| $B$         | 1729.1972   | -1824.6403  | 515.9028   | -37.7617 |
| $C^2D^2$    | -3648.1521  | 3017.1415   | -795.8842  | 67.5684  |
| $C^2D$      | 2072.6618   | -1645.7120  | 424.8206   | -36.4804 |
| $C^2$       | -268.1996   | 209.5025    | -54.8183   | 4.9326   |
| $CD^3$      | -4172.9902  | 3801.8255   | -1057.3326 | 87.1069  |
| $CD$        | 3148.4384   | -2908.2295  | 818.6345   | -68.2110 |
| $C$         | -521.9749   | 489.1567    | -139.7461  | 11.8942  |
| $D^3$       | 140.1179    | -8.3992     | -14.1078   | 1.1023   |
| $D$         | 54.9715     | -134.5670   | 46.2524    | -3.3622  |
| 1           | -39.8132    | 51.6017     | -15.2822   | 2.5156   |
Table S12. For $21 \leq \text{MLT} < 3$ when $\mathbf{A}^e > 300$ nT.

| $\mathbf{A}^2$ | $\mathbf{A}^2$ | $\mathbf{A}$ | $\mathbf{1}$ |
|----------------|----------------|-------------|--------------|
| $-4767.3972$   | $2931.6324$    | $-186.9034$ | $-13.7966$   |
| $8849.6518$    | $-3612.6320$   | $-73.7785$  | $71.8434$    |
| $-3941.6253$   | $1194.0994$    | $218.7614$  | $-49.3895$   |
| $-237.0725$    | $120.0989$     | $-38.6316$  | $2.3034$     |
| $-1389.1577$   | $1306.8915$    | $-298.5480$ | $21.8833$    |
| $1803.1691$    | $-1589.1820$   | $386.2908$  | $-28.9243$   |
| $-2120.6692$   | $1732.8122$    | $-434.2252$ | $33.8314$    |
| $4825.2319$    | $-4080.9516$   | $1061.9505$ | $-85.3927$   |
| $-2359.9272$   | $2116.9879$    | $-582.8724$ | $47.8042$    |
| $-1790.6478$   | $39.7558$      | $24.5674$   |              |
| $-7322.6675$   | $2447.2619$    | $347.6579$  | $-91.0338$   |
| $3323.3825$    | $-761.7557$    | $-309.4931$ | $54.8654$    |
| $29.1433$      | $-12.5949$     | $30.9798$   | $-3.0094$    |
| $1803.2254$    | $-1493.6573$   | $299.5264$  | $-18.8906$   |
| $-2002.5882$   | $1667.8647$    | $-381.6669$ | $26.9726$    |
| $2285.5556$    | $-1890.1988$   | $478.2440$  | $-37.4343$   |
| $-5197.1076$   | $4430.9076$    | $-1158.6021$| $93.2198$    |
| $2535.1290$    | $-2279.0977$   | $627.2383$  | $-51.3663$   |
| $76.2616$      | $-136.5977$    | $51.8708$   | $-4.7539$    |
| $-163.8528$    | $276.4589$     | $-101.8775$ | $9.0071$     |
| $-25.5844$     | $-36.2580$     | $19.3630$   | $-1.5412$    |
| $-117.7559$    | $131.5090$     | $-44.7371$  | $4.1161$     |
| $248.3031$     | $-299.9492$    | $106.9029$  | $-10.2377$   |
| $-57.1891$     | $106.1994$     | $-45.2434$  | $4.7107$     |
| $-192.7063$    | $172.2809$     | $-44.3984$  | $3.3900$     |
| $475.4294$     | $-421.9168$    | $108.1886$  | $-8.2028$    |
| $-241.3631$    | $214.4019$     | $-54.5530$  | $5.2218$     |

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