MHD simulation on magnetic compression of field reversed configurations with NIMROD

Y. Ma\textsuperscript{1}, P. Zhu\textsuperscript{1,2,*}, B. Rao\textsuperscript{1,*} and H. Li\textsuperscript{3}

\textsuperscript{1} International Joint Research Laboratory of Magnetic Confinement Fusion and Plasma Physics, State Key Laboratory of Advanced Electromagnetic Engineering and Technology, School of Electrical and Electronic Engineering, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China
\textsuperscript{2} Department of Engineering Physics, University of Wisconsin-Madison, Madison, WI 53706, United States of America
\textsuperscript{3} College of Physics and Optoelectronic Engineering, Shenzhen University, Shenzhen 518060, China

E-mail: zhup@hust.edu.cn and borao@hust.edu.cn

Received 6 October 2022, revised 18 January 2023
Accepted for publication 21 February 2023
Published 6 March 2023

Abstract

Magnetic compression has long been proposed a promising method for plasma heating in a field reversed configuration (FRC). However, it remains a challenge to fully understand the physical mechanisms underlying the compression process, due to its highly dynamic nature beyond the one-dimensional (1D) adiabatic theory model (Spencer \textit{et al} 1983 \textit{Phys. Fluids} \textbf{26} 1564). In this work, magnetohydrodynamics simulations on the magnetic compression of FRCs using the NIMROD code (Sovinec \textit{et al} 2004 \textit{J. Comput. Phys.} \textbf{195} 355) and their comparisons with the 1D theory have been performed. The effects of the assumptions of the theory on the compression process have been explored, and the detailed profiles of the FRC during compression have been investigated. The pressure evolution agrees with the theoretical prediction under various initial conditions. The axial contraction of the FRC can be affected by the initial density profile and the ramping rate of the compression magnetic field, but the theoretical predictions on the FRC’s length in general and the relation $r_s = \sqrt{2}r_o$ in particular hold approximately well during the whole compression process, where $r_s$ is the major radius of FRC separatrix and $r_o$ is that of the magnetic axis. The evolutions of the density and temperature can be affected significantly by the initial equilibrium profile and the ramping rate of the compression magnetic field. During the compression, the major radius of the FRC is another parameter that is susceptible to the ramping rate of the compression field. Basically, for the same magnetic compression ratio, the peak density is higher and the FRC’s radius $r_s$ is smaller than the theoretical predictions.

Keywords: field reversed configuration, compact toroid, magnetic compression, MHD simulation

(Some figures may appear in colour only in the online journal)
1. Introduction

The field reversed configuration (FRC) is a compact toroid that is dominated by the poloidal magnetic field [1]. The advantages of FRC as a magnetic confinement of high-temperature plasma include the high averaged beta [2], the linear device geometry, the stability exceeding expectations from the magnetohydrodynamics (MHD) theory [3, 4], and the natural divertor structure [5]. Besides these, an FRC plasma can maintain its magnetic structure during the supersonic translation [6, 7] and survive the violent collision and merging process [8, 9]. FRC is considered a promising candidate for compact fusion reactor due to its simplicity, high power density, and robustness [10]. To achieve fusion through the path of FRC, one approach is to maintain a steady state FRC in the high performance regime [11] using the advanced biasing control [12] and neutral-beam injections [13]. Another approach is to compress FRC in a short time [14–17]. Recently, an FRC device based on the collision merging is under construction in Huazhong University of Science and Technology (‘HFRC’) [18, 19], where preliminary experiments on the magnetic compression of the merged FRC are planned.

The magnetic compression has long been explored as a potentially effective method for the heating of FRCs [3]. The FRX-C/LSM (Field-Reversed Experiment-C/Large ‘S’ Modification) experiments show that the plasma temperature increases about four times through magnetic compression [20], and experiments on the IPA (Inductive Plasma Accelerator) device show that the FRC can be compressed to keV [17]. However, the short time scale of the compression process (typically on the order of tens of microseconds) and the drastically dynamic behaviors of the FRC plasma make it difficult to develop effective diagnoses. Previously a one-dimensional (1D) adiabatic theory [21] developed to predict the scalings of FRC compression has been found consistent with the radially averaged temperature and density in the initial stage of FRC compression in the FRX-C/LSM experiments [20]. Later a NIMROD simulation of the fast magnetic compression of FRC plasma shows that the rise of peak pressure follows the theory prediction within the initial 10 μs in general [22].

Despite these earlier studies, the applicability of the theory remains to be more systematically examined in simulations. In particular, the 1D theory is based on several assumptions that may over-simplify the more realistic situations. For example, the theory assumes a quasi-static and highly elongated FRC during compression and temperature or density profile effects, as well as the curvature of field lines in a 2D magnetic configuration are neglected. The exact extent to which these assumptions are valid or not is yet to be further assessed.

In this work, we perform simulations on the magnetic compression of the FRC using the resistive MHD model implemented in the NIMROD code [23]. To provide a more comprehensive view of the magnetic compression process based on the Spencer theory [21] while confirming some key scaling laws of the analytical model predictions, the numerical model is chosen to be more comparable with the theory than the experiments, and the physical parameters are set as close to the analytical model as possible. Although the single MHD model may not possess the necessary non-ideal or kinetic physics elements for an accurate evaluation of the FRC plasma stability, the analytical theory and scaling laws developed by Spencer [21] from a 1D single MHD model nonetheless agree approximately with the previous FRC compression experiments [20], which indicates the adequacy and significance of MHD model for capturing the dominant order physics of the FRC compression process. In simulations the plasma pressure and \( r_s = \sqrt{2} r_o \) relation agree well with the theoretical predictions. Under various conditions, the axial contraction of the FRC may slightly differ but is still overall consistent with the theory. By contrast, the evolutions of the radius, temperature and density are condition-sensitive. In general the radius is smaller and the peak density is higher than the theoretical predictions.

The paper is organized as follows. The analytical model and the numerical approach are introduced in section 2. The simulation results are presented in section 3. Starting with a representative simulation case (section 3.1), the effects of the initial density profile (section 3.2) and the effects of the ramping rate of the compression magnetic field (section 3.3) are discussed. Finally, the discussion and conclusions are presented in section 4.

2. Analytical model and numerical model

2.1. Analytical model

For the purpose of comparison, we briefly review the 1D adiabatic model of FRC compression developed by Spencer [21]. Assuming the quasi-static condition, the FRC compression process is approximated as a sequence of equilibrium states. For a highly elongated FRC, its 2D equilibrium solution follows the theory prediction within the initial 10 μs in general [22].

Despite these earlier studies, the applicability of the theory remains to be more systematically examined in simulations. In particular, the 1D theory is based on several assumptions that may over-simplify the more realistic situations. For example, the theory assumes a quasi-static and highly elongated FRC during compression and temperature or density profile effects, as well as the curvature of field lines in a 2D magnetic configuration are neglected. The exact extent to which these assumptions are valid or not is yet to be further assessed.

In this work, we perform simulations on the magnetic compression of the FRC using the resistive MHD model implemented in the NIMROD code [23]. To provide a more comprehensive view of the magnetic compression process based on the Spencer theory [21] while confirming some key scaling laws of the analytical model predictions, the numerical model is chosen to be more comparable with the theory than the experiments, and the physical parameters are set as close to the analytical model as possible. Although the single MHD model may not possess the necessary non-ideal or kinetic physics elements for an accurate evaluation of the FRC plasma stability, the analytical theory and scaling laws developed by Spencer [21] from a 1D single MHD model nonetheless agree approximately with the previous FRC compression experiments [20], which indicates the adequacy and significance of MHD model for capturing the dominant order physics of the FRC compression process. In simulations the plasma pressure and \( r_s = \sqrt{2} r_o \) relation agree well with the theoretical predictions. Under various conditions, the axial contraction of the FRC may slightly differ but is still overall consistent with the theory. By contrast, the evolutions of the radius, temperature and density are condition-sensitive. In general the radius is smaller and the peak density is higher than the theoretical predictions.

The paper is organized as follows. The analytical model and the numerical approach are introduced in section 2. The simulation results are presented in section 3. Starting with a representative simulation case (section 3.1), the effects of the initial density profile (section 3.2) and the effects of the ramping rate of the compression magnetic field (section 3.3) are discussed. Finally, the discussion and conclusions are presented in section 4.

2. Analytical model and numerical model

2.1. Analytical model

For the purpose of comparison, we briefly review the 1D adiabatic model of FRC compression developed by Spencer [21]. Assuming the quasi-static condition, the FRC compression process is approximated as a sequence of equilibrium states. For a highly elongated FRC, its 2D equilibrium solution follows the theory prediction within the initial 10 μs in general [22].

Despite these earlier studies, the applicability of the theory remains to be more systematically examined in simulations. In particular, the 1D theory is based on several assumptions that may over-simplify the more realistic situations. For example, the theory assumes a quasi-static and highly elongated FRC during compression and temperature or density profile effects, as well as the curvature of field lines in a 2D magnetic configuration are neglected. The exact extent to which these assumptions are valid or not is yet to be further assessed.

In this work, we perform simulations on the magnetic compression of the FRC using the resistive MHD model implemented in the NIMROD code [23]. To provide a more comprehensive view of the magnetic compression process based on the Spencer theory [21] while confirming some key scaling laws of the analytical model predictions, the numerical model is chosen to be more comparable with the theory than the experiments, and the physical parameters are set as close to the analytical model as possible. Although the single MHD model may not possess the necessary non-ideal or kinetic physics elements for an accurate evaluation of the FRC plasma stability, the analytical theory and scaling laws developed by Spencer [21] from a 1D single MHD model nonetheless agree approximately with the previous FRC compression experiments [20], which indicates the adequacy and significance of MHD model for capturing the dominant order physics of the FRC compression process. In simulations the plasma pressure and \( r_s = \sqrt{2} r_o \) relation agree well with the theoretical predictions. Under various conditions, the axial contraction of the FRC may slightly differ but is still overall consistent with the theory. By contrast, the evolutions of the radius, temperature and density are condition-sensitive. In general the radius is smaller and the peak density is higher than the theoretical predictions.

The paper is organized as follows. The analytical model and the numerical approach are introduced in section 2. The simulation results are presented in section 3. Starting with a representative simulation case (section 3.1), the effects of the initial density profile (section 3.2) and the effects of the ramping rate of the compression magnetic field (section 3.3) are discussed. Finally, the discussion and conclusions are presented in section 4.

2. Analytical model and numerical model

2.1. Analytical model

For the purpose of comparison, we briefly review the 1D adiabatic model of FRC compression developed by Spencer [21]. Assuming the quasi-static condition, the FRC compression process is approximated as a sequence of equilibrium states. For a highly elongated FRC, its 2D equilibrium solution reduces to the 1D relation

\[
p_m = p + \frac{B_s^2}{2 \mu_0} = \frac{B_w^2}{2 \mu_0},
\]

where \( p_m \) is the maximum pressure and \( B_w \) is the magnetic field outside the FRC plasma, and the cylindrical coordinate system \((r, \theta, z)\) is introduced. From the ansatz that the pressure \( p \) is only a function of flux \( \Psi = 2\pi \int_0^r B_s r' dr' \) and \( \Psi \) is an even function of \( r^2 - r_o^2 \), the following relation between the major radius of separatrix \( r_s \) and that of the magnetic axis \( r_o \) can be derived from equation (1) [24],

\[
r_s = \sqrt{2} r_o.
\]

Considering that in the axial direction there is no plasma at \( z = 0 \), and the conservation of magnetic flux at \( z = 0 \) and \( z \gg 0 \), the volume averaged plasma beta \( \langle \beta \rangle = \frac{1}{\pi r_s^2} \int_0^{r_s} \frac{p}{B_s^2 / (2\mu_0)} 2\pi r dr = 1 - \frac{x_s^2}{2} \) (3)

where the \( x_s = r_s / r_o \) is the ratio of the separatrix radius to the wall radius. Under the quasi-static assumption, the plasma velocity \( \vec{u} = 0 \), and the total energy within the separatrix is
2.2. Numerical model

Simulations in this work have been performed using the NIMROD code [23]. The resistive single-fluid MHD equations solved in simulations are as follows

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{u}) = 0 \tag{5}
\]

\[
\rho \left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = \vec{J} \times \vec{B} - \nabla p - \nabla \cdot \vec{\Pi} \tag{6}
\]

\[
\frac{n}{\gamma - 1} \left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) \cdot \nabla T = -p \nabla \cdot \vec{u} + Q \tag{7}
\]

\[
\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \tag{8}
\]

\[
\vec{E} = -\vec{u} \times \vec{B} + \eta \vec{I} \tag{9}
\]

where \( n \) is the plasma number density, \( \rho \) the mass density, \( p \) the pressure, \( T = T_1 + T_2 \) the total temperature, \( \vec{u} \) the plasma velocity, \( \vec{J} \) the current density, \( \vec{\Pi} \) the viscosity tensor, \( \vec{B} \) the magnetic field, \( \vec{E} \) the electric field, \( \eta \) the resistivity and \( Q = \eta \vec{J}^2 \) represents the resistive heating source. For comparison with Spencer’s scaling laws, the ideal MHD model of FRC compression is approximated with the above resistive MHD simulation model. In particular, the Spitzer’s resistivity model is used

\[
\eta(t) = \eta_0 \left( \frac{T(t)}{T_0} \right)^{-\frac{2}{3}} \tag{10}
\]

where \( T_0 \) is a constant that is set to the maximum temperature at \( t = 0 \), \( \eta_0/\mu_0 = 1 \text{ m}^2/\text{s} \), which is much smaller than the typical resistivity of FRCs [19, 28, 29]. Although the ohmic heating effect is not considered in the analytical model, the ohmic heating energy induced in the simulations is two orders of magnitude smaller than the internal energy and thus negligible. A parallel viscosity of \( 10^4 \text{ m}^2/\text{s} \) is also used following previous FRC simulations [28, 30]. These dissipation parameters are introduced to maintain the numerical stability of the simulations, where only the minimal range of values are kept to ensure the numerical convergence of the simulation results. The time step of simulations is set to \( \Delta t = 5 \times 10^{-9} \text{ s} \) to resolve the Alfvénic scale of the dynamic compression process.

To apply the self-consistent boundary conditions in the NIMROD simulations, the normal component of \( \vec{B} \) and the tangential component of \( \vec{E} \) are specified and other components of \( \vec{B} \) and \( \vec{E} \) are calculated self-consistently. The NIMROD code implements the finite element discretization in the poloidal plane and the Fourier series decomposition in the toroidal direction. For the magnetic compression simulations, both electric and magnetic boundary conditions are assumed axisymmetric, so that they are applied to the \( N = 0 \) Fourier component only. Similar to the previous simulations [28], the vector potential at boundary is set as

\[
\vec{A} = A_0 \hat{\theta} = r_w B_z f(t) g(t) \hat{\theta}, \tag{11}
\]

\[
f(z) = \frac{1}{1 + e^{(z-z_i)/\lambda} + e^{(z-z_f)/\lambda}} \tag{12}
\]

\[
\vec{E} = -\frac{\partial \vec{A}}{\partial t} = -r_w B_z f(z) g(t) \hat{\theta} \tag{13}
\]

and the magnetic field can be calculated according to \( \vec{B} = \nabla \times \vec{A} \). By specifying the \( E_\parallel \) and \( B_z \), respectively, it is expected to have a compression magnetic field \( B_z(z) \) ramping with time, which also satisfies the divergence-free condition at boundary.

3. Simulation results

3.1. The dynamic process of magnetic compression

The initial FRC equilibrium before the start of compression is prepared using the NIMEQ code [31, 32]. The axial length of the simulation domain is 4 m and the radius is \( r_w = 0.3 \text{ m} \) as commonly designed for a modern FRC device. In the \((r,z)\) poloidal plane, the grid consists of \(16 \times 72 \) finite elements with the sixth-order polynomial basis functions along each direction. The plasma pressure outside the separatrix is assumed to be constant. The simulation starts from the simplest uniform number density profile, and the effects of the initial
density profile will be discussed in section 3.2. The initial magnetic field strength at wall $B_w$ is about 0.4 T, density is $n_0 = 1 \times 10^{21} \text{ m}^{-3}$ and the maximum total temperature is $T_0 = 422$ eV. For higher efficiency, a large $x_s$ is often adopted in the magnetic compression experiment [17], and as an ideal limit case, the simulation starts with $x_s \sim 1$ in this work. The magnetic compression boundary condition discussed in section 2.2 is applied from $z = -2 \text{ m}$ to $z = 2 \text{ m}$ at $r = r_w$ as shown as figure 1, and all boundaries are assumed to be the no-flow solid walls.

For a linear ramping of the applied magnetic field $B_w$ at wall, the consequent FRC evolution during the magnetic compression process is obtained from NIMROD simulations. The comparisons between the NIMROD results and the 1D theory are shown in figure 2, where the simulation cases are performed with different maximum toroidal mode number $N = 0, 1$ and 2 respectively without any initial perturbation internal to the simulation domain. As can be seen from figure 2, the simulation results with different maximum toroidal mode numbers are exactly the same, and no $N > 0$ instability is observed, which confirms the numerical convergence and accuracy of the simulation.

For comparison with the 1D theory, the maximum pressure $p_m$, maximum total temperature $T_m$, maximum plasma number density $n_m$ and maximum $B_z$ at the $z = 0$ plane are plotted in figure 2 respectively, where $x_s = r_s/r_w$ is the relative radius and $l$ is the length of the FRC. The subscript ‘1’ and subscript ‘2’ represent the initial equilibrium state at $t = 0$ and the final compressed state at the corresponding time respectively. The curves of theoretical predictions are plotted using scaling laws shown in table 1. The simulation results for all $p_m$, $T_m$ and $l$ are in good agreement with the theory during the compression.

The FRC shrinks at a faster rate in the radial direction and the maximum number density at the middle plane rises more rapidly compared to the theoretical predictions, which are consistent with the experimental results [20]. Close inspection of the figure 2 shows that the majority of the volume reduction occurs during $B_{w2}/B_w1 < 3$, after which the rates of shrinkage in both the axial and radial directions decrease. The simulation runs until the magnetic compression ratio $B_{w2}/B_{w1} \sim 8$. This is because, on the one hand, at the late stage of the compression process the length of the FRC shrinks to less than 0.3 m when the single-fluid MHD model becomes less applicable since the finite Larmor radius and kinetic effects may no longer be negligible; on the other hand, the mesh does not get finer accordingly as the FRC shrinks, so that the numerical error becomes more significant. The agreement between the simulation results and the theory for the curves of $p_m$, $T_m$ and $l$ exceeds expectation, because the FRC is quickly compressed from elongated to oblate through a dynamic process in the simulation different from the quasi-static condition assumed in the theory. The comparison suggests that the theory appears applicable beyond the limitation from its own assumptions. The applicability of the theory will be further checked in sections 3.2 and 3.3.

The radial profiles of the pressure, temperature, density and the axial magnetic field along the middle plane $z = 0$ at different times can be used to illustrate the compression process (figure 3). The radii of zero crossings of the $B_z$ shown in figure 3(d) are those of the magnetic axis or $O$-point of FRC, i.e. $r_o$ at corresponding times. It is noted that the pressure and the temperature profiles continue to steepen on both sides of $r_o$, and form a narrow region of sharp gradients by the last stage of the compression process. Furthermore,
Figure 2. Comparisons of the theory (blue lines) and representative simulation results with the toroidal mode number $N = 0$ (green rectangles), $N = 0, 1$ (purple circles) and $N = 0, 1, 2$ (red plus signs), and (a) the compression field $B_w$ variation with time, for the (b) pressure ratio $p_2/p_1$, (c) temperature ratio $T_2/T_1$, (d) density ratio $n_2/n_1$, (e) radius ratio $x_{s2}/x_{s1}$ and (f) length ratio $l_2/l_1$ as functions of the magnetic compression ratio $B_{w2}/B_{w1}$. The majority of the volume reduction occurs during the stage $B_{w2}/B_{w1} < 3$ (yellow shade).

The temperature outside $r_o$ is higher than the inside instead of peaking around $r_o$ as in the case of pressure, which can also be seen from the contours of the pressure and temperature in figure 4. The possible reason for forming this kind of pressure and temperature profiles is that the FRC compression rate is slower than the maximum acoustic speed, while the thermal conduction terms especially the parallel thermal conduction are not taken into account in the adiabatic model for pressure in simulations. The pressure and temperature contours in the poloidal plane at a sequence of time show the expected FRC shrinking toward the center and the transition from elongated to oblate shape during the compression process (figure 4). Moreover, the FRC maintains a good symmetry about the middle plane at $z = 0$ in the poloidal plane.
Figure 3. Radial profiles along $z = 0$ of the (a) pressure, (b) temperature, (c) density and (d) $B_z$ at different times during the compression process respectively. The simulation is same as the $N = 0$ case shown in figure 2.

throughout the compression process, and no $N = 0$ instability such as the Roman candle instability [1] occurs. In contrast the radial density profile loses its initial hollow structure after $t = 10 \mu s$, and the maximum density accumulates at the cylinder axis at $r = 0$ (figure 3), which however does not increase monotonically with time but decreases slightly after $t = 35 \mu s$. 

Figure 4. (a) Pressure and (b) temperature contours in the $r - z$ plane of the same simulations as shown in figure 2 at $t = 0 \mu s$, $t = 15 \mu s$, $t = 25 \mu s$, and $t = 50 \mu s$ respectively.
The maximum values of pressure, temperature and density are not at the same location as shown in figure 3, which may be why they could be higher than the theoretical predictions at certain given time (figure 2).

For all the simulations in absence of initial perturbation, not only that no MHD instability is observed, but also that the magnetic field structure in the radial direction remains essentially same when higher toroidal components \((N = 1, 2)\) are included. The \(r_s\) and the \(\sqrt{2} r_o\) are approximately equal during the compression process, where the major radius of the magnetic axis \(r_o\) is the radius of the zero crossing of the \(B_z\) at \(z = 0\), and the radius of the separatrix \(r_s\) is the position where the flux \(\Psi = 0\) outside the \(r_o\) at \(z = 0\) (figure 5), which confirms the theory prediction in equation (2) that \(r_s = \sqrt{2} r_o\). In spite of the supersonic region near the separatrix inside the FRC, figure 5 implies that the quasi-static equilibrium relation in equation (1) is approximately satisfied at the middle plane and the axisymmetry holds well during the compression.

### 3.2. Effects of the initial number density profile

Section 3.1 starts with the simplest uniform number density profile assumption. However, the measured number density profile of FRC equilibrium is usually nonuniform [33–36]. In order to study the effects of number density profile on the compression process, a density profile similar to the rigid rotor (RR) equilibrium is adopted for comparison purpose

\[
n(\Psi) = n_0 \left( \frac{p(\Psi)}{p_0} \right)^\Gamma, \quad (14)
\]

where \(\Psi\) is the poloidal flux, \(n_0 = 1 \times 10^{21} \text{m}^{-3}\) is the initial density used in section 3.1, \(p_0\) is the pressure at the magnetic axis and \(\Gamma\) is the profile index where \(\Gamma = 0\) implies a uniform initial density distribution as the case in section 3.1. For \(\Gamma = \frac{1}{2}\) used in this section, the following radial density profile from the RR model with \(k = 0.9\) at the \(z = 0\) plane [3, 24]

\[
n(r) = n_m \text{sech}^2 \left[ k \left( \frac{2 r^2}{r_s^2} - 1 \right) \right], \quad (15)
\]

can be essentially reproduced, where \(k\) is the shape parameter and \(n_m\) is the maximum density. Similar number density profile is measured on FRX-L (Field-Reversed Experiment-Liner) [37], even though different density profiles are also found in other FRC experiments [38–40]. The specific initial 1D profile and 2D distribution of number density are shown in the figure 6. Compared with the simulation in section 3.1, the equilibrium pressure profile and boundary conditions remain the same, and only the initial density and temperature distributions are changed.

Figure 7 shows the simulation results with nonuniform initial density compared with the 1D theory and the simulation results with the uniform initial density. The simulation results with the uniform initial density are same as the case in section 3.1. Because the \(B_w\) is the sum of the magnetic field externally applied at the boundary condition and induced by the plasma, the \(B_w\) of the two cases shown in figure 7(a) are slightly different after \(t > 25 \mu s\), even though the externally applied components are the same. As shown in figure 7, the peak pressure ratio \(p_2/p_1\) curves of the two simulation cases are basically identical. The \(L_2/L_1\) curves of the two simulation cases differ slightly in the interval of \(B_{w2}/B_{w1} > 2.5\) and \(B_{w2}/B_{w1} < 4\), and the simulation case with nonuniform initial density shrinks slightly slower than the uniform initial density case in the radial direction after \(B_{w2}/B_{w1} > 3\). The evolutions of the temperature and density during the compression process are mostly affected by the initial density profile. Compared to the case with uniform initial density, the maximum temperature for nonuniform initial density profile rises more rapidly after \(B_{w2}/B_{w1} > 2.5\), and the maximum density rises more slowly after \(B_{w2}/B_{w1} > 2\).

A close inspection of the density profile is shown in figure 8. Similar to the simulation results in section 3.1, the radial density profile loses the hollow density structure before \(t = 10 \mu s\) during the compression process, and the density profile changes from elongated to oblate. Furthermore, as can be seen from figure 9, the time history of \(r_s\) from simulation confirms the theoretical prediction \(r_s = \sqrt{2} r_o\), similar to the case with uniform initial density, which implies that the quasi-static equilibrium relation in equation (1) at the middle plane and
both the externally applied boundary and the plasma induced ear with time, for the reason explained earlier that it includes the increment of as the ramping rates of other three cases are approximated (green circles in figure 10). Simulation cases with four different ramping rates of the compression magnetic field are compared (figure 10). The simulation case with an approximate ramping rate of 0.05 T µs⁻¹ (green circles in figure 10) is same as presented in section 3.1. The ramping rates of other three cases are approximated as 0.03 T µs⁻¹, 0.06 T µs⁻¹ and 0.08 T µs⁻¹ respectively. The increment of $B_w$ is approximately but not exactly linear with time, for the reason explained earlier that it includes both the externally applied boundary and the plasma induced contributions. A faster ramping rate of the $B_w$ basically implies a more intense dynamic process. As the ramping rate of the compression field increases, the peak temperature rises more slowly, the peak density rises more rapidly after $B_{w2}/B_{w1} > 3$, and the radius of the FRC decreases faster with $B_{w2}/B_{w1}$. The difference in ramping rate has a slight effect on the axial length variation in the range $2 < B_{w2}/B_{w1} < 4$, but has no effects on the pressure evolution. The 1D theory prediction for the pressure evolution remains in good agreement with the simulation results despite the dynamic effects considered in the simulation.

The correctness of the $p_m$ scaling law and the relationship $r_t = \sqrt{2} r_o$ both relies on the 1D static equilibrium relation (1). The deviations from the relation (1) are shown in figure 11. For most part of the compression process, the radial average of deviation from the relation (1) in each of the four simulation cases is less than 30%, and can reach almost zero at times. The deviation can be attributed to the dynamic radial motion and the curvature effects of an oblate shape. In general, the slower the ramping rates, the smaller is the deviation as expected. In all cases, the errors in static force balance in equation (1) initially increase with the radial motion to their maximums, and then both drop to their minimums before $B_{w2}/B_{w1} < 4$. Afterwards the elongation ratios $E_2/E_1$ of the four simulation cases decrease monotonically from 0.9–1.0 to 0.4–0.5 as the separatrix shape changes from elongated to oblate, and in the meantime the residual errors see slight increase, where the elongation $E = l/(2r_o)$, $E_1$ is the elongation at $t = 0$ and $E_2$ is the elongation at the corresponding time during the compression. The $x_s$ scaling law in the 1D model, however, relies on not only the 1D equilibrium relation (1) but also the condition of magnetic flux conservation $d\Psi = 0$ inside the separatrix [21].

**Figure 6.** (a) The 1D density profile along $z = 0$ of the initial equilibrium (blue dashed line) and the RR model with the shape parameter $\kappa = 0.9$ (red solid line). (b) The 2D contour of the initial density.
Figure 7. Simulation results with nonuniform initial density (purple circles) in comparisons with the 1D theory (blue lines) and the simulation results with the uniform initial density that is same as the $N = 0$ case shown in figure 2 (green rectangles). (a) The time dependence of the compression field $B_w$. (b) The pressure ratio $p_2/p_1$. (c) Temperature ratio $T_2/T_1$. (d) Density ratio $n_2/n_1$. (e) Radius ratio $x_2/x_1$. and (f) Length ratio $l_2/l_1$ as functions of the magnetic compression ratio $B_{w2}/B_{w1}$ for different initial density profiles.
Figure 8. (a) The 1D radial density profile along \( z = 0 \) at different times during the compression. (b) The density contours at different times during the compression. (c) The 1D radial plasma temperature profile along \( z = 0 \) at different times during the compression. (d) The plasma temperature contours at different times during the compression. The simulation is same as the case shown in figure 6.

Figure 9. The time history of the \( r_s \) of the theoretical prediction (red solid line) and the same simulation as shown in figure 7 (blue dashed line).
Figure 10. The comparison of the 1D theory (blue lines) and four simulation cases with different ramping rates of the compression magnetic field. The simulation with the ramping rate $\sim 0.05 \text{T} \mu\text{s}^{-1}$ is same as the $N = 0$ case shown in figure 2. (a) The compression field $B_w$ as a function of time. (b) The pressure ratio $p_2/p_1$, (c) temperature ratio $T_2/T_1$, (d) density ratio $n_2/n_1$, (e) radius ratio $x_{s2}/x_{s1}$, and (f) length ratio $l_2/l_1$ as functions of the magnetic compression ratio $B_{w2}/B_{w1}$ for different ramping rates of the $B_w$. 
Figure 11. (a)–(d) The elongation ratio $E_2/E_1$, normalized radial velocity $|V_r|$ at $r_s$ and the deviations from the relation $p + \frac{B^2}{2\mu_0} = \frac{B^2_w}{2\mu_0}$ as functions of the magnetic compression ratio $B_{w2}/B_{w1}$ for the same four simulation cases shown in figure 10, where the $|V_r|$ is normalized by its maximum value during the compression and the error% is the radial average value of $|LHS - RHS|/RHS$ for the relation $p + \frac{B^2}{2\mu_0} = \frac{B^2_w}{2\mu_0}$ from $r = 0$ to $r = r_s$ in the $z = 0$ middle plane.

Figure 12. The poloidal fluxes $\Psi$ as functions of the magnetic compression ratio $B_{w2}/B_{w1}$ for the same four simulation cases shown in figure 10.

The flux dissipation in the four simulation cases with different compression field ramping rates are shown in figure 12, where the faster compression field ramping rate leads to more flux loss, which corresponds to the faster decreasing rate of $x_s$, as well as the corresponding higher rate of density increase.

4. Discussion and conclusions

In summary, the simulations on the magnetic compression of FRCs using the NIMROD code, and their comparisons with the Spencer’s adiabatic theory [21] have been presented. The
single-fluid MHD model is adopted, and the physical parameters are set as close to the analytical model for the purpose of comparison. A set of self-consistent boundary conditions have been implemented to model the effects of the externally applied magnetic field for compression. The simulation results for the pressure evolution agree well with the theory prediction for various initial conditions. The axial contraction of the FRC is slightly faster than the theoretical prediction after \( B_{w2}/B_{w1} > 3 \). In the range \( B_{w2}/B_{w1} > 2 \) to \( B_{w2}/B_{w1} < 4 \), the evolution of the axial length is somewhat influenced by the initial density profile and the ramping rate of the compression field. Nevertheless, the theoretical prediction on the FRC’s length and the relation \( r_s = \sqrt{2} r_o \) hold approximately well during the entire compression process.

The initial density profile of the FRC and the ramping rate of the compression magnetic field both have significant effects on the evolutions of the temperature and density. Compared to the case with uniform initial density, for the case with nonuniform initial density profile, the peak density rises more slowly after \( B_{w2}/B_{w1} > 2 \), and the peak temperature rises more rapidly after \( B_{w2}/B_{w1} > 2.5 \). Based on the simulation results, a more central-peak initial density profile and a relatively slower ramping rate for the compression field are required for a more efficient heating. The major radius of the FRC is another parameter that is susceptible to the ramping rate of the compression field. A faster ramping rate of the compression field leads to a faster decrease in FRC’s radius. In general, for the same magnetic compression ratio, the peak density is higher and the radius is smaller than the theoretical predictions, which are consistent with the experimental results [20].

During the compression process, the pressure and temperature maintain a hollow structure, while their gradients increase on both radial sides of the magnetic axis at \( r_o \). Moreover, the temperature does not have a peaked profile near the \( r_o \) as the pressure, and the temperature outside \( r_o \) is higher than the inside. Unlike the pressure and temperature profiles, the radial density profile loses its hollow structure during the compression.

For the purpose of theory comparison, our simulations are based on the single-fluid MHD model, and the simulation results have reproduced the bulk part of the scaling laws of the FRC compression process as measured from the experiments and predicted by the Spencer theory, thus further confirming the significance of MHD model, likely due to the primarily axisymmetric and macroscopic dynamics that govern the compression process. For simulations to be more experimentally relevant, non-ideal and kinetic effects should also be taken into account. For examples, the finite-Larmor radius and the two-fluid effects are to be included next, along with more realistic values of resistivity and anisotropic thermal conductivities. When the non-axisymmetric perturbations and the toroidal flow are included, both MHD instability and the non-ideal, kinetic effects are expected to play significant roles in the compression process.

Acknowledgment

The authors thank Profs. C.R. Sovinec, Bihe Deng and Zhijiang Wang for helpful discussions. This work was supported by the National Key Research and Development Program of China (Grant Nos. 2017YFE0301805 and 2019YFE0305004), the Fundamental Research Funds for the Central Universities at Huazhong University of Science and Technology (Grant No. 2019kfyXJJS193), the National Natural Science Foundation of China (Grant No. 51821005), and the U.S. Department of Energy (Grant Nos. DE-FG02-86ER53218 and DE-SC0018001). The computing work in this paper was supported by the Public Service Platform of High Performance Computing by Network and Computing Center of HUST. The authors are very grateful for the supports from the NIMROD team and the J-TEXT team.

ORCID iDs

P. Zhu  https://orcid.org/0000-0002-5773-8861
H. Li  https://orcid.org/0000-0002-7050-1295

References

[1] Tuszewski M. 1988 Field reversed configurations  Nucl. Fusion 28 2033
[2] Lin M., Liu M., Zhu G., Shi P., Zheng J., Lu Q. and Sun X. 2017 Field-reversed configuration formed by in-vessel \( \theta \)-pinch in a tandem mirror device  Rev. Sci. Instrum. 88 093505
[3] Steinhauser L.C. 2011 Review of field-reversed configurations  Phys. Plasmas 18 070501
[4] Schwarzmeier J.L. et al. 1983 Magnetohydrodynamic equilibrium and stability of field-reversed configurations  Phys. Fluids 26 1295
[5] Steinhauser L.C. 1996 FRC 2001: a white paper on FRC development in the next five years  Fusion Technol. 30 116
[6] Sekiguchi J., Asai T. and Takahashi T. 2018 Super-Alfvénic translation of a field-reversed configuration into a large-bore dielectric chamber  Rev. Sci. Instrum. 89 013506
[7] Kobayashi D. and Asai T. 2021 Experimental evidence for super-Alfvénic acceleration of the field-reversed configuration due to a magnetic pressure gradient  Phys. Plasmas 28 022101
[8] Asai T. et al. 2021 Observation of self-organized FRC formation in a collisional-merging experiment  Nucl. Fusion 61 096032
[9] Asai T. et al. 2019 Collisional merging formation of a field-reversed configuration in the FAT-CM device  Nucl. Fusion 59 056024
[10] Dettrick S.A. et al. 2021 Simulation of equilibrium and transport in advanced FRCs  Nucl. Fusion 61 106038
[11] Binderbauer M.W. et al. 2015 A high performance field-reversed configuration  Phys. Plasmas 22 056110
[12] Tuszewski M. et al. 2012 Field reversed configuration confinement enhancement through edge biasing and neutral beam injection  Phys. Rev. Lett. 108 255008
[13] Gota H. et al. 2019 Formation of hot, stable, long-lived field-reversed configuration plasmas on the C-2W device  Nucl. Fusion 59 112009
[14] Slough J. et al 2016 Staged magnetic compression of FRC targets to fusion conditions ALPHA Annual Review
[15] Bol K., Ellis R.A., Eubank H., Furth H.P., Jacobsen R.A., Johnson L.C., Mazzucato E., Stodiek W. and Tolnas E.L. 1972 Adiabatic compression of the tokamak discharge Phys. Rev. Lett. 29 1495
[16] Intrator T. et al 2004 A high density field reversed configuration (FRC) target for magnetized target fusion: first internal profile measurements of a high density FRC Phys. Plasmas 11 2580
[17] Slough J., Votroubek G. and Pihl C. 2011 Creation of a high-temperature plasma through merging and compression of supersonic field reversed configuration plasmas Nucl. Fusion 51 053008
[18] Zhang M., Xuan J., Yang Y., Rao B., Xiao J., Zhang Y., Peng Y., Zhao Y., Yu K. and Pan Y. 2020 Design of high voltage pulsed power supply for HFRC IEEE Trans. Plasma Sci. 48 1688
[19] Peng Y., Yang Y., Jia Y., Rao B., Zhang M., Wang Z., Wang H. and Pan Y. 2022 Simulation on formation process of field-reversed configuration Nucl. Fusion 62 066037
[20] Rej D.J., Taggart D.P., Baron M.H., Chrien R.E., Gribble R.J., Tuszewski M., Waganaar W.J. and Wright B.L. 1992 High-power magnetic-compression heating of field-reversed configurations Phys. Fluids B 4 1909
[21] Spencer R.L. et al 1983 Adiabatic compression of elongated field-reversed configurations Phys. Fluids 26 1564
[22] Woodruff S., Macnab A.I.D. and Mattor N. 2008 Adiabatic compression of a doublet field reversed configuration (FRC) J. Fusion Energy 27 128
[23] Sovinec C.R., Glasser A.H., Gianakon T.A., Barnes D.C., Nebel R.A., Kruger S.E., Schnack D.D., Plimpton S.J., Tarditi A. and Chu M.S. 2004 Nonlinear magnetohydrodynamics simulation using high-order finite elements J. Comput. Phys. 195 355
[24] Armstrong W.T. et al 1981 Field-reversed experiments (FRX) on compact toroids Phys. Fluids 24 2068
[25] Intrator T.P., Siemon K.E. and Sieck P.E. 2008 Adiabatic model and design of a translating field reversed configuration Phys. Plasmas 15 042505
[26] Intrator T.P. et al 2008 Physics basis and progress for a translating FRC for MTF J. Fusion Energy 27 57
[27] Tuszewski M. 1988 A semiempirical formation model for field-reversed configurations Phys. Fluids 31 3754
[28] Milroy R.D., Kim C.C. and Sovinec C.R. 2010 Extended magnetohydrodynamic simulations of field reversed configuration formation and sustainment with rotating magnetic field current drive Phys. Plasmas 17 062502
[29] Guo H., Hoffman A.L., Milroy R.D., Steinhauser L.C., Brooks R.D., Deards C.L., Grossnickle J.A., Melnik P., Miller K.E. and Vlases G.C. 2008 Improved confinement and current drive of high temperature field reversed configurations in the new translation, confinement and sustainment upgrade device Phys. Plasmas 15 056101
[30] Macnab A.I.D., Milroy R.D., Kim C.C. and Sovinec C.R. 2007 Hall magnetohydrodynamics simulations of end-shorting induced rotation in field-reversed configurations Phys. Plasmas 14 092503
[31] Howell E.C. and Sovinec C.R. 2014 Solving the Grad–Shafranov equation with spectral elements Comput. Phys. Commun. 185 1415
[32] Li H. and Zhu P. 2021 Solving the Grad–Shafranov equation using spectral elements for tokamak equilibrium with toroidal rotation Comput. Phys. Commun. 260 107264
[33] Cobb J.W., Tajima T. and Barnes D.C. 1993 Profile stabilization of tilt mode in a field-reversed configuration Phys. Fluids B 5 3227
[34] Okada S., Kiso Y., Goto S. and Ishimura T. 1989 Reduction of the density profile of a field-reversed configuration plasma from detailed interferometric measurements J. Appl. Phys. 65 4625
[35] Gota H. et al 2018 Internal magnetic field measurements of translated and merged field-reversed configuration plasmas in the PAT-CM device Rev. Sci. Instrum. 89 103114
[36] Gota H. et al 2021 Overview of C-2W: high temperature, steady-state beam-driven field-reversed configuration plasmas Nucl. Fusion 61 106039
[37] Renneke R.M. 2007 Global power balance on high density field reversed configuration for use in magnetized target fusion PhD Thesis Purdue University
[38] Lee K.Y. 2020 Generalized radial profile of field-reversed configurations based on symmetrical properties Nucl. Fusion 60 046010
[39] Steinhauser L.C. and Intrator T.P. 2009 Equilibrium paradigm for field-reversed configurations and application to experiments Phys. Plasmas 16 072501
[40] Steinhauser L.C. 2008 Equilibrium rotation in field-reversed configurations Phys. Plasmas 15 012505