Shape of the inflaton potential and the efficiency of the universe heating

A.D. Dolgov, A.V. Popov, and A.S. Rudenko

Novosibirsk State University, Novosibirsk, 630090, Russia
ITEP, Bol. Cheremushkinskaya ul., 25, 113259 Moscow, Russia
Dipartimento di Fisica e Scienze della Terra, Università degli Studi di Ferrara
Polo Scientifico e Tecnologico - Edificio C, Via Saragat 1, 44122 Ferrara, Italy
Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation, Troitsk, Moscow, 142190 Russia
Budker Institute of Nuclear Physics, Novosibirsk, 630090, Russia

It is shown that the efficiency of the universe heating by an inflaton field depends not only on the possible presence of parametric resonance in production of scalar particles but also strongly depends on the shape of the oscillations of the inflaton around its equilibrium point. In particular, when the inflaton oscillations deviate from pure harmonic one towards a succession of step functions, the production probability rises by several orders of magnitude. This in turn leads to a higher temperature of the universe after inflaton decay. An example of the inflaton potential, which leads to such type of the field behavior and sufficiently long inflation, is presented.

I. INTRODUCTION

The cosmological inflation included, roughly speaking, the following two epochs. The first one was a quasi-exponential expansion, when the Hubble parameter, $H$, slowly changed with time and the universe expanded by a huge factor, $e^N$ where

$$N = \int H dt \gg 1.$$ \hspace{1cm} (1.1)

During this period the Hubble parameter exceeded the inflaton mass or, better to say, the condition was fulfilled

$$H^2 > \left| \frac{d^2U(\phi)}{d\phi^2} \right| \equiv |U''(\phi)|,$$ \hspace{1cm} (1.2)

where $U(\phi)$ is the potential of the inflaton field, $\phi$. Due to the large Hubble friction (see eq. (3.3)) during this time, the field $\phi$ remained almost constant slowly moving in the direction of the "force" ($-U'$).

The second stage began when $H^2$ dropped below $|U''|$ and lasted till the inflaton field reached the equilibrium value at $U'(\phi_{eq}) = 0$. It is usually assumed that $\phi_{eq} = 0$ and $U(\phi_{eq}) = 0$ to avoid nonzero vacuum energy. During this period $\phi$ oscillated around $\phi_{eq}$ producing elementary particles, mostly with masses below the frequency of the inflaton oscillations. This was a relatively short period which may be called big bang, when the initial vacuum-like state exploded, creating the primeval cosmological plasma.

The process of the universe heating was first studied in refs. [1–3] within the framework of perturbation theory. A non-perturbative approach was pioneered in refs. [4, 5]. In both these papers a possibility of excitation of parametric resonance, which might grossly enhance the particle (boson) production rate was mentioned. In the model of ref. [4] parametric resonance could not be effectively induced because of the red-shift and scattering of the produced particles which were dragged out of the resonance zone. In this work was dedicated to non-perturbative production of fermions. However, the resonance may be effective if it is sufficiently wide. In this case the particle production rate can be strongly enhanced [5, 6].

As is well known, parametric resonance exists for bosons only. In quantum language, it can be understood as Bose amplification of particle production due to presence of identical bosons in the final state, the same phenomena as the laser induced radiation. For bosons there can be another phenomenon leading to very fast and strong excitation of the bosonic field coupled to inflaton, if the effective mass squared of such field would be negative (tachyonic situation). It might naturally happen for sufficiently large and negative product $g\phi$, see below eqs. (2.1, 2.2). This is the Higgs-like effect, when vacuum state becomes unstable.

Both phenomena are absent in the case of fermion production. Imaginary mass of fermions breaks hermicity of the Lagrangian, so tachyons must be absent. As for parametric resonance, it is not present in the fermionic...
equations of motion. The latter property is attributed to the Fermi exclusion principle. Non-perturbative study of the fermion production shows that the production probability rises when the effective mass of fermions crosses zero and may be significant \cite{1, 2}.

A recent review on the mentioned above subjects can be found e.g. in ref. \cite{3}.

In this paper, we study how the efficiency of the parametric resonance excitation depends on the shape of the signal $\phi(t)$, where $\phi$ is an inflaton field. We argue that for some non-harmonic oscillations the resonance could be excited considerably faster; the cosmological particle production by the end of inflation would be much more efficient, and the temperature of the created plasma became noticeably higher. In sec. \textbf{II} we consider this problem in flat space-time, neglecting cosmological expansion, to get a feeling of a proper choice of the inflaton potential that could generate the signal $\phi(t)$ most efficient for particle production. In the next sec. \textbf{III} we study the evolution of the inflaton field in cosmological background for different potentials trying to make the form of $\phi(t)$ close to the signal in flat space-time for which the resonance is faster and stronger excited than it does for purely harmonic $\phi(t)$. We also comment there on the properties of inflationary cosmology with such modified inflaton potentials. In this section the scalar particle production by inflation is also considered. The results are compared to the particle production rate for the "harmonic" inflaton. In sec. \textbf{IV} we conclude.

We have chosen the sign and the amplitude of the initial inflaton field to avoid or to suppress tachyonic amplification of the produced field $\chi$. This case will be studied elsewhere.

\section{II. Parametric Resonance in Flat Space-Time}

Let us consider at first an excitation of parametric resonance in the classical situation, when the space-time curvature is not essential and the Fourier amplitude of the would-be resonating scalar field $\chi$ satisfies the equation of motion:

$$\ddot{\chi} + (m^2 + k^2 + g\phi) \chi = 0,$$

(2.1)

where $m_{\chi}$ is the mass of $\chi$, $k$ is its momentum, and $g$ is the coupling constant between $\chi$ and another scalar field $\phi$ with the interaction:

$$L_{int} = -g\phi \chi^* \chi.$$

(2.2)

This other field is supposed to be homogeneous, $\phi = \phi(t)$, and satisfying the equation of motion:

$$\ddot{\phi} + U'(\phi) = 0.$$

(2.3)

Our task here is to determine $U(\phi)$, so that the parametric resonance for $\chi$ would be most efficiently excited. An optimal meander form of the inflation field $\phi$ is suggested by the phase parameter approach in the theory of parametric resonance \cite{9}. Here we demonstrate that just a slight shift from the standard Mathieu model towards optimal inflaton potential leads to a drastic increase of the particle production rate. We compare the modified results with the standard case when the potential of $\phi$ is quadratic $U(\phi) = m^2\phi^2/2$ and so equation (2.3) has a solution $\phi(t) = \phi_0 \cos(mt + \theta)$, where amplitude $\phi_0$ and phase $\theta$ can be found from initial conditions. One can choose the moment $t = 0$ in such a way that $\theta = 0$, i.e. $\phi(t) = \phi_0 \cos mt$. Substituting this expression into eq. (2.1), we come to the well-known Mathieu equation:

$$\ddot{\chi} + \omega_0^2 (1 + h \cos mt) \chi = 0,$$

(2.4)

where $\omega_0^2 = m^2 + k^2$ and $h = g\phi_0/\omega_0^2$. When $h \ll 1$ and the value of $m$ is close to $2\omega_0/n$ (where $n$ is an integer), equation (2.4) describes a parametric resonance, when field $\chi$ oscillates with an exponentially growing amplitude. For $h \ll 1$, the solution of eq. (2.4) can be presented as a product of slowly (but exponentially) rising amplitude by a quickly oscillating function with frequency $\omega_0$:

$$\chi = \chi_0(t) \cos(\omega_0 t + \alpha).$$

(2.5)

The amplitude $\chi_0$ satisfies the equation:

$$-\dot{\chi}_0 \cos(\omega_0 t + \alpha) + 2\omega_0 \dot{\chi}_0 \sin(\omega_0 t + \alpha) = h\omega_0^2 \chi_0 \cos mt \cos(\omega_0 t + \alpha).$$

(2.6)

Let us multiply eq. (2.6) by $\sin(\omega_0 t + \alpha)$ and average over the period of oscillations. The right hand side would not vanish on the average, if $m = 2\omega_0$. In this case $\chi_0$ would exponentially rise if $\alpha = \pi/4$:

$$\chi_0 \sim \exp \left( \frac{1}{4} h\omega_0 t \right).$$

(2.7)
In this way we recovered the standard results of the parametric resonance theory.

The rise of the amplitude of $\chi$ is determined by the integral:

$$\frac{1}{2} \chi^2 + \frac{1}{2} \dot{\phi}^2 = -g \int dt \chi \dot{\phi},$$

as one can see from eq. (2.1). It can be shown that the maximum rate of the rise is achieved when $\phi$ is a quarter-period meander function (an oscillating succession of the step-functions with proper step duration), ref. [9].

Let us note that the behavior of the solution for $\chi$ would dramatically change with rising $h$. For a large $h$ the eigenfrequency squared of $\chi$ noticeably changes with time. It may approach zero and, if $|h| > 1$, it would even become negative for a while, see eq. (2.4). In the latter (tachyonic) case $\chi$ would rise much faster than in the case of classical parametric resonance. We postpone the study of tachyonic case for a future work, while here we confine ourselves to non-tachyonic situation.

To demonstrate an increase of the excitation rate for anharmonic oscillations we choose the potential for the would-be inflaton field $\phi$ satisfying the following conditions: at small $\phi$ it approaches the usual harmonic potential, $U \rightarrow m^2 \phi^2/2$, while for $\phi \rightarrow \infty$ it tends to a constant value. It is intuitively clear that in such potential field $\phi$ would live for a long time in the flat part of the potential and quickly change sign near $\phi = 0$. This behavior can be rather close to a periodic succession of the step-functions which we mentioned above. As an example of such potential we take

$$U(\phi) = \frac{1}{2} m^2 \phi^2 - \frac{1}{2} \lambda_2 (\phi / m_{Pl})^4,$$

where $m, \lambda_0, \lambda_2, \lambda_4$ are some constant parameters, with $m$ having dimension of mass, $\lambda_2$ being dimensionless. Here $m_{Pl}$ is the Planck mass. Observational data on the density perturbations induced by the inflaton demand $m \approx 10^{-15} m_{Pl}$ in the model with the inflaton potential $U(\phi) = m^2 \phi^2/2$, so we use $m = 10^{-8} m_{Pl}$ throughout the paper. We also take $\lambda_0 = 85, \lambda_2 = 4, \lambda_4 = 1$. The plots of potentials are shown in Fig. 1. Here and below the red (or dashed) curves are for the quadratic potential $U(\phi) = m^2 \phi^2/2$ and the blue ones are for potential (2.9). For our choice of $\lambda$ parameters the plots cross each other at points $\phi = 0, \pm 9 m_{Pl}$.

![FIG. 1: Potential of the inflaton $U(\phi)$. Red dashed line is quadratic potential $U(\phi) = m^2 \phi^2/2$, and blue line is potential (2.9). The parameters are $m = 10^{-8} m_{Pl}, \lambda_0 = 85, \lambda_2 = 4, \lambda_4 = 1$. Field $\phi$ is measured in units of $m_{Pl}$ and $U(\phi)$ is measured in units of $10^{-6} m_{Pl}^2$.](image)

We solved equation (2.1) with potential (2.9) numerically and compared this solution, $\dot{\phi}_U(t)$, with the harmonic solution, $\dot{\phi}_H(t) = \phi_0 \cos \omega t$. The results are presented in Fig. 2 for the initial conditions $\phi(0) = 4.64 \, m_{Pl}$, $\dot{\phi}(0) = 0$. The value of amplitude $\phi_0 = 4.64$ is chosen because in this case the frequencies of $\dot{\phi}_U(t)$ and $\dot{\phi}_H(t)$ are approximately equal.

For to the chosen shape of potential (2.9), function $\dot{\phi}_U(t)$ differs from cosine towards the step function. Therefore, one can expect that parametric resonance would be excited faster for potential (2.9) than for the quadratic one.

Now we can numerically solve equation (2.1) with the computed $\dot{\phi}(t)$, with the chosen for this example values: $m_\chi = 0, k = 1, g = 5 \cdot 10^{-8}$ (in units of $m$), and the initial conditions $\chi(0) = 1/\sqrt{2 \omega(0)} \approx 0.67, \dot{\chi}(0) = \sqrt{\omega(0)/2} \approx 0.745$, where $\omega^2(0) = k^2 + g \phi(0)$. These initial conditions correspond to solution of eq. (2.1) with constant $\omega$: $\chi(t) = e^{-i \omega t}/\sqrt{2 \omega V}$, where $V = 1$ (in units of $m^{-3}$) is the volume. As a result we obtain function $\chi(t)$ shown in Fig. 3. As expected, the amplitude of oscillations of $\chi$ increases faster in the case of potential (2.9); e.g. at $t = 450 \, m^{-1}$ the amplitude ratio is approximately two.

Since we are going to apply the results for the calculation of the particle production rate at the end of cosmological inflation, it would be appropriate to present the number density of produced $\chi$-particles $n_k(t)$, which is defined as

$$n_k(t) = \left( \frac{\chi^2}{2} + \frac{\omega_2^2 \lambda_2^2}{2} \right) \frac{1}{\omega_k}.$$

(2.10)
where the expression in the brackets is the energy of the mode with momentum $k$, and $\omega_k = \sqrt{k^2 + g\phi}$ is the energy of one $\chi$-particle.

The number densities of the produced $\chi$-particles for the harmonic $\phi$ and slightly step-like one are presented in Fig. 4 by red and blue curves, respectively.

Despite a decrease of particle number densities in some short time intervals, there is an overall exponential rise, which goes roughly as $\chi^2$. Therefore, the ratio of particle numbers for two the types of the potential is approximately equal to the ratio of the amplitude squared which is about four for $t = 450 \, m^{-1}$.

III. RESONANCE IN EXPANDING UNIVERSE

The universe heating after inflation was achieved due to coupling of the inflaton field $\phi$ to elementary particle fields. In this process mostly particles with masses smaller than the frequency of the inflaton oscillations were produced. The decay of inflaton could create both bosons and fermions. The boson production might be strongly enhanced due excitation of parametric resonance in the production process. Hence bosons were predominantly created initially. Later in the course of thermalization they gave birth to fermions. For a model description of the first stage of this process we assume, as we have done in sec. II, that the inflaton coupling to a scalar field $\chi$ has
the form $-g\phi\chi^2\chi$, where $g > 0$ is a coupling constant with dimension of mass. We also assume that the initial value of $\phi$ is positive. With this choice of the parameters the tachyonic situation can be avoided. Otherwise $\chi$ would explosively rise even at inflationary stage. As a result the contribution of $\chi$ into total cosmological energy density would become non-negligible and should be taken into account in the Hubble parameter. This effect may inhibit inflation. These problems will be studied elsewhere.

The equation of motion of the Fourier mode of $\chi$ with conformal momentum $k$ in the FLRW metric has the form:

$$\ddot{\chi} + 3H\dot{\chi} + \left(\frac{k^2}{a^2} + g\phi\right)\chi = 0,$$  \hspace{1cm} (3.1)

where $a = a(t)$ is the cosmological scale factor, $H = \dot{a}/a$ is the Hubble parameter, and field $\chi$ is taken for simplicity to be massless, $m_\chi = 0$. We assume that the universe is 3D-flat and that the cosmological energy density is dominated by the inflaton field, so $H$ is expressed through $\phi$ as:

$$H = \sqrt{\frac{8\pi}{3}} \sqrt{\frac{\dot{\phi}^2/2 + U(\phi)}{m_{Pl}^2}},$$  \hspace{1cm} (3.2)

where $U(\phi)$ is the potential of the inflaton, $m_{Pl} \approx 1.2 \cdot 10^{19}$ GeV is the Planck mass and it is assumed, as usually, that the inflaton field is homogeneous, $\phi = \phi(t)$. Correspondingly the equation of motion for $\phi$ has the form:

$$\ddot{\phi} + 3H\dot{\phi} + U'(\phi) = 0,$$  \hspace{1cm} (3.3)

where $U' = dU/d\phi$.

We study here particle production by $\phi$ which evolves in potential (2.9) described in the previous section, where it is shown that the particle production in flat space-time is strongly enhanced in comparison with the particle production by $\phi$ with the potential $U(\phi) = m^2\phi^2/2$. We do the same thing here.

In expanding universe the liquid friction term $3H\dot{\phi}$ in the equation of motion for $\phi(t)$ (3.1) strongly modifies evolution of $\phi$ and it is necessary that the resonance should be generated faster than $\phi$ significantly dropped down.

We find numerical solutions of equation (3.1) with the same potentials as above, i.e the harmonic one and $U(\phi)$ presented in eq. (2.9). As initial conditions we take $\phi(0) = 0$ and two different values $\phi(0) = 4.64 m_{Pl}$ and $\phi(0) = 9 m_{Pl}$, which we consider in parallel. The first one, $\phi(0) = 4.64 m_{Pl}$, is equal to the value, which we took in the flat universe case (see Fig. 2). However, in this case the initial energies, $U(\phi(0))$, are very much different for the two potentials (see Fig. 1). So we consider also the initial condition $\phi(0) = 9 m_{Pl}$ for which both potentials $U(\phi(0))$ have equal magnitude. The results of numerical solution of equation (3.1) are shown in Fig. 5.

At the first stage (not at too large $t$) the inflaton field, $\phi(t)$, rolls towards the minimum of potential quite slowly due to large “friction” $H$ (see Fig. 2). During this period the scale factor $a(t)$ grows exponentially (see Fig. 7), this process is called inflation. For the successful inflation one needs the condition $\int_{0}^{t_i} H(t) dt > 70$ to be satisfied, where $t_i$ and $t_e$ are the times of beginning and of the end of inflation, respectively. It can be easily seen in Fig. 6 that this condition is fulfilled for both potentials.

After $\phi(t)$ reaches the minimum of the potentials, it has not enough energy to climb high back because of the energy loss due to the friction, so $\phi$ starts to oscillate with decreasing amplitude. The moment $t_0$, when $\phi(t)$ crosses zero for the first time, one can consider as the end of inflation and the onset of oscillations. The Hubble parameter becomes quite small by this moment, $H \lesssim m$, and continues to decrease, so during the oscillations one can neglect $H$ in comparison with $m$.

Equation (3.1) is simplified by the substitution $\phi(t) = \Phi(t)/a^{3/2}(t)$, and in the case of quadratic potential it has a solution $\phi(t) = \phi_0(t) \cos mt$, where $\phi_0(t) \sim a^{-3/2}(t)$. Therefore, equation (3.1) turns into

$$\ddot{\chi} + 3H\dot{\chi} + \frac{k^2}{a^2} \left(1 + \frac{g\phi_0 a^2}{k^2} \cos mt\right)\chi = 0,$$  \hspace{1cm} (3.4)

which is Mathieu equation with friction term, so the condition of parametric resonance (10) is

$$\left|m - \frac{2k}{a}\right| < \sqrt{\left(\frac{g\phi_0 a}{2k}\right)^2 - 9H^2}.$$  \hspace{1cm} (3.5)

Let us consider now equation (3.1) in general case. It is convenient to make the substitution $\chi(t) = X(t)/a^{3/2}(t)$, so one obtains:

$$\ddot{X} + \left(\frac{k^2}{a^2} + g\phi - \frac{3}{4}H^2 - \frac{3\dot{a}}{2a}\right)X = 0,$$  \hspace{1cm} (3.6)
Equation (3.6) has the form of free oscillator equation $\ddot{X} + \omega^2 X = 0$ with frequency depending on time. During the oscillations the terms $H^2$ and $\dot{a}/a$ are relatively small, so one can take:

$$ \omega(t) \approx \sqrt{\frac{k^2}{a^2(t)} + g\phi(t)}. \quad (3.7) $$

If one neglects time dependence of $a(t)$ and takes quite small $g$, then $\omega$ is almost constant and the solution of (3.6) is $X(t) \simeq e^{-i\omega t}/\sqrt{2\omega}$, which corresponds to $\dot{\chi}(t) = e^{-i\omega t}/\sqrt{2\omega} V$, where $V = a^3$ is the comoving volume.

Energy and number of produced particles in comoving volume can be expressed as follows:

$$ E(t) = \frac{\dot{X}^2}{2} + \frac{\omega^2 X^2}{2}, \quad (3.8) $$

$$ N(t) = \left( \frac{\dot{X}^2}{2} + \frac{\omega^2 X^2}{2} \right) \frac{1}{\omega}. \quad (3.9) $$

The scale factor $a(t)$ changes much slower with time than $X(t)$, therefore $\dot{\chi} \approx \dot{X}/a^{3/2}$. Thus the energy and the number densities of the produced $\chi$-particles can be defined as:

$$ \dot{\rho}_\chi(t) = \frac{\dot{X}^2}{2} + \frac{\omega^2 X^2}{2}. \quad (3.10) $$
The first simplified approach, which gives a correct order of magnitude the produced particles on the damping of the inflaton oscillations and to include the contribution of the created heating temperature on the basis of this result. A more precise way is to take into account the back reaction of energy density of the produced particles at this moment as an ultimate one and to estimate the cosmological the model stopped to be self-consistent and we should modify the calculations. The easiest way is to take the $\frac{k}{a}$ as an initial value and to ensure that $\omega$ is always positive during the interesting time interval.

At large time the amplitude of $\phi$ oscillation becomes small and potential (2.9) tends closely to the quadratic one, therefore the condition of parametric resonance (3.5) holds for both potentials. Thus, the resonance occurs in the narrow region when $k/a(t)$ is near $m/2$. In Fig. 8 one can see at what time the resonance occurs for the two potentials of $\phi$ with the chosen parameters.

We assumed here that the inflaton field gives the dominant contribution to the cosmological energy density. Therefore, when the energy density of the produced particles, $\rho_\chi$, becomes comparable to $\rho_\phi = \dot{\phi}^2/2 + U(\phi)$, the model stopped to be self-consistent and we should modify the calculations. The easiest way is to take the energy density of the produced particles at this moment as an ultimate one and to estimate the cosmological heating temperature on the basis of this result. A more precise way is to take into account the back reaction of the produced particles on the damping of the inflaton oscillations and to include the contribution of the created particles into the Hubble parameter. The first simplified approach, which gives a correct order of magnitude estimate of the temperature, is sufficient for our purposes.

The energy densities of the produced particles, $\rho(t)$, for $\phi(0) = 4.64 m_{Pl}$ and $\phi(0) = 9 m_{Pl}$ are presented in Fig. 8. One can see that for the quadratic potential of $\phi$ the parametric resonance is quite weak and the energy density of particles produced during resonance is much less than $\rho_\phi$, which is about $3 \cdot 10^9 m^4$ at the moment of resonance. On the contrary, in potential (2.9) $\rho_\chi$ increases very quickly and becomes comparable to $\rho_\phi$ at $t \approx 2050$ and $t \approx 2330$ in the case of $\phi(0) = 4.64 m_{Pl}$ and $\phi(0) = 9 m_{Pl}$, respectively.

The number densities $n(t)$ calculated according to formula (3.11) and total number $N(t)$ of the produced particles are presented in Fig. 10 and Fig. 11, respectively.

The forms of these plots can be understood as follows. During some time after beginning of $\phi$ oscillations the conditions of parametric resonance are not fulfilled and therefore $\chi$-particles are not produced in considerable amount. Due to the universe expansion the term $(k/a(t))^2$ decreases permanently and at some moment the mode with conformal momentum $k$ enters the resonance region. It is manifested as an exponentially growth of $\rho(t)$, $n(t)$, and $N(t)$. Then the resonance conditions stop to be satisfied once again, therefore $\chi$-particles stop to be

$$n_\chi(t) = \left(\frac{\dot{\phi}^2}{2} + \frac{\omega^2 \chi^2}{2}\right) \frac{1}{\omega}.$$

(3.11)
FIG. 9: Energy density $\rho(t)$ of the produced particles $\chi$. Time $t$ is measured in units of $m^{-1}$ and $\rho(t)$ is measured in units of $m^4$. The upper plots correspond to $\phi(0) = 4.64 \, m_{Pl}$, and the lower ones correspond to $\phi(0) = 9 \, m_{Pl}$. Black lines denote $\rho_{\phi}$.

FIG. 10: Number density of the produced $\chi$-particles $n(t)$. Time $t$ is measured in units of $m^{-1}$ and number density is measured in units of $m^3$. The upper plots correspond to $\phi(0) = 4.64 \, m_{Pl}$ and the lower ones correspond to $\phi(0) = 9 \, m_{Pl}$.

produced, their total number remains constant, while their number density, $n(t)$, decreases as $a^{-3}$.

Suppose that $\chi$-particles reach thermal equilibrium very quickly. The energy density of relativistic particles is simply related to the temperature:

$$\rho_{\chi} = \frac{\pi^2}{30} g_\ast T^4,$$

(3.12)

where $g_\ast \sim 100$ is number of relativistic particle species in the thermalized plasma. Therefore, from Fig. 9 in the case of potential (2.9) one can estimate temperature by the moment when $\rho_{\chi} \sim \rho_{\phi}$ as

$$T \sim 5 \, m \approx 10^{-5} \, m_{Pl} \sim 10^{14} \, \text{GeV}.$$  

(3.13)

Finally it should be noted that production of $\chi$-particles is possible also due to the usual non-resonant decay
\( \phi \rightarrow \chi \chi^* \). However the width \( \Gamma \) of such decay is very small. Indeed, it follows from eq. (2.2) that \( \Gamma = g^2/16\pi m \) for charged massless \( \chi \). Therefore, for our choice of the interaction constant, \( g = 5 \cdot 10^{-4} m \), the typical time of the decay is \( \tau = 1/\Gamma \sim 2 \cdot 10^8 m^{-1} \), which is much longer than the time when the resonance occurs (see Fig. 8). Thus the contribution of non-resonant decay is negligible.

The inflationary model based on potential (2.9) is similar to the well known new inflationary scenario or inflation at small field \( \phi \), see e.g. book [11]. Correspondingly the slow roll parameters satisfy the condition \( \varepsilon \ll |\eta| \). The definition of the parameters and their relation to scalar and tensor perturbations can be found in refs. [11–14]. The inequality mentioned above means that the amplitude of gravitational waves in considered here model must be very small. However, this conclusion is model dependent and possibly may be lifted.

IV. CONCLUSION

We have shown that even a small modification of the shape of the inflaton oscillations could lead to significant increase of the probability of particle production by inflaton and consequently to a higher universe temperature after inflation. The particle (boson) production by inflaton oscillating in harmonic potential was compared to the particle production by inflaton with flat potential at infinity and it was found that the parametric resonance in the latter case is excited much more efficiently.

The inflationary model which is considered above is not necessarily realistic. We took it as a simple example to demonstrate the efficiency of the particle production and the heating of the universe. The impact of the energy density of the produced particles on the cosmological expansion may noticeably change the phenomenological properties of the underlying inflationary model.

We skip on purpose a possible tachyonic amplification of \( \chi \)-excitement to avoid deviation from the main part of the work on amplification of particle production due to variation of the shape of the inflaton oscillations. This case will be studied elsewhere.

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