CHARMED MESON DECAYS TO TWO PSEUDOSCALARS

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A recent update of data on the decays of $D^0$, $D^+$, and $D_s^+$ to pairs of light pseudoscalars calls for a renewed analysis of key decay amplitudes and tests of flavor symmetry. The present data change our previous understanding of relative phases between amplitudes that describe Cabibbo-favored decays of the charmed mesons. The new data also seem to favor a smaller octet-singlet mixing angle for the $\eta$ and $\eta'$ mesons when singly-Cabibbo-suppressed processes are taken into account. We also discuss the effects of the new data on interference between Cabibbo-favored and doubly-Cabibbo-suppressed decays.

I INTRODUCTION

The CLEO collaboration have recently reported new results on charmed-meson decay rates and branching ratios\cite{1}, many with experimental errors less than present world averages\cite{2}. This calls for an update of a previous work\cite{3} on extraction of flavor-topology amplitudes and relative phases between them. SU(3) flavor symmetry, applied here, has been shown useful in finding relative strong phases of amplitudes in $D \to PP$ decays, where $P$ represents a light pseudoscalar meson\cite{4,5,6}.

The diagrammatic approach is once again reviewed in Sec. II. In Sec. III we talk about Cabibbo-favored decays and a new and previously unexpected relative phase between two of the amplitudes in light of the new data. We also perform an analysis of the Cabibbo-favored decay amplitudes using a variable angle for the octet-singlet mixing in $\eta$ and $\eta'$. Sec. IV updates the singly-Cabibbo-suppressed amplitudes and their role in determining amplitudes associated with disconnected diagrams. In Sec. V we discuss doubly-Cabibbo-suppressed decays. We conclude in Sec. VI.

II DIAGRAMMATIC AMPLITUDE EXPANSION

The flavor-topology technique for analyzing charmed meson decays makes use of SU(3) invariant amplitudes. The key amplitudes that describe the physics of Cabibbo-favored decays have been defined in Ref.\cite{3}, and include a color-favored tree ($T$), a color-suppressed tree ($C$), an exchange ($E$), and an annihilation ($A$) amplitude. The Cabibbo-favored (CF) amplitudes are proportional to the product $V_{ud}V_{cs}$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.
We have described octet-singlet mixing in the decaying meson, $\tau$- favored $D|\lambda$ and hence the ratio between $|A|$ closer to one another in phase. The magnitude of $180^\circ \theta$ has been assumed. This conclusion changes when we make use of new branching ratios for Cabibbo-favored, singly-Cabibbo-suppressed and doubly-Cabibbo-suppressed amplitudes by unprimed, primed, and quantities with a tilde, respectively. The relative hierarchy of these amplitudes in terms of the Wolfenstein parameter $\lambda = \tan \theta_C = 0.2317$ [2] is $1 : \lambda : -\lambda : -\lambda^2$, where $\theta_C$ is the Cabibbo angle.

### III CABIBBO-FAVORED DECAYS

Cabibbo-favored $D$ decays have been discussed at length in Refs. [3, 6], where the $C$, $E$ and $A$ amplitudes were found to have large phases relative to the dominant $T$ amplitude. In particular, we found [3] that the amplitudes $E$ and $A$ had a relative phase of approximately $180^\circ$. This conclusion changes when we make use of new branching ratios for Cabibbo-favored $D$ decays [1]. The phases of $C$ and $E$ relative to $T$ are unchanged, but in the favored solution $E$ and $A$ now are no longer separated by a $180^\circ$ phase difference, but are much closer to one another in phase. The magnitude of $A$ is smaller than was found in Ref. [3] and hence the ratio between $|A|$ and $|E|$ is also smaller: $|A| = (0.21 \pm 0.09)|E|$.

In Table I we list the branching ratios $B$ [1] corresponding to the Cabibbo-favored decay modes and the amplitudes extracted using $A = M_D [8\pi Bh/(p^*\tau)]^{1/2}$, where $M_D$ is the mass of the decaying meson, $\tau$ is its lifetime and $p^*$ is the center-of-mass 3-momentum of a final state pseudoscalar meson. We have described octet-singlet mixing in the $\eta$ and $\eta'$ mesons in terms of an angle $\theta_\eta$:

$$
\eta = -\eta_8 \cos \theta_\eta - \eta_1 \sin \theta_\eta \ , \quad \eta' = -\eta_8 \sin \theta_\eta + \eta_1 \cos \theta_\eta \ , \quad \text{where}
$$

$$
\eta_8 \equiv (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6} \ , \quad \eta_1 \equiv (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}
$$

| Meson | Decay mode | $B$ [1] (%) | $p^*$ (MeV) | $|A|$ (10^{-6}GeV) | Rep. | Predicted $B$ (%) |
|------|------------|-------------|-------------|-----------------|-----|-----------------|
| $D^+$ | $K^-\pi^+$ | 3.891 ± 0.077 | 861.1 | 2.52 ± 0.03 | $T + E$ | 3.905 |
| $\bar{K}^0\pi^0$ | | 2.380 ± 0.092 | 860.4 | 1.97 ± 0.04 | $(C - E)/\sqrt{2}$ | 2.347 |
| $\bar{K}^0\eta$ | | 0.962 ± 0.060 | 771.9 | 1.32 ± 0.04 | $C/\sqrt{3}$ | 1.002 |
| $\bar{K}^0\eta'$ | | 1.900 ± 0.108 | 564.9 | 2.17 ± 0.06 | $-(C + 3E)/\sqrt{6}$ | 1.920 |
| $D_s^+$ | $\bar{K}^0K^+$ | 2.98 ± 0.17 | 850.3 | 2.12 ± 0.06 | $C + A$ | 2.939 |
| $\pi^+\eta$ | | 1.84 ± 0.15 | 902.3 | 1.62 ± 0.07 | $(T - 2A)/\sqrt{3}$ | 1.810 |
| $\pi^+\eta'$ | | 3.95 ± 0.34 | 743.2 | 2.61 ± 0.11 | $2(T + A)/\sqrt{6}$ | 3.603 |
Figure 1: Construction of Cabibbo-favored amplitudes from observed processes using a least \( \chi^2 \)-fit. The sides \( C+T \), \( C+A \), and \( E+T \) correspond to measured processes; the magnitudes of other amplitudes listed in Table I are also needed to specify \( T \), \( C \), \( E \), and \( A \). These figures correspond to the \( |T| > |C| \) solution. Left: \( \theta_\eta \) fixed at \( \arcsin(1/3) = 19.5^\circ \) with \( \chi^2 = 1.79 \) for one degree of freedom. Right: exact solution with \( \theta_\eta = 11.7^\circ \) and \( \chi^2 = 0 \).

and have taken \( \theta_\eta = \arcsin(1/3) = 19.5^\circ \) so that [7]

\[
\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}, \quad \eta' = (2s\bar{s} + u\bar{u} + d\bar{d})/\sqrt{6}.
\]  

(3)

We can extract \( T \), \( C \), \( E \), and \( A \) uniquely (up to a complex conjugation) by defining \( T \) to be purely real. The extracted amplitudes, in units of \( 10^{-6} \) GeV, are:

\[
T = 2.927 \pm 0.022 \quad (4)
\]

\[
C = (2.337 \pm 0.027) \exp[i(-151.66 \pm 0.63)\degree] \quad (5)
\]

\[
E = (1.573 \pm 0.032) \exp[i(120.56 \pm 1.03)\degree] \quad (6)
\]

\[
A = (0.33 \pm 0.14) \exp[i(70.47 \pm 10.90)\degree] \quad (7)
\]

These amplitudes are shown on an Argand diagram in the left-hand panel of Fig. 1. They were extracted by a least \( \chi^2 \)-fit to the data, resulting in \( \chi^2 = 1.79 \) for 1 degree of freedom, and update those quoted in Refs. [3, 6].

While the above analysis works fairly well and yields a low value of \( \chi^2 \), one could obtain an exact solution (\( \chi^2 = 0 \)) by introducing one more parameter; the number of constraints (known branching ratios) would then equal the number of unknown variables. There are several possible sources of an extra parameter. One might add singlet amplitudes, but they are expected to be much smaller than the non-singlet ones and would result in too large a parameter space. A plausible new parameter is the angle \( \theta_\eta \) describing octet-singlet mixing in \( \eta \) and \( \eta' \), which was fixed in the above analysis to [7] \( \theta_\eta = 19.5^\circ \). An exact solution is obtained in this case for \( \theta_\eta = 11.7^\circ \). The corresponding amplitudes, in units of \( 10^{-6} \) GeV, have been listed below and plotted on an Argand diagram in the right-hand panel of Fig. 1:

\[
T = 3.003 \pm 0.023 \quad (8)
\]
Table II: Branching ratios and invariant amplitudes for Cabibbo-favored decays of charmed mesons to a pair of pseudoscalars with 2 different values of $\theta_\eta$. ($\phi_1 = 45^\circ - \frac{\phi_2}{2}$ and $\phi_2 = 19.5^\circ$.)

| Meson | Decay mode | $\mathcal{B}$ (%) | Rep. | Predicted $\mathcal{B}$ (%) |
|-------|------------|-------------------|------|-----------------------------|
| $D^0$ | $K^-\pi^+$ | 3.891±0.077 | $T + E$ | 3.891 3.905 |
| $\bar{K}^0\pi^0$ | 2.380±0.092 | $(C - E)/\sqrt{2}$ | 2.380 2.347 |
| $\bar{K}^0\eta$ | 0.962±0.060 | $\frac{C}{\sqrt{2}}\sin(\theta_\eta + \phi_1) - \frac{E}{\sqrt{2}}\cos(\theta_\eta + 2\phi_1)$ | 0.962 1.002 |
| $\bar{K}^0\eta'$ | 1.900±0.108 | $\frac{C}{\sqrt{2}}\cos(\theta_\eta + \phi_1) - \frac{E}{\sqrt{2}}\sin(\theta_\eta + 2\phi_1)$ | 1.900 1.920 |
| $D^+$ | $\bar{K}^0\pi^+$ | 3.074±0.097 | $C + T$ | 3.074 3.090 |
| $D_s^+$ | $\bar{K}^0K^+$ | 2.980±0.17 | $C + A$ | 2.980 2.939 |
| $\pi^+\eta$ | 1.84±0.15 | $T\cos(\theta_\eta + \phi_1) - \sqrt{2}A\sin(\theta_\eta + \phi_1)$ | 1.840 1.810 |
| $\pi^+\eta'$ | 3.95±0.34 | $T\sin(\theta_\eta + \phi_1) + \sqrt{2}A\cos(\theta_\eta + \phi_1)$ | 3.950 3.603 |

Another set of solutions, but one with $|T| < |C|$, was also found in the process of minimizing $\chi^2$ as a function of $\theta_\eta$. This branch has $\chi^2 = 0$ at $\theta_\eta = 18.9^\circ$. Fig. 3 shows the Argand diagram plot for the corresponding amplitudes. Fig. 4 shows the variation of $\chi^2$ and other parameters as functions of $\theta_\eta$. In this case the point at $\chi^2 = 0$ lies very close to the frequently-quoted [7] value of 19.5$^\circ$. The amplitudes and relative phases of $T$, $C$, $E$, and $A$ (including the relative phase between $T$ and $A$) show only slight variations over the range of values considered for $\theta_\eta$ ($9^\circ - 22^\circ$). However the value of $\chi^2$ in the present case shows a much more rapid increase with a change in $\theta_\eta$ as compared to the $|T| > |C|$ case mentioned earlier. The maximum contribution to the $\chi^2$ comes from the process $D^0 \rightarrow \bar{K}^0\eta$ in the $|T| < |C|$ case. To understand this better let us look at the representation for this amplitude (listed in Table II). The part of this representation that depends on $E$ is proportional to $\sin(\theta_\eta - \phi_2)$ which takes the value zero at $\theta_\eta = \phi_2 = 19.5^\circ$. Hence over the range of $\theta_\eta$ values we consider this term doesn’t undergo considerable change. The other term in this representation that depends on $C$ is proportional to $\sin(\theta_\eta + \phi_1)$ which is of the order of unity in this range of $\theta_\eta$ values. Now the $|C|$ vs. $\theta_\eta$ plot in Fig. 4 shows that as we decrease $\theta_\eta$, $|C|$ decreases so
that the amplitude of the $D^0 \to \bar{K}^0 \eta$ process decreases. This leads to a $3 - 4\sigma$ variation in the branching ratio and hence a high $\chi^2$ contribution. In the $|T| > |C|$ case, however, Fig. 2 shows an increase in $|C|$ as we reduce $\theta_\eta$ from $19.5^\circ$ to $9^\circ$. This leads to a reasonably stable value of the branching ratio and hence a very small contribution to the relevant $\chi^2$.

### IV SINGLY-CABIBBO-SUPPRESSED DECAYS

#### A SCS decays involving pions and kaons

Assuming the relative hierarchy between Cabibbo-favored and singly-Cabibbo-suppressed decay amplitudes described in Sec. I, and using $\lambda = \tan \theta_C = 0.2317$ we find, in units of $10^{-7}$
Figure 3: Construction of Cabibbo-favored amplitudes from observed processes using a least $\chi^2$-fit. The sides $C+T$, $C+A$, and $E+T$ correspond to measured processes; the magnitudes of other amplitudes listed in Table I are also needed to specify $T$, $C$, $E$, and $A$. These figures correspond to the $|T| < |C|$ solution with $\chi^2 = 0$ and $\theta_\eta = 18.9^\circ$.

GeV:

\begin{align*}
T' &= 6.96; \\
C' &= -5.25 - 2.78i; \\
E' &= -1.76 + 2.65i; \\
A' &= 0.99 + 0.34i.
\end{align*}

(12) (13) (14) (15)

where we have considered the $|T| > |C|$ solution for $\theta_\eta = 11.7^\circ$. These amplitudes may then be used to predict the branching ratios for singly-Cabibbo-suppressed (SCS) $D$ decays. In Table III we summarize the measured and predicted amplitudes of SCS decays to pions and kaons. As was noted in [3], we predict $\mathcal{B}(D^0 \to \pi^+\pi^-)$ larger than observed and $\mathcal{B}(D^0 \to K^+K^-)$ smaller than observed. This deviation from flavor SU(3) symmetry is due at least in part to the ratios of decay constants $f_K/f_\pi = 1.2$ and form factors $f_+(D \to K)/f_+(D \to \pi) > 1$. Other predictions for singly-Cabibbo-suppressed decays involving pions and kaons are consistent with those in Ref. [3]. Table III also includes branching ratios predicted using the $|T| < |C|$ ($\theta_\eta = 18.9^\circ$) solution that was obtained in the previous section. However the predictions for the branching ratios of $D^+ \to K^+K^0$ and $D_s^+ \to \pi^+K^0$, in this case are an order of magnitude smaller than their measured values.
Figure 4: Behavior of $\chi^2$ and Cabibbo-favored decay amplitudes and relative phases (|$T$|, |$C$|, $\theta_{CT}$, |$E$|, $\theta_{ET}$, |$A$|, $\theta_{AT}$) as functions of $\theta_\eta$ for the $|T| < |C|$ solution.

B SCS decays involving $\eta$, $\eta'$

In Table IV we quote the branching ratios and extracted amplitudes for singly-Cabibbo-suppressed D-meson decays involving $\eta$ and $\eta'$ as reported in [1]. The values of $C'$ and $E'$ obtained in the previous section may be used to determine the relevant parts of the amplitudes for $D^0$ decays involving $\eta$ and $\eta'$. Additional “disconnected” flavor-singlet diagrams $SE'$ and $SA'$ [8] are required. In Table V we show the representations of the above amplitudes as a function of $\theta_\eta$ using Eqs. (1,2).

In Tables VI ($\theta_\eta = 19.5^\circ$) and VII ($\theta_\eta = 11.7^\circ$) we write the amplitudes so that the coefficient of $SE'$ or $SA'$ is always one. As explained in [3, 6] this information may be used to determine $SE'$ and $SA'$ using a plotting technique.

In Fig. 5 we show the construction technique to find the singlet amplitudes for $\theta_\eta = 19.5^\circ$. A $\chi^2$ minimization fit was used to obtain the amplitudes. We find both a small and a large solution for $SE'$ with a small value for $\chi^2$ of around 0.215 for both solutions. In units of $10^{-7}$...
Table III: Branching ratios and invariant amplitudes for singly-Cabibbo-suppressed decays of charmed mesons to pions and kaons.

| Meson  | Decay mode | $\mathcal{B}^\text{[I]}$ $(10^{-3})$ | $p^*$ (MeV) | $|A|$ $(10^{-7}\text{GeV})$ | Rep. | Predicted $\mathcal{B}$ $(10^{-3})$ | $|T| < |C|$ | $|T| > |C|$ |
|--------|------------|-----------------------------------|-------------|------------------|------|-------------------------------|--------------|--------------|
| $D^0$  | $\pi^+\pi^-$ | 1.45 ± 0.05 | 921.9 | 4.70 ± 0.08 | $-(T' + E')/\sqrt{2}$ | 2.24 | 2.24 |
| $\pi^0\pi^0$ | 0.81 ± 0.05 | 922.6 | 3.51 ± 0.11 | $-(C' - E')/\sqrt{2}$ | 1.36 | 1.35 |
| $K^+K^-$ | 4.07 ± 0.10 | 791.0 | 8.49 ± 0.10 | $(T' + E')$ | 1.92 | 1.93 |
| $K^0\bar{K}^0$ | 0.32 ± 0.02 | 788.5 | 2.39 ± 0.14 | 0 | 0 | 0 |
| $D^+$  | $\pi^+\pi^0$ | 1.18 ± 0.06 | 924.7 | 2.66 ± 0.07 | $-(T' + C')/\sqrt{2}$ | 0.88 | 0.89 |
| $K^+\bar{K}^0$ | 6.12 ± 0.22 | 792.6 | 6.55 ± 0.12 | $(T' - A')$ | 0.73 | 6.15 |
| $D_s^+$ | $\pi^+K^0$ | 2.52 ± 0.27 | 915.7 | 5.94 ± 0.32 | $-(\bar{T}' - A')$ | 0.37 | 3.08 |
| $\pi^0K^+$ | 0.62 ± 0.23 | 917.1 | 2.94 ± 0.55 | $-(C' + A')/\sqrt{2}$ | 0.86 | 0.85 |

Table IV: Branching ratios and amplitudes for singly-Cabibbo-suppressed decays of $D^0$, $D^+$ and $D_s^+$ involving $\eta$ and $\eta'$. Here we use the representations quoted in Eqn. 8.

| Meson  | Decay mode | $\mathcal{B}^\text{[I]}$ $(10^{-4})$ | $p^*$ (MeV) | $|A|$ $(10^{-7}\text{GeV})$ | Rep. |
|--------|------------|-----------------------------------|-------------|------------------|------|
| $D^0$  | $\pi^0\eta$ | 6.80 ± 0.70 | 846.2 | 3.36 ± 0.17 | $-\frac{1}{\sqrt{2}}(2E' - C' + SE')$ |
| $\pi^0\eta'$ | 9.10 ± 1.27 | 678.0 | 4.34 ± 0.30 | $\frac{1}{\sqrt{3}}(E' + C' + 2SE')$ |
| $\eta\eta$ | 16.7 ± 1.8 | 754.7 | 5.57 ± 0.30 | $\frac{2\sqrt{2}}{3}(C' + SE')$ |
| $\eta\eta'$ | 10.5 ± 2.6 | 536.9 | 5.24 ± 0.65 | $-\frac{1}{3\sqrt{2}}(C' + 6E' + 7SE')$ |
| $D^+$  | $\pi^+\eta$ | 35.4 ± 2.1 | 848.4 | 4.82 ± 0.14 | $\frac{1}{\sqrt{2}}(T' + 2C' + 2A' + SA')$ |
| $\pi^+\eta'$ | 46.8 ± 3.0 | 680.5 | 6.18 ± 0.20 | $\frac{1}{\sqrt{3}}(T' - C' + 2A' + 4SA')$ |
| $D_s^+$ | $K^+\eta$ | 17.6 ± 3.6 | 835.0 | 5.20 ± 0.53 | $\frac{1}{\sqrt{3}}(T' + 2C' - SA')$ |
| $K^+\eta'$ | 18.0 ± 5.0 | 646.1 | 5.98 ± 0.83 | $\frac{1}{\sqrt{3}}(2T' + C' + 3A' + 4SA')$ |

8
Table V: Representations for singly-Cabibbo-suppressed decays of $D^0$, $D^+$ and $D_s^+$ involving $\eta$ and $\eta'$ for an arbitrary $\eta - \eta'$ mixing angle $\theta_\eta$. Here we use the representations quoted in Eqs. (11, 12). ($\phi_1 = 45^\circ - \frac{\pi}{4}$ and $\phi_2 = 19.5^\circ$).

| Meson | Decay mode | Representation |
|-------|-------------|----------------|
| $D^0$ | $\pi^0\eta$ | $\frac{C_\eta}{\sqrt{2}} \cos(\theta_\eta + \phi_1) - E' \sin(\theta_\eta + \phi_1) - \frac{\sqrt{3} \cdot SE'}{\sqrt{2}} \sin(\theta_\eta)$ |
| | $\pi^0\eta'$ | $\frac{C_\eta}{\sqrt{2}} \sin(\theta_\eta + \phi_1) + E' \cos(\theta_\eta + \phi_1) + \frac{\sqrt{3} \cdot SE'}{\sqrt{2}} \cos(\theta_\eta)$ |
| $\eta\eta'$ | $\frac{\sqrt{3} \cdot C_\eta}{\sqrt{2}} \cos(\theta_\eta + \phi_1) - \frac{3 \cdot E'}{2} \cos(\theta_\eta) \cos(\theta_\eta + 2\phi_1) + \frac{3 \cdot SE'}{2} \sin(2\theta_\eta)$ |
| | $\eta\eta'$ | $-\frac{\sqrt{3} \cdot C_\eta}{\sqrt{2}} \cos(\theta_\eta + \phi_1) + \frac{3 \cdot E'}{2} \sin(2\theta_\eta + 2\phi_1) - \frac{3 \cdot SE'}{2} \cos(2\theta_\eta)$ |
| $D^+$ | $\pi^+\eta$ | $\frac{C_\eta}{\sqrt{2}} \sin(\theta_\eta + \phi_1) + \frac{\sqrt{3} \cdot C_\eta}{\sqrt{2}} \cos(\theta_\eta) + \sqrt{2} \cdot A' \sin(\theta_\eta + \phi_1) + 3 \cdot SA' \sin(\theta_\eta)$ |
| | $\pi^+\eta'$ | $-\frac{C_\eta}{\sqrt{2}} \cos(\theta_\eta + \phi_1) + \frac{\sqrt{3} \cdot C_\eta}{\sqrt{2}} \sin(\theta_\eta) - \sqrt{2} \cdot A' \cos(\theta_\eta + \phi_1) - 3 \cdot SA' \cos(\theta_\eta)$ |
| $D_s^+$ | $K^+\eta$ | $T' \cos(\theta_\eta + \phi_1) + \frac{\sqrt{3} \cdot C_\eta}{\sqrt{2}} \cos(\theta_\eta) + \frac{3 \cdot A'}{2} \cos(\theta_\eta + 2\phi_1) - \sqrt{3} \cdot SA' \sin(\theta_\eta)$ |
| | $K^+\eta'$ | $T' \sin(\theta_\eta + \phi_1) + \frac{\sqrt{3} \cdot C_\eta}{\sqrt{2}} \sin(\theta_\eta) - \frac{3 \cdot A'}{2} \sin(\theta_\eta + 2\phi_1) + \sqrt{3} \cdot SA' \cos(\theta_\eta)$ |

GeV, they are $SE^\prime = -(0.37 \pm 0.20) - (0.56 \pm 0.31)i$ and $SE^\prime = (5.25 \pm 0.29) - (3.43 \pm 0.21)i$. The second is less likely, as it has too large an amplitude when compared to the connected amplitudes. For $SA'$, only a large solution ($SA' = -(5.79 \pm 0.14) + (1.61 \pm 0.17)i$ with $\chi^2 = 2.6$) is found using a $\chi^2$ minimization fit. A similar exercise was carried out for $\theta_\eta = 11.7^\circ$. The corresponding construction is shown in Fig. [6] In this case we once again find two solutions for $SE^\prime$ (one large and one small in magnitude) and only one solution for $SA'$. These are $SE^\prime = -(0.20 \pm 0.20) - (0.81 \pm 0.48)i$ ($\chi^2 = 0.5$), $SE^\prime = (4.25 \pm 0.37) - (3.67 \pm 0.21)i$ ($\chi^2 = 0.5$) and $SA' = -(6.52 \pm 0.13) + (1.13 \pm 0.24)i$ ($\chi^2 = 4.9$). Qualitatively similar conclusions were reached in [3] where we used only the value $\theta_\eta = 19.5^\circ$.

C Sum rules for $D^0 \to (\pi^0\pi^0, \pi^0\eta, \eta\eta, \pi^0\eta', \eta\eta')$

Let us consider the singly-Cabibbo-suppressed decays of the $D^0$, where the final pseudoscalars are $\pi^0$, $\eta$, or $\eta'$. In Tables III and IV we list the flavor topology representations for all such decays. It is interesting to note that there are 5 such amplitudes all of which depend only on $C'$, $E'$ and $SE'$. This means we can algebraically relate two or more of these amplitudes through sum rules, such as

$$\sqrt{2} \, \mathcal{A}(D^0 \to \pi^0\pi^0) + \sqrt{3} \, \sin \theta_\eta \, \mathcal{A}(D^0 \to \pi^0\eta') + \sqrt{3} \, \cos \theta_\eta \, \mathcal{A}(D^0 \to \pi^0\eta) = 0 \, ,$$  \hspace{1cm} (16)

$$1 - 3 \, \sin^2 \theta_\eta \, \mathcal{A}(D^0 \to \pi^0\pi^0) + \mathcal{A}(D^0 \to \eta\eta) - \sqrt{6} \, \sin \theta_\eta \, \mathcal{A}(D^0 \to \pi^0\eta') = 0 \, ,$$  \hspace{1cm} (17)

$$2 \, \cos^2 \theta_\eta \cos(2 \theta_\eta) + \sin^2(2 \, \theta_\eta) \, \mathcal{A}(D^0 \to \pi^0\pi^0) + 2 \, \cos(2 \, \theta_\eta) \, \mathcal{A}(D^0 \to \eta\eta)$$

$$+ \sqrt{2} \, \sin(2 \, \theta_\eta) \, \mathcal{A}(D^0 \to \eta\eta') = 0 \, .$$  \hspace{1cm} (18)

A sum rule relating three amplitudes can be represented by a triangle whose sides are the magnitudes of the corresponding amplitudes. As in [3], these triangles have angles not equal to zero or 180°, showing that the amplitudes have non-zero relative strong phases.
Table VI: Real and imaginary parts of amplitudes for singly-Cabibbo-suppressed decays of $D^0$, $D^+$ and $D_s^+$ involving $\eta$ and $\eta'$ ($\theta_\eta = 19.5^\circ$), in units of $10^{-7}$ GeV, used to generate plots in Fig. 5.

| Amplitude($A$) | Expression | Re  | Im  | $|A_{\text{exp}}|$ |
|----------------|------------|-----|-----|-----------------|
| $-\sqrt{6}A(D^0 \rightarrow \pi^0\eta)$ | $2E' - C' + SE'$ | 1.06 | 8.85 | 8.22 ± 0.42 |
| $\sqrt{2}A(D^0 \rightarrow \pi^0\eta')$ | $\frac{1}{2}(C' + E') + SE'$ | -3.31 | 0.28 | 3.76 ± 0.26 |
| $\frac{3}{2\sqrt{2}}A(D^0 \rightarrow \eta\eta)$ | $C' + SE'$ | -4.77 | -2.57 | 5.91 ± 0.32 |
| $-\sqrt{6}A(D^0 \rightarrow \eta\eta')$ | $\frac{1}{2}(C' + 6E') + SE'$ | -2.27 | 2.32 | 3.17 ± 0.39 |
| $\sqrt{2}A(D^+ \rightarrow \pi^+\eta)$ | $T' + 2C' + 2A' + SA'$ | -2.24 | -3.70 | 8.34 ± 0.25 |
| $-\sqrt{2}A(D^+ \rightarrow \pi^+\eta')$ | $\frac{1}{2}(T' - C' + 2A') + SA'$ | 3.01 | 1.00 | 3.79 ± 0.12 |
| $-\sqrt{3}A(D_s^+ \rightarrow \eta\eta)$ | $-2E' - C' + SE'$ | 1.06 | 8.85 | 8.22 ± 0.42 |
| $-\sqrt{3}A(D_s^+ \rightarrow \eta\eta')$ | $\frac{1}{2}(2T' + C' + 3A') + SA'$ | 2.39 | -0.10 | 3.66 ± 0.51 |

Table VII: Real and imaginary parts of amplitudes for singly-Cabibbo-suppressed decays of $D^0$, $D^+$ and $D_s^+$ involving $\eta$ and $\eta'$ ($\theta_\eta = 11.7^\circ$), in units of $10^{-7}$ GeV, used to generate plots in Fig. 6.

| Amplitude | Coefficients in the expression | Re  | Im  | $|A_{\text{exp}}|$ |
|-----------|-------------------------------|-----|-----|-----------------|
| $-4.03A(D^0 \rightarrow \pi^0\eta)$ | $T' \ C' \ E' \ A' \ SE' \ SA'$ | 0 | 1 | 5.04 | 13.22 | 13.54 ± 0.70 |
| $0.83A(D^0 \rightarrow \pi^0\eta')$ | $T' \ C' \ E' \ A' \ SE' \ SA'$ | 0 | 1 | -3.27 | 0.31 | 3.62 ± 0.25 |
| $1.68A(D^0 \rightarrow \eta\eta)$ | $T' \ C' \ E' \ A' \ SE' \ SA'$ | 0 | 1 | -6.91 | 5.35 | 9.37 ± 0.51 |
| $-0.51A(D^0 \rightarrow \eta\eta')$ | $T' \ C' \ E' \ A' \ SE' \ SA'$ | 0 | 1 | -2.57 | 1.39 | 2.69 ± 0.33 |
| $2.85A(D^+ \rightarrow \pi^+\eta)$ | $T' \ C' \ E' \ A' \ SE' \ SA'$ | 1.47 | 3.42 | 0 | 2.95 | 0 | 1 | -4.80 | -8.50 | 13.74 ± 0.41 |
| $0.59A(D^+ \rightarrow \pi^+\eta')$ | $T' \ C' \ E' \ A' \ SE' \ SA'$ | -0.28 | 0.15 | 0 | -0.57 | 0 | 1 | 3.31 | 0.60 | 3.65 ± 0.12 |
| $-2.85A(D_s^+ \rightarrow \eta\eta)$ | $T' \ C' \ E' \ A' \ SE' \ SA'$ | -1.95 | -3.42 | 0 | 1.81 | 0 | 1 | 6.21 | 10.13 | 14.83 ± 1.52 |
| $0.59A(D_s^+ \rightarrow \eta\eta')$ | $T' \ C' \ E' \ A' \ SE' \ SA'$ | 2.84 | -0.19 | 0 | 0.62 | 0 | 1 | 2.84 | -0.19 | 3.52 ± 0.49 |
Figure 5: Determination of the disconnected singlet annihilation amplitudes $SE'$ (left) and $SA'$ (right) from SCS charmed meson decays involving $\eta$ and $\eta'$ in the solutions with $|T| > |C|$ and $\theta_\eta = 19.5^\circ$. Left: $D^0$ decays to final states as shown; right: $D^+$ or $D_s^+$ decays to final states as shown. The small black circles show the solution regions. Arrows pointing to them denote the complex amplitudes $-SE'$ (left) and $-SA'$ (right).

The sum rule in [3] relating squares of magnitudes of amplitudes,

$$
8|A(D^0 \to \pi^0 \eta')|^2 + 16|A(D^0 \to \pi^0 \pi^0)|^2 = 16|A(D^0 \to \pi^0 \eta)|^2 + 9|A(D^0 \to \eta \eta)|^2,
$$

may be re-evaluated using the data from Tables III and IV. We find, in units of $10^{-14}$ GeV$^2$,

$$
8|A(D^0 \to \pi^0 \eta')|^2 + 16|A(D^0 \to \pi^0 \pi^0)|^2 = 348 \pm 24 \quad (20)
$$

$$
16|A(D^0 \to \pi^0 \eta)|^2 + 9|A(D^0 \to \eta \eta)|^2 = 460 \pm 35 \quad (21)
$$

The deviation from equality by about 2.6$\sigma$ indicates the degree of flavor-SU(3) breaking.

The above sum rule refers to the special case of $\theta_\eta = 19.5^\circ$. A more general sum rule, valid for arbitrary values of $\theta_\eta$, is

$$
8 \sin^2 \theta_\eta \cos^2 \theta_\eta |A(D^0 \to \pi^0 \eta')|^2 + 27 \cos^2 \theta_\eta (3 \cos^2 \theta_\eta - 2) |A(D^0 \to \pi^0 \pi^0)|^2 = 27 \cos^2 \theta_\eta (3 \cos^2 \theta_\eta - 2) |A(D^0 \to \pi^0 \eta)|^2 + 9|A(D^0 \to \eta \eta)|^2,
$$

where we have kept the normalization of the previous sum rule. The left-hand side of this sum rule, in units of $10^{-14}$ GeV$^2$, is $340 \pm 19$, while the right-hand side is $535 \pm 40$. Here the discrepancy rises to 4.4$\sigma$. Allowing $\theta_\eta$ to be a free parameter does not improve the description of SCS decays.
V DOUBLY-CABIBBO-SUPPRESSED DECAYS

Table VIII contains a list of doubly-Cabibbo-suppressed decay branching ratios, amplitudes and their representation in terms of $\tilde{T}$, $\tilde{C}$, $\tilde{E}$, and $\tilde{A}$. The magnitudes of these are obtained by multiplying the corresponding Cabibbo-favored amplitudes listed in Sec. III by $-\lambda^2$. Using $\lambda = 0.2317$ we find $B(D^0 \to K^+\pi^-) = 1.12 \times 10^{-4}$ and $B(D^+ \to K^+(\pi^0, \eta, \eta') = (1.49, 1.06, 1.16) \times 10^{-4}$, where we have used $\theta_\eta = 11.7^\circ$ and the $|T| > |C|$ solution. While the experimental branching ratio for $D^0 \to K^+\pi^-$ remains 29% above the prediction from flavor $SU(3)$, the measured branching ratio for $D^+ \to K^+\pi^0$ matches the predicted value to around $1\sigma$. In Fig. 7 we plot the branching ratios of the predicted doubly-Cabibbo-suppressed decays $D^0 \to K^-\pi^+$ and $D^+ \to K^+(\pi^0, \eta, \eta')$, which are ones for which measurements or upper bounds exist, as functions of $\theta_\eta$.

As described in [3] in general the decays $D \to (K\pi, \bar{K}\pi)$ are related to each other by either a simple U-spin interchange $s \leftrightarrow d$, or by interchanging “Tree” and “Annihilation” amplitudes. In the former case, the extent of $SU(3)$ breaking in the prediction for the decay asymmetry is expected to be very small.

In $D^0$ decays the asymmetry is predicted as a consequence of U-spin:

$$R(D^0) = \frac{\Gamma(D^0 \to K_S\pi^0) - \Gamma(D^0 \to K_L\pi^0)}{\Gamma(D^0 \to K_S\pi^0) + \Gamma(D^0 \to K_L\pi^0)}$$

$$= 2\lambda^2 = 0.107 .$$

and is indeed consistent with the observed value [9]

$$R(D^0) = 0.108 \pm 0.025 \pm 0.024 .$$
Table VIII: Branching ratios and amplitudes for doubly Cabibbo-suppressed decays of $D^0$, $D^+$ and $D_s^+$.

| Meson Decay mode | $B(10^{-4})$ | $p^*$ (MeV) | Representation | Predicted $B$ $\theta_\eta = 11.7^\circ$ |
|------------------|--------------|-------------|----------------|---------------------------------|
| $D^0$ $K^+\pi^-$ | 1.45 ± 0.04  | 861.1       | $T + E$        | 1.12                           |
| $K^0\pi^0$      | 860.4        |             | $(\bar{C} - \bar{E})/\sqrt{2}$ | 0.69                           |
| $K^0\eta$       | 771.9        |             | $\frac{\sqrt{2}}{2} \sin(\theta_\eta + \phi_1) - \frac{\sqrt{3}}{2} \cos(\theta_\eta + 2\phi_1)$ | 0.28                           |
| $K^0\eta'$      | 564.9        |             | $-\frac{\sqrt{2}}{2} \cos(\theta_\eta + \phi_1) - \frac{\sqrt{3}}{2} \sin(\theta_\eta + 2\phi_1)$ | 0.55                           |
| $D^+$ $K^0\pi^+$ | 862.6        |             | $T + E$        | 2.01                           |
| $K^+\pi^0$      | 1.72 ± 0.19  | 864.0       | $(\bar{T} - \bar{A})/\sqrt{2}$ | 1.49                           |
| $K^+\eta$       | < 1.3        | 775.8       | $\frac{\sqrt{2}}{2} \sin(\theta_\eta + \phi_1) - \frac{\sqrt{3}}{2} \cos(\theta_\eta + 2\phi_1)$ | 1.06                           |
| $K^+\eta'$      | < 1.8        | 570.8       | $\frac{\sqrt{2}}{2} \cos(\theta_\eta + \phi_1) + \frac{\sqrt{3}}{2} \sin(\theta_\eta + 2\phi_1)$ | 1.16                           |
| $D_s^+$ $K^0K^+$ | 850.3        |             | $T + C$        | 0.38                           |

Figure 7: Branching ratios in units of $10^{-4}$ of doubly-Cabibbo-suppressed decays $D^0 \rightarrow K^+\pi^-$, $D^0 \rightarrow K^0(\pi^0, \eta, \eta')$, $D^+ \rightarrow K^+(\eta, \eta')$ as functions of $\theta_\eta$ for the $|T| > |C|$ solution.
Similarly for the $D^+$ and $D_s^+$ decays one may construct the following quantities and predict:

\[ R(D^+) \equiv \frac{\Gamma(D^+ \to K_S\pi^+) - \Gamma(D^+ \to K_L\pi^+)}{\Gamma(D^+ \to K_S\pi^+) + \Gamma(D^+ \to K_L\pi^+)} \]

(26)

\[ \equiv 2\lambda^2 \text{Re} \frac{C + A}{T + C} = -0.005 \pm 0.013 \]  
(27)

\[ R(D_s^+) \equiv \frac{\Gamma(D_s^+ \to K_SK^+) - \Gamma(D_s^+ \to K_LK^+)}{\Gamma(D_s^+ \to K_SK^+) + \Gamma(D_s^+ \to K_LK^+)} \]

(28)

\[ \equiv 2\lambda^2 \text{Re} \frac{C + T}{A + C} = -0.0022 \pm 0.0087 \]  
(29)

Here we have made use of the amplitudes obtained from the $\chi^2$ minimum solution that satisfies $|T| > |C|$ and allows for $\theta_\eta = 11.7^\circ$. These represent similarity with respect to the predictions in Ref. [3], $R(D^+) = -0.006^{+0.033}_{-0.028}$ and $R(D_s^+) = -0.003^{+0.019}_{-0.017}$. Experimentally only the first asymmetry is measured [9]:

\[ R(D^+) = 0.022 \pm 0.016 \pm 0.018 \]  
(30)

in agreement with the new prediction just as with the earlier one.

VI CONCLUSIONS

We have re-analyzed the decays of charmed mesons to pairs of light pseudoscalars using flavor SU(3) in the light of new experimental determinations of branching ratios [1] with experimental errors often smaller than previous world averages [2]. The main difference with respect to a previous analysis [3] is that the “annihilation” amplitude $A$, found previously to have a phase of almost $180^\circ$ with respect to the “exchange” amplitude $E$, now has a phase of $\sim (100 \pm 10)^\circ$ with respect to $E$ for the preferred set of amplitudes and phases. While consequences for singly-Cabibbo-suppressed are qualitatively similar to those in [3], similar decay asymmetries involving $D^+ \to K_{(S,L)}\pi^+$ and $D_s^+ \to K_{(S,L)}\pi^+$ are predicted; both are in good agreement with the observed value as well as the ones obtained previously.

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