Membranes from the Material Assuming Nonlinear Elastic Properties if Exposed to Aggressive Environment

V V Erastov1, A V Erastov1, I V Erofeeva2, A A Treshev3, A A Bobryshev4, L N Shafigullin4

1Project Institute "Arkhstroyproekt", Saransk, 430001, Russia
2Research Institute of Building Physics of the Russian Academy of Architecture and Building Sciences, 21 Locomotivny passage, Moscow, 127238, Russia
3Tula State University, Tula, 300012, Russia
4Naberezhnye Chelny Institute (branch) of the Federal State Autonomous Educational Institution of Higher Education "Kazan (Volga) Federal University", Republic of Tatarstan, Naberezhnye Chelny, 423812, Russia

E-mail: ira.erofeeva.90@mail.ru

Abstract. The article considers the behavior of a membrane of an incompressible material under the influence of a two-sided diffusion fracture mechanism, which manifests itself in a change in the initial modulus of elasticity. The object of study is an incompressible material membrane. The relationship between deformations and displacements was established in the framework of the nonlinear theory of elasticity in accordance with the Karman formalism. For the phenomenological model, the maximum tensile stresses arising in a nonlinear elastic membrane are obtained, depending on the level of aggressiveness of the medium and the intensity of the acting uniformly distributed load. The resulting solution reflects the operation of the membrane, taking into account the accumulation of defects affecting the stress state. Nonlinearity of the results is manifested to a greater extent as defects accumulate.

1. Introduction

In the construction of buildings and structures, various thin-walled products and structures are widely used. These include membrane coatings that are made of thin sheets (strips) that carry load-bearing and enclosing structures [1, 2]. They are made on the basis of metal and other materials. Promising are considered to be membranes made on the basis of polymer composite materials [3 – 12].

At the enterprises of the processing industry: (bread baking plants, dairy plants, meat processing plants), in agricultural buildings and various engineering structures during upkeeping, building structures are exposed to an aggressive biological environment. The action of the biological environment is manifested multifaceted and, as a result, is characterized by a change in the physico-mechanical properties of materials, in particular, a decrease in strength properties, a change in the geometric dimensions of structures and individual elements, etc. [13 – 26]. There are some researches devoted to the study of the stress-strain state of thin-walled structures and the theoretical assessment of their durability [27 – 32]. Of considerable interest are studies aimed at establishing the nonlinear elastic properties of materials exposed to aggressive environment.
In this article, a membrane from incompressible material is the object of the study. A sample in the form of a plate experiences two-sided action of an aggressive environment and the change in the modulus of elasticity over the thickness of the plate with time has been studied.

2. Calculation results and discussion

Consider the behavior of a membrane (Figure 1. A.) from an incompressible material under the influence of a two-sided diffusion fracture mechanism that manifests itself in a change in the initial elastic modulus, which is a function of time and coordinate along the thickness of the membrane (Figure 1. B)

\[ E(z,t) = \begin{cases} 
E_0(1 - (S_1(t) - z) / S_1^{\max}) & \text{at } 0 \leq z \leq S_1(t), \\
E_0 & \text{at } S_1(t) < z < h_0 - S_2(t), \\
E_0(1 - (S_2(t) - z) / S_2^{\max}) & \text{at } 0 \leq z \leq S_2(t). 
\end{cases} \]

where \( E_0 \) is the modulus of elasticity of the material in the initial state before the start of degradation.

Given the coincidence of the guide tensors in the process of changing the mechanical properties of the material, we identify the relationship between stresses and deformations \( \sigma = E(z,t) \cdot \varepsilon \) for incompressible materials with the relationship between stress and strain intensities [17]:

\[ \sigma_i = E(z,t) \cdot e_i. \]

![Figure 1](image-url)

**Figure 1.** In this case simply justify the caption so that it is as the same width as the graphic.

In the general case, for the strain intensity there is a known dependence [34]:

\[ e_i = 2(\sqrt{3})^{-1}(B_i + B_2z + B_3z^2)^{1/2}, \]

\[ B_i = e_i^2 + e_i^2 + e_i^2 + e_i^2 + \frac{1}{4} \gamma_{uy}^2, \]

where

\[ B_2 = 2\gamma_x N_x + 2\gamma_y N_y + \frac{1}{2} \gamma_{uy} N_{uy}, \]

\[ B_3 = N_x^2 + N_y^2 + N_{uy}^2 + \frac{1}{4} N_{uy}^2. \]

A feature of the membrane is that it has only tensile forces; therefore, all terms containing curvatures must be set equal to zero.

The relationship between deformations and displacements is established in the framework of the non-linear theory of elasticity in accordance with the Karman formalism, i.e.
The increment of the total energy of the system can be written as:

\[ E_i = U - W = \int_0^t \left[ E(z,t) \cdot e \cdot d_e \cdot dV - \int_A (g_o \cdot \omega + g_x \cdot u + g_y \cdot v) dA \right] \]

where \( g_o \) is the vertical projection of the intensity of the external load; \( g_x \) and \( g_y \) are projection of the intensity of the external load on the corresponding axis.

Under the action of a uniformly distributed load on the membrane perpendicular to its middle surface for functional (5), we have:

\[ E_i = \int_{-a}^{a} \int_{-b}^{b} \frac{2}{3} B_i \varphi_i (t) dx dy - \int_{-a}^{a} \int_{-b}^{b} g_o \omega \cdot dx dy, \]

where

\[ \varphi_i (t) = E_0 (h - \frac{S_i^2 (t)}{2S_{max}^2} - \frac{S_i^2 (t)}{2S_{max}^2}). \]

The further solution of the problem requires setting the displacement function, which we represent in the trigonometric form, i.e.

\[
\begin{align*}
u &= c \sin \frac{\pi x}{a} \cos \frac{\pi y}{2b}, \\
v &= d \sin \frac{\pi y}{b} \cos \frac{\pi x}{2a}, \\
\omega &= \omega_0 \cos \frac{\pi x}{2a} \cos \frac{\pi y}{2b}
\end{align*}
\]

As a result of substituting representations (7) into functional (6), taking into account (3) and (4), we obtain:

\[
E_i = \left\{ c^2 \pi^2 \left( \frac{b}{a} + \frac{a}{16b} \right) + d^2 \pi^2 \left( \frac{a}{b} + \frac{b}{16a} \right) + \frac{8}{3} c d - \omega_0^2 \left[ c \frac{\pi^2}{24a^2 b} (8b^2 - a^2) + d \frac{\pi^2}{24ab^2} (8a^2 - b^2) \right] \right\}
\]

\[
+ \frac{\pi^4}{1024} \frac{9a^4 + 2a^2 b^2 + 9b^4}{a^3 b^3} \int_0^t \varphi_1 (t) - g_o \omega_0 \frac{16ab}{\pi^2}.
\]

We find variables \( c, d \), and \( \omega_0 \) as a result of minimizing the function of the functional \( E_i \) from the following system of equations:
With the known displacement functions (7), it is easy to determine the components of deformations (4) and tensile stresses.

### 3. Example 1

For a membrane with sides $2a=2b=12$ m build the dependence of the deflection function in time if: the membrane thickness $h=0.08$ m; elastic modulus at $t=0$, $E_0=1.2 \times 10^{10}$ Pascal; functions of changes in the elastic modulus in time ($t$ in years) from the upper and lower surfaces of the membrane: $S_1(t) = h[1 - \exp(-0.0156 t)]$ and $S_2(t) = h[1 - \exp(-0.0251 t)]$.

Taking into account symmetry, the first two equations in system (8) are identical:

$$
\begin{align*}
\frac{\partial E_i}{\partial c} &= 0; \\
2\pi^2 \left( \frac{b}{a} + \frac{a}{16b} \right) + \frac{8}{3} d - \omega_0^2 \pi^2 \left( \frac{8b^2 - a^2}{24a^2b} \right) = 0; \\
\frac{\partial E_i}{\partial d} &= 0; \\
2\pi^2 \left( \frac{a}{b} + \frac{b}{16a} \right) + \frac{8}{3} c - \omega_0^2 \pi^2 \left( \frac{8a^2 - b^2}{24ab^2} \right) = 0; \\
\frac{\partial E_i}{\partial \omega} &= 0; \\
\varphi_i(t) \frac{2}{3} \left[ -2\omega_0 \left( \frac{c\pi^2 (8b^2 - a^2)}{24a^2b} + \frac{d\pi^2 (8a^2 - b^2)}{24ab^2} \right) \right] + 4\omega_0 \frac{\pi^4 (9a^4 + 2a^2b^2 + 9b^4)}{1024a^4b^4} - \frac{16g_0 ab}{\pi^2} = 0.
\end{align*}
$$

(8)

From the third equation (8) we find:

$$
c = d = 7\omega_0^2 \pi^2 \left[ 24a \left( \frac{17\pi^2}{8} + \frac{8}{3} \right) \right].$$

Table 1. Calculation results.

| Load kN / m² | Operating time, years |
|-------------|----------------------|
|             | 0  | 5  | 10 | 15 | 20 |
| 1           | 0.809 | 0.816 | 0.837 | 0.872 | 0.922 |
| 10          | 1.742 | 1.759 | 1.804 | 1.878 | 1.986 |
| 25          | 2.365 | 2.387 | 2.448 | 2.549 | 2.695 |
| 50          | 2.979 | 3.007 | 3.085 | 3.212 | 3.396 |
| 100         | 3.753 | 3.789 | 3.887 | 4.047 | 4.279 |
| 250         | 5.094 | 5.142 | 5.275 | 5.492 | 5.807 |
| 500         | 6.418 | 6.479 | 6.646 | 6.920 | 7.317 |
| 50          | 2.979 | 3.007 | 3.085 | 3.212 | 3.396 |
It should be noted that the impact of a biologically active environment must be considered along with other factors affecting the stress-strain state of building structures. These factors are:

- force impacts from both design and non-design-based loads arising during operation;
- alternating effects of temperature fields in autumn and spring on the material of building structures;
- differential settlement of buildings foundations due to the washway of the soil skeleton;
- increased concentration of aggressive environment, having a chemical, electrochemical, electromagnetic or any other environment different from biological character;
- increased indoor humidity, creating a microclimate favorable for the development of microflora.

Despite the variety of factors at the first stage of the study, we will take some parameter $\alpha$ as a measure of damage. It characterizes the degree of damage depending on the level of the stress-strain state, specific energy of deformation, parameters describing the interaction of a biologically aggressive environment with the material of structures, etc. In the general case, the kinetic equation for damage accumulation can be written as [33, 34]:

$$\frac{d\alpha}{dt} = F(\sigma, \dot{\sigma}, t, T, P, a^*, p, ...) \quad (9)$$

Here $\sigma$ is the stress level; $\dot{\sigma}$ - rate of stresses change; $t$ is the time; $T$ is the temperature; $P$ is a parameter characterizing the properties of an aggressive environment; $a^*$ is the specific strain energy; $p$ is the humidity of the environment.

In studying the stress-strain state of a thin membrane, we adopted the following phenomenological model of damage accumulation:

$$\frac{da}{dL_c} = \left[ k \frac{d\sigma}{dL_c} + f(L_c, \sigma) \left[ \frac{1}{1 - \alpha} \right]^n \right], \quad (10)$$

where $L_c$ is the level of the aggressive environment during its action; $\sigma$ is the stress level; $k$ and $n$ are constants; $f(L_c, \sigma)$ is a function satisfying the condition

$$\int_0^\phi f(L_c, \sigma) dL_c = (A + BL_c + CL_c^2 + DL_c^3 + FL_c^4) \int_0^\phi \sigma(L_c) dL_c = k_c(L_c) \int_0^\phi \sigma(L_c) dL_c. \quad (11)$$

Here $A, B, C, D$ and $F$ – are constants determined on the basis of experimental data.

3. Example 2

Let for a membrane with dimensions in the plan of $6 \times 6$ m and the thickness of 0.1 m, a change in the physicomchemical properties under the influence of the biological medium can be characterized by the experimental data shown in Table 2. In addition, the membrane material is susceptible to surface unilateral destruction, described by the dependence

$$h(L_c) = h_0 \exp(-mL_c), \quad (12)$$

Here $m = 0.142$ is a constant value found from the experiment

| Properties change       | Levels of aggressive environment (concentration $x$ action time) |
|-------------------------|---------------------------------------------------------------|
|                         | 0.000  | 0.502  | 1.004  | 1.506  | 2.008  | 2.51   |
| Destructive stresses, MPa|        |        |        |        |        |        |
| 10                      | 12     | 9.5    | 9.2    | 8.7    | 8.4    |        |
| Modulus of elasticity, MPa |      |        |        |        |        |        |
| 90                      | 120    | 140    | 210    | 298.5  | 335    |        |
As a result of computer calculations for the adopted phenomenological model (10), the maximum tensile stresses (in N/m²) (see Table 3) were obtained. They arise in a nonlinear elastic membrane, depending on the level of aggressiveness of the medium and the intensity of the acting uniformly distributed load $g_0$ (N/m²).

Table 3. Calculation results.

| Value loads, N / m² | Levels of aggressive environment (concentration x action time) |
|---------------------|---------------------------------------------------------------|
|                     | 0.000 | 0.502 | 1.004 | 1.506 | 2.008 |
| 10                  | 382.6 | 399.3 | 418.8 | 502.7 | 606.5 |
| 35                  | 757.6 | 874.4 | 965.3 | 1159  | 1398  |
| 60                  | 1085  | 1253  | 1383  | 1660  | 2003  |
| 85                  | 1369  | 1580  | 1744  | 2094  | 2526  |
| 110                 | 1626  | 1876  | 2071  | 2486  | 3000  |

1. The behavior of a membrane made of an incompressible material under the influence of a two-sided diffusion destruction mechanism has been theoretically studied. The resulting solution reflects the operation of the membrane, taking into account the accumulation of defects that affect the stress state.

2. It was established that the nonlinearity of the results is manifested to a greater extent as defects accumulate. So, in the absence of an aggressive medium, the ratio of maximum tensile stresses that occur in the membrane at loads of 110 and 10 (N/m²) is 4.25, and for a level of aggressive medium of 2.008 this ratio is 4.95.

4. References

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