Information-theoretic User Interaction: Significant Inputs for Program Synthesis

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Abstract. Programming-by-example technologies are being deployed in industrial products for real-time synthesis of various kinds of data transformations. These technologies rely on the user to provide few representative examples of the transformation task. Motivated by the need to find the most pertinent question to ask the user, in this paper, we introduce the significant questions problem, and show that it is hard in general. We then develop an information-theoretic greedy approach for solving the problem. We justify the greedy algorithm using the conditional entropy result, which informally says that the question that achieves the maximum information gain is the one that we know least about.

In the context of interactive program synthesis, we use the above result to develop an active program learner that generates the significant inputs to pose as queries to the user in each iteration. The procedure requires extending a passive program learner to a sampling program learner that is able to sample candidate programs from the set of all consistent programs to enable estimation of information gain. It also uses clustering of inputs based on features in the inputs and the corresponding outputs to sample a small set of candidate significant inputs. Our active learner is able to tradeoff false negatives for false positives and converge in a small number of iterations on a real-world dataset of string transformation tasks.

1 Introduction

In recent years, the field of automatic program synthesis of data transformation programs from user-provided example-based specifications has received wide attention from the industrial community [12]. Data transformation programs commonly arise in machine learning [15], healthcare [10], and IT administration [17], as well as in any business analytics that involves data wrangling. The ability to process data by providing examples of the desired transformation not only makes data wrangling approachable to non-programmers, but also simplifies data scientists’ workflows, who commonly dedicate as much as 80% of their time to manual wrangling [15]. This has led multiple software companies to incorporate semi-automatic transformation synthesis in their machine learning

* A note on history: This article was submitted to CAV 2018, 2019, and 2020, and PLDI 2019 and 2020. A recent paper in PLDI 2020 titled “Question selection for interactive program synthesis” (independently) addresses the same problem.
IDEs, including Microsoft’s Azure ML Workbench\(^1\), Google’s Cloud Dataprep\(^2\) (based on Trifacta\(^3\)), Uber’s Michelangelo\(^4\) and Tableau\(^5\).

Specifying programs by input-output examples is notoriously ambiguous. Even an extensively engineered program synthesis system may require as many as 7 examples to correctly identify an intended program \([18]\). In the past, intent ambiguity has typically been addressed by trying to heuristically avoid it: impose a sophisticated ranking on the underlying domain-specific language \([30]\) so that ranking disambiguates user intent. However, heuristics can fail, and in such cases, the user is responsible for finding an input where the synthesized transformation does not match their intent and provide an additional example. This is relatively straightforward when datasets are small where the user can simply eye-ball the data \([11]\), but much more challenging with a large dataset intended for business analytics.

The intent ambiguity challenge has caused the industry to embrace an interactive and predictive approach to program synthesis. Synthesis proceeds in rounds wherein the system proactively makes suggestions to the user, and the user provides information accordingly. For instance, Azure ML Workbench suggests a subset of significant inputs from the data that may best disambiguate the hypothesized transformations \([20]\), and Trifacta Wrangler suggests possible next steps in the desired transformation \([32]\).

While the academic community has long modeled program synthesis as an iterative interactive process \([14, 24]\), proactively generating high-quality examples, or constraints, to optimize convergence to the intended program is still an open problem. With the concrete goal of building an active program synthesizer, we start by formulating the problem of generating optimum user queries in an abstract setting. We show its hardness, and then cast the problem in a probabilistic framework to enable application of information-theoretic methods. We then use the chain rule for conditional entropy to design a greedy algorithm for the problem. Intuitively, the chain rule says that the system can pick the question whose answer will yield the most information gain by finding the question about which the system knows the least.

The abstract greedy information-gain procedure is instantiated to build an active program learner. The active program learner iteratively refines its belief of the intended program. This belief state is just a probability distribution over the program space. We now face two challenges. First, working with probability distributions over program space is intractable. We overcome this challenge by estimating probability distributions using Monte-Carlo methods. In fact, we describe a sampling program learner that uses importance sampling to generate a belief state (and not just one correct program). Second, the number of inputs (that is, the number of possible questions we can ask the user) can be really

\(^1\)https://azure.microsoft.com/en-us/services/machine-learning-services
\(^2\)https://cloud.google.com/dataprep
\(^3\)https://trifacta.com
\(^4\)https://eng.uber.com/michelangelo
\(^5\)https://www.tableau.com/
large. We address this challenge by presenting a clustering-based approach for sampling of input space, where our key idea is to use features from both the input, and the output corresponding to that input.

We evaluate our active program learner based on whether it (a) minimizes the number of synthesis iterations, (b) minimizes the number of false positive queries \(\text{(i.e. extraneous queries when the learned program is already correct)}\), and (c) minimizes the number of false negative queries \(\text{(i.e. missing queries when the learned program is actually incorrect)}\). These criteria are difficult to satisfy simultaneously, but we present a comprehensive evaluation of different techniques to pick a trade-off solution.

2 Significant Questions

Consider a blackbox software system \(bb\). Let us say we want to answer a fixed question \(q\) about \(bb\). To answer the question \(q\), we can ask questions from a predefined set \(QS = \{q_1, q_2, \ldots, q_n\}\) of questions, and we assume there is an oracle (say, a user) that can answer these questions about \(bb\). We are interested in the following problem: which of the \(n\) questions should we ask the oracle? Our goal is to minimize the number of interactions with the oracle required in the process of answering the question \(q\) about the given system \(bb\).

The hypothesis space, \(HS\), is a set \(\{p_1, \ldots, p_N\}\) of all possible values that \(bb\) can take; in other words, \(bb\) is known to belong to the set \(HS\). In general, the set \(HS\) need not contain the concrete programs, but only some abstractions that are sufficient to answer the questions \(q\) and \(q_1, \ldots, q_n\). This distinction is not important here, so for simplicity, assume that \(HS\) contains concrete programs.

The answer space, \(AS\), is a set \(\{a_1, \ldots, a_m\}\) of all possible answers for the questions in \(QS\). A given question-answer pair, \((q_i, a_j)\), can either be consistent with a given hypothesis \(p_k\), or inconsistent with it. The notation \(p_k \models (q_i, a_j)\) denotes that hypothesis \(p_k\) is consistent with \((q_i, a_j)\), and \(p_k \not\models (q_i, a_j)\) denotes it is not.

We are interested in finding a plan for asking questions. A plan is a mapping \(\sigma : (QS \times AS)^* \rightarrow QS \cup \{\perp\}\) that maps a history of question-answer pairs, possibly of length 0, to the next question to ask. A plan \(\sigma\) is terminating if there is a finite number \(k\) such that \(\sigma((QS \times AS)^k) = \perp\) for all \(k' \geq k\).

A sequence of question-answer pairs,

\[
(q_0, a_0), (q_1, a_1), \ldots, (q_i, a_i),
\]

is consistent (with respect to a plan \(\sigma\) and a program \(bb\)) if
(a) \(bb \models (q_i, a_i)\) for all \(i = 0, \ldots, l\); that is, each answer \(a_i\) correctly answers the question \(q_i\) about the program \(bb\), and
(b) \(q_{i+1} = \sigma((q_0, a_0), \ldots, (q_i, a_i))\); that is, the questions in the sequence are picked using the given plan.

A feasible sequence of question-answers, as in (1), is maximal (with respect to terminating plan \(\sigma\)) if \(\sigma((q_0, a_0), \ldots, (q_i, a_i))) = \perp\).
Maximal feasible sequences are important to state the correctness requirement of any plan: Given any maximal feasible sequence of question-answer pairs, we should be able to deduce enough about the unknown program \( bb \) to answer the question \( q \) about it.

**Definition 1 (Significant Questions Problem).** Given a hypothesis space \( HS = \{p_1, \ldots, p_N\} \), a set \( QS = \{q_1, \ldots, q_n\} \) of questions, a set \( AS = \{a_1, \ldots, a_m\} \) of answers, synthesize a terminating plan \( \sigma : (QS \times AS)^* \mapsto QS \cup \{\perp\} \) s.t. given any sequence of the form

\[
(q_0, a_0), (q_1, a_1), \ldots, (q_l, a_l)
\]

that is maximal and feasible with respect to the plan \( \sigma \) and program \( bb \), it is possible to deduce an “\( a \)” s.t. \( bb \models (q, a) \).

Each plan \( \sigma \) can be visualized as a tree: each node in the tree is labeled with a question, each node has as many children as there are answers to its question, the root node is labeled with \( \sigma(\epsilon) \), and every other node is labeled by the question generated by the plan \( \sigma \) based on the question-answer pairs on the path from the root to that node. The process of answering \( q \) about \( bb \) using the plan \( \sigma \) corresponds to traversing a path from a root to a leaf in the tree for \( \sigma \).

The optimum significant questions problem seeks to find a plan that has a minimum value for the worst-case number of questions asked. In terms of the tree visualization, we want the plan whose tree has the least height.

**Example 1 (Inserting element in a sorted list).** We can cast the problem of inserting a number into a given sorted list of \( n \) numbers as an optimum significant questions problem. The unknown \( bb \) here is the input number that has to be inserted (in a known sorted list). The question \( q \) we want answered about \( bb \) is: what is the position where we need to insert \( bb \). The possible answers \( a \) for this question \( q \) are \( \{0, 1, \ldots, n\} \). The set of all questions \( QS \) we are allowed to ask are \( \{q_0, \ldots, q_n\} \), where \( q_i \) asks if \( a \leq i \), and the set of possible answers \( AS \) is \( \{true, false\} \). The goal is to find the answer \( a \) by asking the fewest number of questions (in the worst case). An optimum plan here would correspond to the binary search procedure: the first question would be \( q_{\lfloor \frac{n}{2} \rfloor} \), and depending on the answer, the next question would be \( q_{\lfloor \frac{n}{4} \rfloor} \) or \( q_{\lfloor \frac{3n}{4} \rfloor} \), and so on. The tree visualizing the binary search plan has height \( O(\log(n)) \).

### 3 Information-Guided User Interaction

In this section, we present a greedy approach for solving the optimum significant questions problem. First, we note that achieving optimality is NP-hard: this follows by a reduction from set cover. Hence, we resort to greedy methods. But, before we describe our greedy approach, we need to cast the problem in a general probabilistic framework.

Let \( E \) be a set of question-answer pairs. The unknown artifact \( bb \) can be viewed as a random variable that can take one of the values in \( HS \). Let \( Pr(bb = \)}
\( p_k \mid E \) denote the probability that the blackbox program \( bb \) is \( p_k \) given that \( bb \) is known to be consistent with all the question-answer pairs in \( E \). Clearly, \( Pr(bb \mid E) \) is a probability distribution over the hypothesis space \( HS \).

We can answer any given question about \( bb \) if we know the identity of \( bb \). For the rest of the paper, we shall assume that the question \( q \) we want to answer about \( bb \) is just the identity of \( bb \), and we will use \( q \) as a variable that ranges over the questions in \( QS \) that we can ask the oracle. Our knowledge about the identity of \( bb \) is directly measured by the entropy \( En(Pr(bb)) = \sum_k -Pr(bb = p_k) \log(Pr(bb = p_k)) \) of the probability distribution \( Pr(bb) \). Clearly, our goal is to reduce the entropy of \( Pr(bb) \).

Initially, we do not have answers to any questions, and hence \( E = \emptyset \), and our belief about \( bb \) is given by the probability distribution \( Pr(bb \mid E = \emptyset) = Pr(bb) \).

Now, assume we ask question \( q \) from \( QS \). Let \( Pr(bb \mid q) \) denote the probability distribution on \( bb \) conditioned on knowing the answer to the question \( q \). We view \( q \) also as a random variable which takes values in the answer set \( AS \). Using the chain rule for conditional entropy, we can compute the entropy \( En(Pr(bb \mid q)) \) of the distribution we get after asking the question \( q \) as

\[
En(Pr(bb \mid q)) = En(Pr(bb)) - En(Pr(q)) \tag{2}
\]

where \( Pr(q) \), a probability distribution on the answer space \( AS \), is defined by

\[
Pr(q = a) = \sum_{\{p\mid p = (q,a)\}} Pr(bb = p) \tag{3}
\]

In the information-theoretic user interaction model, we solve the significant questions problem by choosing the next question \( q \) so that it (greedily) minimizes the entropy of \( Pr(bb \mid q) \). Equation 2 shows that the most greedy choice would be the question \( q \) whose entropy \( En(Pr(q)) \) is maximum. We can generalize the observation of Equation 2 to the case when we have already obtained answers to some \( i \) prior questions. If \( E \) denotes a sequence \( (q_1, a_1), \ldots, (q_i, a_i) \) of question-answer pairs that have been obtained so far, then the greedy plan \( \sigma^* \) is given by:

\[
\sigma^*(E) = \begin{cases} 
\bot & \text{if } En(Pr(q^*_E \mid E)) = 0 \\
q^*_E & \text{otherwise}
\end{cases}
\]

where \( q^*_E = \arg\max_q \{En(Pr(q \mid E))\} \).

The following proposition says that the greedy plan is indeed greedy.

**Proposition 1.** Consider an instance of the significant questions problem where the question \( q \) is the same as identity of \( bb \). If this instance has a solution, then the greedy plan \( \sigma^* \) will be a solution. Although the plan \( \sigma \) may not be optimum, it is greedy in every step; that is, whenever \( \sigma^*(E) \neq \bot \), it is the case that \( \sigma^*(E) = \arg\min_q \{En(bb \mid E, q)\} \).

Since the knowledge we have about a random variable is inversely related to its entropy, the above result intuitively states that, to greedily seek knowledge, we should ask the question about which we know the least.
Example 2. Continuing with Example 1, assume that the prior probability distribution (for the index $q$ where the unknown input will be inserted in the sorted list) is a uniform distribution over $\{0, \ldots, n\}$; that is, $Pr(q = i) = 1/(n + 1)$ for all $i$. If we want to minimize the entropy, we know that the optimum choice would be question $q_i$ that maximizes $En(q_i)$. Now, the question $q_i$ has two possible answers, and hence the possible answers for $q$ are partitioned into two clusters, namely, $\{0, 1, \ldots, i\}$ where the answer is true, and $\{i + 1, \ldots, n\}$ where the answer is false. Hence, we have

$$\sigma^*(\emptyset) = \arg\max_{q_i \in QS} \frac{i + 1}{n + 1} \log\left(\frac{i + 1}{n + 1}\right) - \frac{n - i}{n + 1} \log\left(\frac{n - i}{n + 1}\right).$$

We know this entropy is maximized when $i$ is the floor of $(n + 1)/2$. Thus, we get the binary search procedure. Note that the greedy approach also suggests a way to generalize binary search when the (prior) distribution of elements to be inserted is not uniform.

4 Active Program Synthesis

The main application of the greedy approach for user interaction design we pursue in this paper is the significant input problem in interactive program synthesis. The key challenge in implementing the greedy information gain procedure is to find ways to estimate the entropy of the different questions that can be posed to the user. Since the number of programs and the number of inputs can be very large, we use sampling techniques to estimate the various probabilities for computing entropies.

4.1 From Passive to Active Synthesis

Let $I, O$ be sets that denote the domain for the input space and the output space respectively. Let $f : I \to O$ be a fixed function (that is unknown to the program learner, but is known to the user). Let $\Sigma := I \times O$ be the set of all possible input-output examples, and let $\Sigma^*$ denote the set of all finite sequences of these examples. Let $PS$ be the space of all programs (considered by the program synthesizer) that map $I$ to $O$. Note that $f$, and every $p \in PS$, maps $I$ to $O$, but the difference is that elements in $PS$ are computable (executable) descriptions of functions, whereas $f$ is modeling the user.

A passive program learner $ppl$ is a computable function with the signature $ppl : \Sigma^* \times 2^I \to PS$ such that for any input-output example sequence $seq$,

$$seq := [(in_1, f(in_1)), \ldots, (in_k, f(in_k))], \quad (4)$$

and a subset $I_0 \subseteq I$ of the input space, the passive program learner returns a program $p := ppl(seq, I_0)$ that satisfies all the given input-output examples; that is,

$$p(in_j) = f(in_j) \quad \text{for every } j = 1, 2, \ldots, k.$$
Require: $ppl^*$, a modified passive program learner
Require: $I_0$, a subset of inputs
Require: $\epsilon > 0$, an uncertainty threshold

function ActiveProgramLearner($ppl^*$, $I_0$)

Input-output examples $\varphi \leftarrow []$
Prob. Dist. $pd \leftarrow$ domain-dependent prior on $PS$

Entropy (uncertainty) about desired program $un \leftarrow En(pd)$

while $un \geq \epsilon$ do

foreach $i \in I_0$: $Pr_i \leftarrow \lambda a : \sum_{p \in PS, p(i) = a} pd(p)$
\> $Pr_i(a)$ is probability of $a$ being output on $i$

$i \leftarrow \text{argmax}_{i \in I_0} En(Pr_i)$
\> input with greatest uncertainty

$\varphi \leftarrow \varphi \sqcup \langle i, f(i) \rangle$
\> Call oracle $f$ to get $f(i)$ and add $(i, f(i))$ to $\varphi$

$pd \leftarrow ppl^*(\varphi, I_0)$
\> Update belief prob. dist. about intended program

$un \leftarrow En(pd)$
\> Update uncertainty about intended program

end while

return $p_{\text{best}} = \text{argmax}_{p \in PS} pd(p)$

Fig. 1. An active program learner that selects the next significant input (to query the oracle/user) at each iteration greedily based on information gain.

The goal of the passive learner is to find a program $p$ that matches $f$ on all inputs in $I_0$, and not just the inputs in the provided examples.

Existing programming-by-example (PBE) systems can be viewed as passive program learners. They maintain a sequence $\text{seq}$ of input-output examples, which initially is either empty or contains just one input-output pair. They then generate the program $p := ppl(\text{seq}, I_0)$, and ask the user if the outputs $p(I_0)$ match the expected outputs. If not, the user provides a new input-output pair that gets added to $\text{seq}$ and the process repeats.

Finally, we note that the program $p$ returned by $ppl$ is not arbitrary, but one that is ranked highest. The ranker is designed to prefer programs that are most-likely to be the user-intended program. Designing such rankers is not easy: it is often achieved by a combination of machine learning and human tweaking of ranking function parameters based on user feedback.

Active program learner. We now turn the passive learner into an active learner. Procedure $\text{ActiveProgramLearner}$ in Figure 1 uses greedy information gain (Proposition 1) to implement an active program learning method. The procedure maintains its current belief of the intended program as a probability distribution $pd$ on the program space $PS$. The entropy, $En(pd)$ of this distribution is a measure of our uncertainty, and while our uncertainty measure is greater-than a threshold $\epsilon$, we continue to add an input-output $\langle i, f(i) \rangle$ to the set $\varphi$ of input-output examples. The input $i$, which is picked in each iteration as a significant input, is the one that maximizes entropy $En(Pr_i)$. Note that $En(Pr_i)$ is the uncertainty in the output for input $i$ given our current belief $pd$ of the desired program. Once a new input-output pair is added to $\varphi$, we use an enhanced passive learner, $ppl^*$, to update $pd$ in that iteration. When the loop
terminates, we return the program $p$ whose probability $pd(p)$ is maximum as the learnt program.

The active program learner uses an enhanced passive learner, $ppl^*$, as a subroutine. The key difference between $pp$ and $ppl^*$ is that $ppl^*$ returns a probability distribution $pd$ on the program space, and not just a single program. Since computing and representing $pd$ precisely is not feasible, the probability distribution $pd$ is returned in the form of a sampled set of programs (consistent with the input-output examples generated so far) and an assignment of probability to this sampled subset.

### 4.2 Sampling Program Learner

The enhanced passive program learner, $ppl^*$, is implemented as a sampling program learner. A sampling program learner $spl$ is a computable function with the signature $spl : \Sigma^* \times 2^I \times \text{SamplingSpec} \rightarrow (PS \rightarrow [0, 1])$ such that for any input-output example sequence $seq$, subset $I_0$ of inputs, and a sampling specification $sspec$, the returned probability distribution $pd := spl(seq, I_0, sspec)$ is such that

1. if $pd(p) > 0$ then $p$ is consistent with all examples in $seq$,
2. the set $\{p \mid pd(p) > 0\}$ is consistent with the sampling specification $sspec$.

The probability distribution returned by $spl$ is assumed to reflect the current belief about the intended program.

The reason for working with samples is obvious: the space of programs consistent with a given set of input-output examples can be very large. It has been observed that a typical real-life domain-specific language for data transformation may contain up to $10^{20}$ programs consistent with a given single input-output example [30]. While such a program space can be represented symbolically using version space algebras [21, 16, 25] or finite tree automata [34, 35], working with a probability distribution over this space is infeasible and counterproductive. We therefore work with samples.

A sampling specification $(top, random)$ is a pair of numbers that indicate how many top programs to pick and how many random programs to sample from the set of programs consistent with the given input-output examples. The procedure for collecting $k$ top programs poses no significant challenges: most passive program learners that can generate one top program can also generate $k$ top programs. So, we just focus on random sampling here.

First, we observe that a sampling specification is trivial to satisfy when the set of all programs consistent with the correctness specification (input-output examples) is small. We also note that we can sample from state-of-the-art symbolic program set representations, including VSAs [16] and FSAs [35]. We next describe how to randomly sample while performing synthesis using an enumerative [16] or deductive approach [25].

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6 This is more challenging when an underlying program synthesizer is based on a constraint solver, where requesting top $k$ programs is feasible for state-of-the-art optimizing solvers [2] but requesting uniformly random instances is hard.
Consider the program space $PS$ that consists of programs generated by a top-down tree grammar. Let $N := f(N_1, N_2) \mid g(N_3, N_4)$ be two top-down tree automata transitions that say that a program generated by nonterminal $N$ can either be of the form $f(p_1, p_2)$, or of the form $g(p_3, p_4)$, where each $p_i$ is recursively generated from nonterminal $N_i$. While there are many approaches for program synthesis, the preferred inductive synthesis approach is based on decomposing a synthesis problem, $Synth(N, \phi)$, on a nonterminal $N$ and a specification (input-output examples) $\phi$ into subproblems on nonterminals $N_1, \ldots, N_4$ and derived specifications $\phi_1, \ldots, \phi_4$, and subsequently, putting the results of the subproblems together to obtain a solution for the original problem. The key idea behind a sampling program learner is that we can extend this decomposition step to also decompose the sampling specification $ss$. This means that we decompose the learning problem $(N, \phi, ss)$, where $N$ is the nonterminal, $\phi$ is a program correctness specification, and $ss$ is a sampling specification, into subprogram learning problems, $(N_i, \phi_i, ss_i)$, for $i = 1, \ldots, 4$, and after we have recursively solved the subproblems, we obtain a solution for the original problem by composing the solutions together. In particular, for sampling, this means we get samples of subprograms, and we use them to get samples for the top-level program.

Figure 2 recursively defines the function $RandomK(N, \phi)$, which returns $k$ random samples of programs generated by $N$ and consistent with $\phi$. Its definition follows the definition of the passive learner, $Synth$, itself. In particular,

(R1) if $Synth$ decomposes the synthesis problem on $N$ and $\phi$ to synthesis over $f(N_1, N_2)$ and $g(N_3, N_4)$, then we uniformly sample from a set containing $k$ random samples of the form $f(N_1, N_2)$ and $k$ of the form $g(N_3, N_4)$, and

(R2) if $Synth$ decomposes the synthesis problem on $f(N_1, N_2)$ and $\phi$ in terms of subproblems on $N_1$ and $N_2$, and gets its result in the form $f(\bigcup_i P_i, \bigcup_j P_j)$, then $RandomK$ samples equal number from each $P_i$ to get the $k$ random samples of $f$-rooted programs.

Note that random sampling is not uniform over the program set, but uniform over the syntactic classes of programs that are generated during the synthesis process: this is ideal because it ensures that samples are diverse.
Table 1. Clustering inputs: Assume that the three programs \( p_1, p_2, p_3 \) have equal probability (\( \frac{1}{3} \)), and that they produced the shown outputs on the five inputs. Based on features in the input strings, Inputs \( i_1 \) and \( i_2 \) are similar, and even have equal entropy 0.91, and should be clustered. Inputs \( i_{95}, i_{96} \) may get clustered with \( i_1, i_2 \) based on input features, but based on features in the output (columns \( o_1, p_2, \) and \( p_3 \)) they are likely to be in different clusters, which is good since \( i_{96} \) has higher entropy.

| Inputs  | Programs | Entropy |
|---------|----------|---------|
| \( i_1 \) = “foo1bar11baz” | \( p_1 \) | 1 11 \( \sum_{i=1}^{3} - \frac{1}{i} \log \frac{1}{i} \) |
| \( i_2 \) = “foo2bar22baz” | \( p_2 \) | 2 22 |
| \( i_{95} \) = “fooabar1baz” | \( p_3 \) | a a a1 |
| \( i_{96} \) = “fooabar-1baz” | a a -1 |
| \( i_{97} \) = “uvw” | a -1 |
| \( i_{97} \) = “uvw” | \( \epsilon \)\( \epsilon \)\( \epsilon \) | 1.6 |

We omit several low-level details about sampling program learners here, e.g. sub-specifications can be conditioned on other sub-specifications. However, most of these details are easy to extend to sampling specifications by following the approach taken for correctness specifications [25].

**Probability measure function.** Finally, we need to assign probabilities to the sampled programs. We assign a probability to a sampled program that is proportional to its rank order. Note that our sample contains some top-k program and some randomly sampled programs. All (passive) program learners are equipped with a ranking function that assigns a rank to each (synthesized) program; however, these ranks do not directly map to probabilities in any way, but are only coarse indicators of a program’s likelihood to be the intended program. The top-k programs are picked based on this ranking. While the rank (score) itself is not meaningful, the order it induces on the programs remains meaningful, and hence assigning a (slightly higher) probability to samples that are ranked higher is justified.

In Section 4.5 we show the value of the combination of top-k and random sampling by performing evaluation using three specific approaches: (a) no sampling, (b) only top-k programs, and (c) a combination of top-k and uniform sample.

### 4.3 Input Sampling

Input sampling is required because real-life datasets in data wrangling scenarios typically contain tens of thousands of rows, and enumerating them all is counterproductive for multiple reasons. First, a typical UI response time for a user-facing application must stay within 0.5 sec, which is difficult to satisfy when enumerating over the entire dataset. Second, most inputs in a typical dataset have similar distinguishability; ideally we should consider just one representative.

Input sampling aims to reduce the computational cost of the active program learner by restricting it to a sample of the input space \( I \). The naïve uniform
sampling approach is not ideal here, and we use two key ideas for sampling inputs: (1) input-features based clustering and (2) output-features based clustering.

**Input-features based clustering.** The hypothesis here is that *reasonable programs behave similarly on inputs with similar features, and hence, such inputs are likely to have the same uncertainty*, as illustrated in Table 1. Hence, we cluster the inputs based on *string clustering*, and sample equally from each cluster to ensure full coverage of different “shapes”.

A string clustering algorithm takes a dataset \( I \), and returns a *partition* of this set into disjoint clusters. It is parameterized with a similarity measure to cluster the strings in \( I \). Formally, it has the following signature:

\[
\text{cluster}: I \mapsto (I \mapsto \{1, 2, \ldots, M\})
\]

where \( M \) is the (maximum) number of clusters created. A *partition* is a function \( \text{Partition}: I \mapsto \{1, \ldots, M\} \) that maps each input \( \text{in} \) to one of the \( M \) clusters. Let \( I^i = \{\text{in} \mid \text{Partition}(\text{in}) = i\} \) denote the \( i \)-th cluster.

Intuitively, inputs in different clusters should have sufficiently different syntactic shape. Thus, a *diverse* uniform sample \( I^* \) of \( n = |I| \) inputs can be constructed by randomly sampling \( \lceil n \times |I^i|/|I| \rceil \) inputs from the \( i \)-th cluster \( I^i \).

Clustering is parameterized by a similarity measure on the input space, which is based on standard features extracted from strings; see [22] for details.

**Example 3.** Consider the data transformation task where a user has presented one example “12 in” \( \mapsto \) “12”, and the set of other inputs includes “8 in” and “30 cm” (and other strings denoting length in either in or cm). Most candidate programs that are learnt from the one given example are unlikely to perform differently on “30 cm”, and hence it is unlikely that “30 cm” will be a distinguishing input. However, input clustering will clearly identify two separate clusters corresponding to the two units, and “30 cm” should be presented as a significant input to the user.

**Output-feature based clustering.** The hypothesis here is that *if the output on input \( i \) looks sufficiently different from the outputs generated by other inputs, then uncertainty about \( i \) is likely to be high*. Hence, the outputs generated by the current candidate programs can indicate which inputs are potential candidates for being significant. Table 1 shows a 2-d matrix over programs and inputs: output-based clustering partitions inputs based on clustering the values in Column \( p_1 \) (where \( p_1 \) is the top-ranked program). We can optionally also cluster based on Column \( p_2 \) and Column \( p_3 \). Whereas the entropy \( En(i) \) of an input \( i \) is defined by the values in \( i \)’s row, \( En(i) \) having a low value often correlates with \( p_1(i) \) being of a different “shape” than other outputs in Column \( p_1 \) (see Input \( i_{95} \) and \( i_{96} \)).

**Example 4.** Consider a scenario where the user is extracting the year from dates formatted in many different forms. The given input set contains the inputs “05-Feb-2015”, “25 December 2013”, “2010-12-12”, and “9/3/2017”. Clustering the
input space gives a large number of partitions. However, the output set generated by any synthesized candidate program should be relatively uniform (in this scenario, form a single cluster described as $d\{4\}$).

**Null outputs.** Exceptions need to be handled properly when computing the uncertainty $En(i)$ about an input $i$. Specifically, when an input does not satisfy the preconditions of a (synthesized) program, its output defaults to a special value null value, denoted as, say, $\epsilon$. However, not all such values are identical, and hence, when defining uncertainty about an input, we treat every instance of a null value in the output as being different from each other.

**Example 5.** String transformation programs often return a null value when the input is not of the format they expect. For example, the programs “extract the first digit” and “extract the second digit” both return the null value on the string “ABC”. However, it is a stretch to say that these programs behave similarly on the input “ABC”. Therefore, we consider all null values to be unequal when defining the uncertainty, $En(i)$, about (the output on) input $i$. In Table 1, Input $i_{97}$ generates null values on all programs, and hence, with this change, $En(i_{97})$ is not 0 but the larger value 1.6.

### 4.4 Information Gain and Distinguishability

The uncertainty $En(i)$ about an input $i$ is closely related to its ability to distinguish programs in $PS_1$: recall that an input $i$ distinguishes programs $p_1$ and $p_2$ if $p_1(i) \neq p_2(i)$. The following proposition states that optimizing for uncertainty is at least as general as optimizing for distinguishability.

**Proposition 2.** Given two inputs $in_1$ and $in_2$, if $p_i(in_1) \neq p_j(in_1) \implies p_i(in_2) \neq p_j(in_2)$ for all programs $p_i,p_j$, then $En(in_1) \leq En(in_2)$.

### 4.5 Evaluation

We evaluate active program learners in the following way. We enclose the while loop in Procedure `ActiveProgramLearner` inside an outer loop that terminates only when the program $p_{best}$ learnt by inner loop is consistent with all the input-output examples; that is, $p_{best}(i) = f(i)$ for all $i \in I_0$. The outer loop is intended to mimic interaction with the user, which is needed whenever the inner loop terminates, but the outer does not. These cases are counted as *false negatives*, and in such cases, we continue the inner loop by picking an input $i$ where $p_{best}(i) \neq f(i)$ as the next significant input. Active program learners are evaluated based on:

**Number of iterations:** We want to minimize the number of iterations of the inner loop until the outer loop terminates.

**False positives:** Whenever the inner loop generates an input $in$ on which the current program $p_{best}$ and the desired function $f$ agree (that is, $p_{best}(in) = f(in)$), the resulting iteration appears futile to the user. Such inputs $in$ are called false positives, and we want to minimize them.
**False negatives:** False negatives occur when the active learner terminates with an unintended program. We want to minimize the number of false negatives.

The three criteria above differ in importance in different applications. Generally, false negatives are more expensive than false positives since a false negative requires the user to manually find the next distinguishing input in $I_0$, whereas a false positive requires only a confirmation of the current program output. However, they also differ in their cognitive load and user experience implications: a false positive is likely to cause irritation and mistrust in the system, whereas a false negative may lead the user toward ending the interaction prematurely and using an incorrectly synthesized program.

We evaluated 9 different variants of our active program learner on a collection of 791 scenarios. The goal of the variants is to showcase the value of each key idea proposed in this work. These variants are defined by their choice in the (a) program sampling (PS) dimension (“top-$k$”, and “top-$k$ ∪ random”), and the (b) input sampling (IS) dimension (random sampling, input clustering, output clustering, input+output clustering). Apart from the 8 variants obtained from the above choices, we had one baseline version.

The baseline is a passive program learner where we do not use information gain to pick significant inputs, and let the user do the job. That is, the implementation just picks the first input where the output of the current program does not match the intent (mimicking what the user would have to do when interacting with a passive learner). Thus, in this baseline, the number of false positives is 0, but every iteration adds 1 to the number of false negatives. The goal of the (eight variants of the) active learner is to reduce the number of false negatives (the most important criterion) by potentially increasing the number of false positives.

We remark that the baseline is a state-of-the-art and not naive: the input that the user picks to provide an example is, in fact, a distinguishing input. One can argue that a smart user might pick a more informative input, but this paper shows that the active learner can actually mimic such smart users, and thus reduce the cognitive load on such users.

Procedure **ActiveProgramLearner** uses a threshold $\epsilon$ and compares it to the entropy, $En(pd)$, of the probability distribution over possible programs, $pd$, to decide when to terminate. Since computing $En(pd)$ just for this purpose is wasteful, in our implementation, the active learner terminates the session when none of the top-ranked programs are distinguished by the (maximum entropy) significant input.

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**Evaluating program sampling strategies.** The top part of Table 2 shows the change in performance of the active program learner as we change the program sampling technique. The input sampling technique is fixed to random.

Compared to the baseline, where we have a very high number of false negatives and 0 false positives, when using top-$k$ programs as our sample, we increase

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7 Part of the benchmarks were taken from [https://github.com/Microsoft/prose-benchmarks](https://github.com/Microsoft/prose-benchmarks).
| Significant Input | #Iterations | #False Positives | #False Negatives | #Time-Outs |
|-------------------|-------------|------------------|-----------------|------------|
| Algorithm Variant | ≤ 1 | ≤ 2 | ≤ 3 | ≤ 4 | ≤ 32 |
| Baseline          | 11 | 39 | 109 | 184 | 737 | 0  | 4840 | 47 |
| Top-k             | 11 | 39 | 108 | 182 | 738 | 5049 | 42  | 47 |
| Top-k ∪ Random-k  | 11 | 41 | 110 | 184 | 732 | 5082 | 3   | 53 |
| Random            | 11 | 41 | 110 | 184 | 732 | 5082 | 3   | 53 |
| Input Clustering  | 315 | 498 | 609 | 667 | 733 | 724  | 158 | 52 |
| Output Clustering | 409 | 611 | 690 | 717 | 742 | 276  | 193 | 43 |
| Input-Output Clustering | 296 | 481 | 597 | 655 | 733 | 783  | 139 | 52 |
| Baseline (user)   | 11 | 39 | 109 | 184 | 737 | 0  | 4840 | 47 |
| False Positives   | 409 | 611 | 690 | 717 | 742 | 276  | 193 | 43 |
| False Negatives   | 296 | 481 | 597 | 655 | 733 | 783  | 139 | 52 |
| Overall IS        |             |                 |                 |             |

**Table 2.** Number of scenarios (out of 791) solved using up to #Iterations iterations by variants of the active programs learner. We also show the number of false positives (#FalsePositives) and false negatives (#FalseNegatives) generated across all scenario instances for these variants. All runs share a timeout of 60 sec, with a median time of 0.8 sec per iteration and mean time of 1.3 sec per iteration.

The number of false positives (because we generate inputs that distinguish between irrelevant programs), but significantly decrease the number of false negatives. Combining top-\(k\) with random-\(k\) gives enough diversity to the program sample to further reduce false negatives, but adds slightly more false positives. The number of scenarios solved in a given number of iterations does not change substantially. This experiment clearly shows that top-\(k\) and random-\(k\) programs is the best choice for sampling programs.

**Evaluating input sampling strategies.** The middle part of Table 2 shows the effect of changing the input sampling technique on the performance of the active program learner. We fix program sampling to top-\(k\) combined with random-\(k\) here. The “Random” sampling strategy is implemented as follows: if the total number of inputs is less-than a parameter \(M\), then it returns all the inputs, and otherwise it samples \(M\) inputs randomly from the set of all inputs. It turns out that a large percentage of our benchmarks contained a small number of inputs (less-than \(M\)). Consequently, random sampling picked the complete set of inputs, causing very few false negatives. Clustering causes the active learner to consider only a selected number of pertinent inputs: this reduces the number of false positives, but since we are ignoring inputs, it adds more false negatives. Since clustering focuses the active learner on promising inputs, we see a drastic improvement in the number of benchmarks solved with just 1, 2, or 3 iterations. Output clustering aggressively removes inputs, and hence, it reduces false positives dramatically, but at the cost of slightly increasing false negatives. The results show the value of clustering – especially when the number of available inputs is really large – and trade-off between reducing false negatives and false positives.

**Overall Evaluation.** In the bottom part of Table 2 we compare the baseline with the version that optimizes for false positives and the version that optimizes for
false negatives (ignoring the version that use all “Random” for input sampling because they are essentially using all inputs). We see that the best active program learners based on greedy information gain perform much better than what the user is able to achieve interactively, while also significantly reducing the cognitive load (#false negatives) on the user.

5 Related Work

Query filtering in active learning. Since we are not synthesizing inputs, but just picking the “best” input to send as a query to the user, our work falls under the query filtering paradigm of active learning [3]. A particular filter, called query by committee (QBC) [27, 8], works by sampling a committee of (consistent) programs, sampling (randomly) an input (query), and evaluating entropy of the input on that sample (using our terminology) to either pick or reject it. This is similar to our work where we sample the programs to evaluate entropies of inputs. The main difference is that the work on QBC is mostly a theoretical study that makes many assumptions, such as, existence of a uniform sampling algorithm from the version space. Our work shows how the same concepts can be applied to a real program synthesis task. Moreover, we also discuss ways to sample programs and even sample inputs in a way to make the QBC ideas practical in the program synthesis setting. In the field of program synthesis, the QBC paradigm was used very recently to pick queries when synthesizing datalog programs [29]. However, it does not formally cast the program synthesis problem in a probabilistic framework as we do here. Furthermore, the output space is Boolean (unlike in our setting, where it is String, which causes us to introduce novel ideas, such as, output clustering), and allows program sampling to be “complete” in a sense (by picking a most-specific and most-general program from the version space). This is not possible in our more general setting. This difference also manifests in the fact that [29] has a complete procedure.

Input and Output Clustering. We have used novel ideas for sampling inputs based on clustering on features in the input and features in the generated output (by some top-ranked program). The work on synthesis with abstract examples [5] is based on a similar intuition. It recognizes that certain input-output examples are similar enough to be clustered and presented to the user as one abstract example. The goal there is to let the user effectively give a set of concrete examples at once to the synthesizer (by validating an abstract example). In our work, we use clustering of inputs to perform intelligent sampling of inputs. We pick a concrete input from this sample to present to the user.

Distinguishing Inputs. The notion of significant inputs introduced here generalizes distinguishing inputs, introduced in prior work on program synthesis [13, 9]. An input $i$ is distinguishing if there exist two programs $p_1$ and $p_2$ that are both consistent with the current constraints, but produce different outputs $p_1(i)$ and $p_2(i)$ on the input $i$. Thus, $i$ distinguishes between two programs, and hence an additional input-output constraint for $i$ eliminates either $p_1$ or $p_2$. 
In this work we generalize this idea to not just two but a set of programs, and furthermore, emphasize that not all such inputs are equally effective for optimizing synthesis convergence in practice. A distinguishing input is a likely candidate for being significant. However, a significant input must also satisfy three stronger requirements:

1. A significant input is ⊥ when the active learner is confident that it has converged. Hence, it is possible that distinguishing inputs exist, but the active learner nevertheless does not pick any of them as significant.
2. A distinguishing input disambiguates any two programs. In this work, we show that to optimize the convergence of the active learner, a significant input does not treat all programs equal: it prioritizes highly-ranked programs and programs that disagree with the current candidate.
3. In prior work [13, 9], the input space is known a priori, such as the space of all size-\(n\) bitvectors. This allows closed-form formulations of the input selection problem and analysis of its convergence. In the practically-inspired formulation of the significant input problem (Section 2), the input space is not known in closed form.

**Oracle-Guided Inductive Synthesis.** Jha and Seshia recently developed a novel formalism for example-based program synthesis called oracle-guided inductive synthesis (OGIS) [14]. It builds on top of counterexample-guided inductive synthesis (CEGIS), a common paradigm for building interactive synthesis engines [31]. In OGIS, a synthesis engine has access to an oracle, which is parameterized with the types of queries it is able to answer. Typical kinds of queries include class membership, counterexamples, and distinguishing inputs.

The setting studied in this work can be likened to OGIS, with the user playing the role of an oracle providing counterexamples. However, it differs in two major ways. First, we define the notion of significant inputs that try to minimize some convergence criteria, such as the number of iterations. Second, we consider the “active learner” setting where we use information gain to generate significant inputs and present them proactively to the user. Both aspects improve the usability of an interactive system.

**Predictive Interaction.** The notion of proactively interacting with the user during a synthesis session is known as predictive program synthesis or predictive interaction. Mayer et al. [18] established that any form of interaction (such as displaying a paraphrased program candidate or presenting distinguishing inputs) improves the correctness and subjective trust in an example-based data transformation system. Building on their findings, in this work we investigate how particular choices of significant inputs and techniques for selecting them impact the convergence criteria of a synthesis interaction. Similarly, Kandel et al. [15] and Peleg et al. [24] present different settings of predictive interaction, in which the proactively sought constraints and suggestions describe the subexpressions of the desired program, as opposed to its behavior on individual inputs.
Active Learning. The process of using significant inputs to iteratively perform program synthesis – as described in this paper – is an example of active learning [26]. In active learning, data is not available a priori, but the learner queries for data that will help it converge. The question of significant inputs – the next query to make – becomes the core problem of active learning. In fact, since the inputs on which the synthesized program ought to work are also available in our setting, our setting falls under what is known as pool-based active learning [19]. This setting has been studied for classification, version space reduction, and other classic machine learning domains [4, 1], but here we study it in the real-life domain of a string data transformation system.

Software Testing. Significant inputs relate to synthesis in the same way as test inputs relate to verification. The goal of both is to improve confidence in the underlying artifact after these inputs have been used to perform synthesis or verification. Test inputs are picked so that executions on those inputs covers, for example, all possible program paths. Significant inputs are picked so that each one (given as an input-output constraint) eliminates a subspace of programs, and together they eliminate (almost) all unintended programs.

Data-driven invariant learning. There is plenty of work in learning invariants from data [6, 28, 36, 7, 23], but it is mostly in the passive setting. The significant question problem arises when synthesizing invariants using active learning. Jha and Seshia [14] use a synthesis framework to explore theoretical bounds on learning iterations – what we call the optimum significant questions problem – but they do not propose any algorithmic approach, such as the information gain approach here, to achieve the theoretical bounds.

6 Conclusion

The last decade of work in example-based program synthesis, and industrial applications of resulting technologies, have shown that (a) program synthesis in practice proceeds as an iterative interactive session, and (b) the user’s cognitive load and confidence in the synthesis system largely depends on the interaction interface between the user and the system. Proactive resolution of intent ambiguity is paramount to delivering high-quality user experience. In this work, we formally study the general significant questions problem – questions to proactively ask the user – and use information-theoretic notion of entropy to solve it. We instantiate the general approach to develop an active program learner that is shown to minimize the number of synthesis iterations until convergence, as well as control the number of false positive and false negative examples.

While the framework of significant questions and information gain introduced here helps optimize the convergence criteria, it does not directly address the criteria of the user’s confidence in the program and cognitive load. Experimentally measuring the effect of different techniques for generating user interaction on the user experience is an important area of future work. Further exploration of
active invariant learning in program synthesis by examples is also left for future work.

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