Modal Analysis of Double-Bolted Connection under Different Operating Conditions

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Abstract: Based on the finite element analysis and modal analysis, the vibration characteristic parameters and the law of behaviour changes in different bolted connection structures were studied. According to the different contact status of each component and bolt, the 16 typical working conditions were designed and selected, and the subspace iteration method was used to simulate the experiment with the ANSYS Workbench software. The simulation results showed that the natural frequency of the double-bolted connection structure tends to be stable under different working conditions. A certain degree of fluctuation of the natural frequency sometimes occurred, but the fluctuation range was not large. This research thus laid the foundation for subsequent structural health monitoring and diagnosis.

1. Introduction
The bolted connection structure is simple in form, convenient in disassembly and diverse in form, thus it is widely used in industry, such as docking of separation surface between spaceflight structure cabins and main shaft structure of wind generator. According to the distinct operational principle, the bolt connection is mainly divided into bolt connection and reaming bolt connection, with notable differences in the gap between the hole wall and the bolt rod. Although the bolt connection structure can provide a certain connection stiffness and strength for the structure, however, because of the geometric catastrophe and the destruction of the structure to the continuity, the local deformation of the joint will be considerable and the distribution of stress and deformation is uneven. The relaxation of bolt pre-load and the fatigue as well as creep at the joint will impinge on the reliability of the connection. More importantly, due to the contact and friction between the adjacent interfaces, it is vulnerable to rendering structure damping, nonlinearity, uncertainty in the analysis process of bolted connection and even catastrophe [1-5].

In the past decades, scholars across the world had studied the bolted joint structure mainly from the dynamic performs of the bolted joint stiffness of the bolted joint structure and the loosening mechanism. Yao et al. studied the effects of aero-engine bolting loads and structural parameters on joint stiffness [6]. Wang established the corresponding models and verified the nonlinear dynamics of bolted joint structures [7]. Liu analyzed the loosening mechanism of bolted joints under axial load excitation [8]. Klok studied mechanical behavior of bolted joints under impact rates of loading [9]. Tan researched the effect of bolt spacing on the tightness behavior of bolted flange joints [10].

Recent years, structure health monitor had increasingly become prevail. He explored vibration-based structural damage detection with applications to structures with bolted joints [11]. Ma et al. investigated nonlinear vibration characteristics of a rotor system with pedestal looseness fault under
different loading conditions [12]. Zhang et al. researched application of sub-harmonic resonance for the detection of bolted joint looseness [13].

Besides, many scholars took modal analysis as a helpful tool to explore the dynamic performs of the system. Barad et al. found that natural frequency could help crack detection in cantilever beam though experimental study [14]. Tse proposed modal analysis for simplified model of a linked building system [15]. Huang et al built a simplified mode of dynamometer to achieve the theoretical natural frequency and the results indicated that the dimension of the key structure had huge impact on the natural frequency [16]. Wang et al. carried out an optimal design of the dynamic characteristics on the accessory driving system and the results revealed that the first-order natural frequency was sensitive to different tensioner dimensions [17].

The connection status of double-bolted joint has a notable effect on the reliability of the entire structure. Hence, it is indispensable to develop an effective method for health monitoring of bolted joints to detect the loss of the pre-load or the looseness of the bolts, which can trigger failure of bolted joint, causing possible engineering disasters, if not promptly detected. In this paper, based on the finite element analysis and subspace iteration method in the modal analysis, the characteristic parameters of vibration and the law of behavior changes in different double-bolted connection structures were investigated. According to the distinctive contact status of each part and bolt, the 16 typical working conditions were designed, and the ANSYS Workbench software was served as the main simulation platform to support all of these simulation experiments.

2. Calculation of modal analysis

The finite element method is an effective numerical method for solving partial differential equations. Its theoretical basis is calculus of variations. In essence, it is almost a weighted residual method for piecewise interpolation [18]. And it is an effective approximate calculation method, thus the ANSYS Workbench platform is chosen to conduct the corresponding simulation experiments.

Modal analysis is used to determine the vibration characteristics of the design structure or machine parts, namely natural frequencies and modes of structures, which are critical parameters in the design of dynamic load. Meanwhile it is also a necessary procedure for other dynamic analysis problems such as transient dynamic analysis, harmonic response analysis and spectral analysis [19].

The approximate calculation methods of modal analysis are mainly divided into Ritz method, matrix iteration method and subspace iteration method. The matrix iteration method does not require the hypothetical modal vectors, but every iteration of the first order inherent characteristic is required. However, the original high order eigenvalue problem is converted to a lower order eigenvalue problem via Ritz method, greatly reducing the degrees of freedom, and some low order eigenvalues and its eigenvectors can be obtained at the same time, but the precision depends mainly on the accuracy of the hypothetical mode. So the subspace iteration method ingeniously combines the above two methods. On the one hand, Ritz method is applied to shrinking the complexity of degree of freedom, and to determine the corresponding natural frequencies and natural modes. On the other hand, the matrix iteration method is employed to perform iterative operation, which can obtain sufficient accurate hypothesis mode of vibration and improve the accuracy of calculation [20].

The typical process for modal analysis as follows:

1. Establish system vibration differential equation, shown in Equation (1).

\[
M \ddot{x} + C \dot{x} + Kx = f(t)
\]  

(1)

Where \(M\) is the mass matrix, \(C\) is the damping matrix, \(K\) is the stiffness matrix, \(f(t)\) is the exciting force vector and \(x\) is the displacement vector.

2. Then neglect the damping item and let \(f(t) = 0\), then obtain the frequency characteristic equation of the system, shown in Equation (2).

\[
([K] - \omega_n^2[M])\{u\} = 0
\]  

(2)
The above equations were solved by subspace iterative method. Firstly, Ritz method was used to reduce the degree of freedom of the problem. For a system with \( n \) degrees of freedom, in order to calculate the first \( P \)-order inherent properties, \( S \) had to be assumed a certain mode larger than \( P \).

\[
[u]_0 = \{[\varphi]_1, [\varphi]_2, ..., [\varphi]_{n1}\}
\]  

(3)

In order to approximate the real vibration mode, the first iteration mode matrix \([\varphi]_0\) assumed above had to be calculated by the matrix iteration method.

\[
[\varphi]_1 = [D][u]_0 = [\delta][M][u]_0
\]  

(4)

Before the next iteration, \([\varphi]_1\) was performed orthogonalization so that its column vectors converge to the dominant modes of different orders after iterations. And with \([\varphi]_1\) served as a hypothetical mode, the generalized mass matrix and generalized stiffness matrix of reduced dimension subspace for the first iteration was reckoned by using Ritz method.

\[
\begin{bmatrix}
[\tilde{M}]_1 = [\varphi]_1^T[M][\varphi]_1 \\
[\tilde{K}]_1 = [\varphi]_1^T[K][\varphi]_1
\end{bmatrix}
\]  

(5)

Then the eigenvalue equation was obtained, shown in Equation (6).

\[
((\tilde{K})_1 - \tilde{\omega}^2_{m}[\tilde{M}]_1)[Y]_1 = 0
\]  

(6)

The dimension-reduced subspace, i.e., the first iteration result of the \( S \)-orders natural frequencies and the corresponding modal vectors in the original system can be calculated by Equation (6), shown in Equation (7).

\[
[Y]_1 = \{[Y^{(1)}]_1, [Y^{(2)}]_1, ..., [Y^{(S)}]_1\}
\]  

(7)

The above results were then brought into Equation (8).

\[
\{u\} = \{[\varphi]\} [Y]
\]  

(8)

After the above linear transformation, the first iteration result of the first \( S \)-orders modal vectors at the original system could be acquired, shown in Equation (9).

\[
[u]_1 = \{[\varphi]_1, [Y]_1 = \{[u]^{(1)}_1, [u]^{(2)}_1, ..., [u]^{(S)}_1\}
\]  

(9)

Repeated the above iterative calculation until the calculation result and the previous result satisfied with the accuracy requirements.

3. Simulation experiment

As shown in Figure 1, this model can be regarded as a cantilever structure, consisting mainly of two cover plates, two bolts and fixed ends. Plate 1 is above, Plate 2 is below, and the middle is connected by Bolt1 and Bolt 2. The left end face of Plate 1 is approximately fixed at the fixed end.

The sizes of cover plate and bolt are shown in Table 1. And the relevant material parameters are displayed in Table 2.
Figure 1. The schematic of the overall bolted connection structure

Table 1. Dimensions of all components

| Component | Dimension (mm) |
|-----------|---------------|
| Plate 1   | 100*20*5      |
| Plate 2   | 100*20*5      |
| Bolt 1    | M6*16         |
| Bolt 2    | M6*16         |
| Nut 1     | M6*5          |
| Nut 2     | M6*5          |

Table 2. Material properties of all components

| Modulus of elasticity (Pa) | Poisson ratio | Density (Kg/m³) | Volume modulus (Pa) | Shear modulus (Pa) |
|----------------------------|---------------|-----------------|---------------------|--------------------|
| 2e11                       | 0.3           | 7850            | 1.67e11             | 7.7e10             |

First, according to the dimensions shown in Table 1 above, a three-dimensional parameterized model of the two bolted connection structure is set up and the assembly is formed. Then it is stored in a general three-dimensional model format and imported into the Workbench. Secondly, based on the parameters depicted in Table 2, the corresponding parameters of this model is configured: modulus of elasticity denotes 2e11 Pa, Poisson ratio designates 0.3, the volume modulus refer to 1.67e11 Pa, shear modulus indicates 7.7e10 Pa, density connotes 7850 kg/m³, ultimately the entire finite model is meshed into grids, and the total number of meshes is 3242.

In this paper, there are six components, namely Plate 1, Plate 2, Bolt 1, Bolt 2, Nut 1 and Nut 2. All these components have various contact relationships. The cover Plate 1 and the cover Plate 2 are constantly bonded. Then the complex threaded connection between Bolt 1 and Nut 1 is simplified to be bonded. And Bolt 2 and Nut 2 do the same, also set to be bonded. The contact between the lower surface of the Bolt head 1 and the upper surface of the Plate 1 is set to bonded, and the upper surface of the Nut 1 and the lower surface of the Plate 2 are set to be bonded. Similarly, the contact between the lower surface of the Bolt head 2 and the upper surface of the Plate 1 is set to bonded, and the upper surface of the Nut 2 and the lower surface of the Plate 2 are set to be bonded.

In addition, Bolt rod 1 and Plate 1 and Plate 2, as well as Bolt rod 2 and Plate 1 and Plate 2 can be contacted, or cannot contact. In this article, the contact status between Bolt 1 and Plate 1 or Plate 2 can be simply divided into two state: bonded or not bonded, that is, there is clearance between Bolt rod 1 and Plate 1 or Plate 2. Similarly, there is a similar contact relationship between Bolt rod 2 and Plate 1 or Plate 2.

The contact between the Bolt 1 and the inner holes of Plate 1 and Plate 2 is independent. Likewise, the contact of the Bolt 2 with the inner holes of the Plate 1 and the Plate 2 is also unrelated. Therefore, the experimental scheme consists of 16 operating conditions. As shown in Table 3, "0" represents a gap between the bolted rod and the cover plate, whereas "1" embodies no gap between the bolt and the cover plate, i.e., bonded. The 16 working conditions can be represented by four binary digits. For a certain working condition, read data from left to right, then top to bottom. For instance, the working condition 1000 signifies only Bolt 1 rod and Plate 1 hole is bounded, Bolt 1 rod and Plate 2, Bolt 2 rod...
and Plate 1, Bolt 2 rod and Plate 2 all have gaps, i.e., unbounded. Other working conditions are analogous in turn and not in redundancy.

**Table 3.** List of all working operations

| Working condition | Position | Bolt 1 | Bolt 2 | Working condition | Position | Bolt 1 | Bolt 2 |
|-------------------|----------|--------|--------|-------------------|----------|--------|--------|
| Case 1            | Plate 1  | 0      | 0      | Case 9            | Plate 1  | 0      | 1      |
|                   | Plate 2  | 0      | 0      |                   | Plate 2  | 0      | 0      |
| Case 2            | Plate 1  | 0      | 0      | Case 10           | Plate 1  | 1      | 0      |
|                   | Plate 2  | 0      | 1      |                   | Plate 2  | 1      | 0      |
| Case 3            | Plate 1  | 0      | 0      | Case 11           | Plate 1  | 1      | 1      |
|                   | Plate 2  | 1      | 1      |                   | Plate 2  | 0      | 0      |
| Case 4            | Plate 1  | 1      | 0      | Case 12           | Plate 1  | 1      | 1      |
|                   | Plate 2  | 1      | 1      |                   | Plate 2  | 1      | 0      |
| Case 5            | Plate 1  | 1      | 0      | Case 13           | Plate 1  | 0      | 0      |
|                   | Plate 2  | 0      | 0      |                   | Plate 2  | 1      | 0      |
| Case 6            | Plate 1  | 0      | 1      | Case 14           | Plate 1  | 0      | 1      |
|                   | Plate 2  | 1      | 0      |                   | Plate 2  | 0      | 0      |
| Case 7            | Plate 1  | 1      | 0      | Case 15           | Plate 1  | 0      | 1      |
|                   | Plate 2  | 0      | 1      |                   | Plate 2  | 1      | 1      |
| Case 8            | Plate 1  | 1      | 1      | Case 16           | Plate 1  | 1      | 1      |
|                   | Plate 2  | 0      | 1      |                   | Plate 2  | 1      | 1      |

Annotation: “0” represents a gap between the bolted rod and the cover plate, whereas “1” embodies no gap between them.

4. Result analysis

![Figure 2](image)

**Figure 2.** Change trend of natural frequency in 16 working conditions

The first 10-order natural frequencies of 16 operating conditions are shown in Figure 2. As can be seen from Figure 2, although the working conditions are different from each other, their respective first 10-order natural frequencies show certain regularity, both upsurge with the increase of modal order, and their first 10-order natural frequencies range from 150 Hz to 8000 Hz. In the 4th order, 5th order and
8th order modes, the natural frequencies have significant changes, whereas the corresponding natural frequencies basically coincide in other modes.

![Figure 3](image.png)

**Figure 3.** Detailed changes in the natural frequency mutation

The 4th, 5th and 8th order modes of the 16 working conditions are exhibited in Figure 3. From Figure 3 (d), it can be seen that the 5th and 8th order modal frequency values of the 16 conditions are relatively stable, which are about 2300-2500 Hz and 6000-6200 Hz, respectively. The 4th mode frequencies of the 16 working conditions vary fairly sharply, ranging from 2000 to 2500 Hz. It reveals that though the frequency is low, the vibration status of the system will be easily changed.

From Figure 3 (a) - (c), it can be seen that modes of the 16 operating condition vary periodically whatever 4th, 5th and 8th modes. For example, in the 4th order mode, the modal frequency of 4 conditions is 1983.7 Hz, the modal frequency of 4 operating conditions is 2275.1 Hz, the modal frequency of the 4 working conditions is 2315.4 Hz, and the modal frequency of the other 4 operating conditions is 2455.7 Hz. This demonstrations that the vibration is homogeneous and periodic under dissimilar connect status.

5. Conclusion
In this paper, the double-bolted connection structure is taken as the research object. And according to the gap between bolt rod and bolt holes of the cover plate, 16 kinds of connection conditions are designed to execute the finite element experiment. The experimental results indicate that though the frequency is low, it is effortless to trigger the severe vibration of the overall structure. And in a certain mode, modal frequencies at different operating conditions are regular and periodic, which can lay a foundation for the future investigation of vibration characteristics of double-bolted connection structure as well as the location and diagnosis of faults.

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