The Friedmann Paradigm: A critical review

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Abstract

The Friedmann paradigm for a dynamical universe emanating from a spacetime singularity is critically reviewed. Quantum effects, playing the essential role at the very early stages, suggests that the universe may follow different course to that presented by the standard Friedmann solutions. The investigation of the back-reaction effect of the vacuum energy of quantized massless matter fields at finite temperatures shows that the original spatial singularity is avoided and that the universe is maintained all times at a critical density. Instead of having a universe that was created at once we have an emergent universe with energy being created continuously so as to maintain the overall density at its critical value. The calculations presented here provide a basis to construct a dynamical model for the universe where all the known problems of the standard big bang can be avoided from start without the need to assume the occurrence of an inflation phase.
I. INTRODUCTION

The Einstein field equations describes the relation between the geometry of the spacetime and its matter/energy content by the equations

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu}, \]

(1)
where $R_{\mu\nu}$ are the components of the Ricci tensor, $R$ is the Ricci scalar and $T_{\mu\nu}$ are the components of the energy-momentum tensor.

One more set of field equations were suggested by Einstein, which originally were devised in order to freeze the dynamical universe described by (1), these are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = -\frac{8\pi G}{c^4} T_{\mu\nu},$$  

where $\Lambda$ is the so-called cosmological constant. A positive $\Lambda$ represent a repulsive long-range force that may counter balance the gravitational attraction between all parts of the universe. For this reason and with an accurately chosen value of $\Lambda$ Eqs. (2) can produce a static universe. This was the original choice made by Einstein himself in order to manipulate a static universe in accordance with the dominating picture at that time.

The construction of either (1) or (2) requires defining a basic spacetime metric $ds^2$ in some general form and defining a matter/energy distribution $T_{\mu\nu}$ to stand on the right-hand side. Consequently the differential equations can be derived and solutions may be sought within certain boundary conditions. The results enable us understand how such a model universe will behave with time. Parameters calculated according to a given model should be testable by real observations.

Confirmed astronomical observations performed during the second decade of the last century suggested that the universe is expanding. This abandoned the existence of a cosmological constant value that balances the gravitational attraction, but did not necessarily abandon the presence of a parameter like $\Lambda$. This would allow for considering models of the universe with non-zero $\Lambda$ which we will now call the ”cosmological term”. Such models will be discussed later but first let us consider the more standard Friedmann models for the universe.

II. THE FRIEDMANN SOLUTIONS

A homogeneous and isotropic distribution of matter/energy will lead to a spherically symmetric spacetime. This can be best described by the line-element ($c = 1$)

$$ds^2 = dt^2 - S^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right],$$  

where $S(t)$ is called the scale factor.
where $S(t)$ is a scale factor that describes the separation between any two points on the spatial hypersurface

$$d\sigma^2 = \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2).$$

(4)

In (3) the factor $k$ describes the curvature of the space; $k = +1$ describes positively curved space, whereas $k = 0$ describes a flat space and $k = -1$ describes the negatively curved hyperbolic space.

If we set

$$r = \sin \chi$$

where $0 \leq \chi \leq \pi$, then with $k = 1$ we have

$$d\sigma^2 = d\chi^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

This is a 3-dimensional spherical hypersurface ($S^3$) at which the coordinates of any point are described by $(\chi, \theta, \varphi)$. Therefore, the spatial part of the metric in (3) is a 3-dimensional hypersurface in 4-dimensional spacetime.

Alexander Friedmann adopted the metric in (3) and consequently calculated the components of $R_{\mu\nu}$ as

$$R_0^0 = \frac{\ddot{S}}{S},$$

$$R_1^1 = R_2^2 = R_3^3 = \frac{\ddot{S}}{S} + \frac{2\dot{S}^2 + 2k}{S^2},$$

$$R = 6 \left[ \frac{\dddot{S}}{S} + \frac{\dot{S}^2 + k}{S^2} \right],$$

He further assume that the energy momentum tensor is given by

$$T^\mu_\nu = \text{diag} (\rho, -p, -p, -p) ,$$

(5)

where $\rho$ is the energy density and $p$ is the pressure.

Using (5) the time-time component of (1) gives
\[
\left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} = \frac{8\pi G}{3}\rho
\] 

(6)

This is normally called the Friedmann equation. The spacial components yields three identical equations, this is

\[
2\frac{\ddot{S}}{S} + \left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} = -8\pi G p.
\]

(7)

Subtracting (6) from (7) we get yet a third equation

\[
\frac{\ddot{S}}{S} = -\frac{4\pi G}{3}(\rho + 3p).
\]

(8)

If we have to solve the above equations we need an equation which defines the development of the matter/energy density, this could be obtained from the so-called fluid equation which is essentially the law of conservation of energy and momentum. The Friedmann paradigm assumes that

\[
T^\mu_{\nu\mu} = 0.
\]

(9)

This gives the fluid equation

\[
\dot{\rho} + 3\left(\frac{\dot{S}}{S}\right)(\rho + p) = 0.
\]

(10)

The condition in (9) also implies that the spacetime is conserved

\[
R^\mu_{\nu\mu} - \frac{1}{2}g^\mu_{\nu}R_{\mu} = 0.
\]

(11)

This conservation of the spacetime is carried through the varying gravitational field within the spacetime while spacetime is developing, e.g., gravitation become weaker as the universe expand and becomes stronger as the universe contract. Consequently the matter density has to vary like \(1/S^3(t)\). This normally is derived by solving the Friedmann equations for a model of a pressureless dust universe.

In order to obtain the complete solutions of the Friedmann equation we need to specify an equation of state which relates the momentum \(p\) to the energy density \(\rho\), this is usually given by
\[ p = \omega \rho, \quad (12) \]

with \( \omega = 0, 1/3, -1/3 \) and \(-1\) for pressureless dust, em radiation, vacuum and cosmological term respectively.

Details of obtaining the different solutions of the Einstein field equations according to the Friedmann paradigm can be found in [1], [2] or [3].

III. PARAMETERS OF THE FRIEDMANN SOLUTIONS

There are several basic parameters that can be designated for the Friedmann solutions these are

A. The Hubble parameter

This is defined by

\[ H = \frac{\dot{S}(t)}{S(t)}. \]

At present time the value of \( H \) is designated \( H_0 \) and is called the \textit{Hubble constant}. The Hubble parameter defines the rate of the expansion of the universe.

B. The density parameter

One important parameter that is deduced from the Friedmann solutions is the density parameter given by

\[ \Omega = \frac{\rho}{\rho_c}, \quad (13) \]

where \( \rho_c \) is a critical energy density defined by

\[ \rho_c = \frac{3H^2}{8\pi G}. \]

This parameter defines the geometry of the spacetime as being positively curved when \( \Omega > 1 \), flat when \( \Omega = 1 \) and is negatively curved when \( \Omega < 1 \). The Friedmann equation can be re-written in terms of this parameter as
\[ \Omega - 1 = \frac{k}{S^2(t)H^2}. \] (14)

The density parameter do not change throughout the whole course of the development of the universe. This implies that the state of curvature for the universe stays as it is all the time, i.e., no change in the curvature mode.

C. The deceleration parameter

This is defined by

\[ q = -\frac{\ddot{S}(t)}{S(t)H^2}, \] (15)

the larger the value of \( q \), the more rapid is the deceleration.

By adopting the condition (9) the standard model is assuming that all the mass/energy content of the universe was capsulated in a singularity, from which the universe emanated. By solving the Friedmann equation using the equation of state we conclude that the universe emanated rather violently with acceleration at start. This caused to christen the model that was based on the Friedmann paradigm as the big bang model. This is why it is always mentioned in the literature that the universe once was in the state of a singularity with infinite density. However, such a claim is physically unfounded.

IV. THE STANDARD BIG BANG MODEL

During the late fourties of the last century a scenario for the production of natural chemical elements were proposed by Gamow and collaborators. This scenario considered a thermodynamical treatment of a Friedmann universe that was assumed to be initially composed of a hot soup of elementary particles. The scenario was able to explain the natural abundance of light elements only, whereas other elements are found to be synthesized inside massive stars. The standard big bang model predicted the existence of a relic cosmic microwave background radiation (CMB) that was left over from the era when electrons were combined with the nuclei to form atoms. This radiation is thought to be highly homogenous and isotropic. Early calculations deduced that these radiation should have the spectrum of a blackbody at temperature \( T \sim 50K \) but later this figure was refined to be about 3K.
In 1965 Penzias and Wilson discovered the cosmic microwave background radiation. This discovery was considered to be the hard evidence for the credibility of the big bang scenario, and therefore, for the underlaying assumptions including those implied by the Friedmann solutions. This discovery boosted the interest in the big bang model and research in cosmology. Subsequently, this was grown into a sort of a paradigm, which I call the Friedmann paradigm. The whole content of the universe was thought to have popped from nowhere at once with infinitely high temperature and went expanding and cooling, breeding all sorts of elementary particles that were rushing all around in thermodynamical equilibrium. This paradigm dominated modern cosmology during the last three decades and superseded all other considerations like the steady-state theory of Hoyle and collaborators [4]. Refined measurements on a large angular scales confirmed the main features of the CMB but marked higher accuracy in respect to the homogeneity and isotropy.

Recent analysis of the CMB measurements indicates that the spatial part of the universe is nearly flat. This requires that the average density of the universe at the time of recombination be equal to the critical density. Since no enough luminous matter is observed in the universe to cover the required density, cosmologists assumed the existence of dark matter and (later dark energy too) in order to compensate for the missing amount. In fact there is nothing against such an assumption since our exploration of the universe is by no means complete; it’s only remain to find it.

The standard big bang model does not take into consideration any quantum effects; such effects are thought to have played an important role at the very early stages of the universe. As we will see later these effects rules-out the possibility of the existence of a singularity and would define the later course of the development of the universe.

V. FRIEDMANN PARADIGM: A CRITICAL VIEW

The main points that feature the Friedmann cosmology are the followings:

1. The assumption that the total energy content of the universe is conserved throughout the history of the universe allowing for violation only at one initial moment.

2. The existence of an initial singularity in space and time.

3. The assumption that the universe is effectively one component closed system with
no outside; accordingly the total entropy is assumed to be constant $dS = 0$. This assumption implied that the expansion of the universe is fully adiabatic.

4. The big question facing the Friedmann paradigm is how the universe managed to cross its own horizon? A universe created according to the Friedmann paradigm is apt to collapse under its own gravitational attraction. It is not clear how the universe would go on expanding unless there is a driving force within such a universe. Some important bit of physics is missing here. The universe cannot work without a cosmological constant or vacuum energy.

5. No quantum effects were taken into consideration, for this reason the homogeneity and isotropy of the spacetime was assumed to be perfect.

The above features undermined the standard big bang model which was based on the Friedmann paradigm and posed some serious problems like the horizon, flatness and the formation of large cosmic structures and other problems. The resolution of these problems needed the introduction of some sorts of the quantum effects in the treatment of the very early universe. This was introduced through an *ad hoc* solution assuming the existence of *inflation* phase prior to the big bang phase. Technically this required the assumption of a time-dependent scalar field that played the source for the inflation \[5\]. These suggestions were taken further and was developed into full fledge scheme later by Linde and others \[6\], but generally some basic criticism remains to be seriously valid unless a more profound scheme is developed \[7\], \[8\] and \[9\].

VI. QUANTUM EFFECTS

The consideration of the quantum effects requires the quantization of the gravitational field. But since gravity, as best described by the theory of general relativity, is non-linear therefore the standard canonical quantization will not be suitable; the superposition principle is not applicable and perturbation theory will not be consistent. For this reason we have to resort to a semiclassical consideration in which matter fields are to be considered quantized, whereas gravity is taken as a classical field. This approximation has proved to be workable near and above the Planck scale \[10\].
The most interesting quantity to be considered for the quantum field consideration in curved spacetime is the vacuum expectation value of the energy-momentum tensor. The reason is that this quantity stands on the right-hand side of the Einstein field equation and, therefore, would act as a source for the energy in the universe. This is called the back-reaction effect. However, the direct calculations of the vacuum expectation value of the energy-momentum tensor in a time-dependent closed universe, like the RW universe, is cumbersome because of the difficulty in defining the vacuum state in a time-dependent metric \cite{11}, and because of the anomalies that appears in the trace of $T_{\mu\nu}$ \cite{10}. For these reasons the more simpler case of the Einstein static universe were extensively considered and it was found that $\langle 0|T^\nu_{\mu}|0 \rangle$ in this universe is non-zero \cite{12}. By demanding self-consistency for the Einstein field equations, this produces a non-singular universe \cite{13}. This encouraged us to consider the finite temperature corrections to the vacuum expectation value of several other field where it was shown that these corrections produces a distribution of energy that, if utilized requiring self-consistent Einstein field equations, would generate a temperature radius relationship that exhibit some peculiar behavior \cite{17}, \cite{18} and \cite{19}. To have a glance of the calculations here is a short summary.

The general structure of the energy density of the quantum fields at finite temperatures is

$$<T_{00}>_{tot} = <T_{00}>^b_T + <T_{00}>^c_T + <T_{00}>_0,$$  \hspace{1cm} (16)

where $<T_{00}>^b_T$ is the flat space “black-body” term, $<T_{00}>^c_T$ is the correction term arising from the finite size and the field-curvature coupling, and $<T_{00}>_0$ is the zero-temperature vacuum energy density.

In more compact form \cite{16} can be written as

$$<T_{00}>_{tot} = <T_{00}>_T + <T_{00}>_0,$$  \hspace{1cm} (17)

where

$$<T_{00}>_T = \frac{1}{V} \sum_{n} \frac{d_n \epsilon_n}{\exp \beta \epsilon_n \pm 1},$$ \hspace{1cm} (18)
in which $\beta = 1/T$, $\epsilon_n$ and $d_n$ are the energy eigenvalues and degeneracy of the $n^{th}$ state respectively. The plus sign is for bosons and the minus sign is for fermions. Throughout the rest this paper we will use the system of units in which $c = G = \hbar = k = 1$.

The back reaction effect is then calculated by substituting $<T_{00}>_{tot}$ on the right hand side of (2) and require a self-consistent solution for the Einstein field equation. Because of the symmetry properties enjoyed by the Einstein universe the general solution always results in a simple form

$$\frac{3}{a^2} = 16\pi <T_{00}>_{tot}.$$  \hspace{1cm} (19)

A. The conformally coupled scalar field

The conformally coupled massless scalar field satisfies the equation

$$\nabla_\mu \nabla^\mu \Phi - \frac{R}{6} \Phi = 0,$$  \hspace{1cm} (20)

where $\nabla_\mu$ is the covariant derivative and $R$ is the scalar curvature.

The expression for the $T - a$ relationship for the conformally coupled massless scalar field as deduced from the calculations of the back reaction effects is given by

$$a^2 = \frac{8}{3\pi} \sum_{n=1}^{\infty} \frac{n^3}{\epsilon_n/Ta - 1} + \frac{1}{90\pi}.$$  \hspace{1cm} (21)

As is shown in Fig. 1., this relationship exhibit a minimum radius $a_0 = (1/90\pi)^{1/2}l_p$ at $T = 0$, and the transition temperature is $T_{max} = 2.218T_p$, at which the radius of the universe is given by $a_c = 0.072l_p$.

B. The minimally coupled massless scalar field

The minimally coupled massless scalar field satisfies the equation

$$\nabla_\mu \nabla^\mu \phi = 0.$$  \hspace{1cm} (22)
The vacuum energy density at $T = 0$ for this field is zero. Therefore, the result of the back reaction of the field at finite temperatures will contain the contributions from the energy mode-sum only. Accordingly, the $T - a$ relationship is given by

$$a^2 = \frac{8}{3\pi} \sum_{n=0}^{\infty} \frac{(n + 1)^2[n(n + 2)]^{1/2}}{e^{[n(n + 2)]^{1/2}/Ta} - 1}. \quad (23)$$

Results of the back reaction study shows that the universe will be singular in presence of this field. The transition temperature is $T_{max} = 0.6 T_p$, occurring at a radius $a_c = 0.68 l_p$. 

C. The neutrino field

The field equation is given by

$$\gamma_\mu \nabla^\mu \psi = 0, \quad (24)$$

where $\gamma_\mu$ are the Dirac matrices.

The $T - a$ relationship is then given by

$$a^2 = \frac{16}{3\pi} \sum_{n=1}^{\infty} \frac{n(n + 1/2)(n + 1)}{e^{(n+1/2)/Ta} + 1} + \frac{17}{180\pi}. \quad (25)$$

The results obtained from the study of back reaction indicates that this relation exhibit a minimum radius $a_0 = (17/180\pi)^{1/2} l_p$, and transition temperature $T_{max} = 1.076 T_p$ at a critical radius $a_c = 0.204 l_p$.

D. The Photon field

The free photon field satisfies the covariant equation

$$\nabla_\mu F^{\mu\nu} = 0. \quad (26)$$

The study of the back reaction effects resulted in the following $T - a$ relationship

$$a^2 = \frac{16}{3\pi} \sum_{n=2}^{\infty} \frac{n(n^2 - 1)}{e^{n/Ta} - 1} + \frac{11}{45\pi}. \quad (27)$$
This relationship exhibits a minimum radius \( a_0 = (11/45\pi)^{1/2}l_p \), and a transition temperature \( T_{\text{max}} = 1.015T_p \) taking place at \( a_c = 0.34l_p \).

The results of these calculations are depicted in FIG. 1. Generally we notice that with small radii around the Planck scale the temperature rises exponentially with the radius, whereas at large radii the energy of the universe behaves according to the Planck blackbody radiation law. An interesting feature of this temperature-radius relationship is that it exhibit a maximum temperature at the Planck scale. This maximum separates between what we call the \textit{Casimir regime} and the \textit{Planck regime} and indicates a change of the physical state of models with radii above certain values.

To explain the transition from the Casimir regime into the Planck regime and because this is a sort of a phase transition taking place at a very early stage of the universe, we considered the phenomena of Bose-Einstein condensation in an Einstein universe. Consideration of the Bose-Einstein condensation of spin 1 field in the ultra-relativistic limit \footnote{[20]} showed that the critical condensation temperature is the maximum temperature itself obtained in considering the back-reaction effect. This is a remarkable result that would certainly need further investigation.

**VIII. THE COSMOLOGICAL CONSTANT**

As I mentioned earlier, the cosmological constant was first introduced by Einstein in order to construct a static universe that will not collapse against its own gravitational attraction. The discovery of Hubble that the universe may be expanding led Einstein to abandon the idea of a static universe and, along with it the cosmological constant. Recent year have witnessed a resurgence of interest in the possibility that a positive cosmological constant \( \Lambda \) may dominate the total energy density in the universe (for recent reviews see \footnote{[21]} and \footnote{[22]}). At a theoretical level \( \Lambda \) is predicted to arise out of the zero-point quantum vacuum fluctuations of the fundamental quantum fields. Using parameters arising in the electroweak theory results in a value of the vacuum energy density \( \rho_{\text{vac}} = 10^6 \text{ GeV}^4 \) which is almost \( 10^{53} \) times larger than the current observational upper limit on \( \Lambda \) which is about \( 10^{-47} \). This means that \( \text{GeV}^4 \sim 10^{-29} \text{ gm/cm}^3 \). On the other hand, the QCD vacuum is expected to generate a cosmological constant of the order of \( 10^{-3} \text{ GeV}^4 \) which is many orders of magnitude larger than the observed value. This is known as the old cosmological constant.
The new cosmological problem is to understand why $\rho_{\text{vac}}$ is not only small but also, as the current observations seem to indicate, is of the same order of magnitude as the present mass density of the universe.

The value of the cosmological constant for an Einstein universe seem to be trivial. It is directly related with the total energy density. However, since the energy density in an Einstein universe varies inversely with $a^2$ and not with $a^3$, new features are expected in the behavior of the cosmological constant. In what follows we are going to investigate the possible values of the cosmological constant for different radii of the Einstein universe in presence of the massless matter fields. But since different radii of the universe corresponds to different temperature with a non-trivial relationship between the radius and the temperature as was found earlier, the values of the cosmological constant at different temperatures turns out to be non-trivial and is rather of some serious interest as we find a qualitative differences between the conformaly coupled and minimally coupled scalar fields.

Field theorists and particle physicists insist on the value they obtain for the cosmological constant \[23\]. However this is much larger than that obtained by analyzing recent measurements of CMB \[24\]. The reason is that they cannot see a resolution for this discrepancy and have no alternative to their standard model of particle physics. But from dimensional argument $\Lambda$ should be small now. One major point which particle physicists seem to have neglected is the difference between conformaly coupled scalar field and the minimally coupled scalar field. Despite the fact that both fields are alike in the present universe, it may be of some importance to know that this difference is essential in the very early universe. In fact, a minimally coupled field do not exist in nature unless the universe is absolutely flat \[20\].

The results of these studies show that the Einstein ”toy” model considered here can explain the low present-value of the cosmological constant \[18\]. It is found that the cosmological constant takes nearly the same value for small radii and then at certain radius (the value where maximum temperature occurs) solutions shows that the value of the cosmological constant decays quickly for smaller and smaller values.

Contracting Eq. \[2\] and taking into consideration that $R_{00} = 0$ in the static Einstein universe and that $T^{\mu}_{\mu}$ vanishes for massless fields, we obtain

$$\Lambda = \frac{R}{4} = \frac{3}{2a^2}. \quad (28)$$
On the other hand, for the case of Einstein static universe, the field equation reduces to

\[-\frac{3}{a^2} + \Lambda = -8\pi \rho_{tot}, \quad (29)\]

and

\[-\frac{1}{a^2} + \Lambda = \frac{8\pi \rho_{tot}}{3}, \quad (30)\]

where

\[\rho_{tot} = \langle 0| T_{0}^{0} |0 \rangle. \quad (31)\]

Here we will consider \(\rho_{tot} = \rho_{vac} + \rho_{rad}\), but in a more general case one can set \(\rho_{tot} = \rho_{vac} + \rho_{rad} + \rho_{matter}\), with \(\rho_{rad}\) belonging to the massless field filling the spatial part of the universe and \(\rho_{matter}\) belonging to the pressureless dust that may exist. The addition of the energy density of the pressureless matter will not make any qualitative change in the results since \(\rho_{matter}\) in an Einstein universe specifically behaves same as \(\rho_{vac}\) and \(\rho_{rad}\).

Solving (29) and (30) we obtain

\[\Lambda = 8\pi \rho_{tot}. \quad (32)\]

Using and the results obtained in the previous section we obtain the results depicted in FIG. 2. These results show that the conformaly coupled scalar, the neutrino and the photon field have similar behaviors. For these three fields the value of the cosmological constant during the Casimir regime (the vacuum era) is high and is nearly constant, a point which is required by the inflationary models. Also, it is very important to notice that the cosmological constant decays to very small values when the size of the universe is already within the Planck scale. This behavior comes in agreement with what inflation theories is suggesting.

Particle physicists find that the vacuum energy is given by

\[\varepsilon_{vac} \sim \frac{E_P}{r^3_P}, \quad (33)\]

and they think that this value should be constant and independent of the radius of the universe (see for example [3], p.60). But in fact studies of quantum fields in curved spacetime indicates that this belief is not true. In the Einstein universe, at the least, the vacuum energy is directly related to the radius of the universe. There is no reason why should the same
would not apply to the time-dependent RW universe. Particle physicists may need to revisit their theory for some fundamental considerations. For example the scalar field by which typical calculations are carried in the standard model of particle physics is usually taken to be minimally coupled, i.e., the curvature coupling is considered to be zero. However our considerations of quantum fields in curved spacetime \[20\] shows that the minimally coupled scalar field may not be the proper one to consider, especially at the early stages of the universe for the essential effect of the curvature term which acts as a correction to the mass term. Conformal invariance should be taken seriously in seeking proper solutions.

IX. THE VALUE OF EINSTEIN UNIVERSE

The above calculations and results did not seem to attract the attention of physicists working to solve problems with contemporary cosmology. The reason is: the Einstein universe is static, and no one would expect that it would suggest anything of realistic value. Physicist expect that a realistic model for the universe must be dynamical taking into consideration the reality of the expansion of the universe. Moreover, and in analogy with electrodynamics, physicist insist on a dynamical consideration since they expect that some gravitational effects, like particle creation, will be produced consequently, which will certainly back-react on the whole model.

In fact there are enough reasons to support our belief that studies of some physical parameters in an Einstein universe can provide a picture of the development of the universe at very early stages. Such results will, qualitatively at the least, be correct. However the Einstein static universe remained to be of interest to theoreticians since it provided a useful model to achieve better understanding of the interplay of spacetime curvature and of quantum field theoretic effects.

The conformal relationship between the static Einstein universe and the Robertson-Walker universe and the possibility to consider the Einstein universe of a given radius as representative of an instantaneously static Robertson-Walker universe \[12\] and the \((1 - 1)\) correspondence between the vacuum and the many particle states of both universes as established by the work of Kennedy \[14\], suggests that the thermal behavior of a real closed universe is qualitatively similar to the results obtained in this work. Therefore, we feel that the calculations in the Einstein universe are useful in understanding the interplay between
quantum fields and the curvature. Indeed our calculations showed that an Einstein universe with curvature radius of about two order of magnitude larger than the Hubble radius will have the same CMB temperature as the presently measured one. On the other hand the analysis of the most recent observations of CMB spectrum suggests that the curvature radius of the real universe is at least 5 times larger than the Hubble radius \( \text{[25]} \). This is a point in favour of the practical relevance of our calculations.

Despite the fact that the Einstein universe is static we can imagine a series of successive states of Einstein universe developing according to parameters that are related self-consistently according to the Einstein field equations. In this context a dynamically developing universe may be imagined as a series of successive static states each described by the Einstein static model. This obviously is related to the philosophical question of whether infinite divisibility can ontologically exist at all. In such a model the geometro-dynamical effects like particle production by changing geometry is obviously lost. However, because general relativity is a self-consistent theory, therefore in effect such a model is expected to exhibit a consistent overall behavior so that the end result will not be different from those obtained in a fully dynamical model. We claim that this property is specific to the Einstein universe which is conformal to the RWF universe. Indeed, it is the property of the Einstein universe of having matter content without motion, but then the geometry of the universe is strictly related to its matter content such that any larger universe would necessitate an increase in the material content of the universe. In a dynamically analogous model this means that

\[
\dot{M} = a,
\]

which may be taken to compensate for the particle creation in the dynamical state.

I feel that the results presented in this paper, are quite encouraging to construct a new dynamical model that starts from Planck parameters evolving to the present stage with the total density falling like \( 1/a^2 \) rather than \( 1/a^3 \). However, such a model will involve continuous particle creation at a rate proportional to Hubble constant. Obviously a mechanism for the particle creations has to be devised for such a process. In this respect the mechanism suggested by Hoyle-Narlikar (\[26\] and \[27\]) in the context of the steady-state theory may be useful, however the rate of creation will be different from that of the steady state theory.
Furthermore we note that our calculation shows that a universe stemming from the vacuum and developing by the availability of the vacuum energy would be maintained to be at the critical density. Accordingly a justification can be provided be construct a non-singular universe free from the problems of the standard big bang model. We mean that our calculations will provide the necessary justifications for the consideration of Ozer and Taha [28] of a critical density universe. On the other hand this will explain why do we have a flat or nearly flat universe without the need for inflation. However it should be pointed out here that what remains is the remarkable success of the standard big bang scenario (which utilizes the Friedmann paradigm) in explaining the natural abundance of light elements. If any viable alternative to the Friedmann paradigm is to be presented then it should be able to explain the natural abundance of light elements (see [29]).

**Figure Caption:**

**FIG. 1:** The temperature-radius relationship deduced from the finite temperature corrections to the vacuum energy plotted for different matter fields: the conformally coupled scalar field (1), the neutrino field (2), the photon field (3) and the minimally coupled field (4). (see Ref. [30]).

**FIG. 2:** contributions of the conformally coupled scalar field (1), the photon field (2), the neutrino field (3) and the minimally coupled scalar field (4) to the cosmological constant in an Einstein universe at finite temperatures. (see Ref. [30]).

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