Universal Extra Dimension

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This is a brief discussion of the following features of the Universal Extra Dimension (UED) model:
(i) Formulation, (ii) Indirect bounds, (iii) Collider search and the Inverse Problem, (iv) Astrophysical bounds, and (v) UED with two extra dimensions.

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I. INTRODUCTION

The Universal Extra Dimension (UED) model, or models as there are already quite a few interesting variants of the minimal one, has a flat metric (like ADD) and a small compactification radius of $O(\text{TeV}^{-1})$ (like RS). Also, this is the most democratic extra dimension (ED) model in the sense that all Standard Model (SM) fields can propagate in the extra dimension, or bulk. Thus, in essence, it is quite similar to the first-generation ED models of Kaluza and Klein [1]. However, the motivations are quite different.

Why UED? The cons first: this does not solve the gauge hierarchy problem. Both ADD and RS models lower the Planck mass, as seen on our brane, by two very different but very elegant mechanisms. UED does nothing of that sort; in fact, we will see that the model breathes more freely when we do not include gravity. Issues like the stabilisation of the radius of the extra dimension are not addressed. If you think that the fine-tuning of the Higgs mass is the most serious issue in particle physics, you will probably not turn to this model.

Now the pros. First, the dark matter. The dark matter provides almost one-fourth of the energy density of the universe, and UED, among all extra dimensional models, supplies a very good candidate for the cold dark matter (CDM). In fact, it is the most theoretically motivated candidate — e.g., in supersymmetry, we impose the conservation of R-parity by hand and get the neutralino dark matter; but the UED dark matter is a necessary consequence of the formulation. Second, the dark matter constraint and the indirect limits on the compactification radius guarantee a spectrum that is completely within the reach of LHC. The first excited states of the SM particles should be between 400-900 GeV.

As has been pointed out, the collider signals mimic those of supersymmetry; so this is probably the most serious case of the so-called LHC inverse problem — the discrimination of SUSY and UED from signals. In fact, I will discuss the issue of discriminating UED from other NP models too, not confining only to R-parity conserving SUSY.

UED scores more positive points if one considers two extra dimensions. As I will touch upon later, the 6-d UED model answers two very important questions: Why proton lifetime is so large? (This is one of the most challenging problems in the extra dimensional models.) Why there are three generations?

The plan is as follows: I will discuss, in the subsequent sections, (i) Formulation, (ii) Indirect bounds, (iii) Collider search prospects, and discrimination from other models, most notably supersymmetry, (iv) Astrophysical bounds, and inclusion of gravity, and (v) UED with two extra dimensions. Except for the last part, I will concentrate on the minimal version of UED. There are at least three other talks that will focus on several interesting features of the model: the phenomenology of the scalar sector of minimal UED [2] and of 6-d UED [3], and the power-law evolution of the gauge couplings in UED [4].

II. FORMULATION

The basic idea of Appelquist, Cheng, and Dobrescu [2] is very simple: apart from four large dimensions $x^{\mu}$, there is a small dimension $y$ (this I will call the bulk), which is compactified on a circle of radius $R$. The points $y$ and $y+2\pi R$ are identified. This means that the momentum along the fifth direction, $p_5$, is discrete: $p_5 R = n$, where $n$ is any integer. Just like particle in a box, there will be equispaced states, whose masses are given by

$$m_n^2 = m_0^2 + n^2/R^2,$$

where the last term comes from $p_5^2$. The integer $n$ is called the Kaluza-Klein (KK) number; $n = 0$ corresponds to the zero mode. Every SM particle is associated with an equispaced tower of particles of identical quantum numbers. As all particles can access the bulk, momentum along the fifth dimension, and hence $n$, is conserved in any process. Thus, (i) all $n \neq 0$ particles are to be pair produced in collider experiments [3]; (ii) the lightest particle for each $n$ level is stable. Note that if $R^{-1}$ is of the order of a few hundred GeV, the particles for each $n > 0$ level are quasi-degenerate. This degeneracy is somewhat lifted by the radiative corrections [4, 5].

In the minimal 5d UED model, the fermions are necessarily vectorial, even in their zero modes. This can simply be guessed from the fact that $\gamma_5$ is a part of the $\gamma$-matrix set itself: $\Gamma^M = (\gamma^\mu, i\gamma_5)$. To get chiral fermions in zero mode, we need a further $Z_2$ orbifolding, so that the compactified dimension becomes $S_1/Z_2$. 
This is nothing but a fold of the circle along one of its diameters and identification of the points $y$ and $-y$ for $-\pi R \leq y \leq \pi R$ (remember that there is already the identification of $y \rightarrow y + 2\pi R$). Fields can be even or odd under this $\mathbb{Z}_2$: the Higgs field, the first four components of the gauge fields, the right-chiral component of $SU(2)$ singlet fermions, and left-chiral components of $SU(2)$ doublet fermions are all even, while the fifth component of the gauge fields, the left-chiral component of $SU(2)$ singlet fermions, and right-chiral components of $SU(2)$ doublet fermions are all odd.

The 5d fields can be Fourier expanded as

$$
\phi_+(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \phi_+^{(0)}(x^\mu) + \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \cos \frac{ny}{R} \phi_+^{(n)}(x^\mu),
$$

$$
\phi_-(x^\mu, y) = \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \sin \frac{ny}{R} \phi_-^{(n)}(x^\mu),
$$

where $\phi_+$ is even and $\phi_-$ is odd under $\mathbb{Z}_2$. Fields which are odd under the $\mathbb{Z}_2$ orbifold symmetry do not have zero modes. Only even fields have zero modes, which are identified with the SM particles. There are two fixed points on the orbifold that are mapped onto themselves: $y = 0$ and $y = \pi R$. The even and odd fields satisfy $\partial_y \phi_+ = 0$ and $\partial_y = 0$ at the fixed points.

There are two types of radiative corrections to eq. (1). The first, called the bulk correction, comes from the fact that the theory is not Lorentz invariant ($y$ is different from $x^\mu$) and therefore the wavefunction renormalization $Z$ for the large dimensions is not equal to $Z_5$, that for the fifth dimension. This correction, proportional to $Z - Z_5$, occurs for loops that can sense the compactification (i.e., if we flatten out the ED the loop definitely vanishes); it is in general small and zero for fermions. While an exact expression is available in [36], these corrections are subdominant in the determination of the spectrum.

The second correction is called the orbifold correction. This is in general log-divergent. In the presence of the $S_1/\mathbb{Z}_2$ orbifolding, the translational invariance along $y$ is lost. However, there is a discrete $\mathbb{Z}_2$ symmetry, different from the earlier $\mathbb{Z}_2$, of $y \rightarrow y + \pi R$ which is still intact. The conservation of KK-number $n$ breaks down to the conservation of KK-parity, defined as $(-1)^n$, which is a result of this second $\mathbb{Z}_2$. Thus, (i) $n = 1$ particles are to produce in pairs while $n = 2$ particles can be singly produced; (ii) the lightest $n = 1$ particle, often called the Lightest KK Particle (LKP), is the only stable $n \neq 0$ particle. This turns out to be an excellent CDM candidate. In fact, this is completely analogous to the $\mathbb{Z}_2$ symmetry of the underlying theory leading to a dark matter candidate for R-parity conserving supersymmetry and the Little Higgs model with T-parity conservation.

There are terms located on the fixed points that receive large log-divergent contributions. One must include such terms to have a consistent theory [33]. To regulate these terms, one introduces a cut-off, $\Lambda$, up to which the theory is said to be valid; hence the corrections come as $\log(\Lambda^2/\mu^2)$, where $\mu$, the regularisation scale, may be taken to be $n/R$ for $n$-th level particles. $\Lambda$ should be at least of the order of $R^{-1}$; phenomenologically, one takes $\Lambda R$ to be upto 50 or 100. The finite parts of these corrections are undetermined and remain as free parameters of the theory; one may take, as a simplifying assumption, the finite parts to vanish at the cutoff scale $\Lambda$ (that is one of the principal assumptions of the minimal UED).

Thus, the orbifold corrections are of the form

$$
\delta \propto \frac{1}{16\pi^2} f(g_i) \log \frac{\Lambda^2}{\mu^2},
$$

where $f(g_i)$ is a function depending upon the gauge couplings $g_i$ under which the field transforms nontrivially. For the exact expressions, the reader is referred to [36]. Note that just like any ED model, UED is nonrenormalisable and should be treated as an effective theory valid up to the cutoff scale.

![Figure 1](image.png)

**FIG. 1:** $n = 1$ levels for $R^{-1} = 500$ GeV and $\Lambda R = 20$. Taken from [36].

The following points are worth remembering:

- The radiative corrections are positive. Therefore, the doublet quarks, which are nonsinglet under all three SM gauge groups, receive the maximum correction. The $SU(2)$ singlet leptons (whose zero mode is right-chiral but excitations are vectorial, with the $\mathbb{Z}_2$-even component being right-chiral) receive the minimum shift and are closest to the tree-level mass as given in eq. (1).

- While the vectorial mass terms $n/R$ and the radiative corrections do not couple $SU(2)$ singlet and doublet fermions, the Yukawa term does. This is significant only for the top quark and hence the $n = 1$ top quark masses differ from their other charge +2/3 counterparts.

- The gauge fields have five components. The first four, which are even under $\mathbb{Z}_2$, appear as the excited gauge boson. The fifth component is a $\mathbb{Z}_2$-odd
scalar. This mixes with the excitations of the Higgs doublet with the same quantum numbers. For example, the fifth component of $W^\pm$ mixes with the excitation of the charged Goldstone. Out of the two states, one is eaten up by the $n = 1$ gauge boson (the Higgs mechanism for the excited level), while the other one remains in the physical spectrum. For $R^{-1} \gg m_W, m_Z$, the physical particle is almost the excitation of the $n = 0$ Higgs field. Thus, there are four new $n = 1$ scalars: $H^\pm, H^0$ (the excitation of the SM Higgs boson), and $A^0$ (the excitation of the neutral CP-odd Goldstone). The hierarchy $m_{H^\pm} < m_{A^0} < m_{H^0}$ is fixed, but the spacing depends, among other factors, on the SM Higgs mass $m_h$. There is one more term, $m_{\gamma}^2$, a soft term located only at the fixed points, that affects only the scalar spectrum. In the minimal UED, $m_{\gamma}^2$ is taken to be zero. More detailed phenomenology of the scalar sector may be found in [8, 9] and also in the talk [2].

- At the $n = 1$ level, $W^3$ and $B$ mix to give the physical states $Z_1$ and $\gamma_1$. However, the Weinberg angle $\theta_1$ is very small, almost close to zero, so that $(W^3)_1 \approx Z_1$ and $B_1 \approx \gamma_1$. The latter is almost always the LKP. If one includes gravity, the graviton becomes the LKP for $R^{-1} \leq 800$ GeV. Also, for very heavy SM Higgses, the excited charged Higgs $H^\pm$ may turn out to be the LKP, which in any way is cosmologically not viable. Since $\gamma_1$ is neutral and weakly interacting, this is an excellent CDM candidate. As this is the end product of the cascade of any $n = 1$ state, the UED signal at the colliders, at least for the production of $n = 1$ states, is comparatively soft SM particles or jets and large missing energy carried away by the LKP.

III. INDIRECT BOUNDS ON 1/R

All SM particles have their corresponding towers. Thus, we expect finite radiative corrections from these heavy degrees of freedom on low-energy observables, over and above the SM effects. We mention several such observables and corresponding bounds on $R^{-1}$. Unlike collider signals, these effects are not sensitive to the precise value of the cutoff scale $\Lambda$.

UED is a Minimal Flavour Violation (MFV) model. Such models are characterised by the fact that there are no new CP violating phases apart from the one present in the CKM matrix, which, in its $3 \times 3$ form, is still unitary, and no new FCNC operators apart from those occurring in the SM. MFV-type models include two-Higgs doublet model, supergravity with small $\tan \beta$ and no new sources of FCNC (aligned quark and squark mass matrices), gauge mediated SUSY, little Higgs with T-parity, et cetera.

The new physics (NP) contribution to any MFV model is severely restricted. From the direct and indirect measurements of the sides and angles of the Unitarity Triangle (UT), one can construct a so-called Universal Unitarity Triangle, valid for all MFV-type models. The predictions are very close to that of the SM; for example, $\sin(2\beta) = 0.735(0.732) \pm 0.049$, and the tip of the UT $\rho = 0.174(0.187) \pm 0.068(0.059)$, $\eta = 0.360(0.354) \pm 0.031(0.027)$, where the numbers in parenthesis are those for the SM. Thus, just from CP-violating observables, it is almost impossible to detect any evidence of any MFV-type model.

One can have excited gauge bosons, charged Higgs, and quarks, inside the loop for the box diagram of $B^0 - \bar{B}^0$ mixing. This contributes to the mass difference $\Delta M_d$ between the $B_d$ mass eigenstates. While there are potentially infinite number of diagrams, coming from all $n$ upto infinity, it is enough to truncate the series at, say, $n$. Also, for minimal UED, the result after such a truncation is finite and convergent. From the measured value of $\Delta M_d$, the lower bound of $R^{-1}$ is about 250-300 GeV [10, 11]. A similar consideration applies for the partial width of $Z$ to a $b\bar{b}$ final state; the limit is roughly 300 GeV [12].

A rather strong constraint comes from the radiative decay $B \rightarrow X_s + \gamma$ [13]. The experimental number for the branching fraction is $(3.55 \pm 0.24 \pm 0.09 \pm 0.03) \times 10^{-6}$, where the first error is a combination of statistical and systematic errors, the second one comes from the uncertainties in the energy extrapolation, and the third one is due to the subtraction of $B \rightarrow X_d + \gamma$ events. The theoretical number, within the framework of SM and at the NNLO level, is $(2.98 \pm 0.26) \times 10^{-6}$. The uncertainties include higher order perturbative effects, hadronic power corrections, parametric dependences, and the uncertainty in the charm-quark mass. While the numbers agree at about $1.1 \sigma$ level, any model, like minimal UED, that decreases the branching ratio, is forced to be very tightly constrained. For example, Haisch and Weiler [12] found $1/R \geq 600$ GeV at 95% CL. The calculation, however, takes the UED at LO for the matching of the corresponding Wilson coefficients; two-loop effects with UED are yet to be computed. The fact, together with the 99% CL bound, still keeps open a lower value of $R^{-1} \sim 400$ GeV, which may be interesting for a future generation $e^+e^-$ machine.

The last indirect bound comes from the oblique parameters $S, T$ and $U$ [14]. The precision electroweak studies constrain $R^{-1} \geq 600$ GeV for a light SM Higgs at about 115 GeV. However, this bound is strongly sensitive on the SM Higgs mass; for example, a 300 GeV Higgs will keep $R^{-1} \geq 400$ GeV open. The bound is also mildly sensitive to the top mass.
IV. COLLIDER SEARCHES

A. Tevatron and LHC

The role of colliders to investigate the possible nature of spacetime was highlighted by Antoniadis [15]. I will not say much about the Tevatron bounds as they have been superseded by the indirect constraints. The LHC however, is a different story. As we will see later, $R^y$ has a theoretically motivated upper bound of about 0.1 TeV, from dark matter abundance. Thus, the entire parameter space is accessible to LHC; if UED is there, it should be visible unless coming from a transition between $n=1$ excitations that they can be singly produced as $s$-channel resonances, and hence do not require more energy than that needed for the pair production of $n=1$ states. Thus, if the collider is energetic enough to pair produce $n=1$ states, it should produce $n=2$ resonances too.

I have discussed earlier that there must be terms located at the fixed points $y=0$, $\pi R$ that go as $\log(\Lambda^2/\mu^2)$ for the UV completion of the theory. They reduce the conservation of KK-number to the conservation of KK-parity, $(-1)^n$. Thus, terms where an $n=2$ gauge boson is coupled to two $n=0$ fermions are allowed; the coupling is small, suppressed by the boundary-to-bulk ratio, but strong enough to produce the $n=2$ gauge bosons. While the signal of $Z_2$ may be observed as a sharp bump in the dilepton channel (these bosons, in turn, decay mostly into two $n=0$ fermions, so there is no missing energy), $\gamma_2$ will go unobserved. This is because $\gamma_2$ couples almost entirely to $n=0$ quark pairs, and a dijet signal with no missing energy is definitely going to be swamped in the LHC environment.

This is where a next-generation $e^+e^-$ collider, like the International Linear Collider (ILC), may come to our rescue. This is discussed in the next subsection.

An important byproduct is the enhancement of the Higgs production rate at the LHC. This goes mostly through gluon-gluon fusion, and with UED in the picture as the ‘smoking gun’ of UED. However, this question is subtle and we will return to it later. Here, one may note that they can be singly produced as $s$-channel resonances, and hence do not require more energy than that needed for the pair production of $n=1$ states. Thus, if the collider is energetic enough to pair produce $n=1$ states, it should produce $n=2$ resonances too.

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ture, there is an excited top triangle that adds to the SM amplitude. For \( m_H = 150 \text{ GeV} \) and \( R^{-1} = 500 \text{GeV} \), this enhancement can be as high as 80\% \[20\].

**B. ILC**

The International Linear Collider is still in the blueprint stage. This is going to be a single-pass linear collider, with two densely packed beams of electron and positron hitting each other. The initial centre-of-mass energy should be 500 GeV, an with sufficient motivation (read discovery of new particles at the LHC) it can be upgraded to 1 TeV. ILC will provide a clean environment. This fact has motivated a number of studies of UED at the ILC \[21, 22, 23, 24, 25\]. Let me just highlight the major points.

- **ILC can discriminate between UED and SUSY.**
  One just needs to look at the differential decay distributions of the final-state electrons in \( e^+ e^− \rightarrow e_1^+ e_1^- \) or muons in \( e^+ e^− \rightarrow \mu^+_1 \mu^-_1 \). For example, the distribution of \( e^+ e^− \rightarrow \mu^+ \mu^- \) plus missing energy dips at \( \theta = \pi/2 \) for UED and peaks there for SUSY (the total number of events will also be different).

![Graph](image)

**FIG. 4:** Differential distribution for \( e^+ e^− \rightarrow \mu^+ \mu^- \) plus missing energy. From \[22\].

- **ILC can act as a factory for production of the \( n = 2 \) gauge bosons \( \gamma_2 \) and \( Z_2 \) \[24\], in the same vein as LEP-1 which was a Z-factory. If ILC sits on one of these resonances, the production cross-section can be tens of picobarns. This, however, needs a prior knowledge of at least the approximate positions of the peaks, which should be available from the LHC, and planning the ILC design accordingly. Unfortunately, ILC has predetermined centre-of-mass energies that it will operate in, and it may be too late to change that in view of the LHC data.

- **One can still salvage the situation and use the ILC to observe the narrow \( n = 2 \) resonances, even if they are away from the machine \( \sqrt{s} \).** This is because the beam energy is degraded by the QED processes of initial state radiation (ISR) and beamstrahlung; in both the cases, one or more photons is radiated off the incoming particles so that the effective CM energy is less, and there is a possibility that it will hit the resonance. This is nothing but the phenomenon of radiative return, already observed in LEP-1.5. While the luminosities fall as we go away from the CM energy, the much enhanced cross-section at the resonance ensures that the signal is still visible \[25\]. To observe such signals, ILC upgrade running at \( \sqrt{s} = 1 \text{ TeV} \) will be required; a higher energy machine, like the proposed CLIC, will probe further in the parameter space.

- **While it has been established that UED can successfully be discriminated from R-parity conserving SUSY at the ILC, the issue of discrimination from other models should also be investigated.** For example, R-parity violating SUSY, with nonzero \( \lambda \) and \( \lambda' \)-type couplings, can show s-channel peaks from sneutrino resonances, which are analogous to the \( n = 2 \) resonances. Such peaks can be observed from RS gravitons, or one or more extra \( Z' \)s. It has been shown in \[25\] that a simultaneous study of dijets and 4 lepton plus missing energy signals can act as a useful discriminator. The latter signal can come, in UED, from the process \( e^+ e^− \rightarrow Z_1 Z_1 \). Depending on the value of \( \Lambda \), the dijet invariant mass distribution may show both \( \gamma_2 \) and \( Z_2 \) peaks, or they may be fused into one. Whatever the case might be, a sharp excess over the SM background is expected. The 4\( \ell \) plus missing energy signal also shows an excess. Since all the leptons are expected to be soft, one can look for leptons, none of whose \( p_T \) should exceed 40 GeV. This reduces the SM background to a negligible value; also, other competing models would give very different distributions (for example, in a left-right symmetric model with extra \( W' \) and \( Z' \), we do not expect soft leptons). Thus, a simultaneous study of both these signals in the so-called signature space of the LHC should give an unambiguous map to the multi-model parameter space.

**V. ASTROPHYSICAL BOUNDS**

The best motivation of UED is, perhaps, a strong candidate for the cold dark matter, viz., the LKP, which is mostly \( B_1 \), the excitation of the hypercharge gauge boson. This is neutral and weakly interacting, and over a very large part of the parameter space, is the lightest \( n = 1 \) particle (a small portion may have the \( n = 1 \) charged Higgs as the LKP, but being charged, that is not
a good CDM candidate).

The KK excitations, and hence the LKP, were freely produced and annihilated in the early universe, where \( T \gg R^{-1} \). As the universe expands and cools, the LKP decouples and freezes out, and forms a thermal relic. The number density of the LKP can be estimated using the Boltzmann equation, and the LKP self-annihilation rate, whose cross-section is roughly \( 95g_1^4/324\pi m_{\gamma_1}^2 \), where \( g_1 \) is the \( U(1)_Y \) coupling constant \cite{26, 27}. The most accurate result is the one quoted in the last reference of \cite{26}. The relic abundance has been measured by WMAP to be \( \Omega h^2 = 0.110 \pm 0.006 \) \cite{28}, which translates roughly to a window between 850 and 900 GeV for the LKP. If the CDM is not entirely due to the LKP, the lower limit may be relaxed, but the upper limit should be about \( R^{-1} \leq 900 \) GeV. This is the result that makes us confident about the discovery of UED, provided it is the path Nature takes, at the LHC.

There may be other species of \( n = 1 \) particles close to the LKP. The best candidates for this are the excited leptons and neutrinos. If they are sufficiently close, their effects on co-annihilation should be taken into account. The closer they are, the tighter is the upper bound on \( R^{-1} \). This is shown in figure 6. All in all, one can say that the parameter space of UED should conservatively be in the range 400 GeV < \( R^{-1} \) < 900 GeV. For a more detailed review, I would suggest ref. \cite{27}.  

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FIG. 5: Dijet mass distribution for two different values of \( \Lambda \). Also shown is the distribution normalised by the SM expectation. The resolution is taken to be 20 GeV. From \cite{25}.
A. Gravity in UED

If one includes gravity in UED, place must be provided for the KK excitations of gravitons, whose $n$-th level should be simply at $n/R$ [29]. The $n = 1$ graviton immediately becomes the LKP upto $R^{-1} = 810$ GeV, above which the radiative corrections on $\gamma_1$ push it below $G_1$. However, the graviton LKP scenario is not viable. In the graviton LKP region, $\gamma_1$ decays to $G_1$ and a photon. The contribution of this process to the diffuse photon flux is much above the experimental limit, ruling the graviton scenario out. If one includes gravity, there is a small patch of the parameter space, with $R^{-1} > 810$ GeV, where $\gamma_1$ is the LKP and still does not violate the dark matter overproduction constraint. The $n = 1$ excitations should be observable at the LHC but the ILC, even with the upgraded $\sqrt{s} = 1$ TeV option, will draw a blank. The model also becomes rather fine-tuned and loses much of its charm as far as the collider search is concerned.

VI. 6-D UED

The 6-d UED, where there are two extra dimensions accessible to the SM particles, has some strong phenomenological motivations. It is well-known that the proton stability is a problematic issue in any extra dimensional scenario, because of the new higher dimensional operators that may lead to an unacceptably quick decay of the proton. In 6-d UED, the global symmetries of the theory prevents all proton decay operators less than dimension 9 [30]. For example, the decay $p \to e^-\pi^+\pi^+\nu\nu$ has a lifetime of $10^{35}$ years for $R^{-1} = 500$ GeV and $\Lambda R = 5$, and scales as $(R^{-1}/500)^{12}$ and $(\Lambda R/5)^{22}$.

It was shown in [31] that an $SU(2)_L$ global gauge anomaly exists unless the number of $Z_2$-even doublets differ from that of $Z_2$-odd doublets by an integral multiple of six. For each generation, this difference is either 2 or 4; thus, one needs three generations (or a multiple of three) for the anomaly cancellation.

To get chiral fermions, the 6-d UED needs to be compactified on a chiral square [32]. This is a square with adjacent sides identified. Each SM particle has excitations specified by two positive integers $(j,k)$, so that the mass of the $(j,k)$-th excitation, at the tree-level, is given by

$$m^2_{j,k} = m^2_0 + \frac{j^2 + k^2}{\Lambda R^2}.$$  \hspace{1cm} (4)

The scalar sector of 6-d UED is richer than its 5-d counterpart. Each gauge field has 6 degrees of freedom; hence, there is an extra adjoint scalar in the spectrum for each gauge field. The scalar adjoint $B_{1,0}$ is the LKP and turns out to be a good dark matter candidate. For more phenomenological analysis, the reader may look at [33], and also in the talk [3].

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[34] One may question how, in the process of one $n = 0$ particle decaying into two $n = 1$ particles, KK number is conserved. Note that while performing the Fourier expansion of the 5-d field, we summed over all positive integers only. Fields with negative $n$ are identical in all respect with fields with positive $n$. So, while considering the KK-number conservation, one should consider both choices: $|n_1 \pm n_2|$. If one of the combination fits, that’s fine. The complexity arose because KK-number is an additive quantum number and not a multiplicative one like R-parity.
[35] The need for such terms can be understood as follows. One can draw a 0-0-2 (these are KK numbers of the external legs) vertex at one-loop, with $n = 1$ particles flowing in the loop, which conserves KK number at all vertices. However, this vertex is in general divergent. To cancel this divergence, one needs a divergent term itself in the original Lagrangian.