A CATALOG OF MOVING GROUP CANDIDATES IN THE SOLAR NEIGHBORHOOD

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ABSTRACT

Based on the kernel estimator and wavelet technique, we have identified 22 moving group candidates in the solar neighborhood from a sample which includes around 14,000 dwarfs and 6000 giants. Six of them were previously known as the Hercules stream, the Sirius-UMa stream, the Hyades stream, the Caster group, the Pleiades stream, and the IC 2391; five of them have also been reported by other authors. 11 moving group candidates, not previously reported in the literature, show prominent structures in dwarf or giant samples. A catalog of moving group candidates in the solar neighborhood is presented in this work.

Key words: solar neighborhood – stars: abundances – stars: kinematics

1. INTRODUCTION

Since the first discovery of the two clearest examples of moving groups in the solar neighborhood (the Hyades and the Ursa Major) by Proctor (1869), studies of moving groups have been the subject of much attention. There are two main techniques to detect moving groups. One is the convergent point (Brown 1950; Jones 1971) technique for proper motion data; the other is searching for kinematic structure in velocity space. Most works adopted the second technique. For example, Gomez et al. (1990) used for kinematic structure in velocity space. Most works adopted the second technique. For example, Gomez et al. (1990) used the stochastic expectation maximization (SEM) algorithm to decompose the sample into the sum of tridimensional Gaussians in the \((U, V, W)\) velocity, and concluded that the observed distribution of the residual velocity can be explained as the sum of four independent distributions which were related to several open clusters in the solar neighborhood. Chen et al. (1997) developed an algorithm using a nonparametric kernel estimator to describe the stellar distribution in a four-dimensional space (velocities and age), and four moving groups near the Sun (Pleiades, Sirius, Hyades, and IC2391) have been identified without assuming any prior knowledge of moving groups, neither the velocity distribution nor other physical properties.

After the release of the Hipparcos Catalog (ESA 1997), research on moving groups in the solar neighborhood has made great progress since accurate parallaxes and proper motions for a large number of stars are available. Skuljan et al. (1999) use the wavelet transform technique to analyze the distribution derived by an adaptive kernel method and also find several moving groups. Recent works by Famaey et al. (2005b, 2007) have identified five moving groups based on 6000 giants and investigated the mass distribution of the Hyades stream based on the Geneva–Copenhagen survey of 14,000 dwarfs from Nordström et al. (2004). Klement et al. (2008) identified at least four “phase-space overdensities” of stars on very similar orbits in the solar neighborhood using the first Radial Velocity Experiment (RAVE) public data release. Different authors use different techniques and different samples, but they all report the presence of moving groups in the solar neighborhood.

With the newly developed wavelet transform technique by Skuljan et al. (1999), we attempt to identify the stellar moving groups in the solar neighborhood by combining both the dwarf samples from Nordström et al. (2004) and the giant samples from Famaey et al. (2005b). The main goals are to investigate whether the locations in velocity space of those groups derived from different samples are consistent as well as finding new streams based on large samples of stars and the new method.

2. THE METHOD TO IDENTIFY MOVING GROUPS

Since the sample is in the solar neighborhood, the position of the stars does not provide any discriminant information for the detection of moving groups, so the detection in our Letter depends mainly on the two components of the stellar velocity—the \(U\) component pointing toward the galactic and the \(V\) component toward the direction of the galactic rotation. First, the probability density function (pdf), \(f(U, V)\), is estimated by a kernel function, and next the wavelet transform will be carried out, so we can finally recognize the moving groups on the basis of the analysis of wavelet coefficients.

2.1. Density Distribution

We use the kernel function to decide the probability density at any given point. The type of kernel function used in our method is a radial basis function (Equation (1)), where \(h\) is a smoothing parameter and \((\Sigma)\) is the covariance matrix. The probability density of the \((U, V)\) panel is then derived from Equation (2), where \(n\) is the number of stars in our sample:

\[
K(x, x_i) = \exp \left( -\frac{1}{2} \left( x - x_i \right)^T \Sigma^{-1} \left( x - x_i \right) \right) h > 0, \quad (1)
\]

\[
\rho(x) = \frac{1}{n h^2 2\pi |\Sigma|^{1/2}} \sum_{i=1}^{n} K(x, x_i). \quad (2)
\]

2.2. Wavelet Transform

After deriving the density of any given \((U, V)\) point, in order to achieve the \((U, V)\) center and the dispersion of the possible moving groups, we use the two-dimensional wavelet transform technique to analyze them (Skuljan et al. 1999). Wavelet analysis (Daubechies 1988; Chui 1992; Ruskai et al. 1992) is a fantastic tool and is becoming more and more popular in astronomical research.

The wavelet transform provides an easily interpretable visual representation of \(\rho(x)\). Moreover, the continuous wavelet transform can be used in singularity detection. In this Letter, we use it to find some structures working at different scales.
To process a wavelet transform of a function $f(x, y)$, we must define an analyzing wavelet $\psi(x/\sigma, y/\sigma)$ called the mother wavelet in advance. $\sigma$ is the scale variable. The wavelet coefficient of the point $(\mu, \nu)$ then can be derived from:

$$\omega(\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\psi \left( \frac{x - \mu}{\sigma}, \frac{y - \nu}{\sigma} \right) \, dx \, dy.$$  

The actual choice of the analyzing wavelet $\psi$ depends on the particular application. A mother wavelet named Mexican hat (Skuljan et al. 1999) is the second derivative of a Gaussian and generally gets good results when applied to find singularities. So when we intend to search for certain groups from a given data distribution, a two-dimensional Mexican hat is selected as the mother wavelet:

$$\psi(x, y, a) = \left( 2 - \frac{x^2 + y^2}{a^2} \right) e^{-\left(\frac{x^2+y^2}{2a^2}\right)}.$$  

The main characteristic of the function $\psi$ is that the total volume is equal to zero, which is what enables us to detect any overdensities in our data distribution. The wavelet coefficients will all be zero if the analyzed distribution is uniform; but if there is any significant “bump” in the distribution, the wavelet transform will give a positive value at that point.

### 3. RESULTS AND DISCUSSIONS

Two data sets are selected as our samples. One is the 14,000 dwarf (Nordström et al. 2004, hereafter Nord04); the other is 5311 K and 719 M giant (Famaey et al. 2005b). The information on dwarf samples came from the Geneva–Copenhagen survey, in which the ages, metallicities, and kinematic properties are provided, while that of the giant sample was obtained from CORAVEL/Hipparcos/Tycho-2. First, the probability density was derived by the method described in Section 2.1. For Equation (1), the smoothing parameter $h$ should be carefully selected because small values of $h$ emphasize noisy structure, while large $h$ values will smooth out all the details of the distribution. Our selection of the value of $h$ adopts the method of Asiain et al. (1999). The value of $h$ is set to be 0.145 for the dwarf sample and 0.117 for the giant sample. The density distribution in the $(U, V)$ plane has some irregularities and cannot be described by a unique velocity ellipsoid, and some “bumps” can clearly be recognized. Famaey et al. (2008) derived the threshold of noise wavelet coefficients to have a value of $10^{-4}$. Thus, the coefficients smaller than $10^{-4}$ in the present work will be rejected. Figure 1 shows the contour map of the positive wavelet coefficients obtained in the $(U, V)$ space from $a = 4$ km s$^{-1}$.

#### 3.1. Monte Carlo Simulation

After obtaining the probability density of the samples and the wavelet coefficient contour map, the results show several features. However, are these features real or are they caused by some noise? There are several factors that will affect the observed number of stars in each bin, such as the measurement error, statistical fluctuations related to the finite sample, etc. We can expect that the number of observed stars in each bin will be subject to the Poisson distribution with an average of $N$ which can be derived by the pdf multiplied with a factor $nS$, where $n$ is the total number of stars and $S$ is the area covered by a square bin. To investigate how probable are the features shown in Figure 1, we generate a large number ($N = 2000$) of Poisson random copies. For each copy, we get a positive wavelet coefficient map (after rejecting those smaller than $10^{-4}$). Figure 2 shows an example of one copy. The probability of these features is shown in Table 1. Typically, only those features with 90% or better probability are considered to be real.

#### 3.2. The Detected Moving Groups

From Figure 1, it is clear that stars are clumped at different locations in the $(U, V)$ space. Table 1 shows the central $(U, V)$ of 22 possible moving groups detected by either the dwarf sample or the giant sample by using the wavelet technique. Table 2 shows the center velocities corresponding to groups in recent work in the literature. We analyzed the dwarf sample and the giant-star sample separately because the star numbers in the solar neighborhood of dwarf and giant stars are different due to different lifetimes with more dwarf stars than giant stars. In view of this, merging the two types of stars into one sample will reduce the statistically grouped structure in the $(U, V)$ space. In some cases, the giant sample can give better structure than the dwarf sample as we will show later (for group 7). It seems that a more reasonable way is to compare these grouped structures detected from both the dwarf and giant samples and check if they can give consistent results. In this sense, we have found...
Table 1  
Heliocentric Velocities of the Moving Groups Detected by the Dwarf and Giant Samples

| No. | Dw(U, V, W) | σd(U, V, W) | Dw(P) | Gt(U, V, W) | σg(U, V, W) | Gt(P) | [Fe/H] | σ[Fe/H] | Zmax | Moving Group |
|-----|-------------|--------------|-------|-------------|--------------|-------|--------|---------|------|-------------|
| 1   | −38, −18, −10 | 6.6,10       | 100%  | −38, −17, −11 | 6.6,12       | 100%  | −0.09  | 0.17    | 0.1–0.2 | Hyades      |
| 2   | −12, −23, −10 | 6.6,10       | 100%  | −15, −23, −10 | 6.6,12       | 100%  | −0.17  | 0.1    | 0.1–0.2 | Pleiades    |
| 3   | −32, −49, −15 | 5.5,12       | 100%  | −35, −51, −11 | 5.5,15       | 100%  | −0.16  | 0.2    | 0.1–0.2 | Hercules    |
| 4   | 10.3, −11    | 6.6,10       | 100%  | 10.3, −14    | 6.6,13       | 100%  | −0.21  | 0.15   | 0.0–0.1  | Sirius-UMa |
| 5   | −11, −7, −12 | 5.5,10       | 100%  | −13, −6, −10  | 5.5,15       | 97%   | −0.20  | 0.17   | 0.0–0.1  | Coma (or Castor) |
| 6   | 38, −20, −15 | 4.4,12       | 100%  | 37, −25, −12  | 4.4,14       | 98%   | −0.24  | 0.21   | 0.1–0.2  | A08 (new feature) |
| 7   | −57, −45, −16 | 5.5,13       | 100%  | −55, −50, −16 | 5.5,14       | 98%   | −0.22  | 0.29   | 0.2–0.3  | F08 (F3 green) |
| 8   | 57, −5, −10  | 3.3,12       | 93%   | 60, −5, −10  | 5.18         | 87%   | −0.28  | 0.18   | 0.2–0.3  |            |
| 9   | −56,7,1      | 3.3,21       | 97%   | −50,4,3     | 3.3,20       | 90%   | −0.28  | 0.17   | 0.1–0.2  |            |
| 10  | 30, −5, −11  | 3.3,6        | 87%   | 28,0, −11  | 6.6,5        | 86%   | −0.25  | 0.13   | 0.2–0.3  |            |
| 11  | 65, −20, −7  | 5.8,16       | 63%   |           |             |       |        |        |        |            |
| 12  | −53, −19, −12 | 3.3,21       | 92%   | −50, −20, −18 | 3.3,10       | 93%   | −0.20  | 0.24   | 0.2–0.3  |            |
| 13  | 20, −50, −5  | 3.3,22       | 82%   |           |             |       |        |        |        |            |
| 14  | −16, −15, −12 | 4.3,10       | 90%   | −18, −15, −16 | 4.3,5        | 91%   | −0.13  | 0.10   | 0.0–0.1  | IC 2391    |
| 15  | 9, −15, −9   | 5.6,12       | 97%   |           |             |       |        |        |        |            |
| 16  | 42.8, −29    | 3.3,15       | 92%   |           |             |       |        |        |        |            |

Notes. Column 1 gives the sequence number of the groups. Column 2 and 5 give the velocity centers of the groups: Column 2 is that detected by the dwarf sample; Column 5 is that detected by the giant sample. Column 3 is the σ(U, V, W) for dwarfs and Column 6 is that for giants. Column 4 is the detected probability of each moving group during the simulation for the dwarf sample while Column 7 is that for the giant sample; Column 8 is the average [Fe/H] of the groups; Column 9 is the σ[Fe/H] of the groups; Column 10 is the Zmax peak value of each group; the last column is the name of the corresponding moving group identified previously (A08 is from Antoja et al., 2008) and T3 means Table 3 in that paper; F08 is from Famaey et al. (2008) and F3 means Figure 3 in that paper.

Table 2  
Center Velocities Corresponding to Groups in Recent Work in the Literature

| No. | D(U, V) | E(U, V) | F(U, V) | F05(U, V) | F07(U, V) | F08(U, V) | K08(U, V) | A08(U, V) |
|-----|---------|---------|---------|-----------|-----------|-----------|-----------|-----------|
| 1   | −40, −20 | −40.4, −16.0 | −42, −18 | −37, −17  | −35, −18  | −25, −15  | Tab. 3, No. 2 |
| 2   | −12, −22 | −11.6, −20.7 | −13, −19 | −15, −25  | −16, −23  | −25, −15  | Tab. 3, No. 1 |
| 3   | −30, −50  | −35, −50.0  | −42, −51 | −30, −50  | −35, −51  | −20, −50  | Tab. 3, No. 16 |
| 4   | 9.3      | 14.9,1.4   | 9.3      | 6.5,3.9   | 10, −5    | 5,1.5     | 6.4       | Tab. 3, No. 3 |
| 5   | −10, −5  | −3, −4     | −10, −10 | −10, −10  |           |           |           | Tab. 3, No. 4 |
| 6   | 15, −60  | 5.8, −59.6  | −20.8, −15.9 |           |           |           |           | Tab. 3, No. 6 |
| 7   | −55, −51 |           |         |           |           |           |           |           |
| 8   |         |           |         |           |           |           |           |           |
| 9   |         |           |         |           |           |           |           |           |
| 10  |         |           |         |           |           |           |           |           |
| 11  |         |           |         |           |           |           |           |           |
| 12  |         |           |         |           |           |           |           |           |
| 13  |         |           |         |           |           |           |           |           |
| 14  |         |           |         |           |           |           |           |           |
| 15  |         |           |         |           |           |           |           |           |
| 16  |         |           |         |           |           |           |           |           |
| 17  |         |           |         |           |           |           |           |           |
| 18  |         |           |         |           |           |           |           |           |
| 19  |         |           |         |           |           |           |           |           |
| 20  |         |           |         |           |           |           |           |           |
| 21  |         |           |         |           |           |           |           |           |
| 22  |         |           |         |           |           |           |           |           |

Notes. Column 1 is the sequence number of the groups in our work. Column 2 is from Dehnen (1998); Column 3 is from Eggen (1971, 1991, 1992a, 1992b, 1992c, 1996); Column 4 is from Fux (2001); Column 5 is from Famaey et al. (2005); Column 6 is according to Famaey et al. (2007); Column 7 is given by Famaey et al. (2008); Column 8 is from Klement et al. (2008); Column 9 is from Antoja et al. (2008).

* Groups 1 and 2 were unresolved by Klement et al. (2008).

that the central (U, V) values of these groups between the two samples are quite consistent from 12 groups.

Among these groups, groups 1–5 are quite strong and they have been widely reported in the literature to be moving groups. We can compare our results with previous works by Eggen (1991, 1992a, 1992b, 1992c) based on the FK5 catalog and Dehnen (1998) and find good agreement for the Pleiades (group 1), Hyades (group 2), and Sirius (group 4). Specifically,
our values are quite consistent with those of Dehnen (1998),
who gave mean motions of (−12, −22), (−40, −20), and
(9, 3) km s\(^{-1}\) for the Pleiades, Hyades, and Sirius groups,
respectively, based on the sample of 14,369 stars observed by
the \textit{Hipparcos} satellite. Eggen (1991, 1992a, 1992b, 1992c)
showed mean motions of (−11.6, −20.7), (−40.0, −17.0), and
(14.9, 1.3) km s\(^{-1}\) for the Pleiades, Hyades, and Sirius that are
somewhat updated by using \textit{Hipparcos} parallaxes. Our values
are also similar to those values in recent works by Famaey et al.
(2007) and Antoja et al. (2008). Mean motions of group 3 are
also in agreement with the Hercules stream centered at (−35,
−45) found by Fux (2001). It is noted that the mean motions
of group 3 are slightly different between the dwarf and giant
groups. Considering that the giant group shows better structure
than that of the dwarf group in Figure 1, we suggest that the
mean motions of giant groups have more reasonable values for
group 3. Our group 5 with mean motions of (−11, −7) in the
present work is very similar to the Castor group with (−10,
−10), which is classified as a young group by Famaey et al.
(2007) with mean motion of (−10, −12).

Besides the five known moving groups, two moving groups
(groups 6 and 7) with mean motions of (38, −20) and (−57,
−45) are quite significant from both the dwarf and giant samples
and the probability from our simulation is larger than 98%. For
our group 6, Antoja et al. (2008) also suggested the structure
centered at (35, −20) to be a new group. However, since the
structure is weak in their work, they considered it as a part of the
elongation of the Sirius or Coma Berenices structure. However,
group 6 is very prominent and has a well defined shape in the
present work both for the dwarf and giant samples. Moreover,
their mean motions are far away from those of Sirius and the
Coma Berenices structures without significant connections in
the contour plot of Figure 1. Note that the giant sample of
group 6 has some extension to the southwest direction, which
disappears in the simulation contour of Figure 2 after adding
noise while the center part of group 6 persists. It seems that
the center part of group 6 is real while the extension to the southwest
direction may be noise. In this sense, we suggest that group 6 is
real in the giant sample, and it is very significant in the dwarf
sample.

The other five groups (8, 9, 10, 12, 14) also have some features
in both the dwarf and giant samples and the simulation shows
that four of them are significant with probability above 90% of
being real, except for group 10 with probabilities of 86% and
82%. The rest of the groups are shown only in the dwarf sample
(15 and 16) or only in the giant sample (11, 13, 17, 18, 19, 20, 21, 22); but four groups (11, 13, 17, 21) are not
very significant with probability lower than 90% by simulation.
Interestingly, some of these features have been reported in
previous works. For example, our group 14 is consistent with
the IC 2391 stream, which shows a mean motion of (−20.8,
−15.9) by Eggen (1992b) and of (\(U = −20, V = −12\)) by
Chen et al. (1997). Moreover, our groups 18, 20, and 22 may be
the same groups as groups 7, 14, and 13 in Table 3 of Antoja
et al. (2008), which summarizes the possible grouping structures
from Dehnen (1998) and Eggen’s serial papers.

A catalog of 22 possible moving group candidates is given in
Table 1, although some of them are not so significant (10, 11, 13, 17, 21) for a statistical requirement of the probability being
above 90%. Certainly, these groups with low probabilities from
our statistical analysis need further study. The metallicity is only
available in the dwarf sample, based on which we present the
peak and scatter of [Fe/H] for all the candidates in Table 1.

![Figure 2](image-url). Contour map of the positive wavelet transform coefficient for one copy of the simulation.

![Figure 3](image-url). Z\(_{\text{max}}\) distributions of groups 3 and 7.
3.3. The W Velocity Distributions of the Groups

Although previous work to detect the moving groups does not take into account the W velocity, this component may bring some new information. Therefore, we have generated the (V, W) and (U, W) contour plots with the same method. As expected, the distribution of W velocity is limited to a narrow range from $-20 \text{ km s}^{-1}$ to $5 \text{ km s}^{-1}$ for both the dwarf and giant samples. Moreover, the dispersion in the W velocity for these groups is about $10–15 \text{ km s}^{-1}$, which is significantly larger than those of the U and V components of 4–6 km s$^{-1}$ in defining these groups. Thus, the (V, W) and (U, W) contour plots have no advantages in identifying the possible moving groups. Several groups of the 22 candidates actually belong to the same groups in the (V, W) and (U, W) contour plots. Note that groups 9 and 12 have the largest W dispersion of 20 km s$^{-1}$ because they have very weak features in our Figure 1. In general, these moving group candidates based on the (U, V) contour are considered to be more reliable by taking into account the large scatter in the W-component distribution.

Our groups 3 and 7 appear to coincide with the Hercules group identified as green lumps in Figure 1 of Famaey et al. (2008); there they think the green lumps are both the Hercules group. However, the argument is not significant and we suggest that they may be a distinct group. Figure 3 shows the statistical result of the $Z_{\text{max}}$ distribution for $Z_{\text{max}} < 1.0 \text{ kpc}$. It seems that the peak of group 3 in the $Z_{\text{max}}$ distribution is at 0.1–0.2 kpc while there is a large contribution from stars with $Z_{\text{max}}$ of 0.2–0.3 kpc for group 7. Moreover, from Famaey et al. (2004a), it is clear that the $\sigma(U, V, W)$ of the Hercules group is significantly higher than those of the Hyades or Sirius, which could indicate an overlapping of more than one distinct group. In the present work, the $\sigma(U, V, W)$ is nearly the same for groups 3 and 7 as well as other groups. Finally, there is also a hint of [Fe/H] deviation with the average [Fe/H] of $-0.16 \text{ dex}$ in group 3 versus $-0.22 \text{ dex}$ for group 7. Further study of this topic is necessary before firm conclusions can be drawn.

4. CONCLUSIONS

Using the dwarfs and giants in the solar neighborhood, we illustrate a detailed analysis of the UV distribution. This analysis reveals 22 possible grouping structures identified by the kernel estimator and wavelet technique. The locations in velocity space of 12 possible moving groups from both dwarf and giant samples are consistent. 11 groups, including the five well known groups, Pleiades, Hyades, Hercules, Sirius, and Castor streams, are consistent with previous works. Eight groups, not reported by previous works, are presented and most of them are thought to be significant in term of statistics. In summary, a catalog of 22 moving group candidates with the centers (U, V, W), their dispersions, and mean metallicity are given.

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