A novel deterministic model for generating synthetic wind speeds

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Abstract

This work proposes a deterministic methodology to generate synthetic wind-speed time series consistent with both a probability density function and a spectral density function. The method relies on the random-phase multisine signal to generate an initial sequence conforming to the target spectral density. A following iterative rank-reordering procedure rearranges the samples drawn from the target probability distribution so as to match the desired spectral density function. Such a rank-reordering procedure generates a final signal that conforms to a stationary, pseudo-random process. An application of the method is presented along with a comparison with two different state-of-the-art algorithms from the literature.

Keywords— Synthetic wind speeds, random-phase multisine, pseudo-random process

1 Introduction

The availability of synthetic or computer-generated wind data that are consistent with both a probability density function (PDF) and a spectral density function (PSD) can save valuable time in a number of applications related to wind energy. For instance, synthetic wind-speed time series are particularly useful in deriving power time series from power curves to provide a time-varying estimate of wind turbines yield. Such time series can also be used to mimic realistic input conditions for testing and training wind resource assessment applications.

Wind-speed fluctuations are often described statistically, in terms of a random process constrained by statistical descriptors such as the PDF and the autocorrelation function or, equivalently, the PSD. A further simplification is to regard this random process as stationary, where one assumes that the statistical descriptors do not change over time. Despite the stationarity assumption, no exact mathematical solution exists for the problem of fitting an arbitrary sequence to both a non-Gaussian PDF and a given PSD, mainly due to the limitation of linear filtering operations to impose a desired spectral content which inevitably leads to a Gaussian sequence. Wind-speed records do exhibit non-Gaussian marginal distributions with increasing length of the time series, therefore an iterative approach is required to address this problem.

In this work we present a novel method for generating synthetic wind-speed time series by means of a stationary, pseudo-random process for which the user can impose both the power spectral density function and the probability density function. The methodology proposed here relies on the technique for designing broadband excitation signals with a user-imposed power spectrum and probability distribution developed by Schoukens and Dobrowiecki \[1\]. Such a technique makes use of the random-phase multisine signal, a well-known class of signals in the system identification community, to generate a signal consistent with the imposed target spectral density. Subsequently, an iterative rank-reordering process attempts to give the samples of a previously generated sequence with the imposed PDF the same autocorrelation structure that the samples of the sequence consistent with the target PSD possess.

Similar methods have been already proposed for
2 REVIEW OF PREVIOUS WORK

The issue tackled by this paper stems from the more general problem of generating coloured non-Gaussian random sequences which conform to a given PDF and PSD or, equivalently, autocorrelation function. No exact mathematical solution exists for such a problem due to the impossibility of imposing an arbitrary spectral content to a non-Gaussian sequence without altering its marginal distribution towards a normal distribution. Therefore, a number of different approaches have been proposed in the literature to address this problem. In particular, the work of Gujar and Kavanagh [3] seems to be the first to have suggested a scheme based on a linear filtering operation to impose the prescribed power spectral density to an initial white noise Gaussian sequence, followed by a zero-memory nonlinear transformation (ZMNL) to impose the desired PDF to the generated signal without altering considerably its power spectrum. Such a two-step approach has lately become the most commonly employed procedure for the generation of coloured non-Gaussian random signals. Liu and Munson [7] proposed the same procedure where the ZMNL transformation was modelled through an expansion of Hermite polynomials, but providing a more thorough study of the method. Likewise, Grigoriu [5] devised a similar algorithm for simulating stationary non-Gaussian processes in which the ZMNL transformation was defined by means of a translation process [9].

In the method proposed by Filho and Yacoub [10] the imposition of the target non-Gaussian PDF is instead achieved by implementing a rank-ranking process, which concurrently attempts to reproduce the autocorrelation of the sequence possessing the desired PSD. Contrary to the previous methods, this advantageous approach removes the requirement of an analytical expression for the ZMNL transformation to impose the target PDF.

A different class of simulation algorithms for stationary non-Gaussian processes has been introduced by Yamazaki and Shinozuka [11] who proposed an iterative algorithm to match both the target probability distribution and the desired spectral content. In this method, the initial Gaussian sequence with the prescribed PSD is generated by the spectral representation method (SRM), which was first introduced by Shinozuka and Jan [12] for simulating Gaussian stochastic processes, and further developed by Shinozuka and Deodatis [13]. The desired PDF is then imposed on the generated signal and the distortion of the PSD is corrected iteratively. The same algorithm was later improved by Deodatis and Micaletti [14] to match highly skewed non-Gaussian probability distributions, and by Shi and Deodatis [15] to preserve the Gaussian characteristics of the underlying initial sequence throughout the iterative process. The latest development in this class of simulation algorithms can be found in the work of Bocchini and Deodatis [16] along with a critical review of all the previous works.

To date, the algorithm presented by Nichols et al. [17] for generating non-Gaussian coloured stochastic sequences appears to be the one that yields the most accurate numerical results in matching both the target PDF and PSD. Following the approach of [10], they introduced an iterative scheme for the rank-reordering step to simulate the influence of the ZMNL transformation which is equivalent to that put forward by Schreiber [2]. By combining the initial linear filter for generating surrogate data [2], and coloured, non-Gaussian signals [3]. In contrast to those methods, our approach generates the initial sequence conforming with the target PDF in a deterministic way, and introduces a stochastic component by means of the random phases of the multisine signal. Our main goal is to show that this algorithm is suitable for generating synthetic wind-speed time series which are consistent with a target PDF and a prescribed PSD.

The method is first tested for the generation of mean, horizontal wind-speed time series which are Weibull distributed and possess the low-frequency component of the experimental spectral content deduced by Van der Hoven [4]. With this test, we show that the present model can simulate the fluctuation of the mean wind speed, which is associated with the low-frequency region of the power spectrum and can be well approximated by a Weibull-distributed process.

It is shown that for sufficiently long time series both the target probability distribution function and the target spectral content can be reproduced with arbitrarily high accuracy. In addition, to show its capabilities we provide a comparison of the proposed method with two of the most effective algorithms available in the literature to address this problem.

2 Review of previous work

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imposing the target spectral content with this iterative scheme, they showed that a wide number of combinations of non-Gaussian PDF and PSD can be easily reproduced. Also Yura and Hanson [11] obtained good results in terms of reproducibility of probability distribution and autocorrelation function, but with a simpler and non-iterative method. Their algorithm makes use of the spectral representation method to generate the signal with the target spectral content which is followed by a single application of the cumulative distribution function (CDF) mapping employed in [11]. In contrast with Yamazaki and Shinozuka, they showed that an iterative scheme is not necessary to reproduce fairly accurately the target functions for a number of combinations of probability distribution and autocorrelation functions.

It is worth mentioning that an application for the generation of wind-speed time series conforming to a prescribed PSD and a target non-Gaussian PDF by means of a stationary, stochastic process has been recently attempted by Torrielli et al. [17] with the aim of reproducing the macro-meteorological region of a measured horizontal wind speed spectrum. Their method consisted in a revised version of the algorithm developed by Masters and Gurley [18], and they also later proposed a modified version of the Nichols’ algorithm to allow to recover the underlying deterministic component of the wind from the simulated time series [19].

The method of the present paper makes use of the random-phase multisine for designing an initial stochastic sequence possessing the target spectral content:

\[ z(t) = \sum_{k=1}^{N} A_k \sin(2\pi f_0 t + \phi_k). \]  

(1)

The periodic and broadband nature of this type of signal enables to impose a desired spectral content by setting its harmonic amplitudes \( A_k \), and guarantees the generation of a stochastic sequence through the introduction of the random phases \( \phi_k \). Realizations of Eq. (1) therefore result in Gaussian-distributed signals whose mean and variance are imposed by the \( A_k \)'s.

This sequence is then used to iteratively rearrange the samples conforming to the target PDF so that they match the autocorrelation of the multisine signal. This rank-reordering procedure is similar to the one proposed by [11] and [3], and effectively simulates the action of a zero-memory nonlinear transformation.

3 Random-phase multisine iterative method

Suppose that we wish to generate a time series of \( N \) horizontal wind speeds consistent with a desired probability distribution function and which conforms to a prescribed power spectral density function. The initial step to carry out is to generate a sequence of \( N \) samples conforming to the target PSD denoted by \( S_{zz}(f) \). In order to do that, the target one-sided power spectrum \( S_{zz}(f) \) is to be sampled at \( N/2 \) discrete frequencies \( f_k, k = 0, ..., N/2 - 1 \). The sampling frequency \( f_s \), and consequently the frequency resolution \( \Delta f = f_s/N \), are dictated by the Nyquist frequency criterion for the maximum resolvable frequency according to the relation \( f_{NY} = f_s/2 \). Therefore, assuming the highest frequency at which the PSD is estimated as the Nyquist frequency, we can sample the target PSD function at the discrete frequencies \( f_k = 2k f_{NY}/N, k = 0, ..., N/2 - 1 \), which yield the corresponding discrete values of the one-sided target power spectrum \( S_{zz}(f_k) \). These discrete samples are then translated into Fourier amplitudes by taking

\[ Z_k = \sqrt{\frac{N f_s}{2}} S_{zz}(f_k), \quad k = 0, ..., N/2 - 1. \]  

(2)

The initial sequence possessing the target PSD will only serve to shape the final sequence of wind speed samples towards the desired autocorrelation, and it is generated as a random-phase multisine signal directly in the frequency domain. To do that, the sequence of the extracted Fourier amplitudes \( Z_k \) is first extended with \( N/2 \) zeros, \( Z_k = 0, k = N/2, ..., N - 1 \), and the mean of the signal in time domain is removed by setting \( Z_0 = 0 \). Each Fourier amplitude is then multiplied by a random phase \( \theta_k \) drawn from the uniform distribution \( U(0, 2\pi) \); the resulting sequence is inverse Fourier transformed and only the real part is retained so as to generate the real signal \( z_n \) with

\[ z_n = 2\Re\{FT^{-1}(Z_k e^{j\theta_k})\}, \quad n = 0, ..., N - 1. \]  

(3)

At this stage, the randomness introduced in the process by the random phases ensures that an initial stochastic multisine sequence is generated. The next step is to generate a new sequence \( x_n, n = 0, ..., N - 1 \) that is consistent with the target probability density function. This is achieved by applying
a CDF mapping technique on a linear sequence of $N$ samples on the interval $(0, 1)$ denoted by $u_n$, which represents the cumulative distribution function of a uniformly distributed variable, and is defined as

$$u_n = \frac{1 + 2n}{2N}, \quad n = 0, \ldots, N - 1.$$  

(4)

With this technique, one can easily obtain a sequence of $N$ samples, indicated by $x_n$, which is distributed with the desired probability density function. That can be achieved by a single application of the inverse cumulative distribution function of the target PDF to the previously generated sequence $u_n$.

$$x_n = F^{-1}(u_n),$$  

(5)

where $F^{-1}$ is the inverse target CDF. Such a technique is not restricted to cumulative distribution functions that possess analytical inverse. In cases where $F^{-1}$ cannot be analytically determined a nonlinear curve fitting can be performed to obtain a numerical inverse of the target CDF to apply in Eq. (5). Note that the CDF mapping is applied to a deterministic sequence, i.e. $u_n$, which always generates the same sequence $x_n$ from a given target PDF defined by a given set of parameters. This deterministic generation of the samples does not affect, however, the ability of such a technique to reproduce accurately the extreme values of the target distribution, which increase in magnitude with the number of samples $N$ and hence with the length of the generated sequence. Thus, the inverse CDF operation results in a sequence of $N$ samples $x_n$ already sorted in ascending order, which is desired for the reordering procedure of the signal that follows.

In generating $x_n$, we must ensure that the variance of the target PDF equals the variance obtained by integrating the two-sided target power spectral density over its frequency domain with its zero-frequency component removed. This constraint originates as a direct consequence of the Wiener-Khintchine theorem which relates the PDF and the autocorrelation (hence the PSD) of a stationary process through the variance of the signal generated by the process (20 21).

To complete the initialization procedure, the variance and the mean of the previously generated sequence consistent with the target PDF, $x_n$, are imposed to the initial multisine signal $z_n$. This results in the new sequence $\tilde{z}_n$

$$\tilde{z}_n = z_n^0 + \bar{x},$$  

(6)

where $z_n^0 = (\sigma_x/\sigma_z)z_n$ , and $\bar{x} = \frac{1}{N} \sum_n x_n$ is the desired mean from the target PDF. Before starting the iterative rank-reordering procedure, the Fourier amplitudes of $\tilde{z}_n$ are first calculated and then stored, $\tilde{Z}_k = |FT(\tilde{z}_n)|$.

At this stage, the goal is to reorder the samples of the sequence $x_n$, which conform to the target PDF, so as to generate a new sequence which has a power spectral density function matching the target PSD possessed by $\tilde{z}_n$. This is achieved by implementing a rank-reordering iterative procedure which simulates the effect of a zero-memory nonlinear transformation and consists of three steps. Firstly, the already ordered values of the sequence $x_n$ are given the same positions in the new signal as the sorted values of $\tilde{z}_n$ possess within their sequence. That is, the samples of $x_n$ are reordered so that its smallest value, $x_0$, is given the same position in the new sequence as the smallest value of $\tilde{z}_n$ has in its own sequence, and so forth for all the $N$ values. Such reordering will produce a new sequence $\tilde{x}_n$. Secondly, the phases of the Fourier spectrum of this new, reordered sequence $\tilde{x}_n$ are computed by means of the Fourier transform of the signal, which yields

$$\phi_k = \tan^{-1}\left(\frac{\Im\{FT(\tilde{x}_n)\}}{\Re\{FT(\tilde{x}_n)\}}\right),$$  

(7)

where $\Im\{\cdot\}$ and $\Re\{\cdot\}$ indicate respectively the imaginary part and the real part of the transform. Lastly, the sequence $\tilde{x}_n$ is replaced by a new multisine signal generated by taking the real component of the inverse Fourier transform of the stored Fourier amplitudes $\tilde{Z}_k$ multiplied by the last computed phases $\phi_k$

$$\tilde{z}_n = \Re\{FT^{-1}(\tilde{Z}_k e^{j\phi_k})\}.$$  

(8)

In the next iteration this sequence will be employed to perform the successive rank-reordering step of the samples of $x_n$.

This iterative procedure attempts to reproduce the target autocorrelation function by giving the values of $x_n$ the same ranking in the sequence as the ranking that similar values of the sequence consistent with the target PSD, $\tilde{z}_n$, have among each other. At each iteration, the first step will produce a new signal $\tilde{x}_n$ conforming with the target probability distribution and, at the same time, attempts to reorder its samples according to the ranking of the last generated sequence $\tilde{z}_n$, shaping its autocorrelation function towards the target autocorrelation function (i.e. the target PSD).
4 Test case

In performing this rank-reordering step, values from the target PDF are replacing the samples of \( \tilde{z}_n \) to generate \( \tilde{x}_n \). Therefore this new sequence will possess exactly the target PDF but only an approximation of the target Fourier amplitudes \( \tilde{Z}_k \). Before restoring the target Fourier amplitudes, a further step is needed to retain the positional relationship that the samples of \( \tilde{x}_n \) have among each other within their sequence; that is achieved by storing the phases \( \phi_k \) in the second step (Eq.(7)). Finally, the third step will generate the new signal \( \tilde{z}_n \) by performing an inverse Fourier transform on the stored initial target Fourier amplitudes \( \tilde{Z}_k \) and the last computed phases \( \phi_k \). Despite producing a new sequence with the desired Fourier amplitudes (target PSD), Eq.(8) will alter the probability distribution of the resulting sequence towards a Gaussian distribution due to the central limit theorem as a direct consequence of the inverse Fourier transform operation. Therefore, the three-step scheme has to be iterated up to the point where the variations of the phases of the reordered signal with the target PDF, \( \phi_k \), become negligible, i.e. when the reordering step does not alter significantly the Fourier amplitudes of the last generated signal \( \tilde{z}_n \).

The resulting sequence will possess exactly the target PDF since it consists of samples drawn from the sequence with the target probability distribution. As for the deviations from the target PSD, Filho and Yakoub [10] implemented in their work the same rank-reordering iterative process and proved that this algorithm converges to the target PSD as the number of samples \( N \) increases. In the next section, we provide an example with a multimodal power spectrum to show that the algorithm is also capable of reproducing such a class of power spectral density functions.

It is important to point out that, in contrast to [3] and [5], the present methodology simulates a stationary pseudo-random process and can be therefore regarded as deterministic. Here the initial sequence consistent with the target PDF, \( x_n \), is not obtained as a finite number of realizations of a stochastic variable, but rather it is generated in a deterministic fashion by applying the inverse CDF transform to the CDF of a uniform distribution. The only stochastic component is introduced in the method by the use of the random phases \( \theta_k \) which, however, only affect the rearrangement of the samples of the final sequence without altering stochastically the values of the sequence. This has two important implications. First, by simulating the process with the same set of initial phases \( \theta_k \) one always gets exactly the same output signal. Second, by introducing random sets of initial phases \( \theta_k \), a number of different signals consistent with the same target statistical descriptors (i.e. PDF and PSD) can be generated.

4 Test case

In this section, we test the proposed method for the generation of horizontal wind speed timeseries. In particular, we choose to simulate synthetic wind speeds with a sampling period of 10 minutes that possess a spectral content representative of the distribution of kinetic energy associated with the macrometeorological range of frequencies produced by the wind speed fluctuations. To this end, we consider as target PSD the low-frequency component of the spectrum of horizontal wind speeds derived by Van der Hoven [4] from different sets of measurements. Such a broadband content in the low-frequency region of this power spectrum is delimited by the so-called spectral gap and it is therefore associated with the fluctuation of the mean wind speed whose averaging period lies in the spectral gap.

The solid line of Fig. 2 shows the target spectral content selected for this test. We decide to reproduce a spectral window comprised of frequencies ranging from \( f \sim 3 \times 10^{-7} \) Hz, corresponding to a time
scale of approximatively one month, to \( f \sim 8 \times 10^{-4} \) Hz which corresponds to a sampling period of 10 minutes according to the maximum resolvable frequency \( (f_s = 2f_{\text{max}}) \). This spectral window includes the fluctuations of kinetic energy in the wind associated with the passage of large-scale weather systems (synoptic peak at \( f \sim 3 \times 10^{-6} \) Hz), and the variations related to the day and night cycle (diurnal peak at \( f \sim 2.5 \times 10^{-5} \) Hz). The fast fluctuations occurring due to turbulence (high-frequency component peak at \( f \sim 1.5 \times 10^{-2} \) Hz) are not included in the target spectral content. Therefore, such a choice for the upper cutoff frequency ensures that no additional energy due to faster fluctuations of the wind is injected in the simulated timeseries by disregarding the micrometeorological peak and, at the same time, guarantees that the target spectrum is physically consistent with the choice of generating wind speed time series with a sampling period of 10 minutes. The lower end of the spectral window coincides with the lowest available frequency of the selected target spectral content where no annual cycle is present.

As for the target probability distribution, for this test we employ a Weibull PDF since observed horizontal wind speeds averaged over 10 minutes exhibit at many locations frequency distributions that are often well described by this marginal distribution \[22\]. In addition, such a probability distribution has an analytically invertible CDF given by

\[
F^{-1}(x) = A(-\log(1 - x))^{1/k},
\]

where \( A \) and \( k \) are respectively the scale and the shape parameter of the Weibull PDF, and \( x \) is the cumulative distribution value which ranges in the interval \((0, 1)\). Hence, such a PDF is suitable for the application of the present method. The initial sequence \( x_n \) is generated by applying Eq. \[9\] to the sequence \( u_n \) of \( N \) uniformly distributed samples in the interval \((0, 1)\) (inverse CDF transform method of Eq. \[5\]). The simple transformation of Eq. \[9\] does not include any tailored technique for the prediction of long-term extreme winds, and the extreme values of the generated Weibull sequence remain completely determined by the parameters of the distribution and the number of generated samples \( N \).

For this test, \( N = 105120 \) wind speed samples were generated corresponding to 2 years of data with a sampling period of \( \Delta t = 10 \) min. The total length of the sequence so generated is not intended to be representative of the fluctuation of the mean wind speed throughout two full calendar years; rather, it is regarded as a series of 24 months each of them expressing the variation of the mean wind speed consistent with the distribution of the variance imposed by the target spectral content.

The mean wind speed was set to \( \bar{u} = 8 \) m/s. The scale parameter \( A \) and the shape parameter \( k \) were derived by imposing the selected mean wind speed and the variance obtained by integrating the target PSD over the frequency range of interest. This resulted in \( A = 8.95 \) m/s and \( k = 1.67 \), respectively. Fig. \[3\] shows a segment of the time series generated in this test along with the agreement in terms of PDF and PSD with the respective target functions. Since the generated sequence consists of samples produced by means of Eq. \[9\], there is an exact match between the target PDF and the probability distribution of the generated time series. Therefore, one may expect that the deviation in terms of the spectral content must account for the total error introduced by the simulation.
Nevertheless, the number of simulated samples $N$ ensures that the algorithm generates a time series with a PSD that converges to the target spectral content giving a highly accurate match despite the multimodal nature of the target PSD (Fig. 3c).

5 Comparison with selected methods

This section provides a comparison of the proposed algorithm with two different methods for generating coloured, non-Gaussian signals consistent with both a target PDF and a target PSD, which we believe are the ones giving the most accurate results available in the literature. They are respectively the non-iterative algorithm of Yura and Hanson [5] and the iterative method proposed by Nichols et al. [3], and they will be both implemented for the generation of horizontal wind speed time series consistent with the same PDF and PSD target functions employed in the test case. Before presenting the results of the comparison, we provide a brief description of the two methods to point out their most relevant features and differences with our algorithm which are also summarized in Table 1.

5.1 Yura and Hanson method

Yura and Hanson [5] proposed a non-iterative method which simulates a stationary, stochastic process to generate a signal consistent with both a specified probability density function and a power spectral density function. Their method relies on the spectral representation method to generate a signal with the desired spectral content and makes use of a technique based on the inverse CDF transform method to impose the target probability distribution to the generated signal. Their algorithm can be summarized as follows. First, an initial zero-mean, unit-variance Gaussian seed is generated via a random generator. This initial sequence is then coloured in the Fourier domain with the target Fourier amplitudes extracted from the target PSD, and a new sequence is generated by inverse Fourier transforming into the time domain this coloured sequence. As a final step, an inverse CDF transform technique is applied to the CDF of the generated time signal so as to impose the target probability distribution to the final sequence. It is important to note two major differences with the methodology proposed in the present paper. The initial seed here produced is a result of a random process and therefore this method, contrary to ours, possesses an intrinsic stochastic nature. Secondly, no iterative procedure is considered to correct the distortion in the PSD introduced by imposing the desired PDF to the coloured time sequence.

5.2 Nichols et al. method

Although the algorithm put forward by Nichols et al. [3] bears a close resemblance with the present method, there are a few substantial differences that are worth pointing out. To begin with, their initial sequence possessing the desired PDF is generated by means of a random generator, whereas in the proposed method the process to get the initial sequence carrying the target probability distribution is deterministic. Therefore, Nichols’ method relies on multiple realizations of a stochastic process as the main source of variability for the generated signal. On the other hand, in the method of the present paper the variability of the final signal is guaranteed by the random phases of the initial multisine signal which provide a stochastic rearrangement of the deterministic samples. Secondly, in Nichols et al. method the imposition of the variance from the target PSD is carried out on the initial signal with the desired PDF. This introduces a deviation in the target probability density function and requires a further calculation step to estimate the effective parameters of the target PDF which are consistent with the constraint imposed on the variance by the target PSD. In the random-phase multisine method the sequence with the desired PDF is instead designed such that it meets such a constraint allowing to have better control on the target probability density that we want to reproduce (e.g. by setting one parameter of the target distribution and determine the other one accordingly). In contrast with our method, Nichols et al. algorithm does not generate any initial sequence possessing the target spectral content and the extracted target Fourier amplitudes are employed directly in the iterative process to impose the desired PSD on the signal. As for the iterative part, the two methods do share effectively the same rank-reordering iterative process which performs the calculation of the phases of the current signal and generates a new signal by means of the target Fourier amplitudes so as to restore the target power spectrum.
5 COMPARISON WITH SELECTED METHODS

Figure 3: Simulation of a pseudo-random, Weibull distributed process with Random-phase multisine method. (a) Segment of generated time series. (b) PDF agreement. (c) PSD agreement.

Figure 4: Simulation of a stochastic, Weibull distributed process with Yura and Hanson method. (a) Segment of generated time series. (b) PDF agreement. (c) PSD agreement.

Figure 5: Simulation of a stochastic, Weibull distributed process with Nichols et al. method. (a) Segment of generated time series. (b) PDF agreement. (c) PSD agreement.

5.3 Comparison results

The two methods selected for comparison were implemented for generating horizontal wind-speed time se-
5 COMPARISON WITH SELECTED METHODS

| Preprocessing | Algorithm | Postprocessing |
|---------------|-----------|---------------|
| Yura and Hanson | - random seed $\sim N(0,1)$ | - non-iterative | - averaging PSD |
| | - target Fourier amplitudes | - SRM + CDF mapping | | |
| Nichols et al. | - target Fourier amplitudes | - iterative rank-reorder | - restoration of $\bar{x}$ |
| | - random sequence $\sim$ target PDF | | |
| | - imposition of $\sigma^2$ from target PSD | | |
| Random-phase multisine | - target Fourier amplitudes | - iterative rank-reorder | |
| | - random-phase multisine with target PSD | | |
| | - deterministic sequence $\sim$ target PDF | | |
| | - imposition of $\bar{x}$ and $\sigma^2$ from target PDF | | |

Table 1: Summary of the most important features of the compared methods.

ries which are consistent with the same target PDF and PSD described in the test case. A number of $N = 105120$ wind-speed samples were generated for each simulation.

Fig. 4 shows the results produced by the implementation of the non-iterative Yura and Hanson method. The horizontal wind-speed time series generated by their algorithm yields an excellent match in terms of PDF with the target function, showing slight fluctuations around the target PDF as a consequence of the stochastic generation of the initial seed (Fig. 4b). Thus, one can expect the agreement with the target PDF to improve with the number of simulated samples $N$. However, this feature of their method also yields a considerably noisy power spectrum of the generated time series as shown in Fig. 4c. This is an additional shortcoming introduced by the random generation of the initial sequence which guarantees only an asymptotic convergence to the imposed PSD due to the finite length of the generated sample. This means that the PSD derived from the generated time series can only approximate the desired PSD. Therefore, a smoothing technique needs to be applied on the generated PSD in order to assess the agreement with the target spectral content. To accomplish that, we performed Welch’s averaging technique on the generated noisy spectrum which produced the agreement of Fig. 4c. While the shape of the target PSD appears to be correctly reproduced by the generated sequence, Yura and Hanson algorithm seems to fail in imposing the proper scale of power density over all frequencies. The work of [5] only shows agreements in terms of autocorrelation coefficients (or equivalently autocorrelation functions) without discussing the issue of the noisiness of the generated spectrum. Therefore, we believe that a further investigation of the interplaying relation between the noisy power spectrum and the autocorrelation function related through the Wiener-Khintchine theorem must be undertaken in order to provide a proper assessment on the deviations from the target PSD. In Fig. 4a a segment of the generated time series is shown.

The application of the Nichols et al. method produced the results shown in Fig. 5. The agreement with the target PDF is excellent and only a slight fluctuation around the target probability function can be noticed (Fig. 5b). As in Yura, this is due to the stochastic generation of the initial sequence and it is reduced with the sample size. Note that the random-phase multisine method is not affected by this shortcoming as the generation of the initial signal is deterministic. Finally, the rank-reordering procedure implemented in Nichols’ method seems to prevent altogether the generation of a noisy spectrum and yields an excellent agreement with the target PSD which is as good as the one produced by the random-phase multisine (Fig. 5c). When assessed in terms of computational efficiency, the proposed method and the Nichols et al. method show comparable performances provided that they are both run with a fixed number of iterations. For this test, the wall clock time was estimated to be approximately 0.65 s on a 2.3 GHz dual-core Intel Core i5 processor. A segment of the generated time series is shown in Fig. 5a.
6 Discussion and conclusions

A novel method for the generation of synthetic wind-speed time series that are consistent with both a given probability distribution and a desired spectral content has been presented in this paper. Such a methodology provides a computationally fast algorithm based on the random-phase multisine signal to generate a large number of samples that fit with arbitrarily high accuracy the target statistical descriptors.

To show its potential, it has been first tested for the simulation of the macrometeorological frequency content that 10-minute averaged wind-speed measurements typically exhibit over a time scale of approximately one month. The low-frequency component of the Van der Hoven spectrum and the Weibull probability distribution have been chosen respectively as the target PSD and the target PDF to reproduce for this test. The generated time series has shown an exact match with the target marginal distribution and an excellent agreement with the target spectral content, demonstrating that the proposed algorithm is capable of simulating signals characterized by a multimodal PSD with multiple peaks as the ones typically measured for 10-minute averaged wind speeds.

In addition, the model of the present paper has been compared with two state-of-the-art algorithms for the generation of spectrally colored, non-Gaussian signals available in the literature. The comparison has shown that it can outperform those methods in terms of the agreement with at least one of the target statistical descriptors when implemented to simulate wind-speed time series consistent with the same PDF and PSD available in the literature. The comparison has shown that the proposed algorithm is capable of reproducing for this test. The generated time series has shown an exact match with the target statistical descriptors.

As in the methods [5] and [3] employed in the comparison, the simulation is restricted only to stationary time series. This represents an important limitation when it comes to simulating different time scales as the presence of diurnal, seasonal and annual frequency components make the fluctuation of the mean wind speed a highly nonstationary phenomenon whose marginal distribution varies continuously across time scales. With regard to the ability of the present method to simulate realistic wind speeds, it is also important to note that higher-order moments are not taken into consideration in this process. Higher-order statistical descriptors might also be necessary to match in order to generate synthetic time series consistent with measured time series.

In contrast with existing methods, the algorithm here presented simulates a pseudo-random process in which deterministic samples are subjected to a stochastic, iterative rearrangement to match the target autocorrelation function. Thus, for target CDF that possess an analytical inverse, this convenient feature allows to always retrieve an exact match with the target probability distribution for any number N of simulated samples.

References

[1] J. Schoukens and T. Dobrowiecki. Design of broadband excitation signals with a user imposed power spectrum and amplitude distribution. In IMTC/98 Conference Proceedings. IEEE Instrumentation and Measurement Technology Conference. Where Instrumentation is Going (Cat. No. 98CH36222), volume 2, pages 1002–1005. IEEE, 1998.

[2] T. Schreiber and A. Schmitz. Improved surrogate data for nonlinearity tests. Physical Review Letters, 77(4):635, 1996.

[3] J.M. Nichols, C.C. Olson, J.V. Michalowicz, and F. Bucholtz. A simple algorithm for generating spectrally colored, non-gaussian signals. Probabilistic Engineering Mechanics, 25(3):315–322, 2010.

[4] I. Van der Hoven. Power spectrum of horizontal wind speed in the frequency range from 0.0007 to 900 cycles per hour. J. Meteor., 14(2):160–164, April 1957.

[5] H. Yura and S. Hanson. Digital simulation of an arbitrary stationary stochastic process by spectral representation. J. Opt. Soc. Am. A, JOSAA 28(4):675–685, April 2011.

[6] U. Gujar and R. Kavanagh. Generation of random signals with specified probability density functions and power density spectra. IEEE Transactions on Automatic Control, 13(6):716–719, December 1968.

[7] B. Liu and D. Munson. Generation of a random sequence having a jointly specified marginal distribution and autocovariance. IEEE Transactions on Acoustics, Speech, and Signal Processing, 30(6):973–983, December 1982.

[8] M. Grigoriu. Simulation of Stationary Non-Gaussian Translation Processes. Journal of Engineering Mechanics, 124(2):121–126, February 1998.

[9] M. Grigoriu. Crossings of Non-Gaussian Translation Processes. Journal of Engineering Mechanics, 110(4):610–620, April 1984.

[10] J. C. S. Santos Filho and M. D. Yacoub. Coloring Non-Gaussian Sequences. IEEE Transactions on Signal Processing, 56(12):5817–5822, December 2008.

[11] F. Yamazaki and M. Shinozuka. Digital Generation of Non-Gaussian Stochastic Fields. Journal of Engineering Mechanics, 114(7):1183–1197, July 1988.

[12] M. Shinozuka and C. M. Jan. Digital simulation of random processes and its applications. Journal of Sound and Vibration, 25(1):111–128, November 1972.
REFERENCES

[13] M. Shinozuka and G. Deodatis. Simulation of Stochastic Processes by Spectral Representation. Appl. Mech. Rev, 44(4):191–204, April 1991.

[14] G. Deodatis and R. C. Micaletti. Simulation of Highly Skewed Non-Gaussian Stochastic Processes. Journal of Engineering Mechanics, 127(12):1284–1295, December 2001.

[15] Y. Shi, G. Deodatis, and S. Koutsourelakis. A novel approach for simulation of non-gaussian fields: application in estimating wire strengths from experimental data. In 9th ASCE Joint Special Conference on Probabilistic Mechanics and Structural Reliability, American Society of Civil Engineers, Sandia National Laboratories, 2004.

[16] P. Bocchini and G. Deodatis. Critical review and latest developments of a class of simulation algorithms for strongly non-Gaussian random fields. Probabilistic Engineering Mechanics, 23(4):393–407, October 2008.

[17] A. Torrielli, M. P. Repetto, and G. Solari. Long-term simulation of the mean wind speed. Journal of Wind Engineering and Industrial Aerodynamics, 99(11):1139–1150, November 2011.

[18] F. Masters and K. R. Gurley. Non-Gaussian Simulation: Cumulative Distribution Function Map-Based Spectral Correction. Journal of Engineering Mechanics, 129(12):1418–1428, December 2003.

[19] A. Torrielli, M. P. Repetto, and G. Solari. A refined analysis and simulation of the wind speed macro-meteorological components. Journal of Wind Engineering and Industrial Aerodynamics, 132:54–65, September 2014.

[20] N. Wiener. Generalized harmonic analysis. Acta Math., 55:117–258, 1930.

[21] A. Khintchine. Korrelationstheorie der stationären stochastischen prozesse. Mathematische Annalen, 109(1):604–615, Dec 1934.

[22] I. Troen and E. L. Petersen. European Wind Atlas. Riso National Laboratory, 1989.