The running of featureful primordial power spectra

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Abstract. Current measurements of the temperature and polarization anisotropy power spectra of the Cosmic Microwave Background (CMB) seem to indicate that the naive expectation for the slow-roll hierarchy within the most simple inflationary paradigm may not be respected in nature. We show that a primordial power spectra with localized features could in principle give rise to the observed slow-roll anarchy when fitted to a featureless power spectrum. Future CMB missions have the key to disentangle among the two possible paradigms and firmly establish the slow-roll mechanism as the responsible one for the inflationary period in the early universe. From a model comparison perspective, and assuming that nature has chosen a featureless primordial power spectrum, we find that, while with mock Planck data there is only weak evidence against a model with localized features, upcoming CMB measurements may provide strong evidence against such a non-standard primordial power spectrum.
1 Introduction

Inflation is the most elegant and so far successful theory that is able to provide the seeds for the structures we observe today in our universe and solve the main problems of the standard Big Bang Cosmology simultaneously [1–3]. The most economical description of the inflationary paradigm is based on the addition of a single new scalar degree of freedom, dubbed the inflaton, coupled to Einstein Gravity and slowly-rolling down a potential. Models of inflation are usually tested by means of their predictions for the standard inflationary observables, among which we have the tensor-to-scalar ratio $r$, which characterizes the amplitude of the primordial gravitational wave spectrum, and three parameters governing the scale dependence of the power spectrum $P_{\zeta}(k)$: the scalar spectral index $n_s$, its running $dn_s/d\ln k$ and possibly the running of the running $n_{\text{run,run}}$. For a recent appraisal of the constraining power of $dn_s/d\ln k$ and $n_{\text{run,run}}$ in disentangling different inflationary scenarios, see e.g. Refs. [4, 5].

The values of these parameters and their associated 68% confidence level (CL) errors arising from the latest Planck 2015 temperature and polarization TT,TE,EE+lowP [6] data are:

$$n_s = 0.9586 \pm 0.0056,$$

$$dn_s/d\ln k = 0.009 \pm 0.010,$$

$$n_{\text{run,run}} = 0.025 \pm 0.013. \quad (1.1)$$

There are very interesting and slightly suspicious aspects in the measured values of the parameters governing the primordial power spectrum. For instance, there is a mild preference for a positive $n_{\text{run,run}} \sim 10^{-2}$, while the standard single field slow-roll inflationary paradigm typically predicts a negative one. But what is more important and remarkable is the fact that, even if the current errors on both $n_{\text{run,run}}$ and $dn_s/d\ln k$ are still large to deduce any strong conclusion, the mean values of these parameters do not seem to follow the expected hierarchy within the simplest slow-roll expansion. Namely, within this context, one would naively expect that $dn_s/d\ln k \simeq (n_s - 1)^2$ and $n_{\text{run,run}} \simeq (n_s - 1)^3$. These observational findings have previously motivated other works to look for alternative inflationary models in which a different hierarchy is expected, see Ref. [7].

Apart from the canonical single field slow-roll scenario, which will lead to the standard power-law primordial power spectrum, there exist a vast number of inflationary models in which the primordial power spectrum possesses some features, see Ref. [8] for an extensive
review. Examples of possible theoretical scenarios in which a feature in $P(k)$ may arise are, for instance, models in which there are non-canonical kinetic terms in the Lagrangian [9], where the value of the sound speed of the primordial curvature perturbation $c_s$ differs from the value $c_s = 1$ expected in the single-field slow-roll paradigm. Other examples of featured models are those in which the sound speed varies with time [10–15] or those governed by an inflaton potential with a sharp step/feature [16–34], or within the so-called axion monodromy scenarios [35–41] (see also the recent work of Ref. [42]).

In this paper we focus on the possible interpretation of the current cosmological data and of the forecasted constraints arising from future CMB missions in terms of a featured primordial power spectrum shape. Namely, the inflationary mechanism realized in nature could be different from the usual single-field slow-roll paradigm and the reconstructed values of the $dn_s/d\ln k$ and $n_{\text{run,run}}$ parameters could be hinting that.

The structure of the paper is as follows. In Sec. 2 we present a simple, theoretically motivated, featured primordial power spectrum. In Section 3, we describe the method and the cosmological probes exploited in our analysis. The present constraints on the usual slow-roll parameters, obtained when a wrong assumption about the real shape of the power spectrum is made, are described in Sec. 4, where we also explore the disentangling potential expected from future CMB missions. We conclude in Sec. 5.

2 Features in the primordial potential: A toy model

The simplest realization of inflation arise from considering a sufficiently flat and smooth potential in which the slow-roll conditions are satisfied. However, as previously stated, one could also consider models in which the inflationary potential exhibits features that modify the dynamics of the inflaton. One particular option is to consider a class of well-motivated models in which the inflationary potential shows periodic modulations \(^1\). These ripples in the potential have the characteristic signature of enhancing the three-point correlation function of the primordial perturbations, leading to a resonant primordial bispectrum and, thus, generating large non-Gaussianities [43].

A subset of this class of resonant models arises naturally in the framework of string theory. The so-called axion monodromy model makes use of the axion shift symmetry in order to address the problem of Planck-suppressed terms in the effective Lagrangian, as well as explaining the flatness of the inflaton potential. Furthermore, the inflationary potential in these scenarios exhibits modulations whose amplitude and frequency is given by the properties of the moduli fields [36, 37]. A simple realization of a single-field monodromy model leads to an inflationary potential of the form \(^2\):

$$V(\phi) = V_0(\phi) + \Lambda^4 \cos \left( \frac{\phi}{f} \right).$$

Here, $f$ is the axion decay constant and $\Lambda$ is the size of the modulations.

In general, a template for the primordial power spectrum within this class of resonant models reads as [40, 44]

$$P_\zeta(k) = P_0(k) \left\{ 1 + \frac{8 f_{\text{NL}}^\text{NS}}{\omega^2} \cos \left[ \omega \ln \left( \frac{k}{k_*} \right) \right] \right\},$$

\(^1\)Features in the potential will usually break down the slow-roll approximation. For sharp and high frequency features, developments have been made concerning the slow-roll techniques (see, e.g., [30, 41] and references therein).

\(^2\)For more general realizations see e.g. Ref. [40].
where $f_{NL}^{res}$ is related to the amplitude of the resonant non-Gaussianity, $k_*$ is the pivot scale, $\omega$ is the resonance frequency which is related to the parameters of the inflationary potential and $P_0(k)$ is the power-law primordial power spectrum with a scale-invariant tilt, i.e.,

$$P_0(k) = A_s \left( \frac{k}{k_0} \right)^{n_s - 1}.$$

(2.3)

Thus one can see that the properties of the primordial power spectrum are highly correlated with that of the primordial bispectrum. Analyzing the power spectrum, recent studies were able to obtain the following constraint [45]:

$$f_{NL}^{res} \lesssim 10^{-3} \omega^{5/2}.$$  

(2.4)

Naively, for some allowed values of the model parameters, one could expect to find that there is an agreement between the predictions of the primordial power spectrum in Eq. (2.2), and the ones given by the standard slow-roll paradigm, taken into account the non-zero values for the running of the spectral index, $dn_s/d\ln k$, and for the running of the running, $n_{\text{run,run}}$. These two possibilities are therefore expected to provide a similar fit to current data, as we will illustrate in the following sections. Consequently, the primordial power spectrum given by Eq. (2.2) should be regarded as a toy model. Nevertheless this simple model has been extensively proposed in the literature as a compelling alternative to the slow-roll paradigm. We will use this simple model as a working example throughout our study.

3 Methodology and Cosmological data sets

In order to quantify the viability of the resonant toy-model given by Eq. (2.2), we consider the Planck CMB satellite measurements of the temperature and polarization anisotropies (the so-called TT, TE and EE angular spectra), which extend up to a multipole $\ell_{\text{max}} = 2500$. We combine these measurements with Planck low-multipole polarization data, ranging from multipoles $\ell = 2$ up to $\ell = 29$. We make use of the publicly available Planck likelihood code [46], which also includes a number of nuisance parameters, that we treat accordingly to Refs. [46, 47]. To derive the constraints on the different inflationary parameters, we make use of the Boltzmann equations solver CAMB code [48] and apply Markov Chain Monte Carlo (MCMC) methods by means of the latest version of the CosmoMC package [49]. As for current constraints, we consider an extended $\Lambda$CDM model described by the following set of parameters:

$$\{\omega_b, \omega_c, \Theta_s, \tau, \ln (10^{10} A_s), n_s, r, dn_s/d\ln k, n_{\text{run,run}}\},$$

(3.1)

where $\omega_b \equiv \Omega_b h^2$ and $\omega_c \equiv \Omega_c h^2$ represent the physical baryon and cold dark matter energy densities, $\Theta_s$ is the angular scale of recombination, $\tau$ is the reionization optical depth, $A_s$ is the normalization of the primordial power spectrum, $n_s$ is the scalar spectral index, $dn_s/d\ln k$ and $n_{\text{run,run}}$ are the running and the running of the running of the spectral index. The priors for these parameters are shown in Tab. 1, both for the standard MCMC and the PolyChord analyses (see Sec. 4).

We shall also perform forecasted MCMC analyses to estimate the expected constraining power of future CMB data in the context of featured models, generating mock data for a cosmological model described by the parameters above detailed, including $dn_s/d\ln k$ and $n_{\text{run,run}}$. The best-fit values for these parameters are chosen to be those detailed in Ref. [6]. Then, we show the expectations from a Planck-like survey, to compare the results with those
obtained with real Planck data. This could give us an appraisal of how much the forecasted errors within an ideal scenario with perfect foreground subtraction change when the true, real measurements are performed. For future CMB data, we consider a COreE-like mission, following the specifications of Ref. [50].

Then, we repeat the same exercise above but assuming that nature has chosen a featured primordial power spectrum. We have assumed $\omega = 2.3501$ and $f_{\text{res}}^{\text{NL}} = 0.0084$ in Eq. (2.2) as benchmark values, but similar results are obtained with many other possible choices of the parameters, showing that there are many other possibilities to mimic the putatively observed slow-roll hierarchy. The procedure is as follows. We first generate mock data assuming a featured primordial power spectrum as the one given by the resonant model within the axion monodromy scenario, see Eq. (2.2). Then, we fit this (mock data) model to a standard power spectrum following the usual slow-roll expansion, to see whether a non-trivial primordial power spectrum with localized features could be mimicked by the observed values of the running, $dn_s/d\ln k$, and of the running-of-the-running, $n_{\text{run,run}}$, of the scalar perturbations. We present our main findings in the next section.

4 Present and future constraints

The present constraints are shown in Fig. 1, where we show the 68% and 95% CL in the two-dimensional $(n_s, dn_s/d\ln k)$, $(n_s, n_{\text{run,run}})$ and $(dn_s/d\ln k, n_{\text{run,run}})$ planes, as well as the one-dimensional posterior probability distribution for each of the three parameters. We illustrate the allowed contours for three different analyses. The black (blue) curves illustrate the results from an analysis of Planck forecasted (current) TT, TE and EE measurements. The red lines denote the results when the toy resonant model described in Sec. 2, describing axion monodromy inflation scenarios, is fitted to Planck forecasted TT, TE and EE measurements assuming (incorrectly) the slow-roll paradigm. It is very important to notice that, albeit these results have been obtained from a particular choice of the parameters governing Eq. (2.2) ($\omega = 2.3501$ and $f_{\text{res}}^{\text{NL}} = 0.0084$) to generate the mocks that afterwards are fitted to the slow-roll scheme, very similar results are obtained for a large range of the toy-model parameters. This fact shows that, observationally, it is currently very difficult to disentangle among featureless models and the plethora of featured models described by the toy-model explored here. Therefore, one can argue that the apparent slow-roll anarchy is due to the fact that the primordial power spectrum is described by an axion monodromy-like inflaton potential. This statement is further supported by the difference in the best-fit $\chi^2$ values obtained for these

| Parameter Prior PolyChord prior |
|------------------|------------------|------------------|
| $\omega_b \equiv \Omega_b h^2$ 0.005 $\rightarrow$ 0.1 0.02 $\rightarrow$ 0.024 |
| $\omega_c \equiv \Omega_c h^2$ 0.01 $\rightarrow$ 0.99 0.1 $\rightarrow$ 0.14 |
| $\Theta_s$ 0.5 $\rightarrow$ 10 1.035 $\rightarrow$ 1.045 |
| $\tau$ 0.01 $\rightarrow$ 0.8 0.01 $\rightarrow$ 0.2 |
| $\ln (10^{10} A_s)$ 2.7 $\rightarrow$ 4 2.8 $\rightarrow$ 3.4 |
| $n_s$ 0.9 $\rightarrow$ 1.1 0.9 $\rightarrow$ 1.02 |
| $dn_s/d\ln k$ $-0.5$ $\rightarrow$ 0.5 $-0.1$ $\rightarrow$ 0.1 |
| $n_{\text{run,run}}$ $-0.5$ $\rightarrow$ 0.5 $-0.1$ $\rightarrow$ 0.1 |

Table 1. Uniform priors for the cosmological parameters considered in the present analysis.
two possibilities. The modest value of $\Delta \chi^2 \simeq 2$ obtained when fitting the resonant toy model of Eq. (2.2) to the true underlying model, rather than to the slow-roll scenario described by the $dn_s/d\ln k$ and $n_{\text{run,run}}$, suggests that the interpretation of current data in terms of a primordial power spectrum with localized features is perfectly plausible and compatible with the most recent CMB temperature and polarization measurements. Additional constraints arising from bispectrum considerations do not change the main findings above described, as the non-gaussianity parameter $f_{\text{NL}}$ turns out to be negligibly small for the parameter space of interest here.

To further assess the fact that, with the present Planck data, an underlying model with localized features in the primordial power spectrum could be hidden in the form of a slow-roll anarchy in which the slow-roll parameters do not respect the expected hierarchical values, we have run the CosmoMC publicly available code with the PolyChord nested sampler [51]. This will provide us the Bayesian evidence needed to compute the Bayes factor, which will allow for a proper model comparison. Let us label by $M_0$ the model in which the (mock) Planck data is generated with a power spectrum described by the slow-roll expansion and fitted to this very same scenario. We instead refer to model $M_1$ when the (mock) Planck data is generated with a featured primordial power spectrum but it is fitted to a standard power-law model with $dn_s/d\ln k$ and $n_{\text{run,run}}$ different from zero. Then, the value we obtain for the Bayes factor $|\ln B_{01}| = 1.6$ indicates that there is only weak evidence favoring $M_0$ from Planck data (see e.g. Ref. [52]). This result further reinforces the findings quoted above.

Figure 2 shows the analogue of Fig. 1 but for mock CMB data generated accordingly to the future COrie mission specifications [50]. Notice that in this case the allowed contours in the $(n_s, dn_s/d\ln k)$, $(n_s, n_{\text{run,run}})$ and $(dn_s/d\ln k, n_{\text{run,run}})$ planes are clearly separated, if compared to the previous case, dealing with Planck data. Therefore, the parameter $n_{\text{run,run}}$ provides a unique and powerful tool to disentangle between the featureful and featureless models, as the contours using this parameter do not overlap at the $2\sigma$ level. This is the main result of this study, which should however be taken with some caveats. From Fig. 1, one can notice that the constraints from current Planck data are not as good as their forecasted values. However, one can not extrapolate this behaviour to the COrie case, as the impact from e.g. systematics and foreground removals could look completely different in this case. Furthermore, the experience gained with Planck data cleaning will also help in matching the forecasted and real-data results. For the COrie case, we we have also performed a proper model comparison analysis, as previously illustrated for the Planck case. The Bayes factor that we obtain in this case is $|\ln B_{01}| = 7.2$, indicating that, if nature has chosen a featureless power spectrum, there should be strong evidence favoring this model, truly assessing the underlying cosmological scenario.

5 Conclusions

Inflationary theories provide the most compelling solution to the standard cosmological problems (horizon, flatness and generation of primordial perturbations). In its canonical version, inflation is related to the existence of a scalar field, the inflaton, slowly rolling down its potential. This is known as the slow-roll paradigm, and leads to a hierarchy in the parameters governing the primordial power spectrum’s power-law. Namely, the running $dn_s/d\ln k$ of the scalar spectral index ($n_s$) and its running $n_{\text{run,run}}$ are expected to be second and third order in the slow-roll parameters, respectively. However, observationally, this hierarchy is not satisfied, with the current mean value of $n_{\text{run,run}}$ being larger than the corresponding one
Figure 1. 68% and 95% CL in the two-dimensional \((n_s, \text{d}n_s/\text{d}\ln k), (n_s, n_{\text{run,run}})\) and \((\text{d}n_s/\text{d}\ln k, n_{\text{run,run}})\) planes as well as the one-dimensional posterior probability distribution for the \(n_s, \text{d}n_s/\text{d}\ln k\) and \(n_{\text{run,run}}\) parameters. The black/blue curves illustrate the results resulting from an analysis to Planck forecasted/real TT, TE and EE measurements, assuming the standard slow-roll paradigm. The red curves denote the results when the toy resonant model is (wrongly) fitted to Planck forecasted TT, TE and EE measurements assuming slow-roll.

for \(dn_s/\text{d}\ln k\). Even if errors are still very large to draw any definite conclusion, one could look whether alternative inflationary models predict a different hierarchy, closer to present measurements [7]. In this regard, we have asked ourselves whether this observed anarchy could be due to the fact that the primordial power spectrum has some localized features, as in theoretical scenarios with non-canonical kinetic terms, a time-varying sound speed or within the so-called axion monodromy scenarios. We have focused here on this latter case, exploring a toy model which reasonably describes the axion-monodromy inflationary predictions. Indeed, we have shown that when fitting mock Planck data generated assuming a featured toy-model to a featureless power spectrum, the values of the running of the scalar and of its running can mimic the observed anarchy. To reinforce our conclusions, we have also carried out a proper model comparison analysis, and, assuming that nature has chosen
Figure 2. As Fig. 1 but for COrE mock data. Therefore, there are not equivalent curves to those shown in blue in Fig. 1.

a model with the current mean values of $dn_s/d\ln k$ and $n_{\text{run,run}}$. Planck mock data show weak evidence when this model is compared to a model in which a featured primordial power spectrum is fitted to the slow-roll hierarchy one. A model comparison analysis in the COrE case will provide strong evidence against the featured model, assuming that the underlying true cosmology is a model with the standard power-law power spectrum with values for the $dn_s/d\ln k$ and $n_{\text{run,run}}$ parameters equal to their current best-fit values. Future CMB measurements, as those expected to be carried out by the COrE satellite mission, have therefore the key to disentangle among these two possibilities.

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