Nonlinear Regge Trajectories in Theory and Practice

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Abstract. The problems related to nonlinear behavior of Regge trajectories (RT) and their renormalization group invariance are discussed.

Spectroscopy

Since the very first papers on Regge analyses of high-energy behavior of the hadron scattering amplitudes it was a habit to use an extrapolation of the strikingly linear behavior of Regge trajectories at positive invariant mass squared, \( t \), to the scattering region, where \( t < 0 \). Afterwards the famous Veneziano model gave an argument in favor of the linear and exchange degenerated trajectories, at least as a good first approximation, probably subject to minor nonlinear modifications. Fig. 1 illustrates the Chew-Frautschi plot for \( \rho-\omega \) meson family, which seemingly confirms such an idea. However, as was stressed in Ref. [1], the models implying both linearity and exchange degeneracy lead to an awfully bad \( \chi^2/d.o.f. \geq 100 \). Nonetheless, one can weaken the demand on RT giving up, say, exchange degeneracy. Does it mean that (in case if the timelike \((t>0)\) region is described well with approximately linear trajectories) we have to expect the same in the scattering \((t\leq0)\) region?

Theory

Prior to search for an apriory answer it is interesting to try to see what simplest field theoretic models, like \( \phi^3 \)- or \( \phi^4 \)-theories yield for the RT [2]. Fig. 2 exhibits a typical picture of the RT \( t \)-dependence which can be only approximately linear in some interval of positive \( t \), while at negative \( t \) (and large enough positive \( t \)) flattens and tends to some constant, depending on the model.

Figure 1: The data on hadron spectroscopy.

Figure 2: The typical dependence of RT in simple field models.

Certainly, \( \phi^3 \) and \( \phi^4 \) models are not very realistic, and afterwards some phenomenological nonlinear models for RT were suggested. We mention only three
options (of many). In Ref. [3] the trajectory of the form
\[ \alpha(t) = 1 + \gamma(\sqrt{t_0} - \sqrt{t_0 - t}) \] (1)
was considered and applied to quite a successful description of the scattering data. One of the distinctive features of this model is negative and infinite value of \( \alpha(t) \) at infinitely large and negative \( t \), which is difficult to interpret physically. This trajectory is devoid of particles (this is not prohibited, though).

Equation
\[ \alpha(t) = c \ln(b - at), \quad c \sim g^2 \] (2)
represents the attempt to account for some QCD-like features, such as vector gluon exchange [4]. The resulting trajectory is negative at large \( \pm t \) and again devoid of particles on it.

Finally, the equation
\[ \alpha(t) = a + bt + b\sqrt{(t_0 - t)(t_0 - t_0^*)} \] (3)
from Ref. [5] gives us something different: the RT grows linearly at large \( (+t) \) and tends to a constant \(-1\) at large \((-t)\). Linear behavior is normally related to the stringlike behavior of hadrons at higher spins. We remark that \( \alpha(t) \) from Eq. [5] has two complex conjugated branch points, \( t_0 \) and \( t_0^* \).

\section*{QCD}

If we turn to QCD – supposedly the ultimate theory of strong interactions – then we find that in the region of applicability of perturbative methods, i.e. large \((-t)\), the pomeron trajectory tends to \(+1\) at large \((-t)\) [3], while the secondary (meson) trajectory tends to \(0\) in the same direction [7]:

Such an asymptotic corresponds to exchanges of non-interacting gluons and quarks. This seems to be quite natural because at small distances (large \(-t)\) we expect the effects of asymptotic freedom. However, the approach to these asymptotic values appears to be quite nonperturbative (terms of order \( g^{10/3} \) for the pomeron trajectory and \( g^{5/3} \) for the meson one). Both the results were obtained via a Bethe-Salpeter equation with the kernel corresponding to the lowest order in QCD coupling.

What do we get at small \( t \)? Leading singularity \( \alpha_{2g}(0) \) was calculated in Ref. [8] in two first orders in the QCD coupling \( g^2 \),
\[ \alpha_P(0) = 1 + \frac{3 \ln2}{\pi^2} g^2 \left( 1 - \frac{5}{\pi^2} g^2 \right), \] (6)
and does not contain, in contrast to the case of large \((-t)\), nonanalytic terms in \( g \). However, Eq. (6) exhibits a function which depends both on implicit scale dependence of the QCD coupling and the renormalization scheme.

\section*{Renormalization group invariance}

The last circumstance seems quite worrying because the Regge trajectories are generally related to physical amplitudes and spins and masses of observed particles. Hence they cannot depend on any arbitrary choice of renormalization scale and scheme. From the RG invariance it follows that in case of massless QCD (which fairly admits the existence of massive hadrons due to “dimensional transmutation”) RT have a general form
\[ \alpha(t) = \Phi \left( \frac{t}{\mu^2} e^{K(g^2)} \right) \] (7)
with the renormalization scale \( \mu \) and QCD coupling \( g \). \( \Phi \) is some function that has to be defined by the dynamics and \( dK/dg^2 = \beta(g^2) \) is the usual QCD \( \beta \)-function: \( \beta(g^2) = -\beta_0 g^4 - \beta_1 g^6 - \ldots \).

One can immediately derive from Eq. (7) the following important consequences.

1. \( \alpha(0) = \Phi(0) \), i.e. the intercept does not depend on \( g^2 \) at all.
2. If \( \alpha(t) \) is analytic at \( t = 0 \) then \( \alpha(t) = \alpha(0) + \alpha'(0)t + \ldots \) where
\[ \alpha'(0) \big|_{g^2 \to 0} \sim \left( \frac{1}{g^2} \right)^{\gamma_{p}^2} e^{-\gamma_{p}^2}. \] (8)
Eq. (8) shows that the slope of the RT is highly non-perturbative. Let us note that the independence of the intercept of $g^2$ is self-consistent in the sense that if one takes $\mu^2 = -t$, then $\alpha(t)$ becomes the function of the running coupling at $(-t)$:

$$\alpha(t) = \Phi\left(-e^{K(g^2(-t))}\right).$$

From the definition of $K$ it follows that

$$K(g^2(t)) - K(g^2) = \ln\left(\frac{-t}{\mu^2}\right) \to -\infty \quad (t \to 0),$$

so we obtain again $\Phi(0)$. In the case of finite $g^2(0)$ [9] it has to be an IR fixed point: $\beta[g^2(0)] = 0$.

So one can conclude that, first, QCD dictates essentially nonlinear RT, and nonlinearity, as will be seen later, is not negligible at quite low $t$; second, the very RT are not analytic in coupling constants which is a direct consequence of the RG invariance.

**Practice**

Now we turn to practical implementations of these considerations [10]. Fig. 3 shows the $t$-dependence of the hard and soft pomeron trajectories, and that of the $C$-even secondary $f_2$-trajectory which takes into account QCD results. The dashed lines show the linear extrapolations from the positive $t$ region.

Nonlinear trajectories with QCD-motivated behavior were used also for the description of the $pp$ and $\bar{p}p$ scattering. Fig. 4 shows quite a good fit. The dashed lines show what happens if one takes a linear approximation for the RT.

When one deals with hard diffractive processes the soft pomeron appears to be unsufficient. Thereof the need in “hard” pomeron. Fig. 5 demonstrates the use of combined nonlinear hard and soft pomerons with quite a successful quality.

**Problems**

Certainly, there remain a lot of problems. For instance, the $\rho$-meson trajectory which according to QCD has to tend to zero at large-$t$ shows quite indefinite picture (Fig. 6). The problem was addressed in Refs. [11, 12]. More exacerbated situation occurs with $\pi$-meson trajectory which is definitely negative already at $t = 0$.

If to take the QCD-motivated and physically appealing RT behavior depicted at Fig. 7 (parton exchanges at large $(-t)$ and “stringy” behavior at large $t$), then we encounter a possible problem with analyticity, as such trajectories have complex $t$-plane singularities which can be dangerous for the whole amplitude, regular at complex $t$ (on the first sheet). In its turn this can be related to possible microcausality violation [13], one of the pillars of modern physics.
Summary

1. Analyticity of RT in $t$ seems to imply their singular behavior at $g^2 \sim 0$;
2. QCD (partonlike) asymptotics at high ($-t$) and "stringy" asymptotics at high $t$ imply complex singularities of RT in the $t$-plane. Probable clash with microcausality.
3. QCD behavior at high ($-t$) does not match with monotony of RT in $t$ for $\rho$, $\pi$, ..., heavy quarkonia.

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References

[1] P. Desgrolard, M. Giffon, E. Martynov, and E. Predazzi, Eur.Phys.J. C 18 (2001) 555
[2] B.W. Lee and R.F. Sawyer, Phys.Rev. 127 (1962) 2266
[3] R. Fiore, L.L. Jenkovszky, F. Paccanoni, and A. Prokudin, Phys. Rev. D 68 (2003) 014005
[4] D. D. Coon and H. Suura, Phys.Rev. D 10 (1974) 348
[5] P.D.B. Collins and P.J. Kearney, Z.Phys. C 22 (1984) 277
[6] R. Kirschner and L.N. Lipatov, Z.Phys. C 45 (1990) 477
[7] J. Kwiecinski, Phys.Rev. D 26 (1982) 3293
[8] V.S. Fadin and L.N. Lipatov, Phys.Lett. B 429 (1998) 127; M. Ciafaloni and G. Camici, Phys.Lett. B 430 (1998) 349
[9] D.V. Shirkov and I.L. Solovtsov, Phys.Rev.Lett. 79 (1997) 1209
[10] A.A. Godizov and V.A. Petrov, JHEP 0707 (2007) 083; A.A. Godizov and V.A. Petrov, Phys.Rev. D 78 (2008) 034028
[11] S.J. Brodsky, W.-K. Tang, and C.B. Thorn, Phys.Lett. B 318 (1993) 203; A.B. Kaidalov, arXiv: hep-ph/0612358
[12] A.A. Godizov, Phys.Atom.Nucl. 71 (2008) 1792
[13] V.A. Petrov and A.P. Samokhin, Phys.Lett. B 237 (1990) 500