Minimum record length for detecting a prospective uniform sea level acceleration at a tide gauge station

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ABSTRACT
The unambiguous detection of a uniform sea level acceleration at a tide gauge station is important under an increasingly warmer Earth for the long-term coastal risk assessments. Although the timescales needed for detecting mean sea level trends at tide gauges were well investigated in the past, very few studies addressed the same issue for the sea level accelerations in depth. In this study, we demonstrate that the ability to discern a uniform sea level acceleration at a tide gauge station depends on its magnitude, systematic and random sea level variations, the degree of autocorrelation of the random variations, the length of its historical record and the desired statistical significance level of its estimate. We offer a formulary to estimate the required minimum record lengths needed to detect a statistically significant prospective uniform acceleration at a coastal or Island tide gauge station based on the retrospective analyses of its sea level record. To demonstrate its effectiveness, the minimum record lengths required to detect statistically significant prospective uniform accelerations were calculated at 19 globally distributed tide gauge stations with long records.

1. Introduction
Accelerated rate of global sea level rising threatens coastal communities. Although the timescales needed for detecting mean sea level trends at long-term tide gauges were well investigated in the past, very few studies addressed the same issue for the sea level accelerations in depth. An earlier study by Haigh et al. (2014) stated ‘We find that the most important approach to earliest possible detection of a significant sea level acceleration lies in improved understanding (and subsequent removal) of interannual to multidecadal variability in sea level records’. Their study emphasised determining timings for various scenarios at which accelerations might first be recognised in the future. Another study by Jordà (2014) concluded that ‘The main difference is that the length of the records has to be about 40–60 years longer to detect an acceleration than to detect a linear trend leading to an equivalent change after 100 years’.

The conclusions from both investigations were broad, vague and incomplete. Studies by Iz (2014, 2015) have already demonstrated that interannual to multidecadal variability in sea levels can be modelled at tide gauges, TG, with with long records using their proxies. These proxies are natural sea level fluctuations that compound with periodicities of luni-solar forcing and generate sub and super harmonics at various frequencies (see, Keeling & Whorf, 1997; Munk et al., 2002; Yndestad, 2006 for the underlying mechanisms). This top-down modelling approach detects and separates low-frequency sea level variations effectively in TG records and eliminates their confounding effects in detecting a uniform sea level acceleration if present. If the TG station has yet to experience a uniform acceleration, pertinent statistics of its historical records can be used to establish confidence intervals to detect a prospective sea level acceleration. In this study, we offer a formulary to estimate the minimum record length required to detect a prospective uniform acceleration starting at a tide gauge station with long records using the retrospective analyses of its sea level record.

In the following sections, we first present the findings of a previous study by Iz (2017) in which the low-frequency sea level fluctuations are modelled and estimated together with the trends and uniform accelerations experienced at 27 globally distributed TG stations with long records. A formulary is then derived that make use of the statistics generated from their historical records to calculate Minimum Record Lengths, MRL, to detect a uniform1 acceleration at a TG station if there is any. Subsequently, MRL were calculated at nineteen globally distributed tide gauge stations that did not experience any statistically significant uniform accelerations for demonstration.
2. A kinematic model for observed sea level anomalies at a TG station

Studies by Iş (2014), (Iş, 2015), (Iş, 2017) and Iş et al. (2018) demonstrated that low-frequency sea level variations can be effectively represented by the sub and super harmonics generated by compounding of luni-solar forcings with natural variations in sea level acting as proxies. They are incorporated into the following model with their a priori known periodicities to represent the kinematics of the sea level variations,

\[
\begin{align*}
    h_t &= h_0 + v_h (t - t_0) + \frac{a}{2} (t - t_0)^2 \\
    &+ \sum_{h=1}^{17} a_h \cos \left( \frac{2\pi}{P_h}(t - t_0) \right) + \gamma_h \sin \left( \frac{2\pi}{P_h}(t - t_0) \right) \\
    &+ \varepsilon_t
\end{align*}
\]

In this model, the initial epoch of the measurements is shifted to the middle of the series \( t_0 \) to improve the numerical stability of the solutions and control the accumulation of errors. Monthly averaged tide gauge data are represented by \( h_t \), at epochs \( t = 1 \ldots n \) where \( n \) is the number of monthly observations. The unknown sea level reference height is defined at \( t_0 \). The initial velocity at \( t_0 \) is denoted by \( v_h \), and \( a \) is the acceleration/deceleration in sea level. The initial velocity is a relative trend if the measurements are subject to vertical crustal movements at TG stations, which do not affect the estimation of the sea level acceleration. The periodicities \( \{ P_h, h = 1, \ldots, m \} \) consist of 11 sub and super harmonics of node tides and those attributed to the solar radiation are shown in Table 1. Each period introduces two parameters, \( a_h, \gamma_h \), for the sine and cosine components from which the amplitudes \( a_h \) and the phase angles of the periodic terms are determined. In total, the kinematic model includes 37 unknown parameters.

The variable \( \varepsilon_t \) represents the lump-sum effect of random fluctuations with an underlying first order autoregressive AR(1) process together with independent random variations, \( u_t \), in sea level. The AR(1) effects can be quantified from the residuals of an Ordinary Least Squares, OLS, solution to Eqn. (1).

The first-order autoregressive disturbances \( \varepsilon_t \) in monthly averaged TG observations are represented as,

\[
\varepsilon_t = \rho \varepsilon_{t-1} + u_t
\]

In this expression, \(-1 < \rho < 1\) is the unknown autocorrelation coefficient of the first-order AR(1) process.\(^2\) The stochastic processes for the random noise \( u_t \) and autocorrelated \( \varepsilon_t \), have the following assumed distributional properties,

\[ E(u_t) = 0, E(u_t^2) = \sigma_u^2, E(u_t u_{t-1}) = 0 \]

This expression together with Eqn.(2) gives,

\[ E(\varepsilon_t) = 0, E(\varepsilon_t^2) = \sigma_\varepsilon^2 = \frac{\sigma^2}{1 - \rho^2} \]

If the observation equation represented by Eqn.(1) at an epoch \( t-1 \) is multiplied by \( \rho \) and subtracted from the following observation equation at \( t \), the effect of AR(1) is removed,

\[ h_t - \rho h_{t-1} = [h_0(t) - h_0(t_0)] + \left[ v_h(t) - v_h(t_0) \right] + \frac{a}{2} (t - t_0)^2 - \rho \left[ v_h(t-1) - v_h(t_0) \right] + \frac{a}{2} (t-1 - t_0)^2 \\
+ \sum_{h=1}^{17} \left[ a_h \cos \left( \frac{2\pi}{P_h}(t - t_0) \right) - \rho a_h \cos \left( \frac{2\pi}{P_h}(t-1 - t_0) \right) \right] + \gamma_h \sin \left( \frac{2\pi}{P_h}(t - t_0) \right) - \rho \gamma_h \sin \left( \frac{2\pi}{P_h}(t-1 - t_0) \right) \\
+ \left[ \gamma_h \sin \left( \frac{2\pi}{P_h}(t - t_0) \right) - \rho \gamma_h \sin \left( \frac{2\pi}{P_h}(t-1 - t_0) \right) \right] + u_t
\]

The random errors in this representation are identically and independently, i.i.d., distributed with zero expected value, i.e. \( u_t \sim \mathcal{N}(0, \sigma^2_u) \). Hence, the observation equations based on Eqn.(5) can be solved iteratively using OLS method to determine the optimal AR(1) correlation coefficient (Hildreth & Lu, 1960).

3. Retrospective solutions

The model introduced in the previous section enables joint estimation of a linear trend and acceleration together with luni-solar sub and super harmonics acting as proxies to represent natural sea level variations at a TG station. In a previous study by Iş (2017), the records of 27 globally distributed TG stations shown in Figure 1 were analysed.

Pertinent solution statistics are tabulated in Table 2. All listed velocity and acceleration parameters are statistically significant (except those stations with empty cells for accelerations). The Durbin-Watson statistics, DW, for the residuals show that all the solutions are devoid of any unmodeled fluctuations at these stations. AR(1) correlation coefficients are in the range of 0.1–0.7; hence, not all of them are negligible.\(^3\) Listed in Table 2 are also the Root Mean Square Error, RMSE, of the solutions. They vary within the range 33–127 mm. Of 27 TG stations, only eight stations have experienced statistically significant accelerations not exceeding 0.02 mm/yr\(^2\). Note that the acceleration estimates at these TG stations are markedly different than those estimated from globally averaged Satellite Altimetry, SA, measurements, which span only 24 years. Consequently, the effect of low-frequency sea level changes cannot be modelled and removed from the globally averaged records (Iş, 2017; Iş et al., 2018).

| Table 1. Luni-solar periodicities (yr) of luni-solar origin. |
|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| Nodal                   | Nodal                   | Nodal                   | Solar                   | Annual                   | Chandler |
| 74.5                    | 18.6                    | 3.7                     | 11.1                    | 1.00                     | 429.5/365.4 |
| 55.8                    | 9.3                     | 3.1                     | 22.2                    | 0.50                     |          |
| 37.2                    | 6.2                     | 2.6                     | 22.2                    | 0.25                     |          |
| 4.7                     | 2.3                     |                          |                          |                          |          |

\(^2\) For AR(1) processes, the random noise \( u_t \) and autocorrelated \( \varepsilon_t \) have the following distributional properties.

\(^3\) Listed in Table 2 are also the Root Mean Square Error, RMSE, of the solutions.
4. A formula to quantify the uncertainty of a prospective sea level acceleration at a TG station

The following equation can be evaluated prospectively to assess the contribution of a uniform acceleration at a TG station,

\[ h_t^a = \frac{1}{2} a(t-t_0)^2 + \varepsilon_t \] (6)

It represents a prospective acceleration in sea level anomalies at a given TG station after the removal of the datum constant, trend and periodic variations, estimated retrospectively using historical records (Table 2).

The effect of AR(1) is removed if Eqn. (6) at an epoch \( t-1 \) is multiplied by \( \rho \) and subtracted from the subsequent observation equation at \( t \),

\[ h_t^a - \rho h_{t-1}^a = \frac{(t-t_0)^2 - \rho(t-1-t_0)^2}{2} a + \varepsilon_t \] (7)

Table 2. Blank cells refer the acceleration estimates that are not statistically significant at \( \alpha = 0.05 \) significance level, \( \hat{\sigma}_v \) is RMSE of the solutions, DW refers to Durbin-Watson statistic, \( \rho \) is the AR(1) correlation coefficient (Iz, 2017).

| TG Station         | Length yr | \( v_r \) mm/yr | \( \sigma_{v_r} \) mm/yr | \( a \) mm/yr\(^2\) | \( \sigma_a \) mm/yr\(^2\) | \( \hat{\sigma}_v \) | \( \rho \) | DW  |
|-------------------|-----------|----------------|-------------------------|-----------------|-------------------------|----------------|------|-----|
| Annapolis USA     | 87        | 3.50           | 0.10                    | 0.01            | 0.005                   | 98.9           | 0.4  | 2.1 |
| Atlantic City USA | 104       | 3.53           | 0.15                    | 0.01            | 0.005                   | 56.7           | 0.2  | 1.9 |
| Baltimore USA     | 113       | 3.07           | 0.06                    |                |                         | 53.0           | 0.2  | 2.0 |
| Boston USA        | 89        | 2.55           | 0.07                    |                |                         | 43.5           | 0.3  | 2.1 |
| Fermandina USA    | 118       | 2.06           | 0.10                    |                |                         | 76.1           | 0.4  | 2.0 |
| Key West USA      | 102       | 2.27           | 0.07                    |                |                         | 41.0           | 0.4  | 2.0 |
| New York USA      | 159       | 2.85           | 0.04                    | 0.01           | 0.002                   | 54.9           | 0.3  | 2.1 |
| Pensacola USA     | 92        | 2.48           | 0.10                    |                |                         | 40.1           | 0.4  | 2.0 |
| San Francisco USA | 161       | 1.51           | 0.06                    | 0.02           | 0.003                   | 44.7           | 0.6  | 2.1 |
| Honolulu USA      | 110       | 1.37           | 0.10                    |                |                         | 33.0           | 0.7  | 2.0 |
| Cuxhaven DE       | 167       | 2.13           | 0.07                    |                |                         | 142.1          | 0.1  | 2.0 |
| Den Helder DE     | 146       | 1.47           | 0.06                    |                |                         | 92.1           | 0.1  | 2.0 |
| Travemunde DE     | 158       | 1.61           | 0.05                    |                |                         | 77.3           | 0.1  | 2.0 |
| Brest FR          | 205       | 0.97           | 0.04                    | 0.01           | 0.001                   | 72.8           | 0.1  | 2.1 |
| Amstelumden NL    | 144       | 1.87           | 0.07                    | 0.02           | 0.003                   | 88.7           | 0.1  | 1.9 |
| Delfshul NL       | 150       | 1.76           | 0.07                    | 0.01           | 0.004                   | 114.3          | 0.1  | 2.0 |
| Harlingen NL      | 150       | 1.35           | 0.07                    | 0.01           | 0.004                   | 114.6          | 0.1  | 2.0 |
| Terselling NL     | 94        | 1.26           | 0.13                    |                |                         | 99.7           | 0.1  | 2.0 |
| Swinoujscie PL    | 188       | 0.71           | 0.06                    | 0.01           | 0.003                   | 99.7           | 0.3  | 1.9 |
| Landsort SE       | 128       | –2.93          | 0.16                    |                |                         | 124.4          | 0.4  | 1.9 |
| Stockholm SE      | 126       | –3.78          | 0.15                    |                |                         | 127.4          | 0.4  | 1.9 |
| Liverpool UK      | 125       | 0.98           | 0.09                    |                |                         | 81.9           | 0.2  | 2.1 |
| N. Shields UK     | 120       | 1.84           | 0.06                    |                |                         | 55.0           | 0.3  | 2.1 |
| Sydney AU         | 101       | 1.00           | 0.06                    |                |                         | 45.8           | 0.2  | 1.9 |
| Ketchikan CA      | 101       | –0.29          | 0.11                    |                |                         | 72.8           | 0.3  | 2.1 |
| Prince Rupert CA  | 106       | 1.27           | 0.11                    |                |                         | 71.4           | 0.3  | 2.1 |
| Mumbai IN         | 133       | 0.76           | 0.04                    | 0.01           | 0.003                   | 49.0           | 0.2  | 2.0 |
| SA Global         | 24        | 3.13           | 0.12                    | 0.12           | 0.03                    | 1.06           | 0.93 | 2.0 |
This expression can be evaluated for \( t = 1 \cdots n \), where \( n \) is the number of prospective monthly observations at a TG station. If the left-hand side of the observation equations are known, then an OLS solution can be carried out to estimate a prospective sea level acceleration and its uncertainty. For this study, we are only interested in the uncertainty of the acceleration, which is not correlated with the trend estimate. After a simple derivation, it can be shown that the standard error of the acceleration estimate is given by,

\[
\hat{\sigma}_a = 12^2 \hat{\sigma}_u \left( \sum_{t=1}^{n} \left[ \frac{1}{2} \left( t - \frac{n}{2} \right)^2 - \rho \frac{1}{2} \left( t - 1 - \frac{n}{2} \right)^2 \right] \right)^{-1}
\]

(8)

Historical records also provide information about the RMSE, \( \hat{\sigma}_u \), of prospective random variations, \( u_t \). Note that \( 12^2 \) converts monthly rate based on monthly averaged TG records into yearly rate since \( t \) and \( n \) are expressed in monthly units.
Figure 2. (Continued).
For a given $a$, one can obtain the critical value $z_a$ from the density function, assumed to be Normal distribution, and compare it against $z$. If $|z| > z_a$, then the null hypothesis is rejected at $a$ significance level; hence, null hypothesis is rejected. Assessing Eqn.(11) for uniform accelerations for a range of expected magnitudes and different record lengths enables determining the earliest detection record length, MRL.

The required MRL for the TG stations listed in Table 2 are determined in the following section.

6. Required minimum record lengths for TG stations with long records

MRLs were calculated using Eqn. (11) for the TG stations listed in Table 2 that did not experience any accelerations until recently. Their long records permit the removal of low-frequency sea level changes as described in Lz (2014) and provided the necessary statistics to evaluate the formulary. The results are shown in Figure 2 quantify the MRLs using bar charts for prospective uniform accelerations within the interval (0.0–0.15 mm/yr$^2$) with 0.01 mm/yr$^2$ increments.

For instance, Fernandina, USA, TG station exhibits an AR(1) effect of $\rho = 0.4$, $\sigma_a = 76.1$ mm (Table 2). The analyses of its 118-year-long records did not reveal any statistically uniform acceleration until 2017. If the sea level starts accelerating at this station since then, the bar chart in Figure 2 displays the required MRLs for 15 prospective accelerations at $a = 0.05$ significance level.

5. Null-hypothesis testing of a statistically significant prospective uniform sea level acceleration

The null-hypothesis testing for a statistically significant prospective uniform acceleration, $a$, is stated as,

$$H_0 : a = 0 vs. H_1 : a \neq 0$$  \hspace{1cm} (9)

For which a confidence interval for its estimated value $\hat{a}$ is given by,

$$P(\hat{a} - z_c\hat{\sigma}_a < \hat{a} < \hat{a} + z_c\hat{\sigma}_a) = 1 - a$$  \hspace{1cm} (10)

where,

$$z := \frac{\hat{a}}{\hat{\sigma}_a}$$  \hspace{1cm} (11)

Quantifying the above expression requires the following information at a TG station:

- Prospective random sea level variations $\sigma_u$ experienced at the TG station. We estimated historical records at TG stations listed in Table 2 using their historical records as a proxy for the future random variabilities.
- The number of months $n$ ($t = 1 \cdots n$) needed to assess the uncertainty of a prospective acceleration.
- Correlation coefficient, $\rho$, of AR(1) of the error disturbances. Once again, Table 2 provides their estimated values inferred from the historical records of the listed TG stations.

Figure 2. MRLs based on $a = 5\%$ significance level at TG stations listed in Table 2 that have yet to experience a uniform acceleration.
Overall, the TG stations that are in the vicinity of each other require similar MRLs for detecting a prospective uniform sea level acceleration as expected. Other than this commonality, the required MRL is location and station dependent, and it does not lend itself to a single record length to be applied for all the globally distributed TG stations.

A hindsight assessment was made also for globally averaged monthly SA observations using the statistics (Table 2) reported by İz et al. (2020). The uniform acceleration at this station is already statistically significant during 1993–2018. The bar chart for the MRLs included in Figure 2 for the SA records shows that the estimated MRLs are markedly shorter than those from TG records because of the extreme precision of the records (İz et al., 2020). The time span of the MRL confirms that a statistically significant uniform acceleration can be estimated during this period. Yet, this outcome is misleading since the confounding periodicities detected at TG stations were not accounted for in globally averaged SA data. Therefore, the estimated uniform acceleration may as well be due to the globally present low-frequency sea level variations caused by lunar-solar periodicities as demonstrated through a meta-analysis of the TG stations (İz, 2014) and elucidated in İz et al. (2020).

7. Conclusion

In this study, we demonstrated that the ability to discern a uniform sea level acceleration at a tide gauge station is contingent on its magnitude, the level of random sea level disturbances the degree of AR(1) effect, the length of the historical tide gauge record for modelling and removal of systematic sea level periodicities and the desired statistical significance level. Nonetheless, it is quantifiable. We offered a formulary and a null-hypothesis testing procedure to estimate MRLs needed to detect a prospective uniform acceleration at any tide gauge station based on the retrospective analyses of its historical records. We also estimated the MRLs required to detect a uniform sea level acceleration at TG stations listed in Table 2 with long records to demonstrate its effectiveness. The results that are shown in Figure 2 quantifies the MRL for within the interval (0.0–0.15 mm/yr²) with 0.01 mm/yr² intervals. Overall, the TG stations that are in the vicinity of each other require similar MRL for detecting a prospective uniform sea level acceleration as expected. Other than this commonality, the required MRLs are location and station dependent, and there is no single timescale to report a universal MRL to use for all the globally distributed TG stations. The formulary and the necessary protocol for determining the required MRL reported in this study are exhaustive, robust and unambiguous to make an accurate assessment to detect a statistically significant prospective uniform acceleration at a TG station.

Notes
1. Conventionally, sea level variations at TG stations are modelled using quadratics in which the acceleration/ deceleration is uniform throughout the observed series and its magnitude is twice the coefficient of the parabolic term. In this study, the term acceleration refers to a ‘uniform acceleration.’
2. Previous studies have revealed that the autocorrelations experienced at TG stations are always positive (İz, 2014).
3. Expected DW value for the random residuals is 2.0.

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