New Adaptive Beamforming for Coherent Interference based on Covariance Matrix Reconstruction

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Abstract. Conventional beamforming fail to derive the full-rank covariance matrix due to the existent of coherent interference in the spatial domain. Most of the existing methods suppress the correlation interference by sacrificing the degree of freedom (DOF). In this paper, we develop a new adaptive beamforming for coherent interference without the loss of DOF, and the process of this new algorithm is as follows. First, the steering vector of the interference signal is estimated through convex optimization. Then we estimate the interference signal power by decomposing the covariance matrix and reconstruct the interference plus noise covariance matrix (INCM). Finally, the array weights are obtained based on the maximum SNR criterion. The proposed method has no loss of DOF and the array gain is adjacent to the ideal value. The feasibility of the technique is proved by theoretical derivation, and the effectiveness of the method is also verified by numerical results.

1. Introduction

Adaptive beamforming is the main research content of array signal processing field, beamforming is also called spatial filtering, referring to the certain weighted process using spatial characteristics of the received signal. Adaptive beamforming synthesizes input information under certain criteria and widely applies the adaptive algorithm in beamforming to determine the optimal weighted value adaptively, so as to enhance the desired signal and suppress the interference signal and noise [1].

Due to the multipath effect of space propagation, Interference signals which are coherent with the desired signals commonly exist in the received signals of array antenna, and the received signal covariance matrix of array with coherent signals will appear rank deficit. Due to the multipath effect of spatial propagation, some interference signals that are coherent with the desired signal exist in the received signal of the array antenna, thus causing the rank loss of the received signal covariance matrix.

In this case, the conventional beamforming cannot form nulls in the direction of the interference signals, and may even improperly suppress the desired signals, i.e., the signal cancellation phenomenon [2] will occur, resulting in a poor performance of the system.

At present, the beamforming algorithm for coherent signal is principally divided into two kinds according to the actual engineering needs, multipath synthesis and coherent suppression ways. Multipath synthesis [3] refers to the effective combination of the desired signal and the coherent interference signal, and maximizes the SNR of the array output. Literature [4] is a typical coherent receiving method, taking the projection vector of the synthesis direction vector of the desired signal and the coherent signal in the incoherent subspace as the actual array weight, which is only accessible in theory. In practical engineering, coherent reception method is rarely used, because the coherent signal is assumed to be only different in phase and amplitude. However, the coherent signal may also have some difference in frequency in practice.
Beamforming methods for coherent signal suppression are mainly divided into two categories: decoherence and preprocessing. Decoherence algorithm solves the rank loss of covariance matrix by processing the data. Classical algorithms include spatial-smoothing (SS), matrix reconstruction, etc. The spatial smoothing algorithm [5] divides the uniform linear array (ULA) into several overlapping sub-arrays, and replaces the sampling covariance matrix with the mean value of each sub-matrix, effectively solving the rank loss problem and reducing the coherence between the desired signal and the interference.

The spatial smoothing algorithm is simple to operate and applicable in a wide range. On the contrary, it sacrifices partial array aperture and gain, and is only available for equally-spaced arrays. Literature [6][7][8] improve the spatial smoothing algorithm and enhance the robustness. Common matrix reconstruction algorithm includes vector singular value method [9], Toeplitz matrix reconstruction, and etc. Literature [10] averaged the diagonal elements and constructed a Hermitian Toeplitz matrix as the estimation of the sampling covariance matrix. In literature [11], the correlation functions of the data received by each array element and the reference array are sequentially arranged to establish a new Hermitian Toeplitz matrix as the estimation of the sampling covariance matrix. These algorithms enhance the degrees of freedom (DOF) and accomplish decoherence through the reconstruction of covariance data. However, [11] method will cause the chaos of signal and noise subspace, and shift the position of main peak and interference nulls, diminishing the beamforming gain. Literature [12], leverage eigenvalue decomposition on the covariance matrix of received signals to distinguish the eigenvectors corresponding to the desired signals, and the INCM is reconstructed by using the remaining eigenvectors and eigenvalues. This adaptive beamforming method has high beamforming gain and strong robustness to array position errors and coherent interference.

The pre-processing algorithm requires priori information about the arrival direction of desired signal and interference. The signal is pre-processed and the weighted parameters of the array element are solved. Classic algorithms include Complementally Transformed Minimum Variance algorithm (CTMV) [12], and Duvall algorithm [13]-[15]. CTMV simultaneously constrain the incoherent and coherent interference. The transformation matrix is used through non-strong constraint to pre-process the received data, expecting to remove the desired signal, reserve interference and avoid signal cancellation. Duvall algorithm eliminates the desired signal through spatial domain transforming to obtain the INCM. This method performs well in decoherence, but it needs to estimate the arrival angle of the desired signal or interference in advance. Besides, it is sensitive angle estimation error, and the calculation amount is increased significantly. Based on the Duvall algorithm, a robust coherent adaptive beamforming algorithm based on multilevel blocking is suggested in literature [17], enhancing the robustness of the algorithm and improving the beamforming gain simultaneously.

INCM is a key step to improve beamforming performance. The INCM can be reconstructed in two ways: integral reconstruction and sparse reconstruction. Integral reconstruction [18]-[22] requires only the prior information about the desired signal and involves low-complexity calculations; however, the disadvantage is that the performance is reduced. In sparse reconstruction [23]-[26], the INCM is reconstructed by estimating the DOAs and powers of the signal. This method has high computational complexity and its performance is limited by the accuracy of the DOA estimation.

This paper proposes a new adaptive beamforming algorithm aiming at related interference. First, we estimate signal steering vector with optimized algorithm and obtain the coherent matrix of the signal from the covariance matrix by using the steering vector matrix. Then we extract the diagonal elements to estimate the signal power, and further reconstruct the INCM. Finally, the weights of each array element are obtained with MVDR criterion. The proposed method has the advantages of DOF conservation, high utilization rate of array elements and strong robustness to coherent interference.

In this paper, we use uppercase and lowercase boldfaces to denote matrices and vectors, respectively; $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^T$ denote conjugate, transpose, and conjugate transpose respectively; $E(\cdot)$ denotes statistical expectation; $\text{diag}(\cdot)$ denotes diagonalization and $I$ denote the identity matrix.
2. Basic Knowledge

2.1. Signal model

Assuming that the signal sources of the incident array are unrelated to each other and the noise is Gaussian white noise, the received signal of the m-element array is in the following form:

\[ x(t) = \sum_{i=0}^{D-1} a_i s_i(t) + n(t) \]

The incoherent signal array receiving signal model can be obtained by transforming the equation above into matrix form:

\[ x(t) = A s(t) + n(t) \]

Where \( A = [a_0, a_1, \cdots, a_{D-1}] \) denote the signal steering vector. \( a_i = [1, e^{-j2\pi f_i t}, \cdots, e^{-j2\pi f_i t_{D-1}}]^T \), \( i = 0, 1, 2, \cdots, D \), \( n(t) \) is white gaussian noise. \( s(t) = [s_0(t), s_1(t), \cdots, s(t_{D-1})], i = 0, 1, \cdots, D \) is the desired signal waveform. Among the rest, \( s_i(t) \) stands for the desired signal waveform and \( a_0 \) is the corresponding steering vector. Assuming that \( P \) of \( D \) incident signal sources are correlated, for convenience of expression, we tag them as \( s_i(t), i = 0, 1, 2, \cdots, P-1 \), and acquire the equation (3).

\[ s_i(t) = \beta s_0(t), i = 0, 1, 2, \cdots, P-1 \]

The signal model can be defined as:

\[ x(t) = \sum_{i=0}^{P-1} \beta_a s_i(t) + \sum_{i=P}^{D-1} a_i s_i(t) + n(t) \]

By converting the equation (4) into matrix form, the received signal model of the array can be obtained when the relevant signals exist:

\[ x(t) = A_c \beta_b s_c(t) + A_d s_d(t) + n(t) \]

Where \( \beta = [1, \beta_1, \cdots, \beta_{P-1}]^T \) denotes correlation complex coefficient matrix, \( A_c = [a_0, a_1, \cdots, a_{P-1}] \) is the steering vector matrix of correlation signal, \( s_c(t) = [s_0(t), s_1(t), \cdots, s_{P-1}(t)] \) is the correlation signal, \( A_d = [a_p, a_{p+1}, \cdots, a_{D-1}] \) is the guide vector matrix of unrelated signal, \( s_d(t) = [s_p(t), s_{p+1}(t), \cdots, s_{D-1}(t)]^T \) is the independent signal.

Correlation signals in space usually include co-frequency interference and multipath signals. In this paper, multipath signal mainly refers to the strongly correlated signal formed by the desired signal of far-field incident at the receiver's near-field reflection. Therefore, the DOA of each signal varies greatly.

2.2. MVDR algorithm

The minimum variance distortionless response (MVDR) beamforming can optimize the weighted vector with the goal of maximizing the SNR of the output signal when the desired signal is known. Assuming that the desired signal direction \( \theta_0 \) and its corresponding steering vector \( a_0 \) are known, MVDR is equivalent to the following constrained optimization problem:

\[ \min_w \quad w^H R_{\theta} w \]
\[ \text{s.t.} \quad w^H a_0 = 1 \]

The gain of the desired signal is kept constant to minimize the total output power of the interference signal and noise. The optimization model is solved by Lagrange multiplication, which results in the Capon beamforming.
\[ w_{opt} = \frac{R_{xx}^{-1}a_0}{a_0^H R_{xx}^{-1}a_0} \] (7)

\( R_{xx} \) is difficult to obtain directly, thus the sampling covariance matrix of L snapshots is usually used to replace the interference plus noise covariance when the received data does not contain the desired signal. Then we can calculate the projected sample covariance matrix as:

\[ \hat{R}_x = \frac{1}{L} \sum_{t=1}^{L} x(t)x^H(t) \] (8)

However, when the desired signal power is relatively strong, the beamforming performance of the INCM \( \hat{R}_x \) decreases seriously. Therefore, the method to reconstruct the INCM \( \hat{R}_{xx} \) based on the removal of the desired signal component in the sampling covariance matrix is the key to improve the beamforming performance.

3. Proposed method

Accurate reconstruction of the INCM and the desired signal steering vector estimation is the key step to improve the adaptive beamforming algorithm. In this paper, the signal steering vector is estimated by the optimization algorithm, and the signal power is derived by the estimated steering vector, and then the INCM is reconstructed.

3.1. Desired signal steering vector estimation

The received signal covariance matrix of an m-element array can be expressed as:

\[ R_x = E[x(t)x^H(t)] = A\Delta A^H + \delta^2 I \] (9)

We found the following optimization equation to roughly estimate the signal direction and power according to equation (9).

\[
\begin{align*}
\min_{\hat{\Delta}, \hat{\delta}} & \quad \| R_x - \hat{\Delta} \hat{\Delta}^H + \hat{\delta}^2 \delta^2 I \|_F^2 + \xi \| \hat{\Delta} \|_F^2 \\
\text{subject to} & \quad \| \hat{\Delta} \|_F > 0, \hat{\delta}^2 > 0
\end{align*}
\] (10)

Where \( \hat{\Delta} = [\hat{\Delta}_0, \hat{\Delta}_1, \cdots, \hat{\Delta}_m] \) represents the estimated steering vector matrix, \( \hat{\Delta}_i = [\hat{\Delta}_0, \hat{\Delta}_1, \cdots, \hat{\Delta}_m] \) is the estimated interference signal steering vector matrix, and \( \hat{\Delta}_a \) is the estimated desired signal steering vector.

The DOA of the signal can be inferred by searching for the spectral peak of the spatial spectrum under the condition of known array structure. As a rule of thumb, the estimation accuracy is higher when \( \xi \in [0.1, 0.3] \). When there are relevant signals, the estimation of DOA of the above algorithm still maintains strong robustness, but the power estimation will emerge large deviation. The signal power will be precisely estimated in the next section to further accurately reconstruct the INCM.

3.2. Reconstruction of INCM

When the interference is partially coherent with the desired signal, the covariance matrix of the received signal of the element array can be further stated as
\[ R_x = E[x(t)x^H(t)] = ADA^H + \delta_n^2 I \]

\[
= \begin{bmatrix}
\beta_0\beta_0^{*}\delta^2 & \beta_0\beta_1^{*}\delta^2 & \ldots & \beta_0\beta_{P-1}^{*}\delta^2 \\
\beta_1\beta_0^{*}\delta^2 & \beta_1\beta_1^{*}\delta^2 & \ldots & \beta_1\beta_{P-1}^{*}\delta^2 \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{P-1}\beta_0^{*}\delta^2 & \beta_{P-1}\beta_1^{*}\delta^2 & \ldots & \beta_{P-1}\beta_{P-1}^{*}\delta^2 \\
\end{bmatrix} A^H + \delta_n^2 I
\]

\[
\hat{D} = pinv(\hat{\Lambda})(R_x - \hat{\delta}_n^2 I)inv(\hat{\Lambda}^H)
\]

\[
\hat{D} = \begin{bmatrix}
\hat{\beta}_0\hat{\beta}_0^{*}\hat{\delta}^2 & \hat{\beta}_0\hat{\beta}_1^{*}\hat{\delta}^2 & \ldots & \hat{\beta}_0\hat{\beta}_{P-1}^{*}\hat{\delta}^2 \\
\hat{\beta}_1\hat{\beta}_0^{*}\hat{\delta}^2 & \hat{\beta}_1\hat{\beta}_1^{*}\hat{\delta}^2 & \ldots & \hat{\beta}_1\hat{\beta}_{P-1}^{*}\hat{\delta}^2 \\
\vdots & \vdots & \ddots & \vdots \\
\hat{\beta}_{P-1}\hat{\beta}_0^{*}\hat{\delta}^2 & \hat{\beta}_{P-1}\hat{\beta}_1^{*}\hat{\delta}^2 & \ldots & \hat{\beta}_{P-1}\hat{\beta}_{P-1}^{*}\hat{\delta}^2 \\
\end{bmatrix} \hat{\delta}_{D-1}^2
\]

We extract the diagonal elements of \( \hat{D} \) and remove the desired signal components form the coherent matrix of the interference signal \( \hat{D}_i \). The coherent matrix is diagonal matrix, the elements on the diagonal are the power of the interference signal, and the rank of the coherent matrix \( \hat{D}_i \) is the number of the interference signal.

\[
\hat{D}_i = \begin{bmatrix}
\hat{\beta}_i\hat{\beta}_i^{*}\hat{\delta}^2 & 0 & \ldots & 0 \\
0 & \hat{\beta}_i\hat{\beta}_i^{*}\hat{\delta}^2 & \ldots & \vdots \\
\vdots & \vdots & \ddots & \hat{\beta}_{P-1}\hat{\beta}_{P-1}^{*}\hat{\delta}^2 \\
0 & \ldots & 0 & \hat{\delta}_{D-1}^2 \\
\end{bmatrix}
\]

\[
\hat{R}_{in} = \hat{\Lambda}_i\hat{D}_i\hat{\Lambda}_i^H + \hat{\delta}_n^2 I
\]

\( \hat{R}_{in} \) in equation (17) denotes the reconstructed INCM, and the array weight is obtained by the final estimated desired signal steering vector according to MVDR criterion.
\[ w_{\text{pro}} = \frac{\hat{\mathbf{R}}_{\text{Rx}} \hat{\mathbf{a}}_0}{\hat{\mathbf{a}}_0^\dagger \hat{\mathbf{R}}_{\text{Rx}} \hat{\mathbf{a}}_0} \] (18)

For clarity, the main steps of the proposed algorithm are summarized as follows:

- Acquire the covariance matrix of the received signal
- Estimate the signal steering vector using the optimization algorithm
- Estimate noise and interference signal power through eigenvalue decomposition
- Reconstruct INCM based on the estimation of steering vector and signal power
- Obtain the optimal weight of the array by MVDR criterion

4. Numerical examples and performance analysis

In this section, numerical examples are presented to investigate the performance of the proposed adaptive beamforming. A ULA with \( M = 10 \) omnidirectional sensors spaced half a wavelength is considered. Assuming four signals impinging from the direction of \( 20^\circ, -40^\circ, 70^\circ, 50^\circ \). The first signal is desired signal, the next two signals are coherent interference (the multiple correlation coefficient is 1, \( 1 + 0.5i \) respectively) and the last signal is independent interference. The proposed algorithm is compared with (VL-SS), Toeplitz matrix reconstruction (TOP), and Duvall method under the same experimental conditions.

Example 1 analysis the beamforming diagram. The SNR of all the six signals is 20\( \text{dB} \), the snapshot number is 200, and the angular interval is 0.1\(^\circ\). And figure 1 shows the beam diagram of the MVDR algorithm without any decoherence processing, figure 2 displays the beam diagram of the four decoherence algorithms.

Figure 2 demonstrate that the direct use of MVDR algorithm failed to generate null in the direction of coherent interference due to the absence of decoherence processing. And figure 3 shows VL-SS algorithm with variable diagonal loading space smoothing (VL-SS) has a wider main lobe due to the array aperture loss, while TOP algorithm fails to generate null in the 70\(^\circ\) interference direction. Duvall algorithm changes the signal and noise components, resulting in low sidelobe and shallow null. Due to the accurate reconstruction of the INCM without rank loss, the proposed method in this paper generates deep null in each interference direction and effectively realizes the correlation interference suppression.

Figure 1. Beampattern based on MVDR beamforming  
Figure 2. Comparison of beam patterns of different coherent beamforming algorithms
In Example 2, the performance of the four algorithms is compared and analysed according to the changes of output SINR under different input SNR with snapshot number fixed to 200. It is observed from figure 4 that Duvall algorithm perform well at high SNR, but at low SNR, the performance deteriorates seriously. VDL-SS algorithm lost array aperture and thus lost array gain. By contrast, the proposed method has no array aperture loss and accurately estimate the signal noise power. In conclusion, the output SINR of the proposed method is the highest.

Example 3 test the output SINR of the four algorithms versus the snapshot number when the SNR is fixed at 20 dB, and compare and analyse their performance. Figure 4 shows that the algorithms can converge quickly except for Duvall algorithm when the number of snapshots is about 40, and the output SINR of the proposed method is the highest.

5. Conclusion
A new adaptive beamforming for coherent interference based on covariance matrix reconstruction is proposed. Our technique reconstructs INCM through estimated interference signal steering vector and interference noise power. The reconstructed INCM has no rank loss and obtain the optimal weight using the MVDR algorithm. This method can effectively suppress relevant interference and achieve the objective of accurately estimating the noise and signal power. Furthermore, the algorithm has no loss of DOF, with satisfactory array gain which is close to ideal value. Meanwhile, the numerical results also verify that the algorithm in this paper has good performance under low SNR and low snapshot number condition, and is suitable for practical engineering.

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