Flutter suppression of wind turbine blade based on nonlinear block backstepping control

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Abstract. Aiming at solving the nonlinear flutter problem of aeroelastic system of wind turbine blades, a controller combing block backstepping control and variable pitch motion was designed. Based on the typical blade-section model of the spring-mass-damper and the second-order model of the pitch actuator, nonlinear aeroelastic equation of the system under steady aerodynamic was given. Considering the multivariable, strong coupling factors and the unitary pitch control of the wind turbine blade aeroelastic system, a state space equation for a single-input/multiple-output nonlinear model was deduced. Based on the principle of block control, a nonlinear system with block control standard was obtained, which was composed of two subsystems. In order to suppress blade flutter and protect the blade, backstepping control was used to control blade pitch motion. The nonlinear block backstepping controller was designed by constructing Lyapunov function, and time response which can track the dynamic error of system was demonstrated. The simulation results showed that the designed controller can converge the divergent chattering trend to zero, and make the system globally stable.

1. Introduction

The wind turbine blade is prone to flutter under the coupling action of the inertia force, elastic force and complex aerodynamic load force. Classical flutter occurs at the potential flow of the horizontal axis wind turbine in the state of low angle of attack, with the self-excited oscillation caused by the flap-wise motion and twist motion of wind turbine blade, when the flow is basically attached without obvious diversion [1] The flutter theory of flat wings were first proposed by Lobitz [2] for a fixed-wing aircraft, which can be used to qualitatively describe the unsteady aerodynamic force of airflow acting on the non-streamline body. Kallesoe [2] analysed the aerodynamic characteristics of the blade under static attack angle by calculating eigenvalues and eigenvectors. Naiu [2] introduced B-L unsteady aerodynamic model to provide periodic time-varying unsteady potential aerodynamic force for rotating blades at low angels of attack based on the standardized flap-wise/twist vibration interface, so as to research the classical flutter stability characteristics of rotating blades of dynamic time-varying aeroelastic system. In terms of flutter suppression, Tingrui [3] analyzed classical flutter and active control of single-cell thin-walled composite wind turbine blade beam based on piezoelectric actuation, Effects of piezoelectric actuation for classical flutter suppression on wind turbine blade beam subjected to combined transverse shear deformation, warping restraint effect, and secondary warping are investigated. Although there has been no report of classical flutter on wind turbine blades, with the research and development of large wind turbines, the blades are developing towards a slenderer direction. How to suppress classical flutter has become a more important consideration in the blade design process.
Backstepping control is essentially a recursive design method of a class of nonlinear systems, informally speaking, after introducing virtual control, the complex nonlinear aeroelastic system can be decomposed into several systems with lower order [2], and then appropriate Lyapunov function was selected to ensure the robustness of the system [2]. Because the wind turbine control system is a kind of nonlinear, multivariable, strong coupling complex system, the traditional backstepping controller can bring about the problems of a lengthy recursive process and error-prone calculation. In the present study, a control scheme combining block control and backstepping control was proposed, which can simplify the design process. Based on the structure model of wind turbine blades, a mathematical model of bending-twisting coupling of wind turbine blades was established. On the basis of the standard model of block control [2], introducing a new error state vector, according to the principle of backstepping control, a block backstepping controller was designed and simulated.

2. Structural model and aeroelastic model

2.1. Structural model

The typical section model of the blade with a large aspect ratio (the section rotation radius is r), is adopted to study time-domain response curves of vertical bending and elastic torsion of the blade. The mass body of the section is suspended by springs and dampers in the section of Y and Z directions, respectively, as shown in Figure 1. The Y direction stands for the flap-wise direction; the twist angle is denoted by α; the pitching angle is denoted β; V stands for the inflow wind; \( F_y \) and \( M_\alpha \) are expressed as aerodynamic force and moment, respectively.

![Figure 1. Coordinate systems and aerodynamic forces.](image)

By summing the mass item, stiffness and damping items of the system, the equilibrium equation of force and moment was derived. Meanwhile, the influence of static moment was ignored, and the second-order motion differential equations of flapping and torsional direction can be obtained as follows [2]

\[
F_y = \rho \ddot{y} + C_y \dot{y} + K_y y \\
M_\alpha = I_\alpha \ddot{\alpha} + I_\beta \dot{\beta} + C_\alpha \dot{\alpha} + C_\beta \dot{\beta} + K_\alpha \alpha + K_\beta \beta
\]

\[
\text{where the flap motion is denoted by } y, \text{ twist motion denoted } \alpha, \text{ pitching motion denoted } \beta; \text{ C and } K \text{ stand for the stiffness and damping coefficients of flapping, twist and pitching respectively; } \rho \text{ stand for the total linear mass of the section, and } \rho = \int d\rho; \text{ I stand for the mass moment of inertia of twist and pitching, and } I = \int r^2 d\rho. \text{ In order to obtain a non-dimension form suitable for parametric research, the following normalization is introduced:}
\]

\[
\gamma = \frac{I}{\rho}, \quad \zeta = \frac{C_m}{\rho}, \quad \Omega = \frac{K_m}{\rho}, \quad \bar{\omega} = \frac{1}{\rho} \quad \text{MERGEFORMAT (2)}
\]
where $\tilde{\omega}_m$ is the normalized natural frequency, $\tilde{\zeta}_m$ is the damping ratio, $\Omega$ stands for the angular velocity, $m$ refers to the dynamic equation of flapping degree of freedom; $F_y$ and $M_a$ are expressed by force coefficients $c_L$ and moment coefficients $c_M$.

$$F_y = \frac{1}{2} \rho_a c \tilde{V}^2 c_L, \quad M_a = \frac{1}{2} \rho_a c \tilde{V}^2 c_M \quad \text{MERGEFORMAT (3)}$$

where $\rho_a$ is the air density, $\tilde{V}$ is the inflow wind speed, $c$ is the chord length.

### 2.2. Aeroelastic model

Introducing the second-order model of the pitch actuator designed by our research group in the early stage [2]:

$$I_\beta \ddot{\beta} + C_\beta \dot{\beta} + K_\beta \beta = -K_\beta \beta_{\text{ref}} \quad \text{MERGEFORMAT (4)}$$

where $\beta_{\text{ref}}$ stands for the controller request pitching angle, that is, the input of pitching angle in the control system. Other parameters are $I_\beta = 0.2$; $C_\beta = 1.1$; $K_\beta = 1$; $\tilde{\omega}_\beta = 5$; $\tilde{\zeta}_\beta = 0.02$. Combining equations (1) ~ (4), the aeroelastic dynamic equation of the system is defined as:

$$\begin{bmatrix}
\frac{1}{2} R_c c_L \\
\frac{1}{2} R_c c_M \\
\frac{1}{2} K_\beta \beta_{\text{ref}}
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 & 0  \\
0 & 1 & 0 & 0  \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{y} \\
\ddot{a} \\
\ddot{\beta}
\end{bmatrix}
+ \begin{bmatrix}
\kappa \xi \tilde{\omega}_\beta & 0 & 0  \\
0 & \kappa \xi \tilde{\omega}_\beta & \kappa \xi \tilde{\omega}_\beta  \\
0 & 0 & \kappa \xi \tilde{\omega}_\beta
\end{bmatrix}
\begin{bmatrix}
\dot{y} \\
\dot{a} \\
\dot{\beta}
\end{bmatrix}
+ \begin{bmatrix}
\kappa^2 \tilde{\omega}_\beta^2 & 0 & 0  \\
0 & \kappa^2 \tilde{\omega}_\beta^2  \\
0 & 0 & \kappa^2 \tilde{\omega}_\beta^2
\end{bmatrix}
\begin{bmatrix}
\dot{y} \\
\dot{a} \\
\dot{\beta}
\end{bmatrix}$$

\text{MERGEFORMAT (5)}

where $\kappa = \Omega c / \tilde{V}$ stands for the reduced frequency, $R_c = c^2 \rho_a / \tilde{\rho}$ stands for the ratio of the air mass density to the linear density of blade section. In view of the effect of the blade weight, and the primes denote differentiation with respect to the reduced time $\tau = (\tilde{V} / c) t$. Based on the stability analysis of eigenvalues, the force and moment coefficients can be defined as:

$$c_L = \frac{\partial C_L}{\partial (\alpha + \beta)} (\alpha + \beta + \dot{y}) \quad \text{MERGEFORMAT (6)}$$

$$c_M = \frac{\partial C_M}{\partial (\alpha + \beta)} (\alpha + \beta + \dot{y}) \ell$$

where $\partial C_L / \partial (\alpha + \beta)$ stands for the slope of the force coefficients against the angle of attack, and can be considered and defined as a constant value to $-2\pi$; $\ell$ is the chord fraction, which is equal to 0.15.

### 3. Nonlinear block backstepping controller

#### 3.1. State space model

In order to convert the aeroelastic model to the state space model, we might as well assume:

$$x_1 = \begin{bmatrix} x_{11} \ x_{12} \ x_{13} \end{bmatrix}^T = \begin{bmatrix} y \ x \ \beta \end{bmatrix}^T, \quad x_2 = \begin{bmatrix} x_{21} \ x_{22} \ x_{23} \end{bmatrix}^T = \begin{bmatrix} \dot{y} \ \dot{x} \ \dot{\beta} \end{bmatrix}^T, \quad u = \beta_{\text{ref}}$$

\text{MERGEFORMAT (7)}
where $x_i \in \mathbb{R}^n, u \in \mathbb{R}^n$, denoted the subsystem and control input, respectively. Then the nonlinear aeroelastic equation can be transformed into the state equation with single input and multiple output structure as follows.

\[
\dot{x}_i = \dot{x} = x_2 \\
\dot{x}_2 = \ddot{x} = Q^{-1}W\ddot{x} + Q^{-1}Ex - Q^{-1}Ru
\]

where

\[
R = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \\
Q = \begin{bmatrix} 1 & 0 & 0 \\
0 & 1 & 1 \\
0 & 0 & 0.2 \end{bmatrix} \\
W = \begin{bmatrix} -k_\alpha & -k_\beta & 0 & 0 \\
-k_\alpha & -k_\beta & -k_\beta & -R_j \pi \\
0 & 0 & -1.1 \end{bmatrix} \\
E = \begin{bmatrix} -k_\alpha^2 & -R_j \pi & -R_j \pi \\
-R_j \pi & -k_\alpha^2 & -R_j \pi & -R_j \pi \end{bmatrix}
\]

\* MERGEFORMAT (8)

\* MERGEFORMAT (9)

3.2. Design of block backstepping controller

The purpose of the block backstepping controller is to introduce error state vectors so that the trend of blade flutter divergence can track the expected state trajectory gradually and tend to be stable. Define the error state vector for the first subsystem $\sigma$ as:

\[
\sigma = x_{1d} - x_1 = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} x_{11d} - x_{11} \\ x_{12d} - x_{12} \\ x_{13d} - x_{13} \end{bmatrix}
\]

\* MERGEFORMAT (10)

where $x_{1d}, x_{2d}$ stand for the desired state trajectories, then take the derivative of $\sigma$:

\[
\dot{\sigma} = x_{1d} - \dot{x}_1 = \begin{bmatrix} \dot{x}_{11d} - \dot{x}_{11} \\ \dot{x}_{12d} - \dot{x}_{12} \\ \dot{x}_{13d} - \dot{x}_{13} \end{bmatrix}
\]

\* MERGEFORMAT (11)

Construct the Lyapunov function of the first subsystem as:

\[
V_1 = \frac{1}{2} \sigma^T \sigma
\]

\* MERGEFORMAT (12)

The derivative with respect to time:

\[
\dot{V}_1 = \frac{1}{2} \sigma^T \dot{\sigma} = \sigma_1 (\dot{x}_{11d} - x_{21}) + \sigma_2 (\dot{x}_{12d} - x_{22}) + \sigma_3 (\dot{x}_{13d} - x_{23})
\]

\* MERGEFORMAT (13)

According to the Lyapunov stability principle, known $V_1$ :PD, in order to satisfy $\dot{V}_1$ :ND, then assuming $\dot{x}_{1d} - x_2 = -\lambda \sigma, \lambda > 0$, then we can get:

\[
\dot{V}_1 = -\lambda \sigma_1^2 - \lambda \sigma_2^2 - \lambda \sigma_3^2
\]

\* MERGEFORMAT (14)

It is obvious that the above equation satisfies the ND requirement. In order to realize the above equation, we might introduce the virtual error state vector $v$ to make $x_2$ track the expected state trajectory $x_{2d}$ incrementally and stably. Then stability control of $x_1$ can be achieved.

\[
v = x_{2d} - x_2 = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} x_{21d} - x_{21} \\ x_{22d} - x_{22} \\ x_{23d} - x_{23} \end{bmatrix}
\]

\* MERGEFORMAT (15)
The derivative with respect to time:
\[
\dot{\mathbf{v}} = \dot{x}_{2d} - \dot{x}_2 = \dot{x}_{2d} - (Q^T \dot{\mathbf{W}} \dot{x} + Q^{-1} E \dot{x} - Q^{-1} R u) \tag{16}
\]
According to Equation (15), Equation (13) can be further converted:
\[
\dot{V}_1 = \sigma_1 (\dot{x}_{1d} - x_{2id} + v_1) + \sigma_2 (\dot{x}_{1d} - x_{2id} + v_2) + \sigma_3 (\dot{x}_{1d} - x_{2id} + v_3)
\tag{17}
\]
Construct the new Lyapunov function based on Equation (13):
\[
V_2 = V_1 + \frac{1}{2} \mathbf{v}^T \mathbf{v} \tag{18}
\]
and similarly, we can get:
\[
\dot{V}_2 = -\lambda \sigma_1^2 - \lambda \sigma_2^2 - \lambda \sigma_3^2 + \sigma \mathbf{v}^T \mathbf{v} \tag{19}
\]
Known \(V_2: PD\), according to the Lyapunov stability principle, in order to satisfy \(\dot{V}_2: ND\), we might assume that:
\[
H = \sigma^T \mathbf{v} + \mathbf{v}^T \mathbf{v} \tag{20}
\]
If H: ND, then \(\dot{V}_2: ND\), and we can get:
\[
\dot{H} = \nu_1 (\sigma_1 + \dot{v}_1) + \nu_2 (\sigma_2 + \dot{v}_2) + \nu_3 (\sigma_3 + \dot{v}_3) \tag{21}
\]
To ensure the controller design to meets the requirement, we might assume that:
\[
\sigma + \dot{v} = -\mu \mathbf{v}, \quad \mu > 0 \tag{22}
\]
From what has been discussed above, when equation (22) holds, the whole system gradually and steadily converges to the desired state trajectory.
\[
x_{1d} - x_1 + \dot{x}_{2d} - (Q^T \dot{\mathbf{W}} \dot{x} + Q^{-1} E \dot{x} - Q^{-1} R u) = -\mu (x_{2d} - x_2) \tag{23}
\]
then
\[
u = -R^T (Q^{-1})^T \left\{ -\mu (x_{2d} - x_2) + x_1 - (x_{1d} + \dot{x}_{2d}) + Q^T \dot{\mathbf{W}} \dot{x} + Q^{-1} E \dot{x} \right\} \tag{24}
\]
### 4. Simulation and analysis
Four sets of basic structural parameters based on the research cases of NACA 0015 aerofoil for classical flutter analysis are shown in table 1.

| Table 1. Parameters of simulation model. | \(\kappa\) | \(\bar{\omega}_y\) | \(\bar{\omega}_u\) | \(R_1\) | \(\bar{\zeta}_y\) | \(\bar{\zeta}_u\) |
|---|---|---|---|---|---|---|
| Case1 | 0.13 | 4 | 7 | 0.02 | 0.02 | 0.01 |
| Case2 | 0.13 | 4 | 7 | 0.08 | 0.02 | 0.01 |
| Case3 | 0.04 | 4 | 7 | 0.02 | 0.02 | 0.01 |
| Case4 | 0.04 | 4 | 7 | 0.01 | 0.02 | 0.01 |

Given the initial amplitudes of flapping motion and torsional motion, with values being 0.1, the flutter time domain response curves of the four sets of basic structural parameters can be obtained under the action of no pitching paddle, as shown in Figure 2.
It can be seen that Case 2 and Case 3 in Figure 2 show the divergences of flapping flutter and torsional flutter. Since continuous divergence of chattering will lead to blade failure or even fracture, in order to verify the reliability of the controller designed in the present study, the controlled results are demonstrated based on Case 2 and Case 3, with obvious convergence displayed. The controlled results are shown in Figure 3.

5. Summary
The classical flutter analysis in this paper was carried out under the premise of ignoring the bending of the lag. Aiming at solving the nonlinear aeroelastic equation of bending/twisting coupling, the mathematical model of wind turbine blade based on classical steady aerodynamic force was established. Combing block control and backstepping control, a kind of subsystem with multiple variables was designed. In view of the singularity of wind turbine blade control system, the mathematical model was transformed into a state space model with single input and multiple output. Based on backstepping principle, the block backstepping controller was obtained by recursive
operation. Simulation was carried out in Simulink, and the simulation results showed that, under the action of no pitching, the flutter trends of Case2 and Case3 were divergent. Blade fracture failure would take place under the condition of sufficient time without control. After applying the block backstepping controller designed in the present study, the control input was realized through pitching motion. The given initial state gradually converges to the tracking error over time, so as to realize the purpose of flutter instability control. Meanwhile, from the perspective of vibration amplitude, after the backstepping control, the amplitude reached a great attenuation (controlled amplitude was about 1/10 of the original, or even less), and the flutter suppression was realized.

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