Diffuse X-ray emissions from dynamic planetary nebulae

Yu-Qing Lou\textsuperscript{1,2,3,*} and Xiang Zhai\textsuperscript{1,4}

\textsuperscript{1} Department of Physics and Tsinghua Centre for Astrophysics (THCA), Tsinghua University, Beijing, 100084, China
\textsuperscript{2} Department of Astronomy and Astrophysics, University of Chicago, 5640 S. Ellis Avenue, Chicago, IL 60637, USA
\textsuperscript{3} National Astronomical Observatories, Chinese Academy of Sciences, A20, Datun Road, Beijing, 100021, China
\textsuperscript{4} Department of Physics, University of California at San Diego, 9500 Gilman Drive, La Jolla, CA 92037-0350, USA

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ABSTRACT

We present the theoretical results of a piecewise isothermal shock wind model. This was devised to predict the luminosity and surface brightness profile of diffuse X-ray emissions, primarily from the inner shocked downstream wind zone of a planetary nebula (PN) surrounded by a self-similar shocked dense shell and self-similar outer slow asymptotic giant branch wind envelope, both involving self-gravity. We compare and fit our computational model results with the available observations of a few grossly spherical X-ray emitting PNe. By matching the shocked piecewise isothermal self-similar void solutions with the self-gravity of Lou and Zhai for the outer zone and a stationary isothermal fast tenuous wind with a reverse shock for the inner zone across an expanding contact discontinuity, we can consistently construct dynamic evolution models of PNe with diffuse X-ray emissions. On the basis of such a chosen dynamic wind interaction model, both the X-ray luminosity and the radial X-ray brightness profile are determined by three key parameters: the so-called X-ray parameter, $X$, and two radii, $R_{rs}$ and $R_c$, of the reverse shock and the contact discontinuity. We find that the morphologies of X-ray emissions appear in the forms of either a central luminous sphere or a bright ring embedded within optically bright shells. In contrast to previous adiabatic models, the X-ray brightness peaks around the reverse shock, instead of the contact discontinuity surface just inside the outer shocked dense shell. The diffuse X-ray emissions of a few observed PNe appear to support this piecewise isothermal wind–wind dynamic interaction scenario with shocks.

Key words: hydrodynamics – shock waves – stars: winds, outflows – ISM: clouds – planetary nebulae: general – X-rays: ISM.

1 INTRODUCTION

Planetary nebulae (PNe) are produced at the late stage of stellar evolution, when a star transits from the asymptotic giant branch (AGB) phase, where a star blows slow AGB dense wind, to the later phase showing a central compact star (i.e. a hot white dwarf is gradually exposed). As the later tenuous fast wind chases and collides with an earlier circumstellar slow massive AGB wind envelope, a forward shock emerges and runs outwards into the slow AGB wind zone. Meanwhile, a reverse shock travels radially inwards relative to the inner fast wind, and a contact discontinuity in between spatially separates the forward and reverse shocks. Such a dynamic shock interaction of wind–wind collision rearranges the distribution of the AGB wind envelope and eventually results in a quasi-spherical PN (e.g. Kwok, Purton & Fitzgeral 1978; Volk & Kwok 1985; Chevalier 1997; Bian & Lou 2005; Yu et al. 2006; Hu & Lou 2008; Lou & Zhai 2009, hereafter LZ). Ignoring the gravity effect completely, Chevalier (1997) has presented an isothermal dynamic model for PNe, and has constructed spherically symmetric global hydrodynamic solutions to describe the expansion of the outer shocked shell with an inner contact discontinuity of fast wind moving at a constant speed in the inner zone. Hu & Lou (2008) developed a self-similar polytropic shock model with the self-gravity and with a central expanding void in the context of H II regions; in principle, such a formalism allows a central fast tenuous wind inside the cavity and with a reverse shock. LZ have advanced a piecewise isothermal model framework, including the gas self-gravity, and have shown various plausible self-similar dynamic behaviours of outer PN envelopes. The dynamic collision interaction of the inner fast tenuous wind with the outer slow AGB wind envelope is taken into account by including forward and reverse shocks and contact discontinuity in LZ. As an illustrative example of PN application, the isothermal self-similar void (ISSV) solutions of LZ appear to be sensible when fitting the available data of the PN, NGC 7662. Diffuse X-ray emissions as a result of the spectral-line cooling of highly ionized metals (e.g. Kastner et al. 2000; Yu et al. 2009) and probably partial thermal bremsstrahlung from hot electrons reveal...
the important inner morphological features of PNe (i.e. the region enclosed within the dense shell). A powerful reverse shock can heat up the inner tenuous fast wind plasma on the downstream side within an estimated temperature range of \( \sim 10^7 - 10^8 \) K (e.g. Volk & Kwok 1985; Leahy, Kwok & Yin 2000; Akashi, Soker & Behar 2006; Stute & Sahai 2006) and create a reverse shock heated ‘hot bubble’ largely confined by the outer dense shell at an expanding contact discontinuity. For this overall physical scenario, the properties of the diffuse X-ray emissions are mainly determined by the temperature, density, velocity and mass-loss rate of the shocked inner wind. As a result, it offers a unique insight into the shaping processes of PNe and their central tenuous hot bubbles.

In the past decade, advanced instruments in space have allowed us to detect diffuse X-ray emissions from PNe. Together with complementary observations for the dense shells of PNe in optical and infrared bands, we are now able to test the wind–wind dynamic shock interaction scenario on more specific terms. X-ray emissions from three PNe were detected using the ROSAT spacecraft (e.g. Stute & Sahai 2006), and those from nine PNe were detected with the Chandra and XMM–Newton X-ray observatories (e.g. Kastner et al. 2000; Chu et al. 2001; Kastner, Vrtilek & Sokker 2001; Guerrero, Gruendl & Chu 2002; Kastner et al. 2003; Sahai et al. 2003; Montez et al. 2005; Montez et al. 2005; Gruendl et al. 2006; Guerrero 2006). With sufficiently high spatial and energy resolutions as well as detector sensitivity, Chandra and XMM–Newton observations have allowed us to study the extended spatially resolved X-ray structure of PNe. These provide an unprecedented opportunity to examine the wind–wind dynamic interaction scenario and model.

Akashi et al. (2006) qualitatively calculated the morphology of X-ray emissions from PNe, based on the adiabatic self-similar colliding wind solutions for shocked fast wind flows of Chevalier & Imamura (1983) without self-gravity. They showed that the X-ray emission appears in the form of a narrow ring inside the optically bright part of a PN. Montez et al. (2005) have suggested that this is the case of NGC 40. Akashi et al. (2007) have investigated X-ray emissions from PNe by the numerical simulation of adiabatic winds without gravity. They found that in order to explain the observed PNe X-ray properties, a rapidly decreasing mass-loss rate is necessarily required. The adiabatic colliding wind models of Chevalier & Imamura (1983) and Akashi et al. (2007) both indicate that X-ray emissions become brightest very near the dense shell. Therefore, PNe with central luminous X-ray emission profiles cannot be accounted for by their adiabatic colliding wind models. Chevalier & Imamura (1983) also included an isothermal dense shell in their model, in addition to the innermost and outermost stationary isothermal winds.

Recent observations reveal that diffuse X-ray emissions from PNe do generally lie within the interior of the optically bright shells of relatively dense ionized gas, such as PN BD +30\(^\circ\) 3639 (e.g. Kastner et al. 2000; Yu et al. 2009), NGC 40 (e.g. Montez et al. 2005) and NGC 2392 (e.g. Guerrero et al. 2005; Guerrero 2006). From the PN BD +30\(^\circ\) 3639, the ROSAT spacecraft instrument discovered a Gaussian-shape diffuse X-ray emission source (e.g. Leahy et al. 2000). With a high spatial resolution of 0.49 arcsec, Chandra resolved this compact X-ray emission source well, and found a quasi-spherical central bright diffuse X-ray emission blob embedded inside the optical bright shell, with a radius of \( \sim 2 \) arcsec. The X-ray photon flux from the core is \( \sim 10 \) times more intense than that from \( \sim 2 \) arcsec (e.g. Kastner et al. 2000). Moreover, Chandra also found that the X-ray bright centre was slightly concentrated towards the east side. Unlike the PN BD +30\(^\circ\) 3639, the PN NGC 40 shows a distinct annulus X-ray emission (e.g. Montez et al. 2005).

For those PNe of bipolar morphologies that do not clearly present shell-like structures, their diffuse X-ray emissions are also from within the optically bright shells, such as NGC 7027 (e.g. Kastner et al. 2001), NGC 7009 (e.g. Guerrero et al. 2002), NGC 6543 (e.g. Chu et al. 2001), NGC 7026 (e.g. Gruendl et al. 2006) and Menzel 3 (e.g. Kastner et al. 2003). These X-ray observations bear strong implications that wind–wind dynamic interactions or jet–wind interactions are most likely responsible for the inner diffuse X-ray emissions from PNe (e.g. Kastner 2007).

In this paper, we establish a quantitative connection of the self-similar piecewise isothermal dynamic evolution model of PNe with diffuse X-ray emissions, especially the radial distribution of surface brightness (see Hu & Lou 2008 for a relevant self-similar polytropic model development). On the basis of the global ISSV model of LZ for the self-similar evolution of PNe, we show that three key model parameters determine the morphology for diffuse X-ray emissions from PNe. We specifically examine the observations of the PN, NGC 40. In our piecewise isothermal model framework, either ring-like or central bright X-ray morphologies are possible because X-ray emission peaks near the reverse shock instead of the contact discontinuity. Our results lend support to the wind–wind dynamic interaction model for PNe in general. We also compute the luminosity and surface brightness of X-ray emission from PN 7662. We emphasize that this PN should be a highly desirable target source for further X-ray observations with Chandra in the near future.

2 A SPHERICAL WIND–WIND DYNAMIC INTERACTION MODEL FOR PLANETARY NEBULAE WITH SELF-SIMILAR SHELLS

In this section, we first describe our global piecewise isothermal dynamic evolution model for quasi-spherical PNe. At the late stage of stellar evolution, a star enters the AGB phase. During this phase, a star suffers from a high rate of mass loss \( \sim 10^{-6} \) M\(_\odot\) yr\(^{-1}\). Thus, it creates a fairly dense AGB wind shell expanding at a slow radial speed of \( \sim 10 \) km s\(^{-1}\) (e.g. Kwok et al. 1978; Chevalier & Imamura 1983; Chevalier 1997; LZ). The dense gas shell keeps expanding until the stellar hydrogen envelope has been almost depleted and a compact hot white dwarf is gradually exposed at the centre. Meanwhile, a hot tenuous fast wind somehow emerges from the central white dwarf, as indicated by observations. This tenuous fast wind blows at a high speed of \( \gtrsim 10^7 \) km s\(^{-1}\) with a mass-loss rate of \( \sim 10^{-10} - 10^{-7} \) M\(_\odot\) yr\(^{-1}\) (e.g. Cerruti-Sola & Perinotto 1985). This fast inner wind will not diffuse through the outer AGB slow wind, but smash on to the outer dense AGB wind shell (e.g. Kwok et al. 1978). It will then give rise to a forward shock propagating outwards into the slow AGB wind and a reverse shock propagating inwards in the fast wind comoving framework of reference (e.g. Chevalier & Imamura 1983; Akashi et al. 2006). A contact discontinuity naturally forms when the gas pressures of the fast wind and of the AGB wind reach a dynamic balance, with both density and temperature being different on the two adjacent sides. These two shocks, especially the forward shock, rearrange the distribution of gas flow and finally result in a dense shell surrounding the central compact star blowing an inner fast wind (e.g. Kwok 2000; LZ).

A quasi-spherical PN may be roughly divided into four regions (see fig. 1 of Akashi et al. 2006). The tenuous fast wind occupies the inner region from the central compact white dwarf to the reverse shock front at radius \( R_s \). When propagating inwards relative to the fast wind, the reverse shock heats up and slows down the inner wind, and creates a downstream wind region from the reverse shock...
front to the expanding contact discontinuity at radius \( R_c \). These two regions together form the inner zone and are characterized by the fast wind mass-loss rate, wind velocities, densities, temperatures and reverse shock speed. The shocked dense shell region remains between the contact discontinuity and the forward shock at radius \( R_c \).

The fourth region, or the outer AGB slow wind envelope, expands outside the forward shock front at a low speed of \( \sim 10 \text{ km s}^{-1} \). As in LZ, we adopt spherically symmetric non-linear hydrodynamic partial differential equations (PDEs) in spherical polar coordinates \((r, \theta, \phi)\) to model the piecewise isothermal gas dynamics of these four wind regions of a quasi-spherically symmetric PN.

### 2.1 Inner shocked tenuous wind zone featuring a reverse shock

As already noted, a reverse shock emerges and propagates in the inner fast wind zone because of the collisional interaction between the later fast and earlier slow winds. The mass and radial momentum conservations across an isothermal shock front in the shock comoving frame are simply

\[
\rho_\text{d}(u_\text{d} - u_\text{s}) = \rho_\text{s}(u_\text{s} - u_\text{d}), \\
\frac{1}{2} \rho_\text{s} u_\text{s}^2 + \rho_\text{d} u_\text{d}^2 = \frac{1}{2} \rho_\text{d} u_\text{d}^2 + \rho_\text{s} u_\text{s}^2.
\]

Here, \( a \) is the isothermal sound speed, \( \rho \) is the mass density and \( u \) is the radial flow velocity. The two subscripts ‘d’ and ‘s’ denote downstream and upstream sides of an isothermal shock, respectively, and the subscript ‘s’ indicates first association with the shock (e.g. Courant & Friedrichs 1976; Spitzer 1978; Shen & Lou 2004; Bian & Lou 2005; Yu et al. 2006; LZ). The temperature ratio across such a shock front is another parameter to be stipulated (see Shen & Lou 2004).

The reverse shock travels inwards relative to the inner wind, so the inner fast wind region is the upstream side of the shock and the reverse shock heated wind occupies the downstream side until the contact discontinuity at the expanding radius \( R_c \). In the laboratory framework of reference, a reverse shock may move inwards or outwards, or appear stationary in space, depending on various specific situations (e.g. Lou 1996, 1998).

We presume a constant mass-loss rate \( \dot{M}_{\text{bw}} \) for an isothermal inner fast wind. We define \( a_{w,d}(\rho_{w,d}, T_{w,d}) \) and \( v_{w,d}(\rho_{w,d}, \theta_{w,d}) \) as the isothermal sound speed, constant temperature, mass density and constant wind velocity on the downstream (upstream) side of the reverse shock. The sound speed ratio \( \tau_w \equiv a_{w,d}/a_{w,s} = (T_{w,d}/T_{w,s})^{1/2} \) across the shock front characterizes the downstream plasma heating of the reverse shock. The isothermal shock conditions (1) then give

\[
v_{w,s} - v_{w,d} = (1/2) \left[ (v_{w,d} - v_{w,s} + a_{w,d}^2/(v_{w,d} - v_{w,s})) \right]
\]

\[
+ \frac{1}{2} \left\{ \left[ (v_{w,d} - v_{w,s})^2 - a_{w,d}^2 \right]/(v_{w,d} - v_{w,s}) \right\}^{1/2}.
\]

\[
v_{w,d} - v_{w,s} = (1/2) \left[ (v_{w,d} - v_{w,s} + a_{w,d}^2/(v_{w,d} - v_{w,s})) \right]
\]

\[
- \frac{1}{2} \left\{ \left[ (v_{w,d} - v_{w,s})^2 - a_{w,d}^2 \right]/(v_{w,d} - v_{w,s}) \right\}^{1/2},
\]

\[
\rho_{w,d}(R_{w,s}, t) = \rho_{w,d}(R_{w,s}, t)(v_{w,d} - v_{w,s})/(v_{w,d} - v_{w,s}).
\]

Here, \( v_{w,s} \) is the reverse shock velocity in the laboratory reference framework (LZ). Calculations show that positive, negative and zero \( v_{w,s} \) are all physically allowed. For \( v_{w,s} < 0 \), the inner fast wind region shrinks radially inwards until a certain epoch. In other words, the reverse shock heats up the downstream wind plasma and leads to a hot bubble inside the contact discontinuity surface at an expanding radius \( R_c \).

The mass density profile within the central fast wind zone is simply

\[
\rho_{w,a}(r, t) = \rho_{\text{bw}}/(4\pi v_{w,a} r^2).
\]

In the reverse-shocked downstream wind zone, it is

\[
\rho_{w,a}(r, t) \equiv \frac{1 - v_{w,s}/v_{w,a}}{\rho_{w,a}} \frac{\dot{M}_{\text{bw}}}{4\pi r^2} \approx \frac{\dot{M}_{\text{bw}}}{4\pi (v_{w,a} - v_{w,s})},
\]

which remains steady, where the constant \( \beta \) parameter is explicitly defined. This last approximation highlights that the downstream density of shocked inner wind does not sensitively depend on the inner fast wind speed \( v_{w,a} \) because we typically have a small speed ratio of \( v_{w,s}/v_{w,a} \sim 10^{-2} \). Because of the reverse shock, the downwind mass-loss rate just inside the contact discontinuity surface is given by

\[
\dot{M}' = 4\pi R_c^2 \rho_{w,a}(R_c, t) (1 - v_{w,s}/v_{w,a}) \dot{M}_{\text{bw}}.
\]

This is equal to the central mass-loss rate \( \dot{M}_{\text{bw}} \) only when the reverse shock remains stationary in the laboratory reference frame. The difference between \( \dot{M}_{\text{bw}} \) and \( \dot{M}' \) is the mass-loss rate at the reverse shock front as a result of the reverse shock movement.

X-ray observations over the past several years appear to indicate that the temperatures of the inner shocked fast wind, where X-ray photons are released, are a factor of \( 2-10 \) lower than that predicted by simple energy conservation arguments (e.g. Soker & Kastner 2003). In our formulation, we use a chosen sound speed ratio \( \tau_w \equiv a_{w,d}/a_{w,s} = (T_{w,d}/T_{w,s})^{1/2} \) to bridge the two constant temperatures of the upstream and downstream sides across a moving reverse shock.

### 2.2 Dense shell region and outer AGB slow wind envelope

The shocked dense shell region and the outer AGB slow wind envelope, separated by an outgoing forward shock, are described by the ISSV model of LZ. The piecewise isothermal hydrodynamic equations with spherical symmetry are the mass conservation

\[
\frac{\partial M}{\partial r} + u \frac{\partial M}{\partial r} = 0, \quad \frac{\partial M}{\partial r} = 4\pi r^2 \rho,
\]

or equivalently

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) = 0,
\]

and the radial momentum equation

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = -a^2 \frac{\partial \rho}{\partial r} - \frac{GM}{r^2}.
\]

Here, \( u \) is the radial bulk flow speed, \( M(r, t) \) is the enclosed mass within radius \( r \) at time \( t \), \( p(r, t) \) is the mass density, \( a \equiv (p/\rho)^{1/2} = (k_B T/\mu)^{1/2} \) is the isothermal sound speed, \( T \) is the constant gas temperature, \( \mu \) is the mean particle mass, \( p \) is the gas pressure, \( k_B \) is the Boltzmann constant and \( G = 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2} \) is the universal gravitational constant.

Using the known isothermal self-similar transformation

\[
x = r/(at), \quad p(r, t) = (\alpha(x))/(4\pi G r^2),
\]

\[
M(r, t) = a^4 \mu(x)/G, \quad u(r, t) = a \nu(x),
\]

equation (6) leads to

\[
m(x) = x^2 \alpha(x - v).
\]
Then, PDEs (7) and (8) reduce to two coupled non-linear ordinary different equations (ODEs)

\[
\frac{d\rho}{dx} = \left[ \alpha(x - v) - \frac{2}{x} \right] (x - v),
\]

and

\[
\frac{d\rho}{dx} = \left[ \alpha(x - v) - \frac{2}{x} \right] (x - v).
\]

Here, \(\alpha(x), \rho(x)\) are the dimensionless density and velocity corresponding to the mean density \(\rho(r, t)\) and radial flow speed \(u(r, t)\), respectively (Hunter 1977; Shu 1977; Whitworth & Summers 1985; Tsai & Hsu 1995; Shu et al. 2002; Lou & Shen 2004; Shen & Lou 2004; Bian & Lou 2005; LZ). Given proper analytical asymptotic solution conditions at large and small \(x\) as well as in the neighborhood of the sonic critical point, these two coupled non-linear ODEs (11) and (12) can be solved in a straightforward manner. Each dimensionless ISSV solution of these two ODEs gives a behaviour of the dense shell and outer AGB wind envelope outside the contact discontinuity radius \(R_c\).

The forward shock travels outwards in a self-similar manner and divides the shell into the shocked dense shell region and the outer slow AGB wind envelope. We define \(\rho_0, T_0, \alpha_0\) and \(\rho d, T d, \alpha d\) as the sound speed, temperature, reduced mass density and reduced gas radial flow velocity, respectively, on the downstream (upstream) side of the forward shock and the sound speed ratio \(\tau \equiv \alpha d/\alpha_0 \equiv (T d/T_0)^{1/2}\) (Shen & Lou 2004; Bian & Lou 2005; Yu et al. 2006; LZ). In this case, the dense shell region is the downstream side and the outer slow AGB wind envelope is the upstream side of the outgoing forward shock front. We define dimensionless variables \(x_d \equiv V_d/v_d\), and \(x_d \equiv V_d/v_d\), where \(V_d = v_d/t_d\) is the radially outgoing velocity of the forward shock in the laboratory reference framework. Then, the isothermal shock conditions (1) bear the self-similar form of

\[
\frac{\alpha d}{\alpha_0} = (v_d - x_d)/(v_d - x_d),
\]

\[
(v_d - x_d)/(v_d - x_d) - \tau = (\tau v_d - v_d)/(v_d - x_d),
\]

\(\text{(Shen \\& Lou 2004). These isothermal shock jump conditions connect the AGB outer wind envelope with the dense shell region across the forward shock front.}

The analytical asymptotic solution of coupled non-linear ODEs (11) and (12) at large \(x\) is known to be

\[
v = V + \frac{2 - A}{x} + \frac{V}{x^3} + \frac{(A/6 - 1)(A - 2) + 2V^2/3}{x^3} + \ldots,
\]

\[
\alpha = A + \frac{A(2 - A)}{2x^2} + \frac{(4 - A)VA}{3x^3} + \ldots,
\]

where \(V\) and \(A\) are the velocity and mass parameters (Lou \\& Shen 2004; LZ). In our self-similar piecewise isothermal model framework, these two parameters characterize the asymptotic massive AGB wind velocity as

\[
v_{AGB} = V_{Ad}/G,
\]

at very large \(x\) and a constant AGB wind mass-loss rate as

\[
\dot{M}_{AGB} = V_{Ad}^2/G.
\]

These asymptotic relations are used to constrain the range of our piecewise isothermal shock model parameters for PNe.

2.3 Radial expansion of the spherical contact discontinuity interface

The contact discontinuity surface at \(R_c\) expands at a constant speed and spatially separates the inner and outer zones. LZ estimated that

the gravity of the inner tenuous wind zone may be negligible during the dynamic self-similar evolution of the dense shell region and the outer AGB envelope in outer zone. By letting \(m(x_0) = 0\) in equation (10), we then obtain the contact discontinuity surface radius \(R_c = ax_0d\) and its expansion velocity \(v_c = ax_0d\). By doing this, we ignore the self-gravity from the inner shocked wind region within the contact discontinuity radius \(R_c\), but still retain the self-gravity of the dense shell and the AGB slow wind envelope outside the contact discontinuity radius \(R_c\). A contact discontinuity requires that the inner downstream wind velocity \(v_{in}\) and the outer dense shell gas velocity at \(R_c\) be equal to the contact discontinuity expansion velocity, namely

\[
v_{in} = ax_0 = ax_0d(t(x_0)),
\]

which is satisfied automatically when \(m(x_0) = 0\) occurs. Here, \(x_0\) is where an ISSV solution for the dense shell region and the outer envelope starts. Given a proper set of \(\{x_0, \alpha d \equiv \alpha(x_0), x_d\}\), we can readily integrate coupled non-linear ODEs (11) and (12) from \(x_0\) to larger \(x > x_0\) (i.e. from left to right). We can apply self-similar shock jump conditions (equation 13) at \(x_d\) to cross an outgoing shock front once \(a_d\) and \(\alpha d\) are known. Then, we can continue further from \(x_d\) to sufficiently large \(x\) to determine velocity and mass parameters \(V\) and \(A\) with desired accuracies. The flow zone between the contact discontinuity and the forward shock is the dense downstream side and the outer AGB slow wind is the upstream side. The forward shock jump condition connects \(x_d\) and \(x_0\). These solutions have a negligible mass within \(x_0\) and are referred to as ISSV solutions by LZ. In total, eight parameters are involved for constructing a model ISSV solution \(\{x_0, \alpha_0, a_0, x_0, x_d, v_{in}, V, A\}\), with five of these being actually independent.

At the radius \(R_c\) of contact discontinuity, the inner downstream wind plasma pressure is

\[
P_{in}(R_c, t) = \frac{k_B T_{in} a_0}{\bar{m}_d R_c^2} \propto \frac{T_{in}}{t^2},
\]

where \(\bar{m}_d\) is the mean particle mass of the inner downstream wind zone. With the self-similar transformation (9), we have the shocked dense shell gas pressure at the contact discontinuity radius \(R_c\) as

\[
P_{out}(R_c, t) = \frac{k_B T_d a_0}{\bar{m}_d 4\pi G t^2} \propto \frac{T_d}{t^2},
\]

where \(\bar{m}_d\) is the mean particle mass of the shocked dense shell zone. To physically maintain an expanding contact discontinuity at constant speed, the parameters \(M_{in}, v_{in}, v_{in}, v_{in}, T_{in}, a_0\) and \(T_d\) are coupled by the pressure balance condition \(P_{in}(R_c, t) = P_{out}(R_c, t)\) across the contact discontinuity, namely

\[
\bar{m}_d G = \frac{T_d}{\bar{m}_d v_{in}} v_{in} = \frac{T_{in}}{\bar{m}_d (v_{in} - v_{in})} v_{in}^3.
\]

In general, the two mean particle masses \(\bar{m}_d\) and \(\bar{m}_{in}\) are allowed to be different depending on specific situations.

2.4 Comparison with previous models of PNe

Chevalier \\& Imamura (1983) and LZ both invoke a self-similar hydrodynamic wind-interaction phase to explore the global dynamic shock evolution of PNe. In both models, the inner and outer wind zones are characterized by different yet constant mass-loss rates.

More specifically, in the model of Chevalier \\& Imamura (1983), the innermost fast wind zone and the outermost AGB slow wind envelope are both isothermal and expand at constant speeds, with both mass densities scaling as \(\propto r^{-2}\). In between these two wind regions,
both the shocked downstream wind zone and the dense shell are solved separately by adopting a type of adiabatic self-similar transformation for \( \gamma = 5/3 \) hydrodynamic PDEs without gas self-gravity (Parker 1961). These two self-similar dynamic zones are joined by the pressure balance across an outgoing contact discontinuity. Two different Mach numbers are specified respectively for the reverse shock separating the inner \( r^{-2} \) fast wind and self-similar shocked downstream wind zone (hot bubble), and for the forward shock separating the self-similar dense shell and outer \( r^{-2} \) slow AGB wind envelope.

In comparison, the LZ model also assumes an \( r^{-2} \) density profile for a steady piecewise isothermal fast wind zone in the central region around the remnant white dwarf. Differently, the dense shell and the outer AGB slow wind envelope are described by a self-similar dynamic solution with self-gravity and an isothermal shock jump (i.e. the type \( Z \) self-similar solutions of LZ). The self-similar process here differs from that of Chevalier & Imamura (1983) in the following ways: (i) the inclusion of self-gravity; (ii) the piecewise isothermal wind (both the dense shell and the outer AGB slow wind envelope are isothermal but with different temperatures); (iii) a different self-similar transformation because of the presence of self-gravity; (iv) the necessary shock parameters (the forward shock is described by its velocity and sound speed ratio or temperature jump parameter \( r \)). In the inner shocked downstream wind zone, the LZ model has constant temperature and wind speed. For a constant mass-loss rate \( \dot{M}_w = \rho_w v_w d\tau \), where \( \dot{M}_w \) is the proton density (e.g. Sarazin 1986). At the temperature range of \( T \lesssim 3 \times 10^5 \) K, where spectral-line emissions from highly ionized metals are important, a simple approximation is thus \( \Delta (T) \approx 6.2 \times 10^{-9} \) erg cm\(^{-3}\) s\(^{-1}\) (e.g. Sarazin 1986). The X-ray luminosity of a PN can then be calculated by integrating the emissivity over the entire shocked downstream inner wind region:

\[
L_X = \int_{R_n}^{R_e} \frac{\rho_w d(r)}{m_p^2} 4\pi r^2 dr.
\] (21)

Here, \( m_p \) is the proton mass density to proton number density. For an H II region, we simply have \( m_p = m_e \), while \( m_p = 1.18 m_p \) for gas with a helium abundance of H/He \( \approx 0.1/0.9 \) by number. Defining the X-ray parameter \( X \equiv \Lambda \beta^2 / m_p \), we readily obtain the following expression:

\[
L_X = 4\pi X (1/R_n - 1/R_e).
\] (22)

A normalized X-ray surface brightness profile \( B \) can be calculated by integrating over the X-ray luminosity along the line of sight. Let \( R \) be the projected radius from the centre of a grossly spherical PN, then we have the X-ray surface brightness radial profile \( B \) as

\[
B = \frac{1}{2} \left\{ \begin{array}{ll}
\frac{\sqrt{R^2 - R_n^2}}{\sqrt{R^2 - R_n^2}} \left( \frac{\rho_w d(r)}{m_p^2} \right) \frac{\Delta^2}{\mu} r^2 \mu (r) d\tau, & 0 \leq R < R_n \\
\frac{\sqrt{R^2 - R_e^2}}{\sqrt{R^2 - R_n^2}} \left( \frac{\rho_w d(r)}{m_p^2} \right) \frac{\Delta^2}{\mu} r^2 \mu (r) d\tau, & R_n \leq R < R_e \\
0, & R > R_e
\end{array} \right.
\] (23)

By introducing a dimensionless function \( L(z) \) defined as

\[
L(z) = \frac{1}{z^2} [z(1 - z^2)^{1/2} + \arccos z] \quad \text{for} \quad 0 < z < 1,
\] (24)

\( \rho_w \) is the electron number density in the dense shell zone is estimated as \( n_e \approx 3 \times 10^3 \) (1000 yr \( r^{-2} \)) cm\(^{-3}\), where \( t \) is the typical age of a PN, and the thickness of a dense shell zone is estimated as \( \approx 10^{18} \) (1000 yr \( r^{-2} \)) cm. Then, the mean free path of a photon in the dense shell zone would be \( l = l/(n_1 \sigma) \approx 10^{-20} \) (1000 yr \( r^{-2} \)) cm, where \( \sigma = 6.65 \times 10^{-25} \) cm\(^2\) is the electron cross-section for Thomson scattering. Therefore, the dense gas shell should be optically thin for emissions of X-ray photons during the evolution of a PN. We denote the X-ray emissivity of the shock heated downstream inner wind plasma by \( \gamma (T) \) such that \( \gamma (T) \) is the energy emitted per unit time per unit volume, where \( n_p \) is the proton number density (e.g. Sarazin 1986). At the temperature range of \( T \lesssim 3 \times 10^5 \) K, when spectral-line emissions from highly ionized metals are important, a simple approximation is thus \( \gamma (T) \approx 6.2 \times 10^{-9} \) erg cm\(^{-3}\) s\(^{-1}\) (e.g. Sarazin 1986). The X-ray luminosity of a PN can then be calculated by integrating the emissivity over the entire shocked downstream inner wind region:
we immediately arrive at the following expression:

\[ B(R, t) = \begin{cases} 
L \left( \frac{R}{R_c} \right) - \frac{R_{\infty}}{R_c} L \left( \frac{R_{\infty}}{R_c} \right), & 0 \leq R < R_{\infty} \\
L \left( \frac{R}{R_c} \right), & R_{\infty} \leq R < R_c \\
0, & R > R_c.
\end{cases} \]  

Figure 1. Variations of the projected dimensionless X-ray brightness radial profile \( B/[\lambda \beta^2/(m_0^2 R_c^3)] \) within the contact discontinuity radius \( R_c \) for PNe with different radius ratios \( R_{\infty}/R_c \). The projected radius \( R \) is also normalized by the contact discontinuity radius \( R_c \). The range of brightness \( B \) variation spans three orders of magnitudes as \( R_{\infty}/R_c \) varies from 0.1 to 0.9. Different epochs of evolution might be invoked to explain why some PNe are observed to be strong X-ray sources while others of PNe are not. In this figure and caption, we have all \( B \equiv B \) in the main text.

Figs 1 and 2 illustrate several model PN X-ray brightness \( B \) profiles marked with different values of radius ratio \( R_{\infty}/R_c \) within the range for this normalized reverse shock radius \( 0 < R_{\infty}/R_c < 1 \). We emphasize that morphologies of such diffuse X-ray emissions from PNe are determined by this radius ratio \( R_{\infty}/R_c \). The case of \( R_{\infty}/R_c \) close to unity corresponds to a ring-like brightness profile (e.g. the case of PN NGC 40), while a small \( R_{\infty}/R_c \) gives a more spherical appearance for the X-ray surface brightness radial profile with a more luminous central bulge or sphere (e.g. the case of PN BD +30 3639). Both the X-ray luminosity and surface brightness radial profile depend only on the X-ray parameter \( X \) and radius ratio for reverse shock and contact discontinuity. The X-ray parameter \( X \) is a function of \( \Lambda \) (or inner shocked downstream wind plasma temperature \( T_{w,d} \)), \( \beta \) and \( m_0 \), while the parameter \( \beta \) is further determined by the mass-loss rate \( M_{\infty} \) from the central white dwarf, reverse shock speed \( v_{w,s} \), inner fast wind velocity \( v_{w,d} \) and shocked downstream wind velocity \( v_{w,d} \) according to expression (4). As the inner radius of the dense shell, the contact discontinuity radius \( R_c \) can be measured using observations in optical and infrared bands (e.g. Kastner et al. 2000; Guerrero et al. 2004; Montez et al. 2005; Guerrero 2006). Parameters \( X \) and \( R_{\infty} \) can be inferred by fitting an X-ray surface brightness radial profile and the estimate of total X-ray luminosity of a PN. Spectral analysis can measure the innermost fast wind velocity, the dense shell gas velocity \(^1\) and tempera-

tures (four in all) in the inner and outer zones (e.g. Guerrero et al. 2004; Montez et al. 2005; Guerrero 2006). The pressure balance condition (20) across the expanding contact discontinuity radius \( R_c \) further connects the physical parameters in the inner shocked downstream wind region and the dense shell. We emphasize that combined observations in infrared, optical and X-ray bands may be utilized to infer all relevant parameters that characterize a quasisphere PN.

The morphology of diffuse X-ray emission from a PN only depends on the radius ratio \( R_{\infty}/R_c \) (see Figs 1 and 2). Thus, the time evolution of this radius ratio determines how the X-ray morphology evolves with time \( t \). In other words, different epochs of PN evolution may be invoked to explain why some PNe are observed to be strong X-ray sources while others are not. Here, we present straightforward calculations to show the relation between the physical evolution state of PNe with their diffuse X-ray emission morphologies. Suppose the contact discontinuity surface has a radius \( R_0 \) at the time when the central fast wind collides with the slow dense AGB wind envelope. Let \( t^* \) be the time lapse from the time when the forward and reverse shock pair initiated after a wind–wind collision to the current epoch. We may use \( R_0 \) and \( v_c \) to estimate a kinematic time-scale \( t \sim R_0/v_c + t^* \) and regard this time-scale \( t \) as the dynamic age of a PN (see the self-similar transformation equation 9). It follows that \( (t - t^*) \) roughly represents the temporal duration that the central star has depleted its hydrogen envelope but not yet started to launch a fast hot tenuous wind. Then, the radius

\(^1\) Relation (17) indicates that the dense shell gas velocity also gives the inner shocked downstream wind velocity as required by the pressure balance across an expanding contact discontinuity radius \( R_c \).
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s = reaches v3, 0 and 10 km s−1. v ≈ in general, the v408, and 436–447 v(i.e. v → ≈ − is non-negative, then this radius is mainly determined by R = − a v(1 ≈ 3 − km s−1 d d (≈ 4 R =− v). ˙v becomes less important. Otherwise, when the reverse shock d(30) ≈ T(1 ≈ 10 11 v =− R(29) M = = = = =− 1 erg cm−3 can no longer shrink. However, in reality, it is unlikely that X-ray =− ρ ≈− U − v in the long-term evolution for a = inferred by LZ, we find that a(26), ρ R v L ≈− =− 2 per cent of the inner fast wind energy input. Calculations evolve when tified around this stage, then there would be no or very weak X-ray emissions from the hot bubble. As the X-ray luminosity L X =− dEin ≈ 10−3 dUw,d dr ≈ dUw,d dr ≈ 101−3 dUw,dr. (31) The energy conservation then requires that L X ≤ dEin dr − d dr (Uw,u + Uw,d + K + W) ≈ Mfw v2 w,u. (32) The energy conservation requirement (32) must be necessarily met. Using expression (22), we find that equation (32) limits the possible range of the hot bubble temperature T w,d and the reverse shock radius R w. As the X-ray luminosity L X approaches the upper limit Mfw v2 w,u, either the cooling becomes dominantly important so that the temperature T w,d drops rapidly, or the reverse shock radius R w can no longer shrink. However, in reality, it is unlikely that X-ray emissions from the hot bubble reach the upper limit because cooling effects would become extremely important before L X ∼ Mfw v2 w,u. In the above discussion, we have ignored the gravitational potential energy change rate of the shocked downstream zone occupied by the hot bubble.

Fig. 3 illustrates how the radius ratio Rv/Rc, the total X-ray luminosity and the average X-ray surface brightness of PNe evolve with different reverse shock velocities vrs. The solid, dotted and dashed curves in the three panels show how PNe evolve when vrs = −3, 0 and 10 km s−1, respectively. All three models have the same innermost fast wind mass-loss rate Mfw = 10−8 M⊙ yr−1, contact discontinuity radius expansion speed Rv = v∞ = 30 km s−1, inner shocked downstream wind X-ray emissivity Λ = 10−22 erg cm−3 s−1 and R⊙ = 1.3 × 1016 cm (the distance that a wind blows by at 20 km s−1 for ~200 yr). An inner fast wind velocity of ~500 km s−1 is adopted here.

 ratio Rw/Rc evolves with time as

\[
\frac{Rw}{Rc} = \frac{(R⊙ + v∞ t)}{(R⊙ + vt)} = \frac{v∞ t - (v∞ - vrs)t}{v∞ t}.
\] (26)

At the moment that the wind–wind collision occurs and both forward and reverse shocks are initiated, the radius ratio Rw/Rc = 1 and no diffuse X-ray is emitted (see equation 22). If PNe are identified around this stage, then there would be no or very weak X-ray emissions. The time evolution of Rw/Rc is mainly determined by v∞, according to expression (26). If v∞ is non-negative, then this radius ratio has a limiting value of v∞/v∞ in the long-term evolution for a large t (see Fig. 3) during which the initial contact discontinuity radius R⊙ becomes less important. Otherwise, when the reverse shock travels radially inwards, the region inside the contact discontinuity surface tends to be filled with more shock-heated downstream wind plasma. Meanwhile, the gravity, magnetic field and radiation of the central star at the shrinking radius of the reverse shock will eventually become sufficiently strong, and both the asymptotic central wind profile and the shock condition (1) are no longer reliable. We show presently that the energy conservation actually forbids Rw → 0.

We now consider this problem from the perspective of energy conservation. The inner fast wind kinetic energy and thermal energy input per unit time

\[
\frac{dE_{\text{in}}}{dt} = \frac{1}{2} M_{\text{fw}} v_{w,u}^2 + \frac{3}{2} M_{\text{fw}} a_{w,u}^2
\] (27)

should be larger than the kinetic and internal energy increase of the system when the X-ray is emitted. The internal energy change rates of fast wind zone and shocked fast wind zone are, respectively,

\[
\frac{dU_{w,u}}{dt} = \frac{3}{2} 4\pi R_s^2 v_s \rho_w a(R_w)^2 = \frac{3}{2} v_s \rho_w M_{\text{fw}} a_{w,u}^2,
\]

\[
\frac{dU_{w,d}}{dt} = \frac{3}{2} 4\pi R_s^2 v_s \rho_w a(R_w)^2 = \frac{3}{2} 4\pi R_s^2 v_s \rho_w a(R_w)^2
\]

\[
= \frac{3}{2} (1 - v_s/vw) M_{\text{fw}} a_{w,u}^2.
\] (28)

The kinetic energy change rate caused by the reverse shock in the inner wind is

\[
\frac{dK}{dt} = 4\pi R_s^2 \left[(v_{w,d} - v_s) \rho_w a v_{w,d}^2 - (v_{w,u} - v_s) \rho_w a v_{w,u}^2\right] = \frac{M_{\text{fw}}}{2} (1 - v_s/vw) (v_{w,d} - v_{w,u}^2) < 0.
\] (29)

The power of the shocked downstream wind zone (hot bubble) pushing the dense shell outwards is

\[
\frac{dW}{dt} = 4\pi R_s^2 v_w a_R a_{w,u}^2 = M_{\text{fw}} a_{w,u}^2 \frac{(1 - v_s/vw)}{(1 - v_s/vw)}.
\] (30)

Given typically estimated parameter values of a PN of v∞ ≈ 103 km s−1, v∞ = v∞ ≈ 10−2 km s−1, v∞ ≈ 10−1 km s−1, a∞ ≈ 10 km s−1 and a∞ ≈ 10 km s−1 as inferred by LZ, we find that

\[
\frac{dE_{\text{in}}}{dt} \approx 10^{−3} \frac{dU_{w,d}}{dt} \approx \frac{dU_{w,d}}{dt} \approx 10^{−3} \frac{dU_{w,u}}{dt}.
\]

The energy conservation then requires that

\[
L_X \leq \frac{dE_{\text{in}}}{dt} - \frac{d}{dr} \left[U_{w,u} + U_{w,d} + K + W\right] \approx M_{\text{fw}} v_{w,u}^2.
\] (32)

The energy conservation requirement (32) must be necessarily met. Using expression (22), we find that equation (32) limits the possible range of the hot bubble temperature T w,d and the reverse shock radius R w. As the X-ray luminosity L X approaches the upper limit Mfw v2 w,u, either the cooling becomes dominantly important so that the temperature T w,d drops rapidly, or the reverse shock radius R w can no longer shrink. However, in reality, it is unlikely that X-ray emissions from the hot bubble reach the upper limit because cooling effects would become extremely important before L X ∼ Mfw v2 w,u. In the above discussion, we have ignored the gravitational potential energy change rate of the shocked downstream zone occupied by the hot bubble.

Fig. 3 illustrates how the radius ratio Rw/Rc, the total X-ray luminosity and the average surface brightness of PNe evolve with negative, zero and positive reverse shock velocities vrs. While all three conditions give decreasing radius ratio Rw/Rc in general, the temporal evolution of the total X-ray luminosity and the average surface brightness differ from each other significantly. In the case of vrs = −3 km s−1, the reverse shock radius Rw stops at ~10 au, as required by energy conservation (32). We show presently that for both PNe NGC 40 and 7662, X-ray emissions only take up ≤1 per cent of the inner fast wind energy input. Calculations show Rw ≈ 1000 au when L X reaches ~1 per cent of Mfw v2 w,u (i.e. L X = 1011 erg s−1). This appears consistent with the earlier estimate of Frankowski & Soker (2009).
4 OBSERVATIONS OF SAMPLE PLANETARY NEBULAE

4.1 Planetary nebula NGC 40

NGC 40 (PNG120.0+09.8; Acker et al. 1992) is a very low excitation PN powered by a WC8 star evolving towards a white dwarf. Within a ring of nebulosity revealed by optical and near-infrared images of NGC 40, Montez et al. (2005) have detected faint, diffuse X-ray emission distributed as a partial annulus using the data from the Chandra X-ray observatory (ObsID 4480). This annulus-like X-ray profile corresponds to a fairly large ratio of \( R_s/R_c \) and thus a low X-ray luminosity (see equations 22 and 25 and Figs 1 and 2). The actual inferences are X-ray temperature \( T = (8.0 \pm 2.0) \times 10^5 \) K and total luminosity \( L_X \sim 1.5 \times 10^{30} \) \((D\ 1.0 \text{ kpc}^{-1})^2 \) erg s\(^{-1}\) of NGC 40 (Montez et al. 2005). Kastner, Montez & Balick Bruce (2008) reprocessed the X-ray data of NGC 40 and updated the luminosity value to \( L_X \sim 4.0 \times 10^{31} \) erg s\(^{-1}\), which is adopted in our model calculations. The X-ray temperature and luminosity are one of the lowest measured so far for PNe with diffuse X-ray emissions. Here, the distance to NGC 40 (\( D = 1.0 \) kpc) is estimated by Leenhagen, Hamann & Jeffery (1996).

We decompose the X-ray count distribution in terms of cylindrical harmonics in polar coordinates in the plane of the sky and demonstrate the resulting multipole spectrum in Fig. 4. This spectrum reveals a strong polar uniform component (multipole index = 0) and a distinct dipole component (multipole index = 2), which is about one-third times as strong as the uniform component. Although crude, this suggests an exploration for the X-ray brightness radial profile of NGC 40.

The observed X-ray brightness radial profile of NGC 40 is displayed in Fig. 5 (i.e. open circles with error bars). We assume a uniform X-ray emission background approximately and fit our PN model results (i.e. the dotted curve in Fig. 5) with the observed data profile. The best-fitting model gives a radius ratio of \( R_s/R_c \approx 0.8 \) and an X-ray parameter \( X = 2.83 \times 10^4 \) \((D\ 1.0 \text{ kpc}^{-1})^3 \) erg s\(^{-1}\) cm. Using equation (22), these results further lead to an X-ray luminosity from NGC 40 as \( L_X \sim 3.7 \times 10^{31} \) \((D\ 1.0 \text{ kpc}^{-1})^2 \) erg s\(^{-1}\), consistent very well with the observational inference (e.g. Kastner et al. 2008). The model fit also gives a reverse shock radius \( R_s \approx 14 \) arcsec \( D = 2.0 \times 10^{17} \) \((D\ 1.0 \text{ kpc}^{-1}) \) cm and a contact discontinuity radius \( R_c \approx 17 \) arcsec \( D = 2.5 \times 10^{17} \) \((D\ 1.0 \text{ kpc}^{-1}) \) cm, corresponding to the peak and boundary of diffuse X-ray emissions from the shocked hot bubble. In Fig. 6, we circle these model-fitted \( R_s \) and \( R_c \) on the optical and X-ray composite image of PN NGC 40. By visual inspection, \( R_s \) does pass through the X-ray brightest region and \( R_c \) roughly marks the boundary of the diffuse X-ray emission. The LZ piecewise isothermal model claims that the optically bright PN rim is located just outside the contact discontinuity radius \( R_c \) and the density becomes highest near the forward shock radius \( R_s \). Fig. 6 indicates that the forward shock is just outside the contact discontinuity surface so that the optical bright dense shell is very thin (i.e. \( R_s \gtrsim R_c \approx 17 \) arcsec). According to the spectral analysis of Sabbadin et al. (2000), the most optically luminous shell region of NGC 40 still has redshift and blueshift components at radius \( \gtrsim 17 \) arcsec, implying \( R_s \gtrsim 17 \) arcsec. Fig. 6 shows that our model fits the observations very well. The radial separation between the most optically bright radius at \( \sim 17 \) arcsec and the most X-ray bright radius at \( \sim 14 \) arcsec is clearly distinguishable, strongly supporting the LZ piecewise isothermal model. In contrast, the adiabatic model of Chevalier & Imamura (1983) would claim no significant separation between these two radii (e.g. Akashi et al. 2006).

The best-fitting Raymond–Smith thermal plasma emission model (Raymond & Smith 1977) with the X-ray spectrum of NGC 40 indicates a plasma temperature of \( T_X = (8.0 \pm 2.0) \times 10^5 \) K (Montez et al. 2005). At such a temperature, the spectral-line cooling from highly ionized metals is important while the thermal bremsstrahlung emission from free hot electrons is relatively weak, and the total X-ray emissivity is \( \Lambda \approx 2.0 \times 10^{-22} \text{ erg cm}^{-3} \) s\(^{-1}\) (e.g. Sarazin 1986). With \( m_{p} = m_{e} \), we have \( \beta = 3.16 \times 10^{-8} \text{ M}_\odot \text{ yr}^{-1} \) (100 km s\(^{-1}\))\(^{-1}\) using expression (4). We adopt \( v_{\text{wind}} \approx v_{\text{e}} = 30 \text{ km s}^{-1}\) as estimated by optical spectral-line analysis near the contact discontinuity surface (e.g. Sabbadin et al. 2000). Ignoring the duration that the central star stopped to blow slow AGB wind and had not yet begun to launch tenuous fast

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**Figure 4.** Multipole expansion to X-ray counts distribution of NGC 40. Equivalently, this multipole spectrum is the Fourier transformation of the polar angle distribution of X-ray brightness map into \( \phi \)-space, where \( \phi \in [0, \pi, \pi] \). The \( x \)-axis is the index of multipole and the \( y \)-axis is the normalized absolute value of multipole spectrum so that the central strength is 1.

**Figure 5.** Observed X-ray surface brightness profile (circles with error bars) and our model best-fitting curve (dotted curve) for the PN NGC 40. The data from Chandra (ObsID 4480) are taken from Montez et al. (2005). The radial profile was constructed by (i) regarding the X-ray centroid as the centre of NGC 40, (ii) summing the X-ray photon counts in each of a specified set of concentric circular annuli, (iii) taking the error in each annulus for Poisson statistics (square root of the number of photon counts), and (iv) dividing the counts and errors by the area of each annulus bin. The bin width for each annulus is chosen to be \( \sim 3 \) arcsec. A uniform background X-ray flux \( \sim 3.4 \times 10^{26} \) erg s\(^{-1}\) arcsec\(^{-2}\) is estimated in the circular region of radius \( \gtrsim 25 \) arcsec.
wind, we estimate a wind–wind dynamic interaction time-scale of $R_w/v_w \sim 3000$ yr (a comparable time-scale of $\sim 5000$ yr was estimated by Akashi et al. 2006). For the radius ratio $R_{iw}/R_c = 0.8$, we have $v_c - v_{iw} = 6.0$ km s$^{-1}$. Thus, the reverse shock moves outwards in the laboratory framework of reference. Meanwhile, equation (2) gives an inner tenuous fast wind speed $v_{w,a} = 1.1 \times 10^3$ km s$^{-1}$. Equation (4) gives a central star mass-loss rate as $M_{fw} \approx 2.4 \times 10^{-4}$ $M$_{$\odot$} yr$^{-1}$, which is very close to an earlier estimate of $\sim 3 \times 10^{-4}$ $M$_{$\odot$} yr$^{-1}$ by ultraviolet spectral analysis. This falls within the estimated range of $\sim 10^{-6} - 10^{-7}$ $M$_{$\odot$} yr$^{-1}$ (e.g. Cerruti-Sola & Perinotto 1985). By imposing condition (20), the diffuse X-ray emission from the central zone of PN NGC 40 requires $T_d = 1.5 \times 10^4 (10^{-8}/\alpha_8)$ K for the outer shocked dense shell. We note that the X-ray emission only takes up $\lesssim 0.2$ per cent of $M_{fw}v^2_{w,a}$ for PN NGC 40.

However, our best-fitting model in Fig. 5 overestimates the X-ray brightness for smaller radii around the centre. In the LZ model, the temperature of the dense shell region is not high enough to radiate in X-ray bands. In reality, thermal conduction inevitably occurs across the contact discontinuity so that the dense shell region is heated up by the inner shocked downstream wind, and this may lead to partial X-ray emissions. The inner shocked downstream wind zone near the contact discontinuity surface could also become denser and cooler as a result of diffused materials and thermal conduction from the dense shell, especially if the magnetic field is absent (e.g. Steffen, Schönberner & Warmuth 2008). This effect would lead to a relatively higher X-ray flux than our model result around the outer boundary of the hot bubble. Nevertheless, this thermal conduction is expected to be sensitively suppressed even for the presence of a weak magnetic field transverse to the radial direction (e.g. Chevalier 1997; Yu et al. 2006; Wang & Lou 2008; Lou & Hu 2010; LZ), so the total X-ray luminosity of PNe is more or less confined to emissions within radius $R_c$. Within the radius of $\sim 2$ arcsec or $\sim 3.0 \times 10^{16}$ cm or $\sim 2000$ au around the central region, the X-ray brightness of our model is $\sim 75$ per cent greater than the upper limit of the confidence interval of observation (see Fig. 5). Because the reverse shock is at $\sim 14$ arcsec, within which there is no diffused X-ray emission, the projected X-ray photons within radius $\sim 2$ arcsec are actually emitted in the hot bubble at larger radii close to $R_c$ but projected around the centre along the line of sight. If there is a dense dust cloud surrounding the central star, the X-ray photons emitted behind the cloud might be partially absorbed so that the observed central X-ray brightness becomes lower than expected. Both theoretical calculations (e.g. Frankowski & Soker 2009) and observations show that dense gas/cold dust clouds may exist around the central white dwarf of PNe with diameter $\gtrsim 150$ au (e.g. the Helix nebula, dust density $\sim 2.5$ g cm$^{-3}$ by Su et al. 2007). The cross-section of such a dust cloud is about $2 \times 10^3$ au$^2$, which can only absorb $\lesssim 0.5$ per cent X-ray emission within $\sim 2000$ au radius maximally. If this is the case for PN NGC 40, Fig. 5 requires a column electron number density of $\sim 2.7 \times 10^{14}$ cm$^{-2}$ within $\sim 2$ arcsec to absorb $\sim 80$ per cent X-ray emissions behind the central star by Thomson scattering. Such a high column density corresponds to a total cloud mass $\sim 5$ M$_{\odot}$. We thus conclude that X-ray absorption by either a dust or gas cloud cannot explain the unusually low X-ray brightness in the centre of NGC 40. Alternatively, the non-spherical structure of NGC 40 might be the most important factor.

The dipolar structure of the X-ray brightness is noticeable in PN NGC 40. Also, the eastern X-ray region is brighter and seems to be more confined to within the brighter optical rim. As a first approximation, our spherically symmetric treatment modelling here ignores these asymmetries.

### 4.2 Planetary nebula NGC 7662

NGC 7662 appears to be a quasi-spherical PN with a moderately large diameter of $\sim 30$ arcsec or $\sim 4 \times 10^{17}$ cm and with optical surface brightness but without X-ray data available up to the present. Guerrero et al. (2004) probed the structure and kinematics of NGC 7662 based on long-slit echelle spectroscopic observations and on Hubble Space Telescope (HST) archival narrow-band images. They inferred that PN 7662 consists of a central cavity surrounded by two concentric shells (i.e. the shocked dense shell and the outer slow AGB wind envelope). They estimated the temperatures of the two shells as $\sim 1.4 \times 10^4$ K and $\sim 1.1 \times 10^4$ K, respectively. By fitting the radial density profiles of the dense shell zone (see fig. 13 of LZ) and the slow AGB wind envelope with the self-similar dynamic evolution model of LZ for isothermal voids, LZ estimated the relevant physical parameters for PN NGC 7662. These appear to be consistent with optical observations (e.g. Guerrero et al. 2004). LZ have also inferred the properties of the inner fast wind by imposing condition (20) for the pressure balance across the contact discontinuity. The shocked downstream inner wind velocity $v_{w,d} = 25.9$ km s$^{-1}$ was inferred by the outer dense shell expansion. The innermost fast wind mass-loss rate $\dot{M}_{fw} \approx 2 \times 10^{-6}$ M$_{\odot}$ yr$^{-1}$, the inner fast wind speed $v_{w,a} = 1500$ km s$^{-1}$ and the hot bubble (i.e. the shocked downstream inner wind zone) plasma temperature $\sim 6.4 \times 10^4$ K and the reverse shock velocity $v_{rs} = \sim 10$ km s$^{-1}$ represent a plausible set of fitting parameters that can reproduce a PN with more or less the same outer zone of NGC 7662. At the current stage of evolution, we have estimated the two radii $R_i = 5.1 \times 10^{16}$ cm and $R_o = 6.5 \times 10^{15}$ cm (i.e. the radius ratio $R_o/R_i = 0.127$ is
Figure 7. Piecewise isothermal model solution properties of the inner wind zone are explored for the PN NGC 7662. Panel A shows a chosen set of contour curves for \( \log_{10}(M_{fw}/[1 M_\odot/yr]) \) in units of \( M_\odot \) yr\(^{-1} \) in a \( v_{rs} \) versus \( T_{w,d} \) presentation. Panel B shows a chosen set of contour curves for the inner fast wind speed \( v_{w,a} \) in units of 1000 km s\(^{-1} \) in a \( v_{rs} \) versus \( T_{w,d} \) presentation. The \( T_{w,d} \) axes of both panels are in the logarithmic scale. As discussed in the main text, the example model solution with parameters \( v_{rs} = -10 \) km s\(^{-1} \), \( T_{w,d} = 6.4 \times 10^6 \) K, \( M_{fw} = 2 \times 10^{-8} M_\odot \) yr\(^{-1} \) and \( v_{w,a} = 1500 \) km s\(^{-1} \) is marked here by the plus symbol.

Figure 8. Energy solutions of the inner wind zone properties for the PN NGC 7662. Following Fig. 7, here we show the following. Panel A shows a chosen set of contour curves for \( \log_{10}(M_{fw} v_{w,a}^2) \) in units of erg s\(^{-1} \) in a \( v_{rs} \) versus \( T_{w,d} \) presentation. Panel B shows a chosen set of contour curves for \( \log_{10}(\dot{L}_X/(M_{fw} v_{w,a}^2)) \) in a \( v_{rs} \) versus \( T_{w,d} \) presentation. Panel C shows a chosen set of contour curves for \( \log_{10}(\dot{L}_X/(M_{fw} v_{w,a})) \) in a \( v_{rs} \) versus \( T_{w,d} \) presentation. For computing \( \dot{L}_X \), a minimum \( R_{in} = 10 \) au is adopted. The heavy solid curve in panel B marks the upper limit given by ROSAT observations. The bottom-left region to the heavy solid curve gives an X-ray luminosity higher than the upper limit, which is forbidden from the energetic consideration. The special model solution \( v_{rs} = -10 \) km s\(^{-1} \), \( T_{w,d} = 6.4 \times 10^6 \) K, \( M_{fw} = 2 \times 10^{-8} M_\odot \) yr\(^{-1} \), \( v_{w,a} = 1500 \) km s\(^{-1} \) is here marked by the plus symbol.

The total X-ray emissivity is \( \Lambda \approx 5.1 \times 10^{-23} \) erg cm\(^{-3} \) s\(^{-1} \) for a plasma of temperature \( 6.4 \times 10^6 \) K (e.g. Sarazin 1986). For \( m_\perp = m_p \), we then have an X-ray parameter \( X \approx 1.42 \times 10^{10} \) erg s\(^{-1} \) cm\(^{-1} \). This would give a total X-ray luminosity of NGC 7662 as \( \dot{L}_X \approx 2.4 \times 10^{11} \) erg s\(^{-1} \). In short, we predict that an X-ray surface brightness, such as the upper-left plot of Fig. 1, with no distinct reverse shock surface and a total X-ray luminosity similar to that of NGC 40 is anticipated for NGC 7662. This should be detectable by Chandra.

However, without available X-ray observations of NGC 7662, the physical conditions of its inner wind zone confined within the contact discontinuity radius \( R_c \) of the PN NGC 7662 cannot be uniquely determined at this stage of model analysis. In other words, in terms of fitting available observations of NGC 7662 in other bands (e.g. optical, infrared bands, etc.), we can predictively explore X-ray emissions from the inner shocked downstream wind plasma of NGC 7662. In the above procedure, the innermost fast wind mass-loss rate is a key yet adjustable parameter and may vary within a sensible range. We now show the corresponding ranges of our model results for NGC 7662 in Figs 7 and 8. In fact, the LZ ISVV model solution with a parameter set \( u_{rs} = -10 \) km s\(^{-1} \), \( T_{w,d} = 6.4 \times 10^6 \) K, \( M_{fw} = 2 \times 10^{-8} M_\odot \) yr\(^{-1} \), \( v_{w,a} = 1500 \) km s\(^{-1} \) is one of the plausible inner shocked wind solutions that meet the pressure balance condition (20) across the contact discontinuity. By systematically varying the inner downstream shocked wind temperature \( T_{w,d} \) and the reverse shock speed \( u_{rs} \), we readily derive different innermost fast wind velocities \( v_{rs} \) using equation (2) and the mass-loss rates \( M_{fw} \) of the central star with the wind pressure balance condition (20) across the contact discontinuity, as shown in Fig. 7. To be specific, we pick a solution example marked by the plus symbol + in Fig. 7.

Fig. 8 presents the upper limit of X-ray luminosity \( \sim M_{fw} v_{w,a}^2 \) and the X-ray luminosity \( \dot{L}_X \) by equation (22) and the ratio between these two quantities. The ROSAT Position Sensitive Proportional Counter (PSPC) count rate limit is \( \sim 0.03 \) s\(^{-1} \). If NGC 7662 is not detected in the ROSAT All-Sky Survey (RASS), then webPIMMS (http://heasarc.gsfc.nasa.gov/Tools/w3pimms.html) puts the upper limit for the X-ray luminosity at \( \sim 2.2 \times 10^{32} \) erg s\(^{-1} \) (a typical hydrogen column number density \( n_H \approx 10^{21} \) cm\(^{-2} \) and a hot bubble temperature \( T_{w,d} = 2 \times 10^6 \) K are assumed for NGC 7662 at a distance of \( D = 1 \) kpc; e.g. Guerrero et al. 2004). The heavy solid curve in panel B of Fig. 8 marks this upper limit and suggests a possible regime for both \( v_{rs} \) and \( v_{w,a} \) at the upper-right side of the curve. In the regime where \( T_{w,a} \geq 10^6 \) K and \( v_{rs} \geq -10 \) km s\(^{-1} \), the ratio \( \dot{L}_X/(M_{fw} v_{w,a}^2) \) is less than \( \sim 10^{-2} \) per cent. This estimate is also valid for PN NGC 40. Comparing Figs 7 and 8, we can also calculate that the mass-loss rate \( M_{fw} \sim 10^{-7.5} M_\odot \) yr\(^{-1} \) and the fast wind velocity...
$v_{\text{w}} \gtrsim 10^3 \text{ km s}^{-1}$. These estimated results all appear consistent with previous observations and theoretical calculations.

5 SUMMARY AND DISCUSSION

We invoke the recently constructed ISSV hydrodynamic model of LZ to explore the dynamic shock interaction of the inner isothermal fast wind with the outer AGB slow dense wind and to calculate diffuse X-ray emissions from grossly spherical PNe for available observational comparisons. Our piecewise isothermal wind dynamic interaction model involves a forward shock into the outer AGB slow wind envelope, a reverse shock into the inner fast tenuous wind and a contact discontinuity in between these two shocks. Unlike previous dynamic models, such as that of Chevalier & Imamura (1983), the LZ model indicates that the X-ray emission peaks near the reverse shock, instead of at the contact discontinuity surface between the hot bubble and the PN dense shell. Based on the model of Chevalier & Imamura (1983), Akashi et al. (2006) indicate that diffuse X-ray emissions from hot bubbles should be in the form of narrow bright rings. Our piecewise isothermal model calculations show that the morphology of diffuse X-ray emission is determined by the radius ratio $R_{\text{rs}}/R_{\text{c}}$ such that either a ring-like or central bright X-ray morphology can appear. According to our dynamic model analysis, a relatively small $R_{\text{rs}}/R_{\text{c}}$ ratio is suggested for PN BD +30°3639, where a centrally bright diffuse X-ray emission morphology persists (e.g. Kastner et al. 2000; Leahy et al. 2000). In contrast, an adiabatic evolution model of a PN cannot give a morphology for central bright diffuse X-ray emissions. We would expect various morphological cases for a more systematic X-ray survey of PNe.

As current X-ray detectors in space have been powerful enough to observe extended X-ray morphologies of PNe, we offer quantitative line-of-sight X-ray brightness radial profiles for candidate PN sources. In our model framework, the X-ray parameter $X$ (defined in the main text), the reverse shock radius $R_{\text{rs}}$, and the contact discontinuity radius $R_{\text{c}}$ together completely determine both the X-ray luminosity and the surface brightness profile. In summary, X-ray observations together with optical and infrared images of grossly spherical PNe are expected to give specific measurements or estimates for these three key parameters.

The PN NGC 40 radiates intensely in ultraviolet bands, giving a valuable observational platform to test and calibrate our X-ray model results for diagnostics. The results of our piecewise isothermal dynamic wind interaction shock model are fitted well with observations, except that the central X-ray brightness is somewhat higher than that actually observed, as shown in Fig. 5. NGC 40 appears to have a radius ratio $R_{\text{rs}}/R_{\text{c}}$ close to unity with a relatively low temperature of $\sim 10^6$ K in the inner shocked downstream wind plasma. These two facts result in a fairly low X-ray luminosity of $\sim 4.0 \times 10^{31}$ erg s$^{-1}$ from NGC 40 (see Kastner et al. 2008 for this update). Ultraviolet observations estimate a fast wind speed range of $\sim 1800$–2370 km s$^{-1}$ (e.g. Cerruti-Sola & Perinotto 1985; Bianchi 1992). This is about twice our estimated fast wind speed $\sim 1100$ km s$^{-1}$. We also note that the dipolar structure of NGC 40 significantly affects the morphology of diffused X-ray emission. The actual hot bubble of NGC 40 is not very spherical, but is a dumbbell-like shape. Most X-ray photons are emitted from east and west regions, while only a few are from north, south or central regions. Observationally, spectral-line analysis to the optical bright rims shows that the dense shell gas expands at $\sim 40$–50 km s$^{-1}$ in the north–south direction and at $\sim 30$ km s$^{-1}$ in the east–west direction (e.g. Sabbadin et al. 2000). This indicates that the earlier AGB slow wind is denser in the east–west direction so that the collision between the inner fast wind and the slow AGB wind is more violent in the east–west region. This also explains why the dense shell is brighter in the east and west regions. By both quantitative calculation on spherical smoothing and qualitative analysis on dipolar structure, we conclude that wind–wind shock dynamic interaction appears natural for the hot bubble formation and diffused X-ray emissions.

Unlike the PN NGC 40, the PN NGC 7662 is here inferred to have an inward reverse shock velocity of $\sim -10$ km s$^{-1}$. By expression (22), NGC 7662 is predicted to have an increase of X-ray luminosity with increasing time $t$ at an estimated rate of $\sim 0.9 \times 10^{33}$ erg s$^{-1}$ yr$^{-1}$. For example, its X-ray luminosity will reach $\sim 2.1 \times 10^{30}$ erg s$^{-1}$ 20 yr later and $\sim 2.5 \times 10^{30}$ erg s$^{-1}$ 50 yr later. Energy conservation shows a minimum $L_X$ exists so that $L_X$ must be limited by a maximum $\lesssim M_{\text{fw}} v_{\text{w}}^2$ for the time $\lesssim 0.9 \times 10^3$ yr later. The sensible explanation is that the cooling effect becomes very significant even when $L_X$ is still far lower than this upper limit.

At the present epoch, it appears justifiable not to include cooling effects in our piecewise isothermal dynamic model calculations. Akashi et al. (2006) considered the radiative cooling semi-quantitatively for computing the X-ray luminosity based on the self-similar colliding adiabatic wind model of Chevalier & Imamura (1983). They showed that the radiative cooling was unimportant during most of the time of PN dynamic evolution. Nevertheless, as radiative cooling becomes important for the hot bubble evolution (e.g. $\sim 10^3$ yr later for the case of $v_{\text{w}} = -3$ km s$^{-1}$ in Fig. 3), it would be inaccurate to compute X-ray emissions either in our model or in that of Akashi et al. (2006). The X-ray hot bubble will cool down much faster, the contact discontinuity balance condition would be broken and the outer dense shell dynamic behaviour may also be interrupted.

As X-ray luminosity only takes up $\lesssim 1$ per cent of $M_{\text{fw}} v_{\text{w}}^2$, the energy conservation consideration in Section 3 implies that a significant amount of energy should have been converted to other forms during the entire life of the PNe. Otherwise, the temperature of the shocked downstream inner wind should be $\sim 10$ times higher than the observed temperature inferred from diffuse X-ray emissions. However, relatively low temperatures of X-ray emitting gas are generally inferred in PNe at different stages of evolution. We note that the energy conservation calculation in Section 3 is fairly general, where we focus on the mass conservation $M_{\text{fw}} = 4\pi r^2 \rho_{\text{w},d} v_{\text{w},d}$ and choose typical observed parameter values without specifically invoking either isothermal or adiabatic assumptions. Therefore, we suspect that there must be certain long-lasting mechanisms to explain the low temperature of X-ray emitting gas, and such mechanisms are not quantitatively taken into account in either the LZ piecewise isothermal model or the adiabatic model of Chevalier & Imamura (1983). Moreover, such mechanisms have more influence than radiative cooling, so that shocked hot bubbles do not evolve adiabatically, as implied by the observation of central bright X-ray emission in PN BD +30°3639. Thermal conduction and material diffusion might be non-negligible if the magnetic field near the expanding contact discontinuity surface becomes sufficiently weak. Another possibility is that the hot bubble largely between the reverse shock and the contact discontinuity becomes turbulent to partially account for the missing energy. The thermal conduction between the hot bubble and cool dense shell could be considerable (e.g. Kastner et al. 2000; Steffen et al. 2008). The rapid variance of the inner fast wind mass-loss rate of central stars might also be essential to account for the imbalance between the fast wind.
kinetic energy input and the low temperature inferred by diffuse X-ray emissions (e.g. Soker & Kastner 2003; Akashi et al. 2007). In Sections 2 and 3, we did not consider such mechanisms; instead, we have chosen the shocked fast wind temperature $T_{w,d}$ (or $T$) as an adjustable parameter for fitting X-ray observations.

In some PNc with detected X-ray emissions, a very distinct bipolar morphology appears, such as HENIZE 3-1475 (e.g. Sahai et al. 2003), NGC 7026 (e.g. Gruendl et al. 2006), MENZEL 3 (e.g. Kastner et al. 2003), and so forth. Collimated outflows and jets in magnetized gas media might be expected to account for these types of diffuse X-ray emissions. Akashi, Meiron & Soker (2008) performed two-dimensional numerical simulations of jets expanding into the slow wind of AGB stars. They proposed that this jet–wind or outflow–wind interaction model might explain such bipolar morphology of diffuse X-ray emissions.

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