Cosmological imprints of SUSY breaking in models of sgoldstinoless non-oscillatory inflation

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Abstract. In supergravity, the dynamics of the sgoldstino – superpartner of the goldstino superfield associated with the breaking of supersymmetry at low energy – can substantially modify the dynamics of inflation in the primordial Universe. So-called sgoldstinoless models assume the existence of a nilpotency constraint \( S^2 = 0 \) that effectively removes the sgoldstino from the theory. Such models were proposed to realise non-oscillatory inflation scenarios with a single scalar field, which feature a long period of kination at the end of inflation, and therefore a non-standard post-inflationary cosmology. Using effective operators, we propose models in which the sgoldstino is stabilized close to the origin to reproduce the nilpotent constraint. We show that small sgoldstino fluctuations may lead to a sizeable back-reaction on the cosmological history. We study the effect of this back-reaction on the inflation observables measured in the cosmic microwave background and confront the model to a series of constraints including limits on \( \Delta N_{\text{eff}} \). We show that the peculiar form of the potential in the large supersymmetry breaking scale limit can generate peaks in the scalar power spectrum produced from inflation. We study how certain perturbation modes may re-enter the horizon during or after kination and show that a large supersymmetry breaking scale may lead to the formation of primordial black holes with various masses in the early Universe.

Keywords: inflation, supersymmetry and cosmology, primordial black holes

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1 Introduction

In 1973, Volkov and Akulov understood that at energies below the scale of supersymmetry breaking $f_0$, supersymmetry (SUSY) is non-linearly realized [1]. Even though such a realization of SUSY can be understood from a purely geometrical point of view [1, 2], it is also possible to recover non-linear SUSY from a linear realization of SUSY, by imposing certain multiplicative constraints on the superfields of the theory, such as a nilpotency constraint on the Goldstino superfield, $S^2 = 0$ [3–5]. Effectively, the effect of such constraints is to impose the decoupling of heavy states below the scale of supersymmetry breaking, recovering nonlinear supersymmetry in an effective low-energy action. In the context of single-field inflation, such constraints represent a powerful tool to write down simple scalar theories that can be used to describe early cosmology in a simple manner [6–10]. In the context of string theory, the emergence of such constraints has been extensively studied [11–20]. Nevertheless, it is generically difficult in such UV embeddings to restore linear SUSY above the scale $f_0$, meaning that the scalar component of the goldstino $S$ satisfying the constraint $S^2 = 0$ simply does not exist. From a dimension-four quantum field theory point of view, the essence of such constraints remains obscure. The authors of refs. [5, 21] showed that goldstino-less theories, and more generally theories with constrained superfields, can be derived as an effective field theory, by using large non-renormalizable operators. However, it was shown in refs. [22–24] that the large Wilson coefficients limit required in order to obtain an exact nilpotency constraint conflicts with the idea that such operators could originate from an effective field theory. As a matter of fact, the excursions of the goldstino to non-zero field values during inflation can have a drastic impact on the inflationary dynamics and on the subsequent cosmological observables [24, 25].
In the vast majority of inflation models, the inflaton oscillates around the minimum of its potential after inflation, and eventually decays into Standard Model (SM) particles, leading to the hot big bang phase of the Universe’s history. However, there exists a subclass of models in which the inflaton ends up rolling along a very flat portion of its potential after inflation ends. This eternal roll of the inflaton field, which eventually leads to a Slow-Roll (SR) regime due to Hubble friction, may provide an explanation for the nature of dark energy in so-called quintessential inflation models [26–43]. More generally, models of non-oscillatory inflation consider the possibility that the inflaton may simply not oscillate around a minimum at the end of inflation, but, instead, may keep rolling on a flat potential region [37, 44, 45].

Sgoldstino-less models of α-attractors happen to be extremely useful to construct inflation models in SUGRA, in which the inflaton can roll along a flat direction after the end of inflation, and where supersymmetry is naturally broken during and after inflation [26, 46, 47]. However, obtaining the nilpotency constraint on the goldstino superfield from an effective field theory description in the context of non-oscillatory inflation models remains, up to now unexplored. In this paper, we aim to construct effective field theories of supergravity, in which the sgoldstino is naturally stabilized, and where non-oscillatory inflation can be successfully realized. Interestingly, non-oscillatory inflation generally features a long period of kination, an era during which the energy density is dominated by the kinetic energy of the inflaton. We will see that the existence of such an era, combined with the peculiar dynamics of the inflaton field in the presence of SUSY breaking, leads to interesting cosmological signatures. The latter could act as smoking gun evidences of such models in cosmological data.

The paper is organized as follows: In section 2 we introduce the framework in which we work throughout the paper. In particular, we present two classes of inflation models whose cosmological imprints and consequences are studied. In section 3 we exhibit the predictions of these models regarding the perturbation modes in the Cosmic Microwave Background (CMB) and compare them with several constraints, including limits on $\Delta N_{\text{eff}}$. Finally, in Sec. 4 we show that our models can predict the existence of enough primordial black holes to account for the full relic density of dark matter or simply perturb the post-inflationary history of the Universe by dominating the energy density before evaporating.

2 Supergravity model of non-oscillatory inflation with SUSY breaking

In order to easily construct inflation potentials in the presence of a nilpotency constraint $S^2 = 0$ for the goldstino superfield, we use the framework developed in ref. [48].

We parameterize our model by the following superpotential and Kähler potential respectively

$$W = \left(1 - \frac{S}{\sqrt{3}}\right)^3 f(Z)$$  \hspace{1cm} (2.1)  
$$K = K_1(Z, \bar{Z}) - 3 \log \left[1 - \frac{|S|^2}{3} + \frac{|S|^4}{\Lambda^2}\right]$$,  \hspace{1cm} (2.2)

where $f(Z)$ is a holomorphic function that satisfies [48]

$$f(Z) = f(- Z), \quad f(0) \neq 0,$$  \hspace{1cm} (2.3)

and where the chiral superfield $S$ denotes the goldstino superfield, responsible for the breaking of SUSY [48]. The chiral superfield $Z$ contains the inflaton and the UV scale $\Lambda$ stabilizes...
the sgoldstino during the whole history of the Universe, ensuring that the constraint $S^2 = 0$ is approximately satisfied. Throughout the paper, we work in units of reduced Planck mass, where $M_{\text{Pl}} = 1$.

For the models we consider in the next sections, we checked both analytically and numerically that the fields $\text{Re}(Z)$ and $\text{Im}(S)$ have masses larger than the Hubble scale and remain stabilized at zero at all times during the fields evolution. Therefore, we can safely restrict our study to the dynamics of the two remaining real degrees of freedom, $\text{Im}(Z) = z$ and $\text{Re}(S) = s$. Following ref. [48], eq. (2.3) implies that

$$f'(-iz) = -f'(iz), \quad f(-iz) = f(iz), \quad (2.4)$$

where we have used the prime in order to denote the derivative with respect to the argument of the function unless the variable is explicitly stated.

In the purely nilpotent limit, the stabilizing term $|S|^4/\Lambda^2$ is sent to infinity, while the sgoldstino $s$ is assumed to vanish at all times. We will denote by $V_{\text{nil}}(z)$ the potential obtained in this limit. During inflation, SUSY is broken in both the sgoldstino direction and the inflaton direction, whereas after inflation ends, SUSY is broken mainly along the $S$ direction. The gravitino mass squared is then given by

$$m_{3/2}^2 \simeq |f(iz)|^2 = f(iz)^2 \quad (2.5)$$

After one relaxes the pure $S^2 = 0$ constraint, it was shown in ref. [25] that $s$ can acquire a non-zero vacuum expectation value during inflation, which may affect the inflationary dynamics. The value of the sgoldstino vev $\langle s \rangle$ can be expressed as a function of the purely nilpotent potential $V_{\text{nil}}(z)$ as [25]

$$\langle s \rangle = \frac{\Lambda^2 V_{\text{nil}}}{2\sqrt{3} \left[ \Lambda^2 V_{\text{nil}} + 6 m_{3/2}^2 \right]} . \quad (2.6)$$

The scalar potential then acquires a correction from the non-zero vev of $s$ during inflation and the late time expansion, as follows

$$V(z, s) = V_{\text{nil}}(z) + sV_1(z) + s^2V_2(z) + \ldots \quad (2.7)$$

where we have expanded up to the second order in small $s$, and $V_j(z)$ are given by

$$V_1(z) = -2\sqrt{3}V_{\text{nil}}(z) , \quad (2.8)$$

$$V_2(z) = \frac{36 m_{3/2}^2 + 6 \Lambda^2 V_{\text{nil}}}{\Lambda^2} , \quad (2.9)$$

and the sgoldstino mass squared is $m_s^2 = V_2$. Using the non-zero vev of $s$ (2.6), the inflaton effective potential takes the form

$$V_{\text{eff}}(z) = V_{\text{nil}}(z) - \frac{\Lambda^2 V_{\text{nil}}(z)^2}{2 \left[ 6 m_{3/2}^2 + \Lambda^2 V_{\text{nil}}(z) \right]} . \quad (2.10)$$
2.1 Model I — $\alpha$-attractors

The first model we consider is an $\alpha$-attractor, which can be realized by taking the Kähler potential to be of the form

$$K_1 = -\frac{3\alpha}{2} \log \left( \frac{(1 - |Z|^2)}{(1 + Z^2)(1 + \overline{Z}^2)} \right),$$

(2.11)

where $\alpha$ is a dimensionless parameter. In this case, the purely nilpotent potential can be computed using the $s = 0$ limit:

$$V_{\text{nil}} = \frac{(1 - z^2)^2}{3\alpha} |f'(iz)|^2 = -\frac{(1 - z^2)^2}{3\alpha} [f'(iz)]^2.$$

(2.12)

In order to obtain a potential able to describe a non-oscillatory inflation, we consider the function $f(Z)$ by following the method of ref. [48]. Hence we have

$$f(Z) = f_0 - \sqrt{V_0} \log(1 - iZ).$$

(2.13)

Note that this choice is not unique. However, as we will see in what follows, the qualitative consequences arising from the dynamical behaviour of the sgoldstino along the cosmological timeline are relatively generic. Thanks to the set up proposed in ref. [48] that we have chosen here, it is remarkable that the scale of inflation $V_0$ is disconnected from the scale of SUSY breaking at low energy $f_0$ since the inflation potential is mainly sourced by $f'(iz)$, as we will see shortly. In order to normalize the inflaton field, we perform the following redefinition

$$z = \tanh \left( \frac{\varphi}{\sqrt{6\alpha}} \right).$$

(2.14)

Therefore, in the nilpotent limit, the scalar potential takes the form

$$V_{\text{nil}} = \frac{V_0}{3} \left[ \tanh \left( \frac{\varphi}{\sqrt{6\alpha}} \right) - 1 \right] = \frac{4V_0}{3\alpha \left( e^{\sqrt{\frac{2}{6\alpha}} \varphi} + 1 \right)^2},$$

(2.15)

which features two different plateaus with a hierarchical difference of order $V_0$ as can be seen from figure 1. Note that in this case, the plateau located at asymptotically large field values is at zero, meaning that another mechanism should be invoked in order to uplift the potential to a positive cosmological constant value $\Lambda_{\text{DE}} \sim 10^{-120} M_{\text{Pl}}^4$ and address the cosmological constant problem. We do not address this issue in this paper and instead assume that some other mechanism will be responsible for the value of the cosmological constant today. We instead focus on the post-inflationary cosmology, which takes place between the end of inflation and the matter-radiation equality. Nevertheless, it can be noted that a cosmological seesaw mechanism is operating in our model which naturally sources a large energy density $H^2 \sim V_0/3$ during inflation and suppresses the potential energy of the inflationary sector when $\varphi \to 0$. After relaxing the exact nilpotency condition $S^2 = 0$ whilst keeping a finite scale $\Lambda$ in the lagrangian, the scalar potential acquires a correction due to the non-zero vev of $s$ and the effective potential takes the form

$$V_{\text{eff}}(\varphi) = \frac{4V_0}{3\alpha \left( e^{\sqrt{\frac{2}{6\alpha}} \varphi} + 1 \right)^2}$$

$$- \frac{4\Lambda^2 V_0^2}{27\alpha^2 \left( e^{\sqrt{\frac{2}{6\alpha}} \varphi} + 1 \right)^4 \left[ f_0 - \sqrt{V_0} \log \left( \frac{2}{e^{-\sqrt{\frac{2}{6\alpha}} \varphi} + 1} \right) \right]^2 + \frac{2\Lambda^2 V_0}{9\alpha \left( e^{\sqrt{\frac{2}{6\alpha}} \varphi} + 1 \right)^2}}.$$
V_{nil}(\phi), V_{eff}(\phi), f_0 = 10^{-15}
V_{eff}(\phi), f_0 = 10^{-6}

-2 -1 1 2 \phi
2
4
6
8
V(\phi)/V_0

Figure 1. The effective scalar potential of model I, V_{eff}(\phi), in orange and green colors with V_0 = 1.5 \times 10^{-12}, \alpha = 1/6, and changing f_0 with fixing \Lambda = 0.1 in the left panel, while changing \Lambda and fixing f_0 = 10^{-15} in the right panel. The blue dashed curve corresponds to the nilpotent limit. All values are given in the units where M_{Pl} = 1.

2.2 Model II — Canonical Kähler

The second model that we consider simply features a canonical Kähler potential where a shift symmetry is applied in order to to avoid the \eta-problem\footnote{The \eta-problem arises usually in supergravity models of inflation, in which the scalar potential has an overall factor e^K. In the absence of shift symmetry, large field values for the inflaton lead to a large curvature of the inflaton potential, and the inflaton can acquire a mass of the order of the Hubble scale. Hence the slow-roll parameter \eta = V''/V becomes larger than one, which violates the slow-roll conditions needed to inflate the early Universe.}

K_1 = \frac{1}{2} (Z + \bar{Z})^2.

(2.17)

Repeating the procedure described above, one can express the inflaton potential in the exact nilpotent limit as

V_{nil} = |f' (iz)|^2 = -f' (iz)^2,

(2.18)

Using again the method of ref. [48], we choose the function f(Z) to be

f(Z) = f_0 - \sqrt{V_0} \log \left(1 + e^{2\sqrt{3}ibZ}\right).

(2.19)

Again V_0 represents the inflation scale while the SUSY breaking scale is given by f_0, b is a dimensionless parameter, and we normalize the field by redefining z = \frac{\phi}{\sqrt{2}}. In the nilpotent limit, the scalar potential becomes

V_{nil} = \frac{8b^2V_0}{(e^{2b\phi} + 1)^2},

(2.20)

and has again two plateaus, one at the inflation scale V_0 and the other at 0, as depicted in figure 2. After the sgoldstino dynamics is turned on, the effective potential becomes

V_{eff}(\phi) = \frac{8b^2V_0}{(e^{2b\phi} + 1)^2} - \frac{16b^4\Lambda^2V_0^2}{(e^{2b\phi} + 1)^4 \left(\frac{4b^2V_0\Lambda^2}{(e^{2b\phi} + 1)^2} + 3 \left(f_0 - \sqrt{V_0} \log (e^{-2b\phi} + 1)\right)^2\right)}.

(2.21)
3 From cosmic inflation to the present universe

In models of oscillatory inflation, the inflaton is expected to decay into SM particles after it oscillates at the bottom of its potential. In that case, the dynamics of the inflaton scalar field after the time of cosmic reheating may not leave any imprint in the later history of the Universe and can be safely ignored once the Universe has entered its hot big-bang phase. In the case of non-oscillatory inflation, it is extremely important to track the dynamics of the scalar field throughout the whole history of the Universe, as it can contribute significantly to the total energy density at late times and affect, for instance, predictions from Big-Bang Nucleosynthesis (BBN). Even though this possible imprint of the scalar dynamics on post-inflationary cosmology may lead to severe constraints on the inflationary model considered, it is interesting to note that having a global vision of the different phases of evolution of the Universe generally helps fixing the number of $e$-folds of inflation required to accommodate CMB measurements, leading to more precise predictions regarding the inflation observables [24, 49, 50]. In this section, we review the different stages that are endemic to such inflationary models and present the different constraints that can be set during each stage. Throughout this paper, we use the subscripts "end", "kin" and "eq" in order to refer respectively to the end of inflation, the end of kination domination that represents the onset of radiation domination era, and matter-radiation equality representing the beginning of matter domination era.

3.1 Reheating mechanism

The production of hot particles at the end of inflation in non-oscillatory models cannot be realized in a similar manner as in the oscillatory case. Indeed, in the latter case the mass of the inflaton field is given by its value at the minimum and the field undergoes multiple oscillations until $H \sim \Gamma_\phi$ when the inflaton perturbatively decays into lighter particles, where $H$ is the Hubble scale and $\Gamma_\phi$ is the total decay width of the inflaton. In the former case, however, the curvature of the potential goes to zero in the limit $\phi \to \infty$, and as the field rolls, its ability to decay vanishes accordingly. However, successful ways to reheat the Universe exist in the literature, such as instant pre-heating [51, 52], curvaton pre-heating [45, 53, 54] or Ricci reheating [55]. Whereas these different mechanisms require a certain amount of
model building, it is known that SM particles can always be produced out of the vacuum fluctuations at the end of inflation through gravitational production [56, 57]. In order to avoid adding extra ingredients to our scenario, we only consider the latter scenario in the remainder of this work.

Following ref. [32], we denote with the subscript ‘end’ the quantities defined at the time when the slow roll regime ends ($\epsilon(\varphi_{\text{end}}) \equiv 1$, where $\epsilon$ is the slow-roll parameter defined in eq. (4.2)). The energy density of radiation produced through gravitational reheating is therefore given by

$$\rho_{\text{grav}}|_{t=t_{\text{end}}} = \frac{g_{*\text{,end}}q}{1440\pi^2} \left(\frac{H_{\text{end}}}{M_{\text{Pl}}^2}\right)^2 \rho_{\text{end}}, \hspace{1cm} (3.1)$$

where $\rho_{\text{end}}$ is the energy density at the end of inflation, $g_{*\text{,end}}$ is the number of relativistic degrees of freedom present in the thermal bath at the time of reheating and $q$ is an $O(1)$ efficiency factor [32] that we take to be equal to one throughout this paper. The exact value of this parameter depends on the precise dynamics of the inflaton field around the end of inflation. However, an $O(1)$ variation of this parameter is not expected to change our results significantly. Indeed, as we will see in eq. (3.14), the number of $e$-folds of inflation only depends logarithmically on the values of the energy densities, and such variation can be safely neglected.

### 3.2 Kination era

As one can see from eq. (3.1), the fraction of energy injected into SM and DM particles at the end of inflation is much smaller than one. After inflation, the Universe therefore remains dominated by the inflaton energy density. However, because the slow-roll regime is over for $\varphi > \varphi_{\text{end}}$, and because the field simply rolls down a potential that vanishes exponentially fast, the Universe is dominated almost entirely by the kinetic energy of the scalar field. This period of so-called kination is characterized by an equation of state $w_{\text{kin}} \approx 1$ and an energy density redshifting like $a^{-6}$, where $a$ is the scale factor. In the absence of any other source of energy, the equations of motion describing the evolution of the canonically normalized scalar field are given by

$$\ddot{\varphi} + 3H \dot{\varphi} + V_{\varphi} = 0 \hspace{1cm} (3.2)$$

$$H = \sqrt{\frac{\dot{\varphi}^2}{6} + \frac{V(\varphi)}{3}} \hspace{1cm} (3.3)$$

For numerical purposes, it is convenient to define the dimensionless variables

$$x = \frac{\varphi}{M_{\text{Pl}}}, \hspace{0.5cm} y = \frac{\dot{\varphi}}{H_0 M_{\text{Pl}}} \hspace{1cm} (3.4)$$

for which eq. 3.2 splits into two first order differential equations, namely

$$x' = \frac{y}{H}, \hspace{0.5cm} y' = -3y - \frac{\ddot{V}}{H} \hspace{1cm} (3.5)$$

where the prime here denotes the derivative with respect to the number of $e$-folds $N = \ln a$, $H \equiv \frac{\dot{H}}{H^2}$, $V \equiv \frac{V}{H^2 M_{\text{Pl}}^2}$ and the Hubble parameter today is taken to be $H_0 = 67.27 \pm 0.60 \text{ km s}^{-1} \text{ Mpc}^{-1} = 5.891 \times 10^{-61} M_{\text{Pl}}$ [58]. During inflation, it is sufficient to solve this simple system of equations.
After inflation ends, we use eq. (3.1) to estimate the amount of energy produced through gravitational reheating at $\varphi = \varphi_{\text{end}}$. From that point, the matter and radiation energy densities ought to be taken into account in the Friedmann equations, yielding

$$H = \sqrt{\frac{y^2}{6} + \frac{\dot{V}(x(N)) + \dot{\rho}_{\text{rad}}(N) + \dot{\rho}_{\text{m}}(N)}{3}}. \tag{3.6}$$

In this equation, we have defined dimensionless variables for the radiation and matter energy densities:

$$\dot{\rho}_{\text{rad}}(N) = 3 \Omega_{\text{rad},0} e^{-4N}, \tag{3.7}$$
$$\dot{\rho}_{\text{m}}(N) = 3 \Omega_{\text{m},0} e^{-3N}, \tag{3.8}$$

with $\Omega_{\text{rad},0} \sim 10^{-3}$, and $\Omega_{\text{m},0} = 0.3166 \pm 0.0084$, being the current values of the radiation and matter density parameters [58]. At $\varphi = \varphi_{\text{end}}$ we impose that the energy density of particles produced gravitationally given by eq. (3.1) equals $\dot{\rho}_{\text{grav}} = \dot{\rho}_{\text{m}} + \dot{\rho}_{\text{rad}}$ where $\dot{\rho}_{\text{grav}} = \rho_{\text{grav}} / (H^2 M_{\text{Pl}}^2)$. The total energy density is then given by $\dot{\rho}_{\text{tot}} = \frac{y^2}{2} + \dot{V} + \dot{\rho}_{\text{rad}} + \dot{\rho}_{\text{m}}$ and the equation-of-state parameter of the inflaton sector $w_{\varphi}$ is given by

$$w_{\varphi}(N) = \frac{y^2/2 - \dot{V}}{y^2/2 + \dot{V}}. \tag{3.9}$$

As mentioned above, the scalar evolution then proceeds as follows:

- **Kination**: After inflation ends, the scalar field enters a phase of kination where the inflaton kinetic energy is dominant. This corresponds to $w_{\varphi} = 1$, $\rho_{\text{kin}} \propto a^{-6}$ and $a \propto \sqrt{t}$ hence $H = \frac{1}{3t}$. Accordingly, by solving eq. 3.2, we get $\varphi(N)$ during kination

$$\varphi(N) = \varphi_{\text{kin}} + \sqrt{6} M_{\text{Pl}} \left(N - N_{\text{kin}}\right), \tag{3.10}$$

where $N_{\text{kin}}$ and $\varphi_{\text{kin}}$ correspond to the beginning of kination.

- **Radiation**: When SM radiation starts dominating over the scalar field energy density, kination effectively ends and the scalar field evolves in a radiation-dominated Universe. Its evolution can be described by

$$\varphi_{\text{rad}} = \varphi_{\text{kin}} + \sqrt{\frac{2}{3}} M_{\text{Pl}} \ln \left(\frac{H_{\text{kin}}}{H_{\text{rad}}}\right), \tag{3.11}$$

where $H_{\text{rad}}^2 = \frac{2 \dot{\rho}_{\text{rad}}}{3 M_{\text{Pl}}^2}$.

- **Matter domination**: The matter-radiation equality happens at $N_{\text{eq}} = \log \left(\frac{\Omega_{\text{rad,0}}}{\Omega_{\text{m,0}}}\right)$, where the matter energy density begins to dominate the universe.

### 3.3 Late slow-roll and tracking regime

As the Universe energy density decreases during radiation domination (during which $\rho_{\text{rad}}$ decreases like $a^{-4}$), it is possible for the scalar field to enter a new regime of slow roll when $H \sim \sqrt{\dot{V}''(\varphi)}$. In that case, the scalar field behaves like a new component of dark energy, whose relative abundance grows, as compared to matter and radiation. Once this energy
component is close to dominating the energy density of the Universe, this regime of slow-roll ends naturally, and the field dynamics can end up tracking the dynamics of the background energy density, whenever it is made of matter or radiation. This is typically the case if the slope of the scalar potential is still steep enough and the field cannot just behave like a dark energy component only by itself. In models of quintessential inflation, however, the slope of the potential may vanish sufficiently fast, and although the universe enters a tracking regime where \( \rho_\varphi \) tracks radiation or matter for a while, it may start slow-rolling again and behaving like a cosmological constant at present time \( \rho_\varphi \sim \rho_{\Lambda\text{DE}} = \Lambda_{\text{DE}} M_{\text{Pl}}^2 \sim 3.47 \times 10^{-121} M_{\text{Pl}}^4 \).

Tracking solutions have been discussed in the framework of quintessence models [59] as an interesting explanation for two problems, namely, the fine-tuning problem of the dark energy, and the cosmic coincidence problem. Following ref. [60], the criteria for a tracking solution are given by

\[
\Gamma_{\text{track}} \equiv \frac{V''}{(V')^2} \geq 1 \quad \text{and} \quad \left| \frac{d \ln (\Gamma_{\text{track}} - 1)}{d \ln a} \right| \ll 1 \quad (3.12)
\]

Although we do not address the cosmological constant problem in this work, tracking regimes typically occur if the field rolls fast enough to the lower plateau of its potential, quickly allowing the scalar field to contribute up to about 20% of the total energy content of the Universe. As can be seen from figure 3, the scalar sector has an equation of state \( w_\varphi \sim 1/3 \) during radiation domination. Since the scalar field does not interact with SM particles, such a contribution during radiation domination corresponds to dark radiation and can have a dramatic impact on the dynamics of BBN or the emission of the CMB spectrum. In figure (3), we illustrate two situations in which the scalar field starts slow rolling early (left panel) before reaching a tracking regime during the radiation-dominated era, or slow rolls only later (right panel) either at 0 or at small positive values of \( N \) if a cosmological constant is added to the potential.

### 3.4 Scalar field dynamics

When solving numerically the equations of motion described above, one needs to ensure first that the scalar sector is well described by a single-field framework, and secondly that the field does not end up stuck in a metastable minimum.

Along the scalar field trajectory, it is important to keep in mind that the sgoldstino is assumed to track its potential minimum. However, there may exist situations in which the mass of the sgoldstino becomes comparable or smaller than the Hubble parameter. In that case, the sgoldstino oscillations are expected to become sizeable, which would alter the post-inflationary dynamics. In section 2 we have derived the value of the sgoldstino mass as a function of the inflaton scalar field. In our numerical calculations, we will restrict ourselves to the cases where this mass remains larger than the Hubble parameter at all times during the cosmological history.

Furthermore, we have seen that a large SUSY breaking scale can produce a bump along the inflation potential at various field values, with various sharpness and heights, depending on the different parameters of the model, like the UV cutoff scale or the \( \alpha/b \) parameter. In our numerical calculations, we do not consider parameter points for which the inflaton gets stuck because of such a bump.

### 3.5 Inflationary observables

In order to compute the inflation observables, as measured from the CMB angular power spectrum, we need to control how many e-folds of inflation is required since the CMB pivot
mode measured by Planck exited the horizon until the end of inflation, given the post-inflationary history of the Universe we consider. Cosmological scales exit the horizon at \( N_* \), given by the relation \([32]\)

\[
k = a_* H_* \Leftrightarrow e^{N_*} = 2 \frac{H_*}{H_k} \left( \frac{a_{\text{end}}}{a_{\text{kin}}} \right) \left( \frac{a_{\text{eq}}}{a_k} \right),
\]

where \( k = 0.05 \text{ Mpc}^{-1} \) is the pivot scale and \( a_k \) is the value of the scale factor at the time of horizon reentry. This equation can then be re-written under the form

\[
N_* = -\ln \frac{k}{a_0 H_0} + \frac{1}{6} \ln \frac{\rho_{\text{kin}}}{\rho_{\text{end}}} + \frac{1}{3} \ln \frac{\rho_{\text{eq}}}{\rho_{\text{kin}}} + \ln \frac{H_*}{H_{\text{eq}}} + \ln \frac{a_{\text{eq}} H_{\text{eq}}}{a_0 H_0},
\]

which is what we used in our numerical calculations to determine the value of \( N_* \) required for each point in the parameter space.

In figures 4 and 5 we present our results for the tensor-to-scalar ratio \( r \) and spectral index \( n_s \) predicted by Model I and Model II, respectively. To do this, we scanned over the parameters of the model \( \{ \alpha, f_0, \Lambda, V_0, b \} \), and fixed the normalization condition at horizon exit while satisfying eq. (3.14) numerically, to obtain the correct number of e-folds of inflation, and we excluded points for which the inflaton would be stopped in the metastable minimum exhibited in the previous sections. On the left, the logarithmic values of the SUSY breaking scale \( f_0 \) are indicated in the color bar, whereas on the right, the color bar indicates the logarithmic value of scalar relative abundance during BBN. Different points correspond to different values of the UV scale \( \Lambda \) and the parameters \( \alpha \) or \( b \). The results are compared to the most recent constraints combining Planck and Bicep/Kek data \([61]\). Interestingly, demanding that no tracking regime takes place during BBN (by demanding that \( \Delta N_{\text{eff}} < 0.284 \) according

**Figure 3.** Illustration of models of non-oscillatory inflation that lead to a tracking regime (left panel) or behave like kination until the domination of the cosmological constant arises. The Model II is used in these figures with \( f_0 = 5 \times 10^{-8}, \Lambda = 0.1 M_{\text{Pl}}, b = 0.9 \) (left panel), and \( b = 3.5 \) (right panel). Here \( N = 0 \) corresponds to the present time.
Figure 4. Predictions for the inflationary observables, the tensor-to-scalar ratio $r$ and the scalar spectral index $n_s$, in the case of Model I. The left panel indicates in the color bar the value of $\log_{10} f_0/M_{Pl}$ whereas in the right panel the color bar indicates the value of the scalar field relative abundance during BBN.

Figure 5. Predictions for the inflationary observables, the tensor-to-scalar ratio $r$ and the scalar spectral index $n_s$, in the case of Model II. The left panel indicates in the color bar the value of $\log_{10} f_0/M_{Pl}$ whereas in the right panel the color bar indicates the value of the scalar field relative abundance during BBN.

to [58]) rules out most of the points situated near the center of the CMB contours. However in the case of model I this does not set any limit on the value of the corresponding SUSY breaking scale, in the case of Model II one can notice that this imposes a lower bound on the SUSY breaking scale $f_0 \gtrsim 10^{-8}$ which suggests that a large SUSY breaking scale avoids falling in an extended tracking regime during BBN. Finally, in both models, the value of the spectral index is predicted to be larger than $n_s \gtrsim 0.966$. In the case of Model I, another interesting feature is visible, as it appears that the smaller the SUSY breaking scale $f_0$, the larger the tensor-to-scalar ratio $r$. 
4 Primordial black holes

As one can see from figures 1 and 2, one of the main effect of a large SUSY breaking scale on the inflaton potential is to distort the potential and create bumps at a location that is a function of the different parameters of each model. As we have seen in the previous sections, the presence of such a bumpy region along the scalar field trajectory can lead to a significant modification of the number of e-folds of inflation and alter the post-inflationary dynamics of the scalar field. Furthermore, it can also lead to a well-known regime of ultra slow roll that typically can be responsible for the formation of primordial black holes (PBHs) in the early Universe.\footnote{A class of scalar potentials with specific corrections were studied in ref.\cite{62}. These corrections can induce a bump or dip in the scalar potential. These bumps/dips may lead to black hole formation.} In this section, we study this possibility and show that the presence of SUSY breaking in sgoldstino-less inflation models is able to explain the relic density of dark matter under the form of PBHs, or to form a large density of mini-PBHs that might dominate the energy density of the Universe before evaporating and therefore modify the post-inflationary dynamics predicted by the model.

4.1 General principle

The production of a significant fraction of PBHs in the early Universe from inflation is usually achieved by introducing a phase of ultra slow-roll (USR)\cite{63} along the inflation trajectory. During ultra slow-roll, the field velocity decreases rapidly, and so does the first slow-roll parameter $\epsilon$, leading to a peak in the evolution of the second slow-roll parameter $\eta$. If the value of $\eta$ exceeds 3 during that time, the scalar-perturbation power spectrum starts to grow exponentially fast\cite{64}. Whereas the simplest vanilla models of inflation typically do not have any phase of USR along their trajectory, models of so-called inflection-point inflation were constructed in the literature in order to produce the correct amount of dark matter from primordial perturbations\cite{65–67}. In our scenario, the breaking of SUSY at a scale comparable to the inflation scale, which translates into sizeable excursions of the sgoldstino away from the origin, typically produces inflection points along the inflation potential, as we have seen in section 2.1 and 2.2. Even though we have seen in section 3 that the presence of this inflection point can modify the values of the inflation observables, or even lead to the field being stuck in a metastable minimum, there exist situations where the presence of this inflection point leads to an USR regime, leading to the formation of PBHs in the post-inflationary Universe. In what follows, we recall general results about the production of PBHs during a period of kination domination, and exhibit the distributions that can be produced due to the presence of SUSY breaking in our models.

4.2 Primordial perturbations and power spectrum

Primordial black holes typically form when an overdense region of spacetime collapses when it re-enters the Hubble horizon. In order to calculate the number density of PBHs that formed in the early Universe, it is therefore necessary to track the evolution of the spacetime perturbations power spectrum and to evaluate the amount of such overdense regions at the time of horizon reentry for each relevant comoving scale. The evolution of the power spectrum is calculated using the Mukhanov-Sasaki equation\cite{65}

\[ \frac{d^2 u_k}{dN^2} + (1 - \epsilon) \frac{du_k}{dN} + \left[ \frac{k^2}{(e^N H)^2} + (1 + \epsilon - \eta)(\eta - 2) - \frac{d(\epsilon - \eta)}{dN} \right] u_k = 0 , \tag{4.1} \]
where we use the slow-roll parameters

\[ \epsilon = \frac{1}{2} \phi'^2, \quad \eta = \epsilon - \frac{1}{2} \frac{d \ln \epsilon}{dN}, \]  

(4.2)

and the \( u_k \) functions denote the Fourier modes of the so-called Mukhanov-Sasaki variable \( u = a \delta \varphi + a^2 \varphi' \Phi \) (\( \Phi \) being the gravitational potential and \( \delta \varphi \) denoting the quantum perturbation of the inflaton field). The curvature power spectrum can then be calculated after horizon exit for each \( k \) mode using the simple limit \[ P_R(k) = \lim_{k \gg aH} \frac{k^3}{2 \pi^2} \left| \frac{u_k}{e^{N \varphi'}} \right|^2. \]  

(4.3)

Curvature perturbations turn into statistical fluctuations of the density contrast \( \delta \equiv \delta \rho / \rho \). The formation of a primordial black hole typically takes place when this density contrast goes above a critical value \( \delta_c(k) \) [68]. At first-order in perturbation theory, \( \delta \) follows a Gaussian distribution of variance \( \sigma(k) \), and the mass fraction \( \beta \) of PBHs as compared to the total mass of the Universe at the time of formation is given by

\[ \beta(k) = 2 \int_{\delta_c(k)}^{\infty} \frac{1}{\sqrt{2 \pi \sigma^2(k)}} e^{-\delta^2/2 \sigma^2(k)} d\delta, \]  

(4.4)

In this expression, the global factor 2 is needed for normalization reasons (see e.g. ref. [69] and references therein) and the \( k \)-dependency of \( \delta_c(k) \) lies implicitly in the epoch during which the \( k \) mode re-enters the horizon. Indeed, different epochs have different equations of state \( w = p/\rho \), and numerical calculations lead to [70]

\[ \delta_c(k) = \frac{3(1 + w)}{5 + 3w} \sin^2 \left( \frac{\pi \sqrt{w}}{1 + 3w} \right). \]  

(4.5)

The variance \( \sigma(k) \) can be calculated as

\[ \sigma^2(k) = \int_{k_*}^{k_{\text{end}}} 4 \left( \frac{1 + w}{5 + 3w} \right)^2 P_R(q) \left( \frac{q}{k} \right)^4 e^{-q^2/k^2} T \left( \frac{q}{k \sqrt{3}} \right) d \ln q, \]  

(4.6)

where we follow the usual procedure of applying a Gaussian window function that prevents scales vastly different from \( 1/k \) to contribute to the collapse of density contrasts of such a scale [65]. The relation between the comoving curvature and density contrast power spectra at horizon crossing is simply a multiplication by a factor that only depends on the equation of state [71]. Finally, we apply a transfer function \( T \) of the form [72]

\[ T(x) = 3 \frac{\sin x - x \cos x}{x^3}, \]  

(4.7)

that accounts for the sub-horizon density perturbations evolution.

4.3 Mass fraction during kination

For a given post-inflationary chronology, each wavelength re-enters the horizon at a given time and may form PBHs with a given mass. In our case, when inflation ends, according to eq. (3.1) a fraction

\[ \eta = \frac{g_{*\text{, end}} q}{1440 \pi^2} \left( \frac{H_{\text{end}}}{M_{\text{Pl}}} \right)^2, \]  

(4.8)
of the total energy density is produced through gravitational reheating and thereafter redshifts like radiation as $\rho_{\text{rad}} \propto a^{-4}$. For a kination period with equation of state parameter $w > 1/3$, the period of kination domination lasts over a number of $e$-folds

$$\Delta N_{\text{kin}} = \left( \frac{1}{\eta} \right) \left( \frac{1}{3(1+w) - 4} \right).$$

(4.9)

Denoting by $T_{\text{kin}}$ the temperature of the thermal bath at the end of this period, one can write the mass (in Planck units) of a PBH that forms during the kination dominated era as a function of the comoving wavelength $1/k$ as [73]

$$M(k) = 4\pi\gamma \left( \frac{\pi^2 g_{*}^{\text{eq}}}{45} \right)^{1/3} \frac{g_{s}^{\text{eq}}}{g_{s}(T_{\text{kin}})} \frac{3w-1}{3(3w+1)} \left( \frac{a_{\text{eq}} T_{\text{eq}}}{T_{\text{kin}}} \right)^{3(1+w) - 3w} \left( \frac{T_{\text{kin}}}{k} \right)^{3(1+w) - 3w}.$$  

(4.10)

where $\gamma$ is an $O(1)$ parameter that represents the mass proportion of a Hubble patch that effectively collapses into a black hole. For simplicity we take this parameter equal to one in the upcoming numerical calculations. For masses larger than $\gtrsim 10^{16}$g, PBHs have a lifetime longer than the age of the Universe and their contribution to the cold dark matter relic density today is [73]

$$f_{\text{PBH}}(M) = \frac{\Omega_{\text{PBH}}(M)}{\Omega_{c}} = \frac{\gamma}{T_{\text{eq}}} \left( \frac{g_{s}(T_{\text{kin}})}{g_{s}(T_{\text{eq}})} \right)^{1/3} \left( \frac{\Omega_{m} h^2}{\Omega_{c} h^2} \right) \left( \frac{90}{\pi^2 g_{s}(T_{\text{kin}})} \right)^{2w} \left( \frac{4\pi\gamma}{1+w} \right)^{2w} T_{\text{kin}}^{1-3w} \beta(M) M^{-2w}.$$  

(4.11)

After kination ends, the Universe begins to be dominated by radiation again, and the modes that re-enter the horizon then form black holes with the usual mass and dark matter fraction spectrum, which can be obtained from eqs. (4.10) and (4.11) by sending $T_{\text{kin}} \to T_{\text{eq}}$ and $w \to 1/3$.

4.4 Results

Let us now present our results regarding the distributions that can typically be obtained in our models. First of all, let us note that Model I we considered in section 2.1 is different from the second in the sense that the plateau/bump generated from the breaking of SUSY is only located at small values of $\varphi$ around $\varphi = 0$. For this reason, this model tends to cause an USR period that, if it ever exists, only happens towards the very end of inflation. This gives very little room for exploring a significant production of PBHs, and we therefore focus our searches only on Model II, exhibited in section 2.2. In the case of Model II, the location of the inflection point along the inflation trajectory and the value of the slope at the inflection point are non-trivial functions of the different parameters. Importantly, the frequency of the perturbations that are amplified during the USR period, as well as the height of the corresponding peak in the power spectrum depend strongly on these different factors. The masses of the PBHs that may be produced at horizon re-entry and the corresponding mass fractions at the time of formation are extremely sensitive to the different parameters of the model. In table 1 we provide two parameter points that we consider to be of interest. In the first case, we exhibit examples of parameters\textsuperscript{3} that can lead to an USR period at the end of

\textsuperscript{3}We found such examples manually, by searching for parameters that provide the correct amplitude for the power spectrum at horizon exit, while producing the right amount of PBH to constitute 100% of the DM relic abundance today.
Table 1. Parameters used to generate the two distributions presented in figures 6 and 7.

| $f_0$          | $\beta$ | $\Lambda$ | $V_0$      | $M$ [g]   |
|---------------|---------|-----------|------------|-----------|
| $6.34399 \times 10^{-6}$ | 0.955   | 0.64960999 | $15.7 \times 10^{-12}$ | $1.4 \times 10^4$ |
| $1.119038 \times 10^{-5}$ | 0.84    | 1.392     | $3.25 \times 10^{-11}$ | $5.5 \times 10^{20}$ |

Figure 6. Distribution of PBHs generated with the first set of parameters presented in table 1. The gray-shaded area corresponds to PBH masses which would evaporate during or after BBN. The green vertical line stands for masses corresponding to modes exiting the horizon at the end of inflation.

Inflation and therefore produce PBHs with very small masses (of order $10^4$ g in this example). In this case, the entire PBH distribution is produced during the kination dominated era, and one can see from figure 6 that the mass fraction reaches $\sim 0.015$ at the time of formation. With such a large energy fraction at formation and such low masses, it is likely that these PBHs dominate the energy density of the Universe before they evaporate. In such an extreme case, the formation of PBHs would modify the cosmological timeline that we have considered in this paper by splicing a period of early matter domination at the end of kination. This situation goes beyond the scope of this paper and will be investigated in some future work.

In figure 7 we show the results that we obtain for the second set of parameters listed in table 1. On the left panel we show the mass fraction of PBHs at the time of their formation, and on the right panel we present our results for their final relic density fraction. The vertical dashed line on both plots stands for the value of the PBH mass that is formed exactly between kination and radiation domination. Interestingly, the location of this vertical line is uniquely determined by the Hubble rate value at the end of inflation, since the fraction of energy during gravitational reheating depends exclusively on $H_{\text{end}}$ in eq. (3.1) and the duration of the kination period is uniquely given by this energy fraction in eq. (4.9). As we can see from the figure, this line interestingly falls right in the window that happens to be experimentally allowed for PBHs to form the full relic density of cold dark matter in our Universe today (see e.g. refs. [72, 74, 75] for a review of the corresponding constraints depicted in figure 7). It is interesting to note that the PBHs that form during the kination dominated era and those who form in the later radiation dominated era populate the Universe with very different
energy fractions, which explains the sharp drop that both the mass fraction and relic density fraction feature when they cross the vertical dashed lines in figure 7. The reason for the different behavior of the distribution in the two regimes is threefold: (i) the value of the critical density contrast given in eq. (4.5) that is necessary for PBHs to form during kination and radiation domination are different,

\[
\delta_c \approx 0.38 \quad \text{(Kination, } w = 1), \\
\delta_c \approx 0.41 \quad \text{(Radiation, } w = 1/3), \tag{4.12}
\]

which means that PBHs are easier to form during kination domination than radiation domination, and (ii) the evolution of the mass density into the relic density fraction depends on the Universe’s evolution after the PBH formation, as can be seen from eq. (4.11). Finally, (iii) the relation between the contrast and curvature spectra, which can be seen from the \(w\)-dependent coefficient in eq. (4.6) (0.062 vs 0.050) also favours kination. In the example we have chosen, the fact that PBHs form both during kination and the radiation dominated era can be seen from the peculiar form of the distribution, which may be one day ruled out or confirmed experimentally, if such a distribution is ever measured in the future.

Before we conclude, we should finally discuss the value of the inflationary observables corresponding to these different examples. For the first point exhibited in table 1, we find inflation observables with values

\[
r \approx 0.00096 \quad \text{and} \quad n_s \approx 0.969. \tag{4.13}
\]

These values are perfectly consistent with the most recent limits set on the CMB tensor modes [61]. However, we point out the fact that the inflationary observables arising from the second set of parameters, which could in principle produce viable PBH dark matter candidates, feature a spectral index that is below the current experimental limits (\(n_s \approx 0.969\) for the first point).
This feature of the PBH dark matter production from inflationary models was already mentioned in previous work [65]. Accommodating the CMB observables while producing the correct relic density of PBHs at present time thus seems to be challenging, and is likely to require additional ingredients. We let the exploration of such a possibility for future work.

5 Conclusion

In this paper, we have studied the effect SUSY breaking on the inflationary trajectory of sgoldstino-less non-oscillatory inflation models. We have shown that a large value of the SUSY breaking scale $f_0$ can typically bend the inflation potential around the end of inflation which can alter significantly the post-inflationary dynamics and therefore leave visible imprints in the CMB spectrum. Considering a purely gravitational reheating, we have explored how certain regions of the parameter space can lead to periods of tracking where the scalar field energy density tracks the energy density of the background and can contribute to about 20% of the energy density during the radiation dominated era. Such a large contribution to the total energy content of the Universe corresponds to a dark radiation component that is strongly excluded if the tracking takes place during BBN. We also discussed the stability of the sgoldstino during the whole cosmological history and scanned over the parameter space in order to visualise the effect of a large SUSY breaking scale on the CMB observables given the current limits on $\Delta N_{\text{eff}}$. Finally, we have explored the possibility for the effect of SUSY breaking on the inflationary dynamics to lead to a significant production of primordial black holes in the early Universe. We have exhibited two examples where we showed that our models are able to produce PBHs that may either be light and come to dominate the energy density of the Universe before they evaporate at early time, or be heavy enough to survive until present time and constitute a large fraction of dark matter today. To put it in a nutshell, it is clear that the presence of SUSY breaking at very large energy can affect not only the dynamics of inflation, but also the subsequent scalar dynamics that may lead to different cosmological imprints, even if the sgoldstino remains stabilized during the whole cosmological history. Furthermore, similarly to previous attempts at unifying the description of cosmology in particle physics, it is manifest in this work that a full knowledge of the Universe’s evolution, from the time of inflation and cosmic reheating to present time leads to a one-to-one connection between the parameters of the model considered and observational predictions regarding the CMB.

Although the discussion of non-oscillatory inflation in the context of sgoldstino-less models and the corresponding cosmological constraints remains largely model dependent, we believe that this work identified new directions in order to constrain experimentally the post-inflationary history of our Universe. We insist on the fact that there exist many other directions that could be explored in this context. It was demonstrated for example in refs. [55, 76] that the spectrum of gravitational waves produced during inflation may lead to observable signals for gravitational wave detectors when the Universe undergoes a long phase of kination after inflation. Moreover, it is known that the production of primordial black holes usually goes with a secondary spectrum of gravitational waves coming from the next-to-leading order terms in perturbation theory [77–79]. Even if primordial black holes are produced in subdominant amounts, it may be possible for such a spectrum of induced gravitational waves to be visible to future detectors. The production of gravitational waves in the context of sgoldstino-less models of non-oscillatory inflation therefore stands out as an exciting perspective that we will address in a dedicated study in the future.
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