From the Planck to the Photon Scale

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Abstract

Using considerations from the Quantum Zero Point Field and Thermodynamics, we show that the Planck Scale is the minimum (maximum mass) and the Photon Scale is the maximum (minimum mass) Scale in the universe. The arguments also deduce the residual cosmic energy of $10^{-33}eV$ observed lately.

1 Introduction

It was argued by the author from different points of view that the Photon would have a small mass $\sim 10^{-65}gms$ [1, 2]. We will look into this now. This value is within the accepted experimental limits for a Photon mass [3]. It was further argued that it is this Photon mass which is the source of the puzzling residual cosmic energy that has been observed lately[4]. Let us first derive this residual cosmic energy directly from the background Dark Energy. We may reiterate that the ”mysterious” background Dark Energy is the same as the quantum Zero Point Fluctuations in the background vacuum electromagnetic field which is described by harmonic oscillators [5]. Let us now consider, following Wheeler a Harmonic oscillator in its ground state remembering that the background Zero Point Field is a collection of such oscillators [6]. The probability amplitude is

$$\psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\left(m\omega/2\hbar\right)x^2}$$
for displacement by the distance \(x\) from its position of classical equilibrium. So the oscillator fluctuates over an interval 

\[ \Delta x \sim (\hbar/m\omega)^{1/2} \]

The background electromagnetic field is an infinite collection of independent oscillators, with amplitudes \(X_1, X_2\) etc. The probability for the various oscillators to have amplitudes \(X_1, X_2\) and so on is the product of individual oscillator amplitudes:

\[ \psi(X_1, X_2, \cdots) = \exp[-(X_1^2 + X_2^2 + \cdots)] \]

wherein there would be a suitable normalization factor. This expression gives the probability amplitude \(\psi\) for a configuration \(B(x, y, z)\) of the magnetic field that is described by the Fourier coefficients \(X_1, X_2, \cdots\) or directly in terms of the magnetic field configuration itself by

\[ \psi(B(x, y, z)) = P \exp \left( - \int \int \frac{B(x_1) \cdot B(x_2)}{16\pi^3\hbar c r^2_{12}} d^3x_1 d^3x_2 \right) . \]

\(P\) being a normalization factor. Let us consider a configuration where the magnetic field is everywhere zero except in a region of dimension \(l\), where it is of the order of \(\sim \Delta B\). The probability amplitude for this configuration would be proportional to

\[ \exp[-(\Delta B)^2 l^4/\hbar c) \]

So the energy of fluctuation in a region of length \(l\) is given by finally the density [6, 7, 8]

\[ B^2 \sim \frac{\hbar c}{l^4} \]  

(1)

The above energy density corresponds to an energy \(\hbar c/l\) in the volume \(l^3\). This energy is minimum when \(l\) is maximum. Let us take \(l\) to be the radius of the universe \(\sim 10^{28} \text{cms}\). The minimum energy residue of the background Dark Energy or Zero Point Field now comes out to be \(10^{-33}\text{eV}\), exactly the observed value. This observed residual energy is a cosmic footprint of the ubiquitous Dark Energy in the universe a puzzling footprint that, as we noted, has recently been observed [4]. If on the other hand we take for \(l\) the smallest possible length, which has been taken to the Planck length \(l_P\), as
we will see in the sequel, then we get the Planck mass $m_P$.
The minimum mass $\sim 10^{-33}eV$ or $10^{-65}gms$, will be seen to be the mass of
the Photon, which also is the minimum thermodynamic mass in the universe,
as shown by Landsberg from a totally different point of view \cite{9}. So (1) gives
two extreme masses, the Planck mass and the Photon mass.
As an alternative derivation, it is interesting to derive a model based on the
theory of Phonons which are quanta of sound waves in a macroscopic body \cite{10}. Phonons are a mathematical analogue of the quanta of the electromagnetic field, which are the Photons that emerge when this field is expressed
as a sum of Harmonic oscillators. This situation is carried over to the theory
of solids which are made up of atoms that are arranged in a crystal lattice
and can be approximated by a sum of Harmonic oscillators representing the
normal modes of lattice oscillations. In this theory, as is well known the
Phonons have a maximum frequency $\omega_m$ which is given by

$$\omega_m = c \left( \frac{6\pi^2}{v} \right)^{1/3}$$

(2)
in (2) $c$ represents the velocity of sound in the specific case of Photons, while
$v = V/N$, where $V$ denotes the volume and $N$ the number of atoms. In this
model we write

$$l \equiv \left( \frac{4}{3} \pi v \right)^{1/3}$$

$l$ being the inter particle distance. Thus (2) now becomes

$$\omega_m = c/l$$

(3)

Let us now liberate the above analysis from the immediate scenario of atoms
at lattice points and quantized sound waves due to the Harmonic oscillations
and look upon it as a general set of Harmonic oscillators as above. Then we
can see that (3) and (1) are identical as

$$\omega = \frac{mc^2}{\hbar}$$

So we again recover with suitable limits the extremes of the Planck mass and
the Photon mass (and other intermediate elementary particle masses if we
take $l$ as a typical Compton wavelength).
We now examine separately, the Planck scale and the photon mass. We
remark that there were basically two concepts of space which we had inherited from the early days of modern science. The predominant view has been the legacy from the Newtonian world view. Here we consider space-time to form a differentiable manifold. On the other hand Liebniz had a different view of space, not as a container, but rather made up of the contents itself. This lead to a view where space-time has the smallest unit, and is therefore non-differentiable.

Max Planck had noticed that, what we call the Planck scale today,

\[ l_P = \left( \frac{\hbar G}{c^3} \right)^{\frac{1}{2}} \sim 10^{-33} \text{cm} \]  

is made up of the fundamental constants of nature and so, he suspected it played the role of a fundamental length. Indeed, modern Quantum Gravity approaches have invoked \(^\text{[11]}\) in their quest for a reconciliation of gravitation with other fundamental interactions. In the process, the time honoured prescription of a differentiable spacetime has to be abandoned.

There is also another scale too, made up of fundamental constants of nature, viz., the well known Compton scale,

\[ l = \frac{e^2}{m_e c^2} \sim 10^{-12} \text{cm} \]  

where \( e \) is the electron charge and \( m_e \) the electron mass. This had appeared in the Classical theory of the electron unlike the Planck scale, which was a product of Quantum Theory.

The scale \(^\text{[5]}\) has also played an important role in modern physics, though it is not considered as fundamental as the Planck scale. Nevertheless, the Compton scale \(^\text{[5]}\) is close to reality in the sense of experiment, unlike \(^\text{[4]}\), which is well beyond foreseeable direct experimental contact.

### 2 The Planck and Compton Scales

It is well known that String Theory, Loop Quantum Gravity and a few other approaches start from the Planck scale. This is also the starting point in the author’s alternative theory of Planck oscillators in the background dark energy. We first give a rationale for the fact that the Planck scale would be a minimum scale in the universe. Our starting point \(^\text{[11]}\) is the model for the underpinning at the Planck scale for the universe. This is a collection of \( N \)
Planck scale oscillators.

Earlier, we had argued that a typical elementary particle like a pion could be considered to be the result of \( n \sim 10^{40} \) evanescent Planck scale oscillators. We will now consider the problem from a different point of view, which not only reconfirms the above result, but also enables an elegant extension to the case of the entire Universe itself. Let us consider an array of \( N \) particles, spaced a distance \( \Delta x \) apart, which behave like oscillators, that is as if they were connected by springs. We then have [12, 13]

\[
\begin{align*}
  r &= \sqrt{N\Delta x^2} \tag{6} \\
  ka^2 &\equiv k\Delta x^2 = \frac{1}{2}k_B T \tag{7}
\end{align*}
\]

where \( k_B \) is the Boltzmann constant, \( T \) the temperature, \( r \) the extent and \( k \) is the spring constant given by

\[
\omega_0^2 = \frac{k}{m} \tag{8}
\]

\[
\omega = \left( \frac{k}{ma^2} \right)^{\frac{1}{2}} \frac{1}{r} = \omega_0 \frac{a}{r} \tag{9}
\]

We now identify the particles with Planck masses, set \( \Delta x \equiv a = l_P \), the Planck length. It may be immediately observed that use of (8) and (7) gives \( k_B T \sim m_P c^2 \), which of course agrees with the temperature of a black hole of Planck mass. Indeed, Rosen had shown that a Planck mass particle at the Planck scale can be considered to be a Universe in itself. We also use the fact alluded to that a typical elementary particle like the pion can be considered to be the result of \( n \sim 10^{40} \) Planck masses. Using this in (6), we get \( r \sim l \), the pion Compton wavelength as required. Further, in this latter case, using (6) and the fact that \( N = n \sim 10^{40} \), and (7), i.e. \( k_B T = k l^2/N \) and (8) and (9), we get for a pion, remembering that \( m_P^2/n = m^2 \),

\[
k_B T = \frac{m^3 c^4 l^2}{\hbar^2} = mc^2,
\]

which of course is the well known formula for the Hagedorn temperature for elementary particles like pions. In other words, this confirms the conclusion that we can treat an elementary particle as a series of some \( 10^{40} \) Planck mass oscillators. However it must be observed from (7) and (8), that while the
Planck mass gives the highest energy state, an elementary particle like the pion is in the lowest energy state. This explains why we encounter elementary particles, rather than Planck mass particles in nature. Indeed as already noted [14], a Planck mass particle decays via the Bekenstein radiation within a Planck time \( \sim 10^{-42} \text{secs} \). On the other hand, the lifetime of an elementary particle would be very much higher.

In any case the efficacy of our above oscillator model can be seen by the fact that we recover correctly the masses and Compton scales in the order of magnitude sense and also get the correct Bekenstein and Hagedorn formulas as seen above, and get the correct estimate of the mass of the Universe itself, as will be seen below.

Using the fact that the Universe consists of \( N \sim 10^{80} \) elementary particles like the pions, the question is, can we think of the Universe as a collection of \( nN \) or \( 10^{120} \) Planck mass oscillators? This is what we will now show. In fact if we use equation (6) with \( \bar{N} \sim 10^{120} \), we can see that the extent \( r \sim 10^{28} \text{cms} \) which is of the order of the diameter of the Universe itself. Next using (9) we get

\[
\hbar \omega_0^{(\text{min})} \left( \frac{l_P}{10^{28}} \right)^{-1} \approx m_P c^2 \times 10^{60} \approx M c^2
\]

which gives the correct mass \( M \), of the Universe which in contrast to the earlier pion case, is the highest energy state while the Planck oscillators individually are this time the lowest in this description. In other words the Universe itself can be considered to be described in terms of normal modes of Planck scale oscillators (Cf.refs. [15, 16, 13, 17, 18] for details). We do not need to specify \( N \). We have in this case the following well known relations

\[
R = \sqrt{N}l, \quad K l^2 = kT,
\]

\[
\omega_{\text{max}}^2 = \frac{K}{m} = \frac{kT}{ml^2}
\]

In (11), \( R \) is of the order of the diameter of the universe, \( K \) is the analogue of spring constant, \( T \) is the effective temperature while \( l \) is the analogue of the Planck length, \( m \) the analogue of the Planck mass and \( \omega_{\text{max}} \) is the frequency—the reason for the subscript \( \text{max} \) will be seen below. We do not yet give \( l \) and \( m \) their usual values as given in (4) for example, but rather try to deduce these values.
We now use the well known result that the individual minimal oscillators are black holes or mini universes as shown by Rosen \[19\]. So using the well known Beckenstein temperature formula for these primordial black holes \[20\], that is
\[
kT = \frac{\hbar c^3}{8\pi Gm}
\]
in (11) we get,
\[
Gm^2 \sim \hbar c
\]
which is another form of (4). We can easily verify that (12) leads to the value \(m \sim 10^{-5} \text{gms}\). In deducing (12) we have used the typical expressions for the frequency as the inverse of the time - the Compton time in this case and similarly the expression for the Compton length. However it must be reiterated that no specific values for \(l\) or \(m\) were considered in the deduction of (12).

We now make two interesting comments. Cercignani and co-workers have shown \[21, 22\] that when the gravitational energy becomes of the order of the electromagnetic energy in the case of the Zero Point oscillators, that is
\[
\frac{G\hbar^2 \omega^3}{c^3} \sim \hbar \omega
\]
then this defines a threshold frequency \(\omega_{\text{max}}\) above which the oscillations become chaotic. In other words, for meaningful physics we require that
\[
\omega < \omega_{\text{max}}.
\]
Secondly as we saw from the parallel but unrelated theory of phonons \[10, 23\], which are also bosonic oscillators, we deduce a maximal frequency given by
\[
\omega_{\text{max}}^2 = \frac{c^2}{l^2}
\]
In (14) \(c\) is, in the particular case of phonons, the velocity of propagation, that is the velocity of sound, whereas in our case this velocity is that of light. Frequencies greater than \(\omega_{\text{max}}\) in (14) are again meaningless. We can easily verify that using (13) in (14) gives back (12).

Finally we can see from (3) that, given the value of \(l_p\) and using the value of the radius of the universe, viz., \(R \sim 10^{27}\), we can deduce that,
\[
N \sim 10^{120}
\]
In a sense the relation (12) can be interpreted in a slightly different vein as representing the scale at which all energy- gravitational and electromagnetic becomes one. It should also be noted that, a Planck scale particle is a Schwarzchild Black Hole. From this point of view, we cannot penetrate the Planck Scale - it constitutes a physical limit. Thus, in this sense, the Planck scale is indeed the minimum scale while the photon scale is the largest - that is, the concerned masses are respectively the highest and lowest.

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