1. Introduction

Graph cuts have found many applications that address the problem of energy minimization, which occur frequently in computer vision and image processing. One of the most common applications is binary image segmentation, or silhouette extraction. Image segmentation is the process of applying a labeling to each pixel in an image to determine a list of boundaries of objects, areas of interest, or silhouettes of objects in the scene. The resulting pixel labeling enables higher level vision systems to perform complex actions such as gesture recognition for human-computer interfaces Ueda et al. (2003), real-time 3D volume reconstructions Laurentini (1994), and mixed reality applications Takaya et al. (2009). Without accurate and coherent segmentations, these higher end-to-end systems will fail to perform their required functions. This necessitates a method which can correctly identify and extract objects from a scene with high accuracy. However, accuracy usually comes with serious performance penalties, particularly in image processing which examines hundreds of thousands of pixels in each image. This is unacceptable for high-level vision systems, which generally require the processing to be completed in real time.

By creating a graph using information in the image, as in Fig. 1, researchers have recast the problem of energy minimization and image segmentation as a graph theory problem. Then, methods such as flow analysis and finding shortest paths give researchers information about the structure of the underlying image. The results of these methods have been very promising, although the problem of computation complexity (speed) remains. Techniques such as livewire Mortensen & Barrett (1995), normalized cuts Shi & Malik (1997), and graph cuts have been developed to produce more accurate image segmentations and silhouette extractions.

Applying graph cuts to layered frames of a video sequence has shown to be robust for object tracking and object segmentation. In such scenarios, the entire video sequence is treated as a single 3D volume of layered frames, and a cut is performed across the entire sequence at once Boykov & Funka-Lea (2006). In sight of the large size of the graphs, which is increased with the video sequence length, performance of the cut is of paramount importance. To solve this problem, advances in the graph cut algorithm have produced dramatic performance improvements Boykov & Kolmogorov (2004); Juan & Boykov (2006). However, regarding live video applications, there is no prior knowledge of subsequent frames. Techniques that build single graph across the entire video Boykov & Funka-Lea (2006) become then inapplicable in live video scenarios. Instead, a graph must be maintained or...
build at each frame. When considering live real-time video, using faster cut algorithms, is not enough sufficient. The creation of the graph, as well as the structure for utilizing temporal data from previous frames must be reconsidered. Although recent researches have given much attention to graph cuts algorithms, most of them are too computational expensive to reach real-time, limiting their applications to off-line processing. Moreover, with the introduction of the Compute Unified Device Architecture (CUDA), GPUs are no longer exclusively designed for 3D rendering. In CUDA, a GPU can be explicitly programmed as general-purpose shared memory Single Instruction Multiple Data (SIMD) multi-core processors, which allows a high level of parallelism. Thus, many applications that are not yet able to achieve satisfactory performance on Central Processing Units (CPUs), can get benefit from that massive parallelism provided by such devices. Nevertheless, only specific algorithms can be adapted and efficiently designed on GPUs.

In this chapter, we propose the design and implementation of graph cuts that is capable of handling both tracking and segmentation simultaneously, leading to real-time conditions. We first propose to track and segment objects in live video thereby utilizing regional graph cuts and object pixel probability maps, where the graph is dynamically adapted to the motion of objects. We then introduce a fast and generic novel method for graph cuts on GPUs that places no limits on graph configuration, and which is able to reach real-time performance.

2. Background on graph cuts
Graph-based segmentation has a history originally based in mathematics and graph theory. Since graphs are an abstraction of the problem space, they are often applied to many different domains. In addition to image segmentation, graph cuts have been applied to computer networking, selection maximization, and stereo vision. The graph theory that supports
graph-based segmentation methods is described, followed by a survey of frequently used
tools, and a discussion of common graph construction techniques.

Before applications to image segmentation were demonstrated, mathematicians had been
interested in computation of flows through networks. Networks were modeled as directed
graphs with weights (also called capacities) assigned to the edges. The first important
discovery in graph-based segmentation occurred in 1956, when Elias et al. (1956),
and Ford and Fulkerson (1956) independently proved the max-flow min-cut theorem. This theorem proved that solving for the maximum flow across a network is
equivalent to solving for the minimum cut required to separate a network into two disjoint
components.

### 2.1 Max-flow min-cut theorem

The max-flow min-cut theorem defines the relationship between the maximum flow on a flow network, and the minimum cut required to separate the network into two disjoint components. A flow network is a graph \( G = \langle V, E \rangle \) composed of edge set \( E \) and a vertex set \( V \). The vertex set \( V \) contains two labeled vertices, the source \( s \), and the sink \( t \), and each edge \( uv \in E \) has a capacity \( c_{uv} \), and the flow \( f \) between two vertices \( u \) and \( v \) is never more than the capacity, \( f(u, v) \leq c_{uv} \). Then for a path \( P_{u_0u_n} = \{u_0u_1, u_1u_2, \ldots, u_{n-1}u_n\} \) where \( u_0, u_1, \ldots, u_n \in V \), the flow is defined as:

\[
f(P_{uv}) = \min_{ij \in P_{uv}} c_{ij}
\]

(1)

Then the maximum flow of the network \( G \) is defined as the sum of path flows for all paths from \( s \) to \( t \):

\[
f(G) = \sum_{\forall P_{st} \in G} f(P_{st})
\]

(2)

Next, an \( s-t \) cut is defined as a partition of the vertices \( V \) into two sets \( (W, \bar{W}) \) such that \( s \in W \) and \( t \in \bar{W} \) and no edges exists from \( W \) to \( \bar{W} \), as shown in Fig. 2 if the forward edges are
removed. The max-flow min-cut theorem states that the value \( v \) of the maximum flow is
equal to the value of the minimum \( s-t \) cut Papadimitriou & Steiglitz (1998).

![Graph s-t partitioning](https://www.intechopen.com)

While the theorem was proved in 1956, algorithms allowing for efficient computer
computation of very large graphs continued to be a focus of much research. Today, general
categories of algorithms for solving the max-flow/min-cut exist and are often the basis of
additional research.
2.2 Max-flow algorithms
Along with the proof of the max-flow min-cut theorem in Ford & Faulkerson (1956), Ford and Fulkerson presented an algorithm for computing the maximum flow of a flow network. This algorithm forms the basis of a set of algorithms known as Ford-Fulkerson or augmenting paths algorithms. Then, in 1988 Goldberg and Tarjan presented a new way of approaching the computation of flow with their push-relabel algorithm Goldberg & Tarjan (1988). The time complexities for two augmenting paths algorithms and two push-relabel algorithms are presented in Table 1. The two classes of algorithms differ in time complexity based on the number of vertices $V$, or edges $E$. Ford-Fulkerson algorithms would generally perform better on sparse graphs (fewer edges) than do push-relabel algorithms. At the time of writing, one of the fastest (and freely available) implementations is an augmenting paths algorithm utilizing breadth-first-search (BFS) search trees Kolmogorov & Zabin (2004).

| Algorithm                        | Complexity      | Notes                        |
|----------------------------------|-----------------|------------------------------|
| Ford & Faulkerson (1956)         | $O(VE^2)$       |                              |
| Ford & Faulkerson (1956)         | $O(EF)$         |                              |
| Edmonds & Karp (1972)            | $O(V^2E)$       | Only for integer capacities BFS |
| Push-Relabel, Goldberg & Tarjan (1988) | $O(V^2E)$       |                              |
| Push-Relabel w/ FIFO Queue, Goldberg & Tarjan (1988) | $O(V^3)$       |                              |

Table 1. Time Complexity of Maximum Flow Algorithms.

2.3 Augmenting paths algorithm
The augmenting paths algorithm solves the maximum flow problem by iterating searches for paths from $s$ to $t$ and pushing flow equal to the minimum capacity along the path. The algorithm terminates when no paths with remaining capacity exist from $s$ to $t$. The pseudo-code for the algorithm is presented in Algorithm 1.

**Algorithm 1: Augmenting Paths Algorithm.**

```
input: Graph G with flow capacity $c$, source vertex $s$, sink vertex $t$
output: A flow $f$ from $s$ to $t$ which is maximum

1 for all the $uv \in E$ do
2     $f(u,v) \leftarrow 0$
3 end
4 while $\exists$ path $P$ from $s$ to $t$ such that $c_{uv} > 0$ for all $uv \in P$ do
5     $c_f(p) = \min (c_{uv} | uv \in P)$
6     foreach $uv \in P$ do
7         $f(u,v) \leftarrow f(u,v) + c_f(p)$ /* Send flow along the path */
8         $f(v,u) \leftarrow f(v,u) - c_f(p)$
9     end
10 end
```

The difference between the Edmonds-Karp algorithm and the Ford-Fulkerson algorithm in Table 1 lies in the method used to find paths from $s$ to $t$. In step 2 in Algorithm 1, the Edmonds-Karp algorithm uses a BFS search method originating from vertex $s$. Fig. 3 presents a simple case of an augmenting paths algorithm. Fig. 3(a) shows the initial graph. The graph has been constructed with a source vertex and a sink vertex, and all
flow has been initialized to zero. In the next panel, the first path from the source to sink is highlighted in red, and the minimum capacity has been pushed as flow. The result is that the link connecting to the sink has been fully saturated, and this path is no longer usable. This continues through panel three and four, iteratively pushing flow along s-t paths until no unsaturated path remains. Then the cut separates those vertices that still have an unsaturated path back to the source from those that do not.

Fig. 3. Example Augmenting Paths Algorithm Execution.

2.4 Push relabel algorithm

Whereas the augmenting paths algorithm examines entire s-t paths, the push relabel algorithm examines individual vertices and attempts to push flow through the graph network. The push relabel algorithm relies on generating a pseudo-flow (a flow that does not obey the properties of the graph) and finding a convergence on a true flow. The algorithm adds two pieces of information to each vertex, a height $h$ and an excess $e$. The height of the vertex is a distance measurement, reflecting how close the vertex is to the source. The excess is the difference between incoming and outgoing flow of a vertex, or in other words, the flow that is trapped or waiting at the vertex. A vertex with $e > 0$ is said to be overflowing. The algorithm consists of two main operations: push (also called discharge) and relabel (also called raise). Typically a queue of overflowing vertices is checked each iteration, with the first vertex in the list being dequeued and either discharged or relabeled. The loop terminates once there are no further viable discharge or relabel operations to perform (i.e. no remaining overflowing vertices).

Discharging a vertex causes some of the excess to be pushed to neighboring vertices when the incident edge has not completely saturated (has residual capacity). The push decreases the excess in the current vertex, increasing excess in the destination, and decreasing the residual capacity of the edge. The destination vertex is then enqueued into the overflowing list, along with the current vertex if it is still overflowing.

A vertex must be relabeled when it is overflowing but has no valid neighbors. A neighbor is valid if the edge incident to both vertices has remaining capacity, and the destination vertex’s height is lower than the current vertex. The current vertex’s height is increased to one more than the lowest neighbor, which allows further discharging. If a vertex has no remaining outgoing capacity, then it is relabeled to the height of the source plus one, so that the excess may be pushed back to the source.
**Algorithm 2: Push Relabel Algorithm.**

|   |   |
|---|---|
| **input**: Graph $G$ with flow capacity $c$, source vertex $s$, sink vertex $t$ | **output**: A flow $f$ from $s$ to $t$ which is maximum |

1. `list.enqueue(s)`
2. while `not list.isEmpty()` do
   3. $u \leftarrow list.dequeue()$
   4. if $u.excess > 0$ then
      5. $amtPushed \leftarrow discharge(u)$
      6. if $amtPushed == 0$ then
         7. `relabel(u)`
      8. end
   9. end
10. end

In Algorithm 2, the enqueueing of more vertices into the list would occur inside the `discharge` and `relabel` functions. Various techniques to change or improve performance have been proposed, such as pushing to the highest neighbors first, or using a first-in-first-out (FIFO) queue for vertex selection. The general time complexity given in Table 1 would seem to say that the push relabel algorithm is more efficient than augmenting paths, especially for sparse graphs, but in practice this did not seem to be the case, until recently, with parallelized implementations of the algorithm.

Fig. 4 demonstrates the operations of the push relabel algorithm. All vertices have their information printed alongside their label. The source and sink have their height listed after the semicolon, while vertices $u$ and $v$ have their height listed, follow by their current excess (after the slash). The currently active vertex is highlighted in red. There appear to be many more steps than augmenting paths, but this is not necessarily the case, since the path finding steps were not detailed in the previous section.

![Fig. 4. Example Push Relabel Algorithm Execution.](www.intechopen.com)
2.4.1 Parallel push relabel algorithm
As recently as 2008, researchers discovered methods for parallelization of the push relabel family of maximum flow algorithms. Most of the strategies implement methods so that the vertices in the overflowing list can be processed concurrently, vastly speeding up the performance of the algorithm. The parallelization of any algorithm often involves many complex concurrency issues, requiring control structures (often referred to as locks) so that the order of execution of certain statements is guaranteed. However, a lock-less implementation of the push relabel has been presented by Hong Hong (2008), which will be presented here. The algorithm constructs the graph with a pre-flow, effectively saturating all outgoing links from \( s \).

**Algorithm 3: Lock-free, Parallel Push Relabel Algorithm: Master Thread.**

```
input : Graph \( G \) with flow capacity \( c \), source vertex \( s \), sink vertex \( t \)
output: A flow \( f \) from \( s \) to \( t \) which is maximum

/* Initialization, performed by master thread */
1 \( s\).height ← |V|
2 foreach \( u \in V \) − \( \{s\} \) do
   3 \( u\).height ← 0
4 end
5 foreach \( \vec{uv} \in E \) do
   6 \( f(u,v) \leftarrow 0 \)
7 end
8 foreach \( \vec{su} \in E \) do
   9 \( f(s,u) \leftarrow c_{su} \)
10 \( u\).excess ← \( c_{su} \)
11 end
```

As stated by Hong, lines 14-17 must be performed by atomic read-update-write operations. These operations are usually available on multi-core systems, or are implementable via software (though at a cost). Lastly, Hong notes that while a single thread may terminate because of no available operations, it does not mean that the algorithm has terminated. Other vertices may push flow back into the vertex, making it necessary for the thread to continue operations. This problem will be address in the GPU implementation in Section 3.2.

2.5 Building graphs from images
While the max-flow min-cut theorem was first proved in 1956, it was not applied to computer vision and image segmentation for more than 30 years. The key to applying the max-flow min-cut theorem image segmentation is the ability to represent the image as a meaningful flow network, so that the min-cut can be translated back into an image segmentation. One of the first successful image processing applications was developed by Greig et al (1989), which demonstrated success in estimating solutions for noise removal. Then Boykov et al Boykov et al. (2001) demonstrated that the min-cut of a graph can correspond to the maximum a posteriori distribution of a Markov Random Field, which had been used for image segmentation in the past. By formulating image segmentation as an energy minimization problem, and then a max-flow min-cut problem, Boykov et al created graph cuts for image segmentation.
Algorithm 4: Lock-free, Parallel Push Relabel Algorithm: Child Threads.

input: A vertex \( u \)
output: None

/* Main loop performed per thread, one thread per vertex */

1 while \( u.\text{excess} > 0 \) do
2 \( \hat{\varepsilon} \leftarrow u.\text{excess} \)
3 \( \hat{o} \leftarrow \text{null} \)
4 \( \hat{h} \leftarrow \infty \)
5 foreach \( \vec{uv} \in E_f \) do /* edges with capacity */
6 \( \hat{h} \leftarrow v.\text{height} \)
7 if \( \hat{h} < \hat{o}.\text{height} \) then
8 \( \hat{o} \leftarrow v \)
9 \( \hat{h} \leftarrow \hat{h} \)
10 end
11 if \( u.\text{height} \geq \hat{h} \) then /* Discharged */
12 \( d \leftarrow \min (\hat{\varepsilon}, c_{uv} - f(u,v)) \)
13 \( f(u,v) \leftarrow f(u,v) + d \)
14 \( f(v,u) \leftarrow f(v,u) - d \)
15 \( u.\text{excess} \leftarrow u.\text{excess} - d \)
16 \( v.\text{excess} \leftarrow v.\text{excess} + d \)
17 else /* Relabel */
18 \( u.\text{height} \leftarrow \hat{h} + 1 \)
19 end
20 end
21 end

The graph cut creation process consists of splitting the edges of the graph into two categories. All edges that are incident to the source or sink are called \( t \)-links (terminal links), while the other links are called \( n \)-links (neighbor links). The two different types of links are assigned weights (energy) using different methods, and give rise to the energy equation which is to be minimized by the graph cut.

\[
E(A) = \lambda R(A) + B(A) 
\]  
(3)

where

\[
R(A) = \sum_{p \in P} R_p(A_p) 
\]  
(4)

\[
B(A) = \sum_{\{p,q\} \in N} B_{p,q} \cdot \delta_{A_p \neq A_q} 
\]  
(5)

Equation 4 defines the regional energy, which corresponds to the \( t \)-links of the graph. This energy is often a model of the object to be extracted. Research has been conducted into various object models such as Gaussian mixture models Rother et al. (2004) and color histograms Garrett & Saito (2008). One simple method for \( t \)-link energy computation uses histograms. For a given pixel \( p \) and a histogram \( H \), the histogram value for \( p \), \( H(p) \), is divided by
the maximum histogram value to obtain an energy between \([0.0, 1.0]\) as demonstrated in Equation 6.

\[
R(p) = \frac{H(p)}{\max_{q \in V} H(q)}
\]

Equation 5 is the boundary energy, corresponding to the \(n\)-links in the graph. The most common equation used for the boundary energy is the square of the Euclidean distance between two pixels. Equation 7 demonstrates the energy between two RGB pixels \((p^T = [r, g, b])\), normalized to the range \([0.0, 1.0]\). This produces a linear energy model, which is often not optimized for practical use, as the difference between colors captured in real scenes does not exhibit enough variance in the RGB color space. Instead negative exponential functions (Equation 8) can replace the Euclidean difference.

\[
B(\{p, q\} \in V) = 1.0 - \frac{||p - q||}{3.0} (7)
\]

\[
B(\{p, q\} \in V) = e^{-||p - q||} (8)
\]

Fig. 5 demonstrates the differences in boundary energy equations. The x axis is the difference between the pixels \((||p - q||)\), and the y axis is the resultant energy. The exponential function decreases much faster for small differences of pixel intensity earlier than does the linear function.

Finally, graph construction can be summed up by the energy table by Boykov et al Boykov & Funka-Lea (2006), which has been reproduced here. The \(K\) in Table 2 refers to a maximum energy level, computed as:

\[
K = 1 + \max_{p \in P} \sum_{q \in N_p} B_{pq}
\]

This equation defines that links may have hard constraints, limiting them to either the foreground or background by ensuring that their neighbor links are fully saturated due to the large amount of energy in the t-link from the source, or the large drain to the sink.
Table 2. Edge Energy Computations per Edge Type.

| Edge | Weight (Energy) | Condition |
|------|-----------------|-----------|
| $\vec{p}q$ | $B_{pq}$ | $\{p, q\} \in V$ |
| $\vec{ps}$ | $\lambda \cdot R_p(\text{"bkg"})$ | $p \in P, p \notin O \cup B$ |
| $\vec{pt}$ | $\lambda \cdot R_p(\text{"obj"})$ | $p \in P, p \notin O \cup B$ |

3. Proposed system

3.1 Live video extraction

The method for tracking and silhouette extraction is composed of three main steps: building the graph, cutting the graph, and creating temporal seed information for the next frame. These three steps are performed in a cycle, with the temporal seeds being used in the next build phase, shown in Fig. 6. The images on the right show the image representation of data at each step of the algorithm.
Fig. 7. System Operation Flow Chart.

3.1.1 Build the graph
Each edge in the graph requires an energy computation, and as the size of the image increases, the number of edges increases dramatically. For example, an 8-neighborhood graph on a 640x480-resolution video has almost 2 million edges. This results in poor performance on live video because 2 million edges must be recomputed for every frame. If an object is tracked and segmented that does not fill the entire frame, many of the pixels will not need to be computed, such as those far from the object, since they will not likely change unless the object moves closer. To reduce the number of computations required, the graph is restricted to a region-of-interest.
For the very first cut on the graph, the region-of-interest will cover the entire video frame. However, subsequent regions will be smaller, depending on the object size.

3.1.2 Cut the graph
The second step is to cut the graph. For this step, a freely available graph cut implementation by Kolmogorov et al. Kolmogorov & Zabin (2004) was used. This implementation relies on a modified augmenting paths algorithm, as described earlier.

3.1.3 Temporal seeding
Seeding the graph for the next frame is not a simple matter of marking the previous frame’s object pixels as object seeds, and all other pixels as background seeds. If the object were to move within the next frame, then some seeds would be mislabeled, also causing the cut to incorrectly label some pixels. This research describes a new method for seeding pixels based on probabilities of future pixel locations.
When examining any two consecutive frames of a video sequence, many of the pixels of the object will overlap between frames. These will most likely be those pixels closer to the center of the object, as the object must travel farther before these pixels change from object to background. Thus, a probability distribution can be created over the object pixels such that the pixels near the edge will have a lower probability of remaining object pixels in the next video frame. Pixels near the center of the object have a high probability of remaining object pixels in the next video frame. This distribution we call the object pixel probability map.
To compute the pixel probability map, the binary mask of the object segmentation from the previous frame is computed. Then a distance transform is applied to the mask to obtain the pixel probability map, shown on the right hand side of Fig. 7. The distance transform computes the distance from a given white pixel to the closest black pixel. This gives us a gradient from the center of the object (greatest distance) to the cut edge (shortest distance). Using the probability map, the t-links of the next frame’s graph are computed using a modified version of Region Equation R described by Equation 5. Table 3 shows the new energy computations for t-links.
Table 3. Modified Boundary Energy Equation

Where $D_p$ is the value of the probability map for pixel $p$ divided by the largest $D$, and thus $D_p \in [0.0, 1.0]$. The technique also requires that $R_p \in [0.0, 1.0]$ to make sure that the maximum value of the $t$-link weight is $\lambda + 1$ (labeled as $K$ in Table 2).

Finally, a region-of-interest is constructed around the object area of the binary image mask. The region-of-interest is then expanded to contain probable future locations of the object. Metrics for determining the size of potential for the region-of-interest include camera frame rate, previous object velocities, and object scaling speeds.

### 3.1.4 Focused graph cuts

Graph cuts are notoriously computationally expensive. The graph size grows exponentially with image size (image size growth means growing in width and height), and easily includes hundreds of thousands of vertices and millions of edges, as seen in Fig. 8. To alleviate this problem, the proposed system focuses only on those pixels which are most likely to contain the object from the previous frame. By utilizing a dilation of the previous graph cut’s resultant binary image, the operations of function graph cuts can be masked, resulting in a smaller graph on which to operate. Similar methods which use banded graph cuts, cutting only around the boundary of the object, have been proposed Lambaert et al. (2005); Mooser et al. (2007), however these methods have been shown to lose information and misclassify pixels in certain topologies Garrett & Saito (2008); Sinop & Grady (2006).

![Graph Growth Size](image_url)

**Fig. 8.** Graph Growth Size.

Initial timing analysis of graph cuts shown in Fig. 9 demonstrates that the building of the graph consumes a majority of the time, as many floating point operations must be performed (typically slow on CPUs). To speed up the graph cuts even more, the next section details how to harness the parallelization and floating computations of the GPU.
3.2 GPU implementation using CUDA

Driven by the insatiable market demand for real-time, high-definition 3D graphics, the programmable GPU has evolved into a highly parallel, multi-threaded, many-core processor with tremendous computational horsepower and very high memory bandwidth.

The reason behind the discrepancy in floating-point capability between the CPU and the GPU is that the GPU is specialized for compute-intensive, high-parallel computation, and therefore designed such that more transistors are devoted to data processing rather than data caching and flow control.

NVidia’s CUDA allows researchers to easily parallelize their computing task to take advantage of the GPU. CUDA provides a software library used for interfacing with the graphics processor, and a specialized compiler to create executable code to run on the GPU. CUDA presents programmers with two other types of memory access, shared memory and global memory, on top of the texture cache units. Being able to access to all the memory on the GPU (albeit with varying access times) has made it much easier to perform research into GPGPU computing. The method presented in this paper was developed using the CUDA architecture and run on nVidia hardware. There are many technical resources available on the nVidia CUDA homepage nVidia (2009).

The following description of the CUDA model is stated in terms of its hardware and software components. The hardware model, shown in Fig. 10, illustrates how each processor core is laid out in relation to the different memory buses of global memory, texture cache, and shared memory. Device (global) memory is the slowest to access while the texture cache is only available for read operations. Shared memory is used by all of the threads in a block, as described by the software model.

The software model, depicted in Fig. 11, shows the how a block organizes threads, and how the grid of a kernel is organized into blocks. When an invocation of kernel code is to be called, CUDA requires specifying the dimensions of the grid and the size of blocks within the grid (each block is the same size). A collection of threads that will be run in parallel on the SIMD cores is defined as a warp. Operations such as branching will cause threads to diverge into separate warps which must be run serially in time-shared environments. Each thread and each block have a two dimensional ID that can be accessed at any time. This ID is often used to reference memory locations pertaining to the particular thread.
3.2.1 GPU Programming paradigm

To develop a GPU implementation of a traditional algorithm, both data structure layout (the graph representation in memory) and concurrency bottlenecks must be carefully considered to realize the full potential of the GPU. CUDA implementations of graph cuts for image segmentation have traditionally paired the 2D lattice structure of graphs with the 2D grid structure of the CUDA programming model. However, this leads to restrictions on the types of graphs that can be processed by the routine. Furthermore, traditional graph cut algorithms contain too much cyclomatic complexity in the form of branching and looping. In CUDA, divergent code produces poor performance because the SIMD model cannot perform the instructions in parallel, making new techniques for graph cuts necessary.

Vineet and Narayanan presented a method of graph cuts in CUDA that has shown improved performance by pairing the 2D lattice graphs with the CUDA programming model. They conceded that this method would not be feasible for other types of graphs. Vineet & Narayanan
Fig. 12. Conversion of a direct graph into vertex and edge lists.

(2008). In this research, the graphs were structured using a technique similar to that presented by Harish et al Harish & Narayanan (2007) that improved performance in distance calculations on arbitrary graphs using the GPU by representing graph $G(V, E)$ as a vertex array $V_a$ and an edge array $E_a$. Edges in the edge list are grouped so that all edges that originate at a vertex are contiguous, and the vertex contains a pointer to the first edge in the group, as in Fig. 12. Each edge structure holds information about its capacity, current flow, and destination vertex.

3.2.2 Regular graphs
The technique used in this research improved upon the basic implementation of vertex and array lists. To make the list-based graph representation more SIMD friendly, the input graph is made *regular* by adding null edges to any vertex that has fewer edges than the vertex with the highest degree. Second, the code is simplified by only having one kernel, which performs both the discharging and relabeling steps.

Since the GPU is treated as an array of SIMD processors, it is important that the algorithm is able to perform the same operation across multiple data locations. However, this assumes that any kind of graph configuration could be used as input, requiring that each vertex be treated uniquely, as the vertex set is not homogeneous. Particularly in the case of image segmentation, there are three classes of vertices: center vertices, edge vertices, and corners vertices, which have 10, 7, and 5 edges respectively (in an 8-neighborhood graph). Since the vertices must be treated differently, the CUDA framework cannot perform the operations in parallel and it serializes the kernel calls, negating the benefit of using the GPU.

To overcome this problem, the new method used in this research adds extra edges to the graph so that each vertex has the same degree. In Fig. 13, the dotted line of edge $E4$ is an example of a null edge. These null edges point back to the originating vertex and have a capacity of zero. This causes the null edges to be ignored by the graph cut algorithm, as the destination of the edge never has a height lower than the origin (a condition for discharging), and the capacity of the edge restricts the amount of flow discharged to zero.

3.2.3 GPU graph cuts on regular graphs
Algorithm 5 details the steps of the computation that take place on the CPU. The program operates on a list of vertices $V$, and a list of edges $E$. The *initialize* function starts by discharging the maximum possible amount of flow from the source to the neighboring vertices. Then the GPU kernel is invoked so that one thread is executed for each non-source and non-sink vertex. The CPU continues to invoke the GPU kernel until no discharge or relabel operations can be performed.
Algorithm 5: Proposed System Host Graph Cut Loop

1. \texttt{finished} ← false
2. \texttt{Initialize} (\(V, E\))
3. \textbf{while} not \texttt{finished} \textbf{do}
   4. \hspace{1em} // \texttt{GraphCutKernel()} is performed in parallel on the GPU
   5. \hspace{1em} \texttt{finished} ← \texttt{GraphCutKernel} (\(V, E\))
   6. \textbf{end}

The GPU kernel is detailed in Algorithm 6. Since the graph is regular, the loop over the neighbors is fully unrolled. The null edges are ignored because the height of \(u\) will never be less than the current vertex’s height (since they are equal). In addition the discharge can be modified to push a conditionally assigned amount of flow. Conditional assignment allows us to use of the ternary operator, which prevents divergent code since it can be optimized to a zeroing of either operand of an addition.

Algorithm 6: Proposed System GPU Kernel

1. \texttt{tId} ← \texttt{GetThreadId()}
2. \texttt{amtPushed} ← 0
3. \textbf{foreach} \texttt{Neighbor} \(u\) of \(V[\texttt{tId}]\) \textbf{do}
   4. \hspace{1em} if \(u.\text{height} < V[\texttt{tId}].\text{height}\) and \((V[\texttt{tId}], u) \in E\) has capacity > 0 then
   5. \hspace{2em} \texttt{amtPushed} ← \texttt{amtPushed} + \texttt{Discharge} (\(V[\texttt{tId}], u\))
   6. \textbf{end}
   7. \textbf{end}
8. if \texttt{amtPushed} > 0 and \(V[\texttt{tId}].\text{excess} > 0\) then
9. \hspace{1em} \(V[\texttt{tId}].\text{height} = \text{FindLowestNeighbor}(V[\texttt{tId}]).\text{height} + 1\)

4. Experimental results

The GPU experiment analyzes the performance of the GPU push relabel implementation described in Section 3.2. The goal of the implementation is to be able to realize the results of the graph cut faster than the traditional CPU implementations. To measure the performance
benefit of the GPU approach, a pure GPU implementation was compared against the previous CPU implementation and against a hybrid CPU/GPU system. This section begins with the drawbacks of the GPU approach. Then, as remarked at the end of the CPU test analysis, the two areas identified for improvement are the graph building and the graph cutting areas, which are covered in the following two sections respectively.

These tests utilized two video sequences scaled to QVGA (320x240), VGA (640x480), HD 720p (1280x720), and HD 1080p (1920x1080) resolutions.

All tests in this section were performed on an 2.5 GHz Intel Xeon processor with 2 GB of RAM and a nVidia GTX 280 graphics card running Windows XP. This information is important in analyzing the performance results in the following sections, as results are likely to differ with hardware.

4.1 GPU transfer latency

Due to computer hardware architecture, the memory on the GPU is not directly accessible from the CPU. An important limitation of the older GPUs was the time required to transfer the input to the GPU memory and the time required to transfer the GPU output back to the CPU. This latency from transferring data is not applicable to the CPU-only implementations of graph cuts, since all the data resides in the CPU main memory. Traditional Advanced Graphics Port (AGP) technology facilitated fast data transfer to the GPU, but the return path to the CPU was considered unimportant, since the output would then be written to the display device. Even with the new Peripheral Component Interconnect Express (PCIe) bus architecture, at very large image resolutions, minor latency is observed. Fig. 14 demonstrates that for even large HD 1080p (1920x1080) resolution images the absolute maximum frame rate (due to transfer latency) would be approximately 30 frames-per-second.

![Graph of Elapsed Time per GPU Task](image)

**Fig. 14.** Graph of Elapsed Time per GPU Task

4.2 GPU graph building

Due to the parallelism in GPUs, even greater performance benefits can be obtained using CUDA. The GPU performed so well during this task that the histogram bars cannot be seen, and instead the values are written. Furthermore, unlike focused graph cuts, this task is content independent since it works on the entire frame at the same time.

Table 4 gives exact performance numbers for the results shown in Fig. 15. In the final row, the speed up multiplier is computed as the ratio between the GPU implementation and the focused graph cuts implementation. For HD 1080p video, GPU graph building achieves as
Fig. 15. Build Times for Various Image Resolutions

high as a 26x speed increase over the focused graph building, and even greater over traditional graph building methods.

|          | QVGA | VGA | 720p | 1080p |
|----------|------|-----|------|-------|
| Full     | 57.96| 208.91| 638.07| 1090.52|
| Focus    | 15.59| 45.82| 82.99| 477.55|
| GPU      | 0.86 | 3.61 | 8.72 | 17.60 |
| Speedup  | 17.5x| 12.7x| 9.5x | 26.1x |

Table 4. Speed Results for GPU Graph Building.

4.3 GPU push relabel

This section compares the cut speeds of the GPU implementation of the system. Fig. 16 shows that at smaller resolutions, the GPU implementation does not see significant improvement in speed. As the raw clock speed of the CPU is higher than that of the GPU (4x greater), this is to be expected. However, at the higher resolutions (720p and 1080p), the GPUs parallelization capabilities start to have an impact on the performance.

Fig. 16. Cut Times for Various Image Resolutions.

Table 5 gives exact performance numbers for the results shown in Fig. 16.
Table 5. Speed Results for GPU Graph Building.

|        | QVGA | VGA | 720p | 1080p |
|--------|------|-----|------|-------|
| CPU Avg| 20   | 88  | 170  | 503   |
| CPU Min| 15   | 78  | 156  | 453   |
| CPU Max| 32   | 94  | 170  | 503   |
| GPU Avg| 14   | 43  | 107  | 197   |
| GPU Min| 11   | 39  | 91   | 180   |
| GPU Max| 18   | 47  | 117  | 207   |

4.4 GPU Summary

Experimental data show that the GPU implementation is able to perform on average 9 times faster than the CPU implementation for large resolutions (see Table 6). Standard video broadcasts (up to VGA) could be segmented with very little delay as the frame rates approach 15 fps. However, HD video is still beyond the reach of real-time systems, averaging only 3 fps.

Table 6. Summary Results for Full System Performance Tests.

|        | QVGA | VGA | 720p | 1080p |
|--------|------|-----|------|-------|
| CPU    | 150  | 739 | 1739 | 3094  |
| GPU    | 23   | 69  | 186  | 336   |
| Speedup| 6.5x | 10.7x | 9.3x | 9.2x |

Fig. 17 contains sample frames from output video, scaled to fit the page, and scaled relative to each other in order to realize the size difference in resolutions. The segmentation boundary is designated by the red border around the yellow airplane. The ocean and islands are segmented as background, and the airplane is segmented as the object of interest. The videos are samples obtained from the Microsoft WMV HD Content Showcase website Microsoft (2009).

5. Conclusions

This chapter has presented research on a novel system for object silhouette extraction. The originality of the system comes from three distinct method improvements. First, the research developed a method for tracking and segmenting objects across a sequence of images by using temporal information and distance transforms. Second, the method performance was increased by focusing the graph cut on likely image pixels, reducing the graph size to only those pixels that are considered useful. Finally, creating an implementation on the GPU using vertex and edge lists, as opposed to 2D memory arrays, and making the graph regular by adding null edges further improved performance.

Through testing on various image sequences and the standardized Berkeley segmentation data set, the research showed the new system provides fast and accurate results. Future research extensions include alternative color models for objects and new boundary energy equations. Both hold promise for better segmentation in the occlusion case, but must be balanced with computational complexity, otherwise the improvement in speed realized by the system will be negated.

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Fig. 17. Sample Output of GPU Video Sequence Performance Experiment.
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Object tracking consists in estimation of trajectory of moving objects in the sequence of images. Automation of the computer object tracking is a difficult task. Dynamics of multiple parameters changes representing features and motion of the objects, and temporary partial or full occlusion of the tracked objects have to be considered. This monograph presents the development of object tracking algorithms, methods and systems. Both, state of the art of object tracking methods and also the new trends in research are described in this book. Fourteen chapters are split into two sections. Section 1 presents new theoretical ideas whereas Section 2 presents real-life applications. Despite the variety of topics contained in this monograph it constitutes a consisted knowledge in the field of computer object tracking. The intention of editor was to follow up the very quick progress in the developing of methods as well as extension of the application.

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