Effect of interactions on the localization of a Bose-Einstein condensate in a quasi-periodic lattice.

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The transport properties of a Bose-Einstein condensate in a 1D incommensurate bichromatic lattice are investigated both theoretically and experimentally. We observe a blockage of the center of mass motion with low atom number, and a return of motion when the atom number is increased. Solutions of the Gross-Pitaevskii equation show how the localization due to the quasi-disorder introduced by the incommensurate bichromatic lattice is affected by the interactions.

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The intrinsic perfection of lattices made from a standing wave of light surprisingly makes them an excellent candidate for the investigation of disorder in atomic systems. Free from uncontrollable or undesirable defects, precise disorder can be added simply in the form of additional optical lattices [1, 2, 3, 4, 5] or with an optical speckle potential [6, 7, 8, 9]. The combination of optical lattices with a Bose-Einstein condensate (BEC) offers the stimulating complexity of interactions in a setting unimpeded by large thermal fluctuations. The significant observation of the Mott-Insulator phase was realized utilizing a strongly interacting BEC produced in a three-dimensional ordered crystal of light [10]. Extending this work to the strongly disordered regime with the inclusion of quasi-disorder from a bichromatic lattice, recently led to initial experimental evidence of a Bose-glass phase [11].

Another open question remains as to the intermediate behavior between the non-interacting disordered Anderson localized phase and the strongly interacting Bose-glass phase [12].

To begin to address this cross-over regime we have carried out investigations of the transport properties of a condensate in a quasi-periodic lattice. The very complexity provided by the interplay of disorder and interactions that makes disordered condensates a stimulating topic, also can introduce instability inherent in non-linear transport [13]. In fact we find that dynamical instability does occur for transport in a quasi-periodic lattice when the center of mass motion reaches a critical velocity, and has a non-trivial dependence on the interaction strength.

Anderson’s seminal paper in 1958 showed that the wave function of a particle placed in a lattice with disordered on-site energies remains localized when the range of the on-site energies is sufficiently large compared to the hopping energy between neighboring sites [14]. Anderson’s hopping model can be approximated using a quasi-periodic bichromatic lattice, obtained by superimposing a primary optical lattice with a weak secondary lattice of a different incommensurate wavelength. The secondary lattice serves to break the discrete translational invariance of the system, thus favoring localization of the wave functions, however the effect of the quasi ordered may be important depending on the exact parameters of the bichromatic lattice [3, 5].

In our system the 1D incommensurate bichromatic lattice is produced combining the primary optical lattice derived from a Titanium:Sapphire laser operating at a wavelength $\lambda_1 = 830.7(1)$ nm with a secondary lattice obtained from a fiber-amplified diode laser emitting at $\lambda_2 = 1076.8(1)$ nm. Our $^{87}$Rb BEC is produced in aioffe-Pritchard magnetic trap, elongated in the direction of the bichromatic lattice. The trapping frequencies are $\omega_x = 2\pi \times 8.7$ Hz axially and $\omega_L = 2\pi \times 90$ Hz radially. The BEC can be produced in the range of $\simeq 1.5 \times 10^4 - 2 \times 10^5$ atoms. The resulting potential along the lattice axis can be expressed as

$$V(x) = s_1 E_{R1} \sin^2(k_1 x) + s_2 E_{R2} \sin^2(k_2 x) + \frac{m}{2} \omega_x^2 x^2$$  \hspace{1cm} (1)

where $s_1$ and $s_2$ measure the height of the lattice potentials in units of the respective recoil energies $E_{R1} = \hbar^2/(2m\lambda_1^2) \simeq \hbar \times 3.33$ kHz and $E_{R2} = \hbar^2/(2m\lambda_2^2) \simeq \hbar \times 1.98$ kHz, $k_1$ and $k_2$ are the wave numbers of the two lasers, $\hbar$ is the Planck constant and $m$ the mass of $^{87}$Rb.

The possibility of localization with our bichromatic lattice in the absence of interactions is demonstrated by numerical diagonalization of the stationary 1D Schrödinger equation using the potential defined by Eq. (1). A strong primary lattice is chosen, $s_1 = 10$, $s_1 E_{R1}/\hbar = 33$ kHz, and a perturbing secondary lattice of maximum height $s_2 = 2$, $s_2 E_{R2}/\hbar = 4$ kHz. The ground state resulting from the bichromatic lattice is shown in Fig. 1(a) and is contrasted with the ground state of a pure random case in (b). The random potential is simulated using only the primary lattice, $s_1 = 10$, with additional random on-site energies in a box distribution in the range $[0, \Delta]$, where $\Delta/\hbar \leq 4$ kHz. The amount of the disorder is given by either the height of the secondary lattice in (a) or by the maximum on-site energy in the random case (b), and is denoted by $\Delta$ in both (a) and (b). We define $J$ as the tunnelling in the primary lattice. The thin line is the ground state with only the primary lattice, showing the total length of the system in the harmonic trap. The localized states may have a different center from the harmonic trap depending on the disorder real-
 FIG. 1: Non-interacting density profiles in log scale of the ground state with increasing disorder in (a) a bichromatic lattice and (b) a lattice with random on-site energies. The thin line in each graph represents the ground state with only the primary lattice. The last graph in each column shows the on-site energies in the respective potentials with the amount of disorder shown by $\Delta/J \simeq 26$. In all cases the height of the primary lattice is $s_1 = 10$ giving a tunnelling energy of $J/h = 75$ Hz.

In the presence of weak interactions one expects localization effects to persist. In fact an Anderson glass phase has been identified when increasing interactions from an Anderson localized phase to the Bose-glass state, using the superfluid density as the order parameter for the phase transition. Unlike the Bose-glass phase, where both interactions and disorder cooperate to block diffusion, in the Anderson glass phase the interactions tend to delocalize the atoms and the disorder must compete against the interactions [12, 15].

Such behavior can be seen in Fig. 2, showing the ground states in our incommensurate bichromatic lattice with interactions calculated by means of a 1D effective Gross-Pitaevskii equation, namely the nonpolynomial Schrödinger equation (NPSE) [16].

In the presence of interactions the strongly localized ground state of (a) transforms into a state with multiple peaks with partially overlapping tails (b). Upon increasing the interactions, the overlap between these peaks increases until the state eventually becomes extended (c). These results show similar behavior to previous simulations of a BEC in a three-color lattice [6]. Even with the increased interactions, a state with multiple peaks can be recovered by increasing disorder (d)-(f). The crossover behavior between an extended superfluid state and a possible Anderson glass state is very difficult to quantify in this picture. We take a pragmatic approach and characterize the state of the system by investigating the transport properties.

In the experiment, to set in motion our harmonically-trapped condensate we excite dipole oscillations by abruptly shifting the center of the magnetic trap. With a single color lattice, the superfluid BEC oscillates freely, with the dipole frequency modified by the effective mass [17, 18]. Instead, adding an incommensurate bichromatic lattice, we expect the oscillations to be blocked by localization effects. Fig. 3(a) shows the center of mass motion after a shift in the magnetic trap of 6 $\mu$m, with a fixed number of atoms $N = 1.5 \times 10^4$, a fixed height of the primary lattice $s_1 = 10$, and a variable height of the secondary lattice. At $s_2 = 0.1$ ($\Delta/J = 2.6$), the BEC oscillates with some damping. Increasing $s_2$ the motion is strongly damped, until at $s_2 = 0.5$ ($\Delta/J = 13$), the

FIG. 2: Density profile in log scale of the ground state with interactions for (a)-(c) increasing atom number and fixed $\Delta/J = 6.5$, and for (d)-(f) increasing $\Delta/J$ and fixed atom number $N = 1.5 \times 10^4$. In all cases the height of the primary lattice is $s_1 = 10$ giving a tunnelling energy of $J/h = 75$ Hz.

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FIG. 3: (a) Measured dipole oscillations with increasing intensity of the secondary lattice. The height of the primary lattice and atom number are fixed at $s_1 = 10$ and $N = 1.5 \times 10^4$ respectively. (b) Measured dipole oscillations with fixed $s_1 = 10, \Delta/J = 13$ and two different atom number, $N = 1.5 \times 10^4$ and $N = 2 \times 10^5$. The center of mass motion of the atoms was measured after 20 ms of expansion, fitting to the central peak. The dotted lines show damped sinusoidals fitted to the data with the frequency fixed.

BEC does not move. Interestingly, the height of the secondary lattice at which the condensate becomes localized is slightly greater than the non-interacting threshold for localization in the bichromatic lattice shown in Fig 1. The increase of this threshold could be due to the screening effect of interactions shown in Fig. 2.

We extended the investigations to a variable interaction energy effectuated by changing the number of atoms in the condensate. Fig. 3(b) shows the measured center of mass motion of the atoms for two different atom numbers $N = 1.5 \times 10^4$ and $N = 2 \times 10^5$, after an initial harmonic trap shift of 6 $\mu$m, $s_1 = 10$ and $s_2 = 0.5$ ($\Delta/J = 13$). Transport is stopped only for the low number of atoms, although damping is seen with the higher atom number.

Localization due to disorder is not the only physical effect which can block the motion of the atoms. In the case of an incommensurate bichromatic lattice the Bloch theorem cannot be applied, nevertheless the energy spectrum still shows the existence of energy bands. The band structure for the single lattice is complicated by the emergence of ‘mini-gaps’ opening up almost everywhere across the spectrum. However the dominant modification to the first Brillouin zone of the primary lattice, for a weak addition of the secondary lattice, are the extra energy gaps at $k_b$ and $k_1 - k_b$, where $k_1 = 2\pi/\lambda_1$ gives the boundary of the first Brillouin zone of the primary lattice, and $k_0 = 2\pi/\lambda_b$ corresponds to the quasi-periodicity introduced on the larger length scale $\lambda_b = \lambda_1\lambda_2/(\lambda_2 - \lambda_1) = 4.38\lambda_1$ from the beating between the two colors. This simplification to the energy spectrum is particularly true when interactions are introduced, since they can effectively screen the potential varying on length scales larger than the healing length, washing away the energy gaps at smaller $k$. As a consequence, in the presence of interactions one should carefully investigate the possible contribution of dynamical instability that in a single lattice has been observed to block dipole oscillations \cite{12,18} when the quasi-momentum becomes greater than $\simeq 0.5k_1$. In the bichromatic lattice dynamical instability may occur at a small quasi-momentum $\gtrsim 0.5k_b$ corresponding to the beat periodicity, much smaller than the onset of instability with only the primary lattice.

To better understand the contribution of the various physical effects instigating localization, the dipole oscillations in our actual system were simulated using the time-dependent NPSE. Fig. 4(a) shows the center of mass motion using the same parameters as the measurements taken in Fig. 3(b); an initial magnetic trap shift of 6 $\mu$m, $s_1 = 10$, $s_2 = 0.5$, and for both $N = 1.5 \times 10^4$ and $N = 2.0 \times 10^5$. The strongly damped center of mass motion with low atom number and the increased movement with high atom number seen in the experimental measurements is confirmed by the solutions of the NPSE.

From the momentum spectrum it is possible to distinguish when dynamical instability is present. Fig. 4(b) shows the progression of the momentum spectra with $N = 1.5 \times 10^4$, and Fig. 4(c) shows the momentum spectra with $N = 2 \times 10^5$. At $t = 25$ ms, the main components of the momentum spectra correspond to the peaks of the primary lattice at integer multiples of $\pm 2\hbar k_1$, and peaks of the beat periodicity at integer multiples of $\pm 2\hbar k_b$ around the primary peaks. For both high and low atom number, at $t = 50$ ms extra momentum components begin to rapidly grow signifying the onset of dynamical instability \cite{13}. We observe that the instability occurs when the quasimomentum becomes greater than $0.5k_b$. By 75 ms there is a marked difference between the cases of high and low atom number. With $N = 1.5 \times 10^4$ atoms, the initial spectrum has been obscured by the additional momentum components. In contrast, with $N = 2 \times 10^5$ atoms, the momentum spectrum retains much of the original spectrum structure. Intriguingly, the increased nonlinearities inhibit the growth of the instability.

Augmenting the interactions reduces the healing length of the condensate. In previous experiments, with a single lattice of spacing 0.4 $\mu$m and typically a healing length of $\simeq 0.3$ $\mu$m, the lattice spacing is not significantly greater than the healing length, and therefore measurements are largely indifferent to the atom number \cite{13,18}. 

\[ \text{[Equations and figures provided here]} \]
However, in the case of the bichromatic lattice the large spacing of the beating (1.8\,\mu m) allows interactions to effectively smooth over the large scale beat periodicity. The growth of dynamical instability in our bichromatic lattice is governed by the competition between augmentation by increased non-linearity and diminution by screening of the beat periodicity [19].

Consideration of the center of mass motion shown in Fig. 4(a) together with the momentum spectrum helps to unravel the different contributions to localization. The contaminated momenta at 75 ms for low atom number, shown in Fig. 4(b), is reflected in the complete blockage of the center of mass motion. However, most importantly, in the absence of dynamical instability, which is macroscopically observed only after 50 ms, the movement of the atoms is strongly damped with respect to the superfluid case (see dotted line in Fig. 4(a)), suggesting that strong damping of the oscillations is not only due to dynamical instability originating from the beat periodicity of the bichromatic lattice. The presence of interactions and the possible screening of the minigaps at small $k$ from the incommensurate bichromatic lattice renders interpretation of the blocked motion at short times difficult. As already pointed out, with sufficiently strong interactions the energy spectrum could be dominated by the periodicity at $\lambda_n$, and correspondingly the strongly damped motion at short times could be described by a high effective mass. Similarly to the screening behavior seen in Fig. 2, the contribution of the disorder will depend on the relative strength of interactions and the intensity of the disordering potential.

In conclusion, maintaining a minimal number of atoms we have observed a transition from oscillations to blocked motion with increasing intensity of the incommensurate bichromatic lattice. Simulations of our experimental parameters using the NPSE show that the quasi-order inherent in the quasi-periodic bichromatic lattice leads to the onset of dynamical instability, that contributes to the blocked motion only after a critical time. Screening of both localization due to the disorder and dynamical instability due to the beat periodicity was observed with strengthened interactions in the simulations. Increasing the number of atoms in the experiment, we observed a return of oscillating motion.

This work shows that the exact choice of parameters is crucial to separate and isolate the effect of disorder-induced localization, or the non-trivial onset of dynamical instability in a bichromatic lattice. In future measurements one could focus on measurements at small quasimomentum to explore the transport behavior without the complication of dynamical instability, however in an interacting system the contribution of the disorder to blocked center of mass motion cannot be easily distinguished from small amplitude oscillations resulting from an initial high effective mass when the energy spectrum is dominated by the beat periodicity. In the non-interacting limit, the bichromatic lattice is also a very promising tool to investigate Anderson-like localization that could be accessed utilizing fermions or Feshbach resonances, which also provide the important possibility of tuning the interactions.

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