Modelling stress-state dependent nonlocal damage and failure of ductile metals

Marvin Nahrmann¹ and Anton Matzenmiller¹,*

¹ Institute of Mechanics, Dept. of Mechanical Engineering, University of Kassel, Mönchebergstr. 7, 34125 Kassel, Germany

Damage and failure of ductile metals is highly influenced by the stress state. In fact, void growth takes place under hydrostatic tensile loading and void coalescence leads eventually to ductile failure. A completely different damage mechanism occurs for shear dominated loading that results in void elongation within a narrow shear band. In order to account for both failure mechanisms, a stress state dependent damage model is proposed. The model is based on a continuum damage mechanics approach, whereby nonlinear damage evolution is taken into account. The influence of the stress triaxiality and the LODE parameter on damage initiation and failure is considered by the Hosford-Coulomb model. Pathological mesh sensitivity is prevented by an integral type nonlocal formulation. Finally, the proposed stress-state dependent nonlocal damage model is verified by test data for the microalloyed high strength steel HX340LAD for a wide range of stress states.

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1 Constitutive model

The proposed constitutive model includes a plasticity theory, which is coupled with stress-state dependent nonlocal damage in order to represent the material characteristics of ductile metals. It is assumed that plasticity and damage are isotropic. The VON MISÉS yield criterion \( F = 0 \) is employed, whereby the VON MISÉS equivalent stress \( \sigma_{eq} = \sqrt{3\bar{J}_2} \) is compared to the sum of the initial yield stress \( \kappa_0 \) and nonlinear isotropic hardening \( \kappa \).

\[
F = \sqrt{3\bar{J}_2} - (\kappa_0 + \kappa) = 0 , \quad \kappa = \kappa^\infty \left[ 1 - \exp \left( \frac{E_c}{\kappa^\infty E_{pl}} \right) + \alpha_\kappa \left( \frac{E_c}{\kappa^\infty E_{pl}} \right)^{m_\kappa} \right] \tag{1}
\]

The hardening \( \kappa \) is driven by the equivalent plastic strain \( \dot{E}_{pl} = \frac{\int_0^t \sqrt{2/3} \bar{E}_{pl} \dot{\bar{E}}_{pl} dt}{\int \omega \bar{w}(x,y) dy} \), according to (1b), which contains the four parameters \( \kappa^\infty, E_c, \alpha_\kappa, \) and \( m_\kappa \) in the hardening function (1b). Since the second deviatoric stress invariant \( J_2 \) is applied here only, the yield condition is independent of the hydrostatic pressure. The plasticity model is implemented into the finite element software LS-DYNA within a hypoelastic framework considering finite deformations. The CAUCHY stress tensor \( \bar{T} \) is coupled with the nonlocal damage variable \( \bar{D} \) by means of the concept of effective stresses, with the nominal CAUCHY stress tensor in the damaged configuration \( \bar{T}^d \). By applying the integral nonlocal approach according to [1], the nonlocal damage rate \( \dot{D}(x) \) at the reference point \( x \) is obtained by weighting the local damage rate \( \dot{D}(y) \) at neighbouring points \( y \) over a certain region \( \Omega \). Therefore, the weighting function \( w(x,y) \) is employed by the two model parameters \( p \) and \( q \) as well as the characteristic length \( l_c \). Hereby, \( \|x - y\| \) is the EUCLIDEAN norm of the distance between \( x \) and \( y \).

\[
\bar{T}^d = (1 - \bar{D}) \bar{T} , \quad \dot{\bar{D}}(x) = \frac{\int_\Omega \bar{D}(y) w(x,y) dy}{\int \omega \bar{w}(x,y) dy} , \quad w(x,y) = \left[ 1 + \left( \frac{\|x - y\|}{l_c} \right)^p \right]^{-q} \tag{2}
\]

In order to achieve a stress-state dependent nonlocal damage model, the stress triaxiality \( T \) is applied by taking the normalised influence of the mean stress \( \sigma_m \) into account. Moreover, the LODE angle \( \theta_L \) is introduced by considering the third deviatoric stress invariant \( J_3 \). The LODE angle is normalised to the LODE parameter \( L \), which is defined in the range of \(-1 \leq \bar{L} \leq 1\).

\[
T = \frac{\sigma_m}{\sigma_{eq}} , \quad \theta_L = \frac{1}{3} \arccos \left( \frac{3\bar{J}_2}{\frac{3}{2}} J_3 \right) , \quad \bar{L} = 1 - \frac{2}{\pi} \arccos \left( \frac{3\bar{J}_2}{\frac{3}{2}} J_3 \right) \tag{3}
\]

Nonlinear damage growth is modelled by means of an evolution equation for the damage rate according to [2], where the equivalent plastic strain based damage initiation criterion \( \varepsilon_{c}(T, \bar{L}) \) as well as the failure criterion \( \varepsilon_{f}(T, \bar{L}) \) are defined in (4b). Both criteria are functions of the stress triaxiality and the LODE parameter in general. The damage exponent \( n_D \) controls the nonlinearity of the damage evolution behaviour. Considering a stress-state dependent damage and failure criterion, the HOSFORD-COULOMB model, proposed in [3], is applied for the functions \( \varepsilon_{c}(T, \bar{L}) \) and \( \varepsilon_{f}(T, \bar{L}) \).

\[
\bar{D} = \frac{n_D}{\varepsilon_{c}(T, \bar{L}) - \varepsilon_{c}(T, \bar{L})} \left( \frac{\bar{E}_{pl} - \varepsilon_{c}(T, \bar{L})}{\bar{\varepsilon}_{c}(T, \bar{L}) - \varepsilon_{c}(T, \bar{L})} \right)^{n_D-1} \bar{E}_{pl} \cdot \varepsilon_{c}(T, \bar{L}) = \bar{b}_{cf} \left( 1 + \frac{\varepsilon_{c}(T, \bar{L})}{\varepsilon_{c}(T, \bar{L})} \right) ^{\frac{n_D}{2}} \tag{4}
\]

The triaxiality and LODE parameter dependence on the critical strain \( \varepsilon_{c} \) and the failure strain \( \varepsilon_{f} \) is introduced by the functions:

* Corresponding author: e-mail post-structure@uni-kassel.de, phone +49 (0)561 804 2043, fax +49 (0)561 804 2720

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Section 3: Damage and fracture mechanics

\[ g_{c/f}(T, \bar{L}) = \left[ \frac{1}{2} (f_1 - f_2)^{\alpha_c/f} + \frac{1}{2} (f_2 - f_3)^{\alpha_c/f} + \frac{1}{2} (f_1 - f_3)^{\alpha_c/f} \right]^{\frac{1}{\alpha_c/f}} + \frac{1}{2} (2T + f_1 + f_3) \]  

\[ f_1 = \frac{2}{3} \cos \left[ \frac{\pi}{6} (1 - \bar{L}) \right], \quad f_2 = \frac{2}{3} \cos \left[ \frac{\pi}{6} (3 + \bar{L}) \right], \quad f_3 = -\frac{2}{3} \cos \left[ \frac{\pi}{6} (1 + \bar{L}) \right] \]

The HOSFORD-COULOMB approach can be interpreted as a generalised MOHR-COULOMB model with the parameters \( a_c, b_c, c_c \) for damage initiation and \( a_l, b_l, c_l \) for failure. The exponent in (4b) is defined a priori to \( n = 0.1 \).

2 Parameter identification and verification for a wide range of stress states

A stepwise procedure is pursued where the parameters for the ductile steel HX340LAD are identified on the basis of the test data carried out by the Fraunhofer Institute for Mechanics of Materials (IWM), see [4]. At first, the plasticity model is calibrated to tensile test data. Then, the parameters of the nonlocal weighting function are estimated by means of the plastic strain distribution prior to failure. Finally, the HOSFORD-COULOMB model parameters are determined. Therefore, the design of experiments includes flat smooth and notched tensile specimens, various shear specimens as well as a NAKAZIMA biaxial punch test in order to capture a wide range of stress states within the diagram of the triaxiality versus the LODE parameter, see Fig. 1 a). The identified damage initiation and failure surfaces are depicted in Fig. 1 b) and c). The simulation results, obtained by the identified model parameter set in Tab. 1, are compared to test data and shown in Fig. 1 d) and e) for the smooth tensile specimen and the 0° shear test d) and e) for the smooth tensile specimen and 0° shear test for HX340LAD steel.

![Fig. 1: Design of experiments a), damage initiation surface b), failure surface c) and simulation results compared to test data for smooth tensile specimen and 0° shear test for HX340LAD steel d) and e)](image)

Table 1: Model parameters for the HX340LAD microalloyed steel

| Parameter | Value |
|-----------|-------|
| \( E \) [MPa] | 210000.0 |
| \( \nu \) [-] | 0.3 |
| \( \kappa_0 \) [MPa] | 383.69 |
| \( \kappa_m \) [MPa] | 215.37 |
| \( E_m \) [MPa] | 1450.59 |
| \( a_c \) [-] | 0.1 |
| \( m_c \) [-] | 1.2 |
| \( n_0 \) [-] | 2.0 |
| \( l_c \) [-] | 2.0 |

| Parameter | Value |
|-----------|-------|
| \( p \) [-] | 2.0 |
| \( q \) [-] | 2.0 |
| \( \alpha \) [-] | 0.1 |
| \( a_l \) [-] | 1.166 |
| \( b_l \) [-] | 0.9566 |
| \( c_l \) [-] | 0.0443 |
| \( a_l \) [-] | 1.182 |
| \( b_l \) [-] | 1.398 |
| \( c_l \) [-] | 0.07605 |

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