Schwinger type processes
via branes and their gravity duals

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Abstract

We consider Schwinger type processes involving the creation of the charge and monopole pairs in the external fields and propose interpretation of these processes via corresponding brane configurations in Type IIB string theory. We suggest simple description of some new interesting nonperturbative processes like monopole/dyon transitions in the electric field and W-boson decay in the magnetic field using the brane language. Nonperturbative pair production in the strong coupling regime using the AdS/CFT correspondence is studied. The treatment of the similar processes in the noncommutative theories when noncommutativity is traded for the background fields is presented and the possible role of the critical magnetic field which is S-dual to the critical electric field is discussed.
1 Introduction

Schwinger type processes are the simplest nonperturbative phenomena in the field theory. They include famous charge/anticharge pair production in the constant electric field \[1\] as well as the monopole/antimonopole pair production in the magnetic field \[2\]. Natural generalization of these processes in the string theory, concerning the creation of \(p\)-branes in \((p + 2)\)-form fields with the constant curvature has been developed in \[3, 4\]. Another example is the creation of the strings in the constant electric field \[3, 5\] which reduces to the Schwinger pair production in the field theory limit (see \[6\] for a review). One more class of the Schwinger type processes includes so called induced processes, when some external factors such as additional particles or branes, finite temperature or chemical potential affect the probability rate \[3, 6, 7, 8, 9, 10\]. Very interesting phenomena of the related nature has been recently discovered by Myers \[11\].

In this paper we consider Schwinger type processes from the point of view of theories on the brane worldvolumes. It appears that the pair creation has simple brane realization in Type IIB string theory. For instance, Euclidean configuration corresponding to the electrically (magnetically) charged pair creation in four dimensional U(2) gauge theory is described by the worldvolume of F(D)-string stretched between two D3-branes. Exponential dependence of the probability rate is given by the minimum of the F(D)-string effective action. In the simplest case of 1+1 dimensional U(2) gauge theory, which is our basic example, this effective action is just the area of the string surface stretched between two \((p, q)\)-branes (see Fig.2). This leads to the formula (2.20) for the probability rate. When the external field value is small, one gets well-known field theory result. However, as the field becomes stronger, the probability gets modified in accordance with the Bachas-Porrati result \[6\]. We demonstrate that the production of the particles in adjoint and in fundamental representation proceeds in a similar manner.

The power of S-duality in type IIB string theory allows us to treat Schwinger type processes in different electric-magnetic backgrounds on the equal footing and suggests the existence of some interesting processes on the field theory side. For instance, we show that the monopole decays in the electric field (see Fig.1b, Fig.7a) while W-boson decays in the magnetic field (see Fig.7b). Evidently, the probability rates of both processes are exponentially suppressed but such processes could have some cosmological applications. Moreover, from this viewpoint it is natural to consider the critical magnetic field (2.32), which is S-dual to the critical electric field: in the critical magnetic field D-string becomes effectively tensionless, while in the critical electric field fundamental string becomes effectively tensionless.

One more problem addressed in this paper is the interpretation of the Schwinger pair production via the gravity duals of the large \(N\) gauge theories in the spirit of \[15\]. AdS/CFT correspondence gives one the opportunity to take into account quantum effects in the gauge theory by calculating the area of corresponding minimal surface in curved space. We consider two special brane configuration when the bunch of \(N\) D3-branes provides the AdS\(_5\) geometry, while one or two separate probe D3-branes are placed at finite values of the radial coordinate. This corresponds to spontaneous breaking of the gauge group down to \(U(N) \times U(1)\) or \(U(N) \times U(2)\). Then we switch constant electric field on the probe branes and calculate the probability of the pair creation in the strong coupling regime via the corresponding gravity solution. In the case of the \(U(N) \times U(1)\) gauge group we obtained the formula (4.12) for the probability rate. In the case of the \(U(N) \times U(2)\) group we get results in rather implicit form, since the situation turned out to be more complicated there. Note that unlike the Wilson loop
calculation within dual gravity picture \[14, 17, 18, 19\] we have the massive parameter from the very beginning and since we need this parameter to be finite, we do not move probe D3-brane to infinity. Another feature we found is the absence of the Gross-Ooguri type phase transitions \[20, 21\] for the Schwinger type processes at the weak coupling regime and the arbitrary field.

The theories on the D-brane worldvolume with background electric and magnetic fields in a certain limit amount to the NCYM theory \[22\] and NCOS theory \[23, 24\] theories correspondingly. It is natural to discuss the possible role of the Schwinger processes in these theories as the potential sources of their nonperturbative instabilities. We argue that such nonperturbative phenomena are of some importance and especially discuss the possible role of the critical magnetic field.

The paper is organized as follows. Section 2 is devoted to calculation of the pair production probability in the weak coupling regime via the special brane configurations in Type IIB theory. Different types of the electric-magnetic backgrounds are considered. The production of the matter in the fundamental representation as well as the temperature effects are discussed in short. In Section 3 we investigate some interesting processes like decay of the monopole in the electric field and W-boson decay in the magnetic field. These processes can be considered both in the field theory and in the brane setup. In Section 4 we use AdS/CFT correspondence to calculate the amplitudes of the W-boson pair production in \( \mathcal{N} = 4 \) SUSYM theory in the strong coupling limit. Section 5 is devoted to discussion on the nonperturbative stability of NCYM and NCOS theories which are treated as the theories on the D-brane worldvolumes with electric or magnetic backgrounds correspondingly. Some remarks and speculations on the results obtained in the paper and open questions can be found in Conclusion.

## 2 Pair production via branes

### 2.1 Preliminaries

Let us remind the key points concerning the pair production in the field theory. Consider production of the electrically charged pair in the electric field. Schwinger’s calculation \[1\] gives the following result for the probability of this process per unit time and volume in the four-dimensional U(1) gauge theory:

\[
w = (gE)^2 \frac{2s + 1}{8\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{(2s+1)(n+1)}}{n^2} \exp \left( -\frac{n\pi m^2}{gE} \right) \tag{2.1}
\]

where \( s = 0 \) or \( 1/2 \) is the particle spin, \( g \) – charge, \( m \) – mass and \( E \) – constant electric field. Later on we will mainly hunt for the exponential factors. They can be derived by minimizing the simple effective action which is natural in the first-quantized theory:

\[
S_{eff} = mL - gEA \tag{2.2}
\]

where \( L \) and \( A \) are the perimeter and the area of the closed particle trajectory in Euclidean space-time. Then
\[ w \propto \exp(-S_{\text{eff}}^{\text{min}}) \]  

(2.3)

From the symmetry considerations it follows that this trajectory is just a circle of a certain radius \( R \). (see Fig.1a) Therefore, the effective action reads

\[ S_{\text{eff}} = 2\pi mR - \pi gER^2 \]  

(2.4)

and \( R_{\text{min}} = m/(gE) \). Now we come to the answer

\[ w \propto \exp\left(\frac{-\pi m^2}{gE}\right) \]  

(2.5)

for the probability rate, in agreement with the Schwinger formula (2.1).

Fig.1: Field theory effective picture of the Schwinger type processes in Euclidian space-time: (a) \( e^+e^- \) pair creation in constant electric field \( E \). (b) Monopole decay into dyon and anti-charge in constant electric field: \( M \to D + e^- \). Since at weak coupling the dyon is much heavier then \( e^- \), its trajectory is almost the straight line.

In string theory there is a similar process: string pair production in a constant electric field. Generalization of the Schwinger formula (2.1) for this case was obtained by Bachas and Porrati [6]. For the fermionic strings it has the form:
\[ w = \frac{gE}{(2\pi)^{D-1}} \epsilon \sum_S \sum_{n=1}^{\infty} (-1)^{(n+1)(a_S+1)} \left( \frac{|\epsilon|}{n} \right)^{D/2} \exp \left( -\frac{\pi m_S^2}{|\epsilon| n} \right) \] (2.6)

Here \( D \) is a number of non-compact dimensions, \( m_S \) is a mass of the string state \( S \) and

\[ \epsilon = \frac{2}{\pi} \text{arcth}(\pi gE) = \frac{1}{\pi} \log \frac{1 + \pi gE}{1 - \pi gE} \] (2.7)

Below we describe how the exponential dependence of this probability can be obtained from the effective string action \( S_{\text{eff}} \).

### 2.2 Stringy W-boson pair production in U(2) gauge theory

Let us turn to the brane picture behind this process and start with 1+1 dimensional example and U(2) gauge group. The theory without external field is realized on the worldvolume of two D1-branes, displaced at the distance \( v \) in the transverse direction which corresponds to the vacuum expectation value of the scalar. To switch on the electric field one can add standard boundary term [25]:

\[ \oint d\sigma EX^0 \partial_\sigma X^1 \] (2.8)

to the Nambu-Goto action, but in type IIB theory we can consider dyonic \((n,1)\)-strings instead of the D1-branes. Then the value of the electric field in the theory reads as (see, for instance, [26])

\[ E = \frac{ng_{st}}{\sqrt{1 + (ng_{st})^2}} \] (2.9)

where \( g_{st} \) is a string coupling constant, which is related to the field coupling constant \( g \) as \( g_{st} = g^2 \). It follows from this formulae that there exists the critical value of the electric field \( E_{cr} = 1 \). When the field reaches this value the effective tension of the open string becomes zero. From the field theory viewpoint this corresponds to the pair production without exponential suppression since \( S_{\text{eff}}^{\text{min}}[E_{cr}] = 0 \). The W-bosons are represented by the F-strings stretched between two dyonic strings. We assume that with respect to the U(2) gauge group the electric field is chosen as \( \text{diag}(E, -E) \).

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1 It is assumed that \( \alpha' = 1/2 \) in this formula.
Hence we suggest the following Euclidean brane configuration responsible for the pair production in $1+1$ dimensions (see Fig.2). First of all, it should have rotational symmetry. There are two parallel $(n, 1)$-string worldvolumes which naively look like two planes. Later on we will see that in fact their shapes deviate logarithmically from the plane form at infinity. This is a peculiar property of the two dimensional physics. However, this can be interpreted as a renormalization of the W-boson mass and will not affect the form of the final answer for the probability. For this reason we will first assume that the $(n, 1)$-string worldsheet is flat. Two additional ingredients of the picture are: minimal surface formed by the F-string stretched between two $(n, 1)$-string worldsheets and two discs of the radius $R$ representing the worldsheets of $(n-1, 1)$-strings. The reason why these worldsheets must be flat is obvious from the physical point of view since the electric force is acting only on its boundaries.

We choose cylindrical coordinates $(r, \phi, z)$ such that $(n-1, 1)$-string surface is orthogonal to the $z$ axis. The angle $\gamma$ between the minimal F-surface and the $(n-1, 1)$-string worldsheet can be determined geometrically from the junction condition. We suppose that $n$ or $1/g_{st}$ is large enough, then

$$\cos \gamma = \frac{ng_{st}}{\sqrt{1 + (ng_{st})^2}}$$

(2.10)

The angle $\beta$ between $(n, 1)$ and $(n-1, 1)$-string worldsheets in our case is always small since it is given by

$$\sin \beta = \frac{1}{g_{st}(n^2 + 1/g_{st}^2)}$$

(2.11)

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The condition $\cos \gamma = E$ seems to be universal for the problem at hand – as we will see later, it holds in AdS space too.
It follows from (2.10) that when the coupling constant is small, F string surface is orthogonal to 
\((n-1,1)\) string worldsheet and the minimal surface becomes a cylinder. At the large coupling 
F-string should join the dyonic string at zero angle.

Let us briefly comment on how the process looks from the Euclidean time evolution point of view. First, at some point in the Euclidean time two finite pieces of the strings leave D-branes. Later on they touch in the bulk and rearrange into the two strings stretched between the two D-branes. After the materialization in Minkowski space they move apart. This evolution process is presented in Fig.3.

![Fig.3: Euclidean time slices of the strings evolution process in the constant electric field.](image)

To get the rate of the pair production we have to find the minimum of the following action:

\[ S = T_{1,0}A_{1,0} + 2(T_{n-1,1} - T_{n,1})\pi R^2 \]  

(2.12)

where \( R \) is a boundary radius of the F string worldsheet, \( T_{p,q} = \sqrt{p^2 + q^2/g_{str}^2} \) is the \((p,q)\)-string’s tension and \( A_{p,q} \) is the area of the \((p,q)\)-string’s worldsheet:

\[ A_{p,q} = 2\pi \int r(z)\sqrt{1 + (r'(z))^2} \, dz \]  

(2.13)

The action \( A_{p,q} \) has the following integral of motion:

\[ \frac{r(z)}{\sqrt{1 + (r'(z))^2}} = \text{const} \]  

(2.14)

For the F-string there is a minimal radius \( r = r_0 \) where \( r'_0 = 0 \) and therefore const = \( r_0 \). The equation of the minimal surface is

\[ r(z) = r_0 \cosh \frac{z - v/2}{r_0} \]  

(2.15)

Here \( z = 0 \) and \( z = v \) are coordinates of the two dyonic branes. Therefore,

\[ A_{1,0} = \pi r_0 v + 2\pi R\sqrt{R^2 - r_0^2} \]  

(2.16)
Since the condition (2.10) implies
\[ \left. \frac{dr}{dz} \right|_{z=v} = \frac{E}{\sqrt{1 - E^2}} \] (2.17)
we can express the minimal and the maximal radii of the surface as
\[ r_0 = \frac{v}{\log \frac{1+E}{1-E}}, \quad R = \frac{v}{\sqrt{1 - E^2} \log \frac{1+E}{1-E}} \] (2.18)

It is clear that the critical electric field in our approach has quite geometrical interpretation. It is the field when \( r_0 \) vanishes and the minimal surface becomes disconnected. Note that we have no Gross-Ooguri phase transition in our problem at any subcritical electric field. When the electrical field tends to the critical value, \( R \to \infty \). This means that effectively the infinitely long string is produced.

Thus, the minimum of the action (2.12) is given by:
\[ S_{\text{min}} = \frac{\pi v^2}{\log \frac{1+E}{1-E}} \] (2.19)

Finally, after restoring the \( \alpha' \) and \( g \) dependence we obtain the exponential factor
\[ w \propto \exp \left( -\frac{2g^2 \pi^2 \alpha' v^2}{\log \frac{1+2\pi \alpha' gE}{1-2\pi \alpha' gE}} \right) \] (2.20)

which agrees with the Bachas-Porrati result in the following sense: our boundary conditions select the single (ground) state from the whole tower of the stringy modes. That is why our result coincides with the leading term from the Bachas-Porrati probability series where all string states are involved. Note that from the field theory viewpoint the effective charge of the W-boson is \( 2g \) rather then \( g \) since it is in the adjoint representation of the gauge group.

The formulae (2.20) yields the exponential factor in the leading term in the full stringy probability (which would be a sum of the type (2.6)). Let us try to understand how a double summation would arise in our framework. The sum over \( S \) (string states) corresponds to the more complicated extremal surfaces without rotational symmetry. The sum over \( n \) corresponds to the “multi-instanton” contributions, which in our case look like clusters of \( n \) extremal surfaces touching each other on the boundary branes. Preexponential factors can be restored from the consideration of the determinant of the fluctuations around the minimal surface.

**Deformation of the worldsheet in 1+1 dimensional case**

Now let us take into account the effects coming from the deformation of the \((n,1)\)-string worldsheets. We introduce the radius \( R_\infty \gg R \) for the boundaries of this worldsheets and the distance \( v_\infty \) between them at \( R_\infty \). In a moment it will be clear that \( R_\infty \) serves as a IR regulator scale. The action to be minimized is given by
\[ S = T_{1,0}A_{1,0} + 2(T_{n-1,1} - T_{n,1}) \pi R^2 + 2T_{n,1}(A_{n,1} - \pi R_\infty^2) \] (2.21)
The \((n - 1, 1)\)-string surface has the form:

\[
r(z) = R \sin \beta \cosh \frac{z + z_0}{R \sin \beta}
\]  
(2.22)

where the constant \(z_0\) is determined by the relation

\[
R_\infty = R \sin \beta \cosh \frac{z + v_\infty/2}{R \sin \beta}
\]  
(2.23)

We have:

\[
A_{n,1} - \pi R_\infty^2 = \pi R v_\infty \sin \beta - 2\pi R^2 + O(e^{-R/R_\infty})
\]  
(2.24)

Therefore,

\[
S_{\text{min}} = \frac{\pi v^2 (1 + v_\infty/v)}{\log \frac{1-E}{1-E}}
\]  
(2.25)

Since we can rewrite \(v_\infty/v\) as follows:

\[
\frac{v_\infty}{v} = 2g \sqrt{1-E^2} \log \frac{2R_\infty}{R \sin \beta},
\]  
(2.26)

we see that deformation of the dyonic string surface results in additional term in the effective action, which is proportional to the coupling constant \(g\). Such ”quantum” terms are beyond our approximation here, since their appearance is natural in preexponential (”quantum determinant”) factors rather than in exponential factors. Note that from the 1+1 dimensional field theory viewpoint this term can be interpreted as a renormalization of the scalar vacuum value \(v\).

**Pair production in higher dimensions**

To fix the nontrivial field on the brane in higher dimensions, say on the D3-brane in \(d=4\), one has to consider the bound state of the D3-brane with \(N\) F1-strings. The number \(N\) of the F1-branes per unit transverse volume fixes the electric field on the D3-brane worldvolume as follows

\[
N = \frac{E}{\alpha' g_{\text{str}} \sqrt{E^2 - E^2}}
\]  
(2.27)

The pair creation corresponds to the process when one F1-string, initially bound to the one D3-brane, finally becomes stretched between two D3-branes. The calculation of the probability goes similarly to the 1+1 dimensional case but instead of the junction smooth F1-string worldvolume has to be considered. The effective action providing the production amplitude can be derived in the similar manner and involves the surface term and the electric field contribution. This gives the same exponential factors (2.20) as in the 1+1 dimensional case. The differences in higher
dimensions are twofold. First, asymptotically there is no back reaction from the escaping F1-string and therefore there is no logarithmic renormalization of the scalar condensate. Second, there are different preexponential factors in different dimensions which however are beyond our approximation.

### 2.3 Production of the fundamental matter

To consider the production of the pair of particles in the fundamental representation we represent the fundamental matter by the set of branes. In this section for the simplicity U(1) gauge group will be considered. One should distinguish several realizations of the fundamental matter in the different versions of the string theory.

Let us start with IIB/F theory picture. In the simplest IIB realization of the N=4 SUSY theory matter is represented by the fundamental strings connecting the D3 brane where the gauge theory is defined on and $N_f$ D5 branes. In the F theory it is represented by the strings connecting D7 branes wrapped around the torus in (10,11) dimensions and D3 branes. The orientation of the branes, say in F-theory, is as follows: worldvolume coordinates of the D3 brane are $(x_0, x_1, x_2, x_3)$ and of the D7 are $(x_0, x_1, x_2, x_3, x_7, x_8, x_{10}, x_{11})$. The coordinates of the prime interest are $(x_5, x_6)$ since D3 brane is localized at the origin while $k$-th D7 is localized at $z_k = x_{5,k} + i x_{6,k} = m_k$, where $m_k$ is the mass of the $k$-th flavor. It is assumed that all D3 and D7 branes are localized at the origin in $(x_4, x_9)$ directions.

![Brane configuration](image)

**Fig.4: Brane configuration describing production of the fundamental matter.**

Now let us identify the coordinates relevant for the Euclidean configuration describing the Schwinger process. Two coordinates are evident - these are the time and coordinate in the direction of the external field. The metric on the $(x_5, x_6)$ plane is nontrivial since D7 brane amounts to the cosmic string metric around [27]. However, the cosmic string metric locally can be brought into the flat form and the strings connecting D3 and D7 branes become the straight lines. That is why the relevant geometry of the Euclidean solution involves the minimal surface in $\mathbb{R}^3$.

However, there is some difference compared to the adjoint matter production case. The point is that there is no field on the worldvolume of D7 branes. Therefore the minimal surface between the D3 and D7 branes has to join D7 brane at the right angle. To get such configuration we can take the minimal surface relevant for the adjoint case and cut it at $r = r_{\text{min}}$. It is evident that the effective action in this case is twice less then in the adjoint case, in agreement with the field theory result. Hence the exponential factor in the production rate of the fundamentals is
given by

\[ w \propto \exp \left( -\frac{\pi m^2}{gE} \right) \]  

(2.28)

The picture in IIA/M theory looks as follows. In IIA picture one has D4 branes and \( N_F \) D6-branes located at positions \( m_i \) in \( z \) direction. Fundamental matter is represented by the strings connecting D6 and D4-branes. D6-branes amount to the nontrivial Taub-NUT metric around, and therefore one has to consider the minimal surface taking into account this metric. In M-theory picture the field theory is defined on the worldvolume of the M5-brane embedded into \( d = 11 \) manifold whose four coordinates are endowed with Taub-Nut metric. One can derive the same result as in IIB case considering the minimal surface in the Taub-Nut metric.

### 2.4 Monopole pair production

Let us discuss the monopole pair production. To this aim we have to consider stretched D1-branes instead of the F1-branes. The configuration looks similar to the charge case. The probability of the monopole pair production in the field theory limit is

\[ w \propto \exp\left( -\pi m_{\text{mon}}^2 / g_m B \right) \]  

(2.29)

where \( m_{\text{mon}} = v/g \) is the monopole mass and \( g_m \) is the magnetic coupling obeying the quantization condition \( g g_m = 2\pi \). Calculation of the stringy deformed probability amounts to

\[ w \propto \exp\left( -\frac{\alpha' v^2}{\log \frac{14 + g \alpha' B}{1 - g \alpha' B}} \right) \]  

(2.30)

Note that there is a kind of S-duality transformation relating charge and magnetic probabilities. Moreover if there are both electric and magnetic fields, the dyons could be produced.

Let us emphasize that similarly to the electric field case there exists the critical magnetic field value, such that the process of the monopole pair creation in this magnetic field becomes unsuppressed. In the brane picture it corresponds to the field value such that the minimal radius \( r_0 \) of the surface vanishes. Note that in spite of the smallness of the monopole pair production rate compared to the electric pair production rate the dependencies of the critical electric and magnetic fields on the coupling constant are the same

\[ E_{cr} = (2\pi g)^{-1} \alpha'^{-1} \]  

(2.31)

\[ B_{cr} = g^{-1} \alpha'^{-1} \]  

(2.32)

In principle one can expect that the probability of the monopole production amounts from the annulus partition function of the D string in the magnetic background. The probability calculated above is the contribution to the total probability from the single string state. However
the lack of the perturbative description of the D string makes the naive direct calculation of the partition function impossible and we have no formulae from the string theory side to compare with.

The analysis above concerns the 3+1 dimensional theory but there is some funny counterpart in 1+1 dimensions. If we consider the theory on the worldvolume of \((p,q)\) string at large \(p\) then the process of escaping, say \((1,k)\) string due to the junction is possible. Since \(p\) is assumed to be large, the backreaction of the junction on the \((p,q)\) string can be neglected. However since there are no monopoles in the two dimensions the interpretation of the tunneling process is different. Namely, it corresponds to the nonperturbative change of the rank of the gauge group locally. The process looks as follows. First, the small region where the rank of the gauge decreases by one emerges in the Euclidean space. Then it starts to expand in the Minkowski space-time and finally the rank of the gauge group decreases along the whole 1+1 dimensional space-time.

### 2.5 Pair production at finite temperature

Let us consider the periodical Euclidean configuration in the field theory limit. Consider first the situation when \(\beta = 1/T\) is larger then the boundary cylinder radius (Fig.5a). In this case naively there is no \(T\) dependence in the amplitude. When \(\beta\) approaches the radius of the cylinder the configuration changes into the "fish" one (Fig.5b). In the field theory limit we have the periodic array of the of deformed cylinders (Fig.5c).

![Fig.5: Pair production at finite temperature: (a) The region where the rate exponent is independent of the temperature. (b) The critical point where the character of the exponent behaviour changes. (c) The region where the temperature dependence is essential.](image)

The critical temperature, at which the \(T\)-dependence of the tunneling rate drastically changes, can be easily found from the geometry:

\[
T_{cr} = \sqrt{1 - E^2} \log \frac{1 + E}{1 - E}
\]  

(2.33)

It would be interesting to extract such critical temperature from the partition function of the fundamental string at nonzero temperature and the electric field found in [28].
Another interesting process at finite temperature is creation of the pair with nonvanishing total magnetic or electric charges. The simplest geometry of such process which takes use of junctions looks as follows. The array now consists of the deformed cylinders formed from the \((p_1, q_1)\) and \((p_2, q_2)\)-strings while at small temperatures they are connected by the sheets of \((p_1 + p_2, q_1 + q_2)\)-strings (Fig.6). This configuration implies the nontrivial temperature dependence for the corresponding amplitude at any temperature.

3 Decay of BPS particles in the external fields

In this section we consider novel phenomena of the BPS particles decay at rest in the external fields. We concentrate on the examples of the monopole decay in the electric field and the charge decay in the magnetic field. However the consideration of the generic BPS particle decay can be treated in the similar manner\textsuperscript{3}. Note that the case considered here differs from the process of the monopole decay in the external \(B_{\mu\nu}\) field in the bulk with the constant curvature considered in \[13\].

Let us explain the geometry of the process. We have BPS monopole at rest represented by the D1 string stretched between the two D3 branes. The electric field is switched on both D3 branes but evidently D1 brane classically doesn’t feel it. However D1 string can split into F1 string and \((-1, 1)\) string due to junction. After some interval of the Euclidean time virtual strings join into D1 again and the whole Euclidean solution looks like surface with the topology of the cylinder between two semiplanes corresponding to the worldvolumes of D1 strings. Brane configurations corresponding to the monopole and charge decays are presented at Fig.7a-b.

\textsuperscript{3}We call these processes "decay" although a different interpretation is possible. Monopole decay can be considered as a transition of the monopole into dyon and charge, or as an induced emission of the charge by the monopole. Charge decay can be considered as induced production of the dyon-antimonopole pair.
Fig. 7: Induced decay of BPS particles in the external fields in the brane language:
(a) Monopole $(0, 1)$ transition into dyon $(-1, 1)$ and charge $(1, 0)$ in the constant electric field.
(b) Electric charge $(1, 0)$ decay into dyon $(1, 1)$ and anti-monopole $(0, -1)$ in the constant magnetic field.

Let us turn now to calculation of the probabilities of such processes with the exponential accuracy. We start with the monopole decay in the weak electric field. At the weak coupling the tension of the D1 and dyonic string is much larger then the tension of F1 string. Therefore F1 string joins at junction almost at the right angle. Hence in the weak field worldvolume of the virtual F1 string looks as a half of the cylinder while the dyonic string worldvolume can be considered as plane. The sum of the both contributions amounts into the following probability

$$w_{\text{mon}} \propto \exp \left( 2\pi R v (T_{0,1} - T_{-1,1}) - \pi R v T_{1,0} \right)$$

where $R$ is the radius of the Euclidean trajectory of the charge. Since $\delta T$ is proportional to the coupling constant, the first term can be neglected and the probability rate equals to the square root of the spontaneous charge production in the electric field

$$w_{\text{mon}} = \sqrt{w_{\text{spon}}}$$

To get the probability of the charge decay in the magnetic field note that now both virtual strings are heavy at the weak coupling and the Euclidian configuration of the virtual strings almost coincides with the configuration corresponding to the spontaneous production of the monopole pair in the magnetic field. Therefore the probabilities of these two processes coincide as well. Let us emphasize that the probability rates for both processes can be found in the purely field theory approach without any reference to the string theory. The field theory counterpart of the junction is the process when the charged state fills the bound state on the monopole.

One more interesting process in the magnetic field can be considered when the fundamental matter is added. Let us consider the quark in the initial state represented by the string stretched between D3 and D7 brane. The junction amounts to the virtual D1 and dyonic strings forming a cylinder surface. This physical process corresponds to the decay of the quark into the magnetically charged states in the background field.

Let us also suggest the qualitative field theory explanation of the factor $1/2$ in the exponent
of the monopole decay probability. To this aim remind that there are discrete levels of the charge in the monopole background just at the middle of the forbidden zone. Therefore, by taking into account the fact that the charge yields the main contribution to the action, we immediately arrive at the desired factor since the Euclidean path from the discrete level is twice smaller than the path from the lower branch of the Dirac sea.

It is interesting to discuss decay processes at almost critical fields. Since the charge decay process is described by the almost radially symmetric brane configuration it is clear that at the critical magnetic field the surface shrinks at one point and two D1 strings seem to be asymptotic states of the decay. In the monopole decay case the picture is more subtle. Indeed since there is no radial symmetry it is difficult to get the explicit form of the minimal surface. However it seems that the critical electric field such that the shrinking occurs, does exist. The point is that the main contribution to the action comes from the charge and shrinking of the "half" of the surface should amounts into the shrinking of the whole surface.

4 Pair production in gauge theories at large $N$ and AdS/CFT correspondence

4.1 Set-up

In this section we study W-boson pair creation in $\mathcal{N} = 4$ SUSYM theory in strong coupling regime using the AdS/CFT correspondence [15].

Fig.8: AdS/CFT correspondence at work: (a) Type IIB brane picture of the W-boson pair creation induced by the U(1) part of the gauge strength in $\mathcal{N} = 4$ theory with the U(N)$\times$U(1) gauge group. (b) Gravity dual picture: "cap"-like surface in the $AdS_5$ space.

The main idea here is as follows. We start with the bunch of $N$ D3-branes in type IIB
theory and flat background. The low energy worldvolume theory is described by $\mathcal{N} = 4$ U(N) Yang-Mills theory. If we place one additional D3-brane at the distance $v$ from the bunch, this will result in U(N+1) gauge theory, spontaneously broken down to U(N) × U(1). Then, we can turn on constant electric field, corresponding to the U(1) part of the gauge field and study creation of the W-boson pair with the mass $v$ – see Fig.8a. In the large t’Hooft coupling constant limit we can also study this process from the gravity dual point of view. In order to do this we should replace a bunch of the D3-branes with the $AdS_5$ space. $AdS_5$ curvature radius is given by

$$R_{AdS} = (4\pi g_{YM}^2 \alpha'^2 N)^{1/4} \quad (4.1)$$

where $g_{YM}$ is Yang-Mills coupling constant. Instead of the "tube"-like minimal surface in the flat space in Type IIB picture (Fig.8a) now we have "cap"-like minimal surface in curved space (Fig.8b). All consequences of the F1 string interaction with the bunch of N D3-branes (including the strong coupling effects in associated gauge theory) are encoded geometrically in the Euclidean $AdS_5$ space metric:

$$ds^2 = R_{AdS}^2 dr^2 + r^2 d\Omega_3^2 + dz^2 \quad (4.2)$$

where $(r, \Omega_3)$ are the radial and angle coordinates on the D3-brane worldvolume, while $z$ is a coordinate in the transverse (bulk) direction. After the coordinate change in accordance with

$$z = e^\tau \cosh^{-1} \rho \quad r = e^\tau \tanh \rho \quad (4.3)$$

this metric takes the form:

$$ds^2 = R_{AdS}^2 (\sinh^2 \rho \, d\Omega_3^2 + \cosh^2 \rho \, d\tau^2 + d\rho^2) \quad (4.4)$$

which will be useful in further calculations.

An important question concerns the account of the electric field in the gravity picture. One way is to include it directly into the metric (see [29]). Unfortunately, in this case it is hard to find explicit form of the minimal surface due to the lack of the symmetry in the metric. Instead, we add boundary term (2.8) to the effective action:

$$\oint d\sigma F_{01} X^0 \partial_\sigma X^1 \quad (4.5)$$

Here $F_{\mu\nu}$ is the gauge strength. This term is nothing but an ordinary $B_{\mu\nu}$ term, which must be added to the Nambu-Goto action. It will be useful to introduce "scalar" electric field $E$ with the help of the red-shift factor from (4.2):

$$E = F_{01} \frac{z^2}{R_{AdS}^2} \quad (4.6)$$
Then it is straightforward to see that the term (4.5) represents the area $\text{Area}(\Sigma)$ of the disc $\partial \Sigma$, which the minimal surface’s boundary cut out on the D3-brane. Therefore, we have the following effective action:

$$S_{\text{eff}} = T_1 \text{Area}(\Sigma) + E \text{Area}(\partial \Sigma)$$

(4.7)

Of course, is the surface $\Sigma$ has two boundaries, there will be two boundary terms in (4.7). To find the probability rate we will minimize this action in the two steps. First, we will fix the boundary conditions and then find appropriate minimal surface. Second, we will extremize $S_{\text{eff}}$ with respect to the boundary conditions.

Calculation of the probability rate resulting from the ”cap”-like surface (Fig.8b) is presented in section 4.2. It is in fact similar to the circular Wilson loop calculation in $\mathcal{N} = 4$ SUSYM [18, 19], but in our case there are two essential differences. First, there is a constant electric field on the D3-brane, which determines certain angle between F1-string worldsheet and D3-brane. Secondly, we do not move this D3-brane up to infinity, since we need the mass of the W-boson to be fixed.

Another interesting example concerns W-boson pair creation in the case when U(N+2) gauge symmetry is spontaneously broken down to the U(N)$\times$U(2) gauge symmetry. In the gravity dual picture we have two branes in the $\text{AdS}_5$ space and ”tube”-like string surface stretched between them. As we will see in section 4.3, in this case our approach leads to some inconsistency in certain region of parameters. This is presumably due to the fact that flat D3-brane, which we use as a probe, is no more BPS-object when the electric field is turned on, and therefore some curving effects may occur. Probably, it can be more instructive to study corresponding configuration in the $\text{AdS}_3$ case. This issues will be discussed elsewhere [32].

### 4.2 W-boson pair creation in $\mathcal{N} = 4$ SYM theory with the $\text{U}(N)\times\text{U}(1)$ gauge group

The area of the ”cap”-like minimal surface with the circular boundary in the AdS space without electric field was calculated in [30, 31]. The reader can find all necessary details in section 4.3. Since there is a point $r = \rho = 0$ (a ”pole”) on a ”cap”-like surface, we must set $c = 0$ in the integral of motion (4.23). Therefore, equation for this surface in $(\rho, \tau)$ coordinates is just $\tau = \text{const}$. In terms of $(r, z)$ coordinates it has the form:

$$r = a \tanh \rho, \quad z = a \cosh^{-1} \rho, \quad \Rightarrow \quad z^2 + r^2 = a^2$$

(4.8)

The role of the electric field is to determine the angle between this surface and D3-brane. At zero value of the field this angle is equal to $\pi/2$. The action to be minimized is given by

$$S = 2\pi R_{AdS}^2 (\sqrt{1 + x^2} - 1) - \pi R_{AdS}^2 E x^2$$

(4.9)

\footnote{calculated in the $AdS_5$ metric (4.3)}
where \( x = \sinh \rho \) defines the boundary radius of the surface. The extremal value is

\[
x_* = \frac{\sqrt{1 - E^2}}{E} \quad (4.10)
\]

Therefore,

\[
S_{\text{min}} = \pi R_{\text{AdS}}^2 \frac{(1 - E)^2}{E} \quad (4.11)
\]

and after restoring the \( \alpha' \) and coupling constant dependence the probability rate takes the form

\[
w \propto \exp \left( -\sqrt{\pi N \frac{(1 - 2\pi \alpha' g_{YM} E)^2}{E}} \right) \quad (4.12)
\]

Let us emphasize that it does not depend on the W-boson mass or, in other words, on the position of the D3 brane in the AdS space.

### 4.3 W-boson pair creation in \( \mathcal{N} = 4 \) SYM theory with the \( \text{U}(N) \times \text{U}(2) \) gauge group

As a first step we elaborate on the relation between the electric field and the angle, at which the minimal surface touch D3-brane. The effective action in \((r, z)\) coordinates takes the form:

\[
S = \pi R_{\text{AdS}}^2 \left( 2 \int_{z_R}^{z_L} \frac{r}{z^2} \sqrt{1 + (r')^2} dz - E \left( \frac{r_L^2}{z_L^2} + \frac{r_R^2}{z_R^2} \right) \right) \quad (4.13)
\]

Here \( r(z) \) is supposed to solve the equation of motion:

\[
\frac{d}{dz} \frac{rr'}{z^2 \sqrt{1 + (r')^2}} = \frac{\sqrt{1 + (r')^2}}{z^2} \quad (4.14)
\]

with the appropriate boundary conditions: \( r(z_L, R) = r_{L, R} \). In order to find the minimum of (4.13) we should differentiate it with respect to the \( z_{L, R} \) and then set this derivatives to zero. By using the equation of motion we find:

\[
\frac{r'_L}{\sqrt{1 + r_L^2}} = E, \quad \frac{r'_R}{\sqrt{1 + r_R^2}} = -E \quad (4.15)
\]

Therefore,

\[
\frac{dr}{dz} = \frac{E}{\sqrt{1 - E^2}} = \xi \quad (4.16)
\]
At the left (L) brane we have $\xi > 0$ while at the right (R) brane $\xi < 0$. Note that we obtained the same junction (boundary) conditions as in the weak coupling approximation (2.17). Moreover, we see that there exist the same critical value of the electric field as in the 1+1 dimensional case. These are the elementary tests on the consistency of our approach.

As a next simple test we demonstrate how the Schwinger formula arises in the framework of the AdS/CFT-correspondence. For this purpose we consider production of the light W-bosons in the weak electric field\(^5\). The effective action has the form (see below):

$$S = \pi R^2_{AdS} \left( 2 \int_{x_L}^{x_R} \frac{x^2 \, dx}{\sqrt{x^2(x^2 + 1) - c^2}} - E(x^2_L + x^2_R) \right) \quad (4.17)$$

where

$$x_{L,R} \approx c \sqrt{1 + \xi^2 \mp \xi c^2} \quad (4.18)$$

The value of the constant $c$ can be determined from the condition that the distance $v$ between the left and right branes is equal to

$$\frac{v}{R_{AdS}} = \frac{1}{2} \log \frac{1 + x^2_R}{1 + x^2_L} + c \int_{x_L}^{x_R} \frac{dx}{(x^2 + 1)\sqrt{x^2(x^2 + 1) - c^2}} \quad (4.19)$$

Suppose that the constant $c \gg 1$. Then it is easy to show that the logarithm in (4.19) is of order $\xi c$ while the integral is of order $\xi$ and therefore irrelevant. Then,

$$c \approx \frac{v}{2R_{AdS} \xi} \quad (4.20)$$

which implies that the field value and the W-boson mass should be small. Now we can estimate the minimum of the action as follows:

$$S_{\text{min}} \approx \frac{\pi v^2}{2E} \quad (4.21)$$

This gives exactly the Schwinger dependence of the probability rate.

The rest of this section is devoted to the general case of the arbitrary W-boson mass and electric field. More detailed and extended discussion on the results given below will be presented elsewhere ([32]). We take the metric in the form (4.4). In this coordinates the horizon location is at $\tau = \infty$. The surface of rotation can be described in terms of the function $\rho(\tau)$. The area of the minimal surface of rotation with the fixed boundary conditions in the $AdS_5$ space is given by the extremum of the functional:

$^5$we assume that $\xi \ll v/R_{AdS} \ll 1$
\[ A = 2\pi R_{AdS}^2 \int \sinh \rho(\tau) \sqrt{\cosh^2 \rho(\tau) + \dot{\rho}^2(\tau)} \, d\tau \] (4.22)

It has the following integral of motion:

\[ \frac{\sinh \rho \cosh^2 \rho}{\sqrt{\cosh^2 \rho + \dot{\rho}^2}} = c \] (4.23)

where \( c \) is a non-negative constant. Using this integral of motion, we get the following equation for the minimal surface:

\[ \int_{\tau_0}^{\tau_\infty} d\tau = \pm c \int_{\rho_0}^{\infty} \frac{d\rho}{\cosh \rho \sqrt{\sinh^2 \rho \cosh^2 \rho - c^2}} \] (4.24)

Here for simplicity we fix one boundary condition at the horizon (\( \tau_\infty \to \infty, \rho = \infty \)) and consider the area as a function of the other one: \( \rho(\tau_0) = \rho_0 \). The integral in the rhs can be expressed via the elliptic integrals (see, e.g. [33]):

\[ \tau_\infty - \tau_0 = \pm \frac{c}{a} \left( F(a/x_0, ik) - \Pi(a/x_0, -1/a^2, ik) \right) \] (4.25)

where \( x_0 = \sinh \rho_0 \) and

\[ a = \sqrt{\frac{1 + 4c^2}{2}}, \quad k = \sqrt{\frac{1 + 4c^2}{1 + 4c^2} + \frac{1}{1 + 4c^2} - 1} \] (4.26)

Elliptic integrals in (4.25) are defined as

\[ F(x, k) = \int_0^x \frac{dt}{\sqrt{(1 - t^2)(1 - k^2t^2)}} \] (4.27)

and

\[ \Pi(x, n, k) = \int_0^x \frac{dt}{(1 - nt^2)(1 - t^2)(1 - k^2t^2)} \] (4.28)

We can plug (4.24) into (4.22) and obtain

\[ A_{\min}[x_0, c] = 2\pi R_{AdS}^2 \int_{x_0}^{x_\infty} \frac{x^2 dx}{\sqrt{x^2(1 + x^2) - c^2}}, \quad x = \sinh \rho \] (4.29)

It can be also expressed via the elliptic integrals:
\[ A_{\text{min}}[x_0, c] = 2\pi R_{\text{AdS}}^2 \left( x_\infty - \sqrt{1 + x_0^2 - c^2/x_0^2} + aF(a/x_0, ik) - aE(a/x_0, ik) \right) \]  

(4.30)

where

\[ E(x, k) = \int_0^x \frac{\sqrt{1 - k^2t^2}}{\sqrt{1 - t^2}} \, dt \]  

(4.31)

As we discussed before, in order to turn on constant \( U(1) \) part \( E \) of the gauge field on the probe brane, we should include the term

\[ S_E = -2\pi E \int \frac{rdr}{z_0^2} = -ER_{\text{AdS}} \sinh^2 \rho_0 \]  

(4.32)

Therefore, we should minimize the action

\[ S = T_{1,0}A_{\text{min}} - \pi R_{\text{AdS}}^2 E x_0^2 \]  

(4.33)

with respect to the \( x_0 \) assuming that the distance \( v \) between the boundaries of the surface is a constant. The latter is given by

\[ v = R_{\text{AdS}} \int_{z_0}^{z_\infty} \frac{dz}{z} = R_{\text{AdS}} \left( \frac{1}{2} \log \frac{1 + x_0^2}{1 + x_\infty^2} \pm \frac{c}{a} \left( F(a/x_0, ik) - \Pi(a/x_0, -1/a^2, ik) \right) \right) \]  

(4.34)

Then, it is straightforward to rewrite \( dr/dz \) derivative in \( (\rho, \tau) \) coordinates using (4.3, 4.24):

\[ \frac{dr}{dz} = \frac{cx \pm \sqrt{x^2(x^2 + 1) - c^2}}{c \mp x \sqrt{x^2(x^2 + 1) - c^2}} \]  

(4.35)

The equation (4.13): \( dr/dz = \xi \) generally has four solutions:

\[ x^{(\pm, \pm)} = \frac{-1 \pm \sqrt{1 \pm 4\xi c\sqrt{1+\xi^2}}}{2\xi} \]  

(4.36)

The sign which we should take in (4.33) is the same as in (4.24) and is equal to

\[ \pm = \frac{\text{sgn}(1 + \xi x)}{\text{sgn}(\xi - x)} \]  

(4.37)

From the analysis of these branches one can see that if one starts with some value \( E \) of the field and distance \( v \) and then begins to increase them, the regime changes. At certain values of the parameters the solution disappears (becomes complex). However, the effective action do not vanishes at this point. This indicates that our approach fails at certain region of the parameters.
5 Pair production in the noncommutative gauge theory and NCOS theory

5.1 On the pair production in NCYM theory

In this section we examine the Schwinger type processes in the noncommutative theories which can be considered as the certain limits of the theories on D-branes with the background magnetic \cite{22} or electric \cite{23, 24} fields (see \cite{24} for a review). Actually the theories with electric or magnetic backgrounds can be treated as dual to each other and the discussion concerning the relevant duality group can be found in \cite{35, 36}. We would like to consider nonperturbative aspects of the NCYM and NCOS theories. The motivation can be explained recalling the field theory counterpart. It is well known that the electrodynamics in the constant electric field is unstable theory with respect to the nonperturbative pair production. The Schwinger mechanism screens the electric field and finally the stable vacuum state has vanishing electric field. Therefore it is natural to ask if NCYM and NCOS are nonperturbatively stable theories. We will discuss the possible modes of instability in both theories.

Consider first the Dp branes with the magnetic backgrounds which can be obtained from the space $B_{\mu\nu}$ field in the bulk after the appropriate gauge fixing. Another way to represent magnetic field is to assume that there is the nonvanishing density of D1 branes on Dp brane worldvolume. The limit leading to the NCYM theory \cite{22} can be described as follows. Suppose that we start with the flat closed string metric $g_{ij}$ and the weak closed string coupling $g_s$. Then the parameters of the open string theory are

$$G_{ij} = g_{ij} - (2\pi\alpha')^2 (Bg^{-1}B)_{ij}$$ (5.1)

$$\theta^{ij} = -(2\pi\alpha')^2 \left( \frac{1}{g + 2\pi\alpha'B} B \frac{1}{g - 2\pi\alpha'B} \right)^{ij}$$ (5.2)

$$G_s = g_s \left( \frac{\det G_{ij}}{\det(g_{ij} + 2\pi\alpha'B_{ij})} \right)^{1/2}$$ (5.3)

To get the Seiberg-Witten limit leading to the NCYM theory one has to take $\alpha' \to 0$ with $G_{ij}, \theta^{ij}, G_s$ kept fixed \cite{22}. To this aim in the flat space one has either to vanish closed string metric or to take $B \to \infty$. The latter limit is more natural physically however we would like to discuss its reliability. It was assumed in \cite{22} that the limit $B \to \infty$ can be performed safely since there is no critical magnetic field. We would like to argue that it is not the case. We have already meet the critical magnetic field in Section 2 where the nonperturbative production of the monopole pair was discussed. Even more simple argument involves the consideration of the D1 string with the monopole pair at its ends in the magnetic field. The critical magnetic field corresponds to the situation when the force pulling apart the monopoles at the ends exactly
balances the tension of the D1 string. The corresponding critical magnetic field coincides with
that one amounted from the Schwinger process\footnote{The existence of the critical magnetic field has been noted before in \cite{37} in another context.}

Let us show that SW limit encounters the problem indeed. To this aim let us take the
following magnetic field and closed string metric in $(x_2, x_3)$ directions

\[ g_{ij} = g_{cs} \text{diag}(1, 1), \quad B_{ij} = B \epsilon_{ij} \]  \hspace{1cm} (5.4)

The open string parameters are

\[ G_{ij} = G \text{diag}(1, 1), \quad \theta^{ij} = \theta \epsilon_{ij} \]  \hspace{1cm} (5.5)

where

\[ G = (1 + \tilde{B}^2), \quad \theta = \frac{\tilde{B}}{B_{cr}(1 + B^2)}, \quad \tilde{B} = \frac{B}{B_{cr}}. \]  \hspace{1cm} (5.6)

From the relation

\[ 2\pi \tilde{B} \alpha' = G \theta \]  \hspace{1cm} (5.7)

which does not involve small parameter $g_{cs}$ at all it is clear that if there exists the critical
magnetic field the naive decoupling of the stringy degrees of freedom is impossible. Let us
emphasize that the critical magnetic field value does not go to the infinity in the $\alpha' \to 0$ limit
due to the proportionality to $g_{cs}$. Note that the open string coupling at the critical magnetic
field equals to $g_s$.

Therefore we can investigate the pair production in pure noncommutative gauge theory
taking the SW limit of the nonperturbative amplitude in the theory on the D3 brane in the
magnetic background. Let us discuss the possible unstable modes in U(1) NCYM theory. In
this theory there are magnetically charged U(1) noncommutative monopoles \cite{38} which however
have infinite masses and correspond to semiinfinite D1 strings ended on D3 brane as well as open
finite D1 strings. The probability of the monopole production vanishes however it seems that the
probability of the pair of magnetically charged D1 production survives similarly the Bachas-
Porrati description of the annulus partition function. Moreover we could also expect that
similarly to the electric case \cite{3} there could be a sort of classical instability for the magnetically
neutral D1 open string at the critical magnetic field.

The static monopole in U(2) NCYM theory is represented by the tilted D1 string stretched
between two D3 branes. The angle between the D1 and D3 branes is determined by the tension
condition. Unlike the commutative case the monopole state looks as a magnetic dipole whose
size is determined by the noncommutativity \cite{38}. Once again we are looking for the Euclidean
solution corresponding to the monopole pair production. In this case we are producing the pair
of dipoles. It seems that there is no Euclidean configuration corresponding to the production
of U(2) noncommutative monopoles if the magnetic fields on the two D3 branes are different:
$B_1 \neq B_2$.\footnote{The existence of the critical magnetic field has been noted before in \cite{37} in another context.}
5.2 On the nonperturbative processes in NCOS theory

Let us turn to the S-dual of noncommutative SYM theory, namely NCOS theory with nearly critical electric field and remaining stringy degrees of freedom \([24, 23]\). Now take the electric field and closed string metric in \((x_0, x_1)\) directions

\[
g_{ij} = g_{cs} \text{diag}(-1, 1), \quad B_{ij} = E\epsilon_{ij} \tag{5.8}
\]

The open string parameters are

\[
G_{ij} = G \text{diag}(-1, 1) \quad \theta^{ij} = \theta\epsilon^{ij} \tag{5.9}
\]

where

\[
G = (1 - \tilde{E}^2) \quad \theta = \frac{\tilde{E}}{E_{cr}(1 - \tilde{E}^2)} \quad \tilde{E} = \frac{E}{E_{cr}} \tag{5.10}
\]

The relation between the parameters looks similarly to the magnetic case

\[
2\pi \tilde{E}\alpha' = G\theta \tag{5.11}
\]

and the theory near the critical electric field defines the NCOS theory.

The essential difference with the magnetic case is that in the finite \(g_s\) case the effective coupling constant of the NCOS theory vanishes. However one could consider the following limit

\[
g_s \to \infty, \quad g_s^2\alpha' \text{ fixed} \tag{5.12}
\]

The parameters of the NCOS theory are related to the parameters of IIB theory as follows

\[
g_s^2\alpha' = G_s^4\alpha_{eff} \tag{5.13}
\]

\[
\alpha' \text{ tr } F = 1 - \frac{\alpha'_{eff}}{2\alpha_{eff}} \tag{5.14}
\]

where \(F\) is the field strength and \(G_s\) is the coupling of the NCOS theory. In the limit discussed above the field approaches the critical value \(\alpha' \text{ tr } F = 1\).

Consider the possible instability modes. There is the evident classical instability in the electrically neutral sector in U(1) theory. Moreover it is clear that in U(1) theory there is almost unsuppressed Bachas-Porrati probability of the creation which for each stringy mode involves in this limit the factor

\[
w \propto \exp\left(\frac{\text{const}}{\log(E - E_{cr})}\right) \tag{5.15}
\]
In the nonabelian case, for instance with the U(2) gauge group, one could ask about the production of the pair of "W-bosons", that is strings stretched between two D3 branes in d=4 theory. From the consideration in Section 2 it seems that the probability doesn’t vanish if the fields on each U(1) factors are different \( E_1 \neq E_2 \).

The behaviour of the NCOS theories at the nonzero temperature was recently considered in [39, 40]. It was shown that the Hagedorn transition corresponds to the temperature when getting off the neutral strings from the NCOS worldvolume becomes favorable. The quantum instability considered here doesn’t influence the Hagedorn transition for the neutral sector but would affect the transition in the charged sector.

Finally, note that one could add fundamental matter represented, for instance, by the D7 branes. To discuss the Schwinger pair production of the fundamental matter we can use the behaviour of the fundamentals found above. Near the critical electric field the Euclidean configuration responsible for this process looks like an almost noncompact surface approaching D3 brane and having the cusp at the attachment point to D7 brane.

6 Conclusion

In this paper we investigated the Schwinger type amplitudes in the context of the theories on the D-branes worldvolumes. It was shown that in the weak coupling approximation the picture of the particle production is very transparent and the agreement with the string production in the external field was found. We mainly focused in this paper on the theories with the maximal SUSY which corresponds to the field theory on the parallel D branes. However, we expect that the same approach works also for the theories with the less amount of SUSY where the curved geometry of the brane worldvolumes has to be taken into account. A new manifestation of the critical electric and magnetic fields was found. The calculation of the stringy deformed monopole pair production can provide some guess for the derivation of the partition function of the D-string. Some comments concerning the behaviour of the theories at the finite temperature were presented however the complete analysis will be considered elsewhere.

One can consider more general examples of the Schwinger type processes which can involve the branes of different dimensions. For instance, the important generalization of these processes with the additional particles (branes) in the final states has been recently used for the description of the Brane World spontaneous production [41] in the higher rank field. It would be interesting to generalize such processes for the case when the Brane World created along this way carries the additional particles or strings.

We elaborated the pair production processes in the context of AdS/CFT correspondence and found the probability of the process at the strong coupling limit. Surprisingly enough the critical electric field also manifests itself in this calculations. We used the simplified version of AdS/CFT correspondence when the electric field is introduced via a term in the effective action. This works well in AdS\(_3\) case however the exact modification of the metric by the external field has to be taken into account in the higher dimensions. We plan to discuss this point elsewhere.

The noncommutative theories are the natural playground for the application of Schwinger processes hence we investigated the issue of the nonperturbative stability of the NCYM and
NCOS theories. It appears that in both cases the potential modes of instability are indicated. Contrary to the widely accepted viewpoint concerning the absence of the critical magnetic we claim that it does exist and has very clear interpretation. We have mentioned that such critical magnetic field can be the obstacle for the Seiberg-Witten limit to exist. Furthermore, it is possible that Schwinger type processes give nonperturbative (nonanalytical in the field) corrections to the Born-Infeld effective action describing the brane worldvolume theory. However, to make more definite conclusion concerning this point additional analysis is required.

One more interesting outcome of this article is the discovery of a few novel nonperturbative processes of the BPS particles decay in the external fields. We can not exclude that these amplitudes can have some phenomenological implications in the early Universe. For instance the strong magnetic fields could produce the magnetically charged particles from the electrically charged ones with the substantial rate.

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