A Dual Power Law Distribution for the Stellar Initial Mass Function

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ABSTRACT
We introduce a new dual power law (DPL) probability distribution function for the mass distribution of stellar and substellar objects at birth, otherwise known as the initial mass function (IMF). The model contains both deterministic and stochastic elements, and provides a unified framework within which to view the formation of brown dwarfs and stars resulting from an accretion process that starts from extremely low mass seeds. It does not depend upon a top down scenario of collapsing (Jeans) masses or an initial lognormal or otherwise IMF-like distribution of seed masses. Like the modified lognormal power law (MLP) distribution, the DPL distribution has a power law at the high mass end, as a result of exponential growth of mass coupled with equally likely stopping of accretion at any time interval. Unlike the MLP, a power law decay also appears at the low mass end of the IMF. This feature is closely connected to the accretion stopping probability rising from an initially low value up to a high value. This might be associated with physical effects of ejections sometimes (i.e., rarely) stopping accretion at early times followed by outflow driven accretion stopping at later times, with the transition happening at a critical time (therefore mass). Comparing the DPL to empirical data, the critical mass is close to the substellar mass limit, suggesting that the onset of nuclear fusion plays an important role in the subsequent accretion history of a young stellar object.

Key words: accretion – stars: formation – stars: initial mass function

1 INTRODUCTION
The mass distribution of stars at birth, or initial mass function (IMF), is an important observable characteristic of clustered star formation. Observations show that the IMF is fairly independent of heavy-element abundance and has a similar character amongst local field stars and in galactic and extragalactic resolved clusters (Scalo 1986). However, the most distinguishing characteristic of the IMF, the high-mass tail, has measured power law indices that do vary within the range $\alpha \approx 1 - 2$, where the fractional number per logarithmic mass interval $dN/d\ln M \propto M^{-\alpha}$. Frequently quoted values include $\alpha = 1.35$ (Salpeter 1955) and $\alpha = 1.7$ (Kroupa 2002). Significantly, for $\alpha \gtrsim 1$, the vast majority of stars are found at the low mass end. The mass content is fairly evenly distributed, and the bulk of the total stellar luminosity emerges from the high mass end. The functional form of the IMF is key to studying the integrated effect of a collection of stars in a galaxy, e.g., for purposes of modeling the spectrum of a galaxy, its mass-to-light ratio, its star formation rate, or its chemical evolution (Kennicutt 1998). More recent work establishes that the IMF extends into the substellar regime $M < 0.075 M_\odot$ (e.g., Chabrier & Baraffe 2000; Andersen et al. 2008) and also that $dN/d\ln M$ has a peak at $\sim 0.2 M_\odot$ (Kroupa 2001, 2002; Chabrier 2003, 2005). The inclusion of substellar objects that do not achieve a steady-state nuclear fusion epoch (the main-sequence) makes it more important to define the “initial” or “birth” time of an object as the termination of mass accumulation rather than its appearance on the main-sequence in a Hertzsprung-Russell diagram. It is the mass distribution of these substellar objects that is the focus of this paper.

One of the popular ideas for an origin of the IMF is that turbulence in molecular clouds leads to a core mass function (CMF) that reflects properties of the turbulent scaling and also maps onto the IMF (Padoan & Nordlund 2002; Padoan & Nordlund 2004; Hennebelle & Chabrier 2008, 2009). The idea is that the CMF then directly maps onto the IMF since each core forms typically one or two stars...
with a high, and approximately fixed, proportion of the core mass going into the star(s).

While turbulent fluctuations can easily produce cores with mass exceeding the local Jeans mass, which is the minimum mass that can collapse gravitationally, this mechanism is challenged in its ability to create gravitationally collapsing low mass cores. The collapse of a low mass core that is well below the mean Jeans mass requires a very coherent and focused ram pressure due to a turbulent flow (Lomax et al. 2016). Given that the Jeans length is $\lambda_J = (\pi c_s^2/G\rho)^{1/2}$, where $c_s$ is the isothermal sound speed, $G$ is the gravitational constant, and $\rho$ is the density, the Jeans mass is

$$M_J = \rho \lambda_J^3 = 5.5 M_\odot \left( \frac{n}{10^4 \text{ cm}^{-3}} \right)^{-1/2} \left( \frac{T}{10 \text{ K}} \right)^{3/2},$$

where $n$ is the number density. It is very difficult to bring this minimum collapsible mass down to the substellar regime, requiring an extremely high density fluctuation. Thies et al. (2015) made a detailed comparison of observations and turbulent fragmentation models and concluded that an additional formation channel besides the latter is required in order to explain the frequency of substellar objects.

Years of observational surveys of molecular clouds have also revealed only very few candidate protostellar cores that have substellar mass (André et al. 2012; Lee et al. 2013). New high sensitivity observations of young stellar clusters however, reveal an increasing number of faint substellar objects. For example, a study of the large massive star forming cluster RCW 38 by Mužić et al. (2017) using adaptive optics imaging on the Very Large Telescope (VLT) reveals a large number of spectroscopically confirmed substellar objects, with the ratio of substellar to stellar objects estimated to be about 1:2. A new survey of the Orion Nebula Cluster by Drass et al. (2016), also using the VLT, finds a ratio of substellar to stellar objects of about 1:1, although the putative substellar objects are not yet spectroscopically confirmed.

Altogether, the presence of abundant substellar objects in young clusters hints at a mechanism to produce low mass objects from parent cores that have much greater mass. One effective way to do this is through disk instability in centrifugally-supported disks that form around young stars. The high density of disks and their lifetime, that is many orbit times, allows time for the development of gravitational instability, particularly in the early $\lesssim 10^5$ yr of evolution, when accretion from the parent core is significant (Vorobyov & Basu 2006, 2010, 2015). Basu & Vorobyov (2012) presented a scenario in which substellar mass clumps that are formed in the disk are then ejected through multi-body interactions, leading to free-floating proto-brown-dwarfs or proto-giant-planets. Vorobyov (2016) has examined this scenario further, and finds that ejected substellar cores can be distinguished from those formed by direct collapse by their greater specific angular momentum.

In this paper we explore the possibility that the observed IMF is set not so much by the CMF but by the accretion termination processes. Our work is similar in concept to the IMF model of Adams & Fatuzzo (1996) in that it posits that stellar masses are set by accretion terminations, and not related to the Jeans mass of the cloud. Here, we introduce a two phase process for accretion termination, replacing the detailed physics with a simple mathematical model. This process is characterized by a small accretion termination rate in an early phase which then rises to a larger accretion termination rate at a later stage.

In the first phase accretion termination is occurring very rarely and might be due to mechanisms like disk ejection (Basu & Vorobyov 2012; Vorobyov 2016). The transition to the second phase can be associated with the onset of nuclear (deuterium or hydrogen) fusion in the stellar core (Shu et al. 1987; Adams & Fatuzzo 1996). In the second phase the accretion termination probability rises to a greater value when the protostellar outflows have become active. However, other scenarios can also qualitatively fit in this framework. For example, the recent simulations of Lee & Hennebelle (2018a,b) suggest that, in a very dense and turbulent cloud, accretion termination due to the formation of a neighbouring accretor cannot occur in gas that is within the tidal radius of the initial first hydrostatic core. The mass of gas within this tidal radius is estimated to be $\sim 0.1 M_\odot$ in their models, and results in a peak in their calculated IMF at about that mass, and independent of the large scale properties and mean Jeans mass of the initial model cloud.

Our model takes up features of the modified lognormal power law (MLP) distribution elaborated by Basu et al. (2015). The MLP is based on the idea that there is an initial lognormal distribution of protostellar seeds that then grow by accretion from their surroundings at an exponential rate, and that there is an equally likely stopping probability for accretion in every time interval, leading to an exponential distribution of lifetimes. The result is a power-law distribution of final masses at the intermediate and high mass regime, with a power-law index that is equal to the dimensionless ratio of the growth time of accretion to the decay time of the exponential distribution of lifetimes.

In the MLP scenario, an initial delta function distribution of protostellar seed masses (i.e., a lognormal with zero variance) leads to a pure power-law distribution for the IMF. However, if observations show a low mass peak in the IMF, then the MLP model relies on a peak in the initial lognormal distribution of seed masses to lead to a peak in the IMF after accretion evolution. In this paper, we explore whether a peak in the IMF can be set by the accretion history itself, without any need for a peak in the underlying distribution of protostellar seeds.

Put another way, can a delta function of initial protostellar seed masses still yield a peaked IMF? In this paper we explore the scenario that all protostars start their life at a mass much less than their final mass and undergo accretion until it is terminated. This initial mass is likely in the range of $\sim 10^{-3} M_\odot$ to $10^{-2} M_\odot$, based on calculations of cloud collapse, resulting in first hydrostatic cores that then go on to yield second collapse stellar seeds (Larson 1969; Masunaga & Inutsuka 2000). Such seeds are envisioned as the starting point of our model.

The delayed rise of the rate at which accretion is terminated is crucial in our model. The resulting IMF is then naturally peaked and has power laws at both the low and high mass ends. The resulting dual power law (DPL) function differs from lognormal or lognormal-like functions in that at the lowest masses, there will be significantly more objects in the distribution. Distinguishing a lognormal from a power law distribution at the low mass end of the IMF (rather than at the high mass end as done historically) should be an important astronomical target in coming years as very deep
measurements of young stellar clusters are performed with new ground and space based observatories.

2 A MODEL FOR STAR FORMATION

Our model captures some aspects of the random nature of the star formation process while also employing a deterministic accretion process. Here we use it to describe the evolution of mass condensations in gas clouds that undergo accretion until some later time when accretion stops, defining the initial stellar mass.

The IMF has a probability density function denoted as $P_{\text{IMF}}$ which is a function of the mass $M$. It is convenient to work in terms of the function $\xi = \ln m$, where $m \equiv M/M_\odot$ is a normalized mass. Hereafter we use the following notation for probability density functions: $P(\xi) \equiv dN/d\xi = dN/d\ln m$ and $P(m) \equiv dN/dm$. We also often refer to $\xi$ as “mass”.

We are interested in the overall probability density $P_{\text{tot}}(\xi,t)$ to find a condensation of mass $\xi$ at time $t$, which obeys the normalization condition $\int P_{\text{tot}}(\xi,t) d\xi = 1$. It is the sum of the two probabilities, $P_a(\xi,t)$ and $P_s(\xi,t)$. $P_a(\xi,t)$ is the probability to find a condensation of mass $\xi$ still accreting at time $t$, and $P_s(\xi,t)$ is the probability to find a condensation of mass $\xi$ that has ceased accreting and has thus become a member of the IMF:

$$P_{\text{tot}}(\xi,t) = P_a(\xi,t) + P_s(\xi,t). \quad (2)$$

The evolution of $P_a(\xi,t)$ is governed by

$$\partial_t P_a(\xi,t) = \mathcal{L}P_a(\xi,t) - k(\xi,t)P_a(\xi,t), \quad (3)$$

where $\mathcal{L}$ is an operator describing the evolution dynamics, and $k(\xi,t)$ is the accretion-dropout rate, with which the active condensations are becoming inactive, or stationary, and thus a contribution to the IMF. The evolution operator $\mathcal{L}$ could be a Fokker-Planck or a master equation operator, but could also describe just a deterministic accretion of mass. $P_a(\xi,t)$ thus represents the fraction of all condensates for which accretion has ceased,

$$P_a(\xi,t) = \int_0^t k(\xi,t') P_a(\xi,t') dt', \quad (4)$$

where the lower integral limit $t_0$ suggests some presumed start time, which can be extended to $-\infty$ if desired. The IMF is then given by

$$P_{\text{IMF}}(\xi) = \lim_{t \to \infty} P_s(\xi,t) = \int_0^\infty k(\xi,t') P_a(\xi,t') dt'. \quad (5)$$

The evolution equation (3) is supplemented with an initial condition $P_a(\xi,t_0) = P_n(\xi,0)$ and boundary conditions if necessary.

We now study the implications of assumptions made in the literature (Basu & Jones 2004; Basu et al. 2015) that the accretion stops with a constant rate and that the mass accretion rate is proportional to the already accreted mass.

2.1 Constant Accretion-Dropout Rate

Suppose that the evolution operator is only dependent on $\xi$, i.e., $\mathcal{L} = \mathcal{L}[\xi]$, and the rate function is set to a constant, $k(\xi,t) = \delta$, then equation (3) becomes

$$\partial_t P_a(\xi,t) = \mathcal{L}[\xi] P_a(\xi,t) - \delta P_a(\xi,t). \quad (6)$$

It follows from equation (2) that the fraction of still accreting masses,

$$g(t) = \int P_a(\xi,t) d\xi, \quad (7)$$

is smaller than unity and thus $P_a(\xi,t)$ is not a proper probability distribution. But a new distribution $P_t(\xi,t)$ can be introduced via $P_t(\xi,t) = P_a(\xi,t)/g(t)$, that allows $P_t(\xi,t)$ to integrate to unity given a particular $g(t)$. Then $P_t(\xi,t) d\xi$ represents that fraction of the active condensations that can be found in an interval $d\xi$ at $\xi$.

We then use $P_a(\xi,t) = g(t)P_t(\xi,t)$ as an ansatz to decompose equation (6),

$$P_t(\xi,t) \partial_t g(t) + g(t) \partial_t P_t(\xi,t) = g(t)\mathcal{L}[\xi]P_t(\xi,t) - \delta g(t)P_t(\xi,t). \quad (8)$$

With the aim of conserving the probability in $P_t(\xi,t)$, equation (8) can be separated into an ordinary differential equation for $g(t)$ and a partial differential equation for $P_t(\xi,t)$:

$$\partial_t g(t) = -\delta g(t) \quad (9)$$

$$\partial_t P_t(\xi,t) = \mathcal{L}[\xi] P_t(\xi,t). \quad (10)$$

Without loss of generality in the following we set $t_0 = 0$. Utilizing the initial value $g(0) = 1$, which reflects that no condensations have dropped out at $t = 0$, we obtain

$$g(t) = e^{-\delta t}. \quad (11)$$

Inserting this into equation (5) then gives us the IMF as

$$P_{\text{IMF}}(\xi) = \int_0^\infty \delta e^{-\delta t'} P_t(\xi,t') dt'. \quad (12)$$

2.2 Constant Accretion Rate

The accretion process that creates stars is a complicated phenomenon, and can proceed through many phases. It can resemble constant accretion at early times (e.g., Shu 1977) and also exhibit episodic behavior (see Audard et al. 2014, and references within), but must attain greater values to create high mass stars in order to explain the small age spread of stars of different masses in young stellar clusters (Myers & Fuller 1993). Here we use a simplified determinisitic evolution law with the dynamics of the accretion process described by

$$\frac{dm}{dt} = \gamma m, \quad (13)$$

where the accretion rate $\gamma$ gives the characteristic time for this process. Based on this process a condensation with initial mass $m_0$ at time $t_0 = 0$ is transported in time $t$ to $m = m_0 e^{\gamma t}$ and thus the whole initial distribution is shifted to larger masses. With the choice $\xi = \ln m$ the corresponding evolution operator is thus $\mathcal{L}[\xi] = -\gamma \partial_\xi$ and we find

$$\partial_t P_r(\xi,t) = -\gamma \partial_\xi P_r(\xi,t). \quad (14)$$

This is simply the half wave equation, which then propagates the initial value of $P_r(\xi,0)$ forward in time,

$$P_r(\xi,t) = P_{n,0}(\xi - \gamma t). \quad (15)$$

The MLP model (Basu & Jones 2004; Basu et al. 2015)
has been effective in fitting the IMF, but it relies on a log-normal assumption to fit the low mass end,

\[ P_{a,0}(\ln m) d(\ln m) = \frac{\exp \left( -\frac{(\ln m - \mu_0)^2}{2\sigma_0^2} \right)}{\sqrt{2\pi}\sigma_0} d(\ln m), \quad (16) \]

where \( \mu_0 \) and \( \sigma_0^2 \) are the mean value and the variance, respectively. From here on we will use \( \ln m \) instead of \( \xi \) to be close to the original form of the MLP model.

The derivation then proceeds from (5), (11), and (14) to obtain

\[ P_{\text{MLP}}(\ln m) d(\ln m) = \int_0^\infty k(\ln m, t') P_a(\ln m, t') dt' d(\ln m) \]

where \( k_\text{MLP}(\ln m, t') \) is depicted. Note that the product of the two functions shows an initial increase which turns into a slow exponential like decrease close to the crossing point of the two functions.

\[ k_\text{MLP}(\ln m, t') = \frac{1}{\sqrt{2\pi}\sigma_0} \exp \left( -\frac{(\ln m - \mu_0)^2}{2\sigma_0^2} \right) \]

where erfc is the complementary error function and \( a = \delta/\gamma \) is a dimensionless parameter. But the a priori assumption of a lognormal distribution proves unnecessary if the dropout from accretion is simply delayed.

3 DELAYED DROPOUT

As a condensate matures into a star it becomes more efficient at driving off infalling material through increasing radiation and stellar winds, eventually ending accretion.

We envision two regimes: one where accretion is largely not resisted, and a second regime, starting at a later time through some brief transition, when accretion is equally likely to be stopped in any time interval. We allow the dropout rate to be time dependent in that it switches on at a time later than the start of accretion.

3.1 Time-dependent Accretion-Dropout Rate

For simplicity all condensations start with the same seed mass, \( m_0 \). Thus,

\[ P_{a,0}(\ln m) d(\ln m) = \delta(\ln m - \ln m_0) d(\ln m), \quad (19) \]

where \( \delta \) represents the Dirac delta function, in contrast to the unrelated constant \( \delta \) used in this paper. The accretion process then proceeds as in the MLP and thus (14) holds,

\[ P_t(\ln m, t) d(\ln m) = \delta(\ln m - \ln m_0 - \gamma t) d(\ln m). \quad (20) \]

The accretion-dropout rate is chosen to be time dependent: \( k(\xi, t) = k(t) \). With this choice, the same ansatz \( P_a(\xi, t) = g(t) P_t(\xi, t) \) as above can be made. Inserting this into (5) then leads to

\[ P_{\text{IMP}}(\ln m) d(\ln m) = \int_0^\infty k(t') g(t') \delta(\ln m - \ln m_0 - \gamma t') dt' d(\ln m) \]

\[ = \begin{cases} 0 & : m < m_0 \\
\frac{k(\ln m_0) g(t_m)}{\gamma} & : m \geq m_0,
\end{cases} \quad (21) \]

3.2 The DPL Accretion-Dropout Rate

The delay leads to a dual power law (DPL) for the IMF if the delay is characterized through the following choice for \( k(t) \),

\[ k_{\text{DPL}}(t) = \delta(1 + \tanh[(t - t_S)/\eta]), \quad (22) \]

where \( \delta \) is the maximum dropout rate. The rise time \( t_S \) indicates the midpoint of the transition from smallest dropout rate to largest dropout rate. The dropout rise rate \( \eta \) sets the time span \( 1/\eta \) of the transition zone. Also, we note that at the dropout time one finds the greatest rate of increase of the rate function, \( k_{\text{DPL}}(t) \).

The DPL rate is chosen such that for small times it increases slowly from a small but non-zero starting value while for large times it asymptotically approaches \( \delta \) after passing through a transition zone in which it increases rapidly.

Replacing the time independent \( \delta \) in equation (9) with the time dependent dropout rate \( k_{\text{DPL}}(t) \) we obtain

\[ \dot{\delta}_{\text{DPL}}(t) = -k_{\text{DPL}}(t) S_{\text{DPL}}(t) \quad (23) \]

with the solution

\[ g_{\text{DPL}}(t) = e^{-\frac{\dot{\delta}_{\text{DPL}}(t) t}{\gamma}}. \quad (24) \]

In Fig. 1 one sees how the two functions contributing to the IMF complement each other. While \( k_{\text{DPL}}(t) \) provides the increasing part, \( g_{\text{DPL}}(t) \) dominates the decreasing part of the IMF.

We introduce \( m_S \) via \( t_S = (\ln m_S - \ln m_0)/\gamma \), which is the characteristic mass at which the rate function has its steepest increase. Inserting \( t_S \) and \( m_0 \) into equation (21) we find for \( m \geq m_0 \) the truncated dual power law distribution.
$P_{\text{DPL}}$ as

$$P_{\text{DPL}}(m; \alpha, \beta, m_S, m_0) \propto d(\ln m)$$

$$= \frac{k_{\text{DPL}}(m; \alpha, \beta, m_S, m_0)}{d(\ln m)}$$

$$= \frac{\alpha}{2} \exp \left[ -\frac{\alpha}{2} (\ln m - \ln m_0) \right]$$

$$\times \left\{ \cosh[(\ln m_0 - \ln m_S)\beta]/\cosh[(\ln m - \ln m_S)\beta] \right\}^{\frac{\alpha}{\gamma}}$$

$$\times (1 + \tanh[(\ln m - \ln m_S)\beta]) d(\ln m),$$

where $\alpha = \delta/\gamma$ as above and $\beta = \eta/\gamma$ is a further dimensionless parameter.

### 3.3 The DPL Distribution

A careful inspection of equation (25) reveals that $P_{\text{DPL}}$ converges rapidly towards a limiting distribution for $m_0 \to 0$, the dual power law (DPL) distribution:

$$P_{\text{DPL}}(m; \alpha, \beta, m_S) \propto d(\ln m)$$

$$= \frac{\alpha}{2} \exp \left[ -\frac{\alpha}{2} (\ln m - \ln m_S) \right]$$

$$\times \left\{ \cosh(\ln m_S - \ln m) \right\}^{\frac{\alpha}{\gamma}}$$

$$\times (1 + \tanh(\ln m_S - \ln m_S)) d(\ln m).$$

The analysis of the DPL distribution also shows that its decay towards smaller and larger masses are described by power laws $P_{\text{DPL}}(m) \propto m^\alpha$. The exponents $\nu_{m_S < m}$ and $\nu_{m < m_S}$ of those power law tails are related to the model parameters in a simple fashion: $\nu_{m_S < m} = -(\alpha + 1)$ and $\nu_{m < m_S} = 2\beta - 1$.

The surprising and unexpected feature of the DPL function based on the model assumptions made is the power law behavior of the IMF at the low mass tail. Such a power law behavior of the IMF at the low mass end has been suggested in the literature before. Da Rio et al. (2012) performed extensive optical observations of the Orion Nebula Cluster (ONC), and compiled the masses of confirmed cluster members. Their deduced IMF could be fitted by power law functions on both sides of a peak value, when using one of the two considered evolutionary models.

In Fig. 2, we plot the DPL function that utilizes the same values of $\alpha(=1.3)$ and $\beta(=1.205)$ as the fit by Da Rio et al. (2012) (see their table 4). The asymptotic power law behavior is indicated by gray lines on both sides of the peak. The DPL is a continuous function and is shown here for $m_S = 10^{-0.78} = 0.166$. In addition, we plot a MLP with same high mass power exponent $\alpha = 1.3$, $\beta = -2$, and $\gamma = 0.6$. One clearly sees the difference between the DPL and the MLP at the low mass end.

Interestingly the value of $m_S$ that is compatible with the Da Rio et al. (2012) data is about a factor of 2 within the substellar mass limit that has a typical value of 0.075 $M\odot$ (Chabrier & Baraffe 2000) and an estimated range of uncertainty $0.064 M\odot > m_S > 0.087 M\odot$ (Auddy et al. 2016). We also note that $m_S$ is not the mass at which the transition begins, but rather the mass at which a steepest transition in the accretion stopping rate is taking place. This raises the possibility of the physics behind the transition being related to the onset of nuclear fusion. While this is a tempting possibility, this question cannot be answered within the context of the mathematical model presented here.

The importance of a DPL function for the IMF, if verified by future observations, is twofold. It implies the existence of a generative effect for the IMF that operates at low masses, and is not dependent on an imprinted lognormal distribution from stochastic processes. We have presumed that the generative effect is accretion, and the lack of accretion stopping at very early times leads to the power law at low masses. Secondly, if the very low mass IMF is a power law, one can reasonably expect that below some small enough value of $m$, there will be more objects than predicted by a lognormal decay, no matter what is the value of the power law index.

### 4 CONCLUSIONS

Our model uses a simplified scenario for stellar mass accretion and delayed accretion dropout in order to derive an analytic three parameter dual power law (DPL) probability distribution function for the IMF. The two power laws have different exponents. Each has a direct physical interpretation.

The high mass tail has an exponent set by the ratio $\alpha = \delta/\gamma$ of the maximum dropout rate $\delta$ and the accretion rate $\gamma$ as in the MLP model. The low mass tail has an exponent that depends on the ratio $\beta = \eta/\gamma$ of the dropout rise rate $\eta$ and the accretion rate $\gamma$. The third dimensionless parameter of the distribution is related to the characteristic mass $m_S$ at which the dropout rate rises most rapidly.

A remarkable feature of the empirical IMF is that this characteristic mass is so similar to the substellar mass limit. As the empirical IMF of different star clusters vary considerably (Dib 2014), it would be premature to predict that this feature will occur in all of them. Nonetheless, this proximity raises the question of whether the peak of the IMF can be traced to a transition originating from fundamental physics, specifically the onset of nuclear fusion.

Our model provides insight into a possible scenario of generative processes for a dual power law IMF, but does not require that identical values of the power-law indices are present in each star-forming cluster, as these may depend on the specific accretion and dropout histories in that cluster.
What we emphasize is that a dual power law distribution is qualitatively different from a lognormal-like decrease, and has a larger number of substellar objects below a certain mass. If the dual power law is realized in nature, then there could be a large population of currently undetected very low mass substellar objects in the Galaxy.

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