Abstract.
Element settling inside the Sun now becomes detectable from the comparison of the observed oscillation modes with the results of the theoretical models. This settling is due, not only to gravitation, but also to thermal diffusion and radiative acceleration (although this last effect is small compared to the two others). It leads to abundance variations of helium and heavy elements of $\approx 10\%$ below the convective zone. Although not observable from spectroscopy, such variations lead to non-negligible modifications of the solar internal structure and evolution. Helioseismology is a powerful tool to detect such effects, and its positive results represent a great success for the theory of stellar evolution. Meanwhile, evidences are obtained that the element settling is slightly smoothed down, probably due to mild macroscopic motions below the convective zone. Additional observations of the abundances of both $^7\text{Li}$ and $^3\text{He}$ lead to specific constraints on these particular motions.

Key words: Sun : abundances; helioseismology; diffusion processes; element settling

1. Introduction
The importance of element settling inside the stars during their evolution is now widely recognized as a “standard process” (see, for example, Vauclair 1998). As soon as condensed from interstellar clouds, the self-gravitating spheres built density, pressure and temperature gradients which force the various chemical species present in the stellar gas to move with respect to one another. This process, first introduced by Michaud 1970 to account for the chemically peculiar stars, is believed to be the reason for the large abundance variations observed in main-sequence type stars, horizontal branch stars and white dwarfs (Vauclair and Vauclair, 1982).

Inside the convective regions, the rapid macroscopic motions mix the gas components and force their homogenisation. The chemical composition observed in the external regions of cool stars is thus affected by the settling which occurs below the outer convective zones. As the settling time scales vary in first approximation like the inverse of the density, the expected variations are smaller for cooler stars, which have deeper convection zones. While some elements can see their abundances vary by several orders of magnitude in the hottest Ap stars, the maximum expected variations in the Sun are not larger than $\approx 10\%$.

Such variations cannot be observed in the solar atmosphere by spectroscopy. In the present days however, due to helioseismology, we know the
internal structure of the Sun with a high degree of precision. Evidences for the occurrence of element settling are found. Abundance variations of the order of a few percent now become indirectly detectable, by comparisons of the theoretical computations with the results of the inversion of pulsating modes. We have entered a new area in this respect.

2. Theory of element settling

2.1. The Diffusion Equation

What we use to call “microscopic” diffusion of the chemical elements in stars represents a competition between two kinds of processes. First the individual atoms want to move under the influence of the local gravity (or pressure gradient), thermal gradient, radiative acceleration and concentration gradient. Second their motion is slowed down due to collisions with the other ions as they share the acquired momentum in a random way. This competition leads to selective element settling inside the stars.

The computations of this settling process are based on the Boltzmann equation for dilute collision-dominated plasmas. At equilibrium the solution of the equation is the maxwellian distribution function. We consider here situations where the distribution is not maxwellian, but where the deviations from the maxwellian distribution are very small.

Two different methods have been used to solve the Boltzmann equation in the framework of this approximation. The first method relies on the Chapman-Enskog procedure (described in Chapman and Cowling, 1970), using convergent series of the distribution function. This procedure is applied to binary mixtures, leading to expressions with successive approximations for the binary diffusion coefficients. For the diffusion of charged particles in a plasma, a ternary mixture approximation is introduced, including the electrons. This method was widely used in the first computations of diffusion processes in stars (see Vauclair and Vauclair, 1982). More recently, similar methods have still been used by many authors, for example Bahcall and Loeb (1990), Proffitt and Michaud (1991), Michaud and Vauclair (1991), Bahcall and Pinsonneault (1992), Charbonnel, Vauclair, Zahn (1992), Richard et al (1996) (hereafter RVCD). The second method is that of Burgers (1969), in which separate flow and heat equations for each component of a multi-component mixture are solved simultaneously. Descriptions of this method may be found for example in Cox, Guzik and Kidman (1989), Proffitt and VandenBerg (1991), Richer and Michaud (1993), Thoul, Bahcall and Loeb (1994).
In the formalism of RVCD, which has been used for the results given below, the local abundances of the elements are given in terms of their concentrations, solutions of equations of the following type:

\[
\frac{\partial c_i}{\partial t} = D'_i \frac{\partial^2 c_i}{\partial m_r^2} + \left( \frac{\partial D'_i}{\partial m_r} - V'_i \right) \frac{\partial c_i}{\partial m_r} - \left( \frac{\partial V'_i}{\partial m_r} + \lambda_i \right) c_i
\]

where \(c_i\), the concentration of element i, is given in terms of the stellar mass fraction \(m_r\), \(\lambda_i\) is the nuclear reaction rate, and \(D'_i\) is given by:

\[
D'_i = \left(4\pi \rho r^2\right)^2 (D_T + D_{1i})
\]

in which \(D_T\) represents the effective macroscopic diffusion coefficient and \(D_{1i}\) the microscopic diffusion coefficient of element i relative to element 1 (here hydrogen).

\(V'_i\) is given by:

\[
V'_i = \left(4\pi \rho r^2\right) V_{1i}
\]

with:

\[
V_{1i} = -D_{1i} \left( A_i - \frac{Z_i}{2} \right) \left( \frac{m_H GM}{kT R^2} \right) - \alpha_{1i} \nabla \ln T
\]

The thermal diffusion coefficient \(\alpha_{1i}\) is computed using the formalism of Paquette et al. 1986; \(A_i\) and \(Z_i\) represent the atomic mass number and charge of element i and \(m_H\) is the atomic hydrogen mass. \(M\) and \(R\) stand for the stellar mass and radius.

A normalisation condition on the mass fractions of all the elements has to be added to correct for the center-of-mass displacement.

The diffusion equation has to be solved simultaneously for all the considered elements. The order of magnitude of the time scales generally implies the computation of many iterations of the diffusion process for a single evolutionary time step. For each computation of a new model along the evolutionary track, the tables of abundances inside the star have to be transferred for every element, as a function of the internal mass. For the model consistency, these abundance profiles must be taken into account in the interpolation of the opacity tables.

For the Sun, the whole process of complete time evolution has to be iterated several times from the beginning, with small adjustments in the original helium mass fraction and mixing length parameter, to obtain the right Sun and the right age (luminosity and radius with a relative precision of at least \(10^{-4}\)).
2.2. THE TREATMENT OF COLLISIONS

The diffusion time scales are direct functions of the collision probabilities for the considered species. A good treatment of collisions is thus necessary to obtain the abundance variations with a high degree of precision.

For the diffusion of neutral atoms in a neutral gas, the “hard sphere approximation” is used. For ions moving in a neutral medium, or neutrals moving in a plasma, the polarisation of the neutrals have to be taken into account. For collisions between charged ions, problems similar to those encountered for the equations of state have to be solved. The basic question concerns the divergence of the coulomb interaction cross sections. In the first computations of diffusion, the “Chapman and Cowling approximation” was used, assuming a cut-off of the cross section equal to the Debye shielding length. Average values of the shielding factor were used for analytical fits of the resulting diffusion coefficients.

Paquette et al. (1986) proposed a more precise treatment of this problem. They pointed out that the Debye shielding length has no physical meaning as soon as it is smaller than the inter-ionic distance. They proposed to introduce a screened coulomb potential with a characteristic length equal to the largest of the Debye length and inter-ionic distance, and they gave tables of collision integrals which can be used in the computations of diffusion processes in the stellar gases. The Paquette et al. approximation should be generally used in the computations of stellar structure. It may however in most cases be replaced by an analytical expression given by Michaud and Proffitt (1992).

2.3. THE RADIATIVE ACCELERATION

Many authors have computed the gravitational and thermal diffusion of helium and heavier elements in the Sun with various approximations: see, for example, Michaud and Vauclair (1991) and references therein; Cox, Guzik and Kidman (1989); Bahcall and Pinsonneault (1992); Proffitt (1994); Thoul, Bahcall and Loeb (1994); Christensen-Dalsgaard, Proffitt and Thompson (1993); RVCD. In all cases the radiative accelerations were neglected.

For the first time, Turcotte et al (1998) have consistently computed the radiative accelerations on the elements included in the OPAL opacities. They have found that, contrary to current belief, the effect of radiation can, in some cases, be as large as $\approx 40\%$ that of gravity below the solar convective zone. This is important only for metals however, and not for helium. When the radiative accelerations are neglected, the abundances of most metals change by $\approx 7.5\%$ if complete ionisation is assumed below the convection zone, and by $\approx 8.5\%$ if detailed ionisation rates are computed. When the radiative accelerations are introduced, with detailed ionisation, the results
lie in-between. The resulting effect on the solar models is small and can be neglected (while it becomes important for hotter stars).

3. Evidence of Element Settling inside the Sun from Helioseismology

Solar models computed in the old “standard” way, in which the element settling is totally neglected, do not agree with the inversion of the seismic modes. This result has been obtained by many authors, in different ways (see Gough et al. (1996) and references therein). There is a characteristic discrepancy of order one percent, just below the convective zone, between the sound velocity computed in the models and that of the seismic Sun. Introducing the element settling reconciles the two results. Fig.1 shows an example of solar models obtained with and without element settling in the computation. These models have been obtained with the Toulouse code, as described in Charbonnel, Vauclair and Zahn (1992) and RVCD. Some improvements have been introduced in the treatment of the opacities and equation of state, as described in Richard, Vauclair and Charbonnel (1998).

These computations are compared to the results of helioseismology in collaboration with the Warsaw group (Dziembowski et al., 1994). The values of the function $u = P/\rho$ as obtained from the inversion of the solar oscillation modes (seismic Sun) are displayed together with the results of our models.

It could be possible to reduce the discrepancy between the sound velocity in the old solar standard models and in the seismic Sun by adding other effects than element settling. For example changes in the opacities could possibly lead to similar results. However, as already pointed out, element settling must not be considered as a new parameter added in the computations. It represents second order effects in the physics of auto-gravitational spheres, precisely and without any free parameter. Introducing element settling in the standard models means improving the physics. The fact that these new models lie closer to the seismic Sun than the old ones is quite encouraging and may be considered as a proof that the physical improvements are correct.

4. Discussion: necessity of mild mixing, $^7$Li and $^3$He

Although the introduction of pure element settling in the solar models considerably improves the consistency with the “seismic Sun”, some discrepancies do remain, particularly below the convective zone where a ”spike” appears in the sound velocity (see Gough et al. 1996). The helium profiles directly obtained from helioseismology (Basu 1997, Antia and Chitre 1997)
show indeed a helium gradient below the convection zone which is smoother than the gradient obtained with pure settling. Furthermore, standard solar models including element settling do not reproduce the observed abundances of lithium.

The abundance determinations in the solar photosphere show that lithium has been depleted by a factor of about 140 compared to the protosolar value while beryllium is generally believed to be depleted by a factor 2. These values have widely been used to constraint the solar models (e.g. RVCD). However, while the lithium depletion factor seems well established, the beryllium value is still subject to caution. Balachandran and Bell (1998) argue that the beryllium depletion is not real due to an underestimate of the opacity of the continuum in the abundance determinations. Their new treatment leads to a solar value identical to the meteoritic value.

In RVCD, a mild mixing below the convection zone, attributed to rotation-induced shears (Zahn 1992), was introduced to account for the lithium and beryllium depletion. It was shown that such a mixing may also wipe out the spike in the sound velocity, leading to more consistent solar models than the standard ones, computed with pure element settling. In these models the abundance of $^3$He increased in the convection zone, due to a small dredge up from the tail of the $^3$He peak.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Comparison of the $u = P/\rho$ function in the “seismic Sun” and in our models. Dotted line: without element settling; solid line: with settling computed for helium and 14 other elements (from Richard, Vauclair and Charbonnel, 1998)}
\end{figure}
Observations of the $^3\text{He}/^4\text{He}$ ratio in the solar wind and in the lunar rocks (Geiss 1993, Gloecker and Geiss 1996, Geiss and Gloecker 1998) show that this ratio may not have increased by more than $\approx 10\%$ since 3 Gyr in the Sun, which is in contradiction with the results of RVCD. While the occurrence of some mild mixing below the solar convective zone is needed to explain the lithium depletion and helps for the conciliation of the models with helioseismological constraints, the $^3\text{He}/^4\text{He}$ observations put a strict constraint on its efficiency.

Vauclair and Richard (1998) have tried several parametrizations of mixing below the solar convection zone, which could reproduce both the $^7\text{Li}$ and the $^3\text{He}$ constraints. The only way to obtain such a result is to postulate a mild mixing, which would be efficient down to the lithium nuclear burning region but not too far below, to preserve the original $^3\text{He}$ abundance. A mixing effect decreasing with time, as obtained with the rotation-induced shear hypothesis, helps to obtain a $^7\text{Li}$ destruction without increasing $^3\text{He}$ too much, as the $^3\text{He}$ peak itself built up during the solar life. A “cut-off” on the mixing process must however be postulated at some depth just below the $^7\text{Li}$ destruction layer.

The various kinds of mixing processes which may take place below the solar convection zone are summarized in J.P. Zahn’s review (this conference). Here we have tested an effective diffusion coefficient varied as a parameter that we have adjusted to reproduce the observed abundances. We found that the observations of $^7\text{Li}$ and $^3\text{He}$ are well accounted for with a very mild mixing described by a diffusion coefficient not larger than $10^3\text{cm}^2\text{s}^{-1}$, vanishing at about two scale heights below the bottom of the convection zone (figure 2). In this case $^7\text{Li}$ is destroyed by a factor 140 and $^3\text{He}$ is not increased by more than $\approx 5\%$. Meanwhile, beryllium is not destroyed by more than $\approx 20\%$.

In summary the best solar models must include the effect of element settling, which represents an improvement on the physics, without any free parameter added. These models can be considered as the new “standard” models. They cannot however reproduce the $^7\text{Li}$ depletion and they lead to a spike in the sound velocity, compared to the seismic Sun, just below the convective zone. These two observations suggest the presence of some mild mixing in this region of the internal Sun. Adding the very strict constraint on the abundance of $^3\text{He}$ as given by Geiss and Gloecker (1998) leads to a precise description of the allowed profile of the macroscopic diffusion coefficient below the convective zone. This result can be taken as a challenge for the hydrodynamicists.

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Figure 2. Example of profiles of the macroscopic diffusion coefficient below the solar convection zone, which can account for both the $^7$Li and $^3$He constraints (after Vauclair and Richard, 1998). In this case $D_T$ decreases with time. The profiles are shown at 1.22 Gyr (dashed line) and 4.6 Gyr (solid line). $^7$Li is then depleted by a factor 140 while $^3$He does not increase by more than 5% during the whole solar life. In these models, beryllium is not depleted by more than 20%.

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