Numerical modelling of wave barrier in 2D unbounded medium using Explicit/Implicit multi-time step co-simulation

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Abstract. Co-simulation strategies using both Abaqus/Explicit and Abaqus/Implicit are investigated for analysing wave barrier in 2D unbounded soil domain. The Co-simulation is based on the coupling GC method, allowing for coupling different finite element codes with different time integrators and time-scales depending on the partitions of the domain. Absorbing Layers using Increasing Damping (ALID), based on Rayleigh damping, are considered at the boundary to model the semi-infinite medium. The proposed absorbing region is called Hybrid (different time integrators) Asynchronous (different time steps) Absorbing Layers using Increasing Damping (HA-ALID). First, HA-ALID turns out to be more accurate than non-reflective conditions available in Abaqus/Explicit. Second, the time steps for the solid barrier and the HA-ALID are not restrained by the CFL condition imposed by the explicit partition for stability reasons. Third, the critical time step in the domain of interest remains unaffected by the choice of damping characteristics in the HA-ALID, contrary to the case of a full explicit computation. Because of its good accuracy and ability to be realized using only FE Abaqus package, without a third-party software component, this strategy provides a wide range of applications in soil-structure interaction problems.

1. Introduction
The isolation of buildings against shocks and propagating waves in the soil becomes a more and more important problem due to the increasing intensity of machine foundations or human activities such as railway and highway traffic. Thus there is a strong need to reduce vibration experienced by people, structures and equipments using isolation systems. Wave barriers corresponding to various obstacles such as open trenches and solid barriers composed of different materials [1], constitute examples of isolation measures which can be used against soil vibrations. The design of wave barriers motivated the development of analytical and numerical tools for analysing their isolating effect on travelling waves [2, 3]. In most cases, the problem characterized by complex geometry, boundary conditions, nonlinear material behavior laws for the soil, is too difficult to be dealt with an analytical method, requiring the use of well-established numerical methods such as the Finite Element Method (FEM).

One of the critical points of the numerical simulation of wave propagation problems in unbounded domains using the finite element method is how to simulate infinite media. The simplest way is to consider a very large extended numerical mesh, but it leads to important computation times, in particular when long time duration excitations are considered. Hence non-reflective boundary conditions are required at the boundary of the truncated domain for mimicking infinite or semi-infinite media. Several kinds of artificial boundaries in numerical methods have been developed to avoid spurious waves reflected at the boundary, such as the infinite elements (Bettess [4]), absorbing
boundary conditions (Enquist et al. [5]), or PML (Perfect Matched Layers). PML proposed by Bérenger [6] is becoming increasingly used for dealing with infinite media in the context of finite difference and finite element methods. Semblat et al. [7] and Rajagopal et al. [8] introduce more convenient techniques for implementing efficient absorbing conditions into commercial finite element codes, called Absorbing Layers using Increasing Damping (ALID), based on Rayleigh viscous damping matrix associated with an increasing damping ratio in the thickness of the absorbing region. Analytical models enabling to select the ALID parameters are developed by the authors to provide quick and valuable results, satisfying a desired accuracy. However, if an explicit time integration scheme is adopted both in both the domain of interest and Rayleigh absorbing layer, the introduction of Rayleigh matrix will decrease the value of the critical time step. When there are complex structures in the domain of interest, it costs much more computation time. To avoid the critical time step in the domain of interest to be affected by introduction of Rayleigh matrix, the subdomain strategy proposed by Gravouil and Combescure [9-11] can be used to couple different schemes associated with different time steps depending on subdomains. GC method is based on the gluing of velocities at the interface between two finite element partitions using a dual formulation by means of Lagrange multipliers. Co-simulations have been successfully carried out in various applications: structure-structure interaction problem in structural dynamics [12-14], soil-structure interaction [15], and fluid-structure interaction [16]. Here, in the context of analysis of the protective effect of wave barriers against vibrations into soil, the system is decomposed for three partitions: the solid barrier made of concrete, the soil and the absorbing region, using the commercial Abaqus packages including both Abaqus/Standard and Abaqus/Explicit [17]. Abaqus/Explicit is considered for reproducing the wave propagation into the soil, whereas Abaqus/Implicit is considered for the solid wave barrier as well as the absorbing region at the boundary of the truncated mesh. The absorbing region is modelled with an improved ALID strategy: the strong form of wave propagation in a Rayleigh medium, which is a discrete medium characterized by the introduction of a Rayleigh viscous damping matrix, is established, enabling us to derive relationships in order to minimize the wave reflection at interface. 

In this paper, the strong form for the Rayleigh medium is first briefly presented as well as the main relationships for the design of the ALID. Then the weak formulation of the decomposed problem is reminded in order to derive the discrete in space and time algorithm able to couple Abaqus/Explicit with Abaqus/Implicit with their own time step. The derived absorbing region is called Hybrid (different time integrators) Asynchronous (different time steps) Absorbing Layers using Increasing Damping (HA-ALID). In the second section, Lamb’s test is considered to assess the ALID efficiency using Abaqus co-simulation engine. The efficiency of the proposed method is compared with non-reflective conditions available in Abaqus/Explicit, highlighting the very good behavior of the co-simulation strategy. Finally, a simulation of wave barrier problem is carried out using the co-simulation and compared to an Abaqus/Explicit simulation with non-reflective boundary conditions with the same mesh for the domain of interest. It is shown that the co-simulation allows adopting large time steps for the solid barrier and HA-ALID, whereas explicit time integration imposes fine time steps on the whole mesh due to the CFL condition [18] required for the stability of the explicit time integration scheme. Moreover, in comparison to the Abaqus/Explicit simulation, the hybrid (explicit/implicit) asynchronous (different time steps) strategy provides much more accurate results of engineering interest such as the reduction ratio of displacement due to the presence of the barrier.

2. Hybrid asynchronous Rayleigh absorbing layer

2.1. Strong form for the wave propagation into a Rayleigh medium

Rayleigh absorbing layer aims to damp out all the incident waves from the domain of interest while minimizing the spurious waves reflected at the boundary of the truncated domain. For this purpose, the optimal conditions at the interface between a non-dissipative elastic medium $\Omega_1$ and a dissipative Rayleigh medium $\Omega_2$ can be established by considering the continuous problem of wave propagation. A strong form for the wave propagation in the continuous dissipative medium $\Omega_2$, corresponding to the
introduction of the classical viscous Rayleigh damping matrix into the classical semi-discretized of the equation of motion in 3D medium, has been obtained by Zafati et al. [9]. The displacement vector field $u_2$ in the Rayleigh domain $\Omega_2$ is given by the following equations:

$$\rho_2 \partial_2^2 u_2 + \alpha_M \rho_2 \partial_1 u_2 = \text{div} \left( \sigma_2 (u_2) \right)$$  \hspace{1cm} (1)

$$\sigma_2 = \lambda_2 \text{tr} \left( \varepsilon_2 (u_2) \right) + 2 \mu_2 \varepsilon_2 (u_2) + \alpha_K \left( \lambda_2 \text{tr} \left( \varepsilon_2 (\partial_i u_2) \right) + 2 \mu_2 \varepsilon_2 (\partial_i u_2) \right)$$  \hspace{1cm} (2)

$$\varepsilon_2 = \frac{1}{2} \left[ \text{grad} \left( u_2 \right) + \text{grad} \left( u_2^T \right) \right]$$  \hspace{1cm} (3)

Eqs. (1) to (3) constitute the strong form of the propagation into a Rayleigh medium, $\sigma_2$, $\varepsilon_2$, $\lambda_2$, $\mu_2$, being the stress matrix, strain matrix, Lamé coefficients, Young’s modulus and Poisson’s ratio related to the Rayleigh domain $\Omega_2$, respectively.

### 2.2 The design of ALID

The Absorbing Layers using Increasing Damping, called ALID are considered. The main idea is to divide the Rayleigh absorbing medium into several uniform layer, so that the decrements produced by each layer can be multiplied. Because of the logarithmic form of decrement, the total logarithmic decrement can be easily obtained. Due to the difference of damping ratio between subdomains $\Omega_1$ and $\Omega_2$, spurious waves will be produced, it is crucial to control the difference of damping ratios between subdomains. In this paper, a nonlinear increase of damping ratio is adopted to achieve a better accuracy. After analytically solving the interface problem using the previous strong form, the relationship for minimizing the spurious reflections at each interface are shown below:

$$\begin{cases}
E_2^{(i+1)} = \frac{1+\xi_i^2}{1+\xi_i+1} E_2^{(i)} \\
E_2^{(i)} = \frac{1}{1+\xi_i} E_1 \\
\nu_2^{(i)} = \nu_1 \\
\rho_2^{(i)} = \rho_1 \\
\xi_i = f(i) = a \left( \frac{i}{N_e} \right)^M
\end{cases}$$  \hspace{1cm} (4)

where $E_2^{(i)}$ denotes Young’s modulus of each layer in the subdomain $\Omega_2$, $\xi_i$ the damping ratio, $\nu_2^{(i)}$ Poisson’s ratio, $\rho_2^{(i)}$ the density of each layer $i$ in the subdomain $\Omega_2$. The total logarithmic decrement $\delta$ for ALID is written as follows:

$$\begin{cases}
\delta_i = \frac{\omega_0 \xi_i}{V_{1p} N_e} \\
\delta = \sum_{i=1}^{N_e} \delta_i = \frac{\omega_0}{V_{1p} N_e} \sum_{i=1}^{N_e} \xi_i
\end{cases}$$  \hspace{1cm} (5)

where $\delta_i$ represents the logarithmic decrement of each sublayer $i$, $e$ represents the total thickness of the ALID, $N_e$ is the number of layers, the thickness of each sublayer being equal to $e/N_e$. The reflection coefficient $R$ is given by:

$$R = e^{-2\delta}$$  \hspace{1cm} (6)

For example, if the goal is to reach a target logarithmic decrement $\delta=\ln(10)$, this means that 90% of the amplitude of the incident will be absorbed from the interface to the end of the damping layers. Next, the attenuation also occurs for the reflection process from the end of the damping layer towards
the interface. Thus the incident wave is attenuated by 99% in the ALID and the reflection coefficient $R$ is theoretically equal to 1%.

2.3 Weak form and space discretization for the Hybrid Asynchronous ALID

The elastic wave propagation from an elastic non-dissipative medium to a Rayleigh medium should be discretized in space and in time. Let $\Omega$ be a bounded domain belonging to $\mathbb{R}^2$ with a regular boundary. The domain $\Omega$ is divided into two partitions $\Omega_1$ and $\Omega_2$, as shown in Fig.1, such as: $\Omega_1 \cap \Omega_2 = \emptyset$ and $\partial \Omega_1 \cap \partial \Omega_2 = \Gamma_t$. $\Gamma_t$ denotes the interface between the two subdomains, subdomain $\Omega_1$ representing the non-dissipative medium (the domain of interest) and subdomain $\Omega_2$ the Rayleigh medium.

![Figure 1. Subdomain $\Omega_1$ by an explicit scheme with fine time step and subdomain $\Omega_2$ by an implicit scheme with large time step](image)

The subdomain $\Omega_1$ is characterized by its density $\rho_1$, Young’s modulus $E_1$, Poisson coefficient $\nu_1$, $b_1$, the body force, $f_1^B$, the Dirichlet prescribed displacement on $\Gamma_1^D$ and $g_1^N$ the traction force at the Neumann condition on $\Gamma_1^N$. The subdomain $\Omega_2$ is characterized by its density $\rho_2$, Young’s modulus $E_2$, Poisson coefficient $\nu_2$, $b_2$, the body force, $f_2^B$, the Dirichlet prescribed displacement on $\Gamma_2^D$, $g_2^N$ the traction force at the Neumann condition on $\Gamma_2^N$ and the parameters $\alpha_M$ and $\alpha_K$ introduced in the strong form of the wave equation in Eqs (1) and (2). Using a dual Schur formulation, the principle of virtual power for transient dynamics can be written:

$$
\int_{\Omega_1} \rho_1 \ddot{u}_1 \cdot \ddot{u}_1 \, d\Omega + \int_{\Omega_1} \epsilon(\nu_1) \cdot \sigma_1 \, d\Omega + \int_{\Omega_1} \rho_2 \ddot{v}_2 \cdot \ddot{u}_2 \, d\Omega + \int_{\Omega_2} \epsilon(\nu_2) \cdot \sigma_2 \, d\Omega + \alpha_M \int_{\Gamma_2} \rho_2 \ddot{v}_2 \cdot \ddot{u}_2 \, d\Gamma + \int_{\Gamma_2} \nu_2 \cdot \lambda \, d\Gamma + \int_{\Gamma_1} \nu_1 \cdot \lambda \, d\Gamma + \int_{\Gamma_1} \mu \cdot (\ddot{u}_1 - \ddot{u}_2) \, d\Gamma = \int_{\Omega_1} f_1^D \, d\Omega + \int_{\Gamma_1} g_1^B \, d\Gamma + \int_{\Omega_2} f_2^D \, d\Omega + \int_{\Gamma_2} g_2^B \, d\Gamma + \int_{\Gamma_2} g_2^N \, d\Gamma
$$

At the interface between the subdomains, the continuity of velocities is imposed by the following condition:

$$
L_1 \ddot{U}_1 + L_2 \ddot{U}_2 = 0
$$

where $L_1$ and $L_2$ are the Boolean matrices in the case of matching meshes at the interface.

2.4 Time discretization of the Hybrid Asynchronous ALID

For the time discretization, the GC method proposed by Gravouil and Combescure is employed [10, 11]. As illustrated in Fig. 1, an explicit time integrator with a fine time step $\Delta t$ is adopted for the subdomain $\Omega_1$ and an implicit time integrator with a large time step $\Delta t_2$ is used for subdomain $\Omega_2$, with $\Delta t_2 = m \Delta t_1$, $m$ being the time step ratio between two subdomains. In this way, hybrid (different schemes associated) asynchronous (different time steps depending on subdomains) ALID can be obtained, called HA-ALID. The equilibrium of subdomain 1 is prescribed at time $t_0$ at the end of the large time $\Delta t_2$, while the equilibrium of subdomain 2 is prescribed at every time $t_j = j \Delta t_1$ ($j = 1, 2...m)$ at the fine time scale. The gluing of the velocity at the interface is written at the fine time scale. Finally, the weak form given in Eq. (7) with the velocity continuity equation in Eq. (8), can be expressed in the following discrete form in space and time:

$$
M_1 \ddot{U}_1 + K_1 U_1 = F_1^{ext} - L_1^T \lambda_j \quad \text{at time } t = t_j
$$

$$
M_2 \ddot{U}_2 + (\alpha_M M_2 + \alpha_K K_2) U_2 + K_2 U_2 = F_2^{ext} - L_2^T \lambda_m \quad \text{at time } t = t_m
$$
\[ L_1 \ddot{U}_1^j + L_2 \ddot{U}_2^j = 0 \quad \text{at time } t=t_j \]  

(11)

It is important to note that the expression of the classical viscous Rayleigh damping matrix, denoted by \( C = \alpha_M M_1 + \alpha_K K_2 \), is retrieved in the discrete equation of motion of subdomain \( \Omega_2 \). As a result, the \( \alpha_M \) and \( \alpha_K \) parameters introduced in the strong form of the wave represent the classical constant parameters of the viscous Rayleigh damping matrix. Newmark time integration schemes [19] can be adopted for the time discretization. It leads to the equations of motion written as:

\[ \tilde{M}_1 \ddot{U}_1^j = F_1^{\text{ext}} - K_1 U_1^{j-1,p} - L_1^T \dot{b}_j \]
\[ \tilde{M}_2 \ddot{U}_2^m = F_2^{\text{ext,m}} - C_2 U_2^0,p - K_2 U_2^0,p - L_2^T \dot{b}_m \]

(12) (13)

where \( U_1^{j-1,p} \) and \( U_2^0,p \) denote the predictor values in terms of displacement and velocity, classically introduced through the approximate Newmark formulae; they correspond to quantities known at the beginning of the fine step and of the large time step, respectively. The effective stiffness matrices \( \tilde{M}_1 \) and \( \tilde{M}_2 \) related to the two subdomains are defined by:

\[ \tilde{M}_1 = M_1 + \beta_1 \Delta t^2 K_1 \]
\[ \tilde{M}_2 = M_2 + \beta_2 \Delta t^2 K_2 + \gamma_2 \Delta t C_2 \]

(14) (15)

It was demonstrated that the kinematic continuity condition can be expressed as a reduced-size interface problem as follows:

\[ H \lambda^j = b_j \]

(16)

with the interface operator and the right-hand side member vector defined by:

\[ \begin{cases} H = \gamma_1 \Delta t L_1 \tilde{M}_1^{-1} L_1^T + \gamma_2 \Delta t L_2 \tilde{M}_2^{-1} L_2^T \\ b_j = L_1 U_1^{\text{free},j} + L_2 U_2^{\text{free},j} \end{cases} \]

(17)

### 3. Effectiveness of Hybrid Asynchronous Absorbing Layers using Increasing damping

In order to evaluate the effectiveness of hybrid asynchronous Rayleigh absorbing layers (HA-ALID), 2D Lamb’s test has been simulated using Abaqus Explicit/Implicit co-simulation. In Lamb’s test, the concentrated load applied to the surface of an infinite half space medium generates three types of waves propagating through the soil, involving P, S waves and Rayleigh waves. Therefore, 2D Lamb’s test can be considered as a good test for assessing the performance of the HA-ALID using Abaqus co-simulation. Non-harmonic waves are investigated by considering a Ricker incident waves defined by:

\[ R(t, t_p, t_s) = A \left( 2 \pi^2 \frac{(t - t_s)^2}{t_p^2} - 1 \right) \exp \left( -\pi^2 \frac{(t - t_s)^2}{t_p^2} \right) \]

(18)

The chosen values are: \( t_p = 3s, t_s = 3s \) and \( A = 1MN \).

#### 3.1. HA-ALID in Lamb’s test

The HA-ALID for the Lamb’s test is depicted in Figure 2. The soil is assumed to be linear elastic with the following material characteristics: \( \rho_s = 1700 \text{kg/m}^3, E_s = 10 \text{MPa} \) and \( v_s = 0.24 \) for the density, Young’s modulus and Poisson’s ratio, respectively. The material characteristics of the Rayleigh absorbing layer are computed from the optimal conditions given and taking into account a dominant angular frequency corresponding to the fundamental period \( t_p \) of the Ricker wave. We consider 10 sublayers in absorbing region and a size of \( \lambda/10 \) for each sublayer; \( \alpha \) in Eqs (4) is chosen equal to 1.
3.2. Effect of the time ratio on the accuracy of the HA-ALID

The subdomain soil and HA-ALID are integrated with an explicit scheme and an implicit scheme, respectively. A homogeneous time step can be adopted, which satisfies the CFL condition without damping. Indeed, using the implicit time integration for the HA-ALID, we avoid the decrease of the critical time step in the explicit framework due to the introduction of the Rayleigh damping into the discrete equation of motion, as it is noted in Abaqus/Explicit documentation [17]. Moreover, as explained in section 2, it is possible to use a bigger time step in HA-ALID, because we use an unconditionally stable implicit scheme.

In this part, the subdomain soil is integrated with Abaqus/Explicit with a fine time step, whereas the HA-ALID are dealt with Abaqus/Implicit associated with a large time step in order to reduce the computation time. The horizontal and vertical displacements of the observation point with different time step ratios $m$ ($\Delta t_2 = m\Delta t$) equal to 1, 10, 20, and 30, are shown in Figures 3 and 4. The reflected spurious waves recorded at the observation point grow bigger with the increase of the time step ratio $m$. It can be observed that in comparison to the displacements given by reference results, the vertical amplitude of the spurious wave varies from 1% to 2.57% with respect to the vertical amplitude of the
incident wave, while the horizontal amplitude of the spurious wave varies from 0.3% to 5.8% with respect to that of the incident wave. Based on these results, the time step ratio $m$ has to be chosen under 10 without significant influence on the accuracy of HA-ALID. The observed decrease of accuracy as the time step ratio increases can be explained by the following points. First, due to the increase of the time step in the implicit scheme, the numerical errors grow. Second, the GC coupling algorithm is known to be dissipative as soon as heterogeneous time steps are used between the subdomains, generating spurious waves at the interface.

3.3. Comparison between infinite elements and Rayleigh absorbing layers in Abaqus

Infinite elements available in Abaqus/Explicit allows to deal with unbounded media. To further validate the accuracy of the HA-ALID using Abaqus Explicit/Implicit multi time step co-simulation, we compare their performance with infinite elements available in Abaqus/Explicit. Thus, a numerical model is established using Abaqus/Explicit with the same mesh and materials than our previous explicit/implicit simulations: HA-ALID are just replaced with Abaqus infinite elements.

![Figure 5. Vertical displacements at the observation point using different methods](image1)

![Figure 6. Horizontal displacements at the observation point using different methods](image2)

The horizontal and vertical displacements of three numerical models at the observation point are shown in Figures 5 and 6. We can observe that the results obtained by Abaqus Explicit/Implicit co-simulation and reference results are in a good agreement: the reflected spurious wave is 1.47 % in terms of the horizontal displacement, 1.14 % in terms of the vertical displacement when the time step ratio $m$ is equal to 10. In comparison, the reflected spurious wave produced by infinite element is equal to 10.4 % with respect of the vertical amplitude of the incident wave and 17.6% with respect of the horizontal amplitude of the incident wave. In conclusion, HA-ALID have a much better accuracy than Abaqus infinite elements, with the same advantage that the proposed method for modelling an unbounded medium is available in Abaqus packages, without a third party software. In the following section, the relevance of the proposed HA-ALID will be assessed in the case of a wave barrier problem.
4. An application in the numerical simulation of wave barriers

Due to the increasing vibrations caused by human activities, the performance of wave barriers for reducing the distress to adjacent structures and annoyance to people, have been studied for more than 30 years. As illustrated in Figure 7, the wave barrier configuration studied by Al-Hussaini et al. [2] and Beskos et al. [3], is investigated. In this case, it was shown that the major part of the vibration energy is transferred by Rayleigh waves which may cause strong ground motions on nearby structures. An application of HA-ALID is carried out for this case of wave barrier.

Figure 7. Investigated configuration of a 2D soil-barrier system

In this soil-barrier configuration, D is the depth of the barrier, equal to 5 m, W is the width of the barrier, equal to 0.5m, L₁ is the distance from the dynamic load to the barrier equal to 25m and L₂ is the distance from the barrier to the point of interest equal to 25m. The total length of the model is 120m and the depth is 25m. The inclination angle U of the barrier is given as 90°. A dynamic load P with a width of r equal to 1.25 m is applied to the left top surface of the soil, producing a Rayleigh wave to simulate dynamic events such as the compaction, blasting and seismic waves. The dynamic periodic load in the numerical model is

\[ P = P_0 \cos(\omega_0 t), \quad \omega_0 = 100\pi, \quad P = 1000N, \]

The material properties of soil and barrier are shown below. In order to achieve a good accuracy in predicting the propagating waves into the soil, the finite element size is kept as \( \lambda_P/50 \) for both subdomains.

Table 1. Material properties of soil and barrier

| Material | Density \( \rho \) (kg/m³) | Poisson’s ratio \( \nu \) | Young’s modulus \( E \) (GPa) | Damping ratio |
|----------|----------------|----------------|-----------------|--------------|
| Soil     | 1750           | 0.25           | 0.33            | 0            |
| Barrier  | 2397.5         | 0.25           | 11.3            | 0            |

Taking advantage of the partition strategy through Abaqus Explicit/Implicit co-simulation, the 2D soil-barrier system is divided into three partitions integrated in time with their own time integrator and time step: soil subdomain (explicit scheme), barrier subdomain (implicit scheme, with a large time step \( m \) equal to 10), Rayleigh absorbing layers subdomain (implicit scheme, with a large time step \( m \) equal to 10), as shown in Figure 8.

In the solid barrier, the P-wave velocity is equal to 2378.21m/s. In a full explicit computation using Abaqus/Explicit, the time step must satisfy the smallest time step satisfying depending on finite element size and material characteristics. HA-ALID involve large values of damping ratios for absorbing the incident waves. In [17] (Abaqus/Explicit documentation), recommended values are provided so as to guarantee the stability of the explicit time integration scheme. Table 2 resumes the critical time steps depending on the partition under consideration (soil, barrier, ALID). It can be noted that ALID have a large impact on the time step size, leading to an increase of computations time.

Figure 8. Wave-barrier model split in three partitions: soil, solid barrier, ALID.

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Table 2. Critical time steps for each partition in full explicit computations

|             | Soil      | Barrier   | Rayleigh layers | Time step adopted |
|-------------|-----------|-----------|-----------------|-------------------|
|             | 269x10^-6 | 53.8x10^-6 | 27.5x10^-6     | 27.5x10^-6       |

When adopting HA-ALID using Abaqus Explicit/Implicit multi time step co-simulation, the critical time step in the soil is unaffected by the models of the solid barrier and the HA-ALID. Time step sizes for the different partitions are given in Table 3, highlighting the interest of the co-simulation.

Table 3. Critical time steps for each partition in explicit/implicit multi time step co-simulations

|             | Soil      | Barrier   | Rayleigh layers |
|-------------|-----------|-----------|-----------------|
|             | 250x10^-6 | 2500x10^-6| 2500x10^-6      |

Figure 9. Vertical displacements and local zoom at the observation point using HA-ALID with m=10 and infinite elements in a full explicit computation, compared to the reference results (extended mesh)

Figures 9 shows the vertical displacements of the observation point with HA-ALID, compared to the results obtained using infinite elements with the same configuration and the reference results obtained from an extended mesh free of spurious reflected waves from the boundary during the observation period. It turns out that the results of HA-ALID agree well with the reference results, contrary to the full explicit computation with infinite elements: the errors in terms of vertical displacements obtained using infinite elements can be greater than 10%, whereas the biggest error with HA-ALID is equal to 2.7% with respect to the amplitude of reference results.

Figure 10. Reduction ratio of vertical displacements using HA-ALID with m=10 and infinite elements in a full explicit computation, compared to the reference results (extended mesh)

The isolation effect of the installation of the barrier can be assessed by the parameter $A_r$ (amplitude reduction ratio), which provides a quantitative evaluation of the screening effect of the barrier. Its
expression is given by: \( A_r = \frac{A_b}{A_s} \), where \( A_b \) is the displacement amplitude with the barrier and \( A_s \) the displacement amplitude without the barrier. For example, \( A_r = 0.8 \) means that 20% reduction of the vibration has been reached due to the installation of the barrier. The reduction ratio of vertical displacement on the surface beyond the barrier can be plotted in Fig. 10, for the three computations. From this figure, it can be concluded that the reduction ratios obtained by HA-ALID have a better agreement with the reference results than those obtained by infinite elements.

5. Conclusion

In this paper, an improved version of the ALID by tuning the material characteristics of the ALID so as to minimize the wave reflection has been applied. The co-simulation strategy involving different FE codes with their own time integrator and time step, has been employed for the simulation of wave barrier problem using the Abaqus package: the soil partition in which we desire an accurate prediction of the wave propagation has been handled by Abaqus/Explicit with a fine time step, whereas the other partitions, the solid barrier and the ALID, which corresponds to the absorbing region enabling us to model semi-infinite soil medium, have been dealt with Abaqus/Standard with a large time step. The proposed strategy is called Hybrid (different time integrators) Asynchronous (different time steps) Absorbing Layers with Increasing Damping (HA-ALID). The obtained results showed that spurious waves reflected at the boundary of the truncated mesh are much lower than the ones generated by non-reflective conditions available in Abaqus/Explicit for modelling infinite medium (infinite elements).

On the other hand, some interesting benefits of the HA-ALID can be underlined in comparison to the classical ALID. First, large time steps can be adopted in the solid barrier and in the HA-ALID in comparison to the fine time step in the soil medium corresponding to the CFL condition. Second, the critical time step in the soil partition is unaffected by the choice of damping matrix in the layers of the HA-ALID, related to the stiffness or mass matrices, contrary to the case of a full explicit computation for which the critical time step can be significantly reduced. As a conclusion, because of its good accuracy and ability to be realized in Abaqus package without a third-party software, the proposed HA-ALID have a wide range of applications for modelling wave propagation problem involving unbounded domains. The 2D wave barrier problem studied in this paper is a first example. Future work will consider wave propagation in 3D media for soil-structure interaction problems.

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