Dynamic Crack Tip Opening Displacement (DCTOD) as governing parameters for material fragmentation

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Abstract. Fragmentation in metals can be approached either by Mott statistical or by Energy-based fragmentation theory. Recently, Grady showed that the two theories can be reconciled showing that the material parameter that drives tendency to fragmentation and fragment size is the dynamic fracture toughness. Experimental data do not completely agree with these conclusions. In this paper, the dynamic crack tip opening displacement (CTOD) is proposed as fracture parameter which can account for plastic deformation occurring prior fracture. Here, an experimental procedure for determining the critical dynamic CTOD is presented. The circumferential crack bar tension (CCB(T)) was investigated for its use with tensile Hopkinson bar testing equipment. The calibration function in the dynamic range was determined via finite element analysis (FEA). The critical dynamic CTOD was measured using both high speed video recording, with digital image correlation (DIC) technique, and clip gauge at the crack mouth. The proposed procedure has been used to investigate dynamic fracture resistance of high purity copper (99.98%) and to correlate it with available fragmentation data.

1. Introduction

The possibility to characterize material resistance to fragmentation under transient dynamic loading is a requirement in the design of both civil and military applications. Although, full scale test is usually the preferred way to assess material resistance under blast loads, it is expensive, difficult to set-up, often posing safety hazard which limits the possibility as routine testing method. The possibility to measure material resistance to fragmentation with simple laboratory test would be very welcome in material design and selection. In the literature, several models have been presented. In most of the cases, selected parameters (fracture toughness, hardness, yield over ultimate stress ratio, etc.) are phenomenological in nature and suffer the transferability to different classes of materials, different casing geometries, etc.

Mott provided a comprehensive statistical theory to predict fragments number and size distribution in metals and alloys. In Mott’s approach, fragments form as a results of a statistical variability in the material strain to failure. The material is considered elastic-perfectly plastic and fractures are assumed to initiate at the locations where the stress reaches the yield limit stress and strain exceeds local failure strain value. Crack initiation and propagation are neglected as well as fracture dissipated work.

Alternatively to Mott’s statistical theory of fragmentation, Grady developed an energy based fragmentation model, which only apparently differs from Mott’s conclusions [1]. He also demonstrated the possibility to conciliate the two theories. In Grady’s model, the average number and
size of fragments is found to be related to the material dynamic fracture toughness, to the strain rate and material sound speed. Although very elegant, Grady’s theory is found to be inaccurate when dealing with fracture mechanics concepts. Because of this, model predictions not always agree well with experimental data obtained, for instance, with expansion ring test.

In this work, Grady’s theory is reformulated addressing dynamic fracture resistance of ductile materials. In particular, the use of the dynamic crack tip opening displacement (CTOD) as fracture parameter is showed to better describe fragmentation resistance in ductile metals and alloys. In addition, a relatively simple procedure how to measure this parameter is also proposed. Results for cold work copper are presented and discussed.

2. Fragmentation model
Grady’s energy-based fragmentation theory is formulated for an expanding ring or similarly, an infinite uniaxial strip under dynamic tension loading [2].

It is assumed that when the stress reaches the yield stress, pre-existing flaws in form of cracks become active. Under tension, cracks open and fracture propagation occurs when the material dynamic fracture toughness, which accounts for the work of cohesive forces, is exceeded.

In this framework, the following expressions for the average fragment size and number per unit length can be obtained:

\[
x_0 = \left(\frac{\sqrt{2K_c}}{\rho \dot{\varepsilon}}\right)^{2/3},
\]

\[
N = \left(\frac{\rho \dot{\varepsilon}}{\sqrt{12K_c}}\right)^{2/3},
\]

where \(\rho\) is the density, \(\dot{\varepsilon}\) is the strain rate, \(c\) is the sound speed and \(K_c\) is the dynamic fracture toughness.

In Grady’s approach, a number of assumptions, that are not necessarily true, are made. Cracks are assumed to form as soon as the stress reaches the material yield stress and this may not be appropriate for ductile metals and alloys where extensive hardening occurs well before a ductile crack may initiate as a result of void nucleation and growth. This is the case of pure copper and aluminum in which extensive necking prior fragmentation is observed. The strain energy release rate and \(K_c\) recalled by Grady are concepts valid for linear-elastic materials only. When large plastic strain occurs, elastic-plastic fracture concepts, such as J-integral or CTOD, should be used. In ductile metals, crack tip blunting occurs as soon as plastic deformation develops. Blunting disrupts the stress field at the crack tip invalidating fracture mechanics parameters. As a result of this, a dramatic increase of the apparent fracture toughness and delayed crack initiation and growth is usually observed in the experiments. Therefore, linear-elastic fracture concepts, such as the strain energy release rate or the critical stress intensity factor, are not appropriate to describe ductile material resistance to fragmentation.

However, the solution proposed by Grady, which is not only elegant but also simple, can be improved further taking into account for appropriate ductile fracture resistance parameters.

In this work, Grady’s solution has been reformulated in terms of critical crack tip opening displacement (CTOD). The selection of this parameter, to characterize material resistance to fragmentation, offers a number of advantages, one among all, its easy experimental determination. In this work, the procedure for the determination of the critical dynamic CTOD (\(\delta^*_{\text{cr}}\)), as proposed by the authors, is recalled and preliminary results for commercially high purity OFHC copper are presented.

Following Grady’s approach, starting from the momentum imbalance equation we can write,

\[
\rho \dot{\varepsilon} \frac{dx}{dt} = \sigma(\varepsilon; 0) - \sigma(h).
\]

Assuming that a crack is formed (or a flaw becomes active) when the stress reaches the ultimate stress, equation (3) can be rewritten as,
\[ \rho \dot{e} x \frac{dx}{dt} = \sigma_u - \sigma_u \left[ 1 - \left( \frac{\psi}{\delta_c} \right)^m \right], \quad (4) \]

where \( \psi \) is the crack opening coordinate. The right hand side term accounts for cohesive forces at crack initiation which is assumed to start at the ultimate stress. Deriving and solving for \( m = 1 \) (Grady’s cohesive force) for the average fragment size and number per unit length is it possible to get:

\[ x_0 = \left( \frac{6\sigma_u \delta_c}{\rho \dot{e}^2} \right)^{1/3} \quad (5) \]

\[ N = \left( \frac{\rho \dot{e}^2}{6\sigma_u \delta_c} \right)^{1/3}. \quad (6) \]

These expressions are formally similar to Grady’s equations but lead to different results. The most relevant feature of these expressions is that material hardening is accounted for by the ultimate stress (and this is important for strong hardening materials such as annealed copper) and the ductile fracture resistance. Using equivalence expressions, the critical CTOD can be replaced by the critical J-integral, and for the K-dominance condition, by \( K_{lc} \).

\[ \text{Figure 1. Dimension of CCB for use with direct tension Hopkinson bar.} \]

3. Determination of critical dynamic CTOD

The major advantage of critical CTOD fracture criterion with respect to critical J-integral is that it can be directly measured on the sample. For fragmentation purposes, the critical CTOD should be determined under equivalent high strain rate conditions. Several attempts to determine the dynamic fracture toughness have been presented in the literature. Most of the proposed techniques are based on the use of the split Hopkinson pressure bar [3]. When used for determining the critical stress intensity factor, this technique showed a number of issues, mainly related to the need to reverse the compressive stress wave and to the use of standard fracture sample geometries, such as compact tension (C(T)) or single edge notch in bending (SEN(B)), which suffer loss of contact during dynamic loading. The authors have recently proposed the use of circumferentially cracked bar (CCB(T)) sample geometry with direct tension Hopkinson bar [4]. This geometry is derived from standard Hopkinson bar traction sample, figure 1. The critical CTOD can be measured using high-speed video recording since direct tension Hopkinson bar allows visual access to the sample during the test.

In this work, CCB(T) was used to measure both quasi-static and dynamic CTOD in cold-drawn OFHC. The crack was machined by EDM and because of the large plastic deformation occurring before fracture, no fatigue pre-cracking was necessary. The resulting notch radius at the tip was 0.075 mm. Both static and dynamic fracture tests were monitored using high speed camera. For the dynamic test, 65000 fps recording speed with a resolution of 512 x 128 pixel was used.
Figure 2. Definition of critical CTOD: picture frame was extracted from high speed video recording movie. Crack initiation is barely visible as indicated by the arrow.

Figure 3. Comparison of predicted number of fragments in expansion ring test as a function of expanding velocity with equation (6) and experimental data for half-hardened copper.

4. Results and discussion
Numerical simulation of dynamic CCB(T) provided information about the effective strain rate at the tip. The nominal strain rate was 3000 s\(^{-1}\) while, according to numerical simulation, the effective strain rate in the near tip region was found to be 9000 s\(^{-1}\) which is in the same range of the deformation rate of expansion ring tests.

The proposed model was validated comparing predicted number of fragments with experimental data obtained in expanding ring tests. Data reported in the literature are relative to cold drawn high purity copper in as-received conditions. Both quasistatic and dynamic CCB(T) traction tests were
performed on same purity cold drawn copper (99.98%) although present material may differ in the cold work deformation level.

The CTOD at fracture was measured using high speed video recording. With the selected frame rate the moment of crack initiation could be captured accurately as in figure 2. Ductile rupture started at the crack tip. Tests have repeated (five tests per each strain rate) and the observed scatter in the CTOD measure was small (less than ±3%). The measured critical CTOD at rupture is given in table 1. It is worth to be noted that the critical CTOD is found to increase with the strain rate. Apparently, this result may contrast with the fact that fracture toughness $K_{IC}$ is known to decrease with the increase of the strain rate. However, it has to be recalled that this detrimental effect, which is more marked at higher temperature, is observed only below the brittle-ductile transition. In the brittle-ductile transition region and upper shelf regime, the effect of the strain rate on material toughness is reversed. In this temperature regime, the critical stress intensity factor $K_{IC}$ is no longer applicable because of the extensive plastic deformation occurring at the crack tip and consequent loss of K-dominance. In addition, the trend observed in the measured critical CTOD is consistent with the strain rate sensitivity of FCC metals and with the experimental evidence showing a right shift of the dynamic $J_{IC}$ vs temperature fracture toughness curve and an increment of the upper shelf. With the measured critical CTOD, the average number of fragments per unit length as predicted by equation (6) was compared with experimental data provided by in [5].

In figure 3 the comparison is given. To be consistent, the number of fragments using equation (6) should be calculated inferring the material engineering ultimate stress corrected by the strain rate. In this work a very simple strain rate correction factor, as provided by Johnson and Cook strain rate term, was used. The comparison is in a very good agreement with experimental data providing better accuracy that the original Grady expression.

| Strain rate $[s^{-1}]$ | $\delta^*$ [mm] |
|------------------------|--------------|
| $4.25 \cdot 10^{-2}$   | 0.251        |
| $9.0 \cdot 10^{-3}$    | 0.336        |

5. Conclusions
In this work the energy based fragmentation model proposed by Grady for ductile metals and alloys was reformulated. Model considerations about material fracture resistance have been reviewed in the perspective of the validity range of fracture mechanics governing parameters. The use of the critical CTOD as more appropriate fracture parameter, characterizing ductile rupture, has been proposed and the experimental procedure for its determination was presented. Preliminary results on cold drawn high purity copper have been found to agree well with fragmentation data obtained with expanding ring test.

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