THE TRANSITING EXOPLANET HOST STAR GJ 436: A TEST OF STELLAR EVOLUTION MODELS IN THE LOWER MAIN SEQUENCE, AND REVISED PLANETARY PARAMETERS

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ABSTRACT

Knowledge of the stellar parameters for the parent stars of transiting exoplanets is pre-requisite for establishing the planet properties themselves, and often relies on stellar evolution models. GJ 436, which is orbited by a transiting Neptune-mass object, presents a difficult case because it is an M dwarf. Stellar models in this mass regime are not as reliable as for higher mass stars, and tend to underestimate the radius. Here we use constraints from published transit light curve solutions for GJ 436 along with other spectroscopic quantities to show how the models can still be used to infer the mass and radius accurately, and at the same time allow the radius discrepancy to be estimated. Similar systems should be found during the upcoming Kepler mission, and could provide in this way valuable constraints to stellar evolution models in the lower main sequence. The stellar mass and radius of GJ 436 are $M_\star = 0.452^{+0.014}_{-0.012} M_\odot$ and $R_\star = 0.464^{+0.009}_{-0.011} R_\odot$, and the radius is 10% larger than predicted by the standard models, in agreement with previous results from well studied double-lined eclipsing binaries. We obtain an improved planet mass and radius of $M_p = 23.17 \pm 0.79 M_\oplus$ and $R_p = 4.25^{+0.09}_{-0.18} R_\oplus$, a density of $\rho_p = 1.69^{+0.14}_{-0.12} \text{g cm}^{-3}$, and an orbital semimajor axis of $a = 0.02872 \pm 0.00027 \text{AU}$.

Subject headings: planetary systems — stars: evolution — stars: fundamental parameters — stars: individual (GJ 436) — stars: low-mass, brown dwarfs

1. INTRODUCTION

The applications of stellar evolution theory to astrophysics are so widespread, and its validity so often taken for granted, that it is easy to forget that it took decades to develop, and significant effort to validate by comparison with careful measurements, a process that still continues. It is usually only when those theoretical predictions fail that the classical discipline of stellar evolution “makes the headlines”, and even then it draws the attention of relatively few. One such instance has occurred for low mass main-sequence stars. Over the last 10 years or so it has become clear that our understanding of the structure and evolution of these objects is still incomplete. Discrepancies between theory and observation in the radii of stars under 1 $M_\odot$, first mentioned by Hoxie (1973), Lacy (1977), and others, are now well documented for several low-mass eclipsing binaries (see, e.g., Popper 1997, Clausen et al. 1999, Torres & Ribas 2002, Ribas 2003, López-Morales & Ribas 2003). Differences in the effective temperatures have been observed as well. The nature of the discrepancies is such that model radii are underestimated by roughly 10%, while effective temperatures are overestimated.

In recent years stellar evolution has had important applications in the field of transiting extrasolar planets. This is because the planetary parameters of interest (mass $M_p$, radius $R_p$) depend rather directly on those of the star ($M_\star$, $R_\star$), and in most cases models provide the only means of determining the latter. The subject of this paper is GJ 436, a late-type star found by Butler et al. (2004) to be orbited by a Neptune-mass planet with a period of 2.644 days. This object was later discovered by Gillon et al. (2007a) to undergo transits, enabling its size to be determined ($\sim 4 R_\oplus$). As the only M dwarf among the 22 currently known transiting planet host stars, GJ 436 (M2.5V) presents a special challenge for establishing the stellar parameters, because of the disagreements noted above. Little mention seems to have been made of this, and for the most part past studies have relied instead on empirical mass-luminosity ($M$-$L$) relations to set the mass of GJ 436. Radius estimates have often rested on the assumption of numerical equality between $M_\star$ and $R_\star$ for M stars. Despite being the closest transiting planet system (only 10 pc away), it is rather surprising that the mass of the star is only known to about 10% ($\rho_p = 1.69^{+0.14}_{-0.12} \text{g cm}^{-3}$), and an orbital semimajor axis of $a = 0.02872 \pm 0.00027 \text{AU}$.

Given the importance of GJ 436 as the parent star of the only Neptune-mass transiting exoplanet found so far, and hence the closest analog to our Earth with a mass and radius determination, one of the motivations of this paper is to improve the precision of the stellar and planetary parameters by making use of additional observational constraints not used before. Specifically, we incorporate the information on the stellar density directly available from the transit light curve (Sozzetti et al. 2007), which provides a strong handle on the size of the star.

The nature of the discrepancies between evolutionary models and observations for low-mass stars has been examined recently from both the observational and theoretical points of view by López-Morales (2007) and Chabrier et al. (2007). Further progress depends on gathering more evidence to supplement the few available highly accurate mass and radius measurements based on
double-lined eclipsing binaries. Thus, a second motivation for this work, despite the fact that GJ 436 is not a double-lined eclipsing binary, is to present a way of using all observational constraints simultaneously to show that the star presents the same radius anomaly found for the other systems, or more generally, to test the models. Because similar constraints may become available in the future for other M dwarfs given the keen interest in finding smaller and smaller transiting planets, we anticipate that the indirect technique described here may yield valuable information on this problem and eventually help improve our understanding of low-mass stars.

2. CONSTRAINTS AND METHODOLOGY

The basic procedure for establishing the mass and radius of a star that is not in a double-lined eclipsing binary is to place it on an H-R diagram using observational constraints, and compare it with stellar evolution models. We adopt here the models by Baraffe et al. (1998) for a mixing-length parameter of $\alpha_{ML} = 1.0$, which are widely used for low-mass stars. The constraints available for GJ 436 are several. The spectroscopic study by Maness et al. (2007) established the effective temperature to be $T_{\text{eff}} = 3350 \pm 300$ K, and the metallicity was estimated photometrically by Bonfils et al. (2003) to be $[\text{Fe/H}] = -0.03 \pm 0.20$. The well-determined Hipparcos parallax is $\pi_{\text{HIP}} = 97.73 \pm 2.27$ mas. With available visual and near infrared photometry from Legett & Hawkins (1988) and 2MASS, the absolute magnitudes become $M_V = 10.610 \pm 0.051$ and $M_K = 6.048 \pm 0.052$. Additionally, in view of the difficulties in determining effective temperatures for M dwarfs (see, e.g., Maness et al. 2007), we consider as well the infrared color $J - K = 0.802 \pm 0.024$. Following Carpenter (2001) the 2MASS magnitudes have been transformed to the CIT system of Elias et al. (1982) adopted in the Baraffe et al. (1998) models, and averaged with those of Legett & Hawkins (1988), already on that system. The agreement between the two sources is excellent.

Transit light curves for GJ 436 have been obtained from the ground by Gillon et al. (2007a) in the V band, and also at 8 μm by Deming et al. (2007) and Gillon et al. (2007b) using the Spitzer Space Telescope. Aside from minor corrections due to limb-darkening, transit light curves can in general be described using three parameters: the radius ratio between the planet and the star ($R_p/R_*$), the normalized planet-star separation ($a/R_*$), and the impact parameter ($b = a \cos i/R_*$), where $i$ is the inclination angle of the orbit. Seager & Mallén-Ornelas (2003) have shown that $a/R_*$ is directly related to the density of the star, and thus contains valuable information on its size. Sozzetti et al. (2007) have described how $a/R_*$ can be used together with $T_{\text{eff}}$ and stellar evolution models to infer $M_*$ and $R_*$. Briefly, the measured values of $a/R_*$ and $T_{\text{eff}}$ are compared with a fine grid of model isochrones for a wide range of ages and metallicities. Theoretical stellar properties are interpolated along each isochrone using a small step in mass, and all points in the H-R diagram matching the observations within their uncertainties are recorded. The best-fitting mass and corresponding radius are assigned errors based on the full range of model values that are consistent with the observations. Other stellar properties can then be read off the best fitting model. This is the procedure we apply below. For GJ 436 we have restricted the comparison to solar metallicity, given the $[\text{Fe/H}]$ estimate by Bonfils et al. (2003). The values of the light curve parameters we adopt are weighted averages of the results from the ground-based and Spitzer photometry: $a/R_* = 13.34 \pm 0.58$, $R_p/R_* = 0.0834 \pm 0.0007$, and $b = 0.848 \pm 0.010$.

3. MASS AND RADIUS DETERMINATIONS

In previous studies the mass and radius of GJ 436 have been derived in three ways: either 1) the mass has been obtained from the near-infrared ($JHK$) mass-luminosity relations of Delfosse et al. (2000) (giving $M_*=0.44 \pm 0.04\ M_\odot$, Maness et al. 2007) and the radius has been assumed to be numerically equal to the mass (Gillon et al. 2007a, or 2) the mass has been held fixed at the above value and the radius constrained directly from the light curve (Gillon et al. 2007a,b), or 3) both $M_*$ and $R_*$ have been solved for simultaneously subject to the constraint that they be numerically equal and making use of the implicit sensitivity $R \propto M^{1/3}$ between mass and radius in the light curve fitting procedure (Deming et al. 2007, Gillon et al. 2007b). In the second case the radius obtained is near 0.46 $R_\odot$ for $M_* = 0.44\ M_\odot$, and in the third case $M_*=R_*=0.47$ or 0.48 in solar units. Thus, differences of order 0.02 to 0.04 remain between these determinations, depending on the procedure used.

The methodology in the present paper is completely different, and can yield improved precision and also give a better understanding of possible systematics. We initially applied the $a/R_*$ and $T_{\text{eff}}$ constraints as described in using the Baraffe et al. (1998) models, and obtained a mass value near $M_* = 0.50\ M_\odot$, which is considerably larger than previous estimates. The predicted absolute visual magnitude ($M_V = 9.86$) is also much brighter.
than that computed directly from the Hipparcos parallax. This inconsistency strongly suggests a problem with the models, which is not entirely unexpected for a star of this type. As an alternative to \(a/R_\star\), we then experimented using the absolute \(K\) magnitude as a proxy for luminosity, as well as replacing \(T_{\text{eff}}\) with the color index \(J-K\).\(^1\) Figure 1 displays the four constraints in different combinations against solar-metallicity isochrones from 1 to 10 Gyr. The seemingly good fit in all planes belies the serious discrepancies present in other derived quantities that are not shown explicitly. Those results are listed in Table 1. Masses inferred from \(a/R_\star\) are systematically larger than those from \(M_K\), and so are the radii. The masses from \(M_K\) come close to the estimates from the empirical \(M-L\) relations, but the corresponding radii are considerably smaller than expected. This would seem to go in the direction of the results from eclipsing binaries (see Fig. 1).

On the other hand, there is good evidence from various sources that the bolometric luminosities from these models are not seriously in error (Delbosse et al. 2000; Torres & Ribas 2002; Ribas 2006; Torres et al. 2006). This suggests that adjustments to the model radii and temperatures might resolve the discrepancies in Table 1 and allow us to obtain a meaningful result for GJ 436. We explored this by introducing a correction factor \(\beta\) to the radii, and at the same time applying a factor \(\beta^{1/2}\) to the temperatures in order to preserve the bolometric luminosity. We repeated the comparison between the adjusted models and each of the four sets of constraints for a range of \(\beta\) values centered on the value indicated by the eclipsing binary studies. Figure 2 shows the result for several of the key stellar properties. The lines corresponding to the four sets of constraints seem to converge for a value of \(\beta\) near 1.1 (representing a 10% correction to the model radii), which happens to be the typical factor found by the eclipsing binary studies mentioned earlier. For this value of \(\beta\) the models yield essentially the same mass, radius, and luminosity for GJ 436, independently of which set of observational constraints is used, as one would expect from a realistic model. Thus, a self-consistent solution is achieved. To arrive at the best possible values of \(M_\star\) and \(R_\star\) we next applied all the constraints simultaneously, and varied \(\beta\) as before, seeking the best agreement with the measurements. The result is illustrated in the bottom right panel of Figure 2 where the quality of the match as represented by \(\chi^2\) is shown as a function of the correction factor over the restricted range in which the models agree with all four observables within their errors. The best match is again near \(\beta = 1.1\). The resulting mass and radius are \(M_\star = 0.452^{+0.014}_{-0.012}\) \(M_\odot\) and \(R_\star = 0.464^{+0.009}_{-0.011}\) \(R_\odot\). These and other inferred stellar properties for GJ 436 are listed in the top section of Table 2. We emphasize that these quantities are the result of the simultaneous application of the four constraints, and the agreement with some of the values in Table 1 is accidental.

\(^1\) We have refrained from directly applying any constraint based on the \(V\) magnitude because of suspected deficiencies in the models for optical passbands, related to missing molecular opacity sources shortward of 1 \(\mu\)m (see, e.g., Baraffe et al. 1998; Delbosse et al. 2000). Experiments using the \(J-K\) color as an alternative indicate that the \(I\) band is also affected at some level.

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**Fig. 2.—** Stellar properties for GJ 436 derived from the comparison with stellar evolution models by Baraffe et al. (1998), as a function of the adjustment factor applied to the model radii (\(\beta\)). The observational constraints given by \(a/R_\star\), \(T_{\text{eff}}, M_K\), and \(J-K\) are applied and shown in pairwise combinations, as labeled. Icons in the curves are a reflection of the discreteness of some of the quantities tabulated in the models. The simultaneous application of all four constraints results in the \(\chi^2\) curve shown at the bottom right, indicating a best fit for a \(\beta\) value near 1.1 (see text). This value is represented with the vertical dotted line running through this and the other panels.

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4. THE RADIUS DISAGREEMENT WITH THE MODELS

Our mass and radius estimates are compared with measurements for late-type double-lined eclipsing binaries in Figure 3. Only systems with the most accurate determinations are shown (relative errors below 3%), which are taken from the summary by Ribas (2006). For comparison, we include two isochrones from Baraffe et al. (1998) corresponding to ages of 300 Myr (as estimated for two of these binaries) and 3 Gyr (more representative of the field). All eclipsing binary systems are seen to have larger sizes than predicted for their mass. We note also that GJ 436 lies in the gap between masses of 0.43 \(M_\odot\) (for CU Cnc A) and 0.60 \(M_\odot\) (GU Boo B), and thus provides valuable additional information on the radius discrepancies for low mass stars.

López-Morales (2007) has investigated how these discrepancies (\(\Delta R_\star/R_\star\)) depend on metallicity and the strength of the chromospheric activity, quantified in terms of the X-ray luminosity (specifically, \(L_X/L_\text{bol}\)), for the rather limited sample available so far. Both of these factors have been suggested to play a role. The above study examined single M dwarfs as well as M dwarfs in binary systems. GJ 436 is a rather inactive star for its type (Endl et al. 2003, Butler et al. 2004), but was detected nonetheless by ROSAT as an X-ray source because of its proximity. The X-ray luminosity was reported by Hünenberg et al. (1999) to be \(L_X = 0.7 \times 10^{27}\) erg s\(^{-1}\). When combined with the bolometric luminosity in Table 2 we obtain \(L_X/L_\text{bol} = 7.0 \times 10^{-6}\). The metallicity was estimated by Bonfils et al. (2005) to be near solar: [Fe/H] = \(-0.03 \pm 0.20\). Considering GJ 436 as a single star, the value \(\Delta R_\star/R_\star \sim 10\%\) we find is consistent with the overall conclusions of López-Morales (2007) in the
those elements. Fortney et al. (2007) for a 10% fraction of models by Fortney et al. (2007; Gillon et al. 2007b), and agrees very well with the gen/helium envelope (Gillon et al. 2007a; Deming et al. of earlier authors regarding the presence of a hydro-
flection of the better stellar parameters. The slightly
proved precision of these derived parameters is a re-
(2007), who obtain a similar value for
Baraffe et al. (1998) are shown for reference, corresponding to ages as labeled.
sense that it is similar to the offsets for other systems regardless of $L_X/L_{bol}$, and at the same time it seems to follow the trend with [Fe/H] exhibited by other single M dwarfs.

5. PLANET PARAMETERS AND FINAL REMARKS

With the host star properties known, the planet parameters we infer are given in the bottom section of Table 2. The required orbital period and velocity semi-amplitude $K_\star$ are adopted from Maness et al. (2007), along with the eccentricity from Demory et al. (2007), who obtain a similar value for $K_\star$. The improved precision of these derived parameters is a reflection of the better stellar parameters. The slightly larger planet radius than in previous studies confirms with even greater statistical significance the conclusions of earlier authors regarding the presence of a hydrogen/helium envelope (Gillon et al. 2007a; Deming et al. 2007; [Gillon et al. 2007b]; and agrees very well with the models by Fortney et al. (2007) for a 10% fraction of those elements.

In this paper we have shown that GJ 436 provides a valuable test of stellar evolution theory near the bottom of the main sequence, made possible by the fact that it has a transiting planet. Traditional studies in the area of low-mass stars have made the comparison with models by measuring the mass and radius directly for double-lined eclipsing systems containing M dwarfs. In a few other cases angular diameters have been measured interferometrically for single stars, and the mass has been inferred from empirical M−L relations (e.g., [ Lane et al. 2001; Ségransan et al. 2003]). More recently, a variety of constraints and assumptions have been used to infer the mass and radius of the late-type secondaries in F+M systems observed as part of transiting planet surveys (e.g., Bouchy et al. 2005; Pont et al. 2005; Beatty et al. 2007). Though perhaps not as compelling as having actual model-independent mass and radius measurements, the approach in the present work is able to make use of available information for GJ 436 and compare the models directly with the observational constraints without requiring a direct measurement of the mass and radius. The discrepancy in $R_\star$ is derived by parameterizing it in terms of a single adjustment factor to the model radii ($\beta$), assuming the luminosity from theory is accurate, as other observations seem to indicate.

NASA's upcoming Kepler mission, currently slated to launch in early 2009, will emphasize the search for transiting Earth-size planets. These should be easier to detect around late-type stars. Therefore, we anticipate that many systems similar to GJ 436 could be found and become a significant source of information on radii for low-mass stars, since they will have all the observational constraints needed (including trigonometric parallaxes) to test models of stellar evolution in the way we have done here.

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Fig. 3.— Mass-radius relation for all double-lined eclipsing binaries with relative mass and radius errors under 3% (data from Ribas 2006). GJ 436 is shown at the values determined from our modeling (open circle), and is seen to display the same radius discrepancy as the other systems. Two solar-metallicity model isochrones by Baraffe et al. (1998) are shown for reference, corresponding to ages as labeled.
TABLE 1

Stellar parameters for GJ 436 based on different sets of constraints, using the models by Baraffe et al. (1995).

| Parameter          | Observational constraints |
|--------------------|---------------------------|
|                    | $T_{\text{eff}}$ and $a/R_\star$ | $J-K$ and $a/R_\star$ | $T_{\text{eff}}$ and $M_K$ | $J-K$ and $M_K$ |
| $M_\star$ ($M_\odot$) | 0.49$^{+0.003}_{-0.015}$ | 0.515$^{+0.042}_{-0.033}$ | 0.448$^{+0.018}_{-0.008}$ | 0.452$^{+0.014}_{-0.012}$ |
| $R_\star$ ($R_\odot$)  | 0.469$^{+0.000}_{-0.015}$ | 0.484$^{+0.035}_{-0.030}$ | 0.421$^{+0.009}_{-0.009}$ | 0.421$^{+0.009}_{-0.010}$ |
| log $g_\star$ (cgs)     | 4.797$^{+0.032}_{-0.000}$ | 4.798$^{+0.032}_{-0.000}$ | 4.841$^{+0.020}_{-0.009}$ | 4.843$^{+0.011}_{-0.011}$ |
| $a/R_\star$           | ...                        | ...                        | 14.63$^{+0.34}_{-0.19}$  | 14.65$^{+0.31}_{-0.21}$  |
| $T_{\text{eff}}$ (K)    | ...                        | 3684$^{+87}_{-55}$          | ...                        | 3585$^{+19}_{-13}$       |
| $J-K$ (mag)$^a$        | 0.80$^{+0.008}_{-0.001}$  | 0.816$^{+0.002}_{-0.007}$  | ...                        | ...                      |
| $M_V$ (mag)            | 9.86$^{+0.12}_{-0.00}$    | 9.71$^{+0.27}_{-0.36}$     | 10.256$^{+0.070}_{-0.094}$| 10.244$^{+0.082}_{-0.082}$|
| $M_K$ (mag)$^a$        | 5.75$^{+0.000}_{-0.001}$  | 5.66$^{+0.18}_{-0.22}$     | ...                        | ...                      |
| $L_\star$ ($L_\odot$)  | 0.034$^{+0.000}_{-0.035}$ | 0.038$^{+0.0099}_{-0.0069}$| 0.0266$^{+0.0015}_{-0.0016}$| 0.0260$^{+0.0014}_{-0.0017}$|

$^a$ Magnitudes are in the CIT photometric system of Elias et al. (1982).

TABLE 2

Stellar and planetary parameters for the GJ 436 system.

| Parameter     | Value |
|---------------|-------|
| Stellar parameters |       |
| $M_\star$ ($M_\odot$) | 0.452$^{+0.014}_{-0.012}$ |
| $R_\star$ ($R_\odot$)  | 0.464$^{+0.009}_{-0.011}$ |
| $L_\star$ ($L_\odot$)  | 0.0260$^{+0.0014}_{-0.0017}$ |
| log $g_\star$ (cgs)     | 4.843$^{+0.018}_{-0.011}$ |
| Age (Gyr)$^a$           | 6$^{+4}_{-5}$          |
| Planetary parameters   |       |
| $M_p$ ($M_\oplus$)      | 23.17$\pm$ 0.79        |
| $R_p$ ($R_\oplus$)      | 4.22$^{+0.09}_{-0.10}$  |
| $\rho_p$ (g cm$^{-3}$)  | 1.69$^{+0.14}_{-0.12}$  |
| log $g_p$ (cgs)         | 3.107$\pm$ 0.040        |
| $a$ (AU)               | 0.02872$\pm$ 0.00027    |

$^a$ Due to the unevolved nature of GJ 436 the age is essentially unconstrained by the observations. We list this value only for completeness.