UV Extinction as a More Fundamental Measure of Dust than $E(B-V)$ or $A_V$

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Abstract

The gas-to-dust ratio of reddened stars in the Milky Way (MW), the Magellanic Clouds, and in general, is usually expressed as a linear relation between the hydrogen column density, $N(H)$, and the reddening, $E(B-V)$, or extinction in the V-band ($A_V$). If the extinction curve were truly universal, the strength of the relationship and the linearity would naturally be maintained for extinction at any wavelength, as well as for $N(H)$ versus $E(B-V)$. However, extinction curves vary within the MW, and there is no reason, except by chance, why either $E(B-V)$ or $A_V$ would be the most physical measure of dust column density. In this paper, we utilize for the first time full extinction curves to 41 MW sightlines, finding that the scatter between $N(H)$ and extinction is minimized—and the relation becomes linear—for extinction at $2900 \pm 160$ Å. Scatter and nonlinearity increase at longer wavelengths, and are especially large for near-IR extinction. We conclude that near-UV extinction is a superior measure of the dust column density for MW dust. We provide new, nonlinear gas-to-dust relations for various dust tracers. We reduced to 0.7 dex for far-UV extinction, which matches the difference in cosmic abundances of carbon between the two galaxies, and therefore confirms that $(N(C)$ is the preferred measure for gas in the gas-to-dust ratio, even though it may not be a convenient one.

Unified Astronomy Thesaurus concepts: Interstellar dust extinction (837); Interstellar dust (836); Gas-to-dust ratio (638); Extinction (505); Interstellar extinction (841)

1. Introduction

A practical quantification of the amount of dust lying between the observer and a star or galaxy is of great importance for a wide range of Galactic and extragalactic studies, including efforts to model galaxy evolution (Galliano et al. 2018; Salim & Narayanan 2020). Ideally, the dust should be measured in terms of dust column density ($N_d$), and consequently, the gas-to-dust ratio should be expressed as $N(H)/N_d$ (or equivalently $m_H/m_d$), where $N(H)$ represents the hydrogen column density between the observer and the emitter. However, measuring dust column density is not straightforward. In situations where all of the dust lies in front of the reddened object, one can use far-infrared emission to obtain dust column density. IR maps are the basis for the Milky Way (MW) reddening maps of Schlegel et al. (1998), which are widely used to correct (“deredden”) the fluxes of extragalactic objects. However, when the dust is situated both in front of and behind a reddened star in the Galaxy, or when a stellar population is mixed with dust in an external galaxy (galaxy attenuation), IR emission overestimates the dust column. We must then use other methods to estimate it.

Traditionally, reddening ($E(B-V)$, i.e., the difference in extinction between B- and V-bands $A_B - A_V$) is most commonly used as a measure of dust. Consequently, $N(H)/E(B-V)$ is used as a measure of dust-to-gas ratio (e.g., Savage et al. 1977; Bohlin et al. 1978). However, there is no reason why $E(B-V)$ would be a better measure of column density—and, by extension, better correlate with $N(H)$—than $A_V$ (e.g., Zhu et al. 2017), or extinction at some other wavelength ($A_{\lambda}$). If the extinction curve was universal at all wavelengths, i.e., if $A_{\lambda}/A_V = \text{const.}$, the question of the most fundamental measure of dust would be moot. $A_{\lambda}$ would simply be proportional to $E(B-V)$, with the same coefficient of proportionality for all stars. Equivalently, this is to say that the $R_V$ parameter is constant, because by definition $A_V = R_V E(B-V)$. In reality, $R_V$—and therefore the shape of the extinction curve—can vary significantly from one sightline to another, especially for high-opacity regions in the MW ($2 < A_V < 6$; e.g., Cardelli et al. 1989; Valencic et al. 2004). The diversity of extinction curves implies that if extinction in the near-IR is the most physical measure of dust column density for MW dust. We provide new, nonlinear gas-to-dust relations for various dust tracers. We reduced to 0.7 dex for far-UV extinction, which matches the difference in cosmic abundances of carbon between the two galaxies, and therefore confirms that $(N(C)$ is the preferred measure for gas in the gas-to-dust ratio, even though it may not be a convenient one.

It has been asserted by Jura (1980) that there is no reason why either $E(B-V)$ or $A_V$ should be the best measure of dust. Instead, Jura proposed that the fundamentally better-motivated measure of dust may be extinction in the near-IR (NIR), such as $A_{3.6 \mu m}$ (L-band), but this proposal has not been confirmed empirically.

To the best of our knowledge, there has been only one study that attempted to predict the wavelength at which extinction is the best measure of dust. Kim & Martin (1996) used the Kramers–Kronig approach (Purcell 1969), together with dust grain distribution models for different values of $R_V$, to determine the wavelength at which the gas-to-dust ratio is independent from $R_V$—a necessary condition for the minimization of scatter between $A_{\lambda}$ and $N(H)$. For MW dust, they find that this should happen at approximately 3600 Å (near the U-band). However, this result has not been widely accepted or even discussed in subsequent literature, and in any case, the...
question has never been put to an empirical test. The aim of this paper is to do so.

The ancillary goal of this paper is to try to find a unifying framework for disparate gas-to-dust ratios in the MW and SMC. The gas-to-dust ratio, expressed using the column density of hydrogen, is constant to first order within the MW (Bohlin et al. 1978). However, the gas-to-dust ratio is –0.9 dex higher in the SMC, at least where \( E(B-V) \) is used as the tracer of dust. This great discrepancy suggests that the fundamental relation (i.e., one that would not change from galaxy to galaxy) could involve the column density of some element more directly related to the dust, in place of hydrogen (e.g., Clayton & Martin 1985; Mathis 1990; Draine 2003; Welty et al. 2012).

The best candidate for this alternative abundance is carbon, the cosmic abundance of which is 0.6 dex lower in the SMC than in the MW. This difference in abundance goes a significant way toward explaining the difference in \( N(H)/E(B-V) \) gas-to-dust ratios, but is still 0.3 dex short. Here, we investigate if the solution to this discrepancy lies in \( E(B-V) \) not being the most appropriate measure of dust.

In this work, we take advantage of the fact that detailed extinction curves are available in the literature for a relatively large number of MW stars. We use Fitzpatrick & Massa (2007, hereafter FM07) extinction curve parameters, in combination with hydrogen column densities from Gudennavar et al. (2012, hereafter G12), to evaluate, across wavelengths from the UV to the NIR, which \( A_\lambda \) is the most fundamental measure of dust. Section 2 describes these sources of data. Section 3 describes how our sample was created, along with the methods used to determine the best measure of dust. The plausibility of metallicity driving the scatter in the \( N(H) \) versus \( A_\lambda \) relation is also explored. In Section 4 we discuss our results in the context of prior results in the literature. The relation of our results to extinction and metallicity in the SMC is also discussed there.

2. Data and Sample

We draw the main data for this study from two sources: FM07 provides extinction curve parameters, and G12 gives column densities for hydrogen and metals. FM07 represents one of the largest samples of full extinction curves for MW sightlines obtained to date. G12 is the largest compilation of interstellar column density measurements from the literature, containing measurements for over 3000 stars (most in the MW and some in the Magellanic Clouds) across ~50 different species.

2.1. Extinction

FM07 mostly contains B and late O stars for which the authors were able to gather UV via IR data: UV spectro-photometry from the International Ultraviolet Explorer (IUE), \( UBV \) photometry from various sources, and \( JHK \) photometry from the Two-Micron All Sky Survey (2MASS). The original sample observed by IUE was trimmed to remove stars with poorer data, unreddened stars, and unusual stars (Be stars, luminosity class-I stars, and early/luminous O stars), leaving 328 stars in the main FM07 sample.

FM07 introduced an “extinction-without-standards” technique, in which they determine extinction based on a best-fit model of the spectral energy distribution for the observed flux from each star. This model-based method eliminates the need for observations of unreddened comparison stars (the pair method). The accuracy of the pair method can be affected by spectral mismatch between the target and comparison star. Furthermore, the comparison stars themselves are not entirely dust-free, and require an extinction correction.

FM07 published the extinction curves in parameterized form, using a parameterization which is an expanded version of the one given in Fitzpatrick & Massa (1990). In particular, for the UV region (\( \lambda < 2700 \) Å), they use

\[
k(\lambda - V) = \begin{cases} 
    c_1 + c_2x + c_3D(x, x_0, \gamma) & x \leq c_5, \\
    c_1 + c_2x + c_3D(x, x_0, \gamma) + c_4(x - c_5) & x > c_5
\end{cases}
\]

where \( x \equiv \lambda^{-1} \), with \( \lambda \) in micrometers. In Fitzpatrick & Massa (1990), \( c_5 \) was fixed at 5.9 \( \mu m^{-1} \), whereas most values determined using the new parameterization are in that vicinity. \( D(x, x_0, \gamma) \) represents the 2175 Å bump, and is expressed using the Lorentzian-like function

\[
D(x, x_0, \gamma) = \frac{x^2}{(x^2 - x_0^2)^2 + x^2\gamma^2}
\]

where \( x_0 \) specifies the location of the bump, and \( \gamma \) modulates its width.

To describe the optical and near-IR portions of the extinction curve, FM07 employ a cubic spline interpolation. In the optical range, interpolation is based on extinction at 3300, 4000, and 5530 Å. In the near-IR (\( \lambda > 1 \) \( \mu m \)), FM07 assume that the extinction curve is universal (i.e., \( A_\lambda/A_\lambda^{\mu m} = \) constant when \( \lambda > 1 \) \( \mu m \), for all stars) and follows the generally assumed power-law functional form (Rieke & Lebofsky 1985):

\[
A_\lambda \propto \lambda^{-\beta_{NIR}}
\]

with the exponent \( \beta_{NIR} = 1.84 \) (Martin & Whittet 1990). The assumption of a fixed near-IR curve was justified in FM07 based on the use of relatively shallow \( JHK \) data from 2MASS. Fitzpatrick & Massa (2009) added new spectral observations from the Hubble Space Telescope’s (HST’s) Advanced Camera for Surveys High Resolution Camera to investigate the potential for flexibility in the near-IR power-law slope. They demonstrate that, of the 14 stars studied, over half of the NIR power-law fits are significantly improved by allowing \( \beta_{NIR} \) to be a free parameter. They show specifically that the use of the fixed \( \beta_{NIR} = 1.84 \) from Martin & Whittet (1990) is not a good fit for large-\( R_V \) sightlines. Furthermore, the slope appears to be correlated with \( R_V \). Considering the fact that there is still some debate in the literature concerning the nature of IR extinction, we produce two sets of extinction curves. One follows FM07, with a universal extinction curve at \( \lambda > 1 \) \( \mu m \). The other follows FM07 up to 7500 Å, but subsequently follows a power-law parameterization with a variable \( \beta_{NIR} \). The wavelength at which the regime changes is based on the analysis in Fitzpatrick & Massa (2009). The exponent for each sightline is determined according to the relation in Salim & Narayanan (2020), which was constructed from the data reported in Table 3 of Fitzpatrick & Massa (2009):

\[
\beta_{NIR} = -4.20 + 4.59 \left( \frac{A_B}{A_V} \right),
\]
where \( A_{B}/A_V \equiv 1 + 1/R_V \). In other words, the near-IR exponent inversely depends on \( R_V \). The two sets of extinction laws are illustrated in Figure 1 for three stars from the FM07 sample with a range of \( R_V \) values. Note that the extinction curves with non-universal \( \beta_{\text{NIR}} \) show no break in slope around 7500 Å, even though this transition is not smooth by construction; they appear more natural.

We use extinction curves from Gordon et al. (2003) for sightlines to SMC stars. The authors fit reddened stars from both the SMC (wing and bar sample stars) and the LMC (LMC2 supershell and normal samples) to the parameterization given in Fitzpatrick & Massa (1990).

### 2.2. Column Densities

To obtain hydrogen column density measurements, we use the database of literature values for 3008 sightlines to stars compiled in G12. G12 contains two tables: one comprising all of the measurements found in the literature, and a second containing only the most recent measurements for each sightline. We use the latter. G12 reports \( N(H_1) \) and \( N(H_2) \), along with an \( N(H) \) entry, i.e., \( N(H_1 + H_2) \). It is notable that these \( N(H_1 + H_2) \) values sometimes differ somewhat from the combination of individual density values \( (=N(H_1) + 2N(H_2)) \). This is due to the disparate sources from which information was obtained, and the fact that individual \( N(H_2) \) and \( N(H_1) \) measurements have sometimes been reported in the literature more recently than \( N(H) \) for the same sightline. For relative consistency, we use the \( N(H) \) column as opposed to the summed individual values, and refer to it as \( N(H_1 + H_2) \) or \( N(H) \) interchangeably.

Some column density values do not have associated errors, in which case we adopt the mean of the log-errors on the density measurements with reported errors. The notable exception is the sightline toward HD 149757 (ζ Oph), which had an error value on \( N(H) \) that was over an order of magnitude smaller than all the others, and therefore appears erroneous. We reset the error value for this star to the average (as though it did not originally have one).

G12 includes column density measurements for a great many species, of which we use \( N(\text{Fe II}) \), \( N(\text{O I}) \), \( N(\text{O VI}) \), \( N(\text{Mg II}) \), and \( N(\text{Ca II}) \). We pull updated hydrogen column densities for SMC stars from Welty et al. (2012),

### 2.3. Sample Selection

We matched the 328 stars with extinction curves from FM07 to the list of 3008 stars with column density measurements from G12, matching the stars by name. The FM07 stars are mostly \( V < 10 \) mag, with a few slightly fainter; this means that the stars all have names from common catalogs. Ninety-six stars are common to FM07 and G12, of which 70 stars have \( N(H_1) \) measurements, 54 have \( N(H_2) \) measurements, and 50 have \( N(H_1 + H_2) \) values reported. For some stars in the FM07 sample, \( R_V \) was assumed to be 3.1, owing to a lack of IR photometry. For others, the \( O_I \) spline point was not calculated, due to missing \( U \)-band photometry. We omit stars which fit within either category. This leaves us with 41 sightlines in the \( N(H) \) sample, and 55 in the \( N(H_1) \)-only sample.

Pertaining to the analysis of metallicity effects in Section 3.3, of the 41 stars in the \( N(H_1 + H_2) \) sample, 28, 27, 9, 21, and 23 stars have \( \text{Fe II} \), \( \text{O I} \), \( \text{O VI} \), \( \text{Mg II} \), and \( \text{Ca II} \), respectively.

For the SMC analysis in Section 4.3, we first matched the five reddened SMC stars from Gordon et al. (2003)—four bar sample stars and one from the wing sample—with the full G12 list (again using names, as they all have AzV designations). Of the metal species of interest, only \( \text{Ca II} \) had column densities reported (for all but one star, AzV 23). Welty et al. (2012) includes \( N(H) \) measurements for all five stars. Notably, for AzV 23, AzV 214, and AzV 398 (all from bar sample), Welty et al. (2012) determined that there is no significant contribution to \( N(H) \) from \( N(H_2) \).

### 3. Results

The main objective of our analysis is to find the wavelength at which the hydrogen column density \( N(H) \) is best correlated with the extinction \( A_V \), which will reveal the best measure for the dust column in the MW. In previous studies, either \( E(B - V) \) or \( A_V \) has been taken as the measure for dust simply because those measures are most widely available. Furthermore, in many cases where \( A_V \) is used, it has simply been obtained through \( A_V = 3.1E(B - V) \), i.e., assuming a fixed

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1. Obtained from the full data table at https://astro.uchicago.edu/~dwelty/mcplotuv.html.
extinction curve. However, the literature also hints that the true measure of dust may be found somewhere in the NIR extinction curve. However, the literature also hints that the true measure of dust may be found somewhere in the NIR extinction curve. However, the literature also hints that the true measure of dust may be found somewhere in the NIR extinction curve. However, the literature also hints that the true measure of dust may be found somewhere in the NIR extinction curve. However, the literature also hints that the true measure of dust may be found somewhere in the NIR extinction curve.

It is also noteworthy that previous studies have uniformly assumed linearity in the relation between \(N(H)\) and \(E(B-V)\) or \(A_V\). In the case of diverse extinction curves (a range of \(R_V\)), we do expect linearity with respect to the \(A_\lambda\) that is the true measure of dust (which may or may not be \(A_V\)), but not at other wavelengths. Instead of assuming linearity, we allow a power-law dependence of \(N(HI)\) and \(N(HI + H_2)\) on \(A_\lambda\) (and \(E(B-V)\)).

### 3.1. \(N(HI + H_2)\)

Using the FM07 extinction curve parameters for our sample of 41 sightlines, we calculate the extinction \((A_\lambda)\) for each reddened star, for wavelengths in the range 1111–47000 Å, pseudocontinuously sampling 1000 wavelengths from the range logarithmically. Extinctions are calculated using a modified version of the fm07 function from the extinction package in Python (Barbary 2016). We adjust the function such that it can use arbitrary curve parameters, and any \(R_V\). Uncertainty on the resultant \(A_\lambda\) values is calculated by re-running the script with modified \(R_V\) values, once using \(R_V + \sigma_{R_V}\), and again using \(R_V - \sigma_{R_V}\) (where \(\sigma_{R_V}\) is provided in FM07). The resulting range of \(A_\lambda\) for each star should be a good approximation of uncertainty, since the error is dominated by \(\sigma_{R_V}\). After the first run, we also calculated a second set of extinction curves, this time with an \(R_V\)-dependent \(\beta_{NIR}\) redward of 7500 Å.

Figure 2 shows a selection of wavelengths for which \(N(HI + H_2)\) is plotted against \(A_\lambda\); the UV wavelengths are 1500, 2175, and 3000 Å, along with the approximate central wavelengths of the standard photometric bands \(U\) (3700 Å), \(V\) (5500 Å), \(I\) (8000 Å), and \(J\) (1.2 μm). The final panel shows \(E(B-V)\) in place of extinction at a specific wavelength. Values for \(A_I\) and \(A_J\) are based on extinction curves with free \(\beta_{NIR}\). Also shown are the best power-law fits, calculated using the least-squares method:

\[
\log N(HI + H_2) = a \log A_\lambda + b,
\]

where \(a = 1\) would correspond to a linear relation between \(N(HI + H_2)\) and \(A_\lambda\) (in that case, \(b\) gives the gas-to-dust ratio, assuming \(A_\lambda\) as the dust measure). Errors on \(N(HI + H_2)\) from G12 are taken into account in the fit. We calculate a \(\chi^2\) value for each individual fit, which is to first order proportional to the scatter of the residuals around the best fit.

Following the original formulation of Bohlin et al. (1978), most studies assume a linear relation, typically with \(A_\lambda = A_{V}\), or using \(E(B-V)\) as the independent parameter. From Figure 2 we conclude that neither option is optimal. The scatter (\(\chi^2\)) is smaller in the UV and blue optical than at longer wavelengths. In addition, the relation is closer to linear in the UV and blue optical than at longer wavelengths (including near-IR).
and represents \( \lambda \) original, unadjusted FM07 extinction curve parameterization. The red curve is photometric bands are centered on their central wavelengths. Å is tightest for extinctions around 2900 \( \lambda \) from a the FM07-calculated values using an \( R_0 \)-dependent near-IR extinction curve slope (\( \sim \log \); see Section 2.1). The relation between \( N(H_1 + H_2) \) and extinction is tightest for extinctions around 2900 Å. Letters representing common photometric bands are centered on their central wavelengths.

The sum of normalized quadratic residuals (\( \chi^2 \)) between \( \log N(H_1 + H_2) \) and \( A_V \) as a function of wavelength. The \( \chi^2 \) value comes from a fit to Equation (5). There are 39 degrees of freedom. The cyan curve represents fits performed using \( A_V \) values that were calculated using the original, unadjusted FM07 extinction curve parameterization. The dotted line represents a slope of 1 (a linear relation between total hydrogen column density and extinction). The relation between \( N(H_1 + H_2) \) and extinction is exactly linear for extinctions around 3000 Å.

Figure 3. The sum of normalized quadratic residuals (\( \chi^2 \)) between \( \log N(H_1 + H_2) \) and \( A_V \) as a function of wavelength. The \( \chi^2 \) value comes from a fit to Equation (5). There are 39 degrees of freedom. The cyan curve represents fits performed using \( A_V \) values that were calculated using the original, unadjusted FM07 extinction curve parameterization. The dotted line represents a slope of 1 (a linear relation between total hydrogen column density and extinction). The relation between \( N(H_1 + H_2) \) and extinction is exactly linear for extinctions around 3000 Å.

For reference, the best fits are as follows:

\[
\log N(H_1 + H_2) = 1.05 \log A_{2900} + 21.03
\]

(6)

\[
= 0.89 \log A_V + 21.32
\]

(7)

\[
= 1.08 \log E(B - V) + 21.87
\]

(8)

where the free coefficient \( b \) represents the logarithm of hydrogen column density at \( A_{2900} \), \( A_V \), or \( E(B - V) \) of 1. The error on \( N(H) \) determined from these expressions is \( \sim 0.03 \) dex (7%), and is obtained by taking the typical error in \( N(H) \) measurements of 0.1 dex, scaling it by \( \chi^2_{\text{red}} \) (\( \sim 2 \)), and dividing it by the square root of the sample size, resulting in the error of the mean. When compared at \( E(B - V) \) (or \( A_V \)) values appropriate for the respective samples, the \( N(H) \) from our expression is 10% higher than the value given by Bohlin et al. (1978), 6% higher than the value from G12, and 8% lower than the value derived in Zhu et al. (2017), based on their analysis of the whole Anders & Grevesse (1989) sample. These differences are well within the range of statistical error. The advantage of using our nonlinear expressions is that they acknowledge that gas-to-dust ratio is not exactly a constant, particularly when \( E(B - V) \) or \( A_V \) is used to measure the dust.

The sightlines used in this study span the reddening range \( 0.21 < E(B - V) < 1.06 \), with a median value of 0.36 for the \( N(H_1 + H_2) \) sample, and 0.35 for the \( N(H_1) \) sample. At \( E(B - V) \approx 0.08 \), there is a well-known break in the slope of the \( N(H) \) versus \( A_V \) relation (e.g., Savage et al. 1977; Bohlin et al. 1978; Liszt 2014). This can be attributed to \( H_2 \) formation on the surface of dust grains (Hollenbach & Salpeter 1971), which only begins in regions characterized by sufficiently large reddening values (i.e., denser regions). As we do not probe below \( E(B - V) = 0.2 \), we are not sensitive to this change in slope, and do not discuss it further.

3.2. \( N(H_1) \) Only

Here, we carry out the same analysis as above, but using \( N(H_1) \) instead of \( N(H_1 + H_2) \). Since \( N(H_1) \) measurements along Galactic sightlines are more abundant in the literature than those for \( N(H_2) \), we are able to include 55 sightlines here.
It is well-established that the overall correlation with \( E(B - V) \) and \( A_V \) is stronger when using \( N(H_1 + H_2) \) (e.g., Bohlin et al. 1978; Welty et al. 2012). In comparing Figure 5, which shows \( \chi^2 \) values for the fit between \( \log N(H_1) \) and \( A_\lambda \), with the corresponding Figure 3 for \( N(H_1 + H_2) \), one can see that we reproduce the poorer fits. At \( V \), \( N(H_1 + H_2) \) produces \( \chi^2_{\text{red}} \approx 2.1 \), whereas \( N(H_1) \) produces \( \chi^2_{\text{red}} \approx 3.4 \). The \( E(B - V) \) fit is actually significantly worse for \( N(H_1) \), such that \( \chi^2_{\text{red}} \approx 4.6 \), as opposed to \( \chi^2_{\text{red}} \approx 2.3 \) for \( N(H_1 + H_2) \).

It is noteworthy that for \( N(H_1) \), the best fit does not lie in the visible range; rather, the fits gradually improve toward the NIR, stabilizing beyond about 1 \( \mu m \) (see Figure 5). We concluded in Section 3.1 that a minimum value exists at \( \lambda \approx 2900 \) Å, but \( N(H_1) \) does not seem to be connected to the dust in the same way, which is as expected given that \( H_2 \) is known to form on the surface of grains. In Figure 6, we see that the relation never approaches linearity, again in contrast with the case of \( N(H_1 + H_2) \). This behavior can be attributed to the fact that we are not counting all of the hydrogen.

### 3.3. Metallicity Effects on the Gas-to-dust Ratio

Although \( N(H_1 + H_2) \) is strongly correlated with \( A_\lambda \), particularly in the near-UV/blue optical, \( \chi^2_{\text{red}} \) is not 1, indicating excess scatter not described by the errors. Excess scatter has been a consistent feature of such relations in the literature where \( E(B - V) \) and \( A_V \) are used to represent the dust, but even at \( A_{2900} \) (which we claim is the true best dust measure, and therefore any scatter due to differences in the extinction curve is minimized; see Section 4), the errors on \( N(H_1 + H_2) \) do not encapsulate the scatter. A leading possible source of the residual scatter is variation in metallicity (e.g., Welty et al. 2012; Rémy-Ruyer et al. 2014; Kahre et al. 2018). If one accounts for differing metallicity between sightlines, it may be possible to bring the \( \chi^2_{\text{red}} \) value nearer to unity.

We make use of common metallicity indicators (Fe II, O I, O VI, Mg II, and Ca II) in the G12 compilation to assess metallicity effects on our sightlines. To test whether metallicity is in fact correlated with residuals (thereby driving the scatter), we plot the residual at \( \lambda = 2900 \) Å from Section 3.1 (essentially the gas-to-dust ratio) versus the column density of the metal species, normalized by \( N(H_1 + H_2) \).

The results are shown in the top row of Figure 7. We also provide Spearman’s rank correlation coefficient \((-1 < \rho < 1)\) in each panel, this assesses the correlation nonparametrically (i.e., it does not assume linearity). Although most correlations are weak or nonexistent, it is significant that all values of Spearman’s \( \rho \) are negative, implying that a very weak correlation with metal densities is not ruled out.

The failure of metallicity to account for the scatter in the gas-to-dust relation, even when dust is measured as \( A_{2900} \), may seem disappointing. However, it is not clear that one can use the observed metal abundances alone to test the hypothesis in the first place. In fact, many metal species are subject to depletion onto dust grains, so that the gas-phase abundances we measure toward these stars do not represent the true “cosmic” abundance along any given sightline. For Si in particular, depletion is known to essentially entirely offset the observed gas metallicity differences within the MW and the Magellanic Clouds (Roman-Duval et al. 2019). In the interstellar medium, of the species we investigate, Fe and Ca are likely to be substantially depleted, with the Ca depletion depending more strongly on gas density; Mg is largely undepleted in diffuse gas, but depleted in denser gas, and O is not strongly depleted (all inferences from Mathis 1990). If depletion (rather than the intrinsic range of cosmic abundances) is indeed the principal cause of the observed gas metallicity differences between sightlines, then a correlation should exist between depletion and dust density, which may be reflected in a correlation between the observed metallicity and extinction. In the bottom five panels of Figure 7, we show extinction versus observed metallicities. For Fe II and Ca II, there is a relatively clear negative correlation between \( A_{2900} \) and normalized column density—the more dust there is, the more the metals are depleted onto it. The correlation is somewhat weaker, but still present, for Mg II, which once again aligns with Mathis (1990). The two O species seem to be subject to relatively little depletion, although they still exhibit weak negative gradients. Connecting the lower panels with those above, it appears that it is not possible to assess the effects of metallicity on the gas-to-dust ratio within the MW.

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**Figure 5.** Sum of the normalized quadratic residuals \( \chi^2 \) between \( \log N(H_1) \) and \( A_\lambda \) as a function of wavelength. The \( \chi^2 \) value comes from a fit to Equation (5). There are 53 degrees of freedom. The cyan curve represents fits performed using \( A_\lambda \) values calculated using the original, unadjusted FM07 extinction curve parameterization. The relation between \( N(H_1) \) and extinction is tightest for extinctions around 1.7 \( \mu m \), but is much less tight for \( N(H_1 + H_2) \).

**Figure 6.** Slope \((a \text{ from Equation (5)})\) between \( \log N(H_1) \) and \( A_\lambda \), as a function of wavelength. The cyan curve represents fits performed using \( A_\lambda \) values calculated using the original, unadjusted FM07 extinction curve parameterization. The dotted line represents a slope of 1 (a linear relationship between total hydrogen column density and extinction). The relation between \( N(H_1) \) and extinction does not approach linearity.
3.4. SMC

So far, we have focused only on the MW. In the context of hydrogen column density versus extinction/reddening, the SMC has relatively low \( E(B-V) \), but high \( N(\text{HI}+\text{H}_2) \). This would place most SMC sightlines in the upper-left region of the \( N(\text{HI}+\text{H}_2) \) versus \( E(B-V) \) panel in Figure 2, far above the MW best-fit line. To quantify the deviation, we pull the five SMC stars for which Gordon et al. (2003) derived a full extinction curve: four from the bar sample, and one from the wing sample. The wing sample shows extinction features much more similar to the MW than those found in the bar sample, which Gordon & Clayton (1998) propose is a result of weaker star formation in the wing region. It has been suggested that the strong positive offset (for the bar sample in particular) is due to the lower-metallicity environment in the SMC (e.g., Welty et al. 2012; Rémy-Ruyer et al. 2014). We find this offset to be 0.9 dex when \( E(B-V) \) is used as the measure of dust. Here, we wish to determine how much of the offset may be the result of not measuring the dust using a more adequate tracer.

Figure 8 shows the residuals of SMC sightlines with respect to the \( N(\text{HI}+\text{H}_2) \) versus \( A_\lambda \) relations for MW sightlines discussed in Section 3.1. The plot confirms the large offsets in the optical region (around the V-band), particularly with respect to the bar sample stars. However, the offsets are clearly reduced in the UV, albeit at wavelengths shorter than 3000 Å. This lends more weight to our conclusion from Section 3.1 that dust is better measured using extinctions at UV wavelengths. We discuss a possible framework for bringing the MW and SMC gas-to-dust ratios into full agreement in Section 4.3.

4. Discussion

4.1. A Physically Motivated Measure of the Dust Column for the MW

By studying the correlation between \( N(\text{H}) \) and \( A_\lambda \) from the UV to the NIR, we find that the extinction at 2900 Å (\( A_{2900} \)) is more fundamental than \( E(B-V) \) or \( A_V \) as a measure of dust, at least for the MW. Furthermore, the slope of the relation is equivalent to unity around this wavelength, solidifying the result.
Does this result have any theoretical basis? As we pointed out in the introduction, Kim & Martin (1996) mention that $A_V/N(H)$ is not necessarily the best measure of the dust-to-gas mass ratio. Half a century ago, Purcell (1969) suggested an alternative to $A_V/N(H)$, based on the application of the Kramers–Kronig relations to the interstellar medium. For a single grain composition, the integral relation can be derived from the original Kramers–Kronig expressions (Purcell 1969; Martin 1978; Bohren & Huffman 1983). The idea is to integrate the total extinction cross-section of the grains across all wavelengths, via integration of the extinction curve $A_\lambda$ (or $A_w$, where $w \equiv \lambda^{-1}$):

$$\int_0^\infty \frac{A_w}{N(H)w} d\ln w = 1.086 \pi^2 m_H \sum f \frac{M_d}{s_d} M_H$$

(9)

Here, $s_d$ is grain density (which is dependent on composition), and $M_d$ is the column mass density of dust corresponding to the hydrogen column mass density, $M_H = m_H N_H$.

Kim & Martin (1996) plot the integrand from Equation (9) for a combination of silicate and graphite grains, with separate curves for extinction curves with varying values of $R_V$. The authors point out that all curves cross near 2.8 $\mu$m$^{-1}$ (3600 Å). This point of invariance of gas-to-dust mass ratios with $R_V$ coincides roughly with our best-Å$^{-1}$ wavelength of 2900 Å, as given in Section 3.1. While these results do not coincide precisely, their separation is not large, taking into account the modest increase in our Å$^{-1}$ value from 2900 to 3600 Å (see Figure 3). The Kim & Martin (1996) result is also in general agreement with the location at which we find the slope of the $N(H)/A_\lambda$ relation to be 1 ($\sim 3000 \pm 300$ Å). By referring to Figure 4, one can see that the crossing point we find without bootstrapping is quite close to the $U$-band, which is centered very close to 3600 Å.

In a similarly mass-based pursuit, Barbaro et al. (2004) select lines of sight in the solar neighborhood with anomalously high $N(H)/E(B-V)$. They find that the mass density ratio of gas to dust ($\rho_\text{H}/\rho_\text{d}$) is linearly related to $N(H)/E(B-V)$, which demonstrates that $E(B-V)$ cannot be the fundamental measure of dust, which mirrors our findings. They also note that, for anomalous sightlines, a modification of $\rho_\text{H}/\rho_\text{d}$ compared with the Galactic standard is required.

As the above analysis based on the Kramers–Kronig approach also demonstrates, if some $A_\lambda$ is truly the best measure of dust, we expect the gas-to-“dust” ratio (where the extinction $A_\lambda$ represents dust) to be independent of $R_V$. We perform a test for $R_V$-dependence across all wavelengths probed in the main analysis (UV to NIR), and show a selection of wavelengths in Figure 9. Figure 9 shows the wavelength regime where $A_{0.6}/N(H)$ is independent of $R_V$: somewhere around 3000 Å or the $U$-band. This agrees with our result, which was obtained using a different method (minimization of scatter between $N(H)$ and $A_\lambda$). While the switch from slight positive to slight negative correlations in the plots is subtle, it is clear that at $A_\lambda$ and beyond, there is a definite relationship between $A_{0.6}/N(H)$ and $R_V$, providing more evidence for ruling out extinction at those wavelengths as a satisfactory dust measure. Kim & Martin (1996) found that as $R_V$ increased beyond the standard 3.1, their maximum entropy solution did not use up as much material, yielding increased $A_V/N(H)$. These predictions are shown in cyan on the V panel of Figure 9. Draine (2003) constructs an $R_V$-dependent relation for $A_H/N(H)$, using 14 sightlines through translucent clouds from Rachford et al. (2002). The resultant relation is shown in gray on the I panel of Figure 9. Both the Kim & Martin (1996) and Draine (2003) relations agree with our findings.

4.2. The Character of the Near-IR Extinction Curve

In our analysis, we have produced the near-IR parts of the extinction curves by two different methods: (1) assuming universality beyond 1 $\mu$m (following the FM07 parameterization, which imposes it), and (2) modifying the FM07 parameterization beyond 0.75 $\mu$m in order to allow the NIR slope to vary with $R_V$, following the results of Fitzpatrick & Massa (2009). We see that our results are entirely unaffected by this assumption. The reason for considering NIR extinction as a more physically motivated measure lies in the notion that the NIR extinction curve may be universal (Rieke & Lebofsky 1985; Cardelli et al. 1989). A universal curve in the NIR means that the ratio of extinctions for any two wavelengths in the NIR range will be constant, even if the shape of the curve in the UV/optical is variable (and, following Cardelli et al. 1989, is correlated with $R_V$). However, the possibility that the NIR extinction curve is universal only guarantees that any NIR extinction (e.g., $A_\lambda, A_d, A_K$) will be an equally good (or bad) measure of the dust, but not necessarily the best measure. In other words, the question of universality in the near-IR has no bearing on which extinction measure is most closely related to dust column density.

While shedding light on the character of the near-IR extinction curve is not the goal of this paper, we provide a brief overview of the subject. The foundational study by Cardelli et al. (1989) finds that the IR data of the time was consistent with a single extinction law for $\lambda > 0.90 \mu$m, citing several contemporary studies (Jones & Hyland 1980; Koomen 1983; Rieke & Lebofsky 1985; Smith 1987; Whittet 1992), and ultimately using the Rieke & Lebofsky (1985) curve as the basis for the $\beta_{\text{NIR}}$ used in their parameterization, with a value of 1.61. More recently, with the availability of deeper NIR data, higher values have been found: 1.95 (Wang & Jiang 2014), 2.07 Wang & Chen (2019), and upward. Studies with the highest values (e.g., Naoi et al. 2007; Nogueras-Lara et al. 2018) focus on extremely dense regions (such as the Galactic Center and the Coalsack). Steeper slopes may correlate with the density of the regions probed, pointing to non-universality. Indeed, it may be claimed that the wealth of recent works whose results span a wide range of $\beta_{\text{NIR}}$ values themselves contribute to the notion of a variable $\beta_{\text{NIR}}$ as a conglomerate.

As referenced in Section 2.1, the strongest case for nonuniversality may come from Fitzpatrick & Massa (2009). The values they determined for $\beta_{\text{NIR}}$ varied from 0.9 to 2.3, and their Figure 3 reveals a correlation between the near-IR slope and the optical slope (and therefore $R_V$) for the 14 sightlines. Zasowski et al. (2009) shows general agreement with the Fitzpatrick & Massa (2009) results, citing a variance in $\beta_{\text{NIR}}$ with Galactocentric radius, which seems to be consistent with higher values being found toward the Galactic Center. Similarly, Schlafly et al. (2016) find that their extinctions, derived using the standard crayon technique, agree better with the Fitzpatrick & Massa (2009) modification of the FM07 curve than the FM07 curve itself. They fit extinction curves to their reddening vector measurements by computing $dm_\lambda/da$, where $da$ is a small variation in extinction about a typical APOGEE $E(B-V) = 0.65$, and $m_\lambda$ is the observed magnitude in some bandpass $b$. This is performed for bandpasses in both
the optical (grizy) and the NIR (H, K, W1, W2). The resultant fit ($\chi^2 = 26$) for Fitzpatrick & Massa (2009) curves—the only ones tested which allowed $\beta_{\text{NIR}}$ to vary—was quite a bit better than for prior curves more similar to FM07, the closest contender being Fitzpatrick (1999, $\chi^2 = 93$). We point out that Figures 1, 3, 4, and 6 do seem to suggest, but not prove, that a nonuniversal IR law is more natural, because it provides a continuation of the trends seen at shorter wavelengths.

4.3. Dependence of the Gas-to-dust Ratio on Metallicity and the SMC

In the landscape of plots showing $N$(H) versus $E(B-V)$, there is always quite a bit of scatter. Bohlin et al. (1978), found this scatter to be $\sim 30\%$ about the mean. In later studies (such as Rachford et al. 2009), the scatter remains. In all cases, the scatter is larger than can be explained by the errors. In our analysis, the scatter has been partially mitigated by substituting $A_{3000}$ as the measure of dust. However, the resultant $\chi^2_{\text{red}}$ still exceeds unity. We attempted to invoke metallicity effects to explain this in Section 3.3, but as noted, the depletion of various species renders the analysis murky.

The gas-to-dust ratio is widely studied in the literature, both in the Galaxy and beyond, and versus a range of other parameters. Using a variety of Galactic X-ray sources, Zhu et al. (2017) found no correlation between $N$(H)/$A_V$ and hydrogen number density, Galactocentric radius, or distance from the Galactic plane, where the latter two were looked into as a way to explore potential dependence on metallicity. Our results suggest that probing the gas-to-dust ratio using $A_V$, where the residuals are also driven by differences in the extinction curve (Figure 9), dilutes any dependence on metallicity, probably leading to the null result. Vuong et al. (2003) also used Galactic X-ray sources to study the ratio, but in nearby dense clouds, where the ratio (using $A_J$), in the $\rho$ Oph cloud in particular, is found to deviate significantly below the Galactic value. They extend this by finding that the difference between Galactic and local values for the ratio are entirely due to metallicity differences. Kahre et al. (2018) find a power-law relationship between the gas-to-dust ratio and gas-phase oxygen abundance measured from nebular emission in five nearby galaxies, along with a change in this relationship if $H_2$ is excluded from the gas tally. This result, based on different galaxies, does not necessarily contradict our result—the lack of a strong correlation between gas-to-dust ratio and metallicity is found within our Galaxy, where the observed metallicity does not correspond to cosmic abundances as it does for global measurements based on $H_2$ regions.

In Section 3.4, we found that the difference in gas-to-dust ratio between the SMC and the MW, using $A_{1500}$ as the dust measure, is smaller than that found using $A_V$ or $E(B-V)$, but still persists. Interestingly, the remaining difference ($\sim 0.7$ dex) matches the difference in the “cosmic” abundances of carbon between the MW and the SMC (Russell & Dopita 1992; Clayton et al. 2000; Toribio San Cipriano et al. 2017), but not silicon (0.3 dex; Russell & Dopita 1992; Clayton et al. 2000). The relationship between differences in gas-to-dust ratio and
5. Conclusions

In this work, we set out to find the most fundamental measure of dust, and consequently the gas-to-dust ratio. The main conclusions of this study are as follows:

1. The strongest correlation between total hydrogen column density $N$(H1 + H2) and extinction $A_\lambda$ for MW sightlines is found at $\lambda \approx 2900$ Å; furthermore, the correlation is linear at that wavelength. $A_{2900}$ is a superior dust measure to $E(B - V)$ and $A_V$.

2. The result that $A_{2900}$ is a more fundamental measure of dust supports a prediction made by Kim & Martin (1996) via their application of the Kramers–Kronig relation to MW dust. This approach also reveals that the wavelength of the extinction value representing the most physically motivated dust measure is grain size and grain composition specific, but generally lies in the UV.

3. We find no strong correlation between residuals from the $N$(H1 + H2) versus $A_{2900}$ relation and metal column densities. However, we show that such an assessment of the drivers of the gas-to-dust ratio is not possible using the metallicities measured in the same sightlines as the reddened stars due to the depletion of metals onto dust grains.

4. The correlations between $N$(H1) and $A_\lambda$ are uniformly weaker than those for $N$(H1 + H2), and are not linear for any $A_\lambda$.

5. The progression of the quality of the fits at longer wavelengths appears to proceed more naturally if we assume an $R_V$-dependent NIR extinction curve slope ($\beta_{NIR}$) than if we assume a fixed slope (i.e., a universal NIR curve).

6. Extinction at UV wavelengths also helps to reduce the gap between the gas-to-dust ratios measured for the SMC and the MW. This difference is 0.9 dex when dust is measured as $A_V$, and 0.7 dex for $A_{1500}$. The latter value agrees with the difference in cosmic abundances of carbon between the SMC and MW, lending support to the idea that $N$(C) is the more fundamental measure for the gas-to-dust ratio than $N$(H).

Our study provides a new perspective regarding the physical underpinnings of the measurements of dust in our galaxy and beyond, and can be used to inform efforts to model extinction curves, grain size evolution, and attenuation in galaxies.

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Figure 10. Estimated “cosmic” carbon column density for the 41 MW sightlines given in Section 3.1 (gray points), overplotted with 4 sightlines to SMC bar sample stars from Gordon et al. (2003, red triangles), vs. extinction at 1500 Å (left), and $E(B - V)$ (right). $N$(C) is estimated by adjusting the original $N$(H) using constant shifts derived from the $12 + \log$(C/H) values for the MW (solar vicinity; Russell & Dopita 1992) or the SMC (Toribio San Cipriano et al. 2017).
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