On the determination of stress fields and displacements in a thin elastoplastic plate containing elastic inclusion - a shim

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Abstract. The paper is devoted to the determination of the stress-strain state of a mechanical structure, which consists of a thin infinite elastoplastic plate with a hole and a continuous fine elastic inclusion. The complexity of this problem lies in the fact that the shape of the boundary between the elastic and plastic zones in the plate is not known in advance. The small parameter method is used as the solution method, while the small parameter characterizes the deviation of the shape of the contour from the circle and the perturbation of external static boundary conditions. As the zero solution, the axisymmetric elastoplastic state of the plate with inclusion is chosen. Two variants of inclusion fixation in a plate are considered: inclusion is enclosed with tension or soldered. As a result of solving the problem within the framework of ideal plasticity, the distribution of the stress and displacement fields and the shape of the elastoplastic boundary are obtained. To illustrate the case of a plane-stressed state, the results of a numerical experiment on the mathematical model obtained are presented.

1. Introduction

Stressed and deformed states in planar elastoplastic problems have been studied in a number of papers by various authors [1–13, 16–18, 20–23]. In this paper within the framework of a plane stress state the distribution of the stress and displacement fields within a thin plate with a hole close in shape to the regular n-gon is determined by the small parameter method. In this case, a slightly larger continuous inclusion with a similarly shaped contour is pressed into the hole with tension or soldered. The solution in the plastic region is determined according to the condition of plasticity of Tresca-Saint-Venant [6].

At infinity the plate is stretched by mutually perpendicular forces with intensities $P_1$ and $P_2$. It is assumed that the loading scheme is such that the inclusion is in the resilient state, and a plastic deformation zone of the material appears within the plate, which completely encloses the contour of the hole in the plate.

The curve of the boundary between the plate and the inclusion is in the form of a regular smooth polygon. The number of smooth angles is denoted by $n$, when $n = 2$ we have the elliptical shape of the hole in the plate and in the outer boundary of the inclusion.

The solution is in cylindrical coordinates $(\rho, \theta, z)$, while the axis $0z$ is perpendicular to the plane of the plate. We select the origin in the center of the hole.
2. Problem solution
We introduce a small parameter characterizing the deviation of the contour of the hole from
the circle, as well as the perturbation of the static boundary conditions. According to what has
been said, the solution of the problem is sought in the form

\[
\sigma_\rho = \sigma^{(0)}_\rho + \delta \sigma^{(1)}_\rho ,
\]

\[
\sigma_\theta = \sigma^{(0)}_\theta + \delta \sigma^{(1)}_\theta ,
\]

\[
\tau_{\rho\theta} = \tau^{(0)}_{\rho\theta} + \delta \tau^{(1)}_{\rho\theta} ,
\]

\[
\sigma_z = 0,
\]

\[
u_\rho = u^{(0)}_\rho + \delta u^{(1)}_\rho ,
\]

\[
u_\theta = u^{(0)}_\theta + \delta u^{(1)}_\theta ,
\]

\[
r_s = 1 + \delta r_s^{(1)},
\]

where the superscript indicates the approximation number, \( \delta \) is a small parameter, \( \sigma_\rho, \sigma_\theta, \sigma_z, \tau_{\rho\theta}, \nu_\rho, \nu_\theta \) are the components of the stress tensor and the displacement vector, \( r_s \) is the radius of the elastoplastic boundary in the plate.

In the plane perpendicular to the axis \( 0z \), according to [14, 19], we write the equation of the
contour that bounds the hole in the plate before deformation

\[
\rho = \alpha (1 + \delta d_1 \cos n\theta - \ldots),
\]

equation of the contour that limits inclusion prior before deformation

\[
\rho = \alpha_1 (1 + \delta d_1 \cos n\theta - \ldots),
\]

where \( \alpha_1 > \alpha \), \( \alpha \), \( \alpha_1 \) — are the radii of the circular contours in the zeroth approximation, respectively: the holes in the plate, the outer outline of the inclusion, \( \delta \) — the small parameter, \( d_1 \) — the dimensionless constants.

In view of the smallness of the quantity \( \varepsilon \), we take the external boundary of the inclusion
[14, 19] beyond the contact line of the plate and the inclusion, which, in decomposition, is
represented in the form

\[
\rho_{\text{kom}} = R^{(0)} + \delta R^{(1)},
\]

where \( R^{(0)} = \alpha_1 \), \( R^{(1)} = \alpha_1 d_1 \cos n\theta \).

The calculation is carried out with values that has been written in a dimensionless form. So
the quantities having the dimensionality of the stresses are referred to \( 2k \) — doubled value of yield
strength for the shear of the plate material. The displacements and geometric characteristics are
related to the radius of the elastoplastic boundary in the plate \( r_s \) in the zero approximation.
To denote dimensionless quantities, we use their previous notation.
3. **Zero approximation**

For the zero approximation, we choose the axisymmetric state of a thin plate with a circular hole of radius $\alpha$, and a thin resilient continuous inclusion with a circular contour of radius $\alpha_1$. At infinity this construction is stretched by mutually perpendicular forces with intensities $P = (P_1 + P_2)/4k$.

We present the results of solving the problem in the zero approximation separately for the plate and the inclusion.

3.1. **The solution in the zero approximation within the plate**

Following [6], for the zero approximation, we have the solution in the resilient region of the plate

\[
\begin{align*}
\sigma^{e(0)}_\rho &= P - \frac{(q + 1)\alpha}{2\rho^2}, \\
\sigma^{e(0)}_\theta &= P + \frac{(q + 1)\alpha}{2\rho^2}, \\
\tau^{e(0)}_{\rho\theta} &= 0,
\end{align*}
\]

\(u^{e(0)}_\rho = \frac{1}{E_1} \left[ (2 - (q + 1)\alpha)\rho + \frac{3(q + 1)\alpha}{\rho} \right],
\]

\(u^{e(0)}_\theta = 0.\) (5)

Within the plastic zone of the plate we have

\[
\begin{align*}
\sigma^{p(0)}_\rho &= 1 - \frac{(q + 1)\alpha}{\rho}, \\
\sigma^{p(0)}_\theta &= 1, \\
\tau^{p(0)}_{\rho\theta} &= 0,
\end{align*}
\]

\(u^{p(0)}_\rho = \frac{1}{2E_1} \left( \rho + (q + 1)\alpha(1 - 2 \ln \rho) \right),
\]

\(u^{p(0)}_\theta = 0,\) (6)

where $E_1$ is the Young’s modulus of the plate material, $q = q/2k$ is the normal pressure at the interface between the plate and the inclusion.

3.2. **The solution in the zero approximation in the inclusion**

In the elastic inclusion, in the zero approximation, the distribution of the stress and displacement field has the form [15]

\[
\begin{align*}
\sigma^{e(0)}_{B\rho} &= -q, \\
\sigma^{e(0)}_{B\theta} &= -q, \\
\tau^{e(0)}_{B\rho\theta} &= 0,
\end{align*}
\]

(7)
where \( E_2 \) is the Young’s modulus of the inclusion material.

3.3. **Determination of the contact and radius of the elastoplastic boundary in the plate in the zero approximation**

From the conditions for the compatibility of deformations of the plate and the inclusion along the contact line, it follows that

\[
\frac{\partial u_{e(0)}}{\partial \rho} + \frac{\partial u_{e(0)}}{\partial \theta} + \varepsilon = \alpha_1 - \alpha
\]

and from the interfacing conditions on the elastoplastic boundary in the plate

\[
\sigma_{\theta}^{(0)} = \sigma_{\theta}^{e(0)}, \quad \rho = 1
\]

we have the following system of equations for determining the contact pressure \( q \) and the radius of the elastoplastic boundary in the plate in the zero approximation \( r_{s0} \)

\[
r_{s0} \left( 1 - 2 \ln \alpha + 2 \ln r_{s0} + \frac{\alpha_1 E_1}{\alpha E_2} \right) + \frac{\alpha k E_2 - \alpha_1 k E_1 - E_1 E_2 (\alpha_1 - \alpha)}{2k E_2 \left( 1 - \frac{P}{2k} \right)} = 0,
\]

\[
q = 2 \alpha \left( 1 - \frac{P}{2k} \right) - 1.
\]

4. **First approximation**

We now turn to the definition of the first approximation.

The boundary conditions at infinity have the form [14, 19]

\[
\sigma_{\rho} = P - \delta d_2 \cos 2\theta,
\]

\[
\tau_{\rho \theta} = \delta d_2 \sin 2\theta,
\]

where \( P = \frac{P_1 + P_2}{4k}, \delta d_2 = \frac{P_1 - P_2}{4k}, \) where \( d_2 \) is a dimensionless constant.

Linearized interfacing conditions on the elastoplastic boundary in the plate can be presented

\[
\left[ \sigma_{ij}^{(1)} + \frac{d\sigma_{ij}^{(0)}}{d\rho} r_{s}^{(1)} \right]_{\rho=1} = 0,
\]

where \( \sigma_{ij}^{(0)}, \sigma_{ij}^{(1)} \) are the components of the stress tensor for the zero and first approximations, \( r_{s}^{(1)} \) is the ratio describing the contour of the contact boundary in the first approximation.

Along the contact line of the plate and the inclusion, according to [6, 15], we write

1) inclusion is enclosed with tension in the plate

\[
\sigma_{\rho}^{(1)} + \frac{d\sigma_{\rho}^{(0)}}{d\rho} R^{(1)} = \sigma_{e(1)}^{(1)} + \frac{d\sigma_{e}^{(0)}}{d\rho} R^{(1)},
\]

\[
\tau_{\rho \theta}^{e(1)} - \left( \sigma_{B\theta}^{e(0)} - \sigma_{B\theta}^{e(0)} \right) s_1 = 0,
\]
In the resilient region, the plates for the first approximation are solved with the boundary conditions (11), we have the solution with the first approximation within the plate and for a resilient inclusion.

\[ u_{\rho}^{(1)} + \frac{d u_{\rho}^{(0)}}{d \rho} R^{(1)} = u_{B_{\rho}}^{(1)} + \frac{d u_{B_{\rho}}^{(0)}}{d \rho} R^{(1)}, \text{ with } \rho = R^{(0)} \]

2) the inclusion is soldered into the plate

\[
\sigma_{\rho}^{(1)} + \frac{d \sigma_{\rho}^{(0)}}{d \rho} R^{(1)} = \sigma_{B_{\rho}}^{(1)} + \frac{d \sigma_{B_{\rho}}^{(0)}}{d \rho} R^{(1)},
\]

\[ \tau_{B_{\rho}}^{(1)} - (\sigma_{B_{\rho}}^{(0)} - \sigma_{B_{\rho}}^{(0)}) s_1 = \tau_{\rho}^{(1)} - (\sigma_{\rho}^{(0)} - \sigma_{\rho}^{(0)}) s_1, \]

\[ u_{\rho}^{(1)} + \frac{d u_{\rho}^{(0)}}{d \rho} R^{(1)} = u_{B_{\rho}}^{(1)} + \frac{d u_{B_{\rho}}^{(0)}}{d \rho} R^{(1)}, \] (14)

\[ u_{\rho}^{(1)} + u_{\rho}^{(0)} s_1 = u_{B_{\rho}}^{(1)} + u_{B_{\rho}}^{(0)} s_1, \text{ when } \rho = R^{(0)}, \]

where \( R^{(1)} = \alpha_1 d_1 \cos n \theta, s_1 = R^{(1)}/R^{(0)} \Rightarrow s_1 = -n d_1 \sin n \theta. \)

We give the solution of the problem in the first approximation for the resilient and plastic zones of the plate and for a resilient inclusion.

4.1. The solution with the first approximation within the plate

According to [6], taking into account the boundary conditions (11), we have the solution with the first approximation in the resilient region the plates for \( \infty > \rho > 1 \)

\[ \sigma_{\rho}^{(1)} = \left[ \left( 1 - \frac{4}{\rho^2} + \frac{3}{\rho^4} \right) d_2 + \left( -\frac{1}{\rho^4} + \frac{2}{\rho^2} \right) a_{21} + \left( \frac{2}{\rho^4} - \frac{2}{\rho^2} \right) a_{22} \right] \cos 2 \theta + \]

\[ + \frac{1}{2} \left[ \left( -\frac{n}{\rho^2} + \frac{n + 2}{\rho^4} \right) a_{n1} + \left( \frac{n + 2}{\rho^{n+2}} - \frac{n + 2}{\rho^n} \right) a_{n2} \right] \cos n \theta, \]

\[ \sigma_{\theta}^{(1)} = \left[ -\left( 1 + \frac{3}{\rho^2} \right) d_2 + \frac{1}{\rho^4} a_{21} - \frac{2}{\rho^2} a_{22} \right] \cos 2 \theta + \]

\[ + \frac{1}{2} \left[ \left( \frac{n}{\rho^{n+2}} - \frac{n - 2}{\rho^n} \right) a_{n1} + \left( \frac{n + 2}{\rho^{n+2}} - \frac{n - 2}{\rho^n} \right) a_{n2} \right] \cos n \theta, \]

\[
\tau_{\rho \theta}^{(1)} = \left[ \left( 1 - \frac{2}{\rho^2} + \frac{3}{\rho^4} \right) d_2 + \left( -\frac{1}{\rho^4} + \frac{1}{\rho^2} \right) a_{21} + \left( \frac{2}{\rho^4} - \frac{1}{\rho^2} \right) a_{22} \right] \sin 2 \theta + \]

\[ + \frac{1}{2} \left[ \left( -\frac{n}{\rho^{n+2}} + \frac{n + 2}{\rho^n} \right) a_{n1} + \left( -\frac{n + 2}{\rho^{n+2}} - \frac{n}{\rho^n} \right) a_{n2} \right] \sin n \theta, \] (15)

\[ u_{\rho}^{(1)} = \frac{\alpha}{E_1} \left[ \left( \frac{3}{2} \rho - \frac{1}{\rho^2} \right) d_2 + \left( \frac{1}{2\rho^3} - \frac{2}{\rho} \right) a_{21} + \left( -\frac{1}{\rho^3} + \frac{2}{\rho} \right) a_{22} \right] \cos 2 \theta + \]

\[ + \frac{\alpha}{E_1} \left[ \left( \frac{3n}{4(n+1)} \rho^{n+1} + \frac{3n + 2}{4(n-1)} \rho^{n-1} \right) a_{n1} + \right. \]

\[ \left. + \left( -\frac{3(n + 2)}{4(n+1)} \rho^{n+1} + \frac{3n + 2}{4(n-1)} \rho^{n-1} \right) a_{n2} \right] \cos n \theta, \]
\[ u^{(1)}_{\theta} = \frac{\alpha}{E_1} \left[ \left( -\frac{3}{2} \left( \rho + \frac{1}{\rho} \right) + \frac{1}{\rho} \right) d_2 + \frac{1}{2} \left( \frac{1}{\rho^3} + \frac{1}{\rho} \right) a_{21} + \left( -\frac{1}{\rho^3} - \frac{1}{2\rho} \right) a_{22} \right] \sin 2\theta + \right.
\[ + \frac{\alpha}{E_1} \left[ \left( \frac{3n}{4(n+1)} \frac{1}{\rho^{n+1}} - \frac{3n-8}{4(n-1)} \frac{1}{\rho^{n-1}} \right) a_{n1} + \left( -\frac{3(n+2)}{4(n-1)} \frac{1}{\rho^{n+1}} + \frac{3n-8}{4(n-1)} \frac{1}{\rho^{n-1}} \right) a_{n2} \right] \sin n\theta, \]

where \( a_{21}, a_{22}, a_{n1}, a_{n2} \) are constants determined from the interfacing conditions (13) or (14) at the contact boundary between the plate and the inclusion.

In the plastic region of the plate, according to [6, 19], we derive for stresses and displacements

\[ \sigma^{p(1)}_{\rho} = \left[ \frac{a_{21} - 2a_{22}}{\rho} + \frac{2a_{22}}{\rho^2} \right] \cos 2\theta + \left[ \frac{a_{n1} + na_{n2}}{\rho} - \frac{na_{n2}}{\rho^2} \right] \cos n\theta, \]

\[ \tau^{p(1)}_{\rho\theta} = \frac{2a_{22}}{\rho^2} \sin 2\theta - \frac{na_{n2}}{\rho^2} \sin n\theta, \]

\[ u^{p(1)}_{\rho} = \left[ \frac{a_{21} - 2a_{22}}{E_1} \ln \rho + \frac{a_{21} - 2a_{22}}{E_1} - \frac{2a_{22}}{\rho E_1} + \right. \]
\[ +4d_2\alpha - a_{21} \left( \frac{3}{2} \alpha + 1 \right) + a_{22} (\alpha + 4) \right] \cos 2\theta + \]
\[ + \left[ \frac{a_{n1} + na_{n2}}{E_1} \ln \rho + \frac{a_{n1} + na_{n2}}{E_1} + \frac{na_{n2}}{\rho E_1} - a_{n1} \left( \frac{4n+1}{2(n^2-1)} \alpha + 1 \right) + \right. \]
\[ +a_{n2} \left( \frac{3n^2+4n-2}{2(n^2-1)} \alpha - 2n \right) \right] \sin n\theta, \]

\[ u^{p(1)}_{\theta} = \left[ \frac{2a_{22}}{E_1} \ln \rho + \frac{5a_{22}}{2E_1} - 2 \left( 4d_2\alpha - a_{21} \left( \frac{3}{2} \alpha + 1 \right) + a_{22} (\alpha + 4) \right) + \right. \]
\[ +\rho \left[ 6d_2\alpha - 2a_{21} (\alpha + 1) + a_{22} \left( \frac{1}{2} \alpha + 7 \right) \right] \sin 2\theta + \]
\[ + \left. \left[ \frac{a_{n2}}{E_1} \ln \rho - \frac{a_{n2}}{E_1} + 2a_{n1} \left( \frac{4n+1}{2(n^2-1)} \alpha + 1 \right) - a_{n2} \left( \frac{3n^2+4n-2}{2(n^2-1)} \alpha - n \right) + \right. \right. \]
\[ +\rho \left. \left( -a_{n1} \left( \frac{3n+3}{2(n^2-1)} \alpha + 1 \right) + 3a_{n2} \left( \frac{n^2-n-3}{2(n^2-1)} \alpha - n \right) \right) \right] \sin n\theta, \]

\[ = 0, \]
4.2. The solution with the first approximation within the inclusion

According to [8], we obtain the form of the components of the stress tensor and the displacement vector in the first approximation for the resilient inclusion

\[
\sigma_{B\rho}^{(1)} = -2C_1 \cos 2\theta + \left( -n(n-1)C_{1n}\rho^{n-2} - (n-2)(n+1)C_{2n}\rho^n \right) \cos n\theta,
\]

\[
\sigma_{B\theta}^{(1)} = 2 \left( C_1 + 6C_2\rho^2 \right) \cos 2\theta + \left( n(n-1)C_{1n}\rho^{n-2} + (n+1)(n+2)C_{2n}\rho^n \right) \cos n\theta,
\]

\[
\tau_{B\rho\theta}^{(1)} = 2 \left( C_1 + 3C_2\rho^2 \right) \sin 2\theta + \left( n(n-1)C_{1n}\rho^{n-2} + n(n+1)C_{2n}\rho^n \right) \sin n\theta,
\]

\[
EU_{B\rho}^{(1)} = 2 \left( -3C_1\rho + 4C_2\rho^3 \right) \cos 2\theta + \left( 3C_1\rho_1^{n-1} - (3n+2)C_{2n}\rho^{n+1} \right) \cos n\theta,
\]

\[
EU_{B\theta}^{(1)} = 2 \left( 3C_1\rho + 5C_2\rho^3 \right) \sin 2\theta + \left( 3C_1\rho_1^{n-1} + (3n+4)C_{2n}\rho^{n+1} \right) \sin n\theta,
\]

where \( C_1, C_2, C_{1n}, C_{2n} \) are constants determined from the ratios on the interface of the plate and the inclusion.

4.3. Systems of equations for unknown constants determination

The ratio (13) at the contact boundary, in the case when the inclusion is inserted with tension has the form

\[
\frac{a_{21}}{a_1} + 2a_{22} \left( \frac{1}{\alpha_1^2} - \frac{1}{\alpha_1^1} \right) + 2C_1 + a_{n_1} \frac{1}{\alpha_1^1} + a_{n_2} \left( \frac{n}{\alpha_1^1} - \frac{n}{\alpha_1^2} \right) + n(n-1)C_{1n}\alpha_1^{n-2} +
\]

\[+(n-2)(n+1)C_{2n}\alpha_1^n = \frac{(q+1)\alpha d_1}{\alpha_1},\]

\[2C_1 + 6C_2\alpha_1^2 + n(n-1)C_{1n}\alpha_1^{n-2} + n(n+1)C_{2n}\alpha_1^n = 0,
\]

\[
\frac{2a_{22}}{\alpha_1^2} + \frac{n_{a_{n_2}}}{\alpha_1^2} = \frac{(q+1)\alpha d_1}{\alpha_1},
\]

\[+a_{n_1} \left( \frac{\alpha_1}{E_1} + \frac{1}{E_1} - \frac{2n+1}{2(n^2-1)\alpha+1} \right) + a_{n_2} \left( \frac{n\ln \alpha_1}{E_1} + \frac{n}{E_1} + \frac{n}{\alpha_1 E_1} + \frac{3n^2+4n-2}{2(n^2-1)\alpha-2n} \right) +
\]

\[+C_1 \frac{6\alpha_1}{E_2} - C_2 \frac{8\alpha_1^3}{E_2} + 3nC_{1n}\alpha_1^{n-1} + (3n+2)C_{2n}\alpha_1^{n+1} =
\]

\[= -4d_2 \alpha - \frac{1}{E_1} (\alpha_1 - 2\alpha d_1) + \frac{q k \alpha \alpha_1 d_1}{E_2}.
\]

For the soldered inclusion case (14), the system for determining unknown constants has the form

\[
\frac{a_{21}}{a_1} + 2a_{22} \left( \frac{1}{\alpha_1^2} - \frac{1}{\alpha_1^1} \right) + 2C_1 + a_{n_1} \frac{1}{\alpha_1^1} + a_{n_2} \left( \frac{n}{\alpha_1^1} - \frac{n}{\alpha_1^2} \right) + n(n-1)C_{1n}\alpha_1^{n-2} +
\]

\[+a_{n_1} \left( \frac{\alpha_1}{E_1} + \frac{1}{E_1} - \frac{2n+1}{2(n^2-1)\alpha+1} \right) + a_{n_2} \left( \frac{n\ln \alpha_1}{E_1} + \frac{n}{E_1} + \frac{n}{\alpha_1 E_1} + \frac{3n^2+4n-2}{2(n^2-1)\alpha-2n} \right) +
\]

\[+C_1 \frac{6\alpha_1}{E_2} - C_2 \frac{8\alpha_1^3}{E_2} + 3nC_{1n}\alpha_1^{n-1} + (3n+2)C_{2n}\alpha_1^{n+1} =
\]

\[= -4d_2 \alpha - \frac{1}{E_1} (\alpha_1 - 2\alpha d_1) + \frac{q k \alpha \alpha_1 d_1}{E_2}.
\]
To determine the type of constants $a_{21}$, $a_{22}$, $a_{n_1}$, $a_{n_2}$, $C_1$, $C_2$, $C_{1n}$, $C_{2n}$, we use the relations at the interface between the plate and the inclusion (18) or (19). Thus, the obtained relations (15) – (19) completely determine the stressed and strained states in the plate and the inclusion for the first approximation.

### 4.4. Finding the radius of the elastoplastic boundary within the plate

The form of the elastoplastic boundary of the first approximation $r^{(1)}_s$ is determined by the linearized condition [6, 14]

$$r^{(1)}_s = -\left[ \left[ \frac{\partial \sigma^{(1)}_s}{\partial \rho} \right]^{-1} \right]_{\rho=1}. \quad (20)$$

Using (15) and (16), expression (20) gives the radius of the elastoplastic boundary in the plate, which in this case has the form

$$r^{(1)}_s = \frac{(4d_2 + a_{21} - 2a_{22})}{(q + 1)\alpha} \cos 2\theta + \frac{a_{n_1} + 2a_{n_2}}{(q + 1)\alpha} \cos n\theta, \quad (21)$$

where $q$ is the contact pressure at the interface between the plate and the connection.
5. Results
Let $\delta = 0.01$, $\alpha = 0.2$ m, $\alpha_1 = 0.201$ m, $E_1 = 810$ MH/m$^2$, $E_2 = 1100$ MH/m$^2$, $d_1 = 2.79$, $d_2 = 2.9$, $k = 12/\sqrt{3}$ MH/m$^2$, $P_1 = 9.5$ MH/m$^2$, $P_2 = 9.0$ MH/m$^2$, $n = 6$.

In figure 1, curve 1 reflects the dependence $r_s$ on the angle $\theta$, i.e. curve 1 is the shape of the elastoplastic boundary in the plate. The contour 2 corresponds to the contour of the hole in the plate.

The condition on the contact boundary for the pattern corresponds to the condition that the inclusion is enclosed with tension into the plate (13).

![Figure 1. Form of elastoplastic boundary](image)

6. Conclusion
In the paper of the stress and displacement has been in a thin elastoplastic plate and in elastic inclusion. The radius of the elastoplastic boundary in the plate is found. An example is considered and given when the inclusion is enclosed with tension in the plate.

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