The effects of Population III stars and variable IMF on the chemical evolution of the Galaxy

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Abstract

We have studied the effects of a hypothetical initial stellar generation (Population III) containing only massive ($M > 10M_\odot$) and very massive stars ($M > 100M_\odot$, Pair-Creation Supernovae) on the chemical evolution of the Milky Way. To this purpose, we have adopted a chemical evolution model - the two-infall model from Chiappini et al. (1997) - which successfully reproduces the main observational features of the Galaxy. Several sets of yields for very massive zero-metallicity stars have been tested: these stars in fact produce quite different amounts of heavy elements, in particular $\alpha$-elements and iron, than lower mass stars. We have focused our attention on the chemical evolution of $\alpha$-elements, carbon, nitrogen and iron. It was found that the effects of Population III stars on the Galactic evolution of these elements is negligible if only one or two generations of such stars occurred, whereas they produce quite different results from the standard models if they continuously formed for a longer period. Also the effects of a more strongly variable IMF were discussed and to this purpose we have made use of suggestions appeared in the literature to explain the lack of metal-poor stars in the Galactic halo with respect to model predictions. In these cases the predicted variations in the abundance ratios, the SN rates and the G-dwarf metallicity distribution are more dramatic and always in contrast with observations, so we have concluded that a constant or slightly varying IMF remains the best solution. Our main conclusion is that if very massive stars ever existed they must have formed only for a very short period of time (until the halo gas reached the suggested threshold metallicity of $10^{-4}Z_\odot$ for the formation of very massive objects); in this case, their effects on the evolution of the elements studied here was negligible also in the early halo phases. In other words, we cannot prove or disprove the existence of such stars on the basis of the available data on very metal poor stars. Because of their large metal production and short lifetimes very massive primordial stars should have enriched the halo gas to the metallicity of the most metal poor stars known ([Fe/H] $\sim$ $-5.4$) and beyond in only a few million years. This fact imposes constraints on the number of Pair-Creation Supernovae: we find that a number from 2 to 20 of such SNe occurred in our Galaxy depending on the assumed stellar yields.

Key words: Galaxies: chemical abundances and evolution - stars: PopIII, primordial
nucleosynthesis

1 Introduction

There are several theoretical and observational reasons for suggesting that the first stars ("Population III" stars, hereafter PopIII), which formed out of gas of primordial chemical composition, were more massive than today. We recall that the Jeans mass, i.e., the minimum mass necessary for a gas cloud to collapse under its self-gravity and give rise to a protostar, scales as $T^{3/2}$, its effective dependence on temperature being steeper if turbulent phenomena are taken into account (Padoan et al., 1997). Metals are the most effective coolants in gas clouds and they allow for the formation of small mass protostars; in their absence, the cooling of the first stars was based on the rotational-vibrational transitions of molecular hydrogen. Hydrodynamical simulations of the collapse and fragmentation of the primordial gas clouds suggest that the very first stars should have had masses larger than $100M_\odot$ according to Bromm et al. (2004), whereas Abel et al. (2002) give a mass range of $30 - 300M_\odot$. This high-mass biased star formation must have lasted until a certain critical metallicity $Z_{\text{PopIII}}$ was reached, at which point star formation switched to a low-mass dominated normal regime (e.g. a Scalo IMF). The latest calculations constrain this value of $Z_{\text{PopIII}}$ in the range $(10^{-6} - 10^{-4}) \cdot Z_\odot$ (Schneider et al., 2002). The IMF may also depend on the ionization state of the gas and on the background of ionizing photons which could inhibit the formation of H$_2$ and require even bigger halos for the creation of stars (see e.g. Haiman et al., 2000).

Being so massive, a PopIII star should start its main sequence with CNO burning, but since these nucleids were not present in the primordial gas, the star is forced to burn hydrogen via the $p-p$ chain, which is much less effective in counterbalancing the effect of gravity; as a result, the star contracts more with respect to a normal PopI/PopII star, until it makes a trace of $^{12}$C by $3-\alpha$ reaction, at which point it burns H via the CNO cycle. Such a more compact star evolves at a higher surface temperature, thus producing a great deal of ionizing photons. The compactness is also caused by the lower opacity due to the absence of metals. Owing to the lack of metals in their envelope, PopIII stars suffer from less mass loss during their lifetime if compared to massive PopII and PopI stars. This gives way, at the end of their lives, to a more probable formation of massive black holes and the so-called Pair-Creation Supernovae (PCSNe), which leave no remnant after their explosion (Rakavy & Shaviv, 1967; Ober et al. 1982).

From an observational point of view, there are several cosmological and astrophysical problems which could be solved if one or more generations of massive metal-free stars existed. Recently, the WMAP experiment, by observing the large-angle polarization of the CMB, has provided constraints on the number of ionizing pho-
tons produced from either the first massive stars or AGNs at $z \geq 15$ (e.g. Cen, 2003). Furthermore, the Lyman-$\alpha$ forest at $z \approx 4 - 5$, as seen in the QSO light looks considerably metal-polluted (Schaye et al. 2003), suggesting an early stellar population which enriched the intergalactic medium via strong winds.

The problems related to galactic chemical evolution which could be solved by the introduction of PopIII are the pre-enrichment and the necessity of primary nitrogen in the early phases of the Galaxy. In fact, no zero metal stars were ever observed in the Galactic halo: the most metal poor star discovered so far (Frebel et al., 2005) has $\text{[Fe/H]} = -5.4 \pm 0.2$. This may indicate that the first generation of stars in our Galaxy, which enriched the gas out of which were born the Population II stars we observe today, were massive and died soon (even though it cannot be excluded that they were rare enough to be inadequately sampled by surveys so far). If this is the case, we could infer important constraints on their nucleosynthetic properties from the chemical abundances of the most metal-poor stars. As to N production, it is usually assumed that in massive stars nitrogen is a secondary element, i.e., it is created from the seed nuclei of carbon during the CNO cycle; this implies that the abundance of nitrogen should decrease considerably towards lower metallicities. This is at variance with the most recent data of Spite et al. (2005) for N abundances in the Galactic halo, which show a flat $\text{[N/Fe]}$ for $\text{[Fe/H]} < -2.5$. In this respect, PopIII stars could be good candidates for primary production of N from massive stars (see also Marigo et al., 2003; Meynet et al, 2003; Heger et al. 2000).

As far as $\alpha$/Fe ratios in very metal poor stars are concerned there is apparently no need to invoke PopIII star nucleosynthesis (e.g. François et al. 2004), but since PopIII star nucleosynthesis has been invoked to explain some peculiar behaviours of Fe-peak elements (Mn, Co, Cr, Ni, Zn) (see Umeda & Nomoto 2002, 2005) it is interesting to check the effect of PopIII stars also on $\alpha$-elements and Fe. In particular, we aim at testing whether, in spite of the different $\alpha$/Fe ratios predicted for PopIII stars relative to normal stars, the inclusion of these very massive objects in chemical evolution models can still be acceptable. In other words, we aim at investigating if the evolution in time of Galactic chemical abundances can impose constraints on the characteristics and duration of the PopIII phase. Thus we have incorporated the nucleosynthetic products from PopIII stars of $\alpha$ elements, C, N and Fe in a detailed chemical evolution model. To our knowledge this is the first test of this kind since the previous ones had only compared the observed abundance ratios in metal poor stars with the stellar production ratios.

In §2 we describe this model; in §3 we discuss the way we have modelled the PopIII phase; in §4 we show the results; in §5 we briefly examine the effects of a variable IMF similar to that proposed by Larson (1998) (namely, “top-heavy” at the earliest times) and in §6 we draw conclusions.
2 The basic model

The model for the chemical evolution of the Galaxy on which we based our investigation is essentially the two-infall model developed by Chiappini et al. (1997), in which the two main components of the Galaxy (halo/thick disk and thin disk) are supposed to be formed by two separate episodes of accretion of extragalactic gas of primordial chemical composition. During the first episode, the halo is formed on a timescale of 1 Gyr; afterwards, the thin disk forms, with a timescale which is proportional to the Galactocentric distance (“inside-out” formation) such that the timescale for the solar neighbourhood is about 7 Gyr (much longer than for the halo).

It is worth noting that the present model does not refer to the hierarchical structure formation scenario, but it makes use of a backward approach, in the sense that we start from the present-time properties and try to derive the initial conditions which will lead to such properties. This approach is not in contradiction with the cosmological approach which starts from precise initial conditions, but is rather complementary. A comparison of the “forward” and “backward” scenarios can be, in fact, very useful. The reason why we do not adopt the forward approach here is that it still lacks sufficient numerical resolution to resolve single galaxies and contains large uncertainties on basic physical processes. However, we stress that computing galaxy evolution at high resolution and in a cosmological context will ultimately be successful. Recently, Sommer-Larsen et al. (2003) performed simulations of galaxy formation in a standard $\Lambda$CDM cosmology, including star formation and feedback effects and found that the gas infall rate onto the solar cylinder is exponentially declining with time, with an infall timescale comparable to that used in this model and obtained with the backward approach. Moreover, the predictive power of models of chemical evolution, regarding abundances, gas fraction, supernova rates etc., is still greater than that of models taking into account the hierarchical clustering paradigm.

The star formation rate is parametrized as follows:

$$\psi(r, t) = \begin{cases} \tilde{\nu} \sigma_{tot}^{k_2}(r, t) \sigma_{g}^{k_1}(r, t) & \text{if } \sigma_{g}(r, t) > \sigma_{thr} \\ 0 & \text{if } \sigma_{g}(r, t) \leq \sigma_{thr} \end{cases}$$

(1)

where $k_1 = 0.5$ and $k_2 = 1.5$; $\tilde{\nu} = 1 \text{ Gyr}^{-1}$ is the star formation efficiency, $\sigma_g$ and $\sigma_{tot}$ are respectively the surface gas density and the total surface mass density (gas + stars + remnants), $\sigma_{thr} = 7 M_\odot \text{pc}^{-2}$ is a threshold gas density below which star formation stops (Kennicutt, 1989).
The initial mass function (IMF) has the following functional form:

\[
\varphi(m) \propto m^{-(1+x)} \quad x = \begin{cases} 
1.35 & \text{if } 0.1 \leq m/M_\odot \leq 0.6 \\
1.7 & \text{if } 0.6 \leq m/M_\odot \leq 80
\end{cases}
\]

which closely resembles the one proposed, on observational grounds, by Scalo (1986), except that it has two slopes instead of three. Stars do not form outside the mass range \(0.1 \leq M/M_\odot \leq 80\).

The stellar yields were adopted as follows (the choices below correspond to model 5 of Chiappini et al. 2003a, CRM03):

- **Low and intermediate mass stars** \((0.8 \leq M/M_\odot \leq 8.0)\): yields are taken from Van den Hoek & Groenewegen (1997) as a function of initial metallicity. These stars produce He, \(^{12}\)C, N and heavy \(s\)-elements.
- **Massive stars** \((M \geq 10M_\odot)\): these stars give rise to Type II SNe. Their chemical yields are those from Woosley & Weaver (1995) for solar metallicity, extrapolated to the upper limit of the IMF\(^1\); they contribute to the enrichment in \(\alpha\)-elements, some Fe, \(r\)-elements and light \(s\)-elements.
- **Type Ia supernovae**: bearing in mind the W7 (single degenerate) model from Nomoto et al. (1984), yields are taken from Iwamoto et al. (1999) which is an updated version of model W7. These supernovae produce most Fe and traces of light elements.
- **Novae**: nucleosynthesis from nova outbursts is included (José & Hernanz, 1998); novae can contribute to the production of CNO isotopes and Li (Romano & Matteucci, 2003).

This model reproduces well the observational constraints for the solar neighborhood, if we exclude N, overproduced by the yields of Van den Hoek and Groenewegen (1997) for low and intermediate mass stars, and Mg, underproduced by the yields of Woosley and Weaver (1995) for massive stars. The model also reproduces well the chemical evolution of abundance ratios as a function of \([Fe/H]\) for all elements, with the exception of nitrogen (see François et al. 2004); in fact, since the production of nitrogen from massive stars is assumed to be secondary, that model predicts a steep decrease of \([N/Fe]\) vs. \([Fe/H]\) towards lower values of \([Fe/H]\), at

\(^1\) Woosley & Weaver (1995) originally calculated stellar yields for masses in the range \(12 - 40M_\odot\) and found that above this mass a considerable reimplosion may occur with severely reduced yields and likely black hole formation. However, it was shown by François et al. (2004) that if black hole formation occurs in this mass range it is impossible to fit the data at low metallicity; therefore, since very few calculations are available in the mass range \(40 - 100M_\odot\) one has to perform an extrapolation. The models of Woosley & Weaver (1995) do not include mass loss, which is an acceptable assumption for Pop III stars but could be less realistic for later generations.
variance with the new data from Spite et al. (2005) which point towards a roughly constant and solar value for [N/Fe] at very low iron abundances.

3 Modelling Population III

Modelling the PopIII phase requires the choice of a critical metallicity $Z_{\text{PopIII}}$, below which the IMF and the stellar yields are characteristic of metal-free stars. To do that, we need a functional form and a mass range for the IMF and the adoption of zero metallicity stellar yields.

Concerning the first point, we ran models with $Z_{\text{PopIII}}$ in the range $10^{-6} Z_\odot - 0.1 Z_\odot$ (the upper limit of this range is not realistic and has a merely investigative meaning). However, we are showing the results only for $Z_{\text{PopIII}} \geq 10^{-4} Z_\odot$, since we saw no differences in the models with $Z_{\text{PopIII}}$ equal to $10^{-6}$, $10^{-5}$ and $10^{-4} Z_\odot$. This can be understood if we bear in mind that it takes only a few supernovae to enrich the interstellar medium significantly, and as a matter of fact all of these critical metallicities correspond to a unique generation of PopIII stars.

We also ran models with a variety of mass ranges and IMFs for the PopIII phase, but hereafter we will present only models where low and intermediate mass stars were not included in the primordial IMF. This primordial IMF has the same form as in Eq. 2, except that, the turning point being outside the considered mass range, there is a unique slope. In other words:

$$\varphi(m) \propto m^{-(1+x)}, \quad M_L \leq M \leq M_U$$

where $x = 1.7$, $M_L = 10 M_\odot$ and $M_U$ depends on the considered nucleosynthesis, as we shall see next. When $Z_{\text{PopIII}}$ is reached, the IMF switches to the one shown in Eq. 2.

3.1 Nucleosynthesis

The chemical evolution and nucleosynthesis of zero-metal massive stars have been computed since the early eighties (Ober et al., 1983, El Eid et al., 1983) and the work in this field has continued until very recently (Woosley & Weaver, 1995; Heger & Woosley, 2002; Umeda & Nomoto, 2002; Heger et al., 2003; Chieffi & Limongi, 2002 and 2004). Also yields for low (D’Antona, 1982) and intermediate (e.g. Siess et al., 2002) mass stars are available, but we are not using them right now since we are only considering massive stars for the primordial IMF. We supposed that zero metallicity yields hold until $Z_{\text{PopIII}}$ is reached, then the nucleosynthe-
sis becomes the standard one. For our purpose, we have considered four sets of nucleosynthetic yields:

- Ober et al., 1983: these authors computed the yields for zero metallicity stars with main sequence masses in the range $80 - 500M_\odot$. They predicted that these objects should produce mainly oxygen and strongly underproduce nitrogen and Fe-peak nuclei, supposedly because they were lacking the appropriate reaction network in their code to produce iron. They also predicted a substantial mass loss during the main sequence phase, in contrast with all the other considered sets of yields. These calculations are old and likely to have been superceded by the most recent ones; moreover, the lack of iron in their yields leads to a severe overestimation of the [el/Fe] ratios at low values of [Fe/H] with respect to observations for $\alpha$-elements and C (see Fig. 1), even for the lowest values of $Z_{\text{PopIII}}$.

- Woosley & Weaver, 1995 + Heger & Woosley, 2002 (hereafter HW): we have combined two sets of yields, the first one (Woosley & Weaver, 1995, their case A for $M = 12 - 22M_\odot$, case B for $M = 25 - 30M_\odot$, case C for $M = 35 - 40M_\odot$, in order to achieve the maximum enrichment) for stars with main sequence masses in the range $12 - 40M_\odot$, the second one (Heger & Woosley, 2002) for stars with He-core masses in the range $64 - 133M_\odot$, corresponding to main sequence masses of $140 - 260M_\odot$, which explode as PCSNe leaving no remnant (thus, referring to Eq. 3, $M_U = 260M_\odot$). For He-core masses larger than $133M_\odot$, these authors found that a black hole is formed and no nucleosynthesis is produced. They also found that the same situation is likely to occur in zero-metal stars with main sequence masses in the range $40 - 140M_\odot$; however, we also computed models where black hole formation is not considered in this mass range, performing
linear interpolation between 40 and 140$M_\odot$ instead. Pre-SN evolution is calculated at constant mass, so mass loss yields are not taken into account.

- **Umeda & Nomoto, 2002** (hereafter UN): these authors calculated the evolution and nucleosynthesis of PopIII stars for the main sequence mass ranges 13–30$M_\odot$ and 150–270$M_\odot$; the latter includes PCSN progenitors. They compared the abundance ratios of halo stars with the production ratios in PCSNe and concluded that they are not in agreement; however, since this result is based on a simple approach, we did not take it for granted and computed both models where PCSNe are considered ($M_U = 270M_\odot$), and models where they are not ($M_U = 80M_\odot$). Referring to their paper, we adopted the yields for $E_{\gamma 1} = 1$. The authors suppose that black holes form in the mass range 30–150$M_\odot$, but do not rule out a supernova explosion, releasing some small quantity of elements, accompanying black hole formation. Given these uncertainties, we still computed models both with and without black hole formation. In this case as well, mass loss is not taken into account. We notice that the UN yields, contrarily to HW yields, are rather sparse in mass and therefore their incorporation in chemical evolution studies might lead to larger uncertainties.

- **Chieffi & Limongi, 2002-2004** (hereafter CL): this set was also obtained by combining the results of two papers, the first one (Chieffi & Limongi, 2004) for masses in the range 10–35$M_\odot$ and the second one (Chieffi & Limongi, 2002) for stars with masses equal to 50 and 80$M_\odot$. Since these are the only yields computed by these authors, we restricted the mass range to 10–80$M_\odot$, so that no PCSN scenario occurs; in addition, though not explicitly stated by the authors, neither black hole formation nor mass loss are considered. These yields are peculiar in the fact that they adopt a particularly low rate for the reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, which leads to an overproduction of C with respect to other yields. The authors made this choice as a result of a comparison with the chemical composition of an “average” star, representing the halo stars with [Fe/H] ≤ −3.3 (under the hypothesis that they are enriched by a few supernova explosions from the first stars) and bearing in mind that the cross section for that reaction is very poorly constrained. In order to obtain the maximum enrichment, we chose the yields calculated with the lowest mass-cut.

Table 1 summarizes the features of the models we are presenting and discussing here. In Column 1 models are identified as follows: the first letter refers to the adopted yields, the number is associated with $Z_{\text{PopIII}}$ (e.g. “1” stands for $Z_{\text{PopIII}} = 10^{-1}Z_\odot$), the following letter states the mass range if more than one mass range is considered for a given nucleosynthesis (“P” means that PCSNe are included, “M” that they are not); a final “b” means that black hole formation is assumed in a certain mass range which is specified in Column 4. In Column 2 we show the yield source for PopIII stars; in Column 3 we specify the assumed mass range for PopIII stars;

\[2\] Again, linear interpolation may not be the best choice, but due to the lack of available yields in this mass range it would otherwise be impossible to investigate what would be the effect of a non-negligible chemical enrichment from these stars.
finally, in Column 5 we show the adopted threshold metallicity.

4 Results

4.1 Models with PCSNe

We first present the results of all those models which include PCSNe in their primordial IMF; these are all the models with yields of HW (from H1 to H4b in Table 1), for which \( M_U = 260 M_\odot \), and half of the models with yields of UN (from U1P to U4Pb in Table 1) for which \( M_U = 270 M_\odot \).

In Table 2 are shown the solar abundances by mass (i.e., abundances predicted for the gas 4.5 Gyr ago), compared to the same results from our standard model (similar to CRM03 - see the tables for details).

These tables include the duration of the PopIII phase and the number of supernova explosions which occurred before \( Z_{\text{PopIII}} \) was reached in the ISM. Several quantities, such as the star formation rate, the surface gas and star density, the SN rate, etc., calculated in the solar neighbourhood at the present time (i.e., the age of the Galaxy \( t_G = 13.7 \) Gyr) do not suffer from variations with respect to our basic model for any adopted nucleosynthesis, therefore we will not show the results concerning them. This fact is plausible since, as shown in the tables, the PopIII phase is always extremely short (a few Myr) even for the highest critical metallicities and, moreover, most hypotheses which determine these values are left unchanged in our models with respect to CRM03 model.

Our calculations of both the time duration of the PopIII phase and the number of PCSNe which exploded during this phase are a first-approximation estimate and may not be very accurate, since the modeling of the first phases of Galactic collapse may be oversimplified, especially if we consider our adoption of the instantaneous mixing approximation. However, the time duration of the PopIII phase and the corresponding number of PCSNe cannot be much longer than this since it was shown (Oey, 2003) that even if the early evolution took place inhomogeneously (which still has to be verified via measurements of chemical abundances at very low metallicities), it rapidly became essentially homogeneous. In any case, our estimates can be regarded as order-of-magnitude estimates.

One can immediately notice that substantial variations in the predicted solar abundances by mass occur only for Fe, Mg and Si, and in any case the results are very similar (within a factor of 2) to those of CRM03 model. These variations are practically the same, within a given nucleosynthetic set, for every value of the critical metallicity. Focusing on these elements, an improvement of the agreement with ob-
| Model name | Yields | Mass range ($M_\odot$) | BH range ($M_\odot$) | $Z_{\text{PopIII}}$ ($Z_\odot$) |
|------------|--------|------------------------|----------------------|-------------------------------|
| H1         | HW     | 10 – 270               | none                 | 0.1                           |
| H2         | HW     | 10 – 270               | none                 | $10^{-2}$                     |
| H3         | HW     | 10 – 270               | none                 | $10^{-3}$                     |
| H4         | HW     | 10 – 270               | none                 | $10^{-4}$                     |
| H1b        | HW     | 10 – 270               | 40 – 140             | 0.1                           |
| H2b        | HW     | 10 – 270               | 40 – 140             | $10^{-2}$                     |
| H3b        | HW     | 10 – 270               | 40 – 140             | $10^{-3}$                     |
| H4b        | HW     | 10 – 270               | 40 – 140             | $10^{-4}$                     |
| U1P        | UN     | 10 – 270               | none                 | 0.1                           |
| U2P        | UN     | 10 – 270               | none                 | $10^{-2}$                     |
| U3P        | UN     | 10 – 270               | none                 | $10^{-3}$                     |
| U4P        | UN     | 10 – 270               | none                 | $10^{-4}$                     |
| U1Pb       | UN     | 10 – 270               | 30 – 150             | 0.1                           |
| U2Pb       | UN     | 10 – 270               | 30 – 150             | $10^{-2}$                     |
| U3Pb       | UN     | 10 – 270               | 30 – 150             | $10^{-3}$                     |
| U4Pb       | UN     | 10 – 270               | 30 – 150             | $10^{-4}$                     |
| U1M        | UN     | 10 – 80                | none                 | 0.1                           |
| U2M        | UN     | 10 – 80                | none                 | $10^{-2}$                     |
| U3M        | UN     | 10 – 80                | none                 | $10^{-3}$                     |
| U4M        | UN     | 10 – 80                | none                 | $10^{-4}$                     |
| U1Mb       | UN     | 10 – 80                | 30 – 150             | 0.1                           |
| U2Mb       | UN     | 10 – 80                | 30 – 150             | $10^{-2}$                     |
| U3Mb       | UN     | 10 – 80                | 30 – 150             | $10^{-3}$                     |
| U4Mb       | UN     | 10 – 80                | 30 – 150             | $10^{-4}$                     |
| C1         | CL     | 10 – 80                | none                 | 0.1                           |
| C2         | CL     | 10 – 80                | none                 | $10^{-2}$                     |
| C3         | CL     | 10 – 80                | none                 | $10^{-3}$                     |
| C4         | CL     | 10 – 80                | none                 | $10^{-4}$                     |

Table 1
Definition of PopIII models: the first column specifies the model names; the second shows the adopted nucleosynthesis; the third and fourth columns show respectively the considered mass range and the black hole formation range; the last column specifies the critical metallicity marking the end of PopIII phase.
Table 2
Solar abundances by mass predicted by PopIII models with PCSNe and yields from Heger & Woosley (2002) and Woosley & Weaver (1995); data for solar abundances are from Asplund et al. (2004). $\Delta t_{\text{PopIII}}$ is the time duration of the PopIII phase (from the beginning of star formation) and SN# is the estimated number of supernovae which exploded during such phase.

| $X_i$ | Standard | H1 | H2 | H3 | H4 | Obs. |
|-------|----------|----|----|----|----|------|
| C     | 1.6E−3   | 1.6E−3 | 1.7E−3 | 1.7E−3 | 1.7E−3 | 2.1E−3 |
| N     | 9.6E−4   | 9.6E−4 | 9.6E−4 | 9.6E−4 | 9.6E−4 | 5.9E−4 |
| O     | 5.3E−3   | 5.3E−3 | 5.3E−3 | 5.3E−3 | 5.3E−3 | 5.1E−3 |
| Mg    | 2.5E−4   | 2.3E−4 | 2.3E−4 | 2.3E−4 | 2.3E−4 | 5.7E−4 |
| Si    | 8.3E−4   | 5.3E−4 | 5.3E−4 | 5.3E−4 | 5.3E−4 | 6.3E−4 |
| Fe    | 1.1E−3   | 1.1E−3 | 1.1E−3 | 1.1E−3 | 1.1E−3 | 1.1E−3 |
| $Z_{\odot}$ | 1.2E−2 | 1.2E−2 | 1.2E−2 | 1.2E−2 | 1.2E−2 | 1.2E−2 |

| $\Delta t_{\text{PopIII}}$ (Myr) | 10 | 4 | 3 | 2 | - |
| SN# | 2500 | 80 | 8 | 3 | - |

| $X_i$ | Standard | H1b | H2b | H3b | H4b | Obs. |
|-------|----------|-----|-----|-----|-----|------|
| C     | 1.6E−3   | 1.7E−3 | 1.7E−3 | 1.7E−3 | 1.7E−3 | 2.1E−3 |
| N     | 9.6E−4   | 9.6E−4 | 9.6E−4 | 9.6E−4 | 9.6E−4 | 5.9E−4 |
| O     | 5.3E−3   | 5.3E−3 | 5.3E−3 | 5.3E−3 | 5.3E−3 | 5.1E−3 |
| Mg    | 2.5E−4   | 1.7E−4 | 1.7E−4 | 1.7E−4 | 1.7E−4 | 5.7E−4 |
| Si    | 8.3E−4   | 4.9E−4 | 4.9E−4 | 4.9E−4 | 4.9E−4 | 6.3E−4 |
| Fe    | 1.1E−3   | 1.1E−3 | 1.1E−3 | 1.1E−3 | 1.1E−3 | 1.1E−3 |
| $Z_{\odot}$ | 1.2E−2 | 1.1E−2 | 1.1E−2 | 1.1E−2 | 1.1E−2 | 1.2E−2 |

| $\Delta t_{\text{PopIII}}$ (Myr) | 17 | 5 | 3 | 2 | - |
| SN# | 5500 | 200 | 15 | 2 | - |

Observations would be achieved if, with respect to our fiducial model, Mg were overproduced by a factor of about two and Si were slightly underproduced, while for Fe there is no need for variations. In general, models with the yields of HW tend to overproduce Fe and underproduce Mg and Si, the latter effect being enhanced when black hole formation is taken into account; however, the iron overproduction may result from our adoption of those yields which led to the highest enrichment. The PCSN yields of Fe are strongly dependent on the mass cut and a different choice of this parameter could reduce the Fe yield from this population. Instead, when we use the yields of UN, Fe and Si are underproduced, and Mg is overproduced.
with respect to the model without PopIII, if black holes do not form; if they form, all of the three elements are underproduced. Therefore, none of our PopIII models with PCSNe gives a better agreement with observations if compared to our basic standard model. Note that for those elements which are mainly produced by low and intermediate mass stars, which contribute to the chemical enrichment only in the latest stages of Galactic evolution, no variation is seen in the predicted solar abundances with respect to our fiducial model.

Fig. 2 shows the evolution with \([\text{Fe}/\text{H}]\) of the abundance ratios \([\text{X}/\text{Fe}]\) for \(\text{X}\) corresponding to two \(\alpha\)-elements (O and Mg) and to C, for the lowest and the highest value of \(Z_{\text{PopIII}}\). The evolutionary behaviour of N with \([\text{Fe}/\text{H}]\) will be discussed in a separate subsection. For the lowest values of \(Z_{\text{PopIII}}\) (which correspond to very short PopIII phases) the variations of the calculated trends in models with PopIII

\begin{table}[h]
\centering
\begin{tabular}{lcccccc}
\hline
\(X_i\) & Standard & U1P & U2P & U3P & U4P & Obs. \\
\hline
C & 1.6E−3 & 1.6E−3 & 1.7E−3 & 1.7E−3 & 1.7E−3 & 2.1E−3 \\
N & 9.6E−4 & 9.6E−4 & 9.6E−4 & 9.6E−4 & 9.6E−4 & 5.9E−4 \\
O & 5.3E−3 & 5.3E−3 & 5.3E−3 & 5.3E−3 & 5.3E−3 & 5.1E−3 \\
Mg & 2.5E−4 & 3.8E−4 & 3.8E−4 & 3.8E−4 & 3.8E−4 & 5.7E−4 \\
Si & 8.3E−4 & 8.2E−4 & 8.1E−4 & 8.1E−4 & 8.1E−4 & 6.3E−4 \\
Fe & 1.1E−3 & 9.3E−4 & 9.3E−4 & 9.3E−4 & 9.3E−4 & 1.1E−3 \\
\(Z_{\odot}\) & 1.2E−2 & 1.2E−2 & 1.2E−2 & 1.2E−2 & 1.2E−2 & 1.2E−2 \\
\hline
\Delta t_{\text{PopIII}} (\text{Myr}) & - & 10 & 4 & 3 & 2 & - \\
\text{SN#} & - & 2000 & 80 & 8 & 3 & - \\
\hline
\(X_i\) & Standard & U1Pb & U2Pb & U3Pb & U4Pb & Obs. \\
\hline
C & 1.6E−3 & 1.7E−3 & 1.7E−3 & 1.7E−3 & 1.7E−3 & 2.1E−3 \\
N & 9.6E−4 & 9.6E−4 & 9.6E−4 & 9.6E−4 & 9.6E−4 & 5.9E−4 \\
O & 5.3E−3 & 5.3E−3 & 5.3E−3 & 5.3E−3 & 5.3E−3 & 5.1E−3 \\
Mg & 2.5E−4 & 2.1E−4 & 2.1E−4 & 2.1E−4 & 2.1E−4 & 5.7E−4 \\
Si & 8.3E−4 & 5.7E−4 & 5.6E−4 & 5.6E−4 & 5.6E−4 & 6.3E−4 \\
Fe & 1.1E−3 & 9.0E−4 & 9.0E−4 & 9.0E−4 & 9.0E−4 & 1.1E−3 \\
\(Z_{\odot}\) & 1.2E−2 & 1.1E−2 & 1.1E−2 & 1.1E−2 & 1.1E−2 & 1.2E−2 \\
\hline
\Delta t_{\text{PopIII}} (\text{Myr}) & - & 19 & 6 & 3 & 2 & - \\
\text{SN#} & - & 5000 & 250 & 7 & 2 & - \\
\hline
\end{tabular}
\end{table}
relative to our basic model are only observable at the lowest values of $[\text{Fe}/\text{H}]$, as we would expect.

If we arise the value of $Z_{\text{PopIII}}$, obviously major variations occur. For a given element, some models can give a better result than the CRM03 model: for example, if we consider oxygen, models H1b and U1Pb seem to fit the observations better. However, the same models give results at variance with observations for the other $\alpha$-elements. Moreover, as recently shown by François et al. (2004) a better fit for oxygen can be obtained by adopting the oxygen yields of Woosley and Weaver (1995) as a function of metallicity. The same model also improves the agreement between calculations and observations for Mg at low $[\text{Fe}/\text{H}]$. On the contrary, PopIII models seem to better represent the Mg data at low $[\text{Fe}/\text{H}]$, but fail in reproducing them at higher $[\text{Fe}/\text{H}]$.

In conclusion, if the primordial PopIII phase was very short, as seems likely given the low suggested threshold metallicity, the results do not allow us to accept or refute models with PopIII and PCSNe. If instead it lasted longer, the overall trends are better reproduced without invoking PopIII.

4.2 Models without PCSNe

Here we examine what happens in these PopIII models which do not involve the formation of PCSN progenitors. These models have an IMF whose upper limit is given by $M_U = 80 M_\odot$ and include all the models with yields of CL (from C1 to C4 in Table 1) and half the models with yields of UN (from U1M to U4Mb in Table 1). In Table 4 are again shown the solar abundances by mass predicted by the various models compared to the outcome of our fiducial model. The last two rows show the duration of PopIII phase and the number of supernova explosions occurred during the PopIII phase (where present). As expected, also in this case, given the short duration of the PopIII phase, all variations are minimal. Since no change is seen in those quantities calculated at the present time with respect to our fiducial model, these results are not presented here.

As to the solar abundances by mass, the same considerations we made in the previous subsection hold, i.e. we see remarkable differences only for Fe, Mg and Si. Models with yields of UN underproduce Fe and Si, and overproduce Mg with respect to the fiducial model, if black hole formation is not taken into account; when the latter is assumed, all of the three elements are underproduced relative to the standard model. If CL yields are adopted, Mg is overproduced and both Fe and Si are underproduced. Again, none of these models gives a globally better or worse agreement with data. We stress that even if the carbon yields of CL were particularly high, the solar abundance of carbon is dominated by the production by low and intermediate mass stars which occurs in the era following PopIII; so, no variation
Fig. 2. Evolution of $[\text{O/Fe}]$, $[\text{Mg/Fe}]$ and $[\text{C/Fe}]$ with $[\text{Fe/H}]$ in the model of CRM03 (solid line) and in the models with PopIII and PCSNe included in the IMF, for the lowest and highest considered values of the critical metallicity. Data are from Chiappini et al. (1999, and references therein) for the disk phase (open squares); for the halo phase (filled circles) data are from Cayrel et al. (2004) for O and Mg and from Spite et al. (2005) for C.
Table 4
Solar abundances by mass predicted by models with PopIII which do not include PCSNe in their IMF, with yields of Umeda & Nomoto (2002), matched with the results of our fiducial model and the observations. Data for solar abundances are from Asplund et al. (2004). \( \Delta t_{\text{PopIII}} \) is the time duration of PopIII phase (from the beginning of star formation) and SN# is the approximate number of supernovae exploded during that phase.

| \( X_i \) | Standard | U1M | U2M | U3M | U4M | Obs. |
|-----------|----------|-----|-----|-----|-----|------|
| C         | 1.6E-3   | 1.6E-3 | 1.7E-3 | 1.7E-3 | 1.7E-3 | 2.1E-3 |
| N         | 9.6E-4   | 9.6E-4 | 9.6E-4 | 9.6E-4 | 9.6E-4 | 5.9E-4 |
| O         | 5.3E-3   | 5.3E-3 | 5.3E-3 | 5.3E-3 | 5.3E-3 | 5.1E-3 |
| Mg        | 2.5E-4   | 3.8E-4 | 3.8E-4 | 3.8E-4 | 3.8E-4 | 5.7E-4 |
| Si        | 8.3E-4   | 8.2E-4 | 8.1E-4 | 8.1E-4 | 8.1E-4 | 6.3E-4 |
| Fe        | 1.1E-3   | 9.3E-4 | 9.3E-4 | 9.3E-4 | 9.3E-4 | 1.1E-3 |
| Z_{\odot} | 1.2E-2   | 1.2E-2 | 1.2E-2 | 1.2E-2 | 1.2E-2 | 1.2E-2 |
| \( \Delta t_{\text{PopIII}} \) (Myr) | - | 13 | 4 | 3 | 2 | - |
| SN#       | - | 4000 | 150 | 13 | 2 | - |

| \( X_i \) | Standard | U1Mb | U2Mb | U3Mb | U4Mb | Obs. |
|-----------|----------|------|------|------|------|------|
| C         | 1.6E-3   | 1.7E-3 | 1.7E-3 | 1.7E-3 | 1.7E-3 | 2.1E-3 |
| N         | 9.6E-4   | 9.6E-4 | 9.6E-4 | 9.6E-4 | 9.6E-4 | 5.9E-4 |
| O         | 5.3E-3   | 5.4E-3 | 5.3E-3 | 5.3E-3 | 5.3E-3 | 5.1E-3 |
| Mg        | 2.5E-4   | 2.1E-4 | 2.1E-4 | 2.1E-4 | 2.1E-4 | 5.7E-4 |
| Si        | 8.3E-4   | 5.7E-4 | 5.6E-4 | 5.6E-4 | 5.6E-4 | 6.3E-4 |
| Fe        | 1.1E-3   | 9.0E-4 | 9.0E-4 | 9.0E-4 | 9.0E-4 | 1.1E-3 |
| Z_{\odot} | 1.2E-2   | 1.1E-2 | 1.1E-2 | 1.1E-2 | 1.1E-2 | 1.2E-2 |
| \( \Delta t_{\text{PopIII}} \) (Myr) | - | 25 | 8 | 6 | 5 | - |
| SN#       | - | 10000 | 400 | 90 | 20 | - |

is seen in the results for carbon. We can notice that in the models U1Mb–U4Mb, since a large fraction of the forming stars goes into black holes, the metal enrichment is particularly slow and, for a given critical metallicity, the time duration of PopIII phase is about twice the value for other models; such time duration remains very short anyway.

Fig. 3 shows the evolution with \([\text{Fe}/\text{H}]\), for the lowest and the highest values of \( Z_{\text{PopIII}} \) considered, of the same abundance ratios considered in the previous subsection, i.e., the trend of \([X/\text{Fe}]\) vs. \([\text{Fe}/\text{H}]\) for O, Mg, and C (we shall discuss N in
the following subsection). Here as well, the longer the PopIII phase lasts, the more
dramatic its effect is, so that none of these models is able to reproduce the observations
for these elements if we adopt $Z_{\text{PopIII}} = 0.1Z_\odot$, with the exception of model
U1Mb, which however fails to reproduce data for Mg in the whole range of [Fe/H].
For the lowest values of the critical metallicity instead, in most cases only slight
variations are seen with respect to our fiducial model. A remarkable exception is
represented by carbon with the yields of CL. The calculated trend shows an exces-
sively steep increase of the [C/Fe] ratio towards the lowest values of [Fe/H], which
is results from both the low Fe yields from massive stars and the low $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$
reaction rate assumed by these authors.

In general, excluding the PCSNe from the IMF does not lead to a better agreement
with observations: the results are still better for a model which does not include
PopIII stars. It is worth noting that these models predict a minimum [Fe/H]$\approx -6.5$
dex in the halo gas. On the other hand, in the models with PCSNe the predicted
minimum [Fe/H] $\sim -4.0$ dex, even higher than the most metal poor star known
(Frebel et al. 2005). In the models of Figure 3 we can see that for [Fe/H]$< -4.0$
dex remarkable differences among the models are seen also for those cases with
$Z_{\text{PopIII}} = 10^{-4}Z_\odot$. Forthcoming surveys at very low metallicities may thus in prin-
ciple help in putting constraints on the existence of PopIII in the earliest phases of
the Galaxy. However, down to [Fe/H] $\approx -4$ the relatively small spread of available
data (e.g. Cayrel et al., 2004; Spite et al., 2005) seems to support our hypothesis of
instantaneous mixing, while for smaller [Fe/H] there are no available measurements
to prove or disprove such an assumption. Again, low-metallicity surveys could de-
cide whether chemical homogeneities were important or not in the early Galaxy.
Fig. 3. Evolution of [O/Fe], [Mg/Fe] and [C/Fe] with [Fe/H] in our fiducial model (solid line) and in the models with PopIII and with IMF truncated at 80$M_\odot$, thus not including PCSNe included in the IMF, for the lowest and highest considered values of the critical metallicity. Data are from Chiappini et al. (1999, and references therein) for the disk phase (open squares), and for the halo phase (filled circles) from Cayrel et al. (2004) for O and Mg and from Spite et al. (2005) for C. The plots were extended down to [Fe/H] = −6.5 in order to show the differences which arise at the lowest values of [Fe/H] and which could, in principle, allow one to discriminate among the different models once future data are available. Please note that the flat behaviour of the standard model of Chiappini et al. (2003) in the [Fe/H] range between −4.0 and −6.5 is due to the extrapolation of the yields of Woosley & Weaver (1995) to masses up to 80 $M_\odot$, as described in François et al. (2004).
4.3 The nitrogen problem

The main problem with standard models of chemical evolution of the Galaxy resides in the trend of the relative abundance of nitrogen with metallicity. These models usually assume that the N production by massive stars is a secondary process, i.e., N is created starting from seed nuclei of C already present in the gas out of which the star was born. Also low and intermediate mass stars are supposed to produce N in a secondary fashion but some primary N can be produced in intermediate mass stars during the dredge-up episodes in conjunction with hot-bottom burning (Renzini & Voli, 1981). This implies that the abundance of nitrogen should increase with metallicity in the earliest evolutionary phases, and that is what most evolutionary models of the Galaxy predict. Given the lack of data for N (and the large spread in those available) until a few years ago, this trend seemed plausible.
Fig. 5. Evolution with [Fe/H] of the [N/Fe] ratio in our fiducial model (solid line), in the PopIII model with yields of CL and $Z_{\text{PopIII}} = 10^{-4}Z_\odot$ (dotted line) and in model Matt86 (dashed line) where a constant and primary production of $^{14}\text{N}$ is imposed at all metallicities for stars of all masses. Data are from Chiappini et al. (1999, open squares), Spite et al. (2005, filled circles) and Israelian et al. (2004, filled triangles).

However, very recently, high-quality measurements of nitrogen and carbon abundances appeared (Spite et al., 2005; Israeli et al. 2004). These new data indicate that the [N/Fe] ratio is constant and about solar over the whole range of [Fe/H], at variance with the standard model predictions. One solution to this problem could be provided by a primary production of N in PopIII stars. We therefore investigate what is the effect of PopIII on the behaviour of nitrogen. In Fig. 4 is shown the evolution of the [N/Fe] ratio with [Fe/H] with and without including PCSN progenitors in the IMF. One can immediately see that none of the models is able to reproduce the observed constant trend for all values of [Fe/H], with the exception of model C1 which, however, yields very unrealistic results for all the other elements and must therefore be excluded. In general, all models with PopIII stars without rotation predict a trend which is somehow approaching the data at the lowest values of [Fe/H], but if we exclude models with yields of CL, for none of them we can claim a good agreement. If we adopt a higher value for $Z_{\text{PopIII}}$, in most cases the trend flattens well below the data. This fact shows that PopIII stars alone cannot provide a sufficient primary production of N. For some models (e.g. U1P and U1Pb) the plot of [N/Fe] vs. [Fe/H] decreases even more steeply than the CRM03 model towards low [Fe/H].

We now examine more closely the models with yields of CL. The better agreement shown by these models for the [N/Fe] ratio is due to the behaviour of the N/Fe production ratio of PopIII stars. Stars of $30M_\odot$ contribute to the chemical enrichment when [Fe/H] ≈ -4, while stars of $50M_\odot$ explode almost immediately after the beginning of star formation, when [Fe/H] ≈ -4.5. With the yields of CL, the N/Fe production ratio for stars of $30M_\odot$ is about a factor of 1000 greater than that for stars of $50M_\odot$ which leads to an increase of [N/Fe] in the very first Myrs (i.e.
between [Fe/H] = −4.5 and −4). 3 In Fig. 5 it is clear that PopIII stars alone do not solve the problem because, apart from the reasonably good value of [N/Fe] at very low [Fe/H], the predicted [N/Fe] ratio decreases afterwards to increase again for [Fe/H] > −2.0, as expected from the normal secondary N production in massive stars. It is worth noting that even for the lowest values of $Z_{\text{PopIII}}$ these models with CL yields give results for [C/Fe] in contrast with observations. Finally, we compare (always Fig. 5) the results of model C4 with a heuristic model calculated by Matteucci (1986, Matt86); the latter model is still based on the CRM03 model, but assumes that all massive stars produce primary nitrogen. In particular, they yield a constant (0.065$M_\odot$) quantity of $^{14}$N at every metallicity. Although this assumption was made ad hoc, this model seems to fit the data better than the standard model and is useful to understand that the primary production of nitrogen must occur at every metallicity and not only by means of a stellar population confined to the halo phase such as PopIII (in fact, model C4 does not reproduce the data at intermediate [Fe/H]). Therefore, mechanisms other than PopIII must be invoked. So far, we took into account only PopIII yields computed without rotation. For example, Meynet & Maeder (2002) calculated that stellar rotation as a function of metallicity can increase the primary production of nitrogen, and even though it has been shown (Chiappini et al., 2003b) that their rotation yields are not sufficient to obtain the requested trend, this is a mechanism that should be further studied. In other words, we cannot predict the exact amount of primary nitrogen produced by massive stars nor its dependence upon the stellar mass but only suggest that a continuous N production from massive stars is required at every metallicity (see Chiappini et al. 2005, for a more extensive discussion on the N production).

4.4 The [C/O] ratio

Another result for which PopIII has lately been invoked (and, more in detail, PopIII with yields of CL) is the observed trend of [C/O] vs. [O/H], which appears to rise towards low [O/H]. In fact, Akerman et al. (2004) have shown that a chemical evolution model with the assumption of a PopIII phase lasting until a metallicity of $10^{-4}Z_\odot$ was reached, with an IMF including only massive stars (10 − 80$M_\odot$) and where CL yields were adopted (equivalent to our model C4), is able to reproduce the observed trend, whereas our fiducial model does not follow the data at low [O/H] (see Fig. 6). This is mainly due, in CL yields, to the adopted rate for the reaction $^{12}C(\alpha, \gamma)^{16}O$ at the end of He-burning in the core of metal-free stars; the rate being low, the C/O production ratio at low metallicities is increased. However, the same authors (Akerman et al., 2004) in their analysis did not take into account

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3 Of course, this results from the assumption of instantaneous mixing, i.e. that the timescale for N to mix with the environment is much shorter than the difference between the lifetimes of a 30$M_\odot$ and a 50$M_\odot$ star. If this timescale were longer, then such a behaviour would not appear.
Fig. 6. Evolution of the [C/O] ratio with [O/H] in the fiducial model (solid line) and in the PopIII model with yields of CL and $Z_{\text{PopIII}} = 10^{-4} Z_{\odot}$ (dotted line). The data are from Akerman et al. (2004)

non-LTE corrections which depend on metallicity and could tend to systematically lower the measured [C/O] ratio. They also did not exclude the possibility that the apparent trend is an artifact due to limited statistics. Moreover, we have seen in §4.2 that models with yields of CL cannot reproduce the observations of [C/Fe] vs. [Fe/H] for the same reason thanks to which they can well fit the data for [C/O] vs. [O/H] (i.e., the rate of $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$). Instead, if one adopts O yields from Woosley & Weaver (1995) as a function of metallicity (see Chiappini et al. 2005 and François et al., 2004) it is possible to increase the [C/O] ratio at low [O/H]. Thus, even if these data were correct, there is not a unique way to reproduce them, and the models which give the best overall results are those which do not invoke PopIII.

5 The effects of a variable IMF

In this section we briefly explore the effect of a different kind of IMF which could in principle provide a pre-enrichment of the Galactic gas. For this purpose, we adopted the IMF suggested by Larson (1998) which has the following form:

$$\frac{d\varphi(m)}{d\log m} \propto \left(1 + \frac{m}{M_C}\right)^{-x}$$ (4)

where $M_C$ is a “characteristic” mass at which the IMF is peaked, whose scale is given by the Jeans mass of collapsing clouds. By adequately changing the value of $M_C$ as a function of time, one can obtain an IMF which is “top-heavy” in the earliest phases of the Galaxy. In this way we built a “Flat” model and two “Hernandez & Ferrara” models. In the “Flat” (hereafter F) model, the peak of the IMF is shifted towards larger masses during the whole halo phase; this is a heuristic model by means of which we investigate the behaviour of chemical evolution with a strongly
variable IMF. Referring to Eq. 4, for the F model the IMF exponent is \( x = 1.8 \) and \( M_C \) is assumed equal to \( 10M_\odot \) for the first 1Gyr (the duration of the halo phase), then it switches to \( 0.35M_\odot \) and remains constant since then. The choice of \( x = 1.8 \) and \( M_C = 0.35M_\odot \) for the present-time IMF allows us to reproduce the observed solar abundances by mass. In the “Hernandez & Ferrara” (hereafter hfI and hfII) models, the variation of the peak mass follows the suggestion made by Hernandez & Ferrara (2001); these authors explore the predictions of the hierarchical model of galaxy formation about the number and metallicities of metal-poor stars and, comparing these predictions with observational data and assuming a Larson IMF, they conclude that the characteristic mass \( M_C \) should increase with redshift. For the hfI and hfII models, we referred to Fig. 6 of Hernandez & Ferrara (2001), where is shown the variation of \( M_C \) as a function of metallicity, from which we extrapolated the following trends for the characteristic mass:

\[
M_C = \begin{cases} 
10M_\odot & \text{if } Z/Z_\odot \leq 10^{-3.92} \\
13M_\odot & \text{if } 10^{-3.92} < Z/Z_\odot \leq 10^{-3.77} \\
14M_\odot & \text{if } 10^{-3.77} < Z/Z_\odot \leq 10^{-3.58} \\
11M_\odot & \text{if } 10^{-3.58} < Z/Z_\odot \leq 10^{-3.38} \\
6.5M_\odot & \text{if } 10^{-3.38} < Z/Z_\odot \leq 10^{-2.9} \\
2.5M_\odot & \text{if } 10^{-2.9} < Z/Z_\odot \leq 10^{-2.55} \\
0.8M_\odot & \text{if } 10^{-2.55} < Z/Z_\odot \leq 10^{-2.2} \\
0.35M_\odot & \text{if } Z/Z_\odot > 10^{-2.2} \text{ in Model I} \\
0.50M_\odot & \text{if } Z/Z_\odot > 10^{-2.2} \text{ in Model II}
\end{cases}
\]

Moreover, \( x = 1.35 \) for Model I and \( x = 1.8 \) for Model II. These choices are due to the fact that Model I exactly reproduces the IMF suggested in the paper, whereas in Model II, while still adopting the same variation as a function of metallicity, the parameters \( x \) and \( M_C \) are chosen in order to reproduce the observed solar abundances by mass.

The basic model is still the CRM03 best model; excluding the IMF, all the other prescriptions were left unchanged. In particular, the same nucleosynthetic prescriptions hold as in §2 (i.e., the yields adopted for massive stars in this case are those of Woosley & Weaver (1995), solar composition, at all times).

Table 6 shows the results concerning various quantities calculated at the present time (i.e., the Galactic age \( t_G = 13.7 \) Myr) for all the models presented in this section, compared to the results of the basic model and to the observations. With a few exceptions, model results generally agree with the data. Small discrepancies are seen in model F which tends to overestimate the surface mass density of stars, while model hfI strongly overestimates the present-day rate of supernova explosions. This
Table 6
Predictions for models with Larson IMF in the solar neighbourhood matched with the results of the CRM03 model and the observations. Among the quantities calculated at the present time (in brackets are indicated the data sources): $\Psi_0$ is the star formation rate (Rana, 1991); $\dot{F}_0$ is the infall rate (Portinari et al., 1998); $\sigma_g$ is the gas surface mass density (Dickey, 1993); $\sigma_*$ is the star surface mass density (Gilmore et al., 1989); $\sigma_{*,\text{rem}}$ is the surface mass density of stars plus remnants (Mérai et al., 1998); $\sigma_{\text{tot}}$ is the total (gas + stars + remnants) surface mass density (Sackett, 1997); finally, $R_{SNII}$ is the present time rate of Type II SN explosions (Tammann et al., 1994).

| Quantity               | F     | hfl   | hflI  | Standard | Obs. |
|------------------------|-------|-------|-------|----------|------|
| $\Psi_0 (M_\odot pc^{-2} Gyr^{-1})$ | 3.1   | 3.8   | 3.1   | 2.6      | 2 – 5 |
| $\dot{F}_0 (M_\odot pc^{-2} Gyr^{-1})$ | 1.1   | 1.1   | 1.1   | 1.02     | 0.3 – 1.5 |
| $\sigma_g (M_\odot pc^{-2})$ | 7.0   | 8.1   | 7.0   | 7.1      | 7.0   |
| $\sigma_* (M_\odot pc^{-2})$ | 43.3  | 35.6  | 38.5  | 35.4     | 35 ± 5 |
| $\sigma_{*,\text{rem}} (M_\odot pc^{-2})$ | 44.8  | 43.7  | 44.8  | 42.9     | 43 ± 5 |
| $\sigma_{\text{tot}} (M_\odot pc^{-2})$ | 51.8  | 51.8  | 51.8  | 50.0     | 51 ± 6 |
| $R_{SNII} (pc^{-2} Gyr^{-1})$ | 0.018 | 0.056 | 0.019 | 0.014    | 0.020 |

Table 7
Solar abundances by mass calculated by models with a Larson-type variable IMF matched with the results of our fiducial model and the observations. Data for solar abundances are from Asplund et al. (2004).

| $X_i$ | F     | hfl   | hflI  | Standard | Obs. |
|-------|-------|-------|-------|----------|------|
| C     | 2.6E–3| 5.1E–3| 2.9E–3| 1.6E–3   | 2.1E–3 |
| N     | 2.2E–3| 6.2E–3| 2.1E–3| 9.6E–4   | 5.9E–4 |
| O     | 1.0E–2| 2.9E–2| 1.1E–2| 5.3E–3   | 5.1E–3 |
| Mg    | 4.2E–4| 1.4E–3| 4.1E–4| 2.5E–4   | 5.7E–4 |
| Si    | 1.0E–3| 2.7E–3| 1.1E–3| 8.3E–4   | 6.3E–4 |
| Fe    | 1.4E–3| 3.0E–3| 1.6E–3| 1.1E–3   | 1.1E–3 |
| Z⊙    | 2.1E–2| 5.9E–2| 2.1E–2| 1.2E–2   | 1.9E–2 |

is due to the choice of $x = 1.35$ which results in a too flat IMF if compared with other IMFs (Scalo, 1986; Chabrier, 2003) which better reproduce the features of the solar neighbourhood. Such a flat IMF leads to an overproduction of supernova progenitors even at the present time.
In Table 7 we show the calculated solar abundances by mass. If we exclude model hfI, all the predicted values are within a factor of 2 of the observed ones and thus acceptable (with the exception of N). The strong overproduction of metals in model hfI can again be explained by the excessive flatness of the chosen IMF, which favours the overproduction of massive stars.

Fig. 7 shows the evolution of the abundance ratios $[\text{O}/\text{Fe}]$, $[\text{C}/\text{Fe}]$ and $[\text{N}/\text{Fe}]$ with $[\text{Fe}/\text{H}]$ predicted by the models considered here. Model F severely overproduces oxygen and carbon and underproduces nitrogen even at intermediate $[\text{Fe}/\text{H}]$. Model hfI predicts an excessively steep trend of the carbon abundance towards low $[\text{Fe}/\text{H}]$ and slightly underproduces nitrogen. Similar trends are predicted by model hfII which however presents smaller differences relative to the basic model and thus cannot be ruled out on the basis of abundance analysis, although the best overall agreement is still obtained by a constant IMF. The behaviour of other $\alpha$-elements (Mg, Si) is very similar to that of oxygen, so the same conclusions hold.

Fig. 8 shows the temporal evolution of the SNIa rate for model F (the other models do not show remarkable differences relative to the basic model). We can see the pronounced peak during the halo phase which indicates that, because we chose $M_c = 10M_\odot$ as the “most probable” mass in the IMF for the halo phase, the number of Type Ia SN progenitors, which are stars of intermediate mass, is increased by the adoption of this kind of IMF. Such a result also demonstrates how critically the results may depend on the details of the adopted IMF: in fact, the IMF being peaked near the intermediate mass range, the N production is enhanced for all models. The sharp feature in SNIa rate is not seen in models hfI and hfII, owing to the extremely limited time duration of the “top-heavy” phase which ends after 230 Myr.

In Fig. 9 is shown the temporal evolution of the SNII rate for all the models presented here. We notice that the present-day SNII rate crucially depends on the IMF exponent, as already mentioned; this can be seen by the differences between model hfI and the other models which have $x = 1.8$.

Finally, Fig. 10 shows the metallicity distribution of G-dwarfs in the solar neighbourhood predicted by these models; model F and model hfI are not able to fit the data well, predicting an extremely sharp distribution peaked at too high $[\text{Fe}/\text{H}]$. On the other hand, model hfII shows an acceptable agreement with data; however, the distribution peak is still located at $[\text{Fe}/\text{H}]$ higher than that observed.

In conclusion, the effects of a variable IMF which is “top-heavy” in the earliest phases result in an overproduction of metals, an increasing slope for the evolution of abundance ratios towards low $[\text{Fe}/\text{H}]$ and a shift towards larger $[\text{Fe}/\text{H}]$ of the G-dwarf distribution peak. Depending on the particular chosen IMF, there can also be different results in abundance evolution which largely depend on the adopted nucleosynthetic prescriptions. Most results of a variable IMF are at variance with observations, and an overall better agreement is achieved with a constant IMF, un-
Fig. 7. Evolution of [O/Fe], [C/Fe] and [N/Fe] with [Fe/H] in the “Halo” models with variable IMF (dashed-dotted line) compared to the results predicted by model CRM03 (solid line). Data for the disk phase (open squares) are from Chiappini et al. (1999) and references therein; data for the halo phase (filled circles) are from Cayrel et al. (2004) for O, from Spite et al. (2005) for C and N.

less the suggested IMF variations are so small that they are irrelevant (see also Chiappini et al., 2000).

6 Summary and conclusions

In this paper we have explored the effects on the chemical evolution of the solar neighbourhood of one or more PopIII stellar generations, namely massive and very massive stars of primordial chemical composition. To do that we have adopted a recent version of the two-infall chemical evolution model of CRM03 which re-
Fig. 8. Evolution of Type Ia SN rate in the “Halo” models with variable IMF (dashed-dotted line) compared to the results predicted by model CRM03 (solid line). $A$ is the assumed fraction of binary systems in the IMF which evolve to Type Ia SNe.

Fig. 9. Evolution of Type II SN rate for all the models with variable IMF (dashed-dotted line) compared to the results predicted by model CRM03 (solid line). The peak in the halo phase was cut off in model F for graphical reasons. The dark regions seen after 10 Gyr are due to the fast oscillations of SFR when the threshold surface mass density is reached (see Chiappini et al., 1997).

Fig. 10. G-dwarf metallicity distribution in the solar neighbourhood for our fiducial model (solid line) and for the models with variable IMF (dashed-dotted line). Data are from Wyse & Gilmore (1995, dashed histogram) and Rocha-Pinto & Maciel (1996, dotted histogram).
produces the majority of the observed features of the solar neighbourhood and the whole Galaxy. By means of this model we have calculated the evolution of several chemical elements in the Galactic interstellar medium, focusing our attention on C, N, α-elements (O, Mg, Si) and Fe, by adopting several sets of stellar yields calculated for zero-metallicity massive and very massive stars (PCSNe). In particular we have adopted the nucleosynthesis prescriptions from Ober et al. (1983), Heger & Woosley (2002), Umeda & Nomoto (2002) and Chieffi & Limongi (2002, 2004).

We have also tested some suggestions which appeared in the past years in the literature concerning a variable IMF favoring massive stars in the earliest phases of the evolution of the Galaxy. In particular, we adopted the functional forms of the IMF as suggested by Larson (1998) and by Hernandez & Ferrara (2001).

Our main results can be summarized as follows:

- For the suggested values of the threshold metallicity for the formation of very massive PopIII objects, \( Z_{\text{PopIII}} = (10^{-6} - 10^{-4}) \cdot Z_\odot \) (Schneider et al. 2002), the effects of PopIII stars on the predicted abundances and abundance ratios are negligible even in the earliest Galactic evolutionary phases, so that we cannot exclude their existence. Only if we assume that the PopIII regime lasted until \( Z_{\text{PopIII}} = 0.1 Z_\odot \), do we find noticeable differences in the predicted \([X/Fe]\) vs. \([Fe/H]\) trends and the agreement with the observations worsens relative to the standard case without PopIII.

- For the specific elements C and N, we find that in some cases (data relative to C/O from Ackerman et al., 2004) the yields from PopIII can produce a better agreement with the observations. However, in this case, the agreement for all the other elements is worse relative to the standard model. The problem of N has been discussed and in particular the fact that new data at low metallicity seem to suggest a primary origin for this element produced in massive stars. We have shown that rather than invoking N production from PopIII stars, it is necessary to assume a continuous primary production from massive stars at all metallicities. Such a production still needs to be understood, although recent stellar models with rotation (Meynet & Maeder, 2002) predict some primary production of N in massive stars, although not enough to explain the data (see Chiappini et al., 2005 for an extensive discussion on this point).

- We did not discuss the effects of PopIII star nucleosynthesis on the Fe-peak elements (Co, Cr, Ni, Mn) and Zn, whose abundances in very metal poor stars require changes in the standard yields (see also François et al. 2004), since UN02 had already discussed this point and concluded that PopIII stars cannot solve the problem and that nucleosynthesis in high-energy hypernovae seems more promising. However, their analysis was based only on the direct comparison between element production ratios and abundances in very metal poor stars.

- A continuously varying IMF, favoring massive stars in the early stages of Galactic evolution in the manner suggested by Larson (1998) does not produce acceptable results either for the abundance ratios or the G-dwarf metallicity distribu-
tion. Therefore, our conclusion is that a variable IMF in general produces results at variance with observations unless the variation is so small as to be irrelevant. The same conclusion was reached by Chiappini et al. (2000) by studying the evolution of the Galactic disk with the variable IMF proposed by Padoan et al. (1997).

- We computed the number of PopIII SNe required to pollute the Galactic halo to a given $Z_{\text{PopIII}}$ and found that in order to pollute the ISM at the level of $Z_{\text{PopIII}} = 10^{-4} Z_\odot$ we need a number of PCSNe varying from 2 to 20 according to the assumed stellar yields and that the time necessary to reach this situation is always of the order of few Myrs (from 1 to 5). This may explain why we do not see stars with zero-metallicity in the Galactic halo, simply because the number of these stars must have been negligible, even though it cannot be excluded that primordial stars formed in overdense regions and might have ended up in the Galactic bulge (where observations are more challenging) instead than in the halo. In any case, even the standard model of CRM03 with a normal IMF including low mass stars predicts a fraction of stars with $[\text{Fe}/\text{H}] < -5.0$ of the order of $10^{-5}$, in agreement with current estimates.

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