Methods and Results for Quantum Pulse Control on Superconducting Systems

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Abstract—The effective use of current Noisy Intermediate-Scale Quantum (NISQ) devices is often limited by the noise which is caused by interaction with the environment and affects the fidelity of quantum gates. In transmon qubit systems, the quantum gate fidelity can be improved by applying control pulses that can minimize the effects of the environmental noise. In this work, we employ physics-guided quantum optimal control strategies to design optimal pulses driving quantum gates on superconducting qubit systems. We test our results by conducting experiments on the IBM quantum hardware using their OpenPulse API. We compare the performance of our pulse-optimized quantum gates against the default quantum gates and show that the optimized pulses improve the fidelity of the quantum gates, in particular the single-qubit gates. We discuss the challenges we encountered in our work and point to possible future improvements.

Index Terms—quantum optimal control, quantum computing, logical gates, NISQ

I. INTRODUCTION

In the current era of noisy intermediate-scale quantum (NISQ) regime, we expect to see quantum computers with hundreds or thousands of imperfect qubits. Quantum technology companies have made strides towards quantum computers with increasing numbers of qubits. Recently, IBM broke the 100 qubit barrier with a 127 qubit quantum processor, which is a big step towards practical quantum computation. In addition to increasing the number of qubits it is also essential to develop high-fidelity quantum gates to demonstrate the quantum advantage and achieve fault-tolerant quantum computation. However, the current quantum hardware is sensitive to noise and the quantum logic gates often suffer from errors that reduce the fidelity of the respective quantum operations. This in turn affects the ability to reliably carry out large-scale quantum computations. Therefore, there is a need to develop techniques that can improve the quantum gate fidelity as close as possible to unity, and improve the performance of quantum hardware. Previously, Lie group theory and geometric formalism has been implemented for dynamically correcting gates in single-, and multi-qubit systems [1]. Other strategies used for controlling quantum systems are Lyapunov and bang-bang methods [2]–[4]. However, these techniques have a high numerical cost. In recent years, optimal-control-based methods have been widely applied to quantum systems, to maximize the fidelity of the quantum processes that drive the qubit from one state to another. Generally, in optimal control, for a given model of the quantum system such as transmons, ions and neutral atoms etc., the control pulses that minimize the associated cost function are obtained. Some of the earliest optimal-control approaches are Krotov [5] and gradient ascent pulse engineering (GRAPE) [6]. Of late, methods such as chopped random basis optimization (CRAB) [7] and gradient optimization of analytic controls (GOAT) [8] have been proposed. There are several open-source software packages such as C3 [9] and qopt [10] that also utilize optimal control of pulses. So far these methods have been implemented for numerical simulations and not been demonstrated on real quantum hardware. Recently, researchers have been looking into leveraging deep reinforcement learning (DRL) techniques to prepare these target quantum gates from any initial state, and are robust to errors [11], [12]. The DRL algorithm in [12] was executed on the experimental quantum hardware of IBM.

One of the most promising technology to create a full scale quantum computer is through superconducting devices [13]. There are several realizations of superconducting qubits [14]. We explore the impact of quantum optimal control on the gate performance of IBM quantum systems by executing the custom piece-wise-constant (PWC) pulses directly on a superconducting quantum computer using OpenPulse API [15].

This manuscript is organized as follows. First we provide a brief overview of quantum optimal control and its implementation in QuTiP [16]. Second, we outline the implementation of the offline pulses on the real IBM Q hardware. We compare the error-rates of our optimized gates with the default device-gates. We end with a discussion on the challenges we faced and future outlook.

II. BACKGROUND

Quantum optimal control (QOC) is an important tool used extensively in Physics and Chemistry applications where the goal is to steer the time evolution of the quantum system to
a particular target state, unitary operation or a desirable state-to-state transfer. It synthesizes control fields for a particular control target, constraints and time evolution of quantum system. The time-dependent Hamiltonian can be written as $H(t) = H_0 + \sum_{i=1}^{n} u_i(t)H_i$, where the first term is the drift Hamiltonian and second term represents the control Hamiltonian, and $u_i(t)$ are the control functions that describe the strength with which, each of the control Hamiltonian acts as a function of time. The goal is to determine a set of $\{u_i(t)\}$, for each control, to optimize the relevant cost function $\mathcal{C}[\{u_i(t)\}]$. We use QOC method to design the optimal pulses to execute the quantum gate on the quantum computer. The cost function for this particular case is gate infidelity $\mathcal{C} = 1 - \mathcal{F} = 1 - \frac{1}{\sqrt{2}} |\text{Tr}(U_f^\dagger U_t)|^2$, where $U_t$ is the target unitary gate, and $U_f$ is the unitary realized by the control pulses.

Typically, the control problem cannot be solved analytically. Hence one resorts to numerical methods. The main quantum optimal methods used to minimize the cost function are gradient based approaches. One of the oldest techniques is GRAPE (Gradient Ascent Pulse Engineering) [6]. However, this method converges very slowly to the optimal cost function. Also, control pulses based on the CRAB method can be derived from truncated Fourier series. However, the CRAB algorithm utilizes a direct search approach which makes the convergence very slow even for small optimization variables [7]. [17]. A second-order GRAPE method known as L-BFGS-B is a limited-memory algorithm which, as the name suggests, requires much less memory than its precursor and converges faster [18]. We have also tested another optimization method called Simultaneous Perturbation Stochastic Approximation (SPSA) [19]. It is a gradient approximation, but unlike GRAPE it does not measure the gradient of the cost function but the cost function itself. The gradient approximation measures the objective function at only two points. We found that L-BFGS-B converges faster and gives much smaller fidelity error than SPSA. Therefore we will utilize L-BFGS-B as our numerical optimization method of choice.

III. IMPLEMENTATION

A. Pulse Optimization with QuTiP

In this work, we generate optimal pulses using the QuTiP library [16]. QuTiP is an open-source Python package to simulate dynamics of quantum systems, and provides useful tools we need for the pulse optimization. In particular, we use the QuTiP pulseoptim function, which employs the L-BFGS-B as the optimization algorithm [16] and outputs the optimized pulse coefficients for each control term. Our current target quantum architecture is the IBM Q superconducting qubits, since we only have access to them at the time of this work. The Hamiltonian that represents the IBM Q hardware can be constructed based on the information provided by IBM, such as the coupling between qubits and qubit frequencies. The Hamiltonian is then used to generate the control pulses through QuTiP pulseoptim. An example control pulse generated this way for the X gate is given in Fig. 1.

B. Pulse Implementation with Qiskit

We then test the optimized control pulses on the real hardware to see if the quantum gate fidelity has been improved. To do this, we employ the IBM Qiskit-Pulse library, which is a low-level quantum programming tool using pulses [15]. The Qiskit-Pulse is a front-end implementation of the OpenPulse interface [20] which allows the user to modify the pulse parameters and translate the pulse program to an executable circuit which can be implemented on the quantum hardware. The Qiskit-Pulse consists of a pulse shape library, pulse channels, schedules and instructions. The user can input pulse shape, duration, amplitude using the pulse waveform function or by calling any of the existing shapes such as drag (Derivative Removal by Adiabatic Gate) pulse in the pulse library. These variables can be modified to customize the pulse driving any quantum gate.

![Fig. 1: An example of control pulses for X gate generated by the pulseoptim function in QuTiP. The top two panels represent the initial pulses for each control term and the lower two panels show the output pulses after the optimization.](image)

![Fig. 2: (Color online) The control pulse implemented on ibmq_montreal device. Here we use drag pulse shape. D0 is the DriveChannel for qubit 0. The default X gate is replaced by our optimized X gate, which is confirmed in the transpiling process.](image)
it needs to be mapped to the backend using the instruction schedule map. The user can also view whether the pulse has been successfully mapped. The pulse gate can now be implemented at the circuit level. One can view the underlying pulses of the quantum circuit using the schedule function in Qiskit. The circuit is executed on the hardware using this schedule. An example of the pulse schedule for the X gate as implemented on the ibm\textunderscore montreal device is shown in Fig. 2.

C. Benchmarking

To test how well our optimized pulses work, we use two different measures. First, we check the probability distribution of the qubit output state after the gate operation, and see if it is consistent with the expected probability distribution for state $|0\rangle$ or $|1\rangle$. Next, we check the average gate errors using randomized benchmarking (RB) \cite{21}, which is implemented in Qiskit. However, the standard RB procedure in Qiskit does not allow the inclusion of custom gates. Instead, we use the interleaved randomized benchmarking (IRB), which allows us to use the custom gates with the optimized pulses. For an in-depth discussion on IRB please see \cite{22} and Qiskit documentation page. We will explain in more detail how these benchmarks are used in our experiments in Section IV.

IV. Experiments On IBM Q

In the following sections, we present the results for implementing our control pulses using the procedure described in Section III. After a discussion of the details of the IBM Q test systems, we will present our results for the X (not) gate, the $\sqrt{X}$ (square root not) gate, H (Hadamard) gate and the two-qubit CNOT (control not or CX) gate.

A. Details of the IBM Q Test Systems

Our experiments were conducted on ibmq\textunderscore toronto and ibmq\textunderscore montreal systems. The ibmq\textunderscore toronto system has a quantum volume of 32 with 27 qubits. The average $T_1$ is 83.52 $\mu$s. We utilize qubit 0 for our experiments, which has a frequency of 5.225 GHz and average single qubit gate error of $3.068 \times 10^{-4}$. For ibmq\textunderscore montreal the quantum volume is 128 with 27 qubits. The average $T_1$ is 86.76 $\mu$s. Here we again utilize qubit 0 for our experiments, which has a frequency of 4.911 GHz and average single qubit gate error of $4.268 \times 10^{-4}$. We chose qubit 0 for both devices as it is connected to only qubit 1. This simplifies the implementation of the Hamiltonian model numerically. We would also like to mention that both these devices have the same topology \cite{23}.

B. X Gate

First we discuss the results for the X gate, which constitutes one of the basis gates in Qiskit. It is also known as a $\pi$-pulse gate. In IBM Q systems the $\pi$-pulse is regularly calibrated by carrying out a Rabi experiment. More information on this can be found here \cite{24}.

We use the duffing oscillator Hamiltonian for the system, and Pauli X, Pauli Y are the control terms for the control Hamiltonian. We import the values of qubit frequency and dephasing rate from the backend description provided by IBM. The initial pulse shape is chosen to be drag, the amplitude bound is $[0, 1]$. The evolution time for each control term is 52 $\mu$s and the total pulse duration is 480 $\mu$s. The optimized pulse coefficients are obtained using the QuTiP optimal pulse module. The pulses obtained in QuTiP and the control pulse implemented on ibmq\textunderscore montreal are shown in Fig. 2. We test our pulses on the ibmq\textunderscore montreal backend. We first create a circuit to prepare and measure a NOT gate, then reduce it to a pulse schedule before running the pulse job on the device. We measure the qubits, and plot the probability distribution for $|0\rangle$ and $|1\rangle$ state measurements in the bottom panel of Fig. 3. We can see that the output state has $87.3\%$ probability of being in state $|1\rangle$ (up to measurement errors) after implementing the custom X gate in the quantum circuit, while an error-free gate would result in $100\%$ probability.

Next, we characterize our gate using IRB, which gives the estimate of the average error-rate of our custom gate. The IRB
Fig. 4: (Color online) (Top) IRB result for the custom $\sqrt{x}$ gate obtained from optimized pulse controls. The error rate here is $(2.4 \pm 0.8) \times 10^{-4}$. (Middle) IRB result for the original (default) $\sqrt{x}$ gate as implemented in Qiskit. The error rate here is $(6.5 \pm 1.4) \times 10^{-4}$. (Bottom) Probability distribution of the qubit state after applying the pulse-optimized $\sqrt{x}$ gate.

Experiment generates both the standard RB sequence and the interleaved one. This experiment calculates the probabilities to get back to the ground state, fits the probability curve to get $\alpha$ (depolarizing parameter) and $\alpha_c$ (ratio of depolarizing parameter of IRB to RB curve) and calculates the interleaved gate error. In the top and middle panels of Fig. 3 we show the error rates of the custom X gate and the default X gate, using IRB in Qiskit. We find that the error rate for our pulse optimized gate is about 28% lower than the default X gate implemented in Qiskit.

C. Square root NOT ($\sqrt{x}$) gate

Square-root-not gate ($\sqrt{x}$) is also a basis gate in IBM Q systems. Similar to the X gate, we use the duffing oscillator Hamiltonian with Pauli X as the control term for the control Hamiltonian. The optimized pulses are obtained using QuTiP optimal pulse module. The control pulse of length $736 \, dt \sim 162 \, ns$ is implemented on ibmq_montreal.

For open systems the presence of dissipative terms can prevent or enable certain states to be reached. We found that for the $\sqrt{x}$ operation we were not able to reach a global minimum of the cost function. Hence, for the case of $\sqrt{x}$ gate we neglected the decoherence processes during the optimization for computational simplicity.

The error rate for the custom $\sqrt{x}$ gate and the default $\sqrt{x}$ gate is obtained from the results of IRB as shown in Fig. 4. We observe that the error rate of our custom $\sqrt{x}$ gate is 63% lower than the default gate. The bottom panel of Fig. 4 shows the histogram of the qubit final state, which is in approximately equal superposition (up to measurement errors) of $|0\rangle$ and $|1\rangle$ as expected after $\sqrt{x}$ gate-operation.
D. Hadamard (H) gate

Unlike the previous two gates, the Hadamard (H) gate is not part of the basis gates. Instead it is transpiled in terms of $\sqrt{X}$ gate and two $\pi/2$ virtual $Z$ rotations. In our work we directly optimize the controls to implement Hadamard gate on IBM Q system. The control Hamiltonian here consists of Pauli X and Pauli Y terms. The control pulses of total length $1216 \text{ dt} \sim 267 \text{ ns}$ are implemented on ibmq_toronto. Again, we use IRB to calculate the error rate for the custom H gate and the default H gate, shown in the top and middle panels of Fig.[5] We find that the error rate of our custom H gate is higher compared to the default gate, which can be attributed to the longer pulse duration. The sub-optimal custom pulse is also evident from the histogram shown in the bottom panel of Fig.[5] which shows that the probability of the qubit in equal-superposition-state of $|0\rangle$ and $|1\rangle$ is not exactly balanced.

E. Two-qubit gate – CNOT

The CNOT gate is a two-qubit entangling gate which is part of the basis gate set in IBM Q. The CNOT gate is implemented by two-qubit gate known as the cross-resonance (CR) gate. The CR gate drives the target qubit through the control qubit via cross-resonance interaction. The CR Hamiltonian as described in [25] is

$$H_{\text{cr, drift}}^{\text{eff}} = \frac{1}{2} \tilde{w}_1 \sigma_x^{(1)} + \frac{1}{2} \tilde{w}_2 \sigma_x^{(2)} + \Omega(t)_{R,2} \left( \sigma_y^{(1)} \sigma_y^{(2)} \right) + \Omega(t)_{R,1} \left( \sigma_x^{(1)} \sigma_x^{(2)} \right) + \frac{J}{\Delta z_1} \sigma_x^{(1)} \sigma_x^{(2)}. \quad (1)$$

From Eq. 1, the control terms are $\sigma_x^{(1)} \sigma_x^{(2)}$, $\sigma_y^{(1)} \sigma_y^{(2)}$, $\sigma_z^{(1)} \sigma_z^{(2)}$. We first show our results for the “SINE” input pulse shape implemented in the QuTiP optimizer. These results were executed on now retired ibmq_boeblingen and ibmq_rome systems. At the time of running our optimized pulses Qiskit had not released interleaved-randomized benchmarking. We tested our pulses by implementing it in a quantum circuit, and plot the probability distribution of the output states in Fig.[6] On ibmq_boeblingen with the optimized pulses the probability of getting the output state $|11\rangle$ is 79\%, while on ibmq_rome the probability is 87\%, both of which offer little to none improvement over the default CX gate.

Instead of first optimizing CR gate and then inserting single qubit rotations, we directly solve for CNOT gate and instead of first optimizing CR gate and then inserting two-qubit gate known as the cross-resonance (CR) gate. of the basis gate set in IBM Q. The CNOT gate is implemented on the ibmq_montreal system for another set of control pulses with the Gaussian squared pulse shape as the input to the QuTiP solver. The control pulses are shown in Fig.[7] The IRB results for the optimized CX gate are shown in Fig.[8] From our results we see that the average error per gate with the custom CX gate is almost the same as the default CX gate, with the custom CX gate error being only 8\% lower than the default CX gate.

V. DISCUSSION

The properties of an IBM Q system are dynamic and need to be updated at system calibration time. According to the information provided by the IBM Q documentation this occurs at least once over a 24-hour period. These system properties include the qubit frequency, readout error, values of $T_1$ and $T_2$. Hence, this drifting of qubit properties can lead to fluctuations in the performance of qubits over time. We carried out two sets of experiments to study the impact of time on our results. For the first set of experiments we optimized the pulses only once and tested the optimized pulses (for different quantum gates) on IBM Q devices on different days.

In the second set we took into account the frequent calibration of device parameters and optimized the pulses everyday. We then executed the quantum circuit containing the corresponding day-specific pulse gate on the IBM Q device and measured the probability distribution histograms similar to those presented in Section [IV] In both experiments we noted some variations in the performance of the respective gates on certain days. We also compared these measurement results against the results of IRB and noted that the gate errors
the experiments we conducted on the IBM Q system are summarized in Table I. It is worth noting that while we did not see any improvement for the custom H gate with a long 162-ns duration, at a much shorter duration of 31 ns, we were able to improve the average gate error with the custom gate.

We have not tried a shorter gate duration for the CX gate, but will do so in the future.

TABLE I: Comparison of error rate per gate with and without optimized custom pulses as measured by interleaved randomized benchmarking on IBM Q devices for different gate-durations. The default gate duration is fixed at 32 ns. Results for the X, \( \sqrt{X} \), and CX gates were obtained on the ibmq_montreal system. Results for the H gate were obtained on the ibmq_toronto system.

| Gate | Duration (ns) | IRB error rate \((\times 10^{-3})\) |
|------|--------------|----------------------------------|
| X    | 105          | 2.0(5)                           |
| X    | 56           | 2.8(5)                           |
| \( \sqrt{X} \) | 162       | 6.5(1.4)                          |
| \( \sqrt{X} \) | 31        | 6.5(1.4)                          |
| H    | 267          | 5.0(7)                           |
| H    | 28           | 5.0(7)                           |
| CX   | 1193         | 62(13)                           |

VI. CONCLUSION AND OUTLOOK

We have described our methods to improve quantum gate fidelity by optimizing the control pulse that drives the quantum system. We were able to optimize control pulses for quantum gates on superconducting quantum systems using the gradient-based optimization algorithms. We executed these pulses successfully on the IBM Q hardware and verified our results by plotting the output probability histograms and performing interleaved randomized benchmarking of our pulse-optimized gates. While IRB gives some insight into the general gate fidelity, it does not provide quantitative information on how much improvement we can expect for an actual quantum algorithm. Our next step is to apply these custom gates to more complex quantum circuits, and see how much improvement we can obtain on the accuracy of the results. It will also be interesting to see if such optimization techniques will work on other quantum architectures.

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