Cooperative enhancement of channeling of emission from atoms into a nanofiber

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Abstract
We show the possibility of directional guided superradiance from a linear array of distant atoms separated by one or several wavelengths in a line parallel to the axis of a nanofiber. We find that the rate and efficiency of channeling of emission from the atoms into the fiber are cooperatively enhanced by the guided modes.

Keywords: superradiance, distant atoms, nanofiber

Classification numbers: 3.00, 5.16

1. Introduction
Coupling of light to subwavelength structures and its control pose one of the greatest challenges of recent research [1–3]. In the case of dielectric waveguides, it has been shown that a significant fraction of emission from a single atom can be channeled into a nanofiber [2, 3]. The cooperation of two distant atoms via a nanofiber has been discussed [4]. It has been shown that at large distances between the atoms a substantial energy exchange can survive due to the guided modes [4]. In this paper we show the possibility of a directional guided superradiant emission process that can enhance the rate and efficiency of channeling of emission from a linear array of distant atoms into a nanofiber.

Before to proceed we note that superradiance is a problem of fundamental interest [5]. Despite a great deal of research [6], certain aspects of the problem are still not well understood. Recently, the angular distribution of emission from a spatially extended array of atoms in free space has been treated by the quantum trajectory method [7]. The dynamic mode selection has been studied [8]. Superradiant conversion of atomic spin gratings into single photons in an optical cavity has been demonstrated [9].

2. Model
Consider $N$ identical two-level atoms interacting with the quantum electromagnetic field in the vicinity of a nanofiber (figure 1). The fiber has a cylindrical silica core of radius $a$ and refractive index $n_1 = 1.45$, and an infinite vacuum clad of refractive index $n_2 = 1$. We assume that the atomic transition frequency $\omega_0$ is well below the cutoff frequency of the fiber, so the single-mode condition is satisfied for this frequency. In view of the very low losses of silica in the wavelength range of interest, we neglect material absorption. The atoms are located at points $(r_j, \varphi_j, z_j)$, where $j = 1, 2, \ldots, N$ labels the atoms and $(r, \varphi, z)$ are the cylindrical coordinates with $z$ being the axis of the fiber. We assume that the field is initially in the vacuum state. The field can be decomposed into the contributions from the guided and radiation modes, whose quantum expressions are given in [2].

Assume that the characteristic atomic lifetime is large as compared to the optical period $2\pi/\omega_0$ and to the light propagation time between two different atoms. The master equation for the reduced density operator $\rho$ of the atomic system in the electric-dipole, rotating-wave and Born–Markov approximations has been previously derived [4, 6, 7]. In the interaction picture it reads

$$\dot{\rho} = \frac{1}{2} \sum_{i,j=1}^{N} \gamma_{ij} \left(2\sigma_i^+ \sigma_j^- - \sigma_i^- \sigma_j^+ - \sigma_i^+ \rho - \rho \sigma_i^- \sigma_j^+ \right),$$

where $\sigma_j$ and $\sigma_j^+$ are the pseudospin operators that describe the downward and upward transitions of the atoms. The coefficients $\gamma_{ij} = \gamma_{ij}^{(\text{gyd})} + \gamma_{ij}^{(\text{rad})}$, with $i, j = 1, 2, \ldots, N$, characterize the collective spontaneous emission process, where $\gamma_{ij}^{(\text{gyd})}$ and $\gamma_{ij}^{(\text{rad})}$ are the contributions from the guided and radiation modes, respectively [4].
3. Intensity of emission into guided modes

We introduce the total emission intensity \( I = \sum_{\text{all}} h \omega_0 < n_a > \), the intensity of emission into the guided modes \( I_{\text{gyd}} = \sum_{\text{gyd}} h \omega_0 < n_a > \), and the intensity of emission into the radiation modes \( I_{\text{rad}} = \sum_{\text{rad}} h \omega_0 < n_a > \). Here the notation \( \sum_{\text{all}} \sum_{\text{gyd}} \) and \( \sum_{\text{rad}} \) mean the summations over all the modes, the guided modes, and the radiation modes, respectively, and \( \omega_0 \) and \( < n_a > \) are the frequency and mean number, respectively, of photons in a field mode \( \alpha \). We find

\[
I = h \omega_0 \sum_{ij} \gamma_{ij} < \sigma_i^* \sigma_j > = I_{\text{gyd}} + I_{\text{rad}},
\]

where

\[
I_{\text{gyd}} = h \omega_0 \sum_{ij} \gamma_{ij}^{(\text{gyd})} < \sigma_i^* \sigma_j >
\]

and

\[
I_{\text{rad}} = h \omega_0 \sum_{ij} \gamma_{ij}^{(\text{rad})} < \sigma_i^* \sigma_j >.
\]

We note that \( I = -h \omega_0 \dot{P} \), where \( P = \sum_{ij} < \sigma_i^* \sigma_j > \) is the total population of the excited levels of the atoms. The total energy emitted from the atoms is

\[
U = \int_0^\infty I(t) \, dt = U_{\text{gyd}} + U_{\text{rad}},
\]

where

\[
U_{\text{gyd}} = \int_0^\infty I_{\text{gyd}}(t) \, dt, \quad U_{\text{rad}} = \int_0^\infty I_{\text{rad}}(t) \, dt
\]

are the energies emitted into the guided and radiation modes, respectively. The fractions of energy emitted into the guided and radiation modes are given by \( f_{\text{gyd}} = U_{\text{gyd}}/U \) and \( f_{\text{rad}} = U_{\text{rad}}/U = 1 - f_{\text{gyd}} \), respectively.

The decay coefficients \( \gamma_{ij}^{(\text{gyd})} \) and \( \gamma_{ij}^{(\text{rad})} \) have been calculated in [4]. The diagonal decay coefficients \( \gamma_{jj} \) describe the spontaneous decay of individual atoms. The off-diagonal decay coefficients \( \gamma_{jj} \), with the convention \( j \neq j' \), characterize the energy transfer between two atoms. According to [4], the contribution \( \gamma_{jj}^{(\text{gyd})} \) of the guided modes to the transfer rate is periodic in the \( z \)-direction with the period \( \lambda_F = 2\pi/\beta_0 \), where \( \beta_0 \) is the longitudinal propagation constant of the guided modes at the atomic frequency \( \omega_0 \). Meanwhile, the contribution \( \gamma_{jj}^{(\text{rad})} \) of the radiation modes reduces to zero with increasing interatomic distance \( |z_j - z_j'| \).

Therefore, in the limit of large \( |z_j - z_j'| \), the energy transfer coefficient \( \gamma_{jj} \) is mainly determined by the contribution \( \gamma_{jj}^{(\text{gyd})} \) of the guided modes and is almost periodic with the spatial period \( \lambda_F \).

We now assume that the atoms are aligned along a line parallel to the fiber axis, with relatively large atomic separations being equal to integer multiples of the longitudinal wavelength \( \lambda_F \). In other words, we assume that \( r_j = \text{const} \equiv r_0, \psi_j = \text{const} \equiv \psi_0 \) and \( z_{j+1} - z_j = q_j \lambda_F \), with \( q_j \) being nonzero, positive integer numbers. In addition, we assume that the dipoles of the atoms are oriented in the same direction. Under these conditions, the guided energy transfer coefficients \( \gamma_{jj}^{(\text{gyd})} \) achieve their maximum value with respect to the axial direction, \( \gamma_{jj}^{(\text{gyd})} = \gamma_{jj}^{(\text{gyd})} = \gamma_{jj}^{(\text{gyd})} \). Meanwhile, due to the large separations between the atoms, the radiative (unguided) energy transfer coefficients \( \gamma_{jj}^{(\text{rad})} \) are small. In this case we have

\[
\gamma_{jj}^{(\text{gyd})} = \gamma_{jj}^{(\text{gyd})} = \gamma_{jj}^{(\text{gyd})} = \gamma = \gamma_{\text{gyd}} + \gamma_{\text{rad}},
\]

\[
\gamma_{jj}^{(\text{gyd})} = \gamma_{jj}^{(\text{gyd})} = \gamma_{jj}^{(\text{gyd})} = \gamma_{\text{gyd}}.
\]

Here, \( \gamma_{\text{gyd}} \) and \( \gamma_{\text{rad}} \) are the rates of decay into the guided and radiation modes, respectively. They do not depend on the axial coordinate \( z \) of the atoms, but increase with decreasing atom–surface distance \( r - a \). We note that the parameter \( \eta = \gamma_{\text{gyd}}/\gamma_{\text{rad}} \) characterizes the cooperativity of a single atom with the guided modes as well as the fiber-assisted cooperativity between distant atoms.

We calculate the decay rates \( \gamma_{\text{gyd}}, \gamma_{\text{rad}} \) and \( \gamma = \gamma_{\text{gyd}} + \gamma_{\text{rad}} \) and the cooperation parameter \( \eta = \gamma_{\text{gyd}}/\gamma_{\text{rad}} \). In general these characteristics depend on the radius of the fiber, the orientation of the dipole and the position of the atom [2, 4]. We display in figure 2 the numerical results for the case of a radially oriented dipole with the transition wavelength \( \lambda_0 = 852 \text{ nm} \) and the radial dipole orientation are used.

![Figure 1](https://example.com/figure1.png)  
*Figure 1.* Linear array of atoms in the vicinity of a nanofiber.

![Figure 2](https://example.com/figure2.png)  
*Figure 2.* Atomic decay and cooperativity parameters as functions of the fiber radius \( a \) (left panel) and the atom–surface distance \( r - a \) (right panel). (a) The total decay rate \( \gamma \) (solid lines) and the contributions \( \gamma_{\text{gyd}} \) (dashed lines) and \( \gamma_{\text{rad}} \) (dotted lines) from the radiation and guided modes, respectively. The rates are normalized to the atomic natural linewidth \( \gamma_0 \). (b) The cooperativity parameter \( \eta = \gamma_{\text{gyd}}/\gamma_{\text{rad}} \). The transition wavelength \( \lambda_0 = 852 \text{ nm} \) and the radial dipole orientation are used.
for our calculations because it is the wavelength of the cesium D2-line transition, which was used in the experiments on atoms near a nanofiber [3]. We take the radial orientation for the atomic dipole because, according to the results of [4] for a two-level atom, such an orientation leads to the strongest enhancement of the decay rates \( \gamma_{\text{gyd}} \), \( \gamma_{\text{rad}} \) and \( \gamma \). The left panel of figure 2 shows that the decay rate into the guided modes \( \gamma_{\text{gyd}} \) and the cooperativity parameter \( \eta \) achieve their largest values when the fiber radius \( a \) is about 200 nm. This is in agreement with the results of [2] for a realistic cesium atom. The right panel of figure 2 shows that the decay rates \( \gamma_{\text{gyd}} \) and the cooperativity parameter \( \eta \) can be significant when the atom is close enough to the fiber surface. For example, when the atom–surface distance is \( r - a = 100 \text{ nm} \), we obtain \( \gamma_{\text{gyd}} = 0.26 \gamma_0 \), \( \gamma_{\text{rad}} = 1.06 \gamma_0 \) and \( \gamma = 1.32 \gamma_0 \), leading to \( \eta = 0.25 \). Here, \( \gamma_0 \) is the atomic natural linewidth, whose magnitude is about 5.3 MHz in the case of the cesium D2 line. The significant values \( \gamma_{\text{gyd}} = 0.26 \gamma_0 \) and \( \eta = 0.25 \) show that the atom can efficiently radiate into the guided modes and can effectively cooperate with the other atoms in the array over long distances. The rate of decay into the guided modes \( \gamma_{\text{gyd}} \) and the cooperativity parameter \( \eta \) can achieve more substantial values when the atom is closer to the fiber surface. However, when the atom is very near to the surface, the effect of the surface-induced potential must be taken into account. We emphasize that the energy transfer by the equation

\[
\Gamma = \gamma_{\text{rad}} + N \gamma_{\text{gyd}}.
\]

This rate is enhanced [5] by the cooperativity of the atoms via the guided modes. The above solution yields the total excited-state population \( P = e^{-\Gamma t} \) and the intensity of emission into the guided modes

\[
I_{\text{gyd}} = h \omega_0 N \gamma_{\text{gyd}} e^{-\Gamma t}.
\]

Hence the energy emitted into the guided modes is \( U_{\text{gyd}} = h \omega_0 N \gamma_{\text{gyd}} \Gamma' \). Meanwhile, the total emitted energy is \( U = h \omega_0 \). Consequently, the fraction of energy emitted into the guided modes is

\[
f_{\text{gyd}} = \frac{N \gamma_{\text{gyd}}}{\gamma_{\text{rad}} + N \gamma_{\text{gyd}}}.
\]

It is clear that \( f_{\text{gyd}} \) increases with increasing atom number \( N \) and that \( f_{\text{gyd}} \to 1 \) in the limit \( N \to \infty \). Thus the efficiency of channeling of emission from the atoms into the fiber is cooperatively enhanced. We use equation (6) to calculate \( f_{\text{gyd}} \) as a function of \( N \) and display the results in figure 3. For \( N = 100 \) and \( r - a = 100 \text{ nm} \), we obtain \( f_{\text{gyd}} \geq 0.92 \) (see the endpoints of the curves). In particular, for \( N = 100 \) and \( r - a = 100 \text{ nm} \), the factor \( f_{\text{gyd}} \) reaches the value 0.96 (see the endpoint of the dashed curve). Such a high efficiency indicates that the single photon emitted from the atoms is almost entirely directed into the guided modes. A very similar result has been obtained for the superradiance of atoms in an optical cavity [9]. Indeed, in terms of the cooperativity parameter \( \eta = \gamma_{\text{gyd}}/\gamma_{\text{rad}} \) the channeling efficiency \( f_{\text{gyd}} \) given by

\[
\eta = \gamma_{\text{gyd}}/\gamma_{\text{rad}}
\]
equation (6) coincides with the success probability $P = N\eta/(1 + N\eta)$ for conversion in the cavity case [9]. Such a coincidence is due to the fact that the nanofiber mode and the cavity mode have many common features. We note that at the distance of 100 nm from the surface of the 200-nm-radius fiber, the cooperativity parameter is $\eta = 0.25$ (see figure 2). This value is substantially larger than the value $\eta = 6.9 \times 10^{-3}$ for a moderate-finesse cavity [9].

We emphasize that if the initial one-excitation state is not a symmetric, entangled state but simply an asymmetric product state with a single excited atom and $N - 1$ unexcited atoms, then we have $f_{gyd} = \gamma_{gyd}/(\gamma_{rad} + N\gamma_{gyd})$. This factor is reduced with increasing $N$. This is a signature of subradiance in the system.

### 3.2. Coherent product state

We now consider the case where all the atoms are initially prepared in the same coherent superposition state, that is, the initial state of the atoms is the product state $|\Psi\rangle = \prod_j (|e_j\rangle + e^{i\phi} |g_j\rangle)$. Such a state can be prepared by using a plane-wave optical pulse to excite the atoms.

In order to get insight into the case of large $N$ we make an approximation for the last term in equation (3). For the initial product state $|\Psi\rangle$ we have $<\sigma^g_1\sigma^e_j> = -P(N - P)/N^2$ for every pair $j \neq j'$. We assume that this relation is valid for the whole emission process. Such an assumption is reasonable under the condition $N \gg N - P_0 \gg 1$ [6]. With this assumption equation (3) yields

$$\frac{dP}{dt} = -P \left[\gamma + (1 - N^{-1})\gamma_{gyd}(N - P)\right].$$

The solution to the above equation, subject to the initial condition $P(0) = P_0$, is

$$P = N(\kappa + 1)/[\kappa + e^{\Gamma}\gamma],$$

where

$$\kappa = (N - 1)\gamma_{gyd}/\gamma$$

and

$$t_a = \tau \ln[(\kappa + 1)(N/P_0) - \kappa], \quad \text{with} \quad \tau = \Gamma^{-1}$$

The intensity of emission into the guided modes is

$$I_{gyd} = h\omega_0 N\sqrt{\kappa + 1}/\kappa \times \left[\gamma(\kappa + 1) e^{\Gamma t_a}/(\kappa + e^{\Gamma t_a}) - \gamma_{rad}\right].$$

(8)

If $t_a < t_p$, where $t_p = \tau \ln[(1 - N^{-1})(2 + (N - 2)(\gamma_{gyd}/\gamma))]$, then the intensity $I_{gyd}(t)$ has a local peak with the height $I_{gyd}^{\max} = h\omega_0\gamma_{gyd}N^3/4(\gamma_{gyd}/\gamma)$ at the time $t = t_{a} = t_p - t_a$.

Otherwise, the function $I_{gyd}(t)$ monotonically decreases from its initial value $I_{gyd}(0) = h\omega_0 P_0 N\gamma_{gyd} + (N - 1)(1 - P_0/N)$.

It follows from equation (8) that the energy emitted into the guided modes is

$$U_{gyd} = h\omega_0 P_0 \left[1 - \frac{N\gamma_{gyd}}{P_0\gamma} \frac{1}{\kappa + 1} \ln \frac{1}{1 + (1 - P_0/N)\kappa}\right].$$

(9)

### 4. Conclusion

In conclusion, we have shown the possibility of directional guided superradiance from a linear array of distant atoms that are separated by one or several wavelengths in a line parallel to the axis of a nanofiber. The rate of emission is enhanced by the cooperativity of the atoms via the guided modes. The efficiency of channeling of emission into the guided modes...
increases with increasing atom number and approaches unity in the limit of large numbers of atoms.

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