Effects of the Cattaneo–Christov heat flux model on peristalsis

A. Tanveera, S. Hina, T. Hayat, M. Mustafa and B. Ahmadd

*Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan; bDepartment of Mathematical Sciences, Fatima Jinnah Women University, Rawalpindi, Pakistan; cDepartment of Mathematics, National University of Sciences and Technology, Islamabad, Pakistan; dDepartment of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia

**ABSTRACT**

This paper addresses the influence of newly-developed Cattaneo–Christov heat flux model on peristalsis. Analysis has been carried out in a two-dimensional planner channel with wall properties and the Soret effect. An incompressible viscous fluid fills the space inside the channel. The relevant mathematical modeling is developed and a perturbation technique is employed to obtain a series form of solutions about small wave numbers. Expressions of velocity, temperature, concentration and heat transfer are treated graphically, corresponding to elasticity parameters, relaxation time and Prandtl numbers specifically. The graphical results are found distinctive that offers challenging role for further research on the topic. Further, the results of Fourier’s law can be verified when the relaxation time of the Cattaneo–Christov heat flux model is considered absent or concepts of large wavelength and small Reynolds numbers are applied.

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**1. Introduction**

There is growing interest in the study of peristalsis due to its occurrence in extensive environmental, industrial and physiological processes. Some prominent processes, among many others, include the movement of chyme in the digestive tube, the movement of spermatozoa and ova in the cervical canal and fallopian tubes, the carrying of lymph in vessels, the transfer of urine to the bladder, the mixing of food, water transport from the ground to tall trees, and the spontaneous movement of blood vessels. Peristaltic transport is generated through periodic waves travelling beside the walls – thus the peristaltic flow can be developed in tubes or channels. The study of peristaltic activity was initiated by Latham (1966), after which the experimental analysis of Shapiro, Jafferin, and Weinberg (1969) formalized the validity of the theoretical results. Elnaby and Haroun (2008) addressed peristaltic transport with compliant wall effects of viscous fluid, while Vasudev, Rao, Reddy, and Rao (2010) examined the heat transfer phenomenon on peristalsis. In two separate studies, Tripathi and Beg (2014a, 2014b) investigated the peristaltic flow of nanofluid and the peristaltic flow of Berger’s fluid through a porous medium. Abd elmaboud and Mekheimer (2011) presented work on second-order non-linear peristaltic flow, while Ali and Hayat (2007) investigated peristaltic motion in an asymmetric channel. The peristaltic flow comprising copper-water nanofluid has been investigated by Abbasi, Hayat, and Ahmad (2015), while Hayat, Tanveer, Yasmin, and Alsaadi (2015) explored the outcomes of Hall currents and thermal deposition on the peristaltically generated flow of Eyring–Powell fluid. Further, the channel walls possess some elastic properties due to their compliant and damping nature. In physical situations such as the movement of blood through vessels and tissues, tension and the damping of walls play an essential role. The instabilities of a plane channel flow induced by a peristaltic wave bounded by compliant walls have been investigated under a large wavelength for small Reynolds numbers by many researchers in recent years (for some representative studies, see Gad, 2014; Javed, Hayat, & Alsaedi, 2014; Riaz, Nadeem, Ellahi, & Akbar, 2014). The compliant wall effects on peristaltic transport have also been reported by Hina, Hayat, Asghar, and Hendi (2012). Some authors have presented their research on the fluid flows followed by peristaltic activity using a numerical approach. Ali, Javid, Zaman, and Hayat (2016) addressed the heat transfer phenomenon in a curved channel using numerical analysis, while Abbasi, Hayat, and Alsaedi (2015) conducted a numerical study on the Hall current effects with magnetohydrodynamics (MHD) Carreau–Yasuda fluid in a curved flow.
configuration. Mustafa, Abbasbandy, Hina, and Hayat (2014) numerically examined the Soret and Dufour effects of peristaltic flow. Moreover, some interesting studies relevant to computational fluid dynamics (CFD) are mentioned in Zhang, Huang, Zhang, Zou, and Tang (2016), Fu, Uddin, and Curley (2016) and Özkhan, Wenka, Hansjosten, Pfeifer, and Kraushaar-Czarnetzki (2016).

The transfer of heat is a widespread phenomenon that occurs due to the difference in temperature between a system and its environments. Whenever there is a difference in temperature between two objects, heat starts propagating from the higher temperature field to the lower temperature field. Considerable attention has been devoted to studying the behavior of the heat transport mechanism. Some authors have analyzed the heat transfer mechanism through the empirical law of conduction known as Fourier’s law. In particular, Turkylmazoglu and Pop (2013) reported heat transfer effects with Jeffrey fluid, while Jalil, Asghar, and Imran (2013) found the heat transfer mechanism on the moving surface of a free stream. Hayat, Rafiq, Ahmad, and Yasin (2015) looked at the impact of melting heat transfer on peristalsis with thermal radiation and Joule heating, and Tripathi (2013) presented a transient heat flow analysis. Further, Ali, Sajid, Javed, and Abbas (2010) examined the heat transfer phenomenon in curved channel, while Sheikholeslami, Hatami, and Ganji (2014) carried out an analysis on nanofluid heat transfer with a magnetic field. However, the above-mentioned studies addressed the heat transfer mechanism using Fourier’s law. This law was considered to be sufficient to describe heat transfer analysis for two centuries. However, the vector field aspect of heat flux requires that the governing equations for heat transfer involve an objective time derivative. Cattaneo on the basis of Cattaneo’s law proposed a new model by adding a term of relaxation time in Fourier’s law called the modified Fourier heat conduction law. Cattaneo generalizes the modified Fourier heat conduction law by utilizing the Oldroyd’s upper-convected derivative and thus constitutes a single equation for temperature. After the development of the Cattaneo–Christov model, various attempts were made to verify fluid flow according to this law. Christov (2009) investigated the heat equation that describes the frame indifferent expression of the Maxwell–Cattaneo version of heat flux. Straughan (2010) presented the thermal relaxation effects of the Christov heat flux model and found that uniform convection switches to fluctuating convection with narrower channels. Han, Zheng, Li, and Zhang (2014) explored the heat transfer phenomenon of viscoelastic fluid under the Cattaneo–Christov theory, while Haddad (2014) employed the Cattaneo–Christov model to study the thermal instabilities in fluid saturating a porous medium. Mustafa (2015) explored the thermal relaxation aspects of the Cattaneo–Christov model in the flow of a rotating frame, while Khan, Mustafa, Hayat, and Alsaedi (2015) numerically investigated the model with an exponentially stretching surface. Hayat, Imtiaz, Alsaedi, and Almezal (2016) carried out a study for the MHD characteristics of fluid under the Cattaneo–Christov theory with homogeneous–heterogeneous reactions, and Salahuddin, Malik, Hussain, Bilal, and Awais (2016) analyzed the Cattaneo–Christov heat flux model numerically through variable viscosity.

In spite of the considerable importance of the heat transfer mechanism to peristalsis in physical and engineering systems, to date no study has been reported that encountered the heat flux model based on Cattaneo–Christov concepts. The motivation of this work is to predict the behavior of heat flow subject to the Cattaneo–Christov heat flux model under a sinusoidally induced peristaltic wave between compliant walls. An incompressible viscous fluid is employed in a channel to predict the relaxation time effects. Further to exploring the extended heat effects on the concentration field, the Soret effect is employed. Series solutions are obtained for a small wave number and the obtained results are discussed. It is found that convective results (obtained by Fourier’s law) have oscillatory behavior in terms of temperature and concentration. Moreover, in the limiting case, the exact results corresponding to Fourier’s law can be recovered.

2. Mathematical formulation

To begin with, the peristaltically induced flow of an incompressible viscous fluid bounded by horizontal walls with a separating distance of 2\(d\) is considered. The flow configuration is such that the fluid flows with speed \(c\) in the \(x\)-direction along the channel walls whereas the \(y\)-direction is transverse to it. Further, the elastic and damping nature of the wall are also taken into account. Through the following expression, the configuration of the wall can be analyzed:

\[
y = \pm \eta(x,t) = d + a \sin \frac{2\pi}{\lambda}(x - ct),
\]

where \(a\) is the wave amplitude, \(\lambda\) is the wavelength, \(d\) is the distance of the walls from the center of the channel, \(t\) is the time, and \(\eta\) and \(-\eta\) are the displacements of the upper and lower walls, respectively.

The fundamental equations are based on the conservation of mass, momentum, energy and concentration:

\[
\text{div}\mathbf{V} = 0.
\]
The momentum equation is
\[
\frac{\rho \, dv}{dt} = -\nabla p + \text{div} \, \mathbf{S},
\]
\[
\rho \frac{dT}{dt} = \nabla \cdot \mathbf{q},
\]
\[
\frac{dC}{dt} = D \nabla^2 C + \frac{DK_T}{T_m} (\nabla^2 T).
\]

The energy equation \( q \) can be obtained from the heat flux model proposed by Christov (2009), whereas the concentration equation encounters the thermo-diffusion or Soret effect. The mathematical expressions are as follows:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]
\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right),
\]
\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right).
\]

The Cattaneo–Christov heat flux model has the form
\[
q + \lambda_2 \left[ \frac{\partial q}{\partial t} + \nabla q - q \nabla V + (\nabla \cdot V) q \right] = -k \nabla T,
\]
where \( V = (u, v, 0) \) is the two-dimensional velocity of the viscous fluid, \( q \) is the heat flux, \( T \) is the temperature of the fluid, \( k \) is the thermal conductivity and \( \lambda_2 \) is the relaxation time parameter for the heat flux. When \( \lambda_2 = 0 \), the simplified expression of Fourier’s law can be deduced. Since we considered an incompressible fluid, the above expression takes the form
\[
q + \lambda_2 \left[ \frac{\partial q}{\partial t} + \nabla q - q \nabla V \right] = -k \nabla T.
\]

Upon the elimination of \( q \) between Equations (4) and (10), the required energy equation reads
\[
\rho c_p \left( \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) T + \rho c_p \lambda_2 \times \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x \partial y} + \frac{\partial^2 T}{\partial y \partial x} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right).
\]

The concentration equation (Equation (5)) is then formulated as:
\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{DK_T}{T_m} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right),
\]
where \( \rho \) is the fluid density, \( \mu \) is the dynamic viscosity of the fluid, \( v \) is the kinematic viscosity of the fluid, \( C \) is the concentration of the fluid, \( D \) is the mass diffusivity coefficient, \( c_p \) is the specific heat constant, \( k \) is the thermal conductivity, \( K_T \) is the thermal diffusion ratio coefficient and \( T_m \) is the mean temperature of the fluid.

The corresponding conditions at the boundary are given by
\[
T = T_1, \quad C = C_1 \text{ at } y = \eta, \quad (13)
\]
\[
T = T_0, \quad C = C_0 \text{ at } y = -\eta. \quad (14)
\]

Here, \( T_0 \) and \( C_0 \) represent the prescribed values of temperature and concentration at the lower channel walls and \( T_1 \) and \( C_1 \) represent the prescribed values of temperature and concentration at the upper channel walls.

The no-slip condition at the boundary has the form given below:
\[
u = 0, \quad \text{at } y = \pm \eta,
\]
\[
\left[ -\tau \frac{\partial^3 q}{\partial x^3} + m_1^* \frac{\partial^3 q}{\partial x \partial t^2} + d^* \frac{\partial^2 q}{\partial t \partial x} \right] \eta = \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right), \text{ at } y = \pm \eta, \quad (15)
\]

where \( \tau \) is the coefficient of elastic tension, \( m_1^* \) is the coefficient of the mass per unit area, and \( d^* \) is the coefficient of the viscous damping.

On combining equations (7) and (8) by taking partial derivative of equation (7) with respect to \( x \) and partial derivative of equation (8) with respect to \( y \) and then subtracting the resultant expression we get
\[
\rho \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial y} - v \frac{\partial}{\partial x} \right) \left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) = \mu \left[ \frac{\partial}{\partial y} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\partial}{\partial x} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \right]. \quad (16)
\]

The stream function \( \psi(x, y, t) \) and the dimensionless variables are given in the following definitions:
\[
\psi^* = \frac{\psi}{\lambda}, \quad x^* = \frac{x}{\lambda}, \quad y^* = \frac{y}{\lambda}, \quad t^* = \frac{ct}{\lambda}, \quad \eta^* = \frac{\eta}{\lambda}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad \phi = \frac{C - C_0}{C_1 - C_0}, \quad p^* = \frac{d^2 p}{\mu \lambda c}. \quad (18)
\]
The dimensionless equations – Equations (11), (12) and (17) – are obtained as follows:

\[ \delta R \left[ \left( \frac{\partial}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) \left( \frac{\partial^2 \psi}{\partial y^2} + \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right) \right] = 2\delta^2 \frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial y^4} + \delta^4 \frac{\partial^4 \psi}{\partial x^4}, \]  

(19)

\[ \delta Pr \left( \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) + \gamma \psi + \delta^2 Pr R \left[ \left( \frac{\partial^2 \theta}{\partial t^2} + \frac{\partial^2 \psi}{\partial y \partial t} \frac{\partial}{\partial x} + \frac{2}{\delta} \frac{\partial \psi}{\partial t} \frac{\partial^2 \theta}{\partial x^2} - 2 \frac{\partial \psi}{\partial y} \frac{\partial^2 \theta}{\partial x \partial y} \right) + \frac{\partial^2 \psi}{\partial y^2} \frac{\partial \theta}{\partial x} + \frac{\partial^2 \theta}{\partial y^2} \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \theta}{\partial y^2} \right] \right) \times \left( \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y \partial x} \right) = \frac{1}{Sc} \left( \delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right), \]  

(20)

\[ \delta R \left( \frac{\partial \phi}{\partial t} + \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} \right) = \frac{1}{Sc} \left( \delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right), \]  

(21)

with the conditions

\[ \eta = 1 + \varepsilon \sin 2\pi (x - t), \quad \theta = 1, \quad \phi = 1, \quad \text{at} \quad y = \eta, \]  

(22)

\[ \theta = 0, \quad \phi = 0, \quad \text{at} \quad y = -\eta, \]  

(23)

\[ \psi_0 = 0 \quad \text{at} \quad y = \pm \eta, \]  

(24)

\[ \left[ E_1 \frac{\partial^3 \phi}{\partial x^3} + E_2 \frac{\partial^3 \phi}{\partial x \partial t^2} + E_3 \frac{\partial^2 \phi}{\partial x \partial t} \right] \eta = -R\delta \left( \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) \left( \frac{\partial \psi}{\partial y} \right) + \frac{\partial^3 \psi}{\partial y^3} + \delta^2 \frac{\partial^3 \psi}{\partial x^2 \partial y}\right]. \]  

(26)

The solutions of the above equations subject to the corresponding boundary conditions are

\[ \psi_0 = \frac{L}{6} y^3 - \frac{L}{2} \eta^3, \]  

(36)

\[ \theta_0 = \frac{y + \eta}{2 \eta}, \]  

(37)

\[ \phi_0 = \frac{y + \eta}{2 \eta}, \]  

(38)

and the heat transfer coefficient by the definition

\[ Z_0 = \eta_x \frac{\partial \theta_0}{\partial y}. \]  

(39)
It should be noted that Equation (38) is identical to the result in El-Nabawy and Haroun (2008), which was obtained in the absence of heat and mass transfer.

### 3.2. First-order system

\[
R \left[ \left( \frac{\partial}{\partial t} + \frac{\partial \psi_0 \partial}{\partial y \partial x} - \frac{\partial \psi_0 \partial}{\partial x \partial y} \right) \frac{\partial^2 \psi_0}{\partial y^2} \right] = \frac{\partial^4 \psi_1}{\partial y^4}, \tag{40}
\]

\[
R \left[ \left( \frac{\partial}{\partial t} + \frac{\partial \psi_0 \partial}{\partial y \partial x} - \frac{\partial \psi_0 \partial}{\partial x \partial y} \right) \theta_0 \right] = \frac{1}{Pr} \frac{\partial^2 \theta_1}{\partial y^2}, \tag{41}
\]

\[
R \left[ \left( \frac{\partial}{\partial t} + \frac{\partial \psi_0 \partial}{\partial y \partial x} - \frac{\partial \psi_0 \partial}{\partial x \partial y} \right) \phi_0 \right] = \frac{1}{Sc} \left( \frac{\partial^2 \phi_1}{\partial y^2} \right) + \text{Sr} \left( \frac{\partial^2 \theta_1}{\partial y^2} \right), \tag{42}
\]

And the solution

\[
\psi_1 = A_2 y + A_4 y^3 + R \left[ \frac{L \eta_t}{2520} y^2 + \left( \frac{L_t + L^2 \eta_x}{120} \right) y^5 \right], \tag{47}
\]

\[
\theta_1 = R \text{Pr} \left[ \left( \frac{L \eta_t}{4 \eta^2} + \frac{L_t}{12 \eta} \right) \left( y^5 - \eta^4 y^2 \right) \right] - \left( \frac{\eta_t}{2 \eta^2} + \frac{3L \eta_t}{4} - \frac{L \eta}{4} \right), \tag{48}
\]

\[
\phi_1 = -\frac{RScy}{240 \eta^2} \left[ \frac{L \eta(y^2 - 5 \eta^2)^2 + L \eta_t \eta \eta(t + 10 \eta^2 \eta^2 - 35 \eta^4) + \eta_t(20 y^2 + \text{Pr} \eta \eta \eta(t - 2 + 3 \eta^2 y^2) - 60 \eta^2 - 30 \eta^2 \text{Pr} \eta \eta \eta(t + 2 + 3 \eta^2)}{240 \eta^2} \right], \tag{49}
\]

And the heat transfer coefficient

\[
Z_1 = \eta_x \frac{\partial \theta_1}{\partial y}, \tag{50}
\]

where

\[
L = 8 \pi^3 \epsilon \left[ \frac{E_3}{2 \pi} \sin 2 \pi (x - t) - (E_1 + E_2) \cos 2 \pi (x - t) \right],
\]

\[
\eta = 1 + \epsilon \sin 2 \pi (x - t),
\]

\[
A_2 = \frac{R}{2} \left[ \frac{5 L \eta^4 - 23 L \eta^2 \eta^4 + L \eta^3 \eta_t - 7 L \eta^5 \eta_x}{90} \right],
\]

\[
A_4 = -\frac{R}{6} \left[ \frac{L \eta \eta_t + L \eta^2}{2} - \frac{L L \eta^4}{4} - \frac{L^2}{2} \eta^3 \eta_x \right].
\]

Here \(L, L_x, \eta_t, \eta_x, L_{lt}, L_{lx}, \eta_{lt}, \eta_{lx}, \eta_{xt}, \eta_{xx}, A_{lt}, A_{lx}, A_{lt} \), and \(A_{lx} \) represent partial derivatives with respect to the corresponding subscripts.

### 3.3. Second-order system

\[
\frac{\partial^4 \psi_2}{\partial y^4} + 2 \frac{\partial^2 \psi_0}{\partial x^2 \partial y^2} = R \left[ \frac{\partial^3 \psi_1}{\partial t \partial y^2} + \frac{\partial \psi_1}{\partial y} \frac{\partial^3 \psi_0}{\partial x \partial y^2} - \frac{\partial \psi_1}{\partial x} \frac{\partial^3 \psi_0}{\partial y^3} \right] + \frac{\partial \psi_0}{\partial y} \frac{\partial^3 \psi_1}{\partial x \partial y^2} - \frac{\partial \psi_0}{\partial x} \frac{\partial^3 \psi_1}{\partial y^3} \right], \tag{51}
\]

\[
\frac{\partial \psi_0}{\partial y} = \frac{\partial \psi_1}{\partial y} + \gamma \left( \frac{\partial^2 \theta_0}{\partial y \partial t} + \frac{\partial \psi_0}{\partial y} \frac{\partial \psi_0}{\partial x} \right) - 2 \frac{\partial \psi_0}{\partial y} \frac{\partial^2 \theta_0}{\partial t \partial x} - 2 \frac{\partial \psi_0}{\partial y} \frac{\partial^2 \theta_0}{\partial t \partial y} - 2 \frac{\partial \psi_0}{\partial x} \frac{\partial^2 \theta_0}{\partial t \partial x} - 2 \frac{\partial \psi_0}{\partial x} \frac{\partial^2 \theta_0}{\partial t \partial y} \tag{52}
\]

\[
\frac{\partial \psi_0}{\partial y} \frac{\partial^2 \psi_0}{\partial x^2} + \frac{\partial \psi_0}{\partial y} \frac{\partial^2 \psi_0}{\partial x \partial y} + \frac{\partial \psi_0}{\partial x} \frac{\partial^2 \psi_0}{\partial x \partial y} + 2 \frac{\partial \psi_0}{\partial x} \frac{\partial^2 \theta_0}{\partial x^2} + \left( \frac{\partial \psi_0}{\partial x} \frac{\partial^2 \theta_0}{\partial y^2} \right) = \frac{1}{Pr} \left( \frac{\partial^2 \theta_0}{\partial x^2} + \frac{\partial^2 \theta_2}{\partial y^2} \right), \tag{53}
\]

\[
\frac{\partial \psi_0}{\partial y} \frac{\partial^2 \psi_0}{\partial x^2} + \frac{\partial \psi_0}{\partial y} \frac{\partial^2 \psi_0}{\partial x \partial y} + \frac{\partial \psi_0}{\partial x} \frac{\partial^2 \psi_0}{\partial x \partial y} + 2 \frac{\partial \psi_0}{\partial x} \frac{\partial^2 \theta_0}{\partial x^2} + \left( \frac{\partial \psi_0}{\partial x} \frac{\partial^2 \theta_0}{\partial y^2} \right) = 1 \frac{1}{Pr} \left( \frac{\partial^2 \theta_0}{\partial x^2} + \frac{\partial^2 \theta_2}{\partial y^2} \right), \tag{54}
\]

\[
\frac{\partial \psi_2}{\partial y} = 0, \text{ at } y = \pm \eta,
\]

\[
\theta_2 = 0, \phi_2 = 0, \text{ at } y = \eta,
\]

\[
\theta_2 = 0, \phi_2 = 0, \text{ at } y = -\eta,
\]
\[
\frac{\partial^3 \psi_2}{\partial y^3} + \frac{\partial^3 \psi_0}{\partial x^2 \partial y} - R \left[ \left( \frac{\partial}{\partial t} + \frac{\partial \psi_0}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi_0}{\partial x} \frac{\partial}{\partial y} \right) \left( \frac{\partial \psi_1}{\partial y} \right) \right] = 0, \text{ at } y = \pm \eta, \quad (56)
\]

along with the solutions

\[
\begin{align*}
\psi_2 &= \frac{1}{19958400} \times \left[ B_1 y + B_2 y^3 + B_3 y^5 + B_4 y^7 + B_5 y^9 + B_6 y^{11} \right], \quad (57) \\
\theta_2 &= C_1 y + C_2 y^3 + C_3 y^5 + C_4 y^7 + C_5 y^9, \quad (58) \\
\phi_2 &= C_6 y + C_7 y^3 + C_8 y^5 + C_9 y^7 + C_{10} y^9, \quad (59) \\
Z_2 &= \eta_x - \frac{\partial \theta_2(\eta)}{\partial y}. \quad (60)
\end{align*}
\]

The quantities \(B_1 - B_6\) and \(C_1 - C_{10}\) can be determined through algebraic calculations.

4. Results and discussion

To analyze the heat transfer effects on peristalsis we applied the concept of the Cattaneo–Christov heat flux model to the flow of viscous fluid. The stream function equation is comprised of the wall properties, whereas the pair of equations are made for temperature and concentration distribution, and the concentration distribution equation covers the Soret effect. The variations in the fluid velocity \(u\), temperature \(\theta\), concentration \(\phi\) and coefficient of heat transfer \(Z\) are described in this section.

Figure 1 presents the increasing velocity with the wall elastic parameters \(E_1\) and \(E_2\) due to their elastic behavior, whereas an increase in the damping parameter \(E_3\) reduces the velocity. Figures 2–4 show the increase in velocity with increasing values of the wave number \(\delta\), Reynolds number \(R\) and amplitude ratio parameter \(\varepsilon\).

**Figure 1.** Profile of \(u\) for different values of wall parameters when \(\varepsilon = 0.2, x = 0.2, t = 0.1, \delta = 0.1\) and \(R = 0.01\).

**Figure 2.** Profile of \(u\) for different values of wave number when \(\varepsilon = 0.1, x = 0.2, t = 0.1, E_1 = 0.1, E_2 = 0.1, E_3 = 0.01\) and \(R = 3\).

**Figure 3.** Profile of \(u\) for different values of Reynolds number when \(\varepsilon = 0.1, x = 0.2, t = 0.1, E_1 = 0.1, E_2 = 0.1, E_3 = 0.01\), and \(\delta = 0.1\).

**Figure 4.** Profile of \(u\) for different values of amplitude ratio parameter when \(\delta = 0.1, x = 0.2, t = 0.1, E_1 = 0.1, E_2 = 0.1, E_3 = 0.01\) and \(R = 1\).

Figures 5–21 show the results of oscillatory response of graphs corresponding to temperature, concentration and heat transfer coefficient. It is anticipated that the consideration of the extended heat flux model
Figure 5. Profile of $\theta$ for different values of wall parameters when $\varepsilon = 0.2, x = 0.1, t = 0.1, \gamma = 1, \delta = 0.1$ and $R = 1$.

(Cattaneo–Christov model) is responsible for these results. From Figure 5 it can be seen that the behavior of wall properties towards temperature profile is increasing towards $E_1$ and $E_2$, while an increase in $E_3$ lowers the fluid temperature. Figure 6 represents a decrease in

the temperature profile with increasing values of Pr near the upper wall, as larger values of Pr reduce the thermal conductivity of the fluid. The results are found to match well with those obtained by Mustafa (2015) and

Figure 6. Profile of $\theta$ for different values of Prandtl number when $\varepsilon = 0.2, x = 0.1, t = 0.1, \gamma = 1.5, E_1 = 0.1, E_2 = 0.1, E_3 = 0.01, \delta = 0.2$, and $R = 1$.

Figure 7. Profile of $\theta$ for different values of relaxation time parameter when $\varepsilon = 0.2, x = 0.1, t = 0.1, E_1 = 0.1, E_2 = 0.1, E_3 = 0.01, \delta = 0.1, Pr = 1$, and $R = 1$.

Figure 8. Profile of $\theta$ for different values of wave number when $\varepsilon = 0.2, x = 0.1, t = 0.1, E_1 = 0.1, E_2 = 0.1, E_3 = 0.01, \gamma = 1.5, Pr = 1$, and $R = 1$.

Figure 9. Profile of $\theta$ for different values of Reynolds number when $\varepsilon = 0.2, x = 0.1, t = 0.1, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, \gamma = 1.5, Pr = 1$, and $\delta = 0.1$.

Figure 10. Profile of $\phi$ for different values of Reynolds number when $\varepsilon = 0.2, x = 0.1, t = 0.1, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, \gamma = 1.5, Pr = 1, Sr = 2, Sc = 1$ and $\delta = 0.1$. 
Figure 11. Profile of $\phi$ for different values of wave number when $\varepsilon = 0.1$, $x = 0.1$, $t = 0.1$, $E_1 = 0.3$, $E_2 = 0.2$, $E_3 = 0.1$, $\gamma = 1.5$, $Pr = 1$, $Sr = 2$, $Sc = 1$ and $R = 1$.

Figure 12. Profile of $\phi$ for different values of Soret number when $\varepsilon = 0.2$, $x = 0.2$, $t = 0.1$, $E_1 = 0.3$, $E_2 = 0.2$, $E_3 = 0.1$, $\gamma = 1.5$, $Sr = 2$, $\delta = 0.2$, $Sc = 1$ and $R = 1$.

Figure 13. Profile of $\phi$ for different values of Schmidt number when $\varepsilon = 0.2$, $x = 0.2$, $t = 0.1$, $E_1 = 0.1$, $E_2 = 0.1$, $E_3 = 0.01$, $\gamma = 1.5$, $Pr = 1$, $\delta = 0.2$, $Sr = 2$ and $R = 1$.

Figure 14. Profile of $\phi$ for different values of Prandtl number when $\varepsilon = 0.2$, $x = 0.2$, $t = 0.1$, $E_1 = 0.3$, $E_2 = 0.2$, $E_3 = 0.1$, $\gamma = 1.5$, $Sr = 2$, $\delta = 0.2$, $Sc = 1$ and $R = 1$.

Figure 15. Profile of $\phi$ for different values of relaxation time parameter when $\varepsilon = 0.2$, $x = 0.1$, $t = 0.1$, $E_1 = 0.1$, $E_2 = 0.1$, $E_3 = 0.01$, $\delta = 0.2$, $Pr = 1$, $Sc = 1$, $Sr = 2$ and $R = 1$.

Figure 16. Profile of $\phi$ for different values of wall parameters when $\varepsilon = 0.1$, $x = 0.2$, $\delta = 0.2$, $Pr = 1$, $Sc = 1$, $Pr = 1$, $Sr = 2$ and $R = 1$.

Hayat et al. (2016) in terms of a single plate placed towards a positive axis in a semi-infinite domain. However, near the negative side of the channel the results are unique. Figure 7 analyzes the impact of the increasing relaxation time of heat flux $\gamma$ on the channel walls. For increasing values of $\gamma$ opposite behavior is noticed, i.e., an increase in $\gamma$ increases the temperature distribution towards the lower wall and decreases the temperature distribution towards the upper wall when the relaxation
Figure 17. Profile of $Z$ for different values of relaxation time parameter when $\varepsilon = 0.2$, $t = 0.1$, $\delta = 0.1$, $E_1 = 0.1$, $E_2 = 0.1$, $E_3 = 0.01$, $Pr = 1$ and $R = 1$.

Figure 18. Profile of $Z$ for different values of wall parameters when $\varepsilon = 0.1$, $t = 0.1$, $\delta = 0.1$, $\gamma = 1.5$, $Pr = 1$ and $R = 1$.

Figure 19. Profile of $Z$ for different values of Prandtl number when $\varepsilon = 0.2$, $t = 0.1$, $\delta = 0.1$, $E_1 = 0.1$, $E_2 = 0.1$, $E_3 = 0.01$, $\gamma = 1.5$ and $R = 1$.

Figure 20. Profile of $Z$ for different values of Reynolds number when $\varepsilon = 0.2$, $t = 0.1$, $\delta = 0.1$, $E_1 = 0.1$, $E_2 = 0.1$, $E_3 = 0.01$, $Pr = 1$ and $\gamma = 1.5$.

Figure 21. Profile of $Z$ for different values of wave number when $\varepsilon = 0.2$, $t = 0.1$, $E_1 = 0.1$, $E_2 = 0.1$, $E_3 = 0.01$, $\gamma = 1.5$, $Pr = 1$ and $R = 1$.

of the wave number $\delta$ and Reynolds number $R$ at the upper channel wall. Moreover, in absence of $\delta$ and $R$ we get straight lines corresponding to viscous fluid (see Figures 8 and 9). It is noted that by the concepts of a large wavelength and small Reynolds number (Haddad, 2014) the results of Fourier’s law can be retrieved, i.e., the Cattaneo–Christov theory becomes identical to Fourier’s law.

Figures 10 and 11 signify that larger values of $\delta$ and $R$ raise the concentration distribution around the upper wall of the channel. The impact of the Soret number $Sr$ on the concentration distribution is to cause an increase (Figure 12) since higher values of $Sr$ increase the density of the fluid. An increase in the concentration distribution is noticed for the Schmidt number $Sc$ as greater values of $Sc$ result in a decrease in the molecular diffusion that dominates the intermolecular forces between molecules, hence there is increase in the concentration (see Figure 13). The effect of the Prandtl number $Pr$ on
the concentration is to cause an increase because with an increase in Pr the viscosity increases (see Figure 14). Figure 15 exhibits the increasing response of the concentration profile towards the relaxation time \( \gamma \). The wall parameters have an effect on the concentration profile that is opposite to the effect of the temperature (Figure 16), i.e., \( E_1 \) and \( E_2 \) lessen the concentration of the fluid and \( E_3 \) increases it.

Figures 17–19 correspond to the effects of the relaxation time \( \gamma \), wall parameters \( E_1 \), \( E_2 \) and \( E_3 \), and Prandtl number Pr on the heat transfer coefficient \( Z \). The results show an increase in the concentration as a result of these parameters, except for the wall damping parameter \( E_3 \) which results in a decrease in the concentration (see Figure 18). Figures 20 and 21 present the increase in the heat transfer coefficient with growing values of the wave number \( \delta \) and Reynolds number \( R \).

4.1. Concluding remarks

A study of peristaltic motion with flexible walls occupying a viscous fluid composed of thermal convection effects satisfied by the Cattaneo–Christov heat flux model has been presented. In addition, the thermo-diffusion or Soret effect is also factored into the analysis. The results indicate that the thermal relaxation time plays a key role in the heat transfer process. Thus, an interesting relationship between the temperature and concentration has been found. The graphical results encourage further consideration of the Cattaneo–Christov theory with respect to peristalsis. This work can also be used to compare the results of the Cattaneo–Christov heat flux model to other work on peristaltic theory. The main findings are as follows:

- Velocity increases with the wave number and Reynolds number since the speed of the wave as well as its momentum diffusivity increases with increasing values of these parameters.
- The decrease on temperature and increase of concentration is noticed for Prandtl number.
- In contrast to Fourier’s law, the effect of the relaxation time parameter causes a decrease in temperature, but the concentration increases with \( \gamma \).
- The concentration rises where temperature falls with wave number and Reynolds number.
- Due to the generation of fluid viscosity and density, the concentration of the fluid increases as the Schmidt number and Soret number increase.
- The absolute heat transfer coefficient increases with an increase in the relaxation time and the Prandtl number.

Disclosure statement

No potential conflict of interest was reported by the authors.

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