Discrete gauge symmetries in supersymmetric grand unified models

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ABSTRACT

We investigate the presence of discrete gauge symmetries in Grand Unification models based in $SO(10)$ and $E_6$. These models include flipped and unflipped $SU(5)$, $SU(4) \times SU(2)_L \times SU(2)_R$, $SU(3)_C \times SU(3)_L \times SU(3)_R$, and $SU(6) \times SU(2)$. Using the Dynkin formalism we find the $U(1)$ subalgebras contained in the unified groups, give an expression for the Higgs fields that preserve each discrete symmetry, and determine the low-energy matter content implied by chirality. We discuss two $Z_2$ and three $Z_3$ nonequivalent cases. Among the possibilities found, only the usual $Z_2$ matter parity (R-parity) of supersymmetric extensions is consistent with a minimal matter content with no right-handed neutrinos, extra Higgs doublets, or nonstandard $down$-type quarks.
1. Introduction

In supersymmetric (SUSY) extensions of the standard model (SM) with minimal matter content it is possible to define dimension-four operators that violate lepton and baryon number \((L\) and \(B\)) [1]. If present, these terms would produce proton decay mediated by SUSY partners of quarks and leptons and other unobserved processes. To prevent this, one usually assumes the presence of a discrete symmetry of the superpotential known as matter parity [1,2]. The R-parity of the minimal extension (with \textit{even} standard fields and \textit{odd} SUSY partners), in particular, corresponds to the \(Z_2\) matter parity that changes the sign of quark and lepton superfields while leaving the two Higgs doublets unchanged. It has been recently suggested, however, that other \(Z_N\) symmetries forbidding the same trilinears in the superpotential [2], and even discrete symmetries allowing \(L\) or \(B\) violation (baryon and lepton parities [3]), may define phenomenologically consistent models.

A possible explanation for the origin of these discrete symmetries is provided by Grand Unification Theories (GUTs) [4], where they could appear in the following way as a remnant of a gauge symmetry. Suppose that the GUT Lie group contains an extra \(U(1)\), with gauge charges \(Q_i \in \mathbb{Z}\) and \(Q_H \neq 0\) for the standard superfields \(\Phi_i\) and GUT Higgs \(H\), respectively. If \(Q_H = 0 \mod N\), a vacuum expectation value (VEV) of \(H\) will break the \(U(1)\) factor but will imply an effective model still invariant under the gauge transformation \(\Phi_i \rightarrow \exp(i2\pi Q_i/N)\Phi_i\) \[\text{note that } \exp(i2\pi Q_H/N) = 1\], which in general defines a \(Z_N\) discrete symmetry. Moreover, it has been argued that this is the only type of global symmetry not anomalous with respect to gravitational (wormholes, etc.) effects [5].

It is well known that the usual R-parity may be obtained from models containing a \(U(1)_{B-L}\) factor, such as in \(SO(10)\). The conditions (based on the congruence class of the order parameters) for this to happen have been recently discussed in Ref. [6]. In addition, Ibáñez and Ross [3] have classified all the discrete symmetries of phenomenological interest in SUSY models, and have established consistency conditions that must be satisfied if these symmetries are to be a subgroup of a non-anomalous \(U(1)\) gauge symmetry.
In this paper we study some usual GUT scenarios, discussing all the gauged $Z_N$ symmetries that may appear in their effective low-energy limit. As we mentioned above, they are a consequence of the choice of Higgs fields used to break the extra gauge symmetry. We shall make extensive use of the Dynkin formalism [7,8] to identify and classify the GUT Higgs fields leading to each discrete symmetry. We also determine, for each case, the matter content implied by chirality under $SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_N$. Note that nonstandard fields that usually appear in GUT models in vectorlike representations of the SM gauge symmetry (right-handed neutrinos, pairs of down type quarks, etc.) may become here chiral due to the extra $Z_N$ factor.

The next section is devoted to $SO(10)$ and its subgroups [flipped and unflipped $SU(5)$ and the left-right symmetric models $SU(4) \times SU(2)^2$ and $SU(3) \times SU(2)^2 \times U(1)$]. We then consider in Section 3 models based on $E_6$, and make some remarks about other extensions.

2. Models based on $SO(10)$

$SO(10)$ is the simplest GUT gauge group containing the standard $SU(3) \times SU(2) \times U(1)$ group of symmetry with (i) chiral multiplets able to accommodate the spectrum of quarks and leptons and (ii) all its representations free of anomalies [4]. The 16 irreducible representation (or irrep) contains the standard fermions of one family plus a right-handed neutrino, while the electroweak Higgs fields are usually assigned to the vectorlike irreps 10, 120, and/or 126. After the extra symmetry is broken, this scenario is consistent with a minimal matter content of three families of quarks and leptons which are light because of chirality plus two Higgs doublets protected of heavy mass contributions by some other reason (this is the GUT hierarchy problem).

The Cartan subalgebra of $SO(10)$ has dimension five. That means that in $SO(10)$ there are a maximum of five simultaneously diagonalizable generators. In the Dynkin formalism [7,8] these generators label each element of an irrep by a set of five integers, or *weight vector*. The set of weight vectors in an irrep is easily obtained from the highest weight, which specifies the representation, by subtracting a
finite number of roots. The roots (in the basis of fundamental weights) correspond
to rows of the Cartan matrix, that can in turn be derived from the corresponding
Dynkin (or Coxeter-Dynkin) diagram. As we shall see, this formalism is particularly convenient for the analysis of \(U(1)\)'s and gauge discrete symmetries. Hereafter, we shall follow the notation of Ref. [8].

The \(16\) irrep of \(SO(10)\) has highest weight vector \((0 0 0 0 1)\); its weight system
is listed in Table I. We can define five independent charges as linear combinations
of the basis elements of the Cartan subalgebra and embed the SM in \(SO(10)\).
We consider \( A = [1 2 2 1 1] \); \( B = \frac{1}{\sqrt{3}}[-1 0 0 1 -1] \); \( C = \frac{1}{2}[0 0 1 1 1] \);
\( D = \frac{1}{3}[-2 0 3 -1 1] \); \( E = [2 0 2 1 -1] \), where they are specified in the dual basis
(the charge \(Q\) of a weight vector \(\lambda\) will be obtained from the scalar product \(Q \cdot \lambda\)).
The charges \(A\) and \(B\) are the two diagonal generators \(\lambda_3\) and \(\lambda_8\) of \(SU(3)C\), while
\(C\) is the standard weak isospin \(I^W_3\) of \(SU(2)L\). For the weak hypercharge \(Y\) one
may use \(Y = D\) or \(Y = -\frac{1}{5}(D + 2E)\). These two assignations of hypercharge
are equivalent in the sense that they are related by a Weyl reflection [7] of the root
system of \(SU(2)_R \subset SO(10)\), but they lead to different scenarios when \(SO(10)\) is
projected down to models with less symmetry. In particular, they imply unflipped
and flipped \(SU(5)\) [9], respectively. We define \(Y = D\) and, when discussing the
different subgroups of \(SO(10)\), we shall include the Weyl reflection in the projection
matrices (see below). The flavors contained in the \(16\) and \(10 = (1 0 0 0 0)\) irreps
are given in Table I and II.

The nonstandard \(U(1)\) in \(SO(10)\) is generated by any combination of charges
that contains \(E\). Following the procedure of Ref. [3], we use the weak hypercharge
to make zero the nonstandard charge \(Q_1\) of \(q \equiv (u d)\). Conveniently normalized,
\(Q_1\) is then given by

\[
Q_1 = -\frac{3}{5}D - \frac{1}{5}E = [0 0 -1 0 0].
\]  

The charges of quark, lepton, and Higgs superfields in the \(16\) and \(10\) irreps are
obtained from the product \(Q_1 \cdot \lambda\), where \(\lambda\) is the corresponding weight vector. We obtain
\[ Q_1(q, u^c, d^c, l, e^c, N) = (0, 1, -1, 0, -1, 1) , \]
\[ Q_1(h, h', D, D^c) = (-1, 1, 0, 0) , \]

where \( l \equiv (\nu, e) \), \( h \equiv (h^+ h^0) \), and \( h' \equiv (h'^0 h'^-) \). The notation in Eq. (2) should not be confused with that of a weight vector. Higgs doublets in the 120 or 126 irreps would have the same \( Q_1 \) charges.

The \( U(1) \) symmetry generated by \( Q_1 \) contains the generic \( Z_N \) discrete symmetry
\[
\Phi_i \rightarrow \exp \left( \frac{2\pi i}{N} Q_1 \right) \Phi_i ,
\]
whose action on all the fields in the 16 and 10 is given in Table III (there we also give the particular cases with \( N = 2 \) and \( N = 3 \)). The \( Z_N \) symmetry will survive to low energy if the Higgs \( H \) that breaks the extra \( U(1) \) satisfies
\[
Q_1(H) = 0 \mod N .
\]

Note that one needs at least one Higgs with \( Q_1(H) \neq 0 \) to break the \( U(1) \) and reduce the rank from five to four. On the other hand, \( H \) must be neutral with respect to the SM symmetry; imposing zero \( A, B, C, \) and \( D \) charges we find the expression for the weight vector of a generic GUT Higgs in \( SO(10) \):
\[
H_n = (-n \ n \ -n \ 0 \ n) ,
\]
where \( n \) is an integer. The charge \( Q_1 \) [see Eq. (1)] of such a field is
\[
Q_1(H_n) = n .
\]

A VEV \( <H_1> \) would break all of the possible \( Z_N \) symmetries (\( Z_1 \) is the identity). For \( n > 1 \), \( <H_n> \) breaks the extra \( U(1) \) of \( SO(10) \) while leaving unbroken a \( Z_n \) discrete symmetry (or, more precisely, a \( Z_N \) with \( n = 0 \mod N \)).

We find that the smallest (i.e. lowest dimensional) irrep containing \( H_n \) has highest weight
\[
\Lambda_n = (0 \ 0 \ 0 \ 0 \ n) ,
\]
and dimension
\[
\text{dim } \Lambda_n = (1 + n)(1 + \frac{n}{2})(1 + \frac{n}{3})^2(1 + \frac{n}{4})^2(1 + \frac{n}{5})^2(1 + \frac{n}{6})(1 + \frac{n}{7}).
\]  

(8)

Note that each $H_n$ is contained in many different irreps. We list in Table IV some irreps (their highest weight and the dimension) containing $H_n$ for $n = 1, 2, 3$. If, for example, one uses the SM singlet $H_1 = (-1 \ 1 \ -1 \ 0 \ 1)$ in the $16 = (0 \ 0 \ 0 \ 0 \ 1)$ representation to break $SO(10)$, no discrete symmetry survives, whereas using the adequate weights in the $126 = (0 \ 0 \ 0 \ 2)$ and $672 = (0 \ 0 \ 0 \ 3)$ one obtains the $Z_2$ and the $Z_3$ symmetries in Table III, respectively. Obviously, the GUT Higgs field $H_0 = (0 \ 0 \ 0 \ 0)$, like the singlets in the $45 = (0 \ 1 \ 0 \ 0 \ 0)$, $54 = (2 \ 0 \ 0 \ 0 \ 0)$ and $210 = (0 \ 0 \ 0 \ 1 \ 1)$ irreps of $SO(10)$, do not break any discrete symmetry. However, their VEVs do not reduce the rank of the gauge group either (they would lead to intermediate rank-five models).

Each of the $Z_N$ symmetries implies a definite spectrum of light fields and a different set of couplings in the superpotential. In particular, a Majorana mass term for the right handed neutrino in the $16$ transforms under $Z_N$ like $\alpha^{-2}$, and it is forbidden for $N \neq 2$. Consequently, $SO(10)$ models with an unbroken $Z_{N>2}$ symmetry will include one non-weakly-interacting neutrino per family. Note that the down type quarks $D$ and $D^c$ (and, in general, all the fields in the multiplet of the electroweak Higgs) may receive heavy mass contributions of order $<H_n>$ and should not appear in the effective low energy model.

Considering the terms in the superpotential $P$, we find that all the $Z_N$ symmetries allow the standard Yukawa couplings necessary to give mass to quarks and leptons,
\[
P_{\text{MSSM}} = y_u \ qu^c h + y_d \ qd^c h' + y_e \ le^c h' + \mu \ hh',
\]
and forbid the $B$ and $L$ violating terms
\[
P' = y_1 \ u^c d^c d^c + y_2 \ qd^c l + y_3 \ lle^c + \mu' \ lh .
\]

(10)

The $Z_2$ discrete symmetry in Table III is equivalent to the usual matter parity, which would be obtained from the product with the $Z_2$ symmetry in $U(1)_Y$. 

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\((q, u^c, d^c, l, e^c, h, h') \rightarrow (-q, u^c, d^c, -l, e^c, -h, -h')\). The differences between \(Z_N\) models will only appear in higher dimensional operators (effective nonrenormalizable terms) \([2]\) and in the couplings of the extra neutrino \(N\). The model with \(Z_3\) symmetry, for example, will contain in \(P\)

\[ P_3 = y_\nu \, l h N + \lambda \, N N N . \]  

(11)

The couplings \(y_\nu\) must be small enough since they give Dirac masses to the SM neutrinos. The trilinear \(N^3\) violates \(L\) \([lhN\) defines \(L(N) = -1\)] and \(R\)-parity, allowing the \(L\)-violating decay of the lightest SUSY particle (LSP) into quarks and leptons. Some phenomenological implications of models with \(Z_3\) matter parity have been explored in Refs. \([2,10]\).

The group \(SO(10)\) includes \(G_1 = SU(5) \times U(1)\), \(G'_1 = SU(5) \subset G_1\), \(G_1^R = SU(5) \times U(1)_{flipped}\), \(G_2 = SU(4) \times SU(2)_L \times SU(2)_R\), \(G'_2 = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \subset G_2\). Each one of these groups contain the SM symmetry and adequate chiral representations, and could define by themselves suitable GUT models. However, to understand the cancellation of anomalies or the unification of the three gauge couplings, one has to embed them in \(SO(10)\).

A semisimple subgroup \(G_i\) of \(SO(10)\) is specified by the projection matrix \(P_{G_i \subset SO(10)}\) (see details in Ref. \([11]\)), that projects weights \(\lambda_{SO(10)}\) of \(SO(10)\) onto weights \(\lambda_{G_i}\) of \(G_i\),

\[ \lambda_{G_i} = P_{G_i \subset SO(10)} \cdot \lambda_{SO(10)} . \]  

(12)

The dual vectors \(Q\) used to specify the charges satisfy

\[ Q_{SO(10)} = Q_{G_i} \cdot P_{G_i \subset SO(10)} . \]  

(13)

For a sequence of subgroups \(G' \subset G \subset SO(10)\), we have \(P_{G' \subset SO(10)} = P_{G' \subset G} \cdot P_{G \subset SO(10)}\). Also, since we have fixed the embedding of the SM in \(SO(10)\), we shall include in the projection matrices the Weyl reflection \(R\) mentioned above. If \(R\) (written as a matrix) cannot be reduced to a Weyl reflection of the subgroup \(G_i\), then \(P_{G_i \subset SO(10)}\) and \(P_{G_i \subset SO(10)} \cdot R \equiv P_{G_i^R \subset SO(10)}\) will define nonequivalent models.
For non-semisimple subgroups, it is convenient to give the charge of $SO(10)$ that corresponds to the $U(1)$ factor and express the charge of the projected weight. The projection matrices and $U(1)$ charges of the subgroups under consideration are [8]

\[
P_{G_1,G_1' \subset SO(10)} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}, \quad E = [2 \ 0 \ 2 \ 1 \ -1];
\]

\[
P_{G_1^R \subset SO(10)} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}, \quad E^R = [2 \ 0 \ -2 \ 1 \ -1];
\]

\[
P_{G_2 \subset SO(10)} = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ -1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix};
\]

\[
P_{G_2' \subset SO(10)} = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad Q_{B-L} = \frac{1}{3}[-2 \ 0 \ 0 \ -1 \ 1].
\]

The projection to flipped $SU(5)$ $P_{G_1^R \subset SO(10)}$ has been obtained from

\[
P_{G_1^R \subset SO(10)} = P_{G_1 \subset SO(10)} \cdot R_3,
\]

where \(R_3\) is the Weyl reflection with respect to the simple root \(\alpha_3\) of $SO(10)$

\[
R_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}
\]

that exchanges $u^c \leftrightarrow d^c$, $e^c \leftrightarrow N$, and $(h^+ \ h^0) \leftrightarrow (h'^0 \ h'^-)$. The subgroups $G_1$, $G_1^R$, $G_2$, and $G_2'$ still contain the extra $U(1)$ previously considered. The Higgs VEVs leaving a $Z_N \subset U(1)$ unbroken can be found by
projecting the weights $H_n$ of $SO(10)$ in Eq. (5). For $G_1$, $H_n = (0 0 0 0)$ and $E(H_n) = -5n$; for $G_1^R$, $H_n = (-n n -n n)$ and $E^R(H_n) = -n$; for $G_2$, $H_n = (0 n)(0)L(-n)_R$; and for $G_2'$, $H_n = (0 0)(0)C(0)L(-n)_R$ and $Q_{B-L} = n$. In a flipped $SU(5)$ ($G_1^R$) model derived from $SO(10)$, for example,

$$H_1 = (-1 1 -1 1) \in 10 = (0 1 0 0), \quad E^R(H_1) = -1;$$

$$H_2 = (-2 2 -2 2) \in 50 = (0 2 0 0), \quad E^R(H_2) = -2.$$  \hspace{1cm} (17)

Therefore, if the VEVs of the SM singlet in the $10(-1)$ irrep of flipped $SU(5)$ are used to break the extra symmetry, no gauge $Z_N$ based on $SO(10)$ survives, whereas GUT Higgs in the $50(-2)$ leaves a $Z_2$ discrete symmetry (the usual matter parity) unbroken. Analogous conclusions can be extracted for the other subgroups.

The rank-four subgroup $SU(5)$ ($G_1'$) may result from one of the breakings of $SO(10)$ that preserves a $Z_N$; in that case, to break $SU(5)$ and still preserve the $Z_N$ one must avoid certain representations. For instance, the SM singlet $(0 0 0 0)$ in the $24 = (1 0 0 1)$ irrep of $SU(5)$ that results from projecting the $H_1 \in 144 = (1 0 0 1 0)$ of $SO(10)$ (see Table II) would break any possible $Z_N$ discrete symmetry.

3. Models based on $E_6$

The exceptional group $E_6$ defines another anomaly-free GUT with adequate chiral representations. The fundamental irrep $27 = (1 0 0 0 0 0)$ (see Table V) contains a family of quarks and leptons, the two Higgs doublets ($h$ and $h'$) needed in SUSY models, two down-type quarks ($D$ and $D^c$), and two non-weakly-interacting neutrinos ($\nu_4$ and $\nu_5$). Like in $SO(10)$, from a model with three 27 multiplets one may obtain a low-energy limit with minimal matter content, since all the nonstandard fields are in vectorlike representations of the SM symmetry and should become massive at the high scales of gauge symmetry breaking.

The rank-6 group $E_6$ contains $SO(10) \times U(1)$ as a subgroup; the projection matrix and $U(1)$ charge are

$$P_{SO(10) \subset E_6} = \begin{pmatrix}
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0
\end{pmatrix}, \quad F = [1 \ -1 \ 0 \ 1 \ -1 \ 0].  \hspace{1cm} (18)$$
The five charges $A, \ldots, E$ in $E_6$ can be obtained from the corresponding charges in $SO(10)$ [see Eq. (13)], giving $A = [1 \; 2 \; 3 \; 2 \; 1 \; 2]$ ; $B = \frac{1}{\sqrt{3}}[-1 \; -2 \; -1 \; 0 \; 1 \; 0]$ ; $C = \frac{1}{2}[1 \; 1 \; 1 \; 1 \; 0]$ ; $D = \frac{1}{3}[1 \; -1 \; 1 \; -3 \; -1 \; 0]$ ; $E = [-1 \; 1 \; 4 \; 3 \; 1 \; 0]$ . We list in Table V all the weights in the 27 irrep which, under $SO(10) \times U(1)$, decomposes as $27 \rightarrow 16(1) + 10(-2) + 1(4)$. The two nonstandard charges in $E_6$ are, conveniently normalized, $Q_1 = -\frac{3}{5}D - \frac{1}{5}E = [0 \; 0 \; -1 \; 0 \; 0 \; 0]$ , $Q_2 = F + \frac{1}{5}E - \frac{12}{5}D = [0 \; 0 \; 0 \; 1 \; 0 \; 0]$ [Eq. (19)]

$Q_1$ corresponds to the $SO(10)$ charge given in Eq. (1). For the fields in the 27 it gives $Q_1(q, u^c, d^c, l, e^c, h, h', D, D^c, \nu_4, \nu_5) = (0, 1, -1, 0, -1, -1, 1, 0, 0, 1, 0)$ , $Q_2(q, u^c, d^c, l, e^c, h, h', D, D^c, \nu_4, \nu_5) = (0, 1, 0, 1, -1, -1, 0, 0, -1, 0, 1)$ .

The combination of the two $U(1)$s defined by these charges contains a generic $Z_N \times Z_M$ discrete symmetry given in Table VI in terms of $g^{(1)}_N \in Z_N \subset U(1)Q_1$, and $g^{(2)}_M \in Z_M \subset U(1)Q_2$, that represent the generators of the first and second factors of the discrete symmetry group, respectively. Thus an element of $Z_N \times Z_M$ has the generic form $(g^{(1)}_N)^n \cdot (g^{(2)}_M)^m$, where $n < N$ and $m < M$. Among these symmetries, we identify three different $Z_2$ [generated by $g^{(1)}_2$, $g^{(2)}_2$ and $g^{(1)}_2 \cdot g^{(2)}_2$] and four $Z_3$ [$g^{(1)}_3$, $g^{(2)}_3$, $g^{(1)}_3 \cdot g^{(2)}_3$, and $g^{(1)}_3 \cdot (g^{(2)}_3)^{-1}$]. Some of them, however, are related by the Weyl reflection $R = R_3 \cdot R_4 \cdot R_3$ of $SU(3)_R \subset E_6$

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix},$$

which exchanges $Q_1 \leftrightarrow Q_2$ and leaves the SM charges unchanged (for the fields in the 27 irrep, $R$ transforms $d^c \leftrightarrow D^c$, $h^- \leftrightarrow e$, $h'^0 \leftrightarrow \nu$, and $\nu_4 \leftrightarrow \nu_5$ leaving the rest unchanged). Therefore, $g^{(1)}_2$ and $g^{(2)}_2$, as well as $g^{(1)}_3$ and $g^{(2)}_3$, define identical models related by $R$. There are two $Z_2$ and three $Z_3$ non-equivalent discrete gauge
symmetries based on $E_6$ (see Table VI). Each one implies a low energy model with a definite pattern of fields and couplings in the superpotential. In Table VI we named $Z^a_2 = g_2^{(1)}$, $Z^b_2 = g_2^{(1)} g_2^{(2)}$, $Z^a_3 = g_3^{(1)}$, $Z^b_3 = g_3^{(1)} g_3^{(2)}$ and $Z^c_3 = g_3^{(1)} (g_3^{(2)})^{-1}$, where we identify the discrete symmetry group with the element that generates it.

The generic expression for a weight vector of $E_6$ whose VEV respects the SM symmetry is

$$H_{n,m} = (n+m \ -m \ -n \ m \ -m \ n),$$

(22)

where $m$ and $n$ are integers. The $Q_1$ and $Q_2$ charges of such a weight of $E_6$ are

$$Q_1(H_{n,m}) = n$$

(23)

and

$$Q_2(H_{n,m}) = m.$$  

(24)

To reduce the rank of the gauge group from six to four one needs the VEVs of at least two flavors $H_{n,m}$ and $H_{n',m'}$ such that $nm' \neq mn'$. The discrete symmetries $Z^a_2$ and $Z^a_3$, included in $U(1)_{Q_1}$, are left unbroken by Higgs $H_{n,m}$ satisfying $n = 0 \ mod \ 2$ and $n = 0 \ mod \ 3$, respectively. One may obtain a $Z^a_2$ model, for example, combining the VEVs of $H_{0,1}$, the $\nu_5$ flavour in the 27 or in the 351 = ($0 \ 0 \ 1 \ 0 \ 0$) representations, and $H_{2,0}$ in the 351'$ = (2 \ 0 \ 0 \ 0 \ 0)$ or the 2430 = ($0 \ 0 \ 0 \ 0 \ 2$). The symmetry $Z^a_3$ results from VEVs of $H_{1,0}$ and $H_{3,0}$ in the 3003 = ($3 \ 0 \ 0 \ 0 \ 0$) or the 112320 = ($1 \ 1 \ 0 \ 0 \ 1 \ 0$). In both cases these are the lowest dimensional representations required. Note that the same results would be obtained from the Weyl reflection $H_{n,m} \leftrightarrow H_{m,n}$ of VEVs in Eq. (21) that, in particular, exchanges the $\nu_5$ and $\nu_4$ flavours in the 27. Concerning the low energy implications, $Z^a_2$ and $Z^a_3$ are equivalent to the discrete symmetries in $SO(10)$ previously discussed (see Section 2), and they imply the same type of models.

The symmetry $Z^b_2$ in Table VI survives if all the GUT Higgs fields $H_{n,m}$ satisfy

$$n + m = 0 \ mod \ 2.$$  

(25)

This occurs, for example, by taking $H_{1,-1}$ in the 78 = ($0 \ 0 \ 0 \ 0 \ 0 \ 1$) or the 650 = ($1 \ 0 \ 0 \ 0 \ 1 \ 0$) irreps of $E_6$, with $H_{1,1}$ in the 351 = ($0 \ 1 \ 0 \ 0 \ 0 \ 0$) or the 351', or with $H_{2,0}$ or $H_{0,2}$ also in the 351' irrep.
$E_6$ models with an unbroken $Z_2^b$ symmetry contain at low energies one extra pair of doublets $(h, h')$ and of singlets $(D, D^c)$ for each family of quarks and leptons; the neutrinos $(\nu_4, \nu_5)$ are not protected by $Z_2^b$ from heavy mass contributions (see Table VI). Although such a spectrum is compatible with perturbative unification (actually, it has been shown \[12\] that the gauge couplings unify at $\sim 10^{17}\text{GeV}$ with an electroweak mixing angle $\sin^2 \theta_W = 0.23$), these models predict unsuppressed proton decay mediated by the SUSY partners of $d^c$ and $D^c$, since all the trilinears in Eq. (10) are allowed by $Z_2^b$.

The discrete symmetries $Z_3^b$ and $Z_3^c$ survive the VEVs of Higgs fields $H_{n,m}$ such that

$$n + m = 0 \ mod \ 3 \quad (26)$$

and

$$n - m = 0 \ mod \ 3 \ , \quad (27)$$

respectively. The symmetry $Z_3^b$ may result, for example, using the flavours $H_{1,-1}$ (see above) and $H_{1,2}$ in the $\overline{5824} = (1 \ 1 \ 0 \ 0 \ 0 \ 0)$ (which also contains $H_{1,-1}$) or $H_{3,0}$ while for $Z_3^c$ one can take $H_{1,1}$ and $H_{1,-2}$, both in the $\overline{1728} = (0 \ 0 \ 0 \ 0 \ 1 \ 1)$ irrep. Both discrete symmetries protect the quark singlets $(D, D^c)$ and the lepton/Higgs doublets $(h, h')$ in the $27$ representation from acquiring heavy masses. $Z_3^b$ predicts, in addition, one pair of neutrinos $(\nu_4, \nu_5)$ per family. The $Z_3^b$ case seems unrealistic since the presence of trilinears $u^c d^c d^c$ and $qd^cl$ in $P$ would produce too rapid proton decay. In $Z_3^c$ models, although all the dangerous terms in Eq. (10) are absent, there is also an unacceptable proton decay rate due to processes with exchange of squarks $D$ and $D^c$. A possible way to alleviate this problem could be to assume that one of the pairs $(\nu_4, \nu_5)$ remains light at $M_{GUT} \sim 10^{17}\text{GeV}$ and the scalar $\tilde{\nu}_5$ develops an intermediate VEV $\sim 10^{11}\text{GeV}$. Such a VEV would break $Z_3^c$ and make massive all the extra quarks and lepton doublets through terms of type $DD^c\nu_5$ and $hh'\nu_5$. The terms in Eq. (10) would then be present but suppressed by powers of $\frac{<\nu_5>}{M_{GUT}}$, and the required additional suppression seems reasonable.

Note also the absence in $Z_3^c$ models of the dimension five operator $qqql$ \[4\], which in R-parity symmetric models generates a proton decay amplitude suppressed only
by $M_{GUT}^{-1}$.

There are in $E_6$ three types of subgroups which may define satisfactory models:

$G_1 = SO(10) \times U(1)$, $G_2 = SU(3)_C \times SU(3)_L \times SU(3)_R$, and $G_3 = SU(6) \times SU(2)$. $SO(10)$ has been previously considered, and the projection matrix $P_{G_1 \subset E_6}$ is given at the beginning of this section.

For $G_2$, the projection matrix is

$$P_{E_6 \rightarrow G_2} = \begin{pmatrix}
1 & 2 & 2 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 \\
0 & -1 & -1 & -1 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & -1 & 0 & 0
\end{pmatrix}. \quad (28)$$

$G_2$ is a rank-six group, and includes the two extra $U(1)$ factors of $E_6$. The analysis of Higgs flavours that leave a discrete symmetry unbroken can be done straightforwardly by projecting the results obtained for $E_6$. It turns out that

$$H_{n,m} = (0\ 0)_C (0\ n+m)_L (-n\ n-m)_R. \quad (29)$$

The standard matter parity $Z^a_2$, for example, will be obtained combining the VEVs of $H_{0,1}$ in the $(0\ 0)(0\ 1)(1\ 0) = (1, \overline{3}, \overline{3})$ and $H_{2,0}$ in the $(0\ 0)(0\ 2)(1\ 2) = (1, \overline{6}, \overline{15})$.

The projection matrix to $SU(6) \times SU(2)$ is

$$P_{E_6 \rightarrow G_3} = \begin{pmatrix}
-1 & -1 & -1 & -1 & 0 & -1 \\
0 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & -1 & -1 & 0 & 0
\end{pmatrix}. \quad (30)$$

The projected Higgs $H_{n,m}$ has the form

$$H_{n,m} = (-n-m\ 0\ 0\ 0\ 0)(n-m). \quad (31)$$

To obtain a low energy limit with, for instance, $Z^a_2$ discrete symmetry one may use $H_{0,1}$ in the $(0\ 0\ 0\ 0\ 1)(1) = (\overline{6}, \overline{2})$ with $H_{2,0}$ in the $(0\ 0\ 0\ 0\ 2)(2) = (\overline{21}, \overline{3})$.

The projection from $SU(6)$ to $SU(5)$ is done by dropping the first component of the weight vector.
3. Summary and conclusions

We have analyzed the appearance of discrete symmetries of the superpotential as remnants of gauge symmetries in models based on $E_6$ and $SO(10)$. These models include flipped and unflipped $SU(5)$, $SU(4)\times SU(2)_L\times SU(2)_R$, $SU(3)_C\times SU(3)_L\times SU(3)_R$, and $SU(6)\times SU(2)$. We have used the Dynking labelling to identify the GUT Higgs leading to each discrete gauge symmetry and have established the matter content and couplings of the different low energy models. These type of discrete symmetries are not anomalous respect to gravitational effects, and in SUSY models could work as matter parities.

Our results are the following. In models based on $SO(10)$ there is only one generic $Z_N$. The case $N = 2$ corresponds to the usual R-parity of SUSY scenarios, with minimal matter content, an stable LSP, and absence of B and L violating terms in the superpotential. For $N \geq 3$ the trilinears in $P$ involving quarks and leptons are the same, but the presence of right handed neutrinos may introduce LSP decay and L violation. All $Z_N$ symmetries would be broken if one uses as a Higgs the singlet in the 16 of $SO(10)$, which is the smallest representation that reduces the rank of $SO(10)$ while preserving the SM symmetry. The smallest representation containing a GUT Higgs whose VEV breaks the extra $U(1)$ and safes a $Z_N$ symmetry is the $(0 \ 0 \ 0 \ 0 \ N)$, whose dimension is given in Eq. (8).

In models based on $E_6$ the discrete symmetry is a $Z_N \times Z_M$. In addition to the $Z_N$ models of $SO(10)$, we find here another $Z_2$ and two more $Z_3$ nonequivalent cases. All these symmetries imply light nonstandard lepton/Higgs doublets and down type quarks, and it is difficult (although in one of the cases it seems possible) to accommodate a long enough proton lifetime. In general, high dimensional representations are required to obtain these symmetries; using as Higgs the flavors $\nu_4$ and $\nu_5$ in the 27 no gauge discrete symmetry survives.

Obviously, there are other consistent choices of the GUT group. One may consider large $SU(n)$ groups, but in general they require complicated choices of representations to cancel anomalies. One may as well consider non simple groups, but then the perturbative unification of the gauge couplings would be purely acci-
dental. It is also possible to consider the exceptional groups $E_7$ and $E_8$ (with only real representations), $SO(32)$ (which contains the $SU(15)$ model [13]), or family models based on $SO(4n+2)$, but all of them predict the presence at low energies of mirror partners instead of the observed three chiral families. If one thinks of a SUSY extension of the SM embedded in a chiral GUT, then the only models based on discrete gauge symmetries (non anomalous respect gravitational effects) are the ones found here.

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Table I. $16 = (0 0 0 0 1)$ irrep of $SO(10)$ and nonstandard $E$ charge.

|   | $E$     | $E$       |
|---|---------|-----------|
| $u$ : $(0 0 0 0 1)$ | $-1$ | $u^c : (1 0 -1 0 1)$ | $-1$ |
| $d^c : (0 0 1 0 -1)$ | $3$ | $N : (-1 1 -1 0 1)$ | $-5$ |
| $u^c : (0 1 -1 1 0)$ | $-1$ | $e : (1 0 0 0 -1)$ | $3$ |
| $\nu : (1 -1 0 1 0)$ | $3$ | $u : (0 -1 0 0 1)$ | $-1$ |
| $d : (0 1 0 -1 0)$ | $-1$ | $d : (-1 1 0 0 -1)$ | $-1$ |
| $u : (-1 0 0 1 0)$ | $-1$ | $d^c : (0 -1 1 0 -1)$ | $-3$ |
| $d^c : (1 -1 1 -1 0)$ | $3$ | $u^c : (0 0 -1 1 0)$ | $-1$ |
| $e^c : (-1 0 1 -1 0)$ | $-1$ | $d : (0 0 0 -1 0)$ | $-1$ |

Table II. $10 = (1 0 0 0 0)$ irrep of $SO(10)$ and nonstandard $E$ charge.

|   | $E$     | $E$       |
|---|---------|-----------|
| $D : (1 0 0 0 0)$ | $2$ | $D : (0 0 0 1 -1)$ | $2$ |
| $D^c : (-1 1 0 0 0)$ | $-2$ | $h^0 : (0 0 1 -1 -1)$ | $2$ |
| $h^+ : (0 -1 1 0 0)$ | $2$ | $h^{\prime -} : (0 1 -1 0 0)$ | $-2$ |
| $h^{\prime 0} : (0 0 -1 1 1)$ | $-2$ | $D : (1 -1 0 0 0)$ | $2$ |
| $D^c : (0 0 0 -1 1)$ | $-2$ | $D^c : (-1 0 0 0 0)$ | $-2$ |
Table III. Generic $Z_N$ discrete symmetry of models based on $SO(10)$ ($\alpha^N = 1$) and the particular cases with $N = 2$ and $N = 3$ ($\sigma^3 = 1$).

|       | $q$ | $u^c$ | $d^c$ | $l$ | $e^c$ | $N$ | $h$ | $h'$ | $D^c$ | $D$ |
|-------|-----|-------|-------|-----|-------|-----|-----|------|-------|-----|
| $Z_N$ | 1   | $\alpha$ | $\alpha^{-1}$ | 1   | $\alpha^{-1}$ | $\alpha$ | $\alpha^{-1}$ | $\alpha$ | 1     | 1   |
| $Z_2$  | 1   | $-1$  | $-1$  | 1   | $-1$  | $-1$ | $-1$ | 1     | 1     | 1   |
| $Z_3$  | 1   | $\sigma$ | $\sigma^2$ | 1   | $\sigma^2$ | $\sigma$ | $\sigma^2$ | $\sigma$ | 1     | 1   |

Table IV. List of $SO(10)$ irreps which contain a Higgs $H_n$ preserving a $Z_n$ discrete symmetry for $n = 1, 2, 3$.

| $H_1$  | $H_2$  | $H_3$  |
|--------|--------|--------|
| $(-1 \ 1 \ -1 \ 0 \ 1)$ | $(-2 \ 2 \ -2 \ 0 \ 2)$ | $(-3 \ 3 \ -3 \ 0 \ 3)$ |
| $16 = (0 \ 0 \ 0 \ 0 \ 1)$ | $126 = (0 \ 0 \ 0 \ 0 \ 2)$ | $672 = (0 \ 0 \ 0 \ 0 \ 3)$ |
| $144 = (1 \ 0 \ 0 \ 1 \ 0)$ | $1728 = (1 \ 0 \ 0 \ 1 \ 1)$ | $11088 = (1 \ 0 \ 0 \ 2 \ 1)$ |
| $560 = (0 \ 1 \ 0 \ 0 \ 1)$ | $2970 = (0 \ 1 \ 1 \ 0 \ 0)$ | $49280 = (2 \ 0 \ 0 \ 2 \ 1)$ |
Table V. $27 = (1 0 0 0 0)$ irrep of $E_6$ and nonstandard $E$ and $F$ charges.

|      | $E$       | $F$ | $E$     | $F$ |
|------|-----------|-----|---------|-----|
| $u$  | $(1 0 0 0 0 0)$ | $-1$ | $1$     |     |
| $D$  | $(-1 1 0 0 0 0)$ | $2$  | $-2$    |     |
| $d^e$| $(0 -1 1 0 0 0)$ | $3$  | $1$     |     |
| $u^c$| $(0 0 -1 1 0 1)$ | $-1$ | $1$     |     |
| $D^e$| $(0 0 0 -1 1 1)$ | $-2$ | $-2$    |     |
| $v$  | $(0 0 0 1 0 -1)$ | $3$  | $1$     |     |
| $d$  | $(0 0 0 0 -1 1)$ | $-2$ | $1$     |     |
| $h^+$| $(0 0 1 -1 1 -1)$ | $2$  | $-2$    |     |
| $d^c$| $(0 0 1 0 -1 -1)$ | $3$  | $1$     |     |
| $h^{00}$| $(0 1 -1 0 1 0)$ | $-2$ | $-2$    |     |
| $u^c$| $(0 1 -1 1 -1 0)$ | $-1$ | $1$     |     |
| $u$  | $(1 -1 0 0 1 0)$ | $-1$ | $1$     |     |

$D^c$: $(0 1 0 -1 0 0)$
**Table VI.** Generators $g_N^{(1)}$ and $g_M^{(2)}$ of the $Z_N \times Z_M$ discrete symmetry ($\alpha^N = \beta^M = 1$) of models based on $E_6$, and nonequivalent $Z_2$ and $Z_3$ ($\sigma^3 = 1$) particular cases.

|     | $q$ | $u^c$ | $d^c$ | $l$ | $e^c$ | $h$ | $h'$ | $D^c$ | $D$ | $\nu_4$ | $\nu_5$ |
|-----|-----|-------|-------|-----|-------|-----|------|-------|-----|--------|--------|
| $g_N^{(1)}$ | 1 | $\alpha$ | $\alpha^{-1}$ | 1 | $\alpha^{-1}$ | $\alpha^{-1}$ | 1 | 1 | 1 | $\alpha$ | 1 |
| $g_M^{(2)}$ | 1 | $\beta$ | 1 | $\beta$ | $\beta^{-1}$ | $\beta^{-1}$ | 1 | $\beta^{-1}$ | 1 | 1 | $\beta$ |
| $Z_2^a$ | 1 | $-1$ | $-1$ | 1 | $-1$ | $-1$ | 1 | 1 | $-1$ | 1 |
| $Z_2^b$ | 1 | 1 | $-1$ | $-1$ | 1 | 1 | $-1$ | $-1$ | 1 | $-1$ | $-1$ |
| $Z_3^a$ | 1 | $\sigma$ | $\sigma^2$ | 1 | $\sigma^2$ | $\sigma^2$ | 1 | 1 | $\sigma$ | 1 |
| $Z_3^b$ | 1 | $\sigma^2$ | $\sigma^2$ | $\sigma$ | $\sigma$ | $\sigma$ | $\sigma^2$ | 1 | $\sigma$ | $\sigma$ |
| $Z_3^c$ | 1 | 1 | $\sigma^2$ | $\sigma^2$ | 1 | 1 | $\sigma$ | $\sigma$ | 1 | $\sigma$ | $\sigma^2$ |