Introduction to Spin & Lattice Models in the Social Sciences

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Abstract

In recent years, political and financial economists and other social scientists have begun adopting spin and lattice models into their theoretical tool kit. This article introduces examples of how these models are used, and points to some state of the art references. For illustration, a simple dynamical model of how legal rules evolve and propagate in the Anglo-American court system is described.

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1 The Common Law as an Evolutionary System

Almost two centuries ago, Justice Oliver Wendell Holmes wrote “The life of the law has not been knowledge: it has been experience. The felt necessities of the time, ... avowed or unconscious, even the prejudices which judges share with their fellow men, have had a good deal more to do than the syllogism in determining the rules by which men [are] governed.” Likewise, the German legal scholar Friedrich Karl von Savigny wrote that “All law... is first developed by custom and [conventional morality], next by jurisprudence,—everywhere, therefore, by internal silently-operating powers, not by the arbitrary will of a law-giver.”

Holmes’ “felt necessities” and von Savigny’s “internal silently-operating powers” have not gone unnoticed by modern scholars. Modern legal scholars, ranging from Grant Gilmore to Lawrence Friedman, have observed that changes in law at various times are “rapid and violent,” “clean and swift,” and “highly fluctuating” (Yee 2001). In current events, a growing branch of intellectual property, “cyberspace law,” has emerged almost overnight in response to technological innovations and business needs. Moreover, capital markets anticipate such changes in a rational way (Yee 2005, 2006).

Hence, it comes as no surprise that Holmes’ “felt necessities” and von Savigny’s “internal silently-operating powers” drive the basis of an evolutionary framework for understanding the development the common law (Priest 1977; Rubin 1977). The idea is based on Darwinian natural selection: efficient legal rules survive while inefficient rules are culled out by litigation. In the long run,
Figure 1: A Darwinian evolutionary system evolves in time as follows. At any time step \( t \), there is a population of individuals. Mutual competition establishes a fitness rating for each individual. The population then undergoes natural selection based on the fitness rating, selective reproduction, and (usually random) mutations. This process yields a new population of individuals at time \( t + 1 \), the subsequent time step.

The common law contains only rules that survive legal and political challenges. While this paradigm is appealing, it has not yielded empirically refutable predictions which would seriously test the model.

This article introduces a dynamical model of common law evolution originally described in Yee (2001). According to this model, evolution leaves an inevitable trail of paleontological footprints which may be sitting in the Westlaw dunes awaiting empirical identification by legal excavators.

Biological evolution starts with a collection of genes (random degrees of freedom) created by chance chemistry in the earth’s primordial atmosphere.
These early genes competed against each other to survive and replicate. Ultimately, the biological structures we see today, including their inorganic byproducts such as computer software or the price of wheat futures emerged from this competition. Emergence is not imposed exogenously. Rather, structures emerge inevitably from natural selection, which in turn is an inherent consequence of competitive interactions and selective reproduction.

Natural selection and evolution does not only occur in a biological setting. Any system is evolutionary if it has the following four ingredients: set of degrees of freedom or individuals, each ranked by a quality or behavior-based fitness criterion, a selection process based on the fitness ranking, and a mechanism for introducing (usually random) mutations to periodically interject diversity into the reproductive process.

The common law has these four ingredients. As depicted in Figure 2, precedents of the common law are the “individuals” undergoing Darwinian evolution. A precedent’s “economic efficiency” is the degree to which it balances between legal and political forces. Economic efficiency plays the role of Darwinian fitness. The fitness-preferring selection process is provided by the premise that inefficient precedents are subject to more challenges (either in the courts or in the legislature) and hence are more subject to alteration than efficient ones. Mutations are introduced by the volatile nature of litigation or legislation.

As pointed out by many authors, notably Priest (1977) and Rubin (1977), common law precedents tend to evolve towards economic efficiency because inefficient precedents are selectively challenged and ultimately culled away as judges
Figure 2: Interpretation of Figure 1 when the evolving system is the common law (or any precedent-based legal regime). At any time step $t$ is a collection of laws or precedents. As these precedents are applied to regulate social and economic activity, disputes arise concerning their meaning and social desirability. Accordingly, lawsuits (or legislative challenges) seeking to overturn the precedents in controversy are filed. One precedent is more legally “fit” than another if it suffers fewer such challenges. Copies of the unchallenged (and hence the fittest) precedents survive intact to reign at subsequent time step $t + 1$. The challenged precedents, facing judicial modification or termination with each litigation, have a decreased chance of unaltered survival. Moreover, all precedents (challenged and unchallenged) are subject to random mutations stemming from external social pressures which may alter their meaning or applicability at any time. Thus at each new time step is an evolving common law comprised of precedents which are selectively reproduced and mutated replicas of their ancestors.
eventually hit on more efficient and hence less-challenged doctrines even if only by random trial and error. In this view, the common law is a market of doctrines, and a lawsuit is a bid on a specific doctrine. Intensive litigation moves this market towards efficiency whether or not judges consciously choose efficiency because inefficient doctrines will be intensively relitigated until they are efficient.

While this market view is a compelling premise, it is not all there is to the story. Evolution is more than a push towards improvement. Biological evolution has provided a rich history of paleontological footprints: punctuated equilibria, Zipf’s Law, and path-dependency. Accordingly, taking the evolutionary paradigm seriously requires considering its dynamical properties—the paleontological footprints. In Yee (2001) I propose a dynamical model (“CLM”) of common law evolution. Cast as parsimoniously as possible, CLM in its simplest guise is mathematically isomorphic to the Bak-Sneppen models (Bak and Sneppen 1993; see also Yee 1993) and yields interesting paleontology. As discussed, CLM exhibits punctuated equilibrium, Zipf’s Law, path dependency, and a stochastic (but statistically robust) form of efficiency I shall refer to as “smeared” economic efficiency.

2 The Common Law Model (“CLM”)

Economic efficiency of a precedent can change either because (α) the precedent is altered as a result of a court decision or legislation; or its (β) legal environment or (γ) social context changes while the precedent itself remains constant. (β)
recognizes that changes in related laws may induce a change in the economic efficiency of an unaltered precedent. For instance, a modification of traffic laws may distort the economic efficiency of a prevailing “reasonable man” standard in torts without any direct change to the standard itself. Likewise, \((\gamma)\) says that social or cultural changes, perhaps driven by technological innovations, may induce the economic efficiency of a law to change without any changes to the law itself. In this Introduction, I shall assume that changes in economic efficiency are due entirely to \((\alpha)\) and \((\beta)\), not \((\gamma)\).

With this caveat in mind, my CLM has just two ingredients:

- a set of \(N \geq 3\) precedents labeled by integers \(i \in I \equiv \{1, 2, \cdots, N\}\);
- a real-valued economic fitness measure \(e : I \rightarrow [0, 1]\) where \(e = 0\) represents the worst and \(e = 1\) the best possible economic efficiency level.

CLM evolves in time according to the following three rules:

- (I) the precedent \(i\) litigated at each time step is the one with the smallest efficiency value \(e(i)\);
- (II) litigation of precedent \(i\) results in a court ruling which potentially alters not only \(i\) but also \(i\)'s neighboring precedents\(^1\) \(i + 1\) and \(i - 1\);
- (III) litigation outcomes are random in efficiency \(e\), that is courts don’t strive to optimize efficiency. In view of Rule (II), this means a litigation

\(^{1}\text{I assume that precedent space } I \text{ wraps around so that } i = N \text{ and } i = 1 \text{ are next door neighbors. This wrap-around assumption has negligible effect on my results when } N \text{ is sufficiently large (e.g. if } N \gg 10\). In real life, the common law arguably has thousands of precedents.}
of precedent \(i\) results in \(e(i), e(i-1),\) and \(e(i+1)\) each being assigned a new random value in the unit interval.

These three rules drive CLM. As I shall describe, this minimalistic set of rules yield a wealth of interesting evolutionary implications for common law behavior. Rule (I) represents the Rubin-Priest conjecture of selective litigation of the most inefficient laws. Rule (II) is motivated by \((\beta)\), that is precedents do not reign in a vacuum but are interpreted within its legal context at each point in time. A precedent’s meaning and application—and accordingly its economic efficiency—depends on the meaning and application of its cousins in legal logic and, ultimately, on the entire common law. In Rule (II), the multifaceted web of legal relationships between different precedents is represented by the bold simplification that changes in precedent \(i\) directly influences only its next-door neighbors.

Rule (III) assumes that litigation results are entirely random with respect to economic efficiency. While this assumption was selected for its minimalistic nature, it is not as detached from real life as some practitioners are apt to believe. In addition to the everchanging mix of judicial philosophies and idiosyncrasies flowing in and out of the judicial system, judges also make their share of mistakes. As it is, even the Supreme Court’s decisions are demonstrably random in some contexts, such as in at least one area of securities regulation.

A simplified toy model helps elucidate CLM. For the toy model, assume efficiency of each precedent can take on only three values High, Medium, and Low, and litigation of one precedent bears no consequences for its neighbors.
While these assumptions ignore critical elements of CLM—notably Rule II, the interprecedent interaction rule, it serves as a starting point to develop intuition.

In the earlier rounds of the toy model, one third of the precedents will be Low, and they will be litigated first. In each trial, two times out of three the judge will replace the Low precedent with one of a higher score. After enough rounds of litigation, all the Low precedents will be eliminated, if only for one round. When all the Low precedents are eliminated, a random Medium precedent will be litigated in the subsequent time step. There are three possible outcomes. If the judge offers a High result, lawyers will move on to litigate other Medium precedents. If the judge offers a Medium ruling, the common law has not deteriorated. If the judge offers a Low ruling, the Low precedent will be immediately relitigated, again and again if necessary, until it is restored to a Medium or High efficiency. Repeated, instantaneous relitigation of a Low precedent generates a “litigation cluster” in the toy model.

In the toy model, the trend is clear: Low precedents are relitigated relentlessly until they become extinct. Subsequently, Medium precedents are culled until only High precedents remain. Except for short-lived litigation clusters, only High precedents survive in the long run. Therefore, the long run equilibrium state of the toy model is perfectly efficient (except for small litigation clusters). In equilibrium, High precedents live at the “edge” of litigation; that is, as the only surviving precedents, they are randomly exposed to litigation and, when litigated, are relentlessly rehashed until restored to High efficiency.

\[^{2}\] When all the other precedents are High, a litigated precedent has a two thirds chance of being immediately relitigated.
The critical difference between the toy model and CLM is due to Rule II. Rule II, interprecedent interactions, in CLM is responsible for a rich variety of effects in CLM. Because of interprecedent interactions, High precedents are as vulnerable to litigation as their Low or Medium neighbors are. In CLM, High neighbors of a Low precedent are subject to being pulled into the litigation undertow whenever the Low neighbor is litigated. Since the undertow consists of up to three precedents, and only one precedent can be litigated at each step, the efficiency distribution does not collapse into a flat $e = 1$ peak. Instead, litigation of a Low or Medium precedent creates a lasting wave of litigation in the whole surrounding neighborhood. This enhances litigation clusters, creates valleys of inefficient litigation, and smears out the efficiency distribution.

Real life precedents, like in CLM and contrary to the toy model, do not act independently of the remaining body of law. Precedents depend on or complement and reinforce one another in a dynamic Charlotte’s web of legal and logical entanglements. It is precisely the presence of interprecedent entanglements springing from Rule II\(^3\) that differentiates CLM from CK. And, as shown in Yee (2001), it is precisely the dynamics of these entanglements which leave the paleontological footprints of punctuated equilibria, Zipf’s Law, path dependency, and smeared economic efficiency.

For instance, as depicted in Figure 3, a law exhibits rapid violent change interspersed with long periods of quiet. This feature occurs endogenously. In

\(^3\)While one can extend Rule II by introducing next-to-nearest neighbor interprecedent couplings, in keeping with minimalism I shall focus exclusively on nearest-neighbor couplings. With only nearest-neighbor couplings, CLM is similar to a physics model (Bak and Sneppen 1993).
Figure 3: Evolution of a typical precedent’s efficiency value as a function of time in CLM. While equilibrium is reached some time before the $5 \times 10^4$th time step, the precedent continues to evolve and fluctuate in efficiency. Thus, equilibrium is not static: while certain collective properties of the system stabilize at equilibrium, individual precedents continue to evolve and mutate.
CLM, spurts of intense litigation for each precedent are interspersed with irregular periods of inactivity. Figure X depicts the efficiency of a typical precedent as a function of time.

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