Performance Analysis of Intelligent Reflecting Surface-Assisted Wireless System with Non-Ideal Transceiver

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Abstract—Intelligent Reflecting Surfaces (IRSs) have recently emerged as a promising solution for realizing the smart radio environment concept by leveraging the characteristics of metamaterials and large antenna arrays. However, the existing works on IRS do not account for the transceiver impairments while analyzing the performance of IRS assisted wireless systems. This paper investigates the impact of transceiver hardware imperfections on the performance of IRS-assisted wireless systems. Specifically, we have analyzed the spectral efficiency (SE), the energy efficiency (EE) and the outage probability of IRS-assisted wireless systems by deriving their closed-form expressions and verifying them through simulation results. The results show the importance of modeling and compensating for hardware impairment as they significantly restrict the performance of such systems.

Index Terms— Intelligent reflecting surfaces, performance analysis, spectral efficiency, energy efficiency, outage, hardware impairments, relay.

I. INTRODUCTION

High throughput and high energy efficiency are the two key requirements for modern communication systems. The ever increasing demand for high throughput is fueled by ubiquitous mobile services with enhanced quality-of-service (QoS). Further, the innovative applications such as augmented and virtual reality, unmanned mobility, big data analytics, three-dimensional (3D) media, e-health, etc., are highly data intensive [1], [2]. A major characteristic of the futuristic wireless network is the propensity to support such massive traffic. Thus, it requires a very large area spectral efficiency (hundreds of bits/s/Hz/sq. km) and ultra high throughput per user equipment (multiple Gbps) [1]. A scaling of 1000-times in the system capacity has been envisioned in the current wireless system [3]. This explosion of wireless data has continuously driven the advancement in wireless technologies such as millimeter-wave (mmWave) communication, large-scale multiple-input multiple-output (MIMO), capacity-achieving turbo and polar codes. These technologies have also been advocated for the fifth-generation (5G) wireless standard [4]. For further enhancement of data rate, the combination of mmWave with massive MIMO has also been proposed. However, the deployment of large-scale MIMO usually results in a high cost of implementation and increased power consumption. However, the combination of mmWave and massive MIMO needs sophisticated signal processing techniques, which require expensive and energy consuming hardware [5].

Along with high spectral efficiency (SE), the emerging and futuristic wireless networks (5G and beyond) have also raised concerns on the consumption of energy while operating these wireless networks. Further, it is also anticipated that the future wireless network will have to connect over a hundred billion devices in a dense deployment, which includes base stations and access points [6]. Consequently, the bits/Joule energy efficiency (EE) has also emerged as a key performance metric that ensures the sustainability of wireless networks [7], [8]. A recent survey on the implementation of energy efficient wireless networks has recommended multiple joint approaches to limit the overall energy consumption [9]. A few of the prescribed approaches are the usage of renewable energy along with employing energy-efficient hardware, optimal resource allocation, and deployment based on reducing energy consumption. Moreover, deploying a massive number of antennas results in a substantial improvement in the EE of the wireless system, as shown in [10], [11].

Recently, intelligent reflecting surfaces (IRSs) has been proposed as a potential innovative solution to achieve both the goals, i.e., high EE and high SE [5], [12]. IRS has also been acknowledged as one of the critical enablers technology for the sixth-generation (6G) wireless network [13], [14]. IRS is specifically a re-configurable array of passive reflecting elements, wherein each of the elements can independently alter the phase and/or attenuation of the incident signal. Cumulatively, the reflected signal is altered as such that the desired channel response can be realized through proper phase shifting at each of the reflecting elements. The reflected signal can be combined constructively from all the paths to enhance the received signal power or can be combined destructively to mitigate the undesired interference, thus improving the overall performance of the wireless system [15]. The individual unit of IRS can be implemented through reflect-arrays which employ a varactor diode or a PIN diode, where the resonating frequency of these diodes can be controlled electronically [16], [17]. Further, with the advent of Micro-Electrical-Mechanical Systems (MEMS) and eta materials, a real-time reconfiguration of the phase shifts of IRS has also been demonstrated. This real-time reconfiguration of IRS phase-shift has widely enhanced its applicability for deployment across various wireless networks [18]. Moreover, the low hardware footprint of IRS structure facilitates the wider
deployment across the different urban structures in the outdoor environment such as buildings, factory ceilings, roof-tops, etc. Consequently, IRS can easily be integrated into the existing wireless communication environment [19].

Contrary to the conventional relaying, the beamforming gain in the IRS is achieved through intelligent reflection and without consuming any additional energy for re-transmission. IRS simply reflects the incoming signal by suitably adjusting the phase of the reflected signal from each of the reflecting elements in order to have a constructive combination of the signal at the desired receiver. Additionally, the IRS does not require any dedicated time/frequency resource for the re-transmission of the incoming signal. Since IRS does not employ a power amplifier, the circuit power consumption is much less than that in a regular amplify-and-forward (AF) relay [20]. Thus, from an energy consumption standpoint, the IRS is far more energy-efficient as compared to conventional relaying [21].

A. Related Work

In [22], the authors have investigated the approximate achievable data rate assuming that the channels between a transmitter to IRS and IRS to the receiver are independent and Rician distributed. In [23], authors have analyzed the asymptotic uplink sum-rate of a IRS-assisted system in the Rician channel. IRS is adopted for downlink communication in the multi-user scenario for enhancing the EE in [20]. Specifically, in [20], the authors have reported a 300% increase in the EE of IRS vis-a-vis traditional AF relaying based resource allocation. Similarly, the authors in [19], [20], [24] considered the IRS for assisting downlink transmission scenario for multiple users and worked on maximizing the sum-rate. Further, in [21] and [25], the authors have derived the upper-bound for the error performance of IRS-assisted wireless systems. It can be noted that these upper-bound are quite tight for IRS with a large number of reflecting elements; however, the lower bound is not quite accurate when the number of reflecting elements is small. Further, in [26], authors have provided bit error rate (BER) analysis for IRS-assisted non-orthogonal multiple access (NOMA) systems. However, the authors employed the central limit theorem (CLT) to characterize the distribution of the equivalent channel; the results are precise for IRS having a large number of reflecting elements.

In [27], IRS has been employed in simultaneous wireless information and power transfer (SWIPT) aided system, where the energy harvesting is enhanced by the IRS. Further, a self-sustainable IRS have been proposed for a wireless powered communication network (WPCN), where IRS improves downlink energy transfer and uplink information transmission for multiple users [28]. Similarly, in [29], IRS has been proposed to the realm of index modulation (IM), where the author has proposed IRS-space shift keying (SSK) and IRS-spatial modulation (SM) schemes for future wireless communication systems. The joint passive and active beamforming at IRS has been studied in [30] for obtaining the average received power at the user. IRS has also been explored for physical layer security (PLS) in [31]. Lastly, [32] employs IRS for vehicular communications, where the IRS enhances the link-performance of both the vehicle-to-vehicle and vehicle-to-infrastructure link. The seminal work of Renzo et al. comprehensively discussed the concept and state-of-art research for IRSs [33]. Further, it also presented the communication-theoretic framework based on electromagnetic theory for modeling, analysis, optimization and deployment of futuristic IRS empowered smart radio environment. In addition, the work of [15], [17], [21], [34] presented a comprehensive review of IRS empowered communication.

Some recent works have also analyzed the trade-offs of IRS vis-a-vis conventional relaying. For instance, Bjornson et al. have compared the EE performance of IRS-assisted systems against decode-and-forward (DF) relaying assuming the channel to be deterministic [35], while, Renzo et al. discussed the key differences and similarities between IRSs and the relays in [36]. The authors have provided different set of simulations in order to compare the data rate of IRS-assisted system with the relays. Further, in [37] IRS-assisted systems were compared with relay in terms of outage probability, symbol-error rate (SER) as well as ergodic capacity while assuming that the transceivers in both IRS-assisted wireless system and relay-assisted wireless systems were equipped with ideal RF front-ends.

It is evident from above, there is a dearth of literature on the performance of IRS-assisted wireless systems in the presence of non-ideal transceiver hardware, as most of the prior work assume ideal hardware scenario, and thus overlooking the impact caused by the impairment due to non-ideal hardware. In [38], authors have studied the impact of transceiver hardware imperfection in a IRS-assisted system. However, the study was limited in terms of outage probability and upper bound of the SE. The ergodic SE has not been analyzed in [38], further, the authors have also not considered the EE of the system.

B. Motivation and Contributions

Inspired by the current research in IRS, in this paper, we investigate the impact of hardware impairments on the performance of a IRS-assisted wireless system. The major contributions of the proposed work can be summarized as follows:

- In this work, we present a model to characterize the impact of transceiver impairments on IRS-assisted wireless systems taking into account the effect of channel fading as well as the size of IRS. The size is characterized by the number of reflecting elements (REs), $M$, in the IRS.
- Next, we derive the closed-form expressions for the outage probability and SE of the IRS-assisted wireless systems by first obtaining the instantaneous signal-to-distortion-plus-noise-ratio (SDNR) and then utilizing it to derive the above expressions.
- In continuation of the above, we also present the SE analysis for three special cases, viz. a) single reflecting element based IRS i.e., $M = 1$, b) very large sized IRS, i.e., $M >> 1$ and c) high SNR approximation.
- Finally, we present the analytical evaluation of EE for the IRS-assisted wireless system. The results reveal that
in the for an IRS-assisted wireless system, the EE reduces with increasing the number of reflecting elements.

From the derived analytical and simulation results, it has been shown that the hardware impairments severely restrict the performance of IRS-assisted wireless system. Unlike the conventional IRS system wherein the gain due to IRS increases with the number of REs, the SE saturates with increasing the number of REs due to hardware impairments. Specifically, after saturation, no further gain can be achieved in the SE by increasing the number of REs in the IRS. A similar observation has been made while increasing transmit power. Further, it has also been shown that the EE further decreases by increasing the number of reflecting elements. This is due to the fact that increasing the number of reflecting elements raises power consumption, consequently decreasing the overall EE.

Moreover, the result also shows that nonlinear HPA is the major source of distortion. The distortion due to HPA saturates the SE, which consequently degrades the EE of the IRS-assisted system. The impact of nonlinearity can be mitigated by operating the HPA in back-off, thus improving the SE and the outage performance. However, increasing the IBO reduces the efficiency of HPA, which results into reduction in the overall EE of the IRS-assisted system. In comparison to [38], the proposed work provides a complete analytical framework to analyze the EE performance of IRS-assisted systems while considering the impact of hardware impairment. Furthermore, we have also derived the closed-form expressions for the SE of the aforementioned system. At the end, the efficacy of the analytical derivations has been validated through simulations. To the best of our knowledge, this work is the first to theoretically investigate the EE performance of the IRS-assisted system impaired with a non-ideal transceiver.

C. Organization and Notations

The organization of this paper is as follows: Section II provides an overview of the IRS-assisted wireless system, description of transceiver hardware impairments and behavior modeling of HPA. Section III derives the closed-form expressions for SE, EE and outage. The simulation results are presented in Section IV and finally, Section V concludes this paper. Additionally, for convenience, we list the abbreviations presented in Section IV and finally, Section V concludes this paper.

The major notations used in this paper are as follows: The bold lower and upper cases represent vectors and matrices respectively (e.g., $\mathbf{x}$ and $\mathbf{X}$). Complex Gaussian distribution is represented with $\mathcal{CN}(\mu, \sigma^2)$, where $\mu$ is the mean and $\sigma^2$ denotes the variance. The remaining of the notations are summarized in Table III.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a scenario in which a source node (S) communicates with a destination node (D) via IRS. The IRS comprises of $M$ number of reflecting elements (REs). Further, it is assumed that the direct link between the source and destination is not available due to a blockage. The fading channel between the source and the $i$th RE is denoted by $h_i$, whereas $g_i$ is the channel between the $i$th RE and the destination. Both $h_i$ and $g_i$ are assumed to be independent and identically distributed (i.i.d.) and follow the Rayleigh distribution. The above assumptions are inline with the pre-existing works, which consider the fact that even if the line-of-sight links exist between the source and IRS and/or IRS and destination are blocked, there still exist extensive scatterers [20]–[22], [37].

A. Transceiver Impairments

In this subsection, we discuss the non-idealities in the transceiver hardware that causes the signal distortion. The general assumption of ideal hardware is not realistic as the transceiver architecture at the source and destination are prone to various inevitable additive impairments such as I/Q imbalance, phase noise and radio frequency (RF) nonlinearities. Infact, even in the presence of mitigation schemes applied at the transmitter and the receiver, the residual impairments lead to inevitable distortion noise. A block diagram of a typical

| Reference            | Work                    | Contribution                                                                 |
|----------------------|-------------------------|-------------------------------------------------------------------------------|
| Huang et al. [20]    | Energy Efficiency       | Energy-efficient design by optimizing the transmit power and phase shift for   |
| Zhang et al. [22]    | Optimal Phase           | Analytically evaluated the optimal number of required bit for discrete phase   |
| Jung et al. [23]     | Uplink Sum Rate         | Asymptotic analysis of uplink sum-rate distribution                           |
| Basar et al. [21]    | SER                     | Upper bound on the SER in IRS-assisted wireless system                        |
| Thirimualavan et al. | IRS-assisted NOMA       | BER analysis of IRS-assisted NOMA transmission                                 |
| Pan et al. [27]      | SWIPT with IRS          | IRS aided MIMO broadcasting where the energy harvesting capabilities of        |
| Bjornson et al. [33] | IRS v/s DF Relaying     | Compared the energy-efficiency and required transmit power for both IRS and   |
| Renzo et al. [34]    | IRS v/s Relaying        | Analytical Performance Evaluation and Comparison of IRS and AF Relaying       |
| Boulogeorgos et al.  | IRS v/s AF Relaying     | Analyzed the impact of non-ideal hardware on outage, diversity and provided   |
| Boulogeorgos et al.  | Impact of Hardware Impairments | the upper bound on spectral efficiency                                           |

TABLE I
RELATED WORK
TABLE II
LIST OF ACRONYMS

| Abbreviation | Description |
|--------------|-------------|
| AF           | Amplify-and-Forward |
| AM/AM        | Amplitude-to-Amplitude |
| AM/PM        | Amplitude-to-Phase |
| AWGN         | Additive White Gaussian Noise |
| BER          | Bit Error Rate |
| CDF          | Cumulative distribution function |
| CLT          | Central limit theorem |
| DF           | Decode-and-Forward |
| EE           | Energy Efficiency |
| EVM          | Error Vector Magnitude |
| HPA          | High Power Amplifier |
| IBO          | Input Back-Off |
| IM           | Index Modulation |
| IRS          | Intelligent Reflecting Surface |
| LO           | Local Oscillator |
| MEMS         | Micro-Electrical-Mechanical Systems |
| MIMO         | Multiple-Input and Multiple-Output |
| NOMA         | Non-Orthogonal Multiple Access |
| OFDM         | Orthogonal Frequency Division Multiplexing |
| PDF          | Probability Density Function |
| PLS          | Physical Layer Security |
| QoS          | Quality-of-Service |
| RE           | Reflecting Element |
| RF           | Radio Frequency |
| RV           | Random Variable |
| SDNR         | Signal-to-Distortion plus Noise Ratio |
| SE           | Spectral Efficiency |
| SER          | Symbol Error Rate |
| SM           | Spatial Modulation |
| SNR          | Signal-to-Noise Ratio |
| SSPA         | Solid State Power Amplifier |
| SWIPT        | Simultaneous Wireless Information and Power Transfer |
| WPCN         | Wireless Powered Communication Network |
| 5G           | Fifth-Generation |
| 6G           | Sixth-Generation |

TABLE III
NOTATIONS

| Symbol | Definition |
|--------|------------|
| \( Pr[\cdot] \) | Probability |
| \( E[\cdot] \) | Expectation operator |
| \( tr[\cdot] \) | Trace operator |
| \( [\cdot]^H \) | Hermitian operator |
| \( E_1(\cdot) \) | The Exponential integral |
| \( 2F_1(\cdot, \cdot; \cdot) \) | The Gauss hyper-geometric function |
| \( \Gamma(\cdot) \) | Gamma function |
| \( \gamma(\cdot, \cdot) \) | Lower incomplete Gamma function |
| \( K_v(\cdot) \) | Modified Bessel function (second kind, order \( v \)) |
| \( F_v(\cdot) \) | Polygamma function (zero order) |
| \( G^{\nu}_m(\cdot|\cdot) \) | Meijer’s G function |
| \( f_G(x), F_G(x) \) | The PDF and CDF of random variable \( G \) |

Fig. 1. Schematic for the IRS-Assisted Wireless System.

As described in the literature, HPA is one of the most crucial components in the transmit RF-chain. Further, HPA is

Fig. 2. Schematic showing various imperfections at the source transceiver.

transmitter is shown in Fig. 2. The baseband I and Q signals are up-converted to the desired RF after traversing through the modulator. An ideal local oscillator (LO) is intended to provide an exact 90 degrees phase shift between the I/Q branches along with equal gain. However, a phase imbalance occurs due to the non-idealities of the LO, where the phase shift is not exactly 90 degrees. This also results in a gain mismatch, and the combined effect of the gain and phase imbalances is referred to as I/Q imbalance. Similarly, the power amplifier (PA), which is generally the last block in the transmit chain, is a nonlinear device that results in in-band as well as out-of-band distortion. This results in the degradation of the error vector magnitude (EVM) and the spectral regrowth of the transmitted signal.

As described in the literature, these residual impairments can be characterized in terms of distortions, which can be: a) multiplicative, when the received signals are shifted in phase; b) additive, where a distortion noise is added whose variance is proportional to the power of the transmitted signal and received signal; and c) amplified thermal noise [39]. However, for the IRS-assisted wireless system, the PA will have a larger role since only a fraction of power is reflected from the IRS and reaches the receiver.

B. High Power Amplifier (HPA) Model

As described in the literature, HPA is one of the most crucial component in the transmit RF-chain. Further, HPA is
also the major source of nonlinearity in the transmit RF-chain. Since the HPAs are designed to provide maximum efficiency in the saturation region, multi-carrier wideband communication techniques, like the most widely employed orthogonal frequency division multiplexing (OFDM), are highly impacted due to clipping of the peak signal. A lot of literature has been contributed to mitigate the impact of nonlinear distortion due to HPA. Specifically, the proper modeling of HPA is the key to study the impact of nonlinear distortion and mitigating techniques. So, in this subsection, we describe the behavior modeling of the nonlinear HPA present at the source.

A nonlinear HPA is generally characterized by amplitude-to-amplitude (AM/AM) and amplitude-to-phase (AM/PM) distortion. The signal at the output of the HPA can be expressed as [40]
\[
\psi = \Xi_a(\phi) \exp(j(\arg(\phi) + \Xi_p(\phi))),
\]
where, \(\arg(\phi)\) is the phase of the complex input signal to HPA, \(\phi\), and \(\Xi_a(\cdot), \Xi_p(\cdot)\) are the AM/AM and AM/PM characteristics function respectively.

For the solid-state power amplifier (SSPA), a Rapp model\(^1\) can be used for the behavioral characterization, where the AM/AM and AM/PM functions can be given as
\[
\Xi_a(\phi) = |\phi| \left[ 1 + \left( \frac{|\phi|}{A_{sat}} \right)^{2\tau} \right]^{-\frac{1}{2\tau}},
\]
and
\[
\Xi_p(\phi) = \Theta \left( \frac{|\phi|}{A_{sat}} \right)^{4},
\]
where \(A_{sat}\) is the HPA saturation amplitude, \(\tau\) is the smoothness factor which regulates the transition of linear domain to the saturation domain. As \(\tau\) converges to infinity, SSPA effectively converges to the soft-envelope limiter (SEL) model which is characterized by a linear piece-wise model. Further, since \(\Theta\) is typically zero as the AM/AM distortion is much more significant in SSPA [43], (1) can be re-written as
\[
\psi = |\phi| \exp(j(\arg(\phi))) \left[ 1 + \left( \frac{|\phi|}{A_{sat}} \right)^{2\tau} \right]^{-\frac{1}{2\tau}},
\]
or equivalently
\[
\psi = \phi^\mathcal{F}(|\phi|),
\]
with
\[
\mathcal{F}(|\phi|) = \left[ 1 + \left( \frac{|\phi|}{A_{sat}} \right)^{2\tau} \right]^{-\frac{1}{2\tau}}.
\]

Now using the Bussgang theorem [44], which states that the output of a non-linear HPA can be decomposed into two terms: (a) a linear scaling parameter, \(\alpha\), which provides the linear gain of the input signal and (b) a non-linear distortion parameter, \(\delta\), uncorrelated with the input signal with \(\delta \in \mathcal{CN}(0, \xi_\delta^2)\). So, in this case, (5) can be rewritten as
\[
\psi = \alpha \phi + \delta.
\]

\(^1\)Although this work considers the Rapp model, the proposed analytical framework remains valid for other models as well such as polynomial, Saleh Model etc. [41], [42]

Now, we can derive the expressions of \(\alpha\) and \(\xi_\delta^2\) as follows.

The scaling parameter \(\alpha\) can be derived as
\[
\alpha = \frac{E[\phi^2|]\psi]}{E[|\phi|^2]}.
\]

From [45], \(\alpha\) can be evaluated as
\[
\alpha = \frac{A_{sat}}{2\sigma_{in}} \left[ \frac{2A_{sat}}{\sigma_{in}} - \sqrt{\pi} \text{erfc} \left( \frac{A_{sat}}{\sigma_{in}} \right) \exp \left( \frac{A_{sat}^2}{\sigma_{in}^2} \right) \times \left( \frac{2A_{sat}^2}{\sigma_{in}^2} - 1 \right) \right],
\]
where \(\text{erfc}(\cdot)\) is the complementary error function. Similarly the variance of \(\delta\) can be evaluated as
\[
\xi_\delta^2 = E[|\psi|^2] - \alpha E[\phi^2 \psi^*].
\]

Now from [45], \(\xi_\delta^2\) can be expressed as
\[
\xi_\delta^2 = \frac{\sigma_{in}^2}{\kappa_\delta^2} \left[ 1 + \left( \frac{A_{sat}^2}{\sigma_{in}^2} \right) \exp \left( \frac{A_{sat}^2}{\sigma_{in}^2} \right) E[\exp(-\frac{A_{sat}^2}{\sigma_{in}^2})] - \alpha^2 \right],
\]
or equivalently it can be rewritten as
\[
\xi_\delta^2 = \kappa_\delta^{-2} \sigma_{in}^{-2},
\]
where \(\kappa_\delta^2\) represent the scaling parameter for the input signal power, \(\sigma_{in}^2\).

Practically the impacts of distortion due to the HPA non-linearity is mitigated by operating the HPA at input back-off (IBO) from the given saturation level. From the literature, the widely accepted definition of the IBO is given as
\[
\text{IBO} = 10 \log_{10} \left( \frac{A_{sat}^2}{\sigma_{in}^2} \right).
\]

Increasing the IBO shifts the HPA operation towards the linear region from the saturation region. Consequently, the output of HPA is linearized as the nonlinear distortion is reduced. However, it may also be noted that the HPAs are designed to provide maximum efficiency while operating in the saturation region. Thus, increasing the IBO reduces HPA efficiency; consequently, the EE of the overall system suffers.

C. Signal Model

Consequently at the source, the above imperfections will result into a mismatch between the actual transmitted signal and the desired signal, \(x\), where \(x \in \mathcal{CN}(0, \sigma_{in}^2)\). So from (7), the actual transmitted signal, \(\hat{x}\), can be rewritten as
\[
\hat{x} = \alpha x + \delta.
\]

At destination, the baseband equivalent received reflected signal from IRS can be obtained as
\[
y = \sum_{i=1}^{M} h_i g_i r_i \hat{x} + w_d + n,
\]
where, \(n\) denotes the additive white Gaussian noise (AWGN) and which is modeled as a zero-mean complex Gaussian process having a variance of \(\sigma^2\).
Now representing the equivalent source-to-IRS-to-destination channel by $G$, (15) can be represented as
\[ y = Gr_i \bar{x} + w_d + n, \]
\[ y = \sum_{i=1}^{M} |h_i| |g_i|, \]
with
\[ G = \sum_{i=1}^{M} |h_i| |g_i|, \]
where, $w_d$ is the distortion from the receiver hardware imperfections at the destination, and can be modeled as a zero-mean complex Gaussian process\(^2\) with variance
\[ \xi_d^2 = \kappa_d^2 G^2 \sigma^2_{in}, \]
where $\kappa_d$ is a proportionality constant, the value of $\kappa_d$ controls the level of impairment [48].

Moreover, $r_i$ represents the response of the $i$th RE and it can be obtained as
\[ r_i = |r_i| \exp(j \varphi_i), \]
where $\varphi_i$ denotes the phase shift that is applied by the $i$th RE of the IRS.

Further, without losing generality, we assume that $|r_i| = 1$. This assumption is inline with realistic implementations, as discussed in [37]. The IRS can be configured to provide optimal phase shift\(^3\), assuming that the perfect channel estimates are available at IRS. Thus, the optimal phase can be adjusted as
\[ \varphi_i = - (\varphi_{h_i} + \varphi_{g_i}), \]
so, for the optimal phase at IRS, (19) can be rewritten as
\[ r_i = \exp(-j (\varphi_{h_i} + \varphi_{g_i})). \]

Now by substituting (21) into (15), the received signal at the destination can be expressed as
\[ y = G \bar{x} + w_d + n. \]
Thus, the baseband equivalent received signal in (22) can be rewritten by substituting (14) as
\[ y = G \bar{x} + G \delta + w_d + n, \]
\[ = G \bar{x} + \mathcal{W} + n, \]
where, $\mathcal{W}$ denotes aggregated distortion caused by the hardware impairments at the source and the destination.

From (23), it is evident that for a given channel realization, the aggregated impact of source and destination hardware imperfections can be modeled via a zero-mean random variable process with variance
\[ \sigma^2_w = |G|^2 (\kappa^2_\delta + \kappa^2_d) \sigma^2_{in}. \]
\[ \sigma^2_w = \kappa^2_\delta G^2 \sigma^2_{in}, \]
Further from (24), it can be observed that increasing the transmission power increases $\kappa^2_\delta$ and $\kappa^2_d$, consequently the variance of distortion noise due to hardware imperfections increases.

Now, it can also be noted here that (23) reduces to the ideal transceiver case when $\kappa_\delta = \kappa_d = 0$. A similar observation can be made for the case when the impact of hardware imperfections is neglected. Further, from (24), for both these cases $\sigma^2_w = 0$. Similarly, as described in (13), operating the HPA with IBO reduces the impact of nonlinearity, i.e., increasing IBO reduces $\kappa^2_\delta$, this corresponds to the moderately non-linear scenario, where the nonlinearity is controlled through IBO. Thus, the current analytical framework is capable of analyzing various scenarios through appropriate selection of simulation parameter.

III. PERFORMANCE ANALYSIS

In this section, we present the theoretical analysis for the impact of hardware imperfections on the performance of IRS-assisted wireless systems. Specifically, we evaluate the instantaneous end-to-end SDNR, which is further utilized to derive the closed-form expressions for the outage probability. Further, the spectral efficiency is formulated and evaluated for different cases. Specifically, the three special cases, viz. for $M = 1$, large $M$ and high SNR approximation, are also considered. Finally, we present the energy efficiency evaluation where the power modeling is also discussed.

From (23) and (24), the instantaneous SDNR can be obtained as
\[ \rho = \frac{\alpha^2 |G|^2 \sigma^2_{in}}{|G|^2 (\kappa^2_\delta + \kappa^2_d) \sigma^2_{in} + \sigma^2}, \]
\[ \rho_s = \frac{\sigma^2_{in}}{\sigma^2_w}, \]
denotes the transmit signal-to-noise ratio (SNR).

The normalized instantaneous rate $R_{in}$ for the source-to-destination link through IRS can be formulated as
\[ R_{in} = \log_2 \left( 1 + \rho \right), \]
\[ = \log_2 \left( 1 + \frac{\alpha^2 |G|^2}{(\kappa^2_\delta + \kappa^2_d) |G|^2 + \frac{1}{\rho_s}} \right). \]

Next, we provide the statistical characterization of the end-2-end (e2e) channel gain. The probability density function (PDF) and cumulative density function (CDF) of $G$ can be respectively expressed as [37]
\[ f_G(x) = \frac{x^a}{b^{a+1} \Gamma(a+1)} \exp \left( -\frac{x}{b} \right), \]
and
\[ F_G(x) = \frac{\Gamma \left( 1 + \frac{x}{b} \right)}{\Gamma \left( a + 1 \right)}, \]
where
\[ a = \frac{k_1}{k_2} - 1, b = \frac{k_2}{k_1}, \]
with
\[ k_1 = \frac{M \pi}{2}, k_2 = 4M \left(1 - \frac{\pi^2}{16}\right). \] (32)

Special Case: This refers to the specific case where the IRS is just a single RE, i.e. \( M = 1 \). In this scenario, channel gain \( G \) is the product of two independent and identically random variables (RVs) with Rayleigh distribution. Thus, the PDF and CDF for this special case follows a double Rayleigh distribution which can be expressed as below [37]
\[ f_G(x) = 2K_0(x), \] (33)
and
\[ F_G(x) = 1-xK_1(x), \] (34)
where \( K_0(.) \) and \( K_1(.) \) denotes the second kind of modified Bessel functions having the order of 0 and 1 respectively.

A. Outage Probability

The e2e outage from the source-to-IRS and IRS-to-destination can be defined in terms of rate threshold, \( R_{th} \), as below:
\[ P_{out} = \Pr [R_{in} < R_{th}]. \] (35)

Now from (28), the outage can be formulated as:
\[ P_{out} = \Pr \left[ \log_2\left(1 + \frac{\alpha^2|G|^2\sigma_{in}^2}{(\kappa_3^2 + \kappa_4^2)|G|^2 + \frac{1}{\rho_s}}\right) < R_{th}\right], \] (36)
or equivalently
\[ P_{out} = \Pr \left[ \frac{\alpha^2|G|^2}{(\kappa_3^2 + \kappa_4^2)|G|^2 + \frac{1}{\rho_s}} < \rho_{th}\right], \] (37)
where \( \rho_{th} = 2R_{th} - 1 \) is the SNR threshold. After some mathematical manipulation, the outage probability, \( P_{out} \), can be evaluated as:
\[ P_{out} = \Pr \left[ |G|^2 > \frac{\alpha^2\rho_s}{\rho_{th}} - \alpha^2\rho_s(\kappa_3^2 + \kappa_4^2)\right], \]
\[ = \Pr \left[ |G|^2 < \frac{1}{\alpha^2\rho_s(\kappa_3^2 + \kappa_4^2) - \alpha^2\rho_s(\kappa_3^2 + \kappa_4^2)}\right], \]
\[ = \Pr \left[ |G| < T\right], \] (38)
where
\[ T = \frac{1}{\sqrt{\alpha^2\rho_s(\kappa_3^2 + \kappa_4^2) - \alpha^2\rho_s(\kappa_3^2 + \kappa_4^2)}.} \] (39)

Next, from (29) the outage probability, \( P_{out} \), can be evaluated as
\[ P_{out} = \int_0^T f_G(x)dx, \] (40)
or, equivalently
\[ P_{out} = F_G(T), \] (41)
which can be simplified by utilizing (30) and expressed as
\[ P_{out} = \frac{\gamma(a+1, \frac{T}{b})}{\Gamma(a+1)}. \] (42)

Similarly for the special case of \( M = 1 \), the outage probability, \( P_{out} \), in (41) can be evaluated by utilizing (34) as below
\[ P_{out} = 1 - \Upsilon K_1(\Upsilon). \] (43)

B. Spectral Efficiency (SE)

Now, the spectral efficiency (SE) can be defined as
\[ SE = \mathbb{E}[\log_2(1 + \rho)], \] (44)
or equivalently
\[ SE = \int_0^\infty \log_2(1 + \rho) f_\rho(x)dx. \] (45)

Substituting (26) into (45), the SE can be rewritten as
\[ SE = \frac{1}{b^{a+1}\Gamma(a+1) \times} \int_0^\infty \rho \exp \left(-\frac{\rho}{b}\right) \left\{ \ln (1 + c_1\rho_s^2) - \ln (1 + c_2\rho_s^2) \right\} dt, \] (46)
After some mathematical manipulation, (47) can be re-formulated as
\[ SE = \frac{1}{b^{a+1}\ln(2)\Gamma(a+1) \times} \int_0^\infty \rho \exp \left(-\frac{\rho}{b}\right) \left\{ I_1 - I_2 \right\} dt, \] (48)
or equivalently
\[ SE = \frac{1}{b^{a+1}\ln(2)\Gamma(a+1) \times} \left\{ I_1 - I_2 \right\}, \] (49)
where
\[ I_1 = \int_0^\infty \rho \exp \left(-\frac{\rho}{b}\right) \ln (1 + c_1\rho_s^2) dt \] (50)
and
\[ I_2 = \int_0^\infty \rho \exp \left(-\frac{\rho}{b}\right) \ln (1 + c_2\rho_s^2) dt \] (51)
with
\[ c_1 = \alpha^2 + \kappa_3^2 + \kappa_4^2, \] (52)
\[ c_2 = \kappa_3^2 + \kappa_4^2. \] (53)

From [50, ch. 3.2],
\[ \ln(z) = (z - 1) \sum_{i=1}^2 F_i(1, 1; 2; 1 - z), \] (54)
utilizing this relation and substituting (54), the resulting expression in (50) and (51) can be rewritten as
\[ I_1 = c_1\rho_s \int_0^\infty t^{a+2}\exp\left(-\frac{t}{b}\right) F_2(1, 1; 2; -c_1\rho_s t^2) dt, \] (55)
\[ I_1 = \{ 2 b^{a+1} (a-1) \} \Gamma (a) c_1 \rho_s \ln (b\sqrt{c_1 \rho_s}) + 2 (a-1) b^{a+1} \Gamma (a) F_0 (3 + a) - \frac{\pi}{(2 + a) (b c_1 \rho_s)^{\frac{3a}{2} + 1}} \csc \left( \frac{a \pi}{2} \right) \, \text{Erfc} \left( \frac{\sqrt{3}}{2} \right) \, \text{Erfc} \left( \frac{\sqrt{5}}{2} \right) \, \text{Erfc} \left( \frac{\sqrt{7}}{2} \right) \]

\[ I_2 = \{ 2 b^{a+1} (a-1) \} \Gamma (a) c_2 \rho_s \ln (b\sqrt{c_2 \rho_s}) + 2 (a-1) b^{a+1} \Gamma (a) F_0 (3 + a) - \frac{\pi}{(2 + a) (b c_2 \rho_s)^{\frac{3a}{2} + 1}} \csc \left( \frac{a \pi}{2} \right) \, \text{Erfc} \left( \frac{\sqrt{3}}{2} \right) \, \text{Erfc} \left( \frac{\sqrt{5}}{2} \right) \, \text{Erfc} \left( \frac{\sqrt{7}}{2} \right) \]

\[ SE = (a-1) \frac{\Gamma (a)}{\Gamma (a+1)} \{ c_1 \rho_s \log_2 (b^2 c_1 \rho_s) - c_2 \rho_s \log_2 (b^2 c_2 \rho_s) \} + \frac{\pi}{(2 + a) b^{a+2} \ln(2) \Gamma (a+1)} \csc \left( \frac{a \pi}{2} \right) \left\{ \frac{1}{(c_2 \rho_s)^{\frac{a+1}{2} + 1}} \text{Erfc} \left( \frac{\sqrt{3}}{2} \right) \, \text{Erfc} \left( \frac{\sqrt{5}}{2} \right) \, \text{Erfc} \left( \frac{\sqrt{7}}{2} \right) \right\} \]

or equivalently by substituting \( f_0^2 (t) \) from (33)

\[ SE = \int_0^\infty t \log_2 \left( 1 + \frac{\alpha^2 t^2 \rho_s}{(\kappa_3^2 + \kappa_4^2) t^2 \rho_s + 1} \right) K_0 (t) \, dt. \]
and [52, eqn. 17]
\[ z = \frac{\Gamma(c)}{1 + \Gamma(b)} G_{2,2}^{1,1} (z | 1 - a, 1 - b, 0, 1 - c) \]  
(67)

and by using (54), \( I_3 \) and \( I_4 \) can be re-written as
\[ I_3 = \frac{c_1 \rho_s}{2} \int_0^\infty t^3 G_{2,2}^{1,1} \left( c_1 \rho_s t^2 \right) dt \]
(68)

and
\[ I_4 = \frac{c_2 \rho_s}{2} \int_0^\infty t^3 G_{2,2}^{1,1} \left( c_2 \rho_s t^2 \right) dt \]
(69)

Now utilizing [53, ch. 2.3], (68) and (69) can be solved as
\[ I_3 = \frac{1}{4c_1 \rho_s} G_{1,3}^{3,1} \left( \frac{1}{4c_1 \rho_s} | -1, -1, 0 \right) \]
(70)

and
\[ I_4 = \frac{1}{4c_2 \rho_s} G_{1,3}^{3,1} \left( \frac{1}{4c_2 \rho_s} | -1, -1, 0 \right) \]
(71)

Thus, the SE for the special case of \( M = 1 \), as defined in (63), can now be expressed by using (70) and (71) as below
\[ SE = \frac{1}{4 \ln(2) \rho_s} \left\{ \frac{1}{c_1} G_{1,3}^{3,1} \left( \frac{1}{4c_1 \rho_s} | -1, -1, 0 \right) - \frac{1}{c_2} G_{1,3}^{3,1} \left( \frac{1}{4c_2 \rho_s} | -1, -1, 0 \right) \right\} \]
(72)

2) Special Case (Large \( M \)): For the special case when the number of REs in the IRS is quite large i.e., \( M \gg 1 \), the SE can be approximated as below.

From (46), the SE is defined as:
\[ SE = \int_0^\infty \log_2 \left\{ 1 + \frac{\alpha^2 t^2}{(k_3^2 + k_4^2) t^2 + \frac{1}{\rho_s}} \right\} f_G(t) dt, \]
(73)

now when \( M \) is large, the value of \( t^2 \) is large and thus \( (k_3^2 + k_4^2) t^2 \gg \frac{1}{\rho_s} \). So, in (73), \( \frac{1}{\rho_s} \) can be neglected in the denominator.

Thus, after some mathematical manipulation and simplification, the SE for the special case when \( M \) is large can be expressed as
\[ SE = \int_0^\infty \log_2 \left\{ 1 + \frac{\alpha^2 t^2}{(k_3^2 + k_4^2) t^2 + \frac{1}{\rho_s}} \right\} f_G(t) dt, \]
\[ \text{or equivalently, } \]
\[ SE = \int_0^\infty \log_2 \left\{ 1 + \frac{\alpha^2}{(k_3^2 + k_4^2)} \right\} f_G(t) dt, \]
(74)

3) High SNR Approximation: For the special case, when \( \rho_s \) is very large i.e., \( \rho_s \rightarrow \infty \), from (46), the SE can be defined as:
\[ \lim_{\rho_s \rightarrow \infty} SE = \int_0^\infty \log_2 \left\{ 1 + \frac{\alpha^2 t^2}{(k_3^2 + k_4^2) t^2 + \frac{1}{\rho_s}} \right\} f_G(t) dt. \]
(75)

Now, utilizing the fact that \( \frac{1}{\rho_s} \rightarrow 0 \) as \( \rho_s \rightarrow \infty \), we can rewrite the above as
\[ \lim_{\rho_s \rightarrow \infty} SE = \log_2 \left\{ 1 + \frac{\alpha^2}{(k_3^2 + k_4^2)} \right\} \int_0^\infty f_G(t) dt, \]
(76)

or equivalently
\[ \lim_{\rho_s \rightarrow \infty} SE = \log_2 \left\{ 1 + \frac{\alpha^2}{(k_3^2 + k_4^2)} \right\}. \]
(77)

From (74) and (77), it can be observed that for the case when the transmit SNR is very high or the number of REs is large, the SE saturates. Additionally, the SE can not be further enhanced either by increasing the transmit SNR or by increasing the number of REs. This is in contrast with the conventional case where the SE can be enhanced through increase in the transmit SNR. Further, for the case when the number of reflecting elements is larger, the SE saturates even for mild transit SNR. It may be noted here that, the approximation in (77) is the upper bound of SE for high SNR regime.

C. Energy Efficiency (EE)

The EE of the IRS-assisted wireless system can be defined in terms of bits/Joule/Hz as the ratio of the SE and the total power consumed [54],
\[ EE = \frac{SE}{P_{tot}}, \]
(78)

where \( P_{tot} \) represents the sum of power consumed in transmission and the circuitry at the source, IRS and the destination.

Now the power consumption at the source comprises of the \( P_s \), i.e., the source transmit power, \( P_{hpa}^S \), i.e., the power consumed at the HPA, and \( P_c^S \), i.e., the power required for the operation of other circuit components (excluding the power consumed at HPA)\(^4\). The power consumed at the HPA can be modelled as \( P_{hpa}^S = \beta P_s \), where \( \beta = \frac{1}{2} \) and \( \omega \) is the drain efficiency of HPA. Therefore, the total power consumption at the source can be written as
\[ P_{tot} = (1 + \beta) P_s + P_c^S. \]
(79)

Similarly the power consumed at the destination can be written as
\[ P_{tot}^D = P_c^D, \]
(80)

where \( P_c^D \) is the power consumed at the destination circuitry. Since the IRS is equipped with multiple REs, for the ease of exposition and without losing any generality, we assume that

\(^4\)The other circuit power accounts for the power consumed by other blocks such as mixer, a digital-to-analog converter, frequency synthesizer.
the power consumed by each of the RE is identical [20]. So, the power consumed at the IRS can be expressed as

\[ P_{\text{IRS}}^{I} = M P_{c}^{I} \]  \hspace{1cm} (81)

where \( P_{\text{IRS}}^{I} \) is the circuit power consumption at each RE of the IRS.

After taking into account the above, the EE in (78) can be expressed as

\[ EE = \frac{SE}{(1 + \beta) P_{s} + P_{c}^{S} + M P_{\text{IRS}}^{I} + P_{c}^{D}} \]  \hspace{1cm} (82)

From (82), it can be observed that increasing \( M \) increases the power consumption linearly, whereas the SE increases logarithmically with respect to \( M \). Thus, the overall impact will be a compound effect of both, where the EE increases initially with increasing \( M \), but afterward the EE decreases. This has also been verified through simulation results in the next section.

IV. SIMULATION RESULTS

In this section, the theoretical framework provided in Section III is verified through simulations. The simulation results demonstrate the detrimental impact of hardware imperfections on IRS-assisted wireless systems. The main simulation parameters are summarized in Table IV.

### A. Outage

Fig. 3 shows the outage probability as a function of transmit SNR (\( \rho_{th} \)) for different values of \( M \) and \( \rho_{th} \), where \( \rho_{th} \) (\( \rho_{th} = 2^{R_{th}} - 1 \)) is the SNR threshold corresponding to the rate threshold \( R_{th} \). From Fig. 3, the following observations can be made: a) increasing the number of REs significantly improves the outage probability, as can be seen from the plot for \( M = 1 \), 10 and 50, b) increasing \( \rho_{th} \), reduces the outage probability as can be observed from the plot for \( \rho_{th} = 0 \), 5 and 10 dB, and c) increasing the transmit SNR results in a considerable change in outage performance when the number of reflecting elements is large. For instance, it can be viewed from here that for \( M = 10 \) at \( \rho_{th} = 5 \) dB, the outage probability reduces drastically from \( 10^{-1} \) to \( 10^{-7} \) on increasing the transmit SNR by 10 dB.

Fig. 4 shows the impact caused by the level of distortion noise on the performance of outage probability. A significant impact of \( \kappa \) (here \( \kappa = \kappa_{d} \)) can be observed on the outage probability of the IRS-assisted wireless system. Specifically, the impact of distortion is more significant for higher SNR threshold, i.e. \( \rho_{th} = 10 \) dB. Further, for a lower SNR threshold, the outage probability is not significantly affected by the level of distortion noise. This is due to the fact that the lower SNR threshold corresponds to very low transmit SNR, where the contribution of white Gaussian noise is much more significant than the distortion noise. Moreover, increasing the transmit SNR improves the outage probability, which is quite obvious and intuitive as well.

### B. SE and EE

Fig. 5 shows the impact of HPA nonlinearity on the SE and EE of the IRS-assisted wireless system. It can be observed

| Parameter | Simulation Values |
|-----------|-------------------|
| Circuit Power | \( P_{c}^{S} = 10 \) dBm, \( P_{c}^{D} = 10 \) dBm [20] |
| IRS Power Consumption | \( P_{\text{IRS}}^{I} = 10 \) dBm [20] |
| Variance of AWGN Noise | \( \sigma_{d}^{2} = 1 \) |
| Variance of Channel Elements \( h_{i}, g_{i} \) | 1 [37] |
| HPA Power Consumption Factor | \( \beta = 1.25 \) [54] |
that the SE saturates with increasing transmit SNR. Further, the SE can be improved by operating HPA in the back-off region, which shifts the HPA operating point towards the linear region. It can also be observed here that when the number of REs increases, the SE saturates for lower transmit SNR, as can be seen from Fig. 5(a). Thus, it can be noted that in a nonlinear IRS-assisted wireless system with large $M$, the SE cannot be increased by transmitting more power; however, it can be increased by mitigating the nonlinear distortion through operating the HPA with back-off. Further, operating the HPA in the back-off region reduces the power efficiency of HPA, which consequently impacts the overall EE of the system. Thus, there is a trade-off with respect to input back-off, increasing the IBO improves the SE at the cost of EE of the system.

Fig. 5(b) shows the EE for the IRS-assisted wireless system impaired by a nonlinear HPA. As discussed earlier as well, the EE reduces with increasing the transmit SNR. Further, it can also be observed that operating in the back-off improves the EE, which is due to the enhancement in the SE. Also, when the number of REs is increased, the EE further reduces; this is due to the fact that the SE is saturated, whereas the power consumption at the IRS is increasing. Moreover, it can be observed that increasing the number of REs improves the EE of a nonlinear system when there is sufficient back-off.

Fig. 6 shows the impact of HPA nonlinearity on the SE and EE of the IRS-assisted wireless system with respect to increasing the number of REs. Fig. 6(a) shows the SE with
respect to the number of IRS REs, where it can be observed that for a nonlinear system, the SE saturates with the increase in the number of REs. Further, for large $M$, the SE saturates at even lower SNR. Thus either by increasing the transmit power or by increasing the number of REs, the SE saturates, and no further gain can be achieved. However, if the nonlinear distortion impact is reduced by operating in back-off, SE increases with the number of REs. Fig. 6(b) shows the EE of IRS assisted system with respect to increasing the number of REs. As evident from the result, the EE saturates with the increase in $M$. Further, for the higher transmit SNR, the EE starts decreasing with increase in $M$. Also, it can be noted here that the linear system is the most energy-efficient which is quite intuitive. The nonlinear HPA reduces the EE as can be observed from Fig. 6(b). However, operating the HPA with back-off improves the EE of IRS-assisted system.

Fig. 7(a) and (b) shows the SE and the EE of the IRS-assisted wireless system with respect to the transmit SNR, respectively. It can be observed from Fig. 7(a) that SE increases initially with the transmit power; however, it saturates at higher transmit power. It can be noted that after saturation, the SE can not be enhanced by increasing the transmit power. Further, for a fixed transmit SNR, the SE increases when the number of REs increases; however, at a higher transmit SNR, the SE saturates and it can not be improved by increasing the number of REs. Further, it is noted that as the number of REs increases, SE saturates at lower transmit power. As a consequence, it can be inferred that the distortion noise due
to hardware impairments severely limits the SE of the IRS-assisted wireless system.

Fig. 7(b) shows the corresponding EE, where the EE reduces as the transmit power increase, which is quite obvious. It also shows that EE increases with the number of REs, but for a very large number of REs, the EE again decreases. It is interesting to note that increasing the number of REs does not always guarantee an increase in the EE. This can be due to the fact that increasing the number of REs also increases the circuit power consumption, whereas the SE saturates after an initial increment. Thus this compounding impact results in a decrease in the EE for a large number of REs. This can also be observed analytically from (82).

Fig. 8 shows the impact of the varying distortion noise level on the SE and the EE of the IRS-assisted wireless system. Fig. 8(a) shows the SE with respect to transmit power level and varying distortion noise level. The result shows that the SE reduces with a increase in the level of distortion noise. Further, for each distortion noise level, the SE saturates as the transmit SNR is increased. As a performance benchmark, the SE without distortion, i.e., for an ideal linear IRS system, is also plotted.

Fig. 8(b) shows the EE with respect to the transmit power level. As discussed earlier, increasing the transmit power reduces the EE of the system. Further, as can be seen from above, EE degrades with an increase in the level of distortion noise, which is due to the fact that increasing distortion noise level reduces the SE; consequently, the EE also worsens accordingly.

Fig. 9 shows the joint impact of the HPA nonlinearity at the source and hardware distortion at the destination. Fig. 9(a) shows the SE of the IRS-assisted wireless system, whereas Fig. 9(b) shows the EE. As evident from Fig. 9(a), the HPA nonlinearity is more significant and detrimental than the hardware distortion of the destination. Moreover, the joint impact of source and destination nonlinearity further degrades the SE of the IRS-assisted system. However, the HPA nonlinearity can be alleviated by operating the HPA in the back-off region, although it reduces the HPA efficiency, thereby impacting the EE of the system. It can be seen from Fig. 9(a) that increasing the transmit SNR saturates the SE performance irrespective of the distortion that occurs due to destination hardware impairment or source HPA nonlinearity. However, the saturation level of SE can be increased by large input back-off. Similar observations can be made on the EE performance, as evident from Fig. 9(b). Additionally, Table V summarize the comparison of ideal and non-ideal hardware based IRS-assisted wireless transmission system.

It can be inferred that the linear system has the best EE, whereas the joint source HPA nonlinearity and destination hardware impairment has the worse EE. Further, it can also be concluded from the above result, that the HPA is the major source of nonlinear distortion. Although, the impact of HPA distortion can be mitigated through operating in the back-off region. However, operating HPA in the back-off reduces the operating efficiency and consequently, there is trade-off between the SE and EE with respect to HPA back-off. Thus as evident from the above, the non-ideal transceiver significantly impact the performance of IRS-assisted wireless system. So, these contributions can not be neglected in a

![Figure 9](image_url)

**Fig. 9.** Analytical (lines) and simulation (marker points) results of SE and EE with respect to transmit SNR for different IBOs at $M = 10$, assuming $\kappa = 0.1$. 

**TABLE V**

| Simulation Parameter | Ideal Transceiver | Non-Ideal Transceiver |
|----------------------|-------------------|-----------------------|
| SE       | EE | SE | EE | | | | | | | |
| $M = 1, \ \rho_s = 10 \ dB$ | 4.14 | 0.185 | 2.12 | 0.099 |
| $M = 1, \ \rho_s = 20 \ dB$ | 33.64 | 0.152 | 3.89 | 0.018 |
| $M = 10, \ \rho_s = 10 \ dB$ | 11.18 | 0.507 | 3.86 | 0.175 |
| $M = 10, \ \rho_s = 20 \ dB$ | 41.12 | 0.187 | 3.89 | 0.018 |
| $M = 100, \ \rho_s = 10 \ dB$ | 17.90 | 0.777 | 3.89 | 0.169 |
| $M = 100, \ \rho_s = 20 \ dB$ | 47.80 | 0.216 | 3.89 | 0.018 |

...
practical deployment of IRS based wireless systems.

V. CONCLUSION

This work considers the impact of transceiver impairment on the performance of IRS-assisted wireless systems. Specifically, we have analyzed the SE, EE and outage probability by deriving the closed-form expressions. The HPA nonlinearity at the source has been characterized through AM/AM and AM/PM distortions, whereas a generalized additive Gaussian distortion model has been considered to characterize the hardware imperfections at the destination. The results show that the detrimental impact of hardware imperfection cannot be neglected while analyzing the performance of these systems. It can also be observed that after a particular SNR, SE saturates, and any further gain can neither be realized by increasing the number of REs nor by enhancing the transmit power.

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