Cosmological Bounds on an Invisibly Decaying Higgs Boson

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We derive bounds on the Higgs boson coupling $g'$ to a stable light scalar which is regarded as a collisional dark matter candidate. We study the behaviour of this scalar, that we refer to as phion ($\phi$), in the early Universe for different ranges of its mass. We find that a phion in the mass range of 100 MeV is excluded, while if its mass is about 1 GeV, a rather large coupling constant, $g' \gtrsim 2$, and $m_h \lesssim 130$ GeV are required in order to avoid overabundance. In the latter case, the invisible decay mode of the Higgs boson is dominant.

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1 Introduction

We have recently suggested that a light, stable, strongly self-coupled scalar field coupled with the Higgs field would be an interesting candidate for solving the problems of the Cold Dark Matter (CDM) model at galactic scales [1]. In here we discuss some early Universe history of this particle [2]. Our proposal involves a particle physics-motivated model, where the DM particles are allowed to self-interact so as to have a large scattering cross section and negligible annihilation or dissipation. The self-interaction results in a characteristic length scale given by the mean free path of the particle in the halo. This idea was originally proposed to suppress small scale power in the standard CDM model [3, 4] and has been recently revived, in a general context, in order to address CDM difficulties at galactic scale [5]. Our model [1] is a concrete realization of this idea and involves an extra gauge singlet as the self-interacting, non-dissipative cold dark matter particle. Following Ref. [6], we call this scalar particle phion, \( \phi \), and assume that it couples to the Standard Model (SM) Higgs boson, \( h \), with a Lagrangian density given by:

\[
\mathcal{L} = \frac{1}{2} \left( \partial_\mu \phi \right)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{g}{4!} \phi^4 + g' \phi^2 h ,
\]

where \( g \) is the phion self-coupling constant, \( m_\phi \) its mass, \( v = 246 \text{ GeV} \) is the Higgs vacuum expectation value and \( g' \) is the coupling between \( \phi \) and \( h \). A model along these lines have been previously discussed [7]. Clearly the interaction term between the phion and the Higgs boson arises from a quartic interaction \( \frac{g'}{2} \phi^2 H^2 \), where \( H \) is the electroweak Higgs doublet. As shown in [1], the \( \phi \) mass does not arise from spontaneous symmetry breaking since this would yield a tiny scalar self-coupling constant. The phion mass in (1) should be regarded as a phenomenological parameter arising from a more encompassing theory.

As is well known, scalar particles have been repeatedly invoked as DM candidates [8, 9, 10, 11, 12, 13]; however, our proposal has the salient feature that it brings about a connection with the SM Higgs boson which could arise in extensions of the SM. For instance, the hidden sector of heterotic string theories does give rise to astrophysically interesting self-interacting scalars [14]. For reasonable values of \( g' \), the new scalar would introduce a novel invisible decay mode for the Higgs boson. This could, in principle, provide an explanation for the failure in finding the Higgs boson at accelerators sofar [15].

On the astrophysical front, recent observational data on large scale structure, Cosmic Microwave Background anisotropies and type Ia Supernovae suggest that \( \Omega_{\text{tot}} \approx 1 \), of which \( \Omega_{\text{baryons}} \approx 0.05 \) and \( \Omega_\Lambda \approx 0.65 \) [16]; the remaining contribution, \( \Omega_{DM} \approx 0.3 \) (apart from neutrinos that may contribute a small fraction), comes from dark matter (DM), which determines the hierarchy of the structure formation in the Universe. The most prominent theories of structure formation are now \( \Lambda \)CDM and QCDM, which consist, respectively, of the standard Cold Dark Matter (CDM) model supplemented by a cosmological constant or a dark energy, i.e. a negative pressure component.

In the CDM model, initial Gaussian density fluctuations, mostly in non-relativistic collisionless particles, the so-called cold dark matter, grow during the inflationary period
of the Universe and evolve, via gravitational instability, into the structures one observes at present. However, it has been found that the CDM model cannot successfully accommodate the data observed on all scales. For instance, N-body simulations predict a number of halos which is about an order of magnitude greater than the observed number at the level of Local Group [17, 18]. Furthermore, astrophysical systems that are DM dominated, e.g. dwarf galaxies [19, 20, 21] show shallow matter–density profiles with finite central densities. This contradicts high resolution N-body simulations [22, 23, 24], which have singular cores, with $\rho \sim r^{-\gamma}$ and $\gamma$ in the range between 1 and 2. This can be interpreted as an indication of the fact that since cold collisionless DM particles do not have any characteristic length scale they lead, due to hierarchical gravitational collapse, to very dense dark matter halos that present negligible core radius.

On the other hand, recent numerical simulations [25, 26, 27, 28] indicate that the self-interaction of DM particles does bring noticeable improvements on the properties of the CDM model on small scales.

At present, $\phi$ particles are non-relativistic, with typical velocities $v \simeq 200 \text{ km s}^{-1}$, and, therefore, it is impossible to dissipate energy creating more particles in reactions like $\phi\phi \rightarrow \phi\phi\phi\phi$. Thus, as only the elastic channel is kinematically accessible, near threshold, the cross section is given by:

$$\sigma(\phi\phi \rightarrow \phi\phi) \equiv \sigma_{\phi\phi} = \frac{g^2}{16\pi s} \simeq \frac{g^2}{64\pi m_{\phi}^2} \ . \quad (2)$$

A limit on $m_{\phi}$ and $g$ can be obtained by demanding that the mean free path of the particle $\phi$, $\lambda_{\phi}$, is in the interval $1 \text{ kpc} < \lambda_{\phi} < 1 \text{ Mpc}$ [5]. Hence, we have:

$$\lambda_{\phi} = \frac{1}{\sigma_{\phi\phi} n_{\phi}} = \frac{m_{\phi}}{\sigma_{\phi\phi} \rho_{\phi}^h} \ , \quad (3)$$

where $n_{\phi}$ and $\rho_{\phi}^h$ are, respectively, the number and mass density of $\phi$ particles in the halo. Eqs. (2) and (3) imply that

$$\sigma_{\phi\phi} = 2.1 \times 10^3 \left( \frac{m_{\phi}}{\text{GeV}} \right) \left( \frac{\lambda_{\phi}}{\text{Mpc}} \right)^{-1} \times \left( \frac{\rho_{\phi}^h}{0.4 \text{ GeVcm}^{-3}} \right)^{-1} \text{ GeV}^{-2} \ , \quad (4)$$

which, in turn, leads to:

$$m_{\phi} = 13 \ g^{2/3} \left( \frac{\lambda_{\phi}}{\text{Mpc}} \right)^{1/3} \left( \frac{\rho_{\phi}^h}{0.4 \text{ GeVcm}^{-3}} \right)^{1/3} \text{ MeV} \ . \quad (5)$$
We shall next analyse how the requirement that $\Omega_{\phi} h^2 \simeq 0.3$, i.e. that the phion is a suitable DM candidate, and that the phion is able to explain small scale structure, leads to bounds on the couplings $g$ and $g'$.

## 2 Phion density estimate

If the coupling constant $g'$ is sufficiently small, phions decouple early in the thermal history of the Universe and are diluted by subsequent entropy production. In Ref. [1], it was considered out-of-equilibrium phion production via inflaton decay in the context of $N = 1$ Supergravity inflationary models (see. e.g. [29] and references therein) and found that $\Omega_{\phi} h^2 \simeq 0.3$ can be naturally achieved.

On the other hand, for certain values of the coupling $g'$, it is possible that $\phi$ particles are in thermal equilibrium with ordinary matter. In order to determine whether this is the case, we will make the usual comparison between the thermalization rate $\Gamma_{th}$ and the expansion rate of the Universe $H$.

The thermalization rate is given by

$$\Gamma_{th} = n < \sigma_{\text{ann}} v_{\text{rel}} > ,$$

where $n = 1.2 \times T^3/\pi^2$ is the density of relativistic phions and $< \sigma_{\text{ann}} v_{\text{rel}} >$ is the annihilation cross section averaged over relative velocities. On the other hand, the expansion rate is given by:

$$H = \left( \frac{4\pi^3 g_*}{45} \right)^{1/2} \frac{T^2}{M_P} = 1.66 \times g_*^{1/2} \frac{T^2}{M_P} .$$

At temperatures above the electroweak phase transition, $T_{EW} \simeq 300$ GeV, a typical value in many extensions of the SM where one hopes to find the required features to achieve successful baryogenesis, the order parameter (the vacuum expectation value of the Higgs field) vanishes, and hence the $\phi\phi h$ coupling is non-operative. However, this interaction term has its origin in the 4-point coupling, $\phi\phi hh$, which can keep, at high temperatures, the phion-Higgs system into thermal equilibrium. Using the temperature as the center-of-mass energy, the cross section is given by:

$$\sigma_{\text{ann}} v_{\text{rel}} \simeq \frac{g'^2}{32\pi T^2} ,$$

which implies that phions are in thermal equilibrium for temperatures smaller than

$$T_{eq} \simeq \frac{g'^2 M_P}{32\pi^3 g_*^{1/2}} .$$

Therefore, phions cannot be in thermal equilibrium before the electroweak phase transition if $g' \lesssim 10^{-7}$. 

3
Thermal equilibrium can be achieved just below $T_{EW}$, when the trilinear coupling is operative, if $g'$ is such that the thermalization rate, $\Gamma_{th}$, exceeds the Hubble expansion rate. Let us quantify the conditions on $g'$ to satisfy this condition.

The phion annihilation cross section ($T \gtrsim m_h$) is given by the relativistic Breit-Wigner resonance formula:

$$\sigma_{ann v_{rel}} = \frac{4\pi(s/m_h^2)\Gamma(h \rightarrow \phi\phi)\Gamma_h}{(s-m_h^2)^2 + m_h^2\Gamma_h^2}, \quad (10)$$

where $\Gamma_h$ is the total Higgs decay rate. At the resonance peak ($s = m_h^2$) it simplifies to

$$\sigma_{ann v_{rel}} = \frac{4\pi}{m_h^2} BR(h \rightarrow \phi\phi) . \quad (11)$$

From the Higgs decay width into phions

$$\Gamma(h \rightarrow \phi\phi) = \frac{g'^2v^2(m_h^2 - 4m_\phi^2)^{1/2}}{32\pi m_h^2}, \quad (12)$$

we get the decoupling temperature in the limit $m_h \gg m_\phi$:

$$T_D \simeq 150 \frac{\Gamma_h m_h^3}{g'^2M_P v^2} . \quad (13)$$

Thus, in order to have a decoupling temperature of the order of the Higgs mass, the coupling constant should be fairly small:

$$g' \simeq 10^{-10} , \quad (14)$$

where we have introduced the SM value of $\Gamma_h = 3.2$ MeV, obtained from the code HDECAY [31], for a $m_h = 115$ GeV Higgs boson.

Hence, if $g' \geq 10^{-10}$, the phions will be kept into thermal equilibrium after the electroweak phase transition. In this situation, there are two possible scenarios depending whether they decouple while relativistic or otherwise.

In order to study these scenarios, we have to establish the decoupling temperature at $T \simeq m_\phi \ll T_{EW}$, in which case the phion annihilation cross section involves virtual Higgs exchange ($h^*$), as in Figure 1, and is given by [31]:

$$\sigma_{ann v_{rel}} = \frac{8g'^2v^2}{(4m_\phi^2 - m_h^2)^2 + m_h^2\Gamma_h^2} F_X , \quad (15)$$

where

$$F_X = \lim_{m_{h^*} \rightarrow 2m_\phi} \left( \frac{\Gamma_{h^*X}}{m_{h^*}} \right) , \quad (16)$$
and $\Gamma_{h^*X}$ refers to the width for the decay $h^* \rightarrow X$ ($X \neq \phi \phi$, since we are dealing with inelastic scattering only), for $m_{h^*} = 2m_\phi$. For the mass range of interest to us, $m_\phi \sim 10 - 100$ MeV, one finds $F_X \sim 10^{-13}$ \cite{22}.

Under these conditions, the relationship between the coupling constant $g'$ and the decoupling temperature $T_D$ is given by

$$g'^2 = 5.5 \left( \frac{m_h/100 \text{GeV}}{T_D/\text{MeV}} \right)^4. \quad (17)$$

If $g' \leq 0.1$, the phions decouple while relativistic and are as abundant as photons. Since we are interested in stable light phions, it is a major concern avoiding phion overproduction if it decouples while relativistic. Actually, we find that there is an analogue of Lee-Weinberg limit for neutrinos (see e.g. \cite{33}): \begin{equation}
\Omega_\phi h^2 \sim 0.08 \frac{m_\phi}{\text{eV}}, \quad (18)
\end{equation}

yielding a very stringent bound, $m_\phi \lesssim 4 \text{ eV}$, and therefore, $g \lesssim 2.5 \times 10^{-10}$, for the phion self-coupling constant so to solve the small scale structure problem of the collisionless CDM.

In order to ensure that the phions decouple non-relativistically and that their abundance reduces to acceptable levels without fine-tuning the self-coupling constant, $g' > 0.1$ is required. It follows from standard methods that the phion relic abundance \cite{31, 33}, is given by

$$\Omega_\phi h^2 = \frac{1.07 \times 10^9 x_F}{g_*^{1/2} M_P \sigma_{\text{ann}} v_{\text{rel}}}, \quad (19)$$

where $g_*$ denotes the number of degrees of freedom in equilibrium at annihilation and $x_F \equiv m_\phi/T_F$ is the inverse of the freeze-out temperature in units of the phion mass. The relevant cross section is the phion annihilation cross section involving virtual Higgs
exchange, Eq. (17), with $F_X \sim 10^{-13}$ [32]. The freeze-out temperature is set by the solution of the Boltzmann equation

$$x_F \simeq \ln[0.038(g_* x_F)^{-1/2} M_P m_\phi \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle].$$  \hspace{1cm} (20)

For obtaining $x_F \geq 1$ and to apply Eq. (19), it is required that $g' \gtrsim 1.2$, for $m_\phi = 50$ MeV and $m_h = 115$ GeV. However, due to the smallness of the cross section, the relic abundance of the phion is several orders of magnitude larger than the observed value. Therefore, one can conclude that a phion particle with a mass in this range is excluded.

The smallness of the phion annihilation cross section for $m_\phi \simeq 50$ MeV has its origin in the small factor $F_X \sim 10^{-13}$. However, this factor increases significantly with larger phion masses. For $m_\phi \simeq 1$ GeV, $F_X \simeq 10^{-7}$. We find that the requirement $\Omega_\phi h^2 \simeq 0.3$ implies that $m_\phi \gtrsim 500$ MeV and $g' \gtrsim 2$ (which is at the edge of validity of perturbation theory), a solution which holds only for $m_h \lesssim 130$ GeV. Heavier phion and Higgs particles tend to make $\Omega_\phi h^2 > 0.3$. Our results are depicted in Figure 2.

For these large values of the coupling constant, the decay width of the Higgs into phions is given by:

$$\Gamma(h \rightarrow \phi \phi) = 5.23 \left( \frac{m_h}{115 \text{ GeV}} \right)^{-1} g'^2 \text{ GeV}. \hspace{1cm} (21)$$

Thus, the Higgs width is totally dominated by the invisible decay mode and this model can be easily tested at future colliders.
3 Conclusions

In this contribution, we have derived, bounds on $g'$, the coupling constant of the Higgs boson to a stable scalar particle, which contribute to Higgs decay via invisible channels. This particle, the phion, is suitable self-interacting dark matter candidate and allows for a solution of the difficulties of the CDM model on small scales.

We find that, for $g' \lesssim 10^{-10}$, the phions never reach thermal equilibrium and are only produced by out-of-equilibrium decay of the inflaton field [1]. In this scenario, the phion does not contribute to the invisible Higgs boson decay channel. For $g' \gtrsim 10^{-10}$, we have found that, if $g' \lesssim 0.1$, the phion decouples while still relativistic and a limit for its mass, $m_\phi \lesssim 4 \text{ eV}$ can be derived, which, in turn, implies in a strong bound on the phion self-coupling constant, $g \lesssim 10^{-9}$. On the other hand, if $g' \gtrsim 1$, the phion decouples while non-relativistic; but, its abundance is not cosmologically acceptable for phion masses in the range of $50 - 100 \text{ MeV}$ due to the small annihilation cross section. For masses in the range of $0.5 - 2 \text{ GeV}$, we find that abundances of $\Omega_\phi h^2 \simeq 0.3$ require large values of the coupling $g' \simeq 2.5$ and $m_h \lesssim 130 \text{ GeV}$. In this scenario, the Higgs width is dominated by the invisible $h \rightarrow \phi \phi$ mode and can be tested at future colliders.

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References

[1] M.C. Bento, O. Bertolami, R. Rosenfeld and L. Teodoro, Phys. Rev. D62 (2000) 041302.

[2] M.C. Bento, O. Bertolami, R. Rosenfeld, Phys. Lett. B518 (2001) 276.

[3] E.D. Carlson, M.E. Machacek and L.J. Hall, Astrophys. J. 398 (1992) 43.

[4] A.A. de Laix, R.J. Scherrer and R.K. Schaefer, Astrophys. J. 452 (1995) 452.

[5] D.N. Spergel and P.J. Steinhardt, Phys. Rev. Lett. 84 (2000) 3760.

[6] T. Binoth and J.J. van der Bij, Z. Phys. C75 (1997) 17.

[7] V. Silveira and A. Zee, Phys. Lett. B161 (1985) 136.

[8] J.A. Frieman and B.A. Gradwohl, Phys. Rev. Lett. 67 (1991) 2926.
[9] J. McDonald, Phys. Rev. D50 (1993) 3637.

[10] O. Bertolami and F.M. Nunes, Phys. Lett. B452 (1999) 108.

[11] P.J. Peebles, astro-ph/0002493.

[12] J. Goodman, astro-ph/0003018.

[13] T. Matos and L.A. Ureña López, astro-ph/0010226.

[14] A.E. Faraggi and M. Pospolev, hep-ph/0008223.

[15] See, e.g., T. Binoth and J. J. van der Bij, hep-ph/9908250 and references therein.

[16] N. Bahcall, J.P. Ostriker, S. Perlmutter and P.J. Steinhardt, Science 284 (1999) 1481 and references therein.

[17] B. Moore, S. Ghigna, F. Governato, G. Lake, T. Quinn and J. Stadel, Astrophys. J. Lett. 524 (1999), L19.

[18] A.A. Klypin, A.V. Kravtsov, O. Valenzuela and F. Prada, Astrophys. J. Lett. 522 (1999), 82.

[19] B. Moore, Nature 370 (1994) 629.

[20] R. Flores and J.R. Primack, Astrophys. J. 427 (1994) L1.

[21] A. Burkert, Astrophys. J. 477 (1995) L25.

[22] J. Navarro, C.S. Frenk and S.D. White, Astrophys. J. 490 (1997) 493.

[23] S. Ghigna, B. Moore, F. Governato, G. Lake, T. Quinn, J. Stadel, astro-ph/9910166.

[24] B. Moore, T. Quinn, F. Governato, J. Stadel and G. Lake, astro-ph/9903164.

[25] S. Hannestad, astro-ph/9912558.

[26] B. Moore, S. Gelato, A. Jenkins, F.R. Pearce and V. Quilis, astro-ph/0002308.

[27] N. Yoshida, V. Springel, S.D. White and G. Tormen, astro-ph/0002362.

[28] B.D. Wandelt, R. Davé, G.R. Farrar, P.C. McGuire, D.N. Spergel and P.J. Steinhardt, astro-ph/0006344.

[29] M.C. Bento and O. Bertolami, Phys. Phys. B365 (1996) 59.

[30] A. Djouadi, J. Kalinowski and M. Spira, hep-ph/9704448.
[31] C.P. Burgess, M. Pospelov and T. ter Veldhuis, hep-ph/0011335.

[32] J. Gunion, H.E. Haber, G.L. Gordon and S. Dawson, *The Higgs Hunters Guide* (Addison-Wesley, Reading MA, 1990).

[33] E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Reading MA, 1990).