Dynamics of cosmological perturbations and reheating in the anamorphic universe

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Abstract. We discuss scalar-tensor realizations of the Anamorphic cosmological scenario recently proposed by Ijjas and Steinhardt [1]. Through an analysis of the dynamics of cosmological perturbations we obtain constraints on the parameters of the model. We also study gravitational Parker particle production in the contracting Anamorphic phase and we compute the fraction between the energy density of created particles at the end of the phase and the background energy density. We find that, as in the case of inflation, a new mechanism is required to reheat the universe.

Keywords: alternatives to inflation, cosmological perturbation theory, particle physics - cosmology connection, physics of the early universe

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1 Introduction

In the last years we have witnessed the discovery of a lot of precision data concerning the structure of the universe on large scales (see e.g. [2, 3] for recent cosmic microwave background (CMB) anisotropy results). This data can only be explained by invoking a mechanism of structure formation operating in the early universe (see e.g. [4] and [5]). On the other hand, data on the large-scale structure of the universe will then also allow us to probe the physics of the very early universe.

Inflation is the current paradigm for early universe cosmology. The inflationary scenario attempts to solve problems of Standard Big Bang cosmology such as the horizon and flatness problems by invoking a period of rapid accelerated expansion of space [6] (see also [7–12]). Inflation provided a causal explanation for the origin of CMB anisotropies and the large-scale structure of the universe [13] (see also [14]). The inflationary scenario was, in fact, developed before most of the data we now have was in hand. Hence, inflation has been a very predictive scenario. However, inflation also suffers from several problems (see e.g. [15] for an early review). For example, if initial conditions are set at the time when the energy density was given by the Planck scale, then the initial spatial curvature must be tuned to be small in order to enter an inflationary phase (the energy density at that point being several orders of magnitude smaller). In the inflationary scenario there is also a trans-Planckian problem for fluctuations - unless the inflationary phase only lasts close to the minimal amount of time it must in order for the scenario to solve the horizon problem, then all scales which we currently observed had a wavelength smaller than the Planck length at the beginning of inflation, and hence the applicability of Einstein gravity and standard matter actions to the initial development of fluctuations is questionable [16, 17].

Inflation is not the only early universe scenario compatible with cosmological observations [18–22]. There are several alternatives which solve the problems of Standard Big Bang cosmology and which also produce the observed primordial spectrum of fluctuations. One example are bouncing models that involve an initial matter-dominated phase of contraction [23]. In such models, there is no trans-Planckian problem for fluctuations as long as the energy scale of the bounce is smaller than the Planck scale. However, as emphasized in [24], these scenarios suffer from an instability in the contracting phase to the growth of
anisotropies. Among the alternatives to inflation, one which is able to avoid the anisotropy problem, solve the trans-Planckian problem for fluctuations and in which the homogeneous and isotropic background trajectory is an attractor in initial condition space is the *Ekpyrotic scenario* [25]. Ekpyrotic cosmology assumes an early period of ultra-slow contraction which renders the universe spatially flat, smooth (modulo quantum vacuum fluctuations) and isotropic [26], followed by a cosmological bounce which leads to a transition to the expanding phase of Standard Big Bang cosmology. Like in inflationary cosmology, it is assumed that fluctuations originate as quantum vacuum perturbations on sub-Hubble scales which get squeezed and decohere as scales exit the Hubble radius during the contracting phase. However, the induced curvature perturbations obtain a spectrum which is nearly vacuum and hence far from scale-invariant [27, 28] unless entropy fluctuations are invoked [29–32]. Even if entropy fluctuations are introduced, the predicted spectrum of gravitational radiation is highly blue, and generically large non-Gaussianities are induced (see e.g. [33–35] for a review of this topic).

Recently, a new model for the early universe called *Anamorphic cosmology* was proposed by Ijjas and Steinhardt [1, 36, 37]. This scenario combines elements and advantages of both the inflationary and the Ekpyrotic scenarios. It is based on the realization that in scalar-tensor theories of gravity such as dilaton gravity (the low energy limit of string theory) the gravitational parameter is time-dependent in a frame in which the parameters of the matter action (e.g. the mass \( m \) of a particle) are constant (the *string frame*), and on the other hand the matter parameters vary in the frame in which the gravitational parameter (or equivalently the Planck mass \( M_{\text{Pl}} \)) is constant (the *Einstein frame*).\(^1\) The two frames are related via a Weyl transformation. The assumption made in the Anamorphic scenario is that the initial phase of the universe corresponds to Ekpyrotic contraction in the string frame (the frame in which the mass of matter particles is constant), and to inflationary expansion in the Einstein frame. It describes a phase (the *Anamorphic phase*) in which the universe has a smoothing contracting behavior in the string frame, whereas cosmological fluctuations and gravitational waves evolve as in inflationary cosmology. Because the impression of the cosmological background depends on the frame, the authors referred to this class of models as *Anamorphic*. This requires \( m/M_{\text{Pl}} \) to decrease at a particular rate. Then both the spatial curvature and the anisotropies are suppressed, and therefore chaotic mixmaster behavior is avoided, as it is in Ekpyrotic cosmology [26]. On the other hand, like in inflationary cosmology and unlike in the Ekpyrotic scenario, the Anamorphic scenario can directly generate a nearly scale-invariant spectrum of adiabatic cosmological perturbations and gravitational waves using only a single matter scalar field.\(^2\) Note that after the string frame bounce, in the Standard Big Bang cosmology phase of expansion, the string and Einstein frames must coincide. In the following we will explicitly compute the power spectrum of cosmological perturbations in a particular class of Anamorphic models which will allow us to obtain constraints on the model parameters.

Another important aspect to be analyzed in this new scenario is how the late time thermal state emerges (the *reheating* process). In inflationary cosmology, it is typically assumed that there is a separate reheating mechanism which is a consequence of a coupling between the inflaton field (the scalar field driving inflationary expansion) and the Standard Model fields. As studied in [43–48], the energy transfer from the inflaton to Standard Model

\(^1\)Note that this point was the crucial one in the *Pre-Big-Bang cosmology* scenario developed in the early 1990s [38].

\(^2\)Some of these elements were already considered in other works [39–42].
fields is usually very rapid on Hubble time scales and proceeds via a parametric instability in the equation of motion for the Standard Model fields in the presence of an evolving inflaton field. On the other hand, it was shown in [49] that in the matter bounce scenario, gravitational particle production [50, 51] is sufficient to produce a hot early universe. Recently, it was shown that this same mechanism, under some conditions, can also be responsible for the emergence of a hot thermal state in the New Ekpyrotic Model [52, 53], avoiding the need to introduce an additional reheating phase [54].

As previously mentioned, the Anamorphic cosmology combines elements of both inflationary and Ekpyrotic scenarios. In view of this duality, we are interested in analyzing how the generation of the hot universe proceeds in this model. Thus, following the lines of our recent work [54], we compute the energy density produced through the Parker mechanism in the Anamorphic scenario. We only study the contribution to particle production in the Anamorphic phase of contraction, and neglect additional particle production which will occur during the bounce. Hence, we will find a lower bound on the total number of particles produced during the entire cosmological evolution. The goal of this analysis is to understand under which conditions on the model parameters gravitational particle production can be sufficient to reheat the universe, eliminating the need to introduce an extra reheating mechanism, like is done in inflation.

This paper is organized as follows. In section 2, we review the Anamorphic scenario and analyze the background dynamics in a specific class of realizations of this scenario. In section 3, we analyze the dynamics of the cosmological perturbations in the class of models which we consider. We then study Parker particle production during the Anamorphic phase, evaluating the density of produced particles and comparing it to the background density. We conclude in section 5 with a discussion. We will consider background cosmologies described by a Friedmann-Robertson-Walker metric with scale factor $a(t)$ and linearized fluctuations about such a metric.

2 The Anamorphic universe

We begin with a review of the Anamorphic scenario [1]. We consider a theory containing matter with a characteristic mass $m$ (e.g. the mass of a scalar matter field) and with a gravitational constant given by a Planck mass $M_{Pl}$, and we assume that both can be time-dependent. The condition to obtain a smoothing contracting phase from the point of view of the string frame (the frame in which $m$ is independent of time), and in which the Universe is undergoing accelerated expansion from the point of view of the Einstein frame (the frame in which $M_{Pl}$ is constant) can be expressed in terms of frame-invariant dimensionless quantities

$$\Theta_m = \left(H + \frac{\dot{m}}{m}\right) M_{Pl}^{-1}, \quad (2.1)$$

and

$$\Theta_{Pl} = \left(H + \frac{\dot{M}_{Pl}}{M_{Pl}}\right) M_{Pl}^{-1}, \quad (2.2)$$

where $H$ is the Hubble parameter. In order to obtain contraction in the string frame and expansion in the Einstein frame we require $\Theta_m < 0$ and $\Theta_{Pl} > 0$. The quantities on the right hand side of the above eqs. can be written either in the Einstein or Jordan frames as long as both terms are in the same frame.

Particle creation in bouncing cosmologies was also considered in [55] but in a different context.
An Anamorphic phase can be obtained in the context of a generalized dilaton gravity action (dilaton gravity can be viewed as the low energy limit of string theory - if the anti-symmetric tensor field of string theory is set to zero). The action proposed in [1] consists of a single scalar field $\phi$ non-minimally coupled to the Ricci scalar and non-linearly coupled to its kinetic energy density. In addition, we assume the existence of a potential $V_J$ for $\phi$. Specifically, the scalar-tensor theory action of [1] is

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2} M_{Pl}^2(\phi) R - \frac{1}{2} k(\phi)(\partial_{\mu}\phi)^2 - V_J(\phi) + L_m \right),$$

(2.3)

where $g_{\mu\nu}$ is the metric, $R$ is the Ricci scalar and $k(\phi)$ is the non-linear kinetic coupling function. The time dependence of the Planck mass is given by another function $f(\phi)$ via

$$M_{Pl}(\phi) \equiv m_{pl} \sqrt{f(\phi)},$$

(2.4)

with $m_{pl}$ being the reduced Planck mass in the frame where $M_{Pl}$ is time independent. $V_J$ is the potential energy density and $L_m$ is the Lagrangian density of matter and radiation. Unlike what was done in [1], here we will not set the $m_{pl}$ factors to 1. The action is written in the string (or Jordan) frame, and the label J denotes quantities in the Jordan frame. Hereafter we write the Hubble parameter $H$ and perform all our analysis in the Jordan frame. We found this choice appropriated because in this frame, $L_m$ and mass terms that might appear in this contribution are independent of the anamorphic field. Moreover, we are interested in studying the contraction phase which is better described in Jordan frame.

The non-trivial kinetic coupling and $\phi-$dependence of $M_{Pl}$ in the Lagrangian can lead to different signs of $\Theta_m$ and $\Theta_{Pl}$ during the Anamorphic phase ($\Theta_m < 0$ and $\Theta_{Pl} > 0$). There are other requirements in order to obtain an Anamorphic phase of contraction: the rate of contracting has to be sufficiently slow to ensure that spatial curvature and anisotropies are diluted. On the other hand, in the Einstein frame one must obtain an almost exponential expansion in order to obtain a nearly scale-invariant spectrum of cosmological perturbations. In addition, one must ensure that the resulting action is ghost-free (this condition is non-trivial since $k(\phi)$ is negative in the Anamorphic phase). As shown in [1] these condition are satisfied if

$$0 < 3 + 2k(\phi) \frac{f(\phi)}{(m_{Pl}f_{\phi})^2} < \epsilon < 1,$$

(2.5)

where $k(\phi) < 0$ during the smoothing phase. The parameter $\epsilon$, is the effective equation of state, and is defined as

$$\epsilon \equiv -\frac{1}{2} \frac{dln \Theta_{Pl}^2}{dln \alpha_{Pl}},$$

(2.6)

where $\alpha_{Pl} \equiv aM_{Pl}/m_{pl}$, with $a$ being the cosmological scale factor. Another important auxiliary quantity is $K(\phi)$:

$$K(\phi) = \frac{3}{2} \left( \frac{f_{\phi}}{f} \right)^2 + \frac{k(\phi)}{f(\phi)m_{Pl}^2}.$$  

(2.7)

This quantity must always be positive in the Anamorphic phase [1].

\footnote{Here, we use a different notation than in [1] for the time independent reduced Planck mass: $m_{pl}$ in this paper is equivalent to $M_{Pl}^0$ from [1].}
A complete Anamorphic scenario should describe the Anamorphic contracting phase followed by a cosmological bounce leading to the hot expanding phase of Standard Big Bang cosmology during which the Jordan and Einstein frames coincide. It is possible to obtain such a cosmology using the action (2.3), as described in [1]. In order to obtain such a bouncing cosmology it is important that \( k(\phi) \) changes sign. Since we are interested in obtaining a lower bound on the total number of particles produced during the entire cosmological evolution to the present time, we can consider particle production only in the Anamorphic contracting phase. Particle production is due to the squeezing of the fluctuation modes. Such squeezing happens during the Anamorphic phase and continues through the bounce phase. If we neglect the squeezing after the end of the Anamorphic phase we obtain a lower bound on the number of particles produced.

We are going to focus on a specific class of the simple Anamorphic models introduced in [1], where the gravitational coupling during the Anamorphic phase of contraction is

\[
f(\phi) = \xi e^{2A\phi},
\]

and the kinetic coupling is given by

\[
k(\phi) = -e^{2A\phi}.
\]

Furthermore, the potential is taken to be

\[
V_J = V_0 e^{B\phi}.
\]

The parameters \( \xi, A, B \) and \( V_0 \) are positive real numbers. For this special case, the equation of state is approximately constant during almost all the Anamorphic phase and

\[
\epsilon = \frac{1}{2K} (B - 4A)^2, \quad K = 6A^2 - \frac{1}{\xi m_{pl}^2}, \tag{2.11}
\]

and hence the condition (2.5) becomes

\[
0 < \frac{1}{2} (4A - B)^2 < K < A|4A - B|, \tag{2.12}
\]
a strict condition on the model parameters, which however can be satisfied.

In the following, we solve for the background cosmology resulting in this class of models. The Friedmann equation and the equation of motion for \( \dot{\phi} \) yield

\[
\Theta_{Pl}(\phi) = \sqrt{\frac{V_J}{m_{pl}^4 f^2(3 - \epsilon)}}, \quad \dot{\phi}_J(\phi) = \Theta_{Pl} m_{pl} \sqrt{\frac{2\epsilon}{K} f}. \tag{2.13}
\]

From the above equations, and making use of the definition of \( f \), we find that the Jordan frame Hubble parameter during the Anamorphic phase is given by:

\[
H = \left( \sqrt{\frac{K}{2\epsilon}} - A \right) \dot{\phi} \equiv -\alpha_1 \dot{\phi}, \tag{2.14}
\]

where \( \alpha_1 \) is positive because of condition (2.5). The Jordan frame Einstein equations for a spatially flat universe without anisotropies then take the following form in the contracting
Anamorphic phase [1]

\[3H^2M^2 = \frac{1}{2} k(\phi) \dot{\phi}^2 + V_J - 3Hm^2_{\text{pl}} \dot{f},\]

\[\dot{H} = -\frac{1}{2m^2_{\text{pl}}} \frac{k(\phi)}{f} \dot{\phi}^2 + \frac{1}{2} \frac{f_{,\phi}}{f} H \dot{\phi} - \frac{1}{2} \left( \frac{f_{,\phi \phi} \dot{\phi}^2}{f} + \frac{f_{,\phi} \ddot{\phi}}{f} \right).\]  \tag{2.15}

From the above equations it is possible to show that the Hubble parameter in the anamorphic phase \(t < 0\) evolves as

\[H = p/t,\]  \tag{2.16}

with

\[p = -\frac{\sqrt{K/2} - A}{A - |B-4A|/2}.\]  \tag{2.17}

To obtain Ekpyrotic-type contraction in the Jordan frame we require \(0 < p < 1/3.\) In this case, the Anamorphic field comes to dominate the cosmological dynamics during contraction for a wide range of initial conditions, showing that there is no initial condition problem. As explained in [1] this model describes a contracting phase that homogenizes, isotropizes and flattens the universe without introducing initial conditions or multiverse problems and unlike in inflation, initial conditions do not have to be finely-tuned. The reason behind this is because the time-varying masses suppress the anisotropy.

In section 4 of this paper we will study the energy density generated by Parker particle production. Since we will be mainly interested in the relative contribution of these particles to the total energy density, we need to find the expression for the background energy density as a function of time \(t\). Considering that during the Anamorphic phase the Anamorphic field is dominating, the background energy density is given as a function of \(\Theta_{\text{Pl}}\) by

\[\rho_A = 3\Theta_{\text{Pl}}^2 M_{\text{Pl}}^4,\]  \tag{2.18}

and, by using eqs. (2.13), (2.14), (2.17) and (2.16) we have

\[\rho_A \approx 3(p-1)^2 f \frac{m^2_{\text{pl}}}{t^2},\]  \tag{2.19}

where we have used the fact that \(\epsilon \ll 1\). From the second Einstein equation, we obtain the following expression for the time evolution of the Anamorphic field:

\[\ddot{\phi} = \left(A - \sqrt{\frac{K(\phi)\epsilon}{2}}\right) \dot{\phi}^2,\]  \tag{2.20}

and given that \(\dot{\phi} = -p/t\alpha\) we can obtain \(\phi(t)\). Thus, considering the solution for \(\phi(t)\) and the background energy density (2.19) at the end of the Anamorphic phase, namely at \(t = t_{\text{end}}\), we have

\[\rho_{bg}(t_{\text{end}}) \approx 3(p-1)^2 f_{\text{end}} \frac{m^2_{\text{pl}}}{t_{\text{end}}^2},\]  \tag{2.21}

where \(f_{\text{end}}\) is given by \(f_{\text{end}} = \xi e^{2A\phi(t_{\text{end}})}\). If we assume that the bounce occurs at the maximum density \(\rho_{\text{max}} = M^4\), where \(M\) is the mass scale of new physics, (which we suppose to be between the scale of particle physics “Grand Unification” (GUT) and the Planck scale), we can then use \(\rho_{bg}(t_{\text{end}}) \sim \rho_{\text{max}}\) and solve (2.21) for the time \(t_{\text{end}}\).

\(^5\text{Note that we do not require } p \ll 1 \text{ but only } \epsilon \ll 1.\)
3 Cosmological perturbations in the Anamorphic phase

Since in the Einstein frame the Anamorphic phase is one of almost exponential expansion, it allows for the generation of a nearly scale-invariant spectrum of adiabatic curvature modes and gravitational waves with small non-Gaussianities. In fact, the tilts of both the scalar and tensor perturbation spectra are red, whereas in the Ekpyrotic scenario one obtains a blue nearly vacuum spectrum of the fluctuations. In the class of examples considered in the previous section the spectral index $n_s$ of cosmological fluctuations is given by [1]

$$n_s - 1 = - \frac{(B - 4A)^2}{K} = -2 \epsilon,$$

which implies that

$$n_s \approx 1$$

if $\epsilon \ll 1$. In the following we will compute the power spectrum of cosmological perturbations at the end of the anamorphic phase, assuming that the inhomogeneities originate as vacuum fluctuations on sub-Hubble scales in the far past.

In the absence of anisotropic stress (which indeed is not present in the scalar-tensor model which we are considering), it is possible to choose a gauge in which the perturbed Jordan frame metric takes the form

$$ds^2 = -dt^2 + a^2 e^{2\zeta(t,x^i)}dx^i dx^i,$$

where $\zeta$ is the curvature fluctuation variable in comoving gauge.

Following the usual theory of cosmological perturbation, the equations of motion for the linear cosmological fluctuations can be obtained by inserting the above ansatz (3.3) into the full action and expanding to quadratic order in $\zeta$. The result for the quadratic terms is [56–58]

$$S^{(2)} = \frac{m_{\text{pl}}^2}{2} \int d\eta d^3 x \alpha_{\text{pl}}^2 \epsilon [\zeta'^2 - c_s^2(\partial_i \zeta)^2]\$$

$$= \int d\eta d^3 x \frac{z^2}{2} [\zeta'^2 - c_s^2(\partial_i \zeta)^2],$$

(3.4)

where for scalar field matter the speed of sound is $c_s^2 = 1$. Here, $\eta$ is conformal time given by $d\eta = a^{-1}dt$ and the prime indicates a derivative with respect to conformal time. The only difference compared to the equation for fluctuations in Einstein gravity with canonically coupled scalar field matter is in the form of the function $z(\eta)$ which in our model is given by

$$z^2 = 2\alpha_{\text{pl}}^2 \epsilon m_{\text{pl}}^2 = 2a^2 f \epsilon m_{\text{pl}}^2,$$

(3.5)

and in the fact that in (3.4) the scale factor in the integrand is not $a(t)$, but the Einstein frame scale factor $\alpha_{\text{pl}}$ given by

$$\alpha_{\text{pl}}(t) = a(t) \frac{M_{\text{pl}}(t)}{m_{\text{pl}}^2}.$$

(3.6)

In terms of the Mukhanov-Sasaki [56, 57] variable $v_k = z\zeta_k$ the action is that of a canonically normalized scalar field with time-dependent mass. The resulting equation of motion is

$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0,$$

(3.7)

where the comoving momentum is denoted by $k$. 

We must solve this equation in the Anamorphic phase, in which
\[
\frac{z''}{z} = \left[ \frac{2 + \epsilon}{(1 - \epsilon)^2} \right] \frac{1}{\eta^2},
\]
(3.8)
where \(\epsilon\) is the effective equation of state described in the previous section. From this equation, together with the expression for the spectral index in our model \((n_S - 1 \approx -2\epsilon)\), we can see that our effective equation of state parameter, \(\epsilon\), plays an analogous role to the slow-roll parameter in inflationary models in that it will determine the slope of the spectrum.\(^6\)

We now review the computation of the power spectrum. We will solve equation (3.7) in a more general setup which can be applied to several other scenarios (application to the Ekpyrotic and New Ekpyrotic models was done in [54]). In this general setup we re-write \(z''/z\) as
\[
\frac{z''}{z} = \nu^2 - \frac{1}{4}\frac{1}{\eta^2}.
\]
(3.9)
For the Anamorphic phase we have (comparing with (3.8))
\[
\nu = \sqrt{\frac{2 + \epsilon}{(1 - \epsilon)^2} + \frac{1}{4}}.
\]
(3.10)
In an Anamorphic phase, modes start out with wavelength smaller than the Hubble length, they cross the Hubble radius \(l_H(t) \equiv H(t)^{-1}\) at a time \(\eta_H(k)\), and then propagate on super-Hubble scales. The mode evolution on sub-Hubble and super-Hubble scales is very different. On sub-Hubble scales they oscillate, while they are squeezed on super-Hubble scales.\(^7\)

Given a mode with comoving wave number \(k\), the conformal time \(\eta_H(k)\) associated with Hubble radius crossing is given by
\[
k^2 \eta_H^2(k) = \nu^2 - \frac{1}{4}.
\]
(3.11)
For sub-Hubble modes the \(k^2\) term dominates over the \(z''/z\) term and the solution to the equation of motion for the perturbations are oscillatory (fixed amplitude). Assuming that we start in the Bunch-Davies vacuum, the sub-Hubble solution is
\[
v_k = \frac{e^{-ik\eta}}{\sqrt{2k}}.
\]
(3.12)
On the other hand, for super-Hubble modes the \(z''/z\) term dominates over the \(k^2\) term and the perturbations suffer squeezing, resulting in the solution
\[
v_k = c_1(k) \frac{\eta^{1/2-\nu}}{\eta_H(k)^{1/2-\nu}} + c_2(k) \frac{\eta^{1/2+\nu}}{\eta_H(k)^{1/2+\nu}}.
\]
(3.13)
\(^6\)Experts on cosmological perturbation theory will immediately see from the factor 2 in (3.8) that for \(\epsilon \ll 1\) an almost scale-invariant spectrum of fluctuations will result.
\(^7\)The solutions valid for all times are given by Bessel functions, but introducing these tends to obscure the physics.
The coefficients $c_1(k)$ and $c_2(k)$ of the two modes can be found by matching $v_k$ and $v'_k$ at Hubble radius crossing $\eta_H(k)$. This yields

$$c_1(k) = \frac{1}{2\nu} \frac{1}{\sqrt{2k}} e^{-ik\eta_H(k)} \left[ \nu + \frac{1}{2} + ik\eta_H(k) \right],$$

$$c_2(k) = \frac{1}{2\nu} \frac{1}{\sqrt{2k}} e^{-ik\eta_H(k)} \left[ \nu - \frac{1}{2} - ik\eta_H(k) \right].$$

(3.14)

For the value of $\nu$ corresponding to the Anamorphic phase, the first mode is growing, and the second decaying.

The solution for $v_k$, given by the above equations, can be used to compute the power spectrum predicted by the model, as we will study in the following subsection. It can also be used to estimate Parker particle production in the Anamorphic phase, as we discuss in section 4.

3.1 Power spectrum

In this subsection we evaluate the power spectrum of the curvature perturbations produced during the Anamorphic phase. We are interested in modes which are super-Hubble at the end of the Anamorphic phase. In this case we can neglect the contribution of the decaying mode in (3.13) and we obtain

$$|v_k(\eta_{\text{end}})|^2 = \frac{1}{4\nu^2} \frac{1}{2k} [(\nu + 1/2)^2 + k^2\eta_H^2] \left| \frac{\eta_{\text{end}}^{1/2-\nu}}{\eta_H^{1/2-\nu}} \right|^2.\tag{3.15}$$

Therefore, since $\nu \approx 3/2$ (which can be obtained from eqs. (3.8) and (3.9) considering $\epsilon \approx 0$) and by using (3.11) this reduces to

$$|v_k(\eta_{\text{end}})|^2 = \frac{2}{3k^3\eta_{\text{end}}^2} = \frac{(1-p)^2}{p^2} \frac{2H_{\text{end}}^2}{3k^3}.\tag{3.16}$$

With this result we can evaluate the dimensionless power spectrum of curvature fluctuations which for $\nu = 3/2$ (i.e. $\epsilon = 0$) is given by

$$P_\zeta = \frac{k^3}{2} |\zeta_k(\eta)|_{\text{end}}^2 = \frac{k^3}{2} \frac{|v_k^2/z^2|_{\text{end}}}{z^2_{\text{end}}},\tag{3.17}$$

where $\zeta_k = v_k/z$, and $z^2_{\text{end}} = 2f_{\text{end}} \epsilon m_{\text{pl}}^2$. Therefore, we obtain the following power spectrum

$$P_\zeta = \frac{(1/p - 1)^2}{3f_{\text{end}}} \frac{H_{\text{end}}^2}{2\epsilon m_{\text{pl}}^2} = \beta^2 \frac{H_{\text{end}}^2}{2\epsilon m_{\text{pl}}^2}.\tag{3.18}$$

In this limit we obtain an exactly scale invariant spectrum. For $0 < \epsilon \ll 1$ the spectrum obtains a slight red tilt, in the same way that a slight red tilt emerges for simple slow-roll inflation models (the parameter $\epsilon$ in the Anamorphic phase of this model plays the same role as the slow-roll parameter in the inflationary scenario).

The amplitude of the power spectrum of cosmological perturbations is given by a very similar expression as in inflationary cosmology in terms of the dependence on the Hubble parameter and on $\epsilon$. There is a difference in the overall amplitude which is given by the multiplicative factor $\beta$ which depends on two parameters of the model, $p$ and $f$ evaluated at
the end of the Anamorphic phase. The presence of this non-trivial factor in the amplitude will imply that the relative importance of Parker particle production of matter particles will be different in the Anamorphic scenario than in inflationary cosmology.

To fix the model parameters we compare the power spectrum at the end of the Anamorphic phase with the observed value as measured by observations of the Cosmic Microwave Background (CMB). The anamorphic contraction is followed by a second contracting phase in which $\epsilon$ varies considerably, and then by the bounce phase (the reader is referred to figure 2 in [1] for an sketch the overall behaviour of the parameters during the contracting phases). Unlike in [1] in which the so called anamorphic phase last until $\epsilon \approx 1$, here we define the anamorphic phase as the phase in which $\epsilon \approx const$. In principle, the power spectrum could also grow during these two phases. We are here neglecting any such additional growth of fluctuations. If both the pre-bounce phase when $\epsilon$ undergoes rapid change and the bounce phase are short, then this will be an excellent approximation, as shown in detailed studies of the evolution of fluctuations in other nonsingular bounce models [59–66]. If the two additional phases are long (on a time scale set by the maximal value of $|H|$), then the conditions on the model parameters which we derive below in order to obtain agreement with the observed power spectrum will change. What will, however, not change is the relative amplitude of Parker particle production and cosmological perturbations since both will be effected in the same way in the two additional phases.

According to the CMB observations (the latest results being from the Planck team [2]), the amplitude of the power spectrum is $A_\zeta \approx 10^{-10}$. Written in terms of the above power spectrum this yields

$$A_\zeta \sim 10^{-10} \sim \frac{\beta^2}{3} \frac{1}{2\epsilon} \frac{M^4}{m_{pl}^4} ,$$

(3.19)

where we used the fact that $3H_{end}^2/m_{pl}^2 = \rho_{bg}/m_{pl}^4$ and that the background energy density is equal to the maximum density, $\rho_{max} = M^4$, where $M$ is the mass scale of new physics. Given a value of $M$ we can then constrain the parameter $\beta$, like was done for the new Ekpyrotic model in [54]. Given that we know that $0 < p < 1$, this translates into a constraint on the parameter $f_{end}$ that enters in the expression for $\beta$:

$$f_{end} \sim \frac{1}{10^{-10}} \left( \frac{1}{p - 1} \right)^2 \frac{1}{18\epsilon} \frac{M^4}{m_{pl}^4} .$$

(3.20)

Since the value of $\epsilon$ must have the same value as in the inflationary scenario $\sim 10^{-2}$ to yield agreement with the observed slope of the spectrum of cosmological perturbations, we obtain bounds on $f_{end}$ which depend on the ratio of the New Physics and Planck scales.

4 Parker particle production in Anamorphic phase

In this section we are interested in computing the number and energy densities of particles created by squeezing of the modes of a matter scalar field $\chi$ (which we treat as a test scalar field minimally coupled in the Jordan frame) by the time of the end of the Anamorphic phase.

Particle production is a consequence of squeezing. Hence, there are no particles generated for modes with wavelengths smaller than the Hubble length at the end of the anamorphic phase. For super-Hubble modes we must be careful how to interpret the squeezed vacuum state in terms of particles. The particle interpretation only becomes valid when the modes re-renter the Hubble radius during the Standard Model phase of expansion. As we will see,
the energy density in produced particles falls off in the infrared. The integral over modes is hence dominated by modes which have a wavelength only slightly larger than the Hubble radius at the end of the anamorphic phase. These are modes which re-enter the Hubble radius shortly after the beginning of the Standard Model phase of expansion. Hence, the particles energy density which we here compute indeed has an interpretation as particle energy density beginning very early in the expanding phase.

As mentioned at the end of the previous section, what we are computing here is a lower bound on the particle energy density since we are neglecting the squeezing during the bounce and in the phase when $\epsilon$ is rapidly changing before the bounce. However, if the squeezing in those phases were important, it would also be important for the cosmological fluctuations, and the squeezing at the end of the anamorphic phase would have to be smaller. The bottom line is that the effects of the decrease in squeezing at the end of the Anamorphic phase and the extra squeezing after the end of the Anamorphic phase will counteract, and will not substantially effect our final result. Just like in [1] we considered that the power spectrum of cosmological perturbations must be computed in the anamorphic phase. However if that was not the case, then the amplitude of cosmological perturbations would increase in the second phase. This only means that we would have made an unappropriated choice of the normalization of the spectrum. By fixing the normalization to the correct one (corresponding to a smaller amplitude) and considering the increase in the following phase to the observed value would imply in the same final result as we obtained. Since the Parker particle production and the cosmological perturbations are affected in the same way in the two additional phases the same conclusion is valid for the density of the particles produced.

The squeezing of the mode functions in a time-varying cosmological background corresponds to gravitational particle production [50, 51] (see e.g. [67, 68] for textbook treatments). As is the standard approach in quantum field theory on curved space-times, the modes $\chi_k$ of a test scalar field $\chi$ (which can also be the cosmological fluctuations or gravitational waves themselves) can be expanded into positive and negative frequency modes. As described in [50, 51], initial pure positive or negative frequency modes (we are using the Heisenberg representation) whose coefficients are interpreted as creating and annihilation operators become mixed during the time evolution. This corresponds to particle production.

At any time $\eta$, the mode functions of a full solution of the equations of motion can be expanded momentarily\(^8\) into a linear combination of the instantaneous vacuum solutions $\chi_{v,k}$, i.e. in terms of a local positive and negative frequency modes.

$$
\chi_k(\eta) = \alpha_k \chi_{v,k}(\eta) + \beta_k \chi^*_{v,k}(\eta), \\
\chi'_k(\eta) = \alpha_k \chi'_{v,k}(\eta) + \beta_k \chi'^*_{v,k}(\eta).
$$

(4.1)

The time-independent coefficients are the Bogoliubov coefficients that, for bosons satisfy $|\alpha_k|^2 - |\beta_k|^2 = 1$, if both sets of modes are normalized. The quantity

$$
n_k \equiv |\beta_k|^2
$$

(4.2)

is interpreted by a late adiabatic time observer as the particle number in the mode $k$ which has been produced starting from a vacuum initial state. The squeezing which $v_k(\eta)$ undergoes leads to a growth of the expansion coefficients and hence to a growth in the number density $n_k$.

In the following, we shall consider two types of particles. First we will consider particles associated to the adiabatic fluctuation mode (this will be particles associated with the

\(^8\)This means that the field values and their first time derivatives coincide.
Anamorphic scalar field). Then, we will consider $\chi$ as a massless matter field minimally coupled in the Jordan frame. This corresponds to usual matter.

### 4.1 Adiabatic mode particles

In this subsection, we are going to study the energy density in particles associated with the adiabatic fluctuation mode, i.e. $\phi$ particles. We use the solution for the mode function $v_k$ from the previous section, i.e. $\chi_k \equiv v_k$, given by (3.13) and (3.14), that represents the modes that suffered squeezing after crossing the Hubble radius. Using (4.1), we obtain the Bogoliubov coefficients by considering that the solution and its derivative can be expanded in terms of the Bunch-Davis vacuum basis (eq. (3.12)).

Considering only the growing solution of (3.13) on super-Hubble scales, we can evaluate the Bogoliubov coefficient $\beta_k$ and obtain

$$\beta_k(\eta) = \frac{c_1(k)\sqrt{2k}}{2} \left( \frac{\eta}{\eta H(k)} \right)^{1/2 - \nu} \left[ 1 + \frac{1/2 - \nu}{i k \eta} \right].$$  

(4.3)

Recalling the expression for $\nu$ (eq. (3.10)) in the limit $\epsilon \ll 1$, i.e $\nu \approx 3/2$, and substituting the coefficient from (3.14), we have that $\beta_k$ is given by

$$\beta_k(\eta) = \frac{1}{3} e^{-i k \eta H(k)} \left[ 1 + \frac{k \eta H(k)}{2} \right] \left( \frac{\eta}{\eta H(k)} \right)^{-1}.$$  

The number density of produced particles is hence given by

$$n_k(\eta) = \frac{1}{9} \left[ 1 + \frac{(k \eta H(k))^2}{4} \right] \left( \frac{\eta}{\eta H(k)} \right)^{-2}.$$  

(4.4)

Note that this result is the same as we obtained in the New Ekpyrotic scenario [54].

We can now compute the energy density of the particles produced until the end of the Anamorphic phase, i.e. the conformal time $\eta_{\text{end}}$ given by $\rho_p(\eta) \sim (1 - p)^4 \eta_{\text{end}}^2$, and substituting the coefficient from (3.14), we have that $\beta_k$ is given by

$$\rho_p(t_{\text{end}}) \sim (1 - p)^4 t_{\text{end}}^{-4}.$$  

(4.5)

We can compare this density of produced particles with the background density (see eq. (2.21)) in order to see if the particles created are sufficient to lead to a post bounce hot big bang phase

$$\frac{\rho_p(t_{\text{end}})}{\rho_{bg}(t_{\text{end}})} \sim \left( \frac{1}{p} - 1 \right)^2 \frac{H_{\text{end}}^2}{3 f_{\text{end}} m_{\text{pl}}^2} = \left( \frac{1}{p} - 1 \right)^2 \frac{1}{9 f_{\text{end}}^4} M_4^{-1} m_{\text{pl}}^4.$$  

(4.6)

Given the constraint on $f_{\text{end}}$ in order to respect the CMB constraint on the amplitude of the power spectrum we can see that, substituting eq. (3.20) in the above ratio we get

$$\frac{\rho_p(t_{\text{end}})}{\rho_{bg}(t_{\text{end}})} \sim \frac{10^{-12}}{\epsilon} \sim 10^{-10}.$$  

(4.7)

We can see that this ratio is smaller than the one one would get from inflation, since in inflation this is proportional to $M_{\text{GUT}}^4 / m_{\text{pl}}^4 \sim 10^{-12}$. Thus, as in the case of inflation, in this model the energy density in particles created by Parker particle production of the Anamorphic field $\phi$ during the Anamorphic phase is not sufficient to reheat the universe.

---

$^9$We recall that the particle interpretation is only valid after the time of Hubble radius re-entry.
4.2 Matter field dynamics

In the last section we computed the density of particles produced from the curvature perturbations whose main contribution comes from the dominant Anamorphic field. Now let us compute the density of particles produced via the Parker mechanism for a massless matter field minimally coupled to gravity in the Jordan frame in a background driven by the Anamorphic field. This field stands for the matter of the Standard Model of particle physics. It is hence the field of most interest regarding reheating of the universe.

In the Jordan frame, the mass of the matter component is independent of the Anamorphic field and is constant [1]. The dynamics of this field is different from that of the gravitational perturbations since the squeezing term in the equations for the Fourier modes of the matter field and of the perturbations, associated to the Mukhanov-Sasaki variable, are different. After rescaling by the Jordan frame scale factor, the Fourier modes of the matter field obey the equation

\[ \chi''_k + \left( k^2 - \frac{a''}{a} \right) \chi_k = 0, \]  

(4.8)

where during the Anamorphic phase

\[ \frac{a''}{a} = -\frac{p(1 - 2p)}{(1 - p)^2} \frac{1}{\eta^2}. \]  

(4.9)

Analogous to eq. (3.9), we can write the time dependent part of the effective mass as

\[ \frac{a''}{a} \equiv \nu_m^2 - \frac{1}{4} \frac{\eta^2}{\nu_m^2}, \]  

(4.10)

which implies that

\[ \nu_m = \sqrt{\frac{1}{4} - \frac{p(1 - 2p)}{(1 - p)^2}}. \]  

(4.11)

Note that for the range \( 0 < p < 1/3 \) of values of \( p \) which give Ekpyrotic contraction \( \nu_m \) is always real. With that, the equation of motion takes the same form as discussed in the previous sections

\[ \chi''_k + \left[ k^2 - \frac{\nu_m^2 - 1/4}{\eta^2} \right] \chi_k = 0. \]  

(4.12)

We see that for the entire range of values of \( p \) of interest, there is less squeezing for these mode functions than for the cosmological perturbations since the coefficient of the \( \eta^{-2} \) term in (4.12) ranges from 0 to 1/4 whereas it is 2 in the case of cosmological perturbations. This implies that if we start with vacuum fluctuations, the resulting spectrum of matter perturbations will be blue. However, since for particle production the dominant contribution comes from modes which exit the Hubble radius just before the end of the Anamorphic phase, the fact that we have a blue spectrum will not in itself lead to a suppression of the energy density in matter particles relative to the energy density in Anamophic particles computed in the previous subsection. What will lead to a suppression of matter particle energy density relative to Anamorphic particle energy density is the fact that the effective Hubble radius crossing conditions differ. For matter particles, it follows from (4.12) that the Hubble crossing condition is

\[ k^2 \eta_H(k)^2 = \nu_m^2 - \frac{1}{4} \sim p, \]  

(4.13)
instead of
\[ k^2 \eta_H(k)^2 \sim 2, \]
as is the case for the cosmological perturbations.\(^\text{10}\) Hence, the value of \( k_H \) is suppressed by \( \sqrt{\rho} \) for matter fluctuations. Since the energy density is dominated by this ultraviolet scale, it leads to a suppression of the matter energy density relative to the Anamorphic particle energy density.

The analysis of particle production parallels the discussion in the previous subsection. We begin on sub-Hubble scales with the Bunch-Davies vacuum solution,
\[ \chi_k = e^{-ik\eta/\sqrt{2k}}, \]
and while \( k^2 \ll a''/a \) the solution is the same as eqs. (3.13) and (3.14). Thus the Bogoliubov coefficients \( \beta_k \) and the number density of produced particles \( n_k \) are the same that in previous case but with \( \nu_m \) instead \( \nu \). Then
\[ n_k(\eta) = \frac{1}{4\nu_m^2} \left[ \frac{(\nu_m + 1/2)^2}{4} + \frac{(k\eta_H)^2}{4} \right] \left( \frac{\eta}{\eta_H} \right)^{1-2\nu_m}, \]
which leads to following matter density
\[ \rho_p^m(\eta_{\text{end}}) \approx A(\nu_m)(1-p)^{2\nu_m-1} \rho_{m}^{5/2-\nu_m} t_{\text{end}}^{-4}, \]
which is suppressed compared to the contribution of anamorphic particles for the reason discussed in the previous paragraph.

Just like in the previous section, we can now write the ratio between the density of particles produced and the background density in the end of Anamorphic phase, which is given by
\[ \frac{\rho_p^m(t_{\text{end}})}{\rho_{bg}(t_{\text{end}})} \approx A(\nu_m)(1-p)^{-3+2\nu} p^{1/2-\nu} \frac{H_{\text{end}}}{3f_{\text{end}} m_{\text{pl}}^2} = A(\nu_m)(1-p)^{-3+2\nu} p^{1/2-\nu} \frac{1}{9f_{\text{end}}} \left( \frac{M}{m_{\text{pl}}} \right)^4 \]
and substituting our constraint (3.20) we get
\[ \frac{\rho_p^m(t_{\text{end}})}{\rho_{bg}(t_{\text{end}})} \approx 2 \times 10^{-12} \tilde{A}(p), \quad \tilde{A}(p) = A(\nu_m) \left( \frac{\sqrt{\rho}}{1-p} \right)^{5-2\nu_m}. \]
\(^\text{10}\)Hopefully the reader will forgive us for using the same symbol \( \eta_H(k) \) for the two different calculations.
After the Anamorphic phase a second phase of contraction takes place in which $\epsilon$ increases considerably until $\epsilon = 3$. During the beginning of this phase $|H|$ keeps increasing until it reaches its maximum value. After that $|H|$ starts decreasing until it reaches $H = 0$ at the bounce (just after the second phase ends). During the bounce phase $k(\phi)$ changes from negative to positive and the quantity $\Theta_m$ also passes through zero and becomes positive. The $\Theta_m$ bounce, which does not require a $\Theta_{Pl}$ bounce, can occur without violating Null Energy Condition (NEC). After the bounce, the field $\phi$ settles at a minimum of the potential, $f$ and $k$ become fixed and we have $M_{Pl}\Theta_m = M_{Pl}\Theta_{Pl} = H_E$, in agreement with Standard Big Bang evolution in Einstein gravity.

One could argue that during the pre-bounce and bounce phases following after the Anamorphic contracting phases the ratio $\rho_{pm}/\rho_{bg}$ could increase. However, as already mentioned earlier, if there were enhancement of the density produced by Parker mechanism in this second phase, the amplitude of the power spectrum would also increase in this period, and the value calculated here and specially constraint (3.20) would have to be adjusted. These are counteracting effects and will tend to balance themselves out. Also, if the additional phases are short compared to the $|H|^{-1}_{\text{max}}$, the the additional squeezing will have a small effect. Hence, we consider our estimate (4.22) to give a reliable guide.

The bottom line is that matter particle production via the Parker mechanism is, as in the case of inflationary cosmology, not able to effectively reheat the universe quickly. If no additional mechanism is introduced to drain the energy from the Anamorphic field condensate, then the Universe at the beginning of the expanding phase will be dominated for a long time by this condensate. Mechanisms to drain the energy in the condensate include nonlinearities in the action for $\phi$ or couplings between $\phi$ and regular matter.

5 Conclusions

The Anamorphic cosmology recently proposed in [1] corresponds to a new scenario for the early universe which has the interesting property of combining elements and advantages of both the inflationary and the Ekpyrotic scenarios. In the present work we made a further analysis of a realization of this model.

We have studied the growth of cosmological perturbations in the Anamorphic scenario. After reviewing the background dynamics for a specific class of scalar-tensor theories introduced in [1] we have obtained the constraints on the model parameters obtained by demanding that the amplitude of the spectrum of cosmological perturbations agree with observations.

We then studied Parker particle production during the Anamorphic phase of contraction. We studied both the production of Anamorphic particles and of test matter particles (particles minimally coupled in the Jordan frame). We found that, as in the case of inflationary cosmology, Parker particle production is not effective enough to drain a sizeable fraction of the energy density from the Anamorphic field condensate. Thus, as in the case of inflationary cosmology, a new mechanism is required if we want the universe to be dominated by regular matter close to the beginning of the phase of expansion.

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