Normal and Reverse Annual Modulations of Elastic WIMP–Nucleus Scattering Signals

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Abstract

Following our earlier work on the 3-dimensional effective velocity distribution of Galactic WIMPs (not only impinging on our detectors but also) scattering off target nuclei, in this paper, we demonstrate the normal and a “reverse” annual modulations of elastic WIMP–nucleus scattering signals, which could be observed in direct Dark Matter detection experiments. Our simulations show that, once the WIMP mass is as light as only a few tens GeV, the event number and the accumulated recoil energy of WIMP–induced scattering events off both of light and heavy target nuclei would indeed be maximal (minimal) in summer (winter). However, once the WIMP mass is as heavy as a few hundreds GeV, the event number and the accumulated recoil energy of WIMP scattering events off heavy nuclei would inversely be minimal in summer. Understandably, for an intermediate WIMP mass, the event number and the accumulated recoil energy of scattering events off some middle–mass nuclei would show an approximately uniform time dependence.
1 Introduction

Direct Dark Matter (DM) detection experiments aiming to observe scattering signals of Weakly Interacting Massive Particles (WIMPs) off target nuclei by measuring nuclear recoil energies deposited in an underground detector would still be the most reliable experimental strategy for identifying Galactic DM particles and determining their properties [1, 2, 3, 4]. Considering the orbital motion of the Earth around the Sun, the relative velocity of the Earth/laboratory to Galactic halo varies annually, and so the 3-dimensional velocity distribution of incident halo WIMPs impinging on our detectors [5, 6]. Hence, the annual modulation of the event rate for elastic WIMP–nucleus scattering has been proposed for more than three decades as a useful experimental strategy for discriminating annually varying WIMP signals from theoretically uniform backgrounds [7]. In practice, DAMA Collaboration claims their positive observations in the last two decades [8, 9, 10], which can however not be confirmed by other collaborations yet [11, 12, 13, 14].

Meanwhile, in standard material about the annual modulation of WIMP scattering event rate, only the variation of the WIMP incident flux proportional to its incoming velocity in the laboratory reference frame has been taken into account [7, 1, 2, 3, 4]. Nevertheless, in our study on the 3-dimensional effective velocity distribution of Galactic WIMPs (not only impinging on our detectors but also) scattering off target nuclei [6], it was demonstrated that, in addition to the incident flux, the scattering cross section (nuclear form factor) suppression could also affect the scattering probability of incident halo WIMPs moving with different velocities, especially in high recoil energy range as well as when the target nucleus and WIMPs are heavy.

More precisely, while a WIMP moving with a higher incoming velocity can pass more target nuclei (in a unit time) and thus have more opportunities to scatter off one of them, its larger kinetic energy which could transfer to the scattered nucleus would in contrast reduce the scattering probability due to the nuclear form factor suppression [6], especially when heavy target nuclei are used and WIMPs are also heavy. Additionally, we demonstrated in Ref. [6] that the 3-D WIMP effective velocity distribution show two opposite angular distribution patterns for light and heavy target nuclei, respectively, once the mass of incident WIMPs is as heavy as a few hundreds GeV.

These interesting discoveries inspired us to ask whether the elastic WIMP–nucleus scattering event rate would also vary annually in two opposite ways for light and heavy target nuclei, in particular, for the case of heavy WIMPs. Therefore, in this paper, we would like to apply our simulation package for 3-D elastic WIMP–nucleus scattering described in Ref. [15] to study the annual variation of the event number of recorded WIMP scattering signals.

The remainder of this paper is organized as follows. In Sec. 2, we describe briefly our theoretical considerations regarding the annual variation of WIMP–nucleus scattering signals. Then three different types of the annual modulation depending on the WIMP mass and the target nucleus will be presented in Sec. 3. We summarize our observations in Sec. 4.

2 What do we know so far – theoretical considerations

In this section, we summarize our theoretical considerations regarding the annual variation of WIMP–nucleus scattering, which we know so far.
Flux proportionality to the WIMP incident velocity

As mentioned in Introduction, in standard material for direct Dark Matter detection physics, the annual modulation of the WIMP scattering event rate is caused by the yearly variation of the WIMP flux proportional to the WIMP incident velocity due to the Earth’s orbital motion around the Sun \[ \text{Ref. 7 1 2 3 4} \]. This is easy to understand: the higher (lower) the incident velocity of halo WIMPs, the more (fewer) the target nuclei passed by an incident WIMP (in a unit time) and thus the larger (smaller) the scattering opportunity.

However, ...

Cross section (nuclear form factor) suppression

As discussed in detail in Ref. \[ 6 \], for WIMPs moving with low (high) incident velocities and thus carrying small (large) kinetic energies, the maximal transferable recoil energies to target nuclei are small (could be pretty large). Then the cross section (nuclear form factor) suppression and in turn the reduction of the scattering probability are weak or even negligible (could be pretty strong), especially when target nuclei and/or WIMPs are light (heavy) (See Figs. \[ 1 \] for a direct comparison).

Consequently, ...

Forward–backward asymmetry of the 3-D WIMP effective velocity distribution

In a general “3-dimensional” point of view, WIMPs moving (approximately) in the same (opposite) direction as (of) the Galactic movement of our Solar system (more precisely, of the Earth)
would have smaller (higher) relative incident velocity to the Earth/laboratory. Now consider the superposition of the flux proportionality to the WIMP incident velocity and the nuclear form factor suppression. Once WIMPs are as light as only a few tens GeV, since the factor of the flux proportionality dominates, the forwardly–moving WIMPs would have smaller probabilities to scatter off both of light and heavy target nuclei; in contrast, once WIMPs are as heavy as a few hundreds GeV, the factor of the nuclear form factor suppression becomes dominated and the forwardly–moving WIMPs would have larger probabilities to scatter off heavy target nuclei. This can be observed clearly from the angular distribution patterns of the 3-D Galactic velocity of WIMPs scattering off different target nuclei and thus was named as the “forward–backward asymmetry” of the 3-D WIMP effective velocity distribution.

Moreover, our simulations presented in Ref. [6] showed that, once both of WIMPs and target nuclei are light, the average incident velocity of scattering WIMPs are larger than the average velocity of entire halo WIMPs. With the increasing WIMP or target mass, the average incident velocity of scattering WIMPs would be somehow reduced to be smaller than the average of entire halo WIMPs.

Two types of the annual modulation of the 1-D WIMP effective velocity distribution

As mentioned above, it is well known that, due to the Earth’s orbital motion around the Sun, the Earth’s velocity relative to the Dark Matter halo is slightly larger (smaller) in summer (winter). Accompanied with the forward–backward asymmetry of the WIMP effective velocity distribution, this implies that WIMPs moving (approximately) in the opposite direction of the Galactic movement of the Earth would have a–bit–much larger (smaller) relative (incident) velocity to the Earth/laboratory in summer (winter), while the relative (incident) velocity of WIMPs moving (approximately) in the same direction as the Earth’s Galactic movement would averagely be almost unchanged.

Hence, in Ref. [6] we found interestingly that, for cases that WIMPs or our target nuclei are as light as only a few tens GeV, the average Galactic velocity of the scattering WIMPs would be minimal (maximal) in summer (winter), whereas once both of WIMPs and our target nuclei are as heavy as a few hundreds GeV, the average Galactic velocity of the scattering WIMPs would inversely become maximal (minimal) in summer (winter).

3 Simulation results

In this section, we follow basically our earlier works to numerically simulate full 3-D elastic WIMP–nucleus scattering process event–by–event [15, 16]: we generate first a 3-dimensional velocity of an incident WIMP in the Galactic coordinate system according to the theoretical isotropic Maxwellian velocity distribution, transform it to the laboratory coordinate system, and, in the laboratory (more precisely, the incoming–WIMP) coordinate system, we generate an equivalent recoil angle of a scattered target nucleus and validate this candidate scattering event according to the criterion [15]:

\[ f_{N_s}(v_{\chi,\text{Lab}}, \theta_{N_s,\chi_{in}}) = \frac{v_{\chi,\text{Lab}}}{v_{\chi,\text{cutoff}}} \left[ \sigma_0^{SI} F_{SI}^2(Q) + \sigma_0^{SD} F_{SD}^2(Q) \right] \sin(2\theta_{N_s,\chi_{in}}) . \]  

Here \( v_{\chi,\text{Lab}} \) and \( \theta_{N_s,\chi_{in}} \) are the transformed WIMP incident velocity in the laboratory coordinate system and the generated equivalent recoil angle of the scattered target nucleus. \( v_{\chi,\text{cutoff}} \) is a

\[ \text{The elevation of the nuclear recoil direction in the incoming–WIMP coordinate system and the complementary angle of the recoil angle.} \]
cut–off velocity of incident halo WIMPs in the Equatorial/laboratory coordinate systems, which is set as 800 km/s in our simulations, $\sigma_0^{(SI,SD)}$ are the spin–independent/dependent (SI/SD) total cross sections ignoring the nuclear form factor suppressions, and $F_{(SI,SD)}(Q)$ indicate the elastic nuclear form factors corresponding to the SI/SD WIMP interactions, respectively. For the accepted scattering events, we record the numbers and the accumulated recoil energies in different advanced seasons [15].

Four spin–sensitive nuclei: $^{19}$F, $^{73}$Ge, $^{129}$Xe, and $^{183}$W have been considered as our targets, so that our simulation results could basically cover the mass range of almost all nuclei used in direct DM detection experiments. And, as in our earlier works presented in Refs. [16, 17], the SI (scalar) WIMP–nucleon cross section has been fixed as $\sigma_0^{SI} = 10^{-9}$ pb, while the effective SD (axial–vector) WIMP–proton/neutron couplings have been tuned as $a_p = 0.01$ and $a_n = 0.7a_p = 0.007$, respectively. So that the contributions of the SI and the SD WIMP–nucleus cross sections (including the corresponding nuclear form factors) in Eq. (1) are approximately comparable:

$$\sigma_0^{SI} F_{SI}^2(Q) \sim \sigma_0^{SD} F_{SD}^2(Q)$$

(2)

for $^{73}$Ge and $^{129}$Xe nuclei[2].

Moreover, in this paper we assume simply that the threshold energies for all considered target nuclei are negligible, whereas the maximal experimental cut–off energy has been set as $Q_{max} = 100$ keV. [3] 5,000 experiments with 5,000 accepted events on average (Poisson–distributed) in one entire year in one experiment have been simulated.

3.1 Normal annual modulation

We consider two cases performing the normal annual modulation at first.

100-GeV WIMPs off $^{19}$F target nuclei

In Figs. 2, we show the event number (a) and the accumulated recoil energy (b) of $^{19}$F target nuclei scattered by 100-GeV WIMPs. The dashed blue vertical bars indicate the 1σ statistical uncertainties, while the dash–dotted red horizontal lines indicate the yearly average value.

As expected, the event numbers in different seasons show a sinusoidal time dependence. Only unfortunately, due to the relatively small event numbers (833 events/bin on average), the (dashed blue) statistical error bars are still too large to be clearly distinguished from each other. Meanwhile, the seasonal variation of the accumulated recoil energies in Frame (b) shows a relatively better sinusoidal modulation with an $\sim 7.1\% (~1.56\sigma)$ variation amplitude.

20-GeV WIMPs off $^{129}$Xe target nuclei

In Figs. 3, we consider a heavy nucleus $^{129}$Xe as our target and the WIMP mass has been lowered to only 20 GeV. Again, while the seasonal variation of the event number shows a sinusoidal time dependence with large statistical uncertainties, that of the accumulated recoil energies shows a somehow better sinusoidal modulation with an $\sim 6.3\% (~1.35\sigma)$ variation amplitude.

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[1] [2] However, for $^{19}$F and $^{183}$W nuclei, since their masses are either too light or too heavy, the SD or the SI WIMP–nucleus cross section dominates.

[3] Note that, for cases with non–negligible threshold energies and/or narrower energy windows (e.g., 20 or 30 keV), the WIMP–mass and target dependent annual modulation would be more complicated.
Figure 2: The event number (a) and the accumulated recoil energy (b) of $^{19}$F target nuclei scattered by 100-GeV WIMPs. 5,000 accepted WIMP scattering events on average (Poisson-distributed) in one entire year in one experiment have been simulated and binned into 6 bins. The dashed blue vertical bars indicate the 1σ statistical uncertainties, while the dash-dotted red horizontal lines indicate the yearly average value.

### 3.2 Reverse annual modulation

Now we turn to consider two cases of heavy WIMPs scattering off heavy target nuclei.

#### 500-GeV WIMPs off $^{129}$Xe target nuclei

In Figs. 4, we still use the heavy nucleus $^{129}$Xe as our target, but raise the WIMP mass to 500 GeV. As expected, both of the event number and the accumulated recoil energy show roughly reverse sinusoidal variations. However, due to larger recoil energies (transferred from heavy incident WIMPs) and in turn larger statistical uncertainties, the $\sim 0.83\sigma$ ($\sim 2.9\%$) amplitude
of the seasonal variation of the event number would now be a (relatively) better indicator.

200-GeV WIMPs off $^{183}$W target nuclei

As a second example, in Figs. 5, we consider an even heavier nucleus $^{183}$W as our target, but lower the WIMP mass to 200 GeV. Again, we can see two reverse sinusoidal variation curves and the modulation amplitude of the event number is $\sim 0.85\sigma \ (\sim 2.9\%)$.

3.3 Intermediate WIMP/target mass

Combining results shown previously, it would be reasonable to expect that there should be a (target-dependent) turning point on the WIMP mass (more precisely, a turning boundary in the parameter space of the WIMP mass and different interaction couplings), around which the
annual modulation of the WIMP scattering event rate would disappear. To demonstrate this prediction, we consider at the end two cases with an intermediate WIMP or target mass \[5.\]

500-GeV WIMPs off \(^{73}\)Ge target nuclei

At first, in Figs. 6, the WIMP mass has been assumed as heavy as 500 GeV, but a middle–mass nucleus \(^{73}\)Ge has been considered as our target. Kind of as expected, while the event numbers in different seasons might somehow show a reverse sinusoidal time dependence with however a very small variation amplitude (only \(\sim 1.5\%\)), the seasonal variation of the accumulated recoil energy would rather be uniform.

100-GeV WIMPs off \(^{129}\)Xe target nuclei

As a second example, in Figs. 7, we use again the heavy nucleus \(^{129}\)Xe as our target, but assume an intermediate WIMP mass of 100 GeV. Similarly, while the seasonal variation of the event number show a reverse sinusoidal time dependence with an even smaller variation amplitude (only \(\sim 1.4\%\)), that of the accumulated recoil energy show a sinusoidal time dependence with an \(\sim 1.1\%\) variation amplitude.

4 Summary

In this paper, following our earlier work on the 3-dimensional effective velocity distribution of Galactic WIMPs scattering off target nuclei, we demonstrated the normal and the reverse annual modulations of elastic WIMP–nucleus scattering signals, which could be observed in direct Dark Matter detection experiments.

Our simulations show that, once the WIMP mass is as light as only a few tens GeV, the event number and the accumulated recoil energy of WIMP–induced scattering events off both of light and heavy target nuclei would indeed be maximal (minimal) in summer (winter). With \(O(5,000)\) recorded events in one or several consecutive years, the amplitude of the normal annual modulation of the accumulated recoil energy would be \(\sim 1.35\sigma\) (Xe) to \(\sim 1.56\sigma\) (F) (depending on the target and the WIMP masses as well as the analyzed energy window).
Figure 7: As Figs. 3 and 4, the heavy nucleus $^{129}$Xe has been considered, but the WIMP mass has been chosen as 100 GeV.

However, once the WIMP mass is as heavy as a few hundreds GeV, the event number and the accumulated recoil energy of WIMP scattering events off heavy nuclei would inversely be minimal in summer. With $\mathcal{O}(5,000)$ recorded events, the amplitude of the reverse annual modulation of the event number could be smaller than $\sim 1\sigma$. For an intermediate WIMP mass, our simulations demonstrated that the event number and the accumulated recoil energy of scattering events off some middle–mass nuclei would show an approximately uniform or very tiny seasonal variation.

In summary, by applying our simulation package for 3-D elastic WIMP–nucleus scattering, we studied the annual modulation of WIMP scattering signals under a different approach. Hopefully, what we observed in this work could be helpful to our colleagues in analyzing experimental data.

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