The Selfish Higgs and Reheating

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Abstract

We consider the cosmological relaxation of the Higgs mass and the cosmological constant due to the four-form fluxes in four dimensions. We present concrete models of a singlet scalar field with four-form couplings where the Higgs mass is relaxed to a right value as well as the Universe reheats to a sufficiently high reheating temperature after the last membrane nucleation. We also discuss some of interesting features in the cases of singlet scalar fields with non-minimal or minimal couplings to gravity.

*On leave of absence.
1 Introduction

The four-form flux provides an undetermined constant \([1–4]\), enabling the cosmological constant to vary towards a small value. The probability with the Euclidean action \([5]\) may prefer a small cosmological constant among the distribution of values with different flux parameters. Moreover, the four-form flux can be changed in the process of creating membranes \([6]\), so there is a tunneling probability between two configurations with cosmological constants differing by one unit \([7]\).

An interesting proposal was made for relaxing the cosmological constant and the Higgs mass parameter to observed values by the same four-form fluxes \([8, 9]\). A dimensionless coupling between the four-form flux and the Higgs field \([10, 11]\) was introduced such that the flux parameter is scannable in steps of weak-scale value to relax the Higgs mass parameter to a correct value without a fine-tuning, whereas the anthropic argument is relied upon for obtaining the observed cosmological constant \([8, 9, 12]\). The scanning of the Higgs mass parameter stops at a right value for electroweak symmetry breaking as the tunneling probability from the dS phase just after the last membrane nucleation and the AdS phase is exponentially suppressed.

A non-minimal four-form coupling to gravity was introduced recently by the author as the minimal possibility for a successful reheating with the four-form flux \([13]\). Moreover, both the non-minimal four-form coupling to gravity and the four-form coupling to a pseudo-scalar inflaton \([14]\) were considered by the same author for a successful chaotic inflation with spontaneously broken shift symmetry \([15]\).

In this article, we consider the scenarios of the four-form flux for relaxing the Higgs mass parameter and discuss the reheating dynamics in the presence of the additional four-form couplings, such as the non-minimal four-form couplings to gravity or the four-form couplings to a pseudo-scalar field or a complex scalar field. We show how the four-form flux dependent minimum of the scalar potential sets the natural initial condition for a successful reheating after the last membrane nucleation.

The paper is organized as follows. We begin with the relaxation mechanism with the four-form flux in the SM minimally coupled to gravity and describe the effective theory for realizing the reheating process. Then, we present concrete examples for reheating with the non-minimal four-form coupling to gravity or the four-form couplings to singlet scalar fields and give the detailed discussion in each of the examples. Next, conclusions are drawn.

2 The Relaxation mechanism with four-form flux

We consider a three-index anti-symmetric tensor field \(A_{\nu\rho\sigma}\) and its four-form field strength \(F_{\mu\nu\rho\sigma} = 4 \partial_{[\mu} A_{\nu\rho\sigma]}\). Then, the most general Lagrangian with four-form field couplings in the SM are composed of various terms as follows,

\[
\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}} + \mathcal{L}_S + \mathcal{L}_L + \mathcal{L}_{\text{memb}}
\]
with

\[ L_0 = \sqrt{-g} \left[ \frac{1}{2} R - \Lambda - \frac{1}{48} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} - |D_\mu H|^2 - V(H) \right], \]  

(2)

\[ L_{\text{int}} = \frac{c_2}{24} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} |H|^2, \]  

(3)

\[ L_S = \frac{1}{6} \partial_\mu \left[ \left( \sqrt{-g} F_{\mu\nu\rho\sigma} - c_2 \epsilon^{\mu\nu\rho\sigma} |H|^2 \right) A_{\nu\rho\sigma} \right], \]  

(4)

\[ L_L = \frac{q}{24} \epsilon^{\mu\nu\rho\sigma} \left( F_{\mu\nu\rho\sigma} - 4 \partial_\mu A_{\nu\rho\sigma} \right), \]  

(5)

\[ L_{\text{memb}} = \frac{e}{6} \int d^3 \xi \delta^4(x - x(\xi)) A_{\nu\rho\sigma} \frac{\partial x^\nu}{\partial \xi^a} \frac{\partial x^\rho}{\partial \xi^b} \frac{\partial x^\sigma}{\partial \xi^c} \epsilon^{abc}. \]  

(6)

Here, the Higgs potential in the SM is given by

\[ V(H) = -M^2 |H|^2 + \lambda |H|^4. \]  

(7)

We note that \( c_2 \) is a dimensionless parameter for the four-form flux to the Higgs [8–10, 13], taken to be positive in the later discussion without loss of generality. \( L_S \) is the surface term necessary for the well-defined variation of the action with the anti-symmetric tensor field [4], and \( q \) in \( L_L \) (in eq. (5)) is the Lagrange multiplier, and \( L_{\text{memb}} \) is the membrane action coupled to \( A_{\nu\rho\sigma} \) with membrane charge \( e \). Here, \( \xi^a \) are the membrane coordinates, \( x(\xi) \) are the embedding coordinates in spacetime and \( \epsilon^{abc} \) is the volume form for the membrane.

Using the equation of motion for \( F_{\mu\nu\rho\sigma} \) as follows,

\[ F^{\mu\nu\rho\sigma} = \frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} \left( c_2 |H|^2 + q \right), \]  

(8)

and integrate out \( F_{\mu\nu\rho\sigma} \), we obtain the full Lagrangian (1) as

\[ L = \sqrt{-g} \left[ \frac{1}{2} R - \Lambda_{\text{eff}} - |D_\mu H|^2 + M_{\text{eff}}^2 |H|^2 - \lambda_{H,\text{eff}} |H|^4 \right] 
+ \frac{1}{6} \epsilon^{\mu\nu\rho\sigma} \partial_\mu q A_{\nu\rho\sigma} + \frac{e}{6} \int d^3 \xi \delta^4(x - x(\xi)) A_{\nu\rho\sigma} \frac{\partial x^\nu}{\partial \xi^a} \frac{\partial x^\rho}{\partial \xi^b} \frac{\partial x^\sigma}{\partial \xi^c} \epsilon^{abc}. \]  

(9)

\[ M_{\text{eff}}^2(q) = M^2 - c_2 q, \]  

(10)

\[ \Lambda_{\text{eff}}(q) = \Lambda + \frac{1}{2} q^2, \]  

(11)

\[ \lambda_{H,\text{eff}} = \lambda_H + \frac{1}{2} c_2. \]  

(12)

As a result, the equation of motion for \( A_{\nu\rho\sigma} \) makes the four-form flux \( q \) dynamical, according to

\[ \epsilon^{\mu\nu\rho\sigma} \partial_\mu q = -e \int d^3 \xi \delta^4(x - x(\xi)) \frac{\partial x^\nu}{\partial \xi^a} \frac{\partial x^\rho}{\partial \xi^b} \frac{\partial x^\sigma}{\partial \xi^c} \epsilon^{abc}. \]  

(13)

1The membrane tension can be also introduced by \(-T \int d^3 \xi \delta^4(x - x(\xi)) \sqrt{-g^{(3)}} \) where \( g^{(3)} \) is the determinant of the induced metric on the membrane.
The flux parameter $q$ is quantized in units of $e$ as $q = e n$ with $n$ being integer. Whenever we nucleate a membrane, we can decrease the flux parameter by one unit such that both the Higgs mass and the cosmological constant can be relaxed into observed values in the end.

The membrane is located at the boundary between two consecutive dS space configurations that are defined by the flux parameters and differ by one unit. Then, it is argued the tunneling probability between those configurations is given \[7\] by

$$P(n + 1 \rightarrow n) \approx \exp \left( -\frac{24\pi^2 M_P^4}{\Lambda_{n+1}} \right)$$

when $\Lambda_{n+1} \ll T^2/M_P^2$ where $T$ is the membrane tension. Therefore, the probability of changing the flux parameter by one unit becomes large in the early stage of the nucleation, but it becomes extremely suppressed at the last stage, making the Universe entering in a metastable state with a small cosmological constant \[6–9\].

In addition to the relaxation of the cosmological constant with four-form fluxes, the Higgs mass parameter is also scanned at the same time. For $q > q_c$ with $q_c \equiv M^2/c_2$, the Higgs mass parameter $M_{\text{eff}}^2 < 0$, so electroweak symmetry is unbroken, whereas for $q < q_c$, we are in the broken phase. For $c_2 = O(1)$ and the membrane charge $e$ of electroweak scale, we can explain the observed Higgs mass parameter once the flux change stops at $q = q_c - e$ by the previous argument for the tunneling probability \[8,9\]. For $\Lambda < 0$, we can cancel a large cosmological constant by the contribution from the same flux parameter until $\Lambda_{\text{eff}}$ takes the observed value at $q = q_c - e$, but we need to reply on an anthropic argument for that with $e$ being of order weak scale \[12\].

We also remark that there is a need of reheating after the end of the membrane nucleation, because otherwise the Universe would be empty after the continuous exponential expansion in dS phases. The schematic view of the reheating dynamics is shown in the following general form of the effective potential containing a singlet scalar field or inflaton $\phi$,

$$V(H, \phi) = V_{\text{eff}}(H) + (k_1 \phi^n + q + k_2)^2 + V_{\text{int}}(\phi, H)$$

where $V_{\text{eff}}(H) = -M_{\text{eff}}^2|H|^2 + \lambda_{H,\text{eff}}|H|^4$, and $k_1, k_2$ are constant parameters and $n$ is the positive integer, and $V_{\text{int}}(\phi, H)$ is the interaction potential between the SM Higgs and the inflaton. Then, the minimum of the inflaton potential changes after each membrane nucleation, so it is natural to obtain the initial displacement of the inflaton field just before the last membrane nucleation and set the initial condition for reheating. We will discuss some concrete examples for the inflaton potential in the next sections.

### 3 Reheating with non-minimal four-form coupling

In this section, we discuss the minimal possibility for the relaxation of the Higgs mass and the cosmological constant as well as reheating. To that purpose, we add the non-minimal four-form coupling to gravity as well as $R^2$ term \[13\], as follows,

$$\mathcal{L}_{\text{non-minimal}} = -\frac{c_1}{24} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} R + \sqrt{-g} \left( \frac{1}{2} \zeta^2 R^2 \right)$$

3
with the corresponding surface term,
\[ \Delta L_S = \frac{c_1}{6} \partial_{\mu} \left( \epsilon^{\mu\nu\rho\sigma} R A_{\nu\rho\sigma} \right). \]  

### 3.1 Relaxation of Higgs mass

Considering a dual description of the \( R^2 \) term in terms of a real scalar field \( \chi \), the full Lagrangian (1) with eqs. (16) and (17) becomes

\[
\mathcal{L} = \sqrt{-g} \left[ \frac{1}{2} \Omega(H, \chi, q) R - |D_\mu H|^2 + M_{\text{eff}}^2 |H|^2 - \lambda_{\text{eff}} |H|^4 - \Lambda_{\text{eff}} - \frac{1}{2} \chi^2 \right] 
\]

(18)

with

\[
\Omega(H, \chi, q) = 1 + c_1 \left( c_2 |H|^2 + q \right) + \sqrt{\zeta^2 - c_1^2} \chi 
\]

(19)

Furthermore, making the field redefinition by

\[
\sigma = c_2 |H|^2 + q + \frac{\sqrt{\zeta^2 - c_1^2}}{c_1} \chi, 
\]

(20)

we get \( \Omega = 1 + c_1 \sigma \) and rewrite eq. (18) as

\[
\mathcal{L}_1 = \sqrt{-g} \left[ \frac{1}{2} \left( 1 + c_1 \sigma \right) R - |D_\mu H|^2 - V(H, \sigma, q) \right] 
\]

(21)

with

\[
V(H, \sigma, q) = -M_{\text{eff}}^2 |H|^2 + \lambda_{H,\text{eff}} |H|^4 + \Lambda_{\text{eff}} + \frac{c_1^2}{2} \left( \sigma - c_2 |H|^2 - q \right)^2. 
\]

(22)

We impose \( \zeta^2 > c_1^2 \) for the potential for a new scalar field \( \sigma \) to be bounded from below, without a need of a higher dimensional term to stabilize the potential.

Making a Weyl scaling of the metric by \( g_{\mu\nu} = g_{\mu\nu}^E / \Omega \), we get the Einstein frame Lagrangian as follows,

\[
\mathcal{L}_E = \sqrt{-g} \left[ \frac{1}{2} R(g_E) - \frac{3}{4} c_1^2 \Omega^{-2} (\partial_\mu \sigma)^2 - \frac{1}{2} |D_\mu H|^2 - \frac{V(H, \sigma, q)}{\Omega^2} \right]. 
\]

(23)

For \( |c_1 \sigma| \lesssim 1 \), the sigma field kinetic term is canonically normalized by \( \bar{\sigma} = \sqrt{\frac{3}{2}} c_1 \sigma \) and the Einstein-frame Lagrangian becomes

\[
\mathcal{L}_E \approx \sqrt{-g} \left[ \frac{1}{2} R(g_E) - \frac{1}{2} (\partial_\mu \bar{\sigma})^2 - |D_\mu H|^2 - V(H, \bar{\sigma}, q) \right] 
\]

(24)
where
\[ V(H, \bar{\sigma}, q) = -M_{\text{eff}}^2 |H|^2 + \lambda_{H,\text{eff}} |H|^4 + \Lambda_{\text{eff}} + \frac{1}{2} m_\sigma^2 \left( \bar{\sigma} - \sqrt{\frac{3}{2}} c_2 |H|^2 + q \right)^2 \] (25)

with
\[ m_\sigma = \sqrt{\frac{2}{3}} \frac{M_P}{\sqrt{\zeta^2 - c_1^2}}. \] (26)

Thus, in the minimum of the sigma field potential, we get the Higgs potential as in the case with the four-form coupling to the Higgs field only [8,9,13]. The coupling between the sigma and Higgs fields determines the reheating temperature after inflation.

For general field values of \( \sigma \), the canonical sigma field \( \bar{\sigma} \) in Einstein frame is redefined by
\[ \sigma = \frac{1}{c_1} \left( e^{\sqrt{\frac{2}{3}} \bar{\sigma}} - 1 \right), \] (27)

and the Einstein frame Lagrangian becomes
\[ \mathcal{L}_E = \sqrt{-g_E} \left[ \frac{1}{2} R(g_E) - \frac{1}{2} (\partial_\mu \bar{\sigma})^2 - e^{-\sqrt{\frac{2}{3}} \bar{\sigma}} |D_\mu H|^2 - V_E(H, \bar{\sigma}) \right] \] (28)

with
\[ V_E(H, \bar{\sigma}) = \Lambda_{\text{eff}} e^{-2\sqrt{\frac{2}{3}} \bar{\sigma}} + \frac{3}{4} m_\sigma^2 \left( 1 - (1 + c_1 q) e^{-\sqrt{\frac{2}{3}} \bar{\sigma}} - c_1 c_2 e^{-\sqrt{\frac{2}{3}} \bar{\sigma}} |H|^2 \right)^2 \]
\[ + e^{-2\sqrt{\frac{2}{3}} \bar{\sigma}} \left( - M_{\text{eff}}^2 |H|^2 + \lambda_{H,\text{eff}} |H|^4 \right). \] (29)

Here, assuming that the SM Higgs is stabilized at \( \langle H \rangle = v/\sqrt{2} \) in each dS phase, we can rewrite the above sigma field potential as
\[ V_E(\sigma) = V_0(q) + \left[ \frac{3}{4} m_\sigma^2 \left( 1 + c_1 \left( q + \frac{1}{2} c_2 v^2 \right) \right)^2 + \Lambda_{\text{eff}} \right] \left( e^{-\sqrt{\frac{2}{3}} \bar{\sigma}} - e^{-\sqrt{\frac{2}{3}} \sigma \text{m}(q)} \right)^2 \] (30)

where
\[ e^{-\sqrt{\frac{2}{3}} \sigma \text{m}(q)} = \frac{3 m_\sigma^2 \left( 1 + c_1 \left( q + \frac{1}{2} c_2 v^2 \right) \right)}{3 m_\sigma^2 \left( 1 + c_1 \left( q + \frac{1}{2} c_2 v^2 \right) \right)^2 + 4 \Lambda_{\text{eff}}}, \] (31)
\[ V_0(q) = \frac{3 m_\sigma^2 \Lambda_{\text{eff}}}{3 m_\sigma^2 \left( 1 + c_1 \left( q + \frac{1}{2} c_2 v^2 \right) \right)^2 + 4 \Lambda_{\text{eff}}}. \] (32)

We also obtain the approximate form of the potential (30): for \( \Lambda_{\text{eff}} \gg m_\sigma^2 
\[ V_E \simeq \frac{3}{4} m_\sigma^2 + \Lambda_{\text{eff}} \left( e^{-\sqrt{\frac{2}{3}} \bar{\sigma}} - \frac{3 m_\sigma^2}{4 \Lambda_{\text{eff}}} (1 + c_1 q) \right)^2; \] (33)
for $\Lambda_{\text{eff}} \ll m^2_\sigma$,
\begin{equation}
V_E \simeq \frac{\Lambda_{\text{eff}}}{(1 + c_1 q)^2} + \frac{3}{4} m^2_\sigma (1 + c_1 q)^2 \left( e^{-\sqrt{\frac{2}{3}} \sigma} - \frac{1}{1 + c_1 q} \right)^2. \tag{34}\end{equation}

In both limits, away from the minimum, the sigma field dependent potential would become easily dominant, making the sigma field settling into the minimum very quickly. But, there is a crucial difference between the two cases. In the first case with $\Lambda_{\text{eff}} \gg m^2_\sigma$, we can scan mostly the effective mass of the sigma field with the flux parameter. In the second case with $\Lambda_{\text{eff}} \ll m^2_\sigma$, the scanning of the cosmological constant with the flux parameter becomes more apparent. As we decrease $\Lambda_{\text{eff}}$ for the decreasing $q$, it is natural to enter the regime with $\Lambda_{\text{eff}} \ll m^2_\sigma$ and scan the cosmological constant while the sigma field mass is little dependent on the flux parameter.

### 3.2 Reheating

Just before the last nucleation, we need $q = M^2/c_2 \equiv q_c$ and $v = 0$, for which
\begin{equation}
e^{-\sqrt{\frac{2}{3}} \sigma_m(q_c)} \approx \frac{1}{1 + c_1 q_c} \left( 1 + \frac{4eq_c}{3m^2_\sigma (1 + c_1 q_c)^2} \right)^{-1}, \tag{35}\end{equation}
\begin{equation}V_0(q_c) \approx \frac{3m^2_\sigma eq_c}{3m^2_\sigma (1 + c_1 q_c)^2 + 4eq_c} \tag{36}\end{equation}
where we used $\Lambda_{\text{eff}}(q_c - e) = \Lambda + \frac{1}{2}(q_c - e)^2 \approx 0$ in the end, and
\begin{equation}\Lambda_{\text{eff}}(q_c) = \Lambda + \frac{1}{2} q_c^2 = e \left( q_c - \frac{1}{2} e \right) \approx e q_c. \tag{37}\end{equation}

After the last nucleation, we have $V_0 \approx 0$ and
\begin{equation}e^{-\sqrt{\frac{2}{3}} \sigma_m(q_c-e)} \approx \frac{1}{1 + c_1 (q_c - e + \frac{1}{2} c_2 v^2)} \approx \frac{1}{1 + c_1 q_c}. \tag{38}\end{equation}

Suppose that the sigma field settles into the minimum of the potential before the last nucleation. Then, after the last nucleation, the minimum of the potential is shifted from eq. (35) to eq. (38). Taking the initial condition just before the last nucleation to be the minimum of the potential for $q = q_c$, i.e. $\bar{\sigma}_i = \bar{\sigma}_m(q_c)$, we can obtain the sigma field potential after the last nucleation as
\begin{equation}V_E(\sigma) \approx \frac{3}{4} m^2_\sigma \left( 1 + c_1 \left( q_c - e + \frac{1}{2} c_2 v^2 \right) \right)^2 \left( e^{-\sqrt{\frac{2}{3}} \sigma} - e^{-\sqrt{\frac{2}{3}} \sigma_m(q_c-e)} \right)^2 \notag \end{equation}
\begin{equation}\approx \frac{3}{4} m^2_\sigma (1 + c_1 q_c)^2 e^{-2\sqrt{\frac{2}{3}} \sigma_m(q_c)} \left( e^{-\sqrt{\frac{2}{3}} (\sigma - \sigma_m(q_c))} - e^{-\sqrt{\frac{2}{3}} (\sigma_m(q_c-e) - \sigma_m(q_c))} \right)^2 \notag \end{equation}
\begin{equation}= \frac{3}{4} m^2_\sigma \left( 1 + \frac{4eq_c}{3m^2_\sigma (1 + c_1 q_c)^2} \right)^2 \left( e^{-\sqrt{\frac{2}{3}} (\sigma - \sigma_i)} - 1 - \frac{4eq_c}{3m^2_\sigma (1 + c_1 q_c)^2} \right)^2. \tag{39}\end{equation}
As a result, the sigma field starts to oscillate at \( \bar{\sigma} = \bar{\sigma}_i \) with the initial potential energy, given by

\[
V_i \equiv V_E(\bar{\sigma}_i) = \frac{12(eq_c)^2m_\sigma^2}{(3m_\sigma^2(1 + c_1q_c)^2 + 4eq_c)^2}
\]  

(40)

where the latter approximation is made for \( c_1q_c \ll 1 \). Here, we find that: for \( m_\sigma^2 \ll eq_c \), \( V_i \approx \frac{3}{4}m_\sigma^2 \); for \( m_\sigma^2 \gg eq_c \), \( V_i \approx \frac{3}{4}(eq_c)^2/m_\sigma^2(1 + c_1q_c)^2 \). On the other hand, for \( m_\sigma^2 = \frac{2}{3}\sqrt{2}eq_c/(1 + c_1q_c)^2 \), the initial potential energy is maximized to \( V_i \approx (eq_c)/(1 + c_1q_c)^2 \). Thus, the maximum initial potential can be obtained for the inflaton mass of order 1 TeV for \( e \sim (1 \text{ TeV})^2 \) and \( q_c \sim M_P^2 \), but a heavier inflaton mass is favored for a sufficiently high reheating temperature.

Then, the general maximum temperature of the Universe after inflation is given by \( T_{\text{max}} = \left( \frac{99V_i}{\pi^2g_*} \right)^{1/4} \) with eq. (40), thus becoming

\[
T_{\text{max}} \simeq 2.5 \times 10^{10} \text{ GeV} \left( \frac{100}{g_*} \right)^{1/4} \left( \frac{eq_c}{(1 \text{ TeV} \cdot M_P)^2} \right)^{1/4} \times \left( \frac{m_\sigma^2M_P^2}{eq_c} \right)^{1/4} \left( 1 + \frac{3}{4} \left( \frac{m_\sigma^2M_P^2}{eq_c} \right) (1 + c_1q_c/M_P^2)^2 \right)^{-1/2} \]  

(41)

where we have reintroduced the Planck scale for dimensionality. In particular, for \( m_\sigma^2 \gg eq_c \) and \( c_1q_c/M_P^2 \ll 1 \), the maximum reheating temperature becomes

\[
T_{\text{max}} \simeq 1.5 \times 10^9 \text{ GeV} \left( \frac{100}{g_*} \right)^{1/4} \left( \frac{eq_c}{(1 \text{ TeV} \cdot M_P)^2} \right)^{1/2} \left( \frac{380 \text{ TeV}}{m_\sigma} \right)^{1/2}. \]  

(42)

Since the inflaton coupling couples to the SM Higgs through the non-minimal coupling to the four-form flux, the perturbative decay rate of the inflaton into two Higgs bosons is given by

\[
\Gamma_\sigma = \frac{3c_1^2c_2^2 m_\sigma^3}{64\pi M_P^2}. \]  

(43)

Then, the reheating temperature is determined by the inflaton decay to be

\[
T_{\text{RH}} = \left( \frac{90}{\pi^2g_*} \right)^{1/4} (\Gamma_\sigma M_P)^{1/2} = 10 \text{ MeV} \left( \frac{100}{g_*} \right)^{1/4} \left( \frac{c_1}{1} \right) \left( \frac{c_2}{1} \right) \left( \frac{m_\sigma}{380 \text{ TeV}} \right)^{3/2}. \]  

(44)

In this case, the reheating temperature is much smaller than the maximum temperature, due to the double suppressions with the Planck scale and the inflaton mass. But, we can obtain a sufficiently high reheating temperature for the successful BBN. We note that for \( m_\sigma \geq 1.6 \times 10^8 \text{ GeV} \), the reheating temperature becomes identical to the maximum reheating temperature, that is, \( T_{\text{RH}} = T_{\text{max}} \).

In order for the slow-roll inflation to take place before the last membrane nucleation, we need the inflaton mass to be \( m_\sigma \ll H_I = 8 \times 10^{13} \text{ GeV}(r/0.1)^{1/2} \) where \( r \) is the tensor to scalar ratio during inflation. Therefore, from the inflaton mass in eq. (26), we need \( \zeta \gtrsim 2.5 \times 10^4 (0.1/r)^{1/2} \). As a consequence, for \( 2.5 \times 10^4 (0.1/r)^{1/2} \lesssim \zeta \lesssim 5.2 \times 10^{12} \), a slow-roll inflation and an instantaneous reheating is possible at the same time.
4 Reheating with minimally coupled scalar fields

We discuss the relaxation of the Higgs mass and the cosmological constant in the case where a singlet pseudo-scalar or complex scalar with four-form couplings plays a role for reheating.

4.1 Reheating with pseudo-scalar field

We introduce a pseudo-scalar field \( \phi \) with the four-form coupling as

\[
\mathcal{L}_{\text{pseudo-scalar}} = -\frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 + \frac{\mu}{24} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} \phi \phi, \tag{45}
\]

with the corresponding surface term,

\[
\Delta \mathcal{L}_S = -\frac{\mu}{6} \partial_\mu \left( \epsilon^{\mu\nu\rho\sigma} \phi A_{\nu\rho\sigma} \right). \tag{46}
\]

We note that the shift symmetry for the pseudo-scalar field is softly broken by the mass term. Then, after using the equation of motion for \( F_{\mu\nu\rho\sigma} \), we obtain the \( A_{\nu\rho\sigma} \)-independent part of the Lagrangian as

\[
\mathcal{L}_\Pi = \sqrt{-g} \left[ \frac{1}{2} R - \Lambda - |D_\mu H|^2 + M^2 |H|^2 - \lambda_H |H|^4 
- \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{2} (\mu \phi + c_2 |H|^2 + q)^2 \right]. \tag{47}
\]

In this model, for a general flux parameter \( q \), the SM Higgs and the pseudo-scalar are expanded around the vacuum as \( \langle H \rangle = (0, v_H(q) + h)^T / \sqrt{2} \) and \( \langle \phi \rangle = v_\phi + \varphi \), with

\[
v_H(q) = \sqrt{\frac{M^2 - c_2(q + \mu v_\phi)}{\lambda_H + \frac{1}{2} c_2^2}} ,

(48)
\]

\[
v_\phi(q) = -\frac{\mu}{\mu^2 + m_\phi^2} \left( \frac{1}{2} c_2 v_H^2 + q \right). \tag{49}
\]

The minimum of the potential is stable as far as \( m_\phi^2 > c_2^2 \mu^2 v_H^2(q) \), where \( m_\phi^2 = m_\phi^2 + \mu^2 \) and \( m_H^2 = 2\lambda_{H,\text{eff}} v_H^2(q) \). On the other hand, the mass eigenvalues and the mixing angle \( \theta(q) \) are given by

\[
m_{h_{1,2}}^2 = \frac{1}{2} (m_\phi^2 + m_H^2) \pm \frac{1}{2} \sqrt{(m_\phi^2 - m_H^2)^2 + 4c_2^2 \mu^2 v_H^2(q)}, \tag{50}
\]

and

\[
\tan 2\theta(q) = \frac{2c_2 \mu v_H(q)}{m_\phi^2 - m_H^2}. \tag{51}
\]
We find that the critical value of the flux parameter for a vanishing effective Higgs mass parameter or $v_H = 0$ is given by

$$q_c = \frac{1}{c_2} \left( M^2 - c_2 \mu v_\phi(q_c) \right).$$  (52)

Then, solving eq. (52) with eq. (49) for $q_c$, we get

$$q_c = \frac{\mu^2 + m_\phi^2}{m_\phi^2} \frac{M^2}{c_2},$$

$$v_\phi(q_c) = -\frac{\mu}{m_\phi^2} M^2 \equiv v_{\phi,c},$$  (54)

and the cosmological constant at $q = q_c$ is given by

$$V_c = \Lambda + \frac{1}{2} \left( \mu v_\phi(q_c) + q_c \right)^2 + \frac{1}{2} m_\phi^2 v_\phi^2$$

$$= \Lambda + \frac{1}{2} \frac{m_\phi^2}{\mu^2 + m_\phi^2} q_c^2.$$  (55)

On the other hand, electroweak symmetry is broken at $q = q_c - e$, for which

$$v_H(q_c - e) = \sqrt{\frac{|m_H^2|}{\lambda_{H,\text{eff}}}} \equiv v,$$

$$v_\phi(q_c - e) = v_{\phi,c} - \frac{\mu}{\mu^2 + m_\phi^2} \cdot \left( \frac{1}{2} c_2 v^2 - e \right) \equiv v_{\phi,0}$$  (57)

with $|m_H^2| \equiv M^2 - c_2(q_c - e + \mu v_\phi)$, and the cosmological constant at $q = q_c - e$ is tuned to a tiny value as observed,

$$V_0 = \Lambda - \frac{1}{4} \lambda_{\text{eff}} v^4 + \frac{1}{2} \left( \mu v_{\phi,0} + q_c - e \right)^2 + \frac{1}{2} m_\phi^2 v_{\phi,0}^2 \approx 0.$$  (58)

Consequently, we find that the weak scale depends on various parameters in the model, as follows,

$$v^2 = \frac{m_\phi^2}{\mu^2 + m_\phi^2} \left( \frac{c_2 e}{\lambda_{H,\text{eff}} - \frac{1}{2} \frac{c_2^2 \mu^2}{\mu^2 + m_\phi^2}} \right).$$  (59)

In particular, as far as $m_\phi \sim \mu$, the weak scale can be obtained for the membrane charge $e$ of a similar scale, insensitive to the values of $m_\phi$ and $\mu$. But, for $m_\phi \ll \mu$, we can take a larger value of $e$. Moreover, from eqs. (54) and (57), after the last membrane nucleation, the pseudo-scalar VEV is shifted by

$$\Delta v_\phi = v_{\phi,c} - v_{\phi,0} = -\frac{\mu}{\mu^2 + m_\phi^2} \cdot \left( \frac{1}{2} c_2 v^2 - e \right).$$  (60)
As a result, we can make use of the flux-induced deviation of the pseudo-scalar field for reheating, as will be discussed below.

Just after the last membrane nucleation, the full potential can be rewritten as

\[ V(h, \phi) = \frac{1}{4} \lambda_{H, \text{eff}} \left( h^2 - v^2 \right)^2 + \frac{1}{2} (\mu^2 + m_{\phi}^2) (\phi - v_{\phi,0} + \frac{c_2 \mu}{\mu^2 + m_{\phi}^2} (h^2 - v^2))^2. \]  

(61)

Then, setting the initial value of \( \phi \) just before the last nucleation to \( \phi_i = v_{\phi,c} \) and \( \phi = \phi_i + \varphi \), the above potential just after the last nucleation becomes

\[ V(h, \varphi) = \frac{1}{4} \lambda_{H, \text{eff}} \left( h^2 - v^2 \right)^2 + \frac{1}{2} (\mu^2 + m_{\phi}^2) (\varphi - \Delta v_{\phi} + \frac{c_2 \mu}{\mu^2 + m_{\phi}^2} (h^2 - v^2))^2. \]  

(62)

Therefore, at the onset of the pseudo-scalar oscillation, with the SM Higgs frozen to \( h = v \), the initial vacuum energy for reheating is given by

\[ V_i \equiv \frac{1}{2} (\mu^2 + m_{\phi}^2) (\Delta v_{\phi})^2 \]

\[ = \frac{1}{2} \frac{\mu^2}{\mu^2 + m_{\phi}^2} \left( e - \frac{1}{2} c_2 v^2 \right)^2. \]  

(63)

So, the initial vacuum energy is about \( V_i \sim \epsilon^2 \) for \( \mu \sim m_{\phi} \).

Consequently, the maximum temperature of the Universe after inflation would be

\[ T_{\text{max}} = \left( \frac{90 V_i}{\pi^2 g_*} \right)^{1/4} \simeq 55 \text{ GeV} \left( \frac{V_i^{1/4}}{100 \text{ GeV}} \right) \left( \frac{100}{g_*} \right)^{1/4} \]  

(64)

From the \( \varphi \) coupling to the Higgs, \( L \supset -\frac{1}{2} c_2 \mu \varphi h^2 \), and \( m_{\varphi} = \sqrt{m_{\phi}^2 + \mu^2} \), the perturbative decay rate of the pseudo-scalar field into two Higgs bosons is given by

\[ \Gamma_{\varphi} = \frac{c_2 \mu^2}{32 \pi m_{\varphi}} \left( 1 - \frac{4 m_{H}^2}{m_{\varphi}^2} \right)^{1/2}. \]  

(65)

Then, for \( c_2 = O(1) \) and \( \mu \sim m_{\varphi} \gtrsim 0.16 v \) for \( \theta^2 \gtrsim 0.1 \) to be consistent with the Higgs data, we get \( \Gamma_{\varphi} \sim 0.1 m_{\varphi} \gtrsim 0.01 v \), for which \( \Gamma_{\varphi} \gg H \) at \( T_{\text{max}} \), so the reheating is instantaneous. Therefore, the reheating temperature is given by \( T_{\text{max}} \) as in eq. (64).

### 4.2 Reheating with complex scalar field

We introduce a singlet complex scalar field \( \Phi \) with a global or local \( U(1) \) symmetry and the four-form coupling as

\[ L_{\text{complex–scalar}} = -|\partial_{\mu} \Phi|^2 - m_{\Phi}^2 |\Phi|^2 - \lambda_{\Phi} |\Phi|^4 + \frac{\alpha}{24} \epsilon^{\mu \nu \rho \sigma} F_{\mu \nu \rho \sigma} |\Phi|^2, \]  

(66)
with the corresponding surface term,

\[ \Delta \mathcal{L}_S = -\frac{\alpha}{6} \partial_\mu \left( \epsilon^{\mu
u\rho\sigma} |\Phi|^2 A_{\nu\rho\sigma} \right). \] (67)

Then, after using the equation of motion for \( F_{\nu\rho\sigma} \), we obtain the \( A_{\nu\rho\sigma} \)-independent part of the Lagrangian as

\[ \mathcal{L}_{\text{III}} = \sqrt{-g} \left[ \frac{1}{2} R - \Lambda - |D_\mu H|^2 + M^2 |H|^2 - \lambda_H |H|^4 
- |\partial_\mu \Phi|^2 - m_\Phi^2 |\Phi|^2 - \lambda_\Phi |\Phi|^4 - \frac{1}{2} (\alpha |\Phi|^2 + c_2 |H|^2 + q)^2 \right]. \] (68)

For a general flux parameter \( q \), taking \( \alpha > 0 \) and \( m_\Phi^2 < 0 \), the singlet complex scalar field gets a VEV with \( \langle \Phi \rangle = \frac{1}{\sqrt{2}} v_\phi \),

\[ v_\phi(q) = \sqrt{\frac{-m_\Phi^2 - \alpha q - \frac{1}{2} \alpha^2 c_2^2 v_\phi^2(q)}{\lambda_{\Phi, \text{eff}}}}, \] (69)

with \( \lambda_{\Phi, \text{eff}} \equiv \lambda_\Phi + \frac{1}{2} \alpha^2 \), and the Higgs VEV is given by

\[ v_H(q) = \sqrt{\frac{M^2 - c_2 q - \frac{1}{2} \alpha^2 c_2^2 v_\phi^2(q)}{\lambda_{H, \text{eff}}}}. \] (70)

The stability of the minimum is ensured for \( 4 \lambda_{\Phi, \text{eff}} \lambda_{H, \text{eff}} > (\alpha c_2)^2 \). The mass eigenvalues and the mixing angle are given by

\[ m_{h_1,2}^2 = \lambda_{\Phi, \text{eff}} v_\phi^2(q) + \lambda_{H, \text{eff}} v_H^2(q) \mp \sqrt{(\lambda_{\Phi, \text{eff}} v_\phi^2(q) - \lambda_{H, \text{eff}} v_H^2(q))^2 + \alpha^2 c_2^2 v_\phi^2(q) v_H^2(q)}, \] (71)

and

\[ \tan 2\theta(q) = \frac{\alpha c_2 v_\phi(q) v_H(q)}{\lambda_{\Phi, \text{eff}} v_\phi^2(q) - \lambda_{H, \text{eff}} v_H^2(q)}. \] (72)

The critical value of the flux parameter for \( v_H = 0 \) is given by

\[ q_c = \frac{1}{c_2} \left( M^2 - \frac{1}{2} \alpha^2 c_2^2 \right). \] (73)

Then, solving eq. (73) with eq. (69) for \( q_c \), we obtain

\[ q_c = \frac{1}{\lambda_\phi} \left( \frac{\lambda_{\Phi, \text{eff}}}{c_2} M^2 + \frac{\alpha}{2} m_\Phi^2 \right), \] (74)

\[ v_\phi^2(q_c) = -\frac{1}{\lambda_\phi} \left( m_\Phi^2 + \frac{\alpha}{c_2} M^2 \right) \equiv v_{\phi, c}^2. \] (75)
So, for $v^2_\phi > 0$, we need $m^2_\phi < -\frac{\alpha^2}{e^2} M^2$ for $\lambda_\phi > 0$, then $q_c < \lambda_\phi M^2/c_2$; $m^2_\phi > -\frac{\alpha^2}{e^2} M^2$ for $\lambda_\phi < 0$, then $q_c > -|\lambda_\phi| M^2/c_2$. For either $\lambda_\phi > 0$ or $\lambda_\phi < 0$, the magnitude of the flux parameter is bounded from above and the vacuum stability is ensured as far as $\lambda_{\Phi,\text{eff}} = \lambda_\phi + \frac{1}{2} \alpha^2 > 0$. For $q = q_c$, the cosmological constant is given by

$$V_c = \Lambda + \frac{1}{2} q_c^2 - \frac{1}{4} \lambda_{\Phi,\text{eff}} v_{\phi,c}^4$$
$$= \Lambda + \frac{1}{2} q_c^2 - \frac{1}{4 \lambda_{\Phi,\text{eff}}} \left( m_\phi^2 + \alpha q_c \right)^2. \quad (76)$$

On the other hand, when electroweak symmetry is broken at the last membrane nucleation to $q = q_c - e$, the VEVs now become

$$v_H(q_c - e) = \sqrt{\frac{|m_H^2|}{\lambda_{H,\text{eff}}}} \equiv v, \quad (77)$$
$$v_\phi^2(q_c - e) = v_{\phi,c}^2 + \frac{\alpha}{\lambda_{\Phi,\text{eff}}} \left( e - \frac{1}{2} c_2 v^2 \right) \equiv v_{\phi,0}^2 \quad (78)$$

where $|m_H^2| \equiv M^2 - c_2 (q_c - e) - \frac{1}{2} \alpha c_2 v_{\phi,0}^2$. Then, we can determine the electroweak scale in terms of various dimensionless couplings and the membrane charge as

$$v^2 = \frac{\lambda_\phi c_2 e}{\lambda_{\Phi,\text{eff}} \lambda_{H,\text{eff}} - \frac{1}{4} (\alpha c_2)^2}. \quad (79)$$

Therefore, the electroweak scale is of order the membrane charge unless there is a tuning in the dimensionless parameters. As in the previous section, after the last membrane nucleation the singlet scalar VEV is shifted by

$$v_{\phi,c}^2 - v_{\phi,0}^2 = - \frac{\alpha}{\lambda_{\Phi,\text{eff}}} \left( e - \frac{1}{2} c_2 v^2 \right). \quad (80)$$

Just after the last membrane nucleation, the full potential for $\Phi = \frac{1}{\sqrt{2}} \phi$ and the SM Higgs can be rewritten as

$$V(h, \phi) = \frac{1}{4} \lambda_{H,\text{eff}} \left( h^2 - v^2 \right)^2 + \frac{1}{4} \lambda_{\Phi,\text{eff}} \left( \phi^2 - v_{\phi,0}^2 + \frac{\alpha c_2}{\lambda_{\Phi,\text{eff}}} (h^2 - v^2) \right)^2. \quad (81)$$

Then, setting the initial value of $\phi$ just before the last nucleation to $\phi_i = v_{\phi,c}$ and $\phi = \phi_i + \varphi$, the above potential just after the last nucleation becomes

$$V(h, \varphi) = \frac{1}{4} \lambda_{H,\text{eff}} \left( h^2 - v^2 \right)^2 + \frac{1}{4} \lambda_{\Phi,\text{eff}} \left( \varphi^2 + 2 v_{\phi,c} \varphi + v_{\phi,c}^2 - v_{\phi,0}^2 + \frac{\alpha c_2}{\lambda_{\Phi,\text{eff}}} (h^2 - v^2) \right)^2. \quad (82)$$

Therefore, at the onset of the singlet scalar oscillation, with the SM Higgs frozen to $h = v$, the initial vacuum energy for reheating is given by

$$V_i = \frac{1}{4} \lambda_{\Phi,\text{eff}} (v_{\phi,c}^2 - v_{\phi,0}^2)^2$$
$$= \frac{1}{2} \frac{\alpha^2}{\lambda_{\Phi,\text{eff}}} \left( e - \frac{1}{2} c_2 v^2 \right)^2. \quad (83)$$
So, the initial vacuum energy is about \( V_i \sim e^2 \) as in the previous section.

Consequently, the maximum temperature of the Universe after inflation is similarly given by eq. (64). From the \( \varphi \) coupling to the Higgs, \( \mathcal{L} \supset -\alpha c_2 v_{\varphi,c} \varphi^2 \), the perturbative decay rate of the singlet scalar into two Higgs bosons is given by

\[
\Gamma_\varphi = \frac{\alpha^2 c_2^2 v_{\varphi,c}^2}{8\pi m_\varphi} \left( 1 - \frac{4m_h^2}{m_\varphi^2} \right)^{1/2}.
\]

Then, for \( \alpha, c_2 = \mathcal{O}(1) \) and \( m_\varphi \sim \sqrt{2\lambda_{\Phi,\text{eff}}} v_{\varphi,0} = m_\varphi \gg m_h \) to be consistent with the Higgs data, we get \( \Gamma_\varphi \sim 0.1m_\varphi \gtrsim 0.01v \), for which \( \Gamma_\varphi \gg H \) at \( T_{\text{max}} \), so the reheating is instantaneous. Therefore, the reheating temperature is given by \( T_{\text{max}} \) as in eq. (64).

5 Conclusions

We provided new scenarios for solving the hierarchy problem in the SM where the four-form flux can relax not only the Higgs mass but also the cosmological constant to observable values and the reheating mechanism is naturally implemented by the extra four-form couplings. We showed that the non-minimal four-form coupling to gravity or the four-form couplings to singlet scalar fields gives rise to a successful reheating of the Universe at the end of relaxation.

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