Spectroscopy of light baryons with strangeness -1, -2, and -3

Juhi Oudichhya, Keval Gandhi, and Ajay Kumar Rai
Department of Physics, Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat-395007, India
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The present article contains the descriptive study of light strange baryons $\Lambda$, $\Sigma$, $\Xi$, and $\Omega$. The Regge phenomenology with linear Regge trajectories has been employed and the relations between Regge slopes, intercepts, and baryon masses have been extracted. With the aid of these relations, ground state masses are obtained for $\Xi$ and $\Omega$ baryons. Regge parameters have also been estimated to calculate the excited state masses for $\Lambda$, $\Sigma$, $\Xi$, and $\Omega$ baryons in both the $(J, M^2)$ and $(n, M^2)$ planes. The obtained results are compared with available experimental data and various theoretical approaches. We predict the spin-parity of recently observed light baryons and our predictions may be useful in future experimental searches of these baryons and their $J^P$ assignments.

I. INTRODUCTION

Hadron spectroscopy is a tool for understanding the dynamics of quark interactions in composite systems like mesons, baryons, and exotics. The spectroscopic study of baryon with a strange quark(s) will have a great interest because the strange quark would be slightly heavier than up and down quarks and considerably lighter than charm and bottom quarks. In this paper we present a description of hyperons which are $\Lambda$, $\Sigma$ baryons with strangeness -1, $\Xi$ and $\Omega$ baryons with strangeness -2 and -3, respectively. Many experimental facilities around the world have been seeking to study the strange baryons. Recently the ALICE Collaboration observed an attractive interaction between protons and $\Xi^-$ baryon [1]. In 2019 the BESIII Collaboration observed $\Xi(1530)$ in a baryon-antibaryon pair from charmonium decay [2]. At Jefferson Lab, the photoproduction of $\Xi$ has been observed using the CLAS detector [3]. Furthermore, JLab has proposed to investigate the strange hyperon spectroscopy using a secondary KL beam and a GlueX experiment and the results are expected to provide more knowledge of strange hyperons $\Lambda$, $\Sigma$, and $\Xi$ [4]. Extensive research have also been conducted by the BABAR Collaboration [5]. Recently in 2018, the Belle Collaboration observed new excited $\Omega^-$ state as $\Omega(2012)$ through $e^-e^+$ annihilations decaying into $\Xi^0K^-$ and $\Xi^-K^0$ channels [6].

By studying the properties these, so-called, strange baryons or hyperons, hadron physicists will be able to make progress in answering various critical questions, namely; what is the intrinsic structure of these baryons? what are the important degrees of freedom in a baryon? and, are there exotic forms of baryon-like matter? Eventually, addressing these questions helps in shedding light on the deeper fundamental question, namely “how to understand the underlying confinement mechanism?” The anticipated multiplet structure of the baryons must be established experimentally to understand the symmetries and dynamics of the strong interaction, and the details of their excitation spectrum are vital for that. However, experimental information is presently very limited, in particular for $\Xi$ and $\Omega$ baryons with strangeness -2 and -3, respectively. For a large part, the lack of experimental knowledge can be understood by the fact that with the widely usage of electromagnetic probes the production cross section of strange baryons is very limited making it difficult to generate sufficient statistics. The $\Xi$ baryons are produced only at the final state of a decay process and have small cross-sections (typically a few $\mu b$) [7]. Somewhat this comment is also valid for $\Omega^-$ baryons. Also, $\Omega^-$ baryons have zero isospins, which means that $\Omega^- \rightarrow \Omega^{-} \pi^0$ decays are highly suppressed, and this restricts the possible decays of excited states. Therefore, $\Xi K$ being the excepted decay mode for the low-lying $\Omega^-$ states [8]. These decays are similar to the $\Omega^0 \rightarrow \Xi^+ K^-$ decays discovered by the LHCb [8] Collaboration and confirmed by the Belle [9] Collaboration shortly after.

$\textbf{PANDA}$ (antiProton ANnihilations at DArmstadt), the upcoming experimental facility at FAIR (Facility for AntiProton and Ion Research), will have a task to establish the whole spectrum of hyperons through antiproton-proton annihilation [10][11][12]. At the latest, a member of the $\textbf{PANDA}$ group studied the feasibility of the reaction $p\bar{p} \rightarrow \Xi^+ \Xi^-$ and its charge conjugate channel, where the $\Xi^\pm$ denotes the following intermediate resonance states: $\Xi(1530)$, $\Xi(1690)$, and $\Xi(1820)$ [13]. A major goal of the $\Xi$ spectroscopy program at $\textbf{PANDA}$ is the determination of the spin and parity quantum numbers of the $\Xi$ states [14]. Unlike $\Xi$ and $\Omega$, $\Lambda$ and $\Sigma$ baryons have quite a number of experimentally established states. Table I represents the excited $\Xi$ and $\Omega$ baryons listed by the latest version of the Particle Data Group (PDG) [15]. Spin and parity quantum numbers of newly observed $\Xi$ and $\Omega$ states are not yet confirmed and the PDG needs more confirmations. It is crucial to assign the spin-parity ($J^P$) of hadrons which facilitate the determination of properties such as decay width, branching fraction, isospin mass splitting, polarization amplitude, etc.

Since, we have studied the light baryons which are made of the quarks $u$, $d$, and $s$ only. The SU(3) flavour symmetry can be only approximate because the mass of the strange quark is about 0.1 GeV greater than the masses of the up and down quarks, although this mass difference is relatively small compared to the typical QCD binding energy which is of order 1 GeV. The eightfold way and the standard SU(3) Gell-Mann–Okubo...
TABLE I. Masses and $J^P$ values of $\Xi$ and $\Omega$ baryons are listed in PDG [18]. The status is given as poor(*), fair(**), very likely(****), and certain(****).

| Resonance | Mass (MeV) | $J^P$ | Status |
|-----------|------------|-------|--------|
| $\Xi^+$  | 1314.86 ± 0.20 | $\frac{1}{2}^+$ | ****    |
| $\Xi^-$  | 1321.71 ± 0.07  | $\frac{3}{2}^+$ | ****    |
| $\Xi(1530)^0$ | 1531.80 ± 0.32 | $\frac{3}{2}^+$ | ****    |
| $\Xi(1530)^-$ | 1535.0 ± 0.6  | $\frac{3}{2}^-$ | ****    |
| $\Xi(1620)$ | 1620          | *      |        |
| $\Xi(1690)$ | 1690 ± 10     | ****   |        |
| $\Xi(1820)$ | 1823 ± 5      | $\frac{3}{2}^-$ | ****    |
| $\Xi(1950)$ | 1950 ± 15     | ****   |        |
| $\Xi(2030)$ | 2025 ± 5      | $\frac{5}{2}^-$ | ****    |
| $\Xi(2120)$ | 2120          | *      |        |
| $\Xi(2250)$ | 2250          | **     |        |
| $\Xi(2370)$ | 2370          | **     |        |
| $\Xi(2500)$ | 2500          | *      |        |
| $\Omega^-$ | 1672.45 ± 0.29 | $\frac{3}{2}^+$ | ****    |
| $\Omega(2012)^-$ | 2012.4 ± 0.9 | ****   |        |
| $\Omega(2250)^-$ | 2252 ± 9     | ****   |        |
| $\Omega(2380)^-$ | 2380          | **     |        |
| $\Omega(2470)^-$ | 2474 ± 12     | **     |        |

(GMO) formula [19] have played an important role in particle physics. However, the direct generalization of the GMO formula to the charmed and bottom hadrons cannot agree well with experimental data due to higher-order breaking effects. Hence for light baryons the breaking of SU(3) symmetry is minimum.

Several phenomenological and theoretical model have been employed to study the properties of light baryons. The authors of Ref. [20] estimated the mass spectra of strange baryons using the relativistic quark model based on the quasipotential approach and baryons are treated as relativistic quark-diquark bound systems. In the recent study [21,22], the authors employed hypercentral Constituent Quark Model with linear confining potential along with first order correction term to obtain the mass spectra of light strange baryons with strangeness -1,2, and -3. Regge trajectories are explored for the linearity of the calculated masses for $(J,M^2)$ and $(n,M^2)$. Ref. [23] present the complete excited $\Lambda$, $\Sigma$, $\Xi$, and $\Omega$ spectrum in a relativistic quark model based on the three quark Bethe-Salpeter equation with instantaneous two and three body forces. The investigation of instanton-induced effects in the baryon mass spectrum plays a central role in this work. The authors of Ref. [24] present a systematic analysis of spectra and transition rates of strange baryons within the framework of a collective string-like $qqq$ model in which the orbital excitations are treated as rotations and vibrations of the strings.

In the present article, we give a systematic study of strange baryons $\Lambda$, $\Sigma$, $\Xi$, and $\Omega$. Here we employ the Regge phenomenology with the assumption of linear Regge trajectories. We find the relations between the intercept, slope ratios, and baryon masses in both the $(J,M^2)$ and $(n,M^2)$ planes. With the help of these relations, we determine the ground state masses of $\Xi$ and $\Omega$ baryons. We extract the Regge parameters $(a(0)$ and $\alpha^{'})$ to determine the mass spectra of all light strange baryons in both the $(J,M^2)$ and $(n,M^2)$ planes. It is evident that the ground and low-lying resonance states are within a reasonable range for almost all of the models and approaches. However, the higher excited states exhibit huge variations in their mass predictions, which motivated us to study the experimental determination of the spin and parity quantum numbers of these light strange baryons.

The remainder of this paper is organized as follows. After briefing various experimental and theoretical approaches, in Sec. II we describe Regge theory and extract the mass relations. By using these relations we obtain the masses for $\Lambda$, $\Sigma$, $\Xi$, and $\Omega$ baryons in the $(J,M^2)$ plane for natural and unnatural parity states. Further we obtain the Regge parameters for each Regge line and calculate the radial and orbital excited states of these baryons in the $(n,M^2)$ plane. We extend this model and try to determine the remaining states other than natural and unnatural parity states in the $(J,M^2)$ plane. The detailed description of our results obtained is discussed in Sec. III. Finally we concluded our study in Sec. IV.

II. THEORETICAL FRAMEWORK

To study the hadron spectroscopy, the linear Regge trajectory is one of the most effective and widely used phenomenological approach. The plots of Regge trajectories of hadrons in the $(J,M^2)$ plane are usually called Chew-frautschi plots [25]. They used the theory to study the strong quark gluon interaction and observed that the excited states of experimentally missing mesons and baryons exists on the linear trajectories in the $(J,M^2)$ plane. The trajectory of a particular pole is characterized by a set of internal quantum numbers and the hadrons lying on the particular Regge line have the same internal quantum numbers. The most general form of linear Regge trajectories can be expressed as [26-29],

$$ J = a(M) = a(0) + a' M^2, $$

where $a(0)$ and $\alpha'$ are, respectively, the slope and intercept of the Regge trajectory. Regge intercepts and Regge slopes for different flavors of a baryon multiplet can be related by the following relations [30,31],

$$ a_{iiq}(0) + a_{jjq}(0) = 2a_{ijq}(0), $$

$$ \frac{1}{a_{iiq}} + \frac{1}{a_{jjq}} = \frac{2}{a_{ijq}}, $$

where $i,j,q$ represent the quark flavors. Using Eqs. [1]
and \(2\) we obtain,
\[
\alpha'_{ijq} M^2_{ijq} + \alpha'_{jjq} M^2_{jjq} = 2 \alpha'_{ijjq} M^2_{ijjq},
\]
\(4\)

We get two pairs of solutions after combining the relations \(3\) and \(1\) which are expressed as,
\[
\frac{\alpha'_{jjq}}{\alpha'_{ijq}} = \frac{1}{2M^2_{jjq}} \times [(4M^2_{ijq} - M^2_{ijq} - M^2_{jjq}) \pm \sqrt{(4M^2_{ijq} - M^2_{ijq} - M^2_{jjq})^2 - 4M^2_{ijq}M^2_{jjq}}],
\]
\(5\)
and,
\[
\frac{\alpha'_{ijq}}{\alpha'_{ijq}} = \frac{1}{4M^2_{ijq}} \times [(4M^2_{ijq} + M^2_{ijq} - M^2_{jjq}) \pm \sqrt{4M^2_{ijq} - M^2_{ijq} - M^2_{jjq})^2 - 4M^2_{ijq}M^2_{jjq}}],
\]
\(6\)
These are the significant relationships that we have obtained between slope ratios and baryon masses.

Now the above obtained Eq. \(5\) can also be expressed as,
\[
\frac{\alpha'_{jjq}}{\alpha'_{ijq}} = \frac{\alpha'_{kkq}}{\alpha'_{ijq}} \times \frac{\alpha'_{jjq}}{\alpha'_{kkq}},
\]
\(7\)
here \(k\) can be any quark flavor. Thus we have,

\[
\left[ (4M^2_{ijq} - M^2_{ijq} - M^2_{jjq}) + \sqrt{4M^2_{ijq} - M^2_{ijq} - M^2_{jjq})^2 - 4M^2_{ijq}M^2_{jjq}} \right] 
\]

\[
2M^2_{jjq}
\]
\(8\)

This is the general relation we have derived in terms of baryon masses which can be used to estimate the mass of any baryon state if all other masses are known. In the present work, the ground-state \((J^P = \frac{1}{2}^+\) and \(\frac{3}{2}^+\)) masses of the light strange baryons having strangeness -2 and -3 are evaluated using the above equation.

### A. Masses in the \((J, M^2)\) plane

In this section using the relations we have extracted above, we determine the ground-state masses of \(\Xi\) and \(\Omega\) baryons, as well as the orbitally excited state masses of all the light strange baryons \(\Lambda, \Sigma, \Xi,\) and \(\Omega\) for natural \((P = (-1)^{J+\frac{1}{2}})\) and unnatural \((P = (-1)^{J+\frac{3}{2}})\) parities in the \((J, M^2)\) plane. The quark combination of \(\Xi^0\) baryon is \(uus\), hence we put \(i = d, j = s, q = u,\) and \(k = d\) in Eq. \(4\). We obtain the mass expression for \(\Xi^0\) as a function of squared masses of neutron (\(n\)) and \(\Lambda^0\) baryons, which is expressed as

| \(\Lambda\) | \(\Sigma\) | \(\Xi\) | \(\Omega\) |
|----------|----------|----------|----------|
| \(\alpha'\) (\(S = 1/2\)) | 0.8597 | 0.8597 | 0.7420 | - |
| \(\alpha'^+\) (\(S = 3/2\)) | - | 0.8051 | 0.7250 | 0.6585 |

TABLE II. Regge slopes of \(\frac{1}{2}^+\) and \(\frac{3}{2}^+\) trajectories of the light strange baryons. (in Ge\(\text{V}^{-2}\)).

Substituting the masses of \(n\) and \(\Lambda^0\) into Eq. \(4\) we get the ground-state mass of \(\Xi^0\) as 1293 MeV for \(J^P = \frac{1}{2}^+\). Similarly we can get 1537 MeV for \(J^P = \frac{3}{2}^+\). In the same manner we can calculate for \(\Xi^-\) baryon also (see Table \(4\)). Similarly to evaluate the ground state \((J^P = \frac{3}{2}^+)\) mass of \(\Omega^-\) baryon, composed of three strange quarks \((sss)\), we put \(i = u, j = s, q = s,\) and \(k = u\) in Eq. \(4\) we get,
TABLE III. Masses of excited states of Λ baryon in the \((J, M^2)\) plane for natural parities. The numbers in the bold face are the experimental values taken as the input [18] (in MeV).

| States  | Present | Others |
|---------|---------|--------|
| \(N^{S+1} L_J\) | \(\Lambda^0\) | PDG [18] [20] [35] [21] [23] [36] |
| \(1^2 S_{1/2}\) | 1116 | 1116 | 1115 1115 | 1115 1108 1116 |
| \(1^2 P_{3/2}\) | 1552 | 1519 | 1549 1545 | 1534 1508 1650 |
| \(1^2 D_{5/2}\) | 1890 | 1820 | 1825 1890 | 1746 1834 1896 |
| \(1^2 F_{7/2}\) | 2176 | 2100 | 2097 2150 | 1970 2090 |
| \(1^2 G_{9/2}\) | 2429 | 2350 | 2360 | 2204 2340 |
| \(1^2 H_{11/2}\) | 2658 | 2605 |

TABLE IV. Masses of excited states of Σ baryon in the \((J, M^2)\) plane with natural and unnatural parities. The numbers in the boldface are the experimental values taken as the inputs [18] (in MeV).

| States  | Present | Others |
|---------|---------|--------|
| \(N^{S+1} L_J\) | \(\Sigma^+\) | \(\Sigma^0\) | \(\Sigma^-\) | PDG [18] [20] [35] [21] [23] |
| \(1^2 S_{1/2}\) | 1189 | 1193 | 1197 | 1187 1190 | 1193 1190 |
| \(1^2 P_{3/2}\) | 1605 | 1608 | 1611 | 1670 | 1655 1698 1669 |
| \(1^2 D_{5/2}\) | 1934 | 1936 | 1939 | 1915 | 1991 1995 1998 1956 |
| \(1^2 F_{7/2}\) | 2214 | 2216 | 2219 | 2259 | 2245 2318 2236 |
| \(1^2 G_{9/2}\) | 2463 | 2464 | 2467 | 2548 |
| \(1^2 H_{11/2}\) | 2689 | 2690 | 2692 |
| \(1^4 S_{0}\) | 1383 | 1384 | 1387 | 1381 1370 | 1384 1411 |
| \(1^4 P_{1/2}\) | 1776 | 1777 | 1779 | 1775 | 1755 1680 1770 |
| \(1^4 D_{3/2}\) | 2097 | 2098 | 2099 | 2033 | 2060 1962 2070 |
| \(1^4 F_{5/2}\) | 2375 | 2376 | 2377 | 2289 | 2257 2325 |
| \(1^4 G_{7/2}\) | 2623 | 2624 | 2625 | 2620 | 2529 |
| \(1^4 H_{9/2}\) | 2850 | 2851 | 2852 |

TABLE V. Masses of excited states of Ξ baryon in the \((J, M^2)\) plane with natural and unnatural parities. (in MeV).

| States  | Present | Others |
|---------|---------|--------|
| \(N^{S+1} L_J\) | \(\Xi^0\) | \(\Xi^+\) | PDG [18] [20] [35] [21] [23] [37] |
| \(1^2 S_{1/2}\) | 1293 1294 1315 (\(\Xi^0\)) | 1330 1305 | 1310 1322 1317 |
| \(1^2 P_{3/2}\) | 1738 1738 | 1764 1785 | 1780 1871 1801 |
| \(1^2 D_{5/2}\) | 2090 2090 2025 | 2108 2045 | 2013 2234 1959 |
| \(1^2 F_{7/2}\) | 2391 2391 | 2370 | 2460 2355 | 2320 2647 |
| \(1^2 G_{9/2}\) | 2658 | 2658 | |
| \(1^2 H_{11/2}\) | 2900 2900 |
| \(1^4 S_{0}\) | 1537 1533 1532 (\(\Xi^+\)) | 1518 1505 | 1539 1531 1526 |
| \(1^4 P_{1/2}\) | 1934 | 1931 | 1950 | 1853 1900 | 1955 1859 1917 |
| \(1^4 D_{3/2}\) | 2263 2260 | 2250 | 2189 2180 | 2169 2203 2074 |
| \(1^4 F_{5/2}\) | 2550 | 2547 | 2502 | 2505 2588 |
| \(1^4 G_{7/2}\) | 2807 | 2805 |
| \(1^4 H_{9/2}\) | 3043 | 3041 |

TABLE VI. Masses of excited states of Ω baryon in the \((J, M^2)\) plane with unnatural parities (in MeV).

| States  | Present | PDG [18] [20] [35] [21] [22] |
|---------|---------|--------|
| \(N^{S+1} L_J\) | \(\Omega^+\) |
| \(1^2 S_{1/2}\) | 1691 | 1672 | 1678 1635 | 1672 |
| \(1^2 P_{3/2}\) | 2092 | | 2653 2490 | 2528 1970 |
| \(1^2 D_{5/2}\) | 2428 | 2380 | 2369 2295 | 2292 2233 |
| \(1^2 F_{7/2}\) | 2723 | | 2649 | 2606 2521 |
| \(1^2 G_{9/2}\) | 2989 | |
| \(1^2 H_{11/2}\) | 3233 | |

inserting the masses of \(\Xi^0\), \(n_s\) and \(\Lambda^0\) baryons in the above equation we have \(\alpha_\Xi^0/\alpha_N\). Since from Eq. (11) we can have \(\alpha = 2/(M_{J+2}^2 - M_{J}^2)\), hence we get \(\alpha_N = 1.0217\) GeV\(^{-2}\). So we can calculate \(\alpha_\Xi^0 = 0.7420\) GeV\(^{-2}\) for \(1/2^-\) trajectory. Similarly with the help of Eqs. (5) and (6) we can determine the Regge slopes (\(\alpha'\)) of \(\Lambda, \Sigma,\) and \(\Omega\) baryons for both \(1/2^-\) and \(3/2^-\) trajectories. According to the Ref. [23], \(\alpha'_\Lambda \approx \alpha'_\Sigma\), in this work we take this approximation. **Table I** shows the estimated values of \(\alpha'\) and \(\alpha^*\) of the light strange baryons. Now from Eq. (11) one can write,

\[
M_{J+1} = \sqrt{M_J^2 + \frac{1}{\alpha'}},
\]

With the help of values \(\alpha'\) extracted for light-strange baryons, from Eq. (12), the masses of orbitally excited states with \(J^P = \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-, \ldots\) and \(J^P = \frac{5}{2}^-, \frac{7}{2}^+, \frac{9}{2}^-, \ldots\) for natural and unnatural parities respectively, in the
TABLE VII. The values for Regge intercepts ($\beta_0$) and Regge slopes ($\beta$) (in GeV$^{-2}$) of $\Xi^0$ baryon.

| Spin (S) | $\beta_0$ | $\beta$ |
|---------|-----------|---------|
| S-states | 1/2 | 0.11315 | 0.53046 |
| | 3/2 | -0.57199 | 0.66543 |
| P-states | 1/2 | -0.49582 | 0.49520 |
| | 3/2 | -1.19693 | 0.58736 |
| D-states | 1/2 | -0.80656 | 0.41358 |
| | 3/2 | -1.44628 | 0.47768 |

(J, $M^2$) plane can be evaluated. The numerical results are shown in Tables III-VI. Here the spectroscopic notations $N^{2S+1}L_J$ are used to represent the state of the particles, where $N$, $L$, $S$ denote the radial excited quantum number, orbital quantum number, and intrinsic spin, respectively.

B. Masses in the ($n$, $M^2$) plane

In this section the masses for radial and orbital excited states from $1^3S_\frac{1}{2} - 6^2S_\frac{1}{2} 1^4S_\frac{3}{2} - 6^4S_\frac{3}{2}; 1^2P_\frac{1}{2} - 5^2P_\frac{1}{2}; 1^4P_\frac{1}{2} - 5^4P_\frac{1}{2}; 1^2D_\frac{3}{2} - 5^2D_\frac{3}{2}$ and $1^4D_\frac{3}{2} - 5^4D_\frac{3}{2}$ are estimated in the ($n$, $M^2$) plane. The general equation for linear Regge trajectories in the ($n$, $M^2$) plane can be expressed as,

$$n = \beta_0 + \beta M^2,$$  \hspace{1cm} (13)

where $n = 1, 2, 3...$ is the radial principal quantum number, $\beta_0$ and $\beta$ are the Regge intercept and slope of the trajectories. These parameters are extracted for each Regge line for $\Lambda, \Sigma, \Xi,$ and $\Omega$ baryons. Since, the baryon multiplets lying on the single Regge line have the same Regge slope ($\beta$) and Regge intercept ($\beta_0$). Using relation (13) and the values for $\beta_0$ and $\beta$ obtained, we estimated the excited state masses of light strange baryons ($\Lambda, \Sigma, \Xi,$ and $\Omega$) lying on each Regge lines for natural and unnatural parity states.

For instance, using the slope equation, we have $\beta(S) = 1/(M^2_{\Xi(2S)} - M^2_{\Xi(1S)})$ for $\Xi^0$ baryon, where $M_{\Xi(1S)} = 1293$ MeV (calculated above) and taking $M_{\Xi(2S)} = 1886$ MeV from [20] for the 1/2$^+$ trajectory, we get $\beta(S) = 0.53046$ GeV$^{-2}$. From Eq. (13) we can write,

$$\begin{align*}
1 &= \beta_0(S) + \beta(S)M^2_{\Xi(1S)}, \\
2 &= \beta_0(S) + \beta(S)M^2_{\Xi(2S)},
\end{align*}$$  \hspace{1cm} (14)

using the above relations, we get $\beta_0(S) = 0.11315$. With the help of $\beta(S)$ and $\beta_0(S)$, we calculate the masses of the excited $\Xi^0$ baryon for $n = 3, 4, 5...$ Similarly, we can express these relations for $P$ and $D$-wave as,

$$\begin{align*}
1 &= \beta_0(P) + \beta(P)M^2_{\Xi(1P)}, \\
2 &= \beta_0(P) + \beta(P)M^2_{\Xi(2P)}, \\
3 &= \beta_0(D) + \beta(D)M^2_{\Xi(1D)}, \\
4 &= \beta_0(D) + \beta(D)M^2_{\Xi(2D)},
\end{align*}$$  \hspace{1cm} (15)

Table VII shows the values of $\beta_0$ and $\beta$ for the Regge trajectories of $S$, $P$, and $D$ states for spin $S = 1/2$ and $3/2$. In the same manner, we estimated the radial and orbital excited states of other light strange baryons for natural and unnatural parity states. The calculated results are summarized in Tables VII-III.

C. Other states in the ($J, M^2$) plane

So far, we have used conventional formulae to compute the masses of light strange baryons for natural and unnatural parity states. After the successful implementation of this model, now in this section, we try to obtain the remaining other states in the ($J, M^2$) plane by using the same method. Since we have calculated 1$^2P_\frac{1}{2}$ and 1$^4P_\frac{1}{2}$ states earlier, now here we firstly calculate the other three 1$P$ states i.e., 1$^2P_\frac{3}{2}$, 1$^4P_\frac{3}{2}$, and 1$^4P_\frac{1}{2}$ by using the Eq. (5). For $\Xi$ baryon we put $i = d, j = s, q = u$, and $k = d$ in Eq. (5) we have,

$$[\frac{(M_n + M_\Xi)^2}{4M_\Xi^2} - 4M_n^2] = \sqrt{(4M_\Xi^2 - M_n^2 - M_{\Xi(1S)}^2)(4M_n^2 - 4M_{\Xi(1S)}^2)};$$  \hspace{1cm} (16)

After putting the masses in above equation we get masses for 1$^2P_\frac{3}{2}$, 1$^4P_\frac{3}{2}$, and 1$^4P_\frac{1}{2}$ states. Similarly we can determine the other 1$P$ states masses for $\Omega$ baryons. Once we have calculated the 1$P$ states, we extract the Regge slopes for all the light strange baryons in the ($J, M^2$) plane as we have done in previous section. Again using Eq. (13) we have

$$M_{J+1} = \sqrt{M_J^2 + \frac{1}{\alpha^2}}.$$  \hspace{1cm} (17)

Excited state masses can be obtained by using the above relation. Tables X - XVI shows our calculated results for the remaining other states for light strange baryons. We compared our estimated masses with experimental data and other theoretical studies.

III. RESULTS AND DISCUSSION

In the present work, an attempt has been made to obtain the mass spectra of hyperons under the methodology of Regge Phenomenology. Regge slopes were calculated in the ($J, M^2$) plane. With the aid of these
TABLE VIII. Masses of excited states of Λ baryon in \((n, M^2)\) plane. The masses in the boldface are taken as input from Ref. 18 (in MeV).

| \(N^{2S+1}L_J\) | Present | [21]  | [23]  | [35]  | [37]  | [36]  |
|-----------------|---------|-------|-------|-------|-------|-------|
| (S=1/2)         |         |       |       |       |       |       |
| \(1^2S_{1/2}\)  | 1116    | 1115  | 1108  | 1115  | 1113  | 1116  |
| \(2^2S_{1/2}\)  | 1600    | 1592  | 1677  | 1680  | 1606  | 1518  |
| \(3^2S_{1/2}\)  | 1968    | 1885  | 1747  | 1830  | 1880  | 1955  |
| \(4^2S_{1/2}\)  | 2278    | 2202  | 2077  | 2107  | 2173  |       |
| \(5^2S_{1/2}\)  | 2550    | 2540  | 2132  | 2120  |       |       |
| \(6^2S_{1/2}\)  | 2796    |       |       |       |       |       |

| (S=1/2)         |         |       |       |       |       |       |
| \(1^2P_{3/2}\)  | 1552    | 1534  | 1508  | 1545  | 1560  | 1650  |
| \(2^2P_{3/2}\)  | 1690    | 1819  | 1775  | 1770  | 1859  | 1854  |
| \(3^2P_{3/2}\)  | 1818    | 2131  | 2147  | 2185  |       |       |
| \(4^2P_{3/2}\)  | 1937    | 2464  | 2313  |       |       |       |
| \(5^2P_{3/2}\)  | 2049    |       |       |       |       |       |

| (S=1/2)         |         |       |       |       |       |       |
| \(1^2D_{5/2}\)  | 1890    | 1746  | 1834  | 1890  | 1839  | 1896  |
| \(2^2D_{5/2}\)  | 2090    | 2051  | 2078  | 2115  | 2103  |       |
| \(3^2D_{5/2}\)  | 2272    | 2378  |       |       |       |       |
| \(4^2D_{5/2}\)  | 2441    |       |       |       |       |       |
| \(5^2D_{5/2}\)  | 2599    |       |       |       |       |       |

Regge slopes, the masses of the orbitally excited baryons were estimated for both natural and unnatural parity states. After that, the Regge slopes and intercepts were extracted for each Regge line in the \((n, M^2)\) plane, and with the help of these parameters mass spectra of light baryons were obtained successfully. Tables II-V and VI-IX summarizes the calculated masses in the \((J, M^2)\) and \((n, M^2)\) planes respectively, for natural and unnatural parity states along with the other theoretical outcomes and the experimental observations where available. Also, our estimated masses for remaining other states in the \((J, M^2)\) plane are shown in tables X-XIII.

1. For Λ baryon the four star and three star status states Λ(1520), Λ(1820), Λ(1900), Λ(2080), Λ(2100), and Λ(2350) in the PDG 18 are in good agreement with our calculated results with a mass difference of 33-80 MeV. We confirmed the \(J^P\) values of these states in the present work (see tables III and X). We compared our results with the predictions of other theoretical models also and our estimated masses are consistent with the results of Refs. 20, 23, 35 with a mass difference of few MeV as shown in tables III, VIII, and XII.

2. The calculated results in the \((J, M^2)\) and \((n, M^2)\) plane for Σ baryon are summerized in Tables IV IX and XIII. For the case of Σ the masses of well established states; Σ(1670), Σ(1775), Σ(1915), Σ(2030), and Σ(2080) closely matches with the masses of our estimated results, and our model confirmed the spin parity of these states. The PDG states Σ(2455), Σ(2620), and Σ(2250) are two and three starred, their \(J^P\) values are still unknown. The Σ(2250) is very close to our predicted mass 2249 MeV having a mass difference of 1 MeV only (see table XIII). So we predicted this state to be \(1^4F_7^+\) having spin parity \(\frac{5}{2}^-\). The states Σ(2455) and Σ(2620) are fairly close to our estimated masses 2475 MeV and 2624 MeV respectively. So we can say that these two states may belong to \(1G\) state having \(J^P = \frac{3}{2}^+\) and \(\frac{5}{2}^-\) (see tables IV and XIII). The calculated masses for low lying resonance fairly matches with the results of other theoretical predictions 20, 23, 35, but a wide range of mass difference is shown for higher excited states.

3. Our calculated ground-state masses of Ξ baryon 1293 MeV (Ξ0), 1294 MeV (Ξ−) with \(J^P = \frac{1}{2}^-\) and 1537 MeV (Ξ0), 1533 MeV (Ξ−) with \(J^P = \frac{3}{2}^+\) are found to be very close to the experimentally well established masses with a few difference of MeV and also matches very well with the predictions of other theoretical models (see table V). Very few states are confirmed with spin-parity in the Ξ family. The Ξ(1820) is the only negative parity state assigned with \(J^P = \frac{3}{2}^-\) in PDG having mass 1823 MeV which is some what close to our predicted mass 1783 having mass difference of 85 MeV and very close 1825 MeV with a slight difference of 2 MeV only for \(1^2P_{\frac{3}{2}}\) and \(1^4P_{\frac{3}{2}}\) respectively. The Ξ(2030) is assigned with angular momentum having value \(\frac{5}{2}^-\), parity of this state is not confirmed yet. Here we predicted this state with positive parity having mass 2090 MeV belongs to \(1D\) state having \(J^P = \frac{5}{2}^-\). The \(J^P\) value of three stared Ξ(1950) state is still not confirmed in PDG. Our predicted mass 1934 MeV for \(1^4P_{\frac{3}{2}}\) state is near to 1950 Mev with a mass difference of 16 MeV. So, we predicted the spin-parity of this state to be \(\frac{5}{2}^-\) for \(S = 3/2\).
The two starred state Ξ(2250) is close to our estimated mass 2263 MeV with a mass difference of 13 MeV, so we predicted this state as 1D state with $J^P = \frac{7}{2}^-$ for $S = 3/2$. The Ξ(2370) state with two star matches exactly with our predicted mass 2370 MeV (see table [XIV]) and also it is reasonably close to 2391 MeV (see table [X]). So this state may belongs to either $\Sigma^+ F_\frac{3}{2}$ or $\Sigma^0 F_\frac{5}{2}$ having $J^P = \frac{7}{2}^-$. Also, one more state Ξ(2120) with one star is found to be very close to our mass 2115 MeV with a mass difference of 5 MeV (see table [XIV]). Hence we can say that, this may belongs to $1F_\frac{5}{2}$ having $J^P = \frac{5}{2}^+$. Also, one more state Ξ(2120) with one star is found to be very close to our mass 2115 MeV with a mass difference of 5 MeV (see table [XIV]). Hence we can say that, this may belongs to $1F_\frac{5}{2}$ having $J^P = \frac{5}{2}^+$. Also, one more state Ξ(2120) with one star is found to be very close to our mass 2115 MeV with a mass difference of 5 MeV (see table [XIV]). Hence we can say that, this may belongs to $1F_\frac{5}{2}$ having $J^P = \frac{5}{2}^+$. Also, one more state Ξ(2120) with one star is found to be very close to our mass 2115 MeV with a mass difference of 5 MeV (see table [XIV]). Hence we can say that, this may belongs to $1F_\frac{5}{2}$ having $J^P = \frac{5}{2}^+$. Also, one more state Ξ(2120) with one star is found to be very close to our mass 2115 MeV with a mass difference of 5 MeV (see table [XIV]). Hence we can say that, this may belongs to $1F_\frac{5}{2}$ having $J^P = \frac{5}{2}^+$. Also, one more state Ξ(2120) with one star is found to be very close to our mass 2115 MeV with a mass difference of 5 MeV (see table [XIV]). Hence we can say that, this may belongs to $1F_\frac{5}{2}$ having $J^P = \frac{5}{2}^+$. Also, one more state Ξ(2120) with one star is found to be very close to our mass 2115 MeV with a mass difference of 5 MeV (see table [XIV]). Hence we can say that, this may belongs to $1F_\frac{5}{2}$ having $J^P = \frac{5}{2}^+$. Also, one more state Ξ(2120) with one star is found to be very close to our mass 2115 MeV with a mass difference of 5 MeV (see table [XIV]). Hence we can say that, this may belongs to $1F_\frac{5}{2}$ having $J^P = \frac{5}{2}^+$. Also, one more state Ξ(2120) with one star is found to be very close to our mass 2115 MeV with a mass difference of 5 MeV (see table [XIV]). Hence we can say that, this may belongs to $1F_\frac{5}{2}$ having $J^P = \frac{5}{2}^+$. Also, one more state Ξ(2120) with one star is found to be very close to our mass 2115 MeV with a mass difference of 5 MeV (see table [XIV]). Hence we can say that, this may belongs to $1F_\frac{5}{2}$ having $J^P = \frac{5}{2}^+$. Also, one more state Ξ(2120) with one star is found to be very close to our mass 2115 MeV with a mass difference of 5 MeV (see table [XIV]). Hence we can say that, this may belongs to $1F_\frac{5}{2}$ having $J^P = \frac{5}{2}^+$.
TABLE X. Masses of excited states of Ξ baryon in \((n, M^2)\) plane. The masses in the boldface are taken as input from Ref. \[20\] (in MeV).

| N^{2S+1L_J} | Present | Others |
|--------------|---------|--------|
| \(S=1/2\)   |         |        |
| 1^2S_{1/2}  | 1293    | 1294   |
| 2^2S_{1/2}  | 1886    | 1886   |
| 3^2S_{1/2}  | 2333    | 2332   |
| 4^2S_{1/2}  | 2707    | 2706   |
| 5^2S_{1/2}  | 3035    | 3034   |
| 6^2S_{1/2}  | 3331    | 3330   |
| \(S=3/2\)   |         |        |
| 1^4S_{3/2}  | 1537    | 1533   |
| 2^4S_{3/2}  | 1966    | 1966   |
| 3^4S_{3/2}  | 2317    | 2319   |
| 4^4S_{3/2}  | 2621    | 2626   |
| 5^4S_{3/2}  | 2894    | 2900   |
| 6^4S_{3/2}  | 3143    | 3150   |
| \(S=1/2\)   |         |        |
| 1^2P_{3/2}  | 1738    | 1738   |
| 2^2P_{3/2}  | 2245    | 2245   |
| 3^2P_{3/2}  | 2657    | 2657   |
| 4^2P_{3/2}  | 3013    | 3013   |
| 5^2P_{3/2}  | 3331    | 3331   |
| \(S=3/2\)   |         |        |
| 1^4P_{5/2}  | 1934    | 1931   |
| 2^4P_{5/2}  | 2333    | 2333   |
| 3^4P_{5/2}  | 2673    | 2675   |
| 4^4P_{5/2}  | 2974    | 2978   |
| 5^4P_{5/2}  | 3248    | 3253   |
| \(S=1/2\)   |         |        |
| 1^2D_{5/2}  | 2090    | 2090   |
| 2^2D_{5/2}  | 2605    | 2605   |
| 3^2D_{5/2}  | 3034    | 3034   |
| 4^2D_{5/2}  | 3409    | 3409   |
| 5^2D_{5/2}  | 3747    | 3747   |
| \(S=3/2\)   |         |        |
| 1^4D_{7/2}  | 2263    | 2260   |
| 2^4D_{7/2}  | 2686    | 2686   |
| 3^4D_{7/2}  | 3051    | 3053   |
| 4^4D_{7/2}  | 3377    | 3381   |
| 5^4D_{7/2}  | 3673    | 3679   |

\(J^P = \frac{1}{2}^-\). The Ω(2250) state having mass 2252 MeV is very close to our calculated mass 2249 MeV with a slight mass difference of 3 MeV, hence we can say that this state can be a good candidate of D-wave and identified with spin parity \(J^P = \frac{1}{2}^+\) for \(S=3/2\). The two starred state Ω(2380) in PDG is close to both 2338 MeV and 2428 MeV with a mass difference of 42 MeV and 48 MeV respectively. So this state belongs to D-wave and may have spin-parity \(\frac{3}{2}^+\) or \(\frac{5}{2}^+\). The Ω(2470) state is not identified in this work. We compared our results with other theoretical predictions as well and they are in good agreement with the assumptions of Refs. \[20, 23, 35, 37\]. Also we observed a wide range of mass difference for Ω(2012) state with other theoretical results.

**IV. CONCLUSION**

Regge phenomenology has been effectively implemented to study the mass-spectra of light baryons with strangeness -1,-2, and -3. Here our aim of determining the spin parity of experimentally observed states: Σ(2250), Σ(2455), Σ(2620), Σ(1950), Σ(2030), Σ(2120), Σ(2250), Σ(2370), Ω(2012), Ω(2250), and Ω(2380) is accomplished. Also we confirmed the spin parity of many experimentally well established states of Λ and Σ baryons. It is evident that the low-lying resonance masses are in good agreement with other theoretical predictions but for higher excited states we see a wide range of mass difference. This possibly because of the fact that not a single model exactly predicts the spin-parity assignments, and also there is no experimental evidence for the states. As a result, our work with a huge number of spin-parity predictions could be useful for future facilities.
TABLE XI. Masses of excited states of Ω baryon in \((n, M^2)\) plane. The masses in the boldface are taken as input from Ref. [22] (in MeV).

| \(N^{2S+1}L_J\) | Present | [20] | [37] | [23] | [35] |
|----------------|---------|------|------|------|------|
| \((S=3/2)\)   |         |      |      |      |      |
| \(1^4S_{3/2}\) | 1691    | 1678 | 1673 | 1635 |      |
| \(2^4S_{3/2}\) | 2057    | 2173 | 2078 | 2177 | 2165 |
| \(3^4S_{3/2}\) | 2367    |      |      |      |      |
| \(4^4S_{3/2}\) | 2641    |      |      |      |      |
| \(5^4S_{3/2}\) | 2889    |      |      |      |      |
| \(6^4S_{3/2}\) | 3117    |      |      |      |      |
| \((S=3/2)\)   |         |      |      |      |      |
| \(1^4P_{3/2}\) | 2092    | 2653 | 2528 | 2490 |      |
| \(2^4P_{3/2}\) | 2321    |      |      |      | 2534 |
| \(3^4P_{3/2}\) | 2529    |      |      |      |      |
| \(4^4P_{3/2}\) | 2722    |      |      |      |      |
| \(5^4P_{3/2}\) | 2902    |      |      |      |      |
| \((S=3/2)\)   |         |      |      |      |      |
| \(1^4D_{3/2}\) | 2428    | 2369 | 2205 | 2292 | 2295 |
| \(2^4D_{3/2}\) | 2623    |      |      |      |      |
| \(3^4D_{3/2}\) | 2804    |      |      |      |      |
| \(4^4D_{3/2}\) | 2975    |      |      |      |      |
| \(5^4D_{3/2}\) | 3136    |      |      |      |      |

TABLE XII. Masses of other excited states of Λ baryon in the \((J, M^2)\) plane. The numbers in the boldface are the experimental values taken as the input [18] (in MeV).

| \(N^{2S+1}L_J\) | Present | PDG [18] | [21] | [20] | [35] | [37] |
|----------------|---------|----------|------|------|------|------|
| \(1^2P_{1/2}\) | 1670    | 1670     | 1546 | 1406 | 1550 | 1559 |
| \(1^2D_{3/2}\) | 1861    | 1890     | 1769 | 1854 | 1900 | 1836 |
| \(1^2F_{5/2}\) | 2034    | 2082     | 2065 | 2136 | 2180 |      |
| \(1^2G_{7/2}\) | 2193    |          |      |      |      |      |
| \(1^2H_{9/2}\) | 2342    |          |      |      |      |      |

TABLE XIII. Masses of other excited states of Σ baryon in the \((J, M^2)\) plane. The numbers in the boldface are the input (in MeV).

| \(N^{2S+1}L_J\) | Present | PDG [18] | [20] | [35] | [37] | [23] |
|----------------|---------|----------|------|------|------|------|
| \(1^2P_{3/2}\) | 1620    | 1620     | 1630 | 1657 | 1628 |      |
| \(1^4P_{3/2}\) | 1750    |          | 1693 | 1675 | 1746 | 1771 |
| \(1^4P_{3/2}\) | 1709    |          | 1731 | 1750 | 1790 | 1728 |
| \(1^2D_{3/2}\) | 1816    |          | 2025 | 1970 | 1947 | 1961 |
| \(1^4D_{3/2}\) | 2062    |          | 2076 | 2010 | 1993 | 2011 |
| \(1^4D_{3/2}\) | 1997    |          | 2062 | 2030 | 2028 | 2027 |
| \(1^2F_{5/2}\) | 1993    |          | 2347 | 2250 | 2226 |      |
| \(1^4F_{5/2}\) | 2249    |          | 2250 | 2213 | 2265 |      |
| \(1^2G_{7/2}\) | 2155    |          |      |      |      |      |
| \(1^4G_{7/2}\) | 2575    |          |      |      |      |      |
| \(1^2G_{9/2}\) | 2475    |          |      |      |      |      |

TABLE XIV. Masses of other excited states of Ξ baryon in the \((J, M^2)\) plane. (in MeV).

| \(N^{2S+1}L_J\) | Present | PDG [18] | [20] | [35] | [23] | [37] |
|----------------|---------|----------|------|------|------|------|
| \(1^2P_{1/2}\) | 1810    | 1682     | 1755 | 1770 | 1722 |      |
| \(1^4P_{3/2}\) | 1890    | 1758     | 1810 | 1922 | 1894 |      |
| \(1^4P_{3/2}\) | 1825    | 1823     | 1798 | 1880 | 1873 | 1918 |
| \(1^2D_{3/2}\) | 2155    | 2100     | 2065 | 2076 | 1970 |      |
| \(1^4D_{3/2}\) | 2204    | 2121     | 2115 | 2128 | 2065 |      |
| \(1^4D_{3/2}\) | 2115    | 2147     | 2165 | 2141 | 2102 |      |
| \(1^2F_{5/2}\) | 2318    | 2411     | 2350 | 2409 |      |      |
| \(1^4F_{5/2}\) | 2478    | 2385     | 2425 |      |      |      |
| \(1^4F_{5/2}\) | 2370    | 2370     | 2474 | 2425 |      |      |
| \(1^2G_{7/2}\) | 2470    |          |      |      |      |      |
| \(1^4G_{7/2}\) | 2725    |          |      |      |      |      |
| \(1^4G_{9/2}\) | 2600    |          |      |      |      |      |

TABLE XV. Masses of other excited states of Ω baryon in the \((J, M^2)\) plane (MeV).

| \(N^{2S+1}L_J\) | Present | PDG [18] | [20] | [35] | [23] | [37] |
|----------------|---------|----------|------|------|------|------|
| \(1^2P_{3/2}\) | 2030    | 2012     | 2463 | 2410 | 2456 |      |
| \(1^4P_{3/2}\) | 1941    |          | 2537 | 2440 | 2446 |      |
| \(1^4D_{3/2}\) | 2338    | 2380     | 2332 | 2345 | 2287 | 2263 |
| \(1^4D_{3/2}\) | 2249    | 2252     | 2245 | 2312 | 2260 |      |
| \(1^4F_{5/2}\) | 2610    |          |      |      | 2617 |      |
| \(1^4F_{5/2}\) | 2519    | 2599     | 2531 |      |      |      |
| \(1^4G_{7/2}\) | 2856    |          |      |      |      |      |
| \(1^4G_{9/2}\) | 2763    |          |      |      |      |      |

like PANDA, which is intended to explore light strange baryons in depth.
TABLE XVI. Masses of excited states of Ξ baryon in the \((J, M^2)\) plane including natural, unnatural, and other states (in MeV).

| \(N^{2S+1}L_J\) | Present | PDG [18] | [20] | [35] | [23] | [37] |
|-----------------|---------|----------|------|------|------|------|
| \(1^2S_1^+\)    | 1293    | 1315     | 1330 | 1305 | 1310 | 1317 |
| \(1^4S_1^+\)    | 1537    | 1532     | 1518 | 1505 | 1539 | 1526 |
| \(1^2P_2^{-}\)  | 1810    | 1682     | 1755 | 1770 | 1772 |      |
| \(1^2P_0^+\)    | 1738    |          | 1764 | 1785 | 1780 | 1801 |
| \(1^4P_2^{-}\)  | 1890    | 1758     | 1810 | 1922 | 1894 |      |
| \(1^4P_0^+\)    | 1825    | 1823     | 1798 | 1880 | 1873 | 1918 |
| \(1^2D_2^+\)    | 1934    | 1950     | 1853 | 1900 | 1955 | 1917 |
| \(1^2P_2^{-}\)  | 2155    | 2100     | 2065 | 2076 | 1970 |      |
| \(1^2D_0^+\)    | 2090    | 2025     | 2018 | 2042 | 2013 | 1959 |
| \(1^4D_2^+\)    | 2204    | 2121     | 2115 | 2128 | 2065 |      |
| \(1^4D_0^+\)    | 2215    | 2120     | 2137 | 2149 | 2102 |      |
| \(1^2F_2^{-}\)  | 2318    | 2250     | 2189 | 2169 | 2074 |      |
| \(1^2P_2^{-}\)  | 2391    | 2370     | 2460 | 2355 | 2320 |      |
| \(1^2F_0^+\)    | 2478    |          | 2385 | 2425 |      |      |
| \(1^4F_2^+\)    | 2370    | 2370     | 2474 | 2425 |      |      |
| \(1^4F_0^+\)    | 2550    |          | 2502 | 2505 |      |      |
| \(1^2P_0^+\)    | 2470    |          |      |      |      |      |
| \(1^2G_2^+\)    | 2658    |          |      |      |      |      |
| \(1^4G_2^+\)    | 2725    |          |      |      |      |      |
| \(1^4G_0^+\)    | 2600    |          |      |      |      |      |
| \(1^2H_2^+\)    | 2900    |          |      |      |      |      |
| \(1^4H_2^+\)    | 3043    |          |      |      |      |      |

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