More on Probing Branes with Branes

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Abstract

We generalize the Gibbons-Wiltshire solution of four dimensional Kaluza-Klein black holes in order to describe Type IIA solutions of bound states of D6 and D0-branes. We probe the solutions with a D6-brane and a D0-brane. We also probe a system of D2+D0-branes and of a D2-brane bound to a F1-string with a D2-brane. A precise agreement between the SYM and the SUGRA calculations is found for the static force as well as for the \( v^2 \) force in all cases.

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1 Work supported in part by the US-Israel Binational Science Foundation, by GIF - the German-Israeli Foundation for Scientific Research, and by the Israel Science Foundation.
1 Introduction

About a year ago it was conjectured that M-theory in the infinite momentum frame is described by a supersymmetric matrix model [1]. The advantage of this approach is that it provides a regularization scheme for the M-theory at short distances via a SYM description. The success of such a regularization scheme requires, among other things, that at large distances and low energies, where eleven dimensional SUGRA is a good approximation to M theory, the matrix model interactions yield SUGRA interactions. This was the main motivation behind the intensive study in the last year of the correspondence between the SUGRA and SYM descriptions of the long distance interactions of D-branes.

On the SUGRA side the (bosonic) action of a test Dp-brane in the background of a source of Dp-branes is

\[ S = -\frac{1}{g(2\pi)^p} \int d^{p+1}x e^{-\phi} \sqrt{\det h_{\mu\nu}} + A_{01\ldots p} \]  

where \( h_{\mu\nu} \) is the induced metric, \( A_{01\ldots p} \) is a p-form. The action is treated classically and at large distances (compared to the relevant length scale of the source) it can be expanded in powers of \( 1/r \). On the SYM side at low energies the Born-Infeld action of \( Q+1 \) Dp-branes can be approximated by 10D \( N=1 \) SYM with \( U(Q+1) \) gauge symmetry reduced to \( p+1 \) dimensions

\[ S_0 = \frac{1}{g(2\pi)^p} \int d^{p+1}x \frac{1}{4} Tr[F_{MN}F^{MN}] + \text{Fer.}, \]  

where \( M, N \) are ten dimensional indices. In terms of this field theory the presence of a probe Dp-brane at distance \( r \) away from a source of \( Q \) Dp-branes is equivalent to an expectation value to one of the scalars along the flat directions. This expectation value breaks the \( U(Q+1) \) to \( U(Q) \times U(1) \).

At the one loop order the effective SYM action was calculated for a general gauge field background [4, 5]

\[ S_1 = \frac{c_q}{8(2\pi)^{p+2}} \int d^{p+1}x \left( Tr F^4 - \frac{1}{4} Tr (F^2)^2 \right), \]  

where \( F_{MN} \) is the background field, \( c_q = (4\pi)^{q/2-1}\Gamma(q/2) \) and \( r \) is the distance between the Dp-brane probe and the source Dp-branes.

A precise agreement between eq.(3) and the leading term (in \( 1/r \)) of eq.(1) was found in many cases [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. For the D0 case in the infinite momentum frame it was shown that the field theory two loop calculation does not contribute to the leading term interaction [16] but produces precisely the next to leading

\(^2\text{We work in units where } \alpha' = 1.\)
term in $1/r$ obtained from the SUGRA approach [17]. Moreover, non-perturbative effects on the brane were related, in the case of D2-branes, to scattering with momentum transfer along the $x_{10}$ direction in [18] (see also [19, 20]).

The structure of eq. (3) is similar to the structure of the sub-leading term in the expansion of the Born-Infeld action. This led to the conjecture [21, 22] that the effective action generated by the SYM at any order for any Dp-brane will give rise to the corresponding term in the expansion of the Born-Infeld action. Under this assumption the correspondence between SUGRA and SYM was proven to any order in several cases [21, 22].

In this note we examine some more examples supporting the correspondence between SYM and SUGRA. In sec. 2 we probe a bound state of Dp-branes and D(p-2)-branes with a Dp-brane. Most of the results of this section are well known [5, 6, 21]. We believe that we perform the calculation in a somewhat different and hopefully interesting way. In sec. 3 we probe a bound state of a D2-brane and a F1-string with a D2-brane. We use the M-theory description of a D2-brane and a F1-string as unwrapped and wrapped M2-branes respectively to unify different ten dimensional forces. We use T-duality to relate results for different D-brane dimensions. In sec. 4 we generalize the solutions of Gibbons and Wiltshire of four dimensional black holes [23] in order to describe the most general type IIA solution which contains a bound state of D6-branes and D0-branes. We probe this solution with a D6-brane and compare to the SYM result. Applying electric-magnetic duality we relate our result to the result of [13].

In all the cases we have considered a precise agreement between SYM and SUGRA is obtained for the static force as well as for the $v^2$ force. We conclude with a discussion on the possible significance of our result for the case of D6-brane.

## 2 Probing Dp+D(p-2) branes with a Dp-brane

The system of a bound state of a Dp-brane and a D(p-2)-brane and a probe Dp-brane is not supersymmetric. Therefore, a static force as well as a force proportional to $v^2$ between the bound state and the probe Dp-brane are expected. We start with probing a D2-D0 system with a D2-brane and compare the SYM result to the SUGRA result. Then we apply T-duality along some of the transverse directions and show that also the T-dual systems yield the same SYM and SUGRA results.
2.1 The SYM calculation

The effect of a D0 brane being immersed in a D2-brane is to induce a background magnetic field, \( B = F_{12} \), on the D2 world-volume. From eq.(3) we see that the resulting static potential is

\[
U_{\text{stat}} = \frac{3n_2 V_2 B^4}{16r^5},
\]

where \( V_2 \) is the area of the D2. To find the leading velocity dependent potential which is proportional to \( v^2 \) one can use the expression found in [3]

\[
U_{v^2} = \frac{c^7 g_0 v^2 E}{2r^7},
\]

where \( E \) is the total energy on the D-brane, \( E = \int d^2 x T_{00} \). In our case \( E = V_2 B^2 / 8g \pi^2 \) thus

\[
U_{v^2} = \frac{3n_2 V_2 B^2 v^2}{8r^5}.
\]

The \( v^4 \) potential does not depend on the background magnetic field and it is obtained from eq.(3) by taking on the probe \( F_{0i} = \partial_0 x^i = v_i \)

\[
U_{v^4} = \frac{3n_2 V_2 v^4}{16r^5}.
\]

2.2 The SUGRA calculation

We perform the calculation in eleven dimensional SUGRA using the notation of [18]. The eleven dimensional SUGRA solution of a M2-brane with \( x_{10} \) compactified on a circle of radius \( R_{10} \) is [24]

\[
ds^2 = f^{-2/3}(-dx_0^2 + dx_1^2 + dx_2^2) + f^{1/3}(dx_3^2 + ... + dx_{10}^2),
\]

where

\[
f = 1 + r_0^6 \sum_{n=-\infty}^{\infty} \frac{n_2}{(r^2 + (2\pi R_{10}n)^2)^{3/2}}.
\]

The sum is due to the images of the source membrane on the compactified direction \( x_{10} \sim x_{10} + 2\pi R_{10}n \). At large distances \( (r \gg R_{10}) \) we get

\[
f = 1 + \frac{3n_2 r_0^6}{16 R_{10}^5 r^5}.
\]

Adding D0-branes to the D2-brane is the same as boosting the solution in the \( x_{10} \) direction [24]. Then we want to probe the boosted solution with a M2-brane moving in the transverse directions \( x_3, ..., x_9 \) with a constant velocity. Instead we keep the M2 source static and give the probe a velocity along the \( x_{10} \) direction.
The action of the probe in this background is

\[ S = - \int d^3x \tau_2 \sqrt{\det h_{\mu\nu} + \mu_2 H}, \]  

(11)

where the induced metric \( h_{\mu\nu} \) is given by

\[ h_{\mu\nu} = g_{\mu\nu} + \partial_\mu x^i \partial_\nu x^j g_{ij}, \]  

(12)

Expanding in \( v \) we find that the leading term of the probe’s action in this background is

\[ S = \frac{3n_2 V_2 \tau_0^6}{128r_0^5 R_{10}} (v^2 + v_{10}^2)^2 \int dx_0, \]  

(13)

where \( v^2 = \sum_{i=3}^9 v_i^2 \). Since \( \tau_0^6 = 8R_{10} \) [18] and \( B = v_{10} \) [20, 21, 22] we find a precise agreement with the SYM results, eqs.(4, 6, 7).

### 2.3 T-duality

Next we wish to show the correspondence between the SYM and the SUGRA approaches for a bound state of Dp and D(p-2) branes probed by D-p branen for general \( p \). We use the result of sections 2.1 and 2.2 for the \( p = 2 \) case of D2-branes and D0-branes and apply T-duality. We show explicitly that applying T-duality in the SYM description yields the same result as in the SUGRA description.

Let us start with the SYM approach. The description of a bound state of a Dp-brane and a D(p-2)-brane is very similar to the description in the case of D2+D0. Suppose that the world volume of the Dp-brane is along \( x_0, x_1, ..., x_p \) and that the world volume of the D(p-2)-brane is along \( x_0, x_3, x_4, ..., x_p \).

Then the presence of D(p-2)-branes on the Dp-brane is translated, in the SYM theory, to a constant field background \( F_{12} \) on the D-p brane world volume. The calculation for Dp-D(p-2) is a straightforward generalization of the D2-D0 calculation. For instance from eq.(3) it is clear that the ratio between the result for D3-D1 and the result for D2-D0 is

\[ \frac{S_{31}}{S_{20}} = \frac{c_4 V_3 r}{2\pi c_5 V_2} = \frac{2R_3r}{3\pi}. \]  

(14)

Let us turn now to SUGRA calculation. The D3-D1 action can be obtained from the D2-D0 action by T-duality. In order to do so we need to compactify the \( x_3 \) direction, \( x_3 \sim x_3 + 2\pi R_3 \). Then we should take into account also the images of the source along the \( x_3 \) direction. This means that the harmonic function, \( f(r) \), for the compactified case contains also the images of the source. Therefore, the ratio between the action for compactified \( x_3 \), \( S_{20,\text{com}} \), and the action for uncompactified \( x_3 \), \( S_{20} \), is

\[ \frac{S_{20,\text{com}}}{S_{20}} = \sum_{m=-\infty}^{\infty} \frac{r^5}{(r^2 + (2\pi m R_3)^2)^{5/2}}. \]  

(15)
At large distances the sum on the right hand side of eq.(15) can be replaced by an integral and we obtain

\[ \frac{S_{20,}\text{com}}{S_{20}} = \frac{2r}{3\pi R_3}. \]  

(16)

Since T-duality takes \( R_3 \rightarrow 1/R_3 \) we get a precise agreement with eq.(14).

3 Probing D2+F1 with a D2-brane

3.1 The SUGRA calculation

Again we perform the computation in eleven dimensions. The origin of a F1-string and a 2D-brane in M-theory is a wrapped and an unwrapped M2 brane respectively. Therefore, a bound state of a F1-string along, say, \( x_1 \) (smeared in the \( x_2 \) direction) and a D2-brane can be described in M theory as a M2-brane at an angle \( \alpha_{10} \) relative to the probe which contains no F1-string. The angle \( \alpha_{10} \) is given by

\[ \tan \alpha_{10} = \frac{\partial x_{10}}{\partial x_2} = \sigma_{F1}, \]  

(17)

where \( \sigma_{F1} \) can be thought of as the density of the fundamental strings on the D2. To describe the action of the probe in this background we can rotate the source (eqs.(8, 10)) in the \( x_2, x_{10} \) plane while keeping the probe fixed or we can keep the source fixed and rotate the probe. We follow the second approach.

In that case we can use the SUGRA solution eq.(8,10) and eq.(11) to find for \( \alpha, \alpha_{10} \ll 1 \)

\[ U = \frac{3n_2 V_2 (\alpha^2 + \alpha_{10}^2)^2}{16r^5}, \]  

(18)

where \( \alpha^2 = \sum_{i=3}^{9} \alpha_i^2 \) and \( \tan \alpha_i = \frac{\partial x_i}{\partial x_2} \). Note that just like the velocity in the previous section the angles in this section break super-symmetry and hence a static force is generated.

3.2 The SYM calculation

The relation between \( \alpha \) and \( F_{\mu \nu} \) on the D2-brane is given by [26, 27, 28]

\[ E_1 = F_{01} = \frac{\partial x_{10}}{\partial x_2} = \tan \alpha_{10}. \]  

(19)

A trivial generalization (taking \( F_{2i} = \alpha_i \), \( i = 3,...,9 \)) of the result of subsection 2.1 gives for \( \alpha \ll 1 \)

\[ U_0 = \frac{3n_2 V_2 E_1^4}{16r^5}, \]
\[ U_{\alpha^2} = \frac{3n_2 V_2 E_1^2 \alpha^2}{8r^5}, \]
\[ U_{\alpha^4} = \frac{3n_2 V_2 \alpha^4}{16r^3}, \]

which is in a precise agreement with eq. (18).

Note, that T-duality along \( x_1 \) transforms the source into a bound state of D1-brane and F1-string with momentum along the \( x_1 \) direction and the probe into a D1 brane. Then the \( \alpha^4 \) force between the source and the probe, in the D2+F1 case, becomes a \( v^4 \) force in the D1-brane case \cite{24}.

4 Probing D6+D0-branes with a D6-brane

4.1 The SUGRA calculation

There are two possible approaches to compute the Type IIA solution for a bound state of D6-branes and D0-branes. The first one is to start with the six-brane solution in type IIA \cite{29} (not necessarily extremal) and to lift it to eleven dimension, then boost it along the \( x_{10} \) direction (which means that we add D0’s in the type IIA language) and reduce it back to ten dimensions. The second approach is to generalize the Gibbons-Wiltshire (GW) solution \cite{23} of the four dimensional black hole with electric and magnetic charges which solve the five dimensional Einstein-Hilbert vacuum equation. The relation to a D6+D0 bound state solution is anticipated since in type IIA a D0-brane has an electric charge and a D6-brane has magnetic charge.

We shall follow the second approach. Our starting point is the eleven dimensional Einstein-Hilbert action

\[ S = \frac{1}{(2\pi)^8 g^3} \int d^{11}x \sqrt{-g_{11}} R_{11}. \]  

The dimensional reduction

\[ ds_{11}^2 = e^{4\phi/3}(dx_{11} + A_{\mu}dx^{\mu}) + e^{-2\phi/3} ds_{10}^2, \]

leads to the type IIA action (in the string frame)

\[ S = \frac{1}{(2\pi)^7 g^2} \int d^{10}\sqrt{-g_{10}} \left( e^{-2\phi}(R_{10} + 4(\nabla \phi)^2) - \frac{1}{4} F^2 \right). \]

We have kept only fields which are relevant to the solutions considered below.

\(^3\text{We thank A.A. Tseytlin for pointing this out to us.}\)
The spherically symmetric time independent solutions are parameterized by the mass (M), the electric charge (Q) and the magnetic charge (P). P is proportional to $n_0^6$ while Q is proportional to $n_0^0$. The exact relation will be discussed soon. The dilaton charge $\Sigma$ is related to $M, Q, P$ by

$$\frac{8}{3} \Sigma = \frac{Q^2}{\Sigma + \sqrt{3}M} + \frac{P^2}{\Sigma - \sqrt{3}M}. \quad (24)$$

The Type IIA generalization of the GW solution is

$$e^{4\phi/3} = \frac{B}{A},$$

$$A_\mu dx^\mu = \frac{Q}{B} (r - \Sigma) dt + P \cos \theta d\phi, \quad (25)$$

$$g_{\mu\nu} dx^\mu dx^\nu = -\frac{F}{\sqrt{AB}} dt^2 + \sqrt{\frac{B}{A}} (dx_1^2 + \ldots + dx_6^2) + \sqrt{AB/F} dr^2 + \sqrt{AB} (d\theta^2 + \sin^2 \theta d\phi^2),$$

where

$$F = (r - r_+)(r - r_-),$$

$$A = (r - r_{A+})(r - r_{A-}),$$

$$B = (r - r_{B+})(r - r_{B-}), \quad (26)$$

and

$$r_\pm = M \pm \sqrt{M^2 + \Sigma^2 - P^2/4 - Q^2/4},$$

$$r_{A\pm} = \Sigma/\sqrt{3} \pm \sqrt{\frac{P^2\Sigma/2}{\Sigma - \sqrt{3}M}}, \quad (27)$$

$$r_{B\pm} = -\Sigma/\sqrt{3} \pm \sqrt{\frac{Q^2\Sigma/2}{\Sigma + \sqrt{3}M}}.$$ 

Under the electric-magnetic duality (D6↔D0) the solutions are transformed in the following way

$$Q \leftrightarrow P,$$

$$\Sigma \leftrightarrow -\Sigma,$$

$$M \leftrightarrow M. \quad (28)$$

The self-dual ($Q = P$) solution was obtained in [30, 31] and was probed with D-branes in [33].

For the extremal cases (which we focus on in this note) it is convenient to make the coordinate change $r \to r - M$. Then $F = r^2$ and the metric has the simple form

$$ds^2 = -f_1(r) dt^2 + f_2(r) dx_i dx_i + f_1^{-1}(r)(dr^2 + r^2 d\Omega), \quad (29)$$
where
\[ f_1(r) = r^2 / \sqrt{AB}, \quad f_2(r) = \sqrt{B/A}. \] (30)

To determine the exact relation between \( P \) and \( n_6 \) and between \( Q \) and \( n_0 \) we compare these solutions to the well known p-brane solution of type IIA [29]. We start with the pure magnetic extremal solution. Namely, we take
\[ P = 4M, \quad Q = 0, \quad \Sigma = -\sqrt{3}M. \] (31)

This solves eq.(24) and the extremality condition and leads to
\[ e^{-2\phi} = f^{3/2}, \]
\[ A_\mu dx^\mu = 4M \cos \theta d\phi, \]
\[ ds^2_{10} = f^{-1/2}(-dt^2 + dx_1^2 + ... + dx_6^2) + f^{1/2}(dr^2 + r^2d\Omega), \] (32)

where
\[ f = 1 + \frac{4M}{r}. \] (33)

This is obviously a D6-brane carrying magnetic charge. Since in type IIA \( f = 1 + gn_6/2r \) we find for the Kaluza-Klein monopoles
\[ M = \frac{gn_6}{8}, \] (34)

which is indeed the expected relation between the mass and the radius of the compactified direction [32, 33, 34] since \( R_{10} = g \). Using eq.(31) we obtain
\[ P = \frac{gn_6}{2}. \] (35)

Now we turn to the relation between \( n_0 \) and \( Q \). We consider the pure electric extremal solution which is dual to eq.(31),
\[ P = 0, \quad Q = 4M, \quad \Sigma = \sqrt{3}M, \] (36)

and leads to an electric field associated with D0-branes smeared over \( V_6 \)
\[ e^{-2\phi} = f^{-3/2}, \]
\[ A_\mu dx^\mu = \frac{4M}{r}dt, \]
\[ ds^2_{10} = f^{-1/2}(-dt^2 + dx_1^2 + ... + dx_6^2) + f^{1/2}(dr^2 + r^2d\Omega), \] (37)

where again
\[ f = 1 + \frac{4M}{r}. \] (38)
In type IIA, for D0-branes smeared along $V_6$, we have $f = 1 + gn_0(2\pi)^6/2V_6 r$ (see for example section 2.4 in [35]). Thus, we find for the pure electric extremal solution

$$M = \frac{gn_0(2\pi)^6}{8V_6}. \quad (39)$$

To see that this is the expected result for the Kaluza-Klein momentum we recall that we work in units where $l_s = 1$. In units where $l_{11}^P = 1$, $l_s = g^{-1/3}$ and $V_6 \to g^{6/3}V_6$. Then eq. (39) yields the correct KK momentum dependence, $\sim 1/R_{10}$. Now we can use eq. (38) to get

$$Q = \frac{gn_0(2\pi)^6}{2V_6}. \quad (40)$$

It should be emphasized that although eqs. (35, 40) were derived by considering special cases they hold in general simply because $P$ is proportional to $n_6$ and $Q$ is proportional to $n_0$.

Now we are in a position to describe the solution with a large number of D6-branes and a small number of D0-branes ($P \gg Q$). To do so we need to move away from eq. (31) along eq. (24) and the extremality condition. We fix $M$ and take

$$\Sigma = M(-\sqrt{3} + \varepsilon), \quad \varepsilon \ll 1. \quad (41)$$

To leading order in $\varepsilon$ the solutions for $Q(\varepsilon)$ and $P(\varepsilon)$ are

$$Q^2 = \frac{2\varepsilon^3}{3\sqrt{3}} M^2, \quad P = (4 - \sqrt{3}\varepsilon + \frac{\varepsilon^2}{8}) M. \quad (42)$$

Using these relations and eqs. (1, 29) we find that at large distances the static part of the action is

$$S = \frac{-V_6}{(2\pi)^{6}gr} \left(M(-4 + \sqrt{3}\varepsilon) + P\right) \int dt = \frac{-V_6}{8(2\pi)^{6}gr} M\varepsilon^2 \int dt, \quad (43)$$

and the term in the action which is proportional to $v^2$ is

$$S_{v^2} = \frac{\sqrt{3}MV_6\varepsilon}{2(2\pi)^{6}r} v^2 \int dt. \quad (44)$$

### 4.2 The SYM calculation

The gauge theory which lives on the world-volume of the six brane is a $6 + 1$ dimensional theory. The SYM theory in $6 + 1$ is non-renormalizable and therefore ill defined at short distances. Yet we managed to use the one loop effective action of eq. (3) to derive meaningful results for the $r$-dependent part of the force between the source and the probe. We recall that we are considering the case in which the $U(Q + 1)$ gauge symmetry
is broken down to $U(Q) \times U(1)$ with $r$ the expectation value of the appropriate adjoint scalar. The fields with one index in $U(Q)$ and the other in $U(1)$ are becoming massive with $m \sim r$. Thus, at the one loop order the $r$-dependent contribution to the effective action arise through the masses of the states of the open string which are integrated out. The remnants of these integrations are determinants of the form $\det(\partial_\mu \partial^\mu + m_i^2)$ (for the bosonic fields) where $m_i$ depends on $r$ and the background. The $r$ dependent part of these determinants is always well defined.

We use the construction of [36] which describes a bound state of four D6 and D0-branes with no D2 and D4-brane. The background is

$$F_{12} = F_0 \text{diag}(1, 1, -1, -1),$$
$$F_{34} = F_0 \text{diag}(1, -1, -1, 1),$$
$$F_{56} = F_0 \text{diag}(1, -1, 1, -1),$$

(45)

One can check that this background carries no D2-branes ($\int \text{Tr} F \wedge F = 0$) no D4-branes ($\int \text{Tr} F = 0$) but does carry D0-branes ($\int \text{Tr} F \wedge F \wedge F \neq 0$). Plugging this background in eq.(3) we get

$$S = \frac{(6 - 6^2/4)V_6 n_6}{16(2\pi)^6 r} F_0^4 \int dt = \frac{-3V_6 n_6}{16(2\pi)^6 r} F_0^4 \int dt,$$

(46)

where we have embedded the $U(4)$ solution of [36] in $U(n_6)$ ($n_6 > 4$). Since we work in units where $\alpha' = 1$

$$n_0 = \frac{1}{6(2\pi)^6} \int d^6 x \text{Tr} F \wedge F \wedge F$$

(47)

eqs.(33, 40), imply that

$$F_0^3 = \frac{Q}{P}.$$  

(48)

Using eqs.(34, 42, 48) to leading order in $\varepsilon$, a precise agreement with eq.(43) is found.

To find the term which is proportional to $v^2$ we use eqs.(5, 45)

$$S_{v^2} = \frac{3F_0^2 n_6 V_6}{8r(2\pi)^6 r} v^2 \int dt.$$  

(49)

which again agrees with the SUGRA calculation eq.(44).

Now we would like to use the electric-magnetic duality, eq.(28), to make contact with refs. [7, 15]. Under electric-magnetic duality our configuration becomes a D0-brane probe in the background of D0+D6-branes. The action for this probe should be identical to the action of a D0+D6-brane probe moving in the background of a D0-brane which was considered in [15]. Eq.(28) takes $Q \leftrightarrow P$ and hence $F_0 \leftrightarrow 1/F_0$, and $n_6 \leftrightarrow n_0(2\pi)^6/V_6$. It also takes the D6-brane probe to a D0-brane probe and therefore interchanges their
respective tensions \( V_6/g(2\pi)^6 \leftrightarrow 1/g \). Therefore, it takes both eq. (43) and eq. (46) to

\[
S = \frac{3(2\pi)^6n_0}{16V_6F_0^3r} \int dt = \frac{3n_6}{16F_0r} \int dt, \quad (50)
\]

which is in agreement with the result of [7, 15] (where \( n_6 = 4 \)). Acting with electromagnetic duality on the term which is proportional to \( v^2 \), eq. (49), yields

\[
S = \frac{3n_6F_0v^2}{8r} \int dt, \quad (51)
\]

which is in agreement with the result of [7].

5 Conclusions

The agreement between SUGRA and SYM has been verified by now in a multitude of examples. It would be interesting to find the deeper string theoretic reason for this observation. In ref. [21] it is pointed out that although by adding an F-background SUSY is broken the underlying supersymmetry may still lead to the same \( F^4 \) term at small and large distances in string theory. Small distances are controlled by the (weakly coupled) SYM theory while the long distance by the SUGRA theory. It would certainly be important to further explore this agreement.

One might argue that the computation above supports the conjecture that there is a “dual” description (in the sense of \( g \rightarrow 1/g \)) for SYM in 6 + 1 dimensions which decouples from the bulk, perhaps along the lines of [38, 39, 40, 41]. The argument goes as follows: Introducing explicitly the \( \alpha' \) dependence in eq. (2) we get

\[
g_{YM}^2 = (2\pi)^p g \alpha'(p-3)/2. \quad (52)
\]

The low energy limit of superstring theory, which is relevant for our computation of SUGRA is obtained by \( \alpha' \rightarrow 0 \). For the SYM computation we should also keep the gauge coupling constant finite. For \( p > 3 \) this implies that \( g \rightarrow \infty \) which in turn implies that the theory does not decouple from gravity. If this is indeed the case we would expect also the closed string sector associated with gravity to be relevant. Hence, we would not expect, in these cases, the dynamics of the Dp-brane to be described by just the Born-Infeld (or SYM) theory associated with the open string sector. Therefore, there seems to be no reason for the agreement between the results of SUGRA and SYM theories for \( p > 3 \). However such agreements have been found in refs. [13, 7, 15] and in the present

\[\text{There are also } \alpha' \text{ corrections to the Born-Infeld action. However, these corrections involve derivatives of } F_{\mu\nu} \text{ and hence they vanish for the backgrounds considered in the present paper.}\]
The way to explain these agreements bearing in mind the discussion above is to claim that for $p > 3$ there is a “dual” description for the dynamics of the Dp-brane which decouples from the bulk and regulates the SYM theory at short distances. For $p = 4,5$ such theories have been suggested in the closely related investigation of M(atrix) theory compactified on $T^4$ and $T^5$. For $p = 6$ it is not clear whether a “dual” description which decouples from the bulk exists. It is, therefore, natural to argue that the fact that we are accumulating examples which demonstrate that SYM gives results which agree with the SUGRA result also in the $p = 6$ case, support the existence of a “dual” description of the D6-brane which decouple from gravity. This theory will also be the theory describing M(atrix) theory on $T^6$.

We would like to thank A.A. Tseytlin for helpful comments.

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