Precision calculation for nucleon capture by deuteron with Effective Field Theory

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Abstract

We calculate the cross section for radiative capture of neutron by deuteron \( n + d \rightarrow \text{\(^3\)He} + \gamma \) using Effective Field Theory (EFT). The calculation includes N2LO order and we compare our results with available calculated data below \( E=0.2 \) Mev.

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I. INTRODUCTION

Very low-energy radiative capture and weak capture reactions involving few-nucleon systems have considerable astrophysical relevance for studies of stellar structure and evolution and big-bang nucleosynthesis. In the standard Big-Bang deuterons being to be formed through the process $np \rightarrow d\gamma$ and then the following reactions of primordial nucleosynthesis proceed rapidly: $pd \rightarrow 3H e\gamma$, $nd \rightarrow 3H \gamma$, ... . These reactions at the energies relevant for BBN, $0.02 < E < 0.2$ MeV, is not well-measured experimentally and there are significant theoretical uncertainties [1]. Some theoretical calculation has been down for these reactions but all of them show very significant theoretical error.

In this paper we show result with EFT for reaction $nd \rightarrow 3H \gamma$. In next section we show results for two body radiative capture process $np \rightarrow d\gamma$ and in the last section we show our results for three body radiative capture process $nd \rightarrow 3H \gamma$ and then we will compare these results with EFT at N2LO with ENDF/B-VI [2].

II. TWO BODY SECTOR ($np \rightarrow d\gamma$)

An accurate estimation of the capture processes cross sections are essential to the BBN prediction of deuterium and other light element abundances. For example, $np \rightarrow d\gamma$ In the model calculation of Smith, Kawano and Malaney (SKM) an error of 5% was assigned to the cross section for $np \rightarrow d\gamma$ [3]. The SKM result agrees with a slightly earlier evaluation from ENDF/B-VI [2]. The errors in this calculation are not well documented and they could be as large as 10-15%. There is a much earlier calculation (1967) by Fowler, Caughlan and Zimmerman which agrees with the ENDF/B-VI result for energies $E < 0.2$ MeV but disagrees significantly at higher energies.

In model independent calculation or EFT calculation for $np \rightarrow d\gamma$ calculation by Savage and Chen (1999) show 4% theoretical uncertainly [4]. Precision and higher order calculation by Rupak (2000) show 1% theoretical uncertainly for energy $\sim 1$ Mev [5].

III. THREE NUCLEON SYSTEM IN TRITON CHANNEL AND RADIATIVE CAPTURE OF NEUTRON BY DEUTRON

As discussed by Bedaque et. al. [6], the $^2S_\frac{1}{2}$ channel – to which $^3$He and $^3$H belong – is qualitatively different from the other three-nucleon channels. This difference can be traced back to the effect of the exclusion principle and the angular momentum repulsion barrier. In all the other channels, it is either the Pauli principle or an angular momentum barrier (or both) which forbids the three particles to occupy the same point in space.

The spin structure of the matrix elements for $Nd$ radiative capture is complicate also for the low energy interaction, as we have here 3 independent multipole transitions (allowed by the $P-$parity and the total angular momentum conservation): $J^P = \frac{1}{2}^+ \rightarrow M1$ and $J^P = \frac{3}{2}^+ \rightarrow M1$ and $E2$, with the following parametrization of the corresponding contributions to the matrix element:

$$i(t^1N)(\vec{D} \cdot \vec{e}^* \times \vec{k}),$$
where $N$, $t$, $e$, $D$ and $k$ are the 2-component spinors of initial nucleon filed, final $^3He$ (or $^3H$) field, the 3-vector polarization of the produced photon, the 3-vector polarization of deuteron and the unit vector along the 3-momentum of the photons, respectively. The two structures in Eq.(1) correspond to the M1 radiation. The M1 amplitude received contributions from the magnetic moment of nucleon and four-nucleon operators coupling to magnetic fields, which described by the lagrange density involving dibaryon fields:

$$\mathcal{L}_B = \frac{e}{2M_N} N^\dagger (k_0 + k_1 r^3) \sigma \cdot B + e \frac{L_1}{M_N \sqrt{(1s_0)_p(1S_1)}} d_t d_s^\dagger B_j + H.C. \quad (2)$$

where $d_t$ is the $^3S_1$ dibaryon and $d_s$ is $^1S_0$ dibaryon. The coefficient $L_1$ is determined from two body capture experimental data \cite{5}. In Fig. (1) we show diagrams which can be contributed at N2LO, with expansion of the kernel in power of $Q$ and then we can iterate the kernel by inserting into integral equation and regularize at N2LO we finally arrived to cross section of radiative capture of neutron by deuteron.

**FIG. 1:** Diagrams contributing to the M1 amplitude for $nd \to ^3H\gamma$ at N2LO.

**IV. RESULTS**

We show numerical results for cross section in Table(1). This calculation show at LO 10-40% accuracy, at NLO $\approx$ 10% accuracy and at N2LO $\approx$ 1% accuracy for energy 50-70 Kev in comparison with Evaluated Nuclear Data File (ENDF) \cite{2}.
TABLE I: Neutron radiative capture by deuteron in micro barn at N2LO. Last column shows ENDF results for cross section [2].

| Energy (Kev) | $\sigma(\mu b)$ | $\sigma(\mu b)$ | $\sigma(\mu b)$ | ENDF($\mu b$) |
|--------------|-----------------|-----------------|-----------------|---------------|
|              | LO              | NLO             | N2LO            |               |
| 40           | 1.64            | 1.31            | 1.25            | 1.27(0)       |
| 50           | 1.72            | 1.45            | 1.38            | 1.39(0)       |
| 60           | 1.90            | 1.58            | 1.50            | 1.50(0)       |
| 70           | 2.02            | 1.72            | 1.61            | 1.61(0)       |
| 80           | 2.11            | 1.82            | 1.73            | 1.72(0)       |
| 100          | 2.42            | 2.04            | 1.93            | 1.94(0)       |
| 140          | 2.90            | 2.42            | 2.30            | 2.22(9)       |

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