Δ(27) framework for cobimaximal mixing models

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We propose a simple framework based on Δ(27) that leads to the successful cobimaximal lepton mixing ansatz, thus providing a predictive explanation for leptonic mixing observables. We explore first the effective neutrino mass operators, then present a specific model realization based on type I seesaw, and also propose a model with radiative 1-loop seesaw which features viable dark matter candidates.

I. INTRODUCTION

The experimental observation of 3 generations of fermions, with the associated proliferation of parameters (the different masses and mixing angles) constitute the flavour problem, in the Standard Model (SM) these parameters remain free and are simply fitted to observations. Beyond SM theories can be used to attempt explanations of the flavour problem, and be used to predict (or postdict) these parameters, e.g. by providing relations between mixing angles and the Dirac CP phase.

Neutrino oscillation experiments have now measured the leptonic mixing angles with good precision and global fits provide an indication of the Dirac CP phase. The currently observed leptonic mixing pattern can be successfully described by the cobimaximal mixing pattern, which has recently received more attention \[1–13\].

Several extensions of the SM model with extended particle spectrum and discrete flavour groups have been proposed to explain the pattern on lepton masses and mixings. Among these, the discrete group Δ(27) has several nice properties that make it interesting as a family symmetry - the irreducible representations are a triplet and anti-triplet, plus 9 distinct singlets. Δ(27) has been used widely in the literature, often in association with CP symmetries \[10–12, 14–43\]. Δ(27) has been used recently to obtain cobimaximal mixing in \[12\], and in \[10, 11\]. We note however that in \[12\], soft breaking of Δ(27) is invoked, whereas in \[10, 11\] the breaking of Δ(27) employs a generic direction. In contrast, in this paper we will show how Δ(27) is a good family symmetry to construct cobimaximal models with the breaking of Δ(27) following natural directions that are easy to obtain with the group. The layout of the paper is as follows. In section \[II\] we describe two Δ(27) flavour models that lead to the cobimaximal mixing pattern. The implications of those models in lepton masses and mixings are analysed in Section \[III\]. Our conclusions are given in Section \[IV\]. Appendix \[A\] contains a brief description of the Δ(27) discrete group.

II. MODELS

At the effective level we intend to obtain a framework where, in the model building basis for the fermions, the charged lepton Yukawa matrix is diagonal together with a neutrino mass matrix which is diagonalized by the cobimaximal ansatz. Considering the effective operators, such a neutrino mass matrix can be achieved with the following Lagrangian:

\[
\mathcal{L}_Y^{(\nu)} = \frac{\kappa_1}{\Lambda^3} (\overline{l}_L h_u \phi_{23})(l^C_L h_u \phi_{23}) + \frac{\kappa_2}{\Lambda^3} (\overline{l}_L h_u \phi_1)(l^C_L h_u \phi_1) + \frac{\kappa_3}{\Lambda^3} (\overline{l}_L h_u \phi_{123})(l^C_L h_u \phi_{123}) + \frac{\kappa_4}{\Lambda^3} [ (\overline{l}_L h_u \phi_1)(l^C_L h_u \phi_{123}) + (\overline{l}_L h_u \phi_{123})(l^C_L h_u \phi_1) ] + h.c.,
\]

(1)
which requires $\phi_{23}$ to be distinguished by e.g. a $Z_2$ from the other two flavons, $\phi_1$ and $\phi_{123}$. This leads to a cobimaximal form for the neutrino mass matrix, provided a CP symmetry is imposed forcing the coefficients to be real, and that the following VEV patterns for the $\Delta(27)$ triplets SM singlet scalar fields is considered:

$$
\langle \rho \rangle = v_{\rho} (1, 1, 0), \quad \langle \phi_1 \rangle = v_1 (1, 0, 0), \quad \langle \phi_{123} \rangle = v_{123} (1, \omega, \omega^2), \quad \langle \phi_{23} \rangle = v_{23} (0, 1, -1),
$$

where $\omega = e^{i \frac{2\pi}{3}}$, and $\rho$ will be responsible for ensuring the charged lepton Yukawa matrix is diagonal in this basis. Cobimaximal mixing is motivating these VEVs, and in turn these special directions are the motivation for realizing the models in a SUSY framework with a $\Delta(27)$ family symmetry, as they can be easily obtained in SUSY $\Delta(27)$ flavour models through F-term alignment mechanism [32] or D-term alignment mechanism [15]. This is in contrast with the somewhat generic $(r, e^{i\nu}, e^{-i\nu})$ VEV employed in [10, 11]. We therefore consider implicitly extensions of the minimal supersymmetric SM (MSSM), although for our purposes, it is enough to assume that these VEV directions are obtained e.g. through F-term alignment [32], and that the Yukawa Lagrangian arises from an holomorphic superpotential.

In the following subsections we are going to describe two specific models where the cobimaximal mixing pattern is obtained, by adding 3 right-handed (RH) neutrinos.

### A. Model 1.

In this supersymmetric model, the full symmetry $G$ experiences a two-step spontaneous breaking:

$$
G = SU(3)_C \times SU(2)_L \times U(1)_Y \times \Delta(27) \times Z_2 \times Z_{10} \downarrow \Lambda_{int} \downarrow v \quad SU(3)_C \times SU(2)_L \times U(1)_Y \quad SU(3)_C \otimes U(1)_{em}
$$

It is assumed that the discrete groups are spontaneously broken at an energy scale $\Lambda_{int}$ much larger than the electroweak symmetry breaking scale $v = 246$ GeV. In the supersymmetric model under consideration, the scalar sector of MSSM is extended by the inclusion of several gauge singlet scalar fields, whereas the fermion sector is enlarged by considering three very heavy RH Majorana neutrinos. Such heavy RH Majorana neutrinos are crucial for mediating a type I seesaw mechanism that produces the tiny values of the light active neutrino masses. The inclusion of the gauge singlet scalars is necessary for the implementation of the Froggat-Nielsen mechanism that produces the SM charged lepton mass hierarchy and allows to build the neutrino Yukawa terms invariant under the symmetries of the model, that give rise to a predictive cobimaximal neutrino mass matrix texture. The $\Delta(27) \times Z_2 \times Z_{10}$ assignments of fermions and scalars in our model are shown in Table I

| | $i_L$ | $i_{1R}$ | $i_{2R}$ | $i_{13R}$ | $N_{1R}$ | $N_{2R}$ | $N_{3R}$ | $h_u$ | $h_d$ | $\sigma$ | $\rho$ | $\phi_1$ | $\phi_{123}$ | $\phi_{23}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Delta(27)$ | 3 | 10,0 | 10,1 | 10,2 | 10,0 | 10,0 | 10,0 | 1,0 | 1,0 | 1,0 | 1,0 | 1,0 | 3 | 3 |
| $Z_2$ | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $Z_{10}$ | 0 | 5 | 4 | 2 | 0 | 0 | 5 | 0 | 0 | -1 | 0 | 0 | 0 | 5 |

Table I: Leptonic and scalar field assignments under the $\Delta(27) \times Z_2 \times Z_{10}$ symmetry.

Notice that in Table I the numbers in boldface correspond to the $\Delta(27)$ representations and the $Z_N$ charges are written in additive notation.

In this model, the $\Delta(27)$ discrete flavor symmetry is necessary to get a predictive lepton sector through the special VEV directions in Eq. (2). The $Z_2$ symmetry separates the $\Delta(27)$ scalar triplet $\rho$ that participates in the charged lepton Yukawa interactions from the ones $(\phi_1, \phi_{123}, \phi_{23})$ appearing in the neutrino Yukawa terms, thus allowing to treat these sectors independently. The $Z_{10}$ implements the Froggat-Nielsen mechanism that produces the SM charged lepton mass hierarchy, and also distinguishes the $\Delta(27)$ scalar triplet $\phi_{23}$ from the $\Delta(27)$ scalar triplets $\phi_1$ and $\phi_{123}$. Since the spontaneous breaking of the $Z_{10}$ symmetry produces the SM charged fermion mass hierarchy, we set the vacuum expectation values (VEVs) of the different gauge singlet scalars as follows:
where $\lambda = \sin \theta_{13}$, being $\theta_{13}$ is the reactor mixing angle and $\Lambda$ the model cutoff, which can be interpreted as the scale of the UV completion of the model, e.g. the masses of the Froggatt-Nielsen messenger fields.

The Yukawa terms for the lepton sector invariant under the aforementioned symmetries are:

\begin{equation}
\mathcal{L}_Y^{(l)} = y_1^{(l)} (\tilde{T}_L \rho h_d)_{10,0} l_1 \frac{\sigma^8}{\Lambda^9} + y_2^{(l)} (\tilde{T}_L \rho h_d)_{10,2} l_2 R \frac{\sigma^4}{\Lambda^4} + y_3^{(l)} (\tilde{T}_L \rho h_d)_{10,3} l_3 R^2 \frac{\sigma^2}{\Lambda^2} + h.c.,
\end{equation}

\begin{equation}
\mathcal{L}_Y^{(\nu)} = y_1^{(\nu)} (\tilde{T}_L \phi_1 h_u)_{10,0} N_{1R} \frac{1}{\Lambda} + y_2^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{2R} \frac{1}{\Lambda}
+ y_3^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{1R} \frac{1}{\Lambda} + y_4^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{2R} \frac{1}{\Lambda}
+ y_5^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{3R} \frac{1}{\Lambda}
+ m_{N_1} \overline{N}_{1R} N_{1R}^C
+ m_{N_2} \overline{N}_{2R} N_{2R}^C
+ m_{N_3} \overline{N}_{3R} N_{3R}^C
+ m_{N_4} \overline{N}_{1R} N_{2R}^C
+ m_{N_5} \overline{N}_{2R} N_{1R}^C
+ h.c.,
\end{equation}

This is the most general form. We can without loss of generality change the RH neutrino basis such that we choose states where the RH neutrinos $N_1$ and $N_2$ don’t mix (RH neutrino diagonal mass basis) or we can choose states where $N_1$ couples only to $\phi_1$ and $N_2$ couples only to $\phi_{123}$ (RH neutrino flavon basis). In any case, the effective low energy neutrinos will have a term of the form $\frac{\sigma^8}{\Lambda^9} [\langle \overline{1}_L h_u \phi_1 \rangle ] \langle \overline{1}_L h_u \phi_{123} \rangle + [\langle \overline{1}_L h_u \phi_{123} \rangle ] \langle \overline{1}_L h_u \phi_1 \rangle ]$, as required.

**B. Model 2.**

This model is very similar to model 1. The crucial difference here is that the light active neutrino masses are generated from a one loop level radiative seesaw mechanism instead of the tree level type I seesaw mechanism of model 1. To implement such radiative seesaw mechanism in the model 2, we add an extra preserved $Z_2'$ symmetry, under which the right handed Majorana neutrinos and an extra inert scalar singlet $\varphi$, transforming as a $\Delta(27)$ trivial singlet, will be $Z_2'$ charged, as follows:

\begin{equation}
(\varphi, N_{1R}) \rightarrow - (\varphi, N_{1R}), \quad i = 1, 2, 3.
\end{equation}

The whole discrete group of this model will be $\Delta(27) \times Z_2 \times Z_{10} \times Z_{12}'$, where the $\Delta(27) \times Z_2 \times Z_{10}$ group is spontaneously broken as in model 1 whereas the $Z_{12}'$ symmetry is preserved. Due to the preserved $Z_{12}'$ symmetry, our model has scalar and fermionic dark matter candidates. The scalar dark matter candidates will be the lightest of Re$(\varphi)$ and Im$(\varphi)$, while the fermionic dark matter candidate will be the lightest of the RH Majorana neutrinos. The resulting implications of such model in Dark model will be the same as in the model of Ref. [39], which makes our model consistent with dark matter constraints.

The SM charged lepton Yukawa terms will be the same as in model 1, whereas the neutrino Yukawa interactions take the form:

\begin{equation}
\mathcal{L}_Y^{(\nu)} = y_1^{(\nu)} (\tilde{T}_L \phi_1 h_u)_{10,0} N_{1R} \frac{\varphi}{\Lambda^2} + y_2^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{2R} \frac{\varphi}{\Lambda^2}
+ y_3^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{1R} \frac{\varphi}{\Lambda^2} + y_4^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{2R} \frac{\varphi}{\Lambda^2}
+ y_5^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{3R} \frac{\varphi}{\Lambda^2}
+ m_{N_1} \overline{N}_{1R} N_{1R}^C
+ m_{N_2} \overline{N}_{2R} N_{2R}^C
+ m_{N_3} \overline{N}_{3R} N_{3R}^C
+ m_{N_4} \overline{N}_{1R} N_{2R}^C
+ m_{N_5} \overline{N}_{2R} N_{1R}^C
+ h.c.,
\end{equation}

**III. LEPTON MASSES AND MIXINGS**

**A. Model 1.**

After the discrete groups are spontaneously broken, we get the following charged lepton and neutrino Yukawa terms:

\begin{equation}
\mathcal{L}_Y^{(l)} = y_1^{(l)} \tilde{T}_L h_d l_1 R \frac{v_\rho^8}{\Lambda^9} + y_2^{(l)} \tilde{T}_L h_d l_2 R \frac{v_\rho^4}{\Lambda^4} + y_3^{(l)} \tilde{T}_L h_d l_3 R \frac{v_\rho^2}{\Lambda^2} + h.c.,
\end{equation}

\begin{equation}
\mathcal{L}_Y^{(\nu)} = \frac{1}{\Lambda^2} \left[ y_1^{(\nu)} (\tilde{T}_L \phi_1 h_u)_{10,0} N_{1R} \frac{\varphi}{\Lambda^2} + y_2^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{2R} \frac{\varphi}{\Lambda^2}
+ y_3^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{1R} \frac{\varphi}{\Lambda^2} + y_4^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{2R} \frac{\varphi}{\Lambda^2}
+ y_5^{(\nu)} (\tilde{T}_L \phi_{123} h_u)_{10,0} N_{3R} \frac{\varphi}{\Lambda^2}
+ m_{N_1} \overline{N}_{1R} N_{1R}^C
+ m_{N_2} \overline{N}_{2R} N_{2R}^C
+ m_{N_3} \overline{N}_{3R} N_{3R}^C
+ m_{N_4} \overline{N}_{1R} N_{2R}^C
+ m_{N_5} \overline{N}_{2R} N_{1R}^C
+ h.c.,
\right]
\end{equation}
In what regards the neutrino sector, we find that the full $6 \times 6$ charged lepton mass matrix diagonal with the charged lepton masses given by:

$$
L^v = y_1^{(v)} \frac{\nu_1}{\sqrt{2}A^9} + y_2^{(v)} \left( \frac{\nu_1}{\sqrt{2}A^9} + \omega^2 \frac{\nu_1}{\sqrt{2}} \right) + y_3^{(v)} \left( \frac{\nu_1}{\sqrt{2}A^9} + \frac{\nu_1}{\sqrt{2}} \right) + y_4^{(v)} \frac{\nu_1}{\sqrt{2}A^9} + y_5^{(v)} \left( \frac{\nu_1}{\sqrt{2}} \right) + y_6^{(v)} \frac{\nu_1}{\sqrt{2}A^9}
$$

Consequently, the SM charged lepton mass matrix is diagonal with the charged lepton masses given by:

$$
m_e = y_1^{(l)} \frac{\nu_1}{\sqrt{2}A^9} = a_1^{(l)} \frac{\nu_1}{\sqrt{2}}
$$

$$
m_\mu = y_2^{(l)} \frac{\nu_1}{\sqrt{2}A^9} = a_2^{(l)} \frac{\nu_1}{\sqrt{2}}
$$

$$
m_\tau = y_3^{(l)} \frac{\nu_1}{\sqrt{2}A^9} = a_3^{(l)} \frac{\nu_1}{\sqrt{2}}
$$

(11)

where $a_1^{(l)}$, $a_2^{(l)}$ and $a_3^{(l)}$ are real $O(1)$ dimensionless parameters and we have assumed that $v_{h_d} \sim v/\sqrt{2}$, being $v = 246$ GeV the electroweak symmetry breaking scale.

In what regards the neutrino sector, we find that the full $6 \times 6$ neutrino mass matrix read:

$$
M_\nu = \begin{pmatrix}
0_{3 \times 3} & M_{\nu D} \\
M_{\nu D}^T & M_R
\end{pmatrix},
$$

(12)

where the Dirac and Majorana neutrino mass matrices are given by:

$$
M_{\nu D} = \begin{pmatrix}
A_1 + A_2 & B_1 + B_2 & 0 \\
\omega A_2 & \omega B_1 & C \\
\omega^2 A_2 & \omega^2 B_1 & -C
\end{pmatrix},
$$

$$
M_R = \begin{pmatrix}
m_{N_1} & m_{N_4} & 0 \\
m_{N_4} & m_{N_2} & 0 \\
0 & 0 & m_{N_3}
\end{pmatrix},
$$

(13)

with:

$$
A_1 = y_1^{(v)} \frac{v^2}{\sqrt{2}A^9}, \quad A_2 = y_2^{(v)} \frac{v^2}{\sqrt{2}A^9},
$$

$$
B_1 = y_3^{(v)} \frac{v^2}{\sqrt{2}A^9}, \quad B_2 = y_4^{(v)} \frac{v^2}{\sqrt{2}A^9}, \quad C = y_5^{(v)} \frac{v^2}{\sqrt{2}A^9}.
$$

(14)

Assuming that the Majorana neutrino masses are much larger than the electroweak symmetry breaking scale, the light active neutrino masses will be generated from a type I seesaw mechanism. Thus, the light active neutrino mass matrix takes the form:

$$
\tilde{M}_\nu = M_{\nu D} M_R^{-1} M_{\nu D}^T = \begin{pmatrix}
a & d \omega & d \omega^2 \\
d \omega & c & b e^{i\theta} \\
d \omega^2 & c & b e^{-i\theta}
\end{pmatrix},
$$

(15)

where:

$$
a = X (A_1 + A_2)^2 + 2 W (A_1 + A_2) (B_1 + B_2) + Y (B_1 + B_2)^2,
$$

$$
d = (X A_2 + W B_1) (A_1 + A_2) + (Y B_1 + W A_2) (B_1 + B_2),
$$

$$
c = -Z C^2 + X A_2^2 + 2 W A_2 B_1 + Y B_1^2,
$$

$$
b = |Z C^2 + \omega^2 (X A_2^2 + 2 W A_2 B_1 + Y B_1^2)|,
$$

$$
\theta = \text{arg} \left( Z C^2 + \omega^2 (X A_2^2 + 2 W A_2 B_1 + Y B_1^2) \right),
$$

(16)

$$
X = \frac{m_{N_2}}{m_{N_1} m_{N_2} - m_{N_4}^2}, \quad Y = \frac{m_{N_1}}{m_{N_1} m_{N_2} - m_{N_4}^2}, \quad W = -\frac{m_{N_4}}{m_{N_1} m_{N_2} - m_{N_4}^2}, \quad Z = \frac{1}{m_{N_3}}.
$$

(17)

Such light active neutrino mass matrix features a cobimaximal mixing pattern and is diagonalized by the rotation matrix:

$$
U = \begin{pmatrix}
\cos \alpha_{12} & \cos \alpha_{13} & -\cos \alpha_{13} \\
\sin \alpha_{12} & \sin \alpha_{13} & 0 \\
\sqrt{2} \sin \alpha_{12} & \sin \alpha_{13} & 0
\end{pmatrix},
$$

(18)
The physical observables of the neutrino sector, i.e., the three leptonic mixing angles, the CP phase and the neutrino mass squared differences, are given by the following relations:

\[
\begin{align*}
a &= m_3 \sin^2 \alpha_{13} + \cos^2 \alpha_{13} \left( m_2 \sin^2 \alpha_{12} + m_1 \cos^2 \alpha_{12} \right), \\
d &= \frac{\cos \alpha_{13} ( m_1 - m_2 ) \sin 2\alpha_{12} - 2i \sin \alpha_{13} ( m_2 \sin^2 \alpha_{12} + m_1 \cos^2 \alpha_{12} - m_3 )}{2\sqrt{2}}, \\
c &= \frac{1}{8} \left( 4m_3 \cos^2 \alpha_{13} - m_1 \left( 2 \cos 2\alpha_{13} \cos^2 \alpha_{12} + \cos 2\alpha_{12} - 3 \right) + m_2 \left( \cos 2\alpha_{12} - 2 \sin^2 \alpha_{12} \cos 2\alpha_{13} + 3 \right) \right), \\
b &= \left| \frac{1}{8} \left( -4m_3 \cos^2 \alpha_{13} + 4m_1 \sin \alpha_{12} - i \sin \alpha_{13} \cos \alpha_{12} \right)^2 + 4m_2 \left( \cos \alpha_{12} + i \sin \alpha_{12} \sin \alpha_{13} \right)^2 \right|, \\
\theta &= \arg \left[ \frac{1}{8} \left( -4m_3 \cos^2 \alpha_{13} + 4m_1 \sin \alpha_{12} - i \sin \alpha_{13} \cos \alpha_{12} \right)^2 + 4m_2 \left( \cos \alpha_{12} + i \sin \alpha_{12} \sin \alpha_{13} \right)^2 \right].
\end{align*}
\]

The leptonic mixing angles and the CP phase arising from the cobimaximal light active neutrino mass matrix \(\widetilde{M}_\nu\) read:

\[
\begin{align*}
\theta_{13} &= \alpha_{13}, \\
\theta_{12} &= \alpha_{12}, \\
\theta_{23} &= \frac{\pi}{4}, \\
\delta_{CP} &= -\frac{\pi}{2}.
\end{align*}
\]

The physical observables of the neutrino sector, i.e., the three leptonic mixing angles, the CP phase and the neutrino mass squared splittings for the normal mass hierarchy (NH) can be very well reproduced, as shown in Table II, starting from the following benchmark point:

\[
\begin{align*}
a &\approx 3.53\text{meV}, \\
b &\approx 21.51\text{meV}, \\
c &\approx 27.50\text{meV}, \\
d &\approx 5.69\text{meV}, \\
\theta &\approx 178.39^\circ.
\end{align*}
\]

This shows that our predictive model successfully describes the current neutrino oscillation experimental data. Notice that with only five effective parameters, i.e., \(a, b, c, d\) and \(\theta\), we can successfully reproduce the experimental values of the six physical observables of the neutrino sector: the neutrino mass squared differences, the leptonic mixing angles and the leptonic CP phase.

### B. Model 2

The resulting light active neutrino mass matrix arising from radiative seesaw mechanism takes the form:
$$\bar{M}_\nu = \begin{pmatrix} a & d\omega & d\omega^2 \\ d\omega & be^{i\theta} & c \\ d\omega^2 & c & be^{-i\theta} \end{pmatrix}$$  \ (23)

where $a, b, c, d, \theta$ are given by different expressions in terms of the high energy parameters

$$a = X (A_1 + A_2)^2 + 2W (A_1 + A_2) (B_1 + B_2) + Y (B_1 + B_2)^2,$$

$$d = (XA_2 + WB_2) (A_1 + A_2) + (Y B_1 + WA_2) (B_1 + B_2),$$

$$c = -ZC^2 + XA_2^2 + 2WA_2B_1 + YB_1^2,$$

$$b = \left| ZC^2 + \omega^2 (XA_2^2 + 2WA_2B_1 + YB_1^2) \right|,$$

$$\theta = \text{arg} \left( ZC^2 + \omega^2 (XA_2^2 + 2WA_2B_1 + YB_1^2) \right)$$  \ (24)

where the parameters $X, Y, W$ will be loop functions depending on the masses of the $Z'_2$ odd scalars $\text{Re}(\varphi)$ and $\text{Im}(\varphi)$ and $Z'_2$ odd right handed Majorana neutrinos $N_{1R}, N_{2R}$ and $N_{3R}$ similar as in \cite{39, 46}. Given that the light active neutrino mass matrix in model 2 is of the same form as in model 1, the resulting predictions in low energy neutrino observables will be the same in these models and the benchmark point above for $a, b, c, d, \theta$ reproduces well the observed values.

IV. CONCLUSIONS

We have proposed a framework where the cobimaximal mixing pattern is successfully realized, based on the $\Delta(27)$ family symmetry broken by specific directions which are easy to obtain with this group. In this framework, due to the $\Delta(27)$ breaking pattern and in the model-building basis, the Standard Model charged lepton mass matrix is diagonal whereas the light active neutrino mass matrix features a cobimaximal mixing pattern. This generates the experimental values of the neutrino mass squared splittings, the leptonic mixing angles and the leptonic Dirac CP violating phase.

We present two example models within this framework, where $\Delta(27)$ is supplemented by other auxiliary cyclic symmetries. In the first model, the small masses for the light active neutrinos are produced from a tree-level type I seesaw mechanism mediated by three heavy right-handed Majorana neutrinos. In the second model, the light active neutrino masses arise from a radiative seesaw mechanism where three right-handed Majorana neutrinos and a gauge singlet scalar are charged under a preserved $Z'_2$ symmetry, thus having viable dark matter candidates in conjunction with the viable cobimaximal mixing structure.

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Appendix A: The $\Delta(27)$ discrete group

The $\Delta(27)$ discrete group has the following 11 irreducible representations: one triplet 3, one antitriplet $\overline{3}$ and nine singlets $1_{k,l}$ $(k, l = 0, 1, 2)$, where $k$ and $l$ identify how the singlets transform under order 3 generators, corresponding
to a $Z_3$ and $Z'_3$ subgroups of $\Delta(27)$.

$$3 \otimes 3 = \mathbb{F}_{S_1} \oplus \mathbb{F}_{S_2} \oplus \mathbb{F}_A$$

$$\mathbb{F} \otimes \mathbb{F} = 3_S \otimes 3_S \otimes 3_A$$

$$3 \otimes \mathbb{F} = \sum_{r=0}^{2} 1_{r,0} \oplus \sum_{r=0}^{2} 1_{r,1} \oplus \sum_{r=0}^{2} 1_{r,2}$$

$$1_{k,\ell} \otimes 1_{k',\ell'} = 1_{k+k' \mod 3, \ell+\ell' \mod 3} \quad \text{(A1)}$$

Denoting $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ as the basis vectors for two $\Delta(27)$-triplets $3$, one finds:

$$\begin{align*}
(3 \otimes 3)_{S_{\xi_1}} &= \begin{pmatrix} x_1 y_1, x_2 y_2, x_3 y_3 \end{pmatrix}, \\
(3 \otimes 3)_{S_{\xi_2}} &= \frac{1}{2} \begin{pmatrix} x_2 y_3 + x_3 y_2, x_3 y_1 + x_1 y_3, x_1 y_2 + x_2 y_1 \end{pmatrix}, \\
(3 \otimes 3)_{S_A} &= \frac{1}{2} \begin{pmatrix} x_2 y_3 - x_3 y_2, x_3 y_1 - x_1 y_3, x_1 y_2 - x_2 y_1 \end{pmatrix}, \\
(3 \otimes \mathbb{F})_{1, r, 0} &= \begin{pmatrix} x_1 y_1 + \omega^2 r x_2 y_2 + \omega^r x_3 y_3 \end{pmatrix}, \\
(3 \otimes \mathbb{F})_{1, r, 1} &= \begin{pmatrix} x_1 y_2 + \omega^{1+2r} x_2 y_3 + \omega^r x_3 y_1 \end{pmatrix}, \\
(3 \otimes \mathbb{F})_{1, r, 2} &= \begin{pmatrix} x_1 y_3 + \omega^{2+2r} x_2 y_1 + \omega^r x_3 y_2 \end{pmatrix} \quad \text{(A3)}
\end{align*}$$

where $r = 0, 1, 2$ and $\omega = e^{i\frac{2\pi}{3}}$.

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