Lorentz Violation for Photons and Ultra-High Energy Cosmic Rays

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Lorentz symmetry breaking at very high energies may lead to photon dispersion relations of the form $\omega^2 = k^2 + $ new terms suppressed by a power $n$ of the Planck mass $M_\text{Pl}$. We show that first and second order terms of size $|\xi_1| \lesssim 10^{-14}$ and $|\xi_2| \lesssim 10^{-6}$, respectively, would lead to a photon component in cosmic rays above $10^{19}$ eV that should already have been detected, if corresponding terms for $e^\pm$ are significantly smaller. This suggests that LI breaking suppressed up to second order in the Planck scale are unlikely to be phenomenologically viable for photons.

Introduction. Many Quantum Gravity theories suggest the breaking of Lorentz invariance (LI) with the strength of the effects increasing with energy. The most promising experimental tests of such theories, therefore, exploit the highest energies at our disposal which are usually achieved in violent astrophysical processes. If LI is broken in form of non-standard dispersion relations for various particles, absorption and energy loss processes for high energy cosmic radiation can be modified \cite{1}. Conversely, experimental confirmation that such processes occur at the expected thresholds would allow to put strong constraints on such LI breaking effects. This was shown in case of ultra-high-energy cosmic rays producing pions by the Greisen-Zatsepin-Kuzmin (GZK) effect \cite{2} above the threshold at $\sim 7 \times 10^{19}$ eV and in case of pair production of high energy photons with the diffuse low energy photon background \cite{3}.

While the thresholds of electron-positron pair production by high energy $\gamma$-rays on low energy background photons have not yet been experimentally confirmed beyond doubt, constraints on LI breaking for photons have been established based on the very existence of TeV $\gamma$-rays from astrophysical objects \cite{4}.

Here we exploit the fact that if pair production of high energy $\gamma$-rays on the cosmic microwave background (CMB) would be inhibited above $\sim 10^{19}$ eV, one would expect a large fraction of $\gamma$-rays in the cosmic ray flux at these energies, independent on where the real pair production threshold is located. Based on the fact that no significant $\gamma$-ray fraction is observed, we derive limits on LI violating parameters for photons that are more stringent than former limits. These limits do not depend on the poorly known strength of the astrophysical radio background.

Hybrid detectors begin to put constraints on the composition of cosmic rays at highest energies. Particularly it is already possible to put upper limits on the fraction of photons on the 10% level at energies above $10^{19}$ eV using Auger hybrid observations \cite{5}, AGASA \cite{6, 7} and Yakutsk \cite{8, 9} data. Above $10^{19}$ eV, the current upper limit is $\sim 40\%$ \cite{10}. In fact, the latest upper limits from the surface detector data of the Pierre Auger observatory are already at the level of $\sim 2\%$ above $10^{19}$ eV \cite{10}. In the next few years these constraints will improve with statistics: The Pierre Auger experiment can reach a sensitivity of $\sim 0.3\%$ within a few years and $\sim 0.03\%$ within 20 years around $10^{19}$ eV, and a sensitivity at the 10% level around $10^{20}$ eV within 20 years \cite{11}.

Neutral pions created by the GZK effect decay into ultra-high energy photons. They subsequently interact with low-energy background photons of the CMB and the universal radio background (URB) through pair production, $\gamma \gamma \to e^+ e^-$. This leads to the development of an electromagnetic cascade and suppresses the photon flux above the pair production threshold on the CMB of $\sim 10^{15}$ eV. Above $\sim 10^{19}$ eV the interaction length for photons is smaller than a few Mpc, whereas for nucleons above the GZK threshold at $\sim 7 \times 10^{19}$ eV it is of the order of 20 Mpc. As a result, the photon fraction theoretically expected is smaller than $\sim 1\%$ around $10^{19}$ eV, and smaller than $\sim 10\%$ around $10^{20}$ eV \cite{12, 13}, in agreement with experimental upper limits.

The breaking of Lorentz invariance, by modifying the dispersion relation for photons, would affect the energy threshold for pair production. If the change in the dispersion relation is sufficiently large, pair production can become kinematically forbidden at ultra-high energies and such photons could reach us from cosmological distances. As a consequence, at least if ultra-high energy cosmic rays consist of mostly protons, one would expect a significant photon fraction in cosmic rays above $10^{19}$ eV, in con-

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energies in the absence of pair production would overshoot the experimental limits even more than in Figs. 1 and 2.

Therefore, LI violating parameters for photons are constrained by the requirement that pair production be allowed between low energy background photons and photons of energies between $10^{19}$ eV and $10^{20}$ eV. We will assume that pion production itself is not significantly modified and that the modifications of the dispersion relations of electrons and positrons are significantly smaller than for photons. This is consistent since the photon content of other particles is on the percent level [19].

**Formalism.** We denote the 4-momenta with $(\omega, \mathbf{k})$ for the ultra-high-energy photon, $(\omega_b, \mathbf{k}_b)$ for the background photon, and $(E_{\pm}, \mathbf{p}_{\pm})$ for the electron and positron, respectively.

We consider the following modified dispersion relations for photons, electrons and positrons:

$$
\omega^2 = k^2 + \xi_n k^2 \left( \frac{k}{M_{Pl}} \right)^n, \\
E_{\pm}^2 = p_{\pm}^2 + m_e^2 + \eta_n \frac{p_{\pm}}{M_{Pl}} \left( \frac{p_{\pm}}{M_{Pl}} \right)^n, 
$$

with $n \geq 1$ and where $M_{Pl} \approx 10^{19}$ GeV and $m_e$ are the Planck mass and the electron mass, respectively.

Using the exact relation for energy-momentum conservation, the kinematic relation for the decay of a neutral pion of mass $m_\pi$ into two $\gamma$-rays of energy-momentum $(\omega_1, \mathbf{k}_1)$ and $(\omega_2, \mathbf{k}_2)$, respectively, and equal helicity is $2\omega_1 \omega_2 - 2\mathbf{k}_1 \cdot \mathbf{k}_2 = \xi_n (k_1^{n+2} + k_2^{n+2})/M_{Pl}^n = m_\pi^2$. For $|\xi_n| \ll 1$, the absolute values of the LI violating terms are always much smaller than the ones of $\omega_1 \omega_2$ and $\mathbf{k}_1 \cdot \mathbf{k}_2$, which themselves are much larger than $m_\pi^2$ in most of the phase space. Therefore, the kinematics of pion decay is not significantly modified.

This is different for pair production by photons: Exact energy momentum conservation implies that $(\omega + \omega_b)^2 - (\mathbf{k} + \mathbf{k}_b)^2 = (E_+ + E_-)^2 - (\mathbf{p}_+ + \mathbf{p}_-)^2$. The left hand side is maximized for anti-parallel initial photon momenta (head-on collision) and the right hand side is minimized for parallel final momenta of the pair [4, 20]. Writing $p_+ = y k$, $p_- = (1 - y) k$ with $0 \leq y \leq 1$, assuming relativistic leptons and using $\omega \gg \omega_b$, after some algebra we thus obtain at the threshold

$$
\kappa_n k^2 \left( \frac{k}{M_{Pl}} \right)^n + 4 \omega_b y - \frac{m_e^2}{y (1 - y)} = 0, 
$$

where

$$
\kappa_n \equiv \xi_n - \eta_n y^{n+1} - \eta_n (1 - y)^{n+1} 
$$

and the asymmetry $y$ in the final momenta at threshold is determined by maximizing the left hand side of Eq. (2). For example, if $\eta_n^+ = \eta_n^- > -2^{n+3} (m_e/k)^2 (M_{Pl}/k)^n/n(n+1)$, then $y = \frac{1}{2}$.
where Eq. (2) can be rewritten as
\[ x > x_{\text{pair}} \]
pair production is only allowed in the range of energies positive LI breaking term, pair production is allowed for modified dispersion relation, because for photons with a positive LI breaking term, \( \alpha_1 = 2/27 \); Black: photons with unbroken LI, \( \alpha_1 = 0 \); Blue: photons with a negative LI breaking term, with \( \alpha_1 = -6/27, -4/27, -2/27 \), in ascending order. Pair production is kinematically allowed for values of \( x \equiv k/k_0 \) for which the curves are positive.

Introducing \( x \equiv k/k_0 \) with \( k_0 \equiv m_e^2 / [4y(1-y)\omega_b] \), Eq. (2) can be rewritten as
\[ \alpha_n x^{n+2} + x - 1 = 0 , \tag{4} \]
where
\[ \alpha_n \equiv \kappa_n \frac{k_0}{4\omega_b \left( \frac{k_0}{M_{\text{pl}}} \right)^n} . \tag{5} \]

If \( \xi_n = \eta_+ = \eta_- = 0 \) we have \( \alpha_n = 0 \) and \( y = \frac{1}{2} \) and thus the usual threshold for pair production in Lorentz invariant theory, \( k = m_e^2 / \omega_b \). Furthermore, if the LI violating terms in the electron and positron dispersion relations are smaller than the photon terms, \( |\eta_n| \ll \xi_n \), then \( \kappa_n \approx \xi_n \) and we will obtain constraints essentially on the photon terms \( \xi_n \). If not otherwise stated we will make this assumption in the following.

If \( \alpha_n > 0 \), Eq. (4) admits one real positive solution \( x_{\text{pair}}(\alpha_n) < 1 \) for each value of \( \alpha_n > 0 \). Therefore, for photons with a positive LI violating term in the modified dispersion relation Eq. (1), pair production is kinematically allowed above a threshold \( k_0 x_{\text{pair}}(\alpha_n) < k_0 \).

Otherwise, if the coefficient of \( x^{n+2} \) in Eq. (4) is negative, this equation has real solutions only if \( |\alpha_n| \leq \alpha_n^c \equiv (n+1)^{n+1} / (n+2)^{n+2} \). In particular, if \( |\alpha_n| = \alpha_n^c \) there is only one real solution and pair production is kinematically allowed only for a particular value of the momentum of the ultra-high-energy photon. If \( |\alpha_n| < \alpha_n^c \), there are two real solutions, \( 0 < x_{\text{pair}}(\alpha_n) < x_n^b(\alpha_n) \), and thus pair production is only allowed in the range of energies \( k_0 x_{\text{pair}}(\alpha_n) \leq \omega \leq k_0 x_n^b(\alpha_n) \). These two cases are summarized in Fig. 3.

Requiring pair production to be allowed, we obtain constraints only from photons with a negative sign in the modified dispersion relation, because for photons with a positive LI breaking term, pair production is allowed for any value of \( \alpha_n \) above \( k_0 x_{\text{pair}}(\alpha_n) < k_0 \). We also stress that photons with negative LI breaking term are stable against photon decay \( (\gamma \rightarrow e^+ e^-) \) and photon splitting \( (\gamma \rightarrow N \gamma) \).

Requiring the interaction of ultra-high-energy photons, \( 10^{19} \text{eV} \lesssim k \lesssim 10^{20} \text{eV} \), with CMB photons of energy \( \omega_b \simeq 6 \times 10^{-3} \text{eV} \) corresponds to requiring that pair production is kinematically allowed for \( 2.3 \times 10^4 \lesssim x \lesssim 2.3 \times 10^5 \). Since photons with a negative LI breaking term in the dispersion relation have both a lower and an upper energy threshold for pair production, denoted by \( x_{\text{pair}}(\alpha_n) \) and \( x_n^b(\alpha_n) \), respectively, we have the two conditions \( x_{\text{pair}}(\alpha_n) \lesssim 2.3 \times 10^4 \) and \( 2.3 \times 10^5 \lesssim x_n^b(\alpha_n) \). These will lead to constraints on \( \alpha_n \) and thus \( \xi_n \).

Constraints on Lorentz invariance breaking to first order in the Planck mass. In this case \( n = 1 \) and the first condition, \( x_{\text{pair}}(\alpha_1) \lesssim 2.3 \times 10^4 \) is always true if the lower threshold exists, \( \alpha_1 > -\alpha_1^c = -4/27 \). The second condition \( 2.3 \times 10^5 \lesssim x_n^b(\alpha_1) \) is fulfilled if \( \alpha_1 \gtrsim -1.9 \times 10^{-11} \). These two necessary conditions can be translated into a constraint for \( \xi_1 \) using the definition for \( \alpha_n \), Eq. (5), and \( k_0 \simeq 4.4 \times 10^{14} \text{eV} \):
\[ \alpha_1 \equiv \kappa_1 \frac{k_0}{4\omega_b \left( \frac{k_0}{M_{\text{pl}}} \right)} \gtrsim -1.9 \times 10^{-11} ; \quad \kappa_1 \gtrsim -2.4 \times 10^{-15} . \tag{6} \]

For \( n = 1 \) effective field theory implies LI violating terms in the dispersion relation of equal absolute value and opposite sign for left and right polarized photons [21]. Therefore, in order to avoid photon fractions in cosmic rays \( \gtrsim 5 \) times higher than observed above \( \sim 10^{19} \text{eV} \), pair production has to be allowed for both polarizations, and thus for both signs in the dispersion relation. Thus the constraint obtained for \( \kappa_1 \approx \xi_1 < 0 \) is valid also for positive LI violating terms: \( |\xi_1| \lesssim 2.4 \times 10^{-15} \).

Constraints on Lorentz invariance breaking to second order in the Planck mass. In this case \( n = 2 \) and the first condition, \( x_{\text{pair}}(\alpha_2) \lesssim 2.3 \times 10^4 \) is always true if the lower threshold exists, \( \alpha_2 > -\alpha_2^c = -27/256 \). The second condition \( 2.3 \times 10^5 \lesssim x_n^b(\alpha_2) \) is fulfilled if \( \alpha_2 \gtrsim -8.2 \times 10^{-17} \). These two necessary conditions then lead to the following constraint for \( \xi_2 \)
\[ \alpha_2 \equiv \kappa_2 \frac{k_0}{4\omega_b \left( \frac{k_0}{M_{\text{pl}}} \right)^2} \gtrsim -8.2 \times 10^{-17} ; \quad \kappa_2 \gtrsim -2.4 \times 10^{-7} . \tag{7} \]

For interactions with the URB, \( k_0 \simeq 6 \times 10^{19} \text{eV} \), we obtain the constraint assuming the existence of at least one solution with \( x_{\text{pair}}(\alpha_n) \lesssim 2 \). This eventually leads to the conditions \( |\kappa_1| \lesssim 7.2 \times 10^{-21} \) at first order, and \( -\kappa_2 \lesssim 8.5 \times 10^{-13} \) at second order. These are several orders of magnitudes more restrictive than the constraints Eqs. (6) and (7) obtained in the CMB case. Therefore, if the constraints from interactions with the CMB are violated, there would also be no interaction with the URB and so no pair production on any relevant background. Thus
the constraint from pair production with the CMB is not modified by the presence of the URB.

Discussion and Conclusions. To our knowledge, only LI breaking suppressed to first order in the Planck mass have so far been ruled out in the electromagnetic sector [22, 23, 24]. In terms of the dimensionless parameters $\xi_n$, the best upper limit was $|\xi_1| \lesssim 2 \times 10^{-7}$[25], based on frequency dependent rotation of linear polarization (vacuum biprefrince) of optical/UV photons of the afterglow from distant $\gamma$-ray bursts. A former, more stringent constraint, $|\xi_1| \lesssim 2 \times 10^{-15}$[26] was based on polarization of MeV $\gamma$-rays which could not be confirmed [23].

Constraints based on modified reaction thresholds were so far obtained from observations of multi-TeV $\gamma$-rays from blazars at distances $\gtrsim 100$ Mpc, over which such photons are expected to produce pairs on the infrared background. However, given that involved photon energies are much smaller than $10^{19}$ eV, resulting constraints are of the order $|\xi_1| \lesssim 1$[27], much weaker than our constraints $|\xi_1| \lesssim 2.4 \times 10^{-15}$ and $-\xi_2 \lesssim 2.4 \times 10^{-7}$. Our new constraints suggest that LI breaking suppressed up to second order in the Planck scale are unlikely to be phenomenologically viable for photons. Although similar constraints have been obtained in an independent approach based on the absence of vacuum Čerenkov radiation of ultra-high energy protons [19, 28], such constraints depend on the somewhat uncertain partonic structure of these protons.

It is interesting to note that the detection of a photon of $10^{19}$ eV would put strong constraints on any positive LI breaking term in the dispersion relation, $\kappa_1 < 10^{-17}$ for $n = 1$ and $\kappa_2 < 10^{-8}$ for $n = 2$, in order to avoid photon decay.

Our constraints Eqs. (6) and (7) hold for the linear combinations of LI breaking terms for photons, electrons and positrons defined in Eq. (5). They translate directly into constraints on the photon terms $\xi_n$ if the LI breaking terms for electrons and positrons are significantly smaller than the ones for photons. This is typically the case if the only LI breaking terms for electrons/positrons are induced by their photon content [19].

Note that in supersymmetric QED, corrections to the dispersion relation of a particle of mass $m$ are of the form $\xi_n m^2 (k/M)\eta$ and are thus negligible in astrophysical contexts [29]. Therefore, our constraints only apply to the non-supersymmetric case.

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[1] S. R. Coleman and S. L. Glashow, Phys. Rev. D 59, 116008 (1999) [arXiv:hep-ph/9812418].
[2] K. Greisen, Phys. Rev. Lett. 16, 748 (1966); G. T. Zatsepin and V. A. Kuzmin, JETP Lett. 4, 78 (1966) [Pisma Zh. Eksp. Teor. Fiz. 4, 114 (1966)].
[3] R. Aloisio, P. Blasi, P. L. Ghia and A. F. Grillo, Phys. Rev. D 62, 053010 (2000) [arXiv:astro-ph/0001258].
[4] T. Jacobson, S. Liberati and D. Mattingly, Phys. Rev. D 67, 124011 (2003) [arXiv:hep-ph/0209264].
[5] J. Abraham et al. [Pierre Auger Collaboration], Astropart. Phys. 27, 155 (2007) [arXiv:astro-ph/0606619].
[6] K. Shinozaki et al., Astrophys. J. 571, L117 (2002).
[7] M. Risse et al., Phys. Rev. Lett. 95, 171102 (2005) [arXiv:astro-ph/0502418].
[8] G. I. Rubtsov et al., Phys. Rev. D 73, 063009 (2006) [arXiv:astro-ph/0601449].
[9] A. V. Glushkov, D. S. Gorbunov, I. T. Makarov, M. I. Pravdin, G. I. Rubtsov, I. E. Sleptsov and S. V. Troitsky, arXiv:astro-ph/0701245.
[10] M. D. Healy for the Pierre Auger Collaboration, arXiv:0710.0025 [astro-ph].
[11] M. Risse and P. Homola, Mod. Phys. Lett. A 22, 749 (2007) [arXiv:astro-ph/0702652].
[12] G. Sigl, Phys. Rev. D 75, 103001 (2007) [arXiv:astro-ph/0703403].
[13] K. G. Gelmini, O. Kalashev and D. V. Semikoz, arXiv:0706.2181 [astro-ph].
[14] see also http://apcauger.in2p3.fr//CRPropa.
[15] E. Armengaud, G. Sigl, T. Beau and F. Miniati, arXiv:astro-ph/0609375.
[16] K. Shinozaki [AGASA Collaboration], Nucl. Phys. Proc. Suppl. 151, 3 (2006); see also http://www-akeno.icrr.u-tokyo.ac.jp/AGASA/.
[17] R. U. Abbasi et al. [High Resolution Fly’s Eye Collaboration], Phys. Rev. Lett. 92, 151101 (2004) [arXiv:astro-ph/0208243].
[18] T. A. Clark, L. W. Brown, J. K. Alexander, Nature 228, 847 (1970).
[19] O. Gagnon and G. D. Moore, Phys. Rev. D 70, 065002 (2004) [arXiv:hep-ph/0401196].
[20] D. Mattingly, T. Jacobson and S. Liberati, Phys. Rev. D 67, 124012 (2003) [arXiv:hep-ph/0211466].
[21] R. C. Myers and M. Pospelov, Phys. Rev. Lett. 90 (2003) 211601 [arXiv:hep-ph/0301124].
[22] T. Jacobson, S. Liberati and D. Mattingly, Annals Phys. 321, 150 (2006) [arXiv:astro-ph/0505267].
[23] S. Liberati, PoS P2GC, 018 (2007) [arXiv:0706.0142 [gr-qc]].
[24] L. Maccione, S. Liberati, A. Celotti and J. G. Kirk, arXiv:0707.2673 [astro-ph].
[25] Y. Z. Fan, D. M. Wei and D. Xu, Mon. Not. Roy. Astron. Soc. 376, 1857 (2006) [arXiv:astro-ph/0702006].
[26] T. A. Jacobson, S. Liberati, D. Mattingly and F. W. Stecker, Phys. Rev. Lett. 93, 021101 (2004) [arXiv:astro-ph/0309681].
[27] F. W. Stecker, Astropart. Phys. 20, 85 (2003) [arXiv:astro-ph/0308214].
[28] S. Bernadotte and F. R. Klinkhamer, Phys. Rev. D 75, 024028 (2007) [arXiv:hep-ph/0610216].
[29] S. Groot Nibbelink and M. Pospelov, Phys. Rev. Lett. 94, 081601 (2005) [arXiv:hep-ph/0404271].