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How do crude oil futures hedge crude oil spot risk after the COVID-19 outbreak? A wavelet denoising-GARCH-SJC Copula hedge ratio estimation method

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**A B S T R A C T**

In the current paper, we investigate the problem of how do crude oil futures hedge crude oil spot risk after the COVID-19 outbreak. Specifically, given that noise, conditional higher moments and asymmetric tail dependence may exist in crude oil markets, a Wavelet denoising-GARCH-SJC Copula hedge ratio estimation method is proposed to construct hedging portfolios in crude oil markets during the epidemic period. Based on the in-sample and out-of-sample results, the hedging roles of Brent futures and Shanghai crude oil (SC) futures for light and medium crude spots after the COVID-19 outbreak are further researched. The empirical results demonstrate that noise, conditional higher moments and asymmetric tail dependence do exist in crude futures and spots, which have impact on the precision of modeling results. Secondly, the Wavelet denoising-GARCH-SJC Copula hedge ratio estimation method outperforms all control groups, obtaining the best in-sample and out-of-sample hedging effectiveness. Finally, it is reported in the in-sample and out-of-sample hedging results that Brent is the optimal futures to hedge light oil, while SC is the optimal futures to hedge medium oil. The paper provides substantial recommendations for policymakers and investors.

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1. Introduction

As the most essential energy in the world, crude oil is important for national economy and social stability \cite{1,2}. However, the internal & external influence, including oil supply and demand, global economic fundamentals, financial crisis, major public health affairs, geopolitical conflicts and so on, may cause extreme swings in the crude markets \cite{3–6}. Since the outbreak of the novel coronavirus disease (COVID-19), global crude prices have experienced unprecedented extreme fluctuations with significant increases or drops \cite{7–9}. In despite of the increase of global coronavirus vaccine rates and improvement of epidemic prevention measures, the novel coronavirus has been mutating. From the end of 2020, mutant COVID-19 strains, such as Alpha variant, Delta variant and Omicron variant, take turns occurring. During the first five months of 2022, the cumulative confirmed cases exceed 200 million, even attaining the approximate sum of those in 2020 and 2021.\footnote{According to World Health Organization (WHO): https://www.who.int/newsroom/detail/27-04-2020-who-timeline--covid-19.}

Therefore, although the pandemic has continued for more than two years, the huge negative effect of COVID-19 on global economy and crude markets is likely to intensify further. Fig. 1 shows the daily changes of global confirmed cases due to COVID-19.
Hedging is a position taken to reduce the negative effects from changes in the prices of a companion investment [5]. An effective hedging strategy of futures and spot helps investors to reduce adverse risk [10–16]. Therefore, an important and urgent question is raised: How do crude oil futures hedge crude oil spot risk after the COVID-19 outbreak? To resolve the problem, we develop a novel hedge ratio estimation method to construct the effective crude hedging strategy after the outbreak. Based on the in-sample and out-of-sample hedging results, we further investigate the roles of Brent crude futures and SC crude futures as hedges for light and medium crude spots during the epidemic period. The objective of current paper is to hedge different crude spots with matched crude futures through the novel method to obtain the optimal hedging performance after the epidemic outbreak.

The most crucial issue in hedging strategy is how to estimate the optimal hedge ratio, which is the hot focus in academia. Johnson [11] originally proposes the optimal hedge ratio framework. Then, a large number of studies introduce various methods to estimate hedge ratio for constructing effective hedging strategy [11–18]. In early studies, Ordinary Least Squares (OLS) is employed to measure the hedge ratio, such as Lien [12], Huang et al. [13] and Wu [14]. However, residual heteroscedasticity effects and constant hedge ratio may cause poor hedging performance [15,16].

To overcome the shortcomings of OLS, multivariate generalized autoregressive conditional heteroskedastic (GARCH) family models are used to estimate dynamic hedging ratio. For example, Billio et al. [5] develop a Bayesian multi-chain Markov-switching GARCH model for energy spot and futures markets. The results denote that the model is more suitable in extreme periods. Cui & Feng [6] employ multivariate GARCH models to measure the composite hedge ratio with minimum variance and expected utility maximization. The results provide the evidence that the composite ratio outperforms single ratio in most cases. Jie et al. [10] employ a Generalized Orthogonal GARCH (GO-GARCH) to measure the dependence between crude oil futures and spots. The results denote that futures are an effective instrument to hedge spot risk. Lee [17] constructs a Markov regime-switching Cholesky GARCH to estimate hedge ratio. The results demonstrate its superior performance. Lai [18] uses price range information to construct the hedge strategy through DCC model. He finds that price range data helps investors to construct more effective hedging portfolios. However, multivariable GARCH models are limited to measuring the linear dependence between assets [1].

Copula class is the well-known method for describing the nonlinear relationship between variables. Given that dependence across markets may be nonlinear, Copula is widely used in hedging field. Gong et al. [19] develop a GJR-GARCH-Skew t-GAS copula model to hedge spot price risk by crude oil futures. They discover that the method obtains higher hedging efficiency. Nguyen et al. [20] investigate whether gold can be a hedge against the adverse risk value of exchanges during the sub-prime crisis through Copula models. The results show that gold is a robust hedge against the risk of the currencies. Talbi et al. [21] propose a multivariate vine copula-based GARCH model to assess the hedge properties of precious metals. They observe that gold is an effective hedge for G-7 stocks, while silver and platinum are the weak hedge for G-7 stocks. Dai & Zhu [22] employ a DCC-GARCH t-Copula model to hedge stock markets through crude oil and natural gas futures. The empirical results show that crude and gas futures are cheap hedges. Tiwari et al. [23] use a time-varying Markov-switching Copula to hedge Australia’s sectoral stocks with energy futures. They report that gas is a good hedge for all industries except real estate.

With the outbreak and spread of COVID-19, the epidemic has been the focus in hedging researches. For instance, Akhtaruzzaman et al. [24] examine whether gold is a hedge after the COVID-19 outbreak through DCC-GARCH. The results provide the evidence that gold is a hedge during Phase I but not during Phase II. Chemhka et al. [25] employ A-DDC model to investigate the hedging effective of Bitcoin and gold during the COVID-19 period. They find that Bitcoin and gold are good hedge tools against the international portfolio risk. Chkili et al. [26] adopt DCC-FIGARCH to research the role of Bitcoin as a hedge for Islamic stocks after the epidemic outbreak. The empirical results report that although Bitcoin plays a hedge for stocks, however, the hedging strategy involving Bitcoin has higher cost. Choi & Shin [27] use a Vector Autoregression (VAR) model to investigate the hedging effectiveness of Bitcoin after the epidemic happened. They
observe that Bitcoin possesses the inflation-hedging property. Grobys [28], Jiang et al. [29] and Salisu & Shaik [30] do similar research.

In summary, most of existing hedging researches use a variety of hedge ratio estimation methods to examine the hedging effectiveness of different assets for constructing optimal hedging strategy. However, there are still research gaps and limits:

On the one hand, some factors in the crude oil markets may have huge negative impact on the hedging performance but be ignored by previous researches, which are described as follows: (1) Noise has a huge influence on modeling, leading to wrong empirical results [31]. Therefore, a denoising algorithm should be used to filter data. However, most of hedging literature about the crude futures and spots neglects the existence of noise. Although Zheng et al. [32] try to employ empirical mode decomposition (EMD) to construct denoising hedging strategies. However, EMD method has a mode mixing problem, which may affect denoising performance [33]. (2) Volatility modeling is an important prerequisite to ensure successful hedge ratio estimation. GARCH family is the most common method to describe volatility in the hedging modeling, which is limited to describing conditional variance [17–23]. However, conditional higher moments are frequently reported in financial assets [34,35]. GARCH class models are impossible to precisely structure volatility when conditional higher moments (skewness and kurtosis) exist in assets. The existing crude hedging literature neglects the existence of conditional higher moments in assets. (3) Dependence structure is essential to estimating hedge ratio. Previous studies have provided the evidence of asymmetric tail dependence structure between spot and futures [17,36]. However, after the COVID-19 outbreak, up to our knowledge, previous hedging researches ignore asymmetric tail nexus in the crude futures-spot hedging strategy.

On the other hand, the hedging effectiveness of different futures during the COVID-19 period has received enough attention in academia, however, there are still some research gaps and shortcomings: (1) The existing studies have examined the roles of many assets such as Bitcoin, crude and gold futures as hedge tools for spots through constructing in-sample hedging portfolios after the epidemic outbreak [25–29]. However, as Lai [18] and Bali et al. [37] say, how to evaluate the investment performance depends on the out-of-sample tests. For policymakers and investors, out-of-sample investment results have more theoretical and practical implications [1]. Previous in-sample hedging researches are not sufficient to demonstrate the precision and robustness of hedging effectiveness after the pandemic outbreak. Therefore, the out-of-sample hedging effectiveness during the epidemic period should be investigated but be ignored by previous literature. (2) Traditional international crude futures, such as Brent and WTI, use light sour crude as subject matter, however, many crude oil spot such as Oman and Dubai are medium sour crude. Brent futures or WTI futures are difficult to perfectly match medium spots, which may cause poor hedging performance [38]. With the global third largest trading volume, Shanghai crude (SC) futures adopt medium crude oil rather than light crude oil as subject matter. A variety of hedging researches, such as Jie et al. [10], focus on the SC. However, as far as we know, the role of SC as a hedge for light spots and medium spots after the COVID-19 outbreak is still unclear.

Therefore, for filling the previous gaps, a Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method is proposed to construct in-sample and out-of-sample hedging portfolios of crude futures and spots after the COVID-19 outbreak. Based on in-sample and out-of-sample investment results, we further investigate the hedging effectiveness of Brent futures and SC futures. We obtain the following finding. First of all, the current results provide the evidence that noise, conditional higher moments and asymmetric tail dependences exist in crude futures and spots. Besides, the Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method is superior to all control groups in both in-sample and out-of-sample hedging tests. Finally, Brent and SC are the optimal futures to hedge light crude spots and medium crude spots, respectively.

Distinguished from existing literature, the current paper makes notable innovations and contributions as follows: First of all, given the significant impact of noise, conditional higher moments and asymmetric tail dependence on hedging performance, a Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method is developed. After filtering data, it not only describes conditional lower moments and conditional higher moments, but also structures the asymmetric tail dependence between crude spot and futures. Specifically, the novel method is constructed by four phases: (1) Wavelet method overcomes the problem of excessive overlap between noise and useful signal. Therefore, we use the wavelet approach to reserve valuable information and reduce noise in phase I. (2) General Autoregressive Conditional Volatility Skewness Kurtosis (GARCHSK) is able to describe conditional lower and higher moments [36,37]. Therefore, based on denoising data, we employ the GARCHSK to estimate marginal distributions for getting more volatility information in phase II. (3) Based on pairwise marginal distributions, SJC Copula is used to structure the futures-spot nonlinear and asymmetric tail dependence in phase III. (4) Based on the integrated marginal information and joint information, the dynamic optimum hedge ratio is estimated to construct hedging portfolio in phase IV.

Secondly, on the one hand, existing hedging literature only examines the in-sample hedge effectiveness after the epidemic happened [25–30]. However, how to evaluate the investment performance depends on the out-of-sample tests [18]. The hedging performance should be further examined in out-of-sample tests. On the other hand, it is necessary for many medium sour crude spots such as Dubai oil and Oman oil to be hedged by crude futures during the COVID-19 period. However, the traditional crude futures mainly adopt light sour crude spots as subject matters. Unlike light crude
oil futures, SC, as the third largest trading volume crude futures, adopts medium sour spots as the subject matter, which may be a better hedge tool against medium crude spot adverse risk. Therefore, we examine and compare the hedging roles of Brent and SC futures for light crude spots and medium crude spots during the COVID-19 period through in-sample and out-of-sample hedging tests.

The rest of the current paper is organized as follows: Section 2 introduces methodologies. The empirical results and discussion are analyzed in Section 3. Section 4 analyzes the robustness tests. Section 5 concludes the current work.

2. Methodology

2.1. Wavelet denoising method

The early researches such as Goldstein et al. [39], Boudraa et al. [40] and Louka et al. [41] use EMD, Wiener filtering and Kalman filter methods to reduce noise, however, owing to excessive overlap of noise and valuable information, they fail to reduce noise effectively.

Wavelet method is able to denote local signal characteristics, obtaining potential signal features without obeying to special distribution [1]. The wavelet denoising approach decomposes financial series into multiple time scales, overcoming the question of excessive overlap of noise and valuable information [42].

Given a time series with noise, it is expressed by \( x(t) = s(t) + e(t), t = 1, 2, \ldots, N \), where \( s(t) \) and \( e(t) \) show real signal and noise, respectively [43]. The specific denoising work is described as follows:

**Step 1. Decomposition.** Maximal Overlap Discrete Wavelet Transform (MODWT), a modification of Discrete Wavelet Transform (DWT), has translation-invariant and non-dyadic sample length properties, which has been widely used in various fields [44,45]. Therefore, MODWT is employed in the current paper.

Based on MODWT, \( x(t) \) is decomposed to wavelet coefficients \( W_{i,j}(i = 1, 2, \ldots, N, j = 1, 2, \ldots, J) \) at \( J \) scales by following formula:

\[
x = \tilde{w} \tilde{W} = \sum_{j=1}^{J} \tilde{w}_j \tilde{W}_j + \tilde{v}_j \tilde{V}_j
\]

where \( \tilde{W} \) is MODWT coefficient matrix and \( \tilde{w} \) is MODWT matrix, which can be detailed in Zheng et al. [45].

**Step 2. Noise reduction.** Large values of \( W_{i,j} \) are considered as real signal and small values of \( W_{i,j} \) are considered as noise [46]. Given the good continuity and estimation accuracy of soft threshold, it is used to distinguish \( \tilde{W}_{i,j} \) into real signal and noise. The soft threshold method is

\[
\overline{W}_{j,t} = \begin{cases} 
\text{sign}(W_{j,t})(|W_{j,t}| - \lambda) & |W_{j,t}| \geq \lambda \\
0 & |W_{j,t}| < \lambda 
\end{cases}
\]

where original and denoising wavelet coefficients are represented by \( W_{j,t} \) and \( \overline{W}_{j,t} \), respectively. In Eq. (2), \( \lambda \) is the threshold to determine whether \( W_{j,t} \) is noise. If absolute values of \( W_{j,t} \) are larger than \( \lambda \), \( W_{j,t} \) can be retained as \( W_{j,t} \) by \( \text{sign}(W_{j,t})(|W_{j,t}| - \lambda) \), otherwise, \( W_{j,t} = 0 \). In addition, \( \lambda \) is calculated by rigrsure method, which is detailed in Sadooghi & Khadem [47].

**Step 3. Reconstruction.** \( \overline{W}_{j,t} \) is used to reconstruct denoising series \( \{x_t\} \) through Eq. (1).

2.2. GARCHSK

\[
\begin{align*}
\alpha_t & = u_t + \varepsilon_t = \alpha_1 \alpha_{t-1} + \varepsilon_t \\
\varepsilon_t & = h_t^{1/2} \eta_t, \quad \varepsilon_t | h_{t-1} \sim D(0, h_t, s_t, k_t) \\
h_t & = \beta_0 + \sum_{i=1}^{q_1} \beta_{1,i} \eta_{t-i}^2 + \sum_{j=1}^{p_1} \beta_{2,j} h_{t-j} \\
s_t & = \gamma_0 + \sum_{i=1}^{q_2} \gamma_{1,i} \eta_{t-i}^3 + \sum_{j=1}^{p_2} \gamma_{2,j} s_{t-j} \\
k_t & = \delta_0 + \sum_{i=1}^{q_3} \delta_{1,i} \eta_{t-i}^4 + \sum_{j=1}^{p_3} \delta_{2,j} k_{t-j}
\end{align*}
\]

where \( \varepsilon_t | h_{t-1} \) is an information integration at \( t - 1 \). \( D(0, h_t, s_t, k_t) \) contains conditional variance, conditional skewness and conditional kurtosis. In the meanwhile, \( q_1 = q_2 = q_3 = p_1 = p_2 = p_3 = 1 \). In addition, \( \varepsilon_t \) subjects to Gram–Charlier expansion (GCE) distribution:

\[
f (\eta_t | h_{t-1}) = \frac{\phi (\eta_t) \psi^2 (\eta_t)}{I_t}
\]
In Eq. (4), $\psi (\eta) = 1 + \frac{\alpha}{3!} (\eta_1^3 - 3\eta_1) + \frac{\beta}{4!} (\eta_1^4 - 6\eta_2^2 + 3)$, $f_1 = 1 + \frac{\alpha^2}{3!} + \frac{(\alpha-3\beta)^2}{4!}$, where $\phi (\bullet)$ means the density function of standard normal distribution. The logarithm likelihood function without useless constant is defined by Eq. (5).

$$l_t = -\frac{1}{2} \ln h_t - \frac{1}{2} \eta_t^2 + \ln (\psi^2 (\eta_t)) - \ln (f_1)$$  \hspace{1cm} (5)

2.3. SJC Copula

Given variables $x_1, x_2, \ldots, x_n$ with marginal distributions $F_1(x_1), F_2(x_2), \ldots, F_n(x_n)$ and a joint distribution $F(\bullet, \bullet)$, a Copula function $C$ is described as $F(x_1, x_2) = C(F_1(x_1), F_2(x_2))$. The joint density is

$$f(x_1, x_2) = [F_1(x_1)]^{-1} [F(x_1, x_2)]^{-1} f_1(x_1) f_2(x_2)$$  \hspace{1cm} (6)

Elliptical Copula and Archimedean Copula are unable to measure asymmetric tails [3,19,20,36]. However, the asymmetric tail dependence between assets is reported [48]. Unlike above Copula, SJC Copula can structure both asymmetric and symmetric tail dependences. JC Copula is basic to SJC Copula, which can be detailed in

$$C_{JC}(u, v; r^U, r^L) = 1 - \left( 1 - \left[ \frac{1 - (1 - u)^k}{(1 - v)^k} \right]^{-\gamma} + \left[ \frac{1 - (1 - v)^k}{(1 - u)^k} \right]^{-\gamma} - 1 \right)^{-1/\gamma^{1/k}}$$  \hspace{1cm} (7)

where $k = 1 / \log_2(2 - r^U)$, $\gamma = -1 / \log_2(r^L)$, $r^U \in (0, 1)$, $r^L \in (0, 1)$. In addition, $u$ and $v$ are marginal distributions. SJC Copula is described as follows:

$$C_{SJC}(u, v; r^U, r^L) = 0.5 \cdot C_{JC}(u, v; r^U, r^L) + C_{JC}(1 - u, 1 - v; r^U, r^L) + u + v - 1$$  \hspace{1cm} (8)

2.4. Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method

The Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method not only reduces noise and retains real signal, but also obtains more valuable volatility information. It further describes nonlinear & asymmetric tail dependence to achieve a more precise futures-spot nexus. Based on integrated marginal information and joint information, the new approach is able to estimate the optimal hedge ratio. Assume that there are crude oil futures prices $\{P_{f,t}\}$ and crude oil spot prices $\{P_{s,t}\}$, the detailed modeling algorithm of Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method is described as follows:

**Phase 1. Noise reduction.** Given the outstanding performance of wavelet approach, we employ the denoising method based on MODWT to filter crude prices. Then we do the logarithmic and differential treatment to denoising prices, further, the denoising returns $\{r_{f,t}\}$ and $\{r_{s,t}\}$ are obtained.

The choice of wavelet function is essential to ensuring the excellent denoising performance. Therefore, we make a choice among the excellent functions for best denoising performance. Table 1 denotes some common wavelet functions properties. In addition, referring to Li [49], we decompose spot and futures price series into 4 time scales. The specific noise reduction process is detailed in Section 2.1.

**Phase 2. Volatility estimation.** Based on denoising returns, the GARCHSK is used to estimate volatility characteristics and obtain standardized residuals $\eta_t$. Standardized residuals estimated by GARCHSK contain more valuable volatility information than those estimated by GARCH class. Then, the empirical cumulative distribution function (ECDF) is applied to $\eta_t$ for estimating marginal distributions. The specific GARCHSK modeling is detailed in Section 2.2.

**Phase 3. Dependence estimation.** Dependence structure between spot and futures is important and crucial phase for hedging portfolio. Given the nonlinear and asymmetric tail dependence structure, we use SJC Copula to describe futures-spot nexus. The medial correlation coefficient denotes not only nonlinear correlation between variables with tail information but also the dependences below and above the medial level [50,51]. Therefore, based on the parameters estimated by SJC Copula, we measure medial correlation coefficient through Eq. (9).

$$\rho_{medial} = 4C_{SJC Copula}(50\%, 50\%; r^U, r^L) - 1$$  \hspace{1cm} (9)
Phase 4. Hedge ratio estimation. Assume that the investor holds a long spot position and adopts a short futures position to hedge, the hedging portfolio return $r_{h,t}$ at time $t$ is

$$r_{h,t} = r_{s,t} - h_t r_{f,t} \tag{10}$$

where $h_t$ denotes dynamic hedge ratio. In addition, $r_{f,t}$ and $r_{s,t}$ mean the returns of futures and spot, respectively.

Referring to Pan et al. [16], Baillie & Myers [52] and Jeribi & Fakhfekh [53], minimum variance dynamic optimal hedge ratio at time $t$ based on Multivariate GARCH or Copula-GARCH is

$$h_t = \frac{\text{Cov}(r_f, r_s)}{\sigma^2_f} = \rho \frac{\sigma_{s,t}}{\sigma_{f,t}} \tag{11}$$

Eq. (11) is the most common and classic formula to estimate hedge ratio $h_t$ for constructing hedging strategy. In the formula, $\sigma_{s,t}$ and $\sigma_{f,t}$ denote dynamic standard deviation at time $t$. $\text{Cov}(r_f, r_s)$ and $\rho$ is covariance and correlation coefficient. Worthy of note is that whether $\rho_t$ is dynamic or constant, the $h_t$ based on Multivariate GARCH or Copula-GARCH is dynamic.

The modeling process of Wavelet denoising-GARCHSSK-SJC Copula hedge ratio estimation method is visualized in Fig. 2.

3. Empirical results and discussion

3.1. Data

Brent, the famous international futures, is adopted as the crude benchmark in most of global petroleum trading, which uses light sour crude as subject matter [54]. In contrast, SC, the global third largest trading volume crude futures, adopts medium sour crude as subject matter. In the current paper, we employ Brent and SC as crude futures.

Most of hedging researches use Daqing crude oil spot to investigate the role of SC as a hedge. However, Daqing spot is not actually traded in the market [10]. Most of Chinese imported oil is bought from Middle East, and Middle East crude spots such as Oman and Dubai belong to medium sour crude. More importantly, China also authenticates Oman and Dubai as the subject matters for SC. Besides, Brent spot is the underlying asset of Brent futures. Therefore, we employ Oman and Brent as crude spots.

The prices of crude futures and crude spots are collected from the WIND database (https://www.wind.com.cn/). Some cases of novel pneumonia were reported by WHO on December 31, 2019, which has been further identified as COVID-19. Therefore, the COVID-19 period ranges from December 31, 2019 to May 9, 2022, totaling 520 daily transaction closing prices, excluding non-trading days. Furthermore, all prices are dollar-denominated. Besides, MATLAB 2016a is used as the econometric software in this paper.

3.2. Denoising analysis

As Chen et al. [55] say, a high-performing wavelet function should have the properties of orthogonality, compact support, symmetry and certain vanishing moments. For the best denoising results and given the special vanishing moments of coifN, we adopt the wavelet denoising method with haar, sym8 and coif4 wavelet function to filter prices, respectively.

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4 Shanghai futures Exchange: http://www.shfe.com.cn/.
5 In robustness analysis section, we also employ Dubai and WTI spots to exam robustness of the study.
6 Haar wavelet has constant vanishing moments.
Table 2
Denoising performance.

| SNR       | Brent futures | SC futures | Brent spot | Oman spot | Brent futures | SC futures | Brent spot | Oman spot |
|-----------|---------------|------------|------------|-----------|---------------|------------|------------|-----------|
| Haar      | 1.38          | 1.14       | 1.41       | 1.53      | 9.77E−4       | 7.16E−4    | 2.41E−3    | 1.2E−3    |
| Sym8      | 1.81          | 1.28       | 2.45*      | 1.89      | 8.84E−4       | 6.93E−4    | 1.90E−3*   | 1.13E−3   |
| Coif4     | 2.01*         | 1.30*      | 2.42       | 2.03*     | 8.45E−4*      | 6.89E−4*   | 1.91E−3    | 1.09E−3*  |

Note: * means that it outperforms the others at a variable in the aspect.

Table 3
Summary statistics.

|                | Mean  | Std.dev | Skewness | Kurtosis | JB      | ADF     | PP      | Q(1)   | ARCH(1) |
|----------------|-------|---------|----------|----------|---------|---------|---------|--------|---------|
| Brent futures  | 4.48E−4 | 0.01    | −3.18    | 30.59    | 17.332.56*** | −4.95*** | −17.31*** | 121.90*** | 13.08*** |
| SC futures     | 3.55E−4 | 0.01    | −1.18    | 7.69     | 59.721.4*** | −3.67**  | −9.11***  | 283.00*** | 199.19*** |
| Brent spot     | 4.29E−4 | 0.02    | −1.62    | 27.82    | 135.469.1*** | −4.05*** | −20.11*** | 83.18***  | 110.13*** |
| Oman spot      | 4.17E−4 | 0.02    | −1.76    | 27.25    | 129.811.9*** | −4.10*** | −17.88*** | 139.15*** | 17.60*** |

Fig. 3. The time evolutions of denoising and original prices.

After reducing noise work, we obtain the denoising returns through ln(P_t/P_{t−1}), where P_t is the denoising price at day t. Then, we use signal to noise ratio (SNR) and mean square error (MSE) to examine the denoising performance. SNR = 10 \times \log_{10} \left\{ \sum_{n=1}^{N} f^2(n) / \sum_{n=1}^{N} [f(n) - \hat{f}(n)]^2 \right\} and MSE = \frac{1}{N} \sum_{n=1}^{N} [f(n) - \hat{f}(n)]^2, where f(n) and \hat{f}(n) are original and denoising series, respectively. For SNR, the larger value, the better performance, while MSE is the opposite.

We discover in Table 2 that in most cases, coif4 is superior to the other wavelet functions in terms of SNR and MSE. Therefore, we adopt wavelet denoising approach with coif4 to filter data.

The original and denoising crude prices estimated by wavelet denoising method with coif4 are detailed in Fig. 3. It is reported that all denoising prices are obviously smoother than original prices, suggesting that noise in the form of small fluctuations has been significantly reduced. More importantly, the whole tendency of denoising prices are similar to original prices, meaning that the valuable information has been effectively retained. The existence of noise in crude oil markets provides the evidence that the previous researches without denoising approach may obtain error conclusions.

Then, according to the denoising prices estimated by wavelet denoising method with coif4, the denoising returns are obtained by ln(P_t/P_{t−1}), where P_t is the denoising price at day t. The time evolutions of denoising and original returns are detailed in Table 3. The time evolutions of denoising and original returns are detailed in Fig. 4.

Table 3 shows the obvious sumbit and fat tailed skewed features of returns. We observe in Table 3 that all crude oil futures and spots reject null hypothesis of Jarque–Bera test, following non-normal distribution. The results of ADF and PP denote that all returns are stationary without unit root. Furthermore, it is reported in Table 3 that all crude are significantly auto-correlated and subject to ARCH effects.

3.3. GARCHSK and SJC Copula modeling

The GARCHSK is adopted to model the margin for denoising returns. For testing the validity of Wavelet denoising-GARCHSK, the GARCH and GARCHSK based on original returns are employed as control groups. We also employ GARCH

7 In fact, the next modeling works are based on denoising returns, therefore, we show the performance of denoising returns. The conclusion of denoising prices are in line with the current paper.
based on denoising returns as control groups. The modeling results are evaluated by LL, AIC and BIC. Given that the main conclusions about modeling results are consistent and that space is limited, we only show the performance of crude futures.

Table 4 provides the evidence that the volatility methods based on denoising returns outperform those based on original returns in terms of LL, AIC and BIC, which demonstrates that it is necessary to filter data before volatility modeling. In addition, we observe that Wavelet denoising-GARCHSK has the best modeling performance with the largest LL and the smallest AIC & BIC, implying that Wavelet denoising-GARCHSK is the optimum method. The potential reason lies in the fact that after filtering data, the noise in data is efficiently reduced and real information is maximally obtained. Then, GARCHSK structures both lower and higher moments. Therefore, more valuable volatility information and the best modeling performance can be obtained by Wavelet denoising-GARCHSK method.

It is also reported in Table 4 that most of higher moment parameters are significant, verifying that time-varying skewness and time-varying kurtosis exist in the crude market. Fig. 5 shows the time-varying variance, time-varying skewness and time-varying kurtosis of futures. We discover in Fig. 5 that skewness and kurtosis are dynamic, supporting the existence of conditional higher moments. Then, we use ECDF to estimate marginal distributions.

Based on pairwise marginal distributions, SJC Copula is employed to describe the dependence structure across spot and futures, which can be detailed in Table 5. Table 5 demonstrates that there are significantly asymmetric tail futures-spot dependence. Therefore, it is necessary to consider both $\tau^U$ and $\tau^L$ into hedge ratio estimation. The medial correlation coefficient $\rho_{\text{medial}}$ estimated by the Wavelet denoising-GARCHSK-SJC Copula not only captures more valuable volatility information, but also describes nonlinear and asymmetric tail correlations. Therefore, we use the $\rho_{\text{medial}}$ estimated by the Wavelet denoising-GARCHSK-SJC Copula to estimate the crude hedge ratio. In addition, it is reported in Table 5 that the Brent–Brent $\rho_{\text{medial}}$ is larger than Brent–Oman $\rho_{\text{medial}}$, while SC–Oman $\rho_{\text{medial}}$ is larger than SC–Brent $\rho_{\text{medial}}$. The diverse futures-spot relationships mean that Brent futures and SC futures may have different performance as hedges for light and medium crude spots.

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**Table 4**

| Brent futures | Denoising returns | SC futures | Denoising returns |
|---------------|-------------------|------------|-------------------|
| **Original returns** | **GARCH** | **GARCHSK** | **GARCH** | **GARCHSK** | **Original returns** | **GARCH** | **GARCHSK** | **GARCH** | **GARCHSK** |
| $\alpha_1$ | $-0.05^{**}$ | $-0.06^{***}$ | $0.98^{***}$ | $0.89^{***}$ | $0.00^{*}$ | $-0.08^{***}$ | $0.98^{***}$ | $0.84^{***}$ |
| $\beta_0$ | $0.00^{***}$ | $0.00^{***}$ | $0.00^{***}$ | $0.00^{***}$ | $0.00^{***}$ | $0.00^{***}$ | $0.00^{***}$ | $0.00^{***}$ |
| $\beta_1$ | $0.63^{***}$ | $0.67^{***}$ | $0.60^{***}$ | $0.41^{***}$ | $0.79^{**}$ | $0.05^{***}$ | $0.59^{***}$ | $0.21^{***}$ |
| $\beta_2$ | $0.36^{**}$ | $0.01^{*}$ | $0.39^{**}$ | $0.45^{**}$ | $0.15^{**}$ | $0.87^{**}$ | $0.40^{**}$ | $0.04^{**}$ |
| $\gamma_0$ | $-0.33^{*}$ | $-0.50^{***}$ | $-0.33^{***}$ | $-1.00^{***}$ |
| $\gamma_1$ | $0.03^{***}$ | $-0.58^{***}$ | $0.04^{***}$ | $0.01^{***}$ |
| $\gamma_2$ | $-0.86^{***}$ | $-0.34^{***}$ | $0.22^{**}$ | $-0.99^{***}$ |
| $\delta_0$ | $3.18^{***}$ | $4.44^{***}$ | $2.77^{***}$ | $0.04^{***}$ |
| $\delta_1$ | $0.04^{***}$ | $0.15^{***}$ | $0.01^{***}$ | $0.00^{***}$ |
| $\delta_2$ | $0.10^{***}$ | $0.10^{***}$ | $0.00^{***}$ | $0.99^{***}$ |
| **LL** | 2630.02$^#$ | 1112.50 | 2468.95$^#$ |
| **AIC** | $-2242.12$ | $-3104.80$ | $-3103.78$ | $-4917.89$ |
| **BIC** | $-2252.11$ | $-3062.28$ | $-4681.95$ | $-4875.37$ |

**Note:** $^*$, $^{**}$, $^{***}$ demonstrate 10%, 5%, 1% significance levels, respectively. $^#$ denotes that it outperforms the other models in one aspect.

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8 The modeling results of spots can be made available under request addressed to the authors.
Table 5  
SJC Copula modeling.

|          | $\tau^U$ | $\tau^L$ | $k$  | $\gamma$ | $\rho_{modul}$ | LL   |
|----------|----------|----------|------|----------|----------------|------|
| Brent futures | 0.689    | 0.775    | 2.563| 2.719    | 0.617          | 383.42|
| Oman spot  | 0.323    | 0.241    | 1.341| 0.487    | 0.282          | 58.137|
| SC futures | 0.000    | 0.435    | 1.000| 0.8333   | 0.293          | 56.950|
| Oman spot  | 0.473    | 0.461    | 1.638| 0.895    | 0.410          | 138.366|

Fig. 5. Dynamic second moment and dynamic higher moments.

Table 6  
In-sample hedging effectiveness.

|          | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Brent-Brent | 3.49E−1 | 3.50E−1 | 3.42E−1 | 3.48E−1 | 3.37E−1 | 2.99E−1 | 2.98E−1 | 2.99E−1 | 2.99E−1 | 2.98E−1 | 3.63E−1 |
| Brent-Oman | 7.69E−2 | 7.42E−2 | 6.29E−2 | 7.09E−2 | 7.28E−2 | 7.71E−2 | 7.72E−2 | 6.65E−2 | 7.00E−2 | 7.32E−2 | 1.03E−1 |
| SC-Brent   | 1.96E−2 | 1.98E−2 | 1.96E−2 | 1.99E−2 | 1.97E−2 | 2.65E−2 | 2.64E−2 | 2.52E−2 | 2.53E−2 | 2.75E−2 | 1.08E−2 |
| SC-Oman   | 3.67E−2 | 1.95E−2 | 6.11E−2 | 3.80E−2 | 6.37E−2 | 6.45E−2 | 3.93E−2 | 9.10E−2 | 6.33E−2 | 9.34E−2 | 1.65E−1 |

Note: 1~5 denote GARCH-Normal Copula, GARCH-Student t Copula, GARCH-Gumbel Copula, GARCH-Clayton Copula and GARCH-SJC Copula, respectively. 6~10 denote GARCHSK-Normal Copula, GARCHSK-Student t Copula, GARCHSK-Gumbel Copula, GARCHSK-Clayton Copula and GARCHSK-SJC Copula, respectively. 11 denotes Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method. * means a hedge portfolio outperforms the others in terms of hedging effectiveness. Hereinafter the same.

3.4. In-sample hedging performance analysis

Based on the integrated information measured by Wavelet denoising-GARCHSK-SJC Copula, we estimate the optimal hedge ratio to construct in-sample futures-spot hedging portfolios in the crude oil markets. In this section, we begin to research the hedging performance of the novel method and traditional methods. Then, we investigate the differences between Brent futures and SC futures in terms of hedging performance.

We employ GARCH-Normal Copula, GARCH-Student t Copula, GARCH-Gumbel Copula, GARCH-Clayton Copula, GARCHSK-SJC Copula, GARCHSK-Normal Copula, GARCHSK-Student t Copula, GARCHSK-Gumbel Copula, GARCHSK-Clayton Copula and GARCHSK-SJC Copula as control groups. We call all control groups as the traditional methods. Furthermore, according to Pan et al. [16] and Chkilii et al. [26], we use the hedging effectiveness (HE) proposed by Ederington [56] to evaluate the hedging performance, which is described as follows:

$$ HE = \frac{\text{var}_{\text{unhedged}} - \text{var}_{\text{hedged}}}{\text{var}_{\text{unhedged}}} $$

(12)

In Eq. (12), $\text{var}_{\text{unhedged}}$ denotes variance of spot returns and $\text{var}_{\text{hedged}}$ denotes variance of hedging portfolio returns. The larger value of HE, the better hedging performance. Table 6 shows the futures-spot hedging effectiveness after the COVID-19 outbreak.

Table 6 provides the evidence that in most cases, the Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method is superior to all control groups, obtaining the best hedging performance. Compared with traditional hedging methods, the novel approach is able to obtain a variety of marginal and joint valuable information through four-stage
modeling works (from noise reduction to volatility estimation to dependence estimation and finally to hedge ratio estimation). Table 6 demonstrates that the Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method provides the best in-sample hedging effect of futures on spots.

Then, we use the hedging results estimated by Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method in Table 6 to investigate the hedging roles of Brent futures and SC futures. Interestingly, Brent futures are better hedging tool against Brent spot risk than SC futures. However, for Oman spot, the result is the opposite. An effective hedge portfolio needs a high matching of futures and spot [38]. Brent futures use Brent spot as the underlying asset, while Oman spot is medium sour crude, which can be considered as the underlying asset of SC. Therefore, Brent futures have an obviously closer connection with Brent spot than Oman spot, while SC futures are the opposite. The in-sample results demonstrate that we can better hedge crude spots by matched futures. SC rather than Brent should be used to hedge medium spots, while Brent rather than SC should be adopted to hedge light spots.

We also show the dynamic hedge ratio estimated by the novel method in Fig. 6. It is reported in Fig. 6 that before mid-June, 2020 and after early February, 2022, all futures-spot hedgeratios have much volatility phenomena. In fact, the former period corresponds to the COVID-19 initial outbreak, and the latter period corresponds to the COVID-19 mutation outbreak. Within the two periods, the epidemic severely hit global economy, causing extreme changes in crude oil markets, so the Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method frequently change hedgeratio to improve hedging effectiveness.

3.5. Out-of-sample hedging performance analysis

How to evaluate the investment performance depends on the out-of-sample results [18,37]. However, the existing hedging literature ignores the hedging effectiveness outside the sample during the epidemic period. Therefore, we use the novel method and control groups to construct out-of-sample hedge portfolios. Based on the results, we firstly research the hedging performance of the novel method and traditional methods. Then, the differences between Brent futures and SC futures in terms of out-of-sample hedging performance are investigated.

Specifically, the out-of-sample predict work is constructed by a rolling window approach. A rolling window with n observations is adopted to predict N-n one-period-ahead hedge ratio, where N is the length of total sample and n is the length of estimation sample [1,57]. In the current paper, the estimation sample ranges from December 31, 2019 to February 14, 2022, totaling 468 observations, and forecasting sample ranges from February 15, 2022 to May 9, 2022, totaling 52 observations, where the former and the latter make up 90% and 10% of the total sample, respectively.9 The three samples can be detailed in Table 7. The out-of-sample hedging effectiveness after the outbreak can be detailed in Table 8.

It is reported in Table 8 that in most cases, the Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method outperforms all control groups in terms of hedging effectiveness. We discover that the novel method has level-headed and excellent hedging performance. Unlike control groups, after reducing noise and retaining valuable information, the novel method can precisely describe volatility and futures-spot dependence to improve the hedging performance. The results in Tables 6 and 8 testify that an excellent method is essential to the success of hedge portfolio in both in-sample and out-of-sample tests.

9 We try to do a window length robustness. The estimation sample ranges from December 31, 2019 to March 23, 2022, totaling 494 observations, and forecasting sample ranges from March 24, 2022 to May 9, 2022, totaling 26 observations, where the former and the latter make up 95% and 5% of the total sample, respectively. The hedging results are in line with the current paper. The robustness results can be made available under request addressed to authors.
Then, we use the out-of-sample results estimated by Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method to investigate the hedging effectiveness of Brent futures and SC futures. Table 8 provides the evidence that Brent futures and SC futures have obviously different hedging performance for various spots. We observe that the Brent–Brent hedge is superior to the SC–Brent hedge, which suggests that Brent futures are able to better hedge light crude spot than SC futures. On the contrary, it is discovered that the SC–Oman hedge outperforms the Brent–Oman hedge, meaning that SC futures are able to provide medium spot with better hedging effectiveness than Brent futures. The out-of-sample results provide the dependable evidence that it is essential to choose suitable futures for crude spots in hedging portfolio.

### 4. Robustness analysis

WTI and Dubai are the well-known light and medium crude spots, respectively. For proving the reliability of this paper, we use WTI and Dubai crude oil spots to conduct robustness examination, where the conditions remain unchanged. The in-sample results are detailed in Table 9, and the out-of-sample hedging results are detailed in Table 10.

We observe in Tables 9 and 10 that after the COVID-19 outbreak, the Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method has better in-sample and out-of-sample hedging effectiveness than all control groups, in most cases. Furthermore, Brent futures provide WTI spot with better hedging performance than SC futures, while SC futures provide Dubai spot with better hedging performance than Brent futures. The main robustness results are in line with the current study.

### 5. Conclusions

After the COVID-19 outbreak, extreme changes frequently appear in crude markets. It is urgent to hedge diverse crude spots risk by crude futures. In the current paper, we develop a Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method to construct crude hedging portfolios after the COVID-19 outbreak. By the in-sample and out-of-sample results estimated by the novel method, we further study the roles of Brent futures and SC futures in light spot and medium spot hedge portfolios. The conclusions are described as follows:

First of all, the current results provide the evidence that noise exists in crude futures and spots and has a bad influence on modeling performance. The wavelet denoising method with coif4 is the most effective approach to reduce noise and retain valuable information. Besides, Compared with traditional models, the Wavelet denoising-GARCHSK is the optimum
method for describing volatility. Moreover, there are significantly asymmetric tail dependences in crude futures-spot relationships. Therefore, we adopt SJC Copula to describe the crude futures-spot nexus.

Secondly, the in-sample and out-of-sample hedging results after the COVID-19 outbreak demonstrate that in most cases, the Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method outperforms all control groups, obtaining the best hedging effectiveness. Finally, during the epidemic period, Brent futures play a better hedging role for light crude spot than medium crude spot, while SC futures provide better hedging effectiveness for medium crude spot than light crude spot. Robustness tests denote that the current conclusions are robust and reliable.

The current work provides the evidence that based on the Wavelet denoising-GARCHSK-SJC Copula hedge ratio estimation method, the optimal crude hedging performance is obtained through selecting the matched crude futures during the COVID-19 period. The study not only is essential to understanding the crude futures-spot relationships, but also providing new method and references for policymakers and investors to make optimum decisions relating to risk management and asset allocation.

We provide some valuable suggestions for policymakers and investors. Firstly, noise, conditional higher moments and asymmetric dependences have negative impact on the modeling performance. Therefore, these important elements must be factored into risk management and investment decisions. Secondly, the COVID-19 seriously and constantly hit the global economy, causing frequent extreme changes in crude oil markets. In such a complex environment, traditional hedge ratio estimation method is unable to provide crude investors with effective and robust hedging performance. Therefore, the one key of a successful hedge is to choose the optimal hedge ratio estimation method. Thirdly, different futures have obviously diverse hedging performance with different crude oil spots. Brent, the light crude futures, are better to hedge light spots, while SC, the medium crude futures, are better to hedge medium spots. Therefore, the other key of a successful hedge is to choose matched crude futures. In the meanwhile, more crude futures should be introduced to meet the different demands of diverse crude spots.

However, there are some limitations need to be further researched. On the one hand, COVID-19 is far from over, therefore, crude oil markets may experience more extreme swings. We still need to focus on the hedging problem in futures-spot crude during the epidemic period. On the other hand, artificial intelligence technologies, such as deep learning, have quickly developed in recent years. We should use those new methods to construct a better hedging portfolio in the following works.

CRediT authorship contribution statement

Pengfei Zhu: Writing – original draft, Investigation, Conceptualization, Methodology. Tuantuan Lu: Software, Writing – review & editing, Supervision. Shenglan Chen: Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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