Microscopic studies of nuclear matter under diverse conditions of density and asymmetry are of great contemporary interest. Concerning terrestrial applications, they relate to future experimental facilities that will make it possible to study systems with extreme neutron-to-proton ratio. In this talk, I will review recent efforts of my group aimed at exploring nuclear interactions in the medium through the nuclear equation of state. The approach we take is microscopic and relativistic, with the predicted EoS properties derived from realistic nucleon-nucleon potentials. I will also discuss work in progress. Most recently, we completed a DBHF calculation of the Λ hyperon binding energy in nuclear matter.

*Keywords: Nuclear Matter; Equation of State; in-Medium Hadronic Interactions.*

1. Introduction

The properties of hadronic interactions in a dense environment is a problem of fundamental relevance in nuclear physics. Such properties are typically expressed in terms of the nuclear equation of state (EoS), a relation between energy and density (and possibly other thermodynamic quantities) in infinite nuclear matter. The infinite geometry of nuclear matter eliminates surface effects and makes this system a clean “laboratory” for developing and testing models of the nuclear force in the medium.

The project I review in this talk is broad-scoped. We have examined several EoS-sensitive systems/phenomena on a consistent footing with the purpose of gaining a broad overview of various aspects of the EoS. We hope this will help identify patterns and common problems.

Our approach is microscopic, with the starting point being realistic free-space interactions. In particular, we apply the Bonn B1 potential in the Dirac-Brueckner-Hartree-Fock (DBHF) approach to asymmetric nuclear matter as done earlier by the Oslo group. The details of the calculations have been described previously.
it has been known for a long time, the DBHF approximation allows a more realistic description of the saturation properties of symmetric nuclear matter as compared with the conventional Brueckner scheme. The leading relativistic effect characteristic of the DBHF model turns out to be a very efficient saturation mechanism. We recall that such correction has been shown to simulate a many-body effect (the “Z-diagrams”) through mixing of positive- and negative-energy Dirac spinors by the scalar interaction.\(^4\)

In what follows, I will review some of our recent results and discuss on-going work. I stress again that these efforts belong to the broader context of learning more about the behavior of the nuclear force in the medium using the EoS of infinite matter (under diverse conditions of isospin and spin asymmetry) as an exploratory tool. I also emphasize the importance of fully exploiting empirical information, which is becoming more available through collisions of neutron-rich nuclei.

2. Isospin-asymmetric nuclear matter

2.1. Seeking laboratory constraints to the symmetry potential

Reactions induced by neutron-rich nuclei can probe the isospin dependence of the EoS. Through careful analyses of heavy-ion collision (HIC) dynamics one can identify observables that are sensitive to the asymmetric part of the EOS. Among those, for instance, is the neutron-to-proton ratio in semiperipheral collisions of asymmetric nuclei at Fermi energies.\(^5\)

In transport models of heavy-ion collisions, particles drift in the presence of an average potential while undergoing two-body collisions. Isospin-dependent dynamics is typically included through the symmetry potential and isospin-dependent effective cross sections. Effective cross sections (ECS) will not be discussed in this talk. However, it is worth mentioning that they play an outstanding role for the determination of quantities such as the nucleon mean-free path in nuclear matter, the nuclear transparency function, and, eventually, the size of exotic nuclei.

The symmetry potential is closely related to the single-neutron/proton potentials in asymmetric matter, which we obtain self-consistently with the effective interaction. Those are shown in Fig. 1 as a function of the asymmetry parameter \(\alpha = \frac{\rho_n - \rho_p}{\rho}\). The approximate linear behavior, consistent with a quadratic dependence on \(\alpha\) of the average potential energy, is apparent. Clearly, the isospin splitting of the single-particle potential will be effective in separating the collision dynamics of neutrons and protons. The symmetry potential is shown in Fig. 2 where it is compared with empirical information on the isovector part of the optical potential (the early Lane potential\(^6\)). The decreasing strength with increasing energy is in agreement with optical potential data.
Fig. 1. The single-neutron (upper panel) and single-proton (lower panel) potentials as a function of the asymmetry parameter for fixed average density ($k_F = 1.4 fm^{-1}$) and momentum ($k = k_F$).

Fig. 2. The symmetry potential as a function of the nucleon kinetic energy at $k_F = 1.4 fm^{-1}$. The shaded area represents empirical information from optical potential data.

2.2. Summary of EOS results and comparison with recent constraints

In Table 1 we summarize the main properties of the symmetric and asymmetric parts of our EoS. Those include saturation energy and density, incompressibility $K$, the skewness parameter $K'$, the symmetry energy, and the symmetry pressure, $L$. The EoS for symmetric and neutron matter and the symmetry energy are displayed in Figs. 3-4.

A recent analysis to constrain the EoS using compact star phenomenology and
HIC data can be found in Ref.7 While the saturation energy is not dramatically different between models, the incompressibility values spread over a wider range. Major model dependence is found for the $K'$ parameter, where a negative value indicates a very stiff EoS at high density. That is the case for models with parameters fitted to the properties of finite nuclei, whereas flow data require a soft EoS at the higher densities and thus a larger $K'$. The $L$ parameter also spreads considerably, unlike the symmetry energy which tends to be similar in most models. For $L$, a combination of experimental information on neutron skin thickness in nuclei and isospin diffusion data sets the constraint $62 \text{ MeV} < L < 107 \text{ MeV}$.

Overall, our EoS parameters compare reasonably well with most of those constraints. They also compare well with those from other DBHF calculations reported in Ref.7.

The parameters in Table I are defined by an expansion of the energy written as

$$ e = e_s + \frac{K}{18} \epsilon^2 - \frac{K'}{162} \epsilon^3 + ... + \alpha^2 (e_{\text{sym}} + \frac{L}{3} \epsilon + ...), \quad (1) $$

with $\epsilon = (\rho - \rho_s)/\rho_s$, and $\rho_s$ is the saturation density.
Table 1. An overview of our predicted properties for the EOS of symmetric matter and neutron matter.

| Parameter | Predicted Value |
|-----------|-----------------|
| $\varepsilon_s$ | -16.14 MeV |
| $\rho_s$ | 0.185 fm$^{-3}$ |
| $K$ | 259 MeV |
| $K'$ | 506 MeV |
| $\varepsilon_{\text{sym}}(\rho_s)$ | 33.7 MeV |
| $L(\rho_s)$ | 70.1 MeV |

3. Spin-polarized neutron matter

In this section we move on to a another issue presently discussed in the literature with regard to exotic states of nuclear/neutron matter, namely the aspect of spin asymmetry and (possible) spin instabilities.

The problem of spin-polarized neutron/nuclear matter has a long history. Extensive work on the this topic has been done by Vidaña, Polls, Ramos, Bombaci, Mütter, and more (see bibliography of Ref. 8 for a more complete list). The major driving force behind these efforts is the search for ferromagnetic instabilities, namely the existence of a polarized state with lower energy than the unpolarized, which naturally would lead to a spontaneous transition. Presently, conclusions differ widely concerning the existence of such transition and its onset density.

A coupled self-consistency scheme similar to the one described in Ref. 3 was developed to calculate the EoS of polarized neutron matter. The details are described Ref. 8. As done previously for the case of isospin asymmetry, the single-particle potential (for upward and downward polarized neutrons), is obtained self-consistently with the effective interaction. Schematically

$$U_i = \int G_{ij} + \int G_{ii}$$

with $i, j = u, d$. The nearly linear dependence of the single-particle potentials on the spin-asymmetry parameter $^8$

$$U_{u/d}(\rho, \beta) \approx U_0(\rho, 0) \pm U_s(\rho)\beta,$$

with $\beta$ the spin-asymmetry parameter, is reminiscent of the analogous case for isospin asymmetry and may be suggestive of a possible way to seek constraints on $U_s$, the “spin Lane potential”, similarly to what was discussed above for the isovector optical potential. Namely, one can write, for a nucleus,

$$U \approx U_0 + U_\sigma(s \cdot \Sigma)/A,$$

with $s$ and $\Sigma$ the spins of the projectile nucleon and the target nucleus, respectively, and extract an obvious relation with the previous equation. (In practice, the situation for an actual scattering experiment on a polarized nucleus would require a more
Fig. 5. Spin symmetry energy as a function of the neutron density (in fm$^{-3}$).

complicated parametrization than the one above, as normally a spin-unsaturated nucleus is also isospin-unsaturated.)

As already implied by the linear dependence of the single-particle potential displayed in Eq. (3), the dependence of the average energy on the asymmetry parameter is approximately quadratic,

$$e(\rho, \beta) \approx e(\rho, 0) + S(\rho)\beta^2,$$

where $S(\rho)$ is the spin symmetry energy, shown in Fig. 5. The spin symmetry energy can be related to the magnetic susceptibility through

$$\chi = \frac{\rho \mu^2}{2S(\rho)}.$$

The rise of $S(\rho)$ with density shows a tendency to slow down, a mechanism that we attribute to increased repulsion in large even-singlet waves (which contribute only to the energy of the unpolarized state). This could be interpreted as a precursor of spin instability. In Table 2 we show predicted values of the ratio $\chi_F/\chi$, where $\chi_F$ is the susceptibility of a free Fermi gas.

Concerning the possibility of laboratory constraints which may help shed light on these issues, magnetic properties are of course closely related to the strength of the effective interaction in the spin-spin channel, which suggests to look into the $G_0$ Landau parameter. With simple arguments, the latter can be related to the susceptibility ratio and the effective mass as

$$\frac{\chi}{\chi_F} = \frac{m^*/m}{1 + G_0}.$$

Thus, $G_0 \leq -1$ signifies spin instability. (Notice the analogy with the formally similar relation between the incompressibility ratio $K/K_F$ and the parameter $F_0$, where $F_0 \leq -1$ signifies that nuclear matter is unstable.) At this time, no reliable constraints on $G_0$ are available, due to the fact that spin collective modes have not been observed with sufficient strength.
Table 2. Ratio $\chi_F/\chi$ at different densities.

| $\rho/\rho_0$ | $\chi_F/\chi$ |
|---------------|---------------|
| 0.5383        | 2.1455        |
| 0.8886        | 2.3022        |
| 1.3651        | 2.4950        |
| 1.9873        | 2.6411        |
| 2.7741        | 2.6824        |
| 3.7457        | 2.6548        |

In closing this section, we note that we found similar considerations concerning the trend of the magnetic susceptibility to hold for symmetric nuclear matter as well as for neutron matter.

4. Work in progress: non-nucleonic degrees of freedom

There are important motivations for considering strange baryons in nuclear matter. The presence of hyperons in stellar matter tends to soften the EoS, with the consequence that the predicted neutron star maximum masses become considerably smaller. With recent constraints allowing maximum masses larger than previously accepted limits, accurate calculations which include strangeness become important and timely. On the other hand, remaining within terrestrial nuclear physics, studies of hyperon energies in nuclear matter naturally complements our knowledge of finite hypernuclei.

The nucleon and the $\Lambda$ potentials in nuclear matter are the solution of a coupled self-consistency problem, which reads, schematically

$$U_N = \int_{k<k_F^N} G_{NN} + \int_{k<k_F^\Lambda} G_{NA}$$

$$U_\Lambda = \int_{k<k_F^\Lambda} G_{AN} + \int_{k<k_F^\Lambda} G_{AA}$$

To confront the simplest possible scenario, one may consider the case of symmetric nuclear matter at some Fermi momentum $k_F^N$ in the presence of a “$\Lambda$ impurity”, namely $k_F^\Lambda \approx 0$. Under these conditions, the problem stated above simplifies considerably. Such calculation was done in Ref.\textsuperscript{9} within the Brueckner scheme.

We have done a similar calculation but made use of the latest nucleon-hyperon (NY) potential of Ref.\textsuperscript{10} which was provided by the Jülich group. In a first approach, we have taken the single-nucleon potential from a separate calculation of symmetric matter. (Notice that the $\Lambda$ potential is quite insensitive to the choice of $U_N$, as reported in Ref.\textsuperscript{9} and as we have observed as well.) The parameters of the $\Lambda$ potential, on the other hand, are calculated self-consistently with the $G_{NA}$ interaction, which is the solution of the Bethe-Goldstone equation with one-boson exchange nucleon-hyperon potentials. In the Brueckner calculation, angle-averaged
Pauli blocking and dispersive effects are included. Once the single-particle potential is obtained, the value of \(-U_\Lambda(p)\) at \(p=0\) provides the \(\Lambda\) binding energy in nuclear matter, \(B_\Lambda\).

As shown and discussed extensively in Ref.\(^{10}\) there are several remarkable differences between this model and the older NY Jülich potential,\(^{11}\) and those seem to have a large impact on nuclear matter results. The main new feature of this model is a microscopic model of correlated \(\pi\pi\) and \(K\bar{K}\) exchange to constrain both the \(\sigma\) and \(\rho\) contributions.\(^{10}\) With the new model, we obtain considerably more attraction than Reuber et al.,\(^{9}\) about 49 MeV at \(k_F^N=1.35\) fm\(^{-1}\) for \(B_\Lambda\). We have also incorporated the DBHF effect in this calculation (which amounts to involving the \(\Lambda\) single-particle Dirac wave function in the self-consistent calculation through the effective mass) and find a moderate reduction of \(B_\Lambda\) by 3-4 MeV. A detailed report of this project is forthcoming.

The natural extension of this preliminary calculation will be a DBHF self-consistent calculation of \(U_N\), \(U_\Lambda\), and \(U_\Sigma\) for diverse \(\Lambda\) and \(\Sigma\) concentrations.

5. Summary and conclusions

I have presented a summary of recent results from my group as well as on-going work. Our scopes are broad and involve several aspects of nuclear matter, the common denominator being the behavior of the nuclear force, including its isospin and spin dependence, in the medium. I have stressed the importance of seeking and exploiting laboratory constraints. In the future, coherent effort from theory, experiment, and observations will be the key to improving our knowledge of nuclear matter and its exotic states.

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6. References

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