STATUS OF PERTURBATIVE QCD EVALUATION OF HADRONIC DECAY RATES OF THE Z AND HIGGS BOSONS

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We review the current status of the high order perturbative QCD evaluation of the hadronic decay rates of the Z and Higgs bosons. A systematic classification of the various types of QCD corrections to $O(\alpha_s^2)$ and $O(\alpha_s^3)$ is made and their numerical status is clarified.

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1 Introduction

The standard theory of strong interactions, QCD, can be accurately tested based on the precise experiments at LEP/SLC and the recent high order perturbative results for the hadronic width of the Z boson. From the fit of theoretical and experimental results, an accurate value of the strong coupling can be extracted. With the planned experimental precision at LEP, an error of a couple of percent on $\alpha_s$ is expected. In this situation, a further improvement of the theoretical precision for the measured quantities is necessary. In this work, we present a systematic classification of various types of high order QCD contributions to the hadronic decay rates of the Z boson. We also discuss recent results of the evaluation of the $O(\alpha_s^3)$ QCD corrections to the hadronic decay rates of the Standard Model Higgs boson.

2 Hadronic decay width of the Z boson

2.1 Theoretical structure

The decay rate of the Z boson into quark antiquark pair can be written in the following way

$$\Gamma_{Z \rightarrow \text{hadrons}} = \frac{G_F M_Z^2}{8\sqrt{2}\pi} \sum_f \rho_f \left\{ v_f^2 \left[ (1 + 2X_f)\sqrt{1 - 4X_f} + \delta_{\text{QCD}}^V(\alpha_s, X_f, X_t) + \delta_{\text{QED}}^V(\alpha, \alpha_s, X_f) \right] + a_f^2 \left[ (1 - 4X_f)^{3/2} + \delta_{\text{QCD}}^A(\alpha_s, X_f, X_t) + \delta_{\text{QED}}^A(\alpha, \alpha_s, X_f) \right] \right\},$$

where $v_f = 2I_f^{(3)} - 4e_f \sin^2 \theta_W k_f$, $a_f = 2I_f^{(3)}$ are the standard vector and axial couplings. $X_f = m_f^2/M_Z^2$, e.g., $X_b \sim 0.003$. $\delta_{\text{QCD}}^{V/A}$ and $\delta_{\text{QED}}^{V/A}$ stand for the vector and axial parts of the corresponding QCD and QED contributions. The summation $f = u, d, s, c, b$ should be done with a proper care for the singlet contributions, which are included in $\delta_{\text{QCD}}^{V/A}$ (see below). The electroweak self-energy and vertex corrections are absorbed in the factors $\rho_f$ and $k_f$. The current status of the electroweak contributions has been discussed in detail. The small QED corrections in vector and axial channels look like

$$\delta_{\text{QED}}^V = \frac{3}{4} e_f^2 \frac{\alpha}{\pi} [1 + 12X_f + O(X_f^2)] + O(\alpha^2) + O(\alpha \alpha_s),$$

$$\delta_{\text{QED}}^A = \frac{3}{4} e_f^2 \frac{\alpha}{\pi} [1 - 6X_f - 12X_f \log X_f + O(X_f^2)] + O(\alpha^2) + O(\alpha \alpha_s).$$

Corrections $\sim \alpha^2$ and $\sim \alpha \alpha_s$ are negligible at the current level of precision.

2.2 QCD contributions to $O(\alpha_s^3)$

The QCD contributions are represented by the terms $\delta_{\text{QCD}}^{V/A}$ and can be calculated within perturbation theory by evaluating the quantity $\text{Im} \Pi(-s + i0)$ at $s = M_Z^2$, where the function $\Pi$ is defined through a correlation function of two flavor diagonal quark currents

$$i \int d^4x e^{iqx} \langle T j^f_\mu(x) j^f_\nu(0) \rangle = g_{\mu\nu} \Pi(Q^2) - Q_{\mu}Q_{\nu} \Pi'(Q^2).$$

Here, $Q^2$ is a large ($\sim -M_Z^2$) Euclidean momentum and the neutral weak current of quark coupled to Z boson is

$$j^f_\mu = \left( \frac{G_F M_Z^2}{2\sqrt{2}} \right)^{1/2} (v_f \bar{q}_f \gamma_\mu q_f + a_f \bar{q}_f \gamma_\mu \gamma_5 q_f).$$
Because the physical scale ($\sim M_Z$) is much larger than the quark masses involved, we expand the $\Pi$ function in powers of $m_t^2/M_Z^2$. The coefficient functions in this expansion and their imaginary parts can be calculated using the methods of the renormalization group and the computer programs for analytical evaluation of multiloop Feynman diagrams. For a review of the current state-of-art of the high order perturbative QCD calculations for the above quantities see the Ref. 7.

It is convenient to decompose $\delta^{\nu/A}_{\text{QCD}}$ in so called singlet and non-singlet parts. The nonsinglet part is formed from the diagrams with the two quark currents within a single fermionic loop and the singlet part corresponds to the graphs with the quark currents in separate fermionic loops mediated by gluonic states. $\delta^{\nu}_{\text{QCD}} = \delta^{\nu,\text{ns}}_{\text{QCD}} + \delta^{\nu,s}_{\text{QCD}}/\alpha_s^2$, $\delta^{A}_{\text{QCD}} = \delta^{A,\text{ns}}_{\text{QCD}} + \delta^{A,s}_{\text{QCD}}/\alpha_s^2$.

The nonsinglet part in the vector channel to $O(\alpha_s^3)$ reads

$$ \delta^{\nu,\text{ns}}_{\text{QCD}} = \frac{\alpha_s}{\pi} (1 + 12 \overline{X}_f) + \left( \frac{\alpha_s}{\pi} \right)^2 (1.40923 + 104.833 \overline{X}_f + \sum_v F^{(2)}(X_v) + G^{(2)}(X_t)) + \left( \frac{\alpha_s}{\pi} \right)^3 (-12.76706 + 547.879 \overline{X}_f + \sum_v F^{(3)}(X_v) + G^{(3)}(X_t)), \quad (6) $$

where $\overline{X}_f = m_f^2(M_Z)/M_Z^2$. The $O(\alpha_s^2)$ and $O(\alpha_s^3)$ terms in the limit of vanishing light quark masses $m_f = 0$ and infinitely large top mass $m_t = \infty$ have been evaluated. The terms $\sim \overline{X}_f$ represent leading mass corrections. The functions $F^{(2)}(X_v)$ and $G^{(2)}(X_t)$ stand for the contributions from the three-loop diagrams containing the virtual fermionic loop with the massive quark propagating in it. As we will see, these contributions are already small, so there is no necessity to evaluate light quark mass corrections for $F$ and $G$. The function $F$ represents the contribution of light (first five flavors) quarks, while the function $G$ represents the remaining contribution of the decoupled top quark in five flavor effective theory. Numerically,

$$ F^{(2)}(X_v) \approx \overline{X}_v^2 \times \left\{ -0.474894 - \log \overline{X}_v + \sqrt{\overline{X}_v} \left[ -0.5324 + 0.0185 \log \overline{X}_v \right] \right\} \quad (7) $$

is small and can be neglected. Contributions from virtual top quark involve $G(X_t)$ function. We see that $G$ vanishes in the large $X_t$ limit. That is, the heavy quark decouples. The three-loop analytical expressions for the $F$ and $G$ functions were found in Ref. 13. Also, the first two terms in the r.h.s. of eq.(8) have been obtained using the large mass expansion method. At order $\alpha_s^3$, $F^{(3)}(X_f) = -6.12623 \overline{X}_f + O(\overline{X}_f^2)$ [13]. Based on the large mass expansion technique, the following tiny correction has been obtained for the virtual top quark contribution in the limit $X_t \rightarrow \infty$ [13]

$$ G^{(3)}(X_t) \approx \overline{X}_t^{-1} \times \left\{ \frac{44}{675} + \frac{2}{135} \log \overline{X}_t - \sqrt{\overline{X}_t} \left[ 0.001126 + 0.001129 \log \overline{X}_t \right] \right\}. \quad (8) $$

In these formulas, $\alpha_s$ denotes running MS coupling in five flavor theory evaluated at $M_Z$. The transformation relation for different number of flavors and different scales, as well as the relation between the MS running mass and the pole mass can be found in Ref. 16.

The nonsinglet part in the axial channel is very similar to the one in the vector channel, except the light quark mass corrections are different.

$$ \delta^{A,\text{ns}}_{\text{QCD}} = \frac{\alpha_s}{\pi} (1 - 22 \overline{X}_f) + \left( \frac{\alpha_s}{\pi} \right)^2 (1.40923 - 85.7136 \overline{X}_f + \sum_v F^{(2)}(X_v) + G^{(2)}(X_t)) + \left( \frac{\alpha_s}{\pi} \right)^3 (-12.76706 + (\text{unknown}) \overline{X}_f + \sum_v F^{(3)}(X_v) + G^{(3)}(X_t)). \quad (10) $$
The terms \( \propto X_f \) represent leading light quark mass corrections\([2,3,4]\).

In the vector channel, the singlet \( O(\alpha_s^2) \) contribution is identically zero due to Furry’s theorem\([5]\). In the axial channel, at the same order, the light doublet contributions add up to zero in the limit of degenerate quark masses. This is because in the Standard Model, quarks in a weak doublet couple with opposite sign to the Z boson in the axial channel. However, the contribution from the t,b doublet turns out to be significant due to the large mass splitting.\([6]\)

\[
\mathcal{L}_A^{(2)} = \left( \frac{\alpha_s}{\pi} \right)^2 \left[ \left( -\frac{37}{12} - \log X_t + \frac{7}{81} X_t^{-1} + 0.013 X_t^{-2} \right)_{m_b=0} + X_b (18 + 6 \log X_t) - \frac{X_b}{X_t} \left( \frac{80}{81} + \frac{5}{27} \log X_t \right) \right],
\]

where small mass corrections \( \propto X_b \) have been calculated in Ref. 20. At the \( O(\alpha_s^3) \), both channels contribute. The vector channel contribution in the limit of massless light quarks reads\([6]\)

\[
\mathcal{L}_V^{(3)} = \left( \frac{\alpha_s}{\pi} \right)^3 \left[ -0.41318 \left( \sum_{f=u,d,s,c,b} v_f \right)^2 + (0.02703 X_t^{-1} + 0.00364 X_t^{-2} + O(X_t^{-3})) v_t \sum_{f=u,d,s,c,b} v_f \right],
\]

where negligible terms \( \propto X_t^{-1}, X_t^{-2} \) were computed in Ref. 15. In the axial channel, the \( O(\alpha_s^3) \) singlet contribution in the large top mass expansion reads\([6]\)

\[
\mathcal{L}_A^{(3)} = \left( \frac{\alpha_s}{\pi} \right)^3 \left[ -15.98773 - \frac{67}{18} \log X_t + \frac{23}{12} \log^2 X_t \right].
\]

The light quark mass corrections at the \( O(\alpha_s^3) \) for the singlet parts are not yet known. However, at the current experimental precision they are not expected to be detectable. Summarizing the present knowledge for the singlet parts, we write \( \delta_{QCD}^{V,S} = \mathcal{L}_V^{(3)}, \quad \delta_{QCD}^{A,S} = \mathcal{L}_A^{(2)} + \mathcal{L}_A^{(3)} \).

### 2.3 On large “\( \pi^2 \)” terms

Let us briefly mention about the importance of the so called \( \pi^2 \) terms appearing naturally as a result of analytical continuation of the results of perturbative evaluation from Euclidean to Minkowski space. Indeed, because of the relation \( \frac{1}{\pi} \text{Im} \log^3(s + 0) = -3 \log^2 s + \pi^2 \) and relations similar to that, one can trace the appearance of the large contributions due to the \( \pi^2 \) terms starting at \( O(\alpha_s^3) \). For the known quantity \( R(s) \) in electron-positron annihilation at low energies, we write

\[
R = 1 + \frac{\alpha_s}{\pi} d_1 + \left( \frac{\alpha_s}{\pi} \right)^2 d_2 + \left( \frac{\alpha_s}{\pi} \right)^3 \left( d_3 - \pi^2 \beta_0^2 d_1/3 \right) + \left( \frac{\alpha_s}{\pi} \right)^4 \left[ (d_4 - \pi^2 (\beta_0^2 d_2 + 5 \beta_0 \beta_1 d_1/6)) \right. + \left. \left( \frac{\alpha_s}{\pi} \right)^5 \{ d_5 - \pi^2 [2 \beta_0^2 d_3 + 7 \beta_0 \beta_1 d_2/3 + (\beta_0 \beta_2 + (\beta_1^2/2) d_1 \} + \pi^4 \beta_0^3 d_1/5 \}. \right.
\]

The \( \beta \) function coefficients can be found in, e.g., Ref. 16. The \( d_i \) are known up to \( i = 3 \) and are of order 1 (\( d_i = \{ 1, 1.4, -0.7 \} \)). The \( \pi^2 \) terms are large. They can be calculated up to order \( \alpha_s^5 \) using the known \( d_i \) and \( \beta_i \). Numerically, they are \( \{-12.1(\alpha_s/\pi)^3, -89.2(\alpha_s/\pi)^4, -648(\alpha_s/\pi)^5\} \). It is reasonable to expect that the contributions \( \propto \pi^2 \) are dominant at all orders and their resummation seems to be of primary importance. From the above equation, one can also get the idea about the size of the still uncalculated higher order corrections and numerical error estimates.
3 Hadronic decay width of the Higgs boson

The perturbative QCD evaluation of the hadronic decay rates of the SM Higgs boson is very similar to that of the Z boson. In fact, exactly the same set of diagrams have to be evaluated at each order. However, in this case one considers quark scalar current correlators. Also, the calculation involves renormalization of quark mass, even in the massless quark limit. This is because of the explicit quark mass dependence of the Yukawa coupling. We give the expression for the quantity $\Gamma_{H \rightarrow \text{hadrons}}$ to $O(\alpha_s^2)$, without detailing the particular contributions from the singlet and nonsinglet parts.

$$\Gamma_{H \rightarrow \text{hadrons}} = \frac{3\sqrt{2}G_F M_H}{8\pi} \sum_f m_f^2 \left(1 + \frac{\alpha_s}{\pi} e^2 \Delta_{\text{em}}\right) \times (1 + \Delta_{\text{weak}}) \times \left[\left(1 - 4 \frac{m_f^2}{M_H^2}\right)^{3/2} + \frac{\alpha_s}{\pi} \left(5.667 - 40 \frac{m_f^2}{M_H^2}\right) \left(29.147 - 99.725 \frac{m_f^2}{M_H^2} + 12 \sum_{v=u,d,s,c,b} \frac{m_v^2}{M_H^2}\right)\right],$$  \hfill (15)$$

where $m_f$ is the $\overline{\text{MS}}$ quark mass evaluated at $M_H$. The leading electroweak contributions, represented by the terms $\Delta_{\text{em}}$ and $\Delta_{\text{weak}}$, are calculated in Refs. 23,24,3. The leading QCD corrections $\sim \alpha_s G_F m_t^2$ have been calculated in Ref. 25. The $O(\alpha_s)$ QCD correction with the exact quark mass dependence was found in Ref. 26. The $O(\alpha_s^2)$ corrections for the scalar quark current correlator and to the Higgs decay rates have been calculated in Refs. 27,16. The last term in the above expression is due to nonvanishing quark masses from the nonsinglet diagrams containing virtual fermionic loop and the singlet diagrams. Here we assume that the top quark is decoupled (the limit $m_t = \infty$). The correction arising from the virtual top quark was found in Ref. 28 and for the singlet diagrams in Ref. 29. These corrections have to be added to the above expression for a precision numerical analyses. Finally, we note that the similar results exist for the pseudoscalar (MSSM) Higgs boson. We refer the readers to the original papers and review articles for the issues that we were not able to discuss here.

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