Influence of many-particle interactions on slow light phenomena in quantum dots

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Abstract. We investigate the impact of many-particle interactions on group-velocity slowdown achieved via Electromagnetically Induced Transparency (EIT) in quantum dots. Using a ladder scheme we find in the steady-state an increase in maximum slow-down as compared to the non-interacting case, which can be attributed to Coulomb interaction effects. The necessary pump power at which maximum slow down is obtained EIT remains, however, unchanged.

1. Introduction

The subject of slowing down light in a highly dispersive media has gained enormous attention since Hau et al. reported on light propagating at 17 m/s through a vapor of ultra cold Na atoms [1]. A large research effort has been put into extending these results to semiconductor nanostructures, where a major motivational factor lies in the application for optoelectronic devices. Quantum dots (QDs) are a promising candidate for applications exploiting quantum coherence phenomena due to their atom-like properties and large dephasing times [2]. For instance, an all optical buffer based on slow light in quantum dots has been proposed [3].

Most theoretical approaches [3, 4, 5] have taken offset in the atomic description, thus treating the quantum dot as an inhomogeneously broadened three-level system, with strong dephasing effects, thereby disregarding many-particle effects. A physically more realistic model would include other quantum dot states, as well as the continuum of states generated by the wetting layer (WL). Furthermore, the Coulomb interaction that serves to couple discrete and continuum states, redistribute carriers, and renormalize energies and fields, should also be taken into account. Recent publications [6, 7], have addressed these issues, however the treatment is done in a transient regime and for application purposes the steady-state properties are of particular interest.

In this work we study the EIT generated slow light properties of an InAs quantum dot embedded in a GaAs quantum well using the generalized semiconductor Bloch equations for a ladder scheme setup.
2. Theory
The heterostructure we consider, consists of conical InAs quantum dots (radius of 9 nm and height 3 nm) residing on a 1.2 nm thick WL, sandwiched between two slabs of GaAs. The electronic structure is calculated as the solution to the single band Schrödinger equation for the envelope wavefunction in the effective mass approximation, using an approach similar to [8]. We find 6 confined levels for both electrons and holes plus a continuum of delocalized WL states, each level doubly degenerate due to spin. The dipole selection rules allows for a ladder scheme, where a powerful pump beam (polarized along the growth direction of the dots) drives the electronic intraband transition $|e0\rangle \rightarrow |e5\rangle$, while a weak probe beam (polarized in the dot plane) connects the $|e0\rangle \rightarrow |h0\rangle$ interband transition (see Fig. 1).

The linear optical response, i.e. the susceptibility $\chi(\omega)$, whose real and imaginary part is related to refraction and absorption, respectively, is found from the macroscopic polarization $P(\omega)$ as

$$\chi(\omega) = \frac{P(\omega)}{\epsilon_b E_{\text{probe}}(\omega)},$$

where $\epsilon_b$ is the permittivity of the host material, and $E_{\text{probe}}$ is the amplitude of the probe field.

The time resolved macroscopic polarization $P(t)$ is computed from the microscopic polarizations according to semiclassical theory [9]:

$$P(t) = \frac{1}{w} \left( N_{\text{dot}} \sum_{i,j} \mu_{ij} P_{ij}(t) + \frac{1}{A} \sum_k [\mu_k P_k(t) + \text{c.c.}] \right).$$

$P_{ij}$ and $P_k$ are microscopic polarization components of localized dot states $(i,j)$ and diagonal interband polarization of delocalized WL states $(k)$, respectively. In this treatment we disregard polarizations relating to transitions connecting dot and WL states, this is a good approximation when the electron and hole envelopes are not too different, which is the case in the present paper. Dipole matrix elements between localized states are denoted $\mu_{ij}$, whereas $\mu_k$ is the dipole moment relating to WL states. $N_{\text{dot}}$ is the two-dimensional density of the dots in the WL plane, $A$ is the normalization area of the WL, and $w$ is the thickness of the active region.

The microscopic polarizations are the off-diagonal components $\Psi_{\nu_1\nu_2}$ of the reduced density
matrix $\rho_{\nu_1\nu_2}$ and are found by solving the generalized Bloch equations, given by
\[
 i\hbar \frac{\partial}{\partial t} \Psi_{\nu_1\nu_2} = -[\tilde{\epsilon}_{\nu_1}(t) - \tilde{\epsilon}_{\nu_2}(t)] \Psi_{\nu_1\nu_2}(t) - [n_{\nu_2}(t) - n_{\nu_1}(t)] \Omega_{\nu_1\nu_2}(t)
\]
\[
 - \sum_{\nu_3 \neq \nu_1, \nu_2} [\Omega_{\nu_1\nu_3}(t)\Psi_{\nu_3\nu_2}(t) - \Psi_{\nu_1\nu_3}(t)\Omega_{\nu_3\nu_2}(t)]
\]
\[
 = i\hbar S_{\nu_1\nu_2}(t), \quad (3)
\]
\[
i\hbar \frac{\partial}{\partial t} n_{\nu_1}(t) = - \sum_{\nu_3 \neq \nu_1} [\Omega_{\nu_1\nu_3}(t)\Psi_{\nu_3\nu_1}(t) - \Omega_{\nu_3\nu_1}(t)\Psi_{\nu_1\nu_3}(t)]
\]
\[
 = i\hbar S_{\nu_1\nu_1}(t), \quad (4)
\]
where
\[
\tilde{\epsilon}_{\nu} = \epsilon_{\nu} + \sum_{\nu_3 \neq \nu} [V_{\nu\nu_3\nu_4} - V_{\nu_3\nu_4\nu}] \rho_{\nu_3\nu_4}(t)
\]
\[
\Omega_{\nu_1\nu_2}(t) = - e\mu_{\nu_1\nu_2} E(t) + \sum_{\nu_3 \neq \nu_4} [V_{\nu_1\nu_4\nu_3\nu_2} - V_{\nu_1\nu_2\nu_3\nu_4}] \rho_{\nu_3\nu_4}(t), \quad (5)
\]
\[
\Omega_{\nu_1\nu_2}(t) = - e\mu_{\nu_1\nu_2} E(t) + \sum_{\nu_3 \neq \nu_4} [V_{\nu_1\nu_4\nu_3\nu_2} - V_{\nu_1\nu_2\nu_3\nu_4}] \rho_{\nu_3\nu_4}(t), \quad (6)
\]
are the Hartree-Fock renormalized single particle energy and generalized Rabi frequency, respectively. $n_{\nu}$ is the diagonal component of the density matrix, i.e. $\rho_{\nu\nu}$. The term $-e\mu_{\nu_1\nu_2} E(t)$ is the electromagnetic field interaction in the dipole approximation, and the matrix elements of the Coulomb interaction are $V_{\nu_1\nu_2\nu_3\nu_4} = \int \Phi^*_{\nu_1}(\mathbf{r})\Phi^*_{\nu_2}(\mathbf{r}')/4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'| \Phi_{\nu_3}(\mathbf{r}')\Phi_{\nu_4}(\mathbf{r}) d^3r d^3r'$. For the situations considered here, screening effects are disregarded due to low WL densities.

Off-diagonal scattering terms $S_{\nu_1\nu_2}(t)$ are approximated by a temperature dependent effective dephasing rate $\gamma_d$. Diagonal terms representing collision induced particle exchange processes, are mimicked by a population relaxation towards quasiequilibrium Fermi-Dirac functions. The scattering rates are denoted $\gamma_{c-c}$ and $\gamma_{c-p}$ representing carrier-carrier and carrier-phonon scattering. The details of this procedure are described presented in [6].

The non-interacting (atomic) model is found by taking the limit where all Coulomb elements and population scattering rates are set to zero.

The numerical solution involve 78 equations of motion for the dot (66 for each polarization component, and 12 for the populations) together with 300 for the momentum resolved WL states, corresponding to a discretization of the WL using 100 k-points. The full set of equations are solved with a four point Runge-Kutta method.

3. Results

We simulate experiments where two gaussian shaped pulses are incident on the QD sample. Steady state results are obtained by using a sufficiently long pump pulse, i.e. a pulse temporally several times wider than the dephasing time ($\gamma_d^{-1}$) of the system. We then apply a temporally narrow probe beam at the peak of the pump and examine the complex susceptibility seen by the probe. All calculations are performed using a dot density of $N_{\text{dot}} = 5 \cdot 10^{14}$ m$^{-2}$ at a lattice temperature of 200 K, for which the literature [10, 2] gives scattering rates around $\gamma_d = 1.5 \cdot 10^{12}$ s$^{-1}$, $\gamma_{c-c} = 2.0 \cdot 10^{12}$ s$^{-1}$, and $\gamma_{c-p} = 2.0 \cdot 10^{11}$ s$^{-1}$. The resulting optical response for a pump pulse of width (FWHM) 20 ps with a peak intensity of 77 MW/cm$^2$ is shown in Fig. 2 illustrating both the interacting and the non-interacting approach. An obvious difference between the two cases is the change in probe beam energy towards negative detuning. This is due to the excitonic shift of the $|\psi\rangle - |\psi\rangle$ transition. An apparent feature of Fig. 2 is that the peaks of the imaginary part of the susceptibility are higher for the many-particle model. While the distance between the peaks remains the same one can readily see that a larger area is covered.
Figure 2. Complex susceptibility vs. probe energy. The solid line retains to the atomic model, the dashed line to the many-particle model. The energy is measured relative to the single-particle transition energy $\epsilon^{(0)}_{e_0,h_0}$. To facilitate a comparison between the two cases, the non-interacting spectra have been shifted.

by the many-particle spectrum. This is evidence that oscillator strength has been shifted into the $|e_0\rangle - |h_0\rangle$ transition, in other words it has been Coulomb enhanced. Considering the the real part of the susceptibility, the spectrum experiences a general shift upwards; this behavior can again be attributed to a change in the dot oscillator strengths, which tends to take "weight" from high energy transitions and put into lower energy transitions. Investigations have shown that the wetting layer states surrounding the dot plays non-negligible part in this redistribution.

Figure 3. Maximum slowdown factor vs. pump pulse peak intensity. The solid curve contains the independent particle result, the dashed curve is the outcome of including many-body interactions.

The physical consequences of these results can readily be quantified by considering the slowdown factor $S$. It is a measure of the group velocity reduction, which is a figure of merit relevant for optical storage, and is defined by

$$S = \frac{c}{v_g} = n + \omega \frac{\partial n}{\partial \omega},$$

(7)
where \( c \) is the speed of light, and \( n \) is the refractive index. Figure 3 depicts the maximum achievable slowdown as a function of pump peak intensity for both models. The slowdown factor is increased significantly when including interactions, this is due to the enhancement of the probe transition. The slowdown climbs to its peak value faster for interacting case, but the maximum is seen to appear at almost the same value of pump power for both models, which indicates that the pump transition \( |e_0\rangle - |e_5\rangle \) is unchanged by the inclusion of many-particle effects. This is in contrast to other findings [7], where a shift in pump power of several orders of magnitudes was observed, however the results were obtained in a transient regime using a \( \Lambda \)-scheme involving real carrier excitation.

4. Conclusion
We have investigated the slow light properties of InAs quantum dots using a ladder scheme. Emphasis has been put on the differences between using an atomic model or one that includes many-particle interactions on the level of the Hartree-Fock approximation. It is found that due to coulomb enhancement effects the slowdown factor is increased in the many-body case. Higher values of the maximum achievable slowdown factor is found, accompanied by a steeper slope of the slowdown vs. pump power curve. However, for this particular EIT-scheme, there is observed no change in the necessary pump power required to reach maximum slowdown.

Acknowledgments
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