Milli-Hertz Gravitational-wave Background Produced by Quasiperiodic Eruptions

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Received 2021 December 15; revised 2022 March 31; accepted 2022 April 1; published 2022 May 11

Abstract

Extreme-mass-ratio inspirals (EMRIs) are important targets for future space-borne gravitational-wave (GW) detectors, such as the Laser Interferometer Space Antenna (LISA). Recent works suggest that EMRIs may reside in a population of newly discovered X-ray transients called “quasiperiodic eruptions” (QPEs). Here, we follow this scenario and investigate whether LISA could in the future detect the QPEs. We consider two specific models, in which the QPEs are made of either stellar-mass objects moving on circular orbits around massive black holes (MBHs) or white dwarfs (WDs) on eccentric orbits around MBHs. We find that in either case the five QPEs detected so far are too weak to be resolvable by LISA. However, if QPEs are made of eccentric WD–MBH binaries, they radiate GWs over a wide range of frequencies. The broad spectra overlap to form a background that peaks in the milli-Hertz band and has a signal-to-noise ratio of 9–17 even in the most pessimistic scenario. The presence of this GW background in the LISA band could impact future searches for seed black holes at high redshift as well as stellar-mass binary black holes in the local universe.

Unified Astronomy Thesaurus concepts: Gravitational waves (678); Intermediate-mass black holes (816); White dwarf stars (1799); X-ray transient sources (1852)

1. Introduction

An extreme-mass-ratio inspiral (EMRI) consists of a massive black hole (MBH) and a small compact object, such as a stellar-mass black hole (BH), a neutron star, or a white dwarf (WD), moving on a tightly bound orbit (Amaro-Seoane et al. 2007). Because of gravitational-wave (GW) radiation, the orbit decays and the small object eventually coalesces with the MBH. If the mass of the MBH is $10^5–10^7 M_\odot$, the GW radiated during the last few years of the system falls into the sensitive band of the Laser Interferometer Space Antenna (LISA; Amaro-Seoane et al. 2017). During this period, as many as $10^4–10^5$ GW cycles could be accumulated in the data stream, providing rich information about the spacetime geometry close to an MBH (Gair et al. 2013; Berry et al. 2019).

Despite their scientific importance, many basic properties of EMRIs, such as the event rate, are largely unconstrained. The difficulty lies in the lack of a distinctive electromagnetic (EM) signature. For example, the EMRIs containing stellar-mass BHs are considered to dominate the EMRI population (de Freitas Pacheco et al. 2006), but the predicted event rate varies from a dozen per year (within a redshift of $z = 4.5$) to as many as a few $\times 10^5$ per year (see Babak et al. 2017 and Gair et al. 2017 for summaries). The fraction of EMRIs that are resolvable by LISA depends on the redshift distribution, and is also uncertain. If most EMRIs are unresolved, they would form a GW background that is practically indistinguishable from noise (Sigl et al. 2007; Bonetti & Sesana 2020).

Unlike a stellar-mass BH or neutron star, a WD revolving around an MBH could be tidally detonated if the MBH has moderate mass ($10^3–10^6 M_\odot$, e.g., Luminet & Pichon 1989; Rosswog et al. 2009), or it could activate the MBH via Roche-lobe overflow and tidal disruption (e.g., Ivanov & Papaloizou 2007; Zalamea et al. 2010; MacLeod et al. 2014). Therefore, the EMRIs containing WDs are potential targets for joint EM and GW observations (Sesana et al. 2008; Eracleous et al. 2019). In particular, they encode valuable information about the astrophysical environments that lead to the formation of EMRIs. In theory, various dynamical processes could deliver WDs to the vicinities of MBHs, including the dynamical relaxation of star clusters (Hils & Bender 1995), tidal capture (Ivanov & Papaloizou 2007), the partial disruption of red-giant stars (Bogdanović et al. 2014), and the tidal separation of WD binaries (Miller et al. 2005). It is estimated that as many as $10^5$ WD EMRIs could be detected by LISA with a reasonable signal-to-noise ratio ($S/N$; Hils & Bender 1995; Sigurdsson & Rees 1997; Ivanov 2002; Sesana et al. 2008).

Interestingly, the EM counterpart to the above WD EMRI may have been found in a new type of transient, called “quasiperiodic eruptions” (QPEs; Miniutti et al. 2019; Giustini et al. 2020; Arcodia et al. 2021; Chakraborty et al. 2021). Five QPEs have been discovered so far, and they share a distinctive feature: within an hour, the X-ray count rate surges by one to two orders of magnitude, and such an eruption recurs every few hours. The short duration of each outburst and the short recurrence timescale resemble the characteristics of a small object periodically swooping by an MBH along a tightly bound, highly eccentric orbit. The similarity leads to the suggestion that QPEs are powered by eccentric WD EMRIs whose WDs are filling up their Roche lobes and feeding the MBHs during their pericenter passages (King 2020). This interpretation is further supported by observational evidence of earlier tidal disruption events in two of the QPEs (GSN069 and XMMSL1 J024916.6-041244; see Shu et al. 2018; Miniutti et al. 2019; Chakraborty et al. 2021; Sheng et al. 2021), corroborating the picture that partial disruptions of stars could...
deposit their compact cores (such as WDs) close to the vicinities of MBHs.

Further theoretical studies suggest that WD–MBH binaries may be too short-lived to explain the detection rate of QPEs, because the WDs would expand in a runaway fashion as soon as the mass transfer starts (Metzger et al. 2022). One way of alleviating this problem is replacing the WD with a main-sequence star, so that the mass transfer is more stable (Linial & Sari 2017; Zhao et al. 2021). Another solution involves less eccentric orbits for those objects (not necessarily WDs) around MBHs, so that mass transfer can be avoided (Ingram et al. 2021; Metzger et al. 2022; Xian et al. 2021). In these models, QPEs also emit GWs because they are essentially still EMRIs. Alternatively, QPEs may not be associated with EMRIs, due to the instability of the accretion disk (Miniutti et al. 2019; Motta et al. 2020; Sniegowska et al. 2020; Raj & Nixon 2021). In this case, little GW radiation is expected. These models, however, have difficulties in explaining two of the QPEs, which are found in quiescent galaxies showing no signs of accretion disks (Arcodia et al. 2021).

Although QPEs may contain EMRIs, whether they can be detected by LISA is still unclear. Conventionally, the detectability of an EMRI is evaluated by comparing the characteristic strain $h_c$ with the LISA noise curve (see, e.g., Barack & Cutler 2004). This method should be applied to QPEs with caution. First, QPEs, if they are EMRIs, are evolving on a timescale of $10^5–10^4$ yr, according to previous studies (e.g., King 2020; Ingram et al. 2021; Metzger et al. 2022). Such a timescale is much longer than the mission duration of LISA. Consequently, the variation $\Delta f$ of the GW frequency $f$ is small, i.e., $\Delta f \ll f$, during the observational period. In this case the $S/N$, which is essentially an integration of the characteristic strain over the range of frequency shift, is not proportional to $h_c$ (e.g., Zhao et al. 2021), but to $h_c \sqrt{\Delta f/f}$ (Barack & Cutler 2004; Robson et al. 2019). Second, when the orbital eccentricity is high, as would be the case if QPEs are powered by WDs (e.g., King 2020), the GW power is emitted over a wide range of harmonic frequencies (Peters & Mathews 1963). Therefore, the total $S/N$ should count all the harmonics, instead of a single harmonic, as in the case of a circular WD–MBH binary (Sesana et al. 2008; Han & Fan 2018). Third, a recent study of the EMRIs with stellar BHs suggests that although the majority are unresolvable by LISA, together they form a background that may be higher than the instrument noise (Bonetti & Sesana 2020). Whether QPEs produce a similar background deserves investigation. Understanding this background is important because it may impinge on the detection of the seed MBHs at high redshift, as well as the binary BHs (BBHs) that could be the progenitors of the sources already detected by the Laser Interferometer Gravitational-Wave Observatory (LIGO) and the Virgo detectors (e.g., Bonetti & Sesana 2020).

Here, we take the above three factors into account and study the detectability of QPEs by the future LISA mission. The paper is organized as follows. In Section 2, we describe two models proposed for QPEs that contain EMRIs. Based on these models, we calculate the corresponding GW spectra for the five detected QPEs. In Section 3, we compute the GW background formed by QPEs and investigate its detectability by LISA. We also compare it with the GW background resulting from other types of sources and evaluate the impact on the future search for seed BHs and BBHs by LISA. In Section 4, we summarize our results and discuss the caveats. Throughout the paper, we assume a standard $\Lambda$ cold dark matter cosmology with the parameters $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_\Lambda = 0.7$, and $\Omega_M = 0.3$.

2. Models

2.1. EMRIs on Circular Orbits

We first consider a model in which QPEs are produced by stellar-mass objects moving on relatively circular orbits around MBHs (Metzger et al. 2022; Xian et al. 2021). Such an orbit has three parameters: the mass of the MBH $M$, the mass of the stellar-mass object $m$, and the orbital period $P$. For the five QPEs detected so far, we give their parameters in Table 1, which are derived in the following ways.

The orbital period $P$ is determined by the time interval between successive eruptions. Note that in some models, the small object collides with the accretion disk of the MBH twice per orbital period, and hence $P$ is twice the time interval between eruptions (Xian et al. 2021). We neglect this factor of 2 because it does not qualitatively affect the amplitude and detectability of the GWs.

The mass $M$ of the MBH is not an observable and is derived for different QPEs using different methods. For GSN 069 and RX J1130.9+2747, the masses are derived from fitting their X-ray spectra with accretion disk models (adopted from Shu et al. 2017 and Miniutti et al. 2019). The mass of XMMSL1 J0249-041244 is inferred from the correlation between the mass of an MBH and the velocity dispersion of the bulge of the host galaxy (from Wevers et al. 2019). For eRO-QPE1 and eRO-QPE2, since the masses of their host galaxies have been derived in previous works (Arcodia et al. 2021), we use them to estimate the masses of the MBHs according to the empirical scaling relation (Reines & Volonteri 2015):

$$\log(M_{MBH}/M_\odot) = 7.45 + 1.05 \log (M_{stellar}/10^{11} M_\odot).$$

(1)

The mass $m$ of the small object is model-dependent and uncertain. To accommodate various theoretical possibilities, we treat $m$ as a free parameter and vary it between 0.2 $M_\odot$, mimicking WDs or stripped cores of main-sequence stars, and 10 $M_\odot$, accounting for massive main-sequence stars or stellar-mass BHs.

Given the above parameters, and assuming that the orbits are circular, the GW radiation timescale (Peters 1964) is many orders of magnitude longer than the mission duration of LISA, about $t_{LISA} = 4$ yr. In this case, the GW spectrum is essentially monochromatic, and the increment of the frequency $\Delta f$ during the observational period is much smaller than the GW frequency $f$. Note that for circular orbits, $f = 2/P$.

In this situation, the $S/N$, which is an integration of the characteristic strain in the range of frequency shift, reduces to

$$\text{SNR}^2 = \frac{h_c^2 \Delta f}{f^2 S_n(f)},$$

(2)

where

$$h_c = \frac{1}{\pi d} \sqrt{\frac{2GE}{c^3 \dot{f}}},$$

(3)

is the characteristic strain, $d$ is the transverse comoving distance, $E$ is the power of the GW radiation, and $\dot{f} = -2P/P^2$ is the chirp rate defined in the rest frame of the source. In the last equation, $S_n(f)$ is the one-side amplitude.
spectral density of LISA (Robson et al. 2019). According to the last equation, the effective strain, which is directly proportional to the S/N, is

\[ h_{\text{eff}} = h_c \sqrt{\Delta f / f} \]  

(see also Barack & Cutler 2004). It is smaller than the normally adopted characteristic strain \( h_c \) of a fast-chirping signal by a factor of \( \sqrt{\Delta f / f} \). Another way of understanding \( h_{\text{eff}} \) is by noticing that the rms amplitude of the GW is \( h = h_c \sqrt{\Delta f / f} \), so that \( h_{\text{eff}} = h_c \sqrt{h_{\text{LISA}} \Delta f / f} = h_c \sqrt{N_{\text{obs}} / f} \), where \( N_{\text{obs}} \) is the number of GW cycles caught by LISA during the observational period. Therefore, the more cycles observed, the higher the S/N.

The effective strain computed using the above model, and assuming \( h_{\text{LISA}} = 4 \) yr, is shown in Figure 1 as the squares connected by the dashed lines. Each QPE is represented by a vertical line segment because we have allowed \( m \) to vary between 0.2 and 10\( M_\odot \). They are all below the LISA sensitivity curve \( \sqrt{1 / f S_n(f)} \) (and \( S/N \ll 1 \)), suggesting that LISA could not detect such QPEs.

### 2.2. WDs on Eccentric Orbits

We now consider another model, in which the small object in a QPE is a WD, and it is moving along a highly eccentric orbit around the central MBH (King 2020). In this model, GW radiation causes the orbital pericenter to decay until the WD fills the Roche lobe and starts feeding the MBH. Such a system can be characterized by four parameters: besides the \( M \) of the MBH and the \( m \) of the WD, there are also the semimajor axis \( a \) and the eccentricity \( e \) of the orbit. Following King (2020), we derive these parameters using their relationships with the observables of the QPEs. The basic steps are given below.

We first adopt the MBH mass \( M \) and orbital period \( P \) from the previous subsection. The semimajor axis can then be derived from \( P = 2\pi (GM/a^3)^{-1/2} \). To establish a relationship between the remaining two parameters, \( m \) and \( e \), we use the physical requirement that the mass transfer timescale \( t_{\text{MT}} \) equals the decay timescale of the pericenter \( t_{\text{p}} \), where \( M \) is the orbit-averaged accretion rate of the MBH and \( t_{\text{p}} = a(1-e) \) is the pericenter distance. Throughout this paper, the dot symbol denotes the time derivative.

The accretion rate \( \dot{M} \) is determined by the light curve of the eruptions. From the peak luminosity \( L \) of an eruption and its FWHM \( \Delta t \), we get \( \dot{M} = L \Delta t / (\eta \epsilon^2) \), where \( \eta \) is the radiative efficiency and \( \epsilon \) is the speed of light. To write \( t_{\text{p}} / t_{\text{p}} \) in terms of \( a \) and \( e \), we note that the specific angular momentum \( J = \sqrt{G(M+m)a(1-e^2)} \) is proportional to \( \sqrt{t_{\text{p}}} \) when \( e \approx 1 \). Therefore, we can write \( \dot{M} / M \approx [J / J] \). We note that, according to Peters (1964), \( [J / J] \) is proportional to \( (1-e^2)^{5/2} \), not \( (1-e^2)^{3/2} \), which is related to the loss of orbital energy and has been misused in previous works (e.g., King 2020; Zhao et al. 2021).

To close the equations, we need another relationship between \( m \) and \( e \). This is given by the condition of Roche-lobe overflow. This requires that, during the pericenter passage, the WD, which has a size of about \( R_{\text{WD}} \approx 0.013R_\odot (M/m)^{-1/3} \), fills the Roche lobe, whose radius is \( R_{\text{RoC}} \approx 0.46R_\odot (M/m)^{1/3} \) (King 2020). Finally, we find that

\[ m \approx 0.20C^{-15/22}M_\odot, \]  

\[ e \approx 1 - 0.072C^{5/11}P_4^{-2/3}, \]  

where

\[ C = \left( \frac{M}{10^5M_\odot} \right)^{4/15} \left( \frac{L \Delta t}{10^{35}\text{erg}} \right)^{-2/5} \left( \frac{\eta}{0.1} \right)^{2/5} \]  

and \( P_4 = P/(10^4 \text{s}) \).

The values of the observables, \( \Delta t \) and \( L \), as well as the derived physical parameters, \( m \) and \( e \), are given in Table 1. We find that \( m \) falls in the typical mass range of WDs, suggesting that the model is self-consistent. Moreover, \( e \) is higher than 0.9, consistent with the scenario that the WDs are delivered to the MBHs by either partial tidal disruption or binary separation, though the event rate is expected to be low (Metzger et al. 2022).

Because the binary is now eccentric, the GW radiation is spread into a wide range of harmonic frequencies (Peters & Mathews 1963). To compute the characteristic strain of the nth harmonic, which has a frequency \( n/P \) and a chirping rate of

| Source          | \( z \) | \( M/M_\odot \) | \( P/\text{ks} \) | \( \Delta t/\text{ks} \) | \( L/\text{erg s}^{-1} \) | \( m/M_\odot \) | \( e \) | References          |
|-----------------|-------|----------------|----------------|----------------|----------------|----------------|-------|-------------------|
| GSN 069         | 0.018 | \( 4.0 \times 10^2 \) | 31.55          | 2.05           | \( 5.0 \times 10^{14} \) | 0.322           | 0.972 | Minitti et al. (2019) |
| RX J1301.9+2747 | 0.02358 | \( 1.8 \times 10^3 \) | 16.5           | 1.2            | \( 1.4 \times 10^{12} \) | 0.150           | 0.928 | Giustini et al. (2020) |
| eRO-QPE1        | 0.0505 | \( 9.1 \times 10^2 \) | 66.6           | 13.7           | \( 3.3 \times 10^{12} \) | 0.461           | 0.986 | Arcodia et al. (2021)  |
| eRO-QPE2        | 0.0175 | \( 2.3 \times 10^3 \) | 8.64           | 0.8            | \( 1.0 \times 10^4 \) | 0.178           | 0.901 | Arcodia et al. (2021)  |
| XMMSSL1 J024916.6-041244 | 0.019 | \( 8.5 \times 10^4 \) | 9              | 1              | \( 3.4 \times 10^{11} \) | 0.169           | 0.901 | Chakraborty et al. (2021) |

**Figure 1.** The effective GW strain of the QPEs (colored squares and solid dots) vs. the LISA sensitivity curve \( \sqrt{1 / f S_n(f)} \) (gray dashed line). The squares connected by dashed lines correspond to the model in which the QPEs contain circular binaries. The dots refer to the model in which the QPEs are powered by WDs on eccentric orbits around MBHs.
\[ \dot{f}_{n,r} = -n \ddot{P} / P^2, \]
we use the formula
\[ h_{c,n} = \frac{1}{\pi d} \sqrt{\frac{2G\mathcal{E}_n}{c^2f_{c,n}^3}}, \]
derived in Barack & Cutler (2004), where \( \mathcal{E}_n \) denotes the radiative power of the \( n \)th harmonic. The effective strain, which is directly correlated with the \( S/N \), is computed with
\[ h_{c,N} = \sqrt{\sum \frac{\Delta f / f}{f}} \] (Barack & Cutler 2004), and is shown in Figure 1 as the colored dots.

It is clear that each QPE is now emitting a wide GW spectrum. We find that the peak of the spectrum occurs at a frequency of about \( \sqrt{GM/r_p^3} \), which can be understood due to the fact that the strongest GW radiation is produced when the WD passes the orbital pericenter. We also find that the frequency of the peak coincides with the most sensitive band of LISA, around 3 milli-Hertz (mHz). However, the effective strain remains below the sensitivity curve of LISA. The \( S/N \)s of the five QPEs are now \( S/N = (7.3, 6.3, 3.8, 4.3, 1.9) \). Since detecting a signal of comparable complexity (such as that of an EMRI) requires an \( S/N \) of about 20 (e.g., Babak et al. 2010), we conclude that LISA cannot detect these five QPEs.

3. GW Background

3.1. Background Computation

So far, we have shown that an individual QPE is unresolvable by LISA because the \( S/N \) is too low. However, a population of unresolved QPEs may form a background that has a significantly higher \( S/N \). This could happen if QPEs are made of WDs on eccentric orbits around MBHs. In this case, the GW spectrum is broad, as we have seen in the previous section. This broadness increases the chance of signal overlapping in the same frequency band.

To estimate the GW background produced by QPEs, we follow the method presented in Bonetti & Sesana (2020). This method was designed for EMRIs made of stellar-mass BHs, but with slight modification is applicable to QPEs, as we will explain in this section.

We start from the definition of the GW background (Phinney 2001), and write the characteristic strain as
\[ h_{c,gwb}(f) = \frac{4G}{\pi c f^2} \int \frac{dz}{1 + z} \frac{dn_c}{dz} \frac{dE}{d\ln f_c}, \]
where \( f \) is the frequency in the observer’s frame, \( f_c = f(1 + z) \) is in the rest frame of the source, and \( n_c \) denotes the number of WD–MBH mergers per unit comoving volume. The factor \( 1 + z \) on the denominator accounts for the redshift of the energy of the GWs, due to the expansion of the universe. Using Equation (8) to replace the term \( dE / d\ln f_c \) in the last equation, and after some algebra (also see the derivation of Equation (31) in Bonetti & Sesana 2020), we get
\[ h_{c,gwb}(f) = \int [dz \frac{dN}{d\ln P}] \frac{h_{c,n}}{2} \frac{f_n}{f_n^2}, \]
where \( d^2N / (dzd\ln P) \) is the number of QPEs (WD–MBH binaries) per unit redshift and in the bin of \( (\ln P, \ln P + d \ln P) \). Note that \( f_n \) and \( \dot{f}_n \) are defined in the observer’s frame, so that \( f_n(1+z) = n/P \) and \( \dot{f}_n(1+z)^2 = -n \ddot{P} / P^2 \).

The summation over \( n \) harmonics in the last equation reflects the fact that the GW signal in one frequency bin could come from a population of different QPEs, whose periods satisfy \( P = n/[f(1+z)] \). In our calculation, the maximum value of \( n \) is set to \( 10^5 \). Including higher harmonics does not significantly affect our results, because the corresponding QPEs have longer orbital periods. These QPEs, according to the relationship \( \dot{E} \propto r_p^{-7/2} P^{-1} \propto r_p^{-7/2} n^{-1} \) when \( e \sim 1 \), are significantly less powerful than those with shorter periods, i.e., smaller \( n \). The minimum value of \( n \) is normally chosen to be 1. However, when \( f \) is sufficiently large, \( P \) may become so short that the orbital pericenter enters the region where the corresponding Roche radius \( R_{RoC} \) is significantly smaller than the size of the WD \( R_{WD} \). The WD should have been tidally disrupted in this case. To eliminate such systems in the estimation of the GW background, we set \( h_{c,n}(f) = 0 \) whenever the condition \( R_{RoC}(r_p) = R_{WD}/2 \) is met.

Now, we determine the value of the term \( d^2N / (dzd\ln P) \) in Equation (10). We assume for simplicity that each QPE in the universe resembles one of the five detected QPEs shown in Table 1. This allows us to divide the GW background into five parts and calculate each part separately. Then, according to the continuity equation, \( dN/d\ln P \propto -P/P \), we can write
\[ \frac{d^2N}{dzd\ln P} = \sum_{i=1}^{5} C_i n_{c,i} \frac{dV}{dz} \left| \frac{P_i}{P} \right|. \]

where \( i \) denotes one of the five kinds of QPEs, \( C_i \) is a normalization factor for each kind, and \( V \) refers to the comoving volume. Since all the five observed QPEs are found within a redshift of \( z = 0.0505 \), which corresponds to a comoving volume of \( \Delta V \approx 0.041 \text{Gpc}^3 \), we can derive
\[ C_i n_{c,i} \Delta V \tau_i \approx 1 \]
by integrating Equation (11), where \( \tau_i = -\langle P_i / \dot{P}_i \rangle \) is a characteristic timescale given by the observed parameters of the \( i \)th QPE. Note that the product \( C_i n_{c,i} \) has the same dimension as the volumetric merger rate, because \( C_i n_{c,i} = 1 / (\Delta V \tau_i) \). The total volumetric merger rate is
\[ \mathcal{R} = \sum_{i=1}^{5} \mathcal{R}_i = \sum_{i=1}^{5} \frac{1}{\Delta V \tau_i} \approx 2.7 \text{ Gpc}^{-3} \text{yr}^{-1}. \]

In this way, we can compute the terms on the right-hand side of Equation (11) and derive the value of \( d^2N / (dzd\ln P) \).

Now, plugging Equation (11) into Equation (10), and assuming that the volumetric merger rate does not vary significantly with redshift, we find
\[ h_{c,gwb}(f) = \sum_{i=1}^{5} \int dz \frac{dV}{\Delta V \tau_i} \frac{P_i}{P} \left| \frac{h_{c,n,i}}{2} \right| \frac{f_n}{f_n^2}, \]
One can check that this equation is consistent with Equation (31) of Bonetti & Sesana (2020), since the term \( P_i / |P| \tau_i \) in our equation replaces their \( dN / d\ln P \). We note that Bonetti & Sesana performed an additional step of weighing to account for the short evolution timescales of their EMRIs. We can skip this step, because the evolution timescales of our QPEs \( \tau_i = 42, 162, 13, 289, \) and 677 yr) are much longer than the fiducial mission duration of LISA (4 yr).
Equation (14) can be further simplified, because \((1 + z)f_n/f_e = -P/P_e\). Therefore, we finally have

\[
h_{\text{eff}, \text{gwb}}^2(f) = \sum_{n=1}^{5} \int \frac{dV}{dz} \frac{dV_{\text{obs}}}{d\Omega} \frac{h_{\text{eff}}^2(f_{n,e})}{2f_{n,e} \tau_i}.
\] (15)

According to the last equation, we understand that before the WD is completely disrupted, each QPE contributes effectively

\[
h_{\text{eff}, \text{gwb}}^2 = \sum_{n=1}^{5} \frac{h_{\text{eff}}^2(f_{n,e})}{2f_{n,e} \tau_i} \quad \text{(16)}
\]

to the GW background.

### 3.2. Results

Figure 2 shows the GW background \(h_{\text{gwb}}^2\) computed according to Equation (15). The two solid curves correspond, respectively, to maximum redshifts of \(z = 0.5\) and 1 in the integration. The S/Ns of the GW background are calculated with

\[
\text{SNR}_{\text{gwb}}^2 = h_{\text{LISA}} \int \frac{h_{\text{gwb}}^2(f)}{f^2 S_\text{LISA}(f)} df
\] (17)

(Thrane & Romano 2013), and are 9.4 and 17 in the above two cases, if we adopt the fiducial observational time of \(t_{\text{LISA}} = 4\) yr. The S/N does not significantly increase even if we consider the QPEs at higher redshift \((z > 1)\).

We emphasize that the GW background shown in Figure 2 should be regarded as a lower limit. (i) The current QPE sample is by no means complete, because it is compiled from heterogeneous observations. (ii) In our calculation, the event rate of WD–MBH mergers is about 2.7 Gpc\(^{-3}\) yr\(^{-1}\)/\((6 \times 10^9\) galaxies Gpc\(^{-3}\)) \(\sim 4 \times 10^{-7}\) yr\(^{-1}\) per galaxy. Such a rate is roughly consistent with the model in which WDs are delivered to MBHs by two-body relaxation (Sigurdsson & Rees 1997; Freitag 2003). However, it is almost two orders of magnitude lower than the predicted rate if WDs are captured by MBHs due to the partial tidal disruption of red-giant stars (Bogdanović et al. 2014). The partial disruption of red giants, as has been pointed out by MacLeod et al. (2012), could contribute up to 10% to the total tidal disruption rate (a few \(\times 10^{-4}\) yr\(^{-1}\), according to Wang & Merritt 2004) if the MBH is smaller than about \(10^6\) \(M_e\).

If the number density of QPEs is indeed \(10^3\) times higher than the number density used in our model, the GW background would increase by one order of magnitude, since \(h_{\text{gwb}} \propto \sqrt{n_e}\), and the S/N would increase by a factor of \(10^2\), since SNR\(_{\text{gwb}} \propto h_{\text{gwb}}^2\). The result is shown in Figure 3 as the higher black dashed curve. The lower black dashed curve is the same as the background for \(z < 1\) shown in Figure 2. For comparison, we also plot pessimistic and optimistic estimations of the GW background due to EMRIs with stellar-mass BHs (cyan dashed curves; adopted from Bonetti & Sesana 2020). We find that at \(f \gtrsim 2\) mHz, the pessimistic background due to QPEs exceeds the pessimistic one due to stellar-BH EMRIs. Moreover, between 6 and 30 mHz, the optimistic GW background due to QPEs becomes comparable to the optimistic background due to stellar-BH EMRIs. Furthermore, in the most sensitive band of LISA (1–10 mHz), the GW background due to QPEs shows a positive slope, while the background due to EMRIs containing stellar BHs has a negative slope. Such a difference may be used in the future to distinguish the dominant source that is generating the background.

Bonetti & Sesana (2020) pointed out that an excessive GW background in the mHz GW band would impinge on several science goals of LISA, including the search for seed MBHs at \(z \gtrsim 20\) as well as detecting stellar-mass BBHs in their early inspiral phase. To understand the ramifications of our results, we also show in Figure 3 the GW signals of seed MBHs and stellar-mass BBHs, and compare them with the background due to QPEs and stellar-BH EMRIs. On one hand, we find that the pessimistic background due to QPEs and EMRIs becomes higher than the signals produced by stellar-mass BBHs only when the binaries emit low-frequency GWs, e.g., 1 mHz, or have a small chirp mass, e.g., \(10\) \(M_e\). These binaries, however, are too weak to be detected by LISA anyway. Therefore, in the most pessimistic case, the GW background is too weak to affect
the detection of seed BHs or stellar-mass BBHs. On the other hand, the most optimistic background is higher than the LISA noise in the frequency band of (1, 10) mHz. In particular, the combined GW background is comparable to the chirp signal of a seed BH binary at $z = 20$ if the chirp mass is lower than $300 M_{\odot}$ (see the orange curve). Such a mass corresponds to the seed BHs produced by Population III stars (Volonteri 2010). Moreover, the combined optimistic background in the 1–10 mHz band also exceeds the effective strain of a stellar-mass BBH at $z = 0.1$ if the chirp mass of the binary is smaller than $10 M_{\odot}$, or the frequency of the emitted GWs is lower than $3 \text{ mHz}$. The progenitors of many LIGO/Virgo BBHs fall into this region. These results highlight the necessity of compiling observationally a complete sample of QPEs to put a better constraint on the level of the GW background.

It is worth noting that if a large population of QPEs remain undetected so far, some of them may be close enough to be resolvable by LISA. For example, in the above optimistic scenario, where there are 100 times more QPEs within the volume of $z \leq 0.0505$, the nearest QPEs would be on average (100)$^{1/3} \approx 4.6$ times closer than those shown in Table 1, if QPEs are distributed uniformly in the local universe. Correspondingly, the S/N would also increase by 4.6 times. Given a detection threshold of S/N $= 20$ for a signal of this complexity (e.g., Babak et al. 2010), two kinds of QPEs, similar to GSN 069 and RX J1301.9+2747, would become resolvable by LISA. The exact number of QPEs that would be resolvable depends on the details of the population model, which are not well understood at present. A rough estimation suggests that about $100 \times (7.3/20)^{3} \approx 5$ GSN069-like QPEs and $100 \times (6.3/20)^{3} \approx 3$ RXJ1301.1-like ones may be resolvable.

4. Summary and Conclusion

Motivated by the suggestion that the recently discovered QPEs may contain EMRIs, we have calculated in this paper the GW spectra and studied their detectability by LISA. We have investigated two scenarios proposed in the literature, in which the orbits of the EMRIs are, respectively, circular and highly eccentric ($e > 0.9$). We find that in both cases the signal of each detected QPE is too weak to be discernible by LISA (Figure 1).

This conclusion differs from the one made by Zhao et al. (2021), mainly because we noticed that the systems are evolving on a timescale much longer than the mission duration of LISA, so that the number of GW cycles accumulated in the LISA band is greatly suppressed relative to the number of cycles coming from fast-chirping sources. Sesana et al. (2008) and Han & Fan (2018) have also studied the GW signal of a WD around an intermediate–massive BH (IMBH). We notice that the S/N derived by them is much higher than the detection threshold of LISA. This result is caused by the fact that their WDs are only a few years away from the final mergers with the IMBHs, while our QPEs are tens of hundreds of years before the mergers. The GW radiation is much weaker in our case.

More importantly, we find that if the EMRIs in QPEs are eccentric, their broad GW spectra could overlap in a wide range of frequencies, producing a background that has a much higher S/N. In the most pessimistic scenario (Figure 2), the S/N reaches 17 (9) due to those QPEs within a redshift of 1 (0.5). In the most optimistic case, the number density of the QPEs could be 100 times higher than what has been observed so far, and the S/N of the background would also increase by two orders of magnitude. In the latter optimistic case, the QPE background, together with the background produced by those EMRIs containing stellar-mass BHs, would become the dominant source of confusion in the most sensitive band of LISA, i.e., 1–10 mHz. The presence of such a background may affect the future search for seed BHs and stellar-mass BBHs by LISA (Figure 3).

Finally, we point out that whether QPEs are made of WDs moving on eccentric orbits around MBHs is still unclear. Further theoretical work is needed to identify observable signatures that can be used to distinguish different models.

This work is supported by the National Science Foundation of China, grants Nos. 11991053 and 11873022, the National Key Research and Development Program of China, grant No. 2021YFC2203001, and the China Manned Space Project (CMS-CSST-2021-B11). We thank Nick Stone, Xinwen Shu, Zhenyin Zhao, and Fayin Wang for helpful discussions, and the anonymous referee for constructive comments.

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