Baryons and Vector Dominance in Holographic Dual QCD

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The infinite tower of vector mesons encoded in holographic dual QCD bring drastic changes to the soliton structure of the nucleon. The nucleon is given by a point-like instanton in 5D surrounded by a vector meson cloud with the vector dominance restored by the infinite tower. I discuss the possible relevance of this structure in hot and dense hadronic matter.

§1. The objective

I would like to discuss in this talk a particular aspect of baryon structure that arises from holographic dual QCD (hQCD for short) that has an important ramification on some current issues of nuclear/hadronic physics. My talk will be largely motivated by the series of work\(^1,2\) I have done recently with my string theory/particle physics colleagues in Korea. This work addressed broadly two different issues. As nicely exposed in the recent talks by my colleagues\(^3,4\) to string theorists, it is already quite surprising that the notion of gravity/gauge duality can access certain properties of the baryons at such an accuracy, say, \(\sim 10\%\) level. Here string theory purports to first ascertain how well it postdicts the baryon properties well described by QCD proper or rather by its effective field theories and then to address problems that go beyond the standard model, e.g., baryon decay\(^5\) just to cite one.

What I am interested in here is quite different in nature. I would like to see in what way string theory may provide us with something that cannot, at present, be accessed by QCD proper.

I would like to first describe what it is that we would like to understand, describe what hQCD can actually do and then state what needs to be done to enable hQCD to answer the question posed.

§2. The origin of hadron mass

One of the currently active researches in strong interaction physics is to unravel how the ground-state hadrons, \(\rho, \omega, p\) and \(n\), that figure importantly in nuclear physics get most of their masses, given that the masses of the basic constituents, quarks, are tiny on the strong interaction scale. Numerous experiments have been done at various laboratories in the world and will continue in the upcoming facilities at CERN, GSI etc. in search of evidence for the assumed mechanism of the mass generation. At the intuitively simplest level, one may attribute the mass mostly to the spontaneous breaking of chiral symmetry as prescribed by QCD. Taking the \(\rho\)
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(and ω) as the prime example,∗ this would suggest that the vector-meson mass $m_v$ should “run” along the quark condensate $\langle \bar{q}q \rangle$ as the latter slides due to a vacuum change. If correct, this picture6 would predict that when nuclear matter is heated as in heavy-ion collisions or compressed as in compact stars, the mass would “shed”, and would go to zero in the chiral limit, as the critical temperature or density is reached from below since QCD dictates that the order parameter of chiral symmetry $\langle \bar{q}q \rangle \rightarrow 0$. This feature is concisely captured in what is called “BR scaling”.

The signal for BR scaling has been experimentally searched for with dileptons as a snapshot of the vector meson that propagates in hot/dense medium created in heavy-ion collisions. The idea is to map out the spectral functions of the vector meson as a function of invariant mass and look for the “dropping mass” effect below the free-space peak. The results so far obtained are under debate with a clear understanding still largely missing due to a plethora of background processes that are unrelated to the chiral vacuum properties being searched, but the consensus seems that the effect of BR scaling is not visible in the putative spectral functions so far extracted.

One hastily drawn conclusion was that “BR scaling is ruled out by Nature”. This then brings one back to square zero: Where does $m_v$ come from? While this possibility cannot yet be excluded at present, I will take the contrary view recently put forward7 that the dileptons measured in heavy-ion collisions, as they stand, carry no direct information on chiral symmetry. I will then suggest how hQCD with its infinite tower of vector mesons could confirm or refute whether the notion of BR scaling is valid.

§3. Vector dominance and hidden local symmetry

3.1. Vector manifestation

Recent developments indicate that at low energies below the chiral scale, $\Lambda_\chi \sim 4\pi f_\pi$, strong interactions are governed by hidden local symmetry theory denoted HLS$_\infty$ involving an infinite tower of flavor vector fields $v^{(k)}$, $k = 1, \cdots, \infty$ and that the dynamics of hadrons in medium with the possibility of vector meson mass dropping to near zero requires that local gauge symmetry be present. In this section, I will focus on the lowest members $v = \rho$ (and ω) in the presence of the (pseudo-) Goldstone pion fields. This can be done by formally integrating out all vector excitations of the tower except for the lowest. Let me call the resulting theory HLS$_1$. This (nonabelian) flavor gauge theory consisting of the Goldston pion field $\pi$ and the gauge field $\rho$ is considered to be an effective field theory of QCD valid at a scale much less than the chiral scale $\lambda_\chi$ encoding the correct chiral symmetry pattern $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_{L+R}$ and lends itself to a systematic chiral perturbation theory including the gauge degrees of freedom.8 The local gauge invariance in this theory allows one to easily do quantum loop calculations even when $m_v$ becomes comparable to that of the pion, which is zero in the chiral limit, a situa-

∗ The nucleon mass is somewhat subtler, requiring more details, so I will not go into it here. I advertise the monograph6 for discussions on this matter.
tion characterizing our approach which is difficult, if not impossible, to access in the unitary gauge used by other workers, e.g., massive Yang-Mills or tensor formalism.

To the leading order in the derivative expansion and neglecting the quark masses, HLS$_1$ has only three parameters, the gauge coupling $g$, the pion decay constant $F_\pi$ and the decay constant of the longitudinal component of the $\rho$ meson $F_\sigma$ or, more conveniently, the ratio $a = (F_\sigma/F_\pi)^2$. Matching the vector and axial-vector current correlators of HLS$_1$ to the corresponding QCD correlators at a matching scale $\Lambda_M \lesssim \Lambda_\chi$ determines $g$, $F_\pi$ and $a$ at $\Lambda_M$ in terms of the QCD variables $\alpha_c$, $\langle \bar{q}q \rangle$, $\langle G^2_{\mu\nu} \rangle$ etc. at $\Lambda_M$. Quantum one-loop calculations via renormalization group equations (RGE) reveal that the theory possesses a variety of fixed points. However, imposing that the vector and axial vector correlators be equal when chiral symmetry is restored with $\langle \bar{q}q \rangle = 0$ (in the chiral limit), picks one particular fixed point called “vector manifestation (VM)”:\(^8\)

$$g^* = 0, \quad a^* = 1. \quad (3.1)$$

In this theory, there is nothing special about the behavior of the parameter $F_\pi$ under RG flow other than that the physical (on-shell) pion decay constant $f_\pi = F_\pi + \Delta$ (with $\Delta$ pion-loop term) should vanish as the condensate vanishes. What is particularly significant for us is that the fixed point (3.1) is reached when $\langle \bar{q}q \rangle$ is dialled to zero, independently of how the dialing is done. In the case we are concerned with here the dialing is done by temperature (or density), so the VM fixed point can be identified with the chiral restoration point $T = T_c$ (or $n_c$). Very near the fixed point, the constants behave simply as

$$g \propto \langle \bar{q}q \rangle \to 0,$$

$$a - 1 \propto (\langle \bar{q}q \rangle)^2 \to 0. \quad (3.2)$$

This predicts the main properties for the problem at hand:

$$m_v \propto \langle \bar{q}q \rangle \to 0,$$

$$\Gamma \propto (\langle \bar{q}q \rangle)^2 \to 0. \quad (3.3)$$

The simple scalings (3.2) and (3.3) hold only very near the VM fixed point. Away from the “flash point” defined below, the physical properties such as pole mass etc. depend on the condensate in a much more complicated way and cannot be simply used as a signal for chiral symmetry.

3.2. Violation of vector dominance (VD)

In relativistic heavy ion collisions, there are a huge number of sources for dileptons which need to be judiciously taken into account for confronting experiments. For our discussion, we can simply zero in on the $\rho$ channel which is the dominant source of dileptons. We assume that we know how to sort out other sources, and focus on the dipions $\pi^+\pi^-$ and the $\rho^0$ meson as the principal dilepton sources. Now in zero temperature and matter-free space, vector dominance (VD) works very well, so the dileptons are produced via

$$\pi^+ + \pi^- \to \rho^0 \to \gamma^* \to l^+ + l^- \quad (3.4)$$
where \( l = \mu \text{ or } e \). There is no direct \( \gamma\pi^+\pi^- \) coupling — which is what VD means. In HLS\(_1\) theory, the photon coupling to \( \rho \) and \( \pi \) is given by

\[
\delta\mathcal{L} = -2eagF_\pi^2A^\mu_{em}\text{Tr}[\rho_\mu Q] + 2ie\left(1 - \frac{a}{2}\right)A^\mu_{em}\text{Tr}[V_\mu Q],
\]

(3.5)

where \( A^\mu_{em} \) is the photon field, \( Q \) the quark charge matrix and \( V_\mu \) the pionic vector current. The second term describing the photon-pion coupling vanishes when \( a = 2 \) which is the value \( a \) takes in free space, consistent with low-energy theorems. In hot/dense matter, there are thermal/dense loop corrections, but it is clear from (3.5) that VD must break down when temperature/density drives \( a \) to 1 as the second term starts contributing. The first term, when coupled to \( \pi^+\pi^- \), gives rise to a vector-mediated contribution reduced from the VD value by a factor \( \sim 2 \). Note that the photon-\( \rho \) coupling vanishes as \( g \to 0 \), which can be interpreted as the \( \rho \) wave function vanishing at the origin when the \( \rho \) mass vanishes.\(^7\)

When one probes vector-meson properties with the dilepton as a snapshot, one is looking at the first term of (3.5). While the second term as a source for dileptons would be absent if VD held, with the violation, this is no longer the case. The direct photon-pion coupling allows to be produced those dileptons that carry no direct information on the vector meson properties we are interested in.

Medium-dependent corrections to the photon-\( \rho \) coupling \( g_{\gamma\rho} = agF_\pi^2 \) can be calculated readily at one-loop order. The leading medium correction is the pion-loop correction to \( F_\pi^2 \) and this has been computed in temperature although not in density.\(^9\) The temperature-dependent one-loop correction gives, roughly, another factor \( \sim \sqrt{2} \) to the reduction factor.\(^10\) Density-dependent corrections will increase this factor further. Thus in the vicinity of the VM fixed point, there is a reduction factor in the photon-\( \rho \) coupling of \( \sim 2\sqrt{2} \) with respect to vector dominance with \( a = 2 \).

3.3. “Hadronic freedom”

To cover the range of temperature and density involved in the evolution of the hot/dense matter produced in heavy-ion collisions, a useful concept is the “flash point” at which the \( \rho \) meson recovers \( \sim 90\% \) of its free-space mass, its full strong coupling constant and \( a \sim 2 \). The flash point is more or less known in temperature, say, \( T_{\text{flash}} \sim 120 \text{ MeV} \) but not in density. A rough guess is that \( n_{\text{flash}} \sim 2 - 3n_0 \). Although density effects are not well known in HLS\(_1\), there is a strong indication that whenever nucleonic matter is present, \( a \) approaches near 1. This is observed in nucleon EM form factors. In HLS\(_1\), the baryon will appear as a skyrmion, giving a contribution to the form factors via \( V_\mu \) in (3.5) if \( a \neq 2 \). Indeed, phenomenology requires that \( a \) be very near 1,\(^2\) so VD is strongly violated already at one nucleon level. We expect this to be even more so in many-nucleon systems, i.e., nuclei and nuclear matter. This suggests that in baryonic matter between \( T_c \) and \( T_{\text{flash}} \), we may safely set \( a \sim 1 \). This could well be an oversimplification, so needs to be confirmed. Furthermore in this interval, the gauge coupling is assumed to be very weak so that\(^\ast\)

\(^\ast\) Introducing density in HLS\(_1\) is not yet worked out since it would involve introducing baryons as solitons.
we can ignore interactions involving the $\rho$ meson. This region in which hadronic interaction is ignorable is called “hadronic freedom” region.

The notion of the hadronic freedom anchored on vector manifestation of chiral symmetry has been applied to various heavy-ion processes. It is this notion that has been invoked to argue that all past dilepton experiments in heavy ion collisions have failed to see the signal for BR scaling.\(^7\)

§4. Return of vector dominance

4.1. The problem

The problem that hQCD with HLS\(^\infty\) may be exploited to resolve, assuming that it is a better approximation to QCD than HLS\(^1\), can now be precisely stated. What played a crucial role in HLS\(^1\) was that as the vector manifestation fixed point was approached, the $\rho$ mass vanished and the parameter $a$ went to 1 while the physical pion decay constant went to zero. There both the vanishing of the lowest-lying vector-meson mass and the violation of VD figured importantly. The question is: Is this a correct feature of QCD? We would like to know what hQCD can say about this. Let me first describe what we know about a hQCD model that reproduces correctly certain features of chiral symmetry of QCD predicted in the large $N_c$ and ’t Hooft limit.

4.2. HLS\(^\infty\)

The notion that HLS\(^1\) is an emergent gauge symmetry can be extended to an infinite tower of flavor local fields, arriving at a 5D Yang-Mills theory called a “dimensionally deconstructed QCD”.\(^11\) A closely related 5D YM theory arises top-down from string theory. A version that implements the spontaneously broken chiral symmetry and confinement, particularly pertinent to the problem at hand, is that constructed by Sakai and Sugimoto (SS) exploiting D4-D8/D\(^8\) branes.\(^12\) If I were to introduce adequately what goes into this model — which requires a battery of string theory terminologies, I would have no space for what I want to discuss. I will therefore have to skip that task entirely, referring to the publications\(^1),12\) for details. There are in the literature quite a few articles that review the SS model in detail, but too numerous to even cite properly.

I will simply start with the 5D action derived by SS\(^12\) in the form suitable for my purpose,\(^1\)

$$S = S_{YM} + S_{CS},$$

(4.1)

where

$$S_{YM} = - \int dx^4 dw \frac{1}{4e^2(w)} \text{Tr} F_{MN} F^{MN} + \cdots,$$

(4.2)

where $F$ is the 5D field tensors of nonabelian flavor gauge fields, $S_{CS}$ is the Chern-Simons action that encodes anomalies which we won’t need explicitly here and $(M, N) = 0, 1, 2, 3, 4$, $w$ is the fifth coordinate in a conformally flat coordinate and
$e(w)$ is the position-dependent “electric coupling” of the form

$$\frac{1}{e^2(w)} \propto \lambda N_c M_{KK} U(w). \quad (4.3)$$

Here $U(w)$ is the energy scale extended along the 5th ($w$) coordinate, $\lambda$ is the 't Hooft constant $\lambda = g_{YM}^2 N_c$ and $M_{KK}$ is the Klein-Kaluza mass that sets the only scale in the given approximation of the theory. The ellipses in (4.2) stand for higher derivative terms in the expansion of the DBI action for D8 branes in the D4 background, which are ignored.

The important point for what follows is that the gravity action (4.1) with (4.2) and (4.3) is dual to the gauge theory valid in the large $\lambda$ and $N_c$ limit. With the “probe approximation,” i.e., $N_f \ll N_c$, that ignores the back reaction of the flavor on the gluon background, the known classical supergravity solution enters and allows one to obtain the simple form (4.2). This theory may not have a correct UV completion to QCD. But for the problem at hand, what seems to crucially matter is the generic structure of the 5D YM action that is also arrived at bottom-up.

In the given approximations, there are only three parameters in the chiral limit, i.e., $\lambda$, $N_c$ and $M_{KK}$. For $N_c = 3$ required by nature, both $\lambda \approx 10$ and $M_{KK} \approx 0.94$ GeV are fixed by phenomenology in the meson sector.\(^{12}\) So there are no free parameters for the baryon sector that we are interested in.

In order to analyze what (4.1) describes in 4D, the standard procedure is to factorize the 5th coordinate by the Kaluza-Klein decomposition. The fifth dimension then plays the role of energy spread in the sense of RG flow. Choosing a suitable gauge, say, $A_5 = 0$, one finds that the resulting action is given by one in which Goldstone pion fields (in the chiral limit) are coupled to an infinite tower of nonabelian gauge fields with hidden gauge invariance, i.e., HLS\(\infty\).

4.3. Vector dominance in HLS\(\infty\)

This HLS\(\infty\) is found to describe surprisingly — or rather unreasonably — well a variety of meson processes involving vector mesons and pions. What is most significant — and perhaps generic for hQCD — is that there are no direct electroweak (EW) couplings to the pions, rendering all EW form factors of the pion totally vector-dominated. This can be seen immediately in the $A_5 \sim \pi$ gauge. One finds, in particular, the pion charge form factor — which is of isovector — takes the form

$$F_1^\pi(Q^2) = \sum_{k=1}^{\infty} \frac{g_{v(k)} g_{v(k)\pi\pi}}{Q^2 + m_{v(k)}^2}$$

with $Q^2 = -q^2$. Here the quantities $g_{v(k)\pi\pi}$ are the trilinear couplings between pions and the vector mesons denoted $v^{(k)}$ and $g_{v(k)}$ are the photon-vector meson coupling. Equation (4.4) shows that in the presence of an infinite tower, the “old” vector dominance by the lowest $v^{(1)} = \rho$ is replaced by a “new” one in which the infinite tower enters.

There is no surprise in this “new” vector dominance given that there is no direct photon-pion coupling. But what about the photon coupling to the nucleon which we know, is not vector-dominated when only the lowest vector mesons are present?
Here the situation looks very different at first sight because baryons must arise as solitons. In this 5D YM theory (4.2), a baryon must emerge as an instanton.\(^1\),\(^14\) It has been found\(^1\) that the nucleon properties that are reliably calculable in the quenched approximation in lattice QCD simulations can be reproduced well in this soliton model.\(^\star\)

The instanton size is found to go like \(R \sim 1/M_{KK} \sqrt{\lambda}\) modulo a constant factor of order 1, so it is pointlike in the large \(\lambda\) limit. Now the minimal coupling of EM field is holographically related to the minimal coupling of the 5D flavor gauge field, so one expects in the action a term of the form \(\int dx^4 \int dw [-iB \gamma^M (\partial_M - iA_M^{(N)})B]\) where \(B\) is the 5D baryon interpolating field for the instanton. So one would at first sight think that there will be a coupling of the photon to the small size instanton that is not vector-dominated in the usual sense. Such a picture naturally arises, agreeing well with experimental data to a large momentum transfer, in the usual Skyrme model implemented with fluctuating vector mesons,\(^13\) namely, the nucleon form factor described by the photon coupling to the skyrmion and the vector mesons, roughly of the same size as (3.5) with \(a \approx 1\). This skyrmion structure had been interpreted in terms of chiral bag model for the nucleon representing an “intrinsic core” in which the quark degrees of freedom of QCD reside.\(^15\) The infinite tower changes this structure most drastically. By a suitable field redefinition of the flavor gauge fields and exploiting the RG flow in the energy spread, the direct coupling photon-instanton coupling can be absorbed into the tower, rendering the nucleon form factors entirely given by the vector-dominated forms. For instance the isovector form factor is of the form identical to that of the pion (4.4) with only the pion replaced by the nucleon, \(F^N_{1}(Q^2) = \sum_{k=1}^{\infty} \frac{g_{v(k)} \cdot g_{v(k)} N N}{Q^2 + m_{v(k)}^2}. (4.5)\)

Here the infinite tower plays an indispensable role.

There are a number of remarkable features in what we have obtained in this model:

1. The infinite tower encoded in the instantonic structure brings basic changes to the structure of elementary baryon as well as that of dense hadronic matter. Among other things, the physical size of the nucleon is almost entirely given by vector-meson cloud\(^2\) leaving only \(\sim 0.1\) fm for the intrinsic degree of freedom. This is in line with what is found in chiral perturbation theory where the pion cloud plays a dominant role. Applied to many-nucleon systems, one expects the equation of state to be drastically modified.\(^6\)

2. The vector-dominated form factors are found to work well. Even the nucleon form factors come out within \(\sim 10\%\) accuracy for momentum transfers \(Q^2 \leq 1/2\) GeV\(^2\).

3. The sum rules \(F^\pi_{1}(0) = 1\) and \(F^N_{1}(0) = 1\) are both almost completely saturated by the lowest four isovector mesons. In both cases, the lowest \(\rho\) overshoots the sum rule by \(\sim 30\%\) which are mostly compensated by the next lying \(\rho'\).

\(^\star\) E.g., the calculated [experimental] values are: \(g_A \approx 1.30\) [exp: 1.27] and \((\mu_{\pi_n}^0 + \mu_{\pi_n}^0)/\mu_N \approx 0\) [exp: −0.1]. The difference from quenched lattice is expected to arise at \(O(N_c^{-2})\) as explained in 1).
4. By charge conservation, one obtains a new form of universality, \( \sum_{k=1}^{\infty} g_{\nu(k)} \pi \pi \simeq \sum_{k=1}^{\infty} g_{\nu(k)} N N \).

§5. The question for hQCD

To address the problem posed in §4.1, we need to understand how the masses of (at least) the low-lying four-vector mesons and the photon coupling to them and to the baryon, change in medium. The crucial quantity here is the quark condensate. The efforts to introduce the quark condensate and quark masses into the top-down hQCD — i.e., geometrically — are being made but the solution remains unknown. It is known in QCD that unlike the quantities that are well described in the large \( \lambda \) and \( N_c \) limit, the quark condensate is sensitive to \( N_c \): For \( N_c \to \infty \), it is independent of temperature up to \( T_c \) — which is obviously incorrect. This will be the same in hQCD. In HLS\(_1\), the violation of vector dominance and the vanishing of the gauge coupling as given in (3.2) figuring crucially in hadronic freedom are closely tagged to the quark condensate. Clearly one has to figure out how to compute \( 1/N_c \) (and perhaps also \( 1/\lambda \)) corrections.

The question to answer is: What do the HLS\(_1\) properties associated with the VM mean in terms of the infinite tower in HLS\(_\infty\)?

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