$B \rightarrow \phi K_s$ versus Electric Dipole Moment of $^{199}$Hg Atom in Supersymmetric Models with Right-handed Squark Mixing

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Abstract

The correlation between the CP asymmetry in $B \rightarrow \phi K_s$ ($S_{\phi K_s}$) and the chromoelectric dipole moment (CEDM) of strange quark ($d_s^C$), which is constrained by the electric dipole moment (EDM) of $^{199}$Hg, is studied in the supersymmetric (SUSY) models with the right-handed squark mixing. It is known that, if the right-handed bottom and strange squarks have a CP-violating mixing, such as in the SUSY SU(5) GUT with right-handed neutrinos, the induced gluon-penguin diagram may give a sizable contribution to $S_{\phi K_s}$. However, when the left-handed bottom and strange squarks also have a mixing, the conspiracy of the left and right-handed squarks may lead to a sizable $d_s^C$, which is enhanced by $m_b/m_s$. While the estimate for the EDM of $^{199}$Hg, induced by $d_s^C$, might have large uncertainties due to the hadron and nuclear dynamics, the current bound implies the gluon penguin contribution by the right-handed squarks to $S_{\phi K_s}$ should be suppressed and the deviation of $S_{\phi K_s}$ from the Standard Model may not be so large. Also, we discuss the constraint from the EDM of $^{199}$Hg in the SUSY SU(5) GUT with the right-handed neutrinos.
The CP violation in $B \to \phi K_s$ is sensitive to the new physics since $b \to s\bar{s}s$ is a radiative process [1]. Recently, the Belle experiment in the KEK $B$ factory reported that the CP asymmetry in $B \to \phi K_s (S_{\phi K_s})$ is $-0.96 \pm 0.50^{+0.09}_{-0.11}$, and $3.5\sigma$ deviation from the Standard-Model (SM) prediction $0.731 \pm 0.056$ is found [2]. At present the Babar experiment does not observe such a large deviation as $0.45 \pm 0.43 \pm 0.07$ [3]. Thus, the combined result is not yet significant, however, the Belle’s result might be a signature of the new physics.

It is known that the supersymmetric (SUSY) models may predict a sizable deviation of the CP violation in $B \to \phi K_s$ from the SM prediction. If the right-handed bottom and strange squarks have a sizable mixing, the gluon penguin diagram may give a non-negligible contribution to $b \to s\bar{s}s$ in a broad parameter space where the contribution to $b \to s\gamma$ is a sub-dominant. The right-handed squark mixing is well-motivated in the SUSY SU(5) GUT with the right-handed neutrinos, since the tau neutrino Yukawa coupling may induce the large mixing between the right-handed bottom and strange squarks [4]. Nowadays, $B \to \phi K_s$ in the SUSY models is extensively studied [5][6].

In this paper the correlation between the CP asymmetry in $B \to \phi K_s (S_{\phi K_s})$ and the chromoelectric dipole moment (CEDM) of strange quark ($d_s^C$) is studied in the SUSY models with the right-handed squark mixing. In various SUSY models, the left-handed bottom and strange quark mixing is as large as $\lambda^2 \sim 0.04$ since it is induced via the radiative correction by the quark Yukawa coupling with the CKM mixing. In this case the right-handed and left-handed squark mixings between the second and third generations may lead to the non-vanishing CEDM of the strange quark. Thus, $S_{\phi K_s}$ and $d_s^C$ may have a strong correlation in the SUSY models with the right-handed squark mixing. Since $d_s^C$ is constrained by the EDM of $^{199}$Hg, the gluon penguin contribution from the right-handed squark mixing to $S_{\phi K_s}$ should be suppressed.

First, we review the constraint on the CEDM of the strange quark. For the detail, see Ref. [8]. The effective Hamiltonian for the CEDM for quarks is given by

$$ H = \sum_{q=u,d,s} d_s^C i \frac{g_s \notg \notq}{2} \sigma^{\mu\nu} T^A \gamma_5 q C^A_{\mu\nu}. $$

It is known that the CEDMs of the light quarks generate the T-odd nuclear force, $\bar{N}NN' i\gamma_5 N'$, via the T-odd interaction of $\pi^0$ and $\eta$ to nucleons. For the EDM of $^{199}$Hg,
the EDMs of the constituent nucleons are electrically screened and the dominant contribution comes from the T-odd nuclear force [9]. Now the CEDM of the light quarks are strongly constrained by measurement of the EDM of $^{199}$Hg atom as [7]

$$e|d_d^C - d_u^C - 0.012d_s^C| < 7 \times 10^{-27} \text{ecm},$$  \hspace{1cm} (2)

based on the QCD sum rule calculation in Ref. [8]. The coefficient for the CEDM of the strange quark is suppressed compared with those for CEDM of the down and up quarks, $d_d^C$ and $d_u^C$, due to the heavier $\eta$ mass and the smaller T-even $\eta$ coupling than those of pion. From Eq. (2),

$$e|d_s^C| < 5.8 \times 10^{-25} \text{ecm},$$  \hspace{1cm} (3)

if $d_d^C$ and $d_u^C$ are negligible. In this paper we use this constraint on $d_s^C$. In the theoretical estimate of Eq. (2), the uncertainty from the nuclear dynamics is dominant, and it might be large. The neutron EDM may also lead to the constraint on $d_s^C$ comparable to Eq. (3) though it depends on the calculation [10]. Also, the authors in Ref. [11] show from the QCD sum rule that the Peccei-Quinn symmetry suppresses the contribution of $d_s^C$ to the neutron EDM. Thus, we do not use the constraint from the neutron EDM in this paper.

In the SUSY models, when the left-handed and right-handed squarks have mixings between the second and third generations, the CEDM of the strange quark is generated by a diagram in Fig. 1(a), and it is enhanced by $m_b/m_s$. Using the mass insertion technique, $d_s^C$ is given as

$$d_s^C = \frac{c\alpha_s m_{\tilde{g}}}{4\pi m_{\tilde{q}}} \left( -\frac{1}{3}N_1(x) - 3N_2(x) \right) \text{Im} \left[ (\delta_{LL}^{(d)})_{23} (\delta_{LR}^{(d)})_{33} (\delta_{RR}^{(d)})_{32} \right],$$  \hspace{1cm} (4)

where $m_{\tilde{g}}$ and $m_{\tilde{q}}$ are the gluino and averaged squark masses and $c$ is the QCD correction. We take $c = 3.3$. The functions $N_i$ are given as

$$N_1(x) = \frac{3 + 44x - 36x^2 - 12x^3 + x^4 + 12x(2 + 3x) \log x}{(x - 1)^6},$$  \hspace{1cm} (5)

$$N_2(x) = -\frac{10 + 9x - 18x^2 - x^3 + 3(1 + 6x + 3x^2) \log x}{(x - 1)^6}. $$  \hspace{1cm} (6)

The mass insertion parameters $(\delta_{LL}^{(d)})_{23}$, $(\delta_{RR}^{(d)})_{32}$, and $(\delta_{LR}^{(d)})_{33}$ are given by

$$(\delta_{LL}^{(d)})_{23} = \frac{(m_{\tilde{d}_L}^2)_{23}}{m_{\tilde{q}}^2}, \ (\delta_{RR}^{(d)})_{32} = \frac{(m_{\tilde{d}_R}^2)_{32}}{m_{\tilde{q}}^2}, \ (\delta_{LR}^{(d)})_{33} = \frac{m_b(A_b - \mu \tan \beta)}{m_{\tilde{q}}^2},$$  \hspace{1cm} (7)
where \((m_{\tilde{d}_{L(R)}}^2)\) is the left-handed (right-handed) squark mass matrix. In the typical SUSY models, \((\delta_{LL}^{(d)})_{23}\) is \(O(\lambda^2) \simeq 0.04\). From this formula, \(d^C_s\) is estimated in a limit of \(x \to 1\) as

\[
ed^C_s = \frac{\alpha_s}{4\pi} \frac{m_{\tilde{q}}}{m_{\tilde{g}}} \left(-\frac{11}{30}\right) \text{Im} \left[ (\delta_{LL}^{(d)})_{23} (\delta_{LR}^{(d)})_{33} (\delta_{RR}^{(d)})_{32} \right] \tag{8}
\]

\[
= -4.0 \times 10^{-23} \sin \theta \text{ e cm} \left( \frac{m_{\tilde{q}}}{500\text{GeV}} \right)^{-3} \left( \frac{\delta_{LL}^{(d)}}{0.04} \right) \left( \frac{\delta_{RR}^{(d)}}{0.04} \right) \left( \frac{\mu \tan \beta}{5000 \text{GeV}} \right) \tag{9}
\]

where \(\theta = \arg[(\delta_{LL}^{(d)})_{23} (\delta_{LR}^{(d)})_{33} (\delta_{RR}^{(d)})_{32}]\). Here, we neglect the contribution proportional to \(A_b\) since it is sub-dominant. From this formula, it is obvious that the right-handed squark mixing or the CP violating phase should be suppressed. For example, for \(m_{\tilde{q}} = 500\text{GeV}, \mu \tan \beta = 5000\text{GeV},\) and \((\delta_{LL}^{(d)})_{23} = 0.04,\)

\[|\sin \theta(\delta_{RR}^{(d)})_{32}| < 5.8 \times 10^{-4}. \tag{10}\]

In Ref. [12] the neutron EDM is discussed in the SUSY SO(10) GUT, in which the left-handed and right-handed squark mixings are induced. They show that the EDM of the down quark is enhanced by \(m_b/m_d\) due to the diagram similar to Fig. 1(a).

Now, let us discuss the correlation between \(d^C_s\) and \(S_{\phi K_s}\) in the SUSY models with the right-handed squark mixing. As mentioned above, the right-handed bottom and strange squark mixing may lead to the sizable deviation of \(S_{\phi K_s}\) from the SM prediction by the gluon penguin diagram. The box diagrams induced by the right-handed squark mixing also contribute to \(S_{\phi K_s}\), however, they tend to be sub-dominant and do not derive the large deviation of \(S_{\phi K_s}\) from the SM prediction. Thus, we neglect the box contribution in this paper for simplicity. The effective operator inducing the gluon penguin diagram by the right-handed squark mixing is

\[
H = -C^R_8 \frac{g_s}{8\pi^2} m_b s_R \sigma^{\mu\nu} T^A b_L G^A_{\mu\nu}. \tag{11}\]

When the right-handed squarks have the mixing, the dominant contribution to \(C^R_8\) is supplied by a diagram with the double mass insertion of \((\delta_{RR}^{(d)})_{32}\) and \((\delta_{RL}^{(d)})_{33}\) (Fig. 1(b)). Especially, it is significant when \(\mu \tan \beta\) is large. The contribution of Fig. 1(b) to \(C^R_8\) is given as

\[
C^R_8 = \frac{\pi \alpha_s m_{\tilde{q}}}{m_{\tilde{g}}^2 m_b} (\delta_{LR}^{(d)})_{33} (\delta_{RR}^{(d)})_{32} \left(-\frac{1}{3} M_1(x) - 3 M_2(x)\right) \tag{12}\]
up to the QCD correction. Here,
\[
M_1(x) = \frac{1 + 9x - 9x^2 - x^3 + (6x + 6x^2) \log x}{(x - 1)^5},
\]
\[
M_2(x) = -2 \frac{3 - 3x^2 + (1 + 4x + x^2) \log x}{(x - 1)^5}.
\]

In a limit of \(x \to 1\), \(C_R^8\) is reduced to
\[
\begin{align*}
C_R^8 & = \frac{7\pi\alpha_s}{30m_{\tilde{q}}} (\delta^{(d)}_{LR})_{33} (\delta^{(d)}_{RR})_{32}. 
\end{align*}
\]

Comparing Eq. (8) and Eq. (15), we find a strong correlation between \(d_s^C\) and \(C_R^8\) as
\[
d_s^C = -\frac{m_b}{4\pi^2} \frac{11}{7} \text{Im} \left[ (\delta^{(d)}_{LL})_{23} C_R^8 \right]
\]
up to the QCD correction. The coefficient 11/7 in Eq. (16) changes from 3 to 1 for \(0 < x < \infty\).

In Fig. 2, we show the correlation between \(d_s^C\) and \(S_{\phi K_s}\) assuming a relation \(d_s^C = -m_b/(4\pi^2)\text{Im}[(\delta^{(d)}_{LL})_{23} C_R^8]\) up to the QCD correction. Here, we take \((\delta^{(d)}_{LL})_{23} = -0.04, \arg[C_R^8] = \pi/2\) and \(|C_R^8|\) corresponding to \(10^{-5} < |(\delta^{(d)}_{RR})_{32}| < 0.5\). The matrix element of chromomagnetic moment in \(B \to \phi K_s\) is
\[
\langle \phi K_s | \frac{g_s}{8\pi^2} m_b (\bar{s_i} \sigma^{\mu\nu} T^a_{ij} P R b_j) G^a_{\mu\nu} | B_d \rangle = \kappa \frac{4\alpha_s}{9\pi} (\epsilon_{\phi P B}) f_{\phi} m_\phi^2 F_+(m_\phi^2),
\]
and \(\kappa = -1.1\) in the heavy-quark effective theory [6]. Since \(\kappa\) may suffer from the large hadron uncertainty, we take \(\kappa = -1\) and \(-2\). From this figure, it is found that the deviation of \(S_{\phi K_s}\) from the SM prediction due to the gluon penguin contribution should be tiny when the constraint on \(d_s^C\) in Eq. (3) is applied. If we loosen the CEDM constraint by a factor of 100 or more, the large deviation of \(S_{\phi K_s}\) by the gluon penguin diagram with the right-handed squark mixing is possible.

We ignored the constraint from \(b \to s\gamma\) in Fig. 2. If \((\delta^{(d)}_{RR})_{23} \sim O(1)\), the contribution may not be negligible. When the gluino penguin diagrams proportional to \((\delta^{(d)}_{RR})_{23}\) are dominant, \(Br(B \to X_s\gamma)\) is approximately given as
\[
Br(B \to X_s\gamma) = 7.0 \times 10^{-6} \left( \frac{\mu \tan \beta}{5000 \text{GeV}} \right)^2 \left( \frac{m_{\tilde{q}}}{500 \text{GeV}} \right)^{-4} \left( \frac{|(\delta^{(d)}_{RR})_{23}|}{0.04} \right)^2
\]
\[
(18)
\]
for large tan $\beta$. Here, we ignore the SM and other SUSY contributions for simplicity. Imposing that the SUSY contribution in Eq. (18) does not exceed the central value of the experimental branching ratio, $Br(B \to X_s\gamma) = (3.3 \pm 0.4) \times 10^{-4}$ [14], we can put the experimental bound on $(\delta_{RR}^{(d)})_{23}$ as

$$|\langle \delta_{RR}^{(d)} \rangle_{23}| \lesssim 0.27 \left( \frac{\mu \tan \beta}{5000 \text{GeV}} \right)^{-1} \left( \frac{m_q}{500 \text{GeV}} \right)^2,$$

and then, $S_{\phi K_s} \gtrsim (0.2 - 0.3) \ (-(0.3 - 0.4))$ for $\kappa = -1(-2)$. While this estimate of the bound on $(\delta_{RR}^{(d)})_{23}$ is a little rough, it is obvious that the constraint from the CEDM of the strange quark is much stronger than it.

Here, we notice that the above analysis of the CEDM of the strange quark and $S_{\phi K}$ is changed if the strange quark mass, $m_s$, is radiatively induced by the one-loop SUSY diagrams and the tree-level contribution is subdominant. This is because we have to perform a chiral rotation of the strange quark to make the mass term canonical when the induced mass has a phase. However, the $Br(B \to X_s\gamma)$ constraint implies that the radiative contribution to $m_s$ should be suppressed. When both the left-handed and right-handed squarks have mixings, the dominant SUSY contribution to $m_s$ comes from Fig. 3 and it is given as

$$\delta m_s \simeq 3 \text{ MeV} \left( \frac{(\delta_{LL}^{(d)})_{23}}{0.04} \right) \left( \frac{(\delta_{RR}^{(d)})_{32}}{0.04} \right) \left( \frac{\mu m_\tilde{q}}{m_\tilde{d}} \right) \left( \frac{\tan \beta}{50} \right) \left( \frac{m_b}{5 \text{ GeV}} \right) \left( \frac{f(x)}{1/3} \right),$$

where $f(x) = (2 + 3x - 6x^2 + x^3 + 6x \log x)/6/(1 - x)^4$. Thus, when the left-handed squark mixing is determined by the KM matrix as $(\delta_{LL}^{(d)})_{23} \simeq 0.04$, the one-loop correction to $m_s$ cannot dominate over the tree level due to the $Br(B \to X_s\gamma)$ constraint in Eq. (19). There is the logical possibility that there is some cancellation between the contributions to $Br(B \to X_s\gamma)$, in which case the contribution to the strange quark mass may not be small. Even in such a case, the effects on $S_{\phi K_s}$ is still small. The reason is that the quark mass matrix must be re-diagonalized, and the rotation of the right-handed strange quark can remove the contributions to $S_{\phi K_s}$.

Now we showed the strong constraint on $S_{\phi K_s}$ from $d_s^C$ in a case that the right-handed bottom and strange squarks have a mixing. Let us show the loopholes in this argument.

\[\text{This equation is derived from the formula in Ref. [13].}\]
First, $d_C^s$ is induced by both the left-handed and right-handed squark mixings between the second and third generations. Thus, if the left-handed squark mixing $(\delta_{LL}^{(d)})_{23}$ is smaller than $\sim 10^{-4}$, the constraint from the CEDM of the strange quark is not significant. Also, if the left-handed strange squark is heavier than the other squarks, the $d_C^s$ is suppressed while $S_{\delta K_s}$ is not changed. Second, we neglect the CEDM contribution to the EDM of $^{199}$Hg atom from the up and down quarks since it depends on the detail of the model. Thus, it might be possible that the combination $d_C^d - d_C^u - 0.012d_C^s$ is accidentally canceled below the experimental bound. Third, the estimate for the EDM of $^{199}$Hg atom might have large uncertainty due to the hadron and nuclear dynamics. Also, if $\kappa$ in Eq. (17) is extremely large, the sizable deviation of $S_{\phi K_s}$ might is possible. Fourth, the large left-handed squark mixing may supply the large gluon penguin contribution to $S_{\phi K_s}$ even if the right-handed squark mixing is suppressed. However, it that case, $b \to s\gamma$ constraint is strong, and the cancellation among the SUSY diagrams in $b \to s\gamma$ is required.

Finally, we discuss about the constraint from the the EDM of $^{199}$Hg in the SUSY SU(5) GUT with the right handed neutrinos. In the model the tau neutrino Yukawa coupling induces the right-handed down-type squark mixing between the second and third generations radiatively as

$$\left( m_{\tilde{d}_R}^2 \right)_{32} \approx -\frac{2}{(4\pi)^2} e^{i(\varphi_{d_2} - \varphi_{d_3})} U_{33} U_{23}^* \frac{m_{\nu_\tau} M_{\nu_\tau}}{(H_2)^2} (3m_0^2 + A_0^2) \log \frac{M_G}{M_{\text{GUT}}}, \quad (21)$$

and the mixing is enhanced by the large angle of the atmospheric neutrino. Here, we assume for simplicity that the right-handed neutrino mass matrix is diagonal. $m_{\nu_\tau}$ and $M_{\nu_\tau}$ are the left-handed and right-handed tau neutrino masses, $U$ is the MNS matrix, and $M_G$ and $M_{\text{GUT}}$ are the reduced Planck mass and GUT scale. The CP violating phase $e^{i(\varphi_{d_2} - \varphi_{d_3})}$ is inherent in the SUSY SU(5) GUT [4]. Since the phase of $U_{33} U_{23}^*$ is suppressed due to the small $U_{13}$, the phase of $(m_{\tilde{d}_R}^2)_{32}$ comes from the GUT inherent one dominantly. $(m_{\tilde{d}_R}^2)_{32}$, and then $d_C^s$, are proportional to $M_{\nu_\tau}$. From Eq. (21),

$$\left( \delta_{RR}^{(d)} \right)_{32} \approx -1 \times 10^{-3} \times e^{i(\varphi_{d_2} - \varphi_{d_3})} \times \left( \frac{m_{\nu_\tau}}{5 \times 10^{-2}\text{eV}} \right) \left( \frac{M_{\nu_\tau}}{10^{13}\text{GeV}} \right) \left( \frac{U_{33} U_{23}^*}{1/2} \right) \left( \frac{3m_0^2 + A_0^2}{3m_{\tilde{d}}^2} \right). \quad (22)$$

The CEDM of the strange quark is larger than the experimental bound when $M_{\nu_\tau}$ is larger than about $10^{12-13}$ GeV and $(\varphi_{d_2} - \varphi_{d_3})$ is of the order of 1. This means that the
measurement of the EDM of $^{199}$Hg atom is very sensitive to the right-handed neutrino sector in the SUSY SU(5) GUT. The current experimental bound on the EDM of $^{199}$Hg atom is determined by the statistics, and the further improvement is expected [7].

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Figure 1: a) The dominant diagram contributing to the CEDM of the strange quark when both the left-handed and right-handed squarks have mixings. b) The dominant SUSY diagram contributing to the CP asymmetry in $B \rightarrow \phi K_s$ when the right-handed squarks have a mixing.

Figure 2: The correlation between $d_s^C$ and $S_{\phi K_s}$ assuming $d_s^C = -m_b/(4\pi^2)\text{Im}[(\delta_{LL}^{(d)})_{23}C_{8R}]$. Here, $(\delta_{LL}^{(d)})_{23} = -0.04$ and $\text{arg}[C_{8R}] = \pi/2$. $\kappa$ comes from the matrix element of chromo-magnetic moment in $B \rightarrow \phi K_s$. The dashed line is the upper bound on $d_s^C$ from the EDM of $^{199}\text{Hg}$ atom.
Figure 3: The dominant SUSY contribution to the strange quark mass when both the left-handed and right-handed squarks have mixings.