Energy-cost-aware flow-shop scheduling systems with state-dependent energy consumptions

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Abstract. The necessity of managing a production system with less cost and pollution in such a competitive world has enticed the attention of many researchers. This paper deals with a multi-state two-machine flow-shop scheduling problem under Time-Of-Use electricity tariffs. The main issue is to assign a set of jobs to available time slots with different energy costs to minimize the total energy consumption costs. For this purpose, a new linear programming (LP) model is proposed, and several numerical results are solved using CPLEX Studio 12.9.0 solver to evaluate the efficiency of the proposed model for the problem. The obtained results demonstrate that CPLEX solver is not able to find the optimal solution for the problems as large as 20 jobs and 90 periods during the 3600 seconds time limitation.

Keywords: Flow-shop scheduling, Time-Of-Use electricity tariffs, Energy consumptions

1. Introduction

During the past decade, economic and societal developments have led to a rapid increase in energy consumption. Besides, the price of electricity as one of the main energy sources in industry has been constantly increased in most of industrial countries. Therefore, improving electric power efficiency and saving electricity plays a very important role in industry 4.0. Recently, various approaches have been proposed in both academia and industry to integrate energy efficiency into the scheduling problems to improve their efficiency and decrease their environmental pollution simultaneously. For example, Biel et al. 2016 [3] presented a literature review of decision support models for energy-efficient production planning. Merkert et al. 2015 [10] proposed several options to integrate energy management and scheduling on enterprise-wide optimization concepts. Gahm et al. 2016 [5] classified the literature based on three aspects and developed a framework for energy-efficient scheduling. The most popular objectives in the presented work are minimizing the amount of energy consumptions and (or) minimizing the cost of energy consumptions of the system. Generally, the energy consumption of a production system consists of the amount of energy consumed during the non-processing states (start-up, idle states, the transition between different states, and shut down states), and the amount of energy consumed during the processing state.

The other factors, which may modify the energy consumption of the system, are including the kind of machine or job and the processing speed of the machine. Furthermore, in practice, electricity suppliers in different countries propose variable pricing to balance the electricity supply and demand to improve the reliability and efficiency of electrical power grids. These variations of electricity prices affect the total energy consumption costs of any production system. The most usual classification of
time-varying rates is Time-Of-Use (TOU), Critical Peak Pricing (CPP), and Real-Time Pricing (RTP). Therefore, energy-efficient scheduling problem in a manufacturing system can be presented with the objective(s) of the energy consumption minimization, or (and) total energy cost minimization. One of the most popular solutions to minimize the total energy consumption of any system is to deal with the energy consumption of the non-processing states and use a scheduling method to change the processing jobs order and machine’s state during a period by shifting from on-peak periods to off-peak ones. In general, the shop floor configurations include single-machine, flow-shop, parallel-machine, and job-shop systems. The following literature review covers the previous studies that investigated the energy-efficient scheduling problem in a production unit with single-machine or flow-shop system. For the single-machine scheduling problem, Aghelinejad et al. 2018 [1] studied a problem with different possible states for the machine and different energy consumptions. They considered variable energy prices during the horizon time with the objective of minimizing the total energy consumption costs. Two versions of the problem were investigated. The case with a predetermined order for the processing jobs, and the case that searches the optimal schedule for the machine state and the jobs simultaneously. The complexity of several other energy-oriented single-machine scheduling problems with the objective of the total energy consumption costs minimization are addressed in [2]. Wang et al. 2016 [12] presented a bi-objective batch scheduling optimization problem to minimize the makespan and total energy cost of a single machine system.

They assumed the Time-of-use tariffs for electricity, and different energy consumption levels depending on the machine’s temperature level. Gong et al. 2016 [6] addressed a novel production scheduling method to minimize the energy cost considering finite state machine, multiple processes idle modes and time varied electricity prices. Gong et al. 2017 [7] proposed a method for energy-efficient and labor-aware production scheduling at the unit process level under real-time electricity pricing.

For the flow-shop scheduling problem, Masmoudi et al. 2017 [9] considered energy constraints and different energy costs during the planning horizon of a flow-shop system in a lot-sizing problem. Pileroood et al. 2018 [11] studied a two-machine flow shop scheduling problem under time-dependent electricity tariffs when the energy consumption depends on the jobs and the machines. Zhai et al. 2017 [14] proposed a dynamic scheduling approach to minimize the electricity cost of a flow shop system containing m machines with a grid-integrated wind turbine under real-time pricing. They also considered three states for the machines like off, basic and working state. Fazli Khalaf and Wang 2018 [4] addressed a two-stage stochastic flow-shop scheduling problem to minimize the total electricity purchase cost. Liu et al. 2018 [8] investigated a permutation flow shop scheduling problem to minimize the total idle energy consumption of the machines. For the cases with two- machines, they proposed a relaxed Johnson’s algorithm to obtain the optimal schedule, and for the cases with multiple machines (more than 2), they proposed a novel NEH heuristic algorithm to obtain an approximate energy-saving schedule. Zhang et al. 2019 [15] studied a flexible flow shop scheduling problem in the environment of TOU energy prices. They established an energy consumption model of machine tools that involves the processing energy, standby energy, and set-up energy. Wang et al. 2018 [13] considered a two-machine permutation flow shop scheduling problem to minimize the total electricity cost of processing jobs under time-of-use electricity tariffs. They proposed a mixed-integer linear programming model and two heuristic algorithms for the problem.

A comprehensive review of previous researches on production scheduling and energy consumption problems demonstrate that only few of them considered a multi-state flow-shop scheduling problem under time varied energy prices. In this study, we focused on the energy
consumption costs minimization of a two-machine flow shop system by using the scheduling method. The objective is to propose a new mathematical model for this problem and evaluate the performance of the existing solver (CPLEX Studio 12.9.0) for different size of the problem. The remainder of this manuscript is organized as follows. In section 2, the problem is defined and its assumptions are described. A new mathematical model is presented for this problem. In section 3, the numerical experiments based on several randomly generated instances are presented to evaluate the efficiency of the proposed model. Finally, section 4 draws the conclusions as well as the future directions of our study.

2. Problem definition

This research deals with a two-machine flow shop problem with a given set of jobs $J = (1, 2, ..., n)$ that are available to be processed at time zero. The jobs must proceed during a horizon of time with $T$ periods on which the Time-Of-Use electricity tariff is considered ($G_t$: Cost of energy in period $t$). The studied machines have five main states $s$ (OFF, ON, Idle, Ton and Toff). The possible transitions between OFF and ON states for turning on and off the machines are named as Ton and Toff states. To simplify the problem, the five possible states of the machine are considered as integer numbers ($s = 1$ for OFF, $s = 2$ for ON, $s = 3$ for Idle, $s = 4$ for Ton and $s = 5$ for Toff). Each machine $i$ consumes a specific amount of energy ($E_i$) in state $s$ for a specific number of periods which depends on the state ($d_{i,s}$; $s \in \{\text{ON, OFF, Idle, Ton, Toff}\}$). Each job requires to be processed at first on machine 1 and then on machine 2. Each machine can process at most one job per period, and each job can be processed on one machine at most in the same period. Once the processing of a job on a machine has started, it must be completed without preemption. Also, each job must be processed in the same order on every machine. Let $p_{j,1}$ and $p_{j,2}$ be the processing times of job $j$ on machines 1 and 2 respectively.

![Figure 1. The possible states and transitions for machine $i \in \{1,2\}$.](image)

As it is illustrated in Figure 1, the machine must be in OFF state during the initial and final periods. It must be mention that the energy consumption of the machine $i$ in state OFF is considered to be $0$ ($E_{i,1} = 0$). When it is decided to turn on the machine $i$, this transition takes $d_{i,4}$ periods and it consumes $E_{i,4}$ units of energy per period. Then the machine is in ON state and is ready to process a job $j = 1, \ldots, n$, that takes $p_{j,i}$ periods and consumes $E_{i,2}$ units of energy per period. Once the selected job is processed completely, there are three possibilities for the machine: it may stay in ON state and process the next job; it can go to Idle state for one period or more and also, the machine can go to OFF state. Regarding the energy consumption and the unit of energy price, any of these possibilities may be selected. Note that, in this study, the transition time between Idle and ON states, and its energy consumption are neglected. Besides, the transition between Idle and OFF states is not allowed. Therefore, when the machine is in the Idle state, for the next period, it may stay in Idle state or pass to ON state. The objective of this study is to find the most economical production schedule in terms of energy consumption costs during the given time horizon.
Decision variables:

\[ \alpha_{i,s,t} = \begin{cases} 
1 & \text{if machine } i \text{ is in state } s \text{ during period } t \\
0 & \text{otherwise} 
\end{cases} \]

\[ y_{j,i,t} = \begin{cases} 
1 & \text{if job } j \text{ is in processing on machine } i \text{ during period } t \\
0 & \text{otherwise} 
\end{cases} \]

Mathematical model:

\[
\min \sum_{t=0}^{T} c_t \left( \sum_{i=1}^{2} \sum_{s=1}^{5} E_{ls} \cdot \alpha_{i,s,t} \right)
\]

\[ \alpha_{i,1,t} = \begin{cases} 
1 & \forall i = 1,2; \; t \in [0,T] 
\end{cases} \] (2)

\[ \alpha_{i,5,t} = \begin{cases} 
1 & \forall i = 1,2; \; \forall t \in [0,T] 
\end{cases} \] (3)

\[ \alpha_{i,1,t} \leq \alpha_{i,1,t+1} + \alpha_{i,4,t+1} ; \forall i = 1,2; \; \forall t \in [0,T-1] \] (4)

\[ \alpha_{i,2,t} \leq \alpha_{i,2,t+1} + \alpha_{i,3,t+1} + \alpha_{i,5,t+1} ; \forall i = 1,2; \; \forall t \in [0,T-1] \] (5)

\[ \alpha_{i,3,t} \leq \alpha_{i,2,t+1} + \alpha_{i,3,t+1} ; \forall i = 1,2; \; \forall t \in [0,T-1] \] (6)

\[ \alpha_{i,4,t} \leq \alpha_{i,4,t+1} + \alpha_{i,2,t+1} ; \forall i = 1,2; \; \forall t \in [0,T-1] \] (7)

\[ \alpha_{i,5,t} \leq \alpha_{i,5,t+1} + \alpha_{i,1,t+1} ; \forall i = 1,2; \; \forall t \in [0,T-1] \] (8)

\[ \sum_{t'=t+1}^{t+d_{i,4}} \alpha_{i,4,t'} \geq (\alpha_{i,4,t+1} + \alpha_{i,1,t} - 1) \cdot d_{i,4} ; \forall i = 1,2; \; \forall t \in [0,T-d_{i,4}] \] (9)

\[ \sum_{t'=t+1}^{t+d_{i,5}} \alpha_{i,5,t'} \geq (\alpha_{i,5,t+1} + \alpha_{i,2,t} - 1) \cdot d_{i,5} ; \forall i = 1,2; \; \forall t \in [0,T-d_{i,5}] \] (10)

\[ \alpha_{i,s,t} + \alpha_{i,s,t+d_{i,s}} \leq 1 ; \forall i = 1,2; \forall t \in [0,T-d_{i,s}]; \; \forall s \in \{4,5\} \] (11)

\[ \sum_{j=1}^{n} y_{j,i,t} = \alpha_{i,2,t} ; \forall i = 1,2; \forall t \in [0,T] \] (12)

\[ \sum_{j=1}^{n} y_{j,i,t} \leq 1 ; \forall i = 1,2; \forall t \in [0,T] \] (13)

\[ p_{j,1} \cdot y_{j,2,t} \leq \sum_{t'=0}^{t-1} y_{j,1,t'} ; \forall t \in [0,T]; \forall j \in [1,n] \] (14)
\[
\sum_{t'=0}^{t-p_{j,i}} y_{j,i,t'} + \sum_{t'=t+p_{j,i}}^{T} y_{j,i,t'} \leq p_{j,i} \cdot (1 - y_{j,i,t}) \quad \forall i = 1,2; \forall t \in \left[p_{j,i} T - p_{j,i} - 1\right] \\
\quad \forall j \in [1,n] \\
\sum_{t=1}^{T} y_{j,i,t} \geq p_{j,i} \quad \forall i = 1,2; \forall j \in [1,n] \\
\alpha_{i,s,t}, y_{j,i,t} \in \{0,1\} 
\]

In this model, the objective is to minimize the total energy consumption cost of the system, which depends on the unit of electricity price in each period, as well as, the energy consumption of each machine (the machine’s state) in each period (equation (1)). Equation (2) identifies that the machines are in OFF state during the initial and final periods. Equation (3) expresses that in each period the machine must be in one of the possible states (ON, OFF, Idle, Ton, and Toff). Equations (4) to (8) limit the machine’s state in each period regarding its state in the previous period. Equations (9), (10) and (11) identify lower and upper number of required periods for Ton and Toff states. Equation (12) and (13) indicate that each machine may process at most one job per period, and if machine \( i \) processes job \( j \) during period \( t \), it must be in state ON \( (s = 1) \). Equation (14) expresses that each job requires to be processed at first on machine 1 and then on machine 2. Equation (15) translates the non-preemption constraints of the jobs. Equation (16) specifies the processing time of each job on each machine.

3. Numerical experiments

To analyze the performance of the proposed model, several criteria like objective value, the number of variables and constraints as well as computing time are considered. For this purpose, numerous instances are randomly generated which are solved using CPLEX Studio 12.9.0 solver.

For generating the data, the instance size corresponds to the number of processing jobs \( (n) \) and the number of existing periods \( (T) \) in the operating horizon, and for each size of instance, five different instances are generated. Processing time of each job is considered as an integer random number between 1 and 5 periods, and the unit of energy cost per period is generated randomly between 1 and 10. Since the computation time for all the instances is limited to one hour (3600 s), the obtained results for the instances with the size as large as 15 jobs and 70 periods are represented in Table 1.

As can be seen in Table 1, the average number of variables and constraints for these instances are equal to 1663 and 2850, respectively. Moreover, the average computation time for these instances is 825 seconds.

| \((n,T)\) | Obj value | Nb variables | Nb constraints | CPU (s) |
|----------|-----------|--------------|----------------|--------|
| \((5,30)\)_1 | 872       | 620          | 1147           | 1.7    |
| \((5,30)\)_2 | 719       | 620          | 1148           | 0.55   |
| \((5,30)\)_3 | 785       | 620          | 1153           | 0.34   |
| \((5,30)\)_4 | 925       | 620          | 1145           | 8.02   |
| \((5,30)\)_5 | 1338      | 620          | 1140           | 3.33   |
4. Conclusions and future directions
A scheduling problem with two-machine flow-shop system is considered in this paper to minimize the total energy consumption costs. The considered machines have multi-state and the energy costs are time varying within the horizon time. For this problem, a mathematical model is proposed and its performance is evaluated based on several numerical instances. The obtained results demonstrate that, CPLEX solver is not able to find the optimal solution for the problems as large as 20 jobs and 90 periods during the 3600 seconds time limitation.

As a result, for future works, it could be interesting to propose some heuristic and meta-heuristic algorithms that can solve the medium and large size instances of this problem in a reasonable time. Considering the release date and set-up time for each job, and the energy constraint at each period can be proposed as the potential research directions of this study.

5. References
[1] Aghelinejad, M., Ouazene, Y., & Yalaoui, A. (2018). Production scheduling optimisation with machine state and time-dependent energy costs. International Journal of Production Research, 56(16), 5558-5575.
[2] Aghelinejad, M., Ouazene, Y., & Yalaoui, A. (2019). Complexity analysis of energy-efficient single machine scheduling problems. Operations Research Perspectives, 6, 100105.
[3] Biel, K., & Glock, C. H. (2016). Systematic literature review of decision support models for energy-efficient production planning. Computers & Industrial Engineering, 101, 243-259.
[4] Fazli Khalaf, A., & Wang, Y. (2018). Energy-cost-aware flow shop scheduling considering intermittent renewables, energy storage, and real-time electricity pricing. International Journal of Energy Research, 42(12), 3928-3942.
[5] Gähm, C., Denz, F., Dirr, M., & Tuma, A. (2016). Energy-efficient scheduling in manufacturing companies: A review and research framework. European Journal of Operational Research, 248(3), 744-757.
[6] Gong, X., De Pessemier, T., Joseph, W., & Martens, L. (2016, September). A power data driven energy-cost-aware production scheduling method for sustainable manufacturing at the unit process level. In 2016 IEEE 21st International Conference on Emerging Technologies and Factory Automation (ETFA) (pp. 1-8). IEEE.
[7] Gong, X., Van der Wee, M., De Pessemier, T., Verbrugge, S., Colle, D., Martens, L., & Joseph, W. (2017). Integrating labor awareness to energy-efficient production scheduling...
under real-time electricity pricing: An empirical study. Journal of cleaner production, 168, 239-253.

[8] Liu, G. S., Li, J. J., & Tang, Y. S. (2018). Minimizing Total Idle Energy Consumption in the Permutation Flow Shop Scheduling Problem. Asia-Pacific Journal of Operational Research, 35(06), 1850041.

[9] Masmoudi, O., Yalaoui, A., Ouazene, Y., & Chehade, H. (2017). Lot-sizing in a multi-stage flow line production system with energy consideration. International Journal of Production Research, 55(6), 1640-1663.

[10] Merkert, L., Harjunkoski, I., Isaksson, A., S¨aynevirta, S., Saarela, A., & Sand, G. (2015). Scheduling and energy–Industrial challenges and opportunities. Computers & Chemical Engineering, 72, 183-198.

[11] PILEROOD, A. E., HEYDARI, M., & MAZDEH, M. M. (2018). A two-stage greedy heuristic for a flowshop scheduling problem under time-of-use electricity tariffs. South African Journal of Industrial Engineering, 29(1), 143-154.

[12] Wang, S., Liu, M., Chu, F., & Chu, C. (2016). Bi-objective optimization of a single machine batch-scheduling problem with energy cost consideration. Journal of cleaner production, 137, 1205-1215.

[13] Wang, S., Zhu, Z., Fang, K., Chu, F., & Chu, C. (2018). Scheduling on a two-machine permutation flow shop under time-of-use electricity tariffs. International Journal of Production Research, 56(9), 3173-3187.

[14] Zhai, Y., Biel, K., Zhao, F., & Sutherland, J. W. (2017). Dynamic scheduling of a flow shop with on-site wind generation for energy cost reduction under real time electricity pricing. CIRP Annals, 66(1), 41-44.

[15] Zhang, M., Yan, J., Zhang, Y., & Yan, S. (2019). Optimization for energy-efficient flexible flow shop scheduling under time of use electricity tariffs. Procedia CIRP, 80, 251-256.