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To cite this article: V.N. Zirakashvili et al 2019 J. Phys.: Conf. Ser. 1181 012026

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Sources, spectra and composition of ultra-high energy extragalactic cosmic rays

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Abstract. The inverse problem of cosmic ray transport of ultra-high energy cosmic rays is considered. The source spectrum and composition are derived based on the Auger data on energy spectrum, energy dependence of mean logarithm of atomic mass number and its variance. The regularization procedure for considered ill-posed problem and the statistical analysis of experimental data are employed.

1. Introduction
The origin of ultra-high energy cosmic rays is a key problem of cosmic ray astrophysics. The observed suppression the cosmic ray flux at energies above $\sim 5 \times 10^{19}$ eV is most probably associated with two effects: the GZK cutoff [1, 2] and acceleration limit in cosmic ray sources [3, 4]. The isotropy of the highest energy cosmic rays are indicative of their extragalactic origin. The list of potential sources which could give the observed cosmic ray flux includes active galactic nuclei, gamma-ray bursts, fast spinning newborn pulsars, interacting galaxies, large-scale structure formation shocks and some other objects, see reviews [5, 6] and references therein.

The present knowledge about the highest energy cosmic rays was mainly acquired from the Pierre Auger Observatory (Auger), Telescope Array experiment (TA), and from the Yakutsk complex EAS array, see [7, 8, 9]. The mass composition of these cosmic rays remains uncertain. The interpretation of HiRes and TA data favors predominantly proton composition at energies $10^{18}$ to $5 \times 10^{19}$ eV, whereas the Auger data indicate that the cosmic ray composition is becoming heavier with energies changing from predominantly proton at $10^{18}$ eV to more heavy composition at about $5 \times 10^{19}$ eV. The mass composition interpretation of the measured quantities depends on the assumed hadronic model of particle interactions which is based on not well determined extrapolation of physics from lower energies.

The energy spectrum in extragalactic sources is commonly determined by the trail-and-error method when one makes the calculations of the expected at the Earth cosmic ray intensity assuming some shape of the source energy spectrum and the source composition. The calculations follow cosmic ray propagation from the source to the observer, e. g. [10]. The standard assumption is that the source spectrum is a power law on magnetic rigidity up to some maximum rigidity.

In our work [11] we showed how to inverse the procedure and calculate the source function starting from the observed at the Earth spectrum without ad hoc assumptions about the shape
of source spectrum. The only assumption done is that different nuclei follow the same spectral function on the magnetic source rigidity. Simple cases of the source composition that includes protons and Iron nuclei were considered. The case of complex source composition with a given abundances of a few elements was considered in [12]. More complete statistical analysis of cosmic ray data including the fitting of source composition is the subject of the present work.

2. Solution of inverse problem for a system of cosmic-ray transport equations

We use the following transport equation for cosmic ray protons and nuclei in the expanding Universe filled with the background electromagnetic radiation (see [13] for details):

\[
- \frac{\partial}{\partial \varepsilon} \left( \varepsilon \left( \frac{H(z)}{(1 + z)^3} + \frac{1}{\tau(A, \varepsilon, z)} \right) F(A, \varepsilon, z) \right)
- H(z)(1 + z) \frac{\partial}{\partial z} \left( \frac{F(A, \varepsilon, z)}{(1 + z)^3} \right) + \nu(A, \varepsilon, z) F(A, \varepsilon, z)
= \sum_{i=1,2...} \nu(A + i \rightarrow A, \varepsilon, z) F(A + i, \varepsilon, z) + Q(A, \varepsilon)(1 + z)^m \tag{1}
\]

The system of Eqs. (1) for all kinds of nuclei with different mass numbers \(A\) from Iron to Hydrogen should be solved simultaneously. The energy per nucleon \(\varepsilon = E/A\) is used here because it is approximately conserved in a process of nuclear photodisintegration, \(F(A, \varepsilon, z)\) is the corresponding cosmic-ray distribution function, \(z\) is the redshift, \(Q(A, \varepsilon)\) is the density of cosmic-ray sources at the present epoch \(z = 0\), \(m\) characterizes the source evolution (the evolution is absent for \(m = 0\)), \(\tau(A, \varepsilon, z)\) is the characteristic time of energy loss by the production of \(e^-e^+\) pairs and pions, \(\nu(A, \varepsilon, z)\) is the frequency of nuclear photodisintegration, the sum in the right side of Eq. (1) describes the contribution of secondary nuclei produced by the photodisintegration of heavier nuclei, \(H(z) = H_0((1 + z)^3 \Omega_m + \Omega_\Lambda)^{1/2}\) is the Hubble parameter in a flat universe with the matter density \(\Omega_m = 0.3\) and the \(\Lambda\)-term \(\Omega_\Lambda = 0.7\).

The numerical solution of cosmic-ray transport equations follows the finite differences method. The variables are the redshift \(z\) and \(\log(E/A)\).

Let us introduce solution \(G(A, \varepsilon; A_s, \varepsilon_s)\) of Eqs. (1) at \(z = 0\) for a delta-source \(Q(A, \varepsilon) = \delta_{AA_s} \delta(\varepsilon - \varepsilon_s)\). This source function describes the emission of nuclei with mass number \(A_s\) and energy \(\varepsilon_s\) from cosmic ray sources distributed over all \(z\) up to some \(z_{\text{max}}\). The general solution of Eqs. (1) at the observer location \(z = 0\) can now be presented as

\[
F(A, \varepsilon, z = 0) = \sum_A \int d\varepsilon' G(A, \varepsilon; A', \varepsilon') Q(A', \varepsilon'). \tag{2}
\]

The observed all-particle spectrum is determined by the summation over all types of nuclei \(\sum_A F(A, E/A, z = 0)/A\) that is

\[
N(E) = \sum_{A,A'} A'^{-1} \int d\varepsilon' G(A, E/A; A', \varepsilon') Q(A', \varepsilon'). \tag{3}
\]

As noted above, we shall assume that the source spectra of nuclei can be expressed in terms of one function on rigidity:

\[
Q(A, \varepsilon) = k(A) q(\varepsilon A/Z) \tag{4}
\]

Here \(q(\varepsilon)\) is the source proton spectrum and coefficients \(k(A)\) determine the source chemical composition.
The set of discrete values of particle energy $\varepsilon_i$ is defined to solve the transport equation numerically. The grid with constant $\Delta\varepsilon/\varepsilon$ and with 25 energy bins per decade is used in our calculations. Eq. (3) in the discrete form is

$$N_i = \sum_j S_{ij} q_j,$$

$$S_{ij} = \sum_{A,A'} Z(A') k(A') A'A' \Delta\varepsilon G_{ij}(A,E_i/A;A',\varepsilon_j Z(A')/A'),$$

where the subscript indexes $i$ and $j$ denote the corresponding energies $\varepsilon_i$ and $\varepsilon_j$.

The source term $q_j$ can be found from this set of linear Eqs. (5) if the observed all particle spectrum $N(E)$ and chemical composition of the source are known. We considered the case of protons and iron nuclei in the source [11]. It was found that the solutions of equation (5) have a physical meaning only for a limited range of proton to iron ratio. In addition the solution can be unstable relative small deviations of the left hand side of Eq. (5) so that the inverse problem is ill-posed. Below we use the following regularization procedure [14] for the whole set of equations.

Let introduce the functional

$$L = \sum_i \left( 1 - \frac{1}{N_i} \sum_j S_{ij} q_j \right)^2 + \varepsilon_R \sum_j (q_{j-1} - 2q_j + q_{j+1})^2$$

Here $\varepsilon_R$ is the regularization parameter. The first term in this equation is simply the sum of squared relative deviations from the observable spectrum $N(E)$. For $\varepsilon_R = 0$ this functional is minimized by solutions of Eqs. (5) and and its value equals to zero.

Renormalized set of equations is found from the condition $\partial L/\partial q_j = 0$:

$$\sum_j S_{kj}^R q_j = N_k^R, \quad N_k^R = \sum_i \frac{1}{N_i} S_{ik}, \quad S_{kj}^R =$$

$$\sum_i \frac{1}{N_i^2} S_{ik} S_{ij} + \varepsilon_R (6\delta_{kj} - 4\delta_{k,j-1} - 4\delta_{k,j+1} + \delta_{k,j-2} + \delta_{k,j+2}),$$

3. Approximation of experimental data

To simplify calculations and damp the spread of data points in the measured at the Earth cosmic ray spectrum, we use its analytical approximation.

The formula

$$J(E) \propto E^{-3.23}, \quad E < 5 \times 10^{18} \text{eV};$$

$$J(E) \propto E^{-2.63} \times \left[1 + \exp(\log(E/10^{19.63} \text{eV})/0.15)\right]^{-1}, \quad E > 5 \times 10^{18} \text{eV},$$

is adopted in our calculations to approximate the Auger data [15].

4. Results

The minimal value $10^{-3} - 10^{-2}$ of the parameter $\varepsilon_R$ was adjusted to provide the smooth positive source spectrum $q_j$. We found that this method does not work for arbitrary chemical composition. However the diapason of the chemical composition is significantly broader in comparison with the exact solution of Eq. (5).
The results obtained for non-evolutionary sources \((m = 0)\) are shown in Figures 1-5. The value of maximum redshift \(z_{\text{max}} = 3\) was used. The coefficients \(k(A)\) were scanned to minimize the mean sum \(\delta\) of squared deviations from the observed all-particle spectrum, mean logarithm \(A\) and variance of the logarithm \(A\) \(\sigma^2\) measured by Auger collaboration [16].

\[
\delta = \frac{1}{n} \sum_{i=1}^{n} \left( 1 - \frac{1}{N_i} \sum_{j} S_{ij} q_j \right)^2 + \frac{1}{n} \sum_{i=1}^{n} \left( (\ln A)_i - (\ln A)_{i}^{\text{obs}} \right)^2 + \frac{1}{n} \sum_{i=1}^{n} \left( \sigma^2_i - \sigma^2_{i}^{\text{obs}} \right)^2 \tag{9}
\]

The obtained coefficients \(k(A)\) are given in the last line of Table 1. These numbers should be considered as very approximate ones because their moderate variations produce a small change of \(\delta\) (cf. Figure 3).

\[\text{Figure 1.} \quad \text{Calculated source spectrum} \quad q(E/Z) \text{ in arbitrary units.}\]

\[\text{Figure 2.} \quad \text{Calculated spectra of protons and nuclei. The all particle spectrum (thick solid) and the analytical approximation of Auger cosmic ray spectrum (gray solid line) are also shown.}\]

It is evident that our model reproduces the observed all particle spectrum and measured mean logarithm \(A\).

5. Discussion and Conclusion
We showed how one can find average spectrum of extragalactic sources from the cosmic ray spectrum observed at the Earth. This task was formulated as an inverse problem for the system of transport Eqs. (1) that describe the propagation of ultra-high energy cosmic rays in the expanding Universe filled with the background electromagnetic radiation.

Mathematically, the inverse problems for transport equations (1) are ill-posed in the general case that manifests itself in the instability of derived solutions. To avoid this problem we use the regularization procedure (Eqs. 6,7). The best choice of abundances for five groups of nuclei, H, He, CNO, Si and Fe, was found minimizing the value of \(\delta\). The same spectral function on the magnetic rigidity for the source spectra of different nuclei was assumed.

We found that the adjusted chemical composition permits to explain the Auger data [15, 16]. The Auger data favor the transition from a proton source composition to the heavier one as the energy is rising. With our heavy source composition, this case is most closely reproduced.
Figure 3. Dependence of the mean square of deviations $\delta$ on the Helium and CNO abundance in the source.

Figure 4. Calculated mean logarithm $A$ (solid line). The experimentally measured mean logarithm $A$ data of Auger collaboration [16] using EPOS-LHC model of particle interactions in the atmosphere (data with error-bars) are also shown.

Figure 5. Calculated variance of the logarithm $A$ (solid line). The experimentally measured variance of the logarithm $A$ data of Auger collaboration [16] using EPOS-LHC model of particle interactions in the atmosphere (data with error-bars) are also shown.

by the calculations illustrated in Figure 2. The obtained source spectrum is hard (see Figure 1) and resemble the results [17, 18] based on the analysis of direct transport problems with a power law source spectrum. The maximum energy of accelerated particles $(3...5)Z \times 10^{18}$ eV is relatively low in this case that alleviates the problem of cosmic ray acceleration. The calculated
Table 1. Chemical composition of cosmic rays at the sources

|       | H, % | He, % | CNO, % | Si, % | Fe, % |
|-------|------|-------|--------|-------|-------|
| A     | 6    | 4     | 14     | 28    | 56    |
| galactic sources (for comparison) | 92   | 7     | 0.65   | 0.07  | 0.07  |
| extragalactic sources (this work) | 0    | 73    | 24     | 2.5   | 0.8   |

The composition of cosmic rays at the Earth shown in Figures 4,5 is also in accordance with the Auger measurements.

The study of inverse transport problem is a useful tool for the investigation of ultra high energy cosmic rays allowing the abandonment of the standard assumption of power law source spectrum with an abrupt cutoff at some maximum magnetic rigidity as it is usually assumed when the direct problem is considered.

Acknowledgments
The work was partially supported by Russian Fundamental Research Foundation grant 16-02-00255.

[1] Greisen K 1966 Phys. Rev. Lett. 16 748
[2] Zatsepin G T and Kuzmin V A 1966 JETP Lett. 4 78
[3] Allard D 2012 Astropart. Phys. 39 33
[4] Aloisio R, Berezinsky V and Blasi P 2013 arXiv:1312.7459v1
[5] Kotera K and Olinto A V 2011 Ann. Rev. Astron. Astrophys. 49 119
[6] Anchordoque L A 2018 arXiv:1807.09645v1
[7] Berat C for Pierre Auger Collaboration 2017 EPJ Web of Conferences 136 id.02017
[8] Sokolsky P for the TA and HiRes collaborations 2013 EPJ Web of Conferences 52 06002
[9] Saburov A V, Glushkov A V, Pravdin M I et al. 2017 Proc. 35th ICRC 35 552 POS(ICRC2017)552
[10] Allard D 2009 arXiv:0906.3156v1
[11] Ptuskin V S, Rogovaya S I and Zirakashvili V N 2015 JCAP 03 054
[12] Zirakashvili V N, Ptuskin V S and Rogovaya S I 2017 Radiophys. Quant. Electronics 59 859
[13] Ptuskin V S, Rogovaya S I and Zirakashvili V N, 2013 Adv. Space Res. 51 315
[14] Tikhonov A N and Arsenin V Y 1977 Solutions of Ill-Posed Problems (New York: Winston)
[15] Letessier-Selvon A for the Pierre Auger Collaboration 2014 Brazilian J. of Phys. 44 560 arXiv:1310.4620
[16] Aab A et al. (Pierre Auger Collaboration) 2014 Phys. Rev. D 90 122005, arXiv:1409.4809
[17] Aab A et al. (Pierre Auger Collaboration) 2017 JCAP 04 038
[18] Aloisio R, Berezinsky V and Blasi P 2014 JCAP 10 20