Immersed boundary conditions in global, flux-driven, gyrokinetic simulations

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Abstract. A penalization technique is applied in global and flux driven gyrokinetic simulations to model a limiter configuration. The immersed boundary is implemented in terms of a restoring force to a cold distribution function. It acts as a perfect heat absorber. Parallel kinetic propagation of a cold front generated by the limiter is tested with a 1-D 1-V single species model. An analytical expression is derived to describe the initial transient of the density, particle flux and energy profiles, induced by the ballistic propagation of the cold front in phase space \((x, v)\). The same propagation is recovered in the global gyrokinetic code on the distribution function along the poloidal direction. The velocity of the front measured in the code matches the theoretical prediction.

1. Introduction
The tokamak magnetic configuration aims at confining plasma in a toroidal volume. However, plasma wall interaction is both unavoidable and necessary for steady state operation. Two main configurations are used to organize this interaction: limiter and axisymmetric divertor. A major challenge for such components is the heat transfer out of the plasma. In such configurations the plasma volume is usually divided into two regions: the confined core, where magnetic field lines are completely immersed in the plasma volume and is also called ”closed field lines” region, and the external Scrape-Off Layer (SOL), where magnetic field lines intersect the machine wall, alternatively ”open field lines” region. There, transport towards the wall can occur in the direction parallel to the magnetic field lines and the heat is transferred to the wall component in the same direction. In the SOL, therefore, parallel and transverse transport compete and confinement is strongly reduced.

Due to the very different physics characterizing these two regions as well as the complexity of the whole system, core and SOL are usually studied separately. However, the interface between these two regions, namely the plasma edge, is of fundamental importance for plasma energy confinement. The build-up of an edge transport barrier can take place, leading to a significant increase of plasma energy content [1]. The main channel of transverse transport, thus energy loss, is governed by turbulence which appears to generate non-local effects [2]. The possibility of turbulence spreading from the core to the edge [3] and/or inward [4] is currently matter of scientific debate. Moreover, recent works show evidence of turbulence interplay between core and edge regions [5]. These results stress the importance of modeling the system in a global...
framework, from the core all way to the SOL, as already done, for example, in edge modeling [6].

Global and flux-driven gyrokinetic simulations are up to now the most complete model [7, 8] to address turbulence self-organization, which governs energy confinement. However, these models are mostly dedicated to describing turbulent heat transport in the core [9–11] and the majority of the codes cannot handle the edge geometry. More complete codes are being developed and address the challenging interplay [12, 13]. Assuming a non-local behavior of turbulent transport, the choice in edge modeling is potentially important. For most global codes, the radial outer boundary condition is constituted by a poloidally symmetric buffer region extracting heat and quenching turbulence, to ensure numerical stability.

In this work we show how the outer boundary condition of the gyrokinetic global and flux driven code GYSELA is improved to handle a limiter-like configuration. The presence of the wall is modeled with a penalization technique, already implemented in fluid codes to model plasma-wall interaction [14–16]. The plasma boundary is immersed in the numerical domain and acts as a heat sink. Two main advantages of this technique are (i) poloidal asymmetry can be introduced in the boundary without modifying the geometry of the magnetic field and (ii) the shape of the immersed boundary can be easily modified to compare different machine set-ups. In this approach, the Vlasov equation is modified within the penalized region to mimic the interception of the plasma by a cold object protruding from the boundary inside the plasma volume. When poloidal asymmetry is given to the penalized region, an external layer with respect to the plasma volume is naturally created, in which plasma is in contact with the cold boundary via transport in the direction parallel to the magnetic field. Since the parallel transport is normally much higher than the transverse one, when initializing and enforcing a cold spot inside the plasma domain we expect the propagation of a cold front along the parallel direction, which progressively cools the whole SOL towards the limiter temperature. This property is used here to verify the penalized scheme implemented in GYSELA. Even though the code is not able to catch the physics of all the phenomena acting in the SOL, the new boundary introduces one fundamental property which distinguish it from the core. The external layer in which parallel and perpendicular transport compete, acts as a boundary for the core dynamics.

The paper is structured in the following way: in Section 2, we describe in detail the penalized boundary with limiter geometry. Section 3 is dedicated to the analysis of a penalized 1-D kinetic model, with which we study the generation of a cold front and its propagation along the parallel direction. In Section 4, we verify the parallel propagation of the cold front with the penalized limiter implemented in GYSELA.

2. The penalized outer boundary in GYSELA

GYSELA is a gyrokinetic global code presently used for fusion plasma turbulence investigations. A complete description of the code can be found in [17]. GYSELA evolves the Vlasov equation for the 5-D ion distribution function coupled with the quasi-neutrality equation to determine the electric field. Even if the code is able to evolve both electron and ion species, in this work electrons are assumed to respond adiabatically to the electric field. In this approximation, unless more species are considered [18], density is invariant and heat only is transported. The code operates in flux-driven mode: a heat source is present inside the plasma bulk and builds plasma profiles, which in turn drive turbulence. This configuration resembles the experimental situation, where profiles evolve self-consistently with externally prescribed heating conditions. In presence of a heat source, a heat sink is needed to extract the fluxes in steady state conditions. Hence the external boundary condition acts as a heat sink for the system and must therefore mimic the heat extraction at the material boundaries (wall and limiter) of the machine.
2.1. Heat sink penalization
The heat sink at the wall and limiter is implemented as an immersed boundary with a penalization technique. A mask function \( M \) in space defines the shape of the immersed boundary inside the simulation domain. A Krook-type restoring force localized by the mask drives the distribution function \( f \) towards a target value \( g \), at low wall temperature. The evolution equation for the distribution function is thus modified as

\[
\frac{df}{dt} = C_{coll} + S_{source} - \nu M(r, \theta)(f - ng)
\]  

where the notation on the left hand side stands for the total derivative of the gyro-center distribution function and at the right hand side the Krook restoring force has been added to the collision \( C_{coll} \) and source \( S_{source} \) terms. The target distribution function \( g \) is chosen as a Maxwellian characterized by the three quantities density \( n_g \), mean velocity \( \bar{v}_g \) and temperature \( T_g \) respectively \( n_g = 1, \bar{v}_g = 0 \) and \( T_g \sim 10 \) times lower than the characteristic temperature of the core plasma. The lower limit on the cold temperature \( T_C \) is given by the resolution in velocity space. The width of the Maxwellian in velocity space, given right by the temperature, should indeed be large enough to cover several steps in velocity, so that the function \( g \) is well resolved by the numerical grid. To ensure density conservation the target distribution function \( g \) is multiplied by the local density value \( n(r, t) = \int f(r, v_\parallel, \mu, t)dv_\parallel d\mu \) with \( x \) the position and \( v_\parallel, \mu = v_\perp/2B_0 \) \( (B_0 \) is the reference magnetic field) respectively the parallel and perpendicular velocity coordinates used in the gyrokinetic frame. The coefficient \( \nu \) is a constant that defines the strength of the restoring force and the multiplication by the mask function \( M(r, \theta) \), where \( r, \theta \) are radial and poloidal coordinates, determines the position and extent of the wall and limiter regions. Inside this penalized region, the mask function is set equal to one and the restoring force is at its maximum strength \( \nu \). Outside, the mask is set equal to zero and the restoring force is no longer active. The transition interface from 0 to 1 of the mask function is as sharp as possible, compatible with numerical resolution and stability of the code. Typically, 90% of the transition is completed within ~ 5 local larmor radii.

The restoring force drives the distribution towards the cold value \( ng \) at temperature \( T_C \); the more it departs from the target value, the stronger the force. Fluctuations of the distribution function are then damped within the immersed boundary. The restoring force is a perfect heat sink in the limit \( \nu \rightarrow \infty \) as readily seen while taking the second order moment of the Krook term:

\[
\int_{-\infty}^{+\infty} -\nu M(r, \theta)(f - ng)v^2 dv = -2\nu M(r, \theta)(E - E_C)
\]

where \( E = \int \frac{1}{2}fv^2 dv \) and \( E_C = \int \frac{1}{2}ngv^2 dv \) are the actual and target energy value. Since \( E_C \ll E \), given \( T_C < T \), the right hand side of equation 2 is negative and thus corresponds to a heat loss term. The heat generated by the source term, localized in the core, is then transported across the system and is removed when reaching the penalized boundary.

2.2. LIMITER configuration geometry
The magnetic configuration of the GYSELA code (see Appendix A) consists in poloidally concentric flux surfaces of circular cross section, which resemble the geometry of Tore Supra. The limiter configuration is introduced in GYSELA with a poloidally asymmetric mask function, as shown in figure 1-left. A solid object toroidally symmetric and of given poloidal \( \Delta \theta \) and radial \( \Delta \lambda \) extension, namely the limiter, protrudes from the bottom wall into the plasma chamber as in Tore Supra [19]. The separatrix between core and SOL regions, dotted line in figure 1-left, is located at the radial position corresponding to \( r = a \). Inside the separatrix \( (r < a) \), the mask function is equal to zero and the restoring force is not active. Outside the separatrix \( (a < r < a + \Delta \lambda) \), the limiter modeled by the mask function breaks the poloidal symmetry of particle motion and

introduces heat losses in the parallel direction. At the very edge \((r > a + \lambda)\), The restoring force is poloidally symmetric, modeling the external wall of the machine. Therefore within \(a < r < a + \Delta \lambda\), plasma touches the penalized region along both radial and poloidal directions and heat is removed both in parallel (towards the limiter) and in perpendicular (towards the wall) directions. This external plasma layer corresponds to the SOL.

![Figure 1. Left: Poloidal section of the mask function \(M(r, \theta)\). In the figure the three toroidal coordinate are labeled \((r \text{ radial}, \theta \text{ poloidal and } \varphi \text{ toroidal})\) as well as the poloidal extent of the limiter \(\Delta \theta\) and the radial SOL width \(\Delta \lambda\). Right: Cut in radial direction (top) for \(\theta = 3\pi/2\) (solid line) and \(\theta = 0\) (dashed line); cut in poloidal direction (bottom) for radial position \(r > a\).](image)

### 3. Generation of the cold front by penalization

As already introduced in Section 1, the motion of a charged particle along the magnetic field line is much faster than the perpendicular drift. The ratio between the two velocities is of the order of the small parameter \(\rho_* = \rho_i/a\) with \(\rho_i\) the ion larmor radius and \(a\) the minor radius of the torus. Hence the penalized region at low temperature induces the generation and propagation of a cold front along the parallel direction, which cools the SOL at the limiter temperature.

This phenomenon is at first analyzed in a simplified 1-D, 1-V model for a single species motion along the magnetic field lines, without electric field. We thus neglect all forces acting on the particles, except for that yielding the restoring force within the penalization region. The evolution equation for the distribution function in phase space \((x, v)\) reads:

\[
\partial_t f(x, v, t) = -v \partial_x f(x, v, t) - \nu M(x) \left( f(x, v, t) - n(x, t)g(v) \right)
\]  

(3)

The first term on the right hand side corresponds to the advection along the field line, the second is the restoring force. The spatial domain extends between \(0 < x < L\) in the parallel direction. The limiter, of total amplitude \(2x_0\), is placed half at the left edge and half at the right edge of the domain, traced by the mask function \(M(x)\) in figure 2. Therefore, plasma region extends between \(x_0 < x < L - x_0\). Like in GYSELA penalization term, the coefficient \(\nu\) determines the strength of the restoring force and the target distribution function \(g(v) = \exp(-v^2/2T_C)/\sqrt{2\pi T_C}\) is multiplied by the actual density \(n(x, t) = \int f(x, v, t)dv\). Defining the particle flux \(\Gamma(x, t) = \int f(x, v, t)vdv\) and the energy density \(E(x, t) = \frac{1}{2} \int f(x, v, t)v^2dv = \frac{1}{2} \Pi\)
Figure 2. Mask function used in the simplified 1-D, 1-V model. The \( x \)-direction is parallel to the magnetic field lines. The limiter, of total amplitude \( 2x_0 \), is located at the two edges of the \( x \) domain, respectively between \( 0 < x < x_0 \) and \( L - x_0 < x < L \). Therefore, plasma region extends between \( x_0 < x < L - x_0 \).

as the first moments of the distribution function in velocity space and given 3, we obtain the following conservation equations

\[
\partial_t n(x, t) + \partial_x \Gamma(x, t) = 0 \tag{4}
\]

\[
\partial_t \Gamma(x, t) + \partial_x \Pi(x, t) = -\nu M(x) \tag{5}
\]

\[
\partial_t E(x, t) + \partial_x Q(x, t) = -\nu M(x)(E - E_C) \tag{6}
\]

where \( Q = \frac{1}{2} \int f v^3 dv \) is the energy flux and \( E_C = \int n(x) g v^2 dv \) is the target energy.

The distribution function (figure 3) is initialized to be hot \( f_H(v) = \exp(-v^2/2T_H)/\sqrt{2\pi T_H} \) (wide Maxwellian in velocity space) inside the plasma region and cold \( f_C(v) = \exp(-v^2/2T_C)/\sqrt{2\pi T_C} \) (narrow Maxwellian) inside the limiter region as

\[
f(x, v, t = 0) = f_0(x, v) = n_H f_H(v) \ast (1 - M(x)) + n_C f_C(v) \ast M(x) \tag{7}
\]

Here \( n_H, n_C, T_H, T_C \) can be arbitrary chosen, within the constraint \( T_H > T_C \). For this study we have initialized a constant density profile \( n_H = n_C = 1, T_H = 1 \) while \( T_C = 0.4 \).

3.1. Linear advection in phase space

Inside the plasma region, the mask is equal to zero and no force is acting on the system. The trajectory is a uniform motion at constant velocity

\[
\frac{dx}{dt} = v \quad \frac{dv}{dt} = 0 \tag{8}
\]

The evolution of the system is determined by the initial state \( f_0(x, v) \):

\[
f(x, v, t) = f_0(x - vt, v) \tag{9}
\]

The initial \( f_0 \) is shifted in space on the distance \( vt \). Each point of the velocity space is decoupled from the others and follows its own motion at velocity \( v \). Two fronts are thus generated in phase space \( (x, v) \), one from each side of the limiter, which transport the cold distribution function form the limiter to the plasma domain as shown in figure 4-left. The two fronts are propagating one in the positive direction in the half-plane \( v > 0 \), referred in the following as \( v_+ \), and the other
Figure 3. Left: Initial distribution function in phase space \((x, v)\) in logarithmic scale. Right: Cut of the initial distribution function in velocity space for both the hot (squares) and cold (circles) regions.

...
Figure 4. Left: Evolution of the distribution function in phase space \((x,v)\) in logarithmic scale. The red/gray solid line is the front \(v_+\) propagating for \(v > 0\), the black solid line is the front \(v_-\) which propagates for negative velocities. Right: Cut of the distribution function evolution in velocity space starting from the initial hot value (squares), three consecutive times (dotted, dashed and dash-dotted line respectively) and the final cold state (circles).

When the restoring force is weak, a hot front can propagate inside the cold region (Figure 5-left), following the same dynamic as described in Section 3.1, and the whole domain is cooled on much longer time scales. An alternative way of implementing penalization, which avoids such a transient heating inside the limiter, is to apply an infinite restoring force \(\nu \to \infty\). In this case the restoring force is dominant with respect to the advection term inside limiter region, thus when \(M \neq 0\). Setting \(\nu \to \infty\) actually transforms the mask shape into a step function, with transition at the first non zero value of \(M(x)\). Hence, inside the limiter, the advection of the heat flux is negligible and Equation 6 becomes simply

\[
\partial_t \mathcal{E} = -\nu(\mathcal{E} - \mathcal{E}_C)
\]

The analytical solution of the previous Equation 12 is

\[
\mathcal{E}(t) = \mathcal{E}(t = 0)e^{-\nu t} + \mathcal{E}_C(1 - e^{-\nu t})
\]

which for \(\nu \to \infty\) gives \(\mathcal{E}(t) = \mathcal{E}_C\). Inside the limiter region the energy density sticks to the target cold value \(\mathcal{E}_C\) defined by the temperature \(T_C\) and thus constant in time. For given velocity a cold front propagates out of the limiter while no hot propagation takes place into the limiter (figure 5-right). When the cold front has completed one period, the whole domain has been cooled to the limiter temperature \(T_C\). The numerical treatment of the limit \(\nu \to \infty\) is described in the following Section 4.

3.3. Density, flux and energy behavior

In the plasma region the distribution function is subject to the advection term, which is responsible for the propagation of the two fronts \(v_+\) and \(v_-\) as described in Section 3.1. The moments \(M_k\) of the distribution function can thus be computed as the sum of three integrals in velocity space, namely:

\[
M_k = \int_{-\infty}^{v_-} n_C f_C(v) v^k dv + \int_{v_-}^{v_+} n_H f_H(v) v^k dv + \int_{v_+}^{\infty} n_C f_C(v) v^k dv
\]
Figure 5. Distribution function evolution in real space for a positive velocity $v = +3$ in the case of weak (left) infinite (right) restoring force.

The first integral accounts for the cold distribution function transported by the front $v_-$, hence it spans from $-\infty$ to $v_-$ in velocity space; the second integral accounts for the initial hot distribution function, thus between $v_-$ and $v_+$ in velocity space; the last integral accounts for the cold distribution function transported by the front $v_+$ and it is calculated from $v_+$ to $+\infty$ in velocity space.

Considering an infinite restoring force $\nu \to \infty$, thus a step mask function, one can neglect the periodic propagation of the front in velocity space. The analytical solutions for the density, particle flux and energy are therefore:

$$n(x, t) = n_C + \frac{n_H}{2} \text{erf} \left( \frac{v}{\sqrt{2T_H}} \right) \bigg|_{v_-}^{v_+} - \frac{n_C}{2} \text{erf} \left( \frac{v}{\sqrt{2T_C}} \right) \bigg|_{v_-}^{v_+} \quad (15)$$

$$\Gamma(x, t) = n_C \sqrt{\frac{T_C}{2\pi} e^{-\frac{v^2}{2T_C}}} \bigg|_{v_-}^{v_+} - n_H \sqrt{\frac{T_H}{2\pi} e^{-\frac{v^2}{2T_H}}} \bigg|_{v_-}^{v_+} \quad (16)$$

$$\mathcal{E}(x, t) = T_C + \left[ \frac{T_H}{2\pi} \text{erf} \left( \frac{v}{\sqrt{2T_H}} \right) - \sqrt{\frac{T_H}{2\pi} ve^{-\frac{v^2}{2T_H}}} \bigg|_{v_-}^{v_+} \right] - \left[ \frac{T_C}{2\pi} \text{erf} \left( \frac{v}{\sqrt{2T_C}} \right) - \sqrt{\frac{T_C}{2\pi} ve^{-\frac{v^2}{2T_C}}} \bigg|_{v_-}^{v_+} \right] \quad (17)$$

Since the model includes only the advection term, the expressions 15, 16, 17 are valid only within the plasma region. These results exactly fulfill both the charge balance (Equation 4) and the momentum balance (Equation 5). Inside the limiter region the restoring force drives the distribution function to the actual density value $n(x, t)$ and both density and particle flux can evolve, as derived in the following on the basis of Equations 15 and 16. On the other hand, the energy density is fixed inside the limiter to the cold value $\mathcal{E}_C$, as already shown before.

Figure 6 shows the analytical expression for the density in time and space, within the plasma region $x_0 < x < L - x_0$. An initial transient leads to a lowering of the density profile. At asymptotic time the error functions of Equation (15) are all flattened to zero and the density profile is constant and equal to the initial cold value $n_C$. The total number of particles inside the
plasma region, which is the integral of $n(x,t)$ in the interval $[x_0, L-x_0]$ is shown in figure 6-right: it decreases at the beginning of the transient and then tends asymptotically to the initial value. Given Equation 4, the total number of particles $N = \int n(x,t) dx$ in the system is conserved $\partial_t N = 0$. Hence, during the transient, density increases inside the penalized region. This phenomenon is interpreted as condensation: density accumulates in the region at lower temperature. When a constant density is initialized in the domain $n_C = n_H$, indeed, an initial pressure gradient is created in the system due to $T_C < T_H$. In steady state conditions the momentum balance equation within the plasma region reads $\nabla \Pi = 0$ so that during this initial transient the system adapts to reduce the pressure gradient.

Figure 6. Left: Spatial and time evolution of the analytical density in equation (15). Centre: Cut at time $t = 0$ (solid line), $t = 0.3$ (dotted), $t = 1.5$ (dashed), $t = 4$ (dash-dotted line). Right: Relative variation of the number of particles $\bar{N}(t)/\bar{N}(t = 0) = \int_{x_0}^{L-x_0} n(x,t) dx / \int_{x_0}^{L-x_0} n(x,0) dx$ inside the plasma region.

Figure 7. Initial density profile (solid line). Density accumulation transient in the case of strong restoring force $\nu = 1000$ (dashed line) and weak restoring force $\nu = 0.5$ (dotted line).

The behavior of the particle flux mirrors the density: at the early beginning particles are moving towards the limiter to then redistribute much more slowly to the center. The reorganization of the density profile is indeed subject to two different time scales: the fast condensation inside the limiter is governed by the hot thermal velocity $v_{thH} = \sqrt{T_H}$ of the plasma region. Conversely in the limiter, the temperature is cold so that the following redistribution of the density from the limiter towards the plasma region is governed by the cold thermal velocity
\( v_{thC} = \sqrt{T_C} < v_{thH}. \) The effect of the restoring force within the limiter region is not taken into account in the analytical model. When Equation (3) is solved numerically, one can study the effect of the restoring force on the global density profile (figure 7). The restoring force resists to the propagation of a higher density inside the limiter. When the restoring force is strong, indeed, accumulation of density is observed only at limiter sides, where \( M(x) < 1 \) weakens the restoring effect.

\[ v_{thC} = \sqrt{T_C} < v_{thH}. \]

**Figure 8.** *Left:* Spatial and time evolution of the analytical particle flux in equation (16). *Right:* Cut at time \( t = 0.3 \) (dotted), \( t = 1.5 \) (dashed).

\[ v_{thC} = \sqrt{T_C} < v_{thH}. \]

The lowering of the temperature is not linear, as shown in figure 9. This happens as a result of kinetic propagation of the cold front: the bulk of the distribution function is cooled down on much longer time scales than the tail. At first temperature decreases from both sides of the limiter towards an intermediate state. It corresponds to the situation when the tail of the distribution function in velocity space has already stepped to the cold value. Next the system is cooled at the limiter temperature starting from the region farthest from the limiter. This last process is governed by the slower time scales of the bulk velocities.

**Figure 9.** *Left:* Spatial and time evolution of the temperature, computed as \( \mathcal{E}(x,t)/n(x,t) \). *Right:* Cut at \( x = 0.2 \) (dotted line) and midbox \( x = 1 \) (dashed line).
4. Parallel front propagation in GYSELA geometry

In this Section the parallel front propagation generated by penalization is verified in the GYSELA code. A GYSELA simulation is run where only parallel transport is allowed. Infinite penalization is applied. To numerically handle \( \nu \to \infty \) one possible choice is to apply a standard Krook term with a very large \( \nu \). In this work however, the following alternative approach has been pursued: at each time step \( \Delta t \), inside the limiter and wall region, the distribution function is fully replaced by its target value \( n(t)g \) as

\[
f(t + \Delta t) = (1 - M(r, \theta)) \ast f(t) + M(r, \theta) \ast n(t)g
\]  

(18)

Note that \( f(t) \) is the gyrokinetic distribution function in the 5-D phase space and \( n(t) \) depends on real space as well. This formula differs from the application of a standard Krook term with very large but finite \( \nu \) basically only in the transition region, where \( 0 < M(x) < 1 \). Details of this difference are given in Appendix B. The mask function used is the one in figure 1. The motion along the field lines is observed in the \( \theta - \)direction. On figure 10 a cut of the distribution function is shown for \( r > a \), hence in the SOL, and in the plane \((\theta, v_{||})\). The propagation of the cold front in phase space is observed, as predicted from the 1-D 1-V model.

![Figure 10.](image)

**Figure 10.** Cut of the 5D GYSELA distribution function in the \((\theta, v_{||})\) plane for given radial (inside the Scrape-Off Layer), toroidal position and perpendicular velocity. The limiter extends poloidally in the interval between \( \theta = 3/2\pi \pm 0.5 \). Left the initial state. Right at time \( t = 100\Omega_i \).

4.1. Poloidal displacement of the front

The propagation velocity of the front along the poloidal direction can be computed as

\[
\frac{d\theta}{dt} = \frac{d\theta}{ds} \frac{ds}{dt}
\]

where \( s \) is the curvilinear abscissa along the magnetic field lines. The first factor on the right hand side corresponds to the \( \theta \)-projection of the field line, the second is the parallel velocity. Considering the normalization used in GYSELA, the poloidal velocity of the front displacement is

\[
\frac{d\theta}{dt} = \frac{v_{||}}{\sqrt{A_s q_{gs}(r) R_0}}
\]  

(19)
where $A_s$ indicates the mass number of the ion species considered, $q_{Gys}$ a quantity proportional to the safety factor for GYSELA geometry (see Appendix A) and $R_0$ the major radius of the torus. The propagation velocity recovered in GYSELA is shown in figure 11 as well as the theoretical prediction. GYSELA results matches the theoretical value within 1% relative error.

![Figure 11. Displacement of the cold front in the poloidal direction in function of time. Dots correspond to GYSELA measurements and solid line to the theoretical prediction.](image)

5. Conclusion
In this work, a penalization technique has been applied to the radial outer boundary condition of the gyrokinetic global and flux-driven code GYSELA to mimic limiter configurations. Through the mask function, which defines the geometry of the limiter, the immersed boundary acts both as a heat sink and a momentum sink. Indeed the target distribution function in the limiter volume is both as cold as numerically possible and with zero mean velocity. Poloidal asymmetry due to the finite extent of the limiter in the poloidal direction generates an outer layer that mimics the actual Scrape-Off Layer, in which parallel and perpendicular transport compete. In this paper, the parallel propagation of a cold front generated by the penalized limiter is both analyzed in a simplified 1-D kinetic model and verified in GYSELA.

From the analysis of the simplified model, the penalized limiter induces the propagation of a cold front along the parallel direction. This is a kinetic propagation and moves ballistically for each point of velocity space. The initial cold spot imposed inside the plasma volume through the penalized limiter and characterized by a narrower distribution function, propagates in phase space $(x, v)$. The cooling of the SOL region effectively reduces the probability to find particles at high velocity. The transient associated to the front propagation also generates a modification of the density profile. An infinite restoring force can be implemented to ensure a steady state cold spot inside the mask.

In GYSELA simulations, allowing only for parallel transport in the axisymmetric limiter configuration, the parallel propagation of the cold front generated by the limiter is verified on the distribution function in the poloidal direction. The front propagation velocity is in close agreement with the theoretical value.

We have shown that an immersed boundary set as a heat absorber verifies the appropriate properties to stand for the heat loss to a limiter. A Scrape-Off Layer is then created in the simulation domain in which parallel and perpendicular transport compete. This layer will then act as a boundary for the development of core turbulence. The new boundary condition mimics
the experimental set-up such as that of Tore Supra. A more complete version still needs to be
developed to include in the penalized heat sink both ion and electron species. This improved boundary already allows one to investigate turbulence development from the core all way to the edge, avoiding spurious damping of turbulent fluctuations in the edge, as occurs in the more standard buffer regions.

Appendix A. GYSELA magnetic configuration
For completeness, we detail here the GYSELA magnetic configuration. The chosen system of coordinates is toroidal \((r, \theta, \varphi)\), where both \(\theta, \varphi\) are the geometrical poloidal and toroidal angles. The magnetic field is defined as

\[
B = \frac{B_0 R_0}{R(r, \theta)} \left( r \frac{q_{gys}(r) R_0}{R(r, \theta)} e_\theta + e_\varphi \right) \tag{A.1}
\]

Here \(B_0\) and \(R_0\) correspond to the magnetic field and the major radius of the torus computed at the magnetic axis, while \(R(r, \theta) = R_0 + r \cos(\theta)\). The vectors \(e_\theta = r \nabla \theta\) and \(e_\varphi = R \nabla \varphi\) are the unit vectors in poloidal and toroidal periodic directions. The function \(q_{gys}(r)\) depends only on radial direction and is related to the safety factor \(q(r)\). When choosing the geometrical angles \((\theta, \varphi)\) as coordinates, the local field line pitch reads

\[
\frac{B \cdot \nabla \varphi}{B \cdot \nabla \theta} = q_{gys}(r) \frac{R_0}{R(r, \theta)} = q_{gys}(r) \frac{1}{1 + \epsilon \cos(\theta)} \tag{A.2}
\]

with \(\epsilon = r/R_0\). The safety factor results to be the average on a flux surface of the local pitch angle, namely

\[
q(r) = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{1 + \epsilon \cos(\theta)} d\theta = q_{gys}(r) \sqrt{1 - \epsilon^2} \tag{A.3}
\]

Appendix B. Difference between the application of an infinite penalization and a large Krook restoring force
We detail here the difference in the numerical application of an infinite penalization as described in Section 4 and a very strong restoring force of the Krook type.

The numerical algorithm of GYSELA [17] includes a time-splitting technique which allows to solve the time advance due to each operator of Vlasov equation separately. Thus the equation for the Krook term corresponds to

\[
\partial_t f = -\nu M(r, \theta)(f - ng) \tag{B.1}
\]

We here substitute \(\nu M(r, \theta) = \tilde{\nu}(r, \theta)\), thus we allow a spatial dependence of the restoring force strength which does not depend directly on the mask function. The analytical solution of this equation over a time step of amplitude \(\Delta t\), corresponding to the time step of one iteration, is

\[
f(t + \Delta t) = f(t) e^{-\tilde{\nu}(r, \theta) \Delta t} + ng(1 - e^{-\tilde{\nu}(r, \theta) \Delta t}) \tag{B.2}
\]

After \(k\) iterations the solution becomes

\[
f(t + k\Delta t) = f(t) e^{-\tilde{\nu}(r, \theta) k \Delta t} + ng(1 - e^{-\tilde{\nu}(r, \theta) k \Delta t}) \tag{B.3}
\]

On the other hand, when applying an infinite penalization as described in Section 4, the time advance related to the penalization term is given by

\[
f(t + \Delta t) = (1 - M(r, \theta)) f(t) + M(r, \theta) ng \tag{B.4}
\]
In this case, the spatial dependence of the restoring force can be given only by the mask $M(r, \theta)$. After $k$ iterations

$$f(t + k\Delta t) = (1 - M(r, \theta))^k f(t) + (1 - (1 - M(r, \theta))^k) \nu g.$$  \hspace{1cm} (B.5)

The results B.3 and B.5 are equivalent if

$$e^{-\bar{\nu}(r, \theta)k\Delta t} = (1 - M(r, \theta))^k$$ \hspace{1cm} (B.6)

or alternatively

$$-\bar{\nu}(r, \theta)\Delta t = \log(1 - M(r, \theta))$$ \hspace{1cm} (B.7)

The right hand side of Equation B.7 does not depend on time step $\Delta t$ or restoring force strength $\nu$. Thus when defining a mask function, in the case of infinite penalization as in Equation B.4, the effective strength of the restoring force applied $\bar{\nu}(r, \theta) = \log(1 - M(r, \theta))/\Delta t$ will depend on the time step. To rigorously compare different simulations one would thus prefer the application of a very strong restoring force as in Equation B.2 so that once the coefficient $\nu$ is defined the same restoring force is applied, even when changing the time step. Nevertheless for increasing $\nu$ the shape of the coefficient $\bar{\nu}(r, \theta)$ is close to a step function, which can lead to numerical instabilities. In this case, one should first define the maximum gradient numerically acceptable for the function $\bar{\nu}(r, \theta)$ and next, given the restoring force strength $\nu$, recover a modified mask function $\hat{M}(r, \theta) = \bar{\nu}(r, \theta)/\nu$ to use in equation B.1. Substituting $\bar{\nu}(r, \theta) = \nu \hat{M}(r, \theta)$ in the relation B.7 we distinguish three different cases: (i) the case $M(r, \theta) = 0$ corresponds to $\nu = 0$ and the restoring force is not active in both the formulations; (ii) in the case $M(r, \theta) = 1$ the relation B.7 exactly gives the limit $\nu \to \infty$; (iii) the case $0 < M(r, \theta) < 1$, and so $0 < \bar{\nu}(r, \theta) < \nu$, is restricted to a narrow region $\sim 5\rho_i$. Hence, the formulation B.4 can be preferred to the application of a very strong Krook term when one would enforce the limit $\nu \to \infty$, taking into account that the effective restoring force strength depends on the time step only in the narrow transition region.

References

[1] Team A 1989 Nuclear Fusion 29 1959 URL http://stacks.iop.org/0029-5515/29/i=11/a=010
[2] Grcan D and et al 2013 Nuclear Fusion 53 073029
[3] Mattor N and Diamond P H 1994 Phys. Rev. Lett. 72(4) 486–489
[4] Garbet X, Laurent L, Samain A and Chinaradet J 1994 Nuclear Fusion 34 963
[5] Dif-Pradalier G 2017 Plasma and Fusion Research 12 1203012
[6] Baschetti S and et al 2018 these proceedings
[7] Brizard A J 2004 Physics of Plasmas 11 4429–4438
[8] Garbet X, Idomura Y, Villard L and Watanabe T 2010 Nuclear Fusion 50 043002
[9] Goerler T, Lapillon X, Brunner S, Dammert T, Jenko F, Merz F and Told D 2011 Journal Of Computational Physics 230 7053–7071
[10] Idomura Y, Urano H, Aiba N and Tokuda S 2009 Nuclear Fusion 49 065029
[11] Bottino A and et al 2007 Physics of Plasmas 14 010701
[12] Ku S, Chang C and Diamond P 2009 Nuclear Fusion 49 115021
[13] Nori M, H J and Janhunen S 2018
[14] Isard L and et al 2010 Journal of Computational Physics 229 2220–2235
[15] Serre E and Bufferand e a Contributions to Plasma Physics 52 401–405
[16] Paredes A, Bufferand H and et al 2014 Journal of Computational Physics 274 283–298
[17] Grandgirard V and et al 2016 Computer Physics Communications 207 35 – 68 ISSN 0010-4655
[18] Donnel P and et al 2018 Submitted to Computer Physics Communications
[19] Gunn J and et al 2007 Journal of Nuclear Materials 363-365 484 – 490 ISSN 0022-3115 plasma-Surface Interactions-17