Towards a Proof of the Shelah Presentation Theorem in Metric Abstract Elementary Classes.

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Abstract. In [Za11], we proved the following version of Shelah’s Presentation Theorem in the setting of Metric Abstract Elementary Classes:

Theorem. Given an MAEC \((K, \preceq_K)\), there exist an expansion \(L'\) of \(L(K)\), an \(L'\)-theory \(T'\) and a set of \(T'\)-types \(\Gamma\) (in the setting of Continuous Logic) such that \(K = PC_L(T', \Gamma)\) (i.e.: \(K\) is a projective class with omitting types).

In [Za11], we claimed that the new function symbols are not necessarily uniformly continuous. In this paper we provide a proof they are in fact uniformly continuous.

1. Introduction

Shelah and Stern proved that First Order Logic is not a good framework to study (from a Model-Theoretic point of view) classes of Metric Structures -such as Banach spaces-. In fact, they proved that these classes have a behavior similar to Second Order Logic with Predicates (see [ShSt78]). Because of that, there was necessary to study a logic which would satisfy good properties as in First Order Logic (e.g., Downward Löwenheim-Skolem-Tarski Theorem, Compactness Theorem, etc.) and at the same time that would be a suitable logic to study classes of Metric Structures from this point of view. This was the beginning of Continuous Logic (for short, CL; see [BeBeHeUs08]).

Shelah’s Presentation Theorem is a very interesting and important result in (discrete) it Abstract Elementary Classes (for short, AECs; a good framework to study classes of discrete structures which are not axiomatizable in First Order Logic) because this result allows us

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to work with Ehrenfeucht-Mostowski models, and also it allows to prove the existence of arbitrarily large-enough models in $\mathcal{K}$ via the existence of Hanf numbers (see [Ba09]). In fact, this follows from the fact that AECs are Projective Classes (the statement of the classical Shelah’s Presentation Theorem in discrete AECs) and that AECs are controlled in some way by an infinitary logic.

**Metric Abstract Elementary Classes** (for short, MAECs) is a generalization of **Continuous Logic** parallel to the notion of **Abstract Elementary Class** (see [Ba09]), but we work with the completion of the union of an elementary chain instead of working just with such union, and also we work in that setting with density character instead of cardinality. We follow the definitions and terminology given by Åsa Hirvonen and Tapani Hyttinen ([HiHy08]).

Hirvonen and Hyttinen (see [HiHy08]) proved the following weaker version of Shelah’s Presentation Theorem

**Theorem 1.1.** Let $(\mathcal{K}, \preceq_{\mathcal{K}})$ an MAEC of $L$-structures with $|L| + LS^d(\mathcal{K}) \leq \aleph_0$. Then for each $M \in \mathcal{K}$ we can define an expansion $M^*$ with Skolem functions $F^k_n (k, n < \omega)$ such that:

1. If $A \subseteq M^*$ and $A$ is closed under the functions $F^k_n$ then $\overline{A} \upharpoonright L \in \mathcal{K}$ and $\overline{A} \upharpoonright L \preceq_{\mathcal{K}} M$.
2. For all $a \in M$, $A_a := \{(F^\text{length}(a))^n (a) : n < \omega\}$ is such that
   - $\overline{A_a} \upharpoonright L \in \mathcal{K}$ and $\overline{A_a} \upharpoonright L \preceq_{\mathcal{K}} M$,
   - If $b \subseteq a$ (as sets) then $b \in A_b \subseteq A_a$.

However, they do not prove that an MAEC is a Projective Class with omitting types. They used their version of Shelah’s Presentation Theorem for constructing Ehrenfeucht-Mostowski models in this setting.

In [Za11] we refined their argument, providing an explicit theory in Continuous Logic and an explicit set of types which work in a similar way as in the original proof in the AEC setting, proving that an MAEC is in fact a Projective Class. In [Za11] we claimed that the interpretations of the given function symbols in the extended language were not necessarily uniformly continuous, which is a requirement for fitting this theory and this set of omitted types in the setting of Continuous Logic.

In this paper, we will prove that those interpretations are actually
uniformly continuous (claim 3.3). For the sake of completeness, we will provide the proof given in [Za11].

For basic notions of Continuous Logic, we refer the reader to [BeBeHeUs08]. For basic notions of MAEC, we refer the reader to [HiHy08, Za11].

2. Metric Abstract Elementary Classes

Remark 2.1. Through this paper, we call a complete metric space an $L$-structure (in the context of Continuous Logic).

Definition 2.2. The density character of a topological space is the smallest cardinality of a dense subset of the space. If $X$ is a topological space, we denote its density character by $dc(X)$. If $A$ is a subset of a topological space $X$, we define $dc(A) := dc(A)$.

We consider a natural adaptation of the notion of Abstract Elementary Class (see [Gr02] and [Ba09]), but working in the context of Continuous Logic (see [BeBeHeUs08]). We follow the definitions given by Åsa Hirvonen and Tapani Hyttinen (see [HiHy08]).

Definition 2.3. Let $L$ be a language as in [BeBeHeUs08], but without the uniform continuity modulus. A multi-sorted metric $L$-structure is a tuple

$$M := \langle \{ A_i, d_i \}_{i \in I}, \mathbb{R}, \{ c_j \}_{j \in J}, \{ F_k \}_{k \in K}, \{ R_l \}_{l \in L} \rangle,$$

where:

1. Each $(A_i, d_i)$ is a complete metric space.
2. $\mathbb{R}$ is an isomorphic copy of the real field $\langle \mathbb{R}, +, \cdot, 0, 1, \leq \rangle$.
3. Each $c_j$ is a constant in a fixed sort $A_i(j)$.
4. Each $R_k$ is a continuous predicate; i.e., $R_k$ corresponds to a function $R_k : A_{i(k)} \times \cdots \times A_{i(k)} \to [0, 1]$ which is closed, i.e.: if $(\mathbf{\tau})_{n<\omega} \to \mathbf{\tau}$ as tuples, then $(R_k(\mathbf{\tau}_n))_{n<\omega} \to R_k(\mathbf{\tau})$, where $n$ is called the arity of $R_k$.
5. Each $F_l$ is a function $F_l : A_{i(l)} \times \cdots \times A_{i(l)} \to A_{i(l)}$ which is closed (i.e.: if $(\mathbf{\tau})_{n<\omega} \to \mathbf{\tau}$ as tuples, then $(F_l(\mathbf{\tau}_n))_{n<\omega} \to F_l(\mathbf{\tau})$), where $m$ is called the arity of $F_l$.

If it is clear that we are working in a metric context, we just called them $L$-structures.

Definition 2.4. Let $\mathcal{K}$ be a class of $L$-structures as defined in 2.3 above and $\prec_\mathcal{K}$ be a binary relation defined in $\mathcal{K}$. We say that $(\mathcal{K}, \prec_\mathcal{K})$ is a Metric Abstract Elementary Class (shortly MAEC) if:

1. $\mathcal{K}$ and $\prec_\mathcal{K}$ are closed under $\cong$. 

(2) \( \prec_K \) is a partial order in \( K \).
(3) If \( \mathcal{M} \prec_K \mathcal{N} \) then \( \mathcal{M} \subseteq \mathcal{N} \).
(4) (Tarski-Vaught chains) If \( (\mathcal{M}_i : i < \lambda) \) is a \( \prec_K \)-increasing chain then
   (a) the function symbols in \( L \) can be uniquely interpreted on the completion of \( \bigcup_{i < \lambda} \mathcal{M}_i \) such that \( \bigcup_{i < \lambda} \mathcal{M}_i \in K \)
   (b) for each \( j < \lambda \), \( \mathcal{M}_j \prec_K \bigcup_{i < \lambda} \mathcal{M}_i \)
   (c) if each \( \mathcal{M}_i \in K \in \mathcal{N} \), then \( \bigcup_{i < \lambda} \mathcal{M}_i \prec_K \mathcal{N} \).
(5) (coherence) if \( \mathcal{M}_1 \subseteq \mathcal{M}_2 \prec_K \mathcal{M}_3 \) and \( \mathcal{M}_1 \prec_K \mathcal{M}_3 \), then \( \mathcal{M}_1 \prec_K \mathcal{M}_2 \).
(6) (Downward Löwenheim-Skolem) There exists a cardinality \( LS^d(K) \) (which is called Löwenheim-Skolem number) such that if \( \mathcal{M} \in K \) and \( A \subseteq M \), then there exists \( \mathcal{N} \in K \) such that dc(\( \mathcal{N} \)) \( \leq \) dc(\( A \)) + \( LS^d(K) \) and \( A \subseteq \mathcal{N} \prec_K \mathcal{M} \).

Examples 2.5.  (1) Any Continuous Logic Elementary Class with the elementary substructure relation is an MAEC.
(2) Positive bounded theories, where \( \prec_K \) is interpreted by the approximate elementary submodel relation (see [HeIo02]).
(3) Compact Abstract Theories, see [Be03, Be05]
(4) The class of Banach spaces, where \( \prec_K \) is interpreted by the closed subspace relation (see [HiHy08]).
(5) Hilbert Spaces expanded with an unbounded closed selfadjoint operator (see [Ar14]). This example is not necessarily axiomatizable in Continuous Logic.

2.1. Some basic facts. In this section, we mention some basic (and classic) facts towards getting a proof of Shelah Presentation Theorem. This basic facts are also used in the classic proof in the (discrete) Abstract Elementary Classes (for short, AECs), but for the sake of completeness we provide their statements.

Fact 2.6. Let \( (I, \leq) \) be a directed partial order of size \( \lambda \). Then there exists a family \( \{ I_\alpha : \alpha < \lambda \} \) of suborders of \( I \) such that:

1. Each \( I_\alpha \) is a directed order and \( |I_\alpha| < \lambda \)
2. If \( \alpha < \beta < \lambda \), then \( I_\alpha \leq I_\beta \)
3. \( I = \bigcup_{\alpha < \lambda} I_\alpha \).

Reference. [Ma85]

We prove the following fact in a similar way as in (discrete) AECs (mutatis mutandis). In fact, we strongly use the Tarski-Vaught chains axiom (MAEC axiom). Notice that in MAECs, this axiom involves
not just the union of the \(<\kappa\)-chain, we have to take the completion of that union. Despite of the sketch of the proof is almost the same as in (discrete) AECs, for the sake of completeness we provide a proof of this fact.

**Proposition 2.7.** Let \((I, \preceq)\) be a directed partial order and \((M_i : i \in I)\) a \(<\kappa\)-directed system. Then:

(a) \(\bigcup_{i \in I} M_i \in \mathcal{K}\).
(b) \(M_j \preceq \bigcup_{i \in I} M_i\) for each \(j \in I\).
(c) If \(M \in \mathcal{K}\) and \(M_j \preceq M\) for each \(j \in I\), then \(\bigcup_{i \in I} M_i \preceq M\).

**Proof.** Assume this fact holds for \(\alpha < |I|\). By fact 2.6 we have that there exists a family \(\{I_\alpha : \alpha < \lambda\}\) of suborders of \(I\) such that:

1. Each \(I_\alpha\) is a directed order and \(|I_\alpha| < |I|\)
2. If \(\alpha < \beta < |I|\), then \(I_\alpha \leq I_\beta\)
3. \(I = \bigcup_{\alpha < |I|} I_\alpha\).

Define \(M_\alpha := \bigcup_{\alpha < |I|} M_i\). By induction hypothesis (b) we have that \(M_j \preceq M_\alpha\) for every \(j \in I_\alpha\). If \(\alpha < \beta\), and \(I_\alpha \subseteq I_\beta\), then \(M_j \preceq M_\beta\) for every \(j \in I_\alpha\). By induction hypothesis (c) we have that \(M_\alpha := \bigcup_{\alpha < |I|} M_j \preceq M_\beta\).

It is easy to check that \(\bigcup_{\alpha < |I|} M_\alpha = \bigcup_{\alpha < |I|} M_i\), so by definition 4) (a) we have that \(\bigcup_{\alpha < |I|} M_i = \bigcup_{\alpha < |I|} M_\alpha \in \mathcal{K}\). Then (a) holds.

If \(j \in I\), there exists \(\alpha < |I|\) such that \(j \in I_\alpha\), so \(M_j \preceq M_\alpha\) (by induction hypothesis) and by definition 2.1 (b) \(M_\alpha \preceq \bigcup_{\alpha < |I|} M_\alpha = \bigcup_{\alpha < |I|} M_i\). Therefore \(M_j \preceq \bigcup_{\alpha < |I|} M_i\), i.e. (b) holds.

Let \(M\) be an \(L\)-structure in \(\mathcal{K}\) such that \(M_j \preceq M\) for each \(j \in I\). By induction hypothesis (c), for each \(\alpha < |I|\) we have that \(M_\alpha \preceq M\). So, by definition 2.1 (c) we have that \(\bigcup_{\alpha < |I|} M_i = \bigcup_{\alpha < |I|} M_\alpha \preceq M\). So, (c) holds.

\(\Box_{\text{Proposition 2.7}}\)

**Definition 2.8** (directed system). Let \(\mathcal{K}\) be a Category. A functor \(D : (I, \preceq) \to \mathcal{C}\) is said to be a **directed system** if and only if \((I, \preceq)\) is a directed ordered set. Set \(M_k := D(k)\) for every \(k \in I\) and \(f_{i,j} : M_i \to M_j\) the morphism associate to the unique \(I\)-morphism \((i,j) : i \to j\) via \(D\) whenever \(i \preceq j\).

**Definition 2.9** (directed limits). We say that \(\mathcal{K}\) is **closed under directed limits** if for every directed system \(D : (I, \preceq) \to \mathcal{C}\) there exist \(M \in \text{ob} (\mathcal{K})\) and \(\mathcal{C}\)-morphisms \(f_{i,0} : M_i \to M\) \((i \in I)\) such that

1. for any \(i \preceq j\) we have \(f_{i,0} = f_{j,0} \circ f_{i,j}\)
(2) if any \( N \in \text{ob}(C) \) has a system of \( C \)-morphisms \( g_{i,\infty} : M_i \to N \) which satisfies 1. above, then there exists a unique \( C \)-morphism \( h : M \to N \) such that \( g_{i,\infty} = h \circ f_{i,\infty} \).

Such morphisms \( f_{i,\infty} \) are called \textit{canonical morphisms}.

**Corollary 2.10.** An MAEC \( \mathcal{K} \) (viewed as a category with morphisms the \( \mathcal{K} \)-embeddings) is closed under directed limits.

**Proof.** This is a direct consequence of proposition 2.7 and axiom 1. of definition 2.1.

\( \square \) Corollary 2.10

3. \textsc{The main question.}

**Definition 3.1.** Let \( \mathcal{K} \) be a class of \( L \)-models in the continuous logic setting (but with closed functions instead of uniformly continuous functions). We say that \( \mathcal{K} \) is a projective class with omitting types (shortly, PCT \( \Gamma \) class) iff there exist an expansion \( L' \) of \( L \), an \( L' \)-theory \( T' \) and a set of \( L' \)-types \( \Gamma \) such that \( \mathcal{K} = PC_{L}(T',\Gamma) := \{ \mathcal{M} \upharpoonright L : \mathcal{M} \models T' \text{ and } \mathcal{M} \text{ omits every type in } \Gamma \} \).

And finally, we provide a proof of the version of Shelah Presentation Theorem in the setting of MAECs. We have to clarify that although the sketch of the proof is almost the same of the discrete proof, we are working in a metric setting and we have to change lots of details in the proof. This is the proof given in \([Za11]\), which we provide for the sake of completeness. In \([Za11]\) we claimed that the interpretations of the function symbols of the Skolemization are not necessarily uniformly continuous, but in this paper we will prove that they are in fact uniformly continuous (claim 3.3).

**Theorem 3.2** (Shelah’s Presentation Theorem in MAECs). Given an MAEC \((\mathcal{K}, \prec_{\mathcal{K}})\), there exist an expansion \( L' \) of \( L(\mathcal{K}) \), a \( L' \)-theory \( T' \) and a set of \( T' \)-types \( \Gamma \) such that \( \mathcal{K} = PC_{L}(T',\Gamma) \) (i.e.: \( \mathcal{K} \) is a projective class with omitting types).

**Proof.** Let \( L' \) be the language obtained from \( L(\mathcal{K}) \) by adding new \( n \)-ary function symbols \( F^n_i \) \((i < LS(\mathcal{K}))\). Let \( T' \) be the theory which says that \( \sup x_0 \ldots \sup x_{n-1} |F^n_i(x_0,\ldots,x_{n-1}) - x_i| = 0 \) for all \( i < n \). Notice
that all $F^m_i$ are defined as projections, so they are continuous (and therefore, closed).

Take $\mathcal{M} = T'$ and $\bar{a} \in M$. For $b <_{\text{subtuple}} \bar{a}$, define $U_\bar{a} := \{ F^m_i(b) : i < LS(K) \}$, where $m := l(b)$.

Define $\Gamma$ as follows: for each $\mathcal{M} = T'$ and each tuple $\bar{a} \in M'$, $tp(\bar{a}/\emptyset, \mathcal{M}') \in \Gamma$ unless we have the following two conditions:

1. Taking $\bar{b} <_{\text{subtuple}} \bar{a}$, $U_\bar{a}$ is a dense subset of a submodel of $\mathcal{M}' \upharpoonright L$, which we denote by $\mathcal{M}_{\bar{a}}$, and $\mathcal{M}_{\bar{a}} \in \mathcal{K}$.

2. Taking $\bar{b} <_{\text{subtuple}} \bar{a}$, we have $\mathcal{M}_{\bar{a}} <_K \mathcal{M}_{\bar{b}}$.

Take $\mathcal{M} \in PC_L(T', \Gamma)$. So, there exists $\mathcal{M}' = T'$ such that omits all the types in $\Gamma$ and $\mathcal{M} = \mathcal{M}' \upharpoonright L$. Consider the sets $U_\bar{a}$, for each $\bar{a} \in M$. As $\mathcal{M}'$ omits all the types in $\Gamma$, each $U_\bar{a}$ is the universe of a submodel $\mathcal{M}_{\bar{a}}$ of $\mathcal{M}$ such that $\mathcal{M}_{\bar{a}} \in \mathcal{K}$. By proposition 2.7 we have that $\bigcup_{\bar{a} \in M} \mathcal{M}_{\bar{a}} \in \mathcal{K}$.

Since $\mathcal{M}' = T'$, we have that $\bar{a} \in U_\bar{a}$ and so $\bigcup_{\bar{a} \in M} \mathcal{M}_{\bar{a}} = \mathcal{M}$. So, $\mathcal{M} \in \mathcal{K}$.

In the other way, take $\mathcal{M} \in \mathcal{K}$. We define $\mathcal{M}'$ as follows: for $n = 0$, choose $\mathcal{M}_0 <_K \mathcal{M}$ of density character $LS^d(\mathcal{K})$ and let $U_0 := \{ F^0_i : i < LS^d(\mathcal{K})\}$ be an enumeration of a dense subset of $\mathcal{M}_0$. Having done this for $n$, let $\bar{a} \in M$ be of lenght $n + 1$. Choose $\mathcal{M}_{\bar{a}} <_K \mathcal{M}$ of density character $LS^d(\mathcal{K})$ which contains $\bigcup \{ M_{\bar{a}} : b <_{\text{subtuple}} \bar{a} \}$ and $\bar{a}$, and let $U_\bar{a} := \{ F^{n+1}_i(\bar{a}) : i < LS(\mathcal{K})\}$ be an enumeration of a dense subset of $\mathcal{M}_{\bar{a}}$ such that $F^{n+1}_i(\bar{a}) = a_i$ for $i < n + 1$, where $\bar{a} := (a_0, \ldots, a_n)$.

Claim 3.3. $F^{n+1}_i$ is an uniformly continuous function.

Proof. Notice that $F^{n+1}_i : \mathcal{M}^{n+1} \to \mathcal{M}$ defined as above is a projection. The topology in $\mathcal{M}^{n+1}$ is given by the metric defined by $d^*(\bar{b}, \bar{c}) := \max_{j \in \mathbb{N}} \{ d(b_j, c_j) \}$ (where $\bar{b} := (b_0, \ldots, b_n)$, $\bar{c} := (c_0, \ldots, c_n)$ and $d$ is the metric of $\mathcal{M}$). Let $\varepsilon > 0$. Taking $\delta := \varepsilon$, if $d^*(\bar{b}, \bar{c}) := \max_{j \in \mathbb{N}} \{ d(b_j, c_j) \} < \delta$, then

$$d(F^{n+1}_i(\bar{b}), F^{n+1}_i(\bar{c})) = d(b_i, c_i) \leq \max \{ d(b_j, c_j) \} = d^*(\bar{b}, \bar{c}) < \delta = \varepsilon$$
Notice that $\mathcal{M}' \models T'$ and $\mathcal{M}'$ omits every type in $\Gamma$.

\begin{center}
\begin{tikzpicture}
\draw (0,0) -- (4,0);
\draw (0,1) -- (0,5);
\draw (4,1) -- (4,5);
\draw (0,0) -- (4,5);
\draw (0,0) -- (0,5) node[anchor=north west] {$F^n_i(\bar{a})$};
\draw (4,0) -- (4,5) node[anchor=north east] {$\mathcal{M}$};
\draw (2,0) node[below] {$\pi$};
\draw (2,1) node[above] {$F^n_i$};
\draw (2,2) node[above] {$\mathcal{M}_\pi$};
\draw (2,3) node[above] {$n$-tuples in $\mathcal{M}$};
\end{tikzpicture}
\end{center}

\[\Box \text{Theorem 3.2}\]

**Remark 3.4.** Let $L$ be a (first order) language and $\mu := \sup\{|L|, \kappa\}$. Since Shelah proved that any AEC $\mathcal{K}$ is a $\text{PCT}(\kappa, 2^\kappa)$ class (i.e., $\mathcal{K} = \text{PC}_L(T, \Gamma)$ with $|T| = \kappa$ and $|\Gamma| = 2^\kappa$) with $\text{LS}(\mathcal{K}) = \kappa$, as a consequence of the M. Morley’s omitting types theorem -see [Mo65]- we have that there exists a cardinality $H_1 := \mathfrak{d}(2^\mu)$, such that if $\mathcal{K}$ is an AEC of $L$-structures with $\text{LS}(\mathcal{K}) = \kappa$ such that if there exists $M \in \mathcal{K}$ of cardinality $> H_1$ then there exists a model in $\mathcal{K}$ in any cardinality $> H_1$. But its existence strongly depends on the existence of Hanf numbers in (discrete) omiting type classes (which depends on infinitary logics, see [Mo65]). Despite there are some recent works about some intends of providing suitable notions of metric infinitary logics (see [Ea14]), it is still open if this can yields a suitable analysis of Hanf numbers which implies that metric PC classes have a Hanf number, and so MAECs does as well. However, W. Boney proved that Hanf numbers exist for MAECs by using a kind of adjoint functors between the original MAEC and an auxiliary discrete AEC defined by some well-behaved dense subsets closed under function symbols (see [Bo14]).

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