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Abstract. The q-dependent cross-correlation between worldwide stock indexes have been analyzed. The correlations among different fluctuations have different identities. The correlation-based network has been constructed for different magnitude of fluctuations. We found apparent community structure related to geographical location. Those results have deepened our understanding about the cross-correlation between stock markets.

1. Introduction

The collective behaviors of cross-correlations between different financial assets has become extremely attractive \cite{1–3}. The traditional cross-correlation analysis always relied on some linear tools such as Pearson cross-correlation coefficient. But actual linear correlations are very rare in real world situation. In order to take into account the non-linearity and non-stationarity in real-world data, detrended methods have been proposed to tackle the non-linear and non-stationary of real world datasets.

The most popular nonlinear and non-stationary time series analysis method is the detrended fluctuation analysis(DFA) \cite{4}. The generalization of DFA method named detrended cross-correlation fluctuation analysis (DCCA) has been proposed to quantify non-linear cross-correlations between a pair of non-stationary signals \cite{5}. DFA and DCCA are then extended to their multifractal versions: MFDFA and MFDCCA, respectively \cite{6–8}. Those tools have been widely used in a broad range of systems including biological, financial to physical systems \cite{9–17}. Recently a cross-correlation coefficient similar to the Pearson coefficient, which is based on DCCA, the detrended cross-correlation coefficient $\rho(s)$, was introduced in ref. \cite{18}. This coefficient has been applied to non-stationary signals and quantifies the correlations among fluctuations of detrended non-stationary signals at a given detrending scale $s$ \cite{19}.

The DCCA coefficient $\rho(s)$ has been widely employed to study the non-linear cross-correlations among financial time series \cite{10,20–23}. A more recent extension of the DCCA coefficient $\rho(s)$ based on the q-dependent fluctuation function $F_q$ from MFDFA and MFDCCA \cite{6,8,24} has been proposed. Kwapieni et al. \cite{24} recently indicated that this method could be employed to analyze the empirical data from physical, biological, social and financial systems. Our focus here is on worldwide stock indexes.

Here the q-dependent cross-correlation coefficient is used to quantify the cross-correlations among the return time series of worldwide stocks indexes. Although the q-dependent cross-correlation among stock return time series have been analyzed in a very detailed manner for
both high frequency and daily data [16,25]. But the properties of nonlinear cross-correlation among worldwide indexes is still unclear. Here we calculate the q-dependent cross-correlation matrices $C(q,s)$. Then the statistical properties of the matrices at different multifractal orders $q$ and varying time scales $s$ have been calculated. We then use the planar maximally filtered graph (PMFG) method [26] to characterize the cross-correlation matrices as complex networks and to analyze their basic topological features. The PMFG network has been widely used in the research about the cross-correlation matrices [17,25,28]. We use the community detection algorithm to detected the community structure of the PMFG networks [29–31]. The PMFG networks at different multifractal order $q$ and detrended scales $s$ have unique structures and related with very clear geography identities. The paper is organized as follows. In Sec. 2 we introduce the data and methodologies used in this paper. In Sec. 3 we present the main empirical results. The last section provides our conclusion.

2. Data and Methodology

2.1. Data

The dataset we use include 37 indexes all over the world. There are 5 indexes from the US, 15 indexes from the European, 13 indexes from Asia, 5 indexes from America and 1 index from Africa. Every index includes 3381 data points from 2003-01-03 to 2015-12-18.

2.2. q-dependent cross-correlation analysis

We can be obtained the q-dependent cross-correlation coefficient from the following procedure [8]:

(i) For a pair of time series $x_i$ and $y_i$, $i = 1 \ldots l$. The integrated time series are generated first

$$
\chi^x(k) = \sum_{i=1}^{k} x_i - \langle x \rangle, k = 1 \ldots l, \quad (1)
$$

$$
\chi^y(k) = \sum_{i=1}^{k} y_i - \langle y \rangle, k = 1 \ldots l. \quad (2)
$$

(ii) Then $\chi^x(k)$ and $\chi^y(k)$ are divide into $2M_s = 2 \times \text{int}(l/s)$ non-overlapping boxes of length $s$ from the beginning and the end of two integrated time series $\chi^x(k)$ and $\chi^y(k)$. The local trends for each segment $v(v = 1, \ldots, 2M_s)$ can be estimated by a least-square fit and are subtracted from $\chi^x(k)$ and $\chi^y(k)$ to detrend the integrated series. Then the residual signals $X,Y$ equal to the differences between the integrated signals and the $m$th-order polynomials $P^{(m)}_{s,v}$ fitted to these signals are as follows:

$$
X_s(i,v) = \sum_{i=1}^{s} \chi^x(vs+i) - P^{(m)}_{X,s}(i,v), \quad (3)
$$

$$
Y_s(i,v) = \sum_{i=1}^{s} \chi^y(vs+i) - P^{(m)}_{Y,s}(i,v). \quad (4)
$$

The covariance and variance of $X$ and $Y$ in a box $v$ are defined as:

$$
f_{XY,s}(v) = \frac{1}{s} \sum_{i=1}^{s} X_s(i,v)Y_s(i,v), \quad (5)
$$

$$
f_{Z,s}(v) = \frac{1}{s} \sum_{i=1}^{s} Z^2_s(i,v), \quad (6)
$$
here $Z$ represents either $X$ or $Y$.

(iii) We then define the fluctuation functions at multifractal order $q$ and detrending scale $s$ as:

$$F^q_{XY}(s) = \frac{1}{2M_s} \sum_{v=1}^{2M_s} sgn[f_{XY,s}^2(v)]|f_{XY,s}^2(v)|^{q/2}, \quad (7)$$

$$F^q_{ZZ}(s) = \frac{1}{2M_s} \sum_{v=1}^{2M_s} |f_{ZZ,s}^2(v)|^{q/2}. \quad (8)$$

Then the $q$-dependent cross-correlation coefficient between two time series $x_i$ and $y_i$ is defined as:

$$\rho(q,s) = \frac{F^q_{XY}(s)}{\sqrt{F^q_{XX}(s)F^q_{YY}(s)}}. \quad (9)$$

When $q = 2$ we have the original detrended cross-correlation coefficient $\rho(s)$ [18].

The $q$-dependent cross-correlation coefficient is bounded in the range of $[-1, 1]$ when $q \geq 0$. This coefficient has arbitrary value when $q < 0$ [24]. We only use the case when $q > 0$. The exponent $q$ acts as a filter. When $q > 2$ the boxes with large fluctuations contribute to $\rho(q,s)$ the most, but when $q < 2$ the boxes with relatively small values will dominate the fluctuation function. Meanwhile, the detrend scale $s$ represents the different length of trend in the time series.

2.3. Planar maximally filtered graph

Due to the dense representation of the cross-correlation matrix among stock indexes, it’s very hard to distinguish the structural information from the random noise. Thus we need a sparse representation of the cross-correlation matrix to illustrate the structural information. The complex network approach has been widely used to analyze the cross-correlation matrix due to its straightforward interpretation. Here the planar maximally filtered graph (PMFG) method [26,27] is used to construct networks based on cross-correlation matrices $C(q,s)$. The PMFG algorithm converts the dense correlation matrix into a sparse representation $q$-PMFGs. As it is known from ref. [26], PMFG keeps both the hierarchical organizations of the minimum spanning trees (MST) and it also induces cliques, especially those three and four nodes cliques. The basic topological parameters such as clustering coefficient $C$, the shortest-path length $L$ and the assortativity $A$ [32] have also been calculated. A heterogeneity index $\gamma$ is also used to measure the heterogeneity of the network [33]:

$$\gamma = \frac{N - 2}{N - 2\sqrt{N - 1}} \sum_{ij \in \{e\}} (k_i k_j)^{-1/2}.$$

Here $k_i$ and $k_j$ are the degrees of nodes $i$ and $j$ connected by edge $\{e_{ij}\}$.

3. Results

Here in figure 1 we present the basic statistics of the $q$-dependent cross-correlation matrices. We can see that there is a very obvious transition for the first four moments. The mean correlation for small multifractal order $q$ is relatively larger than those for higher order $q$. The shapes of the distributions change from left skew to right ones. Obvious distribution transition gives a hint about the different cross-correlation structures for small and large fluctuations of the world-wide indexes.

Figure 2 gives the $q$-PMFG networks for different multifractal order $q$ and detrending scales $s$. A very interesting phenomena is that those $q$-PMFG networks carry obvious geography
Figure 1. The first four moments of the q-dependent cross-correlation matrices for each multifractal order $q$ and detrending scale $s$.

information. Those stock indexes belong to the same geography location tend to reside in the same cluster. We use the infomap community detection algorithm to detect the community structures of the q-PMFGs [29]. The indexes in the same community are placed in the same circle. Different colors are related to the geography information of the indexes. We find that for large $q$ the indexes are mixed with each other and for small $q$ the community structures are more geography related and separated from each other.

Here in figure 3, we present the basic topology quantities for q-PMFG networks. The clustering coefficient $C$ for small multifractal order $q$ and small detrending scale $s$ is relative small which means the small fluctuations are strong correlated with each other. The heterogeneous index also revealed the heterogeneous network structure for small $q$ and $s$. 
Figure 2. The q-PMFG networks for different multifractal orders \( q \) and detrending scales \( s \). The yellow nodes are those indexes from Asian. The black nodes are those indexes from European. The red ones are those indexes from north America. The sky blue nodes are those indexes from South America. The only blue node is the index from Africa. The location of each node in the network is determined by the community detection algorithm. Each circle in the network represents one community.
Figure 3. The basic topology quantities for q-PMFG at different multifractal orders $q$ and detrending scales $s$. $C$ is the clustering coefficient, $L$ is the shortest path length, $H$ is the heterogeneity index and $A$ is the assortative of the network.
4. Conclusion

In conclusion, the detrended cross-correlation coefficient has been used to analyze the worldwide stock indexes. The fluctuations at different magnitudes have their own identities. The cross-correlation among small fluctuations are stronger than those among large fluctuations. The q-PMFG networks have also been employed to analyze the correlation structures of the q-dependent cross-correlation matrices. The q-PMFGs give a very clear geography related community structures. Whilst the community structures at different fluctuation orders are quite different from each other. The results from this paper have deepened our understanding about the correlation structure among worldwide stock markets.

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